Nonlinear control method of electro-hydraulic system driving Two-DOF robot arm with output constraint

Qing Guo1,2,3,*, Yili Liu1 and Dan Jiang4

1School of Aeronautics and Astronautics, University of Electronic Science and Technology of China, Chengdu, 611731, China
2State Key Laboratory of Fluid Power and Mechatronic Systems, Zhejiang University, Hangzhou, 310027, China
3Center for Intelligent Aircraft Systems Technology and Application, University of Electronic Science and Technology of China, 611731, China
4School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, Chengdu, 611731, China

*E-mail: guoqinguestc@uestc.edu.cn

Abstract. This paper studies tracking control problem for a certain parameter electro-hydraulic system (EHS) to constrain the output cylinder position in prescribed performance. In the control design, a backstepping controller is adopted to guarantee the system stability and the dynamic tracking performance. The dynamic surface control (DSC) is used to eliminate the complexity explosion of virtual control repeatedly derivatives to make the controller and parameter design simple. A barrier Lyapunov function (BLF) is employed in the process of control design to avert the contravention of the time-varying output constraints. The comparative simulation results are given to demonstrate that the effectiveness of the proposed controller. Furthermore the smooth dynamic surface is obtained to guarantee the sufficient stable margin of EHS.

1. Introduction

Electro-hydraulic system is an interdisciplinary subject based on electrical control, hydraulic transmission and computer, which is widely used because of high control accuracy, fast response speed, high output power, flexible signal processing and easy to realize feedback of parameter[1]-[2]. EHS uses electro-hydraulic servo valve to convert small power signal into high power hydraulic power, and realizes the servo control of heavy machinery. With the continuous development of technology and the continuous improvement of the performance requirements, the stability, rapid and accurate response of the EHS become a basic requirement. Therefore, the research on the control of the electro-hydraulic system[3] becomes the inevitable trend.

EHS is a strict feedback model in nonlinear system, and the boundedness of output constraints is an important problem in the control of nonlinear systems. In the actual operation of the EHS, the violation of the constraint may reduce the performance, endanger the system, even damage the hardware. In order to cope with the constraints or limitations of the operating space and hardware capacity, and improve the performance of the actual system, the constraint control becomes one of the key areas of the control theory and engineering research. The existing research in the field of control
has carried out a large number of studies on constraint problems[4]-[5]. A concept of invariant set is proposed to solve the constraint problem, but constraints can be satisfied only when the initial state is constrained to an invariant set[6]-[7]. Model predictive control mainly solves the state and control variables in a reasonable time interval with the optimal control algorithm[8]. Afterwards, the Lyapunov stability theory has been an important method of nonlinear control, then some scholars put forward the barrier Lyapunov function which is a kind of continuous function.

When barrier Lyapunov function[9] is used, as the system order number increasing, the steps in the process of virtual control iterative differential easily differential explosion phenomenon, resulting in a serious computational burden and complex controller design process of control system, it is more difficult to use in actual system. Therefore, the idea of the barrier Lyapunov function method based on the backsteping[10-12], dynamic surface control[13] to solve the time-varying output constraints of nonlinear systems[14] is proposed, this method adopts the first order filtering of the virtual control in the traditional backstepping method and introduces a new state, thus avoiding the emergence of the surge item[15].

Different from the existed work of EHS control method, the main contribution of this paper is given as follows:

1) The barrier Lyapunov function is proposed to handle the output constraint, which restrain the hydraulic cylinder position in desirable boundary.

2) The dynamic surface control is adopted to eliminate the differential expansion in the processing of backstepping controller design which guarantees the system stability and the dynamic tracking performance of EHS.

2. Plant description
As shown in figure 1, the control mechanism of EHS[16] is comprised by a electro-hydraulic servo valve, a hydraulic cylinder, a fixed displacement pump, a relief valve.

