Octet Baryon Masses in Partially Quenched Chiral Perturbation Theory

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Abstract

The mass spectrum of the lowest lying octet baryons is calculated to next-to-next-to-leading order in heavy baryon chiral perturbation theory and partially quenched heavy baryon chiral perturbation theory. We work in the isospin limit and treat the decuplet baryons as dynamical fields. These results are necessary for extrapolating QCD and partially quenched QCD lattice simulations of the octet baryon masses.

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The study of low-energy QCD remains a long-standing and exciting challenge since its conception in the early 1970’s. Over the years, various models and tools have been developed in an attempt to study the non-perturbative regime of the theory of strong interactions. Despite the success some of these ideas have had, an intimate understanding of low-energy QCD and the hadron spectra remains elusive. Recently, progress toward this end is being made at many fronts, one of which is lattice QCD and partially quenched chiral perturbation theory (PQ\(\chi\)PT). Lattice QCD is a first principles method to calculate QCD observables using numerical techniques. While ongoing progress in this area is impressive, one is presently restricted to using light quark masses on the order of the strange quark mass, significantly heavier than those in nature. Hence one needs to understand the quark mass dependence of QCD observables in order to compare lattice simulations with physical QCD. For sufficiently small quark masses, chiral perturbation theory (\(\chi\)PT) can be used to extrapolate unquenched lattice simulations from lattice quark masses to physical quark masses. However, for lattice masses that are presently being used the convergence of the chiral expansion is uncertain. Baryon masses as well as other observables have been computed using QCD and quenched QCD (QQCD) \[1, 2, 3, 4\]. In QQCD the determinant arising from the path integral over the quark fields is set to a constant, which greatly reduces the computing time of lattice simulations. There has been some interest in using chiral effective theories to extrapolate these lattice data to lighter quark masses, for example \[5, 6, 7\], but there is no rigorous limit of QQCD in which one recovers QCD.

Partially quenched QCD (PQQCD) has been formulated as an alternative to QCD and QQCD \[8, 9, 10, 11\]. In PQQCD, the valence quarks, which are coupled to external states, and the sea quarks, which contribute to the quark determinant, are treated as independent fields. Thus the valence and sea quark masses can be different. This freedom is generally exploited to make the valence quark masses much smaller than the sea quark masses. The low energy effective theory of PQQCD is PQ\(\chi\)PT \[12, 13\]. The coefficients of operators appearing in the theory are the low-energy constants (LEC’s) which characterize the short distance physics, while the long distance behavior is characterized by meson loops. The LEC’s of \(\chi\)PT are all contained in PQ\(\chi\)PT which allows rigorous predictions for QCD using PQ\(\chi\)PT. Moreover, the ability to independently vary the valence and sea quark masses greatly increases the allowed parameter space available to PQQCD. This enables a better extraction of the LEC’s appearing in PQ\(\chi\)PT, as they can be fit to a much larger set of PQ lattice calculations.

These last few years have seen much work and progress in calculating properties of baryons in PQ\(\chi\)PT \[14, 15, 16, 17, 18, 19, 20, 21, 22\]. In this work we calculate the masses of the octet baryons to next-to-next-to leading order (NNLO) in \(\chi\)PT. We also provide the first NNLO calculation of the octet baryon masses in PQ\(\chi\)PT. These calculations are performed in the isospin limit of SU(3) and we keep the decuplet baryons as dynamical fields. In the partially quenched calculation we use three valence, three ghost and three sea quarks, with two of the sea quarks degenerate.
II. HEAVY BARYON CHIRAL PERTURBATION THEORY

A. Pseudo-Goldstone Bosons

For massless quarks, QCD exhibits a chiral symmetry, $SU(3)_L \otimes SU(3)_R \otimes U(1)_V$, which is spontaneously broken down to $SU(3)_V \otimes U(1)_V$. Chiral perturbation theory, the low-energy effective theory, emerges by expanding about the physical vacuum state of QCD. In the limit of massless quarks, the pions, kaons and eta emerge as the Goldstone bosons of the broken $SU(3)_A$. Given that the quark masses are small compared to the scale of chiral symmetry breaking, the lowest lying mesons emerge as an octet of pseudo-Goldstone bosons.

The pseudo-Goldstone bosons are collected in an exponential matrix

$$
\Sigma = \exp \left( \frac{2i}{f} \Phi \right) = \xi^2, \quad \Phi = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi_0 + \frac{1}{\sqrt{6}} \eta \\
\pi^- \\
K^-
\end{pmatrix}
$$

(1)

With the above convention, the pion decay constant, $f$, is 132 MeV. Under an $SU(3)_L \otimes SU(3)_R$ transformation these fields transform in the following way

$$
\Sigma \rightarrow L \Sigma R^\dagger, \quad \xi \rightarrow L \xi U^\dagger = U \xi R^\dagger,
$$

(2)

where $U$ is implicitly defined by equations (1) and (2). The effective Lagrangian describing the strong dynamics of these mesons at leading order in $\chi$PT is

$$
\mathcal{L} = \frac{f^2}{8} \text{Tr} (\partial^\mu \Sigma \partial_\mu \Sigma) + \lambda \text{Tr} (m_q^\dagger \Sigma \dagger + m_q \Sigma).
$$

(3)

Leading order (LO) in the power counting is $O(m_q)$ and thus $O(m_q) \sim O(q^2)$, where $q$ is a pion momentum. The quark mass matrix, $m_q$, is given in the isospin limit, $(m_u = m_d = m)$, by

$$
m_q = \text{diag}(m, m, m_s).
$$

(4)

To the meson masses to LO are given by

$$
m^2_\pi = \frac{8\lambda}{f^2} m, \quad m^2_K = \frac{4\lambda}{f^2} (m + m_s), \quad m^2_\eta = \frac{8\lambda}{3f^2} (m + 2m_s).
$$

(5)

B. Baryons

To systematically include the octet and decuplet baryons into the chiral Lagrangian, we use heavy baryon $\chi$PT (HB$\chi$PT) [23, 24, 25, 26], where the baryon fields are redefined in terms of velocity dependent fields.

$$
B_v(x) = \frac{1 + \frac{j}{2} e^{i M v \cdot x} B(x),
$$

(6)

where $v_\mu$ is the four-velocity of the baryon, $B$. This field redefinition corresponds to parameterizing the momentum of a nearly on-shell baryon as

$$
p_\mu = M_B v_\mu + k_\mu,
$$

(7)
where $k_\mu$ is the residual momentum. The effect of this is to eliminate the standard Dirac mass term for the baryons:

$$\bar{B} (i\partial - M_B) B = \bar{B}_v i\partial B_v + \mathcal{O}(\frac{1}{M_B}).$$

(8)

From Eq. (8), it is easy to verify that derivatives acting on $B_v$ bring down powers of $k$, rather than $p$. Thus, higher dimension operators of the heavy baryon field $B_v$ are suppressed by powers of $M_B$. Heavy baryon $\chi$PT is applicable in the limit that pion momenta and the off-shellness of the baryon are small compared to the chiral symmetry breaking scale, $\Lambda_\chi \sim 1\text{GeV}$, i.e.:

$$q, k \cdot v \ll \Lambda_\chi.$$

(9)

In this limit, HB$\chi$PT has a consistent derivative expansion as derivatives are suppressed by powers of $\Lambda_\chi$.

When the spin-$\frac{3}{2}$ decuplet baryons, $T$, are included in the theory, an additional mass parameter, $\Delta = M_T - M_B$, must be included. The meson masses, $m_\phi$, where $\phi$ is a pion, kaon or eta, are much smaller than $\Delta$ close to the chiral limit, and therefore the decuplet resonances can be integrated out of the theory, leaving only the pseudo-Goldstone mesons and the octet baryons. Decuplet effects would then show up in the theory as higher dimension operators suppressed by powers of $C^2/\Delta$ (where $C \sim 1.5$ is the $TB\pi$ coupling). In the real world, $\Delta \sim 300\text{ MeV}$ and $m_\pi/\Delta \sim 1/2$, so $\mathcal{O}(m_\pi/\Delta)$ corrections could be sizeable and spoil the power counting. Additionally, if one is to use present day lattice calculations to fit the free parameters of the theory, $m_{\text{lat}}/\Delta \geq 1$. The situation becomes worse when considering $\mathcal{O}(m_K/\Delta)$ and $\mathcal{O}(m_\eta/\Delta)$ corrections. There is also ample phenomenological evidence in the nucleon sector that suggests the importance of including the lowest lying spin-$\frac{3}{2}$ resonances $^{24, 25, 26, 27, 28, 29}$.

To include the decuplet baryons, one must include $\Delta$ in the power counting. The mass splitting, $\Delta \sim m_\phi$ so $\Delta$ and $m_\phi$ are $\mathcal{O}(q)$, where $q$ is a typical small pion momentum, and the quark mass, $m_q$, is treated as $\mathcal{O}(q^2)$. The baryon mass is treated as $M_B \sim \Lambda_\chi$. As the LEC’s are a priori unknown, we can combine the $1/M_B$ and $1/\Lambda_\chi$ expansions into one expansion in powers of $1/\Lambda_\chi$. There is one exception: constraints from reparameterization invariance determine the coefficients of some of the higher dimension operators arising in the $1/M_B$ expansion. $^{30, 31}$. Thus these $1/M_B$ corrections must be kept distinct to insure the Lorentz invariance of the heavy baryon theory to a given order.

The effective Lagrangian at leading order (LO) in the $1/M_B$ expansion is well known $^{23}$. In this paper we will embed the baryons in rank-3 flavor tensors. The convenience of this choice will become readily apparent when we generalize to PQ$\chi$PT. The $SU(3)$ matrix of the lowest-lying spin-$\frac{1}{2}$ baryon fields is

$$B = \begin{pmatrix}
\frac{1}{\sqrt{6}} \Lambda + \frac{1}{\sqrt{2}} \Sigma^0 \\
\Sigma^- \\
\Xi^-
\end{pmatrix}
\begin{pmatrix}
\Sigma^+ \\
\frac{1}{\sqrt{6}} \Lambda - \frac{1}{\sqrt{2}} \Sigma^0 \\
\Xi^0 - \frac{2}{\sqrt{6}} \Lambda
\end{pmatrix}
\begin{pmatrix}
p \\
n \\
-2 \Xi^0
\end{pmatrix}.$$

(10)

We then embed them in the tensor $^{32}$:

$$B_{ijk} = \frac{1}{\sqrt{6}} \left( \epsilon_{ijl} B^l_k + \epsilon_{ikl} B^l_j \right),$$

(11)
which has the symmetry properties

\[ B_{ijk} = B_{ikj} \quad \text{and} \quad B_{ijk} + B_{jik} + B_{kji} = 0. \]  

(12)

Under chiral $SU(3)$ transformations, this baryon tensor transforms as

\[ B_{ijk} \rightarrow U_i^l U_j^m U_k^n B_{lmn}. \]  

(13)

The spin-$\frac{3}{2}$ decuplet baryons can be described by a Rarita-Schwinger field, $(T^\mu)_{ijk}$, which is totally symmetric under the interchange of flavor indices, and which satisfies the constraint, $\gamma_\mu T^\mu = 0$. We employ the normalization convention that $T^{111} = \Delta^{++}$. Under a chiral $SU(3)$ transformation, the decuplet tensor transforms identically to the octet tensor:

\[ T_{ijk} \rightarrow U_i^l U_j^m U_k^n T_{lmn}. \]  

(14)

The octet and decuplet baryon tensors obey the spin-algebra laid out in \[23\], which can be used to eliminate the Dirac structure of the theory; all Lorentz tensors made from spinors can be written in terms of $v^\mu$ and $S^\mu$, where $v^\mu$ is the four-velocity of the baryon and $S^\mu$ is the covariant spin-vector.

Thus the Lagrangian to leading order in the $1/M_B$ expansion can be written as \(^1\)

\[ \mathcal{L}_{LO} = \left( B \ i v \cdot D \ B \right) + 2\alpha_M ( B B \mathcal{M}_+) + 2\beta_M (B B \mathcal{M}_+) + 2\sigma_M (B B) \text{Tr}({\mathcal{M}_+}) - (\mathcal{B}^\mu [iv \cdot D - \Delta] T_\mu) + 2\gamma_\mu (\mathcal{B}^\mu \mathcal{M}_+ T_\mu) - 2\sigma_\mu (\mathcal{B}^\mu T_\mu) \text{Tr}({\mathcal{M}_+}) + 2\alpha (B S^\mu B A_\mu) + 2\beta (B S^\mu A_\mu B) + 2H (\mathcal{T}^\mu S^\mu A_\mu T_\mu) + \sqrt{\frac{3}{2}} C \left[ (T^\mu A_\mu B) + (BA_\mu T^\nu) \right]. \]  

(15)

Above, $D_\mu$ is the chiral-covariant derivative which acts on the $B$ and $T$ fields as

\[ (D^\mu B)_{ijk} = \partial^\mu B_{ijk} + (V^\nu)_i^l B_{jk} + (V^\nu)_j^l B_{ik} + (V^\nu)_k^l B_{ij}. \]  

(16)

The vector and axial-vector meson fields appearing in the Lagrangian are given by

\[ V_\mu = \frac{1}{2}(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi), \quad A_\mu = \frac{i}{2}(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi), \]  

(17)

and

\[ \mathcal{M}_+ = \frac{1}{2}(\xi^\dagger m_\xi \xi^\dagger + \xi m_\xi \xi). \]  

(18)

In Eq. 15, the brackets, $(\ )$, denote a contraction of the flavor and Lorentz indices. For a matrix $\Gamma_{ij}^\alpha$ acting in spin-space and a matrix $Y_{ij}^\alpha$ acting in flavor space, the contractions are defined as \[32\]

\[ (\overline{B} \Gamma B) = \overline{B}^{\alpha,kji} \Gamma_{ij}^\alpha B_{\beta,ijk}, \quad (\overline{B}^\mu \Gamma T_\mu) = \overline{T}_\mu^{\alpha,kji} \Gamma_{ij}^\alpha T_{\mu\beta,ijk} \]

\[ (\overline{B} \Gamma Y B) = \overline{B}^{\alpha,kji} \Gamma_{ij}^\alpha Y_i^l B_{\beta,lijk}, \quad (\overline{T}_\mu^\alpha \Gamma Y T_\mu) = \overline{T}_\mu^{\alpha,kji} \Gamma_{ij}^\alpha Y_i^l T_{\mu\beta,lijk} \]

\[ (\overline{B} \Gamma B Y) = \overline{B}^{\alpha,kji} \Gamma_{ij}^\alpha Y_i^l B_{\beta,lijk}, \quad (\overline{B} \Gamma Y Y T_\mu) = \overline{T}_\mu^{\alpha,kji} \Gamma_{ij}^\alpha (Y^\mu)_i^l T_{\mu\beta,lijk}. \]  

(19)

\(^1\) For brevity, we will drop the subscript $v$ from the velocity dependent heavy baryon fields.
Such contractions ensure the proper transformations of the field bilinears under chiral transformations. To compare with the coefficients used in the standard two-index baryon formulation \[23, 24, 26\], it is straightforward to show that:

\[
\alpha = \frac{2}{3} D + 2 F , \quad \beta = -\frac{5}{3} D + F , \\
\alpha_M = \frac{2}{3} b_D + 2 b_F , \quad \beta_M = -\frac{5}{3} b_D + b_F , \quad \sigma_M = b_D - b_F + \sigma .
\]  

(20)

### C. Higher Dimensional Operators

The Lagrangian in Eq. (15) contains some, but not all, terms of \( O(q^2) \). To calculate the baryon masses to \( O(m_B^2) \) we must include all relevant \( O(q^2) \) operators, which contribute to the mass via loops, and all relevant \( O(q^4) \) operators which contribute at tree level. The Lagrangian also includes operators of \( O(q^3) \), but they do not contribute to the baryon self-energy.

