Bayesian Zero-Shot Learning
Sarkhan Badirli\textsuperscript{1}, Zeynep Akata\textsuperscript{2}, and Murat Dundar\textsuperscript{3}
\textsuperscript{1}Purdue University
\textsuperscript{2}University of Amsterdam
\textsuperscript{3}Indiana University Purdue University Indianapolis

Abstract

Object classes that surround us have a natural tendency to emerge at varying levels of abstraction. We propose a Bayesian approach to zero-shot learning (ZSL) that introduces the notion of meta-classes and implements a Bayesian hierarchy around these classes to effectively blend data likelihood with local and global priors. Local priors driven by data from seen classes, i.e. classes that are available at training time, become instrumental in recovering unseen classes, i.e. classes that are missing at training time, in a generalized ZSL setting. Hyperparameters of the Bayesian model offer a convenient way to optimize the trade-off between seen and unseen class accuracy in addition to guiding other aspects of model fitting. We conduct experiments on seven benchmark datasets including the large scale ImageNet and show that our model improves the current state of the art in the challenging generalized ZSL setting.

1 Introduction

Natural images exhibit power-law property; hence, in a randomly sampled training set, no training examples are expected to be available for most of the object categories [20, 10, 27]. This restriction becomes more evident in a fine-grained object recognition task. Zero-shot learning (ZSL), which considers training and test classes, i.e. seen and unseen classes, as two disjoint sets, was introduced to mitigate this limitation [11, 17]. The two groups of classes are linked through a shared set of attributes, that characterize high level semantic descriptions of all classes. During the training phase, a mapping between examples of seen classes and their corresponding class-based attributes is learned. This mapping is later used to identify examples of unseen classes during the test phase.

The standard ZSL setting restricts test time search space to only unseen classes. This somewhat unrealistic stipulation was later relaxed in the generalized ZSL (GZSL) setting to include all classes during test phase [24]. In GZSL, side information, i.e. attributes, are as important as perceptual representation of images. Attribute vectors are either manually annotated [11, 5] or derived from free-form text using word embedding [25, 6, 13]. Early line of work in ZSL [11] assumes attribute independence and uses probabilistic classifiers to assign images to test classes.

In this paper, we tackle ZSL by introducing a two layer Bayesian hierarchy manifesting over both seen and unseen classes. Our approach is designed to leverage the implicit hierarchy present among classes, especially evident in fine grained data sets [29, 15, 18]. Unlike earlier approaches, which seek to optimize an embedding between image and
semantic spaces, the proposed approach assumes that there are latent classes that define the class hierarchy in image space and uses semantic information to build the Bayesian hierarchy around these meta-classes.

Our model uses two types of Bayesian priors: global and local. As the name suggests global priors are shared across all classes whereas local priors are only shared among semantically similar classes, which are identified based on the distances between attribute vectors in the Euclidean space. Unlike standard Bayesian models where the posterior predictive distribution establishes a compromise between prior and likelihood, our approach utilizes posterior predictive distributions to reconcile information about local and global priors as well as likelihood to more effectively accommodate the class hierarchy. In this framework, unseen classes are represented by their corresponding meta classes (see Figure 1) and test samples are classified based on posterior predictive likelihoods computed for both seen and unseen classes. Our approach achieves significant improvements on both seen and unseen class accuracies to achieve the best results on a variety of benchmark datasets among the currently published state of the art methods.

Our contributions are as follows. (1) We propose a hierarchical Bayesian model based on the intuition that actual classes originate from their corresponding local priors, each defined by a meta-class of its own. (2) We derive the posterior predictive distribution (PPD) for a two-layer Gaussian mixture model to effectively blend local and global priors with data likelihood. These PPDs are used to implement a maximum-likelihood classifier, which represents seen classes by their own PPDs and unseen classes by meta-class PPDs. (3) Across seven datasets with varying granularity and sizes, we consistently improve upon the state of the art in GZSL settings.

2 Related Work

In this section, we discuss the prior work on zero-shot learning and hierarchical generative models related to ours.

