Gamma-radiation in non-Markovian Fermi systems

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Abstract

The gamma-quanta emission is considered within the framework of the non–Markovian kinetic theory. It is shown that the memory effects have a strong influence on the spectral distribution of gamma-quanta in the case of long-time relaxation regime. It is shown that the gamma-radiation can be used as a probe for both the time-reversible hindrance force and the dissipative friction caused by the memory integral.

PACS numbers: 05.45.+b,02.50.+s,03.65.-w,03.65.Ge
I. INTRODUCTION

The dynamics and the dissipative properties of the many body Fermi system depend in many aspects on the dynamic distortion of the Fermi surface in momentum space. As is well-known [1], the presence of Fermi surface distortion allows the description of so-called collisional mechanism of relaxation and gives rise to the damping of collective motion. An additional one-body mechanism of relaxation exists in the finite system where the particles are placed into the external mean field. The origin of this damping is the collision of the particles with moving potential wall [2]. We will consider below both of them.

On the other hand relaxation of collective motion implies fluctuations in the corresponding collective variables, as follows from the fluctuation-dissipation theorem. Furthermore, the fluctuations in a particle density imply an accelerated motion of charges inside the charged system like a nucleus and lead, therefore, to radiation. The spectral distribution of this fluctuational radiation depends on the relaxation (dissipation) properties of the collective motion, in particular, on the dynamic distortion of the Fermi surface. We therefore suggest that a study of the shape of the radiation spectrum emitted from the heated system provides an opportunity to obtain information on the effects of temperature on dissipative properties and on the transition from the low-temperature (quantum) to the high-temperature (classical) regime in a finite many body system.

In the present paper, we are interested in the spectrum of fluctuations in shape variables. The precise form of such spectra can be expected to depend on the parameters of the model, such as the collision time, and, especially, on the memory effects. Here, we want to study these dependencies as one step to our ultimate goal of determining the model parameters from a comparison with experimental data, as might be possible due to a relation of the above mentioned spectra to $\gamma$-spectra.

In what follows, we combine the thermal and quantum fluctuations by means of the fluctuation-dissipation theorem. Such an approach presents a convenient connection between different regimes of collective motion such as the quantum zero-sound regime at zero temperature and the collisional first-sound regime in a hot system. Such an approach presents a convenient connection between different regimes of radiation such as the quantum regime at zero’s temperature and the thermal black body radiation of a hot system.

This paper is organized as follows. In Sec. II we suggest a proof of the Langevin equation
for the macroscopic collective variables starting from the collisional Landau-Vlasov kinetic equation, including the memory effects in the collisional integral. In Sec. III we review the classical approach to the fluctuational radiation. We adopt a Langevin equation with a random force as a source of the fluctuations. The main features of the dynamic distortion of the Fermi surface are taken into account. In Sec. IV we apply the results of Sec. II to the analysis of the spectral density of the fluctuational radiation. Concluding remarks are presented in Sec. V.

II. SURFACE FLUCTUATIONS IN A FINITE FERMI SYSTEM

To consider the fluctuations which accompany the collective motion in many-body Fermi-system, one can start from the collisional kinetic equation in presence of a random perturbation $y$ [1, 3]

$$\frac{\partial}{\partial t} f + \frac{p}{m} \cdot \nabla_r f - \nabla_r U \cdot \nabla_p f = \text{St}[f] + y,$$

where $f \equiv f(r, p; t)$ is the phase-space distribution function, $U \equiv U(r, p; t)$ is the selfconsistent mean field and $\text{St}[f]$ is the collision integral. The momentum distribution is distorted during the time evolution of the system and takes the following form

$$f(r, p; t) = f_{eq}(r, p; t) + \sum_{\delta f_{lm}} Y_{lm}(\hat{p}).$$

Here, $\epsilon$ is the single particle energy and $(\partial f/\partial \epsilon)_{eq} \sim \delta(\epsilon - \epsilon_F)$, where $\epsilon_F$ is the Fermi energy [1]. Below we will restrict ourselves to the azimuthally symmetric case (longitudinal perturbation) where $\delta f_{lm}$ is $m$-independent.
We will consider a linear response to the external random perturbation $y$. The linearized kinetic equation (1) is given by

$$\frac{\partial}{\partial t} \delta f + \hat{L} \delta f = \delta \text{St}[f] + y \quad (4)$$

where $\delta \text{St}[f]$ is the collision integral linearized in $\delta f = f - f_{\text{eq}}$ and the operator $\hat{L}$ represents the drift term

$$\hat{L} \delta f = \frac{p}{m} \cdot \nabla_r \delta f - \nabla_r U_{\text{eq}} \cdot \nabla_p \delta f - \nabla_r \delta U \cdot \nabla_p f_{\text{eq}}.$$ 

The collision integral $\delta \text{St}[f]$ depends on the transition probability of the two-nucleon scattering with initial momenta $(p_1, p_2)$ and final momenta $(p_1', p_2')$. At low temperatures $T \ll \epsilon_F$ the momenta $(p_1, p_2)$ and $(p_1', p_2')$ are localized near the Fermi surface and the relaxation time approximation can be used, see Refs. [1, 3],

$$\delta \text{St}[f] = - \frac{1}{\tau} \delta f |_{l \geq 1}, \quad (5)$$

where $\tau$ is the collisional relaxation time. The notation $l \geq 1$ means that the perturbation $\delta f |_{l \geq 1}$ in the collision integral includes only Fermi surface distortions with a multipolarity $l \geq 1$ in order to conserve the particle number in the collision processes [1]. The inclusion of the $l = 1$ harmonic in the collision integral of Eq. (5), at variance with the isoscalar case [2], is due to nonconservation of the isovector current, i.e. due to a collisional friction force between counterstreaming neutron and proton flows. The relaxation time $\tau$ depends on the temperature and contains, in the general case, memory effects ($\omega$-dependence) [3]:

$$\tau \equiv \tau(\omega, T) = \frac{4 \pi^2 \beta \hbar}{(\hbar \omega)^2 + \zeta T^2} \quad (6)$$

where $\beta$ and $\zeta$ are constants which are derived by the in-medium nucleon-nucleon scattering. Note that the well-known Landau’s prescription [13] assumes $\zeta = 4 \pi^2$. The parameter $\beta$ in Eq. (6) is rather badly established. It depends mainly on the in-medium nucleon-nucleon scattering cross-section $\sigma_{NN}$. For example, this value was calculated in Refs. [14] and [15] with the results between $\beta = 2.4$ and $\beta = 19.3$ for different assumptions about the scattering cross-section $\sigma_{NN}$.

