A novel two-mode squeezed light based on double-pump phase-matching

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Abstract: A novel two-mode non-degenerate squeezed light is generated based on a four-wave mixing (4WM) process driven by two pump fields crossing at a small angle. By exchanging the roles of the pump beams and the probe and conjugate beams, we have demonstrated the frequency-degenerate two-mode squeezed light with separated spatial patterns. Different from a 4WM process driven by one pump field, the refractive index of the corresponding probe field $n_p$ can be converted to a value that is greater than 1 or less than 1 by an angle adjustment. In the new region with $n_p < 1$, the bandwidth of the gain is relatively large due to the slow change in the refractive index with the two-photon detuning. As the bandwidth is important for the practical application of a quantum memory, the wide-bandwidth intensity-squeezed light fields provide new prospects for quantum memories.

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1. Introduction

Memory for quantum states of light is a necessary component for any future quantum optical computer [1]. In order to extend the storage procedure to squeezed states, we need squeezed light that is resonant to the atomic medium we are using for storage, namely $^{87}$Rb or $^{85}$Rb. The generation of squeezed light at atomic wavelengths have been obtained on the rubidium D1 line [2, 3] and D2 line [4–6]. The squeezed vacuum state on the rubidium D1 line has been stored [7, 8]. Furthermore, the bandwidth is important for the practical application of a quantum memory [9]. The generated wide-bandwidth intensity-squeezed light fields at atomic wavelengths provides new prospects for a quantum memory. Therefore, it is worth initiating a study on how to generate a two-mode squeezed state of wide-bandwidth, especially frequency degenerate two-mode squeezed state of wide-bandwidth.

The first experimental demonstration of squeezed states of light by Slusher et al. [10] was based on four-wave mixing (4WM) in sodium vapor. Since then, many techniques for producing different types of squeezing have been explored, each with its own advantages and limitations for particular applications [11]. Nondegenerate 4WM in a double-$\Lambda$ scheme [12] was identified as a possible scheme to generate a squeezed state or squeezed twin beams, as described in Refs. [13–20].

The generated twin beams by the 4WM process in atomic system with higher squeezing degree were firstly realized by McCormick et al. [19, 20] based on degenerate pump fields, as shown in Fig. 1(a). A single linearly polarized pump beam, $\nu_{\text{pump}}$, is crossed at a small angle with an orthogonally polarized, much weaker probe beam, $\nu_{\text{probe}}$. The 4WM process amplifies the probe and generates a quantum-correlated conjugate beam, $\nu_{\text{conjugate}}$, on the other side of the pump (at a higher frequency), as shown in Fig. 1(b). In this case, a pair of photons of the (single)
pump is transformed, via the 4WM process, into a photon in the probe beam and a photon in the conjugate beam. By modulating the involved ground (excited) state with one (two) laser beam (beams), the gain and squeezing degree can be enhanced [21, 22]. The best initial results for two-mode intensity-difference squeezing at low frequencies seem to be \( \approx 1.5 \text{ kHz} \) [23] to the recently reported \( \approx 700 \text{ Hz} \) [24] or even \( \approx 10 \text{ Hz} \) [25]. The generated entanglement between the probe and conjugate beams can realize quantum imaging [26, 27]. The cascaded 4WM can generate the quantum correlated triple beams [28, 29] and can also be used to realized SU(1,1) interferometers for highly sensitive phase measurements [30, 31]. This 4WM process also supports many spatial modes, making it possible to amplify complex two-dimensional spatial patterns [32–35].

Recently, as shown in Fig. 1(c), a new 4WM process driven by two pump fields of the same frequency crossing at a small angle was realized [36, 37]. Instead of two superimposed rings centered around the pump beams, we find that the output field is satisfied with a two-pump forward phase matching geometry and is two-beam excited conical emission [38]. That is, the light is emitted on the surface of a circular cone centered on the bisector of the two pump beams. In this paper, we further implement frequency-degenerate two-mode squeezed light based on a 4WM driven by two pump fields crossing at a small angle through an optical phase locked loop (OPLL) [39] and give theoretical explanations. By analyzing the gain, we find that the phase matching condition can be achieved under the conditions of \( n_p > 1 \) and \( n_p < 1 \) by an angle adjustment. The theoretical range of angles for achieving different regions is given. In the new region with \( n_p < 1 \), the bandwidth of the gain is relatively large due to the slow change in the refractive index with the two-photon detuning, which is advantageous for realizing wide-bandwidth intensity-squeezed light.

2. Frequency degenerate squeezed light

In our experiment [36], the state \( |g, m\rangle \) (or state \(|1, 2\rangle \)) involves the hyperfine levels \( |S_{\frac{1}{2}}, F = 2, 3\rangle \), where the hyperfine splitting of the ground state is \( \omega_{12} = 2\pi \times 3.035 \text{ GHz} \), and the excited state \( |e\rangle \) (or state \(|3, 4\rangle \)) is \( |S_{\frac{1}{2}}\rangle \) has an excited state decay rate of \( \gamma = 2\pi \times 5.75 \text{ MHz} \). The pump field is blue-detuned approximately 1 GHz to the D1 line of Rb-85 \( S_{\frac{1}{2}} \rightarrow P_{\frac{1}{2}} \). The powers of the pump fields \( E_{P1} \) and \( E_{P2} \) are set to 350 mW, and their waists at the crossing point
When two pump fields \( P_1 \) and \( P_2 \) are generated by a Ti:Sapphire laser, the beams of frequencies \( \nu_{P2} - \nu_{21} \) and \( \nu_{P1} + \nu_{21} \) are the unexpected nonlinear processes. Using this configuration we have investigated the generation of the frequency degenerate squeezed light. The reason is that the noise on the two sides of the atomic line is asymmetrical as Davis et al. [42] pointed out. On the other hand, we reduce the gain by adjusting the temperature from 125 °C to 105 °C, so that the unexpected nonlinear process can be suppressed. Using this configuration we have investigated the generation of the frequency degenerate and spatial nondegenerate twin beams. We use the experimental setup with the two pump fields that are generated by a Ti:Sapphire laser (\( \Delta_1 \sim 1 \) GHz) and a semiconductor laser (\( \Delta_2 \sim 2 \) GHz) to implement this scheme, and the frequency difference of the two pumps is achieved by using an OPLL with the beat frequency of 6.075 GHz as shown in Fig. 3(a). The probe beam is generated by frequency-shift the light form Ti: Sapphire with double-passed 1.52 GHZ acousto-optics modulators (AOM). The AOM
frequency shift, and hence the two-photon detuning $\delta$ is adjusted to optimize performance and change the scheme from frequency non-degenerate to degenerate twin beams. In the experiment of degenerate four-wave mixing, two pump lights with a frequency difference of 6 GHz or more are required to drive at the same time. We use the amplifier lock scheme to generate pump light by frequency shifting. In order to ensure a fixed phase difference between the two pumping light fields, we use a beat frequency interlocking method-OPLL to lock a semiconductor laser and a Ti:Sapphire laser to each other. Since the frequency shift is as small as 0.1 Hz, the lock relative frequency difference is less than 1 Hz.

