Research on Three-Stage Optimization Model for Airline Seat Inventory Control Based on the Callable Mechanism

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Abstract. Due to the randomness of passenger demand for airline seats, it is almost impossible to predict the demand for high class shipping space, which will lead to serious loss of earnings for airlines. Therefore, it is particularly important to study the method of seat inventory control based on the callable mechanism. In this paper, a three-stage sales model based on the callable mechanism is established. Compared with the traditional two-stage and three-stage sales models, it is found that the implementation of the callable mechanism can enable airlines to realize risk hedging and arbitrage. Finally, numerical examples are given to verify the advantages of the optimization model in this paper.

1. Introduction

Airline revenue management [1] is an important technology for airlines to improve corporate earnings. At present, it has become a means to determine the profit and loss of domestic and foreign airlines. The core of air revenue management is seat inventory control, the seat inventory control strategy refers to the airlines difference according to the requirements of passenger, the available seats of plane are divided into different levels (such as high and low prices of cabin seats) to satisfy the demands of passengers, and the key decision is to allocate the number of high and low cabin seats, the seats in right price are allocated to the most appropriate passengers for more revenue [2].

Demand forecasting is the basis of seat inventory control. Because passengers’ demand for airline's high class cabin seats changes is random, it is difficult to predict the demand. There are two possible situations for the inaccurate prediction of the high class demand [3]:

(1) The actual demand is higher than the predicted demand, that is, the booking capacity provided by the airline cannot meet the demand of high class passengers. At this time, airlines generally use overbooking methods, but overbooking may lead to the phenomenon of denied boarding. Therefore, to solve the problem, airlines have to increase additional production cost which will seriously tarnish the image of airlines.

(2) The actual demand is lower than the predicted demand, that is, the booking capacity provided exceeds passenger demand. At this time, there was no expected high-class passengers coming, it will causes spoilage when an aircraft takes off, which means loss of revenue for airline seats.

In order to solve the loss of airline revenue caused by these two situations, Gallego (2004) proposed the flexible control method of air passenger seats, which is called the callable mechanism. The system requires selling callable tickets on low class cabin tickets and selling regular tickets at the same time.
When the demand of high class cabin tickets exceeds the available seats, those sold will be recalled. The recalled passengers can be notified in advance and can be compensated [4].

At present, research on the callable mechanism is still in the exploratory stage at home and abroad. In foreign countries, Gallego and Kou [5] constructed a two-stage revenue optimization model under the callable mechanism, and solved the problem about conditions of optimal allocation number of recalled tickets and the recall compensation price of recalled tickets in the callable mechanism, and pointed out the seat inventory control strategy of the callable mechanism for high-demand routes. In China, Zhu Jinfu, Zhou Yan and Gao Qiang [6] of Nanjing University of Aeronautics and Astronautics studied the recall system of aviation revenue management at the first time. Gui Yunmiao, Gao Qiang and Sun Gang [7-9] established a mathematical model and combined it with the revenue management system, which provided a very favorable decision on the optimal amount allocation of the recalled tickets of the airline.

On the above-mentioned situations, this article will research the callable mechanism combined with three-stages of sales, establish recall mechanisms based on the three-stages of sales model. Comparing the traditional two-stages with three-stages of sales mode analysis, it can be found that the implementation can make the airlines hedge risk arbitrage, and at last a numerical example is given to validate the superiority of the optimization model in this paper.

2. Modelling and Model Solution

2.1. Model Establishment

In this paper, it is assumed that the airline divides the cabin tickets of a certain flight into three types: callable ticket, low class cabin ticket and high class cabin ticket. The three types of tickets can be sold in three stages:

1. The first stage is the period \((0, T_1]\), in which callable tickets are first sold.
2. The second stage is the period \((T_1, T_2]\), in which low class cabin tickets are sold.
3. The third stage is the period \((T_2, T_3]\), in which high class cabin tickets are sold. In the third stage, if actual passengers demand of high class cabin tickets is higher than the rest of the number of shipping space, so the airlines can recall sold tickets in the first stage and will compensate those passengers.

