The irreversibility and classical mechanics laws

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Abstract

The irreversibility of the dynamics of the conservative systems on example of hard disks and potentially of interacting elements is investigated in terms of laws of classical mechanics. The equation of the motion of interacting systems and the formula, which expresses the entropy through the generalized forces, are obtained. The explanation of irreversibility mechanism is submitted. The intrinsic link between thermodynamics and classical mechanics was analyzed.

Introduction.

Irreversibility determines the contents of the second law of thermodynamics in fundamental physics. According to this law there is function S named entropy, which only grows for isolated systems, achieving a maximum in an equilibrium state. But the potentiality of forces for all four known fundamental interactions of elementary particles is a cause of reversibility of the Newton equation [1] and therefore declares that the natural processes also should be reversible. Thus, the fundamental physics includes parts contradicting each other: there are reversible classical mechanics and irreversible thermodynamics.

The first attempt to resolve these contradictions belongs to Boltzmann. From the $H$-theorem it follows that the many-body systems should equilibrates. But at reception of these results Boltzmann used probabilistic principles. Therefore the contradiction between classical mechanics and statistical physics has not been overcome. Great importance of this problem for all physics explains its big popularity among the famous physicists of the world. The history of its solution is very extensive and at times, dramatic. Therefore let us reference only several works, which precisely enough describe a modern condition of a problem of irreversibility [2-4].

For overcoming of the irreversibility problem I. Prigogine has suggested to try to create the expanded formalism of open systems within the framework of classical mechanics laws [4]. Indeed, by analyzing dynamics properties of a hard disks system it turned out that such formalism appears in a process of the solution of a problem of irreversibility [5-9]. For example, as a result of this analyzing the generalized
Liouville equation and equation of the motion for the systems was obtained. Here on the background of results [5-9] the explanations of the mechanism of irreversibility, interrelation of classical mechanics and thermodynamics are offered. All this determines a way of construction of the expanded formalism of open systems within the framework of classical mechanics laws.

Our investigation is based on the next method. The conservative system of interacting elements is prepare in the nonequilibrium way. This system is splitting on subsystems so that they could be accepted as in equilibrium. The subsystems dynamics under condition of their interactions is analyzed.

First of all we investigate the dynamics of the simplest hard-disks systems. For those we find that so-called generalized forces between selected subsystems should depend from velocities of those. This leads to the key idea about impossibility of solution of the problem of irreversibility in terms of canonical Hamilton or Liouville equations. So, we have built the generalized Hamilton and Liouville equations. Then we deduce how the irreversible dynamics follows from the generalized Liouville equation.

After that we analyze the dynamics of Newtonian systems and deduce the equations of the motion for the underlying subsystems. On the base of these equations we answer the question: why and how the velocity dependence for the generalized forces between the subsystems appears for the case of the potential-type forces between the elements. Next point: the intrinsic link between thermodynamics and classical mechanics was analyzed. The general formula for the entropy of the system in terms of generalized forces is obtained which corresponds to irreversible transformation of the interaction energy of subsystems into the internal energy by means of the generalized forces work.

Irreversibility in a hard-disk system.

We used an approach of pair interaction of disks. Their motion equations are deduced with help of the matrix of pair collisions. As complex plane this matrix is given [5]:

\[ S_{kj} = \begin{pmatrix} a & -ib \\ -ib & a \end{pmatrix} \]  

where \( a = d_{kj} \exp(i\vartheta_{kj}) \); \( b = \beta \exp(i\vartheta_{kj}) \); \( d_{kj} = \cos\vartheta_{kj} \); \( \beta = \sin\vartheta_{kj} \); \( i \) - an imaginary unit; \( k \)- and \( j \)- numbers of colliding disks; \( d_{kj} \)- the impact parameter (IP), determined by distance between centers of colliding disks in the Cartesian plane system of coordinates with axes of \( x \) and \( y \), in which the \( k \)-disk swoops on the lying the \( j \)- disk along the \( x \)- axis. The scattering angle \( \vartheta_{kj} \) varies from 0 to \( \pi \). In consequence of collision the transformation of disks velocities can be presented in such form: \( V_{kj}^+ = S_{kj}V_{kj}^- \) (a), where \( V_{kj}^- \) and \( V_{kj}^+ \) - are bivectors of velocities of \( k \) and \( j \)- disks before \((-\))

