Stealth acceleration and modified gravity

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Abstract. We show how to construct consistent braneworld models which exhibit late time acceleration. Unlike self-acceleration, which has a de Sitter vacuum state, our models have the standard Minkowski vacuum and accelerate only in the presence of matter, which we dub ‘stealth acceleration’. We use an effective action for the brane which includes an induced gravity term, and allow for an asymmetric set-up. We study the linear stability of flat brane vacua and find the regions of parameter space where the set-up is stable. The four-dimensional graviton is only quasi-localized in this set-up and as a result gravity is modified at late times. One of the two regions is strongly coupled and the scalar mode is eaten up by an extra symmetry that arises in this limit. Having filtered the well-defined theories we then focus on their cosmology. When the graviton is quasi-localized we find two main examples of acceleration. In each case, we provide an illustrative model and compare it to ΛCDM.

Keywords: dark energy theory, cosmology with extra dimensions, gravity

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1. Introduction

Recent observations of high redshift supernovae suggest that our universe has entered a period of acceleration [1]. Combining these observations with the microwave background data [2], and the results of large scale structure surveys [3], we deduce that the universe is approximately spatially flat, and that the observed acceleration is consistent with standard 4D cosmology if we assume that roughly 70% of the actual energy content of the universe comes from a small positive cosmological constant, $\Lambda/8\pi G \sim 10^{-12} $ (eV)$^4$ [4]. Of the residual matter content, only about 4% corresponds to baryonic matter, the remaining component corresponding to an also as yet unobserved ‘dark matter sector’. Although this simple energy pie chart fits the data extremely well, effective field theory methods have so far failed to give a satisfactory explanation for such a small value of the cosmological constant. Naive arguments predict a cosmological constant of the order of $m_{\text{Pl}}^4$, i.e. $10^{120}$ times larger than the ‘observed’ value, which fine-tuning problem has inspired a search for alternative explanations for the observed acceleration.

An interesting alternative is that supernovae data are actually indicating the presence of a new physical scale where novel gravitational physics kicks in. The scale in question is a classical energy scale corresponding to the current Hubble curvature radius, $H_0 \sim 10^{-34} $ (eV). The braneworld scenario [5,6] has been a natural breeding ground for these ideas (see, for example, [7]–[10]). Typically, gravity becomes weaker in the far infra-red due to gravitational leakage into the extra dimensions. In some cases, this modification of gravity at large distances leads directly to exponential acceleration at late times, even when there is no effective cosmological constant [8,10,11]. Solutions of these types are often referred to as self-accelerating, since their vacuum state corresponds to a de Sitter brane. However, there is by now good evidence that self-accelerating solutions lead to problems with perturbative ghosts [12].

Modifying Einstein’s general relativity in a consistent manner in the infra-red has proven to be a very difficult task. There are three main reasons for this: theoretical
consistency such as ghost-like instability or strong coupling; experimental bounds mostly coming from the solar system; and naturalness, i.e. a restricted number of additional fields and parameters for the modified theory. The first problem, for the time being at least, has been the most stringent one. Indeed, it is common for the vacua of such modified theories to suffer from classical and/or quantum instabilities, at least at the level of linear perturbations. Most candidate braneworld models [7]–[9] have been shown to suffer from such instabilities or strong coupling or both [12, 13]. In the absence of exact solutions, gravity becomes unpredictable at the strong coupling scale due to a breakdown of linear perturbation theory. Whilst this is certainly problematic from a practical point of view, it has been argued that strong coupling is actually a crucial ingredient in these models, enabling them to evade problems with solar system observations [14, 15]. The presence of ghosts (coupled to ordinary fields) represents a far more serious problem. Generically, a ghost mode appears in the perturbative spectrum of the theory at the scale where gravity is modified, effectively driving the acceleration, much like a cosmological constant put in by hand. The problem is that the ghost and the coupled fields will be spontaneously produced in the vacuum, including modes of arbitrarily high 3-momenta that will rapidly destroy the background.

The situation is not better settled for four-dimensional theories that modify the Einstein–Hilbert action directly. Here, early attempts go back to Brans and Dicke [16], who considered a massless scalar–tensor theory with only one additional constant degree of freedom, the kinetic coupling $\omega_{BD}$. Such a theory (which for example meets the naturality condition) has been ruled out by solar system time delay experiments pushing $\omega_{BD} \gtrsim 40,000$ coming from the Cassini satellite [17]. Non-minimal scalar–tensor theories with some potential and hence varying $\omega$ can still be consistent with PPN experiments if we suppose that the scalar field varies between solar system and cosmological scales. Indeed, we remind the reader that the relevant cosmological scale $H_0^{-1}$ and the size of the solar system give a dimensionless quotient of $(1 \text{ AU} H_0) \sim 10^{-15}$ whereas the typical PPN parameter $\gamma$ is 1 with error $10^{-5}$. Vector–tensor theories have also been shown to be plagued by instabilities (the exception being the case studied by Jacobson [18] where the vector has a fixed norm, and Lorentz invariance is broken at the level of the action$^4$). Adding higher order combinations of the Riemann curvature tensor also generically leads to the appearance of ghosts except if one considers functionals of the Ricci scalar curvature $F(R)$ [19]. Then the theory in question can be translated via a conformal transformation to a non-minimal scalar–tensor theory. Solar system constraints are again problematic (though see [20] for recent developments) and often vacuum spacetimes such as Minkowski are not even solutions to the field equations (although perhaps this is not such a big problem in the cosmological setting).

In this paper we present an alternative general class of ghost-free braneworld models which possess accelerating solutions. However, unlike the usual self-accelerating braneworld solutions, and indeed unlike ΛCDM, our models do not necessarily lead to de Sitter cosmologies at late times. Indeed, the final fate of the universe may not even be an accelerating cosmology! Also, unlike the usual models in which even the vacuum brane accelerates, in our models the vacuum state is a stable Minkowski brane, and the Friedmann equation actually implies that while the Minkowskian brane is the vacuum state

$^4$ At the level of cosmological solutions this is of course true since there is no timelike Killing vector; the difference there being that the symmetry is not broken at the level of the action.
for an empty brane, once one has an ‘ordinary’ cosmological fluid, i.e. \( p = w \rho \) with \( w \geq 0 \), then the brane can enter an era of acceleration, even though there is no cosmological constant or dark energy present. Depending on the specific parameters in the model, this acceleration can either persist, or the universe at some stage exits from accelerated expansion into a stiff matter cosmology. In either case, the expansion is power law, rather than exponential.

Clearly an accelerating cosmology is of no interest if it is hampered by the usual consistency problems; therefore we perform a general scan of possible braneworld solutions, filtering out those solutions that we suspect will contain problems with ghosts (such as self-accelerating vacua or regions where we prove the appearance of ghosts). We also further restrict our parameter space by requiring that the bout of acceleration is preceded by a sufficient period of standard 4D cosmology, in order to reproduce standard early universe cosmology, such as nucleosynthesis, the development of structure etc, so that the standard cosmological picture is retained. Note, therefore, that these braneworld models are not suited for explaining primordial inflation. In fact, the basic feature of these models is that it is only at late times in a matter dominated universe that this acceleration can kick in. Indeed, if the parameters of the model were tuned so that the modified gravity scale would become important during the radiation era, then the universe would not accelerate, but merely ‘coast’ [22], before returning to an expanding, decelerating, cosmology. Therefore, our models explain quite naturally why it is only at scales comparable to the current Hubble time (or indeed longer) that acceleration can occur.

The rest of this paper is organized as follows. In the next section we describe our set-up involving a single brane embedded in some five-dimensional bulk spacetime. This will be a combination of the DGP [8], and asymmetric brane [10] models. In other words, we consider an asymmetric set-up, with an induced curvature term on the brane. We will present the equations of motion, and derive the vacuum states corresponding to Minkowski branes. In section 3 we consider vacuum perturbations, focusing on the scalar radion mode. We will derive the effective action for this mode, in order to filter out those solutions that contain a perturbative ghost. In section 4 we introduce a cosmological fluid and derive the cosmological solutions, establishing the conditions required for a consistent cosmology evolving towards a Minkowski vacuum as the energy density decreases. The consistent solutions are then analysed in greater detail in section 5, with special emphasis on the conditions for cosmic acceleration. We analyse two main models in detail, showing how they give rise to cosmic acceleration, and comparing these models with ΛCDM. Finally, we conclude in section 6.

2. Asymmetric braneworlds: formalism and set-up

Consider a single 3-brane, \( \Sigma \), embedded in between two bulk five-dimensional spacetimes \( \mathcal{M}_i \), where \( i = 1, 2 \). The brane can be thought of as the common boundary, \( \Sigma = \partial \mathcal{M}_1 = \partial \mathcal{M}_2 \) of these manifolds. Each spacetime \( \mathcal{M}_i \) generically has a five-dimensional Planck scale given by \( M_i \), and a negative (or zero) cosmological constant given by \( \Lambda_i = -6k_i^2 \). In general we will not consider \( Z_2 \) symmetry and we will allow for the cosmological constants, and even the fundamental mass scales to differ on either side of the brane [10]. Allowing for \( \Lambda_1 \neq \Lambda_2 \) is familiar enough in domain wall scenarios [23]. Here we are also allowing for \( M_1 \neq M_2 \). This is not so familiar, but could arise in a number of ways. Suppose,
for example, that this scenario is derived from a fundamental higher dimensional theory. This theory could contain a dilaton field that is stabilized in different fundamental vacua on either side of $\Sigma$. From the point of view of a 5D effective description, the 5D Planck scales would then differ accordingly. Indeed naive expectations from string theory point towards this asymmetric scenario as opposed to a symmetric one. Different effective Planck scales can also appear on either side of a domain wall that is bound to a five-dimensional braneworld [24].

