Two-point gauge invariant quark Green’s functions
with polygonal phase factor lines

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Abstract
Polygonal lines are used for the paths of the gluon field phase factors entering in the definition of gauge invariant quark Green’s functions. This allows classification of the Green’s functions according to the number of segments the polygonal lines contain. Functional relations are established between Green’s functions with polygonal lines with different numbers of segments. An integrodifferential equation is obtained for the quark two-point Green’s function with a path along a single straight line segment where the kernels are represented by a series of Wilson loop averages along polygonal contours. The equation is exactly and analytically solved in the case of two-dimensional QCD in the large-$N_c$ limit. The solution displays generation of an infinite number of dynamical quark masses accompanied with branch point singularities that are stronger than simple poles. An approximation scheme, based on the counting of functional derivatives of Wilson loops, is proposed for the resolution of the equation in four dimensions.

Keywords: QCD, quark, gluon, Wilson loop, gauge invariant Green’s function

1. Introduction
Path-dependent phase factors are the natural ingredients in gauge theories for the description of parallel transport of gauge covariant quantities in integrated form [1, 2]. They also allow for the construction of gauge invariant Green’s functions, which are the adequate tools for the study of the physical properties of the theory. The Wilson loop [3], which corresponds to a phase factor along a closed contour, is used to set up a criterion for the recognition of the confinement of quarks in QCD [3, 4, 5]. Properties of Wilson loops were thoroughly studied in the past [6, 7, 8, 9, 10, 11] and applications to bound states of quarks were considered [12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

On the other hand, approaches using gauge invariant correlators meet difficulties due to the extended nature of the phase factors and could not up to now provide a complete systematic procedure of solving the theory. In spite of these difficulties, the advantages one might expect from a gauge invariant approach merit continuation of the efforts that are undertaken. Gauge invariant quantities are expected to have an infrared safe behavior, free of artificial or unphysical singularities. For this reason, they are better suited to explore the nonperturbative regime of the theory; in QCD, this mainly concerns the occurrence of confinement. Also, the knowledge of gauge invariant wave functions of bound states facilitates calculations of matrix elements of operators involving phase factors.

The present talk is a summary of recent work of the author [22, 23] trying to deduce exact integrodifferential equations for two-point quark gauge invariant Green’s functions (2PQIGF), in analogy with the Dyson-Schwinger equations of ordinary Green’s functions [24, 25, 26, 27, 28]. The method of approach is based on the use of polygonal lines for the paths of the phase factors. Polygonal lines are of particular interest since they can be decomposed as a succession of straight line segments with mutual junction points. Straight line segments are Lorentz invariant in form and have an unambiguous limit when the two end points approach each other. Polygonal lines can be classified according to the number of segments or sides they contain, which in turn is reflected on the 2PQIGFs.

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2. Green’s functions with polygonal lines

Let \( U(y, x) \) be a path-ordered phase factor along an oriented straight line segment going from \( x \) to \( y \). A displacement of one end point of the rigid segment, while the other end point is fixed, generates also a displacement of the interior points of the segment. This defines a rigid path displacement. Parametrizing the interior points of the segment with a linear parameter \( \lambda \) varying between 0 and 1, such that \( z(\lambda) = \lambda y + (1 - \lambda)x \), the rigid path derivative operations with respect to \( y \) or \( x \) yield \([1, 2, 29, 30]\)

\[
\frac{\partial U(y, x)}{\partial y^\alpha} = -igA_\alpha(y)U(y, x) + ig(y - x)^\alpha \\
\times \int_0^1 d\lambda AU(y, z(\lambda))F_{\beta\alpha}(z(\lambda))U(z(\lambda), x),
\]

(1)

\[
\frac{\partial U(y, x)}{\partial x^\alpha} = +igU(y, x)A_\alpha(x) + ig(y - x)^\alpha \\
\times \int_0^1 d\lambda (1 - \lambda)AU(y, z(\lambda))F_{\beta\alpha}(z(\lambda))U(z(\lambda), x),
\]

(2)

where \( A \) is the gluon potential, \( F \) its field strength and \( g \) the coupling constant.

