INVESTIGATING THE PROCESS OF SHRINKAGE DEPRESSION FORMATION AT THE COMBINED RADIAL-BACKWARD EXTRUSION OF PARTS WITH A FLANGE

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1. Introduction

The machining processes of metals, both hot and cold, are the preparatory base of machine engineering and metallurgy. Given the ability to form a metal at high performance and low waste, thereby improving its mechanical properties, the importance of pressure treatment in modern industry is enormous [1]. This increases the role of effective resource-saving methods for pressure metal treatment such as cold volumetric stamping [2]. Combining traditional schemes of longitudinal extrusion by radial or lateral extrusion methods opens up the possibility of manufacturing parts with a more complex...
configuration over fewer technological transitions [3, 4]. Combining the reverse and radial extrusion schemes in the manufacture of hollow parts with a flange improves technological capabilities of the processes of precision volumetric stamping by reducing the energy and labor intensity of production and increasing a metal utilization factor. However, along with these advantages, there are a series of problems, primarily related to the issues of plasticity and hardening of metal under conditions of the complex stressed-deformed state [5, 6]. In addition, despite the high efficiency of the combined extrusion processes, examples of their practical implementation are limited.

The range of products manufactured by machine building and instrumentation enterprises contains a large number of hollow parts with flanges and protrusions of various shapes [7]. This should encourage research into new effective processes for obtaining this type of components, such as combined extrusion. However, when making them by cold extrusion, under certain conditions there may occur defect formation (failure to maintain the shape and size of a finished product, cracks and ruptures of protrusions, shrinkage depression, etc.). The defect formation processes are a deterrent to the active implementation of these processes in industry. Therefore, they require a thorough study of the complex of geometric and technological factors that contribute to the emergence (or hampering) of defects in the process of deformation. In this case, special attention should be paid to building full-fledged mathematical models for calculating force parameters and shape formation that would make it possible to predict defect formation in the processes of combined extrusion.

2. Literature review and problem statement

Currently, the active implementation of combined extrusion processes in industry is constrained by the lack of detailed recommendations on determining the force parameters of the process, predicting shape change and defect formation [8–17].

A series of papers [8–13] are limited to studies of force parameters and phased shape change in the processes of combined extrusion. Work [8] analyzes the effect of geometric parameters and friction conditions in the process of direct-reverse-radial extrusion based on a finite element method employing the software ABAQUS (ABAQUS Inc., USA). Papers [9, 10] report studying the force parameters and shape change of hollow parts in the process of combined extrusion, which are limited to modeling in Deform-2D (Scientific Forming Technologies Corporation (SFTC), USA). The combined extrusion processes were investigated in studies [11, 12] by the methods of finite elements and the upper estimation (using rigid triangular elements). A comparative analysis of theoretically and experimentally obtained data on the force parameters of the process and on the shape change in workpieces was carried out. Paper [13] analyzed the process of combined radial-backward extrusion in a conical matrix for successive stages of the process. The study relates to the calculations of force parameters of the process, as well as a shape change; it is limited in application. The theoretical analysis was carried out using the method of power balance taking into consideration the hardening of a metal; experimental testing of the results obtained was performed. However, the constructed mathematical models do not make it possible to analyze patterns in the course of the deformation process with defect formation (deviation in the shape of a finished product, formation of shrinkage depression, cracks, etc.). Thus, the issues related to determining the limit of deformity and defect formation in the processes of combined extrusion remained unresolved.

The issues of assessing the unevenness of the deformed state, the deformity of a metal, and the limit of shape change in workpieces at cold extrusion were addressed in papers [14–16]. Study [14] calculated, based on the upper estimate method, the magnitudes of the reduced pressure and accumulated deformation at cold lateral extrusion. The characteristic deformation zones and the presence of a dead zone, as well as the effect of friction conditions on the course of the process, have been defined. Paper [15] estimated the limit of shape change in workpieces made from different materials in the processes of cold volumetric deformation. Study [16] calculated plasticity resource according to various criteria for the process of combined radial-direct extrusion. The derived pathways of material particle deformation make it possible to assess the resource of plasticity and the technological possibilities of the combined radial-direct extrusion process for various materials whose plasticity diagrams are known. Study [16] also examined the stressed-deformed state in a deformation site at the combined extrusion of parts with a flange. Based on the study conducted, the resource of plasticity exhaustion in the flange zone has been evaluated for various materials. Thus, papers [15, 16] addressed the stressed-deformed state, estimated the resource of plasticity and the technological capabilities of the processes of combined extrusion of parts made from various materials.