Further assumptions:

1. When an airline offers callable tickets in first stage, passengers are given the opportunity at the time of purchase to grant the capacity provider the option of recalling their booking at a known recall price \(P_\star\).
2. The airline will only implement the callable mechanism after determining that the expected revenue of the implementation of the callable mechanism is higher than that of the traditional management mode, which is \(P_H + P_K - P_L > P_L\).
3. The passenger demand of low class cabin and high class cabin is subject to normal distribution, \(D_L\) and \(D_H\) are independent of each other. The demand of passengers for callable tickets is randomly distributed, and the probability of passengers’ demand for recalled tickets is 100 percent, that is, the airline can always sell at least one recalled ticket.
4. The prices of airline tickets satisfy condition: \(R_K < P_L < P_H\), and the selling prices of these class seats are exogenous variables, and the airline only allocates the seats number in each class.
5. Air tickets are sold over the internet, which means the cost of callable tickets implement is minimal and negligible.
6. Overbooking, no-shows, go-shows and cancellations situations are not consideration.

The variables used in this article are as follows:

- Total seats number in each class of the airline flight (total tickets number) is denoted by \(c\). First-stage, second-stage and third-stage passengers demands are integer-valued random variables denoted by \(D_K, D_L\) and \(D_H\). respectively. First- stage, second- stage and third- stage ticket prices are denoted by \(P_K, P_L\) and \(P_H\). The number of callable seats allocated by the airline in the First-stage is denoted by \(K\). Sales of callable seats in the first-stage are denoted by \(V_K = \min\{D_K, K\}\). Low-class seats’
allocation in the second-stage are denoted by $L_K = \theta (c - V_K)$, with $\theta \in (0,1)$ and it is determined by the airlines according to the traditional two-stage revenue management method. Sales of low-class cabin seats in the second-stage are denoted by $S_K = \min(D_L, L_K)$. The capacity available for high class cabin seats in the third-stage is $H_K = c - S_K$. The number of units sold at high class cabin seats is $W_K = \min[D_H, H_K]$, and the number of units called is $Q_K = \min[V_K, (D_H + V_K + S_K - c)^+]$.

Based on all the conditions above, three-stage optimization model based on the callable mechanism (TOMCM) can be obtained to implement the callable mechanism of the airline:

$$R_K = P_K V_K + P_L S_K + P_H W_K - P_L Q_K$$
$$V_K = \min[D_K, K]$$
$$S_K = \min[D_L, L_K]$$
$$W_K = \min[D_H, H_K]$$
$$Q_K = \min[V_K, (D_H + V_K + S_K - c)^+]$$

The objective function is the total revenue of a flight based on the callable mechanism. The total revenue includes four parts: the revenue of callable tickets in the first-stage, the revenue of low class tickets in the second-stage, the revenue of high class tickets and the cost of recalling the recalled tickets in the third-stage. The numbers of recalled tickets, low class tickets and high class tickets sold in three stages are defined by constraints (2) to (4), respectively. The number of passengers that can be recalled by the airline in the first stage is defined by constraint (5).

2.2. Model Solving

2.2.1. Comparison between traditional two-stage model and TOMCM. In the traditional revenue management model, there are no recall tickets. The following is a comparative analysis of the traditional model and the callable mechanism. In the traditional revenue management model, when $K = 0$, air tickets are divided into low class cabin tickets and high class cabin tickets. Then the total revenue can be listed:

$$R_0 = P_L S_0 + P_H W_0 = P_L S_0 + P_H \min[D_H, c - S_0]$$

So, compared with the revenue of traditional two-stage sales model, the revenue difference of callable mechanism is:

$$R_K - R_0 = [P_K V_K + P_L S_K + P_H W_K - P_L Q_K] - [P_L S_0 + P_H W_0] = P_K V_K + P_L (S_K - S_0) + P_H (W_K - W_0) - P_L Q_K$$
$$= P_K V_K + P_L (S_K - S_0) + P_H \min[D_H, c - S_K] - \min[D_H, c - S_0] - P_L \min[V_K, (D_H + V_K + S_K - c)^+]$$

