CONSTRAINTS IN COSMOLOGICAL PARAMETER SPACE FROM THE SUNYAEV-ZELDOVICH EFFECT AND THERMAL BREMSSTRAHLUNG

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ABSTRACT

We discuss how the space of possible cosmological parameters is constrained by the angular diameter distance function \(D_A(z)\) as measured using the SZ/X-ray method, which combines the Sunyaev-Zeldovich (SZ) effect and X-ray brightness data for clusters of galaxies. New X-ray satellites and ground-based interferometers dedicated to SZ observations should soon lead to \(D_A(z)\) measurements limited by systematic rather than random error. We analyze the systematic and random error budgets to make a realistic estimate of the accuracy achievable in the determination of \((\Omega_m, \Omega_{\Lambda}, h)\), the density parameters of matter and cosmological constant, and the dimensionless Hubble constant, using \(D_A(z)\) derived from the SZ/X-ray method and the position of the first “Doppler” peak in the cosmic microwave background fluctuations. We briefly study the effect of systematic errors. We find that \(\Omega_m, \Omega_{\Lambda},\) and \(w\) are affected, but \(h\) is not, by systematic errors that grow with redshift. With as few as 70 clusters, each providing a measurement of \(D_A(z)\) with a 7% random and 5% systematic error, \(\Omega_m\) can be constrained to \(\pm 0.2,\) \(\Omega_{\Lambda}\) to \(\pm 0.2,\) and \(h\) to \(\pm 0.11\) (all at 3\(\sigma\)). We also estimate constraints for the alternative three-parameter set \((\Omega_m, w, h)\), where \(w\) is the equation-of-state parameter. The measurement of \(D_A(z)\) provides constraints complementary to those from the number density of clusters in redshift space. A sample of 70 clusters \((D_A\) measured with the same accuracy as before) combined with cluster evolution results (or a known matter density) can constrain \(w\) within \(\pm 0.45\) (at 3\(\sigma\)). Studies of X-ray and SZ properties of clusters of galaxies promise an independent and powerful test for cosmological parameters.

Subject headings: cosmological parameters — cosmology: theory — galaxies: clusters: general — X-rays: galaxies: clusters

1 INTRODUCTION

What set of cosmological parameters characterizes our universe?

According to the most popular cold dark matter (CDM) scenario, the universe consists of baryonic matter and a substantial amount of “dark” matter. A variety of recent measurements have led to the conclusion that the matter density parameter \(\Omega_m \approx 0.3\) (Turner 2000), while cosmic microwave background (CMB) measurements strongly favor a flat spacetime (de Bernardis et al. 2002; Lee et al. 2001), and Type Ia supernovae (SNe Ia) measurements indicate that the universe is accelerating, suggesting a negative pressure (Riess et al. 2000; Perlmutter et al. 1999). Taken together, these pieces of evidence suggest that the baryonic and dark matter content of the universe is supplemented by an additional smooth component with negative pressure \(P_w\),

\[
\rho_w c^2 = -w P_w, 
\]

modeled by the equation of state \(\rho_w c^2 = -w P_w\), where \(\rho_w\) is the density of this component and \(w\) is a dimensionless state parameter on the order of unity (see Huterer & Turner 2001).

Each existing data set constrains, with limited accuracy, some subset of the cosmological parameters. Different measurements and combinations of measurements, such as SNe Ia, CMB fluctuations, IRAS infrared galaxy surveys, classical double radio galaxy properties, 1.2 Jy galaxy redshift surveys, gravitational lensing, cluster X-ray temperature function and cluster number counts, baryon and gas mass fraction, and the Sunyaev-Zeldovich (SZ) effect have been used to constrain cosmological parameters (Jaffe et al. 2001; Balbi et al. 2000; Tegmark & Zaldarriaga 2000; Guerra, Daly, & Wan 2000; Efstathiou et al. 1999; Lasenby, Bridle, & Hobson 2000; Perlmutter et al. 1999; Gawiser & Silk 1998; Lineweaver 1998; White 1998; Pen 1997; Sasaki 1996; Huterer & Turner 2001; Majumdar & Subrahmanyan 2000; Bridle et al. 1999; Diego et al. 2001).

