Double Threefold Degeneracies for Active and Sterile Neutrinos

Ernest Ma\textsuperscript{a} and G. Rajasekaran\textsuperscript{b}

\textsuperscript{a} Physics Department, University of California, Riverside, California 92521

\textsuperscript{b} Institute of Mathematical Sciences, Chennai (Madras) 600113, India

Abstract

We explore the possibility that the 3 active (doublet) neutrinos have nearly degenerate masses which are split only by the usual seesaw mechanism from 3 sterile (singlet) neutrinos in the presence of a softly broken $A_4$ symmetry. We take the unconventional view that the sterile neutrinos may be light, i.e. less than 1 keV, and discuss some very interesting and novel phenomenology, including a connection between the LSND neutrino data and solar neutrino oscillations.
Present experimental data [1, 2, 3] indicate that neutrinos oscillate. Hence they should have small nonzero masses and mix with one another. On the other hand, since neutrino oscillations only measure the difference of mass squares, the possibility that all 3 active neutrinos are nearly degenerate in mass should not be overlooked [4]. There are two canonical ways of making $m_\nu$ nonzero. One is through the small vacuum expectation value (VEV) of a Higgs triplet [5, 6]. The other is through the addition of 3 heavy singlet neutral fermions (usually considered as right-handed neutrinos $N_R$). In that case, a Dirac mass $m_D$ linking the left-handed doublet neutrinos $\nu_L$ with $N_R$ as well as a Majorana mass $M$ for $N_R$ are allowed. Combining the two mechanisms, the following mass matrix

$$M_{\nu N} = \begin{pmatrix} m_0 & m_D \\ m_D & M \end{pmatrix}$$

(1)

is obtained. The eigenvalues are simply $m_0 - m_D^2/M$ and $M$. Without $m_0$ (which comes from the VEV of the Higgs triplet), this is just the famous seesaw mechanism [7] for a small neutrino mass. The singlet $N_R$ is too heavy to be detected experimentally, unless $m_D$ comes from a different Higgs doublet with a suppressed VEV, in which case $M$ may in fact be only a few TeV or less and become observable at future colliders. Using the model of Ref.[8], it has also been shown [9] that the possibly large observed discrepancy of the muon anomalous magnetic moment [10] may be explained, provided that the 3 active neutrinos are in fact nearly degenerate in mass, in order not to conflict with the present experimental bound on $\tau \rightarrow \mu \gamma$.

In this paper we consider the case where both $m_D$ and $M$ are small, but $m_D$ is still less than $M$ by perhaps an order of magnitude. This is in contrast to the pseudo-Dirac scenario [11], i.e. $m_0, M << m_D$, in which case neutrino oscillations would be maximal between active and sterile species, in disfavor with the most recent data [1, 2]. We also supplement our model with a discrete $A_4$ symmetry [12, 13] which maintains the separate degeneracies of the 3 active and 3 sterile neutrinos. This $A_4$ is then broken spontaneously and softly to
allow for realistic charged-lepton masses as well as neutrino mass differences as in Ref. [12].
The new idea here is that the 3 sterile neutrinos could be light and help to account for the LSND data [3] as shown below.

Before discussing the theoretical reasons for $m_0$, $m_D$, and $M$ to be small, consider first the phenomenology of such a possibility. The 3 active neutrinos $\nu_e$, $\nu_\mu$, $\nu_\tau$ are now each a linear combination of 6 light neutrino mass eigenstates. With $m_D$ less than $M$ by an order of magnitude, the mixing of $N$ with $\nu$ is still small; hence the presumably large mixings among the 3 active neutrinos themselves are sufficient to explain the atmospheric [1] and solar [2] neutrino data. This leaves the LSND data [3] to be explained by the mixing of $\nu$ with $N$.

