Alpha-cluster Condensations in Nuclei and Experimental Approaches for their Studies

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Abstract The formation of alpha-clusters in nuclei close to the decay thresholds is discussed. These states can be considered to be boson-condensates, which are formed in a second order phase transition in a mixture of nucleons and $\alpha$-particles. The de Broglie wavelength of the $\alpha$-particles is larger than the nuclear diameter, therefore the coherent properties of the $\alpha$-particles give particular effects for the study of such states. The states are above the thresholds thus the enhanced emission of multiple-$\alpha$s into the same direction is observed. The probability for the emission of multiple-$\alpha$s is not described by Hauser-Feshbach theory for compound nucleus decay.

1 Binding energy of alpha particles

The binding energies of nuclei in their ground states as a function of mass number show a peculiar systematic behavior, explained by the liquid drop model. Deviations from a smooth curve are due to shell effects, and are some times discussed to be related to the formation of $\alpha$-clusters. The specific properties of the nucleon-nucleon force, namely the saturation which occurs if the spin and isospin quantum numbers are both coupled to zero, produces a very strong binding of $\alpha$-particles [1]. In addition, due to the internal structure of the $\alpha$’s an increased (30%) central density is observed compared to the the usual central density in nuclei. The $\alpha$-particle is therefore a unique cluster subsystem in nuclei.

This feature is well known from the early history of nuclear science, and there has been small but steady activity in the field of clustering in nuclei in the last decades. Recently more attention to clustering in nuclei has emerged due to the study of weakly bound nuclei at the drip lines. For these nuclei clustering is very

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important even for the properties of ground states. These are well reproduced in model independent approaches, like in the antisymmetrized Fermionic molecular dynamics (FMD) which uses all degrees of freedom in the nuclear forces, in the approach by Feldmeier et al. [2] [3]. With a related approach, the antisymmetrized molecular dynamics (AMD) with effective N-N forces, Horiuchi and Kanada-En'yo et al. [4] [5] [6] are able to reproduce the ground state properties and a large variety of excited nuclear states, in particular those with molecular structure. In these calculations the density distributions of the nucleons are obtained. Quite spectacular are the results for loosely bound nuclear systems [5] [6], where \( \alpha \)-clusters appear naturally as dominant substructures. This work has established that \( \alpha \)-clusters play a decisive role in the description of light nuclei, in particular for the loosely bound neutron-rich isotopes. For example the extra neutrons are found in covalent molecular orbitals around two \( \alpha \)-particles forming bound molecular two-center systems for the beryllium isotopes [7].

Furthermore, the \( \alpha \)-particle is the most important ingredient in the concept of the Ikeda-diagram [8] [9] [10], where highly clustered states (e.g. linear chains) are predicted at excitation energies around the energy thresholds for the decomposition into specific cluster channels.

![Fig. 1](image)

**Fig. 1** The experimental binding energies per \( \alpha \)-particle in N=Z nuclei, as function of the number of \( \alpha \)-particles, \( N_\alpha \). The lines are drawn to connect the points. The same quantities are shown under the assumption with two different heavy clusters as cores: \( ^{40}\text{Ca} \) and \( ^{52}\text{Fe} \), as indicated (adopted from [11]).

In order to explore the dynamics of \( \alpha \)-clustering in excited states of N=Z nuclei the systematics of binding energies per \( \alpha \)-particle in nuclei \( E_{\text{BA}}/N_\alpha \), has been considered [11]. The experimental masses have been taken from ref. [12], for \( ^{164}\text{Pb} \)
from a theoretical study of the mass A=164 region [13]. With the total binding energy $E^t_B(N,Z)$, the binding energy of all $\alpha$-particles in N=Z nuclei can be obtained, this will again follow the well known curve for the binding energy (per nucleon) properly rescaled. We are interested in the binding energy per $\alpha$-particle, $E_B/\alpha/N_\alpha$, determined from the experimental data, as shown in Fig. 1, there we show the quantity,

$$E_B/\alpha/N_\alpha = \left[ E^t_B(N,Z) - N_\alpha E_\alpha \right]/N_\alpha. \quad (1)$$

Here we have the typical maximum values around Fe-nuclei. The energy for the threshold states can be read from this figure. Searching for the alpha-condensed states we can also include states with a strongly bound core, e.g. a core of $^{16}$O, a core of $^{40}$Ca or $^{52}$Fe, with the appropriate number $N'_\alpha$ of free $\alpha$-particles outside. The binding energy with a core shown in Fig. 1 being:

$$E_B^{40Ca}(N'_\alpha)/(N_\alpha) = \left[ E^t_B(N,Z) - E_B^{40Ca}(N,Z) - (N_\alpha - 10)E_\alpha \right]/(N_\alpha - 10). \quad (2)$$

From this figure we can deduce that the excitation energy, where the value of $E_B/\alpha/N_\alpha$ reaches zero, the threshold for complete decay, becomes lower with the inclusion of a core. For heavier nuclei around $^{100}$Sn and masses approaching $^{164}$Pb the nuclei become unstable relative to single (or multiple) proton or $\alpha$-particle emission. The excitation energies, where the binding energy value for $\alpha$’s approaches zero - these are the values where $\alpha$-particle condensates can form - will be discussed below, sect. 2. Although these $\alpha$-condensed states are at rather high excitation energies in the continuum of nucleonic states, they may have collective properties, which can give them a smaller observable width. We expect the decay into many $\alpha$-particles, a decay not described by the Hauser-Fehsbach formalism for statistical compound nucleus decay (see below).

