Bose-Einstein condensation as an alternative to inflation

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This essay received an Honorable Mention in the
2015 Gravity Research Foundation Essay Competition

It was recently shown that gravitons with a very small mass should have formed a Bose-Einstein condensate in the very early Universe, whose density and quantum potential can account for the dark matter and dark energy in the Universe respectively. Here we show that the condensation can also naturally explain the observed large scale homogeneity and isotropy of the Universe. Furthermore gravitons continue to fall into their ground state at every epoch, accounting for the observed flatness of space at cosmological distances scales. Finally, we argue that the density perturbations due to quantum fluctuations within the condensate give rise to a scale invariant spectrum. This therefore provides a viable alternative to inflation, which is not associated with the well-known problems associated with the latter.

It is believed that our Universe started around the time of the so-called the Big Bang (BB) singularity, continued to expand, and is currently doing so at an accelerating rate, driven by a small cosmological constant or Dark Energy (DE) [1-4]. The latter constitutes about 70% of all matter/energy, while the rest is shared by Dark Matter (DM), about 25% and visible matter, about 5%. The observed incredible degree of homogeneity and isotropy ('horizon problem') and spatial flatness ('flatness problem') of our Universe are normally attributed to inflation, a proposed short but rapid phase of exponential expansion soon after the BB, which smoothed out all un-evenness [5, 6]. Inflation also appears to give a ready explanation for the lack of observed GUT monopoles in the universe. However, a number of serious problems associated with inflation have been pointed out, including the unknown origin and nature of a scalar field and the class of potentials required for the process, which appears improbable, although not impossible, requiring an enormous fine-tuning, and the ill-understood energy transfer mechanism to the current contents of the Universe [7-11]. Furthermore recent analysis of observed data do not seem to support the simplest inflationary models [12-18]. Following some earlier work by the current author and collaborators [19-22], in this article we propose a much simpler explanation of the above puzzles, one that requires few speculations and almost no fine tuning. We argue that quanta of gravity, or gravitons with a tiny mass (but consistent with observations and theory) form a Bose-Einstein condensate (BEC) in our Universe in the earliest epochs. This is described by a single macroscopic quantum ground state of the size of the universe, which as shown in [22] can correctly account for both DM and DE. Further by its very nature, it is homogeneous and isotropic, a property which persists as the Universe expands to its current epoch and beyond. In addition, since its constituent gravitons are all in their ground states with little or no momentum, they do not propagate as force carriers. This manifests itself as zero spatial curvature at any epoch, just as observed. We also argue that the density perturbation spectrum is scale-invariant in this model. Finally, without the need for additional fields, the problem of energy transfer to current contents is absent.

We first review some essential results from [19-22]. Starting with the quantum corrected Raychaudhuri equation obtained in [19] by assuming a fluid or condensate filling our Universe, and described by a wavefunction \( \phi = R e^{iS} (R(x), S(x)) = \text{real functions} \), the quantum (Bohmian) trajectories, defined by the velocity field \( u_a = \hbar \partial_a S / m \), replace the classical geodesics [23], and it was found that the quantum corrected second order Friedmann equation for the scale factor \( a(t) \) is given by

\[
\frac{\dot{a}}{a} = - \frac{4\pi G}{3} (\rho + 3p) + \frac{\Lambda a^2}{3} + \frac{\hbar^2}{3m^2} h^{ab} \left( \Box R / R \right)_{ab} + \frac{\epsilon_1 \hbar^2}{m^2} R_{ab} \; .
\]

(1)

In [20-22] it was postulated that the condensate is nothing but a BEC of gravitons of a tiny mass \(^1\). In fact it was shown in [22] that the critical temperature of such a condensate is given by

\[
T_c = \frac{3}{m^{1/3} a} K
\]

(2)

\(^1\) in [22] axions were also explored. Axions would still solve the homogeneity problem but not the flatness problem. We ignore the possibility of axions here.
such that for $m \lesssim 1$ eV, the critical temperature exceeds the temperature of the Universe, given by $T = 2.7/\Lambda K$, meaning that most gravitons would have dropped to their ground state, and a BEC would have formed in the very early universe. Further while the temperature dropped to their ground state, and a BEC would have formed much earlier. Since BEC has a couple of degrees do not overlap in the standard BB recombination (within the bigger past light cone from a black patch) as explained above. Figure 1(d) depicts if there was no BB or initial singularity, in which the very large, or probable infinite age of the Universe gives ample opportunity for the BEC to form and temperature to equalize throughout.

