Covariant density functional theory for exotic nuclei near the neutron drip-line

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Abstract. The covariant density functional theory with a few number of parameters allows a very successful description of ground-state and excited-state properties for nuclei all over the nuclear chart. The recent progress on the covariant density functional theory and its applications for nuclear structure and astrophysics as well as extensions by the Beijing group and collaborators are summarized. In particular, the halos in exotic nuclei are discussed in details.

1. Introduction

The nuclear physics knowledge we have known today is basically built on our understanding of nuclei along the β-stability line. One of the current nuclear physics frontiers is the understanding of the nuclear structure far from the β-stability line and the origin of the elements. The tool we use for such task is the covariant density functional theory (CDFT). Naturally, the first question to be answered is why the CDFT is chosen.

The density functional theory (DFT) with a small number of parameters allows a very successful description of ground-state and excited-state properties of nuclei all over the nuclear chart. In particular, the covariant version of DFT takes the Lorentz symmetry into account in a self-consistent way, and has received wide attention during the past years due to its successful description of lots of nuclear phenomena [1, 2, 3, 4, 5]. The representations with large scalar and vector fields in nuclei, of the order of a few hundred MeV, provide more efficient descriptions than non-relativistic approaches that hide these scales. Moreover, it is of particular importance that the CDFT includes nuclear magnetism [6], i.e., a consistent description of currents and...
time-odd fields, which plays an important role in the nuclear rotations [7, 8, 9, 10] but is usually
difficult to adjust to experimental data in non-relativistic functionals.

The relativistic Bruekner Hartree Fock theory and the CDFT with density-dependent mesonnucleon couplings can reproduce well the nuclear saturation properties in infinite nuclear
matter [11, 12, 13]. Furthermore, the CDFT can reproduce well the measurements of the
isotopic shifts in the Pb region [14], give naturally the spin-orbit potential, explain the origin
of the pseudospin symmetry [15, 16] as a relativistic symmetry [17, 18, 19, 20, 21, 22] and the
spin symmetry in the anti-nucleon spectrum [23, 24], and is reliable for nuclei far away from the
β-stability line, etc. Obviously, the CDFT is one of the best candidates for the description of
exotic nuclei.

Recent applications of the CDFT and its extensions by the Beijing group include

- a new point-coupling energy functional PC-PK1, which improves the description for isospin
dependence of binding energy along either the isotopic or the isotonic chains [25];
- halo nuclei in the relativistic Hartree-Bogoliubov (RHB) and relativistic Hartree-Fock-
Bogoliubov (RHFB) theory in continuum as well as continuum Skyrme Hartree-Fock-
Bogoliubov (HFB) theory with Green’s function method [26, 27, 28, 29, 30, 31];
- deformed halo nuclei in a deformed RHB model in continuum based on the Woods-Saxon
basis (DRHBWS) [32, 33, 34];
- the first study of r-process nucleosynthesis using relativistic mean-field mass model and its
application in the nucleocosmochronology [35, 36, 37];

In this contribution, only the applications to halos in spherical and deformed nuclei will be
discussed in details due to the limit in length.

2. Covariant Density Functional Theory

The details of CDFT can be found in Refs. [1, 2, 3, 4, 5]. Taking the CDFT with the point-
coupling interaction as an example, the basic building blocks are densities and currents

\[ \langle \bar{\psi} \mathcal{O} \Gamma \psi \rangle, \quad \mathcal{O} \in \{1, \bar{\tau}\}, \quad \Gamma \in \{1, \gamma_\mu, \gamma_5, \gamma_5 \gamma_\mu, \sigma_{\mu \nu}\}, \]

where \( \psi \) is Dirac spinor field of nucleon, \( \bar{\tau} \) is the isospin Pauli matrix, and \( \Gamma \) generally denotes the
4 \times 4 Dirac matrices. There are ten such building blocks characterized by their transformation
characteristics in isospin and Minkowski space [38]. In this proceeding, the isospin vectors and
space ones are denoted by arrows and bold types, respectively.

