Magnon-photon strong coupling for tunable microwave circulators

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We present a generic theoretical framework to describe non-reciprocal microwave circulation in a multimode cavity magnonic system and assess the optimal performance of practical circulator devices. We show that high isolation (> 56 dB), extremely low insertion loss (< 0.05 dB), and flexible bandwidth control can be potentially realized in high-quality-factor superconducting cavity based magnonic platforms. These circulation characteristics are analyzed with materials of different spin densities. For high-spin-density materials such as yttrium iron garnet, strong coupling operation regime can be harnessed to obtain a broader circulation bandwidth. We also provide practical design principles for a highly integratable low-spin-density material (vanadium tetracyanoethylene) for narrow-band circulator operation, which could benefit noise-sensitive quantum microwave measurements. This theory can be extended to other coupled systems and provide design guidelines for achieving tunable microwave non-reciprocity for both classical and quantum applications.

I. INTRODUCTION

Non-reciprocal microwave devices are ubiquitous and important in the classical and quantum information processing, as they protect delicate measurements from reflected signals [1]. The non-reciprocal effect arises from broken time-reversal symmetry, traditionally realized with ferrite materials [2–4]. Recently, a variety of avenues have been reported to realize non-reciprocity without the use of magnetic materials, including optomechanical coupling [5, 6], reservoir engineering [7, 8], non-linear effect [9, 10], and temporal modulation [11]. Those approaches, albeit being non-magnetic, typically require strict phase matching condition and have limited tunability [5–7, 9, 10], with added complexity in experimental implementations. Nowadays, due to the high demand in sensitive microwave signal detections, especially at single-photon level for superconducting quantum circuits, low-loss, tunable, and compact electromagnetic circulator devices are of great interest [1].

Cavity magnonic systems have attracted significant attentions recently [12–23] due to the strong interaction between magnon excitations and microwave photons. Previous studies have demonstrated magnon-photon strong coupling in various resonant microwave systems, such as copper 3-dimensional (3D) cavities [14–18] and coplanar microwave circuits [19, 20, 24]. However, those cavity magnonic systems are in the conventional coherent coupling configuration, where magnons are coupled with a single microwave mode without any non-reciprocal effect. Only a few recent works have investigated non-reciprocal coherent or dissipative magnon-photon coupling in two-port systems to demonstrate isolator devices [25, 26].

In this work, we present a generic theoretical model for non-reciprocal multimode cavity magnonic systems. We show that by harnessing the selective coupling between the magnon mode and microwave modes with different chiralities, as well as the interference effect between different paths in a three-port system, non-reciprocal microwave circulation with high isolation, low insertion loss, and flexible controllability can be achieved. For device implementation, we propose a practical design based on a high-quality factor (Q) superconducting ring resonator which is coupled with a high-Q magnon mode in a low-Gilbert-damping magnetic media [12, 27] under a bias magnetic field. Two exemplary material platforms are discussed: (1) yttrium iron garnet (YIG), a high-spin-density material that can work in the strong coupling regime to obtain broader circulation bandwidth [28], and (2) vanadium tetracyanoethylene (V[TCNE]2), a highly integratable low-spin-density material for narrow-bandwidth operation [29]. Unlike commercial circulators designed for octave broadband operations, this work exploits cavity enhanced circulation effect and trade the circulation bandwidth for high isolation and low insertion loss, which are the most desirable performance parameters for delicate single-photon level quantum measurements.

