Analysis Method of Structural Sensitivity of Dual Mass Flywheel and Power Transmission

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Abstract. Dual mass flywheel (DMF) can effectively attenuate torsional vibration from the engine. Aiming to obtain greater damping performance, the sensitivity analysis method is studied combining with the constraint relation of primary flywheel assembly and secondary flywheel assembly, which is used to determine the key structural parameters of DMF which have the greatest relationship with the natural frequency of the system. A car with a DMF is taken as an object of sensitivity analysis, then the key structural parameters of the DMF is obtained and adjusted, finally, simulation results verify the effectiveness of the analysis method.

1. Introduction

The vehicle vibration noises will directly influence the ride safety, the fuel economy and the NVH (Noise, Vibration and Harshness) performance of vehicles [1]. The power source vibration noises accounts for more than one-half of the vehicle vibration noises, also resulting in the vibration noises of the power transmission [2]. Torsional vibration is the main source of the vibration noises of the power transmission [3]. Dual Mass Flywheel (DMF) is a new kind of vehicle torsional damper, which not only has the function of the single mass flywheel but also the driven plate type clutch torsional vibration damper [4]. With reasonable inertia distribution and torsional stiffness, DMF can make the resonant rotational speed of the power transmission under idling condition lower than the idling rotational speed, which can also attenuate torsional vibrations under driving conditions [5]. DMF is mounted between the engine and the transmission system. The schematic diagram of DMF is shown in Figure 1, which consists of a primary flywheel assembly, a secondary flywheel assembly and a damper. The primary flywheel assembly includes a starting gear ring, a signal ring, a cover and a primary flywheel. The secondary flywheel assembly comprises a flange, a seal disc and a secondary flywheel. The damper is composed of spring and damping elements. DMFs can be divided into several types according to the structure and the form of the springs, in which the circumferential arc spring dual mass flywheels (DMF-CS) is the most widely used type. The primary assembly and the crankshaft are connected by bolts. In addition, the clutch can be connected by the secondary assembly. Thus, power from engine can be transmitted to the primary assembly firstly, and then the flange compresses the arc springs to transmit the power to the secondary assembly. Finally, the power transmitted to the power transmission drives the car.
Hartmut Bach [6], Theodossiades S [7] and Schaper [8] simulated the characteristics of a power transmission with DMF under different operating conditions. They compared the angular accelerations and displacements between the output of engine and the input of gearbox under idling, sliding and accelerating conditions. The simulating results showed that excellent damping performance of DMF was demonstrated in idling and low speed conditions. Tolga Duran and E. Çağrı Sever A [9] analyzed the effects of single mass and dual mass flywheel on endurance limit with the dynamical loadings under engine operating conditions. He found that although the angular acceleration of the crankshaft increased with DMF, the corresponding dynamic load of the crankshaft decreased as the inertia was reduced. Li Guanghui [10] studied the natural torsion characteristic of DMF and proposed the result that the natural frequency would be the minimum when the inertia ratio of the primary and secondary flywheel was 1:1.

Structural sensitivity can reflect the gradient of the structural parameters to the response of the system, which has been widely used. Using structural sensitivity will facilitate the optimization of the dynamic characteristics by modifying the structural parameters. Yue Guiping [11] carried out the acoustic simulations of orifice noise of intake system and then determined the design parameters of quarter wave tuner through sensitivity analysis without the accurate model of the engine. Moreover, Lin J and Parker R G [12] used the well-defined vibration mode properties of tuned planetary gears to calculate eigen-sensitivities and establish exact formulae, which connected natural frequency sensitivity with the modal strain or kinetic energy and provided efficient means to determine the sensitivity to all stiffness and inertia parameters by inspection of the modal energy distribution.

The above literatures show that the main structural parameters of DMF include the inertias of the primary and secondary flywheel assembly and the multi-stage torsional stiffness, which affect the damping performance of DMF. In this paper, the sensitivity analysis method based on the constraint relation of DMF is used to analyze the relationship between the main structural parameters of DMF and the natural frequencies of the system, and then the key structural parameters with the greatest influence on certain natural frequency are determined from the main structural parameters, eventually, the key structural parameters are adjusted for better damping performance based on the sensitivity analysis results.

