Make questions as a stimulus for students to help them carry out their Polya’s step in solving problems

L N Zamnah¹, Zaenuri², Wardono³, Sukestiyarno³

¹Postgraduate Mathematics Education, Universitas Negeri Semarang, Indonesia
²Mathematics Department Faculty of Mathematics and Natural Sciences Universitas Negeri Semarang, Indonesia
³Corresponding author: nailah_lala@yahoo.co.id

Abstract. Problem solving ability is very important to be mastered by everyone. Not only because most of human life will deal with problems that need to be solved, but problem solving can also increase analytical power and can help to solve problems in various other situations. Problem-solving ability is also one of the main goals of learning mathematics. One solution to solve mathematical problems is the Polya’s step. Polya's step is a problem-solving technique which was created by mathematician named George Polya. This literature review aims to discuss to make questions as a stimulus for students to help them carry out their polya’s step in solving problems. Polya's step consists of: 1) understanding the problem; 2) devising a plan; 3) carrying out the plan; and 4) looking back.

1. Introduction

Problem-solving abilities are very important for everyone. Not only because most of human life will face the problems that need to be resolved, but the problem-solving can also increase analytical skills and can help to solve problems in various other situations. Problem solving in mathematics is important [1] which is considered to be the heart of mathematics is problem solving, because it is not only learning mathematics material but also developing thinking skills [2].

Problem-solving abilities cannot be developed properly without activities or efforts to develop these potential abilities. One effort that can be made to develop these potential abilities is through an educational program. One of the educational programs that can develop problem-solving skills is mathematics. Beigie which says that through problem-solving, students can learn about deepening their understanding of mathematical concepts by working through the issues carefully selected which use the application of mathematics to real problems [3].

The curriculum in some countries places mathematical problem-solving as one of the most important things, including the Singapore Curriculum places problem-solving as the main goal of learning mathematics by placing five interrelated components in problem-solving, namely: Skills, Concepts, Processes, Attitudes, and Metacognitions. The Hongkong Curriculum places problem-solving as a tool, so that the problem-solving approach is almost used in every learning process. The British curriculum places problem-solving at the heart of mathematics and as a cycle of processes in which there are representation, communication-reflection, interpretation-evaluation, and the use of analysis-reasoning procedures. The Dutch curriculum places problem solving as a learning approach, so it is known as RME. Australia places problem-solving as a skill or ability to be able to make a choice, to interpret, to formulate models, to investigate problem situations, and to communicate the
solutions effectively. Finland views problem-solving from various points of view: as a tool to advance mathematical thinking, as a tool of problem solving and as a process by which previously acquired data is used in new and unknown situations [4].

The implication of the importance of mathematical problem-solving abilities, during mathematics learning students should be trained to solve mathematical problems and Ulya said that the teacher's attention is needed when the process of thinking about solving problems so that students' problem solving abilities are more developed, both in the real world context and in the context of mathematics [5]. However, to make this happen we are faced with several problems. Among these problems is that there are still wrong opinions regarding problem-solving. Sometimes there is an assumption that solving mathematical problems is by using mathematical formulas, and the problems in mathematics textbooks did not present many mathematical problem-solving questions, but most of the questions presented are only to practice arithmetic skills or the use of formulas [6]. Therefore, in this article we will discuss making mathematical problem-solving questions based on the step of Polya.

2. Methods
The methodology used in this research is narrative literature review, using primary sources as the main book reference, and secondary sources of empirical research results from research results in reputable international journals and reputable national journals.

3. Results and Discussion
Problem-solving is a process that is taken to overcome the difficulties faced in order to achieve what is the goal or desired. Mathematics problem solving is complex and can be challenging, both cognitively and emotionally [7]. In mathematics learning, problem-solving is one of the results to be achieved and an ability that is expected to be mastered by students. In addition, problem solving is one of the thinking skills that teachers use to teach students how to think [8].

Problem-solving is also a process of an activity that prioritizes the importance of procedures, the strategic steps taken by students in solving problems, so that ultimately finding answers to questions. Problem solving is a complex process that requires a variety of skills to be used together [9]. Ozcan also argues that problem-solving is transferring knowledge that is already owned into new situations [10]. Bahar and Maker stated that the concept of problem solving is referred to by scientists as a high-level thinking process consisting of intellectual ability and major cognitive processes [11].

