Research on sampling schemes of Weibull distribution product life test

Zhewen Li\textsuperscript{1,2}, Guixiang Shen\textsuperscript{1,2}, Yingzhi Zhang\textsuperscript{1,2}, Liming Mu\textsuperscript{1,2} and Jun Zheng\textsuperscript{1,2}

\textsuperscript{1} Key Laboratory of CNC Equipment Reliability, Ministry of Education
\textsuperscript{2} School of Mechanical and Aerospace Engineering, Jilin University, Changchun, China

E-mail: zhangyz@jlu.edu.cn

Abstract. According to Weibull distribution timing truncation test, considering that it is difficult to have a strictly defined complete failure samples in engineering practice, so the life sampling schemes of censored failure samples are discussed. Here, we have considered random right censored samples with different censored ratios. Compared with other sampling studies, this paper is not limited to explore more sampling theories, but uses SPSS software to give the specific operation processes of three sampling schemes: simple random sampling, two-stage sampling and Bayes sampling considering the minimum total cost, as well as the discussion on the verification analysis of the sampling accuracy. In order to simplify the estimation of Weibull distribution parameters, the estimated parameters are obtained by the probability plot analysis of Minitab software, thus the error between the sample’s MTBF (Mean Time Between Failure) and the overall MTBF is calculated. The sensitivity of Weibull distribution parameters is obtained by Minitab software. Finally, according to the accuracy of the three sampling schemes with different censored ratios, the optimal life sampling decision is obtained, which may provide sampling reference for the decision makers.

1. Introduction
With the development of quality reliability, higher and higher requirements are put forward for the life test sampling. For example, under the conditions of complete samples and censored samples, the economic requirements and the accuracy of sampling schemes become more critical. The lifetime of engineering machinery products is mostly subject to the two-parameter Weibull distribution. The life sampling and sampling accuracy analysis of Weibull distribution products are important bases for the evaluation of product quality. The research on sampling schemes of different sample types for Weibull distribution products has been a hot topic for scholars in various countries.

The theories of life test design show that, within a certain test time, the timing truncation test is convenient to control the test process and realize the estimation of test costs and resources. For example, Balamurali et al. [1] studied the multi-state life sampling schemes by the timing truncation test. Although the timing truncation test cannot obtain all the failure data, it will significantly reduce the total cost of the test, so as to obtain the economic sampling schemes. However, the traditional timing truncation tests take large samples as the test object, which is difficult to explain the sampling design of small samples.

In recent years, many scholars at home and abroad have improved the experimental design methods and sampling schemes by introducing Bayes theory on the basis of the timing truncation tests, and
obtained good results. Shi et al. [2] designed a Bayes standard sampling scheme based on the exponential distribution, and calculated the shortest test time for the samplings with different failure numbers, but it has fewer constraints on the total cost of the sampling. Based on the historical information of product life, Zhang et al. [3] solved the interval of sample size with the minimum sampling cost as the target. Zeng et al. [4] designed an optimal sampling scheme based on the sampling criterion of minimum expected total cost for life test. Wu et al. [5] also studied the sampling schemes with the minimum total cost, considered the risks of the producer and the user, and carried out sensitivity analysis on the sampling results.

