Abstract—We consider semi-elastic applications such as video conferencing in which user utility is a sigmoid function of the average bit rate over multiple time slots. The goal is to maximize the total expected user utility over time through allocation of downlink power and subcarriers in each time slot. We show that greedy allocation to maximize incremental utility in the current time slot can be implemented in a distributed fashion by an exchange of price and demand amongst users, the network, and an intermediate power allocation module. We then propose resource allocation that considers both the average rate achieved so far and the future expected rate, and show how future expected rate can be estimated by modeling the probability that a user will be allocated a subcarrier in a future time slot. The performance is illustrated using numerical results.

I. INTRODUCTION

In cellular networks, the allocation of wireless resources such as downlink transmission power has long been a central design problem. Fourth generation (4G) cellular networks will integrate voice, video, and data applications using packet switching. As a consequence, resource allocation should take into account the characteristics of applications when best-effort packet switching would not result in acceptable performance.

Multimedia applications, e.g. video streaming and video conferencing, constitute a rapidly increasing proportion of the total traffic on cellular networks. In this paper, we are motivated by the problem of resource allocation for video conferencing applications over cellular networks. For such applications, variable bit rate video encoding algorithms are typically used, e.g. MPEG. For real-time use, such as required to support video conferencing, best-effort service can result in unacceptable performance.

Video encoding algorithms often use a group of pictures as a central concept, and intentionally vary the bit rate over different frames within this group. For networks, this variability poses a problem, as neither constant bit rate service nor best effort service is appropriate. User ratings of video conferencing are often strongly correlated to the average bit rate transmitted over one or more groups of pictures.

User utility is frequently used to guide resource allocation decisions. Utility is often modeled as a function of transmission rate. The shape is usually assumed to be a step function for inelastic constant bit rate voice applications, a concave function for elastic data applications, and a sigmoid function for semi-elastic applications such as video [1]. We posit here

that for video conferencing applications, user utility should be a sigmoid function, not of the instantaneous transmission rate, but of the average rate over one or more groups of pictures.

Resource allocation for video conferencing is thus a challenge. Power allocation for voice applications represented by step utility functions has been well studied [2][3]. Similarly, power allocation for elastic data applications represented by concave utility functions has been well studied [4][5][6][7][8]. Power allocation for semi-elastic applications with utility as a sigmoid function of instantaneous transmission rate is less well-known. Lee et. al. [9] considers both scheduling and connection access control for CDMA systems, and proposes using pricing to select which users are active and to allocate power. Hande et. al. [10] similarly considers sigmoid utility functions under a single resource constraint, and gives conditions under which the Nash equilibrium using marginal cost pricing is optimal. Both of these papers assume that utility is a function of instantaneous rate. We are unaware of any research literature on power allocation when utility is a function of average bit rate over a period of time.

In this paper, we consider allocation of downlink power and subcarriers in OFDM 4G cellular systems for semi-elastic applications. We model user satisfaction by a sigmoid function of the average rate over a time window. The goal is to create algorithms that allocate power and subcarriers in each time slot in a manner that optimizes total average user utility over time. The difficulty with maximizing utility over many time slots is that the system should consider both the current channel and the likely achievable average rates by the end of the time window. For instance, suppose that it is early in the time window and a user currently faces a worse than average channel. Should the network allocate less than average power, figuring that later in the time window it can make up the difference? Does this depend on the auto-correlation of the channel? If late in the time window this user has experienced an average rate that places it in the convex portion of the utility function, should the network respond by increasing the transmission power? Alternatively, should the network give up on this user obtaining decent performance in this time window, and use the power to increase the utility of other users?

The paper proceeds as follows. In section II, we pose the resource allocation problem. In section III, we consider a non-causal system in which at the beginning of each time window the network knows the channel gains of each user in each time slot of the window. While knowing all the future
information is unrealistic, this will serve as an upper bound on causal systems. We illustrate how a dual problem formulation can be used to allocate power and subcarriers in each time slot of the time window. The complexity can be reduced by distributing the resource allocation process amongst users, the network, and intermediate power allocation modules. The network allocates power and subcarriers, and charges the power allocation module a shadow price for power in each time slot. The power allocation module translates this price per unit power into a price per unit rate, and resells the system resources to users. Users choose desired rates based on the cost and the resulting utility. We pose an iterative algorithm that determines near-optimal shadow prices in each time window.

In the remainder of the paper, we consider causal systems which must make resource allocation decisions in each time slot without knowledge of the channel gains of users in future time slots. The challenge is to decide how the resource allocation in the current time slot should be based on the average rate achieved so far in the time window and on the likely achievable rate during the remainder of the time window. In section IV, we design a resource allocation policy that considers the average rate achieved so far and ignores future rate, by only attempting to maximize the total incremental utility of the current slot. We show how this allocation can be iteratively determined in a distributed fashion amongst user, network, and power allocation modules by exchanging shadow prices and desired rates and powers. In section V, we propose a resource allocation policy that considers both the average rate achieved so far and the future expected rate. We show how this future expected rate can be estimated by modeling the probability that a user will be allocated a subcarrier in a future time slot. We show that although maximization of the expected utility due to future rate is prohibitively complex, maximization of the utility of the expected average rate over the time window can be near-optimally solved in a distributed fashion using a similar exchange of price and demand.

