On the Flux Vacua in F-theory Compactifications

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We study moduli stabilization of the F-theory compactified on an elliptically fibered Calabi-Yau fourfold. Our setup is based on the mirror symmetry framework including brane deformations. The complex structure moduli dependence of the resulting 4D $\mathcal{N} = 1$ effective theory is determined by the associated fourfold period integrals. By turning on appropriate $G$-fluxes, we explicitly demonstrate that all the complex structure moduli fields can be stabilized around the large complex structure point of the F-theory fourfold.

I. INTRODUCTION

String theory compactifications to four dimensional spacetime provide a multitude of massless scalar fields. Unless these extra moduli fields are stabilized, one cannot predict anything for low energy physics including gravity. Moreover, the recent observational data for the acceleration of the universe motivated us to construct de Sitter vacua from a UV-complete quantum theory of gravity. Under these circumstances, the moduli stabilization and comprehensive study of flux vacua have become one of the major topics in string theory.

In the moduli stabilization, determination of the scalar potential of 4D $\mathcal{N} = 1$ effective theories arising from spacetime compactifications is of particular interest. In the language of 4D $\mathcal{N} = 1$ supersymmetry, there are two kinds of contributions to the scalar potential of moduli fields, namely the Kähler potential and the superpotential. The main problem of string compactifications is how to derive these quantities quantum mechanically from the geometry of internal compact spaces.

On the other hand, the mirror symmetry in string theory is known to be a useful tool to understand exact properties of moduli fields of geometries as first demonstrated to the quintic Calabi-Yau threefold in [1]. As has been explicitly performed in the literature, mirror symmetry can be applied to consider the closed string moduli stabilization. Inclusion of open string sector in the presence of the brane for the compact Calabi-Yau manifolds was initiated in [2] and has been subsequently applied in many contexts. In this framework, a brane is fixed on a specific submanifold and the system does not have a continuous open string moduli dependence. This means that the effective superpotential due to the wrapped branes cannot be evaluated from this kind of undeformed setup.

For the case of compact Calabi-Yau threefolds, the inclusion of brane deformations was first carried out in [3]. By using a Hodge theoretic approach, they computed the brane superpotential depending on both open and closed string moduli. Thereafter, alternative and more efficient methods to evaluate the brane superpotential has been constructed (see [4–7] for details). Remarkably, these generalizations have led to a duality between open string on a threefold with branes and closed string on a fourfold without branes, which can be naturally incorporated into the framework of the F-theory [8].

The F-theory conjecture implies that the physics of Type IIB string compactifications with branes on a complex three-dimensional Kähler manifold can be encoded in the geometry of an elliptically fibered Calabi-Yau fourfold. In contrast to various string compactifications on Calabi-Yau threefolds, the moduli stabilization of F-theory has not been fully established. The aim of this work is to fill this gap by utilizing the mirror symmetry techniques to study the F-theory vacua in the large complex structure limit, where the dynamics of moduli fields has not been investigated explicitly. For other earlier attempts in a similar spirit, see [9–11] initiated the M-theory and F-theory compactifications with $G_4$ fluxes, [12] investigated the orientifold limit [13, 14] of F-theory and [15] based on the K3 $\times$ K3 backgrounds.

II. F-THEORY COMPACTIFICATIONS INCLUDING FLUXES

First we describe basic ingredients for spacetime compactification in the F-theory framework. For more details, we refer the reader to [16].

A. F-theory on Calabi-Yau Fourfolds

Let us consider a class of 4D $\mathcal{N} = 1$ effective theories arising from F-theory compactified on the elliptically fibered Calabi-Yau fourfold $X_4 \rightarrow B_3$. Here, $B_3$ is a complex three-dimensional Kähler base space with positive curvature. This setup can be also regarded as a Type IIB string theory compactified on $B_3$ with an axio-dilaton which varies over $B_3$ holomorphically.

