EVENT-TRIGGERED ADAPTIVE FAULT-TOLERANT CONTROL FOR MULTI-AGENT SYSTEMS WITH UNKNOWN DISTURBANCES

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ABSTRACT. This paper presents an event-triggered consensus control protocol for a class of multi-agent systems with actuator faults, sensor faults and unknown disturbances. The adaptive neural network compensation control method is introduced to solve the problem of sensor faults. The event-triggered mechanism is developed to reduce the communication burden. In the control design process, the radial basis function neural networks are used to approximate the unknown nonlinear functions, and a nonlinear disturbance observer is used to eliminate the effect of unknown external disturbances. Furthermore, based on the graph theory and Lyapunov stability theory, it is further shown that the consensus tracking errors are semi-globally uniformly ultimately bounded. Finally, the simulation example illustrates the effectiveness of the designed control protocol.

1. INTRODUCTION. Over the last decade, some control problems have been investigated such as finite-time control [4, 12, 15, 21, 39, 51], delay-dependent control [13, 14, 19], networked control [20, 22, 23, 48]. However, the cooperative tracking control problems of multi-agent systems (MASs) have received much more attention [7, 17, 18, 30, 35, 52, 53]. As far as we know, the cooperative tracking control problems of MASs widely exist in the practical applications, such as spacecraft formation [26], animal groups [32] and distributed sensor networks [27]. The consensus problem of MASs is an important topic and the distributed cooperative control problem has been intensively investigated in the past few years, e.g., [5, 6, 9, 37, 49]. In [5], the consensus control problem of MASs with uncertain nonlinear dynamics and external disturbances was addressed, and a robust adaptive control approach was proposed to deal with the approximation errors and external disturbances. Hu et al. [6] considered the consensus control problem of high-order MASs with antagonistic interactions, and a novel control strategy was developed by using the relative state information from its neighbors. The leader-following output consensus control problem for a class of high-order nonlinear MASs was studied in [9]. Zhang et
al. [49] investigated the consensus tracking control problem of nonlinear MASs with state constraints and unknown disturbances. Wu et al. [37] studied the consensus control problem for positive MASs with nonlinear control input, and the global and local consensus control results were analyzed for the case of saturation-type sector input nonlinearities.

Compared with the time-sampling, the event-triggered control strategy can reduce resource consumption and improve resource utilization [3,10,11,40,47,54,55]. Hence, the event-triggered control of MASs has attracted considerable attention. More specifically, Tan et al. [33] investigated the leader-following consensus control problem of MASs by using a distributed event-triggered impulsive control method. You et al. [43] addressed the distributed event-triggered consensus control problem for a class of nonlinear MASs subject to actuator saturation. The output consensus control problem was considered for heterogeneous linear MASs via event-triggered control method in [25]. The event-triggered consensus control problem was studied in [42] for the fractional-order MASs. Wang et al. [34] investigated the tracking consensus control problem for the second-order leader MASs by designing fractional-order follower observer, where a periodic sampled-based data event-triggered control method was employed. In addition, an event-triggered strategy was presented for MASs to solve the cooperative output regulation problem under switching communication topologies in [8]. Zheng et al. [50] studied the event-triggered problem for interconnected switched MASs.

From another point of view, some different fault-tolerant consensus control methods have been studied for MASs. In [44], the output feedback control problem was considered for a class of uncertain nonlinear systems with actuator failures. The problem of consensus tracking control was addressed for MASs with actuator faults under directed networks in [38]. Luo et al. [24] considered the event-triggered consensus control problem for linear MASs with actuator faults. The work in [29] addressed the consensus fault-tolerant control problem for MASs subject to linear fractional transformation uncertain parameters. In [28], the robust fault-tolerant control protocol with actuator saturation was proposed to solve the consensus control problem of MASs. The event-triggered control method for MASs with sensor faults was proposed to solve the consensus control problem [2].

Inspired by the above observations, the event-triggered control problem for MASs with unknown disturbances, sensor and actuator faults are considered in this paper. The main contributions and difficulties of this paper are highlighted as follows:

1) In order to eliminate the effect of unknown external disturbances, the disturbance observer is designed in this paper, and the unknown nonlinear functions are considered in MASs with unknown disturbances, which is a new challenge for the design of the disturbance observer.

2) Different from the works in [24,28,29], the fault problems are considered for the MASs in this paper, which not only contains actuator faults but also includes sensor faults. In addition, the sensor faults problem is solved via the proposed adaptive neural network (NN) compensation control method.

3) An event-triggered fault-tolerant consensus control protocol is provided for MASs. Compared with the results in [41] and [36] that only deal with a limited number of actuator faults, the event-triggered control mechanism is introduced in this paper for MASs with actuator failures which is able to withstand infinite failure times, and the communication resources can also be successfully saved.
The remaining of this paper is arranged as follows. In the next section, we describe the graph theory and give the formulation of the consensus problem of nonlinear MASs. The event-triggered controller is designed in Section 3. The numerical simulation is provided in Section 4, and the conclusions are finally drawn in Section 5.

2. Preliminaries.

2.1. Basic graph theory. The information exchanges among the agents can be modeled by a directed graph $\zeta = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ including a set of agents $\mathcal{V} = \{1, \ldots, N\}$, a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and the relevant adjacency matrix $\mathcal{A} = [a_{i,j}] \in \mathbb{R}^{N \times N}$, which is defined as $a_{i,i} = 0$, $a_{i,j} > 0$ if an edge $(\mathcal{V}_i, \mathcal{V}_j) \in \mathcal{E}$ and $a_{i,j} = 0$, otherwise. The Laplacian matrix $L = (l_{i,j}) = D - A$ and $L \times 1_N = 0$, where $1_N$ is a column vector with 1 for each element, $D = \text{diag}(d_1, \ldots, d_N)$ and the in-degree for agent $i$ is $d_i = \sum_{j=1}^N a_{i,j}$. The $\zeta_c = (\mathcal{V}_c, \mathcal{E}_c)$ is augmented graph with $\mathcal{V}_c = \{0, 1, \ldots, N\}$ and $\mathcal{E}_c \subseteq \mathcal{V}_c \times \mathcal{V}_c$, where 0 is called as leader. The directed graph $\zeta$ is defined as a spanning tree, if there exists at least a directed path from the root to all other agents.

Lemma 2.1. [46] If denote $B = \text{diag}(a_1, 0, \ldots, a_{N,0})$, all eigenvalues of the matrix $L + B$ have positive real part if and only if the graph contains a spanning tree with vertex 0 as the root.

2.2. Problem formulation. For $i = 1, \ldots, N$, the system model of the $i$-th agent is described by

$$
\begin{aligned}
\dot{x}_{i,m} &= x_{i,m+1} + f_{i,m}(x_i) + H_{i,m}(t) \\
\dot{x}_{i,n} &= \sum_{p=1}^M b_{i,p} u_{i,p} + f_{i,n}(x_i) + H_{i,n}(t) \\
y_i &= h_i(x_{i,1})
\end{aligned}
$$

where $x_i = [x_{i,1}, \ldots, x_{i,n}]^T \in \mathbb{R}^n$, $u_i = [u_{i,1}, u_{i,2}, \ldots, u_{i,M}]^T \in \mathbb{R}^M$ and $y_i$ stand for the system state, control input, and the system output, respectively. For $m = 1, 2, \ldots, n - 1$, $f_{i,m}(x_i)$ and $f_{i,n}(x_i)$ are unknown nonlinear functions, $H_{i,m}(t)$ and $H_{i,n}(t)$ are unknown external disturbances, $b_{i,p}$ is a known scalar, and $h_i(x_{i,1}) = k_i(\cdot)x_{i,1} + \rho_i(\cdot)$, where $k_i(\cdot)$ and $\rho_i(\cdot)$ denote the parameters of sensor faults.

