Testing SUSY

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Abstract

If SUSY provides a solution to the hierarchy problem then supersymmetric states should not be too heavy. This requirement is quantified by a fine tuning measure that provides a quantitative test of SUSY as a solution to the hierarchy problem. The measure is useful in correlating the impact of the various experimental measurements relevant to the search for supersymmetry and also in identifying the most sensitive measurements for testing SUSY. In this paper we apply the measure to the CMSSM, computing it to two-loop order and taking account of current experimental limits and the constraint on dark matter abundance. Using this we determine the present limits on the CMSSM parameter space and identify the measurements at the LHC that are most significant in covering the remaining parameter space. Without imposing the LEP Higgs mass bound we show that the smallest fine tuning (1:13) consistent with a relic density within the WMAP bound corresponds to a Higgs mass of 114±2 GeV. Fine tuning rises rapidly for heavier Higgs.

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1 Introduction

As the Large Hadron Collider starts operation, the search for supersymmetry reaches the interesting stage at which candidate models for physics beyond the Standard Model (SM) may be definitively tested. In order to do this it is necessary to quantify the viable range of the SUSY parameters, and in particular limit the mass scale of the supersymmetric spectrum. The bounds on the masses of supersymmetric states come most sensitively from the requirement that supersymmetry solves the hierarchy problem, i.e. it ensures that the electroweak breaking scale is consistent with radiative corrections without undue fine tuning. Fine tuning [1]-[14] is a measure of the probability of unnatural cancellations between soft SUSY breaking terms in the determination of the electroweak breaking scale after including quantum corrections and experimental constraints. The LEP constraints have already placed SUSY under some pressure as the fine tuning is found to be quite large. The reason for this is largely due to the LEP bound on the mass of the lightest Higgs mass, $m_h$, because satisfying it requires large radiative corrections that depend logarithmically on the mass of the top squarks and this in turn requires large stop masses and hence large supersymmetry breaking soft terms. However, even in the simplest implementations of SUSY, there is still a significant area of parameter space to explore and it is of importance to provide a quantitative measure of what needs to be done to fully test SUSY.

In this paper we shall use the fine tuning measure, $\Delta$, to quantify fine tuning. The measure determines the cancellation needed between the independent terms contributing to the electroweak symmetry breaking vacuum expectation value and provides an intuitively reasonable way of quantifying fine tuning. Of course it is necessary to supplement this by imposing an upper bound for $\Delta$ beyond which fine tuning is unacceptable and this is a subjective judgement. For the most part we will just display the range of fine tuning found without imposing such a cut off, except when assessing whether the LHC will be able fully to test the model. As we shall discuss the fine tuning measure allows one to quantify the significance of individual measurements in testing SUSY over the full parameter space and so is useful in correlating the various measurements relevant to SUSY searches. It also allows us to determine the most important measurements for testing SUSY at the LHC.

In order to illustrate the usefulness of the fine tuning analysis we will study in detail the Constrained Minimal Supersymmetric Model (CMSSM) which is the minimal supersymmetric extension of the SM with a restricted set of SUSY breaking soft terms. We later comment on the

\footnote{Light supersymmetric states are also needed for accurate gauge coupling unification. However the dependence on the mass is only logarithmic so gauge unification does not provide the most restrictive bound on the SUSY spectrum.}

\footnote{Unless the Higgs has non-standard decays [15].}
effect of relaxing this assumption and on other effects beyond CMSSM. We compute $\Delta$ to two-loop order, using the results of [16]-[20] and present a scan of the full CMSSM parameter space compatible with current experimental and theoretical constraints. In particular we require:

- radiative generation of electroweak symmetry breaking.
- no colour/charge breaking vacua.
- consistency with current experimental bounds on superpartner masses, electroweak precision data, $b \to s\gamma$, $b \to \mu\mu$ and anomalous magnetic moment constraints.
- a radiatively corrected SM-like Higgs mass in agreement with the current LEPII bound on $m_h$.
- consistency with the thermal relic density constraint.

