Stable and Metastable Cosmic Strings in Heterotic M-Theory

Evgeny I. Buchbinder

School of Natural Sciences, Institute for Advanced Study
Einstein Drive, Princeton, NJ 08540

Abstract

We address the question of finding stable and metastable cosmic strings in quasi-realistic heterotic M-theory compactifications with stabilized moduli and de Sitter vacua. According to Polchinski’s conjecture, the only stable strings in the absence of massless fields are Aharonov-Bohm strings. Such strings could potentially be created in heterotic compactifications as bound states of open membranes, five-branes wrapped on four-cycles and solitonic strings. However, in generic compactifications, the process of moduli stabilization can conflict production of Aharonov-Bohm strings. In this case, heterotic cosmic strings will have to be unstable under breakage on monopoles. We estimate the monopole masses and find that they are big enough so that the strings can be metastable with a sufficiently long lifetime. On the other hand, if we allow one or more axions to remain massless at low energies, stable global strings can be produced.
1 Introduction

In [1], Polchinski proposed a classification of various cosmic strings in the context of superstring theory and conjectured that any theoretically possible decay should be allowed. This conjecture implies that in the framework of string theory there are only two types of absolutely stable cosmic strings. The first type is global strings which are charged under a massless axion. The second type is Aharonov-Bohm strings [2]. These strings can exist only if there is a particle that picks up a fractional phase around the string and does not couple to the low-energy gauge group. For example, in string compactification models with all moduli, including all axions, stabilized, Aharonov-Bohm strings, provided they exist, are the only stable strings. To find a concrete decay mechanism of other types of strings in a given model can be very complicated. Nevertheless, if Polchinski’s conjecture is correct it always exists. For example, in some heterotic $SO(32)$ compactifications, a fundamental string seems stable though it is neither global nor Aharonov-Bohm. However, in [1] it was demonstrated the existence of open heterotic $SO(32)$ strings which in four dimensions end on monopoles. This provides a concrete mechanism how a fundamental $SO(32)$ string can break on monopoles. If this conjecture is correct, it is likely that cosmic strings in compactification and brane models are only metastable. A systematic analysis of of fundamental and Dirichlet cosmic strings was performed in [3]. It was shown that in type IIB models with many Klebanov-Strassler [4] throats it is possible to have brane/anti-brane inflation [5] followed by de Sitter (dS) vacua [6] and production of cosmic F- and D-strings. In fact, the strings produced this way are only metastable. Various properties of these strings were studied in [7, 8, 9, 10].

In this paper, we attempt to describe how stable and metastable cosmic strings can arise in heterotic M-theory [11, 12, 13]. Such compactifications are very attractive from the phenomenological viewpoint. Recent results [14, 15, 16, 17] show the possibility of obtaining the Standard Model spectrum with no extra exotic matter in this framework. Moduli stabilization and inflationary properties of heterotic M-theory were studied in [18, 19, 20, 21, 22, 23, 24, 25]. Cosmic strings in these models were studied in [26, 27]. The natural cosmic string candidates are open membranes, five-branes wrapped on four-cycles in compactification space and solitonic strings as well as their bounds states. All these strings have two major universal sources of instability. The first one is that they bound axionic domain walls. If the axion receives a mass the strings will rapidly collapse [34]. The second instability is the possible breakage on monopoles. The latter instability is not as severe as the former. If the square of the mass of the monopoles is much bigger than the string tension,
the string will be metastable with a long lifetime.

To understand exactly which string or bound states can arise in a given model it is impor-
tant to understand the potential for all the axions. Such potential always arise in stabilizing
moduli. This is why it seems hard to ask the question about stability of various strings
outside of the context of moduli stabilization. In sections 2 and 3, we discuss moduli stabi-
lization in heterotic M-theory. The reason is twofold. First, as just explained, it is important
for understanding what kind of axionic potentials appear in the known methods of moduli
stabilization. Second, the tension of potential cosmic string candidates is moduli dependent.
Therefore, it is important to set-up moduli stabilization procedure to be consistent with
having the tension of possible strings within the observational bound. In [26], it was shown
that it possible to stabilize a five-brane close to the visible sector so that an open membrane
stretched between this five-brane and the visible brane will appear as a string with a small
tension. In section 2, we show that it is possible to stabilize the Calabi-Yau Kahler moduli in
such a way that one or more two-cycles are much smaller than the Calabi-Yau scale. This
implies the possibility that a five-brane wrapped on a four-cycle will also appear as a string
with a small tension. In section 3, we discuss the volume stabilization, Fayet-Iliopoulos (FI)
terms and dS vacua. In section 4, we discuss stability of cosmic strings. First, we consider
the case, when one of the axions remains massless after moduli stabilization. As explained
in section 3, this still might be consistent with stabilizing the remaining moduli in a dS
vacuum. In this case, we show that it is possible to find stable global strings in classification
of [1]. In fact, an open membrane studied in [26] is an example of global strings. However,
depending on exactly which axion remains massless, it might happen that open membranes
become unstable under domain wall formation, whereas five-branes wrapped on a four-cycle
become stable global strings. Then we consider the case when all axions are stabilized. An
open membrane can no longer be stable because the axion which it is not periodic around it
becomes massive. The problem of domain wall formations is partially resolved if the axion
is charged under an anomalous $U(1)$ and gauged away [4]. However, in the $E_8 \times E_8$ theory,
only one linear combination of axions can be gauged [1]. This means that a string can be
stable only if it is charged under this gauged linear combination and uncharged under all
the remaining massive axions. These strings can be formed as bound states of open mem-
branes, five-branes and solitonic strings. It is natural to propose that in a generic heterotic

\footnote{The remaining axion can receive a potential in the low-energy field theory from, for example, QCD
instantons. Moreover, the non-existence of continuous global symmetries in string theory implies that,
eventually, each axion has to receive a potential. We will not discuss it in this paper.}
M-theory compactification, it is always possible to create a bound state with the property stated above. Under some circumstances these bound states are stable Aharonov-Bohm strings. However, we point out that their production might not be consistent with moduli stabilization. Therefore, one can expect production of strings which are not seen by any scalar field in the spectrum. By Polchinski’s conjecture they have be unstable and break on monopoles. We estimate the monopole masses in heterotic M-theory and argue that these strings can have a sufficiently long lifetime.

There are important issues which we will not consider in this paper. First of all, cosmic strings can be detected only if they are produced after inflation. We will not discuss it in this paper. Some reasons why various heterotic cosmic strings can be produced after inflation can be found in [26, 27]. Furthermore, there can be additional instability sources besides domain wall formation and breakage on monopoles. For example, strings can dissolve into flux or break on the visible or hidden brane. Unlike the instabilities studied in this paper, these other processes cannot be studied in a universal manner and very much model dependent.

2 Wrapped Five-Branes as Cosmic Strings and Anisotropic Calabi-Yau Threefolds

2.1 The Tension

Five-branes wrapped on four-cycles in Calabi-Yau threefold can either themselves be viewed as strings in four dimensions or be ingredients of string bound states. To make sure that the strings of interest will have a relatively small tension we have to understand under what condition the four-cycle on the which the five-brane is wrapped can be made small comparing to the Calabi-Yau scale.

We consider a five-brane wrapped on a four-cycle and parallel to the orbifold fixed planes.\(^2\) From the four-dimensional viewpoint such a configuration will look like a string. The tension of this strings was evaluated in details in [27] so we will be very sketchy. The tension behaves as

\[ \mu \sim M_{11}^6 v_4, \quad (2.1) \]

where \(M_{11}\) is the eleven-dimensional Planck scale and \(v_4\) is the volume of the four-cycle. It

\(^2\)Throughout the paper we will refer to one of these planes as to the visible sector and to the other one as to the hidden sector.
is easy to realize that in order for this tension to satisfy the observational bound

\[ G\mu \sim 10^{-7} \]  

(2.2)

the size of the four cycle should be sufficiently smaller than the Calabi-Yau scale. See [27] for details. It is possible to create such an object only if it is possible to stabilize some of the \( h^{1,1} \) moduli in such a way that one or more cycles have a small area. This will be discussed in the subsection 2.3.

