Geometrical methods in loop calculations and the three-point function

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A geometrical way to calculate $N$-point Feynman diagrams is reviewed. As an example, the dimensionally-regulated three-point function is considered, including all orders of its $\varepsilon$-expansion. Analytical continuation to other regions of the kinematical variables is discussed.

1. INTRODUCTION

The analytical structure of the results for $N$-point Feynman diagrams can be better understood if one employs a geometrical interpretation of kinematic invariants and other quantities. For example, the singularities of the general three-point function can be described pictorially through a tetrahedron constructed out of the external momenta and internal masses (see Fig. 1a). This method can be used to derive Landau equations defining the positions of possible singularities [1] (see also in [2]).

In Ref. [3], it was demonstrated how such geometrical ideas could be used for an analytical calculation of one-loop $N$-point diagrams. For example, in the three-point case in $n$ dimensions, the result can be expressed in terms of an integral over a spherical (or hyperbolic) triangle, as shown in Fig. 1b, with a weight function $\cos^{3-n} \theta$, where $\theta$ is the angular distance between the integration point and the point 0, corresponding to the height of the basic tetrahedron (see in [3]). This weight function equals 1 for $n = 3$ (see also in [4]). For $n = 4$, one can get another representation [5, 6], in terms of an integral over the volume of an asymptotic hyperbolic tetrahedron. Here we will discuss the application of the approach of Refs. [3, 7] to the three-point function in any dimension $n$, as well as its $\varepsilon$-expansion ($n = 4 - 2\varepsilon$) within dimensional regularization [8].

2. TRIGONOMETRIC STUFF

We will use notations defined in [3], namely:

$$c_{jl} = \cos \tau_{jl} \equiv \frac{m_j^2 + m_l^2 - k_{jl}^2}{2 m_j m_l} ,$$

(1)

$$\cos \tau_{0i} = \frac{m_0}{m_i} , \quad m_0 = m_1 m_2 m_3 \sqrt{\frac{D^{(3)}}{\Lambda^{(3)}}} ,$$

(2)

$$\Lambda^{(3)} = -\frac{1}{4} \left[ (k_{12}^2)^2 + (k_{13}^2)^2 + (k_{23}^2)^2 - 2k_{12}^2 k_{13}^2 - 2k_{12}^2 k_{23}^2 - 2k_{13}^2 k_{23}^2 \right] = -\frac{1}{4} \lambda \left( k_{12}^2, k_{13}^2, k_{23}^2 \right) ,$$

(3)

$$D^{(3)} = \begin{vmatrix} 1 & c_{12} & c_{13} \\ c_{12} & 1 & c_{23} \\ c_{13} & c_{23} & 1 \end{vmatrix} = 1 - c_{12}^2 - c_{13}^2 - c_{23}^2 + 2 c_{12} c_{13} c_{23} .$$

(4)
Assuming that all $|c_{ij}| \leq 1$, $\Lambda^{(3)} > 0$, and $D^{(3)} > 0$, we get spherical triangles. In other cases, we need to use the hyperbolic space. The transition corresponds to the analytic continuation (see below).

Using the approach of Ref. [3], we split the spherical triangle into three smaller ones (see Fig. 2), so that $\varphi_{12} + \varphi_{23} + \varphi_{31} = 2\pi$. Moreover, it is convenient to split each of the resulting triangles into two rectangular ones, as shown in Fig. 3. By definition, $\frac{1}{2} (\varphi_{12} + \varphi_{12}) = \varphi_{12}$, $\frac{1}{2} (\varphi_{12} + \varphi_{12}) = \tau_{12}$. Let us list useful relations between the sides and angles of the spherical triangle (see also in [3]):

$$
\cos \left( \frac{1}{2} \tau_{12}^+ \right) = \cos \frac{\tau_{01}}{\cos \eta_{12}}, \\
\cos \left( \frac{1}{2} \varphi_{12}^+ \right) = \tan \eta_{12}, \\
\cos \left( \frac{1}{2} \varphi_{12}^- \right) = \frac{\tan \eta_{12}}{\tan \tau_{01}}, \\
\sin \left( \frac{1}{2} \tau_{12}^+ \right) = \sin \tau_{01} \sin \left( \frac{1}{2} \varphi_{12}^+ \right), \\
\sin \left( \frac{1}{2} \tau_{12}^- \right) = \sin \tau_{02} \sin \left( \frac{1}{2} \varphi_{12}^- \right), \\
\tan \left( \frac{1}{2} \tau_{12}^\pm \right) = \sin \eta_{12} \tan \left( \frac{1}{2} \varphi_{12}^\pm \right), \\
\sin \eta_{12} = \sin \tau_{01} \sin \kappa_{12}^+ = \sin \tau_{02} \sin \kappa_{12}^-.
$$

