Two-loop QED Corrections to Bhabha Scattering

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Recent developments in the calculation of the NNLO corrections to the Bhabha scattering differential cross section in pure QED are briefly reviewed and discussed.

1. Status of the NNLO corrections

Bhabha scattering, \(e^+e^- \rightarrow e^+e^-\), is a crucial process in the phenomenology of particle physics. Its relevance is mainly due to the fact that it is the process employed to determine the luminosity \(\mathcal{L}\) at \(e^+e^-\) colliders: in fact, \(\mathcal{L} = \mathcal{N}_{\text{Bhabha}}/\sigma_{\text{th}}\), where \(\mathcal{N}_{\text{Bhabha}}\) is the rate of Bhabha events and \(\sigma_{\text{th}}\) is the Bhabha scattering cross section calculated from theory.

Two kinematic regions are of special interest for the luminosity measurements, since in these regions the Bhabha scattering cross section is comparatively large and QED dominated. At colliders operating at c.m. energies of \(\mathcal{O}(100\text{ GeV})\), the relevant kinematic region is the one in which the angle between the outgoing particles and the beam line is of few degrees. This, for instance, was the case at LEP, where the luminometers were located between 50 and 100 mrad, and it will be also the case at the future ILC (luminometers between 25 and 80 mrad \(\text{[1]}\)). At machines operating at c.m. energies of the order of 1 – 10 GeV, the region of interest is instead the one in which the scattering angle is large; as an example, at KLOE, the luminosity measurement involves Bhabha scattering events that take place at angles between 55\(^\circ\) and 125\(^\circ\) \(\text{[2]}\). Moreover, the large angle Bhabha scattering will be employed at the ILC in order to study the beam effects that lead to a non monochromatic luminosity spectrum \(\text{[3]}\).

Experimentally, Bhabha scattering is measurable with a very high accuracy. At LEP, the experimental error on the luminosity measurement was \(4 \cdot 10^{-4}\) \(\text{[4]}\). At ILC it is expected to be of the same order of magnitude or better (the goal of the TESLA forward calorimeter collaboration is to reach, in the first year of run, an experimental error of \(1 \cdot 10^{-4}\) \(\text{[5]}\)). This remarkable accuracy requires, as a counterpart, an equally precise theoretical calculation of the Bhabha scattering cross section, in order to keep the luminosity error small. Therefore, radiative corrections to the basic process have to be under control.

In order to match the detector geometry and experimental cuts of any particular machine, a Monte Carlo event generator is needed. In the recent past, several groups have been working on Monte Carlo generators for Bhabha scattering, in both the large-angle and small-angle kinematic regions. The LEP theoretical simulations of Bhabha events were based on BHLUMI \(\text{[6]}\), whose theoretical error, mainly due to missing higher order corrections, is estimated to be \(4.5 \cdot 10^{-4}\) (see for instance \(\text{[7]}\)). The KLOE collaboration employs the Monte Carlo event generators BABAYAGA \(\text{[8]}\) and BHAGENF \(\text{[9]}\), which have an estimated theoretical error of \(5 \cdot 10^{-3}\); within the error claimed, they are in agreement with each other. Moreover, BHWIDE \(\text{[10]}\) and MCGPJ \(\text{[11]}\) provided valuable checks. All the mentioned Monte Carlo programs for Bhabha scattering employ the mass of the electron as a
cut-off for collinear divergences; this is to be taken into account when calculating NLO ($\mathcal{O}(\alpha^3)$) and NNLO ($\mathcal{O}(\alpha^4)$) corrections to the cross section.

The complete $\mathcal{O}(\alpha^3)$ corrections to Bhabha scattering, in the full Electroweak Standard Model, have been known for a long time \[12\]. The corrections of $\mathcal{O}(\alpha^4)$ to the differential cross section in the Standard Model are not yet known. In recent years, several papers were devoted to the study of NNLO corrections in pure QED. In \[13,14,15\] the second order radiative corrections, both virtual and real, enhanced by factors of $\ln^n(s/m^2)$ (with $n = 1, 2$, $s$ the c. m. energy squared, and $m$ the mass of the electron) where studied. The complete set of these corrections was finally obtained in \[16\]. This was achieved by employing the QED virtual corrections for mass-less electron and positrons of \[17\] and the results of \[18\]. In \[20\], the complete set of photonic $\mathcal{O}(\alpha^4)$ corrections to the differential cross section that are not suppressed by positive powers of the ratio $m^2/s$ was calculated. The virtual corrections of $\mathcal{O}(\alpha^4)$ involving a closed fermion loop, together with the corresponding soft-photon emission corrections, were obtained in \[21\]: no mass expansion or approximation was employed, and the result retains the full dependence on the electron mass $m$. The calculation was performed by means of the Laporta-Remiddi algorithm \[22\] which takes advantage of the integration by parts \[23\] and Lorentz-invariance \[24\] identities in order to reduce the problem to the calculation of a small set of master integrals. The master integrals were calculated using the differential equations method \[25\]; their expression in terms of harmonic polylogarithms \[26\] is given in \[27\]. Several papers deal with the unapproximated calculation of the master integrals necessary for the evaluation of the photonic NNLO corrections \[28\]: in particular, the reduction to master integrals of these corrections is complete, and only the master integrals related to the two-loop boxes have not yet been all evaluated. The status of the calculation of these master integrals is discussed in \[29\]. In \[30\], finally, the unapproximated calculation of the photonic vertex contributions to the $\mathcal{O}(\alpha^4)$ Bhabha scattering cross section, as well as the calculation of the subset of radiative corrections due to the interference of one-loop diagrams (already considered in \[31\]) was completed. A few papers discuss the leading NNLO weak corrections to Bhabha scattering cross section \[32\].

