Abstract

In this current literature, there are several models of Fully Fuzzy Data Envelopment Analysis (FFDEA) where inputs-outputs data and weights were fuzzy numbers. The main purpose of this study is to evaluate the performance of a set of Decision Making Units (DMUs) and fully ranking DMUs in fully fuzzified environment. In this paper, an FFDEA problem is discussed, and a method for solving this FFDEA problem is also proposed. This method based on the multi-objective linear programming and the simplex method is proposed for computing an optimal fuzzy solution to a FFDEA problem in which fuzzy ranking functions are not used. We acquire fuzzy efficiency scores with solving FFDEA. Afterwards, we have used a ranking function to rank these fuzzy scores. A numerical example is used to demonstrate and compare the results with those obtained using alternative approaches.

Keywords: Data Envelopment Analysis, Efficiency, Fuzzy Numbers, Multi-Objective

1. Introduction

Data Envelopment Analysis (DEA) is systematic approach for the analyzing the performance of organizations and operational processes. It can evaluate the relative efficiencies of homogeneous Decision Making Units (DMUs) without knowing production functions, only by using input and output data. The first model od DEA introduced by charnes et al. To see the other classic models of DEA, the readers can see. The basic DEA results group the DMUs into two sets: one set is efficient DMUs and the other is inefficient DMUs. In many cases, it is necessary to give a full ranking of the DMUs. For this purpose, different methods with different properties to achieve full ranking of DMUs have been proposed. Sexton et al. proposed the ranking of DMUs based on a cross efficiency. The benchmarking, initially developed by Torgersen was employed to rank all the efficient DMUs. Andersen and peterson first developed the most popular ranking method called super-efficiency. Cooper and Tone ranked the DMUs according to scalar measuring of inefficiency in DEA, based on the slack variables. Liu and Peng proposed Common Weights Analysis (CWA) to determine a set of indices for common weights to rank efficient DMUs of DEA. In many situations, such as manufacturing system, a production process or service system, inputs and outputs are volatile and complex so that it is difficult to measure them in an accurate way. Instead the data can be given as a fuzzy variable. Many fuzzy approaches have been introduced in the DEA literature. Guo and Tanaka and Lertsirkul et al. applied possibility measure proposed by Zadeh to the fuzzy DEA model. Lertsirkul et al. show that for the special case, in which fuzzy membership functions of fuzzy data are trapezoidal types, possibility DEA models become LP models. Liu and Liu presented credibility measure in 2002. This paper will extend the CCR model to a fuzzy DEA model based on credibility measure and then give a fuzzy ranking method all the DMUs with fuzzy inputs and outputs. The fuzzy ranking method was developed by Guo and Tanaka. This approach provides fuzzy efficiency for an evaluated DMU at a specified α-level. Meilin Wen et al. proposed a new fuzzy DEA model based on credibility measure as well as a ranking method provided, they designed a hybrid algorithm combined with fuzzy simulation and genetic algorithm to compute the fuzzy model DEA. Hatami and et al. proposed a DEA Method by constructing a FFLP model. They applied the FFLP model developed by Allahviranloo et al. (2008) and transform the DEA model into a fully fuzzified DEA model. Zerafat Angil et al. proposed a six-stage algorithm to rank...
the efficient DMUs using fuzzy concept. Kazemi and et al. used the proposed method of Lumar et al. to solve the mentioned problems. To see the other FDEA models, the readers can see. This paper is organized as follows: In section 2 we present the basic definitions of fuzzy arithmetic. In the next section, is defined ranking function. In section 4 the FFDEA is introduced and solve using the above mentioned multiplication and the properties of the presented linear ranking function with related theorems, are in introduced. Section 5, is devoted to a numerical example.

