\textbf{B^*B\pi(\gamma) couplings and D^* \rightarrow D\pi(\gamma)-decays within a 1/M-expansion in full QCD}

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\textbf{Abstract}

To leading order in $\alpha_s$, we evaluate the leading and non-leading $1/M_b$ corrections to the $B^*B\pi$ and $B^*B\gamma$ couplings using QCD spectral moment sum rules in the full theory. We find that, for large $M_b$ and contrary to the heavy-to-light $B \rightarrow \pi(\rho)l\bar{\nu}$ form factors, which are dominated by the soft light quark vacuum condensate, these couplings are governed by the hard perturbative graph, like other heavy-to-heavy transitions. We also find that for the $B^* \rightarrow B\gamma$, the $1/M_b$ correction is mainly due to the perturbative and light quark condensate contributions originating from the graphs involving the heavy quark part of the electromagnetic current, which are essential for explaining the large charge dependence in the observed $D^{*-} \rightarrow D^{-}\gamma$ and $D^{*0} \rightarrow D^{0}\gamma$ decays. Our best numerical predictions without any free parameters for the $B^*$-meson are: $g_{B^*-B^0\pi^-} \approx 14 \pm 4$, $\Gamma_{B^*-B\gamma^-} \approx (0.10 \pm 0.03)$ keV and the large charge dependence of the ratio: $\Gamma_{B^*-B\gamma^-}/\Gamma_{B^{*0}\rightarrow B^{0}\gamma} \approx 2.5$. For the $D^*$-meson, we find: $\Gamma_{D^{*-0}\rightarrow D^{0}\pi^-} \approx 1.54\Gamma_{D^{*0}\rightarrow D^{0}\pi^0} \approx (8 \pm 5)$ keV, $\Gamma_{D^{*-}\rightarrow D^{-}\gamma^-} \approx (0.09 \pm 0.07)$ keV and $\Gamma_{D^{*0}\rightarrow D^{0}\gamma} \approx (3.7 \pm 1.2)$ keV, where the branching ratios agree within the errors with the present data, while the total widths $\Gamma_{D^{*0}\rightarrow all} \approx (11 \pm 4)$ keV and $\Gamma_{D^{*-}\rightarrow all} \approx (12 \pm 7)$ keV are much smaller than the present experimental upper limits.
1 Introduction and notations

The $B^* B\pi$ and $D^* D\pi$ couplings have been studied by several authors using QCD spectral sum rules combined with the soft pion techniques \cite{4} (see also \cite{5}), light cone sum rules \cite{6} or heavy quark expansion plus soft pion techniques \cite{7}, while the $B^* B\rho$ coupling has been studied recently using QCD double exponential sum rules for the three-point function \cite{5}. However, though, apparently convenient, as one works with the two-point function, the sum rule approach of \cite{4} is quite peculiar due to the presence of the unphysical so-called parasitic term, which can only be eliminated in a more involved combination of sum rules. Moreover, the connection of the light cone sum rule used in \cite{3}, with the standard parametrization for studying its light-cone, see e.g. \cite{8}, though, one cannot use (like currently done in the literature) such a pole parametrization for studying its $M_0$-behaviour at $q^2 = 0$ \cite{4}. Experimental measurements of these couplings are expected to be improved and available in the forthcoming high-statistics $B$- and $\tau$-charm-factory machines from the processes $B \to \pi\tau\nu$, $D^* \to D\pi$ and $B^*(D^*) \to B(D)\gamma$.

In this paper, we shall use the QSSR double moments sum rule approach in order to study the large $M_0$-behaviour of the previous couplings and to estimate their values. Contrary to the popular double exponential sum rule, this approach is quite advantageous in the analysis of the three-point function, as it prevents the blow-up of the QCD series when the heavy quark mass is large but here the number of derivatives remains finite \cite{4}. The couplings are defined as:

$$
\langle B^*(p)B(p')\pi(q) \rangle = g_{B^* B\pi} q_\mu \epsilon^\mu, \quad \langle B^*(p)B(p')\gamma(q) \rangle = -eg_{B^* B\gamma} p_\alpha p'_\beta \epsilon^{\mu\alpha\beta} \epsilon_\mu^\nu, \quad (1)
$$

where $q \equiv p' - p$ and $-Q^2 \equiv q^2 \leq 0$, while $\epsilon_\mu$ are the polarization of the vector particles. We shall be concerned with the vertex function:

