Analysis of Strong Decays of the \( Z_c(4600) \) with the QCD Sum Rules

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Abstract

In this article, we tentatively assign the \( Z_c(4600) \) to be the \([dc]_P[u\bar{c}]_A - [dc]_A[u\bar{c}]_P\) type vector tetraquark state and study its two-body strong decays with the QCD sum rules based on solid quark-hadron duality. The predictions of the partial decay widths can be confronted to the experimental data in the future to diagnose the nature of the \( Z_c(4600) \).

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Key words: Tetraquark state, QCD sum rules

1 Introduction

Recently, the LHCb collaboration performed an angular analysis of the decays \( B^0 \rightarrow \psi K^+\pi^- \) using proton-proton collision data corresponding to an integrated luminosity of 3fb\(^{-1}\) collected with the LHCb detector, studied the \( m(J/\psi\pi^-) \) versus \( m(K^+\pi^-) \) plane, and observed two possible structures near \( m(J/\psi\pi^-) = 4200 \) MeV and 4600 MeV, respectively \cite{1}. The structure near \( m(J/\psi\pi^-) = 4200 \) MeV is close to the exotic state \( Z_c(4200) \) reported previously by the Belle collaboration \cite{2}. The structure near \( m(J/\psi\pi^-) = 4600 \) MeV is in excellent agreement with our previous prediction of the mass of the \([qc]_P[\bar{q}\bar{c}]_A - [qc]_A[\bar{q}\bar{c}]_P\) type tetraquark state with the spin-parity-charge-conjugation \( J^{PC} = 1^{--} \), \( m_V = (4.59 \pm 0.08) \) GeV \cite{3}.

In Ref. \cite{3}, we perform detailed and updated analysis of the \([sc]_S[\bar{sc}]_V + [sc]_V[\bar{sc}]_S\) type, \([sc]_P[\bar{sc}]_A - [sc]_A[\bar{sc}]_P\) type and \([qc]_P[\bar{q}\bar{c}]_A - [qc]_A[\bar{q}\bar{c}]_P\) type vector tetraquark states with the QCD sum rules based on our previous works \cite{4,5}. The predictions support assigning the \( Y(4600) \) to be the \([sc]_P[\bar{sc}]_A - [sc]_A[\bar{sc}]_P\) type vector tetraquark state, assigning the \( Y(4360/4320) \) to be the \([qc]_S[\bar{q}\bar{c}]_V + [qc]_V[\bar{q}\bar{c}]_S\) type vector tetraquark state. In Ref. \cite{5}, we choose the \([sc]_P[\bar{sc}]_A - [sc]_A[\bar{sc}]_P\) type tetraquark current to study the hadronic coupling constants in the strong decays \( Y(4600) \rightarrow J/\psi f_0(980), \eta_c\phi(1020), \chi_{c0}\phi(1020), D_s D_s, D^*_s D_s, D_s D^*_s, \psi'\pi^+\pi^- \), \( J/\psi\phi(1020) \) with the QCD sum rules based on solid quark-hadron duality. The predicted width \( \Gamma(Y(4600)) = 74.2^{+29.2}_{-21.8} \) MeV is in excellent agreement with the experimental data 68\pm11 \pm 1 \) MeV from the Belle collaboration \cite{7}, which also supports assigning the \( Y(4600) \) to be the \([sc]_P[\bar{sc}]_A - [sc]_A[\bar{sc}]_P\) type tetraquark state with \( J^{PC} = 1^{--} \). In the isospin limit, the tetraquark states with the symbolic quark structures \( c\bar{c}d\bar{u}, c\bar{c}d\bar{u}, \alpha \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}, \alpha \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \) have degenerated masses.

In this article, we tentatively assign the \( Z_c(4600) \) to be the \([dc]_P[u\bar{c}]_A - [dc]_A[u\bar{c}]_P\) type vector tetraquark state and study its two-body strong decays with the QCD sum rules based on solid quark-hadron duality by taking into account both the connected and disconnected Feynman diagrams in the operator product expansion \cite{5}, which is valuable in understanding the nature of the vector tetraquark states.

The article is arranged as follows: we obtain the QCD sum rules for the hadronic coupling constants \( G_{Z_c J/\psi\pi}, G_{Z_c \eta_c \rho}, G_{Z_c J/\psi\phi_0}, G_{Z_c \chi_{c0} \rho}, G_{Z_c D^* D^*}, G_{Z_c D D} \) and \( G_{Z_c D^* D} \) in section 2; we present the numerical results and discussions in section 3; section 4 is reserved for our conclusion.

2 The QCD sum rules for the hadronic coupling constants

Now we write down the three-point correlation functions for the hadronic coupling constants \( G_{Z_c J/\psi\pi}, G_{Z_c \eta_c \rho}, G_{Z_c J/\psi\phi_0}, G_{Z_c \chi_{c0} \rho}, G_{Z_c D^* D^*}, G_{Z_c D D} \) and \( G_{Z_c D^* D} \) in the two-body strong decays

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we have assumed that the dominant components of the a₀(980) are two-quark states [9]. We choose those currents shown in Eq.(8) to interpolate the corresponding mesons according to the standard definitions for the current-hadron couplings or the decay constants $f_{J/\psi}$, $f_\rho$, $f_\pi$, $f_{a_0}$, $f_{\chi_c}$, $f_{D}$, $f_{D^*}$, $\lambda_Z$.

\[
\begin{align*}
\langle 0 | J_{J/\psi,\mu}(0) | J/\psi(p) \rangle &= f_{J/\psi} \epsilon_{\mu \nu \rho \sigma} J^{\nu}_{\psi} \xi^\rho_{\mu} J^{\sigma}_{\psi}, \\
\langle 0 | J_{\rho,\mu}(0) | \rho(p) \rangle &= f_\rho m_\rho \xi^\rho_{\mu}, \\
\langle 0 | J_{\eta_0}(0) | \eta_0(p) \rangle &= \frac{f_{\eta_0} m_{\eta_0}^2}{2m_c}, \\
\langle 0 | J_{\eta}(0) | \eta(p) \rangle &= \frac{f_{\eta} m_{\eta}^2}{m_\eta + m_\rho}, \\
\langle 0 | J_{a_0}(0) | a_0(p) \rangle &= f_{a_0} m_{a_0}, \\
\langle 0 | J_{\chi_c}(0) | \chi_c(p) \rangle &= f_{\chi_c} m_{\chi_c}, \\
\langle 0 | J_{D}(0) | D(p) \rangle &= f_D m_D \xi^\mu_D, \\
\langle 0 | J_{D^*,\mu}(0) | D^*(p) \rangle &= f_{D^*} m_D \xi^\mu_{D^*}, \\
\langle 0 | J_{\mu}(0) | Z_c(p) \rangle &= \lambda_Z \xi^\mu_{Z_c},
\end{align*}
\]
where the $\xi_\mu$ are the polarization vectors.

