Consequences of parton’s saturation and string’s percolation on the developments of cosmic ray showers

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Abstract

At high gluon or string densities, gluons’ saturation or the strong interaction among strings, either forming colour ropes or giving rise to string’s percolation, induces a strong suppression in the particle multiplicities produced at high energy. This suppression implies important modifications on cosmic ray shower development. In particular, it is shown that it affects the depth of maximum, the elongation rate, and the behaviour of the number of muons at energies around $10^{17}$–$10^{18}$ eV. The existing cosmic ray data point out in the same direction.

One of the most crucial astrophysical issues of the highest energy cosmic rays (above $\sim 10^{17}$ eV) is that of their composition. This problem is linked to the identification of the origin and possible sources of these cosmic rays. Current theoretical models expect a transition from galactic to extragalactic or galactic halo sources near the region of the ankle which leads to the usual expectation of the changing of composition from heavy to light elements.

Experimentally, measuring the composition at these energies is a challenging task. The very low fluxes involved imply that one has to rely on indirect measurements which depend on simulations of the development of cosmic ray cascades in the atmosphere. These, in turn, are model dependent and, specifically, depend on extrapolations of hadronic models to energies and kinematical regions never measured in the laboratory. There is, therefore,
some degree of uncertainty in the shower development and one may ask what is the effect of this uncertainty in the reconstruction of shower parameters, mainly total energy and mass composition.

To avoid this problem, experimental groups have concentrated on observables which are expected to be more or less independent of the hadronic model used, or which have its dependence under theoretical control. These parameters include the maximum of the cascade, $X_{\text{max}}$, the slope parameter, $\beta = d \log(\rho_\mu(600))/d \log(E)$, where $\rho_\mu(600)$ is the muon density at 600 m from the core, and the elongation rate, $D_{10} = dX_{\text{max}}/d \log_{10}(E)$. Other parameters have been less frequently used, see Ref. [7] for a general review.

Several experimental groups have measured the cosmic ray spectrum and mass composition in the ankle region and beyond using the above mentioned parameters [1–5], see also [7]. The results on mass composition are inconclusive. Fly’s Eye [1] observe a change on the slope of $X_{\text{max}}$ in the region around $3 \times 10^{17}$ eV, which is interpreted as a change on the composition from heavy (iron) dominated to light (proton) dominated. However, AGASA [2] measures a muon component and $\beta$ parameter consistent with iron on that region. Although some part of the discrepancy between AGASA and Fly’s Eye may be due to the use of different hadronic models [8], as pointed out by Nagano et al. [3] the important issue is that AGASA sees no significant change on the muon component of showers all along the ankle region and beyond, from $10^{16.5}$ eV to $10^{19.5}$ eV, and thus no strong change on composition is inferred. HIRES and MIA collaborations [4] have jointly measured both the $X_{\text{max}}$ and the $\beta$ parameter. They observe a strong change of $X_{\text{max}}$ with energy, which implies a large elongation rate, $D_{10} = 95$ gr/cm$^2$ and on the other hand they see no change on the slope of the muonic component, $\beta = 0.73$, measured which is broadly compatible with the AGASA observations. HIRES and MIA however have measured these parameters in a narrow range of energy, from $10^{17}$ to $10^{18}$ eV and with low statistics, only during a limited exposure.

In this paper we will show that under rather general conditions a change on the hadronic interactions at the energies of interest is expected, which may have important consequences for the interpretation of cosmic ray data. Whether this change is enough or not to produce
the observed changes on the cosmic ray data we can not tell at present. On the other hand we can state the necessary conditions for this change to explain the observed data:

i) There should be an abrupt change on the hadronic interactions at the observed energy $E_{\text{lab}} \sim 5 \times 10^{17} \text{ eV}$ for Fe–Air collisions. This corresponds to a CM energy of $\sim 4200 \text{ AGeV}$ and a density of gluons of $\sim 9 \text{ fm}^{-3}$. ii) At this energy the slope of the growth of the multiplicity with energy should vary from $\sim 0.24$ to a essentially flat $\lesssim 0.09$. If i) and ii) are verified then there is no additional need for a change on composition to explain the data.

It is important to point out that, although the change on the multiplicity may or may not be enough to produce the observed results, some effect should always be present and should be taken into account in any realistic simulation of cosmic ray showers. Currently no Monte Carlo code for cosmic ray showers has yet been implemented with these effects.

