FORMATION OF SPIRAL-ARM SPURS AND BOUND CLOUDS IN VERTICALLY STRATIFIED GALACTIC GAS DISKS

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ABSTRACT

We investigate the growth of spiral-arm substructure in vertically stratified, self-gravitating, galactic gas disks, using local numerical MHD simulations. Our new models extend our previous two-dimensional studies, which showed that a magnetized spiral shock in a thin disk can undergo magneto-Jeans instability (MJI), resulting in regularly spaced interarm spur structures and massive gravitationally bound fragments. Similar spur (or “feather”) features have recently been seen in high-resolution observations of several galaxies. Here we consider two sets of numerical models: two-dimensional simulations that use a “thick-disk” gravitational kernel, and three-dimensional simulations with explicit vertical stratification. Both models adopt an isotermal equation of state with $c_s = 7$ km s$^{-1}$. When disks are sufficiently magnetized and self-gravitating, the result in both sorts of models is the growth of spiral-arm substructure similar to that in our previous razor-thin models. Reduced self-gravity due to nonzero disk thickness increases the spur spacing to ~10 times the Jeans length at the arm peak. Bound clouds that form from spur fragmentation have masses $(1-3) \times 10^7 M_\odot$ each, similar to the largest observed GMCs. The mass-to-flux ratios and specific angular momenta of the bound condensations are lower than large-scale galactic values, as is true for observed GMCs. We find that unmagnetized or weakly magnetized two-dimensional models are unstable to the “wiggle instability” previously identified by Wada & Koda. However, our fully three-dimensional models do not show this effect. Nonsteady motions and strong vertical shear prevent coherent vortical structures from forming, evidently suppressing the wiggle instability. We also find no clear traces of Parker instability in the nonlinear spiral arm substructures that emerge, although conceivably Parker modes may help seed the MJI at early stages since azimuthal wavelengths are similar.

Subject headings: galaxies: ISM — instabilities — ISM: kinematics and dynamics — ISM: magnetic fields — MHD — stars: formation

1. INTRODUCTION

The interstellar medium (ISM) contains structure at many scales, and the growth of much of this structure is believed to represent the first step in initiating galactic star formation. In particular, the molecular portion of the ISM is concentrated into dense clouds over a range of scales (Blitz 1993); each of these clouds also contains significant higher density substructure. The largest of these cloud complexes (up to $\sim 10^7 M_\odot$, including atomic envelopes) contain most of the mass in the overall distribution of giant molecular clouds (GMCs). In galaxies with prominent spiral structure, GMCs tend to be concentrated into the spiral arms. Observationally, the higher than average specific star formation rates (i.e., per unit gas mass) within spiral arms (Knapen et al. 1992, 1996) suggest an evolutionary sequence: compression of diffuse gas (and/or collection of smaller clouds) within the arms prompts GMC formation, and star formation is subsequently triggered in the densest portions within GMCs.

From a theoretical point of view, interaction of the ISM with spiral arms should naturally result in bound cloud formation when the mean density is high enough that self-gravitating instabilities are able to grow. The growth of self-gravitating structures in spiral arms has been investigated by many authors, using both linear-theory analyses (Balbus & Cowie 1985; Balbus 1988; Elmegreen 1994; Kim & Ostriker 2002), and more recently, nonlinear hydrodynamic and magnetohydrodynamic (MHD) simulations that yield bound clouds with properties similar to observed GMCs (Kim & Ostriker 2002, hereafter Paper I).

Interestingly, self-gravity acting on the gas within spiral arms can apparently also lead to growth of structures larger than GMCs. These structures take the shape of gaseous spurs that project from the main body of the arm into the interarm region. The first direct numerical simulations showing the development of these trailing spurs (Paper I) occurred contemporaneously with the release of the now-famous Hubble Heritage image of M51 (Scoville & Rector 2001; Scoville et al. 2001), in which strikingly similar structures are evident. Furthermore, a recent Spitzer Legacy image of M51 displays a quasi-regular distribution of thin, trailing dust filaments throughout the interarm regions (Kennicutt 2004). The dust filaments seen in these images could well be the remnants of gaseous spurs initiated inside spiral arms, which have maintained their integrity deep into the interarm regions. A recent archival study of Hubble Space Telescope (HST) galaxy images (Lavigne et al. 2006) has shown that arm substructures of the kind seen in M51 are in fact ubiquitous in the grand design spirals where the global pattern is clean enough to make identification of spiral spur substructures possible. When identified as a regular series of dust lanes extending out of a primary dust lane across the bright stellar arm and into the interarm region, such spur structures have historically been termed “feathers” (Lynds 1970).
In this work we extend the (thin-disk) two-dimensional numerical models of Paper I into three dimensions, in order to study the effects of vertical stratification on the development of condensation instabilities in spiral arms. In our previous work we argued that self-gravity, aided by magnetic fields, is the key to gaseous spur and dense cloud formation. However, it has recently been proposed that Kelvin-Helmholtz instabilities are also able to trigger spiral arm spur formation (Wada & Koda 2004). In addition, a theoretical proposal with a long history is that Parker instability (see Paper I) and the “wiggle instability” of Wada & Koda (2004) can develop, depending on parameters. In this section we briefly summarize the numerical methods and model parameters that we adopt; the reader is referred to Papers I and II for the more complete description.

2.1. Basic Equations and Numerical Methods

We study the nonlinear evolution of self-gravitating gas under the influence of a stellar spiral potential that is tightly wound (i.e., domain \( \ll R_0 \)) involving spiral arms, it is advantageous to construct a local Cartesian frame centered on a position \( R_0, \theta_0 = \Omega_0 t, z_0 = 0 \), that corotates with the stellar spiral pattern. The local Cartesian frame is inclined with respect to the \( \hat{R}, \hat{\phi} \) coordinate directions by an angle \( i \) in such a way that \( \hat{x} \) and \( \hat{y} \) correspond to the directions in the midplane perpendicular and parallel, respectively, to the local segment of the spiral arm, while \( \hat{z} \) denotes the direction perpendicular to the galactic plane (Roberts 1969; Balbus 1988). The simulation domain is a rectangular parallelepiped with size \( L_x \times L_y \times L_z \). The dimensions of the box are \( L_x = 2\pi R_0 \sin i/m \), equal to the arm-to-arm distance for an \( m \)-armed spiral, \( L_y = 2L_0 \), and \( L_z = 8H_0 \), where \( H_0 \) denotes the vertical scale height of the gas distribution when the spiral potential perturbation is absent.

In this local frame, the background velocity arising purely from galactic rotation is approximately given by

\[
v_0 = R_0(\Omega_0 - \Omega_\phi) \sin \theta \hat{x} + \left[R_0(\Omega_0 - \Omega_\phi) - q_0 \Omega_0 \hat{\phi}\right] \hat{y},
\]

where \( \Omega_\phi \) is the angular velocity of gas at \( R_0 \) in the inertial frame and \( q_0 \equiv -(d \ln \Omega_0/d \ln R)_0 \) is the local shear parameter of the background flow in the absence of the spiral arm (Paper I). For a flat rotation curve, \( q_0 = 1 \). We assume that the gas velocity \( v \) induced by the spiral potential is much smaller than \( R_0 \Omega_\phi \) and neglect terms arising from curvature effects in the coordinates (e.g., Goldreich & Lynden-Bell 1965b; Julian & Toomre 1966).

In this local, rotating frame, the MHD equations are written as

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla P + \frac{1}{4\pi \rho} (\nabla \times \mathbf{B}) \times \mathbf{B} + q_0 \Omega_0 \hat{\phi} \hat{y} - 2\Omega_0 \hat{\phi} \mathbf{v} - \nabla (\Phi_s + \Phi_{\text{ext}}),
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}),
\]

\[
\nabla^2 \Phi_s = 4\pi G \rho,
\]

\[
P = c_s^2 \rho
\]

(see Roberts 1969; Roberts & Yuan 1970; Shu et al. 1973; Balbus & Cowie 1985; Balbus 1988), where \( v_T \equiv v_0 + v \) is the total velocity in the rotating frame, \( c_s \) is the isothermal sound speed, \( \Phi_s \) is the self-gravitational potential of the gas, and \( \Phi_{\text{ext}} \) is

The plan of this paper is as follows: In \( \S \) 2 we lay out our numerical methods and the specifications of the models we shall study. In \( \S \) 3 we present results for two-dimensional “thick disk gravity” models, in which both magneto-Jeans instability (see Paper I) and the “wiggle instability” of Wada & Koda (2004) can develop, depending on parameters. In \( \S \) 4 we present the results of three-dimensional stratified models, identifying the conditions under which spurs and clouds form via various different self-gravitating mechanisms (but not through the “wiggle instability”). In \( \S \) 5 we conclude with a summary of our new results and a discussion of their implications within the larger context of current work on galactic structure and star formation.
the external stellar potential. Other symbols have their usual meanings. In all the simulations presented in this paper, for simplicity we adopt an isothermal equation of state in both space and time, as expressed by equation (6).

The imposed external stellar potential \( \Phi_{\text{ext}} \) varies both in the plane and perpendicular to the plane; to lowest order it is separable into two parts as

\[
\Phi_{\text{ext}} = \left( \frac{\pi G \Sigma_z}{c_{\sigma,z}} \right)^2 z^2 + \Phi_{\text{sp}} \cos \left( \frac{2\pi x}{L_x} \right). \tag{7}
\]

The first, quadratic term in \( z \) describes the variation in the gravitational potential near the midplane from a self-gravitating stellar distribution with surface density \( \Sigma_z \) and vertical velocity dispersion \( c_{\sigma,z} \).

Since the bulk of gas remains within 1 scale height of the stellar disk, this is quite a good approximation for the potential in studying the dynamical evolution of the gas. The second, sinusoidal term with amplitude \( \Phi_{\text{sp}} \) in equation (7) represents a local analog of the logarithmic spiral potential used in Roberts (1969) and Shu et al. (1973). Since \( \Phi_{\text{sp}} < 0 \) and \( |x| \leq L_2/2 \), the spiral potential attains its minimum at the center \( (x = 0) \) of the box.

