The Study on Selection of Evaluation Criterion for Slope Instability

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Abstract: How to select an appropriate instability evaluation criterion is a key issue in the application of the finite element strength reduction method. Commonly used evaluation criteria for slope instability mainly include non-convergence criterion of finite element numerical calculation, displacement saltation criterion at characteristic point, and penetration criterion of plastic zone. The researches on these three types of instability criteria are still continuing, but there is no uniform conclusion yet. Based on the different characteristics of the three types of instability evaluation criteria, this paper compares and analyzes the evaluation results of three criteria on the slope stability by trial calculations, and then discusses the characteristics and effectiveness of three types of instability evaluation criteria. In actual numerical calculation, it is necessary to minimize the influence of human factors and comprehensively consider the three types of instability evaluation criteria to improve the consistency of the evaluation results. When the evaluation results are inconsistent, priority should be given to the feature point displacement saltation criterion at characteristic point or penetration criterion of plastic zone. In addition, a reasonable dilatancy angle is needed to accurately consider the influence of dilatancy on slope stability.

1. Introduction

There are many methods about slope stability analysis, and the key lies in how to calculate the safety factor and determine the sliding surface[1]. Zienkiewicz et al.[2] first proposed the concept of using finite element strength reduction method to analyze slope stability. With the continuous development of numerical analysis technology, finite element strength reduction method has been widely used in slope stability analysis. A key issue in the application of the finite element strength reduction method is how to choose an appropriate instability evaluation criterion. The obtained slope stability safety factor varies from different instability evaluation criterion. Commonly used instability evaluation criteria mainly include non-convergence criterion of finite element numerical calculation, displacement saltation criterion at characteristic point, and penetration criterion of plastic zone. Many scholars have discussed the validity and uniformity of different instability evaluation criteria.

Zhao et al.[3] and Liu et al.[4] comparative analyzed the advantages and disadvantages of different instability evaluation criteria. Zhao et al. believed that the non-convergence of finite element numerical calculation can be used as an effective criterion for slope instability. Liu et al., however, held that the error of safety factor obtained by non-convergence criterion may be large in some cases. Griffiths et al.[5] expressed the opinion that the non-convergence of the finite element numerical calculation can only occur after the penetration of plastic zone and displacement mutation of the characteristic point. However, through the research, Wang et al.[6] found that the non-convergence of numerical calculations...
may occur either during or following the displacement saltation of characteristic point. In addition, Cheng et al.\cite{7} discovered that the non-convergence of numerical calculations may occur before and after the penetration plastic zone. Therefore, from the above, the opinions held by these scholars are contradictory.

Pei et al.\cite{8} and Chen et al.\cite{9} analyzed the internal connection of the three types of instability evaluation criteria and proposed that three of them have theoretical unity. Tu et al.\cite{10}, from the perspective of the energy mechanism of slope deformation failure, proved the unity of the three types of instability evaluation criteria, and further put forward that the unequal safety factors obtained in different instability evaluation criteria are mainly caused by human factors such as convergence criterion and grid accuracy. In other words, the approximate solution of numerical calculation is the essential reason for the disunity of different criteria.

In summary, the research on the three types of instability evaluation criteria is still continuing, and a unified conclusion has not been formed. The three types of instability evaluation criteria are mainly put forward from different points such as stress, strain and displacement. However, the complicated action mechanism of geotechnical materials leads to complex stress, strain and displacement during slope deformation failure, which make it difficult to research the instability evaluation criterion. Based on the different characteristics of the three types of instability evaluation criteria, this paper compares and analyzes the evaluation results of different criteria by trial calculations, and then discusses the characteristics and effectiveness of three types of instability evaluation criteria.

2. The evaluation criteria for slope instability

2.1 Finite element strength reduction method

The safety factor is a quantitative index to measure the stability of the soil in slope engineering. Slope safety factor can be defined as the degree of shear strength reduction when the soil just reaches the critical failure state, that is, the ratio of the actual shear strength to the critical failure shear strength\cite{11}.

The basic principle of the finite element strength reduction method is to divide both the shear strength parameters $c$ and $\tan \varphi$ by a reduction factor $F_r$ and obtain the new shear strength parameters $c_m$ and $\varphi_m$:

$$c_m = \frac{c}{F_r}, \quad \varphi_m = \arctan \left( \frac{\tan \varphi}{F_r} \right)$$  \hspace{1cm} (1)

The new material parameters in Eq.(1) are used for finite element analysis. In the numerical calculation, one can assume different strength reduction factors $F_r$ through trial calculations and observe that whether the calculations are converge or not. Increasing the value of $F_r$, when the slope reaches the limit equilibrium state (or critical failure state), the corresponding strength reduction factor $F_r$ is the slope safety factor $F_r$, and the critical fracture surface of the slope can be also obtained.

