Peculiarities of matter excitation by ultra-short electromagnetic pulses

V Astapenko
Moscow Institute of Physics and Technology, Moscow, Russia
E-mail: astval@mail.ru

Abstract. The paper is devoted to theoretical analysis of matter excitation by ultra-short electromagnetic pulses (USP) with controllable shape in the frame of perturbation approach. We calculate the dependence of the total probability of photo-absorption of USP by an atom upon the pulse duration. It is shown that investigated probability has characteristic features which depend upon phase parameters of USP.

1. Introduction

Ultra-short electromagnetic pulses generation with controlled parameters is one of the fastest growing trends in modern experimental laser physics. By now it's possible to obtain pulses of duration 80–100 as in XUV in a line with pulses of a few fs with controlled parameters such as phase and chirp [1 – 7]. These pulses can be generated in teraherz, infrared, visible and far ultraviolet spectra and have duration from sub-cycle up to few cycles at carrier frequency.

This paper is devoted to theoretical study of atomic photo-absorption under the action of USPs with different durations and shapes in the frame of perturbation theory [8].

2. Ultra-short pulses with controllable shape

Consider the impact on the atom pulse of Gaussian shape with electric field varies in time according to the law:

\[ E(t) = E_0 \exp\left(-\frac{t^2}{\Delta t^2}\right) \cos\left(\omega_0 t + \Phi(t)\right), \]

where \( E_0 \) is amplitude of the electric field strength, \( \Delta t \) is parameter proportional to pulse duration (see below), \( \omega_0 \) is carrier frequency, \( \Phi(t) \) is carrier phase with respect to pulse envelope (CE-phase).

Let us consider two cases. In the first case we assume that CE phase is constant: \( \Phi(t) = \varphi = \text{const} \), but from pulse to pulse its value can be changed in controllable way.

Parameter \( \Delta t \) can be expressed in terms of cycle number in the pulse as follows:
\[ \Delta t = \frac{2\sqrt{2\pi} n_c}{\omega_0}, \]  

where \( n_c = \Delta t_p / T = \delta_0 \), \( t_p / 2\pi \) is number of cycles in pulse. Pulse duration \( \Delta t_p \) we define according to the following relation: \( \Delta t_p \leftrightarrow W/w \) (long pulse limit) where \( W \) is total probability of photo-process (during all time action of USP) and \( w \) is the probability per unit time.

In the second case we have: \( \Phi(t) = \kappa t^2 \). Such pulse form is called chirped and parameter \( \kappa \) is temporal chirp. In chirped pulse considered here carrier frequency is linearly dependent on time: \( \omega_c = \omega_0 + \kappa t \).

3. Photo-absorption in the local plasma frequency approximation

Photo-absorption cross-section is calculated here within purely classical approach. Atom is considered as inhomogeneous distribution of electron density \( n(r) \), so that each spatial point has its own local plasma frequency \( \omega_p(r) \). Photon absorption occurs at the distance from atomic nucleus where local plasma frequency matches the frequency of external radiation \( \omega \). Corresponding expression for cross-section has the form [9]:

\[ \sigma_{ph}(\omega) = \frac{2\pi^2 e^2}{mc} \int n(r) \delta(\omega - \omega_p(r)) d^3 r. \]  

Integration in the right-side of equation (3) using delta function gives the photo-absorption cross section within Brandt-Lundqvist approximation:

\[ \sigma_{ph}^{BL}(\omega) = \frac{4\pi^2 \omega}{c} \frac{r_{o_p}^2 n(r_o)}{\left| n'(r_o) \right|} \]  

where \( r_o \) is solution for the equation

\[ \omega = \omega_p(r) = \sqrt{\frac{4\pi e^2 n(r)}{m}}. \]  

