we propose a cosmological braneworld scenario in which two branes collide and emerge as reborn branes whose tensions have signs opposite to the original tensions of the respective branes. in this scenario, gravity on each of the branes is described by a scalar-tensor-type theory in which the radion plays the role of the gravitational scalar, and the branes are assumed to be inflating. however, the whole dynamics is different from those of the usual inflation, due to the non-trivial dynamics of the radion field. transforming the conformal frame to the einstein frame, this born-again scenario resembles the pre-big-bang scenario. thus, our scenario has features of both inflation and pre-big-bang scenarios. in particular, gravitational waves produced from vacuum fluctuations may have a very blue spectrum, while the inflaton field gives rise to a standard scale-invariant spectrum.

§1. introduction

the inflationary universe scenario is a natural solution to fundamental problems of the big-bang model, such as the horizon problem. however, it is not a unique choice. for example, a universe with an era of contraction is also a possibility. the pre-big-bang scenario is a realization of such a case in the superstring context. unfortunately, however, the pre-big-bang scenario suffers from the singularity problem, which cannot be solved without understanding the stringy non-perturbative effects.

one of the remarkable features of superstring theory is the existence of extra dimensions. conventionally, the extra dimensions are considered to be compactified to form a small compact space of the planck scale. however, recent revolutionary progress in string theory has lead to the brane-world picture. in this paper, we consider a system of two branes having tensions of opposite sign, with the intermediate spacetime (bulk) described by an anti-de sitter space (ads5). one of the branes is assumed to be our universe, and there exists an inflaton field that leads to inflation. the other brane is assumed to be vacuum, but with a non-zero cosmological constant (see fig. 1).

assuming the slow roll of the inflaton field, we can regard both branes as vacuum (de sitter) branes. hence, we analyze this case in detail. to this time, mostly the
static de-Sitter two-brane system has been considered in the cosmological context.\textsuperscript{8} However, it is now well-known that a static de-Sitter two-brane system is unstable.\textsuperscript{9} We therefore investigate the non-trivial radion dynamics and focus on its cosmological consequences (See the previous work on the radion dynamics given in Ref. 10). As a result, we find a new scenario of the braneworld, which we call the “born-again braneworld scenario”. We show that the two branes can collide without developing serious singularities, as seen from an observer on either brane, and emerge as reborn branes with the signs of the Lagrangians reversed. We find that our scenario has features common to both the conventional inflationary scenario and the pre-big-bang scenario. In a sense, we can regard it as a non-singular realization of the pre-big-bang model in the braneworld context. (See related works and criticism of them presented in Refs. 11 and 12.) In particular, a flat spectrum for the density perturbation is naturally produced, while background gravitational waves with a very blue spectrum are generated through the collision. It may be possible to detect this using future interferometric gravitational wave detectors.\textsuperscript{13}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Radion as the distance between two branes.}
\end{figure}

\section{Effective action}

We begin by reviewing the effective equations on a brane at low energies, which we derived in previous papers\textsuperscript{14, 15} (see also Refs. 16 and 17)). This effective theory is valid if the energy density on a brane is much smaller than the brane tension. Strictly speaking, we cannot use this action when two branes collide, because the junction conditions lose their meaning in this case. Indeed, the collision process is singular from the 5-dimensional point of view. However, this singularity is relatively mild, and the action is completely regular at the collision point. This leads us to assume that the collision process can be described by this effective action. This assumption is crucial for later analysis.

Our system is described by the action

\[
S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left( R + \frac{12}{\ell^2} \right) - \sum_{i=A,B} \sigma_i \int d^4x \sqrt{-g^i\text{-brane}}
\]
Born-Again Braneworld

\[ + \sum_{i=A,B} \int d^4x \sqrt{-g^i_{\text{brane}}} \mathcal{L}_{\text{matter}}, \]  

(2.1)

where \( \mathcal{R}, g^i_{\text{brane}} \) and \( \kappa^2 \) are the 5-dimensional scalar curvature, the induced metric on the \( i \)-brane, and the 5-dimensional gravitational constant, respectively. We consider an \( S_1/\mathbb{Z}_2 \) orbifold spacetime with the two branes as the fixed points. In the first Randall-Sundrum (RS1) model, the two flat 3-branes are embedded in AdS\(_5\) with the curvature radius \( \ell \) and the brane tensions given by \( \sigma_A = 6/(\kappa^2 \ell) \) and \( \sigma_B = -6/(\kappa^2 \ell) \). Then we have \( g^{A_{\text{brane}}}_{\mu\nu} = e^{2d/\ell} g^{B_{\text{brane}}}_{\mu\nu} \), where \( d \) is the distance between the two branes. We assume this model to be the ground state of our model.

