Phase Transition in Conformally Induced Gravity with Torsion

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We have considered the quantum behavior of a conformally induced gravity in the minimal Riemann-Cartan space. The regularized one-loop effective potential considering the quantum fluctuations of the dilaton and the torsion fields in the Coleman-Weinberg sector gives a sensible phase transition for an inflationary phase in De Sitter space. For this effective potential, we have analyzed the semi-classical equation of motion of the dilaton field in the slow-rolling regime.

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I. INTRODUCTION

Among the four fundamental interactions in nature, the two feeble interactions are characterized by dimensional coupling constants; those are the Fermi’s coupling constant \(G_F = (300 \text{ Gev})^{-2}\) and Newton’s coupling constant \(G_N = (10^{19} \text{ Gev})^{-2}\).

The interactions with dimensional coupling constants of inverse mass dimensions are strongly diverse and non-renomalizable. However, from the success of Weinberg-Salam model, the weak interaction at a fundamental level is actually characterized by a dimensionless coupling constant, and the dimensional nature of \(G_F\) results from a spontaneous symmetry breaking. Indeed \(G_F \approx \frac{v^2}{\omega}\), where \(v_\omega \approx 300\) Gev is the vacuum expectation value of Higgs field. The weakness of the weak interaction comes from the largeness of the vacuum expectation value of Higgs field \([1]\).

In the light of the above remarks, it is considerable that gravity is also characterized by a dimensionless coupling constant and that the weakness of gravity is associated with symmetry breaking at the high energy scale. Similarly to \(G_F, G_N\) is given by the inverse square of the vacuum expectation value of a scalar field, dilaton. It was independently proposed by Zee \([2]\) and Smolin \([3]\) that the Einstein-Hilbert action,

\[
S = -\int d^4x \sqrt{g} \frac{1}{16\pi G_N} R, \tag{1}
\]

can be replaced by the modified action

\[
S = \int d^4\sqrt{g}(-\frac{1}{2}\xi\phi^2 R + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)), \tag{2}
\]

where the coupling constant \(\xi\) is dimensionless. The potential \(V(\phi)\) is assumed to attain its minimum value when \(\phi = \sigma\), then

\[
G_N = \frac{1}{8\pi\xi\sigma^2}. \tag{3}
\]

Through the spontaneous symmetry breaking, the symmetric phase of the scalar field transits to an asymmetric phase of the scalar field. On the analogy of the \(SU(2) \times U(1)\) symmetry of the weak interactions, we can consider a symmetry which is broken through spontaneous symmetry breaking in the gravitational interactions. The most attractive symmetry is the conformal symmetry which rejects the Einstein- Hilbert action Eq.(1), but admits the modified action Eq.(2) with the specific coupling \(\xi = -\frac{1}{6}\) and quartic potential. We can write down a conformally invariant induced gravity action without introducing the torsion field. However, the spontaneous symmetry breaking mechanism does not work for the scalar field theory with \(\xi = -\frac{1}{6}\) in De Sitter space.

In the minimal Riemann-Cartan space, the vector torsion behaves effectively like a conformal gauge field \([1]\). The introduction of this torsion field makes the dimensionless coupling constant in Eq.(2) free in the conformally invariant induced gravity action. Therefore, it is necessary to introduce the torsion field into the conformally induced gravity. Since we expect that the conformal symmetry is broken at very high energy scale, it is natural to consider the conformal symmetry with an inflation scenario \([4]\). We have investigated the quantum behavior of the dilaton and the vector torsion field in the conformally induced gravity. The one-loop effective potential of this action exhibits kind of a phase transition which may be responsible for an inflation scenario \([4, 5]\).
II. CONFORMALLY INDUCED GRAVITY IN MINIMAL RIEMANN-CARTAN SPACE

In this section we construct a conformally invariant induced gravity action with torsion field. Let us start from the condition of conformal invariance of the tetrad postulation

$$D_\alpha e^i_\beta \equiv \partial_\alpha e^i_\beta + \omega^i_{\beta \gamma} e^\gamma_\beta - \Gamma^{\gamma}_{\beta \alpha} e^i_\gamma = 0, \quad (4)$$

to find how the connection and the torsion behave under the conformal transformation,

$$e^i_\alpha' = \exp(\Lambda(x)) e^i_\alpha, \quad (\omega^i_{\beta \gamma})' = \omega^i_{\beta \gamma} \quad (5)$$