2.2 The evaluation criteria for slope instability

At present, there are three main evaluation criteria for judging that whether a slope reaches the critical failure state: (1) the convergence of the finite element calculation, (2) the sudden change of the strain and displacement at the characteristic point, (3) the entirely run-through or penetration of plastic zone.

However, using the non-convergence of the finite element calculation as the failure criterion, which is related to the finite element algorithm, can be affected by some uncertain human factors. The displacement mutation at characteristic point and the continuous penetration of plastic zone also are affected by uncertain human factors in numerical calculation. In addition, the selection of both grid accuracy and convergence criteria would affect the evaluation results of the above three evaluation criteria.
3. Numerical example of slope

3.1. Calculation model
In order to analyze the mechanism of slope deformation failure, the classic model of general homogeneous slope was selected\cite{12, 13}, and the finite element strength reduction method was used to calculate the slope safety factor. The slope calculation model is illustrated in Fig. 1. The slope is regarded as an ideal elastoplastic material, in which the M-C yield criterion is adopted, and the material parameters of the slope are shown in Table 1. Fig. 1 shows the grid division of the slope calculation model, which includes 3400 elements and 3541 nodes. The boundary conditions are: horizontal constraints on the left and right sides, horizontal and vertical constraints on the bottom, and free boundary on the slope.

Table 1  Material parameters for the slope

| Elastic modulus $E$/MPa | Poisson's Ratio $\nu$ / | Bulk density $\gamma$/kN/m$^3$ | Cohesion $c$/MPa | Internal friction angle $\varphi$ /° |
|-------------------------|------------------------|-------------------------------|------------------|-----------------------------------|
| 100                     | 0.3                    | 20                            | 0.042            | 17                                |

Fig. 1  Finite element model of the slope

3.2. Calculation results based on different evaluation criteria
According to the M-C yield criterion, the calculations under the conditions of associated flow ($\phi = \varphi$) and non-associated flow ($\phi = 0$) are respectively carried out. The relationship curve between the displacement at characteristic point of the slope and the reduction factor is shown in Fig. 2, and Fig. 3 is the penetration diagram of the plastic zone. The characteristic points of the slope are selected respectively at the top node A, the middle node B and the bottom node C. The displacement of the characteristic point is the displacement in the X direction.

It can be seen from Fig. 2 that the safety factors under associated flow and non-associated flow conditions, based on displacement saltation criterion at characteristic point, are 1.24 and 1.21, respectively. From Figure 3, it can be found that the safety factors under the two conditions based on penetration criterion of plastic zone are 1.24 and 1.21. When the reduction factors reach 1.28 and 1.22 respectively, the numerical calculations under the two conditions become non-convergent. Therefore, based on the non-convergence criterion of the numerical calculations, the safety factors obtained under associated flow and non-associated flow conditions are 1.28 and 1.22, respectively. Comparing the safety factors of slope under three evaluation criteria, the safety factors under associated flow conditions are slightly greater than those under non-associated flow conditions.
3.3. Discussion
The safety factors under different evaluation criteria are shown in Table 2. By comparing the results in Table 2, it is can be found that the safety factors based on displacement saltation criterion at characteristic point and penetration criterion of the plastic zone are consistent, while the safety factors based on the non-convergence criterion of numerical calculation are slightly larger than the first two.
criteria. This is mainly because the displacement naturally increases rapidly after the penetration of plastic zone, but the numerical calculation does not necessarily become nonconvergent.

In fact, the overall failure of slope results from that the equivalent plastic zone penetrates from the toe to the top of slope, which drives to infinite plastic flow. At the same time, the displacement of characteristic point abruptly increase correspondingly, and the finite element numerical calculation cannot converge due to the infinite growth of the displacement. The three types of instability evaluation criteria have theoretical unity. In addition, the numerical calculation itself is an approximate solution. Due to the influence of human factors such as meshing accuracy and convergence criterion, the evaluation results of three criteria are inconsistent. When the grid accuracy is high enough and the convergence criterion is strict enough, the evaluation results would become consistent. Therefore, in actual numerical calculation, it is necessary to comprehensively consider the three types of instability evaluation criteria to improve the consistency of the evaluation results. When the evaluation results are inconsistent, priority should be given to displacement saltation criterion at characteristic point and penetration criterion of plastic zone.