2. Preliminaries

Definition 1. A fuzzy number \( A = (a_1, a_2, a_3) \) is said to be a triangular fuzzy number if it, s membership function is given by,

\[
u_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \frac{x-a_3}{a_2-a_3} & a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}
\]

Parametric form of a fuzzy number has been introduced and presented by \( A = (A(r), A(r)) \), where \( A(r) = (\bar{A}(r), 0 \leq r \leq 1 \) satisfying the following requirements:

1. \( A(r) \) is monotonically increasing left continuous function.
2. \( A(r) \) is monotonically decreasing left continuous function.
3. \( A(r) \leq A(1) \), 0 \leq r \leq 1
4. \( A(r) = A(0) \), 0 \leq r, 0 \leq 1 \leq r

If \( \bar{A} = (a_1, a_2, a_3) \) then in parametric form is

\( \bar{A} = (a_1 + r(a_2 - a_1), a_3 - r(a_3 - a_2)) \)

Definition 2. A triangular fuzzy number \( \bar{A} = (a_1, a_2, a_3) \) is said to be nonnegative fuzzy number, if and only if \( a_1 \geq 0 \).

Theorem 1. Let \( \bar{A} = (a_1, a_2, a_3) \) and \( \bar{B} = (b_1, b_2, b_3) \) be two triangular fuzzy number, then

(i) \( \bar{A} \oplus \bar{B} = (a_1+b_1, a_2+b_2, a_3+b_3) \)

(ii) \( \bar{A} = (-a_3, -a_2, -a_1) \)

(iii) \( \bar{A} \oplus \bar{B} = (a_1+b_3, a_2+b_2, a_3+b_1) \)

(iv) If \( \bar{B} = (b_1, b_2, b_3) \) be a nonnegative triangular fuzzy number, and then

\[
\bar{A} \otimes \bar{B} = \begin{cases}
(a_1 b_1, a_2 b_2, a_3 b_3) & a_1 \geq 0 \\
(a_1 b_3, a_2 b_2, a_3 b_1) & a_1 < 0, a_3 \geq 0 \\
(a_1 b_3, a_2 b_2, a_3 b_1) & a_1 < 0
\end{cases}
\]

Proof.

Definition 3. Let \( \bar{A} = (a_1, a_2, a_3) \) and \( \bar{B} = (b_1, b_2, b_3) \) be two triangular fuzzy number, then

(i) \( \bar{A} \geq \bar{B} \) if \( a_1 \leq b_1 \), \( a_2 \leq b_2 \), \( a_3 \leq b_3 \)

(ii) \( \bar{A} \leq \bar{B} \) if \( a_1 \geq b_1 \), \( a_2 \geq b_2 \), \( a_3 \geq b_3 \)

(iii) \( \bar{A} \geq \bar{B} \) if \( a_1 \geq b_1 \), \( a_2 \geq b_2 \), \( a_3 \geq b_3 \)

(iv) \( \bar{A} \leq \bar{B} \) if \( a_1 \leq b_1 \), \( a_2 \leq b_2 \), \( a_3 \leq b_3 \)

This implies for example, that if \( F(\bar{A}) > F(\bar{A}_i) \), the fuzzy number \( \bar{A}_i \) is numerically greater than fuzzy number \( \bar{A}_i \) the higher \( \bar{A} \) is, the larger \( F(\bar{A}) \) is.

Here we introduce a linear ranking function that is similar to the ranking function. For an arbitrary fuzzy number \( \bar{A} = (A(r), A(r)) \), we use ranking function as follows:

\[
D(\bar{A}) = 1/2(\int_0^1 A(r) dr + \int_0^1 \bar{A}(r) dr)
\]

For triangular fuzzy number this reduces to:

\[
D(\bar{A}) = a_2 + 1/4(a_3 - a_1).
\]