The brane itself has some vacuum energy, or tension. This will ultimately be fine-tuned against the bulk parameters, $\Lambda_i$ and $M_i$, in order to admit a Minkowski vacuum solution. In other words, there will be no effective cosmological constant on the brane. As in the original DGP model [8], we will also allow for some intrinsic curvature to be induced on the brane. Such terms are rather natural and can be induced by matter loop corrections [25], finite width effects [26] or even classically from higher dimensional modifications of general relativity [27].

Our set-up is therefore described by the general five-dimensional action,

$$S = S_{\text{bulk}} + S_{\text{brane}}.$$  \hfill (2.1)

The bulk contribution to the action is described by

$$S_{\text{bulk}} = \sum_{i=1,2} M_i^3 \int_{M_i} \sqrt{-g} (R - 2\Lambda_i) + 2M_i^3 \int_{\partial M_i} \sqrt{-\gamma} K^{(i)}$$  \hfill (2.2)

where $g_{ab}$ is the bulk metric with corresponding Ricci tensor $R$. The metric induced on the brane ($\partial M_i$) is given by

$$\gamma_{ab} = g_{ab} - n_a n_b$$  \hfill (2.3)

where $n^a$ is the unit normal to $\partial M_i$ in $M_i$ pointing out of $M_i$. Of course, continuity of the metric at the brane requires that $\gamma_{ab}$ is the same whether it is calculated from the left or from the right of the brane. In contrast, the extrinsic curvature of the brane can jump from right to left. In $\partial M_i$, it is defined as

$$K^{(i)}_{ab} = \gamma^{(i)}_a \gamma^{(i)}_b \nabla_c n^d.$$

Its trace appears in the Gibbons–Hawking boundary term in (2.2).

The brane contribution to the action, meanwhile, is described by

$$S_{\text{brane}} = \int_{\text{brane}} \sqrt{-\gamma} (m_{\text{pl}}^2 R - \sigma + L_{\text{matter}})$$  \hfill (2.5)

where $\sigma$ is the brane tension, and $L_{\text{matter}}$ includes any matter excitations. We have also included the induced intrinsic curvature term, $\mathcal{R}$, weighted by a 4D mass scale, $m_{\text{pl}}$. Note that we have taken $m_{\text{pl}}^2 > 0$, as in the original DGP model. There are two reasons for this. Firstly, when it comes to studying the cosmological solutions, we would like this term to dominate the cosmology at early times, in order to reproduce the standard 4D cosmology, as discussed in the introduction. Secondly, allowing $m_{\text{pl}}^2 < 0$ could result in vacuum perturbations containing a spin-2 ghost [28].

The equations of motion in the bulk region, $M_i$, are just the Einstein equations, with the appropriate cosmological constant, $\Lambda_i$.

$$E_{ab} = R_{ab} - \frac{1}{2} R g_{ab} + \Lambda_i g_{ab} = 0.$$  \hfill (2.6)
The equations of motion on the brane are described by the Israel junction conditions, and can be obtained by varying the action (2.1), with respect to the brane metric, $\gamma^{ab}$. This gives

$$\Theta^{ab} = 2 \langle M^3 (K_{ab} - K_{\gamma ab}) \rangle + m^2_{\text{pl}} \left( R_{ab} - \frac{1}{2} R \gamma_{ab} \right) + \frac{\sigma}{2} \gamma_{ab} = \frac{1}{2} T_{ab} (2.7)$$

where $T_{ab} = -(2/\sqrt{-\gamma})(\partial \sqrt{-\gamma} \mathcal{L}_{\text{matter}}/\partial \gamma^{ab})$. The angled brackets denote an averaged quantity at the brane. More precisely, for some quantity $Q_i$ defined on the brane in $\partial M_i$, we define the average

$$\langle Q \rangle = \frac{Q_1 + Q_2}{2} (2.8)$$

Later on we will also make use of the difference, $\Delta Q = Q_1 - Q_2$. Note that the Israel equations here do not use the familiar ‘difference’, because we have defined the unit normal as pointing out of $M_i$ on each side, i.e. the approach is that of the brane as a boundary. Israel’s equations on the other hand were derived for thin shells in GR, i.e. where the brane is a physical, very thin, object, and the normal is thus continuous, pointing ‘out’ on one side of the wall, and ‘in’ on the other.

We will now derive the vacuum solutions to the equations of motion (2.6) and (2.7). This corresponds to the case where there are no matter excitations, and so $T_{ab} = 0$. In each region of the bulk, we introduce coordinates $x^a = (x^\mu, y)$, with the brane located at $y = 0$. We will not be interested in de Sitter solutions, since these will only arise through an excess in vacuum energy, or through ‘self-acceleration’. The former offers no alternative to $\Lambda$CDM, whereas the latter is expected to suffer from a generic ghost-like instability [12]. Therefore, we seek solutions of the form

$$ds^2 = \bar{g}_{ab} dx^a dx^b = a^2(y) \eta_{ab} dx^a dx^b. (2.9)$$

Inserting this into the bulk equations of motion (2.6) gives

$$\left( \frac{a'}{a} \right)^2 = k^2 a^2, \quad \frac{a''}{a} = 2 k^2 a^2 (2.10)$$

where ‘prime’ denotes differentiation with respect to $y$. Note that we have dropped the asymmetry index $i$ for brevity. Equations (2.10) have solution

$$a(y) = \frac{1}{1 - \theta ky} (2.11)$$

where $\theta = \pm 1$. Note that each region of the bulk corresponds to $0 < y < y_{\text{max}}$, where

$$y_{\text{max}} = \begin{cases} 1/k & \text{for } \theta = 1 \\ \infty & \text{for } \theta = -1. \end{cases} (2.12)$$

For $k \neq 0$, this means that when $\theta = 1$ we are keeping the adS boundary (growing warp factor) whereas when $\theta = -1$ we are keeping the adS horizon (decaying warp factor). For $k = 0$, we simply have a Minkowski bulk, in the usual coordinates, and the sign of $\theta$ is irrelevant.

The boundary conditions at the brane (2.7) lead to a finely tuned brane tension

$$\sigma = -12 \langle M^3 \theta k \rangle. (2.13)$$

This fine-tuning guarantees that there is no cosmological constant on the brane, and is equivalent to the (asymmetric) Randall–Sundrum fine-tuning for $\theta_1 k_1 < 0$ and $\theta_2 k_2 < 0$ [6].
3. Linearized vacuum perturbations and asymptotic stability

We shall now consider linearized perturbations, $h_{ab}$ about our background solutions (2.9) and (2.11), so that

$$ds^2 = a^2(y) [\eta_{ab} + h_{ab}(x, y)] \, dx^a \, dx^b.$$  

(3.1)

In the unperturbed spacetime, the gauge was fixed in both $M_1$ and $M_2$ so that the brane was at $y = 0$. However, a general perturbation of the system must also allow the brane position to flutter. In $M_i$, the brane will be located at

$$y = f_i(x^\mu).$$  

(3.2)

Of course, these expressions contain some gauge dependence due to invariance under the following diffeomorphism transformations:

$$y \to y + \eta(x, y), \quad x^\mu \to x^\mu + \zeta^\mu(x, y).$$  

(3.3)

Now, it is convenient and physically relevant to decompose these transformations in terms of the 4D diffeomorphism group. This gives $\zeta^\mu = \xi^\mu + \partial^\mu \xi$, where $\xi^\mu$ is a Lorentz gauge vector satisfying $\partial_\mu \xi^\mu = 0$. We do likewise for the perturbation:

$$h_{\mu\nu} = h_{\mu\nu}^{TT} + 2 \partial_\mu F_\nu + 2 \partial_\nu \partial_\mu E + 2 A \eta_{\mu\nu},$$  

(3.4)

$$B_\mu = B_\mu - \xi'_\mu,$$  

(3.5)

$$h_{yy} = 2 \phi.$$  

(3.6)

Again, $F_\mu$ and $B_\mu$ are Lorentz gauge vectors, whereas $h_{\mu\nu}^{TT}$ is a transverse-trace-free tensor, $\partial_\mu h_{\mu\nu}^{TT} = h_{\mu\nu}^{TT} = 0$. Note that Greek indices are raised and lowered using $\eta_{\mu\nu}$. Under the gauge transformations (3.3), the various components of the perturbation transform as follows:

$$h_{\mu\nu}^{TT} \to h_{\mu\nu}^{TT},$$  

(3.7)

$$B_\mu \to B_\mu - \xi'_\mu,$$  

(3.8)

$$F_\mu \to F_\mu - \xi_\mu,$$  

(3.8)

$$\phi \to \phi - \frac{(\eta a')'}{a}, \quad B \to B - \xi' - \eta, \quad E \to E - \xi, \quad A \to A - \frac{a'}{a} \eta.$$  