In the F-theory perspective, the Kähler potential for complex structure moduli fields in 4D $\mathcal{N} = 1$ effective
theories can be represented by
\[ K = - \ln \int_{X_4} \Omega \wedge \overline{\Omega}, \]  
(1)
where \( \Omega \) denotes a holomorphic \((4, 0)\)-form on \( X_4 \). Here and in what follows, we have adopted the reduced Planck unit \( M_{\text{Pl}} = 2.4 \times 10^{18} \text{ GeV} \approx 1 \). It is also well-known that F-theory admits a superpotential of the form
\[ W = \int_{X_4} G_4 \wedge \Omega, \]  
(2)
in the presence of non-zero four-form fluxes \( G_4 \). This expression is inherited from a duality between F-theory and M-theory [9, 16–18]. To guarantee the compactness expression is inherited from a duality between F-theory and M-theory [9, 16–18]. To guarantee the compactness condition given by
\[ \text{volume of a background and is in general a nontrivial structure [21]. This additional term corresponds to the hypermultiplets in this determination is due to the lack of understanding about the quantum moduli space of hypermultiplets (see [20] for recent developments) and the possibility of its consistent reduction to the 4D \( \mathcal{N} = 1 \) supergravity formulation in general region of the moduli space. Here we simplify the situation and only add a classical term \(-2 \ln \mathcal{V}\) to the Kähler potential and assume that the radius of the background manifold is sufficiently large so that the classical Kähler moduli space has a no-scale structure [21]. This additional term corresponds to the volume of a background and is in general a nontrivial function of the Kähler moduli and the mobile D3-brane moduli [16]. We assume that \( \mathcal{V} \) will be stabilized at a particular constant after the complex structure moduli stabilization, as first demonstrated in [22].

**B. Moduli dependence**

First we will describe general aspects of complex structure moduli dependence in F-theory compactifications.

More concrete expressions based on a fixed background will be presented in the next subsection.

For a Calabi-Yau fourfold \( X_4 \) with \( h^{3,1}(X_4) \) complex structure moduli, the period integrals of holomorphic \((4, 0)\)-form \( \Omega \) defined by
\[ \Pi_i = \int_{\gamma^i} \Omega \]  
(5)
encode a closed string moduli dependence of the system. Here, \( \gamma^i \) with \( i = 1, \ldots, h^{3,1}(X_4) \) denote a basis of primary horizontal subspace of \( H_4(X_4) \). In terms of these fourfold periods, the Kähler potential for the complex structure moduli (1) can be written as
\[ K = - \ln \left( \sum_{i,j} \Pi_i \eta^{ij} \Pi_j \right), \]  
(6)
where we have introduced a moduli independent intersection matrix \( \eta^{ij} \) and a dual basis \( \hat{\gamma}^j \) in \( H^4_H(X_4) \) as
\[ \eta^{ij} = \int_{X_4} \hat{\gamma}^i \wedge \hat{\gamma}^j, \quad \int_{\gamma^i} \hat{\gamma}^j = \delta^{ij}. \]  
(7)

Now we consider turning on a class of \( G_4 \) fluxes whose integer quantum numbers are given by
\[ n_i = \int_{\gamma^i} G_4. \]  
(8)
These fluxes generate a superpotential for the complex structure moduli of the form
\[ W = \sum_{i,j} n_i \Pi_j \eta^{ij}. \]  
(9)

Note that our choice of \( G_4 \) fluxes (8) only involved with \( H^4_H(X_4) \). In general, there exists additional contributions to the system from other subspaces of \( H^4(X_4) \) (see e.g. [23]). More rigorous treatment for the couplings arising from these remaining \( G_4 \) fluxes would be indispensable for studying the stabilization of Kähler moduli fields.

**C. Topological data**

As a simplest example of a fourfold \( X_4 \), we consider an elliptically fibered Calabi-Yau fourfold \( X_4^* \) which has been constructed in [4] from the quintic Calabi-Yau threefold with one toric brane (see also [6]). For details about F-theory fourfold construction, we refer the reader to [5] where a general analysis about the mirror pairs for the elliptic Calabi-Yau fourfolds has been clarified. Note that not every Calabi-Yau threefold can be uplifted to the consistent F-theory fourfold background in these prescriptions. As mentioned in [5], the existence of an elliptic fibration structure in the mirror of the underlying threefold is crucial for the fourfold uplifting.
The period integrals (5) for the fourfold $X^*_f$ have been obtained in [4, 6] by using toric geometry techniques and the result is

$$\Pi_1 = 1, \quad \Pi_2 = z, \quad \Pi_3 = -z_1, \quad \Pi_4 = S,$$
$$\Pi_5 = 5Sz, \quad \Pi_6 = \frac{5}{2}z^2, \quad \Pi_7 = 2z_1^2, \quad \Pi_8 = -\frac{5}{2}S^2 - \frac{5}{3}z_3^3,$$
$$\Pi_9 = -\frac{2}{3}z_1^3, \quad \Pi_{10} = -\frac{5}{6}z_3^3, \quad \Pi_{11} = \frac{5}{6}S^2 z^2 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4,$$  

