Mathematical model of gas dynamics of a nozzle channel with a gas-dynamic method of thrust vector control using a porous insert

N A Brykov, Yu V Kaun and A A Yatsenko
Baltic State Technical University "VOENMEH" D.F. Ustinova, St. Petersburg, 190005, Russia

E-mail: brykovna@ya.ru

Abstract. The ability to change the magnitude and direction of the thrust vector is a fundamental parameter of the propulsion systems of aircraft. A wide range of methods for controlling these quantities has been developed, which are used depending on the design schemes. The article discusses the organization of the gas-dynamic method of thrust vector control, carried out using distributed gas injection through a porous insert.

1. Introduction
The operation of the propulsion system of the aircraft proceeds against the background of changing gas-dynamic parameters both in the combustion chamber and in the environment; therefore, a traditional nozzle with invariable geometric characteristics cannot be optimal over the entire range of flight times. After analyzing the operating mode of the nozzle, it can be seen that it operates either in the overexpansion mode (the degree of off-design $m = p_a/p_{atm} > 1$, where $p_a$ - pressure at the nozzle exit, $p_{atm}$ - ambient pressure), or in underexpanded mode ($p_{atm}m < 1$), which significantly reduces the thrust characteristics of the rocket engine. In this sense, an “ideal” nozzle should track changes in the flow parameters and be able to change the gas-dynamic flow parameters in order to maintain the design flow regime ($m = 1$), which is technically difficult to implement.

A number of researchers propose to use air injection through an adjustable annular slot into the supercritical region of the nozzle to reduce thrust losses during overexpansion flow [1-3]. In this case, external pressure acts through a narrow annular slot on the flow in the nozzle and leads to forced separation of the gas from the nozzle walls. Due to the separation of the flow and, in fact, the formation of a new nozzle, overexpansion of the flow does not occur and the nozzle operates practically in the design mode. In this case, the thrust value also increases by several percent [4]. However, this method is only a one-stage change in the degree of nozzle off-design; therefore, in [5], it was proposed to use several adjustable slots located along the supersonic part of the nozzle.

Technically more perfect is the use of distributed injection of external air into the nozzle flow, which is provided by a gas-permeable nozzle wall. In this case, the gas-permeable wall of the nozzle is possible with the inclusion of inserts made of porous materials. Distributed injection provides practical adiabatic compression of the flow without the formation of shock waves.
2. Mathematical model

Let us formulate a mathematical model of gas movement in the nozzle channel in the case of the presence in it of a mass-supply section made of a porous material.

Consider an arbitrary volume \( dV(t, x) \) (figure 1). Let's select an arbitrary area on the surfaced \( S \) of the volumed \( V \). Mass flows out through this platform at a speed \( \vec{v} \), while the platform itself, in the general case, can move with speed \( \vec{v}_s \). Let us write for the case of a fixed volume (\( \vec{v}_s = 0 \)) the laws of conservation of mass, momentum and energy:

\[
\begin{align*}
\frac{\partial}{\partial t} \iiint_V \rho \, dV &= - \iiint_S \rho \vec{n} \vec{v} \, dS; \\
\frac{\partial}{\partial t} \iiint_V \rho \vec{v} \, dV &= - \iiint_S [\rho \vec{v}^2 + p] \vec{n} \, dS; \\
\frac{\partial}{\partial t} \iiint_V \rho E \, dV &= - \iiint_S [\rho E + p] \vec{v} \vec{n} \, dS,
\end{align*}
\]  

(1)

Where \( \rho \) is the gas density, \( \vec{v} \) is the gas velocity, \( E \) is the total energy per unit mass.

Let us apply the above integral equations to a gas moving in a channel of variable cross-section in a one-dimensional formulation. Consider a portion of gas enclosed between two flow sections perpendicular to the axis and passing through points \( x \) and \( (x + dx) \) - figure 2.

The cross-sectional areas of the flow are respectively denoted: at a point \( x \) through \( S_1 \), and at a point \( x + dx \) through \( S_2 \). The surface that limits the volume under consideration will consist of lateral surfaces \( I_1 \) and \( I_2 \) having an area \( S_1 \) and \( S_2 \), as well as a lateral surface \( I_3 \). The normal component of the velocity to the section \( S_1 \) is equal to \( v_x \), and to the section \( S_2 \) is equal \( v_x (x + dx) \) due to the fact that the external normal to the volume under consideration to the section with the area \( S_2 \) is directed opposite to the direction of the axis \( x \). Due to the smallness of the value \( \Delta x \), we will consider the cosine of the angle between the normal to the lateral surface \( I_2 \) and the axis \( x \) to be small.

The change in mass in the volume \( dV \) occurs due to the inflow/outflow of mass through the surfaces \( I_1 \) and \( I_2 \), as well as, in the case of the presence of a porous insert on the side surface of the nozzle channel, due to the supply of gas through this insert with the surface \( I_3 \). Then the right-hand sides of the conservation laws can be written as:

\[
\iiint_S \rho \vec{n} \vec{v} \, dS = I_1^1 + I_2^1 + I_3^1,
\]  

(2)
where the superscript refers to the number of the equation of system (1), and the subscript refers to the number of the surface.

