Quantum Noise Minimization in Transistor Amplifiers

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General quantum restrictions on the noise performance of linear transistor amplifiers are used to identify the region in parameter space where the quantum-limited performance is achievable and to construct a practical procedure for approaching it experimentally using only the knowledge of directly measurable quantities: the gain, (differential) conductance and the output noise. A specific example of resonant barrier transistors is discussed.

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Heisenberg uncertainty relations restrict the performance of amplifiers and detectors. Derived from rather general properties (canonical commutation relations for signals carried by non-conserved bosons, or the nonequilibrium Kubo formula for other signals) such restrictions specify the best-possible noise performance but do not provide a procedure for obtaining it. For example, a (phase insensitive) linear amplifier must add to the amplified signal a noise power of at least $G^2/2$ per unit bandwidth, where $G^2$ is the power gain. This restriction, referred to below as the 

Heisenberg limit, is very general and applies e.g. to laser amplifiers, parametric RF amplifiers, field effect transistors, single electron transistors and molecular transistors. However, the particular source of the noise varies and therefore also the procedures one needs to follow in order to minimize it. In parametric amplifiers this noise is the 

equilibrium current noise in the idler resistor and therefore this resistor should be cold enough to produce only the zero point fluctuations.

In transistor devices, in which the amplification is performed by a signal on a gate strongly modulating the output current, cooling the device is not sufficient to obtain the ideal noise performance. Such devices manifest 

nonequilibrium noise (called Idling-noise below) in the source-drain current even when the gate voltage is held fixed. When the gate is connected to a signal source having nonzero impedance, fluctuations in the gate potential will arise from fluctuations in the number of charge carriers in the gate region. These gate potential fluctuations cause additional source-drain current fluctuations (called here amplified back-action noise).

Using restrictions on the noise performance of (phase insensitive) transistor amplifiers, we present a procedure for an experimental identification of the region in parameter space where quantum-limited noise performance is allowed (if such a region exists). Constructed for practical purposes, this procedure only makes use of the knowledge of quantities which are 

directly measurable. Neither a knowledge of the hamiltonian of the signal source nor that of the transistor is required. As an example we show how this procedure can achieve the Heisenberg limit in certain resonant barrier transistors.

We begin by introducing the restrictions on the noise performance of transistor amplifiers. Consider a signal carried by a current $I_{in}$ which is flowing out of a source having a differential conductance $g_s$ connected to the amplifier output port. The resulting amplified signal $I_{out}$ is delivered to a load resistor, having a differential conductance $g_L$ and which enters the amplifier input port. The constraints presented below hold for this case. However, the noise minimization procedure which is derived from them holds also in the general case of impedance mismatch. If $I_{out}(t)$ is proportional to $I_{in}(t)$ the amplifier is called linear (and phase insensitive). One can then define the 

power gain

$G = g_L^{1/2}$, of the amplifier by the input-output relation $I_{out}(t) = G g_L^{1/2} I_{in}(t)$. To be valid quantum mechanically, this input-output relation must be augmented to have the form

