Reflection from black holes

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Black holes are presumed to have an ideal ability to absorb and keep matter. Whatever comes close to the event horizon, a boundary separating the inside region of a black hole from the outside world, inevitably goes in and remains inside forever. This work shows, however, that quantum corrections make possible a surprising process, reflection: a particle can bounce back from the event horizon. For low energy particles this process is efficient, black holes behave not as holes, but as mirrors, which changes our perception of their physical nature. Possible ways for observations of the reflection and its relation to the Hawking radiation process are outlined.

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Conventional intuitive arguments attribute two defining properties to black holes. They should absorb every particle that comes close, releasing nothing back. There is, however, a limitation on this intuitive picture. It stems from thermodynamics properties that ascribe the entropy and temperature to black holes. The first indication that gravitational fields could have entropy came when investigation by Christodoulou [1] of the Penrose process [2] for extracting energy from the Kerr black hole showed that there is a quantity that could not go down, which Hawking found [3] to be proportional to the area of the horizon. For low energy particles this process is efficient, black holes behave not as holes, but as mirrors, which changes our perception of their physical nature. Possible ways for observations of the reflection and its relation to the Hawking radiation process are outlined.

It follows from this that the action has a logarithmic singularity on the horizon

\[ S(r) \simeq \mp \varepsilon \ln(r - 1), \quad r \to 1. \]  

The signs minus and plus in (2) and (3) correspond to the incoming and outgoing trajectories respectively; they are discussed in some detail below with the help of Fig. 1. Importantly, the singularity of the action is a covariant property, it persists even in those coordinates that eliminate the singularity of the metric and classical equations of motion. Consider, for instance, the Kruskal coordinates \( U, V \) defined according to

\[ U = - (r - 1)^{1/2} \exp[(r - t)/2], \]
\[ V = (r - 1)^{1/2} \exp[(r + t)/2]. \]

It is known, e.g. Ref. [10], that in \( U, V \) coordinates there are four regions of the space-time that are shown in Fig. 1. Areas I and III represent two identical copies of the outside, asymptotically flat region. Areas II, IV show two copies of the inside region that have different physical properties. All four areas are divided by the horizon that is located at \( U = 0, \) or \( V = 0. \) In the vicinity of the horizon the incoming particle follows the trajectory \( AB \) and, after crossing the horizon, resides in II. There are also the outgoing trajectories; following the trajectory \( CD \) the particle escapes from the inside region \( IV \) into the outside world \( I. \) Importantly, the areas II and IV are not connected. This fact ensures that when the particle comes to the inside region \( II \) it stays there, there is no classical trajectory that would bring it back into the outside world. In Kruskal coordinates the metric is regular on the horizon. The classical equations of motion are also regular. In the vicinity of the horizon

\[ ds^2 = - \left(1 - \frac{1}{r}\right) dt^2 + \frac{dr^2}{1 - 1/r} + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2). \]
they are described by simple equations $V = \text{const}$ for the incoming trajectory, and $U = \text{const}$ for the outgoing trajectory (Fig. 1). In contrast, the singularity of the action persists. It is convenient to present it in terms of the full action $S = -\varepsilon t + S(r)$ (in which we drop now the irrelevant for us angular term $L_\phi$), which in the vicinity of the horizon reads

$$S \simeq -\varepsilon \ln(V^2), \quad U^2 \ll V^2 \ll 1, \quad \varepsilon \ln(V^2), \quad V^2 \ll U^2 \ll 1. \quad (6)$$

We observe an interesting distinction. The classical equations of motion can be made regular on the horizon, while the action has a singularity. Similar difference exists in the Aharonov-Bohm effect [17], where the classical equations of motion do not depend on the flux of the magnetic field, while the action exhibits the singularity. Using the Aharonov-Bohm effect as a motivation, one can anticipate that the singularity of the action should have important consequences for quantum description. Developing this argument, let us describe the radial motion of the probing particle with the help of the semiclassical wave function $\phi(r) \propto \exp[iS(r)]$ [21]. Taking $r$ in the outside region in the vicinity of the horizon and using the action [3] one can write the wave function as

$$\phi(r) = \exp[-i\varepsilon \ln(r-1)] + \mathcal{R} \exp[i\varepsilon \ln(r-1)]. \quad (8)$$