### 2.2 A Topological Obstruction

There is a topological obstruction to having a five-brane wrapped on a four-cycle in the bulk. It comes from the fact the Bianchi identity requires the existence of the flux \( G_{(2,2,0)} \) [13]. The notation is that \( G_{(2,2,0)} \) has two holomorphic, two antiholomorphic indices along Calabi-Yau and no indices along the interval. This flux provides a warping of the metric along the interval [13, 28, 29, 27]

\[
ds^2 = e^{-f(x^{11})} g_{\mu\nu} dx^\mu dx^\nu + e^{f(x^{11})} (g_{\text{CY} mn} dy^m dy^n + dx^{11} dx^{11}) ,
\]

(2.3)

where

\[
e^{f(x^{11})} = (1 - x^{11} G)^{2/3}
\]

(2.4)

and \( G \) is given by

\[
G = \int \omega \wedge G_{(2,2,0)}.
\]

(2.5)

In general, a heterotic M-theory compactification contains five-branes wrapped on holomorphic two-cycles. To simplify language, in this subsection, we will refer to five-branes wrapped on two-cycles as to three-branes and to the five-brane wrapped on a four-cycle as to one-brane. In the absence of three-branes in the bulk, \( G \) in eq. (2.5) is given by [13]

\[
G = \frac{-1}{8\pi V} (\frac{\kappa_{11}}{4\pi})^{2/3} \int \omega \wedge (tr \mathcal{F}^{(1)} \wedge \mathcal{F}^{(1)} - \frac{1}{2} tr \mathcal{R} \wedge \mathcal{R}) ,
\]

(2.6)

where \( \mathcal{F}^{(1)} \) is the instanton on the visible sector and \( \omega \) is the Kahler form. In the presence of the three-branes, the right hand side of eq. (2.6) will be modified by the cohomology class of the three-branes. Since the integral in (2.6) is an integer (up to normalization) and the integral of \( \omega \) over any two-cycle is positive, it follows that if the flux \( G_{(2,2,0)} \) is topologically non-trivial, the integral of \( G_{(2,2,0)} \) over any four-cycle is non-zero. This means that there
is a problem with wrapping a five-brane on a four-cycle. Indeed, on a worldvolume of the five-brane there is a coupling
\[ \int B_2 \wedge G_{(2,2,0)}. \] (2.7)
This coupling can be understood if one compactifies one of the non-compact directions on a circle. Then one obtains a type IIA D4-brane on a four-cycle with the coupling
\[ \int A_1 \wedge G_{(2,2,0)}, \] (2.8)
which is known to be there. From eq. (2.7) it follows that \( G_{(2,2,0)} \) acts like a source for \( B_2 \) which has to be canceled. Otherwise, such a configuration is not allowed.

In this paper, we will restrict ourselves to the case when \( G_{(2,2,0)} \) is cohomologically trivial in some region in the bulk. Then the coupling eq. (2.7) goes away and there is no obstruction to wrapping a five-brane on a four-cycle. Let us discuss under what circumstances it can be achieved. The cohomology class of the \((2,2,0)\) component of the flux is given by \[ G_{(2,2,0)}(x^{11}) = (c_2(V_1) - \frac{1}{2}c_2(X))\epsilon(x^{11}) + (c_2(V_2) - \frac{1}{2}c_2(X))\epsilon(x^{11} - \pi \rho) + \sum I [W_I] \epsilon(x^{11} - x_I). \] (2.9)
Here \( c_2(V_1) \) and \( c_2(V_2) \) are the second Chern classes of the vector bundles on the visible and hidden sectors respectively, \( c_2(X) \) is the second Chern class of the Calabi-Yau threefold \( X \), \( x_I \) is the position of the \( I \)-th three-brane whose cohomology class is \([W_I]\). The function \( \epsilon(x) \) is defined as +1 for \( x > 0 \) and −1 for \( x < 0 \). Let us pick a point along the interval where we would like to place a one-brane. Let \([W_1]\) be the total three-brane class to the left of this point and \([W_2]\) be the total three-brane class to the right. We have
\[ G_{(2,2,0)} = c_2(V_1) - c_2(V_2) + [W_1] - [W_2]. \] (2.10)
We want this to be zero. Therefore, we set
\[ c_2(V_1) - c_2(V_2) + [W_1] - [W_2] = 0 \] (2.11)
This equation should be supplemented by the usual anomaly cancellation condition \[ c_2(V_1) + c_2(V_2) + [W_1] + [W_2] = c_2(X). \] (2.12)
These two equations can be rewritten as
\[ 2c_2(V_1) + 2[W_1] = c_2(X) \] (2.13)
and

\[ 2c_2(V_2) + 2[W_2] = c_2(X). \]  \hspace{1cm} (2.14)

Thus, we are interested in compactifications where eqs. (2.13) and (2.14) are satisfied for some three-brane classes \([W_1]\) and \([W_2]\). The necessary condition for eqs. (2.13) and (2.14) to have a solution is that \(c_2(X)\) must be represented by an even four-form. If this condition is satisfied, then, in general, it should not be difficult to satisfy eqs. (2.13) and (2.14) provided one can allow many three-branes in the bulk. It is important to note that allowing many three-branes in the bulk is consistent with stabilization of the interval in a phenomenological range \([26]\). This will be reviewed in the next subsection. Let us now discuss the condition that \(c_2(X)\) must be even. We will simply present some examples of Calabi-Yau threefolds with even \(c_2(X)\) as an argument for possibility of existence of solutions of eqs. (2.13) and (2.14) for some compactifications. Let us consider Calabi-Yau threefolds elliptically fibered over the Hirzebruch or del Pezzo surfaces. These manifolds have been used for analyzing heterotic GUT vacua in \([30, 31]\). The second Chern class \(c_2(X)\) for such manifolds was computed by Friedman, Morgan and Witten in \([32]\) and was found to be

\[ c_2(X) = \pi^*c_2(B) + 11\pi^*c_1(B)^2 + 12\sigma \cdot \pi^*c_1(B). \]  \hspace{1cm} (2.15)

Here \(c_1(B)\) and \(c_2(B)\) are the first and the second Chern classes of the base \(B\), \(\pi\) is the projection map of \(X\) to the base and \(\sigma\) is the global section. The necessary condition of \(c_2(X)\) to be even is that

\[ c_2(B) + 11c_1(B)^2 \]  \hspace{1cm} (2.16)

is an even number. For the Hirzebruch surfaces one has (see, for example, appendix B of \([30]\) and references therein for properties of the Hirzebruch and del Pezzo surfaces)

\[ c_2(B) = 4, \quad c_1(B)^2 = 8. \]  \hspace{1cm} (2.17)

So \(c_2(X)\) is an even class. For del Pezzo surfaces \(dP_r\) one has

\[ c_2(B) = 3 + r, \quad c_1(B)^2 = 9 + r. \]  \hspace{1cm} (2.18)

Then it follows from eqs. (2.15) and (2.16) that \(c_2(X)\) is even for any \(r\). One can check that \(c_2(X)\) is also even for more complicated Calabi-Yau threefolds with a non-trivial homotopy group which were used in \([14, 15, 16, 17]\) for obtaining heterotic Standard Model vacua.

This shows that it is conceivable to obtain quasi-realistic compactifications with \([G_{(2,2,0)}] = 0\) in some region in the bulk. If we put a one-brane in this region, we obtain a consistent
configuration. Note that the warp-factor is unity in such a region and a one-brane is a BPS object. Also note that the flux $G_{(2,2,0)} = 0$ is discontinuous across three-branes. Therefore, the warp-factor can be unity only in some region between two three-branes. At a generic position along the interval the warp-factor will still be non-trivial.

