A Measurement System for the Phase Retardation of Liquid Crystal Particles Under an Electric Field Effect

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Abstract. The optical characteristics of LCDs, e.g. their brightness, contrast ratio and response time, are sensitive to the structural parameters of the liquid crystal cells. This study develops a phase retardation measurement system using a laser interferometer, a liquid crystal cell and a function generator. When the liquid crystal particles are subjected to an electric field effect, their orientation changes, causing a corresponding change in the optical path length through the liquid crystal cell. By relating the optical path length difference to the change in the particle angle, the relationship between the phase retardation and the applied voltage can be obtained.

1. Introduction

Reinitzer (1888) was the first researcher to observe liquid crystal characteristics when he noticed that the storax acid in cholesterol took on the appearance of a white muddy liquid over a certain temperature range and emitted different optical spectra at different temperatures. In 1960, researchers showed that liquid crystal changes its color not only under different temperatures, but also under different pressures. Since liquid crystal changes its orientation when subjected to an electric field, its birefringence characteristics also change and can be calculated from the measured change in the phase retardation signal. Therefore, it is necessary to develop precise laser-based techniques for the phase retardation measurement of liquid crystal in order to identify the birefringence characteristics of optical elements.

Traditionally, researchers measured the refractive index of optical elements by using a common path type interferometer structure to measure their birefringence and phase retardation characteristics. However, modern researchers generally measure the birefringence characteristics and phase retardation of a waveplate using either the heterodyne interference technique with a lock-in amplifier, or the Electro-Optical (E-O) modulation technique [1]. In 1997, Feng et al. [2] developed a circular heterodyne interferometer system to measure the change of rotation of the principal axis of glucose caused by a variation in its concentration and the phase difference induced by a variation in its optical properties. Cameron et al. [3] proposed the digital closed-loop system shown in Figure 1 to investigate the relationship between the glucose concentration and the angle of the principal axis. In their approach, a Farad rotator driven by the sinusoidal output of a function generator was used to control the rotational polarization. The signal from the Farad rotator was passed through the sample cell and a compensation signal was then calculated in a feed back system containing a second Farad rotator and an analyzer. Following the compensation calculations, the light received from an optical detector was...
used to establish a heterodyne interference signal, which was then transformed using a lock-in amplifier to obtain the change in angle of the principal axis.

![Diagram](image_url)

**Figure 1.** Experimental measurement of variation in glucose concentration using Farad rotator.

The properties of liquid crystals are generally investigated using an experimental arrangement comprising a liquid crystal cell and two polarizers. The optical characteristics of liquid crystals are typically measured using a Newton interferometer, a Fizeau interferometer, or a lateral shearing interferometer. However, obtaining the band center, band number, and band distance measurements required to calculate the parameters of the liquid crystal is challenging using such methods.

Recently, a new measurement method has been proposed, namely the phase shifting interferometer system. The principle of this system is to establish an initial estimate of the reference phase and to modify this reference phase inversely by recording the change of phase. This system is suitable for measuring and monitoring optical components and is commonly employed in the manufacturing of optical communication elements, e.g. to assess the quality of the finished product. In this system, the optical path difference of the reference light beam is continuously changed by a mechanical device such as a stepping motor or a piezoelectric actuator. Although, this system is capable of obtaining measurements with a micron-scale level of accuracy, the problem of mechanical component errors still remains. Consequently, this study uses a He-Ne laser interferometer to measure the change in the optical path length through a liquid crystal cell when an electric field is applied. By relating the optical path length difference to the change in the particle angle, the relationship between the phase retardation and the applied voltage is obtained.

## 2. Measurement System and Mathematical Analysis

### 2.1. Transmissivity experiment

As shown in Figure 2, in the light transmissivity experiment, the liquid crystal cell is positioned between two orthogonal polarizers. When a linear polarized light beam enters the liquid crystal film at an angle of $\theta$ to the director axis of the liquid crystal, it is split into two beams with different polarizations, namely the extraordinary wave (E-light), in which the polarization direction is parallel to the liquid crystal axis, and the ordinary wave (O-light), in which the polarization direction is perpendicular to the axis. Since the E-light and the O-light pass through the liquid crystal with different velocities, their indices of refraction are different. Consequently, a phase difference, $\delta$, exists between the two waves when they emerge from the liquid crystal film, i.e.

$$
\delta = 2\pi d \left( \frac{1}{\lambda_o} - \frac{1}{\lambda_e} \right) = 2\pi d \left( \frac{n_e - n_o}{\lambda_e} \right) = \frac{2\pi d \Delta n}{\lambda_e}
$$

(1)
where $\Delta n$ depends on the applied voltage, the temperature, and the wavelength of the incident light $\lambda_v$, and is given by $\Delta n = n_{\text{eff}} - n_o$.

