The low-energy Goldstone mode in a trapped dipolar supersolid

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A supersolid is a counter-intuitive state of matter that combines the frictionless flow of a superfluid with the crystal-like periodic density modulation of a solid1,2. Since the first prediction3 in the 1950s, experimental efforts to realize this state have focused mainly on helium, in which supersolidity remains unobserved4. Recently, supersolidity has also been studied in ultracold quantum gases, and some of its defining properties have been induced in spin-orbit-coupled Bose–Einstein condensates (BECs)5,6 and BECs coupled to two crossed optical cavities7,8. However, no propagating phonon modes have been observed in either system. Recently, two of the three hallmark properties of a supersolid—periodic density modulation and simultaneous global phase coherence—have been observed in arrays of dipolar quantum droplets9–11, where the crystallization happens in a self-organized manner owing to intrinsic interactions. Here we directly observe the low-energy Goldstone mode, revealing the phase rigidity of the system and thus proving that these droplet arrays are truly supersolid. The dynamics of this mode is reminiscent of the effect of second sound in other superfluid systems28–30 and features an out-of-phase oscillation of the crystal array and the superfluid density. This mode exists only as a result of the phase rigidity of the experimentally realized state, and therefore confirms the supersolidity of the supersolid.

Symmetry breaking is a crucial concept for describing phase transitions in particle16 and condensed matter physics15,16. A spontaneous symmetry breaking occurs when the Hamiltonian of a system is invariant with respect to a certain symmetry while the equilibrium ground state is not. An additional order parameter is therefore necessary to describe the system. A broken continuous symmetry, such as translational invariance or the U(1) symmetry associated with particle number conservation, leads to two types of collective excitation called the Goldstone17 and Higgs18 modes. These modes can usually be identified from the resulting effective potential, which has the shape of a Mexican hat when plotted with respect to the order parameter. A schematic example is shown in Fig. 1a. In this effective potential the gapless Goldstone17 mode and the gapped Higgs18 mode correspond to the phase and amplitude modulation of the complex order parameter at long wavelengths, respectively. In Fig. 1b, c, we schematically show the Goldstone and Higgs modes after breaking the continuous translational symmetry for an infinite system, in which they correspond to a spatial shift (phonons) and an amplitude oscillation of the periodic density modulation. Both Goldstone and Higgs modes have successfully been observed using spectroscopic methods in various platforms including superfluid helium5,9, solid-state systems20–22 and ultracold quantum gases23–27.

A quantum mechanical ground state that simultaneously breaks the global U(1) symmetry and the continuous translational symmetry is the long-sought supersolid. A promising system in which to realize supersolidity is a dipolar quantum gas, which inherently features a preferred length scale for a periodic density modulation owing to its roton-like dispersion relation28–30. This roton wavelength can be tuned through the external trapping potential29. For increasingly dominant dipolar interactions, the roton mode softens and finally becomes imaginary, at which point mean-field theory predicts a collapse. However, this collapse is prevented by beyond-mean-field effects, leading to stable quantum droplets. Supersolid ground states formed by arrays of these droplets have been proposed for suitably chosen combinations of confinement and scattering lengths31,32. Following this, arrays of quantum droplets featuring supersolid properties—specifically, the crystalline density modulation and global phase coherence—have recently been realized experimentally33–35. Simply by tuning the contact interaction, the system undergoes a phase transition from a normal BEC to a supersolid, and then a crossover to an array of isolated droplets. Like a normal crystal, the isolated droplet array has a low compressibility. The coherent array, however, maintains a higher compressibility owing to the compressible BEC component. Distinct from spin–orbit-coupled BECs5 and BECs coupled to two crossed optical cavities7,8, spatial ordering of these droplet arrays arises from intrinsic interactions between the particles, and therefore the crystal can support phonon modes. However, a definite proof of superfluid phase rigidity, a synonym for superfluid stiffness, is still missing and is required in order to verify the supersolid nature of the system.