The higher dimensional operators introduce new LEC’s that, in principle, must be determined from experiments or lattice QCD calculations. However, many of them can be derived exactly using reparameterisation invariance (RI) \[30\]. The baryon momentum parameterization in terms of \( v^\mu \) and \( k^\mu \) is not unique when considering the \( 1/M_B \) corrections. When the velocity and residual momentum are simultaneously transformed in the following way

\[
v \rightarrow v + \frac{\epsilon}{M_B} , \quad k \rightarrow k - \epsilon ,
\]  

(21)

the momentum \( p_\mu \) in Eq. (7) is unchanged. Reparameterization invariance requires the effective Lagrangian to be invariant under such a transformation, which ensures the theory is Lorentz invariant to a given order in \( 1/M_B \). Furthermore, utilizing RI has non-trivial consequences as it connects operators at different orders in the \( 1/M_B \) expansion, exactly fixing the coefficients of some of the higher dimensional operators with respect to the lower ones. We find that the fixed coefficient Lagrangian is \(^2\)

\[
\mathcal{L}_{(2)}^{(2)} = - \left( \frac{D^2}{2M_B} B \right) - \alpha \left( \frac{S}{M_B} B v \cdot A \right) + \alpha \left( \frac{\bar{iD} \cdot S}{M_B} B v \cdot A \right) \\
- \beta \left( \frac{S}{M_B} B v \cdot A \right) B - \beta \left( \frac{\bar{iD} \cdot S}{M_B} B v \cdot A \right) + \left( \frac{D^2}{2M_B} T^\mu T_\mu \right)
\]  

(22)

where \(^3\) \( D_\perp^2 = D^2 - (v \cdot D)^2 \).

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\(^2\) While there are at least three other methods of generating the \( 1/M_B \) Lagrangian, the technique of RI allows an explicit check for redundant or missing operators.

\(^3\) Using \( D_\perp^2 \) instead of \( D^2 \) is a choice as one could use the LO equations of motion to eliminate the \( (v \cdot D)^2 \) term \[33\], (suitably shifting the LECs as well so that observables remain unchanged). This is an aesthetic choice as it gives the familiar form of the first relativistic correction to the propagator, \( \bar{k}^2/2M_B \).
The Lagrangian contains additional $\mathcal{O}(q^4)$ operators as well as $\mathcal{O}(q^4)$ operators which are invariant under the $SU(3)$ chiral transformations. The operators relevant to the self energy of the octet baryons are

$$\mathcal{L}_{c.t.}^{(2,4)} = \frac{1}{(4\pi f)} \left\{ b_1^A \overline{B}^{kji} (A \cdot A)_i^n B_{njk} + b_2^A \overline{B}^{kji} (A \cdot A)_k^n B_{ijn} ight.$$  

$$+ b_3^A \overline{B}^{kji} (A_\mu)_i^n (A^\mu)_j^n B_{lnk} + b_4^A \overline{B}^{kji} B_{ijk} \text{Tr} (A \cdot A)$$  

$$+ b_5^A \overline{B}^{kji} (v \cdot A v \cdot A)_i^n B_{njk} + b_6^A \overline{B}^{kji} (v \cdot A v \cdot A)_k^n B_{ijn}$$  

$$+ b_7^A \overline{B}^{kji} \left( \mathcal{M}_+ \mathcal{M}_+ \right)_i^n B_{njk} + b_8^A \overline{B}^{kji} \left( \mathcal{M}_+ \mathcal{M}_+ \right)_k^n B_{ijn}$$  

$$+ b_9^A \overline{B}^{kji} \left( \mathcal{M}_+ \right)_i^n \left( \mathcal{M}_+ \right)_j^n B_{lnk} + b_{10}^A \overline{B}^{kji} \left( \mathcal{M}_+ \mathcal{M}_+ \right)_k^n B_{ijn}$$  

$$+ b_{11}^A \overline{B}^{kji} B_{ijk} \text{Tr} (\mathcal{M}_+) \text{Tr} (\mathcal{M}_+) \right\} . \quad (23)$$

The operators of the form $(\overline{B} v \cdot A v \cdot AB)$ have not been kept explicit in the literature so far for calculations of the octet baryon masses to $\mathcal{O}(m_\pi^2)$ \cite{24, 26, 34}. These operators have identical flavor structure to the corresponding $(\overline{B} A \cdot AB)$ operators. However, these two sets of operators have different Lorentz structure, which gives rise to different finite $m_\pi$ contributions to the octet baryon self energy calculations. One can choose a renormalization scheme such that the contributions to the octet baryon masses from these different operators can not be distinguished. Therefore, with a suitable redefinition of the $b_{1-4}^A$ and $b_{1-7}^M$ coefficients, the operators with coefficients $b_{1-4}^A$ can be neglected in the baryon mass calculations, as their contributions to the masses can be absorbed by the other operators to this order in the chiral expansion.

However, the operators with coefficients $b_4^A$ and $b_7^A$ can be distinguished in $\pi N \rightarrow \pi N$ scattering at tree level, for example. It is therefore useful to keep both types of operators explicitly in the Lagrangian, allowing them to be distinguished in the octet baryon mass calculation. This also provides a consistent means to determine these LECs, as they are the same coefficients which appear in the Lagrangian used for the calculation of other observables like $\pi N \rightarrow \pi N$ scattering.  

In principle, additional $1/M_B$ operators with the same chiral symmetry properties as those contained in Eq. (23) can be generated. However, these $1/M_B$ operators do not have their coefficients constrained by RI, therefore they can be absorbed by a re-definition of the $b_i^{A,v,A,M}$ coefficients. For example, the Ram-Mabillard field used to describe the decuplet baryons contains un-physical spin-1/2 degrees of freedom which can propagate when the decuplet baryons are off mass-shell. It can be shown that the contribution to the octet baryon masses from these un-physical degrees of freedom is suppressed by $1/M_B$ \cite{35}. Therefore these effects are implicitly included in the operators with coefficients $b_i^{A,v,A}$.  

\footnote{One could also add operators of the form $(\overline{B} S \cdot A S \cdot AB)$, but it is straightforward to show that they are a linear combination of the operators in Eq. (23), and therefore not distinct.}
The full Lagrangian relevant to the calculation of the octet baryon masses to \( \mathcal{O}(m_q^2) \) is

\[
\mathcal{L}_{m_q^2} = \mathcal{L}_{LO} + \mathcal{L}_{(2)}^{(2)} + \mathcal{L}_{c.t.}^{(2,4)}.
\]

(24)

III. BARYON MASSES

The mass of the \( i \)th baryon in the chiral expansion is

\[
M_{B_i} = M_0(\mu) - M_{B_i}^{(1)}(\mu) - M_{B_i}^{(3/2)}(\mu) - M_{B_i}^{(2)}(\mu) + \ldots
\]

(25)

Here, \( M_0(\mu) \) is the renormalized mass of the octet baryons in the chiral limit, is independent of \( m_q \) and also \( B_i \). \( M_{B_i}^{(n)} \) is the contribution to the \( i \)th octet baryon of the order \( m_q^n \), and \( \mu \) is the dimensional regularization scale. For this calculation we use a renormalization scheme of always subtracting terms proportional to

\[
\left[ \frac{1}{\epsilon} - \gamma + 1 + \log 4\pi \right].
\]

The inclusion of the decuplet baryon fields in HB\( \chi \)PT requires additional operators involving powers of \( \Delta/\Lambda_\chi \), as \( \Delta \) is a chiral singlet. Thus all terms in the Lagrangian can be multiplied by arbitrary functions of \( \Delta/\Lambda_\chi \) without altering their symmetry properties under chiral transformations. To the order we are working, we can systematically include the contributions to the octet masses from these additional operators, by treating all constants in the calculation as arbitrary polynomial functions of \( \Delta/\Lambda_\chi \), and expand them to the appropriate order. For example \( \alpha_M, \beta_M \) and \( \sigma_M \) must be expanded out to \( \mathcal{O}(\Delta^2/\Lambda_\chi^2) \) while \( D, F \) and \( C \) must be expanded to \( \mathcal{O}(\Delta/\Lambda_\chi) \),

\[
\alpha_M \rightarrow \alpha_M \left( \frac{\Delta}{\Lambda_\chi} \right) = \alpha_M \left[ 1 + a_1^M \left( \frac{\Delta}{\Lambda_\chi} \right) + a_2^M \left( \frac{\Delta}{\Lambda_\chi} \right)^2 + \mathcal{O} \left( \frac{\Delta^3}{\Lambda_\chi^3} \right) \right],
\]

\[
D \rightarrow D \left( \frac{\Delta}{\Lambda_\chi} \right) = D \left[ 1 + d_1 \left( \frac{\Delta}{\Lambda_\chi} \right) + \mathcal{O} \left( \frac{\Delta^2}{\Lambda_\chi^2} \right) \right].
\]

(26)

Additionally, the LECs are also implicit functions of \( \mu \), to cancel the scale dependence of observables arising from the loop integrals.

In calculating the masses, the leading dependence upon \( m_q \) arises from the terms in our Lagrangian, Eq. (15), with coefficients \( \alpha_M, \beta_M, \) and \( \sigma_M \), which can be determined from experiment or lattice simulations. The \( \mathcal{O}(m_q^{3/2}) \) contributions arise from the one-loop diagrams shown in Fig. 1 which are formed from the operators in the Lagrangian with
coefficients $\alpha, \beta$ and $\mathcal{C}$. The $\mathcal{O}(m_q^2)$ contributions arise from the one-loop diagrams shown in Fig. 2 from the tree-level contributions of the operators with coefficients, $b_i^M$, and from the NLO wave-function corrections.

We find that the contributions to the nucleon mass are \(^5\)

\[
M_N^{(1)} = 2m_0(\alpha_M + \beta_M + 2\sigma_M) + 2m_\pi\sigma_M, \quad (27)
\]
\[
M_N^{(3/2)} = \frac{1}{8\pi F^2} \left[ \frac{3}{2} (D + F)^2 m_\pi^3 + \frac{1}{6} (D - 3F)^2 m_\eta^3 + \frac{1}{3} (5D^2 - 6DF + 9F^2) m_K^3 + \frac{\mathcal{C}^2}{\pi} \left( \frac{4}{3} F(m_\pi, \Delta, \mu) + \frac{1}{3} F(m_K, \Delta, \mu) \right) \right], \quad (28)
\]

consistent with previous calculations \([23, 36]\). The contribution of this work is the NNLO

\[^5\text{We express } \alpha \text{ and } \beta \text{ in terms of } D \text{ and } F, \text{ following the convention set by } [13].\]
mass contribution to the octet baryons in $SU(3)$,

$$- M_N^{(2)} = - \mathcal{L}(m_\pi, \mu) \left\{ \frac{1}{(4\pi f)^3} \left[ \frac{3}{2} b_1^A + \frac{3}{2} b_2^A - b_3^A + 3b_4^A + \frac{3}{8} b_1^A + \frac{3}{8} b_2^A - \frac{1}{4} b_3^A + \frac{3}{4} b_4^A \right] + \frac{1}{M_B(4\pi f)^2} \left[ \frac{27}{16} (D + F)^2 + \frac{5}{2} C^2 \right] \right\}$$

$$- \mathcal{L}(m_\eta, \mu) \left\{ \frac{1}{(4\pi f)^3} \left[ \frac{1}{6} b_1^A + \frac{1}{6} b_2^A + \frac{1}{6} b_3^A + b_4^A + \frac{1}{24} b_1^A + \frac{1}{24} b_2^A + \frac{1}{24} b_3^A + \frac{1}{4} b_4^A \right] + \frac{1}{M_B(4\pi f)^2} \left[ \frac{3}{16} (5D^2 - 6DF + 9F^2) + \frac{5}{8} C^2 \right] \right\}$$

$$+ \mathcal{L}(m_\mu, \mu) \left\{ \frac{1}{(4\pi f)^2} \left[ \frac{6}{(4\pi f)^2} (\alpha_M + \beta_M + 2\sigma_M) \right. \right.$$

$$+ \mathcal{L}(m_\eta, \mu) \left\{ \frac{1}{(4\pi f)^2} \left[ \frac{1}{3} m^A (\alpha_M + \beta_M + 2\sigma_M) + \frac{8}{3} m_s \right] + \frac{1}{(4\pi f)^2} \left[ 2(\overline{m} + m_s) (\alpha_M + \beta_M + 4\sigma_M) + (5D^2 - 6DF + 9F^2) M_N^{(1)} \right. \right.$$

$$- \frac{9}{2} (D - F)^2 M_N^{(1)} - \frac{1}{2} (D + 3F)^2 M_N^{(1)} \right\}$$

