Model Independent Determination of the Top Yukawa Coupling from LHC and LC

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Abstract

We show how a measurement of the process $pp \rightarrow t\bar{t}H + X$ at the LHC and a measurement of the Higgs boson branching ratios $\text{BR}(H \rightarrow b\bar{b})$ and $\text{BR}(H \rightarrow W^+W^-)$ at a future linear electron positron collider can be combined to extract a model-independent measurement of the top quark Yukawa coupling. We find that for $120 \text{ GeV} < m_H < 200 \text{ GeV}$ a measurement precision of 15\% including systematic uncertainties can be achieved for integrated luminosities of 300 \text{ fb}^{-1} at the LHC and 500 \text{ fb}^{-1} at the LC at centre-of-mass energy of 350 GeV.

1 Motivation

The Yukawa coupling of the Higgs boson to the heaviest quark, the top quark, is of great interest for the study of the nature of electroweak symmetry breaking and the generation of masses. While the Yukawa couplings to bottom and charm quarks and to tau leptons and muons are in principle accessible through the Higgs boson decay branching ratios, the Higgs boson decay into top quark pairs is kinematically forbidden for light Higgs bosons as they are favoured by theory and electroweak precision data. The only Standard Model process that probes the top Yukawa coupling at tree level is the associated production of a $t\bar{t}$ pair with a Higgs boson. This process occurs at the LHC (mainly $gg \rightarrow t\bar{t}H^0$) as well as at the LC ($e^+e^- \rightarrow t\bar{t}H^0$). In the latter case the cross section is only significant at centre-of-mass energies in excess of 800 GeV. At the LHC, the final states that have been investigated so far are $t\bar{t}b\bar{b}$ [1–4] and $t\bar{t}W^+W^-$ [5,6], the $t\bar{t}\tau^+\tau^-$ final state is under study [7,8]. At tree level, their production rates are proportional to $g_{tH}^2 \text{BR}(H^0 \rightarrow b\bar{b})$ and $g_{tH}^2 \text{BR}(H^0 \rightarrow W^+W^-)$, respectively. The absolute values of $\text{BR}(H^0 \rightarrow b\bar{b})$ and $\text{BR}(H^0 \rightarrow W^+W^-)$ can be measured accurately in a model independent way at the LC from the corresponding decay branching ratios [9]. These can be measured already at a first phase of the LC ($\sqrt{s}$ between 350 and 500 GeV). Thus, the combination of the measurements of both machines can be used to determine the value of $g_{tH}$ without model assumptions and presumable before a second phase of the LC ($\sqrt{s} \sim 1$ TeV) would come into operation.

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2 Measurements at the LHC

The results from the following ATLAS analyses of the $tt\bar{t}H^0$ process are used:

1. $tt\bar{t}H^0$ with $tt\rightarrow bbq\ell\nu$ and $H^0\rightarrow b\bar{b}$ [1];
2. $tt\bar{t}H^0$ with $H^0\rightarrow W^+W^-$ and two like-sign leptons [5];
3. $tt\bar{t}H^0$ with $H^0\rightarrow W^+W^-$ and three leptons [5].

| $m_H$ (GeV) | $30 \text{fb}^{-1}$ | $300 \text{fb}^{-1}$ |
|-------------|---------------------|---------------------|
| $tt\bar{t}H^0$ | $H^0\rightarrow bb$ | $H^0\rightarrow bb$ |
| signal | background | signal | background |
| 100  | 83.4 | 303.4 | 279.0 | 1101.3 |
| 110  | 63.0 | 275.7 | 232.5 | 1140.6 |
| 120  | 43.0 | 234.1 | 173.1 | 1054.2 |
| 130  | 26.5 | 200.1 | 112.5 | 1015.8 |
| 140  | 13.9 | 178.2 | 62.4 | 947.1 |

Table 1: Expected number of signal and background events for the $tt\bar{t}H^0$ with $tt\rightarrow bbq\ell\nu$ and $H^0\rightarrow b\bar{b}$ analysis at LHC [1].

