Parameter estimation and adaptive control of Euler–Lagrange systems using the power balance equation parameterisation

Jose Guadalupe Romera, Romeo Ortegaa and Alexey Bobtsov b

aDepartamento Académico de Sistemas Digitales, ITAM, Ciudad de México, México; bFaculty of Control Systems and Robotics, ITMO University, St. Petersburg, Russia

ABSTRACT
It is widely recognised that the existing parameter estimators and adaptive controllers for robot manipulators are extremely complicated, stymieing their practical use – in particular, for robots with many degrees of freedom. This is mainly due to the fact that the existing parameterisation includes the complicated signal and parameter relations introduced by the Coriolis and centrifugal forces matrix. In an insightful remark of their seminal paper, Slotine and Li suggested to use the parameterisation of the power balance equation, which avoids these terms – yielding significantly simpler designs. To the best of our knowledge, such an approach was never actually pursued in on-line implementations, because the excitation requirements for the consistent estimation of the parameters are ‘very high’. In this paper, we use a recent technique of generation of ‘exciting’ regressors developed by the authors to overcome this fundamental problem. The result is applied to general Euler–Lagrange systems and the fundamental advantages of the new parameterisation are illustrated with comprehensive simulations of a 2 degrees-of-freedom robot manipulator.

1. Introduction
It is well known that the implementation of on-line parameter identifiers and adaptive controllers for Euler–Lagrange (EL) systems is complex and very computationally demanding (Gautier & Khalil, 1992; Khalil & Dombre, 2002; Niemeyer & Slotine, 1991; Ortega et al., 1998; Spong et al., 2020). This is mainly due to the fact that the parameterisation that is used to obtain the linear regression equation (LRE) needed for their implementation – introduced in Khosla and Kanade (1985) – is based on the full model of the system dynamics, which involves complicated signal and parameter relations introduced in the Coriolis and centrifugal forces matrix. An additional difficulty for the application of these adaptive techniques is that, in order to obtain a linear relation in the LRE, it is necessary to over-parametrise the vector of unknown parameters. This approach has very serious shortcomings, in particular the need of more stringent excitation conditions stemming from the fact that the parameter search takes place in a bigger dimensional space with nonunique minimising solutions – see Ljung (1987), Sastry and Bodson (1989) and the detailed discussion in Ortega, Gromov, et al. (2021, Section 1). This situation has severely stymied the practical implementation of these advanced identification and control techniques in many critical applications, for instance, for robot manipulators (Huang & Chien, 2010; Niemeyer & Slotine, 1991; Zhang & We, 2021).

A computational complexity reduction is achieved restricting ourselves to the estimation of the so-called base inertial parameters introduced in Gautier and Khalil (1998), which exploits the fact that the matrix defining the LRE of Khosla and Kanade (1985) is not full rank, see Sousa and Cortesao (2014) for some recent developments of this approach. Another route, pursued by practitioners, to overcome this difficulty is to replace the complicated expression of the regressor using function approximation, leading to the so-called regressor-free adaptive controllers. See Huang and Chien (2010) for a detailed description of this procedure applied to robot manipulators. Unfortunately, as always with function approximation-based techniques (Ortega, 1996), although they might lead to successful designs, there is no solid theoretical guarantee that the procedure will work – see Huang and Chien (2010, Section 4.5).

In an insightful remark of their seminal paper (Slotine & Li, 1998, Section 2.2), the authors suggested to use the parameterisation of the power balance equation. The main advantage of this approach is that, as mentioned above, the resulting LRE avoids the cumbersome terms related to the Coriolis and centrifugal forces matrix. This is a significant simplification that drastically reduces the complexity and computational demands. To the best of our knowledge, such an approach was never actually pursued, because the excitation requirements for the consistent estimation of the parameters is ‘very high’ – see Ortega, Gromov, et al. (2021, Remark 16). One notable exception where this parameterisation was used is in the work of Niemeyer and Slotine, where it was combined with the classical parameterisation, in a composite adaptive controller that, as is well-known combines the terms of two estimators. In an independent line
of research, the use of the power balance equation for parameter estimation was also suggested in Gautier and Khalil (1998), see Khalil and Dombre (2002, Section 12.6.2) for a detailed description of the model. In contrast with the proposal of Slotine and Li (1998), where in a standard way an LRE is used for on-line estimation, in the aforementioned papers the power balance equation is integrated in a series of intervals to generate an overdetermined set of linear equations, from which they identify the parameters via least-squares minimisation procedures. This approach was also used in Block (1991) to identify the parameters of the pendubot. Interestingly, the author observed that the identification of the friction terms was very problematic with this method—an issue also discussed in Prüfer et al. (1994), where the lack of excitation is identified as the culprit of this problem.