Here the two terms on the right-hand side are constructed from the incoming and outgoing classical trajectories respectively. Hence, the first term represents the proper incoming wave that describes the particle approaching the event horizon. The corresponding classical trajectory is shown by $AB$ in Fig. 1. The second term presents the proper outgoing wave; it is introduced in order to allow for an opportunity of the mixing, interference of the incoming and outgoing waves in the wave function. This wave corresponds to the classical trajectory shown by $CD$ in Fig. 1. The quantity $\mathcal{R}$ in (3) has the meaning of the reflection coefficient. The conservation of the flux of particles implies $|\mathcal{R}| \leq 1$. Moreover, conventional intuitive arguments prompt one to put the reflection coefficient in (3) to zero, $\mathcal{R} = 0$; indeed, if the black hole is a perfect absorber, then only the incoming wave should describe the particle approaching the horizon. However, the wave function (3) is singular on the horizon $r = 1$, while the intuitive arguments are based on the properties of classical trajectories that by themselves are unable to probe a possible singularity at $r = 1$, simply because the trajectories possess none.

Thus, generally speaking, the reflection coefficient $\mathcal{R}$ in (3) may have a nonzero value due to quantum reasons. To verify that this really happens, let us use the analytical continuation of the wave function. Introduce with this purpose a new parameter $\zeta$, $z = r - 1$, considering $\zeta$ as a complex variable. Choose some coordinate in the vicinity of the horizon taking $\zeta > 0$, $\zeta \ll 1$. Rotate now $\zeta$ in the complex $\zeta$-plane clockwise over the angle $2\pi$ (anticlockwise rotation is not justified, see below). This complex transformation brings $\zeta$ back to its original physical value $z > 0$. However, since the function (3) is singular at $z = 0$, it acquires some new value, let us call it $\phi^{(2\pi)}(\zeta)$. From (3) one derives

$$\phi^{(2\pi)}(\zeta) = \varrho \exp[-i\varepsilon \ln(\zeta-1)] + \frac{\mathcal{R}}{\varrho} \exp[i\varepsilon \ln(\zeta-1)], \quad (9)$$

where $\varrho = \exp(-2\pi\varepsilon)$. The analytically continued function $\phi^{(2\pi)}(\zeta)$ satisfies the same equation as the initial function $\phi(r)$. Moreover, one has to expect the wave function $\phi^{(2\pi)}(\zeta)$ to satisfy the same normalization conditions as the initial wave function $\phi(r)$. This requires that one of the coefficients in (9), either $\varrho$, or $\mathcal{R}/\varrho$ should have an absolute value equal to unity, similar to the coefficient unity in front of the first term on the right-hand side of (3). Since $\varrho < 1$, one finds that $|\mathcal{R}|/\varrho = 1$, concluding that

$$|\mathcal{R}| = \exp(-2\pi\varepsilon). \quad (10)$$
This result indicates that the event horizon can reflect particles.

The symmetry of the space-time provides a more general and reliable way leading to the same conclusion. Remember, the areas I and III in Fig.1 describe two identical sets of the outside region. This means that a transformation that brings an arbitrary point A of the region I into the symmetrically located point A' in the region III, see Fig.1 is a symmetry of the wave function. The wave function (5) is presumed to be a scalar, therefore this symmetry can manifest itself only through variation of the phase of the wave function. In other words, the transformation $A \rightarrow A'$ results in the transformation $\phi \rightarrow \phi' = \pm \phi$. Being applied twice, this transformation brings the point $A'$ back to A and should therefore bring the wave function to its initial value. This fact fixes the phase of the space-time up to a sign $\pm$.

There is a convenient way to implement this symmetry. Let us use firstly the complex rotation $z \rightarrow \exp(-2\pi i)z$. Eqs. (4), (5) show that it results in the transformation $U \rightarrow -U, V \rightarrow -V$ that brings the point A to $A''$ in Fig.1. After that we can transform $A'' \rightarrow A'$ using the operation of the time inversion $T$, see (4), (5). The necessity of the time inversion is related to the fact that the arrow of time flow in areas I and III is opposite, see Fig.1. The symmetry of the space-time provides a more general form for the wave function $\phi$ in a more general form

$$|\phi\rangle = |\text{in}\rangle + |\mathcal{R}|\langle\text{out}| .$$

Here the time-dependent factor $\exp(-i\varepsilon t)$ is included in the wave function $|\phi\rangle = \exp(-i\varepsilon t)\phi(r)$. This form allows one to present the incoming and outgoing waves in (15) via convenient Kruskal coordinates (11). (10)$$\langle\text{in}\rangle = \exp[-i\varepsilon \ln(V^2)], \quad |\text{out}\rangle = \exp[i \varepsilon \ln(U^2)].$$

