Dynamic behavior of pile foundations under vertical and lateral vibrations

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ABSTRACT
Pile foundations supporting machines, buildings under seismic effect, and wind turbines are subjected to dynamic loads. Under such conditions, there is a necessity to evaluate the dynamic behavior of piles. Therefore, a 3D numerical modeling technique is needed to consider the complicated dynamic interaction between the piles and soil (pile-soil interaction) and between adjacent piles in the same group (pile-soil-pile interaction). To validate the results of 3D numerical simulation of piles, the results obtained from the numerical model has been compared to the measurements of a selected case study. The 3D finite element model has also been used to evaluate the impedance parameters and induced peak displacements. The study is performed for single piles and pile groups. The different techniques to model the piles are evaluated. In addition, the effect of related parameters, such as excitation force frequency \((f)\), soil modulus of elasticity \((E_s)\), pile slenderness Ratio \((L/D)\), dimensionless spacing ratio \((S/D)\) and pile group size \((n_g)\) have been studied.

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Introduction

In engineering practice, foundations can be subjected to dynamic loads in addition to static loads. In this case, the use of deep foundations instead of shallow foundations may become a necessity. Pile foundations are used to avoid bearing capacity failure, increase the dynamic stiffness of the system, or decrease the dynamic oscillations. However, calculations become more complicated as a full understanding of the dynamic interaction between the pile and the soil (pile-soil interaction) and between adjacent piles (pile-soil-pile interaction) is required. The dynamic interaction between individual piles and soil has been investigated by Tajimi [1] using a continuum model and Penzien [2] using a lumped mass model. However, the complexity of these rigorous approaches kept them from being used in practice. In 1974, Novak introduced an...
approximate approach for the dynamic analysis of single vertical piles undergoing harmonic motion [3]. This approach assumes that the soil is composed of a set of independent infinitesimally thin horizontal layers. These layers extend to infinity and can be viewed as a generalized Winkler’s medium that has inertia and the ability to dissipate energy. This approach was extended by Novak and Aboul-Ella [4] to include the effect of having a soil profile that changes with depth. Furthermore, Novak and Sheta [5] included the effect of the weak zone around the pile that can be used to represent either slippage in the soil-pile interface or the actual weak zone formed around the pile during construction. All these studies show that frequency has a significant impact on the dynamic impedance parameters for single piles. This behavior has also been confirmed by field experiments [5–7]. Field experiments conducted on large scale piles have also shown a non-linear dynamic pile response as a result of soil non-linearity. Consequently, the impedance parameters of a single pile are also dependent on the amplitude of the excitation force.

The static behavior of pile groups was thoroughly discussed in the pioneering work of Poulos [8,9]. These studies have shown that, under static loads, the interaction between piles within the same group leads to the decrease of the group stiffness. Therefore, the settlement of a group pile is larger than the settlement of a single pile. However, studies concerned with the dynamic analysis have shown that the pile group interaction is rather complicated [5,10,11]. Unlike the static response, the dynamic interaction may cause the impedance parameters of the pile group to increase or decrease. Generally, the dynamic interaction between piles can be neglected if the spacing between piles in the group is more than twenty times the pile diameter [12]. The stiffness and damping of the pile group, in this case, can be considered as the linear summation of stiffness and damping of every single pile in the group. However, for smaller spacing, the oscillation of one pile starts to influence the oscillation of the other piles in the group and the interaction between the piles more noticeable. Due to the complexity of considering the pile-soil-pile interaction using rigorous analysis, simplified approaches were introduced. In order to estimate the stiffness of the pile group, it was suggested to use the static interaction coefficients from Poulos and Davis [9], Randolph [13] and Randolph and Poulos [14]. However, the static interaction approach was found to be adequate only for low frequency small and moderately large groups [12]. Furthermore, the static interaction approach does not estimate the damping coefficient of the pile group.

