Mass and Scalar Cross-sections for Neutralino Dark Matter in Anomaly Mediated Supersymmetry Breaking Model

Debasish Majumdar \(^a,\)\(^*\)

\(^a\)Theory Division, Saha Institute of Nuclear Physics, 
1/AF Bidhannagar, Kolkata 700064, India.

We have considered neutralino to be the lightest supersymmetric particle (LSP) in the framework of minimal Anomaly Mediated Supersymmetric (mAMSB) model. We have studied variation of neutralino mass with the supersymmetric parameters. Considering these neutralinos to be the candidates for weakly interacting massive particle (WIMP) or cold dark matter (CDM), we have calculated the neutralino nucleon scalar cross-sections and compared them with DAMA-NaI neutralino direct detection search results. From this study we observe that the mAMSB model results cannot explain the allowed region in WIMP mass and WIMP-nucleon scalar cross-section space obtained from annual modulation signature in DAMA-NaI experiment.

Observational evidence like velocity curves of spiral galaxies, measurement of X-ray emissions of the cluster of galaxies, gravitational lensing and other theoretical calculations strongly suggest the existence of enormous amount of matter in the universe which is invisible to us. This invisible, nonluminous matter or ‘Dark Matter’ constitute major fraction of the mass of the universe. While there may well be more than one type of dark matter, like baryonic or non-baryonic, hot (relativistic) or cold (non-relativistic), large scale structure calculations suggest non-relativistic massive particles (cold) that were decoupled from the plasma when the universe was entering into the epoch of matter dominance, constitute a dominant fraction of dark matter. These cold dark matters should be made up of weakly interacting massive (\(>\mathrm{GeV}\)) particles or WIMPs.

\(^*\)E-mail address: debasish@theory.saha.ernet.in
The favourite candidate for the WIMPs for cold dark matter, are lightest supersymmetric particles or LSP. As generally WIMPs are to be electrically neutral, the widespread choice for LSP is neutralino. Neutralino is a neutral Majorana particle and a superposition of fermionic super-partners of gauge and Higgs bosons namely bino, wino and two neutral higgsinos.

The direct detection of dark matter WIMPs can be realised by investigating their elastic scattering on the target nuclei of the detector. Hence for theoretical prediction of the WIMP detection rate it is important to calculate this cross-section for a choice of dark matter candidate.

The WIMPs in the galactic halo are assumed to have an approximately Maxwellian velocity distribution in the galactic rest frame. But for an observer on the earth (for one who is moving with the velocity of the earth) one has to include earth’s motion with respect to the halo. This has two components. One is sun’s motion in galaxy and the other is earth’s motion with respect to the sun. The directionality of this latter motion changes over the year as earth rotates around the sun. This in turn induces an annual variation of the WIMP speed relative to earth (maximum when earth’s rotational velocity adds up to the velocity of the sun and minimum when these velocities are in opposite directions). This imparts an annual modulation of the direct detection rates of the WIMPs constantly encountered by earth [1, 2]. DAMA- NaI detector at Gran Sasso in Italy [3] looks for this annual modulation signature in WIMPs direct search. They also have published allowed $3\sigma$ confidence level contours for their dark matter search in $M_W - \xi\sigma_p$ plane where $M_W$ is the WIMP mass, $\sigma_p$ is the WIMP -nucleon scattering cross section and $\xi = \rho_\chi/\rho_\ell$, $\rho_\chi$ being the local dark matter density and $\rho_\ell$ is the halo density (taken to be 0.3 GeV/cm$^3$).

In this letter minimal Anomaly Mediated Supersymmetry Breaking (mAMSB) model is considered for generating neutralino ($\chi$) LSP. It is our endeavour here to investigate the mass and scalar cross-sections for these neutralinos within the range of parameters given by mAMSB model. We also investigate whether these mass and scalar cross-sections can explain DAMA-NaI dark matter search results.

In this letter we consider the supersymmetric particle neutralino as a WIMP particle which is a possible candidate for dark matter. Neutralino is the lowest mass eigenstate of linear superposition of photino ($\tilde{\gamma}$), zino ($\tilde{Z}$), and the two Higgsino states ($\tilde{H}_1^0, \tilde{H}_2^0$) [4] and is written as