Let us now discuss the case when a one-brane is placed in a region with non-trivial $G_{(2,2,0)}$. For consistency, the source coming from (2.7) has to cancel. The only possibility to do it is to allow membranes to end on a one-brane. These membranes will look like a string in four non-compact dimensions and stretch along the interval. Their one end will be located on the one-brane and their other end will be located on either a three-brane or one of the orbifold planes. The number of membranes should be such that the source coming from the flux cancels. This membrane/five-brane configuration is totally consistent with string dualities. Indeed, upon compactifying one of the non-compact directions of this string bound state on a circle one obtains a type IIA D4-brane on a four-cycle with fundamental strings attached. This mechanism of canceling the source originating from the flux is totally analogous to the case of branes wrapping $S^5$ in AdS/CFT correspondence [33]. Due to the flux through $S^5$ in order to wrap a brane on $S^5$ one has to attach strings to the brane.

From the four-dimensional viewpoint, the membrane/five-brane configuration looks like a string bound state. It is not BPS since its tension receives an extra contribution from the membranes. In there is only one membrane needed to cancel the flux, the tension is

$$\mu = \mu_5 + \mu_2 = \int_{4\text{-cycle}} d^4 y \sqrt{-g} + \int dx^{11} \sqrt{-g},$$

(2.19)

where the metric is given by eq. (2.3) and the integral over $dx^{11}$ is over the length of the membrane. The dependence of $\mu$ on $x^{11}$, up to a constant, is given by

$$\mu \sim (1 - x^{11} G)^{2/3}.$$  

(2.20)

This function does not have a minimum so this bound state has to move until it coincides with one of the three-branes of one of the orbifold planes. This is not surprising since this state is not BPS. Once a one-brane is on top of a three-brane or an orbifold plane the description in terms of the five-brane on a four-cycle breaks down. Unfortunately, it is not known what happens when two M-theory five-branes come on top of each other. So if a one-brane coincides with a three-brane it is hardly possible to say what kind of object arises. Things are slightly better when a one-brane coincides with an orbifold plane. The arising state should be interpreted as an instanton on $R^{1,3} \times X$. The second Chern class of this instanton should have two indices along $R^{1,3}$ and two indices along $X$. It would be interesting
to find an appropriate solution by, for example, assuming that the instanton is small and can be approximated by an instanton on $R^4$. However, it is very likely that if the scale of this instanton can be stabilized it is due to the existence of compact directions which makes the analysis much more complicated. We will not consider this case in this paper leaving it for future research.

Let us summarize the results of this subsection. We argued that under some circumstances it is conceivable to find a region where the flux is cohomologically trivial. Then there is no obstruction to placing a five-brane wrapped on a four-cycle in this region. In this paper, we will assume that such a region can be found. On the other hand, if one places such a five-brane in a region with non-trivial flux, it tends to move until it coincides with a three-brane or an orbifold plane. In both case the original description in terms of a brane on four-cycle breaks down. We will not pursue this issue in this paper.

2.3 Stabilization of the $h^{1,1}$ moduli

In this subsection, we will discuss the possibility of stabilizing one or more two-cycles in a Calabi-Yau threefolds at a small area. This will create an anisotropy which might be relevant for production of cosmic strings with a small tension by wrapping five-branes on four-cycles of a small volume. For simplicity, we will consider the case when $h^{1,1} = 2$ though the generalization for any $h^{1,1} > 1$ is straightforward. Our aim is to create a potential energy that will tend to produce an anisotropy of the Calabi-Yau threefold. We consider the following system of moduli

$$Z_\alpha, T_1, T_2, Y_I, \quad I = 1, \ldots N,$$  \hspace{1cm} (2.21)

where $Z_\alpha$ are the complex structure moduli, whose actual number will be irrelevant, $T_1$ and $T_2$ are the two Kahler structure moduli, $Y_I$ are the moduli of five-branes wrapped on the same isolated genus zero holomorphic curve. For simplicity, we will assume that we can choose the basis of curves in such a way that the volume of the cycles in different homology classes are controlled exactly by $ReT_i$. All the five-brane will wrap the curve whose volume is controlled by $ReT_1$. The precise structure of the $T_i$ moduli is as follows \[35, 36\]

$$T_i = Rb_i + ip_i.$$  \hspace{1cm} (2.22)

Here $R$ is the size of the interval, $b_i$ are the Kahler moduli of the Calabi-Yau threefold and $p_i$ are internal components of the three-form potential. The moduli $b_i$ are not independent,

$$\sum_{i,j,k} d_{ijk} b_i b_j b_k = 6,$$  \hspace{1cm} (2.23)
where \( d_{ijk} \) are the triple intersection numbers. The five-brane moduli are defined as follows \[37, 38\]
\[
Y_I = \frac{y_I}{\pi \rho} Re T_1 + i(a_I + \frac{y_I}{\pi \rho} Im T_1). \tag{2.24}
\]
Here \( y_I \) is the actual positions of the \( I \)'th five-brane, \( \pi \rho \) is the reference length of the interval and \( a_I \) is the axions on the worldvolume of the \( I \)'th five-brane. This system of moduli should be supplemented by the volume multiplet
\[
S = (V + \ldots) + i\sigma, \tag{2.25}
\]
where \( V \) is the Calabi-Yau volume in the middle of the interval, \( \sigma \) is the axion dual to the antisymmetric tensor and by the ellipsis we indicate that the real part of \( S \) gets modified by in presence of five-branes \[37, 38\]. We will assume that this modification is small enough and can be ignored. All moduli are dimensionless and normalized to the the reference scales \( v_{CY}^{1/6} \sim (10^{16} \text{GeV})^{-1} \) and \( \pi \rho \sim (10^{15} \text{GeV})^{-1} \). By construction, \( Re Y_I < Re T_1 \). To obtain the four-dimensional coupling constants in a phenomenological range, \( V \) and \( R \) have to be stabilized at (or be slowly rolling near) a value of order one. As will be discussed later, stability of various types of cosmic strings depends whether or not there is a potential for the \( S \) modulus. Thus, we will postpone our discussion of stabilization of \( S \) till section 4.

Let us also point out that in a generic compactification the system (2.21) should also be supplemented by the vector bundle moduli. For simplicity, we will ignore them.

We want to understand how to stabilize the system (2.21) in an anti de Sitter (AdS) vacuum with \( b_2 \ll b_1 \). Stabilization of \( S \) and de Sitter vacua will be discussed in the next section. The Kahler potential of this system is given by the standard expression
\[
\frac{K}{M_{Pl}^2} = -\ln \left( -i \int \Omega(Z) \wedge \overline{\Omega}(\overline{Z}) \right) - \ln \left( \frac{1}{6} \sum_{ijk} d_{ijk}(T_i + \overline{T}_i)(T_j + \overline{T}_j)(T_k + \overline{T}_k) \right) \\
+ \sum_I K_1(Y_I + \overline{Y}_I)^2, \tag{2.26}
\]
where
\[
K_1 = \frac{\tau_5}{(T_1 + \overline{T}_1)(S + \overline{S})} \tag{2.27}
\]
and \( \tau_5 \) is the five-brane tension. It was proposed in \[37, 38\] that the \( Y \)-moduli and the \( S \) moduli Kahler potential forms on logarithm
\[
-\ln \left( S + \overline{S} - \sum_I \frac{\tau_5(Y_I + \overline{Y}_I)^2}{T_i + \overline{T}_i} \right), \tag{2.28}
\]
However, in this paper, we will assume that the $Y$-dependent corrections are small enough and it is accurate to expand the logarithm keeping the quadratic terms in $Y$. The superpotential for the system will be

$$ W = W_f + W_{np}. $$

We will assume that the $(3, 0, 1)$ $G$-flux is turned on. It produces the superpotential for $Z_\alpha$ of the form

$$ W_f = \frac{M_{Pl}^2}{v_{CY} \pi \rho} \int d\Sigma^{11} \int_{CY} G \wedge \Omega. $$

