On the ambiguities in the effective action in Lorentz-violating gravity

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 abstract

We investigate the occurrence of ambiguities for Lorentz-violating gravitational Chern-Simons term. It turns out that this term is accompanied by a coefficient depending on an undetermined parameter, due to an arbitrariness in the choice of the conserved current.

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I. INTRODUCTION

The possibility of Lorentz symmetry breaking which is being intensively discussed nowadays, opens a wide range for new physical phenomena [1]. Besides the generation of new Lorentz-breaking couplings, one important consequence of that investigation is the ambiguity found in the finite contributions to the effective action [2]. Recently, this ambiguity was shown to appear in a very specific form, that is, it manifests in a finite but otherwise undetermined extra term in the one-loop effective action of the Lorentz-breaking electrodynamics within the functional integral formalism [3]. However, as in the framework of the functional integral method there are different approaches for calculating the one-loop effective action, a natural question is whether such extra term arises within different schemes of the one-loop calculations.

Another natural problem is the study of the possible Lorentz-breaking terms in gravity models. In this context, the gravitational Chern-Simons term, firstly introduced in [4], was shown to arise as a quantum correction in the theory of a spinor field coupled to gravity in a Lorentz-breaking manner, both in the linearized approximation [5] as well as in the full-fledged theory [6]. Further, a number of physical aspects of this term, such as, properties of the energy-momentum tensor and other conserved currents, and the possible black hole solutions, were studied [7].

In this paper, we carry out the calculation of the gravitational Chern-Simons term using functional integral approach similarly to [3], but securing that the gravitational gauge invariance is respected.

The structure of the paper looks like follows. In the section 2, a contribution to the effective action generated by the modification of an integral measure is discussed. The section 3 is devoted to obtaining of the perturbative contribution to the Chern-Simons action. In the Summary, the results are discussed.

II. GRAVITATIONAL CHERN-SIMONS ACTION

Let us, following [3], describe the functional integral approach for the theory involving spinors coupled to gravity in a Lorentz-breaking manner. The action in which we are interested includes a Lorentz-breaking term proportional to a constant vector \( b^\mu \),

\[
S = \int d^4x e \left( i \frac{e^\mu_a \bar{\psi} \gamma^a \gamma_5 D_\mu \psi - m \bar{\psi} \psi - b_\mu J_\mu^5 \right),
\]

where \( e^\mu_a \) is the tetrad (vierbein) and \( e \equiv \det e^\mu_a \). The covariant derivative is given by

\[
D_\mu \psi = \partial_\mu \psi - i \omega_\mu \bar{\psi},
\]
where $\omega_\mu = \frac{1}{4} \omega_\mu^{abc} \sigma_{bc}$ is the spin connection and $\sigma_{bc} = \frac{i}{2} [\gamma^b, \gamma^c]$. The current is chosen as

$$j_5^\mu = e^\mu_a \bar{\psi} \gamma^a \gamma_5 \psi + K^\mu,$$

(3)

with a convenient definition for $K^\mu$ to be specified later.

Within this work we consider the gravitational field to be purely external. The generating functional for this theory looks like

$$Z = \int D\psi D\bar{\psi} \exp(iS).$$

(4)

Under the chiral transformations of the spinor field

$$\psi(x) \to \exp[i\alpha(x)\gamma_5] \psi(x),$$
$$\bar{\psi}(x) \to \bar{\psi}(x) \exp[i\alpha(x)\gamma_5],$$

(5)

the integral measure $D\psi D\bar{\psi}$ is corrected by the multiplier

$$\Delta = \exp\left\{ -2i \int d^4x \alpha(x) \lim_{M \to \infty} \text{Tr} \left[ \gamma_5 e^{-\left(e^\mu_a \gamma^a D_\mu / M\right)^2} \right] \right\},$$

(6)

where the covariant derivative is given by (2). Proceeding as in [12, 13], we arrive at

$$\Delta = \exp\left( -\frac{i}{384\pi^2} \int d^4x \alpha(x) \epsilon^{\mu\lambda\rho\sigma} R_{\mu\nu a b} R_{\lambda\rho \sigma}^{ab} \right).$$

(7)

It is easy to see (cf. [4]) that

$$\frac{1}{4} \epsilon^{\mu\nu\lambda\rho} R_{\mu\nu a b} R_{\lambda\rho \sigma}^{ab} = \partial_\mu L^\mu,$$

(8)

where

$$L^\mu = \epsilon^{\mu\nu\lambda\rho} \left( \omega_{\nu ab} \partial_\lambda \omega_\rho^{ba} - \frac{2}{3} \omega_{\nu a b} \omega_\lambda^{bc} \omega_\rho c a \right)$$

(9)

is the Chern-Simons current.

