MEASURE OF MODIFIED ROTATABILITY FOR SECOND ORDER RESPONSE SURFACE DESIGNS

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Abstract: A measure enables us to assess the degree of rotatability for a given response surface designs. In this paper, a new measure of modified rotatability for second order response surface designs is suggested. The method is illustrated using central composite designs for \( 2 \leq v \leq 17 \).

Keywords: response surface designs; central composite designs; measure of rotatability.

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1. INTRODUCTION

Response surface methodology is a collection of mathematical and statistical techniques useful for analyzing problems where several independent variables influence a dependent variable. The independent variables are often called the input or explanatory variables and the dependent variable is often the response variable. An important step in development of response surface designs was the introduction of rotatable designs by Box and Hunter [1]. Das and Narasimham [2] constructed rotatable designs using balanced incomplete block designs (BIBD). A design is said

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to be rotatable if the variance of the response estimate is a function only of the distance of the
point from the design centre. Das et al. [3] introduced modified second order response surface
designs. Park et al. [5] introduced measure of rotatability for second order response surface
designs. Victorbabu et al. [14] suggested modified second order response surface designs using
central composite designs. Victorbabu and Surekha [8] developed measure of rotatability for
second order response surface designs using central composite designs. Victorbabu and
Vasundharadevi [11] suggested modified second order response surface designs using BIBD.
Victorbabu and Surekha [9] studied measure of rotatability for second order response surface
designs using BIBD. Victorbabu et al. [13] suggested modified second order response surface
designs, rotatable designs using pairwise balanced design. Victorbabu and Vasundharadevi [12]
studied modified second order response surface designs, rotatable designs using symmetrical
unequal block arrangements (SUBA) with two unequal block sizes. Victorbabu and Surekha [10]
studied measure of rotatability for second order response surface designs using incomplete block
designs. These measures are useful to enable us to assess the degree of modified rotatability for a
given second order response surface designs.

2. CONDITIONS FOR SECOND ORDER ROTATABLE DESIGNS

Suppose we want to use the second order response surface design $D=\{(x_{iu})\}$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_{i} x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i<j}^{v} b_{ij} x_{iu} x_{ju} + e_u$$  \hspace{1cm} (2.1)

where $x_{iu}$ denotes the level of the $i^{th}$ factor ($i=1,2,\ldots,v$) in the $u^{th}$ run ($u=1,2,\ldots,N$) of the
experiment, $e_u$'s are uncorrelated random errors with mean zero and variance $\sigma^2$. $D$ is said to
be second order rotatable design (SORD), if the variance of the estimated response of $\hat{Y}_u$ from
the fitted surface is only a function of the distance ($d^2=\sum_{i=1}^{v} x_i^2$) of the point $(x_1,x_2,\ldots,x_v)$ from the
origin (centre) of the design. Such a spherical variance function for estimation of second order
response surface is achieved if the design points satisfy the following conditions [cf. Box and
Hunter [1], Das and Narasimham [2]].

1. \[ \sum x_{iu} = 0, \quad \sum x_{iu}x_{ju} = 0, \quad \sum x_{iu}x_{ju}x_{ku} = 0, \quad \sum x_{iu}^3 = 0, \quad \sum x_{iu}x_{ju}^3 = 0, \quad \sum x_{iu}x_{ju}x_{ku}x_{lu} = 0; \text{ for } i \neq j, k \neq l \] (2.2)

2. (i) \[ \sum x_{iu}^2 = \text{ constant } = N\lambda_2; \]

(ii) \[ \sum x_{iu}^4 = \text{ constant } = cN\lambda_4; \text{ for all } i \] (2.3)

3. \[ \sum x_{iu}^2x_{ju}^2 = \text{ constant } = N\lambda_4; \text{ for } i \neq j \] (2.4)

4. \[ \sum x_{iu}^4 = c\sum x_{iu}^2x_{ju}^2 \] (2.5)

5. \[ \frac{\lambda_4}{\lambda_2^2} > \frac{v}{(c+v-1)} \] (2.6)

where \( c, \lambda_2 \) and \( \lambda_4 \) are constants and the summation is over the design points.

If the above mentioned conditions are satisfied, the variances and covariances of the estimated parameters become,

\[ V(\hat{b}_0) = \frac{\lambda_4(c+v-1)\sigma^2}{N[\lambda_4(c+v-1)-v\lambda_2^2]}, \]

\[ V(\hat{b}_1) = \frac{\sigma^2}{N\lambda_2}, \]

\[ V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4}, \]

\[ V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4 \left[ \frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2} \right]}, \]

\[ \text{Cov}(\hat{b}_0, \hat{b}_i) = \frac{-\lambda_2\sigma^2}{N[\lambda_4(c+v-1)-v\lambda_2^2]}, \]

\[ \text{Cov}(\hat{b}_{ii}, \hat{b}_{ij}) = \frac{(\lambda_2^2-\lambda_4)\sigma^2}{(c-1)N\lambda_4[\lambda_4(c+v-1)-v\lambda_2^2]} \] (2.7)

and other covariances are zero.
3. **Conditions for Modified Second Order Rotatable Designs**

The most widely used design for fitting a second order model is the central composite design. Central composite designs are constructed by adding suitable factorial combinations to those obtained from \( \frac{1}{2^p} \times 2^r \) fractional factorial design (here \( 2^{(c+v)} = \frac{1}{2^p} \times 2^r \) denotes a suitable fractional replicate of \( 2^r \), in which no interaction with less than five factors is confounded). In coded form the points of \( 2^r (2^{(c+v)}) \) factorial have coordinates \((\pm a, \pm a, ..., \pm a)\) and \(2^r\) axial points have coordinates of the form \(((\pm b, 0, ..., 0), (0, \pm b, ..., 0), ..., (0, 0, ..., \pm b))\) etc., and \(n_0\) central points. The usual method of construction of SORD is to take combinations with unknown constants, associate a \( 2^r \) factorial combinations or a suitable fraction of it with factors each at \(\pm 1\) levels to make the level codes equidistant. All such combinations form a design. Generally, SORD need at least five levels (suitably coded) at \(0, \pm a, \pm b\) for all factors \((0,0,...,0))\)- chosen centre of the design, unknown level \(\pm a\) and \(\pm b\) are to be chosen suitably to satisfy the conditions of the rotatability). Generation of design points this way ensures satisfaction of all the conditions even though the design points contain unknown levels.

