Effects of equation of state on hydrodynamic expansion, spectra, flow harmonics and two-pion interferometry

Danuce M. Dudek, Wei-Liang Qian, Chen Wu, Otávio Socolowski Jr., Sandra S. Padula, Gastão Krein, Yogiro Hama, Takeshi Kodama

1Universidade Federal Fronteira Sul - Campus Realceza, Caixa Postal 253, 85770-000, Realceza, PR, Brazil
2Escola de Engenharia de Lorena, Universidade de São Paulo, 12602-810, Lorena, SP, Brazil
3Faculdade de Engenharia de Guaratinguetá, Universidade Estadual Paulista, 12516-410, Guaratinguetá, SP, Brazil
4Instituto de Matemática Estatística e Física, Universidade Federal do Rio Grande, Caixa Postal 474, 96201-900 Rio Grande, RS, Brazil
5Instituto de Física Teórica, Universidade Estadual Paulista, Caixa Postal 66318, 05389-970 São Paulo, SP, Brazil and Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, 21941-972 Rio de Janeiro, RJ, Brazil

(Dated: Nov. 07, 2016)

We perform an extensive study of the role played by the equation of state in the hydrodynamic evolution of the matter produced in relativistic heavy ion collisions. By using the same initial conditions and freeze-out scenario, the effects of different equations of state are compared by calculating their respective hydrodynamical evolution, particle spectra, harmonic flow coefficients $v_2$, $v_3$ and $v_4$ and two-pion interferometry radius parameters. The equations of state investigated contain distinct features, such as the nature of the phase transition, as well as strangeness and baryon density contents, which are expected to lead to different hydrodynamic responses. The results of our calculations are compared to the data recorded at two RHIC energies, 130 GeV and 200 GeV. It is found that the three equations of state used in the calculations describe the data reasonably well. Differences can be observed among the studied observables, but they are quite small. In particular, the collective flow parameters are found to not be sensitive to the choice of equation of state. Implications of this fact is discussed.

PACS numbers: 47.75.+f, 25.75.-q, 21.65.Mn

I. I. INTRODUCTION

The equation of state (EoS) of strongly interacting matter plays a major role in the hydrodynamic description of the hot and dense system created in heavy ion collisions [1-4]. It governs how the hydrodynamic evolution transforms the initial state conditions and fluctuations into final state femtoscopic correlations and anisotropies in terms of collective flow. Motivated by the lattice QCD simulations, which indicate that the quark-hadron transition has a crossover at zero baryon density [5-10], many different equations of state (EoSs) have been proposed [11] by fitting the lattice data, and combining it with EoSs appropriate for the hadronic phase [12] at low temperatures [13-22]. For a study involving EoSs with a first order phase transition, see Ref. [23].

The assumption of zero baryon density is a reasonable approximation for the initial conditions (IC) of the systems created at RHIC and LHC. However, strongly interacting matter possesses several conserved charges, such as electric charge, net baryon number and strangeness. Studies have shown [24-28] that the thermodynamic properties and phase transitions are modified when the number of conserved charges in the system changes. In the case of a liquid-gas phase transition, for instance, the increase of the number of conserved charges increases the dimension of the binodal surface and the corresponding transition is continuous [29-31]. In view of this, one can expect that in the case of the QCD matter, conserved charges could affect the duration of the hydrodynamic evolution of the system in the transition region and likely would manifest themselves at the stage of hadronization. Therefore, experimental data on multiplicity, ratio of particle yields and their fluctuations need to be analysed through models which handle properly finite baryon density and strangeness. Statistical models with finite chemical potential are capable of describing the data reasonably well [32-35], indicating that it is important for the study of the system evolution to use EoSs providing a reasonable description of the matter produced over a large range of densities and temperatures. Following this line of thought, a compromise was proposed by Hama et al. [36], where a phenonemological critical point is introduced to smoothen the transition region where the baryon density is smaller than that of the critical point. In the model, finite baryon chemical potential is taken into consideration in both the quark-gluon plasma (QGP) and in the hadronic phase. Such phenonemological approach reflects well the main characteristic of a smooth crossover transition while explicitly considering non-zero
baryon density. Unfortunately, in the QGP phase, the model does not accurately reproduce asymptotic properties of the QGP matter.

