Lepton Polarization Asymmetry in $B \rightarrow \ell^+ \ell^-$ Decay beyond the Standard Model

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The lepton polarization asymmetry in the $B \rightarrow \ell^+ \ell^-$ decay, when one of the leptons is polarized, is investigated by using the most general form of the effective Hamiltonian. We find allowed regions for the new scalar Wilson coefficients, assuming that the experimental branching ratio is measured within 10% percent uncertainty. Then using these restrictions to the new coefficients the sensitivity of the lepton polarization asymmetry to them is studied. Moreover, it is observed that there are regions of terms describing the scalar interactions, where lepton polarization asymmetry differs from zero, which can serve as a good test for searching new physics beyond the standard model.

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The study of rare B-decays is one of the most important research areas in the particle physics. These decays induced by flavor changing neutral currents (FCNC) provide a promising ground for testing the structure of weak interactions. These decays are forbidden at tree level in the standard model (SM). Therefore, the analysis of these decays allows us to check the predictions of the SM at quantum levels. Moreover, these decays are very sensitive to the new physics beyond the SM, since loops with new particles can give considerable contribution to the SM result. The new physics effects in rare decays can appear in two different ways, namely via modification of Wilson coefficients exist in the SM or through new operators with new Wilson coefficients absent in the SM.

The rare pure leptonic $B_q \rightarrow \ell^+ \ell^- (q = d, s$ and $\ell = e, \mu, \tau$ ) decays are very good probes to test new physics beyond the standard model, mainly to reveal the Higgs sector. The present experimental bound of $B_s \rightarrow \mu^+ \mu^-$ has obtained the branching ratio (Br) of $(B_s \rightarrow \mu^+ \mu^-) < 5.8 \times 10^{-7}$ at (90% C.L.). Note that the prediction of the SM for the Br is $(3.8 \pm 1) \times 10^{-9}$. 

In this Letter, we investigate the lepton polarization using the most general form of effective Hamiltonian. More precisely our goal is to search whether there are such a regions of $C_{PV}$ and $C_{PS}$ for which lepton polarization differs from SM prediction, i.e. differs from zero.

Note that lepton polarization for $B_s \rightarrow \ell^+ \ell^-$ decay is studied in Ref. [6].

The effective Hamiltonian for the $b \rightarrow s \ell^+ \ell^-$ transition in terms of twelve model independent four Fermi interactions can be written in the following form:

$$H_{\text{eff}} = e \frac{G_{F} \alpha}{\sqrt{2} \pi} V_{ts} V_{tb}^{*} \left[ C_{SL} \bar{\tau} \sigma_{\mu \nu} q^{\nu} L b \bar{\ell} \gamma^{\mu} \ell \right. + \left. C_{BR} \bar{\tau} \sigma_{\mu \nu} q^{\nu} R b \bar{\ell} \gamma^{\mu} \ell + C_{L_{LL}} \bar{\tau} \gamma_{\mu} b L \bar{\ell} L \gamma_{\mu} \ell \right. + \left. C_{R_{LL}} \bar{\tau} \gamma_{\mu} b R \bar{\ell} R \gamma_{\mu} \ell + C_{L_{LR}} \bar{\tau} \gamma_{\mu} b L \bar{\ell} R \gamma_{\mu} \ell \right. + \left. C_{L_{RL}} \bar{\tau} \gamma_{\mu} b L \bar{\ell} L \gamma_{\mu} \ell \right. + \left. C_{R_{RL}} \bar{\tau} \gamma_{\mu} b R \bar{\ell} R \gamma_{\mu} \ell \right. + \left. i C_{TE} \mu_{\alpha \beta} \sigma_{\alpha \beta} b \bar{\ell} \ell \right],$$

where $L$ and $R$ are

$$R = \frac{1 + \gamma_{5}}{2}, \quad L = \frac{1 - \gamma_{5}}{2},$$

$C_{\alpha}$ are the coefficients of the four Fermi interactions, and $q = p_{2} + p_{1}$ is the momentum transfer. Among twelve Wilson coefficients some of them are already exist in the SM. For example, the coefficients $C_{SL}$ and $C_{BR}$ in penguin operators correspond to $-2m_{s}C_{7}^{\text{eff}}$ and $-2m_{d}C_{7}^{\text{eff}}$ in the SM, respectively. The next four terms in Eq. (1) are the vector-type interactions with coefficients $C_{L_{LL}}, C_{L_{LR}}, C_{R_{LR}}$ and $C_{R_{RL}}$. Two of these vector interactions containing $C_{L_{LL}}^{\text{tot}}$ and $C_{R_{LR}}^{\text{tot}}$ do exist in the SM as well in the form of $(C_{9} - C_{10})$ and $(C_{9}^{\text{eff}} + C_{10})$. Therefore we can say that $C_{L_{LL}}^{\text{tot}}$ and $C_{R_{LR}}^{\text{tot}}$ describe the sum of the contributions of the SM and the new physics which can be written as

