Model-independent Constraints on Cosmic Curvature: Implication from Updated Hubble Diagram of High-redshift Standard Candles

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Abstract

The cosmic curvature ($\Omega_k$) is a fundamental parameter for cosmology. In this paper, we propose an improved model-independent method to constrain the cosmic curvature, which is geometrically related to the Hubble parameter $H(z)$ and luminosity distance $D_L(z)$. Using the currently largest $H(z)$ sample from the well-known cosmic chronometers, as well as the luminosity distance $D_L(z)$ from the relation between the UV and X-ray luminosities of 1598 quasars and the newly compiled Pantheon sample including 1048 SNe Ia, 31 independent measurements of the cosmic curvature $\Omega_k(z)$ can be expected covering the redshift range of $0.07 < z < 2$. Our estimation of $\Omega_k(z)$ is fully compatible with flat universe at the current level of observational precision. Meanwhile, we find that, for the Hubble diagram of 1598 quasars as a new type of standard candle, the spatial curvature is constrained to be $\Omega_k = 0.08 \pm 0.31$. For the latest Pantheon sample of SNe Ia observations, we obtain $\Omega_k = -0.02 \pm 0.14$. Compared to other approaches aiming for model-independent estimations of spatial curvature, our analysis also achieves constraints with competitive precision. More interestingly, it is suggested that the reconstructed curvature $\Omega_k$ is negative in the high-redshift region, which is also consistent with the results from the model-dependent constraints in the literature. Such findings are confirmed by our reconstructed evolution of $\Omega_k(z)$, in the framework of a model-independent method of Gaussian processes (GP) without assuming a specific form.

Unified Astronomy Thesaurus concepts: Quasars (1319); Cosmological parameters (339); Hubble diagram (759)

1. Introduction

The spatial curvature of the universe, i.e., whether the space of our universe is open, flat, or closed is one of the most fundamental issues in particle physics and modern cosmology. Its value, or even its sign, is closely related to the fundamental Copernican principle assumption and the Friedmann–Lemaître–Robertson–Walker (FLRW) metric (Cao et al. 2019a; Qi et al. 2019a), an exact solution of the Einstein’s equations obtained under the assumption of homogeneity and isotropy of space. Meanwhile, a possible detection of a nonzero curvature also bears important information of many important problems such as the evolution of our early universe (Ichikawa et al. 2006; Clarkson et al. 2007; Gong & Wang 2007; Virey et al. 2008; Cao et al. 2019b), as well as the accelerating expansion of the late-time universe, which is supported by the observations of Type Ia supernovae (SNe Ia; Riess et al. 1998; Perlmutter et al. 1999) in combination with independent estimates of cosmic microwave background (CMB; Ade et al. 2016), ultra-compact structure in intermediate-luminosity radio quasars (Cao et al. 2017a, 2017b), and strongly gravitationally lensing systems (SGL; Cao & Zhu 2012; Cao et al. 2012, 2015; Ma et al. 2019a). Let us note that, although a spatially flat universe is favored at a very high confidence level by the current popular observations (especially the latest Planck-2016 results of CMB; Ade et al. 2016), the previous measurements of cosmic curvature are indirect and model-dependent, which strongly depend on a specific model for dark energy (e.g., the cosmological constant $\Lambda$; Di Valentino et al. 2020; Handley 2019). However, considering the strong degeneracy between the spatial curvature and the dark energy equation of state, model-independent estimation for the spatial curvature from different popular probes has been performed in the literature (Qi et al. 2019b).

The most straightforward technique to constrain the cosmic curvature is by confronting the theoretical Hubble diagram (reconstructed by the Hubble parameter measurements) with the observed luminosity distances to the objects whose redshifts are known (Clarkson et al. 2008). This test has been fully implemented with updated observational SNe Ia data acting as standard candles (Shafee & Clarkson 2010; Mortsell & Onsson 2011; Sapone et al. 2014; Cai et al. 2016). However, considering the uncertainty caused by nuisance parameters characterizing SNe Ia light curves (Li et al. 2016; Rana et al. 2017; Wang et al. 2017; Wei & Wu 2017), an improved model-independent test of cosmic curvature to $z \sim 3.0$ has recently been performed with ultra-compact structures in radio quasars as standard rulers (Cao et al. 2019b). Meanwhile, Takada & Doreo (2015) proposed that the combined radial and angular diameter distances from the baryon acoustic oscillation (BAO) can be used to constrain the curvature parameter, with the achievable accuracy of such $\Omega_k$ measurement at $\Delta \Omega_k \sim 10^{-3}$. Another method was also put forward to carry out a test in the framework of the sum rule of distances along null geodesics of the FLRW metric, by employing strong lensing observations (Einstein radius or time delays) and supernova distance measurements (Räsänen et al. 2015; Denissenya et al. 2018). More recently, such a method has been applied to the latest data set of strong lensing systems in combination with intermediate-luminosity quasars calibrated as standard rulers (Qi et al. 2019b). It is interesting to note that, considering the cross-correlation between foreground mass and gravitational shear of background galaxies, the assumed lens model has a considerable impact on the cosmic curvature constraint, which slightly favors a spatial closed universe. In addition, some recent studies have also discussed the possibility of extending the above analysis to the simulated data of gravitational waves from future gravitational wave
detectors, which can be considered as a standard siren to provide the information of luminosity distance (Jiménez 2018).