In this case, analysis of the technological capabilities of the combined extrusion processes is considered in the aspect of assessing the limit of shape change and possible defects due to metal destruction and does not imply studying defect formation of the shrinkage depression type and failure to maintain the shape of parts.

The analysis of defect formation in the processes of direct, radial, and radial-backward extrusion was reported in paper [17]. Based on the simulation data from QForm2D/3D («KwantorForm,» Russia), the authors proposed a diagram of regions that are critical in terms of defect formation in the form of shrinkage depression in a bottom part at the combined extrusion of hollow parts with a flange. However, these data are limited to the case of variation in the flange height, ranging from 1 mm to 6 mm, the thickness of the cup’s wall, ranging from 3.5 mm to 14.5 mm, with a matrix radius of 22.5 mm. Paper [17] also considers the possibilities to predict the process of defect formation in the form of shrinkage depression in the bottom part of a cup with a flange using an energy method of the upper assessment. The proposed estimation scheme for a flat-deformed process of combined radial-backward extrusion makes it possible to determine the critical value for the thickness of the cup’s bottom $H_{c}$ in terms of the emergence of shrinkage depression. However, a given estimation scheme is limited by the condition $H_{c}>h_{c}$, where $h_{c}$ is the height of the extruded flange, which significantly narrows the scope of its application. In addition, the authors failed to investigate the influence of friction conditions and other technological solutions as control factors for correcting the emergence of a defect in the form of shrinkage depression.

Thus, one can state that there are certain under-investigated issues related to the analysis of combined extrusion processes in terms of possible defect formation. It is critically needed to construct the estimation schemes for predicting
shape formation and defect occurrence, as well as to develop technological means that could ensure effective control over the flow of a metal. That would improve the efficiency of the combined extrusion processes and contribute to their wide implementation in industry.

3. The aim and objectives of the study

The aim of this study is to construct mathematical models for the process of combined extrusion of hollow parts with a flange that would make it possible to predict defect formation in the form of shrinkage depression. That could provide an opportunity to identify patterns in the process of combined radial-backward extrusion with the emergence of a defect in the form of shrinkage depression in the bottom part of a cup.

To accomplish the aim, the following tasks have been set:
- to develop an estimation scheme of the process of combined extrusion of hollow parts with a flange that would make it possible to predict the emergence of shrinkage depression in the bottom part of a cup throughout the entire process of deformation;
- to establish patterns in the course of the process of combined radial-backward extrusion with the emergence of shrinkage depression in the bottom part of a cup, as well as without defect formation, and to investigate the effect of friction conditions on the process of shrinkage depression formation;
- to experimentally test the feasibility of using the devised estimation scheme to predict defect formation in the form of shrinkage depression in the bottom part of a cup.

4. Theoretical analysis of the processes of combined extrusion of parts with a flange with the formation of shrinkage depression

4.1. Construction of an estimation scheme for the process of combined radial-backward extrusion of parts with a flange with the formation of shrinkage depression

A generally recognized defect, characteristic of cold extrusion processes, which significantly reduces the quality of parts and the technological capabilities of the process, is the shape deviation in the form of shrinkage depression. Depending on the extrusion scheme, defects of this type occur both at different stages of the process and in different zones of the part that is being shaped (Fig. 1). A combined extrusion with multiple degrees of freedom of the flow at different ratios between the geometric and technological parameters of the same process may result in the emergence of shrinkage depression in different zones of the workpiece being shaped (Fig. 1, a, b). For example, during the combined radial-backward extrusion of cups with a flange, at the final stage of the process, with a predominantly reverse flow of a metal, there is a shrinkage depression in the bottom part. The predominantly radial flow is characterized by the emergence of shrinkage depression at the inner wall of the cup, which is in contact with a punch. In addition, at the final stage of the process the defect formation is possible of this type in the axial part of a workpiece in the form of metal waste from the end of the punch (Fig. 1, b, c).

