Abstract

The effect of the Gauss–Bonnet term on the $SU(2)$ non–Abelian regular stringy sphaleron solutions is studied within the non–perturbative treatment. It is found that the existence of regular solutions depends crucially on the value of the numerical factor $\beta$ in front of the Gauss–Bonnet term in the four–dimensional effective action. Numerical solutions are constructed in the $N = 1, 2, 3$ cases for different $\beta$ below certain critical values $\beta_N$ which decrease with growing $N$ ($N$ being the number of nodes of the Yang–Mills function). It is proved that for any static spherically symmetric asymptotically flat regular solution the ADM mass is exactly equal to the dilaton charge. No solutions were found for $\beta$ above critical values, in particular, for $\beta = 1$. 

Stringy Sphalerons and Gauss–Bonnet Term

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Since the Bartnik and McKinnon’s discovery [1] of the regular particle–like solutions to the coupled system of the Einstein–Yang–Mills (EYM) equations there was a growing interest in revealing their possible physical significance. It was shown that these solutions could play at the ultramicroscopic distances a role analogous to that of electroweak sphalerons [2]. Sphaleron interpretation is supported by the existence of the odd–parity YM negative modes [3] (apart from the previously known even–parity ones [4]), as well as the fermion zero modes and the level–crossing phenomenon [5]. Natural question arises whether the EYM sphalerons survive in more sophisticated field models suggested by the theory of superstrings. It was shown recently that regular sphaleron solutions exist within the context of the Einstein–Yang–Mills–Dilaton (EYMD) theory [6], [7], [8], [10], [9]. Remarkably, they have a dilaton charge exactly equal to the ADM mass. This property is similar to that of the extremal dilaton black holes which are likely (at least some) to represent exact solutions of the string theory.

To further investigate possible relevance of the EYMD sphalerons to the string theory we study here the EYMD system with the Gauss–Bonnet (GB) term which is typically present in stringy gravity as the lowest order curvature correction. Similar problem for Abelian dilatonic black holes was studied recently within the perturbative approach [11]. However, for regular solutions, one needs a more precise treatment. In order to see in a continuous way how EYMD solutions are modified by the GB term we introduce into the lagrangian a numerical factor $\beta$ so that $\beta = 0$ corresponds to the pure EYMD system. It turns out that series expansion of the regular solution near the origin is essentially $\beta$–dependent. Also, computing the GB contribution into the effective energy density on the background EYMD solutions, one can observe that the GB effect becomes non–small for $\beta$ of the order of unity. For this reason we avoid any perturbative treatment of the GB term and attack the problem numerically. Starting with $\beta = 0$ we increase gradually the value of this parameter and search (using the shooting strategy) for solutions interpolating smoothly between the regular asymptotic expansion near the origin and an asymptotically flat expansion at infinity. Although the leading terms of expansions near infinity are not modified by the GB corrections, those near the origin are affected substantially. We construct numerical solutions for $N = 1, 2, 3$ and some $\beta \neq 0$ and show that regular solution cease to exist above certain critical values $\beta_N$ depending of the number of nodes $N$ of the YM function. For all solutions found within the domains of existence, modifications due to GB term are relatively small, and all characteristic functions still preserve the typical behaviour they have in the pure EYMD case. We also prove analytically that the dilaton charge of any regular solution (with an exact account for the GB term) is equal to its ADM mass independently on the value of $\beta$. For $\beta = 0$ a stronger relation holds between $g_{00}$ and the dilaton factor everywhere.

We start with the following bosonic part of the heterotic string effective action in four dimensions in the Einstein frame:

$$S = \frac{1}{16\pi} \int \left\{ (-R + 2\partial_\mu \Phi \partial^\mu \Phi) - \alpha' \exp(-2\Phi) (F_{a\mu \nu} F_a^{\mu \nu} - \beta G) \right\} \sqrt{-g} d^4x , \quad \text{(1)}$$

where $\Phi$ is the dilaton, $F$ is the Yang-Mills field strength and $G$ is the Gauss–Bonnet term which can be presented as the divergence of the topological current

$$G = R_{\mu \nu \lambda \tau} R^{\mu \nu \lambda \tau} - 4 R_{\mu \nu} R^{\mu \nu} + R^2 = \nabla_{\mu} K^\mu . \quad \text{(2)}$$
Integrating by parts the GB term in (1) one can rewrite the action in somewhat simpler form (both in (1) and (2) we ignore surface terms which are not relevant for the present analysis)

\[ S = \frac{1}{16\pi} \int \left\{ ((-R + 2\partial_{\mu}\Phi\partial^{\mu}\Phi) - \alpha' e^{-2\Phi}(F_{a\mu\nu} F_a^{\mu\nu} - 2\beta (\partial_{\mu}\Phi)K^\mu) \right\} \sqrt{-gd^4x}. \] (3)

