Collinear acousto-optical filtration of polychromatic Bessel light beams in lithium niobate crystals

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Abstract. The features of collinear acousto-optical filtration of quasi-diffractive Bessel light beams of o- and e-type in uniaxial crystals are investigated. Using the method of overlap integrals, an expression is found for the diffraction efficiency depending on the parameters of the acousto-optical interaction, as well as on the values of the overlap integrals. It is shown that for the zero-order mode of a Bessel light beam for a lithium niobate crystal under conditions of transverse phase synchronism and in the optical spectrum range of 0.4-0.7 \(\mu\)m, the filter bandwidth of \(~0.2\) nm is achievable; with an increase in the order of the mode \(m\geq1\), the increase in the bandwidth is insignificant and is \(\sim0.23-0.24\) nm.

1. Introduction

Currently, for the purposes of acousto-optical (AO) transformation, Bessel light beams (BLBs) propagating in uniaxial crystals are of considerable interest [1, 2]. A number of features of collinear AO interactions of BLBs have been studied in [3, 4]. At the same time, the features of AO transformations of the spatial structure of the BLB of various orders were mainly studied. It should be noted, however, that for a number of important applications, the processes of collinear AO filtration of polychromatic BLBs on ultrasound (US) in anisotropic media are of considerable interest [5]. Such processes are promising for creating narrow-band collinear tunable AO filters [6]. In this regard, for the purposes of AO filtering of broadband optical radiation, it is important to use BLBs, since they have the properties of quasi-diffraction and self-reconstruction of the spatial structure [4].

In this paper, the collinear AO filtration of high-order Bessel polychromatic light beams under collinear AO interaction in uniaxial lithium niobate (LiNbO\textsubscript{3}) crystals is considered using the method of overlap integrals. In this case, as an example, the AO diffraction of BLB in crystals on a shear ultrasonic wave propagating at a certain angle to the optical axis of the crystal is considered. This geometry of the collinear associated AO interaction is most effective and is realized when the diffracted light waves propagate orthogonally to the optical axis of the crystal (\(\theta_o,e = 00\)) [7].

In addition to the usual longitudinal phase matching, BLBs must meet the conditions of transverse phase matching [3, 4]. This agreement is due to the fact that BLBs with different taper angles have different spatial structure and, as a result, different values of the overlap integrals of the diffracted beams. At the same time, the calculation of the overlap integrals (\(g_m\)) allows us to find their maximal values under conditions of transverse synchronism.
2. Theoretical investigations and discussion

Let us consider the geometry of the AO interaction, at which the ultrasonic wave propagates in the LiNbO₃ crystal in the direction of the x-axis and occupies the space between the planes z = 0 and z = l. We assume that the incident wave in a uniaxial crystal has an ordinary "o" polarization and is an o-type Bessel light beam [7]. In turn, the diffracted wave has an unusual "e" polarization and is an e-type Bessel beam (figure 1). The axis of the incident o-type BLB is located in the XZ plane at a certain angle to the optical z-axis of the crystal. In this case, the conditions of spatial \( \mathbf{k}_o + \Delta \mathbf{k} = \mathbf{k}_e \) and temporal \( \omega + \Omega = \omega_d \) synchronism are realized, where \( \mathbf{k}_o \) (\( \mathbf{k}_e \)) is the wave vector of o-type (e-type) Bessel beam, \( \mathbf{K} \) is the wave vector of the ultrasonic wave, \( \Delta \mathbf{K} \) is the phase synchronization disorder (longitudinal phase mismatch). The system of coupled wave equations for the amplitudes of the incident \( (A_o) \) o-type beam and the diffracted \( (A_e) \) e-type beam has the form

\[
\frac{dA_o}{dz} = -i k^2 \frac{2 \pi}{l} \int_0^l R_p (\mathbf{\varepsilon}_o^* \Delta \mathbf{\varepsilon}_e) d \varphi d \rho \frac{A_o e^{-i \Delta k z}}{2k_o \frac{2 \pi}{l} \int_0^l |\mathbf{\varepsilon}_o|^2 d \varphi d \rho},
\]

\[
\frac{dA_e}{dz} = -i k^2 \frac{2 \pi}{l} \int_0^l R_p (\mathbf{\varepsilon}_o^* \Delta \mathbf{\varepsilon}_o) d \varphi d \rho \frac{A_e e^{i \Delta k z}}{2k_e \frac{2 \pi}{l} \int_0^l |\mathbf{\varepsilon}_e|^2 d \varphi d \rho},
\]