Finally, in section VI we illustrate the performance of these algorithms using numerical results. We find that the non-causal algorithm is near-optimal, with some sub-optimality when resources are severely constrained and a significant number of users are unable to achieve rates in the concave portion of the utility curve. We find that the greedy causal algorithm is similar to that of the non-causal algorithm when resources are severely constrained, but lags as resources become more plentiful. The reason is that although the greedy algorithm can react to a poor channel by allocating fewer resources, it does not consider future rates and thus may not decrease allocations enough. We find that prediction of future expected rates closes a portion of this gap by allocating resources in a balanced way based on both current channel and likely future channels.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a single cell downlink OFDM system serving \( K \) users, with \( N \) subcarriers. The bandwidth of each subcarrier is \( B \) which is assumed to be less than the coherence bandwidth of the channel so that the channel response can be considered

\[
r_{k,n,t}(p_{k,n,t}) = B \log_2 \left( 1 + \frac{|H_{k,n,t}|^2}{\delta^2} \right)
\]

where \( p_{k,n,t} \) is the power allocated, \( |H_{k,n,t}| \) is the channel gain, \( I \) is the interference power and \( \delta^2 \) is the noise power. The channel gain is assumed to be a stationary Markov process and the fading on different subcarriers is assumed to be independent to each other. The base station is assumed to know the channel gain of each user on each subcarrier in the current time slot and the conditional density \( f(\|H_{k,n,t}\|^2 | H_{k,n,T}^2) \) for future slots \( \tau \). The total rate of user \( k \) in time slot \( t \) is the sum of the rate over all subcarriers:

\[
R_{k,t} = \sum_{n=1}^{N} r_{k,n,t}
\]

Assume the time window consists of \( T \) time slots. At time \( t \), denote the contribution toward the average rate during the current time window by \( S_{k,t} = \frac{1}{T} \sum_{\tau=1}^{T} R_{k,\tau} \). The utility of user \( k \) is assumed to be a function \( U_k(S_{k,T}) \) which maps the average rate achieved in a time window \( S_{k,T} \) to the level of the satisfaction perceived by the application. Utility is assumed to be a sigmoid function, as pictured in Fig 1, namely there exists an inflection point \( S^{f}_{k,T} \) such that \( U_k \) is convex for \( S_{k,T} < S^{f}_{k,T} \) and concave for \( S_{k,T} > S^{f}_{k,T} \). We denote the rate at the maximum average utility by \( S^{c}_{k,T} \), namely

\[
S^{c}_{k,T} = \arg \max_{S_{k,T}} U_k(S_{k,T})
\]

We pose a maximization problem for each time window as follows. Denote the power allocation by \( p = \{p_{k,n,t}\} \). Each subcarrier can be allocated to at most one user, thus denote the feasible set of power and subcarrier allocations by \( A = \{p \text{ s.t. } \forall t, n, \ p_{k,n,t} > 0 \text{ for at most one user } k\} \). The goal is to maximize the total user utility within the time window \( T \) under constraints that the total transmitted power in each time slot not exceed the power supply \( P \):

\[
U_{tot}^* = \max_{p \in A} \sum_{k=1}^{K} U_k(S_{k,T}) \tag{1}
\]

s.t. \( \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n,t} \leq P \ \forall t; \ \ p_{k,n,t} \geq 0 \ \forall k, n, t \)

III. NON-CAUSAL VERSION

We first consider a non-causal system in which at the beginning of each time window the network knows the channel

![Fig. 1. Sigmoid Utility Function](image-url)
gains of each user in each time slot of the window. While knowing all the future information is unrealistic, this will serve as an upper bound to causal systems.

Because all of the channel gains are known at the beginning of each time window in this version, the resource allocation decisions for power and subcarriers can be made jointly for all users and all time slots within the window. The multiple time slot decisions are thus transformed into a single decision at the beginning of each time window. This will allow full consideration of the variation of channel for each user from time slot to slot and of the average bit rate during the time window achieved as a result of allocations made in each slot.

However, the direct solution of problem (1) requires solving $KNT$ fixed point equations. We are thus motivated to solve a dual problem. The idea, used previously for strictly concave sigmoid utility functions, transforming problem (1) into:

$$J_{d}(\lambda, \mu) = \max_{\lambda, \mu} \sum_{k=1}^{K} U_k(d_k) \text{ s.t. } S_k,T \geq d_k \tag{2}$$

The new problem (2) must have the same solution as the original problem (1), since $U_k$ is an increasing function of $d_k$, and thus at the optimum $d_k = S_k,T$.

However, it is easier to search for the optimal shadow prices and to let them determine the optimal resource allocation than to directly search for the optimal powers and subcarrier allocations. This can be done by posing a dual problem, see e.g. [12]. The Lagrange of (2) is then given by:

$$J_{d}(\lambda, \mu) = \sum_{k=1}^{K} U_k(d_k) + \sum_{t=1}^{T} \mu_t \left( P - \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n,t} \right)$$

$$+ \sum_{k=1}^{K} \lambda_k(S_k,T - d_k) \tag{3}$$

where $\lambda = \{\lambda_k\}$ are the Lagrangian multipliers associated with the rate constraints and $\mu = \{\mu_t, 1 \leq t \leq T\}$ are the Lagrangian multipliers associated with the power constraints.