and after \((+\)) collisions, correspondingly; \( V_{kj} = \{V_k, V_j\} \), \( V_j = V_{jx} + iV_{jy} \) - are complex velocities of the incident disk and the disk - target with corresponding components to the \( x \)- and \( y \)- axes. The collisions are considered to be central, and friction is neglected. Masses and diameters of disks-"d" are accepted to be equal to 1. Boundary conditions are given as either periodical or in form of hard walls. From (a) we
can obtain equations for the change of velocities of colliding disks [5-7]:

\[
\begin{pmatrix}
\delta V_k \\
\delta V_j
\end{pmatrix} = \varphi_{kj}
\begin{pmatrix}
\Delta^-_{kj} \\
-\Delta^+_{kj}
\end{pmatrix}.
\]

(1)

Here, \(\Delta_{kj} = V_k - V_j\) - are relative velocities, \(\delta V_k = V^+_k - V^-_k\), and \(\delta V_j = V^+_j - V^-_j\) - are changes of disks velocities in consequence of collisions, \(\varphi_{kj} = i\beta e^{i\theta_{kj}}\).

That is, eq. (1) can be presented in the differential form [5]:

\[
\dot{V}_k = \Phi_{kj} \delta(\psi_{kj}(t)) \Delta_{kj}
\]

(2)

where \(\psi_{kj} = [1 - |l_{kj}|]/|\Delta_{kj}|\); \(\delta(\psi_{kj})\)-delta function; \(l_{kj}(t) = z^0_{kj} + \int_0^t \Delta_{kj} dt\) - are distances between centers of colliding disks; \(z^0_{kj} = z^0_k - z^0_j\), \(z^0_k\) and \(z^0_j\) - are initial values of disks coordinates; \(\Phi_{kj} = i(l_{kj}\Delta_{kj})/(|l_{kj}||\Delta_{kj}|)\).

The equation (2) determines a redistribution of kinetic energy between the colliding disks. It is not a Newtonian equation because the forces between the colliding disks depend from their relative velocities.

So, we get the generalized Hamilton equation to be applied for studying the subsystems dynamics [6]:

\[
\frac{\partial H_p}{\partial r_k} = -\dot{p}_k + F^p_k
\]

(3)

\[
\frac{\partial H_p}{\partial p_k} = V_k
\]

(4)

These are the general Hamilton equations for the selected \(p\)-subsystem. The external forces, which acted on \(p\)-subsystem, presented in a right-hand side an eq.(3).

Using eq.(3,4), we can find the Liouville equation for \(p\)-subsystem. For this purpose, let’s to take a generalized current vector - \(J_p = (\dot{r}_k, \dot{p}_k)\) of the \(p\)-subsystem in a phase space. From equations (3,4), we find [6,7]:

\[
\frac{df_p}{dt} = -f_p \sum_{k=1}^T \frac{\partial}{\partial p_k} F^p_k
\]

(5)

Eq.(5) is a Liouville equation for \(p\)-subsystem. It has a formal solution:

\[
f_p = \text{const} \cdot \exp \left[ - \int_0^t \left( \sum_{k=1}^T \frac{\partial}{\partial p_k} F^p_k \right) dt \right].
\]

The equation (5) is obtained from the common reasons. It is suitable for any interaction forces of subsystems. Thus, the eq.(5) is applicable to analyze any open nonequilibrium systems. In particularly, it can be used for explanation of irreversibility.

The right side of eq.(5), \(f_p \sum_{k=1}^T \frac{\partial}{\partial p_k} F^p_k\), is the integral of collisions. This integral can be obtained from the motion equations of the systems element. For example, for a hard disks system it can be found with the help of the eq. (2).

