Robust Eco-Drawing Control of Autonomous Vehicles Connected to Traffic Lights

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Abstract—This paper focuses on the speed planning problem for connected and automated vehicles (CAVs) communicating to traffic lights. The uncertainty of traffic signal timing for signalized intersections on the road is considered. The eco-driving problem is formulated as a data-driven chance constrained robust optimization problem. Effective red light duration (ERD) is defined as a random variable, and describes the feasible passing time through the signalized intersections. In practice, the true probability distribution for ERD is usually unknown. Consequently, a data-driven approach is adopted to formulate chance constraints based on empirical sample data. This incorporates robustness into the eco-driving control problem with respect to uncertain signal timing. Dynamic programming (DP) is employed to solve the optimization problem. Simulation results demonstrate that the proposed method can generate optimal speed reference trajectories with 40% less vehicle fuel consumption, while maintaining the arrival time at a similar level when compared to a modified intelligent driver model (IDM). The proposed control approach significantly improves the controller’s robustness in the face of uncertain signal timing, without requiring to know the distribution of the random variable a priori.

Keywords—Data-driven, Eco-driving, Robust control, Traffic signal, CAV.

I. INTRODUCTION

Connected and automated vehicle (CAV) technology is revolutionizing the automotive industry. In particular, CAVs may significantly improve safety, energy economy, and convenience, with the abilities for self-driving, path/velocity planning, vehicle-to-infrastructure (V2I), and vehicle-to-vehicle (V2V) communications [1], [2]. Optimal eco-driving control — a promising technology enabled by CAVs — is defined as a velocity control method to achieve optimal energy economy [3]. Intuitively speaking, optimal eco-driving seeks the best velocity trajectory to minimize energy consumption in a driving mission.

In the literature, optimal eco-driving is also referred as ecological driving, speed trajectory planning, driving advisory or driver assistance systems [4]. One may conceptualize eco-driving as one layer within a multi-layer hierarchical control architecture for CAVs, depicted in Fig. 1 to clarify the scope of this paper. Due to space limitation, this architecture cannot be comprehensive, but it covers the critical blocks required for CAV implementation.

- The eco-routing block determines the most energy-efficient route, given user requirements and route data (such as road grade, traffic speed, intersection delays, fuel or charging facilities [5]). This block outputs the optimal route, i.e. a set of waypoints along with the intersection locations, speed limits, road grade.
- The eco-driving block is the scope of this paper. This block takes route information and computes the energy-optimal speed trajectory. Some constraints depend on the driving context: for instance, passing a signalized intersection green phases. Historical data of signal phase and timing (SPaT) and dedicated short range communications (DSRC) might be required.

Generally, eco-routing and eco-driving dont have hard real-time requirements, and can be implemented in the Cloud. Conversely, the blocks described next are safety-critical, and need to be executed in real-time on the vehicle.

- The real-time planning block solves several planning problems for CAVs, which may include maneuver planning (e.g. decision to stay in a lane or change), path planning, and trajectory planning. These blocks also depend on the driving context, and their boundaries are quite blurred [6].
- The real-time control block tracks the longitudinal and
lateral trajectories, and interfaces with the vehicle actuators. A CAV control architecture may be interfaced to the vehicle powertrain (e.g., controlling engine torque, braking torque and gear shifting) and to its steering system.

The real-time planning and control blocks require feedback from the vehicle, its position relative to the surrounding environment, and predictions of moving obstacles [1]. A CAV may be equipped with a GPS/INS unit for localization, one or more cameras, radars and lidars for perception, and a DSRC unit for V2V and V2I communication. These data are processed by the localization and perception block, that estimates the position of the ego CAV and its surroundings. To cope with pedestrians, cyclists and non-connected vehicles, an environment prediction block predicts the future trajectories of obstacles on the road.

Over the past 5 years in particular, the optimal eco-driving problem has been intensively studied [8], [9]. These efforts can be classified into two groups, with or without the use of traffic, signal phase and timing (SPaT) information.

When SPaT information is not considered, the eco-driving control scenario is simpler. Dynamic programming (DP) based approach is found in [10] for the speed control of an internal combustion engine (ICE) vehicle to obtain the optimal velocity profile. DP is further applied to an electric vehicle (EV) in [11] for eco-driving control. Both of them have demonstrated that properly designed speed profile is able to significantly improve the vehicle energy efficiency. More comprehensively, a cloud-based velocity profile optimization approach is designed in [12] under a spatial domain formulation. Spatial optimization is further adopted by [13] for ecological driving, with a short-term adaptation level added to avoid traffic congestion. Optimal eco-driving has also been integrated to the energy management of hybrid electric vehicles, with Pontryagin’s Minimal Principle used to solve the optimization problem in [14]. From existing studies, heuristic or optimization based strategies have been successfully utilized to gain better fuel efficiency in eco-driving without the SPaT information used [15], [16].

SPaT information is critical in urban driving environment, but greatly increases the complexity of eco-driving problems [17], [18]. Extra constraints are required when an intersection is controlled by traffic signals [19]. In [20] and [21], traffic lights location, signal phase and timing are taken into account. Optimization is conducted to find the optimal driving pace by enforcing that the vehicle only drives through green light windows. DP is combined with a pruning algorithm to reduce the optimization domain [22]. By considering each signalized intersection as one stage, a multi-stage pseudospectral control method is proposed by [23] in an arterial road structure. Hierarchical MPC is adopted in [24], and demonstrated effective online eco-driving control capabilities. [25] considers the car waiting queue in a multi-lane road scenario, and designed an eco-cooperative adaptive cruise control scheme. Assuming the engine operates only on the optimal brake specific fuel consumption (BSFC) line, sequential convex optimization is applied in [26] for eco-driving through multiple signalized intersections.

In the aforementioned eco-driving studies, the SPaT information is assumed to be deterministic and known. However, this assumption does not match well the reality. Uncertainty always exists due to car waiting queues, pedestrians, cyclists, varying patterns of traffic lights and other factors depicted in Fig. 2(a). Moreover, assuming deterministic SPaT in the control design might cause collision risks when unexpected transition happens. We further discuss this point later.

In Mahler and Vahidi [27], SPaT is predicted, probabilistically, based on current phase and averaged timing data, and further used in a deterministic optimal control approach for velocity planning. In this paper, we follow the approach in [28] and propose a robust control approach which considers the probability distribution of SPaT. However, the actual SPaT information distributions at different intersections are difficult to obtain in practice. Overall, the issue of SPaT uncertainty in optimal eco-driving is significant, yet not fully addressed in the existing literature.

