Power Markets for Controlling Smart Matter

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Abstract

Embedding microscopic sensors, computers and actuators into materials allows physical systems to actively monitor and respond to their environments. This leads to the possibility of creating smart matter, i.e., materials whose properties can be changed under program control to suit varying constraints. A key difficulty in realizing the potential of smart matter is developing the appropriate control programs. We present a market–based multiagent solution to the problem of maintaining a physical system near an unstable configuration, a particularly challenging application for smart matter. This market control leads to stability by focussing control forces in those parts of the system where they are most needed. Moreover, it does so even when some actuators fail to work and without requiring the agents to have a detailed model of the physical system.

1 Introduction

Embedding microscopic sensors, computers and actuators into materials allows physical systems to actively monitor and respond to their environments in precisely controlled ways. This is particularly so for microelectromechanical systems (MEMS) [1, 2, 6] where the devices are fabricated together in single silicon wafers. Applications include environmental monitors, drag reduction in fluid flow, compact data storage and improved material properties.

In many such applications the relevant mechanical processes are slow compared to sensor, computation and communication speeds. This gives a smart matter regime, where control programs execute many steps within the time available for responding to mechanical changes. A key difficulty in realizing smart matter’s potential is developing the control programs. This is due to the need to robustly coordinate a physically distributed real-time response with many elements in the face of failures, delays, an unpredictable environment and a limited ability to accurately model the system’s behavior. This is especially true in the mass production of smart materials where manufacturing tolerances and occasional defects will cause the physical system
to differ somewhat from its nominal specification. These characteristics limit the effectiveness of conventional control algorithms, which rely on a single global processor with rapid access to the full state of the system and detailed knowledge of its behavior.

A more robust approach for such systems uses a collection of autonomous agents, that each deal with a limited part of the overall control problem. Individual agents can be associated with each sensor or actuator in the material, or with various aggregations of these devices, to provide a mapping between agents and physical location. This leads to a community of computational agents which, in their interactions, strategies, and competition for resources, resemble natural ecosystems [13]. Distributed controls allow the system as a whole to adapt to changes in the environment or disturbances to individual components [10].

Multiagent systems have been extensively studied in the context of distributed problem solving [4, 7, 15]. They have also been applied to problems involved in acting in the physical world, such as distributed traffic control [17], flexible manufacturing [22], the design of robotic systems [18, 25], and self-assembly of structures [20]. However, the use of multiagent systems for controlling smart matter is a challenging new application due to the very tight coupling between the computational agents and their embedding in physical space. Specifically, in addition to computational interactions between agents from the exchange of information, there are mechanical interactions whose strength decreases with the physical distance between them.

In this paper we present a novel control strategy for unstable dynamical systems based on market mechanisms. This is a particularly challenging problem, for in the absence of controls, the physics of an unstable system will drive it rapidly away from the desired configuration. This is the case, for example, for a structural beam whose load is large enough to cause it to buckle and break. In such cases, weak control forces, if applied properly, can counter departures from the unstable configuration while they are still small. Successful control leads to a virtual strengthening and stiffening of the material. Intentionally removing this control also allows for very rapid changes of the system into other desired configurations. Thus an effective way of controlling unstable systems opens up novel possibilities for making structures extremely adaptive.

2 Dynamics of Unstable Smart Matter

The devices embedded in smart matter are associated with computational agents that use the sensor information to determine appropriate actuator forces. The overall system dynamics is a combination of the behavior at the location of these agents and the behavior of the material between the agent locations. In mechanical systems, displacements associated with short length scales involve relatively large restoring forces, high frequency oscillations and rapid damping. Hence, they are not important for the overall stability [14]. Instead, stability is primarily determined by the lowest frequency modes. We assume that there are enough agents so that their typical spacing is much smaller than the wavelengths associated with these lowest modes. Hence, the lower frequency dynamics is sufficiently characterized by the displacements
Figure 1: An unstable dynamical system. a) The unstable chain with the mass points displaced from the unstable fixed point which is indicated by the horizontal dashed line. The masses are coupled to their neighbors with springs, and those at the end of the chain are connected to a rigid wall. b) A chain of upward-pointing pendulae connected by springs as an example of an unstable spatially extended system.

at the locations of the agents only. The high-frequency dynamics of the physical substrate between agents serves only to couple the agents’ displacements.