One of the efficient theoretical methods for calculating cold extrusion processes, including taking into consideration possible defect formation, is the energy method [8]. Currently, two main modifications of the energy method are actively used: the energy method or the method of power balance (axisymmetric problems) and the upper assessment method (flat problems). To calculate the axisymmetric processes, the deformable volume of a workpiece is split into axisymmetric kinematic modules with the appropriate kinematically possible velocity fields (KPVF). In this case, the modules are considered in the cylindrical system of coordinates r, θ, z, taking into consideration the axial symmetry and the equality to zero of the circular component of velocity \( V_0 = 0 \). After selecting the appropriate set of KPVFs for the process, the main equation of the energy balance in powers is built:

\[
p F v_0 = \sum d_i + \sum n_{i,j} + \sum n_{i,j} \cdot \sigma_i, \quad (1)
\]

where \( N_i \) is the power of forces of the plastic deformation of module \( i; N_{i,j} \) is the power of cut-off forces between the neighboring modules \( i \) and \( j; N_{i,j} \cdot \sigma_i \) is the power of friction forces, generated at the surface of the contact between module \( j \) and tool \( n; V_0 \) is the speed of an active deforming tool; \( p \) is the pressure of deformation; \( F \) is the area of the cross-section of the active deforming tool.

\[
\bar{p} = \sum d_i + \sum n_{i,j} + \sum n_{i,j} \cdot \sigma_i, 
\]

where \( \sigma_i \) is the fluidity strain, MPa.

The resulting magnitude of the reduced pressure makes it possible to calculate, regardless of the brand of a material, the magnitude of deformation pressure \( p \) and the force of deformation \( P \):

\[
p = \bar{p} \cdot \bar{\sigma}, \quad P = p \cdot F. \quad (3)
\]

In the development of new estimation schemes according to the modular approach, it is possible to use, for known kinematic modules, the ready components of reduced pressures of deformation, cutting, and friction [19]. In this case, in order to qualitatively assess the prediction of defect formation in the form of a shrinkage depression at combined extrusion, it is necessary to pay special attention to the character of change in the optimal value for the kinematic parameters in the course of the process.

The previously devised [20] estimation scheme (Fig. 2, the process of deformation at \( H > 0 \)) makes it possible to investigate the force characteristics of the process of combined radial-backward extrusion, shape change in a part, and defect formation in the form of a shrinkage depression in the cup’s bottom part.

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**Fig. 1. Schematics of defect formation, the type of shrinkage depression:**
- **a** – radial-backward extrusion; **b** – reverse-direct extrusion; **c** – radial-direct extrusion

By dividing both parts of the derived equality by a multiplier \( F V_i \sigma_i \), we obtain a formula for the reduced pressure \( \bar{p} \):

\[
\bar{p} = \sum N_i + \sum n_{i,j} + \sum n_{i,j} \cdot \sigma_i, 
\]

where \( \sigma_i \) is the fluidity strain, MPa.

The resulting magnitude of the reduced pressure makes it possible to calculate, regardless of the brand of a material, the magnitude of deformation pressure \( p \) and the force of deformation \( P \):

\[
p = \bar{p} \cdot \bar{\sigma}, \quad P = p \cdot F. \quad (3)
\]
It has been established that the shrinkage depression forms at the final stage of the extrusion process, when the flow of a metal in the radial direction is almost non-existent, and the main part of the metal of the workpiece flows vertically upwards. A given moment is matched with the presence of a minimum in the \( W \) parameter depending on the current thickness of the cup’s bottom \( H_{\text{bottom}} \). In this case, the process, which occurs without defect formation, is characterized by the absence of a minimum point of magnitude \( W \) (a given magnitude decreases during the entire process of deformation).

The kinematic and geometric parameters applied for the improved estimation scheme (Fig. 2) are: \( V_0 \) is the speed of an active deforming tool; \( V_1 \) is the speed at the boundaries of kinematic modules 1 and 2; \( W \) is the rate of metal outflow in the vertical direction that forms the cup’s wall; \( V \) is the rate of metal outflow in the radial direction that forms the flange zone; \( R_1 \) and \( R_2 \) are the punch and matrix radii, respectively; \( h_1 \) is the flange thickness; \( H \) is the magnitude that characterizes the inclination angle of the module 2 boundary; \( l_1 \) is the increment in the cup’s wall; \( l_2 \) is the flange increment.