By choosing $\alpha(x) = -x_\mu b^\mu$ and integrating by parts, we then find

$$\Delta = \exp\left( -\frac{i}{96\pi^2} \int d^4x b_\mu \epsilon^{\mu\lambda\rho} \left[ \omega_{\nu ab} \partial_\lambda \omega_\rho^{ba} - \frac{2}{3} \omega_{\nu a b} \omega_\lambda^{bc} \omega_\rho c a \right] \right).$$

(10)

The argument of the exponential exactly reproduces the form of the Chern-Simons term from [4, 5, 6] (see also [13] for a general discussion of gravitational anomalies).

Under the transformations (5) the action is changed to

$$S = \int d^4x \left[ \frac{i}{2} e^\mu_a \bar{\psi} \gamma^a D_\mu \psi - e^\mu_a (\partial_\mu \alpha) \bar{\psi} \gamma^a \gamma_5 \psi - m \bar{\psi} e^{2i\alpha \gamma_5} \psi - b_\mu j_5^\mu \right].$$

(11)
Let us now obtain the explicit form of the current \( j_5^\mu \) introduced in Eq. (3). It is well known that its first term, that is, \( e^\mu_a \bar{\psi} \gamma^a \gamma_5 \psi \), is gauge invariant. At the same time, the requirement of gauge invariance of the action imposes the restriction that the integral of \( K^\mu \) must be gauge invariant, although \( K^\mu \) itself is not necessarily invariant. It is easy to verify (details of the discussion can be found in [4]), that, in the case of gravity, the acceptable form of this vector is the Chern-Simons topological current

\[
e K^\mu = -C e^{\mu \nu \lambda \rho} \left( \omega_{\nu ab} \partial_\lambda \omega^b_\rho - \frac{2}{3} \omega_{\nu ab} \omega^e_\lambda \omega^b_e \omega^e_\rho \right),
\]

where \( C \) is some constant and \( e^{\mu \nu \lambda \rho} = e e^\mu_a e^\nu_b e^\lambda_c e^\rho_d e^{abcd} \), which evidently will give rise to a gravitational Chern-Simons term added in the action. As a result, the generating functional of the theory, in which the modified measure and the additional term generated by the conserved current are taken into account, can be written as

\[
Z = \exp \left( -\frac{i}{96\pi^2} \int d^4x b_\mu L^\mu \right) \times \exp \left( iC \int d^4x b_\mu L^\mu \right) \times \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left[ i \int d^4x e^{\frac{i}{2} e^\mu_a \bar{\psi} \gamma^a D_\mu \psi - m \bar{\psi} e^{2i x \cdot b \gamma_5} \psi} \right].
\]

By performing the fermionic integration, \( Z = e^{i T r [\omega]} \), the whole one-loop effective action \( \Gamma_{\text{eff}}[\omega] \) of the gravitational field is given by

\[
\Gamma_{\text{eff}}[\omega] = \left( C - \frac{1}{96\pi^2} \right) \int d^4x b_\mu e^{\mu \nu \lambda \rho} \left( \omega_{\nu ab} \partial_\lambda \omega^b_\rho - \frac{2}{3} \omega_{\nu ab} \omega^e_\lambda \omega^b_e \omega^e_\rho \right) + S'_{\text{eff}}[\omega],
\]

with

\[
S'_{\text{eff}}[\omega] = -i T r \ln \left( \frac{i}{2} e^\mu_a \gamma^a \bar{D}_\mu - m e^{2i x \cdot b \gamma_5} \right),
\]

where \( T r \) stands for the trace over Dirac matrices as well as trace over the integration in momentum and coordinate spaces.

**III. PERTURBATIVE INDUCTION OF THE CHERN-SIMONS ACTION**

To complete the calculation of the one-loop gravitational Chern-Simons action, we must compute \( S'_{\text{eff}}[\omega] \) up to first order in the Lorentz-breaking vector \( b_\mu \) which is expressed as

\[
S'_{\text{eff}}[\omega] = -i T r \ln \left( \frac{i}{2} e^\mu_a \gamma^a \bar{D}_\mu - m - 2 i e m x \cdot b \gamma_5 + e^\mu_a \gamma^a \omega_\mu \right).
\]
Here we use the weak field approximation in which the vierbein and the connection are expressed in terms of the metric fluctuation $h_{\mu \nu}$ (which is the only dynamical field in the weak field approximation of gravity) as $e_{\mu a} = \eta_{\mu a} + \frac{1}{2} h_{\mu a}$ and $\omega_{\mu ab} = \frac{1}{2} (\partial_b h_{\mu a} - \partial_a h_{\mu b})$, thus we have