The starting point of the CDFT with the point-coupling interaction is an effective Lagrangian
of the point-coupling form which can be written as a power series in \( \bar{\psi} \mathcal{O} \Gamma \psi \) and their derivatives,

\[
\mathcal{L} = \bar{\psi} (i \gamma_\mu \partial^\mu - m) \psi - \frac{1}{4} F^{\mu \nu} F_{\mu \nu} - e \frac{1}{2} \bar{\psi} \gamma^\mu \psi A_\mu \\
- \frac{1}{2} \alpha_s (\bar{\psi} \psi)(\bar{\psi} \psi) - \frac{1}{2} \alpha_v (\bar{\psi} \gamma_\mu \psi)(\bar{\psi} \gamma^\mu \psi) - \frac{1}{2} \alpha_{TV} (\bar{\psi} \bar{\tau} \gamma_\mu \psi)(\bar{\psi} \bar{\tau} \gamma^\mu \psi) \\
- \frac{1}{3} \beta_s (\bar{\psi} \psi)^3 - \frac{1}{4} \gamma_s (\bar{\psi} \psi)^4 - \frac{1}{4} \gamma_v [(\bar{\psi} \gamma_\mu \psi)(\bar{\psi} \gamma^\mu \psi)]^2 - \frac{1}{2} \delta_s \partial_\nu (\bar{\psi} \psi) \partial^\nu (\bar{\psi} \psi) \\
- \frac{1}{2} \delta_v \partial_\nu (\bar{\psi} \gamma_\mu \psi) \partial^\nu (\bar{\psi} \gamma^\mu \psi) - \frac{1}{2} \delta_{TV} \partial_\nu (\bar{\psi} \bar{\tau} \gamma_\mu \psi) \partial^\nu (\bar{\psi} \bar{\tau} \gamma^\mu \psi),
\]

where all the symbols have their usual meaning as in Ref. [5].

The mean-field approximation leads to the replacement of the operators \( \bar{\psi} (\bar{\mathcal{O}} \Gamma \psi) \) in Eq. (2)
by their expectation values which become bilinear forms of the nucleon Dirac spinor \( \psi_k \),
\[
\langle \bar{\psi} (\bar{\mathcal{O}} \Gamma \psi) \rangle \psi \rightarrow \langle \Phi \psi (\bar{\mathcal{O}} \Gamma \psi) \psi \Phi \rangle = \sum_k v_k^2 \bar{\psi}_k (\bar{\mathcal{O}} \Gamma)_k \psi_k,
\]
where \( i \) indicates \( S, V, \) and \( TV \). The sum \( \sum \) runs over only positive energy states with the occupation probabilities \( v_k^2 \), i.e., the “no-sea” approximation. Based on these approximations, one finds the energy density functional for a nuclear system

\[
E_{\text{DF}}[\tau, \rho_S, j^\mu, A_\mu] = \int d^3r \: \mathcal{E}(r),
\]

with the energy density

\[
\mathcal{E}(r) = \mathcal{E}^{\text{kin}}(r) + \mathcal{E}^{\text{int}}(r) + \mathcal{E}^{\text{em}}(r),
\]

which is composed of a kinetic part

\[
\mathcal{E}^{\text{kin}}(r) = \sum_k v_k^2 \bar{\psi}_k^{\dagger}(r) (\alpha \cdot \mathbf{p} + \beta m) \psi_k(r),
\]

and an interaction part

\[
\mathcal{E}^{\text{int}}(r) = \frac{\alpha_S}{2} \rho_S^2 + \frac{\beta_S}{3} \rho_S^3 + \frac{\gamma_S}{4} \rho_S + \frac{\delta_S}{2} \rho_S \Delta \rho_S + \frac{\alpha_V}{2} j^\mu j^\mu + \frac{\gamma_V}{4} (j^\mu j^\mu)^2 + \frac{\delta_V}{2} j^\mu \Delta j^\mu
\]

\[
+ \frac{\alpha_{TV}}{2} \bar{j}^\mu_{TV} \cdot (\bar{j}^\mu_{TV}) + \frac{\delta_{TV}}{2} \bar{j}^\mu_{TV} \cdot \Delta (\bar{j}^\mu_{TV})
\]

with the local densities and currents

\[
\rho_S(r) = \sum_k v_k^2 \bar{\psi}_k(r) \psi_k(r),
\]

\[
j^\mu_V(r) = \sum_k v_k^2 \bar{\psi}_k(r) \gamma^\mu \psi_k(r),
\]

\[
\bar{j}^\mu_{TV}(r) = \sum_k v_k^2 \bar{\psi}_k(r) \bar{\tau} \gamma^\mu \psi_k(r),
\]

and an electromagnetic part

\[
\mathcal{E}^{\text{em}}(r) = \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - F^{0 \mu} \partial_0 A_\mu + e A_\mu j^\mu.
\]

Minimizing the energy density functional Eq. (4) with respect to \( \bar{\psi}_k \), one obtains the Dirac equation for the single nucleon

\[
[\gamma^\mu (i \partial^\mu - V^\mu) - (m + S)] \psi = 0.
\]