Here we describe the experimental realization of a state that simultaneously shows the three necessary hallmarks of a supersolid—a periodic density modulation, global phase coherence and phase rigidity. In our work, the phase rigidity is proved by studying the low-energy Goldstone mode of the system, which arises from the two broken symmetries. The observed low-energy Goldstone mode features an out-of-phase oscillation between the droplet array and the superfluid density, involving Josephson-like dynamics between the droplets and therefore highlighting the phase rigidity of the state. The out-of-phase Goldstone mode emerges precisely in the supersolid region and provides compelling evidence of the presence of supersolidity.

For an infinite supersolid droplet array that simultaneously breaks the global U(1) symmetry and the continuous translational symmetry, we can define the corresponding two complex order parameters as the condensate wavefunction \( \Psi(k = 0) \) and the periodic density modulation \( \rho(k = \frac{2\pi}{L}) = \mathcal{F} |\Psi(x)|^2 \) with a wavelength \( d \). Accompanying the two order parameters, there are two branches of Goldstone and Higgs modes. This leads to a complex excitation spectrum33,34, featuring two kinds of sound, one corresponding to the compressibility of the supersolid and one corresponding to the superfluid stiffness. Correspondingly, for a trapped supersolid droplet array with a finite size, the two Goldstone modes consist of an in-phase combination of phonons in the BEC and the crystal part of the supersolid (Fig. 2a), and an out-of-phase oscillation of the two (Fig. 2b). The in-phase mode is exactly the centre-of-mass (COM) dipole oscillation of the whole cloud at the trap frequency. However, the out-of-phase mode maintains the COM by a precise interplay of the crystal motion and the superfluid counterflow, which can have a much lower energy. In contrast to the low-energy Goldstone mode, the Higgs mode in our system is expected...
to be strongly damped and therefore has only a negligible effect on our observations.

To thoroughly explore the two modes in trapped droplet arrays, we implement Bogliubov–de Gennes theory based on the extended Gross–Pitaevskii equation, including the relevant beyond-mean-field corrections. In the supersolid region, both the in-phase and out-of-phase modes are found at the long-wavelength limit that corresponds to the sample size (see Methods). As expected, the energy of the in-phase mode exactly corresponds to the trap frequency, while the out-of-phase Goldstone mode has an excitation energy much lower than the trap frequency. The simulated time evolution of the out-of-phase mode in a supersolid array is shown in Fig. 2c. Starting from the symmetric three-droplet ground state, the droplet array moves to one side.

This movement of the droplet array is accompanied by a superfluid atom transport that redistributes the atoms in order to maintain the COM. For large displacements, which are realized for larger excitation amplitudes, we observe an oscillation between a three- and four-droplet state (see Methods). For each time step we confirm that the COM stays unchanged within our simulation resolution. Furthermore, from this simulation we obtain an oscillation frequency of 5.6 Hz for this low-energy mode, using the parameters of the simulation of Fig. 2c, in agreement with the Bogoliubov result.

To maintain the COM, the superfluid flow characterized by the droplet imbalance $\eta$ of the droplet array and the crystal displacement $\Delta x$ must satisfy a certain relation. In an array of three droplets, we define the imbalance as $\eta = (N_1 - N_3)/(N_1 + N_2 + N_3)$, with $N_i$ the atom number in the $i$th droplet, numbered from left to right, and the displacement $\Delta x$ as the arithmetic mean of the positions of the three droplets relative to the COM of the whole cloud, also including a thermal background. Figure 2d shows the strong correlation between $\eta$ and $\Delta x$ extracted from the numerical simulation shown in Fig. 2c, which is found to be the same for different excitation amplitudes. More importantly, the correlation is also robust against a small variation of the scattering length, corresponding to different fractions of the superfluid background. Therefore, the existence of such a correlation acts as a clear signature for the out-of-phase Goldstone mode, and thus proves the supersolidity of the system.