We parametrize the metric of the static spherically symmetric spacetime as

\[ ds^2 = W dt^2 - \frac{dr^2}{w} - R^2(d\theta^2 + \sin^2\theta d\phi^2), \] (4)

where \( W = w\sigma^2 \) and all functions depend on the single variable \( r \). In this case only the radial component of the topological current is relevant

\[ K^r = \frac{4(w\sigma^2)'(wR^2 - 1)}{R^2\sigma^2}. \] (5)

Here and below primes mean derivatives with respect to \( r \).

A magnetic part of the static spherically symmetric \( SU(2) \) Yang–Mills connection can be expressed in terms of the single function of the radial variable \( f(r) \)

\[ A^a_{\mu} T_\alpha dx^\mu = (f - 1)(L_\phi d\theta - L_\theta \sin\theta d\phi), \] (6)

where \( L_r = T_a n^a, L_\theta = \partial_\theta L_r, L_\phi = (\sin\theta)^{-1}\partial_\phi L_r, n^a = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta) \) is the unit vector and \( T_a \) are normalized Hermitean generators of the \( SU(2) \) group.

Integrating out the angular variables in (2) and eliminating some total derivatives one obtains the following reduced effective action

\[ S = \int dtdr (L_g + L_m + L_{GB}), \] (7)

where

\[ L_g = \frac{\sigma}{2}(R'(wR)' + 1) + w\sigma' RR', \] (8)

is the gravitational part,

\[ L_m = -\frac{1}{2}wR^2\Phi' - \frac{\alpha'}{2}Fe^{-2\Phi}, \] (9)

is the matter part,

\[ L_{GB} = 2\alpha'\beta\sigma^{-1}\Phi'W'(wR^2 - 1)e^{-2\Phi}, \] (10)

is the Gauss–Bonnet contribution, and

\[ F = 2wf' + \frac{(1 - f^2)^2}{R^2}. \] (11)

Note that an arbitrary rescaling of the slope parameter \( \alpha' \to k\alpha' \) together with the corresponding rescaling of the radial variable \( r \to \sqrt{kr} \) is a symmetry transformation of the effective action (7). Choosing Planck units \( \alpha' = 1 \) we are left with the only dimensionless parameter \( \beta \).
The equations of motion (including an Einstein constraint) can be obtained by direct variation of (7) over $\sigma, w, R, f, \Phi$. Then fixing the gravitational gauge as $R = r$ one finds the following set of equations

$$\frac{\sigma'}{\sigma} = r\Phi'^2 + \frac{2f'^2 e^{-2\Phi}}{r} + \frac{4\beta}{r} \left( \frac{\Phi'(w - 1)e^{-2\Phi}}{\sigma} \right) - \frac{W'\Phi'}{\sigma^2 e^{-2\Phi}} , \quad (12)$$

$$w' \left( 1 - \frac{4\beta(1 - 3w)e^{-2\Phi}}{r} \right) + \frac{F}{r} e^{-2\Phi} + rw\Phi'^2 = \frac{(1 - w)}{r} \left( 1 - 4\beta w(e^{-2\Phi})' \right) , \quad (13)$$

$$\frac{1}{2} \left( \frac{W'}{\sigma} \right)' + \left( \frac{W}{\sigma} \right)' + \sigma \left( wr\Phi'^2 - \frac{(1 - f^2)^2}{r^3} e^{-2\Phi} \right) + 4\beta \left( \frac{W'\Phi e^{-2\Phi}}{\sigma} \right)' = 0 , \quad (14)$$

$$\left( w\sigma f' e^{-2\Phi} \right)' + \frac{\sigma f(1 - f^2)e^{-2\Phi}}{r^2} = 0 , \quad (15)$$

$$\left( \sigma r^2 w\Phi' \right)' + \sigma F e^{-2\Phi} + 2\beta \left( \frac{W'(1 - w)}{\sigma} \right) e^{-2\Phi} = 0 . \quad (16)$$

