Diagnostics of inhomogeneous stellar jets

Convolutions and data reconstruction

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ABSTRACT

Context. In the interpretation of stellar jet observations, the physical parameters are usually determined from emission line ratios, obtained from spectroscopic observations or using the information contained in narrow band images. The basic hypothesis in the interpretation of the observations is that the emitting region is homogeneous across the line of sight. Actually, stellar jets are in general not homogeneous, and therefore line of sight convolution effects may lead to the main uncertainties in the determination of the physical parameters.

Methods. We used standard techniques to determine the physical parameters, i.e., the electron density, temperature and hydrogen ionisation fraction across the jet, and a multi-Gaussian method to invert the Abel transform and determine the reconstructed physical structure.

Results. When assuming an homogeneous medium the physical parameters, integrated along the line of sight, do not represent the average of the true values, and therefore line of sight convolution effects may lead to the main uncertainties in the determination of the physical parameters.

Key words. ISM: Herbig-Haro objects – ISM: jets and outflows – Techniques: image processing – Methods: data analysis – Stars: pre-main sequence – winds, outflows

1. Introduction

Stellar outflows are generally observed as chains of bright knots with a characteristic spectrum that mainly includes forbidden emission lines (see, e.g., the review by Reipurth & Bally [2001]).

Different approaches are used to extract the information on the outflow excitation conditions from the observed emission lines. To calculate the electron density ne, the standard method involves the line ratio [SII]λ6716/λ6731, which is strongly dependent on ne and less sensitive to the electron temperature T (see, e.g., Osterbrock [1989]). The [SII] ratio, together with the ratios between lines with different excitation temperatures (e.g., [OI]λ5577/λ6300) is used to determine both ne and T. Moreover, the [SII], [OI]λλ6560, 6363 and [NII]λλ6548, 6583 emission lines are often used to calculate, apart from ne and T, the hydrogen ionisation fraction xH and the total density nH (=ne/xH) (Bacciotti & Eisloeffel [1999] hereafter BE99). Additionally, several line ratios in the optical and near infrared wavelength range (e.g., Brugel et al. [1981], Pesenti et al. [2003], Nisini et al. [2005], Podio et al. [2006]) can be combined, with weights representing their respective errors (Hartigan & Morse [2007] hereafter HM07), to determine the physical parameters of the HH objects.

Another possibility consists of obtaining predictions of emission line intensities from plane parallel shock models (e.g., Hartigan et al. [1987], Hartigan et al. [1994], Lavalleey-Fouquet et al. [2000], Pesenti et al. [2003]) or from numerical simulations that include a detailed treatment of the ionisation and recombination evolution of several different species (e.g., Raga [2007]), and compare the results with observations.

It is important to develop strategies to properly extract the maximum possible amount of information from the observed line ratios. Convolution, instrumental and observing conditions (i.e., seeing) affect the derivation of physical parameters from observations. Projection or convolution effects are an obvious consequence of the lack of information on the spatial distribution along the line of sight. Additionally, if the cooling region is not resolved the observed emission lines come from regions with different excitation conditions along the main jet axis. Actually, also in observations where the cooling region is well resolved the determined parameters are a convolution of the volumetric parameters along the line of sight, i.e., across the jet.

Only for an homogeneous medium the mean values of ne, T, xH (integrated within the beam size and along the line of sight) will coincide with the corresponding values calculated from the observed line ratios. In the case of an inhomogeneous medium...
it is necessary to understand what relation holds between the derived and the volumetric mean physical parameters.

This problem has been studied in detail in the context of stellar atmospheres by several authors (e. g. Doschek [1984] Almleaky et al. [1989] hereafter ABS99, Brown et al. [1991] Judge et al. [1997], McIntosh et al. [1998] Judge [2000]. As they show the information contained in a set of line ratios may be used to determine the function $\zeta(n_e)$, defined as the “emission measure differential in density”, given by $n_e^2(z)dz = \zeta(n_e)dn_e$ (e. g. Judge 2000), where $z$ is the position across the stellar atmosphere. If several line ratios are available, and if the geometry of the system is known, it becomes possible to infer the dependence of the density $n_e$ along the line of sight.

Despite its importance, this problem has not been studied in similar detail in the context of stellar outflows. As far as we are aware, only Safer (1992) studied the influence of inhomogeneities on the density diagnostics in outflows from T Tauri stars, applying a simple model of an isothermal medium with a power law radial dependence for the electron and total densities, and showing that different ratios may be used to obtain information on the density structure.

