Theoretical Description of the HERA Data on $F_2$ at Low $Q^2$

A.B. Kaidalov  
ITEP, B. Cheremushkinskaya 25  
117259 Moscow, Russia

C. Merino  
Dpto. de Física de Partículas  
Universidade de Santiago de Compostela  
E-15706 Santiago de Compostela, Spain

Abstract

It is shown that the CKMT model for the nucleon structure function $F_2$ gives a good description of the recent HERA data at low and moderate $Q^2$. Also the fit to the same data obtained with a modified version of the model in which a logarithmic dependence on $Q^2$ has been included, is presented. For moderate values of $Q^2$, in the current available range of $x$, the first parametrization leads to a better description of the data.

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1. Introduction

The CKMT model [1] for the parametrization of the nucleon structure function $F_2$ is a theoretical model based on Regge theory which provides a consistent formulation of this function in the region of low $Q^2$, and that can therefore be used as a safe and theoretically justified initial condition in the perturbative QCD evolution equation, to obtain the structure function at larger values of $Q^2$. Thus, the CKMT model, which gives a good description [1] of all the pre-HERA measurements [2] and of the first small-$x$ experimental data from HERA [3] on $F_2(x, Q^2)$ and $\sigma_{\gamma p}^{tot}(\nu)$, is a useful tool to reach deeper insight on the crucial interplay between soft (low $Q^2$) and hard (high $Q^2$) physics that now for the first time is being studied in the two small-$x$ experiments (H1 and Zeus) at HERA.

In this Letter, we present the description that the CKMT model provides for the more recent published data on $F_2$ at low $Q^2$ by both HERA experiments [4,5], which should contribute to determine the initial condition for the perturbative QCD evolution equation. Also we present, the description of the same experimental data by a modified version of the CKMT model in which a logarithmic dependence on $Q^2$ asymptotically predicted by the perturbative QCD, has been included. Even though both descriptions lead to good descriptions of experimental data in the present experimental range of $x$ for moderate values of $Q^2$, the fit obtained with the non-modified version of the CKMT model shows a better agreement and has more natural values of parameters.

2. The model

The CKMT model [1,6] proposes, taking into account what we know from Regge theory and hadronic interactions, for the nucleon structure functions

$$F_2(x, Q^2) = F_S(x, Q^2) + F_{NS}(x, Q^2), \quad (1)$$

the following parametrization of its two terms in the region of small and moderate $Q^2$. For
the singlet term, corresponding to the Pomeron contribution:

\[ F_S(x, Q^2) = A \cdot x^{-\Delta(Q^2)} \cdot (1 - x)^{n(Q^2)+4} \cdot \left( \frac{Q^2}{Q^2 + a} \right)^{1+\Delta(Q^2)}, \]  

(2)

which \( x \to 0 \) behavior is determined by an effective intercept of the Pomeron, \( \Delta \), which takes into account Pomeron cuts and, therefore (and this is one of the main points of the model) it depends on \( Q^2 \). This dependence was parametrized in [1] as :

\[ \Delta(Q^2) = \Delta_0 \cdot \left( 1 + \frac{\Delta_1 \cdot Q^2}{Q^2 + \Delta_2} \right). \]  

(3)

Thus, for low values of \( Q^2 \) (large cuts), \( \Delta \) is close to the effective value found from analysis of hadronic total cross sections (\( \Delta \sim 0.08 \)), while for high values of \( Q^2 \) (small cuts), \( \Delta \) takes the bare Pomeron value, \( \Delta \sim 0.2-0.25 \). The parametrization for the non-singlet term, which corresponds to the secondary reggeon (f, \( A_2 \)) contribution, is:

\[ F_{NS}(x, Q^2) = B \cdot x^{1-\alpha_R} \cdot (1 - x)^{n(Q^2)} \cdot \left( \frac{Q^2}{Q^2 + b} \right)^{\alpha_R}, \]  

