Modeling influence of fault zone on stress distribution in rock mass with regard to the accumulated elastic energy

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Abstract. Based on the mathematical model of structurally inhomogeneous rock mass, the problem of stress distribution nearby a disjunctive break ahead of the stoping face is considered. The model describes the property of rocks to accumulate elastic energy in the form of internal self-balancing stresses. The problem is solved numerically in the quasi-static formulation using the finite element method. It is shown that the presence of the fault zone can provoke release of the accumulated elastic energy and, thereby, change stress distribution in surrounding rock mass.

1. Introduction
It is common knowledge that stress state of rocks and stability of underground openings are governed by the internal structure of rock mass. One of the essential properties of a structurally non-uniform rock mass is the ability to accumulate external action energy in the form of internal self-balancing stresses. A real rock mass is composed of hierarchical blocks, and such structure can have many equilibriums states. Some rock mass zones can accumulate and then release energy under certain conditions, i.e. they act as potential sources of energy [1–8]. This property has been observed in laboratory-scale experiments and in field studies [9, 10].

As a rule, energy is accumulated at boundaries of blocks or in fault zones when shear stress on the fault edges is balanced by friction between the edges. The balance is disrupted under certain conditions, and the accumulated energy releases. This process can either be stable relaxation or unstable catastrophe [11, 12]. Such events are particularly hazardous when they occur near working face.

The problem of stresses and strains in rock mass adjacent to an underground opening is one of the key problems of rock mechanics. Many researchers have addressed different formulations of this problem, and its solution is in a varying degree used in all technologies of underground mineral mining. Special interest is provoked by formulations on redistribution of stresses in the adjacent rock mass under effect of various disjunctions [13–18]. Taking account of structural hierarchy and internal self-balancing stresses is possible in the framework of different approaches. In [19, 20] the mathematical apparatus and models with internal variables are developed; in [21, 22] the approach based on the method of non-Archimedean analysis is used while in [23, 24] the discrete element method finds application. In this paper, we take mathematical considerations from [8, 21] and analyze numerically deformation of rock mass with a disjunctive break nearby a working face.
2. Mathematical model
Let us consider a two-scale model of rock mass in the initial stress state [8, 21] (plane-strain deformation). At microscale elastic particles (grains) occur at the points of a square lattice. Pore space is filled with binding material with different elastic characteristics than the grains have (Figure 1). Plastic shearing between particles develops nonlinearly, with the stages of strengthening, strength loss and residual strength. The grain boundary sliding diagram relating tangential forces and shears ($\varepsilon_{ij}^R$) is shown in figure 2 (a piecewise linear approximation). External loading causes deformation of grains and shearing between them. This, in turn, results in deformation of pore space. Elastic stresses in grains and in pore space are balanced by shearing forces at the grain contacts and, thus, the element of the medium has a certain energy margin. Internal self-balancing stresses can be considerable even in case of absence of external stresses.

The grain boundary sliding conditions are approximated by a piecewise linear function (figure 2). In this case, the increments in the microstresses and microshears are related as:

$$\Delta \varepsilon_{12}^R = \Delta \tau_{12} / G_i^p, \quad \Delta \varepsilon_{21}^R = \Delta \tau_{12} / G_2^p,$$

(1)

where $G_i^p, G_2^p$ are the present moduli of grain boundary sliding for each family of boundaries. The values $G_i^p$ are defined by the preset constants $\gamma_i, \gamma_i^{max}, \tau_i^{max}, \tau_i^{res}$, where $i = 1, 2$ is the number of a family of boundaries.

$$G_i^p = \begin{cases} G_i^p, & 0 \leq \gamma_i < \gamma_i^p, \\ -G_i^p, & \gamma_i^p \leq \gamma_i < \gamma_i^{max}, \\ 0, & \gamma_i^{max} \leq \gamma_i. \end{cases}$$

The constitutive equations of the model at macroscale relate the macrostress increments $\Delta \sigma_{ij}$ and macrostrain increments $\Delta \varepsilon_{ij}$

$$\begin{pmatrix} \Delta \varepsilon_{11} \\ \Delta \varepsilon_{22} \\ \Delta \varepsilon_{12} \end{pmatrix} = W \cdot \left( (T + R)^{-1} + 2(T + P)^{-1} \right) \cdot W^{-1} \cdot \begin{pmatrix} \Delta \sigma_{11} \\ \Delta \sigma_{22} \\ \Delta \sigma_{12} \end{pmatrix},$$

