Neutrinos: A Glimpse Beyond the Standard Model

P. Ramond

Institute for Fundamental Theory, Department of Physics
University of Florida, Gainesville, Fl 32611

(Neutrino-98, Takayama, Japan, June 1998)

Dedicated to the Memory of Dick Slansky

1 A Short History of Neutrinos

Neutrinos are awesome: of all elementary particles, only neutrinos (not even quarks!) have their own conferences, this year Neutrino-98, on a par with Susy, Strings, Lattices, and the like.

It is sobering to remind ourselves that all weak interaction experiments start out wrong, even when performed by the greatest experimentalists of their times. In 1911-1912, using a magnetic spectrometer and photographic plates, O. Von Bayer, O. Hahn, and L. Meitner [1, 2] were the first to measure the spectrum of electrons in $\beta$ radioactivity. Their conclusion: like $\alpha$ radioactivity, the spectrum of the decay product is discrete!

In 1914, Chadwick [3], performed similar measurements in Geiger’s laboratory in Berlin and came out with a different conclusion, that the spectrum of $\beta$ electrons is continuous. The Great War interrupted the discourse, and the next step in the story were measurements by C. D. Ellis [4] who showed that the discrete lines found earlier were due to internal conversion. Finally in 1927, C.D. Ellis and W. A. Wooster [5] found that the mean energy liberated in $\beta$ decay accounted for only 1/3 of the allowed energy. By that time even Lise Meitner agreed that the electron spectrum was continuous, setting the stage for W. Pauli’s famous letter.

In a December 1930 letter that starts with typical panache, “Dear Radioactive Ladies and Gentlemen...”, W. Pauli proposes a “desperate”way out: there is a companion particle to the $\beta$ electron. Undetected, it must be electrically neutral, and in order to balance the $N - Li^0$ statistics, it carries spin 1/2. He
calls it the *neutron*. It is clear from the letter that Pauli saw no reason why this new particle could not be massive.

In 1933, E. Fermi in his formulation of the theory of $\beta$ decay gave it its final name, the little neutron or *neutrino*, as it is clearly much lighter than Chadwick’s neutron which had been discovered since Pauli’s letter.

The next step in our story is in 1945, when B. Pontecorvo \[6\] puts forward the idea that neutrinos can be detected. It is based on the following observation: an electron neutrino can hit a $^{37}\text{Cl}$ atom and transform it into $^{37}\text{Ar}$. While the Chlorine atoms are plentiful, as in cleaning fluid $C_2\text{Cl}_4$, Argon is an inert gas that does not interact much; furthermore it is radioactive and sticks around just long enough to be detectable through its decay: its abundance can be monitored by patient and careful experimentalists. Pontecorvo did not publish the report, perhaps because of its secret classification, or perhaps because he showed it to Fermi who thought the idea ingenious but not immediately achievable.

In 1953, Cowan and Reines \[7\] proposed a different technique to detect neutrinos, by using a liquid scintillator.

In 1954, Davis \[8\] uses Pontecorvo’s original proposal, by setting up outside a nuclear reactor, and then using radio-chemical techniques to detect the Argon atoms.

In 1956, Cowan and Reines \[9\] announced they had detected $\nu_e$’s through the reaction $\nu_e + p \rightarrow e^+ + n$. Cowan passed away before 1995, the year Fred Reines was awarded the Nobel Prize for their discovery. There emerge two lessons in neutrino physics: not only is patience required but also longevity: it took 26 years from birth to detection and then another 39 for the Nobel Committee to recognize the achievement!

In 1956, motivated by rumors that Davis had found evidence for antineutrinos coming from a pile, Pontecorvo \[10\] reasoned, in analogy to Gell-Mann and Pais, who had just shown how a $K$-meson could oscillate into its antiparticle, that it could be due to a similar effect: an electron neutrino produced in the Savannah reactor could oscillate into its own antiparticle and be detected by Davis. The rumor went away, but the idea of neutrino oscillations was born; it has remained with us ever since, and proven the most potent tool in hunting for neutrino masses.

