A Framework of Meta Functional Learning for Regularising Knowledge Transfer

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Abstract—Machine learning classifiers’ capability is largely dependent on the scale of available training data and limited by the model overfitting in data-scarce learning tasks. To address this problem, this work proposes a novel framework of Meta Functional Learning (MFL) by meta-learning a generalisable functional model from data-rich tasks whilst simultaneously regularising knowledge transfer to data-scarce tasks. The MFL computes meta-knowledge on functional regularisation generalisable to different learning tasks by which functional training on limited labelled data promotes more discriminative functions to be learned. Based on this framework, we formulate three variants of MFL: MFL with Prototypes (MFL-P) which learns a functional by auxiliary prototypes, Composite MFL (ComMFL) that transfers knowledge from both functional space and representational space, and MFL with Iterative Updates (MFL-IU) which improves knowledge transfer regularisation from MFL by progressively learning the functional regularisation in knowledge transfer. Moreover, we generalise these variants for knowledge transfer regularisation from binary classifiers to multi-class classifiers. Extensive experiments on two few-shot learning scenarios, Few-Shot Learning (FSL) and Cross-Domain Few-Shot Learning (CD-FSL), show that meta functional learning for knowledge transfer regularisation can improve FSL classifiers.

Index Terms—Knowledge Transfer, Functional Learning, Meta Learning, Regularisation.

1 INTRODUCTION

The success of current deep architectures benefits a great deal on representation learning, in the sense of learning “big models” of richer representations for many tasks. Recent developments on self-supervised learning, or models trained on very large-scale data [3], [9], seem to suggest that powerful and universal representations could be learned for all tasks in all domains.

Given a universal feature extractor, can a good classifier for a particular task be effectively learned from only a few labelled examples of that task? Having a good universal representation does not guarantee fitting generalisable hypotheses of different individual tasks from a few labelled samples. For a Few-Shot Learning (FSL) task, many researchers had devoted their efforts in addressing the severe overfitting problem resulting in inferior classification accuracy and generalisation on novel categories [39], [54]. Typical FSL settings [7], [10], [14] assume that given a large amount of labelled data on source/base tasks, and few labelled data on target/novel tasks, a FSL algorithm can learn good hypotheses on novel tasks. Moreover, one may further consider Cross-Domain Few-Shot Learning (CD-FSL) when the source and target tasks are from significantly different semantic domains [18], [48].

Given a learned representation from richly labelled data, we consider that the underlying data distribution should follow the continuity, cluster, and manifold assumptions, as in Semi-Supervised Learning (SSL) [6]. Figure 1 illustrates this phenomenon from both SSL and supervised learning. Hypotheses learned from larger amount of examples (richer) are favoured than those trained by fewer examples. Moreover, good hypotheses should prefer geometrically simpler decision-boundaries and encourage points in the same cluster to have the same label. This should be a general principle for task-agnostic patterns of a hypothesis.

In a hypothesis/functionial space, we aim to learn gradually task-agnostic patterns of change in fitting hypotheses to training data from few to many labelled examples. In particular, the latent knowledge of task-agnostic patterns of change in a hypothesis fitting process is to be learned as a functional, estimated from a family of richly labelled data on source tasks that simultaneously satisfies new hypotheses of the same/similar family of functional generalisable to learning new target tasks. To that end, we introduce a framework with meta-learning strategy to learn this functional, called...
Essentially, our MFL framework learns a functional regularisation on how to best fit new hypotheses on scarcely labelled novel tasks according to how to best fit hypotheses on richly labelled base tasks, thus imposing penalties (constraints) on excessive optimisation (overfit) in fitting the novel hypotheses. Particularly, given the task of learning a novel hypothesis from scarcely labelled data, our functional encourages a process of learning the hypothesis by approximating the learning process of richly labelled data, from which it favours to satisfy the underlying data distribution principles of continuity, cluster, and manifold. The functional from MFL captures model learning regularisation knowledge from source data and transfers it to guide the FSL of novel tasks. Our approach to knowledge transfer as learning regularisation (how to learn) differs fundamentally to other existing methods of knowledge transfer on what to learn, e.g. representations in FSL. Figure 1 illustrates our idea of MFL that learns a task-agnostic, transferable and generalisable functional, a function in the functional space, to remit the overfitting problem in hypothesis optimisation given scarcely labelled data.

We explore a meta-learning paradigm to learn a functional of meta-knowledge as the regularisation of learning process. Essentially, MFL first samples many functional episodes to craft a functional set of paired classifiers trained by the corresponding few and many labelled data, respectively. MFL is learned to predict the functional of many labelled data, given the input functional from few labelled data. It achieves the meta-knowledge learning/transfer through functional regularisation. This is the vanilla MFL proposed in our conference version [27].

Based on the understanding of vanilla MFL [27], we generalise it and improve the formulations by introducing new variants: 1) We explore the information from prototypes (MFL-P) as the classes’ examples can provide the relative positional relationship of classes to help the functional learning in functional space; 2) To learn more regularisation knowledge from multiple source information, we additionally introduce a functional in representational space and naturally combine it with the functional in functional space, formulating a Composite MFL (ComMFL); 3) As the functional learned from one block of MFL contains limited capacity for transferring the regularisation knowledge, we employ an iterative update strategy to connect a sequence of basic module blocks (MFL-IU) to progressively learn the generalisable regularisation knowledge.

Moreover, we consider a more challenging learning problem, that is, the functional learning for a multi-class classifier which has higher dimensions of parameters, larger functional space and extra inter-class relationships compared with that for a binary classifier. As a trail, we generalise our MFL methods to multi-class classifiers by introducing an outer loop for MFL to capture a functional from more episodes.

We summarise our contributions as follows.

- We formulate knowledge transfer in few-shot learning as a problem of transfer learning regularisation (how to learn) rather than knowledge transfer in representation (what to learn). This problem is solved by a meta functional learning (MFL) framework.
- We introduce three variants of MFL, i.e. a MFL-P that learning a functional with auxiliary information from prototypes, a ComMFL which learns a composite functional with wider regularisation knowledge from functional space and representational space, and a MFL-IU employing an iterative update strategy for MFL that aims to gradually improve the classifier’s learning ability through the transfer of functional regularisation.
- We generalise our MFL methods to the functional learning from binary classifiers to multi-class classifiers, and introduce a readily ensemble method to improve the robustness of classifiers.
- We apply the MFL to both the standard few-shot learning and the cross-domain few-shot learning problems. We provide comprehensive experiments on miniImageNet, CIFAR-FS, CUB, Cars and Places to validate the effectiveness of MFL and its variants in improving FSL by minimising model overfit.