Under the accepted designations of the improved estimation scheme, the process of deformation proceeds in successive three stages. Stage 1 corresponds to the set of kinematic modules 1, 2, 3, 4. In this case, \( H_{\text{bottom}}=H+h_1 \) and module 2 is trapezoid. At \( H=0 \) (\( H_{\text{bottom}}=h_1 \)), there is a degeneration of the trapezoid module 2 into a rectangular module (this stage II corresponds to the dotted boundary line in Fig. 2). The process of deformation then proceeds to the stage at which the height of module 1 becomes less than the thickness of the flange (stage III, the set of kinematic modules 1, 2, 3, 4). This stage corresponds to ratios \( H_{\text{bottom}}<h_1 \) with a change in the inclination of the boundary of flow divide of trapezoid module 2. In this case, the calculations made earlier corresponded only to the first stage of the deformation process [20]. Consideration of stages II and III could significantly expand the possibilities for studying the emergence of defects in the form of a shrinkage depression in the cup’s bottom part for the final stage of the deformation process at \( 0<H_{\text{bottom}}<h_1 \).

The main changes in the generalized estimation scheme, which takes into consideration the entire process of deformation up to \( H_{\text{bottom}} \rightarrow 0 \), relate to changes in the configuration of module 2. We shall consider KPVF inside a given module according to the selected stages in a deformation process (Table 1).

### Table 1

| Stage | Module shape | KPVF |
|-------|--------------|------|
| 1 I   | 11           | \( v_{r_1} = \frac{W+a v_{r_1}}{r} \) |
|       | 11           | \( v_{z} = \frac{W+a v_{z}}{r} \) |
|       | 11           | \( v_{r_2} = \frac{M+W(R_2^2-r^2)}{2r} \) |
|       | 11           | \( z = a(r+0) \) |
|       | 11           | \( a = \frac{H}{R_a-R_2} \) |
|       | 11           | \( b = l_1-h_1 \) |

Thus, the calculation of force parameters of the process of deformation comes down to the sequentially passage of all three stages. Using the calculations for stage I of deformation at \( H_{\text{bottom}}<h_1 \), we obtain, for the geometric parameters related to \( R_1 \) and kinematic, related to \( V_0 \), the magnitude for the reduced pressure:

\[
P_0 = \frac{1}{\sqrt{3}} \left( \frac{2}{3} \left( \frac{M+\bar{W}(1-\frac{R_1^2}{R_2^2})}{R_2} \right) + \frac{2}{3} \left( \frac{H}{R_2} \right) + \frac{4 \mu_2}{3} \left( \frac{M+\bar{W}(1-\frac{R_1^2}{R_2^2})}{R_2} \right) \right) + \frac{1}{\sqrt{3}} \left( \frac{1}{2} \left( \frac{M+\bar{W}(1-\frac{R_1^2}{R_2^2})}{R_2} \right) \right) + \frac{1}{\sqrt{3}} \left( \frac{1}{2} \left( \frac{M+\bar{W}(1-\frac{R_1^2}{R_2^2})}{R_2} \right) \right) + \frac{1}{\sqrt{3}} \left( \frac{1}{2} \left( \frac{M+\bar{W}(1-\frac{R_1^2}{R_2^2})}{R_2} \right) \right)
\]
where

\[ \dot{e}_{z2} = \frac{\partial v_{z2}}{\partial z} = W + a v_{z2} \]

is the speed of relative linear deformation of module 2;

\[ U_1 = \frac{n_1}{n_1} \rho_{r2} z(r) dr, \quad U_2 = \frac{n_2}{n_2} \rho_{z2} z(r) dr, \quad U_3 = \frac{n_3}{n_3} \rho_{z2} z(r) dr \]

are the magnitudes that determine the highest estimate of the reduced deformation pressure in zone 2 [20].

For the case of the degeneration of trapezoidal module 2 into rectangular (at \( H_{bottom} = h_1 \) or \( H = 0 \)), the point estimate of the reduced pressure in relative parameters, taking into consideration the history of deformation, takes the form:

\[
\bar{p}_2 = \left\{ \begin{array}{l}
1 + \frac{2}{\sqrt{3}} \left[ \left(1 + W \right) \left( R_1 - 1 \right) \right] + \frac{1}{\sqrt{3}} \frac{R_1}{R_2} + \frac{1}{3} \frac{R_1}{(1 + W)} + W \left( \frac{R_1}{R_2} - 1 \right) - \frac{2}{\sqrt{3}} \frac{R_1}{R_2} + \frac{1}{3} \frac{R_1}{(1 + W)} - \frac{2}{\sqrt{3}} \frac{R_1}{R_2} - \frac{1}{3} \frac{R_1}{(1 + W)} + \frac{4\mu_1}{\sqrt{3}} \frac{R_1}{R_2} - \frac{2}{\sqrt{3}} \frac{R_1}{R_2} + \frac{1}{3} \frac{R_1}{(1 + W)} \end{array} \right. \]

