Abstract In this paper, we construct a class of (n+1)-dimensional \( n \geq 4 \) slowly rotating black hole solutions in Brans-Dicke-Maxwell theory with a quadratic potential. These solutions can represent black holes with inner and outer event horizons, an extreme black hole and a naked singularity and they are neither asymptotically flat nor (anti)-de Sitter. We compute the Euclidean action and use it to obtain the conserved and thermodynamics quantities such as entropy which does not obey the area law. We also compute the angular momentum and the gyromagnetic ratio for these type of black holes where the gyromagnetic ratio is modified in Brans-Dicke theory compared to the Einstein theory.

Keywords Black Holes · Thermodynamics · Brans-Dicke

1 introduction

Brans-Dicke theory [1] is perhaps the most alternative theory to the Einstein general relativity. This theory contains both Mach’s principle and Dirac’s large number hypothesis. The theory has recently received interest as it arises naturally as the low energy limit of many theories of quantum gravity such as the supersymmetric string theory or the Kaluza-Klein theory and also found consistent with present cosmological observations [2]. The theory contains an adjustable parameter \( \omega \) that represents the strength of coupling between scalar field and the matter. For certain values of \( \omega \), the BD theory agrees with GR in post-Newtonian limit up to any desired accuracy and hence weak-field observations cannot rule out the BD theory in favor of general relativity [3], although the singularity problem has remained yet.

Shortly after the appearance of this theory one of its authors, C. Brans obtained the statically spherically symmetric solutions [4]. After that many authors have investigated black holes in Brans-Dicke theory [5]. Hawking has proved in four dimensions the stationary and vacuum Brans-Dicke solution is just the Kerr solution with...
constant scalar field everywhere [3]. Cai and Myung have proved that in four dimensions, the charged black hole solution in the Brans-Dicke-Maxwell theory is just the Reissner-Nordstrom solution with a constant scalar field [4] and the Kerr-Newman type black hole solutions which are different from general relativity solutions have been constructed for $-5/2 < \omega < -3/2$ in [8]. Thermodynamics of black holes in Brans-Dicke theory is investigated by some authors [9].

On the other hand, the rotating black hole solutions in higher dimensional Einstein gravity was found by Myers and Perry [10]. The solutions were uncharged and can be considered as generalization of the four dimensional Kerr solutions. Moreover, recently, it has been shown that the gravity in higher dimensions contains much richer dynamics than in four dimensions. As an example, there exists a black ring solution in five dimensions with the horizon topology of $S^2 \times S^2$ [11] which have the same both mass and angular momentum as the Myers-Perry solution and therefore contradicts the uniqueness theorem in five dimensions. Although the nonrotating black hole solutions in the higher dimensional Einstein-Maxwell gravity was found many years ago [12], the analytical solution of charged generalization of Myers-Perry solutions in $(n+1)$-dimensional Einstein Maxwell gravity have not been found yet. Solutions of different kinds of charged rotating black holes in higher dimensions have been discussed in the framework of supergravity and string theory [13, 14, 15]. In [16], the solutions of charged rotating black hole in $(n+1)$-dimensional Einstein-Maxwell theory with a single rotation parameter in the limit of slow rotation have been constructed. In addition, [17] contains a class of charged slowly rotating black hole solutions in Gauss-Bonnet gravity. Rotating black holes in Einstein-Maxwell-Dilaton gravity is discussed in [26].

In this paper we investigate the charged slowly rotating black holes in Brans-Dicke theory by using the conformal transformation between dilaton fields and Brans-Dicke theory. The structure of this paper is as follows. In section 2 we obtain the solution of charged rotating Brans-Dicke black holes and discuss about their casual structure, then in section 3 we obtain the conserved quantities of the finite action by using the Euclidean action. Section 4 contains our results.