In gauge invariant quantities, the end point contributions of the segments are usually cancelled by other neighboring point contributions and one remains only with the interior point contributions of the segments, represented by the integrals above. We introduce for them a shorthand notation:

\[
\frac{\partial U(y, x)}{\partial y^+} = ig(y - x)^\alpha \int_0^1 d\lambda AU(y, z(\lambda)) \\
\times F_{\beta\alpha}(z(\lambda))U(z(\lambda), x),
\]

(3)

\[
\frac{\partial U(y, x)}{\partial x^+} = ig(y - x)^\alpha \int_0^1 d\lambda (1 - \lambda)AU(y, z(\lambda)) \\
\times F_{\beta\alpha}(z(\lambda))U(z(\lambda), x).
\]

(4)

The superscript + or – on the derivative variable takes account of the orientation on the segment and specifies, in the case of joined segments, the segment on which the derivative acts.

The vacuum expectation value (or vacuum average) of a Wilson loop along a contour \( C \) will be designated by \( W(C) \). In the case of a polygonal contour \( C_n \), with \( n \) segments and \( n \) junction points \( x_1, x_2, \ldots, x_n \), it will be designated by \( W_n \) and represented as an exponential functional \([8, 10]\): \[ W_n = W(x_n, x_{n-1}, \ldots, x_1) = e^{F(x_n, x_{n-1}, \ldots, x_1)} = e^{F_n}. \] The 2PGIQGF with a phase factor along a polygonal line composed of \( n \) segments and \((n - 1)\) junction points is designated by \( S_{(n)} \):

\[
S_{(n)}(x, x'; t_0, \ldots, t_1) = -\frac{1}{N_c} \langle \bar{\psi}(x')U(x', t_{n-1}) \\
\times U(t_{n-1}, t_{n-2}) \ldots U(t_1, x)\psi(x) \rangle,
\]

(6)

the quark fields, with mass parameter \( m \), belonging to the fundamental representation of the color gauge group \( SU(N_c) \) and the vacuum expectation value being defined in the path integral formalism. (Spinor indices are omitted and the color indices are implicitly summed.) The simplest such function is \( S_{(1)} \), having a phase factor along a straight line segment:

\[
S_{(1)}(x, x') \equiv S(x, x') = -\frac{1}{N_c} \langle \bar{\psi}(x')U(x', x)\psi(x) \rangle.
\]

(7)

(We shall generally omit the index 1 from that function.)

For the internal parts of rigid path derivatives, we have definitions of the type

\[
\frac{\delta S_{(n)}(x, x'; t_0, \ldots, t_1)}{\delta \psi(x')} = \frac{1}{N_c} \langle \bar{\psi}(x')U(x', t_{n-1}) \\
\times U(t_{n-1}, t_{n-2}) \ldots U(t_1, x)\psi(x) \rangle.
\]

(8)

3. Integrodifferential equation

The above Green’s functions satisfy the following equations of motion concerning the quark field variables:

\[
(i\gamma^\mu \partial_\mu - m)S_{(n)}(x, x'; x_0, \ldots, x_{n-1}) = \imath \delta^\mu_\alpha(x - x') \\
\times e^{F(x, x_{n-1}, \ldots, x_0)} + i\gamma^\mu \frac{\delta S_{(n)}(x, x'; x_0, \ldots, x_{n-1})}{\delta \psi(x')},
\]

(9)

which become for \( n = 1 \)

\[
(i\gamma^\mu \partial_\mu - m) S(x, x') = \imath \delta^\mu_\alpha(x - x') + i\gamma^\mu \frac{\delta S(x, x')}{\delta \psi(x')}\]

(10)

Multiplying Eq. (9) with \( S_{(1)}(t_1, x) \) and integrating with respect to \( x \), one obtains functional relations between various 2PGIQGFs. A typical such relation is:

\[
S_{(n)}(x', x_0, \ldots, x_{n-1}) = S(x, x') e^{\delta F(x', x_0, \ldots, x_{n-1})} \\
+ \frac{\delta S(x, y_1)}{\delta y_1} + S(x, y_1) \frac{\delta}{\delta y_1} \times S_{(n+1)}(y_1, x'; x_{n-1}, \ldots, t_1, x).
\]

(11)