In the last years, a wealth of data coming from HERA and the heavy ion SPS experiments have risen questions about the behaviour of the hadronic interactions at very high energy. We may consider perturbative, gluons, or non perturbative, strings, as the fundamental variables of our description. At high gluon density, the saturation of gluons and/or a strong jet shadowing are expected. In the case of high string density we expect the fusion of strings or colour rope, and probably, above a critical string density the percolation of strings and the formation of quark gluon plasma at the nuclear scale are expected.

One general feature of all these hadronic phenomena is the strong suppression of particle multiplicity compared to the multiplicity expected in their absence. Namely, for central Pb–Pb collisions the charged particle multiplicity expectations in the central rapidity region changes between 1500 (7500) for the relativistic heavy ion collider, RHIC, (the large hadron collider, LHC) for models that do not include these effects to 900 (3000) when they are

*Sibyll version 2.0 incorporates some shadowing effect. However this was done for $pp$ collisions only and does not affect to our reasoning below.
included \cite{14}. As a framework we will use the quark gluon string model (QGS) \cite{15}, a modified version of the Dual Parton Model \cite{16}. The model is based on the large $N$ expansion of QCD but it is largely phenomenological and describes most of the soft hadronic physics rather well. Inclusion of hard, perturbative, physics has been done in various ways. In the quark–gluon string model, multiparticle production is related to the interchange of multiple strings which break and subsequently hadronize.

In this model one can most easily understand the expected changes on the behavior of hadronic collisions at high energies. It is more convenient to work in the plane transverse to the collision. In this plane, strings are seen as small circles of fixed radius, $r$. As the energy increases, the number of strings interchanged increases and the total area occupied by strings increase. At high energy, strings start to overlap and fuse together. For high enough string density, $n_c$, strings may percolate in a second order phase transition, i.e. continuous paths of strings are formed in the collision area. Since the number of independent strings is reduced after the fusion one expects a depletion on the number of particles produced, i.e. a reduction on the multiplicity.

In the QGS the multiplicity grows with energy as $n(s) \sim s^\Delta$, where $\Delta$ is related to the intercept of the soft pomeron \cite{15}. In the case of percolation, the reduction of multiplicity is given by \cite{17,14}

$$n'(s) = n(s)\sqrt{F(\eta)},$$

(1)

where

$$F(\eta) = \frac{1 - e^{-\eta}}{\eta},$$

(2)

and the parameter $\eta$ is the fraction of the total area occupied by strings

$$\eta = \frac{\pi r^2 N_s}{\pi R^2}.$$  

(3)

\footnote{Here we consider minimum bias events, which are relevant for cosmic ray experiments. The parameter $\Delta$ may depend on the centrality of the collision}
Here $N_s$ is the number of strings produced in the collision, $r$ is the string’s transverse size, and $R$ is the total collision area. $N_s$ grows with energy as $N_s \sim s^{\Delta'}$, where $\Delta'$ is the intercept of the soft pomeron [15]. Therefore at large $\eta$ the total multiplicity grows with energy as

$$n'(s) \sim s^{\Delta - \Delta'/2}. \quad (4)$$

This reduction of multiplicity is not exclusive of the percolation of strings. For instance, in perturbative QCD a reduction in the number of jets produced as the energy increases is also expected [10]. At high energy the number of jets produced grows with energy as $n(s, p_t^2) \sim (s/(4p_t^2))^{\Delta_H}$, where $\Delta_H$ is the intercept of the hard pomeron and $p_t$ is the transverse momentum of the jet. A (mini)jet occupies a transverse area of order $\pi/p_t^2$, since the number of jets increases rapidly with energy, saturation occurs when the area occupied by the jets equals the total transverse area [10,14]:

$$\frac{n(s, p_t^2)\pi/p_t^2}{\pi R^2} = 1, \quad (5)$$

which implies

$$n(s, p_t^2) \sim s^{\Delta_H}. \quad (6)$$

Both Eqs. (4,6) have been checked directly in Monte Carlo simulation [11,12]. Surprisingly, for nucleus–nucleus collisions the reduction in the power of multiplicity growth with energy is of the same order both for the case of string fusion and of shadowing. The power changes from $\sim 0.24$ to $\sim 0.19$. In general parton saturation, shadowing, string fusion, or percolation will produce the effect of reduction of the multiplicity although we expect the degree of this reduction to be model dependent.