The 6 GHz beat frequency signal after the frequency locking system is stabilized is shown in Fig. 3(a). The modulating signal peaks appearing in the frequency range of ±1 MHz around both sides of the peak, which is caused by the feedback noise of the OPLL system itself. When the probe and conjugate beams are near degenerate and frequency difference between them $\sim 2.3$ MHz, the intensity-difference noise is shown in Fig. 3(b), and we find the OPLL feedback noise become the main limitation to get better squeezing. When the beat signal is further reduced, as shown by the arrow in Fig. 3(b), we obtain the intensity-difference noise with frequency difference between probe and conjugate beams $< 1$ Hz, as shown in Fig. 3(c), which indicate the twin beams are totally indistinguishable. The inset in Fig. 3(c) shows the process of gradually reducing the beat signal to below 1 Hz, where the black and red lines are phase locked within less than 2 KHz and 1 Hz, respectively.

### 3. Theoretical Model

![Fig. 3. (a) Spectral noise density of the beat signal produced by interference of two phase locked lasers (pump1 and pump2). (b) Intensity-difference noise with frequency difference between probe and conjugate beams $\sim 2.3$ MHz. (c) Intensity-difference noise with frequency difference between probe and conjugate beams $< 1$ Hz.](image)

In this section, we firstly describe the frequency non-degenerate squeezed light based on non-collinear 4WM. As shown in Fig. 4(a), we assume that the two pump fields $E_{P1}$ and $E_{P2}$ couple the transitions $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |4\rangle$, respectively. The probe field couples the transition $|2\rangle \rightarrow |3\rangle$, and the conjugate field couples the transition $|1\rangle \rightarrow |4\rangle$. The transitions $|1\rangle \rightarrow |2\rangle$ and $|3\rangle \rightarrow |4\rangle$ are not dipole allowed transitions. Since the two pump fields $E_{P1}$ and
where transitions, we just swap the pump field of coupling transitions. We add the two sets of conclusions to obtain the final result.

Next, we describe the frequency-degenerate squeezed light based on exchanging the roles of the pump beams and the probe and conjugate beams. As shown in Fig. 4(b), the double-\( \omega \) detuning is different. Therefore, the frequency-degenerate and non-degenerate squeezed lights based on non-collinear 4WM can be described by a same set of equations.

\[
\hat{H} = \hat{H}_{\text{atoms}} + \hat{H}_I, \\
\hat{H}_{\text{atoms}} = \hbar \omega_{41} \sigma_{44} + \hbar \omega_{31} \sigma_{33} + \hbar \omega_{21} \sigma_{22},
\]

and

\[
\hat{H}_I = -\hbar (\Omega_{P1} e^{i(k_{P1} r - \omega_{P1} t)} \sigma_{31} + \Omega_{P2} e^{i(k_{P2} r - \omega_{P2} t)} \sigma_{32} + g_p \hat{E}_p e^{i(k_p r - \omega_p t)} \sigma_{42} + g_c \hat{E}_c e^{i(k_c r - \omega_c t)} \sigma_{41}) + \text{H.c.}
\]

Here, \( \omega_{n1} = \omega_n - \omega_1 \) (\( n = 2, 3, 4 \)), \( \sigma_{nm} = |n\rangle \langle m| \) (\( n, m = 1, 2, 3, 4 \)), 2\( \Omega_{P1} = \mu_{31} E_{P1}/\hbar \) and 2\( \Omega_{P2} = \mu_{42} E_{P2}/\hbar \) are the Rabi frequencies, \( g_n = (n = p, c) \) are the atom-field coupling constants, and \( \hat{E}_p \) and \( \hat{E}_c \) are the slowly varying envelope operators of the probe and conjugate field.

The equations for the atomic operators \( \sigma_{nm} \) (\( n, m = 1, 2, 3, 4 \)) in the Heisenberg picture are given in the Appendix. Using the atomic operators to evaluate the linear and nonlinear components of the polarization at \( \omega_p \) and \( \omega_c \), the polarization of the atomic medium at a particular frequency is given by

\[
\hat{P}_p(\omega_p) = N d_{23} \hat{\sigma}_{23} + \text{H.c.}
\]

and

\[
\hat{P}_c(\omega_c) = N d_{14} \hat{\sigma}_{14} + \text{H.c.},
\]

where \( N \) is the number density of the atomic medium. The polarizations of the medium at frequency \( \omega_n \) (\( n = p, c \)) are given by

\[
\hat{P}_p(\omega_p) = \frac{\hbar \omega_p}{2 \hbar} d_{01} e^{i(k_{P1} r - \omega_{P1} t)} + \frac{\hbar \omega_p}{2 \hbar} d_{01} e^{i(k_{P2} r - \omega_{P2} t)} + \text{H.c.},
\]

and

\[
\hat{P}_c(\omega_c) = \frac{\hbar \omega_c}{2 \hbar} d_{01} e^{i(k_c r - \omega_c t)} + \frac{\hbar \omega_c}{2 \hbar} d_{01} e^{i(k_c r - \omega_c t)} + \text{H.c.}
\]
Here, the two coefficients $\chi_{pp}$ and $\chi_{cc}$ describe the effective linear polarization processes for the probe and conjugate fields, respectively, and unlike the usual linear coefficients, they depend nonlinearly on the pump field. The other two coefficients $\chi_{pc}$ and $\chi_{cp}$ are responsible for the 4WM process. A detailed calculation is given in the Appendix.

Fig. 6. The direct and cross susceptibilities for the probe and conjugate fields as a function of the two-photon detuning $\delta/\gamma$. The solid lines are the real parts, and the dashed lines are the imaginary parts. The excited state decay rate is $\gamma = 2\pi \times 5.75$ MHz, and $\gamma_c = 0.5\gamma$. The hyperfine splitting of the ground state is $\omega_{21} = 2\pi \times 3.035$ GHz. The detuning of pump1 is $\Delta_1 = 174\gamma$. The Rabi frequencies of $\Omega_{P_1}$ and $\Omega_{P_2}$ are $\Omega_{P_1} = 28\gamma$ and $\Omega_{P_2} = 30\gamma$, respectively.