It is useful also to compute an effective energy density as it enters the standard Einstein equations with account for the GB term

$$2T_0^0 = w\Phi'^2 + \frac{F}{r^2} e^{-2\Phi} + \frac{4\beta}{r^2} \left\{ 2w(w - 1)(\Phi'' - 2\Phi'^2) + \Phi'w'(3w - 1) \right\} e^{-2\Phi} . \quad (17)$$

For $\beta = 0$ the system reduces to that of [1] and the corresponding solutions exhibit typical BK structure of the YM function: solutions start from $f = \pm 1$ and goes asymptotically to $\mp 1$ either monotonically ($N = 1$) or after $N - 1$ oscillations around zero.

As a first step of the analysis we calculate the GB term and the corresponding density $\sqrt{-gG}$ substituting the sphaleron solutions found without an account for the GB term. Numerical results are shown on the Fig. 1. One can see that the value of GB term increases with growing number of nodes of the YM function. It can be anticipated that its influence on the sphaleron solutions will increase for higher $N$. We have also calculated the effective energy density (17) for the background EYMD solutions. Fig. 2 clearly shows that relative contribution of the GB term for $\beta = 1$ is not small. This presumably invalidate any attempt to treat the GB term perturbatively, so we are faced with the problem of constructing numerical solutions to the system (12)–(16).

To define the ADM mass $M$ and the dilaton charge $D$ one writes asymptotic expansions for $W$

$$W = 1 - \frac{2M}{r} - \frac{2D^2 M}{r^3} + O\left( \frac{1}{r^4} \right) , \quad (18)$$

and the dilaton

$$\Phi = \Phi_\infty + \frac{D}{r} + \frac{DM}{r^2} + \frac{8M^2 D - D^3}{6r^3} + O\left( \frac{1}{r^4} \right) . \quad (19)$$
The corresponding expansion of \( \sigma \) reads

\[
\sigma = 1 - \frac{D^2}{2r^2} - \frac{4D^2 M}{3r^3} + O\left(\frac{1}{r^4}\right). \tag{20}
\]

To ensure asymptotic flatness it is sufficient (as it is for \( \beta = 0 \)) to have for the Yang–Mills function \( f \)

\[
f = \pm 1 + O\left(\frac{1}{r}\right). \tag{21}
\]

Clearly, GB–induced terms do not influence the leading behaviour of solutions near infinity.

In contrary, an expansion of regular solutions near the origin is affected by the curvature terms. From the system (12)–(16) one finds

\[
\begin{align*}
f &= -1 + br^2 + O(r^4), \\
\Phi &= \Phi_0 + \Phi_2 r^2 + O(r^4), \\
\sigma &= \sigma_0 + \sigma_2 r^2 + O(r^4), \\
W &= W_0 + W_2 r^2 + O(r^4),
\end{align*}
\tag{22-25}
\]

or in terms of \( w \):

\[
w = 1 + w_2 r^2 + O(r^4), \tag{26}
\]

where the following relations hold

\[
W_0 = \sigma_0^2, \quad W_2 = 2\sigma_0 \sigma_2 + w_2 \sigma_0^2. \tag{27}
\]

Let us prove that for any regular solution to the system (12)–(16) (if exists), the ADM mass \( M \) is exactly equal to the dilaton charge \( D \). Combining Eqs. (12), (14) and (16), after some rearrangement one can find the following identity

\[
\left(2\sigma r^2 w \Phi' + \frac{W' r^2}{\sigma}\right)' = 4\beta Q', \tag{28}
\]

where

\[
Q = \sigma^{-1} \{ (w - 1)(W' + 2W \Phi') - 2r \Phi w W \}. \tag{29}
\]

Integrating this relation over the semiaxis with account for (18)–(20) on gets

\[
\left(\frac{W}{\sigma} r^2 \Phi' + \frac{W' r^2}{2\sigma}\right)\bigg|_0^\infty \equiv M - D = 2\beta [Q(\infty) - Q(0)]. \tag{30}
\]