(2)
Coefficients in equation (2) depend on the microproperties of grains (matrix $T$), pore-filling material (matrix $P$), grain boundary sliding moduli (matrix $R$) and the angle $\alpha$ responsible for the orientation of regular packing of grains in the Cartesian coordinate system (matrix $W=W(\alpha)$). In turn, the macrostresses $\sigma_{ij}$ and microstresses $t_{ij}$ of grains and pore-filling material $p_{ij}$ are not mutually independent and fulfill the condition of compatibility [21] given by:

$$\sigma_{ij}^* = t_{ij} + 2m \cdot p_{ij}, \quad i, j = 1, 2;$$

here, $\sigma_{ij}^*$ are the macrostresses at the components of axes in the coordinate system of regular packing of grains (figure 1); the dimensionless value $0 < m < 1$ characterizes porosity [21].

Conditions (3) imply that one and the same macrostresses $\sigma_{ij}^*$ can conform with many stress states of grains $t_{ij}$ and pore space $p_{ij}$. Furthermore, the absence of macrostresses means no equal absence of internal microstresses which can be alternating and totally balancing each other.

The system of constitutive equations (2) and the strain–displacement relation is closed by the equilibrium equations (for increments)

$$\frac{\partial \Delta \sigma_{11}}{\partial x_1} + \frac{\partial \Delta \sigma_{12}}{\partial x_2} + \Delta X_1 = 0, \quad \frac{\partial \Delta \sigma_{12}}{\partial x_1} + \frac{\partial \Delta \sigma_{22}}{\partial x_2} + \Delta X_2 = 0.$$

where $\Delta X_1, \Delta X_2$ are the increments of mass forces.

In view of the nonlinearity of the grain boundary sliding diagram, the problem is formulated in terms of increments. The general solution is constructed by steps of loading with regard to the change in properties during deformation: $\sigma_{ij}^{k+1} = \sigma_{ij}^k + \Delta \sigma_{ij}^k, \quad \varepsilon_{ij}^{k+1} = \varepsilon_{ij}^k + \Delta \varepsilon_{ij}^k$, where the superscript $k$ is a number of iteration.

The analysis of the type of the system (2), (4) shows that for the descending branch of the grain boundary sliding diagram, there exists a critical slope $G^p$ governed by shear moduli and Poisson’s ratios of grains, $\mu^l, \nu^l$, and pore space, $\mu^p, \nu^p$. When $G^p$ is lower than a critical value, the system is elliptical and deformation process is stable. In case that the slope exceeds the critical value, the process of deformation is unstable and jumps of strength loss can take place in the medium. In this study, we limit ourselves to the stable deformation parameters.

Based on the model (2), (4), the finite element-based algorithm and computer program have been developed, which allow solving quasi-static plane boundary value problems.

3. Numerical modeling results
The computational domain models a working face (Figure 3). We select linear distribution of macrostresses: $\sigma_{22}^0 = -\gamma (H - x_2), \sigma_{11}^0 = \xi \sigma_{22}^0, \sigma_{12}^0 = 0$, where $\gamma$ is the specific gravity of rocks mass, $H$ is the depth of mining; $\xi$ is the lateral earth pressure coefficient. Aside from the macrostresses, it is required to find initial microstress state to satisfy relations (3). We choose the distribution of microstresses depending on the parameter $q$: $t_{11}^0 = 0.5 \sigma_{11}^0, \quad t_{22}^0 = 0.5 \sigma_{22}^0, \quad t_{12}^0 = \pm q t_{ij}^{\max}, \quad p_{ij}^0 = (\sigma_{ij}^0 - t_{ij}^0) / 2m, \quad i, j = 1, 2$ and analyze two problems. In the first problem, it is assumed that shear microstresses on grains and, consequently, in pore space are absent in the computational domain, i.e. $q = 0$ everywhere. In the sliding diagram in figure 2, this state conforms with the point $O$ (origin of coordinates). In the second problem, a disjunction break in rock mass adjacent to the working face
(colored grey in figure 3) is considered, and it is assumed that shear microstrains of grains in the fault make a certain part of the critical stress, e.g. \( q = 0.9 \), while it is still \( q = 0 \) in the adjacent rock mass. In the sliding diagram, this state is shown by the points \( A \) or \( B \) depending on the shear microstress sign.

\[ \Gamma_0 \text{ is the stoping void boundary; } 0 \leq \Delta d^k \leq 1 \] is the increment in the loading parameter at a \( k \)-th step. The condition \( \sum_k \Delta d^k = 1 \) means total unloading (absence of macrostresses) of the stoping void boundary. At the computational domain boundary \( \Gamma_1 \), it is assigned that \( \Delta u_n \big|_{\Gamma_1} = 0 \), \( \Delta \tau_n \big|_{\Gamma_1} = 0 \), where \( \Delta u_n \) is the increment in the normal component of displacement vector.