2 Solutions of Slowly Rotating Black Holes In Brans-Dicke-Maxwell theory

The action of the Brans-Dicke-Maxwell theory in $(n+1)$-dimension with a scalar field $\Phi$ and a self-interacting potential $V(\Phi)$ is

$$I_G = \frac{1}{16\pi} \int_{\mathcal{M}} d^{n+1}x \sqrt{-g(\Phi, R - \frac{\omega}{\Phi} (\nabla\Phi)^2 - V(\Phi) - F_{\mu\nu}F^{\mu\nu})}$$

(1)

where $R$ is the Ricci scalar, $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ is the electromagnetic tensor field, $A_{\mu}$ is the vector potential, $\omega$ is the coupling constant and $\Phi$ is the BD scalar field. By varying the the action (1) with respect to the metric $g_{\mu\nu}$, the scalar field $\Phi$ and vector field $A_{\mu}$ one can obtain the following field equations

$$G_{\mu\nu} = \frac{\omega}{\Phi^2}(\nabla_{\mu}\Phi\nabla_{\nu}\Phi - \frac{1}{2}g_{\mu\nu}(\nabla\Phi)^2) - \frac{V(\Phi)}{2\Phi}g_{\mu\nu} + \frac{1}{\Phi}(\nabla_{\mu}\nabla_{\nu} - g_{\mu\nu}\nabla^2\Phi)$$

$$+ 2\frac{1}{\Phi}(F_{\mu\lambda}F_{\nu}^{\lambda} - \frac{1}{2}F_{\rho\sigma}F^{\rho\sigma}g_{\mu\nu})$$

(2)

$$\nabla^2\Phi = -\frac{n-3}{2((n-1)\omega + m)}F^2 + \frac{1}{2((n-1)\omega + m)} \left[ (n-1)\Phi \frac{dV(\Phi)}{d\Phi} - (n+1)V(\phi) \right]$$

(3)
where $G_{\mu \nu}$ is the Einstein tensor. It is not easy to solve the field equations (2)-(4) directly because the right hand side of eq. (2) includes the second derivatives of the scalar field. Fortunately we can transform these field equations to the dilaton field equations by conformal transformations. If one uses the following conformal transformations

$$
\bar{g}_{\mu \nu} = \Phi^{2/(n-1)} g_{\mu \nu}
$$
$$
\bar{\phi} = \frac{n-3}{4\alpha} \ln \Phi
$$

then the action \( I \) takes the form

$$
\bar{I}_G = -\frac{1}{16\pi} \int_M d^{n+1}x \sqrt{-\bar{g}} \left\{ \bar{\mathcal{R}} - \frac{4}{n-1} (\nabla \bar{\phi})^2 - \bar{V}(\bar{\phi}) - e^{-\frac{4\Phi}{n-2}} \bar{F}_{\mu \nu} \bar{F}^{\mu \nu} \right\}
$$

where $\bar{\mathcal{R}}$ and $\nabla$ are the Ricci scalar and covariant derivative corresponding to the metric $\bar{g}_{\mu \nu}$ and $\bar{V}(\bar{\phi})$ is:

$$
\bar{V}(\bar{\phi}) = \Phi^{-(n+1)/(n-1)} V(\Phi)
$$

Action (7) is just the action of $(n+1)$-dimensional Einstein-Maxwell-dilaton gravity where $\bar{\phi}$ is the dilaton field, $\bar{V}(\bar{\phi})$ is a potential for $\bar{\phi}$ and $\alpha$ is a constant which determines the strength of coupling of the scalar and electromagnetic field $\bar{F}_{\mu \nu}$. By varying the action (7) with respect to $\bar{g}_{\mu \nu}$, $\bar{\phi}$ and $\bar{F}_{\mu \nu}$, we obtain

$$
\bar{\mathcal{R}}_{\mu \nu} = \frac{4}{n-1} (\nabla_{\mu} \bar{\phi} \nabla_{\nu} \bar{\phi} + \frac{1}{4} \bar{V}(\bar{\phi}) \bar{g}_{\mu \nu}) + 2e^{-\frac{4\Phi}{n-2}} \bar{F}_{\mu \lambda} \bar{F}^{\lambda \nu} - \frac{1}{2(n-1)} \bar{F}_{\rho \sigma} \bar{F}^{\rho \sigma} \bar{g}_{\mu \nu}
$$
$$
\nabla^2 \bar{\phi} = \frac{n-1}{8} \frac{\partial \bar{V}}{\partial \bar{\phi}} - \frac{\alpha}{2} e^{-\frac{4\Phi}{n-2}} \bar{F}_{\rho \sigma} \bar{F}^{\rho \sigma}
$$
$$
\nabla_{\mu} [\sqrt{-\bar{g}} e^{-\frac{4\Phi}{n-2}} F^{\mu \nu}] = 0
$$