where the asterisk "*" stands for complex conjugation. In equations (1), the vector functions of the polarization of an incident Bessel beam of o-type and a diffracted beam of e-type are given by the following expressions [1, 2]

\[
\mathbf{\varepsilon}_o = e_{o1} \mathbf{\varepsilon}_1 + e_{o2} \mathbf{\varepsilon}_2 + e_{o3} \mathbf{\varepsilon}_3, \quad \mathbf{\varepsilon}_e = e_{e1} \mathbf{\varepsilon}_1 + e_{e2} \mathbf{\varepsilon}_2 + e_{e3} \mathbf{\varepsilon}_3,
\]

\[
e_{o1} = i q_o \cos \theta_o (J_{m-1}(q_o \rho) e^{-iq} + J_{m+1}(q_o \rho) e^{iq}) / 2 + i q_o |\sin \theta_o| J_m(q_o \rho),
\]

\[
e_{o2} = q_o \cos \theta_o (J_{m-1}(q_o \rho) e^{-iq} - J_{m+1}(q_o \rho) e^{iq}) / 2 \sqrt{1 + a^4 t g^4 \theta_o} + q_o \sin^2 \theta_o (J_{m+1}(q_o \rho) e^{iq} - J_{m-1}(q_o \rho) e^{-iq}) / 2,
\]

\[
e_{o3} = q_o \cos \theta_o (J_{m-1}(q_o \rho) e^{iq} - J_{m+1}(q_o \rho) e^{-iq}) a^2 t g \theta_o / 2 \sqrt{1 + a^4 t g^4 \theta_o} + q_o \sin \theta_o (J_{m+1}(q_o \rho) e^{iq} - J_{m-1}(q_o \rho) e^{-iq}) / 4,
\]
Here the following notation is introduced: \( \vec{e}_1, \vec{e}_2, \vec{e}_3 \) are the unit vectors in the crystallographic coordinate system \( X_1, X_2, X_3 \); \( \rho, \varphi \) are the cylindrical coordinates of the BLB, \( \varphi' = \arctan[a \sin(\varphi)] \), \( a = n_0 / n_e ; n_o (n_e) \) is the ordinary (extraordinary) refractive index of the crystal; \( q_{o, e, \rho} = k_{o, e} \sin \gamma_{o, e} \), \( q_{o, e} \) is the parameter of the taper of the BLB; \( \theta_o = 90^0 - \theta_e \); \( \theta_e = 90^0 - \arctan[a \tan(\theta_e)] \); \( \Delta \epsilon \) is the change in the dielectric constant tensor induced by the ultrasonic wave.

It follows from (2) that the incident and diffracted \( o \)-and \( e \)-type beams, respectively, have a complex polarization structure inhomogeneous in the beam cross-section \([1, 2] \). In this case, the polarization vectors \( \vec{e}_o \) and \( \vec{e}_e \) and modes of \( m \)-th order Bessel beam depend on the Bessel functions of five orders \( J_m, J_{m+1}, J_{m+2} \).

Here, by definition, \( m \)-th order Bessel beam has a common phase factor, that is \( e^{i(mz + k_e \rho - \omega t)} \), it propagates in the crystal at a phase velocity \( c / k_e \) without changing the spatial distribution of the transverse component.

The solution of the system of equations of coupled waves (1) is sought using the following boundary conditions: \( A_0(z = 0) = A \), \( A_e(z = 0) = 0 \). Then the efficiency \( \eta = |A_e(z = l)|^2 / |A|^2 \) of AO diffraction is given by the relation:

\[
\eta = \frac{\chi^2 \sin^2 \left( \frac{l_d}{\chi \sqrt{2 + (\Delta k_z/2)^2}} \right)}{\chi^2 + (\Delta k_z/2)^2},
\]

where

\[
\chi = \frac{mn_0^2 \theta_{p0}}{2n_0 \lambda \cos^2 \theta_0 \sqrt{2 \lambda_0}}
\]

\[
\Delta k_z = \left( -\frac{2\pi n_0}{\lambda^2} \right) [\xi(\alpha) - \eta - 1] \lambda_0,
\]