The present work employs different EoSs in an ideal hydrodynamical model to study their effects on particle spectra, flow harmonics and two-pion interferometry. In the following section, we briefly review different EoSs suggested in the literature and then discuss their effects as employed in the present work. In section III we present the results of our hydrodynamical simulations, particle spectra, harmonic coefficients as well as two pion interferometry. The calculations are done for RHIC energies of 130 GeV and 200 GeV, for various centrality windows. Conclusions and perspectives for future work are presented in section IV.

II. EQUATION OF STATE AND HYDRODYNAMICAL MODEL

Many different EoSs compatible with results of lattice QCD simulations have been investigated in the literature. Huovinen [13] proposed an EoS connecting a lattice QCD EoS to another one for a hadronic resonance gas (HRG) model, requiring continuity of the entropy density and its derivatives in the transition region. In a later work, Huovinen and Petreczky [14] improved the parameterization of Ref. [13] by focusing on the trace anomaly, \( \Theta \equiv T^\mu_\mu = e - 3P \). In this case, the lattice EoS at high temperature is adopted and is connected smoothly to an EoS of a HRG model at low temperature by requiring that the trace anomaly, as well as its first and second derivatives, are continuous. In Refs. [16, 37], an EoS was proposed also based on the lattice data and a HRG model. In this EoS, the sound velocity is interpolated in the transition region and is constrained by means of thermodynamical relations to match the lattice QCD entropy density through an integral in temperature. A few other EoSs were proposed along similar lines [17, 18, 20], using the lattice EoS at high temperatures and connecting it to a phenomenological hadronic EoS using different prescriptions. In some of those approaches, however, there are issues of thermodynamic consistency.

On the other hand, instead of interpolating lattice QCD data, some works focused on EoSs with a critical end point in the phase diagram. In Ref. [29], for instance, a phenomenological critical point is introduced via an EoS from the MIT bag model for the QGP phase, connected to a HRG EoS for the hadronic phase. Another attempt was implemented in Ref. [38], where an SU(3) Polyakov-Nambu-Jona-Lasinio (PNJL) model was used for the high temperature phase. A critical end point is naturally obtained by using the Polyakov loop as the order parameter of the deconfinement transition.

We note that most of the EoSs discussed above consider only zero baryon density. Moreover, in the hydrodynamical simulations, usually averaged IC were used, and only a few works previously adopted full three-dimensional (3-D) hydrodynamical simulations. Though it was estimated in Refs. [13, 39] that the effect of finite chemical potential is small, less than a few percent, it is not clear, whether it could increase in the case of event-by-event fluctuating IC. In addition, in most studies, calculations were only done for some specific collision energy and centrality windows. In view of this, it seems worthy to carry out an event-by-event 3-D simulation on the effects of the EoS, covering a broader range of the published data.

In this work, hydrodynamical calculations are investigated by using the full 3-D ideal hydrodynamical code NEXSPheRIO. For a more quantitative treatment of heavy ion collisions, the effect of viscosity should be taken into account. However, the main purpose of this study is to qualitatively investigate the differences resulting from using various EoSs rather than to reproduce the data pre-
In this work, we investigate three different types of EoS:

- (LQCD) A lattice QCD EoS proposed by Huovinen and Petreczky [14] with zero baryon chemical potential,
- (CEP) A lattice QCD inspired EoS [36] with smooth transition and a critical end point, which considers finite baryon chemical potential,
- (FOS) An EoS with first-order phase transition [52] which considers both finite baryon chemical potential and local strangeness neutrality. It is a generalization of the EoS (FO) discussed in Ref. [23].

The first EoS, LQCD, adopts the parameterization “s95p-v1” from [14] which fits the lattice QCD data for the high temperature region, while adopting a HRG model for the low temperature region. This EoS considers vanishing baryon and strangeness densities. In the calculations, the pressure and energy density are obtained through the trace anomaly Θ by the following relations

\[
p(T) = p(T_{\text{low}}) \left[ \frac{T^4}{T_{\text{low}}^4} \right] \int_{T_{\text{low}}}^{T} \frac{d\varepsilon}{T^5} \Theta,
\]

where a sufficiently small lower limit of integration, \(t_{\text{low}}\), is used in practice [14].

