Frequency analysis of Cepheids in the Large Magellanic Cloud: new types of classical Cepheid pulsators

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ABSTRACT

We have performed a detailed systematic search for multiperiodicity in the Population I Cepheids of the Large Magellanic Cloud. In this process, we have identified for the first time several new types of Cepheid pulsational behaviour. We have found two triple-mode Cepheids pulsating simultaneously in the first three radial overtones. In 9 per cent of the first overtone (FO) Cepheids, we have detected weak but well-resolved secondary periodicities. They appear either very close to the primary pulsation frequency or at a much higher frequency with a characteristic period ratio of 0.60–0.64. In either case, the secondary periodicities must correspond to non-radial modes of oscillation. This result presents a major challenge to the theory of stellar pulsations, which predicts that such modes should not be excited in Cepheid variables. Non-radial modes have also been found in three of the fundamental first overtone (FU/FO) double-mode Cepheids, but no such oscillations have been detected in single-mode Cepheids pulsating in the FU mode.

In 19 per cent of double-mode Cepheids pulsating in the first two radial overtones (FO/SO type), we have detected a Blazhko-type periodic modulation of amplitudes and phases. Both modes are modulated with a common period, which is always longer than 700 d. Variations of the two amplitudes are anticorrelated, and maximum of one amplitude always coincides with minimum of the other. We have compared observations of modulated FO/SO Cepheids with predictions of theoretical models of the Blazhko effect, showing that the currently most popular models cannot account for properties of these stars. We propose that the Blazhko effect in FO/SO Cepheids can be explained by a non-stationary resonant interaction of one of the radial modes with another, perhaps non-radial, mode of oscillations.

Key words: methods: data analysis – stars: oscillations – Cepheids – Magellanic Clouds.

1 INTRODUCTION

Classical Cepheids and RR Lyrae stars are among the best studied pulsating variables. For decades, they have been considered primary examples of purely radial pulsators, displaying large amplitude oscillations, with only one or two radial modes excited. This simple picture was challenged in the end of the last century when Olech et al. (1999) discovered low-amplitude non-radial modes in three RR Lyr stars of globular cluster M55. In the following years, it was shown that the presence of secondary periodicities identified with non-radial modes of oscillations is a very common property of RR Lyr variables. They were detected in a substantial fraction of stars in every studied RR Lyr population: in Large Magellanic Cloud (LMC) (Alcock et al. 2000, 2003; Soszyński et al. 2003; Nagy & Kovács 2006), Small Magellanic Cloud (SMC) (Soszyński et al. 2002), Galactic Bulge (Moskalik & Poretti 2002, 2003; Mizerski 2003; Collinge, Sumi & Fabrycky 2006), Galactic field (Wils, Lloyd & Bernhard 2006; Szczygieł & Fabrycky 2007, see also Smith 1995) and most recently in ω Cen (Moskalik & Olech 2008). Depending on the stellar system, non-radial modes are excited in between 5 and 30 per cent of all RR Lyr stars.

Motivated by these findings, we decided to search for non-radial modes in classical (Population I) Cepheids. As far as pulsation properties are concerned, Cepheids are very close siblings of RR Lyr stars. It is therefore very interesting to check if non-radial modes are excited in these pulsators as well. With this goal in mind, we conducted a systematic search for multiperiodicity among Population I (Pop. I) Cepheids of the LMC. The primary source of data for this analysis was the Optical Gravitational Lensing Experiment-II
2 TIME SERIES ANALYSIS OF CEPHEID LIGHTCURVES

We analysed all fundamental mode (FU), first overtone (FO) and double-mode (FU/FO and FO/SO) LMC Cepheids listed in the OGLE-II catalogues (Udalski et al. 1999b; Soszyński et al. 2000), nearly 1300 stars in total. We omitted all variables marked by the authors as FA or BR. The former are most likely Pop. II Cepheids, while status of the latter group is somewhat unclear. We used the I-band OGLE-II data obtained with the difference image analysis (DIA) method. The search for secondary periodicities was conducted with a standard consecutive pre-whitening technique. First, we fitted the data with a Fourier series describing pulsations with the dominant mode:

\[ I(t) = \langle I \rangle + \sum_{k} A_k \sin(2\pi kf_i t + \phi_k) \]  

(1)

The frequency of the mode, \( f_i \), was also optimized. For double-mode Cepheids, we fitted the data with double-frequency Fourier series, representing pulsations in two dominant (radial) modes:

\[ I(t) = \langle I \rangle + \sum_{jk} A_{jk} \sin(2\pi(jf_1 + kf_2) t + \phi_{jk}) \]  

(2)

The residuals of the fit were then searched for secondary frequencies. This was done with the Fourier transform, calculated over the range of 0–6 d\(^{-1}\). If any new periodicity was detected, a new Fourier series, including all frequencies identified so far, was fitted to the data and the fit residuals were examined again. The process was stopped when no new significant frequencies appeared. At this stage, we performed data clipping, rejecting all measurements deviating from the fitted function by more than 5\( \sigma \), where \( \sigma \) is the standard deviation of the fit residuals. OGLE-II photometry was very clean and only for a small number of stars data points were removed by this criterion. After data clipping, the frequency analysis was repeated.

As a final step of analysis, we checked all Cepheids with detected secondary peaks for a possible light-curve contamination. It is a well-known shortcoming of the DIA reduction method that the measured flux of a star can be contaminated by light coming from neighbouring stars (i.e. Mizerski & Beijer 2002; Hartman et al. 2004). As a result, some of the discovered secondary frequencies might originate not in the Cepheid, but in another variable star in the Cepheid’s neighbourhood. In order to remove such spurious detections, we proceeded in the following way. First, we identified all variable stars from OGLE-II catalogue (Zebrun et al. 2001), which are located within \( r = 1 \) arcmin of the studied Cepheid. For each of these stars, we performed frequency analysis and then checked if the detected frequencies or their aliases appear as secondary peaks in Cepheid’s power spectrum. With this procedure, we weeded out 14 contaminated Cepheids. In LMC fields SC1–SC12, we extended the search radius to \( r = 1.5 \) arcmin, but no new cases of contamination were found. The OGLE-II catalogue of variable stars is not complete, though. It misses most small amplitude variables, which can contaminate light curves of nearby stars as well. Therefore, we performed additional contamination check, calculating frequency spectra of all stars (not only known variables) within \( r = 20 \) arcsec of the Cepheid. No more contaminated Cepheids were rejected at this step.

3 NEW DOUBLE-MODE CEPHEIDS

In the course of our analysis, we identified five new ‘canoncal’ double-mode Cepheids, pulsating in two radial modes. In four of them, the fundamental mode and the first overtone are excited (FU/FO type), while in the fifth Cepheid the first and second overtones are excited (FO/SO type). Basic properties of these variables are listed in Table 1. Periods and amplitudes given in the table (as well as in all other tables in this paper) are determined through the least-square fits of appropriate multifrequency solutions to the data.

