GRavitational wave recoil and the retention of intermediate-mass black holes

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ABSTRACT

During the inspiral and merger of a binary black hole, gravitational radiation is emitted anisotropically due to asymmetries in the merger configuration. This anisotropic radiation leads to a gravitational wave kick, or recoil velocity, as large as \( \sim 4000 \) km s\(^{-1}\). We investigate the effect gravitational recoil has on the retention of intermediate-mass black holes (IMBHs) within the population of Galactic globular clusters by simulating the response of IMBHs to black hole mergers. Assuming that our current understanding of IMBH formation is correct and yields an IMBH seed in every globular cluster, we find a significant problem in retaining low-mass IMBHs (\(<1000 \, M_\odot\)) in the typical merger-rich globular cluster environment. Given a uniform black hole spin distribution and orientation and a stellar-mass black hole mass function generated in a low-metallicity system, we find that only three of the Milky Way globular clusters can retain an IMBH with an initial mass of 200 \( M_\odot\). Even if IMBHs have an initial mass of 1000 \( M_\odot\), only 60 would remain within Milky Way globular clusters, and each would reside only in the most massive clusters. Our calculations show that if there are black holes of mass \( M > 50 \, M_\odot\) in a cluster, repeated IMBH–black hole encounters will eventually eject a \( M = 1000 \, M_\odot\) IMBH with greater than 30\% probability. As a consequence, a large population of rogue black holes may exist in our Milky Way halo. We briefly discuss the dynamical implications of this process and its possible connection to ultraluminous X-ray sources (ULXs).

Subject headings: black hole physics — galaxies: nuclei — gravitation — gravitational waves — relativity

1. INTRODUCTION

Ample observational evidence exists for two types of black holes: stellar-mass black holes (BHs), with \( 10 M_\odot \lesssim m \lesssim 10^2 M_\odot \), and supermassive black holes (SMBHs), with \( m \gtrsim 10^6 M_\odot \) (e.g., Kormendy & Richstone 1995). Although the existence of a third black hole class is still under debate, there are observational hints for intermediate-mass black holes (IMBHs) as well, with masses \( 10^2 M_\odot \lesssim m \lesssim 10^5 M_\odot \) (e.g., Gebhardt et al. 2005; Gerren et al. 2002; Filippenko & Ho 2003; Heggie et al. 2006; Trenti et al. 2007a, 2007c; Trenti 2006; Ulvestad et al. 2007; cf. Baumgardt et al. [2003] for an alternative view). These IMBHs may form within dense star clusters and may be the best explanation for ultraluminous X-ray sources (ULXs) in young star-forming regions (Fabbiano 1989; Roberts & Warwick 2000; Ptak & Colbert 2004; Fabbiano & White 2006) and in nearby extragalactic star clusters. If the observational case for IMBHs is still unclear, the exact formation mechanism is perhaps less constrained. Within a globular cluster, there are currently three plausible IMBH formation theories: stellar runaway, compact object mergers, and growth from Population (Pop) III remnants (cf. van der Marel [2004] for a review). Each of these mechanisms starts with very different initial conditions, and the first two require the high stellar densities of a globular or stellar cluster environment. Thus, growing IMBHs over the ensemble of observed globular clusters may require more than a single mechanism. We briefly introduce these mechanisms below.

Recent simulations of core collapse in stellar clusters have shown that stellar collisions could induces rapid stellar runaway growth of an \( \mathcal{O}(10^3 M_\odot) \) IMBH in as little as 3 Myr (Portegies Zwart & McMillan 2002; Gültekin et al. 2006; Freitag et al. 2006; see also Portegies Zwart et al. 2004). However, this runaway growth process requires number densities higher than \( \mathcal{O}(10^6 \) pc\(^{-3}\)), and few (if any) globular clusters in the Milky Way are as dense today. Of course, globular clusters are thought to be more dense at formation (Trenti et al. 2007b; Heggie et al. 2007), but calculations still suggest that the runaway process may occur in only about 20\% of the Milky Way globular cluster system (Baumgardt et al. 2005). By using less extreme central densities and including the effect of primordial binaries, simulations have shown that instead of a single IMBH, stellar runaways generate two or more black holes with masses \( \sim 500 \, M_\odot \) (Gültekin et al. 2006). When these large black holes merge, they form a single IMBH and a strong gravitational wave signal for the Laser Interferometer Space Antenna, or LISA, a planned gravitational wave observatory set to launch in the next decade (Freggeau et al. 2006; Gültekin et al. 2006).

While runaway star collisions seem to be the preferred IMBH formation channel within dense systems, in systems where the relaxation time is long, stellar-mass BH collisions may be a more likely growth mechanism. Here the initial seed IMBH forms naturally from a heavier than average black hole, perhaps a stellar remnant with \( \sim 250 M_\odot \) (Wise & Abel 2005). Then, stellar-mass BHs are captured by the initial seed, and the resulting binary hardens through interactions with other stellar-mass objects. Although these interactions risk ejecting the BH supply before significant growth occurs, one way to funnel more BHs to the growing IMBH is through four-body collisions (O’Leary et al. 2006a; Miller & Hamilton 2002; Gültekin et al. 2004).

