A criterion for identifying a mixed-mode I/II hydraulic fracture crossing a natural fracture in the subsurface

Yijin Zeng1,2, Wan Cheng3, Xu Zhang1,2 and Bo Xiao1,2

Abstract

Hydraulic fracturing has been proven to be an effective technique for stimulating petroleum reservoirs. During the hydraulic fracturing process, the effects of the natural fracture, perforation orientation, stress reorientation, etc. lead to the production of a non-planar, mixed-mode I/II hydraulic fracture. In this paper, a criterion for a mixed-mode I/II hydraulic fracture crossing a natural fracture was first proposed based on the stress field around the hydraulic and natural fractures. When the compound degree \((K_{II}/K_I)\) approaches zero, this criterion can be simplified to identify a pure mode I hydraulic fracture crossing a natural fracture. A series of true triaxial fracturing tests were conducted to investigate the influences of natural fracture occurrence and in situ stress on hydraulic fracture propagation. These experimental results agree with the predictions of the proposed criterion.

Keywords

Hydraulic fracturing, natural fracture, stress interference, crossing criterion, reservoir stimulation

Introduction

Hydraulic fracturing is commonly used in low-permeability reservoir stimulation. The object of the fracturing process is to obtain single or multiple cracks or even fracture

1State Key Laboratory of Shale Oil and Gas Enrichment Mechanisms and Effective Development, Beijing, China
2SINOPEC Research Institute of Petroleum Engineering, Beijing, China
3Faculty of Engineering, China University of Geosciences, Wuhan, China

Corresponding author:
Wan Cheng, Faculty of Engineering, China University of Geosciences, Wuhan, China.
Email: chengwancup@163.com

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networks with sufficient flow conductivity. One of the mechanisms causing hydraulic fracture complexity is the interaction between hydraulic and natural fractures (Cordero et al., 2019).

Whether a hydraulic fracture can cross a natural fracture is affected by many factors. The experiments conducted by Norman and Jessen (1963) and Daneshy (1974) proved that the strength of the weak plane and the principal stress differences have important effects on the interaction between hydraulic and natural fractures. Wang’s experiments (Wang et al., 2018) also showed that hydraulic fracture crossing/diverting behavior depends on the approach angle and the thickness of natural fracture. To predict whether a natural fracture opens or shear slippage occurs (Blanton, 1982), a variety of criteria have been proposed, such as the Warpinski criterion (Warpinski and Teufel, 1987) and the Zhou criterion (Zhou et al., 2008; Zhou and Xue, 2011). These criteria were used to investigate the interaction between hydraulic and natural fractures (Cheng et al., 2017; Gu et al., 2012). Renshaw and Pollard (1995) proposed a simple criterion for predicting whether a pure mode I fracture will cross an unbounded frictional interface orthogonal to the approaching fracture in brittle linear elastic materials. The Renshaw’s criterion (Renshaw and Pollard, 1995) was extended by Gu et al. (2012) to describe a pure mode I hydraulic fracture crossing a natural fracture with cohesion at a non-orthogonal intersection angle. All these criteria and experiments focused on the interaction between two vertical planes, which meant that the strike angle was considered but the dip angle was neglected. Hence, Cheng et al. (2014) developed a new criterion for identifying a mode I hydraulic fracture crossing a natural fracture with arbitrary occurrence in three-dimensional space and proved its validity via true triaxial fracturing testing. Cheng’s criterion (2014) was used in simulating fracture propagation in the naturally fractured reservoirs (Li et al., 2017). Zhao et al. (2019) proposed a new composite criterion through coupling the fluid flow and rock elastic deformation. This new criterion was validated by the published intersection criteria and experiments (Cheng et al., 2014; Gu et al., 2012).

However, all the above criteria are suitable to determine the interaction between a natural fracture and a mode I hydraulic fracture. The influences of natural fractures, perforation orientation, stress interference (Guo et al, 2019a), faults, etc. result in a non-planar hydraulic fracture geometry. The shear stress component induced by the in situ stresses acts on the inclined fracture surface and results in a mode II stress intensity factor ($K_{II}$) on the hydraulic fracture tip. The fluid pressure inside the hydraulic fracture results in a mode I stress intensity factor ($K_I$) at the hydraulic fracture tip. Hence, a non-planar hydraulic fracture generally belongs to a mixed-mode I/II fracture. In this study, a criterion for a mixed-mode I/II hydraulic fracture crossing a natural fracture was first proposed based on the stress field around the hydraulic fracture tip and the natural fracture. Additionally, a series of true tri-axial fracturing tests were conducted to validate the proposed criterion.

