Measuring primordial anisotropic correlators with CMB spectral distortions

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We show that inflationary models with broken rotational invariance generate testable off-diagonal signatures in the correlation between the μ-type distortion and temperature fluctuations of the cosmic microwave background. More precisely, scenarios with a quadrupolar bispectrum asymmetry, usually generated by fluctuations of primordial vector fields, produce a nonvanishing μ-T correlation when |ℓ₁ − ℓ₂| = 2. Since spectral distortions are sensitive to primordial fluctuations up to very small scales, a cosmic variance limited spectral distortion experiment can detect such effects with a high signal-to-noise ratio.

I. INTRODUCTION

Deviations of the cosmic microwave background (CMB) frequency spectrum from a perfect blackbody distribution, generally referred to as “CMB spectral distortions,” can be an important source of information about the thermal history of the early Universe. Spectral distortions are in effect generated by any process that could inject energy in the baryon-photon plasma, starting from very early times, and can thus be used to study and constrain such processes. At present we have no detection of CMB spectral distortions, with the best constraint dating back to the COBE/FIRAS limit, ∆ν/ν ≤ 10⁻⁴ [2]. However, current technological advancements and planned space missions, such as PIXIE [4], will produce dramatic improvements in the near future. For these reasons, spectral distortions have been receiving increasing attention in recent years, and they are the starting point for a growing number of new and original applications.

In this context, it was recently pointed out in Ref. [5] that CMB spectral distortions from dissipation of acoustic waves in the primordial plasma can be used to test the statistical properties of primordial cosmological perturbations. More specifically, the authors of Ref. [5] show that the correlation between the so-called μ distortion parameter and the CMB temperature fluctuations probes the three-point function (bispectrum) of primordial curvature perturbations in the squeezed limit (i.e. for triangle configurations in which one wave number is much smaller than the other two, thus producing a coupling between small and large scales). High-precision measurements of the μ-T correlation can then be used to estimate the bispectrum amplitude parameter fNL, for models predicting non-Gaussianity (NG) signals with a significant contribution from squeezed triangles. Similarly, the μ-μ autocorrelation probes the trispectrum of primordial fluctuations and can be used to infer the τNL parameter [5].

The main interest of this approach to testing primordial NG, in contrast to other methods such as measurements of the CMB temperature bispectrum, is that spectral distortions allow one to reach much smaller scales than what was achievable with temperature anisotropies, where ℓmax is limited by Silk damping. This in turn produces a significant increase of sensitivity for a cosmic-variance-dominated experiment. Different groups have so far studied the possibility to use μ distortions in order to constrain primordial NG of the local type [6,7], eventually including a running of the fNL parameter [7,8], NG generated by models with excited initial states [9], icosequatorial modes [9] or primordial magnetic fields [10,11].

In this paper, we will instead consider primordial scenarios characterized by the breaking of rotational invariance. We consider both Gaussian and non-Gaussian primordial perturbation fields, introducing, respectively, a spatial modulation in the power spectrum and a preferred direction in the bispectrum. Anisotropic bispectra can arise in inflationary models with a U(1) gauge field (e.g., Refs. [13–15]), while Gaussian primordial perturbations characterized by power spectra with a dipolar modulation have been considered in light of the observed hemispherical power asymmetry in CMB data [16–20].

The vast majority of inflationary models with breaking of rotational invariance predict primordial bispectra which peak in the squeezed limit. It is thus reasonable to expect these bispectra to generate non-negligible μ-T correlations. This is indeed what we will find. Moreover, the anisotropic quadrupolar nature of the primordial signal produces off-diagonal couplings, sourcing terms with |ℓ₁ − ℓ₂| = 2. Besides anisotropic inflationary scenarios, which are the main focus of this paper, we also consider
II. CMB µ DISTORTION ANISOTROPY

Heating due to diffusion damping of acoustic waves can induce distortions in the CMB blackbody spectrum. At very early times, when double Compton scattering is efficient \((z \gtrsim z_i \approx 2 \times 10^6)\), any generated distortion is immediately erased due to quick thermalization, and the blackbody spectrum is maintained. On the other hand, for \(z_f \lesssim z \lesssim z_i\) with \(z_f \approx 5 \times 10^4\), double Compton scattering has become inefficient, and Compton scattering mainly thermalizes the system, without changing the number of photons. Hence, due to energy injection via acoustic wave dissipation, a blackbody spectrum is altered to a Bose-Einstein distribution with nonzero chemical potential: \[ \mu \approx \frac{1}{4} \left\langle \delta z_2^2(z) \right\rangle_{z_f} \]

where \(\langle \rangle_p\) denotes a quantity averaged over an oscillation period, and \(\left\langle F(z)^{z_i_{z_f}} \right\rangle \equiv F(z_i) - F(z_f).\) The density fluctuation affecting \(\mu\) is expressed as (see e.g., Refs. [31–34])

\[ \left\langle \delta z_2^2(z) \right\rangle \approx 1.45 \left\langle \sum_{n=1}^{2} \int \frac{d^3k_n}{(2\pi)^3} \zeta k_n \right\rangle \left\langle \Delta \left( k_1 \right) \Delta \left( k_2 \right) \right\rangle_{p} e^{i(k_1+k_2) \cdot x}, \]

where \(\zeta\) denotes primordial curvature perturbation and the photon transfer function is approximately given by \(\Delta \left( k \right) \approx 3 \cos(kr) \exp[-k^2/k_D(z)]\) with \(r(t) \approx 2t/(a\sqrt{3})\). The diffusion damping scales at \(z_i\) and \(z_f\) are, respectively, \(k_D(z_i) \approx 12000\) Mpc\(^{-1}\) and \(k_D(z_f) \approx 46\) Mpc\(^{-1}\). \[ [31–34] \]

