Semiclassical String Solutions on deformed NS5-brane Backgrounds and New Plane wave

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Abstract

We study different Penrose limits of supergravity solution of NS5-brane in the presence of RR field. Although in the case of NS5-brane we get a 4-dimensional plane wave, in the case with RR field we will get two different plane waves; a 4-dimensional and a 3-dimensional one. These plane wave solutions are the backgrounds that a particular string solution feels at one loop approximation. Using the one loop correction one can identify a particular subsector of LST/deformed LST which is dual to type II string theories on these plane waves.
1 Introduction

Little string theory (LST) is one of the most important examples of non-local theories. This theory arises on the worldvolume of NS5-branes in a decoupling limit where $g_s \to 0$ with fixed $\alpha'$ [1, 2]. LST’s share many properties with usual string theory, such as T-duality and Hagedorn behavior of density of states, but are nevertheless non-gravitational theories. To study the theory one could use the holographic dual of LST, given in terms of string propagation in the linear dilaton background [3] along the AdS/CFT correspondence [4].

Recently there has been an improvement in understanding this correspondence beyond the supergravity limit. In fact it has been conjectured [5] that string theory on the maximally supersymmetric ten-dimensional plane wave has a description in terms of a certain subsector of the large $N$ four-dimensional $\mathcal{N} = 4$ $SU(N)$ supersymmetric gauge theory at weak coupling. More precisely this subsector is parametrized by states with conformal weight $\Delta$ carrying $J$ units of charge under the $U(1)$ subgroup of the $SU(4)_R$ R-symmetry of the gauge theory, such that both $\Delta$ and $J$ are parametrically large in the large $\ 't \ Hooft$ coupling while their difference, $\Delta - J$ is finite. Then it has been possible to work out the perturbative string spectrum from gauge theory side. The idea of [5] is based on the observation that the plane wave background with RR 4-form in type IIB string theory is maximally supersymmetric [6], solvable [7, 8] and can be understood as a certain limit (Penrose limit) of $AdS_5 \times S^5$ geometry [9, 5].

On the other hand the authors of [10] identified certain classical solutions representing highly excited string states carrying large angular momentum in the $AdS_5$ part of the metric with gauge theory operators with high spin and conformal dimension which is identified with the classical energy of the solution in the global $AdS$ coordinates.

In this context the BMN operators [5] can also be identified with classical solutions of string in $AdS_5$ with angular momentum in $S^5$ space. Small fluctuations around this classical solution in second order of fluctuation will lead to the corresponding type IIB maximally supersymmetric plane wave [10, 11]. Then the one loop approximation around this classical solution can be used to study the anomalous dimension of the BMN operators. Higher loop approximation can also be used to study the higher order corrections to the anomalous dimension of BMN operators. In fact from string theory point of view there would be corrections controlled by an expansion parameter given by $\frac{1}{R^2}$, where $R^2$ is the radius of the AdS space. From the gauge theory point of view this leads to the $\frac{1}{J}$ perturbative expansion to the anomalous dimensions of the BMN operators.

The idea of [10] has been generalized for other string/M theory backgrounds in several papers including [12, 25].

Physically the new scaling limit defined in [5] opens up new insight about the gauge theory/string theory correspondence. Therefore it would be natural to see whether this strategy can be used for other theories. In fact this is the aim of this
paper to explore this idea for LST theory as well as its noncommutative/dipole deformations. We note, however, that the Penrose limit of NS5-brane and its deformation has partially been studied in the literature, for example see [28, 38, 39].

In this paper we will take the point of view of [10] to further study this theory. In section 2 we shall consider a classical string solution in the background generated by NS5-brane. Using the quantum correction around this classical solution we have been able to identify a subsector of LST theory which is dual to the string theory on a 4-dimensional plane wave. This subsector is parameterized by energy and angular momentum whose difference receives controllable quantum corrections. In section 3 classical string solution in NS5-brane in the presence of nonzero RR field with all indices along the NS5-brane is studied. In this case we find two different classes of classical solutions which lead to two different plane waves. We will then identify the corresponding subsectors of noncommutative LST theory which are dual to these plane waves. The one loop approximation around these classical solutions has also been studied. In section 4 the same analysis has been considered for dipole deformation of LST. The last section is devoted to conclusions.

2 Plane wave from NS5-brane

In this section we shall review the Penrose limit of the NS5-brane supergravity solution in type II string theories [26, 27, 28]. We will show that the corresponding plane wave is the background that a particular classical string solution feels in one loop approximation. This can be used to identify a subsector of LST which is dual to string theory on the obtained plane wave.

The supergravity solution on $N$ NS5-branes is given by

$$
d s^2 = -dt^2 + d\vec{x}^2 + f(dr^2 + r^2 d\Omega_3^2),

\quad e^{2\phi} = g_s^2 f, \quad dB = 2Nl_s^2 \epsilon_3, \quad f = 1 + \frac{Nl_s^2}{r^2},
$$

(1)

where $d\vec{x}$ parameterizes a 5-dimensional flat space and $\epsilon_3$ is the volume of $d\Omega_3^2$. In the decoupling limit where $g_s \to 0$ and $l_s = \text{fixed}^3$, setting $r = g_s u$, the supergravity solution (1) reads

$$
d s^2 = -dt^2 + d\vec{x}^2 + \frac{N}{u^2}(du^2 + u^2 d\Omega_3^2),

\quad e^{2\phi} = \frac{N}{u^2}, \quad dB = 2N \epsilon_3,
$$

(2)

which provides the gravity dual description of LST.