2. Structural sensitivity analysis method of automobile power transmission

With rotational motion, the dynamic model of automobile power transmission is a torsional vibration model. The model without damping is given by

\[ ([K] - \omega_i^2 [J]) \theta_i = 0 \]  

Figure 1. A schematic diagram of DMF
Where $[K]$ and $[J]$ are torsional stiffness matrix and inertia matrix, respectively, $\omega_i$ is the $i$th order natural frequency, and $\theta_i$ is the $i$th order rotational displacement vector.

2.1. Sensitivity of natural frequencies to torsional stiffness

Both $[K]$ and $[J]$ are real symmetric matrix. To simplify the calculation, pre-multiply the Equation (1) by $\theta_i^T$ to obtain Equation (2).

$$\theta_i^T ([K] - \omega_i^2 [J]) \theta_i = 0$$

(2)

$$\theta_i^T [J] \theta_i = M_i$$

(3)

Where $M_i$ the modal mass under the $i$th order.

Let the absolute and relative sensitivities of $\omega_i$ to the torsional stiffness of the $j$th unit be $S_{ab}(\omega_i / K_j)$ and $S_r(\omega_i / K_j)$, respectively. The partial derivative with respect to $K_j$ in Equation (2) is operated to obtain $S_{ab}(\omega_i / K_j)$ and $S_r(\omega_i / K_j)$.

$$S_{ab}(\omega_i / K_j) = \frac{[(\theta)_j - (\theta)_{j+1}]^2}{2 \omega_i M_i}$$

(4)

$$S_r(\omega_i / K_j) = \frac{[(\theta)_j - (\theta)_{j+1}]^2 K_j}{2 \omega_i M_i}$$

(5)

2.2. Sensitivity of natural frequencies to inertias

Let the absolute sensitivity and relative sensitivity of the $i$th natural frequency $\omega_i$ to torsional stiffness of the $j$th unit be $S_{ab}(\omega_i / J_j)$ and $S_r(\omega_i / J_j)$. Seeking the partial derivative with respect to $J_j$ in Equation (2), $S_{ab}(\omega_i / J_j)$ and $S_r(\omega_i / J_j)$ can be obtained as

$$S_{ab}(\omega_i / J_j) = \frac{\omega_i [(\theta)_j]_j}{2 M_i}$$

(6)

$$S_r(\omega_i / J_j) = \frac{J_j [(\theta)_j]_j}{2 M_i}$$

(7)

2.3. Characteristics of structural sensitivity

The expressions of the absolute and relative sensitivities show that the former is appropriate to qualitative analysis and evaluation, and the latter is suitable for quantitative analysis and evaluation. In practical engineering, we can firstly evaluate the gradients between the torsional stiffness and inertias and the target modal natural frequencies using absolute structural sensitivity method to find the structural parameters which have the biggest influence on the natural frequencies. Then we can establish the accurate mathematical relation between the parameters and the natural frequencies using the relative sensitivity method, modifying the structural parameters.

3. Analysis method of DMF and the power transmission based on structural sensitivity

3.1. Analysis method of moment of Inertia of DMF
It is assumed that the inertias of the primary and secondary flywheel assembly are $J_1$ and $J_2$, respectively, and the inertia of the single mass flywheel matched to the engine is $J_3$, which is provided by the engine manufacturer. Because DMF has the function of the single mass flywheel, $\frac{J_1 + J_2}{J_3} = 1$. Furthermore, the inertia ratio of the primary and secondary flywheel assembly can be obtained by the constraints of the inertias, the masses and the installation spaces of the primary and secondary flywheel assembly. With initial conditions, the initial values of $J_1$ and $J_2$ can be determined by

$$\lambda = \frac{J_1}{J_2} \quad J_1 + J_2 = J_3 \quad (8)$$

$$J_1 = \frac{\lambda J_3}{(\lambda + 1)} \quad J_2 = \frac{J_3}{(\lambda + 1)} \quad (9)$$