In this case, by assuming the magnitude \( H_1 = h_1 + H \) in the KPVF of module 2 for stage III where \( H \), in the course of the process, accepts values from 0 (not including) to \( -h_1 \), one can completely duplicate the magnitude of reduced pressure \( \bar{p}_2 \).

Thus, we finally obtain, taking into consideration the degradation of module 2 at \( H = 0 \) and the history of deformation (relative to the stroke), the generalized magnitude of the reduced pressure:

\[
\bar{p} = \begin{cases} 
\bar{p}_1 & \text{at } \bar{p}_X \in [0, R_0 - \bar{R}_1 \cup (R_0 - \bar{R}_1, \bar{R}_L), \\
\bar{p}_2 & \text{at } \bar{p}_X = \bar{R}_L - \bar{R}_1.
\end{cases}
\]

where \( \bar{R}_X \) is the relative stroke; \( \bar{R}_L \) is the relative initial height of a workpiece.

4.2. Analysis of patterns in the course of the process with the emergence of shrinkage depression in the cup’s bottom part and without defect formation

Let us consider the effect of change in the friction conditions on the cup’s wall and punch \( \mu_2 \), responsible for the reverse flow, and \( \mu_1 \), for the outflow of a metal that forms a flange zone in the radial direction, on the possibility of forming shrinkage depression (Fig. 3). At equal values for the magnitudes \( \mu_1 = \mu_2 \), there is a slight shift in the critical course of the punch corresponding to the beginning of the formation of a given defect. The shift of the critical stroke of the punch does not exceed 1 mm for friction conditions \( \mu_1 = \mu_2 = 0.3 \) (curve 1, Fig. 3) and \( \mu_1 = \mu_2 = 0.08 \) (curve 2, Fig. 3). In addition, the relative speed of metal outflow in the vertical direction \( W \), also changes slightly, while at stage III there is actually a transition from the combined extrusion to the reverse extrusion. The increase in the friction factor in the bottom part \( \mu_1 = 0.3 \) at a fixed value \( \mu_2 = 0 \) results in a more significant shift of the critical magnitude \( \Delta H_x \) downwards to 8.5 mm (curve 3, Fig. 3). This leads to the possibility of an earlier moment in the formation of shrinkage depression, which is a negative factor. In this case, the increase in the friction factor on the cup’s wall punch \( \mu_2 = 0.3 \) at a minimum value of \( \mu_1 = 0 \) makes it possible to avoid the emergence of a defect at all stages of the deformation process. This is due to the absence of a minimum point, as the magnitude \( W \) decreases throughout the entire deformation process; it is a favorable factor (curve 4, Fig. 3). Therefore, one can obtain a period of critical values for stroke \( \Delta H_x \), at which it is likely that there will be a shrinkage depression for these geometric parameters of the process, considering the extreme values of friction factors characteristic of cold extrusion.

For the process parameters \( R_1 = 10.5 \) mm, \( R_2 = 22.5 \) mm, \( h_1 = 3 \) mm, and various contact friction conditions, we performed simulation employing QForm2D/3D (Fig. 4). We have analyzed patterns in the staged shape change of a part made from lead alloy, in terms of influence of these technological factors on the critical values for the thickness of the cup’s bottom in the formation of such a defect. The results, based on QForm2D/3D simulation, confirm the significant effect of friction conditions on the possibility of a defect in the form of shrinkage depression and are consistent with the results from theoretical studies based on the estimation scheme.