To study the Penrose limit of the above supergravity solution we will first rescale $t \to \sqrt{N}t$ and consider a null geodesic along $t$ and $\psi$ directions$^4$ and at a fixed point

$^3$Hereafter for simplicity, we set $l_s = 1$.

$^4$Here we parameterize the 3-sphere in (2) as $d\Omega_3^2 = d\eta_1^2 + \cos^2 \eta_1 d\eta_2^2 + \sin^2 \eta_1 d\eta_3^2$. 

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with respect to other coordinates. In fact we consider a particle moving along \( \eta_2 \) direction and sitting at \( u = u_0 \) and \( \eta_1 = 0 \). The geometry close to this trajectory can be obtained by the following rescaling

\[
t = \frac{1}{2}(x^+ + x^-), \quad \eta_2 = \frac{1}{2}(x^+ - x^-), \quad \eta_1 = \frac{z}{\sqrt{N}}, \quad u = \sqrt{N} e^{-\rho_0 + \rho/\sqrt{N}}.
\]  

(3)

In large \( N \) limit keeping \( x^+, x^- \) and \( z \) fixed the solution (2) reads

\[
ds^2 = -dx^+ dx^- - \frac{1}{4} z^2 (dx^+)^2 + d\tilde{z}^2 + d\tilde{y}^2, \quad B_{+z_1} = z_2.
\]  

(4)

with a constant dilaton \( \phi = \rho_0 \), and \( \tilde{y} = (\rho, \vec{x}) \). This provides an exact string background to all orders in worldsheet perturbation theory [29, 30, 31].

Physically what we have considered is a physical system around the following classical string configuration

\[
t = k\tau, \quad \rho = \rho(\sigma), \quad \eta_2 = \omega\tau.
\]  

(5)

and all other coordinates are set to zero. This classical solution represents a string stretched along the radial coordinate and rotating along \( \eta_2 \) direction with the angular velocity \( \omega \). From the LST point of view this corresponds to a subsector of theory parameterized by its energy\(^5\), \( E \) and quantum number \( J \) such that in large \( N \) limit both of them grow as \( N \) while \( E - J \) is finite. Using the classical action of string one can find a relation between energy, \( E \), and the angular momentum along the \( \eta_2 \) direction, \( J \). Doing so in the point like string limit one finds

\[
E^2 = \frac{N\rho_0^2}{4\pi^2} + J^2.
\]  

(6)

The background this state feels is the plane wave (4). To see this let us consider small quantum fluctuations around this classical solution as following

\[
t = k\tau + \frac{\tilde{t}}{\sqrt{N}}, \quad \rho = \rho_0 + \frac{\tilde{\rho}}{\sqrt{N}}, \quad \eta_2 = k\tau + \frac{\tilde{\eta}_2}{\sqrt{N}}, \quad x_i = \tilde{x}_i, \quad \eta_1 = \frac{\tilde{z}_1}{\sqrt{N}}, \quad \eta_2 = \tilde{z}_2
\]  

(7)

Expanding the string action around this classical solution to the second order in fluctuations, one finds:

\[
I_{(2)} = -\frac{1}{4\pi} \int d^2\xi [-\partial_a \tilde{t} \partial^a \tilde{t} + \partial_a \tilde{\eta}_2 \partial^a \tilde{\eta}_2 + \partial_a \tilde{\vec{y}} \partial^a \tilde{\vec{y}} + \partial_a \tilde{z}_1 \partial^a \tilde{z}_1 + k^2 \tilde{z}^2 + 4k\tilde{z}_2 \partial_a \tilde{z}_1].
\]

(8)

This action should be compared with the string action in the plane-wave background (4). To do this let us define the lightcone coordinates \( x^\pm = \eta_2 \pm t \). In these coordinates the expansion must be done around the following classical solution

\[
x^+ = p^+ \tau, \quad x^- = \tilde{y} = z_i = 0, \quad \rho = \rho_0, \quad p^+ = 2k.
\]

(9)

\(^5\)If we want to work with the original coordinate as in (2) the energy \( E \) needs to be scaled as \( \sqrt{NE} \).
One can see that in the quantum gauge, $\tilde{x}^+ = 0$, the action (8) reads

$$I_{(pw)} = -\frac{1}{4\pi} \int d^2 \xi [\partial_\alpha x^+ \partial^\alpha x^- - \frac{1}{4} z_i^2 \partial_\alpha x^+ \partial^\alpha x^- + \partial_\alpha \tilde{y} \partial^\alpha \tilde{y} + \partial_\alpha z_i \partial^\alpha z_i + 2B_{+z_1} \partial_\tau x^+ \partial_\sigma z_1],$$  

(10)

which is the string action on the plane wave background (4) in the lightcone gauge.