2 The formation of alpha condensates

The $\alpha$-particle condensates formed at the thresholds will be unbound states, their decay properties will be one of the most important points in the discussion of these boson states. In lighter nuclei, in particular for the second $0^+_2$ state in $^{12}$C, the Hoyle state [12], which can be considered as the first boson condensate, a gamma-decay is possible, here with the sequence $0^+_2 \rightarrow 2^+_1 \rightarrow 0^+_1$, a process most important for the formation of $^{12}$C in stars. The recent proposal of an $\alpha$-particle condensate wave function (THSR), by Tohsaki, Horiuchi, Schuck and Röpke [15], describes the properties of the Hoyle state very well, whereas even the largest shell model calculations fail completely to reproduce this state [16, 17], see also refs. [19] for the most recent discussion.

In medium size nuclei (Z<20) the $\alpha$-condensates, calculated using the known alpha-alpha-potential with a self-consistent approach (based on the Gross-Pitaevskii
equation), will have Coulomb barriers [18] for the decay into multiple \( \alpha \)’s. With these barriers the states will have sufficiently small width for potential studies by inelastic scattering. However, the heaviest nucleus for which this barrier can create a quasi-bound state [18], is estimated to be around \(^{40}\text{Ca}\). In heavier nuclei these states will be embedded high in the continuum of the fermionic states, their decay is expected to be non-statistical, the most characteristic property to study.

### 2.1 Second order Phase transition

The ground states of nuclei are well described by the shell model with a self-consistent potential of all nucleons. If a cluster model with \( \alpha \)-clusters is used, their strong spatial overlap, the anti-symmetrization of all nucleons destroys their original properties, a fact widely discussed in the literature (see refs. [20] for the most recent discussion). The intrinsic structure of \( \alpha \)-clusters in these cases are very different from that of free \( \alpha \)-particles, still a large variety of molecular resonances connected to clusters are observed in N=Z nuclei [21].

We want to discuss the formation of an \( \alpha \)-particle gas, where the average distance between \( \alpha \)-particles is much larger with rather small spatial overlap. The corresponding nucleon density will be well below normal nuclear densities. In fact, in the theoretical investigation of Bose-Einstein condensates in nuclei Tohsaki, Schuck et al. [15][16][17][18] find, that at the thresholds for multi \( \alpha \)-particle decays, the states with \( \alpha \)-clusters have a much larger radial extension than the ground states (larger \( \alpha-\alpha \) distances). From the viewpoint of the nucleonic fermion gas the appearance of such states will depend on the temperature \((i.e.\) excitation energy, \(E^*_x\)) of the nucleus. The concept of a second order phase transition as in a chemical reaction with two components can be used [11], a concept well established in thermodynamics of composite systems in statistical physics [22]. The basic equation is the “reaction” of four “free” nucleons (two protons and two neutrons coupled to total values of spin and isospin of zero) forming \( \alpha \)-clusters:

\[
(N_1 + N_2 + N_3 + N_4) \leftrightarrow \alpha\text{-particle} + 28.3 \text{ MeV}.
\]

The free nucleons, \( N_i \), should have a definite volume and pressure, in order to define thermodynamic quantities and where the density allows the occurrence of the mentioned reaction. We can assume that the particles interact in a well defined volume created by a self-consistent mean field for the nucleons (the Hartree-Fock approach) and for the \( \alpha \)-clusters with the Gross-Pitaevskii approach for bosons. This latter has been used in the work of Yamada and Schuck [18].

In models like the AMD [6] a certain number of nucleons are confined in a volume with a positive kinetic energy, as suggested in Fig. 2. In this model a cooling method is applied to find the states of the lowest energy and higher density. The energy of the nucleons inside the nucleus is defined by their volume, and their Fermi-energy can be deduced from the nuclear radius, as described in text books [23]. In the AMD approach with the cooling process a certain \( \alpha \)-cluster phase is observed, before the formation of the higher density states with bound fermions, and finally
the ground states are reproduced. At the end the formation of normal nuclei with a binding energy per nucleon of 8.2 MeV or more is observed, a value which is higher than in the $\alpha$-cluster (7.073 MeV). These two values define the difference in the chemical potentials in the two phases (Fig. 2). For less bound nuclei (binding energy per nucleon around 7.073 MeV), the $\alpha$-clusters are obtained in a “natural” way. Starting from the ground states of normal nuclei the nucleons will form an $\alpha$-cluster phase, with increasing temperature of the nucleus, e.g. with increasing excitation energy, see Fig. 2 (“heating”). This excitation energy becomes rather low in neutron-rich light exotic nuclei, where clustering may appear already in the ground states as the dominant structure [6], this may also happen potentially in very heavy N=Z nuclei.