**Intersection criterion of hydraulic and natural fractures**

**Mixed-mode I/II hydraulic fracture**

A hydraulic fracture in a reservoir generally deviates from a plane because of many factors, such as stress interference during multistage fracturing (Guo et al., 2019b), discontinuities in naturally fractured reservoirs, and the use of oriented perforations in the borehole.
The shear stress induced by the remote in situ stresses that act along the non-planar hydraulic fracture can result in a non-zero $K_{II}$ at its tip, while the pressure of fracturing fluid inside the hydraulic fracture contributes to the $K_I$ at its tip. Hence, an inclined hydraulic fracture tip generally reflects a mixed-mode I/II fracture according to fracture mechanics. It is assumed that a natural fracture and hydraulic fracture are vertically distributed in a reservoir and that the propagating hydraulic fracture bends to meet the natural fracture at an arbitrary orientation (Figure 1). The horizontal plane satisfies the plane-strain condition. The angle measured anticlockwise from the natural fracture to the maximum horizontal stress is denoted as $\beta_N$. The angle measured anticlockwise from the propagation direction of the hydraulic fracture to the maximum horizontal stress is denoted as $\beta_H$. The angle between the natural fracture and hydraulic fracture tip is defined as the approaching angle

$$\theta = \beta_N - \beta_H$$

The normal and shear stress components, induced by the remote in situ stress, that act along the natural fracture are expressed by

$$
\begin{align*}
\sigma_{rN} &= \sigma_h \cos^2 \beta_N + \sigma_H \sin^2 \beta_N \\
\tau_{rN} &= -\frac{1}{2} (\sigma_H - \sigma_h) \sin 2\beta_N
\end{align*}
$$

Similarly, the normal and shear stress components at the hydraulic fracture tip are given by

$$
\begin{align*}
\sigma_{rH} &= \sigma_h \cos^2 \beta_H + \sigma_H \sin^2 \beta_H \\
\tau_{rH} &= -\frac{1}{2} (\sigma_H - \sigma_h) \sin 2\beta_H
\end{align*}
$$

**Figure 1.** Sketch of a non-planar hydraulic fracture contacting a natural fracture.
The combined stress field of the remote in situ stresses and hydraulic fracture tip stresses is

\[
\begin{align*}
\tau_{xy} & = -\frac{1}{2} (\sigma_H - \sigma_h) \sin 2\beta_H + \frac{K_1}{\sqrt{2\pi r}} B(\vartheta) + \frac{K_{II}}{\sqrt{2\pi r}} \left[ \cos \frac{\vartheta}{2} - A(\vartheta) \right] \\
\tau_{x} & = \sigma_h \cos \beta_H^2 + \sigma_H \sin \beta_H^2 + \frac{K_1}{2\pi r} \left[ \cos \frac{\vartheta}{2} - A(\vartheta) \right] - \frac{K_{II}}{2\pi r} \left[ 2\sin \frac{\vartheta}{2} + B(\vartheta) \right] \\
\sigma_y & = \sigma_h \cos \beta_H^2 + \sigma_H \sin \beta_H^2 + \frac{K_1}{\sqrt{2\pi r}} \left[ \cos \frac{\vartheta}{2} + A(\vartheta) \right] + \frac{K_{II}}{\sqrt{2\pi r}} B(\vartheta) \\
\tau_{y} & = -\frac{1}{2} (\sigma_H - \sigma_h) \sin 2\beta_H + \frac{K_1}{\sqrt{2\pi r}} B(\vartheta) + \frac{K_{II}}{\sqrt{2\pi r}} \left[ \cos \frac{\vartheta}{2} - A(\vartheta) \right]
\end{align*}
\]

where \( A(\vartheta) = \sin^{\frac{\vartheta}{2}} \cos^{\frac{\vartheta}{2}} \sin^{3\frac{\vartheta}{2}}; B(\vartheta) = \sin^{\frac{\vartheta}{2}} \cos^{\frac{\vartheta}{2}} \cos^{3\frac{\vartheta}{2}} \).

