Experimental and Numerical Investigation on Reinforced Concrete Slab under Low Velocity Impact Loading

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Abstract. The effect of out plane dynamic loading on the reinforced concrete slab is evaluated in order to estimate the impact resistance of reinforced concrete under low velocity impact loading. The experimental study is performed on M40 ($\sigma_c = 48 \, MPa$) concrete slab specimen of size (1200 × 1200 × 50) mm reinforced with 6 $\, mm$ Fe500 bars using pendulum drop weight (60 kg) impacting the center of specimen at a drop height of 370 mm. The impact velocity is calculated using open-source software as 2.7 m/s. The experimental study is evaluated for the impact force, crack pattern and the failure mode of the specimen. The peak force was observed as 36 kN from the experiment. The finite element analysis is performed using concrete damage plasticity (CDP) model available in ABAQUS to validate the model incorporating strain rate effects to accurately predict the behavior under dynamic loading. The constitutive model incorporating strain rate effects successfully predicted the behavior under impact loading and was quite beneficial contrary to the tedious experimental procedures. Overall, the results thus obtained from the finite element analysis found to be closely matching with the experimental results.

1. Introduction

The concrete is a well-known construction material used for protective structures of national importance. Mostly structures experience quasi-static loadings during their lifespan nonetheless, some abnormal loadings such as blast and dynamic loadings are of significant importance. The construction practices involve the risk of unintended accidents from the failure of machinery or man-made mistakes and sometimes natural hazards such as typhoons and landslides. Such incidents may cause falling of objects on underlying structures and pose a threat to the safety of livelihood and durability of structure under impact. Therefore, the effect of falling weight over reinforced concrete slabs are of great importance.

Concrete is a quasi-brittle material which is strain rate sensitive and its characterization under dynamic loadings can be made accurately by taking the effect of strain rates. The most referred works for mechanical properties of concrete under dynamic loadings were reviewed and accumulated by Bischoff and Perry [1] and Fu et al. [2]. Although, concrete is extensively used, however it has a very complex material behavior. Unlike ductile materials which have analogous properties in tension and
compression and can be accurately predicted by von-misses stresses, concrete has complex behavior in compression and tension. The complexity arises from non-linearity in the tensile cracking, different strain hardening/softening in compression as well as tension, stress-strain relation, degradation of stiffness and the bond between the concrete and the rebars in RCC element. Furthermore, the confinement as well as strain rate dependency in dynamic loading conditions may further add to the complexity of modelling its material behaviour. The previous endeavours by investigators were mainly experimental in nature and predicts that mechanical properties of concrete under higher rate of loadings were higher as compared to their quasi-static properties [3–5]. Due to the ambiguity in fact that strain rate dependency is an intrinsic material property or due to inertial forces, it is still an active and controversial topic even today that validates the necessity of further experimental and numerical study.

Kishi et al. [6] performed numerical analysis on rectangular reinforced concrete slabs under falling weight impact loading subjected to different boundaries. The slab with line supports on four sides and particularly a slab with two line supports on two opposite sides-the other two sides free and slab with one line and two corner-point supports were considered. The results indicate that time history response of dynamic responses were well predicted and major crack patterns have been roughly predicted for each support conditions. Iqbal et al. [7] studied concrete plates of 800 × 800 mm, with 10-20% pretensioned impacted by 243 kg steel hammer dropping at centre of the span and it was varying from 500 to 1000 mm height. The drop impact resistance and energy absorption capacity of prestressed reinforced concrete slabs with 20% prestress was found to enhance upto 4.5 and 38 % cause of stiffness enhancement. The displacement was reduced by 28% as compared to non-prestressed concrete plates. Williams [8] found that the important parameter for concrete in tension include the tensile strength and the post cracking tension stiffening behaviour. Neglecting tension-stiffening effects in finite element method (FEM) models by using a simple tension cut off result in tensile stresses in the region of a crack having a spurious dependency on the mesh size. Pajak [9] stated that the dynamic tensile strength of concrete can achieve the value of 13 times its quasi-static strength. The tests conducted using hydraulic and drop hammer machine have lesser scattering than research data obtained using split Hopkinson pressure bar (SHPB). However, due to the lesser test duration the results obtained using technical scopes of the measure can have additional distortions. The continuum damage mechanics (CDM) is very well accepted for the wide range of problems in the industries as well as structural mechanics and it has been used widely in the IRIS 2012 related to the numerical modelling and simulations [10].