Having detected the neutrino, there remained to determine its spin and mass. Its helicity was measured in 1958 by M. Goldhaber \[11\], but convincing evidence for its mass has, up to this meeting, eluded experimentalists.

In 1957, Lee and Yang propose that weak interactions violate parity, and the neutrino is again at the center of the action. Unlike the charged elementary particles which have both left- and right-handed components, neutrinos are purely left-handed (antineutrinos are right-handed), which means that lepton-number is chiral.

In 1962, a second neutrino, the muon neutrino is detected \[12\], (long anticipated by theorists Inoué and Sakata in 1943 \[13\]). This time things went a bit faster as it took only 19 years from theory (1943) to discovery (1962) and 26
year s to Nobel recognition (1988).

That same year, Maki, Nakagawa and Sakata \cite{14} introduce two crucial ideas; one is that these two neutrinos can mix, and the second is that this mixing can cause one type of neutrino to oscillate into the other (called today flavor oscillation). This is possible only if the two neutrino flavors have different masses.

In 1963, the Astrophysics group at Caltech, Bahcall, Fowler, Iben and Sears \cite{15} puts forward the most accurate of neutrino fluxes from the Sun. Their calculations included the all important Boron decay spectrum, which produces neutrinos with the right energy range for the Chlorine experiment.

In 1964, using Bahcall’s result \cite{16} of an enhanced capture rate of $^8B$ neutrinos through an excited state of $^{37}$Ar, Davis \cite{17} proposes to search for $^8B$ solar neutrinos using a 100,000 gallon tank of cleaning fluid deep underground. Soon after, R. Davis starts his epochal experiment at the Homestake mine, marking the beginning of the solar neutrino watch which continues to this day. In 1968, Davis et al reported \cite{18} a deficit in the solar neutrino flux, a result that has withstood scrutiny to this day, and stands as a truly remarkable experimental tour de force. Shortly after, Gribov and Pontecorvo \cite{19} interpreted the deficit as evidence for neutrino oscillations.

## 2 Standard Model Neutrinos

The standard model of electro-weak and strong interactions contains three left-handed neutrinos. The three neutrinos are represented by two-components Weyl spinors, $\nu_i$, $i = e, \mu, \tau$, each describing a left-handed fermion (right-handed antifermion). As the upper components of weak isodoublets $L_i$, they have $I_{3W} = 1/2$, and a unit of the global $i$th lepton number.

These standard model neutrinos are strictly massless. The only Lorentz scalar made out of these neutrinos is the Majorana mass, of the form $\nu_i^\dagger \nu_j$; it has the quantum numbers of a weak isotriplet, with third component $I_{3W} = 1$, as well as two units of total lepton number. Thus to generate a Majorana mass term at tree-level, one needs a Higgs isotriplet with two units of lepton number. Since the standard model Higgs is a weak isodoublet Higgs, there are no tree-level neutrino masses.

What about quantum corrections? Their effects are not limited to renormalizable couplings, and it is easy to make a weak isotriplet out of two isodoublets, yielding the $SU(2) \times U(1)$ invariant $L_i^\dagger L_j \cdot H^\dagger H$, where $H$ is the Higgs doublet. As this term is not invariant under lepton number, it is not be generated in perturbation theory. Thus the important conclusion: The standard model neutrinos are kept massless by global chiral lepton number symmetry. The detection of non-zero neutrino masses is a tangible indication of physics beyond the standard model.
3 Neutrino Mass Models

The present experimental limits on neutrino masses are quite impressive, \( m_{\nu_e} < 10 \text{ eV}, \) \( m_{\nu_\mu} < 170 \text{ keV}, \) \( m_{\nu_\tau} < 18 \text{ MeV} \) [20]. Any model that generates neutrino masses must contain a natural mechanism that explains their small value, relative to that of their charged counterparts.

