An uncertainty principle for star formation – III. The characteristic emission time-scales of star formation rate tracers

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ABSTRACT

We recently presented a new statistical method to constrain the physics of star formation and feedback on the cloud scale. Fundamentally, this new method is only able to recover the relative durations of different evolutionary phases. It therefore requires a “reference time-scale” to convert the relative time-scales into absolute values. The phase over which the star formation rate (SFR) tracer is visible is the logical choice to assign this reference time-scale to, since it can be characterised using stellar population synthesis models. In this paper, we use hydrodynamical disc galaxy simulations to produce synthetic emission maps of several SFR tracers and apply our statistical method to measure the associated characteristic time-scale of each tracer. These cover 12 ultraviolet (UV) filters (from GALEX, Swift, and HST) covering a wavelength range 150−350 nm, as well as Hα. For solar-metallicity environments, we find the characteristic time-scales for Hα with (without) continuum subtraction to be ∼4.3 (6−16) Myr and for the UV filters to be in the range 17−33 Myr, monotonically increasing with wavelength. We find that the characteristic time-scale decreases towards higher metallicities, as well as to lower star formation rate surface densities, if the stellar initial mass function is not well-sampled. We provide fitting functions to the resulting reference time-scales to facilitate observational applications of our statistical method across a wide range of galactic environments. More generally, our results predict the time-scales over which photoionisation and UV heating take place around star-forming regions.

Key words: galaxies: evolution – galaxies: ISM – galaxies: star formation – galaxies: stellar content – H II regions

1 INTRODUCTION

Characterising the time-scales of astrophysical processes is a common problem within astronomy as most of these processes take much longer than a human lifetime. Specifically within the field of star formation, characterising the physical processes involved in the evolution of clouds and star forming regions, as well as obtaining absolute star formation rates (SFRs) from the luminosity at a given wavelength, requires knowledge of the underlying time-scales. The emission in different wavebands is dominated by different types of stars (Hao et al. 2011) and so it is possible to determine relationships between the luminosity at a given wavelength and the SFR (Calzetti et al. 2007; Hao et al. 2011; Murphy et al. 2011; Kennicutt & Evans 2012, among others). We can also associate characteristic time-scales for which the emission is visible from the expected lifetimes of these dominant stars. For example, in order to produce Hα emission, high energy photons are required to ionise the surrounding interstellar medium. These high energy photons must originate from very massive stars which have lifetimes less than 10 Myr (Leitherer et al. 1999; Murphy et al. 2011). Since the stars that are producing the Hα emission live no longer than 10 Myr, the emission itself should have a characteristic time-scale that is of a similar order of magnitude. This characteristic time-scale is an important property to determine, because it characterises how long photoionising feedback can act on the surrounding interstellar medium.

In Kruijssen & Longmore (2014) we put forward a new statistical method, titled “an uncertainty principle for star formation” (hereafter KL14 principle), which can be applied to observational...
data using the HEISENBERG code (Kruijssen et al. 2018). This new method is able to constrain the relative durations of the different star formation phases. However, to turn these into an absolute timeline, it is critical to have access to a known “reference time-scale”. For observational applications of the method, the most direct way of providing such a reference time-scale is by characterising the time-scales of SFR tracers. The statistical method is designed for applications over a wide range of galactic environments and across cosmic time; for a single time-scale to be applicable to all these different environments is unlikely. For example, we show in Figure A1, using STARBURST99 (Leitherer et al. 1999; Vázquez & Leitherer 2005) simulations, that metallicity has a strong impact on the maximum lifetime of H\textalpha emission in star forming regions. Therefore, it is important to understand how these characteristic time-scales change with the galactic environment. Dependences of interest may be on the particular SFR tracer (ideally these cover a range of wavelengths), the metallicity, or the (sampling of the) stellar initial mass function (IMF).

Previous work has attempted to define characteristic time-scales for different SFR tracers, which has shown that there is no single method for doing so. Instead, there exists a range of possible methods – such as a luminosity-weighted mean, a percentage intensity change, or a percentage of the cumulative emission – that can result in differences up to an order of magnitude in time-scale (Leroy et al. 2012; Kennicutt & Evans 2012). We use a self-consistent approach to determine the SFR tracer time-scale; that is, we apply the KL14 principle itself to synthetic emission maps to accurately measure the emission time-scales of the SFR tracers. We can then use these time-scales in future applications of the KL14 principle as reference time-scales.

The structure of this paper is as follows. In Section 2, we discuss the KL14 principle and the practical application of the associated HEISENBERG code. We present the method used for constraining the characteristic time-scales of different SFR tracers with well-sampled IMFs in Section 3 and discuss the results for solar metallicity in Section 4. In Section 5, we demonstrate how the time-scales depend on metallicity. In Section 6, we demonstrate the effects of incomplete IMF sampling in low SFR surface density environments. Finally, we summarise these results and present our conclusions in Section 7.

2 UNCERTAINTY PRINCIPLE FOR STAR FORMATION

The analysis presented in this paper is based on the KL14 principle and its specific realisation in the HEISENBERG code (Kruijssen et al. 2018). Therefore, we first summarise the method here.

The goal of the KL14 principle is to constrain the cloud-scale physics of star formation and feedback using images of galaxies mapped in tracers that track different stages of the star formation process. By determining the duration of successive phases of an evolutionary process, one can gain a better understanding of the physics involved. In Kruijssen & Longmore (2014) and Kruijssen et al. (2018), the method is illustrated with the formation of stars from gas. If we provide HEISENBERG with a galaxy mapped in an appropriate gas tracer (e.g. CO) and SFR tracer (e.g. H\textalpha), HEISENBERG determines, among other quantities, the lifetime of molecular clouds within the observed galaxy. Depending on the combination of tracers one uses, HEISENBERG can constrain the durations of different stages of the star formation timeline.

In basic terms, the KL14 principle represents the galaxy as a collection of independent star-forming regions, where each region is evolving along its timeline independently of the neighbouring regions. The number of regions that are emitting in each of the two tracers (with the possibility that the region is in a transition phase, and so emitting in both) is related to the duration of that phase: the shorter the duration of a phase, the less likely we are to observe a region in that phase. HEISENBERG quantifies this statistically to determine the duration of one phase relative to the other. If the duration of one of these two phases is known a priori, it can be used as a “reference time-scale” to calibrate the timeline. In this paper, the “reference map” and “reference time-scale” refer to the emission map associated to the phase of known duration and its duration, respectively. The role of “reference map” is generally taken up by an SFR tracer emission map because its emission time-scale can be estimated using stellar population synthesis modelling. Calibrating these time-scales is the goal of this paper.

We outline here the procedure used by HEISENBERG and refer the reader to Kruijssen et al. (2018) for the specific details. The method fundamentally relies on the findings of Kruijssen & Longmore (2014), where it is shown that the relative change of the flux ratio is a direct function of the underlying evolutionary timeline. The procedure is as follows.

(i) The user provides HEISENBERG with an input file and two emission maps of the same galaxy tracing two successive phases (hereafter phase 1 and phase 2). The phase 2 map is used as the reference map with reference time-scale, \(t_2\), specified by the user in the input file. As mentioned above, this is usually the SFR tracer map and its associated characteristic time-scale.

(ii) HEISENBERG identifies the location of emission peaks in each input map.

(iii) Each map is convolved using a top hat kernel for a range of aperture sizes (as specified in the input file).

(iv) For each pair of convolved maps, HEISENBERG calculates the flux ratio of the two maps (\(F_{12}\)) relative to the galactic average (\(F_{12,\text{gal}}\)), within an aperture corresponding to the convolution scale.
centred at each identified peak. As the aperture size increases, the calculated flux ratio tends to the galactic average (i.e. \( F_{12}/F_{12,\text{gal}} \) tends to unity).

(v) Each aperture size has two associated flux ratios: the average flux ratio calculated at the locations of the emission peaks in the phase 1 map and another for those in the phase 2 map.

(vi) Showing the change in flux ratios as a function of aperture size produces a “tuning fork diagram” (see Figure 1), where each branch corresponds to the flux ratio when focusing on the locations of emission peaks in the phase 1 and phase 2 maps.

(vii) The models which describe the shapes of the two branches (see Kruĳssen et al. 2018, Equations 81 and 82) have three free parameters: the typical separation length between identified peaks (\( \lambda \)), the relative temporal overlap between the two phases (\( t_{\text{over}}/t_1 \)), and the relative duration of phase 1 (\( t_1/t_2 \)).

(viii) HEISENBERG carries out a reduced-\( \chi^2 \) fit to find the model parameters that best describe the data.

(ix) The reference time-scale specified in the input file (\( t_R \)) is used to recover \( t_{\text{over}} \) and \( t_1 \) as absolute time-scales.

The procedure described above assumes that the reference time-scale is known; however, this is not necessarily true. Without this reference time-scale, HEISENBERG can only determine relative lifetimes. In this paper, we constrain reference time-scales for a series of SFR tracers using the method above; however, the role of the SFR tracer map changes. In normal observational applications, HEISENBERG utilises the SFR tracer map as the reference, along with the associated characteristic time-scale of the SFR tracer, to calibrate the evolutionary timeline. In Section 3, we describe how we use a simulated galaxy to create reference maps from stars within a known age bin (the duration of which is used as \( t_R \)) to calibrate the characteristic time-scale of synthetic SFR tracer emission maps (i.e. the time-scale referred to as \( t_R \) above). As long as the two maps trace successive phases of the evolution, HEISENBERG can be used to constrain their relative lifetimes; the physical quantity used in each map (e.g. luminosity or mass surface density) is not relevant.