Adding the energy momentum tensor to each of the two branes, and allowing deviations from the pure AdS\(_5\) bulk, the effective (non-local) Einstein equations on the branes at low energies take the forms

\[ G^\mu_{\nu}(h) = \frac{\kappa^2}{\ell} T^A_{\mu\nu} - \frac{2}{\ell} \chi^\mu_{\nu}, \]  

(2.2)

\[ G^\mu_{\nu}(f) = -\frac{\kappa^2}{\ell} T^B_{\mu\nu} - \frac{2}{\ell} \chi^\mu_{\nu} \sqrt{\Omega^4}. \]  

(2.3)

where \( h_{\mu\nu} = g^{A_{\text{brane}}}_{\mu\nu} \) and \( \Omega \) is a conformal factor that relates the metric on the \( A \)-brane to that on the \( B \)-brane (specifically, \( f_{\mu\nu} = g^{B_{\text{brane}}}_{\mu\nu} = \Omega^2 h_{\mu\nu} \)), and the terms proportional to \( \chi_{\mu\nu} \) are 5-dimensional Weyl tensor contributions, which describe the non-local 5-dimensional effect. Although Eqs. (2.2) and (2.3) are non-local individually, with undetermined \( \chi_{\mu\nu} \), they can be combined so as to reduce them to local equations for each brane. Since \( \chi_{\mu\nu} \) appears only algebraically, one can easily eliminate \( \chi_{\mu\nu} \) from Eqs. (2.2) and (2.3).

2.1. \( A \)-brane

First, consider the effective equations on the \( A \)-brane. Defining a new field \( \Psi = 1 - \Omega^2 \), we find

\[ G^\mu_{\nu}(h) = \frac{\kappa^2}{\ell\Psi} T^A_{\mu\nu} + \frac{\kappa^2}{\ell\Psi} (1 - \Psi)^2 T^B_{\mu\nu} + \frac{1}{\Psi} \left( \psi^|\mu_{|\nu} - \delta^|\mu_{|\nu} \psi^|\alpha_{|\alpha} \right) \]

\[ + \frac{3}{2\Psi(1 - \Psi)} \left( \psi^|\mu_{|\nu} - \frac{1}{2} \delta^|\mu_{|\nu} \psi^|\alpha_{|\alpha} \right), \]  

(2.4)

\[ \Box \Psi = \frac{\kappa^2}{3\ell(1 - \Psi)} \left( T^A + (1 - \Psi) T^B \right) - \frac{1}{2(1 - \Psi)} \psi^|\mu_{|\mu} , \]  

(2.5)

where “|” denotes the covariant derivative with respect to the metric \( h_{\mu\nu} \). Since \( \Omega \) (or equivalently \( \Psi \)) contains the information of the distance between the two branes, we call \( \Omega \) (or \( \Psi \)) the “radion”.

We can also determine \( \chi^\mu_{\nu} \) by eliminating \( G^\mu_{\nu} \) from Eqs. (2.2) and (2.3). Then, we have

\[ \chi^\mu_{\nu} = -\frac{\kappa^2}{2\Psi(1 - \Psi)} \left( T^A_{\mu\nu} + (1 - \Psi) T^B_{\mu\nu} \right) - \frac{\ell}{2\Psi} \left( \psi^|\mu_{|\nu} - \delta^|\mu_{|\nu} \psi^|\alpha_{|\alpha} \right) \]

\[ + \frac{3}{2(1 - \Psi)} \left( \psi^|\mu_{|\nu} - \frac{1}{2} \delta^|\mu_{|\nu} \psi^|\alpha_{|\alpha} \right), \]  

(2.6)
Note that the index of $T^{B\mu\nu}$ is to be raised or lowered by the induced metric on the $B$-brane, $f_{\mu\nu}$.