| $m_H$ (GeV) | $30 \text{fb}^{-1}$ | $300 \text{fb}^{-1}$ |
|-------------|---------------------|---------------------|
| $tt\bar{t}H^0$ | $H^0\rightarrow WW(2\ell)$ | $H^0\rightarrow WW(3\ell)$ |
| signal | bckgr | signal | bckgr | signal | bckgr | signal | bckgr |
| 120  | 4.4 | 19.6 | 2.7 | 21.2 | 12.7 | 80.6 | 10.5 | 97.6 |
| 140  | 15.0 | 19.6 | 8.7 | 21.2 | 50.0 | 80.6 | 33.7 | 97.6 |
| 160  | 21.1 | 19.6 | 13.0 | 21.2 | 72.3 | 80.6 | 55.3 | 97.6 |
| 180  | 17.3 | 19.6 | 10.3 | 21.2 | 60.9 | 80.6 | 41.7 | 97.6 |
| 200  | 10.5 | 19.6 | 5.7 | 21.2 | 43.2 | 80.6 | 26.4 | 97.6 |

Table 2: Expected number of signal and background events for the $tt\bar{t}H^0$ with $H^0\rightarrow W^+W^-$ (two like-sign leptons and three leptons, respectively) analyses at LHC [5].

The expected numbers of selected signal and background events in the three channels for various Higgs masses and total integrated luminosities of 30 fb$^{-1}$ and 300 fb$^{-1}$ are listed in Tables 1 and 2. The results obtained in this sub-section are based on the anticipated data sample of one LHC detector, with the luminosity per detector quoted above.

From the expected event numbers we first estimate the uncertainty (statistical and systematic) on the measured cross section $\sigma_{data}^{tt\bar{t}H}$. Further uncertainties arise when $\sigma_{data}^{tt\bar{t}H}$ is compared to the theoretical prediction as a function of $g_{tt\bar{t}H}$.

The uncertainty on the observed cross section $\sigma_{data}^{tt\bar{t}H}$ is calculated as

$$\left(\frac{\Delta \sigma^{data}_{tt\bar{t}H}}{\sigma^{data}_{tt\bar{t}H}}\right)^2 = \left(\frac{S+B}{S}\right)^2 + \left(\frac{\Delta B_{syst}}{S}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta \epsilon}{\epsilon}\right)^2.$$ 

Here, $S(B)$ is number of signal (background) events. $\Delta B_{syst}$ is the uncertainty on the background determination from sideband data (10% in the $h \rightarrow bb$ channel at high luminosity, 5% otherwise). $\Delta L$ is the error on the integrated luminosity (5%) and $\Delta \epsilon$ is the error on the determination of the efficiency. This error involves uncertainties on the tagging efficiency for individual b-jets (3%) and leptons (3% from isolation requirement and 2% from reconstruction efficiency) and an overall detector efficiency uncertainty of
Table 3: Expected relative precision on \( \sigma_{ttH} \times BR(H \to X) \) for the various LHC \( t\bar{t}H^0 \) analyses including systematic uncertainties (statistical error only). For \( H^0 \to W^+W^- \) the expected signal and background in the two and three lepton final state have been added.

2% (following [10]). The total value of \( \Delta \epsilon \) is then calculated for each channel individually depending on the number of leptons and b-jets.

The expected error including systematic uncertainties and taking into account only the statistical error of each channel is shown in Table 3. For the \( H^0 \to W^+W^- \) decay mode the signal and background from the two lepton and three lepton channels are added together since their signal contributions are exclusive and the overlap in the background is small. The obtained result is consistent with the study presented in [10].