The above equations yield an expression for the \(\mu\) distortion [5, 6]:

\[ \mu(x) \approx \left\langle \sum_{n=1}^{2} \int \frac{d^3k_n}{(2\pi)^3} \zeta k_n \right\rangle \int d^3k_3 \delta^{(3)} \left( \sum_{n=1}^{3} k_n \right) \]

\[ f(k_1, k_2, k_3) e^{-i(k_1+k_2) \cdot x}, \]

\[ f(k_1, k_2, k_3) \equiv \frac{9}{4} W \left( \frac{k_3}{k_n} \right) e^{-\left( k_1^2+k_2^2 \right)/k_D(z)} z_f^{3}, \]

where \(W(x) \equiv 3j_1(x)/x\) is a top-hat filter function, limiting the scales where the acoustic waves are averaged to give heat as \(k_3/k_1 \lesssim 1\), and we have used an estimate: \(\left\langle \cos(k_1 r) \cos(k_2 r) \right\rangle_p \approx 1/2\). In the following analysis, we take \(k_n\) to be \(k_D(z_f)\), in order to estimate a lower bound on the \(\mu\) distortion. Then, the transfer function \(f(k_1, k_2, k_3)\) filters the squeezed-limit signals, satisfying \(k_1, k_2 > k_D(z_f) > k_3\). As seen in Eq. (3), the \(\mu\) distortion has quadratic dependence on the primordial curvature perturbation (while usual CMB anisotropy is simply proportional to the curvature perturbation), and hence the correlation between the \(\mu\) distortion and the CMB temperature anisotropy or the power spectrum of the \(\mu\) distortion is generated if the bispectrum or trispectrum of curvature perturbations is nonzero [5, 6]. In what follows, we analyze these correlators, including broken rotational invariance.

III. µ-T CORRELATION DUE TO PRIMORDIAL ANISOTROPIC NON-GAUSSIANITY

We here investigate potential imprints on spectral distortions of primordial models which generate a curvature bispectrum with a quadrupolar asymmetry. This can be expressed as \(\left\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \right\rangle = (2\pi)^3 \delta^{(3)} (k_1 + k_2 + k_3) B_{k_1 k_2 k_3}\), with [13]

\[ B_{k_1 k_2 k_3} = 6 \sum_{p} f_{NL} P(k_1) P(k_2) \left[ 1 + \left\langle \sum_{M} \lambda_{2M} \left( Y_{2M}(\hat{k}_1) + Y_{2M}(\hat{k}_2) \right) \right\rangle \right] +(2\permut) \]
peak on squeezed triangles), and the \( \mu-T \) correlation is indeed sensitive to the squeezed limit of the curvature bispectrum \( (k_1 \approx k_2 \gg k_3) \).

The projection of \( \mu(x) \) [Eq. (3)] on the last scattering surface leads to

\[
d_{\ell m} = \int d^2 \mathbf{n} \mu(x_\mathbf{n}) Y_{\ell m}(\mathbf{n}) = 4\pi (-i)^{\ell} \left[ \prod_{n=1}^2 \int \frac{d^3 k_n}{(2\pi)^3} \right] \int \frac{d^3 k_3}{(2\pi)^3} \left( \sum_{n=1}^3 \delta(k_n) \right) Y_{\ell m}^*(\mathbf{k_3}) j_\ell(k_3 x_\mathbf{n}) f(k_1, k_2, k_3), \tag{6}
\]

where \( x = |x|, \mathbf{n} = x/x \), and \( x_\mathbf{n} \) is the conformal distance to the last scattering surface. The correlation between \( a_{\ell m}^\mu \) and the CMB temperature anisotropy \( a_{\ell m}^T = 4\pi i^{\ell+1} \int \frac{d^3 k_3}{(2\pi)^3} T(k_3) Y_{\ell m}^*(\mathbf{k_3}) [T_\ell(k) \text{ being the CMB radiation transfer function}] \) reads

\[
\langle a_{\ell_1 m_1}^\mu a_{\ell_2 m_2}^T \rangle = i^{\ell_2 - \ell_1} \left[ \prod_{n=1}^2 \int \frac{d^3 k_n}{(2\pi)^3} \right] \int \frac{d^3 k_3}{(2\pi)^3} \left( \sum_{n=1}^3 \delta(k_n) \right) Y_{\ell_1 m_1}^*(\mathbf{k_3}) Y_{\ell_2 m_2}^*(\mathbf{k_3}) T_\ell(k_3 j_\ell(k_3 x_\mathbf{n}) f(k_1, k_2, k_3)) \times \frac{12}{5} f_{\text{NL}} P(k_1) P(k_3) \left[ 1 + \sum_M \lambda_{2 M} Y_{2 M}(\mathbf{k_3}) \right]. \tag{7}
\]

The fact that \( f(k_1, k_2, k_3) \) filters the squeezed signals \( k_1 \approx k_2 \gg k_3 \) reduces this to

\[
\langle a_{\ell_1 m_1}^\mu a_{\ell_2 m_2}^T \rangle \approx 4\pi i^{\ell_2 - \ell_1} \int \frac{k_2^2 dk_2}{2\pi^2} \int \frac{d^3 k_3}{(2\pi)^3} \left[ -1 \right]^{m_1} \delta_{\ell_1, \ell_2} \delta_{m_1, -m_2} \times Y_{\ell_1 m_1}^*(\mathbf{k_3}) Y_{\ell_2 m_2}^*(\mathbf{k_3}) T_\ell(k_3 j_\ell(k_3 x_\mathbf{n}) f(k_1, k_1, k_3)) \times \frac{12}{5} f_{\text{NL}} P(k_1) P(k_3) \left[ 1 + \sum_M \lambda_{2 M} Y_{2 M}(\mathbf{k_3}) \right]. \tag{8}
\]

Here, the contribution from \( \sum_M \lambda_{2 M} Y_{2 M}(\mathbf{k_3}) \) has vanished, since \( \int d^2 \mathbf{k_3} Y_{2 M}(\mathbf{k_3}) = 0 \). The \( \mathbf{k_3} \) integrals can be analytically computed, reading

\[
\int d^2 \mathbf{k_3} Y_{\ell_1 m_1}^*(\mathbf{k_3}) Y_{\ell_2 m_2}^*(\mathbf{k_3}) Y_{2 M}(\mathbf{k_3}) = (-1)^{m_1 + m_2} \left[ \delta_{\ell_1, \ell_2} \delta_{m_1, -m_2} \right], \tag{9}
\]

and

\[
\int d^2 \mathbf{k_3} Y_{\ell_1 m_1}^*(\mathbf{k_3}) Y_{\ell_2 m_2}^*(\mathbf{k_3}) Y_{2 M}(\mathbf{k_3}) = (-1)^{m_1 + m_2} \delta_{\ell_1, \ell_2} \delta_{m_1, -m_2} \left[ \lambda_{2 M} \right]. \tag{10}
\]

