Superluminal neutrino energy spectrum of OPERA and MINOS

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Abstract

We analyze the velocity dependence on energy of superluminal neutrino recorded by the OPERA and MINOS collaborations and manage to approximate the energy spectrum by a power law \( E = p + C p^a \) where parameters must be taken in the range \( a = 0.40 \div 1.18 \) and \( C = 1.5 \times 10^{-5} \div 4.15 \times 10^{-4} \) (momentum and energy are expressed in GeV). This rough estimation is constrained by the errors of measurements, and new experimental data are requested.

1 Introduction

Neutrino was believed to be a massless fermion with energy

\[ E = pc \]  

(1)

and group velocity

\[ v = \frac{dE}{dp} = c \]  

(2)

equal to the speed of light \( c = 1 \) (in relativistic units). The modern theory expects, however, that neutrino has finite mass [1]

\[ m = m_\nu < 0.28 \text{eV} \]  

(3)
that implies deviation from the energy spectrum and velocity \( v \neq c \). Recent experiments of the OPERA Collaboration [2] have revealed supraluminal motion of neutrino. The time between signals

\[
t_0 + \delta t = \frac{L}{v} \tag{4}
\]

measured at the baseline \( L = 730 \) km, was less than \( t_0 = L/c \), implying that a small delay \( \delta t \) was negative and the velocity of neutrino

\[
v - 1 = -\frac{\delta t}{L} \tag{5}
\]

was definitely above the speed of light. The velocity shift (5) revealed almost no dependence on energy, and at the average energy

\[
E = 17 \text{ GeV} \tag{6}
\]

the time delay

\[
\delta t = -57.8 \pm 7.8 \quad \text{[stat.]} \quad +8.3 \quad \text{[sys.]} \quad \text{ns} \tag{7}
\]

corresponded to

\[
v - 1 = (2.37 \pm 0.32 \quad \text{[stat.]} \quad +0.34 \quad \text{[sys.]} \quad ) \times 10^{-5} \tag{8}
\]

Superluminal neutrino was also observed by the MINOS Collaboration [3] as well as in supernova explosion SN1987a [4].

This fact is a serious puzzle to the researchers. There is no lack of hypotheses to explain it [5]. However, the value of superluminal velocity (8) imposes a severe constraint on the energy spectrum. The energy dependence of \( v \) was also explored by the OPERA collaboration [2], but it was not possible to warrant solid data because it was beyond the accuracy of the measurement on account of large errors. However, the energy dependence of \( (v - 1) \) on \( E \) can contain very important information that may allow to understand the nature of neutrino. In the present paper we analyze the experimental data of the OPERA [2] and MINOS [3] and try to establish the range of possible energy spectrum of superluminal neutrino.

## 2 Neutrino is not free tachyon

The most natural and plain idea is to treat neutrino as a massive tachyon whose energy spectrum

\[
E = \sqrt{p^2 - m^2} \tag{9}
\]
yields superluminal group velocity

\[ v = \frac{dE}{dp} = \frac{p}{\sqrt{p^2 - m^2}} \]  \hspace{1cm} (10)

estimated as

\[ v \simeq 1 + \frac{1}{2} \frac{m^2}{p^2} \simeq 1 + \frac{1}{2} \frac{m^2}{E^2} \]  \hspace{1cm} (11)

for an ultra-relativistic particle \((E \simeq p \gg m)\). It implies a tiny positive shift above the speed of light. However, according to (3) and (11), we cannot get estimation greater than

\[ v - 1 \simeq 10^{-22} \]  \hspace{1cm} (12)

at \(E = 17\) GeV. Otherwise, we have to expect very large tachyon mass \(m \simeq 120\) MeV that contradicts to the expected upper bound (3). It is clear that superluminal neutrino cannot be a free tachyon with the energy spectrum (8), neither a free tardyon (ordinary particle) with the energy spectrum \(E = \sqrt{p^2 + m^2}\).

However, it is not a problem of the theory because there are many sophisticated arguments explaining the superluminal velocity of neutrino (9). Nevertheless, it is highly desirable to know the energy dependence of quantity

\[ f[E] = v - 1 \]  \hspace{1cm} (13)

that is equivalent to dependence on the momentum \(f[p]\) for an ultra-relativistic particle. The knowledge of this dependence allows to restore the energy spectrum of neutrino

\[ E = p + \int f[p] dp \]  \hspace{1cm} (14)

and test hypotheses of its nature.