Kaynia and Kausel [10] studied the possibility of extending the applicability of the static interaction factors approach into dynamic interaction studies. Good results were obtained from this approach compared to more rigorous three-dimensional analysis. However, it was limited to a narrow range of parameters. Dobry and Gazetas [15] introduced an approximate method for calculating the dynamic interaction factors. This method was simple and
produced satisfactory results. However, it assumed that as a pile vibrates, cylindrical waves emanate simultaneously from that pile and propagate radially outward in the horizontal direction toward other piles in the group. This assumption is realistic for axially loaded piles. However, for laterally loaded piles, it may over-predict the damping and stiffness of large pile groups (number of piles > 16). Moreover, field experiments found that the dynamic response of pile groups is more sensitive to frequency than single piles [11].

The paper consists of four parts. The first part validates the simulation of the piles using the numerical finite element program (PLAXIS 3D software) by comparing the numerical model output with laboratory measurements of a monopile supporting an offshore wind turbine. The second part discusses the different techniques to numerically model the piles. The third and fourth parts discuss the dynamic behavior of single piles and pile groups, respectively. In addition, a parametric study is performed to evaluate the effect of related parameters: excitation force frequency (f), soil modulus of elasticity (E_s), pile slenderness Ratio (L/D), dimensionless spacing ratio (S/D) and pile group size (n_g).

Case study and 3D model validation

The laboratory work conducted by Hetland [16] is used to verify the capability of the numerical model to simulate the dynamic behavior of pile foundations. The work of Hetland [16] was concerned with the dynamic behavior of offshore monopile wind turbines. The laboratory model implemented a simulation of the actual case study with a scale of 1:20, as illustrated in Figure 1. The model consisted of a square concrete tank 4 m wide and 2 m deep, containing sand from a glazifluvial deposit in Hokksund, Norway. Table 1 presents the assigned sand properties according to laboratory measurements [16]. In addition, the soil shear modulus is found to be proportional to the effective overburden pressure using a power rule, thus, the soil shear modulus increases with depth as shown in Figure 2. Meanwhile, the linear Elastic Model used in PLAXIS is only capable of considering a linear increase of stiffness with depth. Therefore, soil profile in PLAXIS is divided into 0.5 m thick sub-layers as shown in Figure 2.

Rayleigh damping is used in PLAXIS to simulate both soil geometric and material energy dissipation. Different soil damping values were assigned to the numerical model based on the applied overburden pressure. The input parameters used to determine the Rayleigh damping values in PLAXIS model are presented in Table 2. It can be noticed that the soil damping ratio decreases with the increase of the overburden pressure. This is related to the increase of the confinement stresses around soil particles, which as a result decreases the soil material damping.

Sand surface was subjected to different overburden pressures which were applied to the model using a membrane and a suction pump. The membrane
was installed at the top of the soil making the system airtight while the suction pump was connected to a drainage pipe existing at the bottom of the tank. Therefore, when the pump operates, the membrane is pulled down applying the required overburden pressure that may vary according to the applied suction power. The overburden pressure is simulated using a surface load covering the entire soil surface (as shown in Figure 3(a)). The value of this load varies according to the overburden pressure applied in the laboratory model between 0 and 40 kPa.

To model the wind turbine monopile, a steel pile is used with an outer diameter of 0.273 m and a wall thickness of 4 mm. The steel pile is installed in

**Table 1. Soil properties for the laboratory test [16].**

| Parameter               | Value | Parameter               | Value |
|-------------------------|-------|-------------------------|-------|
| Friction Angle, \( \phi \) \(^{[\circ]} \) | 38    | Specific gravity, \( G_s \) [-] | 2.71  |
| Void Ratio, \( e \) [-] | 0.664 | Poisson’s Ratio, \( \nu \) [-] | 0.35  |
| Porosity, \( n \) [%]  | 39.9  | Poisson’s Ratio (small strain) [-] | 0.20  |
| Min. Porosity, \( n_{\text{min}} \) [%] | 36.4  | S-wave velocity, \( V_s \) [m/s] | 120 – 200 |
| Max. Porosity, \( n_{\text{max}} \) [%] | 48.8  | P-wave velocity, \( V_p \) [m/s] | 200 – 300 |
| Relative Density, \( D_r \) [%] | 76    | Unit Weight, \( \gamma \) [kN/m\(^3\)] | 16    |
Figure 2. Shear modulus profile with depth at different overburden pressures.