We have introduced a geometrical function:

\[
h_{l_1 l_2 l_3} = \sqrt{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \tag{11}
\]

which vanishes for configurations that do not respect the triangle inequality, \( |l_1 - l_2| \leq l_3 \leq l_1 + l_2 \), and parity symmetry, \( l_1 + l_2 + l_3 = \text{even} \). A final form is written as

\[
\langle a_{\ell_1 m_1}^\mu a_{\ell_2 m_2}^T \rangle = \mathcal{C}_{\ell_1} \left[ (-1)^{m_1} \delta_{\ell_1, \ell_2} \delta_{m_1, -m_2} \right] + (-1)^{m_2} \mathcal{C}_{\ell_2} \left[ (-1)^{m_1} \delta_{\ell_1, \ell_2} \delta_{m_1, -m_2} \right], \tag{12}
\]

where

\[
\mathcal{C}_{\ell_1} = \mathcal{C}_{\ell_1}^\mu T, \quad \mathcal{C}_{\ell_1}^\mu T = \int d^2 \mathbf{k_3} Y_{\ell_1 m_1}^*(\mathbf{k_3}) Y_{\ell_2 m_2}^*(\mathbf{k_3}) T_\ell(k_3 j_\ell(k_3 x_\mathbf{n}) f(k_1, k_2, k_3)) \times \frac{12}{5} P(k_1) P(k_3) \left[ 1 + \sum_M \lambda_{2 M} Y_{2 M}(\mathbf{k_3}) \right]. \tag{13}
\]

One can find that, via the angular integrals, the quadrupolar angular dependence in Eq. (5) produces \( h_{l_1 l_2 l_3} \) in Eq. (13), resulting in nonvanishing off-diagonal components when \( |l_1 - l_2| = 2 \), as well as diagonal ones: \( l_1 = l_2 \). Figure 1 displays a numerical evaluation of \( \langle a_{\ell_1 m_1}^\mu a_{\ell_2 m_2}^T \rangle \). One can see from this figure that nonzero \( \lambda_{2 M} \) creates both diagonal (green dashed) and off-diagonal (blue dotted) lines with different shapes. We will study the detectability of \( \lambda_{2 M} \) in Sec. V.

The primordial models generating Eq. (5) often produce a quadrupolar asymmetry also in the curvature power spectrum (see, e.g., Ref. [14] for a specific inflation model where there is a coupling between the inflaton field and a vector kinetic term \( F^2 \), and see Refs. [33, 36] for reviews). The amplitude of the quadrupolar asymmetry in the curvature power spectrum is generally characterized via the parameter \( y \) [37]. One could expect some signature of this modulation to be imprinted in a \( \mu-\mu \) correlation, since we know that the latter is sensitive to the \( \tau_{\text{NL}} \) trispectrum parameter (see Ref. [5]), which in turn describes large scales modulations of small-scale power [35, 39]. Of course, a standard isotropic primordial power spectrum is also expected to produce a nonvanishing \( \mu-\mu \) correlation.
μ correlation, via its disconnected contribution to the trispectrum. The hope here is that the anisotropic nature of the g_μ signal can again produce off-diagonal couplings, that are not sourced by the isotropic part. However, this is not the case, as we now explicitly show.

We start with a power spectrum in a general anisotropic form: \( \langle \zeta_{k_1} \zeta_{k_2} \rangle = (2\pi)^3 \delta^{(3)}(k_1 + k_2) P(k_1) \). In this form, the g_μ parametrization is given as \( P(k) = P(k)[1 + g_μ (k \cdot \hat{p})^2] \). We are thus now interested in the study of μ-μ correlations arising from the above anisotropic primordial power spectrum:

\[
\left\langle \prod_{n=1}^{2} a_{\ell n m n}^{(\mu)} \right\rangle = 2(-i)^{\ell_1 + \ell_2} \left[ \prod_{n=1}^{2} \int \frac{d^3k_n}{2\pi^2} P(k_n) \right] \times \int d^3k_3 \delta^{(3)} \left( \sum_{n=1}^{3} k_n \right) Y_{\ell_1 m_1}^*(\hat{k}_3) Y_{\ell_2 m_2}^*(-\hat{k}_3) \times j_{\ell_1}(k_3 x_{1s}) j_{\ell_2}(k_3 x_{2s}) f^2(k_1, k_2, k_3). \tag{15} \]

The integrand is strongly peaked on squeezed configurations (\( k_1 \approx k_2 \gg k_3 \)) due to the filtering by \( f(k_1, k_2, k_3) \). The \( k_3 \) integral can then be reduced to \( \int d^2k_3 Y_{\ell_1 m_1}^*(\hat{k}_3) Y_{\ell_2 m_2}^*(-\hat{k}_3) \), and this results in vanishing off-diagonal components due to usual orthonormality of the spherical harmonics. We thus arrive at the conclusion that, in the μ-μ power spectrum, the anisotropic signatures of the curvature power spectrum cannot be differentiated from the isotropic ones.

On the other hand, a connected anisotropic contribution to the curvature trispectrum is expected to create some non-vanishing off-diagonal signals, as well as diagonal ones due to \( \tau_{NL} \). It is known that anisotropic trispectrum signals can also be produced in vector inflation models [40–43], while there may exist other possible models where unknown anisotropic signals are realized. The trispectrum parametrization is still nontrivial and the resultant μ-μ signatures need to be analyzed on a case-by-case basis. We leave this for future work.