Significantly higher mass Pop III stars may yield IMBHs directly. If a Pop III protostar has a mass of \( 10^5 M_\odot \), it will be gravitationally unstable and will collapse to an IMBH before it even enters the main sequence (Baumgarte & Shapiro 1999; Shibata & Shapiro 2002). The probability of an IMBH forming from such a massive Pop III star certainly depends on the initial stellar mass function (IMF), as well as highly uncertain details of zero-metallicity stellar evolution. However, there has been some suggestions that the IMF in the early universe is quite top heavy.
and that stellar mass loss is negligible (Fryer et al. 2001; Heger et al. 2003). These suggestions imply that BHs in a proto–globular cluster environment are more massive than those formed in more recent times (van der Marel 2004). On the other hand, Pop III star formation is thought to take place in dark matter overdensities at $z \sim 12$–20 (Madau & Rees 2001), and only the most massive globular clusters are thought to be embedded in a significant enough dark matter overdensity to allow for Pop III formation.

However the IMBH forms, for the first $\sim 0.5$ Gyr after formation it is dynamically active (Spitzer 1987). Due to mass segregation, the initial environment around an IMBH in a globular cluster is especially rich in BHs (Froeschlé et al. 2002). While many of these BHs are quickly ejected by few-body interactions with the IMBH, enough remain to subject the IMBH to tens to hundreds of mergers with BHs in the primordial globular cluster system (Portegies Zwart & McMillan 2000; O’Leary et al. 2006a; Gültekin et al. 2004).

In light of general relativistic black hole merger simulations, surviving this IMBH-BH merger epoch may be difficult. Recent advances in numerical relativity have at last pinned down the dynamics of black hole mergers—simulating the coalescence, merger, and ring-down of equal-mass circular nonspinning binary black holes in full general relativity (Pretorius 2005; Campanelli et al. 2006; Baker et al. 2006a). In the past year, more astrophysically relevant numerical simulations, including spins and unequal masses, have been published by several groups.

One of the most exciting results of general relativity for structure formation is that binary black hole systems strongly radiate linear momentum in the form of gravitational waves during the plunge phase of the inspiral. This radiation results directly from an asymmetry in the orbital configuration and can generically yield a gravitational wave “kick” velocity as fast as 4000 km s$^{-1}$ (González et al. 2007a; Campanelli et al. 2007b; Schnittman 2007; Schnittman & Buonanno 2007). While rare, kick speeds of such magnitude are dynamically interesting for galaxies at high redshift and at the present epoch (Merritt et al. 2004a; Volonteri & Rees 2006). Even typical kick velocities ($\sim 200$ km s$^{-1}$) are interestingly large when compared to the escape velocity of an average globular cluster ($\sim 50$ km s$^{-1}$; Webbink 1985; F彩色ata et al. 2004; Merritt et al. 2004a; Volonteri & Rees 2007). Hence, regardless of how an IMBH forms, the biggest challenge may be to determine how a globular cluster retains them in the face of a repeated onslaught of gravitational wave kicks from mergers with other black holes. Of course, if IMBHs are formed through mergers of stellar-mass black holes, the formation mechanism itself needs to be able to account for these large kicks as well.

In this paper we explore IMBH retention within young globular clusters after collisions with BHs. We calculate the retention probability under a variety of assumptions for the initial IMBH seed mass, the BH mass distribution, and the initial spin distributions. In addition, for the mass range encompassed by the IMBH formation channels listed above (stellar runaway, stellar-mass BH collisions, and Pop III stars), we simulate the IMBH survival probability after merging with the available BHs over a collision timescale. Given the known current structure of the Milky Way globular cluster system, we can then estimate the fraction of globular clusters that may have allowed a particular IMBH formation channel. Combining these two key estimates, we can determine the maximum expected number of Milky Way globular clusters that could have retained their IMBHs. We outline the approach in $\S$ 2, present the results in $\S$ 3, compare this mechanism to three-body ejections in $\S$ 4, and discuss the caveats, implications, and future directions in $\S$ 5.

2. ASSIGNING KICKS

In order to determine the probability that an IMBH survives the short merger epoch after formation, our simulations consist of 10$^6$ Monte Carlo realizations of an $N$-step black hole merger chain. After assigning the mass, spin, and orientation to the seed black hole, we allow it to experience $N$ black hole mergers. Each of the $N$ encounters and incoming black hole parameters are designed to mimic the predictions of current IMBH growth scenarios within proto–globular and stellar cluster environments.

During a merger, gravitational waves radiate not only linear momentum, but also angular momentum and energy or mass. Fully relativistic numerical simulations suggest that $\sim 25\%$ of the angular momentum (defined at the innermost stable circular orbit) can be radiated during the merger (Pretorius 2005; Campanelli et al. 2006; Baker et al. 2006a; Herrmann et al. 2007a). In addition, the mass of the merger product is only $\sim 95\%$ of the mass of the two progenitor black holes (Pretorius 2005; Campanelli et al. 2006; Baker et al. 2006a; Herrmann et al. 2007a). Hence, after each step within a particular merger tree, we adjust the total spin and mass of the merger remnant to account for these losses. This allows us to follow the IMBH spin and mass self-consistently.