The single-particle effective Hamiltonian contains local scalar \( S(r) \) and vector \( V^\mu(r) \) potentials,

\[
S(r) = \Sigma_S, \quad V^\mu(r) = \Sigma^\mu + \bar{\tau} \cdot \bar{\Sigma}_{TV}^\mu,
\]

where

\[
\Sigma_S = \alpha_S \rho_S + \beta_S \rho_S^2 + \gamma_S \rho_S^3 + \delta_S \Delta \rho_S,
\]

\[
\Sigma^\mu = \alpha_V j^\mu_V + \gamma_V (j^\mu_V)^2 + \delta_V \Delta j^\mu_V + e A^\mu,
\]

\[
\bar{\Sigma}_{TV}^\mu = \alpha_{TV} \bar{j}^\mu_{TV} + \delta_{TV} \bar{\Delta} j^\mu_{TV}.
\]

The Coulomb field \( A_0 \) is determined by Poisson’s equation.
As the translation symmetry is broken in the mean-field approximation, proper treatment of center-of-mass (c.m.) motion is very important, and here the c.m. correction energy is taken into account in a microscopic way,

\[ E_{\text{c.m.}}^{\text{mic}} = -\frac{1}{2m_A} \langle \hat{\mathbf{P}}^2_{\text{c.m.}} \rangle \]  

(13)

with \( A \) mass number and \( \hat{\mathbf{P}}_{\text{c.m.}} = \sum_i^A \hat{\mathbf{p}}_i \) the total momentum in the c.m. frame [39, 40, 41].

For the formulas presented above, we mainly focus on the \( ph \) channel of the CDFT. For the \( pp \) channel, both the BCS and the Bogoliubov methods are widely used. However, it has been proved that for the exotic nuclei near the neutron drip-line, the Bogoliubov method could provide much more reliable results [53].

In a phenomenological construction of the CDFT, the nucleon-nucleon effective interaction is usually adjusted to the ground-state data of finite nuclei. For instance, a new parametrization PC-PK1 for the CDFT with point-coupling interaction is proposed by fitting to the observables of 60 selected nuclei, including the binding energies, charge radii, and empirical pairing gaps [42].

\[ \text{Figure 1.} \text{ Deviations of the binding energies calculated with PC-PK1, DD-PC1, and PC-F1 from the data [43] (upper panels) as well as the corresponding calculated ground state deformations in comparison with data [44] (lower panels) for Yb and U isotopes. Taken from Ref. [42].} \]

As reported in Ref. [42], the binding energies and quadrupole deformations of the ground states for Yb and U isotopes are investigated with PC-PK1. In the upper panels of Figure 1, the deviations of the binding energies calculated with PC-PK1, DD-PC1 [45], and PC-F1 [46] from the data [43] are shown. A systematic underestimation of the binding energies around 3 MeV for both Yb and U isotopes by PC-PK1 can be reduced to within 1 MeV after taking into account the rotational correction energies with the cranking approximation [47]. For PC-F1, the difference between the calculated and the observed binding energy decreases monotonically with the neutron number and the agreements can not be improved by the rotational correction. As almost all the isotopes in Figure 1 are used to adjust DD-PC1, it is not surprising to see its good agreement with the data without taking into account the rotational corrections.

The calculated ground-state deformations are shown in the lower panels of Figure 1 in comparison with the available data [44]. It is found that PC-PK1 can provide also a good description for the deformations as well as their corresponding evolutions with neutron number for both Yb and U isotopes.
3. Halos in spherical nuclei

Since the discovery of a large radius in $^{11}\text{Li}$ [48], the exotic nuclear halo phenomenon becomes one of the most interesting topics close to the nucleon drip-lines. In halo nuclei, the weakly binding leads to many new features, e.g., the coupling between bound states and continuum due to the pairing correlations, and the extended spatial density distributions. For the reliable theoretical description of the halo structures, it is of special significance to have the appropriate asymptotic behavior of nuclear densities at large radial distance and keep the self-consistency in dealing the discrete bound states and the continuum as well as the couplings between them. In fact such goal can be achieve by the relativistic continuum Hartree-Bogoliubov (RCHB) theory [26, 27, 28, 29] which fully takes into account the mean-field effects of the continuum.