### 3 METHOD FOR CALCULATING THE CHARACTERISTIC EMISSION TIME-SCALES OF SFR TRACERS FOR A FULLY-SAMPLED IMF

We present here the steps we take to find the characteristic time-scales for Hα and ultraviolet (UV) SFR tracers (see Table 1 for details) using synthetic emission maps and the HEISENBERG code. As we described in Section 2, HEISENBERG can determine the duration of the first input map from the second by using the latter as a reference map (i.e. the map showing the evolutionary phase of known duration). This means that if we provide HEISENBERG with a galaxy map of one of the SFR tracers (e.g. Hα) along with a reference map, HEISENBERG can provide us with the time-scale associated with that tracer. We follow this approach to measure the SFR tracer time-scale (such as the metallicity, see Section 5, and IMF

#### Table 1. In this paper we find characteristic time-scales for the star formation rate tracers detailed here.

| Telescope      | Instrument | Filter | \( \bar{\lambda}_w \) [nm] |
|----------------|------------|--------|-----------------------------|
| GALEX          | FUV        |        | 153.9                       |
| GALEX          | NUV        |        | 231.6                       |
| Swift          | UVOT       | M2     | 225.6                       |
| Swift          | UVOT       | W1     | 261.7                       |
| Swift          | UVOT       | W2     | 208.4                       |
| HST            | WFC3       | UVIS1 F218W | 223.3              |
| HST            | WFC3       | UVIS1 F225W | 238.0              |
| HST            | WFC3       | UVIS1 F275W | 271.5              |
| HST            | WFC3       | UVIS1 F336W | 335.8              |
| HST            | WPFC2      | F255W  | 259.5                       |
| HST            | WPFC2      | F200W  | 297.4                       |
| HST            | WPFC2      | F336W  | 335.0                       |

(b) The Hα filters we consider.

| Filter | Details |
|--------|---------|
| Hα−W   | Hα emission with continuum subtraction. This is not a true filter but a direct measurement of the hydrogen-ionizing photon emission, see Section 3.3 for details. |
| Hα+W   | A narrow band filter including Hα and the continuum as defined in Equation 3. The total filter width is indicated by \( W \); we consider \( W = \{10, 20, 40, 80, 160\} \) Å. |

The results in this paper are based on the “high-resolution” simulated galaxy from Kruĳssen et al. (2018). We set up the initial conditions for this galaxy using the methods described in Springel et al. (2005). The simulation has a total of \( 4.95 \times 10^9 \) particles: \( 1 \times 10^6 \) in the dark matter halo, \( 2.31 \times 10^6 \) in the stellar disc, \( 1.54 \times 10^6 \) in the gas disc, and \( 1 \times 10^5 \) in the bulge. The dark matter halo particles have a mass of \( 9 \times 10^5 \) \( M_\odot \) and the star and gas particle types both have a mass of \( 2.7 \times 10^4 \) \( M_\odot \). This gives us a \( 9 \times 10^{11} \) \( M_\odot \) halo, \( 1.05 \times 10^{10} \) \( M_\odot \) disc (60 per cent in stars and 40 per cent in gas), and \( 2.7 \times 10^8 \) \( M_\odot \) bulge.

We then evolve the initial conditions for 2.2 Gyr using the smooth particle hydrodynamics (SPH) code P-GADGET-3 (last described by Springel 2005), which makes use of the SPHGA\(L\) hydrodynamics solver. SPHGA\(L\) was implemented by Hu et al. (2014) in order to overcome many of the numerical issues associated with traditional SPH. To be considered for star formation, gas particles require temperatures less than \( 1.2 \times 10^4 \) K and hydrogen particle density more than \( 0.5 \) cm\(^{-3}\). Stars are formed from eligible gas particles stochastically according to the method described in Katz (1992). Supernova explosions return mass, momentum, and thermal energy back to the ISM; these are distributed using a kernel weighting to the 10 nearest gas particles. The result of the simulation is a near-\(L^*\) isolated flocculent disc galaxy. Figure 2 shows a
stellar reference map (Section 3.2) and a synthetic Hα – map (Section 3.3) of this galaxy.

3.2 Generation of the reference maps

The role the reference map plays in the HEISENBERG code is to calibrate the absolute evolutionary timeline of the star formation process. In the context of this paper, it is used to calibrate the characteristic time-scale of the synthetic SFR tracer emission maps.

In our experiments aimed at measuring the SFR tracer emission time-scales, we need to know the reference time-scale exactly. For this reason, we use simulated rather than real galaxies. We produce reference maps from the simulation by generating mass surface density maps of the star particles in a specific age bin. The width of this age bin acts as the reference time-scale, \( t_R \). We smooth the selected star particles using a Wendland C\(^4\) kernel (Dehnen & Aly 2012) (the same kernel SPHGaL introduces into P-Gadget-3) over the 200 nearest neighbouring particles; this produces a realistic reference map (i.e. not a map of point particles).

In principle, we have a free choice over the age bin we use. However, for the best results and the most realistic set-up there are a few restrictions. In Section 2, we note that HEISENBERG is designed such that the reference map corresponds to the second phase of the evolutionary timeline. To avoid any overlap between the evolutionary phases, the minimum age of the star particles used in the reference map \( t_M \) must therefore be at least the duration of the first (SFR tracer emission) phase \( t_{E,0} \), we include the subscript “0” to indicate that this is for a well sampled IMF; this distinction is necessary in Section 6 of the evolutionary timeline. This defines the lower limit of the stellar age bin used to generate the reference map:

\[ t_M \gtrsim t_{E,0} \quad (1) \]

At the same time, it is undesirable to select a value of \( t_M \) much larger than the galactic dynamical time because groups of star particles formed in the same clouds may have dispersed. We therefore prefer using \( t_M \approx t_{E,0} \).

Kruĳssen et al. (2018) show that HEISENBERG provides the most accurate measurement of the underlying time-scales if the duration associated to both of the input maps is similar (within a factor of 10). This finding sets the preferred width of the age bin:

\[ t_R \approx t_{E,0} \quad (2) \]

In Figure 3, we show a schematic timeline of how \( t_M \), \( t_R \), and \( t_{E,0} \) are related.

To quantify (and avoid) any systematic biases of the measured SFR tracer time-scale, we investigate the dependence on the choice of stellar age bin used to generate the reference map. In practice, this means we vary the values of \( t_M \) and \( t_R \). We present the range of values we use for \( t_M \) and \( t_R \) in Table 2. These are guided by the range of possible characteristic time-scales for Hα and far-
We create input reference maps from the star particles that fall within a particular age bin. The age bin, for a given reference map, is defined through $5t_{\alpha} \leq \text{Age} \leq 5t_{\alpha} + t_{\alpha}$. We show here all the values used in this paper for $t_{\alpha}$ and $t_{\alpha}$ when defining these age bins. This results in a $9 \times 9$ array of reference maps (see Figure 4). See Section 3.2 for more details.

| Emission Type | $t_{\alpha}$ and $t_{\alpha}$ [Myr] |
|---------------|-----------------------------------|
| H$\alpha$     | 1 3 5 7 10 15 20 25 30           |
| UV            | 5 10 15 20 25 30 50 70 100        |

ultraviolet (FUV) emission found in Leroy et al. (2012). Leroy et al. use the results of STARBURST99 calculations to determine a characteristic time-scale using several methods: a luminosity-weighted average time, as well as the times at which the tracer emission reaches a particular limit in terms of the total cumulative emission or its instantaneous intensity.

### 3.3 Generation of the emission maps

In order to preform our analysis, we need to produce synthetic emission maps. The simulation that we base this work on (see Section 3.1) contains no information about the expected emission spectrum. We therefore use SLUG2 (da Silva et al. 2012, 2014; Krumholz et al. 2015), a stochastic stellar population synthesis code, to take the age and mass of the star particles and predict the associated emission for the filters specified in Table 1.

With the SLUG2 model, we predict the expected rest-frame emission spectrum for every star particle within the simulation. The code first samples an IMF to construct a simple stellar population of total mass matching that of the star particle and then uses stellar evolution tracks along with the age of the star particle to determine the combined emission of this simple stellar population. SLUG2 then converts the full combined emission spectrum into a single luminosity value for each of the SFR tracers in Table 1 using filter response curves. These single luminosity values are what we assign to our star particles when we produce our synthetic rest-frame emission maps. We use the same smoothing procedure as described in Section 3.2. This means that, even though our star particles are treated as simple stellar populations, the star-forming regions themselves, which are a collection of multiple particles, will have an age spread. An example of a synthetic H$\alpha$– map is shown in Figure 2.

The adopted UV response filters are all included by default in SLUG2 (see Krumholz et al. 2015 for more details). The H$\alpha$ SFR tracers, however, require different steps. For H$\alpha$– we use the hydrogen-ionizing photon emission, $Q(\text{H}^0)$, directly and for H$\alpha$ + $W$ we define the narrow band filter, $\mathcal{F}_{\text{H}\alpha+W}$, as

$$\mathcal{F}_{\text{H}\alpha+W} = \begin{cases} 1 & 6562 - \frac{W}{2} \text{ Å} \leq \lambda \leq 6562 + \frac{W}{2} \text{ Å} \\ 0 & \text{Otherwise} \end{cases}$$

The emission spectrum produced by SLUG2, includes the H$\alpha$ emission line but does not calculate the underlying absorption feature from the stellar continuum. In Appendix A we use STARBURST99 simulations to investigate when the absorption can no longer be neglected. We find that for the time-scales we are considering the absorption is negligible.

For the analysis in Section 4, we use a Chabrier (2005) IMF with Geneva solar-metallicity evolutionary tracks (Schaller et al. 1992) and STARBURST99 spectral synthesis. The SLUG2 model samples the IMF non-stochastically (we use a well sampled IMF) and no foreground extinction is applied. The surrounding material has a hydrogen number density of $10^3$ cm$^{-3}$. We assume that only 73 percent of the ionising photons are reprocessed into nebular emission, which is consistent with the estimate from McKee & Williams (1997); this could be because those photons are absorbed by circumstellar dust, or because they escape outside the observational aperture (the observational effects of these two possibilities are indistinguishable).

