Is the cosmological constant screened in Liouville gravity with matter?

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There has been a proposal that infrared quantum effects of massless interacting field theories in de Sitter space may provide time-dependent screening of the cosmological constant. As a concrete model of the proposal, we study the three loop corrections to the energy–momentum tensor of massless $\lambda \phi^4$ theory in the background of classical Liouville gravity in $D = 2$ dimensional de Sitter space. We find that the cosmological constant is screened, in sharp contrast to the massless $\lambda \phi^4$ theory in $D = 4$ dimensions due to the sign difference between the cosmological constant of the Liouville gravity and that of the Einstein gravity. To argue for the robustness of our prediction, we introduce the concept of time-dependent infrared counter-terms and examine if they recover the de Sitter invariance in the $\lambda \phi^4$ theory in comparison with the Sine–Gordon model, where it was possible.

Subject Index B34, B72, E05, E86

1. Introduction

Recent observation of dark energy in our universe has led to the conviction that there exists a tiny but positive value of the cosmological constant $\Lambda$. This means that our space-time is de Sitter (dS) space with the Hubble constant $H$ being $\sqrt{\Lambda}$. There has been a proposal that the strong infrared (IR) divergence property of the quantum corrections on dS space may explain the smallness of $\Lambda$ in our current universe (the so-called cosmological constant problem). If we are to calculate quantum corrections to the value of the cosmological constant today, we have to deal with quantum field theories on dS space. This can be performed by using the in–in formalism or Schwinger–Keldysh formalism [1–6].

The IR divergence makes it difficult to keep the dS invariance in the propagators of massless fields. The question of whether or not to preserve the dS invariance has been addressed repeatedly [7–21] (for IR effects during cosmological inflation, see [22] for a review). The ambiguity in imposing the boundary condition on the propagators at the horizon has also been discussed in [23–27]. A complete agreement has not been reached yet in the evaluation of loop corrections to the energy–momentum (EM) tensor $T_{\mu\nu}$ in quantum gravity coupled to matter in the four dimensions (4D) [28–33]. In this situation we believe that studying quantum gravity and matter loop effects on the EM tensor...
$T_{\mu\nu}$ in two-dimensional (2D) dS space may help clarify the problem of quantum corrections to the cosmological constant $\Lambda$

Our $D = 2$ dimensional model for quantum gravity coupled to matter fields is based on the 2D Liouville field theory [34] minimally coupled to matter fields. The Liouville field is a Weyl factor of the metric and originally it has no kinetic term at the classical level. The origin of the kinetic term is from the Weyl non-invariant measure of the path integral for quantum gravity such as Weyl anomaly. The resulting Liouville field theory captures the non-perturbative dynamics of the low-energy effective field theory of the 2D quantum gravity [35,36] and hence contains the complete information of the quantum gravity as an ordinary quantum field theory. Once we derive the Liouville field theory as the 2D quantum gravity (Liouville gravity), one may take the classical limit by assuming a large number of matter fields. The Liouville field theory is conformally invariant, and at least classically, there are no subtleties in the dS background. The “coupling constant” of the potential term in the Liouville field theory is related to the cosmological constant of 2D quantum gravity and renormalized by the matter loop effects. Similarly to the higher-dimensional Einstein equation with the dS breaking quantum matter EM tensor, we expect that the subtle quantum IR effects of the interacting massless fields may significantly affect the dynamics of the Liouville gravity.

Constructing a 2D model based on the Liouville field theory is also motivated by (and is related to) an old work by Polyakov for the IR screening of the cosmological constant [37]. The Weyl factor of the metric plays a leading role there. There are attempts to deduce the screening mechanism from the dynamics of the Weyl factor in 4D gravity [38–40] and, more relevant to our work, in 2D dilation gravity [41,42] (including Liouville gravity as a special case) where the cosmological constant is canonically quantized non-perturbatively. If such a mechanism is really at work in de Sitter spacetime, it would significantly affect our mindset in relation to understanding the cosmological constant problem. Since the Liouville gravity coupled to quantum matter is a power-counting renormalizable field theory, we should be able to answer the question unambiguously.\(^1\) To be screened or not to be screened, that is the question.

In this work we restrict ourselves to the perturbative effects from the matter sector. As a concrete example, we choose a scalar field theory with $\lambda\phi^4$ interaction on dS background. Evaluation of the matter loop corrections to $T_{\mu\nu}$ to higher loops is carried out by using the propagator for a massless scalar field $\phi$ with a dS symmetry-breaking term. Hence the EM tensor acquires the logarithmic time dependence which is often referred to as the IR logarithm, $\ln a(t)$. We find that our result shows the screening effect of the effective cosmological constant up to order $\lambda^3$ corrections in $D = 2$ Liouville gravity. We emphasize that it is in sharp contrast to $D = 4$, where the $\lambda\phi^4$ theory shows the anti-screening effect. The difference comes from the treatment of the cosmological constant in the Liouville gravity from the Einstein gravity. We also find that the degree of IR divergence from massless scalar fields in 2D dS space is the same as that in 4D, at least within a perturbative computation.

This conclusion, however, is puzzling for at least one reason. In flat Minkowski space, the IR limit of $\lambda\phi^4$ theory in $D = 2$ dimensions is equivalent to a free massless Majorana fermion (or critical Ising model) from the Landau–Ginzburg construction [43,44]. The free Majorana fermion is conformally

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\(^1\)In [33], the possibility of screening the cosmological constant in non-unitary time-like Liouville theory was discussed. See also footnote 5 in comparison with their approach.
invariant and does not show any IR pathology in dS space. The cosmological constant induced by the free Majorana fermion is never screened.

A similar puzzling situation was reported in the literature [45,46] in relation to the (in)equivalence between the Sine–Gordon model and massive Thirring model in dS background. If we quantize the Sine–Gordon model perturbatively around the massless scalar field theory in dS space, the quantum IR effect of the massless scalar propagator appears as in $\lambda \phi^4$ theory and the dS symmetry is broken. On the other hand, in the dual fermionic picture, there is no such breaking effect at all. In this particular case, however, we will see in the last section that by adding non-conventional dS breaking local counter-terms (with which the quantum dS breaking is cancelled in the final correlation functions), we may recover the dS invariance in the Sine–Gordon model.

Whether such counter-terms are allowed or should be added must be determined from some other principles. If we stick to the dS invariance, there is no reason not to add them unless it is inconsistent with more important principles such as gauge invariance. The mechanism should work in other space-time dimensions, while at this stage we are not certain if the obstruction to recover the dS invariance from IR counter-terms existed for gravitons or gauge fields in higher dimensions.

To argue for the robustness of our prediction, we explore such a possibility in $\lambda \phi^4$ theory as well. Certainly, we may cancel the dS breaking screening effects on the cosmological constant by adding the time-dependent classical IR counter-term by hand. However, unlike the Sine–Gordon model, we do not find systematic ways to remove the dS breaking effects in the other correlation functions by time-dependent but local counter-terms. So within our perturbation theory, the quantum IR effects in $\lambda \phi^4$ theory are real phenomena and it is not natural to cancel only the time-dependence in the cosmological constant.

In the next section, we briefly review the IR divergence originated from non-conformally invariant massless scalar fields in dS space in general space-time dimensions. Introduction of an IR cutoff for momentum integration leads to the IR logarithm $\ln a$ in the coordinate space propagator, which immediately breaks part of the dS symmetry, namely dilatation invariance, $\eta \rightarrow b\eta$, $x^i \rightarrow b x^i$. It makes the cosmological constant time dependent through the Einstein equation. We introduce our 2D model of Liouville gravity and matter loop corrections in Sect. 3. In Sect. 4, we compute the perturbative corrections of order $\lambda^2$ to the EM tensor from massless scalar loops in $\lambda \phi^4$ theory. In Sect. 5, we discuss the possibility of the dS non-invariant counter-terms designed for cancelling the dS breaking IR logarithms in comparison with Sine–Gordon model in dS space. We conclude with a discussion in Sect. 6. In Appendix A, we report the detailed computation of the order $\lambda^2$ corrections to the EM tensor in $\lambda \phi^4$ theory.

2. Cosmological constant problem and infrared effects

2.1. Infrared divergences in de Sitter space

In this paper, we work on quantum field theories on D-dimensional dS space. Among various choices of coordinates, we mainly use the Poincaré coordinate, in which it is manifested that dS space is conformally flat. The dS geometry is expressed by the metric

$$ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x},$$

where the scale factor $a$ is given by Hubble constant $H$, and the conformal time $\eta$ as

$$a = e^{Ht} = -\frac{1}{H\eta}, \quad H(t) \equiv \frac{\dot{a}(t)}{a(t)}.$$

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Here $\eta$ is related to the physical time $t$ by

$$
\eta = -\frac{1}{H} e^{-Ht},
$$

(2.3)

and it runs from $-\infty$ to 0 ($-\infty \leq t \leq \infty$). By using the conformal time $\eta$, the dS metric becomes

$$
\text{d}s^2 = a^2(\eta)(-d\eta^2 + d\vec{x} \cdot d\vec{x}).
$$

(2.4)

This coordinate covers half of the global dS space.

