Research on Target Magnetic Properties Based on Multi-magnetic Dipole Model

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Abstract. It is difficult to accurately analyze and master the magnetic properties of irregular objects with complex internal magnetic structure. In this paper, the multi-magnetic dipole model of magnetic target is established. The magnetic field data around the magnetic target are measured by magnetometer array, and the position of the multi-magnetic dipole is searched by genetic algorithm, and the fitting error is detected at the same time. The experimental results show that the magnetic field model established by this method has high accuracy and the errors of magnetic field and magnetic moment are less than 5%. This method is beneficial to the research and engineering application of magnetic properties of objects.

1. Introduction
Many magnetic objects, such as satellites, bombs, submarines, etc., have a variety of magnetic components in their own or their interior, which show a certain degree of magnetism in space. However, due to the irregular shape of these magnetic objects, complex internal magnetic structure, uneven magnetization and other factors, it is difficult to analyze and control their magnetic properties. In solving such problems, the magnetic dipole model [1,2] has attracted much attention in civil and military fields because of its clear theoretical deduction and simple calculation. It is often used in magnetic target modelling [3,4] and positioning [5,6]. Thus, the problem of multi-magnetic source solution of magnetic objects is transformed into the study of location and magnetic moment of multi-magnetic dipole, which brings convenience to calculation without distortion.

The commonly used methods for solving multi-magnetic dipole target modeling can be divided into two categories: one is linearizing the non-linear problem, using derivative optimization method, also known as local optimization method, which is prone to noise interference in practice and even some abnormal step size. The other is completely non-linear intelligent algorithm, also known as global optimization, in which genetic algorithm is widely used because of its simple operation, high success rate and accuracy [7].

In this paper, the multi-magnetic dipole model of magnetic target is established. In CM2 Laboratory, the magnetic field data around the magnetic target are measured rapidly by magnetometer array. The mean square root of the difference between the measured and simulated values is taken as the objective function, and the position of the multi-magnetic dipole is searched by genetic algorithm.
At the same time, the fitting error is detected. Finally, the magnetic measurement, magnetic modeling and magnetic field calculation of the target object are well completed.

2. Multi-magnetic Dipole Model

In order to describe and calculate the magnetic properties of a magnetic object, it is assumed that there are many magnetic dipoles in it, and the magnetic moment and magnetic field information of a magnetic object are generated by these magnetic dipoles. According to the actual needs, the measurement space of magnetic objects is set up as simple cuboids, and these magnetic dipoles are distributed in the cuboids, as shown in Figure 1.

![Multi-magnetic Dipole Model](image)

Fig. 1. The multi-dipole model of magnetic target.

There are $N$ magnetic dipoles in the interior of a magnetic object with coordinates of $(x_i, y_i, z_i)$, $i = 1, 2, ..., N$. The third component of the magnetic field at the measuring point $P(x_j, y_j, z_j)$ is:

$$
B_{zi} = \sum_{i=1}^{N} \left( a_{zi} M_{zi} + b_{zi} M_{yj} + c_{zi} M_{xj} \right)
$$

$$
B_{yx} = \sum_{i=1}^{N} \left( b_{yx} M_{yi} + b_{yx} M_{xj} + b_{yx} M_{xj} \right)
$$

$$
B_{xz} = \sum_{i=1}^{N} \left( c_{xz} M_{xi} + c_{xz} M_{yj} + c_{xz} M_{yj} \right)
$$

(1)

Where $M_{xi}, M_{yi}$ and $M_{zi}$ are the magnetic moment components of the $i$-th magnetic dipole in the $x, y,$ and $z$ directions respectively. The coefficients $a, b,$ and $c$ are:

$$a_{zi} = \frac{\mu_0}{4\pi} \left[ \frac{3(x_j - x_i)^2}{r^3} - \frac{1}{r^3} \right],$$

$$a_{yx} = \frac{3\mu_0(x_j - x_i)(x_j - y_i)}{4\pi r^5},$$

$$a_{xz} = \frac{3\mu_0(x_j - x_i)(x_j - z_i)}{4\pi r^5},$$

$$b_{yi} = \frac{\mu_0}{4\pi} \left[ \frac{3(y_j - y_i)^2}{r^3} - \frac{1}{r^3} \right],$$

$$b_{yx} = \frac{3\mu_0(y_j - y_i)(y_j - x_i)}{4\pi r^5},$$

$$b_{xz} = \frac{3\mu_0(y_j - y_i)(y_j - z_i)}{4\pi r^5},$$

$$c_{yi} = \frac{\mu_0}{4\pi} \left[ \frac{3(z_j - z_i)^2}{r^3} - \frac{1}{r^3} \right],$$

$$c_{yx} = \frac{3\mu_0(z_j - z_i)(z_j - x_i)}{4\pi r^5},$$

$$c_{xz} = \frac{3\mu_0(z_j - z_i)(z_j - y_i)}{4\pi r^5}.$$
\[ r = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2} \]