We use the recent results from numerical relativity to model the IMBH recoil velocity after each merger. Since this is such a critical piece of our model, we review the current picture of gravitational wave recoil briefly here.

Gravitational recoil estimates of binary black hole mergers have been addressed using both semianalytic methods in the unequal-mass, nonspinning case (Fitchett 1983; Favata et al. 2004; Danour & Gopakumar 2006; Blanchet et al. 2005; Sopuerta et al. 2006) and numerical methods in more general, spinning, unequal-mass scenarios (Herrmann et al. 2007a, 2007b; Baker et al. 2006b; Gonzalez et al. 2007b; Koppitz et al. 2007; Campanelli et al. 2007a, 2007b; Tichy & Marronetti 2007). For nonspinning binaries, the most comprehensive numerical study of kicks is that of Gonzalez et al. (2007b), which was found in agreement with the semianalytic estimates of (Sopuerta et al. 2006), where a maximum kick velocity of $\sim 175 \pm 10$ km s$^{-1}$ was obtained for the mass ratio $q = M_1/M_2 \sim 0.36 \pm 0.02$.

The gravitational recoil is expected to increase with increasing spin (Redmount & Rees 1989; Whitbeck 2006), and this behavior has been confirmed by several numerical relativity groups. Numerical simulations have demonstrated that the radiative linear momentum loss predicted by post-Newtonian studies (Thorne 1980; Kidder 1995) can be used to fit numerical results, yielding a generalized formula for the recoil velocity as a function of the individual black hole’s spin, initial orientation, phase at merger, and mass ratio. We adopt the parameterized fit of Campanelli et al. (2007a), adding the expected $(1 + e)$ contribution for eccentric orbits (Sopuerta et al. 2007) to yield the following formula:

$$v_{\text{kick}} = (1 + \epsilon) \left[ \sqrt{\left( v_m + v_\perp \cos \xi \right)^2 + v_\parallel^2 \sin \xi^2} \right],$$  

(1)

where

$$v_m = A \frac{q^2 (1 - q)}{(1 + q)^5} \left[ 1 + B \frac{q}{(1 + q)^5} \right],$$  

(2)

$$v_\perp = H \frac{q^2}{(1 + q)^5} \left( \alpha_2^p - \alpha_0^p \right),$$  

(3)

$$v_\parallel = K \cos (\Theta - \Theta_0) \frac{q^2}{(1 + q)^5} \left( \alpha_2^p - \alpha_0^p \right),$$  

(4)
where the fitting constants are $A = 1.2 \times 10^4 \text{ km s}^{-1}$, $B = -0.93$, $H = (7.3 \pm 0.3) \times 10^4 \text{ km s}^{-1}$, and $K = (6.0 \pm 0.1) \times 10^4 \text{ km s}^{-1}$, while the subscripts 1 and 2 refer to the first and second BH, respectively. The unit vectors ($\hat{i}$ and $\hat{j}$) are orthogonal to each other and span the initial orbital plane, while the subscripts $\perp$ and $| \rangle$ stand for perpendicular and parallel to the orbital angular momentum. There are four fitting parameters: the mass ratio $q \equiv M_2/M_1$; the reduced spin parameter $a_i \equiv S_i/M_i^2$, where $S_i$ is the spin angular momentum of BH $i$; and the eccentricity $e$. In addition, there are three angles to specify the orientation of the merger: $\Theta$, the angle between the in-plane component of $\delta i \equiv (M_1 + M_2)(S_1/M_2 - S_1/M_1)$ and the infall direction at merger; $\Theta_0$, the angle between $\delta i$ and the initial direction of motion; and $\xi$, the angle between the unequal mass and spin contributions to the recoil in the orbital plane. The merger phase within the orbital plane may also play a role, but it is not included explicitly in this fit. The recoil velocities as given in equation (4) are plotted in Figure 1 as a function of mass ratio and spin parameter.

As we can see from this set of equations, the recoil velocity depends strongly on quantities that describe the symmetry of the merger configuration. Assigning a kick velocity to each step of the merger chain, then, simply amounts to choosing from a distribution of mass ratios, spins, relative spin orientations, and orbital eccentricity. In order to compute the probability that an IMBH remains in its globular cluster, our calculation shall proceed as follows. First, we begin by assuming that an IMBH has formed within a globular cluster with a particular initial mass, $M_{\text{IMBH}}$, spin, and spin orientation, all of which we shall vary. Second, we further assume a certain mass and spin distribution for the BHs in the vicinity of the IMBH, which we shall also vary. Third, we subject this IMBH to a number of mergers expected within a proto–globular cluster environment, each merger with its own mass and spin ratio, orientation, and eccentricity. During each merger, we adjust the IMBH spin and mass to account for gravitational wave losses. Finally, we determine the probability that the kick velocity for the IMBH has remained below the globular cluster escape velocity during the entire chain of mergers. Unless otherwise stated, the globular cluster escape velocity was set to the canonical value of 50 km s$^{-1}$.

