Mathematical model operation of an electric field in the nonuniform mediums at intubation by a direct current

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Abstract. Problems of electroinvestigation are solved for determination of parameters of geophysical structures due to comparison of results of the taken measurements of particular type of physical fields with results of their model operation. If application of electromagnetic fields takes place, then mathematical problem definition consists in searching of coefficients of Maxwell’s equations and is the inverse task. In this work are considered a problem of restitution of electromagnetic parameters of the nonuniform mediums, by results of intubation by a direct current. The modelled fields are described by the system of differential Maxwell’s equations. Tasks are considered in a 3-dimensional approximation. The purpose of work is the solution of a direct task, creation of a program complex for electroinvestigation problem solving and also development of a unique algorithm for the solution of the inverse task, but a basis of a program complex for the solution of a direct task. In work the original algorithm of the solution of the inverse problem of electroinvestigation is constructed by a direct current. For the solution of a direct task the finite element method was used. On the basis of the solution of a direct task the algorithm of searching of electrophysical properties of the nonuniform mediums is developed. For the solution of this task the heuristic algorithm allowing to reduce first of all time of searching of the decision was used. When testing of the developed algorithm it was noted that accuracy of results also is acceptable.

1. Introduction

The task of electrical exploration research in the [1-18]. The purpose of solving geo-prospecting problems is to determine the geometric and physical parameters of different media by comparing the measurement data of some type of physical fields with the corresponding results of their modeling. In the case of applying electromagnetic fields, the mathematical formulation consists in finding the coefficients of the Maxwell equations with respect to some indirect data and refers to the classical inverse problems studied in numerous fundamental works, for example, Matveev B. K. and Zhdanov M.S [8], [9].

One of the most well-known approaches to solving inverse problems of restoring the parameters of the environment model is based on optimization principles that reduce to minimizing the target functional of the deviation of the measured and calculated quantities in the presence of linear or nonlinear constraints on the theoretical values of the environmental parameters as [10], [11]. Thus, the search for the solution of the inverse problem reduces to a multiple solution of the direct problem, which
makes the solution of this problem sufficiently resource-intensive. The peculiarity of these tasks is a large number of variables that are multilevel. In this paper, we consider the distribution of the electric field for solving direct and inverse problems of electromagnetic reconnaissance, characterized by the location of sources and measuring instruments on the surface of the earth. Electrostatic or electromagnetic fields are investigated in different frequency ranges. The simulated fields are described by Maxwell's system of differential equations. The solvable problems for multilayer media are considered in three-dimensional approximation. The goal of research in this area is to determine the geometric characteristics and conductive properties of geophysical structures in the search for minerals.

2. Mathematical formulation of a direct problem.

At points A and B, two electrodes are located on the surface of the earth. To these electrodes let us draw a current with a value $I_k$ and $-I_k$ accordingly. It is required to find the distribution of the electrostatic potential in a given region.

Electromagnetic fields, described by the system of Maxwell's equations are investigated in various applications in papers [6], [7], [15], [16]. In the case of time-constant fields and the absence of volumetric sources, the electrostatic potential in the computational domain $\Omega$ is determined by the equation

$$L\phi = \nabla \sigma \nabla \phi(x) = 0, \quad x \in \Omega$$

where the differential operator $L$ is described in cylindrical coordinates for axisymmetric problems and in Cartesian coordinates for general three-dimensional statements [11]. In the presence of anisotropic media with different conductivities $\sigma_x$, $\sigma_y$, $\sigma_z$ in the directions of the Cartesian axes, we have

$$L\phi = \frac{\partial}{\partial x} \sigma_x \frac{\partial \phi}{\partial x} + \frac{\partial}{\partial y} \sigma_y \frac{\partial \phi}{\partial y} + \frac{\partial}{\partial z} \sigma_z \frac{\partial \phi}{\partial z} = 0$$

To solve this problem, we use the finite element method using the Bubnov-Galerkin method. We rewrite our equation in the following form

$$\int_{\Omega} [N]^T \left( \sigma_x \frac{\partial \phi}{\partial x} + \sigma_y \frac{\partial \phi}{\partial y} + \sigma_z \frac{\partial \phi}{\partial z} \right) d\Omega = 0$$

