Hierarchical structures for a robustness-oriented capacity design

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SUMMARY

In this paper, we study the response of 2D framed structures made of rectangular cells, to the sudden removal of columns. We employ a simulation algorithm based on the Discrete Element Method, where the structural elements are represented by elasto-plastic Euler Bernoulli beams with elongation-rotation failure threshold. The effect of structural cell slenderness and of topological hierarchy on the dynamic residual strength after damage $R_1$ is investigated. Topologically hierarchical frames have a primary structure made of few massive elements, while homogeneous frames are made of many thin elements. We also show how $R_1$ depends on the activated collapse mechanisms, which are determined by the mechanical hierarchy between beams and columns, i.e. by their relative strength and stiffness. Finally, principles of robustness-oriented capacity design which seem to be in contrast to the conventional anti-seismic capacity design are addressed. Copyright © 2010 John Wiley & Sons, Ltd.

KEY WORDS: frames; progressive collapse; robustness; hierarchy

1. INTRODUCTION

Since many decades, design codes ensure a very low probability that a building collapses under ordinary loads, like self weight, dead and live service load, or snow. Nevertheless buildings still do collapse, from time to time. An extremely small fraction of collapses originates from unlikely combinations of intense ordinary load with very poor strength of the building. The majority of structural collapses are due to accidental events that are not considered in standard design. Examples of such extraordinary events are: gross design or construction errors, irresponsible disregard of rules or design prescriptions, and several rare load scenarios like e.g. earthquakes, fire, floods, settlements, impacts, or explosions [1]. Accidental events are characterized by low probability of occurrence, and high potential negative consequences. Since risk is a combination of probability and consequences, the risk related to accidental events is generally not negligible [2].

In 1968 a gas explosion provoked the partial collapse of the Ronan Point building in London. This event highlighted for the first time the necessity of designing robust structures to ensure safety in extraordinary scenarios [3]. Since then, the interest in structural robustness was driven by striking catastrophic collapses [4], until in 2001 the tragic collapse of the World Trade Center renewed the attention in the topic (see e.g. [5] and [6]). The last decades, several design rules aimed at improving structural robustness have been developed (see e.g. [7]).
Accidental events can be classified into identified and unidentified [2, 8]. Identified events are statistically characterizable in terms of intensity and frequency of occurrence. Examples are earthquakes, fire not fueled by external sources, gas explosions, and unintentional impacts by ordinary vehicles, airplanes, trains, or boats. Specific design rules and even entire codes are devoted to specific identified accidental events. Unidentified events comprise a wide variety of incidents whose intensity and frequency of occurrence can not be described statistically, e.g. terrorist bomb attacks, intentional impacts, or gross errors.

The risk related to unidentified accidental events can be mitigated both by structural and nonstructural measures [9]. Nonstructural measures such as barriers and monitoring can reduce the probability that an accidental event affects the structural integrity, others like a wise distribution of plants and facilities can minimize the negative consequences of eventual collapses. Otherwise, structural measures can improve local resistance of structural elements to direct damage, e.g. the design of key elements for intense local load [2], or the application of the Enhanced Local Resistance method [8]. Structural measures can also provide progressive collapse resistance, i.e. prevent spreading of local direct damage inside the structure to an extent that is disproportioned with respect to the initial event. Usual strategies to improve progressive collapse resistance are compartmentalization of structures [10] and delocalization of stress after local damage. Stress delocalization can be obtained exploiting redundancy, plastic stress redistributions (Masoero, Wittel et al., 2010), ties [1], and moment resisting connections [11, 12].

Nowadays several design codes employ the Alternate Load Path Method (ALPM) to evaluate progressive collapse resistance in a conventional way, e.g. [13] and [8]. The method consists in removing one key element, generally a column or a wall, and measuring the extent of subsequent collapse. If the final collapse is unacceptably wide, some of the previously listed measures have to be employed. Hence structures are first designed and subsequently tested to be robust - they are not conceived a priori. This course of action excludes optimizations of the basic structural topology and geometry, that actually play a key role in the response to local damage, considering as an example the very different behavior of redundant and statically determined structures. Anti-seismic design [14] already contains some prescriptions that should be considered before starting a new design, e.g. geometric regularity on the horizontal and on the vertical planes. Furthermore, anti-seismic capacity design requires a hierarchy of the structural elements ensuring that earthquakes can only provoke ductile collapse of the horizontal beams, while failure of columns and brittle ruptures due to shear are inhibited. Differently, for what concerns progressive collapse resistance, optimal overall geometric features are not known, except for the concepts of redundancy and compartmentalization. Furthermore, the idea of hierarchically maximizing progressive collapse resistance is completely absent.