Considering the influence of Weibull distribution parameters on the sampling schemes, Zhang et al. [6] compared the sampling errors of simple random sampling, stratified random sampling and two-stage sampling, and concluded that when the sample size of the two-stage sampling does not exceed half of the simple random samples, the two-stage sampling variance is smaller and the two-stage sampling accuracy is higher. Ma [7] compared the efficiency of the two-stage sampling and stratified sampling, and thought that the efficiency of the two-stage sampling is not high, while the stratified sampling can better reflect the representativeness of samples in the population. Tsai et al. [8] established a sampling scheme with the minimum total cost by means of timing truncation test with known shape parameters and unknown scale parameters varying with batch sizes. Wilrich et al. [9] studied the relationship between Weibull distribution parameters and the sampling schemes by combining the risks of the producer and the user in the timing truncation test. Omari et al. [10] assumed the life obeyed inverse Weibull distribution and solved the minimum sample size under the condition that the average sample life was satisfied. In addition, simple random sampling can also be considered to optimize the sample size. Balamurali et al. [11] considered the setting of shape parameters under the risk of different users, and developed a simple sampling scheme with sample life obeying Weibull distribution. Li et al. [12] used Gibbs sampling principle and Markov chain Monte Carlo (MCMC) method to obtain the maximum likelihood estimation of Bayes prior parameters, simulated the iterative calculation process of Weibull distribution parameters with WinBUGS software, and verified the accuracy of Bayes sampling posterior distribution.

In the research of Weibull distribution product life sampling, most scholars have fully studied the life sampling theory, but the actual sampling operation process is less studied, the existing literature research has not had a thorough discussion on the verification analysis of the sampling accuracy.

The rest of the paper is organized as follows:

In Section 2, Weibull distribution life test sampling schemes of censored failure samples are discussed. Finally, results and discussion, as well as conclusions are given in Section 3 and 4, respectively.

2. Weibull distribution life test sampling schemes of censored samples

Assuming that the sample lifetime obeys a two-parameter Weibull distribution, its distribution function can be expressed as

\[ F(t) = 1 - e^{\frac{-t^m}{\eta}} = 1 - e^{\left(\frac{t}{\eta}\right)^m}, \quad t, m, \eta, \theta > 0 \]  

(1)

In formula (1), \( m \) is the shape parameter, \( \theta \) is the scale parameter, and \( \eta \) is the true scale parameter. In order to verify whether the life sample population obeys Weibull distribution, the probability plot analysis of Minitab software is used to analyze the distribution fitting. The analysis results show that the \( p \) value of Anderson-Darling (AD) statistic in the fitting test is greater than 0.05, indicating that the fitting effect of Weibull distribution is good.

2.1. Replaceable simple random sampling schemes

Without the failure number constraint, this section considers sampling of 20 life samples when the censored ratios of 200 samples are 10%, 20% and 30% respectively, and assumes that the censored samples obey the normal distribution \((0.9, 0.0, 0.2^2)\). The corresponding failure samples are subject to the
same Weibull distribution as the 200 sample population in the Appendix. In order to form 200 sample population, Weibull distribution and normal distribution are used to generate two samples of corresponding quantity.

Enter two columns of variables to store the spindle number and sampling results.

Define data types and determine the sample size.

Complete sampling and arrange in descending order.

**Figure 1.** SPSS simple random sampling procedure.

Weibull distribution probability plot shows that all the three groups of integrated samples obey Weibull distribution. Unless otherwise specified, the numerical units for all samples shall be expressed in thousands of hours (10^3 h). In order to distinguish between life samples and censored samples, the censored samples will be marked with a symbol *. Then, the parameters of Weibull distribution are estimated by Minitab software, so as to calculate the sample’s MTBF.

**Figure 2.** Weibull distribution probability plot with 10% censored ratio.

According to figure 2, the parameters are estimated to be $m = 3.110, \eta = 0.9171, \text{MTBF} \approx 0.8202$.

**Table 1.** $n = 20$ sample lifetime of the simple random sampling with 10% censored ratio.

| count | lifetime | count | lifetime | count | lifetime | count | lifetime |
|-------|----------|-------|----------|-------|----------|-------|----------|
| 1     | 0.205    | 6     | 0.6397   | 11    | 0.8978   | 16    | 0.9683   |
| 2     | 0.435    | 7     | 0.7056   | 12    | 0.914    | 17    | 0.9882   |
| 3     | 0.5299   | 8     | 0.741    | 13    | 0.9554*  | 18    | 1.0562   |
| 4     | 0.5688   | 9     | 0.7456   | 14    | 0.958    | 19    | 1.1629   |
| 5     | 0.5918   | 10    | 0.8697   | 15    | 0.9678*  | 20    | 1.548    |