(4)

where the \( x \to 0 \) behavior is determined by the secondary reggeon intercept \( \alpha_R \), which should be in the range \( \alpha_R = 0.4-0.5 \). The valence quark contribution can be separated into the contribution of the u and d valence quarks by replacing

\[ B \cdot (1 - x)^{n(Q^2)} \to B_u \cdot (1 - x)^{n(Q^2)} + B_d \cdot (1 - x)^{n(Q^2)+1}, \]  

(5)

and the normalization condition for valence quarks fixes \( B_u \) and \( B_d \) at one given value, \( Q^2_0 \), of \( Q^2 \) (we use \( Q^2_0 = 2 \, GeV^2 \) in our calculations). For both the singlet and the non-singlet terms, the behavior when \( x \to 1 \) is controlled by \( n(Q^2) \), with \( n(Q^2) \) being

\[ n(Q^2) = \frac{3}{2} \cdot \left( 1 + \frac{Q^2}{Q^2 + c} \right), \]  

(6)

so that, for \( Q^2=0 \), valence quark distributions have the same power, given by Regge intercepts, as in Dual Parton Model [7], \( n(0)=\alpha_R(0)-\alpha_N(0) \sim 3/2 \), and the behaviour of \( n(Q^2) \) for large \( Q^2 \) is chosen to coincide with dimensional counting rules [8].
The total cross section for real \((Q^2=0)\) photons can be obtained from the structure function \(F_2\) using the following relation:

\[
\sigma_{\gamma p}^{tot}(\nu) = \left[\frac{4\pi^2\alpha_{EM}}{Q^2} \cdot F_2(x, Q^2)\right]_{Q^2=0}.
\] (7)

The proper \(F_2(x, Q^2)\sim Q^2\) behavior at \(Q^2\rightarrow0\), is fulfilled in the model due to the last factors in equations (2) and (4), which were taken in the same form as in references [9] and [10]. Thus, the following parametrization of \(\sigma_{\gamma p}^{tot}(\nu)\) takes place in the CKMT model:

\[
\sigma_{\gamma p}^{tot}(\nu) = 4\pi^2\alpha_{EM} \cdot \left(A \cdot a^{-1-\Delta_0} \cdot (2m\nu)^{\Delta_0} + (B_u + B_d) \cdot b^{-\alpha_R} \cdot (2m\nu)^{\alpha_R-1}\right).
\] (8)

Therefore, the CKMT parametrizes both the nucleon structure functions and \(\gamma p\) total cross-section with a 8-parameter function. Besides the normalizations, 4 of the parameters appear in (8), so the proton structure function contains only 4 extra parameters. Although the parameters are not completely free and they are correlated, some theoretical uncertainty still exists in the determination of their exact values. To solve this uncertainty, comparison with experimental data is needed. From this comparison with experiments, a good description of all available experimental data on total cross-section for real photons and nucleon structure functions from former pre-HERA experiments [2], and from the first HERA measurements [3] was obtained in [1] by using the CKMT model with the values for the different parameters which are listed in Table 1, (a), and which were found without including any HERA data in the fit. It has also to be reminded that the good accuracy of the fit is not very sensitive to changes in the values of the parameters inside the ranges allowed by theory. Moreover, the model provides [11,12] in addition reasonable descriptions of the HERA data on diffraction [13], through the parametrization of the Pomeron structure function, and of the available experimental data on nuclear shadowing [14], by using this parametrization of the Pomeron structure function in the frame of the Gribov theory [15].
3. Description of the HERA data on $F_2$ at low $Q^2$

In the former fit of reference [1], the lack of experimental data at low and moderate $Q^2$ limited the accuracy in the determination of the values of the parameters in the model. Now, the publication of the new experimental data [4,5] on $F_2$ from HERA at low and moderate $Q^2$ gives us the opportunity to include in the fit of the parameters of the model experimental points from HERA experiments from the kinematical region where the parametrization should give a good description without need of any perturbative QCD evolution.