Recently, observations with information on emission profiles across the jet have been presented by several authors (e. g. Beck et al. 2007, Bacciotti et al. 2006, HM07, Coffey et al. 2007). If the emission line profiles are limited to a few observational values across the jet we will show that this information can be used to derive the values of the physical parameters $n_e$, $T$, and $x_H$ assuming a Gaussian profile for the emission lines. If the emission profiles across the jet are well sampled, we will show in this paper how standard tomographic techniques (e. g. Craig & Brown 1986, Brown 1995) may be applied to reconstruct the volumetric mean physical parameters. For instance, electron density $n_e$ can be determined:

$$n_e = \frac{G_{12}}{G_{11}}$$

where $G_{11}$ is the “critical density” and $G_{12}$ is the “sensitive” defined as (ABS89, Osterbrock 1989, Saifer 1992): $G_{ij} = \frac{\int i_i dV}{\int i_j dV}$.

Fig. 1. Schematic representation of a cross section of a stellar jet.

The jet is moving in the direction perpendicular to the plane of the image (the $z$-axis, not shown in the Figure), and is assumed to be axisymmetric. The observer is looking at the jet along the $y$-direction, and the observations are represented by a series of points along the $x$-axis, each one corresponding to the pixel size and obtained integrating the volume intensities along the line of sight and on the beaming size $\Delta x \Delta z$.

$$n_e = \frac{G_{12}}{G_{11}}(G_{12})$$

If the medium is inhomogeneous, the previous formula may also be applied to give a “spectroscopic electron mean density” defined as ABS89:

$$<n_e> = \frac{G_{12}}{G_{11}}\left[\frac{\int i_1 dV}{\int i_2 dV}\right]$$

The volume emissivities for the line $j$ may be written as (e. g. ABS89, Osterbrock 1989, Saifer 1992):

$$i_j = k_j n_e^2 (1 + n_e/n_j)$$

This expression is exact for a two-level atom. In this case, $n_j = A_{21}/(C_{21} + C_{12})$ is the “critical density” and $k_j = \frac{h\nu_{21}}{2\pi}v_{21}(C_{12}/x_H) n_e/n_j$, where $A_{21}$ is the Einstein-A coefficient for the transition, $C_{12}$ and $C_{21}$ are the collisional excitation and de-excitation rates respectively, $h\nu_{21}$ is the transition energy, and

2. Convolution effects on jets

2.1. Convolution effects for a non-resolved jet cross section

We consider an axisymmetric jet in a three-dimensional cartesian reference frame. The $z$-axis of this system is taken along with the main axis of the jet. The observer is looking at the jet along the $y$-axis, and the $x/z$-plane coincides with the plane of the sky (see Fig. 1). The cylindrical radius $r$ is the distance between an arbitrary point and the $z$-axis.

The jet is observed in the [SII] lines 6716, 6731, [OI] lines 6300, 6363 and [NII] lines 6548, 6583 emission lines. These lines give three ratios which depend on the three variables to be determined: $n_e$, $T$, $x_H$. These emission line intensities are observed at a certain number of points $x_i$ (corresponding to the pixel size) in the plane of the sky.

In the following, capital letters will be used to design quantities integrated and convolved (according to the nature of the variable) along the line of sight, and lower case letters for volume quantities. For instance, $i$ represents the volume emissivity (defined by eq. A.1), while $I$ is the intensity (i. e. the emissivity integrated along the line of sight, with units: erg s$^{-1}$ cm$^{-2}$ sr$^{-1}$);
$n_s/n_{th}$ is the population fraction of the species $s$. A fit with eq. 3 is also a very good approximation for many forbidden emission lines, in particular for the red [SII] lines.

Using eq. 3 for replacing $i$ in eq. 4 we then have:

$$< n_e > = \frac{k_1k_2 - G_{12}}{G_{12}/n_1 - k_1/(k_2n_2)}.$$  (4)

This equation is next applied to the case of a stellar jet (see Fig. 1), first considering spectroscopic observations with a long slit along the jet axis, where no spatial information across the jet is available. The observed line emissivity will be given by the integration of the volume emissivity within the jet volume, where $dV = 2\pi r dr dz$, being $dz$ the size of the beam in the $z$ direction.

If the electron density is represented by a Gaussian radial profile, then $n_e = n_0 e^{-r^2/\sigma^2}$, and the spectroscopic mean density defined by eq. 4 becomes:

$$< n_e > = \frac{(n_1 - n_0)(n_0 - n_{ex}) - n_1^2L_1 + n_2^2L_2}{n_1L_1 - n_2L_2}.$$  (5)

where $L_{1,2} = \ln(1 + n_0/n_{1,2})/(1 + n_{ex}/n_{1,2})$, with $n_0$ the electron density on the jet axis and $n_{ex}$ the density evaluated at the jet radius. This is the same expression derived by ABS89 for the case of an exponentially decreasing density in an isothermal stellar atmosphere.

