Remote generation of entanglement for individual atoms via optical fibers

Y. Q. Guo*, H. Y. Zhong, Y. H. Zhang
Department of Physics, Dalian Maritime University, Dalian 116026, P.R.China

H. S. Song
School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian 116023, P.R.China

The generation of atomic entanglement is discussed in a system that atoms are trapped in separate cavities which are connected via optical fibers. Two distant atoms can be projected to Bell-state by synchronized turning off the local laser fields and then performing a single quantum measurement by a distant controller. The distinct advantage of this scheme is that it works in a regime that $\Delta \approx \kappa \gg g$, which makes the scheme insensitive to cavity strong leakage. Moreover, the fidelity is not affected by atomic spontaneous emission.

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Very recently, much attention has been paid to the study of the possibility of quantum information processing realized via optical fibers [1,2]. Generating an entangled state of distant qubits turns out to be a basic aim of quantum computation. It has been pointed out that implementing quantum entangling gate that works for spatially separated local processors which are connected by quantum channels is crucial in distributed quantum computation. Many schemes have been put forward to prepare engineering entanglement of atoms trapped in separate optical cavities by creating direct or indirect interaction between them [3–10]. Some of the schemes involve direct connection of separate cavities via optical fibers, others apply detection of the photons leaking from the cavities. All the implemented quantum gates work in a probabilistic way. To improve the corresponding success probability and fidelity, one must construct precisely controlled coherent evolutions of the global system and weaken the effect of photon detection inefficiency. In the system considered by Serafini et al [5], the only required local control is synchronized switching on and off of the atom-field interaction in the distant cavities. In the scheme proposed by Mancini and Bose [11], a direct interaction between two atoms trapped in distant cavities is engineered, the only required control for implementing quantum entangling gate is turning off the interaction between atoms and the locally applied laser fields. In the present letter, we propose an alternative scheme with particular focus on the establishment of three-qubit entanglement, which is suitable and effective for the generation of three-atom W-type state and two-atom Bell-state. To generate three-atom W-type state, the only control required is synchronized turning off the locally applied laser fields. While, To generate two-atom Bell-state, an additional quantum measurement performed on one of the atoms is needed. We demonstrate that the scheme works in a high success probability, and the atomic spontaneous emission does not affect the fidelity.

The schematic setup of the system is shown in Fig. 1. Three two-level atoms 1, 2 and 3 locate in separate optical cavities $C_1$, $C_2$ and $C_3$ respectively. The cavities are assumed to be single-sided. Three off-resonant driving external fields $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ are added on $C_1$, $C_2$ and $C_3$ respectively. In each cavity, a local weak laser field is applied to resonantly interact to the atom. Two neighboring cavities are connected via optical fiber. The global system is located in vacuum. Using the input-output theory, taking the adiabatic approximation [12] and applying the methods developed in Refs. [11] and [13], we obtain the effective Hamiltonian of the global system as

$$H_{eff} = J_{12}\sigma_i^+\sigma_j^- + J_{23}\sigma_j^+\sigma_k^- + J_{31}\sigma_k^+\sigma_i^- + \Gamma \sum_i (\sigma_i^- + \sigma_i^+),$$

where $\sigma_i^+$ and $\sigma_i^-(\sigma_i^-)$, $i = 1, 2, 3$, are spin and spin raising (lowering) operators of atom $i$. $\Gamma$ represents the local laser field added on the atom. To keep the validity of adiabatic

*Corresponding author: yqguo@newmail.dlmu.edu.cn
approximation, we assume $\Gamma \ll J_{12}(J_{23}, J_{31})$. And

$$J_{12} = 2\kappa^2 \gamma m \left(2\alpha_3 \alpha_2 (Me^{i\phi_{32}} + ke^{i\phi_{12}})/(M^2 - W^2)\right),$$

$$J_{23} = 2\kappa^2 \gamma m \left(2\alpha_1 \alpha_3 (Me^{i\phi_{32}} + ke^{i\phi_{12}})/(M^2 - W^2)\right),$$

$$J_{31} = 2\kappa^2 \gamma m \left(2\alpha_1 \alpha_2 (Me^{i\phi_{32}} + ke^{i\phi_{12}})/(M^2 - W^2)\right),$$

where $\kappa$ is the cavity leaking rate, $\chi = \frac{\kappa}{\Gamma}$, $g$ is the coupling strength between atom and cavity field, $\Delta$ is the detuning. In deducing Eq. (1), the condition $\Delta \ll \kappa \gg g$ is assumed, $M = i\Delta + \kappa, W^3 = k^w e^{i(\phi_{32} + \phi_{13} + \phi_{12})}$. The phase factors $\phi_{21}, \phi_{32}$, and $\phi_{13}$ are the phases caused by the photon transmission along the optical fibers. And

