Charged strange star with Krori–Barua potential in $f(R, T)$ gravity admitting Chaplygin equation of state

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Abstract In present paper, a new compact star model in $f(R, T)$ gravity is obtained, where $R$ and $T$ denote the Ricci scalar and the trace of energy–momentum tensor $T_{\mu\nu}$, respectively. To develop the model, we consider the spherically symmetric spacetime along with anisotropic fluid distribution in the presence of electric field with $f(R, T) = R + 2\gamma T$, where $\gamma$ is a small positive constant. We have used the Chaplygin equation of state to explore the stellar model. The field equations for $f(R, T)$ gravity have been solved by employing the Krori–Barua ansatz already reported in the literature [J. Phys. A, Math. Gen. 8:508, 1975]. The exterior spacetime is described by Reissner–Nordström line element for smooth matching at the boundary. It is worthwhile to mention here that the values of all the constants involved with this model have been calculated for the strange stars 4U 1538-52 for different values of $\gamma$ with the help of matching conditions. The acceptability of the model is discussed in detail both analytically and graphically by studying the physical attributes of matter density, pressures, anisotropy factor, stability, etc. We have also obtained the numerical values in tabular form for central density, surface density, central pressure and central adiabatic index for different values of $\gamma$. The solutions of the field equations in Einstein gravity can be regained by simply putting $\gamma = 0$ to our solution. Moreover, the proposed model is shown to be physically admissible and corroborate with experimental observations on strange star candidates such as 4U 1538-52.

1 Introduction

The study of compact object is a very interesting topic as it plays an important role to relate astrophysics, nuclear physics and particle physics. It is familiar that the neutron stars are built of neutron; on the other hand, strange stars can be composed entirely of strange quark matter (SQM). Neutron stars are bounded by gravitational attraction, and the strange stars are bounded by strong interactions as well as gravitational attractions. In 1915, Albert Einstein ignited the light among the scientific community by presenting one of the greatest achievements of theoretical physics [1–4]. The next year, Karl Schwarzschild [5] presented the first solution to the Einstein’s field equations that describes the neighborhood of a compact object that is spherically symmetric and static having vanishing pressure and density. A lot of pioneer works have been done till date in this direction [6–12].
Bonnor proposed that if the matter present in the sphere carries certain modest electric charge density, a spherical body can remain in equilibrium under its own gravitation and electric repulsion, and no internal pressure is necessary [13]. Stettner [14] studied the stability of a homogeneous distribution of matter containing a net surface and proved that a fluid sphere of uniform density with a modest surface charge is more stable than the same system without charge. According to De Felice et al. [15], the gravitational collapse of a fluid sphere to a point singularity may be avoided in the presence of large amounts of electric charge during an accretion process onto a compact object. Electrostatic repulsion due to the same electric charge along with the pressure gradient counterbalances the gravitational attraction [16,17]. The analysis of Raychaudhuri for charged dust distributions showed that conditions for collapse and oscillation depend on the ratio of matter density to charge density [18]. Di Prisco et al. [19] provided a full and comprehensive analysis of charged, dissipative collapse. A detailed analysis of gravitational collapse in the presence of charged medium was performed by Kouretsis and Tsagas [20] by highlighting the role of Raychaudhuri equation. Maharaj and Takisa utilized an equation of state which is quadratic relating the radial pressure to the energy density.

Alternative gravity is nowadays an extremely important tool to address some persistent observational issues, such as the dark sector of the universe. In current years, several modified theories of gravity have been introduced, but a few theories like $f(R)$, $f(T)$ and $f(R,T)$ have received more attention than any other theories of gravity. The concept of $f(R,T)$ gravity was initially proposed by Harko et al. [21] by considering an extension of standard general relativity in the year 2011, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar $R$ and of the trace $T$ of the stress–energy tensor $T_{\mu\nu}$. They derived the field equations of the model from a variational, Hilbert–Einstein type, principle, and covariant divergence of the stress–energy tensor is also obtained. In that paper, the author proved that the covariant divergence of the stress–energy tensor is nonzero and for this reason, the motion of massive test particles is non-geodesic, and an extra acceleration, due to the coupling between matter and geometry, is always present. It was also proposed by the author that the $f(R,T)$ gravity model depends on a source term, representing the variation of the matter stress–energy tensor with respect to the metric, and its expression can be obtained as a function of the matter Lagrangian $L_m$, and hence, for each choice of $L_m$, a specific set of field equations can be generated. However, Harko and his collaborators have constructed three possible models by taking the functional $f(R,T)$, (i) $f(R,T) = R + 2f(T)$, (ii) $f(R,T) = f(R) + f(T)$ and (iii) $f(R,T) = f(R) + g(R)f(T)$, where $f(R)$, $g(R)$ and $f(T)$ are some arbitrary functions of $R$ and $T$.

Several astrophysical and cosmological models in the background of $f(R,T)$ gravity have been obtained by several authors. Houndjo [22] studied a few distinguished cosmological model in the context of $f(R,T)$ gravity to study a matter influenced era of expanding universe. Yousaf et al. [23] discussed possibility development about relativistic compact star over $f(R,T)$ gravity in framework of Krori and Barua solution and two viable $f(R,T)$ gravity models. Moraes et al. [24] studied hydrostatic equilibrium configurations for the neutron stars and strange stars by using Runge–Kutta 4th-order method to solve the TOV equation in $f(R,T)$ gravity. Das et al. [25] studied spherically symmetric and isotropic compact stellar system in $f(R,T)$ gravity by adopting the Lie algebra with conformal Killing vectors and also presented a model for Gravastar to avoid singularity in the presence of $f(R,T)$ gravity [26]. Jamil et al. [27] have reconstructed some cosmological models in this theory of gravity using the functional form $f(R,T) = R^2 + f(T)$. Sahoo et al. [28] obtained accelerating models in the framework of $f(R,T)$ theory of gravity for an anisotropic Bianchi type III (BIII) universe. Sharif and Zubair [29] investigated perfect fluid distribution and massless scalar
field for Bianchi type I universe. Accelerating cosmological models have been constructed by Sahu et al. [30] in $f(R, T)$ modified gravity theory at the backdrop of an anisotropic Bianchi type III universe. Sahoo, and his collaborators have reconstructed some $f(R, T)$ cosmological models for anisotropic universes [31–35].

