Excitonic and Nematic Instabilities on the Surface of Topological Kondo Insulators

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We study the effects of strong electron-electron interactions on the surface of cubic topological Kondo insulators, such as SmB$_6$. Cubic topological Kondo insulators generally support three copies of massless Dirac nodes on the surface that display a four-fold rotational symmetry, but only two of them are energetically degenerate and exhibit an energy offset relative to the third one. With a tunable chemical potential, when the surface states host electron and hole pockets, strong interactions may drive this system into rotational symmetry breaking nematic and translational symmetric breaking excitonic spin- or charge-density-wave phases, depending on the relative chirality of the Dirac cones. Taking a realistic surface band structure into account we compute the mean field phase diagram for interacting surface states and also analyze the associated Ginzburg-Landau theory. Beyond mean field theory, this system can be described by a two-component isotropic Ashkin-Teller model at finite temperature, and we outline the phase diagram of this model. Our theory provides a possible explanation of recent measurements which detect a two-fold symmetric magnetoresistance and an upturn in surface resistivity with tunable gate voltage in SmB$_6$. Our discussion can also be germane to other cubic topological insulators, such as YbB$_6$.

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I. INTRODUCTION

Time reversal symmetric three-dimensional $Z_2$ (strong) topological insulators (TIs) are characterized by an insulating bulk, but host an odd number of metallic surface states that are protected against time-reversal symmetric perturbations \[1,2\]. In the ten-fold way of classification they belong to class AII \[3\]. In the first generation of $Z_2$ TIs from this class, such as Bi$_2$Se$_3$, electron-electron interactions are negligibly weak. $Z_2$ TIs that belong to class AII, can, nevertheless, be realized in strongly correlated Kondo systems as well, where the hybridization between localized $f$- and conduction $d$-electrons opens up a topologically nontrivial bulk-insulating gap below the Kondo transition temperature \[4\]. The first promising candidate for a topological Kondo insulator (TKI) is SmB$_6$ \[5–11\], which hosts three copies of massless Dirac cones on its surface \[8–10\]. In this paper, we consider the effect of strong electronic interactions on the surface of such TKIs \[12,13\], and demonstrate the possibility of realizing nematic and excitonic density-wave phases. The former instability can be of relevance to a recent experiment on SmB$_6$, which displays a $C_2$ ($C_4$) symmetric magnetoresistance below (above) $5K$ \[14\], which is indicative of nematic ordering at low temperatures.

In the cubic environment of SmB$_6$, the band inversion takes place around the three $X$ points of the bulk Brillouin zone (BZ) \[15,16\]. Consequently, an interface of a cubic TI with the vacuum supports three massless Dirac cones at the $\Gamma$, $X$, and $Y$ points of the surface BZ, which is illustrated in Figs. 1(e) and (f). Throughout the paper, we assume that the surface is cleaved along a high symmetry axis, such as (001). The underlying cubic symmetry enforces equal energies $E_X$ and $E_Y$ of the Dirac nodes at the $X$ and $Y$ points, respectively, and manifests an underlying four-fold rotational or $C_4$ symmetry on the surface. The $\Gamma$ Dirac point, however, is not constrained by this symmetry and generically displays an offset with respect to the $X$ and $Y$ points, i.e., $E_\Gamma \neq E_{X/Y}$. This situation is the same for any cubic TI and a similar band structure can also be realized on the surface of YbB$_6$ \[17,19\]. Therefore, it is conceivable that by ionic liquid gating \[20\] the chemical potential can be placed in between $E_\Gamma$ and $E_{X/Y}$, yielding one hole pocket around the $\Gamma$ point and two electron pockets near the $X$, $Y$ points, as shown in Fig. 1(a). For the sake of simplicity, we assume that $E_\Gamma > E_{X/Y}$. Therefore, as discussed in this paper, strong interactions among the surface states together with electron and hole pockets of comparable sizes induce excitonic spin-density-wave (SDW) or charge-density-wave (CDW) phases on the surface of cubic TKIs. This scenario is consistent with an observed upturn of the surface resistivity in SmB$_6$ with varying gate voltage that may stem from an underlying excitonic instability \[20\]. It turns out that the band structure and the various surface instabilities for TKIs discussed here bear some similarity with iron-based superconductors. Just like in pnictide materials, the surface states are never fully gapped due to the presence of an odd number of Fermi surfaces and one of the electron pockets always remains metallic \[21\].
The precise nature of excitonic pairing depends on the relative chirality between the Dirac cones at the $X/Y$ and $\Gamma$ points. Two distinct situations that are protected by the nontrivial bulk $Z_2$ invariant can arise at the surface: (i) All Dirac cones on the surface have identical chirality. In this case, the excitonic condensate is formed by electrons and holes with opposite spin projection, giving rise to a SDW order, as shown in Fig. 1(e). (ii) If the Dirac points at the $\Gamma$ and $X,Y$ points carry opposite chirality, pairing occurs between particles and holes with same spin projection, leading to CDW order [see Fig. 1(f)]. Our discussion is, however, insensitive to the exact nature of the excitonic ordering. We thus assume equal chirality for all Dirac cones and discuss the SDW instability in the following.

