Unparticle Effects on Top Quark Spin Correlations in $e^+e^-$ Collision

Banu Şahin

Department of Physics, Faculty of Sciences, Ankara University, 06100 Tandogan, Ankara, Turkey

Abstract

We investigate the effects of scalar and vector unparticles on top quark spin correlations via the process $e^+e^-\to t\bar{t}$. In addition to the Standard Model diagrams, there is a new contribution to top-antitop quark production process mediated by unparticle in the s-channel. It is shown that scalar and vector unparticle contribution leads to a considerable deviation of the top spin correlations from the Standard Model one.

PACS numbers: 14.80.-j, 12.90.+b, 14.65.Ha
I. INTRODUCTION

There has been an increasing interest in unparticle scenario which was introduced by Georgi [1, 2]. In this scenario, new physics contains both Standard Model (SM) fields and a scale invariant sector described by Banks-Zaks (BZ) fields [3]. These two sector interact via the exchange of particles with a mass scale \( M_U \). Below this large mass scale interactions between SM fields and BZ fields are described by non-renormalizable couplings suppressed by powers of \( M_U \) [1, 4]:

\[
\frac{1}{M_U^{d_{SM}+d_{BZ}}-1}O_{SM}O_{BZ}
\]  

(1)

Renormalization effects in the scale invariant BZ sector then produce dimensional transmutation at an energy scale \( \Lambda_U \) [5]. In the effective theory below the scale \( \Lambda_U \), the BZ operators are embedded as unparticle operators. The operator (1) is now match onto the following form,

\[
C_{OU} \frac{\Lambda_U^{d_{BZ}-d_U}}{M_U^{d_{SM}+d_{BZ}}-1}O_{SM}O_{U}
\]

(2)

here, \( d_U \) is the scale dimension of the unparticle operator \( O_U \) and the constant \( C_{OU} \) is a coefficient function.

Phenomenological implications of unparticles have been discussed in the literature [6]. In some of these researches mentioned above several unparticle production processes have been studied. A possible evidence for this scale invariant sector might be a missing energy signature. It can be tested experimentally by examining missing energy distributions. Another evidence for unparticles can be explored by studying its virtual effects. Imposing scale invariance, unparticle propagators for spin-0 and spin-1 unparticles are given by [2, 7]:

\[
\Delta(P^2) = i \frac{A_{d_U}}{2\sin(d_U \pi)} (-P^2)^{d_U-2}
\]

\[
\Delta(P^2)^{\mu\nu} = i \frac{A_{d_U}}{2\sin(d_U \pi)} (-P^2)^{d_U-2} \left( -g^{\mu\nu} + \frac{P^\mu P^\nu}{P^2} \right)
\]

(3)

respectively, where
\[ A_{dU} = \frac{16\pi^\frac{5}{2}}{(2\pi)^2 dU} \frac{\Gamma(d_U + \frac{1}{2})}{\Gamma(d_U - 1)\Gamma(2d_U)} \] (4)

In Eq.(3), \((-P^2)^{d_U-2} = |P^2|^{d_u-2}\) in t or u-channel diagrams where \(P^2\) is negative, \((-P^2)^{d_u-2} = |P^2|^{d_u-2}e^{-id_u\pi}\) in s-channel diagrams where \(P^2\) is positive.

Interaction vertices for the scalar and vector unparticles with the SM fermions are given respectively, by

\[ i \frac{\lambda_0}{\Lambda_{dU}^{d_U-1}} - \frac{\lambda_0}{\Lambda_{dU}^{d_U-1}} \gamma^5 + \frac{\lambda_0}{\Lambda_{dU}^{d_U}} \gamma^\mu p_\mu \] (5)

\[ i \frac{\lambda_1}{\Lambda_{dU}^{d_U-1}} \gamma^\mu + i \frac{\lambda_1}{\Lambda_{dU}^{d_U-1}} \gamma^\mu \gamma^5 \] (6)

The top quark is the heaviest fermion in the Standard Model (SM) and its mass is at the electroweak symmetry-breaking scale. Because of its large mass top quark couplings are expected to be more sensitive to new physics than other particles\(^8\). Therefore deviations from the SM expectations in top quark production processes would be a signal for the new physics.

