Distributed Optimal Control With Recovered Robustness for Uncertain Network Systems: A Complementary Design Approach

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Abstract—This article considers the distributed robust suboptimal consensus control problem of uncertain linear multiagent systems, with both $H_2$ and $H_{\infty}$ performance requirements. A novel two-step complementary design approach is proposed. In the first step, a distributed control law is designed for the nominal multiagent system to achieve consensus with a prescribed $H_2$ performance. In the second step, an extra control input, depending on some carefully chosen residual signals indicating the modeling mismatch, is designed to complement the $H_2$ performance by providing robustness guarantee in terms of $H_{\infty}$ requirement with respect to disturbances or uncertainties. The proposed complementary design approach provides an additional degree of freedom for design, having two separate controls to deal with the $H_2$ performance and the robustness of consensus, respectively. Thereby, it does not need to make much tradeoff, and can be expected to be much less conservative than the tradeoff design, such as the mixed $H_2/H_{\infty}$ control method. Besides, this complementary approach will recover the achievable $H_2$ performance when external disturbances or uncertainties do not exist. The effectiveness of the theoretical results and the advantages of the complementary approach are validated via numerical simulations.

Index Terms—$H_2$ control, $H_{\infty}$ control, consensus, cooperative control, distributed control, optimal control, robust control.

I. INTRODUCTION

Optimality and robustness are two main issues and missions in the feedback control theory [1]. Optimality requires an optimal or suboptimal controller to ensure that a closed-loop system satisfies certain predefined performance criteria, with the linear quadratic regulator (LQR) and the linear quadratic Gaussian (LQG)/$H_2$ problems as typical examples. Robustness, on the other hand, characterizes the property that a system still works well in the presence of external disturbances or model uncertainties, which can be addressed in the framework of $H_{\infty}$ control and $\mu$ synthesis. For networked multiagent systems, the optimality and robustness problems encounter new inherent challenges, since the control laws need to be distributed in the sense that only local information between neighboring agents can be utilized and meanwhile the control laws are subject to structural constraints imposed by the network topology [2]. The potential applications of distributed robust optimal control are broad, ranging from unmanned aerial vehicle (UAV) formation to wireless sensor networks and intelligent transportation systems.

In the last two decades, many advances have been reported on optimality issues of networked multiagent systems. In the distributed LQR framework, several approaches (e.g., suboptimal design [3], [4], hierarchical control [5], inverse optimal method [6], and decentralized computation [7]) have been proposed. The $H_2$ consensus problems of multiagent systems were also investigated using distributed protocols in [14] by static state feedback and in [15] by dynamic output feedback. Moreover, through studying the $H_2$ norms of multiagent networks from white noises to the performance variables, the coherence and centrality of networks were formulated and discussed in [8], [9], [10], [11], [12], and [13]. Meanwhile, robustness issues of networked multiagent systems have also attracted much attention. In [14], [16], [17], [18], and [19], disturbance attenuation problems of linear consensus networks were studied from the $H_{\infty}$ control perspective. In [20], [21], and [22], robust synchronization and consensus problems of multiagent systems were investigated, where the agent dynamics are subject to multiplicative or coprime factor uncertainties. In fact, not only the agent dynamics could be perturbed by uncertainties, the interactions among neighboring agents could also be subject to uncertainties. In the cases that the communication channels among agents are subject to multiplicative stochastic uncertainties, robust consensus problems of multiagent systems were considered in [23], [24], [25], and [26], in the sense of mean square and almost sure stability. Moreover, in [27] and [28], robust consensus over deterministic uncertain network graphs was also studied. While distributed $H_2$ control, distributed $H_{\infty}$ control and robust distributed control problems, respectively, have been extensively studied, there are few works that deal with the problem of distributed robust optimal control [35], [41]. It is well known that there is an intrinsic conflict between optimality and robustness in the standard feedback framework [1], [29]. Therefore, in the case of multiobjective design, e.g., the mixed $H_2/H_{\infty}$ control [30] and the $H_{\infty}$ Gaussian control [31], a tradeoff has to be made between the achievable optimal performance and robustness [29]. These tradeoff design approaches suffer from a fundamental drawback of severe conservativeness, since a single controller is developed to address the conflicting requirements simultaneously. For example, the mixed $H_2/H_{\infty}$ control is generally worse than the $H_{\infty}$ control in terms of the robustness and worse than the LQG control in terms of the optimality [31]. Inspired by the structure in [29], a new design paradigm is proposed in [32], consisting of an LQG controller designed for the nominal plant and an operator $Q$ as a separate degree of freedom. The operator $Q$ provides an extra control action to recover the robustness performance for the closed-loop system. This new paradigm is shown to be able to avoid tradeoff and to reduce the conflict between the robustness and achievable suboptimality/optimality. Note that the design paradigms...
in [29] and [32] are applicable only to single-agent systems. So far, novel nontraded off design schemes for multiagent systems have not witnessed significant progress, due to the severe difficulties caused by the requirement of distributed control and structural uncertainties and by constraints imposed by the network graphs.

Motivated by the above, in this article, we consider the distributed robust optimal consensus problems for linear multiagent systems, taking into account both the optimality and robustness at the same time. Specifically, the objective of this article is to present a novel nontraded off complementary design approach to the robust optimal consensus problems for linear multiagent systems. This complementary approach consists of two steps. In the first step, we design a distributed control law for the nominal multiagent system, without considering the disturbances or uncertainties, to ensure that consensus is achieved with a prescribed $H_2$ performance. In the second step, a separate control input, activated by some carefully chosen residual signals indicating the modeling mismatch, will be designed to ensure robustness in terms of the $H_\infty$ requirement. Two cases are considered, namely, the case that relative outputs or absolute outputs of neighboring agents are available. A distinct feature of this complementary approach is that the design of $H_2$ consensus control in this first step is independent of the second step and the extra control action in the second step will complement the $H_2$ performance by providing a robustness guarantee with respect to disturbances or uncertainties.

