Chiral-Odd Structure Function $h_1^D(x)$ and Tensor Charge of the Deuteron.

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Abstract

The chiral-odd structure function $h_1^D(x)$ and the tensor charge of the deuteron are studied within the Bethe-Salpeter formalism for the deuteron amplitude. Utilizing a simple model for the nucleon structure function, $h_1^N$, $h_1^D(x)$ is calculated and the nuclear effects are analyzed.

1 Introduction

There is an increasing interest in the transverse quark distributions in the nucleon, especially the chiral-odd structure function of twist two $[1,2]$, $h_1^N(x)$, and its first moment, the tensor charge of the nucleon $[3,4,5,6,7,8]$. The reasons for the interest are the following. First, $h_1^N(x)$, complimentary to the conventional structure functions $F_{1,2}^N$ and $g_{1,2}^N$, is essential to understand the nucleon spin substructure. At the same time, the tensor charge, $g_T^N$, like the axial charge, is one of the fundamental observables for the nucleon. So far, however, little is known about both $h_1^N(x)$ and $g_T^N$. Second, semi-inclusive deep-inelastic experiments aiming to measure $h_1^N$, and hence the tensor charge of the proton and neutron, $g_T^N$, are being planned $[9]$. Now is the time for the theory to make predictions.

It is important to remember that experimental results for the neutron can only be obtained from the indirect experiments, using the the lightest nuclear targets, the deuteron and $^3He$. Therefore, it is necessary to have a reliable estimate of the nuclear effects, in order to extract dependable information about the neutron from the nuclear data.

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In this letter we study the chiral-odd structure function and the tensor charge of the deuteron, \( h_D^1(x) \) and \( g_T^D \), in the Bethe-Salpeter formalism. The main motivation is to study the nuclear effects in the transverse-polarized deuteron structure function, \( h_D^1 \), and the deuteron tensor charge, \( g_T^D \). Since the nucleon tensor charge measures the net number of transversely polarized valence quarks (quarks minus antiquarks) in a transversely polarized nucleon [4], and so it gets no contribution from the sea-quarks, the neutron isoscalar tensor charge is equal to the one of the proton. As a result, the measurement of the deuteron tensor charge may, in principle, provide a test for the models of the nuclear effects if the proton tensor charge is known precisely. In addition, an independent measurement of the neutron’s \( h_n^1 \) and \( g_T^n \) is obviously useful to understand the transverse spin distribution in the nucleon. Therefore it is essential to account for the nuclear effects in the deuteron for both the chiral-odd structure function \( h_D^1 \) and the tensor charge, \( g_T^D \).

2 Basic formulae

The chiral-odd structure function of the deuteron of twist two, \( h_D^1(x) \), is defined as a simple extension of the case of spin-half hadrons [2]. It can be expressed in terms of a matrix element of the quark bilocal operator:

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P_D S_D | \bar{\psi}(0)i\sigma^{\mu\nu}\gamma_5\psi(\lambda n) | P_D S_D \rangle = 2h_D^1(x) \cdot T^{\mu\nu} + \text{(higher twist terms)}. \tag{1}
\]

An antisymmetric tensor, \( T^{\mu\nu} \), in the rest frame of the deuteron is defined by:

\[
T^{\mu\nu} = (S_D^\mu P_D^\nu - S_D^\nu P_D^\mu), \tag{2}
\]

where \( P_D^\mu \) is the deuteron momentum, \( S_D^\mu \) is the spin of the deuteron [10]:

\[
S_D^\mu(M) = -\frac{i}{M_D} e^{\mu\alpha\beta\gamma} E_\alpha(M) E_\beta(M) P_D^\gamma, \tag{3}
\]

where \( E(M) \) is the deuteron polarization, with \( M = \pm 1, 0 \), \( M_D \) is the deuteron mass. \( S_D \) satisfies the conditions, \( S_D^2 = -1 \) and \( P_D S_D = 0 \).