In Ref. [29], the properties of even-even Ca, Ni, Zr, Sn, and Pb isotopes from the $\beta$-stability line to the neutron drip-line are studied within the RCHB theory with the parametrization NL-SH [49] and a zero range density-dependent pairing force

$$V^{pp}(r, r') = \frac{V_0}{2} \left(1 - P^\sigma \right) \left(1 - \frac{\rho(r)}{\rho_{\text{sat}}} \right) \delta(r-r'),$$

(14)

where all the symbols have their usual meaning as in Ref. [29]. The available experimental two-neutron separation energies $S_{2N}$ are reproduced very well. After careful analysis on the systematic of two-neutron separation energies, single-particle levels, the orbital occupation and nucleon density distributions, the giant halo phenomenon and the contribution from the continuum are demonstrated to appear in Ca isotopes with $A > 60$.

![Figure 2](image-url)

**Figure 2.** (a) The two-neutron separation energies $S_{2N}$ calculated by RCHB theory with NL-SH (Open symbols) in comparison with data available (solid ones) for the even Ca, Ni, Zr, Sn, and Pb isotopes against the neutron number $N$. (b) The root mean square (rms) neutron radii $r_n$ for even Ca, Ni, Zr, Sn, and Pb isotopes by RCHB calculations against the neutron number $N$. The curve $r_0 N^{1/3}$ with $r_0 = 1.139 \text{ fm}$ has been included to guide the eye. Taken from Ref. [29].

In Figure 2, the two-neutron separation energies $S_{2N}$ (a) and neutron rms radii $r_n$ (b) in even Ca, Ni, Zr, Sn, and Pb isotopes calculated by the RCHB theory with NL-SH are compared with the experiment data. In Figure 2(a), the good agreement between experiment and calculation is clearly shown. The experimental magic or submagic numbers $N = 20, 28, \text{ and } 40$ are reproduced. The $S_{2N}$ values for exotic Ca isotopes are extremely close to zero in several isotopes. As discussed in Ref. [29], if taking $^{60}\text{Ca}$ as a stable core, the valence neutrons will gradually occupy the loosely bound states and the continuum above the sub-shell of $N = 40$, particularly for $^{66-72}\text{Ca}$. Similar systematics are also found for the $S_{2N}$ of Zr isotopes beyond $N = 82$ [27]. In fact such typical behavior of $S_{2N}$ can be taken as an evidence of the occurrence of “giant halos” in Ca chain, just
as that in Zr chain [27]. In Figure 2(b), it is very interesting to see that \( r_n \) follows the \( N^{1/3} \) systematics well for stable nuclei although their proton numbers are quite different. Near the drip-line, abnormal behaviors appear at \( N = 40 \) in Ca isotopes and at \( N = 82 \) in Zr isotopes. As pointed in Ref. [29], the nuclei having the abnormal \( r_n \) increase correspond to those having small \( S_{2N} \), which provides another evidence to prove the halo emergence.

In the Hartree level of the CDFT, the important ingredients such as the spin-dependent tensor forces are missing. In the density-dependent relativistic Hartree-Fock-Bogoliubov (DDRHFB) theory [50], the tensor forces owing to \( \pi \) and \( \rho \) meson exchanges can be naturally taken into account and the significant improvements have been made in the consistent description of shell evolution and appropriate conservation of pseudospin symmetry. In Ref. [30], the giant halos are found in Ce isotopes close to the neutron drip-line with the DDRHFB theory using PKA1 [51] and Gogny pairing force D1S [52].

Figure 3. Neutron skin thickness \( (r_n - r_p) \) calculated by DDRHFB with PKA1 and Gogny pairing force D1S for Ca, Zr, Ni, Sn, and Ce isotopes versus \((N - Z) \). Taken from Ref. [30].

Figure 3 displays the isospin dependence of the neutron skin thickness \( (r_n - r_p) \) calculated by the DDRHFB theory with PKA1 for Ca, Ni, Zr, Sn, and Ce isotopes. Against the smooth behavior in the stable region (dotted lines), continuously growing deviations are found in the chains of Ca, Zr, and Ce until the neutron drip-line. As discussed in Ref. [30], this can be considered as an evidence of a halo, which is also found for Ca and Zr isotopes in RCHB calculations [27, 29]. Despite of the existing deviations, Ni and Sn show a similar isospin dependence in both stable and neutron drip-line regions, which may indicate possible formation of the neutron skin.