In the future, the Supernovae/Acceleration Probe (SNAP) project 4 plans to use the SNe Ia method to determine the matter density and the cosmological constant at the few percent level. Even with the next-generation CMB satellites, the Microwave Anisotropy Probe (MAP) and Planck, degeneracies will remain among the cosmological parameters that can be estimated from the results (Efstathiou & Bond 1999; Zaldarriaga, Spergel, & Seljak 1997). The importance of using a wide range of methods, therefore, is twofold. First, a simultaneous consideration of all data sets should allow the best joint estimation of the cosmological parameters. Second, the agreement of different techniques for measuring the cosmological parameters should provide a cross-check of our understanding of the underlying processes and a control against systematic errors. As we extend our analysis of the CMB to more complicated models (tensor fluctuations, finite neutrino masses, etc.), the number of cosmological parameters increases, and it becomes even more important that the widest possible range of data sets is used and that strong controls against systematic errors are in place.

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Many of the techniques of cosmological parameter estimation use clusters as tracer particles. As a result, there is a large number of planned cluster surveys in the two most important nonoptical observational indicators of clustering: the SZ effect and cluster X-ray emission. Sunyaev-Zeldovich effect surveys with dedicated interferometers or receiver arrays will observe hundreds of clusters per year with \( z > 0.5 \) (Browne et al. 2000; Holder et al. 2000; Bartlett 2000). The new X-ray missions (Chandra, XMM) will provide data on hundreds of clusters with high redshift through their deep and medium-deep surveys.

Cluster evolution, the redshift distribution of clusters from SZ and X-ray surveys \([N_{SZ}(z)\) and \(N_X(z)\)] and cluster number counts as a function of X-ray flux \([N_X(S)\)] are important constraints on cosmological parameters. While methods based on the CMB power spectrum and SNe Ia are sensitive to the angular diameter distance, cluster evolution (and number counts) is sensitive to the growth function of matter density fluctuations. Bartlett (2000) estimated the performance of ground-based, arcminute-resolution, SZ surveys and concluded that more clusters will be detected with deep, small-area surveys than shallow, wide-area surveys. Kneissl et al. (2001) studied the performance of the Arcminute MicroKelvin Imager experiment and showed that a set of only about 20 clusters, with redshifts in the range \( z = 0-0.8 \), is needed to measure \( N_{SZ}(z) \) sufficiently well to distinguish between \( \Omega_m = 1 \) and \( \Omega_m = 0.3 \) cosmologies. Carlstrom et al. (2002) discuss a deep SZ ground-based survey and quantify constraints from \( N_{SZ}(z) \) on \( \Omega_m \) and \( \Omega_{\Lambda} \). \( N_{SZ}(S) \) and \( N_{SZ}(z) \) were estimated from the proposed shallower, but all-sky, Planck survey by Diego et al. (2001), who concluded that about 300 clusters (with the necessary optical follow-up to measure redshifts) would suffice to distinguish between open \( \Omega_m = 0.3 \) and \( \Omega_m = 0.3 \) cosmologies at 3 \( \sigma \) confidence. Holder, Haiman, & Mohr (2001) discussed the constraints on the parameter space defined by \( (\Omega_m, \Omega_{\Lambda}, \sigma_8) \) (where \( \sigma_8 \) is the normalization of the matter power spectrum) using cluster evolution. Holder et al. showed that constraints from cluster evolution and SNe Ia observations are highly complementary to each other. Haiman, Mohr, & Holder (2001) discussed the constraints on the \( (\Omega_m, w, h) \) parameter space, assuming a spatially flat geometry \( (\Omega_m = 1 - \Omega_m) \), that follow from an SZ effect survey and a large-angle deep X-ray survey (the Cosmology Explorer). They found that \( N_{SZ}(z) \) and \( N_X(z) \), combined with constraints from CMB or SNe Ia experiments, significantly reduce the degeneracies between \( \Omega_m, w, \) and \( h \). Huterer & Turner (2001) estimated the constraints on \( \Omega_m \) and \( w \) for flat geometry that can be gained by combining results from SNAP, Planck, and SZ and X-ray surveys.