Here in this work, we are interested to study the charge effects within the framework of the $f(R, T)$ gravity in the presence of Chaplygin equation of state (EoS). We have arranged our present paper as follows. The basic field equations of $f(R, T)$ gravity in the presence of charge and the interior spacetime are given in Sect. 2. The field equations have been solved by choosing suitable metric ansatz and a proper choice of an equation of state in Sect. 3. In order to fixed different constants, we have matched our interior spacetime to the exterior Reissner–Nordström line element at the boundary outside the event horizon. The physical attributes of the model in modified gravity with the comparison of Einstein gravity are shown in Sect. 5. The next section describes about the mass radius relationship, and the final section deals with some concluding remarks.

2 Interior spacetime and basic field equations

The Einstein–Hilbert action for $f(R, T)$ gravity in the presence of charged is given by [21],

$$
S = \frac{1}{16\pi} \int f(R, T) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x + \int L_e \sqrt{-g} d^4x,
$$

(1)

where $f(R, T)$ represents the general function of Ricci scalar $R$ and trace $T$ of the energy–momentum tensor $T_{\mu\nu}$ and $L_m$ and $L_e$ are the Lagrangian matter density and Lagrangian for the electromagnetic field, respectively, with $g = \det(g_{\mu\nu})$.

The field equations of the $f(R, T)$ gravity corresponding to action (1) are given by,

$$
f_R R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} f + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f_R = 8\pi (T_{\mu\nu} + E_{\mu\nu})
$$

$$
- f_T (T_{\mu\nu} + \Theta_{\mu\nu}),
$$

(2)

where $f = f(R, T)$, $f_R(R, T) = \frac{\partial f(R, T)}{\partial R}$, $f_T(R, T) = \frac{\partial f(R, T)}{\partial T}$, $\nabla_\nu$ represents the covariant derivative associated with the Levi-Civita connection of $g_{\mu\nu}$, $\Theta_{\mu\nu} = g^{\alpha\beta} \frac{\partial T_{\alpha\beta}}{\partial g^{\mu\nu}}$, and $\Box \equiv \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu)$ represents the D’Alembert operator.

According to Landau and Lifshitz [36], the stress–energy tensor of matter is defined as,

$$
T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta \sqrt{-g} L_m}{\delta g^{\mu\nu}},
$$

(3)

and its trace is given by $T = g^{\mu\nu} T_{\mu\nu}$. If the Lagrangian density $L_m$ depends only on $g_{\mu\nu}$, not on its derivatives, Eq. (3) becomes,

$$
T_{\mu\nu} = g_{\mu\nu} L_m - 2 \frac{\partial L_m}{\partial g_{\mu\nu}}.
$$

(4)

$E_{\mu\nu}$ is the electromagnetic energy–momentum tensor defined by,

$$
E_{\mu\nu} = \frac{1}{4\pi} \left( F^a_\mu F_v^{a\nu} - \frac{1}{4} F^a_\alpha F^{a\beta} g_{\mu\nu} \right),
$$

(5)
where $F_{\mu\nu}$ denotes the antisymmetric electromagnetic field strength tensor, defined by
\begin{equation}
F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu},
\end{equation}
satisfying the Maxwell equations,
\begin{align}
F_{\mu\nu}^{;\nu} &= \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\nu} (\sqrt{-g} F_{\mu\nu}^{\phi}) = -4\pi j^\mu, \\
F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} &= 0,
\end{align}
where $A_\nu = (\phi(r), 0, 0, 0)$ is the four-potential and $j^\mu$ is the four-current vector, defined by
\begin{equation}
j^\mu = \frac{\rho_e}{\sqrt{g_{00}}} \frac{dx^\mu}{dx^0},
\end{equation}
where $\rho_e$ denotes the proper charge density. The only non-vanishing components of the electromagnetic field tensor are $F_{01}$ and $F_{10}$, and they are related by
\begin{equation}
F_{01}^{\phi} = -\frac{\lambda + \nu}{2} q(r) r^2.
\end{equation}
From Eq. (7), the expression for the electric field can be obtained as,
\begin{equation}
F_{01} = -e^{-\frac{\lambda + \nu}{2}} \frac{q(r)}{r^2},
\end{equation}
where $q(r)$ represents the net charge inside a sphere of radius $r$ which can be obtained as,
\begin{equation}
q(r) = 4\pi \int_0^r \rho_e e^{\frac{\lambda + \nu}{2}} r^2 dr.
\end{equation}
Now, the divergence of the stress–energy tensor $T_{\mu\nu}$ can be obtained by taking covariant divergence of (2) (For details, see ref [21,37,38]) as,
\begin{align}
\nabla^\mu T_{\mu\nu} &= \frac{f_T(R, T)}{8\pi - f_T(R, T)} [(T_{\mu\nu} + \Theta_{\mu\nu}) \nabla^\mu \ln f_T(R, T) \\
&\quad + \nabla^\mu \Theta_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \nabla^\mu T - \frac{8\pi}{f_T} \nabla^\mu E_{\mu\nu}] .
\end{align}
From Eq. (12), we can check that $\nabla^\mu T_{\mu\nu} \neq 0$ if $f_T(R, T) \neq 0$. So like Einstein gravity, the system will not be conserved. It can be noted that when $f(R, T) = f(R)$, from Eq. (2), we obtain the field equations of $f(R)$ gravity.

The static and spherically symmetric line element in curvature coordinates $(t, r, \theta, \phi)$ is given by,
\begin{equation}
d\ell^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\Omega^2,
\end{equation}
where $d\Omega^2 = \sin^2 \theta d\phi^2 + d\theta^2$ and the metric coefficients $\nu$ and $\lambda$ are purely radial functions. In the present study, we assume the compact star model with anisotropic fluid. So, the stress–energy tensor of the matter is given by,
\begin{equation}
T_{\mu\nu}^\rho = (\rho + p_r) u^\mu u_\nu - p_t g_{\mu\nu} + (p_r - p_t) \eta^\mu \eta_\nu,
\end{equation}
where $\rho$ is the matter density, $p_r$ and $p_t$ are, respectively, the radial and transverse pressure in modified gravity, and $u^\mu$ is the fluid four-velocity that satisfies the equations $u^\mu u_\mu = 1$ and $u^\mu \nabla_\nu u_\mu = 0$. Now, the matter Lagrangian density, which in the general case could be a function of both density and pressure, $L_m = L_m(\rho, p)$, or of only one of the thermodynamic parameters, becomes an arbitrary function of the density of the matter $\rho$ only, so that $L_m =
L_m(\rho) \ [39]. For our present paper, the matter Lagrangian can be taken as \( \mathcal{L}_m = \rho \) and the expression of \( \Theta_{\mu\nu} = -2T_{\mu\nu} - \rho g_{\mu\nu} \).

