FINAL STATE INTERACTIONS IN $B \to \pi\pi K$ and $B \to K\overline{K}K$ DECAYS

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Analysis of charged and neutral $B$ meson decays into $\pi^+\pi^-K$, $K^+K^-K$, and $K^0\overline{K}^0\overline{K}^0$ is performed using a unitary representation of the $\pi\pi$ and $K\overline{K}$ final state interactions. Comparison of the theoretical model with the experimental data of the Belle and BaBar Collaborations indicates that charming penguin contributions are necessary to describe the $B \to f_0(980)K$ and $B \to \rho(770)^0K$ decays.

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1. Introduction

We report on some studies of three-body $B$ meson decays into $\pi\pi K$ and $K\overline{K}K$ final states. In these reactions one can find an evidence of the direct $CP$ violation similar to that recently discovered in two-body $B^0$ decays into $\pi^+K^-$. Loop-type weak decay diagrams known as penguin terms play an important role in these decays. In particular we study contributions of the charming penguin amplitudes responsible for long-distance effects present in the final state interactions. Strong interactions between pairs of pions and kaons in the final scalar-isoscalar state have been described in Ref. [1]. Here we extend this approach to the $\pi\pi$ interactions in $P$-wave. This enables us to obtain a unitarized description of the final state $\pi\pi$ interactions from threshold up to about 1.2 GeV. The opening of the $K\overline{K}$ threshold near 1 GeV is included in a natural way since both $\pi\pi$ and $K\overline{K}$ $S$-wave channels are coupled in our model.

2. Decay amplitudes

We take into account two components in the weak transition amplitudes governed by $b$-quark decays into $s\bar{u}u$, $s\bar{d}d$, and $s\bar{s}s$, where $u$, $d$ and $s$ denote up, down and
strange quarks, respectively. The first term consists of the amplitude derived in the factorization approximation with some QCD corrections and the second one is a long-distance amplitude with $c$-quark or with $u$-quark in loop. At the hadronic level the second amplitude with the $c$-quark in loop can be interpreted in terms of the intermediate $D_s^{(*)}D^{(*)}$ states which are frequently produced in $B$-meson decays (see Fig. 1).

\[ D_s^{(*)}D^{(*)}\]

Fig. 1. Example of $D_s^{(*)}D^{(*)}$ contribution to $B^-$ decays into $\pi\pi K^-$ and $K\overline{K}K^-$

The charming penguin contribution to the $B^-\rightarrow\pi\pi K^-$ decay amplitude with two pions in $P$-wave is parametrized by the following expression:

\[ \langle (\pi^+\pi^-)_{P}K^- | H | B^- \rangle_{\text{penguin}} = 2 G_F m_\rho C_\rho \Gamma_{\rho\pi\pi}(m_{\pi\pi}) p_\pi p_K \cos \theta, \]

where $G_F$ is the Fermi constant, $m_\rho$ is the $\rho(770)$ mass, $\Gamma_{\rho\pi\pi}(m_{\pi\pi})$ is the $\rho\pi\pi$ vertex function of the $\pi\pi$ effective mass $m_{\pi\pi}$, $p_\pi$ and $p_K$ are the pion and kaon momenta in the $\rho$ rest frame and $\theta$ is the angle between the direction of flight of the $\pi^-$ and the direction of the $\pi^+\pi^-$ system in the $B$ rest frame. The constant amplitude $C_\rho$ reads:

\[ C_\rho = f_K A_0^{B\rightarrow\rho}(m_K^2) \left( V_{ub} V_{us}^* P_u + V_{tb} V_{ts}^* P_t \right). \]

Here $V$’s are the Cabibbo–Kobayashi–Maskawa matrix elements, $f_K$ is the kaon decay constant, $A_0^{B\rightarrow\rho}(m_K^2)$ is the $B\rightarrow\rho$ transition form factor and $P_u$ and $P_t$ are penguin $P$-wave complex parameters to be fitted to the experimental data.

