Comparison of four classical algorithms to determine the apparent thermal diffusivity of heavy clay soil in field and laboratory column experiments

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Abstract. Prediction of soil temperature at an acceptable level of accuracy is essential for managing soil temperature at large scales since its measurement is costly and not practical due to its high spatial and temporal variability. This study was carried out for: 1) Calculate the coefficient of thermal conductivity using classical methods and basing on data from measured soil profile temperatures in column and field conditions and 2) compare the results of the calculated and measured soil temperature, calculated from these data for both conditions. Soil temperature was measured with thermal sensors on both soil profiles in the field and soil column. The measured temperature values were similar in soil column and profile across 0.05, 0.10, and 0.20 m depths, but had the difference for 0.30 cm. The introduction of the second harmonic allows determining with high accuracy the parameters (T0, Ti, εi) of the temperature distribution on the field and column soils surfaces. For example, respectively for the harmonics n = 1 calculated the following statistical characteristics: R² = 86.45%, σ = 3.26, A= 13.07%, UII=0.12 and for n = 2: R² = 98.64%, σ = 1.09, A = 4.20%, UII=0.04. It was found that, mean values for soil thermal diffusivity (k), thermal conductivity (λ), damping depth (d) and heat absorptivity (e), calculated by amplitude, arctangent and logarithm methods, for the column were greater than those for soil profile, in all the cases.

1. Introduction
The thermal regime of the soil is a set and sequence of phenomena of receipt, transfer, accumulation and recoil of heat. It is characterized by temperature at different depths of the soil profile, which has a daily and annual course. The course of change in soil temperature in time and space is called the temperature regime. The temperature regime of the soil plays a significant role in the processes of plant life. Temperature is one of the decisive factors of soil formation process, vital activity of plants and microorganisms. It is associated with the solubility of minerals, gases and their mobility, as well as chemical reactions in soil and plant roots. Soil temperature is closely related to the temperature of the surface layer of the atmosphere. For the vital activity of plants, it is important not how much heat is accumulated by the soil, but how much its intensity, i.e. temperature [1-2].

Soil thermal properties are affected significantly by soil solid fraction, including organic matter content, soil mineral composition, soil water content and air filled porosity [3-6].
Numerous studies of modeling soil thermal properties have been conducted on disturbed and undisturbed soil columns [7].

The main thermal properties of soil are the coefficients of thermal conductivity, thermal diffusivity, heat capacity and heat absorption. Knowledge of these soil characteristics can bring the resolution of such an acute problem of our time as the forecast of the thermal regime of soils [1, 8-9].

However, since soil heat diffusivity and heat conductivity are strongly dependent on the soil water content and soil surface conditions, the results from column studies may not represent the field conditions. In addition, the soil column itself may not represent the soil structure in field condition since soil structure is disturbed more or less through its handling for experiment.

Several methods are available to model soil thermal properties, including apparent diffusivity, from observed soil temperature [9, 10-18]. These researchers and others noted that most of these models are based on solutions of the one-dimensional heat equation with constant diffusion.

When discussing the temperature regime of the soil, it is convenient to use the coefficient of thermal conductivity, since this parameter characterizes the "propagation" of the measured value - temperature [2]. The value of the coefficient of thermal conductivity is of great agronomic importance because it shows the rate of warming or cooling of the soil. The correct assessment of this parameter is very important for the study of thermal processes in the soil.

A lot of work is devoted to the determination of the coefficient of thermal conductivity of the soil, based on the use of solutions to the simplified equation of heat transfer. All these methods are based on the analysis of the course of soil temperature at different depths and over time, and they all assume either the presence of a periodic course of temperature in a certain period, or the real course of temperature.

The objective of this study was to compare results of modeling on field and column studies of the same soil. Heat conductivity and diffusivity were predicted on undisturbed soil column and soil profile by four different classical methods and results were compared. We also compared performance of the methods in predicting soil temperature at both of soil column and soil profile.

2. Material and Methods

2.1. Material

The study area is located between 37°-38° North latitude and 33°-34° East longitude in central Turkey. The elevation of the study area is 1013 m from the sea level. Winters are cold and snowy, summers are hot and dry, and falls and springs are cool and rainy. The mean annual temperature is 11.4°C and precipitation is 318.9 mm. The soils at the experimental site are clay loam (CL). The soils in the study area are Typic Ustifluvents.