First, let us quantify the number of IMBH-BH mergers expected within a young cluster. Even in the absence of an IMBH, BHs eject themselves from globular clusters via standard few-body interactions on a timescale of $\sim 1$ Gyr after the onset of mass segregation (Kulkarni et al. 1993; Sigurdsson & Hernquist 1993; Portegies Zwart & McMillan 2000; O’Leary et al. 2006a). Therefore, due to such Newtonian few-body interactions, the supply of BHs is eventually depleted. With an IMBH, however, this process speeds up impressively, as ejections from interactions with an IMBH become the dominant source of stellar-mass BH ejections (Gültekin et al. 2006). As in most few-body interactions, the ejection of one object tightens the orbit of a remaining bound pair, in this case an IMBH-BH binary, and after several subsequent ejections, the IMBH-BH binary merges. Soon after all the BHs have been evacuated, the short epoch of IMBH-BH mergers ends.

Within this theoretical framework, it is possible to construct a fiducial number of mergers for a proto–globular cluster. This number can be written as

$$N_{\text{merge}} = \frac{N_{\text{BHs}}}{n_{\text{ejections}}/\text{merger}}. \quad (5)$$

Gültekin et al. (2006) predict $N_{\text{merge}} \sim 25$ per IMBH, and we adopt this for the fiducial number of mergers that the IMBH encounters. Although we do vary this parameter in Figure 2, the dependence of the retention probability on the number of mergers is relatively minor, since the IMBH grows in mass over each merger and the kick velocity increases with increasing mass ratio.

For each merger, we randomly select the spin orientation of each black hole, the secondary spin, the secondary mass, and the binary eccentricity from a distribution. We outline the assumptions made for each distribution below. Let us first discuss the issue of the initial spin orientation. Hydrodynamic interactions between a gas disk and a black hole binary are believed to align the spin directions to the angular momentum axis of the binary orbital plane in many active galaxies (Bogdanovic et al. 2007). However, the environment of a globular cluster is not particularly gas-rich, so there is no a priori reason to expect the black hole
spins to be aligned. We therefore assume an isotropic distribution of orientation angles for each encounter.

Let us now discuss the choice of spin magnitude. Most theories predict a nonzero spin for a black hole produced via stellar runaway (Rees & Volonteri 2007) or from a supernova remnant (Fryer et al. 2001). If an IMBH started with zero spin, a merger is likely to spin up the remnant through transfer of orbital to spin angular momentum (Gammie et al. 2004). However, a Kerr black hole can spin down when magnetic field lines thread through the ergosphere to magnetically brake the system (Blandford & Znajek 1977), provided there is a gaseous disk around the remnant. Taking all these considerations into account, we shall explore three cases: (1) the spins magnitude is selected from a uniform initial spin distribution (fiducial case); (2) the initial spin magnitude of the IMBH seed is set to $0.998 M_{\rm IMBH}$; and (3) the spin is initially set to zero. For convenience, we assume the spin of the secondary BH to be randomly selected from a uniform distribution of $[0, 0.998] M_{\text{sec}}^2$, where $M_{\text{sec}}$ is the mass of the secondary BH. Since stellar-mass BHs originate as a supernova remnant, however, the spin is not likely to be uniformly distributed (and is instead very likely to be high (Heger et al. 2003). Therefore, since the kick velocity increases with increasing spin, our choice of a uniform spin distribution shall lead to a conservative survival probability for each merger tree.

The survival probability shall be studied as a function of the initial mass of the seed IMBH. We choose a range of $10$–$3000 M_{\odot}$ to encompass the plausible IMBH formation channels and relevant masses involved. For the three formation channels discussed in §1, the seed masses are likely to be $\sim 1000 M_{\odot}$ for a single stellar runaway (Portegies Zwart et al. 2004); $\sim 100 M_{\odot}$ for the stellar-mass BH collision channel, where the seed IMBH is produced by a massive supernova remnant or a small stellar runaway (Heger et al. 2003); and $\sim 200$–$400 M_{\odot}$ for Pop III remnants within a dense globular cluster (Wise & Abel 2005; Madau & Rees 2001).

One of the biggest uncertainties is determining a proper distribution for the secondary BH masses. As Figure 2 shows, the retention probability depends strongly on the mass ratio between the IMBH and BH. Theoretical black hole mass distributions from solar metallicity field populations tend to peak strongly around 10 $M_{\odot}$ (Fryer & Kalogera 2001a). There are, however, several strong competing effects in a primordial globular cluster that can change this distribution (e.g., low-metallicity stellar evolution, high binary fraction, stellar collisions, mass segregation, and natal kicks). For our fiducial experiment, we assume that the secondary masses are selected from model C1 of Belczynski et al. (2006).

This model includes the effect of mass loss from supernovae and winds for a population of stars with metallicity $Z = 0.0001$, although it has fewer massive remnants from binary mergers (cf. Fig 8 of Belczynski et al. 2006) than is expected for a primordial globular cluster. As an upper limit, we also used a black hole mass function (BHMF) on a Kroupa IMF with a high-mass cutoff of 120 $M_{\odot}$ (Kroupa 2001). Recall that this yields an average stellar mass of about $1 M_{\odot}$, much smaller than the suspected average stellar mass from a zero-metallicity environment. We further assume that each star above $10 M_{\odot}$ evolves directly to a BH with no mass loss. This simplified treatment gives us an average BH mass of $\sim 20 M_{\odot}$. Clearly, mass loss would decrease the average BH mass even in these low-metallicity primordial globulars.