We choose to produce our synthetic emission maps without extinction for a number of reasons. In observational applications of the KL14 principle, there is often some overlap between the first and second phases of the evolutionary timeline. For instance, when applying the method to a molecular gas map (e.g. CO) and an ionised emission map (e.g. H$\alpha$), there will be some non-zero time for which both tracers coexist. When a region resides in this “overlap” phase, the star-forming region may be partially embedded in dust and gas; during this phase the region suffers the most from extinction. We can therefore define the duration of this second phase, $t_2$, as

$$t_2 = t_0 + t_i,$$

where $t_0$ is the duration of the second phase that overlaps with the first, and $t_i$ the duration that is independent. The characteristic timescales we define in this paper are for this independent part, $t_i$, of the second phase. This is where the region is no longer embedded in dust and gas and therefore not suffering from significant extinction. We motivate this by the notion that molecular gas correlates with star formation: as long as CO emission is present, star formation is likely to be ongoing. The “clock” defined by the SFR tracer lifetime only starts when the last massive stars have formed. This does mean that the application of HEISENBERG to tracers other than CO may require a different definition of the reference time-scale. To facilitate this, the HEISENBERG code enables the user to specify if the reference time-scale includes or excludes this overlap phase (see Kruisjes et al. 2018, § 3.2.1).

In addition, it is desirable to exclude extinction for two further reasons. Firstly, the effects of extinction can, in most cases, be significantly reduced if not completely corrected for (e.g. James et al. 2005), meaning in practice the input maps provided to HEISENBERG can be corrected for extinction. Secondly, if we perform our analysis with extincted maps, the results would no longer be generally applicable and would only apply to galaxies that suffer from the same amount of extinction. Our current approach therefore enables constructing a “universal” baseline of extinction-corrected SFR tracer lifetimes. In future work, we aim to consider extinction using galaxy simulations covering a range of gas surface densities (Haydon et al. in prep.).
4 CHARACTERISTIC TIME-SCALES FOR A FULLY SAMPLED IMF AT SOLAR METALLICITY

We constrain the characteristic time-scales for several SFR tracers using the HEISENBERG code. As input to the code, we use synthetic SFR tracer maps and reference maps as described in Section 3. The reference maps are characterised by the ages of the star particles included within them. We investigate how the chosen age bin affects the characteristic time-scale measured by changing the values of $t_\alpha$ and $r_\alpha$ which define the age bin: $t_M \leq \text{Age} \leq t_M + r_M$. This approach does not (initially) result in a single characteristic time-scale for each SFR tracer, but an array spanned by $t_M$ and $r_M$. In what follows, we describe how we reduce these “time-scale arrays” (see Figure 4 for examples) into a single characteristic time-scale for each SFR tracer. However, we first note that HEISENBERG not only outputs the measured time-scale but also its probability density function (PDF). This means that each element of the time-scale array has an associated PDF, which we make use of in the process of defining a single characteristic time-scale.

To define a characteristic time-scale for each SFR tracer from the array of time-scales, we produce $10^6$ realisations of the time-scale array, where the value of each element of each realisation of the time-scale array has been randomly sampled from its associated PDF. For each of the $10^6$ realisations of the time-scale array we calculate the weighted mean of the array. The weighting for an element in the array with time-scale $t_{ij}$ is given by $W_{ij}$:

$$W_{ij} = \frac{w_{ij}^d w_{ij}^u}{\sum_{ij} w_{ij}^d w_{ij}^u}. \quad (5)$$

The $w^d$ weighting favours more strongly elements that satisfy the criteria we describe in Equations 1 and 2 (i.e. the closer $t_{ij}$ is to $t_M$ and $r_M$ the better):

$$w_{ij}^d = \left( \log_{10} \left( \frac{t_{ij}}{t_M} \right) \right)^2 + \left( \log_{10} \left( \frac{r_{ij}}{r_M} \right) \right)^2. \quad (6)$$

The $w^u$ weighting takes the lower and upper uncertainty ($\sigma^d_{ij}$ and $\sigma^u_{ij}$ respectively) on the time-scale $t_{ij}$ (as calculated by HEISENBERG) into consideration:

$$w_{ij}^u = \frac{\sigma^d_{ij} + \sigma^u_{ij}}{2}. \quad (7)$$

This process results in $10^6$ characteristic time-scales, from which we can produce a PDF. We define the characteristic time-scale from the median of the PDF and the uncertainties from the 16th and 84th percentiles.

When applying HEISENBERG to the pairs of reference and SFR tracer maps, we use the default input parameters specified in Tables 1 and 2 of Kruijssen et al. (2018). The only exceptions are as follows. We set $\text{tstar}_{\text{inc1}} = 1$, to indicate that the reference time-scale (i.e. the width of the age bin) also includes the overlapping phase. As we are not making any cuts in galactocentric radius, we also set $\text{cut}_{\text{radius}} = 0$. Finally, we define the range of aperture sizes using a minimum aperture size of $l_{\text{ap, min}} = 25$ pc and a number of $N_{\text{ap}} = 17$ apertures, to produce 17 logarithmically-spaced aperture diameters from 25 – 6400 pc.

In Figure 4, we present, as examples, two time-scale arrays obtained for $H\alpha$ – and WFC3 UVIS F225W SFR tracers. These time-scale arrays only serve as examples, since the elements show the output of HEISENBERG and are not from the $10^6$ realisations. We also show the PDFs associated to the defined characteristic time-scale for $H\alpha$ – and WFC3 UVIS F225W in Figure 5.

We can use the defined characteristic time-scales, $t_{E,0}$, to specify the age bin we use for creating our reference maps in later sections of this paper:

$$t_{E,0} \leq \text{Age} \leq 2t_{E,0}, \quad (8)$$

which complies with the conditions specified in Equations 1 and 2.

In Table 3, we present the characteristic time-scales and associated uncertainties for each of the different SFR tracer filters. We also include the age bins that we will use for the reference maps in later sections; we calculated these using the expression given in Equation 8.

We show the response curves for the UV filters in the upper panel of Figure 6 and also mark the response-weighted mean wavelength, $\lambda_{w}$, against the characteristic time-scale of the young stellar emission in each filter. The figure shows that similar response-weighted mean wavelengths give similar characteristic time-scales. In addition, we see that there is a tight relation between the time scale and the width of the age bin used as the reference time scale and is the total duration of that phase: this includes any overlap.

Table 3. The characteristic time-scales, $t_{E,0}$, obtained for the different SFR tracers (see Table 1 for details) and the corresponding age bins ($t_{E,0} \leq \text{Age} \leq 2t_{E,0}$) for producing reference maps in later sections of this paper. These results are for a well sampled IMF at solar metallicity. The filter order is in increasing filter width ($W$) for $H\alpha$ – and decreasing response-weighted mean wavelength ($\lambda_{w}$) for UV. This table is an extract of Table B1, which includes the characteristic time-scales and age bins for different stellar metallicities ($Z/Z_{\odot}$ = 0.05 – 2).

| Filter        | $t_{E,0}$ [Myr] | Age bin [Myr] |
|---------------|-----------------|---------------|
| $H\alpha$     | 4.3 $^{+1.1}_{-0.5}$ | 4.3 – 8.6 |
| $H\alpha + 10$ Å | 5.6 $^{+1.0}_{-0.5}$ | 5.6 – 11.1 |
| $H\alpha + 20$ Å | 7.3 $^{+1.1}_{-0.5}$ | 7.3 – 14.6 |
| $H\alpha + 40$ Å | 9.3 $^{+1.1}_{-0.5}$ | 9.3 – 18.6 |
| $H\alpha + 80$ Å | 10.7 $^{+1.3}_{-0.5}$ | 10.7 – 21.4 |
| $H\alpha + 160$ Å | 16.4 $^{+1.6}_{-0.5}$ | 16.4 – 32.7 |
| WFC3UVIS1F36W | 33.3 $^{+2.0}_{-0.5}$ | 33.3 – 66.6 |
| WFC3F36W      | 33.1 $^{+2.0}_{-0.5}$ | 33.1 – 66.3 |
| WFC2F36W      | 27.7 $^{+1.6}_{-0.5}$ | 27.7 – 55.4 |
| WFC3UVIS1F275W | 23.5 $^{+1.2}_{-0.5}$ | 23.5 – 47.0 |
| UVOTW1       | 21.8 $^{+1.2}_{-0.5}$ | 21.8 – 43.5 |
| WFC2F255W    | 22.4 $^{+1.2}_{-0.5}$ | 22.4 – 44.7 |
| WFC3UVIS1F225W | 19.6 $^{+1.0}_{-0.5}$ | 19.6 – 39.3 |
| GALEXNUV     | 19.6 $^{+1.0}_{-0.5}$ | 19.6 – 39.1 |
| UVOTM2       | 19.5 $^{+1.0}_{-0.5}$ | 19.5 – 39.0 |
| WFC3UVIS1F218W | 19.4 $^{+1.0}_{-0.5}$ | 19.4 – 38.9 |
| UVOTW2       | 19.0 $^{+1.0}_{-0.5}$ | 19.0 – 38.0 |
| GALEXFUV     | 17.1 $^{+1.0}_{-0.5}$ | 17.1 – 34.2 |

\[4\] Using the average of the lower and upper uncertainty is not technically correct; however, the methods as suggested by Barlow (2003) would have little impact on the final result and so are neglected.

\[5\] This is not in contradiction with what we discuss in Section 3.3. In Section 3.3, we explain that the characteristic time-scales of the SFR tracers we define do not include the overlap phase; and so, when using the characteristic time-scales we present here, one should use $\text{tstar}_{\text{inc1}} = 0$. The
Figure 4. Two examples showing the range of characteristic time-scale values (and associated uncertainties) as determined using HEISENBERG for different reference maps. Top: Ha emission excluding the continuum (Hα→). Bottom: UV emission (WFCC3 UVIS F225W). The reference maps are characterised by the age bin used to select the star particles which are included in the reference map, t_R denotes the minimum age of the star particles and t_G the width of the age bin. The colour-coding is based on the weighting, W, used when calculating the weighted average. All values within the tables are given in Myr.