Alternatively, by putting some restrictions indicating some relation among \( \sum x_{in}^2, \sum x_{in}^4 \) and \( \sum x_{in}^2 x_{ju}^2 \) some equations involving the unknowns are obtained and their solution gives the unknown levels. In SORD the restriction used is \( \sum x_{in}^4 = 3 \sum x_{in}^2 x_{ju}^2 \), i.e., \(c=3\). Other restrictions are also possible through, it seems, not exploited well. We shall investigate the restriction \( (\sum x_{in}^2)^2 = N \sum x_{in}^2 x_{ju}^2 \) i.e. \(\lambda_2^2 = \lambda_4\) (cf. Das et al. [3]) to get another series of symmetrical second order response surface designs, which provide more precise estimates of response at specific points of interest than what is available from the corresponding existing designs. Further, the variances and covariances of the estimated parameters are,

\[
V(b_0) = \frac{(c+v-1)\sigma^2}{N(c-1)}
\]
\[
V(b_i) = \frac{\sigma^2}{N\sqrt{\lambda_4}} \\
V(b_{ij}) = \frac{\sigma^2}{N\lambda_4} \\
V(b_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4} \\
\text{Cov}(b_0, b_{ii}) = \frac{-\sigma^2}{N\sqrt{\lambda_4}(c-1)}
\]

(3.1)

and other covariances are zero. These modifications of the variances and covariances affect the variance of the estimated response at specific points considerably. Using these variances and covariances, variance of estimated response at any point can be obtained. Let \( \hat{Y}_u \) denote the estimated response at the point \((x_{1u}, x_{2u}, \ldots, x_{vu})\). Then,

\[
V(\hat{Y}_u) = V(\hat{b}_0) + d^2[V(\hat{b}_i) + 2\text{Cov}(\hat{b}_0, \hat{b}_{ii})] + d^4V(\hat{b}_{ii}) + (\sum x_{iu}^2x_{ju}^2)(c-3)\sigma^2/(c-1)N\lambda_4
\]

Construction of modified response surface designs is the same as for SORD except that instead of taking \( c=3 \) the restriction \( (\sum x_{iu}^2)^2 = N\sum x_{iu}^2x_{ju}^2 \) is to be used and this condition will provide different values of the unknowns involved.

### 4. Conditions for Measure of Rotatability for Second Order Response Surface Designs

Following Box and Hunter [1], Das and Narasimham [2], Park et al. [5], conditions (2.2) to (2.6) and (2.7) give the necessary and sufficient conditions for a measure of rotatability for any general second order response surface designs. Further we have,

- \( V(b_i) \) are equal for \( i \),
- \( V(b_{ii}) \) are equal for \( i \),
- \( V(b_{ij}) \) are equal for \( i, j \), where \( i \neq j \),
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\[ \text{Cov}(b_i, b_j) = \text{Cov}(b_j, b_i) = \text{Cov}(b_{ii}, b_{ij}) = \text{Cov}(b_{ij}, b_{ii}) = 0 \quad \text{for all} \quad i \neq j, j \neq 1, l \neq i. \]  

(4.1)

Park et al. [5] suggested that if the conditions in (2.2) to (2.6) together with (2.7) and (4.1) are met, then the following measure \( P_v(D) \) given below can be used to assess the degree of rotatability for any general second order response surface design (cf. Park et al. [5], page 661).

\[ P_v(D) = \frac{1}{1 + R_v(D)}, \]  

(4.2)

where

\[ R_v(D) = \frac{6v \left[ V(b_{ij}) + 2 \text{cov} (\hat{b}_i, \hat{b}_j) - 2V(\hat{b}_{ii}) \right]^2 (v-1)}{(v+2)^2(v+4)(v+6)(v+8)g^8} \]  

(4.3)

and \( g \) is the scaling factor.

On simplification, numerator of (4.3), \( [V(b_{ij}) + 2 \text{cov} (\hat{b}_i, \hat{b}_j) - 2V(\hat{b}_{ii})] \) using (2.7) becomes \( (c-3)\sigma^2/(c-1)N\lambda_4 \). Thus \( R_v(D) \) becomes

\[ R_v(D) = \frac{6v [(c-3)\sigma^2]^2 (v-1)}{(v+2)^2(v+4)(v+6)(v+8)g^8} \]  

(4.4)

Note. For SORD, we have \( c = 3 \). Substituting the value of \( c \) in (4.4) and on simplification we get \( R_v(D) \) is zero. Hence from (4.2), we get \( P_v(D) \) is one if and only if a design is rotatable and less than one for a nearly rotatable design.

5. MEASURE OF ROTATABILITY FOR SECOND ORDER RESPONSE SURFACE DESIGNS USING CCD

Following Park et al. [5] and Victorbabu and Surekha [8], the method of measure of rotatability for second order response surface design using CCD is given below.

The central composite design (CCD) consists of \( 2^v \) or a fraction of \( 2^v \) factorial points \((\pm a, \pm a, \ldots, \pm a)\) repeated \( y_1 \) times, \( 2^v \) axial points of the form \((\pm b, 0, \ldots, 0)\) repeated \( y_2 \) times etc., and a centre point \((0, 0, \ldots, 0)\). The central point may be replicated \( n_0 \) times. Thus the
total number of experimental points \((N)\) can be written as \(N = 2^{iv}y_1 + 2vy_2 + n_0\) with level ‘a’ and ‘b’ prefixed then \(c = \frac{2^{iv}y_1a^4 + 2y_2b^4}{2^{iv}y_1}\).