So the magnetic field information can be written in the following matrix form,

\[ \mathbf{B} = \mathbf{G} \cdot \mathbf{M} \]  \hspace{1cm} (2)

Where

\[
\mathbf{B} = \begin{bmatrix}
B_{1x} & B_{1y} & B_{1z} & \cdots & B_{nx} & B_{ny} & B_{nz}
\end{bmatrix}^T,
\]

\[
\mathbf{G} = \begin{bmatrix}
a_{i1} & a_{i2} & a_{i3} & \cdots & a_{i1n} & a_{i2n} & a_{i3n} \\
b_{i1} & b_{i2} & b_{i3} & \cdots & b_{i1n} & b_{i2n} & b_{i3n} \\
c_{i1} & c_{i2} & c_{i3} & \cdots & c_{i1n} & c_{i2n} & c_{i3n}
\end{bmatrix}
\]

\[
\mathbf{M} = \begin{bmatrix}
M_{1x} & M_{1y} & M_{1z} & \cdots & M_{nx} & M_{ny} & M_{nz}
\end{bmatrix}^T
\]

In this way, the magnetic field information \( \mathbf{B} \) becomes a matrix of \( 3m \times 1 \), the magnetic moment \( \mathbf{M} \) becomes a matrix of \( 3n \times 1 \), and \( \mathbf{G} \) becomes a matrix of \( 3m \times 3n \). In this case (2), the linear part and the non-linear part are separated. When solving the problem, only the non-linear part needs to be searched repeatedly, which greatly reduces the difficulty of searching.

By measuring the magnetic field data of each point, calculating the magnetic field of each point in the fitting space and calculating the objective function:

\[
e_e = \sqrt{\sum_i (B_{ix} - B'_{ix})^2 + \sum_i (B_{iy} - B'_{iy})^2 + \sum_i (B_{iz} - B'_{iz})^2} \]  \hspace{1cm} (3)

Where \( i \) is the number of measuring points, \( B(B_{ix}, B_{iy}, B_{iz}) \) is measured value and \( B'(B'_{ix}, B'_{iy}, B'_{iz}) \) is fitted value. Thus, the optimal problem becomes the minimum value of solving \( e_e \).

3. Experimental verification

The experiment was carried out in the zero magnetic environment of CM2 laboratory. A black cuboid object was placed in the center of the laboratory. There were three magnetic sources inside. The measuring space was set as a cuboid, and the magnetic field data of four vertical adjacent planes of the measured object were collected and calculated by using the calibrated magnetometer array. The experiment is shown in Figure 2.

Fig. 2. Magnetic date acquisition.

Through magnetic data measurement and fitting calculation, the experimental target is fitted into five models of magnetic dipole distribution, as shown in Figure 3.
Fig. 3. The optimal model.

The objective function calculation step is set to 8000 steps. After calculation, it is found that the genetic algorithm converges quickly, and it begins to be smooth at 200 steps. The final objective function reaches $2.24 \times 10^{-4}$. The result is shown in Figure 4.

Fig. 4. The optimal process.

After the multi-magnetic dipole modeling of magnetic target is completed, 36 points around the target are compared with the measured values in the three directions of $x$, $y$, and $z$. As shown in Fig. 5, it can be seen that the modeled values are very close to the actual measured values. The calculated magnetic field error is 3.07%, and the magnetic moment error is 3.72%.
Fig. 5. Comparison of modeled values and measured values.

In order to further study the characteristics of the algorithm, five magnetic sources are placed inside the measured object of the black cuboid. The magnetic field data are also collected and fitted. The experimental object is fitted into 16 models of magnetic dipole distribution. The results are shown in Figure 6.
The objective function calculation step is set to 8000 steps. After calculation, it is also found that the genetic algorithm converges quickly, and it begins to be smooth at 200 steps. The final objective function reaches $3.18 \times 10^{-4}$. The result is shown in Figure 7.

After the multi-magnetic dipole modeling of magnetic target is completed, 36 points around the target are compared with the measured values in the three directions of x, y and z. As shown in Fig. 8, it can be seen that the modeled values are very close to the actual measured values. The calculated magnetic field error is 2.78%, and the magnetic moment error is 4.83%.
4. Conclusion
In this paper, the multi-magnetic dipole model of magnetic target is established. The linear calculation part and the non-linear part are separated. The genetic algorithm is used to search for the position of the multi-magnetic dipole by taking the root mean square of the difference between the simulated value and the measured value as the objective function. At the same time, the fitting error is detected, so that the objective function reaches the optimum and the fitting error meets the requirements. The
experimental results show that the magnetic field model established by the multi-magnetic dipole model has high precision and small error of magnetic moment and magnetic field, which is a good magnetic research method.

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