2 RELATED WORK

2.1 Functional Optimisation

Functional optimisation can be regarded as learning to optimise the function. Many works put efforts on learning functional gradients to optimise neural networks by functional gradient optimisation [13], [20], [21], [45] or functional gradient boosting [22], [35], [36]. For example, [21] computes a guide function to optimise the gradient function, formulating a functional gradient optimisation method. Apart from optimisation for gradient, Garg et al. [15] presented a functional optimisation on representation and unifies several self-supervised approaches as a framework to impose a regularisation on the representation via a learnable function using unlabeled data. Rather than learning a functional to optimise the gradient or representation, we aim to meta-learn a functional to regularise the knowledge transfer for classifiers.

2.2 Regularisation

Regularisation is an important technique to improve the generalisation ability of machine learning models in both traditional classification methods [4], [5], [23], [34] and currently popular deep learning methods [12], [16], [24]. Lee et al. [25] and Andrew Y. Ng [34] have investigated the effects of $L_1$ and $L_2$ regularisation for improving the generalisation ability of Logistic Regression (LR) and Support Vector Machine (SVM). Moreover, they give some theoretical proof that regularisation can reduce the generalisation error bound of classifiers. For deep learning methods, some classical regularisation techniques have been widely used, such as weight decay [24] and dropout [12]. MetaReg [1] proposed to explicitly meta-learn a regularisation function for domain generalization. Related to these regularisation techniques, our work is more focused on improving the generalisation ability of traditional classifiers, e.g. LR and SVM, by exploring a learnable and implicit regularisation module equipped with deep learning method.
2.3 Meta Learning

Recently, the idea of meta-learning or learning to learn [44] has been exploited by the machine learning community, as it shows a promise to achieve close to human-level recognition generalisation potential in a controlled setting [29], [53], [55]. In [1], the authors used meta-learning to train a regularisation item for neural network optimisation across domains and demonstrated the benefits to addressing the domain generalization problem. More works [11], [29], [53] are related to few-shot learning. MAML [11] is one of the representatives dealing with few-shot learning task by learning to learn a generalisable initialisation parameters for networks. Different from MAML that only takes meta-learning an initialisation, Meta-SGD [29] presented a method with much higher capacity by additionally learning the meta-learner updating direction and learning rate for few-shot tasks. Rather than learning the optimisation process with meta learner, MeLA [53] is a simpler meta-learner to directly generate model parameters for few-shot tasks. In our work, we also use meta-learning to solve the learning problem with limited labels but aim to train a meta-learner for transferring the implicit regularisation knowledge for few-shot tasks.

2.4 Transfer Learning

Transfer learning aims to leverage the prior knowledge from source training data to address the target tasks where only limited labelled data are available [37], [46]. A typical method [8], [33] for transfer learning is fine-tuning a model pre-trained on a well-labelled base dataset with limited novel target data. Another approach tries to reduce the distance between the distributions of a source domain and the target domain so to better transfer the knowledge learned from the source domain [30] [26]. These transfer methods are widely used for domain adaptation, which assumes that the source and target domain share the same label space. In practice, most source and target domains do not share the same label space, giving rise to the learning problems of open set recognition [41] and few-shot learning [38]. Our work aims to solve some transfer learning problems with limited labelled data, e.g. few-shot learning, cross-domain few-shot learning.

2.5 Few-Shot Learning

Few-shot learning is a task requiring fast recognising novel classes with very limited corresponding labelled samples. Existing FSL methods can be broadly characterized as follows. 1) Metric-based methods learn a common feature space where categories can be distinguished with each other based on a distance metric, and then infer labels for query data with a nearest neighbor classifier [42] or a separate learnable similarity metric [45]. 2) Gradient-based methods design the meta-learner as an optimiser that is learned to update model parameters. These approaches aim to learn good initialised parameters for a network so that the classifiers for novel classes can be learned with several gradient update steps on few labelled examples [11], [28], [39]. 3) Weight generation methods learn to generate classification weights for novel classes. A typical generation method directly predicts the classification weights from the activation statistics of their categories [14], [38]. Besides, some work try to generate better classification weights with denoising auto-encoders for weights reconstruction [15] or looking into the mutual information between generated weights and support/query data [17]. Different from existing work to generate weights from the activations of a feature extractor, we aim to investigate the function learning update dynamics (a functional) which is not limited to backbone training strategies.

2.6 Model Transformation and Composition

Our investigation on knowledge transfer by functional regularisation is related to previous works on model transformation and composition, in particular, a model regression network with MLP architecture for learning a generic, category agnostic transformation from small-sample models to the underlying large-sample models [51]. Subsequently, a MetaModelNet [52] was proposed for transferring the model dynamic from head classes to tail classes in long-tail recognition problem. Functional gradient learning [20] was explored to learn the composition of functions and an incremental strategy was adopted for gradually learning a generator network. Our work is partly inspired by these works but we expand the existing works to a new method of meta functional learning to construct generalisable learning regularisation knowledge capable of guiding ‘infant’ functions to become ‘mature’ functions in a process of function update.

3 Meta Functional Regularisation

3.1 Problem Definition.

Throughout the paper, we use \( I \) to denote the image data, \( y \) represents the corresponding label, we learn a representation function \( \psi : I \rightarrow x \) and \( x \in \mathbb{R}^p \), and a classifier \( f : \psi(I) \rightarrow y \). And the corresponding representational space and functional space are represented as \( \mathcal{H}_\psi \) and \( \mathcal{H}_f \).

In the transfer learning scenario, we consider a large-scale labelled source/base image-label pair set \( D_{\text{src}} = \{(I_j,y_j)\}_{j=1}^M \) with a classifier \( \tilde{f} \) and a small labelled novel/target image set \( D_{\text{nov}} = \{(I_j,y_j)\}_{j=1}^N \) with a novel category \( C_{\text{nov}} \) respectively. On \( D_{\text{src}} \), we learn a representation function \( \psi \), and then we learn a classifier \( f_\phi \), where \( \phi \) is the parameter of \( f \). A common practice in deep learning is end-to-end optimising \( \psi \) and \( f \) by formulating a multi-class classification problem over \( D_{\text{src}} \) with a cross-entropy loss. We utilise this process here to compute a feature representation \( \psi \).

**Functional learning.** Our goal is to learn to fit a functional regularisation, \( T : f_\phi \rightarrow f_\phi \). Specially, the input \( f_\phi(\psi(I)) \) is a classifier fitted by few labelled samples, and \( T \) (\( f_\phi \)) aims at approximating the corresponding function \( f_\phi \) with regularisation knowledge learned from many labelled examples. We use \( \phi \) and \( \tilde{\phi} \) to denote the parameters learned by few and many labelled examples.