In this paper, we will focus on a method that actually delivers estimations of the curvature parameter at different redshifts (Clarkson et al. 2007), using the current observations of standard candle data (quasars, SNe Ia) and standard clock data (Hubble parameters \( H(z) \) inferred from cosmic chronometers). We first reconstruct the function of luminosity distance with respect to redshift \( z \) from two different standard candle data, depending on the parameters characterizing the nonlinear relation between the X-ray and UV luminosities of quasars, as well as the light-curve fitting parameters from the SNe Ia sample. Next, with the Hubble parameter measurements taken into consideration, we directly transform the above observations to \( \Omega_k \) at different redshift, and thus achieve cosmological model-independent constraints on the spatial curvature. Compared with the previous works, measurements of \( \Omega_k \) from observations at different redshift could not only achieve a stringent measurement of the spatial curvature in a direct geometric way, but also call into doubt the FLRW metric and cosmic homogeneity and isotropy (Denissenya et al. 2018; Cao et al. 2019a; Qi et al. 2019a). It is clear that, for the purpose of implementing the method of Clarkson et al. (2007), it would be beneficial to use distance probes covering higher redshifts thus taking advantage of a larger sample of \( H(z) \) observations. For \( H(z) \) data, it can be derived from differential ages of galaxies (a "cosmic chronometer (CC)") and the radial BAO scale in the galaxy distribution. In this analysis, we update the largest distance data through the Hubble diagram of 1598 quasars \((z < 5.100)\) (Risaliti & Lusso 2018) and the Pantheon catalog of 1048 SNe Ia \((z < 2.3)\) (Scolnic et al. 2018), based on which the cosmic curvature at each specific redshift (corresponding to the redshift of each Hubble parameter measurement) could be directly obtained. When the latest quasar sample is used, the spatial curvature is constrained to be \( \Omega_k = 0.08 \pm 0.31 \). For the Pantheon SNe Ia, we obtain \( \Omega_k = -0.02 \pm 0.14 \). These results, in the context of model-independent estimations for spatial curvature, consistently favor a spatially flat universe.

Finally, we use the model-independent method Gaussian processes (GP) to reconstruct the evolution of the curvature of the universe. Our results indicate that a better quality of the observational data sets is also required to detect a tiny cosmic curvature more precisely, which will also be discussed in this paper.

This paper is organized as follows. In Section 2, we give a brief introduction of the theoretical method and the data used in this work. Section 3 investigates the constraints these data put on the cosmic curvature. Finally, the conclusions are presented in Section 4.

2. Method and Observation Data

It is well known that in the FLRW metric, the luminosity distance \( D_L(z) \) can be expressed as

\[
D_L(z) = \frac{c(1 + z)}{H_0} \sin \left[ \sqrt{\Omega_k} \int_0^z \frac{dz'}{E(z')} \right],
\]

where \( H_0 \) denotes the Hubble constant, \( c \) is the speed of light, and \( E(z) = H(z)/H_0 \) is the dimensionless Hubble parameter.

\[3\] Note that the expansion rate measures obtained from BAO observations are possibly dependent on the assumed fiducial cosmological model.

\[4\] In this work, we adopt the prior of the Hubble constant \( H_0 = 67.36 \pm 0.54 \) km s\(^{-1}\) Mpc\(^{-1}\) from the latest Planck CMB observations (Ade et al. 2016).

The curvature parameter \( \Omega_k \) is related to the dimensionless curvature \( K \) as \( \Omega_k = -Ke^2/a_0^2H_0^2 \), where \( a_0 \) is the present value of the scale factor, and \( K = +1, -1, 0 \) corresponds to closed, open, and flat universe. For convenience, we denote \( \sin(x) = \sin(x), x, \sinh(x) \) for \( \Omega_k < 0, = 0, > 0 \), respectively. The derivative of Equation (1) will generate the cosmic curvature \( \Omega_k \), which can be directly determined by using the Hubble parameter and transverse comoving distance as (Clarkson et al. 2007)

\[
\Omega_k = \frac{[H(z)D_L'(z)]^2 - c^2}{[H_0D(z)]^2},
\]

where \( H(z) \) is the expansion rate at redshift \( z \). The luminosity distance \( D_L(z) \) is simply related to the transverse comoving distance \( D_L \) as \( D_L = D_L(z)/(1 + z) \) (Hogg 1999), while \( D_L' = dD_L/(dz) \) denotes the derivative with respect to redshift \( z \). Thus we should use current observational data sets to reconstruct \( D(z) \) and \( D'(z) \), independently, and combine these two reconstructions with independent \( H(z) \) measurements to derive \( \Omega_k \).