In Figs 1 and 2, we present the period ratio versus period diagram (so-called Petersen diagrams) for all double-mode Cepheids detected in the LMC (Alcock et al. 1995, 1999, 2003; Soszynski et al. 2000). For comparison, we also display the FU/FO Cepheids observed in the SMC (Alcock et al. 1997; Udalski et al. 1999a) and in the Galaxy (Antipin 1997a, 1998; Pardo & Poretti 1997; Berndnik & Turner 1998; Wils & Otero 2004).\(^1\) Because of

\[ P_1/P_0 \]

Figure 1. The Petersen diagram for FU/FO double-mode Cepheids in the Galaxy (open squares), LMC (asterisks) and SMC (open circles). New LMC double-mode Cepheids are displayed with filled circles. Dotted lines represent the best linear fits to the period ratios observed in each population.

Table 1. New double-mode Cepheids in OGLE-II LMC sample.

| OGLE ID  | \( P_0 \) (d) | \( P_1 \) (d) | \( P_1/P_0 \) | \( A_1/A_0 \) | Type |
|----------|----------------|----------------|----------------|----------------|-------|
| SC2-263415 | 3.420 377 | 2.455 273 | 0.717 84 | 27.11 | FU/FO |
| SC11-250925 | 7.864 120 | 5.565 078 | 0.707 65 | 11.11 | FU/FO |
| SC17-126402\(^a\) | 3.209 709 | 2.294 942 | 0.715 00 | 24.31 | FU/FO |
| SC20-100852\(^b\) | 3.747 624 | 2.715 290 | 0.724 54 | 16.65 | FU/FO |
| SC20-112788\(^b\) | 0.737 740 | 0.594 287 | 0.805 55 | 0.09 | FO/SO |

\(^a\)Additional mode at \( P = 1.429 \) 530 d detected (see Section 6).
\(^b\)Both radial modes modulated (see Section 7).
different metallicities, the $P_1/P_0$ period ratios in these three stellar systems are somewhat different. Four of the newly identified double-mode Cepheids fit to their respective Petersen diagrams very well. For the fifth object, SC20-100652, the period ratio $P_1/P_0$ is rather high and is close to the value expected for an SMC star. This suggests that SC20-100652 has a lower metallicity than other LMC Cepheids. We also note that the new FU/FO double-mode pulsator SC11-250925 has the longest periods among currently known variables of this class.

4 TRIPLE-MODE CEPHEIDS

In two of the previously known FO/SO double-mode Cepheids, a third strong periodicity was found. The period ratio of $P_3/P_2 = 0.840$, the same in both Cepheids, identifies the new mode as a third radial overtone. In Fig. 3, we present the pre-whitening sequence for one of these stars. The third overtone is detected very securely; in fact it is stronger than the second overtone.

Pulsation properties of the two triple-mode Cepheids are listed in Table 2. In Fig. 4, we display both variables in the Cepheid $P$–$L$ plot constructed for extinction insensitive Wessenheit index, $W_I = I - 1.55(V - I)$ (Madore & Freedman 1991). The stars fit the $P$–$L$ relation very well, proving that apart from multiperiodic pulsations, they are both normal Pop. I Cepheids belonging to the LMC.

Discovery of the triple-mode pulsators listed in Table 2 was originally announced by Moskalik et al. (2004). Three more triple-mode Cepheids have been identified very recently by Soszyński et al. (2008a), bringing the number of objects in the class to five. This
new type of multimode Cepheid pulsations, although extremely rare, is very important for testing stellar models. With three radial modes observed and their periods accurately measured, seismological analysis yields tight constraints on Cepheid parameters (mass, luminosity, metallicity). This imposes constraints on Cepheid evolutionary tracks, including interesting limits on convective overshooting from the core (Moskalik & Dziembowski 2005).

5 ANALYSIS OF FO AND FU CEPHEIDS

5.1 Non-radial modes in first overtone Cepheids

The OGLE-II catalogue of LMC Cepheids (Udalski et al. 1999b) lists 462 first overtone variables. We detected resolved secondary frequencies in 42 of them. This constitutes 9 per cent of the sample. We consider two frequencies to be resolved, if they differ by more than $|\Delta f| = 2/T$, where $T$ is the length of the data. In case of OGLE-II photometry, this means $|\Delta f| > 0.0017\ldots0.0020$ d$^{-1}$, depending on the star. Our criterion is more conservative than usually adopted in other studies (i.e. Alcock et al. 2003; Nagy & Kovács 2006).

The complete inventory of resolved FO Cepheids is presented in Table 3. Following notation originally introduced for the RR Lyr variables (Alcock et al. 2000), we call these stars FO-v Cepheids. Consecutive columns of Table 3 give OGLE number of the star, primary and secondary periods $P_1$ and $P_2$, frequency difference $\Delta f = f_2 - f_1$, period ratio $P_2/P_1$ and amplitude ratio $A_2/A_1$.

In most of the FO-v Cepheids, only one secondary frequency was detected, but in several variables two frequencies were found. In all cases, they have extremely low amplitudes. With the exception of a single star (SC18-208875), the amplitude ratio $A_2/A_1$ is always below 0.1, with the average value of 0.048. We note that the secondary peaks detected in LMC first overtone RR Lyr stars are several times higher, with $A_2/A_1 = 0.31$ on average (Nagy & Kovács 2006).

The secondary frequencies in FO-v Cepheids come in two different flavours. In 37 variables, they are located close to the primary (radial) frequency $f_1$, within $|\Delta f| < 0.13$ d$^{-1}$. Representative examples of such behaviour are displayed in Fig. 5. In 84 per cent of cases, secondary frequencies are lower than the primary one. When two secondary peaks are detected, they always appear on the same side of the primary peak. The observed frequency pattern cannot be explained by amplitude or phase modulation of the radial mode. A periodic modulation always produces an equally spaced frequency multiplet (triplet, quintuplet, etc.), centred on the primary peak. It is also easy to check that measured period ratios are incompatible with those of the radial modes. This implies that the secondary frequencies detected close to the radial mode of the FO Cepheids must correspond to non-radial modes of oscillations. This is the first detection of non-radial modes in this type of stars.

In seven FO-v Cepheids secondary periodicities of a different type are found: we see a high-frequency mode, with characteristic period ratio of $P_2/P_1 = 0.60\ldots0.64$. Examples of this behaviour are displayed in Fig. 6. The two types of secondary modes are not mutually exclusive. In two objects (SC13-165223 and SC17-39517), both a high-frequency secondary peak and a secondary peak close to the primary frequency are present.