In this paper, the feasible passing time of a vehicle through a signalized intersection is formulated in terms of the Effective Red-light Duration (ERD), illustrated in Fig. 2(b). We investigate a data-driven chance constrained eco-driving approach that is robust to uncertain feasible vehicle passing times, with the goal to simultaneously achieve energy economy and safety.
The key novel contributions include:

- Optimal eco-driving is mathematically formulated with ERD as a key novel feature that captures uncertain passing periods through signalized intersections.
- A distributionally robust chance constrained approach is developed to solve the robust optimal eco-driving control problem.
- Based on [29], a sample data based equivalent reformulation of the distributionally robust chance constrained program is given and applied to robust eco-driving.

The remainder of the paper is organized as follows. Section II describes the vehicle and traffic signal model. Section III introduces the robust optimal eco-driving control strategy in a spatial formulation that considers uncertain feasible passing time at signalized intersections. Section IV details a data-driven equivalent reformulation of the distributionally robust chance constrained program. Section V presents the main results, and Section VI summaries the contributions.

II. VEHICLE AND TRAFFIC SIGNAL MODELING

This section details the vehicle and traffic signal mathematical models used for eco-driving speed trajectory optimization.

A. Vehicle Dynamics

The subject vehicle is equipped with a gasoline ICE and a 6-speed gearbox. Since speed planning is the main task in optimal eco-driving control, we consider longitudinal vehicle dynamics and disregard the lateral dynamics. The longitudinal acceleration is calculated by

\[ ma = \frac{r_g T_{eng} - T_{brk}}{R_{whl}} - mg (\cos(\theta) C_r - \sin(\theta)) - \frac{1}{2} \rho A C_d v^2, \]

where \( m \) is the vehicle mass, \( a \) is the acceleration, \( r_g \) is the product of the gearbox and final drive ratios, \( T_{eng} \) is the ICE output torque, \( R_{whl} \) is the wheel rolling radius, \( g \) is the gravitational acceleration, \( \theta \) is the road grade, and \( C_r \) is the rolling resistance coefficient. Parameters \( \rho, A, C_d \) are the air density, frontal area, and air drag coefficient, respectively. Variable \( v \) is the vehicle velocity, \( T_{brk} \) is the braking torque enforced on the wheels, \( C_{r1} \) and \( C_{r2} \) are rolling resistance constants. The longitudinal velocity is computed by

\[ v = \frac{\omega_{eng}}{r_g} \]

where \( \omega_{eng} \) is the ICE rotation speed. The ICE fuel consumption is modeled as a static nonlinear map \( \psi(\cdot, \cdot) \) of the engine torque and speed:

\[ \dot{m}_{fuel} = \psi(T_{eng}, \omega_{eng}) \]

where \( \dot{m}_{fuel} \) is the instantaneous fuel consumption, and \( \psi \) is the pre-stored fuel map (e.g. a look-up table). The transmission efficiency is ignored in this study. Assume \( r_{fd} \) is the final drive ratio. The integrated transmission ratio is formulated as a function of the gear number \( N_{gb} \),

\[ r_{gb} = f(N_{gb}) \cdot r_{fd}, \quad N_{gb} \in \{1, 2, 3, 4, 5, 6\}. \]

B. Fixed Traffic Signal Timing

Traffic signals are space-indexed and time-varying in the optimal eco-driving problem. Assume the total length of the target driving route is \( s_f \). The position of the \( i \)th traffic signal is denoted by \( s^i \). Therefore,

\[ s^i \in [0, s_f], \quad i = \{1, 2, 3, 4, 5...I\} \]

where \( I \) is the total number of traffic signals along the route.

In our formulation, each traffic signal has an independent periodic clock. We denote the absolute time as \( t \in \mathbb{R}^+ \), and the clock period at intersection \( i \) as \( c_i \in \mathbb{R}^+ \). Clock time zero denotes the beginning of the red light phase. Every intersection can have a different clock period. The red-light duration is denoted by \( c_i \),

\[ c_i \in [0, c_f^i]. \]

Denote by \( c_i^0 \) the clock time when the vehicle departs from its origin. Suppose \( t_{f,i} \) is the time at which the subject vehicle passes through the \( i \)th intersection in the traveling time domain. We can compute the corresponding time in the periodic traffic signal clock timing by

\[ c_{f,i} = (c_i^0 + t_{p,i}) \mod c_f^i \]

where \( c_i^0 \) is the vehicle passing time in the signal-cycling clock. The modulo operator allows for conversion from the traveling time domain to the periodic traffic signal clock time domain.

In the case of un-signalized intersections or crossings, the criticality of motion planning and dynamics control would increase. Also, they would necessarily rely on the on-board perception sensors (camera, radar, lidar) to assess the passing conditions in real time - a topic not discussed in this paper.

C. Uncertain traffic signal timing

In the deterministic setting, the SPaT information (i.e. \( c_i^f \) in (15)) is assumed to be deterministic and perfectly known. However, as illustrated in Fig. 2(b), the feasible passing time at a signalized intersection is usually uncertain and random. In this paper, ERD is defined to describe the feasible passing time, denoted as \( c_{i}^{ERD} \):

\[ c_{i}^{ERD} = c_i^f + \alpha^i. \]

Parameter \( c_i^f \) is the base or nominal red-light duration. The random variable \( \alpha^i \) is a comprehensive stochastic time delay at the \( i \)th intersection, caused by traffic signal variation, vehicle waiting queue or other factors. Assume \( \alpha^i \) is distributed over the green light duration, so that \( \alpha^i \in [0, c_f^i - c_i^f] \).

From the real traffic SPaT data collected in [30], it can be seen that the total signal cycling period is not affected by these uncertain factors, meaning \( c_f^i \) is fixed. Therefore in this paper we assume \( c_f^i \) is deterministic and known for all the intersections.

