Parametric oscillations of polaron modes in magnetized semiconductor plasmas

Swati Dubey¹ and S Ghosh
School of Studies in Physics, Vikram University, Ujjain (M.P.), India
E-mail: swati_dubey201@yahoo.com

New Journal of Physics 11 (2009) 093030 (13pp)
Received 27 May 2009
Published 22 September 2009
Online at http://www.njp.org/
doi:10.1088/1367-2630/11/9/093030

Abstract. A simple analytical treatment based on the hydrodynamic model of plasmas is developed to study parametric oscillations of polaron modes in magnetized semiconductor plasmas. Second-order optical susceptibility arising due to nonlinear polarization and necessary threshold pump intensity are estimated using the data of an n-InSb crystal assumed to be irradiated by a frequency doubled 10.6 µm CO² laser. The effect of transverse magnetostatic field on the threshold intensity of the single resonant parametric oscillator has been critically examined. The effects of some important parameters such as the cavity physical length, signal reflectance, conversion efficiency and single pass signal power gain on the threshold intensity are analyzed in detail.

Contents

1. Introduction 2
2. Theoretical formulations 4
   2.1. Second-order optical susceptibility 4
   2.2. SRPO threshold 6
   2.3. Operational characteristics 8
3. Results and discussion 8
References 12

¹ Author to whom any correspondence should be addressed.
1. Introduction

Understanding and controlling the interaction between light and matter are of fundamental importance to a wide range of applications in science and technology. More than a century ago, puzzles over blackbody radiation and atomic line spectra led to the birth of quantum mechanics and there have been innumerable developments since, not least the laser. Since the invention of the laser, a major area of optoelectronics has been devoted to the development of optical devices capable of providing tunable coherent radiation through the electromagnetic spectrum. Laser light can be used to control matter as in optical tweezers to manipulate cells [1]. Surface plasmon polaritons are also being developed as bio-molecule sensors [2]. On the other hand, optical bistable studies explore possible ways to control light with light [3].

Parametric amplification and oscillation in the radio frequency and microwave range [4] were demonstrated before the laser was invented. The same process was expected in the optical region and was actually demonstrated in 1965 by Giordmaine and Miller [5]. It has since become an important effect because it allows the construction of widely tunable coherent infrared sources through the controllable decomposition of the pump frequency. The optical parametric oscillators (OPO) provide a convenient method of generating tunable radiation over broad spectral ranges and one of their very useful characteristics is the low noise level. Further backward OPO was proposed in 1966 by Harris [6], which sparked many efforts to implement it through backward parametric fluorescence [7] and backward difference frequency generation [8].

In optical parametric amplification, a weak signal is made to interact with a strong, higher frequency pump and both the generated difference frequency (known as the idler) and the original signal are amplified. If the idler and the amplified signal are passed through the mixing crystal again, with the proper phase, both are again amplified. In addition, if either one is passed through the crystal again with the proper phase, the result is still a gain in both. Thus, the amplifier can be made into an oscillator by adding the proper feedback (i.e. a resonator) to both the signal and the idler, or by resonating only one. If the gain per path is larger than the loss per path, the signal can build up out of the noise, and the system will oscillate. It is well known that by resonating only the signal or idler in an OPO, both output power stability and the tuning range are improved [9]. Recently major enhancements in the overall performance of continuous wave single resonant parametric oscillator (SRPO) through finite output coupling of the resonant wave [10] and gain width and rise time studies of pulsed unstable OPOs [11] have been reported. Various effective methods of generating eye safe radiations [12] and light in the yellow [13] and blue [14] spectral region based on OPO characteristics are also being developed.

