Scattering of Dirac and Majorana Fermions off Domain Walls

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We investigate the interaction of fermions having both Dirac and left-handed and right-handed Majorana mass terms with vacuum domain walls. By solving the equations of motion in thin-wall approximation, we calculate the reflection and transmission coefficients for the scattering of fermions off walls.

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I. INTRODUCTION

Theoretical arguments [1] and a recent analysis of WMAP data [2] show that the presence of a network of low-tension domain walls in the Universe is not ruled out. Moreover, domain walls could provide a natural and non-exotic alternative to the most popular candidates of dark energy [3]. The evolution of vacuum domain walls in the early Universe is determined by their interaction with the surrounding plasma. The two most relevant effects to be considered are the particle scattering off walls, and the presence of bound states near the walls, the so-called “zero modes”.

The scattering of particles off walls (including the scattering between walls) determines the average velocity $v$ of a wall and thus, in turn, the equation of state of a gas of domain walls, $p_w = (v^2 - 2/3)\rho_w$, where $\rho_w$ and $p_w$ are the energy density and pressure of the gas (see, e.g., [4]). Indeed, when particles scatter off a wall, they generate a frictional force $F = \sum_i n_i R_i \Delta p_i$, where $n_i$ is the number density of particles of species $i$, $R_i$ is their scattering probability (the reflection coefficient), and $\Delta p_i$ is the momentum transfer per collision (see, e.g., [5]). Hence, defining the mean velocity of the walls as $v = \sum_i n_i R_i v_i / \sum_i n_i R_i$, the damping force can be written as $F = \mu v$, where we have defined the frictional coefficient $\mu = (\sum_i n_i R_i \Delta p_i)(\sum_i n_i v_i) / \sum_i n_i R_i$. The mean velocity of a wall is determined by balancing the tension, $f \sim \sigma / r$, where $\sigma$ and $r$ are the surface energy density and the mean curvature radius of a wall, and the friction, $f = F$. The resulting velocity is then $v \sim \sigma / r \mu$. It is clear that a full analysis of the role of domain walls in the Universe imposes the study of their interaction with particles in the primordial plasma.

The presence of zero modes localized on a domain wall has been the object of various papers in the literature (see [5] and references therein, [10, 11, 12, 13, 14, 15, 16]). Since strong evidence for neutrino masses has emerged from various neutrino oscillation experiments in recent years [17], we are motivated to investigate the interaction of Majorana fermions with domain walls (neutrinos are neutral fermions, and then can have both Majorana and Dirac masses). In a recent paper [18], Stojkovic has studied fermionic zero modes in the domain wall background, in the case in which the fermions have both Dirac and left-handed and right-handed Majorana mass terms. The aim of this paper is to study the scattering of such fermions off domain walls.

The plan of the paper is as follows. In Section II we introduce the Lagrangian for a single real self-interacting scalar field $\Phi$, coupled with a fermion $\psi$ having Dirac, left-handed and right-handed Majorana mass terms. We also derive the equations of motion. In Section III we calculate the reflection and transmission coefficients for the scattering of fermions off walls, in both cases in which the coupling to the scalar field $\Phi$ is or not the source of the Majorana mass terms. Finally, we summarize our results in Section IV.

II. LAGRANGIAN, ASYMPTOTIC STATES, AND EQUATIONS OF MOTION

We consider a simplified model in which the kink is an infinitely static domain wall in the $xz$-plane. In this model the scalar sector giving rise to a planar wall is a real scalar field with density Lagrangian

$$\mathcal{L}_\Phi = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{\lambda}{4} (\Phi^2 - \eta^2)^2. \quad (1)$$

In the tree approximation, the set of vacuum states is $\langle \Phi \rangle^2 = \eta^2$, so that one may assume that there are regions with $\langle \Phi \rangle = +\eta$ and regions with $\langle \Phi \rangle = -\eta$. By continuity there must exists a region in which the scalar field is out of the vacuum. This region is a domain wall [10], and the classical equation of motion admits the solution describing the transition layer between two regions with...
different values of $\langle \Phi \rangle$, \[ \Phi(y) = \eta \tanh(y/\Delta), \] where $\Delta = \sqrt{2\lambda/\eta}$ is the thickness of the wall. \[ \text{[54x-883]} \]