The dual function is then given by:

$$J_{d}(\lambda, \mu) = \max_{\lambda, \mu} J(d, \mu)$$

and the dual problem is to optimally choose the Lagrangian multipliers:

$$J = \min_{\lambda, \mu} J(\lambda, \mu) \text{ s.t. } \lambda \succeq 0, \mu \geq 0 \tag{4}$$

The dual function can be decomposed into two pieces, $J(\lambda, \mu) = f_1(\lambda) + f_2(\lambda, \mu)$, where:

$$f_1(\lambda) = \max_{d} \sum_{k=1}^{K} (U_k(d_k) - \lambda_k d_k) \tag{5}$$

The second piece, $f_2(\lambda, \mu)$ can be further decomposed into $NT$ independent problems:

$$f_2(\lambda, \mu) = \sum_{t=1}^{T} \sum_{n=1}^{N} f_{2,n,t}(\lambda, \mu) + \sum_{t=1}^{T} \mu_t P \tag{6}$$

where

$$f_{2,n,t}(\lambda, \mu) = \max_{p \in A} \left( \sum_{k=1}^{K} \frac{1}{2} \lambda_k r_{k,n,t} - \mu_t \sum_{k=1}^{K} p_{k,n,t} \right) \tag{7}$$

According to the first order condition $\partial f_{2,n,t}/\partial p_{k,n,t} = 0$, the solution of (6) is

$$p_{k,n,t} = \left( \frac{B \lambda_k}{T \mu_t \ln 2} - \frac{\delta^2 + 1}{|H_{k,n,t}|^2} \right)^+ \tag{8}$$

Substituting (8) into (6) and simplifying we obtain

$$\Phi_{k,n,t} = \max_{\lambda} \Phi_{k,n,t} \tag{9}$$

where

$$\Phi_{k,n,t} = \frac{\lambda_k}{T} B \left[ \log_2 \left( \frac{B \lambda_k}{T \mu_t \ln 2} \frac{|H_{k,n,t}|^2}{\delta^2 + 1} \right) \right]^-$$

The decomposition indicates a method of distributing the optimization. The determination of rate in (5) indicates a role for each user, while the determination of Lagrangian multipliers $\mu$ in (7) indicates a role for the network. These two roles must be done in coordination. The decomposition suggests to us that there should be an intermediate power allocation module which determines the Lagrangian multipliers $\lambda$ in (7) and determines the powers $p$ in (8). The communication between the users, power allocation module, and network is illustrated in Fig 2.

Each role has a local optimization to accomplish. These can be done iteratively as follows, where the iteration number is denoted by a superscript $i$.

**User $k$ algorithm:** Given $\lambda_i^k$, $d_{k,t+1}^i = \arg \max_{d_k} [U_k(d_k) - \lambda_i^k d_k]$.

**Network algorithm:** Given tentative power and subcarrier allocations $p \in A$, $\mu_{t+1}^i = [\mu_t^i + s_{t+1}^i z_t^i]^+$, where $s_{t+1}^i$ is a
positive scalar stepsize, \([\cdot]^+\) is the projection on \(\mathbb{R}_+\), and \(z^i_k = \text{sgn}(\sum_{k=1}^K \sum_{n=1}^N p^i_{k,n,t} - P)\).

**Power Allocation Algorithm:** Given target rates \(d^t\) and Lagrangian multipliers \(\mu^t\), allocate \(p^t\) using (8) and (9) and update \(\lambda^{i+1} = [\lambda^i + s^i_P z^i_p]^+\), where \(s^i_P\) is a positive scalar stepsize, and \(z^i_p\) is any feasible direction that satisfies \(\text{sgn}(\lambda^{i+1} - \lambda^i) = \text{sgn}(d^i_p - S^i_{k,T})\).

This set of algorithms has an economic interpretation. The Lagrangian multipliers \(\lambda\) can be interpreted as shadow costs for rate. If user \(k\) is charged a price \(\lambda_k\) per unit rate, then (5) states that the system should allocate rate so as to maximize total user surplus, defined as total user utility minus total user cost. The user \(k\) algorithm implements this local optimization for user \(k\). Similarly the Lagrangian multiplier \(\mu\) can be interpreted as a shadow cost for power in time slot \(t\). The network algorithm iteratively adjusts each \(\mu\) by raising it if the demand exceeds the supply, and lowering it if the supply exceeds the demand\(^3\).

The job of the power allocation module is to purchase power in time slot \(t\) at a price \(\mu^t\) per unit power, and to resell it in the form of rate to individual users. The power allocation algorithm purchases power using (8) and (9), and iteratively adjusts each price \(\lambda_k\) by lowering it if the resulting average rate exceeds the user’s purchased rate, and raising it if the purchased rate exceeds the average rate\(^1\). These two actions can be interpreted as an attempt by the power allocation module to maximize profit, defined as revenue from users minus cost for power, as illustrated in (6).

The question is whether these iterations will jointly converge to an allocation \(p, d\). When utility functions are strictly concave, such iterative approaches converge. However, because our utility functions are sigmoid, the solution for (5) is either \(d^i_k = 0\) or \(d^i_k > S^i_{k,T}\). As a result, the algorithms may not jointly converge to an equilibrium point. This typically would occur when the solution to the dual problem includes at least one active user with \(S^i_{k,T} < S^i_{k,T}\). The algorithms must thus be modified to guarantee convergence.