In the relativistic structures, in order to discuss the coupling effects of matter and curvature components in \( f(R, T) \) gravity, let us consider a separable functional form given by,

\[
f(R, T) = f_1(R) + f_2(T), \tag{15}
\]

where \( f_1(R) \) and \( f_2(T) \) are arbitrary functions of \( R \) and \( T \), respectively. Several viable models can be obtained in \( f(R, T) \) gravity by choosing different forms of \( f_1(R) \) along with linear combination of \( f_2(T) \). In our present model, we consider \( f_1(R) = R \) and \( f_2(T) = 2\gamma T \), i.e., we choose

\[
f(R, T) = R + 2\gamma T, \tag{16}
\]

where \( \gamma \) is some small positive constant (which reduces to GR for \( \gamma = 0 \) proposed by Harko et al. \[21\]) to study the effects of curvature–matter coupling. The term \( 2\gamma T \) induces time-dependent coupling (interaction) between curvature and matter. It also corresponds to \( \Lambda CD\)M model with a time-dependent cosmological constant \[40\]. Using (16) into (2), the field equations in \( f(R, T) \) gravity are given by,

\[
G_{\mu\nu} = 8\pi (T_{\mu\nu}^{\text{eff}} + E_{\mu\nu}), \tag{17}
\]

where \( G_{\mu\nu} \) is the Einstein tensor and

\[
T_{\mu\nu}^{\text{eff}} = T_{\mu\nu} + \frac{\gamma}{8\pi} T_{g\mu\nu} + \frac{\gamma}{4\pi} (T_{\mu\nu} - \rho g_{\mu\nu}). \tag{18}
\]

For the line element (13), the field equations in modified gravity can be written as,

\[
8\pi \rho^{\text{eff}} + E^2 = \frac{\lambda'}{r} e^{-\lambda} + \frac{1}{r^2} (1 - e^{-\lambda}), \tag{19}
\]

\[
8\pi p_r^{\text{eff}} - E^2 = \frac{1}{r^2} (e^{-\lambda} - 1) + \frac{\nu'}{r} e^{-\lambda}, \tag{20}
\]

\[
8\pi p_t^{\text{eff}} + E^2 = \frac{1}{4} e^{-\lambda} \left[ 2\nu'' + \nu'^2 - \lambda'\nu' + \frac{2}{r} (\nu' - \lambda') \right]. \tag{21}
\]

The quantity \( q(r) \) actually determines the electric field as,

\[
E(r) = \frac{q(r)}{r^2}, \tag{22}
\]

where \( \rho^{\text{eff}} \), \( p_r^{\text{eff}} \) and \( p_t^{\text{eff}} \) are, respectively, the density and pressures in Einstein gravity where

\[
\rho^{\text{eff}} = \rho + \frac{\gamma}{8\pi} (\rho - p_r - 2p_t), \tag{23}
\]

\[
p_r^{\text{eff}} = p_r + \frac{\gamma}{8\pi} (\rho + 3p_r + 2p_t), \tag{24}
\]

\[
p_t^{\text{eff}} = p_t + \frac{\gamma}{8\pi} (\rho + p_r + 4p_t). \tag{25}
\]

The prime denotes differentiation with respect to ‘\( r \).’ In next section, we shall solve Eqs. (19)–(21) to obtain the model of compact star.
3 Choice of the metric potential and proposed model

In Eqs. (19)–(21), we have three equations with six unknowns. So we have to choose any three of them to make the system solvable. Now, by our knowledge of algebra, we can choose it in \( \binom{6}{3} = 20 \) ways.

For our present model, we choose the coefficient of \( g_{rr} \) and \( g_{tt} \) as,

\[
e^\lambda = e^{Ar^2}, \quad e^\nu = e^{Br^2 + C},
\]

where \( B, A \) are constants of dimension \( \text{km}^{-2} \) and \( C \) is dimensionless quantity. Plugging (26) into (19)–(21), we obtain,

\[
8\pi \rho^{\text{eff}} + E^2 = \frac{1 + e^{-Ar^2}(-1 + 2Ar^2)}{r^2},
\]

\[
8\pi p_r^{\text{eff}} - E^2 = \frac{-1 + e^{-Ar^2}(1 + 2Br^2)}{r^2},
\]

\[
8\pi p_t^{\text{eff}} + E^2 = e^{-Ar^2}(-A + 2B + B(-A + B)r^2).
\]

From Eqs. (27)–(29), one can note that if we choose the suitable expression of \( E^2 \), the expression for \( \rho, p_r \) and \( p_t \) can be obtained from Eqs. (23) and (24). Instead of choosing the expression for \( E^2 \), we consider Chaplygin equation of state,

\[
p_r = \alpha \rho - \frac{\beta}{\rho^n},
\]

where \( \alpha, \beta \) and \( n \) are positive constants. For our present model, we took \( n = 1 \) for the simplicity.

The Chaplygin gas model has a connection with string theory. It can be obtained from the Nambu–Goto action for a D-brane moving in a (D+2)-dimensional spacetime in the light cone parameterization [41–44]. This Chaplygin gas model has been supported different classes of observational tests such as supernovae data [45], gravitational lensing [46,47], gamma ray bursts [48] and cosmic microwave background radiation [49]. Later, the Chaplygin equation of state has been modified to a more generalized Chaplygin gas equation of state. The generalized Chaplygin equation of state has been employed to model the compact objects. Mubasher et al. [50] constructed a stationary, spherically symmetric and spatially inhomogeneous wormhole spacetime supported by a modified Chaplygin gas.