Evaluating the first three moments of Eq. (4) in $p$-space and taking into account the condition [3], we can derive a closed set of equations for the following moments of the distribution function, namely, local particle density $\rho$, velocity field $u_\nu$ and pressure tensor $P_{\nu\mu}$, in the form the continuity and Euler-like equations (for details, see Appendix and Refs.
We will restrict ourselves by the shape fluctuations of Fermi liquid assuming an incompressible and irrotational flow, i.e.,

\[ \nabla \nu u \nu = 0 \tag{7} \]

and assuming also a sharp particle distribution in r-space

\[ \rho = \rho_0 \Theta [R(t) - r] \tag{8} \]

For the description of small amplitude oscillation of a certain multipolarity L we specify the surface as

\[ r = R(t) = R_0 \left[ 1 + \sum_M \alpha_{LM}(t)Y_{LM}(\theta, \phi) \right] \tag{9} \]

The basic continuity and Euler-like equations can be then reduced to the following Langevin equation (see Appendix, Eq. (88))

\[- \omega^2 m_L \alpha_{LM,\omega} + (C_L^{LD} + C_L'(\omega))\alpha_{LM,\omega} - i\omega \gamma_L(\omega)\alpha_{LM,\omega} = \xi_{LM,\omega}, \tag{10}\]

where the index \( \omega \) means the the Fourier transformation for the corresponding values and \( \xi_{LM,\omega} \) is the random force which occurs due to the random perturbation \( y \) in Eq. (1). The left part of Eq. (10) derives the eigenfrequency of surface eigenvibrations of the incompressible Ferm-liquid drop. Namely the corresponding secular equation reads

\[- \omega^2 m_L + C_L^{(LD)} + C_L'(\omega) - i\omega \gamma_L(\omega) = 0 \tag{11} \]

In Eq. (10), the mass coefficient \( m_L \) is given by

\[ m_L = m \int d\mathbf{r} \rho_{eq} \sum_{\nu} |a_{LM,\nu}|^2 = \frac{3}{4\pi L} AmR_0^2 \tag{12} \]

and the static stiffness coefficient \( C_L^{(LD)} \) is derived from the elastic properties of system

\[ C_L^{(LD)} = \frac{1}{4\pi} (L - 1)(L + 2)b_S A^{2/3} - \frac{L - 1}{2L + 1} b_C \frac{Z^2}{A^{1/3}}, \tag{13} \]

where \( b_S \) is the surface energy coefficient appearing in the nuclear mass formula. This definition coincides with the one for the stiffness coefficient in the traditional liquid drop model for the nucleus [21]. We point out, that the nucleon-nucleon interaction, manifested
at the starting equations \([4] \text{ and } [3]\), is presented in Eq. \([10]\) only implicitly through the phenomenological stiffness coefficient \(C_{L}^{(LD)}\). Both coefficients \(b_{S}\) and \(b_{C}\) in Eq. \([13]\) are temperature dependent. We will below assume the following temperature dependence of the surface and Coulomb parameters \([16]\)

\[
    b_{S} = 17.2 \left[ \frac{T_{C}^{2} - 2}{T_{C}^{2} + 2} \right]^{5/4} \text{ MeV}, \quad b_{C} = \frac{3e^{2}}{2\pi r_{C}} (1 - x_{C}T^{2}) \approx 0.55(1 - x_{C}T^{2}) \text{ MeV},
\]

where \(r_{C} = 1.24 \text{ fm} \) \([20]\), the parameter \(x_{C}\) was chosen as \(x_{C} = 0.76 \cdot 10^{-3} \text{ MeV}^{-2}\) and \(T_{C} = 18 \text{ MeV}\) is taken as the critical temperature \(T_{C}\) for infinite nuclear Fermi-liquid \([16]\). The nuclear Fermi-liquid does not exist for temperatures \(T \geq T_{C}\). Using Eq. \([13]\), one can find a limiting temperature \(T_{\text{lim}}^{(LD)}\) where the liquid drop contribution \(C_{L}^{(LD)}\) to the stiffness coefficient vanishes:

\[
    C_{L}^{(LD)} \equiv C_{L}^{(LD)}(T) \bigg|_{T=T_{\text{lim}}^{(LD)}} = 0.
\]

For the parameters used in the present work one obtains \(T_{\text{lim}}^{(LD)} = 7.7 \text{ MeV}\) for quadrupole deformation, \(L = 2\), in \(^{208}\text{Pb}\). For temperatures \(T \geq T_{\text{lim}}^{(LD)}\) there exists always an eigenfrequency with a positive imaginary part giving rise to an exponentially growing deformation.

The additional stiffness coefficient \(C_{L}'(\omega)\) in Eq. \([16]\) is due to the Fermi surface distortion. In the case of quadrupole distortions of Fermi surface, the final form of this coefficient is given by

\[
    C_{L}'(\omega) = d_{L} \frac{(\omega\tau)^{2}}{1 + (\omega\tau)^{2}} P_{\text{eq}},
\]

where

\[
    d_{L} = 2\frac{(L-1)(2L+1)}{L} R^{3}_{0}, \quad P_{\text{eq}} = \frac{1}{3m} \int \frac{gd\mathbf{p}}{(2\pi\hbar)^{3}} \rho^{2} f_{\text{eq}} = \frac{2}{5} \epsilon_{F} \rho_{\text{eq}}.
\]

Both stiffness coefficients \(C_{L}^{(LD)}\) and \(C_{L}'(\omega)\) generate the shape eigenvibrations. The eigenfrequency \(\omega_{L}\) of the undamped (i.e., for \(\gamma_{L}(\omega) = 0\)) eigenvibrations of Fermi-liquid drop is obtained from the implicit equation

\[
    \omega_{L}(\omega) = \sqrt{\left[ C_{L}^{(LD)} + C_{L}'(\omega) \right] / m_{L}}.
\]