The measure of rotatability for second order response surface designs using CCD is

\[
R_v(D) = \left(\frac{(c-3)}{(c-1)}\right)^2 \frac{6v(v-1)}{\lambda_4^2(v+2)^2(v+4)(v+6)(v+8)g^8}
\]

where

\[
g = \begin{cases} 
1, & \text{if } b \geq \sqrt{v} \\
\frac{1}{b}, & \text{if } b < \sqrt{v}
\end{cases}
\]

If \(P_v(D)\) is 1 if and only if the design is rotatable, and it is smaller than one for a nearly rotatable designs.

6. MEASURE OF MODIFIED ROTATABILITY FOR SECOND ORDER RESPONSE SURFACE DESIGNS USING CENTRAL COMPOSITE DESIGNS

The proposed measure of modified rotatability for second order response surface designs using CCD is suggested below.

Consider the following set of points:

(i) \(2^{iv}\) (where \(2^{iv}\) is resolution V fraction of \(2^v\)) points on cube viz., coordinates \((\pm a, \pm a, \ldots, \pm a)\) repeated \(y_1\) times,

(ii) \(2v\) axial points, viz., \((\pm b, 0, \ldots, 0), (0, \pm b, \ldots, 0), \ldots, (0, 0, \ldots, \pm b)\) repeated \(y_2\) times,

(iii) \(n_0\) central points, where \(y_1\) and \(y_2\) are chosen to satisfy the criterion of modified rotatability.

**Theorem 6.1:** The design points,

\[y_1(\pm a, \pm a, \ldots, \pm a) 2^{iv} \cup y_2(\pm b, 0, 0, \ldots, 0)2^1 \cup n_0\]

will give a \(v\)-dimensional measure of modified rotatability for second order response surface designs using central composite designs in
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\[ N = \frac{(2^{(iv)} a^2 y_1 + 2 b^2 y_2)^2}{2^{(iv)} a^4 y_1} \]  

design points, if

\[ c = \frac{2^{(iv)} y_1 a^4 + 2 y_2 b^4}{2^{(iv)} y_1 a^4}, \]  

(6.2)

\[ \left( \frac{b}{a} \right)^4 = \left( \frac{2^{(iv)} y_1}{y_2} \right), \]  

(6.3)

\[ n_0 = \left\{ \frac{(2^{(iv)} a^2 y_1 + 2 b^2 y_2)^2}{2^{(iv)} a^4 y_1} \right\} - y_1 2^{(iv)} - 2 v y_2. \]  

(6.4)

and \( n_0 \) turns out to be an integer.

Proof: For the design points generated from central composite design the conditions in equations (2.2) to (2.6) are satisfied. The conditions in equation (2.3) and (2.4) are true as follows:

\[ \sum x_{n}^2 = 2^{(iv)} y_1 a^2 + 2 y_2 b^2 = N \lambda_2 \]  

(6.5)

\[ \sum x_{n}^4 = 2^{(iv)} y_1 a^4 + 2 y_2 b^4 = c N \lambda_4 \]  

(6.6)

\[ \sum x_{n}^2 x_{m}^2 = 2^{(iv)} y_1 a^4 = N \lambda_4 \]  

(6.7)

From equations (6.6) and (6.7), we have \( 2^{(iv)} y_1 a^4 + 2 y_2 b^4 = c 2^{(iv)} y_1 a^4 \), which on simplification lead to equation (6.2). Using the modified condition \( \lambda_2^2 = \lambda_4 \), from equations (6.5) and (6.7) we get,

\[ N = \frac{(2^{(iv)} a^2 y_1 + 2 b^2 y_2)^2}{2^{(iv)} a^4 y_1}. \]  

Alternatively \( N \) may be obtained directly as

\[ N = 2^{(iv)} y_1 + 2 v y_2 + n_0, \]

where

\[ n_0 = \left\{ \frac{(2^{(iv)} a^2 y_1 + 2 b^2 y_2)^2}{2^{(iv)} a^4 y_1} \right\} - y_1 2^{(iv)} - 2 v y_2. \]

From equations (6.5), (6.6) and (6.7) (taking \( a = 1 \)) and on simplification we get

\[ b^4 = \left( \frac{2^{(iv)} y_1}{y_2} \right), \]

\[ \lambda_2 = \frac{2^{(iv)} y_1 + 2 y_2 b^2}{N}, \]
\[ \lambda_4 = \frac{2^{t(v)} y_1}{y_2}. \]

To obtain measure of modified rotatability for second order response surface designs using central composite designs

\[ P_v(D) = \frac{1}{1 + R_v(D)}. \]

\[ R_v(D) = \left[ \frac{(c-3)}{(c-1)} \right]^2 \frac{6v(v-1)}{\lambda_2^2(v+2)(v+4)(v+6)(v+8)g^8}, \] where \( g \) is a scaling factor

\[ g = \begin{cases} 
\frac{1}{b}, & \text{if } b \geq \sqrt{v} \\
\frac{1}{\sqrt{v}}, & \text{otherwise} 
\end{cases} \]

Example: We illustrate the measure of modified rotatability for second order response surface design for \( v = 5 \) factors in \( N = 36 \) design points (taking \( y_1 = 1, a = 1 \)). From (6.5), (6.6) and (6.7), we have

\[ \sum x_{ju}^2 = 16 + 2y_2 b^2 = N \lambda_2 \]

(6.8)

\[ \sum x_{ju}^4 = 16 + 2y_2 b^4 = cN \lambda_4 \]

(6.9)

\[ \sum x_{ju}^2 x_{ju}^2 = 16 = N \lambda_4 \]

(6.10)

For \( v = 5 \) factors, to make the design modified rotatable, we take \( c = 3 \). From equations (6.9) and (6.10) we get \( b^4 = 16 \Rightarrow b^2 = 4 \Rightarrow b = 2 \) we take \( y_2 = 1 \). From the modified condition \( \lambda_2^2 = \lambda_4 \), we get \( N = 36 \). From equation (6.4) we get \( n_0 = 10 \).