Many authors obtained the solutions of above field equations [18, 19, 20, 21, 22, 23, 24, 25, 26]. Then it will be easy to obtain the solutions of field equations (2)-(4) by applying the conformal transformations (5) to the solution of the field equations (9)-(11). In [26], the solution of field equations (9)-(11) has been obtained for black holes in infinitesimal rotation case to first order in angular parameter $a$. In this case the only term in the metric changes to $O(a)$ is $g_{\phi \phi}$, and $A_\phi$ is the only component of the vector potential that changes, where the dilaton field does not change to $O(a)$. Therefore, for infinitesimal angular momentum up to $O(a)$, we can take the following form for the metric in $(n+1)$-dimension for Einstein-Maxwell-dilaton theory [26]

$$
d s^2 = -U(r) dt^2 + \frac{dr^2}{U(r)} - 2af(r) \sin^2 \theta d\phi
$$
$$
+ r^2 R^2(r) (d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\Omega^2_{n-3})
$$

where $U(r)$, $R(r)$ and $f(r)$ are functions of $r$, and $a$ is a parameter associated with its angular momentum and $d\Omega^2_{n-3}$ denotes the metric of an unit $(n-3)$-sphere. For small
value of \( a \), \( U(r) \) is still a function of \( r \) alone. From equation (11) we can obtain the \( t \) component of the Maxwell equations

\[
\tilde{F}_{tr} = \frac{qe^{4\alpha\Phi/(n-1)}}{(rR)^{n-1}}
\]  

(13)

where \( q \) is an integration constant related to the electric charge of the black hole. By using the definition \( Q = \frac{1}{4\pi} \int \exp[-4\alpha\Phi/(n-1)]F d\Omega \) we obtain the electric charge as

\[
Q = \frac{q\omega_{n-1}}{4\pi}
\]

(14)

where \( \omega_{n-1} \) represents the volume of a constant curvature hypersurface. In general, when we have rotational parameter there is also a vector potential in the form

\[
A_\phi = a g h(r) \sin^2 \theta
\]

(15)

In [26], for \( \tilde{V}(\Phi) \) it is introduced the following function

\[
\tilde{V}(\Phi) = 2 \Lambda^2 \zeta \Phi
\]

(16)

where \( \Lambda, \zeta \) are constants. To obtain the unknown functions \( U(r) \), \( f(r) \) and \( R(r) \) sheykhi suggested the ansatz [26]

\[
R(r) = e^{2\alpha\Phi/(n-1)}
\]

(17)

By substituting eq. (17), the Maxwell fields (13) and (15) and the metric (13) into the field equations (9)-(11), we can obtain

\[
U(r) = - \frac{(n - 2)(\alpha^2 + 1)^2 b^{-2\gamma} r^{2\gamma}}{(\alpha^2 - 1)(\alpha^2 + n - 2)} - \frac{m}{r^{(n-1)(1-\gamma)-1}}
\]

\[
+ 2q^2 (\alpha^2 + 1)^2 b^{-2(\gamma-2)^2^-n}\gamma r^{2(n-2)(\gamma-1)}
\]

(18)

\[
f(r) = m(\alpha^2 + n - 2) b^{(n-3)\gamma} r^{(n-1)(\gamma-1)+1} - 2q^2 (\alpha^2 + 1) b^{(1-n)\gamma} r^{2(n-2)(\gamma-1)}
\]

(19)

\[
\Phi = \frac{(n - 1)\alpha}{2(1 + \alpha^2)} \ln \left( \frac{b}{r} \right)
\]

(20)

\[
b(r) = r^{(n-3)(\gamma-1)-1}
\]

(21)

where \( \gamma = \alpha^2/(1 + \alpha^2) \) and \( b \) is an arbitrary constant. In addition we should have

\[
\zeta = \frac{2}{\alpha(n-1)}, \quad \Lambda = \frac{(n - 1)(n - 2)\alpha^2}{2b^2(\alpha^2 - 1)}
\]

(22)

to full satisfy the field equations.