Remark 1. The actuator fault model is considered in this paper. Therefore, the controller design procedure is more difficult than the ordinary tracking control problem. Moreover, the sensor faults are considered in each agent output, which is a challenge to solve faults problem.

The actuator fault model [16] is described by

$$
\begin{aligned}
u_{i,p}^S(t) &= k_{i,p,g} u_{i,p}(t) + \bar{u}_{i,p,g}(t), \quad t \in [t_{i,p,g}^S, t_{i,p,g}^E] \\
k_{i,p,g} \bar{u}_{i,p,g}(t) &= 0
\end{aligned}
$$

where $k_{i,p,g} \in [0, 1]$, $t_{i,p,g}^S$, $t_{i,p,g}^E$ are all unknown constants, and $\bar{u}_{i,p,g}(t)$ is an unknown, bounded and piecewise continuous signal, where $p = 1, 2, \ldots, M$, $g = 1, 2, \ldots$.

Define

$$
k_i(p) = \begin{cases} k_{i,p,g} & \text{if } t \in [t_{i,p,g}^S, t_{i,p,g}^E] \\ 1 & \text{if } t \in (t_{i,j,g}^S, t_{i,j,g+1}^E) \end{cases}
$$
Lemma 2.2. Choose $W$ where

$$W = \begin{cases} \bar{u}_{i,p}(t) & \text{if } t \in [t_{i,p,g}^S, t_{i,p,g}^E] \\ 0 & \text{if } t \in (t_{i,p,g}^E, t_{i,p,g+1}^S) \end{cases}$$

The actuator faults model can be expressed as

$$u_{i,p}^E(t) = k_{i,p}(t)u_{i,p}(t) + \bar{u}_{i,p}(t)$$

Assumption 1. There exist an unknown constant $\bar{u}_{i,p}$ such that $0 < \bar{u}_{i,p}(t) \leq u_{i,p}$.

Assumption 2. For the $i$-th agent, up to $M - 1$ actuator faults are allowed at the same time, and the desired control objective can still be achieved by the remaining actuators.

In sensor fault model [1], let $f_{si} = (k_i(\cdot) - 1)x_{i,1} + \rho_i(\cdot)$. Therefore, $y_i$ can be rewritten as

$$y_i = x_{i,1} + f_{si}.$$ 

The time derivative of $y_i$ is presented as

$$\dot{y}_i = \dot{x}_{i,1} + f_{psi}$$

where $f_{psi} = \dot{f}_{si}$.

2.3. Radial basis function neural networks. The following radial basis function neural networks (RBF NNs) are used to estimate nonlinear functions [19]:

$$\bar{f}(Z) = W^* S(Z) + F(Z), \quad \forall Z \in \Omega \subset \mathbb{R}^m, \quad |F(Z)| \leq \varepsilon$$

where $F(Z)$ is the approximation error. $S(Z) = [S_1(Z), S_2(Z), \ldots, S_k(Z)]^T$. Furthermore, $S_i(Z)$ denotes the Gaussian basis function

$$S_i(Z) = \exp \left[ -\frac{(Z - \bar{v}_i)^T (Z - \bar{v}_i)}{v_i^2} \right]$$

where $\bar{v}_i = [\bar{v}_{i,1}, \bar{v}_{i,2}, \ldots, \bar{v}_{i,q}]^T$ is the receptive field’s center and $v_i$ represents the width of the Gaussian basis function with $i = 1, \ldots, k$. Additionally, $W^* = [W_1, W_2, \ldots, W_k]^T \in \mathbb{R}^k$ is weight vector which defined as

$$W^* = \arg \min_{W \in \mathbb{R}^k} \left\{ \sup_{Z \in \Omega} |\bar{f}(Z) - W^T S(Z)| \right\}$$

where $W \in \mathbb{R}^k$.

Lemma 2.2. [31] Choose $S(\bar{x}_c) = [S_1(\bar{x}_c), S_2(\bar{x}_c), \ldots, S_k(\bar{x}_c)]^T$ where $\bar{x}_c = [x_1, \ldots, x_c]^T$ is the RBF NNs’ basis function vector. For any positive integer $p \leq c$, the following inequality holds

$$\|S(\bar{x}_c)\|^2 \leq \|S(\bar{x}_p)\|^2$$

2.4. Disturbance observer design. The unknown nonlinear function $f_{i,k}(x_i)$ is estimated by RBF NNs as follows

$$f_{i,k}(x_i) = M_{i,k}^* S_{i,k}(x_i) + v_{i,k}(x_i), \quad \forall x_i \in \Omega_{x_i}$$

where $M_{i,k}^*$ means the weight vector, $S_{i,k}(x_i)$ denotes the Gaussian basis function, and $v_{i,k}(x_i)$ is approximation error satisfying $|v_{i,k}(x_i)| \leq v_{i,k,0}$ with $v_{i,k,0} > 0$. 
Then, we can get
\[
\begin{align*}
\dot{x}_{i,m} &= x_{i,m+1} + M_{i,m}^T S_{i,m}(x_i) + v_{i,m}(x_i) + H_{i,m}(t) \\
\dot{x}_{i,n} &= \sum_{p=1}^{\bar{M}} \bar{b}_{i,p}(k_{i,p} u_{i_p} + \bar{u}_{i_p}) + M_{i,n}^T S_{i,n}(x_i) + v_{i,n}(x_i) + H_{i,n}(t) \\
y_i &= h_i(x_{i,1})
\end{align*}
\]

The disturbance observer is designed as follows
\[
\begin{align*}
\dot{\hat{z}}_{i,k} &= \lambda_{i,k}(x_{i,k} - \hat{Q}_{i,k}) \\
\dot{\hat{Q}}_{i,k} &= x_{i,k+1} + \hat{z}_{i,k} + \bar{M}_{i,k}^T S_{i,k}(x_i)
\end{align*}
\]
where \(\lambda_{i,k} > 0\), \(\bar{M}_{i,k} = M_{i,k}^* - \bar{M}_{i,k}\) and \(x_{i,n+1} = u_i\). Besides, we can separately get the disturbance estimation and the disturbance estimation error as follows
\[
\begin{align*}
\dot{\hat{z}}_{i,k} &= \lambda_{i,k}(\hat{x}_{i,k} - \hat{Q}_{i,k}) \\
&= \lambda_{i,k} \left( H_{i,k}(t) - \hat{z}_{i,k} + \bar{M}_{i,k}^T S_{i,k}(x_i) + \hat{v}_{i,k}(x_i) \right) \\
\dot{\hat{z}}_{i,k} &= -\lambda_{i,k} \hat{z}_{i,k} + \bar{H}_{i,k}(t) - \lambda_{i,k}(\bar{M}_{i,k} S_{i,k}(x_i) + \hat{v}_{i,k}(x_i)) \\
\dot{\hat{z}}_{i,k} &= H_{i,k}(t) - \hat{z}_{i,k}
\end{align*}
\]

3. **Control law design and stability analysis.** The tracking errors are presented as
\[
e_{i,1} = \sum_{j=1}^{N} a_{i,j}(y_j - y_j) + a_{i,0}(y_i - \bar{y}_r) \\
e_{i,\beta} = x_{i,\beta} - a_{i,\beta-1}
\]
where \(i = 1, ..., N\), \(\beta = 2, ..., n\). \(a_{i,\beta-1}\) is the virtual controller, and \(a_{i,0} \geq 0\). Define
\[
\theta_i = \max \left\{ \| W_{i,k}^* \|, k = 1, ..., n \|, i = 1, ..., N \right\}
\]
where \(\theta_i = \theta_i - \hat{\theta}_i\).