At the hadronic side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators into the three-point correlation functions and isolate the ground state contributions to obtain the following results \[10\,11\].

$$\Pi_\mu^3(p, q) = \frac{f_\gamma m_\gamma^2 f_{J/\psi} m_{J/\psi} \xi \gamma_{\mu} G Z_{J/\psi} \gamma_{\nu} \varepsilon_{\mu \nu \sigma} p^\sigma p^\rho}{m_u + m_d (p^2 - m_Z^2) (q^2 - m_Z^2)} \left(-g_\mu^\beta + \frac{p_\mu p_\beta}{p^2}\right) \left(-g_\nu^\sigma + \frac{p_\nu p_\sigma}{p^2}\right) + \ldots$$

$$= \Pi(p^2, p^2, q^2) \left(-\varepsilon_{\mu \nu \sigma} p^\sigma q^\beta\right) + \ldots, \quad (10)$$

$$\Pi_\mu^2(p, q) = \frac{f_{J/\psi} m_{J/\psi} f_\rho m_\rho \xi_\gamma G Z_{J/\psi} \gamma_{\sigma} q^\rho}{2m_c (p^2 - m_Z^2) (q^2 - m_Z^2)} \left(-g_\mu^\beta + \frac{q_\mu q_\beta}{q^2}\right) \left(-g_\nu^\sigma + \frac{q_\nu q_\sigma}{q^2}\right) + \ldots$$

$$= \Pi(p^2, p^2, q^2) \varepsilon_{\mu \nu \sigma} p^\sigma q^\beta + \ldots, \quad (11)$$

$$\Pi_\mu^3(p, q) = \frac{f_{J/\psi} m_{J/\psi} f_{J/\psi} m_{J/\psi} \xi_\gamma G Z_{J/\psi} \gamma_{\sigma} q^\rho}{(p^2 - m_Z^2) (q^2 - m_Z^2)} \left(-g_\mu^\alpha + \frac{p_\mu p_\alpha}{p^2}\right) \left(-g_\nu^\sigma + \frac{p_\nu p_\sigma}{p^2}\right) + \ldots$$

$$= \Pi(p^2, p^2, q^2) g_{\mu \nu} + \ldots, \quad (12)$$

$$\Pi_\mu^4(p, q) = \frac{f_\chi m_\chi f_\rho m_\rho \xi_\gamma G Z_{\chi} \gamma_{\sigma} q^\rho}{(p^2 - m_Z^2) (q^2 - m_Z^2)} \left(-g_\mu^\alpha + \frac{q_\mu q_\alpha}{q^2}\right) \left(-g_\nu^\sigma + \frac{q_\nu q_\sigma}{q^2}\right) + \ldots$$

$$= \Pi(p^2, p^2, q^2) g_{\mu \nu} + \ldots, \quad (13)$$

$$\Pi_\alpha\beta\nu(p, q) = \frac{f_{D^*} m_{D^*} f_{D^*} m_{D^*} \xi_\gamma G Z_{D^*} \gamma_{\sigma} q^\rho}{(p^2 - m_Z^2) (q^2 - m_Z^2)} \left(-g_\mu^\beta + \frac{q_\mu q_\beta}{q^2}\right) \left(-g_\alpha^\nu + \frac{q_\alpha q_\nu}{q^2}\right) + \ldots$$

$$= \Pi(p^2, p^2, q^2) g_{\alpha \beta} q^\nu + \ldots, \quad (14)$$

$$\Pi_\mu^5(p, q) = \frac{f_{D^*} m_{D^*} f_{D^*} m_{D^*} \xi_\gamma G Z_{D^*} \gamma_{\sigma} q^\rho}{m_c (p^2 - m_Z^2) (q^2 - m_Z^2)} \left(-g_\mu^\beta + \frac{q_\mu q_\beta}{q^2}\right) \left(-g_\nu^\sigma + \frac{q_\nu q_\sigma}{q^2}\right) + \ldots$$

$$= \Pi(p^2, p^2, q^2) \left(-p_\nu\right) + \ldots, \quad (15)$$

$$\Pi_\mu^7(p, q) = \frac{f_{D^*} m_{D^*} f_{D^*} m_{D^*} \xi_\gamma G Z_{D^*} \gamma_{\sigma} q^\rho}{m_c (p^2 - m_Z^2) (q^2 - m_Z^2)} \left(-g_\mu^\beta + \frac{q_\mu q_\beta}{q^2}\right) \left(-g_\nu^\sigma + \frac{q_\nu q_\sigma}{q^2}\right) + \ldots$$

$$= \Pi(p^2, p^2, q^2) \left(-\varepsilon_{\mu \nu \sigma} p^\sigma q^\beta\right) + \ldots, \quad (16)$$
Figure 1: The Feynman diagrams for the correlation functions $\Pi_{\mu\nu}^i(p,q)$ with $i = 1, 2, 3, 4$, where the solid lines and dashed lines denote the light quarks and heavy quarks, respectively, the waved lines denote the gluons. Other diagrams obtained by interchanging of the light or heavy quark lines are implied.