One way is to force the algorithms to terminate by placing limits on the shadow costs. For the power allocation algorithm, we propose a subgradient method with bounds to update \(\lambda\):

\[
\lambda^{i+1}_k = \max\left[\min(\lambda^i_k + s^i_P (d^i_k - S^i_{k,T}), \overline{\lambda}_k), \underline{\lambda}\right] \tag{10}
\]

with a suitable choice of step size. The lower bound \(\underline{\lambda}\) can be set to a small suitable constant. The upper bound \(\overline{\lambda}_k\) can be derived from (5) as \(\overline{\lambda}_k = dU_k(S^i_{k,T})/dS_k^i(S^i_{k,T}, S^i_{k,T})\).

When \(\lambda^i_k = \overline{\lambda}_k\), (5) produces a tie between \(d^i_k = 0\) and \(d^i_k = S^i_{k,T}\); we break the tie using \(d^i_k = 0\) if \(S^i_{k,T} = 0\) and \(d^i_k = S^i_{k,T}\) otherwise. The inequality for \(\lambda\) is terminated when:

\[
\lambda^{i+1}_k - \lambda^i_k < \delta \quad \forall k \quad \text{or} \quad S^{i+1}_{k,T} = S^i_{k,T} \quad \forall k \tag{11}
\]

where \(\delta\) is a small constant.

\(^2\)Many optimization methods may be used; below we propose a subgradient method.

\(^3\)Many optimization methods may be used; below we propose a bisection method.

### Table I: Dual Iteration Search

| Every \(T\) slots, initialize \(\mu^0_t = \mu^0_i, \forall t\) and \(\lambda_k = \lambda \quad \forall k\) |
|---|
| Repeat |
| For slot \(t \in [1, T]\), allocate subcarrier and power by (9) and (8) |
| Update \(\mu\) using (12) |
| Until (13) |
| If \(\lambda^i_k = \overline{\lambda}_k\) |
| If \(S^i_{k,T} = 0\) then \(d^{i+1}_k = 0\) Else \(d^{i+1}_k = S^i_{k,T}\) |
| Else calculate \(d^{i+1}_k\) using (5) |
| Update \(\lambda\) using (10) |
| Until (11) |

For the update of \(\mu\) in the network algorithm, we propose a bisection algorithm:

\[
\mu^{i+1}_t = \begin{cases} 
(\mu^i_t + \mu^i_{t+1})/2, & \mu^{i+1}_t = \mu^i_t, \mu^{i+1}_{t+1} = \mu^i_{t+1} \\
(\mu^i_t + \mu^i_{t+1})/2, & \mu^{i+1}_t = \mu^i_{t+1}, \mu^{i+1}_{t+1} = \mu^i_t 
\end{cases} \tag{12}
\]

where the initial lower bound \(\mu^0_t\) can be set to a small suitable constant and the initial upper bound \(\mu^0_t\) can be derived from (8) as \(\mu^0_t = \max_{k,n} (B\lambda_k H_{k,n,T}^2)/(d^2 + 1)\). The iteration for \(\mu_t\) is terminated when:

\[
\mu^{i+1}_t - \mu^i_t < \epsilon \tag{13}
\]

where \(\epsilon\) is a small constant.

We call the resulting algorithm, outlined in Table I, **Dual Iteration Search Non-Causal (DIS NC).** Using these limits on the shadow costs, the algorithm is guaranteed to terminate in finite time. The complexity of the subgradient updates is polynomial in the dimension of the dual problem, and thus the complexity of the DIS NC algorithm is polynomial in the number of users \(K\).

In the rest of the paper, we consider causal systems which must make resource allocation decisions in each time slot without knowledge of the channels gains in future time slots.

### IV. Resource Allocation Based on the Average Rate Achieved So Far

The challenge is to decide how the resource allocation in the current time slot should be based on the average rate achieved so far in the time window and on the likely achievable rate during the remainder of the time window. In this section, we design a resource allocation policy that considers the average rate achieved so far and ignores the future rate. In the following section, we will also consider the expected future rate.

The contribution toward the average rate in the current time window achieved by user \(k\) as of slot \(t - 1\) is given by \(S_{k,t-1}\). This can be taken into consideration by focusing on the incremental utility of user \(k\) in slot \(t\), denoted by \(\Delta U_k(t) = U_k(S_{k,t-1} + R_{k,t}) - U_k(S_{k,t-1})\), that would result from an allocation of power corresponding to a rate \(R_{k,t}\).
A greedy version of the problem (1) would be to maximize the total user incremental utility in each time slot $t$:

$$\max_{\mathbf{p}_k \in \mathbf{A}_k} \sum_{k=1}^{K} \Delta U_{k,t} \quad \forall 1 \leq t < T$$

s.t. \( \sum_{k=1}^{K} p_{k,n,t} \leq P; \ p_{k,n,t} \geq 0 \ \forall k, n \)

where $\mathbf{p}_k = \{p_{k,n,t}, \forall k, n\}$ and $\mathbf{A}_k = \{\mathbf{p}_k \text{ s.t. } \forall n, p_{k,n,t} > 0 \text{ for at most one user } k\}$.