Figure 5. Two examples showing the probability density functions associated to the defined characteristic time-scales. Left: Ha emission excluding the continuum (Hα→). Right: UV emission (WFCC3 UVIS F225W). The vertical line shows the selected time-scale (the median of the distribution); the shaded region, the uncertainty defined by the 16th and 84th percentiles.
Figure 6. Top: The normalised response curves of the UV filters considered in this paper (also see Table 1a). The vertical lines indicate the response-weighted mean wavelengths, $\bar{\lambda}_w$. Bottom: Characteristic emission time-scales for UV filters as a function of response-weighted mean wavelength. The grey curve shows the fit described in Equation 9 and the shaded regions indicates the associated uncertainty.
between the SFR tracer time-scale and wavelength. We describe this relationship between $\lambda_w$ and the UV characteristic time-scale, $t_{E,0}^{\text{UV}}$, by

$$t_{E,0}^{\text{UV}} [\text{Myr}] = \left( 3.20^{+0.30}_{-0.33} \right) \left( \frac{\lambda_w}{225 \text{ nm}} \right)^{\left( 4.21^{+0.22}_{-0.20} \right)} + \left( 16.32^{+0.37}_{-0.32} \right).$$

This relation is obtained by performing a weighted least-squares minimization. The uncertainties on the parameter values are calculated using a Monte Carlo approach. With this relationship it is possible to find the characteristic emission time-scales for UV filters that we have not explicitly considered here.

Similarly, we derive a relation between the H$\alpha$ characteristic time-scale and filter width, $W$, to make it possible to find characteristic time-scales for intermediate $W$:

$$t_{E,0}^{\text{H} \alpha} [\text{Myr}] = \left( 4.8^{+1.1}_{-1.0} \right) \left( \frac{W}{40 \text{ Å}} \right)^{\left( 0.65^{+0.20}_{-0.21} \right)} + \left( 3.8^{+1.1}_{-1.1} \right).$$

We present this relation in Figure 7. We note that the increase in characteristic time-scale with filter width is not due to a change in the H$\alpha$ emission but the result of including more of the long-lived continuum emission.

The UV characteristic timescales that we recover (17.1 – 33.3 Myr) are within the ranges often quoted in the literature (10 – 100 Myr, Kennicutt & Evans 2012; Leroy et al. 2012). The large variation in literature values is due to the fact that there is no single method or distinct set of criteria which one should use in order to constrain the characteristic time-scale of an SFR tracer. With the approach taken in this paper, we have remedied this problem for future observational applications of the KL14 principle.

Leroy et al. (2012) present a table of characteristic time-scales for H$\alpha$ and FUV (at 150 nm). Multiple time-scales are listed for each SFR tracer; these time-scales are defined by the duration required to reach a given percentage limit of the cumulative emission or of the emission intensity relative to the intensity at 1 Myr. We take Leroy et al. (2012, Figure 1) and reverse the process to find the percentage that correspond to the characteristic emission time-scales we determine for H$\alpha$ and GALEX FUV. We list these percentage limits in Table 4.

The percentage limits used in Leroy et al. (2012) are arbitrarily chosen and so we make this conversion into percentage limits to determine if a single percentage of the 1 Myr intensity or cumulative emission can be defined that would correspond to SFR tracer time-scales that we measure. From the results in Table 4, we can see that no single percentage limit can be defined. As there is no consistent limit, the characteristic time-scale for each SFR tracer must be determined individually.

In summary, we see that the characteristic time-scales fall in the range of commonly reported literature values. By comparing these measurements to the time-evolution of the SFR tracer intensities, we find that no fixed percentage of the initial or cumulative emission is capable of matching the obtained characteristic timescales. For this reason, each SFR tracer time-scale must be determined individually using the presented method. However, we do find empirical functions (see Equations 9 and 10) relating the characteristic emission time-scales for UV and H$\alpha$ filters to their filter properties. These relations predict the characteristic time-scale for UV and H$\alpha$ filters we have not considered here.

### 5 THE EFFECTS OF METALLICITY

So far, we have only considered stellar populations of solar metallicity; however, it is well-known that the metallicity affects stellar lifetimes (e.g. Leitherer et al. 1999) and thus the characteristic emission time-scales of SFR tracers. In order to facilitate observational applications of the KL14 principle to the broadest possible range of galaxies, we therefore quantify how the SFR tracer time-scales depend on metallicity. In this section, we repeat the experiments performed in Section 4 but this time we produce synthetic SFR tracer emission maps using evolutionary tracks of metallicities $Z/Z_\odot = 0.05, 0.20, 0.40, 2.00$ (Schaller et al. 1992; Charbonnel et al. 1993; Schaerer et al. 1993a,b).

In Appendix B, we list the characteristic time-scales for a well-sampled IMF for all metallicities (also including the solar metallicity results from Table 3) and the age bins we select for producing reference maps. We see that as the metallicity increases, the characteristic time-scale decreases; we show this $Z - t_{E,0}$ relation in Figure 8 for H$\alpha$ -- Figure 9 for H$\alpha$ filters, and in Figure 10 for the UV filters. We also include empirical fits described respectively.

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**Table 4.** Percentages of the emission intensity relative to its instantaneous value at 1 Myr and of it cumulative value over 100 Myr, evaluated at the characteristic time-scales of the SFR tracers presented in Table 3, based on Figure 1 of Leroy et al. (2012).

| Filter    | H$\alpha$ [%] | FUV [%] |
|-----------|---------------|---------|
| 10 Myr    | 92.4 ± 1.5    | 76.5 ± 0.2 |
| 50 Myr    | 86.6 ± 0.1    | 70.6 ± 0.1 |
| 100 Myr   | 86.6 ± 0.1    | 70.6 ± 0.1 |

*GALEX FUV

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6 This also holds when we perform the analysis for the other metallicities considered: $Z/Z_\odot = 0.05, 0.20, 0.40, 2.00$ (see Section 5 for more details). There is also no consistent percentage for a single tracer across the metallicity range.
As mentioned in Section 4, the Hα− characteristic time-scales are at the higher end of the literature range (if not exceeding), which is due to including more of the long-lived continuum emission. In Figure 9, we show the data for each metallicity and each wavelength along with the fit described by the \((t_{E,0}^{H\alpha}, W, Z)\) surface given in Equation 12 and shown in Figure 11. These figures enable a direct assessment of how well the surface describes the characteristic time-scales as a function of the metallicity and wavelength.

As we did for \(t_{E,0}^{H\alpha}\), we can extend the relation given by Equation 9, which describes \(t_{E,0}^{UV}\) as a function of response-weighted mean wavelength, \(\bar{\lambda}_w\), to also include metallicity. In Figure 10, we show the data for each metallicity and each wavelength along with the fit described by the \((t_{E,0}^{UV}, \bar{\lambda}_w, Z)\) surface given in Equation 13 and shown in Figure 12. The strongest deviations from the fit arise at long (\(\lambda_w > 290 \text{ nm}\) ) wavelengths. For UV filters at these wavelengths, we recommend interpolating the data points (provided in Appendix B) rather than adopting Equation 13. The range of characteristic time-scales found for the UV filters (14.5 – 35.3 Myr ) again fall within the range quoted in literature (10 – 200 Myr, Ken- nicutt & Evans 2012; Leroy et al. 2012); however, they are towards the low end of this range. This is a direct result of the fact that the UV emission from star-forming regions fades with time, and the measured time-scales are naturally biased to the ages of regions from which most UV photons emerge.

In summary, we see that the characteristic time-scales decrease with increasing metallicity. Observational applications of HEISENBERG should therefore use an SFR tracer time-scale appropriate for the metallicity of the observed region. We define empirical relations between the SFR tracer time-scale and the metallicity (for Hα−, Equation 11) and (for Hα+ and UV filters, Equations 12 and 13) the filter properties. For Hα+ and UV SFR tracers, these relations enable the definition of time-scales even for filters that are not explicitly considered here.

6 THE EFFECTS OF IMF SAMPLING

In the previous sections, we determine the characteristic time-scales of SFR tracers using synthetic emission maps where SLUGG fully samples the IMF. In observational applications of the KL14 principle, there is no guarantee (or requirement from HEISEN- BERG) that the regions under consideration have a well sampled IMF. It is therefore important to investigate the impacts of incomplete sampling of the IMF (i.e. a stochastically sampled IMF) on the characteristic time-scales of the SFR tracers, in particular for low-mass star forming regions.

We describe in Section 2 how the abundance of identified regions in each input map reflects the duration associated to that map. Since star formation is traced using emission from massive stars, an SFR tracer emission map consisting of star-forming regions where the IMF is not well sampled will contain fewer identified regions than one with a well sampled IMF. This apparent decrease in the number of star-forming regions is due to the fact that not every region will be able to form stars of sufficient mass to produce the SFR tracer emission. This effect will be particularly important for the Hα ± filters, as Hα emission requires high mass stars (\(>8 \text{ M}_\odot\)) and is dominated by stars of even higher masses. We therefore expect that as the sampling of the IMF becomes more incomplete, the effective characteristic time-scales of the various tracers will decrease, most strongly affecting Hα.