$$\chi = a_1\tilde{\gamma} + a_2\tilde{Z} + a_3\tilde{H}_1^0 + a_4\tilde{H}_2^0.$$ (1)
The supersymmetric model we have chosen here is minimal Anomaly Mediated Supersymmetry Breaking (mAMSB) model [5, 6]. In this model, the observable sector (OS) and the hidden sector (HS) are in two distinct 3-brane separated by a finite bulk distance which is of the order of the compactification radius of a fifth compactified dimension. Unlike in ordinary gravity-mediated supersymmetry breaking where the breaking is transmitted from HS to OS by tree level exchanges, in anomaly mediated supersymmetry breaking model or AMSB model, the supersymmetry breaking is transmitted through loop generated superconformal anomaly. An sparticle spectrum in this model is fixed by three parameters namely \(m_{3/2}\) which is equal to the gravitino mass, the ratio \(\tan \beta\) of the vacuum expectation values of two Higgs fields \(H_1^0\) and \(H_2^0\) and \(\text{sign}(\mu)\) (\(\mu\) being the Higgsino mass). The problem of tachyonic sfermions in this model is remedied by considering a universal mass squared term \(m_0^2\) that will make all sfermion masses positive. This is minimal anomaly mediated supersymmetry breaking or mAMSB model. Therefore, with \(m_0\), we have four parameters in mAMSB model.

In Minimal Supersymmetric Standard Model (MSSM), the \(\chi\) - nucleon scattering cross-sections involve two independent parameters namely neutralino mass and squark mass. In AMSB model both the squark mass and the neutralino mass are determined by \(m_{3/2}\) parameter. For phenomenological reasons, a universal mass term \(m_0\) has to be added to the squark mass. However, the stability of SUSY potential [7] constrains the \(m_0 - m_{3/2}\) parameter space severely. Similar constraints cannot be obtained in MSSM. Hence our calculation has more predictive power.

Recently several authors had used this model for various calculations [7, 8]. In the present calculations, with mAMSB model, we need to fix the range of parameters namely \(m_{3/2}\), \(\tan \beta\), \(m_0\) and the sign of \(\mu\). We have chosen Ref. [7] for this purpose. Datta et al in Ref. [7] give the allowed parameter space for the mAMSB model for \(\tan \beta = 5\) in Fig 1. In Tables 1 and 2 of the same reference, Datta et al furnished some indicative values of the parameters (including lower bounds on \(m_0\) and upper bounds on \(m_{3/2}\)) from their calculations. Using them we have chosen the following ranges for \(\tan \beta\) and \(m_0\).

\[
5 \leq \tan \beta \leq 15
\]
\[
405 \text{ GeV} \leq m_0 \leq 905 \text{ GeV}
\]  

The range of the parameter \(m_{3/2}\) (in TeV) for each choice of \(m_0\) is fixed by the allowed region in Fig 1 of Ref. [7]. The sign of \(\mu\) is taken to be negative.
We attempt here to calculate the scalar cross-section for different WIMP mass. The WIMP here is neutralino dark matter. The neutralino mass and neutralino nucleon scalar scattering cross-sections are calculated in the framework of mAMSB model.

Bottino et al, who have done many pioneering work in dark matter calculations gave an expression for neutralino-nucleon scalar scattering cross-section \([9, 10]\). This expression (Eq. 2 of [9] and Eq. 1 of [10]) reads as

\[
\sigma_{\text{scalar}} = \frac{8G_F^2}{\pi} M_Z^2 m_{\text{red}}^2 \left[ \frac{F_h I_h}{m_h^2} + \frac{F_H I_H}{m_H^2} + \text{squark exchange term} \right]^2
\]  

(3)

The two terms in bracket in the above equations are \(h\) and \(H\) exchange terms where \(h\) and \(H\) are CP even neutral Higgs bosons.

\[
F_h = (-a_1 \sin \theta_W + a_2 \cos \theta_W)(a_3 \sin \alpha + a_4 \cos \alpha)
\]

\[
F_H = (-a_1 \sin \theta_W + a_2 \cos \theta_W)(a_3 \cos \alpha + a_4 \sin \alpha)
\]

\[
I_{h,H} = \sum_q k_q^{h,H} m_q \langle N|\bar{q}q|N \rangle
\]

(4)

In the above \(a_1, a_2, a_3, a_4\) are the coefficients as shown in Eq.(1), the angle \(\alpha\) is the Higgs mixing angle that rotates \(H_1^0\) and \(H_2^0\) into \(h\) and \(H\), \(m_h\) and \(m_H\) are the masses of \(h\) and \(H\) respectively, \(m_{\text{red}}^2\) is the neutralino-nucleon reduced mass. The last term in Eq. (3) can be written as [9]

\[
I_{h,H} = k_{u-\text{type}}^{h,H} g_u + k_{d-\text{type}}^{h,H} g_d
\]

(5)

where \(k_{u-\text{type}}^{h} = \cos \alpha / \sin \beta, k_{d-\text{type}}^{h} = -\sin \alpha / \cos \beta, k_{u-\text{type}}^{H} = -\sin \alpha / \sin \beta, k_{d-\text{type}}^{H} = -\cos \alpha / \cos \beta\) [9]. The values of \(g_u\) and \(g_d\), in terms of pion-nucleon sigma term, \(\sigma_{\pi N}\), are given in Ref. [9, 10]. In the present calculation, for simplicity, we deal only with Higgs mediated terms. Also it is pointed out in Ref. [9] that in general these terms are largely dominant over the squark exchange term.