**Figure 2.** Experimental arrangement for establishing light transmissivity.

From mathematical manipulation [4], it can be shown that the transmissivity of the light, $T$, when the two light axes of the polarized plates are parallel is given by:

$$T = 1 - \sin^2 2\theta \sin^2 \left(\frac{\delta}{2}\right)$$

(2)

When the two light axes of the polarized plates are perpendicular, the transmissivity of the light is given by:

$$T = \sin^2 2\theta \sin^2 \left(\frac{\delta}{2}\right)$$

(3)

2.2. Measurement system

Figure 3 presents a schematic illustration of the current experimental arrangement. As shown, a polarized spectroscope is used to separate the incident laser light into two beams with perpendicular and horizontal polarizations, respectively. The perpendicular polarized light passes through the spectroscope, while the parallel light is refracted. The perpendicular polarized light passes through the liquid crystal cell and is reflected by Corner Reflector 1. Consequently, it passes back through the liquid crystal cell and re-enters the polarized spectroscope. Meanwhile, the parallel polarized light is reflected from Corner Reflector 2, re-enters the polarized spectroscope, and meets the reflected perpendicular polarized light. The two light beams are then reflected from the polarized spectroscope into a receiver, where the difference in their frequencies generates beat-frequency interference. The interferometer uses the beat-frequency to calculate the change in the light path difference on the basis of the difference in frequency of the two light beams. Using this interferometer arrangement, this study calculates the change in the light path difference when electric fields of different magnitudes are applied to the liquid crystal cell.

2.3. Experimental materials

(1) Nematic liquid crystal: ZLI-2950, uniaxial material.

- Liquid crystal phase: -40°C~85°C.
- Dielectric constant: $\varepsilon_\parallel = 7.2$, $\varepsilon_\perp = 3.30$, $\Delta \varepsilon = \varepsilon_\parallel - \varepsilon_\perp = 3.9 > 0$.
- Index of refraction: $n_c = 1.5013$, $n_e = 1.6211$, $\Delta n = n_e - n_c = 0.1198$, at 20°C, $\lambda = 589$ nm.

(2) Liquid crystal cell: Supplied by Mesostate LCD Industries Co., Ltd. Thickness of ITO glass: 1.1mm; thickness of liquid crystal coating: 1.1μm. Liquid crystal cell in planar state.
2.4. Experimental process
In the present experimental interferometer, both corner mirrors are stationary, and the liquid crystal cell is positioned between the polarized spectroscope and the corner reflector. An electric field is then applied to the liquid crystal cell causing the permutation location of the liquid crystal molecules to change. Hence, the length of the light path through the liquid crystal cell also changes. Since the experimental value of the interferometer counter varies as a function of the change in the angle $\theta$, the phase retardation can be obtained.

The major steps in the current experimental procedure can be summarized as follows:

- Liquid crystal is irrigated into the liquid crystal cell, which is then glued on both sides to form a closed liquid crystal cell.
- The orientation of the polarized spectroscope is adjusted such that the perpendicular polarized light is transmitted and the parallel polarized light is reflected.
- The liquid crystal cell is positioned between the spectroscope and the corner reflector, and the light beam is adjusted such that it enters the liquid crystal cell perpendicularly. The interferometer counter is then set to zero.
- A function generator is used to produce a sine wave for the externally applied alternating electric field. The experimental phase retardation value measured by the interferometer is then obtained. This value indicates the optical path difference induced when the electric field is applied to the liquid crystal cell.
- The voltage of the sine wave produced by the function generator is adjusted and Steps 1~4 repeated. The changes in the phase retardation and light path difference are observed.