In this paper, a preliminary study has been conducted both experimentally and numerically to establish the effect of dynamic loadings on reinforced concrete slab. The experiment is carried out on conventional concrete with uniaxial compressive strength as \( f'_c = 48 \, MPa \), reinforced with Fe500 steel bars, see Section 2 for details on experimental campaign. The finite element simulations are performed using finite element software ABAQUS to predict the behavior of the reinforced concrete under varying strain rates, see Section 3. The response history of peak force is evaluated experimentally and numerically validated with the help concrete damage plasticity model (CDP). The parametric study has been conducted for the effect of failure strain for element removal algorithm and tension softening behavior of concrete, see Section 4.

2. Experimental Investigations

The concrete mix design for M40 grade concrete was prepared as per the guidelines of BIS: 10262 2009 [11]. The constituent materials such as cement, fine aggregate and coarse aggregates were tested as per the standard procedures. The mix ratio of (cement: sand: coarse aggregate - 1: 1.423: 1.927) and cement quantity of 500 kg/m³ with water-cement ratio of 0.35 was adopted for making concrete. Ordinary Portland cement (Grade 43) as per IS: 8112- 2013 [12] was used in experimental study. The mix incorporates fine aggregates with specific gravity 2.61 and coarse aggregate of nominal maximum size of 20 mm with specific gravity of 2.68 tested as per IS: 2386-1963 [13] and meets the specifications as per IS: 383-2016 [14]. Potable water was used for mixing and curing of specimen. Superplasticizer Master Galenium 51 (BASF, Mumbai, India) was used to facilitate the required flowability in the mix. To ease the flowability of concrete onto the mould the superplasticizer was employed for 0.25% of
weight of cement. The sample was water cured to a target unconfined compressive strength of $f_{cu} = 48 \text{ MPa}$ at 28 days. The two-way reinforced concrete (RC) slab specimen was designed as per the guidelines of IS: 456 2000 [15]. Fe500 steel bars ($d = 6 \text{ mm}$) was used for reinforcing the concrete in tension and dimension and reinforcement details were presented in Figure 1. The overall dimensions of RC slab were $(1200 \times 1200) \text{ mm}$ with a thickness of 50 mm. The spacing of reinforcement mesh for main and distribution steel was 250 mm and a clear cover of 100 mm was provided.

The experiment was performed using pendulum impact testing frame having a capacity of 250 kN, see Figure 2(a). The test was carried out on RC slab against 60 kg of mass of ogive nose impactor. The specimen was held in place using solid steel rods supporting at the bottom as well as top sides of the specimen is shown in Figure 2(b). The pendulum was released with help of the steel rope which are connected with the winch. The winch was designed for manual operations and having 8 mm diameter ropes. The translational velocity of impactor impacting the RC slab was measured from the experiment using open-source software “Tracker”. The impact force was measured with the help of load cell (250 kN) capacity attached to the impactor as shown in Figure 3.
3. Constitutive behaviour and Numerical modelling

The CDP model was used in the finite element simulation in order to predict the behavior of reinforced concrete slab against low velocity impact loading. Initially, the model was proposed by Lubiner et al. [16], later it was modified by Lee and Fenves [17] in order to predict the behavior of structural systems subjected to cyclic loading as well as dynamic loading. The model able to predict the elasticity and plasticity characteristics of concrete elements, the damage-dependent loading and unloading, stiffness degradation, irreversible strain as well as strain rate effects.

The CDP model considers non-associated plastic flow rule. The eccentricity parameter, m of the flow potential, $G$ with a value of 0.1 has been found out on comparing the experimental data from bi- and triaxial strength results [18,19].

Lim and Ozbakkaloglu [20] calibrated the following equations [Eqn. 1 and 2] for the ratio of the initial equiaxial to uniaxial compressive strength ratio $f'_b / f'_c$ and the second stress invariant parameter $K_c$ as:

$$f'_b / f'_c = 1.57 f_c^{-0.09}$$

$$K_c = 0.71 f_c^{-0.025}$$

(1)

The compressive and tensile damage parameters, $D_c(\varepsilon^{pl}_c)$ and $D_t(\varepsilon^{pl}_t)$ can be characterized using the degraded elastic compressive and tensile stiffness as $(1-D_c)E_c$ and $(1-D_t)E_c$ respectively, where $E_c$ is modulus of elasticity. Li et al. [21] suggested the tensile damage index of the form $\lambda E_c \varepsilon^{cr}_t / (\lambda E_c \varepsilon^{cr}_t + \sigma_t)$ with $\lambda = 0.2$. The compressive damage index with $\lambda = 0.5$ can be taken to yield like results. The damage parameters were defined for the CDP model as:

$$D_c(\varepsilon^{in}_c) = \frac{\varepsilon^{in}_c}{\varepsilon^{in}_c + 2\sigma_c/E_c} < 0 \cdot 8$$

$$D_t(\varepsilon^{cr}_t) = \frac{\varepsilon^{cr}_t}{\varepsilon^{cr}_t + 5\sigma_t/E_c} < 0 \cdot 8$$

