How many SNeIa do we need to detect the effect of weak lensing?

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ABSTRACT

We show that as many as 4000 SNeIa may be required to detect the effect of weak lensing on their flux distribution with a high level of significance. However, if the intrinsic SNeIa magnitude dispersion is unknown one needs an even higher number of SNeIa (an order of magnitude more) to reach a similar level of statistical significance. Moreover, the ability to separate the lensing contribution from the intrinsic scatter depends sensitively on the amplitude of the latter. Using a Kolmogorov–Smirnov (K-S) test we check how the required number of SNeIa changes with level of significance. Our model incorporates a completely analytical description of weak lensing which has been tested extensively against numerical simulations. Thus, future missions such as SNAP may be able to detect non-Gaussianity at a lower significance level of 10\% (through the K-S test) only if the intrinsic scatter is known from external data (e.g. from low redshift observations) whereas ALPACA with 100,000 SNe will definitely detect non-Gaussianity with a very high confidence even if the intrinsic magnitude dispersion is not known \textit{a priori}.

Key words: Cosmology: theory – gravitational lensing – large-scale structure of Universe Methods: analytical – Methods: statistical –Methods: numerical

1 INTRODUCTION

Type Ia supernovae (SNeIa) are powerful probes of the recent expansion of the universe and they provided the main contribution to the discovery of the present acceleration of the universe (Riess et al. 1998; Perlmutter et al. 1999). Indeed, SNeIa are standard candles with a small luminosity dispersion so that by measuring the flux received on the earth one can derive the luminosity distance of the source. Then, by observing many SNeIa one can measure the redshift-distance relation which provides constraints on cosmological parameters (Goobar & Perlmutter 1995). However, even SNeIa are not perfect candles and are affected by various sources of noise such as the magnification produced by gravitational lensing, related to the fluctuations of the matter distribution along the line of sight (Kantowski et al. 1995; Frieman 1997). Flux conservation implies that the random magnification shift is zero (Weinberg 1976) but weak lensing distortions increase the observed SNeIa magnitude scatter and lead to an extended high-luminosity tail (Wambsganss et al. 1997; Valageas 2000). For a flux-limited survey weak lensing also leads to a slight bias towards larger luminosities close to the threshold (Valageas 2000) but this plays no significant role. Then, from the deviation of the magnitude distribution of 63 high redshift SNeIa (Riess et al. 2004) from a Gaussian, Wang (2005) claimed that weak lensing effects may have been detected. Although the distortion agrees at a qualitative level with the expectation from weak lensing magnification (i.e. there are three very bright SNeIa) the statistics was too small to draw firm conclusions. In this Letter we revisit this issue by investigating how many SNeIa are needed to detect with high confidence weak lensing effects. In sect. 2 we describe how weak gravitational lensing by large scale structures affects the apparent magnitude of SNeIa. Next, assuming that the intrinsic magnitude fluctuation (including all sources of noise except lensing) is Gaussian with a known variance we discuss a Kolmogorov–Smirnov test to assess whether a sample of SNeIa may be drawn from such a Gaussian. Then, in sect. 3.1 we compute for a survey such as the proposed SNAP\textsuperscript{1} experiment (Aldering et al. 2004) at which deviation from this Gaussian is detected with a high confidence level. In sect. 3.2 we generalize this procedure to the case where the intrinsic magnitude variance is unknown. We discuss the dependence of our results on the amplitude of the intrinsic

\textsuperscript{1}http://snap.lbl.gov
2 DETECTING WEAK LENSING

If there are no distortions the apparent magnitude \( m_{\text{app}} \) of a supernova at redshift \( z \) is related to its absolute magnitude \( M_* \) and luminosity \( L_* \) by:

\[
m_{\text{app}} = M_* + 5 \log \left( \frac{d_L(z)}{10 \text{pc}} \right) + K(z), \quad M_* = -2.5 \log \left( \frac{L_*}{L_0} \right),
\]