III. ROBUST OPTIMAL ECO-DRIVING

Next, we formulate and solve the robust optimal eco-driving control problem under uncertain signal timing.
A. Problem formulation

The optimal eco-driving control problem is formulated as a nonlinear spatial trajectory optimization problem which seeks to minimize the vehicle’s total fuel consumption and final arrival time. The cost functional $J$ is defined as

$$ J = \int_0^{s_f} \left( \frac{\dot{m}_{\text{fuel}}(s)}{v(s)} + (1 - \lambda) \frac{d}{ds} t(s) \right) ds $$

(10)

$$ = \lambda \cdot m_{\text{fuel}}(s_f) + (1 - \lambda) \cdot t(s_f) $$

where $\lambda$ is a tuning weight, $s$ is traveled distance, $m_{\text{fuel}}(s_f)$ is the cumulative fuel consumption, and $t(s_f)$ is the arrival time at destination. Engine torque, wheel braking torque and transmission gear number are chosen as the control variables

$$ u = [T_{\text{eng}}(s), T_{\text{brk}}(s), N_{\text{gb}}(s)]^T. $$

(11)

The vehicle velocity and travel time are chosen as the state variables

$$ x = [v(s), t(s)]^T, $$

$$ \frac{dv}{ds}(s) = \frac{a(s)}{v(s)}, \quad \frac{dt}{ds}(s) = \frac{1}{v(s)}. $$

(12)

(13)

Here we restrict $v(s) > 0$. The vehicle stops are modeled by assuming $v(s)$ a small constant positive.

The optimization problem is subject to the vehicle dynamics introduced in (1)-(5), traffic light dynamics from (6)-(8), and the following constraints:

$$ T_{\text{eng}}^{\text{min}} \leq T_{\text{eng}}(s) \leq T_{\text{eng}}^{\text{max}}, \quad \forall s \in [0, s_f], $$

(14a)

$$ T_{\text{brk}}^{\text{min}} \leq T_{\text{brk}}(s) \leq T_{\text{brk}}^{\text{max}}, \quad \forall s \in [0, s_f], $$

(14b)

$$ N_{\text{gb}}(s) \in \{1, 2, 3, 4, 5, 6\}, \quad \forall s \in [0, s_f], $$

(14c)

$$ v(0) = v(s_f) = v_{\text{min}}(s), $$

(14d)

$$ a_{\text{min}} \leq a(s) \leq a_{\text{max}}, \quad \forall s \in [0, s_f], $$

(14e)

$$ v_{\text{min}}(s) \leq v(s) \leq v_{\text{max}}(s), \quad \forall s \in [0, s_f], $$

(14f)

$$ t(s_f) \leq t_f. $$

(14g)

A key beneficial feature of a spatial trajectory formulation (as opposed to temporal) is that the final destination arrival times do not need to be known a priori in the global optimization approach. A pre-set maximal arrival time constraint $t_f$ is imposed on the final state variable $t(s_f)$, to balance fuel economy and travel time.

Based on the level of knowledge on the traffic signal timing, three situations are discussed and formulated: (i) traffic signal timing is deterministic and known; (ii) signal timing is random but the probability distribution is known; (iii) signal timing is random and the probability distribution is unknown.

1) Deterministic Timing: For deterministic traffic signal timing, $c_i^s$ is deterministic and known. The following constraint ensures the vehicle passes through signalized intersections during the green phase,

$$ c_i^s \geq c_i. $$

(15)

The optimal eco-driving formulation of (10)-(14) with constraint (15) is denoted as Problem 1.

2) Random Timing with Known Probability Distribution:

Using the definition of ERD in (9), the traffic signal passing constraint in (15) is modified to

$$ c_p^i \geq c_i^{\text{ERD}} = c_i^s + \alpha_i, \quad \forall \alpha_i \in [0, (c_i^s - c_i^f)]. $$

(16)

However, enforcing the constraint above for all values in the support of $\alpha_i$ may be too restrictive. For example, if there exists a small but non-zero probability of $\alpha_i = (c_i^s - c_i^f)$ during heavy congestions, then all feasible solutions will require the vehicle to wait a whole signal cycle before proceeding through the intersection. This is not reasonable.

Instead, traffic signal passing can be enforced up to a certain probability. Mathematically, this amounts to enforcing a chance constraint

$$ \Pr(c_p^i \geq c_i^s + \alpha_i) \geq 1 - \eta, \quad \forall \eta \in [0, 1], $$

(17)

where $\eta$ is the risk level of running red lights. This is typically taken as a small number such as 0.01, or 0.001. Let $F(\alpha_i)$ denote the cumulative distribution function (CDF) of $\alpha_i$. Equation (17) can be re-written as,

$$ \Pr(\alpha_i \leq c_p^i - c_i^s) = F(c_p^i - c_i^s) \geq 1 - \eta, \quad \forall \eta \in [0, 1]. $$

(18)

We assume the CDF $F(\cdot)$ is bijective, and therefore has an inverse function $F^{-1}(\cdot)$. Thus, we can solve for the optimization variable $c_p^i$ to obtain

$$ c_p^i \geq c_i^s + F^{-1}(1 - \eta), \quad \forall \eta \in [0, 1]. $$

(19)

Again, $c_p^i$ is the passing time of the subject vehicle through the $i$th intersection in the signal-cycling clock, which is a function of the control and state variables, $c_p^i(x, u)$. After replacing the constraint (15) with (19), we arrive at the robust optimal eco-driving control formulation. This formulation is denoted as Problem 2, which requires the CDF of $\alpha_i$ to be known to solve the problem.

3) Random Timing with Unknown Probability Distribution:

Assume $\alpha_i$ is a random variable over time $0$ to $(c_i^s - c_i^f)$, whose probability distribution is usually unknown in real life. The distribution of $\alpha_i$ might also vary at different times of the day (rush hour versus 3:00 am), seasons or locations. Thus, $\alpha_i$ may not be accurately modeled by a single probability distribution function.

The optimal eco-driving formulation of (10)-(14) with unknown distribution of $\alpha_i$ is denoted as Problem 3, which we intend to solve via the data-driven chance constrained optimal control in Section IV.

B. Dynamic Programming

Dynamic programming (DP) is employed to solve the non-linear optimal control problems 1 and 2, with an implementation methodology described in [51]. By breaking a complicated problem into simpler sub-problems in a recursive manner, DP is able to solve multi-stage optimal decision making problems.