(10)

where we have ignored further possible corrections to the leading interactions. The complex structure moduli of the fourfold $z, z - z_1, S$ are originated from a bulk quintic modulus, a brane modulus and the axio-dilaton in Type IIB description, respectively.

We redefined a logarithm of a standard complex potential for moduli fields takes a form

$$\eta = \left( \begin{array}{ccccc} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & I_3 & 0 \\ 0 & 0 & \bar{\eta} & 0 & 0 \\ 0 & I_3 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{array} \right), \quad \bar{\eta} = \left( \begin{array}{c} 0 \frac{1}{3} \frac{1}{2} \\ \frac{1}{3} \frac{1}{2} 0 \\ \frac{1}{3} \frac{1}{2} 0 \\ \frac{1}{3} \frac{1}{2} 0 \\ 0 0 -\frac{1}{2} \end{array} \right),$$

(11)

and the Euler characteristic of the background is given by $\chi(X^*_f) = 1860$.

III. ILLUSTRATIVE EXAMPLE OF MODULI STABILIZATION

A. Effective theory for moduli fields

Here we describe the explicit form of the 4D $\mathcal{N} = 1$ effective potentials for moduli fields arising from F-theory compactified on $X^*_f$. Substituting the fourfold data (10) and (11) into (6), one can easily check that the Kähler potential for moduli fields takes a form

$$K = -\ln \left[ -i(S - \bar{S}) \right] - \ln \bar{Y} - 2\ln V,$$  

(12)

where

$$\bar{Y} = \frac{5i}{6}z - \bar{z}^3 + \frac{i}{S - \bar{S}} \left( \frac{5}{12}z - \bar{z}^4 - \frac{1}{6}(z_1 - \bar{z}_1)^4 \right),$$

(13)

and we have added the simplified Kähler moduli sector. Note that our simplification for Kähler moduli fields does not affect the later discussion about the vacuum structure of F-theory compactifications, as long as the masses of Kähler moduli fields are significantly smaller than the other moduli fields.

Similarly, the superpotential can be written as

$$W = n_{11} + n_{10}S + n_8z + n_6Sz + \frac{5}{2} \left( \frac{n_5 + 2n_6}{3} \right)z^2 - \frac{5n_4}{6}z^3 - n_2 \left( \frac{5}{2}S^2 + \frac{5}{3}z_3^3 \right) - n_9z_1 - \frac{n_7}{2}z_1^2$$
$$- \frac{2n_3}{3}z_1^3 + n_1 \left( \frac{5}{6}S^2 z^2 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4 \right),$$  

(14)

and the tadpole cancellation condition takes a form

$$\frac{1860}{24} = n_{12} + n_{11} + n_2n_8 + n_3n_9 + n_4n_{10}$$
$$+ \left( \frac{n_5 + n_6}{5} \right)n_9 - \frac{n_7}{8}.$$  

(15)

Obviously, $n_7$ must be $2+4k$ with $k \in \mathbb{Z}$ in order to satisfy the condition (15) while preserving the integrality of flux quanta. In a similar reason, $n_5 + n_6$ or $n_6$ is constrained to be $5k'$ with $k' \in \mathbb{Z}$.