[Integrations on intermediate variables are implicit and generally are not written throughout this paper. Here, \( y_1 \) is an integration variable.]
Equation (11) expresses $S_{(n)}$ in terms of two quantities: the first one, which plays the role of a driving term, contains the simplest 2PGIQGF, with one straight line segment, together with a Wilson loop average along a polygonal contour with $(n + 1)$ segments; the second term, which appears as a corrective term, is represented by the contribution of a higher-index 2PGIQGF. However, since this equation is valid for any $n \geq 1$, one can use it again in its right-hand side for $S_{(n+1)}$. One therefore generates an iterative procedure that eliminates successively the higher-index 2PGIQGFs in terms of the lowest-index one, $S_{(1)}$. Assuming that the terms rejected to infinity are negligible, one ends up with a series where only $S_{(1)}$ appears together with Wilson loop averages along polygonal contours with an increasing number of sides and rigid path derivatives along the segments. This result shows that among the set of the 2PGIQGFs $S_{(n)}$, $n = 1, 2, \ldots$, it is only $S_{(1)}$, having a phase factor along one straight line segment, that is a genuine dynamical independent quantity. Higher-index 2PGIQGFs could in principle be eliminated in terms of $S_{(1)}$, together with polygonal Wilson loops and their rigid path derivatives.

The construction of $S$ proceeds from the resolution of the equation of motion (10). It is then necessary to evaluate the action of the rigid path derivative on $S$ as it appears in the right-hand side of the equation. This is done by using again the functional relations (11), where the driving term of the right-hand side gives the main contribution. Thus, the rigid path derivative acting along the segment $xt_1$ of $S_{(n)}$, acts in the right-hand side in the first place on the logarithm of the Wilson loop average $F_{n+1}$; it also acts on the remainder containing $S_{(n+1)}$. Using back Eq. (11), one obtains an equation for $\frac{\delta \delta S_{(n)}}{\delta x^{\mu \nu}}$ which expresses the latter as a product of $\delta S_{(n)}$ with $S_{(n)}$ plus a remainder containing the derivative of $S_{(n+1)}$. Continuing the procedure, one factorizes in front of every $S_{(n')}$ ($n' > n$) derivatives of Wilson loop averages.

Selecting in the above set of equations the case $n = 1$, the equation of motion (10) takes at the end the following form:

$$
(i\gamma \partial_{(3)} - m) S(x, x') = i\delta^3(x - x') + i\gamma^\mu \left(K_{1\mu}(x', x) \times S(x, x') + K_{2\mu}(x', x, y_1) S_{(2)}(y_1, x'; x) + \sum_{n=3}^{\infty} K_{n\mu}(x', x, y_1, \ldots, y_{n-1}) \times S_{(n)}(y_{n-1}, x', x; y_1, \ldots, y_{n-2})\right),
$$

where the kernels $K_n$ ($n = 1, 2, \ldots$) contain Wilson loop averages along polygonal contours, at most $(n+1)$-sided, and $(n - 1)$ 2PGIQGFs $S$ and their derivative. The total number of derivatives contained in $K_n$ is $n$, each derivative acting on a different segment. Once the Wilson loop averages and the various derivatives have been evaluated and the high-index $S_{(n)}$s have been expressed in terms of $S$, Eq. (12) becomes an integrodifferential equation in $S$, which is the primary unknown quantity to be solved.

One observes in the right-hand side of the equation the appearance of the whole set of 2PGIQGFs. Gluon propagators are replaced here by Wilson loop averages along polygonal contours and rigid path derivatives acting on them. This ensures gauge invariance of every term of the expansion.

One major difference of the integrals present in the right-hand side of Eq. (12) with those of the Dyson–Schwinger equation is the property that they are not of the convolution type. This is due to the presence of the Wilson loops, whose contours pass by all points of the accompanying terms and do not allow for a convolutive factorization in $x$-space.