To experimentally observe this low-energy Goldstone mode, we prepared ultracold quantum gases of the bosonic dysprosium isotope $^{162}$Dy in an elongated optical dipole trap with trap frequencies $\omega = 2\pi [30(1), 89(2), 108(2)]$ Hz and the magnetic field along the $y$ axis (see Methods). Depending on the final scattering length, the prepared cloud can be in the normal BEC phase ($>100 a_0$), an incoherent droplet array ($<96 a_0$), or a coherent droplet array in between ($a_0$ is the Bohr radius). After reaching the final scattering length, we let the cloud evolve and equilibrate for 15 ms before we probe it using in situ phase-contrast imaging.

Because the oscillation period of the out-of-phase mode is much longer than the droplet lifetime, which is limited by three-body losses, it is currently not feasible to use a standard spectroscopic method to detect the Goldstone mode. However, the low excitation energy of the

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**Fig. 1 | Goldstone and Higgs modes arising from the broken continuous translational symmetry in an infinite supersolid.** (a) Effective potential of an ordered phase as a function of the complex order parameter. Higgs and Goldstone modes correspond to amplitude $\delta a$ and phase $\delta \phi$ modulation of the order parameter $\Psi = Ae^{i\phi}$. The gapless Goldstone mode leads to a degeneracy of the ground state. (b, c) Illustration of the Higgs (b) and Goldstone (c) modes for a broken continuous translational symmetry in an infinite system, for which the modes correspond to an amplitude modulation and a spatial shift of the crystal structure, respectively.

**Fig. 2 | Goldstone modes in a harmonic potential.** (a) and (b) The in-phase (a) and out-of-phase (b) Goldstone modes in a harmonic trap in the long-wavelength limit. The arrows indicate the spatial displacement $\Delta x$ of the droplet array and the flow of the superfluid density. Vertical dashed lines indicate the COM position of the whole cloud. The in-phase mode corresponds to the COM dipole oscillation with the trap frequency, while the out-of-phase mode maintains the COM owing to the counterflow of the droplet array and the superfluid background. As a result, this mode has a very low energy. (c) Simulated dynamics of the out-of-phase Goldstone mode for the ground state, an array of three droplets. The dynamics is illustrated by the normalized line density along the $x$ axis, $n(x)$. The spatial motion of the droplet array is compensated by a superfluid flow of atoms in the opposite direction in order to maintain the COM. (d) Numerically predicted correlation between the imbalance $\eta$ and the displacement $\Delta x$ of the droplet array, together with a linear fit. The error bars shown indicate the uncertainty of the fit used to extract $\eta$ and $\Delta x$. 

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**Methods.** To thoroughly explore the two modes in trapped droplet arrays, we implement Bogliubov–de Gennes theory based on the extended Gross–Pitaevskii equation, including the relevant beyond-mean-field corrections. In the supersolid region, both the in-phase and out-of-phase modes are found at the long-wavelength limit that corresponds to the sample size (see Methods). As expected, the energy of the in-phase mode exactly corresponds to the trap frequency, while the out-of-phase Goldstone mode has an excitation energy much lower than the trap frequency. The simulated time evolution of the out-of-phase mode in a supersolid array is shown in Fig. 2c. Starting from the symmetric three-droplet ground state, the droplet array moves to one side.

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Because the oscillation period of the out-of-phase mode is much longer than the droplet lifetime, which is limited by three-body losses, it is currently not feasible to use a standard spectroscopic method to detect the Goldstone mode. However, the low excitation energy of the
out-of-phase Goldstone mode leads to its excitation within the energy bandwidth of the dynamical formation process. As a result, the mode should always be excited in the prepared samples with a different phase owing to experimental imperfections, such as the non-adiabatic ramping of the scattering length, technical noise and thermal fluctuations. We therefore repeated the experiment many times at each scattering length, in order to statistically map out the correlation between the imbalance $\eta$ and the crystal displacement $\Delta x$.

