Local Lorentz Transformation and “Lorentz Violation”

Ying-Qiu Gu*

School of Mathematical Science, Fudan University, Shanghai 200433, China

Abstract

Some solutions to the anomalies of ultra high energy cosmic-ray (UHECR) and TeV γ-rays require disturbed non-quadratic dispersion relations, which suggest the Lorentz violation. Also, some new theories such as quantum gravity, string theory and the standard model extension imply the Lorentz violation. In this paper, we derive some transformation laws for the classical parameters of nonlinear field system, and then provide their dispersion relations. These dispersion relations also have non-quadratic form but keep the Lorentz invariance. The analysis of this paper may be helpful for understanding the quantum theory and the plausible Lorentz violation.

PACS numbers: 04.20.Gz, 04.20.Cv, 03.50.Kk, 03.65.Pm

Key Words: Lorentz violation, Lorentz transformation, dispersion, mass energy relation, GZK cutoff

1 Introduction

Recently the “Lorentz violation” becomes a hot topic. This situation arises from both experimental and theoretical developments. One is the astronomical observation of the anomalies of ultra high energy cosmic-ray (UHECR) and the TeV γ-rays. The other is the implication of some new physics such as the extended gauge theory, quantum gravity and string theory. In observational aspect, the UHECR protons with energy

*email: yqgu@fudan.edu.cn
$> 10^{20}$ eV were observed\cite{1, 2, 3}. The energy of these protons are beyond the GZK cutoff $5 \times 10^{19}$ eV, which was derived independently by Greisen\cite{4} and Zatsepin, Kuzmin\cite{5} shortly after the discovery of the cosmic microwave background radiation (CMBR). According to the particle theory, the cosmic-ray nucleons with sufficient energy will inelastic collide with photons of CMBR to produce baryons or pions as follows

$$P + \gamma(\text{CMB}) \rightarrow \Delta, \quad P + \gamma(\text{CMB}) \rightarrow p + N\pi,$$

so the nucleons with energies beyond the GZK limit cannot reach us from a source further than a few dozen Mpc. It should be mentioned that, this GZK limit and the physical process are well understood and measured in the laboratory\cite{6}.

Another similar paradox is the TeV $\gamma$-rays. Two photons with energy over $2m_e c^2$ can produce an electron-positron pair $\gamma + \gamma \rightarrow e^- + e^+$. Photons of 10 TeV are most sensitive to 30 $\mu$m infrared photons, so a photon with enough energy propagating in the intergalactic space will interact with infrared background photons, and be exponentially suppressed. However the 10 TeV photons from Mkn501, a BL Lac objects at distance about 150 Mpc\cite{7, 8}, could reach the Earth.

To explain the above anomalies, numerous solutions have been proposed. Most of these solutions are related to slight violation of Lorentz invariance to get a shift of the thresholds of the energy. Glashow believe that the limit speed of a particle depends on its species and should be the eigenvalue of the velocity eigenstates\cite{9}-\cite{15}. Then for particle $P$ the dispersion relation becomes

$$E^2 = p^2 c^2 + m^2 c^2 + P^4 c^4. \quad \text{(1.2)}$$

In \cite{16}, the authors suggested a non-quadratic dispersion relation for photons

$$p^2 c^2 = E^2 (1 + f(\delta)) \quad \text{where} \quad \delta = \frac{E}{E_{QG}} \rightarrow 0 \quad \text{(1.3)}$$

where $E_{QG} \sim 10^{19}$ GeV is an energy scale caused by quantum gravity effect. Then the light speed is perturbed by $E$ as

$$\tilde{c} = \frac{\partial E}{\partial p} \approx c(1 + k\delta). \quad \text{(1.4)}$$

This assumption causes the Lorentz violation and deformed special relativity, which is related to the $\kappa$-Poincaré superalgebra\cite{17}. The Casimir of the $\kappa$-Poincaré superalgebra has a structure similar to (1.3). Nowadays, the deformed relativity or noncommutative geometry is greatly developed\cite{18}-\cite{26}.

In theoretical aspect\cite{27, 28, 29}, the Standard Model Extension and quantum gravity suggest that Lorentz invariance may not be an exact symmetry. The possibility
Lorentz violation has been investigated in different quantum gravity models, including string theory\cite{30, 31}, warped brane worlds\cite{32}, and loop quantum gravity\cite{33}. These models adopt the Lagrangian like the following\cite{34}

\[ \mathcal{L} = \bar{\psi} \left( \frac{1}{2} e^\mu_a \Gamma^a \partial_\mu - M \right) \psi, \]

where \( \psi \) is Dirac spinor, \( e^\mu_a \) is the vierbein, and \( e_a^\mu \) is the inverse,

\[ \Gamma^a = \gamma^a - c_{\mu\nu} e^{\mu a} e^\nu_b \gamma^b - f_{\mu\nu} e^{\mu a} - i k_{\mu} e^{\mu a} \gamma_5 + \cdots, \]

\[ M = m + i \mu \gamma_5 + a_\mu e^\mu_a \gamma^a + b_\mu e^\mu_a \gamma^a \gamma_5 + \cdots. \]

The first right terms of (1.6) and (1.7) correspond to the normal Lorentz invariant kinetic term and mass for the Dirac spinor. But the other coefficients are Lorentz violating coefficients arising from nonzero vacuum expectation values of the coupling tensor fields, which seem to be introduced quite arbitrarily.