Figure 3 demonstrates the effect on the kick velocity distribution between these two black hole mass functions for a $1000 M_{\odot}$
IMBH. Since the distribution of secondary masses is so uncertain, we demonstrated the effect of varying the mass ratio in Figure 2. For a near-solar-metallicity stellar cluster, the entire population of black holes may be less than $M \leq 20 M_\odot$ (Fryer & Kalogera 2001a). Note that our treatment inexplicitly ignores the dynamics of BHs within a cluster environment. In particular, mass segregation will play a huge role in enriching the cluster center with the heaviest BHs; no matter what BHMF we choose, by ignoring mass segregation, we are selecting mergers with more low-mass BHs than we should be for a dynamically evolved system. This will make the retention probability a conservative estimate. Furthermore, globular clusters are thought to be rich in primordial binaries, which interact with other stars and merge to form yet more massive BHs. Some of these primordial binary-generated BHs will be kicked from the cluster, but those that remain are likely to embed the center with more massive black holes than we included in our static BHMFs.

Finally, we must select the eccentricity distribution. If these IMBH-BH mergers were two-body processes, we might expect the eccentricity of the orbit to be very nearly circular right before the black holes merge as gravitational radiation grinds away the orbital angular momentum (Peters & Mathews 1963). However, few-body encounters are much more common within a globular cluster because the interaction cross-section is much larger (Heggie & Hut 2003). Therefore, many BHs are shepherded into mergers with an IMBH through exchange with lower mass black holes (Miller & Hamilton 2002), and the resulting eccentricity can be quite large (Gültekin et al. 2006). In fact, simulations have shown that rare interactions can yield mergers with $e > 0.999$, and such a highly eccentric orbit can become even more eccentric through gravitational radiation emission (Peters & Mathews 1963; Kennefick 1998). Therefore, although rare, highly eccentric binary black hole mergers can take place in astrophysically relevant systems. To assign eccentricities to each merger, we use the simulation results of Gültekin et al. (2006), which empirically characterizes the eccentricity distribution as a function of the mass ratio of the encounter. Note, however, that equation (1) is really valid only in the small-eccentricity regime and, thus, it may be true that the kicks can be even higher than those studied here for such nearly radial orbits.

3. IMBH SURVIVAL AND OCCUPATION FRACTION

The right panel of Figure 5 demonstrates the retention percentage after 25 mergers with black holes selected from the Kroupa BHMF as a function of the initial IMBH mass. This figure indicates that retaining an IMBH of less than 1000 $M_\odot$ occurs less than 33% of the time for the given distribution of black hole masses within a canonical globular cluster. While the large number of the lower mass black holes dominates the total number of mergers, the rare mergers with massive stellar-mass black holes dominate the ejections—and as the IMBH mass increases, ejections are more likely to be caused by the most massive stellar-mass black holes. For example, Figure 4 shows that for a 1000 $M_\odot$ seed, nearly all mergers come from black holes with mass $M > 30 M_\odot$ and that most come from those with mass $M \sim 70 M_\odot$. Naturally, this implies that if $>30 M_\odot$ black holes are extremely rare in primordial globular clusters, the retention fraction increases dramatically (Fryer & Kalogera 2001a), making gravitational recoil ineffective in ejecting massive IMBHs. As expected, the left panel of Figure 5 shows that it is easier to retain an IMBH of less than 1000 $M_\odot$ with the shallower and more realistic Belczynski black hole mass function, with only 30% ejected.

A BH mass function that is tipped more strongly to lower masses would increase the retention rate; note that a Kroupa IMF with no mass loss yields an average BH mass of $\sim 20 M_\odot$, while allowing for mass loss will make high mass ratio mergers less likely. If there are no black holes $>20 M_\odot$ in clusters, then any IMBH seed larger than $M \gtrsim 600 M_\odot$ will remain. This critical mass ratio is comparable to that necessary to avoid ejection from three-body dynamical kicks alone (Gültekin et al. 2004, 2006; O’Leary et al. 2006a; see §4). Since runaway stellar mergers preferentially include the cluster’s most massive stars, which are in turn the precursors to the most massive black holes in the cluster, the most massive black holes used in our calculations may not be present in the cluster.

For a globular cluster of $v_{\text{esc}} = 50$ km s$^{-1}$, we learn that the only way to retain an IMBH during the short BH merger phase is with one rather massive initial seed, such as those produced in a stellar runaway; any process that relies on the formation of $\leq 500 M_\odot$ black holes will be ineffective, as these are ejected from a globular cluster before forming an IMBH. One way to lower the retention mass threshold in a given globular cluster would be to increase its escape velocity. We explore this effect in Figure 2, where we observe that the escape velocity must reach $\sim 80$ km s$^{-1}$ to attain complete retention of 500 $M_\odot$ black holes. This may not be such an unreasonable escape velocity for a young globular cluster (Hénon 1965; Trenti et al. 2007a). To gauge the magnitude of this effect for a cluster with a 100 km s$^{-1}$ escape velocity, we reran the merger chain simulation for a uniform spin distribution and our upper limit (i.e., Kroupa) secondary mass distribution, this time allowing only those kicks larger than 100 km s$^{-1}$ to escape the cluster. We found that retaining a 1000 $M_\odot$ IMBH increases from 33% to 85% if a canonical primordial globular cluster had double its current escape velocity (see Fig. 6). Note, however, that globular clusters with such high escape velocities are rare; it is much more common to observe clusters in the Milky Way with $v_{\text{esc}} \sim 30$ km s$^{-1}$.