Table 2. Damping ratio and laboratory and numerical results.

| Overburden pressure [kPa] | Pile damping ratio, $\zeta_{\text{pile}}$ [-] | Soil damping ratio, $\zeta_{\text{soil}}$ [-] | Laboratory measurements | Hetland model | Current study |
|---------------------------|---------------------------------------------|------------------------------------------|------------------------|---------------|--------------|
| 0                         | 0.005                                       | 50                                       | 7.7                    | 7.9           | 8.05         |
| 20                        | 0.005                                       | 15                                       | 8.5                    | 8.8           | 8.9          |
| 40                        | 0.005                                       | 10                                       | 8.7                    | 9.0           | 9.2          |

Figure 3. PLAXIS 3D model for the laboratory test. a) configuration and loads b) mesh.
the middle of the sand tank. It has an embedded length of 1.4 m and a free length of 4.6 m. The pipe is attached to a flange connection positioned 0.6 m above the soil and another flange is located at the pipe top (Figure 1). The properties of the pipe used in the model are presented in Table 3. The maximum meshing size used in the finite element analysis is determined using the rule established by Kramer [17]. This rule relates the maximum meshing size to the shortest wavelength considered in the analysis as shown in the following equation:

\[
\text{Maximum Meshing Size} = \left(\frac{1}{5} \text{ to } \frac{1}{8}\right) \frac{V_{\text{Wave}}}{f}
\]

where \(V_{\text{wave}}\) is the pressure wave velocity and \(f\) is the dynamic motion frequency. For a natural frequency value of 10 Hz and minimum pressure wave velocity of 200 m/s (Table 1), a maximum meshing size of 2.5 m is allowed. Hence, medium coarse mesh is used in the present analysis with a maximum mesh size of 0.44 m. Moreover, refinement with a local fineness factor of 0.125 is used around the pile. The mesh adopted in the present analysis is shown in Figure 3(b).

The excitation of the pile is introduced using an impact force generated by a hammer striking the pile top. An accelerometer is used to measure the acceleration of the hammerhead to estimate the magnitude of the impact force. For each test, the hammerhead was pulled back to a constant distance of 0.13 m before being released to strike the pile top. Therefore, a horizontal dynamic load of 1 kN (using a dynamic load multiplier) is used to simulate the hammer impact force. The dynamic load multiplier follows the load pattern determined from the accelerometer readings as shown in Figure 4(a). Afterward, natural frequencies \(f_n\) are calculated using the time-displacement curves obtained after applying the impact load as shown in Figure 4(b) using the following equation:

\[
f_n[\text{Hz}] = \frac{1}{T_n}
\]

Where \(T_n\) is the time period required to complete one cycle of oscillation during the stage of free oscillations the application of the impact force. The natural frequencies from the finite element analysis are compared to the laboratory measurements and the results of the 3D model of Hetland [16].

| Parameter                  | Value  | Parameter                  | Value  |
|----------------------------|--------|----------------------------|--------|
| Diameter [m]               | 0.273  | Pile Material Density [kN/m³] | 78.50  |
| Total Length [m]           | 6      | Top Flange Mass [kg]        | 9      |
| Embedded Length [m]        | 1.4    | Connection Flange Mass [kg] | 29     |
| Pile Thickness [mm]        | 4      | Pile Elasticity Modulus, E [kN.m²] | \(2.1 \times 10^8\) |
Table 2 shows a good agreement between the measured and calculated values (differences do not exceed 5%).

In order to conduct the parametric study, a numerical model is constructed using similar procedures to those adopted in the verification model. Soil domain of 14 m x 14 m is used in the analysis with a depth ranging between 12 m and 15 m. Overburden pressure of 20 kPa is placed on the sand surface to simulate the backfill weight above the foundation level. To reduce wave reflections within the model, soil boundaries are placed at a distance ranging between 18 to 23 times the pile diameter. Moreover, viscous boundaries are used at all sides of the soil domain, except the top surface, to absorb incoming waves.