The friction coefficient \(\gamma_{L}(\omega)\) in Eq. \([10]\) is given by

\[
    \gamma_{L}(\omega) = d_{L} \eta(\omega),
\]
where $\eta(\omega)$ is the viscosity coefficient

$$
\eta(\omega) = \frac{\tau P_{eq}}{1 + (\omega \tau)^2}
$$

The secular equation (11) can be used to describe the eigenenergy $E$ and the width $\Gamma$ of the Giant Multipole Resonances (GMR) in cold nuclei. In Fig. 1 and 2 we show the results of calculations and the comparison with experimental data for the case of the isoscalar Giant Quadrupole Resonances (GQR) for the nuclei through the periodic table of elements. The numerical results in Fig. 1 and 2 have been obtained using Eq. (11) and the relaxation parameter $\beta = 0.8$ MeV in Eq. (5) for $T = 0$. Evaluating the LDM stiffness coefficient $C_{L}^{(LD)}$ of Eq. (13), we have used the charge number $Z$ on the beta-stability which is given by [26]

$$
Z = \frac{1}{2} A \left[ 1 - \frac{0.4 A}{A + 200} \right].
$$

As can be seen from Fig. 1 and 2, our approach provides a quite satisfactory description of both the eigenenergies $E_{\text{GQR}}$ and the widths $\Gamma_{\text{GQR}}$ simultaneously. This fact can be used to fit the relaxation parameter $\beta$ in Eq. (6). We will below adopt $\beta = 0.8$ MeV. We point out that the traditional liquid drop model [20] is unable to describe the energy of the GQR, see the dashed line in Fig. 1. In Fig. 2 the dashed lines represent the results for the widths of the GQR obtained by use of simplest Swiatecki’s wall-formula (one-body dissipation) given by [6, 24]

$$
\Gamma_{L,\text{one-body}} = \frac{1}{A} \pi \rho_0 v_F \hbar R_0^2 L \approx 34.3 L A^{-1/3} \text{ MeV}
$$

and the so-called modified wall formula given by [25]

$$
\Gamma_{L,\text{modif}} = \frac{1}{A} \pi \rho_0 v_F \hbar \lambda^2 (L - 1)^2 L \approx 73.9 (L - 1)^2 L A^{-1} \text{ MeV},
$$

where the value of parameter $\lambda^2 \approx 3 \text{ fm}^2$ was obtained from comparisons of calculated and experimental fission-fragment kinetic energies. One can see that the simplest wall formula overestimates significantly the GQR width while the calculation by use of the modified wall-formula is significantly smaller than the experimental results for heavy nuclei.

Coming back to the right part of Eq. (11), note that the random force $\xi_{LM,\omega}$ is derived by the properties for the ensemble averaged correlation function. Namely,

$$
\overline{\xi_{LM,\omega}} = 0, \quad \langle \xi_{LM} \rangle_0 = 2 E(\omega, T) \gamma_L(\omega),
$$
where, see also [12],
\[ E(\omega, T) = \frac{\hbar \omega}{2} \coth \left( \frac{\hbar \omega}{2T} \right) = \frac{\hbar \omega}{2} + \frac{\hbar \omega}{\exp (\hbar \omega/T) - 1}. \quad (21) \]
We have preserved the constant $\hbar$ in Eq. (21) in order to stress the fact that both the quantum and thermal fluctuations are involved into the random force $\xi_{LM,\omega}$.

III. GAMMA-RADIATION CAUSED BY PRESENCE OF RANDOM FORCES

In this section we are going to establish the connection between the spectrum of emitted photons and the equation of motion for the collective variable, the fluctuations of which lead to the former of the radiation spectrum. Let us start from the usual quantum-mechanical definition of the perturbative transition probability per unit time in an energy interval $d(\hbar \omega)$
\[ dW_{fi} = \frac{1}{\hbar^2} \lim_{T \to \infty} \frac{1}{2T} \left| \int_{-T}^{T} dt' \langle \psi_f(t') | V(t') | \psi_i(t') \rangle \right|^2 d\nu_f. \quad (22) \]
Here $V(t)$ is the one-body perturbation field
\[ V(t) = e^{i\omega t} \int dr \, q(r) \hat{\rho}(r) + c.c., \quad (23) \]
where $e\hat{\rho}(r)$ is the charge density operator
\[ e\hat{\rho}(r) = \sum_{i=1}^{A} e_i \delta(r - \vec{r}_i) \quad (24) \]
and $d\nu_f$ is the number of final states in the energy interval $[\hbar \omega, \hbar \omega + d(\hbar \omega)]$. The choice for the function $q(r)$ depends on the problem under consideration.

In a general case the initial and final wave functions, $\psi_i(t)$ and $\psi_f(t)$ respectively, are non-stationary ones and we write
\[ \psi^*_f(t) \psi_i(t) = \varphi^*_f(r_1, ..., r_A) \varphi_i(r_1, ..., r_A) \alpha_{fi}(t). \quad (25) \]
In particular, in a stationary case we have
\[ \alpha_{fi} = e^{i(E_f - E_i)t/\hbar}. \quad (26) \]
and Eqs. (22) and (23) give the usual result for the transition probability per unit time which is $dW_{fi} \sim \delta(E_i - E_f - \hbar \omega)$. 8
We will consider below the electromagnetic $EL$-transitions using for $q(r)$ in (23),

$$q(r) = r^L Y_{LM} \equiv q_{LM}(r).$$  \hspace{1cm} (27)

The usual transformation of Eq. (22) for the case of multipole transitions gives \hspace{1cm} [9, 10]

$$dW_{fi}(EL) = 4 \left( \frac{L+1}{L} \right) \left( \frac{1}{2L+1} \right) \left( \frac{\omega}{\omega_c} \right)^{2L+1} \omega e^2 \left( \frac{\omega}{\omega_c} \right)^{2L+1} \sum_{M,M',M} \left| \left\langle \varphi_f \middle| \int dr q_{LM}(r) \mathcal{\hat{\rho}} \middle| \varphi_i \right\rangle \right|^2 \times \lim_{T \to \infty} \frac{1}{2T} \left| \int_{-T}^{T} dt' e^{i\omega t'} \alpha_{fi}(t') \right|^2 d\omega, $$ \hspace{1cm} (28)

where $\omega_c = \omega$ is the wave number of the photon.