Solving (27)–(29), with the help of (30), we obtain,

\[
\rho^{\text{eff}} = \frac{1}{2(1 + \alpha)} \left[ \frac{(A + B)e^{-Ar^2}}{4\pi} + g_1(r) \right],
\]

\[
p_r^{\text{eff}} = \frac{1}{2(1 + \alpha)} \left[ \frac{(1 + 2\alpha)(A + B)e^{-Ar^2}}{4\pi} - g_1(r) \right],
\]

\[
p_t^{\text{eff}} = \frac{1}{8\pi} \left[ e^{-Ar^2}(-A + 2B + B(-A + B)r^2) + \frac{-1 + e^{-Ar^2}(1 + 2Br^2)}{r^2} \right]
\]

\[
- \frac{4\pi}{1 + \alpha} \left\{ f_1(r) - g_1(r) \right\}.
\]

The above equations provide the expression for matter density and pressures for Einstein gravity.
The expression of $E^2$ is obtained as,

$$
E^2 = \frac{e^{-Ar^2}}{(1+\alpha)(3\gamma + 4\pi)^2} \left[ -2e^{Ar^2} \gamma^2 r^2 g_2(r) 
+ \gamma \left[ 2\alpha(-1 + e^{Ar^2}) + (4\alpha + B + 3\alpha B)r^2 
- 12e^{Ar^2} \pi r^2 g_2(r) 
+ (1 + \alpha)B(-A + B)r^4 + 2(-1 + e^{Ar^2} + Ar^2) \right] 
- 4\pi \left[ 1 + (-A + B)r^2 
- \alpha(-1 + e^{Ar^2} + 2Ar^2) + e^{Ar^2}(-1 + 4\pi r^2 g_2(r)) \right] \right].
$$

Now, we want to solve Eqs. (23)–(25) with the help of (31)–(33) and obtain the expressions for matter density and pressures in modified gravity as,

$$
\rho = \frac{1}{2(1+\alpha)} \left[ \frac{(A + B)e^{-Ar^2}}{\gamma + 4\pi} + g_2(r) \right],
$$

$$
p_r = \frac{1}{2(1+\alpha)} \left[ \frac{(1+2\alpha)(A + B)e^{-Ar^2}}{\gamma + 4\pi} - g_2(r) \right],
$$

$$
p_t = \frac{1}{6\gamma + 8\pi} e^{-Ar^2}(-A + 2B + B(-A + B)r^2) 
+ \frac{-1 + e^{-Ar^2}(1 + 2Br^2)}{r^2} - \frac{2(\gamma + 2\pi)}{1+\alpha} \left\{ \frac{f_1(r)}{\gamma + 4\pi} - g_2(r) \right\}
- \frac{\gamma}{1+\alpha} \left\{ \frac{(A + B)e^{-Ar^2}}{\gamma + 4\pi} + g_2(r) \right\}.
$$

The anisotropic factor $\Delta = p_t - p_r$ is obtained as,

$$
\Delta = \frac{1}{6\gamma + 8\pi} e^{-Ar^2}(-A + 2B + B(-A + B)r^2) 
+ \frac{-1 + e^{-Ar^2}(1 + 2Br^2)}{r^2} - \frac{2(\gamma + 2\pi)}{1+\alpha} \left\{ \frac{f_1(r)}{\gamma + 4\pi} 
- g_2(r) \right\}
- \frac{1}{2(1+\alpha)} \left[ \frac{(1+2\alpha)(A + B)e^{-Ar^2}}{\gamma + 4\pi} - g_2(r) \right],
$$

where $g_1$, $g_2$ and $f_1$ are functions of $r$ given as,

$$
g_1(r) = \sqrt{4(1+\alpha)\beta + \frac{(A + B)^2e^{-2Ar^2}}{16\pi^2}},
$$

$$
g_2(r) = \sqrt{4(1+\alpha)\beta + \frac{(A + B)^2e^{-2Ar^2}}{(\gamma + 4\pi)^2}},
$$

$$
f_1(r) = (1+2\alpha)(A + B)e^{-Ar^2}.$$
4 Boundary condition

In this section to fix the constants $A$, $B$ and $C$, we match our interior spacetime to the exterior spacetime outside the event horizon $r > M + \sqrt{M^2 - Q^2}$, where $Q$ is the total charge enclosed within the boundary $r = R$. The exterior spacetime of the star will be described by the Reissner–Nordström metric [51,52] given by

$$ds^2 = -(1 - \frac{2M}{r} + \frac{Q^2}{r^2})dt^2 + (1 - \frac{2M}{r} + \frac{Q^2}{r^2})^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

Continuity of the metric coefficients $g_{tt}$, $g_{rr}$ and $\frac{\partial g_{tt}}{\partial r}$ across the boundary surface $r = R$ between the interior and the exterior regions gives the following set of relations:

$$1 - \frac{2M}{R} + \frac{Q^2}{R^2} = e^{BR^2+C},$$

$$1 - \frac{2M}{R} + \frac{Q^2}{R^2} = e^{-AR^2},$$

$$\frac{M}{R^2} - \frac{Q^2}{R^3} = BRe^{BR^2+C}.$$  

Junevicus [53] obtained the expressions for $A$, $B$, $C$ from the continuity of the first and second fundamental forms across the surface of the charged fluid sphere in terms of the dimensionless parameters $\frac{M}{R}$ and $\frac{|Q|}{R}$.

Equations (40)–(42) determine the values of the constants $A$, $B$ and $C$ in terms of the total mass $M$, radius $R$ and charge $Q$. By solving the above set of equations, we get

$$A = -\frac{1}{R^2} \ln \left[ 1 - \frac{2M}{R} + \frac{Q^2}{R^2} \right],$$

$$B = \frac{1}{R^2} \left[ \frac{M}{R} - \frac{Q^2}{R^2} \right] \left[ 1 - \frac{2M}{R} + \frac{Q^2}{R^2} \right]^{-1},$$

$$C = \ln \left[ 1 - \frac{2M}{R} + \frac{Q^2}{R^2} \right] - \left[ \frac{M}{R} - \frac{Q^2}{R^2} \right].$$

We also impose the condition $E^2(r = 0) = 0$ which implies,

$$B(\gamma + 3\alpha\gamma - 4\pi) + 2A(2 + 3\alpha)(\gamma + 2\pi) = 2(\gamma^2 + 6\gamma\pi + 8\pi^2)\sqrt{\frac{(A + B)^2 + 4(1 + \alpha)\beta(\gamma + 4\pi)^2}{(\gamma + 4\pi)^2}},$$

and $p_r(r = R) = 0$ gives,

$$\alpha\rho_s^2 = \beta,$$

where $\rho_s$ is the surface density given by,

$$\rho_s = \frac{1}{2(1 + \alpha)} \left[ \frac{(A + B)e^{-AR^2}}{\gamma + 4\pi} + g_2(R) \right].$$
Solving (46) and (47), we get,
\[ \alpha = \frac{3e^{2AR^2}C_1((2A - B)\gamma - 4\pi(A - 2B))}{16(A + B)^2(\gamma + 2\pi)^2 - 9e^{2AR^2}C_1^2}, \]
\[ \beta = \frac{\alpha(A + B)^2e^{-2AR^2}}{(\gamma + 4\pi)^2}. \]

where \( C_1 = (B + 2A)\gamma + 4\pi \).