Thus, we proceed as one had done in [1], but by adding the above mentioned experimental data on $F_2$ from H1 and Zeus at low and moderate $Q^2$, to those [2] from NMC and E665 collaborations, and to data on cross-sections for real photoproduction, into a global fit which allows the test of the model in larger regions of $x$ and $Q^2$. We take as initial condition for the values of the different parameters those obtained in the previous fit [1]. The result of the new common fit to $\sigma_{\gamma p}^{tot}$ and $F_2$ is presented in Figs. 1 and 2, and the final values of the parameters can be found in Table 1, (b). Now, the parameter $\Delta_1$, which was fixed to $\Delta_1=2.0$ in former fits, (a), where also the notation $d$ had been used [1] for the parameter $\Delta_2$, has been left free, and, since the present global fit turns out to be not very sensitive to changes in the original values of the parameters $c$ and $\alpha_R$, we keep them fixed. As it can be seen in the figures, the quality of the description provided by the CKMT model of all the experimental data, and, in particular, of the new experimental data from HERA is very good, with a value of $\chi^2/d.o.f.$ for the global fit, $\chi^2/d.o.f.=106.95/167$, where the statistical and systematic errors have been treated in quadrature, and where the relative normalization among all the experimental data sets has been taken equal to 1.

Also, since the small-$x$ HERA experiments allow for the first time the experimental study of the question of the interplay between soft and hard physics, we have modified our
model, with basically only power dependence on $Q^2$, to include a logarithmic dependence on $Q^2$ as the one predicted asymptotically by perturbative QCD [16]. By doing so, we try to know whether such a modified version of the model provides a smoother matching in the description of both regimes, and whether the available experimental data can distinguish the description obtained with a soft model in which the $Q^2$-dependence saturates, from that of a model which does not present such a saturation.

To include the logarithmic dependence on $Q^2$ in our model, we take into account that the behavior of $F_2$ at small $x$ in QCD, is given by the singularities of the moments of the structure functions [17], the rightmost singularity giving the leading behavior. Thus, we introduce in (1) the following factors [17,18], which correspond to the moments of the structure functions in the language of the OPE expansion, and that can be calculated by making the convolution in rapidity of the hard-upper part with the soft-lower part of the leptonproduction diagram:

$$
\left( \frac{\alpha_s(Q^2_0)}{\alpha_s(Q^2)} \right)^{d_i(n_i)} , \quad i = S, NS,
$$

where the strong coupling constant is taken as

$$
\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \cdot \log \left( \frac{Q^2 + M^2}{\Lambda_{QCD}^2} \right)},
$$

with $M \sim 1$ GeV, a hadronic mass [19] included in (10) to avoid the singularity in $\alpha_s$ when $Q^2 \rightarrow \Lambda_{QCD}^2$, $\Lambda_{QCD} = 0.2$ GeV, and $\beta_0 = 11 - \frac{2}{3}n_f$ (we use in our calculations a number of flavors, $n_f = 3$), and where $d_S(n_S)$ and $d_{NS}(n_{NS})$ are respectively proportional to the largest eigenvalue of the anomalous dimension matrix, and to the anomalous dimension:

$$
d_S(n_S) \sim \frac{d_0}{4(n_S - 1)} - d_1,
$$

with

$$
d_0 = \frac{48}{\beta_0}, \quad d_1 = \frac{11 + \frac{2}{3}n_f}{\beta_0},
$$

$$
\Lambda_{QCD} = 0.2 \text{ GeV}, \quad \beta_0 = 11 - \frac{2}{3}n_f.
$$
and
\[ d_{NS}(n_{NS}) = \frac{16}{33 - 2n_f} \cdot \left( \frac{1}{2n_{NS}(n_{NS} + 1)} + \frac{3}{4} - S_1(n_{NS}) \right), \quad (12) \]
with
\[ S_1(n_{NS}) = n_{NS} \sum_{k=1}^{\infty} \frac{1}{k(k + n_{NS})}. \]