The proposed measure of modified rotatability for second order response surface designs using central composite design at the modified rotatability value \( b = 2, y_2 = 1 \) and \( N = 36 \), we get \( c = 3, \) scaling factor \( g = 0.4472 \), \( R_v(D) = 0 \) and \( P_v(D) = 1 \). Then the design is modified rotatable.

Instead of taking \( b = 2 \) if we take \( b = 2.5 \) for \( v = 5 \) factors from equations (6.9) and (6.10) we get \( c = 5.8828 \). The scaling factor \( g = 0.4, R_v(D) = 5.1237 \) and \( P_v(D) = 0.1633 \). Here \( P_v(D) \)
becomes smaller it deviates from modified rotatability.

The measure of modified rotatability for second order response surface designs using central composite designs are presented in table for $2 \leq v \leq 17$. It can be verified that $P_v(D)$ is 1 if and only if the design is modified rotatable design and it is smaller than one for a nearly modified rotatable design.

**Table.** Measure of modified rotatability for second order response surface designs using central composite designs. (taking $a=1$)

| b  | c   | g   | $R_v(D)$ | $P_v(D)$ |
|----|-----|-----|----------|----------|
| 1  | 1.5 | 0.7071 | 3.6 | 0.2174 |
| 1.3 | 2.4280 | 0.7071 | 0.0642 | 0.9397 |
| 1.414214 | 3 | 0.7071 | 0 | 1 |
| 1.6 | 4.2768 | 0.625 | 0.1630 | 0.8598 |
| 1.9 | 7.5161 | 0.5263 | 2.0395 | 0.3290 |
| 2.2 | 12.7128 | 0.4545 | 9.4338 | 0.9584 |
| 2.5 | 20.5313 | 0.4 | 30.7345 | 0.0315 |
| 2.8 | 31.7328 | 0.3571 | 82.5574 | 0.012 |
| 3.1 | 47.1761 | 0.3226 | 195.1523 | 5.0981x10^-3 |
| 3.4 | 67.8168 | 0.2941 | 420.1218 | 2.3746x10^-3 |
| 3.7 | 94.7081 | 0.2703 | 841.0366 | 1.1876x10^-3 |
| 4  | 129 | 0.25 | 1587.6 | 6.2949x10^-4 |
| 4.3 | 171.94 | 0.2326 | 2854.074 | 3.5025x10^-4 |
| 4.6 | 224.8728 | 0.2174 | 4922.754 | 2.0310x10^-4 |
| 4.9 | 289.2401 | 0.2041 | 8193.337 | 1.2204x10^-4 |
| 5.2 | 366.5808 | 0.1923 | 13219.1 | 7.5642x10^-4 |
| b        | c    | g      | $R_v(D)$ | $P_v(D)$ |
|----------|------|--------|----------|----------|
| 1        | 1.5  | 0.5774 | 24.2369  | 0.0396   |
| 1.3      | 2.4281 | 0.5774 | 0.4320   | 0.6983   |
| *1.414214 | 3    | 0.5774 | 0        | 1        |
| 1.6      | 4.2768 | 0.5774 | 0.4089   | 0.7098   |
| 1.9      | 7.5161 | 0.5263 | 2.7122   | 0.2694   |
| 2.2      | 12.7128 | 0.4545 | 12.5458  | 0.0738   |
| 2.5      | 20.5313 | 0.4    | 40.8729  | 0.0239   |
| 2.8      | 31.7328 | 0.3571 | 109.7906 | 9.0260 × 10⁻³ |
| 3.1      | 47.1761 | 0.3226 | 259.5273 | 3.8384 × 10⁻³ |
| 3.4      | 67.8168 | 0.2941 | 558.7074 | 1.7866 × 10⁻³ |
| 3.7      | 94.7081 | 0.2703 | 1118.47  | 8.9328 × 10⁻⁴ |
| 4        | 129   | 0.25   | 2111.302 | 4.7417 × 10⁻⁴ |
| 4.3      | 171.94 | 0.2326 | 3795.548 | 2.634 × 10⁻⁴ |
| 4.6      | 224.8728 | 0.2174 | 6546.623 | 1.5273 × 10⁻⁴ |
| 4.9      | 289.2401 | 0.2041 | 10896.07 | 9.1768 × 10⁻⁵ |
| 5.2      | 366.5808 | 0.1923 | 17579.69 | 5.6881 × 10⁻⁵ |
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\( v= 4, N= 36, y_1 =1, y_2 =1, n_0 = 12, b = 2 \)

| b  | c  | g  | \( R_v(D) \) | \( P_v(D) \) |
|----|----|----|--------------|--------------|
| 1  | 1.125 | 0.5  | 607.5        | \( 1.6434 \times 10^{-3} \) |
| 1.3 | 1.3570 | 0.5  | 57.1828      | \( 1.7187 \times 10^{-2} \) |
| 1.6 | 1.8192 | 0.5  | 5.6097       | 0.1513       |
| 1.9 | 2.6290 | 0.5  | 0.1400       | 0.8772       |
| *2 | 3 | 0.5  | 0            | 1            |
| 2.2 | 3.9282 | 0.4545 | 0.5815       | 0.6323       |
| 2.5 | 5.8828 | 0.4  | 5.6097       | 0.1513       |
| 2.8 | 8.6832 | 0.3571 | 21.8017     | 0.0439       |
| 3.1 | 12.5440 | 0.3226 | 61.4845      | 0.0160       |
| 3.4 | 17.7042 | 0.2941 | 145.9441     | \( 6.8053 \times 10^{-3} \) |
| 3.7 | 24.4270 | 0.2703 | 309.9039     | \( 3.2164 \times 10^{-3} \) |
| 4  | 33 | 0.25 | 607.5        | \( 1.6434 \times 10^{-3} \) |
| 4.3 | 43.73501 | 0.2326 | 1120.055     | \( 8.9202 \times 10^{-4} \) |
| 4.6 | 56.9682 | 0.2174 | 1965.982     | \( 5.0839 \times 10^{-4} \) |
| 4.9 | 73.06001 | 0.2041 | 3313.174     | \( 3.0173 \times 10^{-4} \) |
| 5.2 | 92.3952 | 0.1923 | 5394.264     | \( 1.8535 \times 10^{-4} \) |
\(v = 5, N = 36, y_1 = 1, y_2 = 1, n_0 = 10, b = 2\)