In this work we analyzed vector and scalar unparticle effects on top quark spin correlations in pair production process \(e^+e^- \rightarrow t\bar{t}\). Since the top quark is very heavy its weak decay time is much shorter than the typical time for the strong interaction to affect its spin\(^9\). Therefore top polarization information is not distributed by hadronization effects and transferred to the decay products. The angular distribution of the top quark decay involves correlations between top decay products and top quark spin:

\[ \frac{1}{\Gamma_T} \frac{d\Gamma}{d\cos\theta} = \frac{1}{2} (1 + A_{\uparrow\downarrow} \alpha \cos\theta) \] (7)

Here the dominant decay chain of the top quark in the standard model \(t \rightarrow W^+b(W^+ \rightarrow l^+\nu,\bar{d}u)\) is considered. \(A_{\uparrow\downarrow}\) is the spin asymmetry and \(\theta\) is defined as the angle between top quark decay products and the top quark spin quantization axis in the rest frame of the top quark. \(\alpha\) is the correlation coefficient and \(\alpha = 1\) for \(l\) or \(\bar{d}\) which leads to the strongest correlation. Therefore top quark polarization can be determined by means of the angular distribution of its decay products.
We take into account top quark spin and antitop quark spin polarizations along the direction of various spin bases. These spin bases are the helicity basis and the incoming beam directions. In the SM there is a spin asymmetry between the produced top-antitop pairs. More specifically, the number of produced top-antitop quark pairs with both spin up or spin down is different from the number of pairs with the opposite spin combinations. Therefore, if the top quark is coupled to a new physics beyond the SM, the top-antitop quark spin correlations could be altered [10].

The top-antitop correlations can be used to search new physics beyond the SM. The unparticle couplings are examples of such new physics. We consider the scalar and vector unparticle interaction terms which are given in Eq.5,6 in addition to the SM contributions. In our calculations we assume that $\lambda_0 = \lambda_1 = 1$.

The research and developments on linear $e^+e^-$ colliders have been progressing and physics potential of these future machines is under study. Linear $e^+e^-$ collider can provide a very useful laboratory to study physics of the top quark. Furthermore this linear colliders have a clean environment and the experimental clearness is an additional advantage of $e^+e^-$ collisions with respect to hadron collisions.

II. TOP QUARK PAIR PRODUCTION AND SPIN CORRELATIONS

Since top quark possesses a large mass its helicity is frame dependent and changes under a boost from one frame to another. The helicity and chirality states do not coincide with each other and there is no reason to believe that the helicity basis will give the best description of the spin of top quarks. Therefore it is reasonable to study other spin bases for top quark.

Top quark spinors are eigenstates of the operator $\gamma_5(\gamma_\mu s^\mu_t)$:

$$[\gamma_5(\gamma_\mu s^\mu_t)]u(p_t, \pm s) = \pm u(p_t, \pm s)$$  \hspace{1cm} (8)

where spin four vector of a top quark is defined by

$$s^\mu_t = \left( \frac{p_t \cdot \vec{s}'_t}{m_t}, \vec{s}'_t \right) + \frac{p_t \cdot \vec{s}''_t}{m_t(E_t + m_t)p_t} \vec{p}_t$$ \hspace{1cm} (9)

here $(s^\mu_t)_{RF} = (0, \vec{s}'_t)$ in the top quark rest frame.
In the presence of unparticles, $e^+e^- \rightarrow t\bar{t}$ process occur via s-channel exchange of unparticles and usual electroweak bosons, $\gamma$ and $Z$. During amplitude calculations one can project the top quark and antitop quark spin to a given spin direction. We consider incoming beam directions and helicity basis. In the top quark rest frame (or in antitop quark rest frame), its spin direction along any beam can be defined as,

$$\vec{s} = \lambda \frac{\vec{p}^*}{|\vec{p}^*|}, \quad \lambda = \pm 1.$$  \hspace{1cm} (10)

where, $\vec{p}^*$ is the particle momentum (positron or electron), observed in the rest frame of the top quark.

