Finite element analysis of silicone rubber based on Yeoh constitutive model and Ogden constitutive model

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Abstract. Based on the two constitutive models of Yeoh constitutive model and Ogden constitutive model, a finite element model of uniaxial compression experiment of silicone rubber was established. The stress-strain curves and stress clouds of different models of silicone rubber were compared to further verify the two models. It is found that the Ogden model (N=3, N=5) has a good fit between the stress-strain curve simulation results and the nominal stress-strain curve when there is only uniaxial compression test, and the model constant Compare solutions.

Keywords: Silicone rubber, Yeoh constitutive model, Ogden constitutive model.

1. Introduction
Rubber materials have super-elasticity and excellent stretchability. Compared with the performance of metal materials, only a few parameters are required, but the characteristics of rubber are more complicated.

The constitutive relation is the basis of studying the mechanical properties of rubber materials. The geometric nonlinear relation of superelastic materials must be considered when establishing the constitutive relation of superelastic materials. According to recent studies, it can be found that mooney-Rivilin model is mainly used in the study of small deformation range of rubber, while Yeoh model is mainly used in the study of large deformation range.

In this paper, the uniaxial compression experiment of silicone rubber material is carried out, and the stress-strain curve is obtained by finite element simulation of uniaxial compression experiment using ABAQUS finite element analysis software to study its uniaxial compression deformation behavior.

2. Introduction to the theory of rubber Yeoh constitutive relationship
The Yeoh model has the advantages of simple form and high accuracy, and its material parameters can be determined by uniaxial compression experiments. When $C_{ij}=0$ ($j\neq0$), special form and $N=3$ in equation, the strain energy density function of Yeoh model is as follows[1,2]:
\[ U = \sum_{i=1}^{3} C_{i0}(I_1 - 3)^i + \sum_{i=0}^{3} \frac{1}{D_i}(J - 1)^{2i} \]

Where \( N \) is the class of the function, \( C_{ij} \) and \( D_i \) are the material constants, usually obtained by experimental tests, and \( I_1 \) and \( I_2 \) are the first-order and second-order strain invariants, respectively.

\[ I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \]

\( \lambda_2 \) and \( \lambda_3 \) are the main elongations. For incompressible materials, \( J=1 \). Recent studies have found that the Yeoh model can better simulate the characteristics of rubber materials at large strains.

### 3. Introduction to rubber Ogden constitutive theory

The strain energy density function of Ogden model is

\[ W = \sum_{i=1}^{N} \frac{2\mu_i}{\alpha_i^2}(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3) + \sum_{i=1}^{N} \frac{1}{D_i}(J - 1)^{2i} \]

In the formula, \( \alpha_i \) and \( \mu_i \) are the parameters of the material model to be determined, and the remaining parameters are consistent with the Yeoh model. When \( N=1 \), the Ogden model is simplified to the Mooney-Rivlin model; when \( N=3 \), the Ogden model is called the Ogden third-order model, which is widely used in engineering [3,4,5].

### 4. Rubber uniaxial compression experiment

#### 4.1. Sample

The shape of the silicon rubber sample is a cylinder of \( \Phi10mm \times 10mm \), and the number of silicon samples is three, and a uniaxial compression experiment is performed.

#### 4.2. Experimental principle

The compression test of the sample adopts the method of GB/T 7757-2009, and the corresponding stress-strain curve is calculated from the compression load-displacement curve of the sample.

The compression experiment was completed by the electronic universal material testing machine model WDW3020, as shown in Figure 1. The main process of the experiment was as follows:

A) Place the rubber sample directly between the upper and lower pressure plates;

B) Pre-test each sample, compress and load each time at a speed of 5 mm/min until the strain reaches 10\%, then unload and make the sample rebound;

C) in the formal experiment, the sample was compressed to at least 40\% strain at a speed of 5 mm/min and a force of 20KN.
4.3. Experimental results

The uniaxial compression test data of silicone rubber is shown in Table 1. In order to reduce the experimental error of the uniaxial compression experiment, the average value of the stresses of the three groups of samples was studied.