II. THEORETICAL MODEL

A. Coupled-Mode Theory

A schematic of the circulator is shown in Fig. 1(a), where a three-port superconducting ring resonator simultaneously supports two degenerate counter-rotating microwave modes. This ring resonator is aligned with a ferrimagnetic disk of similar dimension for optimal mode overlap. Under a static out-of-plane magnetic bias field, the ferrimagnetic disk supports a uniform magnon mode with the resonant frequency linearly proportional to the external field [30], \( \omega_m \approx \gamma \vec{B}_o \), where \( \gamma \approx 28 \text{ GHz/T} \) is the gyromagnetic ratio. The system Hamiltonian can be written as

\[
H = \hbar \omega_{ccw} a_{ccw}^\dagger a_{ccw} + \hbar \omega_{cw} a_{cw}^\dagger a_{cw} + \hbar \omega_m m^\dagger m + H_{int}/\hbar.
\]

(1)

Here, \( a_{ccw} (a_{cw}^\dagger), a_{cw} (a_{ccw}^\dagger), m (m^\dagger) \) are the annihilation (creation) operators for the counter clockwise
(CCW) and the clockwise (CW) microwave mode, and the magnon mode, respectively, with their resonant frequencies denoted as $\omega_{ccw}$, $\omega_{cw}$, and $\omega_m$. Since the CCW and the CW modes are orthogonal, we only need to consider their linear coupling with the magnon mode in our system. So the interaction Hamiltonian is

$$H_{int}/\hbar = -g_{ccw}\left(a_{ccw} + a_{ccw}^\dagger\right)(m + m^\dagger) - g_{cw}\left(a_{cw} + a_{cw}^\dagger\right)(m + m^\dagger),$$

where $g_{ccw}$ and $g_{cw}$ are the coupling strengths between the respective microwave mode and magnon mode.

Under the rotating wave approximation (RWA), the Heisenberg-Langevin equation can be written as

$$\dot{a} = M_1a + Ks_{in},$$

with the input-output relation

$$s_{out} = Cs_{in} + M_2a.$$  \hspace{1cm} (4)

Here $a = \{a_{ccw}, a_{cw}, m\}^T$ is the vector of the cavity field. $s_{in} = \{s_{in1}, s_{in2}, s_{in3}\}^T$ and $s_{out} = \{s_{out1}, s_{out2}, s_{out3}\}^T$ are the input and the output fields at the three ports. Matrix $M_1(3 \times 3)$ is given as

$$M_1 = \begin{pmatrix} -i\omega_{ccw} - \frac{\kappa_{ccw}}{2} & 0 & ig_{ccw} \\ 0 & -i\omega_{cw} - \frac{\kappa_{cw}}{2} & ig_{cw} \\ ig_{ccw} & ig_{cw} & -i\omega_m - \frac{\kappa_m}{2} \end{pmatrix},$$

in which $\kappa_{ccw}$, $\kappa_{cw}$, and $\kappa_m$ are the total dissipation rates for the microwave and magnon modes, respectively. $K$ and $M_2$ are the $(3 \times 3)$ matrices describing coupling of three incoming/outgoing waves with two resonant modes. Based on relation among $K$, $C$, and $M_2$ (see Appendix), the matrix $K(3 \times 3)$ can be in general written as $K = -M_2^{-1}C = \left(\begin{array}{ccc} \sqrt{\kappa_{ccw,1}} & \sqrt{\kappa_{ccw,2}}e^{i\beta} & \sqrt{\kappa_{ccw,3}}e^{i\gamma} \\ \sqrt{\kappa_{cw,1}}e^{i\beta} & \sqrt{\kappa_{cw,2}}e^{i(\alpha + \beta + \pi)} & \sqrt{\kappa_{cw,3}}e^{i(\gamma - \beta + \pi)} \\ 0 & 0 & 0 \end{array}\right).$ \hspace{1cm} (5)

Here $\alpha(\gamma)$ is the relative phase between the excitation port 2(3) and port 1 for mode $a_{cw}$, $\beta$ is the relative phase between the CCW and the CW modes when they are excited by port 1. $\kappa_{ccw,1(2,3)}$ and $\kappa_{cw,1(2,3)}$ represent the external coupling rates of the two microwave modes to the three input/output ports, respectively.

