The construction of student’s thinking transformation: from simple connectivity to productive

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Abstract. The purpose of this study was to identify the process of transforming students’ thinking from simple to productive connectivity at the time of reflection for maximize the mathematical connections that students have constructed in the problem-solving process. The construction of students’ thinking transformation process from simple to productive connectivity was observed based on the completeness of the connective thinking network, which was built on the problem-solving process using Thosio scheme. The purposive sampling technique was used to select three students who had a tendency to transform simple connective thinking to productive. Worksheets and think aloud recording of three students were analyzed by qualitative descriptive approach. In the process of students’ connective transformation construction thinking, from simple to productive, it could be described construction process done by the students which were repairing errors of connection formed in network of simple connective thinking, and build connection not yet complete in network of simple connective thinking so as to form student thinking transformation from simple to productive connective. By and large, there were two construction processes in the transformation of simple connective thinking to productive at the time of reflection.

1. Introduction
The most interesting thing in learning mathematics is that how students construct concepts and build mathematical connections in the process of solving mathematical problems. Mathematical connection is a cognitive process in connecting or associating two or more ideas, concepts, definitions, theorems, procedures, and representations in math, with other disciplines, including real life [1]. Establishing mathematical connections through associating mathematical ideas in the problem-solving process is essential. This is revealed by many researchers that success in solving mathematical problems can not be separated from the ability to build mathematical connections [2,3,4,5,6,7,8]. Students whose thinking structure are suitable to the given problem structure can actually organize ideas in the process of solving mathematical problems [9].

Making association among mathematical ideas when they are connecting mathematical concepts into a form of connective thinking. Connective thinking process takes place in working memory, connecting new information and old knowledge that have the same and interrelated meaning to form a connective thinking scheme [10]. The connections among ideas can actually create a thinking scheme in the form
of organized cognitive networks [11]. The same thing is shown by Bernard and Tall, that the process of positive thinking and structure in the context of interrelated ideas [12]. A scheme is a mental structure used to produce decisions [13]. Students who are not able to develop complete mathematical ideas can not create connective thinking network scheme in response to existing problems. Students in this group are in the category of simple connectivity [7]. Students who have a tendency to think connectively productive can generalize ideas that are built up until the reconstruction phase in the Thosio stages [7].

Making connections among mathematical ideas is an important indicator of one's understanding [14,15]. Through the construction of students' connective thinking transformation from simple to productive, each student can maximize his or her cognitive ability, especially in building connection of mathematical ideas. The results of an empirical study of 72 high school students indicated a connective thinking transformation if students were given the opportunity to perform a process of reflection. Connective thinking transformation occurs through giving the students a chance to do the reflection process. The practical implication of this research result is that through constructive transformation of students' connective thinking from simple to productive, students can maximize their connective thinking ability so that it can be a reliable problem solver.

Studies conducted by some previous researchers only identified the applicability and types of connections that occurred in the general problem-solving process, either through the assignment of individual and group [2,4,6,8,16,17,18]. In addition, in 2015 Susanti has identified three categories of connective thinking: simple, semi-productive and productive. However, from all previous studies, there has been no observation on how to improve the ability of connective thinking through the construction of student's thinking transformation. So this paper is directed to describe how the construction of thinking transforms students from simple to productive connectivity through a process of reflection. This study aims to identify the process of connective thinking construction of students from simple to productive connectivity during the reflection process in order to maximize students' problem solving abilities.

2. Method

The purposive sampling technique was used to select three students of s1, s2, and s3 by giving preliminary test to the data source 72 students at two high schools. Purposive sampling is a technique of data collection with the consideration of such samples can provide the information we need on the observed phenomenon [19]. Therefore, the selection of the three students who have a tendency of simple connective thinking was also considering their ability to communicate. The three students with a simple connective thinking tendency were given a matter of solving connection problem one. The construction of students' connective thinking transformation were observed after the students were given the opportunity to perform a process of reflection and to solve the problem of two connection problems. Supporting instruments were developed from several materials: plane, arithmetic, numerical patterns and algebraic functions packaged in a connection problem-solving to emerge various connector ideas from the students. The outcomes and think aloud recording of the three students before and after reflection were analyzed by qualitative descriptive approach. The semi-structured interview process was conducted to deepen the analysis towards the construction of students' thinking transformation process from simple to productive to obtain the conclusion of research result.