Now, we consider a fermion $\psi$ having Dirac, left-handed and right-handed Majorana mass terms. The source of the Dirac and Majorana mass terms is the Yukawa couplings to the scalar field $\Phi$. In terms of the chiral spinors $\psi_L$ and $\psi_R$ the Lagrangian density of the system is \[ 1:\]

\[ \mathcal{L} = \frac{1}{2} \partial_{\mu} \Phi \Phi^{\mu} - \frac{\lambda}{4} \left( \Phi^2 - \eta^2 \right)^2 + i \bar{\psi}_L \gamma \psi_L + i \bar{\psi}_R \gamma \psi_R - \left( g_D \bar{\psi}_L \psi_R + g_L \bar{\psi}_L \psi_R^c + g_R \bar{\psi}_R \psi_R^c + h.c. \right), \] where $g_D$, $g_L$ and $g_R$ are the Yukawa couplings to the scalar field of the Dirac, left-handed and right-handed Majorana fermions, respectively. In Lagrangian \[ 3:\]

$\psi^c = C \bar{\psi}^T$, where $C = i \gamma^2 \gamma^0$ is the charge conjugation matrix \[ 20:\] and “$T$” indicates the transpose.

In the broken phase (i.e. for $y \to \pm \infty$) where $\Phi$ takes a constant value, $\langle \Phi \rangle = \pm \eta$, the scalar field give mass to the fermions states. It is clear that the chiral fields $\psi_L$ and $\psi_R$ do not have a definite mass, since they are coupled by the Dirac mass term. In order to find the asymptotic states with definite masses, we have to diagonalise the mass matrix in Lagrangian \[ 3:\] or, equivalently, we have to diagonalize the Dirac equation,

\[ (i\partial - G \Phi)\psi = 0, \] where we have introduced the following quantities:

\[ G = \begin{pmatrix} 0 & G \\ G & 0 \end{pmatrix}, \quad g = \begin{pmatrix} g_L & g_D \\ g_D & g_R \end{pmatrix}, \quad \psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \]

In the broken phase, Eq. \[ 4:\] becomes:

\[ (i\partial + M)\psi = 0, \quad \text{if} \; y \to \pm \infty, \] where we have defined the “mass matrix” $M = \eta G$. Diagonalizing the matrix $M$, we get:

\[ (i\partial + \Delta)\Psi_M = 0, \quad \text{if} \; y \to \pm \infty, \] where

\[ \Delta = U^T M U = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & -m_1 & 0 \\ 0 & 0 & 0 & -m_2 \end{pmatrix}, \] $U$ is the unitary transformation which diagonalizes $M$, and $\Psi_M = U^T \Psi$. The eigenvalues of $M$, that is $\pm m_{1,2}$, are given by

\[ m_{1,2} = \frac{1}{2} \left[ m_L + m_R \pm \sqrt{4m_L^2 + (m_L - m_R)^2} \right]. \]

where we have defined

\[ m_D = g_D \eta, \quad m_L = g_L \eta, \quad m_R = g_R \eta. \]

Here, $m_1$ and $m_2$ represent the masses of the free-field propagating degrees of freedom in the theory. It can be showed (see e.g. Ref. \[ 17:\]) that the two massive fermion states are Majorana particles.