Equation (12) can also be analyzed, at least superficially, from the viewpoint of a perturbative expansion. According to Eqs. (3) and (4), each derivative operator results in an insertion of the gluon field strength, leading to the appearance of a valence gluon, accompanied multiplicatively by the coupling constant. In the short-distance regime, where perturbative QCD should be applicable, a naive counting of the number of derivatives would give us an indication about the size of the corresponding term, the leading terms corresponding to those having the least number of derivatives. Here, perturbation theory would be effected in the presence of the polygonal Wilson loops for each term. At large distances, it is expected that Wilson loop averages are saturated by minimal surfaces [8, 21]. Here also, increasing the number of derivatives would lead to less dominant terms. It thus seems reasonable to assume, as a starting hypothesis, that Eq. (12) represents, on practical grounds, a perturbative expansion. The first term of the series, corresponding to a single derivative term, is null for symmetry reasons. Therefore, the leading term of the series would be represented by the two-derivative term with a Wilson loop average along a triangular contour.

In that case, the interaction part of Eq. (10) reduces to the expression

$$
\frac{\delta S(x, x')}{\delta x^{\mu \nu}} \approx - \int d^4y_1 \frac{\delta^2 F_3(x', x, y_1)}{\delta x^{\mu \nu} \delta y_1^{\mu \nu}} \times e^{F_3(x', x, y_1)} S(x, y_1) y^{11} S(y_1, x'),
$$

(13)
which provides the driving term of the kernel of the equation.

4. Two-dimensional QCD

The equations obtained in the previous sections remain also valid in two dimensions and could be analyzed more easily in that case. Two-dimensional QCD in the large $N_c$ limit [31, 32] provides a simplified framework for the study of the confinement properties which are expected to prevail also in four dimensions. Wilson loop averages can be explicitly calculated [33, 34, 35]: for simple contours they are equal to the exponential of the areas enclosed by the contours. In that case, the second-order derivative of the logarithm of the Wilson loop average reduces to a two-dimensional delta-function. Higher-order derivatives give zero, since they act on different segments of the polygonal contour. The case of overlapping self-intersecting surfaces, which give more complicated expressions, should be analyzed separately. A detailed analysis suggests that the residual terms they produce are probably of zero weight under the integrations that are involved. We assume that hypothesis.

In the series of terms of Eq. (12) it is only the second-order derivative that survives [cf. Eq. (15)] and the integrodifferential equation takes the following (exact) expression [23]:

\[
(\gamma \partial_\tau - m)S(x) = i \delta^3(x) - \sigma \gamma^\mu (g_{\mu
u} g_{\nu\rho} - g_{\mu\rho} g_{\nu\sigma}) \chi^\nu \chi^\rho \\
\times \left[ \int_0^1 d\lambda \lambda^2 S((1-\lambda) x) \gamma^\rho S(\lambda x) \\
+ \int_1^\infty d\xi \xi^2 S((1-\xi) x) \gamma^\rho S(\xi x) \right].
\]

(14)

where $\sigma$ is the string tension.

The above equation can be analyzed by first passing to momentum space. Designating by $S(p)$ the Fourier transform of $S(x)$, one can decompose it into Lorentz invariant components:

\[
S(p) = \gamma p F_1(p^2) + F_0(p^2).
\]

(15)

The solution of Eq. (14) can be searched for by using the analyticity properties of the 2PGIQGF, assuming that the quark and gluon fields satisfy the usual spectral and causality properties of quantum field theory [22, 23, 36, 37, 38]. It turns out that the equation can be solved exactly and in analytic form. The functions $F_1$ and $F_0$ are found having an infinite number of branch points located on the positive real axis of $p^2$ (timelike region), starting at thresholds $M_1^2$, $M_2^2$, $\ldots$, $M_n^2$, $\ldots$, with fractional power singularities equal to $-3/2$. Their expressions are [23], for complex $p^2$,

\[
F_1(p^2) = -i \frac{\pi}{2\sigma} \sum_{n=1}^{\infty} \frac{1}{b_n (M_n^2 - p^2)^{3/2}},
\]

(16)

\[
F_0(p^2) = \frac{\pi}{2\sigma} \sum_{n=1}^{\infty} (-1)^n b_n \frac{M_n}{(M_n^2 - p^2)^{3/2}}.
\]

(17)

The Green’s function $S$ [Eq. (15)] then takes the form

\[
S(p) = -i \frac{\pi}{2\sigma} \sum_{n=1}^{\infty} b_n \frac{(\gamma p + (-1)^n+1 M_n)}{(M_n^2 - p^2)^{3/2}}.
\]

(18)

