Spacetime supersymmetry in $AdS_3$ backgrounds

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Abstract

We construct string target spacetimes with $AdS_3 \times X$ geometry, which have an $N = 2$ spacetime superconformal algebra. $X$ is found to be a $U(1)$ fibration over a manifold which is a target for an $N = 2$ worldsheet conformal field theory. We emphasize theories with free field realizations where in principle it is possible to compute the full one-particle string spectrum.

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1 Introduction

The AdS/CFT correspondence\cite{1} relates string theories and M-theory solutions with an $AdS$ factor to the infrared conformal field theory living on the brane system. This was further elaborated upon in Refs. \cite{2,3} where it was explained how one calculates the spectrum and correlation functions of the conformal field theory within a supergravity description. Tests of the conjecture involve comparing the supergravity correlation functions and dimensions of operators with the ones associated to the low energy dynamics of the brane.

In order to construct tests that probe beyond BPS states and the large $g^2 N$ limit, it is necessary to understand the full spectrum of excitations in these backgrounds. The first example\cite{5} was discussed in \cite{5} where a weak coupling regime exists at finite $N$ (see also \cite{6,7}). Further examples have appeared in Refs. \cite{8,9}. In general, in order to probe physics where supersymmetry is less restrictive but still a powerful tool, we want to develop a large collection of spacetimes that are good string backgrounds and that can be understood in a simple fashion. It is the purpose of this paper to give a large class of backgrounds for which this program is possible in principle. Our construction gives rise to spaces with $N = 2$ spacetime supersymmetry, and they are derived from a worldsheet $N = 2$ superconformal field theory.

After we completed this paper, we learned of the results of Ref. \cite{10} which overlaps some of our results.

2 $AdS_3$ backgrounds

We wish to discuss general NSNS backgrounds of Type II superstrings which lead to spacetime supersymmetry on $AdS_3$. We will require that a large radius limit of the geometry exists, such that a semiclassical analysis can be made valid; this is done in order to be able to compare with low energy supergravity. We will restrict ourselves to coset conformal field theories, for which the semiclassical limit corresponds to taking all levels of the corresponding algebras to infinity simultaneously. Moreover, we will be interested in theories with $N = 2$ worldsheet superconformal invariance, which will leads naturally to spacetime supersymmetry.

\footnote{For earlier work, see Refs. \cite{4}.}
These two requirements lead to the following constraints. The spacetime is of the form \( AdS_3 \times X \), with \( X \) a seven-dimensional manifold/orbifold. \( \text{N=2} \) worldsheet supersymmetry is obtained if \( X \) is chosen to be a circle bundle over a complex manifold, which itself is a product of hermitian symmetric spaces constructed as in Kazama-Suzuki\(^\text{[1]}\). (A possible generalization would be to take more general complex manifolds \( X/U(1) \) with constant positive curvature, but this is less well understood.)

This bundle structure allows for a decomposition of the energy momentum tensor in terms of a \( U(1) \) CFT and a hermitian symmetric space coset. This \( U(1) \) pairs with the Cartan of the \( SL(2) \) current algebra into an \( \text{N = 2} \) free system, and the remaining \( SL(2, \mathbb{R})/U(1) \) is also a hermitian symmetric non-compact coset. This splitting of the \( U(1) \) piece of \( X \) is required if we want to bosonize the fermions in the \( SL(2, \mathbb{R}) \) current algebra.

In total, we have a system with \( \text{N = 2} \) worldsheet supersymmetry,

\[
(SL(2, \mathbb{R})/U(1)) \times (X/U(1)) \times U(1)^2
\]

This construction possesses a “natural” GSO projection (preserving the \( \text{N = 2} \) structure) which leads to spacetime supersymmetry. Of course, there are other possible GSO projections, such as that used in the construction of the NS vacua of the \( \text{N = 4} \) geometries considered in \([5, 8, 9]\). Each of these GSO projections will be considered below.

The collection of models constructed in this fashion is a finite family of spaces, and they should be interpreted as conifolds where a set of D1-D5 branes is localized.

A table with a list of compact cosets of dimension \( d \leq 7 \) is provided here, where we include the central charge of the model in terms of the levels of the different groups that form the coset. Other spaces like \( SU(3)/U(1)^2 \) can be constructed as \( SU(3)/(SU(2) \times U(1)) \times SU(2)/U(1) \), so we don’t list them.

Our objective in the next section will be to give a detailed construction of spacetime supersymmetry in these backgrounds, and to show why the standard choice of using the \( \text{N = 2} \ U(1)_R \) worldsheet current is not appropriate.