Let’s consider the important interrelation between descriptions of dynamics of separate subsystems and dynamics of system as a whole. As the expression, \(\sum_{p=1}^P \sum_{k=1}^T F^p_k = 0\), is carried out, the next equation
for the full system Lagrangian, \( L_R \), will have a place:
\[
\frac{d}{dt} \frac{\partial L_R}{\partial V_k} - \frac{\partial L_R}{\partial r_k} = 0
\]
and the appropriate Liouville equation:
\[
\frac{\partial f_R}{\partial t} + V_k \frac{\partial f_R}{\partial r_k} + \dot{p}_k \frac{\partial f_R}{\partial p_k} = 0.
\]
The function, \( f_R \), corresponds to the full system. The full system is conservative. Therefore, we have:
\[
\sum_{p=1}^{R} \text{div} J_p = 0.
\]
This expression is equivalent to the next equality:
\[
\frac{d}{dt} (\sum_{p=1}^{R} f_p) = \frac{d}{dt} (\prod_{p=1}^{R} f_p) = 0.
\]
So, \( \prod_{p=1}^{R} f_p = \text{const} \).

In an equilibrium state we have \( \prod_{p=1}^{R} f_p = f_R \). Because the equality \( \sum_{p=1}^{R} \mathbf{F}_p = 0 \) is fulfilled during all time, we have that equality, \( \prod_{p=1}^{R} f_p = f_R \), is a motion integral. It is in agreement with Liouville theorem about conservation of phase space [10].

Dynamics of strongly rarefied systems of potentially interacting elements is also described by the eq. (2). Therefore for those systems the irreversibility is possible as well [7,8].

The dependence of generalized force from velocities - it is a necessary condition for the irreversibility was really to be occurred. Therefore the question about irreversibility for Newtonian systems is reduced to that about the velocity dependence of forces between subsystems.

For hard disks and strongly rarefied systems of potentially interacting elements the presence of irreversibility is predetermined by equation of motion (2). In these systems the interaction forces between the elements is depending from velocities. Therefore it is clear that the generalized forces will depend from velocities also. But forces between elements for Newtonian systems are potentional. Therefore it is necessary to answer the question: how velocity dependence of generalized force is appearing when forces between the elements are independent from velocities. For this purpose in the next part we will obtain of the motion equation for subsystems.

The evolutionary equation of the motion for subsystem.

Let us to analyze Newtonian systems. We take a system with energy: \( E_N = T_N + U_N = \text{const} \), where \( T_N = \frac{1}{2} \sum_{i=1}^{N} v_i^2 \) - is a kinetic energy; \( U_N(r_{ij}) \) - is a potential energy; \( r_{ij} = r_i - r_j \) - is a distance between \( i \) and \( j \) elements; \( N \) - is a number of elements. Masses of elements are accepted to 1.

The equation of the motion for elements is:
\[
v_i = - \sum_{i=1,j\neq i}^{N} \frac{\partial}{\partial r_{ij}} U
\]
It is reversible Newton equation. The irreversibility in Newtonian systems can be compatible with reversibility of the Newton equation in the case if exists the parameter determining irreversibility, con-
cerning which Newton equation is invariant. Such parameter is a velocity of the center of mass of subsystem. The Newton equation for subsystem elements is invariant in respect to this velocity. But an exchange of energy between subsystems is not invariant to this velocity. These energy exchanges are determined by the work of the generalized forces, which are proportional to the speed of change of the velocity of the center of mass of subsystem.

Easiest explanation of the fact, why the Newton equation does not determine the generalized forces, can be made on the example of system consisting of two balls connected by a spring. Shortly, in the laboratory system of coordinate the energy of each ball simultaneously transforms in kinetic energy of other ball and in the potential spring energy. But the Newton equation describes only transformation of kinetic energy into the potential energy. Therefore to solve this task with the help of the Newton equation, it is necessary to exclude energy of the system motion as the whole. For two bodies it is possible to do by transition to the coordinates system of the center of mass. In this coordinate system the change of kinetic energy of a ball is equally to the change (but with opposite value) of the potential energy. But for three and more bodies it is impossible to do. Therefore \( N \geq 3 \) bodies system in general case cannot be reduced to the independent equations of Newton [4, 8-10].