| \(b\) | \(c\) | \(g\) | \(R_v (D)\) | \(P_v (D)\) |
|------|------|------|-------------|-------------|
| 1    | 1.125| 0.4472| 1354.672    | 7.3764 \times 10^{-4} |
| 1.3  | 1.357013| 0.4472| 127.5127    | 7.7813 \times 10^{-3} |
| 1.6  | 1.8192| 0.4472| 12.5091     | 7.4024 \times 10^{-2} |
| 1.9  | 2.629012| 0.4472| 0.31224     | 0.7620      |
| *2   | 3    | 0.4472| 0           | 1           |
| 2.2  | 3.9282| 0.4472| 0.605       | 0.6231      |
| 2.5  | 5.8828| 0.4   | 5.1237      | 0.1633      |
| 2.8  | 8.6832| 0.3571| 19.9130     | 0.0478      |
| 3.1  | 12.54401| 0.3226| 56.1583     | 0.0175      |
| 3.4  | 17.7042| 0.2941| 133.3013    | 7.446 \times 10^{-3} |
| 3.7  | 24.42701| 0.2703| 283.0576    | 3.5204 \times 10^{-3} |
| 4    | 33   | 0.25  | 554.8737    | 1.799 \times 10^{-3} |
| 4.3  | 43.73501| 0.2326| 1023.028    | 9.7654 \times 10^{-4} |
| 4.6  | 56.9682| 0.2174| 1795.674    | 5.5658 \times 10^{-4} |
| 4.9  | 73.06001| 0.2041| 3026.161    | 3.3034 \times 10^{-4} |
| 5.2  | 92.3952| 0.1923| 4926.971    | 2.0292 \times 10^{-4} |
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\( v = 6, N = 72, y_1 = 1, y_2 = 2, n_0 = 16, b = 2 \)

| b   | c   | g   | \( R_v(D) \) | \( P_v(D) \) |
|-----|-----|-----|-------------|-------------|
| 1.3 | 1.3570 | 0.4082 | 232.6242 | 4.2804 \times 10^{-3} |
| 1.6 | 1.8192 | 0.4082 | 22.8206 | 4.1981 \times 10^{-2} |
| 1.9 | 2.6290 | 0.4082 | 0.5697 | 0.6371 |
| *2  | 3   | 0.4082 | 0.5697 | 0.6371 |
| 2.2 | 3.9282 | 0.4082 | 1.1037 | 0.4754 |
| 2.5 | 5.8828 | 0.4082 | 1.1037 | 0.4754 |
| 2.8 | 8.6832 | 0.3571 | 17.5192 | 0.054 |
| 3.1 | 12.5440 | 0.3226 | 49.4072 | 0.0198 |
| 3.4 | 17.7042 | 0.2941 | 117.2765 | 8.4548 \times 10^{-3} |
| 3.7 | 24.4270 | 0.2703 | 249.0299 | 3.9995 \times 10^{-3} |
| 4   | 33  | 0.25  | 488.1696 | 2.0443 \times 10^{-3} |
| 4.3 | 43.73501 | 0.2326 | 900.0445 | 1.1098 \times 10^{-3} |
| 4.6 | 56.9682 | 0.2174 | 1579.807 | 6.3259 \times 10^{-4} |
| 4.9 | 73.06001 | 0.2041 | 2662.372 | 3.7546 \times 10^{-4} |
| 5.2 | 92.3952 | 0.1923 | 4334.676 | 2.3064 \times 10^{-4} |
\( v = 7, N = 100, y_1 = 1, y_2 = 1, n_0 = 22, b = 2.828427 \)

| \( b \) | \( c \) | \( g \) | \( R_{v}(D) \) | \( P_{v}(D) \) |
|---|---|---|---|---|
| 1 | 1.03125 | 0.3779 | 33744.39 | 2.9634 \times 10^{-5} |
| 1.3 | 1.0893 | 0.3779 | 3896.547 | 2.5657 \times 10^{-4} |
| 1.6 | 1.2048 | 0.3779 | 653.2602 | 1.5284 \times 10^{-3} |
| 1.9 | 1.4073 | 0.3779 | 130.0424 | 7.6311 \times 10^{-3} |
| 2.2 | 1.7321 | 0.3779 | 25.5061 | 3.7727 \times 10^{-2} |
| 2.5 | 2.2207 | 0.3779 | 3.4650 | 0.2240 |
| 2.8 | 2.9208 | 0.3571 | 0.0227 | 0.9778 |
| \*2.828427 | 3 | 0.3536 | 0 | 1 |
| 3.1 | 3.8860 | 0.3226 | 2.8464 | 0.26 |
| 3.4 | 5.1761 | 0.2941 | 17.1698 | 5.5036 \times 10^{-2} |
| 3.7 | 6.8568 | 0.2703 | 53.9352 | 1.8203 \times 10^{-2} |
| 4 | 9 | 0.25 | 130.5361 | 7.6025 \times 10^{-3} |
| 4.3 | 11.6838 | 0.2326 | 273.4281 | 3.6439 \times 10^{-3} |
| 4.6 | 14.9921 | 0.2174 | 521.4529 | 1.914 \times 10^{-3} |
| 4.9 | 19.015 | 0.2041 | 929.9986 | 1.0741 \times 10^{-3} |
| 5.2 | 2.8488 | 0.1923 | 1576.124 | 6.3407 \times 10^{-4} |
MEASURE OF MODIFIED ROTATABILITY