For the nucleons confined in the nuclear volume we apply the concepts of statistical physics for the reaction $4N \longleftrightarrow \alpha$-particle. The rate of the reaction is governed by the free energy, $G$, and the difference in the chemical potentials, $\mu_\alpha$ and $\mu_n$. The chemical potentials are defined as $\mu_i = \delta G / \delta N_i$, $i = n, \alpha$. The thermodynamic free energy depends on the number of nucleons, $N_n$ and on $N_\alpha$, with $G = G(N_n, N_\alpha)$. The change of the free energy becomes

$$\Delta G = \Delta N_\alpha \mu_\alpha + 4 \Delta N_n \mu_n.$$  \hfill (3)

For the phase transition a minimum value of the free energy is needed, this gives the condition $\Delta G = 0$, this feature will be observed at a critical excitation energy $E_{\text{crit}}$. In the nuclear medium $\Delta G$ is the difference between the binding energy of the four nucleons in the free $\alpha$-particle to that in the nuclear medium, as illustrated in Fig. 2.

The kinetic energy of the nucleons determines the temperature, $T$. However, we will use the temperature of the nucleus, $t$, related to its excitation energy. In the normal case of a mixed system of the two species, the relative abundance of $N_\alpha$ to $N_n$ is a function of the temperature (in our case excitation energy) and is obtained through the expression

$$\frac{N_\alpha}{(N_n)^4} = K(t) = \exp\left(\frac{-\Delta G(t)}{RT(t)}\right)$$  \hfill (4)

The value of $K$ is to be determined by experimental observation (the usual coefficient $R$ appears as in statistical physics). For the case of negative $\Delta G(t)$, a decrease of the free energy (corresponding to a large value of the ratio $K$) gives a higher density of the $\alpha$-particles as reaction products. A positive value of $\Delta G$ corresponds to an energetic disadvantage for the reaction creating $\alpha$-particles, resulting in a smaller number $N_\alpha$ as reaction products. In the case of nuclei, the nucleons are embedded in the nuclear medium and are confined in the nuclear potential created by the mean field of all nucleons. The binding energy per nucleon in nuclei is around 8 MeV (dependent on the size of the nucleus and its excitation energy). The nucleons have a larger binding energy in the nuclear medium (in the ground states of stable nuclei) compared to the value in the $\alpha$-clusters. The relative positions of the relevant energies are illustrated in Fig. 2 from ref. [7]. Actually, because the chemical po-
Schematic illustration of the relative values of the energies of free nucleons and alternatively their binding energies in nuclei (8.2 MeV), the latter are generally larger than in $\alpha$-clusters (7.07 MeV). The difference $\Delta G$ between these two binding energies decreases with increasing excitation energy ("heating") in nuclei. At a critical value the binding energies become equal, $\Delta G = 0$, a collective state of bosons (potentially mixed with fermions), the condensed $\alpha$-particle gas can be formed. In the AMD this state is approached by "cooling" from a fermion gas.

The potential of the nucleons will depend on the excitation energy in the nucleus (or on its temperature), we put this dependence in the expression for $\Delta G(t)$.

Alpha-cluster formation is expected if $4E_B/N_n$ is less than or equal to the total binding energy of four nucleons in the $\alpha$-cluster. As the binding energy per nucleon becomes equal or smaller than in the $\alpha$-particle, a new phase will be formed, a strongly interacting Bose gas. For binding energies of the nucleons close to (or larger) that in the $\alpha$-particle it becomes possible to form a mixed phase of $\alpha$-cluster states (liquid) and of nucleons. The binding energy of nucleons in the ground states of nuclei (see Fig. 2) $E_B/N_n = E_{\text{nucleon}}$, is usually larger than in the $\alpha$-particle. The condition for the excitation (condensation) energies is $E_{\text{cond}} \geq E_{\alpha}^{\text{crit}}$. The values for different nuclei relevant to this concept are given in Tab. 1.

We summarize that the $\alpha$-condensation condition is given by $E_B/N_n(E_{\text{crit}}^{\alpha}) \geq 7.07$ MeV.

The value of the critical excitation energy/(per nucleon) in a nucleus, $E_{\text{crit}}^{\alpha}$, should be equal or larger than 7.07 MeV, which is the binding energy of nucleons in the $\alpha$-particle. This statement is the same as the condition $\Delta G(t) = 0$. Alternatively, the phase transition will be achieved at excitation energies of the nucleus, $E^{*}_x$, corresponding to the thresholds where all clusters become unbound, the condition being $E_{\text{BD}}(N,Z) = 0$. This is the original concept of the Ikeda diagram. The Ikeda diagram [8] gives a phenomenological condition for the appearance of clustered states (with the inclusion of other clusters like $^{12}$C, $^{16}$O, etc.) in nuclei. We can state that the Ikeda diagram with $\alpha$-particles can be deduced from thermodynamic considera-
Table 1  Alpha-particle binding and critical excitation energies for the condensation condition in nuclei with N=Z; \( N_\alpha \) - number of \( \alpha \)-particles, \( E_b/N_\alpha \) - binding energy per nucleon, \( E_{\alpha\alpha}/N_\alpha \) - binding energy per \( \alpha \)-particle, \( E_{\text{crit}} \) - condensation energy. The last column shows the values for the case of a \(^{40}\text{Ca}\)-cluster core. All energies in MeV.