We can use these results to predict the occupation fraction of IMBHs within the current Milky Way globular cluster system.
We begin by using estimates of the current central escape velocities of 139 Milky Way globular clusters in the McMaster Globular Cluster Database (Harris 1996); these were calculated by Gnedin et al. (2002) assuming a King model density profile and a mass-to-light ratio in the $V$ band of 3. We then inferred the initial escape velocity of each cluster by assuming that each cluster lost approximately half its mass through tidal stripping within the Galactic potential (Gnedin et al. 1999). We further assume that every globular cluster is seeded with an IMBH seed of mass $M$ (which we vary), that each has a Belczynski stellar black hole mass function, that the black hole spin magnitudes are uniform, and that the spin orientations are isotropic. After subjecting the seed IMBH in each cluster to a million realizations of a 25 stellar-mass black hole merger chain, we calculate the probability that the IMBH survives within the cluster. Figure 7 shows the number of Milky Way globular clusters that should retain IMBHs as a function of the seed IMBH mass.

**Fig. 5.**—Left: Percentage of black hole retention as a function of initial black hole mass within a globular cluster with $v_{\text{esc}} = 50$ km s$^{-1}$. In this experiment, we assumed a black hole mass function described by model C1 of Belczynski et al. (2006) and a uniform distribution of spin orientations and magnitude. A black hole is defined as “retained” if it survives 25 BH collisions with random spin orientations. The three lines on this figure represent different assumptions for the spins of the black hole. Right: Same as left, with a Kroupa BHMF, which should be considered an upper limit.

**Fig. 6.**—Percentage of black hole retention as a function of initial black hole mass for two different globular cluster escape velocities. In this experiment, we assumed a black hole mass function described by model C1 of Belczynski et al. (2006) and a uniform distribution of spin orientations and magnitude.

**Fig. 7.**—Number of Milky Way globular clusters that retain IMBHs as a function of initial IMBH mass. In this experiment, we assumed a black hole mass function described by model C1 of Belczynski et al. (2006) and a uniform distribution of spin orientations and magnitude. The escape velocity for each cluster was set to mimic its initial escape velocity; over time, tidal stripping and shock heating both reduce the escape velocity as the cluster orbits the Galactic potential (Gnedin et al. 2002).
of 200 $M_{\odot}$, we should observe only three globular clusters with IMBHs, each less than $\sim 600 M_{\odot}$, and each occurring in the most massive clusters (with the highest escape velocities). If seed IMBHs form instead with an initial mass of 2000 $M_{\odot}$, roughly 2/3 of the Milky Way globulars could retain them, and the typical current IMBH mass would be $\sim 2400 M_{\odot}$. However, not every Milky Way globular cluster may be capable of hosting such a high-mass seed in the first place, as higher densities may be required to form high-mass IMBH seeds (Portegies Zwart & McMillan 2002; Gürkan et al. 2006; Freitag et al. 2006).

4. COMPARISON TO DYNAMICAL EJECTIONS

In a globular cluster, it is well known that black holes may be ejected through classic three-body encounters between a hard binary and an interloper star (e.g., Mackey et al. 2007), although there is no published result that predicts three-body ejections of black holes more massive than 500 $M_{\odot}$. Nonetheless, to determine whether gravitational wave recoil is more important than three-body recoil in this calculation, we must compare the two processes in a consistent manner. We estimate the recoil from three-body encounters by scaling from Gültekin et al. (2006) with mass ratios relevant to our case here. In a three-body encounter between a hard binary and an interloper with mass $m_{\text{int}}$, the fractional change in binding energy of the binary scales as

$$\Delta E \approx \frac{m_{\text{comp}}}{m_b},$$

where $m_b = m_{\text{IMBH}} + m_{\text{comp}}$ is the mass of the binary made of the IMBH with mass $m_{\text{IMBH}}$ and its companion $m_{\text{comp}}$ (Quinlan 1996). By conservation of momentum and energy after a parabolic encounter, the three-body recoil of a binary with a dominant massive component scales as

$$v_b^2 \approx \frac{m_{\text{comp}}^2 m_{\text{int}}}{m_b} \frac{m_b + m_{\text{int}}}{(m_{\text{int}} + m_b)^2} \approx \frac{m_{\text{comp}} m_{\text{int}}^2}{m_b},$$

where we have used the fact that $m_b \gg m_{\text{comp}}$ and $m_b \gg m_{\text{int}}$. For an escape velocity of 50 km s$^{-1}$ with $m_{\text{comp}} = 10 M_{\odot}$ and $m_b = 10 M_{\odot}$, Gültekin et al. (2006) find that three-body recoils will eject an IMBH 50% of the time for $m_{\text{IMBH}} \approx 100 M_{\odot}$. This probability is averaged over large numbers of encounters and interpolated between mass ratios. It assumes that a single IMBH is interacting with a single population of black holes of mass $10 M_{\odot}$. In our scenario, the typical merging black hole that causes an ejection is $m_{\text{comp}} \approx 80 M_{\odot}$ (Fig. 4), and if we exclude mass segregation, as we have for our gravitational wave recoil calculations, the typical interloping black hole will have the median mass from our black hole mass distribution, $m_{\text{int}} = 11 M_{\odot}$. For a roughly symmetric distribution in recoil velocities (e.g., Sigurdsson & Phinney 1993), the retention probability will have the same dependence on masses as the characteristic velocity (hence the linear regimes in the central parts of all curves in Fig. 12 of Gültekin et al. 2006).