IV. μ-μ CORRELATION DUE TO DIPOLAR POWER ASYMMETRY

We now consider a case in which primordial curvature perturbations depend (mildly) on a spatial position \( \mathbf{x} \) [15–17],

\[
\zeta_\mathbf{x} = \zeta_\mathbf{k} \left[ 1 + A(k) \frac{\mathbf{x} \cdot \hat{p}}{x_{ls}} \right], \tag{16} \]

where \( \hat{p} \) is the direction of a dipolar modulation of CMB power. Models which lead to this type of ansatz for the primordial power spectrum have been proposed to explain the observed power asymmetry in the CMB temperature sky [17,20,47,51]. Again, the dipolar modulation of power motivates the search for off-diagonal signatures in the μ autocorrelation function [note that the correlation between the μ distortion and the temperature CMB anisotropy vanishes in this case, since μ-T is sensitive to the bispectrum, and we assume Gaussianity of the curvature perturbation in Eq. (16)].

One can expand the dipolar modulation in spherical harmonics, \( Y_{1M} \), as

\[
\zeta_\mathbf{k} = \zeta_\mathbf{f} \left[ 1 + \sum_M A_{1M}(k) Y_{1M}(\hat{n}) \right], \tag{17} \]

\[
A_{1M}(k) = \frac{4\pi}{3} A(k) \frac{x_{ls}}{ Membership} Y_{1M}^*(\hat{p}). \tag{18} \]

We now discuss the μ-μ power spectrum in a toy model in which \( A(k) \) is \( const. \). Detailed derivation of the μ-μ correlator induced by a general \( A(k) \) is presented in Appendix A. Substituting Eq. (17) into Eq. (3) and performing the spherical harmonic transform of \( \mu \), projected on the last scattering surface, \( a_{\ell m}^\mu = \int d^2\mu(x_{ls}) Y_{\ell m}^*(\mathbf{n}) \), we obtain

\[
a_{\ell m}^\mu = \bar{a}_{\ell m} + 2(-1)^m \sum_{LM} \bar{a}_{LM}^\mu \times \sum_{M'} A_{1M'} h_{\ell L} \left( \ell - m M 1 M' \right), \tag{19} \]

where we have ignored a \( O(A^2) \) term because of the smallness of \( A \). One can notice that, in ℓ space, the term including the dipolar asymmetry gets convolved with the isotropic part \( \bar{a}_{\ell m}^\mu \), which is expressed in the same form as Eq. (6). Evaluating the autocorrelation in the squeezed limit yields

\[
\langle a_{\ell_1 m_1}^\mu a_{\ell_2 m_2}^\mu \rangle = C_{\ell_1}^\mu (-1)^{m_1+1} \delta_{\ell_1 \ell_2} \delta_{m_1 - m_2} + (-1)^{m_2} C_{\ell_2}^\mu \tag{20} \]

where

\[
C_\ell^\mu = 8\pi \int \frac{k_1^2 dk_1}{2\pi^2} \int \frac{k_3^2 dk_3}{2\pi^2} j_{\ell_1}^2(k_3 x_{ls}) f^2(k_1, k_2, k_3) P^2(k_1), \tag{21} \]

\[
C_{\ell_1 m_1, \ell_2 m_2}^\mu \approx 2(-1)^{m_1} \left( C_{\ell_1}^\mu + C_{\ell_2}^\mu \right) h_{\ell_1 \ell_2} \sum_M A_{1M} \left( \ell_1 \ell_2 1 - m_1 m_2 M \right), \tag{22} \]

with \( P(k) \) denoting the isotropic component of the curvature power spectrum, defined as \( \langle \zeta_{k_1} \zeta_{k_2} \rangle = (2\pi)^3 \delta^{(3)}(k_1 + k_2) P(k_1) \).

The result above tells us that a nonzero dipolar asymmetry creates a distinctive off-diagonal signature, satisfying \( |\ell_1 - \ell_2| = 1 \), in the μ-μ power spectrum. This type of coupling is expected, given the dipolar nature of the modulation. The same type of signature, for this class of models, is seen (on different scales) in the CMB temperature power spectrum [52]:

\[
C_{\ell_1 m_1, \ell_2 m_2}^{TT} = (-1)^{m_1} \left( C_{\ell_1}^{TT} + C_{\ell_2}^{TT} \right) h_{\ell_1 \ell_2} \sum_M A_{1M} \left( -m_1 m_2 1 M \right). \tag{23} \]
Note that $\tilde{C}_\ell^{\mu\mu}$ behaves as a white noise term, since this is sourced by a Gaussian isotropic curvature perturbation, the value of which is evaluated as $\tilde{C}_\ell^{\mu\mu} \lesssim 10^{-28}$ \cite{ref1}.

We know that the dipolar modulation observed in the CMB power spectrum affects only relatively large scales, $\ell \lesssim 60$ (see e.g., Refs. \cite{ref1, ref2}). The choice $A(k) = \text{const}$, that we just made above, is thus unable to explain current observations. A red-tilted $k$ dependence must be introduced in $A(k)$, in such a way as to make the modulation negligible for $k \gtrsim 60/\chi_{\text{ls}} = 0.004 \text{ Mpc}^{-1}$. In this case one can see that the $k_1$ integral in formula (A6) vanishes, since the window function $f(k_1, k_2, k_3)$ is not suppressed only for large $k_1$ ($k_1 > k_D(z_f) \approx 46 \text{ Mpc}^{-1}$, where $k_D$ is the Silk damping scale). It then is not possible to use $\mu-\mu$ to test the possible primordial origin of the observed temperature asymmetry. This reflects the fact that $\mu$ CMB distortions probe very small scales. On the other hand, we used the toy model \cite{ref1} to show that the off-diagonal coupling pointed out above could still be used to build observational tests of rotational invariance at otherwise unaccessible CMB scales.