On the other hand, the mean electron density across the jet is given by:

$$\overline{n_e} = \frac{\int n_e dV}{\int dV} = n_0 - n_{ex} \log(n_0/n_{ex}),$$  (6)

where the right hand side term is obtained as before assuming a Gaussian density profile and $dV = 2\pi r dr dz$.

The ratio between mean spectroscopic density and mean density (i.e. the ratio between the “observed” and the “real” density) is shown in Fig. 2 as a function of the central electron density for the [SII] ratio. Similar curves were calculated by ABS89 for line ratios relevant to stellar atmospheres.

As can be seen from Fig. 2 the ratio between the spectroscopic and the mean density increases for steeper stratifications of $n_e$ across the beam of the jet. At a fixed limb-to-centre electron density ratio $n_{ex}/n_0$, the $< n_e > /\overline{n_e}$ ratio is slowly decreasing. The spectroscopic mean density is always larger than the mean density because denser regions contribute more to the emission. In the low density regime ($n_{ex}/n_0 \ll n_1$) in particular, it is straightforward to show that eq. 5 reduces to $< n_e > /\overline{n_e} \approx (2/3)(\alpha^2 + \alpha + 1)/\alpha$ in $\ln \alpha$ where $\alpha = n_0/n_{ex}$, and $< n_e > /\overline{n_e} \approx 2/3 \ln n_0$ when $n_{ex} \gg n_0$ as in the upper curves of Fig. 2.

The results shown in Fig. 2 are particularly relevant for stellar jets because an important parameter that is determined observationally is the mass flux (and the momentum flux, that has the same behaviour, as described below), defined by $dM \sim \rho v r dr$, where $\rho$ is the mass density ($\propto n_{th} = n_e/n_{th}$) and $v(r)$ is the jet velocity. If velocity and ionization fraction have a “top-hat” profile in the $r$ direction, it may easily be shown that:

$$\frac{\overline{M}}{\langle M \rangle} = \frac{\overline{n_e}}{\langle n_e \rangle}.$$  (7)

where $\overline{M}$ is the mass flux and the $\langle M \rangle$ is the one determined using the spectroscopic mean density. Therefore, values of mass and momentum-flux represent just an upper limit to the real values (see e.g. Cabrit 2002).

![Fig. 2. Ratio between the spectroscopic $n_{sp}$ and average density $n_e$, as a function of the density $n_0$ on the jet axis, for different jet cross sections, characterized by their limb-to-centre electron density ratio $n_{ex}/n_0$.](image)

2.2. Convolution effects for a resolved jet cross section

We now consider the case where information on the emission profile across the jet is available. The volume and integrated emissivities $i(r)$ and $I(x)$ are related to each other by the Abel transform:

$$I(x) = 2 \int_x^R \frac{i(r)r dr}{\sqrt{r^2 - x^2}}.$$  (8)

where $R$ is the jet radius and $x$ is the projected separation between a point in the observed image and the projection of the jet axis on the plane of the sky.

In the case of an homogeneous medium $i = i_0$ and eq. 8 leads to $I(x) = 2i_0 R \sqrt{R^2 - x^2}$. To show the effect of an inhomogeneous medium on plasma diagnostics, in the following we present results obtained numerically integrating eq. 8 for different $n_e$, $T$, $x_{th}$ profiles.

Assuming a profile for $n_e$, $T$, $x_{th}$, for each radial position the synthetic emissivities of the [SII], [SII], 6716, 6731, [NII], 6583, and [OI], 6300 lines are calculated, solving the statistical equilibrium equations using a 5-level atom (with atomic parameters from Mendoza 1983). These spatially dependent emission coefficients are then used to numerically integrate eq. 8. Finally, the physical parameters are determined from the integrated emission line intensities using the BE method (see Appendix A).

The results obtained assuming a Gaussian profile for $n_e$, and constant $T$ and $x_{th}$ cross sections are shown in Fig. 3 The volume and integrated emissivities of [SII], 6716 and [SII], 6731 versus $r/R$ are shown in the top panel, both normalised to their values at $r = 0$. The [SII] ratio is also shown in the central panel of Fig. 3 while the volume and integrated electron densities are shown in the lower panel.

The [SII] ratio determined using the integrated emissivities is higher than the original ratio (continuous and dotted line of Fig. 3, respectively). The difference between $n_e$ and $N_e$ increases from 0 at the external radii to approximately 20% toward the jet axis.

Fig. 4 shows the ratio $n_{ex}/n_0$ between the original and convolved values of the electron density on the jet axis versus the
fraction are assumed to have constant values of 10 to 100 cm⁻³ for different values of $r/R$. Fig. 4.