A similar approach to that used in the previous section can be used. The dual problem for (14) leads to a set of equations that describe the set of actions for users, the power allocation module, and the network in each time slot. Users now maximize incremental utility minus cost, $U_k(S_{k,t-1} + d_k) - \lambda_k d_k$, rather than total utility minus cost $U_k(d_k) - \lambda_k d_k$ as in (5). The network determines Lagrangian multipliers $\mu$ for power, as before, but now does this slot-by-slot. The power allocation module determines the Lagrangian multipliers $\lambda$ for rate, as before, but now only considers the difference between the actual and desired rate in the current time slot. The power allocation module also determines the powers $\mathbf{p}_k$ using (8) and (9), as before.

The communication between the users, power allocation module, and network is as illustrated in Fig. 2, and the allocations can be determined iteratively as follows, where the iteration number is denoted by a superscript $\text{i}$:

**User Algorithm:** Given $\lambda^1_k, d^1_k = \arg \max_{\mathbf{d}} [U_k(S_{k,t-1} + d_k) - \lambda_k d_k]$.

**Network Algorithm:** Given tentative power and subcarrier allocations $\mathbf{p} \in \mathbf{A}_t$, $\mu^{i+1} = [\mu^i + s^i \lambda^{i+1} B^1]$, where $s^i_B$ is a positive scalar stepsize, and $z^i_B = \text{sgn}(\sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n,t} - P)$.

**Power Allocation Algorithm:** Given target rates $d^i$ and Lagrangian multipliers $\mu^i$, allocate $\mathbf{p}_k$ using (8) and (9) and update $\lambda^{i+1} = [\lambda^i + s^i d^i P^1]$, where $s^i_P$ is a positive scalar stepsize, and $z^i_P = \text{sgn}(\sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n,t} - P)$.

We again propose to use subgradient methods for the iteration of $\mu$ and bisection for the iteration of $\lambda$. As above, we place bounds on each shadow cost, except that the upper bound $\lambda^i_k$ is now given by $\lambda^i_k = \max R_{k,t,F}(U_k(S_{k,t-1} + R_{k,t,F} + R_{k,t,T})/(R_{k,t,F} + R_{k,t,T}) = \Delta R_k^i)$ where $\Delta R_k^i = \arg \max_{R_{k,F}} [U_k(S_{k,t-1} + R_{k,F} + R_{k,T}) - U_k(S_{k,t-1})]/(R_{k,F} + R_{k,T})$.

We call the resulting algorithm, outlined in Table II, Dual Iteration Search Greedy (DIS Greedy). Its performance will be examined in section VI.

**V. RESOURCE ALLOCATION BASED ON THE AVERAGE RATE ACHIEVED SO FAR AND ON EXPECTED FUTURE RATE**

In this section, we design a resource allocation policy that considers both the average rate achieved so far and the expected future rate. To estimate allocation in future time slots in the current time window, we assume that the base station knows the channel gain $|H_{k,n,t}|$ for all users and subcarriers in the current time slot $t$ and the conditional density $f((H_{k,n,t}|\tau)|\tau)$ for future slots $\tau$.

Denote the current channel gain by $\mathbf{H}_t = \{H_{k,n,t}, \forall k, n\}$ and the contribution toward the average rate as of time $t - 1$ by $S_{t-1} = S_{k,t-1}, \forall k$. Denote a resource allocation policy that makes decisions on the basis of the current channel and the accumulated rate by $Q(\mathbf{H}_t, S_{t-1})$.

A causal version of the problem (1) would require the determination of a resource allocation policy $Q$ that maximizes the total user utility. Denote the expected incremental utility from the current slot $t$ to the end of the window achieved under policy $Q$ by $V^Q_{t}(\mathbf{H}_t, S_{t-1})$. The optimal resource allocation policy would allocate the power in time slot $t$, $\mathbf{p}_k$, to maximize the incremental utility earned in the current time slot plus the expected incremental utility earned in future time slots. This can be stated recursively as:

$$V^Q_{t}(\mathbf{H}_t, S_{t-1}) = \max_{\mathbf{p}_k \in \mathbf{A}_t} \left\{ \sum_{k=1}^{K} \Delta U_k, t = E \left( V^Q_{t+1}(\mathbf{H}_{t+1}, S_{t-1}) | H_k, S_{t-1} \right) \right\}$$

s.t. $\sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n,t} \leq P; \ p_{k,n,t} \geq 0 \ \forall k, n$

**TABLE II
GREEDY DUAL ITERATION SEARCH**

| Every slot, initialize $\mu^0, \forall k$ and $\lambda_k = \delta_k \ \forall k$ |
|-------------------------------|
| Repeat                        |
| Repeat                        |
| For current slot $t$, allocate subcarrier and power by (9) and (8) |
| Update $\mu$ using (12)       |
| Until (13)                    |
| If $\lambda^i_k = \lambda_k$  |
| If $R_{k,t,F} = 0$ then $d^i_k = 0$ Else $d^i_k = \Delta R^i_k$ |
| Else calculate $d^i_k$ by $\arg \max_{d_k} [U_k(S_{k,t-1} + d_k) - \lambda^i_k d_k]$ |
| Update $\lambda$ using (10)   |
| Until (11)                    |

