Inhibiting the TE\textsubscript{1}-mode diffraction losses in terahertz parallel-plate waveguides using concave plates

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Abstract: We present numerical and experimental results on inhibiting diffraction losses associated with the lowest order transverse electric (TE\textsubscript{1}) mode of a terahertz (THz) parallel-plate waveguide (PPWG) via the use of slightly concave plates. We find that there is an optimal radius of curvature that inhibits the diffraction for a given waveguide operating at a given frequency. We also find that introducing this curvature does not introduce any additional group-velocity dispersion. These results support the possibility of realizing long range transport of THz radiation using the TE\textsubscript{1} mode of the PPWG.

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1. Introduction

Guided-wave propagation of terahertz (THz) radiation has been realized in several types of waveguides, including both metallic and dielectric structures [1–15]. One of the more commonly used structures is the parallel-plate waveguide (PPWG), which has almost exclusively been operated in its transverse-electromagnetic (TEM) mode [4]. This mode has been generally preferred since it has relatively low ohmic losses and no cutoff, and therefore no group velocity dispersion. Recently, we have explored the use of the lowest-order transverse-electric (TE) mode of the PPWG, and predicted the possibility of achieving ultralow ohmic losses in the dB/km range [14]. This loss is three orders of magnitude lower than the lowest loss experimentally demonstrated in the THz range [10], and is comparable to that of telecommunications-grade optical fibers operating at 1.55 µm as well as to low-loss microwave transmission in over-moded circular waveguides [16]. These low ohmic losses could permit long distance transport of THz radiation, significantly longer than what is now feasible. However, with such long propagation distances a new concern arises which is not relevant in either fiber optics or over-moded circular waveguides: energy leakage out of the unconfined sides of the PPWG due to diffraction of the propagating wave. In fact, this would be the dominant loss mechanism in the case under consideration, where the ohmic losses are virtually negligible [14,15].

As a possible approach to mitigating this diffraction loss, we investigate the effects of introducing a slight curvature to the inside plate surfaces based on ideas proposed at other frequencies [17,18]. Some of the analytical principles governing the use of slightly concave plates has already been discussed in our earlier work [14]. In this paper, we present numerical and experimental results to validate this concept.

2. Numerical simulations

First, we present results of a numerical study into the THz propagation behavior, investigating the applicability of this curved-surface waveguide geometry. We use a commercial finite-element-method (FEM) modeling software (COMSOL Multiphysics) to carry out the numerical simulations, and compare the behavior of several curved-surface waveguides to the well-known behavior of a flat-surface one. In the simulation, the outer boundaries of the two plates are assigned perfect-electric-conductor boundary conditions [19]. The simulation space is bounded by enclosing the waveguide within a solid rectangular box of vacuum, the walls of which are assigned low-reflecting boundary conditions to minimize the effects of back reflections. Each of the plates is 3 cm wide and 23 cm long. For each waveguide, the plate separation is 1 cm. In the case of the curved-surface plates, the plate separation is defined to be the plate spacing along the central axis.

To initiate the simulation, a 0.1 THz wave is incident on the input gap between the plates, linearly polarized along the x axis [parallel to the (nominal) plate surfaces] to excite only the TE modes. The input spatial profile is modeled as a Gaussian with an elliptic cross-section, where the major axis is chosen to be 2 cm, which is smaller than the plate width. The minor axis is chosen to be 0.7 cm to optimize the input coupling to the single TE1 mode [14]. The model is solved using an iterative generalized minimal residual solver (GMRES) with Symmetric Successive Over-Relaxation (SSOR) matrix preconditioning.

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Figure 1(a) shows the magnitude of the electric field component oriented in the $x$ direction ($E_x$), along a centralized slice (between the plates) in the $x$-$z$ plane, for a wave propagating inside the flat-surface PPWG. Figure 1(b) shows the $E_x$ distribution for a wave propagating inside a curved-surface waveguide with a radius of curvature of 6.7 cm. (The significance of this value of the radius is explained later.) A side by side qualitative comparison of the two figures indicates that there is less lateral diffraction in the curved-surface waveguide, as the wave propagates along the waveguide.