### 3 Dependence on energy

The OPERA collaboration [2] has also obtained the following data at various energy

\[ E = 13.8\) GeV \hspace{1cm} \delta t = -54.7 \pm 18.4 [\text{stat.}]^{+7.3}_{-6.9} [\text{sys.}] \, \text{ns} \]  \hspace{1cm} (15)

\[ E = 28.2\) GeV \hspace{1cm} \delta t = -61.1 \pm 13 [\text{stat.}]^{+7.3}_{-6.9} [\text{sys.}] \, \text{ns} \]  \hspace{1cm} (16)
\[ E = 40.7 \text{ GeV} \quad \delta t = -68.1 \pm 19.1 \text{ [stat.]} \pm 7.3 \text{ [sys.]} \text{ ns} \quad (17) \]

that together with (6)-(7) can be described by proportionality [6]

\[ \delta t \sim E^{a-1} \quad (18) \]

and, according to (5) and (18), we get [7, 8]

\[ f(E) = v - 1 = AE^{a-1} \quad (19) \]

that is equivalent to

\[ f(p) = v - 1 = Ap^{a-1} \quad (20) \]

for an ultra-relativistic particle. Let us develop this interpretation in detail.

According to (7) and (15)-(17), parameter \( a \) must lay within the range

\[ a = 0.40 \div 2.08 \quad (21) \]

Then, approximation (19) yields the observed velocity (8) within the accuracy of measurement when

\[ A = (0.09 \div 16.6) \times 10^{-5} \quad (22) \]

where \( E \) in (19) is expressed in GeV.

The MINOS Collaboration [3] has recorded superluminal velocity

\[ v - 1 = (5.1 \pm 2.9\text{[stat. + sys.]}) \times 10^{-5} \quad (23) \]

for the low energy neutrino with energy spectrum peaked at approximately \( E = 3 \text{ GeV} \) with a long high-energy tail extending to \( E = 120 \text{ GeV} \) and baseline \( L = 734 \text{ km} \). The energy spectrum taken in the form (28) can yield velocity (23) within the accuracy of measurement when parameter \( a \) is chosen in the range

\[ a = 0.14 \div 1.18 \quad (24) \]

and

\[ A = 1.81 \div 20.6 \quad (25) \]

Velocity (23) is not achieved at \( E = 3 \text{ GeV} \) according to (28) if \( a \) and \( A \) are beyond (24) and (25).

Combining (21), (22), (24) and (25) we have more strict estimation

\[ a = 0.40 \div 1.18 \quad (26) \]
\[ A = (1.81 \div 16.6) \times 10^{-5} \]  \hspace{1cm} (27)

Substituting (20) in (14) we get the energy spectrum of ultra-relativistic neutrino

\[ E = p + C p^a \quad C = \frac{A}{a} = 1.5 \times 10^{-5} \div 4.15 \times 10^{-4} \]  \hspace{1cm} (28)

that satisfies both the OPERA and MINOS experiment. For example, choosing \( a = 1 \) in (28), we obtain velocity (8) and (23) within the accuracy of measurements when \( A = (2.2 \div 3.03) \times 10^{-5} \).

4 Conclusion

Superluminal neutrino observed in experiments [2, 3] cannot be a free tachyon because it mass (3) is not enough to correspond the observed velocity (8).

The dependence (13) of the neutrino velocity \( v \) on the energy \( E \) may give much information about its energy spectrum (14). It can be taken as a power law (28) with parameters (26) and (27). It should be emphasized that parameter \( a \) in (28) must be positive, and the real energy spectrum lays somewhere between \( E = p + 1.5 \times 10^{-5} p^{1.18} \) and \( E = p + 4.15 \times 10^{-4} p^{0.4} \). There is no possibility to establish \( a \) and \( A \) at high accuracy because of errors of experimental data. It is impossible even to clarify whether \( a > 1 \) or \( a < 1 \) and whether function \( v[E] \) is monotonically increasing. Of course, new measurements will reveal the exact energy spectrum of neutrino and clarify its physical nature. Now we can only state that superluminal neutrino is not a free tachyon and that the second term in the right side of (28) may give a hint to nonlinear self-interaction or external field acting on neutrino. It is the subject of further theoretical work.

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