$$
V^{(\mu)}(p, p', q) = -\int d^4x \int d^4y \, e^{i(p'y - px)} \langle T J_L^5(\mu)(x) J_{B^*}^{\nu}(0) J_B(y) \rangle , \quad (2)
$$

where the currents are:

$$
J_L^\mu = \sum_{u,d} e_q \bar{q} \gamma^\mu q + \sum_{c,b} e_Q \bar{Q} \gamma^\mu Q \quad J_L^5 = (m_u + m_d)\bar{u}\gamma^5 d \\
J_{B^*}^\nu = \bar{u}\gamma^\nu b \quad J_B = (m_b + m_d)\bar{d}\gamma^5 b , \quad (3)
$$

and $u$, $d$, $c$, $b$ are the quark fields and $e_q$, $e_Q$ their electric charge in units of $e$. The vertex obeys the double dispersion relation \cite{4}:

$$
V(p, p', q) = -\frac{1}{4\pi^2} \int_{M_0^2}^{\infty} ds \int_{s-p^2}^{\infty} ds' \Im V(s, s') + ... \quad (4)
$$

\footnote{Here, the dispersion relation is done with respect to the two heavy meson momenta, which is not the case of the $B \to \pi(\rho)\bar{\nu}$. This different configuration is important for the $M_0$-behaviour of the different QCD contributions.}
Exploiting the fact that $M_b$ is much larger than the QCD scale $\Lambda$, where the LHS can be evaluated using the Operator Product Expansion à la SVZ \[3\,4\], one can work, in the chiral limit, with the double moment sum rule:

$$\mathcal{M}^{(n,n')} = -\frac{1}{4\pi^2} \int_{M_b^2}^{\infty} \frac{ds}{s^{n+1}} \int_{M_b^2}^{\infty} \frac{ds'}{s'^{n'+1}} \Im V(s,s')$$

(5)

where $n$, $n'$ are finite numbers of derivatives evaluated at $p^2 = p'^2 = 0$.

2 Moment sum rule for the $B^* B\pi$ and $D^* D\pi$ couplings

The perturbative QCD expression of the spectral function reads:

$$-\frac{1}{4\pi^2} \Im V(s,s') = (m_u + m_d)M_b \frac{N_c}{4\pi^2} Q^2 M_b^2 (s + s' + Q^2) - 2ss' \\{(s + s' + Q^2)^2 - 4ss'\}^{3/2},$$

(6)

where $Q^2 \equiv -q^2 \geq 0$ is the pion momentum squared and where the integration limit condition is:

$$(s - M_b^2)(s' - M_b^2) \geq Q^2 M_b^2.$$  

(7)

It is easy to check that, contrary to the case of $B \rightarrow \pi l\bar{\nu}$ form factor, where the light quark condensate is dominant, the light condensate contribution vanishes here after taking the $p^2$ and $p'^2$ derivatives, which is a consequence of the fact that the dispersion relation has been done with respect to the heavy quarks momenta like in the case of a heavy-to-heavy transition. The other remaining effects which are suppressed by $1/M_b^2$ compared to the leading perturbative diagram will be neglected to the approximation we are working \[1\]. The phenomenological side of the sum rule is parametrized using the usual duality ansatz: lowest resonance + QCD continuum from the thresholds $s_c$ and $s'_c$. By transferring this QCD continuum effect into the QCD part of the sum rule, one obtains \[3\]:

$$\mathcal{M}^{(n,n')}_{c} \equiv \frac{\sqrt{2} M_B^2 f_B^*}{M_B^{2(n+1)}} \frac{\sqrt{2} M_B^2 f_B}{M_B^{2(n'+1)}} \frac{\sqrt{2} m_{\pi}^2 f_{\pi}}{m_{\pi}^2 + Q^2} \sim -\frac{1}{4\pi^2} \int_0^{s_c} \frac{ds}{s^{n+1}} \int_0^{s'_c} \frac{ds'}{s'^{n'+1}} \Im V(s,s'),$$

(8)

where the coupling constants are normalized as:

$$\langle 0|J_{L}^\rho|\pi\rangle = \sqrt{2} f_{\pi} M^2_{\pi} \quad \langle 0|J_{\mu}^\rho|\rho\rangle = \sqrt{2} M^2_{\rho} e^\mu,$$

$$\langle 0|J_B|B\rangle = \sqrt{2} f_{B} M^2_{B} \quad \langle 0|J_{B}^\nu|B^*\rangle = \sqrt{2} f_{B^*} M_{B^*} e^\nu,$$