| Nuclide | \( N_\alpha \) | \( E_b \) (MeV) | \( E_{\alpha\alpha}/N_\alpha \) | \( E_{\text{crit}} \) (MeV) | \( E_{\text{crit}}^{(\text{Ca})} \) (MeV) |
|---------|--------------|-----------------|----------------|-----------------|------------------|
| \(^{2}\text{He}\) | 1 | 28.3 | 7.073 | — | — |
| \(^{12}\text{C}\) | 3 | 92.16 | 7.680 | 2.425 | 7.27 |
| \(^{16}\text{O}\) | 4 | 127.6 | 7.976 | 5.609 | 14.44 |
| \(^{20}\text{Ne}\) | 5 | 160.7 | 8.032 | 3.83 | 19.17 |
| \(^{24}\text{Mg}\) | 6 | 197.2 | 8.260 | 4.787 | 28.72 |
| \(^{28}\text{Si}\) | 7 | 236.5 | 8.447 | 5.495 | 38.47 |
| \(^{32}\text{S}\) | 8 | 271.8 | 8.493 | 5.677 | 45.41 |
| \(^{36}\text{Ar}\) | 9 | 306.7 | 8.519 | 5.78 | 52.02 |
| \(^{40}\text{Ca}\) | 10 | 342.0 | 8.551 | 5.910 | 59.10 |
| \(^{12}\text{C}\) | 13 | 344.7 | 8.609 | 6.143 | 79.86 |
| \(^{16}\text{O}\) | 14 | 483.9 | 8.642 | 6.275 | 87.85 |
| \(^{20}\text{Ne}\) | 15 | 607.1 | 8.432 | 5.433 | 97.8 |
| \(^{24}\text{Mg}\) | 16 | 824.5 | 8.371 | 5.192 | 103.8 |
| \(^{28}\text{Si}\) | 17 | 1090.9 | 7.577 | 2.074 | 74.6 |

The level density for the fermionic phase space grows very fast with excitation energies, whereas those for the bosons will grow much slower.

Most important for the properties of the \( \alpha \)-particle gas is, that they do not represent the “ideal” gas, they interact via an interaction which has similarities with a van der Waals interaction, with a strongly repulsive core due to the Pauli principle, see Fig. 3. Two \( \alpha \)-particles form as the lowest state, the ground state of \(^{8}\text{Be}\), a resonance at \( E^*_1 = 92 \text{ keV} \). We can calculate the de Broglie wave length, \( \lambda = \frac{h}{\sqrt{(2\mu E^*_1)}} \) for this case and have \( \lambda = 67 \text{ fm} \) (relative motion between the two \( \alpha \)-particles). If for higher excitation we incorporate the \( 2^+ \) at 3.04 MeV the value of \( \lambda \) is still 12.4 fm. Similarly three \( \alpha \)-particles can form the Hoyle-state just above the three \( \alpha \)-particle threshold in \(^{12}\text{C}\), the \( 0^+ \) at 7.654 MeV (288 keV above the threshold of 7.346 MeV). With these values for three \( \alpha \)-particles we again get a similarly large de Broglie wave length of relative motion. Also the third \( 0^+ \) at 10.3 MeV excitation energy can participate in the formation of a multi-\( \alpha \)-particle correlation.

Overall we have values for \( \lambda \) in the condensed state larger (by factors 2-5) then the radial extension of the nucleus. The multi-\( \alpha \)-particle states will contain the \( \alpha \)-particles mainly in their resonant states in \(^{8}\text{Be}\). The condensed states at the binding energy threshold consisting of \( \alpha \)-particles will form coherent super-fluid states. The resonant states in \(^{8}\text{Be}\) and \(^{12}\text{C}\) act in a similar way as the residual interaction in the formation of the superfluid neutron pairing states, see ref. [1], volume II. The calculations of THSR based on a local \( \alpha-\alpha \) potential reproduce the states of \(^{8}\text{Be}\), and the threshold states in other light nuclei. Inspecting the local potentials in Fig. 5.
for the system of $^{16}$O+$\alpha$-particle, we conclude that alpha-condensates with a $^{16}$O-core can be formed, where this potential will create a common binding potential, for a larger number of alpha’s (e.g. $^{40}$Ca = $^{16}$O+6$\alpha$).