**Lemma 3.1.** [45] Define \(e_1 = \text{col}\{e_{1,1}, e_{2,1}, ..., e_{N,1}\}\), \(y = \text{col}\{y_1, y_2, ..., y_N\}\), \(\bar{y}_r = 1_N \otimes \bar{y}_r\), then
\[
\|y - \bar{y}_r\| \leq \frac{\|e_1\|}{\sigma(L + B)}
\]
where \(\sigma(L + B)\) is minimum singular value.

**Step 1:** The derivative of \(e_{i,1}\) yields
\[
\begin{align*}
\dot{e}_{i,1} &= (d_i + a_{i,0})(x_{i,2} + f_{i,1}(x_i) + H_{i,1}(t) + f_{psi}) \\
&- \sum_{j \in N_i} a_{i,j}(x_{j,2} + f_{j,1}(x_j) + f_{psi} + H_{j,1}(t)) - a_{i,0}\bar{y}_r
\end{align*}
\]
Consider the Lyapunov candidate as
\[
V_{i,1} = \frac{1}{2} e_{i,1}^2 + \frac{1}{2\gamma_i} \bar{\theta}_i^2 + \frac{\bar{z}_{i,1}^2}{2} + \frac{1}{2\eta_{i,1}} \bar{M}_{i,1}^T \bar{M}_{i,1}
\]
where \(\gamma_i > 0\) and \(\eta_{i,1} > 0\). Then, we get
\[
\dot{V}_{i,1} = e_{i,1} [(d_i + a_{i,0})(x_{i,2} + f_{i,1}(x_i) + H_{i,1}(t) + f_{psi}) - a_{i,0}\bar{y}_r]
\]
where \( p \in H \).

By using RBF NNs, the unknown nonlinear function \( \tilde{f}_{i,1}(Z_{i,1}) \) is estimated as follows

\[
\tilde{f}_{i,1}(Z_{i,1}) = (d_i + a_{i,0})(f_{i,1}(x_j) + H_{i,1}(t) + f_{ps}) - \sum_{j \in N_i} a_{i,j}(x_{j,2} + f_{j,1}(x_j)) + f_{ps} + H_{i,1}(t) + \sum_{j \in N_i} a_{i,j}(x_{j,2} + f_{j,1}(x_j))
\]

where \( Z_{i,1} = [x_{i,1}^T, x_{j,1}^T, y_0, \hat{v}_{i,1}, \hat{z}_{i,1}, \hat{z}_{j,1}]^T \) and \( \tilde{f}_{i,1}(Z_{i,1}) \) denotes the estimation error which satisfies \( 0 < |\tilde{f}_{i,1}(Z_{i,1})| \leq \varepsilon_{i,1} \). Based on the Young’s inequality and Lemma 2.2, one has

\[
e_{i,1} \tilde{f}_{i,1}(Z_{i,1}) \leq \frac{\|W_{s,1}\|^2}{2p_{i,1}^2} e_{i,1}^2 S_{i,1}(Z_{i,1})S_{i,1}(Z_{i,1}) + \frac{p_{i,1}^2}{2} + \frac{e_{i,1}^2}{2} + \frac{e_{i,1}^2}{2}
\]

where \( p_{i,1} > 0 \) and \( \tilde{Z}_{i,1} = [x_{i,1}, x_{j,1}, y_0, \hat{y}_0, \hat{v}_{i,1}, \hat{z}_{i,1}, \hat{z}_{j,1}]^T \). Then, it yields,

\[
e_{i,1}(d_i + a_{i,0}) \hat{z}_{i,1} \leq \frac{d_i + a_{i,0}}{2} e_{i,1} + \frac{d_i + a_{i,0}}{2} \hat{z}_{i,1}^2
\]

\[
\sum_{j \in N_i} a_{i,j} \hat{z}_{j,1} e_{i,1} \leq \frac{d_i}{2} e_{i,1} + \sum_{j \in N_i} a_{i,j} \hat{z}_{j,1}^2
\]

\[-\lambda_{i,1} \hat{z}_{i,1} v_{i,1}(x_i) \leq \frac{\lambda_{i,1}^2 v_{i,1}^2}{2}
\]

Define

\[
\alpha_{i,1} = \frac{1}{d_i + a_{i,0}} \left[ -k_{i,1} e_{i,1} - \frac{e_{i,1}}{2} - \frac{\hat{\theta}_i}{2p_{i,1}^2} e_{i,1} S_{i,1}(\tilde{Z}_{i,1})S_{i,1}(\tilde{Z}_{i,1}) \right]
\]

\[
\dot{\hat{M}}_{i,1} = -\eta_{i,1} \lambda_{i,1} \hat{z}_{i,1} S_{i,1}(\tilde{Z}_{i,1}) - \lambda_{i,1} \hat{M}_{i,1}
\]

where \( k_{i,1} > 0 \) and \( \lambda_{i,1} > 0 \). By using the above mentioned RBF NNs, the Young’s inequality, the virtual controller \( \alpha_{i,1} \) and the adaptive law \( \dot{\hat{M}}_{i,1} \), the following formula holds

\[
\dot{\hat{V}}_{i,1} \leq -k_{i,1} e_{i,1}^2 + (d_i + a_{i,0}) e_{i,2} + \hat{\theta}_i \left( \frac{1}{2p_{i,1}^2} e_{i,1}^2 S_{i,1}(\tilde{Z}_{i,1})S_{i,1}(\tilde{Z}_{i,1}) - \frac{\hat{\theta}_i}{\lambda_{i,1}} \right)
\]
Step \( m (m = 2, \ldots, n - 1) \): The derivatives of error \( e_{i,m} \) yields

\[
\dot{e}_{i,m} = x_{i,m+1} + f_{i,m}(x_i) - \dot{\alpha}_{i,m-1} + H_{i,m}(t)
\]

where

\[
\dot{\alpha}_{i,m-1} = \sum_{\vartheta=1}^{m-1} \frac{\partial \alpha_{i,m-1}}{\partial x_{i,\vartheta}} [x_{i,\vartheta+1} + f_{i,\vartheta}(x_i) + H_{i,\vartheta}(t)] + \sum_{\vartheta=0}^{m-1} \frac{\partial \alpha_{i,m-1}}{\partial y_{0}} y_{0}^{(\vartheta+1)}
\]

\[
+ \sum_{\vartheta=1}^{m-1} \sum_{j \in N_i} \frac{\partial \alpha_{i,m-1}}{\partial x_{j,\vartheta}} [x_{j,\vartheta+1} + f_{j,\vartheta}(x_j) + H_{j,\vartheta}(t)] + \frac{\partial \alpha_{i,m-1}}{\partial \theta_{i}} \dot{\theta}_{i}
\]

\[
+ \sum_{\vartheta=1}^{m-1} \lambda_{i,\vartheta}(\dot{z}_{i,\vartheta} + \tilde{M}_{i,\vartheta} \tilde{S}_{i,\vartheta}(x_i) + v_{i,\vartheta}(x_i))
\]

\[
+ \sum_{\vartheta=1}^{m-1} \sum_{j \in N_i} \lambda_{j,\vartheta}(\dot{\tilde{z}}_{j,\vartheta} + \tilde{M}_{j,\vartheta} \tilde{S}_{j,\vartheta}(x_j) + v_{j,\vartheta}(x_j))
\]