In this paper, we propose a procedure to overcome, for the first time, this fundamental problem. Towards this end, we use a recent technique of generation of new LRE with ‘exciting’ regressors developed by the authors in Bobtsov et al. (2021). The result is applied to general EL systems and the significant advantages of the new parameterisation of the power balance equation are illustrated with comprehensive simulations of a 2-degrees-of-freedom (DOF) robot manipulator.

The development of the new LRE of Bobtsov et al. (2021) relies on the use of the following components: (i) the dynamic regressor extension and mixing (DREM) estimator (Aranovskiy et al., 2017), which is a procedure that generates, from a q-dimensional LRE, q scalar LREs, one for each of the unknown parameters; (ii) the parameter estimation based observer (PEBO) proposed in Ortega et al. (2015), later generalised in Ortega, Bobtsov, et al. (2021), that translates the problem of state estimation into a parameter estimation one; (iii) the energy pumping-and-damping injection principle of Yi et al. (2020) to inject excitation to the new regressor. To make the paper self-contained all these derivations are briefly summarised in the Appendix. For the sake of clarity, we restrict ourselves to the case of LRE. However, as indicated in the concluding remarks, the regressor generator proposed in the paper can be applied to the nonlinearly parameterised case.

**Notation.** $I_n$ is the $n \times n$ identity matrix. For $x \in \mathbb{R}^n$, we denote the Euclidean norm as $|x| := \sqrt{x^\top x}$. $\mathcal{L}_1$ and $\mathcal{L}_2$ denote the absolute integrable and square integrable function spaces, respectively.

2. A power balance equation-based parameterisation of EL systems

Following the insightful remark of Slotine and Li (1998, Section 2.2), in this section we derive an LRE for general EL systems using the power balance equation and compare its complexity with the ‘classical’ parameterisation using the EL equations of motion, see also Khalil and Dombre (2002, Section 12.6.2).

2.1 System dynamics

In this paper, we consider $n_q$-DOF, underactuated, EL systems with generalised coordinates $q \in \mathbb{R}^{n_q}$ and control vector $\tau \in \mathbb{R}^m$, with $m \leq n_q$, whose dynamics is described by the EL equations of motion

$$\frac{d}{dt} [\nabla_q \mathcal{L}(q, \dot{q})] - \nabla_q \mathcal{L}(q, \dot{q}) = G(q, \dot{q}) \tau, \quad (1)$$

where $\mathcal{L} : \mathbb{R}^{n_q} \times \mathbb{R}^{n_q} \to \mathbb{R}$ is the Lagrangian function

$$\mathcal{L}(q, \dot{q}) := T(q, \dot{q}) - U(q),$$

with $T : \mathbb{R}^{n_q} \times \mathbb{R}^{n_q} \to \mathbb{R}$ the kinetic co-energy function, $U : \mathbb{R}^{n_q} \to \mathbb{R}$ the potential energy function, $G : \mathbb{R}^{n_q} \times \mathbb{R}^{n_q} \to \mathbb{R}^{n_q \times m}$ is the input matrix, which is assumed known. For $\mathcal{L}$, the transposed gradient, with respect to $q$ and $\dot{q}$ are denoted by $\nabla_q \mathcal{L}$ and $\nabla_{\dot{q}} \mathcal{L}$, respectively. We restrict our attention to simple EL systems, whose kinetic energy is of the form

$$T(q, \dot{q}) = \frac{1}{2} \dot{q}^\top M(q) \dot{q},$$

where $M : \mathbb{R}^{n_q} \to \mathbb{R}^{n_q \times n_q}$, $M(q) > 0$, is the generalised inertia matrix. See Ortega et al. (1998) for additional details on this model and many practical examples and Spong et al. (2020) for a detailed description of robot manipulators.

**Remark 2.1:** It is possible to include in the dynamics (1) the effect of viscous friction terms of the form

$$\frac{d}{dt} [\nabla_q \mathcal{L}(q, \dot{q})] - \nabla_q \mathcal{L}(q, \dot{q}) = G(q, \dot{q}) \tau + R \dot{q},$$

where $R \in \mathbb{R}^{n_q \times n_q}$ a diagonal, positive semidefinite matrix with known coefficients. As shown in Remark 2.2, this effect can also be included in our analysis. However, for the sake of brevity, this additional term is omitted in the sequel.