First of all, it is necessary to transform an equation containing only the first derivatives with respect to x and z. Let us show, for example, a term with a variable x. We use the Green formula

$$\int_{\Omega} \frac{\partial}{\partial x} \sigma_x [N]^T \frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x} \left( [N]^T \sigma_x \frac{\partial \phi}{\partial x} \right) - \int_{\partial \Omega} \sigma_x \frac{\partial \phi}{\partial x} dS$$

Then the first term is transformed to the form

$$\int_{\Omega} [N]^T \frac{\partial}{\partial x} \sigma_x \frac{\partial \phi}{\partial x} d\Omega = \int_{\partial \Omega} \sigma_x \frac{\partial \phi}{\partial x} dS$$

We use the Ostragradsky-Gauss formula. We get

$$\int_{\Omega} \frac{\partial}{\partial x} \left( [N]^T \sigma_x \frac{\partial \phi}{\partial x} \right) d\Omega = \int_{\partial \Omega} [N]^T \sigma_x \frac{\partial \phi}{\partial x} l_x dS$$
Then, having done the same for the remaining terms of the initial equation, we obtain the following expression

$$\int S \left[ N^T \left( \sigma_x \frac{\partial \phi}{\partial x} l_x + \sigma_y \frac{\partial \phi}{\partial y} l_y + \sigma_z \frac{\partial \phi}{\partial z} l_z \right) dS - \int \partial [N^T \sigma_x \frac{\partial \phi}{\partial n} + \partial [N^T \sigma_y \frac{\partial \phi}{\partial n} + \partial [N^T \sigma_z \frac{\partial \phi}{\partial n} d\Omega = 0 \right)$$

in which the integral over the surface is expressed in terms of the derivative along the normal $\frac{\partial \phi}{\partial n}$

$$\int S \left( \frac{\partial [N^T \sigma_x \frac{\partial \phi}{\partial x}}{\partial x} + \frac{\partial [N^T \sigma_y \frac{\partial \phi}{\partial y} \right) d\Omega = \int S [N^T \frac{\partial \phi}{\partial n} dS = 0 \right)$$

The first integral in the resulting expression contributes to the stiffness matrix, the second integral contributes to the column vector of the free terms. An unknown function $\phi$ is defined by the relation.

$$\phi = [N][\Phi]$$

3. Inverse task

The inverse problem can be formulated as an optimization problem with the objective functional

$$F(\sigma_1, ..., \sigma_n) = \sqrt{\sum_{i=1}^{n} (\phi_i - \varphi_i(\sigma_1, ..., \sigma_n))^2} \rightarrow \min$$

Optimization methods for solving inverse problems are applied in various fields [1], [2], [3]. To solve the problem, we use a heuristic algorithm. The essence of the algorithm is in the following: at the first step it is assumed that the computational domain contains one layer with a specific resistance $\sigma$, we obtain a solution for a given computational domain. Further, the calculated region is arbitrarily divided into two parts with a resistivity $\sigma_1$, $\sigma_2$, then we solve the inverse problem for a given computational domain. Further similarly continue to break the calculated area into subregions with specific resistance $\sigma_1$, $\sigma_2$, ..., $\sigma_n$.

The problem of optimization will be solved by the Hook-Jeeves method, and also by the method of complete search. The criterion for stopping the optimization is the difference between the resistivity values at neighboring steps. To stop, the difference is less $\delta = 0.001$. The criterion for stopping the search for the solution of the inverse problem is the difference between the theoretical and experimental values of the potential at the nodal points. If the difference is less $\epsilon = 0.0001$, then the search for the environment parameters is stopped. If the minimum difference between the theoretical and the experimental does not change significantly during the 10 steps, stop searching for the inverse problem solution, and start the algorithm execution first. This algorithm is based on the analysis of existing optimization methods for solving inverse geo-prospecting problems [13-18]. The program implementation was carried out using existing methods [4], [5].

4. Conclusions

The algorithm developed as a result of the work allows solving direct problems of electric probing by the finite element method, and also allows to study inhomogeneous media with different arrangement of layers. In the course of solving the inverse problem, a new heuristic algorithm was developed that allows one to recover the parameters of the medium with high accuracy from the experimental data obtained in solving the direct problem. It is assumed that this method will reduce the search time for environmental parameters.

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