Recall that we find a $\sim 35\%$ retention for an escape velocity of 50 km s$^{-1}$ for $m_{\text{IMBH}} \approx 500 M_{\odot}$ (see Fig. 5, left) under pure gravitational wave recoil. For a $500 + 80 + 11$ three-body interaction, we estimate the retention from three-body interactions to be $\sim 75\%$. Thus, we find gravitational wave recoil to be roughly twice as effective at ejecting IMBHs from their host cluster as dynamical ejections alone.

However, although half of the black holes in the system will have mass smaller than the median mass, the heavier black holes will sink to the center of the cluster and interact with the IMBH more frequently, both via three-body interactions and through direct mergers. Since two-body interactions in a cluster tend toward equipartition of energy, the velocity of the black holes scale as $v \approx m^{1/2}$. In a mass-segregated system, the scale height also scales as $h \approx m^{1/2}$, and, therefore, the scale volume scales as $V \approx m^{3/2}$. In general, the timescale between encounters is $t = n/v\sigma$, where $n = NV^{-1}$ is the number density and the interaction cross section is

$$\sigma \approx \pi r_p^2 + 4\pi r_p G(m_b + m_{\text{int}})/v^2,$$

where $r_p$ is the distance of closest approach. For the case considered here, $m_b \gg m_{\text{int}}$, so that $\sigma \approx m_b v^{-2}$. The rate of three-body encounters with interloping black holes then scales as $R = t^{-1} \sim N m^2$. If the mass of the interloper is instead, e.g., $m_{\text{int}} = 40 M_{\odot}$, then although the number of black holes is $\sim 0.08$ times the number of black holes with mass $m_{\text{int}} = 11 M_{\odot}$, the encounter rates are comparable. With this higher mass interloper, which is more representative of mass segregation, the dynamical ejection is 4 times more efficient, making it roughly twice as efficient as gravitational wave recoil for the $500+80$ coalescence we calculate in Figure 5. Note, however, that the ejection efficiency in Figure 5 does not take into account mass segregation when selecting the stellar-mass black hole. Nonetheless, it may be that gravitational wave ejections are comparable to dynamical three-body ejections. This has the effect of significantly increasing the overall ejection rate, since in cases in which one process fails to eject the IMBH, the other process may succeed.

There are several caveats in making this comparison between three-body encounters and gravitational wave recoil. Our calculations above imply that the IMBH is constantly merging with $m_{\text{comp}} = 80 M_{\odot}$ black holes, which is not the case and could mean that the actual impact of dynamical ejections may be smaller than estimated. The number and number density of black holes of different masses are also time dependent. A full treatment by combining three-body-scattering simulations with gravitational wave recoil and a correct tracking of the black hole population is planned, but it is out of the scope of this paper.

5. CONCLUSION

Our studies indicate that it is a challenge to retain IMBHs during the barrage of BH mergers expected in a fiducial early globular cluster. We find no scenario that guarantees 100% retention for IMBHs up to $3000 M_{\odot}$ when the interacting BHs are selected from a reasonable distribution of spins, orientations, and mass ratios. However, the more massive the initial IMBH seed, the better the retention probability. This may indicate that any IMBH observed in globular clusters today would most likely have originated from an early stellar runaway channel.

The results obtained here might be used to test some IMBH formation mechanisms. For example, Fregene et al. (2006) and Gürkan et al. (2006) find that with a binary fraction above 10%, a single cluster can host multiple runaways, leading to multiple IMBHs. The heaviest two such IMBHs will find their way to the center of the cluster and merge very quickly ($\sim 1$ Myr). Because such IMBHs are nearly equal in mass and likely have high spins, they will escape the cluster upon merger. This leaves the cluster without a seed, unless a third runaway of sufficient mass occurs. The problem is that the third runaway is often much less massive and therefore difficult to retain, as we have shown.

In fact, since lower mass IMBHs so easily escape globular clusters, if globular cluster observations find that they are not rare, it may be possible to constrain the IMBH merger history as well. For example, if many low-mass IMBHs exist within clusters, we may rule out low mass ratio mergers—this would imply that there
is no high-mass tail in the BH IMF. Alternatively, the lower the spin, the better the retention, as Figure 5 shows; if low-mass IMBHs are found in large numbers within clusters and if BHs are found with $\gtrsim 20 M_\odot$, we may have to explore ways in which the black holes spin down and align within a gas-poor globular cluster.