Consider Eq. (1) as a $6 \times 6$ matrix with $m_0$ and $m_D$ representing $3 \times 3$ unit matrices, as required by the $A_4$ symmetry. The soft breaking of $A_4$ means that $M$ may differ slightly from the unit matrix, so that in the basis under which it is diagonal, $M_{\nu N}$ is given by

$$M_{\nu N} = \begin{bmatrix} m_0 & 0 & 0 & m_D & 0 & 0 \\ 0 & m_0 & 0 & 0 & m_D & 0 \\ 0 & 0 & m_0 & 0 & 0 & m_D \\ m_D & 0 & 0 & M_1 & 0 & 0 \\ 0 & m_D & 0 & 0 & M_2 & 0 \\ 0 & 0 & m_D & 0 & 0 & M_3 \end{bmatrix}. \quad (2)$$

The $\nu_e, \nu_\mu, \nu_\tau$ basis is now rotated into the $\nu_{1,2,3}$ basis, and we may assume whatever pattern is suitable for explaining the atmospheric and solar neutrino data. To be specific, consider bimaximal mixing, i.e.

$$\begin{bmatrix} \nu'_1 \\ \nu'_2 \\ \nu'_3 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/2 & -1/2 \\ -1/\sqrt{2} & 1/2 & -1/2 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}. \quad (3)$$

Then the eigenstates of $M_{\nu N}$ are

$$\nu_i = \nu'_i \cos \theta_i - N_i \sin \theta_i, \quad S_i = \nu'_i \sin \theta_i + N_i \cos \theta_i, \quad (4)$$
where \( \sin \theta_i \simeq m_D/M_i \), corresponding to the eigenvalues \( m_0 - m_D^2/M_i \) and \( M_i \) respectively. Since \( M_1 \simeq M_2 \simeq M_3 \) is still assumed, we have

\[
\Delta m_{ij}^2 = \left( m_0 - m_D^2/M_i \right)^2 - \left( m_0 - m_D^2/M_j \right)^2 \simeq \frac{m_\nu m_D^2}{M^2} \Delta M_{ij}^2, \quad (5)
\]

where \( M = (M_1 + M_2 + M_3)/3 \) and \( m_\nu = m_0 - m_D^2/M \).

Consider now the effect of \( S_i \) on \( \nu_\mu \to \nu_e \) oscillations. The well-known expression for this probability is given by

\[
P(\nu_\mu \to \nu_e) = -4 \sum_i U_{\mu i} U_{\nu i} \sum_{j>i} U_{\mu j} U_{e j} \sin^2 \left( \frac{\Delta m_{21}^2 L}{4E} \right). \quad (6)
\]

For the \( L/E \) values appropriate to the LSND experiment, \( \Delta m_{ij}^2 \) is effectively zero between \( \nu_i \) and \( \nu_j \). Naively, we might expect the contribution from \( \Delta m_{ij}^2 \) between \( S_i \) and \( \nu_j \), i.e. \( M^2 - m_\nu^2 \) to be dominant, but that turns out to be negligible. Specifically,

\[
\sum_{j=4,5,6} U_{\mu j} U_{e j} = - \sum_{i=1,2,3} U_{\mu i} U_{e i} = \frac{1}{2\sqrt{2}} (\sin^2 \theta_1 - \sin^2 \theta_2) \simeq \frac{1}{2\sqrt{2}} \left( \frac{\Delta m_{21}^2}{m_\nu M} \right) \simeq 0, \quad (7)
\]

where Eq. (5) has been used. Hence the main contribution to Eq. (6) is actually coming from \( i = 4 \) and \( j = 5 \), i.e.

\[
P(\nu_\mu \to \nu_e) = \frac{1}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \left( \frac{\Delta M_{21}^2 L}{4E} \right). \quad (8)
\]

This means that the neutrino mass difference being probed by the LSND experiment is that between \( S_1 \) and \( S_2 \), and not that between \( \nu_e \) or \( \nu_\mu \) and \( S_i \). To understand this interesting new phenomenon, we note that if the \( M_i \)'s were equal, then \( \nu_e, \nu_\mu, \nu_\tau \) would all be exactly degenerate in mass and there could not be any \( \nu_\mu \to \nu_e \) oscillation; thus any such effect must be proportional to the difference in the \( M_i \)'s and not to the difference between \( M \) and \( m_\nu \).

To fit the LSND data, we take \( \Delta M_{21}^2 \simeq 1 \text{ eV}^2 \) and \( \sin^2 \theta_1 \simeq \sin^2 \theta_2 \simeq 0.06 \), so that \( \sin^2 \theta_{\text{eff}} \simeq 1.8 \times 10^{-3} \). In that case, Eq. (5) relates them to \( \Delta m_{21}^2 \) which may be as large as \( 1.2 \times 10^{-4} \text{ eV}^2 \) as indicated by the solar data. We then obtain \( m_\nu/M \simeq 2 \times 10^{-3} \). If we take
$m_\mu \simeq 0.2 \text{ eV}$, then $M \simeq 0.1 \text{ keV}$. Taking $\Delta m^2_{32}$ to be as small as $1.2 \times 10^{-3} \text{ eV}^2$ as indicated by the atmospheric data, we obtain $\Delta M^2_{32} \simeq 10 \text{ eV}^2$ and $\sin^2 2\theta_{\text{eff}} \simeq 3.6 \times 10^{-3}$ for $\nu_\mu \rightarrow \nu_\tau$ oscillations in the CHORUS [14] and NOMAD [15] experiments, which are just beyond their exclusion boundaries.