However, the solution of this set of equations is computationally intractable. We propose interchanging the utility and the expectation, i.e. maximizing the utility of the expected average rate over the time window instead of maximizing the expected utility. Denote the expected user $k$ rate in time slot $\tau$ under the scheduling policy $Q$ by $E^Q_{t}(\mathbf{H}_t, S_{t-1})$. Then the expected user $k$ rate from time slot $t$ through the end of the time window is given by $R_{k,F} = R_{k,t} + \sum_{\tau=t+1}^{T} E^Q_{t}(\mathbf{H}_t, S_{t-1})$. Denote the incremental utility of the expected average rate over the time window under the scheduling policy $Q$ by $W^Q_{t}(\mathbf{H}_t, S_{t-1})$. This interchange transforms (15) into:

$$W^Q_{t}(\mathbf{H}_t, S_{t-1}) = \max_{\mathbf{p}_k \in \mathbf{A}_t} \left\{ \sum_{k=1}^{K} U_k(S_{k,t-1} + R_{k,F}|T) \right\}$$

s.t. $\sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n,t} \leq P; \ p_{k,n,t} \geq 0 \ \forall k, n$
where the latter constraint places limits on the expected power allocated in future time slots.

A similar approach to that used in the previous sections can be used. The dual problem for (16) leads to a set of equations that describe the set of actions for users, the power allocation module, and the network in each time slot. In addition to Lagrangian multipliers for rate, \( \lambda \), and for power in the current time slot, \( \mu_t \), there are now also Lagrangian multipliers \( \{ \mu_t, \tau \in [t + 1, T]\} \) for power in future time slots. As in the greedy problem considered in the previous section, users maximize incremental utility minus cost, \( U_k(S_{k,t-1} + d_k) - \lambda_k d_k \), and the network determines the Lagrangian multiplier \( \mu_t \) for power in the current time slot. However, now the network must also estimate the Lagrangian multipliers \( \{ \mu_t, \tau \in [t + 1, T]\} \) for power in future time slots. As before, the power allocation module determines the Lagrangian multipliers \( \lambda \) for rate, and determines the powers \( p_t \) using (8) and (9). However, now the Lagrangian multipliers \( \lambda \) are based on the sum of the current rate and the expected future rate. To do this, the power allocation module must estimate the future rate.

The expected future user \( k \) rate is the sum of the expected future user \( k \) rate on each subcarrier:

\[
E(R_{k,t} | H_t) = \sum_{n=1}^{N} E(r_{k,n,t} | H_t) \quad (17)
\]

Denote the probability that under policy \( Q \) subcarrier \( n \) will be assigned to user \( k \) at time slot \( \tau \) as \( Pr\{n \Rightarrow k, \tau\} \). If this probability is known, then the expected future user \( k \) rate on subcarrier \( n \) could be found by:

\[
E(r_{k,n,t} | H_t) = \int_{0}^{+\infty} \text{Log} \left( 1 + p_{k,n,t} \frac{|H_{k,n,t}|^2}{\delta^2 + I} \right) \times f(|H_{k,n,t}|^2) \times d|H_{k,n,t}|^2
\]

since the channel fading is assumed to be a Markov process and the fading on different subcarriers are assumed to be independent to each other. The required probability estimate is given by the following property:

**Property 1:** Let policy \( Q \) be decided by \( \lambda_k \) and \( \mu_t \).

\[
Pr\{n \Rightarrow k, \tau\} = \prod_{k=1, k \neq k}^{K} Pr\{|H_{k,n,t}|^2 > g(|H_{k,n,t}|^2)\}
\]

where

\[
g(|H_{k,n,t}|^2) = (\delta^2 + I)^{T} \frac{B \lambda_k}{T \mu_k} \text{Log} \left( 2 \left( \frac{|H_{k,n,t}|^2}{\delta^2 + I} \right)^{B \lambda_k} \right) \quad (18)
\]

**Proof:** See Appendix.

This probability can also be used to estimate the expected power allocated in future time slots:

\[
E\left( \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n,t} | H_t \right) = \sum_{k=1}^{K} \sum_{n=1}^{N} E(p_{k,n,t} | H_t)
\]

\[
= \sum_{k=1}^{K} \sum_{n=1}^{N} \int_{0}^{+\infty} p_{k,n,t} (|H_{k,n,t}|^2) Pr\{n \Rightarrow k, \tau\} \times f(|H_{k,n,t}|^2) \times d|H_{k,n,t}|^2
\]

(18)

The communication between the users, power allocation module, and network remains as illustrated in Fig. 2, and the allocations can be determined iteratively as follows, where the iteration number is denoted by a superscript \( i \):

**User Algorithm:** Given \( \lambda_k^i, d_k^i = \arg \max d_k [U_k(S_{k,t-1} + d_k) - \lambda_k^i d_k] \).

**Network Algorithm:** Given tentative power and subcarrier allocations \( p_t \in \mathcal{P}_t, \forall t \in [t, T], \mu_{t+1} = \mu_t + s_t^i z_t^i \), where \( s_t^i \) is a positive scalar stepsize, and \( z_t^i = \text{sgn} (\sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n,t} - P) \) when \( \tau = t \) and \( z_t^i = \text{sgn}(E(\sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n,t} | H_t) - P) \) when \( \tau > t \).