Denoting the cost at the $k$th step by $g_k(x_k, u_k)$, the cost function (10) can be approximated by a discrete summation,

$$ J = \sum_{k=0}^{N-1} g_k(x_k, u_k) + g_N(x_N). $$

(20)
According to Bellman’s principle of optimality equation, the objective is to minimize the following cost-to-go function at each stage $k$,

$$V_k(x_k) = \min \{g_k(x_k, u_k) + V_{k+1}(x_{k+1})\}$$

$$= \min \{g_k(x_k, u_k) + V_{k+1}(f(x_k, u_k))\},$$

where $V_k(x_k)$ is the value function (a.k.a. “cost-to-go” function). It represents the minimum cost from stage $k$ to stage $N$, given that the state is initialized to $x_k$ at stage $k$. Note, also, that substitution of general dynamics $x_{k+1} = f(x_k, u_k)$. The terminal cost at $k = N$ is given by the boundary (a.k.a. terminal) conditions in (14).

$$V_N(x_N) = g_N(x_N).$$

Assume the optimal control and state variable trajectories for the optimal control problem are $U^*$ and $X^*$, respectively, then

$$U^* = [u_0^*, u_1^*, u_2^*, ..., u_{N-1}^*]^T,$n

$$X^* = [x_0^*, x_1^*, x_2^*, ..., x_N^*]^T.$$ (23)

Based on terminal condition (22), the optimal control policy can be computed by the following backward recursive algorithm starting from step $(N-1)$ to 0,

$$u_k^* = \arg \min_{u_k \in U_k} \{g_k(x_k, u_k) + V_{k+1}(f(x_k, u_k))\}$$

$$\forall x_k \in X_D,$n

where $U$ and $X_D$ are the feasible control and state variable constraint sets, respectively. Function $f(x, u)$ represents the eco-driving control system dynamics. Interested readers are referred to [22], [33] for a detailed exposition of DP.

IV. DATA-DRIVEN CHANCE CONSTRAINTS

When the probability distribution of $\alpha^i$ is unknown, a data-driven approach is employed to establish equivalent distributionally robust chance constraints for Problem 3.

A. Distributionally Robust Chance Constraints (DCCs)

The optimal eco-driving control solutions could be very sensitive to ambiguous variations of the probability distribution for $\alpha^i$. As such, the solutions may be suboptimal in practice with improperly formulated chance constraints. In this case, an optimization program with distributionally robust chance constraints (DCCs) is proposed, where the constraint in (18) is reformulated into

$$\inf_{\mathcal{P} \in \mathcal{D}} \mathbb{P}\{\alpha^i \leq c_p^i - c_r^i\} \geq 1 - \eta, \quad \forall \eta \in [0, 1],$$ (25)

where $\mathbb{P}$ represents the cumulative probability distribution, and $\mathcal{D}$ is a confidence set of possible probability distributions constructed based on the distributional information. Index $i$ indicates the intersection number, and is omitted in the following equations to simplify notation.

Normally, $\mathcal{D}$ can be easily constructed based on the moment information or other structural properties of the distribution, such as unimodality, symmetry or convexity. However, in practice, distributional information is usually obtained from historical data, empirical experience, or prior knowledge. This is especially true for the stochastic delay $\alpha$ for traffic signal timing. A data-driven chance-constrained approach is adopted from [29] to find an equivalent reformulation of the DCC in (25) based on historical sample data, without any requirement on the distribution properties of $\alpha$.

We use $\phi$-divergence to model the distance between two probability density functions. $\phi$-divergence distance $D_\phi$ is defined as

$$D_\phi(f^* || f) = \int_{\Omega} \phi \left( \frac{f^*(\alpha)}{f(\alpha)} \right) f(\alpha) d\alpha,$$ (26)

where $f^*$ is the true, but usually unknown, probability density function, $f$ is the probability density function estimated from sampled or empirical data, and $\alpha$ is the random variable. We assume $\phi$ to be a convex function defined on $\mathbb{R}^+$, such that $\phi(1) = 0$. Examples include the Kullback-Leibler divergence $\phi_{KL}$, the variance distance divergence $\phi_{VD}$, and the $\chi$ divergence of order 2 $\phi_{\chi^2}$.

$$\phi_{KL}(x) = x \log x - x + 1, \quad \forall x \in [0, +\infty),$$

$$\phi_{VD}(x) = |x - 1|, \quad \forall x \in [0, +\infty),$$

$$\phi_{\chi^2}(x) = (x - 1)^2, \quad \forall x \in [0, +\infty),$$ (27)

To ensure convexity, all the $\phi$-divergence functions above are extended by

$$\phi(x) = +\infty, \quad \forall x \in (-\infty, 0).$$ (28)

Based on the $\phi$-divergence distance metric, we refine constraint (25) by constructing a confidence set of $\mathcal{D}_\phi$ as

$$\mathcal{D}_\phi = \{\mathcal{P} \in \mathcal{P} : D_\phi(f^* || f) \leq d, \phi = \frac{d\mathbb{P}}{d\alpha}\},$$ (29)

where $\mathcal{P}$ represents the set of all probability distributions that $\mathcal{P}$ could be. Scalar $d$ is the allowed distance between the estimated and true distributions, and can be used to tune the risk-aversion level. For a given choice of the $\phi$-divergence function and $d$, the DCC developed in (25) can be converted into

$$\inf_{D_\phi(f^* || f) \leq d} \mathbb{P}\{\alpha^i \leq c_p^i - c_r^i\} \geq 1 - \eta, \quad \forall \eta \in [0, 1].$$ (30)

B. Data-based equivalent reformulation

In Problem 3, the true probability density function $f^*$ in (30) is unknown. However, if empirical data on $\alpha$ is available, then an equivalent reformulation of this DCC can be derived, as proposed and proved by [29]. This equivalent reformulation is adopted in this paper.

Here, let $\hat{\mathbb{P}}$ represent the approximated cumulative probability distribution of $\hat{f}$ estimated from empirical data, then equation (30) is equivalent to

$$\hat{\mathbb{P}}\{\alpha^i \leq c_p^i - c_r^i\} \geq 1 - \eta'_+$$ (31)

where $\eta'_+ = \max\{\eta^i, 0\}$, and $\eta^i$ is estimated from the original risk level and $\phi$-divergence distance. The formulation of $\eta^i$ is given in the Appendix section.
Equations (31), and (41)-(43) comprise the equivalent reformulation of the DCC from (30). For a detailed proof, please refer to Section 3 of [29]. Solving the two-variable nonlinear optimization problem in the reformulated DCC of (31) forms an alternative way to realize robust control.