The soil stiffness profile is divided into 2 m thick sub-layers having a linear increase with depth, as presented in Figure 5. The steel piles are modeled as linear elastic plate elements and their properties are: Young’s modulus $E = 2.1 \times 10^8$ kN/m$^2$ and Poisson’s ratio $\nu = 0.20$. The steel piles are simulated with an outer diameter ($D$) of 0.30 m, wall thickness of 6.35 mm, and length ($L$) of 6 and 9 m. The full length of the piles is embedded inside the soil except for the top 2 cm. The pile top is placed above the soil surface to avoid transmitting the dynamic loads directly from the pile cap into the soil. The pile cap used in the simulation of the pile group is a 5 cm thick steel plate modeled using plate elements with Young’s modulus ranging between $2.1 \times 10^{13}$ to $2.1 \times 10^{16}$ kN/m$^2$. The Young’s modulus of the pile cap is increased by $10^5$ to $10^8$ times the actual value in order to limit the deformation of the pile cap under the dynamic loads. Figure 6 shows the single pile and pile groups used in the analysis.

The dynamic motion of the piles is generated using a harmonic load with a maximum value of 5 kN. This load is applied in the vertical or the horizontal direction at the center of the piles (in case of single piles) or at the center of the pile cap (in case of pile groups). The time-displacement curves obtained from the

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**Figure 4.** a) Dynamic load multipliers b) Natural frequency calculations.
dynamic calculations performed by PLAXIS are used to calculate the impedance parameters (K and C) using the following equations [18].

\[ C = \frac{f_o}{x_o \omega} \sin(\phi) \]

where \( K \) is the stiffness coefficient, \( C \) is the damping coefficient, \( f_o \) is the dynamic force amplitude, \( x_o \) is the dynamic displacement amplitude, \( \omega \) is the
excitation force frequency in (rad/sec), m is the vibrating system mass, and φ is the phase angle.

**Pile modeling**

Two models have been studied to determine the effect of the pile simulation method on the results. The pile may be simulated either as a plate element or as a one-dimensional embedded pile element. The latter one is a structural element offered in PLAXIS 3D which is capable of modeling the behavior of piles using a one dimensional beam element. The beam element interacts with the surrounding soil using a special interface that can simulate the skin friction and the tip resistance of an actual pile as a function in the pile displacement. It assumes the occurrence of soil-pile interaction at the pile center, not at its outer circumference. Instead, an elastic zone is assumed around the embedded pile element with the size of the pile diameter. Within this elastic zone, the plastic behavior of the soil is ignored making the pile behaves more like a volume pile. Figure 7 prevails that the effect of simulating the pile as a one dimensional embedded pile element on results does not exceed 5%. The pile simulation using the plate element represents the actual case which is adopted in the current study; however, the calculation time drops by 75% upon using the embedded pile elements.

**Dynamic behavior of single piles**

A range of excitation frequencies (f) varying between 10 Hz and 60 Hz is adopted to study the dynamic behavior of a single pile. Figures 8 and 9 show the variation of stiffness ($K_s$), damping ($C_s$), and peak displacement for a single pile under the effect of vertical and lateral dynamic loads. The finite element model shows that as the excitation frequency increases, $K_s$ increases and $C_s$ decreases under both types of vibrations. Increasing frequency from 10 Hz to

![Figure 7. Effect of pile modeling technique on peak displacements under a) vertical vibrations b) lateral vibrations (Single Pile – L/D = 20).](image-url)
60 Hz (500% increase) leads to the increase of $K_s$ by 64% (under vertical vibrations) and by 120% (under lateral vibrations). On the other hand, $C_s$ decreases by 40% (under vertical vibrations) and by 26% (under lateral vibrations). These results prove that dynamic pile-soil interaction depends on the excitation frequency \([1–3]\). It is also found that the rate of increase in stiffness is higher than the rate of decrease of damping. Therefore, lower values of peak displacements are noticed at higher frequencies (Figures 8(c) and 9(c)). If a frequency of 60 Hz is applied instead of 10 Hz (500% increase), peak displacement decreases by around 37% under both vertical and lateral vibrations.