(9)

where $f_{\pi} = 93.3$ MeV and $\gamma_{\rho} = 2.56$. In the case where $M_b \rightarrow \infty$ (static limit), it is convenient to work with the non-relativistic variables $E$ and $E'$ defined as:

$$s = (E + M_b)^2 \quad \text{and} \quad s' = (E' + M_b)^2,$$

(10)

\[3\] In this region of $Q^2 \geq 0$, the question of non-Landau and complex singularities and of anomalous thresholds do not arise \[1\].

\[4\] One can however notice that the four-quark condensate contribution behaves like $1/Q^4$ which reflects the fact that the present approach cannot be used at $Q^2 = 0$ as expected.

\[5\] Here and in the following, we shall neglect the contribution of the $\pi'(1.3)$ similarly to previous analysis of the $\omega\rho\pi$- and $\pi NN$-couplings using vertex sum rules \[11\].
and to introduce the new variables:

\[ x = E - E' \quad \text{and} \quad y = \frac{1}{2}(E + E') \, . \]  

(11)

Due to the almost good symmetry between the \( B \) and the \( B^* \), we shall use:

\[ M_{B^*} \simeq M_B, \quad E_c \simeq E_c', \quad n = n' \equiv n_3 \, . \]  

(12)

By keeping the non-leading \( 1/M_b \)-terms in the expansion, we obtain to leading order in \( \alpha_s \):

\[
\mathcal{M}^{(n,n')}_{c} \simeq (m_u + m_d) \frac{M_b^3}{M_B^4(n_3+1)} \frac{N_c}{\pi^2} Q^2 \int_0^{E_c} dx \int_{\frac{y}{\sqrt{x^2 + Q^2}}}^{E_c - \frac{x}{2}} \frac{y \, dy}{(x^2 + Q^2)^{3/2}} \left[ 1 + \frac{1}{M_b} \left[ - \frac{1}{4y} (Q^2 + \frac{3}{2} x^2 + 2y^2) - 2(2n_3 + 1)y + \frac{Q^2}{8M_b^2} \delta(y - \frac{1}{2}\sqrt{x^2 + Q^2}) \right] \right] \, .
\]  

(13)

For consistency, we shall use in our analysis the lowest order expression in \( \alpha_s \) of the decay constants from the moment sum rules [12]:

\[
f_{B^*}^2 \simeq \frac{E_c^3}{2\pi^2 M_B^2} \left( \frac{M_B}{M_b} \right)^{2n_2-1} \left\{ 1 - \frac{3}{2} \frac{(n_2 + 1)}{E_c} \frac{E_c}{M_b} - \frac{\pi^2 \langle \bar{d}d \rangle}{2 E_c^3} \right\}
\]

\[
f_{B}^2 \simeq \frac{E_c^3}{2\pi^2 M_B^2} \left( \frac{M_B}{M_b} \right)^{2n_2+3} \left\{ 1 - \frac{3}{2} \frac{(n_2 + \frac{7}{3})}{E_c} \frac{E_c}{M_b} - \frac{\pi^2 \langle \bar{d}d \rangle}{2 E_c^3} \right\}
\]

(14)

consistent with the normalization of the currents in Eq.(3) and with the definitions in Eq.(9).

One should notice that the overall \( (M_B/M_b) \) factor also brings a \( 1/M_b \)-correction which tends to reduce the apparently huge correction in the curly brackets and leads after the moment sum rules analysis of \( f_B \) to the well-known \( 1 \) GeV/\( M_b \)-correction to this quantity. One can also notice that the \( 1/M_b \)-correction to \( f_{B^*} \) is slightly smaller than the one of \( f_B \) as generally expected. Using the previous formulae in Eq. (14), the emerging effective values of \( E_c \) fixed from the numerical analysis of \( f_B \) and \( f_{B^*} \) including the \( \alpha_s \) corrections are [12, 13]:

\[ E_c^\infty \simeq (1.6 \pm 0.1) \text{ GeV} \quad E_c^B \simeq (1.3 \pm 0.1) \text{ GeV} \quad E_c^D \simeq (1.1 \pm 0.2) \text{ GeV} \, , \]

(15)

which, using Eq. (10), can be parametrized as:

\[ E_c = E_c^\infty \left( 1 - \frac{E_c}{2M_b} \right) \, . \]

(16)

As in [3], we minimize the \( n \)-dependence of the results by requiring that the leading term is \( n \)-independent. This leads to the constraint:

\[ 4n_3 + 1 = 2n_2 + 1 \, , \]