We integrate the time-dependent, ideal MHD equations (2)–(7) using a modified version of the ZEUS code (Stone & Norman 1992a, 1992b). It uses a time-explicit, operator-split, finite-difference scheme for solving the MHD equations on a staggered mesh, and employs the “constrained transport” and “method of characteristics” algorithms to maintain the divergence free condition of magnetic fields as well as to ensure accurate propagation of Alfvén waves. For less diffusive transport of hydrodynamic variables, we apply a velocity decomposition method in updating \( \nu_i \) (Kim & Ostriker 2001). We also employ the Alfvén limiter algorithm of Miller & Stone (2000), setting the limiting speed of the displacement current to \( c_{\text{lim}} = 8c_z \); this value of \( c_{\text{lim}} \) allows a good dynamical range in the low-density, high-\( z \) regions. We solve the Poisson equation by combining the fast Fourier transform method in sheared horizontal coordinates (Gammie 2001) with the Green function method for vertical integration (Miyama et al. 1987). We adopt the shearing box boundary conditions of Hawley et al. (1995) in which the \( x \)-boundaries are shearing-periodic and the \( y \)-boundaries are perfectly periodic, while implementing the outflow boundary conditions of Stone & Norman (1992a) at the \( z \)-boundaries.

We have parallelized the numerical code we use with both OpenMP and MPI, for use on both shared- and distributed-memory platforms. The parallelization is achieved by domain decomposition along the \( z \)-direction, which is convenient for our hybrid technique of solving the Poisson equation. Our high-resolution models in three dimensions have \( 128 \times 256 \times 128 \) zones in \((x, y, z)\).

2.2. Model Parameters

The ISM in galaxies is highly inhomogeneous, turbulent, and multiphase (e.g., Field et al. 1969; McKee & Ostriker 1977; Heiles 2001; Heiles & Troland 2005; Wolfire et al. 2003), and a fully realistic treatment of ISM evolution entails consideration of heating, cooling, and other physical processes that may significantly affect the density and temperature structures of the gas (e.g., Vázquez-Semadeni et al. 2000; Kritsuk & Norman 2004; Piontek & Ostriker 2004, 2005; Audit & Hennebelle 2005). In order to focus on the role of self-gravity in forming intermediate-scale spiral-arm substructure, we instead consider for all models presented in this paper homogeneous initial gas distributions, adopting a simple isothermal equation of state with an “effective” speed of sound \( c_s \). The effects of ISM heating/cooling and the associated multiphase cloudy gas distribution on the interaction with spiral arms will be considered in a subsequent paper.

Our initial disks, in the absence of the spiral perturbation \( \Phi_{\text{sp}} = 0 \) in eq. [7]), are vertically stratified with density \( \rho_0(z) \), have a uniform surface density \( \Sigma_0 = \int \rho_0(z) dz \), and are threaded by magnetic fields \( B_0 = B_0(2\Sigma_0) \) pointing parallel to the spiral arm. The initial equilibrium satisfies force balance between the total (thermal plus magnetic) pressure gradient and the total (self plus external) gravity along the \( z \)-direction. For the strength of the external vertical gravity, we define \( \beta_0 \equiv (\sigma_{\text{sp}}/\Sigma_0)^2/(c_{\sigma,z}^2) \) and take \( r_0 = 1 \). This choice of \( \beta_0 \) corresponds to average galactic-disk conditions of \( \sigma_{\text{sp}} \sim 20 \, \text{km s}^{-1}, \Sigma_0 \approx 35 \, M_\odot \, \text{pc}^{-2} \), and total gas surface density \( \Sigma_0 \approx (11-16) \, M_\odot \, \text{pc}^{-2} \) (e.g., Kuijken & Gilmore 1989; Holmberg & Flynn 2000). Physically, \( \beta_0 \) measures the ratio of gaseous self-gravity to stellar gravity at 1 scale height \( H_0 \) of the gas; \( H_0 \approx \Sigma_0/(2\rho_0(0)) \approx 200 \, \text{pc} \) for the adopted sets of parameters in this paper (see below and Paper II for a discussion of the vertical equilibrium specification).

The three key dimensionless parameters that characterize our simulation models are

\[
Q_0 \equiv \frac{\kappa_0 c_s}{\pi G \Sigma_0} = 1.4 \left( \frac{c_s}{7.0 \, \text{km s}^{-1}} \right) \left( \frac{\Sigma_0}{13 \, M_\odot \, \text{pc}^{-2}} \right)^{-1}, \tag{8}
\]

\[
\beta_0 \equiv \frac{c_s^2}{v_a^2} = \frac{4 \pi \rho_0(z) c_s^2}{B_0^2(z)}, \tag{9}
\]

\[
F \equiv \frac{m_z}{\sin i} \left( \frac{\Phi_{\text{sp}}}{R_0^2 \Omega_0^2} \right), \tag{10}
\]

where \( v_a = B/(4\pi \rho_0)^{1/2} \) is the Alfvén speed. The Toomre stability parameter \( Q_0 \) describes the gas surface density relative to various critical values that mark thresholds for axisymmetric gravitational instability. For a razor-thin disk, the instability threshold is at \( Q = 1 \) (Toomre 1964); for a purely self-gravitating \( (\beta_0 = \infty) \), vertically stratified disk, the threshold is at \( Q \approx 0.676 \) (Goldreich & Lynden-Bell 1965a; Gammie 2001); and for a disk subject to both self-gravity and external gravity with \( \beta_0 = 1 \), the threshold is at \( Q \approx 0.75 \) (Paper II; Kim et al. 2003, hereafter Paper III). The dimensionless parameter \( F \) in equation (10) measures the ratio of the maximum perturbation force from the external spiral potential to the mean radial gravitational force (Roberts 1969). Initially, we take \( \beta_0 \) to be constant everywhere, so that \( \beta_0(z) \approx (\rho_0(z)/c_s)^2 \).

To represent solar neighborhood conditions in dimensional quantities, we use galactocentric radius \( R_0 = 10 \, \text{kpc} \). With \( \Omega_0 = 26 \, \text{km s}^{-1} \, \text{kpc}^{-1} \), the local epicyclic frequency is \( \kappa_0 = (R^{-3}d[R^4 d\Omega^2]/dR)|_{R_0} = (4 - 2g_0)^{1/2} \Omega_0 \approx 36 \, \text{km s}^{-1} \, \text{kpc}^{-1} \) for a nearly flat rotation curve with \( q_0 \approx 1 \) (Binney & Tremaine 1987). The corresponding galactic orbital period is \( t_{\text{orb}} = 2\pi/\Omega_0 = 2.4 \times 10^8 \, \text{yr} \) (\( \Omega_0/26 \, \text{km s}^{-1} \, \text{kpc}^{-1} \)), which we adopt as a fiducial time unit in our presentation. Our results can be rescaled to use other galactic parameters, provided that the dimensionless ratios \( \Omega_{0p}/\Omega_0, L_d/R_0 \), and \( \Omega_{0R}/c_s \) (as well as \( q_0, Q_0, \beta_0 \), and \( F \)) remain the same.

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1 The formula for the gravitational potential of a self-gravitating stellar disk treated as an isothermal fluid is \( \Phi(z) = 2\pi z^2 \ln \cosh(z/H) \), where \( H \equiv \sigma_{\text{sp}}^2/(\pi G \Sigma_0) \) (see Gilmore et al. 1990). The central stellar density is \( \rho_0 = \Sigma_0/(2H) \). For \( z/H < 1 \), \( \Phi(z) \approx (\sigma_{\text{sp}}/H)^2 z^2 = 2\pi G \rho_0 z^2 = (\pi G \Sigma_0/\sigma_{\text{sp}}^2)^2 z^2 \).
For spiral arm parameters, we adopt for all our models pattern speed \( \Omega_p = \Omega/2 \), pitch angle \( \sin i = 0.1 \), and azimuthal wavenumber \( m = 2 \); the corresponding arm-to-arm distance is \( L_x = 2\pi R_0 \sin i/\mu \). For our fiducial parameters, the quasi-radial size of the simulation domain is therefore \( L_x = 3.14 \) kpc.

To simulate various galactic conditions, we select the following three sets of the parameters: \((Q_0, \beta_0, F) = (1.8, \infty, 5\%)\), \((1.5, 10, 5\%)\), and \((1.2, 1, 10\%)\). For our fiducial \( R_0 \) and \( \Omega_0 \), and taking \( c_s = 7 \) km s\(^{-1}\), the corresponding gas surface density, total gas mass \( M_{\text{tot}} \) contained in the simulation box, and scale height \( H_0 \) are \( 11 M_\odot \) pc\(^{-2}\), \( 2.3 \times 10^{5} M_\odot \), and 200 pc for the \( Q_0 = 1.8 \) model with \( \beta_0 = \infty \); \( 13 M_\odot \) pc\(^{-2}\), \( 2.8 \times 10^{5} M_\odot \), and 170 pc for the \( Q_0 = 1.5 \) model with \( \beta_0 = 10 \); and \( 16 M_\odot \) pc\(^{-2}\), \( 3.4 \times 10^{5} M_\odot \), and 180 pc for the \( Q_0 = 1.2 \) model with \( \beta_0 = 1 \). For the \( \beta_0 = 1 \) and 10 models, the respective midplane magnetic field strengths are 4.4 and 1.3 \( \mu \)G, respectively. The model parameter sets are chosen to ensure that all the models are gravitationally stable to quasi-axisymmetric (wave fronts parallel to the spiral arm) perturbations even when the effect of the spiral potential is included. Our objective in this paper is to explore how nonaxisymmetric disturbances in these systems evolve, subject to interactions with spiral arms. We also briefly consider, in \$4.2.4\), development of a model that is unstable to quasi-axisymmetric disturbances.

### 3. Two-dimensional models with thick-disk gravity

Paper I studied the formation of spiral-arm and interarm substructure in two-dimensional disks that were treated as being infinitesimally thin. One of the drawbacks of the thin-disk approximation of Paper I is that it overestimates self-gravity at the disk midplane for modes whose wavelengths approach the disk scale height (e.g., Toomre 1964; Elmegreen 1987). For a given strength of the spiral arm potential, therefore, razor-thin disks tend to have larger density enhancement at the spiral shock than vertically resolved disks, increasing the susceptibility to gravitational instability. Consequently, the critical \( F \) values for stable, one-dimensional spiral shock configurations are lower in razor-thin disks than in more realistic vertically extended disks; to ensure gravitational stability to quasi-axisymmetric perturbations, razor-thin disk models considered in Paper I were limited to \( F \leq 3\% \) for a range of \( Q_0 \) and \( \beta_0 \) values.