We start with the form of the scalar potential responsible for electroweak breaking. With the standard two-higgs doublet notation, it is given by

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - (m_3^2 H_1 \cdot H_2 + h.c.)$$

$$+ \frac{1}{2} \lambda_1 |H_1|^4 + \frac{1}{2} \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1 \cdot H_2|^2$$

$$+ \left[ \frac{1}{2} \lambda_5 (H_1 \cdot H_2)^2 + \lambda_6 |H_1|^2 (H_1 \cdot H_2) + \lambda_7 |H_2|^2 (H_1 \cdot H_2) + h.c. \right]$$

(1)

with the assumption that at the UV scale $m_1^2 = m_2^2 = m_0^2 + \mu_0^2$, and $m_3^2 = B_0 \mu_0$.

The couplings $\lambda_j$ and the soft masses receive one- and two-loop corrections that for the MSSM are found in [19, 20]. We shall use these results to evaluate the overall amount of fine-tuning of the electroweak scale. To this purpose, it is convenient to introduce the notation

$$m^2 = m_1^2 \cos^2 \beta + m_2^2 \sin^2 \beta - m_3^2 \sin 2\beta$$

$$\lambda = \frac{\lambda_1}{2} \cos^4 \beta + \frac{\lambda_2}{2} \sin^4 \beta + \left( \frac{\lambda_3 + \lambda_4 + \lambda_5}{4} \right) \sin^2 2\beta + \frac{\lambda_6 \cos^2 \beta + \lambda_7 \sin^2 \beta}{\sin 2\beta}$$

Minimisation of $V$ gives

$$v^2 = -m^2/\lambda, \quad 2\lambda \frac{\partial m^2}{\partial \beta} = m^2 \frac{\partial \lambda}{\partial \beta}$$

(4)

which fixes $v$ and $\beta$ as functions of the MSSM bare parameters.

The fine-tuning measure, $\Delta$, is defined by [1]
\[ \Delta \equiv \max_{p} |\Delta_{p}| = \{\mu_{0}^{2}, m_{1/2}^{2}, m_{0}^{2}, \lambda_{0}^{2}, B_{0}^{2}\}, \quad \Delta_{p} \equiv \frac{\partial \ln v^{2}}{\partial \ln p} \]  

where all \( p \) are input parameters at the UV scale, in the standard MSSM notation.

We compute the fine tuning at two-loop order including the dominant third generation supersymmetric threshold effects to the scalar potential. The analysis is done in two stages. First a scan is performed over all of parameter space using a slightly simplified two-loop calculation developed to run quickly. Then the analysis is redone using the (slower) SOFTSUSY 3.0.10 package that includes all the effects mentioned above for a set of points that has low fine tuning. It was found important to work at two-loop order as \( \Delta \) changes significantly between one and two-loops. Note that SOFTSUSY uses the measured values of the gauge couplings. The unification scale is determined by the point the \( SU(2) \) and \( U(1) \) couplings unify (of \( O(10^{16} \text{ GeV}) \)). Full details of this procedure and an analysis of the results will appear in a separate publication.

The results are shown in Figure 1. In this the Higgs mass and \( \Delta \) are computed by SOFTSUSY. It agrees to within 0.1 GeV with that determined by Suspect but can differ by \( \pm 2 \text{ GeV} \) from that determined by FeynHiggs. Given this uncertainty, which comes from the higher order terms in the perturbative expansion, the LEP bound should be interpreted as \( m_{h} > 114.4 \pm 2 \text{ GeV} \). In what follows we will usually quote fine tuning computed for the central value. The relic density constraint is tested using MicrOMEGAs 2.2.

The fine tuning distribution shows an interesting structure with a minimum close to the present LEP bound. This structure is largely generated by the \( \lambda \) dependence of \( v \) in eq(1). At tree level the MSSM value for \( \lambda \) is anomalously small, \( \lambda = \frac{(g_{1}^{2} + g_{2}^{2}) \cos^{2}(2\beta)}{8} \), and this is the main reason why fine tuning is large at tree level. Radiative corrections can increase \( \lambda \), reducing the fine tuning. The structure shown in Figure 1 is driven by this effect. For \( m_{h} \) less than the LEP bound the fine tuning is dominated by the \( \mu \) contribution and rises for decreasing \( m_{h} \). This is because the region corresponds to smaller \( \tan \beta \) where the radiatively corrected \( \lambda \) is smaller. Since \( v^{2} = -m^{2}/\lambda \), we see from eq(5) that smaller \( \lambda \) leads to an increase in fine tuning (for an extended discussion see [23]). For \( m_{h} \) greater than the LEP bound the fine tuning is dominated by the \( m_{0} \) contribution because in this region it is necessary to have large positive radiative corrections to the Higgs mass and this requires large stop masses and in turn large \( m_{0} \), greater than the focus point, can control. Since the Higgs mass depends logarithmically on the stop masses, in this region \( m_{0}^{2} \) is roughly proportional to \( e^{m_{S}^{2}} \). The fine