Therefore, for the description of evolution of nonequilibrium system it is necessary to obtain the equation determining generalized forces or the speed of change of the center of mass of system, in results of its interaction with other subsystems. For this purpose we’ll do the following. In the some system of coordinates we shall present kinetic energy of system as energy of subsystem motion as the whole - \( T_{tr}^{N} \), kinetic energy of motion of its elements concerning the center of mass of subsystem - \( \tilde{T}_{ins}^{N} \), a potential energy of subsystems elements - \( \tilde{U}_{ins}^{N} \) and interaction energy with other subsystems. The energy \( E_{ins}^{N} = \tilde{T}_{ins}^{N} + \tilde{U}_{ins}^{N} \) we’ll call a binding energy. At absence of external forces the energy \( T_{tr}^{N} \) and \( E_{ins}^{N} \) are the motion integrals. Their sum is a full energy of the system. We shall assume, that the system is prepared by nonequilibrium way so, that it can be divided into two equilibrium subsystems. Then it is possible to generalize results of their analysis on any nonequilibrium systems.

The following equations for exchange of energy between subsystems have a place [9]:

\[
LV_L \dot{V}_L + \sum_{j=i+1}^{L} \sum_{i=1}^{L-1} \{v_{ij}\left[\frac{\dot{v}_{ij}}{L}\right] + \frac{\partial U}{\partial r_{ij}}\} = - \sum_{j_K=1}^{K} \sum_{i_L=1}^{L} v_{i_L} \frac{\partial U}{\partial r_{i_L,j_K}}
\]  

\[
KV_K \dot{V}_K + \sum_{j=i+1}^{K} \sum_{i=L+1}^{K-1} \{v_{ij}\left[\frac{\dot{v}_{ij}}{K}\right] + \frac{\partial U}{\partial r_{ij}}\} = - \sum_{j_K=1}^{K} \sum_{i_L=1}^{L} v_{j_K} \frac{\partial U}{\partial r_{i_L,j_K}}
\]

Here we take \( LV_L + KV_K = 0 \), \( V_L \) and \( V_K \) is a velocities of the center of mass of \( L \) and \( K \) subsystems; \( L \) and \( K \) are number of elements in subsystems; \( v_{ij} \) - is a relative velocities \( i \) and \( j \) elements; \( L + K = N \). Masses of elements are accepted to 1. Double indexes are accepted for a designation, to what subsystems these elements belong.

The left side in eq.(8,9) determines exchanges of the energy between subsystems. The first members set change kinetic energy of motion of subsystems, \( T_{tr} \) as the whole. The second members describe
transformation of binding energy of the subsystems, $E^{ins}$.

The right side in the equations eq. (8,9) determines potential energy of interaction of subsystems. They cause transformation of kinetic energy of the motion of subsystems, $T^{tr}$, to their binding energy.

In case $L = K$ the equation of the motion for one subsystem can be obtained from eq.(8,9) [9]:

$$
\dot{V}_L = -\frac{1}{V_L}L \sum_{j=i+1}^{L-1} \sum_{i=1}^{L-1} \{ v_{ij} \left[ \frac{\ddot{v}_{ij}}{L} + \frac{\partial U}{\partial r_{ij}} \right] \} - \sum_{j_{jk}=1}^{K} \sum_{i_{jk}=1}^{L} \frac{\partial U}{\partial r_{i_{jk}}} \tag{10}
$$

The equation (10) is determining the generalized force. This force is depending not only from coordinates but from velocities also. Presence of the dependence from the velocities is a necessary condition of irreversibility. Therefore we will call this equation as evolutionary equation. With the help of eq.(2) it is not difficult to show that equation (10) is correct for hard-disks system.

