Harmonic design of piezoelectric energy harvesting device coupled with electric circuit using topology optimization method

Zheqi Lin¹ and Xuansheng Wang²*

¹ Shenzhen Research Institute of Sun Yat-sen University, Shenzhen, Guangdong, 518057, China
² School of Software Engineering, Shenzhen Institute of Information Technology, Shenzhen, Guangdong, 518172, China
*Corresponding author’s e-mail: wxs111111@163.com

Abstract. In this paper, a novel method based on topology optimization is proposed to design the piezoelectric energy harvester connected to the closed electric circuit. The objective function is defined as to maximize the energy conversion efficiency under harmonic dynamic loads. A finite element model which is coupled with piezoelectric-circuit is applied to analyze the piezoelectric energy harvesting systems. Combining the solid isotropic material with penalization (SIMP) method for piezoelectric, an design optimization method that optimizes the piezoelectric material layout was developed. Complex sensitivity analysis was derived and the density function of the SIMP method is updated using the sequence linear programming method. The validity of the proposed approach was demonstrated by numerical example of energy harvester design.

1. Introduction

With the development of modern microelectronic devices, traditional energy supply methods have been unable to adapt to the needs of modern microelectronic devices due to their limited lifetime. Therefore, research on new advanced energy supply technologies has become a hot topic. Researchers are looking for a sustainable power supply method to provide electrical energy sources for electronic devices. One of the most interesting methods is to extract electrical energy from the continuous vibrations in the surrounding environment, namely vibration energy capture technology. Piezoelectric materials can convert mechanical vibration into electrical energy with high energy conversion. Various types of energy harvesting devices using piezoelectric materials have been developed by researchers[1, 2].

Many studies are available on modelling of piezoelectric energy harvesters[3]. The currently common piezoelectric energy harvester is in the form of a cantilever bimorph structure. The two-chip piezoelectric energy harvesting device is formed by sandwiching a layer of metal sheets between two piezoelectric sheets having different polarization directions, and suspending the mass on the energy harvesting device to adjust the natural frequency of the structure. However, the output power of piezoelectric power generation is still very limited, which seriously hinders the wide application of this technology. How to effectively improve the working capacity of piezoelectric energy harvesting devices has become a key issue that needs to be solved urgently[4].
Various types of method have been studied to optimize the size or shape of the piezoelectric and metal sheets. As an effective design tool for structural design, topology optimization studies the optimal distribution of materials in the structure to achieve optimal mechanical and multidisciplinary performance\[5\]. Topology and layout optimization methods based on finite element simulation have been successfully applied to the piezoelectric intelligent structures design (such as actuators\[6\], resonators\[7\], etc.). In terms of piezoelectric energy harvester, Zheng et al. established a topology optimization design model of piezoelectric energy harvester based on SIMP method\[8\]. Nakasone and Silva optimized the topology of the piezoelectric energy harvesters with the goal of maximizing the electromechanical coupling coefficient, and they also considered the dynamic characteristics of the energy harvester\[9\]. Rupp et al. studied the topology optimization design of piezoelectric energy harvester under harmonic excitation and discussed the influence of the thickness ratio of piezoelectric layer and metal layer on the topological form\[10\]. Chen et al. used the level set method to optimize the topology of the piezoelectric energy harvester\[11\].

Generally, three important aspects are required to understand the entire energy harvesting process: firstly, the structural vibrations; secondly, electromechanics of the piezoelectric transduction; and thirdly, the behavior of the electrical circuit. The three aspects are all coupled. The first and second are well-known. Most studies have considered these two coupled relationships in the optimization model of piezoelectric energy harvester. However, the important aspect of coupled behavior of the electrical circuit for this problem is often neglected.

Topology optimization based method is developed to the layout design of the piezoelectric energy harvesting devices connected to the closed electric circuit in this paper. All three coupled effects of the piezoelectric energy harvesting device mentioned above will be considered in the optimization design problem. A finite element model coupled with piezoelectric-circuit is applied to analyze the piezoelectric energy harvesting systems under harmonic dynamic loads. The SIMP method is used for the piezoelectric materials. Updating the design variables of the SIMP method is performed based on complex sensitivity analysis.