The tensor charge of the deuteron is the first moment of \( h_D^1(x) \) and can be readily obtained from eq. (4):

\[
\langle P_D S_D | \bar{\psi}(0)i\sigma^{\mu\nu}\gamma_5\psi(0) | P_D S_D \rangle = 2g_T^D(x)T^{\mu\nu}. \tag{4}
\]

Using the following identities in the rest frame of the deuteron:

\[
T^{\mu\nu}T_{\mu\nu}|_{M=1} = -2M_D^2 \tag{5}
\]

\[
iT^{\mu\nu}\sigma_{\mu\nu}\gamma_5|_{M=1} = -2M_D\gamma_5\gamma_3\gamma_0. \tag{6}
\]
the structure function \( h^D_1(x) \) and the charge \( g^D_T \) are rewritten as the matrix elements of the Lorentz scalar operators:

\[
\begin{align*}
  h^D_1(x) &= \frac{1}{M_D} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P_D S_D \mid \bar{\psi}(0)\gamma_5\gamma_3\gamma_0\psi(\lambda n) \mid P_D S_D \rangle, \\
  g^D_T &= \frac{1}{M_D} \langle P_D S_D \mid \bar{\psi}(0)\gamma_5\gamma_3\gamma_0\psi(0) \mid P_D S_D \rangle,
\end{align*}
\]

(7) (8)

It is impossible to solve eqs. (7) and (8) directly, since the hadron state, \( |P_D S_D\rangle \), still cannot be described in QCD. Instead, we use approach where the “elementary” structure function, \( h^N_1 \), is obtained on the quark level, while the nuclear effects are calculated on the hadron level (e.g. [11, 12] and references therein). We utilize the Bethe-Salpeter formalism, which was previously applied for studying the structure functions \( F^D_{1,2} \), \( b^D_{1,2} \) and \( g^D_1 \) [11, 13]. In what follows, we neglect small possible off-mass-shell effects in the nucleon structure functions [14, 15].

In terms of the Bethe-Salpeter amplitudes \( h^D_1 \) reads:

\[
h^D_1(x) = i \int \frac{d^4p}{(2\pi)^4} h^N_1 \left( \frac{xm}{p_{10} + p_{13}} \right) \frac{\mathrm{Tr} \left\{ \bar{\Psi}_M(p_0, p)\gamma_5\gamma_3\gamma_0\Psi_{M}(p_0, p)(\hat{p}_2 - m) \right\}_{\lambda=1}}{2(p_{10} + p_{13})},
\]

(9)

where \( m \) is the nucleon mass, \( p \) is the relative momentum of nucleons in the deuteron, \( p_{10} \) and \( p_{13} \) are the time and 3-rd components of the nucleon momentum, \( p_1 \), and \( p_{1,2} = P_D/2 \pm p \). \( \Psi_{M}(p_0, p) \) is the Bethe-Salpeter amplitude for the deuteron with the spin projection \( M \) and the isoscalar structure function \( h^N_1 \) is defined as \( h^N_1(x) \equiv (h^D_1(x) + h^N_1(x))/2 \). Note, that there is no explicit mesonic (sea quarks) contribution to the structure function \( h^D_1 \). The mesons manifest themselves by binding the nucleons in the deuteron and, therefore, defining the structure of the amplitude \( \Psi_{M}(p_0, p) \).

Then eq. (9) can be rewritten in the convolution form [17]:

\[
h^D_1(x) = \int \frac{dy}{x} \frac{h^N_1 \left( \frac{y}{x} \right)}{y} H^{N/D}(y),
\]

(10)

where \( H(y)^{N/D} \) is the “effective transverse distribution” of nucleons in the deuteron and its definition is obvious from eq. (9). This is a usual result for the nuclear structure functions calculated in the leading twist approximation, neglecting the off-mass-shell deformation of the nucleon structure function. Eqs. (9) and (10) allow immediately to write the sum rule for the deuteron tensor charge:

\[
\begin{align*}
  g^D_T &= \int_0^1 dx h^N_1(x) \cdot \int_0^{M_D/m} dy H^{N/D}(y) \\
  &= g^N_T \cdot \langle P_D S_D \mid \bar{N}(0)\gamma_5\gamma_3\gamma_0N(0) \mid P_D S_D \rangle,
\end{align*}
\]

(11) (12)
where $N(x)$ is the nucleon field. Eqs. (11) and (12) define the renormalization factor of the nucleon tensor charge by the deuteron structure. This sum rule is similar to the sum rule for the deuteron structure function, $g_1^N$ [13]:

$$
\int_0^1 dx_D g_1^D(x_D) = \int_0^1 dx g_1^N(x) \cdot \langle P_D S_D | \bar{N}(0) \gamma_5 \gamma_3 N(0) | P_D S_D \rangle.
$$

(13)

This sum rule is used for estimating the integral of the neutron structure function, $g_1^n$, from the combined proton and deuteron data.