$$S'_{\text{eff}}[h] = -i \operatorname{Tr} \ln \left[ i \partial - m - 2i m x \cdot b \gamma_5 - \frac{i}{4} h_{\mu \nu} \gamma^\mu \partial^\nu - \frac{i}{16} (h_{\mu a} \partial_\nu h_a^\alpha) \Gamma^{\mu \nu \lambda} \right], \quad (17)$$

where $\Gamma^{\mu \nu \lambda}$ in the antisymmetrized product of three Dirac matrices, i.e., $\Gamma^{\mu \nu \lambda} = \frac{1}{6} \left( \gamma^\mu \gamma^\nu \gamma^\lambda - \gamma^\mu \gamma^\lambda \gamma^\nu + \gamma^\nu \gamma^\lambda \gamma^\mu - \gamma^\nu \gamma^\mu \gamma^\lambda - \gamma^\lambda \gamma^\mu \gamma^\nu - \gamma^\lambda \gamma^\nu \gamma^\mu \right)$. Up to the field independent factor $-i \operatorname{Tr} \ln (i \partial - m - 2i m x \cdot b \gamma_5)$, which may be absorbed in the normalization of the generating functional, we rewrite the Eq. (17) as

$$S_{\text{eff}}^{(n)}[h] = i \operatorname{Tr} \sum_{n=1}^{\infty} \frac{1}{n} \left\{ \frac{1}{i \partial - m - 2i m x \cdot b \gamma_5} \left[ \frac{i}{4} h_{\mu \nu} \gamma^\mu \partial^\nu + \frac{i}{16} (h_{\mu a} \partial_\nu h_a^\alpha) \Gamma^{\mu \nu \lambda} \right] \right\}^n. \quad (18)$$

As we are interested in the radiative induction of the gravitational Chern-Simons action, here and in what follows we restrict ourselves to the second order in $h_{\mu \nu}$ and first order in $b_\mu$. Firstly, let us analyze the terms that come from $n = 1$, as follows

$$S_{\text{CS}}^{(1)}[h] = i \operatorname{Tr} \frac{1}{i \partial - m - 2i m x \cdot b \gamma_5} \frac{1}{16} (h_{\mu a} \partial_\nu h_a^\alpha) \Gamma^{\mu \nu \lambda}. \quad (19)$$

In order to carry out the traces over the integration in space we must use the prescription $i \partial_\mu \rightarrow p_\mu$ and $x_\mu \rightarrow i \frac{\partial}{\partial p_\mu}$, which is more convenient due to the presence of $\alpha(x) = -x \cdot b$, so that we get

$$S_{\text{CS}}^{(1)}[h] = i \int d^4 x \int \frac{d^4 p}{(2\pi)^4} S(p) (-2m) \frac{\partial}{\partial p^\beta} \gamma_5 S(p) \frac{i}{16} (h_{\mu a} \partial_\nu h_a^\alpha) \Gamma^{\mu \nu \lambda}, \quad (20)$$

where the symbol $\operatorname{tr}$ means that the trace is only over Dirac matrices and $S(p) = (\not{p} - m)^{-1}$. Now, by using the identity

$$\frac{\partial}{\partial p^\beta} S(p) = S(p) \gamma_\beta S(p), \quad (21)$$

we have

$$S_{\text{CS}}^{(1)}[h] = i \int d^4 x \ h_{\mu \nu} \Pi^{\mu \nu \beta}_{\alpha} h_{\alpha \beta}, \quad (22)$$

where

$$\Pi^{\mu \nu \beta}_{\alpha} = -\frac{m}{4} \operatorname{tr} \int \frac{d^4 p}{(2\pi)^4} S(p) \gamma_5 S(p) \Gamma^{\mu \alpha \lambda} \frac{\partial}{\partial \lambda} \gamma_\beta. \quad (23)$$