It is possible to analyze the top and antitop quark spin correlations via angular correlations of two charged leptons $l^+l^-$ produced by the top-antitop quark leptonic decay channels. We consider the leptonic decay channels of the top pairs. After integration over azimuthal angles the correlation is given by \cite{10,11},

$$\frac{1}{\sigma} \frac{d^2\sigma}{dcos\theta_{l^+} dcos\theta_{l^-}} = 1 - \frac{A cos\theta_{l^+} cos\theta_{l^-}}{4}.$$  \hspace{1cm} (11)

Here $\sigma$ represents the cross section for the process of the leptonic decay modes, $\theta_{l^+}(\theta_{l^-})$ represents the angle between the top (antitop) spin axis and the direction of motion of the anti-lepton (lepton) in the top (antitop) rest frame. The $A$ coefficient stands for the spin asymmetry between top and antitop pairs and defined as,

$$A = \frac{\sigma(t_1\bar{t}_1) + \sigma(t_2\bar{t}_1) - \sigma(t_1\bar{t}_2) - \sigma(t_2\bar{t}_2)}{\sigma(t_1\bar{t}_1) + \sigma(t_1\bar{t}_2) + \sigma(t_2\bar{t}_1) + \sigma(t_2\bar{t}_2)}.$$  \hspace{1cm} (12)

where $\sigma(t_\alpha\bar{t}_{\alpha'})$ is the cross section of pair top quark production process.

In Table II spin asymmetries are given for various spin bases. One can see from Table II SM spin asymmetry is $A_{SM} = -0.651$ for $\sqrt{s}=0.5$ TeV when top spin and antitop quark spin are in the helicity basis. In unparticle scheme, there are new contributions to pair production process. We calculated spin dependent squared amplitudes including scalar and vector unparticle mediated process. In the scalar unparticle exchange case, spin asymmetry increases and it takes a positive value, $A_S = 0.986$. When we consider vector unparticle exchange, asymmetry takes a negative value, $A_V = -0.727$. It is seen from the tables that
maximum deviation from the SM one occurs in the scalar unparticle case. On the other hand, another useful basis is the incoming beam directions, in which the top quark spin axis is along the positron beam direction in the top rest frame, and antitop quark spin axis is along the electron direction in the antitop rest frame. In this basis, SM asymmetry is

\[ A_{SM} = -0.939 \]  

for \( \sqrt{s} = 0.5 \text{ TeV} \). In scalar unparticle exchange, \( A \) increases and takes a positive value, \( A_S = 0.657 \). In vector unparticle exchange spin asymmetry is, \( A_V = -1 \). It is shown that maximum deviation from the SM observed in the scalar unparticle exchange for the helicity basis. At the tables we considered that \( d_U = 1.1 \) and \( \Lambda_U = 1 \text{ TeV} \).

The influence of the center of the mass energy on the spin asymmetry is shown in Fig.1-4. In Fig.1 we consider the scalar unparticle and helicity basis for top-antitop quark spin. We see from this figure that a sizeable deviation from the SM one occurs at \( d_U = 1.1 \). In Fig.2 we take into account vector unparticle effects. One can see that unparticle contributions leads to a little deviation from the SM at each center of mass energy. Fig.3 shows similar behavior with Fig.1 but for the top spin axis is along the incoming positron beam direction and antitop spin axis is along the incoming electron beam direction. In Fig.4 vector unparticle effects are shown for the same basis with Fig.3. In this figure maximum deviation can be seen at \( d_U = 1.1 \).

In Fig.5 the spin asymmetries as a function of the scale dimension \( d_U \) are given. We consider helicity direction in this figure and it can be seen that scalar unparticle contribution leads to a considerable deviation from the SM. On the other hand, vector unparticle contribution leads to a little deviation. At growing values of \( d_U \) SM, scalar and vector unparticle asymmetries close up. Fig.6 has similar behavior with Fig.5 but in this figure top-antitop spin is along the incoming beam directions.

In our calculations phase space integrations have been performed by GRACE [12] which uses a Monte Carlo routine.