where we have used the following definitions for the hadronic coupling constants,

$$
\langle J/\psi(p)\pi(q)|X(p')\rangle = i\varepsilon^{\alpha\beta\tau\sigma}p_\alpha\xi_{j/\psi}^\alpha\xi_{\pi}^\beta G_{Z_{cJ/\psi\pi}},
$$

$$
\langle \eta_c(p)\rho(q)|X(p')\rangle = i\varepsilon^{\alpha\beta\tau\sigma}q_\alpha\xi_{\eta_c}^\beta p_\tau G_{Z_{c\eta_c\rho}},
$$

$$
\langle J/\psi(p)a_0(q)|X(p')\rangle = i\xi_{j/\psi}^\alpha\xi_{a_0}^\beta G_{Z_{cJ/\psi a_0}},
$$

$$
\langle \chi_{c0}(p)\rho(q)|X(p')\rangle = i\xi_{\chi_{c0}}^\alpha\xi_{c}^\beta G_{Z_{c\chi_{c0}\rho}},
$$

$$
\langle \bar{D}^*(p)D^*(q)|X(p')\rangle = i\varepsilon_{\alpha\beta\tau\sigma}p_\alpha\xi_{\bar{D}^*}^\beta G_{Z_{c\bar{D}^*D^*}},
$$

$$
\langle \bar{D}(p)D(q)|X(p')\rangle = i\varepsilon_{\alpha\beta\tau\sigma}G_{Z_{cDD}},
$$

$$
\langle \bar{D}^*(p)D(q)|X(p')\rangle = i\varepsilon_{\alpha\beta\tau\sigma}p_\alpha\xi_{c}^\beta G_{Z_{c\bar{D}^*D}},
$$

where $G_{Z_{cJ/\psi\pi}}, G_{Z_{c\eta_c\rho}}, G_{Z_{cJ/\psi a_0}}, G_{Z_{c\chi_{c0}\rho}}, G_{Z_{c\bar{D}^*D}}, G_{Z_{cDD}}$ and $G_{Z_{c\bar{D}^*D}}$ are the hadronic coupling constants.

We study the correlation functions $\Pi(p^2, p^2, q^2)$ at the QCD side, and carry out the operator product expansion up to the vacuum condensates of dimension 5 by taking into account both the connected and disconnected Feynman diagrams and neglect the tiny contributions of the gluon condensate. In Fig.1, we draw the Feynman diagrams for the correlation functions $\Pi_{\mu\nu}^i(p,q)$ with $i = 1, 2, 3, 4$ as an example. The connected and disconnected Feynman diagrams correspond to the factorizable and non-factorizable contributions respectively. The factorizable contributions, even the perturbative terms, involve the rearrangements in the color, flavor, and Dirac spinor spaces, which differ greatly from the fall-apart decay of a loosely bound two-meson state.

Now we obtain the hadronic spectral densities and QCD spectral densities through dispersion relation, and write down the correlation functions in the spectral representation,

$$
\Pi_H(p^2, p^2, q^2) = \int_{\Delta_1^2}^{\infty} ds' \int_{\Delta_2^2}^{\infty} ds \int_{\Delta_3^2}^{\infty} du \frac{\rho_H(s', s, u)}{(s' - p^2)(s - p^2)(u - q^2)},
$$

$$
\Pi_{QCD}(p^2, p^2, q^2) = \int_{\Delta_1^2}^{\infty} ds \int_{\Delta_2^2}^{\infty} du \frac{\rho_{QCD}(s, u)}{(s - p^2)(u - q^2)},
$$

(18)
where the subscripts $H$ and $QCD$ denote the hadron side and QCD side of the correlation functions respectively, $\Delta^2 = (m_B + m_C)^2$ (more precisely $\Delta^2 = (\Delta_H + \Delta_B)^2$), the $\Delta^2_H$ and $\Delta^2_B$ are the thresholds. There are three variables $\rho$ and $ds$ where the subscripts $H$ and $B$ respectively, $\Delta^2 = (m_B + m_C)^2$ (more precisely $\Delta^2 = (\Delta_H + \Delta_B)^2$), the $\Delta^2_H$ and $\Delta^2_B$ are the thresholds. There are three variables $s$, $s'$, and $u$ at the hadron side, while there are two variables $s$ and $u$ at the QCD side. We math the hadron side with the QCD side of the correlation functions, and carry out the integral over $ds'$ firstly to obtain the solid duality for the decays $Z \to BC$ [6, 8],

$$
\int_{\Delta^2_H}^{s_B^0} ds \int_{\Delta^2_B}^{u_C^0} du \frac{\rho_{QCD}(s, u)}{(s - p^2)(u - q^2)} = \int_{\Delta^2_H}^{s_B^0} ds \int_{\Delta^2_B}^{u_C^0} du \frac{1}{(s - p^2)(u - q^2)} \left[ \int_{\Delta^2_B}^{s_B^0} ds' \frac{\rho_H(s', s, u)}{s' - p^2} \right],
$$

(19)

where the $B$ and $C$ denote the final states, the $s_B^0$ and $u_C^0$ are the continuum thresholds. Compared to other works on the two-body strong decays of the tetraquark state candidates [12, 13, 14], we do not need to introduce the continuum threshold parameter $s_B^0$ in the $s'$ channel by hand to avoid contamination (such as introducing $\frac{1}{s' - p^2}$, setting $s_B^0 = s_B^0$), or make special assumption of the value of the squared momentum $q^2$ (such as taking the limit $q^2 \to 0$ to obtain the QCD sum rules, calculating the $G_{Z, BC}(Q^2 = -q^2)$ at large $Q^2$ then extracting the $G_{Z, BC}(Q^2)$ to the physical region $Q^2 = -m_C^2$ with highly model dependent functions). In other words, we need only carry out the operator product expansion at large space-like region $-p^2 \to \infty$ and $-q^2 \to \infty$, where the operator product expansion works. For the technical details, one can consult Refs. [6, 8]. We write down the integral over $ds'$ explicitly,

$$
\int_{\Delta^2_B}^{s_B^0} ds' \frac{\rho_H(s', s, u)}{s' - p^2} = \frac{\rho_H(s, u)}{m_B^2 - p^2} + \int_{s_B^0}^{\infty} dt \frac{\rho_{Z'B}(t, p^2, q^2)}{t - p^2} + \frac{\rho_{Z'C}(t, p^2, q^2)}{t - p^2},
$$