In Fig. 4 we illustrate the possible situation for an $\alpha$-condensate in $^{100}$Sn, with a core of $^{40}$Ca and 15 $\alpha$’s. These configurations can be formed in a reaction with a $^{72}$Kr beam and a $^{28}$Si target. At excitation energies of 97 MeV or more (excitation energies discussed earlier) many compound nuclear (CN) states will exist, consisting of different configurations of the $\alpha$-particle gas plus a core. Here again the threshold rules apply with respect to excitation energies. We may expect many overlapping states (with a large decay width), which will interact coherently (see ref. [26]), because the same compound states of the $\alpha$-particle phase can be formed with a different number of $\alpha$-particles. These will interact through the $0^+$ and $2^+$ resonances of $^8$Be and $^{12}$C$^*$, depending on the excitation energy of the state. The decay of such a state (in the figure there is no barrier for the $\alpha$-particles, in difference to the figure shown in ref. [7]) can occur sequentially with different energies in each step, as in CN-decay. However, the most interesting case would be the simultaneous decay, with many $\alpha$-particles with almost equal kinetic energies, a process, which can also be considered as Coulomb explosion [39], see sect. 3.3.
3 Experimental observables

For the observation of states in nuclei, which have spin(parity) = 0(+) and the properties of $\alpha$-particle condensates, there are several characteristic experimental features which we can propose for future studies.

1) The study of the radial extension, e.g. observed in inelastic $\alpha$-scattering and in the form factors from electron scattering experiments.

2) Coherent emission of $\alpha$-particles from compound nuclei in coincidence with large $\gamma$-detection arrays.

3) Fragmentation into multiple $\alpha$-particle channels at GeV/nucleon energies.

4) $\alpha$-$\alpha$-correlations, for CN decay similar to 2.

A further approach which should be mentioned here is the detection of multi-$\alpha$-clusters in a ternary cluster decay as described in refs. [40, 41]. In these cases the coplanar detection of two heavier fragments as in a binary decay, shows missing mass and charge of multi-$\alpha$-clusters. The study of these fission processes indicates that the missing $\alpha$-clusters are emitted from the neck with very small intrinsic excitation and small angular momentum. These multi-$\alpha$-clusters, will be emitted towards very small angles, where they should be detected with charged particle counter-telescopes.
3.1 Inelastic scattering, radial extensions, form factors

The threshold states in nuclei with condensed $\alpha$-particles have spin/parity $J^\pi = 0^+$, these must be populated by monopole excitations (a collective radial density mode). In fact the most important predicted properties of these states, are the larger radial extensions. These can be manifested in inelastic electron and hadron scattering. The inelastic electron scattering on a $^{12}$C-target has been studied repeatedly. The form factor for the transition to the excited state at 7.65 MeV, the Hoyle State, with $J^\pi = 0^+$ is thus well known see refs. [31, 32] and earlier references therein.

![Graph](image)

Fig. 5 The elastic and inelastic scattering of $\alpha + ^{12}$C at 240 MeV with the result of the analysis using the double folding model of Khoa et al. [32]. The vertical line illustrates the shift towards smaller angles for the transition to the 7.65 MeV, the (Hoyle state), due to it’s larger radial extension.
Similarly there are extensive studies of inelastic hadron scattering on $^{12}\text{C}$ using a large variety of projectiles. We concentrate here on the elastic and inelastic $\alpha$-scattering at energies between 104 MeV and 240 MeV, which has recently been analyzed with microscopic transition densities and the double folding approach for the scattering potential \[30\] and with a diffraction model \[33\]. The angular distributions exhibit at smaller angles strong diffraction patterns, and partially also a refractive maximum at larger angles. One feature, known from the early history of nuclear physics, is the Blair phase-rule established in $\alpha$-particle scattering \[32\]. If states are populated in inelastic scattering and sufficiently high energy, diffractive patterns (strong maxima and minima) are observed at forward angles, the structures of the elastic scattering (e.g. the minimum) and the e.g. the maximum for (Fig. 5) in inelastic scattering ($L = 0$) are exactly out of phase if no parity change has occurred. At higher energies the effects of Q-values, their influence on the position of the maxima and minima is small. The position of the diffractive minima depend on the radial extension (e.g. of the excited states).

The result at an incident energy of 240 MeV is shown in Fig. 5, the angular distributions show pronounced diffraction structures. Indeed the diffractive pattern for the inelastic excitation to the $2^+$ state at 4.43 MeV is clearly out of phase with that for the ground state. For the $0^+$ state at 7.65 MeV the diffractive pattern is more pronounced and is shifted by approximately 2 degrees to forward angles relative to the elastic scattering, indicating a larger radius. The calculations, which were performed with the double folding model for the elastic scattering potential as well as for the transition densities \[32\], are also shown in Fig. 5. With this approach and a proper choice of the imaginary potential for the $0^+_2$ state the absolute cross sections are reproduced with a correct value for the $E0$-transition strength. The other inelastic transitions have been calculated, and are perfectly reproduced due to the choice of the transition densities obtained in the folding model. The analysis with a diffraction model \[33\] of such data gives the systematics of the diffraction radius over a large energy range and indicates a 10% larger radius for the $0^+_2$ state compared to the ground state. Similar results will be expected for the $0^+_6$ at 15.1 MeV in $^{16}\text{O}$, which is just above the $4\alpha$-threshold (14.4 MeV), and has been searched for recently \[34\].