3.2 Insights of Functional Regularisation

**Model Dynamics, and Functional Regularisation.** From the learning principles of risk minimization [49] and given a binary classification task with dataset \( D_S = \{x_i, y_i\} \), \( i = 1, \ldots, M \)
According to the number \( k \) algorithm to train a set of classification models/functions set, we adopt a meta-learning strategy here. In principle, \( T \) rather than directly regressing the black lines are the trained classifier boundaries with the training samples from two classes, and the green points present the training samples from two classes, and the black lines are the trained classifier boundaries with the training instances.

For every subset \( S_k \), we can use the same classification algorithm to train a set of classification models/functions \( \{f_1^k, f_2^k, \ldots, f_n^k\} \) by Empirical Risk Minimization (ERM) with Eq. (1).

\[
R_{erm}(f) = \min_{f \in \mathcal{H}_f} \frac{1}{n} \sum_{i=1}^{n} L_c(y_i, f(x_i)),
\]

where \( L_c \) is a loss function to compute the errors. This model can also be optimised by Structural Risk Minimization (SRM) in Eq. (2) which uses a regularisation term \( J(f) \) to increase the model’s generalisation ability and a coefficient \( \lambda \) to balance the learning of \( R_{erm}(f) \) and \( J(f) \).

\[
R_{erm}(f) = \min_{f \in \mathcal{H}_f} \frac{1}{N} \sum_{i=1}^{N} L_c(y_i, f(x_i)) + \lambda J(f). \tag{2}
\]

Suppose that \( m \) is large enough, we can yield an infinite set of optimal functions, which can form a functional space \( \mathcal{H}_f \). In this functional space, vector \( f_0^m \) represents the function without any data training. Thus it can be viewed as a randomly initialised vector in the functional space. With the increase of \( k \), function \( f_k^m \) can be viewed as the model dynamics in the functional space towards the optimal function \( f_m^\ast \). To this end, we simply formulate the model convergence of model dynamics in the functional space.

So what is the ‘implicit’ knowledge of learned by model dynamics in the functional space? Here, we try to intuitively explain it from the perspective of knowledge regularisation. Suppose that we just have two data points in a representation space, \( \{x_1, x_2\} \) and \( \{y_1, y_2\} \). To this end, we simply formulate the model convergence of model dynamics in the functional space.

\[
\lambda = \frac{1}{N} \sum_{i=1}^{N} L_c(y_i, f(x_i)) + \lambda J(f).
\]

1, ..., \( n \), we can obtain \( m \) different subsets \( \{S_1, \ldots, S_k, \ldots, S_m\} \) according to the number \( k \) of training instances. For every subset \( S_k \), we can use the same classification algorithm to train a set of classification models/functions \( \{f_1^k, f_2^k, \ldots, f_n^k\} \) by Empirical Risk Minimization (ERM) with Eq. (1).

4 Meta Functional Learning

In this section, we first introduce a general framework, then develop and analyse the corresponding algorithms.

4.1 Methodology in a Nutshell

As our approach to MFL focuses on learning the functional \( T \) for classifiers on a fixed representational space, we first train a represenator to extract the representations from images. Specifically, we follow the traditional mini-batch training strategy in [50] and use the cross-entropy loss to pre-train a represenator on source dataset \( D_{src} \). After training a represenator, the functional \( T \) can be learned with the following two steps:

**Step 1: Functional Initialization (Sec. 4.1.1).** For a classifier \( f_0 \) trained on task \( T \) containing limited data, the function is not the ideal one due to the over-fitting problem. While the ideal function is hard to compute since the true distribution of task \( T \) is not available. Here we approximately compute the ideal function \( f_0^\ast \) by training classifier with more available data of the classes in task \( T \).

**Step 2: Functional Learning (Sec. 4.1.2).** The functional \( T \) helps the function \( f_0^\ast \) computed with limited label to approach the ideal function. Therefore, one intuitive way is to learn a \( T \) to capture this knowledge guiding the function \( f_0 \) to the approximate ideal function \( f_0^\ast \).

To learn a task-agnostic and generalisable functional \( T \) by using a meta-learning paradigm, a common practice is to optimise the functional by iteratively computing step 1 and 2 process. Unfortunately, such an exhaustive and iterative updating process demands frequently initialising functionals, and thus is difficult for parallel-computing in batches. Therefore, we design a simpler Meta Functional Learning (MFL) framework to train the functional. This alternative pipeline of MFL can be illustrated as Fig. 3: (1) We sample the functional episodes to train \( T \) (Sec. 4.1.1); and (2) we learn \( T \) by different strategies (Sec. 4.1.2).
4.1 Sampling Functional Episodes

Given the trained representator $\psi$, the goal of this step is to craft the paired functional set $F_T = \{F_T^{(b)}\}$ on $D_{src}$ and the class $b \in C_{base}$; and we denote $F_T^{(b)} = \{(f_b^{(b)}, f_b^{(b)}, f_p^{(b)})\}$, where $f_b^{(b)}$ and $f_b^{(b)}$ are the classifiers of class $b$, trained by few and many examples, respectively; and $f_p^{(b)}$ represent the prototypes of the positive class $b$ other negative classes, computed by few labelled examples which are used for training $f_b^{(b)}$.

The sampled functional episodes include different classes in $C_{base}$. This will help our meta functional learning algorithm to learn task-agnostic functional $T$. Specifically, for class $b \in C_{base}$, we compute functional tuple set $F_T^{(b)} = \{(f_b^{(b)}, f_b^{(b)}, f_p^{(b)})\}$. For each tuple, $f_b^{(b)}$ is trained by the set of positive examples $\{(\psi(I_j), y_j = b)\}_i$ i.e. all images in class $b$, and negative examples $\{(\psi(I_j), y_j \neq b)\}$ by randomly sampling from other classes. To obtain the set of tuples, this process is randomly repeated for $M_t$ times.

To compute $f_b^{(b)}$, we take $s$ samples and $k \times s$ samples from class $b$ and other classes. For each $f_b^{(b)}$, we randomly sample samples $M_t$ times and use different hyper-parameters to train the classifiers $f_b^{(b)}$ for increasing their diversity.

4.1.2 Learning from the Scratch

As a vanilla instantiation of our MFL framework, we adopt the $f_b^{(b)}$ by a vanilla binary classifier for class $b$, and the generalised multi-class scenario (one vs. all setting). We utilise the Logistic Regression (LR) classifiers here, and $f_b^{(b)}$ and $f_b^{(b)}$ are the corresponding vectors of LR parameters. For the vanilla MFL, we directly learn $T: f_b \rightarrow f_b$ in functional space.