The second EoS, CEP, considers the following phenomenological parametrization [36] instead of Gibbs conditions for the phase transition

\[
\xi = \Theta + 3p,
\]

where \(p\) and \(\Delta p\) are the pressure in the hadronic and in the QGP phase respectively; \(\xi = \delta(\mu_b)\) is a function of baryon chemical potential \(\mu_b\) which exponentially approaches zero when \(\mu_b\) becomes larger than a critical value \(\mu_c = 0.4\ GeV\) [36]. Eq. (2) has the following solution

\[
p = \xi p^H + (1-\xi)p^Q + \frac{2\delta}{\sqrt{(p^Q - p^H)^2 + 4\delta}} \, ,
\]

where

\[
\lambda = \frac{1}{2} \left[ 1 - (p^Q - p^H)/\sqrt{(p^Q - p^H)^2 + 4\delta} \right] \, .
\]

When \(\mu_b < \mu_c\), Eq. (3) gives a smooth transition from the hadronic phase to QGP phase. On the other hand, it is straightforward to verify that, for small value of \(\delta\), \(p \to p^H\) when \(p^Q < p^H\) and \(p \to p^Q\) when \(p^Q > p^H\). As a result, in the region when \(\mu_b > \mu_c\), due to the smallness of \(\delta\), the phase transition rapidly converges to that of FO [36], namely, \(p \to p^Q\) for the QGP phase, \(p \to p^H\) for the hadronic phase, and \(p \to p^Q = p^H\) in the transition region where the Gibbs conditions are satisfied.

The third EoS, FOS, introduces an additional constraint in the FO, namely, strangeness neutrality, i.e.

\[
\rho_s = 0.
\]

The strangeness chemical potential, \(\mu_s\), is introduced in the EoS, not as an independent variable in the system,
but for increasing the dimension of the binodal surface of the phase transition [26]. It also modifies the phase structure, as discussed below. Before carrying out the hydrodynamical simulations we first discuss qualitatively the differences among the different EoSs.

Fig. 3 shows the phase boundaries of the different EoSs. For LQCD, the deconfinement transition corresponds to the parameterization in the region of $170 \text{ MeV} < T < 220 \text{ MeV}$ on the temperature axis in the plot. For FO and FOS, the phase boundary is determined by the Gibbs
conditions between the quark-gluon and hadronic phases. CEP does not have well-defined phase separation, therefore it is not shown explicitly in the plot. Due to the interpolation scheme, it is almost the same as that of FO beyond the critical point (i.e. for $\mu > \mu_c$), and is smoothed out below that point (i.e. for $\mu < \mu_c$). The top plot shows the phase boundaries in terms of the temperature as a function of baryon density, while the plot in the bottom shows them in terms of the temperature as a function of baryon chemical potential.

We note that FOS possesses an unique feature: the QGP and the hadronic phase boundaries have different

FIG. 4. (Color online) Temporal evolution of the energy density profile on the transverse plane $\eta = 0$ for a given fluctuating event at 200 GeV using different EoS. The arrows show the velocities of the fluid elements on the freeze-out surface. Each panel corresponds to an EoS: LQCD (left column), CEP (middle column) and FOS (right column).
baryon chemical potentials. It can be seen as a result of the strangeness local neutrality condition. This implies that during the phase transition, when the two phases are in equilibrium, it is not necessary that the strangeness density vanishes simultaneously in both phases. This is because in the transition region, the strangeness neutrality condition Eq. (5) reads,
\[
\rho_s = \lambda \rho_s^H + (1 - \lambda) \rho_s^Q = 0.
\]
In other words, in the case of FOS, neither the strangeness density of hadronic phase ($\rho_s^H$) nor that of the QGP phase ($\rho_s^Q$) is necessarily zero in the mixed phase. Therefore, the baryon chemical potential is not fixed during the phase transition, its value depending on the fraction $\lambda$ of the system in the hadronic phase. In general, the baryon chemical potential attains different values at the hadronic phase boundary (with $\rho_s^H = 0$ and $\lambda = 1$) and at the QGP phase boundary (with $\rho_s^Q = 0$ and $\lambda = 0$) as a result of the Gibbs conditions. In the case of FO, the QGP phase boundary coincides with that of the hadronic phase.