\section{V. Fisher Forecast}

We now evaluate the detectability of the signal discussed above through a simple Fisher matrix analysis. In the limit that asymmetries are regarded as tiny modulations of the isotropic part, a minimum-variance estimator for the magnitudes of the asymmetry (in our paper, $h = A_{1M}$ or $\lambda_{2M}$) can be written as \cite{ref1, ref2}: \begin{equation}
\hat{h} = \frac{1}{2} F^{-1} \hat{\alpha} \frac{\delta C}{\delta \hat{\alpha}},
\end{equation}
where $\hat{\alpha} \equiv C_{\ell \mu \mu}^{-1} a$ denotes the observed harmonic coefficients after application of inverse-variance filtering. In the following discussion, we assume that off-diagonal components of the inverse of the covariance matrix are negligibly small; thus the Fisher matrix can be diagonalized.

The variance term $C_{\text{obs}}$ is composed of cosmic variance and instrumental noise. In what follows, we will first investigate an ideal case where $C_{\text{obs}}$ is determined by cosmic variance alone, and then some cases involving instrumental noise levels similar to those of Planck \cite{ref1}, PIXIE \cite{ref1} and CMBpol \cite{ref1}. For $\mu-\mu$ noise spectra, we assume $N_{\mu \mu} = N_{\mu} \exp(\ell^2/\ell_\mu^2)$, with $(N_{\mu}, \ell_\mu) = (10^{-13}, 861)$ (Planck), $(10^{-17}, 84)$ (PIXIE) and $(2 \times 10^{-18}, 1000)$ (CMBpol) \cite{ref1, ref2}. On the other hand, we know that CMB temperature measurements on large scales, which are the only ones to give contributions to the $\mu-T$ correlation signal, are completely cosmic-variance dominated. Thus, we can ignore instrumental noise in $C_{\ell \text{obs}}^{TT}$, so that $C_{\ell \text{obs}}^{TT} \approx C_{\ell}^{TT}$.

\subsection{A. Anisotropic non-Gaussianity}

Assuming $(C_{\ell \text{obs}}^{\mu T})^2 \ll C_{\ell \text{obs}}^{TT} C_{\ell \text{obs}}^{\mu \mu}$, the Fisher matrix for $\lambda_{2M}$ becomes

\begin{equation}
F_{\lambda}^{\mu T} = \frac{F_{\text{NL}}^2}{5} \sum_{\ell_1, \ell_2 = 2} \frac{h_{\ell_1, \ell_2}^2}{C_{\ell_1, \text{obs}}^{\mu \mu} C_{\ell_2, \text{obs}}^{TT}}.
\end{equation}

At first, we focus on a noise-free, cosmic-variance-dominated ideal case, leading to $C_{\ell \text{obs}}^{\mu \mu} = C_{\ell}^{\mu \mu} = \text{const}$. The scaling is evaluated as follows. Since $h_{\ell_1, \ell_2}$ vanishes except for $\ell_2 = \ell_1, \ell_1 \pm 2$, we may drop $\sum_{\ell_2}$ by replacing $\ell_2$ with $\ell_1$. In the Sachs-Wolfe limit $T_\ell(k) \rightarrow -j_\ell(k\chi_{\text{ls}})/5$, and the scale-invariant power spectrum $P(k) \propto k^{-3}$ yields $C_{\ell_1, \ell_1}^{TT} \propto C_{\ell_1}^{TT} \propto \ell_1^{-2}$. These estimates, and $h_{\ell_1, \ell_1}^2 \propto \ell_1$, lead to

\begin{equation}
F_{\lambda}^{\mu T(\text{CV})} \propto \sum_{\ell_1} \frac{(\ell_1^{-2})^2}{\ell_1^2} \propto \ln \ell_{\text{max}},
\end{equation}

where CV stands for cosmic-variance dominated. We have checked that this scaling is in agreement with the full numerical evaluation of Eq. (25) in the ideal case (corresponding to red solid line in Fig. 2). Combining the numerical results with this scaling, we finally obtain an expected $1\sigma$ error bar $1/\sqrt{F}$ as

\begin{equation}
\delta \lambda_{2M}^{\mu T(\text{CV})} \approx \frac{5 \cdot 10^{-3}}{f_{\text{NL}}^2} \left( \frac{C_{\ell}^{\mu \mu}}{10^{-28}} \right)^{1/2} (\ln \ell_{\text{max}})^{-1/2}.
\end{equation}

On the other hand, an expected error obtained from the CMB temperature bispectrum has a different amplitude and scaling of $\ell_{\text{max}}$ as $\delta \lambda_{2M}^{TT(\text{TT})} \approx (2.5/f_{\text{NL}}^2)(2000/\ell_{\text{max}})$ \cite{ref1}. Comparison of these results indicates that the $\mu-T$ correlation can be a much more powerful observable, in case we can remove instrumental uncertainties completely (see Fig. 2).

The expected error bars in the Planck-like, PIXIE-like and CMBpol-like surveys are obtained by computing Eq. (25) with $C_{\ell \text{obs}}^{\mu \mu} = C_{\ell}^{\mu \mu} + N_{\mu \mu}$. The numerical results depicted in Fig. 2 show that, even for potential future surveys like CMBpol, the $\mu-T$ constraint is not competitive with temperature bispectrum measurements. However, an ideal noise-free experiment can achieve extremely tight constraints. This, as usual for this type of signal, is due to the fact that $\mu$ spectral distortions probe (integrated) information up to very high $\ell$. Although achievable only with futuristic surveys and very low noise levels, the potential of this kind of tests is thus large.

\subsection{B. Dipolar power asymmetry}

When we consider our toy model of dipolar asymmetry and estimate $A_{1M} = \text{const}$ from the $\mu-\mu$ power spec-
The Fisher matrix is written as

$$F_A^{(\mu\mu)} = \frac{2}{3} \sum_{\ell_1, \ell_2 = 2}^{\ell_{\text{max}}} h_{\ell_1}^2 h_{\ell_2}^2 \left( C_{\ell_1}^{\mu\mu} + C_{\ell_2}^{\mu\mu} \right)^2. \tag{28}$$