Thus, the conditions of contact friction can be attributed to significant control factors that affect the possibility of defect formation in the process of combined radial-backward extrusion.
5. Analysis of the feasibility of using the estimation scheme to predict defect formation in the form of shrinkage depression in the cup's bottom part

5.1. Experimental study of the process of combined radial-backward extrusion with the formation of shrinkage depression

When implementing cold extrusion processes, one can observe, under certain conditions, the phenomenon of defect formation. For the process of combined radial-backward extrusion, characteristic are the following types of defects:

- failure to maintain the shape and size of a finished product, including the non-fit flange;
- rupture of a flange (evolution occurs in the radial direction to the axis of the extruded part);
- crack at the border of contact between a cup’s wall and the bottom part, leading to the detachment of the latter;
- shrinkage depression in the cup’s bottom part and on the inside of the formed wall of the cup.

Defects of the first type (Fig. 5, a) in the form of curvature of the lateral end occur due to the insufficiently fixed centered punch (co-axiality) in the deformation process, which also contributes to the formation of different thickness of the wall of the resulting cup. The main defects also include the characteristic shape of a flange in the form of a «toe of the boot». Defect formation in the form of flange rupture is typically observed for low-plastic materials or at low flange height for more plastic materials (AA1135, Fig. 5, b). This is facilitated by a rather unfavorable pattern of the stressed state, because the zone of flange formation is dominated by circumferential tensile stresses, and at the peripheral part the stressed state is close to linear stretching.

The extrusion based on the scheme of combined radial-reverse extrusion at the intensive flow of a metal upwards (predominant reverse extrusion at the final stage of the process) may lead to the emergence of shrinkage depression at the cup’s bottom (Fig. 5, c, d). It was experimentally established that the main difference of the process of deformation with defect formation from the process that occurs without the emergence of shrinkage depression is a sharp increase in the magnitude $P$. When studying samples made from lead alloy, the data were obtained on the strength of deformation at the matrix radius $R_2=22.5$ mm and the punch radius $R_1=10.5$ mm. The process of extrusion without the formation of shrinkage depression at the flange height $h_1=4.5$ mm is characterized by a smooth saturation of the magnitude $P$ at the final stage of the deformation process (Fig. 6, point data (2)). In the process of extrusion with the formation of a defect at $H_0=20.5$ mm and the flange height $h_1=3$ mm, the final stage of the process is characterized by a sharp jump in the force of deformation, starting from $Hx=14$ mm (Fig. 6, point data (1)). This occurs at the stage of deformation, corresponding to the moment of the defect’s birth in the form of shrinkage depression with a further increase in the effort of extrusion during the process. This feature in the character of change in the energy-force parameters of the process is associated with a sharp increase in the rate of outflow of a metal in the vertical direction, that is almost with the transition to the scheme with one degree of freedom of flow (radial extrusion is virtually non-existent).
5.2. Comparative analysis of theoretical studies into predicting the formation of shrinkage depression with experimental data

An important stage in the design of the technological process of combined radial-backward extrusion is an analysis of the possibility of a defect in the form of shrinkage depression in the cup's bottom part. In this case, it is necessary to refine the character of change in the relative outflow rate $\bar{W}$ and to determine the critical value of the thickness of the cup's bottom, corresponding to the beginning of shrinkage depression formation. Theoretical study of the character of change in the optimal value $\bar{W}$ was conducted up to $H_{\text{bottom}} \rightarrow 0$.

We have performed a comparative analysis of the experimentally obtained part made from lead alloy with the resulting shrinkage depression and simulation employing QForm2D/3D at $R_1 = 10.5$ mm, $R_2 = 22.5$ mm, $h_1 = 4.5$ mm and the bottom thickness $H_{\text{bottom}} = 2.5$ mm (Fig. 7). The resulting patterns of the part's shape formation are identical. In addition, for these geometric parameters of the process, a change in the relative speed of metal outflow in the vertical direction during the process (Fig. 8) has been investigated. At $H_x = 22.5$ mm ($H_{\text{bottom}} = 4.5$ mm), there is a transition from stage I to stage III; in this case, in close values there is a minimum of the magnitude $\bar{W}$. This indicates the possible beginning of the formation of shrinkage depression, and at $H_x = 24.5$ mm there is actually a transition to the process of reverse extrusion. The onset of shrinkage depression formation according to the QForm2D/3D simulation and the experimental data corresponds to $H_{\text{bottom}} = 2.6$ mm ($H_x = 23.4$ mm). According to theoretical calculations, the onset of shrinkage depression formation is marked by a vertical solid line (T), and, according to experimental data, by a dotted vertical line (E) (Fig. 8). Thus, considering as a criterion value of the minimum point $\bar{W}$ is feasible and produces a slight overestimation of the bottom thickness, corresponding to the start of defect formation. The character of change in the increment of the flange zone during the process of deformation with the formation of shrinkage depression also confirms the patterns in the course of the process. Thus, starting from $H_x = 24.5$ mm and to the end of the deformation process, there is no increase in the flange zone, that is there is no radial extrusion (Fig. 9).