\[
\xi(\theta_0) = \frac{n_0}{\sqrt{n_0^2 \cos^2 \theta_0 + n_e^2 \sin^2 \theta_0}}, \eta = \frac{\lambda_0 f}{n_0 v},
\]

and \( \lambda_0 \) is the central wavelength of light, \( \Delta \lambda \) is the deviation of the wavelength of light from the central one, \( f \) is the frequency US waves, \( v \) is the phase velocity of shear US waves; \( p_{ef} = p_{14} \) ( \( p_{14} \) is the photoelastic constant,
σ is the crystal density, \( I_o \) is the intensity of the ultrasonic wave. The overlap integral \( g_m \) of diffracted waves is found from the relation

\[
g_m = \frac{2\pi \int_0^{R_B} \left| \mathbf{E}_d \cdot \mathbf{E}_e \right|^2 r \, dr \, d\phi \, d\theta}{2\pi \int_0^{R_B} \left| \mathbf{E}_e \right|^2 r \, dr \, d\phi \, d\theta},
\]

where \( R_B \) is the radius of the BLB.

Figure 2 shows the dependence of the overlap integral \( g_m \) on the parameter \( q_n = \Delta q / q_0 \) for the BLB diffraction of different orders \( m = 0 \) (1), 1 (2), 2 (3), 3 (4) (a) and \( m = 10 \) (1), 11 (2), 12 (3), 13 (4) (b) \( m \)-th mode values.

![Figure 2](image)

**Figure 2.** Dependence of the overlap integral \( g_m \) on the parameter \( q_n = \Delta q / q_0 \) for the BLB diffraction of different orders \( m = 0 \) (1), 1 (2), 2 (3), 3 (4) (a) and \( m = 10 \) (1), 11 (2), 12 (3), 13 (4) (b) \( m \)-th mode values.

It follows from figure 2 that the BLB overlap integrals reach their maximal value at the exact transverse synchronization of the diffracted waves (\( \Delta q = 0 \)). For small values of the BLB mode (figure 2(a)) and under the condition of transverse synchronization (\( q_n = 0 \)), the overlap integrals take the following maximal values: \( g_m = 0.67 \) (\( m = 0 \)), \( g_m = 0.84 \) (\( m = 1 \)), \( g_m = 0.96 \) (\( m = 2 \)), \( g_m = 0.99 \) (\( m = 3 \)). For large values of the BLB mode and under the condition of transverse synchronization, there is an intersection of the curves \( g_m(q_n) \) with a subsequent change to the opposite dependence of the overlap integral on the order of the BLB mode.

Figure 3 shows the dependence of the diffraction efficiency \( \eta \) on the transverse \( (q_n) \) synchronization tuning parameter for small (a) and large (b) values of the BLB mode. From figure 3(a) it follows that at ultrasound intensities corresponding to the maximal values of the diffraction efficiency under the condition of longitudinal and transverse synchronization, with an increase (deviation) in the tuning of the transverse synchronization \( q_n \) from the optimal one, the diffraction efficiency decreases. With precise transverse synchronization (\( q_n = 0 \)) and small BLB orders \( m = 0-3 \), the maximal diffraction efficiency is not achieved. For large orders of BLB \( m = 3-30 \) at \( q_n \geq 1 \), the diffraction efficiency is \( \eta \approx 0 \) (figure 3(b)).

Using the relations (3)-(4), we consider the physical characteristics of AO filtration in the optical spectrum range of 0.4-0.7 \( \mu \text{m} \) [8]. For the study, we choose the central wavelength of the studied range \( \lambda_0 = 0.63 \mu \text{m} \) as a central wavelength of the tunable filter. For a given wavelength of light, the refractive indices of the LiNbO\(_3\) crystal are respectively equal to: \( n_o = 2.29 \), \( n_e = 2.2 \) [9]. For the AO interaction length \( l = 10 \text{ cm} \), the maximal diffraction efficiency \( (\eta = 1) \) is achieved under conditions of longitudinal and transverse synchronization at the ultrasonic intensity \( I_o = 0.2 \text{ W/cm}^2 \) [7].
Figure 3. Dependence of the diffraction efficiency \( \eta \) on the transverse synchronism tuning parameter \( q_n \) for \( m = 0 \) (1), 1 (2), 2 (3), 3 (4) (a) and \( m = 10 \) (1), 11 (2), 12 (3), 13 (4) (b) (LiNbO\(_3\) crystal; \( \theta_o, e = 0^\circ; \gamma_o = \gamma_e = 0.5^\circ; R_B = 6 \text{ mm}, I_a = 0.2 \text{ W/cm}^2, f = 570 \text{ MHz}, l = 10 \text{ cm}, \lambda_0 = 0.63 \mu\text{m}).