After post-selection on the total atom number (see Methods), $\Delta x$ and $\eta$ are extracted for each cloud by fitting the in situ density distributions with a sum of four Gaussian functions, corresponding to a BEC or thermal background and three droplets. Some example images with different imbalances and displacements $\Delta x$ for a scattering length of $97.6 a_\text{B}$ are shown in Fig. 3a to visualize how the imbalance changes with respect to the spatial shift of the droplets. Figure 3b shows the measured correlation between $\eta$ and $\Delta x$ for the observed three-droplet states in the supersolid region, where a strong correlation is observed. More importantly, the correlation perfectly coincides with the theoretical prediction (red curve in Fig. 3b) without any free parameters. We interpret the spread in the data around the theoretical correlations as imperfections in the extraction of the imbalance and the displacement, as well as small excitations of higher-lying modes. The existence of the out-of-phase Goldstone mode proves the presence of the low-energy Goldstone mode, and therefore the supersolidity of the system. For comparison, one example of isolated droplet arrays at $91.2 a_\text{B}$ is shown in Fig. 3c, where the correlation is missing, resulting from the lack of superfluid counterflow due to the vanishing superfluid fraction.

To quantify the range of the contact interaction for which we can observe supersolid arrays of quantum droplets, we calculated the variance of the experimental data with respect to the theoretical simulations. A small variance is therefore evidence for the existence of the low-energy Goldstone mode. For comparison, one example of supersolid BEC states is shown in Fig. 3d. We can clearly see a region with smaller variance, and therefore stronger correlation, which coincides exactly with the phase-coherent region in $a$ is the range where we observe the smallest variance, and therefore the strongest correlation, which indicates the existence of the low-energy Goldstone mode. The errors bars indicate one standard error of the experimental data with respect to the correlation obtained from the theoretical simulations.

In conclusion, we have studied the low-energy Goldstone mode in a trapped supersolid droplet array of dipolar dysprosium atoms, which

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**Fig. 3 | Experimental correlation between $\eta$ and $\Delta x$.**

- **a.** Exemplary in situ images of the normalized density $n(x,y)$ with different displacements of the droplet array relative to the total COM of the whole cloud (dashed line). When the droplet array shifts to the right (left) with a positive (or negative) $\Delta x$, atoms flow to the left (or right), resulting in higher atom number on the left (or right) side, in order to keep the COM unchanged. The integrated densities together with the fits (red curves) are also shown.
- **b, c.** Experimental correlations in the supersolid (97.6$a_\text{B}$) (b) and isolated droplet (91.2$a_\text{B}$) (c) regions, respectively. The error bars correspond to the uncertainty of the fits used to extract the position and atom number of each individual droplet. The red curve is the theoretical prediction for a supersolid state without any free parameters, as shown in Fig. 2d. A correlation is clearly demonstrated in the supersolid region, whereas such a correlation is missing in the isolated droplet phase.

**Fig. 4 | The $\eta-\Delta x$ correlation across the phase diagram.**

- **a.** Experimental signature of the phase diagram for our trap geometry. We can identify the different regions by evaluating the in situ density modulation (blue, characterized by the corresponding Fourier weight at the modulation wavelength) and the global phase coherence (red, characterized by averages of the complex fast Fourier transform (FFT) weights for more than 50 repeated time-of-flight interference patterns)\(^6\). The shadowed area is the coherent droplet region we determined. a.u., arbitrary units.
- **b.** Mean variance of the experimental data with respect to the theoretical correlation curve for the supersolid state. Coinciding exactly with the phase-coherent region in $a$ is the range where we observe the smallest variance, and therefore the strongest correlation, which indicates the existence of the low-energy Goldstone mode. The red bars indicate one standard error of the experimental data with respect to the correlation obtained from the theoretical simulations.

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**In conclusion,** we have studied the low-energy Goldstone mode in a trapped supersolid droplet array of dipolar dysprosium atoms, which
enables the simultaneous detection of phase coherence and phase rigidity. The out-of-phase mode features a counterflow of the crystal-like droplet array and the superfluid density, leading to a robust correlation between the imbalance and the displacement of the droplets. This mode therefore directly connects to the broken $U(1)$ symmetry and the continuous translational symmetry, highlighting the supersolid nature of the coherent arrays of dipolar quantum droplets. An extension to our work would be the observation of other collective excitations, especially the Higgs modes, which require additional symmetries to suppress their decay into other lower-energy excitations. Another goal could be to realize a supersolid state with larger droplet numbers, or even two-dimensional supersolid arrays, where an additional spatial symmetry is broken, leading to an even more complex excitation spectrum.

