Tunable Luttinger liquid physics in biased bilayer graphene

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Electronically gated bilayer graphene behaves as a tunable gap semiconductor under a uniform interlayer bias $V_g$. Imposing a spatially varying bias, which changes polarity from $-V_g$ to $+V_g$, leads to one dimensional (1D) chiral modes localized along the domain wall of the bias. Due to the broad transverse spread of their low-energy wavefunctions, we find that the dominant interaction between these 1D electrons is the forward scattering part of the Coulomb repulsion. Incorporating these interactions and the gate voltage dependence of the dispersion and wavefunctions, we find that these 1D modes behave as a strongly interacting Tomonaga-Luttinger liquid with three distinct mode velocities and a bias dependent Luttinger parameter, and discuss its experimental signatures.

Graphene has been the focus of intense research in recent years [1] due to the rich physics of its massless Dirac fermions. Theoretical proposals have shown that spatially modulating an applied gate voltage in monolayer graphene leads to tunable anisotropic transport properties [2]. For bilayer graphene (BLG), it was experimentally demonstrated that applying an electric field perpendicular to the layers, by sandwiching it between two gates, induces a gap in its electronic spectrum that is of the order of the applied interlayer bias $[3,4]$. Biased BLG thus behaves as a semiconductor with a tunable band gap. Such tunability of electronic properties opens up possibilities for graphene based devices [5].

Recently, Martin, Blanter and Morpurgo [7] have proposed a setup where two neighbouring regions of BLG are biased with opposite parity, shown schematically in Fig. 1(a). In this geometry, gapless one dimensional (1D) modes were shown to emerge at the interface where the bias reverses sign (see also Ref. [8]). These modes are analogous to domain wall fermions studied in the context of polyacetylene [9], charge density waves in graphene [10], and field theories in high-energy physics [11]. These modes in BLG may also be viewed as switchable nanowires which can be turned on and off using different gate voltages for a given gate configuration.

In this Letter, we study the effects of interactions on the low-energy modes of a single such wire in the original setup proposed in Ref. [7]. We find that the low-energy wavefunctions of the 1D modes have a broad spread in the direction transverse to the interface — this leads to the dominance of the forward scattering part of the Coulomb interaction between electrons, in a manner akin to large radius carbon nanotubes [12]. Within an abelian bosonization framework, incorporating these forward scattering terms is shown to lead to a strongly interacting Tomonaga-Luttinger liquid [14]. Remarkably, we find the Luttinger parameter of this liquid is tunable by adjusting the gate potential. This results from two competing effects: (i) An increased bias causes further confinement of the wavefunctions to the interface, enhancing the effect of interactions; (ii) An increase in the bias increases the Fermi velocity of the low-energy modes, suppressing the effect of interactions relative to the kinetic energy. The net result is that the Luttinger parameter in the total charge channel, $K_{c+}$, can be varied between 0.15-0.2 by increasing the bias over an experimentally accessible range. At the same time the Luttinger parameter in the transverse charge channel, $K_{c-} \approx 0.63$, is relatively independent of the bias. We thus show that gated BLG can realize a tunable Tomonaga-Luttinger liquid. Such band structure and wavefunction tuning of Luttinger liquids has been suggested in a few other systems recently — in cold atomic gases [13, 14], in magnetic waveguides in graphene [15, 16], in carbon nanotubes in crossed electric and magnetic fields [18, 19], and in gated topological insulators [20]. Such a Luttinger liquid with dominant forward scattering is also expected to arise at charge density wave (CDW) domain walls in single layer graphene, where the CDW involves a weak sublattice density modulation induced by an appropriate substrate [10].

Non-Interacting System. — BLG consists of two Bernal stacked graphene layers as depicted in Fig. 1(b). We label the carbon atoms in the bilayer by a unit cell index $i$, a sub-lattice index $s = a, b$, and a layer index $\ell = 1, 2$ labelling top and bottom layers respectively. The distance between neighboring carbon atoms in the same