### 3.2 Compound nucleus decay, correlated emission of alpha’s

In the formation of N=Z compound nuclei up to mass A = 60-80, the heaviest combination of stable targets and projectiles is $^{40}\text{Ca} + ^{40}\text{Ca}$ giving $^{80}\text{Zr}$ compound states with appropriate excitation energy (see Tab. 1) and coherent $\alpha$-particle states can be formed. For heavy N>Z compound nuclei with a small neutron excess, however, the features discussed below may also apply. For even heavier systems with N=Z, we will have to resort to beams of unstable nuclei, like e.g. a $^{72}\text{Kr}$ beam, which has a good chance of being produced in the future with usable intensities. The compound nucleus with a $^{40}\text{Ca}$-target will be $^{112}\text{Ba}$ ($Q = -52.54$ MeV). Because of the fact, that the heavier compound nuclei are very far off-stability, the reaction $Q$-value
becomes very negative. With an incident energy close to the Coulomb barrier, the final excitation energy (Ex) can be well controlled and moderate values of Ex can be reached (see Tab. 1). These compound nuclei will have also favorable Q-values for the emission of several α-particles. Actually a new collective decay mode, where all alpha-particles share the same kinetic energy, as in Coulomb explosion, can be predicted. However, heavier compound nuclei will be unstable to charged-particle emission (protons and α’s) already in their ground states.

Further we may consider an excess of two or more neutrons (with an isotope with a more intense beam), this would most likely not destroy the special states discussed here. The excess neutrons will be placed in quantum orbits around the emitted clusters, for example as in the ⁹⁻¹⁰Be isotopes forming bound or metastable molecular states and configurations with low nucleon density [7].

We are interested in the multiple α-particle emission. Due to the coherent properties of the threshold states consisting of α-particles interacting coherently with a large de Broglie wave length, the decay of the CN will not follow the Hauser-Feshbach assumption of the statistical model: that all decay steps are statistically independent. If we consider a sequential process, after emission of the first α-particle, the residual nucleus contains the phase of the first emission process; the subsequent decays will follow with very short time delays related to nuclear reaction times (or their inverse, decays), favoring the formation of resonances like ⁸Be(0⁺, 2⁺) and the ¹²C⋆(0⁺, 2⁺) states.

Another view for the α-gas in nuclei is the concept of a collective super-fluid state with a broken symmetry, the α-particle number, a concept much used for neutron pairing in superfluid states in nuclei [1][35]. For the two-neutron pairing states in heavy nuclei, the transfer of neutron pairs between superfluid nuclei [35][36] is strongly enhanced. The analogy to the enhancement of the transfer of correlated neutron-pairs, is the multiple emission of α-particles as a collective transition (changing the particle-number as a collective variable) from compound nuclei with superfluid properties (with α-condensates), i.e. between nuclei with different numbers of α-particles. This feature has been discussed for α-particle transfer between very heavy nuclei in the valley of stability in ref. [37]. Thus the observation of enhanced multiple emissions of α-particles from the compound state can be proposed as the signature for the observation of the collective Bose-gas. These multiple emission should be strongly enhanced relative to the statistical model prediction. For the latter case the emission of several α-particles would be observed into different angles [25].

The coherent emission should occur into the same (identical) angle. This will lead to the situation that the observation of unbound resonances becomes possible, such as ⁸Be(0⁺, 2⁺) and the excited states of ¹²C, the ¹²C⋆(0⁺, 0⁺)-clusters. This feature in fact has been observed in the recent data [24][25][26][27][28][29] discussed in the next section.
3.3 Compound states with multi-α decays

We are interested in the coherent multiple α-particle emission from excited compound nuclei (CN). Due to the coherent properties of the threshold states consisting of α-particles interacting with a large de-Broglie wave length [7], the decay of the CN will not follow the Hauser-Feshbach assumption of the statistical model: a sequential decay and that all decay steps are statistically independent.

After emission of the first α-particle, the residual α-particles in the nucleus contain the phase of the first emission process; the subsequent decays will follow with very short time delays related to nuclear reaction times, and possibly shorter then the $10^{-18}$ seconds of CN decay, actually a simultaneous decay can be considered. This fact should be responsible for the enhanced formation of resonances like $^9$Be and the $^{12}$C*(0⁺, 2⁺) states. An enhanced emission of multiple α-particles is predicted [11]. Very relevant, however, is the larger radial extension of the Boson condensate states, as discussed in refs. [17, 18, 26].

The best way to study such decays is the combination of multi-detector arrays for particle detection with ΔE-E detectors and a “calorimeter” to observe the remaining compound nucleus residue via its γ-decay. Such experiments have been performed with the large γ-detector array GASP at the Legnaro National Laboratory LNL at Padua (Italy), combined with the charged particle detector ball ISIS (details are given in ref. [24]) consisting of 42 ΔE-E telescopes (see Fig. 6).
Fig. 7  Top: This part shows the kinematical situation for the triple pile-up of the signals for 3 α’s in one detector and the emission cone for α’s from the decay of $^{12}$C($^0_2+$). Bottom: Plot of ΔE-E-signals as observed with the ISIS charged particle detector system. The events with the emission of single α’s, of $^8$Be and with three α’s from the state $^{12}$C($^0_2+$) are indicated. The reaction is $^{28}$Si$+^{24}$Mg $\rightarrow^{52}$Fe $\rightarrow^{40}$Ca + X at $E_{\text{lab}}=130$ MeV (courtesy of Tz. Kokalova).