4. Atom (statistical model of Lenz-Jensen)

Electron density in the frame of statistical approach has the form:
\[ n_{LJ}(r) = Z^2 f \left( x = r/r_{TF} \right) a_y^{-3}, \]  
\[ r_{TF} = \frac{b}{\sqrt{2Z}} a_y, \quad b = \frac{9\pi^2}{\sqrt{128}} 0.8853 \]

where \( r_{TF} \) is Thomas-Fermi radius, \( a_y = \hbar^2/m e^2 \) – Bohr radius, \( f(x) \) is universal function of dimensionless distance. Expression of this function depends upon statistical model. Lenz and Jensen proposed the following function \( f_{LJ}(x) \) [10]:

\[ f_{LJ}(x) = 3.7 e^{-\frac{0.79}{9.7 x} \left( 1 + 0.26 \sqrt{9.7 x} \right)^3} \left( \frac{9.7 x}{1+2} \right)^{1/2}. \]

This function we use in our calculations because it describes the electron density distribution more realistic in comparison with Thomas-Fermi function.

4. Total photo-absorption probability

Expression for total probability of photo-process under the action of ultra-short electromagnetic pulses can be obtained in the first order of perturbation theory as was shown in paper [8]. Corresponding expression has the form:

\[ W_{\text{tot}}(n_c) = \frac{c}{(2\pi)^2} \int_0^\infty \sigma(\omega) \left| \frac{E(\omega, n_c)}{\hbar \omega} \right|^2 d\omega, \]  

where \( \sigma(\omega) \) is the spectral cross-section of the process, \( c \) is light velocity, \( E(\omega, n_c) \) is Fourier transform of electric field strength in the pulse.

Normalized total probability of photo-process at monochromatic limit \( (n_c >> 1) \) can be presented in the form:

\[ W_{\text{tot}}^{(\text{mon})}(n_c) = \frac{W_{\text{tot}}^{(\text{mon})}}{\left| E(\omega_0) \right|^2} \rightarrow \frac{c \sigma(\omega_0)}{\hbar \omega_0^2} n_c. \]

Hence, probability of the photo-process in a monochromatic field (long pulse limit) increases linearly with the number of cycles in pulse \( n_c \) as follows from the traditional consideration. Moreover in monochromatic limit the total probability of the photo-process does not depend upon the CE phase \( \varphi \) as one can see from (9).
In the opposite case of zero-duration electro-magnetic pulse (\( n_c \ll 1 \)) for \( \Phi(t) = \varphi = \text{const} \) we have [8]:

\[
W_{\text{tot}}(n_c) = \frac{4\pi}{3} \left( \frac{E_0}{\hbar \omega} \right)^2 n_c^2 \langle 0|\hat{d}^2|0 \rangle \cos^2 \varphi, \tag{10}
\]

where \( \langle 0|\hat{d}^2|0 \rangle \) is average value of squared dipole operator of atomic electrons, \( E_0 \) is amplitude of electric field strength in the pulse. Note that the sum rule was used in the derivation of right-side of equality (10).

It is of the interest to investigate the dependence of total probability of photo-process upon parameter \( n_c \) in the intermediate region (\( n_c : 1 \) and \( n_c \leq 1 \)). For this purpose we used formulas (8) and (4) – (7) for the calculation of USP photo-absorption by Lenz- Jensen atom.

5. Results

The results of our calculations are presented in fig. 1–3 for Lenz-Jensen atom with nucleus charge number \( Z = 30 \) and for normalized total probability of atomic photo-absorption (\( \mathcal{W}_{\text{tot}} \)). The latter quantity is defined by following relation

\[
\mathcal{W}_{\text{tot}} = \frac{W_{\text{tot}}}{|E_0|^2}. \tag{11}
\]

Photo-absorption probability under the action of USP with \( \Phi(t) = \varphi = \text{const} \) is shown in fig. 1 for different values of CE phase and for given carrier frequency. One can see that for sub-cycle pulses there is the strong dependence on CE phase. Namely, photo-absorption probability dependence on cycle number in the pulse has maximum for cosine pulse for \( n_c < 0.5 \). The magnitude of this maximum decreases with the increase of CE phase. Straight lines in fig. 1 – 3 present the total probability of photo-absorption in monochromatic limit when the total probability of photo-absorption \( \mathcal{W}(n_c) \) is linear function of \( n_c \).
Figure 1. Total photo-absorption probability of USP with different values of CE phase calculated for Lenz-Jensen atom: solid line – $\phi=0$, dotted line – $\phi=\pi/4$, dashed line – $\phi=\pi/2$, dashed-dotted line – monochromatic limit, $h\omega_0 = 10$ a.u., $Z=30$.