Schoenfeld's research found that knowledge and behavior are needed for solving mathematical problems and presented four categories as follows: 1) Resources (mathematical knowledge that is owned to solve problems), 2) Heuristics (planning the process of implementing problem solving on problems that are unusual or not standards; rules for solving problems), 3) Control (overall decisions regarding the selection and implementation of resources and strategies), and 4) Belief Systems (a person's "mathematical world view", a set of determinants of individual behavior) [12].

In solving mathematical problems, correct concrete steps are needed so that the answers obtained can be correct. Surya states that problem-solving ability is the ability of students to be able to understand problems through identifying elements that are known, asked, and the adequacy of elements needed to solve the problems, creating or developing strategies to solve problems and represent (with symbols, pictures, graphs, table diagrams, models etc.), Choosing or implementing strategies to find solutions to problems and checking the correctness of the solutions to problems and interpreting it [13].

Four dimensions of the basic framework in problem solving by Wu, namely: 1) reading / obtaining all information from questions; 2) use a real-life approach and common sense to solve problems; 3) mathematical concepts, mathematization and reasoning; and 4) standard and precise counting skills [14].

Polya's steps are some of the mathematical problem-solving steps created by the mathematician, George Polya. Lee Chun-Yi argues that the Polya's method is teacher-centered which has the goal of helping students explain problem-solving ideas related to the lesson [15]. Polya's simple steps and
clear problem-solving stages make Polya’s steps often used to solve mathematical problems [16]. Polya’s Stages have the advantage that students are more careful in understanding each step and the problem-solving process, and can provide a neatly structured framework to solve a long and complex problem so that it can help students organize it as an effort to solve the problems [17]. Gopinath also explained that the application of Polya’s steps requires students to think analytically so that the results of problem-solving become more efficient [18]. The results of Ersoy's research show that polya’s steps in solving mathematical problems have a positive impact on mathematical problem-solving abilities [19].

In his book, Polya describes four step in solving problem-solving, namely: understanding the problem, planning problem-solving, performing calculations to solve problems using a plan that has been compiled and reviewing or re-checking what has been done. In addition, Polya states the process carried out at each step of problem-solving through questions, namely: 1) Understanding the problem, including: is there anything that is not known about the problem?, what data is provided?, Is the data provided sufficient to be used in solving the problem?; 2) devising a plan, this step emphasizes planning for problem-solving. Some questions to make a solution plan include the following: which theory can be used to solve the questions?, from those asked about the question, try to think about whether there are similar or same questions?, can the previous methods be used for problem-solving?, have the existing data been used to solve the questions?, have the important ideas in the questions been taken into account?; 3) carrying out the plan, this step emphasizes the implementation of the planning that has been made at the stage of making a solution plan. The procedures taken are: are each step correct or not?, can you prove that it is correct?; 4) Looking back, this stage emphasizes how to check the correctness of the answers that have been obtained, the procedures that must be considered are: can the refutation be checked?, can the answer be found in other ways? [20]

After describing the stages of solving the problem according to the Polya and how the solving process is, then we will make a problem-solving questions based on the Polya’s Step.

Making problem-solving questions based on Polya can use the instructions from the solving process at each Polya’s Step that have been described. One problem solving question can contain one Polya’s Step or can contain all the Polya’s Step. The following is an example of problem-solving question based on Polya’s Step:

Question: A cube has an edge length 3 times of the height of a cuboid. The length of the cuboid is 4 times of the height of the cuboid, while the height of the cuboid is 0.5 times of the width of the cuboid and the surface area of the cuboid is $175 \, \text{cm}^2$. If you are going to find the surface area of the cube, are the data given sufficient? If sufficient, find the surface area of the cube!

The question is whether the data provided is sufficient if to find the surface area of the cube, the question is to identify the adequacy of the data and at the Polya’s Step is one way to understand the problem. Then for the question, find the surface area of the cube, it is clear that students are required to do calculations and carry out calculations, including at the Polya’s Step.