As discussed in Ref [14], based on the “bouncing-plane-wave” description of the TE$_1$ mode propagation, introducing a slight curvature (transverse to the axis of propagation) of radius $R$ to the inside of the metal plates imparts a lateral focusing effect to the THz beam. The focusing effect at each bounce of the plane wave can be considered as being caused by a
concave mirror with a focal length of $R/2$. This bouncing-plane-wave picture is illustrated in Fig. 2(a), which shows a particular situation where the plane wave undergoes five bounces. The bounce-to-bounce distance is $2d$. It can be shown that the bounce angle is related to the frequency by

$$\theta = \cos^{-1}\left(\frac{c}{v} \right),$$

(1)

where the plate separation $b$ is related to $d$ by $b = 2d \cos \theta$, $v$ is the frequency, and $c$ is the speed of light. After “unfolding” the propagating wave, the repetitive interactions with the curved surface can be simulated by a series of identical thin lenses having a focal length of $R/2$.

Figure 1(c) shows a comparison of the fractional time-averaged power [power inside/(power inside plus outside)] versus distance of propagation for various radii of curvature. These results were extracted from numerical simulation results such as the ones shown in Figs. 1(a) and 1(b). We note that the energy confinement improves as the curvature is increased, and that there is a significant improvement in the energy confinement when $R = 6.7$ cm. This value of $R$ corresponds to the confocal condition at this particular frequency (based on the bouncing-plane-wave argument presented above), which allows maximum power transfer through the optical system [20]. At the confocal condition of $R = 2d$, substituting from Eq. (1), we obtain,

$$R = \frac{b^2 v}{c}.$$  

(2)

This equation can be used to deduce the required radius of curvature that would allow for maximum power transfer, for a given plate separation and preferred operating frequency. For example, when $b = 1$ cm and $v = 0.1$ THz, we can deduce, $R = 6.7$ cm.

3. Experimental techniques and results

Next, we present experimental results showing how a slight curvature on the inside surfaces improves the lateral confinement along the open-ended sides. For the experiment, we fabricate five waveguides using polished aluminum plates. They have the following radii of curvature: $R = 6.7$ cm, $R = 20$ cm, $R = 50$ cm, $R = 100$ cm, and $R = \infty$ (i.e. a conventional flat-surface PPWG). They each have a transverse width of 3.8 cm and a center-to-center plate separation $b = 1$ cm. THz pulses are generated and detected using a conventional THz-time-domain spectroscopy system based on fiber-coupled photoconductive antennas [21]. A schematic that illustrates our measurement technique is shown in Fig. 2(b). The input electric field is polarized parallel to the (nominal) plate surfaces to excite only TE modes. The THz receiver is shown scanning across the output face of one of the curved-surface waveguides. As defined in the figure, $L = 25$ cm, $W = 3.8$ cm, and $b = 1$ cm. The input THz beam is centered on the front face in both the $x$ and $y$ directions and weakly focused using two convex teflon lenses (not shown), to achieve a frequency independent $1/e$ input beam diameter of $=2$ cm. This beam size was chosen to dominantly excite the TE$_1$ mode [14].

Typical THz waveforms measured at the input and output facets of our waveguides are shown in Fig. 3. Figure 3(a) shows the measured THz pulse at the input facet, as well as its amplitude spectrum (inset). Figure 3(b) shows THz pulses measured at the output face of a flat-surface waveguide and a waveguide with a surface curvature of $R = 6.7$ cm.
Fig. 2. (a) Longitudinal cross-section of the PPWG showing the path of the “bouncing plane wave” for five bounces. (b) Schematic of a curved-surface waveguide with its input electric field polarized parallel to the (nominal) inner plate surfaces to excite the TE_1 mode. The THz receiver is shown scanning across the output face.