Under the condition of the slowly varying amplitude approximation, considering nearly co-propagating beams along the $z$ axis, these field equations in the co-moving frame are written
as

\[ \frac{\partial}{\partial z} \hat{E}_p = \frac{ik_p}{2} [\chi_{pp}(\omega_p) \hat{E}_p + \chi'_{pc}(\omega_p) \hat{E}_c^\dagger e^{i\Delta k_z z}], \]

(6)

\[ \frac{\partial}{\partial z} \hat{E}_c = \frac{ik_c}{2} [\chi_{cc}(\omega_c) \hat{E}_c + \chi'_{cp}(\omega_c) \hat{E}_p^\dagger e^{i\Delta k_z z}], \]

(7)

where \( \chi'_{pc} = \chi_{pc} \sqrt{\omega_c/\omega_p} \approx \chi_{pc} \), \( \chi'_{cp} = \chi_{cp} \sqrt{\omega_p/\omega_c} \approx \chi_{cp} \), and \( \Delta k_z \) is the projection of the geometric phase mismatch \( \Delta k = k_P + k_C - k_p - k_c \) on the \( z \) axis. The solutions to the propagation equations (6) and (7) with a medium of length \( L \) are given by

\[ \hat{E}_p = G_1 \hat{E}_p(0) + g_1 \hat{E}_c^\dagger(0), \]

(8)

\[ \hat{E}_c^\dagger = [G_2 \hat{E}_c^\dagger(0) + g_2 \hat{E}_p(0)] e^{-i\Delta k_z L}, \]

(9)

where

\[ G_1 = e^{\delta a L} [\cosh(\xi L) + \frac{a}{\xi} \sinh(\xi L)], \quad G_2 = e^{\delta a L} [\cosh(\xi L) - \frac{a}{\xi} \sinh(\xi L)], \]

\[ g_1 = \frac{a_{pc}}{\xi} e^{\delta a L} \sinh(\xi L), \quad g_2 = -\frac{a_{cp}}{\xi} e^{\delta a L} \sinh(\xi L), \]

(10)

and

\[ a_{pj} = i k_p \chi_{pj}/2, \quad a_{cj} = i k_c \chi_{cj}/2 (j = p, c), \]

\[ a = (a_{pp} + a_{cc} - i \Delta k_z)/2, \quad \xi = \sqrt{a^2 - a_{pc} a_{cp}}, \quad \delta a = (a_{pp} - a_{cc} + i \Delta k_z)/2. \]

(11)

Fig. 7. Theoretical output probe gain \( G_p \) as a function of the two-photon detuning \( \delta/\gamma \) and the geometrical phase match \( \Delta k_z \) with (a) a single pump field and (b) two pump fields. Here, the excited state decay rate is \( \gamma = 2\pi \times 5.75 \text{ MHz} \), the decoherence rate is \( \gamma_c = 0.5\gamma \), the atom density is \( N = 4.5 \times 10^{18} \text{ m}^{-3} \), the length of the medium is \( L = 12.5 \text{ mm} \) and the pump Rabi frequencies are \( \Omega_{P1} = 28\gamma \) and \( \Omega_{P2} = 30\gamma \).

The number operators of the probe beam and conjugate beam are defined as \( \hat{N}_p = \hat{E}_p \hat{E}_p^\dagger \) and \( \hat{N}_c = \hat{E}_c \hat{E}_c^\dagger \), respectively. From the above result, we define the gain of the probe beam in the 4WM process as:

\[ G_p = \frac{\langle \hat{N}_p \rangle_{\text{out}}}{\langle \hat{N}_p \rangle_{\text{in}}} \approx |G_1|^2, \]

(12)
where the initial condition is \( \langle \hat{N}_p \rangle_{in} \gg 1 \) and \( \langle \hat{N}_c \rangle_{in} = 0 \). The 4WM generates a correlated probe and conjugate beams, and the relative intensity fluctuations are reduced for the amplification process. After the 4WM, the relative intensity fluctuation is given by

\[
\Delta^2(\hat{N}_p - \hat{N}_c)_{\text{out}} = (\vert G_1 \vert^2 - \vert g_2 \vert^2) \Delta^2(\hat{N}_p)_{in} + \vert g_1^* G_1 - g_2^* G_2 \vert^2 \langle \hat{N}_p \rangle_{in} + 1. \tag{13}
\]

Hence the beams have been amplified without increasing the relative intensity noise, and they are relative intensity squeezed. The standard quantum limit (SQL) is a differential measurement equal to the total optical power, that is

\[
\langle \hat{N}_p - \hat{N}_c \rangle_{\text{SQL}} \equiv \langle \hat{N}_p + \hat{N}_c \rangle \approx (\vert G_1 \vert^2 + \vert g_2 \vert^2) \langle \hat{N}_p \rangle_{in}. \tag{14}
\]

The noise figure of the process (or “degree of squeezing”) is the ratio of the measured noise to the corresponding shot-noise level for equal optical power. The typically the noise figure is quoted as the noise in decibels relative to the SQL.

4. Phase matching

In this section, we describe the angles \( \theta_1 \) and \( \theta_2 \) between the probe field and the pump fields by phase matching based on the different refractive indices.

As shown in Fig. 5(a), when two pump fields \( E_{P1} \) and \( E_{P2} \) are incident at an angle, the total projection of the wavevector of the pump fields onto the \( z \)-axis is \( 2k_{PZ} \) and becomes smaller; i.e., \( 2 \vert k_{PZ} \vert < \vert k_{P1} \vert + \vert k_{P2} \vert \). The geometric phase matching condition (GPMC) is given by

\[
\Delta k_z = \vert k_{P1} \vert \cos(\theta_{z1}) + \vert k_{P2} \vert \cos(\theta_{z2}) - (\vert k_p \vert + \vert k_c \vert) \cos \theta_3 = 2 \vert k_{PZ} \vert - (\vert k_p \vert + \vert k_c \vert) \cos \theta_3, \tag{15}
\]

where \( \theta_3 \) is the angle between the probe and the projected pump field. In fact, if the 4WM is efficient, the GPMC of Eq. (15) may not be satisfied, but the effective phase matching condition (EPMC) must be met:

\[
k_{P1} + k_{P2} - n_p k_p - n_c k_c = 0, \tag{16}
\]
where the refractive index $n_p = \sqrt{1 + \text{Re}(\chi_{pp})}$, and $n_c = \sqrt{1 + \text{Re}(\chi_{cc})}$. For the case of two pump fields, the Eq. (16) can be written as

$$\cos \theta_3 = \frac{\omega_{P_1} \cos(\theta_{z_1}) + \omega_{P_2} \cos(\theta_{z_2})}{n_p \omega_p + n_c \omega_c}.$$  

(17)

According to $\theta_3$ and $\theta_{z_1}$ ($\theta_{z_2}$), we can determine the angle $\theta_1$ ($\theta_2$) between the probe field and the pump field $P_1$ ($P_2$).

In order to better explain phase matching, we first consider the non-degenerate 4WM case. For the case of two degenerate pump fields, the conservation of energy impose the condition $\omega_p + \omega_c = \omega_{P_1} + \omega_{P_2} = 2\omega_0$, where $\omega_0$ is the frequency of the pump field. Considering $\theta_{z_1} = \theta_{z_2} = \theta_0/2$, the Eq. (17) is written as

$$\cos \theta_3 = \frac{2\omega_0 \cos(\theta_0/2)}{n_p \omega_p + \omega_c},$$  

(18)

where $n_c \approx 1$ due to the conjugate field with a large detuning. For a given angle $\theta_0$, when $n_p = 1$, the EPMC of Eq. (18) imposes $\theta_3 = \theta_0/2$. Under this condition, the GPMC $\Delta k_z = 0$ is also satisfied, which is the phase matching condition in free space, where the beams are required rigorously copropagating as shown in Fig. 5(b).