We note that during the preparation (review process) of this manuscript, we became aware of related complementary studies of higher-lying collective modes.

**Online content**

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41586-019-1569-5.

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METHODS

Sample preparation and experimental details. The complete experimental sequence is described in detail in our previous publications9,38. In short, we prepare a quasiparticle BEC of $^{40}$K in a crossed optical dipole trap formed by two laser beams at 1.064 nm. The degenerate cloud typically contains $4 \times 10^4$ atoms at a temperature below 10 nK. After evaporation, the trap is reshaped within 20 ms to the final trap geometry with trap frequencies of $\omega = 2\pi \times (30(1), 9(2), 108(2))$ Hz. For the measurements presented, the magnetic field $\mathbf{B}$ is oriented along the $y$ direction. Subsequently, we tune the contact interaction from the background scattering length $a_0 = 140(20) a_0$ (refs. 39–41) to approximately 112$a_0$ by ramping the magnetic field closer to the Feshbach resonances near $5.1 G$ (refs. 9,42,43). To reach the droplet region, the magnetic field is further linearly ramped to the final scattering length in the range between 90$a_0$ and 105$a_0$ in 30 ms. Then we let the samples evolve for 15 ms to allow the quantum droplet arrays to form and equilibrate. Subsequently we probe the atomic clouds with our in situ phase-contrast imaging, which is performed along the z axis using a microscope objective with a numerical aperture of 0.3. Our resolution of about 1 $\mu$m allows us to distinguish nearby droplets that are separated by about 3 $\mu$m. By fitting each individual droplet with a Gaussian, we can extract the atom number in each individual droplet, as well as the position of the droplet to a higher precision than our imaging resolution.

To verify the range for which we observe phase-coherent droplet arrays, we implement a time-of-flight interference sequence similar to our previous work9. For this we ramp up the scattering length within 100 $\mu$s to about $a_0$ to accelerate the expansion and then release the atoms from the trap. Owing to the changed geometry the expansion time is now limited to 7.2 ms compared to our previous publication9. Nonetheless, we can observe clear interference patterns and are able to distinguish between phase-coherent and incoherent droplet regimes. Notably, for samples with smaller scattering lengths, which means for magnetic fields closer to the Feshbach resonances, the clouds suffer from more severe three-body loss. The black data points in Extended Data Fig. 1 indicate the corresponding average atom number in the experiment for each scattering length. For our evaluation we post-select realizations in a range $\pm 15\%$ around the average atom number at each scattering length in the statistical evaluation from which the correlation is extracted. The experimental realizations mostly consist of three or four droplets. We analyse these two cases separately after distinguishing them with multiple Gaussian fits. Although the COM is experimentally determined by the whole image, it is dominated by the condensate and thermal background within the BEC and coherent droplet region. For isolated droplets we also observe a thermal background, which then acts as the main contribution to the determined COM of the whole cloud that we use as a reference for the calculation of the displacement.

For the range of scattering lengths studied in this work, the experimentally determined stability of the magnetic field leads to an uncertainty of about 1 $a_0$, while the calibration of the positions and widths of the double Feshbach resonances, which determine the scattering length, results in an uncertainty of about 4 $a_0$. On top of this uncertainty, there is an overall systematic uncertainty due to the absolute value of the background scattering length39–41, which has so far not been measured to high precision. This leads to an uncertainty of all calculated scattering lengths that is of the order of 15%.