The coefficient in front is moduli independent as it is just $M_1^0$. This superpotential is expected, generically, to stabilize all the complex structure moduli. We will assume that it is the case. We will also assume that the complex structure moduli are stabilized at slightly higher scale than all the other moduli so that $W_f$ can be considered constant. The non-perturbative superpotential $W_{np}$ is, approximately,

$$ W_{np} = W_{np}(T_1, Y_I) + W_{np}(T_2). $$

In the presence of the five-branes, the leading contribution to the $T^1$ superpotential is due to open membranes stretched between the adjacent branes

$$ W_{np}(T_1, Y_I) \sim A_1 e^{-\tau_1 Y_1} + A_2 e^{-\tau_1 (Y_2 - Y_1)} + \ldots + A_{N+1} e^{-\tau_1 (T_1 - Y_N)}. $$

The coefficients $A_1, \ldots A_{N+1}$ depend, in general, on the complex structure and vector bundle moduli. For the purposes of this paper they can be taken to be constants. The coefficient $\tau_1$ was estimated in [19] and found to be of order 250. Since we do not have any five-branes wrapped on cycles whose area is controlled by $ReT_2$, the term $W_{np}(T_2)$ is given by

$$ W_{np}(T_2) \sim e^{-\tau_2 T_2}. $$

First, let us consider

$$ D_{T_1} W = 0, \quad D_{Y_I} W = 0 $$

These equations were analyzed in some detail in [26]. The properties of the solution are the following.

1. The distances between the adjacent branes are approximately the same.

3For brevity, we will not discuss the imaginary parts of the moduli. Their stabilization was discussed in [19] [21].
2. Due to existence of the constant (for our purposes) superpotential $W_f$, this distance will be proportional to $-\ln(|W_f|/\tau_1)$.

This is easy to understand intuitively. In the absence of $W_f$, the superpotential Eqs. (2.29), (2.32) will stabilize the relative distance between the adjacent branes. For example, if there is only one five-brane in the bulk it will be stabilized approximately in the middle of the interval. However, the overall interval modulus will run away. The presence of $W_f$ will stabilize the remaining run-away modulus. If the number of five-branes is $N$, the relative distance is approximately $\frac{Rb_1}{N+1}$. Eqs. (2.34) lead to the following equation for the relative distance

$$e^{-\tau_1 \frac{Rb_1}{N+1}} \sim \frac{|W_f|}{\tau_1}.$$  \hspace{1cm} (2.35)

In order this equation to have a solution, the right hand side has to be less than one. First, $W_f$ is quantized in units of $(\frac{\kappa}{11})^{2/3}$. Therefore, in the limit of large volume and large interval, $|W_f|$ is less than one. Second, the order of magnitude of $W_f$ might be reduced by Chern-Simons invariants \[46\]. Third, $\tau_1$ is much greater than one. This guarantees that, generically, the right hand side in eq. (2.35) is less than one. If $N$ is big enough (a simple estimate shows that $N \approx 20$ should suffice) one can always obtain

$$Rb_1 \sim 1.$$  \hspace{1cm} (2.36)

Now let us consider the equation

$$D_{T_2}W = 0.$$  \hspace{1cm} (2.37)

We have

$$e^{-\tau_2 Rb_2} \sim \frac{|W_f|}{\tau_2}.$$  \hspace{1cm} (2.38)

For $\tau \sim 10^{-3} - 10^{-2}$ and generic $|W_f|$ or order, for example, $10^{-2}$ in eleven-dimensional Planck units \[19\] (we are assuming that the scale of $W_{np}$ is set by the eleven dimensional Planck scale) this equation can have a solution only if

$$Rb_2 << 1.$$  \hspace{1cm} (2.39)

For generic triple intersection numbers $d_{ijk}$, our solution

$$Rb_1 \sim 1, \quad Rb_2 << 1$$  \hspace{1cm} (4.40)

is consistent with having

$$R \sim 1, \quad b_1 \sim 1, \quad b_2 << 1.$$  \hspace{1cm} (4.41)
This analysis provides a mechanism for creating an anisotropy of the Calabi-Yau threefold. Creating just one cycle with a small area might not be sufficient to create a four-cycle whose volume is small enough to satisfy the bound estimated in [27]. However, it is straightforward to generalize this analysis for the case $h^{1,1} > 2$. By varying the number of five-branes wrapping cycles in various homology classes one can create enough anisotropy for existence of four cycles with a sufficiently small volume. A five-branes wrapped on such a cycle will look like a string with a small tension in Minkowski space.

3 Volume Stabilization and de Sitter Vacua

As explained in the introduction, in order to understand what kind of strings can be produced and whether or not they are stable, it is important to formulate the set-up for moduli stabilization. In this section, we will complete this set-up by discussing various mechanisms for stabilizing the volume multiplet.

The most common way to stabilize the volume multiplet is to allow the hidden sector gaugino to condense. This produces the superpotential of the form [47, 48]

$$ W_g = h e^{-\epsilon S + ...} $$

(3.1)

Here $h$ is the coefficient whose scale is set by $M_{11}^3$, $\epsilon$ is given by [48]

$$ \epsilon = \frac{2\pi}{b_0 \alpha_{GUT}} $$

(3.2)

where $b_0$ is the coefficient of the one-loop beta function and the ellipsis indicate the threshold corrections depending on other moduli due to fact that the Calabi-Yau volume is warped along the interval. They will not be important for us. Adding $W_g$ to $W$ in eq. (2.29) stabilizes the volume [13, 46]. As the result, the system of moduli [22] supplemented by the volume multiplet $S$ (and, in principle, by the vector bundle moduli, which, for simplicity, have been ignored) can be stabilized in a supersymmetric AdS vacuum.

In order to raise this AdS vacuum to a metastable dS vacuum we will assume that the low-energy gauge group contains a $U(1)$ factor which is anomalous. This anomaly is canceled by the four-dimensional version of the Green-Schwarz mechanism [49, 50]. The axion, in this case, transforms under $U(1)$ as

$$ \sigma \rightarrow \sigma + l. $$

(3.3)

As was pointed out in [51, 1], in $E_8 \times E_8$ compactifications, the existence of the anomalous $U(1)$ implies that the imaginary parts of the $h^{1,1}$ moduli are also charged. The reason is the
following. Due to $E_8$ properties, the anomaly in the $U(1)$ generator $T$ is a linear combination of
\[ \int (TrT^3), \; \int (TrT^2)(TrT^2)(TrF^2), \; \int (TrT^2)(TrT^2)(TrR^2). \]  
(3.4)
This means that the anomaly $T$ exists only if the integral of $TrF$ over at least one two-cycle is non-zero. However, under the same conditions, the anomalous $U(1)$ gauge field $A$ couples to at least one imaginary part of the $h^{1,1}$ moduli. This coupling will come from the term (after we perform the dimensional reduction on $S^1/Z_2$)
\[ \int H \wedge *H, \]  
(3.5)
where
\[ H = dB_2 - (\omega_Y - \frac{1}{2}\omega_L). \]  
(3.6)
The simplest way to proceed is to dualize $dB$ by introducing a six-form $B_6$
\[ dB_6 = *dB. \]  
(3.7)
Then eq. (3.5) will contain a term
\[ \int B_6 \wedge (trF^2 - \frac{1}{2}R^2). \]  
(3.8)
Upon the dimensional reduction to four dimensions this will produce a term
\[ \int p_2 \wedge dA = \int d^4xp\partial_\mu A^\mu, \]  
(3.9)
provided the integral of $F$ over some two-cycle is not zero. Here $p_2$ is the antisymmetric tensor dual to the imaginary part $p$ of some $h^{1,1}$ modulus. Coupling (3.9) guarantees that the axion $p$ is also charge under the anomalous $U(1)$ [50]. If $h^{1,1}$ is bigger than one, in general, more than one axions $p_i$ will be charged under $U(1)$.