2.1. Distance from Quasars and Type Ia Supernovae Observations

As the brightest sources in the universe that can be observed up to redshift \( z \sim 8.0 \), quasars (Mortlock et al. 2011; Bañados et al. 2018) have long been considered as potential candidates for extending the distance far beyond the limits imposed by supernovae \((z \sim 2.0)\). Different from the fundamental property of the cosmological standard candle (standardized luminosity), the nonlinear relation between the X-ray and UV luminosities of quasars can be parameterized as a linear dependence

\[
\log_{10}(L_X) = \gamma \log_{10}(L_{UV}) + \beta,
\]

where \( L_X \) and \( L_{UV} \) are the rest-frame monochromatic luminosities at 2 keV and 2500 Å, while \( \gamma \) and \( \beta \) denote the slope parameter and the intercept. Based on the well-known flux—luminosity relation, the nonlinear relation between the X-ray and UV fluxes of quasars can be written as

\[
\log_{10}(F_X) = \log_{10}(F_{UV}) + 2(\gamma - 1)\log_{10}(D_L) + \beta',
\]

where \( D_L \) is luminosity distance, \( F_X \) and \( F_{UV} \) represent the X-ray and UV flux, and the intercept is rewritten as \( \beta' = (\gamma - 1)\log_{10}(4\pi) + \beta \). One may clearly see that such a relation could provide a potential cosmological probe, i.e., if there is no redshift evolution of the relation, the observed X-ray flux is a function of the observed UV flux, the redshift, and the luminosity distance. Therefore, relevant cosmological parameters can be inferred by fitting this relation to different quasar samples with multiple observations available (Risaliti & Lusso 2015, 2017; Lusso & Risaliti 2016, 2017).

In this paper, we use the newly built Hubble diagram of quasars from a parent sample of 7237 sources, covering the redshift range of 0.036 < \( z < 5.100 \) (Risaliti & Lusso 2018). Note that three “Cleaning” criteria are used to derive the final 1598 sources from the parent sample, in the framework of three filters including X-ray absorption, observational contaminants in the UV, and Eddington bias. The final sample is built merging five groups of quasars: 791 sources from the SDSS-DR7 sample, 612 sources from the SDSS-DR12, 102 sources...
from XMM-COSMOS, 18 sources from the low-redshift sample (Swift), 19 sources from Chandra-Champ, 38 sources from the high-z sample (z > 4), and 18 sources from the new z ∼ 3 sample (XMM-Newton Very Large Program). The final results indicated that such a refined quasar sample could effectively mitigate the large dispersion in the $L_X - L_{UV}$ relation, with more accurate slope determination $\gamma = 0.633 \pm 0.002$ and smaller dispersion $\delta = 0.24$. With a tractable amount of scatter avoiding possible contaminants and unknown systematics, a Hubble diagram of quasars could provide new measurements of the cosmic expansion at higher redshifts ($z < 5.10$), which has never been explored by any other cosmological probes. The scatter plot of 1598 quasars is shown in Figure 1, using the most recent QSO compilation from Risaliti & Lusso (2018). Note that in this analysis, we will treat the slope $\gamma$ and the intercept $\beta$ as two nuisance parameters. The intrinsic dispersion is also taken as a free parameter $\sigma_{int}$ contributing to the intrinsic scatter.

To reconstruct $D(z)$, we use the SNe Ia Pantheon data set released by the Pan-STARRS1 (PS1) Medium Deep Survey, which contains 1048 SNe Ia data ranging from $0.01 < z < 2.3$ (Scolnic et al. 2018). Compared with the previous SNe Ia samples extensively discussed in the previous works, i.e., high-z data ($z > 1.0$) from the SCP survey (Suzuki et al. 2012), the GOODS (Riess et al. 2007) and CAN-DELS/CLASH surveys (Grauer et al. 2014; Rodney et al. 2014; Riess et al. 2018), the Pantheon catalog extends the Hubble diagram to $z = 2.26$, with the combination of the subset of 279 PS1 SNe Ia (Rest et al. 2014; Scolnic et al. 2014) $(0.03 < z < 0.68)$ and useful distance estimates of SNe Ia from SDSS, SNLS, various low-z and HST samples (Scolnic et al. 2018). The observed distance modulus of each SNe Ia is given by

$$
\mu = m_B - M_B + \alpha \cdot X_t - \beta^* \cdot C + \Delta M + \Delta B,
$$