Compared with the tradeoff approach, e.g., the mixed $H_2/H_\infty$ control design, the proposed complementary design has at least two main advantages. First, since the extra control provides an additional degree of freedom for design, the complementary approach has two separate controls to deal with the $H_2$ performance and the robustness of consensus, respectively. Thereby, this approach does not need to make much tradeoff, and can be expected to be much less conservative than the tradeoff approach where one control tackles two conflicting performances. Second, the control action of the second step is proportional to the residual signal which quantifies the modeling mismatch level, thereby having some online “adaptivity” with respect to modeling mismatches. This complementary approach will yield the same achievable $H_2$ performance when modeling mismatches do not exist. By contrast, the tradeoff approach always considers the a priori worst case (for both $H_2$ performance and $H_\infty$ robustness) and still yields the same conservative performance even when disturbances or uncertainties do not exist.

The rest of this article is organized as follows. Some mathematical preliminaries including graph theory and results on $H_2$ and $H_\infty$ performances are summarized in Section II. The $H_2$ and $H_\infty$ consensus problem is formulated in Section III. A two-step complementary design approach is proposed for the $H_2$ and $H_\infty$ consensus problem in Section IV. A simulation example that illustrates the proposed theoretical results are presented in Section V. Finally, Section VI concludes this article.

II. MATHEMATICAL PRELIMINARIES

A. Graph Theory

The information flow among the agents can be conveniently modeled by a graph. An undirected graph is defined by $G = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, N\}$ is the set of nodes (each node represents an agent) and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of unordered pairs of nodes, called edges. An undirected graph is connected, if there exists a path between every two distinct nodes. For an undirected graph $G$, its adjacency matrix, denoted by $A = [a_{ij}] \in \mathbb{R}^{N \times N}$, is defined such that $a_{ii} = 0$, $a_{ij} = 1$ if $(i, j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $L = [L_{ij}] \in \mathbb{R}^{N \times N}$ associated with $G$ is defined as $L_{ii} = \sum_{j=1}^{N} a_{ij}$ and $L_{ij} = -a_{ij}$, $i \neq j$.

Lemma 1 (12): For an undirected graph $G$, zero is an eigenvalue of $L$ with 1 as an eigenvector and all nonzero eigenvalues are positive. Moreover, zero is a simple eigenvalue of $L$ if and only if $G$ is connected.

B. Results on $H_2$ and $H_\infty$ Performances

In this section, we summarize results on $H_2$ and $H_\infty$ performances of linear systems.

Consider the linear system

$$
\dot{x} = Ax + Bu, \quad y = Cx
$$

(1)

where $x \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^m$ is the measured output, and $w \in \mathbb{R}^q$ is the external disturbance.

Let $G(s) = C(sI - A)^{-1} B$ be the transfer function matrix of (1). The $H_2$ norm of $G$ is defined to be $\|G\|_2 = \frac{1}{\pi} \int_{-\infty}^{\infty} \text{tr}(G(j\omega)G(j\omega)^T) d\omega$. We then review the following well-known result on the $H_2$ performance [1].

Lemma 2: Let $\gamma > 0$. The following statements are equivalent.

1) $A$ is stable and $\|G\|_2 < \gamma$.

2) There exists $X > 0$ such that

$$
AX + XA^T + BB^T < 0, \quad \text{tr}(CXC^T) < \gamma^2.
$$

3) There exist $P > 0$ and $Q > 0$ such that

$$
\begin{bmatrix}
ATP + PA & PB \\
BT^TP & -I
\end{bmatrix} < 0,
\begin{bmatrix}
P & C^T \\
C & Q
\end{bmatrix} > 0, \quad \text{tr}(Q) < \gamma^2.
$$

Next, we review the $H_\infty$ performance of (1). If $A$ is stable, the $H_\infty$ norm of (1) is defined to be $\|G\|_\infty = \sup_{\omega \in \mathbb{R}} |\sigma(G(j\omega))|$, where $\sigma(G(j\omega))$ is the maximum singular value of $G(j\omega)$. The following lemma presents a well-known result on the $H_\infty$ performance [1], [33].

Lemma 3: Let $\gamma > 0$. The following statements are equivalent.

1) $A$ is stable and $\|G\|_\infty < \gamma$.

2) There exists $X > 0$ such that

$$
ATX + XA + \frac{1}{\gamma_\infty} XXB^TB + X + C^TC < 0.
$$

III. FORMULATION OF $H_2$ AND $H_\infty$ CONSENSUS PROBLEM

Consider a multiagent system, consisting of $N$ identical uncertain linear agents subject to different noises and external disturbances, the dynamics of the $i$th agent are described by

$$
\begin{align*}
\dot{x}_i &= Ax_i + B_0w_{0i} + B_1w_i + B_2u_i, \\
y_i &= C_2x_i + D_0w_{0i} + D_1w_i, & i = 1, \ldots, N
\end{align*}
$$

(2)

where $x_i \in \mathbb{R}^n$ is the state, $u_i \in \mathbb{R}^p$ is the control input, and $y_i \in \mathbb{R}^{m_1}$ is the measurement output, respectively. In (2), $w_i \in \mathbb{R}^{n_1}$ denotes the external disturbance signal, representing the modeling uncertainty and/or unmodeled dynamics of the $i$th agent, $w_{0i} \in \mathbb{R}^{n_2}$ is a white noise signal with $E\{w_{0i}(t)\} = 0$, and $E\{w_{0i}(t)w_{0j}(t)^T\} = \delta(t - \tau)I$, where $E\{\cdot\}$ and $\delta(t)$ denote the expectation operator and the Dirac function. The matrices $A, B_0, B_1, B_2, C_2, D_0, D_1$ are suitable dimensions. The pair $(A, B_2)$ is assumed to be stabilizable and the pair $(C_2, A)$ is assumed to be detectable. The communication graph among the $N$ agents is represented by an undirected graph $G$.