The anomalous $U(1)$ also leads to FI terms [50]. In heterotic M-theory their scale is approximately the same as that of \( \frac{W^2}{M_{Pl}} \) [21] so that they can be used to raise a supersymmetric AdS vacuum to a dS vacuum [52]. Remarkably, the fact that there is always more than one axion charged under $U(1)$ implies that even in the absence of a superpotential for the multiplet $S$, the volume $V$ can be stabilized in a dS vacuum if FI terms are present. For simplicity, let us consider the case $h^{1,1} = 1$. We will not be very careful about all the factors. They were estimated in [21]. The transformation law (3.3) implies that the Kahler potential for the $S$ multiplet has to be of the form
\[ K(S) = -\ln(S + \bar{S} + \mathcal{V}), \]  
(3.10)
where $\mathcal{V}$ is the anomalous $U(1)$ vector superfield. The term

$$\int d^4x d^4\theta K(S)$$

produces, among other terms, the FI term

$$\int d^4x \frac{D}{\mathcal{V}},$$

where $D$ is the auxiliary field. Then the contribution to the potential energy is

$$\frac{g^2}{\mathcal{V}^2}.$$  \hfill (3.13)

The gauge coupling constant $g^2$ is also moduli dependent

$$g^2 \sim \frac{1}{\mathcal{V} + \gamma},$$

where $\gamma$ stands for the threshold corrections whose precise form is not important. They can be found, for example, in [35]. As was explained, the axion $p$ also transforms the same way which means that the Kahler potential for the $T$-modulus is of the form

$$K(T) = -\ln(T + \bar{T} + \mathcal{V}).$$

This implies that there is a correction to the FI potential energy of the form

$$\frac{g^2}{R^2}.$$ \hfill (3.16)

If $h^{1,1}$ is greater than one the denominator $R^2$ is replaced with some more complicated function of $\text{Re}T_i$. These two contributions imply that the volume-dependent FI contribution to the potential energy is of the form

$$U_{FI} = \frac{B}{\mathcal{V}^2(\mathcal{V} + \gamma)} + \frac{A}{\mathcal{V} + \gamma}.$$ \hfill (3.17)

The coefficient $A$ depends on the $h^{1,1}$ moduli which were stabilized in the previous section and can be considered constants. Thus, approximately, dynamics of $\mathcal{V}$ is governed by the potential\footnote{The FI terms will modify equations of motion for the $h^{1,1}$ moduli and the five-branes. However, since the order of magnitude of FI terms is the same as that of the fluxes, a solution for these moduli will still exist. It is also possible to show that their Kahler covariant derivatives will be shifted by terms proportional to $\frac{1}{\tau_1}$ or $\frac{1}{\tau_2}$ which is much less than one.}

$$U(\mathcal{V}) = e^{K(S)}[G_{SS}^{-1}D_SD_S\bar{W} - 3W\bar{W}] + U_{FI}.$$ \hfill (3.18)
The supergravity term in (3.18) gives just $-\frac{2}{V}$ up to terms quartic on the five-brane positions which will be assumed to be small enough and neglected. Thus, the potential becomes

$$U(V) = -\frac{2}{V} + \frac{B}{V^2(V + \gamma)} + \frac{A}{V + \gamma}. \quad (3.19)$$

Under some conditions on parameters $A, B$ and $\gamma$ this function can have a dS vacuum. See fig. 1. This analysis shows that even in the absence of any superpotentials for the $S$ multiplet, one can stabilize all heterotic moduli in a dS vacuum.

There is one important comment we have to make before finishing this section. In the presence of the anomalous $U(1)$ the non-perturbative superpotentials (2.32), (2.33), (3.1) do not seem gauge invariant. To keep the gauge invariance they must be multiplied by some power a matter field charged under $U(1)$. Let us denote this field by $Q = re^{i\phi}$. Then the variation of the axion will be canceled by the variation of the phase $\phi$. To make sure that the non-perturbative superpotentials stabilize the $h^{1,1}$ moduli, $r$ should receive a vacuum expectation value. Thus, the system of moduli discussed in this and in the previous sections has to be supplemented by the field(s) $Q$. The radius of $Q$, $r$, receives a very complicated potential coming from the both the $F$- and $D$-terms. We will not study it in this paper. Generically, this potential can stabilize $r$ at a non-zero value. We will assume that it is the case and $r$ is stabilized at a value of order one.
4 Axions and Cosmic Strings

4.1 Global Heterotic Strings

After we introduced various moduli stabilizing potentials we can proceed to discuss what kind of strings can be found under different circumstances. There are three natural candidates for cosmic strings in heterotic M-theory: open membranes, five-branes wrapped on four-cycles and solitonic strings. The latter can exist by Kibble’s argument because at least one field charged under the anomalous $U(1)$ receives a vacuum expectation value as pointed out in the previous section. All these three types of strings can potentially have a small tension. An open membrane can have a small tension due to the possibility of stabilizing a five-brane wrapped on an isolated genus zero curve close to the visible sector. This was studied in [26]. A five-brane wrapped on a four-cycle can have a small tension due to the possible Calabi-Yau anisotropy as explained in section 2. A solitonic string can have a small tension because the scale of both the potential and kinetic energy for the charged fields is much less than the four-dimensional Planck scale. The scale of the potential energy is of order $\frac{w_f^2}{M_{Pl}^4}$. This quantity is naturally less than $M_{Pl}^4$ by many orders of magnitude. The scale of the kinetic energy is much less than the Planck scale because the matter fields in heterotic M-theory originate from the orbifold sector. When we normalize the kinetic term the vacuum expectation value of the matter field of interest will be much less than $M_{Pl}$. These arguments suggest that the tension of a solitonic string must be much less than one in four-dimensional Planck units.

The most dangerous source of instabilities of heterotic cosmic strings is formation of axion domain walls of non-zero tension [34]. All three types of strings mentioned above are boundaries of axion domain walls. Once the corresponding axion receives a mass the domain wall will make the string rapidly collapse. A solitonic string is bounded by a domain wall of the phase $\phi$ of the field $Q$. This is a standard field theory result. To have a solitonic string solution means that far away from the string the field $\phi$ is an angular variable. Thus, as we go around the string

$$\int d\phi = 2\pi,$$  \hspace{1cm} (4.1)

meaning that there is a domain wall bounded by the string. An open membrane, like a fundamental string in the weakly coupled $E_8 \times E_8$ theory is bounded by a domain wall of the axion $\sigma$ [31]. For completeness, let us present this argument following Witten [34]. Let us say that we have a string stretched in the $x^3$ direction and localized at $x^1 = x^2 = 0$. Let
Let $\Gamma$ be a contour around the string and $\Sigma$ be a surface bounded by $\Gamma$. Then we have

$$\int_{\Gamma} d\sigma = \int_{\Gamma} *H = \frac{1}{2} \int_{\Sigma} dx^1 dx^2 \partial^\mu H_{\mu 30}. \quad (4.2)$$

Since a fundamental string in the weakly coupled theory and an open membrane in heterotic M-theory couple to $H$, the equations of motion for $H$ implies

$$\partial^\mu H_{\mu 30} \sim \delta(x^1)\delta(x^2). \quad (4.3)$$

Due to the delta-functional source, the integral (4.2) is non-zero meaning that $\sigma$ is not periodic as one goes around the string. The next object of interest is a five-brane wrapped on a four cycle. To simplify our discussion in this section we will consider the case $h^{1,1} = 1$. Though this choice is not quite consistent with section 2, this is not very important as the generalization of what follows for an arbitrary $h^{1,1}$ is straightforward. This five-brane is charged under the axion $p$ which is the imaginary part of the $h^{1,1}$ modulus. In the case $h^{1,1} > 1$, the five-brane will couple to the linear combination of $h^{1,1}$ axions associated to the two-form Poincare dual to the four-cycle on which it is wrapped. Let us show it. Since the five-brane is magnetically charged under $H$ in ten dimensions, we have

$$\int_{\Sigma_3} H \neq 0, \quad \Sigma_3 = \Sigma_2 \times \Gamma, \quad (4.4)$$

where $\Gamma$ is the contour around the string and $\Sigma_2$ is the two-cycle dual to the one on which the five-brane is wrapped. Upon the dimensional reduction

$$H = dp \land \omega, \quad (4.5)$$

where $\omega$ is the Kahler form. Since

$$\int_{\Sigma_2} \omega = 1, \quad (4.6)$$

we obtain

$$\int_{\Sigma_3} H = \int_{\Gamma} dp \neq 0. \quad (4.7)$$

This shows that the five-brane couples to the axion $p$.

