Abstract A new method based on ultra-fast X-ray imaging was developed in this work for simultaneous investigations of the dynamics and the structures of complex two-phase flows. In this paper, cavitation was created inside a millimetric 2D Venturi-type test section, while seeding particles were injected into the flow. Thanks to the phase-contrast enhancement technique provided by the APS (Advanced Photon Source) synchrotron beam, high definition X-ray images of the complex cavitating flows were obtained. These images contain valuable information about both the liquid and the gaseous phases. By means of image processing, the two phases were separated, and velocity fields of each phase were therefore calculated using image cross-correlations. The local vapour volume fractions were also obtained thanks to the local intensity levels within the recorded images. These simultaneous measurements, provided by this new technique, afford more insight into the structure and the dynamic of two-phase flows as well as the interactions between them, and hence enable to improve our understanding of their behaviour. In the case of cavitating flows inside a Venturi-type test section, the X-ray measurements demonstrates, for the first time, the presence of significant slip velocities between the phases within sheet cavities for both steady and unsteady flow configurations.

Keywords Cavitation · Two-phase flows · X-ray imaging · Velocimetry

1 Introduction

Cavitation is one of the most challenging phenomena hydraulic machines encounter, it poses considerable difficulties in both design and maintenance operations. Described as the formation of vapour structures within a liquid when subjected to a sufficiently low pressure, cavitation usually appears in rotating machinery as unsteady clouds which are associated with many undesired effects, such as efficiency loss due to the increase of the hydrodynamic drag, blade erosion, vibrations as well as noise due to the collapse of the vaporous structures. Overall, these phenomena are linked to the complex unsteady mechanisms governing the development and the behaviour of cavitating flows. During the last decades, several research studies were performed in order to comprehend the physical phenomena behind these complex two-phase flows as a crucial step to docile them if not avoid them altogether. This quest relied mostly on experimental studies using cavitation tunnels with simple geometry configurations such as two-dimensional foil sections or 2D Venturi-type sections. Velocity measurements within sheet
cavitating flows, demonstrating, for the first time, the existence of important slip velocities between the phases.

2 Experimental set-up

To perform measurements with X-rays, the test section was aligned with the X-ray source on one side and an X-ray detector on the other. Hollow glass spheres with a silver coating were injected in the flow, and when crossing the beam, they were detected by means of a high spatial resolution detector. A high speed camera was then used to record the flow. To detect microscopic particles moving rapidly, up to 20 m/s, the X-ray beam must have specific characteristics such as a very high intensity and a high spatial coherence. Since no portable X-ray source was found to meet these requirements, the experiments were carried out at the APS synchrotron of the Argonne National Laborotary (Illinois, USA), which resulted in severe restrictions for both the hydraulic test rig and the test section designs.
2.1 Hydraulic test rig

Since the experiments were held at the National Argonne Laboratory, the hydraulic installation had to be portable in order to facilitate its transport. The ‘Venturi-P’ loop was hence designed (figure 1).

During the experiments, the test rig was filled with water recirculating thanks to a ‘Salmon Multi-HE 403’ pump (1) which rotational speed could be regulated up to 3,600 rpm in order to set the mass flow rate. At the pump delivery, a partially filled tank (2) with a free surface is used as a reservoir for flow and pressure fluctuations, as well as a heat exchanger. Indeed, the free surface of the tank acts as a filter to the periodical fluctuations of the flow rate and/or the pressure caused by the passage of the pump blades. Moreover, since the flow tends to heat during its circulation within the test rig, a temperature monitoring system was needed. It consists of a secondary loop (3) connected to an external refrigerating system, insuring the recirculation of cold water within this loop, and a 2,000 W ‘Chromalox’ inline heater (4). The combined actions of the cooling and the heating systems allow to regulate and maintain the flow temperature between 12 and 65 °C. A K-type thermocouple (8) is used to measure the temperature at the inlet of the test section (9).

Three parallel mounted ‘Bürkert - 8032 type’ flow-meters (5, 6 and 7) are used to measure the flow rate. Their respective ranges are 3-16, 8-30 and 16-100 l/min with a 2% reading uncertainty on measurements. Three piezoresistive ‘Keller 10L series’ pressure sensors (10, 11 and 12) are mounted inside cavities within the test section (see figure 2) and calibrated within a range of 0-10 bar with a full scale uncertainty of 0.25% and a frequency response of . The upstream pressure sensor (10) is used to monitor the reference pressure \( P_{\text{ref}} \) and set the flow conditions using the cavitation number \( \sigma \):

\[
\sigma = \frac{P_{\text{ref}} - P_{\text{vap}}}{\frac{1}{2} \rho V^2_{\text{ref}}}
\]

where \( P_{\text{vap}} \) is the vapour pressure at the temperature \( T \) of the flow, \( V_{\text{ref}} \) is the velocity at the entrance of the Venturi profile and \( \rho \) is the liquid density. Decreasing the cavitation number results in higher probability in cavitation occurrence or leads to an increase of the magnitude of the already present cavitation. The accuracy of the pressure and velocity measurements result in a mean uncertainty of 3.5% for the cavitation number. The pressure sensors (11 and 12), mounted downstream from the Venturi throat, are used to measure the pressure fluctuations at the closure regions of the sheet cavities. The adjustment of the pressure in the test rig is performed within the partially filled tank (13), which is connected to a compressor (14) for pressurisation and a vacuum pump for depressurisation (15). This enables to vary the absolute pressure in this tank between 0.1 and 10 bar.

Fig. 1 Schema of the Venturi-P test rig

2.2 Test section

Cavitation was produced in a 2D Venturi profile which design is based upon the water tunnel of the LEGI Laboratory (Grenoble, France) used in numerous experimental and numerical studies [15,16]. It is characterised by a convergent angle of 18° and a divergent angle of 8°, its definition could be found in Coutier-Delgosha et al.[17]. Nonetheless, the use of the APS synchrotron beam has resulted in several requirements for the test section. In fact, due to the small size of the X-ray beam cross-section (see section 3.2), only cavities with small lengths (3-15 mm) could be recorded. Thus, the test section had to be designed with very small dimensions. Furthermore, to ensure a satisfactory signal/noise ratio, both the flow and the test section wall thicknesses, crossed by the beam, had to be reduced as much as possible. These requirements led to a new test section with a geometry scaled down 10 times from the original one.