![Figure 1. The control mechanism electro-hydraulic system.](image)

By analyzing the structure and working principle of electro-hydraulic servo system, the load flow of electro-hydraulic servo valve can be obtained

\[ Q_a = C_d w x_s \sqrt{\frac{1}{\rho} (P_s - \text{sgn}(x_v) P_L) } , \]  

where \( C_d \) is the discharge coefficient , \( w \) is the area gradient of the servo valve spool, \( \rho \) is density of hydraulic oil , \( P_s \) is the supply pressure of the pump , \( P_L = P_a - P_h \) is the load pressure, \( x_v \) is the spool position of servo valve, \( \text{sgn}(\cdot) \) is the sign function.
As the functions appeared in the design of backstepping controller are required to be smooth, the sign function \( \text{sgn}(x_c) \) in (1) is replaced by the hyperbolic tangent function \( \tanh(kx_c) \), where \( k \) is a positive constant.

\[
Q_a = C_d w x \sqrt{\frac{1}{\rho} (p_s - \tanh(kx_c))P_L}.
\]

Hydraulic cylinder flow-pressure continuity equation can be expressed as

\[
Q_a = A_p y + C_n P_L + \frac{V_t \dot{P}_L}{4 \beta_c},
\]

where \( A_p = A_a = A_b \) is annulus area of symmetrical cylinder chamber, \( y \) is output displacement of the hydraulic cylinder, \( C_n \) is the coefficient of the total leakage of the cylinder, \( V_t \) is the half-volume of cylinder, \( \beta_c \) is the effective bulk modulus.

The mechanical dynamic equation is given as

\[
m\ddot{y} = P_L A_p - Ky - b\dot{y} - F_L,
\]

where \( m \) is the load mass, \( F_L \) is the external load on the EHA, \( K \) is the spring constant, \( b \) is the viscous damping coefficient.

The state vector \( x = [x_1, x_2, x_3] = [y, \dot{y}, P_L] \), \( u \) is input voltage of servo valve, then the dynamic model of EHS can be expressed as[17]

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= \frac{1}{m} (-Kx_1 - bx_2 + A_p x_3 - F_L), \\
\dot{x}_3 &= -\frac{4\beta_c A_p}{V_t} x_2 - \frac{4\beta_c C_n}{V_t} x_3 + \frac{4\beta_c C_d w K_{sv} l}{V_t \sqrt{\rho}} \sqrt{p_s - \tanh(ku)x_1 u}, \\
y &= x_1,
\end{align*}
\]

where \( K_{sv} \) is gain of servo valve. The output \( y = x_1 \) is restricted in desirable boundary by feedback control to guarantee the tracking error of cylinder position \( \Delta x_1 = x_1 - yd \) in prescribed performance.

3. Control design

3.1. Useful technical lemmas and definitions

**Remark 1**[18]: The external load \( F_L \) is unknown plant dynamics, which is considered as a structural disturbance of EHS. Although the dynamic value of \( F_L \) depends on the variables \( y, \dot{y}, \ddot{y} \), \( F_L \) is bounded by its upper boundary \( \Delta_{F_L} \), i.e., \( |F_L(t)| \leq \Delta_{F_L} \).

**Remark 2**[19][20]: Generally, the hydraulic parameters \( C_d, \rho, \omega, b, \beta_c, C_n \) are often perturbed by different hydraulic physical characteristic, but the other parameters are known.

**Lemma 1**[21]: Consider a positive constant \( k_x \in \mathbb{R} \), if \( x \in \mathbb{R} \) and \( |x| < |k_x| \), the following inequality holds:

\[
\ln \frac{k_x^2}{k_x^2 - x^2} \leq \frac{k_x^2}{k_x^2 - x^2}.
\]
Lemma 2[22]: For bounded initial conditions, if there exists a \( C^1 \) continuous and positive definite Lyapunov function \( V(x) \), and satisfying \( \kappa_1(\|x\|) \leq V(x) \leq \kappa_2(\|x\|) \), \( \dot{V}(x) \leq -\rho V(x) + c \), where \( \kappa_1, \kappa_2 : \mathbb{R}^n \rightarrow \mathbb{R} \) are class K functions, and \( \rho \) and \( c \) are two positive constants, then the solution \( x(t) \) is uniformly bounded.