In Fig. 2 the pressure is shown as a function of the baryon density, as well as a function of the baryon chemical potential, for different EoSs at a given temperature $T = 150$ MeV. It is worthy noting here that for FOS, neither the baryon chemical potential, nor the strangeness chemical potential is fixed during the isothermal phase transition procedure. As a result, when expressed in terms of pressure versus the chemical potential, the transition region of FO is a point, but it is a curve in the case of FOS, as shown in the top plot of Fig. 2. When expressed in terms of pressure versus baryon density, the transition region corresponds to the horizontal line of constant pressure in the case of FO. On the other hand, the pressure increases during the phase transition in the case of FOS. Therefore, the phase transition in FOS is smoother than that in FO. For the CEP EoS, due to its parameterizations, the transition region is smoothed out as compared to that of FO, the pressure also monotonically increases during the process.

The values of $\epsilon/T^4$ and $3p/T^4$ are plotted as a function of the temperature $T$ for all the EoSs in Fig. 3. In the temperature range 500-600 MeV, the curves for $\epsilon/T^4$ and $3p/T^4$ corresponding to the LQCD EoS have a distinct behavior than the other EoSs, reflecting the fact that it is a fit to the QCD results. In this region, all the other EoS converge to the non-interacting ideal gas limit. On the other hand, in the low temperature limit, all the EoS approach the HRG model, as expected. The differences between CEP, FO and FOS come from the transition region around $T \sim 160$ MeV. Since a first order phase transition of one-component system occurs at a constant temperature, it gives a vertical line in the case of FO. CEP is smoother in comparison with FO due to its phenomenological parameterization. Although the strangeness chemical potential is considered in FOS, it gives exactly the same result as FO. This can be understood by studying the conditions at the phase boundaries.

At vanishing baryonic chemical potential $\mu_b = 0$, it is easy to see that the hadronic phase boundary $\rho_s^H = 0$ and the QGP phase boundary $\rho_s^H$ share the same solution, namely, $\mu_s = 0$, which corresponds to zero baryon density and strangeness neutrality for both phases. This is manifested as the intersection between the phase boundaries and y-axis in the bottom plot of Fig. 1. The middle panel in Fig. 4 shows the results of CEP, FO and FOS for finite chemical potential (hence finite baryon density). The curves at zero chemical potential are also plotted for comparison purposes. It can be seen that $\mu_b = 0.5$ GeV, which is beyond the critical point in the case of CEP ($\mu_c = 0.4$ is the value assumed here for the chemical potential at the critical point), corresponds to a region resembling the first order phase transition, therefore CEP behaves similarly to FO in this case. On the other hand, FOS is slightly different from them since the corresponding transition is not isothermal at finite baryon density.
Nevertheless, all three EoS show very similar features in high and low temperature limits.

III. NUMERICAL RESULTS AND DISCUSSIONS

Here we present results for the spectrum, flow parameters $v_2$, $v_3$ and $v_4$ and two pion interferometry using the EoSs discussed above as input into the hydrodynamical model NEXSPheRIO. The same IC generated by NeXuS and freeze-out criterion are used in all cases. An input IC consists of the energy and baryonic density in the co-moving frame and the flow velocity at the initial $\tau = \tau_0 = 1.0$ in hyperbolic coordinates.

For illustrating the hydrodynamical evolution, plots for the energy density and entropy density are shown in Figs. 3 and 3. In Fig. 3 the energy density for a selected random fluctuating event is shown. The temporal evolution of the energy density in the transverse plane is calculated considering $\eta = 0$ for the three EoSs, LQCD, CEP and FOS. The smoothed IC can be obtained in our case by averaging over different fluctuating IC in the same centrality window. Since it is expected that event by event fluctuations can lead to significant effects on the anisotropic flow components $v_2$, $v_3$ and $v_4$, the calculations in this work are performed using such fluctuating IC. In the ideal hydrodynamic scenario, the total entropy of the system is conserved. To see the above results more quantitatively, the snapshots of the entropy density for the same event in Figs. 4 and 5. In Fig. 4 the energy density for a treated event by event fluctuations can lead to significant effects of Au+Au collision at 200 GeV for 0 - 50% centrality window are shown in Fig. 3 at typical time instants and the results are depicted in Fig. 3. Due to the differences in the EoSs, the same IC may give different total entropy, so the entropy is plotted in percentiles of its initial value, instead of using the absolute values.

| Centrality (%) | FOS (MeV) | CEP (MeV) | LQCD (MeV) |
|---------------|----------|----------|------------|
| 0 - 5         | 128      | 135      | 145        |
| 5 - 10        | 130      | 136      | 145        |
| 10 - 20       | 132      | 138      | 146        |
| 20 - 30       | 135      | 140      | 147        |
| 30 - 40       | 138      | 142      | 148        |
| 40 - 60       | 143      | 146      | 149        |

TABLE I. Centrality (%) vs. freeze-out temperature $T_f$ for Au+Au 130 GeV.