We begin by defining our conventions. In general, we will have an \( SL(2, \mathbb{R})_k \) superconformal current algebra \( (J^a, \psi^a) \), realized by free fields. There is a free \( U(1)_L \) system, with currents \( (K, \chi) \). Finally, there is a coset \( X/U(1) \) with an \( \text{N = 2} \ U(1)_R \) current which we will denote by \( J_c \). We require the total central charge \( c_T = 15 \) such that it may be coupled to the standard superconformal ghost system to define a consistent string theory.
| Coset | Central charge | Group | Dimension |
|-------|----------------|-------|-----------|
| $S^1$ | $\frac{3}{2}$ | U(1) | 1         |
| $\mathbb{Q}^1$ | $3 - \frac{6}{k}$ | $SU(2)_k/U(1)$ | 2         |
| $S^3$ | $\frac{9}{2} - \frac{6}{k}$ | $SU(2)_k$ | 3         |
| $\mathbb{Q}^2$ | $6 - \frac{18}{k}$ | $SU(3)_k/(SU(2)_k \times U(1))$ | 4         |
| $S^5$ | $\frac{15}{2} - \frac{18}{k}$ | $SU(3)_k/SU(2)_k$ | 5         |
| $T^{pq}$ | $\frac{15}{2} - \frac{6}{k} - \frac{6}{k'}$ | $(SU(2)_k \times SU(2)_{k'}/U(1))$ | 5         |
| $\mathbb{Q}^3$ | $9 - \frac{36}{k}$ | $SU(4)_k/(SU(3)_k \times U(1))$ | 6         |
| $S^7$ | $\frac{21}{2} - \frac{36}{k}$ | $SU(4)_k/SU(3)_k$ | 7         |
| $T^{pq}$ | $\frac{21}{2} - \frac{6}{k} - \frac{6}{k} - \frac{6}{k''}$ | $(SU(2)_k \times SU(2)_{k'} \times SU(2)_{k''}/U(1)^2$ | 7         |

The total $U(1)_R$ current may then be written as

$$J_R = (\psi^0 \chi) + \left( : \psi^+ \psi^- : + \frac{2}{k} J^0 + J_c \right) = i \partial H_1 + i \partial H_2$$

(2)

We have bosonized the currents in parenthesis in favor of two bosons $H_1, H_2$. This is the canonical choice for generic $X$, where we require the bosons to be integral.

It is straightforward to verify that the following OPEs are regular

$$J_R(z)J^0(z') \sim 0$$

(3)

$$J_R(z)K(z') \sim 0$$

(4)

and moreover

$$J_R(z)J_R(z') \sim \frac{5}{(z - z')^2}$$

(5)

$$J^0(z)J^0(z') \sim -\frac{k/2}{(z - z')^2}$$

(6)

$$K(z)K(z') \sim \frac{k'}{(z - z')^2}$$

(7)

$^2$Note the normalization $H_2(z)H_2(0) = -4 \ln z$.
$^3$In special cases, $X$ will be such that $J_R$ splits integrally in terms of more bosons. In these cases, spacetime supersymmetry will be enhanced.
where \( k \) is the level of the \( SL(2) \) current algebra and \( k' \) is related to the radius of compactification of the free boson leading to the \( U(1) \) supercurrent. In deriving (5), we have used \( c = 15 \) (and thus \( c(X/U(1)) = 9 - 6/k \)).

The canonical choice for spacetime supercharges is

\[
Q^{\pm\pm} = \oint dz \, e^{\pm i H_1/2 \pm i H_2/2} e^{-\phi/2}
\]

where \( \phi \) is the superconformal ghost. These supercharges have the property that they are BRST invariant, while mutual locality requires that we keep only \( Q^{+\pm} \). Modulo picture changing, we find the commutator

\[
\{Q^{++}, Q^{+-}\} = J^0 - K
\]

Thus, (8) leads naturally to spacetime supersymmetry, but unfortunately, the expected spacetime bosonic symmetries (such as \( SL(2) \)) are not recovered. (problems of this nature were discussed in [5]; there the problems were much worse, as the algebra was doubled, etc.) We’ll consider this in more detail below, and give the correct construction.

