Magnetohydrodynamic Simulation of Solar Supergranulation

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Abstract. Three-dimensional magnetohydrodynamical large eddy simulations of solar surface convection using realistic model physics is conducted. The effects of magnetic fields on thermal structure of convective motions into radiative layers, the range of convection cell sizes and penetration depths of convection is investigated. We simulate a some portion of the solar photosphere and the upper layers of the convection zone, a region extending 30 x 30 Mm horizontally from 0 Mm down to 18 Mm below the visible surface. We solve equations of the fully compressible radiation magnetohydrodynamics with dynamical viscosity and gravity. For numerical simulation we use: 1) realistic initial model of Sun and equation of state and opacities of stellar matter, 2) high order conservative TVD scheme for solution magnetohydrodynamics, 3) diffusion approximation for solution radiative transfer 4) calculation dynamical viscosity from subgrid scale modelling. Simulations are conducted on horizontal uniform grid of 320 x 320 and with 144 nonuniformly spaced vertical grid points on the 128 processors of supercomputer MBC-1500 with distributed memory multiprocessors in Russian Academy of Sciences.

1. Introduction

Convection near solar surface has strongly non-local and dynamical character. Hence numerical simulation provide useful information on the structure spatial scales by convection and help to construct consistent models of the physical processes underlying the observed solar phenomena. We investigate effects of compressibility and weak of magnetic field on formation non-local structure of convection using realistic physics and conservative TVD numerical scheme of Godunov type. The previous simulations were confined by small computational domain and studied processes on scales order size of granulation [Stein & Nordlund (2006)]. In order to investigate collective interaction of convective modes different scales and process of formation of supergranulation we conducted calculation in three dimensional computational box by size 30 Mm in horizontal direction and by size 18 Mm in vertical direction.

2. Numerical method

We take distribution of the main thermodynamic variables by radius due to Standard Solar Model [Christensen-Dalsgaard (2003)] with parameters \((X, Z, \alpha) = (0.7385, 0.0181, 2.02)\), where \(X\) and \(Y\) are hydrogen and helium abundance by mass, and \(\alpha\) is the ratio of mixing length to pressure scale height in convection.
region. We use OPAL opacities and equations of state for solar matter [Iglesias (1996)].

We solve fully compressible nonideal magnetohydrodynamics equations:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0
\]

\[
\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot \left[ \rho \vec{v} \vec{v} + \left( P + \frac{B^2}{8\pi} \right) I - \frac{\vec{B} \cdot \vec{B}}{4\pi} \right] = \rho \vec{g} + \nabla \cdot \tau
\]

\[
\frac{\partial E}{\partial t} + \nabla \cdot \left( \vec{v} \left( E + P + \frac{B^2}{8\pi} \right) - \frac{\vec{B} \cdot \vec{B}}{4\pi} \right) = 1
\]

\[
\frac{\partial \vec{B}}{\partial t} + \nabla \cdot \left( \vec{v} \vec{B} - \vec{B} \vec{v} \right) = -\nabla \times \left( \eta \nabla \times \vec{B} \right)
\]

where \( E = e + \rho v^2/2 + B^2/8\pi \) is the total energy, \( Q_{rad} \) is the energy transferred by radiation and \( \tau \) is viscous stress tensor.

We assume that small scales are independent of resolved scales (Large Eddy Simulations) and rate dissipation is defined from buoyancy and shear production terms [Canuto et al. (1994)]. The numerical method that we used was an explicit Godunov-type conservative TVD difference scheme [Yee et al. (1990)]

\[
U_{i,j,k}^{n+1} = U_{i,j,k}^n - \Delta t L(U_{i,j,k}^n),
\]

where \( \Delta t = t^{n+1} - t^n \) and operator \( L \) is

\[
L(U_{i,j,k}) = \frac{\tilde{F}_{i+1/2,j,k} - \tilde{F}_{i-1/2,j,k}}{\Delta x_i} + \frac{\tilde{G}_{i,j+1/2,k} - \tilde{G}_{i,j-1/2,k}}{\Delta y_j}
\]

\[
+ \frac{\tilde{H}_{i,j,k+1/2} - \tilde{H}_{i,j,k-1/2}}{\Delta z_k} + S_{i,j,k}
\]

Flux along each direction, for example \( x \), was defined by local-characteristic method as follows

\[
\tilde{F}_{i+1/2,j,k} = \frac{1}{2} \left[ F_{i,j,k} + F_{i+1,j,k} + R_{i+1/2} W_{i+1/2} \right]
\]

where \( R_{i+1/2} \) is matrix whose columns are right eigenvectors of \( \partial F/\partial U \) evaluated at generalized Roe average for real gases of \( U_{i,j,k} \) and \( U_{i+1,j,k} \). The \( W_{i+1/2} \) is the matrix of numerical dissipation. Term \( S_{i,j,k} \) is accounted effect of gravitation forces and radiation.