We revisit two-dimensional disk models here, now taking into account the geometrical dilution of self-gravity due to finite disk thickness. Instead of the standard kernel appropriate for a thin disk, we use a “thick-disk” gravitational kernel such that the potential and density Fourier modes with wavenumber \( k \) at the midplane are related by

\[
\Phi_s(k) = -\frac{4\pi G \rho(k)}{|k(1 + |k|H_0)|}, \quad \text{for } k \neq 0, \quad (11)
\]

and \( \Phi_s(k) = 0 \) for \( k = 0 \), where the disk scale height \( H_0 \) is held fixed in both space and time. Equation (11) is in fact exact for an exponential density distribution \( \varpi \propto e^{-|z|/H_0} \) (e.g., Elmegreen 1987), and generally yields a result within 15% of the exact solution of the three-dimensional Poisson equation for self-consistent vertical equilibria (Paper II). For our two-dimensional models, the dependences of all fluid variables on the vertical coordinate are neglected. The results of these two-dimensional disk models allow us to quantify the impact of thick-disk gravity on the formation of arm/interarm substructure without the potential influence of dynamical instabilities that critically involve the vertical dimension.

We have run three sets of models with differing numerical resolutions. The model parameters and simulation results are given in Table 1. Column (1) labels each run. Columns (2) and (3) list the Toomre \( Q_0 \) parameter (eq. [8]) and the field strength in terms of the plasma parameter \( \beta_0 \) (eq. [9]) of an initial disk when a spiral arm perturbation is absent, while column (4) gives the amplitude of the spiral potential in terms of \( F \) (eq. [10]). Column (5) indicates the numerical resolution \( N_x \times N_y \) of the model. Since our simulation box has \( L_x = 2L_y \), \( N_x = 2N_y \) makes each cell square in the \( x-y \) plane. The peak surface density \( \Sigma_{sp} \) of the resulting spiral shock configuration is given in column (6), while column (7) gives the corresponding local value \( Q_{sp} = Q_0(\Sigma_{sp}/\Sigma_0)^{-1/2} \) at the spiral density peak, this scaling follows from the constraint of potential vorticity conservation (Hunter 1964; Balbus 1988; Gammie 1996; Paper I). The width \( W \) of the gaseous spiral arm defined at \( \Sigma = (\max \Sigma + \min \Sigma)/2 \) is listed in column (8). Finally, the mean separation \( \lambda_{sp} \) of the structures that develop in each model is given in column (9) in terms of the arm-to-arm distance \( L_x \) and in column (10) in terms of the normalized wavenumber \( K_y \equiv \lambda_{sp}/L_x \), where the local, thin-disk Jeans wavelength at the arm density peak is defined by

\[
\lambda_{sp} = \frac{c_s^2}{G\Sigma_{sp}} = 160 \text{ pc} \left( \frac{c_s}{7 \text{ km s}^{-1}} \right)^2 \left( \frac{\Sigma_{sp}}{70 M_\odot \text{ pc}^{-2}} \right)^{-1}. \quad (12)
\]
Note that structures that form from magneto-Jeans instability (MJI) in a featureless, uniform, low-shear, razor-thin disk favor \( K_y = 0.50-0.75 \) (Kim & Ostriker 2001), while MJI-driven fragmentation occurring inside spiral arms in the razor-thin limit was found to have \( K_y \approx 0.45-0.54 \) (Paper I).

To initiate our models, we begin with a disk having a uniform shear profile expressed by equation (1), and uniform surface density and magnetic fields corresponding to the chosen values of \( Q_0 \) and \( \beta_0 \). Using a one-dimensional grid in \( \tilde{x} \), we turn on the external spiral potential and slowly increase its amplitude up to a desired level, \( F \). This yields a one-dimensional equilibrium spiral shock profile, which we then use to initialize our two-dimensional simulations. On top of the background profile, we apply density perturbations created by a Gaussian random field with flat power for \( 1 \leq kL_y/2\pi \leq 128 \) and zero power for \( kL_y/2\pi > 128 \). The standard deviation of the density perturbations is fixed to be 3% in real space.\(^2\) We then follow the two-dimensional flow as perturbations evolve and grow through interaction with the background spiral shock, eventually forming self-gravitating substructures. Note that an alternative initialization procedure is simply to start with a uniformly shearing disk and slowly turn on the spiral forcing term; this is the procedure we follow for our three-dimensional models (see § 4), and we have also tested two-dimensional models with this method. We find, for our two-dimensional models, essentially the same end results regardless of the initialization procedure.

### 3.1. Magneto-Jeans Instability inside Spiral Arms

In this subsection we describe the evolution of model ME2d1 (and its lower resolution counterparts ME2d2-3), which have equipartition magnetic fields. In these simulations, gaseous spurs and gravitationally bound clouds form as a direct consequence of MJI operating within spiral arms. The phrase “bound clouds” in this paper refers to gaseous clumps that would collapse in a featureless, uniform, low-shear, razor-thin disk if not for the spiral arm (Paper I).

Figure 1 plots time evolution of the maximum surface density of model ME2d1 (together with other two-dimensional models with different field strength), while Figure 2 shows snapshots of model ME2d1 at \( t/t_{orb} \approx 2.5 \) and 3.2. The perturbations introduced into the flow shear around and relax initially (\( t/t_{orb} \leq 0.4 \)), and then various modes begin to grow due to self-gravity (\( t/t_{orb} \approx 2 \)). In the absence of magnetic fields, growth of perturbations under the reversed-shear conditions within the arm can be inhibited by Coriolis forces; these usually reduce the mass-collapsing effect of self-gravity in a rotating system. However, magnetic tension forces from embedded field lines counteract Coriolis forces, so that potential vorticity is no longer conserved and perturbations can grow rapidly (Lynden-Bell 1966; Elmegreen 1987; Kim & Ostriker 2001). The MJI process works best when shear is weak. The spatially varying sense of shear inside a spiral arm (reversed, then “normal”) keeps the overall shear rate close to zero, therefore providing favorable conditions for the development of MJI within the arm (Paper I).

Since the perturbations in model ME2d1 initially have low amplitude, they remain in the linear regime through \( t/t_{orb} \approx 1.5 \). It is not until \( t/t_{orb} \approx 2 \), when perturbations have crossed the spiral arm twice (since \( t_{cross} = t_{orb}/[\Omega(1 - \Omega_p/\Omega)] \)), that the most dominant MJI mode emerges and shapes the condensed gas flowing into the interarm region into spur structures. Figure 2a shows the surface density (in logarithmic scale) and velocity vectors at \( t/t_{orb} = 2.5 \) viewed from a frame corotating with the spiral arm. We identify four spurs that protrude, at fairly regular intervals, perpendicularly from the main spiral arm, and then sweeping back into a trailing configuration within the interarm region. The normalized wavenumber corresponding to the spur separation \( \lambda_q \) in model ME2d1 is \( K_y = \lambda_q/\lambda_s = 0.11 \). These spurs move in the \( y \)-direction with a speed \( v_y \approx 0.50 R_0 \Omega_0 \) relative to the spiral arm, implying that in the inertial frame they follow very closely the background galaxy rotation at the arm center. Since the lower resolution model ME2d2 with 128 \( \times \) 256 zones also forms four spurs, while model ME2d3 with 128 \( \times \) 128 zones results in three spurs (see Table 1), we conclude that the number of spurs that form in our two-dimensional thick-disk models is independent of numerical resolution as long as the \( y \)-dimension of the simulation domain has 256 zones or more.

Figure 2b draws selected gas streamlines (in red) seen in the stationary-spru frame as well as the surface density of model ME2d1 at \( t/t_{orb} = 2.5 \). In this figure, coordinates are transformed such that the left boundary corresponds to the initial location of the spiral shock front. Because the background flow is shearing and expanding/contracting, the \( x \)-wavenumber of Lagrangian perturbations that move away from the shock front varies as \( k_x = -T k_y \) with

\[
T \equiv \frac{1}{R} \left[ \frac{k_0^2 \Sigma_{sp}}{2 \Omega_0^2 \Sigma_0} - 2 \int_0^R \int_0^\infty \frac{k_y(0)}{k_x} d\tau \right],
\]

where \( R = \Sigma_{sp}/\Sigma_0 \) is the local surface density expansion factor, \( \tau \equiv \int_0^\infty v_x / dv_{x} \) is a dimensionless elapsed time that is measured
from the shock location (or density peak), \( x_{sp}, k_y = 2\pi/\lambda_y \) with \( \lambda_y \) corresponding to the spur spacing, and \( k_x(0) \) the initial \( x \)-wavenumber at \( \tau = 0 \) (Paper I; see also Balbus 1988).

Figure 2h also plots (in red) the theoretical wave fronts of spurs given by \( dy/dx = -k_y/k_x = T \) with an initial condition \( K_x(0) \equiv k_x(0)/k_{x,sp} = 0.45 \), where \( k_{x,sp} = 2\pi/\lambda_{x,sp} \). The fact that the shape of spurs matches quite well with the theoretical prediction suggests that the former simply reflects the shearing and expanding properties of the background flow (Paper I). Note that the gas streamlines rather quickly converge to the spur wave front as they move downstream from the spiral shock, indicating that spurs grow stronger by gathering material mainly along the \( y \)-direction (parallel to the spiral arm).

When the density within the spurs has grown sufficiently, self-gravity causes them to fragment into gravitationally bound condensations. Figure 2c shows the density and magnetic field lines at \( t/t_{orb} = 3.2 \) of model ME2d1. The magnetic fields roughly parallel the arm overall, although they pinch inward within the spurs and are strongly twisted locally in the vicinity of the bound clumps. Bending of field lines is most severe in interarm regions, where the gas moves faster. Model ME2d1 forms four clumps with mass \( M \sim 3.3 \times 10^7 M_\odot \) each; this is about an order of magnitude larger than the clumps that formed in razor-thin disk models of Paper I. Roughly 40% of the total mass, therefore, is collected into bound clumps. These condensations are magnetically supercritical, with the mean mass-to-flux ratio \( M/\Phi_B \sim 2.0G^{-1/2} \), where \( \Phi_B \) is the magnetic flux that passes through each clump and \( G \) is the gravitational constant.\(^3\) The numerical box of model ME2d1 initially has a mass-to-flux ratio of \( M/\Phi_B = 3.6G^{-1/2} \), about twice as supercritical as the clumps that form. This result is consistent with Vázquez-Semadeni et al. (2005), who found that self-gravitating substructures formed in turbulent MHD simulations are in general less supercritical than the parent system, presumably because fragmentation occurring along the flux tubes reduces the cloud mass while preserving magnetic flux (see also Li et al. 2004). Various physical properties of bound clouds that form in two-dimensional thick-disk models are quite similar to those in full, three-dimensional models; we defer detailed discussion to § 4.2.3, where the vertical stratification of disks is explicitly taken into account.