$$+ \mathcal{J}(m_\pi, \Delta, \mu) \left\{ \frac{4C^2}{(4\pi f)^2} \left[ M_N^{(1)} + M_\Delta^{(1)} \right] + \mathcal{J}(m_K, \Delta, \mu) \frac{C^2}{(4\pi f)^2} \left[ M_N^{(1)} + M_\Sigma^{(1)} \right] \right\}$$

$$+ m_A^A \left\{ \frac{1}{(4\pi f)^3} \left[ \frac{3}{16} b_1^A + \frac{3}{16} b_2^A - \frac{1}{8} b_3^A + \frac{3}{8} b_4^A \right] + \frac{1}{M_B(4\pi f)^2} \left[ \frac{45}{32} (D + F)^2 + \frac{9}{4} C^2 \right] \right\}$$

$$+ m_\eta^A \left\{ \frac{1}{(4\pi f)^3} \left[ \frac{1}{48} b_1^A + \frac{1}{48} b_2^A + \frac{1}{48} b_3^A + \frac{1}{8} b_4^A \right] + \frac{1}{M_B(4\pi f)^2} \left[ \frac{5}{32} (D - 3F)^2 \right] \right\}$$

$$+ m_K^A \left\{ \frac{1}{(4\pi f)^3} \left[ \frac{1}{8} b_1^A + \frac{1}{8} b_2^A + \frac{1}{2} b_4^A \right] + \frac{1}{M_B(4\pi f)^2} \left[ \frac{5}{16} (5D^2 - 6DF + 9F^2) + \frac{9}{4} C^2 \right] \right\}$$

$$+ m_K^2 \left\{ \frac{1}{(4\pi f)^2} \left[ \frac{2}{3} (5D^2 - 6DF + 9F^2) M_N^{(1)} - 3(D - F)^2 M_\Sigma^{(1)} \right. \right.$$

$$- \frac{1}{3} (D + 3F)^2 M_\Lambda^{(1)} + C^2 \left[ M_N^{(1)} + M_\Sigma^{(1)} \right] \right\}$$

$$+ m_\eta^2 \left[ b_1^M + b_2^M + b_3^M + 2b_4^M + 2b_5^M + 2b_6^M + 4b_7^M \right]$$

$$+ \frac{m_\eta m_s}{(4\pi f)} \left[ b_5^M + b_6^M + 4b_7^M \right] + \frac{m_s^2}{(4\pi f)} \left[ b_1^M + b_7^M \right]$$

(29)

The LO octet and decuplet masses given in the above expression can be found in Appendix B.

We have made use of several abbreviations for the functions entering from loop contributions.
namely:

\[
\mathcal{L}(m_\phi, \mu) = m_\phi^2 \log \left( \frac{m_\phi^2}{\mu^2} \right),
\]

\[
\mathcal{E}(m_\phi, \mu) = m_\phi^4 \log \left( \frac{m_\phi^2}{\mu^2} \right),
\]

\[
\mathcal{F}(m_\phi, \Delta, \mu) = \left( m_\phi^2 - \Delta^2 \right) \left[ \sqrt{\Delta^2 - m_\phi^2} \log \left( \frac{\Delta - \sqrt{\Delta^2 - m_\phi^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m_\phi^2 + i\epsilon}} \right) - \Delta \log \left( \frac{m_\phi^2}{\mu^2} \right) \right] - \frac{1}{2} \Delta m_\phi^2 \log \left( \frac{m_\phi^2}{\mu^2} \right),
\]

\[
\mathcal{J}(m_\phi, \Delta, \mu) = \left( m_\phi^2 - 2\Delta^2 \right) \log \left( \frac{m_\phi^2}{\mu^2} \right) + 2\Delta \sqrt{\Delta^2 - m_\phi^2} \log \left( \frac{\Delta - \sqrt{\Delta^2 - m_\phi^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m_\phi^2 + i\epsilon}} \right). \tag{30}
\]

It should be stressed that the above expansion, Eq. (27), (28) and (29), is a quark mass expansion. The meson masses are replacements for the quark masses, given by Eq. (5). If an expansion in terms of the physical meson masses is desired, then to be consistent to \( \mathcal{O}(m_\phi^2) \sim \mathcal{O}(m_\phi^4) \), one needs to use the NLO expression for the meson masses in Eq. (27) and the LO expression in Eq. (28) and Eq. (29). These results build on previous work \[26, 34, 37, 38\]. The results for the \( \chi \)PT calculation of the \( \Sigma, \Lambda \) and \( \Xi \) baryons are located in Appendix B.

IV. PQ\( \chi \)PT

In PQQCD Lagrangian is

\[
\mathcal{L} = \sum_{j,k=1}^{9} \bar{Q}^j [i\not{D} - m_Q]_j^k Q_k. \tag{31}
\]

This differs from the \( SU(3) \) Lagrangian of QCD by the inclusion of six extra quarks; three bosonic ghost quarks, \( \tilde{u}, \tilde{d}, \tilde{s} \), and three fermionic sea quarks, \( j, l, r \). The nine quark fields transform in the fundamental representation of the graded \( SU(6|3) \) group. They are combined in the nine-component vector

\[
Q = \begin{pmatrix} q \\ q_{\text{sea}} \\ \tilde{q} \end{pmatrix}. \tag{32}
\]

Here we have separated the quark field vector into the valence, sea and ghost sectors:

\[
q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad q_{\text{sea}} = \begin{pmatrix} j \\ l \\ r \end{pmatrix}, \quad \tilde{q} = \begin{pmatrix} \tilde{u} \\ \tilde{d} \\ \tilde{s} \end{pmatrix}. \tag{33}
\]
The quark fields obey the equal-time commutation relation

\[ Q_i^\alpha(x) Q_j^{\beta\dagger}(y) - (-1)^{\eta_i \eta_j} Q_j^{\beta\dagger}(y) Q_i^\alpha(x) = \delta^{\alpha\beta} \delta_{ij} \delta^3(x - y), \]  

(34)

where \( \alpha, \beta \) are spin and \( i, j \) are flavor indices. Analogous graded equal-time commutation relations can be written for two \( Q \)'s and two \( Q^{\dagger} \)'s. The grading factors,

\[ \eta_k = \begin{cases} 1 & \text{for } k = 1, 2, 3, 4, 5, 6, \\ 0 & \text{for } k = 7, 8, 9 \end{cases}, \]  

(35)

take into account the different fermionic and bosonic statistics for the quark fields. In the isospin limit the quark mass matrix of \( SU(6) \) is given by

\[ m_Q = \text{diag}(m, m, m, m, m, m). \]  

(36)

Setting the ghost quark masses equal to the valence quark masses results in an exact cancellation in the path integral between the valence quark determinant and the ghost quark determinant. The sea quark determinant is unaffected. Thus one has a way to vary the valence and sea quark masses independently. QCD is recovered in the limit \( m_j \to m \) and \( m_r \to m_s \).

### A. Pseudo-Goldstone Mesons

For massless quarks, the theory corresponding to the Lagrangian in Eq. (31) has a graded \( SU(6|3)_L \otimes SU(6|3)_R \otimes U(1)_V \) symmetry which is assumed to be spontaneously broken down to \( SU(6)_V \otimes U(1)_V \) in analogy with QCD. The effective low-energy theory obtained by perturbing about the physical vacuum state of PQQCD is PQ\( \chi \)PT. The result is 80 pseudo-Goldstone mesons whose dynamics can be described at LO in the chiral expansion by the Lagrangian

\[ \mathcal{L} = \frac{f^2}{8} \text{str} \left( \partial^\mu \Sigma^\dagger \partial_\mu \Sigma \right) + \lambda \text{str} \left( m_Q \Sigma + m_Q^{\dagger} \Sigma \right) + \alpha_\Phi \partial^\mu \Phi_0 \partial_\mu \Phi_0 - m_0^2 \Phi_0^2, \]  

(37)

where

\[ \Sigma = \exp \left( \frac{2i\Phi}{f} \right) = \xi^2, \quad \Phi = \begin{pmatrix} M \chi^\dagger \\ \chi \end{pmatrix}. \]  

(38)

The operation \( \text{str}(\ ) \) in Eq. (37) is the supertrace over flavor indices. The quantities \( \alpha_\Phi \) and \( m_0 \) are non-vanishing in the chiral limit. \( M \) and \( \tilde{M} \) are matrices containing bosonic mesons while \( \chi \) and \( \chi^{\dagger} \) are matrices containing fermionic mesons:

\[ M = \begin{pmatrix} \eta_u & \pi^+ & K^+ & J^0 & L^+ & R^+ \\ \pi^- & \eta_d & K^0 & J^- & L^0 & R^0 \\ K^- & K^0 & \eta_s & J_s^- & L_s^0 & R_s^0 \\ J^+ & J_s^+ & \eta_{\tilde{s}} & Y_{\tilde{s}}^+ & Y_{\tilde{s}}^0 & \eta_{\tilde{r}} \\ L^- & L_s^0 & Y_{\tilde{s}}^+ & \eta_{\tilde{r}} & Y_{\tilde{r}}^0 & \eta_{\tilde{r}} \\ R^- & R_s^0 & Y_{\tilde{s}}^+ & \eta_{\tilde{r}} & Y_{\tilde{r}}^0 & \eta_{\tilde{r}} \end{pmatrix}, \quad \tilde{M} = \begin{pmatrix} \bar{\eta}_u & \tilde{\pi}^+ & \tilde{K}^+ \\ \tilde{\pi}^- & \bar{\eta}_d & \tilde{K}^0 \\ \tilde{K}^- & \tilde{K}^0 & \bar{\eta}_s & \tilde{J}_s^- & \tilde{L}_s^0 & \tilde{R}_s^0 \\ \tilde{J}^+ & \tilde{J}_s^+ & \bar{\eta}_{\tilde{s}} & \tilde{Y}_{\tilde{s}}^+ & \tilde{Y}_{\tilde{s}}^0 & \bar{\eta}_{\tilde{r}} \\ \tilde{L}^- & \tilde{L}_s^0 & \tilde{Y}_{\tilde{s}}^+ & \bar{\eta}_{\tilde{r}} & \tilde{Y}_{\tilde{r}}^0 & \bar{\eta}_{\tilde{r}} \\ \tilde{R}^- & \tilde{R}_s^0 & \tilde{Y}_{\tilde{s}}^+ & \bar{\eta}_{\tilde{r}} & \tilde{Y}_{\tilde{r}}^0 & \bar{\eta}_{\tilde{r}} \end{pmatrix}. \]  

(39)

\[ \chi = \begin{pmatrix} \chi_{\eta_u} & \chi_{\pi} & \chi_{K} & \chi_{J}^0 & \chi_{L}^0 & \chi_{R}^+ \\ \chi_{\pi} & \chi_{\eta_d} & \chi_{K} & \chi_{J} & \chi_{L}^0 & \chi_{R}^0 \\ \chi_{K} & \chi_{\pi} & \chi_{\eta_s} & \chi_{J} & \chi_{L}^0 & \chi_{R}^0 \\ \chi_{\pi}^+ & \chi_{\eta_d} & \chi_{K} & \chi_{J}^0 & \chi_{L}^0 & \chi_{R}^+ \\ \chi_{\pi}^- & \chi_{\eta_d} & \chi_{K} & \chi_{J} & \chi_{L}^0 & \chi_{R}^0 \\ \chi_{K} & \chi_{\pi} & \chi_{\eta_s} & \chi_{J} & \chi_{L}^0 & \chi_{R}^0 \end{pmatrix}. \]  

(39)
Thus the octet baryons are contained as an \(8\) been explicitly constructed in [13]. Under the interchange of flavor in dices, one finds [32]:

\[
\text{valence, sea or ghost quarks. One decomposes the irreducible representations of } B
\]

The upper \(3 \times 3\) block of \(M\) is the usual octet of pseudo-scalar mesons and the remaining components are mesons formed with one or two sea quarks.

The flavor singlet field is defined to be \(\Phi_0 = \text{str}(\Phi)/\sqrt{6}\). PQQCD has a strong axial anomaly, \(U(1)_A\), and therefore the mass of the singlet field, \(m_0\) can be taken to be the order of the chiral symmetry breaking scale, \(m_0 \rightarrow \Lambda_{\chi}[39]\). In this limit, the \(\eta\) two-point correlation functions deviate from the simple, single pole form. The \(\eta_a\eta_b\) propagator with \(2 + 1\) sea-quarks and \(a, b = u, d, s, j, l, r, \bar{u}, \bar{d}, \bar{s}\) is found to be at leading order:

\[
G_{\eta_a\eta_b} = \frac{i\epsilon_a\delta_{ab}}{q^2 - m_{\eta_a}^2 + i\epsilon} - \frac{i\epsilon_a\epsilon_b (q^2 - m_{\eta_j}^2)}{3(q^2 - m_{\eta_a}^2 + i\epsilon)(q^2 - m_{\eta_b}^2 + i\epsilon)(q^2 - m_{\eta_j}^2 + i\epsilon)},
\]

where

\[
\epsilon_a = (-1)^{1+\eta_a}
\]

The mass, \(m_{xy}\), is the mass of a meson composed of (anti-)quarks of flavor \(x\) and \(y\), while the mass, \(m_X\) is defined as \(m_X = \frac{1}{3} (m_{jj}^2 + 2m_{rr}^2)\). This can be compactly written as

\[
G_{\eta_a\eta_b} = \epsilon_a\delta_{ab}P_a + \epsilon_a\epsilon_b\mathcal{H}_{ab}(P_a, P_b, P_X),
\]

where

\[
P_a = \frac{i}{q^2 - m_{\eta_a}^2 + i\epsilon},\ P_b = \frac{i}{q^2 - m_{\eta_b}^2 + i\epsilon},\ P_X = \frac{i}{q^2 - m_X^2 + i\epsilon}
\]

\[
\mathcal{H}_{ab}(A, B, C) = -\frac{1}{3} \left[ \frac{(m_{jj}^2 - m_{\eta_a}^2)(m_{rr}^2 - m_{\eta_a}^2)}{(m_{\eta_a}^2 - m_{\eta_b}^2)(m_{\eta_a}^2 - m_X^2)} A - \frac{(m_{jj}^2 - m_{\eta_b}^2)(m_{rr}^2 - m_{\eta_b}^2)}{(m_{\eta_b}^2 - m_{\eta_a}^2)(m_{\eta_b}^2 - m_X^2)} B, \right.
\]

\[
\left. + \frac{(m_X^2 - m_{jj})^2}{(m_X^2 - m_{\eta_a}^2)(m_X^2 - m_{\eta_b}^2)} \frac{(m_X^2 - m_{rr})}{(m_X^2 - m_{\eta_a}^2)(m_X^2 - m_{\eta_b}^2)} C \right].
\]