where \( d_L(z) \) is the luminosity distance, \( K(z) \) the “K-correction” which describes the redshift of the flux spectrum with respect to the observing filter and \( L_0 \) the zero-point of the magnitude system. Therefore, inverting eq. (1) observers who measure the flux from distant supernovae can derive \( d_L(z) \) which provides constraints on cosmological parameters. However, in practice one needs to take into account the intrinsic dispersion \( \sigma_{\text{int}} \) of supernovae magnitudes, which is due to the dispersion of SNeIa luminosities themselves as well as to measurement noises and absorption along the line of sight. This implies a statistical analysis to extract the mean distance modulus \( m_{\text{app}} - M_* \). Another distortion is due to weak lensing which can magnify the luminosity of distant supernovae. Therefore, the apparent magnitude shows a fluctuation \( \delta m \) around its average \( \langle m_{\text{app}} \rangle \):

\[
\delta m = \delta m_{\text{int}} + \delta m_{\text{lens}}, \quad \langle \delta m \rangle = 0,
\]

where we separate the intrinsic fluctuation \( \delta m_{\text{int}} \) and the gravitational lensing distortion \( \delta m_{\text{lens}} \). Note that since the mean weak lensing magnification is unity (as weak lensing only modifies the trajectory of light rays and does not change their energy) gravitational lensing does not bias the average apparent magnitude which allows one to derive \( d_L \) and to put constraints on cosmology. In this work, we focus on the fluctuating part \( \delta m \) and we investigate how many SNeIa are required to detect weak lensing through the statistics of the fluctuations \( \delta m \), which depend on \( \delta m_{\text{lens}} \). In the weak lensing regime which is appropriate for lensing by large scale structure that we consider here the magnification \( \mu \) is related to the usual weak lensing convergence \( \kappa \) by:

\[
\frac{L_{\text{obs}}}{L_{\text{true}}_{\text{lens}}} = \mu \simeq 1 + 2\kappa,
\]

which can be written in terms of the density contrast \( \delta \) along the line of sight as:

\[
\kappa = \frac{3\Omega_m H_0^2}{2c^2} \int_0^{\chi_*} d\chi \frac{P(\chi)D(\chi)}{D(\chi_*)} (1+z)\delta(z).
\]

Here \( H_0 \) is the Hubble constant, \( \chi \) is the radial distance along the line of sight and \( D \) the angular diameter distance. From the definition of magnitudes in eq. (1) we obtain for the apparent magnitude fluctuation:

\[
\delta m = \delta m_{\text{int}} + \delta m_{\text{lens}} = \delta m_{\text{int}} - \frac{5\kappa}{\ln 10}.
\]

We shall assume in the following that \( \delta m_{\text{int}} \) is Gaussian with variance \( \sigma_{\text{int}} \). Then, as in Valageas et al. (2005) the probability distribution of \( \delta m \) can be written in terms of its generating function \( \varphi_{\delta m} \) as:

\[
P(\delta m) = \int_{-\infty}^{+\infty} \frac{dy}{2\pi(\delta m^2)} e^{[\varphi_{\delta m}(y)/\delta m^2]}
\]

with:

\[
\varphi_{\delta m}(y) = \frac{1 + \rho}{\rho} \varphi_{\delta m_{\text{lens}}}(\frac{y}{1 + \rho}) - \frac{1}{1 + \rho} \frac{y^2}{2}
\]

Here we introduced the generating function \( \varphi_{\delta m_{\text{lens}}} \) of the lensing fluctuation and we defined the ratio \( \rho \) of intrinsic and lensing variances by:

\[
\rho = \frac{\langle \delta m_{\text{int}}^2 \rangle}{\sigma_{\text{int}}^2} = \left( \frac{5}{\ln 10} \right)^2 \langle \kappa^2 \rangle.
\]

Finally, we use the model described in Barber et al. (2004) or Munshi et al. (2004) to obtain the generating functions of the convergence \( \kappa \) whence of the lensing magnitude fluctuation \( \delta m_{\text{lens}} \), taking into account the redshift distribution of the sources (here SNeIa) as in Valageas et al. (2005).