In Section 6.1, we explain how we expect the characteristic time-scale of the SFR tracers to change when the IMF is not well sampled. As before, we determine the free parameters using a weighted least-squares minimization and the uncertainties through Monte Carlo methods. With these relations, it is straight forward to recover the characteristic time-scale for a given the metallicity and filter properties without having to repeat the analysis performed here.

Figure 8 shows that the characteristic time-scales of Hα− changes by less than 2 Myr over the metallicity range \([0.05 Z_\odot, 2Z_\odot]\). The ranges of characteristic time-scales (3.9 – 5.5 Myr for Hα−) fall within the range of literature values (1.7 – 10 Myr, Kennicutt & Evans 2012; Leroy et al. 2012).
Figure 9. Characteristic time-scales of \(H\alpha +\) filters as a function of metallicity (left) and filter width (right). The symbols show the results of applying the HEISENBERG code to synthetic \(H\alpha +\) maps. The grey curve shows the fit from Equation 12 and the shaded region indicates the associated uncertainty. The symbols and colours in the right-hand panels correspond to those used on the left.
Figure 10. Characteristic time-scales of UV SFR tracers as a function of metallicity (left) and wavelength (right). The symbols show the results of applying the HEISENBERG code to synthetic UV maps. The grey curve shows the fit from Equation 13 and the shaded region indicates the associated uncertainty. The symbols and colours in the right-hand panels correspond to those used on the left.
time-scales to change as a result of incomplete IMF sampling based on purely analytical considerations. In Section 6.2, we show how these expectations can be tested experimentally and in Section 6.3, we present the results of these tests.

6.1 Theoretical expectation

In this section, we quantify the relationship between how well the IMF is sampled and the characteristic time-scale of the SFR tracer. As mentioned, the characteristic time-scale of the SFR tracer is related to the number of star-forming regions that HEISENBERG can identify in the emission map. We therefore estimate the relative change of the effective SFR tracer time-scale as the fraction of star-forming regions that do contain sufficiently massive stars to emit in the tracer of interest. This approach will be tested below. In practice, this means we need to estimate how many stars, \( N_{\text{min}} \), of at least some minimum mass, \( M_{\text{min}} \), are expected to form within a star-forming region of mass \( M_r \). We consider \( M_{\text{min}} \) to characterise the stellar mass at which the SFR emission becomes noticeable and not the mass contributing the most. The mass of the star-forming region, \( M_r \), can then act as a proxy for how well the IMF is sampled: smaller values of \( M_r \) will result in a region with an IMF that is less well sampled.

We can calculate the probability, \( P \), of producing a minimum number of stars \( N_{\text{min}} \) of at least some minimum mass \( M_{\text{min}} \) in a given star-forming region through a Bernoulli (i.e. binomial) trial. If the region can produce a sufficient number of stars of sufficient mass, then the region is identifiable in the SFR tracer; therefore, in our binomial trial, we define a “success” as producing a star of mass \( M \) which satisfies the condition of \( M_{\text{min}} \leq M < M_r \). The probability of success is given by \( p \), \( N_r \) is the total number of stars within the star-forming region, and \( N \) counts the number of “successful” stars.

The binomial distribution gives the probability of \( k \) successful stars:

\[
P(N = k) = \frac{N_r!}{k!(N_r - k)!} p^k (1 - p)^{N_r - k}.
\]

The probability that we wish to calculate (at least \( N_{\text{min}} \) stars of a mass of \( M_{\text{min}} \) or higher) is given by

\[
P(N \geq N_{\text{min}}) = 1 - P(N < N_{\text{min}}) = 1 - \sum_{k=0}^{N_{\text{min}}-1} P(N = k)
\]

The IMF, \( dN/dm \), describes the distribution of mass amongst the stars within a star-forming region; this means we can use the IMF to determine the values of \( p \) and \( N_r \) and therefore to calculate \( P(N = k) \). In a star-forming region with a well-sampled IMF, \( p \) is the fraction of stars that satisfy the condition \( M_{\text{min}} \leq M < M_r \) and \( N_r \) is the total number of stars within the region:

\[
p = \nu \int_{M_{\text{min}}}^{M_r} \frac{dN}{dm} dm ; \quad N_r = \mu \int_{M_{\text{min}}}^{M_r} \frac{dN}{dm} dm
\]

The normalisation constants \( \nu \) and \( \mu \) are evaluated through

\[
1 = \nu \int_{0}^{M_r} \frac{dN}{dm} dm ; \quad M_r = \mu \int_{0}^{M_r} \frac{dN}{dm} dm
\]

In order to convert the probability value, \( P(N \geq N_{\text{min}}) \), into an estimate for the characteristic time-scale, \( t_E \), we assume a Chabrier (2005) IMF and use the characteristic time-scales we find for a fully sampled IMF, \( t_{E,0} \), (see Appendix B) in the following equation

\[
t_E = t_{E,0} \times P(N \geq N_{\text{min}})
\]
The following set of equations describe the form of Figure 13.

We present the parameter values for all the approximate curves plotted in Figure 13 and described by Equations 21 to 23 where

\[ f(M_t) = 1 + a_1 \exp\left( b_1 \frac{M_t}{M_0} \right) + a_2 \exp\left( b_2 \frac{M_t}{M_0} \right), \]

\[ P(N \geq N_{\text{min}}) \approx \begin{cases} 0 & f(M_t) \leq 0 \\ f(M_t) & 0 < f(M_t) < 1 \\ 1 & f(M_t) \geq 1 \end{cases} \]

where \( a_i, b_i \) for \( i = 1, 2 \) are four parameters that we determine through least-squares minimization.

We create the reference maps in the same way as before: the reference maps are mass surface density maps of the star particles within the age bins specified in Table 3. The emission maps, however, undergo one additional step. As mentioned previously, we can use the mass of the star-forming region, \( M_s \), as a proxy for how well the IMF is sampled; in this case \( M_s \) is the mass of the star particles. We therefore scale these star particle masses by some mass scaling factor, \( F_m \), before S1UG2 predicts the expected emission, this time using its stochastic IMF sampling module. The values of \( F_m \) range from 0.01 – 100, where a lower mass scaling factor means the IMF will be less well sampled. We then use HEISENBERG to determine the characteristic time-scale, as in Section 4. The characteristic time-scale we associate to each mass scaling factor is the average of three characteristic time-scales determined from three independently generated stochastic realisations of the synthetic emission maps.

To relate the relative change of the SFR tracer time-scale due to IMF sampling to observables, we define an average star-forming region mass, \( \mathcal{M}_t \), as

\[ \mathcal{M}_t = \Sigma_{\text{SFR}} \times \pi \left( \frac{\lambda}{2} \right)^2, \]

which uses the SFR surface density, \( \Sigma_{\text{SFR}} \), and quantities that HEISENBERG measures: the total duration of the evolutionary

![Figure 13](image_url)

**Figure 13.** Curves showing how the probability of forming at least \( N_{\text{min}} \) stars of mass \( M_{\text{min}} \) or higher changes with star-forming region mass, \( M_t \). The grey dashed lines indicate the approximate fit to the full calculation. See Section 6.1 for details and Table 5 for fit parameter values. **Left:** Constant \( N_{\text{min}} \). **Right:** Constant \( M_{\text{min}} \).

**Table 5.** Values for the free parameters, \( a_i \) and \( b_i \), in the analytical models plotted in Figure 13 and described by Equations 21 to 23

| \( N_{\text{min}} \) | \( M_{\text{min}} \) | \( a_1 \) | \( b_1 \) | \( a_2 \) | \( b_2 \) |
|------------------|------------------|----------|----------|----------|----------|
| 1.000            | 1.000            | -1.001   | -0.251   | -0.384   | -0.632   |
| 1.000            | 10.000           | -1.016   | -0.137   | -0.146   | -0.338   |
| 1.000            | 10.000           | -1.021   | -0.085   | -0.071   | -0.235   |
| 1.000            | 31.623           | -1.000   | -0.048   | -0.056   | -0.233   |
| 1.000            | 100.000          | -1.000   | -0.008   | -2.393   | -5.405   |
| 1.000            | 3.162            | -1.020   | -0.098   | -0.088   | -0.257   |
| 3.162            | 7.000            | -1.930   | -0.118   | -0.966   | -1.555   |
| 10.000           | 7.000            | -4.816   | -0.235   | -0.987   | -1.638   |
| 31.623           | 7.000            | -38.108  | -0.426   | -1.016   | -0.891   |
| 100.000          | 7.000            | -1000.000| -0.743   | -1.016   | -0.889   |

We adapt the method we present in Section 3 to investigate the effects of a stochastically sampled IMF and thus test experimentally if we recover the same behaviour as described in Section 6.1.
timelines, τ, and the typical separation length of independent star-forming regions, λ, (for details see Kruisjes et al. 2018).

At a fixed total duration of the evolutionary timeline and region separation length, the degree of IMF sampling is controlled by \( \Sigma_{\text{SFR}} \). We calculate the value of \( \Sigma_{\text{SFR}} \) as

\[
\Sigma_{\text{SFR}} = \sum_{\mathcal{E} \in \phi} \frac{m_i}{\mathcal{E}_i} \times F_{\mathcal{E}},
\]

where \( \sum m_i \) is the total mass of all the star particles that fall within the age bin appropriate for the filter, i.e. \( 0 \leq \text{Age} \leq \mathcal{E}_i \) (see Appendix B for the values of \( \mathcal{E}_i \)), which is then scaled by the mass scaling factor \( F_{\mathcal{E}} \), the width of that age bin, and \( \mathcal{E} \) is the radius of the galaxy being studied (for our simulated galaxy \( r = 10 \) kpc, as determined from a visual inspection of the synthetic emission maps).