Now from the expansions (18)–(21) and (22)–(26) it can be found that both above boundary values of \( Q \) are equal to zero, what proves the exact equality \( M = D \). Remarkably, this property of regular EYMD solutions observed first in [6], remains true with account for the GB term for any value of \( \beta \). There is an important difference, however. In the case \( \beta = 0 \) a stronger identity

\[
W = \exp(-2\Phi) \tag{31}
\]
holds, which is similar to the well-known relation for the extremal magnetic dilatonic black holes, where it ensures regularity of the metric in the string frame. When GB term is taken into account this is no longer true while the relation \( M = D \) exhibiting the validity of (31) in the asymptotic region still holds.

Similarly to the system of Einstein–Yang–Mills–Dilaton equations [6], [7], [8], [9] without Gauss–Bonnet term there are three independent parameters in the series solutions of the system (12–16) near the origin: \( b, \Phi_0 \) and \( \sigma_0 \). From them the quantity \( \Phi_0 \) is somewhat trivial because of the symmetry of the system under a dilaton shift accompanied by suitable rescaling of the radial coordinate (if desired, \( \exp(-2\Phi_0) \) may be absorbed into redefinition of parameters in (22)–(26)). However, there is a substantial complication as compared with the pure EYMD theory. In order to fulfill the system (12–16) in the first leading order, the coefficient \( \Phi_2 \) has to be one of the real roots of the following algebraic equation of the forth order

\[
(\Phi_2 + 2b^2e^{-2\Phi_0}) \left(1 + 16\beta \Phi_2 e^{-2\Phi_0}\right)^3 + 32\beta b^4 e^{-6\Phi_0} \left(1 + 8\beta \Phi_2 e^{-2\Phi_0}\right) = 0. \tag{32}
\]

Once \( \Phi_2 \) is found, two other coefficients \( w_2 \) and \( \sigma_2 \) can be obtained as

\[
w_2 = \frac{-4b^2 e^{-2\Phi_0}}{1 + 16\beta \Phi_2 e^{-2\Phi_0}}, \tag{33}
\]
\[
\sigma_2 = \frac{\sigma_0 e^{-2\Phi_0}(4b^2 + 4\beta w_2 \Phi_2)}{1 + 16\beta \Phi_2 e^{-2\Phi_0}}. \tag{34}
\]

It is convenient to regard the Eq. (32) as giving the value of \( \Phi_2 \) as a function of \( b \), while \( \Phi_0 \) is fixed. In fact, a dilaton shift

\[
\Phi_0 \rightarrow \Phi_0 + \delta \Phi_0 \tag{35}
\]

leads to a solution related with the initial one by a radial rescaling. Physically the normalization \( \Phi_\infty = 0 \) is preferable since it ensures a unique mass scale for all solutions. But technically is is convenient to solve the system first by fixing \( \Phi_0 \) arbitrarily, say, \( \Phi_0 = 0 \). Then the rescaled solution will result from

\[
b \rightarrow b \exp(2\delta \Phi_0), \quad \Phi_2 \rightarrow \Phi_2 \exp(2\delta \Phi_0), \quad \sigma_0 \rightarrow \sigma_0. \tag{36}
\]

At the final stage of the calculation we rescaled solutions imposing the condition \( \Phi_\infty = 0 \) in order to fix a unique mass scale for all of them.