The effective action for the $A$-brane that gives Eqs. (2.4) and (2.5) is
\[
S_A = \frac{\ell}{2\kappa^2} \int d^4x \sqrt{-h} \left[ \Psi R(h) - \frac{3}{2(1 - \Psi)} \psi^\alpha \psi^\alpha \right] + \int d^4x \sqrt{-h} \mathcal{L}^A + \int d^4x \sqrt{-h} (1 - \Psi)^2 \mathcal{L}^B.
\] (2.7)

2.2. B-brane

Using the same procedure as that above (but exchanging the roles of $h_{\mu\nu}$ and $f_{\mu\nu}$) also yields the effective equations on the $B$-brane. Defining $\Phi = \Omega^{-2} - 1$, we obtain
\[
G^{\mu\nu}(f) = \frac{\kappa^2}{\ell \Phi} T^{B\mu\nu} + \frac{\kappa^2}{\ell \Phi} (1 + \Phi)^2 T^{A\mu\nu} + \frac{1}{\Phi} (\Phi^\mu_{\nu;\alpha} - \frac{1}{2} \delta^\mu_{\nu} \Phi^{\alpha;\alpha})
\]
\[- \frac{3}{2(1 + \Phi)} \left( \Phi^{\mu}_{\nu;\alpha} - \frac{1}{2} \delta^\mu_{\nu} \Phi^{\alpha;\alpha} \right),
\] (2.8)
\[
\Box \Phi = \frac{\kappa^2}{3\ell} (1 + \Phi) \left\{ T^B + (1 + \Phi) T^A \right\} + \frac{1}{2(1 + \Phi)} \Phi^{\mu}_{\nu;\mu}.
\] (2.9)

Here, “;” denotes the covariant derivative with respect to the metric $f_{\mu\nu}$. Note that the index of $T^{A\mu\nu}$ is raised or lowered by $h_{\mu\nu}$. Because $\Phi$ is equivalent to $\Omega$ or $\Psi$, we also call $\Phi$ the “radion”.

We can also express $\chi^{\mu}_{\nu}$ in terms of quantities on the $B$-brane. We find
\[
\chi^{\mu}_{\nu} = -\frac{\kappa^2}{2\Phi(1 + \Phi)} \left\{ T^{B\mu}_{\nu} + (1 + \Phi) T^{A\mu}_{\nu} \right\} - \frac{\ell}{2\Phi(1 + \Phi)^2} \left[ \Phi^{\mu}_{\nu;\alpha} - \frac{1}{2} \delta^\mu_{\nu} \Phi^{\alpha;\alpha} \right]
\]
\[- \frac{3}{2(1 + \Phi)} \left( \Phi^{\mu}_{\nu;\alpha} - \frac{1}{2} \delta^\mu_{\nu} \Phi^{\alpha;\alpha} \right).
\] (2.10)

The effective action for the $B$-brane is given by
\[
S_B = \frac{\ell}{2\kappa^2} \int d^4x \sqrt{-f} \left[ \Psi R(f) + \frac{3}{2(1 + \Phi)} \Phi^{\alpha}_{\nu;\alpha} \right] + \int d^4x \sqrt{-f} \mathcal{L}^B + \int d^4x \sqrt{-f} \mathcal{L}^A (1 + \Phi)^2.
\] (2.11)

§3. Radion dynamics

Most inflationary models are based on a slow-roll inflation that has a sufficiently flat potential. In this section, we consider the dynamics of branes with vacuum energy as a first-order approximation of a slow-roll inflation model. Qualitative features of the brane cosmology can be understood with this simplified vacuum brane model.

We take the matter Lagrangians to be $\mathcal{L}^A = -\delta\sigma^A$ and $\mathcal{L}^B = -\delta\sigma^B$ in our effective action (2.7) or (2.11). The effective action on the $A$-brane in this case reads
\[
S_A = \frac{\ell}{2\kappa^2} \int d^4x \sqrt{-h} \left[ \Psi R - \frac{3}{2(1 - \Psi)} \psi^\alpha \psi^\alpha \right] - \delta\sigma^A \int d^4x \sqrt{-h}
\]
Because our theory is a scalar-tensor-type theory, we call this original action the “Jordan-frame effective action”. In order to study the dynamics of the radion, it is convenient to move to the Einstein frame, in which the action takes the canonical Einstein-scalar form. Applying the conformal transformation defined by $h_{\mu\nu} = \frac{1}{\Psi} g_{\mu\nu}$ and introducing the new field

$$\eta = -\log \left| \frac{\sqrt{1 - \Psi} - 1}{\sqrt{1 - \Psi} + 1} \right| ,$$

we obtain the Einstein-frame effective action as

$$S_A = \frac{\ell}{2\kappa^2} \int d^4x \sqrt{g} \left[ R(g) - \frac{3}{2} \nabla^\alpha \eta \nabla_\alpha \eta \right] - \int d^4x \sqrt{-g} V(\eta) ,$$

where $\nabla$ denotes the covariant derivative with respect to the metric $g_{\mu\nu}$, and the radion potential now takes the form

$$V(\eta) = \delta \sigma^A \left[ \cosh^4 \frac{\eta}{2} + \beta \sinh^4 \frac{\eta}{2} \right] , \quad \beta = \frac{\delta \sigma^B}{\delta \sigma^A} .$$