In contrast with the above theories of Lorentz violation, the torsion theory is the most natural one in logic\cite{35, 36}, which is derived from the fact that the connection can compatibly introduce an antisymmetric part, namely, the torsion.

\[ \tilde{\Gamma}_{\alpha\beta}^\mu = \Gamma_{\alpha\beta}^\mu + T_{\alpha\beta}^\mu, \]

where \( \Gamma_{\alpha\beta}^\mu \) is the Christoffel symbol, and \( T_{\alpha\beta}^\mu = -T_{\beta\alpha}^\mu \) is the torsion of the spacetime. Different from all matter fields like electromagnetic field, torsion is a geometrical interaction similar to gravity, which uniformly interacts with all matter in an accumulating manner. So to test torsion, it seems more effective to measure the movement of heaven body rather than atoms.

However, in contrast with the natural essence and deep philosophical meanings of the Lorentz invariance, the violation theories seem to be somehow artificial\cite{37}. Some experiments have been elaborated to test the Lorentz violation, but all results gave negative answers in high accuracy\cite{38}-\cite{44}, so one should take a little conservative attitude towards the Lorentz violation. Then how to explain the threshold anomalies of UHECR and TeV \( \gamma \)-rays? Here we present another scenario based on the nonlinear field theory, which also provides non-quadratic mass-energy relations or dispersion relations similar to (1.3), but strictly keeps the Lorentz invariance\cite{45, 46, 47}. In what follows we examine the local Lorentz transformation for some classical parameters defined from nonlinear fields and establish their relations.
2 Local Lorentz transformation and non-quadratic dispersion relation

2.1 Local Lorentz transformation for classical parameters

Taking the Minkowski metric as $\eta_{\mu\nu} = \text{diag}[1, -1, -1, -1]$, we consider the field systems of nonlinear spinor $\psi$ and scalar $\phi$. For the nonlinear spinor $\psi$, the dynamic equation is given by

$$\alpha^\mu (\hbar i \partial_\mu - e A_\mu) \psi = (\mu - F') \gamma \psi,$$

(2.1)

where the $4 \times 4$ Hermitian matrices are defined by

$$\alpha^\mu = \begin{cases} 
I & 0 \\
0 & I 
\end{cases}, \quad \bar{\sigma} = \begin{cases} 
0 & \sigma^k \\
\sigma^k & 0 
\end{cases}, \quad \gamma = \begin{cases} 
I & 0 \\
0 & -I 
\end{cases}$$

(2.2)

with Pauli matrices

$$\bar{\sigma} = (\sigma^k) = \begin{cases} 
0 & 1 \\
1 & 0 
\end{cases}, \quad \begin{cases} 
0 & -i \\
i & 0 
\end{cases}, \quad \begin{cases} 
1 & 0 \\
0 & -1 
\end{cases}.$$

(2.3)

$F = F(\check{\gamma})$ is a positive function of the quadratic scalar $\check{\gamma} \equiv \psi^+ \gamma \psi$.

For scalar field $\phi(x^\mu)$, the dynamic equation is given by

$$\partial_\mu \partial^\mu \phi = K \phi,$$

(2.4)

where $K = K(|\phi|^2, \partial_\mu \phi^+ \partial^\mu \phi)$ is a smooth real function.

For both spinor $\psi$ and scalar $\phi$, the current conservation law holds due to the gauge invariance of their dynamic equations,

$$\partial_\mu \rho^\mu = 0,$$

(2.5)

where the current is defined respectively by

$$\rho^\mu = \begin{cases} 
\psi^+ \alpha^\mu \psi & \text{for spinor,} \\
\kappa \Im(\phi^+ \partial^\mu \phi) & \text{for scalar.} 
\end{cases}$$

(2.6)

$\kappa$ is a normalizing constant. By (2.5) we have the normalizing condition

$$\int_{R^3} \rho^0 d^3x = 1.$$ 

(2.7)

For the nonlinear equations (2.1) and (2.4), their solutions have particle-wave duality [48-52]. In [46, 47, 53, 54], the local Lorentz transformations were widely used for the classical parameters without proof. Here we set the transformations on a solid base at first, and then derive the non-quadratic dispersion relations for some cases. The conditions for these results are helpful to understand the relation between
classical mechanics and quantum theory. To clarify the status of the field system, we defined

**Definition 1** For the field system $\psi$ or $\phi$, we define the central coordinate $\vec{X}$ and drifting speed $\vec{v}$ respectively by

$$\vec{X}(t) = \int_{\mathbb{R}^3} \vec{x} \rho^0 d^3x, \quad \vec{v} = \frac{d}{dt} \vec{X},$$

(2.8)

where $t = x^0$. The coordinate system with central coordinate $\vec{X} = 0$ is called the **central coordinate system** of the field.