It can be inferred from the plots that, at both $\sqrt{s} = 130$ MeV and 200 MeV, the lifetime of the system is the smallest when EoS LQCD is used. This can be understood using Fig. 8 where for a given energy density, the pressure for LQCD is quite different from those of CEP, FO and FOS. In particular, for FO with a first order phase transition, the pressure remains unchanged during the transition process meanwhile the system expands. In the case of CEP and FOS, the phase transition is smoother. One observes that for LQCD, the system takes less time to completely freeze-out than for the other three EoSs, which is more evident for the results at 130 GeV for RHIC energy. This is probably due to its bigger value of the pressure, as can be seen in Fig. 8 while the differences between CEP, FO and FOS are relatively small.

Next, particle spectra, collective flow and two pion interferometry are evaluated using 4000 NeXuS events for each centrality window shown in Tables I and II respectively for Au-Au collisions at 130 and 200 GeV. Balancing between a good statistics and efficiency, only 200 events are used for the calculation of particle spectra and two pion interferometry, but all 4000 events are used to evaluate the flow coefficients. At the end of each event, a Monte-Carlo generator is invoked 100 times for decoupling. There are two free parameters in the present simulation, namely, an overall normalization factor to reproduce closely the multiplicity distribution at each energy and the thermal freeze-out temperature which is adjusted to the slope of the transverse momentum spectra.

The results of the hydrodynamic simulations for the spectra, the flow parameters and the two-pion interferometry are shown in Figs. 6 to 12. Results for the $p_T$ spectra for all charged particles, as well as for identified particles are shown in Fig. 6 and Fig. 7. The particle spectra are calculated by the Cooper-Frye formula

$$E \frac{dN}{d^3p} = \int_{\Sigma} f(x, p) p^\mu d\sigma_{\mu} = \sum_{j} \frac{v_j n_{j\mu} p^\mu}{s_j} f(u_{j\mu} p^\mu),$$

(7)

where $\Sigma$ is a freeze-out hypersurface and $f(x, p)$ is a distribution function. In the PHSP representation, the summation $j$ is taken over all the PHSP particles, which should be taken at the points where they cross the hyper-surface $T = T_f$, and $n_{j\mu}$ is the normal to this hyper-surface.

For each EoS, a normalization factor is introduced for correctly reproducing the $dN/dn$ yields of all charged particles. Our results are compared with PHOBOS’ Au+Au data at 130 GeV [56] and 200 GeV [57]. For the $p_T$ spectra at 130 GeV, a pseudo-rapidity interval $-1 < \eta < 1$ is used in the calculations, which is then compared with the STAR data, where the corresponding pseudo-rapidity intervals are $-0.5 < \eta < 0.5$ [58] and $0.5 < |\eta| < 1$ [59].
The freeze-out temperatures are taken as a function of the centrality to fit the $p_T$ spectrum slope, as shown in Table II for centrality windows other than those listed in the table, the spline interpolation is used. In our calculations, different freeze-out temperatures were fitted to the spectra corresponding to the different EoSs.

For Au+Au collisions at 200 GeV, the pseudo-rapidity interval $0.2 < y < 1.4$ used in the calculations of $p_T$ spectra is the same as in the PHOBOS data. Again, the freeze-out temperature is taken as a function of the cen-
trality, in the ranges shown in Table I. It can be noted that the freeze-out temperature is practically insensitive to the incident energy, which is consistent with other studies [61, 62]. All three EoSs (CEP, FOS, LQCD), reproduce the measured $\eta$ (not shown in the figures) and $p_T$ spectra reasonably well, although some deviations start occurring at $p_T \sim 3$ GeV for peripheral ($\gtrsim 40\%$) collisions.