The maximum hoop stress criterion indicates that the fracture propagates along the direction perpendicular to the maximum tensile stress. The propagation angle is expressed as

\[
\vartheta_0 = \begin{cases} 
\begin{align*}
& f(\Omega) = \arccos \frac{3\Omega^2 + \sqrt{1 + 8\Omega^2}}{1 + 9\Omega^2}, & & \Omega = \frac{K_{II}}{K_1}, & & K_1 \neq 0 \\
& 70.6^\circ, & & K_1 = 0
\end{align*}
\end{cases}
\]

The equivalent stress intensity factor at the fracture tip is defined as

\[
K_e = \frac{1}{2} \cos \frac{\vartheta_0}{2} \left[ K_1 (1 + \cos \vartheta_0) - 3K_{II} \sin \vartheta_0 \right]
\]

Fracture mechanics assumes that materials yield in the fracture process zone \((r < r_c)\), within which stresses mainly depend on the micro-mechanical deformation mechanisms acting at the tip. The stresses within the process zone region do not surpass the stresses at \(r = r_c\). The maximum hoop stress of a mixed-mode I/II hydraulic fracture tip occurs at \(\vartheta = \vartheta_0\) and is expressed as

\[
\sigma_\vartheta |_{\vartheta = \vartheta_0} = \frac{K_e}{\sqrt{2\pi r_c}} + S_R
\]

where \( S_R = \sigma_h \cos^2 (\vartheta_0 - \beta_H) + \sigma_H \sin^2 (\vartheta_0 - \beta_H) \).

There are two conditions that need to be met for a hydraulic fracture to cross a natural fracture: (1) the maximum hoop stress at the hydraulic fracture tip must equal the tensile strength of the rock on the opposite side of the natural fracture

\[
\sigma_\vartheta |_{\vartheta = \vartheta_0} = T_0
\]

(2) There is no shear slippage at the natural fracture surface

\[
|\tau_\vartheta| + \mu \sigma_\vartheta - \tau_0 < 0
\]
Substituting equation (7) into equation (8) yields

$$\sqrt{2\pi r_c} = \frac{K_c}{T_0 - S_R}$$  \hspace{1cm} (10)

Substituting equation (10) into the polar coordinate stress field of the hydraulic fracture tip, the normal and shear stress components acting on the right wing ($\theta = -\theta$) of the natural fracture due to the hydraulic fracture tip stress field can be expressed as

$$\begin{align*}
\sigma_{\text{tip,-}} &= \frac{K_1}{2K_c} (T_0 - S_R) (1 + \cos\theta + 3\Omega \sin\theta) \cos \frac{\theta}{2} \\
\tau_{\text{tip,-}} &= \frac{K_1}{2K_c} (T_0 - S_R) [-\sin\theta + \Omega (3\cos\theta - 1)] \cos \frac{\theta}{2}
\end{align*}$$  \hspace{1cm} (11)

Similarly, the normal and shear stress components acting on the left wing ($\theta = \pi - \theta$) of the natural fracture due to the hydraulic fracture tip stress field can be written as

$$\begin{align*}
\sigma_{\text{tip,\pi-}} &= \frac{K_1}{2K_c} (T_0 - S_R) (1 - \cos\theta - 3\Omega \sin\theta) \sin \frac{\theta}{2} \\
\tau_{\text{tip,\pi-}} &= \frac{K_1}{2K_c} (T_0 - S_R) [\sin\theta - \Omega (3\cos\theta + 1)] \sin \frac{\theta}{2}
\end{align*}$$  \hspace{1cm} (12)

Superposing equations (11) and (2), both the total normal and shear stresses at the right wing of the natural fracture can be obtained by

$$\begin{align*}
\sigma_{-\theta} &= \frac{K_1}{2K_c} (T_0 - S_R) (1 + \cos\theta + 3\Omega \sin\theta) \cos \frac{\theta}{2} + \sigma_h \cos^2 \beta_N + \sigma_H \sin^2 \beta_N \\
\tau_{-\theta} &= \frac{K_1}{2K_c} (T_0 - S_R) [-\sin\theta + \Omega (3\cos\theta - 1)] \cos \frac{\theta}{2} - \frac{1}{2} (\sigma_H - \sigma_h) \sin 2\beta_N
\end{align*}$$  \hspace{1cm} (13)