2.2 Derivation of the new regression equation

A first step in the design of parameter estimators is the derivation of an LRE for the unknown parameters of the EL system. Towards this end, we introduce the following parameterisation of the inertia matrix $M(q)$ and the potential energy $U(q)$

$$M(q) = \sum_{i=1}^\ell m_i(q) \theta_i^M, \quad U(q) = \sum_{j=1}^r U_j(q) \theta_j^U, \quad (2)$$

with known matrices $m_i : \mathbb{R}^{n_q} \to \mathbb{R}^{n_q \times n_q}$ and functions $U_j : \mathbb{R}^{n_q} \to \mathbb{R}$ and $w := \ell + r$ unknown parameters $\theta_i^M, \theta_j^U$, that we group together in a single vector as

$$\theta := \text{col}(\theta_1^M, \ldots, \theta_\ell^M, \theta_1^U, \ldots, \theta_r^U) \in \mathbb{R}^w. \quad (3)$$

We are in position to present the following key derivation motivated by the insightful remark of Slotine and Li (1998, Section 2.2).

**Proposition 2.1:** Define the vector $\Omega \in \mathbb{R}^w$ via

$$\dot{z} = -\lambda(z + \omega)$$

$$\Omega = z + \omega$$

(4)
More precisely, it satisfies the power balance equation

$$
\dot{\omega} := \begin{bmatrix}
\frac{1}{2} q^T m_1(q) \dot{q} \\
\vdots \\
\frac{1}{2} q^T m_r(q) \dot{q} \\
U_1(q) \\
\vdots \\
U_r(q)
\end{bmatrix} \in \mathbb{R}^m.
$$

(5)

The EL system (1) satisfies the LRE

$$
y = \Omega^T \theta
$$

(6)

where

$$
\dot{y} = -\lambda y + q^T G \tau,
$$

(7)

and $\theta$ is defined via (3).

**Proof:** As shown in Ortega et al. (1998, Proposition 2.5), EL systems define a passive operator $G(q, \dot{q}) \tau \mapsto \dot{q}$ with storage function

$$
\mathcal{E}(q, \dot{q}) := T(q, \dot{q}) + U(q).
$$

(8)

More precisely, it satisfies the power balance equation

$$
\dot{\mathcal{E}} = \dot{q}^T G \tau.
$$

(9)

Now, applying the LTI filter

$$
H(p) = \frac{1}{p + \lambda}
$$

(10)

where $p := \frac{d}{dt}$, to both sides of (9) we get

$$
pH(p)[\dot{\mathcal{E}}] = y,
$$

(11)

with the filter state realisation given in (7). On the other hand, using the parameterisation (2), the energy function (8) can be written as

$$
\mathcal{E}(q, \dot{q}) = \frac{1}{2} \sum_{i=1}^\ell q_i^T m_i(q) \dot{q}_i + \sum_{j=1}^r U_j(q) \dot{\theta}_j
$$

$$
= \omega^T \theta
$$

(12)

where we used (3) and (5). The proof is completed noting that (4) is a state realisation of the filter equation

$$
\Omega := pH(p)[\omega].
$$

**Remark 2.2:** In relation to Remark 2.1, the effect of the viscous friction terms in the power balance equation (9) is as follows:

$$
\dot{\mathcal{E}} = q^T G \tau - \dot{q}^T R \dot{q}.
$$

Hence, the incorporation of the unknown matrix $R$ in the LRE (6) leads to the new LRE

$$
y = \Omega^T \theta + \Omega_R \theta_R,
$$

where $\theta_R := \text{col}(R_1, R_2, \ldots, R_{n_q})$ are the diagonal elements of the matrix $R$ and

$$
\Omega_R := H(p)[\text{col}(\dot{q}_1^2, \dot{q}_2^2, \ldots, \dot{q}_{n_q}^2)].
$$

Given the relevance of the friction effects on the parameter identification problem indicated in Section 1, it is of interest to investigate the applicability of the results on friction compensation recently reported in Franco (2021) and Gandarilla et al. (2020).

**Remark 2.3:** It is important to note that the parameters $\theta$ in (2) are not the physical parameters of the system. But they are obtained overparameterising the truly physical ones to obtain a linear parameterisation. This fact will become clear in the 2-dof example treated below. See also Ortega, Gromov, et al. (2021) where a procedure to identify the true physical parameters, using a nonlinear parameterisation, is proposed.

### 2.3 Comparison with the ‘classical’ parameterisation

The fact that it is possible to use the power balance equation (9) to obtain a parameterisation of the robot manipulator dynamics was indicated in an insightful remark in Slotine and Li (1998), but was not further elaborated. To the best of the authors’ knowledge all results on parameter estimation and adaptive control of this kind of systems have relied on the far more involved parameterisation of the full dynamics (2), first proposed in Khosla and Kanade (1985) and cleverly exploited in Slotine and Li (1988), which we briefly review below. The main reason why this parameterisation was not used is because of the stringent excitation requirements that it imposes. The main contribution of our paper is to show that using the DREM procedure (Aranovskiy et al., 2017) and the new LRE proposed in Bobtsov et al. (2021) it is possible to generate alternative LRE – with exciting regressors – to overcome this drawback.