The system we studied, illustrated in Fig. 1a, consists of $n$ mass points connected to their neighbors by springs. In addition a destabilizing force proportional to the displacement acts on each mass point. This force models the behavior of unstable fixed points: the force is zero exactly at the fixed point, but acts to amplify any small deviations away from the fixed point. This system can be construed as a linear approximation to the behavior of a variety of dynamical systems near an unstable fixed point, such as the inverted pendulae shown in the Fig. 1b. In the absence of control, any small initial displacement away from the vertical position rapidly leads to all the masses falling over. In this case, the lowest mode consists of all the pendulae falling over in the same direction and is the most rapidly unstable mode of behavior for this system. By contrast, higher modes, operating at shorter length scales, consist of the masses falling in different directions so that springs between them act to reduce the rate of falling.

The system’s physical behavior is described by

1. the number of mass points $n$
2. the spring constant $k$ of the springs
3. a destabilizing force coefficient $f$
4. a damping force coefficient $g$

We also suppose the mass of each point is equal to one. The resulting dynamics of the unstable chain is given by

\begin{equation}
\begin{align*}
\frac{dx_i}{dt} &= v_i \\
\frac{dv_i}{dt} &= k(x_{i-1} - x_i) + k(x_{i+1} - x_i) + fx_i - gv_i + H_i
\end{align*}
\end{equation}

\footnote{We used a standard ordinary-differential-equation solver [19] to determine the controlled system’s behaviors.}
where \( x_i \) is the displacement of mass point \( i \), \( v_i \) is the corresponding velocity, and \( x_0 = x_{n+1} = 0 \) is the boundary condition. The \( H_i \) term in Eq. (1) is the additional control force produced by the actuator attached to mass point \( i \). We suppose the magnitude of this control force is proportional to the power \( P_i \) used by the actuator. For reasons of simplicity we use a proportionality factor of 1.

For these systems, the long time response to any initial condition is determined by the eigenvalues of the matrix corresponding to the right hand side of Eq. (1). Specifically, if the control force makes all eigenvalues have negative real parts, the system is stable [11]. The corresponding eigenvectors are the system’s modes. Thus to evaluate stability for all initial conditions, we can use any single initial condition that includes contributions from all modes. If there are any unstable modes, the displacements will then grow. We used this technique to evaluate stability in the experiments described below.

### 3 A Power Market for Control

The control problem is how hard to push on the various mass points to maintain them at the unstable fixed point. This problem can involve various goals, such as maintaining stability in spite of perturbations typically delivered by the system’s environment, using only weak control forces so the actuators are easy and cheap to fabricate, continuing to operate even with sensor noise and actuator failures, and being simple to program, e.g., by not requiring a detailed physical model of the system.

Computational markets are one approach to this control problem [3, 5, 12, 14, 16, 21, 23, 24]. As in economics, the use of prices provides a flexible mechanism for allocating resources, with relatively low information requirements [9]: a single price summarizes the current demand for each resource.

In designing a market of computational agents, a key issue is to identify the consumers and producers of the goods to be traded. Various preferences and constraints are introduced through the definition of the agents’ utilities. This ability to explicitly program utility functions is an important difference from the situation with human markets. Finally, the market mechanism for matching buyers and sellers must be specified.

In the market control of smart matter treated here, actuators, or the corresponding mass points to which they are attached, are treated as consumers. The external power sources are the producers and as such are separate from consumers. All consumers start with a specified amount of money. All the profit that the producers get from selling power to consumers is equally redistributed to the consumers. This funding policy implies that the total amount of money in the system will stay constant.

In the spirit of the smart matter regime, where control computations are fast compared to the relevant mechanical time scales, we assume a market mechanism that rapidly finds the equilibrium point where overall supply and demand are equal. Possible mechanisms include a centralized auction or decentralized bilateral trades or arbitrage. This equilibrium determines the price and the amount of power traded.
Each actuator gets the amount of power that it offers to buy for the equilibrium price and uses this power to push the unstable chain.

The utility function for using power $P$ reflects a trade-off between using power to act against a displacement and the loss of wealth involved. While a variety of utility functions are possible, a particularly simple one for agent $i$, expressed in terms of the price of the power, $p$, and the agent’s wealth, $w_i$, is:

$$U_i = -\frac{1}{2w_i} pP^2 + bP|X_i|$$  \hspace{1cm} (2)

where

$$X_i = \sum_{j=1}^{n} a_{ij}x_j$$  \hspace{1cm} (3)

is a linear combination of the displacements of all mass points that provides information about the chain’s state. The parameter $b$ determines the relative importance to an agent of responding to displacements compared to conserving its wealth for future use.