4. Fully Fuzzy DEA Model

Suppose the evaluation of the efficiency of n DMUs is desirable. Each DMU, \( j = 1, 2, ..., n \) consumes \( m \) inputs \( x_{ij} (i = 1, 2, ..., m) \) to produces outputs \( y_{ij} (r = 1, 2, ..., s) \).
The following multiplier from of CCR model can be applied to assess the efficiency score of DMU \( a \):

\[
\text{Max} \quad \theta = \sum_{r=1}^{s} u_r y_{ro} \\
\text{s.t} \quad \sum_{i=1}^{m} v_i x_{io} = 1 \\
\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, \ldots, n \\
u_r \geq 0, r = 1, \ldots, s \\
v_i \geq 0, i = 1, \ldots, m
\]

Where \( u_r (r = 1, 2, \ldots, s) \) and \( v_i (i = 1, 2, \ldots, m) \) are the weights assigned to the \( r \)th output and \( i \)th input, respectively.

Consider the following fully fuzzy DEA model with fuzzy inputs, fuzzy outputs and fuzzy weights may be formulated as follows:

\[
\text{Max} \quad \tilde{\theta} = \sum_{r=1}^{s} \tilde{u}_r \otimes \tilde{y}_{ro} \\
\text{s.t} \quad \sum_{i=1}^{m} \tilde{v}_i \otimes \tilde{x}_{io} \leq \tilde{1} \\
\sum_{r=1}^{s} \tilde{u}_r \otimes \tilde{y}_{rj} \otimes \sum_{i=1}^{m} \tilde{v}_i \otimes \tilde{x}_{ij} \leq \tilde{0}, j = 1, \ldots, n \\
\tilde{u}_r \geq 0, r = 1, \ldots, s \\
\tilde{v}_i \geq 0, i = 1, \ldots, m
\]

Let the parameters \( \tilde{u}_r, \tilde{v}_i, \tilde{y}_{rj}, \tilde{x}_{ij}, \tilde{1} \) and \( \tilde{0} \) be the triangular fuzzy numbers \( \tilde{u}_r = (u_r^{(1)}, u_r^{(2)}, u_r^{(3)}), \tilde{v}_i = (v_i^{(1)}, v_i^{(2)}, v_i^{(3)}), \)
\( \tilde{y}_{rj} = (y_{rj}^{(1)}, y_{rj}^{(2)}, y_{rj}^{(3)}), \tilde{x}_{ij} = (x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)}), i = 1, \ldots, m, r = 1, \ldots, s, j = 1, \ldots, n, \tilde{1} = (1, 1, 1), \tilde{0} = (0, 0, 0) \) respectively. Then the problem (2) can be written as follows:

\[
\text{Max} \quad \theta = \sum_{r=1}^{s} (u_r^{(1)} y_{ro}, u_r^{(2)} y_{ro}, u_r^{(3)} y_{ro}) \\
\text{s.t} \quad \sum_{i=1}^{m} (v_i^{(1)} x_{io}, v_i^{(2)} x_{io}, v_i^{(3)} x_{io}) \leq (1, 1, 1) \\
\sum_{r=1}^{s} (u_r^{(1)} y_{rj}, u_r^{(2)} y_{rj}, u_r^{(3)} y_{rj}) \\
+ \sum_{i=1}^{m} (-v_i^{(3)} x_{ij}, -v_i^{(2)} x_{ij}, -v_i^{(1)} x_{ij}) \leq (0, 0, 0), j = 1, \ldots, n \\
(u_r^{(1)}, u_r^{(2)}, u_r^{(3)}) \geq 0, r = 1, \ldots, s \\
(v_i^{(1)}, v_i^{(2)}, v_i^{(3)}) \geq 0, i = 1, \ldots, m
\]

Now, we apply the FFLP model developed by Pandian\textsuperscript{35}, namely level-sum method and transform the fuzzy DEA model to the following model:

