Photonic scheme for implementing quantum square root controlled Z gate using phase and intensity encoding of light

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Abstract
Quantum logic gate operates on a number of qubits, where a controlled Z gate operates on two qubit data. Similarly, the square root of controlled Z (SRCZ) operates also on two qubit data. The SRCZ gate is very much advantageous in quantum computing. Different quantum gates are implemented by using the physical properties of light such as phase, polarization, intensity, and frequency. Here, the authors propose a photonic scheme for implementing quantum SRCZ logic gate using phase as well as intensity encoding. The non-linear property of Pockels material with proper external biasing voltage is used to develop the scheme.

1 | INTRODUCTION
Quantum logic works on Boolean observable-based data obeying the laws of quantum mechanics. In quantum computing, the quantum logic gate operates on qubits instead of bits. The qubits are the building blocks of a quantum computer. All-optical quantum systems offer very fast and secured computation, communication and data processing due to its large storage capacity, fast response time, high bit error rate, low loss, inherent parallelism etc., and the use of quantum nature of data.

Quantum computation generally requires single-qubit gate, two-qubit gate as well as multi-qubit gate. A single-qubit system basically offers the operations based on two states either |0⟩ or |1⟩. In general, the state of a qubit in quantum mechanics is the superposition of the both two states. The |0⟩ state is represented by a 2 × 1 matrix, expressed as \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \), and |1⟩ state is represented as \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \). The gate matrix of a single-qubit system offers a 2 × 2 unitary matrix. In a two-qubit system, the gate matrix is represented by 4 × 4 unitary matrix. In a two-qubit system, there are four quantum logic states. They are represented by 4 × 1 column matrices such as, |00⟩ = \( \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \), |01⟩ = \( \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \), |10⟩ = \( \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \) and |11⟩ = \( \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \).

Quantum logic gate, whether it is in a single- or a two-qubit system, is always reversible.

Controlled Z (CZ) gate acts on two qubit data, where the first qubit acts as control input and the second qubit acts as target input. The matrix representation of CZ gate is,

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}.
\]

A light polarized with a particular direction when propagates through a non-linear Pockels medium like lithium niobate (LNBO), potassium dihydrogen phosphate (KDP) etc., the refractive index of the material is changed with external biasing signal. The change in refractive index is linearly proportional to the applied electric field. This effect is called Pockels effect and the cell is called Pockels cell. Using this property of the material, different types of optical switches are generated.

Quantum computing depends upon universal set of reversible quantum gates [1,2]. All-optical square roots of Pauli X gate and Pauli Z gate are already implemented using phase and polarization encoding by Dey et al. [3,4]. The quantum C-NOT gates are implemented in different ways by using polarization encoding mechanism, photons and controlled-phase gate of electron spins via quantum dot microcavity coupled system, path encoding mechanism, frequency encoding and so on [5–9]. Depending upon the tremendous computational power and high speed, so many quantum single-qubit and two-qubit logic gates such as CZ gate, quantum
controlled phase gate, efficient Z gate are developed using the optical properties of some materials [10–16]. All-optical integrated Pauli X, Y and Z gates with optical switches are implemented by Sarkar et al. [17]. All-optical photonic devices are developed using electro-optic modulator [18,19]. All-optical Pauli Y gate is implemented by using integrated phase and polarization encoding [20]. Optical quantum computing is enriched with the implementation of various quantum logic gates in different encoding mechanism [21–28]. To attain any feasible quantum computing, a unique three-qubit quantum gate called Deutsch gate, Toffoli gate, two input CNOT gate are developed by Shi [29]. Also, so many all-optical quantum logic gates such as hyper-parallel Toffoli gate, linear optical quantum Toffoli gate, hybrid Fredkin gate are also implemented [30–32]. There are so many logic gates developed using the cross Kerr non-linearity. Deterministic CNOT gate is also developed using single photon source [33]. An architecture is established for implementation of deterministic quantum circuits operating on single-photon qubits [34]. The realization of quantum control gates including the C-NOT gate, Fredkin gate, Toffoli gate, arbitary controlled U gate, arbitrary multi-controlled U gates are proposed [35]. Universal entangler is established with photon pairs in arbitrary states using a parity gate with weak cross-Kerr non-linearity [36]. Polarization multi-photon controlled one-photon unitary gate is developed assisted by spatial and temporal degrees of freedom [37]. Construction of a nearly deterministic polarization Toffoli gate, single-photon controlled mult-photon unitary gate, two-photon polarization controlled arbitrary phase gate are also established using the weak cross Kerr non-linearities [38–40].