The transition probability $dW_{fi}$ allows us to evaluate the power $dP_{fi}$ radiated in the energy interval $d(\hbar \omega)$ as

$$dP_{fi}(EL) = \hbar \omega dW_{fi}(EL).$$  \hspace{1cm} (29)

The classical result the radiated power $dP_{\text{class}}(EL)$ can be obtained from the quantum mechanical one, Eqs. (28) and (29), by using the correspondence principle for the transition density:

$$\left\langle \psi_f(t)|\mathcal{\hat{\rho}}|\psi_i(t)\right\rangle \equiv \alpha_{fi}(t) \left\langle \varphi_f |e\mathcal{\hat{\rho}}| \varphi_i \right\rangle \Rightarrow \epsilon \delta \rho(r, t) = \alpha(t) \epsilon \delta \rho(r).$$ \hspace{1cm} (30)

Here $\epsilon \delta \rho(r, t)$ is the variation of the classical charge density in the external field $V(t)$, Eq. (23). Thus, we have from Eqs. (28)-(30)

$$dP_{\text{class}}(EL) = R_L(\omega) \sum_M |Q_{LM}|^2 \lim_{T \to \infty} \frac{1}{2T} \left| \int_{-T}^{T} dt' e^{i\omega t'} \alpha(t') \right|^2 d\omega,$$ \hspace{1cm} (31)

where

$$R_L(\omega) = 4 \left( \frac{L+1}{L} \right) \left( \frac{1}{2L+1} \right) \omega e^2 \left( \frac{\omega}{\omega_c} \right)^{2L+1}$$ \hspace{1cm} (32)

and

$$Q_{LM} = \int dr q_{LM}(r) \delta \rho(r).$$ \hspace{1cm} (33)

Let us rewrite the time double integral in Eq. (31) as

$$\lim_{T \to \infty} \frac{1}{2T} \left| \int_{-T}^{T} dt' e^{i\omega t'} \alpha(t' \right| = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} dt \int_{-T}^{T} dt' e^{i\omega t'} \alpha(t) \alpha(t + t')$$ \hspace{1cm} (34)
Here the time average
\[
\overline{\alpha(t)\alpha(t+t')} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} dt \alpha(t)\alpha(t+t') = \langle \alpha(t)\alpha(t+t') \rangle
\] (35)
can be considered as an ensemble average \(\langle \ldots \rangle\) for an ergodic system.

Finally, we shall rewrite Eq. (31) as
\[
dP_{\text{class}}(EL) = R_L(\omega) \sum_M Q_{LM}^2 \langle \alpha^2 \rangle_\omega d\omega,
\] (36)
where \(\langle \alpha^2 \rangle_\omega\) is the spectral correlation function \[12\].
\[
\langle \alpha^2 \rangle_\omega = \int_{-\infty}^{\infty} dt' e^{i\omega t'} \langle \alpha(t)\alpha(t+t') \rangle.
\] (37)

As it was demonstrated in the previous section, the dynamics of small fluctuations of the collective variable \(\alpha(t)\) coupled to a heat bath can be described by a Langevin equation of the form
\[
-m_L\omega^2 \alpha_{LM,\omega} - i\gamma_L(\omega)\omega \alpha_{LM,\omega} + m_L\omega_L^2(\omega)\alpha_{LM,\omega} = \xi_{LM,\omega}.
\] (38)
Using Eqs. (36) and (38) we can derive an expression for the emitted fluctuational power in terms of the spectral correlation function \(\overline{(\xi_{LM})^2}_\omega\) of the random force \(\xi_{LM}(t)\):
\[
dP_{\text{class}}(EL) = R_L(\omega) \sum_M Q_{LM}^2 \overline{(\xi_{LM})^2}_\omega / m_L^2 \left[ \omega^2 - \omega_{LM}^2(\omega) \right]^2 + \omega^2 \gamma_L^2(\omega) d\omega.
\] (39)
The function \(\overline{(\xi_{LM})^2}_\omega\) depends on the dissipative properties of the system. In the previous section we have derived this function as well as explicit expressions for the transport coefficients \(m_L, \omega_L\) and \(\gamma_L\) for the case of collective particle excitation, see Eq. (20).

**IV. SPECTRAL DENSITY OF RADIATION. NUMERICAL RESULTS**

Equation (10) for the shape oscillations of a Fermi-liquid drop together with Eqs. (20) and (39) can be used for the analysis of the spectral density
\[
J_L(\omega) = dP_{\text{class}}(EL) / d\omega
\] (40)
of fluctuational radiation. We thus have
\[
J_L(\omega) = \sum_M R_L(\omega) Q_{LM}^2 \overline{(\xi_{LM})^2}_\omega / m_L^2 \left[ \omega^2 - \omega_{LM}^2(\omega) \right]^2 + \omega^2 \gamma_L^2(\omega).
\] (41)
In the case of shape oscillations of $L$ multipolarity we have from Eqs. (30), (8) and (9)

$$\delta \rho(r, t) = -\rho_0 R_0 \delta(r - R_0) \sum_M \alpha_{LM}(t) Y_{LM}(\theta, \phi)$$  \hspace{1cm} (42)

and from Eqs. (27), (30) and (33)

$$Q_{LM} = -\rho_0 R_0^{L+3}.$$  \hspace{1cm} (43)

Collecting Eqs. (41), (20) and (43) we finally find

$$J_L(\omega) = H_L \frac{E(\omega, T) \gamma_L(\omega)}{m_L^2 \left[ \omega^2 - \omega_L^2(\omega) \right]^2 + \omega^2 \gamma_L^2(\omega)}$$

$$= \left[ \frac{1}{2} \hbar \omega + \frac{\hbar \omega}{\exp(\hbar \omega/T) - 1} \right] \frac{H_L \gamma_L(\omega)}{m_L^2 \left[ \omega^2 - \omega_L^2(\omega) \right]^2 + \omega^2 \gamma_L^2(\omega)},$$  \hspace{1cm} (44)

where

$$H_L = 8 e^2 \rho_0^2 \omega \left( \frac{\omega}{c} \right)^{2L+1} \frac{L + 1}{L} \frac{R_0^{2L+6}}{[(2L + 1)!!]^2}. $$  \hspace{1cm} (45)

Note, that presence of term $\hbar \omega/2$ in Eq. (44) reflects a general problem of zero energy contribution. This term provides an unphysical infinite contribution to the total energy of radiation and must be thereby excluded. We preserve this term to provide the correct transition to the quantum regime at $T \to 0$, where this term manifests the zero-point fluctuations.