$$\alpha_1 = \frac{M^2 \epsilon_1 + 2\kappa^2 \epsilon_{31} e_{32} + M \kappa \epsilon_{31} e_3}{M^2 - W^3},$$

$$\alpha_2 = \frac{M^2 \epsilon_2 + 2\kappa^2 \epsilon_{31} e_{32} + M \kappa \epsilon_{31} e_1}{M^2 - W^3},$$

$$\alpha_3 = \frac{M^2 \epsilon_3 + 2\kappa^2 \epsilon_{31} e_{32} + M \kappa \epsilon_{31} e_2}{M^2 - W^3}.$$

We assume that $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_0$, $\phi_{21} = \phi_{32} = \phi_{13} = \phi_0$. This leads to

$$a_1 = a_2 = a_3 = a_0,$$

$$J_{12} = J_{23} = J_{31} = J_0.$$  

The Hamiltonian in Eq. (1) is now written as

$$H_{eff} = H_{zz} + H_s,$$

where

$$H_{zz} = J_0 (\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y + \sigma_1^z \sigma_2^z),$$

$$H_s = \Gamma \sum_i (\sigma_i^x + \sigma_i^y).$$  

Eq. (5) represents the Hamiltonian of an Ising ring model. The entanglement of the ground state of the above Hamiltonian has already been discussed. Here, we study the entanglement of the evolved system state governed by the Hamiltonian. Under the condition $\Gamma \ll J_0$, the secular part of the effective Hamiltonian can be obtained through the transformation $UU^{-1}, U = e^{-iH_{zz}J_0}$, as

$$\hat{H} = \frac{\Gamma}{2} [\sigma_1^x (1 - \sigma_2^z \sigma_2^x) + \sigma_2^x (1 - \sigma_2^z \sigma_1^x) + \sigma_2^z (1 - \sigma_2^x \sigma_2^y)].$$  

The straightforward interpretation of this Hamiltonian is: the spin of an atom in the Ising ring flips if and only if its two neighbors have opposite spins.

For the initial states that one or two of the atoms are excited, the system state is restricted within the subspace spanned by the following basis vectors

$$|\phi_1\rangle = |eg\rangle, |\phi_2\rangle = |eeg\rangle, |\phi_3\rangle = |eg\rangle,$$

$$|\phi_4\rangle = |gee\rangle, |\phi_5\rangle = |eg\rangle, |\phi_6\rangle = |ege\rangle.$$  

The Hamiltonian in Eq. (7) can be written as

$$\hat{H} = \begin{bmatrix}
0 & \Gamma & 0 & 0 & 0 & 0 \\
0 & \Gamma & 0 & 0 & 0 & 0 \\
0 & \Gamma & 0 & 0 & 0 & 0 \\
0 & \Gamma & 0 & 0 & 0 & 0 \\
0 & 0 & \Gamma & 0 & 0 & 0 \\
0 & 0 & 0 & \Gamma & 0 & 0 \\
\end{bmatrix}.$$  

The eigenvalues of the Hamiltonian can be obtained as $E_{12} = \pm \Gamma, E_{34} = \pm \Gamma, E_{56} = \pm 2\Gamma$, and the corresponding eigenvectors are

$$|\psi_{12}\rangle = \frac{1}{2} (|\phi_1\rangle \pm |\phi_2\rangle \pm |\phi_3\rangle - |\phi_4\rangle),$$

$$|\psi_{34}\rangle = \frac{1}{2} (|\phi_1\rangle \pm |\phi_2\rangle \mp |\phi_3\rangle \mp |\phi_4\rangle + |\phi_5\rangle),$$

$$|\psi_{56}\rangle = \frac{1}{\sqrt{6}} (|\phi_1\rangle \pm |\phi_2\rangle + |\phi_3\rangle \pm |\phi_4\rangle \pm |\phi_5\rangle).$$ 

For initial system state $|\Psi(0)\rangle = \sum c_i (0) |\phi_i\rangle$, the evolving system state can be written as $|\Psi(t)\rangle = i \sum c_i(t) |\phi_i\rangle$, where the coefficients $c_i(t)$ are given by

$$c_i(t) = \frac{1}{S^{-1}} (S |\phi_i\rangle - \phi_i |\phi_i\rangle) e^{-iE_i t},$$ 

where $c(0) = [c_1(0), c_2(0), c_3(0), c_4(0), c_5(0), c_6(0)]^T$, and $S$ is the $6 \times 6$ unitary transformation matrix between eigenvectors and basis vectors.