$$I_{out}(t) = G \sqrt{g_L} I_{in}(t) + I_N(t)$$

where $I_N(t) = e^{i H_{tot} t} I_{out}(0) e^{-i H_{tot} t}$, $I_{in}(t) = e^{i H_s t} I_{in}(0) e^{-i H_s t}$, $H_{tot} = H_a + H_s + \gamma H_{a,s}$ is the total hamiltonian, $H_a$ is the hamiltonian of the signal source, $H_s$ is that of the amplifier, and $\gamma$ is that of the interaction between them. $I_N(t)$ is called the noise current operator and is a function of operators related to the amplifier degrees of freedom and therefore commutes with $I_{in} : [I_N(t), I_{in}(t)] = 0$. $I_N$ is called 'noise' because according to Eq. 1 if the source is prepared in an eigenstate of $I_{in}$ with an eigenvalue $i_{in}$, a single measurement of $I_{out}$ would yield the value $G \sqrt{g_L} i_{in}$ plus an additional random contribution from the amplifier, the fluctuations of which are given by $\Delta I_{out}^2$ where $\Delta I^2 \equiv \langle I^2 \rangle - \langle I \rangle^2$ (the average is taken with respect to the amplifier state).
If the signal source and the amplifier are initially prepared in stationary states and if after switching on the coupling they remain in stationary states, although modified ones, and if the amplifier remains approximately impedance matched, then
\[
\Delta I_n^2 \geq (G^2 - 1)\hbar \omega g_t \Delta \nu
\]
where \(\Delta \nu \equiv \Delta \omega/(2\pi)\) is the detection bandwidth and \(\Delta \omega\) is a narrow spread of frequencies around the center frequency \(\omega_0\) of the band in which the detection is performed. This inequality is a constraint on the total amplifier noise. Defining the idling-noise current by \(I_0 \equiv I_N(\gamma = 0)\) and the amplified back-action noise current by \(I_n \equiv I_N(\gamma) - I_0\) and assuming these two contributions have zero mean (for \(\omega_0 \neq 0\) and are uncorrelated, \(\langle I_0 I_n \rangle = 0\), one has \(\Delta I_n^2 = \Delta I_0^2 + \Delta I_a^2\), so that the above inequality restricts the sum of the two types of noise. Assuming that \(I_n \sim \gamma^2\) it is shown below that their product is restricted by the condition [7, 11, 22]:
\[
\Delta I_0(t) \Delta I_n(t) \geq G^2 \frac{\hbar \omega_0}{4} g_t \Delta \nu
\]
which implies that the Heisenberg limit for transistor amplifiers with a large gain, \(G^2 \gg 1\), is achieved if and only if
\[
\Delta I_0^2 = \Delta I_n^2 = G^2 \frac{\hbar \omega_0}{4} g_t \Delta \nu
\]
Eq. (2) resembles constraints derived for general linear detectors [4] and [3] or specific ones [8, 10]. It differs from these results in that it contains only directly measurable quantities: the noise contributions one would measure at the output, the gain and the conductance. Eq. (2) has several nontrivial consequences. It shows that the initial idling-noise \(\Delta I_0^2(t)\) should not be made too small since coupling a device with vanishing idling-noise to a signal will result in the appearance of an amplified back-action noise \(\Delta I_a^2(t)\) which will diverge in order to maintain the inequality in Eq. (2). In particular, for ideal operation of the amplifier at a given gain, the amplified back-action noise and the idling-noise should be each equal to half of the amplified zero point fluctuations of the amplifier.

Before presenting a way to reach the condition Eq. (2), in practice, we outline the derivation of Eq. (2) (for details see Ref. [5]). Applying the nonequilibrium Kubo formula [14, 15] to the amplifier and the source one has:
\[
\int_{-\infty}^{\infty} dt e^{i\omega t} \langle [I_n(t), I_a(0)] \rangle = 2\hbar \omega g_a, \quad \alpha = a, s.
\]
\(g_a = g_t\) is the source-drain differential conductance of the amplifier. \(I_a\) is the unperturbed current signal (i.e. the source current in the absence of coupling to the amplifier). \(I_n\) is the current that would flow out of the amplifier if the load resistor is replaced by a short [4]. The impedance matching implies that \(I_s = 2I_n\) and \(I_a = 2I_{out}\). Denoting \(I(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega I(t)e^{-i\omega t}\), and
\[
\tilde{I} (\omega_0) = \int_{-\infty}^{\infty} + \frac{1}{2} \Delta \omega I(\omega) e^{-i\omega t} d\omega
\]
and using Eqs. (1), (3), and the fact that \(I_m\) and \(I_N\) commute, one has
\[
\langle [I_N(\omega_0), \tilde{I}_N(\omega_0)] \rangle \equiv -\frac{n}{2} G^2 \hbar \omega_0 g_t \Delta \nu.
\]
Subtracting Eq. (5) written for \(\gamma > 0\) from itself written for \(\gamma = 0\) and neglecting terms higher order than \(\gamma^2\) one obtains
\[
\langle [I_N(\omega_0), \tilde{I}_N(\omega_0)] + h.c. \rangle = -\pi G^2 \hbar \omega_0 g_t \Delta \nu.
\]
Written as an expectation value of a commutator, it follows that
\[
\langle [I_N(\omega_0), \tilde{I}_N(\omega_0)] \rangle + \langle [I_0(\omega_0), \tilde{I}_0(\omega_0)] \rangle = -i \pi G^2 \hbar \omega_0 g_t \Delta \nu,
\]
this leads to the uncertainty relation Eq. (4).