The agents in (2) are said to achieve consensus if there exist control laws $u_i$ such that, given $w_{0i} = 0$ and $v_i = 0$, $x_i - x_j \to 0$ as $t \to \infty$ for all $i, j = 1, \ldots, N$. In this article, output variables $z_i, i = 1, \ldots, N$, as defined in (3) (see also [14], [16] and [2]), are adopted to quantify the consensus performance

$$
z_i = \frac{1}{N} \sum_{j=1}^{N} C_1(x_i - x_j), \quad i = 1, \ldots, N
$$

(3)

where $z_i \in \mathbb{R}^{m_2}$, and $C_1 \in \mathbb{R}^{m_2 \times n}$ is a given constant weighting matrix. Note that other candidate performance variables could also be applied, for example, those depending on the specific network topology $G$ as in [15] and [34].
Let \( x = [x_1^T, \ldots, x_N^T]^T, u = [u_1^T, \ldots, u_N^T]^T, y = [y_1^T, \ldots, y_N^T]^T, z = [z_1^T, \ldots, z_N^T]^T, w_0 = [w_{01}^T, \ldots, w_{0N}^T]^T \), and \( w = [w_1^T, \ldots, w_N^T]^T \). The agents in (2) can be written in compact form as

\[
\dot{x} = (I \otimes A)x + (I \otimes B_0)w_0 + (I \otimes B_1)w + (I \otimes B_2)u
\]

\[
y = (I \otimes C_2)x + (I \otimes D_0)w_0 + (I \otimes D_1)w
\]

\[
z = (M \otimes C_1)x
\]

(4)

where \( M = I - \frac{1}{N}11^T \). Let \( T_{wa}(s) \) and \( T_{w}(s) \) denote the closed-loop transfer function matrices of (4) from \( w \) to \( z \) and from \( w_0 \) to \( z \), respectively, under a distributed feedback control law \( u \). The following main problem to be addressed in this article can then be formulated as follows.

**Main problem:** For the multiagent system in (2) and (3), given constants \( \gamma_2 > 0 \) and \( \gamma_\infty > 0 \), find a distributed feedback control law \( u \) such that \( \|T_{wa}(s)\|_2 < \gamma_2 \) and \( \|T_{w}(s)\|_\infty < \gamma_\infty \) and the agents in (2) achieve consensus, that is, \( x_i - x_j \to 0 \) as \( t \to \infty \) for all \( i, j = 1, \ldots, N \) if both \( w_{0i} = 0 \) and \( w_i = 0 \).

We shall call this distributed robust optimal control formulation “\( H_2 \) and \( H_\infty \) Consensus Problem,” referring to \( \|T_{wa}(s)\|_2 < \gamma_2 \) as \( H_2 \) consensus and \( \|T_{w}(s)\|_\infty < \gamma_\infty \) as \( H_\infty \) consensus, respectively.

One method to solve the \( H_2 \) and \( H_\infty \) consensus problem is the standard tradeoff mixed \( H_2/H_\infty \) design [35], [36], i.e., using one single control law such that both performance criteria are satisfied. However, it is well understood that there is an intrinsic conflict between the \( H_2 \) performance and \( H_\infty \) robustness in the mixed \( H_2/H_\infty \) design [1], [29]. To the best of author’s knowledge so far, there has been no effective solution to the consensus problem for multiagent systems that could guarantee noncompromised \( H_2 \) and \( H_\infty \) performance.

In the present article, we will propose a novel nontradeoff complementary design for obtaining distributed control laws that address the said \( H_2 \) and \( H_\infty \) consensus problem. The proposed design contains two steps. In the first step, a distributed control law is proposed, which achieves \( H_2 \) consensus for the controlled multiagent system. In the second step, an extra distributed control law is designed which achieves \( H_\infty \) consensus for the overall network. In particular, we will provide two design methods for obtaining such distributed control laws that solve the \( H_2 \) and \( H_\infty \) consensus problem, based on relative output feedback and absolute output feedback, respectively.

Note that the complementary design can be expected to be much less conservative than the tradeoff approach, as it has two separate controls to deal with the \( H_2 \) performance and the \( H_\infty \) robustness of consensus, respectively.

**IV. TWO-STEP COMPLEMENTARY APPROACH TO THE DISTRIBUTED \( H_2 \) AND \( H_\infty \) CONTROL PROBLEM**

In this section, we will provide two design methods for obtaining distributed control laws that solve the \( H_2 \) and \( H_\infty \) consensus problem with the proposed nontradeoff complementary design, based on relative output feedback and absolute output feedback, respectively.

**A. Relative Output Feedback Case**

In this section, we consider the case where only the relative output information of the neighboring agents is accessible to each agent. In this case, the structure of the proposed complementary design is shown in Fig. 1.

1) **Step One:** In the first step, we consider the \( H_2 \) consensus problem for the case with nominal agent dynamics, i.e., we consider only the noise \( w_{0i} \) (without considering external disturbances or uncertainties \( w_i \), i.e., we let \( w_i = 0 \)). Relying on the relative output information of neighboring agents, we employ the following distributed observer-based protocol [2], [20]:

\[
\dot{v}_i = (A - GC_2)v_i + \sum_{j=1}^{N} a_{ij} (B_2 F(v_i - v_j) + G(y_i - y_j))
\]

\[
u_{2i} = Fv_i, \quad i = 1, \ldots, N
\]

(5)

where \( v_i \in \mathbb{R}^n \) is the protocol state, \( u_{2i} \) is the input of the \( i \)th agent in this step, the matrices \( F \) and \( G \) are feedback gains to be designed. The coefficient \( a_{ij} \) is the \( ij \)th entry of the adjacency matrix of the communication graph among the agents. Since in this step, we only take care of the influence of the noise \( w_{0i} \) on the performance outputs \( z_i \), we consider only the outer loop in Fig. 1. The control input \( u_i \) of agent \( i \) in this case is equal to \( u_{2i} \), with \( w_{\infty} = 0 \). Define the error variables

\[
e_i \triangleq v_i - \sum_{j=1}^{N} a_{ij} (x_i - x_j), \quad i = 1, \ldots, N
\]

(6)

We then have

\[
\dot{e}_i = (A - GC_2)e_i + (GD_0 - B_0) \sum_{j=1}^{N} a_{ij} (w_{0i} - w_{0j}).
\]

(7)

Therefore, if \( G \) is chosen such that \( A - GC_2 \) is Hurwitz, \( v_i \) in (5) is actually an estimate of \( \sum_{j=1}^{N} a_{ij} (x_i - x_j) \) for agent \( i \). That is, \( DO(s) \) in Fig. 1 is in fact represented by the distributed observer in (5).