It is shown in fig. 1 that total photo-absorption probability goes to the straight line for $n_c \geq 1$. Thus in considered case only sub-cycle pulses causes nonlinear dependence of $\mathcal{P}(n_c)$ as a function of $n_c$.

Figure 2. Total photo-absorption probability of cosine USP by Lenz-Jensen atom for two different carrier frequencies: solid line – $h\omega_0 = 500$ eV, dashed-dotted line – $h\omega_0 = 1000$ eV; straight lines show corresponding monochromatic limits.
Fig. 2 shows total photo-absorption probability dependence upon parameter $n_c$ for cosine pulse and different values of carrier frequency. From this figure one can see that in case of Lenz-Jensen atom with charge $Z=30$ probability function $\tilde{W}(n_c)$ in linear limit has smaller angle (with respect to abscissa axis) for greater photon energy.

Total probabilities of photo-absorption of chirped pulses by Lenz-Jensen atom are shown in fig. 3. In this case the dependence on cycle number $n_c$ has oscillating character with maxima decreasing when parameter $n_c$ increases (if dimensionless chirp $\alpha = \kappa \Delta t^2 \neq 0$).

![Figure 3](image_url)

**Figure 3.** Total photo-absorption probability by Lenz-Jensen atom for chirped USP, $\hbar \omega_b = 10$ a.u., $Z=30$: solid line: $\alpha=0$, dotted – $\alpha=0.5$, dashed line – $\alpha=1$, dashed-dotted line – $\alpha=2$, solid straight line – monochromatic limit.

Coincidence of photo-absorption probability for few-cycle chirped pulse and monochromatic one persists. However, the number of cycles at which this coincidence occurs is determined by the chirp parameter $\alpha$. The larger chirp parameter the greater number of cycles should contain pulse for the same value of photo-absorption probability as in monochromatic case.

One can see from fig. 3 that the maxima of $\tilde{W}(n_c)$ function grow with increase of chirp value and shifts in the range of longer pulses.
6. Conclusion

In the frame of perturbation theory the analysis of atomic photo-absorption of electromagnetic USP with controllable phase parameter is made in the intermediate region of pulse durations where number of cycles in the pulse \( n_c \) satisfies the following conditions \( n_c : 1 \) and \( n_c \leq 1 \). It is shown that in this region the total probability of photo-absorption \( P^{\text{ph}}(n_c) \) displays peculiarities as a function of parameter \( n_c \).

So in the case of USP with constant CE phase cosine pulse photo-absorption probability has a maximum for \( n_c \leq 0.5 \) while in the case of sine USP such maximum is absent. In the case of chirped pulses photo-absorption probability has oscillating character as a function of cycle number per pulse in the duration range \( n_c : 1 \). Maxima of photo-absorption function \( P^{\text{ph}}(n_c) \) grow in magnitude with the increase of the chirp value and shifts in the range of longer pulse durations.

Acknowledgments

Work was supported by Russian Foundation for Basic Research (Grant No. 10-02-01095-a).

References

[1] Goulielmakis E., Schultze M., Hofstetter M. et al 2008 Science 320 1614
[2] Morgner U. 2010 Nature Photonics 4 14
[3] Krausz F., Ivanov M. 2009 Rev. Mod. Phys. 81 163
[4] Bandulet H.-C., Comtois D., Bisson E. 2010 Phys. Rev. A 81, 013803
[5] Kahra S., Leschhorn G., Kowalewski M. 2012 Nature Physics 8 238
[6] Znakovskaya I., von den Hoff P., Zherebtsov S. 2009 Phys. Rev. Lett. 103002
[7] Jepsen P. U., Cooke D. G. and Koch M. 2011 Laser Photon. Rev. 5 124
[8] Astapenko V.A. 2010 Physics Letters A 374 1585
[9] Brandt W., Lundqvist S. 1965 Phys. Rev. 139 A612
[10] Gombas P. 1949 Die Statistische Theorie des Atoms und ihre Anwendungen (Wien: Springer-Verlag)