In this paper, we make a first step to cover this deficiency, showing that progressive collapse resistance can be improved by a hierarchy either at the level of the overall geometry (topological hierarchy), and at the strength and stiffness of horizontal and vertical structural elements (mechanical hierarchy). Our approach incorporates the simulation of progressive collapse of regular 2D frames made of reinforced concrete (RC) subjected to the sudden removal of structural elements, following the ALPM framework. We first describe the analyzed frame structures and briefly sketch the approach that is based on the Discrete Element Method (DEM). After the model description, we present the results of the simulations, with focus on the effect of geometry and hierarchy on the activated collapse mechanisms and, consequently, on progressive collapse resistance.

2. HIERARCHICAL STRUCTURES AND DAMAGE

We consider two representative sets of regular 2D framed structures in Figure 1(a). Each set consists of three frames with identical total width \( L_{tot} \) and different topological hierarchical level \( 1/n \), where \( n^2 \) is the number of structural cells in a frame. The horizontal beams, excluded those of the secondary structure, carry a uniform load per unit length \( q_{ext} \). The frames are made of RC with typical mechanical parameters of concrete and steel, as shown in Table I. The total height \( H_{tot} \) of
the structure is kept constant, and two different height-bay aspect ratios \( \lambda = H/L \) of the structural cells are considered (\( \lambda = 0.75 \) and \( \lambda = 1.33 \)).

![Diagram of hierarchical structures](image)

Figure 1. (a) Map of the studied frames. The cohesion of the elements inside the dotted damage area is suddenly removed to represent the initial accidental damage. (b) Cross sections and rebars of beams (left) and columns (right).

### Table I. Mechanical properties of reinforced concrete and steel.

| Parameter                          | Symbol | Units | Value  |
|-----------------------------------|--------|-------|--------|
| Reinforced concrete               |        |       |        |
| Specific weight                   | \( \gamma_{RC} \) | kg/m\(^3\) | 2500   |
| Young modulus                     | \( E_c \) | N/m\(^2\) | 30·10^9 |
| Compressive strength (high)       | \( f_c \) | N/m\(^2\) | 35·10^6 |
| Compressive low (low)             | \( f_c \) | N/m\(^2\) | 0.35·10^6 |
| Ultimate shortening               | \( \varepsilon_{u,c} \) | - | 0.0035  |
| Steel                             |        |       |        |
| Young’s modulus                   | \( E_s \) | N/m\(^2\) | 200·10^9 |
| Yield stress                      | \( f_y \) | N/m\(^2\) | 440·10^6 |
| Ultimate strain                   | \( \varepsilon_{u,s} \) | - | 0.05    |

There exist several ways of introducing hierarchy into the topology of framed structures; here we call a structure “hierarchical” if it is composed of few massive structural elements, that support a secondary structure whose stiffness and strength is neglected. The frames with \( n = 2 \) and \( n = 5 \)
can be seen as reorganizations of those with \( n = 11 \). In detail, each column of the frames with \( n = 5 \) corresponds to two columns of the frames with \( n = 11 \), and the same is valid for the beams, disregarding the first floor beam of the frames with \( n = 11 \), which is simply deleted (see Figure 1(a)). Analogously, the geometry of the frames with \( n = 2 \) can be obtained starting from the frames with \( n = 5 \).

The cross sections of columns are square (see Figure 1(b)), with edges \( h_c = b_c \) proportional to \( H \) with factor \( \lambda_c = 1/10 \). The beams have rectangular cross section whose height \( h_b \) is proportional to \( L \) with factor \( \lambda_b = 1/10 \), and whose base \( b_b \) is proportional to \( h_b \) with aspect ratio \( \delta_b = 2/3 \). The reinforcement is arranged as shown in Figure 1(b), with area \( A_s \) proportional to the area of the cross section by factor \( \rho_{s,c} = 0.0226 \) for the columns (i.e. \( 8\phi 18 \) when \( n=11 \)), and \( \rho_{s,b} = 0.0029 \) for the beams (i.e. \( 4\phi 14 \) when \( n=11 \)).