To obtain the ‘classical’ parameterisation, it is necessary to write the dynamics of the EL system (1) as

$$
\frac{d}{dt} [M(q) \dot{q}] - \frac{1}{2} \nabla_q \left( \dot{q}^T M(q) \dot{q} \right) + \nabla U(q) = G(q) \tau.
$$

(13)

We are in position to present the following well-known result (Khosla & Kanade, 1985), which is given here for the sake of completeness.

**Proposition 2.2:** The EL system (13) satisfies the vector LRE

$$
y = \Psi \theta,
$$

(14)

with $\theta$ defined in (2) and (3),

$$
\dot{y} = -\lambda y + G \tau,
$$

(15)

and the $n_q \times w$ regressor matrix is defined as

$$
\Psi = -\lambda \Psi + \begin{bmatrix}
p m_1(q) \dot{q} - \frac{1}{2} \nabla_q (\dot{q}^T m_1(q) \dot{q}) \\
\vdots \\
p m_r(q) \dot{q} - \frac{1}{2} \nabla_q (\dot{q}^T m_r(q) \dot{q}) \\
\nabla U_1(q) \\
\vdots \\
\nabla U_r(q)
\end{bmatrix}^T.
$$

(16)
Proof: Applying the LTI filter (10) to both sides of (13) we get
\[
pH(p)[M(q)\dot{q}] - \frac{1}{2}H(p)\left[\nabla_q(q^\top M(q)\dot{q})\right]
+ H(p)[\nabla U(q)] = y,
\]
where we have used (15). Now, using the parameterisation (2), the left-hand side of (17) can be written as
\[
\Psi \theta = \sum_{i=1}^{\ell} H(p) \left[p[m_i(q)\dot{q}] - \frac{1}{2} \nabla_q(q^\top [m_i(q)\dot{q}])\right] \theta_i^M
+ \sum_{j=1}^{r} H(p)[\nabla U_j(q)] \theta_j^U,
\]
where we used (3). This completes the proof.

Remark 2.4: The reduction of the computational complexity of the new LRE (6), with respect to the one of the ‘classical’ LRE (14), can hardly be overestimated. It suffices to compare the \(w\)-dimensional vector regressor (5) with the \(n_q \times w\) regressor matrix (16).

3. Generation of ‘exciting’ regressors for the new parameterisation

As mentioned above, the main drawback of the new parameterisation (4)–(7) is that the excitation requirements for consistent estimation are very ‘high’. In this section, we apply the procedure proposed in Bobtsov et al. (2021) to generate new LRE where the regressor has ‘improved’ excitation properties. The generation of new scalar LREs proceeds along the following steps.

(S1) Apply DREM to the original LRE to generate scalar LREs – one for each of the one parameters to be estimated. This procedure is summarised in Proposition A.1, where the dynamic extension is done following Kreisselmeier’s suggestion (Kreisselmeier, 1997).

(S2) Construct the new LRE following the procedure proposed in point P1 of Proposition A.2.

(S3) Select the “input” signals \(u_i\) of the LRE generator as suggested in point P2 of Proposition A.2.

Once the new LREs have been generated the estimator design is completed applying – for instance – a simple gradient descent-based parameter adaptation algorithm like the one suggested in Proposition A.3.

In summary, the proposed estimator with the new LRE proceeds from the original LRE – that is the one obtained from the power balance equation (6) or the classical LRE (14) – then implements the dynamic extension (A1), (A3) and (A5) and wraps up the design with the gradient estimator (A8).

The following remarks pertaining to the convergence properties of the estimator based on the new LRE are in order – see Bobtsov et al. (2021) for additional details.

(R1) The condition (A6) can, in principle, be easily satisfied with a suitable definition of the function \(\alpha(t)\). However, this restriction on this design parameter will, in general, limit our ability to generate an ‘exciting’ regressor \(\Phi_{21}\) for the new LRE (A4), affecting the performance of the estimator.

(R2) Point P2 of Proposition A.3 guarantees the asymptotic convergence of the estimator, provided the condition \(\lim_{t \to \infty} \Phi_{11}(t) \neq \sqrt{2\beta}\) is satisfied. As explained in Bobtsov et al. (2021), this is a technical condition needed to ensure the boundedness of all the signals, and it is ‘generically true’.

4. Indirect adaptive control

In this section, we combine – in a certainty-equivalent way – the parameter estimator proposed in the previous section with a globally stabilising controller.

The formulation of the adaptive control problem requires the following stabilisability condition for the case of known parameters.

Assumption 4.1: Given a desired bounded trajectory for the state vector \((q_\star(t), \dot{q}_\star(t)) \in R^{n_q} \times R^{n_q}\). Define the state tracking error \(\col(q, \dot{q}) := \col(q - q_\star, \dot{q} - \dot{q}_\star)\). There exists a mapping \(\beta : R^{n_q} \times R^{n_q} \times R^m \times R \rightarrow R^\star, whose origin is globally exponentially stable.