Zero/Few-Shot Learning. There has been an increasing interest in classifying fine-grained and large-scale image datasets [18, 29, 15, 20]. This, in turn, led to a surge of interest in ZSL as labeling them is extremely costly. [11] tackles ZSL by implementing a probabilistic classifier for each attribute and then classifying test cases by aggregating attribute probabilities for each class. This approach treats attributes as independent, which is a fairly strong assumption for most real-world data sets. This work was followed by a large body of work that seeks to optimize a mapping from image space, i.e., feature vectors, onto semantic space, i.e., attribute vectors. This line of work can be categorized into two according to whether the mapping is bilinear [6, 9, 19, 1, 2] or non-linear [25, 30]. Related to ours, [34, 16, 22] first maps image and semantic space into an intermediate space and represents unseen classes as a mixture of seen classes. Besides these mainline ZSL studies, a recent study evaluates an extended version of a few-shot learning algorithm for ZSL [26]. This approach learns a deep metric to query images with few shot samples. Extension to ZSL is achieved by replacing few-shot samples with one-shot class attribute vectors.

Generative models for zero-shot learning. Although most of the early work focused on discriminative models there are a few studies that use generative models to tackle ZSL [14, 28]. The study in [14] uses Normal distributions to model both image features and semantic vectors and learns a multimodal mapping between two spaces. This mapping is optimized by minimizing a similarity based cross-domain loss function. In a similar fashion the study in [28] utilizes a regression model to optimize a mapping between class attributes and parameters of class conditional distributions. A comprehensive review of these techniques and their performance on several benchmark data sets can be found in [31].

Bayesian models. In this paper, we offer a hierarchical Bayesian perspective on ZSL as a promising alternative to earlier approaches. Although hierarchical Bayesian mixture models have been previously explored for a variety of clustering problems [8, 3, 4, 33] their extension to ZSL comes with two distinct features that could help the proposed model prevail over the large body of early work in ZSL. First, as a Bayesian model ours offers a systematic approach to sharing information between seen and unseen classes as well as within each group through the utilization of local and global priors. Global priors are defined by hyperparameters whereas local priors are defined by the parameters of the meta classes, which are estimated from corresponding seen classes. Second, as a hierarchical model it can better accommodate data sets with different levels of class abstraction, i.e., fine-grained vs. coarse-grained data sets, which is particularly appealing for large-scale classification. A hierarchical Bayesian model was previously studied in a one-shot learning setting [21]. Our proposed approach differs from this model in two essential aspects. First, unlike our proposed approach, no semantic information was used when establishing the Bayesian hierarchy in [21] and class discovery was performed in a fully unsupervised fashion. Second, our approach introduces the notion of local prior, which becomes highly instrumental in defining meta-classes and modeling dispersion of classes.

Our work. Unlike the vast majority of early work, which seeks to optimize a mapping between image features and attribute vectors our approach readily models class distributions in the feature space by exploiting both local and global priors defined over the parameters of these distributions. Local priors are defined by meta-classes. In the proposed approach attribute vectors only come into play when determining meta-class memberships of actual classes. Classes with similar attribute vectors are pooled together to derive
local priors.

3 Bayesian Zero Shot Learning

Bayesian classification places a shared prior over the parameters of class distributions, which are assumed to be generated independently conditioned on the prior. Imposing the same prior across all classes creates dependencies among them, enabling information propagation and regularization at the same time during model inference. However, in real-world applications the classes are often not generated independently, indeed for a large number of classes different levels of abstraction is expected. On the other hand, availability of semantic side information suggests that there is a deeper level of hierarchy among existing classes than a single global Bayesian prior can explain.

Images from semantically similar classes are embedded close to each other due to their shared latent parameters. When such similarities are not accounted for in the classification model, sample estimates of class parameters derived based on independence assumption among classes become nullified. In other words, knowing the parameters of the global prior may not be sufficient for achieving independence as latent parameters define deeper level hierarchical relationships among classes. Our model resolves this problem by introducing a layer of meta-classes between global prior and actual classes, paving the way for independence and enabling information sharing and propagation across classes.

3.1 Generative Model

Our approach to ZSL employs class similarities by a two layer generative model. As shown in Figure 2, our model identifies meta-classes that determine groupings among classes. These meta-classes play a key role by acting as a local prior for individual classes, i.e. both seen and unseen classes that belong to the same meta-class inheriting the same local prior. In our framework, the data points with the same local prior that do not belong to any of the seen classes can be considered from unseen classes. If the class groupings can be arranged such that there is only one unseen class associated with each local prior then unseen classes can be uniquely identified. Associating each unseen class with a different local prior forms the basis of our approach.