where $m_B$ is the apparent $B$-band magnitude, $M_B$ is the absolute $B$-band magnitude, $C$ is the color parameter quantifying the relation between luminosity and color, and $X_t$ is the light-curve shape parameter quantifying the relation between luminosity and stretch. Note that distance corrections based on the host-galaxy mass ($\Delta M$) and predicted biases from simulations ($\Delta B$) are also taken into account. Based on the new approach called BEAMS with Bias Corrections (BBC; Kessler & Scolnic 2017), the nuisance parameters in the Tripp formula (Tripp 1998) were retrieved and the observed distance modulus is simply reduced to $\mu = m_B - M_B$. We transform the distance modulus $m_B - M_B$ given in the data set to $D_L$ using

$$
D_L(z) = 10^{\mu(z)/5-5}(\text{Mpc}).
$$

Following the strategy of Scolnic et al. (2018), six sources of uncertainties are included in the distance modulus in the Pantheon data set, i.e., the uncertainty from the photometric error ($\sigma_{\text{phot}}$), the mass step correction ($\sigma_{\text{mass}}$), the distance bias correction ($\sigma_{\text{bias}}$), the peculiar velocity uncertainty and mass measurement uncertainty ($\sigma_{\text{vel}}$), the lensing uncertainty due to the LOS mass distribution ($\sigma_{\text{lens}}$), and the intrinsic scatter ($\sigma_{\text{int}}$). For this analysis, the total statistical uncertainty is modeled as

$$
\sigma^2_{\text{SN}} = \sigma^2_{\text{phot}} + \sigma^2_{\text{mass}} + \sigma^2_{\text{bias}} + \sigma^2_{\text{vel}} + \sigma^2_{\text{lens}} + \sigma^2_{\text{int}} \quad \text{(Scolnic et al. 2018)}.
$$

The Pantheon SNe Ia sample and the above error strategy has been widely applied to place stringent limits on cosmological parameters (Qi et al. 2018), provide accurate measurements of the speed of light (Cao et al. 2018) and the cosmic opacity at higher redshifts (Ma et al. 2019b; Qi et al. 2019c). Moreover, in Figure 2, we illustrate the dependence of apparent $B$-band magnitude on redshifts, derived from 1048 SNe Ia data covering the redshift range of $0.01 < z < 2.3$.

Obtaining these observational data points of $D_L(z)$, we can use different methods to reconstruct $D(z)$ and its derivative $D'(z)$. In order to achieve model-independent estimation for the cosmic curvature, we perform an empirical fit to the luminosity distance measurements, based on a third-order logarithmic polynomial of Risaliti & Lusso (2018),

$$
D_L(z) = \ln(10)c/H_0(x + a_1x^2 + a_2x^3),
$$

where $x = \log(1 + z)$, and $a_1$ and $a_2$ are the two constants that need to be optimized and determined by flux measurements of quasar data and apparent $B$-band magnitudes of SNe Ia data. Then, we carry out the Markov Chain Monte Carlo method to obtain the best-fit values and their uncertainties of parameters by using a Python module called emcee (Foreman-Mackey et al. 2013)\(^5\). For the Hubble diagram of the quasar, the parameters ($\gamma$, $\beta$, and $\delta$ characterizing the $L_X - L_{UV}$ relation, $a_1$ and $a_2$ characterizing the luminosity distance) are optimized by

\(^5\) https://pypi.python.org/pypi/emcee
minimizing the $\chi^2$ objective function

$$\chi^2_{\text{QSO}} = \sum_{i=1}^{1598} \left[ \log_{10}(F_X) - \Phi([F_{\text{UV}}], D_L[z_i]) \right]^2 / \sigma^2_{\text{QSO}},$$

(8)

where $\Phi([F_{\text{UV}}], D_L[z_i])$ is defined as

$$\Phi([F_{\text{UV}}], D_L[z_i]) = \gamma \log_{10}(D_L) + 2(\gamma - 1) \log_{10}(D_L) + \beta'.$$

(9)

The variance $\sigma^2_{\text{QSO}} = \delta^2 + \sigma^2_i$ is given in terms of the global intrinsic dispersion ($\delta$), and the $i$th measurement error of ($F_X$).

For the Hubble diagram of SNe Ia, the parameters ($M_B$ characterizing the distance modulus, $\alpha_1$ and $\alpha_2$ characterizing the luminosity distance) are optimized by minimizing the $\chi^2$ objective function

$$\chi^2_{\text{SNe}} = \sum_{i=1}^{1048} \frac{[m_{\text{ps}} - m_{\text{th}}]^2}{\sigma^2_{\text{SNe}}},$$

(10)

where $\sigma_{\text{SNe}}$ accounts for error in SNe Ia observations propagated from the covariance matrix (Scolnic et al. 2018). The marginalized probability distribution of each parameter and the marginalized 2D confidence contours are presented in Figures 3 and 4.