$$C_{L_{LL}}^{\text{tot}} = C_{L_{LL}}^{\text{eff}} - C_{10} + C_{LL},$$

$$C_{R_{LR}}^{\text{tot}} = C_{R_{LR}}^{\text{eff}} + C_{10} + C_{LR}.$$

The terms with coefficients $C_{L_{LR}}, C_{R_{LL}}, C_{L_{RL}}$ and $C_{R_{RL}}$ describe the scalar type interactions. The last two terms with the coefficients $C_{T}$ and $C_{TE}$, obviously, describe the tensor-type interactions. The amplitude of exclusive $B_{s} \rightarrow \ell^+ \ell^-$ decay is obtained by sandwiching of the effective Hamiltonian between meson and vacuum states. It follows from Eq. (1) that in order to calculate the amplitude of the $B_{s} \rightarrow \ell^+ \ell^-$ decay, the following matrix elements are needed:

$$\langle 0 | \bar{\tau} \gamma_{\mu} \gamma_{5} b | B \rangle = -i f_{B_{s}} \bar{p}_{\ell},$$

$$\langle 0 | \bar{\tau} \gamma_{5} b | B \rangle = i f_{B_{s}} \frac{m_{2}}{m_{b} + m_{s}}.$$
All remaining matrix elements $\langle 0|s\Gamma_\ell b|B\rangle$, where $\Gamma_\ell$ is one of the Dirac matrices $I$, $\gamma_\mu$ and $\sigma_{\alpha\beta}$, are equal zero.

For the matrix element of $B \to \ell^+\ell^-$ decay we obtain

$$M = i f_{B s} \frac{G_F a}{2\sqrt{2}\pi} V_{ts} V_{tb} \left[ C_{PV}^\text{tot} \bar{\ell}_\tau \gamma^\nu \ell + C_{PS} \bar{\ell}_\ell \ell \right],$$  

(3)

where pseudovector coefficient $C_{PV}^\text{tot}$ and pseudoscalar coefficient $C_{PS}$ are as follows:

$$C_{PV}^\text{tot} = C_{PV} + C_{SM} \quad (C_{SM} = -2mC_{10}),$$

$$C_{PV} = m_2 (\frac{C_{LL} - C_{LR} - C_{RL} + C_{RR}}{2(m_b + m_\ell)} - C_{RLLR} - C_{RLRR} + C_{RLRL}),$$

$$C_{PS} = \frac{m_2^2}{2(m_b + m_\ell)} (C_{RLLR} - C_{RLRR} + C_{LRRL}),$$

(4)

After some calculation we obtain the following expression for the unpolarized $B \to \ell^+\ell^-$ decay width

$$I_0 = f_{B s}^2 \frac{1}{16\pi m_B} \left| \frac{G_F a}{2\sqrt{2}\pi} V_{ts} V_{tb} \right|^2 \left\{ 2(C_{PV}^\text{tot})^2 m_B^2 + 2C_{PS}m_B^2 v^2 \right\},$$

(5)

where $v = \sqrt{1 - m_\ell^2 / m_B^2}$ is the final lepton velocity.

Now let us calculate the lepton polarization asymmetry. In the rest frame of final leptons, the unit vectors of each lepton polarization can defined as

$$s^\mu = (0, e^\tau_L, (0, \frac{p_\ell}{|p_\ell|}),$$

(6)

where $p_\ell$ is the tree momenta of $\ell^-$($\ell^+$) and subscript $L$ means the longitudinal polarization. Boosting these unit vectors to the dilepton c.m. frame by using Lorentz transformation we obtain

$$s^\mu_{\ell^\pm} = \left( \frac{p_{\ell^\pm}}{m_\ell}, \pm \frac{E_{\ell_{\ell^\pm}}}{m_\ell} \frac{p_{\ell^\pm}}{m_\ell} \right),$$

(7)

where $E_{\ell_{\ell^\pm}}$ the lepton energy, and the upper sign corresponds to the lepton and the lower sign to the antilepton ones.

The decay width of the $B \to \ell^+\ell^-$ decay can be written in the following form:

$$\Gamma = \frac{1}{2} I_0 (1 + P_L^\pm e_L^\mp \cdot n^\mp),$$

(9)

where $P_L$ is the longitudinal lepton polarization asymmetry. It is defined as follows:

$$P_L^\pm = \frac{\Gamma(n^\mp = e^\tau_L^\mp) - \Gamma(n^\mp = -e^\tau_L^\mp)}{\Gamma(n^\mp = e^\tau_L^\mp) + \Gamma(n^\mp = -e^\tau_L^\mp)}.$$  

(11)

After simple calculations we set the following expression for the longitudinal polarization asymmetry:

$$P_L^\pm = \frac{2 \text{Re} (C_{PV}^\text{tot}^* C_{PS}^* \prod^\nu \nu)}{C_{PS}^2 \prod^\nu \nu + (C_{PV}^\text{tot})^2}.$$  

(12)

From this expression it is obvious that in the SM lepton polarization asymmetry $P_L^\pm = 0$, since in SM $C_{PS} = 0$ (see Eq. (4)).