### 3 Target space consistency

If \( X/U(1) \) is a direct product \( \sim \bigoplus_i A_i \), where each factor has central charge \( c_i = b_i - a_i/k_i \), then the condition \( c = 15 \) forces the constraint

\[
\frac{6}{k} = \sum_i \frac{a_i}{k_i}
\]

which relates the level of the \( SL(2) \) algebra to the level of the other cosets in the construction, and it is easy to see that taking \( k_i \to \infty \) for all \( i \) forces \( k \to \infty \). This means that we have a good geometric picture of the coset as a target space, since in the large \( k_i \) limit the cosets behave semiclassically. In writing (10), we have assumed that \( X \) has classical dimension seven. Changing this would modify (10) by terms of order one, so the AdS spacetime would never become semiclassical.

Now let us analyse the spacetime isometries. We would like to see that the isometries have appropriate commutators with the spacetime supercharges. In particular, given the worldsheet \( SL(2) \) currents, we can construct charges that should generate spacetime symmetries; consider the spacetime \( SL(2) \)
raising operator $\mathcal{J}^+ = \oint J^+ dz$. This operator has the following OPE with $J_R$,

$$\mathcal{J}^+ J_R(z) \sim -\psi^+ \chi + 2\psi_0 \psi^+ - \frac{2}{k} J^+ \tag{11}$$

and the last term implies that the OPE $\mathcal{J}^+ Q^{++}$ is non-local. Also, the $Q$’s do not carry $U(1)_R$ charges, which is in conflict with the spacetime $N = 2$ algebra if we want to identify $\oint K$ as the $U(1)_R$ current. That is, the existence of the operator $Q^{++}$ leads to the wrong target space symmetries.

Instead of doing this, we will modify the supersymmetry generator in order that the spacetime Virasoro algebra is recovered. It is simple to see that the problem comes from the factor of $J_0/2k$ in the $U(1)_R$ current of $SL(2, \mathbb{R})/U(1)$. To solve the problem then, it is sufficient to redefine the worldsheet $U(1)_R$ current

$$\tilde{J}_R = J_R + \frac{1}{2k} J^0 = i\partial H_1 + i\partial \tilde{H}_2 \tag{12}$$

However, the OPE $\tilde{J}_R(z)\tilde{J}_R(z') \sim (5 - \frac{1}{4k})/(z - z')^2$ implies that the spin operator

$$\Sigma^{++} = \exp\left(+\frac{iH_1}{2} + \frac{i\tilde{H}_2}{2}\right) \tag{13}$$

has the wrong ($\neq 5/8$) conformal dimension. This means we need to correct again $\tilde{J}_R$. That is, we define

$$J'_R = \tilde{J}_R + \frac{F}{2k} \tag{14}$$

with $F$ a current such that $F(z)F(z') = -J^0(z)J^0(z')$ and that is orthogonal to $J_0$, so that the bosonization of $J'$ leads to the correct conformal weight for the spin operator. We have a canonical choice for $F$ in our models, as we have the extra free $U(1), K$.

Hence, if $K$ has the same level as $SL(2)$, then

$$J'_R = J_R + \frac{J^0 + K}{2k} \tag{15}$$

will lead to the correct conformal weight for the spin operator. Notice that the current $J^0+K$ is null, and orthogonal to $J_R$, so that the natural spin operators

\footnote{To see this, it is useful to realize $Q \sim e^{\oint J_R}$.}
constructed from $J'_R$ have automatically the correct conformal dimension. This also gives a charge to the spin operators under the spacetime $U(1)_R$ generated by $\oint K$.

The full current is a sum of three integral pieces now

$$J'_R = (\psi_0 \chi) + (\psi^+ \psi^-) + \left( \frac{K}{2k} + J_c \right) = i\partial(H'_1 + H'_2 + H'_3)$$

(16)

Let us check that we get the $N = 2$ superconformal algebra in spacetime.

The BRST invariant spin operators will be given by

$$S = \exp(\epsilon_i \frac{iH'_i}{2}) \exp(-\phi/2)$$

(17)

with $\epsilon_i = \pm 1$. Mutual locality and BRST invariance lead to the constraint $\Pi \epsilon_i = 1$, and the spacetime supersymmetry operators are given by

$$Q^{+\pm} = \oint \exp \left( \frac{iH'_1}{2} \pm \frac{iH'_2}{2} \pm \frac{iH'_3}{2} \right)$$

(18)

$$Q^{-\pm} = \oint \exp \left( -\frac{iH'_1}{2} \pm \frac{iH'_2}{2} \pm \frac{iH'_3}{2} \right)$$

(19)

The spacetime supersymmetry algebra (modulo picture changing) is now seen to be given by

$$\{Q^{++}, Q^{-+}\} = J^+$$

(20)

$$\{Q^{+-}, Q^{--}\} = J^-$$

(21)

$$\{Q^{-+}, Q^{--}\} = J^0 + K$$

(22)

$$\{Q^{++}, Q^{++}\} = J^0 - K$$

(23)

which is exactly the $NS$ sector of two dimensional $N = 2$ supersymmetry. That is, this provides a general construction of $N = 2$ spacetime supersymmetry whenever we have $N = 2$ worldsheet supersymmetry. Motions in the $U(1)$ fiber are the generators of the $R$ symmetry spacetime current, and the modifications we made give charge to the supersymmetry generators, which is a requirement of the algebra.