![FIG. 1: (a) Schematic diagram of external gates. $V_T + V_B$ controls the bilayer doping while $V_T - V_B$ controls the gap via the depicted perpendicular electric field. (b) Structure of bilayer graphene with a ‘bias kink’ in $V_{g} = V_{T} - V_{B}$ at $y = 0$.](image)
layer and on the same sublattice is $d \approx 2.46\text{Å}$, while the interlayer distance is $d_{\perp} \approx 3.34\text{Å}$. The minimal tight-binding model for electrons in BLG consists of a nearest-neighbor hopping amplitude $t \approx 3eV$ within each layer, and an interlayer hopping amplitude $t_{\perp} \approx 0.15eV$ between sites $(i, s=a, \ell=1)$ and $(i, s=b, \ell=2)$. Henceforth, we use units where $\hbar = 1$ for convenience, we set the interlayer distance at the mid-gap. For later convenience, we set $V_n, s, \ell = \sqrt{\mu_n}$ which is determined by the Fermi velocity of the two bands ($V_F(y) = \sqrt{\mu_n}$), are a consequence of the symmetry that relates the two bands by an inversion about the $K (-K)$ point at low energies.

Effective 1D Hamiltonian. — To derive the effective low-energy 1D Hamiltonian, we assume a suitable energy cutoff that is smaller than the bulk gap and focus on those single particle states that lie within this energy window and are confined to the 1D interface region. To do this, we first expand the field operators in the complete basis, $\hat{\Psi}_{R, \sigma} = \sum_{k, \alpha} e^{i k n a} \varphi_{\alpha}^0(r) \psi_{k, \sigma}$, where $\varphi_{\alpha}^0(r)$ is the wavefunction of the state in band-$\alpha$ with momentum $k$.

We then restrict the bands to the set $\alpha = \{0, \pi\}$ and consider only momenta in the vicinity of the four Fermi points, $\pm k_F^x$ and $\pm k_F^y$. An additional simplification is made by neglecting the small momentum dependence of the wavefunctions since $\varphi_{\alpha, k_F^x + q}^0(r) \approx \varphi_{R/L}^\alpha(r)$ for small momenta $q$. Here, $\varphi_{R/L}^\alpha(r)$ is the zero-energy wavefunction at $\pm k_F^x$. In doing so, the $r$-dependence of the wavefunction can be separated to yield the low-energy field operators projected to the 1D subspace via

$$\hat{\Psi}_{R, \sigma} \approx \sum_{r=\pm \alpha = \{0, \pi\}} \varphi_{\alpha}^0(r) e^{i k_F^x r} \psi_{r, \sigma}(x),$$

where $r$ is the label R/L for left/right movers and takes the values $+/-$ in the expression, and $\psi_{r, \sigma}(x)$ are slowly varying field operators exclusively dependent on the position along the interface (which we now denote by the continuous variable $x = nd$).

We now rewrite the entire Hamiltonian in terms of operators in the reduced 1D subspace. The free part is simply linearized to give $\sum_{q} \varepsilon_q \psi_{R, \sigma}^d(q) \psi_{r, \sigma}^d(q)$. The effective interaction between fermions in the 1D channel is obtained by a straightforward substitution of Eqn. (4) into the Coulomb term, $\frac{1}{2} \sum_{\sigma, \sigma'} \sum_{RR'} \Psi^d_{R, \sigma} \Sigma^d_{R', \sigma'} U(R, R') \Psi_{R', \sigma'} \Psi_{R, \sigma}$, fol-
followed by a summation over \( r \). This gives rise to various
scattering terms, many of which are rapidly oscillating
and can be dropped. The effective Hamiltonian obtained
contains many terms of the general form
\[
\frac{V_{ij}^{(i)}}{2} \sum_\sigma \hat{\psi}_{\sigma r}^\dagger(x) \hat{\psi}_{\sigma r}^\dagger(x) \hat{\psi}_{\sigma r}^\dagger(x) \hat{\psi}_{\sigma r}^\dagger(x). (2)
\]
Here, \( V_{ij}^{(i)} \equiv V_{ij}^{(i)}(r) \) is the Fourier component of the effective 1D potential
\[
\hat{V}_{ij}^{(i)}(x-x')=\sum_{r,r'} U(r, r') \phi_{\sigma r}^\dagger(r') \phi_{\sigma r}^\dagger(r').
\]
The effective interaction Hamiltonian contains all terms of the form in Eqn. (2) that have a combination of \( R/L \)
and band indices that conserve (crystal) momentum. The index \( i \) classifies the scattering processes using standard
g-ology notation \( \{(22,22)\} \) (see Fig. (1)): \( i = 1 \) refers
to backscattering, \( i = 2 \) to forward-scattering involving
both right and left movers, and \( i = 4 \) to forward-scattering
involving only right or only left movers. The number of distinct processes is greatly reduced by the
fact that all interband scattering terms with parallel spin
merely renormalize the coefficients of a corresponding in-
terband term with parallel spin.