As has been realized, the angular diameter distance-redshift relation \( D_A(z) \) is at the heart of many of these techniques and is sensitive to some important combinations of cosmological parameters while being degenerate under others (Jaffe et al. 2001; Tegmark & Zaldarriaga 2000; Efstathiou et al. 1999; Lasenby et al. 2000; Perlmuter et al. 1999; White 1998). Recently, White (1998) estimated constraints on the pairs of quantities \( (\Omega_m, \Omega_{\Lambda}) \) and \( (\Omega_m, w) \) (the latter in a flat universe) from the \( D_A(z) \) function based on current SNe Ia data combined with CMB first-peak constraints. The analysis shows that the constraint on parameters based on \( D_A(z) \) is nearly orthogonal to the constraint based on the position of the first peak in the CMB fluctuation spectrum. These two data sets are thus highly complementary and form a particularly powerful pair of measurements (see also Tegmark et al. 1998).

The shape and normalization of the observed angular diameter distance function constrains several cosmological parameters (the standard formulae for distance in Friedmann-Robertson-Walker universes are given in, e.g., Peebles 1993). The distance-redshift function \( D_A(z) \) in CDM models depends on the matter density, cosmological constant, Hubble constant, and any other particle density that contributes to the curvature of spacetime. The slope of the distance-redshift function at low redshift is a measure of the Hubble constant, while the shape of the function depends on the curvature and the different densities. In Figure 1 we show the fractional difference in \( D_A(z) \) with fixed matter density and Hubble constant (\( \Omega_m = 0.3 \), \( h = 0.65 \)) but various values cosmological constants (\( \Omega_{\Lambda} = 0.7 \), 0.6, 0.3; solid, dashed, and dot-dashed lines, respectively) relative to a model with zero cosmological constant (\( \Omega_{\Lambda} = 0 \)). It can be seen from this figure that \( D_A(z) \) is most sensitive to the value of the cosmological constant at redshift about unity and quite insensitive to that at small or high redshifts. In the redshift interval from \( z = 0.5 \) to \( z = 1.8 \) the angular diameter distance for a flat (\( \Omega_m = 0.3 \), \( \Omega_{\Lambda} = 0.7 \)) model is more than 10% different from a model with the same matter density but zero cosmological constant (\( \Omega_m = 0.3 \), \( \Omega_{\Lambda} = 0 \)).

We can expect high-precision data from hundreds of clusters of galaxies in the near future. With the present instrument suite, the statistical errors on individual measurements will be small, and so the usefulness of the data will be limited by their systematic errors. In this paper we evaluate the error budget of distance determination based on the SZ effect and X-ray measurements (assuming that the X-ray output is dominated by thermal bremsstrahlung, as is appropriate for the hot clusters in which the SZ effect is strong) and provide a realistic estimate of errors achievable.

![Fig. 1.—Deviation of the angular diameter distance in universes with \( \Omega_m = 0.3 \) and \( \Omega_{\Lambda} = 0.7, 0.6, \) and 0.3 relative to the angular distance function for a universe with \( \Omega_m = 0.3 \) and \( \Omega_{\Lambda} = 0 \), as a function of \( z \) (with \( h = 0.65 \)). The solid, short-dashed, and dot-dashed lines represent percentage deviations with \( \Omega_{\Lambda} = 0.7, 0.6, \) and 0.3, respectively.](image-url)
in the angular diameter distance. We estimate how well one will be able to constrain the two parameter sets \( (\Omega_m, \Omega_b, h) \) and \( (\Omega_m, w, h) \) (assuming a spatially flat geometry: \( \Omega_b = 1 - \Omega_m \)) if this \( D_A(z) \) function is combined with the position of the first Doppler peak in the angular power spectrum of CMB fluctuations.

