Increasing the resolution of the aberrated optical system based on quadratic amplitude apodization

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Increasing the resolution of the aberrated optical system based on quadratic amplitude apodization

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Abstract. Optical imaging systems have a resolution limit due to the diffraction nature of light. Moreover, point spread function is distorted and blurred in the presence of both the wavefront aberrations, and the optical system. One way to overcome the diffraction limit is the amplitude or phase apodization of the optical system. However, as a rule, apodization allows not only reducing the size of the light spot, but also leads to the appearance of side lobes which degrade depicting properties. Thus, it is necessary to observe a compromise between reducing the size of the light spot and the side lobe level, which requires appropriate studies. In this paper we simulate the formation of spread functions from two closely spaced point light sources. We carried out a study of the change in the resolving power of a focusing system with its amplitude apodization by a quadratic function in the absence and with various aberrations.

1. Introduction

Typically, optical imaging systems, such as optical communication systems, microscopes, as well as human vision systems, suffer from resolution limitations. This is related to the nature of light diffraction, as well as on the presence of aberrations both in the wavefront and in the optical system [1-7]. The main causes of wavefront aberrations are: nonideality of the forms of the optical systems elements, errors in the alignment of the system, etc. A generally accepted representation of the wave front is the basis of the Zernike polynomials [8-10]. Earlier, for direct measurement of wavefront aberration coefficients, multichannel DOEs [11-13] were approved, which are coordinated with a set of Zernike polynomials [14-17]. The coefficients of the wavefront expansion in the Zernike polynomials allow us to determine the magnitude of the deviation from the ideal front and the types of aberrations that are present in the distortion. We note that the Zernike polynomials are a convenient analytic approximation of the eigenfunctions of the correlation operator [18], known as Karhunen-Loève functions [19-20].

One way to overcome the diffraction limit is the amplitude or phase apodization of the optical system [21-27]. However, as a rule, apodization allows not only to reduce the light spot size, but also leads to the appearance of side lobes [28-31], which degrade depicting properties. Thus, it is necessary to observe a compromise between reducing the size of the light spot and the side lobes’ level, which requires appropriate studies.

In particular, when asymmetric apodization [21-23] is introduced into the pupil plane, it is possible to eliminate low-frequency side lobes in the distribution of the incident field. This leads to the solution of
a practical problem known as two-point resolution [32], when two closely located point light sources are observed in the presence of geometric aberrations.

2. Theoretical foundations
In this paper we consider the formation of the point spread functions (PSF) from two closely spaced point light sources. A study was also made in the change of the system’s resolution with its amplitude apodization by the quadratic function in the absence and in the presence of various aberrations.

The wavefront is usually described as follows:

\[
W(r,\varphi) = \exp[2\pi i \psi(r,\varphi)],
\]

where \( \psi \) is the phase of the wavefront. We define the phase as follows:

\[
\psi(x) = \begin{cases} 
0, & \psi(x) > 0 \\
\pi, & \psi(x) < 0 
\end{cases}, \quad \varphi(x) = \cos(\alpha x)
\]

To construct the pictures of the spread function we used the simple optical system Fourier-correlator. The simulation results are shown in figure 1.

![Figure 1](image1.png)

**Figure 1 (a, b, c).** The amplitude and phase (\( \alpha=30, x8 \)): (a) wavefront phase \( \psi \), (b) wavefront \( W \), (c) Fourier plane

![Figure 2](image2.png)

**Figure 2 (a, b, c, d, e).** The amplitude in Fourier plane (\( x8 \)), \( \alpha \): (a) 1, (b) 2, (c) 3, (d) 4, (e) 5

From the series of figures (Fig. 2 (a) - (e)), which are presented above, it is evident that the larger the variable parameter \( \alpha \), i.e. the more diffraction orders are encoded in the phase, the further apart from each other are the maximum peaks of the point spread function.