Fig. 3. Effect of the convolution on inhomogeneous jets. The dotted lines represent integrated quantities, while the continuous lines represent volume quantities. The electron density has a Gaussian profile ranging between $10^3$ cm⁻³ on the axis and 100 cm⁻³ at the jet radius. The temperature and the ionisation fraction are assumed to have constant values of $10^4$ K and 0.1, respectively. Top: [SII]λ6716, λ6731 integrated and volume intensities. Centre: [SII]λ6716/λ6731 integrated and volume ratio. Bottom: volume and integrated electron density.

Fig. 4. Ratio between convolved and volumetric densities for different values of $n_0/n_{ex}$.

convolved electron density $N_0$, for different values of $n_{ex}/n_0$. As is evident from the figure, the convolution effect increases when the observed density approaches the critical density (of $\sim 10^4$ cm⁻³) for the [SII] ratio. For high density stratifications, the $n_0/N_0$ ratio converges to a maximum ratio, which depends on the value of $N_0$.

Fig. 5 shows the results obtained for different initial stratifications in $n_e$, $T$ and $x_H$, corresponding to radial profiles with the form $10^{-\beta r/\beta}$ where $\beta = 2, 1, 0.5$ for the top, centre and bottom panels of Fig. 5, respectively. It is clear from Fig. 5 that effects similar to the one observed for $n_e$ in Fig. 3 are present also for $T$ and $x_H$. Additionally, Fig. 5 and 6 show that convolution effects increase with the plasma stratification (as expected).

3. Data reconstruction

3.1. Reconstruction of the jet structure using a Gaussian fit to the observed intensities

The volume emission coefficient projected on the sky plane is also weighted on the beam size $\Delta x\Delta z$. Therefore, the observed emissivity $I(x_i)$ is given by:

$$I(x_i) = 2\Delta z \int_{x_{i-1/2}}^{x_{i+1/2}} dx \int_{i/\Delta}^{R} i(r) dr \frac{r}{\sqrt{r^2 - x^2}}.$$  \hspace{1cm} (9)

where $i = -N, ..., 0, ..., N$, there are $2N + 1$ observational values (with $N = 0, 1, 2, ...$), and $i_{(N+1)/2} = R$. All the relevant quantities are assumed to have negligible variations in the $z$ direction.

A simple scaling between $i(r)$ and $I(x)$ may be obtained in the case of an inhomogeneous medium, using the linearity of the Abel transform of a Gaussian distribution. In fact, if the volume emissivity has the form $i(r) = i_0 e^{-r^2/\sigma^2}$, eq. 8 leads to:

$$I(x) = \sqrt{\pi \sigma i(x) \text{erf} \left( \frac{1}{\sqrt{2}} \left( 1 - \left( \frac{x}{R} \right)^2 \right) \right)}.$$  \hspace{1cm} (10)

where

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt.$$  \hspace{1cm} (11)

is the error function, and $i_0$ and $i_{ex}$ are the volume emissivity at the jet centre and limb, respectively.

The property of the error function: $\text{erf}(x) \approx 1$ when $x \gg 1$ implies that

$$I(x) \approx \sqrt{\pi \sigma i(x)}.$$  \hspace{1cm} (12)

if $x \leq R$ and $i_0 \gg i_{ex}$.

Using $I(x)$ as given by eq. 12, eq. 9 becomes:

$$I(x_i) = \pi \sigma^2 i_0 \left( \text{erf} \left( \frac{x_{i+1/2}}{\sigma} \right) - \text{erf} \left( \frac{x_{i-1/2}}{\sigma} \right) \right) \Delta z.$$  \hspace{1cm} (13)

If only one value of $I$ is observed ($x_{i+1/2} = -x_{i-1/2} = R$), the integrated emissivity will be simply given by:

$$I = \pi \sigma^2 i_0 (1 - e^{-R^2/\sigma^2}) \Delta z \approx \pi \sigma^2 i_0 \Delta z.$$  \hspace{1cm} (14)

Given eq. 14, the relation between the integrated ratio $G$ and the corresponding volume ratio $g$, in the case of Gaussian profile emissivities, is given by:

$$G = \eta g \quad \text{where} \quad \eta = \left( \frac{\sigma_1}{\sigma_2} \right)^2.$$  \hspace{1cm} (15)

If $\sigma_1 \neq \sigma_2$, the integrated and volume ratios will be different. Therefore, the physical parameters determined from the integrated ratio will be different with respect to the one obtained...
Fig. 5. The dotted lines represent convolved quantities, and the continuous lines represent volume quantities. The curves correspond to the integrated and volume densities, obtained using the following initial profiles: \( n \sim 10^{-3} r^2 \), \( T = 10000 \) K, \( x_H = 0.1 \) where \( \beta = 2, 1, 0.5 \) in the upper, central and lower panel respectively. The volume electron density is 1000 cm\(^{-3}\) on the axis and 100 cm\(^{-3}\) at the jet radius.