Fig. 3. (a) Input THz pulse, and (b) output THz pulse for a flat-surface PPWG and a curved-surface waveguide with curvature radius $R = 6.7$ cm, measured on axis. The corresponding amplitude spectra are shown as insets.

In both cases, during each measurement, the receiver is centered in both the $x$ and $y$ directions. A qualitative comparison of both output signals indicates that the dispersion introduced by the curved-surface waveguide is comparable to that introduced by the flat-surface one. Furthermore, the respective spectra (shown in the inset) indicate that there is relatively more low-frequency content in the output signal corresponding to the curved-surface waveguide. We note that this is consistent with an apparent reduction in the
diffraction losses in the vicinity of 0.1 THz (design frequency) for the curved-surface waveguide, as predicted by the theoretical results.

We also image the electric field at the output of the waveguides in a plane perpendicular to the axis of propagation. The THz receiver is raster-scanned in a $20 \times 60 \text{ mm}^2$ grid, in steps of 1 mm. The detected time-domain waveforms obtained from raster-scanning were Fourier transformed and their field amplitudes were used to plot two-dimensional false color plots at specific frequencies. These plots shown in Fig. 4 give the electric field distribution for a flat-surface waveguide, and for curved-surface waveguides with curvature radii of $R = 6.7 \text{ cm}$, 20 cm, and 50 cm.

Panels 4(a) and (b) correspond to the frequencies of 0.1 THz and 0.3 THz, respectively. It is evident that at each frequency, the waveguide with the radius of curvature $R = 6.7 \text{ cm}$ exhibits the most energy confinement. In Fig. 4(a), at a frequency of 0.1 THz, we observe the best improvement in energy confinement with $R = 6.7 \text{ cm}$, as predicted by our numerical simulation results and analytical derivations. Furthermore, we note that in general, the improvement in energy confinement diminishes with increasing frequency.
In Fig. 5, we plot one-dimensional line profiles corresponding to horizontal cuts along the central axis of the two-dimensional color plots shown in Fig. 4. These profiles show the normalized electric field amplitude at the output of the waveguides, when the receiver is scanned across the $x$-axis centered between the plates, as shown inset. Again, we observe that the waveguide with the radius of curvature $R = 6.7$ cm displays the least amount of diffraction at each frequency shown.

These results, along with those of Fig. 4, also indicate that this curvature has the effect of confining a relatively broad range of frequencies, although the confocal condition discussed above applies to only one particular frequency.

In Fig. 6, we plot the full-width-at-half-maximum (FWHM) of the profiles (given in Fig. 5) versus the inverse of the radius of curvature of the waveguide plates. The measured FWHM are plotted as discrete symbols, while the solid lines are least-square fits. We note that the (negative) slope of the FWHM decreases as a function of frequency, and is almost zero at 0.5 THz, indicating that the curved plates have less effect on the output beam size as the frequency increases. To quantify this effect, we show the slope of these lines versus frequency in the inset. The gradient is largest at low frequencies, which is not surprising since the radius of curvature (of $R = 6.7$ cm) was optimized for maximum power transfer at the
lowest frequency of 0.1 THz. This result indicates the bandwidth over which the concave plates have a significant confining effect on the propagating mode. Even though the confocal condition [Eq. (2)] suggests that this mode confinement strategy is a narrow-band effect, it is clear from our results that the mode confinement is effective over a bandwidth of at least several hundred GHz.

![Graph](image)

Fig. 6. Measured FWHM of the electric field profile at the central x axis of the output face of the waveguide as a function of $1/R$. The discrete symbols are the measured values at several frequencies and the lines are least-square fits. The error bars shown for the 0.1 THz data points are representative for all the data points. The inset shows a plot of the slope of each fitted line shown in the main figure, versus frequency, and the corresponding least-square fit.