Because of the constraints of the primary and secondary flywheel assembly, any change of the moment of inertia of the flywheel assembly will cause the change of the other. Thus, the moment of inertia ratio $\lambda$ should be used instead of $J_1$ and $J_2$ as the structural parameter to conduct the calculations, when analysing the gradient relationship between the change of the primary and secondary flywheel assembly and that of the natural frequency. Therefore, the Equation (7) involving $\lambda$ can be rewritten as

$$S_n(\omega / \lambda) = \frac{\partial \omega / \lambda_0}{\partial \lambda} = \frac{\lambda \theta_j \frac{\partial [J]}{\partial \lambda}}{2M_j} \quad (10)$$

Let $j$th and $(j+1)$th units be primary flywheel assembly and secondary flywheel assembly, respectively, then

$$[J] = \begin{bmatrix} J_1 & J_2 & \cdots & 0 \\ J_2 & \frac{\lambda J_3}{(\lambda + 1)} & \cdots & J_3 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & J_n \end{bmatrix} \quad (11)$$

Substituting Equation (11) into Equation (10), it can be rewritten as

$$S_n(\omega / \lambda) = \frac{\lambda}{2M_j (\lambda + 1)} \left[ (\theta_j)_0^2 - (\theta_j)_j^2 \right] \quad (12)$$

According to $S_n(\omega / \lambda)$, the mathematical relationships between the $i$th order natural frequency and $\lambda$ can be established as

$$\Delta \lambda = \frac{\Delta \omega / \lambda}{S_n(\omega / \lambda)} \quad (13)$$

Where $\Delta \lambda$ is the variation based on the initial value of $\lambda$ , $\Delta \omega$ is the variation of the $i$th order natural frequency $\omega_j$ based on the initial conditions. Thus, $J_1$ and $J_2$ are obtained as

$$J_1 = \frac{\lambda + \Delta \lambda}{(\lambda + \Delta \lambda + 1)} \quad J_2 = \frac{J_3}{(\lambda + \Delta \lambda + 1)} \quad (14)$$
\[ \omega_i \] can be determined by the difference between the actual value and the ideal value of \( \omega_i \).

### 3.2 Analysis method of torsional stiffness of DMF

DMF usually has two stage torsional stiffness, then let the first stage stiffness and the second stage stiffness be \( K_1 \) and \( K_2 \), respectively. Usually the first stage stiffness is working under idling condition and low rotational speed, moreover, the second stage stiffness is taking effect under driving condition with high rotational speed, so \( K_1 \) can both transfer the engine power and adjust the system natural frequency. Let the operating angle of DMF at \( K_1 \) and \( K_2 \) be \( \theta_1 \) and \( \theta_2 \), respectively. Generally, the total torsion angle of the DMF springs being \( \theta \) is about 65°-70° [5], thus, \( \theta_1 + \theta_2 = \theta \). \( \theta_1 \) can be primarily valued as

\[
\theta_1 = \frac{T_1}{K_1}
\]

(15)

Where \( T_1 \) is the output torque of the power transmission under idling condition, which is related to the inertias of the secondary flywheel assembly, the clutch and the input shaft of the transmission and the angular accelerations of the starting motor. Accordingly, \( K_2 \) can be primarily calculated as

\[
K_2 = \frac{T_{\max}}{\theta - \theta_1}
\]

(16)

Where \( T_{\max} \) the maximum is torque from the engine, and \( \xi \) is the torque backup coefficient, which is related to the real car.

According to \( S_{\xi}(\omega / K_1) \), the mathematical relationships between the \( i^{th} \) order natural frequency and \( K_i \) can be established as

\[
\begin{align*}
\Delta K_i &= \frac{\omega_1 / \omega_1}{S_{\xi}(\omega / K_i)} K_i \\
K_i &= K_1 + \Delta K_i
\end{align*}
\]

(17) (18)

The second stage stiffness \( K_2 \) can be obtained by the same method.