**Definition 2** If a field is a localized wave pack drifting smoothly without emitting and absorbing energy quantum, that is, it is at the energy eigenstate in its central coordinate system, we call it at the **particle state**. Otherwise, the field is in the process of exchanging energy quantum with its environment, we call it in the **quantum process**.

By the current conservation law (2.5), we have

$$\vec{v} = \int_{\mathbb{R}^3} \vec{x} \partial_0 \rho^0 d^3x = -\int_{\mathbb{R}^3} \vec{x} \nabla \cdot \vec{d}^3x = \int_{\mathbb{R}^3} \vec{d}^3x.$$

(2.9)

For the field at particle state with mean radius much less than the characteristic length of its environment, by (2.8) and (2.9) we have the classical approximation

$$\rho^\mu \rightarrow u^\mu \sqrt{1 - v^2} \delta(\vec{x} - \vec{X}),$$

(2.10)

where

$$u^\mu \equiv (\xi, \xi \vec{v}), \quad \xi = \frac{1}{\sqrt{1 - v^2}}.$$

(2.11)

(2.10) is the precondition for validity of classical mechanics [46, 47, 53]. For such system at particle state, we can clearly define the classical parameters such as “momentum”, “energy” and “mass”, and derive the classical mechanics. From [46, 47], we learn that a system at particle state can be described by classical mechanics in high accuracy, whereas for the system in the quantum process, we must describe it by quantum theory or by the original equation (2.1) or (2.4). The quantum process is an unstable state, which is usually completed in a very short time.

In what follows, we examine the local Lorentz transformation for the classical parameters. Since the rotation transformation is trivial, we only consider the boost one. For the case of flat spacetime, assume $x^\mu$ is the Cartesian coordinate. Consider the central coordinate system of the field with coordinate $\vec{X}^\mu$, which moves along $x^1$ at speed $v$ with $\vec{X}^k(k \neq 0)$ parallel to $x^k$, and $\vec{X}^k = 0$ corresponds to the central coordinate of the field $X^k(t)$. Then the Lorentz transformation between $x^\mu$ and $\vec{X}^\mu$ in the form of matrix is given by

$$x = L(v)\vec{X}, \quad \vec{X} = L(v)^{-1}x = L(-v)x$$

(2.12)
where \( x = (t, x^1, x^2, x^3)^T, \bar{X} = (\bar{X}^0, \bar{X}^1, \bar{X}^2, \bar{X}^3)^T \) and

\[
L(v) = \text{diag} \left[ \begin{pmatrix} \xi \xi^v \\ \xi^v \xi \end{pmatrix}, 1, 1 \right] = (L^\mu_\nu).
\]

(2.13)

Assume \( S, P^\mu \) and \( T^{\mu\nu} \) are any scalar, vector and tensor defined by the real functions of \( \phi \) or \( \psi \) and their derivatives, such as \( S = |\phi|^2, T^{\mu\nu} = \Re(\psi^+ \alpha^\mu \partial^\nu \psi) \). For the field at particle state, all these functions are independent of proper time \( \bar{X}_0 \). Thus in the central coordinate system, the spatial integrals of these functions define the proper classical parameters of the field, and these proper parameters are all constants. Their Lorentz transformation laws are given by

**Theorem 1** For a field system at particle state, the integrals of covariant functions \( S, P^\mu \) and \( T^{\mu\nu} \) satisfy the following instantaneous Lorentz transformation laws under the boost transformation (2.12) between \( x^\mu \) and \( \bar{X}_\mu \) at \( dt = 0 \),

\[
I \equiv \int_{R^3} S(x) d^3x = \sqrt{1 - v^2} \bar{I}.
\]

(2.14)

\[
I^\mu \equiv \int_{R^3} P^\mu(x) d^3x = \sqrt{1 - v^2} L^\mu_\nu \bar{I}^\nu.
\]

(2.15)

\[
I^{\mu\nu} \equiv \int_{R^3} T^{\mu\nu}(x) d^3x = \sqrt{1 - v^2} L^\mu_\alpha L^\nu_\beta \bar{I}^{\alpha\beta}.
\]

(2.16)

Where \( \bar{I}, \bar{I}^\mu, \bar{I}^{\mu\nu} \) are the proper parameters defined in the central system

\[
\bar{I} = \int_{R^3} S(\bar{X}) d^3\bar{X}, \quad \bar{I}^\mu = \int_{R^3} \bar{P}^\mu d^3\bar{X}, \quad \bar{I}^{\mu\nu} = \int_{R^3} \bar{T}^{\mu\nu} d^3\bar{X}.
\]

(2.17)

**Proof** We take (2.15) as example to show the relations. For the field at the particle state, by the transformation law of vector, we have

\[
P^\mu(x) = L^\mu_\nu \bar{P}^\nu(\bar{X}) = L^\mu_\nu \bar{P}^\nu(\bar{X}^1, \bar{X}^2, \bar{X}^3) = P^\mu(\xi(x^1 - vt), x^2, x^3).
\]

(2.18)

So the integral can be calculated as follows

\[
I^\mu = \int_{R^3} P^\mu(x) d^3x \bigg|_{dt=0}
\]

\[
= \int_{R^3} P^\mu(\xi(x^1 - vt), x^2, x^3) \sqrt{1 - v^2} d(\xi(x^1 - vt)) dx^2 dx^3
\]

(2.19)

\[
= \int_{R^3} L^\mu_\nu \bar{P}^\nu(\bar{X}) \sqrt{1 - v^2} d^3\bar{X} = \sqrt{1 - v^2} L^\mu_\nu \bar{I}^\nu.
\]

The proof is finished.