The damage areas, (dotted in Figure 1(a)) contain the structural elements that are suddenly removed to represent an accidental damage event, as suggested by the ALPM framework. The damage is identical for frames with the same \( \lambda \), and is defined by the breakdown of one third of the columns on a horizontal line. The columns and beams removed from frames with \( n = 11 \) correspond to the structural elements removed from frames with \( n = 5 \) and \( n = 2 \). This kind of damage is employed to represent accidental events with a given amount of destructive energy or spatial extent, like explosions or impacts. In this work we do not explicitly simulate very local damage events like gross errors, which would be better represented by the removal of single elements. Nevertheless, we will generalize our results to consider also localized damage events.

3. DEM MODEL

We employ the Discrete Element Method (DEM) to simulate the dynamic response of the frames to the sudden damage [15, 16]. The system dynamics in terms of equations of motion is integrated by means of a 5th order Gear predictor-corrector scheme, with time increments ranging between \( 10^{-6} \)s and \( 10^{-5} \)s, as described for the simulation of collapse of 3D structures [17]. To reduce the 3D code to two dimensions, only the two displacements and one rotation in the vertical plane are activated for the 2D frames. A detailed description of the algorithm can be found in [17, 7], together with a discussion on the applicability. In [18], the DEM model is employed to study the response of a continuous beam to the sudden removal of a support, showing excellent agreement with results based on an alternative approach via energy conservation. Therefore we review only the essentials, and focus on details specific to the application on the analyzed 2D frames.

3.1. Structural representation

In a first step, the structure needs to be assembled by discrete elements and beams. More precisely, Figure 2(a) shows that the frames are represented by four types of Spherical Discrete Elements (SDE). Columns and beams are made of 9 SDEs, respectively with diameter \( D_c = 0.8h_c \) and \( D_b = 0.8h_b \). Constrained SDEs and connection SDEs have same diameter \( D_c \) as column SDEs. The constrained SDEs are clamped to a plane that represents the ground by means of the Hertzian contact model, discussed further in this section.

3.2. Euler-Bernoulli beam elements

Pairs of SDEs are connected by Euler-Bernoulli beam elements - (EBE) that, when deformed, transmit forces and moments to their edge nodes, locally labeled 0 and 1 (see Figure 2(b)). The mass of an EBE is \( M^e = \gamma_{RC} A^e L^e \), where \( A^e \) is the cross sectional area of the structural element corresponding to the EBE. \( M^e \) is concentrated in equal portions at the edge nodes of the EBE, and consequently at the corresponding SDEs. The external load \( q_{ext} \) is introduced adding a mass \( q_{ext} L^e / g \) to the beam SDEs. \( q_{ext} \) is not treated directly as a force to avoid downward accelerations of the SDEs greater than gravity \( g \) during free fall.

For sufficiently small deformations, the EBEs are linear elastic and exert a force \( N \) proportional to the elongation \( \varepsilon \) and directed along the \( \overline{01} \) segment, a shear force \( T \) proportional to the sum of the
In this way, eventual tension \( N > B \) forces and moments directed opposite to \( N \) are both \(|N| \geq |B|\). When yielding in bending occurs, plastic rotations are added at the edge nodes of the EBE. If only tension, i.e. \( N < |B| \), with \( i = 0 \), is greater than \( B \), then only \( \varphi^p_0 \) is applied to restore \( |B_i| = B \). Differently, if both \( |B_0| \) and \( |B_1| \) are greater than \( B \), both \( \varphi^p_0 \) and \( \varphi^p_1 \) are applied to restore \( |B_0| = |B_1| = B \).

We also consider that the ideally plastic regime in bending is entered when \(|B| \geq B_y\). We obtain the bending yield threshold \( B_y \) and the corresponding yielding effective rotation \( \varphi^{eff}_y \), neglecting the contribution of concrete and assuming a lever arm between upper and lower reinforcement equal to the height \( h \) of the cross section:

\[
B_y = t_s \rho_s A^c f_y h + \Delta B_y \quad \text{and} \quad \varphi^{eff}_y = \frac{B_y}{E_c T_e}.
\]

We obtain \( B_y \) from the cross sectional moment of inertia of the EBE, and \( t_s \) is the fraction of reinforcement in tension, i.e. \( 3/8 \) for columns and \( 1/2 \) for beams (see Figure 1(b)). \( \Delta B_y \) considers the beneficial effect of compression inside the EBE. We set \( \Delta B_y \), assuming that bending is entirely carried by the reinforcement, and that the strain \( \varepsilon_s(\Delta B_y) \) in the reinforcement put under tension by \( \Delta B_y \) equals the compressive strain \( \varepsilon_s(N) \) due to \( N < 0 \), namely:

\[
\varepsilon_s(N) = \varepsilon_s(\Delta B_y) \rightarrow -\frac{N}{A^c E_c} = \frac{\Delta B_y}{t_s \rho_s A^c h E_s}.
\]
the sake of simplicity, we assume yielding in bending uncoupled from yielding in axial direction. Furthermore, we neglect yielding due to shear because small plastic deformations are generally associated with shear.