(20)

where the ground state hadronic spectral densities $\rho_H(s, u)$ are known, the transitions $\rho_{Z'B}(t, p^2, q^2) + \rho_{Z'C}(t, p^2, q^2)$ between the higher resonances (or continuum states) and the ground states B, C are unknown, we have to introduce the parameters $C_{Z'B}$ and $C_{Z'C}$ to parameterize the net effects,

$$
C_{Z'B} = \int_{s_B^0}^{\infty} dt \frac{\rho_{Z'B}(t, p^2, q^2)}{t - p^2},
$$

$$
C_{Z'C} = \int_{s_B^0}^{\infty} dt \frac{\rho_{Z'C}(t, p^2, q^2)}{t - p^2}.
$$

(21)

Then we set $p^2 = p^2$, or $4p^2$, and perform the double Borel transform with respect to $P^2 = -p^2$ and $Q^2 = -q^2$ respectively to obtain the QCD sum rules for the hadronic coupling constants,

$$
\int_{m^2}^{2m^2} f_{J/\psi} m_{J/\psi} \frac{m_{J/\psi}}{m_B^2 - m_{J/\psi}^2} \lambda_Z G_{Z_{J/\psi\pi}} \left[ \exp \left( -\frac{m_{J/\psi}^2}{T^2_1} \right) - \exp \left( -\frac{m_Z^2}{T^2_1} \right) \right] \exp \left( -\frac{m_{J/\psi}^2}{T^2_2} \right)
$$

$$
+ (C_{Z', J/\psi} + C_{Z', \pi}) \exp \left( -\frac{m_{J/\psi}^2}{T^2_2} - \frac{m_{Z'}^2}{T^2_2} \right)
$$

$$
= \frac{m_c(\bar{q}q) f_{J/\psi}}{2\sqrt{2}\pi^2} \int_{4m_{\pi}^2}^{\infty} ds \sqrt{1 - \frac{4m_{\pi}^2}{s}} \exp \left( -\frac{s}{T^2_1} \right)
$$

$$
+ \frac{m_c(\bar{q}q_2Gq)}{24\sqrt{2}\pi^2} \int_{4m_{\pi}^2}^{\infty} ds \sqrt{\frac{s}{s - 4m_c^2}} \frac{1}{s - 2m_{\pi}^2} \exp \left( -\frac{s}{T^2_1} \right),
$$

(22)
\[
\frac{f_{\eta c} m_{\eta c}^2 f_{\rho} m_{\rho} \lambda Z G_{Zc, \eta c}^\rho}{2 m_c} \left[ \exp \left( -\frac{m_c^2}{T^2_1} \right) - \exp \left( -\frac{m_c^2}{T^2_2} \right) \right] = (C_{Z'c} + C_{Z'\rho}) \exp \left( -\frac{m_c^2}{T^2_1} - \frac{m_c^2}{T^2_2} \right) \\
= \frac{m_c \langle \bar{q}q \rangle}{2 \sqrt{2} \pi^2} \int_{4m_c^2}^{s_c} ds \sqrt{1 - \frac{4m_c^2}{s}} \exp \left( -\frac{s}{T^2_1} \right) + \frac{m_c \langle \bar{q}g \sigma Gq \rangle}{6 \sqrt{2} \pi^2 T^2_2} \int_{4m_c^2}^{s_c} ds \sqrt{1 - \frac{4m_c^2}{s}} \exp \left( -\frac{s}{T^2_1} \right) \\
- \frac{m_c \langle \bar{q}g \sigma Gq \rangle}{24 \sqrt{2} \pi^2} \int_{4m_c^2}^{s_c} ds \frac{1}{\sqrt{s} (s - 4m_c^2)} \exp \left( -\frac{s}{T^2_1} \right),
\]

(23)

\[
\frac{f_{J/\psi} m_{J/\psi} f_{a_0} m_{a_0} \lambda Z G_{Zc, J/\psi a_0}^\rho}{m_Z^2 - m_{J/\psi}^2} \left[ \exp \left( -\frac{m_{J/\psi}^2}{T^2_1} \right) - \exp \left( -\frac{m_{J/\psi}^2}{T^2_2} \right) \right] \exp \left( -\frac{m_{a_0}^2}{T^2_2} \right) + (C_{Z'c} + C_{Z'\rho}) \exp \left( -\frac{m_{J/\psi}^2}{T^2_1} - \frac{m_{a_0}^2}{T^2_2} \right) \\
= -\frac{1}{32 \sqrt{2} \pi^4} \int_{4m_c^2}^{s_{J/\psi}} ds \int_{4m_c^2}^{s_{a_0}} du \sqrt{1 - \frac{4m_c^2}{s}} \left( 1 + \frac{2m_c^2}{s} \right) \exp \left( -\frac{s}{T^2_1} - \frac{u}{T^2_2} \right),
\]

(24)

\[
\frac{f_{\chi c \rho} m_{\chi c a_0} f_{\rho} m_{\rho} \lambda Z G_{Zc, \chi c a_0}^\rho}{m_Z^2 - m_{\chi c a_0}^2} \left[ \exp \left( -\frac{m_{\chi c a_0}^2}{T^2_1} \right) - \exp \left( -\frac{m_{\chi c a_0}^2}{T^2_2} \right) \right] \exp \left( -\frac{m_c^2}{T^2_2} \right) + (C_{Z'c} + C_{Z'\rho}) \exp \left( -\frac{m_{\chi c a_0}^2}{T^2_1} - \frac{m_c^2}{T^2_2} \right) \\
= \frac{1}{32 \sqrt{2} \pi^4} \int_{4m_c^2}^{s_{\chi c}} ds \int_{4m_c^2}^{s_{\rho}} du \sqrt{1 - \frac{4m_c^2}{s}} \left( 1 - \frac{4m_c^2}{s} \right) \exp \left( -\frac{s}{T^2_1} - \frac{u}{T^2_2} \right),
\]