**B. Baryons**

In PQ\(\chi\)PT the baryons are composed of three quarks, \(Q_iQ_jQ_k\), where \(i - k\) can be valence, sea or ghost quarks. One decomposes the irreducible representations of \(SU(6)[3]_V\) into irreducible representations of \(SU(3)_{\text{val}} \otimes SU(3)_{\text{sea}} \otimes SU(3)_{\text{ghost}} \otimes U(1)\). The method for including the octet and decuplet baryons into PQ\(\chi\)PT is to use the interpolating field [13, 32]:

\[
B_{ijk}^\gamma \sim \left( Q_i^{\alpha,a}Q_j^{\beta,b}Q_k^{\gamma,c} - Q_i^{\alpha,a}Q_j^{\gamma,c}Q_k^{\beta,b} \right) \epsilon_{abc}(C\gamma_5)_{\alpha\beta}.
\]

We require that \(B_{ijk} = B_{ijk}\), defined in Eq. (11), when the indices, \(i, j, k\) are restricted to \(1 - 3\). Thus the octet baryons are contained as an \((8, 1, 1)\) of \(SU(3)_{\text{val}} \otimes SU(3)_{\text{sea}} \otimes SU(3)_{\text{ghost}} \otimes U(1)\) in the \(240\) representation. In addition to the conventional octet baryons composed of valence quarks, \(B_{ijk}\) also contains baryon fields composed of sea and ghost quarks. In this paper we only need the baryons which contain at most one sea or ghost quark, and these states have been explicitly constructed in [13]. Under the interchange of flavor indices, one finds [32]:

\[
B_{ijk} = (-)^{1+n_{\eta_a}} B_{ikj} \quad \text{and} \quad B_{ijk} + (-)^{1+n_{\eta_b}} B_{jk i} + (-)^{1+n_{\eta_j}+n_{\eta_k}+n_{\eta_k}} B_{kji} = 0.
\]
Similarly, one can construct the spin-$\frac{3}{2}$ decuplet baryons which are embedded in the $138$, and have an interpolating field

$$T_{ijk}^{\alpha\mu} \sim \left( Q_i^{\alpha,a} Q_j^{\beta,b} Q_k^{\gamma,c} + Q_i^{\beta,b} Q_j^{\gamma,c} Q_k^{\alpha,a} + Q_i^{\gamma,c} Q_j^{\alpha,a} Q_k^{\beta,b} \right) \epsilon_{abc} \left( C \gamma^\mu \right)_{\beta\gamma}. \quad (46)$$

We require that $T_{ijk} = T_{ijk}$, when the indices $i, j, k$ are restricted to $1 - 3$. Under $SU(3)_{val} \otimes SU(3)_{sea} \otimes SU(3)_{ghost} \otimes U(1)$ they transform as a $(10, 1, 1)$. In addition to the conventional decuplet resonances composed of valence quarks, $T_{ijk}$ contains fields with sea and ghost quarks, which have also been constructed in [13].

Under the interchange of flavor indices, one finds that

$$T_{ijk} = (-)^{1+\eta_i} T_{jik}, \quad T_{ijk} = (-)^{1+\eta_j} T_{ikj}. \quad (47)$$

Under $SU(6|3)_V$, both $B_{ijk}$ and $T_{ijk}$ transform as $[32]

$$B_{ijk} \rightarrow (-)^n(n_i+n_j)(n_k+n_m) U_i^1 U_j^m U_k^n B_{lmn}. \quad (48)$$

C. Baryon-Meson Lagrange Density

The utility of using 3-index baryon tensors in the QCD calculation is fully realized when one extends the $\chi$PT Lagrangian to PQ$\chi$PT. It has recently been shown that in the meson sector there are extra operators in PQ$\chi$PT, that do not arise in $\chi$PT [40]. The Cayley-Hamilton identity for traceless $3 \times 3$ matrices, allows one to reduce the number of operators in the meson sector of $\chi$PT. There is no such identity for the supertraceless matrices of $SU(6|3)$, allowing for extra operators. Use of the $SU(3)$ matrix of baryon fields, Eq. (10), raises similar concerns at higher orders in the expansion of the Lagrangian, when extending to PQ$\chi$PT. However, when using three-index baryon tensors, the flavor contractions involving the baryon fields can not be expressed as a trace, as with the more familiar form [23, 24, 25, 26, 34, 36]. Therefore, there is no danger of inadvertently neglecting operators when extending from $SU(3)$ HB$\chi$PT to $SU(6|3)$ PQHB$\chi$PT. Furthermore, when employing the three-index notation for the baryon fields the coefficients of the HB$\chi$PT operators are the same as the coefficients of the corresponding PQHB$\chi$PT operators [39]. This is useful as for the foreseeable future, reliable calculations of baryonic matrix elements will be performed with PQQCD lattice simulations.

To write down the PQ$\chi$PT Lagrangian, we must also include the appropriate grading factors. The flavor contractions are now defined as [13]

$$B_{ijkl} \Gamma B = B_{i,j,k,l}^{\alpha,\gamma,i,l} \Gamma_{\alpha} \Gamma_{\beta} B_{\beta,ijl},$$
$$B_{ijkl} \Gamma Y B = B_{i,j,k,l}^{\alpha,\gamma,i,l} \Gamma_{\alpha} Y_{i,l} B_{\beta,ijl},$$
$$B_{ijkl} \Gamma B Y = (-)^{(n_i+n_l)(n_k+n_l)} B_{i,j,k,l}^{\alpha,\gamma,i,l} \Gamma_{\alpha} Y_{k,l} B_{\beta,ijl},$$
$$B_{ijkl} \Gamma Y^\mu T_\mu = B_{i,j,k,l}^{\alpha,\gamma,i,l} \Gamma_{\alpha} (Y^\mu)_{i,l} \Gamma_{\beta} T_{\mu,ijkl}. \quad (49)$$
The leading order PQ Lagrangian is given by
\[
\mathcal{L}_{LO}^{PQ} = \left( \mathcal{B} i v \cdot D \mathcal{B} \right) + 2\alpha_M \left( \mathcal{B} \mathcal{B} \mathcal{M}_+ \right) + 2\beta_M \left( \mathcal{B} \mathcal{M}_+ \mathcal{B} \right) + 2\sigma_M \left( \mathcal{B} \mathcal{B} \right) \text{str}(\mathcal{M}_+) \\
- \left( \mathcal{B} \mathcal{B} \right) \frac{1}{\alpha} \mathcal{T}^\mu \left[ i v \cdot D - \Delta \right] \mathcal{T}_\mu + 2\gamma_M \left( \mathcal{T}^\mu \mathcal{M}_+ \mathcal{T}_\mu \right) - 2\sigma_M \left( \mathcal{T}^\mu \mathcal{T}_\mu \right) \text{str}(\mathcal{M}_+) \\
+ 2\alpha \left( \mathcal{B} \mathcal{S}^\mu \mathcal{A}_\mu \right) + 2\beta \left( \mathcal{B} \mathcal{S}^\mu \mathcal{A}_\mu \mathcal{B} \mathcal{S}^\nu \mathcal{A}_\nu \right) + 2\mathcal{H} \left( \mathcal{T}^\nu \mathcal{S}^\mu \mathcal{A}_\mu \mathcal{T}_\nu \right) \\
+ \sqrt{\frac{3}{2}} \mathcal{C} \left[ \left( \mathcal{T}^\mu \mathcal{A}_\mu \mathcal{B} \right) + \left( \mathcal{B} \mathcal{A}_\nu \mathcal{T}^\nu \right) \right].
\] (50)

The fixed coefficient and higher dimensional operators Lagrangians are given by
\[
\mathcal{L}_{PQ}^{\frac{D_i}{M_B}} = - \left( \mathcal{B} \frac{D_i^2}{2M_B} \mathcal{B} \right) - \alpha \left( \mathcal{B} \frac{S \cdot \mathcal{D}}{M_B} \mathcal{B} v \cdot A \right) + \alpha \left( \mathcal{B} \frac{\mathcal{D} \cdot S}{M_B} \mathcal{B} v \cdot A \right) \\
- \beta \left( \mathcal{B} v \cdot A \frac{S \cdot \mathcal{D}}{M_B} \mathcal{B} \right) + \beta \left( \mathcal{B} \frac{\mathcal{D} \cdot S}{M_B} v \cdot A \right) + \left( \mathcal{T}^\mu \frac{D_i^2}{2M_B} \mathcal{T}_\mu \right),
\] (51)

and
\[
\mathcal{L}^{(2,4)}_{PQ} = \frac{1}{(4\pi f)} \left\{ \begin{array}{l}
b_1 \mathcal{B}^{kji} (A \cdot A)_i^n B_{njk} + b_2 (-)^{(n+\mu)(n+\nu)} \mathcal{B}^{kji} (A \cdot A)_i^n B_{ijkl} \\
b_3 (-)^{(n+\mu)(n+\nu)} \mathcal{B}^{kji} (A\mu)_i^n B_{nk} + b_4 (-)^{(n+\mu)(n+\nu)} \mathcal{B}^{kji} (v \cdot A)_i^n B_{nj} \\
b_5 (-)^{(n+\mu)(n+\nu)} \mathcal{B}^{kji} (\mathcal{M}_+ \mathcal{M}_+)_i^n B_{njk} + b_6 (-)^{(n+\mu)(n+\nu)} \mathcal{B}^{kji} (\mathcal{M}_+ \mathcal{M}_+)_i^n B_{ijkl} \\
\end{array} \right\}. (52)
\]

All the coefficients appearing in Eq. (50), Eq. (51) and Eq. (52), have the same numerical values as in \(\chi PT\). The situation is similar to the \(\chi PT\) case considered in Section II B. All the operators in the Lagrangian can be multiplied by arbitrary functions of \(\Delta/\Lambda_\chi\). We include these effects by treating all the coefficients in the Lagrangian as arbitrary polynomial functions of \(\Delta/\Lambda_\chi\), and expand out to the appropriate order (see Section III). All the coefficients appearing in this expansion also have the same numerical values as in the \(\chi PT\) case.

V. BARYON MASSES IN PQ\(\chi PT\)

The chiral expansion of the octet baryon masses in PQ\(\chi PT\) has the same form as in \(\chi PT\).
\[
M_{B_i} = M_0 (\mu) - M_{B_i}^{(1)} (\mu) - M_{B_i}^{(3/2)} (\mu) - M_{B_i}^{(2)} (\mu) + \ldots
\] (53)

\footnote{It is interesting to note operators of the form \(\mathcal{B}[A_{\mu}, A_{\nu}][S^\mu, S^\nu]B\) that vanish in \(\chi PT\) don’t vanish in PQ\(\chi PT\) due to grading factors. However, they don’t contribute to the octet baryon masses.}
FIG. 3: In addition to the one-loop diagrams in Fig. M\textsubscript{B_i}^{(3/2)} also receives contributions from the singlet (hairpins) in PQ\chi PT. Single and double lines correspond to 240-baryons and 138-baryons respectively. The crossed dashed line denotes a hairpin propagator. The filled squares denote the axial coupling given in Eq. (50).

\[ M^1_N = 2m(\alpha_M + \beta_M) + 2\sigma_M(2m_j + m_r) \]  
\[ M^{(3/2)}_N = \frac{1}{8\pi f^2} \left[ 4D(F - \frac{1}{3}D)m^3 \pi + \frac{1}{3}(5D^2 - 6DF + 9F^2)(2m^3_{ju} + m^3_{ru}) \right. \]  
\[ + (D - 3F)^2 M^3(m_{\pi}, m_{\pi}) + \frac{2C^2}{3\pi} F(m_{\pi}, \Delta, \mu) \]  
\[ \left. + \frac{2C^2}{3\pi} F(m_{ju}, \Delta, \mu) + \frac{C^2}{3} F(m_{ru}, \Delta, \mu) \right] \]

consistent with [13]. The contribution of this work is the NNLO mass contribution to the

However, we must also include hairpin contributions from the eta field propagators. See Fig. 3 and Fig. 4.