Then, from the observed distribution of apparent magnitudes, whence of \( \delta m \), one can recover the statistics of \( \kappa \). In this fashion, from the tails of the observed magnitude distribution Wang (2005) claimed that weak lensing effects may have been detected. However, the sample was too small (67 high redshift SNeIa) to draw definite conclusions. Here we reconsider this question by computing how many SNeIa are needed to get a clear detection of weak lensing from SNeIa.

To this order, we use a Kolmogorov-Smirnov (K-S) test (Press et al. 1986, Kendall & Stuart 1969) as follows. From a sample of \( N \) supernovae the observer can compare their magnitude distribution with a Gaussian \( P_G \) of variance \( \sigma_G \) through the K-S distance \( d \) defined by:

\[
d = \max_{\delta m} \left| S_N(\delta m) - P_G(\delta m) \right|.
\]

Here \( P_G(\delta m) \) is the trial cumulative Gaussian whereas \( S_N(\delta m) \) is the discrete cumulative distribution obtained from the data. Thus, \( S_N(\delta m) \) is merely the fraction of observed SNeIa with a magnitude fluctuation smaller than \( \delta m \). As is well known, the interest of the K-S estimator \( d \) is that its distribution is universal when the data is compared with its parent distribution (null hypothesis), whatever it is.

Thus, the cumulative distribution of \( d \) writes in this case:

\[
P(>d) = Q_{KS}(\sqrt{N}d) \quad \text{with} \quad Q_{KS}(\lambda) = -2 \sum_{j=1}^{\infty} (-1)^j e^{-2j^2\lambda^2}.
\]

As expected, eq. (10) shows that the distance \( d \) between the discrete cumulative distribution \( S_N \) and its continuous parent distribution scales as \( 1/\sqrt{N} \). Then, by computing the distance \( d \) of the sample with respect to a trial Gaussian from eq. (9), one can obtain from eq. (10) the probability \( P(>d) \) that a distance of this size or larger would be observed if the trial Gaussian is the true parent magnitude distribution. Therefore, if this significance level \( P(>d) \) is smaller than a threshold \( P_* \ll 1 \) one can conclude with good confidence that this trial Gaussian is not the true parent distribution (disproof of the null hypothesis). In our case, this means that weak lensing has been detected since we assume that this is the only source of distortion from the Gaussian of intrinsic variance \( \sigma_G \) with \( \sigma_G = \sigma_{\text{int}} \).

Thus, to find out for which \( N \) such a disproof of the Gaussian can be obtained with a high significance level we first choose a threshold \( P_* \ll 1 \) (for instance \( P_* = 5\% \)).
and compute from eq. [10] the scaled variable $\lambda_-$ such that $Q_{KS}(\lambda_-) = P_-$. (for $P_- = 5\%$ this yields $\lambda_- = 1.34$). Then, we draw a large number $N_{\text{sim}}$ of samples of $N$ supernovae magnitude fluctuations $\delta m_i$ ($i = 1, ..., N$), for some value of $N$. As explained above, the distribution of these magnitudes is obtained from eq. [8]. Next, we compute for each sample $k$ ($k = 1, ..., N_{\text{sim}}$) the distance $d_k$ to the Gaussian of variance $\sigma_G = \sigma_{\text{int}}$, using eq. [8]. From this set $\{d_k\}$ we obtain the probability $P(> \lambda_-)$ to measure a distance $d$ larger than our threshold $d_- = \lambda_- / \sqrt{N}$. This is simply the fraction of realizations among our $N_{\text{sim}}$ simulations with $d_k > d_-$. Then, we can repeat the same procedure for various $N$ which provides the curve $P(> \lambda_-; N)$ as a function of $N$ (at fixed $\lambda_-$. Since the parent distribution [8] is different from the Gaussian of variance $\sigma_{\text{int}}$, this probability $P(> \lambda_-; N)$ increases with $N$ and goes to unity at large $N$: for sufficiently large $N$ we are sure to detect the difference between both PDF. Finally, we select a second threshold $P_+ \simeq 1$ (for instance $P_+ = 95\%$) and we find above which $N_*$ the probability $P(> \lambda_; N)$ becomes larger than $P_+$. This value of $N_*$ is the number of supernovae needed to detect with a high probability ($P_+$) a weak lensing signature (defined as a distance $d$ from the Gaussian which is more rare than $P_-$).