The effect of changing soil stiffness ($E_s$) is also investigated. Upon reducing the value of soil stiffness ($E_s$) by 50%, the numerical model shows a corresponding reduction in the impedance parameters up to 33% for $K_s$ and 12% for $C_s$ (under vertical vibrations) and up to 43% for $K_s$ and 27% for $C_s$ (under lateral vibrations). Consequently, an increase in the peak displacements is observed ranging between 30% and 45% under vertical vibrations and between 54% and 79% under lateral vibrations (Figures 8(c) and 9(c)). This behavior is expected, as decreasing the soil stiffness ($E_s$) leads to the decrease of the soil resistance around the pile, consequently, a reduction in the system stiffness and damping occur.
Figure 10 shows that the pile slenderness ratio (L/D) has a negligible impact on the dynamic behavior of single piles subjected to either vertical or lateral vibrations. Increasing the pile slenderness ratio from 20 to 30 leads to a decrease in peak displacements by less than 10% (under vertical vibrations) and 3% (under lateral vibrations). This agrees with the results obtained by Novak [3] where the influence of increasing the slenderness ratio is insignificant for long flexible piles especially under lateral motion. This occurs because the soil mass contributing to the dynamic resistance of the system is limited to a certain depth regardless of the total length of the pile.

**Dynamic behavior of pile groups**

The 3D finite element model is used to investigate the dynamic behavior of pile groups. Figures 11 and 12 show the variation of group stiffness (K_g), group damping (C_g), and peak displacement for a pile group (n_g = 4) under the effect of vertical and lateral dynamic motions. K_g increases with a fluctuating rate as the frequency increases. Meanwhile, C_g slightly increases until reaching f = 30 Hz to 40 Hz. Beyond this point, C_g decreases again. Consequently, the peak displacement decreases until becoming almost
constant beyond 45 Hz. The same trend was observed by Dobry and Gazetas [15]. It is also found that the response of a pile group is more sensitive to frequency than the response of a single pile [11].

The effect of changing the soil stiffness ($E_s$) on the dynamic behavior of a pile group of four is also examined as shown in Figures 11 and 12. As a result of decreasing the soil stiffness ($E_s$) by 50%, the finite element model shows

**Figure 10.** Effect of pile slenderness ratio under a) vertical vibrations b) lateral vibrations (Single Pile – 100% $E_s$).

**Figure 11.** Vertical dynamic behavior of pile group: a) stiffness, b) damping, c) peak displacement ($n_0 = 4$, $L/D = 20$, $S/D = 5$).
a non-uniform reduction in $K_g$. Under vertical vibrations, the reduction varies from as low as 4% (between 25 Hz and 35 Hz) to as high as 39% at $f = 60$ Hz. Meanwhile, Figure 11(b) shows that $C_g$ slightly increases (by less than 5%) between 10 Hz and 27 Hz. For frequencies larger than 27 Hz, $C_g$ decreases by a value of up to 39%. Accordingly, a general increase in peak displacements is depicted. However, this increase is variable along the studied frequency range and is found to vary between 8% and 52% (Figure 11(c)). Under lateral vibrations, the reduction varies from as low as 24% at $f = 35$ Hz to as high as 42% at $f = 60$ Hz. Meanwhile, Figure 12(b) shows that $C_g$ decreases by a value of up to 35%. Accordingly, a general increase in peak displacements is depicted. However, this increase is variable along the studied frequency range and is found to vary between 28% and 85% (Figure 12(c)). The same trends have been reported by many researchers [10,15]. The length of a stress wave traveling through a soil media depends on its velocity through this media and the excitation frequency. Therefore, changing the stiffness of the soil ($E_s$) alters the compression and shear wave velocities. Consequently, the length of the stress waves traveling between adjacent piles is modified. Therefore, the phase in which the stress waves are emitted from the vibrating piles and reaching the adjacent piles changes. This change can be in phase or
out of phase with the adjacent piles movement. As a result, the impedance parameters may decrease, increase or inhabit almost no changes.