We now give the theoretical details of our model. In addition to $A_4$, we define 2 global $U(1)$ symmetries: $U(1)_L$ and $U(1)_S$. Under these symmetries, the leptons transform as follows: $(\nu_i, l_i)_L \sim (3, 1, 0), \ l_1R \sim (1', 1, 0), \ l_2R \sim (1'', 1, 0), \ N_iR \sim (3, 1, 1)$. There are 4 scalar doublets: $\Phi_i = (\phi_i^+, \phi_i^0) \sim (3, 0, 0)$, $\eta = (\eta^+, \eta^0) \sim (1, 0, 1)$, and 1 scalar triplet: $\xi = (\xi^+, \xi^0, \xi^0) \sim (1, -2, 0)$. The soft terms $\Phi_i^\dagger \eta$ break $U(1)_S$ and $A_4$, $\xi^\dagger \Phi_i \Phi_j$ break $U(1)_L$ and $A_4$, $N_iN_j$ break $U(1)_L$, $U(1)_S$, and $A_4$, hence they may all be naturally small [16]. Note that the smallness of $M$ is protected by both $U(1)_L$ and $U(1)_S$. Assuming $m_\xi^2$ and $m_\eta^2$ to be positive and large, we then obtain $\langle \xi^0 \rangle$ and $\langle \eta^0 \rangle$ to be small [6, 8] for the terms $m_0$ and $m_D$ respectively.

Since the 3 sterile neutrinos have masses of order 0.1 keV and their decay lifetimes (through their mixing with the active neutrinos) are much greater than the age of the Universe, they would overclose the Universe unless their relic abundance is greatly reduced. This may be achieved in our scenario because $N_iR$ have no gauge interactions and the only Yukawa couplings they have are given by

$$L_{\text{int}} = fN_iR(\nu_i \eta^0 - l_i \eta^0) + h.c.$$  \hspace{1cm} (9)

Hence they decouple from the standard-model particles at the scale $M_\eta$ which we take to be 1 TeV, and whereas $\nu_{e,\mu,\tau}$ are heated by the subsequent annihilations of nonrelativistic particles, $N_iR$ are not [17]. Thus the number densities of the latter are greatly suppressed until the onset of electroweak symmetry breaking. However, the mixing of $N_iR$ with $\nu_i$ is then large enough to produce the former through active-sterile neutrino oscillations and we are faced again with the abundance problem, together with the nucleosynthesis bound of the
effective number of neutrinos $n_\nu$ which is restricted to be less than 4 \[18\]. To evade these cosmological problems, we need $N_{IR}$ to decay quickly as proposed previously \[19\], but at the expense of the unnatural fine tuning of parameters.

In conclusion, we have constructed in this paper a specific model of 3 active (doublet) and 3 sterile (singlet) neutrinos, which are separately threefold degenerate in mass approximately. We find the new and interesting result that neutrino oscillations between active species are governed by the mass differences among the 3 lighter neutrinos and the parallel mass differences [see Eq. (5)] among the 3 heavier neutrinos, but not the mass difference between the two groups. If bimaximal mixing is assumed for explaining the atmospheric and solar neutrino oscillations (using the matter-enhanced solution for the latter), then a possible explanation of the LSND data is the nonnegligible mixing ($\sin^2 \theta \simeq 0.06$) between active and sterile neutrinos. This would then imply sterile neutrino masses of order 0.1 keV (if active neutrino masses are around 0.2 eV) and $\Delta m^2 \simeq 10 \text{ eV}^2$ for the CHORUS and NOMAD experiments with $\sin^2 \theta_{eff} \simeq 3.6 \times 10^{-3}$, which is just barely consistent with their exclusion limits. New data from future long-baseline and medium-baseline neutrino-oscillation experiments will be decisive in confronting this possible 3+3 scenario.

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