Horizon problem

Eq. (2) shows that $T_c(a) \gg T(a)$, and that a BEC spreading across the Universe would have formed almost right after the BB, if there was indeed a BB (when both $T_c, T \to \infty$, but the former still far exceeds the latter). On the other hand, if there was no BB, the scale factor was very small but never zero, and the age of our Universe is infinite, a possibility suggested in [21], then the BEC would have formed much earlier. Since BEC has infinite heat conductivity, total thermal equilibrium and temperature equalization would have been established almost simultaneously throughout the (tiny) Universe. Causality is never violated, the state of the Universe before BEC formation no longer matters, and the BEC can be considered as the initial state for all practical purposes, which evolves eventually to our current Universe. This was also noted earlier in [33]. Also as noted in [20–22], the resulting highly homogeneous and isotropic condensate can be described by a macroscopic wavefunction, e.g. suitably approximated by a shallow Gaussian [51] or having a shallow parabolic profile [52]. Furthermore, also as noted in [22], as the temperature of the Universe falls, more and more gravitons fall in the condensate forming the relation $N_B/N = 1 - (T/T_C)^3$, where $N_B$ and $N$ are the number of gravitons in the condensate and the total number of gravitons (inside + outside the condensate) respectively. $T_C \gg T$ implies $N \approx N_B$, i.e. there are hardly any gravitons left outside the condensate at cosmological scales. The condensate continues to grow, maintaining homogeneity and isotropy at every instant. In other words, no fine-tuned initial conditions for our observable Universe are required. The situation is depicted in Figure 1. The past light cones of CMBR photons at recombination (within the bigger past light cone from a point $O$ in the current epoch) separated by more than a couple of degrees do not overlap in the standard BB

FIG. 1: (a) Big Bang without inflation, (b) Big Bang with inflation, (c) Big Bang with condensate formation, and (d) No Big Bang with condensate formation. Conformal time along the vertical.

Flatness problem

In a quantum theory of gravitation, gravitons are the force carriers, just as photons are for the electromagnetic force in quantum electrodynamics. Indeed, as was shown e.g. in [53] and [54], curvature of spacetime (hence gravitational force) can be entirely built up by graviton fluctuations and their consistent self-couplings, such that they contribute to the full divergence-free energy-momentum tensor. Then since as argued above, the overwhelming majority of gravitons are ‘frozen’ in their ground states in the BEC with little or no momenta (they only have rest energies, are cold and can be identified with DM), they do not propagate, there is no gravity nor gravity waves, and space is flat at any given instant of time. This can also be expressed in terms of perturbations $h_{ij}$ over the flat metric (no gravity), or ‘ripples in spacetime’ which are absent at large scales, and the 3-dimensional Riemann tensor

$$R_{ijkl}^{(3)} = \frac{1}{2} \left( h_{ij,kl} + h_{jk,il} - h_{ik,jl} - h_{il,jk} \right) \equiv 0 , \quad (3)$$

i.e. it vanishes identically. That the spatial part of Einstein equations are trivial can also be seen from [53–54]. This is also consistent with the well-known result that the amplitude of gravity waves is related to the Weyl tensor [55, 56], which is zero for the FRW Universe. The over-
all expansion (preserving homogeneity, isotropy and flatness) is of course driven by the effective cosmological constant due to the wavefunction of the graviton condensate, as explained earlier. Like homogeneity and isotropy, the flatness of space at cosmological scales is also preserved at every instant of time including at present, and no fine-tuning in early epochs are needed. Observed spacetime curvatures and gravity waves at smaller (astrophysical) scales on the other hand are due to local gravitational effects at the level of stars and galaxies. Similarly, any gravity waves at cosmological scales can be attributed to the residual gravitons outside the BEC.