The author proposes a photonic scheme for implementing quantum Square Root Controlled Z (SRCZ) logic gate using phase as well as intensity encoding of light jointly. The SRCZ gate when acts twice on a two-qubit input, it gives the same operation with the CZ gate operate once on the same input. To realize SRCZ gate, one can use the phase change of light as per the applied biasing potential in Pockels cell.

2 | DIFFERENT COMBINATIONS OF QUANTUM OPTICAL SRCZ GATE

The gate matrix of SRCZ gate is assumed to be a $4 \times 4$ unitary matrix, which satisfies the following relation. $\text{SRCZ} \times \text{SRCZ} = \text{CZ} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}$.

By realizing the above criteria, 16 different matrices can be obtained, where each of them follows SRCZ operation. The 16 SRCZ matrices are

$$
\text{SRCZ 1} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & i
\end{pmatrix}
$$

$$
\text{SRCZ 2} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
$$

$$
\text{SRCZ 3} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & i
\end{pmatrix}
$$

$$
\text{SRCZ 4} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
$$

$$
\text{SRCZ 5} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & i
\end{pmatrix}
$$

$$
\text{SRCZ 6} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
$$

$$
\text{SRCZ 7} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & i
\end{pmatrix}
$$

$$
\text{SRCZ 8} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
$$

$$
\text{SRCZ 9} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & i
\end{pmatrix}
$$

$$
\text{SRCZ 10} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
$$

$$
\text{SRCZ 11} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & i
\end{pmatrix}
$$

$$
\text{SRCZ 12} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
$$

$$
\text{SRCZ 13} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & i
\end{pmatrix}
$$

$$
\text{SRCZ 14} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
$$

$$
\text{SRCZ 15} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & i
\end{pmatrix}
$$

$$
\text{SRCZ 16} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
$$
2.1 SRCZ 14 = \[
\begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
\]

SRCZ 15 = \[
\begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & i
\end{pmatrix}
\]

SRCZ 16 = \[
\begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
\]

These SRCZ matrices operate on different qubit nature of inputs. To implement the SRCZ quantum gates, the required logic states are ‘0’, ‘1’, ‘-1’, ‘+i’ and ‘-i’. As complex bits and sign bits both are there, therefore, we have taken the advantage of using phase encoding and intensity encoding jointly to represent the qubits. After the logic operation, the qubit values may be 0, 1, -1, +i and -i. For that reason, it is seen that if the each input bit of qubits is replaced by two qubits (i.e. 4 × 1 input qubit matrix representation is changed to 4 × 2 matrix representation) the implementation becomes easier and suitable for conducting the gate operation. So the 4 × 4 gate matrices and two-qubit input systems are preferred. Different combinations of input are now possible to represent easily by taking 4 × 4 gate matrices and 4 × 2 input qubit matrices. Four different two-qubit inputs are taken here.

They are \[
\begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1
\end{pmatrix}, \begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 1 \\
1 & 0
\end{pmatrix}, \begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 0
\end{pmatrix}
\]

2.2 SRCZ 4 gate operation

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}\begin{pmatrix}
0 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0
\end{pmatrix} = \begin{pmatrix}
0 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0
\end{pmatrix}
\]

2.3 SRCZ 13 gate operation

\[
\begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & i
\end{pmatrix}\begin{pmatrix}
0 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0
\end{pmatrix} = \begin{pmatrix}
0 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 1
\end{pmatrix}
\]
2.4 | SRCZ 16 gate operation

\[
\begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1
\end{pmatrix}
= 
\begin{pmatrix}
0 & 0 \\
0 & -1 \\
-1 & 0 \\
-i & -i
\end{pmatrix}
\]

\[
\begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1
\end{pmatrix}
= 
\begin{pmatrix}
0 & 0 \\
0 & -1 \\
-1 & 0 \\
-i & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -i
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1
\end{pmatrix}
= 
\begin{pmatrix}
0 & -1 \\
-1 & 0 \\
-1 & 1 \\
0 & 0
\end{pmatrix}
\]