Denote \( u_2 = (u_{21}, u_{22}, \ldots, u_{2N})^T \). The distributed observer-based protocol can be written in compact form as

\[
\dot{v} = (I \otimes (A - GC_2))v + (L \otimes B_2 F)v + (L \otimes G)y
\]

\[
u_2 = (I \otimes F)v
\]

(8)

Denote \( v = [v_1^T, \ldots, v_N^T]^T \) and \( \xi = [x^T \quad v^T]^T \). By substituting (5) into (2), the closed-loop network dynamics can then be written in compact form as

\[
\dot{\xi} = \mathcal{A} \xi + \mathcal{B}_0 w_0
\]

\[
z = \mathcal{C}_1 \xi
\]

(9)

where

\[
\mathcal{A} = \begin{bmatrix}
I \otimes A & I \otimes B_2 F \\
L \otimes GC_2 & I \otimes (A - GC_2) + L \otimes B_2 F
\end{bmatrix}
\]

\[
\mathcal{B}_0 = \begin{bmatrix}
I \otimes B_0 \\
L \otimes GD_0
\end{bmatrix}, \quad \mathcal{C}_1 = \mathcal{M} \otimes \begin{bmatrix} C_1 \quad 0 \end{bmatrix}
\]

(10)
The following theorem provides a necessary and sufficient condition for the $H_2$ suboptimal consensus problem.

**Theorem 1:** Assume that the graph $G$ is a connected undirected graph with the Laplacian matrix $L$. Let $\gamma_2 > 0$. Then, the distributed protocol (5) achieves $H_2$ consensus for the network (9) if and only if the following $N - 1$ subsystems

$$
\dot{\xi}_i = \begin{bmatrix}
A & \lambda_i B_2 F \\
\lambda_i G C_2 & A - G C_2 + \lambda_i B_2 F \\
\end{bmatrix} \xi_i + \begin{bmatrix}
B_0 \\
G D_0 \\
\end{bmatrix} \tilde{w}_{0i}
$$

$$
\ddot{z}_i = [C_1 \ 0] \dot{\xi}_i, \ i = 2, \ldots, N
$$

are internally stable and $\sum_{j=2}^{N} \| \hat{T}_{00j,i} \|_2^2 < \gamma_2^2$, where $\lambda_2, \ldots, \lambda_N$ are the nonzero eigenvalues of $L$, and $\hat{T}_{00j,i}$ denotes the transfer function matrix of (11) from $\tilde{w}_{0i}$ to $\dot{z}_i$.

**Proof:** The result can be proved by following similar lines in [2] and [15]. The key steps are sketched here for clarity. First, we apply the unitary transformation $U \otimes I$ onto the dynamics of the consensus error $(M \otimes I)\xi$, where $U$ is a unitary matrix such that $UTL = \text{diag}(0, k_2, \ldots, k_N)$. Note that $UT^2M U = \text{diag}(0, 1, \ldots, 1)$, see, e.g., [2]. Next, by observing that the $H_2$ norm is invariant under unitary transformations, we can get that the $H_2$ suboptimal consensus problem is solved if and only if the following $N - 1$ subsystems

$$
\dot{\xi}_i = \begin{bmatrix}
A & B_2 F \\
\lambda_i G C_2 & A - G C_2 + \lambda_i B_2 F \\
\end{bmatrix} \xi_i + \begin{bmatrix}
B_0 \\
\lambda_i G D_0 \\
\end{bmatrix} \tilde{w}_{0i}
$$

$$
\ddot{z}_i = [C_1 \ 0] \dot{\xi}_i, \ i = 2, \ldots, N
$$

are internally stable and $\sum_{j=2}^{N} \| \hat{T}_{00j,i} \|_2^2 < \gamma_2^2$. Now, by letting $\xi_i = [I \ 0] \dot{\xi}_i$, evidently the subsystems in (12) are equivalent to those in (11). $\square$

Before moving forwards, we need to make the following assumption and introduce a lemma.

**Assumption 1:** The system matrices in (2) satisfy that $D_0 B_0^T = 0$ and $D_0 D_0^T = I$.

Note that Assumption 1 is made to simplify the notation and can be easily removed, see e.g., [15] and [37].

**Lemma 4 (see [15, 37]):** Suppose Assumption 1 holds. Consider the $i$th subsystem in (11) with $\lambda_i = 1$. Let $P > 0$ and $Q > 0$, respectively, satisfy the following inequalities:

$$
(A + B_2 F)^T P + P(A + B_2 F) + C_1^T C_1 < 0
$$

$$
A Q + A^T Q - Q C_2^T C_2 Q + B_0 B_0^T < 0.
$$

If the inequality

$$
\text{tr}(C_2 Q P Q C_2^T) + \text{tr}(C_1 C_1^T) < \gamma^2
$$

holds, then $\hat{T}_{00j,i}$, with $G = QC_2^T$ and $\lambda_i = 1$, satisfies that $\| \hat{T}_{00j,i} \|_2 < \gamma$.

The following theorem provides a design method for obtaining distributed protocols (5) that achieves $H_2$ suboptimal consensus.

**Theorem 2:** Assume that Assumption 1 holds and the graph $G$ is a connected undirected graph with the Laplacian matrix $L$. Let $\gamma_2 > 0$. Let $Q > 0$ be a solution to (14). Let $P > 0$, $W > 0$, $\tau > 0$ be solutions to the following LMI:

$$
\begin{bmatrix}
P A^T + AP - \tau B_2 B_2^T & \bar{P} C_2^T \\
\bar{P} C_2 & -I
\end{bmatrix} < 0
$$

$$
\begin{bmatrix}
P & C_2 Q \\
\bar{P} & W
\end{bmatrix} > 0
$$

$$
\text{tr}(W) + \text{tr}(C_1 C_1^T) < \frac{\gamma^2}{N - 1}.
$$

Then, the protocol (5) with $G = QC_2^T$, $F = -c B_2^2 P^{-1}$, and $c \geq \frac{\gamma^2}{\omega^2}$ achieves $H_2$ consensus.

