Evolutionary and Hydrodynamic Models of Short-Period Cepheids

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Abstract—The evolutionary calculations for population I stars with masses on the main sequence $5 \, M_\odot \leq M_0 \leq 6.1 \, M_\odot$ and initial fractional abundances of helium $Y_0 = 0.28$ and heavier elements $Z_0 = 0.02$ were carried out to the stage of central helium exhaustion. Selected core helium-burning models were used as initial conditions for solution of the equations of hydrodynamics and time-dependent convection describing radial pulsations of Cepheids. In the Hertzsprung–Russel diagram the evolutionary tracks are shown to cross the red edge of the instability strip for $M_0 > 5.1 \, M_\odot$. The grid of hydrodynamic models of core helium-burning Cepheids on the stages of the second and the third crossings of the instability strip was computed. The pulsation period $\Pi$ and the rate of period change $\dot{\Pi}$ were determined as a continuous function of star age for each evolutionary sequence of first-overtone pulsators. Results of calculations agree with observational estimates of $\dot{\Pi}$ recently obtained for the short-period Cepheids V532 Cyg, BG Cru and RT Aur.

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INTRODUCTION

Pulsating variable stars of Population I with nearly symmetrical light curves of the small amplitude ($\Delta V \leq 0.5$ mag) and with periods shorter than 7 day are classified as short-period Cepheids. The General Catalogue of Variable Stars (Samus et al. 2017) lists as many as 50 stars satisfying this criterion where they are designated as DCEPS type variables. The short-period Cepheids belong to the group of least luminous and least massive Population I Cepheids. Most of the DCEPS variables are assumed to be the first overtone pulsators.

The intermediate-mass stars become the Cepheid variables during the core helium-burning stage when the star leaves the red giant domain and evolves in the Hertzsprung–Russel (HR) diagram along the loop crossing the pulsation instability strip (Hofmeister et al. 1964; Iben 1966). Contraction and subsequent expansion of the star during this stage of evolution is due to changes of the mean molecular weight and opacity of the stellar matter in the envelope surrounding the convective core (Walmswell et al. 2015). Excretion of the loop in the HR diagram decreases with decreasing stellar mass and is less sensitive to variations of the Cepheid chemical composition (Pietrinferni et al. 2006; Bertelli et al. 2009). Ambiguity in estimates of the Cepheid lower mass limit is due to uncertainties arising from comparison of the theoretically computed bolometric luminosity $L$ and effective temperature $T_{\text{eff}}$ with empirical instability strip edges expressed in terms of the absolute magnitude $M_V$ and the intrinsic color $(B-V)_0$ (Tammann et al. 2003).

The history of systematic light measurements of many Cepheids is as long as 120 yr. For stars with periods shorter than 7 day this interval encompasses more than $6 \times 10^3$ pulsation cycles, so that the rates of secular period change $\dot{\Pi}$ can be reliably determined from analysis of $O - C$ diagrams (Turner et al. 2006). The goal of the present study is to determine the fundamental parameters of short-period Cepheids with recent observational estimates of the period change rate $\dot{\Pi}$. To this end we employ the method based on consistent calculations of stellar evolution and non-linear stellar pulsations. Earlier this method was employed for determination of the fundamental parameters of long-period Cepheids (Fadeyev 2018b). In this study of evolutionary and hydrodynamic models of short-period Cepheids we also try to evaluate the lower mass limit of Cepheids.

METHODS OF COMPUTATION

Evolutionary sequences of the Cepheid models were computed using the program MESA version 12115 (Paxton et al. 2018). The nucleosynthesis kinetic equations were solved for the reaction...
network ‘pp_cno_extras_o18_ne22.net’ consisting of 26 isotopes from hydrogen \( ^1\text{H} \) to magnesium \( ^{24}\text{Mg} \) coupled by 81 reactions. The rates of nuclear reactions were computed using the database JINA Reactlib (Czyburt et al. 2010). The mass loss rate \( \dot{M} \) was calculated according to Reimers (1975) with parameter \( r_R = 0.3 \). Convective mixing of the stellar matter was treated in the framework of the standard theory by Böhm–Vitense (1958) with mixing length to pressure scale height ratio \( \alpha_{\text{MLT}} = \lambda/H_P = 1.6 \). Extra mixing due to overshooting beyond the boundaries of convective zones was computed following the approach developed by Herwig (2000):

\[
D_{\text{ov}}(z) = D_0 \exp\left(-\frac{2z}{fH_P}\right),
\]

where \( D_0 \) is the diffusion coefficient (Langer et al. 1985) inside the convection zone on the distance \( 0.004H_P \) from the convection zone boundary, \( z \) is the spatial coordinate measured from the boundary of convective instability, \( f = 0.016 \) is the overshooting parameter.