$C$ is the $(3 \times 3)$ matrix describing direct coupling of incoming and outgoing waves. Due to energy conservation, $C$ must be unitary $C^\dagger C = I$. But the specific expression for $C$ depends on the physical implementation of the excitation ports. For example, in the case of a waveguide end-coupling scheme with negligible crosstalk between ports, $C = I$ for open-ended (capacitive) coupling; and $C = -I$ for short-ended (inductive) coupling.

In this paper, we assume that the waveguides and the ring are inductively coupled with no dissipative coupling across different ports, namely,

$$C = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$ \hspace{1cm} (7)

With the expressions of matrices in Eq. 3 & 4, the equation of motion can be solved in frequency domain to obtain the scattering matrix (defined by $s_{out}[\omega] = S[\omega]s_{in}[\omega]$)

$$S[\omega] = C + M_2[-i\omega I - M_1]^{-1}K.$$ \hspace{1cm} (8)

### B. Circulation Under Three-fold Symmetry

Based on the generic coupled mode theory described above, we now focus our discussion on the system circu-
Finally, using Eq. 8, we can obtain the scattering matrix $S$ modes, only

chirality. Therefore, for our two circular microwave

would only couple with microwave mode with the same

total external coupling rates, respectively. At the same
time, we have the relative excitation phase difference be-

between port 2 and 1 as $\alpha = -\gamma = -2\pi/3$, where $n$ is the

integer represents the mode number. Here, we focus on

the two degenerate fundamental circulation microwave

modes ($n = 1$), for which we have $\omega_{ccw} = \omega_{cw}$ and $\alpha =

-\gamma = -2\pi/3$. Since these two modes are orthogonal to each

other, it can be shown that the relative excitation

phase $\beta$ between $a_{ccw}$ and $a_{cw}$ does not contribute to the

final expression of the scattering matrix. Hence, we can

set $\beta = 0$ for simplicity.

Due to the selective coupling rule, the magnon mode

would only couple with microwave mode with the same

chirality. Therefore, for our two circular microwave

modes, only $g_{ccw} = g$ is significant and $g_{cw} \approx 0$. It’s

worth pointing out that a different eigenmode basis can

be chosen for the two degenerate microwave modes by

applying a unitary rotation $U(\theta) =

\begin{pmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{pmatrix}.$

Then the new $M_1$ matrix will become the general form in

Eq. 5 with $g_{ccw} = g \sin \theta$ and $g_{cw} = -g \cos \theta$. This rotation

of the basis, nevertheless, won’t change the physical

results of our analysis.

Under the three-fold geometrical symmetry, the coupling matrix $K$ is given by

$$K = \sqrt{\frac{\kappa_e}{3}}
\begin{pmatrix}
e^{-\frac{i\pi}{3}} & e^{\frac{i\pi}{3}} & 0 \\
e^{-\frac{2i\pi}{3}} & e^{\frac{2i\pi}{3}} & 0 \\
0 & 0 & 0
\end{pmatrix}.$$  (9)

Finally, using Eq. 8, we can obtain the scattering matrix elements

$$S_{11} =
S_{22} = S_{33} = -1 + \frac{\kappa_e}{3(\kappa_{cw}/2 - i(\omega - \omega_{cw}))}
+ \frac{\kappa_e(\kappa_m/2 - i(\omega - \omega_m))}{3(g^2 + (\kappa_m/2 - i(\omega - \omega_m))(\kappa_{cw}/2 - i(\omega - \omega_{cw})))}$$  (10)

$$S_{12} =
S_{23} = S_{31} = \frac{e^{-\frac{2i\pi}{3}} \kappa_e}{3(\kappa_{cw}/2 - i(\omega - \omega_{cw}))}
+ \frac{e^{-\frac{2i\pi}{3}} \kappa_e(\kappa_m/2 - i(\omega - \omega_m))}{3(g^2 + (\kappa_m/2 - i(\omega - \omega_m))(\kappa_{cw}/2 - i(\omega - \omega_{cw})))}$$  (11)