The mysterious period ratio of $0.60\ldots0.64$ cannot be explained in terms of radial modes either. Observationally determined period ratio of the third and first radial overtones is $P_3/P_1 = 0.6766\ldots0.6773$ (see Table 2; Soszyński et al. 2008a). The value of $P_3/P_1$ can be estimated theoretically. Assuming validity of $p$-mode asymptotic relation $f_s - f_3 = f_3 - f_2$, we get $P_3/P_1 = (2 - P_3/P_2)^{-1}$. With $P_3/P_2 = 0.840$ (see Table 2), we find $P_3/P_1 = 0.862$, thus $P_3/P_1 = 0.5835$. This estimate agrees nicely with $P_3/P_1 = 0.5898$ derived from tentative detection of the fourth overtone in one of the LMC Cepheids (Soszyński et al. 2008a). With $P_3/P_1 = 0.60\ldots0.64$, the high-frequency secondary peaks in FO-v Cepheids cannot correspond to either the third or the fourth radial overtone. They have to be attributed to non-radial modes. These modes appear at frequencies $0.02\ldots0.06$ d$^{-1}$ below the frequency of the (unobserved) fourth radial overtone. Interestingly, this frequency difference is comparable to $\Delta f$ for the non-radial modes detected in the vicinity of the first overtone.

In Fig. 7, we show the distribution of frequency differences, $\Delta f$, for the LMC FO-v Cepheids and, for comparison, for the LMC first overtone Blazhko RR Lyr stars (RRc-BL stars). The two distributions are very similar. FO-v Cepheids show stronger preference for negative $\Delta f$, but otherwise, in both types of overtone pulsators non-radial modes are found in similar distances from the radial mode. The only difference between the two histograms is the presence of high-frequency secondary modes ($P_3/P_1 = 0.60\ldots0.64$), which were detected in FO Cepheids, but not in RRc stars.
Remarks.

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photometry (Allsman & Axelrod 2001), which offered a much
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5.2 First overtone Cepheids with unresolved residual power

In 23 FO Cepheids, we found after pre-whitening a significant residual

d∝
of OGLE-II data. We reanalysed all these stars with the MACHO

b photometry (Allsman & Axelrod 2001), which offered a much

longer time base of 2700–2800 d. MACHO data were cleaned be-

fore analysis. We removed all data points with formal errors more

than five times larger than the average formal error in the data

set. We also rejected all measurements differing from the mean
brightness by more than 5 standard deviations (of the unphased data). After cleaning, the data were analysed in a standard way. Remnant power was resolved with MACHO photometry only in one additional Cepheid, SC13-165223. This object is included in Table 3. Remaining 22 Cepheids with unresolved residual power are listed in Table 4. Following notation introduced by Alcock et al. (2000), we call these variables FO-PC Cepheids.

For closer examination of slow amplitude/phase variability in FO-PC Cepheids, we divided the data into 10 subsets, each spanning 10 per cent of the total time base, and then fitted equation (1) to each subset separately. This way, we were able to follow the time evolution of amplitude and phase of the radial mode. In this procedure (also used in Sections 7.2 and 7.3), pulsation frequency $f_1$ was kept fixed. In eight stars, only parabolic trends in pulsation phases were found. These Cepheids most likely undergo a secular period change. In remaining 14 FO-PC Cepheids, variability of amplitudes is clearly seen, in most cases associated with variability of phases. In Fig. 8, we show typical examples of both behaviours.

With currently available data, we cannot make any firm statement about the cause of the observed long-term trends. We only note that in all FO-PC Cepheids, the observed changes of phases and amplitudes are orders of magnitude too fast to be explained by stellar evolution. We recall that fast phase variations of non-evolutionary origin are not unique to Cepheids of the LMC; they are also observed in many FO Cepheids in the Galaxy (Berdnikov et al. 1997, their fig. 3). On the other hand, secular amplitude variations are rare in Galactic FO Cepheids, with Polaris being the only well-documented case (e.g. Bruntt et al. 2008; Lee et al. 2008).

5.3 Fundamental mode Cepheids

The OGLE-II catalogue of LMC Cepheids (Udalski et al. 1999b) lists 718 fundamental mode pulsators. We searched all of them for secondary periodicities. We found no non-radial modes in the FU Cepheids of the LMC. This result is highly significant considering that the FU Cepheid sample is much larger than the FO Cepheid
5.4 Incidence rate of non-radial modes

In Table 5, we present the inventory of different types of pulsators identified in our survey of FO and FU Cepheids in the LMC. For each type of variability, we give the number of stars found, \( N_i \), and the estimated incidence rate in the population with its statistical error. The errors are calculated from the assumption of Poisson distribution (cf. Alcock et al. 2003):

\[
\sigma_{N_i/N} = \frac{1}{N} \sqrt{\frac{N_i(N - N_i)}{N}},
\]

where \( N \) is the total number of FO or FU Cepheids.

The most significant result of our survey is the detection of non-radial modes in LMC Cepheids, but only in those which pulsate in the first overtone. In Fig. 9, we display all Cepheids with non-radial modes on the Wessenheit index \( P - L \) diagram. All variables classified as FO-\( \nu \) Cepheids are indeed firmly located on the first overtone sequence. Their distribution with the pulsation period is not uniform, though. FO-\( \nu \) pulsators are not found for \( P_1 < 1.2 \) d. This suggests that the incidence rate of non-radial modes might depend on the period. To test this hypothesis, we divided the FO Cepheid population into several period bins and estimated the incidence rates in each bin separately. Results are displayed in Fig. 10. The incidence rate of FO-\( \nu \) pulsators systematically increases with period, reaching \( 19 \pm 5 \) per cent for \( \log P_1 > 0.5 \). We think that this is most likely a selection effect. The non-radial modes, which have very small amplitudes, are progressively more difficult to detect in Cepheids with shorter periods, i.e. with lower luminosities. This interpretation implies that the true incidence rate of FO-\( \nu \) Cepheids might be significantly higher than 9 per cent given in Table 5.

5.5 Comparison with RR Lyrae stars

In the last decade, non-radial modes were detected in many RR Lyr stars belonging to various stellar systems. Since pulsations of Cepheids and of RR Lyr stars are in many ways similar, it is interesting to compare the properties of non-radial modes in these two types of variables. One similarity has already been discussed in Section 5.1: in both types of pulsators, non-radial modes appear in essentially the same frequency distances from the radial mode (Fig. 7). However, there are also striking differences between the two groups of stars.

(i) Amplitudes of non-radial modes in classical Cepheids are very low, on average almost an order of magnitude lower than in RR Lyr stars.

Figure 6. Pre-whitened power spectra of a sample of FO-\( \nu \) Cepheids with \( P_1/P_1 = 0.60 \– 0.64 \). Frequencies of removed radial modes indicated by dashed lines.
Cepheids and for LMC RRc-BL stars (from Nagy & Kovács 2006).