In dS space, the IR divergence property of (non-conformally invariant) massless fields is different from that in Minkowski space because large distances are affected by the dS curvature. One can easily see such a property by considering the vacuum loop graphs of massless scalar fields. They are obtained by integrating over the loop momentum $P$ where $P$ is the physical momentum. Let us follow the explanation given in [47,48]. It is convenient to divide the integration region into two, the UV region (sub-horizon) $|P| > H$ and the IR region (super-horizon) $|P| < H$. For example, in 4D space-time,

$$
\int d^3P = \int_{|P|>H} d^3P + \int_{|P|<H} d^3P.
$$

(2.5)

The mode function of a massless minimally coupled scalar field in the Bunch–Davies vacuum [49] is given by

$$
\phi_k^{D=4}(\eta) = \frac{H\eta}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta},
$$

(2.6)

where $k$ is a comoving momentum which is related to physical momentum $P$ as $P = k/a(\eta) = -kH\eta$. In dS space the fluctuations of the massless fields (scalars and gravitons) have a scale-free spectrum which behaves as $1/P^3$ at super-horizon scale due to the second term in (2.6). The loop integral $\int_{|P|<H} d^3P P^{-3}$ then gives rise to a logarithmic divergent contribution at $P \to 0$.

Let us repeat the analysis in $D = 2$, which is the main focus of this paper. The mode function of the massless scalar field is given by

$$
\phi_k^{D=2}(\eta) = -\frac{i}{\sqrt{2k}} e^{-ik\eta}.
$$

(2.7)

This is completely the same form as that in Minkowski space because minimally coupled free massless scalar fields in 2D space-time are conformally invariant and we have a conformal vacuum as a dS invariant vacuum. The vacuum loop is in this case given by

$$
\int_0^\infty dP \frac{1}{2P} = \int_H^\infty dP \frac{1}{2P} + \int_0^H dP \frac{1}{2P},
$$

(2.8)

where we again make use of physical momentum $P = k/a(\eta)$ and divide the integral into $P < H$ and $P > H$ as in the $D = 4$ case, even though there is no distinction between the behavior of the sub-horizon and super-horizon modes. The divergence structure is the same as in $D = 2$ Minkowski space.

As we have seen in the above two examples, the origin of the IR divergence lies at (i) zero comoving momentum $k \to 0$ or (ii) the infinite future $\eta \to 0$. Here we regularize the IR divergence from (i)
by truncating the Hilbert space at some comoving momentum \(k_0\) as an IR cutoff. As a result of this prescription, the second term in (2.5), and (2.8), give a factor \(\ln(a(\eta)/k_0)\).\(^3\)

There is a little subtlety in putting IR and UV cutoffs for momentum in the vacuum loop. To obtain the IR logarithm from the vacuum loop, the UV cutoff of the first integral in (2.5) has been taken implicitly to be physical \(P = \Lambda_{\text{UV}} = \text{const.}\), which makes

\[
\int_H^{\Lambda_{\text{UV}}} dP
\]

constant. On the other hand, the IR cutoff in the second integral is taken to be comoving, \(k_0\), and accordingly, \(P = k_0/a(t)\) is not a constant. It amounts to saying that the UV cutoff of the theory does not change due to the cosmic expansion; on the other hand, the number of IR modes (\(P(t) < H\)) increases with time. If we put the IR cutoff to be physical \(P = \Lambda_{\text{IR}} = \text{const.}\), the time dependence disappears from the vacuum loop.

Even if the IR divergence is regularized once, owing to the exponential expansion of the universe, the dS space distance grows with time and at the same time the physical wavelengths are all red shifted. Eventually, as the conformal time \(\eta\) approaches 0 (\(t \to \infty\)), IR divergence of kind (ii) appears due to \(\ln(a(\eta))\) which is often referred to as the (dS breaking) IR logarithm. From detailed study of this IR logarithm with dS breaking, we can learn the characteristic effect of the massless fields and its consequence in dS space.

The same type of IR divergence as in the case of the vacuum loop appears in the coordinate space propagator. In Sect. 4, we will adopt the dimensional regularization for UV divergence in the loop computations while we adopt the cutoff regularization for IR divergence. The propagator of a massless minimally coupled scalar field in \(D\)-dimensional dS space is obtained in [50,51]. By setting \(D = 2 - \omega\) with \(\omega > 0\), the propagator is given by

\[
i \Delta(x, z) = \alpha \{\gamma(y) + \beta \ln(a(\eta)\alpha(\eta_z))\},
\]

where

\[
y = \frac{- (\eta - \eta_z)^2 + (\vec{x} - \vec{z})^2}{\eta \eta_z},
\]

\[
\alpha = \frac{1}{4\pi} \left(\frac{H^2}{4\pi}\right)^{-\omega}, \quad \beta = \frac{\Gamma(1 - \omega)}{\Gamma(1 - \omega/2)},
\]

and

\[
\gamma(y) = -\frac{\Gamma(1 - \omega/2)}{\omega} \left(\frac{y}{4}\right)^\omega - \frac{\Gamma(2 - \omega/2)}{1 + \omega/2} \left(\frac{y}{4}\right)^{1 + \omega/2} + \beta \delta \\
+ \sum_{n=1}^{\infty} \left[ \frac{\Gamma(1 - \omega + n)}{n\Gamma(1 - \omega/2 + n)} - \frac{\Gamma(2 - \omega/2 + n)}{(1 + \omega/2 + n)(n + 1)!} \left(\frac{y}{4}\right)^{2 + \omega/2} \right] \left(\frac{y}{4}\right)^n + O(k_0^2),
\]

\(^3\) For later use we note that the IR cutoff \(k_0\) is related to the cutoff for the initial time \(t_i\) \([23–27,47,48]\) since the largest comoving scale is given by \(l_0 = k_0^{-1}\), and an identification of \(l_0\) leads to that of \(t_i\) through \(a(t_i)l_0 = L_i\), with \(L_i\) the initial (physical) size of the universe. We adopt \(a(t_i) = 1 (t_i = 0)\) with \(L_i = l_0 = H^{-1}\) as a reference time for the renormalization conditions in Sect. 4 and an initial time cutoff for the vertex integrals in Appendix A.
with
\[\delta = -\pi \cot \left( \pi - \frac{\alpha}{2} \right) + C, \tag{2.13}\]
\[C = \frac{1}{2} \ln \left( \frac{H}{k_0} \right) + \psi \left( 1 - \frac{\alpha}{2} \right) - \psi \left( 1 - \frac{\alpha}{2} \right) + \psi(1 - \omega) - \gamma. \tag{2.14}\]

Here an IR cutoff \(k_0\) for comoving momentum has been introduced. \(\psi(x)\) is the digamma function and \(\gamma\) the Euler–Mascheroni constant. The distance in dS space is commonly denoted by \(y\). The propagator (2.9) has a simple structure such that the first term is manifestly dS invariant because it depends only on the distance \(y\), which respects the dS symmetry in Poincaré coordinates (a dilatation, \(D - 1\) dimensional spatial rotations, \(D - 1\) dimensional spatial translations and \(D - 1\) dimensional special conformal transformations [11–14,45,46]). On the other hand, the second term breaks the dS invariance (the dS isometry \(\eta \rightarrow b\eta\) and \(x \rightarrow bx\) especially) due to the IR logarithm.