Given the functional sets $F_T$, we design a meta func-
tional learning mechanism to learn the functional regulari-
sation $T$. For any given class $b$, the objective of our MFL is to approximate the ground-truth output $f_b^{(b)} = T(f_b^{(b)})$.

We introduce Mean Square Error (MSE) to measure the difference of parameter vectors $(f_b^{(b)}, f_b^{(b)})$ as,

$$l_p = E(f_b, f_b) \rightarrow_T \|f_b^{(b)} - T(f_b^{(b)})\|^2 \tag{3}$$

Model implementation. The functional $T$ is implemented as a deep network, with the model architecture in Fig. 3 (2).

Algorithm 1 Meta Functional Learning (MFL).

Require: Embeddings $\Psi_{src} = \{\psi(I_j), y_j \in C_{base}\}$ of $D_{src}$; Classifier $f_b$; Sampling time $M_t$, $M_f$; Hyper-parameter set $H$; Shot number $s$, $s \times k$; Train epochs $T$;

Ensure: Functional set $F_T$; Functional regularisation $T$;

1: // Sampling Functional Episodes
2: $F_T = \emptyset$; $F_T^{(b)} = \emptyset$, $b \in C_{base}$
3: for all $b \in C_{base}$ do
4: Sample episode $E_t = \{(\psi(I_j), y_j = b)_{i=1}^{N_t} \cup \{\psi(I_j), y_j \neq b\}_{i=1}^{2\times N_t}\}$ from $\Psi_{src}$ and train $f_p^{(b)}$ on $E_t$;
5: Randomly sample sub-episode $E_s$ including $s(s \times k)$ $\psi(I_j)$ with $y_j = (\neq b)$ from $E_t$ and train $f_b^{(b)}$ on $E_s$;
6: Compute $f_b^{(b)}$ including the prototypes of $\psi(I_j)$ with $y_j = b$ and $y_j \neq b$ in $E_s$;
7: $F_T^{(b)} = F_T^{(b)} \cup (f_b^{(b)}, f_b^{(b)}, f_p^{(b)})$;
8: Repeat line 5-7 using $f_c$ with $h$ in $H$;
9: Repeat line 5-8 for $M_f$ times;
10: Repeat line 5-9 for $M_t$ times;
11: $F_T = F_T \cup F_T^{(b)}$;
12: end for
13: // Learning from the Scratch
14: while $t < T$ do
15: Randomly split mini-batches with size $n$ from $F_T$;
16: for each mini-batch do
17: Predict functions $T(f_b, f_p)$ with $T$;
18: Compute the loss in Eq. (3);
19: Update the parameters of $T$;
20: end for
21: end while

It consists of a residual block, where the LeakyReLU activation function is used to learn the nonlinear mapping from fully connection layers. We employ BatchNorm and dropout to improve the generalisation of $T$. The skip connection is used to keep the scale of classifiers’ parameters and avoid the degradation of learning. The pseudo-codes of sampling functional episodes and MFL are shown in Alg. 1.

4.2 Generalised Forms of MFL

We present a vanilla MFL method [27] by learning regularisation knowledge with the input of classifier’s parameters for a binary classifier in Sec. 4.1.1 and Sec. 4.1.2. To further
exploit the potentialities of our MFL framework, we further propose several generalised forms of MFL. Particularly, the vanilla MFL is extended to learning from examples (Sec. 4.2.1), multiple information source (Sec. 4.2.2) and with iterative updates (Sec. 4.2.3). And we further consider MFL in the wider applications: 1) learning functional for multi-class classifiers (Sec. 4.2.4); 2) ensemble classifiers during inference phase (Sec. 4.2.5).

4.2.1 Learning from the Examples

In vanilla MFL, we learn functional $\mathcal{T}$ only using the classifier’s parameter $f_\phi$. The prototype $f_p$ from the representational space is ignored whilst it can provide important category-related information, to help $\mathcal{T}$ better learn the category agnostic knowledge in the meta training episodes. Therefore, we improve the vanilla MFL by learning an extended form with prototypes (MFL-P), i.e. $\mathcal{T} : (f_\phi, f_p) \rightarrow f_\phi$, where $f_p$ is a vector by concatenating the positive and negative prototypes, which are computed by averaging the embeddings of samples from corresponding classes. In MFL-P, the objective is to approximate the ground-truth output $f_\phi^{(b)} = \mathcal{T}(f_\phi^{(b)}, f_p)$. We still use Mean Square Error (MSE) to measure the difference of parameter vectors $(f_\phi, f_p)$ as,

$$l_\tau = E_{(f_\phi, f_p) \sim \mathcal{T}_\tau} \| f_\phi - \mathcal{T}(f_\phi, f_p) \|^2$$

4.2.2 Learning from Multiple Information Source

A vanilla MFL learns regularisation knowledge in the functional space and MFL-P further uses the prototypes as auxiliary knowledge for functional learning. However, these two types of MFL both focus on learning in the functional space whilst the classifier’s function can also be learned from the representational space, that is, the prototypes. To learn better functions with comprehensive knowledge from both representational space and functional space, we propose a Composite MFL (ComMFL) by modifying the model of MFL-P. As in Fig. 5, we use vanilla MFL to obtain a classifier’s function by learning functional knowledge in functional space and additional a model to learn function in representational space. The objective function of ComMFL is formulated as:

$$l_\tau = E_{(f_\phi, f_p, f_\phi) \sim \mathcal{T}_\tau} \| f_\phi - (\mathcal{T}(f_\phi) + \mathcal{T}(f_p)) \|^2$$

4.2.3 MFL with Iterative Updates

The functionals in vanilla MFL, MFL-P and ComMFL are all optimised by a residual-based block with a MSE loss. This one-step process with one module block may limit the capacity of functional to regularise FSL models, especially those trained with extremely-scarce labelled data. Therefore, we employ an iterative update strategy on MFL (MFL-IU) to progressively learn the functional by a sequence of residual-based blocks. Specifically, as illustrated in Fig. 6, MFL-IU$x$ has $x$ residual-based blocks and each block is optimised with a MSE loss. MFL-IU$x$ represents the output of $x$th basic block, i.e. $\mathcal{T}_x (\mathcal{T}_{x-1} \cdots (\mathcal{T}_1 (f_\phi, f_p)))$. A simple version is MFL-IU1 by only using one block for vanilla MFL. The training process of MFL-IU$x$ is illustrated in Alg. 2. Besides, we can employ this iterative update strategy on MFL-P and ComMFL, obtaining MFL-P-IU and ComMFL-IU.