3 NUMERICAL RESULTS

We assume a concordance $\Lambda$CDM cosmology with $\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.7$, $\sigma_8 = 0.88$ and $H_0 = 70$km/s/Mpc. We also adopt a redshift distribution of SNeIa as expected for the SNAP mission (Table 1 of Kim et al. 2004) which plans to observe about 6000 supernovae, of which 2000 may be used for cosmological purposes between redshifts of 0.1 and 1.7 (Aldering et al. 2004). We also use throughout an intrinsic magnitude dispersion $\sigma_{\text{int}} = 0.1$ mag.

We show in Fig. 1 the probability distribution $P(\delta m)$ of the SNeIa magnitude fluctuation $\delta m$ (solid line) from eq. [4]. We also plot for comparison the Gaussian $P_G$ of variance $\sigma_G = 0.1$. Thus, we see that weak-lensing effects increase the dispersion $(\langle \delta m \rangle^2)$ and distort the shape of the distribution with an extended bright tail.

Figure 1. The probability distribution $P(\delta m)$ (solid line) of the SNeIa magnitude fluctuation from the mean, as given by eq. [4]. The dashed line shows the Gaussian $P_G$ of variance $\sigma_G = \sigma_{\text{int}} = 0.1$ mag which corresponds to neglecting weak-lensing effects.

Figure 2. The cumulative probability distribution $P(> \lambda)$ to measure a distance larger than $d = \lambda / \sqrt{N}$ from the Gaussian of variance $\sigma_G = \sigma_{\text{int}} = 0.1$ mag. The dot-dashed curves correspond to $N = 100, 250, 500, 1000, 2000, 3000, 4000, 5000$ from left to right. The left solid curve shows for reference the cumulative probability distribution of the distance from the parent distribution [9]. It is equal to $Q_{KS}$ in eq. [10].

3.1 Known intrinsic variance

We first apply in this section the K-S test as described above in sect. 2. Thus, we show in Fig. 2 the cumulative probability distribution $P(> \lambda)$ to measure a distance larger than $d = \lambda / \sqrt{N}$ from the Gaussian of variance $\sigma_G = \sigma_{\text{int}}$. These PDF are obtained for each $N$ from the distribution of distances $\{d_k\}$ ($k = 1, ..., N_{\text{sim}}$) associated with our $N_{\text{sim}}$ realizations of $N$ supernovae. As $N$ increases the cumulative probability $P(> \lambda)$ develops a plateau which extends to larger values of $\lambda$ as it is easier to detect the deviation of the parent distribution [9] from the trial Gaussian of variance $\sigma_{\text{int}}$. Therefore, the probability $P(> \lambda_-)$ grows with $N$.

Thus, we show in Fig. 3 the curves $P(> \lambda_; N)$ as a function of $N$, for three different thresholds $P_- = 10\%, 5\%$ and $2.5\%$ (corresponding to $\lambda_- = 1.20, 1.34$ and 1.46) from top down to bottom. We can check that for low $N$ the statistics is too small to obtain a clear detection of weak lensing and as $N$ increases the probability to measure the deviation from the Gaussian due to weak lensing effects grows to reach unity at $N \to \infty$. Thus, we find that for significance levels $\{P_- = 10\%, P_+ = 90\%\}$, $N_* = 2000$ supernovae are sufficient to detect weak lensing. Higher levels $\{P_- = 5\%, P_+ = 95\%\}$ and $\{P_- = 2.5\%, P_+ = 97.5\%\}$ require $N_* = 3000$ and $N_* = 4000$ supernovae.