In Equation 25, we consider \( \Sigma_{\text{SFR}} \) as the galaxy average SFR surface density. If there are no strong large-scale morphological features, as is the case here, this galaxy average SFR surface density is appropriate to use in the calculation of \( \mathcal{M}_\tau \). Otherwise, the expression in Equation 25 should be updated to include a factor \( \mathcal{E}_{\text{star, glob}} \), indicating the ratio of the mass surface density\(^7\) on a size scale of \( \lambda \) to the (reference) map average value and therefore accounting for a non-uniform spatial distribution of star-forming regions across the galaxy (see Kruisjes et al. 2018, § 3.2.9 for more details).

By introducing a “mass scaling factor”, \( F_{\mathcal{E}} \), we are able to test experimentally how the characteristic time-scale of different SFR tracers change when the IMF becomes less well sampled. We will use the experimental results to see if we observe the behaviour predicted in Section 6.1.

\[\text{6.3 Characteristic time-scales for a stochastically sampled IMF}\]

We now present the results of a set of experiments aimed at testing how the characteristic time-scales of H\(\alpha\) and several UV SFR tracers change as a result of incomplete IMF sampling. We produce synthetic emission maps, for which we adjust the sampling of the IMF by scaling the masses of the star particles before SLUG2 calculates the emission. We use the HEISENBERG code along with these stochastically sampled IMF synthetic emission maps to find the associated characteristic time-scales and see how they are impacted by incomplete IMF sampling.

In Figure 14, we present the solar-metallicity results for H\(\alpha\) and WFC3 UVIS F225W as examples of how the characteristic time-scales change as a function of the average mass of an independent star-forming region, \( \mathcal{M}_\tau \). The quantity \( \mathcal{M}_\tau \) characterises (chiefly through \( \Sigma_{\text{SFR}} \), see Equation 24) how well the IMF is sampled: lower values of \( \mathcal{M}_\tau \) result in a more stochastically sampled IMF. Each data point\(^8\) in the two left-hand panels of Figure 14 corresponds to a different mass scaling factor, \( F_{\mathcal{E}} \). The characteristic time-scale associated to a given \( F_{\mathcal{E}} \) is the average of three time-scales determined from three independently generated synthetic emission maps. This accounts for the spread in time-scales that results due to the stochastic nature in which the synthetic emission maps are produced. The quantity shown on the vertical axis, \( \mathcal{E}_i/\mathcal{E}_{i,0} \), is the factor by which the measured characteristic time-scale is reduced, compared to the characteristic time-scale for a well sampled IMF (see Table B1 for \( \mathcal{E}_{i,0} \) values.), as a result of incomplete IMF sampling at small region masses or SFR surface densities. We describe the relation between this conversion factor, \( \mathcal{E}_i/\mathcal{E}_{i,0} \), and \( \mathcal{M}_\tau \) through \( P(N \geq N_{\text{min}}) \) as in Equation 20 (see Section 6.1).

The purple curves in Figure 14 indicate the best-fitting form of \( P(N \geq N_{\text{min}}) \). The analytical form that describes \( P(N \geq N_{\text{min}}) \) has two free parameters \( N_{\text{min}} \) and \( M_{\text{min}} \). We constrain the values for these two free parameters using a brute-force approach: we calculate the value of \( \mathcal{E}_{\text{red}}^2 \) for a range of \( N_{\text{min}} \) and \( M_{\text{min}} \) and use the minimum \( \mathcal{E}_{\text{red}}^2 \) to indicate the best-fitting parameter values. We use the method described in Ocar (1982) to determine \( \mathcal{E}_{\text{red}}^2 \) when uncertainties are present on both the abscissa and the ordinate. We consider \( N_{\text{min}} \) values ranging from 1 to 4 in integer steps and \( M_{\text{min}} \) from 0 to 120 M\(\odot\) in steps of 0.05 M\(\odot\). In the right-hand panels of Figure 14, we show the dependence of \( \mathcal{E}_{\text{red}}^2 \) on \( N_{\text{min}} \) and \( M_{\text{min}} \) for the two example filters. Table 6 lists the best-fitting values of \( M_{\text{min}} \) (\( N_{\text{min}} = 1 \) in all cases) for the full range of metallicities (\( Z/Z_\odot = 0.05, 0.20, 0.40, 1.00, 2.00 \)) for the H\(\alpha\) filters.

The data points used in the fitting process (and included in Figure 14) are those that survive a selection cut: we rejected data points for which the calculated characteristic time-scale exceeds the characteristic time-scale for a well sampled IMF by more than 1\(\sigma\) (i.e. \( \mathcal{E}_i/\mathcal{E}_{i,0} > 1 \)). We choose to remove the data that do not satisfy this criterion because they indicate contamination by the continuum emission from (low-mass) stars. For low mass scaling factors (typically \( F_{\mathcal{E}} \approx 0.01 \)), the emission from the continuum dominates over the SFR tracer. This results in characteristic time-scales that describe the long-lived continuum emission and therefore can be orders of magnitude higher than \( \mathcal{E}_{i,0} \).

For the UV filters, we find that data points associated with low mass scaling factors (\( F_{\mathcal{E}} \approx 0.01 \)) are excluded by our data selection criterion. This results in the turn off from \( \mathcal{E}_i/\mathcal{E}_{i,0} = 1 \) being very poorly sampled (see Figure 14 for an example). This means that we cannot reliably distinguish between different \( N_{\text{min}} \) and \( M_{\text{min}} \); therefore, we conclude that UV emission is not significantly affected by IMF sampling and exclude the UV \( M_{\text{min}} \) values from Table 6.

Table 6 shows that there is no clear relation between \( M_{\text{min}} \) and metallicity or filter width, but generally speaking, smaller filter widths have higher \( M_{\text{min}} \). Higher values of \( M_{\text{min}} \) imply higher star-forming region masses below which IMF sampling cannot be neglected (i.e. where \( P(N \geq N_{\text{min}}) \leq 1 \)). For H\(\alpha\) (\( \mathcal{E}_{\text{red}}^2 \)), \( M_{\text{min}} \) ranges from 10.45 – 13.00 (5.20 – 13.90) M\(\odot\). From these \( M_{\text{min}} \) ranges, we obtain region masses below which incomplete IMF sampling affects the SFR tracer time-scales, that is, these are the lowest region masses according to Equation 24 for which IMF sampling can be neglected. For H\(\alpha\) (\( \mathcal{E}_{\text{red}}^2 \)), this range is \( \mathcal{M}_\tau \geq 600 – 800 \) (200 – 900) M\(\odot\). For a region separation length of \( \lambda = 200 \) pc and a total timeline duration of \( \tau = 30 \) Myr, these characteristic region mass limits correspond to \( \Sigma_{\text{SFR}} \gtrsim (6 – 9) \times 10^{-4} \) M\(\odot\) yr\(^{-1}\) kpc\(^{-2}\) for H\(\alpha\) and \( \Sigma_{\text{SFR}} \gtrsim (2 – 10) \times 10^{-4} \) M\(\odot\) yr\(^{-1}\) kpc\(^{-2}\) for H\(\alpha\).

Figure 14 demonstrates that it is important to consider the effects of IMF sampling at low SFR surface densities, when constraining the characteristic time-scale for the H\(\alpha\) ± filters. This is because at low SFR surface densities, the massive stars required to produce H\(\alpha\) emission are not always present. If we ignore this fact, the H\(\alpha\) ± characteristic time-scales will be overestimated; as a result, the evolutionary time-line would be incorrectly calibrated and the time-scales obtained with HEISENBERG would also be overestimated. The agreement between the results of these experiments and

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\(^7\) The quantity \( \mathcal{E}_{\text{star, glob}} \) represents a mass surface density ratio because the reference maps show the mass surface density. In typical observational applications, \( \mathcal{E}_{\text{star, glob}} \) would be a flux density ratio.

\(^8\) For details on the error calculation on \( \mathcal{M}_\tau \), see Appendix C.
In summary, we have shown that the effects of IMF sampling can have a considerable impact on the characteristic time-scales of the Hα filters. The change of time-scales with the characteristic star-forming region mass $M_*$ is well-described by the analytical description of IMF sampling from Section 6.1. This provides the correct SFR tracer time-scale and enables applications of HEISENBERG even in regions of low SFR surface densities. By contrast, the characteristic time-scale of UV tracers is found to be largely insensitive to IMF sampling, implying that no such correction factor is needed. We reiterate that all SFR tracers do still carry an important dependence on the metallicity that should always be accounted for.