In a polar material, the electric field interacts with both optical phonons and electrons. The scattered radiation consists generally of two parts: a single particle portion and a collective part. Single particle scattering is caused by individually moving electrons in the plasma and is nearly elastic. This portion of the spectrum can be used to determine the electron velocity distribution and presents a great interest in transport theory. The collective mode of scattering is due to plasma waves in the electron gas. In solid state plasma, the collective modes are strongly coupled with the longitudinal optical (LO) mode of the lattice and one observes a frequency shift with increasing free carrier concentration [15]. The coupling of LO phonons and free carrier collective excitations by macroscopic longitudinal electric fields in polar semiconductors has been treated theoretically by a number of investigators [16]. This coupled
wave approach can also be generalized to include waves other than electromagnetic. However, in noncentrosymmetric solids, we may replace the idler electromagnetic wave by an optically excited coherent collective mode (such as acoustical and optical phonon modes, polaron and polariton), and using the coupled mode scheme a strong tunable electromagnetic Stokes wave may be achieved as a signal wave at the expense of the pump wave. A polaron is a quasi-particle that arises due to the conduction electron (or hole) together with its self-induced polarization in an ionic crystal or in a polar semiconductor [17]–[19]. This polarization is local in character and is due to the displacements of ions from the equilibrium positions caused by the field produced by the electron density. The resulting interaction with the polarized lattice changes both the energy and mass of the electron [20].

Thus, in the present work, the choice of a weakly polar III–V compound semiconductor viz., n-InSb is made to have a polar optical scattering mechanism so that coherent polaron modes may be excited through ultra fast excitations. It will enable us to investigate the influence of polaron effects on the various optical properties of the medium. Further by using a coupled wave analysis a strong tunable electromagnetic Stokes wave may be achieved as a signal wave at the expense of the pump wave. With the help of hydrodynamic model of plasmas and coupled mode theory, we have analytically investigated the feasibility of achieving a polaron-induced SRPO through an n-InSb/CO\(_2\) laser system with a transverse magnetostatic field. Nonetheless, the present investigation has been made under the following assumptions:

1. All waves are infinite plane waves.
2. The nonlinear medium is semi-infinite, with a plane boundary surface.
3. Depletion of energy from the pump wave is being neglected.
4. The interaction is phase matched.
5. The nonlinear crystal has the same linear optical properties as free space and completely fills the resonator, which has mirrors at \(x = 0\) and at \(x = L\).
6. The crystal sample is shined by a laser with photon energy well below the band gap energy of the semiconducting crystal, so that only free charge carriers influence the optical properties of the material.
7. Effects on optical properties of material due to photo-induced interband transition mechanisms are neglected throughout the analysis.

Thus, in the present paper, we report analytical results obtained while studying threshold and operational characteristics of SRPO in a transversely magnetized semiconductor plasma. We have shown that the proper selection of magnitude of magnetic field effectively lowers the threshold intensity for the onset of SRPO. Hence, the presence of magnetic field is found to be favourable for the phenomenon under study. Threshold intensity is also found to decrease with increasing cavity length and increasing reflectivity of the signal wave. We have also determined conversion efficiency and single pass signal power gain of proposed SRPO and critically examined the influence of input intensity on both of them. Numerical estimations of threshold pump intensity, conversion efficiency and single pass signal power gain have been made with a set of data appropriate for a polar semiconductor crystal duly irradiated by a frequency doubled CO\(_2\) laser to establish the validity of the present work.
2. Theoretical formulations

In its simplest form, the proposed OPO consists of a nonlinear crystal placed within an optical resonator and irradiated by an intense optical pump field at \( \omega_p \). The pump field gives rise to a pair of signal and idler fields at \( \omega_s \) and \( \omega_{pl} \) such that \( \omega_p = \omega_s + \omega_{pl} \). The excited fields at \( \omega_s \) and \( \omega_{pl} \) can be simultaneously amplified as they propagate through the nonlinear crystal. Gain at \( \omega_s \) and \( \omega_{pl} \) is provided through parametric interaction of the three optical fields within the nonlinear medium. This interaction is a result of the nonlinear polarization exhibited by the material, which requires a lack of inversion symmetry in its crystalline structure. For a crystal of length \( L \), we will determine some operational characteristics such as single pass power gain, conversion efficiency and threshold characteristics of the proposed oscillator.

The theory of nonlinear interaction has been discussed by many authors both from classical and quantum mechanical viewpoints. If there are many photons in the radiation field it can be properly described by classical waves. In the treatment of coupled wave problems, the classical description is even more appropriate since then the decay or amplification of the waves depends on the relative phases among them, whereas in the quantum description, if the number of quanta is prescribed, the phases will be undetermined as required by uncertainty principle. Thus, here, we consider the hydrodynamic model of homogeneous semiconductor plasma. It restricts our analysis to be valid only in the limit \( kl \ll 1 \) (\( k \) is the wavenumber and \( l \) is the carrier mean free path). In order to study single resonant parametric oscillation due to excitation of polaron mode in magnetized semiconductor plasmas, we consider the propagation of a pump wave

\[
\vec{E}_p = \hat{x} E_p \exp[i(k_p x - \omega_p t)]
\]

in a homogeneous semiconductor plasma immersed in an external magnetostatic field \( \vec{B} \) along the y-axis.