Our generative model is designed as follows:

\[
\begin{align*}
    x_{jik} &\sim N(\mu_{ji}, \Sigma_j) \\
    \mu_{ji} &\sim N(\mu_j, \Sigma_j \kappa^{-1}) \\
    \mu_j &\sim N(\mu_0, \Sigma_j \kappa_0^{-1}) \\
    \Sigma_j &\sim W^{-1}(\Sigma_0, m) \quad (1)
\end{align*}
\]

with the meta-class index \( j \), the actual class index \( i \), the image index \( k \). We assume that images \( x_{jik} \) come from a Gaussian with mean \( \mu_{ji} \) and covariance matrix \( \Sigma_j \). They are generated independently conditioned not only on the global prior but also on their corresponding meta-class.

Each meta-class is characterized by the parameters \( \mu_j \) and \( \Sigma_j \). \( \mu_0 \) is the mean of the Gaussian prior defined over the mean vectors of meta-classes, \( \kappa_0 \) is a scaling constant that adjusts the dispersion of the centers of meta classes around \( \mu_0 \). A smaller value for \( \kappa_0 \) suggests that class centers are expected to be farther apart from each other whereas...
a larger value suggests they are expected to be closer to each other. On the other hand, $\Sigma_0$ and $m$ dictate the expected shape of the class distributions, as under the inverse Wishart distribution assumption the expected covariance is $E(\Sigma|\Sigma_0, m) = \frac{\Sigma_0 m}{m+D}$, where $D$ is the dimension of image feature space. The minimum feasible value of $m$ is equal to $D + 2$, and the larger the $m$ is the less individual covariance matrices will deviate from the expected shape.

On the other hand, $\kappa_1$ is a scaling constant that adjusts the dispersion of the actual class means around their corresponding meta-class means. A larger $\kappa_1$ leads to smaller variations in class means compared to the mean of their corresponding meta classes, suggesting a fine-grained relationship among classes sharing the same meta-class. On the other hand, a smaller $\kappa_1$ dictates coarse-grained relationships among classes sharing the same meta-class. In this model, classes with the same meta-class also share the same covariance matrix $\Sigma_j$ to preserve conjugacy of the model.

To classify test examples, we need the posterior predictive distributions (PPD) of seen and unseen classes which we will explain next. More details about the derivation are provided in the supplementary.

### 3.2 Posterior Predictive Distribution

In our model, the posterior predictive distribution (PPD) incorporates three sources of information: the data likelihood that arises from the current class, the local prior that results from other classes sharing the same meta class as the current class, and global prior defined in terms of hyperparameters. The derivation in six steps are outlined in Figure 2(b) and Algorithm 1 describes a pseudo code on deriving PPD for both seen and unseen classes. Class sufficient statistics are summarized by $\bar{x}_{ji}$, $S_{ji}$ and $n_{ji}$ which represent sample mean, scatter matrix and size of class $i$ of meta-class $j$, respectively. The notations $\omega_{jc}$ and $\omega_i$ used in the Algorithm 1 represents the current seen class and unseen class, whose PPD is being derived.

In step 1, we establish the link between class sample mean $\bar{x}_{ji}$ and its corresponding meta-class mean $\mu_j$ by marginalizing out the intermediate class mean $\mu_{ji}$. As all of these are Gaussians, this marginalization yields a Gaussian:

$$P(\bar{x}_{ji}|\mu_j, \Sigma_j, \kappa_1) = N(\bar{x}_{ji}|\mu_j, \Sigma_j(\frac{1}{n_{ji}} + \frac{1}{\kappa_1}))(2)$$

In step 2, we use Bayes rule to obtain the posterior distribution of the meta-class mean vector $\mu_j$:

$$P(\mu_j|\mu_0, \Sigma_j, \kappa_0, \kappa_1, \{\bar{x}_{ji}\}_{t_j=j}) = N(\mu_j|\tilde{\mu}_j, \tilde{\Sigma}_j^{-1}\Sigma_j)$$

$$\tilde{\mu}_j = \frac{\sum_{i:t_j=j} \frac{n_{ji}\kappa_1}{n_{ji}+\kappa_1} \bar{x}_{ji} + \kappa_0 \mu_0}{\sum_{i:t_j=j} \frac{n_{ji}\kappa_1}{n_{ji}+\kappa_1} + \kappa_0} \tag{3}$$

$$\tilde{\kappa}_j = (\sum_{i:t_j=j} \frac{n_{ji}\kappa_1}{n_{ji}+\kappa_1} + \kappa_0) + \kappa_0$$

where $t_j$ is the meta-class indicator for class $i$. Note that the mean $\tilde{\mu}_j$ is the weighted average of the prior mean and class means share the same meta-class.