To generate neutrino masses without new fermions, we must break lepton number. This requires adding to the standard model Higgs fields which carry lepton number, as one can arrange to break lepton number explicitly or spontaneously through their interactions.

To impart Higgs with lepton number, they must be coupled to standard model leptons. From invariance requirements, we see that there can be only three such fields with two units of lepton number: An isotriplet Higgs, \( \vec{T} \), and two isosinglets, one positively charged, \( S^+ \), the other doubly charged, \( S^{--} \), with renormalizable couplings

\[
\vec{T} \cdot L_i \bar{\tau} L_j ; \quad S^+ L_i \bar{\tau} L_j ; \quad S^{--} \bar{\tau} \bar{\tau} L_j .
\]

The curvy brackets denote flavor-symmetrization, the square ones flavor antisymmetrization.

With these fields we can construct three types of cubic interactions that break lepton number: \( H \vec{T} H \), \( S^+ S^+ S^{--} \), and \( \vec{T} \cdot \vec{T} S^{--} \), which introduce through their couplings an unknown scale at which lepton number is violated. There are no quartic interactions that violate lepton number.

The Higgs isotriplet has a neutral component; it can be arranged to get a vacuum value, breaking lepton number spontaneously. This leads to a Nambu-Goldstone boson, called the Majoron. Since it is part of an isotriplet, it couples to the Z boson, whose measured width rules out isotriplet breaking of lepton number. One needs electroweak singlet scalars with lepton number to devise Majoron models that are not in manifest conflict with experiment.

Perhaps the simplest way to give neutrinos masses is to introduce for each one an electroweak singlet Dirac partner, \( \vec{N}_i \). These appear naturally in the Grand Unified group \( SO(10) \). Neutrino Dirac masses are generated by the couplings \( L_i \vec{N}_j H \) after electroweak breaking. Unfortunately, these Yukawa couplings yield masses which are too big: they are along the electroweak breaking parameter, of the same order of magnitude as the masses of the charged elementary particles \( m \sim \Delta I_w = 1/2 \). The situation is remedied by introducing Majorana mass terms \( \vec{N}_i \vec{N}_j \) for the right-handed neutrinos. The masses of these new degrees of freedom is arbitrary, since it has no electroweak quantum numbers, \( M \sim \Delta I_w = 0 \). If it is much larger than the electroweak scale, the neutrino masses are suppressed relative to that of their charged counterparts by the ratio of the electroweak scale to that new scale: the mass matrix (in \( 3 \times 3 \) block form) is

\[
\begin{pmatrix}
0 & m \\
m & M
\end{pmatrix} \ .
\]
leading to one small and one large eigenvalue

\[ m_\nu \sim m \cdot \frac{m}{M} \sim \left( \Delta I_w = \frac{1}{2} \right) \cdot \left( \Delta I_w = \frac{1}{2} \right) . \quad (3) \]

This seesaw mechanism \cite{21} provides a natural explanation for the smallness of the neutrino masses as long as lepton number is broken at a large scale \( M \). With \( M \) around the energy at which the gauge couplings unify, this yields neutrino masses at or below the eV region.

The flavor mixing comes from two different parts, the diagonalization of the charged lepton Yukawa couplings, and that of the neutrino masses. From the charged lepton Yukawas, we obtain \( U_e \), the unitary matrix that rotates the lepton doublets \( L_i \). From the neutrino Majorana matrix, we obtain \( U_\nu \), the matrix that diagonalizes the Majorana mass matrix. The 6 \( \times \) 6 seesaw Majorana matrix can be written in 3 \( \times \) 3 block form

\[ M = V_\nu^T \tilde{D} V_\nu \sim \begin{pmatrix} U_\nu & dU_{N\nu} \\ dU_{N\nu}^T & U_N \end{pmatrix}, \quad (4) \]