The basic formalism to calculate the correlation functions in time-dependent backgrounds is called the in–in formalism. In the in–in formalism, two copies of time sheets, named + and −, are prepared and the time path is then closed: \(\int_C d\eta = \int_0^\infty d\eta_+ + \int_0^{-\infty} d\eta_-\). All vertices in the loop diagrams are assigned + or − type. The expectation value of operator(s) \(O(x)\) is given by
\[\langle \Omega | O(x) | \Omega \rangle = \left\{ T \left\{ O(x)e^{-i \int_C H_{\text{int}} d\eta} \right\} \right\}
= \left\{ \tilde{T} \left\{ e^{i \int_0^0 H_{\text{int}} - d\eta} \right\} T \left\{ O(x)e^{-i \int_0^0 H_{\text{int}} + d\eta} \right\} \right\}. \tag{2.15}\]

where \(T\) and \(\tilde{T}\) stand for the usual time-ordering operator and the anti-time-ordering operator, respectively. Here, \(\Omega\) in the first line is the vacuum of the interacting theory, and the \(\langle O(x) \cdots \rangle\) is the expectation value in the free field theory that can be computed by the Wick contraction. We introduced \(\eta\) as an initial time and assumed \(x\) to be + type in the second equality. Depending on the types of vertices, all \(y\) have one of four types [50]:
\[y_{++}(x, x') \equiv a(\eta)a(\eta')H^2[(\vec{x} - \vec{x}')^2 - (|\eta - \eta'| - i\epsilon)^2],\]
\[y_{+-}(x, x') \equiv a(\eta)a(\eta')H^2[(\vec{x} - \vec{x}')^2 - (\eta - \eta' + i\epsilon)^2],\]
\[y_{-+}(x, x') \equiv a(\eta)a(\eta')H^2[(\vec{x} - \vec{x}')^2 - (\eta - \eta' - i\epsilon)^2],\]
\[y_{--}(x, x') \equiv a(\eta)a(\eta')H^2[(\vec{x} - \vec{x}')^2 - (|\eta - \eta'| + i\epsilon)^2],\]
\[y_{ab}(x, x') \text{ stands for } y(x_a, x'_b) \text{ (}a, b = \pm\text{) with } \epsilon \text{ a positive infinitesimal. By substituting each distance } y, \text{ we can construct the four propagators used in the in–in formalism. We denote them by}\]
\[i \Delta_{++}(y_{++}) \equiv \left\{ T \left\{ \phi(x_+)|\phi(x'_+|) \right\} \right\},\]
\[i \Delta_{+-}(y_{+-}) \equiv \left\{ \phi(x_-)|\phi(x'_+) \right\},\]
\[i \Delta_{-+}(y_{-+}) \equiv \left\{ \phi(x_-)|\phi(x'_-) \right\},\]
\[i \Delta_{--}(y_{--}) \equiv \left\{ \tilde{T} \left\{ \phi(x_-)|\phi(x'_-|) \right\} \right\}. \tag{2.17}\]

The short distance (\(y \rightarrow 0\)) limit of the propagator (2.9) is regularized by \(\omega\) and is independent of the labels +, −, as is seen:
\[\lim_{x \rightarrow z} i \Delta(x, z) = \alpha \beta \left( 2\ln(a(\eta)) + \frac{2}{\omega} + C + \gamma + O(\omega) \right). \tag{2.18}\]
2.2. Infrared effects on the cosmological constant $\Lambda$

In space-time dimensions $D > 2$, the Einstein equation describes the relation between the space-time Ricci tensor $R_{\mu\nu}$ and the EM tensor $T_{\mu\nu}$ due to the presence of matter,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \quad (2.19)$$

where $R$ is scalar curvature, $\kappa = 8\pi G$ with $G$ being Newton’s constant, $\Lambda$ is the cosmological constant. In the vacuum states where $T_{\mu\nu}$ is proportional to the metric, we may transfer $T_{\mu\nu}$ to the left-hand side of (2.19), and the vacuum Einstein equation takes the form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda_{\text{eff}} g_{\mu\nu} = 0, \quad (2.20)$$

where

$$\Lambda_{\text{eff}} = \Lambda - \frac{\kappa}{D} T^\rho_\rho. \quad (2.21)$$

The vacuum contribution of $T_{\mu\nu}$ is now combined with $\Lambda$ to define the effective cosmological constant.

In view of this expression we may wonder if a large value of $T_{\mu\nu}$ may cancel the large value of $\Lambda$, yielding the tiny value of $\Lambda_{\text{eff}}$ that we observe today. For such a cancellation, the quantum corrections to $T_{\mu\nu}$ are essential. This idea may or may not address the cosmological constant problem because we have yet to know what the bare cosmological constant $\Lambda$ should be (see [52] for a review).

The situations in dS space with massless interacting fields are much more complicated. As mentioned previously, the massless scalar propagator in dS space has IR divergence. This IR divergence is regulated by the IR cutoff and renormalized. The IR cutoff, however, introduces a dS invariance breaking term from the IR logarithm $\ln a$. Then the expectation value of $T_{\mu\nu}$ of the massless interacting fields in dS space explicitly depends on the IR logarithm $\ln a$ and it becomes non-dS-covariant. As a consequence, the effective cosmological constant becomes time dependent from (2.21). This time-dependent screening effect proposed in the literature may cause the drastic effects in the fine-tuning problem of the cosmological constant.

In perturbation theory, both matter loops and gravity loops may provide sources of corrections to the cosmological constant $\Lambda$. Quantum effects of gravity in 4D dS space have been studied extensively for a long time (see [53,54] for reviews). Due to the difficulty of keeping the dS invariance in the massless propagators and the ambiguity in taking account of the boundary conditions at the horizon, complete agreement has yet to be reached in the evaluation of loop effects on $T_{\mu\nu}$ even after extensive studies. In the semi-classical limit, or in the large number of matter fields limit, the matter loop corrections will dominate over the gravity loop corrections, so we may treat the Einstein gravity classically while replacing $\Lambda$ by the quantum expectation value of the matter contributions in the fixed dS background. Although such a limit is purely academic in our $D = 4$ universe, we may still learn important lessons on screening of the cosmological constant from quantum IR effects.

The goal of this paper is to calculate quantum effects in lower dimensions. In particular, we would like to understand the dS breaking effects, and its impact on the cosmological constant. One thing to be noticed is that IR divergence in lower dimensions is stronger than that in higher dimensions in Minkowski space. A question is whether we observe similar enhancement of the IR effects in $D = 2$ dS space. We also address the question of whether the dS invariance may be recovered from the IR counter-terms. While we demonstrate the possibility in $D = 2$, the same argument may apply in higher dimensions, too.
In $D = 2$, the classical Einstein gravity becomes trivial, and the discussion in this section must be replaced by another model of gravity. We opt to use the Liouville gravity that is induced by the quantum fluctuation of the Weyl mode of the metric. Again, in the large number of matter fields limit, one may treat the Liouville degrees of freedom classically while replacing the effective cosmological constant term from the matter contributions in the fixed Liouville background. The details will be described in the next section.

3. 2D model for quantum gravity with matter

3.1. 2D Liouville theory

In $D = 2$, the Einstein gravity with the Einstein–Hilbert action

$$S[g_{\mu \nu}]=\int d^2x \sqrt{-g} \left( \frac{1}{2\kappa} (R - 2\Lambda) \right)$$  \hspace{1cm} (3.1)

has no dynamical degrees of freedom because the Einstein–Hilbert term is topological due to the Gauss–Bonnet theorem. However, at the quantum level, the Weyl mode $\Phi$ of the metric $g_{\mu \nu} = e^{2\Phi} \hat{g}_{\mu \nu}$ becomes dynamical and the quantum gravity in $D = 2$ dimensions is described by the dynamical Liouville field theory. Here, $\hat{g}_{\mu \nu}$ is the fiducial metric that we can choose arbitrarily. Because of this arbitrariness, the Weyl invariance (i.e. $\hat{g}_{\mu \nu} \to e^{2\sigma} \hat{g}_{\mu \nu}$ and $\Phi \to \Phi - \sigma$) of the Liouville gravity is automatically guaranteed.$^4$

We briefly recapitulate the Liouville theory. We begin with the 2D gravity field (metric) $g_{\mu \nu}$ coupled to “matter fields” collectively called $X$. The action and the partition function are given by

$$S_{2D}[g_{\mu \nu}, X] = \int d^2x \sqrt{-g} \left( \frac{1}{2\kappa} (R - 2\Lambda) \right) + S_{\text{mat}}[g_{\mu \nu}, X],$$ \hspace{1cm} (3.2)

$$Z = \int DX Dg_{\mu \nu} e^{iS_{2D}[g_{\mu \nu}, X]}.$$ \hspace{1cm} (3.3)

In $D = 2$, we may (locally) parametrize the gravity fluctuation by the Liouville degrees of freedom $g_{\mu \nu} = e^{2\Phi} \hat{g}_{\mu \nu}$ with the fiducial metric $\hat{g}_{\mu \nu}$. In this conformal gauge, the path integral over $g_{\mu \nu}$ is replaced by the path integral over the Liouville field $\Phi$ with the appropriate measure factor. Since the Einstein–Hilbert term only gives a topological contribution, we drop the $\frac{R}{2\kappa}$ term in the following.