Thus, all three types of strings have a potential instability after we stabilize the moduli. To prevent an axion domain wall formation, the axion has to be charged under an anomalous $U(1)$ gauge group \cite{3} as in eq. (3.3). Then the axion is, effectively, gauged away from the

\footnote{We compactified the theory on the interval and consider the three-form strength $H$ instead of the four-form strength $G$.}
spectrum. However, as pointed out in [1] and reviewed in the previous section, in the $E_8 \times E_8$ theory there are always more than one axions charged under the anomalous $U(1)$. So one can gauge away only one linear combination. All that indicates that it is important to understand what kind of axion potentials can arise in the process of moduli stabilization which was discussed in the previous two sections. Let us understand which stable strings can be found in the set-up of [26] when the Kahler structure moduli are stabilized by non-perturbative effects but the volume multiplet receives no superpotential. For this we have to understand in detail which axions receive a potential. In the simplest case $h^{1,1} = 1$ there are three axions $\sigma, p$ and $\phi$. In the presence of the anomalous $U(1)$, which is always assumed throughout the paper, all three of them are charged under it. This means that one linear combination of the axions can be gauged away. Without loss of generality, we can assume that this linear combination is $\sigma + p + \phi$. Therefore, no domain walls of the axion $\sigma + p + \phi$ can be formed. The non-perturbative superpotential for the $h^{1,1}$ modulus gives a potential for the linear combination of $p$ and $\phi$. Without loss of generality, we can assume that this linear combination is $p - \phi$. On the other hand, since, by construction, no non-perturbative superpotential for the volume multiplet is turned on, the uncharged linear combination of $\sigma$ and $\phi$, which we will take to be $\sigma - \phi$ has no potential. The outcome is that $\sigma + p + \phi$ is eaten by the broken $U(1)$ vector field, $p - \phi$ is massive and $\sigma - \phi$ massless. As the result, a solitonic string is unstable because the massive field $p - \phi$ is not periodic as we go around the string. For the same reasons, a five-brane on a for-cycle is also unstable. However, an open membrane is stable because the only field which is not gauged and has a jump around it is $\sigma - \phi$ which is massless. In other words, in the given model, an open membrane studied in detail in [26] is stable because it is a global string in Polchinski’s classification [1]. In this model, there is one more global string. One can have a bound state of one $p$-string and one $\phi$ string so that $p - \phi$ will be periodic as we go around the bound state. The massless field $\sigma - \phi$ has a jump as we go around it so this string is also a global string. On the other hand, a bound state of one open membrane and one solitonic string is unstable since the massive field $p - \phi$ is not periodic around this bound state. Similarly, a bound state of an open membrane and a five-brane is also unstable.

Let us emphasize that the analysis of stability of various strings presented above depended on the exact structure of the potential for various axions. One can imagine the situation when the volume multiplet receives a superpotential due to a gaugino condensate in the hidden sector but one or more of the $h^{1,1}$ moduli receives no superpotential. In this case, the real parts of the corresponding $h^{1,1}$ moduli might be possible to stabilize by FI-terms.
and certain linear combinations of the $h^{1,1}$ axions and the phase $\phi$ will be massless. Now open membranes will bound non-zero tension domain walls and will be unstable, however five-branes on four-cycles will now be stable global strings. In any case, if after moduli stabilization one or more axions remain massless, stable global strings can be produced.

4.2 Aharonov-Bohm Heterotic Strings

In this and the next subsections, we will consider the case when all moduli multiplets receive a superpotential. As the result, after moduli stabilization there are no massless pseudoscalars left. The strings which were global and, thus, stable in the previous section will now bound domain walls of non-zero tension and quickly collapse. In the example considered above, we have the axion $\sigma + p + \phi$ eaten by the massive $U(1)$, whereas the $p - \phi$ and $\sigma - \phi$ are both massive. An open membrane is now unstable since it bounds a domain wall of the field $\sigma - \phi$ which is now massive. Similarly, a generic bound state of the three types of strings will also be unstable. A candidate for a stable string should be uncharged under the both massive fields. The only field which is allowed not to be periodic around such a candidate is $\sigma + p + \phi$ which is gauged. In the given example, it is easy to propose a bound state with this property. The bound state of one open membrane, one five-brane and one solitonic string, all properly oriented, can respect periodicity of both $\sigma - \phi$ and $p - \phi$. The only non-periodic field around this bound state is $\sigma + p + \phi$. This string is stable under domain wall formation. It is natural to propose that in any heterotic M-theory compactification, regardless of details, it is always possible to construct a bound state which is charged only under the linear combination of axions which is gauged. Therefore, all such strings are potentially stable.

However, there is one more universal instability for all heterotic cosmic strings, namely, breakage on monopoles. In \cite{P}, Polchinski conjectured that in string theory every potential decay should be allowed. This implies that in the absence of any massless particles the only stable strings are Aharonov-Bohm strings which have an Aharonov-Bohm phase with respect to a particle (more generally, a collection of particles) neutral under the low-energy gauge group. The conjecture implies that the bound state constructed above should be unstable unless it is an Aharonov-Bohm string. Breaking on monopoles is a natural decay process. In general, a concrete decay mechanism can be very complicated. In our case, the mechanism is qualitatively simple. To illustrate it, let us consider first a toy example of the Abelian

\footnote{In this discussion, we are ignoring the axion dependent threshold corrections to the gaugino condensation superpotential. Taking them into account would make our analysis more complicated without introducing any conceptual novelty.}
Higgs model (for a review see, for example, [55]). In this model, we have a $U(1)$ vector field $A$ coupled to a complex scalar $f$ with the potential of the form $(|f|^2 - \eta^2)^2$. The equations of motion will support a string solution with a finite tension. The $U(1)$ is broken in the presence of this solution and the phase of $f$ is gauged. The string contains a magnetic flux tube with quantized flux

$$
\int F = \int_{\Sigma} A = \frac{2\pi n}{e}.
$$

Let us now assume that the $U(1)$ comes after we break some non-abelian gauge group so that the theory also has monopoles. In the presence of the string the $U(1)$ is broken. Therefore, there is no long-range force and the monopoles have to be confined. Since a solitonic string contains a flux tube it can end on monopoles, thus confining them. This process prevents strings from growing to cosmic sizes. In heterotic Standard Model-like models one also gets monopoles after the GUT gauge group is broken to the Standard Model by Wilson lines. So the situation is not very different from the Abelian Higgs Model. Like a solitonic string, a $(\sigma, p, \phi)$-bound state supports a magnetic flux. Therefore, it can break on monopoles. In principle, one can imagine that there are no monopoles in our theory with minimal charge. Then only two or more strings can end on a monopole. However, Polchinski conjectured in [1] that in string theory the charge quantization is always saturated.

