Abstract

The $d(e, e'\pi^+)$ reaction in the parallel kinematics has been investigated using a dynamical model of pion electroproduction on the nucleon. A unitary $\pi NN$ model has been used in order to examine the effects due to the final two-nucleon interactions, pion rescattering from the second nucleon, and the intermediate $NN$ and $N\Delta$ interactions. It has been found that these $\pi NN$ mechanisms are small, but they can have significant contributions to the $d(e, e'\pi^+)$ cross sections through their interference with the dominant impulse term. For the longitudinal cross sections, the effects due to the interference between the pion pole term and other production mechanisms are also found to be very large. Our findings clearly indicate that these interference effects must be accounted for in any attempt to determine from the $d(e, e'\pi^+)$ data whether the pion form factor and/or $\pi NN$ vertex of the pion pole term are modified in the nuclear medium.
Dynamical Study of the \( d(e, e'\pi^+ \right) \) reaction

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I. INTRODUCTION

It has been suggested that the pion form factor and/or the \( \pi NN \) vertex could be modified by the nuclear medium. One possible way to investigate this interesting question is to study the electroproduction of charged pions on the nuclei in the kinematic region where the outgoing pions are in the direction of the exchanged virtual photon. For this so-called parallel kinematics, the pion pole term (i.e the pion-exchange production amplitude of Fig.2a) is expected to be dominant and hence the measured cross sections could be used to explore the medium effects on the pion form factor and/or the \( \pi NN \) vertex. The above consideration has motivated an experiment at Saclay in 1990 [1] and another [2] at Jefferson Laboratory (JLab). The objective of this work is to address some theoretical questions concerning the interpretation of the data from these experimental efforts. Here, we consider the simplest \( d(e, e'\pi^+) \) reaction.

We use the dynamical model developed by Sato and Lee [3] (called the SL model). The aim of the SL model is to interpret the data of pion photoproduction and electroproduction in terms of quark sub-structure of hadrons. The outcome of this effort has two aspects: (i) to establish the interpretation of the \( \gamma N \rightarrow \Delta \) excitation in terms of constituent quark models. (ii) to have a dynamical model which gives a fairly accurate description of all of the data of pion photoproduction and electroproduction, which can be used to perform various nuclear calculations. We make use of the second aspect of the SL model in this investigation.

The SL model is illustrated in Fig.1. It consists of a production term, illustrated in Fig.2, and a term involving \( \pi N \) scattering. The focus of the experiments with parallel kinematics
is the pion pole term, Fig. 2a, which depends on the pion form factor and the $\pi NN$ vertex. Clearly, the effects due to the other terms in Figs. 1-4 must be carefully investigated before the medium effects on the pion pole term can be determined. This will be the ultimate goal of this work.

The second problem we want to address is the extent to which the effect due to the pion pole term will be masked by the hadronic final state interactions. For the $d(e, e'\pi^+)$ reaction, the final $\pi NN$ interactions can in principle be calculated by using the unitary formulation developed in Refs. 4,5. This is a rather complex numerical task and will not be pursued in this work. Instead we consider only the leading terms of a multiple scattering expansion of the scattering amplitude defined by the $\pi NN$ model of Ref. 5.

We thus calculate the $d(e, e'\pi^+)$ cross sections from the four leading mechanisms illustrated in Fig. 3. The impulse term (Imp), illustrated in Fig. 3a, is due to the production on a nucleon in the deuteron. The other three terms are due to final two-nucleon interactions, pion rescattering from the second nucleon ($\pi NN$), and intermediate $NN$ and $N\Delta$ interactions. The amplitudes of these reaction mechanisms can be evaluated straightforwardly from using the SL model and the $\pi NN$ model developed in Refs. 5. Thus our calculations of the $d(e, e'\pi^+)$ cross sections will be free of adjustable parameters.

In section II, we present the formulation for calculating the cross sections of $d(e, e'\pi)$ reaction. The results are presented and discussed in section III.