For RRc-BL variables with frequency triplets, only higher side-peak are taken into account. Shaded area marks variables with $P_2/P_1 = 0.60–0.64$.

(ii) In RR Lyr variables, non-radial modes are detected both in fundamental (RRab) and in first overtone pulsators (RRc). In fact, in all studied stellar systems they are found in the RRab stars either equally frequently (Soszyński et al. 2002) or much more frequently (Mizerski 2003; Soszyński et al. 2003). In sharp contrast, in classical Cepheids non-radial modes are detected only in the first overtone pulsators. The difference between fundamental mode Cepheids and RR Lyr stars is even more striking, considering that for overtone pulsators incidence rates of non-radial modes are roughly the same: 9.1 per cent in FO Cepheids and 9.6–12.1 per cent in RRc stars (Mizerski 2003; Nagy & Kovács 2006).

(iii) When two secondary frequencies are found in an RR Lyr star, they usually form together with the primary frequency an equally spaced triplet, centred on the primary (radial) peak. Such equidistant triplets are never observed in Cepheids.

(iv) In RR Lyr stars, non-radial modes are detected only in close vicinity of the primary pulsation mode. This is also the case for most of the FO-ν Cepheids, but in several variables a high-frequency mode with $P_2/P_1 = 0.60–0.64$ is found. Such high-frequency modes were not detected in RR Lyr pulsators.

5.6 Can secondary frequencies result from blending?

In Section 2, we describe how to identify spurious secondary frequencies, which have appeared through a contamination of DIA photometry with light of neighbouring variables. This procedure cannot identify cases of real blending, resulting from the angular resolution limit of the observations. This raises a legitimate question whether secondary periodicities discussed in this paper originate in Cepheids or whether they are introduced by blending of Cepheids with other variable stars. Although the latter possibility cannot be ruled out in any individual case, we are convinced that the majority of non-radial modes detected in FO-ν Cepheids cannot be explained by blending. Our arguments are statistical. First, observed distribution of frequency differences, $\Delta f \sim f_2 - f_1$, is not random (Fig. 7). There is no reason why blending should introduce frequencies only in the vicinity of a radial mode or at any particular narrow range of period ratios. Secondly, we do not find non-radial modes in Cepheids which pulsate in the fundamental mode. There is no reason why blending should affect stars pulsating in one mode but not in the other mode. Thirdly, blending should be more substantial (i.e. relative flux contribution should be larger) in case of fainter Cepheids (Mochejska 2002). Therefore, secondary periodicities introduced by blending should be found preferentially in short-period pulsators (where their amplitudes should be higher). The statistics of FO-ν Cepheids show just the opposite trend (Fig. 10). Finally, probability of blending should increase proportionally to the density of stars in the field (Kiss & Bedding 2005). This is not the case for the FO-ν Cepheids. In eight densest LMC fields (SC2-SC9; 189 FO Cepheids in total) and in eight least dense LMC fields (SC12, SC14, SC15, SC17-SC21; 157 FO Cepheids in total), the incidence rates of FO-ν pulsations are 10.6 ± 2.2 and 8.9 ± 2.3 per cent, respectively. The two values are statistically the same within 0.6σ, despite twofold difference in average number of stars per field. From all these evidences, we conclude that secondary frequencies detected in the FO-ν Cepheids do not result from blending, but are intrinsic to Cepheids themselves.

![Figure 7. Distribution of frequency differences $\Delta f = f_2 - f_1$ for LMC FO-ν Cepheids and for LMC RRc-BL stars (from Nagy & Kovács 2006). For RRc-BL variables with frequency triplets, only higher side-peak are taken into account. Shaded area marks variables with $P_2/P_1 = 0.60–0.64$.](https://academic.oup.com/mnras/article-abstract/394/3/1649/1071553)

| OGLE ID   | $P_1$ (d) | $A_1$ (mag) | Remarks |
|-----------|-----------|-------------|---------|
| SC1-158032 | 2.169 982 | –           | a,b,c   |
| SC1-324896 | 2.209 283 | 0.173       | a,b,s1  |
| SC2-248540 | 1.823 789 | –           | a,d     |
| SC4-152290 | 1.887 862 | 0.082       | a,b     |
| SC8-76179  | 2.462 125 | 0.183       | b,s2    |
| SC9-153398 | 0.947 292 | 0.087       | a,b     |
| SC10-269402 | 1.912 859 | 0.114       | a,b     |
| SC11-338308 | 1.743 443 | 0.053       | a,b     |
| SC13-74204 | 1.369 660 | 0.079       | a       |
| SC14-148402 | 2.193 230 | 0.209       | b       |
| SC15-31558 | 1.554 964 | 0.046       | a       |
| SC15-158999 | 1.100 853 | 0.266       | b       |
| SC16-999240 | 2.437 482 | 0.152       | b       |
| SC17-15475  | 2.012 359 | 0.200       | a,b     |
| SC17-117748 | 3.572 147 | 0.160       | b       |
| SC17-122399 | 2.783 800 | 0.067       | a       |
| SC17-214859 | 1.954 504 | 0.206       | b,s3    |
| SC18-141019 | 2.270 553 | 0.192       | b       |
| SC18-202349 | 2.432 908 | 0.179       | b,s4    |
| SC21-85282  | 3.372 482 | 0.167       | a,b     |
| SC21-106503 | 2.535 978 | 0.184       | a,b     |
| SC21-136158 | 3.774 050 | 0.153       | a,b     |

Remarks: (a) amplitude variability; (b) phase variability; (c) no residual power detected in MACHO data; (d) no MACHO data available; (s1) same as SC16-57446; (s2) same as SC9-372261; (s3) same as SC18-33591; (s4) same as SC19-48662.
Frequency analysis of LMC Cepheids

Figure 8. Amplitude and phase variations of sample of FO-PC Cepheids. ±1σ error bars shown larger than the symbols. In SC4-152290 and SC11-338308 both amplitudes and phases change in time. In SC15-158999 and SC18-141019 amplitudes are constant and only pulsation phases vary.

Table 5. Variability types in FO and FU Cepheids of OGLE-II LMC sample.

| Type   | Description                              | Number | Inc. rate (per cent) |
|--------|------------------------------------------|--------|----------------------|
| FO-S   | Single-periodic overtone Cepheid         | 397    | 85.9 ± 1.6           |
| FO-ν   | FO with non-radial modes                 | 42     | 9.1 ± 1.3            |
| FO-PC  | FO with variable phase/amplitude        | 22     | 4.8 ± 1.0            |
| FO-Misc| FO with some miscellany                  | 1      | 0.2 ± 0.2            |
| FU-S   | Single-periodic FU Cepheid               | 716    | 99.6 ± 0.2           |
| FU-ν   | FU with non-radial modes                 | –      | –                    |
| FU-PC  | FU with variable phase/amplitude        | 1      | 0.1 ± 0.1            |
| FU-Misc| FU with some miscellany                  | 1      | 0.1 ± 0.1            |

Figure 9. $W_I$ index $P$–$L$ diagram for LMC Cepheids. Dominant (radial) periods of FO-ν Cepheids are displayed with black symbols.