The dependence of the diffraction efficiency \( \eta \) on the spectral bandwidth \( \Delta \lambda \) of the acousto-optic tunable filter (AOTF) for the central wavelength \( \lambda_0 = 630 \text{ nm} \) is shown in figure 4. The width of the AO filtration band was calculated at the level of 50% of the maximal value of the diffraction efficiency. Under conditions of longitudinal and transverse synchronism, the bandwidth was \( \Delta \lambda_{1/2} = 0.01 \text{ nm} \) \((m = 0)\), \( \Delta \lambda_{1/2} = 0.02 \text{ nm} \) \((m = 1)\), \( \Delta \lambda_{1/2} = 0.022 \text{ nm} \) \((m = 2)\), \( \Delta \lambda_{1/2} = 0.023 \text{ nm} \) \((m = 3-30)\).

Figure 4. Dependence of the diffraction efficiency \( \eta \) on the bandwidth \( \Delta \lambda \) under the condition of transverse synchronism for different orders of BLB \( m \): 0 (1), 1 (2), 2 (3), 3-30 (4) (crystal LiNbO\(_3\); \( \theta_o,e = 0^\circ; \gamma_o = \gamma_e = 0.5^\circ; R_B = 6 \text{ mm}, L = 0.2 \text{ W/cm}^2, f = 570 \text{ MHz}, l = 10 \text{ cm}, \lambda_0 = 0.63 \mu\text{m}).

In the experimental conditions, along with the considered spectrum width due to the Bragg synchronism conditions, it is also necessary to take into account the change in the spectrum width associated with the broadening of the ultrasonic and light beams [10, 11]. The calculation of these spectral widths for the ultrasonic \((\Delta \lambda_S)\) and light \((\Delta \lambda_L)\) beams is achieved using the relations [10]:

\[
\Delta \lambda_S = \lambda_0 \varphi_S^2 / 4,
\]

where \( \varphi_S \) is the divergence of the ultrasonic beam, \( \varphi_L \) is the divergence of the light beam. Thus, the total width of the spectrum is:

\[
\Delta \lambda_{tot} = \Delta \lambda_{1/2} + \Delta \lambda_S + \Delta \lambda_L.
\]

It should be noted that for quasi-diffractive BLBs, the natural relations are fulfilled: \( \Delta \lambda_L \ll \Delta \lambda_{1/2} \), \( \Delta \lambda_L \ll \Delta \lambda_S \). In this case, the use of BLB for AO filtering is more preferable, for example, than Gaussian beams [10]. Supposing, for example, \( \Delta \lambda_S = 0.2 \text{ nm}, \Delta \lambda_L = 0.02 \text{ nm} \), we get \( \Delta \lambda_{tot} = 0.230 \text{ nm} \) \((m = 0)\), \( \Delta \lambda_{tot} = 0.240 \text{ nm} \) \((m = 1)\), \( \Delta \lambda_{tot} = 0.242 \text{ nm} \) \((m = 2)\), \( \Delta \lambda_{tot} = 0.243 \text{ nm} \) \((m = 3-30)\). The spectral resolution of AO filtering is given by the ratio: \( N_\lambda = \delta \lambda / \Delta \lambda_{tot} \), where \( \delta \lambda \) is the width of the spectrum range under study [1]. For the considered range of the spectrum \( N_\lambda \approx 1300 \) \((m = 0)\), \( N_\lambda \approx 1200 \) \((m \geq 1)\).
3. Conclusion
Thus, in uniaxial crystals in a wide range of the optical spectrum, collinear AO filtering of polychromatic Bessel light beams of α- and e-types is possible for different m modes under conditions of transverse synchronism of diffracted waves. For lithium niobate crystals in the optical spectrum range of 0.4–0.7 μm, the bandwidth of the AO filter for the zero-order Bessel mode can be ~0.23 nm. At the same time, the resolution of AO filtering is ~1300. For Bessel modes, the strand \( m \geq 1 \) has a bandwidth of ~0.24 nm and a resolution of ~1200.

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