3.2. Dynamic surface control with output constraint

Suppose all the states and parameters are measurable, let

\[
\begin{align*}
    d_L &= -\frac{F_i}{m}, f_2 = -\frac{K}{m}x_1 - \frac{b}{m}x_2, g_2 = \frac{A_p}{m}, f_3 = -\frac{4\beta_iA_p}{V_i}x_2 - \frac{4\beta_i C_d}{V_i}x_3, \\
    g_3 &= \frac{4\beta_i C_d w}{V_i} \sqrt{P - \tanh(ku)x_1}.
\end{align*}
\]  

(7)

In this case, we can assure \( f_2, f_3, g_2, g_3 \) smooth, according to Remarks 1-2, simplify the dynamic model of EHS, the dynamic model of EHS is rewritten as follows

\[
\begin{align*}
    \dot{x}_1 &= x_2 \\
    \dot{x}_2 &= f_2(x_1, x_2) + g_2 x_3 + d_L \\
    \dot{x}_3 &= f_3(x_2, x_3) + g_3 u \\
    y &= x_1
\end{align*}
\]  

(8)

In the process of designing the backstepping controller, the frequent derivation of the differential surge is inevitable, it is difficult to realize in practical situations. To solve this problem, we add dynamic surface control to eliminate the differential expansion to make the controller and parameter design simple. In order to ensure that the closed-loop signal in the closed loop is bounded and the output constraint is not violated, we use BLF to avert the contravention of the time-varying constraints.

The system state errors \( z_i(t) \) \((i = 1, 2, 3)\) are defined as follows

\[
\begin{align*}
    z_i &= x_i - x_{id} \\
    z_{i+1} &= x_{i+1} - \alpha_i, i = 1, 2.
\end{align*}
\]  

(9)

where \( \alpha_i \) is stable virtual control variable.

According to the dynamic surface control method, a filter virtual function \( \beta_i, i = 1, 2 \) can be introduced. Let the stable virtual function \( \alpha_i \) pass through a first order filter, it can be expressed as

\[
\begin{align*}
    \tau_i \dot{\alpha}_i + \alpha_i &= \beta_i, i = 1, 2
\end{align*}
\]  

(10a)

where \( \tau_i \) as the time constant, \( \alpha_i(0) = \beta_i(0), i = 1, 2. \)

So the filter error of the first order filter is \( S_i = \alpha_i - \beta_i, i = 1, 2 \). Then we have

\[
\begin{align*}
    \dot{\alpha}_i &= \frac{\beta_i - \alpha_i}{\tau_i} = -\frac{S_i}{\tau_i}.
\end{align*}
\]  

(10b)

Choose the barrier Lyapunov functions as

\[
V = \frac{1}{2} \log \frac{k^2_a}{k^2_a - z^2_i} + \frac{1}{2} z^2_i + \frac{1}{2} z^2_j + \frac{1}{2} S^2_i + \frac{1}{2} S^2_j.
\]  

(11)

If the two filter virtual functions are designed as
\[ \beta_1 = -(k_a^2 - z_1^2) k_1 z_1 + \dot{x}_{id} \]
\[ \beta_2 = \frac{1}{g_2} \left( -f_2 - k_2 z_2 - \frac{z_1}{k_a^2 - z_1^2} + \dot{\alpha}_1 \right), \] (12)

and the controller is given by
\[ u = \frac{1}{g_3} \left( -f_3 + \dot{\alpha}_2 - k_3 z_3 - g_2 z_2 \right), \] (13)

then
\[ \dot{V} \leq -\Gamma \ln \left( \frac{k_a^2}{k_a^2 - z_1^2} \right) - \Omega \dot{S}_1^2 - \Gamma_2 \dot{S}_2^2 - \Omega_2 \dot{S}_3^2 - k_3 \dot{z}_3^2 + \frac{\sigma_1}{2} + \frac{\sigma_2}{2} + \frac{d_{l\text{max}}^2}{2}, \] (14)

where
\[ \Omega_1 = \frac{1}{\tau_1} - \frac{1}{2(k_a^2 - z_1^2)} - \frac{M_1^2}{2 \sigma_1}, \quad \Gamma_1 = k_1 \left( \frac{k_a^2 - z_1^2}{2} \right) - \frac{1}{2}, \quad \Gamma_2 = k_2 - \frac{|g_2|^2}{2} - \frac{1}{2}, \quad \Omega_2 = \frac{1}{\tau_2} - \frac{|g_2|^2}{2} - \frac{M_2^2}{2 \sigma_2}, \]

are positive constants, \( k_i (i = 1, 2, 3) \) are the controller gains.