In addition, it is found that the safety factor under the condition of associated flow ($\phi = \phi$) is slightly greater than that of non-associated flow ($\phi = 0$). Fig.4 shows the critical slip surface under different flow conditions. From Fig.3 and Fig.4, one can easily obtain that the position of plastic zone varies from different flow rules, and the position of the critical slip surface is also different. The critical slip surfaces under two flow rules both are arc-shaped, but the surface of associated flow has the larger arc at the toe and the closer position to slope corner at the top, which results from the consideration of dilatancy. Due to the influence of dilatancy, the safety factor is improved, and thus the overall stability of the slope is enhanced, which leads to different positions of the slope plastic zone and critical slip surface. However, in actual engineering, the dilatancy angle of soil is generally smaller than the internal friction angle, that is, $\phi > \phi > 0$. Therefore, a reasonable dilatancy angle is needed to accurately consider the influence of dilatancy on slope stability.

| Flow rules        | Penetration criterion of plastic zone | Displacement saltation criterion at characteristic point | Non-convergence criterion of numerical calculation |
|-------------------|---------------------------------------|--------------------------------------------------------|---------------------------------------------------|
| Associated flow   | 1.24                                  | 1.24                                                   | 1.28                                              |
| Non-associated flow | 1.21                                  | 1.21                                                   | 1.22                                              |

4. Conclusions

(1) The three types of instability evaluation criteria have the unity of internal theory. Numerical calculation is an approximate solution. Due to the influence of human factors such as meshing accuracy and convergence criterion, the evaluation results of three criteria are inconsistent.
(2) In the actual numerical calculation, it is necessary to minimize the influence of human factors, and comprehensively consider the three types of instability evaluation criteria to improve the consistency of the evaluation results. When the evaluation results are inconsistent, priority should be given to displacement saltation criterion at characteristic point and penetration criterion of plastic zone.

(3) The safety factor under associated flow condition is slightly greater than that under non-associated flow condition. In numerical calculations, a reasonable dilatancy angle is required to accurately consider the influence of dilatancy on slope stability.

(4) In order to improve the calculation accuracy of safety factor, the appropriate meshing accuracy and convergence criterion should be selected, so as to optimize the calculation efficiency.

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References
[1] Deng D., Li L. (2017) Three-dimensional limit equilibrium method for slope stability based on assumption of stress on slip surface. Rock and Soil Mechanics, 38(1): 189-196.
[2] Zienkiewicz O.C., Humpheson C., Lewis R.W. (1975) Associated and non-associated visco-plasticity and plasticity in soil mechanics. Géotechnique, 25(4): 671-689.
[3] Zhao S., Zheng Y., Zhang Y. (2005) Study on slope failure criterion in strength reduction finite element method. Rock and Soil Mechanics, 26(2): 332-336.
[4] Liu J., Luan M., Zhao S., et al. (2005) Discussion on criteria for evaluating stability of slope in elasto-plastic FEM based on shear strength reduction technique. Rock and Soil Mechanics, 26(8): 1345-1348.
[5] Griffiths D.V., Lane P.A. (1999) Slope stability analysis by finite elements. Geotechnique, 149(3): 387-403.
[6] Wang D., Nian T., Chen Y. (2007) Three problems in slope stability analysis with finite element method. Rock and Soil Mechanics, 28(11): 2309-2313, 2318.
[7] Cheng C., Luo F., Qi C., et al. (2012) Comparative analysis of slope stability by strength reduction method. Rock and Soil Mechanics, 33(11): 3472-3478.
[8] Pei L.J., Qu B.N., Qian S.G. (2010) Uniformity of slope instability criteria of strength reduction with FEM. Rock and Soil Mechanics, 31(10): 3337-3341.
[9] Chen L., Jin X. (2012) Study on the applicability of three criteria for slope instability using finite element strength reduction method. China Civil Engineering Journal, 45(9): 136-146.
[10] Yi-Liang T.U., Liu X.R., Zhong Z.L., et al. (2018) The unity of three types of slope failure criteria. Rock and Soil Mechanics, (1): 173-180.
[11] Duncan J.M. (1996) State of the art: Limit equilibrium and finite-element analysis of slopes. Journal of Geotechnical Engineering, 123(9): 577-596.
[12] Zheng Y. (2012) Development and application of numerical limit analysis for geological materials[J]. Chinese Journal of Rock Mechanics and Engineering, 31(7):1297-1316.
[13] Dai Z., Liu Z., Liu C., et al. (2008) Numerical analysis of soil slope stability considering tension and shear failures. Chinese Journal of Rock Mechanics and Engineering, 27(2):375-382.