**Proof:** In light of Theorem 1, network (9) achieves $H_2$ consensus if the $N - 1$ subsystems in (11) are internally stable and $\| \hat{T}_{00j,i} \|_2^2 < \frac{\gamma_2^2}{N - 1}$. According to Lemma 4, the $i$th subsystem in (11) is internally stable and $\| \hat{T}_{00j,i} \|_2^2 < \frac{\gamma_2^2}{N - 1}$; if there exist $Q > 0$ satisfying (14) and $P > 0$ such that

$$
(A + \lambda_i B_2 F)^T P + P(A + \lambda_i B_2 F) + C_1^T C_1 < 0
$$

and

$$
\text{tr}(C_2 Q P Q C_2^T) + \text{tr}(C_1 C_1^T) < \frac{\gamma^2}{N - 1}.
$$

Let $P = P^{-1}$. Multiplying on both sides of (18) by $P$ and in light of Schur Complement Lemma [38], we obtain that (18) and (19) hold if and only if

$$
\begin{bmatrix}
P(A + \lambda_i B_2 F)^T + (A + \lambda_i B_2 F) P + PC_2^T C_1^T P^{-1}
- I
\end{bmatrix} < 0
$$

and the inequalities (16) and (17) hold at the same time. Evidently, if we choose $F = -c B_2^2 P^{-1}$ and $c \geq \frac{\gamma^2}{\omega^2}$, then (15) implies (20) and thereby (19).

**Remark 1:** The separation property of observed-based controllers shown in [15] and [37] is employed in this theorem. The observer gain matrix $G$ and the feedback gain $F$ are designed in a decoupled way. Moreover, the feasibility of (15) is equivalent to that of (20). Note that by letting $FP = V$ and $\lambda_i = 1$, we know that (20) holds, then

$$
P A + A^T P + B_2 V + V^T B_2^T + P C_2^T C_1 P < 0
$$

which, in light of Finsler’s Lemma [2], [39], is equivalent to that there exist $P > 0$ and $\tau > 0$ such that (15) holds. Therefore, (20) implies (15). The converse was shown in the proof.

**2) Step Two:** In the second step, we design an additional regulating control input $u_{w_i}$ to deal with the external disturbances $w_i$ and to guarantee the $H_2$ robustness while not significantly compromising the $H_2$ performance. Note that, in the second step, we only consider the effect of $w_i$.

Under the $H_2$ consensus protocol (5) in the first step, the augmented agent dynamics are described by

$$
\dot{x}_i = A x_i + B_{2w_i} + B_1 w_i
$$

$$
\ddot{v}_i = (A - G C_2) v_i + \sum_{j=1}^{N} a_{ij} (B_2 (v_i - v_j)) + G C_2 (x_i - x_j) + G D_1 (w_i - w_j)
$$

$$
u_i = u_{21} + u_{w_i}
$$

$$
u_{21} = F v_i, \ i = 1, \ldots, N
$$

where the gain matrices $F$ and $G$ are designed in the first step.

The residual signal $f = [f_1, \ldots, f_N]^T$ in Fig. 1 is used in the second step to activate the inner loop, which is defined as the stacked error between the actual relative outputs of neighboring agents and their observed ones given by a distributed observer.

Therefore, the residual signal $f$ builds on the protocol (5) and is given by the following:

$$
f_i = C_2 v_i - \sum_{j=1}^{N} a_{ij} (y_i - y_j)
$$

$$
= C_2 e_i - D_1 \sum_{j=1}^{N} a_{ij} (w_i - w_j), \ i = 1, \ldots, N
$$

where $e_i$ is defined as in (6).
In this step, we consider a distributed protocol of the form
\[
\dot{u}_i = A_i n_i + B_i f_i \\
u_\infty = C_e n_i + D_e f_i
\]
where \(n_i \in \mathbb{R}^n\) is the state of the protocol, and \(A_i, B_i, C_e, \) and \(D_e\) are protocol matrices to be designed. In this case, \(Q(s) = \left[ \begin{array}{c} A_i \\ B_i \\ C_e \end{array} \right] \) is a general dynamic controller with \(f_i\) as its input. Special forms such as observer-based ones in [20] and [21] can be also considered.

Note that the error \(e_i\) in the current case satisfies
\[
\dot{e}_i = \left( A - GC_2 \right) e_i + (GD_1 - B_1) \sum_{j=1}^{N} a_{ij} (w_i - w_j).
\]  
(24)

Evidently, if the external disturbances \(w_i\) are absent, then it follows from (24) and (22) that \(e_i\) and subsequently \(f_i\) will asymptotically converge to zero. If the external disturbances \(w_i\) are not equal to zero, the inner loop will be activated as \(f_i\) are bounded noise signals and subsequently \(u_\infty\) will be implemented to recover robustness.

Denote \( e = [e^T_1, \ldots, e^T_N]^T, \quad n = [n^T_1, \ldots, n^T_N]^T, \) and \( \zeta = [\xi^T, \eta^T]^T. \) Using (21) and (23), we obtain the closed-loop network dynamics in compact form as
\[
\dot{\zeta} = \left[ \begin{array}{c} A \zeta + B \xi + C \eta \\ \xi + \left( B_2 \zeta + L \xi \right) \end{array} \right] + \left[ \begin{array}{c} B_3 \eta + D \xi \\ \eta + \left( B_4 \xi + D \eta \right) \end{array} \right] w
\]
(25)

\[
z = \left[ \begin{array}{c} \zeta \\ n \end{array} \right]
\]
where \( \zeta \) is defined in (10), and
\[
\dot{\xi} = \left[ \begin{array}{c} \dot{\xi}_1 \\ \dot{\xi}_2 \end{array} \right] = \left[ \begin{array}{c} A_1 \xi_1 + B_1 \xi_2 + C_1 \eta_1 \\ B_1 \xi_2 + C_1 \eta_2 \end{array} \right] + \left[ \begin{array}{c} D_1 \xi_2 + E_1 \eta_2 \\ \eta_2 + G_1 \xi_1 \end{array} \right] \tilde{w}_i
\]
(27)

with
\[
\begin{align*}
\mathcal{A}_i &= \begin{bmatrix} A + \lambda_i B_2 F & B_2 F & 0 \\ 0 & A - GC_2 & 0 \end{bmatrix}, \\
\mathcal{B}_2 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
\mathcal{C}_e &= \begin{bmatrix} A_e \end{bmatrix}, \\
\mathcal{D}_e &= \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \\
\mathcal{D}_1 &= \begin{bmatrix} 0 \end{bmatrix}, \\
\lambda_i &= \lambda_i (GD_1 - B_1), \\
\mathcal{A}_{ii} &= \begin{bmatrix} \lambda_i (GD_1 - B_1) \\ 0 \end{bmatrix}, \\
\mathcal{D}_{ii} &= \begin{bmatrix} 0 \end{bmatrix}, \\
\mathcal{D}_{1i} &= \begin{bmatrix} 0 \end{bmatrix}, \\
\mathcal{C}_{ii} &= \lambda_i (GD_1 - B_1), \\
\mathcal{A}_{1i} &= \begin{bmatrix} \lambda_i (GD_1 - B_1) \\ 0 \end{bmatrix}, \\
\mathcal{D}_{1i} &= \begin{bmatrix} 0 \end{bmatrix}.
\end{align*}
\]

are internally stable and the associated transfer function matrices \(T_{\tilde{w}_i, \tilde{z}_i}\) from \(\tilde{w}_i\) to \(\tilde{z}_i\) satisfy \(\|T_{\tilde{w}_i, \tilde{z}_i}\| < \gamma_{\infty}\), where \(\lambda_2, \ldots, \lambda_N\) are the nonzero eigenvalues of the Laplacian matrix \(L\).