The X-ray test section is 30 cm long and comprises several parts manufactured in polymethyl methacrylate material (Perspex or Plexiglas) with a tolerance of ±0.05. As it could be seen in figure 2, the main part (1) constitutes the bottom and the side walls of the flow passage except beside the Venturi profile where additional inserts complete the walls, it also contains three cavities designed to host the pressure sensors (see section 2.1). The bottom insert (2) forms the convergent/divergent floor of the Venturi section, whereas two horizontal inserts (3) constitute the side walls at the cavitation area. A final insert (4) caps the test section forming its ceiling, and by using a spacer insert between the main part and the cap, the height of the channel could be adjusted. In this work, two test section configurations were tested corresponding to a height at the Venturi entrance \( h_{\text{ve}} \) of 5 and 17 mm. At the entrance of the test section, a 45 mm long channel with a height \( h_{\text{ve}} \) of respectively 19 mm and 31 mm ensures the transition between the pipe’s circular cross-section and the Venturi profile.

The use of inserts presents several advantages. i) the side walls and the floor could be replaced at moderate cost for

\[
\sigma = \frac{P_{\text{ref}} - P_{\text{vap}}}{\frac{1}{2} \rho V^2_{\text{ref}}}
\]
The 5 mm height configuration were steady. This finding explains hence why all cavitating flows obtained with heights greater than 18 mm, influences the creation of unsteady shedding. It was demonstrated that small geometry scales, especially small throat height, were critical for unsteady cavitation. Small geometry scales, especially small throat heights, easily though, the preliminary analysis of the results showed that this type of geometry, with a throat height of 4 mm. Thanks to all inserts, two passages for the X-ray beam exist, providing top and side views of the cavity sheet. Nevertheless, this paper focuses only on the latter.

2.3 Test conditions

In the present study, two series of measurements with different configurations were performed. In the first case, the test section height at the entrance of the Venturi profile $h_{ce}$ was 5 mm (height at the throat: $h_{th}=3.34$ mm). Flow rates of 8, 10, 14 and 16 l/min were tested while the obtained cavities had approximately the same length ($L_{cav} \approx 10$ mm), i.e. the same cavitation number (see Lecoffre [18], p.49). Unexpectedly though, the preliminary analysis of the results showed that the cavities had an unpredictable behaviour and no periodical cloud shedding were produced, although several works have shown that this type of geometry, with a 18° convergent and 8° divergent, produces unsteady cavitating flows with periodical shedding. A joint study [19] with the university of Ljubljana (Slovenia) was then conducted using high speed imaging. The spectral analysis on the cavity fluctuations observed from the images revealed the presence of a scale effect on the behaviour of sheet cavities. It demonstrated that small geometry scales, especially small throat height, influences the creation of unsteady shedding. This finding explains hence why all cavitating flows obtained with the 5 mm height configuration were steady.

To obtain unsteady cavitation with a periodical behaviour, a second series of measurements were performed using a higher Venturi profile thanks to a 12 mm spacer insert. In this configuration, the height at the entrance of the profile was 17 mm ($h_{th}=15.34$ mm at the throat), which allows the development of unsteady periodical cloud shedding. In this configuration, several tests were performed at different cavitation numbers ($\sigma=1.85, 1.97$ and 2.1) and several flow rates ($Q=35.09, 47.84$ and 55.5 l/min). Although this paper treats only one operating point, with $\sigma=1.97$ and $Q=35.09$ l/min, which corresponds to a velocity at the inlet of the Venturi profile of 8.6 m/s.

The flow temperatures for both configurations were constant throughout the experiments and were set respectively to 20°C and 17±0.5°C.

3 X-ray imaging

3.1 Methodology

In the present work, X-ray measurements were based upon two different mechanisms: absorption and phase-contrast enhancement. Commonly employed in numerous studies of two-phase flows using X-rays, absorption allows to obtain flow structures by means of energy attenuation. In fact, when an X-ray travels through the test section, its energy is partially absorbed by each traversed material: section side walls, liquid, vapour and seeding particles, in the present case. This attenuation depends on the materials absorption coefficients as well as their depth.

Since the vapour has a small absorption coefficient compared to the liquid, the more a ray encounters vaporous structures while crossing the flow, the higher energy it still has when it reaches the X-ray detector. On a radiograph (an X-ray image), high energy rays are translated into bright pixels, whereas low energy rays are converted into low intensity pixels. Thus, using this attenuation mechanism, the vapour volume fraction could be determined. Beer-Lambert law (or Beer’s law) relates the attenuation of photons energy to the attenuation coefficients, the densities and the thicknesses of all materials traversed by the beam. For N attenuating materials, Beer’s law could be expressed in terms of the transmittance $T$ as follow:

$$T = \frac{I}{I_0} = e^{-\sum_{i=1}^{N} \mu_i l_i}$$

where $I_0$ is the intensity of the incident X-ray beam, $I$ is the intensity of the transmitted beam after the sample, $\mu_i$ is the attenuation coefficient of a crossed material $i$ and $l_i$ is its depth. In the case of X-ray measurements of cavitating flows, the local intensity that reaches the X-ray detector and
therefore recorded on the image could be written as:

$$I = I_0 e^{-\sum_j \mu_j l_j e^{-(\mu_{\text{liq}} l_{\text{liq}} + \mu_{\text{vap}} l_{\text{vap}})}}$$

$$= I_0 e^{-\mu_{\text{liq}} l_{\text{liq}} - \mu_{\text{vap}} l_{\text{vap}}}$$  \hspace{1cm} (3)

where \( j \) represents an attenuating material between the X-ray source and the detector, excluding the flow (liquid and/or vapour). \( \mu_{\text{liq}} \) and \( \mu_{\text{vap}} \) are respectively the attenuation coefficients of the liquid and the vapour phases whereas \( l_{\text{liq}} \) and \( l_{\text{vap}} \) represent their depths. \( I_0 \) is the intensity of the beam at the detector when the test section is empty, i.e. without air.