(17)

where \( n_2 \simeq 4 - 5 \) is the value where \( f_B \) from the two-point function has been optimized [12]. By evaluating numerically the different integrals, we obtain to a good approximation:

\[
g_{B^* B^*} \simeq g_{B^* B^*}^{LO} \left\{ 1 + \frac{3}{2} \frac{E_c}{M_b} + \frac{\pi^2 \langle \bar{u}u \rangle}{2 E_c^3} \right\}
\]

(18)

where:

\[
g_{B^* B^*}^{LO} \simeq \frac{N_c}{\sqrt{2} f_\pi} \frac{m_u + m_d}{m_\pi^2} M_B \left\{ \mathcal{I}_0 = \frac{Q^4}{E_c^3} \int_0^{E_c - \frac{x}{2}} \frac{y \, dy}{\sqrt{x^2 + Q^2} (x^2 + Q^2)^{3/2}} \right\}
\]

(19)
The analytic expression of the integral $I_0$ is:

$$I_0(\rho \equiv Q/E_c) = \frac{Q}{2\rho} \left[ 1 + \frac{3\rho^2}{\sqrt{1 + \rho^2}} - \rho \right],$$

which exhibits a broad maximum in the range $1-3$ GeV$^2$. At $Q^2 \simeq 2$ GeV$^2$, where the absolute maximum is obtained, its numerical value for different $E_c$ can be parametrized by the interpolating formula:

$$I_0 \simeq (0.119 \pm 0.001)E_c.$$ 

Therefore, using Eq. (16), we finally obtain:

$$g_{B^*B\pi} \simeq \frac{2M_B}{\sqrt{2}f_\pi} g^\infty \left\{ 1 + \frac{E^B_c}{M_b} + \frac{\pi^2}{2} \frac{\langle \bar{u}u \rangle}{(E^B_c)^3} \right\},$$

where we have introduced the static coupling $g^\infty$:

$$g^\infty \equiv \frac{N_c}{2} \left( \frac{m_u + m_d}{m_\pi^2} \right) (0.119E^\infty_c),$$

which controls the interaction of the pion with infinitely heavy fields in the effective Lagrangian approach:

$$L_{\text{int}} = \frac{i}{2} g^\infty \text{Tr} H \gamma^\mu \gamma_5 (\pi^\dagger \partial_\mu \pi - \pi \partial_\mu \pi^\dagger) \hat{H},$$

where $H$ and $\pi$ are the heavy and pion fields. The $M_b$-behaviour obtained here, which is dictated by the one of $f_B^2$, is in agreement with current expectations [1]-[4]. The agreement with the one in [1] (the sum rule used in [1] is very similar to the light-cone sum rule in the treatment of the pion) and the light-cone sum rule [3] can be mainly due to the fact that, in the present process, the hard perturbative diagram gives the leading contribution in $1/M_b$, where the present version of the light-cone sum rule approach, which is dominated, (by construction), by the hard perturbative diagram, is appropriate. In the case where the soft process is dominant, like e.g. in the analysis of the $B \rightarrow \pi(\rho)$ semi-leptonic processes [8], one should need a modified version of the light-cone sum rule approach in order to take properly into account the dominant non-perturbative $\langle \bar{u}u \rangle$ condensate contribution. We expect that, in the moment sum rules, the $\alpha_s$ correction is much smaller than the one in the non-relativistic exponential one, as here $\alpha_s$ is evaluated at a larger scale of about $M_b/\sqrt{n}$, while in the exponential sum rule the scale is much lower at about 1 GeV. Therefore, we expect that the expression in Eq. (22) with the value of $E_c$ in Eq. (15) gives a good approximation of the physical result. However, we consider, as an intrinsic error of the approach, the known 30% effect due to $\alpha_s$ in the decay constants from the moment sum rules [12]. We shall use $\langle \bar{u}u \rangle (1\text{GeV}) = (12.5 \pm 2.5) \text{ MeV}$ [14] rescaled at $Q^2 = 2 \text{ GeV}^2$, and the corresponding value of the quark condensate. Then, we deduce:

$$g^\infty \simeq (0.15 \pm 0.03).$$

where the error takes into account the effect of 30% by the radiative correction to the value of $f_B$ [12]. Our prediction in Eq. (25) is in agreement with the range of values obtained in [1], though the authors in [1] use a too low value of $f_B^{\text{static}} \simeq 1.45f_\pi$, while we use here the two-loop