We assume non-dimensional values of parameters (all stress dimensions are related to the peak shear stress \( \tau_{1}^{\text{max}} \) in Figure 2; length dimensions — to the mineral bed thickness \( h \) in Figure 3):

\[ \mu' = 2.5 \cdot 10^3, \quad v' = 0.2, \quad \mu^p = 0.7 \cdot 10^3, \quad v^p = 0.3, \quad \gamma^* = \gamma^*_2 = 5 \cdot 10^{-4}, \quad \gamma^*_1 = \gamma^*_2 = 10^{-3}, \]

\[ \tau_{1}^{\text{max}} = \tau_{2}^{\text{max}} = 1, \quad \tau_{1}^{\text{res}} = \tau_{2}^{\text{res}} = 0.4, \quad \gamma H = 32.5, \quad \Delta X_1 = \Delta X_2 = 0, \quad \xi = 0.4, \quad m = 0.5, \quad \alpha = 0, \quad h = 1, \]

\[ a_1 = 3.3, \quad a_2 = 5, \quad b_1 = 2.3, \quad b_2 = 2.7, \quad c = 0.4, \quad d = 2, \quad \beta = 27^0, \quad H = 300 \quad \text{and perform calculations until total unloading of the stoping void boundary.} \]

Figure 4 demonstrates the calculated results for the first problem without a fault zone. Figure 4a shows the evolution of the plastic deformation zones (light-grey color marks the local strength loss zones, which corresponds to the descending branch in the grain boundary sliding diagram; dark-grey — residual strength zones, which conforms with the horizontal branch in Figure 2). Figures 4b and 4c present, respectively, contour lines of the maximum shear stress \( \tau_0 = 0.5 \sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2} \) and maximum shear \( \gamma_0 = \sqrt{(\varepsilon_{11} - \varepsilon_{22})^2 + 4\varepsilon_{12}^2} \). The contour lines of the macrostresses \( \sigma_{11}, \sigma_{22}, \sigma_{12} \) are shown in figures 4d–4f, respectively.

It is seen that plastic shears propagate from the natural stress raisers at the stoping void boundary depthward rock mass in the vertical direction predominantly.

Figure 5 presents the calculation results for the second problem with a disjunctive break. Apparently (Figure 5a), in the course of deformation, material inside the fault situated directly above the stoping front comparatively rapidly passes to plasticity and affects stress state of adjacent rock mass (figures 5b–5f). Sliding of the edges of the fault has the most pronounced influence on the distribution of the tectonic (\( \sigma_{11} \)) and shear (\( \sigma_{12} \)) stresses in rock mass.
In whole, under one and the same initial macrostresses, the release of the accumulated energy in the form of internal self-balancing microstresses in a relatively narrow zone of a disjunctive break under the influence of the stoping front advance results in the macroshear of the fault edges and changes the macrostress state of surrounding rock mass.

Figure 4. First problem calculation data without a fault: a) plastic deformation zones; b) contour lines of the maximum shear stress $\tau_0$, c) contour lines of the maximum shear $\gamma_0$, d), e), f) contour lines of $\sigma_{11}$, $\sigma_{22}$, $\sigma_{12}$, respectively.

Figure 5. Second problem calculation data with a fault: a) plastic deformation zones; b) contour lines of the maximum shear stress $\tau_0$, c) contour lines of the maximum shear $\gamma_0$, d), e), f) contour lines of $\sigma_{11}$, $\sigma_{22}$, $\sigma_{12}$, respectively.
It is expected that the influence of the disjunctive break will become even stronger in case of unstable deformation with the uncontrollable dynamic liberation of the accumulated elastic energy.

4. Conclusions
- The developed approach makes it possible to solve problems on deformation of structurally inhomogeneous geomaterials with regard to internal self-balancing stresses.
- Stoping near a disjunctive break can provoke release of internal self-balancing stresses and, thus, affect stress state of surrounding rock mass.

Acknowledgments
This study was supported by the Russian Science Foundation, Project No. 16-17-10121.

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