2.5. Mathematical analysis
Since the nematic liquid crystal used in this study does not undergo any post-processing treatment, the E-light optical axis is parallel to the glass substrate after irrigating the liquid crystal cell, and its direction is the same as that of the vibration of the transmitted perpendicular polarized light. Hence, the length of the initial light path through the liquid crystal cell is given by:

$$ L_i = n_g \times d + n_e \times r + n_g \times d $$

(4)

where $n_g$ is the index of refraction of the two glass plates, $n_e$ is the index of refraction of the E-light, $d$ is the thickness of the glass plates, and $r$ is the thickness of the liquid crystal layer. After applying an electric field to the liquid crystal cell, as shown in Figure 4, the liquid crystal layer can be regarded as a birefringence crystal rotated through an angle, and the optical path length through the liquid crystal cell is modified to:
The relationship between the light path difference, i.e. $L_1 - L_2$, and the change in angle of the liquid crystal, $\delta$, is given by:

$$L = L_1 - L_2 = r \times \left( \frac{n_{\text{eff}}}{\cos(\theta_1 - \theta)} - n_g \right) + n_g \times d \times \left( \frac{1}{\cos \theta_2} - 1 \right)$$  \hspace{1cm} (6)

where $n_{\text{eff}}$ is the efficient index of refraction, i.e.

$$n_{\text{eff}} = \left( \frac{\cos^2 \theta + \sin^2 \theta}{n^2_g + \sin^2 \theta} \right)^{-1}$$  \hspace{1cm} (7)

where $\theta$ is the change in angle of the liquid crystal induced by the applied electrical field, $\theta_1$ is the angle of refraction when the light passes through the first glass plate of the liquid crystal cell into the liquid crystal layer, and $\theta_2$ is the angle of refraction when the light passes from the liquid crystal layer into the second glass plate.

When the light enters the liquid crystal layer from the first glass plate, Snell’s law gives:

$$n_g \sin \theta = n_{\text{eff}} \sin \theta_1$$  \hspace{1cm} (8)

where $n_g$ is the index of refraction of the glass plate. Then:

$$\sin \theta_1 = \frac{n_g \sin \theta}{n_{\text{eff}}}, \quad \cos \theta_1 = \left[ 1 - \left( \frac{n_g \sin \theta}{n_{\text{eff}}} \right)^2 \right]^{0.5}$$  \hspace{1cm} (9)

$$\theta_1 = \sin^{-1}\left( \frac{n_g \sin \theta}{n_{\text{eff}}} \right)$$  \hspace{1cm} (10)

When the light enters the second glass plate from the liquid crystal layer, the included angle with the axis of the incident plane is given by $(\theta_1 - \theta)$. From Snell’s law:

$$n_{\text{eff}} \sin(\theta_1 - \theta) = n_g \sin \theta_2$$  \hspace{1cm} (11)

$$\theta_2 = \sin^{-1}\left( \frac{n_{\text{eff}}}{n_g} \sin(\theta_1 - \theta) \right)$$  \hspace{1cm} (12)

$$\sin(\theta_1 - \theta) = \frac{n_g \sin \theta}{n_{\text{eff}}} \left[ 1 - (\sin \theta)^2 \right]^{0.5} \times \sin \theta$$  \hspace{1cm} (13)
cos(θ₁ - θ) = \left[ 1 - \frac{n_g \sin θ}{n_{eff}} \left( \frac{n_{eff}}{n_g} \right)^{0.5} \right] \left[ 1 - \left( \frac{n_g \sin θ}{n_{eff}} \right)^{0.5} \right]^{0.5 - 0.5}

Therefore:

\[ \cos(θ) = \left( \frac{n_{eff}}{n_g} \right)^{0.5} \left[ 1 - \left( \frac{n_g \sin θ}{n_{eff}} \right)^{0.5} \right]^{0.5} \]

As discussed previously, the corner reflector in the laser interferometer reflects the perpendicular polarized light back through the liquid crystal cell. Since a path difference is generated each time the light passes through the liquid crystal cell, the calculated optical path difference must be divided by a factor of two. Therefore, the path difference, \(EL\), is given by:

\[ EL = 0.5 \times L = 0.5 \times \left( r \times \left( \frac{n_{eff}}{\cos(θ) - θ} \right) - n_g \times d \times \left( \frac{1}{\cos(θ) - 1} \right) \right) \]

Eqs. (7), (14) and (15) are substituted into Eq (16), and the polynomial of θ obtained. Based on the values shown from the interferometer and Eq. (16), θ can be obtained numerically and the phase retardation effect derived.