2.2. Methods

2.2.1. The field study. The specification of the soil profile is described in more detail in [19]. Soil temperature was measured with a water-proof portable thermal Sensor (ThermoChro the iButton DS1921G). The sensor registers and stores temperature measurements in its memory from which can be downloaded by users. The thermal sensors were placed at depths of 0, 2, 5, 10, 15, 20, and 30 cm in the soil profiles.

2.2.2. The Column Study. An undisturbed soil column (figure 1) with 60 cm in length and 10 cm id was extracted from nearby sensors-placed soil profile (figure 1,a, b). The soil column was placed in a plastic column with 60 cm length and 23.6 cm id and the space between soil column and plastic pipe was filled with perlite (Fig1,c). Holes were open in the side of the column at 2, 5, 10, 15, 20, 25, and 30 cm vertical distances and temperature sensors were placed in the column through these holes. The sensors were connected to a 16 Channel PC-Compatible Elimko 680 Data Logger (Analog/Digital...
convertor, which can convert analog signals to digital ones and save on a computer (figure 1, d). The apparatus was set to measure and record hourly temperature.

Figure 1. Preparation of soil column experiment: a) isolating undisturbed soil with column, b) opening the holes in side of soil column and placing the sensors through the holes, c) sensors-placed soil column sealed with perlite and d) the final look of column experiment setup with each of placed sensor connected to a data logger.

3. Calculation of thermal diffusivity of soils

The one-dimensional distribution of the temperature field in an isotropic medium is described by the following classical equation of heat conduction [10]:

\[ C_v(z,t) \frac{\partial T(z,t)}{\partial t} = \frac{\partial}{\partial z} \left[ \lambda(z,t) \frac{\partial T(z,t)}{\partial z} \right] \]  

(1)

where \( T(z,t) \) – is the soil temperature (K or °C) at depth \( z \) (m), \( t \) is time (sec); \( C_v \) – is the volumetric heat capacity of the soil (J m\(^{-3}\) K\(^{-1}\)); and where \( \lambda \) – is the thermal conductivity (W m\(^{-1}\) K\(^{-1}\)).

Assuming that a soil is vertically homogeneous, in this case, that \( C_v \) and \( \lambda \) are independent of depth and time, provides [1-19]

\[ \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} \]  

(2)

where \( \kappa \) – is the apparent soil thermal diffusivity (m\(^2\)/s) and \( \kappa = \lambda / C_v \).

The equation (2) must be solved analytically or numerically to find the temperature change at a given time \( t \) and at depth \( z \) of the soil. To solve equation (2), initial and boundary conditions are needed that reflect the natural process of heat transfer in soil.

The initial conditions correspond to the state of the variable at the zero (initial) time moment. For the theoretical description of the quasi-stationary regime problem (e.g., the daily or annual variation of the soil temperature), the initial condition is absent (so-called problems without an initial condition).

The possibility of such problem setting follows from the experimentally found strict periodicity in the daily and annual temperature rhythms; therefore, any moment can be considered as separated from the beginning of the process by an infinitely long time interval [9-10, 13, 15].

In nature, the course of soil temperature, as a rule, differs from a strict sinusoid. In this case, more accurately, the course of the soil temperature can be approximated by two (or more) harmonics, when the daily variation is represented as two sinusoids — one with a period equal to days, and the second - to half a day. This approach allows us to find a more accurate expression for \( \kappa \), although somewhat more cumbersome.

The most convenient characteristic that can appear as a boundary condition of the 1st kind is the dynamics of the soil surface temperature in the form of the well-known function of time, i.e., a trigonometric polynomial [10, 17]:

\[ T(0,t) = T_0 + \sum_{j=1}^{\infty} T_j \cos(j \cdot \omega t + \varepsilon_j) \]  

(3)

in which \( T_0 \) – average daily (or annual) temperature of the active soil surface (°C or K); \( T_j \) – amplitude of the wave at the surface level for the \( j \)th harmonic (°C or K); \( j \) – index of the harmonic in the series; \( \omega = 2\pi / \tau_0 \) – is the angular daily (or annual) frequency; where \( \tau_0 \) – is the temperature wave period (days
or years), for \(\tau=24\) hours: \(\omega=7.27221.10^{-5}\) (rad/s); \(t\) – time (hours); \(\varepsilon_i\) – phase angle of the wave at the surface level for the \(j\)th harmonic (radians), and \(m\) is the harmonic number.