2.1. Second-order optical susceptibility

We apply the coupled mode scheme to obtain simplified expression for the second-order optical susceptibility via nonlinear polarization at Stoke’s field. We consider that in the presence of two mutually perpendicular fields, the motion of each free electron gives rise to a macroscopic depolarization field normal to the surface of the crystal. The induced depolarizing field strongly couples the longitudinal and transverse degrees of freedom of the plasma and shifts the natural frequency away from the cyclotron frequency and hence induces the collective cyclotron excitation with the resonance frequency \( \omega_{cc} = (\omega_p^2 + \omega_c^2)^{1/2} \), where \( \omega_c = (-eB/m_e) \) is the cyclotron frequency, \( \Omega_p = (n_0 e^2/m_e \epsilon) \) is the plasma frequency and \( \epsilon(=\epsilon_0\epsilon_s) \) is the dielectric constant of a semiconductor, in which \( \epsilon_s \) is the high frequency dielectric constant of the medium and \( m_e \) is the effective mass of electron.

The basic equations considered in the present analysis are:

\[
\frac{\partial^2 \vec{r}}{\partial t^2} + (\Omega_p^2 + \Gamma_e^2)\vec{r} + 2\Gamma_e \frac{\partial \vec{r}}{\partial t} = -\frac{e}{m_e} \left( \vec{E}_p + \frac{\partial \vec{r}}{\partial t} \times \vec{B} \right), \tag{2a}
\]

\[
\frac{\partial^2 \vec{S}}{\partial t^2} + (\omega_L^2 + \Gamma_{ph}^2)\vec{S} + 2\Gamma_{ph} \frac{\partial \vec{S}}{\partial t} = \frac{q}{M} \left( \vec{E}_p + \frac{\partial \vec{S}}{\partial t} \times \vec{B} \right). \tag{2b}
\]
Equations (2a) and (2b) are the linearized zeroth order momentum transfer equations of the oscillatory electron fluid and lattice ions, respectively. \( E_p \) and \( B \) are the pump electric fields and applied magnetostatic field respectively. \( \mathbf{S} = u^+ - u^- \) is the relative displacement of positive and negative ions and \( \omega_L \) is the longitudinal optical phonon frequency. \( \Gamma_e \) represents electron–electron collision frequency, while \( \Gamma_{ph} \) takes into account the optical phonon decay constant. These loss parameters have been introduced phenomenologically and do not vary with the pump and the magnetostatic fields. These equations are similar to plasmon–phonon coupled mode equations [21]. The motion of the induced collective cyclotron vibration produces a polarization \( P_e (\omega = -\omega_r) \), the induced vibrations also influence the oppositely charged ions of a diatomic crystal for the oscillation and finally the oscillations result in an induced polarization \( P_s = NqS \) of the medium. The effective charge of the lattice polarization can be given as

\[
q = \omega_L [MN^{-1} \varepsilon_0 (\varepsilon_1^{-1} - \varepsilon_0^{-1})]^{1/2},
\]

where \( M \) and \( N (= a^{-3}) \) are the reduced mass of the diatomic molecule and number of unit cells per unit volume, respectively, and \( a \) is the lattice constant of the crystal. We assume that the electric fields associated with the electronic and lattice polarizations are parallel to each other and the polarizabilities of the electron and ion systems are additive. Thus, the simultaneous excitation of collective cyclotron vibrations and optical phonons results into coupling between them. The resulting coupled vibrations appear in the form of a new mode known as polaron mode. The equation of motion of a polaron mode is given by

\[
\frac{\partial^2 \mathbf{R}}{\partial t^2} + \omega_{0,pl}^2 \mathbf{R} + 2 \Gamma_{pl} \frac{d\mathbf{R}}{dt} = (NM)^{1/2} \left( -\frac{e}{m_e} + \frac{q}{M} \right) \left[ \mathbf{E}_p + \frac{d\mathbf{W}}{dt} \times \mathbf{B} \right],
\]