We have used Latin indices for the tangent space and Greek indices for the curved space. From the above requirement, the asymmetric affine connection and the torsion which is the antisymmetric part of the connection transform as follows:

$$\Gamma^{\gamma}_{\beta \alpha}' = \Gamma^{\gamma}_{\beta \alpha} + \delta^{\gamma}_{\beta} \partial_\alpha \Lambda, \quad (6)$$

$$T^{\gamma}_{\beta \alpha}' = T^{\gamma}_{\beta \alpha} + \delta^{\gamma}_{\beta} \partial_\alpha \Lambda - \delta^{\gamma}_{\alpha} \partial_\beta \Lambda, \quad (7)$$

$$T^{\gamma}_{\gamma \alpha}' = T^{\gamma}_{\gamma \alpha} + 3 \partial_\alpha \Lambda. \quad (8)$$

Therefore, the contracted torsion $T^{\gamma}_{\gamma \alpha}$ is effectively playing the role of a conformal gauge field. We can separate the torsion into two components;

$$T^{\alpha}_{\beta \gamma} = A^{\alpha}_{\beta \gamma} - \delta^{\alpha}_{\gamma} S_{\beta} + \delta^{\alpha}_{\beta} S_{\gamma}, \quad (9)$$

$$(S^{\alpha})' = S^{\alpha} + \partial_\alpha \Lambda, \quad (A^{\alpha}_{\beta \gamma})' = A^{\alpha}_{\beta \gamma}. \quad (10)$$

To avoid the unnecessary complexity, we adopt the conformally invariant torsionless condition

$$A^{\alpha}_{\beta \gamma} \equiv 0. \quad (11)$$

Because this condition is the conformally invariant extension of the torsionless condition in Riemann space $T^{\alpha}_{\beta \gamma} \equiv 0$, we call this space as the minimal Riemann-Cartan space. For this space, the affine connection is solved in terms of $g_{\mu \nu}$ and $S^{\alpha}$;

$$\Gamma^{\alpha}_{\beta \gamma} = \{\alpha_{\beta \gamma}\} + S^{\alpha} g_{\beta \gamma} - S_{\beta} \delta^{\alpha}_{\gamma}, \quad (12)$$

Let us define the conformally invariant connection, $\Omega^{\alpha}_{\beta \gamma}$;

$$\Gamma^{\alpha}_{\beta \gamma} = \Omega^{\alpha}_{\beta \gamma} + \delta^{\alpha}_{\beta} S_{\gamma}, \quad (13)$$

$$\Omega^{\alpha}_{\beta \gamma} = \{\alpha_{\beta \gamma}\} + S^{\alpha} g_{\beta \gamma} - S_{\beta} \delta^{\alpha}_{\gamma} - S_{\gamma} \delta^{\alpha}_{\beta}. \quad (14)$$

The curvature tensor of the affine connection $\Gamma^{\alpha}_{\mu \nu}$,

$$R^{\alpha}_{\beta \mu \nu}(\Gamma) = \partial_\mu \Gamma^{\alpha}_{\beta \nu} - \partial_\nu \Gamma^{\alpha}_{\beta \mu} + \Gamma^{\alpha}_{\sigma \nu} \Gamma^{\sigma}_{\beta \mu} - \Gamma^{\alpha}_{\sigma \mu} \Gamma^{\sigma}_{\beta \nu}, \quad (15)$$

can be expressed in terms of $\Omega^{\alpha}_{\beta \gamma}$ and $S^{\alpha}$ using Eq. (14);

$$R^{\alpha}_{\beta \mu \nu}(\Omega) = R^{\alpha}_{\beta \mu \nu}(\Omega) + \delta^{\alpha}_{\beta} H_{\mu \nu}, \quad (16)$$

$$R_{\alpha \nu}(\Gamma) = R_{\alpha \nu}(\Omega) + H_{\alpha \nu}, \quad (17)$$

where $H_{\mu \nu} = \partial_\mu S_{\nu} - \partial_\nu S_{\mu}$ is the conformal gauge field strength. With the help of Eqs.(12) and (14), we obtain

$$\sqrt{g} R(\Omega) = \sqrt{g} R(\{\}) + 6 \sqrt{g}(\nabla_{\alpha} S^{\alpha} - S_{\alpha} S^{\alpha}), \quad (18)$$
where $\nabla_\alpha$ is the ordinary covariant derivative in Riemann space.