4.2.4 MFL on Multi-class Classifiers

For a binary classifier, the task-agnostic knowledge learned by functional is simplified as a learning problem at a category-level. That is, assuming that the learned regularisation knowledge is category-agnostic and can be transferred across classes. In this way, the functional set are sampled according to different positive classes, and its scale linearly increases with the number of classes in source dataset. However, for multi-class classifiers, the functional should be capable of capturing the regularisation knowledge of $N$ different classes, as well as their relationships. Obviously, learning MFL for a multi-class classifier is a more challenging task than that for a binary classifier.

Algorithm 2 MFL with Iterative Updates.

Require: Functional set $\mathcal{F}_\tau$; Iterations $X$; Train epochs $T$;
Ensure: Functional regularisation $\mathcal{T} = \{\mathcal{T}_1, ..., \mathcal{T}_X\}$
1: \textbf{while} $t < T$ \textbf{do}
2: \hspace{1em} Randomly split mini-batches with size $n$ from $\mathcal{F}_\tau$;
3: \hspace{1em} \textbf{for} each mini-batch \textbf{do}
4: \hspace{2em} for $x < X$ \textbf{do}
5: \hspace{3em} Predict functions $\mathcal{T}_x(f_\phi, f_p)$ with $\mathcal{T}_x$;
6: \hspace{3em} Compute the loss in Eq. (3);
7: \hspace{3em} Update the parameters of $\mathcal{T}_x$;
8: \hspace{2em} \textbf{end for}
9: \hspace{1em} \textbf{end for}
10: \textbf{end while}
To solve the increased complexity in the functional learning for a multi-class classifier, we extend our MFLs to this scenario by sampling more tuples in a functional set. Specifically, we adopt an outer loop strategy on the MFLs for a binary classifier, and the inner loop is a complete training of MFLs. We train inner loop \( I_{out} \) times and the algorithm is detailed in Alg. 3. By this way, we learn the functional capturing more tasks while avoiding the excessive increase of the storage cost for a functional set.

4.2.5 MFL as an Ensemble of Classifiers

Ensemble method is a machine learning technique that combines several base models in order to produce one optimal predictive model. Generally, the simple ensemble methods, e.g. average the weights or prediction scores of every base-classifier, prefer to yield a moderate prediction results compared to the base-classifiers. However, inferior base-classifier might introduce noisy predictions, resulting negative affect on the ensemble model. We introduce a MFL method that can improve both the base-classifiers and further benefit the ensemble results. Specifically, during the training phase, the MFL gradually captures the converges behaviour of classifiers trained with different hyper-parameters since we sample them into functional set. Thus, this MFL can be used as an ensemble of classifiers with different hyper-parameters; especially for the hyper-parameter-sensitive classifier trained with limited data. So the integrated classifier by MFL can be formulated as

\[
f_{T}^{\text{ens}} = \frac{1}{C} \sum_{c=1}^{C} \lambda_{c} \mathcal{T}(f_{c}),
\]

where \( \mathcal{T} \) is the MFL module, \( f_{c} \) is the classifier trained on limited data with hyper-parameters \( c \) and \( \lambda_{c} \) is the corresponding weight for \( \mathcal{T}(f_{c}) \). In this work, we simply use \( \lambda_{c} \) as 1 for every classifier.

5 Experiments

To evaluate the effectiveness of MFL, we tested MFL on two data-scarce learning problems: \( N \)-way \( K \)-shot classification, i.e. a task aiming to discriminate between \( N \) classes with \( K \) labelled samples of each class, by (1) standard FSL and (2) Cross-Domain FSL (CD-FSL). In particular, we adopted a binary classifier as a vanilla classifier and generalised it to multi-way classification scenario with one vs. all manner. We first evaluated MFL on basic 2-way FSL tasks and then investigated whether the learning pattern of MFL can be generalised to multi-way FSL tasks. We also evaluate our MFL on multi-class classifiers for corresponding multi-way FSL tasks. Furthermore, the experiments on CD-FSL were carried out for learning tasks with different shot numbers to investigate the model generalisation capacity to multi-shot FSL tasks.

Datasets. We employed three FSL datasets: 1) MiniImageNet is a subset of the ILSVRC-12 [40] dataset and contains 100 classes with 600 images per class. We followed the split 1 in [39] and used 64, 16 and 20 classes as base, validation and novel sets, respectively. 2) CIFAR-FS is a dataset with lower-resolution images, and it contains 100 classes with 600 instances in each class. Following the split in [2], we used 64 classes to construct the base set, 16 and 20 for validation and novel set. 3) CUB is a fine-grained dataset which consists of 200 bird categories with 11788 images in total. We used 100, 50 and 50 classes for base, validation and novel sets with the previous setting in [19], and we conducted all experiments with the cropped images provided in [47]. 4) Cars [23] contains 16,185 images of 196 classes of cars. We follow the split in [48] and used 98, 49 and 49 classes as base, validation and novel sets. 5) Places [56] is a dataset for scene recognition with 365 categories and 8 millions of images. We used 183, 91 and 91 classes as base, validation and novel sets following the split in [48].

Implementation. We used Conv4 as the backbone for learning a feature representation. The architecture of this Conv4 network is provided by [42] and it contains four convolutional blocks. Each block comprises a 64-filter \( 3 \times 3 \) convolution, batch normalization layer, a ReLU nonlinearity and a \( 2 \times 2 \) max-pooling layer. For training the representator, we randomly split the images from base classes into (90\%, 10\%) partition as (train, validation) sets. We trained the backbone over 120 epochs. We use SGD optimizer with a momentum of 0.9 and a weight decay of 1e-4. We set batch size as 64 and the learning rate is initialized as 0.01 and decayed with a factor of 0.1 by three times. For training MFL and its variants, we employed BatchNorm (0.1), dropout (0.9) and LeakyReLU (0.01), and the parameters for the first and second fully connected layers are 600 and 1601 respectively. Moreover, we trained MFL and its variants over 30 epochs with batch size (256), and the learning rate is initialised as 0.01 and decay to 1e-3 after 20 epochs. We adopted the Logistic Regression (LR) function as a base binary classifier or multi-class classifier. The hyper-parameters for sampling functional episodes for binary classifiers and multi-class classifiers are shown in Tab. 1. Specifically, we set \( s = \{1, 2, 3, 4, 5\} \) to construct functional tuple sets for \( s \)-shot learning scenarios in FSL. In all experiments, we selected the best model by evaluating them on a validation set and

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
#Outer loop & 1 & 5 \\
#Many-shot model \( M_{1} \) & 5*64 & 500 \\
#Few-shot model \( M_{s} \) & 100 & 200 \\
Negative samples \( k \) & \{1, 2, 3, 4\} & \{way-1\} \\
Hyper-parameter set \( H \) & 1e\{-2, −1, 0, 1, 2\} & 1e\{-2, −1, 0, 1, 2\} \\
#Functional episodes & 5*64*100*5*5 & 5*500*200*1*5 \\
\hline
\end{tabular}
\caption{Hyper-parameters for sampling functional episodes. #way represents the number of classes in multi-class classifier.}
\end{table}
evaluated all methods with 600 episodes randomly selected from the novel classes in the corresponding dataset.