3 | OPTICAL IMPLEMENTATION OF SRCZ GATE USING ELECTRO-OPTIC Pockels CELL

Different optical properties of light wave such as intensity, frequency, phase, polarization etc., are used to encode the qubits in the quantum optical computation. Eight paths are used as input to implement each SRCZ quantum gate. Eight different polarized light rays are passing differently through each input path. No ray interferes with any other. Quantum SRCZ gate consists real positive and negative qubits as well as complex positive and negative qubits. So to encode the qubits, phase and intensity of light are jointly used. The qubit ‘0’ is encoded as no light, ‘1’ state is encoded as light with initial zero phase difference, ‘i’ state is encoded as light with a \(\pi/2\) phase lead and ‘−i’ state is encoded as light with a \(\pi/2\) phase lag. ‘−1’ state is encoded as light with \(\pi\) phase difference.

To implement the quantum SRCZ gate, both phase and intensity encoding principles are used, where LiNbO\(_3\)-based Pockels cells act as optical switch. The Pockels cells are operated with external biasing voltages \(V_{\pm\pi/2}\) and \(V_\pi\). Thus, the cells offer \(\pi/2\) phase lead or lag and \(\pi\) phase lead, respectively, when light propagates through it.

The schematic diagram of SRCZ 1 gate is shown in Figure 1. Here, the first, second and third pairs of input terminals are directly connected to the first, second and third pairs of output terminals, respectively. A LiNbO\(_3\)-based Pockels cell is placed between the fourth pair of input and output terminals. This causes \(\pi/2\) phase lead at the output terminals of the fourth pair with respect to the fourth pair of input terminals. An external biasing voltage \(V_{\pi/2}\) is applied to the Pockels cell. Here, \(q_{01}\), \(q_{00}\) are the input qubits and \(q_{11}, q_{10}\) are the output qubits.

The schematic diagram of quantum SRCZ 2 gate is shown in Figure 2. The first, second and third pairs of input terminals are directly connected to the first, second and third pairs of the output terminals, respectively. A LiNbO\(_3\)-based Pockels cell is used between the fourth pair of input and output terminals. An external biasing voltage \(V_{-\pi/2}\) is applied to the Pockels cell which makes a phase lag of \(\pi/2\) at the output terminals with respect to the input terminals.

To implement SRCZ 3 quantum gate, two Pockels cells are needed. One Pockels cell is used between the third pair of input and output terminals. An external biasing voltage \(V_\pi\) is applied to it. The Pockels cell changes the phase of the input light signal at the output with an amount \(\pi\). Now, another Pockels cell is applied between the fourth pair of input and output terminals with an external biasing voltage \(V_{\pi/2}\). For the light rays of the fourth pair, \(\pi/2\) phase lead is received at the output with respect to input. The first and second pair of input qubits are directly connected to the first and second pair of
output qubits, respectively. The schematic diagram of SRCZ 3 gate is shown in Figure 3.

Two Pockels cells are used in the implementation of SRCZ 4 quantum gate. One is used between the third pair of input and output terminals with an applied biasing voltage $V_x$. It makes a phase change $\pi$ at the third pair of output light with respect to the third pair of input. The other Pockels cell is used between the fourth pair of input and output terminals. An external biasing voltage $V_{-\pi/2}$ is applied to the Pockels cell. It lags the output signal with phase $\pi/2$ with respect to the input signal. The first and second pairs of input terminals are directly connected to the first and second pair of output terminals, respectively. The schematic diagram of SRCZ 4 gate is shown in Figure 4.

The operation of SRCZ 5 gate is shown in Figure 5. Here, two LiNbO$_3$-based Pockels cells are used. One Pockels cell is placed between the second pairs of input and output terminals. An external biasing voltage $V_x$ is applied to this Pockels cell. Due to this applied biasing voltage, a phase difference occurs which leads the output signal with $\pi$ phase with respect to the input signal. Another Pockels cell is used between the fourth pair of input and output terminals. The applied biasing voltage to the Pockels cell is $V_{\pi/2}$ which leads the output signal with a phase $\pi/2$ with respect to the input signal.