Furthermore, the local volume fraction of vapour \( \beta \) could be expressed in terms of depths of the phases traversed by the beam as:

$$\beta = \frac{l_{\text{vap}}}{l} = \frac{l_{\text{vap}}}{l_{\text{vap}} + l_{\text{liq}}}$$  \hspace{1cm} (4)

where \( l = l_{\text{vap}} + l_{\text{liq}} \) represents the width of the inner channel of the test section. From equations 4 and 5, the vapour volume fraction of a cavitating flow in a test section could hence be written as follow:

$$\beta = 1 - \frac{\ln \frac{l_{\text{vap}}}{T_{\text{vap}}}}{\ln \frac{I_{\text{cav}}}{I_{\text{liq}}}}$$  \hspace{1cm} (5)

\( I_{\text{cav}} \) is the local beam intensity at the X-ray detector after crossing a test section with a two-phase cavitating flow, whereas \( I_{\text{liq}} \) and \( I_{\text{vap}} \) are respectively the local intensities while the test section is filled with pure liquid and pure vapour. In practice, \( I_{\text{liq}} \) and \( I_{\text{vap}} \) are determined from calibration images which are recorded at the same optical conditions as the cavitation tests, i.e. photons energy, detector-to-section distance,...(see section 3.2). These images were hence taken while the test section was filled respectively with water and air.

In fact, since it is practically impossible to reproduce cavitating pure vapour conditions in the test section, air was used in the calibration process. The graph ‘a’ of figure 3 shows a comparison between the linear absorption coefficients of liquid water, water vapour and air, whereas graph ‘b’ presents the absorption ratio between the vapour and air. For a monochromatic 13keV beam, this ratio is 0.37%. One could hence assume that air and vapour have the same absorption coefficient.

![Fig. 3](image)

The measurements uncertainties of the vapour volume fraction could be estimated using the following expression:

$$\delta \beta^2 = \left[ \frac{\ln \frac{l_{\text{vap}}}{l_{\text{liq}}}}{I_{\text{vap}} \ln \frac{l_{\text{vap}}}{l_{\text{liq}}}} \right]^2 \delta I_{\text{vap}}^2 + \left[ \frac{\ln \frac{l_{\text{vap}}}{l_{\text{liq}}}}{I_{\text{liq}} \ln \frac{l_{\text{vap}}}{l_{\text{liq}}}} \right]^2 \delta I_{\text{vap}}^2 + \left[ \frac{1}{I_{\text{cav}} \ln \frac{l_{\text{vap}}}{l_{\text{liq}}}} \right]^2 \delta I_{\text{cav}}^2$$  \hspace{1cm} (6)

Note that the attenuation coefficient \( \mu \) of equation 2 is considered for a mono-chromatic beam (see section 3.2). Hence, the errors due to hardening effect [21, 10] are neglected.

The second mechanism involved in the present X-ray measurements is the phase-contrast enhancement mechanism. Although it could be produced by three main techniques [22], its use in fluid mechanics is not conventional. In the case of synchrotron radiations, the phase contrast is the result of Fresnel diffraction via propagation technique [23] where the diffraction can be described by the Fresnel-Kirchoff integral (see Born [24](pp. 417-420)). The contrast, in this case, oc-
sions of high speed cavitation flows, with velocities attaining 20 m/s. For this purpose, high frequency data acquisition was necessary in order to detect the fluctuations of the sheet cavities.

The experiments were conducted at the station 32-ID-B of the APS synchrotron using an X-ray white beam, with a cross section of 1.7 × 1.3 mm². The source generates two types of pulses: a primary flash, used in this study, with a duration of 500 ns and a secondary pulse with a duration of 10 ps. Each type of pulse is generated at a frequency rate of 277.78 kHz, i.e. 3.6 µs separates the beginning of two similar pulses. The flux of incident photons on the sample supplied in this station is 10¹³ ph/s whereas the maximum energy set during the experiments was 30 keV with an average energy of 12.28 keV. The beam is assumed mono-chromatic with this average energy and the attenuation coefficient µ₁ of equation 2 is also considered for this energy. The beam energy should be high enough to provide clear images with high contrast between the flow different structures, including bubbles/liquid interfaces, but take nevertheless into consideration the image saturation level, which should not be reached to enable accurate measurements of the vapour volume fractions (see sections 3.1 and 5). A 400 µm thick cadmium tungstate (CdWO₄) scintillator was placed at approximately 50 cm from the test section to convert flow crossing X-rays into visible lights. The converted beam was then recorded using a ‘APX-RS Photon’ high-speed camera with an acquisition frequency set to 12,070 fps and a spatial resolution of 704 × 688 px² with a pixel size of only 2 µm.

The beamline is equipped with two mechanical shutters, as illustrated in figure 5-a. The first one is slow and operates at a frequency of 1 Hz with an opening duration of 30 ms every second. It has the function of protecting the equipments, such as the detector and the test section, by limiting the dose delivered to them. The second one is a chopper disc developed by Gembicky et al.[26] at the APS to drive time-resolved acquisitions while rotating at a speed reaching up to 32,000 rpm. The radial slots at the chopper disc allow a triangular opening phase of the shutter (figure 5-b).

The main issue related to ultra-fast acquisitions is the synchronisation between the X-ray pulses, the opening of the fast shutter and the camera frames, in order to obtain appropriate image pairs suitable for cross-correlation analysis. The frequency of the fast shutter was set to obtain an opening time close to 9 µs, so that only two primary X-ray pulses (2) and (4) were included in each opening window. The camera was triggered so that the frame change was located at the middle of the triangular window, in order to obtain identical illuminations of both images. Note that the secondary X-ray pulses (1) and (5) could not be avoided, which results in a second illumination in the center of both images, i.e. a bright band in the middle of each image (see section 4). In addition, although it is theoretically possible to avoid the middle secondary flash (3), since the frame change lasts few hundreds of nanoseconds, it was practically inevitable, given that the synchronisation could hardly be perfectly achieved. This results in an asymmetry in the brightness of images: i) one
image has an extra illumination with respect to the other of the same pair; ii) the brightness and the width of the middle bright band, caused by the secondary pulses, changes with the image. These irregularities in image intensities had to be taken into account during the measurements of the vapour volume fractions. Though, thanks to image processing, their effects were minimised (see section 5).