Worth noting is

$$
\cos \eta_{12} = \frac{m_0 \sqrt{k_{12}^2}}{m_1 m_2 \sin \tau_{12}}.
$$

Therefore,

$$
\cos \left( \frac{1}{2} \tau_{12}^+ \right) = \frac{m_2 \sin \tau_{12}}{\sqrt{k_{12}^2}}, \\
\sin \left( \frac{1}{2} \tau_{12}^+ \right) = \frac{m_2^2 - m_2^2 + k_{12}^2}{2m_1 \sqrt{k_{12}^2}}, \\
\cos \left( \frac{1}{2} \tau_{12}^- \right) = \frac{m_1 \sin \tau_{12}}{\sqrt{k_{12}^2}}, \\
\sin \left( \frac{1}{2} \tau_{12}^- \right) = \frac{m_2^2 - m_1^2 + k_{12}^2}{2m_2 \sqrt{k_{12}^2}}.
$$

In a triangle with the sides $m_1$, $m_2$ and $\sqrt{k_{12}^2}$, the angles $\frac{1}{2} \tau_{12}^+$ and $\frac{1}{2} \tau_{12}^-$ are those between the height of the triangle and the sides $m_1$ and $m_2$, respectively. Therefore, the point $T_{12}$ in Fig. 3 corresponds to the intersection of this face height and the sphere.

3. HYPERGEOMETRIC STUFF

Eqs. (3.38)–(3.39) of [3] yield

$$
J^{(3)}(n; 1, 1, 1) = -\frac{i \pi^n/2 \Gamma \left( 3 - \frac{n}{2} \right)}{m_0^{\frac{n}{2}} \sqrt{\Lambda^{(3)}}} \Omega^{(3,n)},
$$

where $\Omega^{(3,n)}$ is an integral over the solid angle $\Omega^{(3)}$ (corresponding to triangle 123 in Fig. 1b),

$$
\Omega^{(3,n)} = \int_{\Omega^{(3)}} \frac{\sin^n \theta \, d\theta \, d\phi}{\cos^{n-2} \theta}. 
$$
According to Fig. 2 and Fig. 3, $\Omega^{(3;n)}$ can be presented as a sum of six contributions:

$$\Omega^{(3;n)} = \omega\left(\frac{\phi}{2}, \eta_{12}\right) + \omega\left(\frac{\phi}{2}, \eta_{23}\right) + \omega\left(\frac{\phi}{2}, \eta_{23}\right) + \omega\left(\frac{\phi}{2}, \eta_{31}\right) + \omega\left(\frac{\phi}{2}, \eta_{31}\right),$$

with (see Refs. [9][10][11])

$$\omega\left(\frac{\phi}{2}, \eta\right) = \frac{1}{2\varepsilon} \int_0^{\phi/2} d\phi \left[ 1 - \left(1 + \frac{\tan^2 \eta}{\cos^2 \phi}\right)^{-\varepsilon} \right],$$

where $\varepsilon = \frac{1}{2}(4-n)$. Defining $\tan \frac{\phi}{2} = \sin \eta \tan \frac{\phi}{2}$, we can obtain another useful representation,

$$\int_0^{\phi/2} d\phi \left(1 + \frac{\tan^2 \eta}{\cos^2 \phi}\right)^{-\varepsilon} = \sin \eta \cos^{2\varepsilon} \eta \int_0^{\pi/2} d\psi \cos^{2\varepsilon} \psi.$$  

The remaining $\phi$-integral in Eq. (10) can be calculated using a substitution $\phi = \arctan \left(\frac{\sqrt{n}}{\sin \eta}\right)$. The result can be presented in terms of Appell’s hypergeometric function $F_1$,

$$\omega\left(\frac{\phi}{2}, \eta\right) = \frac{1}{2\varepsilon} \left[ \frac{\phi}{2} - \tan \frac{\phi}{2} \cos^{2\varepsilon} \eta \times F_1\left(\frac{\phi}{2}, 1, \varepsilon; \frac{3}{2}; - \tan^2 \frac{\phi}{2}, - \tan^2 \frac{\phi}{2}\right) \right].$$