The masses $M_n (n = 1, 2, \ldots)$ are positive, greater than the free quark mass $m$ and ordered according to increasing values. For massless quarks they remain positive. The masses $M_n$ and the coefficients $b_n$, the latter being also positive, satisfy, for general $m$, an infinite set of algebraic equations that are solved numerically. Their asymptotic values, for large values of $n$ such that $n \gg m^2/(\pi \sigma)$, are

\[
M_n \approx \pi n \sigma, \quad b_n \approx \frac{\sigma^2}{M_n + (-1)^n m}.
\]

(19)

The functions $(M_n^2 - p^2)^{-3/2}$ are defined with cuts starting from their branch points and going to $+\infty$ on the real axis; they are real below their branch points on the real axis down to $-\infty$.

The expressions (19) and (17) are represented by weakly converging series. The high-energy behavior of the functions $F_1$ and $F_0$ is obtained with a detailed study of the asymptotic tails of the series and the use of the asymptotic behaviors of the parameters $M_n$ and $b_n$ [Eqs. (19)]. One finds that they behave as in free field theories, which is here a trivial manifestation of asymptotic freedom [39]:

\[
F_1(p^2) \sim \frac{i}{p^2 + i\epsilon},
\]

(20)

\[
F_0(p^2) \sim \frac{im}{p^2 + i\epsilon}, \quad \text{for } m \neq 0,
\]

(21)

\[
F_0(p^2) \sim -\frac{4i\sigma F_0(x = 0)}{(p^2)^2}, \quad \text{for } m = 0.
\]

(22)

In summary, the solution of Eq. (14) is nonperturbative and infrared finite. The masses $M_n$ are dynamically generated, since they do not exist in the Lagrangian of the theory. They could be interpreted as dynamical masses of quarks with, however, the following particular features. First, they are infinite in number. Second, they do not appear as poles in the Green’s function, but
rather with stronger singularities. In \(x\)-space the latter do not produce finite plane waves at large distances and therefore quarks could not be observed as free asymptotic states. Nevertheless, the above singularities being gauge invariant should have physical significance and would show up in the infrared regions of physical processes involving quarks. Finally, the fact that they appear only in the timelike region of real \(p^2\) is an indication that even in the nonperturbative regime the spectral and causality properties of quantum field theory are still satisfied.

Expression (13) of the Green’s function \(S\) can be interpreted as fitting a generalized form of the Källén-Lehmann representation \(\frac{24}{23}\), where the denominator of the dispersive integral has now a fractional power, while the spectral functions are saturated by an infinite series of dynamically generated single quark states with alternating parities. The latter still satisfy Lehmann’s positivity conditions \(\rightarrow 23\).

5. Conclusion

The use of polygonal lines for the paths of the phase factors allow a classification of the two-point quark Green’s functions and a systematic investigation of the properties of the latter through the functional relations they satisfy. An equation similar to the Dyson-Schwinger equation has been obtained for the quark Green’s function with a path made of one straight line segment, in which the kernels are represented by a series involving Wilson loop averages along closed polygonal contours with increasing complexity. Arguments have been developed justifying the treatment of the series perturbatively with respect to the number of functional derivatives acting on the Wilson loops. In that case the leading term of the kernels is provided by the Wilson loop with the simplest contour, corresponding to a triangle, with two functional derivatives.

The above equation has been solved exactly and analytically in the case of two-dimensional QCD in the large-\(N_c\) limit. The solution displays the presence of an infinite number of dynamically generated quark masses, accompanied with branch point singularities of degree \(-3/2\) (stronger than simple poles) located on the positive real axis of the momentum squared variable (timelike region). The qualitative feature that one deduces from these results is that in spite of the strong singularities that have emerged in the exact solution, quark and gluon fields continue satisfying the spectral and causality properties of quantum field theory.

The resolution of the integrogriddifferential equation in the two-dimensional case provides a positive signal for the continuation of the analysis in four dimensions following similar lines.

Acknowledgements

I thank Professor Daya S. Kulshreshtha and the Organizing Committee of the Light Cone Delhi 2012 Conference for the pleasant and stimulating atmosphere created during the Conference and for their warm hospitality. This work is partially supported by the EU I3HP Project ”Study of Strongly Interacting Matter” (acronym HadronPhysics3, Grant Agreement No. 283286).

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