The following theorem provides a design method for obtaining the distributed control law (23).

**Theorem 4:** Assume that \(G\) is a connected undirected graph with the Laplacian matrix \(L\) and that \(B_2\) is of full column rank. Let \(\gamma_{\infty} > 0\). Then, network (25) achieves \(H_{\infty}\) consensus if there exist positive definite matrices \(S_1\) and \(S_2\), and a matrix \(V_1\) such that
\[
S = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}, \quad V = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}
\]
(28)
satisfying the following LMI's:
\[
\begin{bmatrix}
\gamma_i \mathbf{I} \\
(S \mathbf{B}_1 + V \mathbf{D}_1) \mathbf{I} - \gamma_i \mathbf{I}
\end{bmatrix} < 0
\]
(29)
for \(i = 2, N, \) where
\[
\begin{align*}
\mathbf{Y}_i &= \begin{bmatrix} \mathbf{Y}_i \end{bmatrix}, \\
\mathbf{Y}_2 &= \begin{bmatrix} \mathbf{Y}_2 \end{bmatrix}, \\
\mathbf{S} &= \begin{bmatrix} \mathbf{S} \end{bmatrix}, \\
\mathbf{V}_1 &= \begin{bmatrix} \mathbf{V}_1 \end{bmatrix}, \\
\mathbf{V}_2 &= \begin{bmatrix} \mathbf{V}_2 \end{bmatrix}, \\
\mathbf{U}_i &= \begin{bmatrix} \mathbf{U}_i \end{bmatrix}, \\
\mathbf{B}_1 &= \begin{bmatrix} \mathbf{B}_1 \end{bmatrix}, \\
\mathbf{B}_2 &= \begin{bmatrix} \mathbf{B}_2 \end{bmatrix},
\end{align*}
\]
and \(T\) is a nonsingular matrix such that \(T \mathbf{B}_2 = \mathbf{I}\). Then, the system matrix \(\mathcal{R}\) of (23) is given by
\[
\mathcal{R} = \mathcal{A}_1 \mathcal{B}_1 \mathcal{C}_e + \mathcal{A}_2 \mathcal{B}_c \mathcal{C}_e + \mathcal{C}_e \mathcal{D}_c + \mathcal{C}_c \mathcal{D}_c.
\]
(30)

**Proof:** In virtue of Theorem 3 and Lemma 3, it follows that the \(N - 1\) subsystems in (27) are internally stable and \(\|\tilde{T}_{\tilde{w}_i, \tilde{z}_i}\| < \gamma_{\infty}\) if and only if there exist matrices \(S_i > 0\) such that
\[
\begin{align*}
&\begin{bmatrix} \mathcal{A}_i + \mathcal{B}_2 \mathcal{C}_2 \\ \mathcal{C}_e \end{bmatrix} \mathbf{S}_i + \begin{bmatrix} \mathcal{A}_i + \mathcal{B}_2 \mathcal{C}_2 \end{bmatrix} \mathbf{S}_i = \begin{bmatrix} \mathcal{B}_i \mathbf{S}_i + \mathcal{C}_e \mathbf{S}_i + \mathcal{C}_c \mathbf{S}_c \end{bmatrix}, \\
&\begin{bmatrix} \mathcal{A}_i + \mathcal{B}_2 \mathcal{C}_2 \end{bmatrix} \mathbf{S}_i + \begin{bmatrix} \mathcal{A}_i + \mathcal{B}_2 \mathcal{C}_2 \end{bmatrix} \mathbf{S}_i = \begin{bmatrix} \mathcal{B}_i \mathbf{S}_i + \mathcal{C}_e \mathbf{S}_i + \mathcal{C}_c \mathbf{S}_c \end{bmatrix},
\end{align*}
\]
(31)

with
\[
\begin{align*}
\mathbf{S}_i &= \begin{bmatrix} \mathbf{S}_i \\ \mathbf{S}_i \end{bmatrix}, \\
\mathbf{B}_2 &= \begin{bmatrix} \mathbf{B}_2 \\ \mathbf{B}_2 \end{bmatrix}, \\
\mathbf{C}_e &= \begin{bmatrix} \mathbf{C}_e \\ \mathbf{C}_e \end{bmatrix}, \\
\mathbf{C}_c &= \begin{bmatrix} \mathbf{C}_c \\ \mathbf{C}_c \end{bmatrix},
\end{align*}
\]
(32)

Since the matrix \(B_2\) is of full column rank, there exists a matrix \(T\) such that
\[
T \mathbf{B}_2 = \begin{bmatrix} \mathbf{I} \\ 0 \end{bmatrix}.
\]
(33)

By premultiplying \(T = \begin{bmatrix} \mathbf{I} \\ 0 \end{bmatrix}\) and postmultiplying \(T^T\) on (32), it follows that (32) holds if and only if
\[
\begin{bmatrix} \Phi_{1i}^T \\ \Phi_{2i}^T \end{bmatrix} - \begin{bmatrix} \Phi_{1i}^T \\ \Phi_{2i}^T \end{bmatrix} < \begin{bmatrix} \Phi_{1i}^T \\ \Phi_{2i}^T \end{bmatrix} - \begin{bmatrix} \Phi_{1i}^T \\ \Phi_{2i}^T \end{bmatrix} < 0, \quad i = 2, \ldots, N
\]
(33)
Now, let \( S = S_t \) and \( S \bar{B}_2 \bar{R} = V \). Recall that (28), then (33) holds if (29) holds for \( i = 2, \ldots, N \). In this case, due to

\[
\begin{bmatrix}
S_{11} & 0 \\
0 & S_{22}
\end{bmatrix}
\begin{bmatrix}
I \\
0
\end{bmatrix} \bar{R} =
\begin{bmatrix}
V_1 \\
0
\end{bmatrix}
\]

the system matrix is given by (30).