The first and third pairs of the input terminals are directly connected to the first and third pairs of output terminals, respectively.

To implement the SRCZ 6 quantum gate, also two Pockels cells are required. One is placed between the second pair of input and output terminals with a biasing voltage $V_x$. This Pockels cell offers a phase lead of $\pi$ to the output signal in reference to the input signal. The other one is placed between the fourth pair of input and output terminals with a biasing voltage $V_{-\pi/2}$. This offers a phase lag of $\pi/2$ to the output signal in reference to the input signal. The first and third pairs of input terminals are directly connected to the first and third pairs of output terminals, respectively. The schematic diagram of SRCZ 6 gate is shown in Figure 6.

Three Pockels cells are required to implement the quantum SRCZ 7 gate. First pair of input terminals is directly connected to the first pair of output terminals. Two Pockels cells with biasing voltage $V_x$ are connected between second pair of input and output terminals and third pair of input and output terminals.

As both offer a $\pi$ phase change to the output signal with respect to input signal, the use of two different Pockels
cell can be replaced by a single one. The third one is connected between the fourth pair of input and output terminals with an external biasing voltage $V_{x/2}$. This leads to the output signal with phase $\pi/2$ compared with the input signal. The schematic diagram of SRZ 6 quantum gate is shown in Figure 7.

SRZ 8 quantum gate is implemented by using three Pockels cell. First pair of input terminals is directly connected to the first pair of output terminals. Two Pockels cell with external biasing voltage $V_x$ are kept between the second pair of input and output terminals and between the third pair of input and output terminals. The third one is placed between the fourth pair of input and output terminal with external biasing voltage $V_{x/2}$. This lags the output signal with phase $\pi/2$ in comparison to the input signal. The schematic diagram of SRZ 8 gate is shown in Figure 8.

Here, also one can use single Pockels cell instead of using two in the second and third pairs of terminals as both uses electric triggering signal $V_{\pi}$.

### 4 | Biasing Voltage Required for SRZ Gate Operation

The refractive index of a LiNbO$_3$ crystal (when a light polarized in a particular direction $[X]$ normal to the optic axis [propagation direction $Z$] is passing through the crystal and a biasing voltage $V_{z}$ is applied parallel to $Z$ direction to the crystal) is given by: $n_{\pi} = n_0 - \frac{1}{2}\beta_0 r_{13} E_{2z}$, where $n_0$ is the constant refractive index term of the LiNbO$_3$ crystal and $r_{13}$ is the electro-optic coefficient of the crystal.

When the light wave propagates along the $Y$ direction (which is normal to both $X$ and $Z$ directions), then the expression of the electric field of the light at the output is

$$E_{\text{output}} = E_0 \exp \left[ i \left( \omega t - k_0 n_0 l + \frac{1}{2} k_0 n_0 r_{13} V_x \left( \frac{l}{d} \right) \right) \right]$$

where `$l$' and `$d$' are the dimensions of the crystal along $Z$ and $Y$ directions, respectively.

So, the phase difference between the input and output is

$$\Phi = \frac{1}{2} k_0 n_0 r_{13} V_x \left( \frac{l}{d} \right)$$. The voltage required to attain $\pi$ phase difference is $V_{\pi} = \frac{\lambda d}{2 n_0 r_{13}}$. To receive a $\pi/2$ phase difference, the required biasing voltage is $V_{\pi/2} = \frac{\lambda d}{4 n_0 r_{13}}$.

Using the value of the electro-optic (LiNbO$_3$) coefficient, $r_{13} = 8.6 \times 10^{-12}$ V/m and $\lambda_0 = 633$ nm, $l = 10$ cm, $d = 2$ mm, $n_0 = 2.286$, and the value of $V_{\pi}$ and $V_{\pi/2}$ is obtained as $V_{\pi} = 123.18$ V, $V_{\pi/2} = 61.59$ V. These voltages are not very high and easily usable.
5 | SRCZ-BASED CZ OPERATION

The CZ matrix when operates on any of the two qubit inputs
\[
\begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
= \begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
\]
become
\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
= \begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
\]
\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
= \begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
\]

When SRCZ gate operates on the two-qubit input twice then one can get CZ operation. In case of SRCZ 1 gate, the operations are shown as follows:
\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & i \\
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
= \begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
\]

Now if the output is used as the input again, it gives
\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & i \\
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
= \begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
\]
Here, it is clearly seen that the SRCZ 1 gate is not reversible.