Under the conformal transformations, the scalar field in 4-dimensions transforms as follow:

$$\phi'(x) = \exp(-\Lambda)\phi(x). \quad (19)$$

Finally, the conformally invariant Lagrangian function $c\phi^2R(\Omega)$ up to total derivatives can be expressed as follows:

$$\sqrt{g}\phi^2R(\Omega) = \sqrt{g}\phi^2 R(\{\}) - 6\sqrt{g}\phi^2 S_\alpha S^\alpha - 6\sqrt{g} S^\alpha \partial_\alpha \phi^2. \quad (20)$$

Defining the conformally covariant derivative $D_\alpha$,

$$D_\alpha \phi = \partial_\alpha \phi + S_\alpha \phi, \quad (21)$$

we have the following expression of the conformally invariant induced gravity action in terms of $g_{\alpha\beta}, S_\alpha$ and $\phi$:

$$S = \int d^4x \sqrt{|g|} \left[ \frac{-\xi}{2} R(\Omega) \phi^2 + \frac{1}{2} D_\alpha \phi D^\alpha \phi - \frac{1}{4} H_{\alpha\beta} H^{\alpha\beta} - \frac{\lambda}{4!} \phi^4 \right], \quad (22)$$

where we have excluded the curvature square terms. The parameter $\xi$ and $\lambda$ are dimensionless constants. Using Eq.(20) we can rewrite this action in terms of Riemann curvature scalar $R(\{\})$:

$$S = \int d^4x \sqrt{|g|} \left[ \frac{-\xi}{2} R(\{\}) \phi^2 + \frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi - \frac{1}{4} H_{\alpha\beta} H^{\alpha\beta} + (1 + 6\xi) S^\alpha (\partial_\alpha \phi) \phi + \frac{1}{2} (1 + 6\xi) S_\alpha S^\alpha \phi^2 - \frac{\lambda}{4!} \phi^4 \right]. \quad (23)$$

Here we are interested in the $\xi$ range, $-\frac{1}{6} < \xi < 0$.

### III. THE ONE-LOOP EFFECTIVE POTENTIAL IN DE SITTER SPACE

In this section, we have found the one-loop effective potential of the above action in De Sitter space using the background field method and zeta-function regularization in Ref. [10,11].

We consider the quantum fluctuations of the scalar field $\phi$ and the torsion field $S_\alpha$, and treat the metric $g_{\mu\nu}$ as a classical background field;

$$g_{\mu\nu} = g^b_{\mu\nu}, \quad S_\mu = \tilde{S}_\mu, \quad \phi = \phi_b + \phi_t. \quad (24)$$

Let us expand the action Eq.(23) around the background fields, then we have the quadratic action of the quantum fluctuations:

$$I_2 = \int d^4x \sqrt{|g|} \left[ \frac{-\xi}{2} R(\{\}) \tilde{\phi}^2 + \frac{1}{2} \partial_\alpha \phi_t \partial^\alpha \phi_t - \frac{\lambda}{4} \phi_t^2 \phi_t^2 - \frac{1}{4} H_{\mu\nu} H^{\mu\nu} - (1 + 6\xi) \phi_t \nabla_\alpha \tilde{S}^\alpha \phi_t + \frac{1}{2} (1 + 6\xi) \tilde{S}_\alpha \tilde{S}^\alpha \phi_t^2 \right]. \quad (25)$$

For the sake of convenience, we define a potential $V$ as follows;

$$V \equiv \frac{1}{2} \xi R(\{\}) \phi_t^2 + \frac{\lambda}{4} \phi_t^4 = \tilde{V}(\phi_b) + \frac{1}{2} \tilde{V}''(\phi_b) \tilde{\phi}^2. \quad (26)$$

By the Hodge decomposition theorem, one form $S^\mu$ in De Sitter space can be decomposed into two parts, a co-closed form $S^\mu_T$ and an exact form $S^\mu_L$ because there is no harmonic one-form;

$$S^\mu = S^\mu_T + S^\mu_L, \quad (27)$$

where the co-closed form $S^\mu_T$ satisfies $\nabla_\mu S^\mu_T = 0$, and the exact form $S^\mu_L$ can be written as $S^\mu_L = \partial^\mu \chi$ for a function $\chi$.

