Are black holes in an ekpyrotic phase possible?

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Abstract The ekpyrotic phase (a slow contraction cosmological phase before the current expansion phase) manages to solve the main problems of the standard cosmology by means of a scalar field interpreted as a cosmic fluid in the Friedmann equation. Moreover, this phase generates a nearly scale-invariant spectrum of perturbations in agreement with the latest data. Then, the ekpyrotic mechanism is a serious possibility to the inflationary model. In this work—by using the approach of deforming metrics at linear level—we point out that it is impossible to generate a black hole with spherical symmetry supported by an isotropic fluid in this cosmological scenario.

Keywords Black holes, Cosmology

1 Introduction

The inflationary mechanism is still the most popular alternative to solve the main problems of the standard cosmology: the flatness, homogeneity, horizon and isotropy problems. But this mechanism is not the only one. The ekpyrotic mechanism manages to solve these problems by means of a previous phase of contraction before the current phase of expansion (see Lehners (2008) for a review). In this slow contraction phase, the universe is dominated by a stiff fluid with a determined equation of state (EoS): \( w \equiv \frac{p}{\rho} > 1 \) (\( p \) and \( \rho \) are pressure and energy density, respectively). With this fluid, the universe became flat, homogeneous and isotropic before the expansion phase, and the seeds of the structure formation were generated during this contraction phase. These seeds, generated by quantum fluctuations, are nearly scale-invariant and are in agreement with the latest data (Hinshaw et al. 2013; Ade et al. 2014, 2015a,b; Lehners and Steinhardt 2013).

Among the researches of the ekpyrotic universe, there exists the belief that this mechanism solves the problems of the standard cosmology as well as avoids the problems of the inflationary paradigm (Ijjas and Steinhardt 2013). According to these researches, the inflationary mechanism has two important problems: the initial conditions and multiverse-unpredictability. On the other hand, the ekpyrotic cosmology does not have both problems and—with the aid of other ingredients, for example, the Galileon cosmology and ghost condensate (Osipov and Rubakov 2013; Battarra et al. 2014; Qiu et al. 2013)—manages to avoid another issue in standard cosmology: the problem of the initial singularity. As we can see, the ekpyrotic cosmology offers us an interesting option to the inflation.

Moreover, the ekpyrotic cosmology may be extended to a cyclic cosmology. In this cyclic scenario (Steinhardt and Turok 2002), one has an endless sequence where the universe expands and contracts. There is no beginning for the time—therefore, the problem of initial conditions is solved in this extended version. The ekpyrotic mechanism is able to remove debris generated in a previous cycle, such as black holes (BHs). Thus the existence of such a mechanism is crucial to build cyclic cosmologies.

Any modern scientific cosmology has the isotropy as a feature. From the beginning of the modernity, the cosmology assumes that there is no favourite place or direction in the universe (contrary to both the Plato’s cosmology in his Timaeus and the Aristotle’s view in On the Heavens). The Physics, from this time, describes the universe as isotropic at large scales. Hence,
the current scientific cosmology, an Einsteinian cosmology, is assumed to be isotropic.

In this work, by using a determined approach, we intend to show that BHs supported by an isotropic fluid exclude an EoS given by the ekpyrotic phase. From the mechanism developed in Casadio et al. (2002), Bronnikov et al. (2003, 2013), Molina and Neves (2010), Molina and Neves (2012), Neves and Molina (2012), Molina et al. (2014) and generalized in Molina (2013), we study, using the Schwarzschild metric as ansatz, the family of deformed BHs with spherical symmetry supported by an isotropic fluid obtained in Molina et al. (2011). We point out that these solutions are forbidden when supported by a stiff fluid ($w > 1$). This result may be interpreted as an indirect indication of the ekpyrotic phase as a period that removes inhomogeneities, anisotropies and debris of a previous cosmic phase in cyclic cosmologies that use the ekpyrotic mechanism.

The structure of this paper is presented in the following: in Section 2 we presented the ekpyrotic mechanism and its main features; in Section 3 we show the solutions, supported by a isotropic fluid, constructed by means of deformations in the Schwarzschild solution; in Section 4, the final remarks. We adopt the metric signature \textit{diag}($-+++$) and $G = c = 1$, where $G$ is the Newtonian constant, and $c$ is the speed of light in vacuum.