The output is used as the input of SRCZ 1 gate, and the operation is as follows:
\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & i \\
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
= \begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1 \\
\end{pmatrix}
\]

It is seen that the gate is not reversible and the output is the same as that obtained after single time operation on CZ gate:
\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & i \\
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
1 & 0 \\
1 & 1 \\
0 & i \\
\end{pmatrix}
= \begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 1 \\
0 & 0 \\
\end{pmatrix}
\]

The output is taken as the input of the SRCZ 1 gate and the operation is as follows:
\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & i \\
\end{pmatrix}
\begin{pmatrix}
0 & 0 \\
1 & 0 \\
1 & 1 \\
0 & i \\
\end{pmatrix}
= \begin{pmatrix}
0 & 0 \\
0 & 1 \\
1 & 1 \\
0 & 0 \\
\end{pmatrix}
\]

Two SRCZ 1 gates are connected in series as shown in Figure 9. In case of SRCZ 1 gate, the first three pairs of input terminals are directly connected to the first three pairs of output terminals, respectively. Only a phase change \(\pi/2\) occurs at the fourth pair of output terminals. So, when two SRCZ 1 gates are connected serially, then a phase change \(\pi\) occurs at the fourth pairs of output terminals of the second SRCZ 1 gate. So basically, the final output obtained here is the same as the output of a CZ operation on the input once.

The same nature operation is happened for SRCZ 2 to SRCZ 16 quantum gates.

6 | RESULT AND DISCUSSION

The gate matrices of quantum SRCZ gate are first developed. There are 16 possible SRCZ gates received from the fundamental mathematical analysis. The possible gates are SRCZ 1 to SRCZ 16. The SRCZ quantum gates are successfully implemented by using the intensity and phase encoding jointly. To encode the qubits by the proper phase change of light electro-optic Pockels cells are used here. The Pockels cells offer different phase changes with different external biasing signals. To implement the different SRCZ gate, the arrangement of Pockels cells is
different. SRCZ 1 and SRCZ 2 quantum gate is implemented by using a single Pockels cell at the fourth pair of the qubit. Each of SRCZ 3, SRCZ 4, SRCZ 5, SRCZ 6, SRCZ 9 and SRCZ 10 quantum gates are implemented by using two Pockels cells. SRCZ 7, SRCZ 8, SRCZ 11, SRCZ 12, SRCZ 13 and SRCZ 14 quantum gates are implemented by using three Pockels cell between different pairs of the input and output terminals. There are four Pockels cells required to implement the SRCZ 15 and SRCZ 16 quantum gates, but the number of Pockels cells can be minimized.