Similarly, superposing equations (12) and (2), both the total normal and shear stresses at the left wing of the natural fracture can be given by

$$\begin{align*}
\sigma_{\pi-\theta} &= \frac{K_1}{2K_c} (T_0 - S_R) (1 - \cos\theta - 3\Omega \sin\theta) \sin \frac{\theta}{2} + \sigma_h \cos^2 \beta_N + \sigma_H \sin^2 \beta_N \\
\tau_{\pi-\theta} &= \frac{K_1}{2K_c} (T_0 - S_R) [\sin\theta - \Omega (3\cos\theta + 1)] \sin \frac{\theta}{2} - \frac{1}{2} (\sigma_H - \sigma_h) \sin 2\beta_N
\end{align*}$$  \hspace{1cm} (14)

A mixed-mode I/II hydraulic fracture will cross the natural fracture if both equations (13) and (14) satisfy equation (9). Hence, equations (9), (13), and (14) constitute a new criterion for identifying a mixed-mode I/II hydraulic fracture crossing a natural fracture.

**Pure mode I hydraulic fracture**

It is easy to find that the compound degree ($\Omega$) of a mixed-mode I/II fracture significantly affects the total normal and shear stresses that act on the natural fracture, which
predominantly determines whether the natural fracture slips. If the compound degree decreases to zero, this mixed-mode I/II fracture intersection criterion will be simplified to a mode I fracture intersection criterion. Below is the proof

\[
\lim_{\Omega \to 0} \vartheta_0 = \lim_{\Omega \to 0} f(0) = 0
\]  

Substituting equations (1) and (15) into equations (13) and (14), respectively, yield the following expressions

\[
\begin{align*}
\lim_{\Omega \to 0} \sigma_{\pi - \theta} &= (T_0 - \sigma_h \cos^2 \beta_H - \sigma_H \sin^2 \beta_H) \cos^3 \frac{\beta_N - \beta_H}{2} + \sigma_h \cos^2 \beta_N + \sigma_H \sin^2 \beta_N \\
\lim_{\Omega \to 0} \tau_{\pi - \theta} &= -(T_0 - \sigma_h \cos^2 \beta_H - \sigma_H \sin^2 \beta_H) \sin \frac{\beta_N - \beta_H}{2} \cos \frac{\beta_N - \beta_H}{2} - \frac{1}{2} (\sigma_H - \sigma_h) \sin 2 \beta_N \\
\end{align*}
\]

(16)

If the hydraulic fracture tip is orthogonal to the minimum in situ stress for some length after deflection, i.e. \( \beta_H = 0^\circ \), equations (16) and (17) are rewritten as

\[
\begin{align*}
\sigma_{\pi - \theta} &= (T_0 - \sigma_h \cos^2 \beta_H - \sigma_H \sin^2 \beta_H) \sin \frac{\beta_N - \beta_H}{2} + \sigma_h \cos^2 \beta_N + \sigma_H \sin^2 \beta_N \\
\tau_{\pi - \theta} &= -(T_0 - \sigma_h \sin^2 \beta_H) \sin \frac{\beta_N - \beta_H}{2} \cos \frac{\beta_N - \beta_H}{2} - \frac{1}{2} (\sigma_H - \sigma_h) \sin 2 \beta_N \\
\end{align*}
\]

(18)

As shown in Figure 2, if a hydraulic fracture propagates along the direction orthogonal to the minimum in situ stress, \( K_H = 0 \) and \( \beta_H = 0 \). Equations (13) and (14) will be reduced into equations (18) and (19), respectively. If equations (18) and (19) satisfy equation (9), the mode I hydraulic fracture will cross the natural fracture. Otherwise, the mode I hydraulic fracture will not cross the natural fracture. Therefore, equations (9), (18), and (19) constitute a new criterion for identifying a mode I hydraulic fracture crossing a natural fracture. Cheng et al. (2014) developed a new criterion for identifying a mode I hydraulic fracture crossing a natural fracture with arbitrary occurrence (including an arbitrary strike and dip angle) in three-dimensional space. Cheng's (2014) criterion is also suitable to identify the interaction between a mode I hydraulic fracture and a vertical
Assuming $\alpha_N = 90^\circ$, Cheng’s criterion is the same as this criterion. When the cohesion and tensile strength along the natural fracture both equal zero, this criterion is similar to the Gu’s criterion (Gu et al., 2012). At different approaching angles, whether a hydraulic fracture crosses a natural fracture is described in Figure 3 in terms of two regions divided by a boundary line. In the upper right region of this plot, the hydraulic fracture will cross the natural fracture. In the lower left region of this plot, the hydraulic fracture will not cross the natural fracture. The interaction between hydraulic and natural fractures is usually divided into three types: crossing, arrest, and deflection. Arrest and
deflection will occur in the lower left region. In addition, if \( \beta_H = 90^\circ \), i.e. \( \theta = 90^\circ \), this criterion will be simplified to

\[
\frac{\sigma_H}{\sigma_h} > 0.3536 \frac{1 + \mu}{\mu}
\]  