The above decomposition is orthogonal;

$$\int d^4x \sqrt{|g|} S^\mu_T S^\nu_L = 0. \quad (28)$$

Choosing the following gauge fixing term,

$$\triangle I_2 = \frac{1}{2} \alpha \int d^4x \sqrt{|g|} \left( \nabla_\mu \tilde{S}^\mu_L + \alpha^{-1} (1 + 6\xi) \phi_b \tilde{\phi}^2 \right)^2, \quad (29)$$

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and adding the gauge fixing term to the quadratic action Eq.\((34)\) whose independent quantum fields are \(\{\hat{S}_L^\mu, \tilde{S}_T^\mu, \tilde{\phi}\}\), we have the following gauge fixed quadratic action of the quantum fluctuations;

\[
I_2 + \triangle I_2 = \frac{1}{2} \int d^4x \sqrt{|g|} \hat{S}_L^\mu(-\Box_{\mu\nu} + R_{\mu\nu} + (1 + 6\xi)\phi_b g_{\mu\nu})\hat{S}_L^\nu + \alpha\hat{S}_L^\mu(\nabla_{\mu} \nabla_{\nu} + \alpha^{-1}(1 + 6\xi)\phi_b^2 g_{\mu\nu})\hat{S}_L^\nu + \tilde{\phi}(-\Box - \tilde{V}'(\phi_b) + \alpha^{-1}(1 + 6\xi)^2\phi_b^2)\tilde{\phi}.
\]

The one-loop generating functional in the landau gauge in which \(\alpha\) goes to the infinity \((\alpha \rightarrow \infty)\) is

\[
Z_1 = \frac{\text{det} \mu^{-2}Q}{\text{det} \mu^{-2}(W + (1 + 6\xi)\phi_b^2)} \text{det} \mu^{-2}(Q - \tilde{V}'(\phi_b))^{1/2},
\]

where \(Q = -\Box\) without zero mode, \(W = -\Box + \frac{\bar{R}}{2}\), and \(\bar{R}\) is the constant scalar curvature in De Sitter space. We have dropped the spurious zero mode integrations in the path integral because the zero mode of conformal factor can be absorbed into the fixed constant background of dilaton field.

One-loop effective potential for the quantum fluctuations of the torsion vector and the scalar field in Coleman-Weinberg sector \([12]\) (we assume that \(\lambda\) is order of \((1 + 6\xi)^2\)) can be obtained using the zeta-function regularization \([11,13]\);

\[
\tilde{V}_1(\phi) = \tilde{V}(\phi) + \frac{1}{2\Omega} \ln \text{det} \mu^{-2}(W + (1 + 6\xi)\phi_b^2),
\]

where \(\Omega = \frac{8\pi^2}{3}\) is the volume of De Sitter space, and \(\mu\) is a parameter with mass dimension.

In the large radius limit, \((1 + 6\xi)a^2\phi^2 >> 1\), the above effective potential becomes

\[
\tilde{V}_1(\phi) = \tilde{V}(\phi) + \frac{3(1 + 6\xi)^2}{64\pi^2} \phi^4(\ln \frac{(1 + 6\xi)\phi^2}{\mu^2} - \frac{3}{2}) + \frac{(1 + 6\xi)}{64\pi^2} \bar{R}\phi^2(\ln \frac{(1 + 6\xi)\phi^2}{\mu^2} - 1).
\]

**IV. SEMICLASSICAL EQUATION OF MOTION**

In this section we will analyze the semiclassical equation of motion for the scalar field and the metric considering the effective one-loop potential which has been obtained in the previous section. We have found the effective Lagrangian density as follows;

\[
\sqrt{g}L_{\text{eff}} = \sqrt{g}\left[ -\frac{1}{2\lambda}\phi^4 + \frac{1}{2}a^4 \phi^3(\ln \frac{(1 + 6\xi)\phi^2}{\mu^2} - \frac{3}{2}) + \frac{(1 + 6\xi)}{64\pi^2} \bar{R}\phi^2(\ln \frac{(1 + 6\xi)\phi^2}{\mu^2} - 1) + \rho_v \right],
\]

where

\[
V_{\text{eff}}(\phi, a) = \frac{\lambda}{4!}a^4 + \frac{3(1 + 6\xi)^2}{64\pi^2} \phi^4(\ln \frac{(1 + 6\xi)\phi^2}{\mu^2} - \frac{3}{2}) + \frac{(1 + 6\xi)}{64\pi^2} \bar{R}\phi^2(\ln \frac{(1 + 6\xi)\phi^2}{\mu^2} - 1) + \rho_v
\]