Furthermore, the test conditions (flow rate and pressure) produce cavitating flows with average cavity lengths comprised between 3 mm and 13 mm. However, since the beam has a cross-section of 1.7×1.3 mm², each sheet cavity needed to be divided into several windows recorded successively but not simultaneously (figure 6). In an explicit way, N image pairs of the first window (position 1), which corresponds to the beginning of the cavity, were recorded. The test section was then shifted parallel to the divergent floor to position 2 where the same number of pairs were recorded and so forth until the end of the two-phase cavity. The displacement of the test section was ensured by a motorised platform (the pipes at the inlet and the outlet of the test section are flexible). Note that the window shifting was performed with an overlap of 0.1 mm. This overlap is important to preserve information at the boundaries of each window for velocity and void fraction measurements as well as for their validation.

3.3 Seeding particles

The quality of velocity measurements relies considerably on the characteristics of the tracers used during the experiments, which is all the more true in the case of two-phase flows given that particle detection is usually complex. The choice of seeding particles has been discussed in detail in a preliminary work [27]. Numerous particles have thus been investigated to determine the best tracers for the liquid phase: radio-opaque particles such as iron, copper and silver powders of various sizes (from 6 to 25 μm) have been tested in cavitating flow configurations. In addition to their high density (2, 3 and 10 g/cm³ respectively) compared to the liquid (1 g/cm³ for water), the image analysis has shown that some of these particles conglomerate and form large structures (over 40 μm).

Therefore, an other type of particles was tested. They consist of silver-coated hollow glass spheres with average diameters of 10 and 17 μm. These particles show several advantages over radio-opaque ones: i) their density is closed to the water’s (1.4 g/cm³), which minimises the effect of gravity as well as avoids particles from being trapped at the bottom of the horizontal tank; ii) they do no agglomerate iii) unlike metallic powders, they are spherical and their identification on the images is eased since their hollowness reduces the X-ray attenuation, whereas the silver coating improves the phase-contrast mechanism at the particle/liquid interface (figure 7).

4 Image processing

As discussed previously, the images acquired during this work contain simultaneous information about the flow structures and the dynamics of each phase. In order to exploit these information, an image processing-based algorithm was developed to separate the seeding particles and the vapour. In fact, from each X-ray radio-graph, three new images are created: i) an image of particles for measurements of the liquid phase velocities, ii) image of bubbles for vapour phase...
velocity measurements, and iii) image of vapour structures with neither interfaces nor particles used for vapour volume fraction measurements.

The first step of this algorithm consists on adjusting the non-uniformities of image intensities caused by the beam’s secondary pulses (figure 8-a). This preliminary step is essential for the good functioning of the phase separation method. Consequently, the average grey level of each line of pixels is replaced by the image mean intensity, so all lines have the same mean intensity value, the one of the image (see figure 8-b). Hence, for a given pixel, the intensity $I_1$ is replaced in the adjusted image by the intensity $I_2$ according to the following expression:

$$I_2 = I_1 \frac{\bar{I}_{im}}{\bar{I}_{pl}}$$

where $\bar{I}_{im}$ and $\bar{I}_{pl}$ represent respectively the average grey levels of the image and the line to which the pixel belongs. Although this process alters the global contrast of images, it was found very effective and necessary to correct the disparities in their intensities, without which the phase separation method would not be effective. Note that the middle band brightness and size differ from image to image without any clear pattern.

4.1 Image processing for liquid velocity measurements

The identification of tracers, in this processing, relies mostly on the detection of their edges. Enhancing the contrast of images would therefore make this recognition more efficient. This task was performed by eliminating the image background, i.e. the liquid phase. The method that was chosen for the liquid suppression was based on the vapour volume fraction measurements technique using X-ray absorption (see section 3.1). Indeed, for each pixel of the pre-adjusted image, the equation 6 was applied, while the calibration intensities of each phase were obtained from liquid and gas images which have been subjected to the same preliminary processing of grey level adjustments. Note that the obtained results, comprised between 0 (pure liquid) and 1 (pure vapour) were multiplied by $2^{16}$ to convert them to 16 bit images. Also they do not represent the local vapour volume fractions, given that the grey levels of initial images were altered. Nonetheless, this method presents the advantage of suppressing the liquid phase efficiently and therefore enhancing the contrast while both particles and bubbles remain as it is shown in figure 8-c.

Prior to the interface detection process, an intermediate step, which consists of removing large areas of vapour structures by means of a low frequency local filtering, is applied. The prime goal of this step is to lower the number of edges and consequently facilitate the detection and suppression processes. In fact, a 2D moving average filter is firstly applied on the vapour/particle image obtained from the previous processing, this results in a high frequency filtered image which is subsequently subtracted from the vapour/particles image. It has to be pointed out that the window size for the filtering should be larger than particles diameter to ensure that tracers would not be inappropriately filtered out. Figure 8-d shows the result of this processing. This step has also the advantage of increasing further the contrast between gaseous structures and particles, enabling a better distinction between them.

Interface identification is performed using Canny’s method of edge detection [28] which is based on image intensity gradients. To do so, the image is firstly de-noised thanks to a 2D Gaussian filter, then the intensity gradient of each pixel of the obtained image is determined. An intensity gradient map is hence defined, and from which only local maxima are conserved whereas all other (non-maxima) gradients are deleted. Finally, thresholds are applied to eliminate all the edges that have low potential to belong to an interface. Figure 8-e shows the result of this method on the detection of particles and the remaining vapour interfaces.