Our treatment is complementary to previous work of Kneissl et al. (2001), Carlstrom et al. (2002), Diego et al. (2001), Holder et al. (2001), Haiman et al. (2001), and Huterer & Turner (2001), who used \( N_{SZ}(S) \), \( N_{SZ}(S) \), and \( N_X(z) \) to constrain cosmological parameters, since we discuss the importance of \( D_A(z) \). It also complements the work by White (1998), who used \( D_A(z) \) determined from existing SNe Ia data to constrain \( (\Omega_m, \Omega_b, h) \) and \( (\Omega_m, w, h) \) (flat); the errors from the SZ/X-ray technique have significantly different characteristics, and we are concerned with the limitations that will be encountered with future survey data.

In the next section we briefly describe the well-known method of angular distance determination based on the SZ effect and thermal bremsstrahlung with an emphasis on how measurements are used and how their error propagates to the angular diameter distance. In §3 we give a detailed analysis of the error budget, and in §4 we discuss the constraints on cosmological parameters. Finally, in §5 we summarize our conclusions.

2. DETERMINATION OF ANGULAR DIAMETER DISTANCE USING CLUSTERS

Distance determinations using the SZ effect and X-ray emission from the intracluster medium (hereafter the SZ/X method) are based on the fact that these processes depend on different combinations of physical parameters of the clusters. The SZ effect (Sunyaev & Zeldovich 1980; for recent reviews, see Birkinshaw 1999 and Rephaeli 1995) is a result of inverse Compton scattering of CMB photons off hot electrons in the intracluster (IC) gas. The number of photons is conserved, but on average, the photons gain energy and thus generate a decrement in the Rayleigh-Jeans part of the spectrum and an increment in the Wien region. The amplitude of the SZ effect does not depend on the redshift of the cluster. We will discuss static thermal and kinematic thermal SZ effects in this paper, where the "static" effect is present in all clusters, and the "kinematic" effect is only present for those clusters with a nonzero line-of-sight (LOS) peculiar velocity relative to the Hubble flow. A typical Rayleigh-Jeans decrement of the static SZ effect is about 50 times that of the kinematic SZ effect. The static SZ effect is proportional to the LOS pressure integral of the IC gas,

\[
\Delta T \propto g(\nu) \int dz n_e(r) T_e(r) \, ,
\]

where \( g(\nu) \) is the frequency dependence of the effect \( (g \rightarrow -2 \) in the nonrelativistic Rayleigh-Jeans limit) and \( n_e(r) \) and \( T_e(r) \) are the electron density and temperature, respectively, as functions of position within the cluster. The central X-ray surface brightness of a cluster is the emission-weighted LOS average of \( n_e(z)^2 T_e(z) \):

\[
S_X \propto \frac{1}{(1+z)^4} \int dz n_e(z)^2 T_e(z) \Lambda(T_e, Z_{ab}) \, ,
\]

where \( \Lambda(T_e, Z_{ab}) \) is the cooling function integrated over the deprojected energy band of observations and \( Z_{ab} \) is the metal abundance of the gas.

On any given LOS, equations (1) and (2) and the emission-weighted X-ray temperature \( T_e \) (from spectroscopy) provide three independent integral constraints for the two functions, \( n_e(r) T_e(r) \) and \( T_e(r) \), although these functions cannot be determined uniquely. If instead one assumes parameterized functional forms for \( n_e(r) T_e(r) \) and \( T_e(r) \), these three equations can be used to constrain the controlling parameters. One important parameter that can be found from fitting the data is a characteristic LOS physical size of the system \( R_{LOS}^{char} \), and the angular diameter distance of the cluster can then be determined if this size is compared with the corresponding angular size \( \theta_{LOS}^{char} \). However, we can measure only the apparent characteristic angular size \( \theta_{LOS}^{char} \) of the system as it appears projected into the plane of the sky (with corresponding physical size \( R_{LOS}^{char} \)). The main difficulty in the SZ/X method is the deprojection of the cluster from its two-dimensional images, that is, the problem of finding the value of \( \theta_{LOS}^{char} \) from the measured value \( \theta_{LOS}^{char} \). If this can be done, one can determine the angular diameter distance as

\[
D_A = \frac{R_{LOS}^{char}}{\theta_{LOS}^{char}} \, .
\]