where \( \mathbf{W} = \mathbf{f} + \mathbf{S} \), \( \Gamma_{pl} = \Gamma_e + \Gamma_{ph} \) and \( n = n_0 + n_1 \exp[i(\mathbf{k}r - \omega t)] \). We considered that the motion of polarons takes place with respect to a new origin shifted by some distance from the original position. This consideration allows us to express the displacement of polaron mode in terms of new parameter \( R \) defined as \( \mathbf{R} = (NM)^{1/2} \mathbf{W} \) [22]. Polaron wave frequency (\( \omega_{0,pl} \)) at the center of Brillouin zone (zero wavevector mode) is given by [16]

\[
2\omega_{0,pl}^2 = \Omega_p^2 + \omega_c^2 + \Omega_{ph}^2 \pm [2(\Omega_p^2 + \omega_c^2 + \Omega_{ph}^2) - 4(\Omega_p^2 \omega_c^2 + \omega_c^2 \Omega_{ph}^2)]^{1/2},
\]

where \( \omega_T \) is the transverse optical phonon frequency.

Other basic equations governing parametric oscillations of polaron mode in the medium are

\[
\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} + n_1 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial n_1}{\partial x} = 0,
\]

\[
\frac{\partial E_{pl}}{\partial x} = -\frac{n_1 e}{\varepsilon_0} + \left( \frac{n_0 e}{\varepsilon_0} - \frac{Nq}{\varepsilon_0} \right) \frac{dR}{dx}.
\]

Conservation of charge is represented by the continuity equation (equation (6)) in which \( n_0 \) and \( n_1 \) are the equilibrium and perturbed electron densities respectively. \( v_0 \) and \( v_1 \) are the oscillatory fluid velocities of electrons of effective mass \( m_e \). Effective polaron electrostatic field \( E_{pl} \) arising due to the induced electronic and lattice polarizations is determined from Poisson’s equation (7) in which \( \varepsilon_0 \) is the permittivity of free space. We also assume that the energy transfer between the pump, polaron and Stoke’s waves satisfy phase matching conditions which are \( \omega_s = \omega_p - \omega_{pl} \) and \( k_s = k_p - k_{pl} \).
Following the procedure adopted by Neogi and Ghosh [23] and using equations (1)–(7), we may obtain second-order susceptibility via nonlinear polarization at Stokes field considered as $P_{nl}(\omega_s) = \varepsilon_0 \chi^{(2)} E_p E^*_p$:

$$\chi^{(2)} = \frac{ek_pl}{m_e\varepsilon_0} \frac{\omega_c A_1 A_4 \delta p X_1}{m_e^2 P_2 F_3},$$

(8)

where

$$A_1 = 1 + \frac{(\omega_c^2 + \Omega_p^2 + \Gamma_c^2)}{(\Gamma_c + i\omega_s)^2 + \Omega_p^2 + \omega_c^2}, \quad A_2 = 1 + \frac{(\omega_c^2 + \Omega_p^2 + \Gamma_e^2)}{(\Gamma_e - i\omega_p)^2 + \Omega_p^2 + \omega_c^2},$$

$$A_3 = 1 - \frac{\omega_c^2}{(2\Gamma_e - i\omega_p)^2 + \omega_c^2}, \quad A_4 = 1 + \frac{\left(\frac{n_{eq}}{M} + \frac{N_{eq}}{m_e} - \frac{n_{eq}^2}{m_e} - \frac{N_{eq}^2}{M}\right)}{F_1},$$

$$F_1 = (-\omega_p^2 + 2i\Gamma_p \omega_p - \omega_{0,pl}^2), \quad F_2 = (-\omega_p^2 + 2i\Gamma_p \omega_p + \Omega_p^2 A_1),$$

$$F_3 = (-\omega_s^2 + 2i\Gamma_e \omega_s + \Omega_p^2 A_1), \quad X_1 = \frac{\omega_c^2}{(2\Gamma_p + i\omega_p)^2 + \omega_c^2},$$

and

$$\delta p = \frac{(2\Gamma_e - i\omega_p)}{(2\Gamma_e - i\omega_p)^2 + \omega_c^2}.$$

