Constraint on nuclear symmetry energy from nuclear charge dependent neutron skin thickness of nuclei

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Abstract

An alternative way to constrain the density dependence of the symmetry energy with the slope of the neutron skin thickness of nuclei which shows a linear relation to both the isospin asymmetry and the nuclear charge with a form of $Z^{2/3}$ is proposed. The linear dependence of the neutron skin thickness on the nuclear charge and isospin asymmetry is systematically studied with the data from antiprotonic atom measurement experimentally and the extended Thomas-Fermi approach incorporated the Skyrme energy density functional theoretically. An obviously linear relationship between the slope parameter $L$ of the symmetry energy and the slope of the neutron skin thickness on the isospin asymmetry can be found by adopting 66 Skyrme interactions in the calculation. Combining the available experimental data, the constraint of $16 \lesssim L \lesssim 66$ MeV on the slope parameter of the symmetry energy is obtained. The Skyrme interactions satisfying the constraint are selected.

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The nuclear symmetry energy $S(\rho)$ is the difference in energy per nucleon between pure neutron matter and symmetric nuclear matter, which is the key ingredient of the nuclear equation of state (EoS) for asymmetric nuclear matter. It governs the important properties of nuclei and neutron star. It also plays a significant role in the nuclear reaction dynamics and the stability of the phases within the neutron star and the interior cooling process within it. It is well known that the density dependence of $S(\rho)$ for cold nuclear matter predicted from different models is extremely variant. Acquiring more accurate knowledge of the density dependence of the symmetry energy has become one of the main goals in nuclear physics at present and in the near future and has stimulated many theoretical and experimental studies[1–5]. To characterize the density dependence of the symmetry energy, $S(\rho)$ is expanded near saturation density ($\rho_0$) as

$$S(\rho) = S(\rho_0) + \frac{L}{3}\left(\frac{\rho - \rho_0}{\rho_0}\right) + \frac{K_{\text{sym}}}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + ...,$$

with the slope parameter $L = 3\rho_0 \frac{dS(\rho)}{d\rho}|_{\rho_0}$ and the curvature parameter $K_{\text{sym}} = 9\rho_0^2 \frac{d^2S(\rho)}{d\rho^2}|_{\rho_0}$.

The calibration of neutron skin thickness of nuclei defined by $\Delta R_{np} = \langle r_n^2 \rangle^{\frac{1}{2}} - \langle r_p^2 \rangle^{\frac{1}{2}}$ has attracted a lot of attention in recent years because of the sensitivity of $\Delta R_{np}$ to the density dependence of the symmetry energy. The calculations in either non-relativistic or relativistic mean-field models show a well-defined linear correlation between the $\Delta R_{np}$ of heavy nuclei and the slope parameter $L$ of the symmetry energy at the saturation density [6–8]. Thus, $\Delta R_{np}$ of nuclei can be used as a powerful observable to constrain the density dependence of the symmetry energy at $\rho_0$ and lower densities. The difficulty in the calibration of neutron skin thickness of nuclei stems from the difficulty in the measurement of neutron distribution. The main methods to measure the neutron distribution or neutron skin thickness include hadron scattering[9–12], $\pi^-$ elastic scattering[13], antiprotonic atoms[14–17], excitation of giant dipole[18–20] and spin-dipole resonances[21, 22] on inelastic alpha scattering. Unfortunately, the obtained values of $\Delta R_{np}$ from different experimental method depend on the used analysis model and sometimes are not totally consistent with each other. It is also hard to judge the model dependence of the systematic error in different experimental method. The parity-violating electron scattering[23, 24] will be a hopeful option to measure the neutron distribution with unprecedented precise of 1% in a model independent way. However, it is still not available up to now. In this case, it will be difficult to accurately and consistently constrain the symmetry energy by directly using the data of neutron skin thickness. We
notice that the $\Delta R_{np}$ of 26 stable nuclei all over the periodic table (from $^{40}\text{Ca}$ to $^{238}\text{U}$) have been accumulated from antiprotonic atom measurement. In Ref.\cite{14}, the dependence of $\Delta R_{np}$ on the isospin asymmetry $\delta = (N - Z)/A$ for these 26 nuclei was extracted from experimental data of antiprotonic atom measurement, which reads

$$\Delta R_{np} = (-0.03 \pm 0.02) + (0.90 \pm 0.15)\delta.$$  \hfill (2)

Recently, Warda et al. represented this relationship in droplet model with surface width dependence\cite{25}. However it is already known that the antiprotons are only sensitive to the tail of the neutron distribution. An assumed shape for the neutron density is needed to extract the rms radius. Therefore the uncertainty in the value of $\Delta R_{np}$ is unavoidable in this approach.