Max \( \theta^1 = \sum_{r=1}^{s} u_r^{(1)} y_{r0} \)
Max \( \theta^2 = \sum_{r=1}^{s} u_r^{(2)} y_{r0} \)
Max \( \theta^3 = \sum_{r=1}^{s} u_r^{(3)} y_{r0} \)

\[
\text{s.t} \quad \sum_{i=1}^{m} v_i^{(1)} x_{i0} \leq 1 \\
\sum_{i=1}^{m} v_i^{(2)} x_{i0} \leq 1 \\
\sum_{i=1}^{m} v_i^{(3)} x_{i0} \leq 1 \\
\sum_{r=1}^{s} u_r^{(1)} y_{rj} - \sum_{i=1}^{m} v_i^{(3)} x_{ij} \leq 0, j = 1, 2, \ldots, n \\
\sum_{r=1}^{s} u_r^{(2)} y_{rj} - \sum_{i=1}^{m} v_i^{(2)} x_{ij} \leq 0, j = 1, 2, \ldots, n \\
\sum_{r=1}^{s} u_r^{(3)} y_{rj} - \sum_{i=1}^{m} v_i^{(1)} x_{ij} \leq 0, j = 1, 2, \ldots, n \\
\theta^1 \leq \theta^2 \\
\theta^2 \leq \theta^3
\]

Based on Pandian\textsuperscript{35}, if \( \tilde{u}_r = (u_r^{(1)}, u_r^{(2)}, u_r^{(3)}), \tilde{v}_i = (v_i^{(1)}, v_i^{(2)}, v_i^{(3)}), \)
\( r = 1, 2, \ldots, s, i = 1, 2, \ldots, m \) is an efficient solution to the MOLP problem (4). Then, \( \tilde{u}_r^* = (u_r^{(1)}, u_r^{(2)}, u_r^{(3)}), v_i^* = (v_i^{(1)}, v_i^{(2)}, v_i^{(3)}), \)
\( r = 1, 2, \ldots, s, i = 1, 2, \ldots, m \) is an optimal solution to the problem (3). We run MOLP model (4) and obtain the following optimal fuzzy solution to the FDEA problem (3):

\[
\tilde{u}_r^* = (u_r^{(1)}, u_r^{(2)}, u_r^{(3)}), v_i^* = (v_i^{(1)}, v_i^{(2)}, v_i^{(3)}), \quad r = 1, 2, \ldots, s, i = 1, 2, \ldots, m. 
\]

Finally, the objective function of model (4) is used to computing the optimal fuzzy efficiency of each DMU as follows:

\[
(\theta^1, \theta^2, \theta^3) = \left( \sum_{r=1}^{s} u_r^{(1)} y_{r0}, \sum_{r=1}^{s} u_r^{(2)} y_{r0}, \sum_{r=1}^{s} u_r^{(3)} y_{r0} \right)
\]
5. Numerical Example

In this section we have a numerical example from Guo and Tanaka study that its data are on Table 1. By putting the data of Table 1 on the proposed model, we will have model (5). This model is written to calculate the first decision making units efficiency and 4 other models have to be written to calculate the efficiency of other units. Now, the MOLP problem to the given fully fuzzy DEA problem is given below:

Max $\theta^1 = 2.4 u_1^{(1)} + 3.8 u_2^{(1)}$
Max $\theta^2 = 2.6 u_1^{(2)} + 4.1 u_2^{(2)}$
Max $\theta^3 = 2.8 u_1^{(3)} + 4.4 u_2^{(3)}$