Let \( c = \min \{ 2\Gamma_1, 2\Gamma_2, 2\Omega_1, 2\Omega_2, 2k_3 \} \), then
\[ \dot{V} = -c V + \sigma, \quad \sigma = \frac{\sigma_1}{2} + \frac{\sigma_2}{2} + \frac{d_{l\text{max}}^2}{2}. \] (15)

Integrating two sides of (15), \( V \) yields
\[ V(t) \leq V(0)e^{-ct} + \sigma (1 - e^{-ct})/c. \] (16)

Now according to (16), as let \( t \to \infty \), the error convergence mainly depends on the item \( \sigma/c \). Thus, the increased control gains \( k(i = 1, 2, 4) \) and the reduced constant \( c \) can arbitrarily shrink this error convergence. This implies that the Lyapunov function \( V \) is ultimate bounded. Thus, the system state errors \( \ddot{z}_1, \ddot{z}_2, \ddot{z}_3 \) are stable.

4. Simulation

The block diagram of the proposed method is shown in figure 2. The simulations are carried out by Matlab / Simulink software. The tracking error boundary of output cylinder position is \( ka = \pm 2 \text{mm} \). Some known hydraulic parameters are \( x_{\text{max}} = 5 \text{ mm}, L_{\text{max}} = 58 \text{ mm}, ps = 40 \text{ bar}, Ap = 4.91 \text{ cm}^2, Vt = 8.74 \times 10^5 \text{ m}^3, C_d = 0.62, w = 0.024 \text{ m}, Ctl = 2.5 \times 10^{-11} \text{ m}^3/(\text{sPa}), \beta e = 7000 \text{ bar}, \rho = 850 \text{ kg/m}^3, K = 10 \text{ N/m}, b = 50 \text{ Ns/m}, Ksv = 5 \times 10^{-4} \text{ m/V}, Tsv = 10 \text{ ms}. \) The load mass is \( m = 3.347 \text{ kg} \). The control gains are set as \( k_1 = 1000, k_2 = 200, k_3 = 1000, \) and \( k_4 = 1 \). To illustrate the advantage of the proposed method, the traditional PI controller is given by
\[ u = k_p(y_d - y) + k_i \int_0^t (y_d - y) dt, \] where the control gain \( kp = 150, ki = 10 \).

The comparison results with PI control method are shown in figure 3-6. The position demands of hydraulic cylinder is selected as \( y_d = 50 \sin(2\pi t) \text{ mm} \). Figure 3 illustrates the dynamic tracking response of cylinder position. The dynamic tracking accuracy of the proposed BLF method is better than PI as shown in figure 4, whose output error is constrained in prescribed boundary \( |Ax| < k_a = 2 \text{ mm} \). The two dynamic surface errors S1 and S2 are shown in figure 5, which are smooth.
to avoid the complexity explosion caused by the virtual control repeatedly derivatives. Then the final control variable \( u \) is shown in Figure 6.

![Diagram](image)

**Figure 2.** Block diagram of the proposed method.

![Graph](image)

**Figure 3.** The dynamic response \( x_1 \).

**Figure 4.** Tracking error \( \Delta x_1 \).

![Graph](image)

**Figure 5.** Two dynamic surface errors \( S_1 \) and \( S_2 \).

**Figure 6.** The control variable \( u \).

5. Conclusion

In this paper, a backstepping controller is proposed for electro-hydraulic system to drive Two-DOF robotic arm. To eliminate the differential expansion, the dynamic surface control is employed. A barrier Lyapunov function is used to avert the contravention of the time-varying output constraints. The comparison simulation results with PI control have indicated that the presented control scheme can make the output well follow the target trajectory while ensuring the constraints satisfaction.
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