From the SRCZ 1 gate operation, it is seen that the first, second and third pairs of input qubits are directly connected to the first, second and third pairs of output qubits, respectively, and no electro-optic modulator is required between these three pairs of input and output terminals. So, there is no change of phase and intensity between the first, second and third pair of input and output terminals. An electro-optic Pockels cell with an applied biasing voltage \( V_{\pi/2} \) is placed between the fourth pair of input and output qubits only, in this case. So, \( \pi/2 \) phase change occurs at the output qubits with respect to the input qubits. Similarly, from the gate operation of SRCZ 2, it is seen that the input and output qubits are directly connected respectively with the first, second and third pairs of input and output terminals. So, no changes of phase were seen as well as intensity occurring between the input and output qubits. The only change is occurred for the fourth pair of the qubits. An electro-optic Pockels cell with an applied biasing voltage \( V_{\pi/2} \) is placed between the fourth pair of input and output qubits. The phase of the output qubits, thereby, is changed. It makes a lag of \( \pi/2 \) phase in respect to the input qubits. For the SRCZ 3 gate operation, there is no change of phase and intensity between the first and second pairs of input and output qubits. In the third pair, each of the output qubits leads by a phase \( \pi \) with respect to the input qubits. Similarly, a phase change occurs also at the fourth pair of output qubits. The output signal leads by a phase \( \pi/2 \) with respect to the input signal of the fourth pairs. From the implementation of SRCZ 4 gate, it is seen that there is no change of phase and intensity at the output signal with respect to input of the first and second pairs. In the third pair, each of the output qubits leads by a phase \( \pi \) in reference to the respective input qubits. It is also seen that the output qubits of the fourth pair lag by a phase \( \pi/2 \) with respect to the input qubits. The difference between SRCZ 3 and SRCZ 4 gates is only seen in the fourth pair of output qubits. In case of SRCZ 3 gate, the output qubits of the fourth pair lead over input qubits and in case of SRCZ 4, the output qubits of the fourth pair lag over input qubits. For the SRCZ 5 gate operation, it is seen that the first pair and third pair of input and output qubits are the same, no changes of phase and intensity occur here. The output qubits of the second pair lead by a phase \( \pi \) with respect to the input qubits and the output qubits of the fourth pair lead by a phase \( \pi/2 \) in reference to the input qubits. From the gate operation of SRCZ 6, it is seen that there are no change of phase and intensity at the output signal with respect to the input signal of the first and third pairs. In the second pair of output qubits, a phase change occurs, where

the output qubits lead by a phase \( \pi \) in reference to input qubits. Similarly, in the fourth pair the output qubits lag by a phase \( \pi/2 \) with respect to the input qubits. From the gate operation of SRCZ 7, it is seen that there is no change of phase and intensity only in the first pair of input and output qubits. The output qubits of the second and third pairs lead by a phase \( \pi \) with respect to the input qubits. The output qubits of the fourth pair lead by a phase \( \pi/2 \) with respect to the input qubits. In the SRCZ 8 gate operation, it is seen that the first pair of input qubits and output qubits are the same, no phase and intensity change is observed here. The second and third pairs of output qubits lead by phase \( \pi \) in reference to the respective input qubits. In the fourth pair, the output qubits lag by phase \( \pi/2 \) in reference to the input qubits. The basic difference between the SRCZ 7 gate and SRCZ 8 gate is, in case of SRCZ 7 gate the output qubits lead by phase \( \pi/2 \) with reference to the input signal in the fourth pairs and in case of SRCZ 8 gate, it lags by phase \( \pi/2 \) with reference to the input signal also in the fourth pairs.

It is also seen from the implementation of different SRCZ gate that they are constructed by changing the position of the LiNbO\(_3\)-based Pockels cell with an external biasing voltage \( V_\pi \) between the first, second and third pairs of input and output terminals. Only the number of required Pockels cells is varying. To implement all the 16 SRCZ gates, a Pockels cell with an applied biasing signal either \( V_{\pi/2} \) or \( V_{-\pi/2} \) is always kept between the fourth pair of input and output terminals.

7 | CONCLUSION

In this work, 16 different SRCZ quantum gates are developed. They are implemented by using LiNbO\(_3\)-based electro-optic Pockels cells. Phase and intensity of light are jointly used to encode the qubits. When the SRCZ gates are operated on the input qubits, then basically phase shift occurs at the output qubits. The output signals lead with \( \pi \) or \( \pm\pi/2 \) with respect to the input signal in different SRCZ gates. The required electrical voltage (\( V_\pi \) and \( V_{\pi/2} \)) used in the Pockels cell is low and can be reduced also by changing the dimension of the used LiNbO\(_3\)-crystal-based Pockels cell, if the LiNbO\(_3\) is used in transverse mode. The implementation of the scheme is all-optical, so the speed of operation is very high, that is of the order of THz limit. As the used input and output signals are photonic, so the qubits are formed by the quantum states of light and this satisfies the basic requirement of a quantum gate. From our above discussion, it is seen that \( V_\pi \) and \( V_{\pi/2} \) can be made more low, if the electro-optic crystal having high electro-optic co-efficient is used. Organic polymer, photonic band gap crystal etc. can be used for making the \( V_\pi \) and \( V_{\pi/2} \) in order of low voltage.

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