We now present a noise minimization procedure aimed at obtaining the two equalities in Eq. (3) in devices in which the Heisenberg limit is achievable. This procedure requires certain practical conditions to hold, the main one being that the coupling \(\gamma\) between the signal source and the transistor gate can be smoothly controlled over a wide range of values. It is also taken for granted that the source-drain bias voltage \(V\) is well controlled. The control of the coupling can be achieved, for example, by a control of the gate capacitance. The procedure involves only the knowledge of measurable quantities - there is no need to calculate in advance the \(V\) and \(\gamma\) dependence of the noise. The procedure consists of two simple steps which we refer to as noise balancing and gain matching. In the first step, one varies the coupling and the bias voltage until they reach two values, \(\gamma_1\) and \(V_1\) where the two types of noise reach the same value:
\[
\Delta I_0^2(V_1, \gamma_1) = \Delta I_n^2(V_1)
\]
The functional dependence of the idling-noise on \(V\) and \(\gamma\) differs from that of the amplified back-action noise (e.g., \(I_0 \sim \gamma^0\) while \(I_n \sim \gamma^2\)). Equating the two types of noise should therefore be possible by varying either \(\gamma\) or \(V\). The variation of both (and of other controllable parameters) is in general necessary in order to maintain the linearity of the amplifier. The noise balancing does not imply noise minimization and the total noise may even increase during this step. In order to describe the step that follows noise balancing, two power gains are defined: The first, the signal power gain \(G_1^2(V_1, \gamma_1)\), is determined by a direct gain measurement. The second, the noise power gain \(G_2^2(V_1)\), is calculated using the relation:
\[
\Delta I_0^2(V_1) = G_2^2(V_1) \frac{\hbar \omega_0}{4} g_t \Delta \nu.
\]
\(\frac{\hbar \omega_0}{4} g_t \Delta \nu\) is half the power delivered by the zero point fluctuations of the amplifier to the load. Therefore, \(G_2^2\) is the idling-noise referred to this power. The second step consists of matching the two gains by varying the bias voltage and the coupling until \(G_2^2(V) = G_2^2(V, \gamma)\). This should be done while maintaining the condition
\[
\gamma G(\gamma, V) = \text{const.}
\]
If $G$ (as is often the case) $\sim \gamma V$, Eq. (8) means that the gain matching is performed while keeping the product of $\gamma^2$ and the voltage constant: $\gamma^2 V = \gamma^2 V \text{1.}$ Eq. (8) ensures that the gain matching is performed while keeping the idling-noise and amplified back-action noise balanced as in Eq. (9) and therefore, the condition given by Eq. (9) (and thus also the Heisenberg limit) is achieved.

It remains to explain why the condition Eq. (9) ensures that the two types of noise remain equal while the gains are matched. For this, we consider the origin of the amplified back-action noise. Due to the linear coupling, a current fluctuation of order $\Delta I_0$ in the transistor induces a fluctuation of order $\Delta I_0 \Delta I_0$ in the signal source. This fluctuation is amplified and contributes a noise power $\sim \gamma^2 G^2 \Delta I_0^2$ to the output signal. This extra noise is the amplified back-action, $\Delta I_0^2$. Thus,

$$\frac{\Delta I_n^2}{\Delta I_0^2} \sim \gamma^2 G^2,$$

(9)

which means that the ratio of the idling-noise and amplified back-action noise remains constant if $\gamma^2 G^2$ does.

A typical example is where the idling noise is a shot-noise i.e., it results from the partitioning of charges between the two sides of a tunnelling barrier in the source-drain current path. The transfer of a fraction of this noise into the signal source stems from transitions enabled by the appearance of new scattering channels in the presence of the signal source where passing electrons transfer a quantum of $\hbar \omega_0$ to the signal source. The total contribution of these processes is proportional to the number of electrons in the transistor which can participate in such transitions. At zero temperature, and if $\hbar \omega_0 \ll eV$, all electrons in the nonequilibrium energy window created by the voltage $V$ may undergo such transitions and therefore the number of these transitions is $\sim V$. Thus, the power emitted into the source is $\sim \gamma^2 V$. After amplification, the contribution of these additional fluctuations in the signal current, is $\Delta I_n^2 \sim \gamma^2 V G^2$. On the other hand, the (low frequency) shot-noise power is $\Delta I_0^2 \sim \hbar^2 \omega_0 \gamma^2 V$. These two estimates confirm Eq. (9).