When $n_p > 1$, the EPMC of Eq. (18) is established to require that $\theta_3 > \theta_0/2$, which means that the GPMC of Eq. (15) cannot be satisfied and will occur $\Delta k_z > 0$, as shown in Fig. 5(c).

Considering $\theta_1 = \theta_2 = \theta$, using the law of cosines we obtain the angle requirement between the probe field and the pump fields:

$$\theta > \cos^{-1} \left[ \frac{1 + \cos \theta_0}{2} \right].$$  

(19)

If $n_p < 1$, similarly, the EPMC of Eq. (18) requires that $\theta_3 < \theta_0/2$ and imposes $\Delta k_z < 0$, as shown in Fig. 5(d). In addition, the generated probe and conjugate beams have separate directions, which requires that the angle $\theta_3 > 0$. Furthermore, using the minimum value of the refractive index $\min(n_p)$ according to Eq. (18), we obtain

$$0 < (\theta_3)_{\min} < \theta_3 < \frac{\theta_0}{2},$$  

(20)

where $(\theta_3)_{\min} = \cos^{-1} \left[ 2\omega_0 \cos(\theta_0/2) / (\min(n_p) \omega_p + \omega_c) \right]$ and correspondingly,

$$\frac{\theta_0}{2} < \theta_{\min} < \theta < \cos^{-1} \left[ \frac{1 + \cos \theta_0}{2} \right],$$  

(21)

where $\theta_{\min} = \cos^{-1} \left[ \omega_0(1 + \cos \theta_0) / (\min(n_p) \omega_p + \omega_c) \right]$. Compared to the single pump field case, this is a new region. Because for the single pump field case, the angle $\theta_0$ here is equivalent to 0, where the condition $\theta_3 < \theta_0/2$ cannot be satisfied because $\theta_3$ cannot be less than 0. That is, when $n_p < 1$, the EPMC cannot be satisfied for 4WM driven by a single pump field.

For degenerate case, the form of Eqs. (15-17) is the same except for the magnitude of the wave vectors. With two strong beams with the frequency of the probe and conjugate beams, and along the direction of them, and a week beam having the frequency and direction of the previous pump, we can generate of the frequency degenerate and spatial nondegenerate twin beams by changing the detunings of $\Delta_1$ and $\Delta_2$. Similar to non-degenerate case, if $n_p + n_c = 2$, then GPMC $\Delta k_z = 0$ is satisfied. If $n_p + n_c > 2$ or $n_p + n_c < 2$, then the corresponding GPMC $\Delta k_z > 0$ or $\Delta k_z < 0$ is also obtained.
5. Data analysis

In this section, we numerically analyze the characteristics of the squeezed light produced in this new region with $n_p < 1$.

Fig. 6(a-b) show the direct susceptibilities $\chi_{pp}$ and $\chi_{cc}$ for the probe and conjugate fields as a function of the two-photon detuning $\delta/\gamma$, and we obtain that $\chi_{cc}$ is far less than $\chi_{pp}$ due to large detuning. In Fig. 6(a), when $\delta < 0$, the real part of $\chi_{pp}$ is effectively responsible for the index of refraction of the probe for the single pump field case [43, 44]. However, for our two pump fields input case, the phase matching condition can also be satisfied on the other side $\delta > 0$.

According to Eq. (12), we plot the probe gain $G_p$ as a function of the two-photon detuning $\delta$ and the geometric phase mismatch $\Delta k_z$ in the presence of a single pump field and two pump fields, as shown in Fig. 7. One can see that the maximum gains are obtained on the side $\delta < 0$ with $\Delta k_z > 0$, for both cases. When $\Delta k_z < 0$, the probe gain $G_p$ does not exist for the single pump field input case and occurs for two pump fields input case. The bandwidth of the probe gain is relatively large due to the slow change in the refractive index with the two-photon detuning.

![Theoretical simulation vs Experimental data for probe gain and squeezing](image)

Fig. 9. (a) The gain of the probe field and (b) the squeezing as a function of the two-photon detuning $\delta/\gamma$. The parameters are as follows: $\gamma = 2 \pi \times 5.75$ MHz, $\gamma_c = 0.5 \gamma$, $N = 4.5 \times 10^{18}$ m$^{-3}$, $L = 12.5$ mm, $\Omega_{p1} = 28 \gamma$, $\Omega_{p2} = 30 \gamma$, $\theta_0/2 = 0.615^0$, and $\theta_1 = \theta_2 = 0.861^0$.

The theoretical output probe gain $G_p$ as a function of the two-photon detuning $\delta/\gamma$ and the probe-pump angle $\theta$ with (a) a single pump field and (b) two pump fields is shown in Fig. 8, where we consider $k_{p1} = k_{p2} = k_p$ and $\theta_1 = \theta_2 = \theta$. It can be seen from that for a single pump field, the gain and 4WM process is on the $\delta < 0$ side as the angle $\theta$ increases due to phase matching. For the case of two pump fields, the gain and 4WM process can be achieved on the left side (Line $L_2$) or the right side (line $L_0$ or line $L_1$) by choosing the angle $\theta$ between the probe field and the pump fields, for a given angle $\theta_0$.

The area intersecting the dashed line $L_1$ in Fig. 8(b) is the area selected by our experimental parameters, where $\theta_0/2 = 0.615^0$, and $\theta = 0.861^0$. According to the minimum value $\min(\text{Re}(\chi_{pp})) = -1.609 \times 10^{-4}$ in Fig. 6, using Eq. (21), we obtain

\[
0.7630^0 < \theta < 0.8697^0. \quad (22)
\]

If we only choose the angle $\theta$ based on the bandwidth, we choose line $L_0$ [Fig. 8(b)] because it has the largest bandwidth. However, in the experiment, the angle is finely adjusted according to the degree of squeezing, and the optimum value of the angle $\theta$ is different. If the angle $\theta$ is chosen as $1.2^0$ of line $L_2$ in Fig. 8(b), the 4WM process driven by two pump fields will also be observed on the $\delta < 0$ side due to the large gain. As shown in Fig. 8(b), the strong conjugate field $c$ and two weak conjugate fields $c_1$ and $c_2$ may all occur because of their gains [37]. However,
in this region, the absorption is also large, which will affect the degree of squeezing of the two generated beams.

Here, $\theta_0$ has a fundamental effect on wavevector matching in the new 4WM process, thus opening up a region in which high-intensity-difference squeezed light can be obtained over a wide bandwidth with low loss and moderate gain. The gain curve in Fig. 8(b) shifts upward as the angle $\theta_0$ increases, because the minimum value of the angle $\theta$ is greater than the angle $\theta_0/2$.

The gain of the probe field and the squeezing as a function of the two-photon detuning $\delta$ is shown in Fig. 9, where the square represents experimental data and the solid line is a theoretical simulation. The theoretical simulations and experimental data of the gain of the probe field are in good agreement as shown in Fig. 9(a). The squeezing degree is affected by the spatial mode mismatch, optical absorption by atomic system, optical loss in the light path, and atomic decoherence. Fig. 9(b) shows the theoretical simulations and experimental data of squeezing, where the theoretical squeezing curve is reduced by 0.56 times and the agreement is not very good because these effects are not included in our model in order to clarify the physics picture concisely. On the new $\delta > 0$ side as shown in Fig. 9(a), the bandwidth of the gain is relatively larger than that for the single pump field case, which is advantageous for realizing wide-bandwidth frequency-degenerate and nondegenerate intensity-squeezed light. These light fields can be widely used in quantum information and other fields.