For the calculation of scalar cross-sections (Eq. 3-5) one has to compute \(a_1, a_2, a_3, a_4\), the Higgs mixing angle \(\alpha\), the Higgs masses \(m_h\) and \(m_H\), the neutralino mass \(m_\chi\). These are calculated in the framework of mAMSB model for the range of supersymmetric parameters discussed above, using the code ISAJET 7.48 [11]. We have checked that the neutralino is the LSP. In Fig. 1 we show the variation of neutralino mass \(m_\chi\) with \(\tan \beta\) for three sets of values of \(m_0\) and \(m_{3/2}\) with \(\mu < 0\). Fig. 1 shows that there are no significant variations of \(m_\chi\) with \(\tan \beta\). In Fig. 2 we plot \(m_0\) (in GeV) vs \(m_\chi\)
for three fixed sets of values of \( m_{3/2} \) and \( \tan \beta \). These three sets are chosen by fixing \( m_{3/2} \) at a lower value 31.5 TeV with three values of \( \tan \beta \) namely 5.0, 10.0 and 15.0. It is observed from Fig. 2 that although the plot corresponding to \( \tan \beta = 5 \) is rather flat the plots for \( \tan \beta = 10 \) and 15.0 show some variations of neutralino mass with \( m_0 \). More so the mean difference of \( m_\chi \)'s corresponding to \( \tan \beta = 5 \) and \( \tan \beta = 10 \) is almost double than that of \( \tan \beta = 10 \) and \( \tan \beta = 15 \). The variation of \( m_\chi \) is therefore a collective effect of variation of the three mAMSB model parameters.

For calculation of neutralino-nucleon scalar cross-sections using Eq. (3-5), we express all the quantities in the units of GeV and the cross-sections are expressed in pico barn (pb). The values of \( g_u \) and \( g_d \) (in Eq. 5) are obtained from [9] and they are \( g_u = 123 \text{ GeV}, \ g_d = 288 \text{ GeV} \). The results of calculated scalar cross-sections, \( \sigma_{\text{scalar}} \) (in pb) for various neutralino masses (\( m_\chi \) in GeV) are shown as scatter plot in Fig. 3. The particular pattern of this scatter plot for cross-section calculations using mAMSB model is reflective of the shape of the allowed region for \( m_0 - m_{3/2} \) parameter space for mAMSB model as shown in Fig 1. of Datta et al [7].

In order to study whether these results can represent the DAMA-NaI results stated above, we also show in Fig. 3, the results obtained from DAMA-NaI experiment. They are shown as contour plots in Fig. 3. The contours in Fig. 3 (continuous plots) are obtained from Fig. 4b of Ref. [3] where they have given 3\( \sigma \) C.L. allowed regions for experiments in four different yearly cycles (DAMA-NaI-1 to DAMA-NaI-4, with total statistics of 57986 kg.day) alongwith DAMA-NaI-0 results [12]. The plots suggest that although the neutralino mass range falls within that obtained from DAMA-NaI experiments but the cross-sections are about an order of magnitude below than the DAMA contour. An attempt to find out \( \xi = \rho_\chi / \rho_\ell \) where \( \rho_\ell = 0.3 \text{ GeV/cm}^3 \) [3] using the relation \( \xi = (\xi \sigma)_{\text{DAMA}} / \sigma_{\text{calc.}} \) leads to values of \( \rho_\chi \sim 3 \text{ GeV/cm}^3 \) which is much more than the halo density \( \rho_\ell \) and thus is unphysical.

