Thermoelectric spin diffusion in a ferromagnetic metal

Moosa Hatami\textsuperscript{a}, Gerrit E. W. Bauer\textsuperscript{a}, Saburo Takahashi\textsuperscript{b}, Sadamichi Maekawa\textsuperscript{b,c}

\textsuperscript{a}Kavli Institute of NanoScience, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands
\textsuperscript{b}Institute for Materials Research, Tohoku University, Sendai, Miyagi 980-8577, Japan
\textsuperscript{c}CREST, Japan Science and Technology Agency, Tokyo 100-0075, Japan

Abstract

We present a semiclassical theory of spin-diffusion in a ferromagnetic metal subject to a temperature gradient. Spin-flip scattering can generate pure thermal spin currents by short-circuiting spin channels while suppressing spin accumulations. A thermally induced spin density is locally generated when the energy dependence of the density of states is spin polarized.

Key words: A. Metallic ferromagnets; D. thermoelectrics; D. spin diffusion; D. spin caloritronics

\textit{PACS:} 72.15.Jf; 75.60.Jk; p85.75.-d; 75.30.Sg

1. Introduction

Thermoelectric properties attracted new interest in recent years due to the improved performance of nanometer-scale miniaturization of electronics, thus deserves to be studied in magnetic systems. The Seebeck effect refers to conversion of a thermal differential into an electric voltage, for instance in a thermocouple. A simple Stoner model with energies $\varepsilon_k = \varepsilon_0 + \Delta$ for spins $\pm \uparrow$, where $\varepsilon_0$ is the kinetic energy for momentum $k = |\mathbf{k}|$, $\Delta$ ($T$) the temperature dependent exchange splitting, and $\mu_0$ the ground state chemical potential. The Boltzmann equation describes evolution of the distribution of conduction electrons in the presence of external fields and random scattering processes and for a steady state reads

$$\left( \mathbf{v}_k \cdot \nabla_r + \frac{\mathbf{F}^{(\alpha)}(\mathbf{k})}{\hbar} \cdot \nabla_k \right) f^{(\alpha)}(\mathbf{k}, r) = \left( \frac{\partial f^{(\alpha)}(\mathbf{k})}{\partial t} \right)_{\text{scatt.}}$$

where $\mathbf{v}_k = \nabla_k \varepsilon_k$ is the electron velocity and $f^{(\alpha)}(\mathbf{k})$ its distribution as a function of momentum $\mathbf{k}$, position $\mathbf{r}$ and spin $\alpha$. The forces $\mathbf{F}^{(\alpha)}(\mathbf{r}) = \varepsilon \nabla_r \phi - \alpha (\partial \Delta / \partial T) \nabla_r \Delta$ originate from the gradients of the electric potential $\phi$ and exchange splitting $\Delta$, where the latter may include an external (Zeeman) magnetic field. $e = |e|$ is the modulus of the electron charge and $\hbar = \hbar / 2\pi$ Planck’s con-
The charge and energy currents must be conserved, *i.e.* \( \sum_{(\alpha)} \nabla_{\tau} \cdot \mathbf{J}_{(\alpha)}(\mathbf{r}) = 0 \) and \( \sum_{(\alpha)} \nabla_{\tau} \cdot \mathbf{J}_{E_{(\alpha)}}(\mathbf{r}) = 0 \).

The problem is simplified by linearizing Eq. (6). We disregard the higher order term \( \mathbf{F}^{(\alpha)}g^{(\alpha)} \) and the temperature dependence of the material parameters. Expanding \( f^{(\alpha)} \) around the equilibrium distribution function \( f_0^{(\alpha)} \), we obtain for the local distribution function in the spin-flip terms

\[
f^{(\alpha)}(\varepsilon_k, \mathbf{r})|_{T=\text{const.}} \approx f_0^{(\alpha)}(\varepsilon_k) + \left( -\frac{\partial f_0^{(\alpha)}}{\partial \varepsilon_k} \right) \mu^{(\alpha)}(\mathbf{r}),
\]

and for the divergence

\[
\nabla_{\tau} f^{(\alpha)}(\varepsilon_k, \mathbf{r}) \approx \left( -\frac{\partial f_0^{(\alpha)}}{\partial \varepsilon_k} \right) \nabla_{\tau} \mu^{(\alpha)} + \left( \varepsilon^{(\alpha)} - \mu^{(\alpha)} - e\phi \right) \nabla_{\tau} T.
\]