After having considered the asymptotic fermion states, it is now clear that the particle content of Lagrangian \[ 3:\] consist of two Majorana fermions with masses $m_1$ and $m_2$ interacting with a vacuum domain wall (described by the scalar field $\Phi$). The aim of this paper is to study the scattering of this two states off a wall. To this end, we use the following representation of the Dirac matrices,

\[ \gamma^0 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}, \gamma^1 = \begin{pmatrix} i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{pmatrix}, \gamma^2 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & i\sigma^1 \end{pmatrix}, \gamma^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \]

where $\sigma^k$, $k = 1, 2, 3$, are the Pauli matrices. In this representation, a four-component fermion has left-handed and right-handed component of the form

\[ \psi_L^T = (\alpha, \beta, -\alpha, -\beta), \quad \psi_R^T = (\gamma, \delta, \gamma, \delta). \]

We will concentrate on the solution describing the motion of fermions perpendicular to the wall, i.e. along the $y$-axis, and then we suppose that

\[ \Phi = \Phi(y), \quad \psi_L = \psi_L(y,t), \quad \psi_R = \psi_R(y,t). \]

The Lagrangian \[ 3:\], together with Eqs. \[ 12:\] and \[ 13:\], implies the equations of motion

\[ \Phi'' - \chi \Phi (\Phi^2 - \eta^2) = 4g_D \text{Re}(\alpha^* \gamma - \beta^* \delta), \]

and

\[ \beta' + i\alpha = g_D \Phi \gamma + g_L \Phi \beta^*, \]

\[ \alpha' - i\beta = g_D \Phi \beta + g_L \Phi \alpha^*, \]

\[ \delta' + i\gamma = g_R \Phi \delta + g_R \Phi \gamma^*, \]

\[ \gamma' - i\delta = g_R \Phi \gamma + g_R \Phi \delta^*, \]

where $\text{Re}(x)$ is the real part of $x$ (here, and throughout, a prime and a dot will denote differentiation with respect to $y$ and $t$, respectively).

In the following we shall analyze the simple case in which the back-reaction of the fermion field $\psi$ on the domain wall configuration is null. Indeed, we make the
ansatz: $\beta = \alpha^*$ and $\gamma = \delta^*$, which is compatible with Eq. \textbf{(15)}, and makes null the right hand side of Eq. \textbf{(14)}. [This, in turns, means that the wall profile is given by Eq. \textbf{(2)}.] Moreover, writing $\alpha$ and $\delta$ as a sum of positive and negative energy states,
\[ \alpha(y,t) = \alpha_+(y)e^{-iEt} + \alpha_-(y)e^{iEt}, \]
\[ \delta(y,t) = \delta_+(y)e^{-iEt} + \delta_-(y)e^{iEt}, \]
and inserting into Eq. \textbf{(15)} we get
\[ \alpha^*\delta' + E\alpha_+ = g_0\Phi\delta^* + g_L\alpha_+, \]
\[ \delta^*\alpha' - E\delta_+ = g_0\Phi\alpha^* + g_R\delta_+, \]
\[ \alpha^*(\delta') - E\alpha_+ = g_0\Phi\alpha^* + g_R\alpha_+, \]
\[ \delta^*(\alpha') - E\delta_+ = g_0\Phi\delta^* + g_R\delta_+. \]
Starting from Eq. \textbf{(17)}, we will calculate, in the next Section, the reflection and transmission coefficients for the scattering of fermions off walls.

III. SCATTERING

We will work in the “thin-wall approximation”\textsuperscript{2}, that is to say we suppose that the thickness of the wall is vanishingly small, $\Delta \rightarrow 0$. In this case, the wall profile takes the simple form $\Phi = \eta \text{sgn}(y)$, where $\text{sgn}(x)$ is the sign-function. The thin-wall approximation is valid whenever the wavelength of scattered particles is much greater than the thickness of the wall. This approximation allows us to find analytical solutions to the equations of motion and does not affect the main results of our analysis.

In thin-wall approximation, the solution of the system \textbf{(17)} for $y > 0$ is easily found:
\[ \alpha_+ = c_1e^{ip_1y} + c_2e^{ip_2y} + c_3e^{-ip_1y} + c_4e^{-ip_2y}, \]
\[ \alpha^* = i x_1c_1e^{ip_1y} + i x_2c_2e^{ip_2y} - i x_3c_3e^{-ip_1y} - i x_4c_4e^{-ip_2y}, \]
\[ \delta_+ = i x_3c_1e^{ip_1y} + i x_4c_2e^{ip_2y} - i x_5c_3e^{-ip_1y} - i x_6c_4e^{-ip_2y}, \]
\[ \delta^* = x_5c_1e^{ip_1y} + x_6c_2e^{ip_2y} + x_5c_3e^{-ip_1y} + x_6c_4e^{-ip_2y}. \]