Here we have shifted the vacuum energy by \(\rho_v\) which might be attributed to quantum corrections of other fields we have not considered. By varying the action Eq.\((34)\), we get two equations of motion for the scalar field and the metric;

\[
\Box \phi + \xi R(\{\}) \phi = -\frac{\partial V_{\text{eff}}}{\partial \phi},
\]

\[
\xi\phi^2(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) = \partial_{\nu} \phi \partial_{\nu} \phi - \frac{1}{2}g_{\mu\nu} \partial_{\alpha} \phi \partial_{\alpha} \phi - \xi(g_{\mu\nu} \Box \phi^2 - \nabla_{\mu} \partial_{\nu} \phi^2) + g_{\mu\nu} V_{\text{eff}}(\phi),
\]

where we have not considered the backward contribution of the curvature dependence of the effective potential into the Einstein equation \((37)\).
To investigate the symmetry-breaking equation in this model, let us look for the solution of these equations with φ = σ = constant. The scalar equation of motion Eq.(36) is reduced to the following:

\[ \xi R(\{\}) = -\frac{1}{\sigma} \frac{\partial V_{\text{eff}}}{\partial \phi} \bigg|_{\phi=\sigma} . \]  

(38)

The trace of Einstein Eq.(37) is

\[ -\xi R(\{\}) \phi^2 + \partial_\alpha \phi \partial^\alpha \phi - 4V_{\text{eff}} + 3\xi \Box \phi^2 = 0 , \]  

(39)

which implies for the constant φ = σ

\[ \xi R(\{\}) = -\frac{4}{\sigma^2} V_{\text{eff}}(\sigma) . \]  

(40)

With the help of Eqs.(38) and (40), we have the symmetry-breaking equation;

\[ \left( \frac{\partial V_{\text{eff}}}{\partial \phi} - \frac{4V_{\text{eff}}(\phi)}{\phi} \right) |_{\phi=\sigma} = 0 . \]  

(41)

Therefore, the symmetry breaking equation for the induced gravity is different from the usual \( \frac{\partial V_{\text{eff}}}{\partial \phi} |_{\phi=\sigma} = 0 \) in the scalar theory with Einstein-Hilbert action.

Presently, it is assumed that we are in the broken symmetry phase with φ = σ. If \( V_{\text{eff}}(\sigma) \neq 0 \), this uniform background energy density acts like a cosmological constant in Einstein equation. By the requirement of the vanishing of the cosmological constant in the true vacuum of flat space, the constant part of \( V_{\text{eff}}(\sigma, a) \) can be determined

\[ \rho_v = \frac{3(1 + 6\xi)^2 \sigma^4}{128\pi^2} . \]  

(42)

From the Eq.(41), we can express the parameter \( \mu \) in terms of \( \sigma \) as follows;

\[ \ln \frac{(1 + 6\xi)\sigma^2}{\mu^2} = 1 - \frac{8\pi^2\lambda}{9(1 + 6\xi)^2} . \]  

(43)

In De Sitter space, the metric can be written as

\[ ds^2 = dt^2 - e^{2H_0 t} d\bar{x}^2 , \]  

(44)

and the scalar curvature is \( \tilde{R} = -12H^2 \). Using Eqs.(42) and (43), the effective potential Eq.(33) for the dilaton field in De Sitter becomes

\[ \tilde{V}_{\text{eff}}(\phi_s) = \frac{3}{64\pi^2} \phi_s^4 \left( \ln \phi_s^2 - \frac{1}{2} \right) + \frac{3}{128\pi^2} - \frac{3}{2} H^2 \phi_s^2 \left( \frac{1}{8\pi^2} \ln \phi_s^2 - \frac{\lambda_s}{9} \right) - \frac{6\xi}{(1 + 6\xi)^2} H^2 \phi_s^2 , \]  

(45)

where, for the sake of convenience, we have defined

\[ \phi_s \equiv \sqrt{(1 + 6\xi)} \phi, \quad \sigma_s \equiv \sqrt{(1 + 6\xi)} \sigma, \quad \lambda_s \equiv \frac{\lambda}{(1 + 6\xi)^2} . \]  

(46)

and chosen the unit \( \sigma_s = 1 \). This effective potential governs the evolution of the dilaton field in De Sitter space through the equation of motion

\[ \Box \phi = -\frac{\partial \tilde{V}_{\text{eff}}(\phi)}{\partial \phi} . \]  

(47)

It is found that the effective potential (45) shows a phase transition which is sensible for an inflationary scenario. The critical radius \( 1/H \) of the phase transition has been obtained at \( \frac{1}{H} \approx 19 \) in the plotting of the one-loop effective potential (45) varying the radius \( \frac{1}{H} \) with the fixed \( \lambda_s = 1.0 \) and \( (1 + 6\xi) = 0.1 \).