$$S_{21} =
S_{13} = S_{32} = \frac{e^{-\frac{2i\pi}{3}} \kappa_e}{3(\kappa_{cw}/2 - i(\omega - \omega_{cw}))}
+ \frac{e^{-\frac{2i\pi}{3}} \kappa_e(\kappa_m/2 - i(\omega - \omega_m))}{3(g^2 + (\kappa_m/2 - i(\omega - \omega_m))(\kappa_{cw}/2 - i(\omega - \omega_{cw})))}$$  (12)

We see that the scattering matrix satisfies relations $S_{11} = S_{22} = S_{33}, S_{12} = S_{23} = S_{31},$ and $S_{21} = S_{13} = S_{32},$ as expected from the rotational symmetry. Without the

three-fold symmetry, for example, if the microwave ex-
ternal coupling rate for three waveguides are different,

the reflection coefficients could be in general different

$S_{11} \neq S_{22} \neq S_{33}$. Such degeneracy will also be broken

for other transmission parameters. Next, we can see from

Eq. (10, 11, 12), if the microwave CCW and CW modes are non-degenerate in the frequency domain, the system has the non-reciprocal signal transmission with-

out the coupling to the magnon mode. This is because

once the signal propagation direction is reversed, for the

same propagation path, the CCW and CW modes effect-

ively swap, leading to different phase interference. The

origin of this non-reciprocity is the different phase shift

between mode $a_{ccw}$ and $a_{cw}$ at the output ports. Thus,

although the microwave CCW and CW mode are degene-

rate due to the structural symmetry, selective coupling

between magnon mode and the two rotational microwave

modes gives rise to a non-reciprocal scattering matrix. If

there is no magnon-photon coupling ($g = 0$), $S_{12}$ and $S_{21}$

do not have the amplitude non-reciprocity. In the situation

when ($g \neq 0$), the signal circulation starts to occur

due to magnon-coupling-induced interference, the isolation

ratio between the scattering matrix parameters $S_{12}$

and $S_{21}$ depends on both $g$ and the magnon resonance
detuning ($\omega_{ccw/cw} - \omega_m$).

III. MATERIAL PLATFORMS

In this section, we are interested in implementing circu-
lators via both high spin density and low-spin density

ferrimagnetic materials. To achieve circulators with low

insertion loss and high isolation, an ideal ferrimagnetic

material should have the low Gilbert damping factor $\alpha$.

A wide exploited high density ferrimagnetic materials is

single crystal ferrimagnetic material yttrium iron gar-

net (YIG, $\alpha_{\text{YIG}} \sim 3 \times 10^{-5}$) [27, 31–35] whereas a par-

ticularly interesting low density ferrimagnetic material

is organic-based ferrimagnet vanadium tetracyanoethy-

lene thin films ($V[\text{TCNE}]_2, \alpha_{[\text{VTCNE}]_2} \sim 3.8 \times 10^{-5}$)

[29, 36, 37]. The relevant material parameters of bulk

and thin-film YIG as well as $V[\text{TCNE}]_2$ are listed in table

I [26, 31, 36].
A. YIG-based Circulator

Yttrium iron garnet (YIG) is widely used as the magnon media for its excellent magnetic properties such as long spin lifetime and wide tunability. YIG has a relatively large saturation magnetization ($4\pi M_s = 1750$ G [38]), which is about 20 times higher than that of V[TCNE]$_2$, thus can be regarded as a high-spin-density material [27, 29, 31–37]. Recently, YIG thin films and spheres have been used to study the coherent coupling between magnons and microwave photons. The reported coupling strengths range from several megahertz to a few gigahertz by engineering the mode overlap factor, mode volume and resonant frequencies [14–20, 24]. Both YIG thin film and YIG bulk can be promising candidates for building high performance circulators, due to their low Gilbert damping factors. The key difference is that, for YIG thin film, when magnetized in the out-of-plane direction, the bias field needs to overcome the demagnetization field ($\sim 1750$ Oe) to fully saturate and effectively excite the magnon resonances ($\omega_m = \gamma |\vec{B}_0 - \vec{H}_d|$). On the other hand, the demagnetization field for the bulk is only around tens of oersted. Thus, compared with bulk, using YIG thin-film as the magnon media may introduce extra flux in the superconducting microwave resonator [39, 40].