II. FORMULATION

In the rest frame of the initial deuteron, the differential cross section of the $d(e, e'\pi)$ reaction can be calculated from

$$
\frac{d^6\sigma}{d\Omega_{e'}dE_{e'}d\Omega_{\pi}dk} = \sigma_M \left[ W_2 - 2W_1 \tan^2\left(\frac{\theta_e}{2}\right) \right]
= \Gamma_v [\sigma_T + \varepsilon\sigma_L],
$$

(1)

where $\theta_e$ is the angle between the outgoing and incoming electrons, $E_{e'}$ is the outgoing electron energy, and $\vec{k}$ is the outgoing pion momentum. The Mott cross section is defined
by
\[
\sigma_M = \frac{\alpha^2 \cos^2(\frac{\theta_e}{2})}{4E_e \sin^4(\frac{\theta_e}{4})},
\]
(2)
where \(\alpha = \frac{e^2}{4\pi} = \frac{1}{137}\). If the photon four-momentum is denoted as \(q = (\omega, \vec{q})\), the other kinematic factors in Eq.(1) are
\[
\varepsilon = \frac{1}{1 + 2|\vec{q}|^2 \tan^2(\frac{\theta_e}{2})},
\]
\[
\Gamma_v = \alpha \frac{E_e' K}{2\pi^2 E_e Q^2} \frac{1}{1 - \varepsilon}.
\]
Here we have defined \(Q^2 = -q^2 = |\vec{q}|^2 - \omega^2\) and \(K = \frac{W^2 - m_d^2}{2m_d}\) with \(m_d\) being the deuteron mass and \(W = [(\omega + m_d)^2 - q^2]^{1/2}\) being the invariant mass of the \(\gamma d\) system. From the above definitions, one can show that the longitudinal cross section \(\sigma_L\) and transverse cross section \(\sigma_T\) in Eq.(1) are related to the structure functions \(W_1\) and \(W_2\) by
\[
\sigma_T = \frac{4\pi^2 \alpha}{K} [-W_1],
\]
\[
\sigma_L = \frac{4\pi^2 \alpha}{K} \left[\frac{|\vec{q}|^2}{Q^2} W_2 + W_1\right].
\]
The structure functions can be calculated from
\[
W_1 = -\frac{1}{2} \sum_{\lambda=\pm 1} (2\pi)^6 k^2 \int d\vec{p}_1 \delta(\omega + m_d - E_\pi(\vec{k}) - E_N(\vec{p}_1) - E_N(\vec{p}_2))
\times \frac{1}{2J_d + 1} \sum_{\lambda = \pm M_d, m_{s1}, m_{s2}} I_{m_{s1}, m_{s2}, \lambda, M_d}(\vec{k}, \vec{p}_1, \vec{p}_2; \vec{q}, \omega),
\]
(7)
\[
W_2 = \frac{Q^4}{|\vec{q}|^4} (2\pi)^6 k^2 \int d\vec{p}_1 \delta(\omega + m_d - E_\pi(\vec{k}) - E_N(\vec{p}_1) - E_N(\vec{p}_2))
\times \frac{1}{2J_d + 1} \sum_{M_d, m_{s1}, m_{s2}} I_{m_{s1}, m_{s2}, 0, M_d}(\vec{k}, \vec{p}_1, \vec{p}_2; \vec{q}, \omega) - \frac{Q^2}{|\vec{q}|^2} W_1,
\]
(8)
where \(\lambda\) is the photon polarization, \(m_{s_i}\) and \(M_d\) are the z-component of the \(i\)-th nucleon and the deuteron respectively. The pion momentum is \(\vec{k}\) and the momenta of the outgoing two nucleons are \(\vec{p}_1\) and \(\vec{p}_2\). The energy variables are defined as \(E_\pi(\vec{k}) = \sqrt{m_\pi^2 + \vec{k}^2}\) for the pion
and $E_N(\vec{p}) = \sqrt{m_N^2 + \vec{p}^2}$ for the nucleon. The isospin variables are suppressed to simplify the presentation.

For the considered reaction mechanisms illustrated in Fig. 3, the total amplitude in Eqs.(7)-(8) is

$$I_{m_{s_1},m_{s_2},\lambda,M_d} = \langle \vec{p}_1 m_{s_1}, \vec{p}_2 m_{s_2}; \vec{k} \mid I \mid q\lambda, \Psi^{J_d M_d} \rangle,$$

where $\Psi^{J_d M_d}$ is the deuteron wavefunction, and

$$I = I^{(Imp)} + I^{(FSI)} + I^{(Resc)} + I^{(BB)}.$$  

(10)

Explicitly, the impulse term (Fig. 3a) is defined by

$$I^{(Imp)} = \sum_{i=1,2} A(i),$$

(11)

where $A(i)$ is the pion electroproduction operator on the $i$–th nucleon. The Nucleon-Nucleon $NN$ final state interaction term (Fig. 3b) is

$$I^{(FSI)} = \sum_{i=1,2} T_{NN,NN}(E - K_\pi) G_{NN}(E - K_\pi) A(i).$$

(12)