In a general case, we have to take into account the radiation friction effects in Eq. (44) to guarantee the asymptotic convergency of the spectral density $J_L(\omega)$ at $\omega \to \infty$. It can be done by the following substitution for $\gamma_L(\omega)$ in the denominator of Eq. (44) (see Ref. 9)

$$\gamma_L(\omega) \to \gamma_L(\omega) + (\omega/\omega_L)^2 \gamma_L'(\omega).$$  \hspace{1cm} (46)

Here $\gamma_L'(\omega)$ is the radiation friction coefficient

$$\gamma_L'(\omega) = \Gamma_L/\omega,$$  \hspace{1cm} (47)

$\Gamma_L$ is the radiation width of the surface excitation

$$\Gamma_L = (1/\omega) \pi H_L \alpha_{L,0}^2$$  \hspace{1cm} (48)

and $\alpha_{L,0}$ is the zero-point amplitude

$$\alpha_{L,0}^2 = \frac{\hbar}{2 \sqrt{C_L m_L}} = \frac{\hbar}{2 \omega_L m_L}.$$  \hspace{1cm} (49)
The formula (44) for the radiation is valid for arbitrary collision times $\tau$ and thus describes both the quantum and the high temperature limit as well as the intermediate cases. From it one can obtain the leading order terms in the different limits mentioned.

(1) High temperature limit: $\omega \tau \rightarrow 0, T \gg \hbar \omega$

The contribution from the dynamic distortion of the Fermi surface can be neglected in this case and we have from Eq. (16)

$$C_L'(\omega_L) \approx 0. \quad (50)$$

The eigenfrequencies $\omega_L$ of the shape oscillations are determined here by the usual liquid drop model as

$$\omega_{L,0} = \sqrt{C_L/m_L}. \quad (51)$$

In the high temperature regime, the Fermi liquid viscosity $\eta(\omega)$, Eq. (19), approaches the classical expression

$$\eta_{\text{class}} = \frac{1}{5} \rho_0 p_F^2 \tau_0. \quad (52)$$

where $p_F$ is the Fermi momentum and $\tau_0 \equiv \tau(\omega = 0, T)$. The spectral correlation function $(\xi_{LM})^2_\omega$ of the random force can be found from Eqs. (20), (21) and (18)

$$\langle \xi_{LM} \rangle^2_\omega = 2 \gamma_{L,0} T. \quad (53)$$

where $\gamma_{L,0} = \gamma_L(\omega = 0)$. This correlation function is independent of $\omega$, i.e., it corresponds to a white noise.

In this limit, the spectral density of radiation, Eq. (44), is given by

$$J_L(\omega) = H_L \frac{\gamma_{L,0} T}{m_L^2 (\omega^2 - \omega_{L,0}^2)^2 + \omega^2 \gamma_{L,0}^2}, \quad (54)$$

where $\omega_{L,0} = \sqrt{C_L/m_L}$. The spectral density (54) is proportional to the temperature $T$ as expected for a classical thermal emission of radiation (18). In the high temperature limit $T \rightarrow \infty$ we have $\gamma_{L,0} \rightarrow 0$ and

$$\lim_{T \rightarrow \infty} \frac{\gamma_{L,0}}{m_L^2 (\omega^2 - \omega_{L,0}^2) + \omega^2 \gamma_{L,0}^2} = \frac{\pi}{2 m_L \omega_{L,0}^2} \left[ \delta(\omega - \omega_{L,0}) + \delta(\omega + \omega_{L,0}) \right]. \quad (55)$$

Thus, the spectral density of the radiation is given at high temperature by

$$J_L(\omega) = \pi H_L \alpha_{L,\text{therm}}^2 \delta(\omega - \omega_{L,0}), \quad (56)$$
where $\alpha^2_{L,\text{therm}}$ is the square of the thermal oscillation amplitude

$$\alpha^2_{L,\text{therm}} = \frac{T}{2m_L \omega_{L,0}^2} = \frac{T}{2C_L}. \tag{57}$$

The result of Eqs. (56) and (57) recovers the Rayleigh–Jeans law for the black body radiation.

(2) Quantum regime: $\omega \tau \to \infty$, $T \ll \hbar \omega$

The contribution to the stiffness coefficient from the dynamic distortion of the Fermi surface is now given by (see Eq. (16))

$$C'_L(\omega) \approx \tilde{C}'_L = d_L P_{eq}. \tag{58}$$

We note that, in a cold Fermi system at $L \neq 1$, $\tilde{C}'_L$ provides the main contribution to the stiffness coefficient. The viscosity coefficient $\eta(\omega)$, Eq. (19), can be approximated in this limit by

$$\eta(\omega) = (P_{eq}/4 \pi^2 \beta \hbar)[1 + \zeta(T/\omega)^2]. \tag{59}$$

The spectral correlation function $(\xi_{LM})^2_\omega$ is obtained from Eqs. (20), (18) and (21) to be

$$(\xi_{LM})^2_\omega = \hbar \omega \tilde{\gamma}_L, \tag{60}$$

where

$$\tilde{\gamma}_L = d_L P_{eq}/4 \pi^2 \beta \hbar \tag{61}$$

does not depend on $\omega$. The spectral correlation function (60) now corresponds to a blue noise.

The spectral density of radiation $J_L(\omega)$ can be found from Eqs. (44) and (60) to have the form

$$J_L(\omega) = H_L \frac{\hbar \omega \tilde{\gamma}_L}{m_L^2 (\omega^2 - \tilde{\omega}_L^2)^2 + \omega^2 \tilde{\gamma}_L^2}, \tag{62}$$

where $\tilde{\omega}_L$ is the eigenfrequency

$$\tilde{\omega}_L = \sqrt{(C_L + \tilde{C}'_L)/m_L}$$

of the zero sound mode in the case of no damping.
Similarly to the high temperature regime, for low temperatures the spectral density takes the form of a sharp peak in the limit of small damping, i.e. for $\tilde{\gamma}_L \to 0$:

$$J_L(\omega) = \lim_{\tilde{\gamma}_L \to 0} H_L \frac{\hbar \omega \tilde{\gamma}_L}{m_L^2 (\omega^2 - \tilde{\omega}_L^2)^2 + \omega^2 \tilde{\gamma}_L^2} = H_L \frac{\pi \hbar}{2 m_L \tilde{\omega}_L} \delta(\omega - \tilde{\omega}_L) = \pi H_L \tilde{\alpha}_{L,0}^2 \delta(\omega - \tilde{\omega}_L),$$

where (compare with Eq. (57))

$$\tilde{\alpha}_{L,0}^2 = \frac{\hbar \tilde{\omega}_L}{2 m_L \tilde{\omega}_L^2} = \frac{\hbar \tilde{\omega}_L}{2 (C_L + \tilde{C}_L')}$$

is the square of renormalized zero-point amplitude (compare with Eq. (57)). In this limit the expression for the spectral density coincides with the usual quantum-mechanical result for the photon emission associated with shape oscillations of the charge $Z$. We recall that the quantum-mechanical result was obtained from the classical approach, Eq. (39). It is due to the fact that the quantum fluctuations have been incorporated into the correlation function (20) through the factor $E(\omega, T)$, Eq. (21), see also [12].