If one assumes spherical symmetry, \( \theta_{LOS}^{char} = \theta_{LOS}^{char} \), and a deprojection is possible without further assumptions about the functional forms of \( n_e(r) T_e(r) \) and \( T_e(r) \) (Silk & White 1978). This method, however, needs high signal-to-noise ratio and high angular resolution SZ and X-ray images of the cluster, and thus the analytic deprojection has not been used so far. Most commonly, the distribution of gas in clusters is described by the spherical isothermal \( \beta \)-model (Cavaliere & Fusco-Femiano 1976). This model seems to give a good fit to X-ray data (Sarazin 1988), and thus it is usually assumed that the IC gas follows a spherical isothermal \( \beta \)-model, where the IC gas is isothermal and in hydrostatic equilibrium in the gravitational potential of the cluster. The electron concentration in the cluster atmosphere is then \( n_e(r) \propto \left[ 1 + (r/r_c)^2 \right]^{-3\beta/2} \), where \( r_c \) is the core radius (a suitable characteristic size of the cluster). The shape parameter \( \beta \) describes how the kinetic energy is distributed between the galaxies and IC gas. Typically, \( r_c = 0.1 \ h^{-1} \) Mpc and \( \beta \approx 1/4 \) (Sarazin 1988). In practice, the IC gas temperature is determined from X-ray spectroscopy, and the core radius projected in the plane of the sky \( \theta_{FIT}^{\beta} \) and the shape parameter \( \beta^{FIT} \) are determined from fitting the \( \beta \)-model to a high-resolution X-ray image. Finally, equations (1) and (2) are used to determine the angular diameter distance using equation (3)

\[
D_A = \frac{R_{FIT}^{\beta}}{\theta_{FIT}^{\beta}} = \frac{R_{LOS}^{char}}{\theta_{LOS}^{char}} \, ,
\]

which is crucially dependent on the assumption of spherical symmetry; \( \theta_{FIT}^{\beta} = \theta_{LOS}^{char} \).

If we adopt the \( \beta \)-model representation, we can write the angular diameter distance as a function of measurable quantities for a cluster as

\[
D_A \propto \frac{1}{(1+z)^4} S_{X0} N(H) \Lambda(T_e, Z_{ab}) \frac{\Delta T_0^2 \Lambda(T_e, Z_{ab})}{\eta^2 \theta_e} \, ,
\]

where \( \Delta T_0 \) and \( S_{X0} \) are the central SZ effect and X-ray surface brightness (unabsorbed), respectively, \( N(H) \) is the
absorbing column density, and $F(\beta)$ is a known function of $\beta$ (or other parameters if a different description of the structure of the cluster is used). Some of the measured (or estimated) quantities on the right-hand side of this equation are highly correlated, so a careful error analysis is needed; the errors on individual quantities cannot be added in quadrature. No bootstrapping is required in using equation (5) (it is a direct method, with no need for using the usual cosmic distance ladder), as was realized by Sunyaev & Zeldovich (1980), but the strong assumptions of spherical symmetry and isothermality are only crude approximations of the real situation.

It has been pointed out recently that relativistic effects become important for high-temperature clusters (Rephaeli 1995). Relativistic corrections to the inverse Compton scattering have been calculated and used to interpret observations in terms of the Hubble constant (Rephaeli 1995; Birkinshaw 1999; Birkinshaw & Hughes 1994; Holzapfel et al. 1997, and references therein). Relativistic corrections to the thermal bremsstrahlung cooling function are also needed (Hughes & Birkinshaw 1998; Rephaeli & Yankovitch 1997). These corrections are on the order of 3% in the central SZ decrement $\Delta T_\text{c}$ since for most X-ray observations to date the X-ray emissivity $\Lambda_e(T_e, Z_{\text{ab}})$ is a slowly varying function of $T_e$. Other uncertainties, from the dependence of $\Lambda_e$ on the abundance of metals in the intracluster medium (ICM) and the absorbing column on the LOS, are less important, and we can also neglect the contribution to the error in the angular diameter distance from the redshift.