## 2 The ekpyrotic phase

Such as the standard Big Bang cosmology, the ekpyrotic cosmology assumes a Friedmann-Lemaître-Robertson-Walker (FLR W) metric as a solution of the gravitational field equations to describe the spacetime fabric. In the first version Khoury et al. (2001), a brane world model was used. But, some years later, a four-dimensional version, an effective model, was introduced in Buchbinder et al. (2007). Assuming the FLR W metric

$$ds^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right],$$  \hspace{1cm} (1)

where $a(t)$ is the scale factor, which depends on the cosmic time $t$, $k$ determines the spatial curvature ($k = 0$, flat universe; $k = -1$, open universe; $k = 1$, closed universe), and the cosmic matter-energy content is described by a perfect fluid $T_{\mu}^\nu = \text{diag}(-\rho, p, p, p)$, the generalized Friedmann equation reads

$$H^2 = \frac{1}{3} \left( -\frac{3k}{a^2} + \frac{\rho_m}{a^3} + \frac{\rho_\phi}{a^4} + \frac{\sigma^2}{a^6} + \ldots + \frac{\rho_\phi}{a^{6(1+w_\phi)}} \right),$$

with $H = \dot{a}/a$, where dot represents a derivative w.r.t. time $t$; $\rho_m$ and $\rho_\phi$ mean the energy densities of non-relativistic matter (including dark matter) and radiation, respectively. The term which depends on the $a^{-6}$ indicates the energy density of anisotropies. Lastly, $\rho_\phi$ denotes the energy density of the scalar field that generates the ekpyrotic phase—interpreted as a perfect fluid in the Friedmann equation. Such a field has the action

$$S_\phi = \int d^4x \sqrt{-g} \left( -\frac{1}{2} g^{\mu\nu} \partial_{\nu} \phi \partial_{\mu} \phi - V(\phi) \right),$$  \hspace{1cm} (3)

where $g$ is the metric determinant, and an EoS written as

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{1}{3} \sigma^2 + V(\phi).$$  \hspace{1cm} (4)

Typically, the potential is given by $V(\phi) = -V_0 e^{-c \phi}$, with $V_0$ and $c$ playing the role of constants. A very steep potential of this type produces an EoS for the ekpyrotic phase with a dominant fluid given by $w_\phi > 1$. Such a matter content is dominant in Eq. (2) during a contraction phase ($a^{-n}$, with $n > 6$). The term that describes the anisotropies, $\sigma^2/a^6$, is not dominant over the last term in Eq. (2). Then, the slow contraction phase, where the stiff fluid is dominant over other matter contents, produces a homogeneous and isotropic universe, free of the Belinsky-Khalatnikov-Lifshitz (BKL) instabilities, before the current expansion phase. The authors in Garfinkle et al. (2008) have shown that the ekpyrotic mechanism works even when the initial conditions of this slow contraction phase are highly inhomogeneous and anisotropic.

However, the standard ekpyrotic mechanism with one scalar field is considered tachyonically unstable (see, for example, Mingzhe (2013)). To solve this problem, another scalar field is introduced to generate a stable mechanism with nearly scale-invariant perturbations and small non-Gaussianity (Ijjas et al. 2014).

In the next section, we will show how to build BH solutions supported by an isotropic fluid. To obtain these BHs, the approach which deforms BHs will be presented. Contrary to the work Garfinkle et al. (2008), in this work, we do not study the initial conditions of the ekpyrotic phase and its evolution. In this article, we assume an isotropic fluid (as a linear constraint in the deformation approach) and show that an ekpyrotic phase does not support these deformed BHs. That is, an isotropic universe with EoS $w > 1$, or an ekpyrotic phase where the universe is already isotropic (close to the bounce), does not support the deformed geometries. The novel part of this work is the use of the deformation approach to show these results.
3 Constructing isotropic black holes

It is possible to build isotropic BH solutions by means of deformations. From the standard singular solutions, such as Schwarzschild, Reissner-Nordström, Schwarzschild-(A)-dS and others (Casadio et al. 2002; Bronnikov et al. 2003; Casadio and Molina 2010, 2012; Neves and Molina 2012; Molina et al. 2011), and regular solutions (Neves 2015a, b), at linear level, deformed BHs are obtained by imposing a linear constraint in the energy-momentum tensor (the approach was generalized in Molina (2013), where other constraints were used). These deformed solutions are close to the standards BHs. A linear constraint in the energy-momentum tensor to generate isotropic solutions means:

\[ p_r = p_t, \]  

where

\[ R_{\nu}^{\mu} - \frac{1}{2} R \delta_{\nu}^{\mu} = 8 \pi T_{\nu}^{\mu} = 8 \pi \begin{pmatrix} -\rho & p_r \\ p_r & p_t \end{pmatrix}. \]  

(5)

The subscript \( \text{lin} \) stands for linear, and \( C - 1 \) may be seen as the first-order expansion coefficient. Then, we want to obtain deformed solutions until the linear level (in Molina et al. (2011) there is a discussion on the possibility of generating deformed solutions beyond the linear level). According to Molina (2013), the constant \( C \) fixes basically three kind of solutions: for \( C > 0 \) one has either regular or singular black holes, \( C < 0 \) may determine wormholes, and for \( C = C_0 \), one has extremal black holes. For \( C = 1 \) the standard cases are restored.