Assumption 4.1, although satisfied in several practical examples, is quite strong for underactuated systems. Notice, however, that it can be relaxed but at the price of requiring a more complicated analysis for the proof of the adaptive stabilisation claim.

The control objective is then to design a parameter estimator such that the (certainty-equivalent) adaptive control \(\tau = \beta(q, \dot{q}, \dot{\theta}, t)\) ensures global asymptotic tracking, that is,

\[
\lim_{t \to \infty} \col(q_\star(t), \dot{q}(t)) = 0,
\]

with all signals bounded.

We are in position to present the main result of this section. The proof exploits the fact that we have consistent estimates, and it follows verbatim the proof of Ortega, Bobtsov, et al. (2021, Proposition 6) – see also Ortega, Bobtsov, et al. (2021, Proposition 8) – hence, it is omitted for brevity.

Proposition 4.1: Consider the EL system (13) verifying Assumption 4.1 in closed-loop with the certainty-equivalent adaptive control

\[
\tau = \beta(q, \dot{q}, \dot{\theta}, t),
\]

where the estimated parameters are generated, proceeding from the LRE (6), as suggested in the previous section. Assume the
conditions of Propositions A.2 and A.3 (that ensure a consistent estimation) hold. Under these conditions, (19) holds with all signals bounded.

Remark 4.1: For fully actuated systems, i.e. \( m = n_q \), the mapping \( \beta(q, \dot{q}, \theta, t) \) can be chosen as the Slotine–Li Controller that, in the known parameter case, is given by (Slotine & Li, 1988)

\[
\beta(q, \dot{q}, \theta, t) = M(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + g(q) + K_1s,
\]

where we defined the signals

\[
\dot{q}_r := \dot{q} - K_2\ddot{q}, \quad s := \dot{q} + K_2\ddot{q},
\]

with diagonal, positive definite gains \( K_1, K_2 \in \mathbb{R}^{n_q \times n_q} \). The closed-loop system is then

\[
M(q)s + [C(q, \dot{q}) + K_1]s = 0, \quad \dot{\ddot{q}} + K_2\ddot{q} = s,
\]

that – as indicated in Ortega et al. (1998, Remark 4.5) – has a globally exponentially stable equilibrium at the origin.

5. Application to a fully actuated 2-dof robot manipulator

In this section, we present the two LRE derived above for the classical fully actuated 2-dof robot manipulator (Craig, 2009).

5.1 Derivation of the new (eqn6) and classical (eqn14) LREs

The equation of motion for the robot is given by (13) with

\[
M(q) = \begin{bmatrix}
\theta_1 + 2\theta_2 \cos(q_2) & \theta_3 + \theta_2 \cos(q_2) \\
\theta_3 + \theta_2 \cos(q_2) & \theta_3
\end{bmatrix},
\]

\[
U(q) = \theta g \begin{bmatrix} 1 + \sin(q_1 + q_2) \end{bmatrix} + \theta_3 g \begin{bmatrix} 1 + \sin(q_1) \end{bmatrix},
\]

with \( G = I_2 \), with \( g \) the gravitational constant and the unknown parameters

\[
\theta = \begin{bmatrix}
\ell_2^2 m_2 + \ell_1^2 (m_1 + m_2) \\
\ell_1 \ell_2 m_2 \\
\ell_1^2 m_2 \\
\ell_1 (m_1 + m_2)
\end{bmatrix},
\]

where \( \ell_i \) is the length of the link \( i \) with mass \( m_i \) for \( i = 1, 2 \). Now, following (2) we define

\[
m_1 := \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad m_2(q_2) := \cos(q_2) \begin{bmatrix} 2 & 1 \end{bmatrix}, \quad m_3 := \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad U_1(q) := g[1 + \sin(q_1 + q_2)], \quad U_2(q_1) := g[1 + \sin(q_1)].
\]

Thus the LRE (6) holds with

\[
y = H(p)[q^\top \tau],
\]

and the regressor vector

\[
\Omega = pH(p) \begin{bmatrix}
\frac{1}{2} q_1^2 \\
\frac{1}{2} q_2^2 + q_1 q_2 \\
g(1 + \sin(q_1 + q_2)) \\
g(1 + \sin(q_1))
\end{bmatrix}.
\]

On the other hand, the classical LRE (14) is given by \( y = H(p)\tau \) and

\[
\Psi = H(p) \begin{bmatrix}
p \dot{q}_1 \\
p \cos(q_2) (2\dot{q}_1 + \dot{q}_2) + p \cos(q_2) \dot{q}_1 + \sin(q_2)(\dot{q}_1^2 + \dot{q}_2) \\
p \dot{q}_2 \\
g \cos(q_1 + q_2) \\
g \cos(q_1) \\
g \cos(q_1 + q_2)
\end{bmatrix}^\top.
\]

5.2 Comparative simulation results

In this section, we present comparative simulations of the parameter estimators and adaptive control using the classical (14) and the new parameterisation (6), with and without DREM and using the new LRE or not. First, we give the results for open-loop parameter identification for different input signals. Then, we present the ones obtained for the Slotine–Li adaptive controller.