**Power Allocation Algorithm:** Given target rates \( d^i \) and Lagrangian multipliers \( \mu_t^i \), \( \forall t \in [t, T] \), allocate \( p_t \) using (8) and (9). Calculate \( E(R_{k,t} | H_t) \) \( \forall t \in [t, T] \) using (17). Update \( \lambda_{k+1}^i = [\lambda_k^i + s_t^i z_t^i]^+, \) where \( s_t^i \) is a positive scalar stepsize, and \( z_t^i \) is any feasible direction that satisfies \( \text{sgn}(\lambda_{k+1}^i - \lambda_k^i) = \text{sgn}(d_k^i - R_{k,F}/T) \).

We again propose to use subgradient methods for the iteration of \( \mu_t \) and bisection for the iteration of \( \lambda_k \). As above, we place bounds on each shadow cost, except that the upper bound \( \lambda_k \) is now given by \( \lambda_k = dU_k(S_{k,t-1} + R_{k,F}/T) / (R_{k,F}/T) \).

We call the resulting algorithm, outlined in Table III, Dual Iteration Search with Prediction (DIS Prediction). Its performance will be examined in section VI.

**Table III**

| Dual Iteration Search with Prediction |
|--------------------------------------|
| Every slot, initialize \( \mu_t^i = \mu_0^i, \forall t \) and \( \lambda_k = \lambda \) \( \forall k \) |
| Repeat |
| Repeat |
| For current slot \( t \), allocate subcarrier and power by (9) and (8) |
| For future slot \( \tau \), calculate \( E_{\mu_t^i} (\sum_{n=1}^{N} E(p_{k,n,t} | H_t)) \) by (18) |
| Update \( \mu_t^i \) using (12) |
| Until (13) |
| If \( \lambda_{k+1}^i = \lambda_k^i \) |
| If \( R_{k,F} = 0 \) then \( d_k^i+1 = 0 \) else \( d_k^i+1 = \Delta R_{k,F}^j \) |
| Else calculate \( d_k^i+1 = \arg \max d_k [U_k(S_{k,t-1} + d_k) - \lambda_k^i d_k] \) |
| Update \( \lambda_k^i \) using (10) |
| Until (11) |

| VI. SIMULATION RESULTS |

In this section, we examine via simulation the performance of the three algorithms. For purposes of the simulation, we set the following parameters: \( B = 20\text{KHz} \), \( N = 100 \), \( I = 0.5 \), \( \delta = 0.5 \), \( T = 4 \) and \( P \) varies from 0.05 to 0.14. We simulate \( K = 10 \) users, with identical sigmoid utility functions given by:

\[
U_k(S_{k,T}) = \begin{cases} a(S_{k,T}/4), & \text{if } S_{k,T} < 20\text{kbps} \\ (S_{k,T}/4 + b)^{1/3}, & \text{else} \end{cases}
\]
where $a = (5/6)^{1/3}/25$ and $b = -25/6$. The utility function has a rate at the maximum average utility at average rate $S_{k,T} = 25$ kbps.

The channel fading $H_{k,n,t} \sim CN(0, 1)$. Thus the channel gain $|H_{k,n,t}|$ has a Rayleigh distribution, and the power gain $|H_{k,n,t}|^2 \sim Exp(1)$ [13]. The channel is Markov, with a correlation coefficient of $|H_{k,n,t}|^2$ in consecutive time slots equal to $\rho = 0.5$. The power gain can thus be generated by:

$$|H_{k,n,t+1}|^2 = \rho |H_{k,n,t}|^2 + \sqrt{1-\rho^2}v$$

where $v \sim Exp(1)$ is independent of $H_{k,n,t}$.

The total utility of all users per time window under each algorithm is illustrated in Fig 3, as a function of the total downlink power $P$. As should be expected from any reasonable resource allocation strategy, the total utility is an increasing concave function of the power $P$ for all three algorithms.

We first consider the performance of the non-causal algorithm denoted DIS NC. Recall that at the beginning of each time window, this algorithm has knowledge of the channel gains of each user in each time slot of the window. However, DIS NC is not optimal, since it solves the dual problem (2) instead of the primal problem (1). Sub-optimality will occur when there is a duality gap. To quantify the amount of sub-optimality, we calculate an upper bound (denoted in Fig 3 as Upperbound) by substituting the solution of DIS NC into (3)\(^4\).

We see that at low powers there is a significant performance gap between DIS NC and the upper bound. This gap is caused by users who under DIS NC are allocated average rates lower than $S_{k,T}$, the rate at the maximum average utility. In general, it is inefficient for users to have average rates below $S_{k,T}$, since the required resources could often be assigned in a manner that raises many of these users above the inefficient convex portion of the utility curve. As the base station power increases, the performance gap between DIS NC and the upper bound decreases, reflecting that all users now obtain rates above $S_{k,T}$ resulting in a duality gap of zero.

We turn next to the causal greedy algorithm denoted DIS Greedy. Recall that this algorithm only takes into account the current user channels on each subcarrier and each user’s average rate achieved so far in the time window. As expected, the performance of DIS NC serves as an upper bound to that of DIS Greedy. When the base station power is low, the performance gap between the two is minimal. This indicates that a greedy approach that maximizes only the incremental utility in the current time slot is sufficient. Consideration of future achievable rate, even if known at the beginning of the time window, does not help. At higher base station powers, however, the performance gap becomes significant. Knowledge of future channels is now helpful.