III. CONCLUSION

In this paper we have studied the top spin correlations with the unparticle effects in \( e^+e^- \) collision. We calculate spin dependent cross sections to obtain top and antitop spin asymmetry. It is shown that existence of scalar or vector unparticle leads to a significant deviation of the spin asymmetry from the its SM value. Variations of the spin asymmetry
are reflected by the angular distribution of the top and antitop decay products. Therefore spin correlations provide us useful information to test the new physics beyond the SM.

[1] H. Georgi, Phys. Rev. Lett. 98, 221601 (2007).
[2] H. Georgi, Phys. Lett. B650, 275 (2007).
[3] T. Banks and A. Zaks, Nucl. Phys. B196, 189 (1982).
[4] K. Cheung, W.-Y. Keung and T.-C. Yuan, arXiv:0706.3155.
[5] S. Coleman and E. Weinberg, Phys. Rev., D7, 1888 (1973).
[6] M. Luo and G. Zhu, arXiv:0704.3532.
   C.H. Chen and C. Q. Geng, arXiv:0705.0689;
   Y. Liao, arXiv:0705.0837;
   G.J. Ding and M.L. Yan, arXiv:0705.0794;
   T.M. Aliev, A.S. Cornell and N. Gaur, arXiv:0705.1326;
   X.Q.Li and Z.T. Wei, Phys. Lett. B651, 380 (2007);
   M.A. Stephanov, Phys. Rev. D76,035008 (2007);
   N. Greiner, arXiv:0705.3518
   S.L. Chen and X.G. He, arXiv:0705.3946
   H. Davoudiasl, arXiv:0705.3636
   T.M. Aliev, A.S. Cornell and N. Gaur, JHEP 0707, (2007);
   P. Mathews and V. Ravindran, arXiv:0705.4599;
   S. Zhou, arXiv:0706.0302;
   G.J. Ding and M.L. Yan, arXiv:0706.0325;
   C.H. Chen and C.Q. Geng, arXiv:0706.0850
   Y. Liao and J.Y. Liu, arXiv:0706.1284;
   M. Bander, J.L. Feng, A. Rajaraman and Y. Shirman, arXiv:0706.2677
   T.G. Rizzo, arXiv:0706.3025
   S.L. Chen, X.G. He and H.C. Tsai, arXiv:0707.0187;
   R. Zwicky, arXiv:0707.0677;
   T. Kikuchi and N. Okada, arXiv:0707.0893
   R. Mohanta and A.K. Giri, arXiv:0707.1234
C.S. Huang and X.H. Wu, arXiv:0707.1268
N.V. Krasnikov, arXiv:0707.1419
A. Lenz, arXiv:0707.1535
D. Choudhury and D.K. Ghosh, arXiv:0707.2074
T.A. Ryttov and F. Sannino, arXiv:0707.3166
M. Neubert, arXiv:0708.0036
G. Bhattacharyya, D. Choudhury and D.K. Ghosh, arXiv:0708.2835
Y. Liao, arXiv:0708.3327
A. T. Alan and N. K. Pak, arXiv:0708.3802
A. B. Balantekin and K. O. Ozansoy, arXiv:0710.0028
A. T. Alan, N. K. Pak and A. Senol, arXiv:0710.4239
I. Sahin, B. Sahin, arXiv:0711.1665
K. Cheung, C. S. Li, T.-C. Yuan, arXiv:0711.3361
R. Mohanta, A. K. Giri, arXiv:0711.3516
K. Huitu, S. K. Rai, arXiv:0711.4754
S. Dutta, A. Goyal, arXiv:0712.0145
J. R. Mureika, arXiv:0712.1786
Y. Wu, D.-X. Zhang, arXiv:0712.3923
T. Kikuchi, N. Okada, M. Takeuchi, arXiv:0801.0018
X.-G. He, S. Pakvasa, arXiv:0801.0189
C.-H. Chen, C. S. Kim, Y. W. Yoon, arXiv:0801.0895
K. Cheung, T. W. Kephart, W. Y. Keung, T.-C. Yuan, arXiv:0801.1762
I. Sahin, arXiv:0801.1838
S. Dutta, A. Goyal, arXiv:0801.2143
C.-F. Chang, K. Cheung, T.-C. Yuan, arXiv:0801.2843
V. Barger, Y. Gao, W.-Y. Keung, D. Marfatia, V. N. Senoguz, arXiv:0801.3771
H.-F. Li, H. Li, Z.-G. Si, Z.-J. Yang, arXiv:0802.0236
M. J Aslam, C.-D. Lu, arXiv:0802.0739
[7] K. Cheung, W.-Y. Keung and T.-C. Yuan, arXiv:0704.2588.
[8] R.D. Peccei, X. Zhang, Nucl.Phys. B337, 269 (1990);
R.D. Peccei, S. Peris and X. Zhang, Nucl.Phys. B349, 305 (1991);
[9] I. Bigi, Y. Dokshitzer, V. Khoze, J. Kühn and P. Zerwas, Phys.Lett. B181, 157 (1986).