In order to study further about the possible halos in Ce isotopes, in Fig. 4, the nuclear matter distributions and neutron canonical single-particle configurations for Ce isotopes closed to the drip-line are plotted. As shown in Fig. 4(a), the neutron densities become more and more extended after the isotope \(^{186}\text{Ce} (N = 128)\), which is evidence for halo. From Fig. 4(b) it is clear that such extremely extended matter distribution, for example, in \(^{198}\text{Ce}\), is mainly caused by the low-\( l \) states, namely, the halo orbits \( \nu 4s_{1/2}, \nu 3d_{5/2} \), and \( \nu 3d_{7/2} \). In Fig. 4(c) it is found that the halo orbits \( \nu 4s_{1/2}, \nu 3d_{5/2} \), and \( \nu 3d_{7/2} \) are located around the particle continuum threshold when they are gradually occupied. For the isotopes beyond \(^{184}\text{Ce} (N = 126)\), the Fermi levels approach the continuum threshold such closely that the stability of these halo isotopes becomes sensitive to pairing effects. Near the low-\( l \) states, the high-\( l \) states \( \nu 2f_{9/2} \) and \( \nu 2g_{7/2} \) are found. As discussed in Ref. [30], because of the relatively large centrifugal barrier for \( g \) orbits, they do not contribute much to the extended neutron distributions. Nonetheless, the existence of high-\( l \) states near halo orbits is of particular significance, because it leads to a rather high level density around the Fermi surface, and evidently the pairing effects are enhanced to stabilize the halo isotopes. Seen from the occupations of the halo orbits \( N_{\text{halo}} \) in Fig. 4(d), the halos in \(^{186}\text{Ce}, \)
4. Deformed halos

It is well known that most of the nuclei observed are deformed. Therefore the existence of the deformed halos is an interesting question. If the deformed halos exist, what new features are

Figure 4. (a) Neutron and proton densities, (b) relative contributions of different orbits to the full neutron density in $^{198}$Ce, (c) neutron canonical single-particle energies, occupation probabilities (x-axis error bars), and Fermi energies $E_F$ (open circles), and (d) neutron numbers filling the halo orbits $4s_{1/2}$, $3d_{5/2}$, and $3d_{3/2} (N_{halo})$ and those lying beyond the spheres with radii $r = 10$, 11, 12, 13, 14, 15, and 16 fm, respectively. Results were calculated by DDRHF with PKA1 plus the Gogny pairing force D1S. The box radius adopted was $R = 28$ fm. Taken from Ref. [30].

$^{188}$Ce, and $^{190}$Ce are ordinary, while $^{192}$Ce, $^{194}$Ce, $^{196}$Ce, and $^{198}$Ce have giant halos as more than two neutrons occupy the halo orbits. Similar conclusions can be drawn from the neutron numbers beyond $r = 10$ fm [$N_r > 10$ fm in Fig. 4(d)], which is large enough for halos.

In the above RHB and RHFB theories, and also most of the non-relativistic HFB approaches, the continuum states are approximated by discretization, thus the proper asymptotic behavior for the continuum state needs a big box, and the information for the width of the quasiparticle resonance is missing. In Ref. [31], the self-consistent continuum Skyrme HFB theory using the Green’s function technique in coordinate space is established. In this approach, the densities needed in the energy density functional can be obtained by the contour integral of the Green’s function, which is constructed by the wave functions with proper asymptotic behavior for the continuum states. From quasiparticle spectrum of the occupation and pair number densities, one can obtain the widths of the quasiparticle resonances.

Taking neutron-rich nucleus $^{128}$Mo as an example, the neutron density $\rho(r)$ and pair density (also called the pairing tensor) $\tilde{\rho}(r)$ calculated by Skyrme HFB theory with Green’s functional method and “discretized” method using different box sizes $R$ are shown in Fig. 5. The densities from Green’s function method exponentially decay in the asymptotic region, while the densities from “discretized” method drop artificially at the edge of the box due to the box boundary condition, i.e., the wave function equals zero at the edge of the box. Therefore, compared with the “discretized” method, the Green’s function approach with proper boundary condition for the continuum states can provide the appropriate asymptotic behavior of the density distribution.

So far, the Green’s function method has only been used in the spherical non-relativistic HFB approach, which has already been used to investigate the ground states of the nuclei near the neutron drip-line [31]. One of the future topics will be the development of the CDFT theory based on the Green’s function method and also for deformed cases.
Figure 5. Neutron density $\rho(r)$ and pair density $\tilde{\rho}(r)$ for neutron-rich nucleus $^{128}$Mo calculated by Shyrme HFB theory with “discretized” method and Green’s function method with different box sizes $R$. Take from Ref. [54].

expected?