Now we discuss the evolving system state for initial state that only one atom is excited, that is, $|\Psi(0)\rangle = |\phi_1\rangle$, which leads to $c(0) = [1, 0, 0, 0, 0, 0]^T$, we can obtain

$$c_1(t) = \frac{2}{3} \cos \Gamma t + \frac{1}{3} \cos 2\Gamma t,$$

$$c_2(t) = \frac{1}{3} \sin \Gamma t - \frac{1}{3} \sin 2\Gamma t,$$

$$c_3(t) = \frac{1}{3} \cos \Gamma t + \frac{1}{3} \cos 2\Gamma t,$$

$$c_4(t) = \frac{1}{3} \sin \Gamma t - \frac{1}{3} \sin 2\Gamma t,$$

$$c_5(t) = \frac{1}{3} \cos \Gamma t + \frac{1}{3} \cos 2\Gamma t,$$

$$c_6(t) = \frac{1}{3} \sin \Gamma t - \frac{1}{3} \sin 2\Gamma t.$$  

It should be noted that the Hamiltonian in Eq. (6) or Eq. (7) remains invariant under the permutation of atoms 1, 2 and 3. We also note that the initial state $|\Psi(0)\rangle$ has exchange symmetry for atoms 2 and 3. So, there is no doubt that $c_3(t) \equiv c_3(t)$ and $c_6(t) \equiv c_2(t)$.

Eqs. (12) lead to an resolvable analyzing of the three-atom or two-atom entanglement nature of the involving system state. The entanglement of three-particle pure states can be measured by intrinsic three-party entanglement which is defined as

$$C_{abc} = C_{a(bc)} - C_{ab}^2 - C_{ac}^2,$$

where $C_{a(bc)}$, which represents the tangle between a subsystem $a$ and the rest of the global system (denoted as $b, c$), is represented as

$$C_{a(bc)} = 4 \text{Det} \rho_a = 2(1 - \text{Tr} \rho_a^2),$$

and $C_{ab} (C_{ac})$ is the well known Concurrence that is used for entanglement measurement of qubits $a$ and $b$ ($a$ and $c$) [17]

$$C_{ab} = C(\rho_{ab}) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4),$$

(15)
where $\rho_{ab}$ is the reduced density matrix of qubits $a$ and $b$, $\lambda_1, \lambda_2, \lambda_3$ and $\lambda_4$ are four non-negative square roots of the eigenvalues of the non-Hermitian matrix $\rho_{ab}(\sigma_z \otimes \sigma_y)\rho_{ab}^*(\sigma_z \otimes \sigma_y)$ in decreasing order.

The entanglement is described in Fig. 2. The solid line represents three-atom intrinsic entanglement, the dotted line represents the entanglement of atoms 2 and 3, and the dashed line represents the tangle between atom 2 and the rest two atoms.

In most region of the time interval $(0, 2)$, the tangle between atom 2 and the rest two atoms does not alter too much. It seems that two-atom entanglement makes the largest contribution to the variety of three-atom entanglement, since the three-atom entanglement is expressed by the difference between the tangle and the two-atom entanglement [see Eq. (13)]. The peak entanglement of atoms 2 and 3 is much larger than that of atoms 1 and 2. These may suggest the following physical picture: for the initial state only one atom is excited, the interaction between distant atoms can generate strong and relatively steady entanglement shared by one atom and the rest. Also, the interaction can cause strong entanglement shared by any two atoms, while only the atoms that are initially in ground state share the largest two-atom entanglement.

In detail, two-atom entanglements $C_{12}$ and $C_{23}$ approach their maximum at $\Gamma_{11} = (2k + 1)\pi$ ($k = 0, 1, 2, 3 \cdots$), where three-atom entanglement $C_{123}$ turns out to be zero. It is clearly shown in Fig. 2 that $C_{23}$ is always larger than $C_{12}$ in the whole region. In fact, it can be analytically proved that $C_{23} = 2C_{12}$ at $\Gamma_{11}$. At $\Gamma_{23} = (2k+1)\pi + \frac{1}{4}\pi$, three-atom entanglement $C_{123}$ periodically reaches a maximum, the corresponding two-atom entanglement is zero.

At the points, the initial state evolves into the following states

$$\Psi(t) = -\frac{1}{2}(egg) + \frac{2\sqrt{2}}{3} |\Psi_{123}\rangle,$$

$$\Psi(t) = -\frac{1}{2}(egg) + \frac{\sqrt{3}}{2} |gee\rangle,$$

where $|\Psi_{123}\rangle = |g\rangle_1(|e\rangle_2 + |g\rangle_2) + |g\rangle_1 |\Psi^+\rangle_{23}$. We can name $|\Psi_{123}\rangle$ as a Bell-correlated state.