[10] M. Arai, N. Okada, K. Smolek, V. Simak, Phys.Rev.D70, 115015 (2004).

[11] T. Stelzer, S. Willenbrock, Phys.Lett. B374 169 (1996);
    A. Brandenburg, Phys.Lett. B388 (1996) 626;
    D. Chang, S.C. Lee, A. Soumarokov, Phys. Rev. Lett. 77 (1996) 1218;
    G. Mahlon, S. Parke, Phys. Rev. D53 (1996) 4886;
    G. Mahlon, S. Parke, Phys. Lett. B411 (1997) 173;

[12] T. Kaneko in “New Computing Techniques in Physics Research”, ed. D. Perret-Gallix, W. Wojcik, Edition du CNRS, 1990;
    MINAMI-TATEYA group, “GRACE manual”, KEK Report 92-19, 1993;
    F. Yuasa et al., Prog. Theor. Phys. Suppl. 138 (2000) 18.
FIG. 1: Spin asymmetry as a function of center of mass energy in the helicity basis for the scalar unparticle. The scale dimension is $d_U = 1.1, 1.9$. $\Lambda_U = 1 \text{ TeV}$

TABLE I: Spin asymmetries of the top quark pair for the process $e^+ e^- \rightarrow t\bar{t}$ in the helicity basis. $\Lambda_U = 1 \text{ TeV}$, $d_U = 1.1$ and $\sqrt{s} = 0.5 \text{ TeV}$.

|                  | $A_{SM}$ | $A_{S}$  | $A_{V}$   |
|------------------|----------|----------|-----------|
| SM               | -0.651   | 0.986    | -0.727    |
| Scalar Unparticle|          |          |           |
| Vector Unparticle|          |          |           |

TABLE II: Spin asymmetries of the top quark pair for the process $e^+ e^- \rightarrow t\bar{t}$. Top quark spin axis is along the positron beam direction and antitop quark spin axis is along the electron beam direction. $\Lambda_U = 1 \text{ TeV}$, $d_U = 1.1$ and $\sqrt{s} = 0.5 \text{ TeV}$.

|                  | $A_{SM}$ | $A_{S}$  | $A_{V}$   |
|------------------|----------|----------|-----------|
| SM               | -0.939   | 0.657    | -1        |
| Scalar Unparticle|          |          |           |
| Vector Unparticle|          |          |           |
FIG. 2: The same as Fig1 but for the vector unparticle.

FIG. 3: Spin asymmetry as a function of center of mass energy for the scalar unparticle. Top quark spin axis is along the positron beam direction and antitop quark spin axis is along the electron beam direction. The scale dimension $d_U = 1.1, 1.9$. $\Lambda_U = 1 TeV$. 
FIG. 4: The same as Fig3 but for the vector unparticle.

FIG. 5: Spin asymmetry as a function of scale dimension $d_U$ in the helicity basis for the scalar and vector unparticle. $\sqrt{s} = 0.5$ TeV, $\Lambda_U = 1$ TeV
FIG. 6: Spin asymmetry as a function of scale dimension $d_U$ for the scalar and vector unparticle. Top quark spin axis is along the positron beam direction and antitop quark spin axis is along the electron beam direction. $\sqrt{s} = 0.5$ TeV, $\Lambda_U = 1$ TeV