Here $c_i$ are integration constants,
\[ p_{1,2} = \sqrt{E^2 - m_i^2}, \]
with $m_{1,2}$ given by Eq. \textbf{(9)}, and
\[ x_{1,2} = \frac{p_{1,2}(E + m_{1,2})}{(E + m_{1,2})(E + m_R) - m_D^2}, \]
\[ x_{3,4} = \frac{p_{1,2}[(E + m_{1,2})(m_R - m_L \pm m_1 \mp m_2) - 2m_D^2]}{2m_D[(E + m_{1,2})(E + m_R) - m_D^2]}, \]
\[ x_{5,6} = \frac{m_R - m_L \pm m_1 \mp m_2}{2m_D}. \]

The solution in the case $y < 0$ is obtained from Eq. \textbf{(18)}, by the substitutions $c_1 \rightarrow d_1$ and $x_1 \rightarrow y_1$, where $d_i$ are new integration constants and
\[ y_{1,2} = \frac{p_{1,2}(E + m_{2,1})}{(E - m_L)(E - m_R) - m_D^2}, \]
\[ y_{3,4} = \frac{p_{1,2}[(E - m_{1,2})(m_R - m_L \mp m_1 \pm m_2) + 2m_D^2]}{2m_D[(E - m_{1,2})(E - m_R) - m_D^2]}, \]
\[ y_{5,6} = x_{5,6}. \]

Returning to the expression for the chiral spinor fields $\psi_L$ and $\psi_R$, we have
\[ \psi_L = \psi_L^+(+) + \psi_L^-(+), \quad \psi_R = \psi_R^+(+) + \psi_R^-(+), \]
where $\psi_L^\pm$ and $\psi_R^\pm$ are explicitly given by
\[ \psi_L^\pm = \left( \begin{array}{c} \alpha_{z,\pm} \\ \alpha_{x,\pm} \\ -\alpha_{z,\pm} \\ -\alpha_{x,\pm} \end{array} \right) e^{\pm iEt}, \quad \psi_R^\pm = \left( \begin{array}{c} \delta_{z,\pm} \\ \delta_{x,\pm} \\ -\delta_{z,\pm} \\ -\delta_{x,\pm} \end{array} \right) e^{\pm iEt}. \]

For definiteness we consider solutions for which we have incident fermion states from the left ($y < 0$) which are scattered into reflected waves going to the left and transmitted waves going to the right ($y > 0$). Therefore, the fermions are represented by incoming and reflected waves to the left of the wall and by transmitted waves to the right. Hence, taking into account Eqs. \textbf{(18)} and \textbf{(23)} we obtain the transmitted, incident, and reflected left-handed wave functions:
\[ (\psi_L^\pm)^{\text{trans}} = (c_1u_{L,1}^\pm e^{\pm ip_1y} + c_2u_{L,2}^\pm e^{\pm ip_2y}) e^{\mp iEt}, \]
\[ (\psi_L^\pm)^{\text{inc}} = (d_1v_{L,1}^\pm e^{\pm ip_1y} + d_2v_{L,2}^\pm e^{\pm ip_2y}) e^{\mp iEt}, \]
\[ (\psi_L^\pm)^{\text{refl}} = (d_3v_{L,3}^\pm e^{\pm ip_1y} + d_4v_{L,4}^\pm e^{\pm ip_2y}) e^{\mp iEt}, \]
with the condition $c_3 = c_4 = 0$. Here, we have introduced the spinors
\[ (u_{L,1}^\pm)^T = (1, ix_1, -1, -ix_1), \]
\[ (u_{L,2}^\pm)^T = (1, ix_2, -1, -ix_2), \]
\[ (u_{L,3}^\pm)^T = (1, -ix_1, -1, ix_1), \]
\[ (u_{L,4}^\pm)^T = (1, -ix_2, -1, ix_2). \]