The energy, $E$ and the angular momentum, $J$ of the point like string up to the second order will read:

$$E = \frac{1}{2\pi} \int d\sigma (k + \partial_\tau \tilde{t}),$$

$$J = \frac{1}{2\pi} \int d\sigma (k + \partial_\tau \tilde{\eta}_2 - k \tilde{z}^2 - \tilde{z}_2 \partial_\sigma \tilde{z}_1),$$  

(11)

therefore we get

$$E - J = \frac{1}{4\pi} \int d\sigma (-2\partial_\tau \tilde{x}^- + 2k \tilde{z}^2 + 2\tilde{z}_2 \partial_\sigma \tilde{z}_1).$$  

(12)

By making use of the Virasoro constraint,

$$2k\partial_\tau \tilde{x}^- - \partial_\tau \tilde{t} \partial_\sigma \tilde{t} + \partial_\alpha \tilde{y} \partial^\alpha \tilde{y} + \partial_\alpha \tilde{z}_i \partial^\alpha \tilde{z}_i + \partial_\alpha \tilde{z}_i \partial^\alpha \tilde{z}_i + \partial_\tau \tilde{z}_i \partial^\tau \tilde{z}_i + \partial_\sigma \tilde{z}_i \partial_\sigma \tilde{z}_i + \partial_\tau \tilde{\eta}_2 \partial_\sigma \tilde{\eta}_2 + \partial_\sigma \tilde{\eta}_2 \partial_\sigma \tilde{\eta}_2) - k^2 \tilde{z}^2 = 0,$$

(13)

one can solve $\partial_\tau \tilde{x}^-$ in terms of the second order fluctuations, and thus we find

$$E - J = \frac{1}{4\pi} \int d\sigma \left[ \partial_\tau \tilde{x}^+ \partial_\tau \tilde{x}^- + \partial_\alpha \tilde{x}^+ \partial_\alpha \tilde{x}^- + \partial_\tau \tilde{y} \partial_\tau \tilde{y} + \partial_\alpha \tilde{y} \partial^\alpha \tilde{y} + \partial_\tau \tilde{z}_i \partial_\tau \tilde{z}_i + \partial_\alpha \tilde{z}_i \partial^\alpha \tilde{z}_i + \partial_\sigma \tilde{z}_i \partial_\sigma \tilde{z}_i + k^2 \tilde{z}^2 + 4k \tilde{z}_2 \partial_\sigma \tilde{z}_1 \right].$$  

(14)

The expression in the right hand side is the 2-dimensional transverse Hamiltonian of the worldsheet. By making use of this expression one can find the quantum correction to the classical relation between $E$ and $J$. In fact one finds

$$E - J = \sum_n \left( \frac{N|n|}{J} N_n^{(6)} + \omega_n^+ N_n^+ + \omega_n^- N_n^- \right),$$

$$\omega_n^\pm = |1 \pm \frac{Nn}{J}|,$$

(15)

where $N_n^{(6)}$, $N_n^+$ and $N_n^+$ are occupation numbers along the 6-dimensional flat space and directions $z_1$ and $z_2$ when they are written in normal modes.

Therefore one might conclude that type IIA string theory on the plane wave background (4) is dual to a subsector of LST theory which is parameterized by energy, $E$ and angular momentum, $J$ such that both of them grow like $N$ in large $N$ limit while $E - J$ is finite and given by (15).
3 Plane wave from noncommutative deformation of NS5-brane

In this section we shall consider the Penrose limit of noncommutative deformation of LST theory. This will be done by making use of the supergravity solution of NS5-brane in the presence of nonzero RR field. We will first consider the case where all indices of RR fields are along the NS5-brane worldvolume. In fact these are decoupled theories (ODp) on the worldvolume of type II NS5-branes in the presence of nonzero $p$-form RR field \[32, 33\]. The excitations of these theories include light open Dp-branes. The gravity description of these theories have been studied in \[36, 37\]. We could also consider a case where one leg of the RR fields is in the transverse direction which corresponds to the dipole deformation of LST. This case will be studied in the next section.

The gravity description of ODp theory is given by \[36\]

$$ds^2 = (1 + a^2 r^2)^{1/2} \left[ -dt^2 + \sum_{i=1}^p dx_i^2 + \frac{\sum_{j=p+1}^5 dx_j^2}{1 + a^2 r^2} + \frac{N}{r^2} (dr^2 + r^2 d\Omega_3^2) \right],$$

$$A_0...p = \frac{1}{\tilde{g}} a^2 r^2,$$

$$A_{(p+1)...5} = \frac{1}{\tilde{g}} \frac{a^2 r^2}{1 + a^2 r^2},$$

$$e^{2\phi} = \frac{\tilde{g}^2 (1 + a^2 r^2)^{(p-1)/2}}{a^2 r^2},$$

$$dB = 2N \epsilon_3,$$

$$a^2 = \frac{l_{\text{eff}}^2}{N},$$

(16)

where $l_{\text{eff}}$ and $\tilde{g}$ are effective string tension and effective string coupling of the theory which are the parameters of the theory after taking the decoupling limit \[32\]. Here $\epsilon_3$ is the volume of $S^3$ part of the metric.