Thermonuclear helium burning in the intermediate–mass stars occurs in the convective core so that the helium abundance jump appears on the upper boundary of the convection zone. During stellar evolution the amplitude of this jump increases due to gradual decrease of the central helium abundance The core growth and displacement of the upper boundary of the convection zone along the Lagrangian mass zones is accompanied by irregular ingestion of helium–rich material which is responsible for sharp increases of nuclear energy production and appearance of spurious loops on the evolutionary tracks. In general the effect of core breathing pulses substantially prolongates the core helium–burning evolutionary stage (Constantino et al. 2016) and leads to significant mistakes in theoretical estimates of the Cepheid period change rates. In the present study to avoid the abrupt changes of central helium abundance during the core helium–burning stage we employed the method which restricts the mass flux on the outer boundary of the convective core (Spruit 2015; Constantino et al. 2017). Evolutionary calculations of the core helium–burning stage were carried out with the time step limit \( \Delta t \leq 10^2 \text{yr} \) and the number of Lagrangian mass zones of the stellar model was \( \approx 1.5 \times 10^4 \).

Selected models of evolutionary sequences corresponding to the core helium–burning Cepheid stage were used as initial conditions in solution of the equations of radiation hydrodynamics and time–dependent convection describing radial stellar oscillations. The inner boundary of hydrodynamic models was considered as the rigid permanently emitting sphere of radius \( r_0 = 0.1R \), where \( R \) is the stellar radius. Initial values of mesh variables required for the solution of the Cauchy problem were determined by nonlinear interpolation of the evolutionary model mesh variables. The method of the solution of the Cauchy problem is described in our earlier paper (Fadeyev 2015). Hydrodynamic models were computed on the nonuniform Lagrangian mesh consisting of \( N = 500 \) mass zones. The Lagrangian mass intervals increase inward as the geometrical progression with the constant factor \( q \approx 1.03 \). The oscillation period \( \Pi \) of each hydrodynamic model was determined using the discrete Fourier transform of the kinetic energy of pulsation motions \( E_K \) (Fadeyev 2018a).

**RESULTS OF COMPUTATION**

In the present study the initial conditions were presented by 12 evolutionary sequences of stars with initial masses \( 5 \, M_\odot \leq M_0 \leq 6.1 \, M_\odot \) that were computed with the initial mass step \( \Delta M_0 = 0.1 \, M_\odot \). The initial helium and metal fractional abundances on the main sequence \( (t_{\text{ev}} = 0) \) were assumed to be \( Y_0 = 0.28 \) and \( Z_0 = 0.02 \), respectively. Each evolutionary sequence was computed to helium exhaustion in the stellar core \( (Y_c \leq 10^{-4}) \).

Results of consistent stellar evolution and nonlinear stellar pulsation calculations are illustrated in Fig. 1, where evolutionary tracks of stars with initial masses \( M_0 = 5.3 \, M_\odot, 5.5 \, M_\odot, 5.7 \, M_\odot, \) and \( 5.9 \, M_\odot \) are shown in vicinity of the Cepheid instability strip of the HR diagram. Dotted lines indicate the stage of evolution corresponding to positive growth rate of the pulsation kinetic energy \( (\eta = \Pi^{-1}d\ln E_K/\Pi dt > 0) \). Here \( t \) is time connected with stellar pulsations, \( E_K \) is the maximum of the kinetic energy of pulsation motions. Radial oscillations of Cepheids are described with good accuracy in terms of standing waves, so that the kinetic energy \( E_K \) reaches its maximum \( E_K \) twice per period \( \Pi \).

The age of the star \( t_{\text{ev},0} \) on the instability strip edge \( (\eta = 0) \) was determined by linear interpolation between two adjacent models with opposite signs of the growth rate \( \eta \) (Fadeyev 2013). For considered evolutionary sequences the lower mass limit of Cepheids is in the range \( 5.1 \, M_\odot < M_0 < 5.2 \, M_\odot \). In particular, the turning point of the evolutionary track with \( M_0 = 5.2 \, M_\odot \) corresponds to the effective temperature \( \log T_{\text{eff}} = 3.742 \) which exceeds the effective temperature of the red edge by \( \Delta \log T_{\text{eff}} = 3.4 \times 10^{-3} \). At the turning point the star is the fundamental mode pulsator with period \( \Pi \approx 5.3 \) day.

Evolution of the core helium–burning stars shown in Fig. 1 proceeds clockwise along the loop where the
vertical dashes indicate the oscillation mode switch. As in our earlier paper (Fadeyev 2019) in this study we assume that the time of mode switch is much less in comparison with nuclear evolution time of the Cepheid. The star age $t_{\text{ev,sw}}$ at the mode switch was evaluated as a mean age of two adjacent models pulsating in different modes. Oscillations in the fundamental mode take place in stars with lower effective temperatures near the red edge of the instability strip whereas oscillations in the first overtone occur in stars with higher effective temperatures located in the HR diagram near the blue edge of the instability strip.