The perturbed risk level $\eta'$ for KL divergence, $\chi^2$ divergence are given below for implementation purposes, denoted as $\eta'_{KL}$, $\eta'_{VD}$ and $\eta'_{\chi^2}$, respectively. Assume the original risk level $\eta$ is a small number, therefore,

$$\eta'_{VD} = \eta - \frac{d}{2}, \quad (32)$$

$$\eta'_{\chi^2} = \eta - \frac{\sqrt{d^2 + 4d(\eta - \eta^2)} - (1 - 2\eta)d}{2d + 2}, \quad (33)$$

For KL divergence, the perturbed risk level is

$$\eta'_{KL} = 1 - \inf_{x \in (0,1)} \left\{ \frac{e^{-d g(x)\eta} - 1}{x - 1} \right\}, \quad (34)$$

which can be computed by bisection line search.

C. Implementation procedure

The work-flow to establish an equivalent reformulation of the distributionally robust chance constraint based on collected empirical data is summarized in Fig. 3. The chance reliability $\eta$ and $\phi$-divergence distance $d$ are set at the design stage by the control engineer.

![Work-flow to construct an equivalent formulation of the DCC based on empirical or historical data.](image)

The approximated cumulative probability function $\hat{F}$ and the density function $\hat{f}$ are computed from an empirical database. The perturbed risk level $\eta'$ can be found via bisection line searching once the chance reliability $\eta$ and the $\phi$-divergence distance $d$ have been selected.

The equivalent reformulated DCC can be obtained by following the procedure in Fig. 3. Appending this reformulated DCC to Problem 3 yields an optimization program for the optimal eco-driving problem with random $\alpha$ with unknown probability distribution. The only requirement is access to empirical or historical data on $\alpha$.

D. Parameter selection

With the reformulated distributionally robust chance constraint, we can calculate the inverse function of $F^{-1}(1-\eta')$ in (31) without knowing any structural properties of the random variable distribution. Several questions could be asked with respect to the practical implementation of this approach.

- What is the proper size of the historical sample data $N$ for the random variable $\alpha$?
- How to choose the distance between the estimated true distribution $d$, using the $\phi$-divergence criterion?
- What is the relationship between $N$ and $d$?

Intuitively, the three questions above are closely related to each other. The distance $d$ can decrease toward 0 as the sample data size $N$ tends to infinity. Due to limited data, however, $d$ is always selected to be greater than 0 to ensure conservatism. The dimension of the random variable is also critical in deciding a proper size for $N$.

A simulation experiment is conducted in this section to test the performance of the reformulated DCC, and numerically answer the above questions. Here, we assume $\alpha$ is a truncated Gaussian random variable, whose true probability distribution is unknown to the control engineer:

$$\alpha \sim \mathcal{N}_{[0,30]}(6, 16) \in [0, 30]. \quad (35)$$

We set the chance reliability parameter $\eta = 0.03$. The value
of inverse function $F^{-1}(1 - \eta)$ with true distribution is calculated as a benchmark. According to the reformulated equivalent DCC, an alternative inverse function $F^{-1}(1 - \eta')$ is used to replace $F^{-1}(1 - \eta)$. Given randomly generated sample data with size $N_s$ and a pre-defined distance $d$, inverse function $F^{-1}(\cdot)$ can be calculated. The results are demonstrated in Fig.

As can be seen, when $\eta = 0.00001$ and 0.0001, the alternative estimates $F^{-1}$ gradually converge to the true value as $N_s$ grows. When the sample data size $N_s$ is greater than 500, the marginal improvement is trivial. As $\eta$ increases to 0.001, the $\chi^2$ and KL divergence estimates of $F^{-1}$ exceed the true value when $N_s$ is greater than 200, indicating that $F^{-1}(1 - \eta')$ tends to be more conservative when $d$ is relatively larger. This phenomenon is further illustrated in the $\eta = 0.01$ case. Here KL, $\chi^2$ and VD divergence $F^{-1}$ converge to nearly 14.8, 14.3 and 13.7, respectively. These results are all greater than the true value of 13.5. Moreover, we can see that when the sample data size is small (below 200 for example), then $F^{-1}$ converges faster to the true value with greater values of $d$.

Based on the above numerical simulation results and the definition of $d$, we formulate $d$ as a function of the sample data size density $N_s/R_c$, where $R_c$ is the dimension of the sample data,

$$d = f \left( \frac{N_s}{R_c} \right) = a_1e^{b_1 \frac{N_s}{R_c}} + a_2e^{b_2 \frac{N_s}{R_c}} + c, \quad (36)$$

where $a_1$, $b_1$, $a_2$, $b_2$ and $c$ are coefficients that can be fit via regression methods based on sample data. The first term in (36) is used to control the convergence trend, and the second term is used to control the decay rate. The regression model (36) partially answers the three questions above regarding parameter selection in the equivalent DCC. Still, proper selection of these parameters requires decision makers to have adequate empirical knowledge of the random variable.

V. SIMULATIONS

Three optimal eco-driving control approaches for different traffic signal scenarios are simulated in this section. The three approaches include (i) deterministic control via Problem 1; (ii) robust control via Problem 2; and (iii) data-driven chance constrained robust control via Problem 3.

A. Setup and Baseline Model

1) Vehicle and route parameters: The vehicle parameters and engine fuel map used for simulation are extracted from Autonomie, and summarized in Table I. The maximum and minimal velocity limits are set to 16 and 0 m/s, respectively.

Two sample driving routes with 3 and 7 signalized intersections are considered, and named as route 1 and route 2, respectively. All of the full cycling periods $c_f^i$ and red-light durations $c_r^i$ are set to 60s and 30s, respectively. The beginning time $c_b^i$ is arbitrarily selected between 0 and 30s. Other realistic selections of the full cycling time and red-light duration can also be incorporated in the proposed optimal eco-driving control strategy. The position and timing information for the two sample routes are summarized in Table II.