3.1 4-dimensional plane wave

The Penrose limit of background (16) has been studied in \[38, 39\] (see also \[40, 41, 42\]). The obtained plane wave in this case corresponds to the background which a point like string moving in the 3-sphere feels. In fact to get the corresponding plane wave we first rescale $t \to \sqrt{N} t$ and then consider the following classical string configuration

$$t = k\tau, \quad \rho = \rho(\sigma), \quad \eta_2 = \omega \tau,$$

(17)

where $\rho = \ln(ar)$. The Nambu-Gotto action for this classical string configuration reads

$$I = -\frac{1}{2\pi} \sqrt{N(k^2 - \omega^2)} \int d\rho \sqrt{1 + e^{2\rho}},$$

(18)

which in the point like string limit leads to the similar result as (6). Actually the plane wave background seen by the small quantum fluctuations around this classical

\[For p = 1, 2 see also \[34, 35\].\]
solution can be obtained from a null geodesic around \( \rho = \rho_0 = \text{constant} \) [39]. In this case we perform the following coordinate transformations:

\[
\begin{align*}
\rho &= \rho_0 + (1 + e^{2\rho_0})^{-1/4} \frac{r}{\sqrt{N}}, \\
t &= \frac{1}{2}(1 + e^{2\rho_0})^{-1/4}(x^+ + \frac{e}{N}), \\
\eta_1 &= (1 + e^{2\rho_0})^{-1/4} \frac{z}{\sqrt{N}}, \\
x_j &= (1 + e^{2\rho_0})^{1/4}w_j, \quad (j = p + 1, \ldots, 5), \\
x_i &= (1 + e^{2\rho_0})^{-1/4}y_i, \quad (i = 1, \ldots, p),
\end{align*}
\]

(19)

and \( \eta_3 \) is kept fixed. One then obtains in the Penrose limit \( (N \to \infty) \) a plane wave background as following

\[
ds^2 = -dx^+dx^- - \frac{z^2}{4}dx^+dx^+ + dz^2 + dy^2,
\]

(20)

where \( dy^2 = dr^2 + dw^2 + dz^2 \). Here we have also rescaled the longitudinal coordinates by \( x^\pm \to x^\pm (1 + e^{2\rho_0})^{\pm 1/4} \). In this limit, one gets also a nonvanishing \( p + 1 \) form field and nonzero B field:

\[
\begin{align*}
dA_{+ry_1 \cdots y_p} &= \frac{1}{g} e^{2\rho_0} (1 + e^{2\rho_0})^{-(p+1)/4}, \\
A_{(p+1) \cdots 5} &= \frac{1}{g} \frac{e^{2\rho_0}}{1 + e^{2\rho_0}}, \\
B_{+z_1} &= (1 + e^{2\rho_0})^{-1/2}z_2,
\end{align*}
\]

(21)

and the constant dilaton after the Penrose limit is given by:

\[
e^{2\phi} = g^2 \frac{(1 + e^{2\rho_0})^{(p-1)/2}}{e^{2\rho_0}}.
\]

(22)

To compare this plane wave solution with that obtained from NS5-brane supergravity background it is constructive to study the small quantum fluctuations around the above classical solution. Consider the following small fluctuations around the classical solution (17)

\[
t = k\tau + \tilde{t}, \quad \eta_2 = k\tau + \tilde{\eta}_2, \quad \rho = \rho_0 + \tilde{\rho}, \quad x_i = \tilde{y}_i, \quad x_j = \tilde{y}_j, \quad \eta_i = \tilde{z}_i,
\]

(23)

with \( i = 1, \ldots, p \) and \( j = p + 1, \ldots, 5 \). For simplicity we first rescale the coordinates as follows:

\[
\begin{align*}
\tilde{t} \to \frac{(1 + e^{2\rho_0})^{-1/4}}{\sqrt{N}}\tilde{t}, \quad \tilde{x}_i \to (1 + e^{2\rho_0})^{-1/4}\tilde{y}_i, \quad \tilde{x}_j \to (1 + e^{2\rho_0})^{1/4}\tilde{y}_j, \\
\tilde{\rho} \to \frac{(1 + e^{2\rho_0})^{-1/4}}{\sqrt{N}}\tilde{\rho}, \quad \tilde{\eta}_1 \to \frac{(1 + e^{2\rho_0})^{-1/4}}{\sqrt{N}}\tilde{z}_1, \quad \tilde{\eta}_2 \to \frac{(1 + e^{2\rho_0})^{-1/4}}{\sqrt{N}}\tilde{\eta}_2,
\end{align*}
\]

(24)

and \( \tilde{z}_2 \) is also rescaled by \( (1 + e^{2\rho_0})^{-1/4} \). We can now proceed as in the previous section to write the string action up to the second order in fluctuations. The result is:

\[
I_2 = -\frac{1}{4\pi} \int d^2\xi \left[ - \partial_\alpha \tilde{t} \partial^\alpha \tilde{t} + \partial_\alpha \tilde{\eta}_2 \partial^\alpha \tilde{\eta}_2 + \partial_\alpha \tilde{y}_i \partial^\alpha \tilde{y}_i + \partial_\alpha \tilde{z}_i \partial^\alpha \tilde{z}_i \right.
\]

\[
\left. + \partial_\alpha \tilde{\rho} \partial^\alpha \tilde{\rho} + \partial_\alpha \tilde{\eta}_1 \partial^\alpha \tilde{\eta}_1 + \partial_\alpha \tilde{z}_1 \partial^\alpha \tilde{z}_1 \right].
\]
\[ k^2 z^2 + \frac{4k}{(1 + e^{2\rho_0})^{1/2}} \tilde{z}_2 \partial_{\sigma} \tilde{z}_1 \, , \]  

(25)

where \( \vec{y} = (\rho, x_i, x_j) \). To compare this action with the string action on plane wave solution (20) we define the lightcone coordinates as \( x^\pm = \eta_2 \pm t \). In these coordinates the small fluctuations must occur around the following classical solution

\[ x^+ = p^+ \tau \, , \, x^- = 0 \, , \, x_i = x_j = 0 \, , \, \rho = \rho_0 \, , \, z_i = 0 \, , \, p^+ = 2k \, . \]  