v=8, N=100, y_1 =1, y_2 =1, n_0=20, b=2.828427

| b  | c    | g    | R_v(D)   | P_v(D)      |
|----|------|------|----------|-------------|
| 1  | 1.0313 | 0.3536 | 49612.5 | 2.0156×10^{-5} |
| 1.3 | 1.0893 | 0.3536 | 5728.877 | 1.7452×10^{-4} |
| 1.6 | 1.2048 | 0.3536 | 960.4523 | 1.0401×10^{-3} |
| 1.9 | 1.4073 | 0.3536 | 191.1942 | 5.2031×10^{-3} |
| 2.2 | 1.7321 | 0.3536 | 37.5001 | 2.5974×10^{-2} |
| 2.5 | 2.2207 | 0.3536 | 5.0944 | 0.1641 |
| 2.8 | 2.9208 | 0.3536 | 0.0213 | 0.9792 |
| *2.828427 | 3 | 0.3536 | 0 | 1 |
| 3.1 | 3.8860 | 0.3226 | 2.4531 | 0.2896 |
| 3.4 | 5.1761 | 0.2941 | 14.7975 | 6.3301×10^{-2} |
| 3.7 | 6.8568 | 0.2703 | 46.4829 | 2.1060×10^{-2} |
| 4  | 9     | 0.25  | 112.5 | 8.8106×10^{-3} |
| 4.3 | 11.6838 | 0.2326 | 235.6487 | 4.2226×10^{-3} |
| 4.6 | 14.1121 | 0.2174 | 449.4039 | 2.2567×10^{-3} |
| 4.9 | 19.015 | 0.2040816 | 801.501 | 1.2461×10^{-3} |
| 5.2 | 23.8488 | 0.19230 | 1358.351 | 7.3564×10^{-4} |
\( v = 9, N = 200, y_1 = 1, y_2 = 2, n_0 = 36, b = 2.828427 \)

| \( b \) | \( c \) | \( g \) | \( R_v (D) \) | \( P_v (D) \) |
|-------|-------|-------|-------------|-------------|
| 1.03125 | 0.3333 | 68470.9 | 1.4605 \times 10^{-5} |
| 1.0893 | 0.3333 | 7906.502 | 1.2646 \times 10^{-4} |
| 1.2048 | 0.3333 | 1325.534 | 7.5385 \times 10^{-4} |
| 1.4073 | 0.3333 | 263.8698 | 3.7754 \times 10^{-3} |
| 1.7321 | 0.3333 | 51.75445 | 1.8956 \times 10^{-2} |
| 2.2207 | 0.3333 | 7.030896 | 0.1245 |
| 2.9208 | 0.3333 | 0.0293 | 0.9715 |
| 3.8860 | 0.3226 | 2.1136 | 0.3212 |
| 5.17605 | 0.2941 | 12.74948 | 7.2374 \times 10^{-2} |
| 6.8568 | 0.2703 | 40.0496 | 2.4361 \times 10^{-2} |
| 9 | 0.25 | 96.9298 | 1.0211 \times 10^{-2} |
| 11.68375 | 0.2326 | 203.0345 | 4.9011 \times 10^{-3} |
| 14.1121 | 0.2174 | 387.2057 | 2.5759 \times 10^{-3} |
| 19.015 | 0.2041 | 690.5719 | 1.4459 \times 10^{-3} |
| 23.8488 | 0.1923 | 1170.353 | 8.5371 \times 10^{-4} |
MEASURE OF MODIFIED ROTATABILITY

\( \nu=10, \ N=200, \ y_1=1, \ y_2=2, \ n_0=32, \ b=2.828427 \)

| b    | c      | g      | \( R_\nu(D) \) | \( P_\nu(D) \) |
|------|--------|--------|----------------|----------------|
| 1.0  | 1.0313 | 0.3162 | 90122.22       | 1.1096 \times 10^{-5} |
| 1.3  | 1.0893 | 0.3162 | 10406.63       | 9.6083 \times 10^{-5} |
| 1.6  | 1.2048 | 0.3162 | 1744.683       | 5.7284 \times 10^{-4} |
| 1.9  | 1.4073 | 0.3162 | 347.3085       | 2.8710 \times 10^{-3} |
| 2.2  | 1.7321 | 0.3162 | 68.1198        | 1.4468 \times 10^{-2} |
| 2.5  | 2.2207 | 0.3162 | 9.2542         | 9.7521 \times 10^{-2} |
| 2.8  | 2.9208 | 0.3162 | 0.0386         | 0.9628          |
| *2.828427 | 3     | 0.3162 | 0              | 1              |
| 3.1  | 3.8860 | 0.3162 | 2.1401         | 0.3185          |
| 3.4  | 5.1761 | 0.2941 | 11.0100        | 8.3263 \times 10^{-2} |
| 3.7  | 6.8568 | 0.2703 | 34.5855        | 0.2810          |
| 4.0  | 9      | 0.25   | 83.7054        | 1.1806 \times 10^{-2} |
| 4.3  | 11.6838| 0.2326 | 175.3338       | 5.6711 \times 10^{-3} |
| 4.6  | 14.9921| 0.2174 | 334.3779       | 2.9817 \times 10^{-3} |
| 4.9  | 19.015 | 0.2041 | 596.3549       | 1.6740 \times 10^{-3} |
| 5.2  | 23.8488| 0.1923 | 1010.678       | 9.8846 \times 10^{-4} |
\( v=11, N=200, y_1 = 1, y_2 = 2, n_0 = 28, b^* = 2.828427 \)