Now to obtain the solutions of the field equations (2)-(4) for Brans-Dicke-Maxwell theory, we take a metric of the form

\[
ds^2 = - A(r)dt^2 + \frac{dr^2}{B(r)} - 2ag(r)\sin^2 \theta dtd\phi
\]

\[
+ r^2 H^2(r)(d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\Omega_{n-3}^2)
\]

(23)
then to determine the unknown functions $A(r)$, $B(r)$, $g(r)$ and $H(r)$, we apply the con- 
formal transformations (21), (24) and (28) to the eqs. (27), (28) and (29). Then we have

$$A(r) = -\frac{(n-2)(\alpha^2 + 1)^2b^{-2\gamma(\frac{\gamma-3}{\gamma-5})}r^{2\gamma(\frac{\gamma-3}{\gamma-5})}}{(\alpha^2 - 1)(\alpha^2 + n - 2)} + \frac{2q^2(\alpha^2 + 1)^2b^{2\gamma(2-n+\frac{\gamma-2}{\gamma-1})}}{(n-1)(\alpha^2 + n - 2)}r^{2(n-2)(\gamma-1)-\frac{4\gamma}{\gamma-1}} = \frac{m b^{-\frac{4\gamma}{\gamma-1}}r^{\gamma(n-1)+\frac{4\gamma}{\gamma-1}}}{r^{(n-2)}}$$

(24)

$$B(r) = -\frac{(n-2)(\alpha^2 + 1)^2b^{-2\gamma(\frac{\gamma-3}{\gamma-5})}r^{2\gamma(\frac{\gamma-3}{\gamma-5})}}{(\alpha^2 - 1)(\alpha^2 + n - 2)} + \frac{2q^2(\alpha^2 + 1)^2b^{2\gamma(n-2+\frac{\gamma-2}{\gamma-1})}}{(n-1)(\alpha^2 + n - 2)}r^{2(n-2)(\gamma-1)+\frac{4\gamma}{\gamma-1}} = \frac{m b^{-\frac{4\gamma}{\gamma-1}}r^{\gamma(n-1)+\frac{4\gamma}{\gamma-1}}}{r^{(n-2)}}$$

(25)

and

$$g(r) = \frac{m(\alpha^2 + n - 2)b^{(n-3+\frac{n}{\gamma-1})}\gamma}{\alpha^2 + 1}r^{(n-1)(n-\gamma)+1-\frac{4\gamma}{\gamma-1}} - \frac{2q^2(\alpha^2 + 1)b^{(1-n+\frac{1}{\gamma-1})}\gamma}{n-1}r^{2(n-2)(\gamma-1)-\frac{2\gamma}{\gamma-1}}$$

(26)

$$H(r) = \left(\frac{b}{r}\right)^\gamma\frac{n-3}{n-5}$$

(27)

$$\Phi(r) = \left(\frac{b}{r}\right)^{2\gamma}\frac{n-1}{n-3}$$

(28)

$$F_{fr} = \frac{gb^{(3-n)}}{r^{(n-3)(1-\gamma)+2}}$$

(29)

$$V(\Phi) = 2A^2\frac{n+\frac{n}{\gamma-1}-4}{n-2\gamma}$$

(30)
At this point, it is worthwhile to investigate the physical properties of these solutions. We can show that the Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverges at $r = 0$ and it is finite for $r \neq 0$ and goes to zero as $r \to 0$. So we find that there is an essential singularity at $r = 0$. In addition, the solution is ill-defined for $\alpha = 1$ and the cases $\alpha > 1$ and $\alpha < 1$ should be considered separately. For $\alpha > 1$, there exists a cosmological horizon (fig. 1) and there is no cosmological horizon for $\alpha < 1$, and in this case the solution eq. (24) contains a wide variety of casual structure which depends on the values of the metric parameters $\alpha, m, q$ and $k$ (fig. 2-3).