Finally, note that the inequalities in (29) are linear matrix inequalities with respect to the unknown variables, we need to check only the two LMI s in (29) for \( i = 2 \) and \( N \), and the other \( N - 3 \) LMI s in (29) corresponding to \( i = 3, \ldots, N - 1 \), also hold, with their variables chosen to be some convex combinations of those satisfying the two LMI s in (29) for \( i = 2, N \). This completes the proof.

Remark 2: In the novel control structure in Fig. 1, the design of \( H_2 \) consensus control of the outer loop is independent of the control \( Q(s) \) in (23) of the inner loop. The extra control \( Q(s) \) relies on the residual signal \( f \), which is the stacked error between the actual relative outputs of neighboring agents \( \sum_{j=1}^{N} a_{ij} (y_i - y_j) \) and their observed ones \( C_{ij} v_i \) given by the distributed observer (5). The extra control action of the inner loop, activated by residual signal \( f \) in the presence of external disturbances or uncertainties \( u_{li} \), will complement the \( H_2 \) performance by providing \( H_\infty \) robustness guarantee with respect to \( u_{li} \). This is the reason why this approach is called a complementary design approach.

Remark 3: The two-step complementary approach proposed in this section, compared to the tradeoff approach, has at least two main advantages:

1) The extracontrol \( Q(s) \) in (23) provides an additional degree of freedom for design. Therefore, the current complementary approach has two separate control inputs to deal with the \( H_2 \) performance and the \( H_\infty \) robustness of consensus, respectively, thereby does not need to make much tradeoff, and can be expected to be much less conservative, while the tradeoff design has only one control to tackle two conflicting performance simultaneously. Although Theorems 2 and 4 in this section are conservative; however, it should be pointed out that the conservatism of these results is not caused by the complementary approach. On the contrary, it in fact highlights the difficulty of distributed control.

2) The control action of the inner loop is proportional to the residual signal which quantifies the modeling mismatch level, thereby having some online “adaptivity” with respect to modeling errors. This complementary approach will yield the same achievable \( H_2 \) performance when modeling mismatches do not exist, because in this case the inner loop will be deactivated. By contrast, the tradeoff approach always considers the a priori worst case and produce the same conservative performance even when disturbances or uncertainties disappear.

Remark 4: It should be mentioned that the order of the overall control law designed by the complementary approach is higher than the tradeoff approach, since both distributed protocols (5) and (23) are required in the former approach while only one dynamic controller is needed in the latter. This is the price to provide more degree of design freedom and it is actually not a big issue considering the abundance of cheap storage and computing resources. Besides, the extra control action of the inner loop, providing robustness guarantee, will inject some white noises into the closed-loop network dynamics, and thereby will make certain compromise of the \( H_2 \) performance.

B. Absolute Output Feedback Case

In this section, we consider the case where absolute output information of each agent is available. In this case, we adopt local observers for the agents, instead of the distributed observers as in the previous section. The structure of the complementary design approach in this case is shown in Fig. 2.

1) Step One: Based on the absolute output \( y_i \), we propose for each agent the following Luenberger observer:

\[
\dot{\hat{v}}_i = A \hat{v}_i + B u_i + \hat{G} (y_i - C_2 \hat{v}_i) \tag{34}
\]

where \( \hat{G} \) is the observer gain to be designed. In the first step, we consider only the outer loop, therefore, \( u_i = u_{2i} \). For the \( H_2 \) consensus problem, we design the following protocol:

\[
u_{2i} = \hat{F} \sum_{j=1}^{N} a_{ij} (\hat{v}_i - \hat{v}_j) \tag{35}
\]

where \( \hat{F} \) is the feedback gain to be designed. Denote \( \hat{\xi} = \left[ \hat{v}_1^T, \ldots, \hat{v}_N^T \right]^T \) and \( \tilde{\xi} = \left[ x^T - \hat{v}^T \right]^T \). The closed-loop network dynamics in this case can be written in compact form as

\[
\dot{\hat{\xi}} = \begin{bmatrix} I \otimes A & \mathcal{L} \otimes B \hat{F} \\ I \otimes \hat{G} C_2 & I \otimes (A - \hat{G} C_2) + \mathcal{L} \otimes B_2 \hat{F} \end{bmatrix} \hat{\xi} + \begin{bmatrix} I \otimes B_0 \\ I \otimes \hat{G} D_0 \end{bmatrix} w_0
\]

where \( \mathcal{L} = M \otimes \left[ C_1 \ 0 \right] \hat{\xi} \). (36)

It is easy to verify that the \( H_2 \) consensus problem of (36) can be reduced to the same condition as in Theorem 1. Then, Theorem 2 can also be used to design the protocol (35).

2) Step Two: In the second step, we define the residual signals \( \tilde{f}_i \) as follows:

\[
\tilde{f}_i = C_2 \hat{v}_i - y_i, \quad i = 1, \ldots, N
\]

(37)

which is actually the local estimated output error. Therefore, \( \tilde{f}_i \) quantifies the difference between the actual plant and the ideal plant, and we can see that \( \tilde{f}_i \to 0 \) as \( t \to \infty \) if there exist no disturbances or uncertainties.

