Spin dynamics in a two dimensional quantum gas

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We have investigated spin dynamics in a 2D quantum gas. Through spin-changing collisions, two clouds with opposite spin orientations are spontaneously created in a Bose-Einstein condensate. After ballistic expansion, both clouds acquire ring-shaped density distributions with superimposed angular density modulations. The density distributions depend on the applied magnetic field and are well explained by a simple Bogoliubov model. We show that the two clouds are anti-correlated in momentum space. The observed momentum correlations pave the way towards the creation of an atom source with non-local Einstein-Podolsky-Rosen entanglement.

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Since the optical trapping of Bose-Einstein condensates (BECs) enabled the investigation of quantum gases with multiple spin components, spinor condensates have become a particularly rich research field [1, 2]. While initial work focused on an understanding of the ground state and dynamical properties of spinor condensates [3, 4], recent experiments have started to exploit their properties for applications in other fields. In particular, the production of entangled states through spin dynamics [5, 6] has spawned interest in spin dynamics for their applications in precision metrology [7, 8].

Spin dynamics in a trapped quantum gas are strongly influenced by the geometry of the confining potential. In particular, highly asymmetric optical traps provide a way to reduce the dimensionality of a trapped quantum gas, both with respect to the motional and spin degrees of freedom [9]. Thus, tailored confining potentials offer new avenues for exploiting spin dynamics, e.g., the generation of correlated pairs of atoms in well-defined motional states [10], similar to work on four-wave-mixing of ultra-cold atoms in an optical lattice [11, 12].

In this Rapid Communication, we investigate spin dynamics in a quantum gas confined to two dimensions (2D) by an optical lattice. We show how the spin excitation modes in the 2D potential lead to ring-shaped density distributions with superimposed angular density modulations. The density distributions depend on the applied magnetic field and are well explained by a simple Bogoliubov model. We show that the two clouds are anti-correlated in momentum space. The observed momentum correlations pave the way towards the creation of an atom source with non-local Einstein-Podolsky-Rosen entanglement.

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of 285 mG in a horizontal direction, which is maintained throughout the experiment. Following preparation in the lattice, the cloud has a finite thermal component: on average, we obtain $N_{\text{BEC}} = (4.2 \pm 0.7) \times 10^4$ atoms in the BEC and $N_{\text{th}} = (1.2 \pm 0.2) \times 10^5$ in a thermal fraction at temperature $\sim 120$ mK. The thermal component arises from a technical limitation in switching off our magnetic trap. Spin dynamics are initiated by applying an additional magnetic field along the vertical direction. This field is turned on with a linear ramp over 1 ms and adds vectorially to the magnetic bias field to set $q$; the resulting magnetic field is held constant for a variable evolution time. The spin dynamics are brought to an end by switching off the optical lattice.

We probe the result of the spin dynamics by Stern-Gerlach separation and absorption imaging along the vertical direction after 20 ms time-of-flight. To avoid saturating the optical depth of our imaging system for the $|0\rangle$ cloud, a third microwave pulse is applied just after the lattice is switched off to transfer part of the $|0\rangle$ population to $|F = 1, m_F = 0\rangle$, which is transparent to the imaging light [18]. To obtain accurate atom numbers, the imaging system was calibrated following [19, 20]. Finally, the point-spread function of the imaging system is well-described by a Gaussian function with $1/e^2$ waist 5.72 $\mu$m. A typical absorption image is shown in Fig. 1(a).

For this image the peak optical depth of atoms in state $|0\rangle$ was reduced to 1.5. The $|\pm1\rangle$ clouds have an interesting ring structure with a number of peaks on the circumference and will be discussed in detail below.