Thus, (1) is modified in the following way:
\[ F_2(x, Q^2) = \left( \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{d_S(n_{S})} \cdot F_S(x, Q^2) + \left( \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{d_{NS}(n_{NS})} \cdot F_{NS}(x, Q^2). \quad (13) \]

The exponents of the new factors in (13), \( d_S(n_{S}) \) and \( d_{NS}(n_{NS}) \), give us the singularities in \( n_i, i=S,NS \), of the momenta, which, as we mentioned above, control the QCD small-\( x \) behavior of \( F_2 \). Therefore, in our model, these exponents have to be evaluated, see (2) and (4), at \( n_S = 1 + \Delta(Q^2 \to \infty) = 1 + \Delta_0(1 + \Delta_1) \) and \( n_{NS} = \alpha_R \), respectively. We consider in our expressions only the LO behavior, and again we can use the relation (3) to write the total cross-section for real photoproduction from (13).

At this point, two main differences between our approach and the one of reference [18] which also provides good fits of the experimental data, can be mentioned. First, while in our model just one Pomeron with \( Q^2 \)-dependent intercept is present, in the approach of ref. [18] two components, a constant soft Pomeron and a hard Pomeron with very large intercept \( \Delta \sim 0.5 \), are used. Secondly, the proper \( Q^2 \) behavior (see also [10]) of \( F_2 \) for \( Q^2 \to 0 \) is provided in a natural way. So when the logarithmic dependence on \( Q^2 \) is included in the model, a justified choice [19] of the effective mass \( M^2 \) in \( \alpha_s(Q^2) \) (see equation (10)) provides a non-singular behavior of \( \alpha_s(Q^2) \) throughout the \( Q^2 \) range. In [18], on the contrary, one particular value of \( M^2 = \Lambda_{QCD}^2 \) has to be taken in (10) in order to get the required behavior of \( F_2 \) for \( Q^2 \to 0 \), thus resulting in a singular behavior \( \alpha_s(Q^2) \to \infty \), when \( Q^2 \to 0 \).

Then, we use this modified version of the CKMT parametrization of \( F_2 \) to repeat the fit of the same experimental data, including the HERA data on \( F_2 \) at small and moderate
$Q^2$. As starting point for the QCD evolution, we take the same value $Q_0^2=2.\text{GeV}^2$ we use to fix the normalization of the valence component. The result of this second fit is also presented in Figures 1 and 2, and the final values of the parameters in the model are given in Table 1, (c). Now, only the parameter $c$ has been kept fixed to its original value in (a).

As it can be seen in the figures, the quality of this second fit is also reasonable, although the value of $\chi^2/d.o.f.$ ($\chi^2/d.o.f.=453.19/167$), is now appreciably higher than in the fit obtained with the non-modified version of the CKMT model.

One common feature of the two fits is that both give higher values of the total cross-section for real photons than the experimental ones, in the region of large $W$ where the experimental error bars are large. More accurate measurements in this region should clarify this point. Concerning the final values of the parameters, it has to be noticed that $\Delta_0$ takes in (b) and (c) similar values, both slightly larger than the original one in (a), providing a stable asymptotic behavior for the effective $\Delta(Q^2\to\infty)\sim0.25-0.3$ which includes some effects of perturbative QCD evolution.

4. **Conclusions**

In conclusion, the CKMT model for the parametrization of the nucleon structure functions provides a very good description of all the available experimental data on $F_2(x, Q^2)$ at low and moderate $Q^2$, including the more recent small-$x$ HERA points. Also the fit to the same data obtained with a modified version of the model in which a logarithmic dependence on $Q^2$ is included, has been presented. Even though the quality of this second description is also reasonable, its $\chi^2/d.o.f.$ is appreciably higher than that corresponding to the fit obtained with the non-modified version of the CKMT model.
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| CKMT model | (a)    | (b)    | (c)    |
|------------|--------|--------|--------|
| A          | 0.1502 | 0.1301 | 0.1188 |
| a          | 0.2631 | 0.2628 | 0.07939|
| Δ₀         | 0.07684| 0.09663| 0.1019 |
| Δ₁         | 2.0    | 1.9533 | 1.2527 |
| Δ₂         | 1.1170 | 1.1606 | 0.1258 |
| c          | 3.5489 | 3.5489 (fixed) | 3.5489 (fixed) |
| b          | 0.6452 | 0.3840 | 0.3194 |
| α₀         | 0.4150 | 0.4150 (fixed) | 0.5872 |