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\(^{3}\)We have also studied the contribution to \( \Delta \) coming from the uncertainty in the top Yukawa coupling and the strong coupling. Using the modified definition of \( \Delta \) appropriate to measured parameters we find them to be sub-dominant.

\(^{4}\)Strictly this has been derived for the SM only. However it also applies to the CMSSM for viable parameter choices.
Figure 1: Two-loop fine tuning vs Higgs mass for the scan $2 \leq \tan \beta \leq 55$. The solid line is the minimum fine tuning with $(\alpha_s, M_t) = (0.1176, 173.1 \text{ GeV})$. The dashed and dotted lines in (a) have $(\alpha_s, M_t) = (0.1156, 174.4 \text{ GeV})$ and $(0.1196, 171.8 \text{ GeV})$ respectively to account for the $1\sigma$ experimental errors. In (b), the red (lighter) points saturate the non-baryonic relic density within the $1\sigma$ WMAP bounds, $\Omega h^2 = 0.1099 \pm 0.0062$. The blue (darker) points have a thermal relic density, $\Omega h^2 < 0.1037$.

tuning measure in this region is in turn approximately proportional to $m_0^2$ and hence grows exponentially with the Higgs mass. The two-loop corrections increases this growth slightly, explaining the sharp rise seen in Figure 1 at large $m_h$. The position of the dip in fine tuning comes from the lower bounds on the SUSY masses; as they increase the dip moves to higher masses and $\Delta$ at the minimum increases. If it rises beyond an unacceptable level one may conclude that the CMSSM solution to the hierarchy problem has been fully tested and found to fail. At present we are far from this point with regions of SUSY parameter space having fine tuning less than 1 part in 8.8.

Not yet included in the analysis is the constraint coming from the dark matter abundance. In Figure 1(b) we show that there are points in parameter space that populate the regions with small fine tuning and have a dark matter density consistent with providing all or some of the dark matter. Note that this favours the part of the dip around the LEP Higgs mass bound (c.f. [30], [31]) even though this bound has not been included in the analysis! Taking into account the theoretical uncertainty in determining $m_h$ we conclude from that the most likely mass for the Higgs consistent with the observed dark matter abundance is $m_h = 114 \pm 2 \text{ GeV}$ corresponding to a fine tuning of $1 : 13$. For a saturation of the WMAP bound within $1\sigma$, one finds $m_h = 116 \pm 2 \text{ GeV}$ corresponding to a fine tuning of $1:19$. For $m_h = 121 \text{ GeV}$ this rises to $1 : 100$ fine tuning and at $m_h = 126 \text{ GeV}$ to $1 : 1000$ fine tuning.

The fine tuning measure can readily be used to establish the remaining allowed range of the SUSY parameters by plotting the contribution to fine tuning of the various components.
Figure 2: Dependence of minimum fine tuning on SUSY parameters ($\mu > 0$, relic density unrestricted). The solid, dashed and dotted lines are as explained in Fig 1. In (b)-(d), the darker shaded regions are eliminated when $m_{h} > 114.4$ GeV is applied for the case with the central ($\alpha_s, M_t$) values. In (a) and (e), $m_{h} > 114.4$ GeV is applied, and the points in (e) are only for the central ($\alpha_s, M_t$) values.
This is shown in Figure 2. From these graphs one may see that, once one chooses an upper limit for the fine tuning measure $\Delta$, the allowed range for the parameters is defined. For the case that we define $\Delta = 100$ as the upper limit beyond which we consider SUSY has failed to solve the hierarchy problem, we obtain the bounds:

\begin{align*}
    m_h &< 121 \text{ GeV} & 5.5 < \tan \beta < 55 \\
    \mu &< 680 \text{ GeV} & 120 \text{ GeV} < m_{1/2} < 720 \text{ GeV} \\
    m_0 &< 3.2 \text{ TeV} & -2 \text{ TeV} < A_0 < 2.5 \text{ TeV} \\
\end{align*}