(25)
\[
\frac{f_{D^a D^b} f_{D^+ D^+} + \lambda_Z G_{Z^* D^* D^*}}{4 (\tilde{m}_D^2 - m_{D^0}^2)} \left[ \exp \left( -\frac{m_{D^0}^2}{T_1^2} \right) - \exp \left( -\frac{\tilde{m}_Z^2}{T_2^2} \right) \right] \exp \left( -\frac{m_{D^+}^2}{T_2^2} \right) \\
+ (C_{Z^* D^*} + C_{Z^* D^*}) \exp \left( -\frac{m_{D^0}^2}{T_1^2} \right) - \frac{m_c}{64 \sqrt{2} \pi} \int_{s_0}^{0} ds \int_{m_c^2}^{s_1} du \left( \begin{array}{c}
2u + m_c^2 \\
1 - \frac{m_c^2}{s}
\end{array} \right) \left( 1 - \frac{m_c^2}{u} \right) \exp \left( -\frac{m_c^2}{T_1^2} - \frac{u}{T_2^2} \right) \\
- \frac{\langle \bar{q} q \rangle}{24 \sqrt{2} \pi^2} \int_{m_c^2}^{s_1} du \left( 1 - \frac{m_c^2}{u} \right) \exp \left( -\frac{m_c^2}{T_1^2} - \frac{u}{T_2^2} \right) \\
- \frac{m_c^2}{8 \sqrt{2} \pi^2} \int_{m_c^2}^{s_1} ds \left( 1 - \frac{m_c^2}{s} \right)^2 \exp \left( -\frac{m_c^2}{T_1^2} - \frac{m_c^2}{T_2^2} \right) \\
+ \frac{\langle \bar{q} q \sigma G \rangle}{288 \sqrt{2} \pi^2} \int_{m_c^2}^{s_1} du \left( 2u + m_c^2 \right) \left( 1 - \frac{m_c^2}{u} \right) \left( 1 - \frac{m_c^2}{s} \right) \left( 4 + \frac{3m_c^2}{u} \right) \exp \left( -\frac{m_c^2}{T_1^2} - \frac{u}{T_2^2} \right) \\
+ \frac{m_c^4}{32 \sqrt{2} \pi^2} \int_{m_c^2}^{s_1} ds \left( 1 - \frac{m_c^2}{s} \right)^2 \exp \left( -\frac{m_c^2}{T_1^2} - \frac{m_c^2}{T_2^2} \right) \\
+ \frac{\langle \bar{q} q \sigma G \rangle}{96 \sqrt{2} \pi^2} \int_{m_c^2}^{s_1} du \exp \left( -\frac{m_c^2}{T_1^2} - \frac{u}{T_2^2} \right) \\
- \frac{\langle \bar{q} q \sigma G \rangle}{96 \sqrt{2} \pi^2} \int_{m_c^2}^{s_1} du \exp \left( -\frac{m_c^2}{T_1^2} - \frac{m_c^2}{T_2^2} \right) \\
+ \frac{\langle \bar{q} q \sigma G \rangle}{96 \sqrt{2} \pi^2} \int_{m_c^2}^{s_1} ds \exp \left( -\frac{s}{T_1^2} - \frac{m_c^2}{T_2^2} \right), \\
\right.
\]
\[
\frac{f_{D+}m_{D+}^2 f_{D^o}m_{D^o} f_{D^*}m_{D^*}^2}{m_c^2} \frac{\lambda_Z \bar{G}_{Z,DD}}{4 (m_{D}^2 - m_{D^o}^2)} \left[ \exp \left( -\frac{m_{D^o}^2}{T_1^2} \right) - \exp \left( -\frac{\bar{m}_Z^2}{T_1^2} \right) \right] \exp \left( -\frac{m_{D^*}^2}{T_2^2} \right) + (C_{Z^*D} + C_{Z^*D}) \exp \left( -\frac{m_{D^o}^2}{T_1^2} \right) - \exp \left( -\frac{\bar{m}_Z^2}{T_1^2} \right) \right] \exp \left( -\frac{m_{D^*}^2}{T_2^2} \right) \\
= \frac{3m_c}{64\sqrt{2\pi}^4} \int_{m_c^2}^{s_0^b} \frac{d s}{d u} \int_{m_c^2}^{s_0^b} d u \left( 1 - \frac{m_c^2}{u} \right)^2 \left( 1 - \frac{m_c^2}{s} \right)^2 \exp \left( -\frac{m_c^2}{T_1^2} \right) \exp \left( -\frac{m_c^2}{T_2^2} \right) \\
- \frac{\langle \bar{q}q \rangle}{8\sqrt{2\pi}^2} \int_{m_c^2}^{s_0^b} \frac{d s}{d u} \left( 1 - \frac{m_c^2}{u} \right)^2 \exp \left( -\frac{m_c^2}{T_1^2} \right) \exp \left( -\frac{m_c^2}{T_2^2} \right) \\
- \frac{m_c^2 \langle \bar{q}q \rangle}{8\sqrt{2\pi}^2} \int_{m_c^2}^{s_0^b} d s \left( 1 - \frac{m_c^2}{s} \right)^2 \exp \left( -\frac{m_c^2}{T_1^2} \right) \exp \left( -\frac{m_c^2}{T_2^2} \right) \\
- \frac{m_c^2 \langle \bar{q}q \rangle}{32\sqrt{2\pi}^2} \int_{m_c^2}^{s_0^b} d s \left( 1 - \frac{m_c^2}{s} \right)^2 \exp \left( -\frac{m_c^2}{T_1^2} \right) \exp \left( -\frac{m_c^2}{T_2^2} \right) \\
- \frac{m_c^2 \langle \bar{q}q \rangle}{16\sqrt{2\pi}^2} \int_{m_c^2}^{s_0^b} d s \left( 1 - \frac{m_c^2}{s} \right)^2 \exp \left( -\frac{m_c^2}{T_1^2} \right) \exp \left( -\frac{m_c^2}{T_2^2} \right) \\
- \frac{\langle \bar{q}g \rangle \sigma G_q}{192\sqrt{2\pi}^2} \int_{m_c^2}^{s_0^b} d s \left( 1 - \frac{m_c^2}{u} \right)^2 \exp \left( -\frac{m_c^2}{T_1^2} \right) \exp \left( -\frac{m_c^2}{T_2^2} \right) \\
- \frac{\langle \bar{q}g \rangle \sigma G_q}{96\sqrt{2\pi}^2} \int_{m_c^2}^{s_0^b} d s \left( 1 - \frac{m_c^2}{s} \right)^2 \exp \left( -\frac{m_c^2}{T_1^2} \right) \exp \left( -\frac{m_c^2}{T_2^2} \right) \\
- \frac{\langle \bar{q}g \rangle \sigma G_q}{96\sqrt{2\pi}^2} \int_{m_c^2}^{s_0^b} d s \left( 1 - \frac{m_c^2}{s} \right)^2 \exp \left( -\frac{m_c^2}{T_1^2} \right) \exp \left( -\frac{m_c^2}{T_2^2} \right) \\
+ (C_{Z^*D} + C_{Z^*D}) \exp \left( -\frac{m_{D^o}^2}{T_1^2} \right) - \exp \left( -\frac{\bar{m}_Z^2}{T_1^2} \right) \right] \exp \left( -\frac{m_{D^*}^2}{T_2^2} \right)
\end{align*}