Actuator $i$ always pushes in the opposite direction of $X_i$, i.e., it acts to reduce the value of $X_i$. In this paper we focus on the simple case of purely local control where $a_{ij} = 1$ when $i = j$ and is 0 otherwise. Thus, consumer $i$ considers only its own displacement $x_i$. For simplicity, we use an ideal competitive market in which each consumer and producer acts as though its individual choice has no affect on the overall price, and agents do not account for the redistribution of profits via the funding policy. Thus a consumer’s demand function is obtained by maximizing its utility function as a function of power:

$$\frac{dU_i}{dP} = -\frac{P}{w_i} + b|X_i| = 0 \Rightarrow P_i(p) = b|X_i|w_i$$  \hspace{1cm} (4)

This demand function causes the agent to demand more power when the displacement it tries to control is large. It also reflects the trade-off in maintaining wealth: demand decreases with increasing price and when agents have little wealth. The overall demand function for the system is just the sum of these individual demands, giving

$$P^{\text{demand}}(p) = \frac{b}{p} \sum |X_i|w_i$$  \hspace{1cm} (5)

Similarly, each producer tries to maximize its profit $\rho$ given by the difference between its revenue from selling power and its production cost $C(P)$: $\rho = pP - C(P)$. To provide a constraint on the system to minimize the power use, we select a cost function for which the cost per unit of power, $C(P)/P$ increases with the amount of power. A simple example of such a cost function is

$$C(P) = \frac{1}{2a} P^2$$  \hspace{1cm} (6)

The parameter $a$ reflects the relative importance of conserving power and maintaining stability. We obtain the producer’s supply function by maximizing its profit:

$$\frac{d\rho}{dP} = p - \frac{dC}{dP} = 0 \Rightarrow P(p) = ap$$  \hspace{1cm} (7)
This is the same for all producers, so the overall supply function is then just

\[ P_{\text{supply}}(p) = nap \]  \hspace{1cm} (8)

From this the price and amount of traded power is determined by the point where the overall supply and demand curves intersect, i.e., \( P_{\text{demand}}(p) = P_{\text{supply}}(p) \). For our choices of the utility and cost functions, this condition can be solved analytically to give

\[ p_{\text{trade}} = \sqrt{\frac{b}{na} \sum_{i=1}^{n} |X_i|w_i} \]  \hspace{1cm} (9)

Given this equilibrium price, agent \( i \) then gets an amount of power equal to \( P_i(p_{\text{trade}}) \) according to Eq. (4) and the resulting control force is directly proportional to received power.

We can also consider the case where the amount of power available to the system is limited to \( P_{\text{max}} \). This hard constraint has the effect of limiting the overall supply function when the price is high so it becomes

\[ P_{\text{supply}}(p) = \begin{cases} 
nap & \text{if } p < P_{\text{max}}/na \\
\frac{P_{\text{max}}}{n} & \text{otherwise}
\end{cases} \]  \hspace{1cm} (10)

The final aspect of the market dynamics is how the wealth changes with time. This is given by

\[ \frac{dw_i}{dt} = -pR_i(p) + \frac{1}{n}pP_{\text{demand}}(p) \\
= -b|X_i|w_i + \frac{b}{n} \sum_{j=1}^{n} |X_j|w_j \]  \hspace{1cm} (11)

because we use the funding policy that all expenditures are returned equally to the agents in the system.

### 4 Comparing with Local Controls

As a simple comparison for the market behavior, we also study a local control method. In this case, each actuator \( i \) pushes with a strength that is proportional to the displacement of its respective mass point, and ignores the displacements of all other mass points. Specifically, the local controls are simply given by \( H_i = -c x_i \). Other control strategies \[11\] can try to estimate the amplitude of the lowest modes and push only against these modes, since these are the ones most important for stability.

For comparison with the market, we restrict ourselves to the case where the amount of available power is limited. This is useful for evaluating the ability of different control methods to maintain stability using only weak forces. We distinguish two ways the power could be limited for the local control. In the first, each actuator is separately limited to use no more than \( P_{\text{max}} \) power (local control 1), which corresponds to a situation where each actuator has a separate power source such as its own battery. Any actuator that requests more power than this maximum has its
control force reduced to require only $P_{\text{max}}$, i.e., $|H_i| = P_{\text{max}}$. The second local control allows available power to be moved among the different actuators and is limited only in that all actuators together cannot use more than $P_{\text{global}} = nP_{\text{max}}$ power (local control 2). This overall limit is implemented by comparing the total power requested according to the local control, i.e., $P_{\text{request}} = \sum_i |x_i|$ to the maximum available. If the requested amount exceeds the maximum, each agent has its power reduced by the factor $P_{\text{max}} / P_{\text{request}}$ so that the overall used power equals the global limit. The corresponding market has a total available power of $P_{\text{global}} = nP_{\text{max}}$.