The unknown parameters of the robot are taken as \( l_1 = 0.7 \, \text{m} \); \( l_2 = 0.8 \, \text{m} \); \( m_1 = 1.5 \, \text{kg} \); and \( m_2 = 0.5 \, \text{kg} \). The filter (10) is implemented with \( \lambda = 1 \) and zero initial conditions. For all simulations, the initial estimates are \( \dot{\theta}_i(0) = 0 \), initial velocities \( \dot{q}(0) = \text{col}(0, 0) \) and initial positions \( q(0) = \text{col}(0.6\pi, 0.7\pi) \) rad.

For the new LRE generator of Proposition A.1, we set \( \beta = \frac{1}{4} \), and the free function \( \alpha \) is selected as \( \alpha(t) = \sin(\frac{1}{2}t) \).

5.2.1 Open loop parameter estimation

To evaluate the effect of the richness content of the input signal on the estimator performance, we consider the following signals \( \tau \):

\[
\begin{align*}
\tau_a(t) &= \text{col} \left( e^{-0.4t}, e^{-0.5t} \right) \\
\tau_b(t) &= \begin{cases} 
\text{col}(1, 3) & t \in [0, 2] \, \text{s} \\
0 & t > 2 \, \text{s}
\end{cases} \\
\tau_c(t) &= \frac{\cos(4t)}{2 + t} \text{col}(1, 3).
\end{align*}
\]

Notice that these signals are interval exciting, but clearly not persistently exciting (PE). Therefore, there is no guarantee that the standard gradient estimator\(^5\) (Sastry & Bodson, 1989, Equation (2.0.17)) for the classical LRE (14), which is given by

\[
\dot{\theta} = \Gamma \Psi^\top (y - \Psi \hat{\theta}),
\]

with \( \Gamma = \Gamma^\top \in \mathbb{R}^{5 \times 5} \), positive definite, will ensure parameter convergence. Actually, we show in the simulations that this estimator has in all cases a steady-state error. On the other hand, we
show that using DREM the estimated parameters converge but after a certain time a drift away from the true value is observed for all three input signals. The latter undesired behaviour, which is probably due to the loss of excitation and the follow-up accumulation of numerical integration errors, is avoided if we additionally use the new LRE. In these simulations we set $\Gamma_1 = 25I_5$ for the estimator (28) and $\gamma_i = 25$ for the DREM ones.

The result of these simulations is presented in Figures 1–3, where we also show the behaviour of the regressor signals $\Delta$ and $\Phi_{21}$. As seen from the figures, in the three cases, $\Delta(t) \to 0$ loosing excitation, while $\Phi_{21}(t) \neq 0$, guaranteeing PE and consequently exponential convergence. Moreover, we show in the figures the regressors $\Phi_{21,i}$ of the five elements of the vector theta, but the plots are almost overlapped, therefore, hardly distinguishable. This behaviour explains the long-term drift of DREM with the original LRE, which is avoided by the use of the new LRE.

In the next series of simulations, we used the power balance equation-based parameterisation, with the adaptation gains set as $\Gamma = 100I_5$ for the estimator (28) and $\gamma_i = 100$ for the DREM ones. The result of the simulations is depicted in Figures 4–6, which shows a similar scenario as the classical parameterisation. One notable difference between the two parameterisations is that with the new one the excitation of $\Phi_{21}$ is lost. Also, notice that using only DREM – without the new LRE – the parameters do not converge, revealing the critical importance of this modification.

### 5.2.2 Adaptive control

Our last series of simulations pertains to the implementation of the Slotine–Li adaptive controller of Remark 4.1 in regulation and tracking. In both cases, we selected the control gains $K_1 = 7I_2$ and $K_2 = 4I_2$ and the adaptation gain $\Gamma = 25I_5$ and $\gamma_i = 10$ for the regulation case and $\gamma_i = 25$ for the tracking problem. We propose as desired equilibrium point $q_* = \text{col}(2\pi, 0.3\pi)$, and...
for the tracking problem

\[ q_* = \begin{bmatrix} 0.4\pi \sin(0.4t) + 0.3\pi \sin(0.3t) + 0.2\pi \\ 0.3\pi \cos(0.3t) - 0.1\pi \cos(0.5t) + 0.3\pi \end{bmatrix}. \]