### 4. Example and simulation

#### 4.1 Analysis Example

A car matching a CVT is taken as example, where \( J_1 \) denotes the moment of inertia and \( K_i \) denotes the torsional stiffness linking the two lumped masses. The structural parameters of the power transmission are listed in Table 1, where the unit of moment of inertia and torsion stiffness are \( \text{kg}\cdot\text{m}^2 \) and \( \text{Nm}/\text{rad} \), respectively.
Table 1. Structural parameters of the power transmission

| Names of Elements                  | Inertia | Value of Inertia | Torsional Stiffness | Value of Torsional Stiffness |
|-----------------------------------|---------|------------------|---------------------|-------------------------------|
| Driven part of rubber damper      | $J_1$   | 4.795E-3         | $K_1$               | 14320.0                       |
| Driving part of rubber damper     | $J_2$   | 2.038E-3         | $K_2$               | 74636.1                       |
| Accessories                       | $J_3$   | 9.74E-5          | $K_3$               | 356080.5                      |
| Cylinder 1                        | $J_4$   | 4.669E-3         | $K_4$               | 358856.9                      |
| Cylinder 2                        | $J_5$   | 4.712E-3         | $K_5$               | 360638.8                      |
| Cylinder 3                        | $J_6$   | 4.712E-3         | $K_6$               | 359640                        |
| Cylinder 4                        | $J_7$   | 4.686E-3         | $K_7$               | 1871080                       |
| Primary flywheel assembly         | $J_8$   | 0.08             | $K_8$               | 160.43; 733.39                |
| Secondary flywheel assembly       | $J_9$   | 0.012            | $K_9$               | 98731.62                      |
| Input shaft of CVT                | $J_{10}$| 7.312E-3         | $K_{10}$            | 48483.78                      |
| Driving cone of CVT               | $J_{11}$| 0.0268368        |                     |                               |

The car is fitted with a DMF. The inertias of the primary and secondary flywheel assembly are 0.08 kg∙m$^2$ and 0.012kg∙m$^2$, respectively, and the first stage stiffness and second stage stiffness are 160.43Nm/rad ($\theta_1=45.25^\circ$) and 733.39Nm/rad ($\theta_2=15^\circ$), respectively. In addition, the hollow travel angle is 4.75°, and the total torsional angle is 65°, that is, $\theta=65^\circ$. Based on the above data, the modal analysis is carried out, and the natural frequencies under idling and driving conditions are listed in Tables 2 and 3.

| Table 2. Natural frequencies under idling condition (Hz) |
|-------------------------------------------------------|
| $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | $f_9$ |
| 15.8  | 239.8 | 603.8 | 742   | 1053  | 1731  | 2481  | 3602  | 10667 |

| Table 3. System natural frequency under driving condition (Hz) |
|---------------------------------------------------------------|
| $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | $f_9$ | $f_{10}$ |
| 22.5  | 239.8 | 300   | 603.8 | 821.79| 1053  | 1731  | 2481  | 3602  | 10667   |

The absolute sensitivities of the 1$^{st}$ order natural frequency to the inertias and torsional stiffness can be obtained based on torsional vibration model under driving condition, as shown in Tables 4 and 5.

| Table 4. The absolute sensitivities of the 1$^{st}$ order natural frequency to inertias under driving condition |
|-------------------------------------------------------------|
| $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_6$ | $J_7$ | $J_8$ | $J_9$ | $J_{10}$ | $J_{11}$ |
| -221  | -218  | -217  | -217  | -216  | -216  | -215  | -215  | -1104 | -1122    | -1150     |
Table 5. The absolute sensitivities of the 1st order natural frequency to torsional stiffness under driving condition

| \( K_1 \) | \( K_2 \) | \( K_3 \) | \( K_4 \) | \( K_5 \) |
|----------|----------|----------|----------|----------|
| 5.62E-07 | 4.18E-08 | 1.89E-09 | 5.18E-09 | 1.01E-08 |

| \( K_6 \) | \( K_7 \) | \( K_8 \) | \( K_9 \) | \( K_{10} \) |
|----------|----------|----------|----------|----------|
| 1.68E-08 | 9.29E-10 | 0.101543 | 3.1E-06  | 7.97E-06 |

For inertias, the absolute sensitivities show that the inertias of the secondary flywheel assembly and the CVT component have the greatest influence on the 1st order natural frequency under driving condition and the natural frequency will decrease with the increase of inertias. For torsional stiffness, the torsional stiffness of DMF at driving stage has the greatest influence on the 1st order natural frequency and the natural frequency will increase with the increase of torsional stiffness. Since the structural parameters of the CVT cannot be modified, the key structural parameters will be the inertia ratio \( \lambda \) of the primary and secondary flywheel assembly and the second stage stiffness \( K_8 \).