Define

\[
V_{i,m} = V_{i,m-1} + \frac{e_{i,m}^{2}}{2} + \frac{\dot{z}_{i,m}^{2}}{2} + \frac{\tilde{M}_{i,m}^{T} \tilde{M}_{i,m}}{2\eta_{i,m}}
\]

where \( \eta_{i,m} > 0 \). The \( \dot{V}_{i,m} \) is derived as

\[
\dot{V}_{i,m} = e_{i,m}[x_{i,m+1} + f_{i,m}(x_{i,m}) + H_{i,m}(t) - \dot{\alpha}_{i,m-1}]
\]

\[
+ \dot{z}_{i,m}(-\lambda_{i,m} \dot{z}_{i,m} + \tilde{H}_{i,m} - \lambda_{i,m} \tilde{M}_{i,m}^{T} \tilde{S}_{i,m}(x_{i}))
\]

\[-\lambda_{i,m} v_{i,m}(x_{i}) - \frac{1}{\eta_{i,m}} \tilde{M}_{i,m}^{T} \tilde{M}_{i,m} + \dot{V}_{i,m-1}
\]

\[
= \dot{V}_{i,m-1} + e_{i,m}(e_{i,m+1} + \alpha_{i,m} + \tilde{f}_{i,m}(Z_{i,m}) + \tilde{z}_{i,m}) - \tilde{m}_{i,m} e_{i,m}^{2}
\]

\[-\dot{e}_{i,m} e_{i,m}^{2} - e_{i,m} \sum_{\vartheta=1}^{m-1} \sum_{j \in N_i} \left( \frac{\partial \alpha_{i,m-1}}{\partial x_{j,\vartheta}} + \frac{\partial \alpha_{i,m-1}}{\partial \tilde{z}_{j,\vartheta}} \lambda_{j,\vartheta} \right) \dot{z}_{j,\vartheta}
\]

\[+
\sum_{\vartheta=1}^{m-1} \left( \frac{\partial \alpha_{i,m-1}}{\partial x_{i,\vartheta}} + \frac{\partial \alpha_{i,m-1}}{\partial \tilde{z}_{i,\vartheta}} \lambda_{i,\vartheta} \right) \dot{z}_{i,\vartheta} - \lambda_{i,m} \tilde{z}_{i,m} v_{i,m}(x_{i})
\]

\[-\frac{1}{\eta_{i,m}} \tilde{M}_{i,m}^{T} \tilde{M}_{i,m} \dot{\theta}_{i} + e_{i,m}(-\varphi_{i,m}(Z_{i,m}) - \frac{\partial \alpha_{i,m-1}}{\partial \theta_{i}}) \dot{z}_{i,m} - \lambda_{i,m} \tilde{z}_{i,m} + \tilde{H}_{i,m}
\]

with

\[
\tilde{f}_{i,m}(Z_{i,m}) = f_{i,m}(x_i) - \sum_{\vartheta=1}^{m-1} \frac{\partial \alpha_{i,m-1}}{\partial x_{i,\vartheta}} [x_{i,\vartheta+1} + f_{i,\vartheta}(x_i)] - \sum_{\vartheta=0}^{m-1} \frac{\partial \alpha_{i,m-1}}{\partial y_{0}} y_{0}^{(\vartheta+1)}
\]

\[-\sum_{\vartheta=1}^{m-1} \sum_{j \in N_i} \frac{\partial \alpha_{i,m-1}}{\partial x_{j,\vartheta}} [x_{j,\vartheta+1} + f_{j,\vartheta}(x_j)] - \sum_{\vartheta=1}^{m-1} \sum_{j \in N_i} \frac{\partial \alpha_{i,m-1}}{\partial x_{j,\vartheta}} \dot{z}_{j,\vartheta}
\]
where for \( m = 2 \), \( \tilde{d}_i + \tilde{a}_i,0 = d_i + a_i,0 \), and for \( m > 2 \), \( \tilde{d}_i + \tilde{a}_i,0 = 1 \). Define \( c_i > 0 \) and \( m_i > 0 \), let

\[
\begin{align*}
\hat{c}_{i,m} &= \sum_{\vartheta=1}^{m-1} \left( \frac{1}{4c_i} \right) \left[ (\frac{\partial \alpha_{i,m-1}}{\partial x_i, \vartheta}) + (\frac{\partial \alpha_{i,m-1}}{\partial z_j, \vartheta}) \lambda_{i,\vartheta} \right]^2 \\
\hat{m}_{i,m} &= \sum_{\vartheta=1}^{m-1} \sum_{j \in \mathcal{N}_i} \left( \frac{1}{4m_i} \right) \left[ (\frac{\partial \alpha_{i,m-1}}{\partial x_j, \vartheta}) + (\frac{\partial \alpha_{i,m-1}}{\partial z_j, \vartheta}) \lambda_{j,\vartheta} \right]^2
\end{align*}
\]

According to Young's inequality, it has

\[
\begin{align*}
-e_{i,m} \sum_{\vartheta=1}^{m-1} \left( \frac{\partial \alpha_{i,m-1}}{\partial x_i, \vartheta} + \frac{\partial \alpha_{i,m-1}}{\partial z_j, \vartheta} \lambda_{i,\vartheta} \right) \tilde{z}_{i,\vartheta} &\leq \hat{c}_{i,m} e_{i,m}^2 + \sum_{\vartheta=1}^{m-1} c_{i,\vartheta} \tilde{z}_{i,\vartheta}^2 \\
-e_{i,m} \sum_{\vartheta=1}^{m-1} \sum_{j \in \mathcal{N}_i} \left( \frac{\partial \alpha_{i,m-1}}{\partial x_j, \vartheta} + \frac{\partial \alpha_{i,m-1}}{\partial z_j, \vartheta} \lambda_{j,\vartheta} \right) \tilde{z}_{j,\vartheta} &\leq \hat{m}_{i,m} e_{i,m}^2 + \sum_{\vartheta=1}^{m-1} \sum_{j \in \mathcal{N}_i} n_{i} \tilde{z}_{j,\vartheta}^2 \\
e_{i,m} \tilde{z}_{i,m} &\leq \frac{e_{i,m}}{2} + \frac{\tilde{z}_{i,m}}{2} + \frac{\lambda_{i,m}^2 \nu_{i,m}^2}{2}
\end{align*}
\]

The function \( \varphi_{i,m}(Z_{i,m})(2 \leq m \leq n - 1) \) is used to estimate \( \frac{\partial \alpha_{i,m-1}}{\partial \theta_i} \), where

\[
\begin{align*}
\varphi_{i,m}(Z_{i,m}) = - \rho \frac{\partial \alpha_{i,m-1}}{\partial \theta_i} - \sum_{\vartheta=2}^{m} e_{i,m} \frac{\gamma_i}{2p_i^m} \left( \partial \alpha_{i,\vartheta-1} \right) \frac{\partial \alpha_{i,\vartheta-1}}{\partial \theta_i} + \sum_{\vartheta=1}^{m-1} \frac{\partial \alpha_{i,m-1}}{\partial \theta_i} \frac{\gamma_i}{2p_i^m} e_{i,m}^2 S_{i,\vartheta}^T S_{i,\vartheta}
\end{align*}
\]