Criteria based on the shape and the size of the detected edges were applied to delete all interfaces which are not susceptible to belong to particles: i) the number of pixels belonging to one edge should not surpass the size of particles circumference; ii) the large diameter of an edge must not exceed the particles diameter (20 $\mu$m); iii) the minimum-to-maximum diameter ratio of an edge should be greater than 80% to ensure its roundness, given that particles are almost perfectly spherical. In addition to this last criterion, Hough transform method of shape recognition [29,30] was applied to select circular interfaces with the given particles diameter. The result of the edge selection processing is shown in figure 8-f.

All particles are finally restored from the remaining edges, using the intensities from the vapour/particles image figure 8-c. In order to ensure a better particle detection and reduce the vapour noise due to spurious edge selection and/or particle restoration, the processing steps showed in figures 8-e,f were reapplied on the resulted images with slightly more strict thresholds. The final result is presented in (figure 8-g).

4.2 Image processing for vapour velocity measurements

The second processing aims to suppress seeding particles from the X-ray images in order to perform measurements of the gaseous phase velocities. To accomplish this task, the particle detection process performed in the first stage of the phase separation algorithm is used to locate the seeding tracers on the initial images after intensity adjustments (figure 8-b). Each particle is subsequently deleted and replaced by the local intensities of its neighbouring. A local 2D filter was
finally applied to reduce the noise caused by the particle replacement process. The result of this method is shown in figure 8-h.

The particle detection and/or suppression processes developed in the present work are not flawless seeing the complexity of the two-phase flow structures within the recorded X-ray images. Indeed, during the image processing, structures of gas could be mistakenly taken for particles and hence remain on the tracers images, and vice versa. Furthermore, the qualities of both particles and vapour images could be affected have the liquid tracers been suppressed inadvertently, not to mention the local filtering performed to reduce the noise which could also have some effect on the results. These imperfections have inevitably, an influence on the accuracy of the calculations of both velocities, the reason why much efforts have been devoted to estimate the errors and validate through this means the method of image processing discussed in this paper (see section 6).

5 Vapour volume fraction

Several difficulties were encountered during the measurements of the local vapour volume fractions, due to the limitations of the imaging technique of the APS synchrotron and the complexity in the synchronisation of the recording devices (see section 3.2): i) the size of the bright band in the center of the recorded images, attributed to the secondary X-ray flashes, is not constant and changes with the images; ii) the global intensities of the images of the same pair differ from one image to another (see figures 9-a and 9-b).

Additionally, in order to ensure accurate measurements of the vapour volume fractions, the intensities of all original images should be conserved, so no grey level adjustments had to be performed on either flow or calibration images. Thus, for each pair of recorded images (figures 9-a and 9-b), the measurements were performed using an average image of the two X-ray radiographs as shown in figure 9-c. Note that particles were filtered out from the initial images by applying the same processing method presented in section 4.2. Thanks to image averaging, the disparities in the size and the brightness of the middle band as well as in the global intensities, due to the encountered impediments in synchronisation, were almost cancelled out (more details could be found in [31]). Furthermore, the calibration images for liquid and vapour were obtained by averaging a large number of non-
The local vapour volume fractions $\beta$ of cavitating flows were calculated using equation 6, derived from Lambert-Beer’s law, where $I_{\text{vac}}$, $I_{\text{vap}}$ and $I_{\text{liq}}$ represent the local intensities obtained respectively from images c, d and e shown in figure 9. The result presented in figure 9-f shows that the effect of the spurious band has been neutralized thanks to the image averaging. However, low values of $\beta$ are found as a consequence of X-rays diffraction at the vapour/liquid interfaces. To correct these aberrant values, each interface was detected using Canny’s method of edge detection (see section 4.1) and was subsequently replaced by the $\beta$ mean values of its surrounding. Figure 9-f shows the final result of the vapour volume fraction measurements.

To estimate the accuracy of the results, the above method was applied on three different types of images: i) X-ray images of a non-cavitating flow with seeding particles; ii) X-ray images of air; iii) X-ray images artificially adjusted to simulate a void fraction equal to 0.5. For each case, image pairs were averaged and the vapour volume fractions were measured using the same calibration images. In addition, particle suppression technique was applied to ensure the filtering of tracers. The deviations from the expected values (0 for non-cavitating flow, 1 for vapour and 0.5 for artificial images) varied between 0 and $\pm$4.5%. These errors correspond mostly to the remaining disparities caused by the synchronisation problems encountered during the experiments at the APS. Note that the errors have exceeded $\pm$15% when measurements of $\beta$ were performed using initial images without pair averaging. Furthermore, the equation 7, used to estimate the uncertainties of $\beta$ includes additional errors of approximately $\pm$2%. The maximum total errors in the measurements of the vapour volume fractions are hence estimated to $\pm$6%.

6 Velocity measurements

6.1 Image cross-correlations

Unlike in standard 2D2C-PIV (two-dimensional two-component particle image velocimetry), where only a thin section of the flow is illuminated by a laser sheet allowing 2D velocity measurements within a plane, X-ray imaging provides radiographs of the whole width of the flow crossed by the beam. Therefore, it’s the space-averaged behaviour and/or structure over the flow width that are available. Nonetheless, thanks to the 2D geometry and the small thickness (4 mm) of the Venturi profile, the 3D effect of cavitation inside the test section were highly minimized and hence the flow could be considered two-dimensional.

The liquid and the vapour velocities were obtained by means of image cross-correlations using MatPIV 1.6.1 open source toolbox for Matlab® software, originally developed by Sveen [32] from the university of Oslo (Sweeden) and modified within the LML laboratory by Cuvier et al. [33]. Four successive passes with rectangular (rather than square) interrogation windows of different sizes were performed for each velocity field. The rectangular shape, with the large side in the flow direction, has the advantage of reducing the gradient effect at the shear areas between the cavity sheet and the flowing liquid above it. In the final pass, a one-dimensional Gaussian fit based on three points was applied to ensure a sub-pixel accuracy of displacements. The window deformation technique [34] (pp. 148-154) was also employed to improve the measurements accuracy in regions with high velocity gradients. Figure 10 shows some examples of velocity fields of liquid and vapour phases. It has to be noted that the measurements of vapour velocities were performed only in regions with local vapour volume fractions higher than 7%. This was possible thanks to a mask used to remove non-physical measurements, based here on $\beta$ values in each interrogation window.