where \( \epsilon \) is the tiny ratio of the electroweak to lepton number violating scales, and \( \tilde{D} = \text{diag}(\epsilon^2 D_{\nu}, D_N) \), is a diagonal matrix. \( D_\nu \) contains the three neutrino masses, and \( \epsilon^2 \) is the seesaw suppression. The weak charged current is then given by

\[ j_{\mu}^\pm = e_i^\dagger \sigma_{\mu} U_{MNS} \nu_j \quad (5) \]

where

\[ U_{MNS} = U_e U_\nu^T \quad (6) \]

is the matrix first introduced in ref \cite{14}, the analog of the CKM matrix in the quark sector.

In the seesaw-augmented standard model, this mixing matrix is totally arbitrary. It contains, as does the CKM matrix, three rotation angles, and one CP-violating phase, and also two additional CP-violating phases which cannot be absorbed in a redefinition of the neutrino fields, because of their Majorana masses (these extra phases can be measured only in \( \Delta L = 2 \) processes). All are additional parameters of the seesaw-augmented standard model, to be determined by experiment.

Their prediction, as for for the quark hierarchies and mixings, necessitates further theoretical assumptions. Below we present such a framework, which predicts maximal mixing between \( \nu_\mu \) and \( \nu_\tau \) \cite{22} and a thrice Cabibbo suppression of \( \nu_e \) into \( \nu_\mu,\tau \).

### 4 A Neutrino Mixing Model

This model \cite{23} follows from the Cabibbo suppressions of the Yukawa couplings of the standard model. Using the well-known Cabibbo suppressions in the quark
sector, we identify family symmetries on the quarks that reproduce the patterns. We generalize this symmetry to the leptons, using grand-unified groups in a very simple way, and then use the lepton assignments to produce Cabibbo suppressions in the lepton sectors. Using special properties of the seesaw mechanism, we find a unique lepton mixing matrix, with the properties already described.

We assume that the Cabibbo suppression comes about because of extra family symmetries in the standard model. A standard model invariant operator, such as \( Q_i \overline{d}_j H_d \), if not invariant under the additional symmetry, cannot be present at tree-level. Assuming the existence of an electroweak singlet field \( \theta \), which serves as the order paramater for this new symmetry, the interaction

\[
Q_i \overline{d}_j H_d \left( \frac{\theta}{\Lambda} \right)^{n_{ij}}
\]

(7)

can appear in the potential as long as the family charges balance under the new symmetry. When \( \theta \) acquires a vev, this leads to a suppression of the Yukawa couplings of the order of \( \lambda^{n_{ij}} \) for each matrix element, where \( \lambda = \theta / \Lambda \) is assumed to be like the Cabibbo angle, and \( \Lambda \) is the natural cut-off of the theory. This is a natural mechanism in the context of an effective low energy theory with cut-off \( \Lambda \).

As a consequence of the charge balance equation

\[
X_{ij}^{[d]} + n_{ij} X_\theta = 0 ,
\]

(8)

the exponents of the suppression is related to the charge of the standard model invariant operator. That charge is the sum of the charges of the fields that make up the invariant. Let us now apply this mechanism to the invariants in the seesaw mechanism.

We start with the charged lepton Yukawa couplings of the form \( L_i \overline{N}_j H_u \), with charges \( X_{L_i} + X_{N_j} + X_H \), which gives the Cabibbo suppression of the \( ij \) matrix element. It follows that we can write the orders of magnitude of these couplings in the form

\[
\begin{pmatrix}
\lambda^{l_1} & 0 & 0 \\
0 & \lambda^{l_2} & 0 \\
0 & 0 & \lambda^{l_3}
\end{pmatrix}
Y
\begin{pmatrix}
\lambda^{p_1} & 0 & 0 \\
0 & \lambda^{p_2} & 0 \\
0 & 0 & \lambda^{p_3}
\end{pmatrix}
\]

(9)

where \( Y \) is a Yukawa matrix with no Cabibbo suppressions, \( l_i = X_{L_i} / X_\theta \), \( p_i = X_{N_i} / X_\theta \). The first matrix will form the first half of the MNS matrix in the charged lepton current.

