Stackelberg scheme in vendor – buyer inventory model for imperfect quality product with partial backordering

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Abstract. In this paper, several new mathematical analysis approaches in probabilistic inventory model which incorporate both imperfect quality items existence and lead-free demands are proposed using non-cooperative games. Considered the monopolistic relationship between vendor and buyer in some real inventory problem, the vendor is set to be a leader who determines the Stackelberg’s optimal move after observing the optimal responses from the buyer as a follower. Optimum condition using Karush – Khun – Tucker conditions is applied with Stackelberg scheme to get optimum value of the system.

1. Introduction
Multi echelon model has been used supply chain analysis by many authors, for example [6-11]. Furthermore, there are many kinds of result about game theory framework in inventory management model, for example result from, Chachon and Nettesine, Alei et al, Elyasi et al, Esmaelli, Setiawan and Esmaelli et al [1,3-5,13-16]. Chen et al. [2] discuss a dual-channel supply chain with a manufacturer as a Stackelberg leader and the manufacturer as the follower. To get more realistic model, we present our model as a two-echelon vendor-buyer inventory non-cooperative games under a probabilistic condition in which both the vendor and the buyer are assumed as a player. Adopted from the real situation in the industrial relationship, which the vendor holds a monopolistic policy for their buyer in common circumstances, especially if they have many big firms and strong influence in their business relationship, so Stackelberg equilibrium would be applied in that system. We take into account strongly possibility of the appearance of deteriorating item in a lot of arrival items in buyer’s side. That may be caused by imperfect processes of production, imperfect transportation process from the vendor’s side to the buyer’s side. Research in this paper deals with statistical conditions such as imperfect quality, lead-free demands and partial back-ordering in the decentralized model when the monopolistic condition holds.

2. Assumptions and Notations
This research is based to the two-echelon inventory with one vendor and one buyer with one type of item in supply chain system. Vendor as a leader and buyer as a follower and follow Stackelberg process. Shortage conditions are allowed and partially back-ordered with fraction of the demand $\beta, \beta \in [0,1]$. The reorder point $r = \text{expected demand during lead time} + \text{safety stock} (S)$ and $S = k \times (\text{standard deviation of lead time})$, so $r = DL + k\sigma \sqrt{L}$, where $k$ is safety factor. The lead time length $L$ consists of $m$ mutually independent components. The $i^{th}$ component has the normal duration $b_i$, the minimum duration $a_i$ and the crashing cost per unit time $c_i$ and the lead time crashing cost per cycle $C(L)$ for a given $L \in (L_i, L_{i-1}]$ is given by $C(L) = c_i (L_{i-1} - L) + \sum_{j=1}^{m} c_j (b_j - a_j)$.
If we adding we have

\[ ETC_b = \frac{(y-1)^{-1}(C(I) + S_b + nF + c_{ib} Q + [\pi + \pi_0 (1-\beta)]E[(X-r)])D}{Q} \]

\[ + h_{b1} \left( \frac{(y-1)Q(\theta-1)}{2} + k\sigma \sqrt{L} - (\beta - 1)E[(X-r)] \right) + h_{b2}Q\gamma\theta \]

\[ \frac{(h_{b1}D - h_{b2}Q)\gamma\theta}{2x(1-\gamma)(1-\theta)} \]

\[ \Leftrightarrow \]

\[ ETC_v = \left( S_v + nF + c_{vb} Q + C(L) + \left( \frac{\sqrt{1+k^2 - k}}{2} \right) \frac{\pi_0 \sqrt{L}}{D} \right) \frac{(y-1)Q(\theta-1)}{2} \]

\[ + h_{b1} \left( \frac{(y-1)Q(\theta-1)}{2} + k\sigma \sqrt{L} - (\beta - 1)E[(X-r)] \right) + h_{b2}Q\gamma\theta \]

\[ \frac{(h_{b1}D - h_{b2}Q)\gamma\theta}{2x(1-\gamma)(1-\theta)} \]

Average level inventory for vendor can be obtained as follow

\[ -(2P)^{-1}n^2Q^2 + nQ^2P^{-1} + (2D)^{-1}(n^2 - n)(Q^2 - \gamma Q^2)(1-\theta)n \]

we have holding cost term in cost function of the vendor

\[ h_v \left( \frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + (2D)^{-1}(n^2 - n)(Q^2 - \gamma Q^2)(1-\theta) \right) \]