We consider an EBE failed when excessive $\varepsilon^{pl}$ and $\varphi^{pl}$ are cumulated. For this purpose, the coupled breaking criterion:

$$\frac{\varepsilon^{pl}}{\varepsilon_{th} - \varepsilon_y} + \max_i \left( \frac{\varepsilon^{pl}}{\varphi_{th}^{pl_i}} \right) \geq 1 \quad \text{if} \quad \varepsilon^{pl} > 0 \quad \text{and}$$  

$$\frac{-\varepsilon^{pl}}{\varepsilon_{cy} - \varepsilon_{cth}} + \max_i \left( \frac{\varphi^{pl}}{\varphi_{th}^{pl_i}} \right) \geq 1 \quad \text{if} \quad \varepsilon^{pl} < 0 ,$$  

is employed. $\varepsilon_{th}$, $\varepsilon_{cth}$, and $\varphi_{th}$ are the maximum allowed plastic elongation, shortening, and rotation in uncoupled conditions. We consider high plastic capacity of the structural elements setting $\varepsilon_{th} = 2\varepsilon_{u,s}$, $\varepsilon_{cth} = 2\varepsilon_{u,c}$, and $\varphi_{th} = 0.2\text{rad}$ (see Table I). Failed EBEs are instantly removed from the system. We neglect ruptures due to shear assuming that, in agreement with a basic principle of capacity design, a sufficient amount of bracings ensures the necessary shear strength.

3.3. Inter-sphere contact

The Hertzian contact model is employed for the SDEs to consider collisions between structural elements. A repulsive force arises when two SDEs partially overlap. This force is proportional to the overlapping volume with stiffness coefficient $Y$. Overlapping is damped by an additional force proportional and opposite to the overlapping velocity with coefficient $\gamma_n$. We also set forces that are tangent to the surface of colliding SDEs. The tangential forces are proportional to the relative tangential velocity of the overlapping SDEs with coefficient $\gamma_t$, with an upper bound given by a Coulomb friction force proportional to the repulsive force with factor $\mu$. Tangential forces induce moments to the SDEs, and rotations which are damped as well by moments proportional to the relative rolling velocity with coefficient $\gamma_w$. We also employ Hertzian contacts for SDEs colliding with the ground plane.

Table II. Hertzian contact coefficients employed for the 2D simulation.

| Parameter                  | Symbol | Units     | Value |
|----------------------------|--------|-----------|-------|
| Sphere – sphere            |        |           |       |
| Contact stiffness          | $Y$    | $10^7\text{N/m}^3$ | 10    |
| Normal damping             | $\gamma_n$ | $10^6\text{Ns/m}$ | 10    |
| Coulomb tangent damping    | $\mu$  | $10^4\text{Ns/m}$ | 1     |
| Dynamic tangent damping    | $\gamma_t$ | $10^4\text{Ns/m}$ | 1     |
| Rolling damping            | $\gamma_w$ | $\text{Nms}$     | 50    |
| Sphere – ground            |        |           |       |
| Contact stiffness          | $Y^g$  | $10^7\text{N/m}^3$ | 5     |
| Normal damping             | $\gamma_n^g$ | $10^6\text{Ns/m}$ | 5     |
| Coulomb tangent damping    | $\mu^g$ | $10^4\text{Ns/m}$ | 5     |
| Dynamic tangent damping    | $\gamma_t^g$ | $10^4\text{Ns/m}$ | 5     |

In granular dynamics, the contact parameters are generally set referring directly to the material of the grains [15]. In our model, the SDEs represent large portions of structural elements including concrete, steel bars and bracings, with imperfections and cracks. Therefore, instead of setting the contact parameters a priori, we search for values providing a qualitatively realistic dynamics of rubble (see Table II). In detail, we set the damping parameters to prevent rebounding of colliding SDEs and rolling of rubble stacking on the ground. Furthermore, we consider $E_c$ as the upper bound of $Y$, while the lower bound is the maximum value that would allow SDEs to pass through each other. In between these bounds, we choose the maximum $Y$ that does not allow rubble to cut
columns at the base, because superficial fragmentation processes inhibit this phenomenon in real structural collapses.