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\footnote{We have checked using the complete perturbative expressions of the three- and two-point functions that the higher order $1/M$ corrections are small and do not spoil the validity of the following approximate formula even at the charm mass.}
value \( f_B^{\text{static}} \approx 2f_\pi \) from [12] and from the recent lattice results [15]. For the physical \( B \)-meson, and including the \( 1/M_b \) correction which is of the order of \(+28\%\) at the \( b \)-mass, we obtain:

\[
g_{B^*B\pi} \approx 14.5 \pm 3.3, \tag{26}\]

where again the error takes into account the effect of radiative corrections to \( f_B \). We have used the two-loop non-relativistic pole masses [10]:

\[
M_b = (4.7 \pm 0.03) \text{ GeV} \quad M_c = (1.45 \pm 0.05) \text{ GeV}, \tag{27}\]

consistent with the present use of non-relativistic sum rules. One can notice that, like in the case of \( f_B \), the \( 1/M_b \)-correction is large (28\% at the \( b \)-quark mass and 76\% at the \( c \)-quark mass). This feature can make the extrapolation of the result to the \( D \)-meson quite risky. However, as already mentioned earlier, our explicit evaluation of the complete perturbative expression of the three- and two-point correlators indicate that higher order corrections in \( 1/M \) remain small. Therefore, we can deduce, with a quite good confidence, the estimate from the moments:

\[
g_{D^*D\pi} \approx 7.1 \pm 1.6. \tag{28}\]

We cross check the validity of the previous result by invoking semi-local duality sum rules for the two- or three-point functions which correspond respectively to the particular cases where \((n_3, n_2) = (−1/2, −1)\) or \((n_3, n_2) = (−1, −2)\), and which have been discussed extensively in the case of the QCD two-point functions for light [18] and heavy quarks [19, 13]. In these particular cases, the \( 1/M \) corrections to the leading term of the three-point function are much smaller and result by a correction of about \(+(9-11)\%\) and \(+ (23-26)\%\) for the coupling respectively at the \( b \) and \( c \) quark masses, giving [7]:

\[
g_{B^*B\pi} \approx 12.7 \pm 2.9, \quad g_{D^*D\pi} \approx 5.0 \pm 1.1. \tag{29}\]

By considering the previous results, we conclude that the most conservative estimate of the couplings from the sum rule is:

\[
g_{B^*B\pi} \approx 14 \pm 4, \quad g_{D^*D\pi} \approx 6.3 \pm 1.9. \tag{30}\]

Our final results are in good agreement with the ones in [1] but are much smaller than the ones from [3] and to the indirect determination of [20], which is correlated to a higher input value of the \( B \to \pi\ell\nu \) form factor. Taking into account that the \( 1/M_b \)-correction to \( f_{B^*} \) is approximately equal in strength but opposite in sign with the one for \( g_{B^*B\pi} \), one obtains with a good accuracy:

\[
R_{BD} \equiv \frac{g_{B^*B\pi} f_{B^*} \sqrt{M_B}}{g_{D^*D\pi} f_{D^*} \sqrt{M_D}} \approx 1, \tag{31}\]

as expected from an alternative analysis [1]. Our numerical values of the physical couplings are in better agreement with the results in [1] including the radiative corrections than with the ones in [3], which is higher than ours by a factor 2. \footnote{One should also notice that the use of the Laplace sum rules leads to a small value of the optimization scale \( \tau \), which is \textit{practically} similar to the semi-local duality sum rules used here.} \footnote{The agreement with [1] for the physical \( B \)-meson is due to the approximately same value of \( f_B \) used here \((1.5f_\pi)\) and in [1] \((1.36f_\pi)\), though we, originally, do not start with the same value of \( f_B^{\text{static}} \). In our analysis, the value of \( f_B^{\text{phys}} \approx 1.5f_\pi \), at the physical \( B \)-meson mass has been deduced from \( f_B^{\text{stat}} \approx 2f_\pi \), after taking into account the \( 1/M_b \)-correction. In Ref. [1], this \( 1/M_b \)-effect seems to be much smaller than currently expected as, there \( f_B^{\text{stat}} \approx 1.45f_\pi \approx f_B^{\text{phys}} \).} The coupling in Eq. (30) leads to the prediction:

\[
\Gamma_{D^*\to D^0\pi^-} = \frac{g^2_{D^*D\pi}}{24\pi M_{D^*}} |q_\pi|^3 \approx 1.54 \Gamma_{D^0\to D^0\pi^0} \approx (8 \pm 5) \text{ keV}, \tag{32}\]