In step 3, we obtain the local prior for the class mean vector $\mu_{jc}$ by propagating the information from other classes sharing the same meta-class as the current class $c$. This is achieved by integrating out the meta-class mean vector $\mu_j$.

$$P(\mu_{jc}|\mu_0, \Sigma_j, \kappa_0, \kappa_1, \{\bar{x}_{ji}\}_{t_j=j}) = N(\mu_{jc}|\tilde{\mu}_j, \Sigma_j(\tilde{\kappa}_j^{-1} + \kappa_1^{-1})) \tag{4}$$

In step 4, we derive the posterior of the current class mean vector $\mu_{jc}$ by combining current class sample mean $\bar{x}_{jc}$ from step 1 and the local prior from step 3.

$$P(\mu_{jc}|\mu_0, \Sigma_j, \kappa_0, \kappa_1, \{\bar{x}_{ji}\}_{t_j=j}, \bar{x}_{jc}) = N(\mu_{jc}|\frac{n_{jc}\bar{x}_{jc} + \kappa_0 \tilde{\mu}_j}{n_{jc} + \kappa_0}, \Sigma_j(\tilde{\kappa}_j^{-1} + \kappa_0^{-1}))$$

$$\tilde{\kappa}_j = (\sum_{i:t_j=j} \frac{n_{ji}\kappa_1}{n_{ji}+\kappa_1} + \kappa_0) + \kappa_0 + \kappa_1 \tag{5}$$

In step 5, we derive the posterior distribution of the covariance matrix $\Sigma_j$ by combining the local prior of the covariance matrix $P(\Sigma_j|\Sigma_0, m)$ with the distribution of the scatter matrices of the classes associated with meta-class $j$.
$S_{ji}$ and current class $S_{jc}$:
\[
P(\Sigma_j \{ S_{ji} \}_{i:j}, S_{jc}) = IW(\Sigma_j \bar{S}_c, m + \sum_{i:i=j} (n_{ji} - 1) + n_{jc})
\]
\[
\bar{S}_c = \sum_{i:i=j} S_{ji} + S_{jc} + S_{\mu}
\]
\[
S_{\mu} = \frac{n_{jc} \bar{S}_c}{\tilde{k}_j + n_{jc}}
\]

In step 6, we derive the posterior predictive distribution by integrating out meta-class mean vector $\mu_j$ and covariance $\Sigma_j$ in the form of a Student-t distribution as follows.
\[
P(x|\{ \bar{x}_{ji}, S_{ji} \}_{i:j}, \bar{x}_{jc}, S_{jc}, \mu_0, \kappa_0, \kappa_1) = T(x|\bar{\mu}_{jc}, \bar{\Sigma}_{jc}, \bar{v}_{jc})
\]
\[
\bar{\mu}_{jc} = \frac{n_{jc}\bar{x}_{jc} + \tilde{k}_j \bar{\mu}_j}{n_{jc} + \tilde{k}_j}
\]
\[
\bar{v}_{jc} = n_{jc} + \sum_{i:i=j} (n_{ji} - 1) + m - D + 1
\]
\[
\bar{\Sigma}_{jc} = \frac{\Sigma_0 + \sum_{i:i=j} S_{ji} + S_{jc} + S_{\mu}}{(n_{jc} + \tilde{k}_j + 1)}
\]  
(7)

where, $\bar{\mu}_j$, $\tilde{k}_j$ and $S_{\mu}$ are defined as in Equation (3), (5) and (6) respectively. The index $c$ in Equation (7) represents the current seen class, whose PPD is being derived. Top $K$ most similar seen classes are identified as the ones with the smallest Euclidean distance to the current class in the attribute space. If the current class is a seen class, PPD takes the form in Equation (7). When it is an unseen class with no images available in training, the sample statistics of the current class in (7) drops and PPD becomes:
\[
P(x|\{ \bar{x}_{ji}, S_{ji} \}_{i:j}, \mu_0, \kappa_0, \kappa_1) = T(x|\bar{\mu}_j, \bar{\Sigma}_j, \bar{v}_j)
\]
\[
\bar{v}_j = \sum_{i:i=j} (n_{ji} - 1) + m - D + 1
\]
\[
\bar{\Sigma}_j = \frac{(\Sigma_0 + \sum_{i:i=j} S_{ji}) (\tilde{k}_j + 1)}{\tilde{k}_j \bar{v}_j}
\]  
(8)

where $\bar{\mu}_j$ and $\tilde{k}_j$ are defined as in Equation (3) and (5), respectively. In this setting, a new image is labeled by evaluating PPDs for seen and unseen classes and assigning the image to the class that generates the maximum likelihood.