4. SIMULATING PROGRESSIVE COLLAPSE

The simulations are organized into two steps: first the structure is equilibrated under the effect of $q_{ext}$ and gravity, then the EBEs inside the damage area are suddenly removed, and the subsequent dynamic response is simulated. Our aim is to quantify three collapse loads:

(i) $q_u^I$: maximum static load that the intact structure can carry;
(ii) $q_c$: critical load, i.e. the minimum load that causes dynamic collapse after damage. $q_c$ is applied statically to the intact structure first, and then is kept constant during the dynamic response triggered by the initial damage;
(iii) $q_{p,t}$: minimum load corresponding to total collapse after damage. By definition, $q_c \leq q_{p,t} \leq q_u^I$.

In our DEM model we do not have a straightforward unique measure of load, because the mass of the SDEs depends on the external load $q_{ext}$ and on the self weight of the structural elements. Therefore we introduce an equivalent load $q_{eq}$ that, applied to the massless structure, produces the same static effect at critical points where collapse is triggered. More details about the equivalent loads are summarized in the Appendix.

For each analyzed structure, we first apply the entire structural mass. In a subsequent step, the external load $q_{ext}$ is increased until the intact structure collapses in static conditions, and we employ the most adequate equivalent load to evaluate $q_u^I$. Then we slightly decrease $q_{ext}$, equilibrate, introduce the damage, and calculate whether progressive collapse is triggered and to what an extent. Performing several simulations with progressively smaller $q_{ext}$, the final extent of collapse changes from a total to a partial one, and we employ again an adequate equivalent load to compute $q_{p,t}$. It may happen that the structure collapses even when $q_{ext}$ is reduced to zero. In this case, we start reducing the specific weight of the structural elements, and thus the structural mass. When the structure does not collapse during the dynamic response to the initial damage, an adequate equivalent load provides $q_c$. Once we obtain the collapse loads, we estimate the progressive collapse resistance referring to the residual strength fraction $R_1 = q_c/q_u^I$. Actually, progressive collapse resistance is more directly related to $q_c$, but the advantage of $R_1$ is that it can not be improved by simply strengthening the structural elements, which would increase both $q_c$ and $q_u^I$. Robustness-oriented structural optimization is required to increase $R_1$, which therefore is a good indicator to compare different structural solutions.

4.1. Bending collapse

In our model, the bending yield threshold $B_y$ does not depend on the strength of concrete $f_c$. Therefore, setting the high value $f_c = 35\text{N/mm}^2$, the mainly compressed columns get much stronger than the horizontal beams, that fail in bending (see Figures 3 and 4). The resulting collapse mechanisms resemble triple-hinge and four hinges mechanisms, reflecting the large plastic capacity of the structural elements.

Frames with $n=2$ can only undergo total collapse if the initial damage triggers a bending mechanism. On the other hand, the bending mechanism initiates local collapse in frames with lower hierarchical level $1/n$, that can still lead to total collapse (see Figure 5). This can happen if the applied load and the plastic capacity of the structural elements are large enough to let the falling central part of the structure dynamically drag down the lateral portions [17]

The collapse loads, expressed in terms of equivalent loads $q_{eq}$, are summarized in Figure 6 as a function of the hierarchical level $1/n$, for different slenderness of the structural cells $\lambda$. In Figure 6, superscript $B$ indicates bending collapse mechanism. We employ equivalent loads referring to perfectly brittle or perfectly plastic bending failure (see the Appendix). The collapse loads decrease with $\lambda$, i.e. a slender structure seems weaker, and increase with $1/n$, i.e. hierarchical frames are
Figure 3. Static bending collapse mechanism before damage. Time $t = 0$ s corresponds to the first breaking of an EBE.

Figure 4. Dynamic bending collapse mechanism after damage. Time $t = 0$ s corresponds to the application of the initial damage.

stronger. The residual strength fraction $R_1$ does not depend on $\lambda$, while hierarchical structures with low $n$ are more robust than homogeneous ones (see Figure 6). In fact, the concentration of bending moment at the connection between a beam hanging above the damage area and the first intact column depends on the number of removed columns. In the simulations, we remove a constant fraction of one third of the columns on a horizontal line (see Figure 1). Therefore homogeneous structures lose more columns and are less robust toward the bending collapse mechanisms. On the other hand, since the number of removed columns is decisive, we expect that the hierarchical level does not influence the $R_1$ toward bending collapse in case of single column removal. Finally we consider the generic 2D frame as part of a regular 3D structure and divide the collapse loads in Figure 6 by $L$, i.e., by the tributary length of the beams in the direction perpendicular to the frame. In this way, collapse loads per unit area are obtained in Figure 7 and show that: $\lambda$ does not influence $q_c/L$ and $q^u/L$, structures with slender cells are less likely to undergo total collapse after damage, $q^l/L$ is independent from the hierarchical level, and $q_c/L$ is proportional to $1/n$. 