From Eqs. (43)–(45) and (49)–(50), it can be easily checked that for a particular stellar model, by fixing the ratio of mass to the radius and net charge to the radius, one can obtain the values of \( A, B, C, \alpha \) and \( \beta \) for different choices of \( \gamma \). It can also be checked that \( \alpha \) and \( \beta \) depend on \( \gamma \), but \( A, B \) and \( C \) do not.

## 5 Physical analysis of the present model

In this section, we shall check different physical attributes one by one both analytically and with the help of graphical representation.

### 5.1 Metric potential

For our present model, the metric potentials are chosen as,
\[ e^\nu = e^{Br^2 + C}, \quad e^\lambda = e^{Ar^2}, \]
We note that \( e^\nu |_{r=0} = e^C > 0 \) and \( e^\lambda |_{r=0} = 1 \); moreover,
\[ (e^\nu)' = 2Be^{C+Br^2}r, \quad (e^\lambda)' = 2Ae^{Ar^2}r. \]
Therefore, the form of the metric potential chosen here ensures that the metric function is non-singular, continuous and well behaved in the interior of the star. On a physical basis, this is one of the desirable features for any well-behaved model.

### 5.2 Density and pressure

The central density and central pressure for modified gravity are obtained as,
\[ \rho_c = \frac{1}{2(1 + \alpha)} \left[ \frac{A + B}{\gamma + 4\pi} + D \right], \]
\[ p_c = \frac{1}{2(1 + \alpha)} \left[ \frac{(1 + 2\alpha)(A + B)}{\gamma + 4\pi} - D \right], \]
where \( D \) is a constant depending on \( \gamma \) and its expression is given as,
\[ D = \sqrt{4(1 + \alpha)\beta + \frac{(A + B)^2}{(\gamma + 4\pi)^2}}. \]

The density and pressure gradients are obtained by taking the differentiation of Eqs. (35)–(37) with respect to \( r \) as,
\[ \frac{d\rho}{dr} = -\frac{A(A + B)e^{-2Ar^2}r}{(1 + \alpha)(\gamma + 4\pi)^2} \left[ e^{Ar^2}(\gamma + 4\pi) + h(r) \right]. \]
The matching condition of the metric potentials $e^\lambda$ and $e^\nu$ is shown against radius for the compact star 4U1538-52 by taking the values of the constants $A$, $B$ and $Q$ mentioned in Table 1.

The matter density $\rho$ is plotted against $r$ inside the stellar interior for different values of $\gamma$ mentioned in the figure.

\[
\frac{dp_r}{dr} = \frac{A(A + B)r e^{-2Ar^2}}{(1 + \alpha)(\gamma + 4\pi)^2} \left[ (1 + 2\alpha)e^{Ar^2}(\gamma + 4\pi) - h(r) \right],
\]
\[
\frac{dp_t}{dr} = \frac{r}{6\gamma + 8\pi} \left[ 2B(-A + B)e^{-2Ar^2} - \frac{2A(A + B)e^{-2Ar^2}}{(1 + \alpha)(\gamma + 4\pi)^2} \left( e^{Ar^2}(\gamma + 4\pi) - h(r) \right) + \frac{4A(A + B)e^{-2Ar^2}(\gamma + 2\pi)}{(1 + \alpha)(\gamma + 4\pi)^2} (1 + 2\alpha)e^{Ar^2} \times (\gamma + 4\pi) - h(r) \right] - \frac{2e^{-Ar^2}(A - 2B + 2ABr^2)}{r^2} + 2Ae^{-Ar^2} \left( A - 2B + (A - B)Br^2 \right) + \frac{2\left(1 - e^{-Ar^2}(1 + 2Br^2)\right)}{r^4},
\]

where $h(r) = \frac{A + B}{\sqrt{4(1 + \alpha)\beta + \frac{(A + B)^2e^{-2Ar^2}}{(\gamma + 4\pi)^2}}}$. 

The behavior of the metric potentials $e^\nu$ and $e^\lambda$ is shown in Fig. 1. The matter density $\rho$ and both radial and transverse pressures $p_r$, $p_t$ and the anisotropic factor $\Delta$ and the electric field $E^2$ versus the radial coordinate $r$ for the compact star 4U 1538 – 52 are shown in Figs. 2 and 3 for the numerical values of the parameters mentioned in Table 1. Both the anisotropic
factor and electric field start from zero at the center. The anisotropic factor increases toward the surface of the star; on the other hand, the electric field increases toward the surface of the star up to about 6 km, and then, it decreases toward boundary but at the surface of the star $E^2(R) > 0$, $\Delta(R) > 0$. The radial pressure vanishes at the surface, and neither the transverse pressure nor the matter density vanishes there. The similar nature of the electric field $E^2$ was obtained by Maharaj et al. [54] to describe the models for quark stars with a linear equation of state (Fig. 4).

5.3 Velocity of sound

The radial and transverse velocity of sound $V_r$ and $V_t$, respectively, for our present model is obtained as,

$$V_r = \sqrt{\frac{dp_r}{d\rho}}, \quad V_t = \sqrt{\frac{dp_t}{d\rho}}.$$

A model of compact star will be physically acceptable if both $V_r$, $V_t < 1$ which is known as causality condition. On the other hand, according to Le Chatelier’s principle, the speed of sound must be positive. Combining the previous two cases, one can get, $0 < V_r$, $V_t < 1$. For our present model of compact object, the square of the radial and transverse speed of sound is obtained as,

$$V_r^2 = \frac{(1 + 2\alpha)e^{Ar^2}(\gamma + 4\pi) - h(r)}{e^{Ar^2}(\gamma + 4\pi) + h(r)},$$

$$V_t^2 = \frac{(1 + \alpha)(\gamma + 4\pi)^2e^{2Ar^2}}{A(A + B)(6\gamma + 8\pi)} \left[ 2B(A - B)e^{-Ar^2} - \frac{2A(A + B)e^{-2Ar^2}\gamma}{(1 + \alpha)(\gamma + 4\pi)^2}(e^{Ar^2}(\gamma + 4\pi) + h(r)) - \frac{4A(A + B)e^{-2Ar^2}(\gamma + 2\pi)}{(1 + \alpha)(\gamma + 4\pi)^2} \left( (1 + 2\alpha)e^{Ar^2} \times (\gamma + 4\pi) - h(r) \right) - \frac{2e^{-Ar^2}(A - 2B + 2ABr^2)}{r^2} - 2Ae^{-Ar^2}(A - 2B + (A - B)Br^2) - \frac{2(1 - e^{-Ar^2}(1 + 2Br^2))}{r^4} \right].$$