3. Formula Analysis and Compensation

In order to reduce the influence of environmental noise on the experimental results and to increase the degree of measurement accuracy, the present experiments were performed on an optical experimental platform to suppress vibrations and optical sensors were used to detect the environmental temperature, pressure, and humidity. However, two further factors must also be considered, namely the influence of the azimuthal anchoring force on the permutation of the liquid crystal particles and the deviation of the incidence angle. Both factors affect the validity of the experimental results and hence their influence must be carefully analyzed. Appropriate compensation methods for these factors are introduced in the paragraphs below.

3.1. Effect of azimuthal anchoring force

In 1972, Berreman found that the surface geometry structure of a liquid crystal cell exerts a significant influence on the permutation of the liquid crystal molecules within the cell. Specifically, he found that if regular grooves were introduced on the surface of the liquid crystal, the surface geometry structure provided a liquid crystal molecule azimuthal anchoring force, \(w_a\), along the direction of the grooves, i.e.

\[ w_a = \frac{2\pi^3 A^2 K}{\Lambda^3} \]

where \(A\) is the depth of the groove, \(K\) is the elastic coefficient of the liquid crystal, and \(\Lambda\) is the distance between the grooves. Since the liquid crystal is permuted from layer to layer, the permutation of the liquid crystal layers near the glass base will be affected by the azimuthal anchoring force when an AC electric field is applied. Hence, it is assumed that the optical axis of the liquid crystal layer at a distance of approximately \(r/15\) from the glass plates remains unchanged when an electric field is applied.

3.2. Deviation of angle of incidence

When the polarized direction of the incident light is parallel or perpendicular to the optical axis of the waveplate, the light is E-light or O-light, respectively, after it enters the waveplate. In practice,
however, it is not easy to adjust the optical path. The relative errors are explained below and appropriate compensation methods proposed.

The present measurement system assumes that the polarized direction of the incident light is fully parallel to the optical axis of the liquid crystal. However, due to the influence of external operating conditions, the light may not actually enter the liquid crystal cell perpendicularly and a deviation angle \( a \) is caused, as shown in Figure 5.

![Figure 5. Deviation of incident light in liquid crystal cell.](image)

Initially, the vector of the optical axis of the liquid crystal \( \vec{n}_b \) is:

\[
\vec{n}_b = \sin a \vec{j} + \cos a \vec{k}
\]  

(18)

The vector of the optical axis of the liquid crystal changes to \( \vec{n}_\theta \) after the liquid crystal particles rotate an angle \( \theta \) as a result of the applied electric field, i.e.

\[
\vec{n}_\theta = -\sin \theta \cos a \vec{i} + \sin a \vec{j} + \cos \theta \cos a \vec{k}
\]  

(19)

Taking the inner product of Eq. (19) and the z-axis shown in Eq. (20):

\[
\vec{n}_\theta \cdot \vec{n} = |\vec{n}_\theta| |\vec{n}| \cos \theta = \cos a \cos \theta
\]  

(20)

\[
\theta_r = \cos^{-1} [\cos a \cos \theta]
\]  

(21)

where \( \theta_r \) is the included angle between \( \vec{n} \) and \( \vec{n}_\theta \). The effective index of refraction is then modified to:

\[
\frac{1}{n_{\text{eff}}^2} = \frac{\cos^2(\theta_r)}{n_0^2} + \frac{\sin^2(\theta_r)}{n_e^2}
\]  

(22)

3.3. Determining parameters and compensation of formulas

From the discussions presented in the two sections above, it is clear that the actual optical path is affected by the inclined angle \( a \) of the incident plane. The original light path, \( L_1 \), is therefore given by:

\[
L_1 = n_g \times \frac{d}{\cos \theta} + n_g (a) \times \frac{r}{\cos \theta} + n_e \times \frac{d}{\cos \theta^*}
\]  

(23)

where \( \theta^* \) is the angle of refraction when the laser light passes from air into the first glass plate in the liquid crystal cell, and \( \theta_1 \) is the angle of refraction when the laser light passes from the first glass plate into the liquid crystal layer.