We find that the mass expansion of the nucleon is:

We find that the mass expansion of the nucleon is:
octet baryon masses.

\[ -M_N^{(2)} = \mathcal{L}(m_\pi, \mu) \left\{ \frac{1}{(4\pi f)^3} \left[ \frac{b_{3\pi}^A}{3} + \frac{b_{4\pi}^A}{8} \right] - \frac{1}{M_B(4\pi f)^2} \left[ \frac{3D(3F-D)}{2} + \frac{5C^2}{4} \right] \right\} \\
-\mathcal{L}(m_\pi, m_\mu, \mu) \left\{ \frac{1}{(4\pi f)^3} \left[ b_1^A + b_2^A + b_3^A + \frac{1}{2} b_1^{vA} + \frac{1}{2} b_2^{vA} \right] + \frac{1}{M_B(4\pi f)^2} \left[ \frac{3(5D^2 - 6DF + 9F^2)}{4} + \frac{5C^2}{4} \right] \right\} \\
-\mathcal{L}(m_{ju}, \mu) \left\{ \frac{1}{(4\pi f)^3} \left[ b_1^A + b_2^A + \frac{1}{4} b_1^{vA} + \frac{1}{4} b_2^{vA} \right] + \frac{1}{M_B(4\pi f)^2} \left[ \frac{3}{8}(5D^2 - 6DF - 9F^2) + \frac{5}{8}C^2 \right] \right\} \\
-\mathcal{L}(m_{ru}, \mu) \left\{ \frac{1}{(4\pi f)^3} \left[ b_4^A + b_5^A + \frac{1}{4} b_4^{vA} + \frac{1}{4} b_5^{vA} \right] \right\} \\
+2\mathcal{L}(m_{nj}, m_{nj}, \mu) + \mathcal{L}(m_{nj}, m_{nj}, \mu) + \mathcal{L}(m_{nj}, m_{nj}, \mu) + \mathcal{L}(m_{nj}, m_{nj}, \mu) + \mathcal{L}(m_{nj}, m_{nj}, \mu) \right\} \\
+\mathcal{L}(m_\pi, m_\pi, \mu) \frac{4\pi_m(\alpha_M + \beta_M)}{(4\pi f)^2} + \frac{8m_\pi\sigma_M}{(4\pi f)^2} \left[ 2\mathcal{L}(m_{nj}, \mu) + \mathcal{L}(m_{nj}, m_{nj}, \mu) \right] \\
+\frac{4\sigma_M}{(4\pi f)^2} \left\{ 2(m_j + m_r)\mathcal{L}(m_{jr}, \mu) + m_r \left[ \mathcal{L}(m_{nj}, \mu) + \mathcal{L}(m_{nj}, m_{nj}, \mu) \right] \right\} \\
+\mathcal{L}(m_{ju}, \mu) \frac{1}{(4\pi f)^2} \left\{ 4(\pi_m + m_j)(\alpha_M + \beta_M) + 2(5D^2 - 6DF + 9F^2)M_N^{(1)} \\
-9(D - F)^2M_{\Sigma_j}^{(1)} - (D - 3F)^2M_{\Sigma_j}^{(1)} \right\} \\
+\mathcal{L}(m_{ru}, \mu) \frac{1}{(4\pi f)^2} \left\{ 2(\pi_m + m_r)(\alpha_M + \beta_M) + (5D^2 - 6DF + 9F^2)M_N^{(1)} \\
- \frac{9}{2}(D - F)^2M_{\Sigma_r}^{(1)} - \frac{1}{2}(D - 3F)^2M_{\Sigma_r}^{(1)} \right\} \\
+\mathcal{J}(m_\pi, \mu) \frac{2C^2}{(4\pi f)^2} \left[ M_N^{(1)} + M_{\Delta}^{(1)} \right] + \mathcal{J}(m_{ju}, \mu) \frac{2C^2}{(4\pi f)^2} \left[ M_N^{(1)} + M_{\Sigma_j}^{(1)} \right] \\
+\mathcal{J}(m_{ru}, \mu) \frac{C^2}{(4\pi f)^2} \left[ M_{\Sigma_j}^{(1)} + M_{\Sigma_{\pi}}^{(1)} \right] + \frac{b_4^{vA}}{2(4\pi f)^3} \left[ m_{nj}^A + m_{jr}^A + \frac{1}{4} m_{nj}^4 \right] \\
-\frac{1}{4\pi f^3} \frac{b_3^{vA}}{16} + \frac{1}{M_B(4\pi f)^2} \left[ \frac{5D(3F-D)}{4} + \frac{9C^2}{8} \right] \\
+\frac{b_1^{vA} + b_2^{vA}}{4(4\pi f)^3} - \frac{5(5D^2 - 6DF + 9F^2) + 9C^2}{8M_B(4\pi f)^2} \right\} \\
+\frac{b_1^{vA} + b_2^{vA}}{8(4\pi f)^3} - \frac{5(5D^2 - 6DF + 9F^2) + 9C^2}{16M_B(4\pi f)^2} + \text{cont.}
\[ + \mathcal{M}^4(m_\pi, m_\pi) \left\{ \frac{b_1^A + b_2^A + b_3^A}{8(4\pi f)^3} - \frac{15(D - 3F)^2}{16M_B(4\pi f)^2} \right\} \\
+ \frac{b_4^A}{8(4\pi f)^3} \left[ 2\mathcal{M}^4(m_{\eta_j}, m_{\eta_j}) + \mathcal{M}^4(m_{\eta_p}, m_{\eta_p}) \right] \\
+ m_\pi^2 \left[ \frac{2}{(4\pi f)^2} \left( \frac{2}{3}(5D^2 - 6DF + 9F^2)M_N^{(1)} - 3(D - F)^2M_{\Sigma_j}^{(1)} \right) \\
- \frac{1}{3}(D + 3F)^2M_{\Lambda_j}^{(1)} + C^2 \left( M_N^{(1)} + M_{\Sigma_j}^{(1)} \right) \right] \right\} \\
+ m_{\pi u}^2 \left[ \frac{2}{(4\pi f)^2} \left( \frac{2}{3}(5D^2 - 6DF + 9F^2)M_N^{(1)} - 3(D - F)^2M_{\Sigma_j}^{(1)} \right) \\
- \frac{1}{3}(D + 3F)^2M_{\Lambda_j}^{(1)} + C^2 \left( M_N^{(1)} + M_{\Sigma_j}^{(1)} \right) \right] \right\} \\
+ \frac{\overline{m}_2^2}{(4\pi f)} (b_1^M + b_2^M + b_3^M) + \frac{\overline{m}_{m_j}}{(4\pi f)} (2b_5^M + 2b_6^M) + \frac{m_2^2}{(4\pi f)} (2b_4^M + 4b_7^M) \\
+ \frac{\overline{m}_r}{(4\pi f)} (b_5^M + b_6^M) + \frac{m_jm_r}{(4\pi f)} 4b_r^M + \frac{m_r^2}{(4\pi f)} (b_4^M + b_7^M), \tag{56} \]

where the functions not already defined in Eq. (30) are,

\[ \mathcal{M}^n(m_\phi, m_\phi') = \mathcal{H}_{\phi\phi'}(m_\phi^n, m_\phi', m_\chi^n) \]

\[ \mathcal{L}(m_\phi, m_\phi') = \mathcal{H}_{\phi\phi'}(\mathcal{L}(m_\phi, \mu), \mathcal{L}(m_\phi', \mu), \mathcal{L}(m_\chi, \mu)) \]

\[ \overline{\mathcal{L}}(m_\phi, m_\phi') = \mathcal{H}_{\phi\phi'}(\overline{\mathcal{L}}(m_\phi, \mu), \overline{\mathcal{L}}(m_\phi', \mu), \overline{\mathcal{L}}(m_\chi, \mu)). \tag{57} \]

We stress that these mass expressions, Eq. (54)–(56), are quark mass expansions. If a meson mass expansion is desired, one can express these quark masses in terms of the lattice meson masses. To do this consistently, one must replace the quark masses in Eq. (56) with the NLO relation to the PQ\(\chi\)PT meson masses, while the quark masses in Eq. (55) and Eq. (56) can be replaced by the LO relation to the PQ meson masses. It is easy to show that these expressions reduce to the \(\chi\)PT expressions, Eq. (27)–(29), in the limit that \(m_j \to \overline{m}\) and \(m_r \to m_s\). The results for the NNLO mass calculations for the remaining octet baryons can be found in Appendix C.

VI. CONCLUSIONS

We have calculated the \(\mathcal{O}(m_\pi^2)\) contribution to the masses of the octet baryons in the isospin limit of \(\chi\)PT and PQ\(\chi\)PT, keeping the decuplet baryons as dynamical intermediate states. Working to this order in the chiral expansion introduces a large number of LEC’s. In order for the calculation to have any predictive power, these LEC’s must be fit from various experiments and or lattice results. Lattice calculations will eventually allow first principles determination of these constants, and thus predictions of QCD observables. However, for the time being, to make rigorous predictions of the baryon mass spectra arising from QCD, one needs to perform PQQCD lattice simulations and extrapolate these results using PQ\(\chi\)PT.
To date, there are limited PQQCD lattice calculations of the octet baryon masses. We hope this work will help to further interest an execution of these calculations. This is the first calculation to $\mathcal{O}(m_q^2)$ in the baryon sector using PQ$\chi$PT. Calculations of this order are necessary for reducing the uncertainty in the chiral extrapolations and systematically studying the chiral expansion of both PQ$\chi$PT and $\chi$PT.

Note Added. While this work was being completed, an independent analysis by Frink and Meissner appeared [41]. Their work considers the octet baryon masses in $\chi$PT only.

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APPENDIX A: CHIRAL LOGS AND OTHER FUNCTIONS

For the readers convenience, here we list all the functions arising in the calculation of the octet baryon masses, including the previously defined ones (which have been left unnumbered).

$$
\mathcal{L}(m_\phi, \mu) = m_\phi^2 \log \left( \frac{m_\phi^2}{\mu^2} \right)
$$

$$
\overline{\mathcal{L}}(m_\phi, \mu) = m_\phi^4 \log \left( \frac{m_\phi^2}{\mu^2} \right)
$$

$$
\mathcal{F}(m_\phi, \Delta, \mu) = (m_\phi^2 - \Delta^2) \left[ \sqrt{\Delta^2 - m_\phi^2} \log \left( \frac{\Delta - \sqrt{\Delta^2 - m_\phi^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m_\phi^2 + i\epsilon}} \right) - \Delta \log \left( \frac{m_\phi^2}{\mu^2} \right) \right] - \frac{1}{2} \Delta m_\phi^2 \log \left( \frac{m_\phi^2}{\mu^2} \right)
$$

$$
\mathcal{J}(m_\phi, \Delta, \mu) = (m_\phi^2 - 2\Delta^2) \log \left( \frac{m_\phi^2}{\mu^2} \right)
+ 2\Delta \sqrt{\Delta^2 - m_\phi^2} \log \left( \frac{\Delta - \sqrt{\Delta^2 - m_\phi^2 + i\epsilon}}{\Delta + \sqrt{\Delta^2 - m_\phi^2 + i\epsilon}} \right)
$$
\[ M^n(m_\phi, m_{\phi'}) = \mathcal{H}_{\phi\phi'}(m^n_\phi, m^n_{\phi'}, m^n_X) \]
\[ \mathcal{L}(m_\phi, m_{\phi'}) = \mathcal{H}_{\phi\phi'}\left(\mathcal{L}(m_\phi, \mu), \mathcal{L}(m_{\phi'}, \mu), \mathcal{L}(m_X, \mu)\right) \]
\[ \mathcal{Z}(m_\phi, m_{\phi'}) = \mathcal{H}_{\phi\phi'}\left(\mathcal{Z}(m_\phi, \mu), \mathcal{Z}(m_{\phi'}, \mu), \mathcal{Z}(m_X, \mu)\right) \]
\[ \mathcal{F}(m_\phi, m_{\phi'}) = \mathcal{H}_{\phi\phi'}\left(\mathcal{F}(m_\phi, \Delta, \mu), \mathcal{F}(m_{\phi'}, \Delta, \mu), \mathcal{F}(m_X, \Delta, \mu)\right) \]
\[ \mathcal{J}(m_\phi, m_{\phi'}) = \mathcal{H}_{\phi\phi'}\left(\mathcal{J}(m_\phi, \Delta, \mu), \mathcal{J}(m_{\phi'}, \Delta, \mu), \mathcal{J}(m_X, \Delta, \mu)\right) \]

(A1)

(A2)

And \( \mathcal{H}_{ab}(A, B, C) \) is defined in Eq. (13).

**APPENDIX B: OCTET BARYON MASSES IN \( \chi PT \)**

The mass of the \( i^{th} \) baryon in the chiral expansion is

\[ M_{B_i} = M_0(\mu) - M^{(1)}_{B_i}(\mu) - M^{(3/2)}_{B_i}(\mu) - M^{(2)}_{B_i}(\mu) + \ldots \quad \text{(B1)} \]

The LO and NLO mass corrections to the octet baryons are listed for completeness. The LO mass corrections are given by,

\[ -M^{(3/2)}_{B_i} = \frac{-2}{(4\pi f)^2} \sum_\phi \left[ (C_{BB\phi})^2 \pi m_\phi^3 + C^2(C_{TB\phi})^2 \mathcal{F}(m_\phi, \Delta, \mu) \right] , \]

The sum on \( \phi \) is over loop mesons with mass \( m_\phi \). The \( C_{BB\phi} \) and \( C_{TB\phi} \) are Clebsch-Gordon coefficients between the Axial field and two octet baryons or an octet and decuplet baryon. The sum of these coefficients are listed in Table I

Results of this work are the NNLO mass corrections in \( \chi PT \) to the octet baryons given by the expression,

\[ -M^{(2)}_{B_i} = -\frac{1}{(4\pi f)^2} \sum_\phi \left[ C_{BB\phi}^A + \frac{1}{4} C_{BB\phi}^A \right] \mathcal{L}(m_\phi, \mu) + \frac{1}{(4\pi f)^3} \sum_\phi C_{BB\phi}^A \frac{1}{8} m_\phi^4 \]

\[ -\frac{1}{(4\pi f)^2} \frac{3}{8M_B} \sum_\phi \left\{ (C_{BB\phi})^2 \left[ 5\mathcal{Z}(m_\phi, \mu) + \frac{5}{2} m_\phi^4 \right] \right\} + C^2(C_{TB\phi})^2 \left[ \mathcal{J}(m_\phi, \Delta, \mu) + m_\phi^2 \right] \]

\[ + \frac{3}{(4\pi f)^2} \left\{ M^{(1)}_{B_i} \sum_\phi (C_{BB\phi})^2 \left[ \mathcal{L}(m_\phi, \mu) + \frac{2}{3} m_\phi^2 \right] \right\} - \sum_{\phi,j} (C_{BB\phi})^2 M^{(1)}_{B_j} \left[ \mathcal{L}(m_\phi, \mu) + \frac{2}{3} m_\phi^2 \right] \right\} + 3C^2 \]

\[ + \frac{3}{(4\pi f)^2} \left\{ M^{(1)}_{B_i} \sum_\phi (C_{TB\phi})^2 \left[ \mathcal{J}(m_\phi, \Delta, \mu) + m_\phi^2 \right] \right\} + \sum_{\phi,j} (C_{TB\phi})^2 M^{(1)}_{T_j} \left[ \mathcal{J}(m_\phi, \Delta, \mu) + m_\phi^2 \right] \right\} + \frac{1}{(4\pi f)^2} \sum_\phi C_{BB\phi}^M \mathcal{L}(m_\phi, \mu) - \frac{1}{(4\pi f)^2} \sum_{i,j} C_{B_{ij}} m_i m_j \]  

(B2)
TABLE I: The LO Octet and Decuplet Baryon Masses in \( \chi PT \).