Our consideration so far was restricted by the outside region. Eqs. (10), (17) provide now an opportunity to verify that Eqs. (15), (10) remain valid in the inside region as well. To justify this claim let us use the symmetry (11) rewriting in the following form

$$\phi^{(2\pi)}(V,U) = \pm \phi(U,V),$$

where $\phi(U,V) \equiv \langle U,V|\phi\rangle$ and $\phi^{(2\pi)}(U,V)$ is defined with the help of the transformation $z \rightarrow \exp(-2\pi i)z$ applied to the function $\phi(U,V)$ in which the arguments $U,V$ are functions of $z$. Eqs. (12), (17), (18) show that, indeed, Eqs. (15), (10) for the wave function remain valid inside the horizon. (Thus Eqs. (15), (10) give a general form for the wave function, valid inside and outside the horizon.)

An interesting implication arises for a particle confined in the inside region of a black hole. Classically this particle is localized in the region II in Fig.1. However, quantum result (15) shows that the wave function of this particle necessarily incorporates an admixture of the second term, which is the outgoing wave. This wave corresponds to outgoing classical trajectories, see CD in Fig.1. In other words, the wave function of the confined particle necessarily incorporates an admixture of the term that is located in the inside region IV. Remember that the two internal regions II and IV are completely isolated from each other in the classical approximation. In the quantum picture they prove to be related because events that take place in these two regions interfere in wave function (15). Therefore an attempt to localize the particle
exclusively in the region II fails; there is a finite probability to find this particle in the region IV. From this region the confined particle can escape into the outside world following the classical outgoing trajectory, see CD in Fig.1.

We see that confinement inside a black hole cannot be absolute, the locked in particle has a chance to run away. Hence, a black hole necessarily creates a flux of particles escaping from its inside region. It is natural to identify this flux with the Hawking radiation. The validity of this identification is supported by the following arguments. Consider the probability of the escape, which equals \( W_{\text{rad}} = \mathcal{P}(1 - \mathcal{P}) \). The first factor here \( \mathcal{P} = R^2 \) describes the probability to populate the outgoing state in the wave function \( |1\rangle \), while the second factor \( 1 - \mathcal{P} \) refers to the probability to cross the horizon; it is less than 1 due to reflection from the horizon. Comparing this result with the probability of absorption of the incoming particle \( W_{\text{abs}} = 1 - \mathcal{P} \) (where again the reflection is responsible for the factor \( 1 - \mathcal{P} \)), one finds that the ratio of the two probabilities satisfies relation

\[
W_{\text{rad}}/W_{\text{abs}} = \mathcal{P} = \exp(-\varepsilon/T), \tag{19}
\]

which shows that the black hole remains in equilibrium with the field of radiation if the latter has the temperature \( T \). Consequently, the black hole radiates as a black body with the temperature \( T \), in agreement with the Hawking effect. Thus Eq.15 provides a new transparent way to explain the radiation process. The radiation takes place simply because particles escape from the inside region of black holes.

Quantum corrections described by \( [4, 5, 6] \) amend basic properties of black holes. Classically an incoming particle freely crosses the event horizon going inside, but after that cannot go out. Quantum corrections make possible the reflection of the incoming particle from the horizon. For low energy particles, \( \varepsilon < T \), the reflection is strong, a black hole behaves as a mirror. For experimental observation of this effect there should be found a strong source of low frequency radiation located closely to a black hole to make the signal reflected by the black hole observable \( [22] \). The reflection takes place even for strong gravitational fields created by small black holes with the mass comparable with the Planck mass. There are known few quantum phenomena for strong gravitation fields, including the Hawking radiation and a suggestion to quantize the black hole spectrum \( |n\rangle \); the reflection from black holes is a new entrant in the strong-field area. Quantum corrections also make impossible complete confinement; a particle locked in inside the horizon maintains an opportunity to run away into the outside world, providing a new simple explanation for the phenomenon of the Hawking radiation. Our discussion was restricted by the Schwarzschild metric \( [10] \). Very similar conclusions hold for charged rotating black holes as will be discussed later.

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[21] It can be shown that on the horizon \( r \to 1 \) the semiclassical description is asymptotically accurate; in particular, the preexponential factor in relation \( \phi(r) \propto \exp[iS(r)] \) is a constant.
[22] To mention one opportunity, consider the collapse in a binary system that consists of a star and a black hole. The gravitational and electromagnetic waves that take the energy out of this system have the necessary low frequency spectrum and are powerful at latest stages of the collapse. One should expect that the reflection of these waves from the black hole should manifest itself in intensity as well as in spectral and angular distributions of the radiation.