**Remarks 1** The Lorentz transformation laws (2.14), (2.15) and (2.16) are valid for the varying speed \( v(t) \), because the integrals are only related to the simultaneous condition \( dx^0 = dt = 0 \), and the relations only related to algebraic calculations.

**Remarks 2** When the field is not at the particle state, the covariant integrands will depend on the proper time \( \bar{X}_0 \), then the calculation (2.19) can not pass through,
and the relations (2.14)-(2.16) are usually invalid, except the integrand satisfies some conservation law similar to (2.3).

In some text books, the relations (2.14)-(2.16) are directly derived via Lorentz transformation of the integrands and volume element relation

\[ d^3x = \sqrt{1 - v^2} d^3\bar{X}. \]  

(2.20)

As mentioned by remark 2, the calculation will provides wrong result if the field is not at the particle state, because (2.20) is just spatial volume, it suffers from the problem of simultaneity.

Usually, the proper parameters have very simple form. For any true vector, it always takes \( \bar{I}^\mu = (\bar{I}^0, 0, 0, 0) \), then

\[ I^\mu = \sqrt{1 - v^2} \bar{I}^0 u^\mu. \]  

(2.21)

For some cases of tensor \( I^{\mu\nu} \), it can be expressed as

\[ I^{\mu\nu} = \sqrt{1 - v^2} \left( Ku^{\mu}u^{\nu} + J\eta^{\mu\nu} \right), \]  

(2.22)

where \( K, \bar{J} \) are constants.

In the curved spacetime with orthogonal time coordinate, if the radius of curvature in the neighborhood of the center of the field is much larger than the mean radius of the field, the above calculations and relations can be parallel transformed into the curved spacetime. In this case, let \( \bar{X}^\mu \) be the central Cartesian coordinate of the tangent spacetime at the center of the field. \( X^\mu \) is an inertial coordinate system in the tangent spacetime fixed with the curved spacetime. \( \bar{X}^\mu \) instantaneously moves along \( X^1 \) at speed \( v \). Then it is easy to check the local Lorentz transformation laws (2.14), (2.15) and (2.16) also hold, as long as the field is at the particle state.

### 2.2 Non-quadratic dispersion relation for spinor

For the dark spinor, \( e = 0 \) in (2.1). From [46, 47], we get the momentum and mass-energy relation of the spinor as follows

\[ p^\mu = \left( m_0 + W \ln \frac{1}{\sqrt{1 - v^2}} \right) u^\mu, \]  

(2.23)

\[ E = \frac{m_0}{\sqrt{1 - v^2}} + W \left( \frac{1}{\sqrt{1 - v^2}} \ln \frac{1}{\sqrt{1 - v^2}} + \sqrt{1 - v^2} \right), \]  

(2.24)

where \( W \ll m_0 \) is the proper energy provided by the nonlinear potential. The mass-energy relation or dispersion relation in the usual form is given by

\[ (E - W \sqrt{1 - v^2})^2 = \vec{p}^2 + \left( m_0 + W \ln \frac{1}{\sqrt{1 - v^2}} \right)^2. \]  

(2.25)
Referring to electron, by estimation we have $\frac{W}{m_0} \sim 10^{-6}$. For a spinor moving at speed $1 - 10^{-n}$, for $n > 1$ we have

$$\ln \frac{1}{\sqrt{1 - v^2}} \approx 1.15n - 0.35. \quad (2.26)$$

The Lagrangian of the particle becomes

$$L = -\left( (m_0 + W) + W\ln \frac{1}{\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}} \right) \sqrt{g_{\mu\nu}\dot{x^\mu}\dot{x}^\nu}, \quad (2.27)$$

where $\dot{x}^\mu = \frac{dx^\mu}{dt}$. So the nonlinear term leads to a tiny departure from the geodesic.