Equation (3) is equivalent to:

\[T(0, t) = T_0 + \sum_{j=1}^{m} A_j \cos(j \cdot \omega t) + B_j \cos(j \cdot \omega t)\]  

(4)

where \(A_j\) and \(B_j\) are the amplitudes (\(\circ\)C or K) of the cosine and sine terms, respectively. Their values are determined by the method of least squares.

The lower boundary condition is:

\[
\lim_{x \to \infty} T(x, t) = T_0
\]

(5)

The solution of equation (2), with the boundary conditions (3) and (5), is as follows [10]:

\[T(y, \tau) = T_0 + \sum_{j=1}^{m} T_{aj} \cdot \cos\left[j\theta \tau + \alpha_j\left(\varepsilon_j, y\right)\right]\]

(6)

where, \(y = z / L, \tau = \alpha \tau / L^2, b_j = \sqrt{j^2 \theta^2 / 2}, \theta = \alpha \tau L^2 / \kappa, T_{aj} = T_j \cdot e^{-b_j y}\) — is the amplitude of temperature fluctuations at the dimensionless depth \(y\), \(\alpha_j(\varepsilon_j, y) = \varepsilon_j - b_j y\) — initial phase at dimensionless depth \(y\). Solution (6), usually with \(m = 1\) or 2, was used by various researchers to estimate the thermal diffusivity parameter \(\kappa\) [9-10, 13-15].

The main thermal characteristics of soil are the coefficients of volumetric heat capacity, thermal conductivity, thermal diffusivity, thermal absorptivity (assimilability), heat flow on the soil surface and the damping depth of diurnal temperature waves.

The volumetric heat capacity \((C_v)\) of the soil is calculated from the conventional formulas [2]:

\[C_v = C_{m,s} \cdot \rho_b + C_{v,w} \cdot \theta = \left[C_{m,org} \cdot \frac{m_{org}}{m} + C_{m,\min} \left(1 - \frac{m_{org}}{m}\right)\right] \cdot \rho_b + C_v \cdot \rho_b \cdot \rho_u \cdot \theta\]

(7)

where, the \(C_{m,s}\) is the specific heat of the soil's solid part, J/(kg·°C); \(\rho_b\) is the soil bulk density, kg/m\(^3\); \(C_{v,w}\) is the volumetric heat capacity of the soil moisture equal to 4186.6·10\(^6\) kJ/(m\(^3\)·°C); \(C_v\) is the specific heat of water, J/(kg·°C); \(\rho_u\) is the water density, kg/m\(^3\); \(\theta\) is the volumetric moisture content (m\(^3\)/m\(^3\)); \(C_{m,\text{org}}\) and \(C_{m,\text{min}}\) are the specific heats of the organic and mineral components of the soil solid phase respectively, J/(kg·°C); \(m_{org}\) is the mass of soil organic matter, kg; \(m\) is the soil mass (kg); \(m_{org} / m\) is the content of organic substance in soil, %.

Thermal conductivity \((\lambda)\) is the product of volumetric heat capacity \((C_v)\) of the soil and thermal diffusivity \((\kappa)\) and is given as:

\[\lambda = \kappa C_v\]

(8)

The soil depth \(d\) (m) at which temperature oscillations are attenuated is determined by the thermal diffusivity of soil and is calculated from the following formula [2 and 10]:

\[d = \sqrt{\tau_0 \kappa / \pi}\]

(9)

A no less important thermal characteristic of the soil is heat absorptivity, which determines the degree of its heat accumulation, depends on the thermal conductivity \(\lambda\), the thermal diffusivity \(\kappa\), and the heat capacity per unit volume \(C_v\), and is calculated from the following relations [9]:

\[e = \sqrt{\kappa C_v} = \lambda / \sqrt{\kappa} = C_v \sqrt{\kappa}\]