The relative sensitivities of the 1st order natural frequency to torsional stiffness driving condition are shown in Table 6, where \( S_n(\alpha / K_8) = 0.495 \), and it can be calculated that \( S_n(\alpha / \lambda) = 0.062 \). According to vibration-absorption mechanism, analysis method of structural sensitivity and the initial conditions, the modified results can be obtained, that are, \( J_8 = 0.077 \) kg.m\(^2\), \( J_9 = 0.015 \) kg.m\(^2\), \( K_8 = 189 \) Nm/rad \( (\theta_1 = 37.75^\circ) \) (the first stage stiffness), \( K_8 = 511 \) Nm/rad \( (\theta_2 = 21.5^\circ) \) (the second stage stiffness), \( \theta_0 = 4.75^\circ \) (the hollow travel angle) and \( \theta = 59.25^\circ \) (the total torsion angle).

Table 6. The relative sensitivities of the 1st order natural frequency to torsional stiffness under driving condition

| \( K_1 \) | \( K_2 \) | \( K_3 \) | \( K_4 \) | \( K_5 \) |
|----------|----------|----------|----------|----------|
| 5.35E-05 | 2.08E-05 | 4.47E-06 | 1.24E-05 | 2.42E-05 |

| \( K_6 \) | \( K_7 \) | \( K_8 \) | \( K_9 \) | \( K_{10} \) |
|----------|----------|----------|----------|----------|
| 4.03E-05 | 1.16E-05 | 0.495229 | 0.002033 | 0.00257 |

After modification, the natural frequencies of the power transmission under idling and driving conditions are obtained as shown in Tables 7 and 8. The 1st order natural frequency is 17.9Hz and the corresponding resonance speed is 1074rpm under idling condition, which is far from the idle speed. Furthermore, the 1st order natural frequency is 18.5Hz and the corresponding resonance speed is 1110rpm under driving condition, which can avoid resonances in low speed region.

Table 7. Natural frequencies under idling condition (Hz)

| \( f_1 \) | \( f_2 \) | \( f_3 \) | \( f_4 \) | \( f_5 \) | \( f_6 \) | \( f_7 \) | \( f_8 \) | \( f_9 \) |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 17.9     | 240.1    | 605.1    | 713.4    | 1053.6   | 1731.9   | 2481.7   | 3604.7   | 10667.8   |

Table 8. System natural frequency under driving condition (Hz)

| \( f_1 \) | \( f_2 \) | \( f_3 \) | \( f_4 \) | \( f_5 \) | \( f_6 \) | \( f_7 \) | \( f_8 \) | \( f_9 \) | \( f_{10} \) |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 18.5     | 240.1    | 284      | 605.1    | 803      | 1053.6   | 1731.9   | 2481.7   | 3604.7   | 10667.8 |


4.2 Simulation

The dynamic models under idling condition and driving condition built by ADAMS software are shown as Figure 2 and 3, respectively.

Under idling condition, the frequency of excitation is 12.5Hz and the frequency of excitation is 25Hz under driving condition. For the power transmission matching the DMF without modification, angular acceleration curves of engine and transmission under idling condition and driving condition are shown as Figure 4 and 6, respectively. For the power transmission matching the DMF with modification, angular acceleration curves of engine and transmission under idling condition and driving condition are shown as Figure 5 and 7, respectively. Where red curve stands for angular acceleration of engine and the blue curve stands for angular acceleration of transmission. After modification, angular acceleration of transmission is attenuated from 0.05rad/s^2 to 0.04 rad/s^2 under idling condition. However, angular acceleration of transmission is still greater than that of engine because the resonant speed is higher than the idle speed. Moreover, angular acceleration of transmission is attenuated from 1.25rad/s^2 to 0.4 rad/s^2 under driving condition and angular acceleration of transmission is smaller than that of engine, which means the vibration from the engine is isolated.
5. Conclusion

The structural sensitivity analysis results show the rotational inertia coefficient and torsional stiffness of DMF are the most important factors affecting the first order natural frequencies of the powertrain under idling and driving conditions, so the method is convenient and purposeful to adjust structural parameters of DMF for improving the damping performance of DMF, which can also compensate the inaccuracy of the structural parameters of some components in modelling process to some extent.

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