### 3.3 Meta-class Formation

Meta-class for each unseen class is formed by finding $K$ most similar seen classes to the current unseen class using $\mathcal{L}^2$ distance between the attribute vectors $\phi$ of that unseen class and of seen classes. In the case of tie, the least similar class among selected seen classes ($K^{th}$) is replaced by the next one until tie is broken. These define a local prior in the PPD of the unseen class. Meta-class formation for a seen class follows the same procedure. We use the $\mathcal{L}^2$ distance between the current seen class attribute and other seen class attributes to find $K$ most similar classes. As we have access to seen class samples, the PPD of the seen class (Equation 7) uses class samples in addition to local and global priors from its meta-class. An illustration for the formation of the meta-class associated with an unseen class blue whale, from AWA dataset, is shown in Figure 1. $\phi$(blue whale) is compared against $\phi$(seen) in the semantic space, humpback and killer whale are identified as the two closest matches. Using humpback and killer whale class samples, the meta-class for blue whale is formed as a local prior in the PPD for blue whale.

### 4 Experiments

We evaluate the performance of the proposed approach on several benchmark data sets and compare the results with the current state of the art in ZSL.

**Datasets & specifications.** Experiments are evaluated on ZSL datasets widely used for benchmarking. Among those, CUB [29], FLO [15] and SUN [18] are medium scale, fine-grained datasets. AWA1 [12] and AWA2 [32] and aPY [5], on the other hand, are coarse-grained datasets. Finally we evaluate our model on ImageNet [20] with more than 14 million images and 21K classes. SUN, AWA1, AWA2, aPY and CUB datasets come with visual attributes whereas FLO uses sentences and ImageNet uses word embeddings as class vectors. We use the publicly available image embeddings of [32], i.e., 2048-dimensional top-layer pooling units of the 101-layered ResNet [7] as feature vectors. Additional information about each dataset including the number of images, number of attributes, and sizes of train, validation, and test class splits are present in Table 1.

| Dataset     | #imgs | Type | #att | $|Y^{all}|$ | $|Y^s|$ | $|Y^u|$ |
|-------------|-------|------|------|------------|--------|--------|
| FLO         | 8,189 | fine | 102  | 102        | 62 + 20| 20     |
| SUN         | 14,340| fine | 102  | 102        | 580 + 65| 72     |
| CUB         | 11,788| fine | 312  | 200        | 100 + 50| 50     |
| AWA1        | 30,475| coarse| 85   | 50         | 27 + 13| 10     |
| AWA2        | 37,322| coarse| 85   | 50         | 27 + 13| 10     |
| aPY         | 15,339| coarse| 64   | 32         | 15 + 5| 12     |
| ImageNet    | 14M   | large| 500  | 21K        | 1K     | 20K    |

Table 1: Specifications of all datasets used in our experiments. $|Y^{all}|$, $|Y^s|$, and $|Y^u|$ denote the number of classes in all, seen and unseen classes, respectively. To clarify the numbers in last 3 columns, we give an illustration on FLO dataset: FLO has total of 102 classes of which 62 are training, 20 are validation (both seen during training) and 20 are test classes (unseen during training).
For ImageNet following the benchmark in [32] we use all of the images from 1K classes, i.e. seen classes, for training so that we do not violate the zero-shot assumption as ResNet-101 [7] is trained on the same 1K classes from ImageNet. We evaluate the proposed technique in nine different configurations as proposed in [32], all of which differs according to how test class subsets are chosen.

**Evaluation criteria.** We use the same evaluation procedure employed in [32] as described below. The standard practice in ZSL literature is to evaluate classification performance by Top-1 accuracy. To avoid large classes dominating the overall accuracy, Top-1 accuracy is separately calculated for each class and the mean of individual class accuracies is used for evaluation. GZSL setting includes both seen and unseen classes in the test phase, hence the search space includes all the classes, i.e. $|Y^\text{all}|$. Hence, first seen and unseen class accuracies are separately computed and then their harmonic mean is used as the final score for evaluation. For ImageNet, the final score is the average Top-1 accuracy over the images of unseen classes (although the search space is still $|Y^\text{all}|$) as no images from seen classes are available during testing phase.