s.t $2.4 u_1^{(1)} + 3.8 u_2^{(1)} - 4.5 v_1^{(1)} - 2.3 v_2^{(1)} \leq 0$; (5)
2.2 $u_1^{(1)} + 3.3 u_2^{(1)} - 2.9 v_1^{(1)} - 1.6 v_2^{(1)} \leq 0$
2.7 $u_1^{(1)} + 4.3 u_2^{(1)} - 5.4 v_1^{(1)} - 3 v_2^{(1)} \leq 0$
2.5 $u_1^{(1)} + 5.5 u_2^{(1)} - 4.8 v_1^{(1)} - 2.4 v_2^{(1)} \leq 0$
4.4 $u_1^{(1)} + 6.5 u_2^{(1)} - 7.1 v_1^{(1)} - 4.6 v_2^{(1)} \leq 0$
2. $u_1^{(2)} + 4.1 u_2^{(2)} - 4 v_1^{(2)} - 2.1 v_2^{(2)} \leq 0$
2.2 $u_1^{(2)} + 3.5 u_2^{(2)} - 2.9 v_1^{(2)} - 1.5 v_2^{(2)} \leq 0$
3.2 $u_1^{(2)} + 5.1 u_2^{(2)} - 4.9 v_1^{(2)} - 2.6 v_2^{(2)} \leq 0$
2.9 $u_1^{(2)} + 5.7 u_2^{(2)} - 4.1 v_1^{(2)} - 2.3 v_2^{(2)} \leq 0$
5.1 $u_1^{(2)} + 7.4 u_2^{(2)} - 6.5 v_1^{(2)} - 4.1 v_2^{(2)} \leq 0$
2.8 $u_1^{(3)} + 4.4 u_2^{(3)} - 3.5 v_1^{(3)} - 1.9 v_2^{(3)} \leq 0$
2.2 $u_1^{(3)} + 3.7 u_2^{(3)} - 2.9 v_1^{(3)} - 1.9 v_2^{(3)} \leq 0$
3.7 $u_1^{(3)} + 5.9 u_2^{(3)} - 4.4 v_1^{(3)} - 1.9 v_2^{(3)} \leq 0$
3.3 $u_1^{(3)} + 5.9 u_2^{(3)} - 3.4 v_1^{(3)} - 1.9 v_2^{(3)} \leq 0$
5.8 $u_1^{(3)} + 8.3 u_2^{(3)} - 5.9 v_1^{(3)} - 1.9 v_2^{(3)} \leq 0$
3.5 $v_1^{(1)} + 1.9 v_2^{(1)} \leq 1$
4 $v_1^{(2)} + 2.1 v_2^{(2)} \leq 1$
4.5 $v_1^{(3)} + 2.3 v_2^{(3)} \leq 1$
$u_1^{(1)} - u_2^{(2)} \leq 0$
$u_1^{(2)} - u_2^{(3)} \leq 0$

We consider the following LP problem (6) related to the above MOLP problem:

Max $2.4 u_1^{(1)} + 2.6 u_2^{(1)} + 2.8 u_1^{(3)} + 3.8 u_2^{(3)}$ + 4.1 $u_1^{(2)} + 4.4 u_2^{(3)}$