From Snell’s law:
\[ \theta^* = \sin^{-1}\left(\frac{n_{sa}}{n_g}\sin a\right), \quad \theta_i = \sin^{-1}\left(\frac{n_g}{n_{eff}(a)}\sin\theta^*\right) \] (24)

After the liquid crystal rotates an angle \( \theta \) as a result of the applied electric field, the length of the optical path through the liquid crystal cell changes to:

\[ L_2 = n_g \times \frac{d}{\cos\theta} + n_{eff}(a) \times \frac{r}{15\cos\theta} + n_{eff}(\bar{\theta}) \times \frac{13\gamma}{15} \times \frac{1}{\cos(\theta_2 - \theta)} + n_{eff}(a) \times \frac{r}{15\cos\theta_4} + n_\theta \times \frac{d}{\cos\theta_5} \] (25)

where \( \bar{\theta} \) is the angle of rotation of the liquid crystal after the electric field is applied, \( \theta_1 \) is the angle of refraction when the light passes from the first glass plate into the liquid crystal layers subjected to the azimuthal anchoring force, \( \theta_2 \) is the angle of refraction when the light passes from the liquid crystal layers subjected to the azimuthal anchoring force into the liquid crystal layers subjected to the applied electric field, and \( \theta_3 \) is the angle of refraction when the light passes from the liquid crystal layers subjected to the applied electric field into the liquid crystal layers subjected to the azimuthal anchoring force.

From Snell’s law:

\[ \theta_2 = \sin^{-1}\left(\frac{n_{eff}(a)}{n_{eff}(\bar{\theta})}\sin(\theta + \theta_1)\right) \] (26)

\[ \theta_3 = \sin^{-1}\left(\frac{n_{eff}(\bar{\theta})}{n_{eff}(a)}\sin(\theta_2 - \theta)\right) \] (27)

\[ \theta_4 = \sin^{-1}\left(\frac{n_{eff}(a)}{n_\theta}\sin\theta_3\right) \] (28)

Therefore, the optical path difference, \( L_2 \), is modified to:

\[ L = L_2 - L_1 = n_{eff}(a) \times r \times \left[ \frac{1}{15\cos\theta_1} + \frac{1}{15\cos\theta_4} \right] + n_{eff}(\bar{\theta}) \times \frac{13\gamma}{15} \times \frac{1}{\cos(\theta_2 - \theta)} + n_\theta \times \frac{d}{\cos\theta_5} \] (29)

Substituting Eqs. (24) and (26)–(28) into Eq. (29), the relationship between the optical path difference and the rotation angle \( \theta \) of the liquid crystal particles can be obtained, as illustrated graphically in Figure 6.

\[ \text{Figure 6. Relationship between optical path difference and rotation angle.} \]
4. Results and Error Analysis

4.1. Results

In the present experiments, a He-Ne laser interferometer is used to measure the change in the optical path through a liquid crystal cell when an electric field is applied. By relating the optical path length difference to the change in the particle angle, the relationship between the phase retardation and the applied voltage is obtained. The experimental parameters relating to the laser and the liquid crystal cell are presented in Table 1.

Table 1. Parameter properties of liquid crystal.

| Parameter                  | Value             |
|----------------------------|-------------------|
| Wavelength of laser λ      | 633nm             |
| Refraction of glass n_g    | 1.50041475        |
| Thickness of glass d_g     | 1.1mm             |
| Refraction of liquid crystal n_e | 1.6211 |
| Cell gap                   | 1.1μm             |

Initially, the function generator was assigned a frequency of 4Hz, and voltages in the form of a sine wave with magnitudes of 2.72 V, 3.09 V, 3.47 V, and 3.72 V, respectively, were applied to the liquid crystal cell. The laser path differences obtained by the interferometer at the different applied voltages are presented in Table 2.