To understand how such knowledge helps, we focus on one user when $P = 0.13$. The cumulative rate of this user, normalized by $T$, is illustrated in Fig. 4 for each of 4 time slots. In the first time slot, this user’s channel is below average on most subcarriers and worse than almost all other users’ channels. As a consequence, both DIS NC and DIS Greedy assign no system resources. In the second time slot, this user’s channel is above average on most subcarriers, and as a consequence both DIS NC and DIS Greedy do assign system resources. However, DIS NC also knows that the channel of this user in future time slots will not be as good (relative to other users) as it is now, and consequently assigns a higher rate in the second time slot than DIS Greedy.

Finally we turn to the causal algorithm that includes future rate prediction denoted DIS Prediction. Recall that this algorithm takes into account not only the current user channels on each subcarrier and each user’s average rate achieved so far in the time window, but also the expected future rate based on the conditional distribution of the channel in future time slots. As expected, the performance of DIS Prediction exceeds that of DIS Greedy but falls short of that of DIS NC. When the base station power is low, all three algorithms achieve similar performance. At higher base station powers, however, DIS Prediction starts to outpace DIS Greedy by using its prediction of future rates. Returning to Fig. 4, we can see an example of how prediction helps. In time slots 1 and 2, DIS Prediction makes similar decisions as DIS Greedy. In time slot 3, however, the two algorithms differ. The user has a decent channel. In response, DIS Greedy assigns moderate system resources. In contrast, DIS Prediction also takes into account the predicted final utility for this user, which is poor, and consequently chooses not to allocate resources to this user in time slot 3.

\(^4\)Direct solution of (1) is computationally intractable.
Even though prediction of future expected rates results in increased performance, it still does not achieve the performance resulting from exact knowledge of future channels. In the second time slot, DIS NC assigns more resources than DIS Prediction, because DIS NC also knows that the channel of this user in future time slots will not be as good (relative to other users) as predicted. Although the user eventually obtains almost the same average rate (by slot 4) under both algorithms, DIS NC has accomplished this more efficiently, resulting in the ability to increase the utility of other users.

VII. CONCLUSION

We considered power and subcarriers allocation in OFDM cellular systems for semi-elastic applications whose utility is a sigmoid function of the average bit rate over multiple time slots. We showed that a greedy allocation that maximizes incremental utility in the current time slot can be implemented in a distributed fashion by an exchange of price and demand amongst users, the network, and an intermediate power allocation module. Users purchase rate in each time slot by maximizing the difference between utility and cost. The network prices power in each time slot so that the demand for power equals the base station supply. A power allocation module can be used to efficiently rectify the decisions of users and the network. It transforms the price for power into a price per unit rate for each user, so that users modify their desired rate to match available resources. The power allocation module also assigns the power and subcarriers to users in a manner that maximizes total user utility.

We then proposed an improved resource allocation policy that considers both the average rate achieved so far and the future expected rate. We show how future expected rate can be estimated by modeling the probability that a user will be allocated a subcarrier in a future time slot. Users are unaware of this prediction, but it requires additional work by the network and by the power allocation module. The network must estimate future prices for power. The power allocation module must use these price estimates to estimate future rate.

The performance of each algorithm is illustrated using numerical results. When the base station power is low, both algorithms have similar performance, although neither makes optimal decisions for users who can not achieve good average rates. When the base station power is moderate or high, the algorithm that uses the prediction of future rates outperforms the greedy algorithm by taking into account both expected future channels and expected final average rate.

VIII. APPENDIX

Proof of Property 1:

According to (6), user $k$ obtains subcarrier $n$ if and only if the following condition is satisfied for all other users $k \neq k$:

$$\frac{\lambda_k r_{k,n,\tau}}{T} - \mu_T p_{k,n,\tau} > \frac{\lambda_{\hat{k}} r_{k,n,\tau}}{T} - \mu_T p_{k,n,\tau}$$

Substituting for $p_{k,n,\tau}$ from (8), this occurs if and only if:

$$\frac{\lambda_k r_{k,n,\tau}}{T} - \mu_T (2r_{k,n,\tau}/B - 1) > \frac{\lambda_{\hat{k}} r_{k,n,\tau}}{T} - \mu_T (2r_{k,n,\tau}/B - 1)$$

Substituting $H^2_{k,n,\tau}/(\delta^2 + 1) = 2r_{k,n,\tau}/B \mu_T \ln 2 / (B\lambda_k/T)$ into the above equation and rearranging terms:

$$\lambda_k > \frac{r_{k,n,\tau} - B}{B ln 2} \left( 1 - \frac{1}{2\lambda_k r_{k,n,\tau}/B} \right)$$

(19)

We expand one term using a Taylor expansion:

$$\frac{1}{2r_{k,n,\tau}/B} \approx 1 + \ln 2 \left( -\frac{r_{k,n,\tau}}{B} \right) + \frac{(\ln 2)^2}{2} \left( -\frac{r_{k,n,\tau}}{B} \right)^2$$

(20)

Substituting (20) into equation (19), user $k$ obtains subcarrier $n$ if and only if $\lambda_k/\lambda_{\hat{k}} > r_{k,n,\tau}/r_{k,n,\tau}$ for all $\forall k \neq k$, i.e. if and only if $\forall \hat{k} \neq k$.

$$|H_{k,n,\tau}|^2 > (\delta^2 + I) T \mu_T \ln 2 / (B\lambda_k)$$

(21)

Property 1 directly follows.

IX. ACKNOWLEDGEMENT

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