(10)

To properly describe the temperature changes in the near surface soil layer, one must account for the redistribution of fluxes in the soil. If the thermal conductivity of the soil is known, we can determine the heat flux through the surface deep into the soil according to Fourier’s Law [1, 5, 9-10]. The vertical conductive heat flux in a soil, \(G\) (W/(m\(^2\)·°C)), at depth \(z\) and time \(t\) is given by:

\[G(z, t) = -\lambda(z) \left[\frac{\partial T(z, t)}{\partial z}\right]\]

(11)

Using (6) for \(m = 1\) in the formula (11), we can easily show that, the analytical solution for soil heat flux at any depth \(z = h\) and time \(t\) is
4. Classical methods for determining the coefficient of apparent thermal diffusion

The determination of the coefficient of thermal diffusivity of the soil has been discussed in many theoretical and experimental works. On the basis of analytical solutions (3) of equation (1), various authors derived formulas and methods for determining the apparent thermal diffusivity coefficient [9-16]. In the present work, the four algorithms were used to estimate the apparent thermal diffusivity \( \kappa \), based on solving equation (3).

Below are the formulas that are used for the case when the daily temperature variation on the soil surface is represented by one and two harmonics.

\[
G(z = h, t) = \left( \sqrt{2} \frac{\lambda}{d} \right) \cdot e^{-\frac{h}{d}} 
\sum_{i=1}^{4} T_i \cdot \cos \left( \omega t + \varepsilon - \frac{h}{d} + \frac{\pi}{4} \right), \quad \left( d = \sqrt{2 \kappa / \omega} \right)
\]

All the above parameters are calculated mainly through the soil diffusion coefficient (\( \kappa \)).

4.1. Method 1 (Amplitude Equation)

This method has been developed based on Fourier's first law, which expresses soil temperature change by depth as a steadily decreasing soil temperature amplitude, as described by Equation (16) [9-10]:

\[
\kappa = \frac{\omega}{2} \cdot (z_2 - z_1)^2 \cdot \ln \left[ \frac{T_{\text{max}}(z_1) - T_{\text{max}}(z_2)}{T_{\text{max}}(z_2) - T_{\text{max}}(z_1)} \right]
\]

where; \( T_{\text{min}}(z) \) and \( T_{\text{max}}(z) \) — are the minimum and maximum temperature values during the measurement at the depths \( z=z_1 \) and \( z=z_2 \) respectively, \( \tau_0 \)— period of heat wave (e.g., 24 hours, for diurnal calculations).

This method has the advantage that only four temperature observations (minimum and maximum) at two depths \( z=z_1 \) and \( z=z_2 \), are required for the determination of the apparent thermal diffusivity \( \kappa \). The next method in this group is the so-called phase equation which is based on recording the time interval between occurrences of maximum temperatures at two depths.

4.2. Method 2 (Phase Equation)

The value of the thermal diffusivity \( \kappa \) of the soil can be found from the equality: \( \alpha (\varepsilon, z) = \varepsilon - bz \), which expresses the initial phase at a depth \( z \). For this, knowledge of the initial phases at both depths \( z_1 \) and \( z_2 \) is necessary. Then we have [9, pp. 237]:

\[
\kappa = \frac{\pi}{\tau_0} \left( \frac{z_1 - z_2}{\phi_1 - \phi_2} \right)^2
\]

4.3. Method 3 (Arctangent Equation)

In nature, changes in temperature of the surface of the soil, as a rule, differ from a strict sinusoid. This approach allows us to find a more accurate expression for \( \kappa \), although somewhat more cumbersome. Using such approximations, various formulas for two and four observation times are obtained. They are listed below. For two harmonics the apparent thermal diffusivity can be determined from temperature measurements at two depths as an arctangent equation in the form of [9 and 13]:

\[
\kappa = \frac{0.5 \cdot \omega (z_2 - z_1)^2}{\arctan^2 \left[ \left( \frac{T_1(z_1) - T_2(z_1)}{T_2(z_2) - T_2(z_1)} \right) \left| T_2(z_1) - T_4(z_1) \right| \left| T_2(z_2) - T_4(z_2) \right| \right]}
\]

where \( T_i(z_j) \) and \( T_i(z_j) \) are the soil temperature at the depths \( z=z_j \) and \( z=z_2 \), respectively, at the time at the time moment \( t_i = i \cdot \tau_0 / 4 \) (for our example \( \tau_0=24 \) h and \( t_1=6, \ldots \) and \( t_4=24 \) hours).