where \( \bar{m}_Z^2 = \frac{m_c^2}{T_1^2} \), the \( T_1^2 \) and \( T_2^2 \) are the Borel parameters, the unknown functions \( C_{Z^*J/\psi} + C_{Z^*\pi}, C_{Z^*\eta_c} + C_{Z^*\phi}, C_{Z^*J/\psi} + C_{Z^*a_0}, C_{Z^*\chi_c} + C_{Z^*\rho}, C_{Z^*D^*} + C_{Z^*D}, C_{Z^*D} + C_{Z^*D} \) and \( C_{Z^*D} + C_{Z^*D} \) parameterize the higher resonances or continuum state contributions. In calculations, we observe that there appears divergence due to the endpoint \( s = 4m_c^2 \), we can avoid the endpoint divergence with the simple replacement \( \frac{1}{\sqrt{s - 4m_c^2}} \rightarrow \frac{1}{\sqrt{s - 4m_c^2 + 4m_c^2}} \) by adding a small squared s-quark mass \( 4m_c^2 \).
Now we give an example to illustrate how to carry out the operator product expansion. The correlation function $\Pi_{\mu\nu}(p, q)$ at the QCD side can be written as,

$$\Pi_{\mu\nu}(p, q) = -\frac{i}{\sqrt{2}} \varepsilon^{ijk} \varepsilon^{imn} \int d^4xd^4ye^{ipx}e^{iqy} \left\{ \text{Tr} \left[ \gamma_\mu S^{ak}_c(x)CS^T_b(y)C\gamma_5CS^T_d(y)xCS^T_b(-y)cS^a_c(-x) \right] \\
+ \text{Tr} \left[ \gamma_\mu S^{ak}_c(x)\gamma_\nu CS^T_b(y)C\gamma_5CS^T_d(y)xCS^a_c(-x) \right] \right\}, \quad (29)$$

where the $i, j, k, m, \cdots$ are color indexes, the $S^{ak}_c(x)$ and $S^{bj}_c(y)$ are the full c and $u/d$ quark propagators, respectively $[11][16][17]$. We carry out the integrals over $d^4x$ and $d^4y$, and take into account all terms proportional to the tensor structure $\varepsilon_{\mu
u\lambda\beta}p^\lambda q^\beta$, irrespective of the perturbative terms, quark condensate terms and mixed quark condensates terms, in other words, we calculate all the Feynman diagrams shown in Fig.1. However, not all terms (or Feynman diagrams) have contributions due to the special tensor structure $\varepsilon_{\mu
u\lambda\beta}p^\lambda q^\beta$.

### 3 Numerical results and discussions

At the QCD side, we take the vacuum condensates to be the standard values $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{GeV})^3$, $\langle \bar{q}g_sGq \rangle = m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = (0.8 \pm 0.1) \text{GeV}^2$ at the energy scale $\mu = 1 \text{ GeV}$ $[10][11][13]$, and take the $\overline{MS}$ masses $m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}$ and $m_s(m_s) = 0.959 \text{ GeV}$ from the Particle Data Group $[19]$. Moreover, we take into account the energy-scale dependence of the quark condensate, mixed quark condensate and $\overline{MS}$ masses from the renormalization group equation,

$$\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{\pi^2} t},$$

$$\langle \bar{q}g_sGq \rangle(\mu) = \langle \bar{q}g_sGq \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{\pi^2} t},$$

$$m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{\pi^2} t},$$

$$m_s(\mu) = m_s(2\text{GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})} \right]^{\frac{12}{\pi^2} t},$$

$$\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0^2} + \frac{b_2 (\log^2 t - \log t - 1) + b_3 b_2}{b_0^4 t^2} \right], \quad (30)$$

where $t = \log \frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi}$, $b_2 = \frac{2887-203n_f + 32n_f^2}{128\pi^2}$, $\Lambda = 210 \text{ MeV}$, 292 MeV and 332 MeV for the flavors $n_f = 5, 4$ and 3, respectively $[19][20]$, and evolve all the input parameters to the optimal energy scale $\mu$ with $n_f = 4$ to extract the hadronic coupling constants. In this article, we take the energy scales of the QCD spectral densities to be $\mu = \frac{n_f}{2}$, which is acceptable for the mesons $D$ and $J/\psi$ $[6][17]$. In this article, we neglect the small u and d quark masses.