(6)

It is clear that there is still a wide range of parameters that needs to be explored when testing the CMSSM. Will the LHC be able to cover the whole range? To answer this note that one must be able to exclude the upper limits of the mass parameters appearing in Table 1. Of course the state that affects fine tuning most is the Higgs scalar and one may see from Figure 1 that establishing the bound $m_h > 121 \text{ GeV}$ will imply that $\Delta > 100$. However the least fine tuned region corresponds to the lightest Higgs consistent with the LEP bound and this is the region where the LHC searches rely on the $h \rightarrow \gamma \gamma$ channel which has a small cross section and will require some $30 \, fb^{-1}$ at $\sqrt{s} = 14 \text{ TeV}$ to explore. Given this it is of interest to consider to what extent the direct SUSY searches will probe the low fine tuned regions.

For $\mu$ the $\Delta = 100$ upper bound corresponds to a Higgsino mass of $O(0.5 \text{ TeV})$ while for $m_{1/2}$ the bound corresponds to a gluino mass of $O(1.5 \text{ TeV})$ and a Wino or neutralino mass of $O(300 \text{ GeV})$. On the other hand, due to the focus point behaviour, the bound on $m_0$ is very weak corresponding to squark and slepton masses in the multi TeV region. Thus we expect that in testing SUSY the most significant processes at the LHC will be those looking for gluinos, winos and neutralinos. In Table 1 we give the upper limits on the mass of these states corresponding to $\Delta = 100$. For comparison we show the limit on the stop and sbottom masses, much larger due to the weak limit on $m_0$. The remaining states of the MSSM spectrum are dominated by the $m_0$ soft mass term and are bounded by $O(3$ to $4.4 \text{ TeV})$. All these upper mass limits scale approximately as $\sqrt{\Delta_{\text{min}}}$, so may be adapted depending on how much fine tuning the reader is willing to accept.

| $\tilde{g}$ | $\chi_1^0$ | $\chi_2^0$ | $\chi_3^0$ | $\chi_4^0$ | $\chi_1^\pm$ | $\chi_2^\pm$ | $\tilde{t}_1$ | $\tilde{t}_2$ | $\tilde{b}_1$ | $\tilde{b}_2$ |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 1720       | 305        | 550        | 660        | 665        | 550        | 670        | 2080       | 2660       | 2660       | 3140       |

Table 1: CMSSM upper mass limits on superpartners (in GeV), such that $\Delta < 100$ remains possible.

Studies of SUSY at the LHC $^{32}$ have shown that in the LHC experiments have a sensitivity to gluinos of mass $1.9 \text{ TeV}$ for $\sqrt{s} = 10 \text{ TeV}$, $2.4 \text{ TeV}$ for $\sqrt{s} = 14 \text{ TeV}$ and luminosity
10 fb$^{-1}$. Of relevance to the first LHC run the limit is 600 GeV for $\sqrt{s} = 10$ TeV and luminosity 100 pb$^{-1}$. These correspond to probing up to $\Delta = 120, 180$ and 14 respectively. As we have discussed charginos and neutralinos can be quite light, but their signal events are difficult for LHC to extract from the background, owing in part to a decreasing $M_{\tilde{W}} - M_{\tilde{Z}}$ mass gap as $|\mu|$ decreases [33, 34]. An Atlas study [35] of the trilepton signal from chargino-neutralino production found that 30 fb$^{-1}$ luminosity at 14 TeV is needed for a $3\sigma$ discovery significance for $M_{\tilde{2}} < 300$ GeV and $\mu < 250$ GeV [36].