The one step of time integration is defined by Runge-Kutta method [Shu & Osher (1988)] as
\[
U^{(1)} = U^n + \Delta t L(U^n) \\
U^{(2)} = \frac{3}{4} U^n + \frac{1}{4} U^{(1)} + \frac{1}{4} \Delta t L(U^{(1)}) \\
U^{n+1} = \frac{1}{3} U^n + \frac{2}{3} U^{(2)} + \frac{2}{3} \Delta t L(U^{(2)})
\]

The scheme is second order by space and time. For approximation viscous terms we used central differences. For evaluation radiative term in energy equation we used the diffusion approximation

\[
Q_{\text{rad}} = \nabla \cdot \left[ \frac{4acT^3}{3k\rho} \nabla T \right]
\]

We use uniform grid in x-y directions and nonuniform grid in vertical z one. We apply periodic boundary condition in horizontal planes and choose on the top and bottom as follows

\[
v_{z,k} = -v_{z,k+1}, v_{x,k} = v_{x,k+1}, v_{y,k} = v_{y,k+1} \\
\frac{dp}{dz} = \rho g_z, p = p(\rho), e = \text{const} \\
B_x = B_y = 0, dB_z/dz = 0
\]

In initial moment magnetic field equal 50 G and has just one vertical component. For the magnetic diffusitivity we take constant value \( \eta = 1.1 \times 10^{11} \text{cm}^2\text{sec}^{-1} \)

3. Results

On the figures 1-4 results of development convection after 12 solar hr MHD numerical simulation are shown. We founded that magnetic field concentrate on the boundary of convective cells in forms magnetic flux and sheets. Diverging of convective flows from centre supergranular expell weak magnetic field on the edges of convective cell. Average size of supergranular cells is 10-20Mm with lifetime about 8-10 hours. We apply procedure averaging by interval time two hour and find value of velocity from centre supergranular equal to 1-1.5 km/sec. In places action of strong magnetic field with strength 700-900 G we observe effect of suppression of convection and decreasing of fluctuation of temperature. Magnetic pressure in regions concentration of magnetic flux prevent inflow of matter. Transfer of radiation energy in these places is suppressed. We founded from simulation that maximum of value of magnetic field in computational domain equal to 1300 G.

Inside of supergranular we have usual picture of evolution of convection on scale of granulation with average sizes of cells about 1-2Mm and lifetimes about 10 minutes. Here we see wider upflows of warm, low density, and entropy neutral matter and downflows of cold, converging into filamentary structures, dense material. We observe continuous picture formation and destruction of granules. Granules with highest pressure grow and push matter against neighboring granules, that then shrink and disappear. Ascending flow increases pressure in center of granule and upflowing fluids decelerates motion. This process reduce
heat transport to surface and allow material above the granule to cool, become denser, and by action gravity to move down. We observe formation new cold intergranule lane splitting the original granule.

From figure 4 we see distinctly existence three different regions development of convection. In near of solar surface to the depth 4 Mm we founded zone of turbulent convection. In this region cold blobs of matter move down with velocity in maximum about 4 km/sec with maximum Mach number equal 1. The downdrafts has different and very complicate vertical structure. Some ones travel small distance from surface and become weak enough to be broken up by the surrounding fluid motion. Other ones conserve motion with high velocity and move on distance about 6 Mm. We observe that different nature such behavior is due to initial condition of formation downdrafts. In place of confluence of convective cells to one point more energy is released.

In region from 5Mm to 8Mm of depth we reveal more quiet character of convective flow than in turbulent zone. Below 8Mm we see clear separate large scale density fluctuations and streaming flow of matter similar jets with average velocity about 1km/sec. The magnetic field in these places has values about 300G. Distance beetween different narrow jets gives size of supergranular cells. Places of intersections of path jets with horizontal plane are vertices of huge convective cells. From distribution iso-surface of magnetic field with strength 500 G we have discovered that pumping of magnetic field occupy more part of all computational domain up to bottom boundary.

Magnetic field near solar surface in our numerical simulation has very complicate structure. Inside supergranular cells we see formation, growth and evolution with time many arising to surface loops of magnetic field. There is places in vertices of huge cells with vorticity motion that provide quick rate magnetic helicity transport across solar photosphere(Fig.5,6). On the boundary supergranular cell magnetic field has just vertical component. In these parts we observe quick changing sign and big values of current helicity(Fig.7).

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MHD simulation of solar convection

Figure 1. Image of contours of the vertical velocity in horizontal plane near solar surface.

Figure 2. Image of contours of the vertical component of magnetic field at the same plane as that for Figure 1.
Figure 3. Image of contours of the temperature in the horizontal plane near solar surface

Figure 4. Image of contours of the fluctuations density and field of velocity in vertical plane.
Figure 5. Image of contours of the magnetic helicity transport rate in horizontal plane near solar surface as that for Figure 1.

Figure 6. Image of contours of the magnetic helicity transport rate and field of velocity in small part computational domain at the same plane as that for Figure 5.
Figure 7. Image of contours of the current helicity at the same plane as that for Figure 1.