5.1 Meta Functional Learning

5.1.1 MFL for Binary Classifier

Competitors. We compared our methods against existing models for N-way 1-shot FSL tasks from three perspectives: 1) Comparison with the base classifier: We used Logistic Regression (LR) as a typical classifier. As in Tab. 2, the Vanilla LR represents a naive LR classifier trained on labelled data, while Vinilla MFL, Vinilla MFL-IU3, MFL-P-IU3 and ComMFL-IU3 are the predicted functions with corresponding models. 2) Comparison with typical FSL methods: Baseline [7] ProtoNet [42], and MAML [11]; 3) Comparison with a model transformation method: MetaModelNet [52]. Since no official results are provided on these comparison methods in N-way classification FSL, we re-ran the released code in [7] for evaluating existing FSL methods and evaluated MetaModelNet with our re-implemented model following [52].

Results and analysis. Table 2 shows the comparative results on miniImageNet and CIFAR-FS. We can see that: (1) Our methods can effectively transfer the regularisation knowledge to benefit the naive functions, i.e. Vanilla LR, yielding more robust and accurate functions with significant performance improvement on 2/3/4/5/10/20-way 1-shot FSL; (2) Our methods significantly outperform three typical FSL methods, achieving the potentially smooth and discriminative hypotheses on a fixed embedding space; (3) MetaModelNet can improve the performance of the Vanilla LR in low-way (1-5 way) FSL tasks, while the improvement in higher way (10/20 way) FSL tasks is limited. In contrast, our methods performed well in all N-way 1-shot FSL tasks. This verifies that our methods on binary classifiers are more robust and generalisable to multi-way FSL tasks.

Effects of generalised forms of MFL. In Tab. 2, we observe that all forms of MFL, i.e. vanilla MFL, vanilla MFL-IU3, MFL-P-IU3 and ComMFL-IU3, are effective in improving the performance of the Vanilla LR. In particular, vanilla MFL-IU3 performed better than vanilla MFL due to the benefit from the progressively increasing functional regularisation knowledge provided by the iterative update strategy. Essentially, involving the information of examples can benefit the functional learning of regularisation knowledge. As expected, the results of MFL-P-IU3 and ComMFL-IU3 show better performance on N-way 1-shot FSL tasks compared with vanilla MFL-IU3. We note that the two ways to explore the information from examples perform slightly differently. That is, ComMFL-IU3 obtains slightly better performance than MFL-P-IU3, suggesting that the composition of different functionals, i.e. a functional from examples in the representational space and a functional from functions in the functional space, is a better choice to improve the learning of generalisable regularisation knowledge.

5.1.2 MFL for Multi-class Classifiers

In 5.1.1, we present extensive experimental results to verify the effectiveness of the various forms of MFL for improving a binary classifier with few labelled data. To further evaluate the generalisation ability of our methods on a multi-class classifier, we conducted experiments on 3/4/5-way 1-shot FSL tasks by learning functional regularisation on corresponding 3/4/5-class classifiers. In particular, the hyper-parameters for sampling functional episodes are in Tab. 1. Note that the number of functional episodes for a multi-class classifier is larger than that for a binary classifier to satisfy the requirements of larger functional space.

Binary classifier vs. Multi-class classifier. As we illustrated in Sec. 4.2.4, MFL for multi-class classifiers is more challenging due to the functional space for multi-class classifiers is larger and hard to capture. The results in Tab. 2 and Tab. 3 valid this assumption and we observe that the functional learning on a binary classifier is more effective than that on a multi-class classifier. In particular, for the 1-shot 5-way FSL tasks, the ComMFL-IU3 on a binary classifier obtains 52.03% whilst that on a multi-class classifier get an inferior result 48.88%, and this observation is similar in the 3/4-way 1-shot FSL tasks. Interestingly, with an auxiliary information from examples, MFL-P-IU3 performs inferior to vanilla MFL-IU3. This observation is reverse to the results on binary classifiers, which is counterintuitive and indicates that the samples might guide a biased learning for the regularisation functional on the functional space for a multi-class classifier. Additionally, ComMFL-IU3 and vanilla MFL-IU3 obtain competitive results on a multi-class classifier. This benefits from the individually networks to learn the functional on the functional space and the representational space, such the examples would not directly affect the functional learning on the functional space.

5.1.3 MFL as an Ensemble of Classifiers

We conducted a simply average strategy on the predicted functions by the MFL-regularised classifiers using different hyper-parameters, i.e. $C = 0.1, 1, 10$, and we compute a more accurate functions compared with each classifier with MFL. The favour of ensemble method is preferring to yields a moderate results compared to best base-classifier, and this also occurs in the few-shot learning tasks shown in the Tab. 4 for the ensemble of Vanilla LR. With a weight averaging strategy, the ensemble method performs competitively well compared with the best base-classifier, achieving the same recognition result (46.18%). We also use this weight averaging strategy to integrate the weights of functions predicted by our MFLs. Table 4 shows that the averaged results on the functions predicted by MFL perform better than those of each base-classifiers. This suggests that it is a good choice of using ensemble methods after our MFLs. We conjecture that our MFLs can transform the inferior classifiers trained with limited labels to more accurate ones, so that the ensemble method on the transformed classifiers can compute a more robust classifier and remitting the negative effects from the inferior classifiers without MFLs.