The configuration of velocity measurements of the vapour is undoubtedly different than the one of standard PIV, given that no seeding particles are used to trace the gaseous phase. In the present configuration, the cross-correlations within vapour are based on the variation of grey levels between regions of high intensities (bubbles) and others with much lower intensities such as the liquid/vapour interfaces. Figure 10-c shows an example of a typical correlogram obtained...
from cross-correlations within the vapour phase. We can observe that it is characterised by the same Gaussian shape as the ones usually found in standard PIV, although its peak is slightly larger. The analysis of vapour correlograms shows that the correlation peaks have a width of approximately 12 pixels which signifies that the intensity shades within each interrogation window behaves as a distinct small tracer, with a diameter of 12 pixels, rather than the whole window acting as one large tracer. In order to estimate the accuracy on the velocity measurements of both liquid and vapour, several validation methods have been developed.

6.2 Validation

In particle image velocimetry, the total uncertainties $\epsilon_{\text{tot}}$ are generally divided into two components: bias errors $\epsilon_{\text{bias}}$ based on the standard deviation from the exact solution and random errors $\epsilon_{\text{rms}}$ obtained from the root mean square (RMS) of the deviation:

$$
\epsilon_{\text{tot}} = \epsilon_{\text{bias}} + \epsilon_{\text{rms}}
$$

The accuracy of measurements is influenced by numerous parameters such as particles size, interrogation window shifting, velocity gradients... [35]. Though, in the present work, the images on which the cross-correlations are performed are obtained after a complex image processing which generates additional effects on the accuracy of the results and add supplementary errors to the measurements. In consequence, new validation methods of the velocity measurements are proposed for each phase.

6.2.1 Cross-correlation accuracy within liquid

The uncertainty estimation method for the liquid phase relies on generating synthetic images of a cavitating flow with given particle displacements. The image processing method for particle detection is then applied, and finally the displacements of the liquid tracers are calculated. The error estimation is obtained by comparing the results of particle image cross-correlations and the imposed initial displacements.

Figure 11 shows an example of the process used to generate the synthetic images: from an initial X-ray image of particles in a non-cavitating flow (figure 11-a), a second image was created by shifting each tracer with a known displacement. In order to obtain a good estimation of errors, real velocity fields such as the one presented in figure 10-a were used to shift the particles. These displacements were performed with a sub-pixel precision thanks to a b-spline interpolation. Note that real pairs of images of a non-cavitating flow could be used instead of artificially displaced images, however, the use of imposed displacements has the advantage of including the influence of parameters such as velocity gradients and particles size, besides the effects of image processing. After suppressing the background (i.e. liquid) from non-cavitating flow images (both real and displaced), particles were added to X-ray images of a cavitating flow without seeding tracers as shown in figure 11-b. Therefore, synthetic image pairs of cavitating flow including particles with known displacements were created (figure 11-c).

The comparison between imposed and measured values throughout the sheet cavity resulted in a total deviation from the imposed values $\epsilon_{\text{tot}}$ of $\pm0.85$ px which could be broken down as $\pm0.35$ px of bias error $\epsilon_{\text{bias}}$ and $\pm0.5$ px of random error $\epsilon_{\text{rms}}$. This corresponds in terms of velocity to an accuracy in measurements of around $\pm0.46$ m/s, i.e. between 3 and 4% of the maximum velocities of the studied flows. Furthermore, in regions of pure liquid and/or with low void fractions such as the wake region, the total estimated errors of the displacements were only $\pm0.2$ px, i.e. $\pm0.11$ m/s of errors on velocity measurements. These errors are mostly due to the influence of particles diameter (approximately 8-9 pixels) larger than the optimal size of 2 to 3 pixels as suggested by Raffel et al. [34].

6.2.2 Cross-correlation accuracy within vapour

Two validation methods are proposed to estimate the uncertainties in velocity measurements within vapour. The first one is chiefly similar to the one presented for the liquid phase in the above section §6.2.1. Displacements were imposed on vaporeous structures of a cavitating flow whereas seeding particles were artificially added to vapour images (both original and displaced). The process of particle detection and filtering were therefore applied, and the displacements of vapour...
were then calculated by means of image cross-correlations on the gaseous structures.

Figure 12-a shows a comparison between imposed and the measured displacements in two configurations: i) cavitating flow without particles (figure 11-b); ii) synthetic images subjected to the above processing (figure 11-c).

In the first case, a total deviation of approximately ±0.2 px was obtained. This uncertainty represents mostly the errors ensued from cross-correlations on vapour, and is very similar to the one observed in the case of particles velocities in non-cavitating conditions. This confirms that the different shades of vapour structures behave as separate tracers with relatively small size, as it was previously noticed on the correlation presented in figure 10-c. In the second configuration, using processed images of a cavitating flow with filtered particles, ±0.45 px (±0.24 m/s) of errors have been obtained. This demonstrates that the process of particle filtering has a slight impact on the results accuracy (less than ±0.3 px, i.e. ±0.16 m/s).

The second method of validation was implemented as follow: i) image cross-correlations were firstly applied on image pairs of a cavitating flow after processing; ii) the first image (figure 12-b) was then shifted by half of the obtained displacement (figure 12-d) whereas the second one (figure 12-c) was displaced by the same amount but in the opposite direction (-1/2 of obtained displacements). A new pair of images were hence obtained (figures 12-e and f) iii) cross-correlations are finally reapplied in order to estimate the deviation between both images of the shifted pair (figure 12-g). Theoretically, the displaced images should be identical and any non-zero value of displacements between them is considered as a measurements error. Note that such a method takes into account the errors due to vapour expansion or reduction due to bubbles growth or shrinking between images of the same pair, although this effect is highly reduced given the very short time between two successive images (3.68 μs). Nevertheless, the maximum estimated errors observed using this second method were ±0.4 px, which are practically the same to those obtained from the first method.