6. Conclusion

We have studied that a novel two-mode squeezed light is generated from a 4WM process driven by two pump fields crossing a small angle, where the twin beams are generated with a new phase matching condition. Different from 4WM realized by a single pump field where the gain peak can only be achieved on the $\delta < 0$ side, the new 4WM process is implemented from the $\delta < 0$ side to the $\delta > 0$ side by an angle adjustment. The refractive index of the corresponding probe field $n_p$ can be converted from $n_p > 1$ to $n_p < 1$, which can also be used to convert between slow light [45] and fast light [46]. Based on slow light and fast light of the probe field, two different time-order output pulses can be achieved. On the new $\delta > 0$ side, the refractive index $n_p$ changes slowly with the two-photon detuning $\delta$ over a large range, which leads to a relatively large gain bandwidth. With two strong beams with the frequency of the probe and conjugate beams, and along the direction of them, and a weak beam having the frequency and direction of the previous pump, we have generated the frequency degenerate and spatial nondegenerate twin beams with tuning the detuning of $\Delta_1$ and $\Delta_2$. This type of twin beams can be combined and interfered directly on the beam splitter. These wide-bandwidth intensity-squeezed light fields can be applied in quantum information and quantum metrology.

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A. Susceptibilities with two pump fields crossing a small angle input

As shown in Fig. 3, we assume that the two pump fields $E_{P_1}$ and $E_{P_2}$ couple the transitions $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |4\rangle$, respectively. The probe field couples the transition $|2\rangle \rightarrow |3\rangle$, and the conjugate field couples the transition $|1\rangle \rightarrow |4\rangle$. The transitions $|1\rangle \rightarrow |2\rangle$ and $|3\rangle \rightarrow |4\rangle$ are not dipole allowed transitions.
Consequently, we obtain the following set of equations for the populations $\sigma_{nn}$:

$$\frac{\partial \sigma_{11}}{\partial t} = \Gamma_{13}(\sigma_{33} + \Gamma_{14}(\sigma_{44} + i(\Omega P_{1}e^{-ikP_{1}r}\sigma_{13} + g_{c}\hat{E}_{c}e^{-ik_{p}r}\sigma_{14} - \Omega P_{1}e^{ik_{p}r}\sigma_{31})
- g_{c}\hat{E}_{c}e^{ik_{p}r}\sigma_{41}),$$

(S1)

$$\frac{\partial \sigma_{22}}{\partial t} = \Gamma_{23}(\sigma_{33} + \Gamma_{24}(\sigma_{44} + i(\Omega P_{2}e^{-ikP_{2}r}\sigma_{24} + g_{p}\hat{E}_{p}e^{-ik_{p}r}\sigma_{23} - \Omega P_{2}e^{ik_{p}r}\sigma_{42}
- g_{p}\hat{E}_{p}e^{ik_{p}r}\sigma_{32}),$$

(S2)

$$\frac{\partial \sigma_{33}}{\partial t} = -\Gamma_{3}(\sigma_{33} + i(\Omega P_{1}e^{ikP_{1}r}\sigma_{31} + g_{p}\hat{E}_{p}e^{ik_{p}r}\sigma_{32} - \Omega P_{1}e^{-ikP_{1}r}\sigma_{13}
- g_{p}\hat{E}_{p}e^{-ik_{p}r}\sigma_{23}),$$

(S3)

$$\frac{\partial \sigma_{44}}{\partial t} = -\Gamma_{4}(\sigma_{44} + i(\Omega P_{2}e^{ikP_{2}r}\sigma_{42} + g_{c}\hat{E}_{c}e^{ik_{p}r}\sigma_{41} - \Omega P_{2}e^{-ikP_{2}r}\sigma_{24}
- g_{c}\hat{E}_{c}e^{-ik_{p}r}\sigma_{34}).$$

(S4)

where $\Gamma_{nm}$ is the population decay rate from the level $n$ to level $m$, and we introduce slowly varying matrix elements in time: $\sigma_{13} = \bar{\sigma}_{13}e^{-i\omega_{np}t}$, $\sigma_{14} = \bar{\sigma}_{14}e^{-i\omega_{np}t}$, $\sigma_{24} = \bar{\sigma}_{24}e^{-i\omega_{np}t}$, and $\sigma_{23} = \bar{\sigma}_{23}e^{-i\omega_{np}t}$. In addition, the set of equations for $\bar{\sigma}_{nm}$ ($n \neq m$) are given by

$$\frac{\partial \bar{\sigma}_{24}}{\partial t} = [i(\delta - \Delta_{2}) - \gamma_{42}]\bar{\sigma}_{24} - i[\Omega P_{2}e^{-ik_{p}r}\sigma_{22,44} - g_{p}\hat{E}_{p}e^{-ik_{p}r}\sigma_{43} + g_{c}\hat{E}_{c}e^{-ik_{p}r}\sigma_{34}],$$

(S5)

$$\frac{\partial \bar{\sigma}_{41}}{\partial t} = -(i\Delta_{2} - \gamma_{41})\bar{\sigma}_{41} + i[\Omega P_{1}e^{-ik_{p}r}\sigma_{43} - g_{c}\hat{E}_{c}e^{-ik_{p}r}\sigma_{14,44} - \Omega P_{2}e^{-ik_{p}r}\sigma_{12}],$$

(S6)

$$\frac{\partial \bar{\sigma}_{32}}{\partial t} = [i(\delta - \Delta_{1}) - \gamma_{32}]\bar{\sigma}_{32} + i[\Omega P_{2}e^{-ik_{p}r}\sigma_{22,33} - \Omega P_{1}e^{-ik_{p}r}\sigma_{22,12}],$$

(S7)

$$\frac{\partial \bar{\sigma}_{31}}{\partial t} = -(i\Delta_{1} + \gamma_{31})\bar{\sigma}_{31} - i[\Omega P_{1}e^{-ik_{p}r}\sigma_{14,13} - g_{c}\hat{E}_{c}e^{-ik_{p}r}\sigma_{33} + g_{p}\hat{E}_{p}e^{-ik_{p}r}\sigma_{21}],$$

(S8)

$$\frac{\partial \bar{\sigma}_{43}}{\partial t} = [i(\Delta_{1} - \Delta_{2}) - \gamma_{43}]\bar{\sigma}_{43} + i[\Omega P_{1}e^{ik_{p}r}\sigma_{41} + g_{p}\hat{E}_{p}e^{ik_{p}r}\sigma_{42} - \Omega P_{2}e^{ik_{p}r}\sigma_{23}
- g_{c}\hat{E}_{c}e^{ik_{p}r}\sigma_{13}],$$