The reduced self-gravity in disks with finite thickness results in a smaller number of spurs compared to a razor-thin disk with properties otherwise the same. In order to check whether our simulation results are consistent with the linear theory prediction, we perform a linear stability analysis in a Lagrangian frame comoving with the background flow through a spiral arm. Assuming that the perturbed quantities are well described by plane waves with sinusoidal variations on scales \( \ll R \), one can show that equations (2)–(6) and (11) lead to the following set of linearized equations:

\[
\frac{d\delta \sigma}{d\tau} = K_x(T \delta u - \delta v),
\]

\[
\frac{1}{R} \frac{d(R \delta u)}{d\tau} = 2\delta v - \alpha K_x T \times \left[ 1 - \frac{1}{RK(1 + KH_0 k_{x,sp})} \right] \delta \sigma - \frac{\alpha \Sigma_{sp}}{\beta_0 \Sigma_0} K^2 \delta m,
\]

\(^3\) The critical mass-to-flux limit is \( 0.16G^{-1/2} \).
as a function of \( ME2d1 \) is indicated by the box near left. The most unstable mode evident in the simulation results from model \( /C14/C27 \) where \( 1.8 \) from outside to inside, show final amplification magnitude the corresponding growth times \( \text{(see Paper I).} \)

We choose \( H_0 = 180 \) pc for thick-disk gravity and adopt the equilibrium density and velocity profiles \( \text{(i)n} \) of model \( ME2d1 \) as a background state. By taking \( \delta \sigma = 1 \) and \( \delta u = \delta v = \delta m = 0 \) as an initial condition and varying \( K_y \) and \( k_0(0)/u_0 \), we integrate the perturbed equations in time. The resulting amplification magnitude \( \Gamma \equiv \log \left[ \delta \Sigma_{\text{max}}/\Sigma \right] \) and the growth time \( t_{\text{grow}} \) at which maximum amplification occurs are plotted in Figure 3 as solid and dotted contours, respectively. Figure 3 also marks as a rectangular box the parameters \( (K_y[0] = 0.45 \) and \( K_y = 0.11) \) that give the best fit to the shape and spacing of spurs formed in the simulation of model \( ME2d1 \) \( \text{[the size of the rectangle represents the uncertainty determining} \ K_y(0) \text{or} \ K_y \).}

Figure 3 shows that \( K_y = 0.11 \) is in good agreement with the expectations from linear theory for the wavelength parallel to the spiral arm of the dominant mode. However, the spurs that dominate the simulation in model \( ME2d1 \) have an \( x \)-wavenumber at the spiral shock front that is a factor 2.5 times larger than that which would yield the largest amplification, based on a linear-theory analysis. This is because the initial perturbations in model \( ME2d1 \) have such low amplitudes that two traversals of an arm are required before the perturbations grow into the nonlinear regime. Over this time, \( K_y \) of any initial perturbation varies

\[
\frac{d\delta v}{d\tau} = -\frac{\Sigma_{\text{sp}} \delta u}{\Sigma_0} \frac{1}{R} + \alpha K_y \left[ 1 - \frac{1}{R K(1 + KH_0 k_{1,\text{sp}})} \right] \delta \sigma, \\
\frac{d\delta m}{d\tau} = \frac{\delta u}{R},
\]

where \( \delta \sigma \equiv \delta \Sigma/\Sigma_0, \delta u \equiv i \delta v, k_{1,\text{sp}}/\Omega_0, \delta v \equiv i \delta v, k_{1,\text{sp}}/\Omega_0, \delta m \) is the normalized perturbed vector potential, \( K_y \equiv k_y/k_{1,\text{sp}}, K \equiv k_y/(1 + T^2)^{1/2} \) is the total wavenumber, and \( \alpha \equiv (c_s k_{1,\text{sp}}/\Omega_0)^2 \) (see Paper I).

To quantify how finite disk thickness is expected to affect the development of MJI inside spiral arms, we vary \( H_0 \) and search for \( K_y,0(0) \) and \( K_y,\text{max} \) that give based on linear theory—the maximum amplification magnitude \( \Gamma_{\text{max}} \) for a given value of \( H_0 \). Figure 4 plots the resulting \( K_y,0(0) \) and \( K_y,\text{max} \) as solid lines as well as \( \Gamma_{\text{max}} \) as a dotted line. The equilibrium surface-density profile of model \( ME2d1 \) \( \text{[but with varying} \ H_0 \text{is again used]. The linear theory suggests that the MJI inside spiral arms in a disk with} \ H_0 = 180 \text{pc requires} \ K_y,\text{max} \text{to be} 2.6 \text{times smaller than in a disk with} \ H_0 = 0. \text{The corresponding amplification magnitude} \ \Gamma_{\text{max}} \text{is} 2.3 \text{times smaller for thick disks compared to thin disks. For comparison, we note that in the razor-thin models of Paper I, spurs identified in simulations with} \beta_0 = 1 \text{typically have} \ K_y \sim 0.45-0.54 \text{(or} \lambda_y \sim 1.8 \lambda_{J,\text{sp}}-2.2 \lambda_{J,\text{sp}}, \text{with larger} \ K_y \text{(or smaller} \lambda_y \text{) corresponding to smaller} \ Q_0 \text{miles. Since the amplification factor is enormous when} \ H_0 = 0, \text{the modes that come to dominate in razor-thin disks tend to be selected by compromise between maximum amplification and earliest growth (Paper I), and thus have} \ K_y \text{larger than that corresponding to} \ K_y,\text{max}.}

In the thick-disk models with their lower amplification, on the other hand, early growth gives little advantage over other modes. Model \( ME2d1 \) therefore chooses the mode corresponding to the largest total amplification, whose wavelength turns out to be \( \lambda_y \sim 9 \lambda_{J,\text{sp}} \), which is \( \sim 4-5 \text{times larger (in terms of} \lambda_y/\lambda_{J,\text{sp}} \text{than that found for razor-thin disk models in Paper I. Models with thick-disk gravity have peak surface densities twice as large as their razor-thin counterparts with the same} F \text{and} Q_0, \text{which reduces the Jeans length by about a factor of} 2. \text{Thus, for cloud}

Fig. 3.—Prediction of spiral arm MJI development from linear theory, with thick-disk gravity, for parameters of model ME2d1. The gaseous scale height \( H_0 = 180 \) pc is taken as fixed. Solid contours, spaced at 0.4, 0.6, 0.8, . . . . 1.6, 1.8 from outside to inside, show final amplification magnitude \( \text{[and} \ K_y(0) = 0.45.}

Fig. 4.—Dependence on the gaseous scale height \( H_0 \) of the wavenumbers \( K_{y,\text{max}}(0) \) and \( K_{y,\text{max}} \) \( \text{(solid} \text{lines, left} \ y \text{-axis} \text{) and} \ \Gamma_{\text{max}} \text{[dotted} \text{line, right} \ y \text{-axis} \text{) of the dominant MJI mode in spiral shocks predicted from linear theory. The parameters for model ME2d1 are used.}

\[ \text{[Fig.} \text{3]}

\[ \text{[Fig.} \text{4]}

mass scaling as $k^2 / C_6 \rho / (k y / k J)^2 / C_6 / C_0^2$, one can expect that bound clouds resulting from spur fragmentation in thick-disk gravity models should be $\sim 8-10$ times more massive than in razor-thin disks with the same $F$ and $Q_0$, or $\sim (3-4) \times 10^7 M_\odot$ in dimensional units. This is entirely consistent with the results of our numerical simulations.

3.2. The Wiggle Instability

We now turn to model MS2d1 (and its lower resolution counterparts MS2d2-3), with $Q_0 = 1.5$, $F = 5\%$, and $\beta_0 = 10$ (i.e., $v_A = c_s / \sqrt{10}$), and unmagnetized model H2d1 (with counterparts H2d2-3) having $Q_0 = 1.8$, $F = 5\%$, and $\beta_0 = \infty$. As Figure 1 shows, in models MS2d1 and H2d1 perturbations immediately begin to grow almost exponentially over time, while strong growth in model ME2d1 does not begin until after two successive passages through spiral arms. The growth rates of the maximum surface density in models MS2d1 and H2d1 are almost identical, $\sim 0.70 / \gamma$, suggesting that moderate magnetic fields are not important at least in the linear stages of growth in these simulations.

The two models MS2d1 and H2d1 have similar evolution, but this is quite distinct from the behavior of equipartition-magnetization model ME2d1. Unlike model ME2d1, in which the spiral shock remains relatively straight during the initial phase of MJIG growth, models MS2d1 and H2d1 show small-scale instability that wiggles the spiral shock front and forms structures with high vorticity. The left panel of Figure 5 plots the surface density map in logarithmic gray-scale of model MS2d1 at $t / t_{\text{orb}} = 1.6$. The dotted line marks the location of the spiral shock front, while the dashed line represents the position where shear in the background flow vanishes ($d \rho / dx = 0$). Evidently, the vortical clumps that emerge are closely connected to the shock discontinuity. Column (9) in Table 1 shows that the hydrodynamic models H2d1-3 form more clumps than the weakly magnetized models MS2d1-3, and that the number of vortical clumps produced depends rather sensitively on numerical resolution.