In the noise-free ideal case, we can write $C_{\ell,\text{obs}}^{\mu\mu} = \bar{C}_\ell^{\mu\mu} = \text{const}$. This reduces the Fisher matrix to

$$F_A^{(\mu\mu\text{CV})} = \frac{8}{3} \sum_{\ell_1, \ell_2 = 2}^{\ell_{\text{max}}} h_{\ell_1}^2 h_{\ell_2}^2 = \frac{2}{\pi} (\ell_{\text{max}} + 3)(\ell_{\text{max}} - 2), \tag{29}$$

and hence an expected 1σ error bar $1/\sqrt{F}$ is estimated as

$$\delta A_{1M}^{(\mu\mu\text{CV})} \approx \frac{1.3}{\ell_{\text{max}}}. \tag{30}$$

This can be compared to the results estimated from the temperature power spectrum. The Fisher matrix $F_A^{(TT)}$ becomes the same form of $F_A^{(\mu\mu)}$, aside from a prefactor 4 due to the difference of factor 2 between $C_{\ell_1 m_1, \ell_2 m_2}^{\mu\mu}$ in Eq. (22) and $C_{\ell_1}^{TT}$ in Eq. (25). Since $C_{\ell}^{TT}$ and $\bar{C}_\ell^{TT}$ take almost identical values, the Fisher matrix for the ideal cosmic-variance-limited case can be simplified like Eq. (29), and we finally find $\delta A_{1M}^{(TT\text{CV})} \approx 2\delta A_{1M}^{(\mu\mu\text{CV})}$.

The errors for the cases with more realistic noise levels are computed with $C_{\ell,\text{obs}}^{\mu\mu} = \bar{C}_\ell^{\mu\mu} + N_{\ell}^{\mu\mu}$. The results plotted in Fig. 2 are consistent with the expectation from Eq. (28) that, for $\ell_{\text{max}} \leq \ell_{\mu}$, the error bars simply get bigger by a factor $N_{\ell}/\bar{C}_\ell^{\mu\mu}$ with respect to the noise-free ideal case. We stress again that we are considering here just a toy model with a constant modulation extending up to very high $\ell$. The dipolar asymmetry that is found in the CMB extends up to at most $\ell \sim 100$, and it thus leaves no signature in $\mu-\mu$.

Before concluding this section, we would like to stress that the forecasts presented here do not take into account important issues, like foreground subtraction. Such issues play, of course, a crucial role in actual measurements, and would most likely affect the expected error bars, plotted in Figs. 2 and 3, in a survey-dependent way (with e.g. PIXIE expected to be significantly more efficient than Planck or CMBpol at component separation for $\mu$ reconstruction). An accurate investigation of these effects is beyond the scope of this simple analysis. Our main goal in this section was just to show that, while interesting information on isotropy-breaking signatures is present in the $\mu-T$ and $\mu-\mu$ correlations, only future surveys will be able to extract it (as is the case also for many other NG signatures, such as primordial NG of the local type, or signatures induced by primordial magnetic fields).

**VI. CONCLUSIONS**

We studied CMB $\mu$ spectral distortion signatures arising from dissipation of acoustic waves in models characterized by primordial breaking of rotational invariance. We find that such anisotropic primordial scenarios predict distinctive off-diagonal signatures in the $\mu-T$ correlation matrix.

More specifically, for NG models generating bispectra with a quadrupolar asymmetry, as generally predicted by anisotropic inflationary scenarios, we find nonvanishing $\mu-T$ correlations with a distinct $|\ell_1 - \ell_2| = 2$ signature. The coupling we find is consistent with what we intuitively expected, due to the quadrupolar nature of the rotation-invariance-breaking terms, and since these
trispectrum signature is seen in the \( \mu \)-bispectra, we also studied Gaussian toy models with a study the effects mentioned here, the information content with very high sensitivity are necessary in order to surveys with very large wave numbers, where temperature anisotropies have been erased by Silk damping. It temperature, as a coupling between different motions up to very large wave numbers, where tempera-

fact that \( \mu \) distortions are sensitive to primordial fluctuations at very small scales. This, as originally pointed out in Ref. [5] for local bispectrum distortions are sensitive to primordial fluctuations at very small scales, and does not leave any imprint on \( \mu \)-\( \mu \). However, they show that \( \mu \) distortions allow one in principle to test statistical isotropy on scales much smaller than what is actually probed by CMB tempera-

A spherical harmonic transformation of \( \mu(x) \) [Eq. (3)] leads to

\[
d^\mu_{\ell m} = \left[ \prod_{n=1}^{2} \int \frac{d^3k_n}{(2\pi)^3} \right] \int d^3k_3 \delta^{(3)} \left( \sum_{n=1}^{3} k_n \right) \nonumber \\
\times f(k_1, k_2, k_3) \int d^2 \hat{n} Y_{\ell m}^*(\hat{n}) e^{-i k_3 \cdot x_{12} \hat{n}} \nonumber \\
\times \left(1 + \sum_{M'} [A_{1M'}(k_1) + A_{1M'}(k_2)] Y_{1M'}(\hat{n}) \right)(A1)
\]

where we have dropped the \( \mathcal{O}(A^2) \) term. With an identity

\[
e^{-i k_3 \cdot x_{12} \hat{n}} = \sum_{LM} 4\pi(-i)^{L} j_L(k_3 x_{12}) Y_{L,M}^*(\hat{k}_3) Y_{L,M}(\hat{n})(A2)
\]

the above \( \hat{n} \) integrals are computed as

\[
\int d^2 \hat{n} Y_{\ell m}(\hat{n}) Y_{L,M}(\hat{n}) Y_{1M'}(\hat{n}) = \delta_{\ell \ell} \delta_{m,M} \quad \text{and}
\]

and we finally reach \( d^\mu_{\ell m} = d^\mu_{\ell m} + a^\mu_{\ell m} \), where

\[
da^\mu(d)_{\ell m} = (-1)^m \sum_{LM} 4\pi(-i)^{L} \left[ \prod_{n=1}^{2} \int \frac{d^3k_n}{(2\pi)^3} \right] \nonumber \\
\times \int d^3k_3 \delta^{(3)} \left( \sum_{n=1}^{3} k_n \right) \nonumber \\
\times Y_{L,M}^*(\hat{k}_3) j_L(k_3 x_{12}) \nonumber \\
\times f(k_1, k_2, k_3) \sum_{M'} [A_{1M'}(k_1) + A_{1M'}(k_2)] 
\]

(1 + \sum_{M'} [A_{1M'}(k_1) + A_{1M'}(k_2)]) \left( \ell_1 \frac{L}{M} \frac{1}{M'} \right). (A3)

Appendix A: Derivation of \( \mu-\mu \) correlator generated from dipolar power asymmetry

We here present the details of the computations on the \( \mu-\mu \) power spectrum induced by the dipolar modulation \( [10] \) or \( [17] \). A formula obtained here is applicable to \( A(k) \) with any scale dependence.