The hadronic parameters are chosen as $m_{J/\psi} = 3.0969 \text{ GeV}$, $m_{\pi} = 0.13957 \text{ GeV}$, $m_{\rho} = 0.77526 \text{ GeV}$, $m_{\eta_c} = 2.9839 \text{ GeV}$, $m_{a_0} = 0.980 \text{ GeV}$, $m_{D^+} = 1.8695 \text{ GeV}$, $m_{D^0} = 1.86484 \text{ GeV}$, $m_{D^{*+}} = 2.01026 \text{ GeV}$, $m_{D^{*0}} = 2.00085 \text{ GeV}$, $m_{\chi_b} = 3.41471 \text{ GeV}$, $\sqrt{s^T_D} = 2.4 \text{ GeV}$, $\sqrt{s^T_{D^*}} = 2.5 \text{ GeV}$, $\sqrt{s^T_{J/\psi}} = 3.6 \text{ GeV}$, $\sqrt{s^T_{\eta_c}} = 3.5 \text{ GeV}$, $\sqrt{s^T_{a_0}} = 3.9 \text{ GeV}$ $[19]$, $f_{J/\psi} = 0.418 \text{ GeV}$, $f_{\eta_c} = 0.387 \text{ GeV}$ $[21]$, $f_{\rho} = 0.215 \text{ GeV}$, $\sqrt{s^T_{\rho}} = 1.3 \text{ GeV}$ $[22]$, $f_{a_0} = 0.214 \text{ GeV}$, $\sqrt{s^T_{a_0}} = 1.3 \text{ GeV}$ $[23]$, $f_D = 0.208 \text{ GeV}$, $f_{D^*} = 0.263 \text{ GeV}$ $[24]$, $f_{\chi_b} = 0.359 \text{ GeV}$ $[25]$, $m_Z = 4.59 \text{ GeV}$ $[3]$, $\lambda_Z = 6.21 \times 10^{-2} \text{ GeV}^5$ $[9]$, $f_{\pi} m_{\pi}^2/(m_u + m_d) = -\frac{2}{3}(\bar{q}q)/f_{\pi}$ from the Gell-Mann-Oakes-Renner relation.
Table 1: The Borel windows, hadronic coupling constants, partial decay widths of the $Z_c(4600)$. 

| Decay                     | $T^2$ (GeV$^2$) | $|G|$          | $\Gamma$(MeV) |
|---------------------------|-----------------|---------------|---------------|
| $Z_c^+(4600) \to J/\psi\pi^-$ | 4.0 - 5.0       | 0.90$^{+0.20}_{-0.18}$ GeV$^{-1}$ | 41.4$^{+0.0}_{-0.0}$ |
| $Z_c^+(4600) \to \eta_c\rho$ | 4.1 - 5.1       | 1.01$^{+0.03}_{-0.02}$ GeV$^{-1}$ | 41.6$^{+0.0}_{-0.0}$ |
| $Z_c^+(4600) \to J/\psi a_0(980)$ | 3.4 - 4.4       | 2.37$^{+0.01}_{-0.02}$ GeV | 10.2$^{+0.0}_{-0.0}$ |
| $Z_c^+(4600) \to \chi_{c0}\rho^*$ | 3.1 - 4.1       | 1.35$^{+0.02}_{-0.03}$ GeV | 3.5$^{+0.0}_{-0.0}$ |
| $Z_c^+(4600) \to D^{\ast0}D^{\ast+}$ | 2.1 - 2.8       | 1.58$^{+0.01}_{-0.02}$ MeV | 39.5$^{+0.0}_{-0.0}$ |
| $Z_c^+(4600) \to D^0D^+$     | 1.7 - 2.4       | 1.05$^{+0.01}_{-0.02}$ GeV | 6.6$^{+0.0}_{-0.0}$ |

We set the Borel parameters to be $T^2 = T_{\min}^2 = T^2_{\max}$ for simplicity. The unknown parameters are chosen as $C_Z^{J/\psi} + C_Z^{\pi} = -0.0014$ GeV$^6$, $C_Z^{\eta_c} + C_Z^{\rho} = 0.0015$ GeV$^6$, $C_Z^{J/\psi} + C_Z^{a_0} = -0.0119$ GeV$^8$, $C_Z^{\chi_{c0}} + C_Z^{\rho} = 0.0066$ GeV$^8$, $C_Z^{D^*} + C_Z^{D^{*+}} = 0.0031$ GeV$^7$, $C_Z^{D^*} + C_Z^{D^{*0}} = 0.0022$ GeV$^7$, $C_Z^{D^*} + C_Z^{D^{*0}} = 0.0003$ GeV$^6$ to obtain platforms in the Borel windows, which are shown in Table 1. The Borel windows $T_{\max}^2 - T_{\min}^2 = 1.0$ GeV$^2$ for the charmonium decays and $T_{\max}^2 - T_{\min}^2 = 0.7$ GeV$^2$ for the open-charm decays, where the $T_{\max}$ and $T_{\min}$ denote the maximum and minimum of the Borel parameters. We choose the same intervals $T_{\max}^2 - T_{\min}^2$ in all the QCD sum rules for the two-body strong decays [6, 15], which work well for the decays of the $X(4140)$, $X(4274)$ and $Y(4660)$. In Figs.2-3, we plot the hadronic coupling constants $G$ with variations of the Borel parameters $T^2$ at much larger intervals than the Borel windows. From the figures, we can see that there appear platforms in the Borel windows indeed.

We take into account the uncertainties of the input parameters, and obtain the hadronic coupling constants, which are shown in Table 1 and Figs.2-3. Now it is easy to obtain the partial decay widths of the two-body strong decays $Z_c(4600) \to J/\psi\pi^-, \eta_c\rho, J/\psi a_0, \chi_{c0}\rho, D^*D^*, DD, D^*D$ and $DD^*$, which are also shown in Table 1. From Table 1, we can see that the partial decay width $\Gamma(Z_c^+(4600) \to J/\psi\pi^-) = 41.4^{+20.0}_{-14.5}$ MeV is rather large, which can account for the observation of the $Z_c^+(4600)$ in the $J/\psi\pi^-$ mass spectrum, although it is just an evidence.