The RBF NNs are used to compensate \( \hat{f}_{i,m} \) with \( \bar{Z}_{i,m} = \left[ x_i^T, z_j^T, \hat{y}_0, \hat{\theta}_i, \hat{z}_m^T, \hat{z}_j^T \right]^T \) \((j \in \mathcal{N}_i)\). Hence, one has

\[
\begin{align*}
\hat{f}_{i,m} &= W_{i,m}^{*T} S_{i,m}(Z_{i,m}) + F_{i,m}(Z_{i,m})
\end{align*}
\]

where \( 0 < |F_{i,m}(Z_{i,m})| \leq \varepsilon_{i,m} \). According to Young's inequality, we can get

\[
\begin{align*}
e_{i,m} \hat{f}_{i,m}(Z_{i,m}) &\leq \frac{\theta_{i,m}^2}{2p_i^m} e_{i,m}^2 S_{i,m}^T(\bar{Z}_{i,m}) S_{i,m}(\bar{Z}_{i,m}) \\
&\leq \frac{\mu_i^2}{2} + \frac{\varepsilon_{i,m}^2}{2} + \frac{\bar{\varepsilon}_{i,m}^2}{2}
\end{align*}
\]
where \( p_{i,m} > 0 \) and \( \hat{Z}_{i,m} = [x_{i,m}^T, x_{j,m}^T, \theta_0, \theta_i, z_{i,m}^T, z_{j,m}^T]^T \). Design

\[
\alpha_{i,m} = -k_{i,m}e_{i,m} - \frac{e_{i,m}}{2} - \frac{\hat{\theta}_i}{2p_{i,m}}e_{i,m}S_{i,m}(\hat{Z}_{i,m})S_{i,m}(\hat{Z}_{i,m})
\]

\[
\dot{M}_{i,m} = -\eta_{i,m}\hat{\lambda}_{i,m}\hat{\theta}_{i,m}S_{i,m}(x_i) - l_{i,m}\dot{M}_{i,m}
\]

where \( k_{i,m} > 0 \), \( p_{i,m} > 0 \) and \( l_{i,m} > 0 \). Similarly, one obtains

\[
\dot{V}_{i,m} \leq e_{i,m}e_{i,m+1} - \sum_{\vartheta = 1}^{m}k_{i,\vartheta}e_{i,\vartheta}^2 + \sum_{\vartheta = 1}^{m}(\varphi_{i,\vartheta}^2 + \frac{\varphi_{i,\vartheta}^2}{2}) + \sum_{\vartheta = 1}^{m-1}e_{i,\vartheta}(m - \vartheta)e_{i,\vartheta}^2
\]

\[
+ \frac{\hat{\theta}_i}{2} \sum_{\vartheta = 1}^{m} \frac{\gamma_{i,\vartheta}}{2} e_{i,\vartheta}^2 S_{i,\vartheta}(\hat{Z}_{i,\vartheta})S_{i,\vartheta}(\hat{Z}_{i,\vartheta}) - \hat{\theta}_i) + \sum_{\vartheta = 1}^{m-1} \sum_{j \in N_i} (m - \vartheta)m_{i,j}\vartheta_{i,j,\vartheta}^2
\]

\[
+ \sum_{\vartheta = 1}^{m} \frac{\varphi_{i,\vartheta}}{2} (\lambda_{i,\vartheta}\varphi_{i,\vartheta} + \frac{1}{2} \varphi_{i,\vartheta}^2 + H_{i,\vartheta}) + \sum_{\vartheta = 1}^{m} \frac{l_{i,\vartheta}}{2} M_{i,\vartheta}^T M_{i,\vartheta}
\]

\[
+ \sum_{\vartheta = 1}^{m} e_{i,\vartheta}[\varphi_{i,m}(Z_{i,m}) - \frac{\partial \alpha_{i,m-1}}{\partial \theta_i}]\]

**Step n:** The dynamics of \( \varphi_{i,n} \) is shown as

\[
\dot{\varphi}_{i,n} = \sum_{p=1}^{M} b_{i,p}(k_{i,p}u_{i,p} + \bar{u}_{i,p}) + f_i,n(x_i) + H_{i,n}(t) - \dot{\alpha}_{i,n-1}
\]

where

\[
\dot{\alpha}_{i,n-1} = \sum_{\vartheta = 1}^{n-1} \sum_{j \in N_i} \frac{\partial \alpha_{i,n-1}}{\partial \tilde{z}_{j,\vartheta}} \lambda_{i,\vartheta}(\tilde{z}_{i,\vartheta} + M_{j,\vartheta}^T S_{i,\vartheta}(x_j) + v_{j,\vartheta}(x_j))
\]

\[
+ \sum_{\vartheta = 1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial \tilde{z}_{i,\vartheta}} \lambda_{i,\vartheta}(\tilde{z}_{i,\vartheta} + M_{i,\vartheta}^T S_{i,\vartheta}(x_i) + v_{i,\vartheta}(x_i))
\]

\[
+ \sum_{\vartheta = 1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial x_{i,\vartheta}} [f_i,\vartheta(x_i) + H_{i,\vartheta}(t)] + \sum_{\vartheta = 0}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial \theta_{\vartheta}^{(\vartheta+1)}} y_0^{(\vartheta+1)}
\]

Then, the triggering event is defined for the \( p \)-th actuator as

\[
u_{i,p}(t) = \omega_{i,p}(t^p_k), \quad \forall t \in [t^p_k, t^p_{k+1})
\]

\[
t^p_{k+1} = \inf \{t > t^p_k || \Lambda_{i,p} \geq \Delta_{i,p} | u_{i,p} + \epsilon_{i,p} \}
\]

where the mechanism error is \( \Lambda_{i,p} = \omega_{i,p}(t) - u_{i,p}(t), 0 < \Delta_{i,p} < 1 \) and \( \epsilon_{i,p} > 0 \) with \( p = 1, ..., M \).

**Remark 2.** For \( \forall t \in [t^p_k, t^p_{k+1}) \), the \( p \)-th actuator’s input signal holds \( \omega_{i,p}(t^p_k) \). If (4) is triggered, the time instant will be marked as \( t^p_{k+1} \).
Furthermore, $u_{i,p}$ can be presented as

$$u_{i,p} = \frac{\omega_{i,p} - \tau_{i,2}^p U_{i,p}}{1 + \tau_{i,1}^p \Delta_{i,p}}$$

where $|\tau_{i,1}^k(t)| \leq 1$ and $|\tau_{i,2}^p(t)| \leq 1$. Let $\Pi_i = \inf_{t \geq 0} \sum_{p=1}^m b_{i,p} k_{i,p}$, $\Upsilon_i = (\frac{1}{\Pi_i})$, $P_i = \Upsilon_i \sup_{t \geq 0} \|\varpi_i\|$ with $\varpi_i = \left[ b_{i,1} u_{i,1} - \frac{b_{i,p} \tau_{i,2}^p U_{i,p} z_i}{1 + \tau_{i,1}^p \Delta_{i,p}}, \ldots, b_{i,p} u_{i,p} - \frac{b_{i,p} \tau_{i,2}^p U_{i,p} z_i}{1 + \tau_{i,1}^p \Delta_{i,p}} \right]^T$. Moreover, define $\tilde{I} = [1, \ldots, 1]^T$. The Lyapunov function $V_{i,n}$ is chosen as

$${V_{i,n}} = V_{i,n-1} + \frac{1}{2} \tilde{I}^T \tilde{P} \tilde{I} + \sum_{p=1}^m b_{i,p} \frac{\omega_{i,p}}{1 + \tau_{i,1}^p \Delta_{i,p}} + e_{i,n}(f_{i,n}(x_i) + H_{i,n}(t) - \alpha_{i,n}) - \dot{\tilde{z}}_{i,n}(\lambda_{i,n} \tilde{z}_{i,n} + \tilde{H}_{i,n} - \lambda_{i,n} \tilde{z}_{i,n}) - \frac{\Pi_i}{\tilde{h}_{i,1}} \tilde{Y}_i \tilde{\dot{Y}}_i$$