(S9)

$$\frac{\partial \bar{\sigma}_{21}}{\partial t} = -(i\Delta_{2} + \gamma_{21})\bar{\sigma}_{21} + i[\Omega P_{1}e^{ik_{p}r}\sigma_{23} + g_{c}\hat{E}_{c}e^{ik_{p}r}\sigma_{24} - \Omega P_{2}e^{ik_{p}r}\sigma_{41}
- g_{p}\hat{E}_{p}e^{ik_{p}r}\sigma_{31}],$$

(S10)

where $\sigma_{22,44} = \sigma_{22} - \sigma_{44}, \sigma_{14,44} = \sigma_{11} - \sigma_{44}, \sigma_{22,33} = \sigma_{22} - \sigma_{33}$, and $\sigma_{14,13} = \sigma_{11} - \sigma_{33}$, the single-photon detunings are $\Delta_{1} = \omega_{p1} - \omega_{31}$ and $\Delta_{2} = \omega_{p2} - \omega_{21} + \delta$, the two-photon detuning is $\delta = \omega_{p1} - \omega_{p2} - \omega_{21}$, and the slowly varying matrix elements are $\sigma_{43} = \bar{\sigma}_{43}e^{i(\omega_{p1} - \omega_{p2})t}$, $\sigma_{31} = \bar{\sigma}_{31}e^{i(\omega_{p1} - \omega_{p2})t}$, and $\gamma_{nm}$ gives the dephasing rate of the $\sigma_{nm}$ coherence, and $\gamma_{nm} = (\Gamma_{m} + \Gamma_{n})/2 + \gamma_{nm}^{*}$, where $\Gamma_{n}$ is the total decay rate out of level $n$ and $\gamma_{nm}^{*}$ is the dephasing rate due to any other source of decoherence.

Now we are in a position to solve the properties of the system. For convenience, we let $\Gamma_{3} = \Gamma_{4} = \gamma$ and $\Gamma_{13} = \Gamma_{14} = \Gamma_{23} = \Gamma_{24} = \gamma/2$, and the complex decay rates are

$$\xi_{42} = -\gamma/2 + i(\delta - \Delta_{2}), \xi_{41} = -i(\gamma/2 + i\Delta_{2})$$

(S11)

$$\xi_{32} = -\gamma/2 + i(\delta - \Delta_{1}), \xi_{31} = -i(\gamma/2 + i\Delta_{1})$$

(S12)

$$\xi_{43} = -\gamma + i(\Delta_{1} - \Delta_{2}), \xi_{21} = -(\gamma_{21} + i\delta)$$

(S13)
In order to obtain analytical expressions, we assume that the pump fields propagate without depletion, and the steady-state expectation values for the zeroth-order atomic operators $\sigma_{33}$ and $\sigma_{44}$ are equal to

$$\langle \sigma_{33} \rangle = \langle \sigma_{44} \rangle = \frac{|\Omega_{P1}|^2 |\Omega_{P2}|^2}{|\Omega_{P2}|^2 |\xi_{31}|^2 + |\Omega_{P1}|^2 |\xi_{42}|^2 + 4 |\Omega_{P1}|^2 |\Omega_{P2}|^2}.$$  \hspace{1cm} (S14)

Then, the population differences are given by

$$\langle \sigma_{11,33} \rangle = \langle \sigma_{11,44} \rangle = \frac{|\xi_{31}|^2 |\Omega_{P2}|^2}{|\Omega_{P2}|^2 |\xi_{31}|^2 + |\Omega_{P1}|^2 |\xi_{42}|^2 + 4 |\Omega_{P1}|^2 |\Omega_{P2}|^2},$$  \hspace{1cm} (S15)

$$\langle \sigma_{22,33} \rangle = \langle \sigma_{22,44} \rangle = \frac{|\xi_{42}|^2 |\Omega_{P1}|^2}{|\xi_{31}|^2 |\Omega_{P2}|^2 + |\xi_{42}|^2 |\Omega_{P1}|^2 + 4 |\Omega_{P1}|^2 |\Omega_{P2}|^2}.$$  \hspace{1cm} (S16)

We also assume that the probe and conjugate fields are weak fields, such that we only keep terms to first order in $\Omega_p$ and $\Omega_c$. The steady-state density matrix elements $\tilde{\sigma}_{23}$ and $\tilde{\sigma}_{14}$ are given by

$$\tilde{\sigma}_{23} = \frac{i \xi_{32}}{D} \times \langle \sigma_{11,33} \rangle \left( \frac{\xi_{43}}{\xi_{31}} + \frac{|\Omega_{P1}|^2}{\xi_{31} \xi_{41}} \right) \langle \sigma_{33} \rangle - \frac{(\xi_{43} |\Omega_{P2}|^2 + 2 \xi_{41} |\Omega_{P1}|^2)}{\xi_{31}} \langle \sigma_{22,33} \rangle + \frac{\xi_{21} |\Omega_{P2}|^2}{\xi_{31}} \langle \sigma_{11,44} \rangle + \frac{\xi_{21} \xi_{43}}{\xi_{31} \xi_{41}} \langle \sigma_{22,44} \rangle \langle \sigma_{11,33} \rangle e^{i (\Delta k_p \cdot r_p \hat{E}_p^\dagger \hat{E}_{pc})},$$  \hspace{1cm} (S17)

$$\tilde{\sigma}_{14} = \frac{i \xi_{42}}{D} \times \langle \sigma_{11,44} \rangle \left( \frac{\xi_{31}}{\xi_{42}} + \frac{|\Omega_{P1}|^2}{\xi_{42} \xi_{32}} \right) \langle \sigma_{44} \rangle - \frac{(\xi_{43} |\Omega_{P2}|^2 + 2 \xi_{42} |\Omega_{P1}|^2)}{\xi_{42}} \langle \sigma_{22,44} \rangle + \frac{\xi_{21} |\Omega_{P2}|^2}{\xi_{42}} \langle \sigma_{11,33} \rangle \langle \sigma_{33} \rangle e^{i (\Delta k_c \cdot r_c \hat{E}_c^\dagger \hat{E}_{pc})},$$  \hspace{1cm} (S18)

where

$$D = \xi_{41} \xi_{21} \xi_{43} \xi_{32} \xi_{31} \xi_{42} \xi_{43} \xi_{32} |\Omega_{P1}|^2 + (\xi_{41} \xi_{43} + \xi_{41} \xi_{32}) |\Omega_{P2}|^2 + |\Omega_{P1}|^2 |\Omega_{P2}|^2.$$  \hspace{1cm} (S19)