3.2 Marginalizing over observed variance

The procedure used in the previous section assumes that the intrinsic variance $\sigma_{\text{int}}$ is exactly known so that any deviation from the Gaussian of variance $\sigma_{\text{int}}$ is interpreted as a detection of weak lensing. However, in practice the variance $\sigma_{\text{int}}$ is only known up to some finite accuracy. Moreover, high redshift SNeIa may exhibit a somewhat different variance (because of the evolution of SNeIa metallicities, absorption by dust along the line of sight, etc.). Therefore,
Figure 3. The curves $P(\lambda; N)$ for three different thresholds $\lambda = 10\%, 5\%$ and $2.5\%$ (i.e. $\lambda \approx 1.20, 1.34$ and $1.46$) from top down to bottom, as a function of $N$. At large $N$ one detects almost surely ($P(\lambda; N) \approx 1$) a large deviation from the trial Gaussian (so large that it would have occurred with probability $P \approx 20000$ if the latter Gaussian had been the true parent distribution).

Figure 4. Same as Fig. 2 but with marginalization over the observed variance. The dot-dashed curves show for various $N$ the cumulative probability distribution $P(\lambda)$ to measure a distance larger than $d_{\text{min}} = \lambda_{\text{min}}/\sqrt{N}$ from the closest Gaussian among Gaussians of any variance (it is sufficient to span the range $0.09 < \sigma_g < 0.14$). They correspond to $N = 10000, 20000, 30000, 40000, 50000, 60000$ from left to right. The left solid curve shows for reference the distance from the parent distribution $P$ and obeys eq. (10). For each of these studies statistics are constructed from 1000 simulations. The intrinsic variance is again $\sigma_{\text{int}} = 0.1$ mag.

Figure 5. The curves $P(\lambda; N)$ for three different thresholds $\lambda = 10\%, 5\%$ and $2.5\%$ from top down to bottom as in Fig. 3 but using the distance to the closest Gaussian displayed in Fig. 3. A range of Gaussian PDFs were used (see text for more details). Triangles are actual estimates from our simulations whereas the solid lines are fitting functions.

in this section we marginalize over the variance of the observed sample (keeping $\sigma_{\text{int}} = 0.1$ mag for the true parent distribution), which implies that detection of weak lensing only depends on non-Gaussianities. Thus, for each realization $k$ of $N$ supernovae we compute all distances $d_{k,p}$ of this data set from an ensemble of trial Gaussians $P_{G,p}$ of different variances $\sigma_{G,p}$. From these $d_{k,p}$ we obtain the minimum distance $d_{\text{min},k} = \min_p \{d_{k,p}\}$. Thus $d_{\text{min},k}$ is the minimum distance between this realization and any Gaussian. In practice we use a grid of variances $\sigma_{G,p}$ which spans the range $0.09, 0.14$ with a step of 0.002. Obviously the distance $d_{k,p}$ increases at very small or very large variance $\sigma_g$ and it is minimum for $\sigma_g \approx \sigma_{\text{int}}$. For the cosmology and the redshift distribution that we use in this work we find that the minimum distance corresponds to $\sigma_g \approx 0.125$. Then, from the distribution of minimum distances $d_{\text{min},k}$ provided by our $N_{\text{sim}}$ realizations of $N$ supernovae magnitudes we obtain the cumulative probability distribution $P(\lambda; N)$ to observe a distance larger than $d_{\text{min}}$ from the closest possible Gaussian distribution. We show in Fig. 4, this cumulative probability distribution. Of course, we can check that for a given $N$ the typical distance $d_{\text{min}}$ is smaller than the distance $d$ to the fixed Gaussian of variance $\sigma_g = \sigma_{\text{int}}$ used in Fig. 2. Therefore, a larger number of SNeIa is needed to detect weak lensing.