The spinors $u_{L,i}^\pm$ are obtained from $u_{L,i}$ by the replacements $x_i \rightarrow y_i$, while $u_{L,i}^\pm = C^i_{L,1}u_{L,1}^\pm$ and $u_{L,i}^\pm = C^i_{L,2}u_{L,2}^\pm$, where $i = 1, 2, 3, 4$, and $C$ is the charge conjugation matrix. The transmitted, incident, and reflected right-handed wave functions are obtained from Eq. \textbf{(24)} by
the substitutions $u_{L,i}^{(±)} \rightarrow u_{R,i}^{(±)}$, $v_{L,i}^{(±)} \rightarrow v_{R,i}^{(±)}$, where

$$
(u_{R,i}^{(±)})^T = (x_5, ix_3, x_5, ix_3), \\
(u_{R,2}^{(±)})^T = (x_6, ix_4, x_6, ix_4), \\
(u_{R,3}^{(±)})^T = (x_5, -ix_3, x_5, -ix_3), \\
(u_{R,4}^{(±)})^T = (x_6, -ix_4, x_6, -ix_4).
$$

(26)

The spinors $v_{R,i}^{(±)}$ are obtained from $u_{R,i}^{(±)}$ by the replacements $x_i \rightarrow y_i$, while $u_{R,i}^{(-)} = C v_{R,i}^{(±)}$ and $v_{R,i}^{(-)} = C v_{R,i}^{(±)}$. By imposing continuity of $\alpha(x)$ and $\delta(y)$ in $y = 0$, we get

$$
c_1 + c_2 = d_1 + d_2 + d_3 + d_4, \\
x_1c_1 + x_2c_2 = y_1d_1 + y_2d_2 - y_1d_4 - y_2d_4, \\
x_3c_1 + x_4c_2 = y_3d_3 + y_4d_2 - y_3d_4 - y_4d_4, \\
x_5c_1 + x_6c_2 = y_5d_1 + y_6d_2 + y_5d_3 + y_6d_4.
$$

(27)

Solving the above system with respect to $c_1$, $c_2$, $d_3$, $d_4$, we obtain:

$$
c_1 = \left(1 + \frac{m_1}{E}\right) d_1, \\
c_2 = \left(1 + \frac{m_2}{E}\right) d_2, \\
c_3 = \frac{m_3}{E} d_1, \\
c_4 = \frac{m_4}{E} d_2.
$$

(28)

The total current is defined in terms of the asymptotic fermion states discussed in Section II:

$$
J^\mu_{(±)} = \Psi_M^+(±)\gamma^\mu \gamma^\nu \psi_M^-(±) = \Psi_L^+(±)\gamma^\mu \gamma^\nu \Psi_R^-(±) = (\psi_L^{(±)})^\dagger (\gamma^\nu \gamma^\mu) (\psi_R^{(±)})^\dagger = \left( \begin{array}{cccc}
\gamma^\mu & 0 & 0 & 0 \\
0 & \gamma^\mu & 0 & 0 \\
0 & 0 & \gamma^\mu & 0 \\
0 & 0 & 0 & \gamma^\mu
\end{array} \right) (\begin{array}{c}
\psi_L^{(±)} \\
\psi_R^{(±)} \\
\psi_L^{(±)} \\
\psi_R^{(±)}
\end{array}),
$$

(29)

where the second equality holds because $U$ is a unitary matrix. Here “±” refer to positive and negative energy states, and $\Psi_M^{(±)} = U_T \Psi^{(±)}$. It should be noted that the conjugate wave functions are $(\psi_L^{(±)})^{(±)} = C(\psi_L^{(±)})^T$ and $(\psi_R^{(±)})^{(±)} = C(\psi_R^{(±)})^T$.