2.2. SRPO threshold

In this section, we have developed a model based on slowly varying envelope approximation (SVEA) to determine the threshold intensity of an SRPO including the effects of attenuation loss due to absorption and scattering in the cavity plus the mirror transmission losses. The standard wave equation in terms of nonlinear polarization arising due to electronic and lattice polarizations is written as

$$\nabla^2 \vec{E} = \frac{1}{C^2} \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \vec{P}_{nl}}{\partial t^2}.$$  

(9)

This wave equation can be simplified by considering propagation of light in the $x$-direction only and using the assumption that the laser pulse envelope varies slowly in space and time compared to the period and wavelength of light. This approximation is called the slowly varying envelope approximation (SVEA). By adopting SVEA the equations describing the collinear OPO fields in terms of complex amplitudes, assuming no pump depletion and no wavevector mismatch conditions can be represented by

$$\frac{\partial E_s}{\partial x} + \alpha_s E_s = -i\beta_s E_p E^*_p,$$

(10)

$$\frac{\partial E_{pl}}{\partial x} + \alpha_{pl} E_{pl} = -i\beta_{pl} E_p E^*_s.$$  

(11)
where the \( \alpha s \) are field absorption coefficients and defined as \( \alpha_{s,pl} = (\omega_{s,pl}/\eta C) \chi^{(1)}_s \); \( \eta \) being the refractive index of the crystal and \( \chi^{(1)}_s \) is the imaginary part of linear optical susceptibility. \( \beta s \) are dimensionless coupling parameters defined as \( \beta_{s,pl} = (\omega_{s,pl}/\eta C) \chi^{(2)}_s \); \( \chi^{(2)}_s \) being the second-order optical susceptibility of the crystal at a finite magnetostatic field.

We assume here that the pump passes through the crystal once per round trip. Therefore \( E_p(x = 0) \) is equal to the incident pump field. Solving equations (11) and (12) under the initial boundary conditions of the two waves such as

\[
E_s(x = 0) = E_s(0) \ll E_p(0),
\]

\[
E_{pl}(x = 0) = E_{pl}(0) = 0.
\]

The solutions of equations (10) and (11) that satisfy the boundary conditions given by equation (12), can be assumed as

\[
E_{pl} = C_1 e^{i\gamma_{1}x}, \quad \text{ (13a)}
\]

\[
E_s = C_2 e^{i\gamma_{2}x}, \quad \text{ (13b)}
\]

\( \gamma_{1} = \frac{ig}{2} \) and \( g = \sqrt{\frac{4\alpha_{p,pl}^2g^2E_s^2}{k_0p_0}} \) is the parametric gain coefficient and \( K = \frac{2\pi}{C} \chi^{(2)}_s \).

We assume that the cavity length is equal to the crystal length \( L \). As the electromagnetic wave bounces back and forth between the two reflectors, it passes through the laser medium and is amplified. If the amplification exceeds the losses caused by imperfect reflection in the mirrors and scattering in the medium, the field energy stored in the resonator will increase with time. This causes the amplification constant to decrease as a result of gain saturation. The oscillation level will keep increasing until the saturated gain per pass just equals the losses. At this point the net gain per pass is unity and no further increase in the radiation intensity is possible, that is, steady state oscillations are obtained. For parametric oscillations, a single pass gain must be equal to the round trip loss. The round trip attenuations of the two fields are \( \exp(-\alpha_{pl}^\prime \eta L/C) \) and \( \exp(-\alpha_{s}^\prime \eta L/C) \), respectively, where \( \alpha_{pl}^\prime \) and \( \alpha_{s}^\prime \) are the damping constants at \( \omega_{pl} \) and \( \omega_{s} \). They constitute the attenuation loss due to absorption and scattering in the cavity plus the mirror transmission loss. \( \alpha_{pl}^\prime \) and \( \alpha_{s}^\prime \) can be obtained from the following relation [24]:

\[
\alpha_{s,pl}^\prime = \frac{C}{\eta} \left[ \ln \frac{R_{s,pl} L}{R_{s,p}} - \alpha_{s,pl} \right], \quad \text{(14)}
\]

where \( R_{s} \) and \( R_{pl} \) are the reflectivities corresponding to signal and polaron wave frequencies. Then, we have

\[
E_{pl}(0) = e^{-\alpha_{pl}^\prime \eta L/C} \left[ E_{pl}(0) \cosh\left(\frac{1}{2}gL\right) + iE_s^\ast(0) \sinh\left(\frac{1}{2}gL\right) \right], \quad \text{ (15a)}
\]

\[
E_s^\ast(0) = e^{-\alpha_{s}^\prime \eta L/C} \left[ E_s^\ast(0) \cosh\left(\frac{1}{2}gL\right) - iE_{pl}(0) \sinh\left(\frac{1}{2}gL\right) \right]. \quad \text{ (15b)}
\]