Herrera and collaborators elaborately discussed the concept of cracking for self-gravitating isotropic and anisotropic matter configurations in a series of lectures [55–57]. In 1992, Herrera...
Table 1 The values of $\alpha$, $\beta$ and some physical quantities for the compact star 4U 1538-52 by assuming $M = 0.87 \, M_\odot$, $R = 7.8$ km, $Q = 0.078$

| $\gamma$ | $\alpha$ | $\beta$ | $\rho_c$ (gm · cm$^{-3}$) | $\rho_s$ (gm · cm$^{-3}$) | $p_c$ (dyne · cm$^{-2}$) | $\Gamma_{r_0}$ | $M_\text{eff}$ | $z_{s_3}^\text{eff}(R)$ |
|--------|----------|--------|------------------|------------------|------------------|-------------|-------------|----------------|
| 0.0    | 0.15916  | 5.08405 × 10$^{-8}$ | 1.05601 × 10$^{15}$ | 7.62612 × 10$^{14}$ | 7.23777 × 10$^{34}$ | 3.4221 | 1.28325 | 0.220819 |
| 0.03   | 0.15539  | 4.93998 × 10$^{-8}$ | 1.05503 × 10$^{15}$ | 7.60796 × 10$^{14}$ | 7.08216 × 10$^{34}$ | 3.4029 | 1.28758 | 0.221829 |
| 0.06   | 0.151689 | 4.79945 × 10$^{-8}$ | 1.05404 × 10$^{15}$ | 7.58988 × 10$^{14}$ | 6.9286 × 10$^{34}$ | 3.38409 | 1.29184 | 0.222827 |
| 0.09   | 0.148057 | 4.66236 × 10$^{-8}$ | 1.05305 × 10$^{15}$ | 7.57189 × 10$^{14}$ | 6.77705 × 10$^{34}$ | 3.36564 | 1.29604 | 0.223814 |
| 0.12   | 0.144492 | 4.5286 × 10$^{-8}$ | 1.05204 × 10$^{15}$ | 7.55399 × 10$^{14}$ | 6.62747 × 10$^{34}$ | 3.34757 | 1.30019 | 0.224789 |
Fig. 4 $E^2$ is shown against radius for different values of $\gamma$ mentioned in the figure.

Fig. 5 (left) Square of the radial sound velocity $V_r^2$, (middle) square of the transverse sound velocity $V_t^2$ and (right) the stability factor $V_t^2 - V_r^2$ are plotted against $r$ for the strange star candidate 4U 1538-52 by taking different values of $\gamma$.

[55] introduced the concept of cracking (or overturning) that approach is useful to identify potentially unstable anisotropic matter configurations. He examined that inside the stellar interior, fluid elements, at both sides of the cracking point, are accelerated with respect to each other. Later, Herrera along with his collaborators [7] showed that even small deviations from local isotropy may lead to drastic changes in the evolution of the system as compared with the purely locally isotropic case. Now, it is easy to verify that,

$$\frac{\delta \Delta}{\delta \rho} \sim \frac{\delta (p_t - p_r)}{\delta \rho} \sim \frac{\delta p_t}{\delta \rho} - \frac{\delta p_r}{\delta \rho} \sim V_t^2 - V_r^2.$$  

Moreover, it is clear that for physically reasonable models, the magnitude of perturbations in anisotropy should always be smaller than those in density since for physically acceptable stellar configuration,

$$|V_t^2 - V_r^2| \leq 1 \rightarrow |\frac{\delta \Delta}{\delta \rho}| \leq 1 \rightarrow |\delta \Delta| \leq |\delta \rho|.$$  

These perturbations lead to potentially unstable models when $\frac{\delta \Delta}{\delta \rho} > 0$. To check the causality as well as the potentially stability criterion, we have shown the profiles of $V_t^2$, $V_r^2$ and $V_t^2 - V_r^2$ in Fig. 5. The figures show that $0 < V_t^2$, $V_r^2 < 1$ holds everywhere inside the stellar interior and in the same time, $V_t^2 - V_r^2 < 0$ ensures the potential stability of the present model.

5.4 Equilibrium under forces

Using Eqs. (19)–(21), the generalized Tolman–Oppenheimer–Volkoff (TOV) equation for our present model in $f(R, T)$ gravity can be obtained as,
$$\frac{-v'}{2}(\rho + p_r) - \frac{dp_r}{dr} + \frac{2}{r}(p_t - p_r) + \frac{\gamma}{8\pi + 2\gamma}(\rho' + p'_r + 2p'_t)$$

$$+ \frac{8\pi}{8\pi + 2\gamma} \frac{q}{4\pi r^4} \frac{dq}{dr} = 0. \quad (58)$$

In Eq. (58), for $\gamma = 0$ we regain the conservation equation in Einstein gravity with charged distribution. Now, the above equation can be denoted by,

$$F_g + F_h + F_a + F_e + F_m = 0, \quad (59)$$

where $F_g$, $F_h$, $F_a$, $F_e$ and $F_m$, respectively, denote the gravitational force, hydrostatics force, anisotropic force, electric force and force related to modified gravity and the expressions of the forces are given as,

$$F_g = -\frac{v'}{2}(\rho + p_r)$$

$$= -\frac{B(A + B)e^{-Ar^2}}{\gamma + 4\pi}, \quad (60)$$

$$F_h = -\frac{dp_r}{dr}$$

$$= \frac{A(A + B)r e^{-2Ar^2}}{(1 + \alpha)(\gamma + 4\pi)^2} \left[ (1 + 2\alpha)e^{Ar^2}(\gamma + 4\pi) - h(r) \right], \quad (61)$$

$$F_a = \frac{2}{r}(p_t - p_r) = \frac{2}{r}\Delta$$

$$F_e = \frac{8\pi}{8\pi + 2\gamma} \frac{q}{4\pi r^4} \frac{dq}{dr}$$

$$= \frac{1}{4\pi + \gamma} \left( \frac{2}{r} E^2 + \frac{1}{2} \frac{d}{dr} (E^2) \right), \quad (62)$$

$$F_m = \frac{\gamma}{8\pi + 2\gamma} (\rho' + p'_r + 2p'_t), \quad (63)$$

where the expression for $E^2$, $\rho'$, $p'_r$ and $p'_t$ is given in Eqs. (34), (53)–(55), respectively. The above-mentioned forces for different values of $\gamma$ are shown in Fig. 6, and this figure verifies the equilibrium condition of the model of compact star.