At $t / t_{\text{orb}} \approx 1.7$, the growth of vortical structures in model MS2d1 saturates due to magnetic tension from bent field lines as well as nonlinear effects. (The flow properties inherent in the background spiral shock may also prevent the clumps from growing further.) At this time, the clumps are not sufficiently self-gravitating to experience gravitational collapse due to their own weight. The clumps wander slightly inside the postshock regions and collide with each other to form bigger clumps. As Figure 5 (right) shows, mergers of 10 low-density clumps at $t / t_{\text{orb}} = 1.6$ result in
five high-density, gravitationally bound clumps at $t/t_{\text{orb}} = 2.7$. These bound clumps have an average mass $\sim 1 \times 10^7 M_{\odot}$, a factor of 3 smaller than that originating from MJI in model ME2d1, and are supercritical with an average mass-to-flux ratio $M/\Phi_0 \sim 3G^{-1/2}$. The interarm spur structures prominent in model ME2d1 (see Fig. 2) are not clearly visible in models MS2d1 and H2d1.

What causes the spiral shocks to wiggle and form vortical structures in models MS2d1 and H2d1? Figure 6 shows the temporal evolution of gas surface density and magnetic field lines in the rectangular section shown in the left frame of Figure 5. Note that the vortex generation and evolution of the magnetic field topology in model MS2d1 are similar to those that result from Kelvin-Helmholtz instabilities in unmagnetized (e.g., Corcos & Sherman 1984) or magnetized (e.g., Malagoli et al. 1996; Frank et al. 1996) shear layers. This suggests that the vortical clumps in model MS2d1 may arise from Kelvin-Helmholtz instability in the sheared flow—i.e., azimuthal streaming—associated with spiral shocks. Wada & Koda (2004) performed non-self-gravitating global simulations of razor-thin, unmagnetized disks and found that when spiral perturbations are strong and have large pitch angles, the shock front that develops wiggles and forms discrete clumps bearing remarkable resemblance to those found in our models. Wada & Koda termed this process a “wiggle instability.” Based on the low Richardson numbers in the postshock regions, they argued that the formation of these clumps could be due to Kelvin-Helmholtz instability. As Wada & Koda noted, however, the Richardson number criterion is only a necessary condition for stability (Chandrasekhar 1961), and should not be regarded as an instability criterion. The critical value of $\text{Ri}$ may, in addition, be affected by rotation. Thus, while it is likely that the wiggle instability is indeed a manifestation of Kelvin-Helmholtz instability in the shear flows produced by spiral shocks, it is not yet certain.

Since models considered in Wada & Koda (2004) are non-self-gravitating and unmagnetized, the wiggle instability, whether it is related to Kelvin-Helmholtz instability or not, relies neither on magnetic fields nor self-gravity. Paper I found that gas in razor-thin disks with $F \leq 3\%$ is subject to either the MJI or the swing amplifier, but is stable to the wiggle instability; i.e., non-self-gravitating models at these values of $F$ are expected to be stable. We have run a number of two-dimensional simulations of non-self-gravitating, unmagnetized disks with varying $F$ (not listed in Table 1), and found that the wiggle instability becomes manifest when $F \geq 4\%$. This suggests that spiral shocks must be fairly strong in order to trigger the wiggle instability.\footnote{Wada & Koda (2004) in fact set up extremely strong spiral shocks whose amplitudes amount to $F \approx 2\% \sim 110\%$ for the parameters $\epsilon = 0.1$ and $i = 10^7\%$ in their model A; the arm/interarm contrasts reached 2 orders of magnitude.}

Since non-self-gravitating gas behind a spiral shock is locally Kelvin-Helmholtz stable due to the expanding radial velocity inside the spiral arm (Dwarkadas & Balbus 1996), the wiggle instability is most likely a consequence of interaction between preshock gas and compressed postshock flows that stream at large relative velocity parallel to the shock front. It is also possible that spiral shocks are so strong that the postshock velocity perpendicular to the arm is vanishingly small, limiting the stabilization to Kelvin-Helmholtz modes. In any event, we do not believe the wiggle instability is likely to be important in spiral arms of real galaxies, because as we show in the next section it does not occur in three-dimensional models, in which the fluid variables are not constant with height.

4. THREE-DIMENSIONAL MODELS

To investigate the effects of disk stratification and other vertical variations on the formation of spiral-arm substructures, we have performed three sets of three-dimensional numerical simulations. The chosen parameters for the three-dimensional models listed in Table 2 are identical to those of the two-dimensional models presented in the previous section. We first set up an axisymmetric disk with equilibrium density distribution $\rho_0(z)$ consistent with a chosen set of $Q_0$, $\beta_0$, and $s_0$. We then apply initial perturbations to the background density $\rho_0(z)$, using a spatially uncorrelated, Gaussian random field that has flat power for $1 \leq kL_c/2\pi \leq 64$ and zero power for $64 < kL_c/2\pi$. For the amplitude of the perturbations, we fix the standard deviation of perturbed density in real space to be 3% of the initial midplane density. Next, we slowly turn on a spiral potential perturbation (the second term in eq. [7]) such that it acquires the full strength $F$ at $t/t_{\text{orb}} = 1.5$.

4.1. Vertical Structure of Spiral Shocks

Figure 7 shows evolution of the maximum density in high-resolution three-dimensional models ME3d1, MS3d1, and H3d1. As the strength of the spiral potential increases, a density wave grows rapidly in the gas, until the initially sinusoidal profile steepens into a shock. During this phase, the growth of small-scale...
The imposed spiral potential perturbation. At $t/t_{orb} \approx 1.6$ to 1.8, slightly after the spiral potential attains its full strength, the spiral shock reaches its maximum strength, leaving density and velocity fields that fluctuate around the respective average configuration. At this stage, the spiral shock still remains almost axisymmetric.

Paper I and §3 of the present paper showed that when vertical degrees of freedom are ignored, quasi-axisymmetric, steady state profiles of gaseous spiral shocks can be readily obtained through time-dependent calculations in an external spiral potential (see also Woodward 1975). This implies that one-dimensional spiral shock profiles in the galactic midplane represent stable shock equilibria (provided that gaseous self-gravity and external spiral forcing are not too strong: see Paper I). When we allow for vertical degrees of freedom in the fluid variables, however, we find that the profiles that develop are not in general quasi-steady. In the initial stages of all the three-dimensional models studied in this work, quasi-axisymmetric, two-dimensional spiral shocks develop in the $x-z$ plane (hereafter XZ spiral shocks) that are in general nonstationary, swaying loosely back and forth in the direction perpendicular to the arm.

Based on our simulation results, the “flapping” motions of XZ spiral shocks are strongest at high altitudes, and stronger in model ME3d1 than in models MS3d1 and H3d1. We measure the maximum shock front excursion from its mean location as about $0.06L_x$, $0.02L_z$, and $0.03L_z$ at $z/H_0 = 1$ in models ME3d1, MS3d1, and H3d1, respectively. The nonsteady nature of XZ spiral shocks that we find is in fact commonly seen in other numerical models (e.g., Martos & Cox 1998; Gomez & Cox 2004). Although Soukup & Yuan (1981) constructed stationary shock profiles in a vertically extended disk, their models neglected the effects of the Coriolis force, self-gravity, and magnetic fields. The flapping motions of XZ spiral shocks may just be overshooting of gas caused by instantaneous force imbalance across the shock, or may represent a dynamical instability. Detailed discussion on the physical origin of the spiral shock flapping, and the characteristics of the associated velocity and density fields, will be presented in a subsequent paper.

Since spiral shocks in three dimensions exhibit temporal fluctuations, it is useful to take space-and-time averages in order to visualize the shock structures in the $x-z$ plane. Figure 8 illustrates this using model MS3d1, with the density and velocity fields averaged along the $y$-direction and also over an orbital period from $t/t_{orb} = 1.7$ to 2.7. Figure 8a shows gas density in logarithmic scale and selected streamlines. The streamlines run almost parallel to the galactic midplane in the interarm region, but sharply plunge toward the midplane in the arm. The location of the averaged shock front is also indicated by a heavy line. The low vertical velocities of the gas in the interarm regions implies that vertical force balance is fairly well maintained there. As the material enters the spiral arm region, it is shocked and compressed. The increase of gas density due to the shock compression is largest at the midplane, which in turn produces strong vertical gravity and pulls high-altitude gas toward the midplane. The vertically plunging gas further increases the midplane density after the shock, setting up repulsive pressure gradients that cause it to rebound back to high-altitude regions. The furrow in the streamlines behind the shock front reflects this vertical dive and bounce.
Fig. 8.—Mean spiral shock solutions in the radial-vertical (XZ) plane of model MS3d1, based on spatial average along azimuth (γ) and time average over \(t_{\text{orb}} = 1.7-2.7\). (a) Gas streamlines (thin solid lines) are overlaid on the gas density \([\log (\rho / \rho_0), \text{in gray scale}].\) The heavy vertical line marks the averaged shock front. (b) Average surface density profile (solid line) based on vertical integration of density in \(a\) is compared to its two-dimensional thick-disk counterpart from model MS2d2 (dotted line). (c) The profile of mean gaseous scale height for models MS3d1 (solid line) and MS2d2 (dotted line). The gaseous scale height is generally smaller inside spiral arms than in interarm regions.

Figures 8b and 8c plot as solid lines the surface density profile \(\Sigma(x) = \int_{-\infty}^{\infty} \rho(x, z) \, dz\) and the corresponding gas scale height \(H(x) = \Sigma(x) / (\Omega P(x, 0))\) obtained from the averaged XZ spiral shock shown in Figure 8a. Also plotted as dotted lines are the \(\Sigma(x)\) and \(H(x)\) values of the thick-disk two-dimensional model MS2d2 that has exactly the same parameters and numerical resolution as model MS3d1, but fixed \(H_0 = 170\) pc. Although the surface density in model MS2d2 rises slightly more steeply at the shock than in model MS3d1, they are overall in excellent agreement. This proves that the gravitational kernel given by equation (11) accurately handles the dilution of self-gravity in an extended disk.

As Figure 8c shows, the gas within spiral arms is more compressed toward the midplane than in interarm regions. The tendency for gas to compress vertically in the arms increases as the magnetic field strength decreases. Column (8) of Table 2 indicates that the ratio of arm to interarm scale heights is about 0.5 for models with \(\beta_0 = \infty\) or 10, and 0.8 for \(\beta_0 = 1\) models. This is because models with weaker magnetic fields have higher arm density enhancement, and thus relatively stronger self-gravity that vertically compresses spiral arm gas. We discuss the relationship of this finding to observations in § 5.2.