As per boundary conditions, the idler field is assumed to be zero at the entrance to the crystal. The signal field at the end of the crystal of length \( L \) is given by

\[
E_s(L) = E_s(0) \exp\left(-\frac{\alpha_{s}^\prime \eta L}{C}\right) \exp\left(\frac{1}{2}gL\right). \quad \text{ (16)}
\]

Now for SRPO, only \( gL/2 \) and \( \alpha_{s}^\prime \eta L/C \) are much smaller than 1, but \( \exp(-\alpha_{s}^\prime \eta L/C) \gg 1 \) because of the large transmission loss of the mirrors than the threshold condition will be

\[
g_{th}^2 \approx \frac{8\alpha_{s}^\prime \eta L}{C}, \quad \text{ (17)}
\]
where \( g_{th} \) gives the threshold gain. Corresponding threshold field and threshold pump intensity required to attain SRPO can be determined by the following relations:

\[
E_{pth} = \sqrt{\frac{2\alpha' s L K_k}{\omega_p^2 C K^2}},
\]

\[
I_{pth} = \frac{1}{2} \eta \varepsilon_0 C |E_{pth}|^2.
\]

### 2.3. Operational characteristics

We now address the analytical investigation of operational characteristics such as conversion efficiency and single pass signal power gain of the proposed SRPO.

The conversion efficiency \( \eta_k \) defined as the ratio of the external signal plus idler output energy to incident pump energy, can be written with the help of equations \((15)\) and \((16)\) as

\[
\eta_k = \eta |E_p(L)|^2 + \left(\frac{2\alpha' s C}{\varepsilon_0 C}\right) \eta |E_s|^2.
\]

If the excitation intensity \( (I_p) \) is well above the threshold intensity, we can obtain the single pass signal power gain \( SG \) from the following relation:

\[
SG = \frac{|E_s(L)|^2}{|E_s(0)|^2} = \left| \frac{E_s(0) \exp\left(-\frac{\alpha' s L}{\varepsilon_0 C}\right) \exp\left(\frac{1}{2} g L\right)}{|E_s(0)|^2} \right|^2
\]

Equations \((20)\) and \((21)\) may be used to determine the conversion efficiency and single pass signal power gain, respectively.

### 3. Results and discussion

We now address a detailed numerical analysis of the threshold and operational characteristics of SRPO in a transversely magnetized semiconductor plasma. The following set of parameters has been used to perform numerical appreciation of the results obtained and the crystal is assumed to be irradiated by a CO\(_2\) laser (both CW and pulsed depending upon the intensities required):

\( m_e = 0.014 m_0; \) \( m_0 \) being the free electron mass, \( M = 2.7 \times 10^{-29} \text{ kg}, \) \( N = 3.7 \times 10^{27} \text{ m}^{-3}, \)

\( \varepsilon_s = 17.54, \) \( \varepsilon = 15.68, \) \( q = 3.2 \times 10^{-20} \text{ C}, \) \( \Gamma_e = 10^{11} \text{ s}^{-1}, \) \( \Gamma_{ph} = 10^{2} \omega_{ph}, \) \( \omega_{p} = 1.78 \times 10^{14} \text{ s}^{-1} \)

and \( L = 20 \mu \text{m}. \) The results are plotted in figures 1–5.

The magnitude of the second-order susceptibility is found to be of the order of \( 10^{-6} \) SI units at around 7 Tesla of magnetic field with carrier density \( n_0 = 5 \times 10^{18} \text{ m}^{-3}. \) The magnitude of the second-order susceptibility agrees well with the experimentally observed \([25]\) and theoretically quoted values \([26]\). It is found from equations \((18)\) and \((19)\) that the threshold field for optical parametric oscillation is strongly affected by the external magnetic field and carrier concentration of the medium. The external magnetic field reduces the threshold field to an appreciable extent in a highly doped medium.