4.4. Method 4 (Logarithmic Equation)
Using the assumption made above for Method 3, Kolmogorov (1950) showed that the apparent thermal diffusivity can be calculated, for four observation times, with the following equation [14]:

\[
\kappa = \frac{4\pi \cdot (z_2 - z_1)^2}{\tau_0 \cdot \ln^2 \left( \frac{\left[T_1(z_1) - T_3(z_1)\right]^2 + \left[T_2(z_1) - T_4(z_1)\right]^2}{\left[T_1(z_2) - T_3(z_2)\right]^2 + \left[T_2(z_2) - T_4(z_2)\right]^2} \right)}
\]

where \(T_i(z_j)\) and \(T_i(z_j)\) are the soil temperature at the depths \(z=z_i\) and \(z=z_j\), respectively, at the time moment \(t = t_i \cdot \tau_0 / 4\) \((i = 1, 2, 3, 4)\).

Methods 3 and 4 are analogous to Methods 1 and 2 but take advantage of a greater number of temperature observations to approximate a potentially nonsinusoidal behavior. In should be noted that methods 3 and 4 are similar to methods 1 and 2 in their mathematical approach, but they have an advantage of the larger number of measured soil temperatures \(T_i\) \((i = 1, 2, 3, 4)\). Therefore, a more stable solution can be found, and the thermal diffusivity \(\kappa\) can be determined more reliably [15].

4.5. Comparison of Methods

The performance of four methods was evaluated by Pearson’s Correlation Coefficient (\(r\)), Coefficient of Determination \((R^2)\), Root Mean Squared Error \((RMSE, \sigma)\), Mean Absolute Percentage Error \((MAPE, A)\), and Theil’s U Statistic \((UI)\).

5. Results and Discussion

Vertical changes of soil properties in soil profile and soil column are depicted in Table 1. The soil profile and the column were similar in vertically distributed soil properties, except bulk density between 0.1 and 0.2 m depth. The volumetric water content, \(\theta\), is always greater in soil profile than soil column, which may have an important effect on heat diffusivity. Also, volumetric heat capacity was lower in soil column, resulted from the lower \(\theta\).

| Depth (z), m | Clay, % | Silt, % | Sand, % | BD, kg m\(^{-3}\) | OM\(^b\), % | \(\theta\), m\(^3\) m\(^{-3}\) | \(C_v\), kJ m\(^{-3}\) \(\circ\)C |
|-------------|--------|--------|--------|----------------|-------------|----------------|------------------|
| Soil profile |        |        |        |                |             |                |                  |
| 0.00-0.10   | 38.69  | 54.15  | 7.16   | 1.2187         | 0.14        | 0.30177        | 2283.9           |
| 0.10-0.20   | 40.52  | 55.58  | 3.91   | 1.4314         | 0.12        | 0.34551        | 2645.1           |
| 0.20-0.30   | 40.31  | 53.93  | 5.75   | 1.2908         | 0.11        | 0.30546        | 2359.7           |
| 0.00-0.30   | 39.84  | 54.55  | 5.61   | 1.3136         | 0.12        | 0.31758        | 2429.6           |
| Soil column |        |        |        |                |             |                |                  |
| 0.00-0.10   | 38.98  | 53.97  | 7.06   | 1.200          | 0.15        | 0.21577        | 1908.2           |
| 0.10-0.20   | 39.88  | 55.33  | 4.79   | 1.299          | 0.13        | 0.19618        | 1909.1           |
| 0.20-0.30   | 41.50  | 56.07  | 2.43   | 1.362          | 0.11        | 0.22080        | 2064.9           |
| 0.00-0.30   | 40.12  | 55.12  | 4.76   | 1.287          | 0.13        | 0.21092        | 1960.7           |

\(^a\)Bulk density; 
\(^b\)Organic matter; 
\(^c\)Volumetric water content.