In a limit $N \gg 1$ for enough fine-grained splitting system, the equation (10) determines a field of generalized forces between the subsystems, caused by their relative motion. Irreversibility is connected with decrease of the generalized forces in a result of transition of kinetic energy of motion of a subsystem in the binding energy of subsystems. For any splitting of the equilibrium system the relative velocities of subsystems and the energy flow between them are equal to zero [9, 11].

Let us explain, why the generalized forces depend from velocities of the elements. Inherently, this force determines the change of velocity of the center of mass. But this velocity is determined by distribution of elements velocities. Therefore the interaction force of subsystems should depend not only from the forces of potential interaction of elements, but also from distribution of their velocity.

Laws of conservation of energy and momentum do not forbid for equilibrium system of spontaneous occurrence of an inequality $\dot{T}^{tr} > 0$ for some of subsystems. This inequality means infringement of equilibrium [10, 11] and transition from canonical Hamilton formalism systems description to the description of system inside the framework of the generalized equations. It is connected with appearance of the generalized forces between subsystems. But canonical Liouville equation cannot be transformed by spontaneous way to the generalized type. It is impossible because of conservation of the canonical Hamilton’s formalism. From the physical point of view it is impossible because of stability of an equilibrium state for mixed systems [8]. Such stability is caused by aspiration to zero of the generalized forces arising at a deviation of system from equilibrium [8, 11, and 14]. Hence, the system, having come to equilibrium, never leaves this condition.

**Thermodynamics and classical mechanics.**

Let’s consider interrelation of classical mechanics with thermodynamics. Existence two invariant of the motion-$E^{ins}$ and $T^{tr}$, and also character of their transformation at interaction of the subsystems, determined by the equation (10), allow to catch deep analogy between the equations (8-10) and the basic equation of thermodynamics [14]:

$$
dE = dQ - PdY \tag{11}
$$
Here, according to the common terminology \[14\], \(E\) is internal energy of a subsystem; \(Q\) is a thermal energy; \(P\) is a pressure; \(Y\) is a volume.

The change of energy of the selected subsystem is caused by work of external forces. Therefore to change of full energy of a subsystem corresponds to \(dE\).

The change of kinetic energy of motion of a subsystem as the whole, \(dT^{tr}\), corresponds to the member \(PdY\). Really, \(dT^{tr} = VdV = V\dot{V}dt = Vd\tau = PdY\)

Let’s us determine, what the member in the eq.(11) corresponds to the change of the binding energy of a subsystem. From the virial theorem \[8\] it follows that if potential energy is homogeneous function of the second degree from all radiuses-vectors, then \(\bar{E}^{ins} = 2\bar{T}^{ins} = 2\bar{U}^{ins}\). Feature means averaging on time. Earlier we have obtained, that the binding energy, \(E^{ins}\) increases due to the energy, \(T^{tr}\). But opposite process is impossible. Therefore the member \(Q\) from the eq.(11) is conforming to the energy \(E^{ins}\).

Let the subsystem will consist of \(N\) elements. Then average energy of each element, \(\bar{T}^{ins}_{0} = E^{ins}/N\). Let the binding energy increases on the value \(dQ\). In connection with virial theorem, keeping members of the first order, we shall have:

\[
dQ \approx \bar{T}^{ins}_{0}[d\bar{E}^{ins}/\bar{T}^{ins}_{0}] = \bar{T}^{ins}_{0}[dv/v_0],\]

where \(v_0\) is average velocity of an element, and \(dv\) its change. As the condition the subsystems equilibrium is accepted, we have \(dv/v_0 \sim d\Gamma_m/\Gamma_m\). Here \(\Gamma_m\) is a phase volume of a subsystem, and \(d\Gamma_m\) is increasing \(\Gamma_m\) due to increasing of the subsystem energy on the value, \(dQ\). By keeping members of the first order we’ll have:

\[
dQ \approx \kappa \bar{E}^{ins}_{0}d\Gamma_m/\Gamma_m = \kappa \bar{E}^{ins}_{0}d\ln\Gamma_m,\]

By definition \(d\ln\Gamma_m = dS^{ins}\), where \(S^{ins}\) is entropy \([10,14]\). So, we have near equilibrium: \(dQ \approx \bar{T}^{ins}_{0}dS^{ins} = \kappa \bar{E}^{ins}_{0}dS^{ins}\).