The effect of changing the dimensionless spacing ratio \( S/D = \text{spacing between piles/pile diameter} \) is examined using values of 3, 5 and 10 for a pile group of four under vertical vibrations. Figure 13(a) shows that the impedance parameters and the peak displacement may decrease or increase as a result of increasing the dimensionless spacing ratio \( S/D \). If \( S/D \) increases from 5 to 10, \( K_g \) increases for frequency values ranging between 10 Hz and 39 Hz. For \( f > 39 \text{ Hz} \), the trend is reversed and \( K_g \) decreases at higher \( S/D \) values. The same trend is depicted for both \( C_g \) and peak displacement curves (Figure 13(b,c)). The stress waves emitted from a vibrating pile reach the adjacent vibrating piles at a certain phase. If the spacing between piles increases, the phase in which the stress waves reach the adjacent vibrating piles changes. This change causes the stress waves to be in phase or out of phase with the vibrating adjacent piles, which may cause the impedance parameters to decrease or increase. Moreover, the peaks and dips of the impedance parameters curves become more noticeable as \( S/D \) increases (Figure 13(a,b)).

The effect of pile group size is investigated via the group stiffness/damping efficiency. Group stiffness/damping efficiency is defined as the ratio of the

![Figure 13](image-url)

**Figure 13.** Effect of dimensionless spacing ratio on vertical dynamic behavior: a) stiffness, b) damping, c) peak displacement \( (n_g = 4, L/D = 20, S/D = 5) \).
average stiffness/damping per pile in the group to the stiffness/damping of a comparable single pile. Figure 14 shows the variation of group stiffness efficiency, group damping efficiency and peak displacement for groups of four and nine piles under vertical dynamic motion. Based on the numerical model, the group stiffness/damping efficiency may decrease or increase as a result of increasing the group size ($n_g = 9$ instead of 4). Furthermore, the group stiffness/damping efficiency may achieve values above unity (up to 1.17 and 2.36 for stiffness and damping efficiency, respectively). This takes place because the stress waves traveling between the piles can be in phase or out of phase with the adjacent piles movement. Therefore, changing the number of piles in a group may cause the impedance parameters of the piles to decrease or increase. However, a reduction in peak displacement values ranging between 41% and 52% is attained as the pile group size increases (Figure 14(c)).

Figure 15 shows that the pile slenderness ratio (L/D) has a slight impact on the dynamic behavior of a pile group subjected to either vertical or lateral vibrations. Increasing the pile slenderness ratio from 20 to 30 leads to a decrease in peak displacements by less than 5% (under vertical vibrations) and less than 7% (under lateral vibrations).

**Figure 14.** Effect of group size on vertical dynamic behavior: a) stiffness group efficiency, b) damping stiffness group efficiency, c) peak displacement (100%$E_s$, L/D = 20, S/D = 5).
Conclusions

This study reveals that the complicated behavior of piles under dynamic loads can be numerically simulated using the 3D finite element program PLAXIS. The results of the numerical model are validated using laboratory measurements during the testing of offshore monopile wind turbines under dynamic loads. During the modeling process, one dimensional embedded pile element can be used instead of plate element to simulate piles. Using one dimensional embedded pile element reduces the calculation time significantly while the difference in results does not exceed 5%. The study shows that the dynamic behavior of pile foundations depends on the frequency which accordingly affects the values of the induced displacements. The effect of frequency is more pronounced on the dynamic response of pile groups. It is also found that the slenderness ratio has a negligible effect on the dynamic response of long single piles and pile groups. The results show that upon reducing soil stiffness, the numerical model shows a decrease in the impedance parameters and an increase in the peak displacements. The change in peak displacements is almost uniform for single piles. However, in the case of pile groups, this change is variable along the studied frequency range. It is noted that increasing the spacing between piles or the group size may cause the impedance parameters to increase or decrease. This contradicts the behavior of pile groups under static loading conditions. As a result, the dynamic efficiency of pile groups can achieve values higher than unity. In the studied cases, the dynamic efficiency of the piles may reach values up to 1.17 for stiffness and 2.36 for damping under vertical vibrations.

Disclosure statement

No potential conflict of interest was reported by the authors.
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