In this session, we show the use of YIG bulk disk as the magnon media to achieve microwave circulation under the condition where magnon and photon are strongly coupled ($g > \kappa_{ccw}, \kappa_{cw}, \kappa_m$), and the superconducting microwave ring resonator is over-coupled ($\kappa_e > \kappa_s$). In this scenario, the magnon resonance can be detuned from the microwave resonance into the dispersive coupling regime to reduce loss induced by ferrimagnetic damping. The inherent tunability of the magnon resonance offers extra functionalities for this system to achieve adjustable isolation ratio and switchable signal propagation directions.

To be compatible with the relatively strong bias magnetic field, superconductors with high critical field such as NbTi can be used for the microwave circulator fabrication [41]. The proposed device should be designed to maintain the three-fold structural symmetry with three identical waveguides inductively coupled with ring resonator at the $2\pi/3$ angle difference. The resonant frequencies of CCW and CW mode are determined by the superconducting resonator geometry, and are degenerate ($\omega_{ccw} = \omega_{cw}$) in absence of the magnon media. The intrinsic microwave loss $\kappa_i/2\pi$ for both microwave modes can be estimated based on the previous literature to be around several megahertz [42, 43], when the static magnetic field is much lower than the NbTi critical field. The total external coupling rate for the ring resonator can be adjusted by changing the impedance of the input waveguides. Here, based on previous magnon-photon coupling studies within microwave coplanar resonators [19, 25], the external coupling rate $\kappa_e/2\pi$ can be engineered from tens of megahertz to several gigahertz. To explore both magnon-photon strong and weak coupling regimes, we will set $\kappa_e/2\pi$ to a moderate value (600 MHz), which can be achieved experimentally via the impedance design. Given the system’s structural symmetry, the external coupling rate should be identical for both CCW and CW modes.

Figure 2(a) and (b) show a mapping of the transmission $|S_{21}|^2$ and $|S_{12}|^2$ respectively as a function of $\Delta_m$ and $\Delta_c$, under strongly coupled conditions with $\kappa_e/2\pi = 600$ MHz, $2g/2\pi = 1,200$ MHz. The mode splitting at the on-resonance condition is around $2g$. Under different detunings $\Delta_m/2\pi = \pm 0.87$ GHz, the transmission spectra are plotted in (c)/(d) when YIG is biased with magnetic resonance lower/higher than the microwave resonance.

| Name         | $4\pi M_s$ | $\Delta f @ \sim 10$ GHz | Thickness |
|--------------|------------|--------------------------|-----------|
| YIG bulk     | 1750 G     | 3 MHz                    | $\sim$1 mm|
| YIG thin film| 1750 G     | 3 MHz                    | 1-5 $\mu$m|
| V[TCNE]$_2$  | 90 G       | 2 MHz                    | 1-5 $\mu$m|