Using the atomic operators to evaluate the linear and nonlinear components of the polarization at $\omega_p$ and $\omega_c$, the polarization of the atomic medium at a particular frequency is given by

$$\hat{P}_p(\omega_p) = N d_{23} \hat{E}_p e^{i k_p \cdot r} + \hat{P}_c(\omega_c) = N d_{14} \hat{E}_c e^{i k_c \cdot r} + \text{H.c.},$$  \hspace{1cm} (S20)

$$\hat{P}_c(\omega_c) = \frac{\hbar \omega_c}{2 \hbar V} \hat{E}_c e^{i k_c \cdot r} + \frac{\hbar \omega_p}{2 \hbar V} \hat{E}_p e^{i k_p \cdot r} + \text{H.c.},$$  \hspace{1cm} (S21)
where the two coefficients $\chi_{pp}$ and $\chi_{cc}$ are given as follows:

$$\chi_{pp} = \frac{iN|d_{23}|^2\xi_{41}^2}{\epsilon_0\hbar D}|\Omega_{P1}|^2 \left[ \frac{\xi_{41}}{\xi_{31}} + \frac{|\Omega_{P1}P_2|^2}{\xi_{31}\xi_{41}} \right] \sigma_{11,33} - \left( \frac{\xi_{41}}{\xi_{41}} \right) |\Omega_{P1}|^2$$

$$+ \frac{\xi_{21}(\xi_{41})}{\xi_{42}} \sigma_{22,33} + |\Omega_{P2}|^2 \left[ \frac{\xi_{21}}{\xi_{42}} - \frac{|\Omega_{P1}P_2|^2}{\xi_{42}\xi_{41}} \right] \sigma_{22,44} \right] \right)^2,$$

(S22)

$$\chi_{cc} = \frac{iN|d_{14}|^2\xi_{42}}{\epsilon_0\hbar D^*}|\Omega_{P1}|^2 \left[ \frac{\xi_{42}}{\xi_{31}} + \frac{|\Omega_{P1}P_2|^2}{\xi_{31}\xi_{42}} \right] \sigma_{11,33} - \left( \frac{\xi_{42}}{\xi_{42}} \right) |\Omega_{P1}|^2$$

$$+ \frac{\xi_{21}(\xi_{42})}{\xi_{44}} \sigma_{11,44} + |\Omega_{P2}|^2 \left[ \frac{\xi_{21}}{\xi_{44}} - \frac{|\Omega_{P1}P_2|^2}{\xi_{44}\xi_{42}} \right] \sigma_{22,44} \right] \right)^2.$$

(S23)

The two coefficients $\chi_{pc}$ and $\chi_{cp}$ are given by

$$\chi_{pc} = \frac{iNd_{23}d_{14}\xi_{41}^2\xi_{41}^2|\Omega_{P1}|^2P_2}{\epsilon_0\hbar D^*} \left[ \frac{\xi_{41}^2}{\xi_{31}} \sigma_{11,44} + \left( \frac{\xi_{43}}{\xi_{42}} \right) + \frac{|\Omega_{P1}P_2|^2}{\xi_{42}\xi_{41}} \right] \right]^2,$$

(S24)

$$\chi_{cp} = \frac{iNd_{23}d_{14}\xi_{42}^2\xi_{42}^2|\Omega_{P1}|^2P_2}{\epsilon_0\hbar D} \left[ \frac{\xi_{42}^2}{\xi_{31}} \sigma_{22,33} + \left( \frac{\xi_{43}}{\xi_{42}} \right) + \frac{|\Omega_{P1}P_2|^2}{\xi_{42}\xi_{42}} \right] \right]^2.$$

(S25)

The two coefficients $\chi_{pp}$ and $\chi_{cc}$ describe the effective linear polarization processes for the probe and conjugate fields, respectively, and unlike the usual linear coefficients, they depend nonlinearly on the pump field. The other two coefficients $\chi_{pc}$ and $\chi_{cp}$ are responsible for the 4WM process.

Since the two pump fields have the same polarizations and frequencies, the two pump fields $E_{P1}$ and $E_{P2}$ also couple the transitions $|2\rangle \rightarrow |4\rangle$ and $|1\rangle \rightarrow |3\rangle$ with probabilities of $\frac{1}{2}$. Due to coupling another set of transitions with the same effective electric dipole, we just swap the Rabi frequencies $\Omega_{P1}$ and $\Omega_{P2}$ in the above 4 coefficients $\chi_{pp}$, $\chi_{cc}$, $\chi_{pc}$ and $\chi_{cp}$ to obtain a new set of transitions.