In the next step the uncertainty on \( g^2_{ttH} \times BR(H \to b\bar{b}/W^+W^-) \) which arises when the observed \( \sigma_{ttH} \times BR(H \to b\bar{b}/W^+W^-) \) is compared to its theoretical prediction. These uncertainties consist of the uncertainties in the proton structure functions (5% [11, 12]) and uncertainties in the calculation of the production cross section. Recent full NLO calculations estimate the uncertainty of the cross section prediction to be approximately 15% from a variation of the hard scale [13–16]. The total theoretical uncertainty \( \Delta \sigma_{ttH}^{\text{theo}} \) is obtained by adding the above two sources in quadrature.

Finally, the total uncertainty \( \Delta (g^2_{ttH} \times BR(H \to b\bar{b}/W^+W^-)) \) is obtained according to

\[
\Delta (g^2_{ttH} \times BR(H \to b\bar{b}/W^+W^-))^2 / \sigma_{ttH}^2 = \left( \frac{\Delta \sigma_{ttH}^{\text{theo}}}{\sigma_{ttH}^{\text{theo}}} \right)^2 + \left( \frac{\Delta \sigma_{ttH}^{\text{data}}}{\sigma_{ttH}^{\text{data}}} \right)^2.
\]

### 3 Measurements at the LC

At the LC, the decay branching ratios into b quark pairs and W boson pairs can be measured at \( \sqrt{s} = 350 \text{ GeV} \) to the precision listed in Table 4 [9]. The precise model-independent measurement of Higgs boson branching ratios exploits the measurement of the Higgs-strahlung process \( e^+e^- \to HZ \). Events from specific Higgs decays, e.g. \( H \to b\bar{b} \) and \( H \to W^+W^- \) can be cleanly identified. The branching ratio is determined by normalizing the observed rate for a specific Higgs decay to the total Higgs-strahlung rate. The latter can be measured from the selection of \( Z \to \ell\ell \) events where the invariant mass of the recoil system is consistent with the Higgs mass, independent of the Higgs decay.

| \( m_H \) (GeV) | \( 30 \text{ fb}^{-1} \) | \( 300 \text{ fb}^{-1} \) |
|-----------------|----------------|----------------|
| \( H^0 \to b\bar{b} \) | \( H^0 \to W^+W^- \) | \( H^0 \to b\bar{b} \) | \( H^0 \to W^+W^- \) |
| 100 | 0.398(0.236) | 0.249(0.133) |
| 110 | 0.476(0.292) | 0.287(0.159) |
| 120 | 0.598(0.387) | 1.023(0.974) | 0.345(0.202) | 0.732(0.611) |
| 130 | 0.840(0.568) | 0.524(0.492) | 0.488(0.299) | 0.362(0.295) |
| 140 | 1.444(0.997) | 0.370(0.339) | 0.804(0.509) | 0.252(0.193) |
| 160 | 0.287(0.254) | 0.196(0.137) |
| 180 | 0.331(0.300) | 0.221(0.163) |
| 200 | 0.486(0.454) | 0.282(0.222) |
4 Results

For an extraction of the top quark Yukawa coupling at each Higgs mass we combine the LHC rate measurement of $ttH^0$ with $H^0 \rightarrow b\bar{b}$ or $H^0 \rightarrow W^+W^-$ with the corresponding measurement of the branching ratio at the LC. We make the tree level assumption that the cross section $\sigma_{ttH}$ is proportional to $g_{ttH}^2$. Thus, the relative error on $g_{ttH}$ is simply given by $\Delta g_{ttH}/g_{ttH} = 0.5 \Delta \sigma_{ttH}/\sigma_{ttH}$. The relative error on $\sigma_{ttH}$ is obtained by adding in quadrature the statistical and systematic uncertainties as described above and the error of the LC branching ratio measurement. The combination of the $b\bar{b}$ and $W^+W^-$ final states is performed by

$$\left(\frac{\Delta g_{ttH}}{g_{ttH}}\right)_{\text{comb.}}^2 = \left(\frac{\Delta g_{ttH}}{g_{ttH}}\right)_{W^+W^-}^2 + \left(\frac{\Delta g_{ttH}}{g_{ttH}}\right)_{b\bar{b}}^2.$$

The relative accuracies on the top quark Yukawa coupling achievable are summarised in Table 5. In Figure 1 the relative accuracy from the $H \rightarrow b\bar{b}$ and $H \rightarrow WW$ channels are shown individually and combined both for low and high luminosity at the LHC. Also shown is the results which would be obtained when all systematic errors were neglected.