(26)

Then in the quantum gauge \( \tilde{x}^+ = 0 \) the action (25) reads

\[ I_{(pw)} = -\frac{1}{4\pi} \int d^2 \xi \left[ \partial_\sigma x^+ \partial^\sigma x^- - \frac{1}{4} z^2 \partial_\sigma x^+ \partial^\sigma x^+ + \partial_\sigma y_i \partial^\sigma y_i + \partial_\sigma z_i \partial^\sigma z_i + 2B_{\pm z_1} \partial_\sigma x^+ \partial_\sigma z_1 \right] \, , \]  

(27)

with \( B_{\pm z_1} = (1 + e^{2\rho_0})^{-1/2} z_2 \). We recognize this action as the string action on the plane wave solution (20).

As in the previous section one can write the Virasoro constraint and solve it for \( \partial_\tau \tilde{x}^- \) in terms of the second order fluctuations. This can be used to find an expression for the energy and angular momentum up to second order in fluctuations. Using the obtained expressions for energy and angular momentum one finds

\[ E - J = \frac{1}{4\pi k} \int d\sigma \left[ \partial_\sigma \tilde{x}^+ \partial_\sigma \tilde{x}^- + \partial_\sigma \tilde{x}^+ \partial_\sigma \tilde{x}^- + \partial_\sigma \tilde{y}_i \partial_\sigma \tilde{y}_i + \partial_\sigma \tilde{z}_i \partial_\sigma \tilde{z}_i + k^2 \tilde{z}^2 + \frac{4k}{(1 + e^{2\rho_0})^{1/2}} \tilde{z}_2 \partial_\sigma \tilde{z}_1 \right] \, . \]  

(28)

Note that the right side in the above equation is the worldsheet Hamiltonian of the transverse fluctuations. Therefore we get\(^7\)

\[ E - J = \sum_n \left[ \sqrt{1 + e^{2\rho_0}} \frac{N|n|}{J} N_n^{(6)} + \omega_n^+ N_n^+ + \omega_n^- N_n^- \right] \, , \]

\[ \omega_n^+ = \sqrt{1 + (1 + e^{2\rho_0}) \frac{N^2 n^2}{J^2} + 2 \frac{N n}{J} \right] \, , \]  

(29)

where \( N_n^{(6)}, N_n^+ \) and \( N_n^- \) are occupation numbers along the 6-dimensional flat space and directions \( z_1 \) and \( z_2 \) when they are written in normal modes, respectively. Thus string theory on the plane wave background (20) is dual to a subsector of the noncommutative deformation of LST which is parameterized by energy \( E \) and a quantum number \( J \) such that in large \( N \) limit both of them grow as \( N \) while \( E - J \) is finite and given by (29).

\(^7\)In writing this expression we have used the relation between string theory parameters and worldvolume parameters as \( P^+ = (1 + e^{2\rho_0})^{-1/2} \frac{2J}{N} \).
3.2 3-dimensional plane wave

In comparison with the NS5-brane background the deformed NS5-brane background (16) has an extra parameter corresponding to the RR fields. This parameter can also be thought of as the deformation parameter $a$. The presence of RR field will also break the Lorentz symmetry. Therefore it is natural to consider a boost along the direction in which the RR field is defined. This can be thought of as an other Penrose limit of the background. Physically, what we are considering is a fast moving particle along those directions where the RR field is defined. Then we can look at the theory close to the trajectory of this particle which leads to a new plane wave background. In fact what we really have to do is to consider a classical string configuration which is stretched along the radial coordinate and moves in a direction where the RR-field is defined. In the point like limit this would be the physical system we are looking for.

Let us first obtain the corresponding plane wave and then study the semiclassical string solution leading to this plane wave. To find the corresponding plane wave we consider the following rescaling

$$ t = \frac{1}{2} \left( \frac{x^+}{a} + ax^- \right), \quad x_5 = \frac{1}{2} \left( \frac{x^+}{a} - ax^- \right). \tag{30} $$

In the limit of $a \to 0$ keeping $g_0 = \tilde{g}/a$ fixed the supergravity solution (16) for $p \neq 5$ reads

$$ ds^2 = -dx^+dx^- - \frac{r^2}{4}(dx^+)^2 + \frac{N}{r^2} dr^2 + dy^2 + N d\Omega_3^2, \quad dB = N \epsilon_3, $$

$$ A_{+1 \ldots p} = \frac{1}{2g_0} r^2, \quad A_{(p+1) \ldots 4+} = \frac{1}{2g_0} r^2, \quad e^{2\phi} = \frac{g_0^2}{r^2}. \tag{31} $$

The case of $p = 1$ has first been studied in [36] in the context of lightlike noncommutative geometry. In fact the dual theory we get from this Penrose limit is the lightlike noncommutative deformation of LST. We note also that the obtained plane wave provides a string theory background in which the string theory can be exactly solved. Actually this consists of level $N SU(2)$ WZW, a three dimensional Liouville plane wave plus 4 free fields theory. The Liouville plane wave background in string theory has recently been considered in [43, 44] as a background in which the string theory can be exactly solved.