Next, the results for the harmonic coefficients $v_2$, $v_3$ and $v_4$ are presented. The collective flow is understood as the response to the initial geometric fluctuations, and can be used to extract information on the eccentricities of the initial conditions and transport properties of the
For Au+Au collisions, the calculated \( v_2 \) is shown in Fig. 8 as a function of \( p_T \) and compared with STAR \(^{65}\) results (top) and PHOBOS \(^{66}\) results at 200 GeV (bottom). Similar calculations of \( v_2 \) as a function of \( \eta \) for Au+Au collisions are carried out at these two energies and compared with data from these two collaborations in Fig. 10. In the calculations of \( v_2 \) as a function of \( p_T \) at 200 GeV, only particles in the interval \(-1.0 < \eta < 1.0 \) are considered, whereas the intervals \(-3.9 < \eta < -1.0 \) and \( 1.0 < \eta < 3.9 \) are used to evaluate the event planes in the forward and backward directions. The results are compared to the data from the PHENIX collaboration \(^{67, 68}\) in Fig. 9. When calculating \( v_2 \) as function of \( p_T \), a cut in pseudo-rapidity \(|\eta| < 1.3\) was implemented. There is no momentum cut in the calculations of \( v_2 \) as function of \( \eta \). In both cases, the freeze-out temperatures are from the Tables \(^{11, 12}\). In our calculations, the obtained \( v_n \) is corrected by using the event plane resolution \( \Delta \Psi_n \), in accordance with the two sub-event method \(^{64}\). We use 1000 events for 0-10% centrality window, 4000 events for 20%-30% and the Monte Carlo generator was invoked 500 times for each given IC.

The calculations are also carried out for identified particles as well. In the left column of Fig. 11 the results are shown for \( v_2 \) vs. \( p_T \) for pions, kaons and protons from Au+Au collisions at 130 GeV. The calculations were performed in the 0-50% centrality window. The experimental data are from the STAR collaboration \(^{65, 70}\). In the right column, we present the corresponding results of \( v_2 \) vs. \( p_T \) for these same particles at 200 GeV. The calculations were done for the 0-50% centrality window and are compared to the STAR data corresponding to 0-80% \(^{71}\) and 0-70% \(^{72}\).

From these plots, it is seen that at small \( p_T \) the measured elliptic flow coefficients can be reasonably well reproduced by using all three EoSs. Overall, different EoSs give roughly similar results, and all of them fail to describe the kaon data at 200 GeV when \( p_T \) increases beyond \( \sim 2 \) GeV, probably reflecting the limitations of the ideal hydrodynamics employed in this study. It can be seen that the results of LQCD are slightly different from those of CEP and FOS for \( p_T < 1 \) GeV. This can be understood as follows. For an EoS featuring a first order phase transition, such as FO, the pressure gradient vanishes when the system enters this region. In the present case, although CEP and FOS describe smooth phase transitions, their properties are more similar to that of FO than to those of LQCD. For LQCD, it could be inferred from Fig. 8 that the pressure gradient is bigger than those from other EoSs in the high temperature region (\( T \geq 0.3 \) GeV). As a result, in the case of LQCD, the initial spatial eccentricity of the system would be transformed into momentum anisotropy with the biggest amplified magnitude at high temperature as well as during the hadronization process. At 200 GeV, higher initial temperatures could be achieved, consequently, the matter in evolution spends more time in the QGP phase where most eccentricity is developed, therefore the asymptotic behaviour of LQCD at high temperature plays an in-

---

**System**

The flow anisotropy coefficients of the angular distribution of the emitted particles are defined as

\[
\frac{dN}{d\phi} = \frac{N}{2\pi} \left[ 1 + 2 \sum_{n=1}^{\infty} \left( n^2 \int_0^{\pi} \cos n(\phi - \Psi_n) \, d\phi \right) \right],
\]

where \( N \) is the number of particle created, \( \phi \) is the azimuthal angle, \( \Psi_n \) the events plane (EP) angles \(^{63, 64}\), defined as

\[
\Psi_n = \frac{1}{n} \arctan \left( \frac{\langle \sin(n\phi) \rangle}{\langle \cos(n\phi) \rangle} \right).
\]

The EP is considered the best approximation to the reaction plane, defined by the collision direction and the collision impact parameter, which is not directly accessible experimentally.