2.2. The Expansion Rate Measurements $H(z)$

The expansion rate at any redshift, i.e., the Hubble parameter, is defined as $H(z) = \dot{a}/a$, where $a$ denotes the scale factor and $\dot{a}$ represents its derivative with respect to cosmic time $t$. Jiménez & Loeb (2002) proposed a model-independent method to calculate the expansion rate of the universe by the differential age evolution of passively evolving galaxies

$$H(z) = \frac{\dot{a}}{a} = -\frac{1}{1 + z} \frac{dz}{dt}. $$

(11)

By measuring the age difference between the two galaxies at different redshifts, the Hubble parameter $H(z)$ can be directly obtained from the so-called differential age (DA) or cosmic chronometer approach (in which one calculates the value of $dz/dt$). Actually, $H(z)$ data can also be obtained through the detection of radial BAO from galaxy clustering in redshift surveys (Gaztañaga et al. 2009; Blake et al. 2012; Busca et al. 2013; Samushia et al. 2013; Xu et al. 2013; Font-Ribera et al. 2014; Delubac et al. 2015). However, some recent studies indicated that the expansion rate measurements obtained from BAO observations are possibly dependent on the assumed fiducial cosmological model and the prior for the distance to the last scattering surface from CMB observations (Li et al. 2016). Therefore, in our analysis, we use only the latest 31 DA $H(z)$ measurements in the redshift range of $0.070 < z < 1.965$, which is compiled and presented in Figure 5.

Now we combine the reconstructions of $D(z)$ and $D'(z)$ with $H(z)$ measurements, which can be applied to the derivation of $\Omega_k(z)$ in Equation (2). We stress again that the cosmic curvature test is model independent, so we need not assume any cosmological model, and the two data sets of cosmic chronometers and standard candles are also independent of each other.

2.3. The Gaussian Processes (GP)

In order to reconstruct the evolution of the cosmic curvature $\Omega_k(z)$, the Gaussian process (GP) method will be adopted in the following analysis, with which one can perform a reconstruction of a function and its derivatives from a given data set without assuming any cosmological models. Such an approach, which was originally proposed by Seikel et al. (2012) and extensively applied in various studies in the literature,
Here f(z) is the function that depends on the mean value μ(z) and the covariance function Cov(f(z), f(\tilde{z})) = k(z, \tilde{z}). In the framework of such a mathematical formalism, a random function f(z) without any observations can be generated using the covariance matrix from the GP, based on the observed data and the values of the corresponding slopes at z. For a set of input points \( Z = \{z_i\} \), one can generate a vector of function values at \( Z^* \) as

\[
f^* = N(\mu^*, K(Z^*, Z^*))\tag{12}
\]

Similarly, the observational data can be written as

\[
Y = N(\mu, K(Z, Z) + C), \tag{13}
\]

where C is the covariance matrix of the data. Using the values of y at Z, one can reconstruct the mean and covariance of \( f^* \) as

\[
\tilde{f}^* = \mu^* + K(Z^*, Z)[K(Z, Z) + C]^{-1}(y - \mu) \tag{14}
\]

and

\[
\text{Cov}(f^*) = K(Z^*, Z^*) - K(Z^*, Z)[K(Z, Z) + C]^{-1}K \times (Z, Z^*), \tag{15}
\]

Note that the derivative of the function f(z) can also be calculated through the covariance function.

The crucial task in Gaussian process techniques is to determine the covariance function, with which one can derive the quantities at some redshifts at which they have not been directly measured. In this paper, we focus on the squared exponential covariance function to correlate the values of cosmic curvature at the two different redshifts (z and \( \tilde{z} \)):

\[
k(z, \tilde{z}) = \sigma^2_f \exp\left(-\frac{(z - \tilde{z})^2}{2\ell^2}\right). \tag{16}
\]

Here \( \ell \) quantifies the characteristic length in the x-direction to get a significant change in f(z), whereas \( \sigma_f \) denotes the corresponding typical change in the y-direction. The two hyperparameters (\( \ell \) and \( \sigma_f \)) characterizing the bumpiness of the function can be constrained from the observational data. It should be pointed out that compared with other choices of covariance functions (the Matérn and Cauchy covariance function), the advantage of the squared exponential covariance function lies in its effective reconstruction of the derivative of a function (Seikel et al. 2012). Such an issue has been extensively discussed in the recent studies of Zheng et al. (2020), which also found the insignificant differences between reconstructions performed with different covariance functions (the Matérn, Cauchy, and the squared exponential covariance function). Therefore, in the following analysis, the zero mean function and the squared exponential covariance function will be applied to obtain the reconstructed \( \Omega_k(z) \). Such a model-independent method is executed in the publicly available code called GaPP (Gaussian Processes in Python; Seikel et al. 2013).