This paper is organized as follows: Firstly, the energy harvester optimization description is proposed. Secondly, complex sensitivity analysis for objective function is carried out. Finally, the proposed method is validated by the given numerical example.

![Figure 1. A piezoelectric energy harvester.](image)

2. Optimization formulation

2.1. Finite element model

The coupled governing equations for formulating the harmonic dynamic behaviour in the framework of finite element method are given as follows\[12\]:

\[
\begin{bmatrix}
\mathbf{M}_{uu} & 0 \\
0 & \mathbf{M}_{\Phi\Phi}
\end{bmatrix}
\begin{bmatrix}
\ddot{\mathbf{u}} \\
\ddot{\Phi}
\end{bmatrix}
+
\begin{bmatrix}
\mathbf{C} & 0 \\
0 & \mathbf{C}_{\Phi}
\end{bmatrix}
\begin{bmatrix}
\dot{\mathbf{u}} \\
\dot{\Phi}
\end{bmatrix}
+
\begin{bmatrix}
\mathbf{K}_{uu} & \mathbf{K}_{u\Phi} \\
\mathbf{K}_{\Phi u} & \mathbf{K}_{\Phi\Phi}
\end{bmatrix}
\begin{bmatrix}
\mathbf{u} \\
\Phi
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{F} \\
\mathbf{Q}
\end{bmatrix}
\] (1)
Where, $M_{uu}$, $C$ and $K_{uu}$ are mass matrix, damping matrix and stiffness matrix; $K_{\phi \phi}$ and $K_{e \phi}$ are piezoelectric matrix; $\varepsilon_{\phi}$ is permittivity matrix; $F$ and $Q$ are mechanical loads and electric charges. By assuming $[F \; Q] = [F_0 \; Q_0]e^{j\omega t}$, the coupled equations in frequency domain are obtained as follows:

$$
\begin{bmatrix}
\omega^2 M_{uu} & 0 \\
0 & 0 \\
\end{bmatrix} + j\omega C + \begin{bmatrix}
K_{uu} & K_{e \phi} \\
K_{e \phi} & K_{\phi \phi} \\
\end{bmatrix} \begin{bmatrix}
u \\
\phi \\
\end{bmatrix} = \begin{bmatrix}
F_0 \\
Q_0 \\
\end{bmatrix}
$$

(2)

To simulate an energy harvesting system connected to a closed electric circuit, as shown in figure 1, the Eq. (2) can be modified as follows:

$$
\begin{bmatrix}
\omega^2 M_{uu} & 0 \\
0 & 0 \\
\end{bmatrix} + j\omega C + \begin{bmatrix}
K_{uu} & K_{e \phi} \\
K_{e \phi} & K_{\phi \phi} - \frac{j}{\omega R} \\
\end{bmatrix} \begin{bmatrix}
u \\
\phi \\
\end{bmatrix} = \begin{bmatrix}
F_0 \\
Q_0 \\
\end{bmatrix}
$$

(3)

where $R$ is the resistor.

2.2. Material model
Since topology optimization method is used in the energy harvester design, for simplicity of derivations, the SIMP model is used and the material properties in the design domain is described as:

$$
c = \rho(x)^{p_1} c_0 \\
m = \rho(x)^{p_1} m_0 \\
e = \rho(x)^{p_2} e_0 \\
\varepsilon = \rho(x)^{p_2} \varepsilon_0
$$

(4)

where $c$, $m$, $e$ and $\varepsilon$ denote the stiffness, mass, piezoelectric and dielectric properties, respectively; $\rho(x)$ is the volume density of each element and $0 \ll \rho(x) \ll 1$; $p_1$ and $p_2$ are penalty coefficients to the material.