| Octet Baryons | Decuplet Baryons |
|---------------|------------------|
| \( M_N^{(1)} \) | \( 2m(\alpha_M + \beta_M + 2\sigma_M) + 2m_s\sigma_M \) | \( M_T^{(1)} \) | \( 2m(\gamma_M - 2\sigma_M) - 2m_s\sigma_M \) |
| \( M_N^{(1)} \) | \( m(\frac{2}{3}\alpha_M + \frac{1}{3}\beta_M + 4\sigma_M) \) | \( M_T^{(1)} \) | \( \frac{1}{3}(2m + m_s)(\gamma_M - 3\sigma_M) \) |
| \( M_N^{(1)} \) | \( m(\frac{1}{2}\alpha_M + \frac{1}{2}\beta_M + 2\sigma_M) \) | \( M_T^{(1)} \) | \( \frac{2}{3}m(\gamma_M - 6\sigma_M) + \frac{2}{3}m_s(2\gamma_M - 3\sigma_M) \) |

TABLE II: The octet-octet-axial and decuplet-octet-axial coupling coefficients in \( \chi PT \). The coefficients have been grouped according to loop mesons with mass \( m_\phi \) for each octet baryon.

| \( \phi \) | \( \pi \) | \( \sum_\phi(C_{BB\phi})^2 \) | \( K \) | \( \sum_\phi(C_{TB\phi})^2 \) | \( K \) |
|------------|--------|-----------------|-----|-----------------|-----|
| \( N \)    | \( \frac{3}{2}(D + F)^2 \) | \( \frac{1}{6}(D - 3F)^2 \) | \( \frac{1}{3}(5D^2 - 6DF + 9F^2) \) | \( \frac{1}{2} \) | \( 0 \) | \( \frac{1}{2} \) |
| \( \Sigma \) | \( \frac{2}{3}(D^2 + 6F^2) \) | \( \frac{2}{3}D^2 \) | \( 2(D^2 + F^2) \) | \( \frac{2}{3} \) | \( \frac{1}{3} \) | \( \frac{10}{3} \) |
| \( \Lambda \) | \( 2D^2 \) | \( \frac{3}{2}D^2 \) | \( \frac{3}{3}(D^2 + 9F^2) \) | \( 1 \) | \( 0 \) | \( \frac{3}{2} \) |
| \( \Xi \)   | \( \frac{3}{2}(D - F)^2 \) | \( \frac{1}{6}(D + 3F)^2 \) | \( \frac{1}{3}(5D^2 + 6DF + F^2) \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) | \( 1 \) |

TABLE III: The coefficients of 2-octet 2-axial couplings in \( \chi PT \). The coefficients have been grouped according to loop mesons with mass \( m_\phi \) for each octet baryon.

| \( \phi \) | \( \pi \) | \( \sum_\phi C_{BB\phi\phi}^{(1)} \) and \( \sum_\phi C_{BB\phi\phi}^{(2)} \) | \( K \) |
|------------|--------|-----------------|-----|
| \( N \)    | \( \frac{2}{3}b_4^4 + \frac{2}{3}b_2^4 - \frac{1}{3}b_3^4 + 3b_1^4 \) | \( \frac{1}{6}b_1^4 + \frac{1}{6}b_2^4 + \frac{1}{6}b_3^4 + b_4^4 \) | \( b_4^4 + b_2^4 + 4b_3^4 \) |
| \( \Sigma \) | \( \frac{1}{3}b_1^4 + \frac{1}{3}b_2^4 + \frac{1}{3}b_3^4 + 3b_4^4 \) | \( \frac{1}{6}b_1^4 + \frac{1}{6}b_2^4 - \frac{1}{6}b_3^4 + b_4^4 \) | \( \frac{5}{3}b_1^4 + \frac{7}{6}b_2^4 - \frac{2}{3}b_3^4 + 4b_4^4 \) |
| \( \Lambda \) | \( \frac{2}{3}b_1^4 + \frac{2}{3}b_2^4 - \frac{2}{3}b_3^4 + 3b_4^4 \) | \( \frac{1}{6}b_1^4 + \frac{1}{6}b_2^4 - \frac{1}{6}b_3^4 + b_4^4 \) | \( b_1^4 + \frac{2}{3}b_2^4 + 4b_4^4 \) |
| \( \Xi \)   | \( b_1^4 + \frac{1}{3}b_2^4 + 3b_4^4 \) | \( \frac{1}{6}b_1^4 + \frac{1}{6}b_2^4 - \frac{1}{6}b_3^4 + b_4^4 \) | \( \frac{4}{3}b_1^4 + \frac{11}{6}b_2^4 - \frac{2}{3}b_3^4 + 4b_4^4 \) |
TABLE IV: The $C_{BB\phi \phi}^M$ coefficients in $\chi$PT. The coefficients have been grouped according to loop mesons with mass $m_\phi$ for each octet baryon.

|       | $\pi$       | $\eta$       | $K$                                               |
|-------|-------------|-------------|--------------------------------------------------|
| $N$   | $6m(\alpha_M + \beta_M + 2\sigma_M)$ | $\frac{2}{3}m(\alpha_M + \beta_M + 2\sigma_M)$ | $2(m + m_s)(\alpha_M + \beta_M + 4\sigma_M)$ |
| $\Sigma$ | $m(5\alpha_M + 2\beta_M + 12\sigma_M)$ | $m\left(\frac{5}{3}\alpha_M + \frac{2}{3}\beta_M + \frac{4}{3}\sigma_M\right)$ | $(m + m_s)\left(\frac{5}{3}\alpha_M + \frac{10}{3}\beta_M + 8\sigma_M\right)$ |
| $\Lambda$ | $3m(\alpha_M + 2\beta_M + 4\sigma_M)$ | $m\left(\frac{1}{3}\alpha_M + \frac{2}{3}\beta_M + \frac{4}{3}\sigma_M\right)$ | $(m + m_s)(3\alpha_M + 2\beta_M + 8\sigma_M)$ |
| $\Xi$  | $m(\alpha_M + 4\beta_M + 12\sigma_M)$ | $m\left(\frac{1}{3}\alpha_M + \frac{4}{3}\beta_M + \frac{4}{3}\sigma_M\right)$ | $(m + m_s)\left(\frac{11}{3}\alpha_M + \frac{8}{3}\beta_M + 8\sigma_M\right)$ |

TABLE V: The coefficients for the internal octet mass insertion and the wavefunction correction in $\chi$PT. The coefficients have been grouped according to loop mesons with mass $m_\phi$ for each octet baryon.

|       | $M_{B_i}^{1(1)} \sum_\phi (C_{BB\phi})^2 - \sum_\phi, j (C_{BB\phi})^2 M_{B_j}^{1(1)}$ | $K$                                               |
|-------|------------------------------------------------|--------------------------------------------------|
| $N$   | $0$                                               | $\frac{1}{7}(5D^2 - 6DF + 9F^2)M_N^{1(1)}$         |
| $\Sigma$ | $\frac{2}{7}D^2(M_\Sigma^{1(1)} - M_\Lambda^{1(1)})$ | $-\frac{3}{7}(D - F)^2 M_\Sigma^{1(1)} - \frac{1}{7}(D + 3F)^2 M_\Lambda^{1(1)}$ |
| $\Lambda$ | $2D^2(M_\Lambda^{1(1)} - M_\Sigma^{1(1)})$ | $\frac{2}{7}(D^2 + 9F^2)M_\Lambda^{1(1)}$         |
| $\Xi$  | $0$                                               | $\frac{1}{7}(5D^2 + 6DF + 9F^2)M_\Xi^{1(1)}$         |
|       | $0$                                               | $-\frac{3}{7}(D + 3F)^2 M_\Sigma^{1(1)} - \frac{1}{7}(D - 3F)^2 M_\Lambda^{1(1)}$ |

TABLE VI: The coefficients for an internal decuplet mass insertion and the wavefunction correction in $\chi$PT. The coefficients have been grouped according to loop mesons with mass $m_\phi$ for each octet baryon.

|       | $\sum_\phi, j (C_{TB\phi})^2 M_j^{1(1)} + M_{B_i}^{1(1)} \sum_\phi (C_{TB\phi})^2$ | $K$                                               |
|-------|------------------------------------------------|--------------------------------------------------|
| $N$   | $\frac{4}{7}(M_\Lambda^{1(1)} + M_\Sigma^{1(1)})$ | $\frac{1}{7}(M_\Sigma^{1(1)} + M_\Lambda^{1(1)})$ |
| $\Sigma$ | $\frac{2}{7}(M_\Sigma^{1(1)} + M_\Xi^{1(1)})$ | $\frac{4}{7}(M_\Sigma^{1(1)} + M_\Xi^{1(1)})$ | $\frac{2}{7}(5M_\Sigma^{1(1)} + 4M_\Delta^{1(1)} + M_\Xi^{1(1)})$ |
| $\Lambda$ | $M_\Lambda^{1(1)} + M_\Sigma^{1(1)}$ | $0$ | $\frac{2}{7}(M_\Lambda^{1(1)} + M_\Xi^{1(1)})$ |
| $\Xi$  | $\frac{1}{3}(M_\Xi^{1(1)} + M_\Sigma^{1(1)})$ | $\frac{1}{3}(M_\Xi^{1(1)} + M_\Sigma^{1(1)})$ | $\frac{1}{3}(3M_\Xi^{1(1)} + 2M_\Delta^{1(1)} + M_\Sigma^{1(1)})$ |
TABLE VII: The $C_{B_i}$ coefficients in $\chi$PT. The coefficients are grouped by products of quark masses for each octet baryon.

|   | $\bar{m}^2$ | $\sum_{i,j} C_{B_i} m_i m_j$ | $m_s^2$ |
|---|-------------|---------------------------------|---------|
| $N$ | $b_1^M + b_2^M + b_3^M + 2b_4^M + 2b_5^M$ | $b_5^M + b_6^M + 4b_7^M$ | $b_4^M + b_8^M$ |
| $\Sigma$ | $\frac{1}{3}b_1^M + \frac{1}{6}b_2^M + \frac{1}{6}b_3^M + 2b_4^M$ | $\frac{5}{6}b_3^M + \frac{5}{6}b_5^M + \frac{4}{6}b_6^M + 4b_7^M$ | $\frac{2}{3}b_1^M + \frac{1}{6}b_2^M + b_4^M + \frac{4}{3}b_5^M$ |
| $\Lambda$ | $b_1^M + \frac{1}{2}b_2^M + \frac{3}{2}b_3^M + 2b_4^M$ | $\frac{5}{6}b_3^M + b_5^M + \frac{2}{3}b_6^M + 4b_7^M$ | $\frac{2}{3}b_1^M + \frac{1}{6}b_2^M + b_4^M + \frac{5}{6}b_5^M$ |
| $\Xi$ | $\frac{2}{3}b_1^M + \frac{1}{2}b_2^M + 2b_4^M$ | $\frac{5}{6}b_3^M + \frac{4}{3}b_5^M + \frac{11}{6}b_6^M + 4b_7^M$ | $\frac{1}{3}b_1^M + \frac{5}{6}b_2^M + \frac{1}{6}b_5^M + b_4^M$ |

APPENDIX C: OCTET BARYON MASSES IN PQ$\chi$PT

The mass of the $i^{th}$ baryon in the chiral expansion of PQHB\(\chi\)PT has the same form as in HB\(\chi\)PT.

$$M_{B_i} = M_0 (\mu) - M_{B_i}^{(1)} (\mu) - M_{B_i}^{(3/2)} (\mu) - M_{B_i}^{(2)} (\mu) + \ldots$$  \hspace{1cm} (C1)

The LO and NLO mass corrections to the octet baryons in PQ$\chi$PT are listed here for completeness, and are consistent with [13]. The LO mass corrections for the 240- and 138-baryons are listed in Table VIII, Table X, and Table IX. The NLO mass corrections are,

$$- \ M_{B_i}^{(3/2)} = \frac{-2}{(4\pi f)^2} \sum_\phi \left[ (C_{BB\phi}^{PQ})^2 \pi m_\phi^3 + C^2 (C_{TB\phi}^{PQ})^2 \mathcal{F}(m_\phi, \Delta, \mu) \right]$$

$$- \frac{2}{(4\pi f)^2} \sum_{\phi, \phi'} \pi C_{B\phi \phi'} \cdot \mathcal{M}(m_\phi, m_{\phi'}) + C^2 \mathcal{C}_{T\phi \phi'} \mathcal{F}(m_\phi, m_{\phi'}) \right].$$  \hspace{1cm} (C2)

The sum on $\phi$ is over loop mesons with mass $m_\phi$. The $C_{BB\phi}^{PQ}$ and $C_{TB\phi}^{PQ}$ are PQHB\(\chi\)PT Clebsch-Gordon coefficients between the axial field and the baryon fields. The sums on $\phi, \phi'$ are over contributions from the hairpin eta field interactions. The sum is over pairs of eta fields, with no double counting.
The main results of this work are the NNLO PQHB\(\chi\)PT octet mass calculations.