**Table 2.** $n = 20$ sample lifetime of the simple random sampling with 20% censored ratio.

| count | lifetime | count | lifetime | count | lifetime | count | lifetime |
|-------|----------|-------|----------|-------|----------|-------|----------|
| 1     | 0.2345   | 6     | 0.5059   | 11    | 0.8847   | 16    | 1.1612*  |
| 2     | 0.2821   | 7     | 0.5582   | 12    | 0.9145   | 17    | 1.1964   |
| 3     | 0.2895   | 8     | 0.6333   | 13    | 0.9713   | 18    | 1.2567   |
| 4     | 0.3698   | 9     | 0.6688   | 14    | 1.0535   | 19    | 1.3769   |
| 5     | 0.4331   | 10    | 0.8305   | 15    | 1.1446   | 20    | 1.5750   |

**Table 3.** $n = 20$ sample lifetime of the simple random sampling with 30% censored ratio.

| count | lifetime | count | lifetime | count | lifetime | count | lifetime |
|-------|----------|-------|----------|-------|----------|-------|----------|
| 1     | 0.0522   | 6     | 0.6649*  | 11    | 0.8326*  | 16    | 1.2898*  |
| 2     | 0.4016   | 7     | 0.6671   | 12    | 0.8727*  | 17    | 1.2949   |
| 3     | 0.4524   | 8     | 0.6714   | 13    | 0.9319*  | 18    | 1.591    |
| 4     | 0.4863   | 9     | 0.7691   | 14    | 1.0431*  | 19    | 1.6267   |
| 5     | 0.6249*  | 10    | 0.8048*  | 15    | 1.2396   | 20    | 1.6688   |
For the sampling of 20% censored ratio, we can gain that $m = 2.269, \eta = 0.9248, \text{MTBF} \approx 0.8192$.
For the sampling of 30% censored ratio, we can obtain that $m = 2.134, \eta = 1.007, \text{MTBF} \approx 0.8918$.

2.2. Replaceable two-stage sampling schemes

Suppose that 200 spindle samples come from 20 CNC (Computer numerical Control) machine tools in 5 production bases. In the first stage, 14 machine tools are selected from 5 production bases according to approximate equal probability sampling, and then stratified according to the production bases. There are 3, 6, 4, 3 and 4 machine tools for spindle life sampling in 5 production bases respectively, so the minimum number of sampling units in the second stage is 3 and the maximum is 6. In the second stage, 20 spindle life samples are extracted from 140 samples with approximately equal probability.

Input production base, machine tool and ID, the first stage adopts simple random sampling. 
Set the number and proportion of units in the second stage, and select simple random sampling.
Set the seed number of simple random sampling and complete the second stage sampling.

Figure 3. SPSS two-stage sampling procedure.

Considering that the sample size of spindle samples after the first stage sampling is 140, in order to meet the requirement that 20 samples are taken out from the second stage sampling and the sampling result is an integer, two ratios of 10% and 15% are used for the second stage sampling, and Weibull distribution parameters of 20 samples are estimated to the arithmetic mean of the two types of parameters approximately. Weibull distribution probability plot shows that all the three groups of integrated samples obey Weibull distribution. The following sampling results are given in scatter chart, and the censored samples are specially marked.

Figure 4. Two-stage sampling scheme with censored ratio of 10%.

Therefore, the estimated Weibull distribution parameters of two-stage sampling scheme are $m = 2.2105, \eta = 0.96655$. The MTBF is calculated as $\text{MTBF} \approx 0.856$.

Figure 5. Two-stage sampling scheme with censored ratio of 20%.
As is shown in figure 5, for the sampling with 20% censored ratio, according to the Minitab probability plot, we can obtain two parameters of Weibull distribution as \( m = 2.4645, \eta = 0.9273 \). Therefore, we can calculate that MTBF \( \approx 0.8225 \).