Here $T_{NN,NN}(\omega)$ is the $NN$ scattering operator and $K_\pi$ is the free energy operator for the pion. The $NN$ propagator is defined by

$$G_{NN}(\omega) = \frac{1}{\omega - K_N(1) - K_N(2) + i\epsilon},$$

(13)

where $K_N(i)$ is the free energy operator for the $i$–th nucleon. The pion rescattering term (Fig. 3b) is defined by

$$I^{(res)} = \sum_{i\neq j} t_{\pi N}(E - K_N(j), i) G_{\pi NN}(E) A(j),$$

(14)

where $t_{\pi N}(\omega, i)$ is the $\pi N$ scattering operator on the $i$–th nucleon, and the $\pi NN$ propagator is defined by

$$G_{\pi NN}(E) = \frac{1}{E - K_N(1) - K_N(2) - K_\pi + i\epsilon}.$$
The baryon-baryon interaction term (Fig.3d) is
\[ I^{(BB)} = \sum_{i=1,2} \sum_{B,B'=N,\Delta} h_{\pi N,B}(i) G_{BN}(E) T_{BN,B'N}(E) G_{B'N}(E) F_{B,\gamma N}(i), \]  
where the vertex interactions \( h_{\pi N,B} \) and \( F_{B,\gamma N} \) describe the \( B \leftrightarrow \pi N \) and \( B \leftrightarrow \gamma N \) transitions respectively. In addition to \( G_{NN}(E) \) defined by Eq.(13), Eq.(16) also depends on the \( \Delta N \) propagator. It is defined by
\[ G_{\Delta N}(E) = \frac{1}{E - K_N(1) - K_\Delta(2) - \Sigma_\Delta(E)}, \]  
where \( K_\Delta \) is the free energy operator for the \( \Delta \), and \( \Sigma_\Delta(E) \) is the \( \Delta \) self-energy evaluated in the presence of a spectator nucleon.

In our calculations, the matrix element of \( A(i) \) is generated from the SL model. The matrix elements of the \( \pi N \) scattering operator \( t_{\pi N} \), the baryon-baryon scattering operator \( T_{BN,B'N} \) with \( B, B' = N, \Delta \) are generated from the \( \pi NN \) model developed in Ref. [5]. Therefore, our calculations of the total amplitude Eq.(9) and the \( d(e,e'\pi^+) \) cross sections are free of adjustable parameters.

III. RESULTS AND DISCUSSIONS

We first consider the data from Saclay [1] in 1990. In Fig.4, we show the relative importance between the four amplitudes illustrated in Fig.3. We see that the impulse term (solid curve) dominates. The final \( NN \) interaction (dashed curve) gives comparable contributions only at energies very close to the threshold. This is due to the fact that in this region the energy of the outgoing \( NN \) state is very low and the final \( NN \) interaction is dominated by the attractive \( ^1S_0 \) force. The contributions from the pion rescattering mechanism (dot-dashed curve) and the \( BB \) mechanism (dotted curve) are clearly very weak.

In Fig.5, we compare the predicted cross sections (solid curve) with the data [1]. We see that the general feature of the data is reproduced. However, the discrepancy is rather significant in the region where the missing mass is close to 1900 MeV. This could be due to
the deficiency of the SL model or the higher order reaction mechanisms which are neglected in this work. In the same figure, we also show the results (dashed curve) from the impulse term only. It is seen that the $\pi NN$ interaction mechanisms, illustrated in Figs. (3b-3d), yield an about 10% effect in the region near the peak position. We have found that this is mainly due to the interference between the impulse amplitude and the FSI amplitude. The effects due to pion rescattering and $BB$ interactions are negligible.

We next consider a recent experiment at JLab [2]. The calculated cross sections from each mechanism illustrated in Fig. 3 are compared in Fig. 6. Here the $BB$ interaction term (dotted curve) and the pion rescattering term (dot-dashed curve) become comparable. However, the impulse term (solid curve) is much larger than the other terms. Accordingly, the effects due to the $\pi NN$ (rescattering) mechanisms is much less. This is shown in Fig. 6. In the same figure, we also see that the predicted cross sections are about 20% lower than the data. This perhaps is mainly due to the deficiency of the SL model in this $Q^2 = 0.4\,\text{(GeV/c)}^2$ region. However, it could be due to the neglect of higher order reaction mechanisms.