Figure 1(b) shows the time evolution of the population in $|\pm 1\rangle$ for $q = -67$ Hz in terms of the relative population $(N_{-1} + N_1)/(N_{-1} + N_0 + N_1)$. The data have been fitted by an exponential function $\propto \exp(t/\tau)$ from 0-8 ms. This dependence is expected because Eq. (1) describes an initial exponential amplification analogous to parametric down conversion in non-linear optics [21]. The fitted value of the time constant was $\tau = 3.0 \pm 0.2$ ms. After 8 ms, the population deviates from exponential growth, indicating the breakdown of the ‘linear regime’ in which depletion of $|0\rangle$ can be neglected. Additionally, we begin to observe a small population in $|\pm2\rangle$ at this time: at 8 ms, the relative population in $|\pm2\rangle$ is on average 0.25%, i.e., on the order of our resolution limit.

The spin dynamics that transfer atoms from $|0\rangle$ to $|\pm1\rangle$ exhibit a clear dependence on $q$, as shown in Fig. 1(c). The data show a broad peak centered at $q = -100$ Hz and for higher $|q|$, the population in $|\pm1\rangle$ drops gradually to zero. The $q$-dependence is also evident in the density distributions after time-of-flight. Figure 2(a) shows absorption images along the vertical axis for several values of $q$. The spatial structure of the clouds changes from a singly-peaked distribution at low $|q|$ to a ring-shape with density modulations around the circumference at higher $|q|$.

Typically, ring-shaped density distributions in such time-of-flight images indicate the presence of orbital angular momentum [22], and the appearance of density modulations around the circumference could be interpreted as the matter-wave interference pattern of two or more angular momentum eigenstates [11, 23]. For large $|q|$, one may obtain a good understanding of the ring formed in a simplified, ‘free-space’, picture. In this picture, one can regard the spin-changing collision as scattering two atoms from the stationary BEC into $|\pm 1\rangle$ in a ring in momentum space with radius $p_{\text{rms}} = \sqrt{2 m E} / q$. Momentum conservation requires that the two scattered atoms propagate in opposite directions, as indicated in Fig. 2(a) for $q = -382$ Hz. Although, in principle, the system exhibits polar symmetry, bosonic stimulation breaks the symmetry leading to azimuthal modulations, which result in the forma-
tion of counter-propagating wave packets. Interestingly, these wave packets should comprise an Einstein-Podolsky-Rosen (EPR) entangled pair in momentum $|\pm \rho_{\text{rms}}\rangle$ and spin state $|\pm 1\rangle$.\cite{24,25}

To elaborate on the qualitative picture presented so far, we investigate the excitation spectrum of the $|\pm 1\rangle$ states that arises from the Hamiltonian in Eq. (1). In the Thomas-Fermi approximation, $n_0$ in each lattice site has the shape of the inverted confining potential. Accordingly, atoms in $|\pm 1\rangle$ experience a flat potential bottom plus a small parabolic repulsion $U_1 n_0$.\cite{26} Outside the $|0\rangle$ BEC, the potential rises steeply and the effective potential may be approximated by a cylindrical box $\{1\}$. Figure 3(a) shows the effective potential experienced by $|\pm 1\rangle$ and the cylindrical box approximation. This motivates the simplified Hamiltonian for excitations of the system

$$
H_{ex} = \sum_{m=\pm 1} \int d^2 r \delta \dot{\psi}_m^\dagger \left( -\frac{\hbar^2}{2m} \nabla^2 + U_1 n_0 + q \right) \delta \dot{\psi}_m \\
+ U_1 \int d^2 r n_0 \left( \delta \dot{\psi}_1^\dagger \delta \dot{\psi}_{-1}^\dagger + \delta \dot{\psi}_1 \delta \dot{\psi}_{-1} \right). \tag{2}
$$