Figure 10. Incidence rate of LMC FO-ν Cepheids versus first overtone period.
Table 6. FU/FO-ν double-mode Cepheids in OGLE-II LMC sample.

| OGLE ID      | P₀ (d)   | P₁ (d)   | P₂ (d)   | Δf (d⁻¹) | Pₙ/P₁ | Aₙ/A₁ |
|--------------|----------|----------|----------|----------|-------|-------|
| SC6-6        | 4.841 143| 3.453 214| 3.581 206| -0.010 35| 1.037 06| 0.057 |
| SC15-69667   | 1.904 133| 1.376 951| 1.517 867| -0.067 42| 1.102 34| 0.052 |
| SC17-126402  | 3.209 709| 2.294 942| 1.429 530| 0.263 79 | 0.622 90| 0.040 |

Figure 11. Power spectra of FU/FO-ν double-mode Cepheids after pre-whitening with frequencies of the two radial modes and their linear combinations. Removed radial frequencies indicated by dashed lines.

6 NON-RADIAL MODES IN FU/FO DOUBLE-MODE CEPHEIDS

OGLE-II catalogue of double-mode LMC Cepheids (Soszyński et al. 2000) lists 19 FU/FO pulsators. Discovery of four new members (Section 3) brings the total number of Cepheids in this class to 23. We found non-radial modes in three of them. These are the first detections of non-radial modes in the FU/FO double-mode Cepheids. In the following, we will call these variables FU/FO-ν Cepheids. The stars are listed in Table 6. In the fifth column of the table, we give the frequency difference between the non-radial mode and the first radial overtone, Δf = f₂ - f₁. The pre-whitened power spectra of the FU/FO-ν variables are displayed in Fig. 11. In the first two Cepheids, the secondary mode appears very close to the first overtone radial mode. The values of Δf are very similar to those observed in the FO-ν Cepheids. In the third star, the secondary mode is found at a high frequency, with P₀/P₁ = 0.6229. This is the same puzzling period ratio which has been found in several FO-ν pulsators. Clearly, non-radial modes excited in the FU/FO-ν Cepheids are somehow related to the first radial overtone and their frequencies seem to be drawn from the same distribution as in the case of the FO-ν Cepheids.

7 BLAZHKO EFFECT IN FO/SO DOUBLE-MODE CEPHEIDS

Including one new variable discovered in the current paper (Section 3), there are 56 FO/SO double-mode Cepheids in the OGLE-II catalogue (Soszyński et al. 2000). In one-third of these stars, we detected residual signal close to one or both of the primary (radial) pulsation frequencies. In all cases, however, the secondary frequencies were not resolved from the primary ones. Therefore, the frequency analysis of all FO/SO pulsators was repeated with MACHO b photometry (Allsman & Axelrod 2001), which provided ~2.5 times better frequency resolution than OGLE-II. The MACHO data were cleaned in a manner described in Section 5.2 and then analysed in a standard way. At this stage, the OGLE-II sample was supplemented by 51 additional LMC FO/SO double-mode Cepheids discovered by MACHO outside the area covered by the OGLE-II survey (Alcock et al. 1999, 2000). Our final FO/SO Cepheid sample consisted of 107 objects.

With the MACHO data, residual power close to the primary pulsation frequencies was detected in 37 FO/SO double-mode Cepheids (35 per cent of the sample). In 20 of these stars, which constitute 19 per cent of the sample, we were able to resolve this power into individual frequencies. Two frequencies are considered resolved if they differ by Δf > 0.000715 d⁻¹ (i.e. 1/Δf < 1400 d). All resolved FO/SO Cepheids are listed in Table 7.

7.1 Frequency domain

Resolved FO/SO double-mode Cepheids display a very characteristic frequency pattern. In most cases, in the vicinity of a radial frequency, we detect two secondary peaks. They are located on the opposite sides of the primary (radial) peak and together with the primary peak they form an equally spaced frequency triplet. Such
Table 7. The Blazhko FO/SO double-mode Cepheids in combined OGLE-II + MACHO LMC sample.

| OGLE ID or MACHO ID | \( P_1 \) (d) | \( P_2 \) (d) | \( \Delta f_1 \) (d\(^{-1}\)) | \( \Delta f_2 \) (d\(^{-1}\)) | \( \Delta P \) (d) | \( A_1 \) (mag) | \( A_1^- \) (mag) | \( A_1^{++} \) (mag) | \( A_2 \) (mag) | \( A_2^- \) (mag) | \( A_2^{++} \) (mag) | Remarks |
|---------------------|------------|------------|-----------------|-----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|--------|
| SC1-44845           | 0.951981   | 0.766013   | 0.00127         | 0.00125        | 794         | 0.165       | 0.021       | 0.046       | 0.021       | –           | –           | –       |
| SC-1-285275         | 0.856624   | 0.689234   | 0.00112         | 0.00009        | 902         | 0.177       | 0.031       | 0.040       | 0.022       | 0.008       | –           | –       |
| SC1-335559          | 0.749802   | 0.603647   | 0.00121         | 0.00128        | 789         | 0.227       | 0.019       | 0.062       | 0.023       | –           | –           | –       |
| SC2-55596           | 0.932530   | 0.751387   | 0.00130         | –              | 768         | 0.146       | 0.007       | 0.039       | –           | –           | –           | a       |
| SC6-142093          | 0.896289   | 0.722058   | 0.00091         | 0.00091        | 1103        | 0.154       | 0.028       | 0.043       | 0.017       | 0.012       | –           | –       |
| SC6-267410          | 0.888530   | 0.716773   | –              | 0.00117        | 857         | 0.120       | –           | 0.036       | –           | –           | –           | –       |
| SC8-10158           | 0.689991   | 0.555699   | 0.00086         | 0.00094        | 1082        | 0.175       | 0.011       | 0.031       | 0.019       | – c, s1     | –           | –       |
| SC11-233290         | 1.217532   | 0.978435   | 0.00099         | 0.00104        | 991         | 0.186       | 0.019       | 0.043       | 0.017       | 0.006       | –           | –       |
| SC15-16385          | 0.990417   | 0.795736   | 0.00084         | 0.00088        | 1185        | 0.258       | 0.017       | 0.049       | 0.020       | –           | –           | –       |
| SC20-112788         | 0.737740   | 0.594287   | 0.00072         | 0.00072        | 1390        | 0.165       | 0.062       | 0.019       | 0.015       | 0.016       | –           | –       |
| SC20-138333         | 0.859788   | 0.692171   | 0.00127         | 0.00126        | 793         | 0.189       | 0.013       | 0.060       | 0.018       | –           | –           | –       |
| SC20909.67          | 1.084120   | 0.869989   | 0.00099         | 0.00098        | 1012        | 0.216       | 0.012       | 0.055       | 0.013       | – a, b      | 0.009       | –       |
| 13.58355            | 0.898730   | 0.722801   | 0.00094         | 0.00095        | 1061        | 0.244       | 0.037       | 0.040       | 0.026       | 0.012       | a           | –       |
| 14.95854            | 0.935802   | 0.752812   | 0.00091         | 0.00092        | 1098        | 0.139       | 0.024       | 0.036       | 0.025       | 0.012       | –           | –       |
| 17.246343           | 0.762933   | 0.614047   | 0.00094         | 0.00093        | 1072        | 0.235       | 0.018       | 0.051       | 0.013       | d           | –           | –       |
| 18.223943           | 1.364186   | 1.093324   | 0.00142         | 0.00142        | 706         | 0.230       | 0.035       | 0.063       | 0.017       | – e         | –           | –       |
| 22.5230.61          | 0.633077   | 0.510128   | –              | 0.00124        | 805         | 0.221       | –           | 0.045       | 0.016       | –           | –           | –       |
| 23.2393.45          | 0.734350   | 0.591775   | 0.00129         | 0.00125        | 792         | 0.207       | 0.023       | 0.055       | 0.031       | 0.015       | a           | –       |
| 23.3184.74          | 0.841151   | 0.677788   | 0.00089         | 0.00093        | 1099        | 0.160       | 0.015       | 0.048       | 0.011       | –           | –           | –       |
| 80.70802.618        | 0.715873   | 0.577961   | 0.00109         | 0.00110        | 914         | 0.168       | 0.010       | 0.055       | 0.006       | – d         | –           | –       |

