Keywords: General Relativity, Dark energy, extra dimensions, gravitational waves.

Abstract: Background In several previous papers [1,2] it was exposed the hypothesis that matter could be considered as consisting of gravitational pulses in a six-dimensional space with anisotropic curvature. These pulses interact between them through various mechanisms, one of which is based on time velocity gradient due to gravitational time dilation. As time dilation can also be achieved by speed, physical consequences of this effect are explored.

Results Universe expansion, which can be expressed as Hubble's law, \( v = H_0 \) causes a frame-dragging and an effective time velocity gradient, resulting in a repulsive force between any particle-pulsation. This force is mass-independent and causes an acceleration that can be expressed by the relationship

\[
\frac{d^2r}{dt^2} = H_0^2 r + H_0^2 r \left(1 - \frac{H_0^2}{c^2} r^2 \right)^{-1} \approx 2H_0^2 r
\]

(approximation valid for distances up to 1000Mpc)

Since our best estimate of Hubble's constant value is 71 km/s/Mpc we can estimate the present value of the cosmological constant \( \Lambda \) at a value of \( 1.178 \times 10^{-52} \) m\(^{-2}\). This number is consistent with current observations.

1. Background.

The hypothesis "Matter as gravitational waves" is based on a six-dimensional space with anisotropic curvature, with three extended spatial dimensions, two compacted spatial dimensions (which form approximately an ellipse with a size of about 10-11 m) and one temporal dimension. These dimensions can be described by a system of cylindrical-elliptic coordinates: the extended dimensions by Cartesian coordinates \( x, y, z \) and the plane of compacted dimensions by elliptic coordinates: \( x y h \). The \( x \) dimension is related to the inverse of the mass of elementary particles and the \( h \) dimension is identified with the Minkowski's imaginary coordinate of space-time.

The solutions to the Einstein's field equations in the weak field approximation take the form of waves when applied to a space like this. These waves travel in helical paths due to confinement produced by the curvature of the two compacted dimensions. This confinement provides them all the properties of a particle. Therefore we can refer to this waves as pulse-particles.

2. Frame-dragging due to Universe expansion

The frame-dragging due to expanding Universe can be derived from Hubble's law: \( v = H_0 \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} = H_0 \cdot r \)

and taking time derivative we have

\[
\frac{d^2r}{dt^2} = H_0 \cdot \frac{dr}{dt} = H_0 \cdot H_0 \cdot r = H_0^2 r \rightarrow a = H_0^2 r
\]

Hence, there is a repulsive force between particles due to the expanding universe. Notice that the acceleration is mass-independent.
3. Time gradient due to Universe expansion.

Time dilation in Special Relativity is given by 

\[ t' = \left(1 - \left(\frac{v}{c}\right)^2\right)^{\frac{1}{2}} t \]

where \( t \) is the static observer's time and \( t' \) is the time for an object which is in motion relative to the observer with constant speed \( v \). Because of Universe expansion there is a velocity gradient given by Hubble's law:

\[ v = H_0 r \]

where \( H_0 \) is the Hubble's constant and \( r \) is the distance from the observer.

Because of this velocity gradient there is a time velocity gradient or time gradient, thus the more distant is an object, slower time would flow compared to any observer. This is given by:

\[ t' = \left(1 - \left(\frac{H_0 r}{c}\right)^2\right)^{\frac{1}{2}} t \]

Because of this time gradient there is an apparent refractive index gradient, thus slower the time is slower pulsations would flow compared to an static observer. The apparent refractive index becomes equal to the ratio between observer's time velocity and the time for an object which is at a distance \( r \) from the observer. This is given by:

\[ n(r) = \frac{c}{v(r)} = \frac{t}{t'(r)} = \left(1 - \left(\frac{H_0 r}{c}\right)^2\right)^{-\frac{1}{2}} \]

(2)

4. Acceleration due to time gradient.

Suppose an static observer that study an static pulse-particle in the expanding Universe. Because of the apparent refractive index the pulse-particle is deflated and moves away from observer in a helicoidal way, as we can see from the figure. (This is illustrated by just the 2 compacted dimensions and one extended dimension for better comprehension)

Since \( n(r) \) is known we can use the ray approximation in order to obtain the curvature radius of any radiation due to the apparent refractive index.