We now illustrate our results for the specific case of a signal amplified by a resonant transistor coupled capacitatively to a continuum of LC resonators (quantum harmonic oscillators) that models a resistive signal source. The model is similar in many features to those analyzed in Refs. 8, 9, 17. The total Hamiltonian is

$$H_{\text{tot}} = \sum_{i=1,2} \int_0^\infty d\omega \omega_i (\hat{a}_i^\dagger \hat{a}_i + h \omega_A \hat{b}_i^\dagger \hat{b}_i + \frac{\hbar}{2\pi} \int d\omega \omega_i \hat{b}_i^\dagger \hat{b}_i)$$

$$+ \frac{A^\dagger \omega \hat{Q}_s}{C_g} + \int_B d\omega \hbar \omega \gamma V (\hat{a}(\omega) + \hat{a}^\dagger(\omega))$$

(10)

where $\hat{Q}_s = \Delta Q(\omega_0) \int_B d\omega \omega (a(\omega) + a^\dagger(\omega))$ is the total charge on the capacitors in the LC oscillators and where $B = [\omega_0 - \Delta \omega / 2, \omega_0 + \Delta \omega / 2]$. The $b_i$’s, $A$’s and $a(\omega)$’s satisfy respectively continuous fermionic, discrete fermionic and continuous bosonic commutation relations. $b_i$ annihilates an electron in bath $i = 1, 2$. $A$ annihilates an electron in the resonance level which is located at energy $\hbar \omega_A$. $k(\epsilon)$ is the tunnelling amplitude between the baths and the resonance level. $\epsilon$ is the single-electron energy. $k^2(\epsilon)$, which is the resonance width, is taken to be wider than $eV$ so that the second derivative of the transmission with respect to $\epsilon$, (but not the first), can be neglected. It is also assumed that $k^2(\epsilon)$ is small compared to $\hbar \omega_A$ and the Fermi energy. $C_g$ is the gate capacitance of the amplifier and $\Delta Q(<\omega)$ is the typical charge fluctuation in one of the oscillators in its ground state, $\Delta Q = \sqrt{\hbar \omega_A C / 2}$ where $C$ is the capacitance in each one of the LC circuits. Denoting the coupling constant by $\gamma \equiv e \Delta Q / (C_g k^2)$ and assuming $\gamma \ll 1$, the coupling term in $H_{\text{tot}}$ can be written as $A^\dagger A \hat{Q}_s / C_g = k^2 A^\dagger A \hat{Q}_s / \Delta Q$ which plays the role of $\gamma B_{a,s}$ above. The principle of operation of this transistor amplifier is the following: the signal modulates the position of the resonant level and hence the transmission. In the classical picture this modulates the output current. In the quantum picture, this creates inelastic components for the transmitted electrons which lead to a structure (proportional to the square of a large bias voltage) mirroring the signal power spectrum in the output current power spectrum.

The transistor is taken to be in a zero-temperature stationary state with bath 1 and 2 having chemical potentials $\mu + eV$ and $\mu$ and thus occupation numbers $n_1(\epsilon) = \Theta(\epsilon) \Theta(\mu + eV - \epsilon)$ and $n_2(\epsilon) = \Theta(\epsilon) \Theta(\mu - \epsilon)$. The transistor current operator is defined by the rate of change in the charge of the two baths: $I_n(t) = \frac{1}{2} (\hat{Q}_1(t) - \hat{Q}_2(t))$

(11)

where $Q_i(t) = \int_0^\infty d\epsilon b_i^\dagger(\epsilon, t) b_i(\epsilon, t)$ is the total charge in bath $i$. Solving the Heisenberg equations of motion to second order in $\gamma$ we find (recall: $I_{\text{out}} \equiv \frac{1}{2} I_n$)

$$I_{\text{out}}(t) = I_0(t) + G \int_B \frac{\gamma t}{g_s} \tilde{I}_{\text{in}}(t) + I_n(t) + O(\gamma^3)$$

(12)

where $\tilde{I}_{\text{in}} \equiv \frac{1}{2} \omega_0 Q_s(t)$ and

$$I_0(t) = \frac{e}{4\pi} \int_{\pm B} d\omega e^{-i\omega t} \int_{-\infty}^\infty d\omega' \times$$

$$\left( t^* (\omega') b_1^\dagger (\omega') b_- (\omega' + \omega) + t (\omega') b_1^\dagger (\omega' - \omega) b_+ (\omega'), \right)$$

(13)

is the $\gamma = 0$ current, $\pm B = [-\omega_0 - \Delta \omega, -\omega_0 + \Delta \omega] \cup [\omega_0 - \Delta \omega, \omega_0 + \Delta \omega]$, $b_+ = \frac{1}{\sqrt{2}} (b_1 + b_2)$, $b_- = \frac{1}{\sqrt{2}} (b_1 - b_2)$,

$$G = \frac{e V}{\hbar \omega_0} \sqrt{2(1 - T)},$$

(14)
implies that \( \gamma G \) not be operating at the Heisenberg limit. To achieve this satisfying the constraint Eq. (2) as an equality, may still be satisfied as an equality, may still be satisfied as an equality, and \( \tilde{e} \) one sees that the actual device performs linear amplification of \( I_s \) instead of \( I_s = Q_s \). This is a consequence of the capacitive coupling \( H_{an} = eA/\Delta Q_s/C_{gs} \). However, Eq. (1) is valid also for \( I_s \) and so are all the above results - the only modification one needs to apply is the replacement of \( g_s \) by \( \tilde{g}_s \) as done in Eq. (12).