Making some very simple assumptions for the primordial globular cluster environment, such as the BH IMF and central density structure, we have estimated that less than three globular clusters hosted an initial low-mass IMBH seed. Naturally, there are many uncertainties folded into this estimate, such as the shape of the primordial globular cluster IMF, the degree of mass loss in low-metallicity systems and its effect on the BH IMF, and the detailed role that few-body–BH interactions play in shaping early globular cluster structure. As more work is done on these areas, we can revise our estimates using more generic results. For example, if the BH IMF were instead narrowly peaked around 10 $M_\odot$ (Fryer & Kalogera 2001a) and if the seed black hole were a massive supernova remnant, our calculations indicate that about 10% of the Milky Way globular clusters could retain black holes as small as $\sim 250 M_\odot$. An IMBH of a few hundred $M_\odot$ may not leave an electromagnetically observable signal on the surrounding globular cluster, as the dynamical effects on the surrounding stars may also be produced by a high binary fraction (Trenti et al. 2007b). However, when stellar-mass compact objects merge with these smaller mass IMBHs, they will produce strong gravitational wave signals that should be detectable with Advanced LIGO (Mandel et al. 2007).

Since this paper was submitted, a new study by Baker et al. (2008) correctly points out that in the presence of only 10 $M_\odot$ black holes, the minimum mass needed to avoid ejection from a cluster with escape speed 50 km s$^{-1}$ for the gravitational wave recoil fit used here is $\sim 350 M_\odot$. The value of 10 $M_\odot$, however, is likely small. While the median mass from our distribution (11 $M_\odot$) is very close to this, the mean is 15 $M_\odot$ (increasing the minimum unejectable mass to 525 $M_\odot$). This mean roughly corresponds to the mean of the favored distribution of O’Leary et al. (2006b). Of course, the typical mergers will not be the mergers that are responsible for ejection. The most massive mergers of spinning black holes have the most potential to eject the IMBH from the cluster. If the maximum mass of stellar-mass black holes is 20 $M_\odot$ (e.g., Fryer & Kalogera 2001b), then the minimum unejectable mass becomes 700 $M_\odot$. There is, however, evidence for black holes more massive than this ($M > 23 M_\odot$; Bulik et al. 2008), and there are theoretical reasons to expect higher. For example, Miller & Hamilton (2002) expect the most massive stellar-mass black hole in a dense globular cluster to be $\sim 50 M_\odot$.

which would raise the minimum unejectable mass to $\sim 1700 M_\odot$.

Belczynski et al. (2004) show that under certain assumptions, such as in the low-metallicity environment of a globular cluster 10 Gyr ago, the maximum stellar-mass black hole can be as large as $\sim 80 M_\odot$, making the minimum unejectable mass 2800 $M_\odot$, for an escape velocity of 50 km s$^{-1}$. For a more typical escape velocity of 30 km s$^{-1}$, this increases to 3500 $M_\odot$.

Although we have focused on IMBHs in the Galactic globular cluster population, the same processes may occur in other galaxies. Extragalactic ULXs, which may be powered by $\sim 10^5 M_\odot$ IMBHs, are frequently found near, but not in, young stellar clusters (e.g., Fabbiano et al. 2001; Liu & Bregman 2005). Note, however, that the stellar clusters associated with ULXs are not always the dense stellar systems required by the IMBH formation models considered here (Liu et al. 2007). If such IMBHs did form within the nearby clusters, they may be ejected from gravitational wave kicks coming from mergers with stellar-mass black holes, especially as they would merge with the most massive black holes first. While this would explain their separation from the cluster center, it would not explain the fact that ULXs are accreting sources; it is not clear how an IMBH would pick up a companion on its way out of the cluster, and it is unlikely to retain a stellar companion close enough to overfill its Roche lobe. An IMBH with a stellar companion, however, may be ejected from the host cluster through few-body Newtonian dynamical kicks that harden the binary until it begins accreting (Gültekin et al. 2004, 2006; O’Leary et al. 2006a; Blecha et al. 2006).

Even if the ejected IMBH is not accreting gas as a ULX, electromagnetic observations may still detect rogue black holes. For instance, if the IMBH were to carry a few massive stars along as it is ejected, our results indicate a kinematically fast subpopulation of massive stars near globular clusters. The ejected black holes may leave a temporary imprint on the globular cluster as well. Since they are ejected from the system impulsively, it is likely that the globular cluster core would temporarily expand (Merritt et al. 2006a; Blecha et al. 2006). Direct simulations remain to be done to determine how the globular cluster responds to the ejection of an IMBH.

The consequences of these large recoil velocities may also affect SMBH assembly. The most likely candidates for SMBH seeds are $\sim 10^2 M_\odot$. Pop III stellar remnants at redshifts $z \gtrsim 8$ (Heger et al. 2003; Volonteri et al. 2003; Islam et al. 2003; Wise & Abel 2005; Micic et al. 2007). These relic seeds are predicted to form at the centers of low-mass dark matter halos ($\sim 4 \times 10^8 M_\odot$). As dark matter halos hierarchically merge to assemble the galaxy, the seed black holes sink to the center through dynamical friction and eventually merge. With kick velocities in the range $\sim 10^2$–$10^3$ km s$^{-1}$, it may also be difficult to retain seed SMBHs in high-redshift low-mass dark matter halos. We plan to explore black hole retention and possible kick suppression mechanisms at the low-mass end of the halo mass function using high-resolution cosmological $N$-body simulations in our next paper.

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