If electron and hole pockets are perfectly nested, see Fig. 1(b), the surface states can undergo a BCS-like excitation instability in the (c) SDW, (f) CDW channel.

FIG. 1: (Color online) Top row: Offset among the Dirac points near the $\Gamma$ and $X/Y$ points [(a)], and deviation from perfectly nesting [(b)], due to (c) unequal sizes of the pockets, (d) ellipticity in the electron pocket, parametrized by $\mu$ and $\delta$, respectively. Bottom row: Two possible chiralities of electron (blue) and hole (red) pockets on the surface of cubic TKIs, leading to an excitonic instability in the (e) SDW, (f) CDW channel.

The Keldysh-Kopaev mechanism [22] has been exploited to address the SDW instability in Cr [24] and iron-based superconductors [25, 27]. Antiferromagnetic ordering for weak Hubbard interaction in monolayer [28, 29] and bilayer [30] graphene, and 2D Kondo insulators [31] when placed in an in-plane magnetic field. In this paper, we take into account realistic band structure effects and study excitonic and nematic instabilities on the surface of strongly interacting TKIs. Starting from a discussion of the mean-field phase diagram, followed by a classical Ginzburg-Landau (GL) analysis, we show that this system of coupled order-parameters (OPs) is described by a two-flavor isotropic Ashkin-Teller model. This model exhibits a pair of Ising phase transitions into first a nematic phase, and subsequently to a translational symmetry breaking SDW phase.

The rest of the paper is organized as follows. In the next section we introduce the microscopic description of interacting surface states of a cubic TKI. In Sec. III, we present a mean field phase diagram for the interacting surface states for electron and hole pockets of comparable sizes. Section IV is devoted to a discussion of the Ginzburg-Landau theory, where we also take into account the effects of symmetry breaking terms that may arise due to the presence of an underlying lattice and thermal fluctuations. In Sec. V, we summarize our findings and present a discussion of related topics.

II. MICROSCOPIC HAMILTONIAN FOR INTERACTING SURFACE STATES

This section introduces the microscopic description of the interacting surface states. The noninteracting Hamiltonians describing the helical Dirac fermionic excitations near the $\Gamma, X$ and $Y$ points take the form [setting $\hbar = 1$]

$$H_j = v_x^j k_x \sigma_x - v_y^j b_y \sigma_y,$$

where $j = \Gamma, X, Y$, with $v_x^\Gamma = v_y^\Gamma = v$ as the Fermi velocity of the isotropic Dirac cone near the $\Gamma$ point. The underlying $C_4$ symmetry of the surface BZ implies $v_x^X = v_y^Y$ and $v_y^X = v_x^Y$. The ellipticity of the Dirac cones near the $X$ and $Y$ points is captured by defining $v_x^X = v(1 + \delta)$ and $v_y^X = v(1 - \delta)$.

Excitonic SDW ordering arises from a repulsive interaction between fermions with opposite spin projections in the $\Gamma$ and $X,Y$ pockets. Such a particle-hole pairing instability can be taken into account by adding a repulsive short-ranged spin-spin interaction

$$H_{\text{int}} = -\frac{U_0}{2} \sum_{j=X,Y} \int \frac{d^2q}{(2\pi)^2} s_j^\dagger q \sigma_j^q,$$