\[- M_{B_i}^{(2)} = \frac{1}{(4\pi f)^3}\left\{ \sum_{\phi} \left[ \frac{C_{\phi\phi}}{C_{\phi\phi\phi}^{\phi}} + \frac{1}{4} C_{\phi\phi\phi}^{\phi} \right] \mathcal{L}(m_\phi, \mu) \right. \\
+ \sum_{\phi\phi'} \left[ C_{\phi\phi\phi}^{\phi} + \frac{1}{4} C_{\phi\phi\phi}^{\phi} \right] \mathcal{L}(m_\phi, m_{\phi'}) \right\} \\
+ \frac{1}{(4\pi f)^3}\left\{ \sum_{\phi} \frac{1}{8} C_{\phi\phi}^{\phi\phi} m_\phi^4 \right. \\
\left. + \sum_{\phi\phi'} \frac{1}{8} C_{\phi\phi}^{\phi\phi'} m_{\phi'}^4 \right\} \\
- \frac{1}{M_B(4\pi f)^2} \frac{3}{8}\left\{ \sum_{\phi} (C_{\phi\phi}^{\phi\phi})^2 \left[ 3 \mathcal{L}(m_\phi, \mu) + \frac{5}{2} m_\phi^4 \right] \right. \\
\left. + \sum_{\phi\phi'} \mathcal{L}(m_\phi, m_{\phi'}) \right\} \\
- \frac{C^2}{M_B(4\pi f)^2} \frac{3}{8}\left\{ \sum_{\phi} (C_{\phi\phi}^{\phi\phi})^2 \left[ 5 \mathcal{L}(m_\phi, \mu) + \frac{9}{2} m_\phi^4 \right] \right. \\
\left. + \sum_{\phi\phi'} \mathcal{L}(m_\phi, m_{\phi'}) \right\} \\
+ \frac{3}{(4\pi f)^2}\left\{ M_{B_i}^{(1)} \sum_{\phi} (C_{\phi\phi}^{\phi})^2 \left[ \mathcal{L}(m_\phi, \mu) + \frac{2}{3} m_\phi^2 \right] \right. \\
\left. - \sum_{j,\phi} M_{B_i}^{(1)} (C_{\phi\phi}^{\phi})^2 \left[ \mathcal{L}(m_\phi, \mu) + \frac{2}{3} m_\phi^2 \right] \right\} \\
+ \frac{3C^2}{(4\pi f)^2}\left\{ M_{B_i}^{(1)} \sum_{\phi} (C_{\phi\phi}^{\phi})^2 \left[ J(m_\phi, \Delta, \mu) + m_\phi^2 \right] \right. \\
\left. + \sum_{j,\phi} M_{B_i}^{(1)} (C_{\phi\phi}^{\phi})^2 \left[ J(m_\phi, \Delta, \mu) + m_\phi^2 \right] \right\} \\
+ \frac{3C^2}{(4\pi f)^2}\left\{ M_{B_i}^{(1)} \sum_{\phi\phi'} C_{\phi\phi'} \left[ M^2(m_\phi, m_{\phi'}) + J(m_\phi, m_{\phi'}) \right] \right. \\
\left. + \sum_{j,\phi\phi'} M_{B_i}^{(1)} C_{\phi\phi'} \left[ M^2(m_\phi, m_{\phi'}) + J(m_\phi, m_{\phi'}) \right] \right\} \\
+ \frac{1}{(4\pi f)^2}\left\{ \sum_{\phi} C_{\phi\phi}^{M,\phi\phi} \mathcal{L}(m_\phi, \mu) \right. \\
\left. + \sum_{\phi\phi'} C_{\phi\phi}^{M,\phi\phi'} \mathcal{L}(m_\phi, m_{\phi'}) \right\} \\
- \frac{1}{(4\pi f)} \sum_{i,j} C_{B_i j}^{i} m_i m_j \]  

(C3)

The coefficients for the NLO and NNLO octet baryon masses are listed in Table XI through Table XXI. It is straightforward to show that in the limit \(m_j \to m\) and \(m_r \to m_s\), these expressions reduce to the \(\chi\)PT expressions listed in Appendix B.
TABLE VIII: The LO \(240\) \((B)\) and \(138\) \((T)\) baryon masses composed only of valence quarks in PQ\(\chi PT\).

|                | Octet Baryons | Decuplet Baryons |
|----------------|---------------|------------------|
| \(M_{240}^{(1)}\) | \(2\overline{m}(\alpha_M + \beta_M) + 4m_j\sigma_M + 2m_r\sigma_M\) | \(M_{240}^{(1)}\) |
| \(M_{138}^{(1)}\) | \(M_{138}^{(1)}\) | \(2\overline{m}\gamma_M - 4m_j\sigma_M - 2m_r\sigma_M\) |
| \(M_{240}^{(1)}\) | \(M_{138}^{(1)}\) | \(2\overline{m}\gamma_M + 2m_s\gamma_M - 4m_j\sigma_M - 2m_r\sigma_M\) |
| \(M_{138}^{(1)}\) | \(M_{138}^{(1)}\) | \(2m_s\gamma_M - 4m_j\sigma_M - 2m_r\sigma_M\) |

TABLE IX: The LO masses of the \(138\) \((T)\) baryons composed of 2 valence quarks and 1 sea or ghost quark in PQ\(\chi PT\).

|                | \(138\) Baryons with one sea quark | \(138\) Baryons with one ghost quark |
|----------------|------------------------------------|-------------------------------------|
| \(M_{138}^{(1)}\) | \(M_{138}^{(1)}\) | \(M_{138}^{(1)}\) |
| \(M_{138}^{(1)}\) | \(M_{138}^{(1)}\) | \(M_{138}^{(1)}\) |
| \(M_{138}^{(1)}\) | \(M_{138}^{(1)}\) | \(M_{138}^{(1)}\) |
| \(M_{138}^{(1)}\) | \(M_{138}^{(1)}\) | \(M_{138}^{(1)}\) |
| \(M_{138}^{(1)}\) | \(M_{138}^{(1)}\) | \(M_{138}^{(1)}\) |
TABLE X: The LO masses of the 240 (B) baryons composed 2 valence quarks and 1 sea or ghost quark in PQ\(\chi\)PT.

| \(M_{\Sigma_j}(1)\) | \(\overline{m}(\frac{2}{3}\alpha_M + \frac{2}{3}\beta_M) + 2m_r\sigma_M + m_j(\frac{1}{3}\alpha_M + \frac{1}{3}\beta_M + 4\sigma_M)\) | \(M_{\Sigma_u}(1)\) | \(\overline{m}(\frac{2}{3}\alpha_M + \frac{2}{3}\beta_M) + 2m_r\sigma_M\) |
| \(M_{\Lambda}(1)\) | \(\overline{m}(\frac{2}{3}\alpha_M + \frac{2}{3}\beta_M) + m_j(\alpha_M + 4\sigma_M) + 2m_r\sigma_M\) | \(M_{\Lambda_u}(1)\) | \(\overline{m}(\frac{2}{3}\alpha_M + \frac{2}{3}\beta_M) + m_j(\alpha_M + 4\sigma_M)\) |
| \(M_{\Xi_j}(1)\) | \(\overline{m}(\frac{2}{3}\alpha_M + \frac{2}{3}\beta_M) + 4m_j\sigma_M + m_r(\frac{1}{3}\alpha_M + \frac{1}{3}\beta_M + 2\sigma_M)\) | \(M_{\Xi_u}(1)\) | \(\overline{m}(\frac{2}{3}\alpha_M + \frac{2}{3}\beta_M) + 4m_j\sigma_M\) |
| \(M_{\Omega_j}(1)\) | \(m_s(\frac{2}{3}\alpha_M + \frac{2}{3}\beta_M) + 2m_r\sigma_M + m_j(\frac{1}{3}\alpha_M + \frac{1}{3}\beta_M + 4\sigma_M)\) | \(M_{\Omega_u}(1)\) | \(m_s(\frac{2}{3}\alpha_M + \frac{2}{3}\beta_M) + 2m_r\sigma_M\) |

TABLE XI: The 240-240-Axial coupling coefficients in PQ\(\chi\)PT. The coefficients are grouped by the loop mesons with mass \(m_\phi\) for each of the octet baryons.

| \(\sum_\phi(C_{BB_\phi}^{PQ})^2\) | \(\pi\) | \(K\) | \(\eta_s\) |
| \(\frac{4}{3}(5D^2 - 6DF + 9F^2)\) | \(D(3F - D)\) | 0 | 0 |
| \(-\frac{2}{3}(D^2 - 6DF + 3F^2)\) | \(\frac{2}{3}(3F^2 - D^2)\) | \(-\frac{3}{2}(D^2 - 6DF + 3F^2)\) | 0 |
| \(\frac{2}{3}(D^2 - 12DF + 9F^2)\) | \(-\frac{7}{6}(D^2 - 12DF + 9F^2)\) | \(-\frac{7}{6}(D^2 - 12DF + 9F^2)\) | 0 |
| \(\frac{2}{3}(D^2 - 12DF + 9F^2)\) | \(-\frac{7}{6}(D^2 - 12DF + 9F^2)\) | \(-\frac{7}{6}(D^2 - 6DF + 3F^2)\) | \frac{2}{3}(3F^2 - D^2) |

| \(\frac{4}{3}(5D^2 - 6DF + 9F^2)\) | \(\frac{4}{3}(5D^2 - 6DF + 9F^2)\) | \(\frac{4}{3}(5D^2 - 6DF + 9F^2)\) | \(\frac{4}{3}(5D^2 - 6DF + 9F^2)\) |
| \(\frac{4}{3}(D^2 + 3F^2)\) | \(\frac{4}{3}(D^2 + 3F^2)\) | \(\frac{4}{3}(D^2 + 3F^2)\) | \(\frac{4}{3}(D^2 + 3F^2)\) |
| \(\frac{4}{3}(7D^2 - 12DF + 9F^2)\) | \(\frac{4}{3}(7D^2 - 12DF + 9F^2)\) | \(\frac{4}{3}(7D^2 - 12DF + 9F^2)\) | \(\frac{4}{3}(D + 3F)^2\) |
| \(\frac{4}{3}(D + 3F)^2\) | \(\frac{4}{3}(D + 3F)^2\) | \(\frac{4}{3}(D + 3F)^2\) | \(\frac{4}{3}(D + 3F)^2\) |

| \(\frac{4}{3}(D^2 - F^2)\) | \(\frac{4}{3}(D^2 - F^2)\) | \(\frac{4}{3}(D^2 - F^2)\) | \(\frac{4}{3}(D^2 - F^2)\) |
| \(\frac{4}{3}(D^2 + 3F^2)\) | \(\frac{4}{3}(D^2 + 3F^2)\) | \(\frac{4}{3}(D^2 + 3F^2)\) | \(\frac{4}{3}(D^2 + 3F^2)\) |
TABLE XII: The coefficients for meson loops containing hairpin insertions with an internal 240-baryon.

| | | \( \sum_{\phi<phi} C_B \phi \phi' \) |
|---|---|---|
| \( N \) | \( (D - 3F)^2 \) | \( \eta_u \eta_s \) |
| \( \Sigma \) | \( 4F^2 \) | \( \eta_u \eta_s \) |
| \( \Lambda \) | \( \frac{4}{3}(2D - 3F)^2 \) | \( \eta_s \eta_s \) |
| \( \Xi \) | \( (D - F)^2 \) | \( \eta_s \eta_s \) |

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\text{TABLE XIII: The 240-138-Axial coupling coefficients and the coefficients for meson loops containing hairpin insertions with an internal 138-baryon in PQχPT. The coefficients are grouped by the loop mesons with mass \( m_\phi \) and by the participating \( \eta \) fields in the hairpin interaction, respectively.}
\]

| \( \pi \) | \( K \) | \( \eta_s \) | \( \sum_{\phi} (C_{T \phi \phi})^2 \) | \( \eta_u \eta_u \) |
|---|---|---|---|---|
| \( N \) | \( \frac{2}{3} \) | \( 0 \) | \( \frac{2}{3} \) | \( \frac{1}{3} \) |
| \( \Sigma \) | \( \frac{4}{3} \) | \( 0 \) | \( \frac{4}{3} \) | \( \frac{1}{3} \) |
| \( \Lambda \) | \( \frac{1}{3} \) | \( 0 \) | \( \frac{1}{3} \) | \( \frac{1}{3} \) |

\[
\text{TABLE XIV: The } C_{BB \phi \phi}^{A} \text{ and } C_{BB \phi \phi}^{\nu A} \text{ coefficients in PQχPT. The coefficients are grouped by the loop mesons with mass } m_\phi.\]

| \( \pi \) | \( K \) | \( \eta_s \) | \( \sum_{\phi} C_{BB \phi \phi}^{A} \) | \( \sum_{\phi} C_{BB \phi \phi}^{\nu A} \) | \( \eta_u \eta_u \) |
|---|---|---|---|---|---|
| \( N \) | \( -\frac{1}{2} b_3 \) | \( 0 \) | \( 2b_1^4 + 2b_2^4 \) | \( b_1^3 + b_2^3 \) | \( 0 \) |
| \( \Sigma \) | \( \frac{1}{2} b_3 \) | \( 0 \) | \( \frac{4}{3} b_1^4 + \frac{4}{3} b_2^4 \) | \( \frac{4}{3} b_1^3 + \frac{4}{3} b_2^3 \) | \( \frac{2}{3} b_1^3 + \frac{2}{3} b_2^3 \) |
| \( \Lambda \) | \( -\frac{1}{2} b_3 \) | \( 0 \) | \( 2b_1^4 + b_2^4 \) | \( b_1^3 + \frac{1}{2} b_2^3 \) | \( b_2^3 \) |

\[
\text{TABLE XV: The } C_{BB \phi \phi}^{A} \text{ and } C_{BB \phi \phi}^{\nu A} \text{ coefficients in PQχPT. The coefficients are grouped by the } \eta \text{ fields participating in the hairpin interaction.}
\]

| \( \pi \) | \( \eta_u \eta_u \) | \( \sum_{\phi<phi} C_{BB \phi \phi'}^{A} \) | \( \sum_{\phi<phi} C_{BB \phi \phi'}^{\nu A} \) |
|---|---|---|---|
| \( N \) | \( b_1^4 + b_2^4 + b_3^4 \) | \( 0 \) | \( 0 \) |
| \( \Sigma \) | \( \frac{1}{2} b_1^4 + \frac{1}{2} b_2^4 + \frac{1}{2} b_3^4 \) | \( \frac{2}{3} b_3^4 \) | \( \frac{2}{3} b_1^4 + \frac{2}{3} b_2^4 \) |
| \( \Lambda \) | \( b_1^4 + \frac{1}{2} b_2^4 + \frac{1}{2} b_3^4 \) | \( \frac{1}{2} b_2^4 \) | \( \frac{1}{2} b_2^4 \) |
| \( \Xi \) | \( \frac{2}{3} b_1^4 + \frac{1}{3} b_2^3 \) | \( \frac{2}{3} b_3^4 \) | \( \frac{2}{3} b_1^4 + \frac{2}{3} b_2^4 + \frac{2}{3} b_3^4 \) |
TABLE XVI: The $C_{BB\phi\phi}^{M,PQ}$ coefficients in PQ\(\chi^T\). The coefficients are grouped by the loop meson with mass \(m_\phi\).