Table I. Material parameters for YIG and V[TCNE]$_2$. Figure 2. (a) & (b) are the mapping of the transmission $|S_{21}|^2$ and $|S_{12}|^2$ respectively as a function of $\Delta_m$ and $\Delta_c$, under strongly coupled conditions with $\kappa_e/2\pi = 600$ MHz, $2g/2\pi = 1,200$ MHz. The mode splitting at the on-resonance condition is around $2g$. Under different detunings $\Delta_m/2\pi = \pm 0.87$ GHz, the transmission spectra are plotted in (c)/(d) when YIG is biased with magnetic resonance lower/higher than the microwave resonance.
sion spectra indicate the strong coupling between the microwave photon and the magnon, with clear asymmetric transmission under positive and negative magnon resonance detuning $\Delta_m$. As we can see in the Fig. 2 (c) & (d), $|S_{21}(-\Delta_m)|^2$ equals $|S_{12}(+|\Delta_m)|^2$. At the same time, the optimal circulation with minimal insertion loss and large isolation ratio ($|\text{iso.}| = 20\log_{10}|S_{21}|/|S_{12}|$) can be achieved by optimizing the magnon resonance detuning $\Delta_m$. As shown in Fig. 2 (c) & (d), when $\Delta_m = \pm 0.87 \text{ GHz}$, the insertion loss ($|\text{IL}|$) can be as low as $0.05 \text{ dB}$, with the isolation ratio reaching $56 \text{ dB}$. Noticeably, the directionality of the signal propagation can be easily switched by changing $\Delta_m$ without reorienting the external magnetic field. When the magnon resonance is optimized, the maximal isolation ratio can be achieved in a system is limited by intrinsic losses $\kappa_i$ and $\kappa_m$. If the system dissipation losses can be optimized to $\kappa_i/2\pi = \kappa_m/2\pi = 0.5 \text{ MHz}$ by the fabrication process optimization and/or operating at ultra-low temperatures, the isolation ratio can be further enhanced to $63 \text{ dB}$, with insertion loss being reduced to $0.02 \text{ dB}$.

Next, we study the controllability of the coupled system based on other tuning parameters. Figure 3(a) shows a mapping of 20 dB isolation bandwidth as a function of the magnon-photon coupling strength $g$, and the microwave total dissipation rate $\kappa = \kappa_e + \kappa_i$. The degenerate microwave resonant frequency is $10 \text{ GHz}$ for this calculation. The contour lines delineate $20 \text{ MHz}$, $50 \text{ MHz}$, $100 \text{ MHz}$ and $200 \text{ MHz}$ circulation bandwidth when $|\text{iso.}| = 20 \text{ dB}$, respectively. The parameter spaces can be divided into four regimes. Region I represents weak coupling regime ($g \ll \kappa$) where the coupling strength is too small to induce significant phase modulation at output ports. Similarly, in region IV, at the ultra strong coupling limit when $g \gg \kappa$, the weak signal circulation can be understood as the CCW and CW microwave mode splitting in the frequency domain is much larger than its linewidth, due to the strong coupling with the magnon mode. Thus, the CCW and CW modes have minimal effective overlap in the frequency domain to enable the effective circulation. In region III, when the system is near/at the strong coupling regime $g \sim \kappa$, the $20 \text{ dB}$ isolation bandwidth increases linearly with $g$ and $\kappa$. The optimal circulation happens when the magnon mode is detuned away from microwave modes. Lastly, in region II, when magnon and photon are weakly coupled $g < \kappa$, the isolation bandwidth also yields a linear relation as $g$ and $\kappa$ are increasing linearly. Compared with region III when the system is near strongly coupled, in region II, the magnon mode is tuned close to the microwave mode to achieve strong enough phase modulation for broad band circulation. These properties show that for different applications, various isolation bandwidths can be engineered by tuning magnon resonant frequency, magnon-photon coupling strength, and microwave total dissipation rate.