Table 2. Variation of light path difference with applied voltage.

| Applied voltage (Vrms) | Light path difference L |
|------------------------|-------------------------|
| 2.72V                  | 1.622984769×10^-7       |
| 3.09V                  | 1.98130562×10^-7        |
| 3.47V                  | 2.053667736×10^-7       |
| 3.72V                  | 2.54357235×10^-7        |

The optical path difference values were then substituted into Eq.(16), and the corresponding rotation angles and phase retardations of the liquid crystal calculated. The corresponding results are presented in Table 3. The relationship between the applied voltage and the optical transmissivity was then substituted into Eq. (3) to obtain the theoretical phase retardation value. This value was then substituted into Eq. (1) to obtain the theoretical change of the liquid crystal angle. This theoretical value was then compared with the experimental result, as shown in Table 4.

Table 3. Variation of angle of rotation and phase retardation with applied voltage.

| Applied voltage V | Angle of rotation θ | Phase retardation δ |
|-------------------|----------------------|---------------------|
| 2.72              | 13.8541°             | 0.196π              |
| 3.09              | 12.0837°             | 0.1549π             |
| 3.47              | 11.7514°             | 0.1478π             |
| 3.72              | 10.4622°             | 0.1216π             |
Table 4. Variation of rotation angle and phase retardation with different transmissivities and applied voltages.

| Transmissivity | Applied voltage | Variation of angle $\overline{\theta}$ | Phase retardation $\delta$ |
|----------------|-----------------|----------------------------------------|-----------------------------|
| 90%            | 2.72v           | 27.2384°                               | 0.7952π                    |
| 50%            | 3.09v           | 21.3771°                               | 0.5π                       |
| 20%            | 3.47v           | 16.3121°                               | 0.2952π                    |
| 10%            | 3.72v           | 13.5505°                               | 0.2048π                    |

Tables 5 and 6 present the angle and phase difference errors, respectively. Meanwhile, Figures 7 and 8 plot the experimental and theoretical results for the rotation angle and phase retardation, respectively, against the applied voltage. It is observed that the experimental curve is flatter in both cases. A large discrepancy exists between the experimental rotation angle and the theoretical value, and hence the variation of the experimental phase retardation fails to attain the theoretical value.

Table 5. Errors between experimental and theoretical rotation angle at different applied voltages (1).

| Applied voltage | Experimental value $\theta$ | Theoretical value $\overline{\theta}$ | Error $|\overline{\theta} - \theta| / \overline{\theta}$ |
|-----------------|-----------------------------|----------------------------------------|------------------------------------------------|
| 2.72v           | 13.8541°                    | 27.2384°                               | 49.138%                                      |
| 3.09v           | 12.0837°                    | 21.3771°                               | 43.474%                                      |
| 3.47v           | 11.7514°                    | 16.3121°                               | 27.96%                                       |
| 3.72v           | 10.4622°                    | 13.5505°                               | 22.79%                                       |

Table 6. Errors between experimental and theoretical phase retardation at different applied voltages (1).

| Applied voltage | Experimental value $\delta$ | Theoretical value $\overline{\delta}$ | Error $|\overline{\delta} - \delta| / \overline{\delta}$ |
|-----------------|-----------------------------|----------------------------------------|------------------------------------------------|
| 2.72v           | 0.2155π                     | 0.7952π                                | 72.9%                                         |
| 3.09v           | 0.1633π                     | 0.5π                                   | 67.34%                                        |
| 3.47v           | 0.1564π                     | 0.2952π                                | 47.02%                                        |
| 3.72v           | 0.1216π                     | 0.2048π                                | 40.63%                                        |

4.2. Error analysis and compensation
In examining the failure of the experimental results to attain the theoretical values, there are two factors to be considered: (1) temperature effects and (2) variations in the laser energy. Both factors influence the validity of the experimental results, and hence appropriate compensation methods must be applied.
4.2.1. Temperature effects. Of the many factors which influence the physical properties of liquid crystal, temperature is one of the most significant. The temperature effect is discussed in two parts in the paragraphs below.