Table 6. The functional form of the conversion factor, $P(N \geq N_{\text{min}})$, between the characteristic time-scale measured for a well-sampled IMF and a stochastically sampled IMF has two parameters, $N_{\text{min}}$ and $M_{\text{min}}$. We use $N_{\text{min}} = 1$ and show here the values of $M_{\text{min}}$.

| $Z_\odot$ | $0.05Z_\odot$ | $0.20Z_\odot$ | $0.40Z_\odot$ | $1.00Z_\odot$ | $2.00Z_\odot$ |
|---------|--------------|--------------|--------------|--------------|--------------|
| Hα      | 11.50        | 11.95        | 13.00        | 11.55        | 10.45        |
| Hα+10   | 10.75        | 10.05        | 12.25        | 13.90        | 12.00        |
| Hα+20   | 12.10        | 12.05        | 12.40        | 10.35        | 9.95         |
| Hα+40   | 9.65         | 9.45         | 7.95         | 8.35         | 10.45        |
| Hα+80   | 9.20         | 8.70         | 8.85         | 6.25         | 8.00         |
| Hα+160  | 10.20        | 5.20         | 7.80         | 8.60         | 9.35         |

the theoretical model also demonstrate that the IMF sampling theory presented in Section 6.1 accurately describes how the characteristic time-scale of Hα changes due to incomplete IMF sampling. This means that observational applications of the KL14 principle can use the expressions provided in Equations 20 to 24 to derive an SFR tracer time-scale corrected for IMF sampling. For the UV tracers, however, the characteristic time-scales are mostly insensitive to the effects of incomplete IMF sampling and so these effects can be largely ignored. The constancy of the UV characteristic time-scales over the range of $M_*$ is a result of stars being able to produce UV emission at low masses. This means that the emission will almost always be present in star-forming regions.
7 CONCLUSIONS

We have applied a new statistical method (the HEISENBERG code, which uses the “uncertainty principle for star formation”, Kruijssen & Longmore 2014; Kruijssen et al. 2018) to constrain the characteristic emission time-scales of SFR tracers, i.e. the durations over which Hα and UV emission emerges from coeval stellar populations. We expect these time-scales to be critical in a variety of future studies. Firstly, observational applications of HEISENBERG will enable the empirical characterisation of the cloud lifecycle across a wide range of galactic environments, by measuring e.g. the molecular cloud lifetime and the time-scale for cloud destruction by feedback. However, these require a “reference time-scale” for turning relative time-scales into absolute ones. This reference time-scale is provided by the SFR tracer time-scales obtained in this work. Second, the emission time-scales obtained here and their dependence on metallicity and filter properties provide important input for studies of photoionisation feedback and UV heating.

To obtain the SFR tracer emission time-scales, we generate synthetic SFR tracer emission maps of a simulated near-L* isolated flocculent disc galaxy using the stochastic stellar population synthesis code SLUG2 (da Silva et al. 2012, 2014; Krumholz et al. 2015). We then apply HEISENBERG to combinations of these synthetic maps and “reference maps”, which show the star particles from the simulation in specific, known age bins. With this approach, we self-consistently measure the characteristic time-scales for Hα emission (with and without continuum subtraction), as well as 12 different UV filters.

For stellar populations at solar metallicity and with a fully sampled IMF we find the characteristic time-scales for Hα with (without) continuum subtraction to be 4.3±0.3 (5.6 – 16.4) Myr and for the UV filters to be in the range 17.1 – 33.3 Myr. When considering stellar population with different metallicities (Z/Z⊙ = 0.05, 0.20, 0.40, 1.00, 2.00) the range of characteristic time-scales becomes greater: 3.9 – 5.5 (5.1 – 16.4) Myr for Hα with (without) continuum subtraction and for the UV filters, 14.5 – 33.3 Myr. We define empirical power-law relations that provide the characteristic time-scale as a function of metallicity (Equations 11 to 13). We extend this empirical relation to include the response-weighted mean wavelength, \( \lambda_{\text{res}} \), for UV filters and the filter width, \( \Delta \lambda \), for the Hα + filters. This allows us to use a single relation to determine the characteristic time-scale for all UV and Hα + SFR filters from the filter properties, and the metallicity of the environment.

We also investigate the effects of a stochastically sampled IMF on the characteristic time-scales. Incomplete IMF sampling is found to affect the obtained characteristic emission time-scales in low-SFR galaxies. We quantify this dependence by stochastically sampling from the IMF prior to generating the synthetic SFR tracer emission maps and then measuring the characteristic time-scales with HEISENBERG. We use a Chabrier (2005) IMF to calculate the probability, \( P \), of forming at least \( N_{\text{min}} \) stars of mass \( M_{\text{min}} \) or higher given a star-forming region mass \( M_\bullet \). This probability provides the ratio between the characteristic time-scale for a stochastically sampled IMF, \( t_{\text{SFR}} \), and well sampled IMF, \( t_{\text{id}} \). Therefore, it defines a relation between \( t_{\text{id}}/t_{\text{SFR}} \) and the characteristic mass of independent star-forming regions, \( M_\Sigma \). Given an SFR surface density (from which the characteristic mass can be derived), this relation quantifies the relative change of the SFR tracer time-scale due to IMF sampling.

For UV tracers, the impact of IMF sampling on the characteristic time-scale is minimal (< 30 per cent) and can therefore be ignored (over all metallicities). However, incomplete IMF sampling has a significant effect on the characteristic time-scales of Hα emission. At lower SFR surface densities, the Hα emission time-scale decreases due to IMF sampling effects. Depending on the metallicity and whether the continuum emission has been subtracted, the characteristic time-scale for a well sampled IMF can be used for \( M_\bullet \geq 200 – 900 M_\odot \), which for a region separation of \( \lambda = 200 \) pc and a total lifetime duration of \( t = 30 \) Myr corresponds to \( \Sigma_{\text{SFR}} \geq (2 – 10) \times 10^{-3} M_\odot \text{ yr}^{-1} \text{ kpc}^{-2} \). We derive fitting functions describing the change of the Hα time-scales as a function of the average independent star-forming region mass, \( M_\Sigma \), as parametrised by the minimum region mass required for Hα emission, \( M_{\text{min}} \), which we tabulate as a function of metallicity (Equations 20 to 25 and Table 6).

In summary, we have measured the characteristic emission time-scales of SFR tracers as a function of metallicity and (for UV and Hα+) filter properties, as well as their sensitivity to IMF sampling, which effectively expresses their dependence on the SFR surface density. This spans the range of key environmental factors that affect the time-scales of Hα and UV emission, and provides important constraints on the duration of photoionisation feedback and UV heating. In addition, observational applications of the “uncertainty principle for star formation” will be able to use these time-scales as “reference time-scales” for turning the relative durations of evolutionary phases into an absolute timeline. This is an important step towards constraining the cloud-scale lifecycle of ISM evolution, star formation, and feedback.

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REFERENCES

Barlow R., 2003, in Lyons L., Mount R., Reitmeyer R., eds, Statistical Problems in Particle Physics, Astrophysics, and Cosmology. p. 250 (arXiv:physics/0401042)
Calzetti D., et al., 2007, ApJ, 666, 870
Chabrier G., 2005, in Corbelli E., Palla F., Zinnecker H., eds, Astrophysics Problems in Particle Physics, Astrophysics, and Cosmology. p. 250 (arXiv:physics/0409465), doi:10.1007/978-1-4020-3407-7
Chatzou C., Meynet G., Maeder A., Schaller G., Schaerer D., 1993, A&AS, 101, 415
Dehnen W., Aly H., 2012, MNRAS, 425, 1068
Hao C.-N., Kennicutt R. C., Johnson B. D., Calzetti D., Dale D. A., Moustakas J., 2011, ApJ, 741, 124
APPENDIX A: Hα ABSORPTION AND EMISSION FEATURES

We produce synthetic emission maps by passing the age and mass information of all the star particles from our simulation to SLUG2 (da Silva et al. 2012, 2014; Krumholz et al. 2015). SLUG2 then calculates the predicted emission spectrum for each particle, to which we apply UV and Hα+ filters (Hα− comes directly from the hydrogen-ionizing photon emission). However, the emission spectrum that SLUG2 produces does not include the underlying Hα absorption from the stellar continuum. In this appendix, we use STARBURST99 (Leitherer et al. 1999; Vázquez & Leitherer 2005) to investigate when the Hα absorption feature can no longer be neglected.

We ran STARBURST99 for an instantaneous burst of star formation for the five standard Geneva evolutionary tracks using a Kroupa (2001) IMF and output the data in 0.1 Myr time steps for 20 Myr. We otherwise used the default settings.

The equivalent width of the Hα emission is taken directly from the STARBURST99 output files. To determine the equivalent width of the absorption feature, we model the continuum (straight line) and the absorption feature (Voigt profile) of the high resolution spectral data in the wavelength range 6482 Å ≤ λ ≤ 6642 Å.

In Figure A1, we show the change in the equivalent width of the absorption and emission feature over time; the change in the difference between the two equivalent widths is also included. We see that the emission feature is dominant up to at least 10 Myr and longer for the lower metallicities; this is at least 5 Myr longer than the Hα− time-scales we measure (see Table B1) which are also marked in Figure A1.

We can see from Figure A1 that the Hα time-scales we are considering fall comfortably within the emission-dominant regime and conclude that the absorption feature can safely be neglected for our analysis.

APPENDIX B: COMPLETE SET OF SFR TRACER EMISSION-TIME-SCALES

In Table B1, we list the complete set of SFR tracer emission-time-scales constrained in this paper. This contains the characteristic time-scales of Hα± and all 12 UV filters, for the five different metallicities Z/Z⊙ = 0.05, 0.20, 0.40, 1.00, 2.00. In addition, we include the age intervals that we adopt to define the stellar reference maps used when measuring the SFR tracer time-scales with the HEISENBERG code. For more details on the calculations, see Section 4.

APPENDIX C: ERROR PROPAGATION

In Section 6.2, we calculate the average independent star-forming region mass as

\[ M_i = \sum_{j} (\rho_{emi} + \rho_{ref} - \tau_{lower}) \times \pi \left(\frac{\lambda}{2}\right)^2. \]  

(C1)

This equation uses the SFR surface density, \( \Sigma_{SFR} \), and the duration of the reference map, \( \tau_{ref} \), along with quantities that the HEISENBERG code measures: the typical separation length of independent star-forming regions, \( \lambda \); the duration of the emission map, \( \tau_{emi} \); and the duration of the overlap between the emission and reference phases, \( \tau_{lower} \). We note that Equation C1 and Equation 24 are equivalent through the definition

\[ \tau \equiv \tau_{emi} + \tau_{ref} - \tau_{lower}. \]  

(C2)

Here we describe how we propagate the uncertainties on these quantities into an uncertainty on the characteristic region mass \( M_i \).