We can also start from the effective action on the $B$-brane to obtain the same Einstein-frame effective action. By applying the conformal transformation defined by $f_{\mu\nu} = \frac{1}{\Phi} g_{\mu\nu}$ and introducing the new field

$$\eta = -\log \left| \frac{\sqrt{\Phi} + 1 - 1}{\sqrt{\Phi} + 1 + 1} \right| ,$$

we also arrive at Eq. (3.3).

We are now ready to examine the radion dynamics (see Fig. 2). Notice that the two branes are infinitely separated when $\eta = 0$ ($\Psi = 1$), and they collide when $\eta = \infty$ ($\Psi = 0$). For definiteness, let us assume $\delta \sigma_A > 0$. If $\delta \sigma^A + \delta \sigma^B > 0$,
Ψ will move towards unity; i.e., the branes will move away from each other. If \( \delta \sigma^A + \delta \sigma^B < 0 \), the potential has a maximum at \( \Psi_c = 1 + 1/\beta \), and the behavior depends on whether \( \Psi > \Psi_c \) or \( \Psi < \Psi_c \). If \( \Psi > \Psi_c \), the branes will become infinitely separated. If \( \Psi < \Psi_c \), the branes will approach each other and eventually collide.

The static two de-Sitter brane solution corresponds to the unstable point \( \Psi = \Psi_c \). In fact, considering the fluctuations around \( \Psi_c \), we find an instability characterized by the equation

\[
\delta \ddot{\Psi} + 3H \dot{\Psi} - 4 \left( H^2 + \frac{K}{a^2} \right) \delta \Psi = 0 . \quad (3.6)
\]

We see that the mass square, \(-4(H^2 + K/a^2)\), is negative, in accordance with the previous linear perturbation analysis.\(^9\)

As we mentioned above, in the case \( \Psi < \Psi_c \), the two branes collide. From the 5-dimensional point of view, this is certainly a singularity, where the spacetime degenerates to 4 dimensions. However, as far as observers on the branes are concerned, nothing seems to go wrong. In fact, the action \((2.7)\) is well-defined even in the limit \( \Psi \to 0 \). Let us assume that \( \Psi \) smoothly becomes negative after collision. Then replacing \( \Psi \) as \( \Psi \to -\tilde{\Psi} \) in the action \((2.7)\), we find

\[
- S_A = \frac{\ell}{2\kappa^2} \int d^4x \sqrt{-h} \left[ \tilde{\Psi} R(h) + \frac{3}{2(1+\tilde{\Psi})} \tilde{\Psi}^{\alpha\beta} \tilde{\Psi}_{\alpha\beta} \right] + \int d^4x \sqrt{-h} ( -L^A )
\]

\[
+ \int d^4x \sqrt{-h} \left( 1 + \tilde{\Psi} \right)^2 (-L^B) . \quad (3.7)
\]

This is the same as the effective action on the \( B \)-brane, given in Eq. \((2.11)\), except for the overall change of sign and the associated changes of sign of the matter Lagrangians. This fact can be interpreted as follows. After collision, the positive tension brane becomes a negative tension brane, together with the sign change of the matter Lagrangian, and vice versa for the initially negative tension brane. This implies that, if we live on either of the branes, our world transmutes into quite a different world, and so do we without much damage to the world. That is, we are born again!

This procedure might cause a serious problem when we consider quantum theory. A similar issue arises in string theory if there exists a negative tension brane. However, string theory has the potential ability to overcome this difficulty. For the time being, we can only hope that our prescription has an appropriate interpretation in the context of string theory.

§4. Born-again braneworld

After the collision, if our world had initially been a positive tension brane, we would now be on the negative tension brane. However, the theory described by the action \((2.11)\) with any value of \( \Phi \) contradicts observation. Therefore we assume that we were initially on the negative tension brane \((B\)-brane\) before the collision.