Polchinski’s conjecture says that a $(\sigma, p, \phi)$-bound state is stable only if it is an Aharonov-Bohm string. That is, if there exist particles charged under the anomalous $U(1)$ which pick up a phase $\frac{2\pi}{q}$, $q > 1$ around the string. Let us see if a $(\sigma, p, \phi)$-bound state can be an Aharonov-Bohm string. Recall that we gauge a linear combination of axions. In a generic compactification, it is very likely that this combination will have a charge $q$ greater than one. Any $(\sigma, p, \phi)$-bound state of interest is charged only under this linear combination. Then, if there is a particle of charge one under the anomalous $U(1)$, it will pick up a phase $\frac{2\pi}{q}$ as desired. Indeed, an axion $a$ of charge $q$ couples to the $U(1)$ through the term in the action $$(\partial_\mu a - qA_\mu)^2 \text{ [50].}$$

To minimize the energy of the string we should set

$$
\partial_\mu a - qA_\mu = 0.
$$

As we go around the string, the axion $a$ changes by one. This means that far away from the core of the string, $a$ is the angular variable which we denote by $\theta$. To satisfy (4.9), we have

$$
A_\theta \sim \frac{1}{q}.
$$

This means that any particle of charge one will pick up a phase $\frac{2\pi}{q}$ as in the usual Aharonov-Bohm effect. Because of this argument, one can think it is very likely that a $(\sigma, p, \phi)$-bound
state can be Aharonov-Bohm and, thus, stable. Unfortunately, there is a serious obstruction against it coming from the process of moduli stabilization. As was discussed, to turn on various non-perturbative superpotentials, at least one charged under the anomalous $U(1)$ scalar field should receive a vacuum expectation value. This field has to be single-valued around the string which is possible only if its charge divides $q$. This is a strong restriction which is unlikely to be satisfied in a generic compactification. However, if it is satisfied and there is another particle (or collection of particles) charged only under the anomalous $U(1)$ whose charge is not divisible by $q$, then a $(\sigma, p, \phi)$-bound state is a stable Aharonov-Bohm string. Otherwise, $q (\sigma, p, \phi)$-strings should form a bound state to make sure that a particle with a non-zero vacuum expectation value is single-valued around the string. This bound state, in principle, can still be an Aharonov-Bohm string if there is a particle with a fractional charge which does not receive a vacuum expectation value. In general, one can expect that this string is not charged under any scalar in the spectrum. This happens if, for example, every scalar charged under the anomalous $U(1)$ receives a vacuum expectation value. Since it contains a magnetic flux tube, this string will break on monopoles.

### 4.3 Metastable Heterotic Strings

Despite the fact, that a heterotic cosmic string can break on monopoles, its lifetime can be sufficiently long. The process of breakage of a string can be described by formation of a hole in the Euclidean worldsheet whose boundary is the monopole worldline. The decay rate is then governed by the Euclidean action

$$S_E[X^1, X^2] = m \int d\ell - \mu \int_{\text{hole}} dS.$$  \hspace{1cm} (4.11)

Here the first term is the length of the worldline, the second term is the area of the hole, $m$ is the monopole mass, $\mu$ is the string tension and $X^1$ and $X^2$ are the coordinates on the worldsheet. A straightforward calculation shows that $S_E[X^1, X^2]$ is minimized by a circular worldline of the radius

$$\frac{m}{\mu}.$$  \hspace{1cm} (4.12)

Substituting it to the action (4.11), we find that the decay rate behaves as

$$e^{-\pi m^2/2\mu}.$$  \hspace{1cm} (4.13)

It follows that, if the square of the monopole mass is much bigger than the string tension, the lifetime of the string will be very long. In this subsection, we will estimate the masses of heterotic monopoles and show that they are indeed very large.
There are two sources of monopoles in heterotic M-theory. The first source is open membranes beginning on one of the orbifold planes, ending on a five-brane and wrapping a one-cycle of non-trivial $\pi_1$ in Calabi-Yau space. For these particle-like states to be magnetically charged, the five-brane should carry one or more $U(1)$ gauge groups. This means that this five-brane should be wrapped on a holomorphic curve of genus one or higher. To avoid the problem of breaking on these monopoles it is enough to require the absence of such five-branes in the bulk. Even if there are such five-branes, these monopoles will have very large masses comparing to the scale of the string tension as long as the five-branes are far away from the orbifold fixed planes. This follows from a simple estimation. The mass of this type of monopoles behaves as

$$m_1 \sim M_{11}^3 \ell_{\text{CY}} \ell,$$

where $\ell_{\text{CY}}$ is the length of a one-cycle of non-trivial $\pi_1$ whose scale is set by the Calabi-Yau radius and $\ell$ is the distance between the five-brane and one of the orbifold planes. The scale of $\ell$ is set by the interval scale. It is easy to evaluate that for strings whose tension is within the bound (2.2),

$$\frac{m_1}{\sqrt{\mu}} \sim 10^5.$$

This implies that it will take a very long time for the strings to break.

The second source is the traditional breaking of the GUT gauge group by Wilson lines. The mass of monopoles of this type is of order

$$m_2 \sim \frac{M_{\text{GUT}}}{g_{\text{GUT}}^2}.$$

This means that

$$\frac{m_2}{\mu} \sim \frac{10}{g_{\text{GUT}}^2} \sim 4 \cdot 10^6.$$

It follows from this equation that the decay rate (4.13) is suppressed. Thus, the strings discussed at the end of the previous subsection have a good chance of being metastable with a sufficiently long lifetime.

5 Acknowledgements

The author is very grateful to Juan Maldacena and Joe Polchinksi for helpful discussions and explanations. The work is supported by NSF grant PHY-0503584.
References

[1] J. Polchinski, “Open Heterotic Strings,” hep-th/0510033.

[2] M. G. Alford and F. Wilczek, “Aharonov-Bohm Interaction of Cosmic Strings with Matter,” Phys.Rev.Lett. 62, 1071 (1989).

[3] E. J. Copeland, R. C. Myers and J. Polchinski, “Cosmic F- and D-strings,” JHEP 0406 (2004) 013 hep-th/0312067.

[4] I. R. Klebanov and M. J. Strassler, “Supergravity and a Confining Gauge Theory: Duality Cascades and χSB-Resolution of Naked Singularities,” JHEP 0008 (2000) 052 hep-th/0007191.

[5] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister and S. P. Trivedi, “Towards Inflation in String Theory,” JCAP 0310 (2003) 013 hep-th/0308055.

[6] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, “de Sitter Vacua in String Theory,” Phys.Rev. D68 (2003) 046005 hep-th/0301240.

[7] G. Dvali, R. Kallosh and A. Van Proeyen, “D-term strings,” JHEP 0401 (2004) 035 hep-th/0312005.

[8] P. Binetruy, G. Dvali, R. Kallosh and A. Van Proeyen, “Fayet-Iliopoulos Terms in Supergravity and Cosmology,” Class.Quant.Grav. 21 (2004) 3137-3170 hep-th/0402046.

[9] J. J. Blanco-Pillado, G. Dvali and M. Redi, “Cosmic D-strings as Axionic D-term Strings,” hep-th/0505172.

[10] S. S. Gubser, C. P. Herzog and I. R. Klebanov, “Symmetry Breaking and Axionic Strings in the Warped Deformed Conifold,” JHEP 0409 (2004) 036 hep-th/0405282.

[11] P. Horava and E. Witten, “Heterotic and Type I String Dynamics from Eleven Dimensions,” Nucl.Phys. B460 (1996) 506-524 hep-th/9510209.

[12] P. Horava and E. Witten, “ Eleven-Dimensional Supergravity on a Manifold with Boundary,” Nucl.Phys. B475 (1996) 94-114 hep-th/9603142.

[13] E. Witten, “Strong Coupling Expansion Of Calabi-Yau Compactification,” Nucl.Phys. B471 (1996) 135-158 hep-th/9602070.
[14] V. Braun, Y.-H. He, B. A. Ovrut and T. Pantev, “A Heterotic Standard Model,” Phys.Lett. B618 (2005) 252-258 [hep-th/0501070].