Remarks: (a) unresolved residual power at \( f_1 \); (b) unresolved residual power at \( f_2 \); (c) \( f_1^- \) marginally detected; (d) \( f_2^- \) marginally detected; (e) additional peak at \( \Delta f_2 = -0.00076 \) (d\(^{-1}\)); (s1) same as SC9-286128.

A structure is usually found around both radial modes. An example of this pattern is shown in Fig. 12. In several cases, instead of triplets we find only doublets. This happens only in objects in which secondary frequencies are detected with the lowest signal-to-noise ratio. Therefore, it is most likely that these doublets are in fact parts of triplets, with the missing third component hidden in the noise. When signal-to-noise ratio is high, we encounter an opposite situation. In two FO/SO Cepheids, we find equally spaced frequency quintuplets centred on the second overtone. Consecutive pre-whitening reveals incomplete quintuplets in six other stars, including three variables where they appear around the first overtone. In Fig. 13, we display representative examples of triplets and quintuplets observed in the FO/SO pulsators.

The secondary peaks are always very small, with amplitudes of 15 mmag on average and exceeding 40 mmag only in one star. The side peaks of the multiplets are usually not equal. For the first overtone, the low-frequency components of triplets/quintuplets are always higher than the high-frequency ones. This is also true for 74 per cent of multiplets centred on the second overtone. The separations of components in the two triplets/quintuplets, \( \Delta f_1 \) and \( \Delta f_2 \), are also very small and never exceed 0.00143 d\(^{-1}\). This corresponds to the beat (or modulation) period of \( P_b = 1/\Delta f_1 > 700 \) d. The lower limit for \( \Delta f / (\text{upper limit for } P_b) \) is currently given by resolution of the data. The physical upper limit for the beat (modulation) period \( P_b \) is unknown.

When present around both radial modes, the two multiplets have nearly the same frequency spacings. Specifically, the difference between the two spacings, \( |\Delta f_2 - \Delta f_1| \), is always below \( 8 \times 10^{-5} \) d\(^{-1}\). According to Monte Carlo simulations of Alcock et al. (2000, 2003), for MACHO data such a difference is well within deviations expected from observational noise. In other words, triplets/quintuplets around both radial modes of a FO/SO
Cepheids have the same frequency separation within accuracy of the data.

Frequency separation $\Delta f$ can take any value below 0.00143 $d^{-1}$, but some values are more likely than others. This is clearly visible in Fig. 14. Although statistics are rather small, the distribution of $\Delta f$ shows a bimodal shape with pronounced maxima at $\sim 0.00095$ and $\sim 0.00125$ $d^{-1}$. These correspond to preferred beat (modulation) periods of $P_B \sim 1050$ d and $\sim 800$ d.

7.2 Time domain

The observed triplet/quintuplet frequency pattern is a Fourier representation of a periodic modulation of both radial modes, with the common period $P_B = 1/\Delta f$. In Fig. 15, we display this modulation for one of the FO/SO Cepheids. The plot has been constructed by dividing the data into 10 subsets, each covering 10 per cent of the modulation phases, and then fitting double-frequency Fourier series equation (2) to each subset separately (keeping both frequencies fixed). Fig. 15 shows that both amplitudes and phases of the radial modes undergo modulation. The amplitude variability is much stronger for the second than for the first overtone ($\pm 66$ versus $\pm 17$ per cent change). Minimum amplitude of one mode coincides with maximum amplitude of the other mode. Maximum phase of the first overtone occurs just before maximum of its amplitude. For the second overtone, maximum phase coincides with minimum amplitude. We note that the amplitude of the second overtone is modulated in a distinctively non-sinusoidal way. This explains the presence of quintuplets in the frequency spectrum of this mode.

Characteristic pattern of periodic modulation displayed in Fig. 15 is common to all FO/SO Cepheids of Table 7. The phenomenon is strikingly similar to the Blazhko modulation observed in the RR Lyrae stars (see e.g. Kurtz et al. 2000). Therefore, variables of Table 7 will be called the Blazhko double-mode Cepheids or FO/BL Cepheids.

7.3 FO/SO Cepheids with unresolved residual power

In 17 FO/SO double-mode Cepheids (16 per cent of the sample), we find after pre-whitening a significant remnant power unresolved from the primary pulsation frequencies. This is a signature of slow changes of amplitudes and/or phase of the modes, not resolved within the length of the data. We call these stars FO/SO-PC Cepheids. Their list is given in Table 8.

The amplitude and phase variability of each FO/SO-PC Cepheid was examined with method described in Section 5.2: we divided the data into 10 subsets, each spanning 10 per cent of the total time base and then fitted equation (2) to each subset separately. In some stars, we found only changes of pulsation phases, but in 10 variables we also detected clear changes of both amplitudes. In Fig. 16, we show typical example of such behaviour. The pattern of amplitude and phase variability is very much the same as that displayed in Fig. 15. In particular, the amplitudes of the two radial modes vary in opposite directions. This similarity suggests that most of the FO/SO-PC Cepheids experience the same type of periodic modulation as the Blazhko FO/SO Cepheids listed in Table 7, but on time-scales longer than the current data.