Figure 6. Two-stage sampling scheme with censored ratio of 30%.

As is shown in figure 6, for the sampling with 30% censored ratio, according to the Minitab probability plot, we can obtain two parameters of Weibull distribution as \( m = 2.4315, \eta = 0.96325 \). Therefore, we can calculate that MTBF \( \approx 0.8541 \).

2.3. Replaceable Bayes sampling schemes considering the minimum total cost

The cumulative probability distribution function of two-parameter Weibull distribution is

\[
F(t) = \int_0^t \frac{m}{\eta} \left( \frac{u}{\eta} \right)^{m-1} e^{-\left( \frac{u}{\eta} \right)^m} \, du
\]  

(2)

If \( z = u^m, \theta = \eta^m, x = t^m \), Weibull distribution can be transformed into exponential distribution \( f(x) = \theta e^{-\theta x} \). If the failure sample size is \( r \), for the life sample \( t_1, t_2, \ldots, t_r, \ldots, t_n \), if \( U = \prod_{i=1}^r u_i \), \( X^r = \sum_{i=1}^r t_i \), the likelihood function is expressed as \( L(t; m, \theta) = \theta^r m^{m-1} e^{-\theta X^m} \). Assume that the prior distribution of the shape parameter obeys uniform distribution, for exponential distribution parameter \( \theta \), the conjugate prior distribution is taken as gamma distribution. Thus the joint posterior probability density function of parameters \( m, \theta \) can be expressed.

According to the posterior distribution function of \( m \), the estimated value of shape parameter is

\[
m = \int_{m_{\min}}^{m_{\max}} m \pi(m | t) \, dm = \int_{m_{\min}}^{m_{\max}} \frac{m^{r+1} U^{m-1} (X^m + b)^{m+a}}{\int_{m_{\min}}^{m_{\max}} m' U^{m-1} (X^{m'} + b)^{m'+a} \, dm'} \, dm
\]  

(3)

We can obtain the estimated true scale parameter \( \eta = \left( \frac{X^m + b}{a + r} \right)^{1/m} \), and MTBF can be calculated.

For Bayes sampling scheme with the minimum total cost, it is assumed that the total sampling cost includes factory cost \( A \), total inspection cost \( R \) and sampling inspection cost \( S \). The prior information can be obtained by the probability density function of the unqualified product rate. If the batch of test samples is \( N \), then the total factory cost is \( NA \), the total inspection cost is \( NR \).
Suppose that the probability of accepting and rejecting the sampling scheme is \( p_1, p_2 \). The total expected inspection cost is expressed as \( C_0 = nS + (N-n) \int_0^1 (Ap \cdot p_1 + R \cdot p_2) f(p) dp \). The critical value to determine whether to inspect satisfies \( \bar{p} = R \), when it does not need to be inspected, the revised total cost is expressed as \( C_1 = N \int_0^\bar{p} Apf(p) dp + N \int_{\bar{p}}^1 Rf(p) dp \). The average cost is expressed as \( C_2 = \int_0^\bar{p} Apf(p) dp + \int_{\bar{p}}^1 Rf(p) dp \). The upper limit of cost saving for sampling inspection is \( N \min \left( \bar{p}, R \right) - nS - \left( N-n \right) C_2 \).

Therefore, the minimum total cost sampling scheme can be transformed into the following model.

\[
\begin{align*}
\min C_0 &= nS + (N-n) \int_0^1 (Ap \cdot p_1 + R \cdot p_2) f(p) dp \\
&= \left\{ \begin{array}{ll}
N \min \left( \bar{p}, R \right) - nS - \left( N-n \right) C_2 > 0 & \\
0 & \text{otherwise}
\end{array} \right.
\end{align*}
\]

(4)

Suppose that \( N = 200, n = 20, S = 2000, R = 1500, A = 10000 \). According to the historical data, the prior distribution of the unqualified product rate obeys binomial distribution \( \beta(8, 64) \), and the subsequent probability density function and the expectation are both obtained.