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Figure 5. Snapshots of partial ($q_{eq}=15\text{kN/m}$) and total ($q_{eq}=30\text{kN/m}$) bending collapse after damage of a frame with very strong columns, $n=11$, and $\lambda = 1.33$. Time $t = 0\text{s}$ corresponds to the application of the initial damage.

Figure 6. Equivalent collapse loads and residual strength fraction $R_1$ for frames that undergo bending progressive collapse.

4.2. Pancake collapse

Progressive compressive failure of the columns, also called pancake collapse, occurs when we set the compressive strength of concrete to a small value $f_c = 0.35\text{N/mm}^2$ (see Figure 8). The same could be obtained by reducing the cross section of the columns or by increasing the reinforcement.
inside the beams. Progressive compressive failure of the columns initiates from the columns that are closer to the damage area and then spreads to the outside. We employ equivalent loads $q_{eq}$ referring to the two limit cases of local and of global pancake collapse. Local pancake collapse occurs when the bending stiffness of the beams is very low and when the compressive failure of the columns is very brittle. In this case, the overload after damage is entirely directed to the intact columns that are closer to the damage area, and collapse propagates by nearest neighbor interactions. On the other hand, high stiffness of the beams and large plastic capacity of the columns induce democratic redistribution of overload between the columns. Consequently, the columns crush simultaneously triggering global pancake collapse. The collapse dynamics recorded in our simulations resembles that of global pancake. Note that in the studied framed structures, all the columns have identical compressive strength without disorder. Therefore, once the first two columns crush, pancake collapse can not be arrested. Nevertheless, if $f_c > 0.35 \text{N/mm}^2$, our frames can experience partial collapse because the progressive failure of the columns can be arrested by the initiation of bending collapse.

Figure 7. Collapse loads in Figure 6 divided by $L$, considering the 2D frames as part of regular 3D structures.

Figure 8. Snapshots of pancake collapse after damage for frames with $n=5$, $\lambda = 0.75$, and $n=11$, $\lambda = 1.33$.

Time $t=0$ s corresponds to the application of the initial damage.

Figure 9, where superscript $C$ indicates pancake collapse, shows that the collapse loads increase with the slenderness of the structure $\lambda = H/L$, because the columns have tributary area related to $L^2$ and compressive strength proportional to $H^2$. Furthermore, hierarchical structures with small $n$
Figure 9. Equivalent loads and residual strength fraction for the studied structures undergoing pancake collapse.

appear to be stronger than homogeneous ones both in terms of $q_u$ and of $q_c$. Finally, the residual strength fraction $R_1$ toward pancake collapse is remarkably higher than that toward bending collapse (cf. Figure 6), and is neither influenced by the hierarchical level $1/n$, nor by $\lambda$. In fact, the $R_1$ toward global pancake mode is related to the fraction of columns that are initially removed at one story. In our simulations, we always remove one third of the columns at one story, and obtain the constant value $R_1 \approx 0.6$, slightly smaller than a theoretical 2/3 because of dynamics.

5. CONCLUSIONS

We showed how the dynamic strength after damage of 2D frames, i.e. the critical load $q_c$, depends on the activated collapse mechanism. Bending collapse provokes a local intensification of bending moments at the connections between the transfer beams above the damage area and the first intact column. Consequently, $q_c$ and the residual strength fraction $R_1$ decrease with the number of removed columns. In analogy with fracture mechanics, structures that are prone to bending collapse correspond to notch sensitive materials, while the number of removed columns corresponds to the crack width [19]. If global pancake collapse is triggered, $q_c$ and $R_1$ decrease with the fraction of removed columns, which is analogous to plastic failure of materials that are not notch sensitive. Consistently, after initial damage, $R_1$ corresponding to global pancake collapse is remarkably larger than that corresponding to bending collapse.