When $F = \min[D_H, c - S_K] - \min[D_H, c - S_0]$,

$$F = \begin{cases} 
0, & D_H \leq c - S_0 \leq c - S_K \\
D_H - (c - S_0), & c - S_0 \leq D_H \leq c - S_K \\
S_0 - S_K, & c - S_0 \leq D_H \leq c - S_K \leq D_H 
\end{cases}$$

Then, $F = \min \left\{ S_0 - S_K (D_H - (c - S_0))^+ \right\}$

$$R_K - R_0 = P_K V_K + P_L (S_K - S_0) + P_H \min \left\{ S_0 - S_K (D_H - (c - S_0))^+ \right\} - P_L \min[V_K, (D_H + V_K + S_K - c)^+]$$
$$\geq P_K (S_0 - S_K) + P_L (S_K - S_0) + P_H (S_0 - S_0) - P_L (S_0 - S_K) = (P_K - P_L + P_H - P_L) (S_0 - S_K) \geq 0$$
By comparison of the formulas above, it can be concluded that the revenue of TOMCM is increased compared with the traditional revenue management mode for an airline flight. Moreover, as can be seen from the theoretical analysis above, the increase of this kind of revenue is almost risk-free for airlines, which can be interpreted as an arbitrage behavior. In other words, after the introduction of TOMCM, a flight of an airline can obtain a physical option to hedge its risk compared with the traditional revenue management model. According to the above analysis, the airline will choose to recall only when the potential revenue of the callable tickets are higher than the traditional one, that is \( (P_K - P_L + P_H > P_L) \), and then the airline will implement callable mechanism to increase its revenue.

2.2.2. Comparison between traditional Three-stage model and TOMCM. According to traditional three-stage sales model, the tickets are divided into three levels and three stages of sales. When the callable mechanism doesn’t exist, the total revenue can be listed:

\[
\bar{R}_K = P_K V_K + P_L S_K + P_H \min\{D_H, c - V_K - S_K\}
\]  

(7)

So, compared with the revenue of traditional three-stage sales model, the revenue difference of callable mechanism is:

\[
R_K - \bar{R}_K = P_K V_K + P_L S_K + P_H \min\{D_H, c - V_K - S_K\} - P_L \min\{V_K, (D_H + V_K + S_K - c)^+\} - \left[ P_K V_K + P_L S_K + P_H \min\{D_H, c - V_K - S_K\} \right]
\]

\[
= P_H \left[ \min\{D_H, c - S_K\} - \min\{D_H, c - V_K - S_K\} \right] - P_L \min\{V_K, (D_H + V_K + S_K - c)^+\}
\]

2.2.2. Comparison between traditional Three-stage model and TOMCM. According to traditional three-stage sales model, the tickets are divided into three levels and three stages of sales. When the callable mechanism doesn’t exist, the total revenue can be listed:

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(7)

So, compared with the revenue of traditional three-stage sales model, the revenue difference of callable mechanism is:

\[
R_K - \bar{R}_K = P_K V_K + P_L S_K + P_H \min\{D_H, c - V_K - S_K\} - P_L \min\{V_K, (D_H + V_K + S_K - c)^+\} - P_L \left[ \min\{D_H, c - V_K - S_K\} \right]
\]

When \( F = \min\{D_H, c - S_K\} - \min\{D_H, c - V_K - S_K\} \),

\[
F = \begin{cases} V_K, & D_H \geq c - S_K \geq c - V_K - S_K \\ D_H - (c - V_K - S_K), & c - S_K \geq D_H \geq c - V_K - S_K \\ 0, & c - S_K \geq c - V_K - S_K \geq D_H \end{cases}
\]

Then, \( F = \min\{V_K, (D_H + V_K + S_K - c)^+\} \)

\[
R_K - \bar{R}_K = (P_H - P_L) \min\{V_K, (D_H + V_K + S_K - c)^+\} \geq 0
\]

By comparison, it is also found that TOMCM can create risk-free profits for the airlines and increase the profits of the airlines.