The result of the simulations is depicted in Figures 7–11 for the case of regulation and Figures 12–16 for tracking, where we also show the behaviour of the input signal. From Figure 7, we see that for the classical parameterisation the regulation objective is achieved without parameter convergence, while the new parameterisation is unsuccessful. This is consistent with the well-known fact that there is no unique set of controllers gains that achieve the regulation objective. In Figure 8, we observe that the addition of DREM to the classical parameterisation corrects the lack of parameter convergence for the classical parameterisation but it is of no use for the new one – for both cases the excitation is lost, as shown in Figure 9. The performance for both parametrisations is drastically improved with the use of the new LRE as shown in Figure 10, ensuring in both cases PE as depicted in Figure 11.

The same pattern as in the case of regulation is observed for tracking: Figure 12 shows the good behaviour of the old parameterisation while the new one does not achieve neither the control objective nor parameter convergence. Again, the addition of the new LRE significantly improves the behaviour for both parameterisations underscoring (see Figures 13–15), once again, the critical importance of this modification.

Summarising: The simulation results confirm our claim that the new parameterisation – which is of interest for its reduced computational complexity – is not applicable without DREM and the new LRE. But it becomes a feasible practical solution including these two modifications. It should be highlighted that,
even though the addition of DREM and the new LRE entails additional computations, they are of much smaller magnitude than the ones required for the implementation of the classical parameterisation.

6. Conclusion and future research

We have proven, for the first time, that the significantly simpler parameterisation obtained from the power balance equation of EL systems can be used for their identification and adaptive control even in a scenario with insufficient excitation. The key modification that is required is the use of new LREs, which are generated following the procedure proposed in Bobtsov et al. (2021). We believe this contribution paves the way for a wider utilisation of these advanced control techniques in critical areas like robotics.

The main result of the paper can be verbatim extended to the very broad class of passive nonlinear systems with linearly parameterised storage function. Indeed, in this case the system

$$\dot{x} = f(x, u_p) \tag{29}$$

with $x(t) \in \mathbb{R}^n$, $u_p(t) \in \mathbb{R}^m$ and $y_p(t) \in \mathbb{R}^m$, verifies the relation

$$\dot{\mathcal{H}} = u_p^\top y_p,$$

where the storage function $\mathcal{H} : \mathbb{R}^n \to \mathbb{R}_+$ may be expressed as

$$\mathcal{H}(x) = \phi^\top(x)\theta + b(x),$$
Figure 7. Transient behaviour of the signals $\tilde{q}_i$ and $\tilde{\theta}_i$.

Figure 8. Transient behaviour of the signals $\tilde{q}_i$ and $\tilde{\theta}_i$.

Figure 9. Transient behaviour of the input control $\tau$ and the signal $\Delta$. 
Figure 10. Transient behaviour of the signals $\tilde{q}_i$ and $\tilde{\theta}_i$.

Figure 11. Transient behaviour of the input control $\tau$ and exciting signal $\Phi_{21}$.

Figure 12. Transient behaviour of the signals $\tilde{q}_i$ and $\tilde{\theta}_i$. 
Figure 13. Transient behaviour of the signals $\tilde{q}_i$ and $\tilde{\theta}_i$.

Figure 14. Transient behaviour of the input control $\tau$ and the signal $\Delta$.

Figure 15. Transient behaviour of the signals $\tilde{q}_i$ and $\tilde{\theta}_i$. 
with known functions $\phi: \mathbb{R}^n \to \mathbb{R}^q$ and $b: \mathbb{R}^n \to \mathbb{R}$, and $\theta \in \mathbb{R}^q$ a vector of unknown parameters. It is clear that Proposition 2.1 applies immediately to this case with the definitions

$$y := H(p)[u_p^\top y_p] - pH(p)[b(x)]$$

$$\Omega := pH(p)[\phi(x)].$$

Current research is under way to apply this result to electromechanical systems.

Another immediate extension is to the case of separable non-linear parameterisations, that is the case when the regression equation has the form

$$y(t) = \phi^\top(t)G(\theta),$$

with known functions $G: \mathbb{R}^\ell \to \mathbb{R}^q$, with $\ell < q$, of the unknown parameters $\theta \in \mathbb{R}^\ell$. See Ortega, Gromov, et al. (2021) for additional details.

Another line of research that we are currently investigating pertains to the choice of the tuning parameters in the proposed identification method. Experience has shown that the transient performance of the estimator is strongly dependent on their suitable selection that, unfortunately, it is now done via trial-and-error.

Notes

1. We refer the interested reader to the long citation list given in the recent book (Zhang & We, 2021) for additional references.
2. See Ortega et al. (2020) and Ortega, Aranovskiy, et al. (2021) for other methods to construct the dynamic extension.
3. Of course, other identification methods (Ljung, 1987; Sastry & Bodson, 1989) could be used, but we select this one because of its simplicity and widespread popularity.
4. To simplify the notation we omit the subindex $i$ in the proposition A.2.