According to the prior distribution, the unqualified initial value \( x \) is taken to meet the expectation, and the critical number \( c \) is determined until the expectation exceeds the average unqualified rate.

In this case, for \( n = 20 \) and \( c = 3 \), the total cost of Bayes sampling is the minimum. The average unqualified product rate is \( \bar{p} \approx 0.108 \). The average cost without inspection is \( C_2 \approx 384.45 \). Bayes sampling scheme is expressed as \( (n, c) = (3, 4, \ldots, 86; 3) \).

Suppose the upper MTBF limit of 200 sample population is taken as the upper MTBF limit of the integrated sample. The process of solving the MTBF interval of 200 sample population is as follows:

First, 200 life samples are arranged in ascending order, and the next item is subtracted with the previous item of two adjacent items to get a set of failure interval time arranged as \( t_1 < t_2 < \cdots < t_{200} \). Without special explanation, \( t_i \) only represents the lifetime of \( i \)-th sample. The upper confidence limit \( p_{ui} \) and lower confidence limit \( p_{li} \) of failure probability with confidence level \( \alpha \) can be described by F distribution. Therefore, Weibull distribution function can be transformed linearly. We can obtain that \( m_{uc} = 5.5732, \eta_{uc} = 0.9306, m_{lc} = 4.8078, \eta_{lc} = 0.9017 \). And approximately, the upper limit \( \theta_0 \) and lower limit \( \theta_0 \) of MTBF are \( \theta_0 \approx 0.8260, \theta_0 \approx 0.8599 \). Therefore, take the super parameter of gamma distribution as \( a = 4, b = 5\theta_0 = 4.2995 \).

Here, we use the equidistant sampling method to determine 20 samples of Bayes sampling. It is defined that the sample distance \( k \) is the ratio of the total number of samples to the number of sample units, \( k = 200 / 20 = 10 \). If the first takes the sample \( w \), then it takes the sample \( 2k - w, 2k + w, 4k - w, \ldots, 20k - w \). The Bayes sampling result of random selection is \( w = 6 \). Bayes sampling schemes with censored ratios of 10%, 20%, and 30% are shown in figure 7 to figure 8.

In order to apply the random starting point isometric sampling grouping, the samples above are supplemented with the sample ID = 200. Set the random starting point \( w = 94 \) and \( k = 7 \).

For the samples with censored ratio of 10%, the 26th, 94th and 174th samples are taken out as unqualified products. Given \( m = 0, m = 2 \), the estimated parameters are \( m = 1.594, \eta = 2.812, \theta = 5.197, \text{MTBF} = 2.522 \). Similarly, for the samples with censored ratio of 20%, the estimated distribution parameters are \( m = 1.714, \eta = 3.071, \theta = 6.842, \text{MTBF} = 2.738 \). For the samples with censored ratio of 30%, the estimated parameters are \( m = 1.579, \eta = 2.762, \theta = 4.794, \text{MTBF} = 2.439 \).
Figure 7. Bayes sampling with censored ratios of 10% (a) and 20% (b).

Figure 8. Bayes sampling with censored ratio of 30%.

In order to apply the random starting point isometric sampling grouping, the samples above are supplemented with the sample ID = 200. Set the random starting point \( w = 94 \) and \( k = 7 \).

For the samples with censored ratio of 10%, the 26th, 94th and 174th samples are taken out as unqualified products. Given \( m_1 = 0, m_2 = 2 \), the estimated parameters are \( m = 1.594, \eta = 2.812, \theta = 5.197 \), \( \text{MTBF} \approx 2.522 \). Similarly, for the samples with censored ratio of 20%, the estimated distribution parameters are \( m = 1.714, \eta = 3.071, \theta = 6.842 \), \( \text{MTBF} \approx 2.738 \). For the samples with censored ratio of 30%, the estimated parameters are \( m = 1.579, \eta = 2.762, \theta = 4.794 \), \( \text{MTBF} \approx 2.439 \).