The path integral measure of the 2D quantum gravity also contains the diffeomorphism ghost factors in the conformal gauge, but we will ignore them for our purpose since it has little to do with our interest in the geometric dynamics of the Liouville field (except for the balance of the Weyl anomaly). As for the measure factors of the Liouville field, it is expected to be ultralocal and gives the kinetic term of the Liouville field. We note that the kinetic term of the Liouville action is also induced by the Weyl anomaly

$$\langle T^\text{mat}_\rho \rho \rangle = \frac{c_{\text{mat}}}{24\pi} R$$ \hspace{1cm} (3.4)

of the matter action, where $c_{\text{mat}}$ is the matter central charge. Indeed, the Liouville action may be regarded as the local Wess–Zumino-like term for the Weyl anomaly.

$^4$More precisely, the quantization must respect the symmetry.
Collecting all these quantum contributions to the Liouville degree of freedom, the matter–gravity action can be reformulated as

$$Z \sim \int D\Phi DX e^{iS_L(\Phi) + iS_{\text{mat}}[\Phi, X]},$$

where

$$S_L = -\int d^2x \sqrt{-\hat{g}} \left( \frac{1}{4\pi b^2} \hat{g}^\mu_\nu \partial_\mu \Phi \partial_\nu \Phi + \frac{Q}{4\pi b} \Phi R(\hat{g}) + \frac{\Lambda_{\text{ren}}}{\kappa} e^{2\Phi} \right). \quad (3.6)$$

Here, $\Lambda_{\text{ren}}$ is the renormalized cosmological constant. We will drop the subscript ren in the following. $Q$ is the background charge given by $Q = b + b^{-1}$ (see, e.g., [34]) so that the total action is quantum mechanically conformally invariant. In the classical limit ($b \to 0$) that we will discuss below, we have the value $Q = b^{-1}$.

If the matter action is conformally invariant, the Liouville field does not appear in the matter action $S_{\text{mat}}[\Phi, X]$. We will discuss the matter coupling in the next subsection, and we focus on the Liouville part for now. The Liouville field theory is a conformal field theory in a fixed background $\hat{g}_{\mu\nu}$. The path integral over $\Phi$ is non-trivial, but we may use the trick of the large number of matter fields limit again. When the number of matter fields becomes larger, the induced Liouville kinetic term is larger and larger,\(^5\) so the quantum fluctuation of the Liouville field becomes suppressed. In (3.6), $b^2$ becomes smaller for the larger number of matter fields, and the Planck constant becomes smaller. Therefore, although the origin of the Liouville action is purely quantum mechanical, we may treat it as if it is classical in the limit of large number of matter fields.

In analogy to the dS solution in the Einstein gravity, our interest is the dS solution of the Liouville gravity. There are two alternative viewpoints. One is to choose the background fiducial metric $\hat{g}_{\mu\nu}$ to be dS space. Then we see that the classical equations of motion of the Liouville field become $2\Lambda_1 \kappa^{-1} e^{2\Phi} = -\frac{Q}{4\pi b} H^2$ for constant $\Phi$. With a convenient choice of $\Phi = 1$, the Hubble constant and the 2D cosmological constant (or Liouville coupling constant) is related. Note that the value of $H$ is not that important in the physical metric $g_{\mu\nu} = e^{2\Phi} \hat{g}_{\mu\nu}$ because it is cancelled by the factor $\Phi$ from the Liouville equation. At this point, it is important to remind ourselves that the negative value of $\Lambda$ corresponds to dS space in Liouville gravity (see also the discussion in the next subsection).

The other viewpoint is to consider the Liouville equation in the flat Minkowski space with $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$ so that it becomes $\square \Phi = 4\pi b^2 \Lambda_1 \kappa^{-1} e^{2\Phi}$. The Liouville field cannot become constant, and the simplest solution is $\Phi = -\ln(-H\eta)$, which again gives rise to the physical dS metric $g_{\mu\nu} = e^{2\Phi} \eta_{\mu\nu}$. In whichever viewpoint, the matter action couples to the physical metric $g_{\mu\nu}$, so we may only consider the matter action in the dS space.

Our de Sitter solution is consistent in the sense that it satisfies the Virasoro constraint as well. The easiest way to see this is to take the de Sitter fiducial metric. Due to the de Sitter invariance, the total energy–momentum tensor is proportional to the metric. Then, the Virasoro constraint reduces to the cancellation of the trace anomaly. As discussed in footnote 5, we have fixed the Liouville central charges so that this is the case.

As far as the Liouville mode is concerned, most of the unstable modes are removed from the Virasoro constraint. The unstable zero mode may be classically removed by the future boundary.

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\(^5\) It is determined by the Weyl anomaly of the matter as $6b^{-2} \sim -c_{\text{mat}}$. Actually, it is negatively smaller for larger $c_{\text{mat}}$, and the kinetic term becomes negative. To avoid the difficulty, we may add “non-unitary” matter with large negative central charges. For our discussions, we always keep $b^2$ to be positive. Note that this regime is opposite to the one studied in [33], where $b^2$ was chosen to be negative.
condition. Note that the unstable mode may also be interpreted as a “cosmological” solution in terms of the physical metric. It is a delicate question as to how the quantization may alter the situation, and one proposal based on the analytic continuation from the positive cosmological constant indeed suggests a possible quantum instability of the de Sitter vacuum in Liouville gravity [55]. However, the subject is controversial and beyond the scope of this paper, whose aim is the quantization of the matter on the classical de Sitter solution in Liouville gravity.

So far in this section we have treated the matter contribution as if it preserves the dS invariance. When the matter EM tensor breaks the dS invariance, the classical Liouville equation is modified and the screening effects of the Liouville coupling constant may occur. This is analogous to the matter screening of the effective cosmological constant discussed in the last section, and we will study it in the following.

3.2. The coupling of Liouville gravity and matter

Our main interest is to evaluate the quantum effects of gravity and matter in the IR region by making use of Liouville field theory. The 2D cosmological constant has two faces, one as the coupling of the Liouville potential in terms of the Liouville action, one as the trace of the EM tensor. Let us start with the action

\[ S_{L+\text{mat}}(\Phi, \phi) = -\int d^2x \sqrt{-g} \left[ \frac{1}{4\pi b^2} \hat{g}^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{Q}{\kappa} R \Phi + \frac{\Lambda}{\kappa} + \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right]. \] (3.7)

After taking the Weyl transformation to metric, we obtain

\[ S_{L+\text{mat}}(\Phi, \phi) = -\int d^2x \sqrt{-\hat{g}} \left[ \frac{1}{4\pi b^2} \hat{g}^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{Q}{4\pi b} \hat{R} \Phi + \frac{\Lambda}{\kappa} e^{2\phi} + \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + e^{2\phi} V(\phi) \right]. \] (3.8)

\( \Phi \) is the Liouville field and \( \phi \) is a matter field. The fifth term describes the interaction term between the 2D Liouville gravity and matter.

The above argument is purely classical in the Liouville degrees of freedom. As advocated before we are working in the classical Liouville regime with a large number of matter fields in mind. We will only focus on one particular degree of freedom of the matter (i.e. a scalar with \( \lambda \phi^4 \) interaction), but we always assume the extra large numbers of spectator matter fields to make the classical treatment of the Liouville field theory valid.

The dS symmetry plays an important role in the determination of the trace of the EM tensor together with the conformal symmetry in Liouville field theory. However, we have seen in Sect. 2 that if there is a massless scalar field, IR divergence will arise and break part of the dS symmetry. The existence of a dS invariant vacuum then becomes ambiguous, at least from the perturbative point of view. In this case we have additional time-dependent contributions to the effective cosmological constant.

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6 We have assumed that the Weyl anomaly cancels among the Liouville part, matter part and the ghost part, which we have not written down explicitly.
As in the Einstein gravity case discussed in the previous section, the effective cosmological constant is given by

\[ \Lambda_{\text{eff}} = \Lambda + \kappa \langle V(\phi) \rangle \]

\[ = \Lambda - \frac{\kappa}{2} \langle T_{\mu}^{\mu} \rangle. \quad (3.9) \]

Then the effective Liouville equation takes the form

\[ - \frac{1}{2\pi b^2} \Phi + \frac{Q}{4\pi b} \hat{R} = -2\Lambda_{\text{eff}} \kappa^{-1} e^{2\Phi}, \quad (3.10) \]

where \( \Lambda_{\text{eff}} \) may contain the effects of the IR dS breaking from the matter contributions in (3.9). If this is the case, the Liouville field can no longer be constant with the fiducial dS metric \( \hat{g}_{\mu\nu} \). Then the physical metric \( g_{\mu\nu} \) is not dS invariant in the semi-classical limit. In this sense, the screening of the cosmological constant gives the similar effects in the Liouville gravity to the Einstein gravity in the higher dimensions.