5.2 Learning to Cross Domain

We employed our MFL methods on a more challenging task, CD-FSL. We followed the miniImageNet $\rightarrow$ CUB setting in [7], where $D_{src}$ and $D_{nov}$ are the images from the base classes of miniImageNet and the novel classes of CUB, respectively. Moreover, we generalise this setting to another two datasets, i.e. Cars and Places. For comparison, we
Few-Shot Learning Evaluation: Comparison to Vanilla LR and prior work on minilimageNet and CIFAR-FS with Conv4 backbone. Mean accuracies (%) with 95% confidence intervals results are reported on N-way 1-shot FSL. (·) represent the experimental results with the released codes and (†) are our re-implemented results with the corresponding paper. **Bold**: the best scores.

| Dataset       | Methods                  | 2-way         | 3-way         | 4-way         | 5-way         | 10-way        | 20-way        |
|---------------|--------------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| miniImageNet  | Baseline† [7]            | 70.09±1.13    | 55.74±0.99    | 46.33±0.79    | 40.41±0.68    | 26.50±0.38    | 16.09±0.21    |
|               | ProtoNet† [42]           | 73.76±1.34    | 59.34±1.14    | 51.24±0.95    | 45.22±0.81    | 29.04±0.44    | 18.09±0.23    |
|               | MAML† [11]               | 73.56±1.38    | 62.21±1.16    | 52.44±0.94    | 48.29±0.83    | 31.41±0.47    | -             |
|               | Vanilla LR               | 72.86±1.13    | 59.51±0.93    | 51.05±0.83    | 46.18±0.77    | 31.04±0.44    | 21.09±0.24    |
|               | MetaModelNet† [52]       | 76.34±1.36    | 62.54±1.14    | 53.91±0.97    | 47.99±0.85    | 31.02±0.46    | 19.23±0.24    |
|               | vanilla MFL (Ours)       | 76.09±1.15    | 64.27±1.10    | 54.37±0.86    | 48.88±0.80    | 33.15±0.46    | 22.42±0.25    |
|               | vanilla MFL-IU3 (Ours)   | 77.60±1.23    | 65.62±1.03    | 56.40±0.88    | 50.87±0.82    | 34.43±0.45    | 23.22±0.27    |
|               | MFL-P-IU3 (Ours)         | 78.41±1.21    | 65.47±1.02    | 56.77±0.90    | 51.46±0.83    | 34.58±0.46    | 23.64±0.26    |
|               | ComMFL-IU3 (Ours)        | 78.83±1.23    | 65.90±1.03    | 57.56±0.92    | 52.03±0.83    | 35.27±0.46    | 23.72±0.23    |
| CIFAR-FS      | Baseline† [7]            | 72.66±1.14    | 59.44±1.06    | 50.77±0.85    | 46.16±0.77    | 32.46±0.46    | 22.04±0.26    |
|               | ProtoNet† [42]           | 73.36±1.13    | 60.45±1.20    | 51.87±1.01    | 47.04±0.91    | 31.41±0.51    | 20.48±0.25    |
|               | MAML† [11]               | 75.82±1.35    | 63.06±1.23    | 56.82±1.03    | 50.15±0.94    | 39.52±0.60    | -             |
|               | vanilla MFL (Ours)       | 80.11±1.14    | 68.99±1.03    | 61.10±0.96    | 55.90±0.88    | 42.35±0.53    | 30.62±0.28    |
|               | vanilla MFL-IU3 (Ours)   | 81.39±1.17    | 71.60±1.09    | 63.88±0.99    | 59.38±0.93    | 45.25±0.58    | 32.78±0.29    |
|               | MFL-P-IU3 (Ours)         | 82.68±1.12    | 72.37±1.08    | 64.71±0.99    | 59.88±0.93    | 45.25±0.58    | 32.78±0.29    |
|               | ComMFL-IU3 (Ours)        | 82.64±1.18    | 72.83±1.10    | 65.15±1.02    | 60.33±0.94    | 45.67±0.59    | 33.31±0.29    |

**Table 2**

Multi-class classifier evaluation: Mean accuracies (%) of Vanilla LR and LR with MFL and MFL-IU3 on 3/4/5-way 1-shot tasks from minilimageNet. **Bold**: the best scores.

| #shot | 3-way         | 4-way         | 5-way         |
|-------|---------------|---------------|---------------|
| Vanilla LR | 59.69±0.93    | 51.16±0.83    | 46.22±0.77    |
| vanilla MFL | 61.72±0.99    | 53.01±0.85    | 47.82±0.77    |
| vanilla MFL-IU3 | 62.97±1.00    | 54.34±0.85    | 48.72±0.75    |
| MFL-P-IU3 | 61.85±0.97    | 53.46±0.82    | 47.82±0.76    |
| ComMFL-IU3 | 63.00±0.98    | 54.63±0.85    | 48.88±0.77    |

**Table 3**

MFL as an ensemble: Mean accuracies (%) of our methods with Conv4 backbone on 5-way 1-shot tasks from minilimageNet. **Bold**: the best scores. **Underline**: the second best scores.

| #C   | 0.1 | 1.0 | 10 | Weight Ave. |
|------|-----|-----|----|-------------|
| Vanilla LR | 45.87±0.77 | 46.13±0.77 | 46.18±0.77 | 46.18±0.77 |
| vanilla MFL | 49.40±0.76 | 49.94±0.78 | 48.88±0.80 | 49.33±0.79 |
| vanilla MFL-IU3 | 49.39±0.77 | 50.50±0.78 | 50.98±0.81 | 51.11±0.81 |
| MFL-P-IU3 | 50.75±0.84 | 51.94±0.82 | 51.48±0.81 | 52.33±0.77 |
| ComMFL-IU3 | 51.13±0.85 | 52.06±0.85 | 52.03±0.83 | 52.37±0.84 |

adopted the same competitors in Sec. 5.1.1 and carried out experiments on CD-FSL by using 5-way 1/5-shot settings referring to [7].

**Analysis.** Table 5 shows the results with the following observations: (1) By directly using the learned representation trained on minilimageNet, the three existing FSL methods give inferior performance on CD-FSL. (2) MetaModelNet, the model transformation method, improved the Vanilla LR on FSL but failed on CD-FSL, resulting in a poorer transformed classifier than Vanilla LR. (3) Our methods are able to improve the Vanilla LR by transferring the regularisation knowledge in model learning across domains, yielding a more accurate classifier with 1%-3% increase of classification accuracy on 5-way K-shot CD-FSL under scenarios of mini→CUB and mini→Places. Additionally, we note the improvement on mini→Cars is limited, which might be due to that the embedding space pre-trained on minilimageNet is less-discriminative for Cars, such the assumption of continuity, cluster and manifold distributions for regularisation knowledge transfer is less effective.

5.3 Ablation Study

**Visualisation** To validate our hypothesis, i.e. the regularisation knowledge transfer with MFL, we adopted T-SNE [32] to visualise the classification results of Vanilla LR and MFL-P-IU3 on 2-way 1-shot tasks from the novel classes of minilimageNet. Specifically, we showed three typical data distributions, i.e. continuity, cluster and manifold, for comprehensively describing the regularisation behaviors with the learned functional regularisation knowledge. Figure 7 shows: (1) In a specific feature space, the data distributions fit the characters of continuity, cluster or manifold (Fig. 7(a)); (2) The few-shot classifiers easily overfit to the labelled data, resulting in hypotheses lacking of regularisation and inferior classification results (Fig. 7(b)); (3) Our MFL-P-IU3 can remit this limitation via imposing the functional regularisation knowledge into classifiers, achieving more reasonable hypotheses with superior classification results (Fig. 7(c)).