The classical string solution representing a string moving along $x_5$ direction and stretched in the radial coordinate is

$$ t = k\tau, \quad r = r(\sigma), \quad x_5 = p\tau. \tag{32} $$

For point like strings and in the limit where $a \to 0$ one find a relation between energy $E$ and momentum $P$ which is similar to (6). We can also study the small quantum fluctuations around this classical solution. Basically the procedure is very
similar to what we have considered in the previous cases. The only point in this case is that the WZW part of the action remains unchanged in the procedure of the expansion. In fact the bosonic part of the action takes the following form

\[ I_{\text{total}} = I(t, r, x_i, x_j) + I_{\text{WZW}} \]

(33)

up to an “a” dependent coefficients in WZW part which is one in \(a \to 0\) limit. The first term is also given by

\[ I = -\frac{1}{4\pi} \int d^2\xi \left[ (1 + a^2 r^2)^{1/2} \left( -\partial_\alpha t \partial^\alpha t + \partial_\alpha x_i \partial^\alpha x_i + \frac{\partial_\alpha x_j \partial^\alpha x_j}{1 + a^2 r^2} + \frac{N}{r^2} \partial_\alpha r \partial^\alpha r \right) \right], \]

(34)

which up to second order in the fluctuations around the above classical solution, \(t = k\tau + \tilde{t}, x_5 = k\tau + \tilde{x}_5, r = \tilde{r}\), reads

\[ I_{(2)} = \int d^2\xi \left[ -\partial_\alpha \tilde{t} \partial^\alpha \tilde{t} + \partial_\alpha \tilde{x}_5 \partial^\alpha \tilde{x}_5 + \frac{N}{r^2} \partial_\alpha \tilde{r} \partial^\alpha \tilde{r} + \partial_\alpha \tilde{y}_i \partial^\alpha \tilde{y}_i + m^2 \tilde{r}^2 \right], \]

(35)

where \(y_i, i = 1, \ldots, 4\) represents the four transverse directions to the plane wave. Note also that to get the above action we have taken into account the limit of \(a \to 0\) while kept \(m := ka\) fixed. It is easy to see that this action plus the WZW part is the bosonic part action of string in the plane wave background (31). One could proceed to compute \(E\) and \(J\) and thereby to find the quantum correction to \(E - J\). This would of course get corrections from different parts of the action; contributions from 4 free fields theory, 3-dimensional Liouville plane wave and \(SU(2)\) WZW. It would be interesting to find each contribution explicitly.

4 Plane wave from dipole deformation of NS5-brane

The dipole deformation of LST can be described by the supergravity solution of type IIB NS5-branes in the presence of an RR 2-form potential with one leg along the brane and the other along the transverse directions\(^8\). We can also make a series of T-duality transformations to produce a new supergravity solution. This supergravity solution describes type II NS5-branes in the presence of RR \((6 - p)\)-form, for \(p = 0 \ldots 4\), with one leg along the transverse directions and \((5 - p)\) legs along the NS5-branes worldvolume. The corresponding supergravity solution in the decoupling limit is given by [45]

\[
\begin{align*}
  ds^2 &= (1 + r^2 L^2)^{1/2} \left[ dt^2 - \sum_{i=1}^{p} dx_i^2 - \frac{\sum_{j=p+1}^{5} dx_j^2}{1 + r^2 L^2} \\
  &\quad - \frac{N}{r^2} \left( dr^2 + r^2 d\Omega_3 - \frac{r^4 L^2}{1 + r^2 L^2} (a_1 d\theta_1 + a_2 d\theta_2 + a_3 d\theta_3)^2 \right) \right],
\end{align*}
\]

\(^8\)For definition of dipole field theory and its relevance to string theory see, for example, [46]-[49].
\[ e^{2\phi} = \frac{N}{r^2} (1 + r^2 L^2)^{(p-2)/2}, \]

\[ \sum_{a=6}^{9} A_{(p+1)\ldots 5\theta_a} d\theta_a = \frac{r^2 L}{1 + r^2 L^2} (a_1 d\theta_1 + a_2 d\theta_2 + a_3 d\theta_3), \]  

(36)

where \( \theta_i \)'s are angular coordinates parameterizing the sphere \( S^3 \) transverse to the NS5-branes\(^9\), and

\[ a_1 = \cos \theta_2, \quad a_2 = -\sin \theta_1 \cos \theta_2, \quad a_3 = \sin^2 \theta_1 \sin^2 \theta_2. \]  

(37)

Here \( L \) is the effective dipole moment. There is also a two form \( B \) field representing the charge of the NS5-branes which is given by \( dB = 2N \epsilon_3 \). Note that the above solution is maximally supersymmetric which means that the solution preserves 8 supercharges.

### 4.1 4-dimensional plane wave

Let us first study the system close to the trajectory of a point like string moving in a direction of 3-sphere transverse to the NS5-brane. The background this state feels would be a plane wave solution. To find this plane wave we first rescale \( t \rightarrow \sqrt{N} t \) and define a new radial coordinate \( \rho \) as \( L r = e^\rho \). In these coordinates the supergravity solution (36) reads

\[

ds^2 = (1 + e^{2\rho})^{1/2} \left[ -Nd^2 + \sum_{i=1}^{p} dx_i^2 + \sum_{j=p+1}^{5} dx_j \right] \\
+ N d\rho^2 + N d\Omega_3 - \frac{Ne^{2\rho}}{1 + e^{2\rho}} (a_1 d\theta_1 + a_2 d\theta_2 + a_3 d\theta_3)^2, \\

e^{2\phi} = g_s^2 \frac{(1 + e^{2\rho})(p-2)/2}{e^{2\rho}}, \\
\sum_{a=1}^{3} A_{(p+1)\ldots 5\theta_a} d\theta_a = \frac{e^{2\rho}}{L(1 + e^{2\rho})} (a_1 d\theta_1 + a_2 d\theta_2 + a_3 d\theta_3). \]  

(38)

Now we will consider a null geodesic in \((t, \theta_3)\) directions around \( \rho = \rho_0 = \text{constant} \).