There is one subtle but important distinction between the Einstein gravity and the Liouville gravity that we would like to point out. In the Einstein gravity, if the energy of the universe is positive then the space-time allows the classical dS solution. This is the meaning of the positive cosmological constant in the expanding universe. However, in the Liouville gravity, the opposite is true. If the universe has the negative energy then the Liouville equation allows the classical dS solution (or sphere in the Euclidean signature). This difference yields an interesting consequence in the non-perturbative Liouville cosmology with meta-stable vacua [56,57]. In our study, the sign difference makes the IR effects of the massless \( \lambda \phi^4 \) theory screen rather than anti-screen the cosmological constant in \( D = 2 \) in sharp contrast to the situations in \( D > 2 \).

### 4. Quantum corrections — 2D matter

By fixing the value of the Liouville field \( \Phi \) to its classical configuration, (3.8) is equal to the matter action in a fixed gravitational background. In what follows we concentrate on the dynamics of the matter field \( \phi \). The purpose of this section is to evaluate the massless matter loop corrections to the EM tensor in the massless \( \lambda \phi^4 \) theory. The loop corrections from massless matters are an interesting problem in their own right. Our main interest is the IR logarithms which are particular for massless scalar fields (and graviton) in dS space. We are going to show explicitly that the cosmological constant indeed receives the time-dependent corrections through the dS breaking expectation value of the EM tensor in the way we have discussed in Sects. 2 and 3.

We shall work with a 2D massless minimally coupled scalar field theory with \( \lambda \phi^4 \) interaction. The Lagrangian is given by

\[ \mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi \sqrt{-g} - \frac{1}{4!} \lambda \phi^4 \sqrt{-g} + \Delta \mathcal{L}, \quad (4.1) \]

where \( \Delta \mathcal{L} \) consists of the counter-terms

\[ \Delta \mathcal{L} = -\frac{1}{2} \delta Z g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi \sqrt{-g} - \frac{1}{4!} \delta \lambda \phi^4 \sqrt{-g} - \frac{1}{2} \delta m^2 \phi^2 \sqrt{-g} \]

\[ + \delta \xi \left( R - D(D - 1)H^2 \right) \phi^2 \sqrt{-g} - \frac{\delta \Lambda}{8\pi G} \sqrt{-g}. \quad (4.2) \]
The matter EM tensor is given by

\[ T_{\mu\nu}^{\text{mat}}(x) = (1 + \delta Z) \left( \delta_{\mu}^{\rho} \delta_{\nu}^{\sigma} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \right) \partial_\rho \phi \partial_\sigma \phi - g_{\mu\nu} \left( \frac{\lambda + \delta \lambda}{4!} \phi^4 + \frac{1}{2} \delta m^2 \phi^2 + \frac{\delta \Lambda}{8\pi G} \right) - 2 \delta \xi \left[ g_{\mu\nu} \left( (D - 1) H^2 \phi^2 + (\phi^2)^\rho_{\rho} \right) - (\phi^2)_{\cdot \mu\nu} \right], \]  

(4.3)

where \( \delta \xi \) denotes the covariant derivative with respect to the dS background \( g_{\mu\nu} = a(\eta)^2 \eta_{\mu\nu} \). The first term is the effect of the kinetic term and the second, third and fourth terms are the effects of the potential terms. The last term proportional to \( \delta \xi \) is the conformal counter-term.

We will calculate the vacuum expectation value (VEV) of the EM tensor by using the in–in formalism. Our calculation is regarded as a 2D analogue of that in 4D performed first in [50]. Regarding the IR logarithm, \( \ln a \), we expect that the leading contribution to the EM tensor comes from the potential term. This is because the degree of IR divergence is weakened by derivatives: derivatives acting on the propagators reduce the number of IR logarithms. Hence the contributions of the IR logarithms from the kinetic terms are weaker than from the potential terms at each order of perturbative calculation. More detailed discussions including the issue of conservation of the EM tensor are given in [47,48]. In the following we will focus on the potential term as the leading contribution to the EM tensor and neglect the kinetic term contribution.

Our renormalization prescription in this section follows [50] in the sense that we only introduce the dS invariant counter-terms. This is motivated by the need to keep the equations of motion of the \( \lambda \phi^4 \) theory intact. As in \( D = 4 \), the dS symmetry will be broken by the renormalization. We will ask if the dS breaking counter-terms may or may not rescue the situation in the next section.

The EM tensor deriving from the potential term is

\[ T_{\mu\nu}^{\text{pot.}}(x) = -g_{\mu\nu} \left( \frac{\lambda + \delta \lambda}{4!} \phi^4 + \frac{\delta m^2}{2} \phi^2 + \frac{\delta \Lambda}{8\pi G} \right) - 2 \delta \xi \left[ g_{\mu\nu} \left( (D - 1) H^2 \phi^2 + (\phi^2)^\rho_{\rho} \right) - (\phi^2)_{\cdot \mu\nu} \right]. \]  

(4.4)

To evaluate its expectation value, we expand the time-evolution operator as

\[ \langle \Omega | T_{\mu\nu}^{\text{mat}}(x) | \Omega \rangle \simeq \left[ T \left\{ T_{\mu\nu}^{\text{mat}}(x) \left( 1 + i \int_C \sqrt{-g} d^2 z L_{\text{int}} \right) \right\} \right], \]  

(4.5)

with \( L_{\text{int}} \) made of the order \( \lambda \) terms in (4.1), in order to take into account the first-order effects in the perturbation theory. The resulting expectation value of the EM tensor includes terms of order \( \lambda^2 \). At the first order in \( \lambda \), the expectation value of the EM tensor (4.4) can be evaluated in the free vacuum, as we will see.

We begin with the evaluation of the following terms because we know they will determine the rest of (4.4).

\[ -g_{\mu\nu} \left\{ \langle \Omega | \phi_4(x) \left( \frac{\lambda}{4!} \phi^4(x) + \frac{1}{2} \phi^2(x) \right) | \Omega \rangle \right\} \]

\[ = -g_{\mu\nu} \left[ \frac{\lambda}{4!} \phi_4(x) + \frac{1}{2} \phi^2(x) \right] + i \left( \frac{\lambda}{4!} \right)^2 \int d^2 z \sqrt{-g} \{ T \{ \phi_4(x) \phi^4(z) \} \}

- i \left( \frac{\lambda}{4!} \right)^2 \int d^2 z' \sqrt{-g} \{ \phi_4(x) \phi_4(z') \} + i \frac{\lambda \delta m^2}{2} \int d^2 z \sqrt{-g} \{ T \{ \phi_5(x) \phi^4(z) \} \} \]
We are working in the in–in formalism. The two copies of the vertices on the so-called + and \(-\) coordinates have been introduced: \(z\) and \(z'\) should be regarded as the vertices of the + and \(-\) types, respectively. The space-time point \(x\) at which the EM tensor operator is inserted is now assumed to be on a + coordinate.

**4.1. Order \(\lambda\) potential contributions**

Let us consider the first two terms in (4.6) that are order \(\lambda\) corrections to the matter EM tensor. The two diagrams corresponding to those terms are shown in Fig. 1. The first quantity to be calculated is the mass counter-term \(\delta m^2\) which is determined by the renormalization condition that the renormalized mass is zero at the initial time \(\eta = -1/H(t_i = 0)\). This renormalization condition is the same as in [50, 58].

The one-loop diagrams shown in Fig. 2 give the one-loop scalar squared mass \(M_{1-loop}^2\).

\[
-i M_{1-loop}^2(x, x') = -i \left[ \frac{\lambda}{2} \Delta(x, x) + \delta m^2 \right] \delta^2(x - x').
\]

(4.7)

Substitution of (2.18) into (4.7) yields

\[
M_{1-loop}^2(x, x') = \left[ \frac{\lambda}{2} \alpha \beta \left( 2 \ln(a(\eta)) + \frac{2}{\omega} + A' \right) + \delta m^2 \right] \delta^2(x - x'),
\]

(4.8)

where \(A' = C + \gamma\). The renormalization condition for the mass mentioned above reads

\[
\delta m^2 = -\frac{\lambda}{2} \alpha \beta \left( \frac{2}{\omega} + A' \right) + O(\lambda^2).
\]

(4.9)
It follows that
\[ M^2_{1\text{-loop}}(x, x') = \frac{\lambda}{4\pi} \ln(a(\eta)) \delta^2(x - x'). \] (4.10)