5.5 Relativistic adiabatic index

Initially, Chandrasekhar [58] did the pioneer work in this era to examine the stable/unstable regions for spherical stars and explored the role of the adiabatic index. The adiabatic index $\Gamma$ for an isotropic fluid sphere was proposed by Chan et al. [59] as $\Gamma = \frac{\rho + p_r}{p_r} \frac{dp}{d\rho}$. The expression for adiabatic index in case of pressure anisotropy changes as,

$$\Gamma = \frac{\rho + p_r}{p_r} V^2_{r}. \quad (64)$$

The stability occurs if the adiabatic index is greater than $4/3$ as pointed out by Bondi [60]. For the complexity of the expression of $\Gamma$, we will check this condition with the help of graphical representation. The profile of $\Gamma$ for different values of $\gamma$ is shown in Fig. 7. We see that $\Gamma$ takes the values more than $4/3$ everywhere inside the fluid sphere.
5.6 Energy conditions

There should be some restrictions among the model parameters $\rho$, $p_r$, $p_t$ and $E^2$ which play a crucial role in understanding the nature of matter [61]. For our anisotropic charged model, the four types of the energy conditions are satisfied if and only if the following inequalities hold everywhere inside the fluid sphere.

- **Weak energy condition (WEC):**
  \[ \rho + p_r \geq 0, \quad \rho + p_t + \frac{E^2}{4\pi} \geq 0, \quad \rho + \frac{E^2}{8\pi} \geq 0, \]

- **Strong energy condition (SEC):**
  \[ \rho + p_r \geq 0, \quad \rho + p_t + \frac{E^2}{4\pi} \geq 0, \quad \rho + p_r + 2p_t + \frac{E^2}{4\pi} \geq 0, \]

- **Dominant energy condition (DEC):**
  \[ \rho - p_r + \frac{E^2}{4\pi} \geq 0, \quad \rho - p_t \geq 0, \quad \rho + \frac{E^2}{8\pi} \geq 0, \]

- **Null energy condition (NEC):**
  \[ \rho + p_r \geq 0, \quad \rho + p_t + \frac{E^2}{4\pi} \geq 0, \]

The NEC is a minimum requirement from SEC and WEC, i.e., if NEC is violated, then both SEC and WEC will not be satisfied. The violation of these inequalities ensures the presence of exotic matter which occurred to describe the model of wormhole in the context of the Einstein’s general theory of relativity [62]. The existence of ordinary matter is confirmed, if these conditions are satisfied. For the proposed star models, the validity of these conditions is checked graphically in Fig. 8. It is found that our charged anisotropic model in $f(R, T)$
Fig. 8 All the energy conditions are plotted inside the stellar interior for the strange star 4U 1538-52 for different values of $\gamma$ mentioned in the figure

Fig. 9 $p_r/\rho$ and $p_t/\rho$ are shown against radius for different values of $\gamma$ mentioned in the figure.

gravity satisfies all the above-mentioned energy conditions for different values of $\gamma$ which ensures the presence of the ordinary matter inside the compact stars.

5.7 Equation-of-state parameter

The equation-of-state parameters $\omega_r$ and $\omega_t$ are dimensionless quantity which describes the relation between matter density and pressures, and these also represent the state of matter under a given set of physical conditions. When $\omega_r$, $\omega_t$ lie between 0 and 1, it corresponds to the radiation era [63]. Using Eqs. (35)–(37), the equation-of-state parameters $\omega_r$ and $\omega_t$ for our present model are obtained as,

$$\omega_r = \frac{p_r}{\rho}, \quad \omega_t = \frac{p_t}{\rho}$$

The profiles of both $\omega_r$, $\omega_t$ are plotted in Fig. 9. Both the profiles are monotonic decreasing function of $r$ and also lie in the range $0 < \omega_r$, $\omega_t < 1$. So our present model of compact star in $f(R, T)$ gravity describes the radiating nature. It is also important to find out a relationship between the pressure and density which is known as the equation of state. To develop our model, we have assumed a nonlinear equation of state between the radial pressure and matter density, but still we have no information about the relationship between the transverse pressure
and the matter density. The behavior of radial and transverse pressure with respect to the matter density is shown graphically in Fig. 10.

6 Mass radius relationship and surface redshift

In our proposed charged model, the gravitational effective mass within the radius \( r' \) can be obtained from the following formula [64]

\[
m_{\text{eff}} = 4\pi \int_0^{r'} \rho_{\text{eff}}(\tilde{r}) \tilde{r}^2 d\tilde{r} + \frac{q^2}{2r} + \frac{1}{2} \int_0^{r'} \frac{q(\tilde{r})^2}{\tilde{r}^2} d\tilde{r},
\]

\[
= m + \frac{\gamma}{2} \int_0^{r'} (\rho - p_r - 2p_t)(\tilde{r}) \tilde{r}^2 d\tilde{r} + \frac{q^2}{2r} + \frac{1}{2} \int_0^{r'} \frac{q(\tilde{r})^2}{\tilde{r}^2} d\tilde{r}.
\] (65)

Where \( m = 4\pi \int_0^{r} \rho(\tilde{r})\tilde{r}^2 d\tilde{r}, \) from Eq. (65), it is clear that for \( \gamma = 0, \) both \( m_{\text{eff}} \) and \( m \) coincide. However, by performing the above integration the effective mass function inside the radius \( r' \) of the charged fluid sphere can be obtained as,

\[
m_{\text{eff}} = \frac{r}{2} \left[ 1 - e^{-Ar^2} - \frac{e^{-Ar^2}}{(1 + \alpha)(3\gamma + 4\pi)} \left[ 2e^{Ar^2} \gamma^2 r^2 g_2(r) \right. \right.
\]

\[
- \gamma \left\{ 2\alpha(-1 + e^{Ar^2}) + (4A\alpha + B + 3\alpha B)r^2 
\right. \]

\[
- 12e^{Ar^2} \pi r^2 g_2(r) + (1 + \alpha)B(-A + B)r^4 + 2(-1 + e^{Ar^2} + Ar^2) \right\}
\]

\[
+ 4\pi \left\{ 1 + (-A + B)r^2 
\right. \]

\[
- \alpha(-1 + e^{Ar^2} + 2Ar^2) + e^{Ar^2}(-1 + 4\pi r^2 g_2(r)) \right]\].
\] (66)

The effective mass function is regular at the center as \( m_{\text{eff}} \to 0 \) as \( r \to 0. \) The profile of the effective mass function is plotted in Fig. 11.