Let’s consider the connection of the field of forces of interaction of subsystems with entropy. As the sum of all forces of interaction of subsystems at any moment is equal to zero, the condition, \(\sum_{m=1}^{R} \dot{T}^{tr}_{m} = 0\) have a place. I.e. energy \(T^{tr}\) is redistributed between subsystems and goes on increasing of the binding energy. It is equivalent to entropy increasing. Relative velocities of subsystems go to zero, and the system equilibrates. Near to equilibrium the right side of the eq.(5) is equal to zero. So, entropy deviation from equilibrium state can be determined with the help of following formula:

\[
\Delta S = \sum_{l=1}^{R} \left( \frac{m_l}{E^{m_l}} \sum_{k=1}^{m_l} \int_s \sum F_{ks}^{m_l} dr_k \right) \]

(12)

Here \(E^{m_l}\) is a kinetic energy of subsystem; \(m_l\) is a number elements in subsystem -"l"; \(R\)- is a number of subsystems; \(s\)- is external disks which collided with internal \(k\) disk along trajectory. The integral is determining the work of the force \(F_{ks}^{m_l}\) during equilibrating.

The eq.(12) connects the generalized force with entropy. Similar connection is established by the formula for KS - entropy in which entropy is expressed through Lyapunov’s exponents \([3]\). Inherently the eq.(12) corresponds to the formula for entropy: \(S = \sum_{l=1}^{R} S_l(E^{ins}_{l} + T^{tr}_{l})\), (see, \([11]\)). Really, if \(E^{ins}_{l} \gg T^{tr}_{l}\), then \(dS = \sum_{l=1}^{R} \frac{\partial S_l}{\partial T^{tr}_{l}} dT^{tr}_{l}\) that corresponds to eq.(12).

Summarizing.
Thus, the dynamics of the open and nonequilibrium systems is determined by the generalized Liouville equation and the evolutionary equation of the subsystems motion. The necessary condition of irreversibility follows from generalized Liouville equation. This condition is dependence of the generalized forces from relative velocities of subsystems of any non-equilibrium system. So, due to the work of the generalized forces, the energy of relative motion of subsystems is transformed to their binding energy by irreversibly way and goes on entropy increasing. But the binding energy cannot be transformed to the energy of relative motion of subsystems. This is essence of the mechanism of equilibration [5-9, 11].

Though dynamics of hard disks can be described without use of potential energy, the essence of the mechanism of equilibration for them is identical to the mechanism for Newtonian systems. Difference only in that that in Newtonian systems the energy of relative motion of subsystems can be transformed to their binding energy due to the potential interaction of subsystems.

In equilibrium we have $\dot{T}_{tr} = 0$ and $V_L = V_K = 0$, therefore the work of interaction force of each subsystem, and also energy $T_{tr}$ are equal to zero. It is equivalent to conditions: $\dot{E}_{ins}_L = \dot{E}_{ins}_K = 0$. Hence, the closeness of system to equilibrium means the closeness to zero of the generalized forces between subsystems. The criterion of equilibration is applicability of the canonical Hamilton and Liouville equations for the description of dynamics of subsystems for any splitting of full system. Only in equilibrium the dynamics is reversible for both the subsystems and for the system as a whole. The dynamics is reversible also for the weak fluctuations because the entropy has a second infinitesimal order in respect to deviations of a system from equilibrium [9, 11, 13].

Criterion for the sufficiently fine-grained splitting of system is the closeness of relation $T_{tr}/E_{ins}$ to zero. It is obvious that the number of elements in a subsystem should be large enough. Otherwise fluctuations become commensurable with errors of a field of generalized forces [11, 13].