Figure 2 gives the daily behavior of surface and soil temperatures at different depths for field and column.

To determine the parameters of the soil’s active surface \((T_0, T_i, \text{ and } \epsilon_i)\), we adopted one and two harmonics in condition (3). Using the measurement results for \(x = 0\), that is, \(T \ (x = 0, t_i)\), using the
least squares method, we determined the parameters of the temperature distribution of the surface of studied soils.

As can be seen from the expressions of \( T(0, t) \) for \( m = 1 \) and \( m = 2 \), the introduction of the second harmonic makes it possible to determine with a high degree of accuracy the parameters of temperature distribution on the soil active surface.

Table 2 gives results of calculation of the parameters \( (T_0, T_j, \varepsilon_j) \), and also statistical characteristics of approximation between \( T(0, t) \)– the initial data, and \( T'(0, t) \)– the data computed from formula (3) for \( n = 1 \) and \( n = 2 \). As can be seen from Table 2, the introduction of the second harmonic makes it possible to determine with high accuracy the parameters \( (T_0, T_j, \varepsilon_j) \) of the soil surface.

**Table 2.** Parameters of the field and column soils surfaces and model performance.

| The Parameters at the soil surface | Study Area | Numbers of harmonics |
|-----------------------------------|------------|----------------------|
|                                   | \( m=1 \)  | \( m=2 \)            |
| Mean Temperature at soil surface (\( T_0 \)), \(^\circ\)C | Field | 22.7708 | 22.7708 |
|                                   | Column | 21.750 | 21.750 |
| Amplitude of oscillations of the soil surface temperature (\( T_1 \)), \(^\circ\)C | Field | 10.6215 | 3.9884 |
|                                   | Column | 11.1119 | 3.2132 |
| Phase shift | Field | \( \varepsilon_1 \) | 2.3784 | -0.6212 |
|                                   | Column | 2.3181 | -0.7892 |

| Statistical approximation parameters | Field | Column |
|--------------------------------------|-------|--------|
| Coefficient of Determination, %      | \( R^2 \) | 86.45 | 98.64 |
|                                      | Field | 91.58 | 99.24 |
| Root Mean Squared Error (RMSE) \( T \) in \( t \) | \( \sigma \) | 3.2572 | 1.0877 |
|                                      | Field | 3.2099 | 0.8278 |
| Mean Absolute Percentage Error (MAPE), % | \( A \) | 13.0738 | 4.2024 |
|                                      | Field | 10.6595 | 3.3925 |
| Normalized Standard Error or Theil’s U Statistic | Field | ULI | 0.1231 | 0.0390 |
|                                      | Column | 0.1025 | 0.0308 |

Mean values for soil thermal diffusivity (\( \kappa \)), thermal conductivity (\( \lambda \)) and damping depth (\( d \)), calculated by amplitude, arctangent and logarithm methods, are given in table 3. The values for \( \kappa \), \( \lambda \), and \( d \) predicted for the column were greater than those for soil profile, in all the cases.
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Table 3. The mean values of the thermal diffusivity ($\kappa$), thermal conductivity ($\lambda$), damping depth ($d$), and heat absorptivity ($e$) of the studied soils calculated by four different methods.

| Type of methods | Study Area | $C_v$ kJ/(m$^3$·°C) | $10^{-6}$ $\kappa$ m$^2$/s | $\lambda$ W/(m·°C) | $d$ m | $e$ W·h$^{0.5}$/(m$^2$·°C) | B W/(m$^2$·°C) |
|-----------------|-----------|---------------------|-------------------|----------------|------|---------------------|----------------|
| Amplitude       | Field     | 2429.6              | 0.3618            | 0.8790         | 0.0997 | 24.3563            | 12.4622        |
|                 | Column    | 1960.7              | 1.1579            | 2.2703         | 0.1785 | 35.1641            | 17.9920        |
| Arctangent      | Field     | 2429.6              | 0.3157            | 0.7671         | 0.0932 | 22.7538            | 11.6423        |
|                 | Column    | 1960.7              | 0.6048            | 1.1858         | 0.1290 | 25.4133            | 13.0032        |
| Logarithm       | Field     | 2429.6              | 0.7209            | 1.7515         | 0.1408 | 34.3816            | 17.5918        |
|                 | Column    | 1960.7              | 1.2905            | 2.5303         | 0.1884 | 37.1225            | 18.9943        |
| Phase           | Field     | 2429.6              | 0.1610            | 0.3913         | 0.0666 | 16.2500            | 8.3145         |
|                 | Column    | 1960.7              | 0.2561            | 0.5022         | 0.0839 | 16.5379            | 8.4619         |