(20)

Again, equation (20) is similar to the Renshaw’s (Renshaw and Pollard, 1995) criterion. The comparison between this new criterion and Renshaw’s (Renshaw and Pollard, 1995) criterion and Gu’s (2012) criterion indicates that there are some minor differences between the three curves (Figure 4) due to the intersection condition of equation (8). The tensile stress at \( \varphi = 90^\circ \) is compared with the tensile strength of the material in the Renshaw criterion, which is not reasonable because this direction is rotated \( 90^\circ \) from the fracture tip propagation direction. The principal stress at the fracture tip is compared with the tensile strength of the material in the Gu’s (2012) criterion. The hoop stress at \( \varphi = 90^\circ \) is compared with the tensile strength of rock in equation (8), which is believed to be more reasonable than the Gu and Renshaw criteria.

**Experimental validation**

**Experimental setup**

The experiments were conducted in a true triaxial hydraulic fracturing system. A cubic specimen was positioned between steel platens, and the stress states were controlled by three pumps and a hydraulic voltage stabilizer. The three confining stresses remain stable during the experimental period. The hydraulic fluid injection pressure can reach 140 MPa. A gel solution with tracer (yellow–green powder) was used as the fracturing fluid in these
experiments. Its flow rate and viscosity were 0.326 ml/s and 16.5 mPa s (600 r/min), respectively, in all the experiments.

**Specimen preparation**

The experimental specimen was a cubic concrete block with dimensions of 300 mm × 300 mm × 300 mm, as shown in Figure 5(a). Cement and quartz sand (40–80 mesh) were mixed in a mass ratio of 1:1. The water–cement mass ratio is 0.312. The fracturing fluid was injected into a casted central pipe (simulated wellbore), which had an outer diameter of 16 mm and an inner diameter of 10 mm. The length of the open hole was 20 mm, and the depth of the hole was 140 mm. A piece of paper (the yellow plane in Figure 5(a)) with dimensions of 200 mm × 150 mm × 0.1 mm was vertically positioned to simulate a planar natural fracture in the specimen. Another piece of paper (the blue plane in Figure 5(a)) with dimensions of 100 mm × 150 mm × 0.1 mm was symmetrically and vertically placed around the two sides of borehole, which was expected to produce a mixed-mode I/II hydraulic fracture if hydraulic fracture propagates along this pre-existing fracture. After many times of the rock mechanics test, the average mechanical properties of the specimen are: the elastic modulus is 8.4 GPa; Poisson’s ratio is 0.23; the uniaxial compressive strength is 28.34 MPa, the cohesion of the rock matrix is 2.85 MPa; the uniaxial tensile strength is 2.55 MPa; the frictional coefficient of the rock matrix is 0.75; the cohesion and tensile strength along the pre-existing fracture are both zero; the friction coefficient of the pre-existing fracture is 0.65. The confining stresses satisfy the condition of \( \sigma_v > \sigma_H > \sigma_h \). In this paper, the pre-existing fracture used to simulate the natural fracture is denoted as PF–NF, and that used to simulate the hydraulic fracture is denoted as PF–HF. The simplified schematic of the rock specimen is given in Figure 5(b).

**Experimental scheme and analyses**

Table 1 gives the experimental parameters and results. The results of the specimens in group 1 were from Zhou et al.’s (2008) paper, and results of the specimens in group 2 were from Gu’s (2012) paper. “C” represents crossing, while “A” represents no-crossing.
The hydraulic fracture initiates and propagates along the pre-existing fracture, which is symmetrically positioned along the borehole. $K_I$ and $K_{II}$ can be obtained by

$$
K_I = \frac{P_w - \sigma_h \cos^2 \beta_H - \sigma_H \sin^2 \beta_H}{\sqrt{\pi a}} \sqrt{\pi a}
$$

