Diffraction Patterns in Deeply Virtual Compton Scattering

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Abstract. We report on a calculation to show that the Fourier transform of the Deeply Virtual Compton Scattering (DVCS) amplitude with respect to the skewness variable $\zeta$ at fixed invariant momentum transfer squared $t$ gives results that are analogous to the diffractive scattering of a wave in optics. As a specific example, we utilize the quantum fluctuations of a fermion state at one loop in QED to obtain the behavior of the DVCS amplitude for electron-photon scattering. We then simulate the wavefunctions for a hadron by differentiating the above LFWFs with respect to $M^2$ and study the corresponding DVCS amplitudes in light-front longitudinal space.

Keywords: Deeply Virtual Compton Scattering, Light-front wave functions

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INTRODUCTION

Measurements of Deeply Virtual Compton Scattering (DVCS) cross sections with specific proton and photon polarizations can provide comprehensive probes of the spin as well as spatial structure of the proton at the most fundamental level of QCD. The DVCS process involves off-forward hadronic matrix elements of light-front bilocal currents. The DVCS quark matrix elements can be computed from the off-diagonal overlap of the boost-invariant light-front Fock state wavefunctions (LFWFs) of the target hadron [1,2]. The longitudinal momentum transfer to the target hadron is given by the ‘skewness’ variable $\zeta = Q^2 / 2 \vec{p} \cdot \vec{q}$.

Fourier transform (FT) of the off-forward or generalized parton distributions (GPDs) with respect to the transverse momentum transfer $\Delta^\perp$ at zero skewness and fixed longitudinal momentum fraction $x$ gives the parton distributions in the impact parameter ($b_\perp$) space [3,4]. Also, a 3D picture of the proton has been proposed in [5] in terms of a Wigner distribution for the relativistic quarks and gluons inside the proton.

We introduce a coordinate $b$ conjugate to the momentum transfer $\Delta$ such that $b \cdot \Delta = \frac{1}{2} b^+ \Delta^- + \frac{1}{2} b^- \Delta^+ - b_\perp \cdot \Delta_\perp$. Note that $\frac{1}{2} b^- \Delta^+ = \frac{1}{2} b^- P^+ \zeta = \sigma \zeta$ where we have defined the boost invariant variable $\sigma$ which is an ‘impact parameter’ in the longitudinal coordinate space. The Fourier transform of the DVCS amplitude with respect to $\zeta$ allows one to determine the longitudinal structure of the target hadron in terms of the variable $\sigma$. 
FIGURE 1. Fourier spectrum of the imaginary part of the DVCS amplitude of an electron vs. \( \sigma \) for \( M = 0.51 \text{ MeV}, \ m = 0.5 \text{ MeV}, \ \lambda = 0.02 \text{ MeV} \), (a) when the electron helicity is not flipped; (b) when the helicity is flipped. The parameter \( t \) is in MeV^2.

DVCS AMPLITUDE IN \( \sigma \) SPACE

In order to illustrate our general framework, we will present an explicit calculation of the \( \sigma \) transform of virtual Compton scattering one-loop order [6]. The GPDs in impact parameter space have been calculated in this model [7]. One can generalize this analysis by assigning a mass \( M \) to the external electrons and a different mass \( m \) to the internal electron lines and a mass \( \lambda \) to the internal photon lines with \( M < m + \lambda \) for stability.

The light-front Fock state wavefunctions corresponding to the quantum fluctuations of a physical electron can be systematically evaluated in QED perturbation theory. The state is expanded in Fock space and there are contributions from \( |e^{-}\gamma\rangle \) and \( |e^{-}e^{-}e^{+}\rangle \), in addition to renormalizing the one-electron state at one loop level [1]. We use the handbag approximation. In the domain \( \zeta < x < 1 \), there are diagonal 2 \( \rightarrow \) 2 overlap contributions, [1]. The GPDs \( H_{(2\rightarrow2)}(x,\zeta,t) \) and \( E_{(2\rightarrow2)}(x,\zeta,t) \) are zero in the domain \( \zeta - 1 < x < 0 \), which corresponds to emission and reabsorption of an \( e^{+} \) from a physical electron. The contributions in the domain, \( 0 < x < \zeta \) come from overlaps of three-particle and one-particle LFWFs [1]. A summary of our main results is given in [8] and details of the calculation are given in [9].