(3.9)

We immediately see that the tensor component, $h_{\mu\nu}^{TT}$ is gauge invariant. We can also construct the following vector and scalar gauge invariants in the bulk:

$$X_\mu = B_\mu - F'_\mu, \quad X = A - \frac{a'}{a} (B - E'), \quad Y = \phi - \frac{[a(B - E')]'}{a}.$$  

(3.10)

We will now consider vacuum fluctuations, so that the bulk equations of motion and the Israel junction condition are given by $\delta E_{ab} = \delta \Theta_{ab} = 0$. At this point we assume that the tensors, vectors and scalars do not mix with one another, so that their equations of motion can be taken independently. We will come back and check the validity of this hypothesis against our results later on. Focusing on the scalars, we find that the bulk
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equations of motion give \( \delta E_{ab}^{\text{scalar}} = 0 \), where

\[
\begin{align*}
\delta E_{\mu \nu}^{\text{scalar}} &= -\partial_{\mu} \partial_{\nu} + 2 \eta_{\mu \nu} \left[ \frac{\alpha'}{a} \right] \left( X' - \frac{\alpha'}{a} Y \right) \quad (3.11) \\
\delta E_{\mu y}^{\text{scalar}} &= -3 \partial_{\mu} \left( X' - \frac{\alpha'}{a} Y \right) \quad (3.12) \\
\delta E_{yy}^{\text{scalar}} &= 3 \partial_{\mu}^{2} X + 12 \frac{\alpha'}{a} \left( X' - \frac{\alpha'}{a} Y \right) . \quad (3.13)
\end{align*}
\]

These equations are easily solved on each side of the brane (we remind the reader that we have dropped the asymmetry index \( i \)) to give

\[
X = \frac{U(x)}{a^2}, \quad Y = -\frac{2U(x)}{a^2} \quad (3.14)
\]

where \( \partial^{2} U = 0 \). We can identify \( U(x) \) as the bulk radion mode. It represents two degrees of freedom, \( U_{i}(x), \; i = 1, 2 \), one on each side of the brane. In addition to this, we have the brane bending degrees of freedom, given by \( f_{1} \) and \( f_{2} \). Given that we have two boundary conditions at the brane, namely continuity of the metric and the Israel equations, we expect the number of physical scalar degrees of freedom to be at most two, as we shall now demonstrate.

To impose the boundary conditions at the brane it is convenient to work in the brane-GN (Gaussian-normal) gauge. In this gauge, we have \( B = \phi = 0 \) and the brane is fixed at \( y = 0 \). It follows that

\[
E = W(x) + V(x)Q(y) - U(x)Q(y)^{2}, \quad A = \frac{U(x)}{a^2} - \frac{\alpha'}{a^2} \left[ V(x) - 2U(x)Q(y) \right] \quad (3.15)
\]

where

\[
Q(y) = \int_{0}^{y} \frac{dy_{1}}{a(y_{1})} = \frac{1}{2 \theta k} \left[ 1 - a^{-2}(y) \right] . \quad (3.16)
\]

The brane bending degrees of freedom are now encoded in the fields, \( V_{1} \) and \( V_{2} \). In contrast, \( W_{1} \) and \( W_{2} \) merely reflect the freedom to choose the gauge along the brane, and as such, can be consistently set to zero. To see this, we evaluate our solution at \( y = 0 \) to derive the scalar part of the brane metric (3.4)

\[
h_{\mu \nu}^{\text{scalar}}(x, 0) = 2 \partial_{\mu} \partial_{\nu} W + 2[U - \theta kV]\eta_{\mu \nu}. \quad (3.17)
\]

The pure gauge part, \( 2 \partial_{\mu} \partial_{\nu} W \), and the remainder \( 2[U - \theta kV]\eta_{\mu \nu} \) need to be well defined independently of one another. This means that \( \Delta W = 0 \), and it is easy to see that we can continuously gauge away \( W \) on both sides of the brane, so that \( W = 0 \).

The continuity condition at the brane and the Israel equations now require that

\[
\Delta [U - \theta kV] = \left( M^{3}V + m_{\text{pl}}^{2}[U - \theta kV] \right) = 0. \quad (3.18)
\]

Therefore, as predicted, we are left with at most two scalar degrees of freedom as perceived by a four-dimensional observer. Actually, it is easy to see why this is so if one introduces regulator branes at (say) \( y = y_{*} < y_{\text{max}} \). The remaining degrees of freedom essentially correspond to the fluctuations in the proper distance in between branes. Actually, the introduction of regulator branes is particularly useful when trying to impose boundary
conditions in the bulk, and in terms of calculating a 4D effective action. We shall now develop this in more detail.

In the background, the extrinsic curvature, $K^s_{\mu\nu}$, of the regulator brane satisfies the following:

$$K^s_{\mu\nu} + \frac{a'(y_s)}{a^2(y_s)}\gamma^s_{\mu\nu} = 0 \quad (3.19)$$

where $\gamma^s_{\mu\nu}$ is the induced metric on the regulator brane. We require that this equation also holds in the perturbed scenario. This time, it is easiest to work in GN coordinates relative to the regulator brane (we shall call these regulator-GN coordinates). The regulator brane is now fixed at $y = y_s$, and $B = \phi = 0$. Again, it follows that

$$E = W_s(x) + V_s(x)Q_s(y) - U(x)Q_s(y)^2, \quad A = \frac{U}{a^2} - \frac{a'}{a^2}[V_s(x) - 2U(x)Q_s(y)] \quad (3.20)$$

where

$$Q_s(y) = \int_y^{y_s} \frac{dy_1}{a(y_1)} = \frac{1}{2\theta k} \left[ a^{-2}(y_s) - a^{-2}(y) \right]. \quad (3.21)$$

As before we can set $W_s = 0$. Applying the boundary condition (3.19) we deduce that $V_s = 0$.

It is very important to realize that the brane-GN coordinates and the regulator-GN coordinates are not necessarily the same. In particular, in brane-GN coordinates the true brane is at $y = 0$, but the regulator brane might not be at $y = y_s$. Of course the two sets of coordinates must be related by a gauge transformation. Starting in regulator-GN gauge, we transform to brane-GN gauge with the following coordinate change:

$$x^\mu \to x^\mu + D^\mu \xi, \quad y \to y + \eta \quad (3.22)$$

where

$$\xi = U \left[ Q(y)^2 - Q_s(y)^2 \right] - VQ(y), \quad \eta = \frac{V}{a} - \frac{2U}{a} \left[ Q(y) - Q_s(y) \right]. \quad (3.23)$$

From this we can deduce that in brane-GN coordinates the regulator brane is at $y = y_s + \eta(x, y_s)$ where

$$\eta(x, y_s) = \frac{V - 2UQ(y_s)}{a(y_s)}. \quad (3.24)$$

We would now like to calculate the proper distance between the branes. To do this carefully, it is best to work in a gauge in which both the true brane and the regulator brane are fixed. Clearly neither brane-GN nor regulator-GN gauge will do. Instead, we start off in brane-GN coordinates, and transform to fixed brane coordinates by letting

$$x^\mu \to x^\mu, \quad y \to y + \epsilon(x, y), \quad (3.25)$$

so that

$$\phi \to -\frac{(\epsilon a)'}{a}, \quad B \to -\epsilon, \quad E \to E, \quad A \to A - \frac{a'}{a}\epsilon. \quad (3.26)$$

If we set $\epsilon(x, 0) = 0$ and $\epsilon(x, y_s) = -\eta(x, y_s)$, both branes will be fixed, at $y = 0$ and $y = y_s$ respectively. The proper distance between them is given by

$$z_s = \int_0^{y_s} dy \sqrt{g_{yy}} = \bar{z}_s + \delta z_s \quad (3.27)$$
where
\[ \bar{z}_* = -\frac{1}{\theta k} \ln(1 - \theta k y_*) \] (3.28)
is the proper distance between branes in the background, and
\[ \delta z_* = V - 2Q(y_*)U \] (3.29)
is the fluctuation. A fixed wall gauge is also useful for calculating the effective action. If we assume that the boundary conditions hold at the true brane and at both regulator branes, the effective action (to quadratic order) is simply given by the following bulk integral:
\[ S_{\text{eff}} = \frac{1}{2} \int_{\text{bulk}} \sqrt{-g} M^3 \delta g^{ab} \delta E_{ab}. \] (3.30)
Choosing the fixed brane gauge in both the left hand bulk and the right hand bulk, we find
\[ S_{\text{eff}} = -6 \int d^4x \left\langle M^3 \delta z_* \partial^2 U(x) \right\rangle \] (3.31)
where \( U \) can be related to the fields \( \delta z_* \) using the boundary conditions at the brane (3.18). Now consider what happens, as we remove the regulators by taking the limit \( y \to y_{\text{max}} \), or equivalently \( \bar{z}_* \to \infty \). For \( U \neq 0 \),
\[ \delta z_* \to \begin{cases} -A/k & \text{whenever } \theta k > 0 \\ \infty & \text{whenever } \theta k \leq 0. \end{cases} \] (3.32)
Therefore, if \( \theta k \leq 0 \), the fluctuation in the proper distance between branes diverges. This reflects the fact that the gauge invariant mode, \( U \), is non-normalizable. Since we must have normalizable boundary conditions in order to obtain a local 4D effective theory on the brane, we require that \( U = 0 \) whenever \( \theta k \leq 0 \). The result is that the scalar degree of freedom coming from a given bulk region only survives the single-brane limit when the corresponding regulator is taken to the adS boundary as opposed to the adS horizon. This makes sense for the following reason. If an observer on the brane shines a light ray into a bulk region towards the adS boundary, he/she sees it reflected back from the boundary in a finite proper time. However, if the ray is shone into the bulk towards the adS horizon, it will never be reflected back. One is therefore able to detect fluctuations in the proper distance between the brane and the adS boundary, but not the brane and the adS horizon.