Then, we have

$$\begin{align*}
\dot{V}_{i,n} &\leq \dot{V}_{i,n-1} + e_{i,n} \sum_{p=1}^m b_{i,p} \frac{\omega_{i,p}}{1 + \tau_{i,1}^p \Delta_{i,p}} + e_{i,n} \left[ f_{i,n}(x_i) + H_{i,n}(t) - \alpha_{i,n-1} - \dot{\tilde{z}}_{i,n}(\lambda_{i,n} \tilde{z}_{i,n} + \tilde{H}_{i,n} - \lambda_{i,n} \tilde{z}_{i,n}) - \frac{\Pi_i}{\tilde{h}_{i,1}} \tilde{Y}_i \tilde{\dot{Y}}_i \right] \\
&\quad - \dot{e}_{i,n} \frac{\hat{m}_{i,n} e_{i,n}}{2} - e_{i,n} \sum_{p=1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial x_{i,\theta}} + \frac{\partial \alpha_{i,n-1}}{\partial \tilde{z}_{i,\theta}} \tilde{z}_{i,\theta} \\
&\quad + \sum_{\theta=1}^{n-1} \sum_{j \in N_i} \left( \frac{\partial \alpha_{i,n-1}}{\partial x_{j,\theta}} + \frac{\partial \alpha_{i,n-1}}{\partial \tilde{z}_{j,\theta}} \tilde{z}_{j,\theta} \right) - \lambda_{i,n} \tilde{z}_{i,n} \varpi_{i,n}(x_i) \\
&\quad - \frac{1}{\tilde{h}_{i,1}} \tilde{M}_{i,n} \tilde{\dot{Y}}_i - e_{i,n} \varpi_{i,n} \tilde{e} \tilde{I} + e_{i,n} \tilde{f}_{i,n}(Z_{i,n})
\end{align*}$$

with

$$\begin{align*}
\tilde{f}_{i,n}(Z_{i,n}) &= f_{i,n}(x_i) - \sum_{\theta=1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial x_{i,\theta}} [x_{i,\theta+1} + f_{i,\theta}(x_i)] - \sum_{\theta=1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial \tilde{z}_{i,\theta}} \tilde{z}_{i,\theta}
\end{align*}$$
According to Young’s inequality, it has further

\[ -\varphi_{i,n}(Z_{i,n}) \leq \sum_{\vartheta=1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial x_{j,\vartheta}} \left[ x_{i,\vartheta+1} + f_{i,\vartheta}(x_{j}) \right] - \sum_{\vartheta=1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial y_0} y_0^{(\vartheta+1)} \]

Further, one has

\[ e_{i,n} \varpi_i \leq \Pi_i P_i |e_{i,n}| = \Pi_i \hat{P}_i |e_{i,n}| + \Pi_i \hat{P}_i |e_{i,n}| \]

Then, according to \( 0 < |g| - F \tanh(g/\nu_i) \leq 0.2785\nu_i \), one has

\[ \Pi_i \hat{P}_i |e_{i,n}| \leq \Pi_i (e_{i,n} \hat{P}_i \tanh(e_{i,n} P_i) + 0.2785\nu_i) \]

where \( \nu_i > 0 \) and \( g \in \mathbb{R} \). It can be obtained that

\[ e_{i,n} \sum_{p=1}^{M} b_{i,p} \frac{\omega_{i,p}}{1 + \tau_{i,p}^2 \Delta_{i,p}} \leq \Pi_i e_{i,n}(\hat{Y}_i \alpha_{i,n} \tanh(\frac{\hat{Y}_i \alpha_{i,n}}{\nu_i}) + \hat{P}_i \tanh(\frac{e_{i,n} \hat{P}_i}{\nu_i})) \]

\[ \leq \Pi_i \hat{Y}_i |e_{i,n} \alpha_{i,n}| + 0.2785\nu_i \Pi_i - \Pi_i e_{i,n} \hat{P}_i \tanh(\frac{e_{i,n} \hat{P}_i}{\nu_i}) \]

Furthermore, we have

\[ -\Pi_i \hat{Y}_i |e_{i,n} \alpha_{i,n}| = \Pi_i \hat{Y}_i |e_{i,n} \alpha_{i,n}| - |e_{i,n} \alpha_{i,n}| \]

Let

\[ \hat{c}_{i,n} = \sum_{\vartheta=1}^{n-1} \frac{1}{4e_i} \left[ \left( \frac{\partial \alpha_{i,n-1}}{\partial x_{i,\vartheta}} \right)^2 + \left( \frac{\partial \alpha_{i,n-1}}{\partial z_{i,\vartheta}} \right)^2 \right] \]

\[ \hat{m}_{i,n} = \sum_{\vartheta=1}^{n-1} \frac{1}{4m_i} \left[ \left( \frac{\partial \alpha_{i,n-1}}{\partial x_{j,\vartheta}} \right)^2 + \left( \frac{\partial \alpha_{i,n-1}}{\partial z_{j,\vartheta}} \right)^2 \right] \]

According to Young’s inequality, it has

\[ -e_{i,n} \sum_{\vartheta=1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial x_{i,\vartheta}} \hat{z}_{i,\vartheta} \leq \hat{c}_{i,n} e_{i,n}^2 + \sum_{\vartheta=1}^{n-1} c_i \hat{z}_{i,\vartheta}^2 \]

\[ -e_{i,n} \sum_{\vartheta=1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial x_{j,\vartheta}} \hat{z}_{j,\vartheta} \leq \hat{m}_{i,n} e_{i,n}^2 + \sum_{\vartheta=1}^{n-1} m_i \hat{z}_{j,\vartheta}^2 \]

\[ e_{i,n} \hat{z}_{i,n} \leq \frac{e_{i,n}^2}{2} + \frac{z_{i,n}^2}{2} \]

\[ \leq \frac{1}{2} \left( e_{i,n}^2 + \frac{z_{i,n}^2}{2} \right) \]
The function $\varphi_{i,n}(Z_{i,n})$ is used to compensate $(\frac{\partial \alpha_{i,n-1}}{\partial \hat{\theta}_i})\hat{\theta}_i$, where

$$\varphi_{i,n}(Z_{i,n}) = -\rho_1 \frac{\partial \alpha_{i,n-1}}{\partial \hat{\theta}_i} - \sum_{\vartheta=2}^{n} \frac{e_i,n_{\vartheta_i}^2}{2\rho_i,n_{\vartheta_i}^2} \frac{\partial \alpha_{i,\vartheta-1}}{\partial \hat{\theta}_i} + \sum_{\vartheta=1}^{n-1} \frac{\partial \alpha_{i,n-1}}{\partial \hat{\theta}_i} \frac{\gamma_i}{2\rho_i,n_{\vartheta_i}^2} e_i,n_{\vartheta_i}^2 S_{i,\vartheta}^T S_{i,\vartheta}$$