**Implementation details.** We implement two versions of our model: unconstrained (UBZSL) and constrained (CBZSL) Bayesian ZSL. For large data sets, e.g. ImageNet, our model in Eq.1 suffers from the large memory requirement due to the unconstrained structure of the class covariance matrices. To alleviate this problem we developed a scalable version of our model where the covariance matrices are constrained to have diagonal forms. The only difference between these two models is that constrained version uses an Inverse Gamma prior on the diagonal entries of the covariance matrix as opposed to an Inverse Wishart in the unconstrained version. With this revision the generative model in Eq.1 is updated as follows.

\[
x_{j,ik}^d \sim N(\mu_{ji}^d, \Sigma_{ji}^d)
\]

\[
\mu_{ji}^d \sim N(\mu_j^d, \Sigma_j^{-1})
\]

\[
\mu_j^d \sim N(\mu_0^d, \Sigma_j^{-1})
\]

\[
\Sigma_j^d \sim IG(a_0, b_0)
\]

where the superscript $d$ is added to refer to the $d^{th}$ component of each parameter. The Inverse Wishart parameters $m$ and $\Sigma_0$ are replaced with the scale ($a_0$) and shape ($b_0$) parameters of the Inverse Gamma distribution. The derivation of PPD for the constrained model is in the supplementary.

The hyperparameters of the model are coarsely tuned to maximize the harmonic mean score on the validation set for all datasets but ImageNet. The training, test and validation set splits for these datasets are done according to [32] to maintain a fair comparison. As hyperparameter tuning for ImageNet can be computationally unmanageable and to demonstrate the robustness of the model we used the hyperparameters of the SUN dataset for ImageNet. For CBZSL we utilize all 2048 ResNet features whereas for UBZSL we applied PCA to reduce the dimensionality to 500.

Both UBZSL and CBZSL have four hyperparameters: $\kappa_0, \kappa_1, m, s, K$. Here, K is the selected number of classes most similar to the current class in the attribute space. To simplify the parameter tuning process, we set prior mean, $\mu_0$, to the average of class means. We set $\Sigma_0$ to the average of class scatter matrices scaled by a constant $s$. Our implementation will be made publicly available upon the completion of the review process.

### 4.1 Model Evaluation

In this section, we evaluate our model through an ablation study and investigate the tradeoff between seen and unseen class accuracies.

**Model ablation.** Our model formulates zero-shot learning in the framework of hierarchical Bayes. Towards this end, we validate the necessity of each component in our model by eliminating one component at a time and investigating the performance of the model with remaining components on several benchmark datasets.

Our observations from Table 2 are as follows. (1) If we break the hierarchy by removing the meta-class layer, then actual classes are directly linked to the global prior and same PPD is assigned to all unseen classes. Thus, unseen
Table 3: GZSL results achieved by the proposed approach (CBZSL and UBZSL) along with results of several other techniques from the literature on SUN, CUB, FLO, AWA1, AWA2, aPY datasets. We measure per-class averages top-1 accuracy on seen classes (tr), unseen classes (ts) and their harmonic mean (H).

---

4.2 Comparison with State of the Art

Results obtained by the proposed CBZSL and UBZSL models on SUN, CUB, FLO, AWA1, AWA2, aPY datasets are presented in Table 3. In addition to all SoA techniques reported in [31] we also included results of a more recently published technique [26] in this comparison. These results
Arguably it constitutes the most natural setup to evaluate ZSL learning performance as it contains 22K classes (1K of which are used to train state of the art deep neural networks) and most of these classes are sparsely populated.

Table 4 summarizes ImageNet results under nine different test set configurations. Our unconstrained model (UBZSL) improves over the state of the art in 2/3 Hop and highly populated test classes. Of particular importance is the highly competitive performance by the constrained model (CBZSL) that improves the current state of the art in all test configurations with respect to Top-1 accuracy (3.9% vs 2.18% on 2Hop, 7.51% vs 2.9% on Mp500, 4.78% vs 2.34% on Mp1000). Our model achieves the best results in eight of the nine test configurations for Top-5 and seven of the nine for Top-10 accuracies. Especially in the least populated (Lp500) classes the accuracy improvement is four percentage points in Top-5 and Top-10 accuracies. In most populated classes (Mp500) the accuracy gets almost doubled, i.e. 25.20% vs 14.86% on Top-5.

These results show that as the number of classes and the average number of samples per class (1300 in ImageNet vs 700 in benchmark datasets) increase, the explicit hierarchy across classes becomes more evident leading to more informative local priors. ImageNet contains both coarse- and fine-grained classes. The results suggest that our technique can be equally effective on datasets with hybrid granularity.

## 5 Conclusion

In this study, we proposed a Bayesian approach to zero-shot learning (ZSL) that relies on the consideration that classes in real-world datasets emerge at different levels of abstraction and there are meta-classes that inherently organize class hierarchy in the semantic space. We introduced concepts of local and global priors and showed that knowledge transfer from seen classes to unseen ones can be effectively carried out in the image space by a two-layer GMM.