Another critical performance parameter of a circulator is the insertion loss ($|\text{IL}|$). In Fig. 3(b), we study the contribution of the microwave intrinsic loss to the insertion loss at different microwave external coupling rates with the magnon-photon coupling rate fixed at $g/2\pi = 1,200 \text{ MHz}$. As $\kappa_e/2\pi$ is varied from 300 to $1,200 \text{ MHz}$, with the increase of $\kappa_i$, the $|\text{IL}|$ increases correspondingly. Thus, the high $Q$ superconducting microwave resonator is ideal for achieving ultra low loss circulation. Fig. 3(b) also indicates that at same microwave intrinsic loss rate $\kappa_i$, the insertion loss can be reduced by increasing the microwave external coupling rate $\kappa_e$. This is because that when $\kappa_e$ is dominating in the total microwave dissipation rate, less microwave signal in the resonator dissipates into the intrinsic loss channel, thus, resulting into the low insertion loss. The analyses above establish that many desirable features of a circulator — high isolation, low insertion loss and tunable bandwidth — can be achieved simultaneously.
**B. V[TCNE]₂-based Circulator**

In this session, we focus on microwave circulation based on the low-spin density ferrimagnetic material V[TCNE]₂, which is an organic-based high quality ferrimagnetic semiconductor \((E_g = 0.5\text{ eV}, \sigma = 0.01\text{ S/m})\) exhibiting room temperature magnetic ordering \((T_c > 600\text{ K})\) \([29, 44, 45]\). V[TCNE]₂ has very low Gilbert damping factor on the similar level as single crystal YIG for both continuous and micro-patterned films \([37]\). Particularly, this material can be integrated onto various substrates by chemical vapor deposition while maintaining excellent magnetic properties. Considering high quality YIG can only be grown on lattice-matched substrates, V[TCNE]₂ can be an alternative solution for highly compact integrated magnonic devices. The saturation magnetization of V[TCNE]₂ (90 G) is over an order of magnitude smaller compared with that of YIG (1750 G), leading to significantly reduced bias magnetic field which is desired for many applications \([40, 46]\). On the other hand, the lower material spin density weakens the magnon-photon coupling strength \(g\) under the same microwave circuit design and magnon mode volume, making it difficult to reach strong coupling.

Here, we discuss the microwave circulation when the system is weakly coupled \((k_m < g < \kappa)\). For low spin density material, we assign the magnon-photon coupling strength \(g/2\pi\) to be 100 MHz, with microwave dissipation rates \(\kappa_i/2\pi = 2\text{ MHz}, \kappa_e/2\pi = 600\text{ MHz}\), and magnon resonant linewidth \(\kappa_m/2\pi = 2\text{ MHz}\). The mapping of the transmission scattering parameters is shown in Fig. 4(a) & (b). Similar directional transmission between \(|S_{12}|^2\) and \(|S_{21}|^2\) is observed, with a Lorentzian-shaped transparency window that shows the magnon resonance. A line cut of the transmission map at a fixed bias field, indicated by the white dash line in Fig. 4(a) & (b), is reproduced in 4(c) as a function of the excitation frequency. The low insertion loss \((0.09\text{ dB})\) and high isolation ratio \((77\text{ dB})\) can be achieved with optimized detuning \((\Delta_m = 0.16\text{ GHz})\). Due to the small magnon-photon coupling strength, the 20 dB isolation bandwidth is around 0.5 MHz, much narrower compared to that of YIG. This narrow-bandwidth circulator nevertheless has promising applications in circuit QED systems where it can serve as the filtering function for multiplexed superconducting qubits and resonators \([47, 48]\).

Also, in the under-coupled scenario, the magnon resonance needs to be tuned close to the microwave resonance, so the overall insertion loss is more sensitive to the magnon linewidth \(\kappa_m\) than the strongly coupled system. By operating at cryogenic temperatures with reduced \(\kappa_m/2\pi = 0.5\text{ MHz}\), the \(|IL|\) can be further decreased to 0.04 dB, while maintaining \(|ISO| > 50\text{ dB}|\).

Recently, a technique for micro-patterning of V[TCNE]₂ thin films has been developed for creating high fidelity lithographically defined structures without observable deterioration of magnetic properties \([37]\). With further improvement of the magnon-microwave mode overlap and reduction of magnon mode volume, higher magnon-photon coupling strength can be realized even with low spin density V[TCNE]₂ system, giving rise to a broader operating bandwidth.