3. Result and discussion

Weibull distribution parameters and MTBF of the censored samples are shown in Table 4 to Table 6.

Table 4. Weibull distribution parameters and MTBF of 10% censored ratio sample.

| Sampling scheme | Shape | Scale | MTBF |
|-----------------|-------|-------|------|
| Sample population | 2.121 | 0.856 | 0.8233 |
| Simple random | 3.110 | 0.7640 | 0.8202 |
| Two-stage | 2.2105 | 0.9276 | 0.8560 |
| Bayes | 1.594 | 5.197 | 2.522 |

Table 5. Weibull distribution parameters and MTBF of 20% censored ratio sample.

| Sampling scheme | Shape | Scale | MTBF |
|-----------------|-------|-------|------|
| Sample population | 2.215 | 0.7402 | 0.6556 |
| Simple random | 2.269 | 0.8375 | 0.8192 |
| Two-stage | 2.4645 | 0.7551 | 0.8303 |
| Bayes | 1.714 | 6.842 | 2.738 |
Table 6. Weibull distribution parameters and MTBF of 30% censored ratio sample.

| Sampling scheme     | Shape  | Scale  | MTBF  |
|---------------------|--------|--------|--------|
| Sample population   | 2.148  | 0.8214 | 0.8161 |
| Simple random       | 2.134  | 1.0150 | 0.8918 |
| Two-stage           | 2.4315 | 0.9130 | 0.8541 |
| Bayes               | 1.579  | 4.794  | 2.479  |

It should be pointed out that “Sample population” represents 200 sample data with different censored ratios, and “Sample population” is a benchmark method to measure sampling accuracy, while simple random sampling, two-stage sampling and Bayes sampling are used as comparison methods, and MTBF estimation error between the extracted sample and sample population is taken as comparison index.

Therefore, the best life sampling schemes with 10%, 20% and 30% censored ratio are simple random sampling, simple random sampling and two-stage sampling, respectively.

In order to further discuss the sensitivity of Weibull distribution parameters with different sample sizes and censored ratios, the integrated samples with censored ratios of 0.1, 0.2, 0.3, 0.4 and 0.5 are established respectively. The sample sizes are 10, 20, 30, 40 and 50 respectively. It is assumed that the life samples obey Weibull distribution \( m=1.5, \theta=1 \) and the censored samples obey normal distribution \((0.9,0.2)\). In particular, when the difference between AD statistics of the optimal distribution and Weibull distribution is very small, it is considered that Weibull distribution is approximately obeyed. On the premise of right censoring, the estimated Weibull distribution parameters are calculated by Minitab software, as shown in figure 9.

![Figure 9](image.png)

Figure 9. Variation of shape parameter (a) and scale parameter (b) of Weibull distribution.

As shown in figure 9 (a), if the censored ratio is small (10% ~ 20%), the shape parameter increases with the increase of the censored ratio; if the censored ratio is large (30% ~ 50%), the shape parameter decreases with the increase of the censored ratio. In particular, for the case of small sample size \( n=10 \), the variation law of shape parameter is not obvious, so it is not considered. Under the same censored ratio, the shape parameter decreases first but then increases with the increase of sample size.

As shown in figure 9 (b), under the same sample size, if the censored ratio is small (10% ~ 20%), the scale parameter decreases with the increase of the censored ratio; if the censored ratio is large (30% ~ 50%), the change trend of scale parameter is not obvious. In particular, for the sample size \( n=20 \), there is a sharp increase in the later change of scale parameter, because there is only one available life sample, which is not contradictory to the change rule of scale parameter. Under the same censored ratio, the scale parameter first increases but then decreases with the increase of sample size.