2.3. Topology Optimization formulation
The piezoelectric materials converts mechanical energy into electrical energy. Zheng et al. defined the energy conversion factor under static force to measure the performance of the energy harvester, and used it as the objective function to optimize the structure[8]. Referring to this definition, when the energy harvester is excited by a harmonic of a certain frequency, the energy conversion factor of the piezoelectric energy harvester can be used to describe its working efficiency in a vibrating environment, expressed as

$$
\eta = \frac{\frac{1}{2} \Phi^T K_{\phi \phi} \Phi}{\frac{1}{2} u^T (\omega^2 M_{uu} + K_{uu}) u + \frac{1}{2} \Phi^T K_{\phi \phi} \Phi}
$$

(5)

where, the variables with upper bars are the complex conjugates of the variables. $\frac{1}{2} \Phi^T (\omega^2 M_{uu} + K_{uu}) u$ is mechanical energy; $\frac{1}{2} \phi^T K_{\phi \phi} \Phi$ is electric energy. Then the optimization problem can be formulated as follows:
where, $\Pi^E = \frac{1}{2} \bar{\Phi}^T K_{\Phi \Phi} \Phi$, $V^*$ is the upper limit of the material.

3. Sensitivity analysis

In this paper, the topology optimization model is solved by the Sequence Linear Programming (SLP) method. In the optimization iteration process, the sensitivity of the objective function and the constraint function to the design variables needs to be calculated. The complex sensitivity analysis should be derived due to the complex solutions of the governing equation. For convenient formulation, let

$$K = \begin{bmatrix} -\omega^2 M_{uu} & 0 \\ 0 & 0 \end{bmatrix} + i\omega C + \begin{bmatrix} K_{uu} & K_{u\Phi} \\ K_{\Phi u} & K_{\Phi \Phi} \end{bmatrix} \begin{bmatrix} i \\ -i \end{bmatrix}$$

(7)

Then, the governing equation can be rewritten as:

$$K \begin{bmatrix} u \\ \Phi \end{bmatrix} = \begin{bmatrix} F_0 \\ Q_0 \end{bmatrix}$$

(8)

For a general formulation, an energy function form can be defined as follows:

$$\Psi = \begin{bmatrix} \bar{u}^T \\ \bar{\Phi}^T \end{bmatrix} A \begin{bmatrix} u \\ \Phi \end{bmatrix}$$

(9)

where, $A$ is a general matrix. By differentiating the general energy function, the following equations are obtained

$$\frac{\partial \Psi}{\partial \bar{\rho}} = \begin{bmatrix} \partial \bar{u} / \partial \bar{\rho} \\ \partial \bar{\Phi} / \partial \bar{\rho} \end{bmatrix}^T A \begin{bmatrix} u \\ \Phi \end{bmatrix} + \begin{bmatrix} \bar{u}^T \\ \bar{\Phi}^T \end{bmatrix} \partial A \begin{bmatrix} u \\ \Phi \end{bmatrix} + \begin{bmatrix} \bar{u}^T \\ \bar{\Phi}^T \end{bmatrix} A \begin{bmatrix} \partial u / \partial \bar{\rho} \\ \partial \Phi / \partial \bar{\rho} \end{bmatrix}$$

(10)

the following equations can be obtained by differentiating the governing equation,

$$\frac{\partial u}{\partial \Phi} = -K^{-1} \frac{\partial K}{\partial \bar{\rho}} \begin{bmatrix} u \\ \Phi \end{bmatrix}$$

(11)

$$\frac{\partial \bar{u}}{\partial \bar{\Phi}} = -K^{-1} \frac{\partial K}{\partial \bar{\rho}} \begin{bmatrix} \bar{u} \\ \bar{\Phi} \end{bmatrix}$$

(12)

Then, the differentiations of freedoms of (10) are replaced by the Eq. (11) and Eq.(12). The complex adjoint variables are defined as follows by some algebraic manipulation:
\[
\begin{pmatrix}
-\omega^2 & M_{uu} & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
+ i\omega C +
\begin{bmatrix}
K_{uu} & K_{u\phi} \\
K_{\phi u} & K_{\phi\phi}
\end{bmatrix}
- \frac{i}{\omega R}
\begin{bmatrix}
\lambda_u \\
\lambda_\phi
\end{bmatrix}
\end{pmatrix}^T
\begin{bmatrix}
\lambda_u \\
\lambda_\phi
\end{bmatrix}
= -A^T \begin{bmatrix}
u \\
\Phi
\end{bmatrix}
\]  