For cosmic ray showers, the rate of change of the multiplicity with energy is directly related to the change of the elongation rate. This has been known for a long time as the elongation rate theorem [18]. The elongation rate theorem can be deduced easily, as it follows from a scaling argument. Let $X_{\text{max}}(E)$ be the maximum depth of the shower produced by a primary of energy $E$. On average, the first interaction occurs at depth $\lambda$, the mean free path
of the initial particle. In this first interaction the initial particle splits into \( n(E) \) particles each carrying an average energy \( E/n(E) \). Therefore, we have

\[
X_{\text{max}}(E) = \lambda + X_{\text{max}}(E/n(E)).
\]  

(7)

Assuming that \( X_{\text{max}}(E) \) depends logarithmically on energy we get

\[
X_{\text{max}}(E) = A \log_{10}(E/n(E)) + B,
\]  

(8)

where \( A = X_0 \ln 10 \) and \( B \) are constants. \( X_0 = 37 \text{ gr/cm}^2 \) is the electromagnetic radiation length. If we now assume that \( n(E) \propto E^\Delta \), we get

\[
X_{\text{max}}(E) = A(1 - \Delta) \log_{10}(E) + B'.
\]  

(9)

This is the elongation rate theorem. We can now directly read the elongation rate from the above equation \( D_{10} = A(1 - \Delta) \). As stated previously, a change in the multiplicity growth with energy implies a change in the elongation rate.

In Fig.(1) we show \( X_{\text{max}} \) as a function of energy for the Fly’s Eye and HIRES-MIA experiments. Data have been taken from references [1,4]. The errors shown are only statistical. An additional systematic error of \( \sim 20 \text{ gr/cm}^2 \) must be included in the data. The dash line represents our calculation for the slope parameter, \( D_{10} = 65 \text{ gr/cm}^2, (\Delta = 0.24 \text{ from our simulations}) \) for Fe–Air collision without fusion, normalized with the data at \( 6 \times 10^{17} \text{ eV} \). The dotted curve has a slope parameter \( D_{10} = 78 \text{ gr/cm}^2 \), which would imply a maximum reduction in the slope of growing of multiplicities: from \( \Delta \sim 0.24 \) to \( \Delta \sim 0.09 \). The data from HIRES–MIA is not completely consistent with the Fly’s Eye data. Statistical uncertainties are larger. The elongation rate obtained by the HIRES-MIA collaboration is very large, 95 gr/cm². Notice that the elongation rate theorem predicts an elongation rate always less than 85 gr/cm², an elongation rate larger would imply multiplicities decreasing with energy. Therefore, if the HIRES-MIA result is confirmed, a change of the composition is necessary to explain the data. In the figure we also show the energy region in which a phase transition is expected in Fe–Air collision using the string fusion model [19]. This
region corresponds to the range obtained from percolation theory, $1.1 \leq \eta \leq 1.2$, where $\eta$ is given by Eq. (3).

A word of caution is necessary in the use of the elongation rate theorem. The elongation rate theorem is based on the assumption that the energy is equally shared between the secondaries in the hadronic interaction. From this assumption immediately follows the logarithmic dependence of $X_{\text{max}}$ on the multiplicity. In realistic cases this assumption does not hold and one has to resort to simulations since no analytical formula is known for the $X_{\text{max}}$. We have parameterized for a number of models the dependence of $X_{\text{max}}$ on the change of multiplicity, see Ref. [19] for details. The results of a full Monte Carlo agree with our qualitative discussion. With this in mind we can conclude that to be able to explain the change on the elongation rate we need a change on the slope of the multiplicity from $\Delta \sim 0.24$ to an essentially flat $\Delta \sim 0.09$. The energy region for such change must be around $5 \times 10^{17}$ eV.

The experimental situation with the lateral distribution of muons $\rho_{\mu}(r)$ is clearer. Both in simulations and experiments it is found that the shape of the lateral distribution function for muons is rather independent of the primary’s energy and composition. Therefore, at fixed distance to the core, $r_0$, $\rho_{\mu}(r_0)$ is proportional to the total number of produced muons in the shower. Under rather general arguments this number scales with energy [21]

$$\rho_{\mu}(r_0) \propto N_{\mu} = AE^\beta,$$

where $A$ is a normalization constant and $\beta$ is the slope parameter. As mentioned previously, the slope parameter is found to be constant over a wide range of energies. This result is consistent with Yakutsk, Haverah Park [20], and with all the lower energy experiments.