Finally we return to the intriguing fact that minimum fine tuning plus correct dark matter abundance corresponds to a Higgs mass just above the LEP bound. As we have noted above this point is fixed by the current bounds on the SUSY spectrum and not by the current Higgs mass bound which is not included when doing the scans leading to Figure 1. One may interpret the SUSY parameters corresponding to this point as being the most likely given our present knowledge and so it is of interest to compute the SUSY spectrum for this parameter choice as a benchmark for future searches. This is presented in Table 2 where it may be seen that it is somewhat non-standard with very heavy squarks and sleptons and lighter gauginos. This has some similarities to the SPS2 scenario [37].

|        |       |       |       |       |       |       |
|--------|-------|-------|-------|-------|-------|-------|
| $h^0$  | 114.5 | $\tilde{\chi}^0_1$ | 79    | $\tilde{b}_1$ | 1147  | $\tilde{u}_L$ | 1444  |
| $H^0$  | 1264  | $\tilde{\chi}^0_2$ | 142   | $\tilde{b}_2$ | 1369  | $\tilde{u}_R$ | 1466  |
| $H^\pm$ | 1267  | $\tilde{\chi}^0_3$ | 255   | $\tilde{\tau}_1$ | 1328  | $\tilde{d}_L$ | 1448  |
| $A^0$  | 1264  | $\tilde{\chi}^0_4$ | 280   | $\tilde{\tau}_2$ | 1368  | $\tilde{d}_R$ | 1466  |
| $\tilde{g}$ | 549   | $\tilde{\chi}^\pm_1$ | 142   | $\tilde{\mu}_L$ | 1406  | $\tilde{s}_L$ | 1448  |
| $\tilde{\nu}_\tau$ | 1366  | $\tilde{\chi}^\pm_2$ | 280   | $\tilde{\mu}_R$ | 1406  | $\tilde{s}_R$ | 1446  |
| $\tilde{\nu}_\mu$ | 1404  | $\tilde{\ell}_1$ | 873   | $\tilde{e}_L$ | 1406  | $\tilde{c}_L$ | 1444  |
| $\tilde{\nu}_e$ | 1404  | $\tilde{\ell}_2$ | 1158  | $\tilde{e}_R$ | 1406  | $\tilde{c}_R$ | 1446  |

Table 2: A favoured CMSSM spectrum ($\Delta = 14.7$). Masses are given in GeV.

The results we have presented apply to the case of the CMSSM. It is natural to ask how these results are modified when one considers additional effects beyond CMSSM or relaxes some of its constraints. For example, one could consider different UV boundary conditions for scalar masses, non-universal gaugino masses or UV threshold corrections to the gauge couplings. Let us discuss briefly how these can change our findings. Universal scalar masses lead to a minimum of fine tuning through a focus point [29] and the minimum fine tuning found here corresponds to this focus point. Thus we expect fine tuning will increase if the universal scalar
mass assumption is relaxed. For the case of non-universal gaugino masses, it is known that a reduction of the amount of fine tuning can be obtained [6] below that of the CMSSM case. However this condition is not always sufficient to reduce $\Delta$ on its own, and depends on other parameters values such as $\mu$ or $m_0$, which usually dominate the fine tuning [23]. For a recent discussion of the reduction in fine tuning that can result from non-universal gaugino masses see [8]. Regarding UV scale threshold corrections to gauge couplings, if one imposes gauge coupling unification, they can affect the predictions for the low energy values of the couplings ($\alpha_s(M_Z), \ldots$) [28, 39, 40] and hence, c.f. Figure 1, the fine tuning. However the bottom-up SOFTSUSY approach used here uses the measured values for the gauge couplings and hence does not need to know the UV scale threshold corrections. The fine tuning due to the running of the soft SUSY breaking parameters is only mildly sensitive to the UV scale threshold corrections.

In summary we have made a detailed study of the possibility of testing the CMSSM as a solution to the hierarchy problem using a fine tuning measure. This provides a way of quantifying and correlating the effects of the numerous experimental measurements sensitive to SUSY and for determining the region of parameter space that needs to be explored when testing the model. Using this analysis, new measurements providing stricter bounds on the CMSSM spectrum will further limit the viable parameter space. Similar analyses could (and should) be applied to more general SUSY extensions of the SM and indeed to all proposed models claiming to solve the hierarchy problem.

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*It is hoped to provide in the near future a simple computer package to allow the analysis to be easily updated. In the meantime the authors will be happy to provide the updated information on request.*
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