The off-forward matrix elements for both 2 \( \rightarrow \) 2 and 3 \( \rightarrow \) 1 overlaps are calculated using the analytic form of the LFWFs. There can be two types of contributions, one in which the helicity of the target electron is not flipped, in this case both the GPDs \( H \) and \( E \) will contribute; in the other contribution the helicity is flipped and only \( E \) contributes. There are real and imaginary parts of the DVCS amplitude, both of which can be accessed in the experiment. The imaginary part receives contribution only when \( x = \zeta \), because of the delta function coming from the propagator in the hard part of the amplitude. The other values of \( x \) contribute to the real part. The GPDs are continuous at \( x = \zeta \), and a principal value prescription is used for the propagator in the real part. However, there are potential divergences coming from \( x \rightarrow 1 \) region as well as small \( x \),
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\textbf{1. Introduction}

This study aims to investigate the properties of particle production in high-energy collisions, specifically focusing on the behavior of the DVCS amplitude. The DVCS (Deeply Virtual Compton Scattering) process is a powerful tool to probe the structure of the nucleon and the underlying parton dynamics.

\textbf{2. Wavefunction Approximation}

The wavefunctions used in this study are approximations of the bound-state wavefunctions and are constructed using the LFWF (Light-Cone Wavefunction) approach. The LFWFs are defined as

\begin{equation}
A_{\lambda,\lambda'}(\sigma, t) = \frac{1}{2\pi} \int_{e_2}^{1-e_2} d\zeta \, e^{i\sigma\zeta} M_{\lambda,\lambda'}(\zeta, \Delta_\perp);
\end{equation}

where $M_{\lambda,\lambda'}$ is the DVCS amplitude, $\lambda$ ($\lambda'$) are the helicities of the initial (final) electron. $\sigma = \frac{1}{2} p^+ b^-$ is the (boost invariant) longitudinal distance on the light-cone and the FTs are performed at a fixed invariant momentum transfer squared $-t$. We have imposed cutoffs $e_2 = 0.002$ for the numerical calculation. Fourier transforms have been performed by numerically calculating the Fourier sine and cosine transforms and then calculating the resultant by squaring them, adding and taking the square root, thereby yielding the Fourier Spectrum (FS).

In Fig. 1 and 2 respectively, we have shown the FS of the imaginary and real parts of the DVCS amplitude. Apart from the imaginary part of the helicity-flip amplitude, they show a diffraction pattern in $\sigma$. The helicity non-flip part depends on the scale $Q$, which we took to be 10 MeV.

In the 2- and 3-body LFWFs, the bound-state mass squared $M^2$ appears in the denominator. Differentiation of the LFWFs with respect to $M^2$ increases the fall-off of the wavefunctions near the end points $x = 0, 1$ and mimics the hadronic wavefunctions. One has to note that differentiation of the single particle wave function yields zero and thus there is no $3 - 1$ overlap contribution to the DVCS amplitude in this hadron model. Also, the imaginary part of the amplitude vanishes in this model. We show the amplitudes in Fig. 3. We have also computed the DVCS amplitude and the corresponding diffraction pattern in $\sigma$ using the AdS/QCD framework [9]. This model provides a useful first approximation to hadron wavefunctions which has confinement at large distances and conformal behavior at short distances [10].
From the plots, we propose an optics analog of the behavior of the Fourier Spectrum of the DVCS amplitude. The diffractive patterns in $\sigma$ sharpen and the positions of the first minima typically move in with increasing momentum transfer. For fixed $-t$, higher minima appear at positions which are integral multiples of the lowest minimum, similar to the diffraction pattern in optics. Thus the invariant longitudinal size of the parton distribution becomes longer and the shape of the conjugate light-cone momentum distribution becomes narrower with increasing $|t|$. If one Fourier transforms in $\zeta$ at fixed $\Delta_{\perp}$ and then Fourier transforms the change in transverse momentum $\Delta_{\perp}$ to impact space $b_{\perp}$ [3], then one would have the analog of a three-dimensional scattering center. In this sense, scattering photons in DVCS provides the complete Lorentz-invariant light front coordinate space structure of a hadron.

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