(13)

Then, the following sensitivity analysis of general energy function can be derived:

\[
\frac{\partial \Psi}{\partial \rho} = \begin{bmatrix}
\tilde{u}^T \\
\Phi
\end{bmatrix}^T \frac{\partial A}{\partial \rho} \begin{bmatrix}
u \\
\Phi
\end{bmatrix} + 2\text{real} \left( \begin{bmatrix}
\lambda_u \\
\lambda_\phi
\end{bmatrix}^T A \begin{bmatrix}
u \\
\Phi
\end{bmatrix} \right)
\]  

(14)

In this paper, we define the mechanical energy as follows:

\[
\Pi^s = \frac{1}{2} \begin{bmatrix}
\tilde{u}^T \\
\Phi
\end{bmatrix}^T \begin{bmatrix}
\omega^2 M_{uu} + K_{uu} & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
u \\
\Phi
\end{bmatrix}
\]  

(15)

and electric energy as follows:

\[
\Pi^s = \frac{1}{2} \begin{bmatrix}
\tilde{u}^T \\
\Phi
\end{bmatrix}^T \begin{bmatrix}
0 & 0 \\
0 & K_{\phi\phi}
\end{bmatrix} \begin{bmatrix}
u \\
\Phi
\end{bmatrix}
\]  

(16)

The sensitivity analyses of the above energies and the objective function can be easily obtained and the design variables are updated with the sequence linear programming method.

---

**Table 1. Material properties of PZT.**

| C_{11} | e_{13} | e_{11} |
| 115.65 GPa | -12.31 C/m² | 8.93×10^{-9} F/m |
| 64.89 GPa | -12.31 C/m² | 8.93×10^{-9} F/m |
| 62.29 GPa | 20.76 C/m² | 6.92×10^{-9} F/m |
Table 2 Comparison of sensitivity of objective function

|   | \( \frac{\partial \eta}{\partial \rho} \) | \( \Delta \rho \) | \( \frac{\partial \eta}{\partial \rho} / \Delta \rho \) |
|---|---|---|---|
| A | -14.8740 | -14.8731 | 100.006% |
| B | 3.5451 | 3.5453 | 99.994% |
| C | 0.8973 | 0.8975 | 99.975% |
| D | -8.8491 | -8.8484 | 100.008% |
| E | 3.0567 | 3.0569 | 99.993% |
| F | -0.141096 | -0.141086 | 100.007% |

Figure 4. Optimal topology.

Figure 5. Optimization iteration.

4. Numerical example

In this section, to illustrate the effectiveness of the proposed formulation and numerical techniques, the layout of three-dimensional piezoelectric bimorph energy harvester is optimized. The design domain of the piezoelectric energy harvester under an off-plane force is shown in figure 2. We placed
three electrodes at the bottom, middle and top surface of this model respectively. A potential ground is set in the middle electrode. The top and bottom electrodes are connected to a resister. PZT is employed for the piezoelectric energy harvester and its material properties are given in table 1. The upper bound of volume constraint is set to be 40% of the total volume of the design domain. The resistance is set as 100kΩ. The design of energy harvester subjected to 500Hz excitation are presented in this section using the proposed topology optimization method.

A finite element model coupled with piezoelectric-circuit is applied to analyze the piezoelectric energy harvesting systems, which consider three coupled effects mentioned in the previous session. An 8-node brick element was selected for modelling the PZT bimorph and a circuit element was used for the resistor connected between the top and bottom electrodes.

A 40 by 20 finite element model is built for each layer of the design domain and the proposed methodology is used to design the piezoelectric energy harvester. Some arbitrary elements, as shown in figure 3, are selected for purpose of sensitivity verification. The analytical sensitivities of the objective are compared with the finite difference ones, as shown in table 2. \( \frac{\partial \eta}{\partial \rho} \) denotes the analytical sensitivities, while \( \Delta \eta / \Delta \rho \) represents the finite difference approximations of sensitivities with a perturbation value of 0.0001. The last columns in both tables show the percent agreements between the finite difference and the analytical sensitivities. It can be observed that, for all the selected design variables A–F, highly accurate sensitivities are obtained.