Similarly, the mass matrix for the right-handed neutrinos, \( \overline{N}_i \overline{N}_j \) will be written in the form

\[
\begin{pmatrix}
\lambda^{p_1} & 0 & 0 \\
0 & \lambda^{p_2} & 0 \\
0 & 0 & \lambda^{p_3}
\end{pmatrix}
M
\begin{pmatrix}
\lambda^{p_1} & 0 & 0 \\
0 & \lambda^{p_2} & 0 \\
0 & 0 & \lambda^{p_3}
\end{pmatrix}
\]

(10)
The diagonalization of the seesaw matrix is of the form
\[ L_i H_u \overline{N}_j \begin{pmatrix} 1 \\ \overline{N} \end{pmatrix} \overline{N}_k H_u L_i , \] (11)
from which the Cabibbo suppression matrix from the \( \overline{N}_i \) fields cancels, leaving us with
\[ \begin{pmatrix} \lambda^1 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & \lambda^3 \end{pmatrix} \mathcal{M}' \begin{pmatrix} \lambda^1 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & \lambda^3 \end{pmatrix} , \] (12)
where \( \mathcal{M}' \) is a matrix with elements of order one. The Cabibbo structure of the seesaw neutrino matrix is determined solely by the charges of the lepton doublets! As a result, the Cabibbo structure of the MNS mixing matrix is also due entirely to the charges of the three lepton doublets. This general conclusion depends on the existence of at least one Abelian family symmetry, which we argue is implied by the observed structure in the quark sector.

The Wolfenstein parametrization of the CKM matrix [24],
\[ \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} , \] (13)
and the Cabibbo structure of the quark mass ratios
\[ \frac{m_u}{m_t} \sim \lambda^8 \quad \frac{m_c}{m_t} \sim \lambda^4 ; \quad \frac{m_d}{m_b} \sim \lambda^4 \quad \frac{m_s}{m_b} \sim \lambda^2 , \] (14)
are reproduced by a simple charge assignment on the three quark families, namely
\[ X_{Q, \overline{Q}, \overline{\pi}} = B(2, -1, -1) + \eta_{Q, \overline{Q}, \overline{\pi}}(1, 0, -1) , \] (15)
where \( B \) is baryon number, \( \eta_{\overline{Q}} = 0 \), and \( \eta_{Q} = \eta_{\overline{\pi}} = 2 \). Two noteworthy features emerge: the charges of the down quarks associated with the second and third families are the same, and the \( \eta \) values for both \( Q \) and \( \overline{Q} \) are the same.

Theoretical prejudices based on grand unified quantum numbers determine for us the family charges of the leptons from those of the quarks. In grand unified extensions of the standard model, baryon number generalizes in \( SO(10) \) to \( B - \mathcal{L} \), where \( \mathcal{L} \) is total lepton number, and the standard model families split under \( SU(5) \) as \( 5 = \overline{3} + L \) and \( 10 = Q + \overline{3} + \overline{\rho} \). Thus a natural assignment is to assign \( \eta = 0 \) to the lepton doublet \( L_i \), and \( \eta = 2 \) to the electron singlet \( \overline{\rho} \). In this way, the charges of the lepton doublets are simply \( X_{L_i} = -1(2, -1, -1) \). As we have just argued, these charges determine the Cabibbo structure of the MNS mixing matrix to be
\[ U_{MNS} \sim \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix} . \] (16)
We therefore expect no Cabibbo suppression in the mixing between $\nu_\mu$ and $\nu_\tau$. This mixing scheme is consistent with the preliminary results of SuperKamiokande announced at the 1977 ITP workshop [25], and also consistent with the small angle MSW [26] solution to the solar neutrino deficit.