s.t $2.4 u_1^{(1)} + 3.8 u_2^{(1)} - 4.5 v_1^{(3)} - 2.3 v_2^{(3)} \leq 0$
2.2 $u_1^{(1)} + 3.3 u_2^{(1)} - 2.9 v_1^{(1)} - 1.6 v_2^{(3)} \leq 0$
2.7 $u_1^{(1)} + 4.3 u_2^{(1)} - 5.4 v_1^{(1)} - 3 v_2^{(3)} \leq 0$
2.5 $u_1^{(1)} + 5.5 u_2^{(1)} - 4.8 v_1^{(1)} - 2.4 v_2^{(3)} \leq 0$
2. $u_1^{(2)} + 4.1 u_2^{(2)} - 4 v_1^{(2)} - 2.1 v_2^{(2)} \leq 0$
2.2 $u_1^{(2)} + 3.5 u_2^{(2)} - 2.9 v_1^{(2)} - 1.5 v_2^{(2)} \leq 0$
3.2 $u_1^{(2)} + 5.1 u_2^{(2)} - 4.9 v_1^{(2)} - 2.6 v_2^{(2)} \leq 0$
2.9 $u_1^{(2)} + 5.7 u_2^{(2)} - 4.1 v_1^{(2)} - 2.3 v_2^{(2)} \leq 0$
5.1 $u_1^{(2)} + 7.4 u_2^{(2)} - 6.5 v_1^{(2)} - 4.1 v_2^{(2)} \leq 0$
2.8 $u_1^{(3)} + 4.4 u_2^{(3)} - 3.5 v_1^{(3)} - 1.9 v_2^{(3)} \leq 0$
2.2 $u_1^{(3)} + 3.7 u_2^{(3)} - 2.9 v_1^{(3)} - 1.9 v_2^{(3)} \leq 0$
3.7 $u_1^{(3)} + 5.9 u_2^{(3)} - 4.4 v_1^{(3)} - 1.9 v_2^{(3)} \leq 0$
3.3 $u_1^{(3)} + 5.9 u_2^{(3)} - 3.4 v_1^{(3)} - 1.9 v_2^{(3)} \leq 0$
5.8 $u_1^{(3)} + 8.3 u_2^{(3)} - 5.9 v_1^{(3)} - 1.9 v_2^{(3)} \leq 0$
3.5 $v_1^{(1)} + 1.9 v_2^{(1)} \leq 1$
4 $v_1^{(2)} + 2.1 v_2^{(2)} \leq 1$
4.5 $v_1^{(3)} + 2.3 v_2^{(3)} \leq 1$
$u_1^{(1)} - u_2^{(2)} \leq 0$
$u_1^{(2)} - u_2^{(3)} \leq 0$
$u_1^{(3)} - u_2^{(3)} \leq 0$
$v_1^{(1)} - v_2^{(2)} \leq 0$
$v_1^{(2)} - v_2^{(3)} \leq 0$
$v_2^{(2)} - v_2^{(3)} \leq 0$
$u_1^{(1)} \geq 0, u_2^{(1)} \geq 0, v_1^{(1)} \geq 0, v_2^{(1)} \geq 0,$

Table 1. Example of Guo and Tanaka

| DMU | Input 1 | Input 2 | Output 1 | Output 2 |
|-----|---------|---------|----------|----------|
| 1   | (3.5,4.4,5) | (1.9,2.1,2.3) | (2.4,2.6,2.8) | (3.8,4.1,4.4) |
| 2   | (2.9,2.9,2.9) | (1.4,1.5,1.6) | (2.2,2.2,2.2) | (3.3,3.5,3.7) |
| 3   | (4.4,4.9,5.4) | (2.2,2.6,3.3) | (2.7,3.2,3.7) | (4.3,5.1,5.9) |
| 4   | (3.4,4.1,4.8) | (2.2,2.3,2.4) | (2.5,2.9,3.3) | (5.5,5.7,5.9) |
| 5   | (5.9,6.5,7.1) | (3.6,4.1,4.6) | (4.4,5.1,5.8) | (6.5,7.4,8.3) |
Efficiencies $(0.5453, 0.5906, 0.6360)$ is an optimal fuzzy solution.

Hatami-Lertworasirikul (2002) shown to conform both the results of this study and the units three and one take the score efficiency orders. Table 2 and Table 3 all methods are assessed DMU the results of methods are nuanced together. But generally, efficiency DMUs differ together, thus it’s naturalized that tively. As regards all presented methods for evaluating of DMU classified in first rank and DMU of third Table, we will notice that DMU Hatami-Marbini approach after the units three and one have the highest efficiency. units two, four and five have the highest efficiency and of Lertworasirikul $(0.867,0.999,1.132)$.