We want to complete the construction with the rest of the Virasoro algebra in spacetime. That is, we can write the rest of the modes of the spacetime supercurrent by applying the other Virasoro generators to the spin fields \[.]
It is a straightforward exercise to show that the algebra closes with a central charge given by \( c_{st} = kp \), where \( p \) is the winding number of the strings located at infinity.

It is also clear that the fact that the level of the \( U(1) \) is chosen to be equal to that of the \( SL(2) \) current implies a quantization condition on the radius of the \( U(1) \) bundle to be of the same order of magnitude as the radius of the AdS spacetime.

**4 Comments on \( AdS_3 \times S_3 \times T^4 \) and \( AdS_3 \times S^3 \times S^3 \times U(1) \)**

Now that we have a general recipe, we can go back and analyze previous results \[3, 9\] in a new light. We confine ourselves to a few comments.

In particular, the current \( K \) is chosen in a very special way. For example, in \( AdS_3 \times S^3 \times T^4 \), we could have chosen either the Cartan of \( SU(2) \), or one of the free \( U(1) \)'s. Since we have only analyzed the effects of the spacetime raising operator \( J^+ \), we can always guarantee that we get the spacetime Virasoro algebra with either choice. On the other hand, if we want to keep the full \( SU(2) \) isometries of the target space, we actually have to also eliminate a similar problem with the locality of the \( SU(2) \) raising operator, and the modification in the currents is such that both effects get cancelled simultaneously if we choose to pair the Cartan element of \( SU(2) \) with the one of \( SL(2, \mathbb{R}) \), so the choice of null field solves both problems at the same time.

For \( AdS_3 \times S^3 \times S^3 \times U(1) \), there is one linear combination which we want to add to the Cartan of \( SL(2, \mathbb{R}) \) such that all of the problems with the isometries go away simultaneously. This choice is the diagonal \( U(1) \) which cancels the terms from both \( AdS_3 \) terms, and a posteriori ends up having the correct quantization condition. This chooses a complex structure on the four free \( U(1) \) directions.

Finally, let us remark on \( AdS_3 \times S^3 \times \mathbb{CP}^2 \), where \( \mathbb{CP}^2 \) is understood as the coset \( SU(3)/(SU(2) \times U(1)) \). In this case, it can be seen that the Cartan of the \( SU(2) \) coming from \( S^3 \) can not cancel the non-locality of the raising operator of \( SU(2) \), so that we do not preserve the isometries, and the correct quantization condition squashes the sphere, so it leaves us with \( N = 2 \) spacetime supersymmetry as opposed to \( N = 4 \) which one may have naively guessed.
5 Vertex Operators and free fields.

We have shown that it is straightforward to compute the spacetime NS sector of supersymmetry. Naturally, as we have a field of conformal dimension zero, represented by $\gamma$, we can multiply $Q$ by any fractional powers of $\gamma$. BRST invariance will choose a linear combination of $Q\gamma^\alpha$ with the same spacetime quantum numbers.

Closure of the fermionic algebra on the bosonic generators provides that $\alpha \in \mathbb{Z}$ or $\alpha \in \mathbb{Z} + 1/2$, but not both simultaneously, as then the bosonic generators will not be single valued when $\gamma \rightarrow e^{2\pi i} \gamma$. That is, we have a choice of two super selection sectors for the supersymmetry algebra. One is to be taken as the Neveu-Schwarz sector as we already saw, and the other is the Ramond sector of the spacetime supersymmetry.

This is a feature of the free field realization for the $SL(2)$ current algebra, and it actually lets us write vertex operators for the $SL(2)$ current with free fields. The same can be done with the coset constructions, as they admit free field realizations. Now, we want to analyze the chiral GSO projection. This needs to be done for each case, as we are not using the $N = 2$ structure of the worldsheet CFT directly, but a modified version of it.

Let us fix some notation.