Splitting the system onto the equilibrium subsystems is necessary reception for studying the dynamics laws of the many-body systems. It allows setting a field of the generalized forces and to describe it’s with the help of the evolutionary equation (7). In particular, only due to this reception it is possible to find out, that their work is goes by irreversible way on increase of the binding energy of a subsystem.

The basic results received here do not contradict results of the analysis of systems, which was obtained in earlier works by similar method: splitting into the subsystems and studying its dynamics near equilibrium. So, in [11] the macroscopical motion (it is similar to the subsystems motion in our case) was considered. There by using of the method of multipliers Lagrange it has been strictly proved that equilibrium is possible only when $T_{tr} = 0$. Also the validity of the equation for entropy (12) is follows from it.

Why till now it was impossible to solve a problem of irreversibility? From our point of view the explanation is consists in the following. As a rule it was accepted without doubt that the full description of evolution of nonequilibrium systems could be realized on the basis of the Newton equation and canonical Liouville equation. But we find, that the canonical Liouville and Newton equations are not
applicable for describing work of the generalized forces because in nonequilibrium systems they are
dependent from velocities.

Indeed, let us try to determine the generalized force with the help of Newton eq.(7) by summation
of all forces acting on elements of a subsystem. Then we obtain the forces independent from velocities
due to invariancy of the Newton equation concerning Galileo transformation. So, the Newton equation,
does not describe a field of the generalized forces.

Let’s to emphasize, that the substantiation of irreversibility mechanism and connection classical
mechanics and thermodynamics offered here, follows only from laws of conservation of energy and a
momentum.

The evolutionary equation of the motion subsystem along with the generalized Hamilton and Liouville
equations given above, and also the established relations between classical mechanics and thermodynam-
ics, can be put into the base of theoretical description of evolution of the open systems.

References

[1] T. Hooft, G.: Gauge theories of the forces between elementary particles Sci. American, June (1980),
v 242, 90-116.
[2] Lebowitz, J.L.: Boltzmann's entropy and time’s arrow Physics Today(1993), September, 32-38.
[3] Zaslavsky, G.M.: Chaotic dynamic and the origin of Statistical laws, Physics Today August, Part
1, (1999), 39-45.
[4] Petrosky, T. & Prigogine, I.: The Extension of classical Dynamics for unstable Hamiltonian systems,
Computers Math. Applic., V. 34. No. 2-4. (1997), 1-44.
[5] Somsikov, V.M.: Non-recurrence problem in evolution of a hard-disk system, Intern. Jour. Bifurc.
And Chaos, 11, No 11, (2001), 2863-2866.
[6] Somsikov, V.M.,& Matesov, D. S.: The force field evolution into the hard-disks system Izv. MON
RK. Ser. Phys, (2001), 4, 92-105.
[7] Somsikov, V.M.: Some approach to the Analysis of the Open Nonequilibrium systems, AIP, 20,
(2002), 149-156.
[8] Somsikov, V.M.: The approach to the analysis of the dynamic of non-equilibrium open systems and
irreversibility. Internet Preprint arXiv: cond-mat/0311185 v.1 8 Nov., (2003), 11p;
[9] Somsikov, V.M. Evolutionary equation of the motion for interaction systems and thermody-
namics.: Journal of the evolution of the open systems. V 6,N 1, Almaty, (2004), 40-46.
[10] Lanczos, C.: The variation principles of mechanics, Second edition University of Toronto Press,
1962.
[11] Landau, L.D. & Lifshits, Ye.M.: *Statistical physics* Part 1. Nauka, Moscow, 1976.

[12] Landau, L.D. & Lifshits, Ye.M.: *Mechanics*, Nauka, Moscow, 1973.

[13] Lifshits, Ye.M. & Pitaevsky, A.P.: *Phys. kinetics*, Nauka, Moscow, 1979.

[14] Rumer, Yu.B. & Ryvkin, M.Sh.: *Thermodynamics. Stat. Physics and Kinematics*. Moscow, 1977.

[15] Klimontovich, Yu.L.: *Statistical theory of the open system* Moscow, 1995.