These experiments were performed in a study of γ-decays of compound nuclei selected with a particular particle decay [24]. The large opening angle of the individual ISIS-ΔE-E telescopes, which was 27°, allows to select the spontaneous decay of the weakly unbound states, namely of $^8$Be into two α’s and the $^{12}$C($^0_2+$), into three α-particles. With the rather modest kinetic energy of these fragments and the small decay energies of a few 100 KeV the opening angles between the α’s are in the range of 10° - 25°, which fit into these solid angles. Therefore these prompt multiple α-decays are observed by the pile-up of the signals produced by individual alpha-particles in one of the ΔE-E telescopes. This is shown in Fig. 7. The corresponding
Coincident $\gamma$-spectra gated with the particles from $\Delta E$-telescopes with the emission of three random $\alpha$'s at different angles in different detectors (upper panel), in comparison with that obtained by the $^{12}$C$^{0+}$-gate (lower panel). The reaction is $^{28}$Si + $^{24}$Mg $\rightarrow$ $^{52}$Fe $\rightarrow$ $^{40}$Ca + 3$\alpha$ at 130 MeV. Note the additional lines for $^{36}$Ar in the lower panel (courtesy of Tz. Kokalova).

coincident (particle gated) $\gamma$-decays are compared with the spectra obtained from statistical emission (with the same $\alpha$-multiplicity) into different $\Delta E$-E telescopes (see Fig. 8).

The comparison of the two $\gamma$-spectra with different triggers is shown in Fig. 8 for the reaction $^{28}$Si + $^{24}$Mg $\rightarrow$ $^{52}$Fe $\rightarrow$ $^{40}$Ca + 3$\alpha$, an experiment designed for the spectroscopy of $^{40}$Ca. The spectrum gated with three $\alpha$'s in one telescope shows additional $\gamma$-transitions in $^{36}$Ar, connected with an emission of an additional $\alpha$-particle, it is a dramatic effect, because these transitions are completely absent in the other spectrum gated by random directions of the 3 $\alpha$'s. Initial attempts to explain these differences by parameters of the statistical compound nucleus decay failed, see ref. [25]. A subsequent analysis [26], which uses the features of a $\alpha$-condensed state, namely the larger diffuseness and the larger radial extension gave as an important effect a strong lowering by 10 MeV of the emission barrier for the
emission of $^{12}\text{C}^*_0$. This fact explains, that the energies of the $^{12}\text{C}^*_0$ are concentrated at much lower energies as compared to the summed energy of 3 $\alpha$-particles under the same kinematical conditions. In this way the residual nucleus ($^{40}\text{Ca}$) attains a much higher residual excitation energy [25].

I also show the results of the previous study of the reactions $^{32}\text{S} + ^{24}\text{Mg}$ for the $\gamma$-spectroscopy of $^{48}\text{Cr}$ with $^8\text{Be}$-emission [27, 28, 29] performed with the same mentioned ISIS-GASP-combinations at the LNL in Legnaro. In Fig. 9 we show the identification of $^8\text{Be}$, and on the right side the comparison of the energy spectra, under the same kinematical conditions, for $^8\text{Be}$ and the sum energy of the two $\alpha$'s. We note that the energy spectrum of the $^8\text{Be}$ is shifted to smaller energies, as in the previous case. This again must be explained by a larger diffuseness of the CN-state (an $\alpha$-condensed state) and a lowered Coulomb barrier for the $^8\text{Be}$ emission. The $\gamma$-spectra with the two possible particle gates are shown in Fig. 10. It shows the case of $^8\text{Be}$-emission compared with the statistical emission of two $\alpha$'s in two different detectors, the latter representing the usual statistically independent decay into two different detection angles. Again we found that the particular channel with $^8\text{Be}$ carries less energy and less angular momentum, therefore more subsequent decays are observed. In this case a subsequent neutron and proton emission is observed, the $^{46}\text{Ti}$-channel is strongly increased for the $^8\text{Be}$-gate. Attempts to explain these differences in terms of parameters of CN-decay, (discussed in ref. [28]) gave no conclusive result. At that time the concept of condensed $\alpha$-particle states in the CN was not considered.
Fig. 10 Coincident γ-spectra gated with the particles from ΔE-E-telescopes with the emission of two random α’s in different detectors (upper panel), in comparison with that obtained by the $^8\text{Be}$-gate (lower panel). The reaction is $^{32}\text{S} + ^{24}\text{Mg} \rightarrow ^{56}\text{Ni} \rightarrow ^{48}\text{Cr} + 2\alpha$ at 130 MeV (courtesy of S. Thummerer).