In Fig. 3 we have plotted the spectral density of gamma-quanta emission $J_L(\omega)$ as obtained from Eq. (44) for two temperatures $T = 3 \text{ MeV} < T_{\text{lim}}^{(LD)}$ and $T = 8 \text{ MeV} > T_{\text{lim}}^{(LD)}$ in the case $\beta = 0.8 \text{ MeV}$. The dashed line is for the statistical $\gamma$-quanta emission given by

$$J_L(\omega) = \text{const} \omega^{2L+1} \exp \left( -\frac{\hbar \omega}{T} \right),$$

where the value of ”const” is normalized to the same integral emission as is obtained from Eq. (44). For low temperature $T = 3 \text{ MeV}$ we observe a well defined maximum (solid curve 1) which corresponds to the GQR excitation (zero-sound regime).

An increase of $T$ leads to a shift of the maximum of $J_L(\omega)$ to lower frequencies and to an increase in the width. The shape of the curves near the zero-sound maximum in Fig. 3 is a non-Lorentzian one and depends, in particular, on the retardation effects in the friction coefficient, Eq. (18), and, consequently, on the parameters $\beta$ and $\zeta$ in the relaxation time, Eq. (6). Increasing the temperature we do not find a first sound peak centered at low frequency. We point out an interesting phenomenon. For a large enough value of $\beta$, namely $\beta \geq 0.5 \text{ MeV}$, there is, in principle, a possibility for a resonance-like structure of $J_L(\omega)$ at temperatures $T > T_{\text{lim}}^{(LD)}$ which is due to the pure Fermi-surface vibrations in the momentum space. For these values of $\beta$ there exists a temperature region where $C_L^{(LD)}(T) \leq 0$ but...
$C_L(\omega_L) > 0$, simultaneously. This implies the existence, in this high temperature region, of a particular eigenmode of the Fermi liquid drop where the restoring force is exclusively due to the dynamic Fermi-surface distortion. Note that the non-monotonic behavior of solid curve 2 in Fig. 3 occurs just due to the combination of polynomial $\sim \omega^{2L+2}$ and the Planck’s $\sim [\exp (\hbar \omega / T) - 1]^{-1}$ multipliers in Eq. (44). Note also that the statistical gamma-quanta emission given by Eq. (64) does not exist at high temperatures $T > T_{\text{lim}}^{(LD)}$ because of $C_L^{(LD)}(T) \leq 0$ and a drop is unstable for this temperature regime. That means that the dashed line in Fig. 3 does not occur for $T > T_{\text{lim}}^{(LD)}$.

Some peculiarities of forming of the resonance eigenenergy $E_{\text{GQR}}$ and the corresponding width $\Gamma_{\text{GQR}}$ for the nucleus $^{208}\text{Pb}$ are shown in Figs. 4 and 5. The eigenfrequencies $\omega$ are derived by the secular equation (11). In general, the eigenfrequency $\omega$ depends on both the liquid drop stiffness coefficient $C_L^{(LD)}$ and the specific one $C'_L(\omega)$ caused by the Fermi surface distortions. The dashed line in Fig. 4 represents the result for the classical liquid drop, i.e., with $C'_L(\omega) = 0$. The Fermi-liquid eigenfrequencies $\omega$ (solid lines in Fig. 4) are shifted up with respect to the liquid drop solution (dashed line) due to the strong enhancement of the stiffness coefficient caused by the Fermi-surface distortion (FSD) effect. A shift down of the line 1 at $T = 0$ for a small value of relaxation parameter $\beta$ occurs because of a strong hindrance of the FSD effect in the frequent collision regime. The liquid drop eigenfrequency (dashed line in Fig. 4) disappears at the limiting temperature $T_0 \approx T_{\text{lim}}^{(LD)} = 7.7$ MeV, see Eq. (13). An increase of the relaxation parameter $\beta$ provides a significant contribution $C'_L$ to the stiffness coefficient caused by the Fermi-surface distortion effect. Due to this fact the resonance eigenfrequency $\text{Re}\omega$ exists for temperatures $T_0$ higher than the limiting one $T_{\text{lim}}^{(LD)}$.

The threshold for the Fermi-liquid drop eigenfrequencies $\text{Re}\omega$ depends significantly on the relaxation parameter $\beta$ (see the existence regions for the curves 1, 2 and 3 in Fig. 4).

For each $\beta$ there are few solutions to Eq. (11). One of them, $\omega^{(1)}$, is purely imaginary (see dotted line in Fig. 5). Two of them $\pm \text{Re}\omega - i\text{Im}\omega$ are located symmetric with respect to the imaginary axis. Note also that the solution $\omega^{(1)}$ has the positive imaginary part for $T_0 > T_{\text{lim}}^{(LD)}$ giving rise to an exponentially growing deformation (unstable mode). The motion becomes overdamped in the temperature regions where $\text{Re}\hbar\omega = 0$ (see the dashed paths in Fig. 5).
V. CONCLUSIONS

Starting from the collisional Landau-Vlasov kinetic equation with a random force, we have derived the Langevin-like equation for the surface fluctuations of the particle distribution in a Fermi system. The main feature of these fluctuations is that the higher multipole modes \((L \geq 2)\) are strongly influenced by the Fermi-surface distortion effects: the stiffness coefficient contains an additional contribution \(C'_L(\omega)\) (see Eq. (10)) and the friction coefficient \(\gamma_L\), Eq. (18), includes the collisional relaxation phenomena. We have obtained the random-force correlation function (20) for the general case where we also take into account retardation and memory effects in the relaxation time \(\tau(\omega,T)\). Accounting of the retardation and memory effects plays an important role in order to obtain a correct transition from the quantum mechanical regime in cold system to the classical regime at high temperatures.

The effects of the dynamic distortion of the Fermi surface on the collective motion lead to the peculiarities of the random-force correlation function which do not occur in a classical system. The spectral correlation function (20) is independent of \(\omega\) and corresponds to a white noise in the high temperature regime at \(\omega \tau \to 0\) whereas in the opposite quantum regime at \(\omega \tau \to \infty\) it corresponds to a blue noise (60). The behavior of the radiation spectral density \(J_L(\omega)\) at different temperatures reflects the above mentioned peculiarities of the random-force correlation function. We predict a strong dependence of the shape of the curves \(J_L(\omega)\) on the retardation effects (\(\omega\)-dependence) in the friction coefficient (18) and, consequently, on the parameters \(\zeta\) and \(\beta\) at \(L \geq 2\).