(1) Haller’s empirical formula is used to compensate the relationship between the temperature and the birefringence index of the liquid crystal, i.e.

$$
\Delta n = \Delta n_0 \times (1 - \frac{T}{T_c})^\eta
$$

(30)
where $\Delta n_0$ is the difference in the birefringence index at $T=0 \, ^\circ C$, $T_c$ is the temperature of the clearing point of the liquid crystal structure, and $\beta$ is the material parameter of the liquid crystal. In the current measurement process, the environmental temperature is controlled such that it remains at a constant 25 $^\circ C$ and the temperature of the liquid crystal is set as 20 $^\circ C$. Hence, it can be shown that:

$$\frac{\Delta n_{25}}{\Delta n_{20}} = \left(\frac{12}{13}\right)^{\beta}$$  \hspace{1cm} (31)

Substituting a value of $\beta=0.1986$ into Eq. (31) gives $\frac{\Delta n_{25}}{\Delta n_{20}}=0.9842307$. Therefore, it can be seen that the phase retardation of the liquid crystal reduces as the temperature increases.

(2) It is found that the temperature not only changes the properties of the liquid crystal, but also changes the influence of the applied electric field. The corresponding relationship is given by:

$$\frac{dV}{dT} = 9.0 \frac{mV}{^\circ C}$$  \hspace{1cm} (32)

Eqs. (31) and (32) can be applied to the theoretical values of the rotation angle and the phase retardation. The experimental results for the rotation angle and the phase retardation at different magnitudes of applied voltage can then be compared to these compensated theoretical values, as shown in Tables 7 and 8, respectively. It is apparent that the discrepancy between the experimental and theoretical results is reduced compared to the original results presented in Tables 5 and 6.

| Applied voltage | Experimental value $\theta$ | Theoretical value $\tilde{\theta}$ | Error $|\tilde{\theta} - \theta|$ |
|-----------------|-----------------------------|-----------------------------------|---------------------------------|
| 2.72v           | 13.8541°                    | 26.8488°                          | 48.4%                           |
| 3.09v           | 12.0837°                    | 21.0906°                          | 42.705%                         |
| 3.47v           | 11.7514°                    | 16.1345°                          | 27.166%                         |
| 3.72v           | 10.4622°                    | 10.8862°                          | 3.895%                          |

| Applied voltage | Experimental value $\delta$ | Theoretical value $\tilde{\delta}$ | Error $|\tilde{\delta} - \delta|$ |
|-----------------|-----------------------------|-----------------------------------|---------------------------------|
| 2.72v           | 0.2155\pi                   | 0.7701\pi                         | 72.02%                          |
| 3.09v           | 0.1633\pi                   | 0.4851\pi                         | 66.34%                          |
| 3.47v           | 0.1564\pi                   | 0.2868\pi                         | 45.47%                          |
| 3.72v           | 0.1216\pi                   | 0.2024\pi                         | 39.92%                          |

4.2.2. Laser energy deviation. The energy of the He-Ne laser used in this study may vary as a result of environmental effects. To prevent variations in the laser energy from inducing excessive measurement errors, this study takes great care to ensure that the deviation of the incident light on the liquid crystal cell is reduced. Furthermore, the light path is specifically designed such that the laser light beam passes through the sample twice. At larger values of the light path difference, the influence of deviations of the incident light on the measuring results is reduced.

5. Conclusion
This study has developed a robust interferometer system for investigating the phase retardation of liquid crystal cells subjected to an applied electric field and has proposed an improved method for compensating for the errors caused by the external environment.

The major findings of this study can be summarized as follows:
- As the applied voltage is increased, the experimental and theoretical values of the rotation angle and phase retardation are reduced, and the optical transmissivity decreases.
An increasing temperature reduces the phase retardation of the liquid crystal. The temperature effect not only changes the properties of the liquid crystal, but also changes the influence of the applied electric field. The errors between the experimental and theoretical results are successfully reduced by the proposed temperature effect compensation method.

Variations in the He-Ne laser energy induce errors in the measurement results. These errors can be reduced by minimizing deviations of the incident light on the liquid crystal cell and designing the light path such that it passes through the sample twice.

References
[1] M.H. Chiu, C.D. Chen and D.C. Su 1996 *J. Opt. Soc. Am. A* **13** 1924
[2] C.M. Feng, Y.C. Huang, J.G. Chang, M. Chang and C. Chou 1997 *Opt. Commun.* **141** 314
[3] B.D. Cameron and G.L. C’ote 1997 *IEEE Transactions on Biomedical Engineering* **44** 1221
[4] I.C. Khoo 1995 *Liquid Crystals-Physical Properties and Nonlinear Optical Phenomena* (John Wiley & Sons, Inc.)