The eigenstates of the cylindrical box potential are \cite{26},

$$
\varphi_{nl}(\rho, \phi) = \frac{1}{\sqrt{\pi \rho_0}} J_{|l|+1}(\beta_{nl}) \frac{J_{|l|}((\beta_{nl} \rho)/\rho_0)^{2/3}}{(2M \rho_0^2)} e^{i\phi}, \tag{3}
$$

where $\rho_0$ is the Thomas-Fermi radius of the cloud. $J_{|l|}$ are Bessel functions of the first kind, and $\beta_{nl}$ is the $n$th zero of $J_{|l|}$. The eigenstates $\varphi_{nl}$ have energies $\epsilon_{nl} = \hbar^2 \beta_{nl}^2/(2M \rho_0^2)$. The substitution of $\delta \dot{\psi}_m = \sum_{nl} \varphi_{nl} \dot{\varphi}_{nlm}$ into Eq. (2) leads to the matrix elements $\langle \varphi_{n'l'}|\varphi_{nl}\rangle$. Cylindrical symmetry of $n_0$ and orthogonality of $\varphi_{nl}$ for different $l$ yields $\langle \varphi_{n'l'}|n_0\rangle|\varphi_{nl}\rangle = 0_{l'l'} \langle \varphi_{nl}|n_0\rangle|\varphi_{n'l'}\rangle$. Additionally, a numerical evaluation of the matrix elements shows that $\langle \varphi_{n'l'}|n_0\rangle|\varphi_{n'l'}\rangle / \delta_{n'n'} \delta_{l'l'} \approx \delta_{n'n'} \delta_{l'l'} (n_0|\varphi_{nl}\rangle$ is a good approximation. With this simplification, Eq. (2) may be put into a symmetric form and diagonalized through a Bogoliubov transformation. The excitation energies are

$$
E_{nl}^\pm = \pm i \sqrt{(U_1(n_0|\varphi_{nl}\rangle)^2 - (\epsilon_{nl} + U_1(n_0|\varphi_{nl}\rangle + q)^2}. \tag{4}
$$

The eigenvalues $E_{nl}^\pm$ are either real or imaginary depending on the interplay of $\epsilon_{nl}$, $U_1(n_0|\varphi_{nl}\rangle$, and $q$: a real $E_{nl}^\pm$ determines the standard phase evolution of an eigenstate; an imaginary value describes unstable evolution, i.e., growth or decay in the amplitude of an eigenstate. In the following, we focus on the unstable evolution and refer to $E_{nl}^\pm |h$ as instability rates.

For our experimental parameters, the instability rates form a dense ‘forest’ of overlapping resonances. The density of atoms sets the width and peak value of each resonance, and it is the extreme compression along the lattice symmetry axis that causes many modes to overlap. For $N_{\text{BEC}} = 4.2 \times 10^4$, the central site has $9 \times 10^3$ atoms, and the peak value of the mean field repulsion $U_1 n_0$ is 55 Hz (see Fig. 3(a)). To illustrate how the modes change in shape as a function of $l$, Fig. 3(b) shows $E_{nl}^\pm$ for $n = 1, l = 0, 1, \ldots, 9$ in the central lattice site. The peak instability rate is reached by the mode $(1, 0)$ due to its large overlap with the BEC in $|0\rangle$. Figure 3(c) shows the instability rates for all relevant modes in the range $q = 0$ to $-500$ Hz: it is clear that a single $q$ value supports many unstable modes.

We may quantify the multi-mode character of the spin dynamics by investigating the instability rates at a given $q$. We focus on the high $|q|$ “free-space” regime. Figure 3(d) shows the instability rates in a narrow interval around $q = -400$ Hz, for which 19 modes are unstable. We quantify multi-mode amplification by the ratio $\delta E/E$, where $E/h$ is the mean value of the instability rate of the unstable modes, and $\delta E/h$ is the mean difference of instability rate between modes. For $\delta E/E \ll 1$, all modes grow in population almost equally on a timescale $(4\pi E/h)^{-1}$, as though degenerate in instability rate. In the case of the most unstable modes in Fig. 3(d), $\delta E/h \sim 2$ Hz and $E/h \sim 30$ Hz, for which the time scale for population growth is 2.7 ms, consistent with the experimentally measured value of 3.0 ms (see Fig. 1(b) and associated discussion). Only on a timescale $(4\pi E/h)^{-1} \sim 40$ ms will the evolution of different unstable modes become resolvable, but this lies in the non-linear regime, well outside the 8 ms evolution time we employ (see Fig. 1(b)).