4. Conclusion

In summary, for the sampling problem of 20 samples extracted from 200 censored sample population, the following conclusions are drawn: for the sample population with low censored ratio, the precision
of simple random sampling is higher; for the sample population with higher censored ratio, the precision of two-stage sampling is higher. Two-stage sampling has better stability than simple random sampling for sample population with low and high censored ratios.

For sensitivity analysis of Weibull distribution parameters, under the same sample size, the shape parameter first increases but then decreases with the increase of the censored ratio; the shape parameter decreases at first but then increases with the increase of sample size under the same censored ratio.

Under the same sample size, the scale parameter decreases with the increase of censored ratio, and the change trend is not obvious; under the same sample size, the scale parameter increases first but then decreases with the increase of sample size. Above all, the results of life test sampling give the best sampling decision that the decision maker should take to some extent.

**Appendix**

**Table.** The 200 life samples of machine tool spindles in the timing truncation test in Section

|        | 0.07924 | 0.34582 | 0.49204 | 0.62136 | 0.70746 | 0.82122 | 0.91689 | 1.12655 | 1.38094 |
|--------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|        | 0.08532 | 0.35233 | 0.49242 | 0.63505 | 0.71336 | 0.82129 | 0.91781 | 1.13043 | 1.38342 |
|        | 0.13771 | 0.36487 | 0.49959 | 0.63539 | 0.71657 | 0.82442 | 0.93494 | 1.13189 | 1.39096 |
|        | 0.15248 | 0.36605 | 0.50098 | 0.63662 | 0.72547 | 0.83876 | 0.95343 | 1.14917 | 1.3983  |
|        | 0.17266 | 0.3687  | 0.52596 | 0.64168 | 0.72969 | 0.8398  | 0.95873 | 1.15216 | 1.43312 |
|        | 0.17877 | 0.37945 | 0.53756 | 0.64436 | 0.73893 | 0.8403  | 0.96632 | 1.17267 | 1.51455 |
|        | 0.19869 | 0.38147 | 0.54068 | 0.65561 | 0.74041 | 0.84815 | 0.96821 | 1.17348 | 1.5691  |
|        | 0.23443 | 0.39552 | 0.54502 | 0.65672 | 0.74171 | 0.84884 | 0.97239 | 1.18113 | 1.58663 |
|        | 0.24844 | 0.40614 | 0.54983 | 0.67186 | 0.75441 | 0.85665 | 1.00996 | 1.18741 | 1.61414 |
|        | 0.25449 | 0.40615 | 0.55126 | 0.673   | 0.76739 | 0.86588 | 1.01081 | 1.23476 | 1.62602 |
|        | 0.25928 | 0.42624 | 0.55594 | 0.67815 | 0.76796 | 0.86947 | 1.01195 | 1.23624 | 1.68776 |
|        | 0.26361 | 0.4284  | 0.56567 | 0.67889 | 0.7701  | 0.87593 | 1.01728 | 1.24404 | 1.77512 |
|        | 0.28102 | 0.43447 | 0.57003 | 0.68032 | 0.77446 | 0.88325 | 1.01871 | 1.25693 | 1.79614 |
|        | 0.28815 | 0.45111 | 0.57572 | 0.68273 | 0.78184 | 0.88341 | 1.02051 | 1.25852 | 1.90719 |
|        | 0.29586 | 0.4534  | 0.57658 | 0.68799 | 0.78284 | 0.88479 | 1.02423 | 1.29077 | 1.96619 |
|        | 0.30377 | 0.45347 | 0.57696 | 0.69028 | 0.78942 | 0.89317 | 1.04112 | 1.30008 | 2.08395 |
|        | 0.30529 | 0.46618 | 0.58916 | 0.69233 | 0.79318 | 0.89614 | 1.0516  | 1.30482 |          |
|        | 0.31204 | 0.46935 | 0.60106 | 0.69316 | 0.79488 | 0.90218 | 1.06653 | 1.31859 |          |
|        | 0.32087 | 0.46989 | 0.60301 | 0.69461 | 0.79586 | 0.90561 | 1.06807 | 1.32858 |          |
|        | 0.33964 | 0.47444 | 0.60557 | 0.69531 | 0.80436 | 0.90676 | 1.06851 | 1.3473  |          |
|        | 0.34187 | 0.4857  | 0.6076  | 0.69744 | 0.81308 | 0.91018 | 1.09049 | 1.35411 |          |
|        | 0.34195 | 0.48579 | 0.61533 | 0.69767 | 0.81704 | 0.91161 | 1.10686 | 1.35455 |          |
|        | 0.3444  | 0.4887  | 0.61948 | 0.70073 | 0.81879 | 0.91292 | 1.10687 | 1.36012 |          |