To calculate the uncertainty on \( M_i \) we start with the general expression: the uncertainty on a quantity \( f, \sigma_f \), which is a function of \( N \) variables \( f(x_1, \ldots, x_N) \) is given by (Hughes & Hase 2010)

\[ \sigma_f^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \frac{\partial f}{\partial x_i} \cdot \frac{\partial f}{\partial x_j} \cdot \rho_{ij} \cdot \sigma_i \cdot \sigma_j \right). \]  

(C3)

where \( \sigma_i \) represents the uncertainty on variable \( x_i \) and \( \rho_{ij} \) represents correlation coefficients between variable \( x_i \) and \( x_j \). In order to simplify our expressions and to use the same notation as in Equation C3, we define the following

\[ \kappa \equiv \Sigma_{SFR} \cdot \pi \left(\frac{\lambda}{2}\right)^2, \]  

(C4a)

\[ x_1 \equiv \lambda, \]  

(C4b)

\[ x_2 \equiv \tau_{emi}, \]  

(C4c)

\[ x_3 \equiv \tau_{lower} \]  

(C4d)

and Equation C1 becomes

\[ M_i = \kappa \cdot x_2 \tau_{emi} x_1^2. \]  

(C5a)

We note that \( \Sigma_{SFR} \) and \( \tau_{ref} \) are considered to be without error and do not need to be included as variables. The derivatives we need in order to calculate \( \sigma_{M_i} \) are

\[ \frac{\partial}{\partial x_1} M_i = 2 \kappa \tau_{emi} x_1^2, \]  

(C5b)

\[ \frac{\partial}{\partial x_2} M_i = \kappa x_1^2, \]  

(C5c)

\[ \frac{\partial}{\partial x_3} M_i = -\kappa x_1^2. \]  

(C5d)

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Figure A1. The results of \textsc{Starburst99} simulations for an instantaneous burst of star formation at 0 Myr. We show the change in equivalent width of the H$\alpha$ absorption and emission feature; we also include the difference between the two equivalent widths (H$\alpha$ Observed). The H$\alpha$ – time-scale is marked for comparison.

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Table B1. A summary of all the characteristic time-scales and corresponding age bins (for producing reference maps in Sections 5 and 6), for the different star formation rate tracers (see Table 1 for details). These results are for a well sampled IMF. The filter order is in increasing filter width (W) for Hα+ and decreasing response-weighted mean wavelength (λ*ν) for UV. Table 3 is included in these tables under the 1.00 Z⊙ heading.

(a) Characteristic time-scales, tE0:  

|          | 0.05Z⊙ | 0.20Z⊙ | 0.40Z⊙ | 1.00Z⊙ | 2.00Z⊙ |
|----------|--------|--------|--------|--------|--------|
| Hα−      | 5.5±0.2 | 5.1±0.1 | 4.8±0.3 | 4.3±0.3 | 3.9±0.3 |
| Hα+ 10 Å | 6.2±0.3 | 6.3±0.3 | 5.9±0.3 | 5.6±0.2 | 5.1±0.1 |
| Hα+ 20 Å | 8.0±0.3 | 8.0±0.3 | 7.7±0.4 | 7.3±0.4 | 7.3±0.4 |
| Hα+ 40 Å | 9.0±0.2 | 10.2±0.3 | 9.6±0.5 | 9.3±0.3 | 9.1±0.3 |
| Hα+ 80 Å | 12.1±0.2 | 11.7±0.2 | 11.9±0.2 | 10.7±0.2 | 10.7±0.3 |
| Hα+ 160 Å | 15.1±0.2 | 15.0±0.2 | 15.5±0.3 | 16.4±0.3 | 16.2±0.3 |
| WFC3UVIS1F336W | 31.4±0.3 | 29.7±0.2 | 30.6±0.4 | 33.3±0.4 | 25.3±0.3 |
| WFCPC2F363W | 31.2±0.3 | 30.2±0.3 | 30.3±0.3 | 33.1±0.3 | 25.3±0.3 |
| WFCPC2F300W | 31.6±0.3 | 28.0±0.2 | 30.0±0.2 | 27.7±0.2 | 21.2±0.2 |
| WFC3UVIS1F275W | 30.7±0.3 | 27.9±0.6 | 28.3±0.6 | 23.5±0.4 | 19.1±0.2 |
| UVOTW1 | 30.4±0.3 | 28.0±0.6 | 26.5±0.7 | 21.8±0.2 | 18.6±0.4 |
| WFCPC2F255W | 30.1±0.3 | 27.4±0.5 | 26.3±0.3 | 22.4±0.2 | 18.4±0.3 |
| WFC3UVIS1F225W | 29.8±0.2 | 26.5±0.4 | 24.7±0.3 | 19.6±0.2 | 17.2±0.3 |
| GALEXNUV | 29.5±0.3 | 26.3±0.3 | 23.6±0.2 | 19.6±0.2 | 17.1±0.2 |
| UVOTM2 | 29.5±0.3 | 26.1±0.3 | 23.3±0.2 | 19.5±0.2 | 16.8±0.3 |
| WFC3UVIS1F218W | 29.2±0.4 | 26.0±0.3 | 23.3±0.2 | 19.4±0.2 | 16.9±0.2 |
| UVOTW2 | 29.3±0.3 | 24.8±0.2 | 22.2±0.2 | 19.0±0.2 | 16.5±0.3 |
| GALEXFUV | 26.7±0.3 | 21.4±0.2 | 19.7±0.2 | 17.1±0.2 | 14.5±0.8 |

(b) Age bins, tE0 ≤ Age ≤ 2tE0:  

|          | 0.05Z⊙ | 0.20Z⊙ | 0.40Z⊙ | 1.00Z⊙ | 2.00Z⊙ |
|----------|--------|--------|--------|--------|--------|
| Hα−      | 5.5 − 11.0 | 5.1 − 10.1 | 4.8 − 9.5 | 4.3 − 8.6 | 3.9 − 7.9 |
| Hα+ 10 Å | 6.2 − 12.3 | 6.3 − 12.7 | 5.9 − 11.7 | 5.6 − 11.1 | 5.1 − 10.2 |
| Hα+ 20 Å | 8.0 − 16.1 | 8.0 − 16.1 | 7.7 − 15.5 | 7.3 − 14.6 | 7.3 − 14.6 |
| Hα+ 40 Å | 9.0 − 17.9 | 10.2 − 20.3 | 9.6 − 19.3 | 9.3 − 18.6 | 9.1 − 18.2 |
| Hα+ 80 Å | 12.1 − 24.1 | 11.7 − 23.4 | 11.9 − 23.7 | 10.7 − 21.4 | 10.7 − 21.3 |
| Hα+ 160 Å | 15.1 − 30.1 | 15.0 − 30.0 | 15.5 − 31.1 | 16.4 − 32.7 | 16.2 − 32.4 |
| WFC3UVIS1F336W | 31.4 − 62.8 | 29.7 − 59.5 | 30.6 − 61.2 | 33.3 − 66.6 | 25.3 − 50.6 |
| WFCPC2F363W | 31.2 − 62.3 | 30.2 − 60.4 | 30.3 − 60.7 | 33.1 − 66.3 | 25.3 − 50.6 |
| WFCPC2F300W | 31.6 − 63.1 | 28.0 − 55.9 | 30.0 − 60.0 | 27.7 − 54.5 | 21.2 − 42.5 |
| WFC3UVIS1F275W | 30.7 − 61.3 | 27.9 − 55.9 | 28.3 − 56.5 | 23.5 − 47.0 | 19.1 − 38.3 |
| UVOTW1 | 30.4 − 60.8 | 28.0 − 56.0 | 26.5 − 53.1 | 21.8 − 43.5 | 18.6 − 37.2 |
| WFCPC2F255W | 30.1 − 60.3 | 27.4 − 54.8 | 26.3 − 52.6 | 22.4 − 44.7 | 18.4 − 36.8 |
| WFC3UVIS1F225W | 29.8 − 59.5 | 26.5 − 53.0 | 24.7 − 49.3 | 19.6 − 39.3 | 17.2 − 34.5 |
| GALEXNUV | 29.5 − 59.1 | 26.3 − 52.5 | 23.6 − 47.3 | 19.6 − 39.1 | 17.1 − 34.1 |
| UVOTM2 | 29.5 − 58.9 | 26.1 − 52.2 | 23.3 − 46.7 | 19.5 − 39.0 | 16.8 − 33.5 |
| WFC3UVIS1F218W | 29.5 − 59.0 | 26.0 − 52.0 | 23.3 − 46.7 | 19.4 − 38.9 | 16.9 − 33.9 |
| UVOTW2 | 29.3 − 58.7 | 24.8 − 49.5 | 22.2 − 44.3 | 19.0 − 38.0 | 16.5 − 33.0 |
| GALEXFUV | 26.7 − 53.5 | 21.4 − 42.9 | 19.7 − 39.4 | 17.1 − 34.2 | 14.5 − 29.0 |

Combining Equation 6 with Equation 3 we find the expression for the uncertainty on $M_\star$, $\sigma_{M_\star}$:

$$\left[ \frac{\sigma_{M_\star}}{M_\star} \right]^2 = \frac{4\sigma_1^2}{\chi_1^2} + \frac{(\sigma_2^2 + \sigma_3^2 - 2\rho_{23}\sigma_2\sigma_3)}{\tau_2^2} + \frac{4(\rho_{12}\sigma_1\sigma_2 - \rho_{13}\sigma_1\sigma_3)}{\tau_1^2}. \quad (C7)$$

With this expression, we can take into account the associated uncertainty on the value of $M_\star$ as part of our error analysis and $\chi^2$ calculations when investigating the effects of incomplete IMF sampling on the characteristic time-scales of SFR tracers.

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