To conclude, we have considered minimal Anomaly Mediated Supersymmetry Breaking model for studying the variation of neutralino mass with the supersymmetric parameters, namely \( \tan \beta, m_0 \) and \( m_{3/2} \). We have calculated the neutralino nucleon scalar cross-sections and studied their variations with neutralino mass. Considering neutralino to be cold dark matter candidate, we went on to compare the results with those obtained from DAMA-NaI detection of annual modulation of dark matter signal. It appears that the mAMSB model calculations cannot reproduce DAMA-NaI experimental findings for WIMPs. Although earlier results from calculations within MSSM
frame-work can represent the DAMA results to some extent, for mAMSB model it is
not so and it is due to the fact that the parameter space of mAMSB model is somewhat
different from MSSM. As discussed earlier, mAMSB model has only four parameters
and \( m_0 - m_3/2 \) parameter space is severely constrained. This affects the calculation of
the coefficients \( a_1, a_2 \) etc. and the value of \( \alpha \). The scalar cross-sections thus obtained
using Eqs. (3 - 5) for different \( m_\chi \)’s are therefore different from those obtained
in MSSM. We repeat our calculations within the framework of mAMSB model with dif-
ferent \( g_u, g_d \) values given in Table 2 of a more recent work by Bottino et al [10]. We find
these cross-sections fall by orders of magnitudes from those calculated using [9]. These
results are further off from the results of DAMA-NaI experiments. We have plotted
these results in Fig. 4 for demonstration. The calculation of neutralino relic density in
this model requires the calculations of neutralino-neutralino annihilation cross-sections
and these will constitute our future work.

The author is grateful to Anindya Dutta for introducing him to the computer code
ISAJET and for many valuable discussions. The author also thanks Amitava Datta
for some useful discussions.

References

[1] K.A. Drukier et al., Phys. Rev. D33 (1986) 3495.

[2] K. Freese et al, Phys. Rev. D37 (1988) 3388.

[3] R. Bernabei et al, Phys. Lett. B480 (2000) 23 and references therein.

[4] H.E. Haber and G.L. Kane, Phys. Rep. 117 (1985) 75.

[5] L. Randall, R. Sundrum, Nucl. Phys. B557 (1999) 79.

[6] G.F. Giudice, M.A. Luty, H. Murayama, R. Rattatzi, JHEP 12 (1998) 027; A.
Pamoral, R. Rattazzi, ibid 9905 (199) 013; R. Rattazzi, A Strumia, J.D. Wells,
Nucl. Phys. B576 (2000) 3.

[7] A. Datta, A. Kundu, A. Samanta, Phys. Rev. D64 (2001) 095016, e-print no.
hep-ph/0101034 and references therein.

[8] D.K. Ghosh, A. Kundu, P. Roy, S. Roy, e-print no. hep-ph/0104217.
[9] A. Bottino, F. Donato, N. Fornengo, S. Scopel, Phys. Rev. D59 (1999) 095003.

[10] A. Bottino, F. Donato, N. Fornengo, S. Scopel, e-print no. hep-ph/0111229.

[11] H. Baer, F.E. Paige, S.D. Protopopescu, X. Tata, BNL-HET-99-43, FSU-HEP-991218, UH-511-952-00, (1999), e-print no. hep-ph/0001086.

[12] R. Bernabei et al, Phys. Lett. B389 (1996) 757
Figure Captions

Fig. 1 Variation of neutralino mass $m_\chi$ (in GeV) with $\tan \beta$ in mAMSB model for three sets of values of $m_0$ (in GeV) and $m_{3/2}$ in TeV. $\mu < 0$.

Fig. 2 Variation neutralino mass $m_\chi$ (in GeV) with $m_0$ in GeV for three sets of values of $m_{3/2}$ and $\tan \beta$. See text for detail.

Fig. 3 Variation of neutralino nucleon scalar cross-section $\sigma_{\text{scalar}}$ in pb (pico barn) with neutralino mass $m_\chi$ in (GeV) (scatter plot) and their comparison with DAMA-NaI results (see text for details). For DAMA-NaI results (closed contours) $\xi\sigma$ in pb is plotted along the Y-axis while for the results of present calculations $\sigma_{\text{scalar}}$ in pb are plotted along Y-axis.

Fig. 4 Comparison of cross-section ($\sigma_{\text{scalar}}$ in pb) calculations for different sets of values for $g_u$ and $g_d$. The symbols 'X' and circle corresponds to data sets (a) and (b) respectively in Table 2 of [10], the symbol '.' (dots) are for the same sets as in Fig. 3.
Fig. 1

- \( m_0 = 755.0 \text{ GeV}, \ m_{3/2} = 43.5 \text{ TeV} \)
- \( m_0 = 605.0 \text{ GeV}, \ m_{3/2} = 36.5 \text{ TeV} \)
- \( m_0 = 455 \text{ GeV}, \ m_{3/2} = 31.5 \text{ TeV} \)
Fig. 3
Fig. 4