The error in the spatial fit is dominated by the uncertainty in the central SZ decrement $\Delta T_\text{c}$ and central X-ray surface brightness $S_{X0}$. Both errors are about 8%–10% in the data obtained using BIMA and ROSAT. The total uncertainty in a $D_A$ estimate from spatial fitting to X-ray imaging and SZ interferometric measurements (with BIMA or the RT) is about 14%–18%. We can expect a dramatic improvement in the accuracy of $S_{X0}$ in the near future because of the larger collecting area and higher angular resolution of XMM and Chandra and because the improved imaging will also allow a better choice of models for the ICM. A substantial improvement in the central SZ effect is also likely as the first generation of dedicated SZ interferometers becomes available. The overall statistical error in these parameters should drop to about 4%–5%.

The statistical errors in measurements of electron temperature based on ASCA and ROSAT observations are up to about 15%–20% (Reese et al. 2000; Grainge et al. 1999).

The principal identified systematic errors in the measurements arise from the absolute calibration of the radio and X-ray observations. Errors in the effective area of the ROSAT PSPC and HRI introduce an error of about 10% in $D_A$. This error is greatly reduced (to 1%–2%) for the instruments on Chandra and XMM-Newton. The absolute calibration error of radio interferometers is good to 4%–5% but could be improved to about 1% through extensive observations of point sources, tied to the planetary flux density/brightness temperature scale (Birkinshaw 1999).

Ground-based interferometric observations are also subject to systematic errors due to the removal of background (and cluster) radio sources, which may be imperfect if the sources are variable or have significant angular extent. Imperfect calibration of the phase and amplitude of the detector system may also cause errors near brighter contaminating sources. Fortunately, interferometers are insensitive to large-scale gradients in emission from the ground and atmosphere, and so this source of systematic offset signals from single-dish data is largely removed in interferometric work (see, e.g., Carlstrom et al. 2002).

3. ERROR BUDGET IN THE DETERMINATION OF THE ANGULAR DIAMETER DISTANCE

Errors in the determination of the angular diameter distance may be cast into two major categories: errors from measurements and errors from theoretical modeling. Measurement errors can be statistical or systematic, while errors from theoretical modeling are systematic by nature. The systematic errors can be further classified as “random” or “nonrandom” depending on whether they average out or not for a statistical sample of clusters. In what follows we therefore use the expressions “random modeling errors” when discussing errors that introduce only scatter in the distance determination when averaged over an unbiased sample of clusters and “nonrandom modeling errors” when discussing errors that introduce a bias in the distance determinations even for an unbiased sample of clusters.

In this section we summarize the statistical and systematic errors associated with the SZ and X-ray observations, following the discussions in reports of the latest interferometric observations (from BIMA; Reese et al. 2000; and from the Ryle Telescope [RT]; Grainge et al. 1999) and in review articles (Birkinshaw 1999). We also use some additional references to compile a detailed list of error sources.

3.1. Errors from Measurements

Statistical errors from measurements in the angular diameter distance are dominated by counting statistics in X-ray images and spectra and by Gaussian measurement uncertainties in the SZ measurements. These statistical errors propagate to errors in $\theta_e$ and $\beta$ through fitting for the spatial distribution and to errors in $T_e$, $\Lambda_e$, $Z_{\text{ab}}$, and $N_H$ through fitting the X-ray spectrum. A simultaneous fit of a $\beta$-model on interferometric SZ and X-ray images determines $\theta_e$, $\beta$, $S_{X0}$, and $\Delta T_\text{c}$ (see, e.g., Reese et al. 2000) but causes the errors in these parameters to be strongly correlated.

The most important error sources in a determination of the angular diameter distance are the error in $S_{X0}$ and the error in $T_e$ (eq. [5]). The electron temperature enters the estimate of $D_A$ roughly as $T_e^{-2}$ since for most X-ray observations to date the X-ray emissivity $\Lambda_e(T_e, Z_{\text{ab}})$ is a slowly varying function of $T_e$. Other uncertainties, from the dependence of $\Lambda_e$ on the abundance of metals in the intracluster medium (ICM) and the absorbing column on the LOS, are less important, and we can also neglect the contribution to the error in the angular diameter distance from the redshift.
bution and in this section discuss the errors introduced by deviations from this model. As before, we can identify random and nonrandom systematic modeling errors.