Moreover we can obtain some information about casual structure by considering the temperature of the horizons. By using the definition of hawking temperature on the outer horizon $r^+$ which may be obtained through the definition of surface gravity

$$T_+ = \frac{1}{2\pi} \sqrt{-\frac{1}{2}(\nabla_{\mu}\chi_{\nu})(\nabla_{\mu}\chi_{\nu})}$$  \hspace{1cm} (31)$$

where $\chi$ is the Killing vector $\partial_t$, we can write

$$T_+ = \frac{(\alpha^2 + n - 2)m}{4\pi(\alpha^2 + 1)} + \frac{(n - 2)(\alpha^2 + 1)b^{-2\gamma}}{2\pi(1 - \alpha^2)} + \frac{1}{2\gamma - 1}$$  \hspace{1cm} (32)$$

we see from above equation that the temperature is invariant under coformal transformations [26], this is a result of the regularity of conformal parameter at the horizon. We find from eq. (32) for $\alpha > 1$, the temperature is negative. As we argued above in this case we have cosmological horizons and therefore the cosmological horizons have negative temperature. Numerical calculations show that the temperature of the event horizon goes to zero as the black hole approaches an extreme one. In addition we can show that for $(\alpha < 1)$:

$$m_{ext} = \frac{2(n - 2)(\alpha^2 + 1)^2b^{-2\gamma}}{(n - \alpha^2)(\alpha^2 + n - 2)} \frac{(2 - n)(\gamma - 1) + \gamma}{r^+}$$  \hspace{1cm} (33)$$

In summary, the metric (24) can represent a rotating black hole with two inner and outer horizons located at $r_+$ and $r_-$ provided that the mass parameter $m$ be greater
than \( m_{\text{ext}} \), an extreme black hole when \( m = m_{\text{ext}} \), and a naked singularity when \( m < m_{\text{ext}} \).

The electric potential \( U \), measured at infinity with respect to the horizon is defined by \( U = A_\mu \chi^\mu \mid_\infty - A_\mu \chi^\mu \mid_0 \) (34)

where \( \chi \) is the null generator of event horizon. Therefore we obtain:

\[
U = \frac{q b^{\gamma(3-n)}}{\Xi T r^+} \tag{35}
\]

where \( \Gamma = \gamma(3-n) + n - 2 \).

3 Euclidean action and conserved quantities

The ADM (Arnowitt-Deser-Misner) mass \( M \), entropy \( S \) and electric potential \( U \) of the topological black hole can be calculated through the use of the Euclidean action method [28]. In this approach, initially the electrical potential and the temperature are fixed on a boundary with a fixed radius \( r_+ \). The Euclidean action has two parts; bulk and surface. The first step to make the Euclidean action is to substitute \( t \) with \( i\tau \). This makes the metric positive definite:

\[
ds^2 = + A(r)dt^2 + \frac{dr^2}{B(r)} - 2ag(r)\sin^2 \theta dt d\phi \\
+ r^2 H^2(r)(d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\Omega^2) \tag{36}
\]

There is a conical singularity at the horizon \( r = r_+ \) in the Euclidean metric. To eliminate it, the Euclidean time \( \tau \) is made periodic with period \( \beta \), where \( \beta \) is the inverse of Hawking temperature. Now we try to calculate the Euclidean action of \( (n+1) \)-dimensional Brans-Dicke-Maxwell theory. It can be obtained analytically and continuously changing of action (1) to Euclidean time \( \tau \), i.e.,

\[
I_{GE} = -\frac{1}{16\pi} \int_M d^{n+1}x \sqrt{g} \left( \Phi R - \frac{\omega}{\Phi} (\nabla \Phi)^2 - V(\Phi) - F_{\mu\nu} F^{\mu\nu} \right) \\
+ \frac{1}{8\pi} \int d^n x \sqrt{\Phi} (K - K_0), \tag{37}
\]

where \( K \) represents the extrinsic curvature on the induced metric \( h \), and \( K_0 \) is the extrinsic curvature on the metric \( h \) for flat space-time, which must be added so that it can normalize the Euclidean action to zero in flat space-time. Using the metric (37), we find

\[
R = -g^{-1/2} (g^{1/2} U' V/U)' - 2G_0 + O(a^2) + O(a^4)
\]

\[
K = -g^{-1/2} (g^{1/2} V^{1/2})' \quad K_0 = -\frac{n-1}{r} \tag{38}
\]

where \( G_0 \) is the 0-0 component of the Einstein tensor. By substituting eq. (38) in action (37) and using eqs. (24)-(30), we obtain