In this case we perform the following coordinate transformations

\[
\rho = \rho_0 + (1 + e^{2\rho_0})^{-1/4} r, \quad x_j = (1 + e^{2\rho_0})^{1/4} w_j, \quad (j = p+1, \ldots, 5), \\
\theta_\alpha = (1 + e^{2\rho_0})^{-1/4} \frac{z_\alpha}{\sqrt{N}}, \quad x_i = (1 + e^{2\rho_0})^{-1/4} y_i, \quad (i = 1, \ldots, p), \\
t = \frac{1}{2}(1 + e^{2\rho_0})^{-1/4} (x^+ + \frac{x^-}{N}), \quad \theta_3 = \frac{1}{2}(1 + e^{2\rho_0})^{-1/4} (x^+ - \frac{x^-}{N}). \]  

(39)

\(^9\)Note that here we used a parameterization such that in terms of \( \theta_i \)'s the metric of 3-sphere is given by \( d\Omega_3^2 = d\theta_1^2 + \cos^2 \theta_1 d\theta_2^2 + \cos^2 \theta_1 \cos^2 \theta_2 d\theta_3^2. \)
with \( \alpha = 1, 2 \). In the limit of \( N \to \infty \) we find the following plane wave from the supergravity solution (38)

\[
d s^2 = -dx^+ dx^- - \frac{1}{4} \left( (1 + e^{2\rho_0}) z_1^2 + z_2^2 \right) (dx^+)^2 + d\vec{z}^2 + dy^2,
\]
\[
e^{2\phi} = g_s^2 \frac{(1 + e^{2\rho_0})^{(n-2)/2}}{e^{2\rho_0}}, \quad B_{+z_1} = z_2,
\]

(40)

where \( dy^2 = dr^2 + dy^2 + dw^2 \). This 4-dimensional plane wave is very similar to that in LST case. Of course unlike the LST case, in this case \( z_1 \) and \( z_2 \) have different masses which is the effect of dipole deformation.

The corresponding classical string solution is

\[
t = k \tau, \quad \rho = \rho(\sigma), \quad \theta_3 = \omega \tau,
\]

(41)

which in the point like limit leads to the above plane wave at second order in fluctuations. To see this we consider the small fluctuations around this classical solution. In the one loop approximation one gets

\[
I_2 = -\frac{1}{4\pi} \int d^2 \xi \left[ \partial_\alpha \bar{x}^+ \partial^\alpha \bar{x}^- + \partial_\alpha \bar{y} \partial^\alpha \bar{y} + \partial_\alpha \bar{\theta}_i \partial^\alpha \bar{\theta}_i + k^2 \left( (1 + e^{2\rho_0}) \bar{\theta}_1^2 + \bar{\theta}_2^2 \right) \right] + \frac{1}{2} \partial_\tau \partial_\tau \bar{\theta}_1,
\]

(42)

which is equivalent to the bosonic part of the string action on the plane wave background (40). As in the previous cases one can also write the expressions for energy and angular momentum in terms of the transverse modes. Using the Virasoro constraint in the one loop approximation one finds:

\[
E - J = \frac{1}{2\pi k} \int d\sigma \left[ \partial_\tau \bar{x}^+ \partial_\tau \bar{x}^- + \partial_\tau \bar{y} \partial_\tau \bar{y} + \partial_\tau \bar{\theta}_i \partial_\tau \bar{\theta}_i + \partial_\sigma \bar{\theta}_i \partial_\sigma \bar{\theta}_i + k^2 \left( (1 + e^{2\rho_0}) \bar{\theta}_1^2 + \bar{\theta}_2^2 \right) + 4k \partial_\tau \partial_\tau \bar{\theta}_1 \right].
\]

(43)

We note that the right hand side is the Hamiltonian of the transverse fluctuations. Therefore we find

\[
E - J = \sum_n \left[ \sqrt{1 + e^{2\rho_0}} \frac{N_6}{J} \right] N_n^{(6)} + \omega^+_n N^+_n + \omega^-_n N^-_n,
\]

(44)

where \( N_n^{(6)} \) is the occupation number of the 6 transverse directions and \( N_n^\pm \) is occupation number along \( z_1 \) and \( z_2 \) when they are written in normal mode representation. The \( \omega^\pm \) is given by

\[
\omega^\pm_n = \left( 1 + e^{2\rho_0} \right) + \left( 1 + e^{2\rho_0} \right) \frac{N_n^2}{J^2} \pm \sqrt{\left( 1 + e^{2\rho_0} \right) \frac{4N_n^2}{J^2} + e^{4\rho_0} / 4} \right)^{1/2}.
\]

(45)

Thus one may conclude that string theory on the plane wave background (40) is dual to a subsector of dipole deformation of LST parameterized by energy, \( E \) and angular momentum, \( J \) which at large \( N \) limit both of them grow as \( N \) while \( E - J \) is finite and given by (44).
4.2 3-dimensional plane wave