It is rather simple to calculate the slope parameter, $\beta$, for a pionic cascade from a scaling argument similar to the elongation rate theorem. The number of muons is proportional to the number of charged pions at the maximum. The number of pions, at maximum, produced by a primary of energy $E_0$ is given by

$$N_\pi(E_0) = f_\pi n \int_0^1 dx P(x) N_\pi(xE_0),$$

7
where \( f_\pi = 2/3 \) is the charged pion fraction, \( n \) the total pion multiplicity, and \( P(x) \) is the probability of producing a pion with a fraction of energy \( x \) of the primary energy. Assuming a scaling form, \( N_\pi = AE^\beta \), we get

\[
1 = f_\pi n \int_0^1 dx P(x)x^\beta = f_\pi Z(\beta),
\]

(12)

where \( Z(\beta) \) is the spectrum-weighted momentum [21]. For a given \( P(x) = 1/n \frac{dn}{dx} \), the above equation gives an implicit equation for \( \beta \). It reduces to the textbook’s expression if we assume energy equipartition, i.e. \( P(x) = \delta(x - 1/n) \), which gives \( \beta = (1 + \log(f_\pi))/\log(n) \sim 0.82 \), for \( n \sim 10 \). For realistic models the slope parameter \( \beta \) ranges between 0.7–0.9. In Eq.(12) the multiplicity enters explicitly in the left hand of the expression but also enters implicitly since the probability \( P(x) \) must verify total probability and energy conservation. Since \( Z(\beta) \) is a monotonically decreasing function of \( \beta \) for reasonable choices of \( \frac{dn}{dx} \), a reduction of multiplicity induces a reduction on \( \beta \). Indeed this is what is observed for the \( \frac{dn}{dx} \) calculated for the model with and without fusion. The DPM model gives a value of \( \beta \sim 0.89 \) which agrees with that of the QGSJET model [3]. In the presence of fusion the slope parameter is reduced and we get \( \beta \sim 0.72–0.77 \) depending on the specific implementation of the fusion model. This number was calculated both using Eq.(12) and by direct calculation with a Monte Carlo code.

In Fig.(2), we show the density of muons at 600 m, \( \rho_\mu(600) \), as a function of the energy for the AGASA measurements given as a parameterization and the HIRES-MIA measurements, shown as triangles. Also shown are the QGSJET results for pure iron and proton. Notice that the slope parameter measured by HIRES-MIA agrees with our slope parameter for the case of fusion. Our result, a change of slope parameter from that of pure iron to 0.77 at the energy where percolation is expected, is shown for comparison only. We can see that again it is consistent with the data. The slope measured by AGASA is different from the one calculated for either proton or iron for the QGSJET model. A rapid change on composition, as suggested by the HIRES-MIA data on \( X_{\text{max}} \), would imply a kink in the data for \( \rho_\mu(600) \) at the same energy which is not seen. Instead, our result points towards a mild change on
the slope parameter, from 0.9–0.8 to 0.77 which would be hardly seen given the error bars in the data.

There are a number of additional predictions in our scenario. The average $p_t$ in hadronic collisions should increase in the case of string fusion by about 10 – 20 %. This would produce flatter lateral distribution for the muon densities which could be observed. A particularly well–suited place to look for this effect would be in inclined showers. Inclined showers are composed essentially of high energy muons, and therefore are more sensitive to changes on the first hadronic interactions [22]. Given the current systematical and statistical errors we can not conclude that indeed cosmic ray experiments are observing a saturation of gluons or percolation of strings in hadronic interactions. High quality data with large statistics, as the expected from HIRES and the Pierre Auger observatories, are needed. However it is suggestive that all cosmic ray existing data are consistent with such interpretation. RHIC and LHC will measure the total multiplicity in the relevant energy region and ascertain whether a strong reduction of multiplicity takes place or not. But in any case, even if the change of composition is real, these effects must be taken into account in a complete simulation of cosmic ray showers.

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FIG. 1. Depth of shower maximum as a function of the logarithm of the primary energy as measured by Fly’s Eye (circles) and HIRES–MIA (triangles). Full lines are fits to the data. Dashed and dotted lines are our prediction for a strong reduction on the multiplicity at an energy of $\sim 6 \times 10^{17} \text{ eV}$. Arrows mark the expected region where percolation occurs for Fe–Air collisions in the string fusion model.
FIG. 2. Logarithm of the muon density at 600 m as a function of the logarithm of the energy as measured by HIRES–MIA (triangles) and AGASA (dashed line). Dotted lines are the errors of the AGASA parameterization. Also shown the prediction for the QGSJET model for pure iron (upper full line) and proton (lower full line) and our prediction for a change of slope as given in the text (marked). The arrows mark the position where percolation occurs for Fe–Air collisions in the string fusion model.