In the subsequent topology optimization, the optimization process converges after 60 iterations. The optimal layout of the piezoelectric material is shown in figure 4. We can see that the piezoelectric material tends to distribute to the supporting area where the strain is larger and help to produce more electric energy. Figure 5 shows the iteration history of the optimization process, from which we can see that the energy conversion factor of the piezoelectric energy harvester increases gradually and achieves a value of 6.1% in the final design.

5. Conclusion

A topology-optimization-based approach to improve the energy conversion efficiency of piezoelectric energy harvester coupled with electric circuit under harmonic excitation loads is proposed. A finite element model coupled with piezoelectric-circuit is applied to analyze the piezoelectric energy harvesting systems. The material pseudo-densities are taken as the design variables, and a penalization model on the structural stiffness matrix and piezoelectric matrix is employed. The complex sensitivity analysis for objective function is derived and the design variables are updated by the Sequence Linear Programming method. Through numerical examples, it is demonstrated that the presented approach is efficient in the layout design of piezoelectric energy harvester coupled with electric circuit for maximizing energy conversion efficiency.

Acknowledgments

The support of the Shenzhen Basic Research Program (JCYJ20170307141601162, JCYJ20170306095959113) is gratefully acknowledged.

References

[1] Kim, H. S., Kim, J. H., Kim, J. (2011) A review of piezoelectric energy harvesting based on vibration. International Journal of Precision Engineering and Manufacturing. 12: 1129-1141.
[2] Saadon, S., Sidek, O. (2011) A review of vibration-based MEMS piezoelectric energy harvesters. Energy Conversion and Management. 52: 500-504.
[3] Priya, S., Song, H. C., Zhou, Y., Varghese, R., Chopra, A., Kim, S. G. (2017) A Review on Piezoelectric Energy Harvesting: Materials, Methods, and Circuits, Energy Harvesting and Systems. 4: 1-37.
[4] Bowen, C. R., Kim, H. A., Weaver, P. M., Dunn, S. (2014) Piezoelectric and ferroelectric materials and structures for energy harvesting applications. Energy & Environmental Science.
7: 25-44.

[5] Rozvany, G. I. N. (2009) A critical review of established methods of structural topology optimization. Structural and Multidisciplinary Optimization. 37: 217-237.

[6] Wang, Y., Luo, Z., Zhang, X., Kang, Z. (2014) Topological design of compliant smart structures with embedded movable actuators. Smart Materials and Structures. 23: 045024.

[7] Wang, X., Lin, Z., Ren, Y. (2017) Topology optimization of piezocomposite resonator for maximizing excitation strength and synthesizing desired eigenmodes. Acta Mechanica Solida Sinica. 30: 531-539.

[8] B. Zheng, C.-J. Chang, and H. C. Gea, "Topology optimization of energy harvesting devices using piezoelectric materials," Structural and Multidisciplinary Optimization, vol. 38, pp. 17-23, March 01 2009.

[9] Nakasone, P. H., Silva, E. C. N. (2009) Design of piezoelectric energy harvesting devices and laminate structures by applying topology optimization. in SPIE Smart Structures and Materials & Nondestructive Evaluation and Health Monitoring, pp. 11.

[10] Rupp, C. J., Evgrafov, A., Maute, K., Dunn, M. L. (2009) Design of Piezoelectric Energy Harvesting Systems: A Topology Optimization Approach Based on Multilayer Plates and Shells," Journal of Intelligent Material Systems and Structures. 20: 1923-1939.

[11] Chen, S., Gonella, S., Chen, W., Liu, W. K. (2010) A level set approach for optimal design of smart energy harvesters. Computer Methods in Applied Mechanics and Engineering. 199: 2532-2543.

[12] Lerch, R. (1990) Simulation of piezoelectric devices by two- and three-dimensional finite elements. IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control. 37: 233-247.