7.4 Incidence rate of the Blazhko effect in FO/SO double-mode Cepheids

Periodic amplitude and phase modulation in FO/SO double-mode Cepheids is by no means a rare phenomenon. According to our analysis, its incidence rate is at least 18.7 $\pm$ 3.8 per cent. The true rate can be perhaps even as high as 35 per cent, if FO/SO-PC variables are confirmed to undergo the Blazhko modulation with very long periods, as we suspect.

In Fig. 17, we present the Petersen diagram for our sample of FO/SO double-mode Cepheids of the LMC. The Blazhko FO/SO...
pulsators and FO/PC pulsators are marked with filled and open circles, respectively. The Blazhko FO/PC variables are not detected at all for $P_1 < 0.6$ d, but for longer periods they intermingle with ‘normal’ double-mode Cepheids. The same can be said about FO/PC variables, except that the short-period cut-off is at $P_1 = 0.5$ d. Apart from amplitude and phase variability, the Blazhko pulsators and PC pulsators do not differ from the rest of FO/PC double-mode Cepheids.

Fig. 17 shows, however, that the incidence rate of the Blazhko effect might depend on pulsation period. To address this point, we display in Fig. 18 the histogram of first overtone periods for our sample of FO/PC Cepheids. The incidence rate of the Blazhko effect peaks at $0.8 < P_1 < 1.0$, where it reaches 48 per cent. It sharply declines below 15 per cent for both longer and shorter periods. While the decline of the incidence rate towards shorter periods (i.e. lower luminosities) can be attributed to the selection effects, the decline towards longer periods must be real.

7.5 What causes the modulation?

Any model of the Blazhko FO/PC double-mode Cepheids has to explain two most basic properties of these stars.

(i) Both radial modes are modulated with the same period.
(ii) The amplitude variations of the two modes are anticorrelated: maximum of one amplitude coincides with minimum of the other.

Many different ideas have been put forward to explain the Blazhko modulation in monoperiodic RR Lyrae stars (see Stothers 2006 for most recent summary). Two primary contenders, which are most popular nowadays, are the oblique magnetic pulsator model (Shibahashi 1995, 2000) and the 1:1 resonance model (Nowakowski & Dziembowski 2001). Another very different scenario has been proposed recently by Stothers (2006). We will argue that all these models fail to account for modulation observed in the FO/PC double-mode Cepheids.
Shibahashi’s oblique magnetic pulsator model assumes the presence of a dipole magnetic field inclined to the rotation axis of the star. The field introduces quadruple distortion to the radial mode. As the star rotates, the distortion is viewed from different aspect angles, which leads to variation of an apparent pulsation amplitude. Somewhat similar idea has been considered by Kovács (1995) (see also Balázs 1959). In his scenario, the distortion is caused by an $\ell = 1$ non-radial mode, which is aligned with a magnetic axis of the star. The non-radial mode is excited through a 1:1 resonance with the radial pulsation. Consequently, frequencies of both modes are exactly synchronized. Both in Shibahashi’s and in Kovács’s models, physical amplitudes of pulsation are constant. Modulation of the observed amplitudes is caused by geometrical effect of rotation. The models naturally explain why both radial modes in FO/SO Cepheids are modulated with the same period. However, in the above scenario both modes should reach maximum amplitudes simultaneously, because they are both distorted in the same way. This is not what we observe. Thus, the Blazhko effect in FO/SO double-mode Cepheids cannot be reproduced by the oblique magnetic pulsator.

The model proposed by Kovács (1995) (see also Nowakowski & Dziembowski 2001) assumes a 1:1 resonant coupling of the radial mode to a ‘pair of non-radial modes’ of $\ell = 1$ and $m = \pm 1$. Such a mechanism generates a triplet of equally spaced frequencies. The physical amplitudes of the modes are constant, but beating of their equidistant frequencies leads to a slow periodic modulation of apparent amplitude and phase of pulsation. The 1:1 resonance model naturally explains triplets observed in frequency spectra of the Blazhko FO/SO Cepheids. Quintuplets can also be easily understood by generalizing the model to coupling with modes of $\ell = 2$. However, in the scenario of Nowakowski & Dziembowski (2001), modulation of each radial mode is an independent process. Therefore, there is no reason why both radial modes should be modulated with exactly the same period (although modulation periods should not be very different). There is also no reason why amplitude variations of the two modes should be in any specific phase relation to...
Table 8. FO/SO-PC double-mode Cepheids in combined OGLE-II + MACHO LMC sample.

| OGLE/MACHO ID | \( P_1 \) (d) | \( P_2 \) (d) | \( A_1 \) (mag) | \( A_2 \) (mag) | Remarks |
|---------------|--------------|--------------|----------------|----------------|---------|
| SC4-176400    | 1.108 872    | 0.895 139    | 0.089          | 0.023          | a,b     |
| SC4-220148    | 0.740 833    | 0.595 133    | 0.222          | 0.063          | a,b     |
| SC7-120511    | 1.251 130    | 1.003 698    | 0.133          | 0.028          | a,b     |
| SC10-204083   | 0.526 261    | 0.422 879    | 0.245          | 0.030          | b       |
| SC16-266808   | 1.352 920    | 1.081 016    | 0.167          | 0.025          | b       |
| SC17-186042   | 0.610 998    | 0.491 213    | 0.179          | 0.039          | a,b     |
| SC20-188572   | 1.015 249    | 0.814 350    | 0.167          | 0.032          | a,b     |
| SC21-12012    | 1.341 463    | 1.074 876    | 0.181          | 0.057          | a,b     |
| 6.6934.67     | 0.920 169    | 0.740 010    | 0.181          | 0.050          | a,b     |
| 11.9348.78    | 0.737 799    | 0.594 411    | 0.200          | 0.041          | a,s1    |
| 14.9098.35    | 0.727 783    | 0.586 544    | 0.208          | 0.032          | a,b     |
| 15.10428.60   | 0.651 848    | 0.525 346    | 0.240          | 0.044          | a       |
| 23.3061.82    | 0.595 737    | 0.480 997    | 0.166          | 0.038          | a,b     |
| 24.2853.69    | 0.708 844    | 0.571 663    | 0.269          | 0.030          | b       |
| 24.2855.80    | 0.593 778    | 0.478 320    | 0.178          | 0.020          | b       |
| 47.2127.102   | 0.578 454    | 0.466 643    | 0.227          | 0.024          | a,b     |
| 80.7079.62    | 1.347 888    | 1.076 366    | 0.177          | 0.042          | b       |

Remarks: (a) residual unresolved power at \( f_1 \); (b) residual unresolved power at \( f_2 \); (s1) same as 14.9348.3191.