TABLE I. SUBJECT VEHICLE PARAMETERS

| Parameter (Unit) | Value | Parameter (Unit) | Value |
|------------------|-------|------------------|-------|
| $m$ (kg)         | 1745  | $r_f$ (rad/s)   | 3.51  |
| $R_{w, i}$ (m)   | 0.341 | $\omega_{ma}^i$ (rad/s) | 600  |
| $A$ (m$^3$)      | 2.841 | $T_{max}^i$ (Nm) | 240  |
| $p$ (kg/m$^2$)   | 1.1985| gearbox ratios  | 4,584, 2,964 |
| $C_d$            | 0.356 | $C_{z}$         | 0.0084, 1.2e-4 |
| $C_s$            |       |                  | 1.0, 7.4 |

TABLE II. POSITION AND SPAT INFORMATION OF SAMPLE ROUTES

| No. | Type | $D^1$ (m) | $c_f^i$ | $D^2$ (m) | $c_f^i$ | $c_r^i$ | $c_r^i$ |
|-----|------|-----------|---------|-----------|---------|---------|---------|
| 1   | signal | 200       | 10      | signal    | 200     | 0       | 60      |
| 2   | signal | 400       | 30      | signal    | 400     | 20      | 60      |
| 3   | signal | 600       | 0       | signal    | 600     | 0       | 60      |
| 4   | stop   | 800       | –       | signal    | 800     | 20      | 60      |
| 5   | stop   | 1000      | 0       | –         | 0       | 60      |
| 6   | signal | 1200      | 25      | –         | 25      | 60      |
| 7   | signal | 1400      | 10      | –         | 10      | 60      |
| 8   | stop   | 1600      | –       | –         | 60      |

* Arbitrarily selected values.

2) Modified intelligent driver model: A modified intelligent driver model (IDM) is introduced as a baseline model for comparison purposes. The basic IDM is based on the computation of a desired distance between the eco-vehicle and the front vehicle. When there is no preceding vehicle, the road speed limit is used as a velocity reference.

Assuming that the desired distance between the subject vehicle and front vehicle is $D_{des}$, then

$$D_{des} = D_{min} + \frac{v \cdot t_{hw}}{2\sqrt{a_{max}a_c}} \quad (37)$$

where $D_{min}$ is the minimal vehicle distance, $t_{hw}$ is the desired time headway to the preceding vehicle, $D_{sf}$ is the real distance between the subject vehicle and preceding vehicle, $a_{max}$ is the maximal vehicle acceleration ability, and $a_c$ is the preferred deceleration for comfort.

The vehicle acceleration at each time step is computed by comparing the desired gap distance with the current distance. An additional speed limit term is added to ensure safety,

$$a = a_{max} \left[ 1 - \left( \frac{v}{v_{max}} \right)^4 - \left( \frac{D_{des}}{D_{sf}} \right)^2 \right] \quad (38)$$

To interact with traffic signals or stop signs, we modified the IDM by enabling the driver model to preview the traffic signal status at a human-vision distance $D_v$. Assuming that the current location of the vehicle is $s$, the driver model in (38) becomes

$$a = \left\{ \begin{array}{ll} a_{max} \left[ 1 - \left( \frac{v}{v_{max}} \right)^4 - \left( \frac{D_{des}}{D_{sf}} \right)^2 \right], & \text{if } S_{tss}(s + D_v) = 0, \\ -\frac{v^2}{2D_{sf}}, & \text{if } S_{tss}(s + D_v) = 1. \end{array} \right. \quad (39)$$

where $S_{tss}$ is the traffic signal status, with 1 meaning the traffic signal is red, and 0 meaning the traffic signal is green or yellow. Variable $D_{sf}$ here indicates the distance to
the traffic light when there is no preceding vehicle. In the subsequent simulations, the preview-vision distance is set to $D_v = 100$ meters.

### B. Deterministic Optimal Eco-Driving

Two approaches are compared in this section by tuning the cost function weight $\lambda$.

- Time emphasized eco-driving with $\lambda = 0$, representing minimal arrival time control;
- Fuel emphasized eco-driving with $\lambda = 1$, representing minimal fuel consumption control.

For route 1, the vehicle velocity and traveling time results from the modified IDM and the two eco-driving control approaches are plotted in Fig. 5(a)-(b). In the modified IDM approach the driver starts decelerating the vehicle at $s=100m$ when it “sees” a red traffic signal in front. Due to the lack of full SPaT information, modified IDM is not able to incorporate future signal dynamics into its control. About 10 seconds later, it switches from deceleration to acceleration at $s=170m$, as the signal turns green. This behavior wastes fuel. At the second traffic signal the red light blocks the intersection. The modified IDM waits for 20 seconds until the light turns green. A similar scenario happened at the third signalized intersection, but with a shorter waiting time. The vehicle eventually arrives at the stop sign (the destination) at $t=117s$.

Considering the final arrival time for modified IDM is 117s, we set the maximum arrival time constraint $t_f$ in (14) to be 120s for route 1. This ensures the optimal eco-driving approaches achieve commensurate arrival times.

In Fig. 5(b), the vehicle with eco-driving smoothly drives through the first two intersections by adjusting the velocity between 7-9m/s. A deeper deceleration happens just before driving through the 3rd signalized intersection ($s=600m$) to wait for a green light. After that, the vehicle velocity is restored to a higher level to ensure it arrives at the final destination within the time constraint.

The main difference between the time emphasized and fuel emphasized approaches is that the former one applies higher speeds during the 600-800m range, which is after the last signalized intersection. In this case, the time emphasized approach uses less time to complete the driving route. The total driving time in the time emphasized case is $t=107.6s$, which is about 3s shorter than the fuel emphasized case.

The vehicle velocity and traveling time results for route 2 are shown in Fig. 5(c)-(d), where similar phenomena are observed. The modified IDM avoids complete stops at 3 signalized intersections out of 7, with a final arrival time of 226s. Here, $t_f$ is defined as 250s for optimal eco-driving control. As expected, the eco-driving approaches avoid aggressive accelerations of the vehicle, and cross most of the signalized intersections at lower speeds without any complete stops. The time emphasized and fuel emphasized eco-driving vehicle arrive at the destination at $t=218.6s$ and 228.5s, respectively. Not surprisingly, this arrival time advantage is achieved by imposing higher speed while driving between the last two intersections. Based on the above analysis, we can draw the following remark.

**Remark.** Traffic signal timing awareness can help the vehicle avoid unnecessary deceleration or idles, without sacrificing the arrival time at the destination.

Time dependent vehicle velocity, acceleration, engine speed, and engine torque results for route 1 are illustrated in Fig. 6. In the optimal eco-driving cases, the vehicle acceleration...
Fig. 6. Vehicle velocity, acceleration, engine speed, torque results versus time for route 1.