The behaviour of threshold intensity \( I_{pth} \) as a function of magnetostatic field \( B \) is shown in figure 1 at reflectivity \( R_s = 95\% \) and at carrier concentration \( n_0 = 5 \times 10^{18} \text{ m}^{-3}. \) Initially with
the application of magnetic field, \( I_{\text{th}} \) starts decreasing sharply from a very large value of the order of \( 10^{10} \text{ W m}^{-2} \). Gradually it achieves a minimum value at \( B \approx 3.2 \text{ Tesla} \). This dip may be attributed to the fact that at \( B = 3.2 \text{ Tesla} \) resonance between cyclotron and polaron wave frequency occurs. Further increase in \( B \) causes increase in threshold intensity due to resonance between cyclotron frequency and optical phonon frequency giving a peak at around 7.8 Tesla. The magnetic field \( B > 7.8 \text{ Tesla} \) causes decrease in \( I_{\text{th}} \) because in this range cyclotron frequency starts approaching Stokes frequency. At around \( B = 11 \text{ Tesla} \) the cyclotron frequency becomes nearly equal to the Stokes frequency and hence the linear absorption coefficient becomes very large resulting in a very low threshold intensity. Further tuning in \( B \) increases \( I_{\text{th}} \) but gives a dip again at \( B = 13 \text{ Tesla} \). Beyond this point cyclotron frequency approaches pump frequency and due to this threshold intensity increases abruptly and achieves a maximum value \( 10^{10} \text{ W m}^{-2} \) at \( B = 14.2 \text{ Tesla} \), where cyclotron frequency becomes exactly equal to pump frequency. Hence it can be concluded that resonant enhancement of the threshold input intensity occurs when the electron cyclotron frequency is equal to the angular frequency of pump wave, whereas resonant decrement takes place when cyclotron frequency approaches towards signal and polaron wave frequency. It is clear from the present study that the proper selection of magnetic field magnitude may cause reduction in the threshold intensity for the onset of SRPO by about an order of 6–8.

Figure 2 shows the influence of conversion efficiency on pump intensity. The maximum conversion efficiency of 16.5\% was limited by available pump power rather than damage or intracavity losses. Conversion efficiency increases with increasing pump intensity well above the threshold intensity required for the onset of optical parametric oscillations. Such a behaviour agrees well with the calculated values [27, 28] and experimental values [29, 30].
Figure 2. Variation of conversion efficiency ($\eta_\kappa$) with pump intensity ($I_p$) at $L = 1 \mu m$, $n_0 = 5 \times 10^{28} \text{ m}^{-3}$, $R_s = 95\%$, $B = 7 \text{ Tesla}$.

Figure 3. Variation of single pass signal power gain (SG) with pump intensity ($I_p$) at $L = 1 \mu m$, $n_0 = 5 \times 10^{18}$, $R_s = 95\%$, $B = 7 \text{ Tesla}$. 
Figure 4. Variation of threshold intensity ($I_{\text{th}}$) with mirror reflectivity ($R_s$) at $L = 10 \mu m$, $n_0 = 5 \times 10^{28} \text{ m}^{-3}$, $B = 7 \text{ Tesla}$.

Figure 5. Variation of threshold pump intensity ($I_{\text{pth}}$) with cavity length ($L$) at $n_0 = 5 \times 10^{18}$, $R_s = 95\%$, $B = 7 \text{ Tesla}$.
Figure 3 demonstrates the influence of input pump intensity on the single pass signal power gain of proposed optical parametric oscillator. Single power gain increases linearly with respect to pump intensity of CO\textsubscript{2} laser. Hence, the single pass signal power gain can be enhanced effectively on increasing input pump intensity well below damage threshold of n-InSb crystal, i.e. $4.2 \times 10^{12}$ W m\textsuperscript{−2}.

The behaviour of threshold intensity as a function of mirror reflectivity for signal waves is shown in figure 4. Threshold intensity required for the onset of SRPO decreases quite sharply as the reflectivity increases. The increase in mirror reflectivity increases the net gain per round trip at the signal wavelength resulting in lower oscillation threshold intensity for higher reflectivity. These results are in agreement with experimental observations [28, 30].