3.4 Inelastic excitation and fragmentation

The last entry in Tab. I the last column for the $^{40}\text{Ca}$-core has a negative sign for $^{164}\text{Pb}$, indicating that this nucleus, as well as lighter nuclei (actually above Z=72), are unstable in their ground state to single and multiple proton or multi-α-particle decay. For lighter N=Z nuclei, at excitation energies above $E_{\text{crit}}^\ast$ another decay mode (as already mentioned) becomes possible, which we call Coulomb explosion.
The condensed states are radial monopole excitations with respect to the ground state. The monopole states located at high excitation energies can best be excited by Coulomb excitation at the highest projectile energies. Coulomb excitations of the GQR (giant quadrupole resonance) or the GDR (giant dipole resonance) have been studied up to 300 MeV/Nucleon. The highest cross sections, also for the monopole excitations, are expected at projectile energies above 1 GeV/nucleon. Because of the larger step in excitation energy the increment for the dynamical matching becomes optimum at these highest energies. Such studies exist for some of the lighter N=Z nuclei ($^{12}$C, $^{16}$O, $^{20}$Ne) and heavier. In these studies nuclear emulsions have been used, the silver nuclei (Ag) acting as target nuclei for Coulomb excitation.

![4.5A GeV/c 16O Coherent Dissociation with 8Be like fragmentation](image)

**Fig. 11** Break-up of $^{16}$O at 4.5 GeV/nucleon with the emission of 4 $\alpha$’s, registered in an emulsion. Details of the decay can be seen, e.g. the more narrow cone of two $\alpha$’s, due to the emission of $^8$Be. Different stages of the decay, registered down stream in the emulsion are shown in consecutive panels. P. Zarubin private communication and ref. [42].

The results were obtained at the JINR in Dubna with beams from the Nucletron accelerator. The reaction products are registered in nuclear emulsions, the Coulomb break-up being induced by the heavy target nuclei (Silver, Ag) of the material. In this way very characteristic multiple tracks after break-up have been observed (see Figs. 11 and 12). In the case of $^{16}$O we observe two $\alpha$’s and a $^8$Be, this fact points to the previous discussions of a coherent emission, two $\alpha$’s must be emitted in a close correlation (in energy and space) in order to be able to form a $^8$Be resonance. We expect the formation of $^8$Be from the internal structure of the condensate state in $^{16}$O, but also in the case of a simultaneous (coherent) emission (and only in this case) the interaction of the two $\alpha$’s can form $^8$Be.

The result for the break-up of $^{20}$Ne is shown in Fig. 12 among the different observed break-up’s the emission of 5-$\alpha$’s is observed with remarkable intensity. Again at least one pair of $\alpha$’s is observed, indicating coherent emission with strong correlations, which allow the formation of the low lying $^8$Be resonances.

In this context we mention that Coulomb explosion has been observed in highly charged atomic van der Waals clusters and is discussed by Last and Jortner. In our case the simultaneous emission of many $\alpha$-particles is expected, a decay
process very different from standard statistical compound nucleus decay. In fact in this decay mode the $\alpha$-particles must have all the same energy.

### 3.5 $\alpha$-$\alpha$ correlations

There have been numerous studies of particle-particle correlations in higher energy nuclear reactions around 50-100 MeV/Nucleon [43], as well as for reactions at relativistic energies, where pion-pion correlations have been studied [44]. From this work we find that the spatial and time extension of the source can be studied in these correlations. A specific feature appears here, that the correlations of bosons will exhibit a maximum at the smallest angles and smallest relative momenta. However, with two $\alpha$-particles the Coulomb interaction and the resonances in the $\alpha$-$\alpha$ channel, states in the $^8$Be nucleus dominate the correlations [43]. With an experimental set-up consisting of $\Delta E$-$E$ telescopes like the detector ball EUROSIB (a new development after ISIS described before) consisting of finer granulated detectors, which contain sufficiently small angular resolution, and a $\gamma$-detector ball as in the experiments described in sect. 3.3, the $\alpha$-$\alpha$ correlations should be studied in coincidence with $\gamma$-transitions of the residual N=Z compound nucleus (minus two $\alpha$-particles). With the use of inverted kinematics, the heavier projectiles on a lighter target, the rather low energies of the $\alpha$-particles in the cm system of the compound nucleus will have sufficiently high energy in the laboratory system to be registered in $\Delta E$-$E$-telescopes (an absorber has to be used to block the heavy projectiles). A correlation matrix with $(E_{\alpha_1}, E_{\alpha_2})$, can be constructed. Such correlation matrices (for two $\gamma$-rays) have been constructed in $\gamma$-spectroscopy [45] with $\gamma$-detector balls.
The correlations and the resonances can then be constructed over a wide range of relative and absolute momenta.

4 Conclusions

The data presented here show clearly experimental features, which point to the existence of \( \alpha \)-condensed states giving rise to coherent multi-\( \alpha \)-particle states in excited \( N=Z \) nuclei. With the most recent experimental developments, we can expect that important new features of such states can be observed. These can potentially establish the existence of Bose-Einstein condensates in nuclei, a very promising field of research for future studies.

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