Moreover, using the transformation formula

$$F_1(a, b, c; x, y) = (1-x)^{-a}(1-y)^{-b} \times F_1\left(c-a, b, c; \frac{x}{x-1}, \frac{y}{y-1}\right),$$

the result can be presented as

$$\omega\left(\frac{\phi}{2}, \eta\right) = \frac{1}{2\varepsilon} \left[ \frac{\phi}{2} - \sin \frac{\phi}{2} \cos \frac{\phi}{2} \cos^{2\varepsilon} \tau_0 \times F_1\left(1, 1, \varepsilon; \frac{3}{2}; \sin^2 \frac{\phi}{2}, \sin^2 \frac{\phi}{2}\right) \right],$$

with $\cos \tau_0 = \cos \eta \cos \frac{\phi}{2}$. Similar $F_1$ functions occurred in Refs. [12][13] (cf. also Ref. [14]).

An important formula (shift $\varepsilon \to 1 + \varepsilon$, or $n \to n - 2$) reads:

$$F_1\left(1, 1; \frac{3}{2}; x, y\right) = \frac{y}{x} F_1\left(1, 1 + \varepsilon; \frac{3}{2}; y\right) + \left(1 - \frac{y}{x}\right) F_1\left(1, 1 + \varepsilon; \frac{3}{2}; x, y\right).$$

It can be supplemented by a Kummer relation:

$$(1 - 2\varepsilon) F_1\left(1, \varepsilon; \frac{3}{2}; y\right) = 1 - 2\varepsilon(1 - z) F_1\left(1, 1 + \varepsilon; \frac{3}{2}; y\right).$$

Each of the three functions in Fig. 2 may be associated with a specific three-point function $J^{(3)}(n; 1, 1, 1)$. According to Eq. (3.45) of [3], the result of such splitting reads

$$J^{(3)}(n; 1, 1, 1) = \frac{m_2^2 m_3^2 F_1(3)}{\Lambda(3)} \sum_{i=1}^{3} \frac{F_1(1, 1, 1)}{m_i^2} J^{(3)}(n; 1, 1, 1),$$

where $F_1(1, 1, 1) = \frac{2}{m_2^2} F_1(3; D_3)$ (see also in Ref. [4]).

The geometrical meaning of $F_i$ was discussed in [3]. In particular,

$$m_2^2 m_3^2 F_1(3) + m_1^2 m_2^2 F_1(3) + m_1^2 m_3^2 F_1(3) = \Lambda(3)$$

means that the volume of the basic tetrahedron equals the sum of the volumes after splitting.

For each of the integrals $J^{(3)}$, only one two-point function appears in the reduction formulae. For instance, using Eqs. (12) and (13) we get

$$(n-2)\pi^{-1} J^{(3)}(n+2; 1, 1, 1) = -2m_2^2 J^{(3)}(n; 1, 1, 1) - J^{(3)}(n; 1, 1, 0),$$

and similarly for $J^{(3)}(1)$ and $J^{(3)}(2)$. This yields a geometrical way to derive the recursion in $n$: just take Eq. (13), shift $n \to n + 2$ and substitute Eq. (16). The result is

$$J^{(3)}(n+2; 1, 1, 1) = \frac{\pi m_2^2 m_3^2 m_3^2}{(n-2)\Lambda(3)} \left\{ 2D(3) J^{(3)}(n; 1, 1, 1) \right. + \frac{F_1^{(3)}}{m_2^2} J^{(3)}(n; 0, 1, 1) + \left. \frac{F_1^{(3)}}{m_3^2} J^{(3)}(n; 1, 1, 0) \right\},$$

in agreement with Refs. [15][18] (see also [16][17]).

4. SPECIAL VALUES OF $n$

4.1. $n = 3$ ($\varepsilon = \frac{1}{2}$)

In this case, we can use the known reduction formula for the $F_1$ function,

$$F_1\left(a, b, b'; b + b'\mid x, y\right)$$

and its derivatives, which are given in Ref. [12].
Taking into account that

\[ 2F_1 \left( 1, 1; \frac{3}{2} \mid z \right) = \frac{\arcsin \sqrt{z}}{\sqrt{z(1-z)}}, \]

we get

\[ F_1 \left( 1, 1; \frac{3}{2} \mid \sin^2 \frac{x}{2}, \sin^2 \frac{y}{2} \right) = \frac{\pi - 2\kappa}{\sin \varphi \cos \tau_0}, \]

with \( \cos \kappa = \sin \frac{x}{2} \cos \eta \) and \( \cos \tau_0 = \cos \frac{y}{2} \cos \eta \). Therefore,

\[ \omega \left( \frac{1}{2}, \varphi, \eta \right) \big|_{n=3} = \frac{\varphi}{2} - \frac{\pi}{2} + \kappa. \]  