To eliminate UV divergence from the EM tensor completely, we need to introduce the counter-term for the cosmological constant \( \delta \Lambda \) in addition to the mass counter-term. The diagrams in Fig. 1 \( a1 \) and \( a2 \) with the mass counter-term (4.9) give
\[ \frac{\lambda}{4!} \phi^4(x) + \frac{1}{2} \delta m^2 \phi^2(x) = \frac{\lambda}{4} \cdot 3 \Delta^2_{++}(x, x) + \frac{1}{2} \delta m^2 \Delta_{++}(x, x) \]
\[ = \frac{\lambda}{2} \left( \frac{1}{4\pi} \left( \frac{H^2}{4\pi} \right)^2 \frac{\Gamma(1 - \omega)}{\Gamma(1 - \omega/2)} \right)^2 \]
\[ \times \left( -\frac{1}{\omega^2} - \frac{1}{\omega} A' + \ln^2 a(\eta) - \frac{A^2}{4} \right). \] (4.11)

Accordingly, \( \delta \Lambda \) is determined by the requirement that
\[ -g_{\mu\nu} \left[ \frac{\lambda}{4!} \phi^4(x) + \frac{1}{2} \delta m^2 \phi^2(x) \right] - \frac{\delta \Lambda}{8\pi G} g_{\mu\nu} = \text{(finite)}. \] (4.12)

Thus we have
\[ \frac{\delta \Lambda}{8\pi G} = \frac{\lambda}{32\pi^2} \left( \frac{H^2}{4\pi} \right)^{-\omega} \frac{\Gamma^2(1 - \omega)}{\Gamma^2(1 - \omega/2)} \left[ \frac{1}{\omega^2} + \frac{1}{\omega} A' \right] + \frac{\delta \Lambda_{\text{fin}}}{8\pi G}, \] (4.13)

where \( \delta \Lambda_{\text{fin}} \) is the finite part of the counter-term for which we shall choose \( \delta \Lambda_{\text{fin}} = A^2/4 \) at this stage. As a result, the EM tensor at order \( \lambda \) is obtained as
\[ \langle T^{\text{mat}}_{\mu\nu} \rangle_{\text{pot. } O(\lambda)} = -g_{\mu\nu} \frac{\lambda}{32\pi^2} \ln^2 a(\eta). \] (4.14)

As we have mentioned in the last subsection, the order \( \lambda \) contribution corresponds to the zeroth-order result in the perturbative expansion. In the next subsection and Appendix A, we consider the order \( \lambda^2 \) contribution in order to include the effect from the interaction vertices.

### 4.2. Order \( \lambda^2 \) potential contributions

In order to evaluate the renormalization of the cosmological constant at the \( \lambda^2 \) order, we must deal with the three-loop diagrams. We note that the counter-terms for the coupling constant \( \delta \lambda \) and the conformal coupling \( \delta \xi \) are absent in our computation. This is because the one-loop correction to the \( \lambda \phi^4 \) interaction term is not UV divergent in 2D and we do not have the terms proportional to the mixing of UV and IR divergent term \( \omega^{-1} \cdot \ln a(\eta) \) that are supposed to be cancelled by \( \delta \xi \) [50]. The detail of the calculation of the three loop diagrams is presented in Appendix A. The leading contribution to the EM tensor at this order is
\[ \langle T^{\text{mat}}_{\mu\nu} \rangle_{\text{pot. } O(\lambda^2)} \sim -g_{\mu\nu} \frac{1}{8\pi} \frac{\lambda^2}{(4\pi)^2 H^2} \ln^4 a(\eta). \] (4.15)

The dimensionless expansion parameter can be regarded as \( \lambda/H^2 \) and our perturbative computation is valid as long as \( \ln a(\eta) < H \lambda^{-1/2} \).
From \((4.14)\) and \((4.15)\), we finally obtain the effective cosmological constant at the order \(\lambda^2\),

\[
\Lambda_{\text{eff}} = \Lambda - \frac{\kappa}{2} \langle T_{\rho}^{\text{mat} \rho} \rangle
\]

\[
\sim \Lambda + \text{(Weyl anomaly)} + \frac{\kappa \lambda}{32 \pi^2} \ln^2 a(\eta) + \frac{1}{8\pi} \frac{\kappa \lambda^2}{(4\pi)^2 H^2} \ln^4 a(\eta),
\]

\((4.16)\)

where the Weyl anomaly is given by \(\langle T_{\rho}^{\text{mat} \rho} \rangle = R/(24\pi)\) \([59]\). The effective cosmological constant has time dependence as expected, and it increases as time passes. We recall that in the Liouville gravity the dS vacuum corresponds to a negative value of \(\Lambda\). Therefore, the cosmological constant evolves from the negative value toward zero, leading to the Minkowski space (within our approximation). This means that the massless \(\lambda \phi^4\) theory in \(D = 2\) shows the IR screening effect on the cosmological constant. As we noted in Sect. 3, it crucially relies on the nature of the classical Liouville gravity. For comparison, see for instance \([47,48]\) where massless \(\lambda \phi^4\) theory in 4D dS space has been investigated and the effective cosmological constant shows the anti-screening effect at the perturbative level.

5. Do IR counter-terms recover the dS invariance? A brief look at Sine–Gordon model and \(\lambda \phi^4\) theory

The conclusion that the \(\lambda \phi^4\) theory screens the cosmological constant in late time is puzzling in \(D = 2\) dimensions. In flat Minkowski space, the IR limit of \(\lambda \phi^4\) theory in \(D = 2\) dimensions is equivalent to a free Majorana fermion from the Landau–Ginzburg construction \([43,44]\). The free Majorana fermion is conformally invariant and does not show any IR pathology in dS space.

A similar but slightly simpler question was addressed in the equivalence between the Sine–Gordon model and the massive Thirring model in the literature. In \([45,46]\), it was claimed that the equivalence is lost in dS space due to the quantum IR effects of the massless scalar propagator used in the perturbative construction of the Sine–Gordon model. However, as we presently show, all the dS breaking effects are local and, if we allow the dS non-invariant local counter-terms, these effects can be completely removed. Then the equivalence between the Sine–Gordon model and the massive Thirring model still holds in dS space and both are dS invariant. It is worth asking if we could apply the same technique in \(\lambda \phi^4\) theory to recover the dS invariance. In this paper, we give a brief report on this program and leave the details to a future publication.

We compare the \(\lambda \phi^4\) theory with the Sine–Gordon model in dS background,

\[
S_{\text{SG}} = \frac{1}{2} \int d\eta dx \left( \partial_\eta \phi \partial_\eta \phi - \partial_x \phi \partial_x \phi - \frac{\lambda}{(H \eta)^2} \cos(\beta \phi) \right).
\]

\((5.1)\)

We treat the Sine–Gordon term \(\lambda \cos(\beta \phi)\) as a perturbation around the free massless scalar theory similarly to the perturbative treatment of the \(\lambda \phi^4\) theory we have discussed.

In perturbation theory with respect to \(\lambda\), if we use the propagator with the IR cutoff regularization given by

\[
G_{12} = \langle \phi(\eta_1, x_1) \phi(\eta_2, x_2) \rangle = -\frac{1}{4\pi} \ln \left( \frac{-(\eta_1 - \eta_2)^2 + (x_1 - x_2)^2}{H^{-2}} \right),
\]

\((5.2)\)

the dS invariance is broken in the Sine–Gordon model \([45,46]\) precisely as in \(\lambda \phi^4\) theory. To obtain the dS invariance, the expression must be written by using the dS invariant length

\[
y = \frac{-(\eta_1 - \eta_2)^2 + (x_1 - x_2)^2}{\eta_1 \eta_2},
\]

\((5.3)\)
but then simple replacement in the propagator such as
\[ G_{12}^{\text{inv}} = \langle \phi(\eta_1, x_1)\phi(\eta_2, x_2) \rangle = -\frac{1}{4\pi} \ln \left( \frac{-(\eta_1 - \eta_2)^2 + (x_1 - x_2)^2}{\eta_1 \eta_2} \right) \] (5.4)
does not solve the massless equations of motion \( \Box \phi = 0 \) (except at \( \eta = -\infty \)).

We, however, realize the following alternative possibility. If we modify the action with the time-dependent coupling constant
\[ \kappa_{SG}^{\text{modified}} = \frac{1}{2} \int d\eta dx \left( \partial_\eta \phi \partial_\eta \phi - \partial_x \phi \partial_x \phi - \frac{\lambda(H\eta)^{\beta^2/4\pi}}{(H\eta)^2} \cos(\beta \phi) \right), \] (5.5)
then the de Sitter invariance in the correlation functions is recovered. The time-dependent coupling constant precisely cancels the dS non-invariant IR regularization in the massless scalar propagator (i.e. the IR logarithm). Therefore we may always use the IR counter-terms to remove all the dS breaking effects in the Sine–Gordon model by declaring that (5.5) is our dS invariant renormalized model.