The pulsation period as a continuous function of evolutionary time was determined using the cubic interpolating splines within the whole interval of $t_{\text{ev}}$ where the star is either the fundamental mode or the first overtone pulsator. The plots of evolutionary changes of the pulsation period in Cepheid models with initial masses $M_0 = 5.3$ and $5.9 \, M_\odot$ are shown in Fig. 2, where for the sake of graphical representation the horizontal coordinate is set to zero at the turning point of the evolutionary track where the stellar radius reaches its minimum.

The initial and the final points of each plot in Fig. 2 represent the cross of the red edge of the instability strip by the evolutionary track, whereas the abrupt change of the period by a factor of $\approx 1.5$ is due to pulsation mode switch. The gap in the dependence $\Pi(t_{\text{ev}})$ for the first overtone oscillations of the evolutionary sequence $M_0 = 5.9 \, M_\odot$ is due to the fact that the turning point of the evolutionary track is beyond the blue edge of the instability strip. As is seen, increase of the Cepheid mass is accompanied by increasing ratio $\Delta t_{\text{ev,3}}/\Delta t_{\text{ev,2}}$, where $\Delta t_{\text{ev,2}}$ and $\Delta t_{\text{ev,3}}$ are Cepheid lifetimes during the second and the third crossings of the instability strip.

For evolutionary sequences considered in the present study the phenomenon of mode switch between the fundamental mode and the first overtone occurs in models with radius $R \leq 52 \, R_\odot$. In particular, in evolutionary sequences with initial masses $5.3 \, M_\odot \leq M_0 \leq 5.9 \, M_\odot$ mode switching occurs during both the second and the third crossings of the instability strip, whereas for $6.0 \, M_\odot \leq M_0 \leq 6.1 \, M_\odot$ mode switching occurs only during the second crossing since on the stage of the third crossing the stellar radius is greater than the threshold radius: $R > 52 \, R_\odot$.

The region of excitation of stellar pulsations in Cepheids locates in the layers of partial helium ionization so that the necessary condition for the first
overtones pulsation is that the radius of the helium ionizing zone remains larger than the radius of the node of the corresponding eigenfunction. Unfortunately, the general condition for mode switching cannot be defined by simple relationships. Below we will use the mean density of stellar matter $\langle \rho \rangle = M/(\frac{4}{3} \pi R^3)$ as a parameter connected with the threshold period values of the fundamental mode $\Pi_0$ and the first overtone $\Pi_1$ corresponding to mode switching.

The threshold values $\Pi_0$ and $\Pi_1$ were determined for all evolutionary sequences and are described with good accuracy by relations

$$\Pi_0 = -23.486 - 7.065 \log\langle \rho \rangle,$$

$$\Pi_1 = -14.773 - 4.504 \log\langle \rho \rangle,$$

(2)

(3)

where the pulsation period is expressed in days. Plots of $\Pi_0$ and $\Pi_1$ versus mean density $\langle \rho \rangle$ as well as relations (2) and (3) are shown in Fig. 3.

**COMPARISON WITH OBSERVATIONS**

The diagram period–period change rate provides the best way to compare the results of theoretical calculations with observational estimates of $\Pi$ and $\dot{\Pi}$ (Turner et al. 2006; Fadeyev 2014). Approximation of $\Pi(t_{ev})$ by cubic interpolation splines allows us to determine the period change rate as a continuous function of evolutionary time and finally to express $\dot{\Pi}$ as a function of $\Pi$. Plots of $\dot{\Pi}$ versus $\Pi$ for the Cepheid models pulsating in the first overtone during the second crossing of the instability strip ($\dot{\Pi} < 0$) are shown in Fig. 4. Stellar evolution proceeds from left to right with decreasing period. Each plot represents variation of $\dot{\Pi}$ from the point of mode switching to the blue edge of the instability strip ($M_0 \geq 5.8 \, M_\odot$) or to the turning point of the evolutionary track ($M_0 \leq 5.6 \, M_\odot$) where $\dot{\Pi} = 0$.

Also shown in Fig. 4 are the recent observational estimates of $\Pi$ and $\dot{\Pi}$ for short-period Cepheids DX Gem (Berdnikov 2019a), V532 Cyg (Berdnikov 2019b) and BG Cru (Berdnikov et al. 2019). As is seen, the results of computations agree with observations of V532 Cyg and BG Cru, whereas the observational estimate of $\dot{\Pi}$ in DX Gem is several times higher than it can be predicted from our evolutionary sequences. In order to avoid the discrepancy one should assume that the blue edge of the instability strip of the star with mass $M \approx 6 \, M_\odot$ is displaced to higher effective temperatures and therefore to shorter pulsation periods.