For a spinor with interaction like electromagnetic field, the mass-energy relation of the particle will be much complicated. More generally, we consider the system with following Lagrangian\[47\]

$$L = \psi^\dagger \alpha^\mu (i \partial_\mu - e A_\mu) \psi - \mu \gamma + F(\gamma) - s \gamma G$$

$$-\frac{1}{2} \kappa (\partial_{\mu} A_{\nu} \partial^{\mu} A^{\nu} - a^2 A_{\mu} A^{\mu}) - \frac{1}{2} \lambda (\partial_{\mu} G \partial^{\mu} G - \beta^2 G^2), \quad (2.28)$$

where $A^\mu$ and $G$ are potentials produced by $\psi$ itself, $\kappa = \pm 1$ and $\lambda = \pm 1$ stand for the repulsive or attractive self interaction. Then the 4-dimensional momentum $p^\mu$ and energy $E$ of the system at particle state are respectively defined by

$$p^\mu \equiv \int_{R^3} \psi_\dagger (i \partial^\mu - e A^\mu) \psi d^3x,$$  

$$E \equiv \int_{R^3} \left( \sum_{x_f} \frac{\partial L}{\partial (\partial_t f)} \partial_t f - L \right) d^3x = p^0 + E_F + E_A + E_G, \quad (2.30)$$

where the classical parameters are given by

$$p^\mu = \left( m_0 + s W \gamma + W_F \ln \frac{1}{\sqrt{1 - v^2}} \right) v^\mu, \quad (2.31)$$

$$E_F = W_F \sqrt{1 - v^2}, \quad (2.32)$$

$$E_A = W_A \left( 1 + \frac{v^2}{3} \right) + W_a \frac{v^2}{\sqrt{1 - v^2}}, \quad (2.33)$$

$$E_G = W_G \left( 1 - \frac{v^2}{3} \right) - W_b \frac{v^2}{\sqrt{1 - v^2}}, \quad (2.34)$$

in which the proper parameters are calculated by

$$W_F = \int_{R^3} (F' \dot{\gamma} - F) d^3 \bar{x} > 0, \quad W_\gamma = \int_{R^3} \dot{\gamma} d^3 \bar{x}, \quad (2.35)$$

$$W_A = \frac{\kappa e^2}{8\pi} \int_{R^6} \frac{e^{-ar}}{r} \left( |\psi(\bar{x})|^2 |\psi(\bar{y})|^2 + \bar{\rho}(\bar{x}) \cdot \bar{\rho}(\bar{y}) \right) d^3 \bar{x} d^3 \bar{y}, \quad (2.36)$$

$$W_G = \frac{\lambda s^2}{8\pi} \int_{R^6} \frac{e^{-br}}{r} \dot{\gamma}(\bar{x}) \dot{\gamma}(\bar{y}) d^3 \bar{x} d^3 \bar{y}, \quad (2.37)$$

$$W_a = \frac{\kappa}{3} \left( \frac{ae}{4\pi} \right)^2 \int_{R^3} \left( \int_{R^3} \frac{e^{-ar}}{r} |\psi(\bar{y})|^2 d^3 \bar{y} \right)^2 d^3 \bar{x}, \quad (2.38)$$

$$W_b = \frac{\lambda}{3} \left( \frac{bs}{4\pi} \right)^2 \int_{R^3} \left( \int_{R^3} \frac{e^{-br}}{r} \dot{\gamma}(\bar{y}) d^3 \bar{y} \right)^2 d^3 \bar{x}, \quad (2.39)$$
and \( r = | \vec{x} - \vec{y} | \). By (2.30)-(2.34) we get the dispersion relation as follows

\[
(E - E_F - E_A - E_G)^2 = \vec{p}^2 + \left( m_0 + sW_sG + W_F \ln \frac{1}{\sqrt{1 - v^2}} \right)^2. \tag{2.40}
\]

By (2.25) and (2.40), we find that the interaction term can result in very complicated dispersion relation. How to use these relation to explain the threshold paradoxes of the high energy cosmic rays is involved in the interpretation of parameters, which will be discussed elsewhere. By the way, the scalar interaction may be absent in the nature, because it manifestly appears in proper mass of fermions, see (2.31) and (2.40).

3 Discussion and conclusion

From the above analysis, we find that nonlinear field theories do include the non-quadratic dispersion relation such as (2.25) and (2.40). So the explanation for threshold paradoxes of UHECR and TeV \( \gamma \)-rays by dispersion relation does not definitely require Lorentz violation. It should be mentioned that, all interactions are actually related to nonlinearity, for instance, the charge density of the electromagnetic interaction \( \psi^+ \sigma^\mu \psi \) is nonlinear, which contributes proper energy \( W_A \) for the particle as described by (2.36). This part of energy satisfy the energy-speed relation (2.33).

By (2.31)-(2.34), we learn different interaction term leads to different energy-speed relation. So the experiments towards such relations should bring us important information from each interaction, and the high energy cosmic rays may be the useful materials. On the other hand, the disturbance of nonlinear effect may influence our astronomical observation. For the movement of a heaven body, by (2.27), we learn that the order of the relative deviation from geodesic is \( \text{Err} = \frac{Wv^2}{m_c} \), where \( W \) is the energy contribution of all interactions, \( m \) is the usual mass. The typical values for a proton are \( W \sim 1\text{MeV} \) and \( m_p \sim 10^3\text{MeV} \), so we have \( \frac{W}{m} \sim 10^{-3} \). In a galaxy, the typical speed of heaven bodies relative to the CMB is 300km/s \(^{[55]}\), so we have the typical nonlinear deviation for galactic system \( \text{Err}|_{\text{galaxy}} \sim 10^{-9} \). In the solar system, the typical speed of the planets is about \( v \sim 30\text{km/s} \), so we have the typical nonlinear deviation for solar system \( \text{Err}|_{\text{planet}} \sim 10^{-11} \). Thus, before the nonlinear effects are clearly worked out, an astronomical measurement with relative error less than \( \text{Err} \) is difficult. The precession of the perihelion of Mercury is 43 seconds of arc per century \( 43/(100 \times 360 \times 3600) \approx 3.3 \times 10^{-7} \), so the nonlinear effects have not influence on this result.