Note that in the static limit the matrix elements on the r.h.s. of eqs. (12) and (13) are equal and approximate formulae for the deuteron structure functions are:

$$
g_1^D(x_D) \simeq \left(1 - \frac{3}{2} w_D\right) g_1^N(x),
$$

(14)

$$
h_1^D(x_D) \simeq \left(1 - \frac{3}{2} w_D\right) h_1^N(x),
$$

(15)

where $w_D$ is weight of the d-wave component in the deuteron wave function.

3 Numerical calculations

The method to calculate numerically expressions like (3) is discussed in ref. [13]. The most important details of the calculations are:

1. A realistic model for the Bethe-Salpeter amplitudes is essential for a realistic estimate of the nuclear effects. We use a recent numerical solution [11] of the ladder Bethe-Salpeter equation with a realistic exchange kernel [16].

2. The Bethe-Salpeter amplitudes and, therefore, eq. (3) have a nontrivial singular structure. These singularities must be carefully taken into account [18, 19, 20].

3. The BS amplitudes are numerically calculated with the help of the Wick rotation. Therefore, the procedure of the numerical inverse Wick rotation must be applied.

4. An essential ingredient of the calculations, the nucleon structure function $h_1^N(x)$, is unknown both theoretically and experimentally. A reasonable estimate for using in our calculations should be found.

The effects of the nuclear structure, the Fermi motion and binding of nucleons, is encoded in the effective distribution function $H^{N/D}(y)$ in eq. (10). The result of calculation of $H^{N/D}(y)$
with the Bethe-Salpeter amplitude of the deuteron \([\text{I}], \text{[II]}\) is shown in Fig. 1 (solid line). Two other effective distributions for the deuteron are also shown in Fig. 1 for comparison. The first (dashed line) is \(\vec{f}_{N/D}(y)\), which defines the spin-dependent structure function \(g_{1D}\). Explicit expression for \(\vec{f}_{N/D}(y)\) is similar to the one for \(H_{1N/D}(y)\) (see eqs. (9) and (10)), but with the operator \(\gamma_5(\gamma_0 + \gamma_3)\) instead of \(\gamma_5\gamma_3\gamma_0\). If we again replace the operator by \((\gamma_0 + \gamma_3)\), we get another distribution, \(f_{N/D}(y)\) (dotted line), which defines the spin-independent structure function \(F_{2D}\). Each of the three effective distribution functions \((H_{1N/D}(y), \vec{f}_{N/D}(y)\) or \(f_{N/D}(y))\) would represent the correspondent structure functions of the deuteron \((h_{1D}, g_{1D}\) or \(F_{2D}\)) if the nucleons in the deuteron would be the elementary fermions.

Note that the function \(H_{1N/D}(y)\) is smaller at the maximum than \(\vec{f}_{N/D}(y)\), and also slightly smaller in its normalisation:

\[
\int_0^{M_D/m} dy H_{N/D}^{N/D}(y) = \langle P_D S_D | \bar{N}(0)\gamma_5\gamma_3\gamma_0 N(0) | P_D S_D \rangle = 0.9208, \tag{16}
\]

\[
\int_0^{M_D/m} dy \vec{f}_{N/D}^{N/D}(y) = \langle P_D S_D | \bar{N}(0)\gamma_5\gamma_3 N(0) | P_D S_D \rangle = 0.9215. \tag{17}
\]

This is opposite to the case of the nucleon, where \(h_{1N}^N\) is expected \([2]\) to be larger than \(g_{1N}^N\) (after exclusion of the quark charge factors). The effect is different, since in the polarized deuteron both nucleons are essentially polarized along the same direction, while in the polarized nucleon one quark is polarized in opposite direction and somehow cancels contribution of one of the quarks polarized along the nucleon polarization. The difference between \(H_{1N/D}(y)\) and \(\vec{f}_{N/D}(y)\), and between the tensor and axial charges, \([\text{II}], \text{[III]}\) and \([\text{IV}], \text{[V]}\), is extremely small since the deuteron in essentially a nonrelativistic system and matrix elements of the operators \(\gamma_5\gamma_3\) and \(\gamma_5\gamma_3\gamma_0\) are \textit{exactly} the same in the static limit.