Let us first investigate the cosmological evolution of the \( B \)-brane in the original Jordan frame. We consider the spatially isotropic and homogeneous metric on the
brane

\[ ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^idx^j, \]

(4.1)

where \(a(t)\) is the scale factor and \(\gamma_{ij}\) is the metric of a maximally symmetric 3-space with comoving curvature \(K = 0, \pm 1\). Using Eqs. (2.8) and (2.9), the field equations on the \(B\)-brane can be written

\[
-3\left(H^2 + \frac{K}{a^2}\right) = -\frac{\kappa^2}{\ell} \frac{1}{\Phi} \delta\sigma^B - \frac{\kappa^2}{\ell} \frac{(1 + \Phi)^2}{\Phi} \delta\sigma^A + 3H \frac{\dot{\Phi}}{\Phi} + 3 \frac{\dot{\Phi}^2}{4 \Phi(1 + \Phi)}, \\
-2 \left(\dot{H} - \frac{K}{a^2}\right) - 3 \left(H^2 + \frac{K}{a^2}\right) = -\frac{\kappa^2}{\ell} \frac{1}{\Phi} \delta\sigma^B - \frac{\kappa^2}{\ell} \frac{(1 + \Phi)^2}{\Phi} \delta\sigma^A + \frac{2}{\Phi} \frac{\dot{\Phi}}{\Phi} - \frac{3}{4 \Phi(1 + \Phi)} \frac{\dot{\Phi}^2}{\Phi}, \quad (4.3)
\]

\[ \ddot{\Phi} + 3H \dot{\Phi} = \frac{4\kappa^2}{3\ell} (1 + \Phi) \left[ \delta\sigma^B + (1 + \Phi) \delta\sigma^A \right] + \frac{1}{2} \frac{1}{1 + \Phi} \frac{\dot{\Phi}^2}{\Phi}. \quad (4.4)\]

Note that Eq. (4.2) is the Hamiltonian constraint. Eliminating \(\ddot{\Phi}\) from Eq. (4.3) by using Eq. (4.4) and combining the resulting equation with Eq. (4.2), we obtain

\[ \dot{H} - \frac{K}{a^2} = -2 \left(H^2 + \frac{K}{a^2}\right) - \frac{2\kappa^2}{3\ell} \delta\sigma^B. \quad (4.5)\]

Integrating this equation, we obtain the Friedmann equation with dark radiation,

\[ H^2 + \frac{K}{a^2} = -\frac{\kappa^2}{3\ell} \delta\sigma^B + \frac{C}{a^4}. \quad (4.6)\]

Note that this is just a cosmological version of the non-local Einstein equations on the brane, given in Eq. (2.3), in which the \(\chi^\mu\nu\) term gives the dark radiation \(C/a^4\).

Comparing Eq. (4.2) with Eq. (4.6), we find the following relation between the radion and the dark radiation:

\[ \frac{\kappa^2}{3\ell} \delta\sigma^B + 1 + \Phi \left[ 1 + \frac{(1 + \Phi)}{\beta} \right] - \frac{\dot{\Phi}}{\Phi} - \frac{1}{4} \frac{1}{1 + \Phi} \frac{\dot{\Phi}^2}{\Phi} = \frac{C}{a^4}. \quad (4.7)\]

This gives, in particular, the relation between the initial conditions of the radion and the sign of the dark radiation.

Assuming \(\delta\sigma_B < 0\), the Friedmann equation yields the dependence of \(H\) on the dark radiation. Setting \(H_*^2 = (\kappa^2/3\ell)(-\delta\sigma_B)\), we find

\[ H_*^2 K^2 - 4C > 0 : \]

\[ H = H_* \sqrt{\frac{H_*^2 K^2 - 4C}{\sqrt{H_*^2 K^2 - 4C} \sinh 2H_* t}}, \quad (4.8)\]

\[ H_*^2 K^2 - 4C = 0 : \]

\[ H = H_* \frac{2 e^{2H_* t}}{2 e^{2H_* t} + H_*^2 K}, \quad (4.9)\]
\[ H^2 K^2 - 4C < 0 : \]
\[ H = H_c \frac{\sqrt{4C - H_c^2 K^2} \cosh 2H_c t}{\sqrt{4C - H_c^2 K^2} \sinh 2H_c t + H_c^2 K} . \]  

(4.10)

To realize the born-again braneworld scenario, we consider the case of colliding branes. For simplicity, we assume \( K = 0 \). A numerical solution of \( \Phi \) is displayed in Fig. 3. We indeed see that \( \Phi \) passes through zero smoothly and approaches \(-1\); i.e., the reborn branes will eventually be infinitely separated.