\[
\text{for the physical } B \text{-meson due to the approximately same value of } f_B \text{ used here (1.5f}_\pi) \text{ and in } (1.36f_\pi), \text{ though we, originally, do not start with the same value of } f_B^{\text{static}}. \text{ In our analysis, the value of } f_B^{\text{phys}} \approx 1.5f_\pi, \text{ at the physical } B \text{-meson mass has been deduced from } f_B^{\text{stat}} \approx 2f_\pi, \text{ after taking into account the } 1/M_b \text{-correction. In Ref. [1], this } 1/M_b \text{-effect seems to be much smaller than currently expected as, there, } f_B^{\text{stat}} \approx 1.45f_\pi \approx f_B^{\text{phys}}.\]
where we have assumed isospin invariance for the couplings. Using the observed branching ratios \cite{17}, one can also predict the radiative decays:

$$\Gamma_{D^0\to D^0\gamma} \simeq (3.0 \pm 1.2) \text{ keV} \quad \Gamma_{\bar{D}^*\to D^-\gamma} \simeq 13^{+33}_{-11} \text{ keV} \ .$$

(33)

and the total widths:

$$\Gamma_{D^*\to all} \simeq (12 \pm 7) \text{ keV} \quad \Gamma_{D^0\to all} \simeq (8 \pm 3) \text{ keV} \ .$$

(34)

The predictions for the total widths are much smaller than the present experimental upper limits. An improved measurement of the $D^*$-total widths in the next tau-charm factory machine should provide a decisive test for the validity of these extrapolated predictions.

3 Moment sum rule for the $B^*B\gamma$ and $D^*D\gamma$ couplings

The QCD expression can be decomposed into a light ($q \equiv u, d, s$) and heavy ($Q \equiv c, b$) quark parts. For the corresponding vertex function, the one available in \cite{23} agrees with our recomputation apart for the relative sign between the perturbative and quark condensate contributions in the heavy quark component of the electromagnetic current. After a systematic $1/M_b$-expansion of the full QCD expression, one can inspect that the dominant contribution comes from the perturbative graph related to the light quarks coupled to the electromagnetic current. The heavy quark contribution is $1/M_b$-suppressed compared to the light quark one. However, the perturbative and light quark condensate contributions are of the same order in $1/M_b$ in this heavy quark component. By keeping the $1/M_b$-correction, the QCD part of the sum rule reads:

$$\mathcal{M}^{(n,n')}_{QCD} \simeq \frac{M_b}{M_b^{4(n+1)}} \left[ M_b \frac{N_c}{\pi^2} Q^2 \int_0^{E_c} dx \int_0^{E_c - \frac{1}{2} \sqrt{x^2 + Q^2}} \frac{y \, dy}{(x^2 + Q^2)^{3/2}} \right] + \frac{1}{M_b} \left[ \frac{1}{4y} \left( Q^2 + \frac{x^2}{2} - 10y^2 \right) - 2(2n + 1)y + \frac{Q^2}{8M_b} \delta \left( y - \frac{1}{2} \sqrt{x^2 + Q^2} \right) \right]
- \frac{16}{9} \frac{\alpha_q \langle \bar{u}u \rangle^2}{Q^4} + \frac{e_q}{M_b^{4(n+1)}} \left[ \frac{N_c}{3\pi^2} E_c^3 - \langle \bar{u}u \rangle + O(1/M_b) \right] ,$$

(35)

where $e_q(Q)$ is the charge of the light (heavy) quark in units of $e$. The phenomenological side of the sum rule can be parametrized as:

$$\mathcal{M}^{(n,n')}_{\text{phen}} \simeq g_{B^*B\gamma} \sqrt{2} M_{B^*} f_{B^*} \sqrt{2} M_B^2 f_B \ .$$

(36)

Using an approach similar to the one done for $B^*B\pi$, we deduce the sum rule:

$$g_{B^*B\gamma}(Q^2) \equiv g_{B^*B\gamma}(Q^2) + g_{B^*B\gamma}^H(Q^2)$$

(37)

where:

$$g_{B^*B\gamma}^L(Q^2) \simeq e_q \left[ \frac{N_c\mathcal{I}_0}{Q^2} \left( 1 + \frac{1}{2} \frac{E_c}{M_b} \right) - \frac{16}{9} \frac{\alpha_q \langle \bar{u}u \rangle^2}{(E_c^2)^3} \right] ,$$

$$g_{B^*B\gamma}^H(Q^2) \simeq \frac{e_q}{M_b} \left[ \frac{N_c}{3} - \frac{\pi^2 \langle \bar{u}u \rangle}{(E_c^2)^3} \right] ,$$