The numerical value in eq. (16) is the deuteron structure factor renormalizing the nucleon tensor charge in the deuteron, calculated in a particular model of the \(NN\)-interaction \([\text{VI}]\). Taking into account the differences among existing models, we can estimate a possible “model” error in (16):

\[
\langle P_D S_D | \bar{N}(0)\gamma_5\gamma_3\gamma_0 N(0) | P_D S_D \rangle = 0.93 \pm 0.015.
\]

To calculate the realistic structure function \(h_{1D}^D(x)\) we need the nucleon structure functions \(h_{1N}^N(x)\). However, so far there is no existing experimental data for this function, and very little is known about the form of \(h_{1N}^N\) in theory. In the present paper we follow the ideas of ref. \([2]\) to estimate \(h_{1N}^N\). Since the sea quarks do not contribute to \(h_{1N}^N\), its flavor content is simple:

\[
h_{1N}^N(x) = \delta u(x) + \delta d(x), \tag{18}
\]
where $\delta u(x)$ and $\delta d(x)$ are the contributions of the u- and d-quarks, respectively [2, 3]. Since the matrix elements of the operators $\propto \gamma_5 \gamma_3$ and $\propto \gamma_5 \gamma_3 \gamma_0$ coincide in the static limit, as a crude estimate we can expect that

$$\delta u(x) \sim \Delta u(x), \quad \delta d(x) \sim \Delta d(x),$$

where $\Delta u(x)$ and $\Delta d(x)$ are contributions of the u- and d-quarks to the spin of the nucleon, which is measured through the structure function $g_1^N$. Subsequently, the simplest estimate for $h_1^N$

$$h_1^N(x) = \alpha \Delta u(x) + \beta \Delta d(x),$$

with $\alpha = \beta = 1$ (21)

should not be too unrealistic. In fact, the bag model calculation shows that difference between $\delta q$ and $\Delta q$ is typically only a few percent [2]. This analysis is mostly a qualitative one, since it is limited by the case with one quark flavor and does not pretend to describe phenomenology.

To evaluate possible deviations from the simplistic choice of $h_1^N$, (20) with (21), we suggest:

$$\alpha = \delta u / \Delta u, \quad \beta = \delta d / \Delta d,$$

(22)

where $\delta q$ and $\Delta q$ are the first moments of $\delta q(x)$ and $\Delta q(x)$, respectively ($q = u, d$). For $\delta u$ and $\delta d$ we can adopt the results from the QCD sum rules and the bag model calculations [3, 4]. As to $\Delta u$ and $\Delta d$, we can use the experimental data analysis [21] or theoretical results, e.g. the QCD sum rules results [22]. Thus, we estimate

$$\alpha = 1.5 \pm 0.5, \quad \beta = 0.5 \pm 0.5,$$

(23)

at the scale of $Q^2 = 1$ GeV$^2$.