The numerical strategy consists in solving the system (12)–(16) starting from the series solution (22)–(26) near the origin. The crucial role is played by the parameter \( b \) which should take a discrete sequence of values. For \( \beta = 0 \) the solution of Eq. (32) reads \( \Phi_2 = -2b^2 \exp(-2\Phi_0) \), and clearly this does not impose any restriction on this parameter. But for \( \beta \neq 0 \) it turns out that real solutions for \( \Phi_2 \) do not exist in some region of \( b \). Hence, in addition to the problem of “quantization” of \( b \) one has to ensure that \( b \) belongs to region where the real roots of the Eq. (32) exist. It happens that if \( \beta \) is greater than some (\( N \)-dependent) critical value \( \beta_N \), the allowed region of \( b \) does not contain those quantized values for which regular solutions exist. Only for \( \beta < \beta_N \) regular solutions exist and exhibit behaviour similar to that of the EYMD solutions.
Real roots of the algebraic equation (32) form two branches as shown on the Fig. 3a,b in terms of the quantities \( \tilde{b} = b \exp(-2\Phi_0) \), \( \tilde{\Phi}_2 = \Phi_2 \exp(-2\Phi_0) \). For roots from the second branch (Fig. 3b) we didn’t find any solution, they seem to correspond to \( b \) outside the above quantization domain. Note that this branch does not contain the EYMD root corresponding to \( \beta = 0 \). The first branch 3a has a solution for \( \beta = 0 \), while for any \( \beta \neq 0 \) there are two negative solutions with absolute values \( \Phi_2^{\text{max}}(b) \) and \( \Phi_2^{\text{min}}(b) \). From these two, it is just the second one, \( \Phi_2^{\text{min}}(b) \), which has the limiting value \( \Phi_2^{\text{min}} = -2\tilde{b}^2 \exp(-2\tilde{\Phi}_0) \) when \( \beta \to 0 \). No regular solutions to the system (12)–(16) corresponding to \( \Phi_2^{\text{max}}(b) \) were found neither.

Starting with the known \( \beta = 0 \), \( N = 1 \) EYMD solution \( [6] \) we increased gradually the value of \( \beta \) searching for the desired quantized \( b \) related to \( \Phi_2^{\text{min}}(b) \). Numerical integration of the system (12)–(16) was done using the Runge–Kutta fourth order scheme. The values of the parameters for \( N = 1 \) case, found numerically for some \( \beta \) together with the corresponding ADM mass \( M = D \) are given in the Table 1. The solutions were rescaled to ensure \( \Phi_{\infty} = 0 \).

### Table 1. N=1.

| \( \beta \) | \( b \) | \( \Phi_0 \) | \( \sigma_0 \) | \( \Phi_2^{\text{min}} \) | \( \Phi_2^{\text{max}} \) | \( M = D \) | \( w_2 \) |
|---|---|---|---|---|---|---|---|
| 0. | 1.073 | 0.9311 | 0.3936 | -0.3576 | — | 0.578 | -0.7153 |
| 0.1000 | 1.026 | 0.9199 | 0.3840 | -0.3523 | -3.390 | 0.573 | -0.7344 |
| 0.2000 | 0.9866 | 0.9122 | 0.3744 | -0.3566 | -1.475 | 0.568 | -0.7697 |
| 0.3000 | 0.9619 | 0.9120 | 0.3597 | -0.3833 | -0.8376 | 0.563 | -0.8496 |
| 0.3700 | 0.9657 | 0.9231 | 0.3421 | -0.4938 | -0.5198 | 0.560 | -1.0933 |

One can observe that with increasing \( \beta \) two real roots \( \Phi_2^{\text{min}}(b) \) and \( \Phi_2^{\text{max}}(b) \) converge and merge together for a limiting value \( \beta_1 \) approximately equal to 0.37. For \( \beta > \beta_1 \) there are no such \( b \) which could generate asymptotically flat solutions with \( N = 1 \) compatible with the existence of the real root \( \Phi_2(b) \) of the Eq. 32.

Similar situation was encountered for higher-\( N \) solutions. Numerical results for \( N = 2 \) and \( N = 3 \) are presented in the Tables 2, 3. Figures 4–8 depict the corresponding numerical curves for some values of \( \beta \) and \( N \).

### Table 2. N=2.

| \( \beta \) | \( b \) | \( \Phi_0 \) | \( \sigma_0 \) | \( \Phi_2^{\text{min}} \) | \( \Phi_2^{\text{max}} \) | \( M = D \) | \( w_2 \) |
|---|---|---|---|---|---|---|---|
| 0. | 8.3612 | 2.9399 | 0.1665 | -3.8796 | — | 0.685 | -7.760 |
| 0.1000 | 7.1902 | 1.7481 | 0.1529 | -3.5165 | -15.982 | 0.673 | -7.558 |
| 0.2000 | 6.4017 | 1.7297 | 0.1370 | -3.6597 | -5.7461 | 0.660 | -8.161 |
| 0.2208 | 6.3344 | 1.7343 | 0.1320 | -4.2904 | -4.3127 | 0.657 | -9.478 |