Eq. (13) implies that a large gain, \( G^2 \gg 1 \), requires a stronger assumption than \( eV \gg \hbar \omega \) namely, \( eV \gg \hbar \omega \gamma^{-1} \). We also note that when solving the Heisenberg equations, the coefficient before \( Q_s \) in Eq. (12) turns out to be an operator, \( \tilde{G} \) (Eq. 2, with \( G \rightarrow \langle \tilde{G} \rangle \) is still valid in this case). However, for a narrow bandwidth signal, \( \hbar \Delta \omega \ll eV \), the quantum fluctuations of this operator are negligible \( \Delta G^2 \ll \langle G^2 \rangle \equiv G^2 \). This allows us to replace it by its expectation value.

From Eqs. (12) - (14) one obtains the idling-noise:

\[
\Delta I_0^2 = T(1 - T)\frac{e^3 V}{4\pi \hbar} \Delta \nu, \tag{15}
\]

A lengthier calculation yields the amplified back-action noise

\[
\Delta I^2 = \frac{\gamma^4}{4}T^5(1 - T)\frac{e^3 V}{\hbar \omega} \frac{2\hbar}{\pi \hbar} \Delta \nu. \tag{16}
\]

One also finds that these noise sources are indeed uncorrelated \((I_n I_0) = 0\). Eqs. (15), (16) and (17) yield

\[
\Delta I_0(t) \Delta I_n(t) = \frac{1}{4}G^2\hbar \omega g s \Delta \nu, \tag{17}
\]

Eqs. (15), (16) and (17) demonstrate how an amplifier satisfying the constraint Eq. (12) as an equality, may still not be operating at the Heisenberg limit. To achieve this limit, the noise balancing should be performed. Equating \( \Delta I_0^2 = \Delta I^2 \) yields the condition

\[
\frac{\gamma^2 eV}{\hbar \omega} T^2 = 1. \tag{18}
\]

By Eqs. (8), (7), and (11), any pair of \( \gamma \) and \( V \) satisfying Eq. (18) results in performance at the Heisenberg limit (i.e., here, \( G_N = G \)). By Eq. (14), Eq. (18) implies that \( \gamma G = \text{const} \), confirming Eq. (10). To identify all possible values for the gain at the Heisenberg limit, \( G_H \), we insert Eq. (18) into Eq. (14) and find \( G(H) = \sqrt{2(1 - T)/\gamma T} \). One should recall the assumption that the second derivative of the transmission vanishes which is strictly true only when \( T = 3/4 \). Thus,

\[
G(H) = \frac{2\sqrt{\gamma}}{3}. \tag{19}
\]

To summarize, we presented a practical procedure for finding the region in parameter space where transistor amplifiers achieve the optimum noise performance allowed by quantum mechanics for linear phase insensitive amplifiers. The procedure should be experimentally feasible for linear devices for which such a parameter region exists even if the precise Hamiltonian of the device is unknown. We then verified the validity of this procedure in the case of a resonant barrier transistor amplifier coupled to a resistive signal source modelled as a continuum of LC resonators.

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The transmission amplitude at energy \( E = |t| \), \( I_n(t) \) is the amplified back action noise current the explicit expression for which will not be given here. Note that Eq. (12) is an operator input-output relation and therefore enables one to calculate expectation values of any function of \( I_n \). \( g_s = T e^2/2\pi \hbar \) and \( g_s = \pi \Delta Q^2/\hbar \). \( g_s \) is the differential linear response of the "current" \( \tilde{I}_s = \omega_0 Q_s \). Comparing Eqs. (11) and (12) one sees that actually the device performs linear amplification of \( I_s \) instead of \( I_s = Q_s \). This is a consequence of the capacitive coupling \( H_{ac} = eA/\Delta Q_s/C_{gs} \). However, Eq. (1) is valid also for \( I_s \) and so are all the above results - the only modification one needs to apply is the replacement of \( g_s \) by \( \tilde{g}_s \) as done in Eq. (12).
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