The determination of the mass values is more complicated, as it not only depends on the relative interfamily charge assignments but also on the overall intrafamily charges. Here we simply quote the results from a particular model [23]. The masses of the right-handed neutrinos are found to be of the following orders of magnitude

$$m_{N_e} \sim \Lambda \lambda^{13}; \quad m_{N_\mu} \sim m_{N_\tau} \sim \Lambda \lambda^7,$$

(17)

where $\Lambda$ is the cut-off. The seesaw mass matrix for the three light neutrinos comes out to be

$$m_0 \begin{pmatrix} a\lambda^6 & b\lambda^3 & c\lambda^3 \\ b\lambda^3 & d & e \\ c\lambda^3 & e & f \end{pmatrix},$$

(18)

where we have added for future reference the prefactors $a, b, c, d, e, f$, all of order one, and

$$m_0 = \frac{v_u^2}{\Lambda \lambda^3},$$

(19)

where $v_u$ is the vev of the Higgs doublet. This matrix has one light eigenvalue

$$m_{\nu_e} \sim m_0 \lambda^6.$$  

(20)

Without a detailed analysis of the prefactors, the masses of the other two neutrinos come out to be both of order $m_0$. However, the mass difference inferred by the superKamiokande result [25] (up to this conference) can be reproduced, but only if the prefactors are carefully taken into account. The two heavier mass eigenstates and their mixing angle are written in terms of

$$x = \frac{df - e^2}{(d + f)^2}, \quad y = \frac{d - f}{d + f},$$

(21)

as

$$\frac{m_{\nu_2}}{m_{\nu_3}} = \frac{1 - \sqrt{1 - 4x}}{1 + \sqrt{1 - 4x}}, \quad \sin^2 2\theta_{\mu\tau} = 1 - \frac{y^2}{1 - 4x}. $$

(22)

If $4x \sim 1$, the two heaviest neutrinos are nearly degenerate. If $4x \ll 1$, a condition easy to achieve if $d$ and $f$ have the same sign, we can obtain an adequate split between the two mass eigenstates. For illustrative purposes, when $0.03 < x < 0.15$, we find

$$4.4 \times 10^{-6} \leq \Delta m^2_{\nu_2 - \nu_\mu} \leq 10^{-5} r m eV^2,$$

(23)

which yields the correct non-adiabatic MSW effect, and

$$5 \times 10^{-4} \leq \Delta m^2_{\nu_2 - \nu_\tau} \leq 5 \times 10^{-3} eV^2,$$

(24)
for the atmospheric neutrino effect. These were calculated with a cut-off, $10^{16} \text{ GeV} < \Lambda < 4 \times 10^{17} \text{ GeV}$, and a mixing angle, $0.9 < \sin^2 2\theta_{\mu-\tau} < 1$. It is satisfying that these values are compatible not only with the data but also with the gauge unification scale, and the basic ideas of Grand Unification. With poetic justice, we note that Grand Unification with its prediction of proton decay motivated the building of large underground water Čerenkov counters. The serendipitous detection of neutrinos from SN1987A by the IMB, Kamiokande, and other collaborations, established these detectors as major tools for the discovery of neutrino properties.

5 Outlook

The present field of neutrino physics is being driven by many experimental findings that challenge theoretical expectations. All can be explained in terms of neutrino oscillations, implying neutrino masses and mixing angles, but one should be cautious as evidence for neutrino oscillations has often been reported, only to either be withdrawn or else contradicted by other experiments.

The reported anomalies associated with solar neutrinos [27], neutrinos produced in cosmic ray cascades [25], and also in low energy reactions [28], cannot all be correct without introducing a new type of neutrino which does not couple to the $Z$ boson, a sterile neutrino.

Small neutrino masses are naturally generated by the seesaw mechanism, which works because of the weak interactions of the neutrinos. A similar mass suppression for sterile neutrinos involves new hitherto unknown interactions, resulting in substantial additions to the standard model, for which there is no independent evidence. Also, the case for a heavier cosmological neutrino in aiding structure formation may not be as pressing, in view of the measurements of a small cosmological constant.