Finally, let us single out the terms coming from $n = 2$, given by

$$S_{\text{CS}}^{(2)}[h] = \frac{i}{2} \operatorname{Tr} \frac{1}{i \partial - m - 2i m x \cdot b \gamma_5} \frac{i}{i \partial - m} \frac{i}{4} h_{\mu \nu} \gamma^\mu \partial^\nu - \frac{1}{i \partial - m} \frac{i}{4} h_{\alpha \beta} \gamma_\alpha \partial_\beta \gamma^\beta + \frac{i}{2} \operatorname{Tr} \frac{1}{i \partial - m} \frac{i}{4} h_{\mu \nu} \gamma^\mu \partial^\nu - \frac{1}{i \partial - m} \frac{i}{4} h_{\alpha \beta} \gamma_\alpha \partial_\beta. \quad (24)$$
Besides the above prescription, we also use the identity \([8, 9, 10, 11]\)

\[
h_{\mu\nu}(x)S(p) = S(p - i\partial)h_{\mu\nu}(x)
\]  

in order to disentangle the traces over \(x_\mu\) and \(p_\mu\), and thus we arrive at

\[
S^{(2)}_{CS}[h] = \frac{i}{2} \int d^4x \ h_{\mu\nu} \left( \Pi_1^{\mu\alpha\beta} + \Pi_2^{\mu\alpha\beta}\right) h_{\alpha\beta},
\]

where

\[
\Pi_1^{\mu\alpha\beta} = \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p)(2p^\mu - i\partial^\mu)\gamma^\nu S(p - i\partial)(2p^\alpha - i\partial^\alpha)\gamma^\beta,
\]

and

\[
\Pi_2^{\mu\alpha\beta} = \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p)(2p^\mu - i\partial^\mu)\gamma^\nu S(p - i\partial)(2p^\alpha - i\partial^\alpha)\gamma^\beta.
\]

By applying the momentum derivative on the \(p\)-functions to the right, \(\Pi_1^{\mu\alpha\beta} = \Pi_1^{\mu\alpha\beta} + \Pi_2^{\mu\alpha\beta} + \Pi_3^{\mu\alpha\beta} + \Pi_4^{\mu\alpha\beta}\) and \(\Pi_5^{\mu\alpha\beta} = \Pi_5^{\mu\alpha\beta} + \Pi_6^{\mu\alpha\beta}\), with

\[
\Pi_1^{\mu\alpha\beta} = \frac{m}{8} \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p)\gamma_5 S(p)\bar{S}(p)(2p^\mu - i\partial^\mu)\gamma^\nu S(p - i\partial)(2p^\alpha - i\partial^\alpha)\gamma^\beta,
\]

\[
\Pi_2^{\mu\alpha\beta} = \frac{m}{8} \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p)\gamma_5 S(p)\bar{S}(p)(2p^\mu - i\partial^\mu)\gamma^\nu S(p - i\partial)(2p^\alpha - i\partial^\alpha)\gamma^\beta,
\]

\[
\Pi_3^{\mu\alpha\beta} = \frac{m}{8} \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p)\gamma_5 S(p)(2p^\mu - i\partial^\mu)\gamma^\nu S(p - i\partial)\bar{S}(p - i\partial)(2p^\alpha - i\partial^\alpha)\gamma^\beta,
\]

\[
\Pi_4^{\mu\alpha\beta} = \frac{m}{8} \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p)\gamma_5 S(p)(2p^\mu - i\partial^\mu)\gamma^\nu S(p - i\partial)\bar{S}(p - i\partial)(2p^\alpha - i\partial^\alpha)\gamma^\beta,
\]

\[
\Pi_5^{\mu\alpha\beta} = \frac{m}{8} \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p)(2p^\mu - i\partial^\mu)\gamma^\nu S(p - i\partial)\gamma_5 S(p - i\partial)\bar{S}(p - i\partial)(2p^\alpha - i\partial^\alpha)\gamma^\beta,
\]

\[
\Pi_6^{\mu\alpha\beta} = \frac{m}{8} \text{tr} \int \frac{d^4p}{(2\pi)^4} S(p)(2p^\mu - i\partial^\mu)\gamma^\nu S(p - i\partial)\gamma_5 S(p - i\partial)(2p^\alpha - i\partial^\alpha)\gamma^\beta.
\]

The procedure used to evaluate the integrals \([23], [27], \) and \([28]\) follows the method developed in \([14, 15]\) for the calculation of the axial anomaly in electrodynamics, which has also been employed in the calculations of the axial anomaly in gravity \([16]\) and in supergravity \([17]\). Namely, we first write the most general tensor structure for \(\Pi^{\mu\alpha\beta} = \Pi_1^{\mu\alpha\beta} + \frac{1}{2} \Pi_2^{\mu\alpha\beta} + \frac{1}{2} \Pi_3^{\mu\alpha\beta} + \frac{1}{2} \Pi_4^{\mu\alpha\beta} + \frac{1}{2} \Pi_5^{\mu\alpha\beta} + \frac{1}{2} \Pi_6^{\mu\alpha\beta}\), in momentum space, as follows

\[
S_{CS}[h] = i \int \frac{d^4k}{(2\pi)^4} h_{\mu\nu}(k) \Pi^{\mu\alpha\beta} h_{\alpha\beta}(-k),
\]