|   | \(j u\) | \(ru\) | \(js\) |
|---|---------|---------|--------|
| \(N\) | \(4(m + m_j)(\alpha_M + \beta_M)\) | \(2(m + m_r)(\alpha_M + \beta_M)\) | 0 |
| \(\Sigma\) | \((m + m_j)(\frac{10}{3}\alpha_M + \frac{4}{3}\beta_M)\) | \((m + m_r)(\frac{4}{3}\alpha_M + \frac{8}{3}\beta_M)\) | \((m_j + m_s)(\frac{2}{3}\alpha_M + \frac{8}{3}\beta_M)\) |
| \(\Lambda\) | \((m + m_j)(2\alpha_M + 4\beta_M)\) | \((m + m_r)(\alpha_M + 2\beta_M)\) | \(2(m_j + m_s)\alpha_M\) |
| \(\Xi\) | \((m + m_j)(\frac{2}{3}\alpha_M + \frac{8}{3}\beta_M)\) | \((m + m_r)(\frac{10}{3}\alpha_M + \frac{4}{3}\beta_M)\) | \((m_j + m_s)(\frac{10}{3}\alpha_M + \frac{4}{3}\beta_M)\) |

| \(r s\) | \(\eta_j\) | \(jr\) | \(\eta_r\) |
|---|---|---|---|
| \(N\) | 0 | \(16m_j\sigma_M\) | \(8(m_j + m_r)\sigma_M\) | \(4m_r\sigma_M\) |
| \(\Sigma\) | \((m_s + m_r)(\frac{10}{3}\alpha_M + \frac{4}{3}\beta_M)\) | \(16m_j\sigma_M\) | \(8(m_j + m_r)\sigma_M\) | \(4m_r\sigma_M\) |
| \(\Lambda\) | \((m_s + m_r)\alpha_M\) | \(16m_j\sigma_M\) | \(8(m_j + m_r)\sigma_M\) | \(4m_r\sigma_M\) |
| \(\Xi\) | \((m_r + m_s)(\frac{5}{3}\alpha_M + \frac{2}{3}\beta_M)\) | \(16m_j\sigma_M\) | \(8(m_j + m_r)\sigma_M\) | \(4m_r\sigma_M\) |

TABLE XVII: The $C_{BB\phi\phi'}^{M}$ coefficients in PQ\(\chi^T\). The coefficients are grouped by the \(\eta\) fields participating in the hairpin interaction.

|   | \(\eta_u\eta_u\) | \(\eta_s\eta_s\) | \(\eta_j\eta_j\) | \(\eta_r\eta_r\) |
|---|---|---|---|---|
| \(N\) | \(4m(\alpha_M + \beta_M)\) | 0 | \(8m_j\sigma_M\) | \(4m_r\sigma_M\) |
| \(\Sigma\) | \(m(\frac{10}{3}\alpha_M + \frac{4}{3}\beta_M)\) | \(m_s(\frac{2}{3}\alpha_M + \frac{8}{3}\beta_M)\) | \(8m_j\sigma_M\) | \(4m_r\sigma_M\) |
| \(\Lambda\) | \(m(2\alpha_M + 4\beta_M)\) | \(2m_s\alpha_M\) | \(8m_j\sigma_M\) | \(4m_r\sigma_M\) |
| \(\Xi\) | \(m(\frac{2}{3}\alpha_M + \frac{8}{3}\beta_M)\) | \(m_s(\frac{10}{3}\alpha_M + \frac{4}{3}\beta_M)\) | \(8m_j\sigma_M\) | \(4m_r\sigma_M\) |
TABLE XVIII: The coefficients for internal $^{138}$-baryon mass insertions and the wavefunction corrections. The coefficients are grouped by the loop mesons with mass $m_\phi$ for each octet baryon.

|     | $\pi$ | $K$ | $\eta_s$ |
|-----|-------|-----|----------|
| $N$ | $\frac{2}{9}(M_N^{(1)} + M_\Delta^{(1)})$ | 0 | 0 |
| $\Sigma$ | $\frac{2}{9}(M_{\Sigma}^{(1)} + M_{\Sigma^*}^{(1)})$ | $\frac{1}{9}(5M_{\Sigma}^{(1)} + 4M_{\Delta}^{(1)} + M_{\Sigma^*}^{(1)})$ | 0 |
| $\Lambda$ | $\frac{1}{3}(M_{\Lambda}^{(1)} + M_{\Sigma^*}^{(1)})$ | $\frac{1}{3}(M_{\Lambda}^{(1)} + M_{\Xi^*}^{(1)})$ | 0 |
| $\Xi$ | 0 | $\frac{1}{9}(5M_{\Xi}^{(1)} + 4M_{\Omega^*}^{(1)} + M_{\Xi^*}^{(1)})$ | $\frac{1}{9}(M_{\Xi}^{(1)} + M_{\Xi^*}^{(1)})$ |

TABLE XIX: The coefficients for hairpin interactions with internal $^{138}$-baryon mass insertions and wavefunction corrections. The coefficients are grouped according to the $\eta$ fields participating in the hairpin interaction.

|     | $\eta_u\eta_s$ | $\eta_u\eta_s$ | $\eta_s\eta_s$ |
|-----|----------------|----------------|----------------|
| $N$ | 0 | 0 | 0 |
| $\Sigma$ | $\frac{2}{9}(M_{\Sigma}^{(1)} + M_{\Sigma^*}^{(1)})$ | $-\frac{4}{9}(M_{\Sigma}^{(1)} + M_{\Sigma^*}^{(1)})$ | $\frac{2}{9}(M_{\Sigma}^{(1)} + M_{\Sigma^*}^{(1)})$ |
| $\Lambda$ | 0 | 0 | 0 |
| $\Xi$ | $\frac{2}{9}(M_{\Xi}^{(1)} + M_{\Xi^*}^{(1)})$ | $-\frac{4}{9}(M_{\Xi}^{(1)} + M_{\Xi^*}^{(1)})$ | $\frac{2}{9}(M_{\Xi}^{(1)} + M_{\Xi^*}^{(1)})$ |
TABLE XX: The $C_{Bij}^{PQ}$ coefficients in PQ$\chi$PT. The coefficients are grouped by products of quark masses.

|       | $m^2$     | $\bar{m}m_j$ | $\sum_{i,j} C_{Bij}^{PQ} m_j^2$ | $\bar{m}m_s$ | $\bar{m}m_r$ |
|-------|-----------|---------------|----------------------------------|---------------|---------------|
| $N$   | $b_1^M + b_2^M + b_3^M$ | $2b_5^M + 2b_6^M$ | $2b_4^M + 4b_7^M$ | 0             | $b_5^M + b_6^M$ |
| $\Sigma$ | $\frac{1}{3}b_1^M + \frac{2}{3}b_2^M + \frac{1}{3}b_3^M$ | $\frac{2}{3}b_5^M + \frac{5}{3}b_6^M$ | $2b_4^M + 4b_7^M$ | $\frac{5}{3}b_3^M$ | $\frac{1}{3}b_5^M + \frac{5}{3}b_6^M$ |
| $\Lambda$ | $b_1^M + \frac{1}{2}b_2^M + \frac{1}{2}b_3^M$ | $2b_5^M + b_6^M$ | $2b_4^M + 4b_7^M$ | $\frac{1}{2}b_3^M$ | $b_5^M + \frac{1}{2}b_6^M$ |
| $\Xi$ | $\frac{2}{3}b_1^M + \frac{1}{3}b_2^M$ | $\frac{1}{3}b_5^M + \frac{1}{3}b_6^M$ | $2b_4^M + 4b_7^M$ | $\frac{5}{3}b_3^M$ | $\frac{2}{3}b_5^M + \frac{1}{3}b_6^M$ |

|       | $m_jm_s$ | $m_jm_r$ | $m_sm_r$ | $m_j^2$ | $m_s^2$ | $m_r^2$ |
|-------|-----------|-----------|-----------|----------|----------|----------|
| $N$   | 0         | $4b_7^M$ | 0         | 0        | $b_4^M + b_2^M$ | $b_4^M + b_2^M$ |
| $\Sigma$ | $\frac{4}{3}b_5^M + \frac{1}{3}b_6^M$ | $4b_7^M$ | $\frac{2}{3}b_5^M + \frac{1}{3}b_6^M$ | $\frac{2}{3}b_4^M + \frac{1}{3}b_2^M$ | $b_4^M + b_2^M$ | $b_4^M + b_2^M$ |
| $\Lambda$ | $b_1^M$ | $4b_7^M$ | $\frac{1}{2}b_5^M$ | $\frac{1}{2}b_6^M$ | $\frac{1}{2}b_2^M$ | $\frac{1}{2}b_2^M$ |
| $\Xi$ | $\frac{2}{3}b_5^M + \frac{5}{3}b_6^M$ | $4b_7^M$ | $\frac{1}{3}b_5^M + \frac{5}{3}b_6^M$ | $\frac{1}{3}b_4^M + \frac{5}{3}b_2^M + \frac{1}{3}b_3^M$ | $b_4^M + b_3^M$ | $b_4^M + b_3^M$ |
TABLE XXI: The coefficients for internal 240-baryon mass insertion and the wavefunction corrections. The coefficients are grouped according to the loop mesons with mass $m_{\phi}$ for each octet baryon.

|     | $\pi$                                                                 | $K$                                                                 | $\eta_s$                                                                 |
|-----|------------------------------------------------------------------------|----------------------------------------------------------------------|---------------------------------------------------------------------------|
| $N$ | $\frac{1}{2}(D-F)^2 M_{\Sigma}^{(1)} - \frac{3}{2}(D+3F^2) M_{\Lambda}^{(1)}$ | $(D-F)^2 M_N^{(1)} + \frac{1}{2}(D-F)^2 M_{\Sigma}^{(1)} + \frac{1}{5}(D+3F)^2 M_N^{(1)}$ | $0$                                                                      |
| $\Sigma$ | $-\frac{2}{3}(D^2 + 6F^2)M_{\Sigma}^{(1)} - \frac{2}{3}D^2 M_{\Lambda}^{(1)}$ | $-\frac{2}{3}(D^2 - 6DF + 3F^2)M_{\Sigma}^{(1)} - (D+F)^2 M_{\Lambda}^{(1)}$ | $0$                                                                      |
| $\Lambda$ | $\frac{3}{4}(D-F)^2 M_{\Sigma}^{(1)} + \frac{3}{4}(D+3F^2) M_{\Sigma}^{(1)}$ | $\frac{3}{2}(D-F)^2 M_{\Sigma}^{(1)} + \frac{3}{16}(D+3F^2) M_{\Sigma}^{(1)} - \frac{3}{4}(D-3F^2) M_{\Sigma}^{(1)}$ | $0$                                                                      |
| $\Xi$ | $0$                                                                    | $\frac{1}{2}(D+3F^2) M_{\Xi}^{(1)} + (D-F)^2 \left( M_{\Xi}^{(1)} + M_{\Sigma}^{(1)} \right) - \frac{1}{5}(D-3F^2) M_{\Lambda}^{(1)}$ | $\frac{1}{2}(D-F)^2 M_{\Xi}^{(1)} + \frac{3}{8}(D+3F^2) M_{\Xi}^{(1)}$ |

|     | $j_u$                                                                 | $r_u$                                                                 |                                                                            |
|-----|------------------------------------------------------------------------|----------------------------------------------------------------------|---------------------------------------------------------------------------|
| $N$ | $\frac{1}{2}(5D^2 - 6DF + 9F^2)M_N^{(1)}$                               | $\frac{1}{2}(5D^2 - 6DF + 9F^2)M_N^{(1)}$                            | $\frac{7}{3}(5D^2 - 6DF + 9F^2)M_N^{(1)}$                                |
| $\Sigma$ | $-3(D-F)^2 M_{\Sigma}^{(1)} - \frac{3}{5}(D+3F) M_{\Lambda}^{(1)}$     | $-\frac{3}{5}(D-F)^2 M_{\Sigma}^{(1)} - \frac{3}{5}(D+3F)^2 M_{\Lambda}^{(1)}$ | $-\frac{5}{3}(D-3F) M_{\Lambda}^{(1)}$                                   |
| $\Lambda$ | $\frac{1}{3}(7D^2 - 12DF + 9F^2)M_{\Lambda}^{(1)}$                      | $\frac{1}{3}(7D^2 - 12DF + 9F^2)M_{\Lambda}^{(1)}$                   | $-\frac{2}{5}(D-3F)^2 M_{\Lambda}^{(1)} - \frac{1}{15}(D+3F)^2 M_{\Lambda}^{(1)}$ |
| $\Xi$ | $2(D-F)^2 (M_{\Xi}^{(1)} - M_{\Sigma}^{(1)})$                          | $(D-F)^2 (M_{\Xi}^{(1)} - M_{\Sigma}^{(1)})$                        | $(D-F)^2 (M_{\Xi}^{(1)} - M_{\Sigma}^{(1)})$                            |

|     | $j_s$                                                                 | $r_s$                                                                 |                                                                            |
|-----|------------------------------------------------------------------------|----------------------------------------------------------------------|---------------------------------------------------------------------------|
| $N$ | $0$                                                                    | $0$                                                                  | $0$                                                                      |
| $\Sigma$ | $2(D-F)^2 (M_{\Sigma}^{(1)} - M_{\Xi}^{(1)})$                         | $(D-F)^2 (M_{\Sigma}^{(1)} - M_{\Xi}^{(1)})$                        | $(D-F)^2 (M_{\Sigma}^{(1)} - M_{\Xi}^{(1)})$                            |
| $\Lambda$ | $\frac{1}{3}(D+3F)^2 (M_{\Lambda}^{(1)} - M_{\Xi}^{(1)})$             | $\frac{1}{3}(D+3F)^2 (M_{\Lambda}^{(1)} - M_{\Xi}^{(1)})$            | $\frac{1}{3}(D+3F)^2 (M_{\Lambda}^{(1)} - M_{\Xi}^{(1)})$               |
| $\Xi$ | $-3(D-F)^2 M_{\Xi}^{(1)} - \frac{3}{5}(D+3F)^2 M_{\Xi}^{(1)}$           | $-\frac{3}{5}(D-F)^2 M_{\Xi}^{(1)} - \frac{3}{5}(D+3F)^2 M_{\Xi}^{(1)}$ | $-\frac{2}{5}(D-F)^2 M_{\Xi}^{(1)} - \frac{1}{15}(D+3F)^2 M_{\Xi}^{(1)}$ |
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