Table 2. Comparison of result of different methods

| DMU   | Lertwo Rasirikul (2002) | Zerafat and et al. (2010) | Wen et.al (2010) | Hatami - Marbini And et al. (2011) | Kazemi and Et al. (2014) | Efficiencies Our study |
|-------|-------------------------|---------------------------|------------------|-----------------------------------|-------------------------|--------------------|
| 1     | 0.855                   | 0.915                     | 0.907            | $(0.789,0.855,0.921)$              | $(0.624,0.747,0.899)$   | $(0.543,0.5906,0.6360)$ |
| 2     | 1                       | 1                         | 1                | $(0.981,0.999,1.017)$              | $(0.834,0.882,0.935)$   | $(0.7515,0.7760,0.7786)$ |
| 3     | 0.861                   | 0.948                     | 0.907            | $(0.726,0.861,0.996)$              | $(0.555,0.747,1)$       | $(0.5117,0.6065,0.7013)$ |
| 4     | 1                       | 1                         | 1                | $(0.964,0.999,1.034)$              | $(0.854,0.924,1)$       | $(0.6930,0.7182,0.7434)$ |
| 5     | 1                       | 0.991                     | 1                | $(0.867,0.999,1.132)$              | $(0.631,0.799,1)$       | $(0.6322,0.7229,0.8319)$ |

Table 3. Ranking of fuzzy efficiency scores

| DMU                | Hatami And et al. (2011) | Rank | Kazemi and Et al. (2014) | Rank | Efficiencies | Rank |
|--------------------|--------------------------|------|--------------------------|------|--------------|------|
| 1                  | 0.888                    | 5    | 0.8157                   | 5    | 0.6133       | 5    |
| 2                  | 1.008                    | 3    | 0.9072                   | 2    | 0.782        | 1    |
| 3                  | 0.9285                   | 4    | 0.8582                   | 4    | 0.6539       | 4    |
| 4                  | 1.0165                   | 2    | 0.9605                   | 1    | 0.7308       | 3    |
| 5                  | 1.0652                   | 1    | 0.8912                   | 3    | 0.781        | 2    |

By solving the problem (5), based on the level-sum method proposed by Pandian et al\(^{35}\),

\[
\hat{u}_1 = (0.2144, 0.2144, 0.2144), \quad \hat{u}_2 = (0.0081, 0.0081, 0.0081), \quad \hat{v}_1 = (0.2222, 0, 0.2222), \quad \hat{v}_2 = (0, 0.222, 0) \quad \text{and} \quad \hat{f} = (0.5453, 0.5906, 0.6360)
\]

is an optimal fuzzy solution to the given initial fully fuzzy DEA problem.

By model (4), fuzzy efficiency of DMUs is calculated and its result is on column 7 of Table 2. In all of methods of Lertworasirikul\(^{34}\), Zerafat Angiz and et al\(^{10}\), Wen et al\(^{2}\), units two, four and five have the highest efficiency and after the units three and one have the highest efficiency.

We applied fuzzy ranking function (1) to the results of Hatami-Marbini approach\(^{33}\), Kazemi and et al. approach\(^{31}\) and our approach, the results are presented in Table 3.

If we consider the decimal numbers in seventh column of third Table, we will notice that DMU\(_2\), DMU\(_4\) and DMU\(_5\) will be classified in first rank and DMU\(_3\) in second rank. Also the DMU\(_1\), DMU\(_4\) are located in third, fourth ranks, respectively. As regards all presented methods for evaluating of efficiency DMUs differ together, thus it’s naturalized that the results of methods are nuanced together. But generally, as for Table 2 and Table 3 all methods are assessed DMU\(_2\), DMU\(_4\), DMU\(_5\) best decision making units and after the units three and one take the score efficiency orders. Table 2 shown to conform both the results of this study and the results of other methods.

6. Conclusion

In this paper, an FFDEA problem has been presented. Also, an approach has been given to solve it. We are using the proposed method of Pandian\(^{35}\) to find an optimal fuzzy solution to a FFDEA problem. The main advantage of the proposed method is that the FFDEA problems can be solved by any LP solver using the level-sum method since it’s based on only simplex method. With using of this method, we obtain fuzzy efficiency scores and DMUs are ranked.

7. Reference

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