Finally, we remark that our models do not exhibit the “hydraulic jump” behavior that Martos & Cox (1998) suggested may develop, under certain circumstances, at spiral shock fronts. While hydraulic jumps (which yield an increase in the gas scale height where it is compressed in arms) can occur when the equation of state is fairly stiff, Martos & Cox (1998) note that when \(\gamma = 1\) there are only shocks. Magnetic fields parallel to the arm may provide extra stiffness, but self-gravity tends to draw gas toward the midplane in the postshock region; in our (isothermal) models this results in a decrease of \(H\) within arms. It is not currently known whether the tendency of self-gravity to reduce \(H\) would overcome the tendency of a sufficiently stiff equation of state to increase \(H\) in arms, since the studies of Martos & Cox (1998) did not include self-gravity.

4.2. Nonaxisymmetric Evolution of Three-dimensional Disks

4.2.1. Absence of the Wiggle Instability in Three-dimensional Disks

Based on our two-dimensional thick-disc models, we found that perturbations in spiral shocks can grow, either as a result of the MJI or the wiggle instability. One may naively expect that three-dimensional models would also be prone either to MJI or to wiggle instability, not to mention to other potential three-dimensional instabilities. Surprisingly, however, we find that the wiggle instability seen in some of our two-dimensional models is absent in three-dimensional models, while MJI still operates in three-dimensional models. As we show below, our three-dimensional model MS3d1 with \(\beta_0 = 10\) is subject to MJI, whereas the corresponding two-dimensional model MS2d2 (with identical in-plane resolution to model MS3d1) was found to be unstable to the wiggle instability. In addition, the unmagnetized two-dimensional models H2d1-H2d3 developed vortical clumps via the wiggle instability that later merged and collapsed, but as Figure 7 shows, the corresponding three-dimensional model H3d1 does not develop strongly overdense clumps over the entire run. The peak density in model H3d1 fluctuates with period \(~0.29t_{\text{orb}}\), comparable to the period associated with epicyclic motion, \(~(2\Sigma_0/\Omega_0)^{-1/2}t_{\text{orb}} \approx 0.31t_{\text{orb}}\), within the arm peak.

The wiggle instability seen in two-dimensional disks thus appears to be stabilized in three-dimensional disks, possibly due to a combination of strong vertical shear and nonsteady flow. Figure 9 shows the vertical distribution and gradients of the azimuthal- and time-averaged spiral shock velocities at \(x/L_x = -0.05\) (upstream), 0.05 (at the midplane density peak), and 0.22 (far downstream), from model MS3d1. The velocity shear along the vertical direction amounts to \(q_{\| z} \sim 1\) and \(q_{\| z} \sim 5\) at \(z/H \sim 0.5\) and \(q_{\| z} \sim 2\) at \(z/H \sim 1-2\), where \(q_{\| z} \equiv \Omega_0 1/|d v_z / dz|\) and where \(q_{\| z} \equiv \Omega_0 1 |d v_\phi / d z|\). These shear rates can be compared with \(q_0 = 1\), the radial shear rate in the background azimuthal flow, and with the maximum radial shear in the quasi-azimuthal flow within the arm, \(q = 2 - (2 - q_0)\Sigma_0 / \Sigma_0 \approx 4.3\). Perturbations in the spiral shock of model MS3d1 might try to develop nonaxisymmetric vortical flows similarly to those produced by the wiggle instability in two-dimensional thick-disk models. However, these vortical structures could not remain coherent in \(z\) because of the strong vertical shear in horizontal velocities; vertical motions would then mix dissimilar vorticity, so that the vortices would be lose their integrity. In addition, nonsteady flapping of the shock front would also help prevent coherent structures from forming. Overall, the various three-dimensional effects combine to suppress nonlinear development of the wiggle instability.

Since our three-dimensional models explore cases with \(F = 5\% - 10\%\), our results are not directly comparable to cases in which spiral shocks are as strong as in Wada & Koda (2004)'s models. Strong spiral shocks in two dimensions are more favorable for promoting wiggle instability, but in three dimensions they would also result in stronger shock flapping motions and stronger vertical shear of the in-plane velocities, which tend to
to suppress the wiggle instability. Thus, it would require direct three-
dimensional simulations with very large $F$ to determine whether
increasing shock strength overall stabilizes or destabilizes spiral
shocks to “wiggle” modes. However, we consider this question
of limited practical importance, since real astronomical systems
would in general become self-gravitating if very strong shocks
were present. In this sense, the non-self-gravitating treatment of
Wada & Koda (2004), with $F/C^{110\%}$, appears unrealistic given
that the spiral shocks in their models have density jumps of a
factor 100. Contrasts in real galaxies are far smaller; e.g., the
notoriously strong spiral structure in M51 has arm/interarm con-
trasts of $5–6$ (Garcia-Burillo et al. 1993; see also Rix & Zaritsky
1995; Patsis et al. 2001). This is because equilibrium shock pro-
files are only possible at moderate values of $F$ (Paper I). For
instance, when $Q_{0}=1.8$ and $\beta=\infty$, quasi-axisymmetric shock
equilibria with thick-disk gravity exist only if $F \leq 20\%$, beyond
which shock profiles are highly transient or suffer from immediate
quasi-axisymmetric gravitational instability (see § 4.2.4).

4.2.2. Swing Amplification

When shear is strong (with $q \geq 0.3$) and magnetic fields are
absent, nonaxisymmetric modes are not subject to MJII but can
grow via swing amplification (Goldreich & Lynden-Bell 1965b;
Toomre 1981; Kim & Ostriker 2001). The growth of wave am-
plitudes via swing amplification is moderate unless the $Q_{0}$ value
of the background medium is sufficiently small. Kim & Ostriker
(2001) showed that swing amplification in a razor-thin, feature-
less disk with $Q_{0} \geq 1.3$ yields low-density fluctuations and cannot
form gravitationally bound clumps. Paper I further found that the
swing amplifier produces substructure growth in spiral arms only
if the background spiral perturbation is relatively weak (with
$F \leq 1\%$ in a razor-thin disk). In this case, the density enhance-
ment in the shock is quite low and all the gas has normal shear,
as opposed to the locally reversed shear which occurs wherever
$\Sigma_{\Sigma_{0}} > 2$.

With $F = 5\%$, model H3d1 in the present work has strong
shear reversal corresponding to $q_{\text{min}} = -4$ inside a spiral arm.
In addition, a relatively large value of $Q_{0} = 1.8$ and vertically
diluted self-gravity make swing amplification in model H3d1
quite ineffective. The resulting nonaxisymmetric structures in the
surface density of model H3d1 at the end of the run ($t/t_{\text{orb}} = 6.4$)
have the maximum power in the $\lambda_{y} = L_{y}/3$ mode, which is only
$\sim 0.26\%$ relative to the axisymmetric mode; the associated max-
imum surface density is 1.46 times the mean surface density. We
thus conclude that the swing amplification mechanism is unlikely
to prompt the formation of substructure within spiral arms in real

![Fig. 9.—Vertical distributions (left) and gradients of spiral shock velocities (right). Measurements are slightly upstream from the midplane shock at $x/L_{x} = -0.05$ (dotted line), slightly downstream from the shock at $x/L_{x} = 0.05$ (solid line), and in the interarm region at $x/L_{x} = 0.22$ (dashed line). Profiles are based on the averaged XZ shock configuration of model MS3d1. Shear in the horizontal velocities is very strong for $0.2 \leq |z/H| \leq 2.$](image-url)
For fiducial parameters, the simulation box in the visualization in Fig. 12. We discuss this alternative fragmentation process in § 4.2.3. This fragmentation in general first develops via quasi-axisymmetric instability, so the process is physically distinct from direct swing amplification. We discuss this alternative fragmentation process in § 4.2.4.

4.2.3. Magnetor-Jeans Instability in Three-dimensional Disks

In the preceding subsections, we have shown that the wiggle instability disappears in three-dimensional extended disk models, and that swing amplification is only moderate in unmagnetized, three-dimensional disks that have sizable spiral arm density contrasts but nevertheless are stable to quasi-axisymmetric self-gravitating perturbations. When a system is magnetized and has weak net shear, however, it is expected that perturbations should be able to grow via MJI.

Figure 7 shows that the density in our magnetized models ME3d1 and MS3d1 indeed undergoes exponential growth, with the simulations eventually producing gravitationally bound condensations. As we discuss below, typical wavelengths of the most unstable modes in models ME3d1 and MS3d1 are very similar to that of the MJI in two-dimensional, thick disks; these wavelengths are much larger than those of the wiggle instability in two-dimensional models (see Tables 1 and 2). In addition, spur morphologies and the physical properties of bound clouds that form from fragmentation of the spurs in three-dimensional models are similar to those due to MJI in two-dimensional thick-disk models. This supports the case that structure forms in our three-dimensional models as a result of MJI. Since gravity is a long-range force and insensitive to density structure at small scales, MJI (unlike the wiggle instability) can grow in spite of the unsteady flows of the background XZ spiral shock. The vertical velocity shear and flapping may delay the onset of bound cloud formation to some extent, but it is MJI that drives three-dimensional systems with magnetic fields into eventual runaway.

As in two-dimensional models, the background flow kinematics in our three-dimensional models sculpt the growing perturbations into spurs that branch out nearly perpendicularly from the main spiral arm and become trailing in interarm regions. Figure 10 displays surface density snapshots at the density slices in logarithmic color scale and the velocity vectors with $\lambda_z \equiv L_z/2 = 3.14 \text{kpc}$. The rectangle in (a) indicates the sector viewed as a three-dimensional visualization in Fig. 12.

Fig. 10.—Snapshots of vertically integrated surface density $[\log (\Sigma/\Sigma_0), \text{in gray scale}]$ of $\beta_0 = 10$ model MS3d1 at (a) $t/t_{\text{orb}} = 5.6$, (b) $t/t_{\text{orb}} = 6.0$, and (c) $t/t_{\text{orb}} = 6.3$. For fiducial parameters, the simulation box in the $x$-$y$ plane has a size of $L_x = L_y/2 = 3.14 \text{kpc}$. The rectangle in (a) indicates the sector viewed as a three-dimensional visualization in Fig. 12.