Since friction conditions can be attributed to factors that significantly affect the duration of shrinkage depression formation, the following diagram can be proposed as a preliminary estimate of the defect formation (Fig. 10). A given diagram was derived from theoretical calculations of the moment of shrinkage depression formation, corresponding to the point of minimum in the magnitude $\bar{W}$, when the relative flange thickness $h_1/R_2$ ranges up to 0.35, the relative thickness of the cup's wall $S/R_2$ up to 0.75 at the border sets $\mu_1 = 0$, $\mu_2 = 0.3$ и $\mu_3 = 0.3$, $\mu_4 = 0$. Let us describe the algorithm of using this diagram.

Initially, one needs to find the relative values for the flange height and the thickness of the cup's wall $S = R_2 - R_1$ by marking them in the coordinate system $(h_1/R_2, S/R_2)$ on the right-hand side of the diagram. Next, one needs to proceed, considering the relative value for the cup's bottom thickness $t = H_{\text{bottom}}$ to the left-hand side of the diagram by marking the final position of point $(t/R_2, S/R_2)$. In this case, if the point $(t/R_2, S/R_2)$ is located above the top broken line, separating the region of defect formation (Fig. 10, (I)), the shrinkage depression for these parameters of the process does occur,
below the bottom broken line – does not (Fig. 10, (2)). In addition, in the band of critical values (the region of the plane between broken lines), it is possible to control the emergence of shrinkage depression by changing friction conditions within the range of $\mu_1=0+0.3$, $\mu_2=0+0.3$. In this case, increasing the height of the flange while maintaining the rest of the extrusion process parameters contributes to a later onset or the absence of shrinkage depression, and thus expands the region for obtaining parts without defect formation. Increasing the thickness of the cup's wall while maintaining the rest of the extrusion process parameters, on the contrary, contributes to the earlier onset of shrinkage depression and is a negative factor.

The diagram has a point (1) corresponding to the process of deforming a workpiece made from lead alloy at $h_1=4.5$ mm, $S=12$ mm, $t=3.6$ mm with the formation of shrinkage depression (experimentally obtained data). This point falls into the region of the diagram above the top broken line, that is in the region of shrinkage depression formation, obtained from theoretical data. The point (2) corresponds to the process of deforming a workpiece made from lead alloy at $h_1=3$ mm, $S=8.5$ mm, $t=6.5$ mm without the formation of shrinkage depression. This point falls into the region of the diagram below the bottom broken line, that is the region without the formation of shrinkage depression, derived from theoretical data.

Thus, a comparative analysis of the experimentally obtained data for $R_2=22.5$ mm, $S=12$ mm and $S=8.5$ mm confirmed the possibility of using the built diagram both for the process with the formation of shrinkage depression and without defect formation. The proposed diagram (Fig. 10) for the chosen range of variations in geometric and technological parameters is informative enough and easy to use. Based on theoretical data, it is possible to build diagrams similar to this one, for other ratios of the combined radial-backward extrusion process.

6. Discussion of results of studying the processes of combined extrusion based on the method of kinematic modules

Our study has confirmed the effectiveness of the devised generalized estimation scheme for the process of combined radial-backward extrusion. Three stages in the process progress have been identified and the magnitude for the reduced pressure has been obtained to calculate the deformation process up to $H_{bottom} \rightarrow 0$ (considering the final stage of deformation at $H_{bottom}<h_1$). A change in the character of the relative outflow rate in the vertical direction has been investigated for processes with the formation of shrinkage depression and without a given defect. It is feasible to consider, as a criterion magnitude (corresponding to the beginning of defect formation), the point of minimum $W$.

We have analyzed the effect of change in friction conditions on the cup's wall punch $\mu_2$ responsible for the reverse flow, and $\mu_1$ – for the outflow of a metal forming a flange zone in the radial direction, on the possibility of shrinkage depression formation. It is noted that at equal values for magnitudes $\mu_1=\mu_2$ there is a slight shift in the critical stroke of the punch corresponding to the beginning of the formation of this defect. The boundary friction conditions (maximum value $\mu_1$ at minimum value $\mu_2$, and vice versa) have a much greater impact on the duration of the shrinkage depression formation.