5 Results

We studied a chain composed of 27 mass points, all of them having unit mass and connected by springs with a spring constant of value 1 and damping coefficient 0.1. The destabilizing force coefficient is 0.2, which is sufficient to make the system unstable when there is no control force. All agents start with an initial wealth of 50 money units and we are using the values $a = 0.05$ and $b = 0.001$ in the cost and utility functions. For definiteness, we chose an initial condition where the single element in the middle of the chain had a unit displacement and all other values were at zero. This configuration includes a contribution from all the modes of the system, which are just sinusoidal waves in this chain with uniform masses and spring constants [8]. For the local control, we used $c = 0.2$, which is more than sufficient to ensure stable control when power is unlimited [11].

With $P_{\text{max}} = 0.012$, Fig. 2 compares the performances of both local and market controls. We show both the total power use $\sum_i P_i$ and the average displacement of the chain $\sum_i |x_i|/n$. As can be seen, for the chosen parameter values the market is able to control the unstable chain in spite of the fact that the power is limited to a global maximum. This limit is reached several times. The local controls (1 and 2), on the other hand, fail in both cases, as seen in the figure. These results were obtained in a simulation run that lasted 20 time units. A longer simulation shows that the overall power usage and average displacements decrease with time for the market control while displacement continues increasing for the local controls.

Since the power cost function $C(P)$ does not change, the overall supply curve never changes, as shown in Fig. 3 which displays the supply curve and some demand curves for different times. The demand curves depend on the displacements and wealth of the agents. Since these are dynamical variables, overall demand curve changes in time. In addition to the times I, II and III marked in Fig. 2a, we also plot the overall demand curves for later times IV, V and VI. This shows that the amount of traded power decreases with time while the unstable chain is controlled by the market.

To demonstrate how robust the market mechanism is, we show in Fig. 4 the system’s response when an actuator breaks down. In this case we slightly increase the amount of power available compared to the simulation used for Fig. 2 to $P_{\text{max}} = 0.015$, so that the local controller 2 can also control the unbroken system. With the system initially functioning properly, we turned off the actuator in the middle of the chain after 10 time units and observed the consequent evolution. As can be seen, the market
Figure 2: a) Time development of the overall power usage for a market control (solid) and local controls 1 and 2 (dashed) in the case of limited available power. With the same power limit in all three cases, the market is the only one that can control the unstable chain. Points I, II and III mark the times at which the supply and demand curves intersections are shown in Fig. 3. b) Corresponding time development of the average displacement for a market control and local controls 1 and 2. The market reduces the average displacement with time whereas the local control is not able to prevent it from growing.
is still able to control the system whereas the local control fails to do so.

6 Discussion

In this paper we presented a novel mechanism of controlling unstable dynamical systems by means of a multiagent system using a market mechanism. We described how we defined consumers' and producers' utility functions that lead to the overall supply and demand curves and evaluated the price and amount of traded power within the system.

We showed that the market approach is able to control an unstable dynamical system in the case of limited power whereas a traditional local control strategy fails under the same assumptions. We also demonstrated that a market control adapts better to cases when an actuator breaks during the controlling process. These results show that a market control can be more robust than a local control when operating with given power constraints by focussing the power in those parts of the system where it is most needed. This not only reduces total power use but, more importantly, also allows control with weaker, and thus easier to fabricate, actuators.

The power of market approaches to control lies in the fact that relatively little knowledge of the system to be controlled is needed. This is in stark contrast to traditional AI approaches, which use symbolic reasoning with extremely detailed models of the physical system. However, while providing a very robust and simple design methodology, a market approach suffers from the lack of a high level explanation for its global behavior. An interesting open issue is to combine this approach with the more traditional AI one.

Although we have chosen particular forms of utility, supply and demand func-
Figure 4: Comparison of a market control and a local control in the case where one actuator breaks after 10 time units. Both control strategies would be able to control the system when all actuators would work perfectly. The market is still able to control the system although one actuator is broken but the local controller fails. a) Overall used power vs. time. b) Average displacement vs. time.
tions, there are many other functional forms that can also control the system. These could include additional goals, such as faster recovery from sudden changes and minimizing the number of active actuators. Furthermore, different funding strategies are possible, where profits are shared unequally among agents or the funds are allocated by an external agent. A very promising approach is the possibility of improving the performance of the system by having different market organizations that change in time. In our system, this corresponds to the agents learning to use information on the displacements or velocities of their neighbors when making their control decisions. In this way the multiagent system would take advantage of the fact that markets are a simple and powerful discovery process: new methods for selecting trades can be tried by a few consumers or producers and, if successful relative to existing approaches, gradually spread to other agents. Such a learning mechanism could help the system discover those organizational structures that lead to improved performance and adaptability.

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