We can saturate the width with the two-body strong decays and obtain the total decay width,

$$\Gamma(Z_c^+(4600)) = 144.8^{+50.6}_{-33.9} \text{MeV},$$

which is reasonable for the tetraquark state. The present predictions can be confronted to the experimental data in the future, which may shed light on the nature of the $Z_c(4600)$. The two possible structures near $m(J/\psi\pi^-) = 4200$ MeV and 4600 MeV have not been confirmed by other experiments, and their quantum numbers, such as the spin-parity $J^P$, have not been measured yet. They may originate from statistical fluctuations, unsuitable cuts or subtractions, etc, and do not exist at all. In previous work [3], we observed the $[q\bar{c}]_A[p\bar{q}]_A - [q\bar{c}]_A[q\bar{c}]_A$ type tetraquark state with the $J^{PC} = 1^{--}$ had a mass $(4.59 \pm 0.08)$ GeV, which happens to coincide with the LHCb data. It is interesting to assign the $Z_c(4600)$ to be $[q\bar{c}]_A[p\bar{q}]_A - [q\bar{c}]_A[q\bar{c}]_A$ type vector tetraquark state, and study its two-body strong decays to explore its structures. In this article, we obtain the partial decay widths of the $[q\bar{c}]_A[p\bar{q}]_A - [q\bar{c}]_A[q\bar{c}]_A$ type tetraquark state with $J^{PC} = 1^{--}$, which are valuable in both theoretical and experimental exploring the structures and properties of the tetraquark states, even if the $Z_c(4600)$ does not exist.

Now we make a crude estimation for the systematic uncertainties of the present QCD sum rules. For the correlation functions $\Pi_{\mu\nu}(p, q)$ with $i = 1, 2, 3, 4$, we set $p^2 = 0$, while for the correlation functions $\Pi_{\alpha\beta\nu\mu}(p, q)$ with $i = 4, 5, 6$, we set $p^2 = 4p^2$, then perform the Borel transform with respect to $P^2 = -p^2$ to obtain the QCD sum rules. In fact, we can set $p^2 = \alpha p^2$ by introducing an additional parameter $\alpha$, for example, the left side of the QCD sum rules in Eq.(22) is changed...
Figure 2: The hadronic coupling constants with variations of the Borel parameters $T^2$, where the $A$, $B$, $C$ and $D$ denote the $G_{Z, J/\psi \pi}$, $G_{Z, \eta \rho}$, $G_{Z, J/\psi a_0}$ and $G_{Z, \chi_c \rho}$, respectively.
Figure 3: The hadronic coupling constants with variations of the Borel parameters $T^2$, where the $A$, $B$ and $C$ denote the $G_{Z,D\cdot D}$, $G_{Z,D\bar{D}}$ and $G_{Z,\bar{D}\cdot D}$, respectively.
to

\[
\frac{f_x m^2_x}{m_u + m_d} \frac{f_{J/\psi} m_{J/\psi}}{\alpha^2 (\tilde{m}_Z^2 - m^2_{J/\psi})} [\exp \left( -\frac{m^2_{J/\psi}}{T_1^2} \right) - \exp \left( -\frac{\tilde{m}_Z^2}{T_1^2} \right)] \exp \left( -\frac{m^2_\pi}{T_2^2} \right)
+ \left( C_{Z',J/\psi} + C_{Z',\pi} \right) \exp \left( -\frac{m^2_{J/\psi}}{T_1^2} - \frac{m^2_\pi}{T_2^2} \right),
\]

(32)

where \( \tilde{m}_Z^2 = \frac{m^2_Z}{\alpha^2} \). If we choose \( \alpha^2 = 1.2^2 = 1.44 \), we can obtain the value \( |G_{Z_{cJ/\psi}}| = 0.88 \text{GeV}^{-1} \) by choosing suitable \( C_{Z,J/\psi} + C_{Z',\pi} \), the additional uncertainty is about 2%. If we choose \( \alpha^2 > 1.48 \), say \( \alpha^2 = 1.3^2 = 1.69 \), no stable QCD sum rules can be obtained. Accordingly, if we take \( \alpha = 2.2 \) (\( \alpha^2 = 4.84 \)) to calculate the \( G_{Z_cD\bar{D}} \ flavour coupling constants, we can obtain a value \( G_{Z_cD\bar{D}} = 1.01 \), the additional uncertainty is about 4%. In the heavy quark limit, we can make a crude approximation \( m_Z = m_{J/\psi} = 2m_D/m_D^* \), the ideal values of the \( \alpha \) for the correlation functions \( \Pi_{\mu\nu/\alpha\beta\nu/\nu}(p,q) \) with \( i = 1, 2, 3, 4 \) and 5, 6, 7 are \( \alpha = 1 \) and 2, respectively. In practical calculations, we observe that if there are larger deviations from \( \alpha = 1 \) or 2, no stable QCD sum rules can be obtained.

4 Conclusion

In this article, we tentatively assign the \( Z_c(4600) \) to be the \([dc]_P[\bar{u}\bar{c}]_A - [dc]_A[\bar{u}\bar{c}]_P \) type vector tetraquark state, and obtain the QCD sum rules for the hadronic coupling constants \( G_{Z_cJ/\psi}, G_{Z_cJ/\psi}, G_{Z_cJ/\psi}, G_{Z_cJ/\psi}, G_{Z_cJ/\psi}, G_{Z_cJ/\psi}, G_{Z_cJ/\psi}, G_{Z_cJ/\psi} \) based on solid quark-hadron duality by taking into account both the connected and disconnected Feynman diagrams in the operator product expansion. The predictions of the partial decay widths for the decays \( Z_c(4600) \to J/\psi\pi, \eta_c\rho, J/\psi\rho, \chi_{c0}\rho, D^*D^*, DD, D^*D, D^*D, D^*D \) can be confronted to the experimental data in the future to diagnose the nature of the \( Z_c(4600) \).

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