This Hamiltonian is qualitatively similar to that ob-
tained for Hubbard ladders \( \{(22,22)\} \) with two significant
differences. First, both the wavefunctions and \( V_F \) are
sensitive to changes in the applied gate bias. The mod-
ified wavefunctions alter the distance dependence of the
effective Coulomb interaction, while the change in \( V_F \)
adjusts the relative interaction strength parameterized by
the fine-structure constant \( \alpha \rightarrow \alpha \frac{\Phi}{V_F} \). Second, we note
that the long-range nature of the Coulomb interactions,
together with the large spread of the low-energy wave-
functions, causes the small momentum forward scattering
processes to dominate. This is reminiscent of large ra-
dius single wall carbon nanotubes, where the extension of the wavefunctions around the tube radius suppresses the
bare backscattering \( \{(14)\} \). We have checked that the bare
values of these backscattering and interband scattering
terms are very small, consistent with this argument. For
instance, \( V_{0000}^{(1)}/V_{0000}^{(2)} \sim 10^{-3} \) and \( V_{0000}^{(4)}/V_{0000}^{(2)} \sim 10^{-2} \) at
\( V_g = 0.02t \), so that such processes are expected to be
important only at very low energy and temperature. We
therefore first focus on the forward scattering processes.

**Bosonization.** Using the standard abelian bosoniza-
tion procedure \( \{(14)\} \), we introduce the bosonic field \( \phi_{\alpha r}(x) \)
and the phase \( \theta_{\alpha r}(x) \) whose spatial derivative \( \partial_x \theta_{\alpha r} = \Pi_{\alpha r} \)
is conjugate to \( \phi_{\alpha r}(x) \). Fermion operators can be
represented in terms of these boson fields via \( \psi_{\alpha r}(x) \sim e^{i(\phi_{\alpha r}(x)-\theta_{\alpha r}(x))} \). It is a simple matter to rewrite the
density-density interactions in the boson representation
by means of the relations \( \partial_x \phi_{\alpha r} = -\pi (\hat{\rho}_{\alpha r} + \hat{\rho}_{L\alpha r}) \) and
\( \partial_x \theta_{\alpha r} = \pi (\hat{\rho}_{\alpha r} - \hat{\rho}_{L\alpha r}) \). In addition, the symmetry
between the bands allows us to lighten our notation by defining
\( V_A \equiv V_{\alpha \alpha \alpha \alpha}^{(4)} = V_{\alpha \alpha \alpha \alpha}^{(4)} \)
and \( V_B \equiv V_{\alpha \alpha \alpha \alpha}^{(2)} = V_{\alpha \alpha \alpha \alpha}^{(4)} \).

This leads to the Hamiltonian,
\[
H_1 = \frac{1}{2\pi} \int dx (\partial_x \phi)^T \hat{K}^{-1} (\partial_x \phi) + (\partial_x \theta)^T \hat{K} (\partial_x \theta),
\]
with
\[
\hat{K}^{-1} = V_F \mathbf{1} + \frac{V_F}{2\pi} \begin{pmatrix} g_A & g_B & g_A & g_B \\
 g_B & g_A & g_B & g_A \\
g_B & g_A & g_B & g_A \\
g_A & g_B & g_B & g_A \end{pmatrix}
\]
\[
\hat{K} = V_F \mathbf{1}.
\]
Here \( g_A/B \equiv (2V_A/B)/V_F \), and \( \Phi = (\phi_{\alpha \uparrow}, \phi_{\alpha \downarrow}, \phi_{\alpha \downarrow}, \phi_{\alpha \downarrow})^T \)
with a similar definition for \( \Theta \).