With this viewpoint, let us reconsider the \( \lambda \phi^4 \) theory in 2D dS space. The question is if we could remove all the dS breaking effects by introducing the IR counter-terms as in the Sine–Gordon model. As for the cosmological constant, we could always cancel the dS breaking effects by introducing the time-dependent IR counter-terms of the cosmological constant by
\[ -\delta \Lambda(\eta) = \frac{\kappa}{2} \left( T^\mu_\mu \right) = \frac{\lambda}{32\pi^2} \ln^2 a(\eta) + \frac{1}{6\pi^2} \frac{\lambda^2}{(4\pi^2)^2} \ln^4 a(\eta) + \cdots. \]
However, such cancellation seems ad hoc. Rather, the real question is if we could systematically set up the dS invariant perturbation theory by combining the dS breaking quantum IR effects in the propagator and the dS breaking time-dependent local counter-terms. If this is the case, we have a physical motivation to preserve the dS invariance at the sacrifice of the naive equations of motion.

Within the perturbative approach in \( \lambda \phi^4 \) theory, we find that the similar construction is non-obvious in contrast with the Sine–Gordon model. To illustrate this point, suppose that the dS invariant perturbation theory is achieved by adding the IR local counter-terms in our computation of the screening of the cosmological constant. At the first order in \( \lambda \), we would expect terms like
\[ \lambda(G_{11}^{\text{inv}})^2 = \lambda(\lambda^2 + \frac{1}{16\pi} \log(H\eta_1))^2 \] (from diagram a1 of Fig. 1). At this order, the necessary counter-term to reproduce the dS invariant VEV \( \lambda(G_{11}^{\text{inv}})^2 \) is \( \lambda \phi^2 \log(H\eta) \). At the second order in \( \lambda \), we would like to recover, e.g.,
\[ \lambda^2(G_{12}^{\text{inv}})^4 = \lambda^2(G_{12} + \frac{1}{4\pi} \log(H\eta_1) + \frac{1}{4\pi} \log(H\eta_2))^4 \] (from diagram b3 of Fig. A1 in Appendix A). However, in order to reproduce it from the local counter-terms, we would require the counter-terms \( \lambda \phi^2 \log^2(H\eta) \) and \( \lambda \phi^2 \) among others. This is inconsistent with what we saw in the first-order result. Thus, there are no local counter-terms to fully recover the dS invariant perturbation theory, unlike in the Sine–Gordon example.\(^7\)

6. Discussion
In this paper we have constructed a 2D model of quantum gravity coupled to matter in dS space to explore the quantum IR effects in lower-dimensional dS space. The model (3.8) is described by the Liouville field theory coupled to matter which is minimally interacting with the Liouville field \( \Phi \) through the physical metric. Once the fiducial metric is taken to be dS space, the classical Liouville...
field equation has a constant solution, and in this case the model reduces to an ordinary matter theory in the fixed dS background. One eminent feature of our model is that the cosmological constant in 2D dS space has the negative sign that follows from the Liouville field equation. This property of the cosmological constant is opposite to the case of the Einstein gravity in $D > 2$.

As a concrete matter Lagrangian, we have studied a massless scalar field theory with $\lambda\phi^4$ interaction minimally coupled to Liouville gravity. In dS space, the massless scalar propagator contains the IR divergence in the long wavelength limit and the IR logarithm appears due to the cutoff regularization of the IR divergence. Based on the in–in formalism, we have computed the VEV of the EM tensor of order $\lambda^2$. The resulting VEV (4.15) has a time dependence through the IR logarithms, and as a consequence, the effective cosmological constant shows the screening effect at late time such that the absolute value decreases with time. This should be in contrast with the situations in $D > 2$, in which the cosmological constant is anti-screened in the $\lambda\phi^4$ theory.

The degree of IR divergence in 2D, however, has turned out to be the same as that in 4D [50]. If it were in Minkowski space, the degree of IR divergence in 2D would have been stronger than that in 4D. We do see the IR logarithms $\ln a$ in both 2D and 4D dS space, but we do not observe the enhanced degree of IR divergence in the VEV of the energy–momentum tensor. In dS space, the leading IR logarithm at the two-loop order comes directly from the coincident propagator and it is easy to understand why it is more or less the same both in 2D and 4D because the propagator itself has the IR logarithm in both cases. The situation at the three-loop order is much more non-trivial due to the space-time integral involved. Indeed, not only does this integral generate the extra contributions to the log divergence, but it makes the Hubble parameter appear in the denominator, which is the remnant of the stronger IR divergence in 2D. Based on this observation, we may expect that the same argument for the power-counting of the leading term of the IR logarithm in 4D dS space applies to our 2D case as well. According to [47,48,50], at $L$-loop order, the VEV of the energy–momentum tensor scales as

$$\langle T_{\mu\nu}^{\text{mat}} \rangle \sim -g_{\mu\nu} \left( \frac{1}{H^2} \right)^{L-2} \left( \lambda \ln^2 a \right)^{L-1},$$

where the $L$ dependence of the power law of the Hubble constant compensates the mass dimension coming from the dimensionful coupling constant $\lambda$. Then we may apply the known methods to resum the leading IR logarithms [60,61] in our $D = 2$ case, but we leave the detailed study for a separate work.

Once the effective cosmological constant is time dependent due to the matter quantum effects, the classical Liouville field dynamics will be affected as in the case of 4D Einstein gravity. The matter dynamics modify the classical Liouville equation through the time-dependent matter EM tensor and it prevents the physical metric possessing the dS solution. We would like to investigate the dynamics of the subsequent Liouville field and its quantum effects in the back-reacted solution in a future work.

In order to claim that the observed dS breaking effects are physical, we have to ask if they may or may not be removed from the local counter-terms. Here, we should discuss rather unfamiliar time-dependent IR counter-terms. This possibility plays a crucial role in understanding the (in)equivalence between the Sine–Gordon model and the massive Thirring model in dS space, where we have shown that we are indeed able to recover the dS invariance by adding time-dependent IR counter-terms to the naive perturbative computations using the dS breaking propagator. Within the perturbation theory we have studied, however, we do not see that a similar mechanism works in $\lambda\phi^4$ theory. This fact supports the claim that the observed screening mechanism of the cosmological constant should be physical. It is desirable to establish the non-perturbative argument for further support because the perturbation
becomes unreliable in later times with a smaller effective cosmological constant. Further discussion on this sensitive issue will be found in our future publication.

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Appendix A. Order $\lambda^2$ corrections to the energy–momentum tensor

In this appendix we outline the calculation of order $\lambda^2$ loop corrections to the EM tensor. From (4.6), we start with

$$-g_{\mu\nu} \left\langle \frac{\lambda}{4!} \phi^4(x) + \frac{1}{2} \delta m^2 \phi^2(x) \right\rangle_{O(\lambda^2)}$$

$$= -g_{\mu\nu} \left[ i \left( \frac{\lambda}{4!} \right)^2 \int d^2 z \sqrt{-g(z)} T\{\phi^4(x)\phi^4(z)\} - i \left( \frac{\lambda}{4!} \right)^2 \int d^2 z' \sqrt{-g(z')} T\{\phi^4(z')\phi^4(x)\} ight.$$ 

$$+ i \frac{\lambda}{4!} \frac{\delta m^2}{2} \int d^2 z \sqrt{-g} T\{\phi^2(x)\phi^4(z)\} - i \frac{\lambda}{4!} \frac{\delta m^2}{2} \int d^2 z' \sqrt{-g} T\{\phi^2(z')\phi^4(x)\}$$ 

$$+ i \left( \frac{\delta m^2}{2} \right)^2 \int d^2 z \sqrt{-g} T\{\phi^2(x)\phi^2(z)\} - i \left( \frac{\delta m^2}{2} \right)^2 \int d^2 z' \sqrt{-g} T\{\phi^2(z')\phi^2(x)\} \right].$$

(A1)

Note that we chose $x$ and $z$ as $+$ type vertices and $z'$ as the $-$ type vertex, as explained in Sect. 4. The Feynman diagrams are shown in Fig. A1, where the second line of (A1) to $b1$–$b4$, the third line to $c1$ and $c2$, the fourth line to $d1$ and $d2$, and the last line to $e1$ and $e2$.