**Acknowledgments**

This work was supported by Jilin Province Science and Technology Development Plan Project (Grant No. 20190302104GX), and the National Science and Technology Major Project of the Ministry of Science and Technology of China (Grant No. 2015ZX04005005). Yingzhi Zhang is the corresponding author of this paper.

**References**

[1] Balamurali S, Jeyadurga P and Usha M 2017 Optimal designing of a multiple deferred state sampling plan for Weibull distributed life time assuring mean life *American Journal of Mathematical and Management Sciences* **36**(2) (Krishnan: Elsevier) 150-161

[2] Z Shi, Z L Kuang and C Yang 2009 Design of Bayes standard timing truncation test scheme for exponential distribution *Reliability and environmental test of electronic products* **27**(2) (Guangzhou: CNKI) 6-9
[3] S J Zhang and Z G Liu 2017 Sampling inspection method of small batch products based on Bayes theory *Statistics and decision* 11 (Shijiazhuang: CNKI) 24-27
[4] Y Zeng, L Xiang and Y X Yan 2014 Life test sampling scheme based on the minimum expected total cost under exponential Weibull distribution *Practice and understanding of mathematics* 44(13) (Sichuan: CNKI) 201-209
[5] S J Wu and S R Huang 2012 Progressively first-failure censored reliability sampling plans with cost constraint *Computational Statistics & Data Analysis* 56(6) (Taipei: Elsevier) 2018-30
[6] H B Zhang, Z X Jia and A M Xi 2008 Research on sampling method of reliability test data of CNC machine tools *Reliability and environmental test of electronic products* 26(1) (Beijing: CNKI) 30-33
[7] J Y Ma 2018 Comparison of efficiency of two-stage sampling and stratified sampling *Enterprise science and technology and development* (Shanxi: CNKI) 1 65-67
[8] T R Tsai, Y T Lu and S J Wu 2008 Reliability sampling plans for Weibull distribution with limited capacity of test facility *Computers & Industrial Engineering* 55(3) (Taipei: Elsevier) 721-728
[9] Wilrich P T 2018 *Sampling Inspection by Variables Under Weibull Distribution and Type I Censoring Frontiers in Statistical Quality Control* 12 (Berlin: Springer)
[10] Omari A and Ibrahim A The transmuted generalized inverse Weibull distribution in acceptance sampling plans based on life tests *Transactions of the Institute of Measurement and Control* 40(16) (Mafraq: ResearchGate) 4432-43
[11] Balamurali S, Jeyadurga P and Usha M 2018 Designing of tightened-normal-tightened sampling scheme under Weibull and gamma distributions for mean life assurance *Communication in Statistics- Simulation and Computation* 2 (Krishnan: Elsevier) 1-26
[12] Y Q Li and J Jiang 2017 Research on the distribution of marine diesel engine wear based on MCMC method *Computer knowledge and technology* 13(10) (Beijing: CNKI) 190-4