To conclude, experimental neutrino physics is in a most exciting stage, as it provides in the near future the best opportunities for finding evidence of physics beyond the standard model.

References

[1] Much of the following follows the excellent article by P. Radvanyi, From Becquerel to Pauli in Neutrinos, Dark Matter and the Universe, T. Stolarcyk, J. Tran Thanh Van, F. Vanucci, editors (Editions Frontières, France).

[2] O. Von Bayer, O. Hahn, and L. Meitner, Phys. Zeitschr. 12, 273(1911); ibid 13, 273(1911); 13 264(1912).

[3] J. Chadwick, Verh. d. D. Phys. Ges., 16, 383(1914).

[4] C. D. Ellis, Proc Royal Soc. A99, 261(1921).
[5] C. D. Ellis and W. A. Wooster, Proc. Royal Soc. A117, 109(1927).
[6] B. Pontecorvo, Chalk river Report PD-205, November 1946, unpublished.
[7] C. L. Cowan and F. Reines, Phys.Rev. 90, 492(1953).
[8] Raymond Davis Jr., Phys Rev 97, 766(1955).
[9] C.L. Cowan, F. Reines, F.B. Harrison, H.W. Kruse, A.D. McGuire, Science 124, 103(1956).
[10] B. Pontecorvo, JETP (USSR) 34, 247(1958).
[11] M. Goldhaber, L. Grodzins, A.W. Sunyar, Phys.Rev. 109, 1015(1958).
[12] G. Danby, J.M. Gaillard, K. Goulianos, L.M. Lederman, N. Mistry, M. Schwartz, J. Steinberger, Phys.Rev.Lett. 9, 36(1962).
[13] S. Sakata and T. Inoue, Prog. Theo. Physics, 1, 143(1946).
[14] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theo. Physics, 28, 247(1962).
  B. Pontecorvo, Zh. Eksp. Teor. Fiz. 53, 1717(1967).
[15] J. Bahcall, W. A. Fowler, I. Iben and R. L. Sears, Astrophysics. J. 137, 344(1963).
[16] J. Bahcall, Phys. Rev. Lett. 12, 300(1964).
[17] Raymond Davis Jr., Phys. Rev. Lett. 12, 303(1964).
[18] Raymond Davis Jr., D. Harmer and K. Hoffman, Phys. Rev. Lett. 20, 1205(1968).
[19] V. Gribov and B. Pontecorvo, Phys. Lett. B28, 493(1969).
[20] Particle Data Group, R. M. Barnett et al., Phys Rev D54, 1(1996).
[21] M. Gell-Mann, P. Ramond, and R. Slansky in Sanibel Talk, CALT-68-709, Feb 1979 (unpublished), and in Supergravity (North Holland, Amsterdam 1979). T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe, KEK, Japan, 1979.
[22] Maximal mixing was recognized long ago as a feature of the grand-unified group SO(10): see J. A. Harvey, P. Ramond and D. B. Reiss, Nucl. Phys. B199, 223(1982)
[23] N. Irges, S. Lavignac and P. Ramond, Phys. Rev. D58, 035003(1998).
[24] L. Wolfenstein, Phys. Rev. Lett. 51, 1945(1983).
[25] E. Kearns, presented at the ITP conference on Solar Neutrinos: News about SNU’s, Dec 1997.

[26] L. Wolfenstein, Phys. Rev. D17, 2369 (1978); S. Mikheyev and A. Yu Smirnov, Nuovo Cim. 9C, 17 (1986).

[27] See N. Hata and P. Langacker, Phys. Rev. D56, 6107(1997).

[28] C. Athanassopoulos et al., Phys. Rev. Lett. 75, 2560(1995); 77, 3082(1996); nucl-ex/9706006.