[15] V. Braun, Y.-H. He, B. A. Ovrut and T. Pantev, “A Standard Model from the $E_8 \times E_8$ Heterotic Superstring, JHEP 0506 (2005) 039 [hep-th/0502155].

[16] V. Bouchard and R. Donagi, “An $SU(5)$ Heterotic Standard Model,” [hep-th/0512149].

[17] V. Braun, Y.-H. He, B. A. Ovrut and T. Pantev, “The Exact MSSM Spectrum from String Theory,” [hep-th/0512177].

[18] G. Curio and A. Krause, “G-Fluxes and Non-Perturbative Stabilisation of Heterotic M-Theory,” Nucl.Phys. B643 (2002) 131-156 [hep-th/0108220].

[19] E. I. Buchbinder and B. A. Ovrut, “Vacuum Stability in Heterotic M-Theory,” Phys.Rev. D69 (2004) 086010 [hep-th/0310112].

[20] M. Becker, G. Curio and A. Krause, “De Sitter Vacua from Heterotic M-Theory,” Nucl.Phys. B693 (2004) 223-260 [hep-th/0403027].

[21] E. I. Buchbinder, “Raising Anti de Sitter Vacua to de Sitter Vacua in Heterotic M-Theory,” Phys.Rev. D70 (2004) 066008 [hep-th/0406101].

[22] E. I. Buchbinder, “Five-Brane Dynamics and Inflation in Heterotic M-Theory,” Nucl.Phys. B711 (2005) 314-344 [hep-th/0411062].

[23] K. Becker, M. Becker and A. Krause, “M-Theory Inflation from Multi M5-Brane Dynamics,” Nucl.Phys. B715 (2005) 349-371 [hep-th/0501130].

[24] L. Anguelova and D. Vaman, “$R^4$ Corrections to Heterotic M-theory,” [hep-th/0506191].

[25] J. Ward, “Instantons, Assisted Inflation and Heterotic M-theory,” [hep-th/0511079].

[26] E. I. Buchbinder, “On Open Membranes, Cosmic Strings and Moduli Stabilization,” Nucl.Phys. B728 (2005) 207-219 [hep-th/0507164].

[27] K. Becker, M. Becker and A. Krause, “Heterotic Cosmic Strings,” [hep-th/0510066].

[28] G. Curio and A. Krause, “Four-Flux and Warped Heterotic M-Theory Compactifications,” Nucl.Phys. B602 (2001) 172-200 [hep-th/0012152].
[29] G. Curio and A. Krause, “Enlarging the Parameter Space of Heterotic M-Theory Flux Compactifications to Phenomenological Viability,” Nucl.Phys. B693 (2004) 195-222 [hep-th/0308202].

[30] R. Donagi, A. Lukas, B. A. Ovrut and D. Waldram, “Holomorphic Vector Bundles and Non-Perturbative Vacua in M-Theory,” JHEP 9906 (1999) 034 [hep-th/9901009].

[31] E. I. Buchbinder, B. A. Ovrut and R. Reinbacher, “Instanton Moduli in String Theory,” JHEP 0504 (2005) 008 [hep-th/0410200].

[32] R. Friedman, J. Morgan and E. Witten, “Vector Bundles And F Theory,” Commun.Math.Phys. 187 (1997) 679-743 [hep-th/9701162].

[33] E. Witten, “Baryons And Branes In Anti de Sitter Space,” JHEP 9807 (1998) 006 [hep-th/9805112].

[34] A. Vilenkin and A. E. Everett, “Cosmic Strings and Domain Walls in Models with Goldstone and Pseudogoldstone Bosons,” Phys.Rev.Lett. 48 (1982) 1867.

[35] A. Lukas, B. A. Ovrut and D. Waldram, “On the Four-Dimensional Effective Action of Strongly Coupled Heterotic String Theory,” Nucl.Phys. B532 (1998) 43-82 [hep-th/9710208].

[36] A. Lukas, B. A. Ovrut, K. S. Stelle and D. Waldram, “Heterotic M-theory in Five Dimensions,” Nucl.Phys. B552 (1999) 246-290. [hep-th/9806051].

[37] J.-P. Derendinger and R. Sauser, “A Five-brane Modulus in the Effective N=1 Supergravity of M-Theory,” Nucl.Phys. B598 (2001) 87-114 [hep-th/0009054].

[38] G. Moore, G. Peradze and N. Saulina, “Instabilities in heterotic M-theory induced by open membrane instantons,” Nucl.Phys. B607 (2001) 117-154 [hep-th/0012104].

[39] S. Gukov, C. Vafa and E. Witten, “CFT’s From Calabi-Yau Four-folds,” Nucl.Phys. B584 (2000) 69-108; Erratum-ibid. B608 (2001) 477-478 [hep-th/9906070].

[40] E. Witten, “Non-Perturbative Superpotentials In String Theory,” Nucl.Phys. B474 (1996) 343-360 [hep-th/9604030].

[41] K. Becker, M. Becker and A. Strominger, “Fivebranes, Membranes and Non-Perturbative String Theory,” Nucl.Phys. B456 (1995) 130-152 [hep-th/9507158].
[42] E. Lima, B. A. Ovrut, J. Park and R. Reinbacher, “Non-Perturbative Superpotentials from Membrane Instantons in Heterotic M-Theory,” Nucl.Phys. B614 (2001) 117-170 [hep-th/0101049].

[43] E. Lima, B. A. Ovrut and J. Park, “Five-Brane Superpotentials in Heterotic M-Theory,” Nucl.Phys. B626 (2002) 113-164 [hep-th/0102046].

[44] E. I. Buchbinder, R. Donagi and B. A. Ovrut, “Superpotentials for Vector Bundle Moduli,” Nucl.Phys. B653 (2003) 400-420 [hep-th/0205190].

[45] E. I. Buchbinder, R. Donagi and B. A. Ovrut, “Vector Bundle Moduli Superpotentials in Heterotic Superstrings and M-Theory,” JHEP 0207 (2002) 066 [hep-th/0206203].

[46] S. Gukov, S. Kachru, X. Liu and L. McAllister, “Heterotic Moduli Stabilization with Fractional Chern-Simons Invariants,” Phys.Rev. D69 (2004) 086008 [hep-th/0310159].

[47] P. Horava, “Gluino Condensation in Strongly Coupled Heterotic String Theory,” Phys.Rev. D54 (1996) 7561-7569 [hep-th/9608019].

[48] A. Lukas, B. A. Ovrut and D. Waldram, “Gaugino Condensation in M-theory on $S^1/Z_2$,” Phys.Rev. D57 (1998) 7529-7538 [hep-th/9711197].

[49] E. Witten, “Some Properties of $O(32)$ Superstrings,” Phys.Lett. B149, 351 (1984).

[50] M. Dine, N. Seiberg and E. Witten, “Fayet-Illiopulos Terms in String Theory,” Nucl. Phys. B 289 (1987) 589.

[51] R. Blumenhagen, G. Honecker and T. Weigand, “Loop-Corrected Compactifications of the Heterotic String with Line Bundles,” JHEP 0506 (2005) 020 [hep-th/0504232].

[52] C. P. Burgess, R. Kallosh and F. Quevedo, “de Sitter String Vacua from Supersymmetric D-terms,” JHEP 0310 (2003) 056 [hep-th/0309187].

[53] T. W. B. Kibble, “Topology of Cosmic Domains and Strings,” J. Phys. A 9 (1976) 1387.

[54] E. Witten, “Cosmic Superstrings,” Phys. Lett. B 153, (1985) 243.

[55] M.B. Hindmarsh and T.W.B. Kibble, “Cosmic strings,” Rept.Prog.Phys. 58 (1995) 477-562 [hep-ph/9411342].