This Hamiltonian is diagonal in the total/transverse
density basis defined via \( \phi_{\alpha \pm} = \phi_{\alpha \uparrow} \pm \phi_{\alpha \downarrow}, \phi_{\alpha \mp} \phi_{\alpha \uparrow} \pm \phi_{\alpha \downarrow} \).
In this basis, the spin and charge sectors decouple. The spin modes are unaffected by interactions, the Luttinger
parameters \( K_{\pm} = 1 \) and the velocities \( u_{\pm} = V_F \). The
charge modes have renormalized velocities and nontrivial
Luttinger parameters, given by
\[
K_{\pm} = (1 + y_{\pm})^{-\frac{1}{2}},
\]
where \( y_{\pm} = 2(V_A \pm V_B)/\pi V_F \). At the Gaussian level, the
only effect of the interactions is thus to strongly modify
\( K_{\pm} \), \( u_{\pm} \). Fig. (3) shows these parameters plotted for
various gate voltages. As seen from the figure, \( K_{\pm} \) can
be tuned significantly by the external bias; by contrast,
\( K_{\pm} \approx 0.63 \) (not shown) is relatively bias independent.

**Observable consequences.** — The strong interactions
in the charge channel lead to three different velocities for
the spin \((\hat{V}_F)\) and charge \(u_{\pm}\) modes in the Luttinger
liquid. Mapping out these dispersing modes, as has been
done in semiconductor heterostructures \( \{(24)\} \), appears
to be challenging in the biased BLG system. A more accessi-
ble signature of the Luttinger liquid physics is the energy
dependence of the single particle density of states (DOS).
We expect \( n(\epsilon) \sim \epsilon^\alpha \), with \( \alpha > 0 \). Such a suppression
of the DOS is expected to lead to a tunneling conduc-
tance \( G \sim T^\alpha \) (for voltages \( eV \ll k_B T \) or a nonlinear
differential conductance \( dI/dV \sim V^\alpha \) (for \( eV \gg k_B T \)). We find \( \alpha_{\text{bulk}} = \frac{1}{2}(K_{c+}^{-1} + K_{c-}^{-1} + K_{c+}^{-1} + K_{c-}^{-1} - 4) \) and \( \alpha_{\text{edge}} = \frac{1}{2}(K_{c+}^{-1} + K_{c-}^{-1} - 2) \), so that bias dependent tunneling exponents are expected to be observed. Various charge density, spin density and superconducting tunneling exponents are expected to be observed in this intermediate energy Luttinger liquid regime. We find that charge and spin density wave operators at \( 2k_F^0 \) and \( 2k_F^\perp \) are most strongly enhanced by interactions in this regime, decaying along the interface as \(|x-x'|^{-2(k_{F+}+k_{F-})}/2\), with the precise lattice scale modulation pattern in \( (n,s,t) \) being determined by the prefactors set by the wavefunctions \( \varphi_n^a(r)/R_n \) from Eq.\([\text{1}]\).

Modulations at \( k_F^0 \pm k_F^\perp \) are subdominant.

**Discussion.** — We have shown that BLG in a suitable gate geometry can realize a tunable Luttinger liquid with four gapless modes. This is expected to break down once backscattering and interband scattering terms become important; since the bare values of these interactions are small and that they are all marginal, and thus flow slowly, there is an intermediate energy window where the physics discussed above should be observable. At very low energy (or temperature), however, we expect that such processes will gap out all sectors except the \( c+ \) channel, which should still exhibit Luttinger liquid physics. Turning to the effect of additional interlayer hopping terms \( (\gamma_3) \) between the \( a2 \) and \( b1 \) sites, we have checked that this renders \( V_F^0 \neq V_F^\perp \). This asymmetry is small for moderate bias voltages; further, interband scattering tends to equalize the velocities \([22, 27]\) so that small velocity asymmetries are expected to be unimportant. We have assumed that the Fermi level is tuned, via \( V_T + V_B \), to be precisely in the middle of the gap - small deviations which tend to slightly dope the interface states while leaving the bulk gap intact will not qualitatively alter the physics discussed here. While the ‘domain wall’ modes discussed here are topologically protected \([7]\) independent of the precise bias profile transverse to the wire, disorder along the wire direction will lead to backscattering. Backscattering is somewhat mitigated by the wavefunction spread, but we expect \([1] \) will lead to insulating behavior at very low energy \([30]\).

Fourier transform scanning tunneling spectroscopy would then be useful to uncover the underlying Luttinger liquid physics \([31]\).

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