Considering structural topology, in case of bending collapse hierarchical structures are more robust toward initial damage with fixed spatial extent (e.g. explosion, impact), and as robust as homogeneous structures toward single column removal (e.g. design error). This is a consequence of the fact that $R_1$ referring to bending collapse decreases with the number of removed columns in one story. Furthermore, the higher damage tolerance of hierarchical structures confirms the analogy with
fracture mechanics, where notch sensitive hierarchical materials are tougher than homogeneous ones [20]. On the other hand, considering global pancake collapse, structural hierarchy does not influence $R_1$ toward initial damage with fixed spatial extent, while hierarchical structures are more sensitive than homogeneous structures to single column removals. This is due to the fact that $R_1$ referring to global pancake collapse decreases with the fraction of removed columns. Figure 9 shows that frames undergoing global pancake collapse after damage can carry the $R_1 \approx 60\%$ of the static ultimate load of the intact structure $q_{uc}$. Since well designed structures can carry a $q_{uc}$ remarkably greater than the environmental load expected when an accidental event occurs, the $R_1$ related to global pancake collapse is large enough to ensure structural robustness for most of the practical cases [7]. On the other hand, $R_1$ related to bending collapse can be small enough to make a structure vulnerable to accidental damage.

In conclusion, hierarchical structures made of few massive elements and with strong beams and weak columns should be preferred. Hierarchy is needed to maximize $R_1$ toward bending collapse, while employing strong beams and weak columns leads to pancake rather than bending collapse. Furthermore, strong beams also improve resistance toward falling rubble, representing a possible vertical compartmentalization strategy for tall buildings. Structural solutions where columns fail before beams are a novel feature of robustness-oriented capacity design, but are also in striking disagreement with one of the principles of anti-seismic capacity design, that requires plastic failure of the beams to occur before rupture of the columns (see e.g. [21]). Overcoming this contradiction is a challenge toward optimizing structural behavior in exceptional situations. Hence the necessity to couple seismic analyses, where horizontal loads can be preeminent, with progressive collapse assessment in presence of damage and vertical loads. The simultaneous presence of horizontal and vertical actions can result into further and more complex collapse mechanisms, involving concurrent bending rupture of beams and crushing of columns.

Despite the simplifications about regular geometry, employed design rules, and model assumptions, our conclusions regarding structural hierarchy have general validity. Nevertheless, one factor that can be significant and that should be considered in future works is shear ruptures of the beams. Neglecting them is reasonable for slender structural elements at small scales, but at large scales, e.g. when $n = 2$, shear ruptures can significantly reduce $q_{uc}$. Therefore, while this paper suggests that structural robustness indefinitely grows with topological hierarchy, considering shear ruptures could indicate an intermediate optimal hierarchical level.

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APPENDIX: EQUIVALENT LOADS

In our model there are two load contributions: the self weight of beams and columns, and the external load \( q_{\text{ext}} \) applied to the beams only. Since the aim of this paper is to study the transitions from collapse to resistance as the load decreases, it is convenient to express the load by a unique parameter, the equivalent load \( q_{\text{eq}} \), incorporating both \( q_{\text{ext}} \) and the self weight. \( q_{\text{ext}} \) is uniformly distributed along each horizontal beam, while the self weight gives an additional load to the beams \( q_{g,b} = gh_b \gamma_{\text{RC}} \) analogous to \( q_{\text{ext}} \). Furthermore, vertical forces \( F = gh_c \gamma_{\text{RC}} \) represent the self weight of a column between two adjacent stories. \( q_{\text{eq}} \) is defined as the load that produces the same static effect as \( q_{\text{ext}}, q_{g,b}, \) and \( F \) on the massless structure. In the following, we summarize analytical expressions of \( q_{\text{eq}} \) that are relevant for our analyses, pointing out what are the static effects that we refer to.

**Intact frame, bending equivalence**  Consider bending collapse \( B \) of the intact structure \( I \). If the beams are brittle we refer to the maximum elastic bending moment in a beam as static effect to define the \( q_{\text{eq}} \). Otherwise, if the beams are ideally plastic, we refer to the work done by a triple-hinge collapse mechanism as static effect to reproduce with \( q_{\text{eq}} \). Nevertheless, in the intact frames, the self weight of the columns \( F \) is directly and entirely carried by the columns themselves. Therefore, independent from the plastic capacity of the beams, the load that is responsible for the possible bending failure is simply:

\[
q_{\text{eq}}^{I,B} = q_{\text{ext}} + q_{g,b} .
\]  

**Intact frame, compression equivalence**  With respect to pancake collapse of the intact structure, we employ the compression \( C \) at the base of the generic column as static effect for the definition of \( q_{\text{eq}} \). Considering the intact massless structure subjected to \( q_{\text{eq}} \) and assuming democratic load sharing between the \( n_c \) columns, leads to:

\[
C_{\text{eq}} = q_{\text{eq}} L n_s \frac{n_c - 1}{n_c} .
\]  