to the free Hamiltonian, where $U_0 > 0$ and

$$s_j^q = \int \frac{d^2k}{(2\pi)^2} c_{j,k+q}^\dagger \sigma_3 c_{j,k}.$$
is the spin operator. $c^{\dagger}_{j,k\alpha}$ creates a fermion in the $j = \Gamma, X, Y$ pocket with momentum $k$ and spin $\alpha$. The momentum of the $X$ and $Y$ excitation is measured relative to the nesting vectors $Q_X = (\pi, 0)$ and $Q_Y = (0, \pi)$. In Eq. (3), a summation over the spinor indices $\alpha$ and $\beta$ is implied. Within the same framework, CDW ordering can be studied by simply replacing the Pauli matrix $\sigma_3$ by $\sigma_0$ in Eq. (3) and changing the sign of one matrix in $H_{X/Y}$ or $H_T$ in Eq. (4), without qualitatively changing the results.

III. MEAN FIELD PHASE DIAGRAM

To gain quantitative insight into the phase diagram of the surface theory, we analyze it in the mean field approximation. Introducing the mean field excitonic condensates $\Delta_{X/Y} = U_0(c^{\dagger}_{\Gamma,k\alpha}(\sigma_3)_{\alpha\beta}c_{X/Y,k\beta})/2$, where $\langle \ldots \rangle$ denotes the thermal expectation value, the free energy density reads

$$F = \frac{2}{U_0}(|\Delta_X|^2 + |\Delta_Y|^2) - \frac{1}{2} \sum_{i=1}^{6} \int \frac{d^2k}{(2\pi)^2} \ln \left[ 2 \cosh \frac{\beta E_i}{2} \right],$$

where $\beta$ is the inverse temperature and $E_i$ are the six eigenvalues of the quadratic single-particle Hamiltonian

$$H_{HS} = \begin{bmatrix} H_T - \lambda_- \sigma_0 & -\Delta_X \gamma_3 & -\Delta_Y \gamma_3 \\ -\Delta_X \gamma_3 & H_X + \lambda_+ \sigma_0 & 0 \\ -\Delta_Y \gamma_3 & 0 & H_Y + \lambda_+ \sigma_0 \end{bmatrix}. \quad (5)$$

In the above equation, we set $\lambda_{\pm} = \lambda \pm \mu$ as illustrated in Fig. 1(a). The SDW OPs $\Delta_{X/Y}$ display a valley U(1) symmetry associated with their phases $\Delta_j = |\Delta_j|e^{i\phi_j}$. As is characteristic for two-dimensional Dirac systems, the free energy in Eq. (4) diverges linearly due to large-momentum contributions, which, however, can be absorbed in a renormalization of the effective interaction strength

$$\frac{1}{U_0} = \frac{1}{U} - \frac{2}{v^2} \Lambda,$$

where $U > 0$ is the renormalized interaction and $\Lambda$ is an ultraviolet cutoff in momentum space $[32, 33]$, which in real systems corresponds to the band gap. Consequently, physical quantities only depend on $U$ but not on the non-universal cutoff scale $\Lambda$ or the bare coupling $U_0$.

In Fig. 2 (left), we present the phase diagram as a function of chemical potential $\mu$ and temperature $T$. For the parameters $\lambda = 2U^{-1}$ and the ellipticity of the $X/Y$ surface pockets $\delta = 0.2$ as obtained by minimizing the free energy in Eq. (4). At small chemical potential, the ground state displays a two-fold rotational or $C_2$ symmetry, where electrons from either $X$ or $Y$ pocket pair with holes from the $\Gamma$ point, respectively, yielding $|\Delta_X| \neq 0$ and $|\Delta_Y| = 0$ or vice versa. The appearance of $C_2$ SDW order naturally introduces nematicity (characterized by either $\Delta_X = 0$ or $\Delta_Y = 0$) in the system. As the temperature is increased, there is a transition from the SDW phase to the paramagnetic (PM) phase, which is continuous (second order) in the mean field approximation. Within the same approximation, transitions to nematic and SDW phases occur at the same temperature. However, as will be discussed in the following, once thermal fluctuations are incorporated, the transition temperatures for these two instabilities are generically different. If the chemical potential (and hence the Fermi surface mismatch) is increased, the direct $C_2$-normal transition at low temperature is masked by an intermediate phase in which the $C_4$ rotational symmetry is restored and all Fermi pockets participate in the excitonic pairing. Figure 2 (right) shows the complete phase diagram as a function of $\mu$, $\delta$, and $T$. Increasing the ellipticity $\delta$ pushes the critical chemical potential for the $C_2$-$C_4$ transition to smaller values but only mildly affects the subsequent $C_4$-PM transition. Hence, while a small ellipticity favors the $C_2$ phase at small $\mu$, the region of the phase diagram with $C_4$-symmetry increases when the Fermi surfaces are strongly anisotropic. We point out that the structure of the BZ in iron-based superconductors is qualitatively similar to the one for the surface states of cubic TKIs. Interestingly, the phase diagrams of these two systems bear some qualitative similarities $[21, 34, 35]$. In particular, the $C_2$-$C_4$ phase transition that can be tuned by doping has been observed experimentally in pnictide materials $[33]$.