(3)

Where $\varepsilon^{in}_c = \varepsilon_c - \sigma_c / E_c$ is the inelastic compressive strain; $\varepsilon^{cr}_t = \varepsilon_t - \sigma_t / E_c$ the cracking strain; and $E_c$ the modulus of elasticity:
\[ E_c = \left(3320\sqrt{f_c^2} + 6900\right)\left(\frac{\rho_c}{2320}\right)^{1.5} (MPa) \]  

(5)

Based on the lateral strain and axial strain relation, Li et al. [21] provided the dilation angle to be:

\[ \beta = \tan^{-1}\left(\frac{6(n_0 - n_c)}{3E_c \varepsilon_{cp} f_c^2 + 2(n_0 - n_c) - 3}\right) \]

(6)

Where \( n_0 = 8 \times 10^{-6} f_c^2 + 0.0002 f_c^0 + 0.138 \), the Poisson’s ratio [22] at the elastic condition whereas, \( n_c = 0.5 \) is the Poisson’s ratio of concrete at peak engineering stress [23]. The \( \varepsilon_{cp} \), strain at the peak compressive stress as follows:

\[ \varepsilon_{cp} = \mu_a \left(\frac{f_c^0}{E_c}\right)^{0.75} \left(\frac{\rho_c}{2320}\right) \]

(7)

Where \( \mu_a = 4.26 \) for the crushed aggregates or \( \mu_a = 3.78 \) for the rounded aggregates [24] and \( \varphi = 0.3 \). The viscosity parameter was used as 0.0001 to filter spurious high-frequency fluctuations of strain rate [25]. The constitutive parameters used for CDP model based on Eqns. (1), (2) and (5-7) are shown in Table 1.

The tensile behaviour under uniaxial compression was based on the stress-strain relationship proposed by Aslani and Jowkarmimandi [26] takes the form:

\[ s_c (\varepsilon_c) = \frac{n}{n - 1 + (\varepsilon_c / \varepsilon_{cp})^n} \]

(8)

Where \( s_c \) and \( \varepsilon_c \) are the engineering stress and strain, \( n = n_t = (1.02 - 1.17 \frac{f_c^0}{E_c \varepsilon_{cp}})^{-0.71} \) for \( \varepsilon_c \leq \varepsilon_{cp}, \) and \( n = n_t + a + 28b \) for \( \varepsilon_c > \varepsilon_{cp} \) in which \( a = 3.5(12 - 0.0166 f_c^0)^{0.46} \) and \( b = 0 \cdot 83\exp(- \frac{911}{f_c^0}) \), and \( \varepsilon_{cp} = (\frac{f_c^0}{E_c})\left(\frac{r}{r-1}\right) \), as strain at peak compressive stress in which \( r = \frac{f_c^0}{f_p} + 0.8 \).

The dynamic increase factor (DIF) under compression in light of uniaxial strain rate, \( \dot{\varepsilon} \), proposed by [27] is given below;

\[ DIF = \begin{cases} 1.026 \varepsilon_{6.35} \geq 1.0 & \dot{\varepsilon} \geq 30 s^{-1} \\ \gamma_s (\dot{\varepsilon} / \dot{\varepsilon}_0)^{1/3} & \dot{\varepsilon} \leq 30 s^{-1} \end{cases} \]

(9)

Where \( \alpha_s = \frac{1}{5 + 9(f_s / f_{sa})} \) and \( \log \gamma_s = 6.156 \alpha_s - 2 \), \( \dot{\varepsilon} = \) strain rate; \( f_s = 10 MPa \); and \( \dot{\varepsilon}_0 = -30 \times 10^{-6} \) s\(^{-1}\). The compressive behaviour defined in the CDP model using Eqn. 8 is shown in Figure 4(a) and compression damage parameters calculated using Eqn. 3 is shown in Figure 4(b). The DIF of concrete under uniaxial compressive is calculated using Eqn. 9 as 1.25 and 1.32 for \( \dot{\varepsilon} = 1 \) s\(^{-1}\) and 30 s\(^{-1}\), respectively and plotted in Figure 4(a).