After removing the regulators, and imposing the boundary conditions (3.18), we find that there are no normalizable scalar degrees of freedom left if \( \theta_1 k_1, \theta_2 k_2 < 0 \). This does not come as a surprise since in this case the bulk includes the adS horizon on both sides of the brane, and so one would not expect there to be any normalizable radion degrees of freedom for the reasons outlined in the previous paragraph. In all other cases, the effective action (3.31) is given by
\[ S_{\text{eff}} = 6 \left[ m_{\text{pl}}^2 - \left\langle M^3 \theta \right\rangle \right] \int d^4x (\partial A)^2 \] (3.33)
where the field \( A = A(x, 0) = U - \theta k V \) measures conformal rescalings of the metric on the brane. It represents the only scalar degree of freedom that survives, since it is the only
scalar in the induced metric that cannot be locally gauged away. Physically, however, we might have expected there to be two radion degrees of freedom since both sides of the bulk included the adS boundary. Careful inspection, however, shows that this is not the case. Indeed reintroducing the regulator branes, we can split the two degrees of freedom into a centre of mass motion of the brane and a relative motion [29]. The latter is shown to drop out as we push the regulators out to the boundary. Furthermore, note that even the centre of mass motion drops out if we have a zero cosmological constant on either side of the brane. This is because in flat spacetime we have an extra translation Killing vector in our coordinate chart (with respect to adS) that permits us to consistently gauge away the remaining scalar mode.

Before we start to derive conclusions about when there is a scalar ghost in the spectrum of perturbations, let us pause for a moment to discuss the reliability of these results. In deriving the scalar effective action we assumed that the scalars did not mix with the vectors or tensors. Now, we need not worry about vectors, because the vector contribution can always be locally gauged away on the brane. The same cannot be said for the tensors, which are gauge invariant, so there is a danger that they could mix with the scalars, as was the case for the self-accelerating DGP solution [12]. To see whether this can happen here, we write the scalar gauge-invariant piece in Gaussian-normal gauge as follows:

$$h^{(U)}_{\mu\nu} = -\frac{1}{2k^2\alpha^4}\partial_\mu \partial_\nu U.$$  (3.34)

Because \(\partial^2 U = 0\), we can view \(h^{(U)}_{\mu\nu}\) as a massless transverse-trace-free tensor. Therefore, we can trust our analysis provided there are no massless modes in the tensor sector. This is guaranteed if the background bulk has infinite volume. The volume of the background is finite if and only if \(\theta_1 k_1\) and \(\theta_2 k_2\) are both strictly negative. This corresponds to a generalized Randall–Sundrum scenario where the warp factor decays into the bulk. Then there is indeed a tensor zero mode, and we have mixing between tensors and scalars at zero mass. Although our analysis may therefore be unreliable in this instance, it is well known that in this case there is actually no radion degree of freedom. In fact, the mixing actually ensures that the graviton propagator has the correct 4D tensor structure [30] as in the original RS scenario. It also follows that there can be no infra-red modification of gravity since the graviton zero mode always guarantees Einstein-like behaviour at large distances and at late times. This will not be so interesting from an infra-red cosmological perspective, so from now on, we will drop the case \(\theta_1 k_1, \theta_2 k_2 < 0\).

In all other cases, the bulk volume is infinite and there is no normalizable zero-mode graviton in the spectrum and hence no mixing. However, we do want a quasi-localized graviton dominating up to some energy scale in the graviton spectrum as one gets for DGP [8] or GRS [9]. This is guaranteed by the induced curvature on the brane entering with the ‘correct’ sign \(m^2_{\text{pl}} > 0\), as we will demonstrate in the next section. Whenever the quasi-localized zero mode dominates, the higher mass modes and the radion mix giving some type of well-defined generalized scalar–tensor gravity. At large distances, however, the quasi-localized nature of the graviton disappears giving a continuum of massive modes with no mass gap. Then the radion mode (3.34) no longer mixes and is dangerous in the sense that it can be a ghost. Whenever \(k_1 k_2 = 0\) we have already discussed how the radion mode disappears. However, if \(k_1 k_2 \neq 0\), the radion is present, and we see from (3.33) that
it is not a ghost, provided
\( \chi = m_{\text{pl}}^2 - \left< \frac{M^3 \theta}{k} \right> \leq 0. \) (3.35)

It is interesting to note the competition between the bulk term in \( \chi \), and the induced gravity term: the ‘correct’ sign for the DGP term \( (m_{\text{pl}}^2 > 0) \) always contributes to a ghost-like radion (as does a localized warp factor). This behaviour is caused by the so-called conformal ghost that appears in 4D Einstein gravity for perturbations about Minkowski space (see for example [31]). In that case the scalar mode is harmless since it provides the correct tensor structure in the propagator mixing with the zero-tensor mode. Strictly speaking each individual mode does not exist independently: it is only their linear combination which has physical meaning. However, in our case the absence of a graviton zero mode ensures that there is no mixing between modes and so a radion ghost always signals a vacuum instability.

Of particular interest are the limits \( \chi \to 0 \), and \( \chi \to \infty \). For small/large \( \chi \), the radion couples very strongly/weakly to the trace of the energy–momentum tensor, since schematically we have
\[ -\chi \partial^2 A \sim T. \] (3.36)

In the \( \chi \to \infty \) limit, the radion completely decouples. This corresponds to the case where \( k_1 k_2 = 0 \) so at least one side of the bulk is Minkowski. The radion decouples in this case because it costs no effort to translate the brane toward the Minkowski side, and hence no brane bending can be detected.

The \( \chi \to 0 \) limit might also be referred to as the conformal limit, since the brane can only support conformal matter sources \( (T = 0) \) to linear order in perturbation theory. If we introduce some non-conformal matter, the linearized theory breaks down because of strong coupling, and the geometry responds non-linearly. This behaviour reflects the onset of a new symmetry. It is reminiscent of the partially massless limit \( (m^2 = 2H^2) \) of a massive graviton propagating in de Sitter space [32]. In that theory an extra symmetry kicks in that eliminates the scalar degree of freedom. In our case, the linearized field equations, \( \delta E_{ab} = 0 \) and \( \delta \Theta_{ab} = \frac{1}{2} T_{ab} \), become invariant under the transformation \( h_{ab} \to h_{ab} + h^{(f)}_{ab} \) where
\[ h^{(f)}_{\mu\nu} = (1 - a^{-2}) \partial_\mu \partial_\nu f - 2k^2 f \eta_{\mu\nu}, \quad h^{(f)}_{yy} = h^{(f)}_{yy} = 0. \] (3.37)

This is pure gauge in the bulk, but not on the brane. The transformation therefore encodes an extra symmetry, beyond the usual diffeomorphisms, that eliminates the radion degree of freedom when \( \chi = 0 \).

In summary, if the bulk has infinite volume, we have a theory without a normalizable tensor zero mode, for which the radion will dominate at large distances. In certain limits, the radion either decouples \( (\chi \to \infty) \), or is eliminated by a new symmetry \( (\chi \to 0) \). Otherwise, the radion is present, and will render the vacuum unstable on large scales unless \( \chi \leq 0 \).