![Fig. 3. The time evolution of \( \Phi \). The evolution is completely regular at the collision point.](image)

![Fig. 4. The evolution of the Hubble constant in the Jordan frame. The solution rapidly approaches the de-Sitter spacetime. We also plotted the pre-big-bang solution in the Einstein frame.](image)

Let us analyze this collision. We denote the Hubble constant at the time of collision \( t = t_c \) by \( H_c \). Applying Eq. (4.7) to the vicinity of the time of collision, we find

\[ \Phi = -2(1 - \sqrt{\gamma})H_c(t - t_c) ; \quad \gamma = 1 - \frac{H_c^2}{H_c^2} \left( 1 + \frac{1}{\beta} \right) . \]  

(4.11)

As expected, \( \Phi \) behaves perfectly smoothly around the time of collision. The brane
geometry is, of course, perfectly regular as well. In fact, the Friedmann equation continues to hold without a hint of collision.

Now, we transform these quantities into the Einstein frame. Because \( \Phi \rightarrow -1 \) eventually, we can regard our present universe to be described by the Einstein frame. The relation between the Einstein frame and the Jordan frame is

\[
ds_{E}^{2} = -dt_{E}^{2} + b^{2}(t_{E})\delta_{ij}dx^{i}dx^{j}
\]

\[
= |\Phi|[-dt_{J}^{2} + a(t_{J})^{2}\delta_{ij}dx^{i}dx^{j}], \tag{4.12}
\]

where we have attached the subscripts \( E \) and \( J \) to the time coordinates to denote the cosmic time in the Einstein frame and the Jordan frame, respectively. Thus we have

\[
b = \sqrt{|\Phi|}a, \quad dt_{E} = \sqrt{|\Phi|}dt_{J}. \tag{4.13}
\]

Therefore, the Hubble parameter in the Einstein frame behaves in the vicinity of collision as

\[
\frac{\dot{b}(t_{E})}{b(t_{E})} = \frac{1}{3t_{E}} + \frac{H_{c}}{(3(1 - \sqrt{\gamma})H_{c}|t_{E}|)^{1/3}}, \tag{4.14}
\]

where the collision time in the Einstein frame is set to be \( t_{E} = 0 \).

We note that in the Einstein frame, the universe contracts rapidly just before the collision, and the Hubble parameter diverges to \(-\infty\) at collision. Then, the universe is reborn with an infinitely large Hubble parameter, which looks like a big-bang singularity. Thus, because there exists no singularity in the Jordan frame, the pre-big-bang phase and the post-big-bang phase in the Einstein frame are successfully connected. That is, our scenario is indeed a successful realization of the pre-big-bang scenario in the context of the braneworld (see Fig. 4).

§5. Observational implication

As we can see from Eq. (4.6), the universe will rapidly converge to the quasi-de-Sitter regime, while the radion can vary, as long as the relation (4.7) is satisfied. In the Jordan frame, because the metric couples with the radion, the non-trivial evolution of the radion field affects the perturbations. This possibility discriminates our model from the usual inflationary scenario. On the other hand, the inflaton does not couple directly with the radion field. Hence, the inflaton fluctuations are expected to give adiabatic fluctuations with a flat spectrum. This feature of our model is an advantage it has over the pre-big-bang model.

5.1. Radion fluctuations

To study the behavior of the radion fluctuations, it is convenient to work in the Einstein frame. We express the metric perturbation in the Einstein frame as

\[
ds_{E}^{2} = b^{2}[-(1 + 2A)\delta^{2} + 2\partial_{i}Bdx^{i}d\tau + ((1 + 2R)\delta_{ij} + 2\partial_{i}\partial_{j}E)dx^{i}dx^{j}] \tag{5.1}
\]
The action for a curvature perturbation $\mathcal{R}$ on the $\delta \eta = 0$ (i.e., radion-comoving) slice reads (for a concise review, see Appendix B of Ref. 20))

$$S = \frac{1}{2} \int d\eta d^3x z^2 \left[ R'_c^2 - R_c^2 \mathcal{R}_c |_{\mathcal{R}_c} \right], \quad (5.2)$$

where $\mathcal{H} = b'/b$ and

$$\mathcal{R}_c = \mathcal{R} - \mathcal{H} \frac{\delta \eta}{\eta'}, \quad z = \sqrt{\frac{3\ell}{2\kappa^2}} \frac{bn'}{\mathcal{H}}. \quad (5.3)$$

The equation of motion for $\mathcal{R}_c$ is

$$\mathcal{R}_c'' + 2 \frac{z'}{z} \mathcal{R}_c' - \left\langle \Delta \mathcal{R}_c \right\rangle = 0. \quad (5.4)$$