In this letter, we suggest an alternative way to constrain the density dependence of the symmetry energy by the slope of a nuclear charge dependent linear relation of $\Delta R_{np}$ vs $\delta$ based on the 26 experimental data from the antiprotonic atoms obtained up to now. In this way, only the tendency of $\Delta R_{np}$ changing with the isospin asymmetry $\delta$ is required to get the information of the symmetry energy. The systematic uncertainty due to the experimental method itself is therefore expected to be largely reduced. The more data of $\Delta R_{np}$ for different nuclei with the same experimental method, the more accurate constraint on the density dependence of the symmetry energy can be obtained. One should take into account the nuclear charge dependent linear relation of $\Delta R_{np}$ vs $\delta$. Because the neutron skin thickness is the difference of the neutron and proton rms radii, it should depend not only on the symmetry energy but also on the Coulomb interaction which is far from clear.

Let us first study the systematic behavior of the nuclear charge dependence of $\Delta R_{np}$. The approach of semi-classical extended Thomas-Fermi approximation\cite{26, 27} up to the second order (ETF2) incorporated a potential energy density functional including the standard Skyrme energy density and Coulomb energy density with the Coulomb exchange term is applied in the calculations, in which the nuclear surface diffuseness is self-consistently taken into account. We calculate the proton and neutron density distributions of nuclei (See \cite{28} for details) with a spherical symmetric Fermi functions by means of restricted density variational method\cite{27, 29, 31}. With the density distributions determined in this way, the ground state properties such as the energy and the nuclear charge radii of series of nuclei have been calculated. The corresponding experimental data can be reasonably well reproduced\cite{32}. 

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FIG. 1: Linear correlation between the neutron skin thickness and the isospin asymmetry for Ca, Ni, Zr, Sn, Yb, Pb and U elements. Scatter symbols denote the calculation results. Solid lines denote the linear fitting results with a form $C_Z Z^{2/3} + C_\delta \delta$.

Based on the calculated rms radii of proton and neutron, we get the neutron skin thickness $\Delta R_{np}$. The effective Skyrme interaction SLy4 is adopted in this work since SLy4 is very successful in describing the bulk properties and surface properties of nuclei\cite{33}.

With this approach we calculate the $\Delta R_{np}$ for the isotopes with charge number in the range of $20 \leq Z \leq 92$, only the even-even nuclei are taken into account. Fig.1 shows the correlations between $\Delta R_{np}$ and $\delta$ for Ca, Ni, Zr, Sn, Yb, Pb and U isotopes. It is seen from the figure that the $\Delta R_{np}$ and the isospin asymmetry $\delta$ of nuclei are linearly correlated for Ca, Ni, Zr, Sn, Yb, Pb and U isotopes, respectively. The fitting lines are nearly parallel with each other and the slopes of $\Delta R_{np}$ as a function of $\delta$ for all selected elements is about 1.10 fm per unit $\delta$ within the range of $0 \leq \delta \leq 0.24$. The intercepts of the lines depends on the nuclear charge and roughly has a form of $\propto Z^{2/3}$. We have varied the form of the charge number dependence from $Z^{1/3}$ to $Z^{4/3}$ and find that only the form of $\propto Z^{2/3}$ can describe the charge dependence of $\Delta R_{np}$. To understand the $\propto Z^{2/3}$ dependence of $\Delta R_{np}$, we investigate the relation between the squared charge mean radii and the charge number $Z$ for the nuclei with same isospin asymmetry. We find that the squared charge mean radii
FIG. 2: Measured charge mean square radii $\langle r^2_{ch}\rangle$ as a function of $Z^{2/3}$ for selected nuclei with the same $\delta$.

for the nuclei with same isospin asymmetry relate to $Z^{2/3}$ linearly. Fig.2 shows a monotonic increasing tendency of the experimental data of $\langle r^2_{ch}\rangle$ with $Z^{2/3}$ for nuclei with $\delta = \frac{1}{9}$, $\frac{1}{6}$, and 0.2, respectively. Obviously, increasing the charge number for the nuclei with same isospin asymmetry enlarges the $\langle r^2_{ch}\rangle$ and thus reduce the neutron skin thickness.