For $\delta = 0$ (circular Fermi surfaces) the quadratic Hamiltonian in Eq. (5) manifests a U(1) symmetry among the excitonic OPs $\Delta_X$ and $\Delta_Y$, and consequently the free energy depends only on the magnitude $\Delta^2 = |\Delta_X|^2 + |\Delta_Y|^2$. Thus, in the limit $\delta = 0$, there is no distinction between the $C_2$ and the $C_4$ symmetric phases. At zero temperature, the free energy takes the particularly simple form $F = \mu^2 - \Delta_0^2/2$, where $\Delta_0$ represents the SDW OP at $T = 0$ and $\mu = 0$, which implies a first-order transition between condensed and normal phase at the standard critical Clogston-Chandrasekhar value $\mu_{\text{crit}} = \Delta_0/\sqrt{2}$, which can also be seen in Fig. 2 (right).

IV. GINZBURG-LANDAU THEORY

To further elaborate the phase diagram presented above, we consider the GL theory of the ordered state. It can be constructed by systematically expanding the free energy $F$ in powers of $\Delta_X$ and $\Delta_Y$, yielding

$$F(\Delta_i) = K \left[ (|\nabla \Delta_X|^2) + (|\nabla \Delta_Y|^2) + \alpha (|\Delta_X|^2 + |\Delta_Y|^2) \right] + \beta \frac{1}{2} (|\Delta_X|^2 + |\Delta_Y|^2)^2 + \gamma |\Delta_X|^2 |\Delta_Y|^2. \quad (7)$$

The last term (proportional to $\gamma$) plays an important role in determining the symmetry breaking in the ordered phase. As discussed above, for $\gamma = 0$, the free energy is degenerate for fixed $|\Delta_X|^2 + |\Delta_Y|^2$. If $\gamma > 0$, surface states develop finite expectation value either of the OP
Thus, the SDW state breaks the surface under \( X \) we must treat the low energy approximation for the surface states and \( U(1) \) symmetry. This, however, is only an artifact of discrete \( C_4 \) symmetry on the left panel. It is worth pointing out that the phase diagrams we obtain here are qualitatively similar to the one extracted experimentally for iron pnictides, which also share similar structure of the BZ.

\( |\Delta_X| \) or \( |\Delta_Y| \), but not both. Such a phase manifestly breaks the \( C_4 \) rotational symmetry down to \( C_2 \). On the other hand, when \( \gamma < 0 \), the system minimizes the free energy by simultaneously condensing \( |\Delta_X| \) and \( |\Delta_Y| \) at the same temperature.

In terms of the microscopic parameters, \( \gamma \) reads as

\[
\gamma = \text{Tr} [\hat{G}_T \hat{G}_X \hat{G}_T \hat{G}_Y + \hat{G}_Y \hat{G}_Y \hat{G}_T \hat{G}_X - 2 \hat{G}_T \hat{G}_X \hat{G}_T \hat{G}_Y], \quad (8)
\]

where \( \text{Tr} \) implies a summation over momentum, Matsubara frequency, and spinor indices, and

\[
\hat{G}^{-1}_T = -i\omega + H_T - \lambda_-, \quad \hat{G}^{-1}_{X/Y} = -i\omega + H_{X/Y} + \lambda_+. \quad (9)
\]

If all bands are perfectly circular (\( \delta = 0 \)), \( \hat{G}_X = \hat{G}_Y \) and concomitantly \( \gamma = 0 \), which remains true even if the bands are not perfectly nested, i.e., \( \mu \neq 0 \). For realistic cases, however, with elliptic \( X \) and \( Y \) Fermi pockets (i.e., \( \delta \neq 0 \)), we have \( \gamma \neq 0 \). For small ellipticity, expanding all the quartic terms in \( F \) in powers of \( \delta \), we obtain

\[
\gamma = \delta^2 g(T, \mu) \text{ is a positive function close to } T_c.
\]

Thus, the SDW state breaks the \( C_4 \) symmetry on the surface under \( X \leftrightarrow Y \). In the limit of large ellipticity, we must treat \( \delta \) non-perturbatively as discussed in the previous section, which gives the same result as long as \( \mu \) remains small, as shown in Fig. 2.