3. Results and Discussion

By applying the abovementioned procedure to the quasar distance reconstruction and 31 Hubble parameter measurements, we obtain the results shown in Figure 6 to determine the \( \Omega_k(z) \) at each point z, which we want to reconstruct. The uncertainty of these measurements is calculated from propagated uncertainties of D(z), \( D'(z) \) and H(z). In principle, the function \( \Omega_k(z) \) can be reconstructed from observations, and the FLRW metric is ruled out if \( \Omega_k(z) \) is not constant. However, at lower redshifts, the errors become large due to the poor reconstructions of D(z) and \( D'(z) \) in that region. The accuracy of these measurements improves with increasing redshift z. In Figure 6, it is shown that all of the reconstructed \( \Omega_k(z) \) is consistent with the vanishing cosmic curvature within the 1σ limit. Therefore, estimation of the spatial curvature using H(z) and quasars is fully compatible with flat universe at the current level of observational precision. Then we can give the weighted mean of the present value of curvature density parameter, based on the most straightforward and popular way of summarizing multiple measurements, i.e., inverse variance.
D(z) + D′(z) reconstruction and thus the effectiveness of this model-independent test (a tiny change in the reconstructed D(z) and D′(z) would result in a very significant change to the nearby Ω_k measurements). In order to investigate the impact of sample incompleteness on Ω_k estimation, we also carry out the analysis by dividing the full sample into different subsamples given their redshifts and fitting a constant Ω_k in each subsample. The redshifts of the H(z) data span from z = 0.07 to z = 1.965, so we divide the H(z) measurements into five groups with z < 0.5, 0.5 < z < 1.0, 1.0 < z < 1.5, and 1.5 < z < 2.0. The first group has 19 H(z) measurements with redshifts z < 0.5, the second group has 5 H(z) with redshifts 0.5 < z < 1.0, the third group has 4 H(z) with redshifts 1.0 < z < 1.5, and the fourth group contains 3 H(z) with 1.5 < z < 2.0. The cosmic curvature parameter can be obtained as Ω_k = 0.34 ± 0.03, Ω_b = 0.03 ± 0.02, Ω_b = 0.31 ± 0.02, and Ω_k = −0.22 ± 0.19 at the 68.3% confidence level, respectively. Note that the derived curvature is negative in the high-redshift region, which is also consistent with the results from the model-dependent constraints in the literature (Cai et al. 2016).

Finally, an accurate reconstruction of Ω_k(z) can considerably improve our understanding of the inflation models and fundamental physics (Cai et al. 2016; L’Huillier & Shafieloo 2017; Shafieloo et al. 2018). In order to investigate the evolution of Ω_k(z) without assuming a specific form, a model-dependent method of Gaussian processes (GP; Seikel et al. 2012) can be employed to reconstruct the cosmic curvature from the observational data in a straightforward manner, without any parametric assumption regarding the cosmological model. Figure 8 shows the Ω_k parameter as a function of redshift for the two different cases, derived from the combined data sets of H(z)+QSO (upper) and H(z)+SNe Ia. One could note that a universe with zero curvature (spatially flat geometry) is strongly supported by the available observations. This is the most unambiguous result of the current data sets. Moreover, the accuracy of these measurements strongly depends on redshift z and the quality of the observational data, including the Hubble parameter measurements, quasar and SNe Ia sample. We expect that as the precision of the future data improves, especially at higher redshifts, our approach will yield an even more accurate determination of Ω_k. Interestingly, although the constraint by using quasar flux measurement data is not obvious improving compared to using SNe Ia, it help us have a deeper understanding of the cosmic curvature at an earlier stage of the universe. Such an issue has been extensively discussed in many previous works (Cao et al. 2019b; Qi et al. 2019).

4. Conclusions

In this paper, we have used a new model-independent method to test the cosmic curvature, based on the current H(z) observations from the well-known cosmic chronometers, as well as the luminosity distance D_L(z) from the relation between the UV and X-ray luminosities of quasars (Risaliti & Lusso 2018) and the newly compiled SNe Ia data (Pantheon sample; Scolnic et al. 2018). Our results show that 31 independent measurements of the cosmic curvature can be expected, covering the redshift range of 0.07 < z < 2 and the approach initiated in Clarkson et al. (2007) can be further developed. First, we reconstruct a function of luminosity distance D_L(z) and its derivative D′(z), with the currently largest compilation of two types of standard

\[ \Omega_k = \sum \left( \frac{\Omega_k,i}{\sigma_{\Omega_k,i}} \right)^2, \]

\[ \sigma_{\Omega_k}^2 = \sum \frac{1}{\sigma_{\Omega_k}^2}, \quad (17) \]

where \( \Omega_k \) stands for the weighted mean of cosmic curvature and \( \sigma_{\Omega_k} \) is its uncertainty. We find that, from the quasar and cosmic chronometer observations, model-independent estimation for the spatial curvature is \( \Omega_k = 0.08 \pm 0.31 \). This is fully in agreement with the constraints obtained from the latest Planck CMB measurements (Ade et al. 2016). Moreover, one issue that should be discussed is the comparison of our cosmological results with those of earlier studies done using alternative probes. More specifically, the precision of this estimation is comparable to that derived from the current estimation of the cosmic curvature from the recently compiled set of 120 intermediate-luminosity quasars (ILQSO) observed in a single-frequency VLBI survey (Cao et al. 2019b; Qi et al. 2019b). Such a conclusion is also consistent with the recent analysis of Räisänen et al. (2015), which discussed constraints on cosmic curvature by combining the strong lensing and supernova distance measurements, in the framework of another model-independent test based on the distance sum rule.