Fig. 6. Effect of the convolution on inhomogeneous jets. The dotted lines represent integrated quantities, and the continuous lines represent volume quantities. Thin lines correspond to the ionisation fraction, and thick lines to the temperature. The curves are obtained using the following profiles: \( T_e, x, n_e \sim 10^{-3} r^2 \), where \( \beta = 2, 1, 0.5 \) in the upper, central and lower panel, respectively.

Fig. 7. [SII]6716/6731 ratio (points) and fits (lines) with eq. 16 corresponding to different temperatures. The fitting parameters are the following: \( n_0 = 1841.89, 2457.4, 2883.9 \) cm\(^{-3}\), \( a_1 = 0.678, 0.687, 0.695 \), \( a_2 = 2.295, 2.333, 2.336 \) for \( T = 5000, 10000, 15000 \) K, respectively.

We first show how to apply eq. 15 to the [SII] ratio to determine \( n_e \) and later the results will be generalised to more line ratios, to determine all of the physical parameters.

A very good approximation to the relation between the [SII] ratio and \( n_e \) may be obtained from eq. 4 and is given by:

\[
g = \frac{n_e + n_0}{n_e a_2 + n_0 a_1},
\]

where the parameters \( n_0, a_1 \) and \( a_2 \) are in general temperature dependent. This formula is exact for a two-level atom, and in general is just a fit to the \( g = g(n_e) \) curve at constant \( T \) and \( x_H \).

Fig. 7 shows the [SII] ratio and the fit calculated using eq. 16. The fitting parameters are reported in the Figure caption as a function of temperature.

In eq. 16 \( n_e \) and \( g \) represent the volume electron density and ratio respectively. A similar relation will also link the integrated quantities \( N_e \) and \( G \):

\[
G = \frac{N_e + n_0}{N_e a_2 + n_0 a_1}.
\]

Using eq. 15, 16 and 17 it is possible to obtain a relation between \( n_e \) and \( N_e \):

\[
n_e = \frac{N_e (\eta a_2 - a_1) + n_0 a_1 (1 - \eta)}{N_e a_2 (1 - \eta) + n_0 (a_2 - \eta a_1)}.
\]

Fig. 8 shows the ratio between the volumetric and the observed densities \( n_e/N_e \) (obtained from eq. 13) as a function of \( \eta \), for different values of \( \eta \). For electron densities \( N_e \geq 5000 \) cm\(^{-3}\) and \( N_e \leq 100 \) cm\(^{-3}\), the \( n_e/N_e \) ratio becomes increasingly larger. These density ranges correspond to the high and low-density regimes (see Fig. 7). Also, for densities around \( 10^3 \) cm\(^{-3}\), the differences may be as high as a factor of 2 or 3, depending on the value of \( \eta \).

This procedure may naturally be generalised to include the calculation of \( T \) and \( x_H \):

- for each line with spatial information across the jet, the value of \( \sigma \) is obtained with a Gaussian fit to the observed emission profile.

using the volume ratio. Furthermore, the simple scaling given by eq. 15 may be used to reconstruct the volume intensities given the integrated intensities and the values of \( \sigma_1 \) and \( \sigma_2 \), and from there to determine the radial dependence of the physical parameters.
The volumetric line ratios are reconstructed using eq.[15] as $g_{1,2} = \frac{l_1}{l_2} = \frac{\sigma_2}{\sigma_1} G_{1,2}$.

The reconstructed ratios are used to determine the physical parameters using any of the available methods (e. g. the BE method, see Appendix [A]).

If the jet is observed through a narrow slit placed along the main jet axis, eq. [13] has to be used instead of eq. [14]. Therefore, $\eta$ will be defined in this case as:

$$\eta = \left(\frac{\sigma_1}{\sigma_2}\right)^2 \frac{\text{erf} \left(\frac{\Delta x}{\sigma_1}\right)}{\text{erf} \left(\frac{\Delta x}{\sigma_2}\right)},$$

where $\Delta x$ is the beam size along the $x$-direction.