7 Results

As it was presented in previous sections, phase-contrast X-ray imaging allows, after some processing, to have a simultaneous access to instantaneous velocities of liquid and gas of two-phase flows as well as to their structures. To illustrate the point, the results obtained for steady and unsteady configurations of cavitating flows are presented in this section.

7.1 Steady flows

Velocity fields of each phase were calculated for several steady flow configurations. Figure 13-a presents examples of time-averaged velocity fields of liquid in two separate positions: i) the beginning of the cavity (left figure), ii) the center of the flow (right figure). As it could be clearly observed, it exists a narrow zone above the cavity within which the flow have high speed compared to all other regions inside the flow. This area of overspeed is located close to the throat region at the boundary between the liquid and the two-phase mixture. This characteristic of the flow, which (at what we are aware of) hasn’t been mentioned in the literature, was enabled thanks to the very high resolution of the images and to the use of rectangular interrogation windows for cross-correlations (see section 6.1). The acceleration of the liquid is attributed to the circumvention of the arriving flow to the two-phase mixture: the cavitation pocket behaves as an obstacle leading to a local acceleration of the liquid in order to conserve the flow rate. This flow behaviour could also be observed in the time-averaged velocity profile within position 1 presented in figure 13-c. In this figure, liquid and vapour mean velocity profiles in the flow direction at several distances from the Venturi throat are presented. As one can observe, there are significant differences in velocities between the phases throughout the cavity. These deviations in velocities depend on the position within both ‘x’ and ‘y’ directions as shown in figure 13-c. This figure shows profiles of the mean slip velocities according to the ‘x’ direction, i.e. the flow main direction, $\overline{U}_l$, with $U_j = U_l - U_v$; $U_l$ and $U_v$ are respectively the liquid and the vapour velocities within the ‘x’ direction.

At the beginning of the cavity, the new formed vapour fails to keep pace with the high speed flowing liquid, resulting in a non-negligible slippage between these two phases.
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Fig. 12 Validation processes of vapour velocities - First method: (a) comparison between imposed and measured displacements - Second method: (b) and (c) respectively first and second image of a cavitating flow (d) displacement field obtained by cross-correlations between images, (e) shifted image of (b) with 1/2 of the obtained displacement, (f) shifted image of (c) with -1/2 the displacement, (g) displacement profile between shifted images (e) and (f) representing measurements errors

(positions 1 and 2). At the center of the cavity, the slip velocity becomes less important due to the widening of the Venturi profile causing the reduction of the flow speed.

To gain more insight into the steady flow behaviour, time-averaged dimensionless slip velocities \( \overline{U}_{ds} \), with \( \overline{U}_{ds} = (\overline{U}_l - \overline{U}_g) / \overline{U}_l \), are presented in figures 14 for several positions throughout the attached cavity as well as the closure and wake regions. As it could be observed in figure 14-a, the profiles are divided into two segments, the upper one corresponds to the liquid moving towards in Venturi downstream whereas the lower part represents the dimensionless slippage of the re-entrant jets. Note that for a better representation, \( \overline{U}_{ds} \) values corresponding to liquid velocities \( \overline{U}_l \) between -0.5 and 1 m/s are not represented: \( \overline{U}_l \| \ll 1 \Rightarrow \| \overline{U}_{ds} \| \to \pm \infty \), hence the discontinuity of the profiles. Inside the sheet cavity, one can notice the resemblance of all dimensionless slip velocity profiles. In fact, these profiles would practically superimpose, were they translated with respect to each other according to the cavity height (i.e. ‘y’ direction). This shows that the vapour drive by the liquid varies very little throughout the length of the attached cavity. This could be easily observed in the shear area between the two-phase mixture and the liquid where the dimensionless slip velocities are minimum (6-10%). As one gets closer to the center of the cavitation pocket, the slippage becomes much more important (≥ 20%). At the cavity downstream (figure 14-b), a progressive straightening up of the profiles is observed, which could be attributed to the disappearance of the re-entrant jet and the shear zone.

Beside the presented case, other configurations of steady cavitation at different flow rates (8, 10, 14 and 16 l/min) and the same cavitation number (a cavity length equal to 10 mm), were tested. Figure 15 shows a comparison of time-averaged dimensionless slip velocities inside the attached cavity (position 3) and in the shedding zone (position 7). It is clear that, for the same cavitation number, slippage between phases is similar within the flows two-phase mixtures, and this regardless of the flow rate. This finding highlights thus the presence of a similitude law governing the slippage between the phases of cavitating flows. Further analysis has still to be undertaken to quantify this behaviour, nonetheless from the obtained velocity profiles, one can conclude the following behaviours: i) entrained by the fast flowing liquid, the vapour cloud lags behind, leading to the observed slippage between the two phases throughout the cavity. ii) at the closure region (positions 5-6), liquid and gas velocities have opposite signs, while the liquid is moving downstream, vapour is moving back underneath the cavity towards the flow upstream, hence the mean re-entrant jet in this area is mainly formed of vapour. This could be explained if we consider that the re-entrant jet is firstly initiated by the liquid which starts its ascent towards the Venturi throat, entraining the vapour at the end of the cavity, which begins re-entering with a time lag due to their inertia. iii) The X-ray measurements of steady cavitation show also that the slippage between liquid and vapour depends on the position according to the ‘y’ direction, the structure of the vapour, ranging from a cloud at the cavity/liquid interface to distinct bubbles at the center of the cavity, may play a role into this behaviour. Analysis of the vapour phase structure based on the size of vapour bubbles and vapour volume fractions is currently performed to get more information about the dynamic/structure interaction.
7.2 Unsteady flows