The realistic form of the distributions $\Delta u(x)$ and $\Delta d(x)$ can be taken from a fit to the experimental data for $g_1^N$. In our calculations we use parametrization from ref. [23]. At this point we have to realize that, in spite of the expected relations (19), distributions $\delta q$ and $\Delta q$ are very different in their nature. Especially at $x \lesssim 0.1$, where $\Delta q$ probably contains a singular contribution of the polarized sea quarks, but $\delta q$ does not. Therefore we expect eq. (20) to be a reasonable estimate in the region of the valence quarks dominance, say $x \gtrsim 0.1$ (see also [7]). For completely consistent analysis, the parameters $\alpha$ and $\beta$, and the distributions $\Delta u(x)$ and $\Delta d(x)$ should be scaled to the same value of $Q^2$. However, for the sake of the unsophisticated estimates we do not go into such details.
The results of calculation of the nucleon and deuteron structure functions, \( h_1^N \) (solid lines) and \( h_1^D \) (dashed lines), are shown in Fig. 2. The group of curves 1 represents case (21), which is a possible lower limit for \( h_{1,N,D}^{} \) in accordance with our estimates (23). Curves 2 represent the case \( \alpha = 1.5, \ \beta = 0.5 \), which is close to the midpoint results of the bag model and the QCD sum rules. The upper limit corresponding to the estimates (23) is presented by curves 3. For all cases the deuteron structure function is suppressed compared to the one for the nucleon, mainly due to the depolarization effect of the D-wave in the deuteron. This is quite similar to the case of the structure functions \( g_1^N \) and \( g_1^D \). To illustrate this similarity we show the ratio of the structure functions \( h_1^D / h_1^N \) (solid line) in Fig. 3 together with the ratio \( g_1^D / g_1^N \) (dotted line). The straight dash-dotted line in Fig. 3 represents the value of the matrix elements (16) and (17), which are not distinguishable on this scale. This line approximately corresponds to eqs. (14) and (15).

Note that our estimate of the nucleon structure function \( h_1^N \), (23), gives systematically a larger value for the function than naive suggestion (21), the curves 1 in Fig. 2, which essentially corresponds to the estimate \( h_1^N \simeq (18/5) g_1^N \), neglecting possible negative contribution of the s-quark sea [21]. The large size of the effect suggests that it can be detected in future experiments with the deuterons [9]. The ratio of the structure functions \( h_1^D / h_1^N \) in this case is also shown in Fig. 3 (dashed line).

4 Summary

We have considered the chiral-odd structure function of twist two, \( h_1^D(x) \), and its first moment, the tensor charge of the deuteron. In particular,

1. The structure function of the deuteron is calculated in the Bethe-Salpeter formalism for the deuteron amplitude. Explicit analytical and numerical results for the effective distribution function of the nucleons in the deuteron, defining the deuteron structure function \( h_1^D \) are obtained.

2. It is shown that the deuteron structure function \( h_1^D(x) \) exhibits deviation from the nucleon structure function \( h_1^N \) similar to the case of the usual spin-dependent structure functions \( g_1^D \) and \( g_1^N \). The deuteron structure factor renormalizing the tensor charge of the nucleon in the deuteron is calculated.

3. A simple phenomenologically motivated model for the evaluation of the nucleon function \( h_1^N \) is suggested, using the available results of the QCD sum rules and the bag model for
$h_1^N$. The estimates show that the nucleon structure function, $h_1^N(x)$, can be significantly larger than the spin-dependent structure function after extraction of the averaged quark charge, i.e. $(18/5)y_1^N(x)$. This effect definitely can be tested experimentally in the semi-inclusive deep inelastic scattering and, therefore, can bring a better understanding of the spin structure of the nucleon.

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Figure captions

Figure 1: The effective distribution function $H^{N/D}(y)$ (solid line) for the nucleon contribution to the deuteron structure function $h^D_1(x)$. Other effective distributions: spin-dependent $\bar{f}^{N/D}(y)$ (dashed line) and spin-independent $f^{N/D}(y)$ (dotted line).

Figure 2: The structure functions $h^N_1(x)$ (solid lines) and $h^D_1(x)$. Groups of curves: 1 - $\alpha = 1.0, \beta = 1.0$ (“lower limit”); 2 - $\alpha = 1.5, \beta = 0.5$ (“mid point”); 3 - $\alpha = 2.0, \beta = 0.0$ (“upper limit”) (see eqs. (20) and (23)).

Figure 3: The ratio of the deuteron and nucleon structure functions $h^D_1(x)/h^N_1(x)$. Curves: solid line - $\alpha = 1.5, \beta = 0.5$; dashed line - $\alpha = 1.0, \beta = 1.0$. The dotted curve presents the ratio of the spin-dependent structure functions $g^D_1(x)/g^N_1(x)$. The dash-dotted line approximately corresponds to eqs. (14) and (15).
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Fig. 3. A. Umnikov et al, The chiral-odd structure function...