As is well known, coset conformal field theories admit free field realizations [13, 14]. For each of the raising operators of the algebra we use a $B, C$ system, with OPE given by

$$B(z)C(z') = \frac{1}{z - z' - \theta \theta'} = \frac{1}{Z - Z'}$$  \hspace{1cm} (24)

describing how these can be super-bosonized into a set of two null (lightcone) scalars $T, U$ with OPE

$$T(z)U(z') = \log(Z - Z')$$  \hspace{1cm} (25)

by taking $B = aDU \exp(a^{-1}T)$, $C = \exp(-a^{-1}T)$.

To the Cartan elements we associate free supersymmetric bosons $H$. The total collection of free bosons will be labeled by $\phi$, and the fermions by $\psi$.

For the bosons we want to take operators (in the left moving sector let say) which have an odd number of free fermion insertions in the $[-1]$ picture.

$$\mathcal{O}(\psi^\alpha, \partial\phi, \partial) e^{ip\phi}$$  \hspace{1cm} (26)

In this basis the embedding of the group theory (allowed lattice of values for $p$) is not manifest.
This requirement corresponds to the chiral GSO projection. The mass shell condition

\[ p \cdot (p - \lambda) + N = \frac{1}{2} \]  \hspace{1cm} (27)

may be equated to

\[ -\frac{j(j+1)}{2k} + m^2 + 1/2, \]

where \( j \) is the spacetime \( SL(2) \) quantum number and \( m^2 \) is to be understood as the \( AdS_3 \) mass of the state. \( \lambda \) is the curvature coupling of the free scalars. The GSO projection implies that we get a positive value for \( j \); that is, the theory does not have tachyons.

The number of free superfields is ten, and the total field with a worldsheet curvature coupling of the \( AdS \) and \( X \) theories combined together is a null field. In this sense, all theories have the same underlying structure. It is the choice of lattice (modular invariant) which makes them different from one another.

The ten dimensional massless vertex operators are the most interesting as they predict the supergravity spectrum of the compactification. In this case the polynomial \( S \) reduces to one free fermion operator. Amongst these, we will find all the chiral operators of the spacetime conformal field theory which can be described by string vertex operators.

The fermion vertex operators are constructed by acting with the spacetime supersymmetry generators on the spacetime boson vertex operators that we described. As we have a choice of super selection sector, they will look different in each of the cases. It is also clear that this difference is in the powers of \( \gamma \), and therefore the OPE of two fermion vertex operators closes on the ones of bosonic type.

The advantage of having a free field realization is that we have a choice of ten free fermions \( \psi^i \), which can be bosonized into five scalars, and writing spin vertex operators is straightforward. One has to remember that in order to get the right charges for the supersymmetries, they will be multiplied by powers of \( e^{ip \cdot \phi} \) with \( p^2 = 0 \).

From the bosonization, we find that each \( B,C \) system contributes two lightcone scalars, i.e., their signature is \((1,1)\). Hence the lattice that we obtain has a signature of \((n,m)\) with \( n + m = 10 \), and \( n,m \geq 2 \). The difficulty in writing the partition function lies in finding the constraints on the lattice and the screening operators of the system, so that at the end we can recover unitarity and modular invariance.
6 Discussion

We have given a complete construction of $N = 2$ spacetime supersymmetry on $AdS_3$ spacetime for type II NS NS backgrounds, which are constrained to admit a version of $N = 2$ supersymmetry on the worldsheet. It is clear that as we have exact conformal field theories on the worldsheet, these are solutions to all orders in $\alpha'$ of the string equations of motion. Moreover, usually when we have enough spacetime supersymmetry, this is what we need to guarantee that there are no non-perturbative corrections to the target space. In principle, if we ask questions that can be answered perturbatively, this approach should give a complete set of calculational tools.

On the other hand, further work is required. In particular, the coset model predictions certainly go beyond supergravity and compute the full spectrum of the one-particle states of the string theory propagating on these spacetimes, and therefore, we can get a good idea of what the spacetime conformal field theory might be, even when we are not required to be in any large radius limit. This should shed light on the $\frac{1}{N}$ corrections to the conformal field theories.

The detailed form of the string partition function is not known, and it is of course important to construct it and check modular invariance. This is not clear, as we can not go to a lightcone gauge where everything is manifestly unitary. Also, as we have constructed these theories with free fields, it is likely that everything is complicated by the screening charges, so that a very good knowledge of how to extract the real physical degrees of freedom is required. Although some progress has been made in [15], it is far from complete.

Spaces with non-compact cosets [16, 17] are also interesting, but their spacetime CFT description is bound to be complicated by the non compactness, as the meaning of the conformal boundary comes into question, and if we get a spacetime CFT it seems naturally to give rise to a continuous spectrum of states which is certainly more difficult to analyze.

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