The proposed two-layer GMM offers extreme flexibility in modeling datasets with different characteristics through tuning of its hyperparameters, each of which models a different aspect of the data. We performed extensive experiments with benchmark datasets (fine-grained, coarse-grained and large-scale) to demonstrate the utility of the proposed Bayesian approach for ZSL, the results of which favor the proposed approach over other state-of-the-art ZSL techniques. On all six of the benchmark datasets, the unconstrained version of the proposed model outperformed the state-of-the-art techniques by a wide margin by achieving solid improvements especially in unseen class accuracies. On the large scale ImageNet dataset the constrained model proved to be more competitive than not only the unconstrained version of the proposed model but also the best competing state-of-the-art technique under a variety of test configurations.

Table 4: ImageNet results in nine different test phase configurations. Lp and Mp 500, 1K, 5K refer to least and most populated 500, 1000 and 5000 classes respectively. 2/3 Hop represents the classes that are 2/3-hops away from 1K training classes according to the ImageNet label hierarchy. Finally All appears for all 21K ImageNet classes. The results are in top-K accuracy.

| Split | UBZSL | CBZSL | SoA from [31] |
|-------|-------|-------|--------------|
|       | Top-1 | Top-5 | Top-10       |
| 2Hop  | 2.6   | 13.1  | 20.3         |
|       | 3.9   | 15.0  | 22.8         |
| 3Hop  | 0.8   | 4.1   | 6.9          |
|       | 1.0   | 4.1   | 6.9          |
| Lp500 | 1.8   | 5.1   | 8.6          |
|       | 2.5   | 10.2  | 14.3         |
| Lp1K  | 1.2   | 4.6   | 7.3          |
|       | 2.3   | 7.3   | 10.7         |
| Lp5K  | 0.5   | 2.0   | 3.5          |
|       | 0.6   | 2.4   | 4.0          |
| Mp500 | 3.4   | 17.4  | 26.5         |
|       | 7.5   | 25.2  | 35.0         |
| Mp1K  | 2.4   | 13.0  | 20.2         |
|       | 4.8   | 17.3  | 25.5         |
| Mp5K  | 1.1   | 6.1   | 9.9          |
|       | 1.5   | 6.6   | 10.5         |
| All   | 0.3   | 1.8   | 3.0          |
|       | 0.4   | 1.8   | 2.9          |
|       | 0.3   | 2.0   | 3.4          |

4.3 Large-Scale Experiments on ImageNet

ImageNet is currently the most challenging dataset for ZSL. Arguably it constitutes the most natural setup to evaluate...
References

[1] Z. Akata, F. Perronnin, Z. Harchaoui, and C. Schmid. Label-embedding for image classification. *TPAMI*, 2016. 2, 7

[2] Z. Akata, S. Reed, D. Walter, H. Lee, and B. Schiele. Evaluation of output embeddings for fine-grained image classification. In *CVPR*, 2015. 2, 7

[3] M. Dundar, F. Akova, H. Z. Yerebakan, and B. Rajwa. A non-parametric bayesian model for joint cell clustering and cluster matching: identification of anomalous sample phenotypes with random effects. *BMC Bioinformatics*, 15(1):314, 2014. 2

[4] M. Dundar, H. Z. Yerebakan, and B. Rajwa. Batch discovery of recurring rare classes toward identifying anomalous samples. In *SIGKDD*. ACM, 2014. 2

[5] A. Farhadi, I. Endres, D. Hoiem, and D. Forsyth. Describing objects by their attributes. In *CVPR*, 2009. 1, 5

[6] A. Frome, G. S. Corrado, J. Shlens, S. Bengio, J. Dean, M. A. Ranzato, and T. Mikolov. Deive: A deep visual-semantic embedding model. In *NIPS*, 2013. 1, 2, 7

[7] K. He, X. Zhang, S. Ren, and J. Sun. Deep residual learning for image recognition. In *CVPR*, 2016. 5, 6

[8] S. Kim and P. Smyth. Hierarchical dirichlet processes with random effects. In *NIPS*, 2007. 2

[9] E. Kodiroy, T. Xiang, and S. Gong. Semantic autoencoder for zero-shot learning. In *CVPR*, 2017. 2, 7

[10] A. Krizhevsky, I. Sutskever, and G. E. Hinton. Imagenet classification with deep convolutional neural networks. In *NIPS*, 2012. 1