\[
I_{GE} = \frac{\beta \omega_{n-1}}{16\pi} \left( \frac{b^{(n-1)\gamma (n-1)}}{(a^2 + 1)} \right) - \frac{\omega_{n-1}}{4l^{n-2}} \left( b^{(n-1)\gamma r_+^{(n-1)(1-\gamma)}} \right) \\
- \frac{\beta \omega_{n-1}}{8\pi} \left( \frac{n - \alpha^2}{n(n-2)(\alpha^2+1)^2} \right) \frac{m a^2}{b^2(n-2)\gamma \omega_{n-1}} - \frac{\beta q^2}{8\pi T r_+} \tag{39}
\]
where \( \Gamma = (n-3)(1-\gamma) + 1 \). According to Ref. [29], the thermodynamical potential can be given by \( I_{GE} \), we get

\[
I_{GE} = \beta M - \beta S - \beta U q - \beta \Omega J
\]

(40)

where \( M \) is the ADM mass, \( S \) the entropy and \( U \) the electric potential corresponding to the conservation charge \( q \) and \( \Omega = a \) in this case. Comparing eqs. (39) and (40), we find

\[
M = \frac{b(n-1)}{16\pi} \frac{n-1}{1 + \alpha^2 n^{(n-1)m}},
\]

(41)

\[
S = \frac{b(n-1)1(n-1)(1-\gamma)}{4(n-2)}
\]

(42)

and

\[
Q = \frac{q\omega(n-1)}{4\pi}
\]

(43)

\[
J = \frac{(n-\alpha^2)(\alpha^2 + n-2)b^{2(n-2)}\gamma \omega_{n-1}^m}{8\pi n(n-2)(\alpha^2 + 1)^2}
\]

(44)

We can see from above equations that the ADM mass, entropy and electric potential are invariant under the conformal transformation [18]. In addition, in the context of BD gravity, where we have the additional gravitational scalar degree of freedom, the entropy of the black hole does not follow the area law. This is due to the fact that the black hole entropy comes from the boundary term in the Euclidean action formalism.

In addition the charge which is calculated in eq. (43) is the same as the one which was calculated in eq. (14). By combining eqs. (41) and (44), we can write:

\[
J = \frac{2Ma}{n-1}
\]

(45)

For calculating the gyromagnetic ratio of this type of black hole, first we need magnetic dipole moment for this slowly rotating black hole, i.e., \( \mu = Qa \), then the gyromagnetic ratio is given by

\[
g = \frac{2\mu M}{QJ} = \frac{n(n-1)(n-2)(\alpha^2 + 1)}{(n-\alpha^2)(\alpha^2 + n-2)b^{(n-3)\gamma}}
\]

(46)

As our solutions are neither asymptotically flat nor (A)dS, we get \( g \geq 2 \) in four dimension, in contrast to asymptotically flat or (A)dS which have \( g \leq 2 \) in four dimension [60]. In the absence of a non-trivial dilaton \( (\alpha = \gamma = 0) \), the gyromagnetic reduces to:

\[
g = n - 1
\]

(47)

4 Conclusions

Here we construct the solutions of slowly rotating black holes in \((n + 1)\)-dimensional Brans-Dicke-Maxwell theory with a liouville-type potential in the limit of slow rotation parameter, with an arbitrary value of coupling constant \( \omega \). Our solutions are neither asymptotically flat nor (A)dS, in contrast to the rotating black holes in the Einstein-Maxwell theory. The solutions are ill-defined for \( \alpha = 1 \) and for \( \alpha > 1 \) we have cosmological horizons and there is not any cosmological horizon for \( \alpha < 1 \). In the latter case \( (\alpha < 1) \), we can have a black hole with inner and outer event horizons if \( m > m_{ext} \), an extreme black hole if \( m = m_{ext} \) and a naked singularity for \( m < m_{ext} \).
The cosmological horizons have a negative temperature for $\alpha > 1$. Then we computed the Euclidean action and obtained the thermodynamics and conserved quantities. We computed temperature and entropy for this type of black hole and we found they did not change to $O(a)$ from the static case, in addition entropy does not follow the area law. Moreover we obtained angular momentum and gyromagnetic ratio for this rotating Brans-Dicke black hole, the gyromagnetic ratio is modified in this theory.

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