$$
K_{II} = -\frac{1}{2} (\sigma_H - \sigma_h) \sin^2 \beta_H \sqrt{\pi a}
$$

If $K_e > K_{IC}$, the hydraulic fracture will initiate. If the boundary conditions satisfy the mixed-mode I/II fracture intersection criterion, the interaction between the hydraulic and natural fractures is predicted. The interaction between the hydraulic and pre-existing fractures is also divided into three types: crossing, arrest, and deflection. In contrast to crossing, it is difficult to distinguish arrest from deflection in these experiments because fracturing liquid generally leaks into the natural fracture for these two cases. However, they both result in the same phenomenon, i.e. no crossing. As shown in Figure 6, the PF–HF crossed the PF–NF as a vertical plane, and it deflected continuously toward the direction orthogonal to $\sigma_h$. The predictions by this criterion agree well with the experimental results listed in Table 1.

Table 1. Experimental parameters and results (all the specimens satisfy $\alpha_N = 90^\circ$).

| Group | Test | $\beta_N$ ($^\circ$) | $\beta_H$ ($^\circ$) | $\theta$ ($^\circ$) | $\sigma_N$ (MPa) | $\sigma_H$ (MPa) | $\sigma_h$ (MPa) | $\Delta \sigma$ (MPa) | Test result | This criterion |
|-------|------|----------------------|----------------------|---------------------|----------------|----------------|----------------|----------------|--------------|---------------|
| 1a    | 1    | 90                   | 90                   | 20                  | 10             | 5              | 5              | A              | A            |               |
|       | 2    | 30                   | 30                   | 20                  | 10             | 5              | 5              | A              | A            |               |
|       | 3    | 90                   | 90                   | 20                  | 10             | 3              | 7              | C              | C            |               |
|       | 4    | 60                   | 60                   | 20                  | 10             | 3              | 7              | C              | A            |               |
|       | 5    | 30                   | 30                   | 20                  | 8              | 5              | 3              | A              | A            |               |
|       | 6    | 60                   | 60                   | 20                  | 13             | 3              | 10             | C              | C            |               |
|       | 7    | 30                   | 30                   | 20                  | 13             | 3              | 10             | A              | A            |               |
|       | 8    | 60                   | 60                   | 20                  | 8              | 5              | 3              | A              | A            |               |
| 2b    | 9    | 90                   | 90                   | 27.7                | 13.8           | 6.9            | 6.9            | C              | C            |               |
|       | 10   | 90                   | 90                   | 27.7                | 7.6            | 6.9            | 0.7            | A              | A            |               |
|       | 11   | 75                   | 75                   | 27.7                | 17.6           | 6.9            | 10.7           | C              | C            |               |
|       | 12   | 75                   | 75                   | 27.7                | 8.3            | 6.9            | 1.4            | A              | A            |               |
|       | 13   | 45                   | 45                   | 27.7                | 17.2           | 6.9            | 10.3           | A              | A            |               |
|       | 14   | 45                   | 45                   | 27.7                | 8.3            | 6.9            | 1.4            | A              | A            |               |
| 3     | 15   | 90                   | 90                   | 20                  | 14             | 4              | 10             | C              | C            |               |
|       | 16   | 90                   | 15                   | 75                  | 20             | 14             | 4              | 10             | C            | C            |
|       | 17   | 90                   | 30                   | 60                  | 20             | 14             | 4              | 10             | C            | A            |
|       | 18   | 90                   | 45                   | 45                  | 20             | 14             | 4              | 10             | A            | A            |

*After Zhou et al. (2008).

*After Gu et al. (2012).*
Discussion

In a naturally fractured reservoir or in unconventional oil and gas reservoirs, such as shale gas, shale oil, and coalbed methane reservoirs, the interaction between hydraulic and natural fractures is rather complex and remains poorly understood. Thus, it is critically important to evaluate the influence of natural fractures on the initiation and propagation of hydraulic fractures for the design and diagnosis of hydraulic fracturing treatments to maximize the productivity of reservoirs. The observed phenomena (such as crossing and no-crossing) or objective criteria (such as the mixed-mode I/II and mode I fracture intersection criteria) are also expected to contribute to the understanding and prediction of the interactions between a propagating hydraulic fracture and other discontinuities (bedding plane, fault, cleat, etc.) in the subsurface (Huang and Liu, 2018; Jiang and Cheng, 2018). A natural fracture network is composed of many natural fractures, and every natural fracture can independently affect the hydraulic fracture. Thus, this criterion can also be useful to understand the formation of complex fracture networks by hydraulic fracturing and can be applied to reservoir simulation after some scaling transformation.
When $\Omega \to 0$, the mixed-mode I/II fracture intersection criterion will be simplified to be a pure mode I fracture intersection criterion, i.e. the latter criterion is a particular case of the former. If a hydraulic fracture is a mode II fracture: $\vartheta_0 = 70.6^\circ$. This hydraulic fracture will become a mixed-mode I/II fracture quickly after it starts to propagate. Hence, this article does not deduce the criterion for a mode II hydraulic fracture crossing the natural fracture. The criterion in this paper can be used to judge whether a mixed-mode I/II or a pure mode I hydraulic fracture crosses a natural fracture. It can also be used to investigate the effects of in situ stresses, natural fracture occurrence, etc. on the initiation and propagation of a hydraulic fracture in a naturally fractured formation and to predict the formation of fracture networks. The applicability of this criterion is much broader than that of Gu’s criterion (2012) and Cheng’s criterion (2014).