[11] C. Lampert, H. Nickisch, and S. Harmeling. Learning to detect unseen object classes by between-class attribute transfer. In *CVPR*, 2009. 1, 2

[12] C. Lampert, H. Nickisch, and S. Harmeling. Attribute-based classification for zero-shot visual object categorization. *TPAMI*, 36(3):453–465, 2013. 5

[13] T. Mikolov, K. Chen, G. Corrado, and J. Dean. Efficient estimation of word representations in vector space. In *ICLR*, 2013. 1

[14] T. Mukherjee and T. Hospedales. Gaussian visual-linguistic embedding for zero-shot recognition. In *EMNLP*, 2016. 2

[15] M. Nilsback and A. Zisserman. Automated flower classification over a large number of classes. In *ICCVGI*, 2008. 1, 2, 5

[16] M. Norouzi, T. Mikolov, S. Bengio, Y. Singer, J. Shlens, A. Frome, G. Corrado, and J. Dean. Zero-shot learning by convex combination of semantic embeddings. In *ICLR*, 2014. 2

[17] M. Palatucci, D. Pomerleau, G. E. Hinton, and T. M. Mitchell. Zero-shot learning with semantic output codes. In *NIPS*, 2009. 1

[18] G. Patterson and J. Hay. Sun attribute database: Discovering, annotating, and recognizing scene attributes. In *CVPR*, 2012. 1, 2, 5

[19] B. Romera-Paredes and P. H. Torr. An embarrassingly simple approach to zero-shot learning. In *ICML*, 2015. 2, 7

[20] O. Russakovsky, J. Deng, H. Su, J. Krause, S. Satheesh, S. Ma, Z. Huang, A. Karpathy, A. Khosla, M. Bernstein, A. C. Berg, and L. Fei-Fei. Imagenet large scale visual recognition challenge. *IJCV*, 115(3):211–252, 2015. 1, 2, 5

[21] R. Salakhutdinov, J. Tenenbaum, and A. Torralba. One-shot learning with a hierarchical nonparametric bayesian model. In *JMLR workshop*, 2012. 2

[22] S. Changpinyo, W.-L. Chao, B. Gong, and F. Sha. Synthesized classifiers for zero-shot learning. In *CVPR*, 2016. 2, 7

[23] W. J. Scheirer and T. E. Boult. Statistical methods for open set recognition. In *CVPR Tutorial*, 2016. 8

[24] W. J. Scheirer, A. Rocha, A. Sapkota, and T. E. Boult. Towards open set recognition. *TPAMI*, 36, 2013. 1

[25] R. Socher, M. Ganjoo, C. D. Manning, and A. Ng. Zero-shot learning through cross-modal transfer. In *NIPS*, 2013. 1, 2

[26] F. Sung, Y. Yang, L. Zhang, T. Xiang, P. H. Torr, and T. M. Hospedales. Learning to compare: Relation network for few-shot learning. In *CVPR*, 2018. 2, 7, 8

[27] C. Szegedy, W. Liu, Y. Jia, P. Sermanet, S. Reed, D. Anguelov, D. Erhan, V. Vanhoucke, and A. Rabinovich. Going deeper with convolutions. In *CVPR*, 2015. 1

[28] V. K. Verm and P. Rai. A simple exponential family framework for zero-shot learning. In *ECML*, 2017. 2, 7

[29] P. Welinder, S. Branson, T. Mita, C. Wah, F. Schroff, S. Belongie, and P. Perona. Caltech-ucsd birds 200, 2010. Caltech, Technical Report, CNS-TR-2010-001. 1, 2, 5

[30] Y. Xian, Z. Akata, G. Sharma, Q. Nguyen, M. Hein, and B. Schiele. Latent embeddings for zero-shot classification. In *CVPR*, 2016. 2, 7

[31] Y. Xian, C. Lampert, B. Schiele, and Z. Akata. Zero-shot learning - a comprehensive evaluation of the good, the bad and the ugly. *TPAMI*, 2018. 2, 7, 8

[32] Y. Xian, C. H. Lampert, B. Schiele, and Z. Akata. Zero shot learning a comprehensive evaluation of the good, the bad and the ugly. In *CVPR*, 2017. 5, 6

[33] H. Z. Yerebakan, B. Rajwa, and M. Dundar. The infinite mixture of infinite gaussian mixtures. In *NIPS*, 2014. 2

[34] Z. Zhang and V. Saligrama. Zero-shot learning via semantic similarity embedding. In *ICCV*, 2015. 2