Note, that the numerical values of ADM mass/dilaton charge monotonically decrease with growing \( \beta \) for each fixed \( N \). Also, it can be observed that the limiting values of \( \beta_N \) decrease with the increasing \( N \): \( \beta_1 = 0.37, \beta_2 = 0.22, \beta_3 = 0.21, ... \). It can be
Table 3. N=3.

| $\beta$  | $b$    | $\Phi_0$ | $\sigma_0$ | $\Phi_2^{\text{min}}$ | $\Phi_2^{\text{max}}$ | $M = D$ | $w_2$  |
|---------|--------|----------|------------|------------------------|------------------------|---------|--------|
| 0.      | 53.8351| 2.6920   | 0.0678     | -26.600                | —                      | 0.7042  | -53.202|
| 0.2000  | 36.5453| 2.5930   | 0.0504     | -22.348                | -30.543                | 0.6744  | -49.817|
| 0.21117 | 36.2683| 2.5956   | 0.0492     | -25.151                | -25.212                | 0.6726  | -55.558|

anticipated that $\beta_N$ has a limiting value $\beta_\infty$ as $N \to \infty$, which would presumably give an absolut bound of the existence of static spherically symmetric regular EYMD–Gauss–Bonnet solutions. It is also interesting to note, that although the contribution of the GB terms to the energy density for $\beta$ close to $\beta_N$ is not small (as it is shown on Fig. 5), the behaviour of $f$ and metric functions is very similar to that of pure EYMD solutions. It has also to be noted that, when the limiting value $\beta_N$ is approached, neither singularities no other numerical problems arise; so the only reason for the absence solutions when $\beta$ exceeds the above critical value is an intrinsic incompatibility of the series expansion near the origin.

We conclude with the following remarks. When Gauss–Bonnet term is included, the total number of derivatives in the system of equations increases, as well as the degree of its non–linearity. However, in a limited region of the numerical factor $\beta$ the behaviour of solutions remains qualitatively the same as in the pure EYMD case. Moreover, the remarkable equality of the ADM mass to the dilaton charge remains unaffected by the GB term for any $\beta$. However it is likely that EYMD sphalerons are destroyed by the Gauss–Bonnet term for sufficiently large values of $\beta$. The most persistent is the $N = 1$ solution, which exists up to $\beta = 0.37$. Higher $N$ solutions cease to exist for lower $\beta$, the limiting value is likely to be of the order of 0.2.

This work was supported in part by the ISF Grant M79000 and by the Russian Foundation for Fundamental Research Grant 93–02–16977.

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Figure Captions

Fig. 1. GB term (B) and GB density (A), calculated for pure EYMD $N = 1$ solutions, curves (C) and (D) – GB density for $N = 2, 3$ EYMD solutions.

Fig. 2. Contributions to the energy density $r^2 \ast T_{00}$, from YMD ($\beta = 0$) and GB parts ($\beta = 1$) calculated using EYMD solutions : (A): $N = 1$, YMD; (B): $N = 1$, GB; (C): $N = 2$, YMD; (D): $N = 2$, GB.

Fig. 3a,b. Real roots of Eq. 32 (two different branches), $\beta = 0.1, 0.2, 0.37, 0.5, 1$ in terms of $\bar{b} = b \exp(-2\Phi_0)$, $\bar{\Phi}_2 = \Phi_2 \exp(-2\Phi_0)$.

Fig. 4. “Gauss–Bonnet” mass distribution (contribution to ADM mass from $\beta$ – dependent terms) for solutions with $\beta = 0.2, N = 1, 2, 3$.

Fig. 5. Energy density for $N = 3, \beta = 0.2$. (A): total energy density; (B): contribution from $\beta$ -independent terms; (C): GB contribution.

Fig. 6. Yang-Mills function $f$ for $N = 1, 2, 3$. Solid lines: solutions with GB term ($\beta = 0.2$), dashed lines: purely EYMD solutions.

Fig. 7. Metric function $W = g_{00}$ (dashed lines) and $\exp(-2\Phi)$ (solid lines) for $N = 1, 2, 3, \beta = 0.2$.

Fig. 8. Metric function $\sigma$ for $N = 1, 2, 3$. Solid lines: solutions with GB term ($\beta = 0.2$); dashed lines: purely EYMD solutions ($\beta = 0$).
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