Comparison of Calculation Methods for Cracking Width of Recycled Aggregate Concrete Beam Under Short-term Loads

Yuying Wang1, Jiajun Tang2* and Qingming Zhao2

1School of Engineering, the Tourism College of Changchun University, Changchun, Jilin, 130607, China
2School of Civil Engineering, Changchun Institute of Technology, Changchun, Jilin, 130012, China
*Corresponding author’s e-mail: tangjiajun@ccit.edu.cn

Abstract. In order to study the formula for calculating the maximum crack width of recycled aggregate concrete beams under short-term load, four methods of calculating the maximum crack width in the existing literature were compared and analyzed through examples. The results show that Yin Lei's calculation method based on bond-slip theory has smaller error and lower data dispersion. The calculated value of the formula agrees well with the experimental value, which can better meet the requirements of safety, applicability and durability.

1. Introduction
Recycled aggregate concrete (RAC) is a kind of environmental friendly green concrete material. It makes waste concrete into recycled aggregate by crushing, screening and cleaning, which is used to replace part of natural aggregate. It makes construction waste be reused and solid waste be recycled, which has significant social and economic benefits and environmental benefits [1-2]. On July 23, 2019, the Ministry of science and technology of China issued the notice on the application guide of key special projects such as solid waste recycling of national key R & D plan in 2019 on its website, aiming to strengthen the disposal of solid waste and garbage, and promote the comprehensive conservation and recycling of resources.

At present, there are many researches on the mechanical properties of RAC and the flexural, crack and shear properties of RAC beams. For example, Zhang Xuebing et al. [3-4] found that when the steel fiber content increased from 0 to 140 kg/m3, the compressive strength, splitting tensile strength, flexural strength, tension compression ratio and folding compression ratio of RAC increased by 9.8%, 59.2%, 37.5%, 45.6% and 26.0% respectively. Cao Wanlin et al. [5] changed the replacement rates of recycled coarse aggregate (RCA) and recycled fine aggregate (RFA) to test the bending performance of 7 groups of beams. The results show that no matter what the replacement ratio of RCA and RFA is, the loading experience of RAC beam is the same as that of normal strength concrete (NSC) beam, which has four characteristics of elasticity, cracking, yielding and failure; the section strain obeys the assumption of plane section; the influence of the replacement ratio of RCA and RFA on the flexural performance of RAC beam is very small, and the calculation formula of flexural capacity of NSC beam is still applicable to RAC beam. Based on the measured cracking data of 20 groups of RAC beams in China and the code for design of hydraulic concrete structures (SL 191-2008), Tang Jiajun et al. [6] gave the calculation formula of RAC beam crack resistance, and verified the applicability of the formula through reliability analysis. Yan Guoxin et al. [7-8] made statistics on a large number of RAC
beam test data, and then combined with GB 50010-2002 and GB 50010-2010 to study and reanalyze the shear bearing capacity calculation formula of RAC beam.

Considering that most of the research on the calculation formula of RAC beam is focused on the bending capacity, shear capacity and cracking moment, and the calculation formula of the maximum crack width of RAC beam is relatively less. Based on this, this paper first introduces the existing calculation method of the maximum crack width in the related research, and then verifies the applicability of the formula through the actual example and makes a comparison. Due to the lack of measured data of crack width of RAC beam under long-term load, this paper only studies the crack width of RAC beam under short-term load.

2. Introduction of several calculation methods

2.1. Code for design of concrete structures (GB 50010-2010)

Code for design of concrete structures (GB 50010-2010) [9] stipulates that the maximum crack width of reinforced concrete flexural members with various sections considering long-term action can be calculated according to the following formula:

\[ \omega_{\text{max}} = \alpha_c \psi \sigma_s \left( 1.9c_s + 0.08 \frac{d_{eq}}{\rho_w} \right) \]  
\[ \psi = 1.1 - 0.65 \frac{f_k}{\rho_w \sigma_s} \]  
\[ \rho_w = \frac{A_s}{0.5bh} \]  
\[ \sigma_s = \frac{M}{0.87A_s h_0} \] 

Where: \( \omega_{\text{max}} \) is the maximum crack width under long-term load, mm; \( \alpha_c \) is the force characteristic coefficient of the component, dimensionless; \( \psi \) is the strain non-uniformity coefficient of longitudinal tensile reinforcement between cracks, dimensionless; \( \sigma_s \) is the stress of longitudinal reinforcement, MPa; \( E_s \) is the elastic modulus of the reinforcement, MPa; \( c_s \) is the distance from the outer edge of the outermost longitudinal tensile reinforcement to the bottom edge of the tensile zone, mm; \( d_{eq} \) is the equivalent diameter of longitudinal reinforcement, mm; \( \rho_w \) is the effective reinforcement ratio of longitudinal reinforcement, dimensionless; \( f_k \) is the standard value of concrete axial tensile strength, MPa; \( A_s \) is the sectional area of the reinforcement, mm\(^2\); \( b \) is the section width, mm; \( h \) is the section height, mm; \( h_0 \) is the effective height of the section, mm.