Collecting results for all six triangles, we reproduce Eq. (5.4) of [3] (see also in [3]),

\[ \Omega^{(3;3)} = \Omega^{(3)} = \psi_{12} + \psi_{23} + \psi_{31} - \pi. \]  

4.2. \( n = 2 \) (\( \varepsilon = 1 \))

In this case, the function \( F_1 \) reduces to

\[ F_1 \left( 1, 1, 1; \frac{3}{2} \mid x, y \right) = \frac{1}{x-y} \left[ \sqrt{x} \arcsin \sqrt{x} \right] - \frac{\sqrt{y} \arcsin \sqrt{y}}{\sqrt{1-y}} \].

In this way, we get

\[ F_1 \left( 1, 1, 1; \frac{3}{2} \mid \sin^2 \frac{x}{2}, \sin^2 \frac{y}{2} \right) = \frac{\varphi - \tau \sin \eta}{\sin \varphi \cos^2 \tau_0} \]

and, therefore,

\[ \omega \left( \frac{1}{2}, \varphi, \eta \right) \big|_{n=2} = \frac{1}{4} \tau \sin \eta. \]  

Collecting results for all six triangles, we get

\[ \Omega^{(3;2)} = \frac{1}{3} \left( \tau_{12} \sin \eta_{12} + \tau_{23} \sin \eta_{23} + \tau_{31} \sin \eta_{31} \right). \]

Recalling that the two-point integral in two dimensions is proportional to \( \tau / \sin \tau \) (see Eq. (4.3) of Ref. [3]), we see that the three-point integral (with \( n = 2 \)) is a combination of three-point integrals, with coefficients proportional to \( \sin \tau_{jl} \sin \eta_{jl} \) (cf. Eq. (10) of Ref. [4]).

4.3. \( n = 5 \) (\( \varepsilon = -\frac{1}{2} \))

In this case, we obtain

\[ \omega \left( \frac{1}{2}, \varphi, \eta \right) \big|_{n=5} = \frac{\pi}{2} \frac{\varphi}{2} - \kappa \]

\[ + \frac{1}{2} \tan \eta \ln \left( \frac{1 + \sin \frac{\varphi}{2}}{1 - \sin \frac{\varphi}{2}} \right). \]

Collecting results for all six triangles, we get

\[ \Omega^{(3;5)} = - \left( \psi_{12} + \psi_{23} + \psi_{31} - \pi \right) \]

\[ + \tan \eta_{12} \ln \left( \frac{m_1 + m_2 + \sqrt{k_{12}}}{m_1 + m_2 - \sqrt{k_{12}}} \right) \]

\[ + \tan \eta_{23} \ln \left( \frac{m_2 + m_3 + \sqrt{k_{23}}}{m_2 + m_3 - \sqrt{k_{23}}} \right) \]

\[ + \tan \eta_{31} \ln \left( \frac{m_1 + m_3 + \sqrt{k_{13}}}{m_1 + m_3 - \sqrt{k_{13}}} \right). \]

In other words, the five-dimensional three-point integral can be expressed in terms of the three-dimensional three- and two-point integrals (see Eqs. (5.4) and (4.6) of [3]), in agreement with Eq. (14).

4.4. \( n = 4 \) (\( \varepsilon \to 0 \))

In this case, we need to expand the \( F_1 \) function up to the term linear in \( \varepsilon \) (it is easy to see that the \( \varepsilon^0 \) term cancels the \( \frac{\varphi}{2} \) contribution). The result can be presented in terms of Clausen function, see Eq. (5.21) of [3]. Collecting three contributions of this type, we get \( 6 \times 3 = 18 \) Clausen functions that can be analytically continued in terms of 12 dilogarithms [13]. In Ref. [6] the result is presented in terms of 15 Clausen functions.