Here all the calculations are done by using the event plane method \(^{64}\), and the results for \( v_n \) are obtained as a function of pseudo-rapidity, the transverse momentum. For Au+Au collisions, the calculated \( v_2 \) is shown in Fig. 8.
FIG. 9. The anisotropic flow components $v_2$, $v_3$ and $v_4$ are shown as a function of $p_T$ for all charged particles, corresponding to the FOS, CEP and LQCD EoSs, for Au+Au collisions at 200 GeV for the 0 - 10% (l.h.s.) and for the 20% - 30% centrality window (r.h.s.). The data are from the PHENIX Collaboration.

Increasingly important role, which makes it distinct from all other EoSs. On the other hand, at a lower incident energy, 130 GeV, the system develops relatively more anisotropy in the transition region, therefore the properties of the phase transition become more important.

This makes the curve of CEP to become closer to that of LQCD.

The last part of this paper is dedicated to the investigation of the influence of different EoSs on two-particle quantum statistical correlations, also known as GGLP
A systematic study on the role of EoS on hydrodynamical evolution of the system is carried out and discussed here. By adopting the parameters tailored to each specific EoS, which consist of an overall renormalization factor and freeze-out temperatures, the particle spectra, harmonic flow coefficients and two-pion interference are calculated by NEXSPheRIO code. The calculations cover a wide range of centrality windows at two different RHIC
energies. It is found that all EoS successfully reproduce the particle spectra and elliptic flow at small $p_T$ region.
The hydrodynamical evolution of the system is affected by the EoS, which consequently leads to some small differences observed in collective flow. In the case of HBT radii, all the EoSs give reasonable but not exact description of the data in all $p_T$ range.

Hydrodynamics assumes local thermal equilibrium, based on which the dynamical properties of the hot and dense system are expressed in terms of the EoS. The calculations carried out in this work show that the results are weakly dependent on the EoS. On the other hand, re-
sults based on transport models such as AMPT or PHSD reveal that, on event-by-event basis a state close to the local thermal equilibrium seems to occur only during a very short time scale and in a very small domain. Such models can also provide reasonable description of the event averaged (or those calculated by cumulants) $v_2$ data. This observation, combined with the results shown here based on the NeXSPheRIO code, suggest that the collective flow parameters may not be sensitive to the genuine non-linear hydrodynamic response, but they may merely reflect the initial density fluctuations, when averaged. In fact, such speculation is also supported by the almost linear dependence of event averaged $v_n$ to $\epsilon_n$. Therefore, the reasons mentioned above seem to weaken the assumption of the local thermal equilibrium in hydrodynamics on an event-by-event basis to be an essential requirement when one discusses the collective flow parameter in event average.

V. ACKNOWLEDGMENTS

The authors are thankful for valuable discussions with Wojciech Florkowski and Tamás Csorgó. We gratefully acknowledge the financial support from Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), Fundação de Amparo à Pesquisa do Estado de Minas Gerais (FAPEMIG), Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES).