Since \( \sum_{j=1}^{N} a_{ij} (\hat{x}_i - \hat{x}_j) \) is the consensus error and \( \hat{v}_i \) is the estimate of \( x_i \) for agent \( i \), we can see that \( \sum_{j=1}^{N} a_{ij} (\hat{f}_i - \hat{f}_j) \) denotes the estimated output error of the consensus error. Therefore, in this case we design the control input \( u_{sci} \) for the inner loop based on \( \sum_{j=1}^{N} a_{ij} (\tilde{f}_i - \tilde{f}_j) \) instead of \( \hat{f}_i \) as in the previous section. Specifically, we consider a distributed protocol of the form

\[
\begin{aligned}
\dot{\hat{n}}_i &= A_c n_i + \hat{B}_c \sum_{j=1}^{N} a_{ij} (\tilde{f}_i - \tilde{f}_j) \\
u_{sci} &= \hat{C}_c n_i + \hat{D}_c \sum_{j=1}^{N} a_{ij} (\tilde{f}_i - \tilde{f}_j) \tag{38}
\end{aligned}
\]

where \( n_i \in \mathbb{R}^n \) is the state of the protocol, and \( \hat{A}_c, \hat{B}_c, \hat{C}_c, \) and \( \hat{D}_c \) are protocol matrices to be designed. In this case, \( Q(s) = \begin{bmatrix} \hat{A}_c & \hat{B}_c \\ \hat{C}_c & \hat{D}_c \end{bmatrix} \) in Fig. 2.
Let \( \delta_t = \bar{n}_t - x_t \). Denote \( \bar{\delta} = [\bar{\delta}_1^T, \ldots, \bar{\delta}_6^T]^T, \bar{\xi} = [x^T, \bar{\delta}^T]^T \) and \( \bar{n} = [\bar{n}_1^T, \ldots, \bar{n}_6^T]^T \). Then, it follows from (2), (35), and (38) that the closed-loop network dynamics are given by

\[
\begin{bmatrix}
\dot{\bar{\xi}} \\
\dot{\bar{n}}
\end{bmatrix} = \begin{bmatrix}
\bar{\mathcal{A}} + \bar{\mathcal{B}} \bar{\mathcal{G}} \bar{\mathcal{C}}_2 & \bar{\mathcal{B}}_2 \bar{\mathcal{G}}_2 \\
0 & \bar{\mathcal{A}}_c
\end{bmatrix} \begin{bmatrix}
\bar{\xi} \\
\bar{n}
\end{bmatrix} + \begin{bmatrix}
\bar{\mathcal{B}} - \bar{\mathcal{B}}_c \\
-\bar{\mathcal{D}}_c
\end{bmatrix} + \begin{bmatrix}
\mathds{1} \\
0
\end{bmatrix} w
\]

where \( \mathds{1} \) is a vector of ones.

\[ z = [\bar{\mathcal{C}}_1 \ 0] \begin{bmatrix}
\bar{\xi} \\
\bar{n}
\end{bmatrix} \tag{39} \]

Similarly as in Theorems 3 and 4, the control law (38) can be constructed to achieve \( H_\infty \) consensus with prescribed index. The details are omitted here for conciseness.

V. SIMULATION EXAMPLE

In this section we will use a simulation example to illustrate the proposed complementary \( H_2 \) and \( H_\infty \) design by dynamic output feedback, as in Theorems 2 and 4 in Section IV-A, for obtaining distributed protocols.

Consider a multiagent system that consists of six agents. The dynamics of each agent is given by (2), where

\[
A = \begin{bmatrix}
-2 & 2 \\
-1 & 1
\end{bmatrix}, \quad B_0 = \begin{bmatrix}
0 & 0 \\
0.5 & 0
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
1 & 0 \\
0.6 & 1
\end{bmatrix}, \quad B_2 = \begin{bmatrix}
1 & -1 \\
0 & 1
\end{bmatrix}
\]

\[
C_1 = \begin{bmatrix}
1 & 0
\end{bmatrix}, \quad C_2 = \begin{bmatrix}
0.8 & 1
\end{bmatrix}, \quad D_1 = 0.1, \quad D_0 = \begin{bmatrix}
0 & 1
\end{bmatrix}.
\]

The communication graph among the agents is shown in Fig. 3, which is a connected undirected graph with the Laplacian matrix \( \mathcal{L} \). The smallest nonzero and the largest eigenvalues of \( \mathcal{L} \) are computed to be \( \lambda_2 = 1.35820 \) and \( \lambda_N = 5.30289 \).

In the first step of the complementary design, we obtain via Theorem 2 a distributed control law that takes care of the \( H_2 \) performance. We choose \( \gamma_2 = 2 \) and compute the control gains \( \bar{F} = [0.0898, 2.0360] \) and \( \bar{G} = [0.5584, 0.7792]^T \). Next, in the second step, we obtain via Theorem 4 a distributed control law that deals with the \( H_\infty \) consensus. We compute the protocol gains to be \( \bar{A}_c = \begin{bmatrix}
-0.5468 & 0 \\
0 & -0.5468
\end{bmatrix}, \quad B_c = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}, \quad C_c = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}, \quad D_c = -1.1570. \) The associated computed upper bound for the \( H_\infty \) robustness is \( \gamma_{\infty, \text{min}} = 1.79822. \) In Fig. 4, we have plotted the state trajectories of the agents. It can be seen that indeed the proposed distributed protocols together achieve consensus.

As a comparison, we will next compare the output performance of the proposed complementary approach with that of the mixed \( H_2 \) and \( H_\infty \) control design in [41], in presence of the external noise and disturbance.

In particular, we choose the external noise \( w_{\text{ext}} \) to be a uniformly distributed signal, generated by MATLAB command \( \text{rand}() \). We choose the disturbance \( w_t \) to be \( w_1 = 3 \sin(110t) + 0.2, w_2 = 3 \sin(30t) + 0.4, \ w_3 = -2 \sin(110t) + 0.6, \ w_4 = 3 \sin(30t) - 0.6, \ w_5 = -2 \sin(0.1t) - 0.4, \) and \( w_6 = \sin(0.1t) - 0.2. \) The plots of the trajectories of the performance outputs \( z_i \) are given in Fig. 5. It can be seen that, in the presence of noises and disturbances, indeed the proposed complementary approach guarantees a better performance than the mixed design.

VI. CONCLUSION

In this article, we have presented a novel complementary approach to the distributed \( H_2 \) and \( H_\infty \) consensus problem of multiagent systems. Through introducing an extra control input that depends on some carefully chosen residual signals, which indicates the modeling mismatch, the complementary approach provides an additional degree of freedom for control design and complements the \( H_2 \) performance of consensus by providing the \( H_\infty \) robustness guarantee. This complementary approach does not involve much tradeoff, and can be expected to be much less conservative than the tradeoff design. Extending the proposed complementary approach to the case of directed graphs or switching topology is an interesting direction for future research [42], [43].

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