After integration over momenta and invoking the Sommerfeld expansion, we obtain the particle diffusion equation

\[
\nabla_{\tau}^2 \left( \tilde{\mu}^{(\alpha)} - eS^{(\alpha)}T \right) = \frac{\mu^{(\alpha)} - \mu^{(-\alpha)}}{\left| f_0^{(\alpha)} \right|^2},
\]

where \( f_0^{(\alpha)} = \sqrt{D_0^{(\alpha)} \tau_{sf}^{(\alpha)}} \), \( \tilde{\mu}^{(\alpha)} = \mu^{(\alpha)} - e\phi \), and the spin-dependent Seebeck coefficients \( S^{(\alpha)} = -eL_0 T \partial_{\varepsilon_k} \ln \sigma^{(\alpha)} |_{\mu_0} \) (Mott’s formula) in terms of the Lorenz constant \( L_0 = (\pi^2/3) k_B / e^2 \) have been introduced. By multiplying Eq. (6) by \( \varepsilon_k^{(\alpha)} \) and integrating, we obtain the heat diffusion equation

\[
\nabla_{\tau}^2 \left( -S^{(\alpha)} \tilde{\mu}^{(\alpha)} + eL_0 T \right) = S_{sf} \frac{\mu^{(\alpha)} - \mu^{(-\alpha)}}{\left| f_0^{(\alpha)} \right|^2},
\]

where due to the detailed balance of spin-flip scattering \( S_{sf} = -eL_0 T \partial_{\varepsilon_k} \ln \left( N^{(\alpha)} / \tau_{sf}^{(\alpha)} \right) |_{\mu_0} \) does not depend on spin.

From Eqs. (12) and (13)

\[
\nabla_{\tau}^2 \left( \frac{S^2}{L_0} (1 - P^2) P_S \mu_s + \left( 1 + \frac{S^2}{L_0} \right) T \right) = 0,
\]

where \( \sigma = \sigma^{(1)} + \sigma^{(1)} \), \( P = (\sigma^{(1)} - \sigma^{(-1)}) / \sigma \), \( \sigma S = \sigma^{(1)} S^{(1)} + \sigma^{(1)} S^{(-1)} \), \( P_S = (S^{(1)} - S^{(-1)}) / S \), and \( \mu_0 = \mu^{(1)} - \mu^{(1)} \). For metals the dimensionless parameter \( S^2 / L_0 \ll 1 \). By letting \( S^2 / L_0 \to 0 \) the temperature gradient becomes constant and the conventional diffusion equations for the spin \( \mu_s \) and charge accumulation \( \tilde{\mu}_c = \left( \tilde{\mu}^{(1)} + \tilde{\mu}^{(1)} \right) / 2 \) are recovered [10]:

\[
\nabla_{\tau}^2 \mu_s(\mathbf{r}) - \mu_s(\mathbf{r})/\tau_{sf}^2 = 0,
\]

\[
\nabla_{\tau}^2 [\tilde{\mu}_c(\mathbf{r}) + P \mu_s(\mathbf{r}) / 2] = 0.
\]
with \( l_{sd} = \left( \left( l_{sd}^{(L)} \right)^{-2} + \left( l_{sd}^{(R)} \right)^{-2} \right)^{-1/2} \).

Let us consider a conducting magnetic wire with length \( \Lambda \) in the \( x \) direction. When we fix the charge and spin distribution functions at the left(right) ends to be \( \mu_{sL(R)} \) and \( \mu_{sL(R)} \), respectively,

\[
\mu_{s}(x) = \frac{1}{2} (\mu_{sL} + \mu_{sR}) + \frac{P}{4} (\mu_{sL} + \mu_{sR}) + \left[ (\mu_{sR} - \mu_{sL}) + \frac{P}{2} (\mu_{sR} - \mu_{sL}) \right] \frac{x}{\Lambda} - \frac{P}{2} \mu_{s}(x),
\]

\[
\mu_{s}(x) = \mu_{sL} + \frac{\sinh (\lambda/2 - x/l_{sd})}{\sinh \lambda} + \mu_{sR} + \frac{\sinh (\lambda/2 + x/l_{sd})}{\sinh \lambda},
\]