![Figure 4](image)

**Figure 4.** (a) & (b) are the mapping of the transmission \(|S_{21}|^2\) and \(|S_{12}|^2\) respectively as a function of \(\Delta_m\) and \(\Delta_c\). The V[TCNE]₂ - based circulator is weakly coupled with \(g/2\pi = 100\text{ MHz}\) and \(\kappa_e/2\pi = 600\text{ MHz}\). Figure (c) plots the full scattering parameters, showing low \(|IL|\) and high \(|ISO|\) being achieved simultaneously.

**IV. CONCLUSION**

In summary, we have theoretically investigated a high-performance microwave signal circulation based on a multiport coupled magnon-photon system. The non-reciprocity arises from the interference between the two counter propagating microwave modes, introduced by the chirality-dependent coupling with the magnon excitation. The device implementations are analysed using practical parameters of the superconducting microwave circuit and low Gilbert damping factor materials (YIG & V[TCNE]₂). High performance microwave circulation with low insertion loss \(< 0.05\text{ dB}\) and high isolation ratio \((> 56\text{ dB})\) can be achieved with both high/low spin density materials. Although the obtainable isolation bandwidth in low spin density material (V[TCNE]₂)
is narrower than high spin density platform (YIG), it is beneficial for applications that desire frequency selective isolation such as superconducting quantum computing systems. Additional advantages of this magnon-photon system include great tunability and directional switchability. The proposed theory is general and can be applied to study multimode induced non-reciprocity in other hybrid systems, such as the quantum optical circulators based on chiral atom-light coupling. [49, 50].

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Appendix A

1. Proof of $M_2^\dagger M_2 = \Gamma_e$

We denote the external and intrinsic dissipation matrices for the microwave signal as $\Gamma_e = \begin{pmatrix} \kappa_{ccw,e} & 0 & 0 \\ 0 & \kappa_{cw,e} & 0 \\ 0 & 0 & 0 \end{pmatrix}$, and $\Gamma_i = \begin{pmatrix} \kappa_{ccw,i} & 0 & 0 \\ 0 & \kappa_{cw,i} & 0 \\ 0 & 0 & \kappa_m \end{pmatrix}$. Consider a special case with no incident wave ($s_{in} = 0$) and no intrinsic loss ($\Gamma_i = 0$). Due to energy conservation, the decay of the intra-cavity energy should equal to the output power $\frac{d(a^\dagger a)}{dt} = s_{out}^\dagger s_{out}$. From Eq. 3, we have

$$\frac{d(a^\dagger a)}{dt} = \frac{da^\dagger}{dt} a + a^\dagger \frac{da}{dt} = a^\dagger \left(M_1^\dagger + M_1\right) a = -a^\dagger \Gamma_e a.$$  (A1)

On the other hand, the output power is

$$s_{out}^\dagger s_{out} = a^\dagger M_2^\dagger M_2 a.$$  (A2)

Comparing Eq. A1 & Eq. A2 above, we get

$$M_2^\dagger M_2 = \Gamma_e.$$  (A3)

2. Proof of $-C^\dagger M_2 = K^\dagger$

From energy conservation, we can predict that $s_{in}^\dagger s_{in} - s_{out}^\dagger s_{out} = \frac{d(a^\dagger a)}{dt} + a^\dagger \Gamma_i a$. Combining Eq. 3 Eq. 4 and Eq. A3, we can have

$$s_{in}^\dagger C^\dagger M_2 a + a^\dagger M_2^\dagger C s_{in} = -s_{in}^\dagger K^\dagger a - a^\dagger K s_{in}. $$  (A4)

Therefore, we can get $-C^\dagger M_2 = K^\dagger$.

According to the relation among $K$, $C$, and $M_2$, we can drive the expression of matrix $K$ and shown in Eq. 6, and the full scattering matrix can be calculated from Eq. 8.

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