In summary, the complete NNLO corrections to Bhabha scattering in the full Standard Model are still far from being completely known. Concerning the pure QED contributions, from a phenomenological point of view, all the numerically relevant corrections to the NNLO differential cross section are known, with the only exception of the ones arising from the production of soft pairs, for which only the terms enhanced by $\ln^n(s/m^2)$ with $n = 1, 2$ are present. If we consider instead the exact fixed order calculation, the situation is less satisfactory. The unapproximated two-loop QED photonic box contributions are still missing. Moreover, it would be interesting to evaluate the corrections arising from diagrams with closed loops of heavier fermions (like muons or taus).

2. The Small Mass Limit

At the colliders mentioned above, the mass of the electron is very small in comparison with the c. m. energy. Therefore, it is reasonable to expect that it can be safely ignored except in the...
terms where it acts as a cut-off for collinear divergencies, as it was done in obtaining the results of [20]. For the set of corrections obtained in [21] and [30], for which unapproximated analytic results are available, it is possible to determine the numerical relevance of the terms suppressed by positive powers of the ratio \( m^2/s \) as a function of the beam energy.

This analysis is performed in [30]. The three contributions taken into account are the one arising from graphs with a closed fermion loop (\( N_F = 1 \)), the photonic corrections involving at least a vertex graph (\( \text{Vertices} \)), and the interference of one-loop box diagrams (\( \text{Box Box} \)). For each contribution, the quantity

\[
D_i = \left( \frac{\alpha}{\pi} \right)^2 \left| \left( \frac{d\sigma_2^{(i)}}{d\Omega} - \frac{d\sigma_2^{(j)}}{d\Omega} \right) \right|_L \times \left( \frac{d\sigma_0}{d\Omega} + \left( \frac{\alpha}{\pi} \right) \frac{d\sigma_1}{d\Omega} \right)^{-1},
\]

with \( i = (N_F = 1) \), \( \text{Vertices} \), \( \text{Box Box} \), is plotted as a function of the beam energy \( E \). In Eq. (1) \( d\sigma_2^{(i)}/d\Omega \) is the unapproximated \( \mathcal{O}(\alpha^4) \) correction to the cross section taken into account. It includes the contribution of the virtual diagrams and the one of the corresponding diagrams with the emission of up to two soft photons (with energy smaller than the cut-off \( \omega \)). \( d\sigma_2^{(i)}/d\Omega|_L \) is the same quantity as \( d\sigma_2^{(i)}/d\Omega \), aside for the fact that the terms proportional to positive powers of the ratio \( m^2/s \) are neglected.

In Figs. 1, 2 and 3 the functions \( D_i \), evaluated for two different sample angles (10° and 90°), are shown. It is clear that the approximation in which terms proportional to positive powers of the ratio \( m^2/s \) are neglected is extremely good already at energies that are significantly smaller than the ones encountered in \( e^+e^- \) experiments.

The NNLO corrections arising from the two-loop photonic boxes are not taken into account in this analysis, since unapproximated result for that set of corrections are not available. Nevertheless, it is reasonable to expect in the \( m^2/s \to 0 \) limit a behavior similar to the one of the other NNLO corrections.

3. Results

In Fig. 4 all the QED contributions to the \( \mathcal{O}(\alpha^4) \) Bhabha scattering cross section known at the moment are plotted as a function of the scattering angle. Terms suppressed by positive powers of the ratio \( m^2/s \) were neglected.

The dotted line represents the photonic corrections [16,20]. The corrections of \( \mathcal{O}(\alpha^4(N_F = 1)) \) [21] (dashed line) have, for this choice of \( \omega \) (\( \omega = E \)), an opposite sign with respect to the photonic ones. Moreover they include large terms propor-
R. Bonciani and A. Ferroglia

Figure 4. Photonic, $N_F = 1$, and total contributions to the cross section at order $\alpha^4$. The beam energy is chosen equal to 0.5 GeV and the soft-photon energy cut-off $\omega$ is set equal to $E$.

Concerning the latter, only the terms proportional to $\ln(n(s/m^2))$ ($n = 1, 2, 3$) are known (see [15]). The dashed-dotted line in the figure represents the sum of the $O(\alpha^4(N_F = 1))$ cross section with the known terms of the pair production corrections\(^2\). The solid line, finally, is the complete order $\alpha^4$ QED Bhabha scattering cross section, including photonic, $N_F = 1$ and pair production contributions.

To conclude, we presented the status of the NNLO QED corrections to the Bhabha scattering differential cross section, paying particular attention to the phenomenological relevance of the terms that are suppressed by positive powers of the ratio $m^2/s$. It turns out that, in view of the precision required by present and future experiments, the NNLO QED corrections can not be neglected, and should be included in the Monte Carlo event generators.

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Two-loop QED Corrections to Bhabha Scattering

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