3. Numerical Experiment

According to the established model, it is assumed that the total number of cabin C of an airline flight is 300. The plan of cabin seats inventory control is as follows:

**Table 1. Status of the three-stage sales cabin**

| Stage | Price | Space allocation | Passenger demand |
|-------|-------|------------------|------------------|
| 1     | 350   | 70               | 50               |
| 2     | 700   | 180              | 210              |
| 3     | 1200  | 90               | 110              |

According to these data, the model established above is used to solve the problem of total sales revenue about TOMCM, traditional two-stage sales model and traditional three-stage sales model of the airline company respectively. The results are shown in table 2:

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Table 2. Sales revenue of various models

|                      | Revenue | 1  | 2  | 3  | total |
|----------------------|---------|----|----|----|-------|
| TOMCM                | 17500   | 126000 | 104000 |    | 247500|
| traditional two-stage sales model | --     | 182000 | 48000  |    | 230000|
| traditional three-stage sales model | 17500 | 126000 | 84000   |    | 227500|

According to the formula (1) to formula (5) above and data in table 1, the revenue of TOMCM can be calculated. According to formula (6), the revenue of traditional two-stage sales model can be calculated. According to formula (7), the revenue of traditional three-stage sales model can be calculated.

It can be seen from table 2 that the total revenue of TOMCM is 247,500 Yuan, the total revenue of implementation of traditional two-stage sales mode is 230,000 Yuan, and the revenue of implementation of traditional three-stage sales mode is 227,500 Yuan. The total revenue of TOMCM is the highest. Comparing with the traditional two-stage sales model, the total revenue of TOMCM increased to 7.61%. Compared with the traditional three-stage sales model, the total revenue of TOMCM increased to 8.79%. Therefore, it can be found that even compared with traditional two-stage and three-stage sales models, the total revenue of the airlines to implement TOMCM is significantly increased.

4. Conclusion

This paper combines the callable mechanism model with the three-stage sales model. So, a three-stage sales cabin control model based on the recall mechanism is established, comparing with the traditional two-stage and three-stage sales model, it is found that the implementation of the callable mechanism can enable airlines to realize risk hedging and arbitrage. Finally, numerical examples are given to verify the superiority of this optimization model. However, the model has some shortcomings in this paper, such as overbooking, no-shows, go-shows and cancellations, which are not considered. Multiple (four or more) sales stages based on the callable mechanism are also the direction of further research.

Acknowledgments

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References

[1] Littlewood K. Special Issue Papers: Forecasting and control of passenger bookings. Journal of Revenue & Pricing Management 2 (2005) 111-123.
[2] Xu hong, hu yunquan and li jun. Research on dynamic pricing model and seat allocation of airlines. System engineering theory and practice 12 (2004) 44-48.
[3] zhu jinfu, zhou yan and yao yun. Earnings analysis of flexible control for reservation of air passenger cabin. Forecast 5 (2006) 70-74.
[4] Gallego G. Flexible and callable revenue management. Available from <http://xueshu.baidu.com> (2004-10-15).
[5] Gallego G, Kou S G, Phillips R. Revenue Management of Callable Products. Management Science 3 (2011) 550-564.
[6] Zhou yan. Research on the recall system of seat inventory control in earnings management (Ph.D.). Nanjing university of aeronautics and astronautics. (2005).
[7] Gui yunmiao, gao qiang and sun gang. Study on the optimization method of the number of recalled tickets in earnings management. East China economic management 5 (2012) 159-160.
[8] Gui yunmiao. Study on air cargo revenue management and process optimization (Ph.D.). Nanjing university of aeronautics and astronautics. (2007)
[9] Gao qiang. Research on cabin control in aviation revenue management (Ph.D.). Nanjing university of aeronautics and astronautics. (2006)