where \( \lambda = \Lambda/l_{sd} \) measures the spin-flip scattering. When \( \lambda \gg 1 \) and \( |x/l_{sd} - \lambda| \gg 1 \) we find that \( \mu_{s}(x) = \lambda^{(L)} \mu_{sL} e^{-x/l_{sd}} + \mu_{sR} \sigma^{(R)} / \sinh \lambda \) is appreciable only in proximity of the edges. The vanishing of \( \mu_{s} \) in the bulk is independent of the thermoelectric forces applied by a temperature or chemical potential difference over the wire. By iteratively reintroducing the heat generation by spin-flip processes that are of order \( S^2/\mathcal{L}_0 \) we observe that their effect is confined to the edges where \( \nabla^2 \mu_{s} \) is significant. Gradients of the exchange potential and external Zeeman magnetic fields do not appear explicitly at all. We conclude that the spin accumulation cannot persist over distances longer than the spin-flip diffusion length. Metallic spin diffusion therefore cannot explain the observed spin-Seebeck signal in permalloy with \( L/l_{sd} \sim 10^5 \).)

3. Thermoelectric transport and thermal spin currents

The local Fermi-Dirac spin distribution functions are defined by spatially and spin-dependent chemical potentials. Temperatures depend on position, but are not spin-dependent in the presence of the sufficiently strong inelastic scattering assumed here. In this Section we consider transport of the conduction electron spins in a magnetic metal experiencing thermoelectric forces, viz. an external electric field and gradients of the local chemical potentials and temperature. Using the Sommerfeld expansion in Eqs. (17, 18) we obtain for the local spin particle and heat currents \( \hat{Q}_{s}^{(\alpha)} = J_{s}^{(\alpha)}(r) - \mu_{0} J^{(\alpha)}(\varepsilon, r) \), valid for \( \mathcal{L}_0 T^2 |\partial^2 \sigma(\varepsilon)|_{\varepsilon} \ll \sigma(\varepsilon) \),

\[
\left( \begin{array}{c}
J^{(\alpha)} \\
\hat{Q}^{(\alpha)}
\end{array} \right) = \sigma^{(\alpha)} \left( \begin{array}{c}
1 \\
S^{(\alpha)} T
\end{array} \right) \left( \begin{array}{c}
\nabla_{r} \bar{\mu}^{(\alpha)} / \varepsilon \\
\nabla_{r} T
\end{array} \right).
\]

The spin-dependent thermal conductivities obey the Wiedemann-Franz law \( \kappa^{(\alpha)} \approx \mathcal{L}_0 T \sigma^{(\alpha)} \) in the limit \( S^{(\alpha)} \ll \sqrt{\mathcal{L}_0} \) and the total thermal conductivity \( \kappa = \kappa^L + \kappa^R = \mathcal{L}_0 T \sigma \). Eq. (19) can be rewritten in terms of the charge, spin and heat currents, \( J_{c(s)} = J^{(1)} \pm J^{(\uparrow)} \) and \( \hat{Q} = \hat{Q}^{(1)} + \hat{Q}^{(\uparrow)} \), respectively,

\[
\left( \begin{array}{c}
J_{c} \\
J_{s} \\
\hat{Q}
\end{array} \right) = \sigma \left( \begin{array}{ccc}
P & 1 & S \\
P & 1 & P^2 S \\
ST & P^2 ST & \mathcal{L}_0 T
\end{array} \right) \left( \begin{array}{c}
\nabla_{r} \bar{\mu}/ \varepsilon \\
\nabla_{r} \mu_s / 2 \varepsilon \\
-\nabla_{r} T
\end{array} \right),
\]

in which \( P \) and \( P^2 \) stand for the spin-polarization of conductivity \( \sigma^{(s)} \) and its energy derivative \( \partial_{\varepsilon} \sigma^{(s)} \).

In the following we concentrate on the expressions for charge and spin chemical potential distributions induced in a one-dimensional magnetic wire by a constant temperature gradient, \( \Delta T/\Lambda \), which is part of an open electric circuit, i.e. in the absence of a charge current. The three unknowns \( \sigma_{sL(R)} \) and \( \Delta T \) are determined by boundary conditions.