**Table 1**: Values of the parameters in the CKMT model obtained in former fits, (a), in the fit in which also the low \( Q^2 \) HERA data [4,5] have been included, (b), and in the fit to the same data obtained with the modified version of the CKMT model in which a logarithmic dependence of \( F_2 \) on \( Q^2 \) has been taken into account, (c). All dimensional parameters are given in \( GeV^2 \). The valence counting rules provide the following values of \( B_u \) and \( B_d \), for the proton case, when fixing their normalization at \( Q^2 = 2. GeV^2 \): (a) \( B_u = 1.2064, B_d = 0.1798 \); (b) \( B_u = 1.1555, B_d = 0.1722 \); (c) \( B_u = 0.6862, B_d = 0.09742 \). In previous fits, (a), the parameter \( Δ₁ \) had been fixed to a value \( Δ₁ = 2 \).
Figure Captions

Fig. 1: $\sigma_{\gamma p}^{total}$ and $\sigma_{\gamma^*p}^{total}$ (in $\mu$barns) vs W (in GeV) for different values of $Q^2$. Theoretical fits have been obtained with the CKMT model (full line) and the modified version of the CKMT model (dashed line). Points at (a), $Q^2=0.0 GeV^2$ (*8.), (b), $Q^2=0.15 GeV^2$ (*6.), (c), $Q^2=0.25 GeV^2$ (*5.), (d), $Q^2=0.5 GeV^2$ (*4.), (e), $Q^2=0.8 GeV^2$ (*3.), (f), $Q^2=1.5 GeV^2$ (*2.), and (g), $Q^2=3.5 GeV^2$ (*1.). Experimental points for $F_2 (\sigma_{\gamma^*p}^{total})$ are from references [4], (black circles), and [5], (crosses), and experimental data on $\sigma_{\gamma p}^{total}$ are from references [20,21,22].

Fig. 2: $F_2(x,Q^2)$ vs $Q^2$ (in $GeV^2$) for different values of $x$. Theoretical fits have been obtained with the CKMT model (full line) and the modified version of the CKMT model (dashed line). Experimental points at (a), from left to right, $x=0.42 \cdot 10^{-5}$, $x=0.44 \cdot 10^{-5}$, and $x=0.46 \cdot 10^{-5}$ (*8.); (b), from left to right, $x=0.85 \cdot 10^{-5}$, $x=0.84 \cdot 10^{-5}$, $x=0.83 \cdot 10^{-5}$, and $x=0.86 \cdot 10^{-5}$ (*6.); (c), from left to right, $x=0.13 \cdot 10^{-4}$, and three points at $x=0.14 \cdot 10^{-4}$ (*5.); (d), $x=0.5 \cdot 10^{-4}$ (*4.); (e), $x=0.8 \cdot 10^{-4}$ (*3.); (f), $x=0.2 \cdot 10^{-3}$ (*2.); (g), $x=0.5 \cdot 10^{-3}$ (*1.). Experimental points for $F_2$ are from references [4], (black circles), and [5], (crosses).
Figure 1

\[ \sigma_{\text{proton}}^{\text{total}} (\mu\text{barn}) \]

\[ Q^2 (\text{GeV}^2) \]

(a) \( Q^2 = 0. (*8.) \)

(b) \( Q^2 = 1. (*1.) \)

(c) \( Q^2 = 2. \)

(d) \( Q^2 = 3. \)

(e) \( Q^2 = 4. \)

(f) \( Q^2 = 5. \)

(g) \( Q^2 = 6. \)

W (GeV)
Figure 2