![Figure 1: Achievable precision on the top Yukawa coupling from 30 fb$^{-1}$ at the LHC and 500 fb$^{-1}$ at the LC (left), and from 300 fb$^{-1}$ at the LHC and 500 fb$^{-1}$ at the LC (right). The red curve shows precision obtainable from the $H \rightarrow b\bar{b}$ final state, the green from the $H \rightarrow W^+W^-$ final state and the blue curve from the combination of the two. The dashed lines show the expected precision taking into account only statistical errors.](image)

Table 4: Relative precision on the branching ratio for $H^0 \rightarrow b\bar{b}$ and $H^0 \rightarrow W^+W^-$ expected for a LC running at $\sqrt{s} = 350$ GeV with 500 fb$^{-1}$.
Table 5: Expected relative error on the top Yukawa coupling $g_{ttH}$ from the rate measurement including all systematic uncertainties (statistic errors only) at the LHC and from the branching ratio measurement at the LC.

| $m_H$ (GeV) | $30 \text{ fb}^{-1}$ | $300 \text{ fb}^{-1}$ |
|------------|-------------------|---------------------|
|            | bb | WW | bb+WW | bb | WW | bb + WW |
| 100        | 0.22(0.12)   |          |       | 0.15(0.07)   |          |       |
| 110        | 0.25(0.15)   |          |       | 0.17(0.08)   |          |       |
| 120        | 0.31(0.19)   | 0.52(0.49) | 0.27(0.18) | 0.19(0.10)   | 0.38(0.31) | 0.17(0.10) |
| 130        | 0.43(0.28)   | 0.28(0.25) | 0.23(0.19) | 0.26(0.15)   | 0.20(0.15) | 0.16(0.11) |
| 140        | 0.72(0.50)   | 0.20(0.17) | 0.19(0.16) | 0.41(0.26)   | 0.15(0.10) | 0.14(0.09) |
| 150        |              | 0.18(0.14) |          | 1.88(1.21)   | 0.14(0.08) | 0.14(0.08) |
| 160        |              | 0.16(0.13) |          |              | 0.13(0.07) |
| 170        |              | 0.17(0.13) |          |              | 0.13(0.07) |
| 180        |              | 0.18(0.15) |          |              | 0.14(0.08) |
| 190        |              | 0.22(0.19) |          |              | 0.15(0.10) |
| 200        |              | 0.26(0.23) |          |              | 0.16(0.11) |

Figure 2: Achievable precision on the top Yukawa coupling from 300 fb$^{-1}$ at the LHC and 500 fb$^{-1}$ at the LC at 350 GeV taking into account all systematic uncertainties (solid red curve) and using only statistical errors (dashed green curve). For comparison, the expected precision from 1000 fb$^{-1}$ at the LC at 800 GeV alone (dotted blue curve) is also shown.
For 300 fb$^{-1}$ at the LHC and 500 fb$^{-1}$ at the LC the obtainable relative uncertainty is approximately 15% for a Higgs boson mass between 120 and 200 GeV. The purely statistical uncertainty ranges from 7% to 11% as shown in Fig. 2. The size of the obtained uncertainties is comparable to those obtained for the LHC alone [10] but in contrast to the latter no model-dependent assumptions are made. In Fig. 2 we also show the precision which can be achieved at the LC alone if operated at 800 GeV center-of-mass energy [17] from the measurement of the $e^+e^-\rightarrow t\bar{t}H^0$ process with $H^0\rightarrow b\bar{b}$ and and $H^0\rightarrow W^+W^-$ combined.

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