According to the above investigation, we propose an empirical expression for the description of the neutron skin thickness of a nucleus as

$$\Delta R_{np} = C_z Z^{2/3} + C_\delta \delta,$$

(3)

We have performed a $\chi^2$ analysis in order to obtain the optimized $C_z$ and $C_\delta$ from the experimental data of $\Delta R_{np}$ from antiprotonic atom measurement. But the accumulated data of $\Delta R_{np}$ are not enough to get a precise constraint on the parameters. Thus we obtain the parameters $C_z$ and $C_\delta$ by fitting the expression (2) obtained from experimental data of antiprotonic atom measurement. Firstly we use the data $\Delta R_{np}$ for $^{40}$Ca to determine the range of $C_z = (-0.0041 \pm 0.0027)$fm because for $^{40}$Ca the $\delta$ is equal to zero. Then we adopt $C_z = -0.0041$ fm to fit $\Delta R_{np}$ for all 26 experimental data available up to now, in which the weight from error bar are taken into account to obtain the optimized $C_\delta$. The fitting
FIG. 3: (Color online) (a) Comparison of the results with \((-0.0041 \pm 0.0027)Z^{2/3} + (1.05 \pm 0.08)\delta\) to the experimental neutron skins from antiprotonic measurements\(^{14}\). Results of DM are also shown. The dashed gray lines denote the upper and lower limits of Eq.(2) and the solid red lines denote those of Eq.(4). (b) to (e) Comparison of the results with the fitting linear formula to the experimental data from antiprotonic measurements for Sn, Zr, Te and Cd isotope chains.

\(C_\delta\) of 1.05 fm is obtained. The range of \(C_\delta = (1.05 \pm 0.08)\) fm is determined through two limits in the expression (2) extracted from experimental data. We finally obtain the neutron skin thickness as a function of both the nuclear charge and the isospin asymmetry of nuclei, extracted from the available data of \(\Delta R_{np}\) given by antiprotonic atom measurement as

\[
\Delta R_{np} = (-0.0041 \pm 0.0027)Z^{2/3} + (1.05 \pm 0.08)\delta. \tag{4}
\]

The uncertainty of \(C_\delta\) determined in this approach is considerably reduced from 0.15 fm in Eq.(2) to 0.08 fm when the charge dependence of neutron skin thickness is taken into account. The comparison between the \(\Delta R_{np}\) obtained with this expression and experimental data is
FIG. 4: (Color online) The correlation between $L$ and the parameters $C_Z$ and $C_\delta$, respectively. The full circles are the calculated $C_Z$ and $C_\delta$ for each Skyrme interaction. The solid black vertical lines denote the extracted value of $C_Z$ and $C_\delta$ from experiment. The dashed black vertical lines denote the extracted upper and lower limits of them.

given in Fig.3(a). The results of droplet model (DM)\textsuperscript{[25]} are also shown in Fig.3(a) for comparison. One can see that Eq.(4) can reproduce most of the experimental data well and also is in good agreement with the DM calculations. In the sub-Figs (b),(c),(d),(e) of Fig.3, we pick out Sn, Zr, Te, Cd isotope chains from the available 26 experimental neutron skin thickness data and compare with the results of Eq.(4). One can see that the experimental data are reproduced reasonably well.