[1] P. F. Kolb and U. W. Heinz, (2003), arXiv:nucl-th/0305084.
[2] E. Shuryak, Prog.Part.Nucl.Phys. 53, 273 (2004), arXiv:hep-ph/0312227.
[3] U. W. Heinz and R. Snellings, Annu. Rev. Nucl. Part. Sci. 63, 123 (2013), arXiv:1301.2826.
[4] C. Gale, S. Jeon, and B. Schenke, Int.J.Mod.Phys. A28, 1340011 (2013), arXiv:1301.5893.
[5] Z. Fodor and S. Katz, JHEP 0203, 014 (2002), arXiv:hep-lat/0106002.
[6] F. Karsch, Nucl.Phys. A698, 199 (2002), arXiv:hep-ph/0103314.
[7] Y. Aoki, G. Endrodi, Z. Fodor, S. Katz, and K. Szabo, Nature 443, 675 (2006), arXiv:hep-lat/0611014.
[8] Y. Aoki, Z. Fodor, S. Katz, and K. Szabo, JHEP 0601, 089 (2006), arXiv:hep-lat/0510084.
[9] S. Borsanyi et al., JHEP 1011, 077 (2010), arXiv:1007.2580.
[10] S. Borsanyi et al., Phys.Lett. B730, 99 (2014), arXiv:1309.5258.
[11] A. Bazavov et al., Phys.Rev. D80, 014504 (2009), arXiv:0909.4379.
[12] P. Braun-Munzinger, K. Redlich, and J. Stachel, (2003), arXiv:nucl-th/0304013.
[13] P. Huovinen, Nucl.Phys. A761, 296 (2005), arXiv:nucl-th/0505036.
[14] P. Huovinen and P. Petreczky, Nucl.Phys. A837, 26 (2010), arXiv:0912.2541.
[15] P. Huovinen and P. Petreczky, J.Phys. G38, 124103 (2011), arXiv:1106.6227.
[16] M. Chojnacki and W. Florkowski, Acta Phys.Polon. B38, 3249 (2007), arXiv:nucl-th/0702030.
[17] M. Chojnacki, W. Florkowski, W. Broniowski, and A. Kisiel, Phys.Rev. C78, 014905 (2008), arXiv:0712.0947.
[18] H. Song and U. W. Heinz, Phys.Rev. C78, 024902 (2008), arXiv:0805.1756.
[19] T. S. Biro and J. Zimanyi, Phys.Lett. B650, 193 (2007), arXiv:hep-ph/0607079.
[20] A. Chaudhuri, Phys.Lett. B681, 418 (2009), arXiv:0909.0391.
[21] J. Steinheimer et al., Phys.Rev. C81, 044913 (2010), arXiv:0905.3099.
[22] S. De, D. K. Srivastava, and R. Chatterjee, J.Phys. G37, 115004 (2010), arXiv:1008.1475.
[23] Y. Hama, T. Kodama, and O. Socolowsi Jr., Braz.J.Phys. 35, 24 (2005), arXiv:hep-ph/0407264.
[24] B. Lukacs, J. Zimanyi, and N. Balazs, Phys.Lett. B183, 27 (1987).
[25] K.-S. Lee and U. W. Heinz, Phys.Rev. D47, 2068 (1993).
[26] H. Muller and B. D. Serot, Phys.Rev. C52, 2072 (1995), arXiv:nucl-th/9505013.
[27] W. L. Qian, R.-K. Su, and P. Wang, Phys.Lett. B491, 90 (2000), arXiv:nucl-th/0008057.
[28] W. Qian, R.-K. Su, and H. Song, Phys.Lett. B520, 217 (2001), arXiv:nucl-th/0107036.
[29] L. Yang, W. L. Qian, R.-K. Su, and H. Q. Song, Phys.Rev. C70, 045207 (2004), arXiv:nucl-th/0311017.
[30] W. L. Qian, R.-K. Su, and H. Q. Song, J.Phys. G30, 1893 (2004), arXiv:nucl-th/0409063.
[31] L. Yang, S. Y. Yin, W. L. Qian, and R.-k. Su, Phys.Rev. C73, 025203 (2006), arXiv:nucl-th/0506060.
[32] A. Andronic, P. Braun-Munzinger, and J. Stachel, Nucl.Phys. A772, 167 (2006), arXiv:nucl-th/0511071.
[33] ALICE Collaboration, B. Abelev et al., Phys.Rev.Lett. 109, 252301 (2012), arXiv:1208.1974.
[34] V. Begun, M. I. Gorenstein, M. Hauer, V. Konchakovski, and O. Zozulya, Phys.Rev. C74, 044903 (2006), arXiv:nucl-th/0606036.
[35] J. Fu, Phys.Lett. B722, 144 (2013).
[36] Y. Hama et al., Nucl.Phys. A774, 169 (2006), arXiv:hep-ph/0510096.
[37] W. Broniowski, M. Chojnacki, W. Florkowski, and A. Kisiel, Phys.Rev.Lett. 101, 022301 (2008), arXiv:0801.4361.
[38] K. Fukushima, Phys.Lett. B591, 277 (2004), arXiv:hep-ph/0310121.
[39] F. Karsch and E. Laermann, (2003), arXiv:hep-lat/0305025.
[40] K. Dusling, G. D. Moore, and D. Teaney, Phys.Rev. C81, 034907 (2010), arXiv:0909.0754.
[41] M. Luzum and J.-Y. Ollitrault, Phys.Rev. C82, 014906 (2010), arXiv:1004.2023.
PHENIX Collab. \( \sqrt{s_{\text{NN}}} = 200 \text{ GeV} \)

\( \frac{(h^+ + h^-)}{2} \)

- 0%-10%
- EoS FOS \( T_F = 129 \text{ MeV} \)
- EoS CEP \( T_F = 135 \text{ MeV} \)
- EoS LQCD \( T_F = 145 \text{ MeV} \)

\( p_T(\text{GeV}) \)

\( V_2 \)