Table 1. Experimental data of uniaxial compression of silicone rubber

| strain/| Sample 1 | Sample 2 | Sample 3 | Average value |
|--------|----------|----------|----------|---------------|
|        | Stress/MPa| Stress/MPa| Stress/MPa| Stress/MPa    |
| 0      | 0         | 0        | 0        | 0             |
| 5      | 0.2928    | 0.289    | 0.3119   | 0.2903        |
| 10     | 0.4456    | 0.4393   | 0.48     | 0.4414        |
| 15     | 0.6035    | 0.5921   | 0.643    | 0.5959        |
| 20     | 0.769     | 0.7461   | 0.8085   | 0.7537        |
| 25     | 0.9511    | 0.9167   | 0.9995   | 0.9282        |
| 30     | 1.165     | 1.1204   | 1.2376   | 1.1353        |
| 35     | 1.4515    | 1.4006   | 1.5712   | 1.4176        |
| 40     | 1.8818    | 1.8169   | 2.097    | 1.8385        |
| 45     | 2.6407    | 2.535    | 3.0392   | 2.5702        |
| 50     | 4.1444    | 3.9534   | 4.8994   | 4.0171        |
5. ABAQUS simulation results
In this paper, the finite element analysis software ABAQUS is used to perform the finite element simulation of the Yeoh model and the Ogden model on the silicone rubber according to the uniaxial compression experimental method. The Mises stress cloud diagram obtained from the experiment in ABAQUS is as follows:

![Mises stress cloud diagram of Yeoh model of silicone rubber](image)

**Figure 2.** Mises stress cloud diagram of Yeoh model of silicone rubber

![Mises stress cloud of silicone rubber Ogden model (N=3)](image)

**Figure 3.** Mises stress cloud of silicone rubber Ogden model (N=3)

The average nominal stress-strain curve of silicone rubber is compared with the stress-strain curve simulated by Yeoh model and Ogden model, as shown in the following figure:
By comparing the nominal stress-strain curve of silicone rubber with the stress-strain curve obtained by fitting ABAQUS using Yeoh model and Ogden model N=3 and N=5, it was found that when there is only uniaxial compression test, simulation by Ogden model The obtained stress-strain curve and the nominal stress-strain curve have a higher degree of coincidence, and when N=5 than N=3, the simulated curve fit is higher, that is, the higher the model function order, the better the curve fit The higher. However, the Ogden fifth-order model's calculation increment step in the later stage of strain is obviously smaller, and even non-convergence occurs.

The parameters of Yeoh model and Ogden model obtained by abaqus simulation fitting are as follows:

Table 2. Yeoh model and Ogden model parameters

|            | I  | μ       | α     | D    |
|------------|----|---------|-------|------|
| Ogde (N=3) | 1  | 0.7845  | 2.0555| 0    |
|            | 2  | 3.7655  | 20.3664| 0    |
|            | 3  | -1.8836 | -10.1800| 0    |
| Ogde (N=5) | 1  | 69.8417 | 5.1010| 0    |
|            | 2  | -132.6554| 6.7142| 0    |
|            | 3  | 66.8723 | 9.0262| 0    |
|            | 4  | 5.2615  | -2.4110| 0    |
|            | 5  | -6.6506 | -5.5635| 0    |
| Yeoh      |    | C10     | C20   | C30  |
|           |    | 0.6465  | -0.3968| 0.2040|

6. Conclusion
The research in this paper is based on the Yeoh constitutive model and Ogden constitutive model commonly used in engineering. Through uniaxial compression tests on silicone rubber materials and the use of ABAQUS finite element simulation software, Yeoh constitutive model and Ogden constitutive model are used to obtain two The parameters of the three models and the stress-strain curves are compared, and the following conclusions are obtained:
1. It is proved that Ogden model is more suitable for rubber compression deformation behavior than Yeoh model only in uniaxial compression experiment;
2. It is proved that the higher the end of the Ogden model function, the higher the fit between the simulated stress-strain curve and the nominal stress-strain curve.
3. The parameters of Yeoh constitutive model and Ogden constitutive model of silicone rubber were fitted by ABAQUS.

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