(38)
where $I_0$ is the integral defined in Eq. (19). For $M_b \to \infty$, the coupling is given by the light quarks contribution and remains constant. The $1/M_b$-correction in the light quark contribution is much smaller than the one for $g_{B^0 \pi}$ since there is an almost cancellation of the $1/M_b$-correction with the one from the $E_c$-dependence of $I_0$ as can be deduced from Eq. (16), while the one due to the heavy quark is important at the $c$-quark mass. The light quark coupling exhibits a typical monopole behaviour for $Q^2 \geq M^2_\rho$, while the heavy quark coupling is $Q^2$-independent. Therefore, we use a light vector meson dominance for the estimate of the light quark coupling, which can be related to the $B^* B^0$-coupling $(V \equiv \rho, \omega)$ as:

$$g^L_{B^* B^0 \gamma}(Q^2) \equiv \left( \frac{\sqrt{2}M_V^2}{2\gamma_V} \right) \frac{e_q}{Q^2 + M^2_V} \frac{\alpha}{\sqrt{2}} g^L_{B^* B^0 \rho} .$$

A sum rule analysis of the $B^* B^0$-coupling similar to the one for $B^* B \pi$, shows a very good stability for $0.4 \leq Q^2 \leq 2.2$ GeV$^2$, where the optimal value obtained for $Q^2 \approx 1$ GeV$^2$ reads:

$$g^L_{B^* B^0 \rho} \approx (0.84 \pm 0.10)/\left( \frac{\sqrt{2}M^2_V}{2\gamma_V} \right) .$$

We have used $\alpha_s \langle \bar{u}u \rangle^2 \approx (5.8 \pm 0.9) 10^{-4}$ GeV$^4$ [21, 10], which shows a negligible contribution of the four-quark condensate in the range of $Q^2$-stability. Therefore, we deduce:

$$g^L_{B^* \gamma}(Q^2 = 0) \approx \left[ e_q(1.14 \pm 0.15) + e_q \frac{(0.90 \pm 0.16) \text{ GeV}}{M_b} \right] \text{ GeV}^{-1} ,$$

$$g^L_{D^* \gamma}(Q^2 = 0) \approx \left[ e_q(1.11 \pm 0.24) + e_q \frac{(0.90 \pm 0.16) \text{ GeV}}{M_c} \right] \text{ GeV}^{-1} .$$

For the $B$-meson, the heavy quark contribution is relatively small. One obtains:

$$\Gamma_{B^* \to B^{-}\gamma} = g^2_{B^* \gamma} \frac{\alpha}{3} |q_{\gamma}|^3 \approx (.10 \pm .03) \text{ keV} .$$

and with a better accuracy for the ratio:

$$\frac{\Gamma_{B^* \to B^{-}\gamma}}{\Gamma_{B^0 \to B^0\gamma}} \approx 2.5 ,$$

which deviates strongly from the naïve static limit ($M_b \to \infty$) expectation $(e_u/e_d)^2 = 4$. For the $D$-meson, the heavy quark contribution is relatively important. One should notice that the $\Upsilon$-contribution has been completely ignored in the phenomenological analysis of [5], which can explain their opposite prediction of this ratio with respect to us and to the data. We deduce within the previous approximations:

$$\Gamma_{D^0 \to D^0\gamma} \approx (8.0 \pm 2.7) \text{ keV}$$

$$\Gamma_{D^* \to D^{-}\gamma} \approx (.01 \pm .08) \text{ keV} .$$

If one instead works with the semi-local duality like-sum rule using the complete expressions of the perturbative contributions, one finds that the heavy quark contribution to the coupling is reduced by 30% and leads to:

$$\Gamma_{D^0 \to D^0\gamma} \approx (6.7 \pm 2.4) \text{ keV}$$

$$\Gamma_{D^* \to D^{-}\gamma} \approx (.04 \pm .08) \text{ keV} .$$

Therefore, the most conservative sum rule estimate is:

$$\Gamma_{D^0 \to D^0\gamma} \approx (7.3 \pm 2.7) \text{ keV}$$

$$\Gamma_{D^* \to D^{-}\gamma} \approx (.03 \pm .08) \text{ keV} .$$
which, despite the large error, shows that the heavy quark contribution acts in the right direction for explaining the large charge dependence of the observed decay rates [7]. One can combine these results with the ones in Eq. (32), for an attempt to deduce the ratios of rates:

\[
\frac{\Gamma_{D^{*0}\rightarrow D^0\pi^0}}{\Gamma_{D^{*0}\rightarrow D^0\gamma}} \quad \frac{\Gamma_{D^{*+}\rightarrow D^0\pi^-}}{\Gamma_{D^{*+}\rightarrow D^-\gamma}},
\]