Another consideration involves the possibility that concave plates could introduce appreciable group velocity dispersion. In order to quantify and compare the dispersion behavior, we consider the fundamental equation governing the input and output relationship of the experimental system. Assuming single-mode propagation, this can be written in the frequency domain as

$$E_{out}(\omega) = E_{in}(\omega)TC \exp(-j \beta L) \exp(-\alpha L),$$  \hspace{1cm} (3)

where $E_{out}(\omega)$ and $E_{in}(\omega)$ are the complex spectral components at angular frequency $\omega$ of the output and input electric fields, respectively; $T$ is the total transmission coefficient, which takes into account the impedance mismatch at the entrance and exit faces; and $C$ is the total amplitude coupling coefficient, which takes into account the spatial-mode mismatch at both the entrance and exit faces. $L$ is the distance of propagation, $\alpha$ is the attenuation constant, and $\beta$ is the phase constant.

The experimental group velocity can be estimated using $V_{\text{g(exp)}} = (d\beta_{\text{exp}} / d\omega)^{-1}$, where $\beta_{\text{exp}}$ corresponding to each waveguide can be derived using the measured input and output signals and substituting in Eq. (3). In this derivation, we assume that there is no phase information in the product $TC$. Based on classical waveguide theory [22], the theoretical
group velocity for the TE$_1$ mode in a conventional flat-surface PPWG is given by,

$$V_{g(\text{theory})} = \frac{c \beta}{k_0}, \quad \beta = \sqrt{(k_0)^2 - \left(\frac{\pi}{b}\right)^2} \quad \text{and} \quad k_0 = \frac{w}{c}.$$ We plot $V_{g(\text{exp})}$ for $R = 6.7$ cm, 50 cm, and $\infty$, along with $V_{g(\text{theory})}$, as a ratio with respect to $c$, in Fig. 7. We find that in all three experimental cases the group velocity dispersion is negligible throughout the spectrum, except at the very low-frequency end, and comparable to the theoretical curve.

This minimal dispersion behavior is due to the TE$_1$-mode cutoff frequency [given by $c/(2b)$] of 15 GHz being very close to the low end of the input spectrum. We also observe that within the noise level, there is no appreciable additional dispersion due to the surface curvature.

4. Conclusion

We have demonstrated that it is possible to inhibit diffraction losses for the TE$_1$ mode of a PPWG operating in the THz region, by utilizing plates with slightly concave surfaces. Using a simple “bouncing plane wave” analysis, we demonstrate how to determine an ideal radius of curvature for a waveguide operating at a given THz frequency. We show both experimentally and theoretically that for a waveguide with a plate separation of 1 cm, one can inhibit the diffraction at (and around) a frequency of 0.1 THz, when the surface has a radius of curvature of 6.7 cm. These results support the possibility of realizing long range transport of THz radiation via PPWGs, as predicted in Ref [14]. We also note that a recent study has proposed the use of slightly concave plates even for the TEM mode of the PPWG [23].

We would like to emphasize that the goal of this work was to demonstrate that one could inhibit the inherent lateral diffraction in the PPWG by using slightly concave surfaces, such that the waveguide geometry is not significantly perturbed from a PPWG. We note that as long as Eq. (2) holds, one could keep on reducing $R$ below 6.7 cm and achieve the confocal condition for frequencies lower than 0.1 THz for $b = 1$ cm. However, a significant reduction
in $R$ will perturb the PPWG to the point that the above bouncing-plane-wave derivation no longer holds, implying Eq. (2) no longer holds. On the other hand, for a given $b$ (and $W$), when one keeps reducing $R$, it is logical to assume that the energy confinement will naturally improve since this automatically minimizes the peripheral openings. In fact, one could eventually achieve 100% confinement when the side-edges come into contact, resulting in a fully enclosed metallic waveguide with an elliptic-like cross section. However, this resulting waveguide geometry will not exhibit the desirable low loss and low dispersion properties associated with the PPWG.

Finally, we could also imagine a scenario where only one plate (one inner surface) is curved. In this case, based on the bouncing-plane-wave description, the curved plate would still provide a focusing effect, while the planar one simply acts as a planar mirror. Therefore, based on Eq. (2), to achieve the confocal condition at a given frequency, the required radius of curvature would be double that of when both plates are curved, which is somewhat counter intuitive.

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