Simulation details. Our theory is based on numerically solving the extended Gross–Pitaevskii equation31,44,45, including quantum fluctuations as beyond-mean-field corrections, which prevents the system from collapse and leads to stable quantum droplets3,46,47. We obtain the ground states by performing imaginary time evolution48 of the extended Gross–Pitaevskii equation including a harmonic potential with $\omega = 2\pi \times (30, 90, 110)$ Hz, similar to the experiment. In this trap we obtain a three-droplet ground state for scattering lengths $a_s \geq 91 a_0$ as shown in Extended Data Fig. 3. To identify the phase-coherent droplet arrays, we use the same indicator of the nearest minimum divided by the central maximum as an estimation of the overlap between the droplets, as in our previous publication9. The result is shown in Extended Data Fig. 1, where we can successfully identify the three different regions, with the coherent region being located at $94a_0$–$97a_0$. Compared to the experimental data, the coherent region obtained from the simulations is shifted by $3a_0$, in line with recent results with erbium atoms showing a similar deviation11,15.

To calculate the excitation spectrum, we follow the Bogoliubov–de Gennes theory by linearizing the extended Gross–Pitaevskii equation around the ground state44,45. The resulting Bogoliubov–de Gennes equations are then solved numerically to obtain the excitation modes49. Examples of the dispersion relations obtained are shown in Extended Data Fig. 2 for three different scattering lengths. Owing to finite sample size, the excitation spectra become discrete, with each mode corresponding to a spread out momentum. To visualize the spectra we take the eigenvector structure factor50,51, which indicates the dynamic response of the cloud. The higher the amplitude of this structure factor, the stronger the density response of the system to the corresponding mode. In this work, we ignore the modes with excitation energies higher than about 90 Hz, at which point excitations along the other trap axes start to have a role, making the spectrum much more complex.

In the BEC phase (Extended Data Fig. 2a), there is only one excitation branch, featuring an increasing excitation energy with increasing momentum $q$. By decreasing the contact interaction strength we can observe the appearance of the roton mode in Extended Data Fig. 2b. Lowering the scattering length further, the roton mode softens and we enter the supersolid regime, where an additional excitation branch with lower energy appears, owing to breaking of the continuous translational symmetry. This low-energy mode corresponds to the out-of-phase Goldstone mode, whose correlation we directly observe in the experiment. Close to the phase transition from BEC to supersolid, the calculations clearly reveal a large energy splitting between the low-energy mode and any other collective mode, especially the in-phase COM mode at the trap frequency. For smaller contact interaction strength we reach the regime of isolated droplets, where the low-energy out-of-phase mode decreases in energy. At the same time, we observe the emergence of a clear periodicity in the excitation spectrum owing to the underlying crystal structure.

Dynamics of the out-of-phase Goldstone mode. From the numerically solved Bogoliubov–de Gennes equations, we can also obtain the phase pattern of each excitation mode. In Extended Data Fig. 3 we show the calculated one-dimensional line cut through the three-dimensional phase pattern of the out-of-phase Goldstone mode, together with a density cut through the calculated density profile of the three-droplet ground state. As in Josephson dynamics, the phase gradient is directly proportional to the particle flow. Distinct from the phase pattern of the in-phase dipole mode, which is a constant phase gradient over the cloud, the out-of-phase mode has, with some phase pattern, with each step coinciding with a dimer or droplet. Whereas the whole BEC background thus experiences a phase gradient with a particular direction, the droplets always experience a gradient in the opposite direction. This leads to the counterflow between the BEC background and the crystal that characterizes this mode. By numerically imprinting the phase pattern of a specific collective excitation onto the ground state, we can directly simulate the excitation dynamics of each individual mode by performing a real-time evolution of the extended Gross–Pitaevskii equation. Doing this for the out-of-phase Goldstone mode, we obtain the dynamical time evolution of this low-energy mode, as shown in Fig. 2c.

We simulate the real-time dynamics of the system for different scattering lengths in the supersolid range. Decreasing the scattering length, we observe an increase of the oscillation period, in line with the results obtained from the calculated dispersion relations. This decrease of the oscillation frequency is accompanied by a decrease in the oscillation amplitude. Looking at the correlation between the imbalance $\eta$ and displacement $\Delta x$ that we use as an indication of the presence of the low-energy Goldstone mode, we observe that it remains unchanged within the uncertainty of our evaluation, even though the oscillation frequency nearly doubles in the range of the scattering length that we studied.