The compactification factor inside the radius \( r \) for our present model is obtained as,

\[
u_{\text{eff}} = \frac{m_{\text{eff}}}{r}.
\] (67)

As addressed by Giuliani and Rothman [65], the problem of finding a lower bound on the radius \( R \) of a charged sphere with mass \( M \) and total charge \( Q \) is given by \( Q < M, \) and in
this case, collapse always takes place at a critical radius $R_c$ outside the outer horizon, and as $Q \to M$, this value approaches the horizon. The upper bound of the mass of charged sphere was generalized by Andréasson \cite{66} as

$$\sqrt{M} \leq \sqrt{\frac{R}{3}} + \sqrt{\frac{R}{9}} + \frac{Q^2}{3R}, \quad (68)$$

by assuming the inequality $\rho - p_r - 2p_t \geq 0$. Equation (68) equivalently gives,

$$\frac{M}{R} \leq \left(\frac{1}{3} + \sqrt{\frac{1}{9} + \frac{Q^2}{3R^2}}\right)^2, \quad (69)$$

One can easily check that Eq. (69) obeys the Buchdahl’s limit \cite{67} $\frac{2M}{R} < \frac{8}{9}$ for uncharged case.

On the other hand, Böhmer and Harko \cite{68} proposed the lower bound of mass to the radius for charged fluid sphere as,

$$\frac{3Q^2}{4R^2} \frac{1 + \frac{Q^2}{18R^2}}{1 + \frac{Q^2}{12R^2}} \leq \frac{M}{R}, \quad (70)$$

and combining (69) and (70), we get,

$$\frac{3Q^2}{4R^2} \frac{1 + \frac{Q^2}{18R^2}}{1 + \frac{Q^2}{12R^2}} \leq \frac{M}{R} \leq \left(\frac{1}{3} + \sqrt{\frac{1}{9} + \frac{Q^2}{3R^2}}\right)^2. \quad (71)$$

In Eq. (71), $R$ represents the radius of the fluid distribution, and $m_{\text{eff}}(r = R) = M$, $q(r = R) = Q$, $M$ and $Q$ are, respectively, the gravitational mass and total charge inside the fluid sphere.

From the above table, we see that the inequality given in Eq. 71 is satisfied by our present model for different values of $\gamma$ (Fig. 12).
7 Discussion

In the present work, in the context of modified theory of gravity, we have obtained a new model of charged compact star. To explore the model, we have considered the functional forms of $f(R, T)$ as $f(R, T) = R + 2\gamma T$, where $R$ and $T$ are, respectively, the Ricci scalar and the trace of energy–momentum tensor $T_{\mu\nu}$, respectively. For our present analysis, we have chosen $\gamma$ as a small positive constant since for negative values of $\gamma$ produces negative value of the electric field $E^2$, we have exclude this case by considering the physical acceptability of the present model. The model has been developed by taking Krori–Barua (KB) ansatz since it is well known that KB metric produces a singularity free model. All numerical calculations and plots have been done for the strange star candidate 4U 1538-52 whose observed mass and radius are given by $(0.87 \pm 0.07) M_\odot$ and $7.866 \pm 0.21$ km, respectively.

To obtain the result in closed form, instead of choosing ad hoc expression for electric field $E^2$, we have chosen nonlinear equation of state as Chaplygin form: $p_r = \alpha \rho - \frac{\beta}{\rho}$. Here, $\alpha$ and $\beta$ are small positive constants. From our analysis, we have shown that the values of $\alpha$ and $\beta$ depend on the coupling constant $\gamma$ and their numerical values have been obtained for the compact star 4U 1538-52 in Table 1. It is clear from the table that both $\alpha$ and $\beta$ decrease if $\gamma$ increases and it is to be noted that the effect of $\beta$ is very small to the model compared to alpha. The variation of $\beta$ with respect to $\alpha$ is shown in Fig. 13. We have also obtained the numerical values of the central density, surface density and central pressure in the order of $10^{15}$ gm cm$^{-3}$, $10^{14}$ gm cm$^{-3}$ and $10^{34}$ dyne cm$^{-2}$, respectively. The numerical values of the central density, surface density and central pressure all decrease with the increasing value of $\gamma$. On the contrary, the numerical values of the effective mass and surface redshift increase as $\gamma$ increases. The present model is potentially stable, and the causality condition is satisfied. All the energy conditions are verified for our model with the help of graphical representation. The effect of coupling parameter $\gamma$ on the different physical parameters like density, pressure, anisotropic factor, sound velocity, compactness factor, mass function has been widely discussed.

| $\gamma$ | Value of lower limit of Eq. (71) | $u^{\text{eff}}(R)$ | Value of upper limit of Eq. (71) |
|----------|---------------------------------|---------------------|---------------------------------|
| 0.0      | 0.00749792                      | 0.164519            | 0.666717                        |
| 0.03     | 0.0260527                       | 0.165074            | 0.667271                        |
| 0.06     | 0.035939                        | 0.16562             | 0.667816                        |
| 0.09     | 0.0435352                       | 0.166159            | 0.668353                        |
| 0.12     | 0.0499014                       | 0.166691            | 0.668881                        |

Fig. 12 The variation of $\rho - p_r - 2p_t$ with respect to radius is shown for different values of $\gamma$ for the compact star 4U1538-52
The variation of $\beta$ with respect to $\alpha$ has been depicted.

The surface stress energy ($\sigma$) and surface pressure ($P$) for our present model are obtained as,

$$\sigma = -\frac{1}{4\pi R} \left[ \sqrt{1 - \frac{2M}{R} + \frac{Q^2}{R^2}} - e^{-\frac{aR^2}{2}} \right],$$

$$P = \frac{1}{8\pi R} \left[ \frac{1 - \frac{M}{R}}{\sqrt{1 - \frac{2M}{R} + \frac{Q^2}{R^2}}} - (1 + BR^2) e^{-\frac{aR^2}{2}} \right].$$

From our entire analysis, we can conclude that the present model of compact star, more or less, behaves like GR.

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