In order to describe the halo phenomena in deformed nuclei, the deformed RHB theory in continuum based on the Woods-Saxon basis (DRHBWS) has been developed recently [56, 32, 33, 57]. In Ref. [32], with the parametrization NL3 [58] and zero range density-dependent pairing force in Eq. (14), the deformed neutron-rich and weakly bound nucleus $^{44}$Mg is investigated within the DRHBWS theory and an interesting shape decoupling between the core and halo is found.

Figure 6. Density distributions of $^{44}$Mg with the $z$ axis as the symmetry axis. (a) The proton density ($x < 0$) and the neutron density ($x > 0$), (b) the density of the neutron core, and (c) the density of the neutron halo. Taken from Ref. [32].

Figure 7. Single-particle spectrum around the Fermi level (dotted line) in the canonical basis for $^{44}$Mg as a function of the occupation probability $v^2$. Taken from Ref. [32].
In Fig. 6(a) the density distributions of all protons and neutrons in $^{44}$Mg are shown. Owing to the large neutron excess, the neutron density not only extends much farther in space but also shows a halo structure. The neutron density is decomposed into the contribution of the core in Fig. 6(b) and that of the halo in Fig. 6(c). It is found that the core of $^{44}$Mg is prolate and the halo has a slightly oblate deformation. As pointed in Ref. [32], this indicates the decoupling between the deformations of core and halo.

Following the discussion in Ref. [32], the single-particle spectrum around the Fermi level for the ground state of $^{44}$Mg is shown in Fig. 7. The orbitals are labeled by the third component of total angular momentum and parity $\Omega^\pi$. The $n$ numbers characterize the different orbitals appearing in this figure according to their energies. In Fig. 7, there is a considerable gap between the two levels with $n = 2$ and $n = 3$. As discussed in Ref. [32], the levels with $\varepsilon_{\text{can}} < -2.5$ MeV contribute to the "core" and the other remaining weakly bound and continuum orbitals with $\varepsilon_{\text{can}} > -1$ MeV naturally form the "halo". Therefore, the neutron density $\rho_n(r)$ is decomposed into two parts, one part coming from the orbitals with canonical single-particle energies $\varepsilon_{\text{can}} < -2.5$ MeV (called "core") and the other from the remaining weakly bound and continuum orbitals (called "halo"). A further decomposition shows that the two weakly bound orbitals, that is, the third ($\Omega^\pi = 1/2^-$) and the fourth ($\Omega^\pi = 3/2^-$), contribute the most to the halo. If the deformed wave functions of the two weakly bound orbitals are decomposed in the spherical Woods-Saxon basis, it turns out that in both cases the major part comes from $p$ waves, as indicated on the right-hand side of Fig. 7. It is known that the angular distribution of $|Y_{10}(\theta, \phi)|^2 \propto \cos^2 \theta$ with a projection of the orbital angular momentum on the symmetry axis $\Lambda = 0$ is prolate and those of $|Y_{1\pm 1}(\theta, \phi)|^2 \propto \sin^2 \theta$ with $\Lambda = 1$ are oblate. It is found that in the third ($\Omega^\pi = 1/2^-$) orbital, both $\Lambda = 0$ and $\Lambda = 1$ components contribute and the latter dominates. Therefore, this orbital has a slightly oblate shape. For the fourth ($\Omega^\pi = 3/2^-$) state, there is only the $\Lambda = 1$ component from the $p_{3/2}$ wave, and thus an oblate shape is also expected.

5. Conclusion

In summary, the recent progress in the applications of the CDFT as well as its extensions by the Beijing group are summarized. In particular, its applications to halos in spherical and deformed nuclei are discussed in details. For spherical halo nuclei, the properties of even Ca, Ni, Zr, Sn, and Pb isotopes from the $\beta$-stability line to the neutron drip-line have been studied by RCHB theory. It has been suggested that the giant halo phenomena appear in Ca isotopes with $A > 60$ apart from Zr isotopes predicted earlier. Considering the tensor forces owing to $\pi$ and $\rho$ meson exchanges, halo has been studied by DDRHFB and the giant halos have been found in Ce isotopes close to the neutron drip-line. The self-consistent continuum Skyrme HFB theory using the Green’s function technique in coordinate space is introduced. It is found that the Green’s function approach with proper boundary condition for the continuum states can provide the appropriate asymptotic behaviors of the density distributions, which does not depend on the space size. For deformed halo nuclei, $^{44}$Mg has been investigated within DRHBWS theory in continuum and an interesting shape decoupling between the core and halo has been found.

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