The tensile behaviour of concrete for cracking stress (\( \sigma_t \)) and crack opening displacement (\( w \)) relationship proposed by Reinhardt et al. [28] was obtained based on static test as follows:

\[ \frac{\sigma_t}{f_t} = \left(1 + \left(c_1 \frac{w}{W_c}\right)^3\right) \exp\left(-c_2 \frac{w}{W_c} - \frac{w}{W_c} (1 + c_2)^3 \exp(-c_2)\right) \]

(10)

Where \( c_1 = 3; c_2 = 6.93 \) and \( W_c \) the critical crack opening displacement with free of stress is determined by \( W_c = 5.136 G_F / f_t \) in which \( G_F = 2.5 f_t \), is the total area under stress-crack opening displacement curve. The proposed fracture energy of concrete \( G_F \), can be obtained by fitting the experimental results, read [29] for details.

From the experimental data of dynamic stress versus crack opening displacement relationships by Weerheijm and Vegt [30] and a constant slope softening is more appropriate for concrete cracking response under high strain rates, as
\[
\sigma_t \frac{f_t}{f_c} = 1 - w / w_{ec}
\]

\( (11) \)

**Figure 4** Response of concrete (a) under compression. (b) Response of concrete under compression damage relation.

Where \( w_{ec} = 2G_f / f_t \). The crack strain \( \varepsilon_{cr}^T = w / L_c \), where \( L_c \) is element characteristic length is used to regularize the mesh-objectivity. \( L_c = 10 \text{ mm} \) is used in present study which satisfies the following condition, \( L_c < \frac{0.4 \Delta E_f}{G_f} \). The DIF under uniaxial tension presented by Malwar and Crawford [31] is used in this study. The DIF of concrete under uniaxial tension is calculated using [31] as 1.42 and 4.41 for \( \varepsilon = 1 \text{ s}^{-1} \) and 30 \text{ s}^{-1}, respectively. The tensile softening behaviour is presented in Figure 5(a) for linear softening using Eqn. 11 and 5(b) for exponential softening using Eqn. 10. The tensile damage relationship following the Eqn. 4 is shown in Figure 5(c) and 5(d).

**Table 1** Materials parameters of concrete.

| Elastic Parameters | Density \( \rho \) | Modulus of Elasticity \( E \) | Poisson’s Ratio \( \nu \) |
|--------------------|---------------------|-------------------------------|-----------------------------|
| Elastic Parameters | 2240 kg/m\(^3\)     | 28.368 GPa                    | 0.166                       |

**CDP Parameters**

| \( f_0^p \) / \( f_c^p \) | \( \beta \) | Eccentricity, \( m \) | \( K_c \) | Viscosity Parameter |
|---------------------------|------------|----------------------|--------|---------------------|
| 1.108                     | 39.023     | 0.1                  | 0.644  | 0.0001              |

**Table 2** Material properties of reinforcement.

| Elastic Properties | Density \( \rho \) | Modulus of Elasticity \( E \) | Poisson’s Ratio \( \nu \) |
|--------------------|---------------------|-------------------------------|-----------------------------|
| Elastic Properties | 7850 kg/m\(^3\)     | 200 GPa                       | 0.3                         |

**Average stress and plastic strain relationship of reinforcement**

| Plastic strain | Average stress (MPa) |
|----------------|----------------------|
| Strain rate 0/s| 296                  |
| Strain rate 1/s| 429                  |
| Strain rate 5/s| 455                  |
| Strain rate 15/s| 473                 |
| 0.1            | 605                  |
| 0.2            | 647                  |
| 655            | 661                  |
The plastic behaviour of steel reinforcement was incorporated using the classical von Mises yield criterion. The proposed bilinear average stress versus strain curve for reinforcement steel bars embedded in concrete by fitting experimental data, refer [32] for more details. The DIF and tensile strength of the steel reinforcement in light of strain rate proposed by Malvar [33] is used in present study. The input parameters for steel rebars has been presented in Table 2.