### 3.2. Reconstruction of the jet structure using tomographic techniques

The Abel transform (eq. 8) has been largely studied and applied in science, and complex methods were developed to solve it (e. g. Craig & Brown [1986]). The analytical solution of the Abel transform is also well known:

$$i(r) = -\frac{1}{\pi} \int_r^\infty \frac{dI}{dx} \frac{dx}{\sqrt{x^2 - r^2}}.$$  

(20)

The determination of $i(r)$ represents an “ill-posed” problem in the sense that small noise in $I(x)$ produces large errors in the determination of $i(r)$, due to the numerical derivative present in the integral. Any numerical method solving eq. (20) using noisy observed data is therefore necessarily unstable.

A method that is less affected by numerical instabilities is obtained by fitting the data by a smooth function and using eq. (20) to invert the fitted curve (e. g. Simonneau [1993]). In this context, we use the multi-Gaussian method developed by Bendinelli [1991], which has been mainly used for deprojection of galaxy surface brightness distribution (e. g. Bendinelli [1991] Emsellem et al. [1994] Bendinelli & Parmeggiani [1995] Cappellari [2002]). This method basically consists of producing a multi-Gaussian expansion fit of the data, and inverting the Gaussian series. Actually, this approach becomes very simple once the fit has been obtained, due to the already mentioned linearity of the Abel transform when applied to a Gaussian distribution. This implies that for a fit to a data series with a sum of Gaussians (centred on the same point):

$$I(x) = \sum_{i=1}^n a_i e^{-x^2/\sigma_i^2}.$$  

(21)

the Abel transform may be inverted analytically (eq. 20) to give:

$$i(r) = \sum_{i=1}^n \frac{1}{\sqrt{\pi} \sigma_i} a_i e^{-r^2/\sigma_i^2},$$  

(22)

where $r$ is the distance from the jet axis.

### 3.3. Application to HH30

We now apply the techniques derived above to the HH30 jet. HH30 is an ideal candidate for this analysis. It moves nearly on the sky plane, has a clear side-to-side symmetry in the region close to the central star, and the cooling region is resolved spatially with Hubble Space Telescope (HST) observations (e. g. Burrows et al. [1996] Ray et al. [1996] Bacciotti et al. [1999] HM07). The data used for this analysis were taken by HST using the slit-less spectroscopy technique, and presented by HM07. Using this powerful technique, the observations by HM07 resolve the cross section of the HH30 jet with $\sim 10 - 20$ pixels.

One position along the jet axis is considered here, at an angular separation of $3.9''$ from the HH30 source, corresponding to a region close to the central star-disk system.

First, to calculate the centre $x_0$ of the Gaussian fit, i. e. the projection of the jet axis on the plane of the sky, all the observed profiles are added. The obtained emissivity is then fitted with an increasing number of Gaussians. The values of $x_0$ and the variance $\sigma$ obtained using up to 5 Gaussians are shown in Tab. 1. Fig. 9 shows the data and a fit with 3 Gaussians (top), and the residual (bottom). From Tab. 1 and Fig. 9 it is evident that a very good approximation to the data is obtained already with 3 Gaussians.

The results obtained are shown in Fig. 10 and Fig. 11. Each emissivity profile is fitted with a multi-Gaussian fit, and using eq. 22 the intensities as a function of the radial position, and the corresponding ratios are reconstructed.

Fig. 10 shows the observed data, the multi-Gaussian fit (top) and the reconstructed data (bottom). The ratios calculated using the reconstructed data differ from the original, observed ratios (see for example the [SII] $\lambda 6716$ curves with respect to the [NII] in the upper and lower panels of Fig. 10).

The physical quantities are shown in Fig. 11. In the upper panel $n_e$ is shown. The points correspond to the observed values, while the continuous and dotted lines correspond to the multi-Gaussian fit and the reconstructed electron density values, re-
observed temperature in the central panel of Fig. 11). All of these
expect a much steeper profiles using deconvolved data. Also, the
e jet presents some degree of asymmetry (see e. g. the wings of the
structure of the HH30 jet are left for a second paper.

In this paper we have studied the influence of inhomogeneities
affected by the fact that the [SII] ratio is close to the high den-
sity regime (see Fig. 8). Other line ratios (see the discussion by
HM07) should be used to deconvolve the jet beam closed to the
source.

Some interesting features may be noted from Fig. 11. First
all, it is evident that HH30 does not have a top hat cross sec-
tion. The reconstructed \( n_e \), \( T \) and \( \lambda_{TH} \) cross sections are much
steeper than the measured cross sections. The reconstructed elec-
don density and ionisation fraction cross sections have strong,
on-axis peaks, and the temperature has an on-axis valley. The
total densities \( (n_{H1} = n_e/\lambda_{TH}) \) are larger by a factor of two and
four on the jet axis with respect to the jet radius, for the inferred
from observation and reconstructed case respectively. The fact
that we have lower temperatures in the denser regions is qualita-
tively consistent with the stronger cooling that one would expect
in higher density regions. The fact that the ionisation fraction is
higher in the denser regions, however, is an effect that escapes
simple qualitative arguments, and should be compared directly
with predictions of jet formation and propagation.