The RBF NNs are employed to approximate $\bar{f}_{i,n}$:

$$\bar{f}_{i,n} = W_{i,n}^T S_{i,n}(Z_{i,n}) + \delta_{i,n}(Z_{i,n})$$

where $0 < |\delta_{i,n}(Z_{i,n})| \leq \varepsilon_{i,n}$, $Z_{i,n} = \left[x_i^T, x_j^T, \dot{\theta}_i, \ddot{\theta}_i, \dddot{\theta}_i, \dddot{x}_i, \dddot{x}_j \right]^T (j \in N_i)$. Similarly, it yields,

$$e_{i,n} \bar{f}_{i,n}(Z_{i,n}) \leq \frac{\theta_{i,n}}{2\rho_i,n_{\vartheta_i}^2} e_i,n_{\vartheta_i}^2 S_{i,n}(\bar{Z}_{i,n}) S_{i,n}(\bar{Z}_{i,n}) + \frac{P_{i,n}^2}{2} + \frac{\varepsilon_{i,n}^2}{2}$$

where $\rho_{i,n} > 0$ and $Z_{i,n} = \bar{Z}_{i,n}$. The adaptive laws are given as

$$\dot{\hat{\theta}}_i = \sum_{\vartheta=1}^{n} \frac{\gamma_i}{2\rho_i,n_{\vartheta_i}^2} e_i,n_{\vartheta_i}^2 S_{i,\vartheta}^T (Z_{i,\vartheta}) S_{i,\vartheta}(Z_{i,\vartheta}) - \rho_1 \hat{\theta}_i$$

$$\dot{\hat{M}}_{i,n} = -\eta_{i,n} l_{i,n} \ddot{z}_{i,n} S_{i,n}(x_i) - l_{i,n} \hat{M}_{i,n}$$

$$\dot{\hat{\gamma}}_i = h_{i,1} |e_{i,n}\alpha_{i,n}| - h_{i,2} \hat{\gamma}_i$$

$$\dot{\hat{P}}_i = \gamma_{i,1} |e_{i,n}| - \gamma_{i,2} \hat{P}_i$$

(7)

where $\eta_{i,n}$, $l_{i,n}$, $\rho_i$, $h_{i,1}$, $h_{i,2}$, $g_{i,1}$ and $g_{i,2}$ are positive parameters. The following inequality holds

$$\dot{V}_{i,n} \leq -\sum_{\vartheta=1}^{n} k_{i,\vartheta} e_{i,\vartheta}^2 + 0.557 \nu_i \Pi_i + \frac{h_{i,2} \Pi_i}{h_{i,1}} \dddot{\hat{\gamma}}_i + \frac{(d_i + a_{i,0}) + 1}{2} \dddot{z}_{i,1}^2$$

$$+ \frac{\gamma_{i,1}^2}{\gamma_i} \dot{\hat{P}}_i \hat{P}_i + \frac{\rho_i}{\gamma_i} \hat{\theta}_i \dddot{\gamma}_i + \sum_{\vartheta=1}^{n-1} (n - \vartheta) c_{i,n} \dddot{z}_{i,\vartheta}^2 + \frac{1}{2} \sum_{j \in N_i} a_{i,j} \dddot{z}_{j,1}^2$$

$$+ \sum_{\vartheta=1}^{n-1} (n - \vartheta) r_{i,n} \dddot{z}_{i,\vartheta}^2 + \sum_{\vartheta=1}^{n-1} \dddot{z}_{i,\vartheta} (-\lambda_{i,n} \dddot{z}_{i,\vartheta} + \dddot{H}_{i,\vartheta}) + \sum_{\vartheta=2}^{n} \dddot{z}_{i,\vartheta}^2$$

$$+ \sum_{\vartheta=1}^{n} \frac{P_{i,n}^2}{2} + \frac{\varepsilon_{i,n}^2}{2} + \frac{\lambda_{i,0}^2 \dddot{v}_{i,0}^2}{2} + \sum_{\vartheta=1}^{n} l_{i,n} \hat{M}_{i,\vartheta}^T \hat{M}_{i,\vartheta}$$

(8)

**Theorem 3.2.** Considering the MASs with actuator faults, sensor faults and unknown disturbances (1), under the event-triggering rules (3), (4), the event-triggered controller (5), and the adaptive laws (7), then, it can be realized that all the signals are semi-globally uniformly ultimately bounded and the tracking errors arrive at a bounded compact set of the origin.

**Proof.** Design the Lyapunov function as

$$V = \sum_{i=1}^{N} V_{i,n}$$
According to the Young's inequality, we have
\[ \tilde{\theta}_i \leq -\frac{1}{2} \tilde{\theta}_i^2 + \frac{1}{2} \tilde{\theta}_i^2 \]
\[ M_i^T \tilde{M}_i,\theta \leq -\frac{1}{2} M_i^T \tilde{M}_i,\theta + \frac{1}{2} M_i^T M_i^* \]
\[ h_i \tilde{P}_i \tilde{Y}_i \leq \frac{h_i \Pi_i}{2 h_i} \tilde{Y}_i^2 - h_i \Pi_i \tilde{Y}_i^2 \]
\[ \gamma_i \tilde{P}_i \tilde{P}_i \leq \frac{\gamma_i \Pi_i}{2 \gamma_i} \tilde{P}_i^2 - \frac{\gamma_i \Pi_i}{2 \gamma_i} \tilde{P}_i^2 \]

The derivative of \( V \) can be described as
\[ \dot{V} \leq \sum_{i=1}^{n} \sum_{\theta=1}^{n} \left\{ \lambda_i - \sum_{j \in N_i} \frac{a_{i,j}}{2} - (n-1)c_i - \sum_{j \in N_i} (n-1)r_j - \frac{d_i + a_{i,0} + 1}{2} \right\} \frac{\tilde{z}_{i,1}^2}{\tilde{z}_{i,1}} \]
\[ + \sum_{i=1}^{n} \sum_{\theta=1}^{n} \left[ \frac{l_i,0}{2 \eta_i} M_i^T \tilde{M}_i,\theta + \sum_{i=1}^{n} \sum_{\theta=1}^{n} l_i,\theta M_i^T M_i^* + \sum_{i=1}^{n} \sum_{\theta=1}^{n} \tilde{z}_{i,\theta} \tilde{H}_i,\theta \right] \]

Define the following inequality
\[ \lambda_{i,1} > \sum_{j \in N_i} \frac{a_{i,j}}{2} + (n-1)c_i + \sum_{j \in N_i} (n-1)r_j + \frac{d_i + a_{i,0} + 1}{2} \]
\[ \lambda_{i,q} > 1 + (n-q)c_i - \sum_{j \in N_i} (n-q)r_j \]
\[ \lambda_{i,n} > 1 \]
when \( \| \tilde{z}_{i,\theta} \| \geq \frac{\| \tilde{H}_i,\theta \|}{\lambda_{i,0}} \), and
\[ \lambda_{i,0} = \min \left\{ \lambda_{i,1} - \sum_{j \in N_i} \frac{a_{i,j}}{2} - (n-1)c_i - \sum_{j \in N_i} (n-1)r_j - \frac{d_i + a_{i,0} + 1}{2}, \right\} \]
\[ \lambda_{i,q} - (n-q)c_i - \sum_{j \in N_i} (n-q)r_j - 1, \lambda_{i,n} - 1 \}