Future work is required to propose a criterion to identify a mixed-mode I/II hydraulic fracture crossing a natural fracture with arbitrary occurrence in three-dimensional space, since all the fractures in the solid material are three-dimensional fractures to some extent. Furthermore, the mechanical properties, such as roughness and aperture, of a pre-existing fracture are generally different from those of a natural fracture, fault, bedding plane, etc., so more work is required to determine their differences.

Conclusions

This paper studies the mechanical interaction between a mixed-mode I/II hydraulic fracture and a natural fracture. Some interesting findings are listed as follows:

1. A new criterion for identifying a mixed-mode I/II hydraulic fracture crossing a natural fracture was proposed. This criterion can be simplified into a criterion for a mode I hydraulic fracture crossing a natural fracture. The experiments agreed well with the predictions of this criterion. The applicability of this criterion is much broader than that of Gu’s criterion and Cheng’s criterion.

2. This criterion is expected to contribute to the understanding and prediction of the interactions between a propagating hydraulic fracture and other discontinuities in the subsurface, although more work is required to research the differences between these discontinuities. Future work is required to propose a criterion to identify a mixed-mode I/II hydraulic fracture crossing a natural fracture with arbitrary occurrence in three-dimensional space.

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Appendix

Notation

- $r_c$: the radius of the process zone around the fracture tip (m)
- $K_e$: equivalent stress intensity factor at the fracture tip (MPa m$^{0.5}$)
- $K_n$: horizontal stress difference coefficient, dimensionless
- $K_1$: stress intensity factor of mode I fracture (MPa m$^{0.5}$)
- $K_{II}$: stress intensity factor of mode II fracture (MPa m$^{0.5}$)
- $r, \theta$: polar coordinate system with the origin at the fracture tip
- $T_0$: tensile strength of the rock (MPa)
- $\alpha_N$: dip angle of the natural fracture
- $\Delta \sigma$: horizontal stress difference (MPa)
- $\theta$: approaching angle (rad)
- $\mu$: natural fracture surface friction coefficient, dimensionless
- $\sigma_H$: horizontal maximum principal stress (MPa)
- $\sigma_h$: horizontal minimum principal stress (MPa)
- $\sigma_{tH}, \tau_{tH}$: normal and shear stress components acting on the hydraulic fracture induced by the remote stress state (MPa)
- $\sigma_{tN}, \tau_{tN}$: normal and shear stress components acting on the natural fracture induced by the remote stress state (MPa)
- $\sigma_{tip,\theta}, \sigma_{tip,\pi-\theta}$: normal stress components acting at the right and left wings of the natural fracture surface when the approaching angle is $\theta$ (MPa)
- $\sigma_V$: vertical principal stress (MPa)
- $\sigma_x, \sigma_y$: $x$ and $y$ stress components acting at the hydraulic fracture tip (MPa)
- $\sigma_\theta$: total normal stress acting on the natural fracture surface (MPa)
- $\tau_{tip,\theta}, \tau_{tip,\pi-\theta}$: shear stress components acting at the right and left wings of the natural fracture surface when the approaching angle is $\theta$ (MPa)
- $\tau_{xy}$: shear stress component acting at the hydraulic fracture tip (MPa)
- $\tau_0$: cohesion of natural fracture surface (MPa)
- $\tau_\theta$: total shear stress acting on the natural fracture surface (MPa)
- $\psi_0$: propagation angle of a mixed mode I/II fracture (°)
- $\Omega$: compound degree of hydraulic fracture, dimensionless