Because we are considering the motion of fermions perpendicular to the wall, the relevant currents are those perpendicular to the kink, i.e., $J^2_{(±)}$. Taking into account the expressions for the chiral wave functions and Eq. (29), we get the transmitted, incident and reflected currents:

$$
(J^2_{(±)})^{\text{tran}} = 8 \left[ (x_1 + x_3 x_5) c_1^2 + (x_2 + x_4 x_6) c_2^2 \right] = \chi^T T \chi,
$$

$$
(J^2_{(±)})^{\text{inc}} = 8 \left[ (y_1 + y_3 y_5) d_1^2 + (y_2 + y_4 y_6) d_2^2 \right] = \chi^T \chi,
$$

$$
(J^2_{(±)})^{\text{refl}} = -8 \left[ (y_1 + y_3 y_5) d_3^2 + (y_2 + y_4 y_6) d_4^2 \right] = -\chi^T T \chi,
$$

(30)

where we have introduced the vector $^2$

$$
\chi^T = 2\sqrt{2} (\sqrt{y_1 + y_3 y_5} d_1, \sqrt{y_2 + y_4 y_6} d_2),
$$

and the “reflection and transmission matrices”

$$
R = \begin{pmatrix}
m_1^2/E^2 & 0 \\
0 & m_2^2/E^2
\end{pmatrix},
T = \begin{pmatrix}
p_1^2/E^2 & 0 \\
0 & p_2^2/E^2
\end{pmatrix}.
$$

(32)

Note that $R + T = 1$. For an incident particle, the reflection and transmission coefficients are given as the ratios of the corresponding reflected and transmitted currents. From Eq. (30) we get

$$
R = \frac{(J^2)^{\text{refl}}}{(J^2)^{\text{inc}}} = \frac{\chi^T T \chi}{\chi^T \chi},
T = \frac{(J^2)^{\text{tran}}}{(J^2)^{\text{inc}}} = \frac{\chi^T T \chi}{\chi^T \chi}.
$$

(33)

Taking into account Eq. (29), the unitary relation, $R + T = 1$, follows immediately. For $m_L = m_R = 0$, it is straightforward to check that $R = m_D^2/E^2$, as it should be.

It should be noted that, since $d_1^2$ and $d_2^2$ are directly proportional to the amplitudes of the free-field incident wave functions (i.e. the incident asymptotic fermion states), by a suitable normalization of wave functions we can take $d_1$ and $d_2$ such that $d_1^2 + d_2^2 = 1$. Let us observe that the two incident fermion states of momenta $p_1$ and $p_2$ are scattered in different way. Indeed taking $d_1 = 0$ we get $R = m_R^2/E^2$, while for $d_2 = 0$ we have $R = m_L^2/E^2$. We see that the interaction with vacuum domain walls is able to produce a local asymmetry in the distribution of the two Majorana fermions states of masses $m_1$ and $m_2$.

In the upper panel of Fig. 1 we plot the reflection coefficient versus the energy at fixed $d_1$, $d_2$, $m_D$ and $m_L$, for different values of $m_R$. In the middle (lower) panel we fix $m_L$ ($m_D$) and $m_R$, and vary $m_D$ ($m_L$). These figures show that the reflection coefficient rapidly decreases as the energy of the incident particles increases, as expected. Indeed, from Eq. (30) we get that $R \simeq A/E^2$ for $E \gg m_{1,2}$, where $A$ is a constant depending on $d_1$, $d_2$, $m_1$, and $m_2$. If $m_L \ll m_D \ll m_R$ and $d_1 = d_2$, then $A = m_D^2$. Moreover, at fixed energy, fixing two of the three masses $m_D$, $m_L$, $m_R$, the reflection coefficient is an increasing function of the remaining mass parameter.

The essential properties of $R$ above discussed does not change if we take $d_1 \neq d_2$. Indeed, the only effect to take $d_1 > d_2$ ($d_1 < d_2$) is, at fixed energy and mass parameters, to increase (decrease) the reflection coefficient.