Previous studies have shown that the maximum crack width under long-term load can be obtained by multiplying the maximum crack width under short-term load by the crack expansion factor [10], then

\[ \omega_{s,\text{max}} = \omega_{\text{max}} \frac{\tau_i}{\tau_f} \] 

Where: \( \omega_{s,\text{max}} \) is the maximum crack width under short-term load, mm; \( \tau_i \) is the crack expansion coefficient, dimensionless.
2.2. Calculation method of reference [11]

In 2011, Wu Jin et al. [11] fitted the crack width test data of 27 groups of RAC beams by changing the concrete strength grade, reinforcement ratio and concrete cover thickness, and obtained the following formula:

\[
\omega_{\text{max}}^R = 1.1 \left( 0.99 - 0.31 \frac{f_{\text{ck}}}{\rho_{\text{ce}} \sigma_s} \right) \frac{\sigma_s}{E_s} l_m \tag{6}
\]

\[
l_m = 2.3c + 0.06 \frac{d_{\text{eq}}}{\rho_{\text{ce}}} \tag{7}
\]

\[
\sigma_s = \frac{M}{\eta A_i h_i} \tag{8}
\]

\[
\eta = \begin{cases} 
1 & \left( \sqrt{\alpha_e \rho} \leq 0.2 \right) \\
1.38 - 1.90 \sqrt{\alpha_e \rho} & \left( 0.2 < \sqrt{\alpha_e \rho} < 0.3 \right) \\
0.8 & \left( \sqrt{\alpha_e \rho} \geq 0.3 \right)
\end{cases} \tag{9}
\]

Where: \( \omega_{\text{max}}^R \) is the short-term maximum crack width of RAC beam, mm; \( l_m \) is the average crack spacing of RAC beam, mm; \( c \) is the thickness of concrete cover, mm; \( \eta \) is the internal force arm coefficient, dimensionless; \( \alpha_e \) is the ratio of elastic modulus of reinforcement to RAC, dimensionless; \( \rho \) is the reinforcement ratio of longitudinal reinforcement, dimensionless; the meanings of other symbols are the same as above.

2.3. Calculation method of reference [12]

In 2012, Yin Lei et al. [12] proposed the calculation formula of the maximum crack width of RAC beams under short-term load based on the bond slip theory and combined with the measured data of 10 groups of RAC beams (parameters are longitudinal reinforcement ratio and recycled aggregate replacement ratio).

\[
\omega_{\text{max}}^R = 0.23\psi \frac{M}{h_i E_i A_i} \frac{d_{\text{eq}}}{\rho_{\text{ce}}} \tag{10}
\]

\[
\psi = 1.05 - 0.59 \frac{f_{\text{ck}}}{\rho_{\text{ce}} \sigma_s} \tag{11}
\]

Where: \( \omega_{\text{max}}^R \) is the short-term maximum crack width of RAC beam, mm; the meanings of other symbols are the same as above.

2.4. Calculation method of reference [13]

In 2014, Liu Chao et al. [13] conducted regression analysis on important coefficients based on test data, and established the calculation formula of RAC beam short-term maximum crack width based on comprehensive method.

\[
\omega_{\text{max}}^R = \tau \alpha \psi \frac{M}{\eta h_i A_i E_i} l_e \tag{12}
\]

\[
\eta = 0.93 - 0.56 \sqrt{\alpha_e \rho} \tag{13}
\]

\[
l_e = \beta \left( 2.5c + 0.058 \frac{d_{\text{eq}}}{\rho_{\text{ce}}} \right) \tag{14}
\]
where:

\[ \eta = 1 - 0.45 \frac{f'_t}{\rho_c \sigma_s} \]  

3. Example illustration

Five groups of experimental data in reference [14] are selected as the calculation examples of this study. As shown in Figure 1, the beam length is 2000 mm, the clear span is 1800 mm, the pure bending section is the same as the shear bending section, which is 600 mm, and the section size is 150 mm × 300 mm. As shown in Table 1, there are 3 C12, 2 C18 and 2 C18 + 1 C14 reinforcement modes for longitudinal reinforcement, with corresponding reinforcement ratios of 0.85%, 1.28% and 1.67% respectively. The grade of reinforcement is HRB400, the replacement rate of RCA includes 30%, 70% and 100%, and the thickness of concrete cover is 20 mm.