4. Cosmological solutions

Let us now consider what happens when we introduce a cosmological fluid to the Minkowski braneworlds derived in section 2. In order to preserve homogeneity and isotropy...
on the brane, we must assume that the bulk metric is a warped product of the form \cite{33}
\begin{equation}
\text{ds}^2 = \mathcal{A}^2(t, z)(-dt^2 + dz^2) + \mathcal{B}^2(t, z)dx^2_k
\end{equation}
where for \( \kappa = 1, 0, -1 \), \( dx^2_k \) is the metric on the unit 3-sphere, plane, and hyperboloid respectively. It turns out that we have enough symmetry to render the bulk equations of motion \eqref{2.6} integrable, and a generalized form of Birkhoff’s theorem applies \cite{33}. When \( \Lambda = \kappa = 0 \), the bulk solution is just a portion of Minkowski spacetime, whereas in all other cases the bulk is a portion of a black hole spacetime with cosmological constant \( \Lambda = -6k^2 \), and horizon geometry parametrized by \( \kappa \):
\begin{equation}
\text{ds}^2 = -V(r)dr^2 + \frac{dr^2}{V(r)} + r^2dx^2_k, \quad V(r) = k^2r^2 + \kappa - \frac{\mu}{r^2}.
\end{equation}
In order to construct the brane, we glue a solution in \( \mathcal{M}_1 \) to a solution in \( \mathcal{M}_2 \), with the brane forming the common boundary. Let us describe this in more detail. Assume for the moment that the bulk solution on both sides of the brane takes the form \eqref{4.2}. In \( \mathcal{M}_1 \), the boundary, \( \partial\mathcal{M}_1 \), is given by the section \((t_i(\tau), r_i(\tau), x^\mu)\) of the bulk metric. The parameter \( \tau \) is the proper time of an observer comoving with the boundary, so that
\begin{equation}
-V_i(r_i)\dot{t}^2 + \frac{\dot{r}^2}{V_i(r_i)} = -1,
\end{equation}
where the overdot corresponds to differentiation with respect to \( \tau \). The outward pointing unit normal to \( \partial\mathcal{M}_1 \) is now given by
\begin{equation}
n_a = \theta_i(\dot{r_i}(\tau), -\dot{t_i}(\tau), 0)
\end{equation}
where \( \theta_i = \pm 1 \). For \( \theta_i = 1 \), \( \mathcal{M}_i \) corresponds to \( r_i(\tau) < r < \infty \), whereas for \( \theta_i = -1 \), \( \mathcal{M}_i \) corresponds to \( 0 \leq r < r_i(\tau) \). The signs of \( \theta \) are consistent with the analysis in the previous two sections.

The induced metric on \( \partial\mathcal{M}_1 \) is that of an FRW universe,
\begin{equation}
\text{ds}^2 = -d\tau^2 + r_i(\tau)^2dx^2_k.
\end{equation}
Since the brane coincides with both boundaries, the metric on the brane is only well defined when \( r_1(\tau) = r_2(\tau) = R(\tau) \). The Hubble parameter on the brane is now defined by \( H = \dot{R}/R \) where \( R = R(\tau) \) is the brane trajectory in the bulk spacetime.

The brane equations of motion are again given by the Israel equations \eqref{2.7}. In addition to the finely tuned tension, we introduce a homogeneous and isotropic fluid on the brane, with energy density, \( \rho \), and pressure, \( p \). We will assume that these satisfy the strong energy condition, so that \( \rho \geq 0 \) and \( \rho + 3p \geq 0 \). This guarantees that any cosmic acceleration that may occur is entirely due to modified gravity. The Israel equations now give the following:
\begin{equation}
\frac{\sigma + \rho}{6} = m_{\text{pl}}^2 \left( H^2 + \frac{\kappa}{R^2} \right) - 2 \left( M^4 \theta \sqrt{H^2 + \frac{V(R)}{R^2}} \right)
\end{equation}
where the tension, \( \sigma \), is given by equation \eqref{2.13}. Note that this formula is general and holds for all values of parameters, even those involving Minkowski backgrounds, for which we simply set \( V(R) = 0 \) in \eqref{4.6}.
Stealth acceleration and modified gravity

For simplicity, we will assume from now on that the bulk spacetime is maximally symmetric by setting the mass term as $\mu = 0$. One can confirm that this term, which has the behaviour of dark radiation, does not play an important role at late times, when it is subdominant to the matter content on the brane. In addition, since observations of the first acoustic peak in the cosmic microwave background demonstrate that the universe is very nearly flat [2], we set $\kappa = 0$. The modified Friedmann equation then takes the compact form

$$\rho = F(H^2)$$

where

$$F(H^2) = 6m_{pl}^2 H^2 - 12 \left( M^3 \theta \left( \sqrt{H^2 + k^2} - k \right) \right).$$

Now, as $H \to 0$, it is easy to check that the energy density $\rho \to 0$. This serves as a consistency check that the finely tuned tension (2.13) guarantees that the Minkowski brane corresponds to a possible vacuum solution. There might be other vacuum solutions with $H_0 > 0$, corresponding to self-accelerating vacua with $F(H_0^2) = 0$. These however are suspected to contain perturbative ghosts [12] (although we will not attempt to show this explicitly here). We will return briefly to discuss how and when such vacua arise presently. Also note that switching off $M_i$ we get the usual Friedmann equation, and switching off $m_{pl}^2$ gives the usual Randall–Sundrum modified Friedmann equation [34].

Since we wish to study the possibility of cosmic acceleration in these models, it is natural to examine the cosmic deceleration parameter,

$$q = -\frac{\ddot{R}}{\dot{R}^2}.$$ 

Assuming a constant equation of state, $p = w\rho$, it follows from (4.7) and conservation of energy

$$\dot{\rho} = -3H(\rho + p)$$

that the deceleration parameter is given by

$$q = -1 + \frac{3}{2}(1 + w) C(H^2).$$

Here the functional

$$C(H^2) = \frac{F(H^2)}{H^2 F'(H^2)},$$

will vary during the course of the cosmological evolution whereas for GR we have simply $C = 1$. It follows that the deceleration parameter will also vary during the cosmological evolution, in contrast to what happens in four-dimensional Einstein gravity where it is constant for constant $w$. For cosmic acceleration to occur, obviously the deceleration parameter must become negative. For ordinary forms of matter satisfying the strong energy condition $1 + 3w \geq 0$, it is easy to check that acceleration can only be achieved if $C(H^2)$ falls below 1.

It is useful to define an effective equation of state parameter for the universe, $\gamma_{\text{eff}}$, which is related to the deceleration parameter $q = 1 + 3\gamma_{\text{eff}}$, from which we deduce that

$$1 + \gamma_{\text{eff}} = C(1 + w).$$

Again, as $C$ varies during the cosmological evolution, so must the effective equation of state. Note however that this should not be confused with a dark energy component with
Figure 1. An example of two possible functions $\rho = F(H^2)$ where there is a Minkowski vacuum (the dotted line), and where the Minkowski vacuum is isolated (solid line). These examples correspond to the normal and self-accelerating branches of pure DGP theory respectively.

a varying equation of state (see e.g. [35]), in which there are two main components of the energy of the universe: matter and the time-varying dark energy. Here, we assume that there is only one dominant cosmological fluid, and $\gamma_{\text{eff}}$ provides a simple, effective way of tracking the gravitational effect of that fluid. It is clear that the cosmological behaviour is completely determined by the functional, $F$, and its derivatives, with the combination $C = F/F' H^2$ proving particularly important when asking questions about cosmic acceleration. To emphasize the difference between taking dark energy, in which we modify the matter in the Friedmann equation to $H^2 = F(\rho)$, and light gravity in which we modify the geometry: $\rho = F(H^2)$, consider the $\Lambda$CDM model. If we regard this in its conventional dark energy way, we have the dark energy with a constant equation of state: $w = -1$. If, however, we regard the cosmological constant as part of the gravitational sector, then we obtain $\gamma_{\text{eff}}^{(\Lambda CDM)} = -\Omega_{\Lambda} H_0^2 / H^2$ for a matter cosmology.

As the universe expands, it is clear from the strong energy condition, and energy conservation (4.9), that the energy density of the universe must decrease. As the energy density is diluted, we ought to approach a vacuum state, which will correspond either to a Minkowski brane or to a self-accelerating brane. We are interested in the case where we approach the Minkowski vacuum. Now, in the absence of any phase transitions, we expect $H$ to vary continuously during the cosmological expansion. It follows that our brane cosmology must be consistent over a range $0 \leq H < H_{\text{max}}$.

To be consistent, our cosmology must adhere to certain rules. To begin with, the strong energy condition requires that the energy density is non-negative. It only vanishes for the vacuum brane, which we have taken to be the Minkowski solution. We therefore require that $F(H^2) > 0$ over the range $0 < H < H_{\text{max}}$ (see figure 1). It is easy to check that this rule implies that

$$F'(0) = 6 \chi \geq 0. \quad (4.13)$$

The alternative scenario, where $F'(0) < 0$, is also shown in figure 4, later. Here there is a small region close to $H_c > 0$ for which $F(H_c) < 0$ and therefore $\rho_c < 0$. This is an unphysical regime. The only way to get a physical vacuum is for $F'$ to change sign, so that we have a self-accelerating vacuum with $H = H_0$. The Minkowski brane in this instance
represents an *isolated* vacuum which will never be approached, even as the energy density is diluted.

Thus, condition (4.13) guarantees that the Minkowski vacuum is the physical one. However, recall from the previous section that unless the radion decoupled for one reason or another, we required $\chi \leq 0$ in order to avoid a ghost. We then arrive at the following important and somewhat surprising conclusion: the case of a physically safe, i.e. perturbatively stable, radion mode corresponds to the unphysical cosmological case. Therefore if we want to stick to a Minkowski vacuum then it follows that the radion must be absent for the solution to be consistent. This corresponds to the strongly coupled case ($\chi = 0$), and the cases for which decoupling occurs ($\chi \to \infty$). We will study the corresponding cosmological solutions for these ‘radion-free’ cases in the next section.