Lorentz invariance includes two fold meanings: One is the covariance of the universal physical laws, which is a problem of philosophy. The other is property of the spacetime, namely the measurement of line element. Whether the spacetime manifold
is measurable is also a philosophical problem, but how to measure the distances is a problem of geometry. The philosophical problem involves the most fundamental and universal postulates which could only be acceptable as faith, because we can neither test all particles whether they satisfy the covariant equation everywhere and every time, nor can we check whether all parts of the spacetime are measurable, including the singularity inside a black hole and each points at a line of Planck length. Even though the Lorentz violation theories can not manifestly violate the covariance, see the forms of (1.5)-(1.7) as example. One may argue the thermodynamics is not covariant, the answer is that it is just a conditional theory but not a universal one, thus to abuse its concepts and laws, such as entropy and the second law, without restriction is inadequate and leads to confusion.

We once measured the spacetime with rule $ds_t = |dt|, ds^2 = dx^2 + dy^2 + dz^2$ and solved infinite practical problems. Today we measure the universe with $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ then we achieved beauty and harmony. There are infinite rules to measure length consistently, but only the spacetime with rule of quadratic form has wonderful properties and potential. If accepting the quadratic rule, the local Lorentz invariance certainly holds for the spacetime.

There are also fields satisfy the covariance but violate the Lorentz invariance. Their Lagrangian always includes term as follows

$$L_{LV} = (g^{\mu\nu} + \tau^{\mu\nu})\partial_{\mu}\partial_{\nu}\phi + \psi^+(\alpha^\mu + a^\mu)\partial_{\mu}\psi + \cdots.$$  

For such fields, the propagating speed is not pure geometrical, which depends on fields $\tau^{\mu\nu}$ and $a^\mu$. Similar to the case of Navier-Stokes equation in fluid mechanics, the nonlinearity is much worse than that of (2.28). Although for adequately small value $\tau^{\mu\nu}$ and $a^\mu$, the solutions to the dynamical equation will also be finite, but the world including such fields will become a mess, because each atom of the same element have different spectrum depending on coordinates, crystal lattices are distorted, the solar system has turbulence and chaos, and the double-helix of DNA has disordered knots. So ‘the coefficients of the highest order derivatives in the Lagrangian must be constants’ should be a fundamental postulate. This postulate is related to the quaternion structure of the world, only of such excellent structure the world becomes so luxuriant but so harmonious. In percipience of such opinion, the modification of general relativity with terms $\phi R, f(R)$ is also doubtful.

Some confusions in physics are caused by ambiguous concepts or circular relations. For example, to the relation between classical mechanics(CM) and quantum mechanics(QM), the common answer must be that: “transform the classical parameters, Hamiltonian and the energy equation of CM into operators, we get QM. Contrarily, the limit of QM as $\hbar \to 0$ provides CM.”
At first, by the answer, it seems that both CM and QM are equivalent theories in logic. In fact they are different theory suitable for different status of a system. The basic concepts such as ‘coordinate’, ‘momentum’ have different meanings in each mechanics, although they have close relations. The uncertainty relation is the typical plausible laws making puzzles and paradoxes[56, 57]. Clearly they should be unified in a higher level theory[45, 46, 50, 52]. secondly, \( \hbar \) is a universal constant acting as a unit to measure other physical parameters, we have not a concept \( \hbar \rightarrow 0 \)? Whether a physical system should be described by CM or QM is obviously not determined by external conditions such as \( \hbar \rightarrow 0 \) and CM or QM, but determined by the status of the system itself. This is the meaning of the Definition 2.

Spacetime and fields are different components of the world. They paly different roles with different characteristics, and satisfy completely different postulates and measurement rules. So it is hard to understand the motivation of the quantum gravity. Why we should modify a well defined and graceful theory, without any definite violation of experiments, by an ambiguous and incomplete theory? Why not modify the quantum field theory by general relativity? The mission of a physical theory is to find out the intrinsic truth and beauty and harmony of the nature. But at its best, besides some ill defined concepts as foam, wormhole, Lorentz violation, what can quantum gravity actually provide us?

In the spinor theory of general relativity context, there exists the pseudo-violation of Lorentz invariance[58, 59], which is caused by the derivatives of the vierbein or local frame. The vierbein is defined in the tangent spacetime of a fixed point in spacetime manifold, and the Lorentz transformation is just an algebraic operation in this fixed tangent spacetime. Whereas the derivatives of the vierbein must involve the tangent spacetimes of different points in some sense, so it violates the local Lorentz invariance. In strict sense, the equivalence principle only holds for the linear tensors, where the influences of vierbein and nonlinear fields are absent. However, the fundamental dynamic equations of the field system should be intrinsically Lorentz invariant.