Question: Ahmad will make a play box out of plywood. The game box is in the form of a cuboid with a length of 3 times of the height while the height is 0.5 times of the width of the game box. The volume of the game box is $6000 \, \text{cm}^3$. Is the data given sufficient to find the required plywood area? If not sufficient, what data is needed? How do you find the area of the plywood? Find the area of plywood needed!

The question, is the data provided is sufficient if to find the required plywood area, If not sufficient, what data is needed is to identify sufficient data to find the required plywood area and at the Polya’s Step this is one way to understand the problem. Then for the question of how to find a plywood area is to plan how to find a solution and at the Polya’s Step is devising a plan. The question of finding the required plywood is definitely the question is to do calculations with a predetermined plan and at the Polya’s step is carrying out the plan.

Question: When studying in groups, Eva found the following problem: A pencil box in the shape of a block with a base area of $180 \, \text{cm}^2$. The length of the pencil case is equal to $1 \frac{1}{4}$ times of it width and
the height is \( \frac{2}{4} \) times of it width. Eva calculated and obtained the pencil case length is 15 cm, the width is 12 cm and the height is 9 cm. Yuli helped calculate it and got the pencil case length is 18 cm, the width is 10 cm and the height is 7.5 cm. Whose answer is correct?

From this question, it can be seen that there are two children who have obtained the length, the height and the width of a pencil box in the shape of a block and the area of the base is known. But the results of the calculations of the two children are different, so the question arises whose answer is correct. To find out whose answer is correct, it must be checked again the answers/results obtained from the two children by counting it again. This type of question can indirectly train students to reviewing the results obtained or at the Polya’s Stage which called looking back.

4. Conclusion

Questions as a stimulus for students to help them carry out their polya steps in solving problem can be made from the process carried out at each step of problem-solving through developed questions and in one mathematical problem solving question can contain one or more or even all the Polya’s Step.

Reference

[1] Özreçberoğlu N and Çağanağa Ç K, 2018 Eurasia J. Math. Sci. Technol. Educ. 14 (4) 1253
[2] Pimta S, Tayruakham S and Nuangchale P, 2009 J. Soc. Sci. 5 (4) 381
[3] Beigie D 2008 Math. Teach. Middle Sch. 13 352
[4] Anderson J 2009 "Mathematics Curriculum Development and the Role of Problem Solving”. In ACSA Conference pp. 1–9
[5] Ulya H and Retnoningsih A 2014 Int. J. Educ. Res. 2 (10) 577
[6] Sugiman, Kusumah Y S and Sabandar J 2010 Pythagoras 16 (1) 1
[7] Di Leo I and Muis K R 2020 Contemp. Educ. Psychol. 62, May p. 101879.
[8] Carson J, 2007 Math Educ 17 (2) 7
[9] Lisesi Ç K M T A 2017 E-Int. J. Educ. Res. 8 (2) 91
[10] Özcan Z Ç, 2016 Int. J. Math. Educ. Sci. Technol. 47 (3) 408
[11] Bahar A and June Maker C 2015 Eurasia J. Math. Sci. Technol. Educ. 11(6) 1531
[12] Thiangthung Y 2016 J. Adv. Humanit. Soc. Sci. 2 (5) 261
[13] Surya E, Putri F A and Mukhtar 2017 J. Math. Educ. 8 (1) 85
[14] Indriyana E and Tirta I M 2017 Int. J. Soc. Sci. Humanit. Invent. 4 (8) 3774
[15] Lee C Y and Chen M J 2015 Eurasia J. Math. Sci. Technol. Educ. 11 (6) 1547
[16] Hayyulbathin I, Winarni R and Murwaningsih T 2018 "Modification Of Polya’s Step To Solve Math Story Problem" In ICRIEMS p. 119–126.
[17] H. Apryanti, F. Ismail, and Y. Fitrianti 2015 Jurnal Pendidikan Matematika RAFA, 1(2) p. 224-243
[18] Gopinath S and Lertlit S 2017 Suranaree J. Soc. Sci. 11 (1) 47
[19] Ersoy E 2016 Online J. New Horizons Educ. 6 (2) 79
[20] Polya G 2014 “How to Solve It” list. In How to Solve It