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One can easily see that, for \( A(k) = \text{const} \), this recovers Eq. [19].

The power spectrum modulation due to \( A(k) \) is given as

\[
C^{\mu\mu}_{\ell_1 m_1, \ell_2 m_2} = \left\langle a^\mu_{\ell_1 m_1} a^{\mu*}_{\ell_2 m_2} \rightangle + \left\langle a^{\mu*}_{\ell_1 m_1} a^\mu_{\ell_2 m_2} \right\rangle
\]

After computing the trispectrum of curvature perturbations, the first term becomes

\[
\left\langle a^\mu(d)_{\ell_1 m_1} a^{\mu*}_{\ell_2 m_2} \rightangle = (-1)^{m_2} \sum_{LM} 2^{L+2}(-i)^{L} \left[ \prod_{n=1}^{2} \int \frac{d^3k_n}{2\pi^2} \right] \nonumber \\
\times \int d^3k_3 \delta^{(3)} \left( \sum_{n=1}^{3} k_n \right) \nonumber \\
\times j_L(k_3 x_{12}) j_L(k_3 x_{12}) f^2(k_1, k_2, k_3) P(k_1) P(k_2) h_{k_1 L_1} \nonumber \\
\times \sum_{M'} [A_{1M'}(k_1) + A_{1M'}(k_2)] \left( \ell_1 \frac{L}{M} \frac{1}{M'} \right). (A5)
\]

Then, the fact that the transfer function \( f(k_1, k_2, k_3) \) filters the squeezed-limit signals \( k_1 \approx k_2 \gg k_3 \) simplifies the \( k_3 \) integral as

\[
\int d^2 k_3 Y_{L,M}^*(k_3) Y_{\ell_2 m_2}(k_3) =
\]
\[ \delta_{L, \ell_2} \delta_{M, m_2} \] Evaluating the second term in the same way, we finally obtain
\[
C_{\ell_1 m_1, \ell_2 m_2}^{\mu \mu} \approx (-1)^{m_1} 16 \pi \int \frac{k_1^2 \, dk_1}{2 \pi^2} \int \frac{k_2^2 \, dk_2}{2 \pi^2} \left\{ \begin{array}{l}
\{ j_1^2 (k_3 x_{1b}) + j_2^2 (k_3 x_{1a}) \}
\{ f^2 (k_1, k_1, k_3) P^2 (k_1) h_{\ell_1 \ell_2} \}
\sum_{M} A_{1M} (k_1) \left( \begin{array}{c}
\ell_1 \\
\ell_2 \\
m_1 \\
m_2 \\
M \end{array} \right) \right. \right.
\]
\[ (A6) \]

Obviously, this recovers Eq. [22] for \( A(k) = \text{const.} \)