3. Result and Discussion

In order to discover new ideas, reflection is very important in the problem-solving process [20]. Reflection is a mental mechanism in the form of a reorganization scheme to assimilate the problem situation into pre-existing knowledge [21,22]. Construction of connective thinking transformation in this study is to see the process of changing the categories of students' connective thinking from simple to productive after reflection. The process of changing connective thinking categories was observed through the development of connection ideas which were built at each stage of thosio: the stages of cognition, inference, formulation, and reconstruction after reflection. The following is a connection-solving figure that has been completed with the student's answer before performing the reflection process.
Figure 1 above shows that students are in the category of simple connective thinking. Students had not been able to complete the connection problem-solving tables correctly. The student mistakenly identified the number indicated the number of triangles in the origami ornament with flanks’ length on two units and the number of trapezoidal legs containing three unit triangles on each level of origami ornaments. Consequently, the students were eventually unable to determine the general formula for tribe to-n term for the origami n-level ornament. After students were given the opportunity to do the reflection process, students were able to identify the characteristics of the unit triangle that compose each level of origami ornaments. As shown in the following interview transcript:

Q: After reviewing, what kinds of unit triangle characteristics do you know?
s1: The unit triangle is an equilateral triangle with flanks’ length on one unit which draw up each level of origami ornaments

Based on the results of the work and interview after the students did the reflection process. Students were able to find a new pattern that could form a triangle flanks’ length on one unit on each level of origami ornaments. Students built a new idea after looking back at the origami ornaments and understood the characteristics of the triangle of the origami ornaments. In this process students performed the stage of cognition and inference where students understood the problem by looking for logical information that could be used to identify the number of triangles with flanks’ length on two units on each level of origami ornaments[13]. Through this process, students also built connections that should be built from the very first place. In the process of reflection, the student realized his mistake in interpreting the same trapezoidal feet that contained three unit triangles. The student identified the number of trapezoids that could be formed at each level of the origami ornaments based on the flang of the trapezoidal base, while the what was expected in the questions, the trapezoidal it self should have formed from the combination of three unit triangles coincided one to another.

The student’s mistake in interpreting the trapezoid caused the student made a miss connection. Therefore the students were unable to develop a suitable idea in identifying any trapezoidal pattern containing three unit triangles. However, in the process of reflection students successfully constructed each trapezoidal pattern that could be formed at each level of origami ornaments and performed the formulation stage, which was calculating the amount of trapezoid that can be formed at each level of origami ornaments. As shown in the following figure:

| On ornament origami | General formula of origami ornaments tribe to n |
|---------------------|-----------------------------------------------|
|                     | degree | degree | degree | degree | degree | degree | n-degrees |
| The number of triangles on origami ornaments with side lengths on two units | 0       | 1       | 3       | 6       | 10      | 15      | 21       |
| The number of trapezoidal legs that contain three unit triangles | 0       | 1       | 3       | 6       | 10      | 15      | 21       |
Figure 2. The result of trapezoidal pattern construction contains of three unit triangles.

Figure 2 shows when the reflection process was going on, the students were constructing new ideas after looking back the picture of origami ornament and understanding the character of unit triangles, the origami ornaments’ organizer. Based on the construction process, students could determine the number of trapezoidal legs that contained three unit triangles at each level of origami ornaments and perform formulation stages. Therefore in the reflection phase, students were also able to construct the \textit{tribe-n} formula from origami n-level ornaments that indicated the number of triangle's flanks' length on one unit. The process is illustrated by the following students' works:

Figure 4 shows that in the reflection process, the students were able to construct the tribe to-n formula for origami ornament n-level. The construction process was applied by constructing students’ ideas of connections from the series of numbers indicating the number of triangles with the flanks’ length on two units at each level of origami ornaments. Before the reflection process, the students identified the wrong trapezoid pattern so that the student had not been able to arrive at the formulation stage which was to verify the trapezoidal pattern and to perform the calculation to find the constant difference of the number series showing the number of triangles with the flanks’ length on two units.
Based on the investigation of the research result, it was concluded that the students did constructive thinking constructions through the fixation against built miss connection and filled in the noting connections that should be built in the early stages before reflection. Both of these construction processes resulted in the formation of a more complete connective thinking network resulting in the transformation of students' connective thinking from simple to productive. As Subanji explained that errors in making connections or the lack of mathematical connections that should be constructed by students in the problem-solving process could be detected through the construction process [23]. The same thing stated by Evitts that the strong connection of each student could be observed through the construction process by identifying the connections of concepts and procedures built into the problem-solving process [24].

The reflection process made students to be able to optimize their connective thinking skills so that there would be a connective thinking transformation from simple into productive one. Students were able to construct connector ideas at the reflection stage so that students would be finally able to solve the problem. As explained by Barmby, et al that to examine the problem solving ability of a person, the important thing to take into account was the completeness of the built connections [25]. Reflection could be a solution to overcome the incompleteness of built connections [26]. Therefore by giving students the opportunity to complete and improve the connections built through the reflection process then the students' mathematical thinking will develop. The findings of this study were supported by the results of previous researchers’ statements that students who were given the opportunity to conduct the process of reflection could lead to the occurrence of ideas reorganization, which came to new ideas in shaping a knowledge scheme [27]. The same view was explained by some experts that the idea-rich cognitive unit was a form of connection that could be expanding to form a new and stronger relationship to the structure of the mental network [11,28].

4. Conclusion
The construction of the student's connective thinking transformation from simple to productive takes place during the process of reflection by going through two stages of repairing connection errors and building the emptiness of connections that should be built. Therefore, during the process of reflection students can optimize their connective ability to think in order to solve the problem.

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