Due to the spherical symmetry of the problem the apparent refractive index gradient is given by:

\[ \nabla n^2 = \frac{d}{dr}\left(1 - \left(\frac{H_0 r}{c}\right)^2\right)^{-\frac{3}{2}} = -\frac{1}{2} \frac{H_0^2}{c^2} \frac{1}{2r} \left(1 - \left(\frac{H_0^2}{c^2} r^2\right)\right) \]
And then:

\[ \frac{1}{\rho} = \frac{1}{c^2 - \frac{H_0^2}{c^2} r^2} \left[ \frac{H_0^2}{c^2} r \left( 1 - \frac{H_0^2}{c^2} r^2 \right)^{-\frac{3}{2}} \right] \frac{1}{r(H')} N \]

operating and taking into account that \( \frac{1}{\rho} = \frac{d^2 r}{ds^2} \) we can write:

\[ \frac{1}{\rho} = \frac{d^2 r}{ds^2} = \left[ \frac{H_0^2}{c^2} r \left( 1 - \frac{H_0^2}{c^2} r^2 \right)^{-1} \right] \frac{1}{r(H')} N \]

For a distant pulse-particle we can take: \( \frac{r}{(r')} N \approx 1 \) and therefore:

\[ \frac{d^2 r}{ds^2} = \frac{H_0^2}{c^2} r \left( 1 - \frac{H_0^2}{c^2} r^2 \right)^{-1} \]

we can apply the chain rule in order to transform the derivative respect to arc into derivative respect to time.

\[ \frac{d^2 r}{dt^2} = \frac{d^2 r}{ds^2} \left( \frac{ds}{dt} \right)^2 + \frac{d r}{ds} \frac{d^2 s}{dt^2} = \frac{d^2 r}{ds^2} \cdot c^2 \]

Replacing:

\[ \frac{d^2 r}{dt^2} \frac{1}{c^2} = \frac{H_0^2}{c^2} r \left( 1 - \frac{H_0^2}{c^2} r^2 \right)^{-1} - \frac{d^2 r}{dt^2} = H_0^2 r \left( 1 - \frac{H_0^2}{c^2} r^2 \right)^{-1} \]

(3)

5. Analysis and Conclusions.

5.1 The term \( \left( 1 - \frac{H_0^2}{c^2} r^2 \right)^{-1} \).

If we graph the second term of expression (3) over distance using a value of \( H_0=71 \) km/s/Mpc we can obtain the figure below:

Notice that this factor is almost equal to one until more or less 1000 Mpc while it tends to infinity at a r value of \( c/H_0 = 1.3 \times 10^{26} \text{ m} = 13,731 \text{ million light-years} \).
Because of Hubble's law a static observer can only obtain information from a sphere of radius equal to $c/\text{Ho}$. But as the universe expands faster than the speed of light when an object exceeds this distance is inaccessible forever (infinite acceleration). Viewed from another perspective, in an expanding universe the matter density must be diminishing, therefore to any observer it would seem that there is a force that gradually draws matter from the observable volume.

5.2 Cosmological constant.

Therefore the acceleration value could be given by:

$$\frac{d^2 r}{dt^2} \sim 2H_0^2 r.$$  

So Einstein was right when he postulated a gravity law given by

$$g = -\frac{A}{r^2} + Br = -\frac{GM}{r^2} + 2H_0^2 r^2 g = -6.6710^{-11} \frac{M}{r^2} + 1.05810^{-35} r.$$  

This gives us a cosmological constant value of

$$\Lambda = \frac{2H_0^2}{c^2} = 1.17810^{-52} m^{-2}.$$  

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5.3 Conclusions

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This value is compatible with observations. Therefore this effect could be the origin of Dark Energy. Notice that an hypothesis selected in order to extend General Relativity to Quantum Mechanics gave us an explanation to the apparently accelerated Universe expansion.
References
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