References

1. C. Simon, et al. “Quantum memories. A review based on the European integrated project "Qubit Applications (QAP)",” Eur. Phys. J. D 58, 1 (2010).
2. T. Tanimura, D. Akamatsu, Y. Yokoi, A. Furusawa, and M. Kozuma, “Generation of squeezed vacuum resonant on a rubidium D1 line with periodically poled KTiOPO4,” Opt. Lett. 31, 2344-2346 (2006).
3. G. Hetet, O. Glockl, K. A. Pilypas, C. C. Harb, B.C. Buchler, H. A. Bachor, and P. K. Lam, “Squeezed light for bandwith-limited atom optics experiments at the rubidium D1 line,” J. Phys. B: At. Mol. Opt. Phys. 40, 221-226 (2007).
4. E. S. Polzik, J. Carri, and H. J. Kimble,”Spectroscopy with squeezed light,” Phys. Rev. Lett. 68, 3020 (1992).
5. F. Marin, A. Bramati, V. Jost, and E. Giacobino,”Demonstration of high sensitivity spectroscopy with squeezed semiconductor lasers,” Optics Commun. 140, 146 (1997).
6. S. Burks, J. Ortalo, A. Chiummo, X. Jia, F. Villa, A. Bramati, J. Laurat, and E. Giacobino,”Vacuum squeezed light for atomic memories at the D2 cesium line,” Opt. Express 17, 3777 (2009).
7. K. Honda, D. Akamatsu, M. Arikawa, Y. Yokoi, K. Akiba, S. Nagatsuka, T. Tanimura, A. Furusawa, and M. Kozuma, “Storage and Retrieval of a Squeezed Vacuum,” Phys. Rev. Lett. 100,093601 (2008).
8. J. Appel, E. Figueroa, D. Korytov, M. Lobino, and A. I. Lvovsky, “Quantum memory for squeezed light,” Phys. Rev. Lett. 100, 093602 (2008).
9. J. Guo, X. Feng, P. Yang, Z. Yu, L. Q. Chen, C.-H. Yuan, W. Zhang, “High-performance Raman quantum memory with optimal control,” Nature Communications 10, 148 (2018).
10. R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Merz, and J. F. Valley, “Observation of squeezed states generated by four-wave mixing in an optical cavity,” Phys. Rev. Lett. 55, 2409 (1985).
11. R. Schnabel, “Squeezed states of light and their applications in laser interferometers,” Phys. Rep. 684, 1 (2017).
12. P. R. Hemmer, D. P. Katz, J. Donoghue, M. Cronin-Golomb, M. S. Shariari, and P. Kumar, “Efficient low-intensity optical phase conjugation based on coherent population trapping in sodium,” Opt. Lett. 20, 982 (1995).
13. M. D. Lukin, P. R. Hemmer, M. Löffler, and M. O. Scully, “Resonant enhancement of parametric processes via radiative interference and induced coherence,” Phys. Rev. Lett. 81, 2675 (1998).
14. A. S. Zibrov, M. D. Lukin, and M. O. Scully, “Nondegenerate parametric self-oscillation via multiwave mixing in coherent atomic media,” Phys. Rev. Lett. 83, 4049 (1999).
15. V. Balic, D. A. Braje, P. Kolchin, G. Y. Yin, and S. E. Harris, “Generation of paired photons with controllable waveforms,” Phys. Rev. Lett. 94, 183601 (2005).
16. P. Kolchin, S. Du, C. Belthangady, G. Y. Yin, and S. E. Harris, “Generation of narrow-bandwidth paired photons: use of a single driving laser,” Phys. Rev. Lett. 97, 113602 (2006).
17. J. K. Thompson, J. Simon, H. Loh, and V. Vuletic, “A high-brightness source of narrowband, identical-photon pairs,” Science 313, 74 (2006).
18. C. H. van der Wal, M. D. Eisaman, A. Andrê, R. L. Walsworth, D. F. Phillips, A. S. Zibrov, and M. D. Lukin, “Atomic memory for correlated photon states,” Science 301, 196 (2003).
19. C. C. McCormick, V. Boyer, E. Arimondo, and P. D. Lett, “Strong relative intensity squeezing by four-wave mixing in rubidium vapor,” Opt. Lett. 32, 178 (2007).
20. C. C. McCormick, A. M. Marino, V. Boyer, and P. D. Lett, “Strong low-frequency quantum correlations from a four-wave-mixing amplifier,” Phys. Rev. A. 78, 043816 (2008).
21. Z. Zhang, F. Wen, J. Che, D. Zhang, C. Li, Y. Zhang, and M. Xiao, “Dressed gain from the parametrically amplified four-wave mixing process in an atomic vapor,” Sci. Rep. 5, 15058 (2015).
22. D. Zhang, C. Li, Z. Zhang, Y. Zhang, Y. Zhang, and M. Xiao, “Enhanced intensity-difference squeezing via energy-level modulations in hot atomic media,” Phys. Rev. A 96, 043847 (2017).
23. C. Liu, J. Jing, Z. Zhou, R. C. Pooser, F. Hudelist, L. Zhou, and W. Zhang, “Realization of low frequency and controllable bandwidth squeezing based on a four-wave-mixing amplifier in rubidium vapor,” Opt. Lett. 36, 2979 (2011).
24. R. Ma, W. Liu, Z. Qin, X. Su, X. Jia, J. Zhang, and J. Gao, “Compact sub-kilohertz low-frequency quantum light source based on four-wave mixing in cesium vapor,” Opt. Lett. 43, 1243 (2018).
25. M.-C. Wu, B. L. Schmittberger, N. R. Brewer, R. W. Speirs, K. M. Jones, P. D. Lett, “Twin-beam intensity-difference squeezing below 10 Hz,” Opt. Express 27, 4769 (2019).
26. V. Boyer, A. M. Marino, and P. D. Lett, “Generation of spatially broadband twin beams for quantum imaging,” Phys. Rev. Lett. 100, 143601 (2008).
27. V. Boyer, A. M. Marino, R. C. Pooser, and P. D. Lett, “Entangled images from four-wave mixing,” Science, 331, 544 (2008).
28. Z. Qin, L. Cao, H. Wang, A. M. Marino, W. Zhang, and J. Jing, “Experimental generation of multiple quantum correlated beams from hot rubidium vapor,” Phys. Rev. Lett. 113, 023602 (2014).
29. Z. Qin, L. Cao, and J. Jing, “Experimental characterization of quantum correlated triple beams generated by cascaded four-wave mixing processes,” Appl. Phys. Lett. 106, 211104 (2015).
30. F. Hudelist, J. Kong, C. Liu, J. Jing, Z. Y. Ou, and W. Zhang, “Quantum metrology with parametric amplifier-based photon correlation interferometers,” Nat. Commun. 5, 3049 (2014).
31. W. Du, J. Jia, J. F. Chen, Z. Y. Ou, and W. Zhang, “Absolute sensitivity of phase measurement in an SU(1,1) type interferometer,” Opt. Lett. 43, 1051 (2018).
32. N. V. Corzo, A. M. Marino, K. M. Jones, and P. D. Lett, “Noiseless optical amplifier operating on hundreds of spatial modes,” Phys. Rev. Lett. 109, 043602 (2012).
33. C. S. Embrey, M. T. Turnbull, P. G. Petrov, and V. Boyer, “Observation of localized multi-spatial-mode quadrature squeezing,” Phys. Rev. X 5, 031004 (2015).
34. L. H. Wang, C. Fabre, and J. Jing, “Single-step fabrication of scalable multimode quantum resources using four-wave mixing with a spatially structured pump,” Phys. Rev. A 95, 051802 (2017).
35. L. Cao, J. Qi, J. Du, and J. Jing, “Experimental generation of quadruple quantum-correlated beams from hot rubidium vapor by cascaded four-wave mixing using spatial multiplexing,” Phys. Rev. A 95, 023803 (2017).
36. F. Jia, W. Du, J. F. Chen, C.-H. Yuan, Z. Y. Ou, and W. Zhang, “Generation of frequency degenerate twin beams in 85Rb vapor,” Opt. Lett. 42, 4024 (2017).
37. E. M. Knutson, J. D. Swaim, S. Wyllie, and R. T. Glasser, “Optimal mode configuration for multiple phase-matched four-wave-mixing processes,” Phys. Rev. A 98, 013828 (2018).
38. M. Kauranen, J. J. Maki, A. L. Gaeta, and R. W. Boyd, “Two-beam-excited conical emission,” Opt. Lett. 16, 943 (1991).
39. R. W. Fox, “Trace detection with diode lasers,” Ph. D. Thesis, University of Colorado, Boulder (1995).
40. D. A. Steck, “Rubidium 85 D Line Data,” http://steck.us/alkalidata
41. N. Corzo, A. M. Marino, K. M. Jones, and P. D. Lett, “Multi-spatial-mode single-beam quadrature squeezed states of light from four-wave mixing in hot rubidium vapor,” Opt. Express, 19, 21358 (2011).
42. W. V. Davis, M. Kauranen, E. M. Nagasako, R. J. Gehr, A. L. Gaeta, and R. W. Boyd, “Excess noise acquired by a laser beam after propagating through an atomic-potassium vapor,” Phys. Rev. A 51, 4152–4159 (1995).
43. M. Jasperse, L. D. Turner, and R. E. Scholten, “Relative intensity squeezing by four-wave mixing with loss: an analytic model and experimental diagnostic,” Opt. Express 19, 3765 (2011).
44. M. T. Turnbull, P. G. Petrov, C. S. Embrey, A. M. Marino, and V. Boyer, “Role of the phase-matching condition in nondegenerate four-wave mixing in hot vapors for the generation of squeezed states of light,” Phys. Rev. A. 88, 033845 (2013).
45. V. Boyer, C. F. McCormick, E. Arimondo, and P. D. Lett, “Ultraslow Propagation of matched pulses by four-wave mixing in an atomic vapor,” Phys. Rev. Lett. 99, 143601 (2007).
46. R. T. Glasser, Ulrich Vogl, and Paul D. Lett, “Stimulated Generation of superluminal light pulses via four-wave mixing,” Phys. Rev. Lett. 108, 173902 (2012).