The link between a consistent cosmology, and the absence of a perturbative ghost can be traced back to the absence of a tensor zero mode. This means that the large distance or late time behaviour is dominated by the scalar radion (if it exists). Now, if one uses the Gauss–Codazzi equations to find the projection of the Einstein tensor on the brane [36], then after linearizing, we find that

$$\delta \left( R_{\mu \nu} - \frac{1}{2} R \gamma_{\mu \nu} \right) = \frac{1}{2 \chi} T_{\mu \nu}.$$  (4.14)

We have left out the explicit contribution from the bulk Weyl tensor since it is only expected to behave like a dark source of radiation [37], and is actually zero for the cosmological branes considered here. Now the effective equation of motion (4.14) would follow from an effective action of the form

$$S_{\text{eff}} = \int \sqrt{-\gamma} \left( \chi R + L_{\text{matter}} \right).$$  (4.15)

At late times, we now have a natural interpretation for $\chi$: it is the effective 4D Planck scale. Taking this to be positive, it is clear that we will have a consistent 4D cosmology. However, in the absence of a tensor zero mode, the conformal mode has nothing to mix with in the far infra-red, giving rise to a dangerous ‘conformal ghost’.

Note that the condition $F'(0) \geq 0$ is actually required for the whole cosmology

$$F'(H^2) \geq 0, \quad \text{for } 0 \leq H < H_{\text{max}}$$  (4.16)

because otherwise the strong energy condition will fail to hold\(^5\). The condition (4.16) also guarantees that we never enter a phase of super-inflation, with $\dot{H} > 0$. This can easily be checked using equation (4.7) and energy conservation (4.9). Super-inflation is usually associated with phantom cosmologies, which will typically lead to ghost-like instabilities [38].

Last, but not least, we must demand that our cosmology passes through a standard 4D phase at some earlier time; otherwise we will run into problems with nucleosynthesis [34]. If $H_{\text{max}}$ is sufficiently large, then when $H \sim H_{\text{max}}$, the induced curvature on the brane will dominate, so that $\rho \sim 6m_p^2 H^2$. For the accelerating cosmologies to be discussed in the next section, we will find that $H_{\text{max}}$ can be taken to be arbitrarily large.

\(^5\) To see this, note that our requirement that $F''$ is positive in a neighbourhood of the origin means that if $F'$ becomes negative, then it must have a zero at finite $H$. This in turn implies that $(\rho + p) = 0$, from (4.9), thus violating strong energy.
5. Radion-free cosmologies

We saw previously that the only cosmological solutions that were universally consistent were those for which the radion field was absent from the vacuum perturbations. This included the conformal limit ($\chi = 0$), and the decoupling limits. The latter corresponds to two possibilities: a generalized Randall–Sundrum scenario or a Minkowski bulk on at least one side of the brane. We will look at both of these cases, as each contains an example of an accelerating cosmology.

5.1. Acceleration with a decoupled radion

The generalized Randall–Sundrum scenario corresponds to the case where we retain the adS horizon on either side of the brane. The scalar excitations about the vacuum decouple. We also have a consistent cosmology for $0 \leq H < \infty$, defined by

$$F(H^2) = 6m_{pl}^2H^2 + 12 \left(M^3 \left(\sqrt{H^2 + k^2} - k\right)\right).$$

(5.1)

One can easily prove that $C(H^2) = \frac{F}{F'}H^2 \geq 1$ for all values of $H > 0$. As discussed in the previous section, acceleration can only occur for ordinary matter satisfying the strong energy condition if $C(H^2)$ falls below 1. We conclude that we can never enter a phase of cosmic acceleration in the generalized Randall–Sundrum scenario for ordinary forms of matter. This comes as no surprise, since the presence of a graviton zero mode guarantees standard 4D behaviour at large distances, and prevents any infra-red modification of gravity.

Now consider the case where we have a Minkowski bulk on at least one side of the brane, and without loss of generality take $k_2 = 0$. Again, all scalar excitations about the vacuum decouple. Having not yet specified the values of $\theta_1$, $\theta_2$, and $k_1$, the cosmology is, in general, defined by

$$F(H^2) = 6m_{pl}^2H^2 - 6M_2^3\theta_1 \left(\sqrt{H^2 + k_1^2} - k_1\right) - 6M_2^3\theta_2H.$$

(5.2)

If both sides of the bulk are Minkowski, i.e., $k_1 = 0$, then

$$F(H^2) = 6m_{pl}^2H^2 - 12\langle M^3\theta \rangle H.$$

(5.3)

This is the DGP model [8]. Whenever $\langle M^3\theta \rangle = 0$, we see that the brane cosmology receives no contribution from the bulk, and behaves exactly as for four-dimensional GR, without a cosmological constant. As is well known, this will not give us any acceleration. Now consider what happens when $\langle M^3\theta \rangle \neq 0$. For small $H > 0$, we have

$$F(H^2) \sim -12\langle M^3\theta \rangle H + \mathcal{O}(H^2).$$

(5.4)

Since $\rho \geq 0$, we clearly require $\langle M^3\theta \rangle < 0$; otherwise the flat vacuum would represent an isolated vacuum, as discussed in the previous section. However, note that when $\langle M^3\theta \rangle < 0$, we have $C(H^2) \geq 1$, for all values of $H > 0$, and there is never cosmic acceleration for ordinary forms of matter.

The first example of an accelerating cosmology occurs when we have Minkowski space on one side of the brane, and adS space on the other. This corresponds to the case $k_1 \neq k_2 = 0$, with the cosmology defined by (5.2). Now for small $H > 0$, we have

$$F(H^2) \sim -6M_2^3\theta_2H + \mathcal{O}(H^2).$$

(5.5)
In order to avoid isolating the flat vacuum, we clearly require that $\theta_2 = -1$. If, in addition, we take $\theta_1 = -1$, it is easy to check that $C(H^2) \geq 1$ for all values of $H > 0$, and so there can never be any acceleration for ordinary forms of matter. Perhaps we should not be surprised that acceleration is impossible when the warp factor decays away from the brane on the $\text{adS}$ side, since this will induce a degree of localization. In contrast, when the warp factor grows away from the brane, we might expect more interesting dynamics, since the graviton will ‘want’ to localize away from the brane, near the $\text{adS}$ boundary. This does indeed happen, and, as we shall now demonstrate in detail, cosmic acceleration can occur for ordinary matter if we set $\theta_1 = 1$.

In the rest of this subsection, we will focus our attention on the case where the $\text{adS}$ boundary is included in $M_1$, and we have Minkowski space in $M_2$. The resulting cosmology is defined by

$$F(H^2) = 6m_{pl}^2H^2 - 6M_1^3\left(\sqrt{H^2 + k_1^2} - k_1\right) + 6M_2^2H. \quad (5.6)$$

Now if $M_1 \leq M_2$, it is relatively easy to check that $C(H^2) \geq 1$ for all $H > 0$, and as such, acceleration can never occur for ordinary matter. We can understand this result as follows. Although the warp factor grows away from the brane in $M_1$, the graviton is not so strongly localized on the $\text{adS}$ boundary, since a small value of $M_1$ makes it easier for the graviton to propagate towards the brane. Thus, the degree of delocalization away from the brane on the $\text{adS}$ side is not as severe as it might have been for larger values of $M_1$.

Now let us focus in detail on the case where the degree of delocalization is more severe, so that cosmic acceleration is maximized, by taking $M_1 > M_2$. It follows that $F'(H^2)$ has one local minimum in $H > 0$, where it takes the value

$$F_{\text{min}}' = 6m_{pl}^2 - 3\frac{M_1^3}{k_1}\left(1 - \frac{M_2^2}{M_1^2}\right)^{3/2}. \quad (5.7)$$

Recall that the cosmological phase of interest corresponds to $F'(H^2) > 0$ for non-zero $H$. Therefore, if $F_{\text{min}}' \leq 0$, we can only have a consistent cosmology for $0 \leq H < H_{\text{max}}$ where $H_{\text{max}}$ is finite. For $0 \leq H < H_{\text{max}}$, we can use the fact that $F''(H^2) \leq 0$ to show that $\mathcal{C}$ is an increasing function, and so $\mathcal{C}(H^2) \geq \mathcal{C}(0) = 1$; thus cosmic acceleration can never occur for ordinary matter if $F_{\text{min}}' \leq 0$. In contrast, if $F_{\text{min}}' > 0$, then it follows that $F'(H^2) > 0$ for all $H > 0$. There is only one cosmological phase, so we can take $H_{\text{max}}$ to be infinite. This condition on $F_{\text{min}}'$ enables us to place the following lower bound on $m_{pl}^2$:

$$m_{pl}^2 > \frac{M_1^2}{2k_1}\left(1 - \frac{M_2^2}{M_1^2}\right)^{3/2}. \quad (5.8)$$

Given that our cosmology is consistent for arbitrarily large values of $H$, it is instructive to consider the asymptotic behaviour of $\mathcal{C}$:

$$\mathcal{C} = 1 - \left[\frac{M_1^3 - M_2^3}{m_{pl}^2}\right]H^{-1} + \mathcal{O}(H^{-2}). \quad (5.9)$$

There are two things to note here. First, at very large $H$, $\mathcal{C} \approx 1$ to leading order, so the standard 4D cosmology is reproduced at early times. This is due to the induced
curvature term on the brane dominating the UV behaviour. Second, the first correction demonstrates that $C$ starts to fall below 1, since $M_1 > M_2$, which is precisely the sort of behaviour we hope to see in order to get cosmic acceleration from ordinary matter. Quite how much acceleration can be obtained depends on the nature of the cosmological fluid (the value of $w$), and the minimum value of $C(H^2)$. It is the latter that measures the degree to which modified gravity is contributing to the acceleration.