When the Pantheon SNe Ia is used, it would increase the chance of finding significantly different \( \Omega_k \) at different redshifts, in the case when the FLRW metric breaks down on some large scale. The results are shown in Figure 7. More importantly, we obtain that the spatial curvature is model independently constrained to be \( \Omega_k = -0.02 \pm 0.14 \), which suggests that there is no significant signal to indicate the deviation of the cosmic curvature \( \Omega_k \) from zero at the current observational data level (H(z) and SNe Ia). Compared with what was obtained from the quasars, there is an improvement in precision when the Pantheon SNe Ia is considered, in the context of model-independent testing of the cosmic curvature. However, there are several sources of systematics we do not consider in the above analysis and that remain to be clarified for this methodology. In particular, it is apparent that sample incompleteness, which affects the number of available H(z) measurements, will also play an important role in the

Figure 7. Thirty-one measurements of the spatial curvature parameter \( \Omega_k \) and its details (upper right) from the Hubble diagram of Pantheon SNe Ia sample and expansion rate measurements of cosmic chronometers.
of GP (Seikel et al. 2012) is employed to reconstruct the cosmic curvature from the observational data in a straightforward manner. Our results show that a universe with zero curvature (spatially flat geometry) is strongly supported by the available observations, which is the most unambiguous result of the current data sets.

Future observations will improve the constraints on the cosmic curvature. On the one hand, a properly calibrated UV—X-ray relation in quasars has a great potential of becoming an important and precise distance estimator in cosmology. A more accurately measured quasar sample observed by SDSS (Shen et al. 2011; Paris et al. 2017) and XMM (Rosen et al. 2016), particularly at high redshifts, should provide an even more stringent constraint on $\Omega_k$. On the other hand, more accurate measurements of Hubble parameters may also improve the effectiveness of our approach in the future. For instance, the Extended Baryon Oscillation Spectroscopic Survey (eBOSS) will compile 250,000 new, spectroscopically confirmed luminous red galaxies, which yield measurements of $H(z)$ with 2.1% precision (Dawson et al. 2016). These upcoming improvements on the precision of cosmic curvature estimation will be of great significance for understanding the evolution of the universe and the nature of dark energy.

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**References**

Ade, P. A. R., Aghanim, N., Arnaud, M., et al. (Planck Collaboration) 2016, A&A, 594, A13