Scale invariant spectrum

The commonly held view is that the minute quantum fluctuations of the scalar inflaton field and that of the spacetime metric coupled to it, and the ‘freezing’ of long wavelength modes outside the horizon during inflation, and their subsequent re-entry at a later time are responsible for the scale invariant spectrum of perturbations over the uniform background, and also the observed large scale structures in our Universe [57–59]. But as pointed out in [60], slow-roll conditions are not essential for the above. In lieu of the inflaton field, now the wavefunction \( \varphi \), as well as its fluctuations satisfy the relativistic Gross-Pitaevskii equation (with self-interaction strength \( g \)) \( \Box + m^2 + g|\varphi|^2 \varphi = 0 \) [60, 61], or a suitable generalization thereof. As noted earlier, due to its infinite heat conductivity, the BEC with uniform temperature throughout is established rapidly, both inside and outside the horizon radius. Therefore quantum fluctuations also occur almost simultaneously throughout, and modes outside the horizon are frozen. Subsequently as the Hubble radius expands, more and more of such modes enter the horizon. For a mode \( k \) entering the horizon when the scale factor is \( a_B \), and the Hubble parameter \( H_B \), such that \( k/a_B = H_B \), one has

\[
(\Delta \phi_k)^2 \approx \frac{1}{2a_B^3(k/a_B)} \approx \frac{H_B^2}{k^3},
\]

(4)

corresponding to a scale invariant spectrum, just as observed in CMBR anisotropies [60]. This is also seen from the perturbation equations. The formation of modes throughout almost simultaneously implies the relation \( \eta \simeq -1/a_B H_B \) between conformal time and expansion rate, leading to the equation for perturbations [57]

\[
\ddot{v} + \left( k^2 - \frac{2}{\eta^2} \right) = 0,
\]

(5)

(where \( \dot{v} \) denotes scalar or tensor perturbations, and over-dot signifies derivative with respect to the conformal time) which again predicts a scale invariant spectrum. This is consistent with the fact that inflation although sufficient, is not necessary for a scale invariant spectrum [57].

Conclusions

Starting with just one reasonable assumption, namely that gravitons have a tiny mass, consistent with theory and experiments, we have shown that the corresponding critical temperature for condensate formation is extremely high. Thus they will form a condensate at the earliest epochs. The density of the BEC accounts for the DM in our Universe, while an effective cosmological constant derived from the quantum potential of the condensate accounts for the observed DE. Furthermore a macroscopic wavefunction describing the BEC made up of gravitons in their ground states is homogeneous and isotropic by nature, and in the absence of propagating gravitons, space is effectively flat for any time-slice. No fine tuning is required, and the laws of thermodynamics are satisfied; in fact the latter require the formation of the condensate. Our model appears to be at least as effective as inflation in explaining the above observations, but with far fewer independently verifiable assumptions. Other problems associated with inflation, such as reheating, trans-Planckian problem, or that of self-reproduction or multiverses are avoided here as well. Further due to the continuous growth of the cosmic BEC, our model predicts that homogeneity, isotropy and flatness should be preserved at \( \text{all} \) epochs, past and future, unlike inflation, which does not rule out spatial curvatures at very early and late epochs. This is a prediction which may be used to distinguish it from the inflationary paradigm. Since the BEC is present forever (unlike the inflaton, which decays soon after the inflationary era), there should be other testable predictions as well. For example, a comparison of density profiles generated within a BEC at galactic scales may be compared with those that are suggested by observations and simulations of DM. Finally, we note that since the BEC forms right after the BB (or earlier, if there was no BB), there may have not been sufficient time for GUT monopoles to form, even if the theories which predict their production are taken to be correct. Therefore this provides a viable solution of the so-called monopole problem as well.

Acknowledgment

I thank A. F. Ali and R. K. Bhaduri for useful discussions. This work is supported by the Natural Sciences and Engineering Research Council of Canada.

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