4. Comparative analysis of calculation methods

Formulas (1-5, 6-9, 10-11, 12-15) are respectively used to calculate the short-term maximum crack width of five groups of RAC beams in reference [14]. The results are shown in Table 2. According to the analysis of Table 2, the average value of the ratio of the test value to the calculated value in Yin Lei et al. [12] is 0.997, the error is only 0.3%, the standard deviation and coefficient of variation are the smallest, and the data dispersion is low. In order to better meet the requirements of safety, applicability and durability, it is suggested to use formula (10-11) to calculate the maximum crack width of RAC beam under short-term load. In GB 50010-2010, the average value of the ratio of the test value to the calculated value is 1.007, the error is 0.7%, the standard deviation and the coefficient of variation are 0.052 and 0.051 respectively, and the calculation accuracy is second only to the reference [12], indicating that the calculation method of the maximum crack width of NSC beam is also applicable to RAC beam. The error of calculation method in Liu Chao et al. [13] is 1.4%, and the standard deviation and coefficient of variation are small. Formula (12-15) can be used to calculate the maximum crack width of RAC beam under short-term load. However, the calculated value of Wu Jin's
formula in [11] has a large deviation from the experimental value, with an error of 19.7%, and the degree of data dispersion is also high. This may be because the protective layer thickness of 25 mm and 30 mm used in fitting the formula in [11] is different from that of 20 mm in this example. When the thickness of concrete cover is large, it is mostly suitable for building structures with poor environment and low concrete strength.

### Table 2. Calculation results of different methods

| Specimen number | M /kN·m | $w_t$/mm | $w_c$/mm | $w_t/w_c$/mm | $w_t$/mm | $w_c$/mm | $w_t/w_c$/mm | $w_t$/mm | $w_c$/mm | $w_t/w_c$/mm |
|-----------------|---------|-----------|-----------|----------------|-----------|-----------|----------------|-----------|-----------|----------------|
| ZS-3-12         | 33.9    | 0.26      | 0.250     | 1.041          | 0.204     | 1.274     | 0.262          | 0.993     | 0.254     | 1.022          |
| ZS0.7-3-12      | 34.5    | 0.28      | 0.259     | 1.081          | 0.210     | 1.336     | 0.271          | 1.033     | 0.262     | 1.069          |
| ZS0.3-3-12      | 33.0    | 0.24      | 0.241     | 0.997          | 0.198     | 1.214     | 0.252          | 0.951     | 0.246     | 0.975          |
| ZS-2-18         | 49.8    | 0.28      | 0.283     | 0.990          | 0.240     | 1.165     | 0.294          | 0.953     | 0.286     | 0.980          |
| ZS-2-18+14      | 62.4    | 0.22      | 0.237     | 0.927          | 0.220     | 0.998     | 0.209          | 1.053     | 0.249     | 0.883          |
| Average value   | —       | 1.007     | —         | 1.197          | —         | 0.997     | —              | 0.986     | —         | —              |
| Standard deviation | —   | 0.052     | —         | 0.115          | —         | 0.041     | —              | 0.062     | —         | —              |
| Coefficient of variation | — | 0.051     | —         | 0.096          | —         | 0.042     | —              | 0.063     | —         | —              |

Note: $M$ is the bending moment, $w_t$ is the test value of crack width, and $w_c$ is the calculated value of crack width.

## 5. Conclusions

This paper first introduces four calculation methods of the maximum crack width of concrete beams under short-term load, and then verifies the applicability of each formula through the analysis of calculation examples, and makes a comparison, and obtains the following conclusions.

1. According to the calculation methods of GB 50010-2010, Yin Lei and Liu Chao, the calculated value of the maximum crack width under short-term load is in good agreement with the experimental value, while the calculated value of Wu Jin’s calculation method deviates greatly from the experimental value.

2. In order to better meet the requirements of safety, applicability and durability, Yin Lei’s calculation method based on bond slip theory is recommended.

3. Due to the lack of experimental data in this study, the calculation method of the maximum crack width of RAC beam under short-term load needs to be further studied.

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