Now let us come to explore the density dependence of the nuclear symmetry energy from the neutron skin thickness of nuclei $\Delta R_{np}$ correlated with both charge and $\delta$ of nuclei. We perform a systematic calculations within the ETF2 approach by using 66 sets of Skyrme interactions like SLy series, SkT series, $v$ series, Skz0~4, MSk1~6, SkM, SkM*, SkM1, SkMP, SII~ SVII, SIII*, SGI, SGII, Zs, Es, Gs, Rs, FitB, BSk1, RATP, SKRA. The slope parameter $L$ of the bulk symmetry energy covers a widely range from nearly $-30$ MeV to 100 MeV for all the 66 Skyrme interactions. For each Skyrme interaction, we calculate the neutron skin thickness of the nuclei for which the neutron skin thickness data from antiprotonic atom measurement are available. Then we get the parameters $C_Z$ and $C_\delta$ in Eq.(3) for each Skyrme interaction. The relationships between the slope parameter $L$ and
$C_Z, C_\delta$ thus can be obtained. Fig.4 displays the correlations between the slope parameter $L$ of the symmetry energy and the parameters $C_Z$ (left panel) and $C_\delta$ (right panel). The full circles are the calculated $C_Z$ and $C_\delta$ for each Skyrme interaction. From the left panel we can see that the parameter $C_Z$ changes from $-0.006$ to $-0.003$ fm which is totally in the range of $[-0.0068, -0.0014]$ fm extracted from the experimental data, and shows an anti-correlation with $L$ of Skyrme interactions. In the right panel of Fig.4, a clearly linear increasing correlation between $L$ and $C_\delta$ can be observed. The slope parameters $L$ for different Skyrme interactions varies with $C_\delta$ in an area limited by two lines $L = (-117.93 \pm 2\sigma) + 151.21C_\delta$ ($\sigma = 6.54$) which are plotted in Fig.4(b) with dashed red lines. With the range of $C_\delta = (1.05 \pm 0.08)$fm extracted from the experimental data, the slope parameter $L$ is constrained in $16 \lesssim L \lesssim 66$ MeV. Within the 66 Skyrme interactions we adopted, the Skyrme interactions satisfying this relation are SLy series, SkM, SkM*, MSk1~2, Skz0~1, SkT3, SkT6~9, SII, SIII*, SGII, RATP. The corresponding symmetry energy coefficients at the saturation density $\rho_0$ are from 28 MeV to 32 MeV for these selected Skyrme interactions. The uncertainty of $L$ obtained from the extracted range of $C_\delta$ is much smaller than that from $C_Z$. It means that the correlation between $\Delta R_{np}$ and isospin asymmetry $\delta$ of nuclei is more sensitive to the density dependence of the symmetry energy than that between $\Delta R_{np}$ and nuclear charge. The obtained constraint on the slope parameter $L$ of the symmetry energy is in consistent with that obtained from the double neutron-proton ratios and isospin diffusion in heavy ion collisions[1].

In summary, we propose an alternative way for constraining the density dependence of the symmetry energy by means of the relation of the neutron skin thickness to the nuclear charge and isospin asymmetry of nuclei. We show that the neutron skin thickness depends on the isospin asymmetry and the charge of nuclei with the form of $Z^{2/3}$. By fitting the available data of neutron skin thickness for a series of nuclei obtained from antiprotonic atom measurement, the parameters for the correlation between $\Delta R_{np}$ and the nuclear charge ($C_z$) and isospin asymmetry ($C_\delta$) can be extracted. Within the framework of the semi-classical extended Thomas-Fermi approximation together with the Skyrme energy density functional and Coulomb energy density, we systematically calculate the neutron skin thickness with 66 Skyrme interactions for the 26 nuclei, for which the experimental data are available, and get the parameters $C_z$ and $C_\delta$ for each Skyrme interaction, respectively. Based on the clearly linear correlation between the slope parameter of the symmetry energy and the slope pa-
parameter $C_{\delta}$ of the neutron skin thickness, we obtained the constraint on the slope parameter of the symmetry energy, i.e. $16 \lesssim L \lesssim 66$ MeV, by using the extracted parameter $C_{\delta}$ from the experimental data. The Skyrme interactions satisfying this constraint are selected. It should be stressed that the spherical symmetry Fermi distribution for proton and neutron density in our calculation is in accordance with that in the analysis in the antiprotonic measurement. It is more suitable and consistent to extract the parameters $C_z$ and $C_{\delta}$ with the data from antiprotonic measurement. We suggest more neutron skin thickness for different element with the same isospin asymmetry to be measured experimentally, so as to extract a more accurate dependence of the neutron skin thickness on the nuclear charge and the isospin asymmetry. With the increasing of the number and the accuracy of data for neutron skin thickness of nuclei based on the antiprotonic atom measurement in future, we believe more accurate constraint on the density dependence of the symmetry energy can be obtained with our proposed method.

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