Because the background behaves as $b \sim (-\tau)^{1/2}$, $\mathcal{H} \sim (2\tau)^{-1}$, and $n' \sim (-\tau)^{-1}$, we have $z \propto b$, and the positive frequency modes for the adiabatic vacuum are given by

$$\mathcal{R}_{c,k} \sim \sqrt{\frac{\pi \kappa^2}{6H_* \ell}} H_0^{(1)}(-k\tau), \quad (5.5)$$

where we have normalized $b$ as $b = |H_*\tau|^{1/2}$. Then we have

$$\left\langle R_c^2 \right\rangle_k \equiv \frac{k^3}{2\pi^2} P(k) = \frac{k^3}{2\pi^2} |\mathcal{R}_{c,k}|^2 \sim \frac{k^3}{H_* M_{pl}^2}, \quad (5.6)$$

where $M_{pl}^2 = \kappa^2/\ell$. Thus the spectrum is very blue. If we define the spectral index by $P(k) \propto k^{n-4}$, this implies $n = 4$.

There are, however, a couple of points that should be mentioned. First, the curvature perturbation $\mathcal{R}_c$ is logarithmically divergent at the instant of collision. This divergence does not disappear even in the Jordan frame. Note that $\mathcal{R}_c$ is defined on the hypersurface on which the radion is uniform, and hence is invariant under the conformal transformation $^{13}$ This suggests the marginal instability of our system. Nevertheless, if we cut off an infinitesimally small time interval around the singularity, say $[-\epsilon, \epsilon]$, and smoothly match $\mathcal{R}_c$ at $\tau = \pm \epsilon$, the result is quite insensitive to the choice of $\epsilon$, as long as $k\epsilon \ll 1$, i.e., the scale is outside the effective Hubble horizon $\mathcal{H}^{-1}$. Thus, we expect the result $(5.6)$ to be valid for all scales of cosmological interest.

The second point, which is a possible drawback, is the following. If inflation (in the Jordan frame) continues for a time of $O(H_*^{-1})$ after collision, the blue spectrum given above should not be called “blue” after all. As can be seen from Fig. 4, or from Eq. $(5.5)$, $|\mathcal{H}|$ is quite symmetric around $\tau = 0$, at least for $|\tau| \lesssim H_*^{-1}$. This implies that all the modes with $k > H_*$ that were once outside the horizon in the pre-big-bang phase came inside the horizon again in the post-big-bang phase by the time

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$^*$ The fact that the perturbation is generically singular when the effective gravitational constant changes sign was first pointed out by Starobinsky.$^{21}$
$\tau \sim H_s^{-1}$. (It should be noted that the conformal time is approximately equal to the cosmic time in the Jordan frame near the collision, with our normalization such that $b = |H_s\tau|^{1/2}$.) Because the evolution near the collision time is approximately time symmetric, the standard vacuum state will re-emerge for the modes that re-enter the horizon in the Einstein frame. These modes will then come outside the horizon again in the inflationary phase, and their spectrum will be a normal scale-invariant one. Therefore, we would obtain a spectrum that is scale-invariant for $k > H_s$ and blue spectrum at $k < H_s$, with the maximum amplitude given by

$$\langle R_c^2 \rangle_{k=H_s} \sim \frac{H_s^2}{M_{pl}^2}. \quad (5.7)$$

For the values of $H_s$ predicted by the standard inflationary models, this is not really a blue spectrum in the observational sense.

This problem can be avoided only if the inflation ends right after the collision, when $\tau \ll H_s^{-1}$. One possibility is to resort to the marginal instability mentioned above. There may be a model in which the marginally divergent spectrum at high frequencies triggers a phase transition to end inflation. It is not clear if it is possible to construct such a model in a natural way. We leave investigation of this point as a future problem.

### 5.2. Gravitational waves

Next, consider the tensor perturbations

$$ds^2 = b^2(\tau) \left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^idx^j\right], \quad (5.8)$$

where $h_{ij}$ satisfy the transverse-traceless conditions $h_{ij,j} = h_{ii} = 0$. For the gravitational tensor perturbations, we have

$$h_{ik}'' + 2\mathcal{H}h_k' + k^2 h_k = 0, \quad (5.9)$$

where $h_k$ is the amplitude of $h_{ij}$. Since $\mathcal{H} \sim (2\tau)^{-1}$, $h_k$ has approximately the same spectrum as $R_c$, including the magnitude. In particular, the spectral index for the gravitational waves is also $n = 4$. (Here, the spectral index is defined by $P_h(k) \propto k^{n-4}$, as in the case of the scalar curvature perturbation. For the tensor perturbation, the conventional definition is $n_T = n - 1$.) Provided that inflation ends right after collision, as discussed in the previous subsection, this gives a sufficiently blue spectrum that can amplify $\Omega_g$ by several orders of magnitude or more on small scales as compared to conventional inflation models. Thus, there is the possibility that it may be detected by a space laser interferometer for low frequency gravitational waves, such as LISA$^{13}$.