The finite element modelling and simulation was performed using ABAQUS/EXPLICIT. The concrete slab is modelled as 3-D deformable body and the impactor and boundary conditions provided were modelled as discrete rigid bodies. The schematic representations of the assembly are shown in Figure 6(a) and the type of element used for mesh generation is shown in Figure 6(b).
The concrete slab is modelled as C3D8R element considering eight node linear brick element with reduced integration and viscous hourglass control with varying element sizes from 7.5 mm at the centre to 10 mm at the corners. The steel reinforcement is modelled as B31 element, a 2-node linear beam in space with an element size of 10 mm and impactor is modelled as R3D4 element with 4-node three-dimensional bilinear rigid quadrilateral element. The analytical rigid supports at the front, back and bottom face rigidly constrained to the reference point as shown in Figure 6(a). “EMBEDDED” constraint was used to define the behaviour of steel reinforcement in concrete slab. The reference points for the rigid bodies have been given boundary condition as “ENCASTRE” to suppress the translational as well as rotational motion. The impactor has rigid body constraint to the attached reference point and inertia and translational velocity has been assigned at the reference point as 60 kg and 2.7 m/s respectively. The impactor is constraint to move only in translation direction in z axis as shown in Figure 6(a). Surface to surface interaction between impactor and RC slab with kinematic contact method was used. For other contacts General contact was used. Tangential behaviour with friction coefficients of 0.1 between impactor and RC slab and 0.5 for general contact has been used. “Hard” contact was used to define the normal behaviour between the surfaces. The dynamic analysis was performed using ABAQUS/EXPLICIT with time step of 0.01 s. The viscosity coefficients $b_1 = 0.06$ for linear term and $b_2 = 1.2$ for quadratic term available by default in ABAQUS, have been used. The bulk viscosity was almost 6% of critical damping of the systems. The field and history outputs were generated at a time increment of 40 μs.

4. Validation of model

The results in terms of impact force were predicted and compared, see Figure 7. The erosion parameters for validation purposes were used as $\varepsilon_t^{pl}$ and $\varepsilon_c^{pl}$ as 0.035 and 0.3 [34]. The failure strain for compressive strain was kept higher to avoid the excessive punching of the impactor onto the RC slab. The erosion strain for steel reinforcement was kept at 0.1. Linear tensile softening (Figure 5(a) and (c)) and exponential softening behavior (Figure 5(b) and (d)) of concrete was used for the validation purposes. The peak force for experimental results was found out to be 37.79 kN whereas the model predicts the peak impact force to be 37.21 kN. The model successfully predicts the peak force during
impact analysis. Using the power softening for tensile behaviour the comparison was made to predict the impact force. On using the power softening using Equation (10) given by [28] the peak force was 34.71 kN in comparison to 37.79 kN during experimental results as shown in Figure 7. The power softening has low effect on the peak force and a deviation of 7% was observed in comparison to experimental results. The brittle behavior of concrete results in lower energy absorption capacity of concrete under dynamic loading. The impact force profile for RC slab generally has primary peak followed by plateau force region. As can be seen that the primary peak in case of numerical simulation matches well with the experimental results, the plateau of impact force profile has lowered value as compared to numerical results. The discrepancies in results were observed owing to the reason that impact force measured by load cell is lower than contact force calculated by numerical model when the mass ratios (defined as ratio of mass of weight above load cell to the mass of head) are lower. During impact, the force measured by load cell is equal to inertial force which depends on mass of weight above load cell subtracted by inertial force due to the weight of head which results in lower values of plateau of force profile measured from experiments as compared to numerical results which calculates contact force at the point of impact [35].

![Figure 7 Comparison of power tensile softening with respect to impact force.](image)

The visualization of cracks at the tension face of the specimen are compared with experimental results in Figure 8. The failure of specimen was mainly due to the tensile failure and was more pronounced at the rear face of the specimen. The front face of the specimen was under compression where the scabbing occurred. The model was able to predict the horizontal macrocracks along the face of the slab. The propagation of cracks in experimental study has slight variation due to the boundary conditions. Owing to infinite stiffness of the rigid parts in the model, the contact between impactor and RC slab is perfectly plastic which has comprehensive amount of energy transfer to the RC slab resulting in excess damage and cracks to occur in numerical studies. More accurate behavior of crack propagation can be predicted by modelling the behavior of impactor and boundary conditions as steel sections.
5. Conclusion
The following conclusions have been drawn based on the experiment and numerical investigations:

- The incorporation of strain rate behaviour in CDP model is recommended for improving the accuracy of the model for dynamic related studies.
- The linear tensile softening behaviour is predicting the peak impact force/first rise more accurately as compared to power softening tensile behavior for dynamic loadings.
- The criteria for erosion strain should be calibrated as per the experimental results, however a general recommendation for 8-10 times the value of failure strain in linear tensile softening has been recommended for moderately reinforced concrete specimen.

6. References
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