The chemical potential and temperature differences over the wire are defined as \( \Delta \mu = \mu_{R} - \mu_{L}, \Delta T = T_{R} - T_{L} \). The open electric circuit boundary condition is \( J_{c}(x) = 0 \). We consider two additional limiting boundary conditions: (i)
the reservoirs at the two ends of the wire are efficient spin sinks so that \( \mu_{L(R)} = 0 \); (ii) spin are not dissipated at the ends of the wire whatsoever, i.e. \( J_s(-\Lambda/2) = J_s(\Lambda/2) = 0 \).

The reality might be somewhere between (i) and (ii). The heat current \( \dot{Q} \) is uniform to leading order in \( S^2/L_0 \).

In the presence of ideal boundary spin sinks (i) the spin accumulation, Eq. (18), vanishes everywhere in the sample. The total chemical potential, Eq. (17), changes linearly in the sample and we find \( \Delta \mu = eS\Delta T \), which implies a uniform thermally excited spin current \( J_s = (P' - P)\sigma S \Delta T/\Lambda \) when \( P' \neq P \) or equivalently \( P_S \neq 0 \). This situation is similar to the classical thermocouple in which the wires of different materials \( (S_A \neq S_B) \) are electrically short-circuited at the ends.

Without boundary spin sinks, (ii) \( J_s(0) = J_s(\Lambda) = 0 \), the spin accumulation distribution in the wire, Eq. (18), becomes

\[
\mu_s(x) = \frac{eP_S S \Delta T}{(1 + PP_S)\lambda} \sinh \left( \frac{x}{l_s} \right) \frac{\lambda}{2},
\]

changes sign crossing the center, \( \mu_s(0) = 0 \), and its modulus increases exponentially at the edges (see Fig. 1).

At the ends \( x = \pm \Lambda/2 \) we find finite spin accumulations

\[
\mu_{sL} = \mu_{sR} = -\frac{eP_S S \Delta T}{(1 + PP_S)\lambda} \tanh \left( \frac{\lambda}{2} \right).
\]

Also this has an analogue with the classical thermopower effect \[9\]: a thermocouple consisting of wires from two different materials, creates a voltage difference in the absence of currents \( (J_{A(B)} = 0) \). The spin accumulation has an effect of the measurable thermopower \( S_e = \Delta \mu/\epsilon \Delta T = (S + \mu_{sR}/\epsilon) \Delta T \). The spin-flip scattering suppresses spin accumulation, but generates a thermally induced spin current

\[
J_s = \frac{\sigma (P - P')}{\lambda} S \Delta T \left( 1 - \frac{\sinh \left( \frac{x}{l_s} \right)}{\cosh \left( \frac{\lambda}{2} \right)} \right),
\]

analogous to the electric current in a short-circuited thermocouple. When \( \lambda \gg 1 \) the spin current equals that in (i) except at the edges, where it exponentially decays. In the opposite limit, in the absence of spin-flip scattering, the spin accumulation adopts a linear relation between contacts \( \mu_s(x) \propto x \Delta T/\Lambda \) and the spin current vanishes.

For efficient spin sinks at the edges (i), the spin current is uniform but the spin accumulation vanishes. The thermally induced spin accumulation in case (ii) is also suppressed sufficiently far away from the terminals. The spin accumulation is proportional to the temperature gradient and does not depend on length of the wire. A spin Seebeck effect should be observable in ferromagnetic wires in proximity of the edges. However, our results do not explain the (approximately) linear inverse spin Hall voltage profile measured by Uchida et al. \[10\] in a permalloy thin film.

4. Temperature dependence of chemical potential and spin density

In this Section we discuss the temperature dependence of the chemical potential and spin density. The strong screening in metals enforces local charge neutrality independent of temperature. The associated variation of the chemical potential vs. temperature is sensitive to the energy dependence of the density of states.