The expansion of excitations in $|\pm 1\rangle$ in terms of states with angular momentum arises naturally from the cylindrical symmetry of the optical lattice, but this choice of basis is arbitrary. In the ‘free-space’ picture, two atoms from the stationary BEC undergo a momentum-conserving spin-changing collision: in the cylindrical basis, one spin state gains positive angular momentum $L$ while the other gains $-L$, and the total angular momentum remains zero.

We simulate the experimental results by forming a superposition state $\psi$ in each lattice site comprised of the set $\{\nu\}$ of modes that have a finite instability rate at a given $q$. In light of earlier work \cite{11,24,26}, we form a general superposition state

![Figure 3](image-url)
structures observed in the experiment at high $q$
for larger $q$, this arises from the $\sim 50$ Hz repulsive bump $U_{1n0}$ (see Fig. 2(a)), and at high $|q|$, the box potential underestimates the energy imparted to the scattered atoms due the finite slope of the walls. One can obtain a simple estimate of the cloud radius in the high $|q|$ regime using the ‘free space’ picture: if each wave packet gains $p_{\text{rms}} = \sqrt{2Mq}$, the position of an atom assuming ballistic expansion is given by $\langle \rho \rangle = \sqrt{\rho_0^2 + (p_{\text{rms}}t/M)^2}$, where $\rho_0$ is the in-trap radius and $t$ is the time-of-flight.

To study the structure around the circumference of the experimental and simulated density distributions, we employ an angular density correlation function. Momentum conservation in the collision process requires that a density peak at angle $\theta$ in a $|\pm 1\rangle$ cloud leads to an anti-correlated density peak located at $\theta' = \theta - \pi$ in the $|\pm 1\rangle$ component (see Fig. 2(a) for $q = -382$ Hz). We define the correlation function $c(\alpha) = \langle \tilde{n}_{-1}\theta \rangle \bar{n}_{+1}(\theta - \alpha) \rangle (\langle \tilde{n}_{-1}\rangle \langle \bar{n}_{+1} \rangle)$, where $\tilde{n}_{\pm 1}$ are the two angular density distributions and the angled brackets denote the mean over $\theta$. The correlation function for $q = -297$ Hz is shown in Fig. 3(b) and exhibits the expected anti-correlation between the two clouds. In order to analyze the $l$-mode composition of the states generated by the spin dynamics, we also show the auto-correlation function for the experimental and simulated density distributions. A comparison of these indicates that only a subset of the energetically allowed $l$ modes contribute to the observed experimental states. Nonetheless, the auto-correlation function confirms that several $l$ states contribute to the observed distributions because $c(\alpha)$ is approximately flat away from the peak at $\alpha = 0$ due to the destructive interference of several frequency components; in the ‘free space’ picture, this amounts to random spontaneous symmetry breaking of the polar symmetry.

In conclusion, we have investigated spin dynamics in a 2D quantum gas and have observed qualitatively new features such as ring-shaped density distributions in time-of-flight. A theoretical analysis of the spin excitation modes in the system provided a qualitative understanding of the ring-shaped clouds and showed their angular structure is due to the interference of several modes. In future work, we will gain a full understanding of the unstable spin modes by studying the dynamics in a single site of the optical lattice. Finally, tailoring the trapping potential further will enable the preparation of anti-correlated wave-packets of quantum degenerate atoms with the aim of producing an atom source with non-local EPR entanglement.

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We model the transfer to the optical lattice by assuming that the atoms are ‘frozen’ by the lattice at $s = 10$. The BEC in the relaxed magnetic trap is taken to have a Thomas-Fermi profile along the lattice symmetry axis and we numerically integrate the profile over successive intervals of $\lambda/2$ to obtain the atom number in each site.