We now turn to discussing how the measured longitudinal cross sections can be used to learn about the medium effects on the pion form factor and/or $\pi NN$ vertex of the pion pole term. First we need to know the effect due to the $\pi NN$ mechanisms. As shown in Fig. 8, this effect only reduces the longitudinal cross section by about 5%. Second, we need to know the contribution from the pion pole term. For the considered parallel kinematics, the pion pole term (Fig. 2a) indeed dominates. On the other hand, the other mechanisms in Figs. 1-2 can still have large effects through their interference with the pion pole term. This is illustrated in Fig. 8 where we see that the pion pole term alone only gives about 50% of the longitudinal cross section. One thus must be cautious in interpreting the measured longitudinal cross sections in terms of the medium effects on pion form factor and/or $\pi NN$ vertex.

In summary, we have investigated the $d(e, e'\pi^+)$ reaction for parallel kinematics by using a dynamical model of pion electroproduction on the nucleon. We have found that the effects due to the final two-nucleon interactions, pion rescattering from the second nucleon
and the intermediate $NN$ and $N\Delta$ interactions to be small. But they can have significant contributions to the $d(e, e'\pi^+)$ cross sections through their interference with the dominant impulse term. For the longitudinal cross sections, the effects due to the interference between the pion pole term and other production mechanisms are also found to be very large. Our findings indicate that these interference effects must be accounted for in any attempt to determine from the $d(e, e'\pi^+)$ data whether the pion form factor and/or $\pi NN$ vertex of the pion pole term are modified in the nuclear medium.

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FIG. 1. Graphical representation of the SL elementary amplitude for $\pi^+$ electroproduction on the proton. Diagram (a) illustrates the production amplitude. Diagram (b) accounts for $\pi N$ scattering.
FIG. 2. Graphical representation of the interaction processes included in the SL elementary amplitude.
FIG. 3. Graphical representation of $\pi^+$ electroproduction on the deuteron Eq.(10). Graph (a), (b), (c) and (d) corresponds respectively to Impulse contribution Eq.(11), $NN$ Final State Interaction Eq.(12), pion Rescattering amplitude Eq.(14), and Baryon-Baryon interaction Eq.(16).
FIG. 4. Cross section for the processes illustrated in Fig. 3 as a function of the missing mass $M_x$ defined as $M_x = (\omega + M_d - E_\pi(k))^2 - (\vec{q} - \vec{k})^2$. The kinematic conditions ($Q^2 = 0.08(\text{GeV/c})^2, W = 1.16 \text{GeV}, E_e = 645 \text{MeV}, E_{e'} = 355 \text{MeV}$) are identical to one setting of Saclay experiment [1]. The full (dashed) curve corresponds to the Imp (Fig.3.a) (FSI (Fig.3.b)) contribution. The dotted-dashed (dotted) curve shows the rescattering (Fig.3.c) (Baryon-Baryon (Fig.3.d)) contribution.
FIG. 5. Data on $d(e, e'\pi^+)$ cross section [1]. The kinematic conditions are the same as in Fig.4.

The dashed curve corresponds to the impulse calculation (Fig.4a). The solid curve shows the effect of inclusion of the two-body interactions (Fig.4b-4c-4d).
FIG. 6. Cross section for the processes illustrated in Fig. 3 as a function of the missing mass. The kinematic conditions ($Q^2 = 0.4(GeV/c)^2, W = 1.16GeV, E_e = 844MeV, E_{e'} = 395MeV$) are identical to one setting of JLab experiment [2]. The full (dashed) curve corresponds to the Imp (FSI) contribution. The dotted-dashed (dotted) curve shows the rescattering (Baryon-Baryon) contribution.
FIG. 7. Data on $d(e, e'\pi^+)$ cross-section [2]. The kinematic conditions are the same as in Fig.6. The dashed curve corresponds to the impulse calculation (Fig.6a). The Solid curve shows the effect of inclusion of two-body interaction (Fig.6b,c,d).
FIG. 8. Dashed curve corresponds to the longitudinal cross section Eq.(6) for one of the JLab kinematics \((Q^2 = 0.4(GeV/c)^2, W = 1.16GeV, E_e = 844MeV, E_{e'} = 395MeV)\) including only the Impulse contribution (Fig.3a). The solid curve shows the effect of inclusion of the two-body interaction(Fig.(3b-3c-3d)). The dotted-dashed curve shows the same observable considering only the pole term (Fig.2a).