3. Numerical modeling
We perform a number of experiments and find the limiting parameter \( \alpha \), according to the Rayleigh criterion. To distinguish between the two points can be considered limiting parameter \( \alpha = 2.5 \) (Fig. 3), because Airy spot radius of the PSF and the distance between the maximum peaks in the Fourier plane are approximately equal. Consequently, it can be concluded that the two points considered in the Fourier plane will be distinguished by the Rayleigh criterion for \( \alpha \geq 2.5 \).
Figure 3 (a, b, c). The amplitude and phase ($\alpha = 2.5$, x16): (a) wavefront phase $\psi$, (b) wavefront W, (c) Fourier plane

Figure 4 (a, b, c, d). (a)-(b) The amplitude of the point spread functions for optical system from a point light source and (c)-(d) two closely spaced point light sources; (b) and (d) – cross-section graphs

To overcome the diffraction limit, we used the quadratic amplitude apodization of the optical system. Apodization of this kind will give an effect similar to what we get in the presence of spherical aberration ($Z_{20}$ - in terms of Zernike, Fig. 5).

Figure 5 (a, b, c). The amplitude and phase of Zernike function $Z_{20} = r^2 \exp[im\varphi]$: (a) wavefront phase $\psi$, (b) wavefront W, (c) PSF

Let’s perform numerical simulation, where we submit a test image - the cross (Fig. 6 (a)) to the input to the Fourier optical scheme, and then add additional spherical aberration to the lens surface (Fig. 6 (b)). We will show how the image will change.

Figure 6 (a, b). The test image - cross: (a) ideal, (b) distorted by spherical aberration

However, distortion of the wave front of the proposed species will bring a positive effect when solving the problem of distinguishing two points. We expect an increase in the distance between the maximum
peaks of the PSF in the Fourier plane. Let us perform a numerical experiment in which the phase of the wavefront will be supplemented by a quadratic function, like a spherical aberration (Fig. 7). Let there be given a field with a phase that is coded as follows: $\psi(x) = r^2 \arg[\cos(\alpha r)]$.

**Figure 7 (a, b).** (a) The amplitude and (b) phase of the field $\psi$

### 4. Results and discussion

As expected, there is an improvement in image quality, from the point of view of overcoming the diffraction limit. From Fig. 8, it can be seen that the distance between the central peaks has increased to a value at which the points are separable according to the Rayleigh criterion.

**Figure 8.** (simple) The amplitude of the point spread functions for optical system without apodization and (bold) with apodization – cross-section graphs

**Figure 9 (a, b, c, d).** The amplitude of the point spread functions for optical system (a)-(b) without apodization and (c)-(d) with apodization; (b) and (d) – cross-section graphs
Similarly, we simulated situations where the wavefront was distorted by various aberrations. In each of the cases (the first four orders of aberration in terms of Zernike) we observe a merger of two peaks in the PSF. Moreover, the next diffraction order is clearly visible (fig. 10).

Figure 10. The amplitude in Fourier plane: $Z_{1-1}$, $Z_{2-2}$, $Z_{20}$, $Z_{3-3}$, $Z_{3-1}$, $Z_{4-4}$, $Z_{4-2}$, $Z_{40}$ (top, left to right), (bottom, left to right) cross-section graphs

5. Conclusion
To overcome the diffraction limit, we used the quadratic amplitude apodization of the optical system. An increase in the distance between the maximum peaks of the PSF was expected. Numerical modeling confirmed the hypothesis of the possibility of distinguishing closely spaced points. In this paper, a study was conducted changing the resolution of the focusing system, adding considered apodization. In a number of experiments, it was found that, in the case of an ideal lens, indistinguishable points can be identified, using the Rayleigh criterion, by modifying the phase features of the wave front. As for the consideration of a nonideal optical system, we simulated situations when the wave front is distorted by some specific aberration. The first four orders of wave aberrations in terms of Zernike were considered. It was found that there is a central peak in the Fourier plane in each of the above cases. If you treat it as a high-frequency noise, the next diffraction order (maximum) is informative in terms of distinguishing between the two points. Thus, it is shown that the considered quadratic amplitude apodization improves resolution of an optical system with small aberrations.

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