The local electron density is the integral of the electron distribution function over all eigenstates, for a normal metal at thermal equilibrium \( n = \sum_k f_k \) (where \( f = f^1 = f^2 \)). Taking into account the energy dependence of the density of states, \( N(\epsilon) \), the electron density in the limit \( (k_BT/\mu_0)^2 \ll 1 \), in which we may use the Sommerfeld approximation, reads \[11\]

\[
n = \int d\epsilon N(\epsilon) f_0(\epsilon; \mu_0, T) = N(\mu_0) \left( \mu_0 - \frac{eS_N T}{2} \right) + \delta n(0),
\]

where \( \delta n(0) = \int^{\mu_0} d\epsilon N(\epsilon) \) is a constant zero-temperature correction. \( S_N = -eC_0 T \partial_\epsilon \ln N(\epsilon) \mu_0 \) can be interpreted as an intrinsic thermopower that does not depend on impurity scattering. Recalling that \( S = e^2N_D \) we observe a relation with the diffuse thermopower as \( S = S_N + S_D \). At thermal equilibrium the charge neutrality, \( n(T) = n(0) \) implies \( \mu(T) = \mu_0 + eS_N T/2 \), i.e., a temperature dependent chemical potential. The Maxwell identity \( \partial S/\partial n|_T = -\partial \mu/\partial T|_n = -eS_N/2 \) relates \( S_N \) to the entropy \( S \).

The above discussion can be extended to a ferromagnetic metal with spin-polarized density of states. The spin-dependent electron densities vary with temperature as

\[
n(\alpha)(\mu, T) = \int d\epsilon N(\alpha)(\epsilon) f_0(\epsilon; \mu_0, T)
\]

\[
= N(\alpha) \left( \mu_0 - \frac{eS_N(T)}{2} \right) + \delta n(\alpha)(0)
\]

Similarly, \( \delta n(\alpha)(0) = \int^{\mu_0} d\epsilon N(\alpha)(\epsilon) \), where \( S_N^{(\alpha)} = -eC_0 T \partial_\epsilon \ln N(\alpha)(\epsilon) \mu_0 \) and \( N(\alpha) = N^{(\alpha)} \) is the density of states at the chemical potential. The transport thermopower is \( S^{(\alpha)} = S_N^{(\alpha)} + S_D^{(\alpha)} \). At finite temperature, the spin-polarized \( eS_N^{(\alpha)} T/2 \) act as intrinsic thermoelectric potentials, resembling the Zeeman splitting of energy bands by an external magnetic field. The charge neutrality condition in the normal metal holds in the ferromagnet as well, i.e. \( \partial(n^{(1)} + n^{(1)})/\partial T = 0 \). Using Eq. (26) this condition implies

\[
\frac{1}{e} \frac{\partial \mu_0}{\partial T} = \frac{N(1)S_N^{(1)} + N(1)S_N^{(1)}}{N(1) + N(1)} = S_N
\]

which reduces to that in the normal metal when \( N(1) = N^{(1)} \). A change in the equilibrium temperature thus shifts the chemical potential (the Fermi level) just as a gate voltage but with magnitude proportional to the intrinsic thermopower \( \delta \mu = eS_N \delta T \). A similar effect exists in the
presence of a uniform magnetic field, where the charge neutrality condition implies a magnetic field-dependent chemical potential in the ferromagnet known as the magneto-Coulomb effect \[12,13\]. Substituting Eq. \[27\] into the expressions for the spin-polarized electron densities, we find for the variation of the spin density \(n_s = n^{(1)} - n^{(0)}\) as a function of temperature as \(\delta n_s / \delta T = e N S_N (P_N - P'_N)\) or, after integration over temperature,

\[
n_s(T) - n_s(0) = \frac{e}{2} N S_n (P_L - P_R) \tag{28}
\]

in which \(P_N\) and \(P'_N\) are the spin polarization of the densities of states \((N^{(0)})\) and its energy derivative \((\partial f^{(0)})/\partial \epsilon\), respectively, and \(N = N^{(1)} + N^{(0)}\). The spin density is thus temperature dependent when \(P_{SN} = (P'_N - P_N)/(1 - P_N P'_N) \neq 0\).

The density of states in a ferromagnet is in general also temperature dependent, \(\partial N / \partial T = (\partial N / \partial \Delta) (\partial \Delta / \partial T) \neq 0\). At thermal equilibrium the spin density and the exchange splitting are closely related to each other \(\Delta(T) \propto n_s(T)\). In the Appendix we show that even far below the Curie temperature this temperature dependence may be significant.