The resulting solutions, derived in the form of a diagram, make it possible to conduct a preliminary assessment of obtaining a part (regardless of the material) without the formation of shrinkage depressions for the following ratios:

- relative thickness of the flange up to 0.35;
- relative thickness of the cup's wall up to 0.75;
- ensuring friction conditions within the range $\mu_1=0+0.3$, $\mu_2=0+0.3$.

The QForm2D/3D simulation and experimental data confirm the feasibility of using the proposed generalized estimation scheme, which makes it possible to predict shrinkage depression formation throughout the entire process of deformation up to $H_{bottom} \rightarrow 0$. The factors that reduce the likelihood of shrinkage depression formation or contribute to its later onset include:

- increasing the thickness of a flange (for example, by introducing a chamfer);
- ensuring the highest friction on the cup’s wall and punch at minimal friction in the bottom and flange zone.

The aim of further research is to extend the possibilities of the energy method for the development of estimation schemes for combined extrusion processes, which would make it possible to effectively predict defect formation in different zones.

7. Conclusions

1. A generalized estimation scheme of the process of combined radial-backward extrusion of hollow parts with a flange has been developed, taking into consideration all stages of deformation. That has made it possible to predict the emergence of shrinkage depression in the cup's bottom part throughout the entire process of deformation.

2. We have investigated a change in the character of relative outflow rate in the vertical direction for processes with the formation of shrinkage depression and without defect formation. It has been proposed to consider, as a criterion magnitude (corresponding to the beginning of defect formation),

![Diagram of dimensions of the parts that can be obtained by combined extrusion without the formation of shrinkage depressions:](image-url)

**Fig. 10.** Diagram of dimensions of the parts that can be obtained by combined extrusion without the formation of shrinkage depressions:

- 1 – experimental data for $R_2=22.5$ mm, $h_1=4.5$ mm, $S=12$ mm, $t=3.6$ mm;
- 2 – experimental data for $R_2=22.5$ mm, $h_1=3$ mm, $S=8.5$ mm, $t=6.5$ mm.

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The diagram has a point (1) corresponding to the process of deforming a workpiece made from lead alloy at $h_1=4.5$ mm, $S=12$ mm, $t=3.6$ mm with the formation of shrinkage depression (experimentally obtained data). This point falls into the region of the diagram above the top broken line, that is in the region of shrinkage depression formation, obtained from theoretical data. The point (2) corresponds to the process of deforming a workpiece made from lead alloy at $h_1=3$ mm, $S=8.5$ mm, $t=6.5$ mm without the formation of shrinkage depression. This point falls into the region of the diagram below the bottom broken line, that is the region without the formation of shrinkage depression, derived from theoretical data.

Thus, a comparative analysis of the experimentally obtained data for $R_2=22.5$ mm, $S=12$ mm and $S=8.5$ mm confirmed the possibility of using the built diagram both for the process with the formation of shrinkage depression and without defect formation. The proposed diagram (Fig. 10) for the chosen range of variations in geometric and technological parameters is informative enough and easy to use. Based on theoretical data, it is possible to build diagrams similar to this one, for other ratios of the combined radial-backward extrusion process.
the point of minimum $\bar{W}$. The effect of different friction conditions on the process of shrinkage depression emergence has been analyzed. It has been established that the greatest influence on the time of shrinkage depression emergence is exerted by the boundary friction conditions (maximum value $\mu_1$ at minimum value $\mu_2$ and vice versa). A later onset (or the absence) of shrinkage depression is contributed to by ensuring the highest friction on the wall of the cup and punch at minimal friction in the bottom and flange zone.

3. The obtained experimental data and the QForm2D/3D simulation confirm the feasibility of using the generalized estimation scheme to predict the formation of shrinkage depression throughout the entire deformation process up to $H_{bottom} \to 0$. Using the proposed diagram is recommended for the relative flange thickness $h_1/R_2$ within the range up to 0.35, the relative thickness of the cup’s wall $S/R_2$ up to 0.75 at the border sets $\mu_1 = 0$, $\mu_2 = 0.3$ and $\mu_1 = 0.3$, $\mu_2 = 0$.

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