Acknowledgments

The author is grateful to his supervisor Prof. Ta-Tsien Li for his encouragement and guidance.
References

[1] M. Takeda, et al, *Extension of the Cosmic-Ray Energy Spectrum Beyond the Predicted Greisen-Zatsepin-Kuzmin Cutoff*, Phys. Rev. Lett. 81 (1998) 1163. astro-ph/9807193

[2] N. Hayashida, et al, *Updated AGASA event list above $4 \times 10^{19}$eV*, Astron. J. 120 (2000) 2190. astro-ph/0008102

[3] A. A. Watson, *Observations of Ultra-High Energy Cosmic Rays*, Astron. Astrophys. 441 (2005) 465-472. astro-ph/0507207

[4] K. Greisen, Phys. Rev. Lett. 16 (1966) 748.

[5] G. T. Zatsepin and V. A. Kuz’min, JETP Lett. 41 (1966) 78.

[6] G. Amelino-Camelia, T. Piran, *Planck-scale deformation of Lorentz symmetry as a solution to the UHECR and the TeV-$\gamma$ paradoxes*, Phys. Rev. D64 (2001) 036005, astro-ph/0008107

[7] S. Coleman, S. L. Glashow, *Cosmic Ray and Neutrino Tests of Special Relativity*, Phys. Lett. B405 (1997) 249-252. hep-ph/9703240

[8] S. Coleman, S. L. Glashow, *Evading the GZK Cosmic-Ray Cutoff*, hep-ph/9808446

[9] F. W. Stecker, S. L. Glashow, *New Tests of Lorentz Invariance Following from Observations of the Highest Energy Cosmic Gamma Rays*, Astropart. Phys. 16 (2001) 97-99. astro-ph/0102226
[15] A. G. Cohen. S. L. Glashow, A Lorentz-Violating Origin of Neutrino Mass?, hep-ph/0605036

[16] G. Amelino-Camelia, et al, Tests of quantum gravity from observations of γ-ray bursts, Nature 393 (1998) 763, astro-ph/9712103

[17] P. Kosinski, J. Lukierski, P. Maslanka, J. Sobczyk, The classical basis for κ-deformed Poincaré (super)algebra and the second κ-deformed supersymmetric Casimir, Mod. Phys. Lett. A10 (1995) 2599, hep-th/9412114

[18] R. Aloisio1, P Blasi, P. L. Ghia, A. F. Grillo, Probing The Structure of Space-Time with Cosmic Rays, Phys. Rev. D62 (2000) 053010, astro-ph/0001258

[19] N. R. Bruno, G. Amelino-Camelia, J. Kowalski-Glikman, DEFORMED BOOST TRANSFORMATIONS THAT SATURATE AT THE PLANCK SCALE, Phys. Lett. B522 (2001) 133-138, hep-th/0107039

[20] Alex Granik, Magueijo-Smolin Transformation as a Consequence of a Specific Definition of Mass, Velocity, and the Upper limit on Energy, hep-th/0207113

[21] R. J. Szabo, Quantum Field Theory on Noncommutative Spaces, Phys. Rept. 378 (2003) 207-299, hep-th/0109162

[22] S. Mignemi, Transformations of coordinates and Hamiltonian formalism in deformed Special Relativity, Phys. Rev. D68 (2003) 065029, gr-qc/0304029

[23] S. Hossenfelder, Deformed Special Relativity in Position Space, Phys. Lett. B 649, 310 (2007), gr-qc/0612167

[24] A. Agostini, et al, Generalizing the Noether theorem for Hopf-algebra spacetime symmetries, hep-th/0607221

[25] M. Daszkiewicz, J. Lukierski and M. Woronowicz, NONCOMMUTATIVE TRANSLATIONS AND *-PRODUCT FORMALISM, hep-th/0701152

[26] S. Ghosh, P. Pal, Deformed Special Relativity and Deformed Symmetries in a Canonical Framework, hep-th/0702159

[27] D. Mattingly, Modern Tests of Lorentz Invariance, Living Rev. Rel. 8 (2005) 5, gr-qc/0502097

[28] R. Bluhm, Overview of the SME: Implications and Phenomenology of Lorentz Violation, hep-ph/0506054
[29] D. Colladay, V. A. Kostelecky, *Lorentz-violating extension of the standard model*, Phys. Rev. D58, 116002

[30] V. A. Kostelecky, S. Samuel, *Spontaneous Breaking Of Lorentz Symmetry In String Theory*, Phys. Rev. D, 39, 683, (1989).