*Fig. 2.* Evolutionary variations of the period of radial pulsations of Cepheids with initial masses $M_0 = 5.3 \, M_\odot$ (solid lines) and $M_0 = 5.9 \, M_\odot$ (dashed lines). Here $t_{ev}(R_{min})$ is the star age at the turning point of the evolutionary track where the stellar radius reaches its minimum.
Fig. 3. Threshold periods of the fundamental mode (filled circles) and the first overtone (filled triangles) versus mean density \langle \rho \rangle. Relations (2) and (3) are shown by dashed lines.

Fig. 4. Period change rate \dot{\Pi} versus period \Pi for Cepheid models pulsating in the first overtone during the second crossing of the instability strip (\dot{\Pi} < 0).
Plots of $\dot{\Pi}$ versus $\Pi$ for the third crossing of the instability strip ($\dot{\Pi} > 0$) are shown in Fig. 5 for evolutionary sequences $M_0 = 5.6, 5.7,$ and $5.8 \, M_\odot$. As is seen, all the plots are in satisfactory agreement with RT Aur (Turner et al. 2007) since its observational estimate of $\dot{\Pi}$ is only one and a half times as high as results of our calculations. Figure 5 shows also the observational estimates of $\Pi$ and $\dot{\Pi}$ for Cepheids CG Cas (Turner et al. 2008) and FF Aql (Berdnikov et al. 2014). Unfortunately, periods of these Cepheids significantly exceed the threshold values $\Pi_1$ determined in the present study.

In Table 1 we list the fundamental parameters of the short-period Cepheids V532 Cyg, BG Cru and RT Aur for which the theoretical estimates of $\dot{\Pi}$ are in agreement with observations.

CONCLUSIONS

In this paper we extended the results of our earlier study (Fadeyev 2014) devoted to the theoretical explanation of the observed secular period changes in Cepheids and where the short-period Cepheids were not considered in detail. In particular, results of the present work demonstrate that significant scatter of the observational estimates of $\dot{\Pi}$ in short-period Cepheids is due to the fact that at the turning point of the evolutionary track the period change rate reduces to zero. Therefore, the near-zero value of $\dot{\Pi}$ allows us to roughly evaluate the Cepheid mass. As is seen in Fig. 1, the turn of the evolutionary track inside the instability strip occurs in stars with masses $M < 5.7 \, M_\odot$.

Results of consistent stellar evolution and nonlinear stellar pulsation calculations allow us to conclude that the lower mass limit of Cepheids on the stage of core helium-burning is $M \approx 5.1 \, M_\odot$. Evolutionary tracks of less massive stars do not cross the red edge of the instability strip and the star remains stable against radial oscillations. One should however to note that this conclusion is based on the results obtained for initial fractional abundances of helium and heavier elements $Y_0 = 0.28$ and $Z_0 = 0.02$. Sensitivity of evolutionary changes of the stellar radius and of the extension of the loop of the evolutionary track to the mean molecular weight (Walmswell et al. 2015) allows us to suppose that the Cepheid lower mass limit depends on the chemical composition of stellar matter.

Among six Cepheids with observational estimates of $\dot{\Pi}$ considered in the present study we succeeded to obtain a good agreement with observations only for three of them. The periods of CG Cas ($\Pi = 4.3656$ day) and FF Aql ($\Pi = 4.4709$ day) are notice-
Table 1. Fundamental parameters of short-period Cepheids

| Cepheids   | \( \Pi \), day | \( t_{\text{ev}}, 10^6 \) yr | \( M, M_\odot \) | \( L, L_\odot \) | \( R, R_\odot \) | \( T_{\text{eff}}, \text{K} \) |
|------------|----------------|----------------------------|----------------|----------------|----------------|----------------|
| V532 Cyg   | 3.2836         | 82.3                      | 5.53           | 2130           | 42.8           | 6000           |
| BG Cru     | 3.3426         | 90.0                      | 5.33           | 1810           | 42.8           | 5760           |
| RT Aur     | 3.7182         | 76.1                      | 5.77           | 2720           | 47.9           | 6030           |

ably greater than the threshold period of the first overtone \( \Pi_1 \) corresponding to switch to the fundamental mode (see Fig. 5). Dependence of the mode switching threshold periods on the mean density (see Fig. 3) implies that the upper first overtone period limit might noticeably depend on the chemical composition of stellar matter. Therefore, to reconcile the theory with observations of short-period Cepheids CG Cas and FF Aql we have to consider more extended grids of evolutionary and hydrodynamic models computed for various initial compositions.

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