In the linear regime, the function \( B_{\text{lin}}(r) \) is determined from

\[ B_{\text{lin}}(r) = A_0(r) \exp \left[ -\int \frac{dr}{rh(r)} \right], \]  

which is solution of Eq. \( (3) \) using the ansatz \( (9)-(10) \), where

\[ h(r) = -\frac{1}{2} A_0(r) - \frac{1}{4} r A_0'(r). \]  

(12)

The zeros of \( h(r) \) are important because may determine the singularities of the spacetime. In this work, \( h(r) \) has one real root, \( r_0 = m \). Using the ansatz \( (9) \) in the Eqs. \( (11)-(12) \), one has

\[ B(r) = A_0(r) \left[ 1 + (C - 1) (r - m)^2 \right], \]  

(13)

and the metric \( (7) \) is completely determined. The zeros of \( B(r) = 0 \) fix the horizons (when a metric is written in the form \( (7) \), the localization of horizons is given by the zeros of \( g^{rr} = B(r) = 0 \)). According to Molina et al. (2011), BHs are possible only with \( C > 0 \) (our value of \( C \) corresponds to the \( C/m^2 + 1 \) in the cited paper).

For \( C > 1 \), the function \( B(r) \) has two zeros, \( r_- < r_+ \). The largest zero corresponds to the event horizon \( r_+ = 2m \). The metric is singular at \( r = r_0 < r_- \). Then, the spacetime structure reads

\[ r_0 < r_- < r_+ < \infty. \]  

(14)

This case corresponds to a singular BH in a noncompact universe.

When \( 0 < C < 1 \), the situation is quite different: there are three zeros in \( B(r) \) (the third zero is not zero of \( A(r) \)). There is a maximum value of \( r \left( r_{\text{max}} > r_+ \right) \). According to Molina et al. (2011), these values of \( C \) lead to a singular BH in a compact universe. The spacetime structure is given by

\[ r_0 < r_- < r_+ < r_{\text{max}}, \]  

(15)
with
\[
\lim_{C\to 1} r_{\text{max}} = \infty.
\] (16)

Then, these are the two types of solutions when the ansatz is given by Eqs. (9)-(10) assuming isotropy.

With \(A(r)\) and \(B(r)\) fixed, the values of the components of the energy-momentum tensor are:
\[
\rho = -\frac{(C - 1)(r - m)(3r - 5m)}{r^2},
\] (17)
\[
p_r = p_t = p = \frac{(C - 1)(r - m)^2}{r^2}.
\] (18)

With these components in the energy-momentum tensor, an EoS is possible:
\[
w = \frac{p}{\rho} = -\frac{r - m}{3r - 5m},
\] (19)

which is almost constant and always negative outside the event horizon (see Fig. 1). For large \(r\), \(w\) is about \(-1/3\). In the case \(C > 1\), one has the simple limit
\[
\lim_{r\to\infty} w = -\frac{1}{3}.
\] (20)

At the event horizon, one has \(w(r_+) = -1\). In this sense, these deformed metrics may not be supported (such as the Schwarzschild-(A)-dS metric is by a fluid with EoS \(w = -1\)) by a stiff fluid \((w > 1)\), dominant in the ekpyrotic phase. Then, at linear level, BHs supported by an isotropic stiff fluid are ruled out. But not only a stiff fluid: as we can see, these metrics are not supported by either a fluid with EoS \(w = 1/3\) (radiation) or a phantom-like fluid \((w < -1)\). Outside the event horizon, the EoS assumes \(-1 < w < -1/3\), which are values corresponding to a quintessence fluid.

![Fig. 1](image-url) \[W = \frac{p}{\rho} \text{ obtained from isotropic deformed solutions.}

As we can see, this EoS is pretty much constant outside the event horizon and rules out a stiff fluid \((w > 1)\), dominant in the ekpyrotic phase. The dashed line indicates the event horizon. In this graphic we use \(m = 11\).

supported by a stiff fluid in the same sense that the Schwarzschild-(A)-dS metrics may be interpreted as solutions with spherical symmetry supported by a fluid with EoS \(w = -1\). We believe that this result may be seen as an indirect indication (it is not a strong proof) of the ability of the ekpyrotic phase to leave the universe free of inhomogeneities and debris (and BHs may be interpreted as debris) in a previous phase before the current phase of cosmic expansion. This result may be important for cosmological cyclic models that assume an ekpyrotic phase.

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4 Final remarks

The ekpyrotic cosmology is an option for the inflationary mechanism to solve the problems of the standard cosmology (including the initial singularity) and to generate nearly scale-invariant quantum fluctuations. This alternative cosmology is characterized by a slow phase of contraction, where the equation of state (EoS) of the matter-energy content in this phase is dominated by a stiff fluid \((w \equiv \frac{p}{\rho} > 1)\).

We show that—by means of a procedure of generating deformed isotropic black holes (BHs) at linear level in \(B(r)\), adopting a linear constraint in the energy-momentum tensor—the metrics with spherical symmetry obtained from the Schwarzschild metric are not supported by a stiff fluid. These metrics are not

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