Our approach to the shape fluctuations and to the corresponding radiation is essentially classical. However, due to the Landau’s ansatz (21), the quantum effects are returned into the fluctuation problem and the correlation functions (20) contain contributions from both quantum and thermal fluctuations. This aspect of the fluctuation theory allows us to reproduce a standard result (63) of the quantum theory for the spectral density of radiation in cold system at zero friction.

Finally, we would like to stress that the fluctuational photon emission, presented in this paper, does not appear as a new additional source of radiation but only as a method for determination of radiation which allows us to include both quantum and thermal emissions of photons in a common consideration.
VI. APPENDIX

Taking three first moments in $\mathbf{p}$-space from Eqs. (4) and (3) we can derive a closed set of equations for the following moments of the distribution function, namely, local particle density $\rho$, velocity field $u_\nu$ and pressure tensor $P_{\nu\mu}$, in the form

$$ \frac{\partial}{\partial t} \delta \rho = -\nabla_\nu (\rho_{eq} u_\nu), \quad (65) $$

$$ m \rho_{eq} \frac{\partial}{\partial t} u_\nu + \rho_{eq} \nabla_\nu \left( \frac{\delta^2 \mathcal{E}}{\delta \rho^2} \right)_{eq} \delta \rho + \nabla_\mu P'_{\nu\mu} = 0, \quad (66) $$

$$ \frac{\partial}{\partial t} P'_{\nu\mu} + P_{eq} (\nabla_\nu u_\mu + \nabla_\mu u_\nu - \frac{2}{3} \delta_{\nu\mu} \nabla_\alpha u_\alpha) = I_{\nu\mu} + y_{\nu\mu}. \quad (67) $$

Here

$$ \delta \rho = \int \frac{g d\mathbf{p}}{(2\pi \hbar)^3} \delta f, \quad u_\nu = \frac{1}{\rho} \int \frac{g d\mathbf{p}}{(2\pi \hbar)^3} \frac{p_\nu}{m} \delta f \quad (68) $$

$g$ is the spin-isospin degeneracy factor, $\mathcal{E}$ is the internal energy density, which is the sum of the kinetic energy density of the Fermi motion and the potential energy density associated with the nucleon-nucleon interaction. The tensor $I_{\nu\mu}$ is the second moment of the collision integral

$$ I_{\nu\mu} = \frac{1}{m} \int \frac{d\mathbf{p}}{(2\pi \hbar)^3} p_\nu p_\mu St[f] \quad (69) $$

and $y_{\nu\mu}$ gives the contribution from the random force

$$ y_{\nu\mu} = \frac{1}{m} \int \frac{d\mathbf{p}}{(2\pi \hbar)^3} p_\nu p_\mu y. \quad (70) $$

The equilibrium pressure, $P_{eq}$, is given by

$$ P_{eq} = \frac{1}{3m} \int \frac{d\mathbf{p}}{(2\pi \hbar)^3} p^2 f_{eq} \quad (71) $$

and $P'_{\nu\mu}$ is the deviation of the pressure tensor from its isotropic part due to the Fermi surface distortion

$$ P'_{\nu\mu} = \frac{1}{m} \int \frac{d\mathbf{p}}{(2\pi \hbar)^3} (p_\nu - mu_\nu)(p_\mu - mu_\mu) \delta f. \quad (72) $$

Using the Fourier transformation for the pressure

$$ P'_{\nu\mu}(t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} P'_{\nu\mu,\omega} \quad (73) $$
and similarly for the other time dependent variables we find the solution to Eq. (67) as

\[
P'_{\nu\mu,\omega} = \frac{i\omega\tau - (\omega\tau)^2}{1 + (\omega\tau)^2} P_{eq}\Lambda_{\nu\mu,\omega} + \frac{\tau}{1 + (\omega\tau)^2}(1 + i\omega\tau) y_{\nu\mu,\omega},
\]

(74)

where we used the symbol

\[
\Lambda_{\nu\mu,\omega} = \nabla_\nu \chi_{\mu,\omega} + \nabla_{\mu} \chi_{\nu,\omega} - \frac{2}{3} \delta_{\nu\mu} \nabla \chi_{\lambda,\omega}
\]

(75)

for this combination of gradients of the Fourier transform \(\chi_{\nu,\omega}\) of the displacement field. The time derivative of \(\chi_\nu(r, t)\) is defined as the velocity field, hence

\[
u_{\nu,\omega} = -i\omega \chi_{\nu,\omega}.
\]

(76)

To obtain Eq. (74) we have also used the fact that the tensor \(I_{\nu\mu}\), Eq. (69), can be reduced to

\[
I_{\nu\mu,\omega} = -\frac{1}{\tau} P'_{\nu\mu,\omega},
\]

(77)

due to our restriction to quadrupole deformation of the Fermi surface.

From Eqs. (65), (66) and (74) we find the equation of motion for the displacement field \(\chi_{\nu,\omega}\) in the form

\[
-\rho_{eq}\omega^2 \chi_{\nu,\omega} + \hat{L}\chi_{\nu,\omega} = \nabla_\mu (\sigma_{\nu\mu,\omega} + s_{\nu\mu,\omega}),
\]

(78)

where the conservative terms are abbreviated by

\[
\hat{L}\chi_{\nu,\omega} = -\frac{1}{m}\rho_{eq} \nabla_\nu \left( \frac{\delta^2 \mathcal{E}}{\delta \rho^2} \right)_{eq} \nabla_\mu \rho_{eq} \chi_{\mu,\omega} - \text{Im} \left( \frac{\omega\tau}{1 - i\omega\tau} \right) \nabla_\mu \frac{P_{eq}}{m} \Lambda_{\nu\mu,\omega},
\]

(79)

\(\sigma_{\nu\mu}\) is the viscosity tensor

\[
\sigma_{\nu\mu,\omega} = -i(\omega/m)\eta(\omega) \Lambda_{\nu\mu,\omega}
\]

(80)

with the viscosity coefficient

\[
\eta(\omega) = \text{Re} \left( \frac{\tau}{1 - i\omega\tau} \right) P_{eq},
\]