Phase-locked averaging was performed for an unsteady configuration in order to investigate the mean evolution behaviour of the flow. To select the different cycles constituting the unsteady flow, the evolution time-signal of the mean local vapour volume fraction within a small area inside the cavity was cross-correlated with a ‘reference’ signal $S_r$ (figures 16-a and b). $S_r$ has a period of $T_{cav} = 1/f_{cav}$, with $f_{cav}$ the flow oscillation frequency. To improve the quality of the phase-locked averaging, only the most representative cavitating cycles were selected thanks to a threshold applied on the results of signals cross-correlation. Finally, the instantaneous velocity fields and the vapour volume fractions corresponding to appropriate image pairs for each cycle step were averaged as shown in figure 16-c. It’s worth noting that the number of steps $n_{stp}$, into which each cycle is divided and averaged, is proportional to the acquisition and the cavity oscillation fre-
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Fig. 15 Time-averaged dimensionless slip velocity profiles $U_{ds}$ with several flow rates ($Q = 8, 10, 14, 16$ l/min) and a cavity length $L_{cav} = 10$ mm within (a) the attached cavity - position 3 (b) wake region - position 7

Fig. 16 Example of phase-locked averaging of unsteady cavitating flow within a Venturi test section: (a) X-ray images of position 1 within the flow, □: area for the vapour volume fractions averaging; (b) phase-locked averaging of vapour volume fraction fields

Fig. 17 Results for an unsteady cavitating flow - $Q = 35.09$ l/min, $\sigma = 1.97$: (a) evolution of the phase-averaged vapour volume fraction fields within 5 stages, ; (b) dimensionless slip velocity profiles within sections I to V

Fig. 18 Instantaneous and phase-locked average cycle of the full flow could be reconstructed.

Fig. 19 Results for an unsteady cavitating flow - $Q = 35.09$ l/min, $\sigma = 1.97$: (a) evolution of the phase-averaged vapour volume fraction fields within 5 stages, ; (b) dimensionless slip velocity profiles within sections I to V

Fig. 20 Instantaneous and phase-locked average cycle of the full flow could be reconstructed.

The instantaneous and the phased-locked averaged velocity profiles presented in figure 18 show that the velocities...
Fig. 18 Profiles of phase-locked averaged velocities (a) and instantaneous velocities (b)

at the re-entrant jet region are very small compared to the those obtained in the steady configurations, partially because the cavitation number is very different in both cases, but also the unsteadiness nature and the structure (at bubble level) of the flow may have a major role in the difference in behaviour. The image observations show that the bubbles shape and size vary significantly with the type of cavitation. Investigations for more flow configurations still need to be performed to confirm and understand the mechanisms behind the different flow dynamics.

8 Conclusion

The need for a better understanding of cavitation has motivated several studies since mid of the previous century [36]. To investigate the dynamic and the structure of cavitating flows, numerous experimental works were carried out using many techniques such as fast imaging, PIV, optical probing or even X-ray densitometry, which allow to comprehend quite a few mechanisms involved into the behaviour of the cavitation phenomenon. They also played a major role in the development of numerical models, since their validation need to be confronted with experimental measurements of velocities and void fractions. Yet, these techniques have shown certain limitations and were not capable of providing simultaneous and instantaneous measurements of the liquid and the vapour velocities, in addition to the local vapour volume fractions, not to mention the restrictions in measurements caused by the complexity of the cavitating flows. Thus, this work makes both a technical and a scientific contribution to the study of complex two-phase flows such as cavitation. Indeed, based on fast X-ray imaging technique provided by the APS synchrotron beam (high energy and spatial coherence), measurements were performed on cavitating flows within a millimetric 2D test section with a Venturi profile. The absorption and the phase-contrast enhancement mechanisms provided unique images of different flow configurations, providing access to their complex two-phase structures. Furthermore, thanks to image cross-correlations on both flow structures and seeding particles, the instantaneous velocities of the gaseous phase and the liquid were respectively obtained. Notwithstanding, to achieve these measurements, image processing methods had to be elaborated to permit the separation of tracers. As complex as it was, this separation process has been validated by means of several techniques developed for each phase, demonstrating a satisfying sub-pixel accuracy on velocity measurements.

To illustrate the utility of such a technique, two configurations of cavitating flows were tested: (i) Steady cavitation: the measurements of the time-averaged velocities highlights the existence of a noteworthy slippage between the liquid and the vapour phases inside sheet cavities. The analysis of the dimensionless slip velocities showed that, regardless of the horizontal position within the attached cavity, the phases behave similarly with respect to each other. Furthermore, for the same cavitation number, the dimensionless slippage was found to be constant within a given position inside the two-phase mixture. Moreover, thanks to the high spatial resolution of the X-ray images, some steady flows behaviour such as the local acceleration of the liquid above the vapour formation region were observed, velocities approximately two times higher were thus measured in this areas. Also, for all steady flows, the findings show that the re-entrant jets, constituted from both liquid and vapour, ascend farther back underneath the cavity until reaching the flow upstream where the vapour is formed.

(ii) Unsteady cavitation: using phase-locked averaging based on the vapour volume fractions, the evolution mean cycle of an unsteady cavity sheet was constructed. The analysis of the phase-locked averaged velocities showed, as for the steady configurations, the existence of important slippage between the two phases, while the dimensionless slippage varies very little with the cavity progression and verti-
13. Some differences were noted in comparison to the steady configurations. Investigations of more cavitating flows in steady and unsteady configurations are being performed to complete the above analysis and allow a finer description of their behaviour, whereas the local volume fraction measurements are supplemented with bubble size estimations.

Moreover, the time-resolved measurements of velocity and vapour volume fractions are currently used to improve the numerical simulation of cavitation using two-equation turbulence models \((k\text{-}\varepsilon, k\text{-}\omega\ldots)\). In fact, the velocity fluctuations and the density of the mixture obtained from the vapour volume fraction measurements are used to estimate the turbulence parameters such as the turbulent shear stress tensor \(\tau\) and the kinetic turbulent energy \(k\), which will provide more information about the interaction between these variables and hence ameliorate the turbulence modelling of such flows. The findings could provide an interpretation to the modification suggested by [37], which although have improved the simulation of cavitating flows behaviour, it remains arbitrary since the density \(\rho\) of the mixture is replaced by a function \(f(\rho)\) in the expression of the turbulent viscosity \(\mu_t\), reducing artificially the viscosity within the two-phase mixture, especially in high liquid fraction regions. The ongoing analysis has led to encouraging primary results which could be used to.

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