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No potential conflict of interest was reported by the author(s).
Appendix. Background material

In this appendix, we present the following preliminary results.

(B1) Derivation of Kreisselmeier’s regressor extension (Ortega et al., 2020, Proposition 3) with the DREM estimator (Aranovskiy et al., 2017, Proposition 1)

(B2) Generation of new LRE (Bobtsov et al., 2021) and excitation injection via energy pumping-and-damping (Yi et al., 2020)

(B3) Properties of the standard gradient estimator for the new LRE.

Proposition A.1: Consider the LRE

\[ Y = \Omega \theta \]

where \( Y(t) \in \mathbb{R}^n \), \( \Omega(t) \in \mathbb{R}^{n \times q} \) are measurable signals and \( \theta \in \mathbb{R}^q \) is a constant vector of unknown parameters. Fix \( \lambda > 0 \) and define the signals

\[ Z = -\lambda Z + \Omega^\top Y \]
\[ \Psi = -\lambda \Psi + \Omega^\top \Omega \]
\[ \mathcal{Y} = \text{adj}(\Psi) Z \]
\[ \Delta = \det(\Psi) \]

The q scalar LRE

\[ \mathcal{Y}_i = \Delta \theta_i, \quad i \in \bar{q} := \{1, 2, \ldots, q\} \]

hold.

Proposition A.2: Consider the scalar LREs\(^4\) (A2) with \( \Delta \) interval exciting (Kreisselmeier & Rietze, 1990), that is such that

\[ \int_0^\kappa \Delta^2(\tau) \, d\tau \geq \delta. \]

for some \( \kappa > 0 \) and \( \delta > 0 \).

Define the dynamic extension

\[ \dot{z} = u_2(\dot{z}) + u_3 z, \quad z(0) = 0 \]
\[ \dot{\xi} = A(\xi) + b(t), \quad \xi(0) = \text{col}(0,0) \]
\[ \dot{\Phi} = A(\Phi), \quad \Phi(0) = I_2, \]

where

\[ A(t) := \begin{bmatrix} 0 & u_2(\Delta(t)) \ u_3(\Delta(t)) \\ -u_1(\Delta(t)) & 0 \end{bmatrix}, \quad b(t) := \begin{bmatrix} -u_1(\Delta(t)) \ 0 \end{bmatrix}. \]

with \( u_1(t), u_2(t), u_3(t) \in \mathbb{R} \) arbitrary signals.

(P1) The new LRE

\[ Y = \Phi_{21} \theta, \]

holds with

\[ Y := z - \xi \]

and \( \Phi_{21}(t) \in \mathbb{R} \) the \( (21) \) element of the matrix \( \Phi \).

(P2) Define the signals

\[ u_1 = -\alpha \Delta \]
\[ u_2 = \alpha \]
\[ u_3 = -\bar{V}(\Phi_{11}, \Phi_{21}), \]

where \( 0 < \beta < \frac{1}{2}, \alpha(t) \in \mathbb{R} \) is a bounded signal such that \( |\alpha(t)\Delta(t)| \neq 0 \)

\[ \alpha(t)\Delta(t) \in L_1, \]

and

\[ \bar{V}(\Phi_{11}, \Phi_{21}) := \frac{1}{2}(\Phi_{11}^2 + \Phi_{22}^2) - \beta. \]

The full state of the LRE generator – that is, \( \Phi, \xi \) and \( z \) – is bounded and either \( \Phi_{21} \notin L_2 \) or

\[ \Phi_{11}(t) + \Phi_{22}(t) \geq 2\beta + \epsilon, \quad \forall \ t \geq 0, \]

for some (sufficiently small) \( \epsilon > 0 \).

Proposition A.3: Consider the scalar LRE (A4) of Proposition A.2 with the normalised gradient estimator

\[ \dot{\hat{\theta}} = y \frac{\Phi_{21}}{1 + \Phi_{21}^2}(Y - \Phi_{11}(\hat{\theta})), \]

with \( y > 0 \). The following implication is true:

\[ \lim_{t \to \infty} \Phi_{11}(t) \neq \sqrt{2\beta} \lim_{t \to \infty} \hat{\theta}(t) = \theta. \]

Proof: The proof follows immediately noting that, on one hand, \( \Phi_{21} \notin L_2 \) ensures parameter convergence (Aranovskiy et al., 2017, Proposition 1). On the other hand, in the light of (A7), the condition \( \lim_{t \to \infty} \Phi_{11}(t) \neq \sqrt{2\beta} \) implies that \( \Phi_{21} \) is PE, yielding exponential convergence.