Determination of changes in mass, momentum and energy across surfaces $I_1$ and $I_2$ do not cause complexity and are described many times in the literature, for example, in [6]. Of interest is the description of flows through a porous structure, that is, a surface $I_3$, which can be written in the following form:

$$
\rho \vec{v} |_{I_3} = \rho_g u_g \frac{\rho^2 u_g^2}{\rho}, \quad \rho E + p |_{I_3} = \rho_g u_g c_v T_g,
$$

where $\rho_g$—is the density of the ejected gas, $u_g$—is the velocity of the ejected gas, $P_g$—is the perimeter of the lateral surface, $I_3, c_v$—is the heat capacity of the gas at a constant volume, $T_g$—is the temperature of the gas.

Ejector parameter spassed through the porous structure of the gas are unknown and requires the formulation of additional relations to determine them. According to Darcy's law the pressure radiant in a flat porous layer at low filtration rates is proportional to the liquid or gas flow rate:

$$
\nabla p = -\frac{\mu}{k} u_f,
$$

where $\mu$—is the dynamic viscosity, $k$—is the permeability coefficient of the porous medium, $u_f$—is the average filtration rate, is the ratio of the volumetric flow rate $Q$ to the area of the openings of the porous structure $F_n$

$$
u_f = \frac{Q}{F_n} = \frac{Q}{kF}.
$$

Physically, the pressure gradient during fluid flow through a porous structure is associated with the loss of momentum of the fluid flow due to viscous dynamic friction. With a strong unsteadiness of the flow, the change in momentum is associated with the acceleration of the flow, and in this case, violation of the upper limit of applicability of Darcy’s law is possible. There are several hypotheses for violation of the linear Darcy’s law at high speeds [7]: the appearance of inertial resistance; the occurrence of a turbulent flow regime; the formation of vortices causing additional vortex resistance, etc.

The critical filtration rate at which the upper limit of Darcy’s law is violated can be estimated by the parameter $Re_{kp} = \frac{u_f a^2 \rho}{\mu a}$, where $a$—is a parameter characterizing the cross-section of pore channels. There are various models for estimating the characteristic filtration rate and intervals of critical values $Re_{kp}$, for example, in [7] Pavlovsky $Re_{kp} = 7.5 \div 9$, and in Shchelkachev $1 \leq Re_{kp} \leq 12$. Such a spread in the ranges is due to the fact that $Re_{kp}$ the parameters, characterizing the geometric microstructure of the porous medium, which correspond to the real one only in the selected approximation sufficient for a given case.

If the upper limit of Darcy’s law is violated, the relationship between the filtration rate and pressure gradient is best described by Forchheimer’s two-term filtration law [8], which expresses a smooth transition from a linear filtration law to a nonlinear one:

$$
-\frac{dp}{dx} = \frac{\mu}{k} u_f + \frac{\nu \rho^2}{\sqrt{k}} u_f^2,
$$

(6)
The dimensionless parameter depending on the structure of the porous medium, $k$ – the coefficient of permeability of the porous material depends on the porosity $\varepsilon$ and the characteristic size of open pores $d_{eff}$, this dependence in general form can be represented as

$$k = f(\varepsilon, d_{eff}) = d_{eff}^2 \text{Sl},$$

(7)

where $\text{Sl}$- Slichter's number. Within the framework of the model of an ideal porous material, it is associated only with the geometric properties of the system of pore channels by the dependence

$$k = \frac{\varepsilon d_{eff}^2}{32},$$

(8)

According to the Kozeny-Karman $[10]$

$$k = \frac{\varepsilon^2}{K(1 - \varepsilon^2)F^2},$$

(9)

where $K$- form factor, $F$- specific pore surface area.

The first term in (2), linear in the filtration rate, dominates at low Reynolds numbers, calculated with respect to the particle size. The second term, quadratic in velocity, is at large Reynolds numbers.

Also, the non-stationary term becomes significant at very low porosity and low velocities, when the energy of intermolecular interaction in the flow turns out to be comparable to the kinetic energy of the flow particles (non-Newtonian liquids), with such filtration flows the lower limit of applicability of Darcy's law is violated.

Thus, the system of equations of one-dimensional gas dynamics of a nozzle channel with gas-permeable insert is written in the form:

$$\frac{\partial \rho S}{\partial t} + \frac{\partial (\rho \bar{v} S)}{\partial x} = \rho_g u_g P_g,$$

$$\frac{\partial \rho \bar{v} S}{\partial t} + \frac{\partial (\rho \bar{v}^2 + p) S}{\partial x} = P_g \rho_g u_g^2 + p \frac{dS}{dx},$$

$$\frac{\partial \rho E S}{\partial t} + \frac{\partial (\rho E + p) \bar{v} S}{\partial x} = \rho_g u_g P_g c_v T_g.$$  

(10)

The considered equations include the values averaged over the cross section; to take into account the uneven distribution of the velocity and density over the cross section, a coefficient $\beta$ is often introduced into (3) characterizing the corresponding correction: $\beta = 1$ in the case of an ideal fluid, with turbulent motion $\beta \approx 1.03 \div 1.1$ [6]. System (3) is closed by the ideal gas equation of state: $p = \rho E$.

To numerically solve the system of equations (3), one can use the Godunov difference scheme $[10]$. The boundary conditions for the lateral surface of the channel in the considered approximation are not explicitly specified, but to solve system (3), it is necessary to know the velocity of the ejected fuel $u_g$, which is determined through Darcy’s or Forchheimer’s law.

3. Conclusion

Thus, a mathematical model of one-dimensional gas dynamics for a nozzle channel was compiled taking into account a porous mass-supply insert.

Acknowledgments

The study was supported by a grant from the Russian Science Foundation No. 21-79-00100, https://rscf.ru/en/project/21-79-00100/.

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