It is important to note that the original data were not decon-
volved with the PSF (with a width of \( \approx 2.5 \) pixels), and we may
expect a much steeper profiles using deconvolved data. Also, the
jet presents some degree of asymmetry (see e. g. the wings of the
observed temperature in the central panel of Fig. 11). All of these
effects, together with a complete study of the two-dimensional
structure of the HH30 jet are left for a second paper.

It should be noted that the results shown in Fig. 11 are af-
fected by the fact that the [SII] ratio is close to the high den-
sity regime (see Fig. 8). Other line ratios (see the discussion by
HM07) should be used to deconvolve the jet beam closed to the
source.

4. Summary and discussion

In this paper we have studied the influence of inhomogeneities
on the electron density, temperature and ionisation fraction val-
ues derived from observed line ratios. Additionally, we have pre-
sented possible methods to analyse the data obtained from high
angular resolution imaging of stellar jets.

Simple scalings are possible when a minimum amount of in-
formation is available. This minimum information is one value
of emission line intensities for each position along the jet, to-
gether with a determination of the jet width \( \sigma \) for all of the dif-
ferent lines used for the plasma diagnostics.

Bacciotti et al. (2000) observed DG Tau with 7 narrow slits
parallel to the main jet axis, and at different positions across the
jet, while Coffey et al. (2007) used one slit placed perpendicular
to the main jet axis. In both cases, they obtained spatial and kin-
ematic information across the jet axis. The approach presented in
§3.1 can be easily applied to these data, calculating the average
physical parameters and jet velocity as a function of position
along the jet. The determination of the jet velocity has not been
explicitly discussed in the present paper, but similar methods to
the ones that we have discussed above can be used in this con-
text.

We have also shown that a more complex tomographic re-
construction (see §3.2) can be used to reconstruct the cross sec-
tion of the flow, obtaining a complete description of the three
dimensional structure of the jet.

The main limitations for the application of these techniques
is that the jet axis has to lie close to the plane of the sky. How-
however, it would be possible (but more complex) to reconstruct
the 3D structure of the flow for arbitrary orientations of the jet
axis. Additionally, the jet has to be nearly axisymmetric. Also,
the applicability of the present deconvolution method is limited
to the nearly axysymmetric region of the jet close to the source.
Farther away from the source, in fact, the interaction of the jet
with the ambient medium produces asymmetries in the intensity
cross-sections.

Finally, while this technique is currently applicable to a few
observed objects, the use of future high resolution instruments of
the new generation of telescopes will make it possible to apply
this technique to a larger sample of objects.

In this paper, preliminary application of our method to recon-
structing the cross section of the HH 30 jet has been presented.
In a second paper, we will describe a much more complete appli-
cation of our method using observations of HH 30 and of other
HH jets.
Fig. 11. Values inferred from observations (points), fitted values (with a multi-Gaussian, continuous line) and reconstructed values (dotted line) of electron density, temperature and ionisation fraction for the upper, central and lower panels respectively.

Appendix A: BE method

The emission coefficient (units: erg s$^{-1}$ cm$^{-3}$ sr$^{-1}$) obtained by the transition from the $i$ to $j$ levels is given by:

$$i_{i,j} = \frac{1}{4\pi} h\nu_{i,j} A_{i,j} \int \frac{n_i}{n_A} \frac{n_A}{n_H}$$

\[\text{(A.1)}\]

where $h\nu_{i,j}$ is the transition energy, $A_{i,j}$ is the spontaneous Einstein coefficient, $n_i/n_A$, $n_A/n_H$ and $n_A/n_H$ are the excitation fraction, the ionisation fraction and the population fraction of the considered species.

The BE method (from BE99) uses a series of line ratios to determine $n_e$, $T$, and $x_{[II]}: [SII] 6731/[SII] 6716$, [OI] 6300/[SII] 6716 + 6731, [NII] 6583/[SII] 6716 + 6731. The [SII] ratio depends on $n_e$ and $T$, while the [OI]/[SII] and [NII]/[SII] ratios depend also on $n_O/n_{[III]}$ and $n_{[II]}/n_{[III]}$ respectively. Additionally, nitrogen and oxygen ionisation fraction are determined assuming charge exchange equilibrium with hydrogen. Therefore $n_O/n_{[III]}$ and $n_{[II]}/n_{[III]}$ become a function of the ionisation fraction and $n_O/n_{[III]}$ and $n_{[II]}/n_{[III]}$ respectively. Finally, the sulphur is supposed to be all single ionised ($n_{[III]} = n_S$) because the photoionisation rate due to diffuse UV radiation (e.g. Tielens 2005 pp. 267) is much larger than the SII radiative recombination rate.