| b   | c        | g  | \( R_v(D) \) | \( P_v(D) \) |
|-----|----------|----|--------------|---------------|
| 1   | 1.0313   | 0.3015 | 114355      | 8.7442 \times 10^{-6} |
| 1.3 | 1.0893   | 0.3015 | 13204.86    | 7.5724 \times 10^{-5} |
| 1.6 | 1.2048   | 0.3015 | 2213.808    | 4.5151 \times 10^{-4} |
| 1.9 | 1.4072   | 0.3015 | 440.6958    | 2.2640 \times 10^{-3} |
| 2.2 | 1.7321   | 0.3015 | 86.4365     | 1.1437 \times 10^{-2} |
| 2.5 | 2.2207   | 0.3015 | 11.7425     | 7.8478 \times 10^{-2} |
| 2.8 | 2.9208   | 0.3015 | 0.0490      | 0.9533        |
| *2.828427 | 3   | 0.3015 | 0       | 1             |
| 3.1 | 3.8860   | 0.3015 | 2.7155      | 0.2691        |
| 3.4 | 5.1761   | 0.2941 | 9.5420      | 9.4898 \times 10^{-2} |
| 3.7 | 6.8568   | 0.2703 | 29.9742     | 3.2285 \times 10^{-2} |
| 4   | 9        | 0.25  | 72.5338     | 1.3597 \times 10^{-2} |
| 4.3 | 11.68375 | 0.2326 | 151.9562    | 6.5378 \times 10^{-3} |
| 4.6 | 14.9921  | 0.2174 | 289.7946    | 3.4389 \times 10^{-3} |
| 4.9 | 19.015   | 0.2041 | 516.8417    | 1.9319 \times 10^{-3} |
| 5.2 | 23.8488  | 0.1923 | 875.9223    | 1.1404 \times 10^{-3} |
**MEASURE OF MODIFIED ROTATABILITY**

\(v=12, \ N=324, \ y_1=1, \ y_2=1, \ n_0=44, \ b=4\)

| b   | c     | g    | \(R_v(D)\)   | \(P_v(D)\)   |
|-----|-------|------|---------------|---------------|
| 1   | 1.0078| 0.2887| 1515172      | 6.5999 \times 10^{-7} |
| 1.3 | 1.0223| 0.2887| 183050       | 5.463 \times 10^{-6}  |
| 1.6 | 1.0512| 0.2887| 33757.97    | 2.9622 \times 10^{-5} |
| 1.9 | 1.1018| 0.2887| 8099.36     | 1.2345 \times 10^{-4} |
| 2.2 | 1.1830| 0.2887| 2296.805    | 4.352 \times 10^{-4}  |
| 2.5 | 1.3052| 0.2887| 718.6723    | 1.3895 \times 10^{-3} |
| 2.8 | 1.4802| 0.2887| 233.4051    | 4.2661 \times 10^{-3} |
| 3.1 | 1.7215| 0.2887| 73.166      | 1.3483 \times 10^{-2} |
| 3.4 | 2.0440| 0.2887| 19.5378     | 4.8691 \times 10^{-2} |
| 3.7 | 2.4642| 0.2703| 5.2857      | 0.1591         |
| *4  | 3     | 0.25 | 0           | 1             |
| 4.3 | 3.6709| 0.2326| 8.2878      | 0.1077        |
| 4.6 | 4.4980| 0.2174| 41.3149     | 2.3632 \times 10^{-2} |
| 4.9 | 5.5038| 0.2041| 115.4141    | 8.59 \times 10^{-3} |
| 5.2 | 6.7122| 0.1923077| 253.7111   | 3.926 \times 10^{-3} |
v=13, N=324, y_1 =1, y_2 =1, n_0 =42, b=4

| b   | c     | g   | R_v(D)  | P_v(D)  |
|-----|-------|-----|---------|---------|
| 1   | 1.0078| 0.2774 | 1824464 | 5.4811×10^{-7} |
| 1.3 | 1.0223| 0.2774 | 220416  | 4.5369×10^{-6} |
| 1.6 | 1.0512| 0.2774 | 40648.98| 2.4600×10^{-5} |
| 1.9 | 1.1018| 0.2774 | 9752.681| 1.0253×10^{-4} |
| 2.2 | 1.1830| 0.2774 | 2765.651| 3.6145×10^{-4} |
| 2.5 | 1.3052| 0.2774 | 865.3747| 1.1542×10^{-3} |
| 2.8 | 1.4802| 0.2774 | 281.05  | 3.5455×10^{-3} |
| 3.1 | 1.7215| 0.2774 | 88.1013 | 1.1223×10^{-2} |
| 3.4 | 2.0440| 0.2774 | 23.526  | 4.0773×10^{-2} |
| 3.7 | 2.4642| 0.2703 | 4.6209  | 0.1779   |
| *4  | 3     | 0.25  | 0       | 1        |
| 4.3 | 3.6709| 0.2325 | 7.2455  | 0.1213   |
| 4.6 | 4.4980| 0.2174 | 36.1187 | 2.6941×10^{-2} |
| 4.9 | 5.5037| 0.2041 | 100.8983| 9.8137×10^{-3} |
| 5.2 | 6.7122| 0.1923 | 221.8014| 4.4883×10^{-3} |
### MEASURE OF MODIFIED ROTATABILITY

$v=14$, $N=324$, $y_1=1$, $y_2=1$, $n_0=40$, $b=4$

| $b$  | $c$   | $g$  | $R_v(D)$   | $P_v(D)$     |
|------|-------|------|------------|--------------|
| 1    | 1.0078| 0.2673| 2155064    | $4.6402 \times 10^{-7}$ |
| 1.3  | 1.0223| 0.2673| 260356.1   | $3.8409 \times 10^{-6}$ |
| 1.6  | 1.0512| 0.2673| 48014.71   | $2.0827 \times 10^{-5}$ |
| 1.9  | 1.1018| 0.2673| 11519.9    | $8.6799 \times 10^{-5}$ |
| 2.2  | 1.1830| 0.2673| 3266.797   | $3.0602 \times 10^{-4}$ |
| 2.5  | 1.3052| 0.2673| 1022.184   | $9.7734 \times 10^{-3}$ |
| 2.8  | 1.4802| 0.2673| 331.9772   | $3.0032 \times 10^{-3}$ |
| 3.1  | 1.7215| 0.2673| 104.0656   | $9.5179 \times 10^{-3}$ |
| 3.4  | 2.0440| 0.2673| 27.78999   | $3.4736 \times 10^{-2}$ |
| 3.7  | 2.4642| 0.2673| 4.4382     | 0.1839       |
| *4   | 3     | 0.25 | 0          | 1            |
| 4.3  | 3.6709| 0.2326| 6.3629     | 0.1358       |
| 4.6  | 4.4980| 0.2174| 31.71892   | 0.03056      |
| 4.9  | 5.5038| 0.2041| 88.6073    | 1.1159 $\times 10^{-2}$ |
| 5.2  | 6.7122| 0.1923| 194.7826   | $5.1077 \times 10^{-3}$ |
\( v=15, N=324, y_1 =1, y_2 =1, n_0=38, b=4 \)