Fig. 7. Engine operating point comparison on the BSFC map for route 1.

variation is generally milder than modified IDM. The engine speed works between 200-300 rad/s. The engine torque is also much smaller than modified IDM, except at time 90-100s and time 0, when the engine generates more power to accelerate the vehicle to higher speed to guarantee the arrival time constraint is respected.

The engine brake specific fuel consumption (BSFC) results for route 1 are shown in Fig. 7. The arrival time, average BSFC and total engine fuel consumption results for route 1 and 2 reported in Table III.

The average BSFC values from the modified IDM case is less than the eco-driving approaches. This might seem counter-intuitive from an engine point-of-view, since lower BSFC values often results in better fuel economy. However, the total fuel consumption in modified IDM is much higher than the eco-driving results. The average BSFC value from time emphasized eco-driving is also lower than the fuel emphasized one, but still consumes more fuel for both test routes. The explanation is as follows: although the average engine fuel efficiency is lower, the greater wheel power requirements for modified IDM and time-emphasized eco-driving produce greater total fuel consumption. The average BSFC of fuel emphasized eco-driving is 19-23% higher than that of modified IDM, while the overall fuel consumption is 50-57% less. The final arrival time of the two approaches is very close, within 6% or less.

Figure 8 shows the sensitivity of the normalized vehicle performance trade-off lines to the parameter $\lambda$ in the cost function. When $\lambda=0$ (time emphasized control), the final arrival time is 2.5% and 4.2% lower than when $\lambda=1$ (fuel emphasized control). However, the latter one consumes 22% and 29% less fuel in the 3-light and 7-light test routes, respectively. This result indicates that with little sacrifice on arrival time, a considerable fuel efficiency improvement can be achieved with optimal eco-driving.

Remark. The optimal velocity trajectories with deterministic traffic signal timing often pass through intersections exactly when the light transitions from red to green. As the switching moment approaches, SPaT uncertainties may require unexpected and sudden stopping.

The remark above can be visualized observing the green circles in Fig. 5. This is an undesirable feature of the deterministic
control formulation. Namely, when the red light duration varies away from the predicted value, a lower layer controller may unexpectedly and suddenly stop the vehicle to ensure safety. This situation endangers passenger comfort and wastes fuel. The data-driven chance constrained robust optimal eco-driving approach is developed to address this problem, with the results presented next.

C. Data-Driven Robust Eco-Driving

In our formulation, \( \alpha \) accounts for the delay caused by all possible uncertainties at an intersection. However, considering the difficulty of measuring car waiting queue and pedestrian passing data in practice, we only utilize traffic signal data to verify the proposed approach.

1) Traffic signal timing data: We adopt an empirical probability distribution for the traffic signal phase duration from an intersection in Montgomery, MD [30] as a case study. The traffic signal is adaptively self-controlled according to the volume of passing vehicles, which is a common design in real life. From the collected data, the stochastic part of ERD, \( \alpha \), follows the probability distribution shown in Fig. 10 (red curves). Similar sample distributions can also be obtained from [36], [37].

To benchmark the eco-driving approach, we assume the distribution in Fig. 10(a) extracted from the collected data is the TRUE one. Artificially generated sample data are used for control implementation and evaluation.

This probability distribution is selected because of its interesting bi-modal characteristic. The most likely red light delays are between 1-2s, possibly due to a small volume of passing vehicles. Another probability peak occurs in the range of 9.5-11.5s of delay. We speculate that this might correspond to rush hour traffic, when the traffic flow along the perpendicular road is much greater than usual.

2) Data-driven Chance Constraint: About 1000 sample data points are generated randomly by following the probability distribution in Fig. 10 used as the empirical data to estimate functions \( \hat{P} \) and \( f \). We set \( \eta = 0.03 \) and \( d = 0.001 \). The sample data histogram, calculated empirical CDF, and resultant \( F^{-1} \) estimates are all visualized in Fig. 10.

In Fig. 10(b), the equivalent \( F^{-1}(1 - \eta^*_f) \) values calculated from \( \chi^2 \), VD and KL divergences are compared with the true \( F^{-1}(1 - \eta) \). We can see the \( F^{-1}(1 - \eta^*_f) \) results...
are slightly greater than the true $F^{-1}$, with VD divergence obtaining the tightest value. This result is consistent to the perturbed risk level formulations of $\eta^*_+ \in (32, 34)$, which mathematically impose some conservatism due to uncertainty on the probability distribution.

**Remark.** The equivalent $F^{-1}(1−\eta^*_+)$ estimates can be very close to the true $F^{-1}(1−\eta)$ with proper data samples and selections for the $\phi$-divergence function and distance $d$.

As analyzed in Section 1V-D, the probability distribution distance $d$ has a crucial effect on the perturbed risk level $\eta^*_+$. A Monte Carlo simulation of $F^{-1}(1−\eta^*_+)$ with $\chi^2$ divergence when distance $d$ varies from 0 to 0.1 is conducted. The results are illustrated in Fig. 8. If $d=0.1$, most of the $F^{-1}(1−\eta^*_+)$ values are between 12.5s and 13.5s, whereas the true $F^{-1}(1−\eta)$ value is about 11.5s. As we gradually reduce $d$, the deviation of $F^{-1}$ decreases tremendously. When $d=0$, the average $F^{-1}(1−\eta^*_+)$ reaches its minimal value of 11.6s, which is very close to the true $F^{-1}$ metric 11.5s. This result is consistent to the inverse function $F^{-1}$ analysis with an assumed truncated Gaussian distribution on $\alpha$ in Fig. 4. When the sample data size $N_x$ is greater than 500 with a small number of $d$, the reformulated perturbed risk level is close to the true value. Reducing $d$ is unable to further eliminate the deviation of $F^{-1}$. Instead, a larger sample data size is needed.

3) Data-Driven Robust Control: In this section, the cost function weight $\lambda$ is always set as 1, meaning only the fuel emphasized control is evaluated here.

Before looking at the results, we recall the hierarchical control architecture introduced in Fig. 1. The optimal eco-driving control layer is used only for velocity planning. In practice, a lower layer controller will adaptively adjust the velocity based on real-time measurements, to ensure safety. In this paper, we only show the velocity planning results without the lower-level controller involved.