5. Thermally induced spin density/polarization

In Section 3, we derived the spin accumulation in a ferromagnetic wire under a temperature gradient, Eq. \[21\]. The chemical potential is a function of the electron density and the temperature, as shown in the previous Section. A local variation of the spin chemical potential from equilibrium \(\delta \mu^{(0)}(x) = \mu^{(0)}(x) - \mu_0\) can be expanded in terms of deviations of non-equilibrium spin-polarized electron densities and temperature \((n^{(0)}, T)\) from their equilibrium values \((n_0^{(0)}, T_0)\),

\[
\delta \mu^{(0)}(x) = \left( \frac{\partial \mu^{(0)}}{\partial n^{(0)}} \right)_T \delta n^{(0)}(x) + \left( \frac{\partial \mu^{(0)}}{\partial T} \right)_{n^{(0)}} \delta T(x).
\]

The derivatives can be found by an extension of Eq. \[29\] by replacing \(f_0\) with the local spin-dependent distribution functions \(f^{(0)}(\epsilon; \mu^{(0)}(x))\), as

\[
\left( \frac{\partial \mu^{(0)}}{\partial n^{(0)}} \right)_{T_0} = \frac{1}{N^{(0)}}, \tag{30}
\]

\[
\left( \frac{\partial \mu^{(0)}}{\partial T} \right)_{n_0^{(0)}} = e S_N^{(0)}. \tag{31}
\]

In a ferromagnet subject to a temperature gradient, the spin chemical potential thus contains two contributions

\[
\mu_s(x) = \delta n^{(1)}(x)/N^{(1)} - \delta n^{(0)}(x)/N^{(0)} + e(S_N^{(1)} - S_N^{(0)}) (T(x) - T_0) \tag{32}
\]

In a bulk ferromagnet with spin-flip scattering we have seen that \(\mu_s(x) = 0\), such that

\[
\delta n^{(1)}(x)/N^{(1)} - \delta n^{(0)}(x)/N^{(0)} = -e(S_N^{(1)} - S_N^{(0)}) (T(x) - T_0), \tag{33}
\]

which is a spin density difference (spin polarization) induced by the temperature gradient. Eq. \[33\], regardless of its position dependence, is equivalent to Eq. \[28\] for the temperature dependence of the spin density at thermal equilibrium. The spin polarization, Eq. \[33\], adopts a linear profile and changes sign in the center of the magnetic wire, where \(T_0 = (T_L + T_R)/2\). It originates from the temperature dependence of the chemical potential in the presence of spin polarization of the intrinsic thermopower in the ferromagnet.

The temperature dependent spin polarization and local exchange splitting in the bulk ferromagnet is a local equilibrium effect that cannot drive a non-equilibrium spin current into laterally contacts. The observed spin signal in the experiments by Uchida et al. \[9\] can therefore not be due to the spin polarization induced by the intrinsic thermopower. A thermally induced spin pumping mechanism due to magnon propagation in the bulk ferromagnet \[14\] may explain the observed spin signals.

6. Conclusions

We presented a theoretical analysis of thermoelectric transport in ferromagnetic metals using an extension of the spin-dependent Boltzmann and diffusion equations. We showed that in a bulk ferromagnetic metal subject to a temperature gradient a pure thermal spin current is generated by spin-flip scattering whereas the spin accumulation vanishes. The temperature dependent spin density arising from an energy-dependent density of states generates a spatially varying local spin polarization along the ferromagnetic wire. The spatial dependence of the exchange splitting induced by a temperature gradient to leading order does not lead to effects on spin transport. This work could help to better understand thermoelectric phenomena in metallic ferromagnets.

Acknowledgments

We thank A. Brataas, J. Ieda, P. J. Kelly, A.H. MacDonald, Y. Tserkovnyak and X. Xiao for fruitful discussions. This work is supported by “NanoNed”, a nanotechnology programme of the Dutch Ministry of Economic Affairs.

Appendix: Temperature dependence of the exchange potential in the Stoner model

Here we analyze the temperature dependence of the chemical potential and equilibrium spin densities based on the Stoner theory of ferromagnetism. The Stoner criterion
\[ N(\mu_0) J = 1, \text{ in which } J \text{ is the Stoner exchange parameter,} \]
describes the onset of ferromagnetism. A stable magnetic ordering leads to an exchange splitting of energy bands for opposite spin directions, which is related to the spin density by \( \Delta(T) = Jn_s(T) \). \( J \) is an essentially atomic-like exchange integral that does not depend on temperature. Here we study the temperature dependence of the chemical potential and the spin density self-consistently.