5. ANALYTIC CONTINUATION

In the integral occurring in Eq. (9), let us substitute \( z \Rightarrow e^{2i\phi} \), so that \( \cos^2 \phi \Rightarrow \frac{(1+z^2)}{4z^2} \) and

\[ 1 + \tan^2 \eta \cos^2 \phi \Rightarrow \frac{(z + \rho)(z + 1/\rho)}{(z + 1)^2}, \]

with

\[ \rho \equiv \frac{1 - \sin \eta}{1 + \sin \eta}. \]

In this way, we get

\[ \frac{\varphi}{2} \int_0^1 d\phi \left( 1 + \frac{\tan^2 \eta}{\cos^2 \phi} \right) \]

\[ \Rightarrow \frac{i}{2} \int_{z_0}^1 \frac{dz}{z} \left[ \frac{(z + \rho)(z + 1/\rho)}{(z + 1)^2} \right]^{-\varepsilon}, \]

\[ \Rightarrow \frac{i}{2} \int_{z_0}^1 \frac{dz}{z} \left[ \frac{(z + \rho)(z + 1/\rho)}{(z + 1)^2} \right]^{-\varepsilon}, \]

\[ \Rightarrow \frac{i}{2} \int_{z_0}^1 \frac{dz}{z} \left[ \frac{(z + \rho)(z + 1/\rho)}{(z + 1)^2} \right]^{-\varepsilon}, \]

\[ \Rightarrow \frac{i}{2} \int_{z_0}^1 \frac{dz}{z} \left[ \frac{(z + \rho)(z + 1/\rho)}{(z + 1)^2} \right]^{-\varepsilon}, \]
Q_j \equiv \int_{z_0}^1 \frac{dz}{z} \ln^j \left[ \frac{(z + \rho)(z + 1/\rho)}{(z + 1)^2} \right]. \quad (25)

The first term, 
\begin{align*}
Q_1 &= \text{Li}_2(-z_0\rho) + \text{Li}_2(-z_0/\rho) - 2\text{Li}_2(-z_0) \\
&\quad + \frac{1}{2} \ln^2 \rho,
\end{align*}

(26)
yields the known result \cite{13} for the three-point function in four dimensions. The r.h.s. of Eq. (26) can also be presented as
\begin{align*}
2\text{Li}_2(z_0) - \text{Li}_2\left(\frac{\rho + z_0}{\rho + z_0^{-1}}\right) - \text{Li}_2\left(\frac{2}{\rho + z_0} z_0^{-1}\right) \\
&\quad + \frac{1}{2} \ln^2 \rho - \frac{1}{2} \ln^2 \left(\frac{z_0\rho(\rho + z_0^{-1})}{\rho + z_0}\right),
\end{align*}

(27)
in agreement with Eq. (5.17) of Ref. \cite{3}.

The second term, \(Q_2\), gives the \(\varepsilon\) term of the three-point function. For \(j = 2\), the integral can be evaluated in terms of polylogarithms,
\begin{align*}
Q_2 &= Q_2^{(1)}(z_0, \rho) \ln \rho + Q_2^{(2)}(z_0, \rho),
\end{align*}

(28)
Furthermore,
\begin{align*}
\frac{\partial}{\partial \rho} Q_2^{(2)}(z_0, \rho) &= - \ln \rho \frac{\partial}{\partial \rho} Q_2^{(1)}(z_0, \rho), \\
\rho \frac{\partial}{\partial \rho} Q_2(z_0, \rho) &= Q_2^{(1)}(z_0, \rho).
\end{align*}

Another useful representation for \(Q_2^{(1)}\) reads
\begin{align*}
Q_2^{(1)} &= 2\text{Li}_2\left(-\frac{\rho + z_0}{1 + \rho z_0}\right) - 2\text{Li}_2\left(-\frac{1 + \rho z_0}{\rho + z_0}\right) \\
&\quad + 2 \ln \left[\frac{\rho}{(\rho + 1)^2}\right] \ln \left(\frac{1 + \rho z_0}{\rho + z_0}\right). 
\end{align*}

The occurring \(S_{1,2}\) functions can be presented in terms of trilogarithms \(L_3\). This result corresponds to Eq. (82) of Ref. \cite{13}. We note that the first calculation of the \(\varepsilon\)-term of the one-loop three-point function was given in Ref. \cite{19} (see also Ref. \cite{20} for the off-shell massless case).

Eq. (26) shows that all higher terms of the \(\varepsilon\)-expansion of the one-loop three-point function can be expressed in terms of one-fold integrals of the products of logarithms of three linear arguments. (For some specific configurations, the \(\varepsilon^2\) terms were studied in Ref. \cite{21}.) The considered representations may be useful to understand the types of generalized functions needed to describe the analytic structure of the results for higher terms of the \(\varepsilon\)-expansion.

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