The dependence of SRPO threshold intensity on cavity length is shown in figure 5. The calculated dependence predicted by our theoretical model clearly shows a sharp decrease in threshold intensity, while increasing cavity length. Hence, the proposed theoretical model agrees well with the experimental observations [28].

It is found that the gap between experimental and theoretical studies is quite large due to the finite size of solid state plasmas. The above discussions reveal that SRPO can be easily achieved in a weakly polar III–V doped semiconductor subjected to a strong transverse magnetostatic field. The present theoretical study provides a model most appropriate for the finite laboratory size solid state plasma and it may be used as a guide to an experimentalist who wants to demonstrate SRPO in n-InSb.

References

[1] Ashkin A 1970 Phys. Rev. Lett. 24 156–9
[2] Homola J 2003 Ann. Bioanal. Chem. 377 528
[3] Szoke A, Danean V, Goldhar J and Kurnit N A 1969 Appl. Phys. Lett. 15 376
[4] Louisell W H 1960 Coupled and Parametric Electronics (New York: Wiley)
[5] Giordmaine J A and Miller R C 1965 Phys. Rev. 14 973
[6] Harris S E 1966 Appl. Phys. Lett. 9 114–6
[7] Yang K H, Richards P L and Shen Y R 1971 Appl. Phys. Lett. 19 320–3
[8] Chemla D S, Batifol E, Byer R L and Herbst R L 1974 Opt. Commun. 11 57–61
[9] Harris S E 1969 Proc. IEEE 57 2096–113
[10] Samanta G K and Ebrahizmazeh M 2008 Opt. Express 16 6883–8
[11] Zou S, Gong M, Liu Q, Yan P and Chen G 2005 Appl. Phys. B 81 1101–6
[12] Vodchits A I, Dashkevich V I, Kazak N S, Pavlenko V K, Pokryshkin V I, Petrovich I P, Rukhovets V V, Kraskovskii A S and Orlovich V A 2006 J. Appl. Spectrosc. 73 285–91
[13] Jensen O B, Brunn-Larsen M, Balle-Petersen O and Skettrup T 2008 Appl. Phys. B 91 61–3
[14] Ruffing B, Nebel A and Wallenstein R 2001 Appl. Phys. B 72 137–49
[15] Mooradian A and Wright G B 1996 Phys. Rev. Lett. 16 999
[16] Kaplan R, Palik E D, Wallis R F, Iwasa S, Burstein E and Sawada Y 1967 Phys. Rev. Lett. 18 159
[17] Devreese J T 1998 Proceedings of the International School of Physics “Enrico Fermi” CXXXVI ed G Iodonisi et al (Amsterdam: IOSS Press)
[18] Dekorsy T, Kutt W, Pfeiffer T and Kurz H 1993 Euro Phys. Lett. 23 223–8
[19] Landau L D 1933 Phys. Z. Sowjet union 3 664
[20] Zhu X-Q, Shi J-J, Liu Z-X and Pan S-H 1997 Z. Phys. B 102 207–16
[21] Kuznetsov A V and Stanton C J 1995 Phys. Rev. B 51 7555
[22] Seeger K 1989 Semiconductor Physics (Springer: Berlin) pp 193–201
[23] Neogi A and Ghosh S 1991 J. Appl. Phys. 69 61

New Journal of Physics 11 (2009) 093030 (http://www.njp.org/)
[24] Shen Y R 1984 *The Principles of Nonlinear Optics* (New York: Wiley) 117–40
[25] Yariv A 1975 *Quantum Electronics* 2nd edn (New York: Wiley) p 58
[26] Fond C Y and Shen Y R 1975 *Phys. Rev. B* **12** 2325
[27] Fan Y X, Eckardt R C, Byer R L, Route R K and Feigelson R S 1984 *Appl. Phys. Lett.* **45** 313
[28] Brosnan S J and Byer R L 1979 *IEEE J. Quantum Electron.* **15** 415
[29] Eckardt R C, Fan Y X, Byer R L, Marquardt C L, Storm M E and Esterowitz L 1986 *Appl. Phys. Lett.* **49** 608–10
[30] Ebrahimzadeh M, Henderson A J and Dunn M H 1990 *IEEE J. Quantum Electron.* **26** 1241