When \( \gamma > 0 \), the SDW order spontaneously breaks the discrete \( C_4 \) rotational as well as the continuous valley \( U(1) \) symmetry. This, however, is only an artifact of the low energy approximation for the surface states and can be reduced if we allow an additional quartic term \( \rho |\Delta_X|^2 |\Delta_Y|^2 [\cos(2\varphi_X) + \cos(2\varphi_Y)] \) in Eq. 7. Such a term can, for example, be generated by pair-scattering processes represented by \( c_4^\dagger c_1^\dagger c_3 c_6 \) and \( c_4^\dagger c_1^\dagger c_6 c_3 \), also known as Umklapp processes, which are allowed in the presence of an underlying lattice. The physical origin of such terms can be appreciated in the following way: the phase degree of freedom of \( \Delta_X \) represents a sliding mode of the SDW order in real space. However, in any material the commensurate density wave will be pinned to the lattice. Hence, we need to take into account such lattice-induced terms to pin density-wave order, that also reduce the (artificial) valley \( U(1) \) symmetry down to \( Z_2 \).

For a large Fermi surface anisotropy, however, the system may choose to condense into an incommensurate density-wave phase. For large chemical doping, on the other hand, various superconducting instabilities may set in. The discussion of these phenomena is beyond the scope of this paper. We focus instead on the regime of small chemical potential, where mean-field theory predicts a \( C_2 \) phase for arbitrary \( \delta \), as shown in Fig 2. In this phase, either \( \Delta_X \) or \( \Delta_Y \) develops a nonzero but real expectation value and thus the surface states simultaneously develop a nematic (due to the breaking of \( C_4 \) symmetry) as well as a translational symmetry breaking commensurate SDW order. Both these orders can be represented by separate Ising-like variables and thus the ground state at \( T = 0 \) displays an exact four-fold degeneracy. However, at finite temperature, thermal fluctuations allow the system to fragment into multiple domains of these degenerate phases.

To understand the role of a domain wall between two configurations, say \( A \) and \( B \), among four possible states with \( \Delta_X > 0 \), \( \Delta_X < 0 \), \( \Delta_Y > 0 \) and \( \Delta_Y < 0 \), we note that the free energy of the domain-wall per unit length is given by \( F = J_{AB} - TS_{AB} \). Here, \( S(J)_{AB} \) is the entropy (energy) per unit length of a single domain wall.
For temperatures $T > J_{AB}/S_{AB}$ we have $F < 0$, and the free-energy is minimized through the proliferation of domain walls between these two phases.

To estimate the result of proliferation of thermal domain walls, we define two Ising-spin variables $s = \text{sgn}(\Delta_X - |\Delta_Y|)$ and $\sigma = \text{sgn}(\Delta_X + \Delta_Y)$. The spin variable $s$ determines the direction of the SDW order, while $\sigma$ represents how the translation symmetry is broken. Therefore, in the nematic phase $s \neq 0$. The energy of the domain walls can be accounted for by an effective exchange Hamiltonian

$$H_{ex} = -\sum_{(i,j)} [J_{XY}s_is_j + J_\pm(1 + s_is_j)\sigma_i\sigma_j],$$

where $J_{XY}$ represents the energy a domain wall between the regions where $|\Delta_X| \neq 0$ and $|\Delta_Y| \neq 0$. $J_\pm$ represents a similar quantity where $\Delta_X$ or $\Delta_Y$ changes the sign without changing the direction of the symmetry breaking (hence the factor $(1 + s_is_j)$). We expect $J_{XY}$ (in units of $J_\pm$) to be proportional to $\delta^2$. In terms of a redefined variable $s \rightarrow \tilde{s} = s\sigma$, the rescaled Hamiltonian assumes the form of a two-component isotropic Ashkin-Teller model

$$\tilde{H}_{ex} = -J_{1/2} \sum_{(i,j)} (\tilde{s}_i\tilde{s}_j + \sigma_i\sigma_j) - J_2 \sum_{(i,j)} \tilde{s}_i\tilde{s}_j\sigma_i\sigma_j,$$

where $J_{1/2} = J_{XY}/T > 0$ and $\tilde{H}_{ex} = H_{ex}/T$. The phase diagram of this model is shown in Fig. 3, which we discuss below qualitatively in terms of the original variables $s$ and $\sigma$.