To maximize cosmic acceleration, we therefore require the minimum possible value of $C(H^2)$:

$$C = C(\dot{H}, \beta, \epsilon) = \frac{(\cos^3 \beta + \epsilon)\dot{H}^2 - 2(\sqrt{1 + \dot{H}^2} - 1) + 2\dot{H} \sin^3 \beta}{(\cos^3 \beta + \epsilon)\dot{H}^2 - \dot{H}^2/\sqrt{1 + \dot{H}^2} + \dot{H} \sin^3 \beta}$$  \hspace{1cm} (5.10)

where we have defined

$$\dot{H} = \frac{H}{k_1}, \quad M_2 = \frac{k_1}{M_1} = \sin \beta, \quad \frac{2m_{pl}^2k_1}{M_1^3} = \cos^3 \beta + \epsilon$$ \hspace{1cm} (5.11)

with $0 \leq \beta < \pi/2$, and $\epsilon > 0$, which is consistent with $M_1 > M_2$ and the bound (5.8). Unfortunately there is no simple analytic minimization of this functional; however, it is not difficult to establish that the minimum occurs along $\epsilon = 0$, i.e. on the boundary of our allowed range of the four-dimensional Planck mass. Therefore, we can never actually attain the maximal value of acceleration; however, numerically, we see that the minimum values that one can typically achieve are of order $C \sim 0.43$. For a matter dominated universe ($w = 0$), we can therefore choose $\beta$ and $\epsilon$ such that the effective equation of state, $\gamma_{\text{eff}}$, falls as low as $\sim -0.57$, and for radiation ($w = 1/3$), $\sim 0.43$.

To see whether this cosmological model is viable, it is not sufficient to demonstrate that the effective equation of state bottoms out at some reasonable negative value, as it may be that the actual cosmological evolution does not spend sufficient time in this negative region to have a significant era of late time acceleration. We therefore need to track the scale factor throughout time to demonstrate that the period and amount of acceleration is at least potentially sufficient. Differentiating the Friedmann equation gives the cosmological evolution of $H$:

$$\frac{d\dot{H}}{dt} = -\frac{3}{2}(1 + w)\dot{H}^2C(\dot{H}^2)$$ \hspace{1cm} (5.12)

where $\dot{t} = k_1t$. This is, in general, a somewhat involved NLDE for $\dot{H}$, and is best integrated numerically. We illustrate acceleration in this model by taking a specific example, for which $C$ is shown in figure 2. There are a number of generic features apparent in this example. At early times, when $H$ is large ($\alpha \approx \pi/2$), $C$ is close to 1, and so we are in the standard 4D regime, because the induced curvature term dominates in the UV. As the universe starts to expand, and cool down, then $C$ starts to fall below 1. If we are in a period of matter domination ($w = 0$), we enter an accelerating phase when $C$ falls below 2/3. The minimum value of $C$ is around 0.5, at which point we have maximal acceleration, whatever the cosmological fluid. $C$ then starts to increase, and we exit the accelerating phase for good. A maximum value of $C$ occurs when $\dot{H} \sim \beta$. Here we are in the phase of greatest deceleration. This maximum value actually diverges when $\epsilon = 0$, at the point at which $F^\gamma$ vanishes. Beyond this phase, at very late times, $C$ approaches its limiting value of 2, and hence the cosmology becomes that of a ‘stiff matter’ universe, where $p = \rho$. 


Stealth acceleration and modified gravity

Figure 2. Plot of $C$ against $\dot{H}$, for $\beta = 0.02$ and $\epsilon = 0.0002$.

Figure 3. Plot of the comoving Hubble radius as a function of time, measured in units of $k_1^{-1}$, for (a) a matter cosmology, and (b) a radiation cosmology, with the same values of $\beta$ and $\epsilon$ as in figure 2.

The evolution of the scale factor can be found by integrating (5.12), and the comoving Hubble radius is plotted in figure 3 for both matter and radiation dominated cosmologies. This shows that while the matter cosmology experiences a significant era of acceleration, the radiation cosmology has only a period of marginal acceleration, which would be more accurately described as ‘coasting’ [22].

Focusing on the matter cosmology, from the plot of the comoving Hubble radius, we see that we enter a period of acceleration at around $t \simeq 0.5k_1^{-1}$, and remain in an accelerating phase until $t \simeq 32k_1^{-1}$. Since we chose a flat universe, our value of $k_1$ is determined by the current value of $\Omega_m$, and the Hubble parameter $H_0$. Since we are using this model for illustrative purposes, we have not performed a best fit of the cosmological parameters in our model to the data; however, choosing some reasonable values of $k_1$ gives the luminosity–redshift plot of figure 4.

Clearly the model gives a plausible fit to the data; however, more work needs to be done to establish its viability, as well as its comparative merits to other models [39]. In particular, we have taken standard typical values of $H_0$ and $\Omega_m$, which of course are the results of a fit of Einstein evolution to various different data sets. It is possible that using the evolution predicted by our model, the best fit values of these parameters may be different, which was the motivation for giving a range of commonsense values of $k_1$. Even
5.2. The conformal cosmology

The remaining ‘radion-free’ cosmology is interesting, because it leads to the greatest amount of acceleration. This is the conformal case, corresponding to

$$\chi = m_{\text{pl}}^2 - \left\langle \frac{M^3 \theta}{k} \right\rangle = 0. \quad (5.13)$$

In this limit, an extra symmetry kicks in to linear order in perturbation theory that eliminates the radion degree of freedom. It also prevents coupling, at linear order, to the trace of the energy–momentum tensor on the brane. If a non-zero trace is present, the geometry responds non-linearly. It is precisely this non-linear effect that gives rise to the acceleration at late times, as we shall soon see.

The first thing to note is that the cosmology is consistent for all values of $H \geq 0$, provided $m_{\text{pl}}^2 > 0$ and $F'''(0) \geq 0$. At early times the induced curvature on the brane guarantees that we experience the standard 4D evolution. The extra dimensions then begin to open up and we enter an accelerating phase at later times. To understand the late time behaviour, let us approximate equation (4.7) by a Taylor expansion near $H = 0$

$$\rho = H^2 F'(0) + \frac{H^4}{2} F''(0) + \frac{H^6}{6} F'''(0) + \mathcal{O}(H^8) \quad (5.14)$$
where

\[ F'(0) = 6\chi, \quad F''(0) = 3 \left\langle \frac{M^3\theta}{k^3} \right\rangle, \quad F'''(0) = -\frac{9}{2} \left\langle \frac{M^3\theta}{k^5} \right\rangle. \quad (5.15) \]

For \( \chi \neq 0 \), \( \rho \) is linear in \( H^2 \) to leading order. Cosmic acceleration in this instance is impossible. It is only as we take the conformal limit, \( \chi \to 0 \), that the linear term goes away, and acceleration becomes possible. This is because \( \rho \) is either quadratic, or even cubic in \( H^2 \) at late times. This will lead to power law acceleration, since

\[ \rho \propto H^2 n = \Rightarrow a(\tau) \sim \tau^{2n/3(1+w)}. \quad (5.16) \]

The quadratic dependence (\( n = 2 \)) occurs when \( F''(0) > 0 \), or equivalently

\[ \left\langle \frac{M^3\theta}{k^3} \right\rangle > 0. \quad (5.17) \]

Note that we cannot have \( \theta_1 = \theta_2 = -1 \), which comes as no surprise since this is just the generalized Randall–Sundrum scenario discussed previously, for which gravity is localized on the brane. To get an idea of how much acceleration occurs at late times, note that for small \( H \), \( C(H^2) = \frac{1}{3} + \mathcal{O}(H^2) \) and so a pressureless cosmological fluid will lead to an effective equation of state, \( \gamma_{\text{eff}} = -1/2 \).

The cubic dependence (\( n = 3 \)) occurs when \( F'''(0) = 0 \), or equivalently

\[ \left\langle \frac{M^3\theta}{k^5} \right\rangle = 0. \quad (5.18) \]

In order to avoid having an isolated vacuum at \( H = 0 \) we must also require that \( F'''(0) > 0 \). This scenario leads to even more acceleration: for small \( H \), \( C(H^2) = \frac{2}{3} + \mathcal{O}(H^2) \) and so a pressureless fluid has an effective equation of state \( \gamma_{\text{eff}} = -2/3 \) at very late times. Naively, we might have expected the greatest acceleration to occur when the adS boundary is retained on both sides, but this turns out not to be the case here. If we assume, without loss of generality, that \( M_1 > M_2 \), then we must also have \( \theta_1 = 1, \theta_2 = -1 \). This means that in \( M_1 \), we retain the adS boundary, whereas in \( M_2 \) we retain the adS horizon.