After determining the parameter values of the thermal diffusivity $\kappa$, by the formulas (13), (15)–(17) for the soil temperature were calculated values $T(z, t)$ for depths 5, 10, 20 and 30 cm.

Table 4. Performance of models to predict soil temperature at four depths of soil profile and soil column.

| Depth, $z$ cm | Type of methods | Study Area | Statistical parameters |
|---------------|-----------------|------------|------------------------|
|               | r, %            | A, %       | $\sigma$ (°C)          | U/I (%)      |
| 5             | Amplitude       | Field      | 96.26                  | 5.76         | 1.4513            | 0.0613         |
|               |                 | Column     | 94.09                  | 9.54         | 2.1923            | 0.0955         |
|               | Arctangent      | Field      | 96.59                  | 5.06         | 1.3197            | 0.0557         |
|               |                 | Column     | 96.26                  | 6.60         | 1.5626            | 0.0681         |
|               | Logarithm       | Field      | 93.65                  | 9.30         | 2.2506            | 0.0950         |
|               |                 | Column     | 93.71                  | 10.04        | 2.2997            | 0.1002         |
|               | Phase           | Field      | 95.96                  | 4.59         | 1.2561            | 0.0530         |
|               |                 | Column     | 97.28                  | 5.65         | 1.4720            | 0.0642         |
| 10            | Amplitude       | Field      | 95.47                  | 4.30         | 1.1326            | 0.0477         |
|               |                 | Column     | 92.83                  | 7.59         | 1.8185            | 0.0800         |
|               | Arctangent      | Field      | 96.52                  | 3.46         | 0.9130            | 0.0384         |
|               |                 | Column     | 97.27                  | 3.80         | 0.9776            | 0.0430         |
|               | Logarithm       | Field      | 86.18                  | 9.03         | 2.3555            | 0.0991         |
|               |                 | Column     | 91.88                  | 8.31         | 1.9940            | 0.0877         |
|               | Phase           | Field      | 92.50                  | 2.69         | 0.7823            | 0.0329         |
|               |                 | Column     | 93.14                  | 7.86         | 1.8939            | 0.0833         |
| 20            | Amplitude       | Field      | 95.07                  | 1.63         | 0.4618            | 0.0194         |
|               |                 | Column     | 86.22                  | 5.27         | 1.3871            | 0.0607         |
|               | Arctangent      | Field      | 96.79                  | 1.11         | 0.3137            | 0.0131         |
|               |                 | Column     | 97.64                  | 1.88         | 0.4920            | 0.0215         |
|               | Logarithm       | Field      | 68.83                  | 5.73         | 1.5094            | 0.0633         |
|               |                 | Column     | 83.34                  | 6.15         | 1.6082            | 0.0704         |
|               | Phase           | Field      | 67.28                  | 2.69         | 0.5163            | 0.0216         |
|               |                 | Column     | 70.22                  | 7.86         | 1.5781            | 0.0691         |
| 30            | Amplitude       | Field      | 86.46                  | 2.75         | 0.7069            | 0.0289         |
|               |                 | Column     | 94.86                  | 2.64         | 0.6933            | 0.0311         |
|               | Arctangent      | Field      | 90.73                  | 2.75         | 0.7000            | 0.0286         |
|               |                 | Column     | 91.67                  | 3.27         | 0.7938            | 0.0356         |
|               | Logarithm       | Field      | 32.63                  | 3.83         | 1.1151            | 0.0456         |
|               |                 | Column     | 92.15                  | 3.21         | 0.8390            | 0.0377         |
|               | Phase           | Field      | 35.50                  | 2.73         | 0.7817            | 0.0320         |
|               |                 | Column     | -5.04                  | 5.59         | 1.4013            | 0.0629         |
Temperature-values were predicted using the $\kappa$-values found by the methods given in table 3. Measured temperature-values were compared with predicted ones to evaluate the performance of the methods. The results are shown in the table 4.