In Eq. (16), $|\Psi(t)\rangle$ is a combination of the initial state and a Bell-correlated state, also it is a W-type state. $|\Psi(t)\rangle$ are linear combinations of the initial state and a state with atomic population inverse with respect to the initial state.

The results imply possible applications in practical distant quantum communication. For example, it can be applied in the preparation of maximally entangled state of distant atoms, and thus acts as an atomic entangling gate. In this case, we assume Alice, Bob, and Charles hold atoms 1, 2, and 3 respectively.

To do this, Alice, Bob, and Charles synchronously turn off their locally applied laser fields at $t_1$. Now, they together have a W-type state $|\Psi(t)\rangle$. Then Alice performs measurement $\sigma_z$ on her atom. She finds her atom is in ground state with probability $\frac{5}{8}$, which is exactly the success probability that the atoms held by Bob and Charles are projected to Bell-state $|\Psi^+\rangle_{23}$, or she finds the system state is recovered to initial state $|egg\rangle$ with probability $\frac{1}{2}$.

The advantage of the scheme is that both Bob and Charles do not need any measurement to entangle their atoms. All the requirement, after the locally applied laser fields are turned off, is a $\sigma_z$ measurement performed by Alice. So, two-atom maximally entangled state can be generated by remote operation. Especially, the measurement performed by Alice does not damage the initial state of the global system if she failed to entangle the others’ atoms. Here, Alice can be regarded as a distant controller, and her atom turns out to be a control-qubit.

In this process, the main obstacle is the spontaneous emission of the atoms and the leakage of optical fibers.

We firstly investigate the affection of atomic spontaneous emission. The evolution of the global system is now described by the non-Hermitian conditional Hamiltonian $H_s = -i\gamma \sum_i |e\rangle_i \langle e| + \bar{H}$, where $\gamma$ denotes the atomic spontaneous emission rate.

In Fig. 3, we plot the success probability $P$ of preparing Bell-state $|\Psi^+\rangle_{23}$ with respect to time for different atomic spontaneous emission rates: $\gamma = 0.001\Gamma$, $\gamma = 0.002\Gamma$, and $\gamma = 0.01\Gamma$. The success probability is undoubtedly sensitive to the atomic spontaneous emission. The maximum probability drops to 0.881, 0.872, and 0.809 respectively. However, for any $\gamma$, the corresponding fidelity can not be affected since $c_3(t) \equiv c_3(t)$ (recall that both the Hamiltonian and the initial state remain invariant under the permutation of atom 2 and atom 3).

The dissipation of the photon leakage along optical fibers can be included in the spin-spin coupling coefficients by the exchange $e^{i\theta} \rightarrow e^{i\theta - \nu L}$, where $v$ is the decay per meter and $L$ is the length of the optical fiber between atoms $i$ and $j$. For
typical fibers \cite{18}, the decay per meter is $\nu = 0.08$. The spin-spin coupling coefficient is now about 90% of that in Eq. (6). The adiabatic approximation $\Gamma \ll J_0$ can still be fully kept. So the entangling gate still works with high fidelity.

Another dissipation is the cavity leakage. While, in the adiabatic approximation, we have assumed $\Delta \approx k \gg g$. The entanglement is then insensitive to the variety of strong leakage rate.

In summary, we have discussed the remote generation of atomic entanglement in a system contains three distant atoms for the initial state that only one atom is excited. The atoms that are initially in ground state share the largest two-atom entanglement. Two-atom entanglement turns out to be the largest contribution to the variety of three-atom entanglement. In an application of preparing entangled state of two atoms, a quantum measurement of $\sigma^z$ performed on the atom that is initially excited at typical time is required after synchronized turning off the locally applied laser fields. The success probability that two atoms are prepared in Bell-state $|\Psi^+\rangle_{23}$ can approach $\frac{1}{2}$. The distinct advantage of this scheme lies in the large detuning and large cavity leakage, that is $\Delta \approx \kappa \gg g$ which loosens the requirement of cavity dissipation. Furthermore, we show that the fidelity of the scheme is not affected by the atomic spontaneous emission. We think this scheme may work as a candidate for scalable long-distance quantum communication or one-way quantum computation \cite{3}.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{Success probability of preparing Bell-state $|\Psi^\pm\rangle_{23}$ as a function of time (in units $\pi\Gamma^{-1}$) for atomic spontaneous emission rates $\gamma = 0.001\Gamma$ (solid line), $\gamma = 0.002\Gamma$ (dotted line), and $\gamma = 0.01\Gamma$ (dashed line).}
\end{figure}

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