Now we study the dependency of $P_L$ on new coefficients $C_{PV}$ and $C_{PS}$. In the present work all new Wilson coefficients are taken to be real. Here we would like to make the following remark. Recent experimental results on the $B$ meson decay into two pseudoscalar mesons indicated that Wilson coefficient $C_{10}$ can have large phase. Therefore in principle the new source for CP violating effects can appear. We will discuss this possibility elsewhere.

Values of the input parameters which we have used in our numerical analysis are: $f_{B s} = 0.245$ GeV, $m_B = (5.2792 \pm 1.8)$ GeV, $m_\mu = 105.7$ MeV, $m_\tau = 1.777$ GeV, $\alpha = \frac{1}{129}$

![Fig. 1. Parametric plot of $C_{PV}$ on $C_{PS}$ when the branching ratio of $B \to \mu^+\mu^-$ measured with 10% uncertainty.](image)

Following from the explicit expressions of the branching ratio and lepton polarization asymmetry, which are dependent on $C_{PV}^\text{tot}$ and $C_{PS}$, $C_{PV}^\text{tot} = C_{PV} + C_{SM}$, so the new Wilson coefficients $C_{PV}$ and $C_{PS}$ are free parameters. In order to obtain an idea about the magnitudes of these coefficients, we assume that in experiments, the branching ratio can be measured within 10% uncertainty. Let us consider the following quantity:

$$R = \frac{Br(B \to \mu^+\mu^-) - Br^{SM}(B \to \mu^+\mu^-)}{Br^{SM}(B \to \mu^+\mu^-)}.$$  

(13)

Obviously, this ratio according to our assumption lies in the region

$$-0.1 \leq R \leq 0.1.$$  

(14)

Using Eqs. (13) and (14) it is very easy to find that allowed values of $C_{PV}$ and $C_{PS}$ lies between two circles of radii $r_1 = 0.92$, and $r_2 = 1.02$ and centers of both the circles coincide with each other (see Fig. 1).

In Fig. 2, we depict the dependence of $P_L$ on $C_{PV}$ and $C_{PS}$, where these coefficients can change in the
allowed region. From this figure we see that there are
regions of $C_{PV}$ and $C_{PS}$, where $P_L$ differs from zero,
and this fact due to the solely with the scalar sector of
the theory.

Therefore, measurement of $P_L$ can serve a good
test for searching new physics beyond the SM due to
the extension of the scalar sector of the SM.

The similar situation takes place for $B \to \tau^+\tau^-$
decay (see Figs. 3 and 4), but allowed regions of $C_{PV}$
and $C_{PS}$ are wider than the previous one. It can be
explained by the fact that $C_{PV}$ is proportional to the
lepton mass. Due to the low efficiency of detecting
$\tau$ leptons it seems that $B \to \mu^+\mu^-$ channel is more
attractive from experimental point of view. Note that
in SM $Br(B \to \mu^+\mu^-) \approx 10^{-9}$ and therefore it is
impossible to observe it in $B$ factories, where only
$5 \times 10^8 \, B\overline{B}$ pair can be produced per year. It is
expected to produce $10^{12} \, B\overline{B}$ pair at LHC. Even if the
efficiency is about $10^{-1} \div 10^{-2}$, then 10–100 event ex-
pected to be seen in LHC. Therefore the measurement of
branching ratio and lepton polarization asymmetry
of $B_s \to \ell^+\ell^-$ decay can be measurable at LHC.

We should also note that the radiative leptonic decay
$^{[1]}$ can contribute at the same order magnitude,
which can be a severe background to the polarization
asymmetry. If we put some lower limit, for photon
energy, i.e. $> 50 \text{eV}$ then detectors can detect hard
photons and lepton pairs. In this case, we have totally
different behaviour for both the decay. Therefore by
taking into account appropriate cut of photon energy
we can eliminate this “background” for pure leptonic
decay. In summary, we have presented an analysis for
the longitudinal lepton polarization using the most
general form of the effective Hamiltonian. It is found
that there are regions of $C_{PV}$ and $C_{PS}$ responsible for
scalar interaction, where lepton polarization asymmetry
differs from zero. Therefore measurement of the
lepton polarization can give invaluable information in
looking for new physics beyond the SM.

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