To calculate the physical quantities, we choose arbitrary values of temperature and ionisation fraction, and invert the [SII] ratio deducing the electron density. Furthermore we use the N/S ratio and the electron density derived previously to determine the ionisation fraction, and finally we use the O/S ratio to find the electron temperature. Finally, we iterate until convergence using the new values of $T$ and $x_{[II]}$.

The atomic parameters used to calculate collisional ionisation, radiative + dielectronic recombination and charge exchange coefficients are from Cox (1970), Aldrovandi & Péquignot (1975, 1976) and Osterbrock (1989).

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References

Aldrovandi, S. M. V., & Péquignot, D. 1973, A&A, 25, 137
Aldrovandi, S. M. V., & Péquignot, D. 1976, A&A, 47, 321
Almleaky, Y. M., Brown, J. C., & Sweet, P. A. 1989, A&A, 224, 328 (ABS89)
Bacciotti, F., & Eisloffel, J. 1999, A&A, 342, 717 (BE99)
Bacciotti, F., & Eisloffel, J., & Ray, T. P 1999, A&A, 350, 917
Bacciotti, F., Mundt, R., Ray, T. P., et al. 2000, ApJ, 537, L49
Beck, T. L., Riera, A., Raga, A. C., et al. 2007, AJ, 133, 1221
Burrows, C. J., Stapelfeldt, K. R., Watson, A. M., et al. 1996, ApJ, 473, 437
Bendinelli, O. 1991, ApJ, 366, 599
Bendinelli, O., & Parmeggiani, G. 1995, AJ, 109, 2
Brown, J. C., Dwivedi, B. N., Sweet, P. A., et al. 1991, A&A, 249, 277
Brown, J. C. 1995, Inverse Problems, 11, 783
Brugel, E. W., Boehm, K. H., & Mannery, E. 1981, ApJS, 47, 117
Cabrit, S., 2002, Constraints on accretion-ejection structures in young stars, ed. J. Bouvier & J.-P. Zahn, EDP Sciences, 3, 147
Cappellari, M. 2002, MNRAS, 333, 400
Coffey, D., Bacciotti, F., & Podio, L. 2007, ApJ submitted
Cox, D. P. 1970, Ph.D. Thesis, Univ. of California, San Francisco, USA
Craig I. J. D., & Brown, J. C., 1986, Inverse problems in astronomy - A guide to inversion strategies for remotely sensed data, Adam Higler, Bristol
Doschek, G. A. 1984, ApJ, 279, 452
Emelson, E., Monnet, G., & Bacon, R. 1995, A&A, 285, 723
Hartigan, P., Raymond, J., & Hartmann, L. 1987, ApJ, 316, 323
Hartigan, P., Morse, J. A., & Raymond, J. 1994, ApJ, 436, 125
Hartigan, P. & Morse, J. 2007, ApJ, 660, 426 (BM07)
Judge, P. G. 2000, ApJ, 531, 585
Judge, P. G., Hubeny, V., & Brown, J. C. 1997, ApJ, 475, 275
Lavalley-Fouquet, C., Cabrit, S., & Dougados, C., 2000, A&A, 356, 41
Mendoza, C. 1983, Planetary Nebulae, ed. D. Flower (Reidel), IAU Symp., 103, 143
 McIntosh, S. W., et al. 1998, A&A, 333, 333
Nisini, B., Bacciotti, F., Giannini, T., et al. 2005, A&A, 441, 159
Osterbrock, D. E., Astrophysics of gaseous nebulae and active galactic nuclei, University Science Books (1988)
Pesenti, N., Dougados, C., Cabrit, S., et al. 2003, A&A, 410, 155
Podio, L., Bacciotti, F., Nisini, B., et al. 2006, A&A, 456, 189
Raga, A. C., De Colle, F., Kajdič, P., et al. 2007, A&A, 465, 879
Ray, T. P., Mundt, R., Dyson, J. E., et al. 1996, ApJ, 468, 103
Reipurth, B., & Bally, J. 2001, ARA&A, 39, 403
Saifir, P. N. 1992, ApJ, 392, 492
Simonneau, E., Varela, A. M., Muñoz-Tuñón, C. 1993, JQSRT, 49, 149
Tielens, A. G. G. M., The Physics and Chemistry of the Interstellar Medium, Cambridge Univ. Press, Cambridge (UK) 2005