**Statistics of the improvements on novel classes.** Essentially, our MFL methods aim to learn task-agnostic knowledge, i.e. the transferable and generalisable functional regularisation knowledge, to improve FSL classifiers. Due to the functional regularisation knowledge is learned from episodes sampled from a base dataset, as a common learning favour of machine learning methods, the learned functional regularisation knowledge prefers to improve the FSL tasks containing the novel classes which are similar to the categories in a base dataset. To investigate this, we designed an experiment on binary classifiers whose parameters are
MFL methods are able to generalised to the FSL tasks with different shot, we conducted experiments on 5-way K-shot (K = 2, 3, 4, 5) FSL with vanilla MFL, vanilla MFL-IU3, MFL-P-IU3 and ComMFL-IU3. Table 6 shows the evaluation results and we can see that: (1) All the forms of MFL can boost the classifiers’ performance on 2/3/4/5-shot FSL; (2) With the number of shot increasing, the improvement of our MFL methods over Vanilla LR decreases. This suggests that the hypotheses can gradually learn regularisation knowledge with the help of available labelled data, yielding more robust hypotheses where the boosting space with regularisation knowledge is narrow, thus the learned functional regularisation knowledge brings less improvement.

**Generalisation to different backbones** We conducted experiments to investigate the generalisation ability of our MFL methods on different backbones. Specifically, we additionally used two backbone networks, i.e. ResNet12 in [50] and recently proposed Shifted window Transformer (Swin Transformer) [31] for learning a representator. In particular, we adopt the small version of Swin Transformer (Swin-S) with the default hyper-parameters in [31] and the image size is resized as $224 \times 224$. As in Tab. 7, our methods show well generalisation ability on different backbones. Noticeable, the improvement on ResNet12 and Swin Transformer is smaller than that on Conv4, we conjecture this may attribute to the shallow architecture of Conv4, yielding less discriminative representation in which the learned vanilla classifiers are easily stuck in the overfitting problem and our MFL can effectively extricate them from this dilemma via the knowledge of functional regularisation.

**Effects on different classifiers.** Essentially, the functional regularisation knowledge improves the FSL classifiers by imposing transferable constraints, and this type of knowledge should be, in principle, generalised to other parametric-classifiers and not limited to the Logistic Regression. With this motivation, we conducted experiments to...
ComMFL-IU3 compared with Vanilla LR. The help of iterative updates strategy. But how do the MFLs, performs superior to vanilla MFL on FSL and CD-FSL with results in Tab. 2 and Tab. 5 show that vanilla MFL-IU3 is expected, the results in Tab. 8 indicates that all forms of MFL can be transferred across different domains for model learning functional spaces between a richly labelled domain and a scarcely labelled domain. We demonstrate that classifiers with less training data can gradually learn the functional regularisation knowledge across domain is less than that within domain, requiring less model capacity provided by the connected blocks with iterative updates.

**TABLE 7**

| Backbone | ResNet-12 | Swin-Transformer |
|----------|-----------|------------------|
| #shot    | 1 5       | 1 5              |
| Vanilla LR | 58.05 77.07 58.56 75.37 |   |
| vanilla MFL (Ours) | 59.45 77.49 59.15 75.52 |   |
| vanilla MFL-IU3 (Ours) | 60.40 77.24 59.35 75.35 |   |
| MFL-P-IU3 (Ours) | 60.46 77.81 59.43 75.24 |   |
| ComMFL-IU3 (Ours) | 60.24 77.20 59.14 75.50 |   |

**TABLE 8**

| Backbone | Conv4 | ResNet-12 |
|----------|-------|-----------|
| #shot    | 1 5   | 1 5       |
| Vanilla SVM | 46.00 62.36 57.88 75.17 |   |
| vanilla MFL (Ours) | 48.85 64.61 57.96 75.41 |   |
| vanilla MFL-IU3 (Ours) | 48.85 64.61 58.95 75.32 |   |
| MFL-P-IU3 (Ours) | 51.25 65.56 60.39 76.26 |   |
| ComMFL-IU3 (Ours) | 51.87 66.80 59.97 75.71 |   |

investigate the generalisation ability of MFL on different base classifiers. Specifically, we additionally used linear Support Vector Machine (SVM) as a base classifier for learning a representation learned with Conv4 and ResNet12. As expected, the results in Tab. 8 indicates that all forms of MFL show clear and consistent improvements over the Vanilla SVM, verifying the generalisation ability of our methods on different classifiers.

**Influence of iterative steps.** As the extensive experimental results in Tab. 2 and Tab. 5 show that vanilla MFL-IU3 performs superior to vanilla MFL on FSL and CD-FSL with the help of iterative updates strategy. But how do the MFLs, i.e. vanilla MFL, MFL-P and ComMFL, perform when we employ more iterative updates. To answer this question, we evaluated vanilla MFL, MFL-P and ComMFL with different iterative updates $x \in \{0, 1, 2, 3, 4, 5, 6\}$ on two typical scenarios, i.e. 5-way 1-shot FSL tasks and 5-way 1-shot CD-FSL tasks.

Figure 9 shows that, as expected, the performance gradually increases with the iterative updates $x$ increasing from 1 to 3. However, when $x$ is too large, i.e. $x > 4$ for FSL and $x > 3$ for CD-FSL, the performance become stable even decreased. Noticeably, the best number of updates for CD-FSL is 3, which is smaller than that for FSL. We conjecture that this is due to the transferable regularisation knowledge across domain is less than that within domain, requiring less model capacity provided by the connected blocks with iterative updates.

**6 Conclusion**

In this work, we explored the idea of knowledge transfer by learning a meta functional of regularisation in the model learning functional spaces between a richly labelled domain and a scarcely labelled domain. We demonstrate that classifiers with less training data can gradually learn the functional regularisation knowledge from a concurrent learning process on more labelled data. Based on this observation, we consider that this functional regularisation knowledge can be transferred across different domains for model learning tasks when training data is scarce. We formulated the MFL framework and generalised it to three different forms, i.e. a MFL with prototypes (MFL-P), a Composite MFL (ComMFL) and a MFL with Iterative Updates (MFL-IU). Extensive experiments on minilImageNet, CIFAR-FS, CUB, Cars and Places, show that the transfer of model learning regularisation knowledge is effective in learning more accurate hypotheses (classifiers) given scarcely labelled data.
Fig. 9. Evaluation on 5-way 1-shot FSL tasks using the MFLs with \( x \) iterative update. \( x = 0 \) represents the result of Vanilla LR.

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