Using a similar procedure, we have also checked that the correlation is independent of the excitation amplitude. This is the case along as long as the oscillation amplitude remains sufficiently small. In our simulations for larger oscillation amplitudes, we can observe that the three-droplet state changes to a four-droplet state for large displacements. As an example, we show two simulated real-time evolutions with higher amplitudes of the low-energy mode in Extended Data Fig. 4. The change from a three-droplet state to a four-droplet state can happen because in our particular trap and at the studied scattering lengths, the energy difference between the two states is small. However, the periodic coherent emergence and disappearance of a new droplet on the edges of the system is further evidence of the presence of superfluid flow. For very large amplitudes, as shown in Extended Data Fig. 4b, the excitation is no longer oscillatory, but travelling in one direction, reminiscent of the Goldstone mode in an infinite system (see Fig. 1c).

Evaluation of four-droplet states. For large displacements in our dynamical simulations of the Goldstone mode, which happen at a large amplitude of the excitation, the three-droplet ground state can change to a four-droplet state. Since the two states with different droplet number are smoothly connected, we also observe four-droplet states in the experiment, owing to strong excitations of the Goldstone mode.

To maintain the COM, this mode should again satisfy a certain relation between the imbalance, which we now define as $n = (N_1 + N_2 - N_3 - N_4)/2$ ($N_1 + N_2 + N_3 + N_4$) for a four-droplet state, and the displacement $\Delta x$, which we again define as the arithmetical mean of all four positions of the individual droplets relative to the COM of the whole cloud. The calculated correlation of a four-droplet state is shown in Extended Data Fig. 5. The data in this plot correspond to the large displacement data that are shown in Extended Data Fig. 4. Again, we obtain a linear relationship, which is robust against variations of the scattering length, as well as the excitation amplitude.

In our experiment we also find a sizeable amount of realizations with four droplets, for which we perform the same statistical analysis as we presented in...
the main text for the three-droplet states. As for the three-droplet state, we find a clear correlation for the supersolid regime (Extended Data Fig. 6a), while the variance of the data are larger for the isolated droplet regime (Extended Data Fig. 6b), indicating that no correlation exists in the isolated droplet region. This can also be seen in the example images of four-droplet states in the supersolid regime with different displacements, which are shown in Extended Data Fig. 6c. Similar to the three-droplet case, we can again calculate the variance of the experimental data with respect to the theoretical correlation curve as an indicator of the existence of the Goldstone mode with respect to the contact interaction strength. Similar to Fig. 4b, the variance of the four-droplet state shown in Extended Data Fig. 7 has a clear minimum in the range where we observe phase-coherent droplet arrays.

**Theory-independent evaluation.** As a test that is independent of our underlying theory, we implement a linear fit to the experimental data at each scattering length and compare the data to this fit, instead of the theoretically obtained correlation. This linear fit allows us to do two complementary tests. First, we can again calculate the spread of our experimental measurements with respect to the linear fit. In agreement with the comparison to the theoretical curve, the calculated variance with respect to the linear fit is again lowest directly in the phase-coherent regime and increases by changing the scattering length in either direction. Second, we can look at the absolute value of the intersection point of the linear fit with the axis of the displacement. For a supersolid state, this intersection should be at zero, corresponding to a symmetric droplet array with a vanishing imbalance at no displacement. For isolated droplets, on the other hand, initial fluctuations during the formation process cannot be compensated, meaning that we can get imbalanced droplet arrays, even if the array is in the centre of the cloud. The intersection points obtained for the observed three- and four-droplet states are shown in Extended Data Fig. 8a, b, respectively. This shows that the intersection point is close to zero only in the supersolid region. Both of these theory-independent checks are additional proof of the existence of a correlation between the imbalance and the displacement arising due from the low-energy Goldstone mode of the system.