Final state interactions in the isospin zero $S$-wave are treated using the unitary model of Ref. 2 in which the $\pi\pi$ and $K\overline{K}$ channels are coupled. In this approach the sum of the several Breit-Wigner terms, usually used in the experimental analyses of the Dalitz plot distributions, is replaced by a set of unitary coupled meson-meson amplitudes. These amplitudes are expressed in terms of phase shifts $\delta_{\pi\pi}$, $\delta_{KK}$ and inelasticity $\eta$ known from other experiments 2. The $P$-wave pion-pion amplitude is well described by a Breit-Wigner term. No arbitrary phases nor relative intensity free parameters for the different resonances are needed. All the resonances appear in a natural way as poles of the meson-meson amplitudes. The scalar resonances $f_0(600)$ and $f_0(980)$ are examples of such poles of a single amplitude. The presence of the $f_0(980)$ is a dominant feature of the experimental $m_{\pi\pi}$ distribution.
3. Comparison with the experimental data

We perform a fit to the data of Belle\cite{3,4} and BaBar\cite{5} Collaborations. Both groups have measured differential $\pi\pi$ effective mass distributions, branching fractions, direct $CP$ violating asymmetries in charged $B$ decays and time-dependent $CP$ violating asymmetry parameters in $B^0$ decays. Moreover, the Belle Collaboration has given helicity angle distributions in the $\rho(770)$ and $f_0(980)$ regions. We obtain a good agreement with the data using only the four complex penguin parameters:

$$S_u = (0.15 \pm 0.10) \exp[i(1.90 \pm 0.71)], \quad S_t = (0.020 \pm 0.002) \exp[-i(0.26 \pm 0.21)]$$

for the $S$-wave and

$$P_u = (1.09 \pm 0.21) \exp[-i(0.98 \pm 0.12)], \quad P_t = (0.065 \pm 0.002) \exp[-i(1.56 \pm 0.08)]$$

for the $P$-wave.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{The $\pi^+\pi^-$ effective-mass distributions in $B^\pm \to \pi^+\pi^- K^\pm$ decays. The data points are taken from Ref. 4 and the solid lines represent the results of our model.}
\end{figure}

In Fig. 2 we show a comparison of our model with the $\pi^+\pi^-$ effective mass distributions measured by Belle for the $B^+ \to \pi\pi K^+$ and $B^- \to \pi\pi K^-$ decays. The large direct $CP$ asymmetry of the order of 0.3, visible in the range of the $\rho(770)$ resonance, is well described by our model. The fit is performed in the $m_{\pi\pi}$ range between 0.60 GeV and 1.06 GeV where the two resonances $\rho(770)$ and $f_0(980)$ dominate the pion-pion mass spectrum. Existence of these resonances leads to interesting interference phenomena seen in Fig. 3 where helicity-angle distributions are plotted for the combined $B^\pm \to \pi\pi K^\pm$ decays. In the $\rho$ range the general behaviour of the data follows the $\cos^2 \theta$ function, characteristic of a polarized $\rho$ decay into $\pi^+\pi^-$. However, in the $f_0(980)$ range the angular dependence is not flat as one can expect for the decay of a $S$-wave resonance. It has an interference component proportional to $\cos \theta$ which originates from the presence of the $\rho$ resonance tail under the $f_0(980)$ peak.
We have also performed calculations of different observables for the $B$ decays into three kaons in which a $K\bar{K}$ pair is in relative $S$-wave. No extra free parameters have been used since all the parameters have been fixed in the fit to the $B \to \pi \pi K$ decays as described above. The averaged branching ratio for the $B^{\pm} \to K^{+}K^{-}K^{\pm}$ $S$-wave channels, integrated over the $K\bar{K}$ effective masses from the threshold up to 1.1 GeV, is equal to $1.7 \times 10^{-6}$ and the corresponding value for the $B^{0} \to K^{+}K^{-}K^{0}$ is $0.9 \times 10^{-6}$. The direct $CP$ asymmetry for the charged $B$ decays is 0.07. The time-dependent asymmetry parameters for the neutral $B$ decays are: $S = -0.80$ and $A = -0.13$.

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