(81)

and \(s_{\nu\mu,\omega}\) is the random pressure tensor

\[
s_{\nu\mu,\omega} = -\frac{\tau(1 + i\omega\tau)}{m(1 + (\omega\tau)^2)} y_{\nu\mu,\omega}.
\]

(82)

The correlation properties of \(s_{\nu\mu,\omega}\) can be obtained for the general case where we also take into account retardation and memory effects in the system, see Ref. [8] for details.
Using the correlation properties of the random tensor $y_{\nu\mu,\omega}$ and the fluctuation-dissipative theorem \[12\], we find for the ensemble average of

$$
\frac{1}{2}[s_{\nu\mu,\omega}(r); s_{\nu'\mu',\omega'}(r')]_+ = \frac{1}{2}[s_{\nu\mu,\omega}(r)s_{\nu'\mu',\omega'}(r') + s_{\nu'\mu',\omega'}(r')s_{\nu\mu,\omega}(r)]
$$

the result

$$
\frac{1}{2}[s_{\nu\mu,\omega}(r); s_{\nu'\mu',\omega'}(r')]_+ = \frac{4\pi}{m^2}E(\omega, T)\eta(\omega)\delta(r - r')\delta(\omega + \omega')[\delta_{\nu\nu'}\delta_{\mu\mu'} + \delta_{\nu\mu'}\delta_{\mu\nu'} - \frac{2}{3}\delta_{\nu\mu}\delta_{\nu'\mu'}], \tag{83}
$$

where

$$
E(\omega, T) = \frac{\hbar\omega}{2}\coth\frac{\hbar\omega}{2T}. \tag{84}
$$

We have preserved the constant $\hbar$ in Eq. \[84\] in order to stress the fact that both quantum and thermal fluctuations are involved in Eq. \[83\].

For the description of small amplitude oscillations of a certain multipolarity $L$ of a liquid drop we specify the liquid surface as

$$
r = R(t) = R_0[1 + \sum_M \alpha_{LM}(t)Y_{LM}(\theta, \phi)]. \tag{85}
$$

We write the displacement field $\chi_{\nu}(r, t)$ for an incompressible and irrotational flow, $\nabla_{\nu}\chi_{\nu} = 0$, as \[6\]

$$
\chi_{\nu}(r, t) = \sum_M a_{LM,\nu}(r)\alpha_{LM}(t), \tag{86}
$$

where

$$
a_{LM,\nu}(r) = \frac{1}{LPR_0^{-2}}\nabla_{\nu}(r^LY_{LM}(\theta, \phi)). \tag{87}
$$

Multiplying Eq. \[78\] by $ma_{LM,\nu}^*$, summing over $\nu$ and integrating over $r$-space, we obtain the Langevin equation for the collective variables,

$$
-\omega^2m_L\alpha_{LM,\omega} + (C_L^{(LD)} + C_L')\alpha_{LM,\omega} - i\omega\gamma_L(\omega)\alpha_{LM,\omega} = \xi_{LM,\omega}. \tag{88}
$$

The collective mass $m_L$ is found to be

$$
m_L = m \int d\rho_{eq} \sum_\nu |a_{LM,\nu}|^2 = \frac{3}{4\pi L}AmR_0^2, \tag{89}
$$

The static stiffness coefficient $C_L^{(LD)}$ is derived from the first term on the right hand side of Eq. \[79\] and is given by \[22\]

$$
C_L^{(LD)} = \frac{1}{4\pi}(L - 1)(L + 2)b_S A^{2/3} - \frac{5}{2\pi L + 1}b_C A^{1/3}. \tag{90}
$$
The random force $\xi_{LM,\omega}$ in Eq. (88) is related to the random pressure tensor $s_{\nu\mu,\omega}$ by

$$\xi_{LM,\omega} = -m \int d\mathbf{r} s_{\nu\mu,\omega} \nabla_{\mu} a_{LM,\nu}^*.$$  
(91)

Using Eqs. (12) and (16) we obtain the spectral correlation function $\langle \xi_{LM,\omega} \rangle$ of the random force $\xi_{LM}(t)$:

$$\langle \xi_{LM,\omega} \rangle^2 = 2 E(\omega, T) \eta(\omega) \int d\mathbf{r} \Lambda^{LM}_{\nu\mu} \nabla_{\mu} a_{LM,\nu}^* = 2 E(\omega, T) \gamma_L(\omega).$$  
(92)

The basic property of the random variable $y$, in Eq. (11), $\bar{y} = \bar{y}_{\nu\mu} = 0$ transfers to both, the random pressure tensor, $s_{\nu\mu,\omega} = 0$, and the random force, $\xi_{LM,\omega} = 0$.

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Figure captions

Fig. 1: The eigenenergies of the isoscalar GQR \((L = 2)\) versus the nuclear mass number \(A\). The results are obtained from the secular equation \([11]\) with \(\beta = 0.8\) MeV. The experimental data are taken from Ref. [23]. The dashed line is for the traditional liquid drop model (LDM) with \(C'_{L}(\omega) = 0\), Ref. [20].

Fig. 2: The same as in Fig. 1 for the widths \(\Gamma\) of the GQR. The dashed lines are for the one body dissipation (wall-formula [24] and modified wall-formula [25]).

Fig. 3: The spectral density of the quadrupole gamma-quanta emission \(J_{L}(\omega)\) for temperatures \(T = 3\) MeV \(< T_{(LD)}^{(lim)}\) (curves 1) and \(T = 8\) MeV \(> T_{(LD)}^{(lim)}\) (curve 2). The solid lines were obtained using Eq. \([44]\) for \(\zeta = 4\pi^{2}\) (Landau’s prescription [13]) and value of relaxation parameter \(\beta = 0.8\) MeV. The dashed line 1 is for the statistical emission of \(\gamma\)-quanta given by Eq. \([64]\) which was normalized to the same integral emission as for solid line 1.

Fig. 4: Dependence of the resonance eigenenergy \(\hbar\omega_{R} = \text{Re}\hbar\omega\) on the temperature \(T\) for three different values of the relaxation parameter \(\beta\) (the solid lines 1, 2 and 3 for \(\beta = 0.8\) MeV, 2.4 MeV and 9.8 MeV, respectively). The dashed line is for the pure liquid drop regime from Eq. \([17]\) with \(C'_{L}(\omega) = 0\).

Fig. 5: The same as in Fig. 4 but for the value of \(\text{Im}\hbar\omega\). The dashed paths are the solutions for the regions where \(\text{Re}\hbar\omega = 0\).