| b   | c       | g   | \( R_\gamma(D) \) | \( P_\gamma(D) \) |
|-----|---------|-----|----------------|-----------------|
| 1.0078 | 0.2582 | 2505114 | 3.9918 \times 10^{-7} |
| 1.0223 | 0.2582 | 302646.2 | 3.3042 \times 10^{-6} |
| 1.0512 | 0.2582 | 55813.82 | 1.7916 \times 10^{-5} |
| 1.1018 | 0.2582 | 13391.09 | 7.4671 \times 10^{-5} |
| 1.1830 | 0.2582 | 3797.427 | 2.6327 \times 10^{-4} |
| 1.3052 | 0.2582 | 1188.218 | 8.4089 \times 10^{-4} |
| 1.4802 | 0.2582 | 385.9008 | 2.5846 \times 10^{-3} |
| 1.7215 | 0.2582 | 120.9691 | 8.1988 \times 10^{-3} |
| 2.0440 | 0.2582 | 32.3028 | 3.0028 \times 10^{-2} |
| 2.4642 | 0.2582 | 5.1591 | 0.1624 |
| 2.5038 | 0.2041 | 78.1599 | 1.263 \times 10^{-2} |
| 6.7122 | 0.1923 | 171.8164 | 5.786 \times 10^{-3} |
### MEASURE OF MODIFIED ROTATABILITY

\(v=16, N=324, y_1 = 1, y_2 = 1, n_0 = 36, b=4\)

| b  | c         | g | \(R_v(D)\) | \(P_v(D)\)       |
|----|-----------|---|------------|------------------|
| 1  | 1.0078    | 0.25 | 2872923 | \(3.480775 \times 10^{-7}\) |
| 1.3| 1.0223    | 0.25 | 347081.7 | \(2.8812 \times 10^{-6}\) |
| 1.6| 1.0512    | 0.25 | 64008.58 | \(1.5623 \times 10^{-5}\) |
| 1.9| 1.1018    | 0.25 | 57357.22 | \(6.5112 \times 10^{-5}\) |
| 2.2| 1.1830    | 0.25 | 4345.978 | \(2.2957 \times 10^{-4}\) |
| 2.5| 1.3052    | 0.25 | 1362.677 | \(7.3331 \times 10^{-4}\) |
| 2.8| 1.4802    | 0.25 | 442.56   | \(2.2545 \times 10^{-3}\) |
| 3.1| 1.72150   | 0.25 | 138.7302 | \(7.1567 \times 10^{-3}\) |
| 3.4| 2.0440    | 0.25 | 37.0456  | \(2.6284 \times 10^{-2}\) |
| 3.7| 2.4642    | 0.25 | 5.9166   | 0.1446           |
| *4 | 3         | 0.25 | 0        | 1                |
| 4.3| 3.6709    | 0.2325 | 4.9722 | 0.1674           |
| 4.6| 4.4980    | 0.2174 | 24.7865 | \(3.88 \times 10^{-2}\) |
| 4.9| 5.5038    | 0.2041 | 69.2414 | \(1.42 \times 10^{-2}\) |
| 5.2| 6.7122    | 0.1923 | 152.2111 | \(6.527 \times 10^{-3}\) |
\[ v=17, N=324, y_1=1, y_2=1, n_0=34, b=4 \]

| \( b \) | \( c \) | \( g \) | \( R_v(D) \) | \( P_v(D) \) |
|-------|------|-----|-------------|-------------|
| 1     | 1.0078 | 0.2425 | 3256956 | 3.0704 \times 10^{-7} |
| 1.3   | 1.0223 | 0.2425 | 3934744.83 | 2.5414 \times 10^{-6} |
| 1.6   | 1.0512 | 0.2425 | 72564.83 | 1.3781 \times 10^{-5} |
| 1.9   | 1.1018 | 0.2425 | 17410.07 | 5.7435 \times 10^{-5} |
| 2.2   | 1.1830 | 0.2425 | 4937.123 | 2.0251 \times 10^{-4} |
| 2.5   | 1.3052 | 0.2425 | 1544.83 | 6.4690 \times 10^{-4} |
| 2.8   | 1.4802 | 0.2425 | 501.7185 | 1.9892 \times 10^{-3} |
| 3.1   | 1.7215 | 0.2425 | 157.2747 | 6.3181 \times 10^{-3} |
| 3.4   | 2.0440 | 0.2425 | 41.9976 | 2.3257 \times 10^{-2} |
| 3.7   | 2.4642 | 0.2425 | 6.7075 | 0.1297 |
| *4    | 3     | 0.25 | 0 | 1 |
| 4.3   | 3.6709 | 0.2326 | 4.4230 | 0.1844 |
| 4.6   | 4.4980 | 0.2174 | 22.0489 | 4.3386 \times 10^{-2} |
| 4.9   | 5.5038 | 0.2041 | 61.5939 | 1.59 \times 10^{-2} |
| 5.2   | 6.7122 | 0.1923 | 135.4 | 7.3 \times 10^{-2} |

*indicates modified rotatability for second order response surface designs using CCD.

(cf. Victorbabu et al. [14]).

**CONFLICT OF INTERESTS**

The authors declare that there is no conflict of interests.
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