Disk galaxies where spiral arms are not so weak. Of course, as the models of Paper I showed, even without magnetic fields, if $Q_0$ is small enough, and/or if $F$ is large enough, spiral arms can become unstable to fragmentation. This fragmentation in general first develops via quasi-axisymmetric instability, so the process is physically distinct from direct swing amplification. We discuss this alternative fragmentation process in § 4.2.4.

As Table 2 indicates, the separation of spurs in our three-dimensional models depends on the numerical resolution. Models ME3d2 and MS3d2 with $N_x = 128$ form three spurs each, while the respective higher resolution model with $N_x = 256$ forms four spurs. Based on the results of two-dimensional models (see Table 1), which show the same number of spurs when $N_x = 256$ or 512, we expect (but have not proved) that our three-dimensional models are converged.
at the midplane ($z = 0$), as well as at the $y = 0.45L_x$ plane; those at the left edge ($x = 0.5L_x$) of the box are displayed in Figure 12b. Clearly, the dense part of the spur sticks out perpendicularly from the main spiral arm. The spur has an aspect ratio of 1:0.35:0.16 in $(x, y, z)$ and a total mass of $\sim 7 \times 10^6 M_\odot$; a similar amount of gas is in the part of the spiral arm shown in Figure 12a. Magnetic fields are overall parallel to the spiral arm, although the variation in the expanding velocity field off the arm causes some degree of radial excursions of magnetic fields. This results in magnetic pressure twice as large in the spur compared to inter-spur regions. As Figure 12b shows, magnetic field lines in the azimuthal-vertical plane are not strongly bent inward at high $z$, indicating that material is accumulated mainly along the (azimuthal) $y$-direction.

As spurs develop further, they concentrate sufficiently to trigger gravitational fragmentation, yielding a few bound condensations. As Figures 10 and 11 show, fragmentation occurs within spiral arms as well as in interarm regions; fragments forming inside the spiral arms linger there, whereas those forming off the spiral arms are carried into the interarm regions and follow galactic rotation. At the end of the runs, model MS3d1 forms 4 bound clouds with an average mass $M \sim 1.2 \times 10^7 M_\odot$, which is respectively about 6 and 1.2 times larger than the local, thin-disk and thick-disk Jeans masses at the arm density peak. These masses are defined by

$$M_{\text{thin}} = \frac{c_s^4}{G^2 \Sigma_{\text{sp}}} = 2.0 \times 10^6 M_\odot \left( \frac{c_s}{7 \text{ km s}^{-1}} \right)^4 \left( \frac{\Sigma_{\text{sp}}}{66 M_\odot} \right)^{-1},$$

$$M_{\text{thick}} = \frac{2\pi c_s^2 H_{\text{sp}}}{G} = 1.0 \times 10^7 M_\odot \left( \frac{c_s}{7 \text{ km s}^{-1}} \right)^2 \left( \frac{H_{\text{sp}}}{120 \text{ pc}} \right)^{-1}.$$

respectively. On the other hand, model ME3d1 produces three bound clouds, one fewer than in the corresponding two-dimensional model ME2d1 with equal magnetic field strength. This is because model ME3d1 with $\beta_0 = 1$ suffers from strong XZ flapping of the spiral shock that tends to work against gravitational mass accumulation. Bound clouds that form in model ME3d1 have an average mass $M \sim 3.0 \times 10^7 M_\odot$, comparable to the result from model ME2d1, which is about 12 times the corresponding thin-disk Jeans mass at the density peak. Roughly speaking, therefore, MJI in three-dimensional, magnetized spiral shocks turns $\sim 20\%$–$30\%$ of the total gas into dense bound clouds in a given epoch of gravitational condensation.

Note that typical masses of bound clouds that form in the current three-dimensional models are very similar to those in two-dimensional thick-disk spiral arm models discussed in §3.1, as well as those in other three-dimensional models without spiral arms (Paper II; Paper III). In addition, bound clouds from different models share similar magnetic and rotational properties. For example, self-gravitating clouds in our three-dimensional models are all magnetically supercritical (by a factor 10 or more) with an average mass-to-flux ratio of $M/\Phi_B \sim 2.4G^{-1/2}$ for model MS3d1 and $\sim 1.1G^{-1/2}$ for model ME3d1. (Mass-to-flux of $<0.16G^{-1/2}$ would be subcritical.) As in two-dimensional cases, these bound clouds in three dimensions are less supercritical than the initial simulation boxes, which have $M/\Phi_B = 7.0G^{-1/2}$ for model MS3d1 and $2.4G^{-1/2}$ for model ME3d1, consistent with the result of Vázquez-Semadeni et al. (2005). These results are remarkably close to that found in two-dimensional...
model ME2d1 in § 3.1 and those resulting from MJI (Paper II) or via swing amplification (Paper III) in three-dimensional disks without spiral features.

Because of efficient magnetic torques exerted by field lines linking a cloud with the surrounding medium (e.g., Gillis et al. 1974; Mouschovias & Paleologou 1979), bound clouds in models MS3d1 and ME3d1 lose a substantial amount of angular momentum and thus rotate relatively slowly. The mean specific angular momentum of bound clouds at the end of the runs is \( J_z \approx 0.2 J_{\text{gal}} \) for both models MS3d1 and ME3d1. Here \( J = \int \rho(\mathbf{r} \times \mathbf{v}) d^3r / \int \rho d^3r \) with the position \( \mathbf{r} \) and velocity \( \mathbf{v} \) are measured relative to the cloud center, and \( J_{\text{gal}} = \Omega_0(M/\Sigma_0)^2/12 \) is the specific angular momentum contained in a square patch of the galactic ISM. A cloud formed from that patch would preserve the value \( J_z = J_{\text{gal}} \) if no angular momentum were lost during its formation (see Paper III). Similar specific angular momenta of clouds have been obtained in our other three-dimensional magnetized simulations (Paper II; Paper III), where we also showed that condensations in unmagnetized models do not lose angular momentum as they evolve, while those in magnetized models do.

Finally, we remark on the effects of Parker (1966) instability on the formation of spiral-arm substructure in our three-dimensional models.
models. The spiral potential compresses the density and magnetic fields, decreasing the average $\beta$ value inside the spiral shock from 10 to 2.5 for model MS3d1 and from 1 to 0.4 for model ME3d1. Taking $\beta \sim 1$ and the radial wavelength $\lambda_p$ equal to the arm width, and neglecting the effect of cosmic-ray pressure, the local dispersion relation of the Parker instability in a rotating disk presented by Shu (1974) yields $k_y \approx 0.5/\bar{H}$ for the most unstable Parker mode (this is insensitive to the specific choices of $\beta$ and $\lambda_p$). The corresponding wavelength is $\lambda_p = 2\pi/k_y = 4\pi H \approx 1.5$ kpc $\approx L_s/4$. This agrees with the separations identified for spurs in models MS3d1 and ME3d1. However, because this wavelength also coincides with the scale predicted to have the largest growth of Parker modes to grow may be suppressed by the strong vertical gradients in $v_r$ and $v_z$ (as seen in Fig. 13 and also Fig. 9). Given the curved nature of the spiral shock front in the XZ plane, high-altitude regions can have large horizontal velocities with respect to the midplane gas below them, such that magnetic fields that begin to buckle vertically cannot maintain the shape required for Parker modes to develop.

Thus, we conclude that magnetic buoyancy effects are probably not of major importance in GMC formation within spiral arms. Even without the suppression of Parker modes by vertical shear that we see here, previous work has shown that Parker instability alone is unable to form high-density clouds in regions away from arms (e.g., Kim et al. 2000; Santillán et al. 2000; Paper II), a consequence of it being a self-limiting process stabilized by tension forces from bent field lines (e.g., Mouschovias 1974). While Parker instability may help seed structure in early stages of gravitational instabilities, and may be essential in removing excess magnetic flux from the disk, it does not appear crucial for massive cloud formation and subsequent star formation.

4.2.4. Fragmentation in Strongly Unstable Arms

We briefly discuss evolution of unmagnetized model HU3d1, with $Q_0 = 1.5$, $F = 5\%$, and $\beta_0 = \infty$. In this model, $Q_0$ is small enough to induce quasi-axisymmetric gravitational instability of a spiral shock. As Figure 7 shows, the maximum density in model HU3d1 at $t/t_{orb} = 1.5$ is larger by a factor of 3 than that in stable model H3d1 with $Q_0 = 1.8$. Soon after the spiral perturbation attains its full strength, the system is dominated by the exponential growth of quasi-axisymmetric modes in the spiral shock. At around $t/t_{orb} = 2.2$, the collapsing postshock layer reaches sufficient density that it begins to undergo nonaxisymmetric gravitational fragmentation. This results in the formation of 13 bound condensations aligned along the shock front, with mass $\sim 1 \times 10^7 M_\odot$ each. Note that while magnetized models MS3d1 and ME3d1 first develop spur structures that subsequently fragment into bound clouds in both arm and interarm regions, the bound clouds in unmagnetized model HU3d1 do not require prior formation of spurs. These clouds form directly in the highest density ridge of the spiral shock. The spacing of bound clouds in model HU3d1 is $\lambda/\lambda_0 \approx 0.5$, where $\lambda_0 = c^2_s/(G\Sigma_0)$ is the Jeans length in the initial featureless disk. This can be compared with $\lambda/\lambda_0 \approx 1.7$ for the spur separation in magnetized model MS3d1 with the same $Q_0$ and $F$ parameters as model HU3d1 (see Table 2). This demonstrates that the presence of magnetic fields makes significant differences in the dynamical evolution of gas in spiral arms.

5. CONCLUSIONS

5.1. Summary

A key unsolved problem in the dynamics and evolution of spiral galaxies lies in understanding the origin of spiral-arm substructure, including gaseous spurs, giant molecular clouds, and complexes of giant H I regions. Various observational and theoretical arguments increasingly support the notion that this arm substructure may originate from gravitational instability of diffuse gas within spiral arms. In Paper I, we studied dynamical evolution of a local segment of a magnetized spiral arm in a razor-thin model of a disk. We showed that magneto-Jeans instability (MJII) initiated within spiral arms naturally yields gaseous spur extending into the interarm region. Paper I further showed that these spurs fragment into bound condensations that could potentially evolve into arm and interarm H I regions.