We can understand this puzzling result as follows. For these conformal cosmologies, the scalar radion is strongly coupled and therefore plays a more important role than the spin 2 graviton mode. While the graviton likes to localize where the warp factor is greatest, the opposite is true for the radion. If the radion is dominant, as is the case here, it is clear that gravity on the brane is weakest when the radion localizes away from the brane, near the adS horizon. Note that in the previous case of a Minkowski bulk, the radion was weakly coupled, and so the graviton played the dominant role.

For comparison with the Minkowski model, we choose a specific example with \( M_2/M_1 = k_2/k_1 = \sin 0.6 \), and show the comoving Hubble scale and a similar subtracted luminosity–redshift plot in figure 5. Once again, we have not performed a ‘best fit’ analysis, but simply taken two reasonable values of the model parameters.

In principle, if we could make \( \rho \) quartic in \( H^2 \) we could achieve yet more late time acceleration. Unfortunately, this would require us to set \( F'(0) = F''(0) = F'''(0) = 0 \), which corresponds here to the trivial case \( F(H^2) \equiv 0 \). One interesting possibility is that the addition of a Gauss–Bonnet term would allow this. The Gauss–Bonnet term is a
higher order curvature correction to the gravitational action which is ghost free (see [40] for
discussions in the context of braneworlds, and [41, 42] for the boundary terms appropriate
to Gauss–Bonnet (GB)). We can therefore view it as a possible first correction in a UV
completion of the gravitational effective action. Such UV completions are sometimes
proposed as a remedy to the problems of the pure DGP model. In this context, the effect
of the GB term is to modify the relation between the constant $k$, and the cosmological
constants on each side of the brane:

$$4\alpha_i k_i^2 = 1 \mp \sqrt{1 + 4\alpha_i \Lambda_i / 3} \quad (5.19)$$

where we allow $\alpha$ to take different values on either side of the brane, in keeping with the
asymmetric set-up. The brane tension is also modified to

$$\sigma = -12 \left( M_3^3 \theta k (1 - \frac{4}{3} \alpha k^2) \right). \quad (5.20)$$

The modified Friedmann equation is again (4.7) but with $F$ given by

$$F(H^2) = 6m_{\text{pl}}^2 H^2 - 12 \left( M_3^3 \theta \left( \sqrt{H^2 + k^2} - k \right) \left( 1 - \frac{4}{3} \alpha k^2 \right) \right) - 32 \left( M_3^3 \theta \alpha H^2 \sqrt{H^2 + k^2} \right). \quad (5.21)$$

From this, we now see that the Gauss–Bonnet term does not significantly modify
our conclusions, although we cannot push the conformal model to stronger late time
acceleration, since this is incompatible with good UV behaviour of the cosmology. Thus,
we are driven to the interesting conclusion that UV corrections in this case appear to
dampen acceleration, although we have not performed a full parameter search to check
whether there are regions of the GB theory with greater acceleration.

6. Discussion

Up until now, braneworld models have offered only one new mechanism for reproducing
cosmic acceleration. These are the self-accelerating de Sitter-like solutions found for DGP
branes [8] that have been shown to be unstable at the level of linearized perturbations [12].
In this paper we have shown that a dynamical alternative mechanism exists whereby the vacuum brane is Minkowski, but when matter is present the universe can undergo a period of acceleration. Quite how much acceleration occurs depends on the details, but we have demonstrated two types of model for which there is a convincing amount of acceleration.

The idea of ordinary matter driving accelerated expansion is not new; for example, the Cardassian model of Freese and Lewis [43] obtained such acceleration by empirically modifying the Friedmann equation, sending $\rho \rightarrow \rho + c \rho^n$, where $n < 2/3$ was some (unknown) parameter. While an interesting empirical model, Cardassian expansion did not provide an explanation of why the matter source contributed to the Friedmann equation in this way. In our model, we use concrete five-dimensional physics to give rise to four-dimensional modifications, where it is the geometry which is altered, rather than the matter source. Of course, one could always rewrite the modified Friedmann equation as

$$H^2 = F(\rho)$$

(6.1)

by algebraically solving $\rho = F(H^2)$; however, as this involves the solution of a quartic in either $H$ or $H^2$, it is messy, and also one has to take great care not to pick up fictitious additional solutions from branch choices. Nonetheless, one could view our models as providing a sound theoretical basis for an effective Cardassian model. For example, the Minkowski–adS model illustrated in section 5.1 is approximately equivalent to Cardassian expansion with $n = 0.5$.

The key feature of our braneworld model for modified gravity is that we have shown that our vacuum solutions are perturbatively stable. The requirement of perturbative stability is obviously crucial. Typically, instabilities appear in the scalar sector so we have focused our attention on scalar perturbations about the Minkowski brane vacuum. It turns out that there is a curious interplay between physically acceptable cosmologies, and stable vacua: if the cosmological solution is well behaved close to the vacuum, then the radion mode is a ghost (and vice versa!) This surprising result can be understood as stemming from the ‘conformal ghost’, as discussed in detail at the end of section 3. Perhaps more importantly, it greatly restricts the set of physically acceptable set-ups: we can only consider those cases for which the radion either decouples completely ($\chi \rightarrow \infty$), or is eliminated by some extra symmetry as in the conformal case ($\chi = 0$).

The conformal cosmology discussed in section 5.2 leads to the greatest amount of acceleration. Although we prefer to regard it as a stand-alone special case of enhanced symmetry, we can gain some understanding of its dynamics by considering the limit of small $\chi \neq 0$. The first thing to note is that the radion coupling scales like $1/\chi$, so we might also think of the conformal case as the strongly coupled limit. Because the radion is so strongly coupled it dominates the dynamics over the spin 2 mode. Therefore the case of greatest acceleration occurs when one has the adS boundary on one side of the brane, and the adS horizon on the other. The presence of the adS boundary ensures that the radion does not decouple, and is able to draw gravity away from the brane by localizing close to the adS horizon!

The case of $\chi$ small and positive may also be of phenomenological interest. The point is that although our linearized analysis suggests that the radion is a ghost, it is so strongly coupled that one cannot fully trust linearized perturbation theory except on the largest
scales. One could therefore imagine a scenario whereby that scale is pushed beyond (say) the size of the observable horizon. Our cosmological solution is a non-linear solution so we can retain the power law acceleration in the presence of matter or radiation.

Naturally, our models, while appearing to give a promising fit to the SN data, are not necessarily preferable to ΛCDM. The main problem is the extension of parameter space—we can explain late time acceleration, but at the cost of an additional parameter. If we suppose that our four-dimensional Planck scale is given, then our models have in general three free parameters\(^6\): \(k_1\), \(M_1\) and \(M_2\) (however, note that the conformal cosmology has only two free parameters, \(k_1\) and \(M_1\), say). This is to be compared to the two parameters of ΛCDM, \(\Omega_m\) and \(\Omega_\Lambda\). Clearly then, our models will in general fail on selection criteria (see e.g. [44] for a discussion of this issue in the current context), unless significant deviations to ΛCDM are found at higher redshift. However, our conformal cosmology, for which the parameter space is not higher dimensional, certainly merits further investigation.

In this paper, we have been primarily concerned with producing a consistent cosmology, i.e. a model which has consistent particle physics (such as no ghosts) and also provides an accelerating cosmology. One question we have not considered is whether tabletop or indeed solar system/astrophysics experiments are consistent. The DGP model has received a great deal of attention, mainly because it appeared to provide a ghost-free way of getting the gravitational phenomenology first discovered in the GRS model; however, more careful analysis uncovered a number of difficulties (such as strong coupling) and a fundamental flaw: the existence of ghosts. Here, we have resolved the fundamental flaw by introducing an asymmetric braneworld, but we have not provided a solution (or indeed an examination) of any of the difficulties. For example, strong coupling, which is an issue in the DGP model, is likely to be an issue here, although as in DGP, it may actually help in evading solar system constraints [14].

Asymmetric braneworlds are an under-explored territory in the world of brane model building, largely because the asymmetry introduces additional complications at the level of the equations of motion. The main rationale for considering symmetric braneworlds is that these correspond to orbifold compactifications in string theory. However, the string landscape is also a popular concept [45], and in the landscape we can expect many different vacua, with many different local cosmological constants [46], and indeed (through warping [47]) the possibility of different local effective Planck masses. Therefore, although the idea of having different Planck masses and cosmological constants on either side of the brane may seem a little strange at first sight, it is in fact a natural, if not generic, occurrence in the context of the string landscape.

We end by re-emphasizing that we do not intend to present this as an improvement on the ΛCDM model. We merely wish to demonstrate what can be achieved consistently in a braneworld set-up. Ultimately, the concordance model is preferable because it provides the simplest fit to the data—i.e. the fit with the smallest number of parameters. However, the definition of ‘simplest’ should also include a measure of how naturally the model fits within an underlying fundamental theory of interactions. Whether it is easier to produce a massively fine-tuned cosmological constant, or discrete vacua with differing parameters (or something else), will no doubt continue to be a source of lively debate for some time!

\(^6\) Note that we are referring here to the parametrization of the matter or geometry, rather than the whole gamut of cosmological parameters.
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