The combination of Eqs.(36) and (39) gives

\[ \phi \Box \phi + \partial_\alpha \phi \partial^\alpha \phi + \phi \frac{\partial V_{\text{eff}}}{\partial \phi} - 4V_{\text{eff}} + 3\xi \Box \phi^2 = 0 . \]  

(48)
From the assumption that $\phi$ is spatially homogeneous, the above Eq.(48) is reduced to
\[
(1 + 6\xi)(\ddot{\phi} + 3H \dot{\phi} + \frac{\dot{\phi}^2}{\phi}) + (V_{\text{eff}}' - \frac{4}{\phi} V_{\text{eff}} f) = 0. \tag{49}
\]
When $\xi = -\frac{1}{6}$, the induced gravity Lagrangian is consistent only if the form of the effective potential $V_{\text{eff}}(\phi)$ is quartic. Therefore, the spontaneous symmetry breaking is impossible for $\xi = -\frac{1}{6}$ in De Sitter space. The trace of Einstein Eq.(39) becomes
\[
12H^2\phi^2 + \dot{\phi}^2 + 6\xi(\ddot{\phi} + \dot{\phi}^2 + 3H \dot{\phi}) - 4V_{\text{eff}}(\phi) = 0. \tag{50}
\]
We are interested in the inflationary solutions of Eqs.(49) and (50), where the expansion rate $H$ is very large in comparison with other quantities, and scalar field changes slowly (slow rollover) \cite{14,15}:
\[
\left| \frac{\dot{\phi}}{\phi} \right| << H, \quad |\ddot{\phi}| << 3H|\dot{\phi}|, \quad |\ddot{\phi}^2| << V_{\text{eff}}(\phi). \tag{51}
\]
In the slow-rolling inflationary regime, Eqs.(49) and (50) are reduced to the followings;
\[
3H \dot{\phi}_s = \frac{4}{\phi_s} V_{\text{eff}}'(\phi_s) - V_{\text{eff}}'(\phi_s), \tag{52}
\]
\[
H^2 = \frac{(1 + 6\xi)}{3\xi \phi_s^2} 3 V_{\text{eff}}(\phi_s), \tag{53}
\]
where
\[
V_{\text{eff}}(\phi_s) = \frac{3}{64\pi^2} \phi_s^4 (\ln \phi_s^2 - \frac{1}{2}) + \frac{3}{128\pi^2} \frac{3}{2} H^2 \phi_s^2 (\frac{1}{8\pi^2} \ln \phi_s^2 - \frac{\lambda_s}{9}). \tag{54}
\]
In the slow-rolling phase, the contribution from the $\frac{4V_{\text{eff}}'(\phi_s)}{\phi_s}$ part of the driving term in the right-hand side of Eq.(52) should be nearly equal to the contribution of the $V_{\text{eff}}'(\phi_s)$ term so that the dilaton field could roll down slowly compared with the expansion rate $H$. This slow rolling inflationary phase surely can not happen at the very center of the potential, but near the origin such that
\[
\ln \phi_s^2 \approx 8\pi^2 \left( \frac{\lambda_s}{9} - \frac{2\xi}{(1 + 6\xi)} \right). \tag{55}
\]
When the scalar filed $\phi_s$ reaches $\phi_s \approx 1$, it is expected that the dilaton field oscillates about the true vacuum with damping because the dilaton field can be coupled to other matter fields through Yukawa couplings $Tr\bar{\psi}\Gamma(\phi \psi)$. Through this dissipation process, the vacuum energy density of the symmetric phase, $\frac{3\sigma_s^2}{128\pi^2}$, is eventually converted into radiation and matters.

V. CONCLUSION

We have considered that the Newton’s gravitational constant $G_N$ is generated through the spontaneous symmetry breaking of a conformal symmetry. It is possible to formulate the conformally induced gravity in Riemann space. However, the spontaneous symmetry breaking via radiative correction does not work for a scalar field with $\xi = -\frac{1}{6}$. We have extended minimally the Riemann space to Riemann-Cartan space to incorporate the torsion vector which is effectively playing the role of a conformal gauge field, then the dimensionless coupling constant $\xi$ is arbitrary. With the introduction of the conformal gauge field, the mechanism of spontaneous symmetry breaking via radiative correction does work as in the case of the massless scalar electrodynamics. The computation of the one-loop effective potential is performed by zeta-function regularization in De Sitter space. Considering this effective potential, we have analyzed the semi-classical equation of motion of the dilaton field. We will consider the case of non-vanishing torsion background and the detail analysis of the effective potential within the context of inflation scenario later.

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