[31] J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos, *Probing models of quantum spacetime foam*, ACT-10/99, CTP-TAMU-40/99, gr-qc/9909085

[32] C. P. Burgess, et al, *Loop-generated bounds on changes to the graviton dispersion relation*, JHEP, 03, 043, (2002), hep-th/0201082

[33] R. Gambini, J. Pullin, *Nonstandard optics from quantum spacetime*, Phys. Rev. D, 59, 124021, (1999), gr-qc/9809038

[34] V. A. Kostelecky, *TOPICS IN LORENTZ AND CPT VIOLATION*, hep-ph/0104227

[35] I. L. Shapiro, *Physical aspects of the spacetime torsion*, Physics Reports 357 (2002) 113C213

[36] A. S. Belyaev, I. L. Shapiro, *Torsion action and its possible observables*, Nucl. Phys. B543 (1999) 20-46, hep-ph/9806313

[37] A. Iorio, *Three questions on Lorentz violation*, physics/0612171

[38] Paul L. Stanwix, M. E. Tobar, P. Wolf, C. R. Locke, E. N. Ivanov, *Improved test of Lorentz invariance in electrodynamics using rotating cryogenic sapphire oscillators*, Phys. Rev. D74, 081101(R) (2006)

[39] P. Wolf, S. Bize, A. Clairon, G. Santarelli, *Improved test of Lorentz invariance in electrodynamics*, Phys. Rev. D70, 051902(R)(2004)

[40] P. Wolf, S. Bize, A. Clairon, A. N. Luiten, G. Santarelli, M. E. Tobar, *Tests of Lorentz Invariance using a Microwave Resonator*, Phys. Rev. Lett. Vol.90(No.6), 060402(2003)

[41] H. Muler, S. Herrmann, C. Braxmaier, S. Schiller, A. Peters, *Modern Michelson-Morley Experiment using Cryogenic Optical Resonators*, Phys. Rev. Lett. Vol.91(No.2), 020401(2003)

[42] C. J. Berglund, et al, *New Limits on Local Lorentz Invariance from Hg and Cs Magnetometer*, Phys. Rev. Lett. Vol.75(No.10), 1879(1995)
[43] T. E. Chupp, et al, Results of a New Test of the Local Lorentz Invariance, Phys. Rev. Lett. Vol.63(No.15), 1541(1989)

[44] E. Fischbach, et al, Lorentz Noninvariance and the Eötvös experiments, Phys. Rev. D32(No.1), 154(1985)

[45] Y. Q. Gu, Canonical Form of Field Equations, Adv in Appl. Clif. Alg. V7(1), 13-24(1997), [http://www.clifford-algebras.org/v7/v71/GU71.pdf](http://www.clifford-algebras.org/v7/v71/GU71.pdf), hep-th/0610189

[46] Y. Q. Gu, New Approach to N-body Relativistic Quantum Mechanics, IJMPA Vol.22(No.11) 2007-2019(2007), [hep-th/0610153](http://www.clifford-algebras.org/v8/82/gu82.htm)

[47] Y. Q. Gu, Mass-Energy Relation of the Nonlinear Spinor, [hep-th/0701030](http://www.clifford-algebras.org/v8/82/gu82.htm)

[48] R. Finkelsten, et al, Phys. Rev. 83(2), 326-332(1951)

[49] M. Soler, Phys. Rev. D1(10), 2766-2767(1970)

[50] Y. Q. Gu, Some Properties of the Spinor Soliton, Adv in Appl. Clif. Alg. 8(1), 17-29(1998), [http://www.cliffordanerrals.org/v8/81/gu81.pdf](http://www.cliffordanerrals.org/v8/81/gu81.pdf)

[51] Y. Q. Gu, Spinor Soliton with Electromagnetic Field, Adv in Appl. Clif. Alg. V8(2), 271-282(1998), [http://www.cliffordanerrals.org/v8/82/gu82.htm](http://www.cliffordanerrals.org/v8/82/gu82.htm)

[52] Y. Q. Gu, Characteristic Functions and Typical Values of the Nonlinear Dark Spinor, [hep-th/0611210](http://www.cliffordanerrals.org/v8/82/gu82.htm)

[53] Y. Q. Gu, A Cosmological Model with Dark Spinor Source, to appear in IJMPA, [gr-qc/0610147](http://www.cliffordanerrals.org/v8/82/gu82.htm)

[54] Y. Q. Gu, Accelerating Expansion of the Universe with Nonlinear Spinors, [gr-qc/0612176](http://www.cliffordanerrals.org/v8/82/gu82.htm)

[55] Y. Sofue, Y. Tsurui, M. Honma, et al, CENTRAL ROTATION CURVES OF SPIRAL GALAXIES, [astro-ph/9905056](http://www.cliffordanerrals.org/v8/82/gu82.htm)

[56] M. Pavisi, The Landscape of Theoretical Physics, Published by Kluwer Academic Publishers, 2001, [gr-qc/0610061](http://www.cliffordanerrals.org/v8/82/gu82.htm)

[57] M. Tegmark, The Mathematical Universe, [arXiv:0704.0646](http://www.cliffordanerrals.org/v8/82/gu82.htm)

[58] Y. Q. Gu, Representation of the Vierbein Formalism, [gr-qc/0612106](http://www.cliffordanerrals.org/v8/82/gu82.htm)

[59] Y. Q. Gu, Simplification of the covariant derivatives of spinors, [gr-qc/0610001](http://www.cliffordanerrals.org/v8/82/gu82.htm)