For weak Fermi surface anisotropy, which corresponds to small values of $J_{XY}/J_\pm(\sim \delta^2)$, there exists a transition (across the dashed line in Fig. 3) from a high temperature disordered phase to a low temperature ordered phase. Along this transition, the exponents change continuously, much like for the Kosterlitz-Thouless transition. In the ordered phase, the surface states break both translational (by SDW order) and rotational (by nematic order) symmetries, and the expectation values of the Ising-spin variables in Eq. 10 are $\langle s \rangle \neq 0$ and $\langle \sigma \rangle \neq 0$. This phase is also known as the Baxter phase [38]. However, for large $\delta$ or $J_{XY}/J_\pm$ (strong Fermi surface anisotropy) transitions associated with these two symmetry breakings bifurcate and occur at distinct temperatures. The system first condenses into the nematic order, where $\langle s \rangle \neq 0$ but $\langle s\sigma \rangle = \langle \sigma \rangle = 0$, and only subsequently enters the ordered (Baxter) phase at lower temperature. This nematic phase is ordered along either $Q_X = (\pi, 0)$ or $Q_Y = (0, \pi)$ in such a way that a large density of sign flips (domain wall) of the order parameter proliferate in the system. In this phase $|\langle \Delta_X \rangle|$ or $|\langle \Delta_Y \rangle|$ is non-zero, but $\langle \Delta_X \rangle = \langle \Delta_Y \rangle = 0$. Consequently, the nematic phase breaks the $C_4$ rotational symmetry, yet still retains the translational invariance of the the system. It is worth mentioning that a similar, but distinct, nematic phase has also been studied for iron-based superconductors [36, 40, 41].

V. SUMMARY AND DISCUSSION

In summary, we discuss excitonic and nematic instabilities on the surface of strongly interacting cubic TKIs. We show that if the chemical potential is placed in between the Dirac points at the $\Gamma$ and $X/Y$ points of the surface BZ, fermions can condense into a nematic phase and at lower temperature also into an excitonic density-wave phase. Otherwise, the excitonic phase may reveal spin- or charge-density wave ordering depending on the relative chirality of the Dirac cones with electron and hole like carriers. Our results provide a possible explanation for the recently observed $C_2$ symmetric magnetoresistance [13] and the upturn in surface resistivity with tunable gate voltage or equivalently the chemical potential [21] in SmB$_6$. By taking into account the effects of symmetry breaking terms and thermal fluctuations, we have argued that this system can be described by a two-component isotropic Ashkin-Teller model and presented a finite temperature phase diagram in Fig. 3.

It should be noted that the surface BZ of cubic TKIs closely mimics the one in pnictides [36, 40, 41]. However, there exist various sharp distinctions between these two systems. For example, due to the strong spin-orbit coupling the SDW order of the surface states breaks only the discrete $Z_2$ symmetry (note that the valley $U(1)$ symmetry of SDW order is only an artifact of the low energy approximation in Eqs. 11 [15] which gets reduced to $Z_2$ due to the presence of an underlying lattice), whereas spin-rotation is a good symmetry and the SDW phase breaks continuous $SO(3)$ symmetry in pnictides [10]. In addition, the Fermi surfaces on the surface of TKIs constitute vortices or anti-vortices in momentum space that in turn encode the bulk topological invariance of the system, while the bands in pnictide materials correspond to trivial non-relativistic regular parabolic bands. Despite such subtle but important differences, we find that the structure of the phase diagram in Fig. 3 for the surface states of TKIs is qualitatively similar to the one calcu-
lated theoretically \[36\] and obtained experimentally \[35\] for iron pnictides. The similarity between such different systems is both surprising and encouraging. Therefore, we expect that our study will initiate future work related to TKIs that may unearth some exotic effects due to the presence of strong electronic correlations in these systems and may as well shed light into the phase structure of iron pnictides.

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