where \( q = 2, \ldots, n-1 \). Define
\[ \mu = \min_{1 \leq i \leq N} \left\{ \sum_{\theta=1}^{n} 2 \eta_i,\gamma_i \Pi_i, h_i \Pi_i, 2(\lambda_{i,1} - (n-1)c_i \right. \]
\[ - \sum_{j \in N_i} (n-1)r_j - \frac{d_i + a_{i,0} + 1}{2} - \sum_{j \in N_i} \frac{a_{j,i}}{2} - \lambda_{i,0} \), \]
\[ 2(\lambda_{i,q} - (n-q)c_i - \sum_{j \in N_i} (n-q)r_j - 1 - \lambda_{i,0}), \]
\[ 2(\lambda_{i,n} - 1 - \lambda_{i,0}), l_{i,\vartheta} \]  
and
\[ \Theta = \sum_{i=1}^{N} 0.557 \nu_i \Pi_i + \sum_{i=1}^{N} \frac{\eta_i}{2} \theta_i^2 + \sum_{i=1}^{N} \sum_{\vartheta=1}^{n} \left( \frac{p_{i,\vartheta,\vartheta}^2}{2} + \frac{e_{i,\vartheta,\vartheta}^2}{2} \right) \]
\[ + \sum_{i=1}^{N} \sum_{\vartheta=1}^{n} \frac{l_{i,\vartheta}}{2 \eta_i,\vartheta} M_{i,\vartheta}^T M_{i,\vartheta} + \frac{h_{i,2} \Pi_i}{2 h_{i,1}} \gamma_i^2 + \frac{\gamma_{i,2} \Pi_i}{2 \gamma_{i,1}} p_i^2 \]

where \( q = 2, \ldots, n - 1 \). Thus, we can obtain
\[ \dot{V}(t) \leq -\mu V(t) + \Theta \]

Then, one has
\[ \frac{1}{2} \epsilon_{i,1}^2 \leq V \leq e^{-\mu t} V(0) + \frac{\Theta}{\mu} (1 - e^{-\mu t}) \]

According to Lemma 3.1, we get
\[ \lim_{t \to \infty} \| y - \bar{y} \| \leq \frac{1}{\sigma(L + B)} \sqrt{\frac{2\Theta}{\mu}} \]

By recalling \( \Lambda_{i,p} = \omega_{i,p}(t) - u_{i,p}(t) \), \( \forall t \in [t_k^p, t_{k+1}^p] \), we can get
\[ \frac{d}{dt} |\Lambda_i(t)| = \frac{d}{dt} (\Lambda_i(t) \times \Lambda_i(t))^{\frac{1}{2}} = \text{sign}(\Lambda_i(t)) \dot{\Lambda_i}(t) \leq |\omega_{i,p}(t)| \]

Since \( \omega_{i,p}(t) \) is differentiable and bounded, \( |\omega_{i,p}(t)| \leq \nabla_{i,p} \), where \( \nabla_{i,p} > 0 \). By noting that \( e_{i,p}(t_p) = 0 \) and \( \lim_{t \to t_{k+1}} e_{i,p}(t_p) = m_{i,p} \), we obtain that the lower bound of inter-execution intervals \( t^* \) must satisfy \( t^* \geq \frac{m_{i,p}}{\nabla_{i,p}} \), the Zeno-behavior is successfully avoided. \( \square \)

**Remark 3.** In this paper, the actuator faults, sensor faults and unknown disturbances have been considered in MASs with event-triggered rules, and the proposed control strategy in this paper can also save communication resources.

**4. Simulation results.** This paper provides the following simulation example to verify the theoretical analysis. In this section, the dynamics of MASs are given as
\[
\begin{align*}
\dot{x}_{i,1} &= x_{i,2} - \sin(x_{i,2})x_{i,1}^2 + H_{i,1} \\
\dot{x}_{i,2} &= \sum_{j=1}^{2} b_{i,j} k_{i,j} u_{i,j} + \sin(x_{i,1}) x_{i,2} + H_{i,2} \\
y_p &= 0.5 \sin(2.46t)
\end{align*}
\]

The following fault model is considered:
\[
\begin{align*}
\begin{cases}
\begin{array}{ll}
u_{i,1,1} & \text{if } t \in [2k, 2k+1) \\
0.5 \nu_{i,1,1} & \text{if } t \in [2k+1, 2k+2) \\
0.3 \nu_{i,1,2} & \text{if } t \in [2k, 2k+1) \\
0 & \text{if } t \in [2k+1, 2k+2)
\end{array}
\end{cases}
\end{align*}
\]
Figure 1. Topology of communication graph

Figure 2. Output trajectories of followers and the leader

Figure 3. The trajectories of tracking errors

\[ u_{2,1}^F = \begin{cases} u_{2,1} & \text{if } t \in [2k, 2k + 1) \\ 0.6u_{2,1} & \text{if } t \in [2k + 1, 2k + 2) \end{cases} \]

\[ u_{2,2}^F = \begin{cases} u_{2,2} & \text{if } t \in [0, 1) \\ 0.2 + 0.2 \sin(t) & \text{if } t \in [1, \infty) \end{cases} \]
Figure 4. The trajectories of event-triggered controllers

Figure 5. The trajectories of errors between disturbances and disturbance observers

Figure 6. The trajectories of errors between disturbances and disturbance observers
where $k = 0, 1, \ldots$ Apparently, the above matrices are given as follows,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

From Fig. 1, we know $B = \text{diag}(1, 0, 1, 0)$. The simulation results are shown by Figs. 2-7 and the correlative design parameters are chosen as $\lambda_{1,1} = 401$, $\lambda_{2,1} = 401.6$, $\lambda_{3,1} = 402.1$, $\lambda_{4,1} = 402$, $\lambda_{1,2} = 410.5$, $\lambda_{2,2} = 400.5$, $\lambda_{3,2} = 400.6$, $\lambda_{4,2} = 410.5$, $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 4.96$, $k_{1,1} = 108$, $k_{2,1} = 255$, $k_{3,1} = 105$, $k_{4,1} = 109$, $k_{1,2} = k_{2,2} = k_{3,2} = k_{4,2} = 66.5$, $\rho_1 = \rho_2 = \rho_3 = \rho_4 = 0.59$, $p_{i,1} = p_{i,2} = 11.3$, $\eta_{i,1} = \eta_{i,2} = 36.4$, $l_{i,1} = 1.7$, $l_{i,2} = 1.5$, $\nu_i = 2.6$, $\Delta_i = 0.28$, $h_{i,1} = 2.71$, $h_{i,2} = 3.45$, $g_{i,1} = 2.9$, $g_{i,2} = 3.2$.

The simulation results are presented in Figs. 2-7. The outputs of followers can track the leader’s signal in Fig. 2, and 3 shows the trajectories of tracking errors. Fig. 4 displays the event-triggered controller. Figs. 5 and 6 describe the errors of the disturbances. The trigger times and the trigger intervals of the agents are drawn in Fig. 7. It can be seen from the above simulation results that the event-triggered adaptive consensus of MASs with actuator faults and unknown disturbances are basically realized.

5. **Conclusion.** The consensus problem in event-triggered MASs with actuator faults and unknown disturbances has been considered in this paper. Furthermore,
the event-triggered controller has been proposed to avoid some unnecessary triggering operations. In addition, a disturbance observer has been proposed to solve the unknown disturbances. Finally, simulation results have been utilized to prove the effectiveness of the proposed method. In our future research, based on the fuzzy logic systems arbitrary approximation property, we will extend the results of this paper to the fuzzy fault detection and switching topology for MASs.

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