### 5.3. Inflaton perturbation

To investigate the inflaton perturbation rigorously, one needs to introduce an inflaton field explicitly and consider a system of equations fully coupled with the radion and the metric perturbation. However, since this is beyond the scope of the present paper, let us just estimate the effect of the metric perturbation induced by radion fluctuations on the inflaton perturbation.
The field equation for the inflaton, $\phi$, in the Jordan frame is
\[
a^{-2}(a^2 \delta \phi')' - \Delta \delta \phi + a^2 \partial_\phi^2 V \delta \phi \\
= -3 \phi' R'_J + a^{-2}(a^2 \phi' A_J)' - a^2 \partial_\phi V A_J - \Delta (E'_J - B_J) \phi',
\]
(5.10)
where the suffix $J$ indicates a quantity in the Jordan frame. Again, let us consider the radion-comoving slice. Then, all the metric perturbation variables in the Jordan frame coincide with their respective counterparts in the Einstein frame.

Ignoring the effect of the inflaton perturbation, the Hamiltonian and momentum constraints on the radion-comoving slice are
\[
\Delta (R'_c - \mathcal{H}(E' - B)_c) = \frac{3}{4} \eta^2 A_c,
\]
\[
R'_c - \mathcal{H} A_c = 0.
\]
(5.11)
Using these equations, Eq. (5.10) reduces to
\[
a^{-2}(a^2 \delta \phi')' - \Delta \delta \phi + a^2 \partial_\phi^2 V \delta \phi \\
= -2 \frac{\mathcal{H}^2 + \mathcal{H}'}{\mathcal{H}^2} R'_c \phi' + \frac{a^2 \partial_\phi^2 V}{\mathcal{H}^2} \left( a^2 \frac{R'_c}{\mathcal{H}} \phi' \right)' - \frac{R'_c}{\mathcal{H}} a^2 \partial_\phi V - \frac{\Delta R'_c}{\mathcal{H}} \phi'.
\]
(5.12)
Since $R'_c/\mathcal{H}$ is finite, we see that the right-hand side of the above is regular and small for the slow-roll inflation. Hence, the inflaton fluctuations are not strongly affected by the radion fluctuations. Thus, the inflaton fluctuations should have a standard scale-invariant spectrum.

§6. Conclusion

In this paper, we proposed a scenario in which two branes collide and are reborn as new branes, called the “born-again braneworld scenario”. Our model has the features of both inflationary and pre-big-bang scenarios. In the original frame, which we call the Jordan frame, because gravity on the brane is described by a scalar-tensor-type theory, the brane universe is assumed to be inflating due to an inflaton potential. From the 5-dimensional point of view, the radion, which represents the distance between the branes and which acts as a gravitational scalar on the branes, has non-trivial dynamics and these vacuum branes can collide and pass through smoothly. After collision, it is found that the positive tension and the negative tension branes exchange their role. Then, they move away from each other, and the radion becomes trivial after a sufficient lapse of time. The gravity on the originally negative tension brane (whose tension becomes positive after collision) then approaches that of the conventional Einstein theory, except for tiny Kaluza-Klein corrections.

We can also consider the cosmological evolution of the branes in the Einstein frame. Note that the two frames are indistinguishable at present if our universe is on the positive tension brane after collision. In the Einstein frame, the brane universe is contracting before the collision and a singularity is encountered at the collision point. This resembles the pre-big-bang scenario. Thus our scenario may be regarded as a non-singular realization of the pre-big-bang scenario in the braneworld context.
Because our braneworld is inflating, and the inflaton has essentially no coupling with the radion field, an adiabatic density perturbation with a flat spectrum is naturally realized. On the other hand, because the collision of branes mimics the pre-big-bang scenario, the primordial background gravitational waves with a very blue spectrum may be produced. This suggests the possibility that we may be able to observe the collision epoch using a future gravitational wave detector, such as LISA.

Admittedly, the collision process must be treated with a more fundamental theory. However, because the singularity at the collision point is very mild, we expect that the qualitative features of our scenario will remain unchanged, even if we include the effect of a (yet unknown) fundamental theory. The born-again scenario surely deserves further investigation.

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