We emphasize that the Stoner model is woefully inadequate at elevated temperatures since it completely neglects spin wave excitations of the magnetic order parameter. Consequently, the critical temperatures are drastically overestimated by the Stoner criterion. Nevertheless we believe that at temperatures sufficiently below the Curie transition, the Stoner model, as a simple implementation of density-functional theory, can provide useful qualitative insights.

The density of states depends on temperature via the exchange splitting. In the rigid-band model \( N^{(a)} = N (\varepsilon + \alpha \Delta) \). The charge neutrality condition \( \partial (n^+ + n^-) / \partial T = 0 \) now implies

\[
\int d\varepsilon \left( \frac{\partial N}{\partial T} + N \frac{\partial f}{\partial T} \right) = 0 \tag{A-1}
\]

where \( N = N^+ + N^- \) and \( f = f^+ = f^- \) at thermal equilibrium. Using the identities

\[ \frac{\partial f}{\partial T} = - \mu \frac{\partial N}{\partial T} + \mu \frac{\partial \mu}{\partial T} \neq \left( \frac{\partial f}{\partial T} \right)_\mu \tag{A-2} \]

and \( \partial N / \partial T = (\partial \Delta / \partial T) (\partial N / \partial \Delta) \), we find for the temperature dependence of the chemical potential

\[
\frac{\partial \mu}{\partial T} = -\chi^{-1} \int d\varepsilon \left( \frac{\varepsilon - \mu}{T} \right) N(\varepsilon, T) \left( - \frac{\partial f}{\partial \varepsilon} \right) - \chi^{-1} \frac{\partial \Delta}{\partial T} \int d\varepsilon \frac{\partial N(\varepsilon, T)}{\partial \Delta} f(\varepsilon) \tag{A-3}
\]

where \( \chi(T) \equiv \int d\varepsilon N(\varepsilon, T) (\partial f / \partial \varepsilon) \) is the (Pauli) susceptibility. The first term above is the intrinsic thermopower \( \varepsilon S_N \), and the second term is the correction by the temperature dependence of the exchange splitting. The above equation can be rewritten as

\[ \frac{\partial \mu}{\partial T} = \varepsilon S_N - \frac{\chi_s \partial \Delta}{\chi \partial T} \tag{A-4} \]

in which \( \chi_s = \chi^1 - \chi^1 \).

Next we are interested in the variation of spin density with temperature. The self-consistency condition reads

\[ \frac{\partial n_s(T)}{\partial T} = \int d\varepsilon \left( \frac{\partial N_s(\varepsilon, T)}{\partial T} f(\varepsilon) + N_s(\varepsilon, T) \frac{\partial f(\varepsilon)}{\partial T} \right) = \frac{1}{J} \frac{\partial \Delta(T)}{\partial T} \tag{A-5} \]

With Eqs. \( \text{[A-2,A-4]} \), the temperature dependence of the spin density becomes

\[ \frac{\partial n_s(T)}{\partial T} = \frac{\varepsilon (S_N - S_{N_s}) \chi_s}{1 - J \left( 1 - (\chi_s / \chi)^2 \right)} \tag{A-6} \]

in which \( S_{N_s} \) and \( \chi_s \) are obtained as intrinsic thermopower and susceptibility but with substituting \( N(\varepsilon, T) \to N_s(\varepsilon, T) \). Equation \( \text{[A-6]} \) is exact within the limits of the validity of the rigid-band Stoner model. Neglect of the interaction correction \( \partial n_s / \partial T \simeq \varepsilon (S_N - S_{N_s}) \chi_s \) is equivalent to our former result Eq. \( \text{[A-28]} \). At the Curie temperature \( J \chi = 1 \) and \( \partial n_s(T) / \partial T \) diverges. At low temperatures, assuming that \( J \chi \) does not change much from the Stoner criterion

\[ \frac{\partial n_s(T)}{\partial T} \to \frac{\varepsilon (P_N - P_N') \chi S_N}{1 - J \chi (1 - P_N')} \tag{A-7} \]

\[ \approx \frac{(P_N - P_N')}{P_N^2} (-e L_0 T) \partial_T N |_{\mu_0}. \tag{A-8} \]

The temperature dependence of the gap follows from Eqs. \( \text{[A-6,A-5]} \), which leads to contributions to the transport thermopower as explained below Eq. \( \text{[19]} \).

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