[1] D. Fixsen, E. Cheng, J. Gales, J. C. Mather, R. Shafer, et al., Astrophys. J. 473, 576 (1996) arXiv:astro-ph/9605054 [astro-ph]
[2] J. C. Mather, E. Cheng, D. Cottingham, R. Eplee, D. Fixsen, et al., Astrophys. J. 420, 439 (1994)
[3] R. Khatri and R. Sunyaev, JCAP 1508, 013 (2015) arXiv:1505.00781 [astro-ph.CO]
[4] A. Kogut, D. Fixsen, D. Chuss, J. Dotson, E. Dwek, et al., JCAP 1107, 025 (2011) arXiv:1105.2044 [astro-ph.CO]
[5] E. Pajer and M. Zaldarriaga, Phys. Rev. Lett. 109, 021302 (2012) arXiv:1201.5375 [astro-ph.CO]
[6] J. Ganc and E. Komatsu, Phys. Rev. D86, 023518 (2012) arXiv:1204.4241 [astro-ph.CO]
[7] M. Biagetti, H. Perrier, A. Riotto, and V. Desjacques, Phys. Rev. D87, 063521 (2013) arXiv:1301.2771 [astro-ph.CO]
[8] R. Emami, E. Dimastrogiovanni, J. Chluba, and M. Kamionkowski, Phys. Rev. D91, 123531 (2015) arXiv:1504.00675 [astro-ph.CO]
[9] A. Ota, T. Sekiguchi, Y. Tada, and S. Yokoyama, JCAP 1503, 013 (2015) arXiv:1412.4517 [astro-ph.CO]
[10] K. Miyamoto, T. Sekiguchi, H. Tashiro, and S. Yokoyama, Phys. Rev. D89, 063508 (2014) arXiv:1310.3886 [astro-ph.CO]
[11] K. E. Kunze and E. Komatsu, JCAP 1401, 009 (2014) arXiv:1309.7994 [astro-ph.CO]
[12] J. Ganc and M. S. Sloth, JCAP 1408, 018 (2014) arXiv:1404.5957 [astro-ph.CO]
[13] N. Bartolo, E. Dimastrogiovanni, M. Liguori, S. Matarrese, and A. Riotto, JCAP 1201, 029 (2012) arXiv:1107.4304 [astro-ph.CO]
[14] N. Bartolo, S. Matarrese, M. Peloso, and A. Ricciardone, Phys. Rev. D87, 023504 (2013) arXiv:1210.3257 [astro-ph.CO]
[15] N. Bartolo, S. Matarrese, M. Peloso, and M. Shiraiishi, JCAP 1507, 039 (2015) arXiv:1505.02193 [astro-ph.CO]
[16] J. Hofuft, H. Eriksen, A. Banday, K. Gorski, F. Hansen, et al., Astrophys. J. 675, 985 (2008) arXiv:0903.1229 [astro-ph.CO]
[17] P. Ade et al. (Planck), Astron. Astrophys. 571, A23 (2014) arXiv:1303.5083 [astro-ph.CO]
[18] Y. Akrami, Y. Fantaye, A. Shafieloo, H. Eriksen, F. Hansen, et al., Astrophys. J. 784, L42 (2014) arXiv:1402.0870 [astro-ph.CO]
[19] P. Ade et al. (Planck), (2015) arXiv:1506.07135 [astro-ph.CO]
[20] S. Aiola, B. Wang, A. Kosowsky, T. Kahlia, and H. Firoozjahi, (2015) arXiv:1506.04405 [astro-ph.CO]
[21] R. A. Sunyaev and Ya. B. Zeldovich, Astrophys. Space Sci. 7, 20 (1970).
[22] A. F. Illarionov and R. A. Sunyaev, Soviet Astr. 18, 413 (1975).
[23] L. Danese and G. de Zotti, A&A 407, 39 (1982).
[24] C. Burigana, L. Danese, and G. de Zotti, A&A 246, 49 (1991).
[25] W. Hu, D. Scott, and J. Silk, Astrophys.J. 430, L5 (1994) arXiv:astro-ph/9402045 [astro-ph]
[26] J. Chluba and R. Sunyaev, Mon.Not.Roy.Astron.Soc. 419, 1294 (2012) arXiv:1109.6552 [astro-ph.CO]
[27] R. Khatri, R. A. Sunyaev, and J. Chluba, Astron.Astrophys. 540, A124 (2012) arXiv:1110.0475 [astro-ph.CO]
[28] J. Chluba, R. Khatri, and R. A. Sunyaev, Mon.Not.Roy.Astron.Soc. 425, 1129 (2012) arXiv:1202.0057 [astro-ph.CO]
[29] R. Khatri and R. A. Sunyaev, JCAP 1206, 008 (2012) arXiv:1203.2601 [astro-ph.CO]
[30] R. Khatri and R. A. Sunyaev, JCAP 1209, 016 (2012) arXiv:1207.6534 [astro-ph.CO]
[31] J. Silk, ApJ 151, 459 (1968).
[32] P. J. E. Peebles and J. T. Yu, ApJ 162, 815 (1970).
[33] N. Kaiser, MNRAS 202, 1169 (1983).
[34] S. Weinberg, Cosmology, by Steven Weinberg. ISBN 978-0-19-855682-7. Published by Oxford University Press, Oxford, UK, 2008. (Oxford University Press, 2008).
[35] E. Dimastrogiovanni, N. Bartolo, S. Matarrese, and A. Riotto, Adv. Astron. 2010, 752670 (2010) arXiv:1001.4049 [astro-ph.CO]
[36] A. Maleknejad, M. Sheikh-Jabbari, and J. Soda, Phys.Rept. 528, 161 (2013) arXiv:1212.9291 [hep-th]
[37] L. Ackerman, S. M. Carroll, and M. B. Wise, Phys.Rev. D75, 083502 (2007) arXiv:astro-ph/0701357 [astro-ph]
[38] R. Pearson, A. Lewis, and D. Regan, JCAP 1203, 011 (2012) arXiv:1201.1010 [astro-ph.CO]
[39] P. Ade et al. (Planck), Astron.Astrophys. 571, A24 (2014) arXiv:1303.5084 [astro-ph.CO]
[40] N. Bartolo, E. Dimastrogiovanni, S. Matarrese, and A. Riotto, JCAP 0911, 028 (2009) arXiv:0909.5621 [astro-ph.CO]
[41] C. A. Valenzuela-Toledo and Y. Rodriguez, Phys.Lett. B685, 120 (2010) arXiv:0910.4208 [astro-ph.CO]
[42] Y. Rodriguez, J. P. Beltran Almeida, and C. A. Valenzuela-Toledo, JCAP 1304, 039 (2013) arXiv:1301.5843 [astro-ph.CO]
[43] M. Shiraiishi, E. Komatsu, M. Peloso, and N. Barnaby, JCAP 1305, 002 (2013) arXiv:1302.3056 [astro-ph.CO]
[44] A. A. Abolhasani, R. Emami, J. T. Firouzjaee, and H. Firouzjahi, JCAP 1308, 016 (2013) arXiv:1302.6986 [astro-ph.CO].

[45] L. Dai, D. Jeong, M. Kamionkowski, and J. Chluba, Phys.Rev. D87, 123005 (2013) arXiv:1303.6949 [astro-ph.CO].

[46] D. H. Lyth, JCAP 1308, 007 (2013) arXiv:1304.1270 [astro-ph.CO].

[47] S. Jazayeri, Y. Akrami, H. Firouzjahi, A. R. Solomon, and Y. Wang, JCAP 1411, 044 (2014) arXiv:1408.3057 [astro-ph.CO].

[48] F. K. Hansen, A. Banday, and K. Gorski, Mon.Not.Roy.Astron.Soc. 354, 641 (2004) arXiv:astro-ph/0404206 [astro-ph].

[49] C. Gordon, W. Hu, D. Huterer, and T. M. Crawford, Phys.Rev. D72, 103002 (2005) arXiv:astro-ph/0509301 [astro-ph].

[50] H. K. Eriksen, A. Banday, K. Gorski, F. Hansen, and P. Lilje, Astrophys.J. 660, L81 (2007) arXiv:astro-ph/0701089 [astro-ph].

[51] C. Gordon, Astrophys.J. 656, 636 (2007) arXiv:astro-ph/0607423 [astro-ph].

[52] D. Hanson and A. Lewis, Phys.Rev. D80, 063004 (2009) arXiv:0908.0963 [astro-ph.CO].

[53] D. Hanson, A. Lewis, and A. Challinor, Phys.Rev. D81, 103003 (2010) arXiv:1003.0198 [astro-ph.CO].

[54] J. Tauber et al. (Planck), (2006), arXiv:astro-ph/0604069 [astro-ph].

[55] D. Baumann et al. (CMBPol Study Team), AIP Conf.Proc. 1141, 10 (2009) arXiv:0811.3919 [astro-ph].