(47)

but, due to the large errors, the comparison of the predictions with the data is not very conclusive. Alternatively, we can combine the predictions in Eq. (46) with the observed branching ratios given in [7]. Then, we predict:

\[
\Gamma_{D^{*0}\rightarrow D^0\pi^0} \simeq (14 \pm 5) \text{ keV} \quad \Gamma_{D^{*+}\rightarrow D^0\pi^-} \leq 18 \text{ keV},
\]

(48)

and:

\[
\Gamma_{D^{*0}\rightarrow all} \simeq (20 \pm 7) \text{ keV} \quad \Gamma_{D^{*+}\rightarrow all} \leq 27 \text{ keV}.
\]

(49)

These results are respectively in fair agreement within the errors with the direct calculation in Eq. (32) and with the prediction in Eq. (34), though the ones for the \( D^{*-} \) have a large error due to the inaccuracy of the measured and predicted \( D^{*-} \rightarrow D^-\gamma \) branching ratio. The agreements between the different results given in this paper is an indication for the self-consistency of the whole approach.

### Conclusion

We have systematically studied the couplings \( P^*P\pi \) and \( P^*P\gamma \) (\( P = B, D \)) using a \( 1/M_b \)-expansion in full QCD with the help of moments sum rules. It is important to notice that, like other heavy-to-heavy transitions, the couplings are dominated by the hard perturbative diagram. This is not the case of the \( B \rightarrow \pi(\rho) l\nu \) and \( B \rightarrow K^*\gamma \) heavy-to-light transitions which are governed by the soft light quark vacuum condensate [3]. Technically, this difference is mainly due to the uses of different dispersion variables for the heavy-to-heavy and heavy-to-light transition processes. We find that, for the \( P^*P\pi \) couplings, the \( 1/M_b \)-corrections due mainly to the perturbative graph are large but they tend to cancel for the quantity \( f_{P\gamma P^*\pi} \) and implies to a good approximation the relation in Eq. (31). For the \( P^*P\gamma \)-coupling, the \( 1/M_b \)-correction is due mainly to the perturbative and light quark condensate contributions from the heavy quark component of the electromagnetic current, which goes in the good direction for explaining the large charge dependence of the ratio of the \( D^{*0} \rightarrow D^0\gamma \) over the \( D^{*-} \rightarrow D^-\gamma \) observed widths. For the \( B^* \)-mesons, our predictions are given in Eqs. (30), (42) and (43), where for experimental interests in the next \( B \)-factory machine:

\[
\Gamma_{B^{*-}\rightarrow B^-\gamma} \simeq (0.10 \pm 0.03) \text{ keV},
\]

\[
\Gamma_{B^{*-}\rightarrow B^-\gamma}/\Gamma_{B^{*0}\rightarrow B^0\gamma} \simeq 2.5,
\]

(50)

where the latter deviates strongly from the naïve static limit (\( M_b \rightarrow \infty \)) expectation \((e_u/e_d)^2 = 4\). By combining the previous different results of the \( D^* \)-meson, our averaged predictions for the different exclusive widths are:

\[
\Gamma_{D^{*+}\rightarrow D^0\pi^-} \simeq 1.54 \Gamma_{D^{*0}\rightarrow D^0\pi^0} \simeq (8 \pm 5) \text{ keV}.
\]

(51)

and:

\[
\Gamma_{D^{*0}\rightarrow D^0\gamma} \simeq (3.7 \pm 1.2) \text{ keV} \quad \Gamma_{D^{*+}\rightarrow D^-\gamma} \simeq (0.09^{+0.40}_{-0.07}) \text{ keV}.
\]

\[
\Gamma_{D^{*0}\rightarrow all} \simeq (11 \pm 4) \text{ keV} \quad \Gamma_{D^{*+}\rightarrow all} \simeq (12 \pm 7) \text{ keV}.
\]

(52)
The branching ratios agree within the errors with the present data though the total widths are well below the experimental upper limits $\Gamma_{D^* \rightarrow \text{all}} \leq 131$ keV and $\Gamma_{D^{*0} \rightarrow \text{all}} \leq 2$ MeV [17]. We urge experimentalists to improve the measurements of these total widths in the near future, as these measurements are necessary for clarifying the present disagreements between different theoretical predictions.

Acknowledgements

One of us (H.G.D) would like to thank the CNRS for a financial support and for the hospitality at the Laboratoire de Physique Mathématique de Montpellier.
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