As pointed out in Ref. [1], the Majorana masses could arise from the coupling to a scalar field which undergoes

\[2\] It is straightforward to check that the quantities $y_1 + y_3 y_5$ and $y_2 + y_4 y_6$ are positive definite.

\[3\] Because the most relevant phenomenological model for neutrino masses is the so-called “see-saw mechanism” in which $m_L = 0$ and $m_D \ll m_R$, in our figures we have taken $m_L \leq m_D \leq m_R$. [14]
a phase transition above the phase transition of the field \(\Phi\). In this case, the source of the Majorana masses is not the coupling with \(\Phi\), and the Majorana mass terms are spatially homogeneous. In this case we set

\[
g_L \Phi \rightarrow m_L, \quad g_R \Phi \rightarrow m_R,
\]

in Lagrangian \((3)\). In thin-wall approximation, the solution of the equations of motion is, for \(y > 0\), equal to Eq. \((18)\), while for \(y < 0\) is:

\[
\alpha_+ = d_1 e^{i p_1 y} + d_2 e^{i p_2 y} + d_3 e^{-i p_1 y} + d_4 e^{-i p_2 y},
\]

\[
\alpha_- = i x_1 d_1 e^{i p_1 y} + i x_2 d_2 e^{i p_2 y} - i x_1 d_3 e^{-i p_1 y} - i x_2 d_4 e^{-i p_2 y},
\]

\[
\delta_+ = -i x_3 d_1 e^{i p_1 y} - i x_4 d_2 e^{i p_2 y} + i x_3 d_3 e^{-i p_1 y} + i x_4 d_4 e^{-i p_2 y},
\]

\[
\delta_+ = -x_5 d_1 e^{i p_1 y} - x_6 d_2 e^{i p_2 y} - x_5 d_3 e^{-i p_1 y} - x_6 d_4 e^{-i p_2 y},
\]

where \(d_i\) are constants of integration, \(p_{1,2}\) and \(m_{1,2}\) are the same as in Eq. \((14)\), and \(x_i\) are given by Eq. \((20)\). Taking into account the expressions for \(\psi_L^{(\pm)}\) and \(\psi_R^{(\pm)}\) (see Eq. \((28)\)), and Eq. \((35)\), we obtain the transmitted, incident, and reflected left-handed wave functions:

\[
(\psi_L^{(\pm)})^{\text{tran}} = (c_1 u_{L,1}^{(\pm)} e^{\mp i p_1 y} + c_2 u_{L,2}^{(\pm)} e^{\pm i p_2 y}) e^{\mp i E t},
\]

\[
(\psi_L^{(\pm)})^{\text{inc}} = (d_1 u_{L,1}^{(\pm)} e^{\mp i p_1 y} + d_2 u_{L,2}^{(\pm)} e^{\pm i p_2 y}) e^{\pm i E t},
\]

\[
(\psi_L^{(\pm)})^{\text{refl}} = (d_3 u_{L,3}^{(\pm)} e^{\mp i p_1 y} + d_4 u_{L,4}^{(\pm)} e^{\pm i p_2 y}) e^{\pm i E t},
\]

where the spinors \(u_{L,1}^{(\pm)}\) are given by Eq. \((24)\). The transmitted, incident, and reflected right-handed wave functions are obtained from Eq. \((39)\) by the substitutions \(u_{L,i}^{(\pm)} \rightarrow u_{R,i}^{(\pm)}\), and \(d_i \rightarrow -d_i\), where the spinors \(u_{R,i}^{(\pm)}\) are given by Eq. \((26)\).

Taking into account Eqs. \((29)\) and \((36)\), we get the transmitted, incident and reflected currents:

\[
(J^{(\pm)}_L)^{\text{tran}} = 8 [(x_1 + x_3 x_5) c_1^2 + (x_2 + x_4 x_6) c_2^2],
\]

\[
(J^{(\pm)}_L)^{\text{inc}} = 8 [(x_1 + x_3 x_5) d_1^2 + (x_2 + x_4 x_6) d_2^2],
\]

\[
(J^{(\pm)}_L)^{\text{refl}} = -8 [(x_1 + x_3 x_5) d_3^2 + (x_2 + x_4 x_6) d_4^2].
\]