Wick contractions and a simple calculation by the use of (2.17) give

$$\left\langle \frac{\lambda}{4!} \phi^4(x) + \frac{1}{2} \delta m^2 \phi^2(x) \right\rangle_{O(\lambda^2)}$$

$$= i \int d^2 z \sqrt{-g} \left[ \frac{\lambda^2}{24} \left( i \Delta^4_{++}(x, z) - i \Delta^4_{+-}(x, z) \right) ight.$$ 

$$+ \frac{1}{2} \left( \frac{\lambda}{2} \Delta(x, x) + \delta m^2 \right) \left( \frac{\lambda}{2} \Delta(z, z) + \delta m^2 \right) \left( i \Delta^2_{++}(x, z) - i \Delta^2_{+-}(x, z) \right) \right]$$

$$\equiv I_1 + I_2.$$
Fig. A1. Order $\lambda^2$ corrections to the EM tensor $T_{\mu\nu}^{pot}$.

where we have collected the integrations into that of $z$ since $z$ and $z'$ are dummy variables, and defined $I_1$ and $I_2$ by

$$I_1(\eta) \equiv i \int d^2z \sqrt{-g} \left[ \frac{\lambda^2}{24} (i \Delta^4_{++}(x, z) - i \Delta^4_{+-}(x, z)) \right].$$  \hfill (A3)

$$I_2(\eta) \equiv i \int d^2z \sqrt{-g} \left[ \frac{1}{2} \left( \frac{\lambda}{2} i \Delta(x, z) + \delta m^2 \right) \left( \frac{\lambda}{2} i \Delta(z, z) + \delta m^2 \right) (i \Delta^2_{++}(x, z) - i \Delta^2_{+-}(x, z)) \right].$$  \hfill (A4)

Let us first consider $I_1(\eta)$. The integrand is expanded as

$$i \Delta^4_{++}(x, z) - i \Delta^4_{+-}(x, z)$$

$$= \left\{ i \Delta^2_{++}(x, z) + i \Delta^2_{+-}(x, z) \right\} \left\{ i \Delta_{++}(x, z) + i \Delta_{+-}(x, z) \right\} \left\{ i \Delta_{++}(x, z) - i \Delta_{+-}(x, z) \right\}$$

$$= \alpha^4 \left\{ \gamma^4(y_{++}) - \gamma^4(y_{+-}) + 4\beta \ln(aa_z)(\gamma^3(y_{++}) - \gamma^3(y_{+-})) \right.\right.$$  

$$\left. + 6\beta^2 \ln^2(aa_z)(\gamma^2(y_{++}) - \gamma^2(y_{+-})) + 4\beta^3 \ln^3(aa_z)(\gamma(y_{++}) - \gamma(y_{+-})) \right\},$$  \hfill (A5)

with abbreviations $y_{++} = y_{++}(x, z)$, $y_{+-} = y_{+-}(x, z)$, $a = a(\eta)$, and $a_z = a(\eta_z)$. It is clear from (2.12) that we can take a limit $\omega \to 0$ safely in (A5) since the terms that include $\omega$ in their denominators offset each other. Thus we have

$$i \Delta^4_{++}(x, z) - i \Delta^4_{+-}(x, z)$$

$$= \left( \frac{1}{4\pi} \right)^4 \left\{ \ln^4 \left( \frac{y_{++}}{4} \right) - \ln^4 \left( \frac{y_{+-}}{4} \right) - (4C + 4 \ln(aa_z)) \left\{ \ln^3 \left( \frac{y_{++}}{4} \right) - \ln^3 \left( \frac{y_{+-}}{4} \right) \right\} \right.$$  

$$\left. + (6C^2 - 2\pi^2 - 36C \ln(aa_z) + 6 \ln^2(aa_z)) \left\{ \ln \left( \frac{y_{++}}{4} \right) - \ln \left( \frac{y_{+-}}{4} \right) \right\} \right\].$$  \hfill (A6)
Next we integrate (A6), noting that \( \sqrt{-g} = a_z^2 \) and
\[
y_{++}(x, z) = a a_z H^2 (r^2 - \Delta \eta^2 + 2i e|\Delta \eta|),
y_{+-}(x, z) = a a_z H^2 (r^2 - \Delta \eta^2 - 2i e \Delta \eta),
\]
with \( r = |\vec{x} - \vec{z}| \) and \( \Delta \eta = \eta - \eta_z \). The cut prescription allows us to write \( \ln y \) as [47,48,50]
\[
\lim_{\eta \to 0} \ln y_{++} = \ln \left[ a a_z H^2 (\Delta \eta^2 - r^2) \right] + i\pi \theta(\Delta \eta^2 - r^2),
\lim_{\eta \to 0} \ln y_{+-} = \ln \left[ a a_z H^2 (\Delta \eta^2 - r^2) \right] - i\pi \theta(\Delta \eta^2 - r^2) \theta(\Delta \eta) - \theta(-\Delta \eta),
\]
and then the interval of integration becomes
\[
\int d^2 z = \int_{\frac{\eta}{\Delta \eta}}^{\eta} d\eta_z \int_{0}^{\frac{\Delta \eta}{\eta}} dr.
\]
This means that the contribution from outside of the past light cone vanishes due to \( i \Delta_{++} = i \Delta_{+-} \)
for either \( r^2 > \Delta \eta^2 \) or \( \Delta \eta < 0 \). Using (A8) and (A9), we have
\[
I_1(\eta) = i \frac{\lambda^2}{24(4\pi)^4} \int_{\frac{\eta}{\Delta \eta}}^{\eta} d\eta_z a_z^2 \Delta \eta \left[ 8\pi i \ln^3 (a a_z H^2 \Delta \eta^2 / 4)
+ \left\{ 8\pi i K_1 - 6\pi i (4C + 4 \ln(a a_z)) \right\} \ln^2 (a a_z H^2 \Delta \eta^2 / 4)
+ \left\{ 8\pi i (K_2 - \pi^2) - 6\pi i K_5 (4C + 4 \ln(a a_z)) \right\} \ln (a a_z H^2 \Delta \eta^2 / 4)
+ 4\pi i (6C^2 - 2\pi^2 - 36 \ln(a a_z) + 6 \ln^2(a a_z)) \ln (a a_z H^2 \Delta \eta^2 / 4)
+ 8\pi i (K_3 - \pi^2 K_4) - (6\pi i K_6 - 2\pi^3 i)(4C + 4 \ln(a a_z))
+ 4\pi i K_4 \left[ 6C^2 - 2\pi^2 - 36 \ln(a a_z) + 6 \ln^2(a a_z) \right]
+ 2\pi i \left\{ - 4C^3 - 12 \ln(a a_z) - 12C \ln^2(a a_z) - 4 \ln^3(a a_z) \right\} \right],
\]
where \( K_n (n = 1, \ldots, 6) \) are some constants which are not important in the subsequent discussions.
For the time integral, it is convenient to change the variable \( \eta_z \) to \( a_z \),
\[
\int_{\frac{\eta}{\Delta \eta}}^{\eta} d\eta_z = \int_{1}^{a} \frac{da_z}{Ha_z}.
\]
The result of integral (A10) has a very long expression and we shall avoid presenting it here in full
because we are interested in the late time (namely \( \eta \ll -1/H \)) behavior of \( I_1(\eta) \) [and \( I_2(\eta) \)].
The leading contributions from \( I_1 \) at that time can be extracted by
\[
I_1(\eta) \sim \frac{\lambda^2}{24\pi (4\pi)^2 H^2} \ln^4 a.
\]
Next we move on to the evaluation of \( I_2(\eta) \). This can be done in a way similar to that used for \( I_1(\eta) \)
and gives a simple expression:
\[
I_2(\eta) = \frac{i}{2} \frac{\lambda^2}{(4\pi)^4} \ln a \int d^2 z \ln a_z 
\times \left\{ \ln^2 y_{++} - \ln^2 y_{+-} - 2(\ln(a a_z) + 2 \ln 2 + C)(\ln y_{++} - \ln y_{+-}) \right\}
= - \frac{2\pi \lambda^2}{(4\pi)^4 H^2} \ln a \int_{1}^{a} da_z \ln a_z \left( -2 - C + 2 \ln \left( a^{-1}_z - a^{-1} \right) \right) \left( a^{-1}_z - a^{-1} \right).
\]
The integral of $az$ gives the result for $I_2(\eta)$,

$$I_2(\eta) = -\frac{2\pi \lambda^2}{(4\pi)^4 H^2} \left[ -\frac{2}{3} \ln^4 a - C \ln^3 a - \left( \frac{\pi^2}{3} - C \right) \ln^2 a ight. \\
-2\text{Li}_2(a^{-1}) - \text{Li}_3(a^{-1}) + \frac{2 + C}{a} + \frac{\pi^2}{3} + 2\text{Li}_3(1) - 2 - C \right] , \quad (A14)$$

where $\text{Li}_n(x)$ denotes the polylogarithm function which decays for small $x$. In this case, the leading contribution to the EM tensor is given by

$$I_2(\eta) \sim \frac{\lambda^2}{12\pi (4\pi)^2 H^2} \ln^4 a . \quad (A15)$$

Then the total contribution is given by (A12) and (A15),

$$I_1 + I_2 \sim \frac{1}{8\pi} \frac{\lambda^2}{(4\pi)^2 H^2} \ln^4 a . \quad (A16)$$

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