If we adding setup and warranty cost for defective items, then \( ETC_v(Q, n) \) is given by

\[ ETC_v = \frac{E[C_v]}{E[n^T]} = \frac{D}{nQ(1-\gamma)(1-\theta)} \left( S_v + c_{vw} Q\gamma\theta + h_v \left( \frac{nQ^2}{P} - \frac{n^2Q^2}{2P} + (1-n)Q^2(\gamma - 1)(\theta - 1)n \right) \right) \]
First, leader may take his/her decision first and then follower reacts by their optimal move. Using optimality condition, for fixed value $n$ and $L$, then individual EOQ of buyer as follower is obtained by the following equation

$$Q_{fl} = \frac{C(L) + D \left( S_b + nF + \frac{\bar{n}\sigma\sqrt{L}}{2} \left( 1 + k_{fl}^2 - k_{fl} \right) \right)}{\sqrt{(\gamma - 1) \frac{\bar{h}_1(1 - \gamma)(1 - \theta)}{2} + \bar{h}_2 \gamma \theta + \frac{(\bar{h}_2 D - \bar{h}_2 D\gamma \theta)}{2(1 - \gamma)(1 - \theta)} (\theta - 1)}}$$ \hspace{1cm} (7)

$$k_{fl} = \sqrt{\frac{A \left( 1 - \frac{\bar{h}_1}{A} \right)^2}{\bar{h}_1 \left( 2 - \frac{\bar{h}_1}{A} \right)}}, \quad A = \left[ \frac{D\overline{\pi}}{2Q_{fl}(1 - \gamma)(1 - \theta)} + \frac{h_{b1}(1 - \beta)}{2} \right]$$ \hspace{1cm} (8)

Actually, Equation (8) is not necessarily optimal value for the inventory system, but EOQ from Equation (7) is used by the leader to determine his optimal move which gives a strong impact on the system and the situation that follower must agree with that move:

$$Q_{ld} = \sqrt{\frac{S_v}{\frac{n_{ld}}{P} \left( \frac{n_{ld}}{2} + 1 \right) + \frac{(n_{ld} - 1)}{2}}}$$ \hspace{1cm} (9)

$$n_{ld} = \frac{2(S_v + c_{yw} \gamma Q)}{h_v \left( 1 + (1 - \gamma)(1 - \theta) \right)}$$ \hspace{1cm} (10)

Then, vendor observe the buyer’s independent move, to get their optimal solution. The form of the optimal solution can be presented in implicit equation only. We have to prepare with numerical solution to get the further information about decision variables

$$\sqrt{\frac{S_v}{\frac{n}{P} \left( \frac{n}{2} + 1 \right) + \frac{1}{2}(n - 1)}} = \frac{D \left( S_b + n^s F + \frac{\bar{n}\sigma\sqrt{L}}{2} \left( 1 + k^s_1^2 - k^s_1 \right) + C(L) \right)}{\sqrt{(\gamma - 1) \frac{1}{2} \frac{h_{b1}(1 - \gamma)(1 - \theta)}{2} + \frac{h_{b2} \gamma \theta + \frac{(\bar{h}_2 D - \bar{h}_2 D \gamma \theta)}{2(1 - \gamma)(1 - \theta)} (\theta - 1)}} (1 - \theta)}$$ \hspace{1cm} (11)

Because there is no safety factor $k$ in vendor’s side, so the information about safety factor $k^s$ is referred to the optimum value of $k$ which taken by the buyer as a follower in the system, or $k^s = k_{fl}$.

$$k^s = \frac{A \left( 1 - \frac{\bar{h}_1}{A} \right)^2}{\bar{h}_1 \left( 2 - \frac{\bar{h}_1}{A} \right)}, \quad A = \left[ \frac{D\overline{\pi}}{2Q_{fl}(1 - \gamma)(1 - \theta)} + \frac{h_{b1}(1 - \beta)}{2} \right]$$ \hspace{1cm} (12)

We can find the other optimum decision variable from Equation (11) that is $n^s$ and $Q^s$ by substitute the value of $k^s$ from Equation (12) into Equation (11).

$$\frac{D \left( C(L) + S_b + n^s F + \frac{\bar{n}\sigma\sqrt{L}}{2} \left( 1 + k^s_1^2 - k^s_1 \right) \right)}{\frac{n}{P} \left( \frac{n}{2} + 1 \right) + \frac{(n - 1)}{2}} = 0$$ \hspace{1cm} (13)

where $V = S_v(1 - \gamma)(1 - \theta) \left[ \frac{h_{b1}(1 - \gamma)(1 - \theta)}{2} + \frac{h_{b2} \gamma \theta + \frac{(\bar{h}_2 D - \bar{h}_2 D \gamma \theta)}{2(1 - \gamma)(1 - \theta)} (\theta - 1)}{2(1 - \gamma)(1 - \theta)} \right]$ and $k^s$ is referred to the optimal value of $n$ on the vendor’s side. Hence, the other formula of $n^s$ as follows
Finally, because the exact or approximation value of \( n^S \) can be obtained from Equation (13), so the value of \( Q^S \) can be obtained from Equation (14). Furthermore, \( (Q^S, n^S, k^S) \) is the Stackelberg value of our system. This value also as optimal value of our system.

4. Conclusion
From our research, several new analysis in probabilistic supply chain using Stackelberg scheme. We use a probabilistic vendor-buyer as a basic model. We follow the lead free demand assumption. Shortages are allowed and will be full by the vendor via the partial back ordering process. In this analysis, we consider with leader-follower inventory system. Stackelberg game is applied to get optimum value decision variable of the system. Based on the mathematical process, we get the mathematical formula of Stackelberg game of the optimum value for decision variable of the system according to the Stackelberg game process. This result can be the preliminary result for further research, like numerical and sensitivity analysis.

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