Now, imposing continuity of \(\alpha^{(\pm)}(y)\) and \(\delta^{(\pm)}(y)\) in \(y = 0\), we get

\[
c_1 + c_2 = d_1 + d_2 + d_3 + d_4,
\]

\[
x_1 c_1 + x_2 c_2 = x_1 d_1 + x_2 d_2 - x_1 d_3 - x_2 d_4,
\]

\[
x_3 c_1 + x_4 c_2 = -x_3 d_1 - x_4 d_2 + x_3 d_3 + x_4 d_4,
\]

\[
x_5 c_1 + x_6 c_2 = -x_5 d_1 - x_6 d_2 - x_5 d_3 - x_6 d_4.
\]

Solving the above system with respect to \(c_1, c_2, d_3, d_4\), and inserting the solution into Eq. \((37)\), we obtain, after some manipulations, the reflection and transmission coefficients:

\[
R = \frac{2 m_L^2 E^2}{2 m_R^2 E^2 + p_1 p_2 (E^2 + p_1 p_2 - m_1 m_2)},
\]

\[
T = \frac{p_1 p_2 (E^2 + p_1 p_2 - m_1 m_2)}{2 m_R^2 E^2 + p_1 p_2 (E^2 + p_1 p_2 - m_1 m_2)}.
\]
The unitary condition follows immediately from Eq. (37) and for \( m_L = m_R = 0 \) we get \( R = m_D^2/E^2 \), as it should be \( \Box \).

It is interesting to note that in the case of spatially homogeneous Majorana mass terms the reflection and transmission coefficients do not depend on the amplitudes of the two incident asymptotic fermion states of momenta \( p_1 \) and \( p_2 \). In fact, the dependence due to the amplitudes of this states factorizes in the expression of the currents in such a way that the reflection and transmission coefficients do not show any explicit dependence on \( d_1 \) and \( d_2 \). Since the two Majorana fermions states of masses \( m_1 \) and \( m_2 \) are scattered in the same way, there is no production of local asymmetry of any kind.

The behavior of \( R \) as a function of one of the tree mass parameters (keeping constant the other two) is the same as in the case of non-constant Majorana mass terms, while for large values of energy, \( E \gtrsim m_{1,2} \), the reflection coefficient decreases as \( R \simeq m_D^2/E^2 \).

We conclude by stressing that the reflection coefficients we found in this paper are important for determining the equation of state of a gas of domain walls which could be present in our Universe. The scattering of Majorana particles off vacuum domain walls, together with the presence of localized zero modes, could strongly influence the cosmic evolution of a gas of domain walls. However, the discussion of this last issue is beyond the aim of this paper and will be the object of future investigations.

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**IV. CONCLUSIONS**

We studied the interaction of fermions having both Dirac and left-handed and right-handed Majorana mass terms with kink domain walls. The source of the Dirac mass term was taken to be the coupling to the scalar field \( \Phi \) that gives rise to a wall. As regards the source of the Majorana mass terms, we analyzed two possible cases. In the first case we assumed that the Majorana masses are generated by the coupling to the scalar field \( \Phi \), in the second one, the Majorana mass terms were taken to be spatially homogeneous.

We found the asymptotic fermion states with definite masses, \( m_1 \) and \( m_2 \), which represent the free-field propagating degrees of freedom in the theory.

By solving the Dirac equation in thin-wall approximation, we calculated the reflection and transmission coefficients for the scattering of such fermions off walls. The peculiar properties of the reflection coefficient \( R \) were analyzed in both cases of non-constant and constant Majorana mass terms.

In the case of non-constant Majorana mass terms, the fermion states with definite masses scatter with different probabilities. Indeed, if the incident state consist of a state of definite mass \( m_1 \) or \( m_2 \), then his scattering probability is \( R = m_1^2/E^2 \) or \( R = m_2^2/E^2 \), respectively. In the case in which the incident state is a superposition of the two definite mass states, then the reflection coefficient has a quite complicated expression. However, for high energy of the incident particles, it is given by \( R \simeq m_D^2/E^2 \) (in the limit \( m_L \ll m_D \ll m_R \)).

In the case of constant Majorana mass terms, the fermion states with definite masses scatter in the same way. We found that for high energy of the incident particles, the reflection coefficient is \( R \simeq m_D^2/E^2 \).

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