B. F-theory flux vacua

Let us study the extremal conditions of moduli fields $\Phi^I = (z, z_1, S)$. The F-term scalar potential of our 4D $\mathcal{N} = 1$ effective theory for moduli fields has a form

$$V = e^K \left( K^{I\bar{J}} D_I W D_J \overline{W} \right),$$

(16)

where $D_I = \partial_I + (\partial_I K)$ and $K^{I\bar{J}}$ is the inverse of the Kähler metric given by $K_{I\bar{J}} = \partial_I \partial_{\bar{J}} K$. Note that the no-scale structure of the Kähler moduli fields [21] is preserved at the classical level and a term proportional to $-3|W|^2$ in the standard 4D $\mathcal{N} = 1$ formula is canceled. Here we also define

$$F^I \equiv K^{I\bar{J}} D_J \overline{W},$$

(17)

for later convenience. In this notation, the extremal conditions for moduli fields become

$$e^{-\frac{K}{\partial^{\phi}}} \frac{\partial V}{\partial^{\phi}} = \left[ K_{I\bar{J}L} - \partial_I K_{J\bar{L}} + K_{I\bar{J}} K_{\bar{L}J} \right] F^I F^\bar{L}$$
$$+ \overline{W} F_{IJ} + (K_{I\bar{J}} - K_{J\bar{I}}) F^{I} \overline{W} + K_{I\bar{J}} F^I W = 0.$$  

(18)

Here we focus on the self-dual $G_4$ fluxes satisfying

$$G_4 = *_{\chi} G_4,$$  

(19)

which correspond to the imaginary self-dual three-form fluxes in Type IIB compactifications. In our model, imposing the self-duality condition is equivalent to set $n_2 = n_3 = n_4 = n_8 = n_9 = n_{10} = 0$. In this setup, our $\mathcal{N} = 1$ effective theory has a solution to the $F$-term.
conditions $F^I = 0$, where the scalar potential becomes zero and the values of the moduli fields are fixed as

\[
\begin{align*}
\text{Re} z &= \text{Re} z_1 = \text{Re} S = 0, \\
\text{Im} z &= \left(\frac{6n_{11}}{5n_1}\right)^{1/4} \frac{2\sqrt{n_6}}{(8n_6(n_5 + n_6) - 5n_z^2)^{1/4}}, \\
\text{Im} z_1 &= \left(\frac{30n_{11}}{n_1}\right)^{1/4} \frac{n_5}{\sqrt{n_6}(8n_6(n_5 + n_6) - 5n_z^2)^{1/4}}, \\
\text{Im} S &= \left(\frac{6n_{11}}{5n_1}\right)^{1/4} \frac{n_5}{\sqrt{n_6}(8n_6(n_5 + n_6) - 5n_z^2)^{1/4}}.
\end{align*}
\]

(20)

For example, there exists a Minkowski vacuum with $n_{D3} = 0$ in the following choice of non-zero $G_4$ fluxes:

\[n_1 = 1, n_5 = 15, n_6 = 10, n_7 = 2, n_{11} = 28.\]  
(21)

The values of the moduli fields (20) in this vacuum are

\[
\begin{align*}
\text{Re} z &= \text{Re} z_1 = \text{Re} S = 0, \\
\text{Im} z &\simeq 2.28, \quad \text{Im} z_1 &\simeq 1.14, \quad \text{Im} S &\simeq 1.71,
\end{align*}
\]

(22)

and the vacuum expectation value of the superpotential becomes $W \simeq -72.97$. One can easily confirm that the mass eigenvalues of moduli fields are positive definite as

\[
\nu^{-2}(91.30, 35.05, 3.94, 2.96, 0.09, 0.07),
\]

(23)

which means that all the complex structure moduli have been completely stabilized.

IV. CONCLUSIONS

It has been known that the effective superpotential and the axio-dilaton dependence of Type IIB compactifications can be reformulated into a geometry and fluxes in F-theory. Meanwhile, exact calculations in such a situation has been studied in a framework of mirror symmetry with or without branes. In this work, we have shown that topological data extracted by mirror symmetry techniques can be directly applied to the F-theory compactifications. Especially we have demonstrated that all the complex structure moduli can be stabilized around the large complex structure point of F-theory fourfold.

Throughout this work, we have only focused on classical interactions of moduli fields. This means that we have not fully utilized the power of mirror symmetry and further quantum corrections to the effective couplings can be also easily calculated. It would be interesting to study the vacuum structure of F-theory including these corrections, which can be also computed as in [24].

Moreover, it would be fascinating to check whether the Kähler moduli can be stabilized as in the LARGE Volume Scenario [25] or the scenario of the Kachru, Kallosh, Linde and Trivedi [22], once our treatment of the Kähler moduli sector is extended.

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