Bahados, E., Venemans, B. P., Mazzucchelli, C., et al. 2018, Natur, 553, 473

Blake, C., Brough, S., Colless, M., et al. 2012, MNRAS, 425, 405

Busca, N. G., Delubac, T., Rich, J., et al. 2013, A&A, 552, A96

Cai, R., Gao, Z., & Yang, T. 2016, PhysD, 93, 043517

Cao, S., Biesiada, M., Jackson, J., et al. 2017a, JCAP, 02, 012

Cao, S., Biesiada, M., Jackson, J., et al. 2017b, A&A, 606, A15

Cao, S., Pan, Y., Biesiada, M., Godlowski, W., & Zhu, Z. H. 2012, JCAP, 03, 016

Cao, S., Qi, J. Z., Biesiada, M., et al. 2018, ApJ, 860, 57

Cao, S., Qi, J. Z., Biesiada, M., et al. 2019a, PDU, 24, 100224

Cao, S., Qi, J. Z., Cao, Z., et al. 2019a, NatSR, 9, 11608

Cao, S., Zheng, X. G., Biesiada, M., et al. 2017b, A&A, 606, A15

Cao, S., & Zhu, Z. H. 2012, A&A, 538, A43

Clarkson, C., Bassett, B., & Lu, T. H. C. 2008, PhRvD, 10, 013010

Clarkson, C., Cortes, M., & Bassett, B. 2007, JCAP, 08, 011

Dawson, K. S., Kneib, J. P., Percival, W. J., et al. 2016, AJ, 151, 44

Delubac, T., Bautista, J. E., Busca, N. G., et al. 2015, A&A, 574, A59
Denissenya, M., Linder, E. V., & Shafieloo, A. 2018, JCAP, 03, 041
Di Valentino, E., Melchiorri, A., & Silk, J. 2020, NatAs, 4, 196
Font-Ribera, A., Kirkby, D., Busca, N., et al. 2014, JCAP, 5, 027
Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, PASP, 125, 306
Gaztañaga, E., Cabré, A., & Hui, L. 2009, MNRAS, 399, 1663
Gong, Y. G., & Wang, A. 2007, PhRvD, 75, 043520
Graur, O., Rodney, S. A., Maoz, D., et al. 2014, ApJ, 783, 28
Handley, W. 2019, PhRvL, submitted (arXiv:1908.09139)
Hogg, D. W. 1999, arXiv:astro-ph/9905116
Ichikawa, K., Kawasaki, M., Sekiguchi, T., & Takahashi, T. 2006, JCAP, 12, 005
Jiménez, J. 2018, JFM, 842, P1
Jiménez, R., & Loeb, A. 2002, ApJ, 573, 37
Kessler, R., & Scolnic, D. 2017, ApJ, 836, 56
L’Huillier, B., & Shafieloo, A. 2017, JCAP, 2017, 015
Li, Z. X., Wang, G. J., Liao, K., & Zhu, Z. H. 2016, ApJ, 833, 240
Liu, T. H., Cao, S., Zhang, J., et al. 2019, ApJ, 886, 94
Lusso, E., & Risaliti, G. 2016, ApJ, 819, 154
Lusso, E., & Risaliti, G. 2017, A&A, 602, A79
Ma, Y. B., Cao, S., Zhang, J., et al. 2019a, EPJC, 79, 121
Ma, Y. B., Cao, S., Zhang, J., et al. 2019b, ApJ, 887, 163
Mortlock, D. J., Warren, S. J., Venemans, B. P., et al. 2011, Natur, 474, 616
Mortell, E., & Onsson, J. 2011, arXiv:1102.4485
Paris, I., Petitjean, P., Ross, N. P., et al. 2017, A&A, 597, 79
Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565
Qi, J. Z., Cao, S., Biesiada, M., et al. 2018, RAA, 18, 66
Qi, J. Z., Cao, S., Pan, Y., Li, J., et al. 2019c, PDU, 26, 100038
Qi, J. Z., Cao, S., Zhang, S. X., et al. 2019b, MNRAS, 483, 1104
Qi, J. Z., Cao, S., Zheng, C. F., et al. 2019a, PhRvD, 99, 063507
Rana, A., Jain, D., Mahajan, S., & Mukherjee, A. 2017, JCAP, 03, 028
Räsänen, S., Bolejko, K., & Inoguenov, A. 2015, PhRvL, 115, 101301
Rest, A., Scolnic, D., Foley, R. J., et al. 2014, ApJ, 795, 44
Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009
Riess, A. G., Rodney, S. A., Scolnic, D. M., et al. 2018, ApJ, 855, 126
Riess, A. G., Strolger, L. G., Casertano, S., et al. 2007, ApJ, 659, 98
Risaliti, G., & Lusso, E. 2015, ApJ, 815, 33
Risaliti, G., & Lusso, E. 2017, AN, 338, 329
Risaliti, G., & Lusso, E. 2018, NatAs, 3, 272
Rodney, S. A., Riess, A. G., Strolger, L. G., et al. 2014, AJ, 148, 13
Rosen, S. R., Webb, N. A., Watson, M. G., et al. 2016, A&A, 590, A1
Samushia, L., Reid, B. A., White, M., et al. 2013, MNRAS, 429, 1514
Sapone, D., Majerotto, E., & Nesseris, S. 2014, PhRvD, 90, 023012
Scolnic, D. M., Jones, D. O., Rest, A., et al. 2018, ApJ, 859, 101
Scolnic, D., Rest, A., Riess, A., et al. 2014, ApJ, 795, 45
Seikel, M., Clarkson, C., & Smith, M. 2012, JCAP, 6, 036
Seikel, M., Clarkson, C., & Smith, M. 2013, GaPP: Gaussian Processes in Python, Astrophysics Source Code Library, ascl:1303.027
Shafieloo, A., & Clarkson, C. 2010, PhRvD, 81, 083537
Shafieloo, A., Kim, A. G., & Linder, E. V. 2012, PhRvD, 85, 123530
Shafieloo, A., L’Huillier, B., & Starobinsky, A. A. 2018, PhRvD, 98, 083526
Shen, Y., Richards, G. T., Strauss, M. A., et al. 2011, ApJS, 194, 45
Suzuki, N., Rubin, D., Lidman, C., et al. 2012, ApJ, 746, 85
Takada, M., & Doré, D. 2015, PhRvD, 92, 123518
Tripp, R. 1998, A&A, 331, 815
Virey, J. M., Talon-Esmieu, D., Ealet, A., Taxil, P., & Tilquin, A. 2008, JCAP, 12, 008
Wang, G.-J., Wei, J. J., Li, Z. X., Xia, J. Q., & Zhu, Z. H. 2017, ApJ, 847, 45
Wei, J. J., & Wu, X. F. 2017, ApJ, 838, 160
Wu, Y., Cao, S., Zhang, J., et al. 2020, ApJ, 888, 113
Xu, X., Cuesta, A. J., Padmanabhan, N., Eisenstein, D. J., & McBride, C. K. 2013, MNRAS, 431, 2834
Zheng, X., Liao, K., Biesiada, M., et al. 2020, ApJ, 892, 103