Applying the same procedure as in sect. 5.4 we can now display in Fig. 5 the curves $P(\lambda; N)$ as a function of $N$ obtained from the minimum distance distributions shown in Fig. 4. We see that an order of magnitude more SNe are needed if the intrinsic variance is not known in advance (e.g. from low redshift studies). About 50,000 SNe are required to detect weak lensing effects in SNeIa studies through the K-S test with a high level of confidence for the redshift distribution that we have considered here. In particular, we now find that the significance levels $\{P_{\text{min}} = 10\%, P_{\text{min}} = 90\%\}, \{5\%, 95\%\}$ and $\{2.5\%, 97.5\%\}$ require $N \geq 30,000, 40,000$ and 45,000 supernovae. Note that the SNAP mission actually plans to observe about 10,000 SNeIa from which about 4000 should be well-characterized (Albert et al. 2005). Therefore, it will be able to detect weak lensing only if the intrinsic dispersion of SNeIa magnitudes is known. On the
Figure 6. The curves \( P(> \lambda_\text{c};N) \) for three different thresholds \( \lambda_\text{c} = 10\%, 5\% \) and 2.5\% from top to bottom. All simulations were performed with 3000 SNe. A set of 1000 simulations were performed to reduce the scatter. Solid lines represent a linear fit were performed with 3000 SNe. A set of 1000 simulations were performed to reduce the scatter. Solid lines represent a linear fit.

3.3 Dependence on intrinsic variance

In these studies we have assumed that the intrinsic variance (which can be unknown) is \( \sigma_{\text{int}} = 0.1 \text{ mag} \). However these results are quite sensitive to \( \sigma_{\text{int}} \). In Fig. 6 we plot for the case of \( N = 3000 \) SNe the cumulative probability \( P(> \lambda_\text{c};N) \) as a function of \( \sigma_{\text{int}} \). We consider the three thresholds used in Figs. 4 and 5 and use 1000 simulations. As in sect. 3.2 we consider the case where the intrinsic variance is known so that the observed SNeIa sample is compared with the trial Gaussian of variance \( \sigma_G = \sigma_{\text{int}} \). Of course, for low \( \sigma_{\text{int}} \) it is easy to detect weak lensing (the probability \( P(> \lambda_\text{c};N) \) goes to unity) since the amplitude of weak lensing effects becomes larger than the intrinsic dispersion of SNeIa magnitudes whereas for high \( \sigma_{\text{int}} \) weak lensing distortions become relatively negligible \( P(> \lambda_\text{c};N) \) goes to zero). We can see that this probability \( P(> \lambda_\text{c};N) \) is quite sensitive to \( \sigma_{\text{int}} \) as it exhibits a fast decrease for larger \( \sigma_{\text{int}} \). This implies that the number of SNeIa required to detect weak lensing signatures through the Kolmogorov-Smirnov test grows quickly with the intrinsic SNeIa magnitude variance. In particular, for \( \sigma_{\text{int}} = 0.16 \text{ mag} \) we find that 7000, 10000 and 20000 SNeIa are required in order to achieve the confidence levels \( \{10\%, 90\%\}, \{5\%, 95\%\} \) and \( \{2.5\%, 97.5\%\} \) (in the case of known intrinsic variance as in sect. 3.2).

4 CONCLUSIONS AND OUTLOOK

In this Letter we have addressed the issue of determining the number of observed SNeIa beyond which weak lensing effects can be detected with a high confidence. For the concordance ΛCDM cosmology, using a model of the large-scale matter distribution which has been checked against numerical simulations, we found that 4000 SNeIa are necessary to distinguish a weak lensing signature with a significance level of 2.5% through a Kolmogorov-Smirnov test. To reach a significance level of 10% we only need 2000 SNeIa. This procedure compares the magnitude distribution of the observed SNeIa with a Gaussian of fixed variance, assuming that the latter describes all sources of noise except for weak lensing magnification. If we consider the variance to be a free parameter (e.g. the intrinsic SNeIa magnitude dispersion or the instrumental noise are not accurately known beforehand) we find that 45,000 supernovae are required to detect with a high confidence (at a 2.5% level) non-Gaussian signatures. Therefore, future experiments such as those planned within the Joint Dark Energy Mission will exhibit clear weak lensing signatures if the intrinsic magnitude dispersion of SNeIa is well known. However to be more confident without any a priori knowledge of \( \sigma_{\text{int}} \) we will have to wait for surveys such as ALPACA. Of course, the possibility of detecting weak lensing effects on SNeIa magnitude distributions also implies that such gravitational lensing effects should be taken into account or used as a complementary tool to constrain cosmology (e.g., Dodelson & Vallinotto 2005).

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