The equivalent $F^{-1}(1−\eta^*_+)$ calculated with $N_x=1000$, $d=0.001$ and $\eta=0.03$, 0.1 and 0.3 are applied in the robust eco-driving approach. The vehicle trajectory and speed profile results compared to the deterministic control result for route 1 are demonstrated in Fig. 11(a)-(b). As can be seen in Fig. 11(a), the deterministic approach only considers the base red-light duration $c^*_i$ of the traffic signal. At the 3rd intersection, the vehicle drives through the intersection right after the base red-light duration ends. With the data-driven chance constraint used, the vehicle intentionally slows down before driving through the 2nd and 3rd intersections, in order to avoid high probability time delays of the red light. When $\eta=0.03$, $F^{-1}(1−\eta^*_+)$ is nearly 12s, meaning a red-light time delay of 12s is considered, which is the longest one in all cases. Consequently, the most conservative tuning ($\eta=0.03$) arrives at the destination the latest among all cases. In Fig. 11(b), we can see that the speed profiles in the four simulations are similar during the first 60s, before driving through the 2nd intersection. The speed profiles diverge thereafter. To ensure the vehicle can arrive at the destination within 120s, the vehicle speed increases to 16m/s (road speed limit) at $t=105$ and 108s when $\eta=0.03$ and 0.1, respectively.

Due to the uncertainty of the red-light durations, there is a probability that the vehicle fails to drive through the intersection with a planned speed profile. We define “Passing Probability” to evaluate the robustness, calculated by

$$\Pr(\text{passing}) = \Pr(c^*_p \geq c^*_i + \alpha^i).$$ (40)

Fig. 11(c) provides the average passing probability for the three intersections for different approaches shown in Fig. 11(a). In the deterministic approach, where red-light duration uncertainty is not considered, the average passing probability of the three signalized intersections is only 54%. With $\eta$ set as
The above passing probability results indicate that, even though we choose $\eta=0.03$, there is still 5.5% probability that the vehicle requires intervention from the lower control layer to avoid a dangerous situation. This point emphasizes the relative roles of the Motion Plan Layer and Eco-Driving Control Layer in Fig. 4. Namely, even with our proposed robust eco-driving control approach, a Motion Plan layer is still necessary to intervene in the presence of local unexpected disturbances experienced in real-time.

D. Fuel consumption and passing probability evaluation

The overall fuel consumption and intersection passing probability of the proposed data-driven chance constrained robust optimal eco-driving control is further evaluated in this section. The sample database size of $\alpha$ is chosen to be 50, 250, 500, 1000, and 2000, respectively. The corresponding $\phi$-divergence distance $d$ is determined based on the analysis conducted in Section IV-D and Fig. 4. The chance reliability $\eta$ is always set to 0.03.

For each control approach, 100 simulations with randomly generated values of $\alpha$ are conducted. The resulting average arrival time (in seconds), fuel consumption (in grams), and passing probability are shown in Table IV. We can see that the modified IDM consumed about 90.6g and 172.7g of fuel on routes 1 and 2, respectively. All the robust eco-driving approaches successfully reduced the fuel consumption to under 53g and 84g, respectively. The corresponding fuel efficiency improvements are 42% and 51%.

With proper selections of $d$, the data-driven chance constrained robust optimal eco-driving control is able to achieve similar performance to the 'True $F^{-1}$' case, where the real distribution of $\alpha$ is assumed to be known as a benchmark. Even when the sample database size is relative small (for example 50), the arrival time and fuel consumption results are nearly equal to the results with large database sizes. The passing probability of modified IDM is always 100%, by construction. This is because modified IDM is, in fact, a deterministic control approach, and is able to observe the traffic signal SPaT in real time to adjust the vehicle speeds. We can see from Table IV that the average passing probability of data-driven chance constrained approaches are generally higher than that of true $F^{-1}$ case. This result indicates the robustness and safety achieved by the equivalent reformulation of DCCs.

VI. CONCLUSIONS

A data-driven chance constrained eco-driving control approach is proposed for velocity trajectory planning of CAVs driving through signalized intersections with uncertain signal timing. We specifically focus on endowing the optimal velocity trajectory with robustness with respect to stochastic red light delays, thus enhancing safety and fuel economy. An equivalent reformulation of the distributionally robust chance constraints is derived based on empirical sample data of the stochastic delay of red light durations. The data-driven chance constrained eco-driving controller is formulated in the spatial domain and solved by DP. Simulation results demonstrate that the fuel consumption can be significantly reduced by sacrificing less than 5% of the arrival time. The data-driven chance constrained control approach is able to effectively improve the control robustness without requiring prior knowledge of the moments or other structural properties of the random variable distribution.

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\[\eta' = 1 - \inf_{z > 0, \theta_0 + \pi z \leq l_0, m(\phi^*) \leq \theta_0 + \pi z \leq m(\phi^*)} \left\{ \frac{\phi^*(\theta_0 + z) - \theta_0 - \eta z + d}{\phi^*(\theta_0 + z) - \phi^*(\theta_0)} \right\} \]

and

\[l_0 = \lim_{x \to +\infty} \frac{\phi(x)}{x} \]

\[\pi = \begin{cases} -\infty, & \text{for } \mu([f_0 = 0]) = 0 \\ 0, & \text{for } \mu([f_0 = 0]) > 0 \\ 1, & \text{otherwise} \end{cases} \]

where \(\mu(\cdot)\) represents the Lebesgue measure on \(\mathbb{R}\) and \([f_0 = 0] := \{\alpha \in \Omega : f_0(\alpha) = 0\}\), \(\phi^*\) is the conjugate function of \(\phi\), \(\phi\) is a convex function defined on \(\mathbb{R}^+\), with \(\phi(1) = 0\) and \(\phi(x) = +\infty\) for \(x < 0\). Variables \(z\) and \(\theta_0\) are the dual variables corresponding to the following constraints, respectively:

\[\int_{\Omega} \phi \left( \frac{f^*(\alpha)}{f(\alpha)} \right) f(\alpha) d\alpha \leq d, \quad \int_{\Omega} f(\alpha) d\alpha = 1. \]

Specifically, for general \(\phi\)-divergence, \(l_0 \geq m(\phi^*)\) always stands. Without any loss of generality, the constraint \(\theta_0 + \pi z \leq l_0\) can be relaxed. Interested readers should consult Appendix 2 of [29].

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