where

\[
\Pi^{\mu\alpha\beta} = \epsilon^{\mu\lambda\rho} b_\lambda k_\rho \left( Ag^{\nu\beta} + Bk^{\nu\beta}\right).
\]
The form factor $A$ is at most logarithmically divergent and $B$ is finite. By employing the Feynman parametrization and evaluating the trace over the Dirac matrices, we obtain

$$A = \frac{3im^2}{4} \int_0^1 dx \int \frac{d^4p}{(2\pi)^4} \frac{(1 - 2x(1 - x))(p^2 - m^2)p^2 + 2x^2(1 - x)^2p^2k^2}{(p^2 - M^2)^4} - \int \frac{d^4p}{(2\pi)^4} \frac{im^2}{(p^2 - m^2)^2}$$

$$B = \frac{3im^2}{4} \int_0^1 dx (1 - 2x)^2 \int \frac{d^4p}{(2\pi)^4} \frac{(1 - 2x)p^2 - (1 - 2x(1 - x))m^2 + 2x^2(1 - x)^2k^2}{(p^2 - M^2)^4},$$

(37)

where $M^2 = m^2 - x(1 - x)k^2$. Notice that $A$ and $-k^2B$ differ at most by a divergent constant which can be adjusted to be zero by a convenient renormalization prescription. This is automatically enforced if gauge invariance holds. Indeed, as the above action must be invariant under the gauge transformation $h_{\mu\nu} \rightarrow h_{\mu\nu} + k_{\mu}\Lambda_{\nu} + k_{\nu}\Lambda_{\mu}$, by imposing the transversality condition

$$k_{\mu}\Pi^{\mu\nu\alpha\beta} = k_{\nu}\Pi^{\mu\nu\alpha\beta} = 0,$$

(38)

we obtain that $A = -k^2B$, so that

$$\Pi^{\mu\nu\alpha\beta} = \epsilon^{\mu\alpha\lambda\rho}b_{\lambda}k_{\rho} \left(-k^2g^{\nu\beta} + k^{\nu}k^{\beta}\right)B.$$

(39)

Finally, by integration over momenta and Feynman parameter, the form factor $B$ results in

$$B = \frac{m^2}{16\pi^2(k^2)^{3/2}} \left[ \sqrt{k^2} - \arctan \left( \frac{k^2}{\sqrt{4m^2 - k^2}} \right) \sqrt{4m^2 - k^2} \right].$$

(40)

The leading contribution in the above expression as $k^2/m^2$ tends to zero is $B = 1/192\pi^2$, which yields

$$S_{CS}[h] = \frac{1}{192\pi^2} \int d^4x b_{\mu} \epsilon^{\mu\nu\lambda\rho} h_{\nu\alpha} \partial_{\lambda} \left( \Box h_{\rho} + \partial^{\alpha} \partial^{\beta} h_{\rho\beta} \right),$$

(41)

in coordinates space. By expressing this Chern-Simons action in terms of the spin connection $\omega_{\mu\nu ab}$, we have

$$S_{CS}[w] = \frac{1}{96\pi^2} \int d^4x b_{\mu} \epsilon^{\mu\nu\lambda\rho} \left( \omega_{\nu ab} \partial_{\lambda} \omega_{\rho b a} - \frac{2}{3} \omega_{\nu ab} \omega_{\lambda bc} \omega_{\rho c a} \right).$$

(42)

Therefore, by taking into account only the Chern-Simons contribution to the effective action $\Gamma_{CS}[\omega]$, we can conclude that we obtain a gravitational Chern-Simons term with an undetermined constant $C$, generated by the undetermined additive term in the current. The ambiguity that we found probably can be interpreted as an implication of the gravitational anomalies.
IV. SUMMARY

We have studied the problem of ambiguities in the linearized gravity. It turns out that the mechanism of generating the ambiguities due to possibility of modifying of the conserved current by additive terms is applicable in the Lorentz-breaking gravity as well as in the Lorentz-breaking electrodynamics, with the ambiguity, depending on an arbitrary constant parameter. In principle, presence of the ambiguity in this theory is also confirmed by the fact that the results obtained in [3] and in [6] for the gravitational Chern-Simons term are different. However, this mechanism of arising of the ambiguity essentially differs from one found earlier for the Lorentz-breaking QED which was based on use of different regularization schemes for the formally divergent integrals. Presently the divergent integrals do not arise at any steps of calculations. We also expect that a similar situation will occur if we consider the complete, non-linearized expression for the gravitational Chern-Simons term.

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