All four methods predicted satisfactory values of temperature well at 0.05, 0.10, and 0.20 m depths of soil profile and soil column as shown in tables 4. For example, for depth $z=5$ and 30 cm the results are shown in figure 3. On the other hand, none of the models performed adequately at 0.30 cm depth of soil profile compared to that of soil column.

![Figure 3 Measured ($T_m$) and amplitude ($T_{p1}$), arctangent ($T_{p2}$), logarithm ($T_{p3}$), and Phase ($T_{p4}$) – predicted soil temperature values at 5 cm and 30 cm soil depths on soil profile and on soil column.](image)

The reason may be attributed to that also, the measured values at 0.30 cm of soil profile were far different from those of soil column (figure 3), which we attributed to the fact that the lower end of the column is open to atmosphere while that of the profile is not, and this would induce the differences in temperature-time relation between soil column and profile at 0.3 m depth. In addition to that, the ambiances of soil column and soil profile are highly different vertically as well as horizontally. The soil column was isolated from the atmosphere with perlite in the outer column, while soil profile as a continuous soil space both vertically and horizontally.

The calculation results showed that, Phase Method (equation 14) outperformed the other methods in predicting soil temperature in profile, for instance for depths 5 and 10 cm, while method Arctangent outperformed the others for soils in column for depths 20 and 30 cm (table 4).

Finally, using the data of tables 2 and 3 and also formula (12) we determined the heat flux on the soil surface ($z = 0$) and at a depth of $z = 5$ cm at the instant of time $t$. Experimental daily behaviors of the temperature and the heat flux for field and column soil are given in figure 4.
From the data presented in figure 4 for \( z=0 \), the heat flux (for soil in field) into the soil attains its maximum \( (Q=400.48 \text{ W/m}^2) \) at 12 hours. The value of the upward-directed heat flux is the largest \( (Q=-400.48 \text{ W/m}^2) \) just midnight, i.e., at 24 hours. An analogous analysis for depth \( z=5 \text{ cm} \) shows that the heat flux into the soil attains its maximum \( (Q=280.14 \text{ W/m}^2) \) at 13 hours. The value of the upward-directed heat flux is the largest \( (Q=-280.14 \text{ W/m}^2) \) at 01.00 hours.

The calculation results for \( z=0 \) showed that the heat flux into soil attains its maximum \( (Q=412.82 \text{ W/m}^2) \) at 12 hours. The upward-directed heat flux is the largest \( (Q=-412.82 \text{ W/m}^2) \) just midnight, i.e., at 24 hours. An analogous analysis for depth \( z=5 \text{ cm} \) shows that the heat flux into the soil attains its maximum \( (Q=316.55 \text{ W/m}^2) \) at 13 hours. The value of the upward-directed heat flux is the largest \( (Q=-316.55 \text{ W/m}^2) \) at 01.00 hours.

6. Conclusions
All four models performed well in predicting soil temperature at all the studied depths both in soil column and profile except at 0.30 m of soil profile. The method comprising heat wave phase amplitude outperformed the rest of the models in predicting soil temperature in soil column for \( z=0.05 \) and 0.10 m, while that comprising the arctangent outperformed the others in the soil profile for \( z=0.20 \) and 0.30 m. Measured values of soil temperature at 0.3 m of soil column were highly different from those of soil profile due to that the lower end of the soil column was open to atmosphere in the latter. The results show that the column studies may not represent the conditions in the field even in the well-prepared experimental conditions since differences in the surrounding media between soil profile and soil column cannot be removed entirely.

The calculated heat flux for soil in field values \( Q_{\text{max}}(0,12h)=400.48 \text{ W/m}^2 \) and \( Q_{\text{max}}(5\text{ cm},12h)=280.14 \text{ W/m}^2 \). For soil in column we have: \( Q_{\text{max}}(0,12h)=412.82 \text{ W/m}^2 \) and \( Q_{\text{max}}(5\text{ cm},13h)=316.55 \text{ W/m}^2 \).

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