**Data availability**
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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**Author contributions** M.G., F.B. and J.-N.S. performed the experiment and analysed the data. J.H. and M.W. performed the numerical analysis. H.P.B., T.L. and T.P. provided scientific guidance in experimental and theoretical questions. All authors contributed to the interpretation of the data and the writing of the manuscript.

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**Additional information**

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Extended Data Fig. 1 | Theoretical phase boundaries. As an indicator of the phase coherence, we show the ratio of the first minimum in the density compared to the central droplet peak density as an indicator of the overlap between the droplets$^{43}$ of the calculated density profile of the ground state in red. The shaded area is the coherent region determined from the overlap. As in the experiment, three regions are identified and the coherent region locates between $94a_0$ and $97a_0$, shifting approximately $3a_0$ from the experimentally obtained phase diagram. These simulations are done for an atom number of $30 \times 10^3$. The black points indicate the measured average atom number in the experiment, with the error bars representing standard deviations for more than ten shots at each scattering length. (a.u., arbitrary units.).
Extended Data Fig. 2 | Dynamic structure factor of the collective excitations. Calculated structure factor $S(\omega, q)$ in the BEC phase for the scattering length $a_s = 100a_0$ (a) and $a_s = 98a_0$ (b) and in the supersolid droplet arrays (c) for $a_s = 96a_0$. Owing to the finite size of the system, long-wavelength modes become discrete and the lowest possible excitation energy is set by the trap frequency (white horizontal line). For decreasing contact interaction strengths we can observe the roton minimum emerging, until finally its gap closes and the supersolid appears. In the supersolid regime we can clearly see the low-energy out-of-phase Goldstone mode and the large gap to all the other modes above the trap frequency of 30 Hz. The colour scale is normalized to the mode with the highest response across the scattering lengths shown.
Extended Data Fig. 3 | Phase pattern of the Goldstone mode. Shown are the density cut through the three-droplet ground state (black) along the x axis and the phase pattern (red) corresponding to the low-energy Goldstone mode.
Extended Data Fig. 4 | Large-amplitude dynamics of the out-of-phase Goldstone mode. Starting from an array of three droplets, the state can change to a four-droplet state for large excitation amplitudes. From there it either oscillates back and forth between the two (a) or the excitation amplitude is so large that we find that the motion is no longer oscillatory, with the excitation instead travelling only in one direction (b). Similar to Fig. 2, $n(x)$ is the normalized line density along the $x$ axis.
Extended Data Fig. 5 | Simulated \( \eta - \Delta x \) correlation of the four-droplet dynamic states. Numerically predicted correlation between the imbalance \( \eta \) and droplet displacement \( \Delta x \) for the four-droplet states appearing at large excitation amplitudes of the low-energy Goldstone mode (blue points, with error bars indicating the uncertainty of the fit used to extract \( \eta \) and \( \Delta x \), as in Fig. 2). The red line is a linear fit.
Extended Data Fig. 6 | Experimental correlation of the observed four-droplet states. Similar to Fig. 3, we show the experimental correlation in the supersolid (97.6a₀, a) and isolated droplet (91.2a₀, b) regions, as well as example in situ images of four-droplet states, with n(x,y) the normalized density (c). For the four-droplet states we observe a clear correlation of imbalance and displacement throughout the supersolid region.
Extended Data Fig. 7 | Variance of the four-droplet data with respect to the theoretical correlation. Similar to Fig. 4b, we find that the variance of the four-droplet data with respect to the theoretical correlation curve (Extended Data Fig. 6) is lowest in the supersolid region. As in Fig. 4a, the shaded area is the coherent region determined experimentally. The error bars indicate one standard error of the experimental data with respect to the theoretical correlation.
Extended Data Fig. 8 | Theory-independent evaluation. Intersection point of the fit with the displacement axis for the three-droplet states (a) and the four-droplet states (b) obtained from a linear fit to the $\eta-\Delta x$ correlation data across the part of the phase diagram we explored. An intersection point close to zero indicates the presence of a superfluid flow that can compensate fluctuations during the formation process. The error bars shown represent the standard error of the fitted intersections. As in Fig. 4a, the shaded area is the coherent region determined experimentally.