Figure 16. Amplitude and phase variations in FO/SO-PC double-mode Cepheid SC17-186042. Symbols are the same as in Fig. 15.

each other, as is observed. Finally, the 1:1 resonance model does not explain why in vast majority of cases modulation is observed either for both radial modes or for none of them.

The new model (or rather an idea) of Stothers (2006) fails to explain the observations of the Blazhko FO/SO Cepheids as well. In this scenario, it is assumed that the turbulent convection in the stellar envelope is cyclically weakened and strengthened by a transient magnetic field generated by dynamo mechanism. As convection has a strong effect on the amplitudes of pulsation (e.g. Feuchtinger 1999), the latter become modulated too. In this picture, we would
resonant mode will not appear as an independent peak in the power spectrum. It is enough that only one of the observed radial modes is modulated by the resonant coupling. This modulation will be carried over to the other radial mode through the so-called cross-saturation effect. Speaking in physical term, the two radial modes compete for the same driving (κ mechanism in the He\(^{+}\) ionization zone). When the amplitude of one mode is suppressed by whatever mechanism (e.g. by a resonance), it allows the other mode to grow. This explains in a very simple and natural way why the two radial modes are modulated with the common period and why their amplitudes are always anticorrelated.

8 CONCLUSIONS

Taking advantage of a large and homogenous photometric data set collected during OGLE-II project, we have performed a systematic frequency analysis of classical Cepheids identified in the LMC (Udalski et al. 1999b; Soszyński et al. 2000). This study has been aimed at finding multiperiodic and non-stationary variables among Pop. I Cepheids and at establishing reliable statistics of different forms of their pulsational behaviour. The main body of our survey has been conducted with OGLE-II photometry, but in cases when frequency resolution turned out to be insufficient, we have resorted to photometry collected by the MACHO experiment.

We have discovered two triple-mode Cepheids which pulsate with three lowest radial overtones simultaneously excited. Triple-mode radial pulsators have been known before, but only among high-amplitude δ Scuti stars (Kovács & Buchler 1994; Antipin 1997b; Handler, Pikull & Diethelm 1998; Wils et al. 2008). This is the first detection of such pulsators among Cepheids. Three additional triple-mode Cepheids have been identified in the LMC by Soszyński et al. (2008a). These rare variables are a very valuable trophy, since their seismological analysis can strongly constrain the stellar evolution theory (Moskalik & Dziembowski 2005).

In 19 per cent of FO/SO double-mode Cepheids, we have detected periodic variability of amplitudes and phases of the two radial modes. Another 16 per cent of FO/SO pulsators are suspected of undergoing the same type of modulation, but on time-scales longer than our data. This phenomenon is analogous to the Blazhko modulation in the RR Lyr stars, which was first discovered a century ago (Blazhko 1907). In the Blazhko FO/SO Cepheids, both modes are modulated with a common period, always in excess of 700 days. The amplitudes of the modes vary in opposite phases, that is, a maximum amplitude of one mode coincides with minimum amplitude of the other. In frequency domain, the modulation manifests itself as splitting of each radial mode into a triplet or sometimes a quintuplet of equidistant peaks, spaced by frequency of the modulation.

The discovery of the modulated FO/SO Cepheids shows that the Blazhko effect and double-mode pulsations are not mutually exclusive. These stars will play a very important role in selecting a proper model to explain the Blazhko phenomenon. Observations of two modulated modes in one star will put any proposed scenario to a very stringent test. Two most popular models, the oblique magnetic pulsator model (Shibahashi 2000) and the 1:1 resonance model (Nowakowski & Dziembowski 2001), have already failed this test and should be ruled out, at least in the case of FO/SO Cepheids.

Perhaps the most exciting result of our survey is the detection of non-radial modes in classical Cepheids. Such modes have been found in 42 overtone pulsators (which constitutes 9 per cent of the sample) and in three FU/FO double-mode pulsators. They have not been found in Cepheids pulsating in the FU mode.
We find two different types of non-radial modes in Cepheids. In most cases, these modes are detected very close to the first radial overtone, but in several stars they appear at considerably higher frequencies. In the latter case, the measured period ratio falls in a narrow range of 0.60–0.64, which places the non-radial mode in the close vicinity of the (unobserved) fourth radial overtone. The observed frequency pattern cannot be explained by any form of amplitude or phase modulation; it is also incompatible with excitation of radial modes. This makes interpretation in terms of non-radial modes the only possibility.

Detection of non-radial modes in classical Cepheids has been claimed before by Kovtyukh et al. (2003). Their claim was based on observations of bumps in spectral line profiles of four Galactic Cepheids. Such structures can be indicative of non-radial oscillations (e.g. Telting 2003). However, the authors have not shown that the observed line profiles vary in a periodic way, or that they vary with their own period different from that of the radial mode. This is an important test, because bumps in the spectral lines can be caused by mechanisms other than non-radial pulsations (e.g. by shocks). Therefore, observations presented by Kovtyukh et al. (2003) are in our opinion inconclusive. The results of frequency analysis discussed in the current paper provide the first solid evidence of non-radial mode excitation in classical Cepheids.

Discovery of non-radial modes in Cepheids poses a major challenge to the stellar pulsation theory. In order to be photometrically detectable, non-radial modes have to be of lower spherical degree, ℓ (Dziembowski 1977). However, linear pulsation calculations predict that in classical Cepheids all modes of ℓ < 4 should be heavily damped (Osaki 1977). This conclusion has recently been confirmed by new calculations of Mulet-Marquis et al. (2007), who have found that all non-radial modes of ℓ < 5 should be damped. We note that this is a very different situation from that encountered in the RR Lyr models, where many modes of ℓ = 1, 2 are linearly excited (Dziembowski & Cassisi 1999).

When writing of this manuscript was almost completed, we learnt about a new preprint on LMC Cepheids by Sozyński et al. (2008b). The authors have analysed data collected during the third phase of the OGLE project (OGLE-III). Among many interesting results, they have also detected secondary periodicities in all subclasses of Pop. I Cepheids. This provides an important confirmation of our results with independent data. The incidence rates given by Sozyński et al. are very similar to ours in case of the double-mode pulsators, but significantly higher than ours in case of the FO Cepheids. This is most likely due to higher number of measurements and much longer time-span of their photometry. Interestingly, Sozyński et al. have detected secondary periodicities in 4 per cent of the FU Cepheids, which we have not. A detailed comparison of incidence rates derived by us and by Sozyński et al. is beyond the scope of the present paper. We only note that such comparison requires some caution, because the latter authors do not discriminate between secondary periodicities which are resolved from the primary frequency and those which are not.

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