Additional to the distributed \( q_{\text{ext}} \) and \( q_{g,b} \), the self weight of the columns for all the \( n_s \) stories adds up to:

\[
C_{\text{DEM}} = (q_{\text{ext}} + q_{g,b}) L n_s \frac{n_c - 1}{n_c} + F n_s .
\]  

Finally, the equivalent load of the intact structure toward compression at the base of the columns is:

\[
C_{\text{eq}} = C_{\text{DEM}} \rightarrow q_{\text{eq}}^{I,C} = q_{\text{ext}} + q_{g,b} + \frac{F}{L \frac{n_c}{n_c} - 1} .
\]  

Plasticity does not affect \( C_{\text{eq}} \) because dynamics is not considered and because static plastic redistributions of loads between the columns are possible only if the columns have different compressive strength, which is not the case in our simulations.
Damaged frame, bending equivalence  We consider that the initial damage can induce perfectly brittle or perfectly plastic bending collapse. Under the hypothesis of perfectly brittle failure, we assume that the columns fully constrain the rotation at the edge of the beams. This is a reasonable assumption, due to the high stiffness of the structural connections, and is also confirmed qualitatively by the simulations. The corresponding bending moment distribution due to $q_{eq}$ and to the DEM loads

![Diagram](attachment:image.png)

Figure 10. (a) Generic beam hanging above the damage area and corresponding static deformed state (dashed line). (b) Distribution of the equivalent load (left hand side) and of the DEM load (right hand side). (c) Simplified scheme and qualitative bending moment diagram for the beam connected to the first intact column. Plastic collapse mechanism of the generic hanging beam under (d) the equivalent load and (e) the load of the DEM simulations.

are plotted in Figure 10(c), where $B_{eq}^{\text{max}}$ and $B_{DEM}^{\text{max}}$ indicate the maximum values. $F_i$ is the self weight of a column hanging above the damage area, equally shared by all the beams hanging above the damage area (refer to Figure 1(a) for $n_s$ and $n_{r,s}$), namely:

$$F_i = \frac{n_s - n_{r,s} - 1}{n_s - n_{r,s}} F .$$  \hspace{1cm} (11)

We compute the equivalent load of the damaged structure $D$ toward brittle bending rupture in the elastic range $el$ by setting $B_{eq}^{\text{max}} = B_{DEM}^{\text{max}}$:

$$q_{eq}^{D,B,el} = q_{ext} + q_{g,b} + \frac{3n_{r,c}}{3n_{r,c} + 1} F_i .$$  \hspace{1cm} (12)
A second definition of $q_{eq}$ is obtained for plastic bending collapse $pl$, by equating the work done by $q_{eq}$ to the work done by the DEM loads during the rigid motion in Figure 10(d) and (e):

$$q_{eq}^{D,B,pl} = q_{ext} + q_{g,b} + \frac{2n_{r,c} - 1}{2n_{r,c}} F_L .$$

(13)

**Damaged frame, compression equivalence** We consider two possible mechanisms of local and global load redistribution between the columns that survive the initial damage. Local redistribution implies that the load above the damage area is supported by the two undamaged columns that are closer to the damage, one on the left and the other on the right. Therefore, after damage, the compression at the base of one of these columns is equal to the compression before damage plus one half of the load above the damage area. Equating the compression due to $q_{eq}$ on the massless structure to the compression due to the DEM loads we obtain $q_{eq}$ for the damaged frame $D$ in terms of compression $C$ after local redistribution $loc$:

$$q_{eq}^{D,C,loc} = q_{ext} + q_{g,b} + \frac{2n_{s} + (n_{s} - n_{r,s} - 1) n_{r,c}}{2n_{s} + (n_{s} - n_{r,s}) n_{r,c}} F_L .$$

(14)

Global redistribution of compressions occurs when all the columns surviving the initial damage share in equal parts the load above the damage area. Equating again the compression due to $q_{eq}$ and to the DEM loads we obtain $q_{eq}$ for the damaged frame $D$ in terms of compression $C$ after global redistribution $gl$:

$$q_{eq}^{D,C,gl} = q_{ext} + q_{g,b} + \frac{n_{s}n_{c} - n_{r,c}(n_{r,s} + 1)}{n_{s}(n_{c} - 1) - n_{r,c}n_{r,s}} F_L .$$

(15)

Equation 14 implicitly assumes that the intact columns closer to the damage area are not external edge columns (see e.g. frames with $n = 2$ in Figure 1). Nevertheless, local and global redistribution coincide and Equation 15 can be employed in that eventuality.