Let us now study a point like string moving in $x_5$ direction and look at the physics close to the trajectory of this point like string. To study the system near this trajectory we perform the following rescaling

$$t = \frac{1}{2} \left( \frac{x^+}{L} + Lx^- \right), \quad x_5 = \frac{1}{2} \left( \frac{x^+}{L} - Lx^- \right).$$

(46)

In the limit of $L \to 0$ the supergravity solution (36) reads

$$ds^2 = -dx^+ dx^- - \frac{r^2}{4} (dx^+)^2 + N \frac{dy^2}{r^2} + N d\Omega_3^2 + \sum_{i=1}^4 dx_i^2,$$

$$e^{2\phi} = \frac{N}{r^2}, \quad \sum_a A_{(p+1)\cdots 4a} + d\theta_a = -\frac{r^2}{2} (a_1 d\theta_1 + a_2 d\theta_2 + a_3 d\theta_3),$$

(47)

and the B field remains unchanged.

This plane wave solution has been studied in [50] in the context of lightlike dipole deformation of LST theory (see also [51]). Note that the metric, dilaton and B field in the plane wave solution (47) are the same as in the plane wave solution coming from lightlike noncommutative deformation of LST theory (31) and the only difference is in the RR fields. This means that as far as the bosonic part of the string theory on these backgrounds is concerned both of them give the same result. But of course the fermionic parts will be different. It is also worth noting that there is a nonzero RR field with one leg along the 3-sphere. This could also deform the $SU(2)$ WZW model as well. Therefore the three different parts of the theory contributing to the correction of $E - J$ are as following: 4 free fields theory, 3-dimensional Liouville plane wave and deformed $SU(2)$ WZW. Since the RR field has only one leg along the 3-sphere one might suspect that the corresponding deformation is dipole deformation. It would be interesting to study this model in more detail.

5 Conclusion

In this paper we have studied different Penrose limits of type II NS5-brane solution in the presence of different RR fields. We have seen that although for NS5-brane we get only one plane wave, for the case with RR field two different plane waves can be obtained. Interestingly enough all of these plane waves lead to exactly solvable string theory backgrounds.

We have also considered the semiclassical string configuration on the backgrounds generated by NS5-brane as well as NS5-brane in the presence of different nonzero RR fields. These string solutions give at one loop approximation the obtained plane wave from the corresponding geometry. This has been used to identified a subsector
of LST/deformed LST which is dual to the string theory in the corresponding plane wave background.

For all cases there is a Penrose limit leading to a 4-dimensional plane wave. In this case the subsector of LST, noncommutative LST and dipole LST is parameterized by energy and angular momentum. In all cases both energy and angular momentum grow as $N$ in large $N$ limit while their differences remain finite. In fact we have written a closed form expression for $E - J$ in all cases. Actually, we have found that the subsector of LST theory is a set of operators carrying angular momentum $J$ of $U(1)$ subgroup of the global symmetry $SO(4)$ with energy $E$ such that

$$
E - J = \sum_n \left[ \sqrt{1 + e^{2\rho_0}} \frac{N|n|}{J} N_n^{(6)} + \omega_n^+ N_n^+ + \omega_n^- N_n^- \right],
$$

$$
\omega_n^\pm = \sqrt{1 + (1 + e^{2\rho_0}) \frac{N^2 n^2}{J^2}} \pm 2 \frac{N n}{J}.
$$

To compare the subsector of deformed LST with the subsector of LST we observe that the factor $1 + e^{2\rho_0}$ in (29) and (44) plays the role of the deformation. For small deformation where $e^{\rho_0} \ll 1$, the corresponding subsectors have the following deviation from the LST

$$
(E - J)_{\text{NC LST}} = (E - J)_{\text{LST}} + \frac{e^{2\rho_0}}{2} \sum_n \left[ \frac{N|n|}{J} N_n^{(6)} + \frac{N^2 n^2}{J^2} \left( \frac{N_n^+}{\omega_n^+} + \frac{N_n^-}{\omega_n^-} \right) \right],
$$

$$
(E - J)_{\text{DLST}} = (E - J)_{\text{LST}} + \frac{e^{2\rho_0}}{2} \sum_n \left[ (E - J)_{\text{LST}} + \frac{N^2 n^2}{J^2} \left( N_n^+ + N_n^- \right) \right]
$$

up to second order in fluctuations and first order in the noncommutative/dipole deformations.

The next step, of course, would be to write the explicit form of the corresponding operators. These operators must be made out of the field content of LST theory. For example consider type IIA NS5-brane. In this case the bosonic part of the theory contains an anti-symmetric self-dual two form $B_{\alpha\beta}$, $\alpha = 0, \cdots, 5$, 2 complex scalars and one real scalar (dilaton). Suppose $Z_1$ is a complex scalar carrying one unit of $U(1)$ charge. Therefore one would expect that the vacuum to be constructed from $\text{Tr}(Z_1^N)$. Of course, since the theory is not conformal one can not identify the energy with dimension of the operator. Nevertheless, following [5] one could guess that the other stringy states can be constructed by inserting either the other scalar or $B_{\alpha\beta}$. It would be quite interesting and also important to study these operators in detail. Among all other things this could increase our knowledge about LST theory. We hope to address this question in future.

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