The thin-disk models of Paper I, however, tend to overestimate midplane self-gravity, which favors small-scale MJII modes. The models of Paper II were also unable to capture potential dynamical consequences of the Parker instability and other three-dimensional instabilities that rely critically on the vertical dimension. Here we have extended Paper I to consider these important effects. We consider two sets of numerical models: two-dimensional disks in which a thick-disk gravitational kernel (eq. [11]) approximately treats the geometric dilution of self-gravity, and full three-dimensional disks in which all fluid variables are allowed to vary with the vertical coordinate. As in Paper I, both models adopt an isothermal equation of state. Our main objectives were to explore how the properties of spurs and bound condensations produced by MJII in vertically extended disks differ from the

\[ 7 \text{ Since in axisymmetry there is no stable shock for the parameters of model HU3d1, we cannot measure cloud spacings in terms of } \lambda_0 (\text{which is undefined}).\]
models of Paper I, and to examine the effectiveness of three-dimensional dynamical instabilities other than MJI in forming spiral-arm substructures. The following summarizes the main results of the present work:

1. In our new models with sufficiently magnetized conditions, spiral shocks give rise to gaseous spurs and bound clouds in a manner similar to the models of Paper I. This is true either for two-dimensional “thick disk” or three-dimensional models. The spur structures themselves are slightly more trailing than those in razor-thin disks. Dilution of self-gravity due to finite disk thickness is significant, causing the typical separation of spurs $x_c$ (along the arm) to be about 10 times the Jeans length $\delta_{3,sp} = c_s^2/(G\Sigma_{sp})$ at the spiral arm peak, which is $3-5$ times larger than the prediction of $x_c/\delta_{3,sp}$ from thin-disk models. The agreement between three-dimensional and two-dimensional “thick disk” models implies that, in this strong-$B$-field case, the mechanism behind spur and clump formation is the MJI. Reduced gravity in thick disks also causes the bound condensations that form when spurs fragment to be more massive than in thin-disk models. The average mass of these clouds is $\sim (1-3) \times 10^7 M_\odot$ corresponding to 6-12 times the thin-disk Jeans mass at the arm peak, and comparable to the thick-disk Jeans mass at the arm peak. The clump masses are comparable to the thin-disk Jeans mass at the mean unperturbed surface density of the disk. These bound clouds are magnetically supercritical with the mean mass-to-flux ratio of $\sim (1-3)G^{-1/2}$, and undergo significant loss of angular momentum (80% of the initial galactic value) via magnetic braking.

2. Before gravitational instability sets in, our three-dimensional models exhibit time-dependent behavior in their spiral shock structure that is quite unlike the rapid approach to steady state that characterizes two-dimensional models. The three-dimensional distributions can be averaged over the $y$-direction (parallel to the spiral arm) to yield an XZ shock profile. In these XZ profiles, the shock front generally moves back and forth relative to its mean position (in quasi-radial coordinate $x$). The flapping period of the XZ spiral shock in an unmagnetized model is $\sim 0.3 t_{\text{orb}}$, comparable to the epicyclic period at the arm peak. The amplitude of the flapping motion tends to be larger at large $z$ in a given model, with radial excursions of $\sim 0.06L_z$ at $|z|/H_0 = 1$ in the $\beta_0 = 1$ model. This amplitude is a factor 3 or 2 times larger than in models with $\beta_0 = 10$ or unmagnetized ($\beta_0 = \infty$) models, respectively. This flapping and other nonsteady motions do not seem to strongly affect development of gravitational instabilities, however, probably because they depend on a mean density enhancement over large scales that need not be strictly coherent for growth.

3. It is informative to consider the time average of the XZ shock profiles. The shock front in this mean profile is in general curved in the $x-z$ plane. In these XZ profiles, spiral arm regions are generally thinner than interarm regions; the ratio of scale heights in arm to interarm regions is about 0.5 for models with $\beta_0 = 10$ or $\infty$ and $\sim 0.8$ for $\beta_0 = 1$ models. The surface density distributions of the time-averaged profiles are similar to those of one-dimensional steady spiral shocks resulting from two-dimensional thick-disk-gravity models averaged over $y$. This suggests that the thick-disk gravitational kernel of equation (11) provides a good approximation for the gravitational potential near the midplane of a three-dimensional disk. It also shows why, due to the right average conditions, gravitational instabilities grow, despite significant stochasticity in the flow.

4. We find that weakly magnetized or unmagnetized two-dimensional models are unstable to the “wiggle instability” described by Wada & Koda (2004) (based on non-self-gravitating models with strong shocks). Wada & Koda (2004) advocated Kelvin-Helmholtz instabilities as an explanation for the wiggle instability, and argued that spiral arm spurs could arise from this mechanism. Indeed, we find the magnetic field topology and the generation of vorticity near the shock front in our two-dimensional unmagnetized or weakly magnetized models are suggestive of Kelvin-Helmholtz instabilities in shear layers. We also find that mergers of vortical clumps in these two-dimensional models can eventually produce self-gravitating clouds with mass $\sim 1 \times 10^7 M_\odot$, although the spur structures are much less prominent than in our two-dimensional models with stronger magnetic fields.

Most importantly, however, we find that the vorticity-generating wiggle instability is absent in full three-dimensional models of all magnetic field strengths. It appears that radial flapping motions of the XZ shock front, combined with strong vertical shear of horizontal velocities, quickly disrupt any coherent vortical structures that would otherwise grow. We therefore conclude that the wiggle instability is an artifact of two-dimensional models within a certain parameter range, and is unlikely to play an important role in forming spiral-arm substructures in real spiral galaxies.

5. While the Parker instability has long been invoked as a primary mechanism for the formation of giant molecular clouds inside spiral arms, our three-dimensional models do not show any noticeable evidence of developing Parker modes. There is no indication of sinuousoidal vertical velocities, or correlation of magnetic hills/valleys with over/under dense regions. Like the wiggle instability, the Parker instability appears to be suppressed by strong vertical shear of in-plane velocities.

6. In addition to MJI, two other mechanisms capable of forming gravitationally bound clouds in disk galaxies are swing amplification and collapse of spiral arms parallel to the shock front (followed by fragmentation). The simulation outcomes of our unmagnetized, three-dimensional models with spiral arms show, however, that the growth of nonaxisymmetric disturbances via swing amplification is very small when $F = 5\%$ and $Q_0 = 1.8$. This is consistent with the results of Paper I, in which we showed that swing amplification in thin disks is efficient only if the background spiral perturbation does not exceed 1%. With reduced self-gravity, swing amplification in three-dimensional spiral arms becomes even more inefficient. Thus, the swing mechanism would appear to apply either in interarm regions or in galaxies without strong spiral arms.

On the other hand, our unmagnetized model with $F = 5\%$ and $Q_0 = 1.5$ was sufficiently unstable that quasi-axisymmetric growth developed as soon as the external potential reached full strength. Fragmentation into closely spaced bound clouds then occurred. Conceivably, this kind of cloud formation process could be important in galaxies that are intrinsically quite close to instability, and experience a tidal encounter that tips them “over the edge.”

5.2. Discussion

The summary above makes clear that our new three-dimensional models uphold the principal conclusions of Paper I—namely, that gravitational instability in the gas component of spiral arms (1) is able to form bound clouds with properties similar to the most massive GMCs; and (2) can lead to the development of interarm extensions similar to features that have been described in the observational literature as spiral arm spurs and feathers. Our models show that gaseous spurs may continue to stand out against the interarm background for a long time after clouds...
form, without being dispersed (see Figs. 10 and 11). Since interarm regions are characterized by low gas density and strong shear, they are normally thought of as being hostile for gas to condense gravitationally. Our model simulations predict, however, that even interarm regions with low mean gas density can be abundant with kiloparsec- and larger scale substructure, arranged in trailing gaseous spurs that may host H II regions. The material that makes up these interarm spurs consists of parcels of gas that were the center of attraction during the most recent spiral arm transit. The concentration of interarm gas into secondary spurs may be crucial in enabling clouds, and hence stars, to form, without being dispersed. In the present models, the timescale for gravitational instability to run away is 2–6 orbits, for a range of values of $Q_{\text{d}}$, $F$, and $\beta_p$. Combining these raw numbers would imply an effective GMC formation timescale from diffuse gas of \(~10^9\) years. However, we believe that in fact this somewhat overestimates the true timescale for the average parcel of diffuse gas to be incorporated in a bound cloud. While the conversion efficiency per GMC formation timescale may be realistic, the time to reach runaway is probably longer than it would be in a real system, because our initial perturbation amplitudes are likely low compared to realistic values. An important direction for future study will be to measure cloud formation rates in simulations where cloud destruction is also incorporated; we plan to pursue studies of this kind. Models of this kind will provide more realistic background conditions from which gravitational instabilities can develop.

While the real ISM has a multiphase structure due to the particular heating and cooling processes that are relevant, we have adopted a much simpler isothermal approximation throughout this paper. The inclusion of gaseous cooling that leads to dense cloud formation at small spatial scales (e.g., Koyama & Inutsuka 2002; Piontek & Ostriker 2004, 2005; Audi & Hennebelle 2005; Heitsch et al. 2005; Vázquez-Semadeni et al. 2006) might indirectly affect formation timescales, mean separations, and masses of the GMCs and other structures that result from self-gravitating instabilities in spiral arms. It is often assumed that the total velocity dispersion in the warm/cold medium can be treated as an effective sound speed; whether this treatment is adequate will be addressed directly in future work. Our three-dimensional isothermal simulations show that the wiggie instability is stabilized by strong vertical shear and nonsteady flows associated with spiral shocks, but the situation could conceivably be different if thermal instability and multiphase gas structure are considered. This outstanding issue will direct our future research as well.

Finally, we remark that, while spiral structure clearly can help gravitational instability and prompt star formation, it may also contribute to limiting these processes. As we have discussed, spiral shocks (especially when strongly magnetized), lead to time dependent motions that may be important in feeding turbulence at scales $\ll H$. The present models are isothermal at $T \sim 8000$ K, so the sound speed $7 \text{ km s}^{-1}$ exceeds turbulent velocity amplitudes. However, in the real ISM there is significant cold gas for which the random turbulent motions far exceed the sound speed $\sim 1 \text{ km s}^{-1}$. The turbulence in the cold ISM component may be key to regulating self-gravitating instabilities. In a future publication, we will discuss results of an investigation focusing on the details of turbulent driving by spiral shocks.

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