Significant random modeling errors arise from the asphericity of the intracluster gas, the peculiar velocity of the cluster, and primordial CMB fluctuations. Serious nonrandom modeling errors can be expected from nonisothermality, cooling flows, clumping, merging, and the finite extent of the cluster. Other issues that may be important are radio point sources and gravitational lensing. We briefly discuss random modeling errors from resolved radio halos in clusters because of their contribution to the measured SZ signal (through synchrotron emission) and to the X-ray signal (through inverse Compton emission), the effect of diffuse free-free emission from cool gas (e.g., in spiral galaxies), and finite optical depth effects.

3.2.1. Random Modeling Errors

Asphericity is one of the most important sources of systematic errors in the determination of the distance to clusters when using a spherically symmetric isothermal $\beta$-model. If the cluster is not spherical, the assumption that $r_{\text{FIT}} = r_{\text{LOS}}$ is invalid. We estimate the extent of this problem by approximating the true structures of clusters as oblate or prolate ellipsoids (or, more generally, as triaxial ellipsoids), while retaining the assumption that the distribution is described by an isothermal $\beta$-model with constant $\beta$. If the symmetry axis is in the LOS, we would assume $r_{\text{FIT}} = r_{\text{LOS}}$ and thus overestimate (underestimate) $D_A$ for prolate (oblate) clusters. Birkinshaw (1999) finds that prolate or oblate clusters with their symmetry axis aligned in the LOS, if assumed spherical, will have a fractional error of

$$\frac{\delta D_A}{D_A} = \frac{a - b}{b}, \quad (6)$$

where $a (= r_{\text{LOS}})$ is the core radius in the LOS and $b (= r_{\text{FIT}})$ is the core radius in the plane of the sky ($a > b$ for a prolate distribution). Clusters often show ellipticity at the level, in projection, of $a/b = 1.25$ (Mohr et al. 1995). If the axis of symmetry is not in the LOS, the error is smaller; therefore, the fractional error in $D_A$ should be less than 25%. The analysis of CL0016+16 of Hughes & Birkinshaw (1998) showed that oblate or prolate distributions may cause smaller than 8% error in $D_A$ if the structure of the ICM is analyzed as spherical. The work of Grainge et al. (1999) implies about a 14% error in $D_A$ for Abell 1413.

$N$-body simulations can be used to understand the details of physics of the cluster geometry and quantify the deviations from the spherical distribution. The numerical simulations of Inagaki, Suginohara, & Suto (1995) show that asphericity causes an error of up to 15% in $D_A$. Cluster merging simulations of Roettiger, Stone, Mushotzky (1997) show that small off-axis merging, in general, causes prolate distributions, while oblate distributions may be caused by large off-axis merging, causing typically less than 20% error in $D_A$. Observationally, Basilakos, Plionis, & Maddox (2000) found that prolate spheroid models fit the Automated Plate Measuring Machine cluster data better than oblate spheroids. Sulkanen (1999) studied a statistical sample of clusters assuming a triaxial $\beta$-model density distribution. Sulkanen’s results indicate that the distance scale obtained assuming a spherical distribution is within 14% of its true value (at 99.7% confidence) based on a sample of 25 clusters with triaxial axes consistent with observations (see also Puy et al. 2000).

In general, oblate or prolate clusters will have their axis randomly distributed in the sky. Zaroubi et al. (1998) studied general deprojections assuming that clusters have axially symmetric density distributions with an arbitrary orientation of the symmetry axis. They found that using SZ and X-ray images, one can determine only a combination of distance and inclination angle. Another image (weak lensing for example) is necessary to decouple these two parameters and determine the distance separately. If only X-ray and SZ data are available, the error that asphericity introduces into the $D_A$ determination for any single cluster cannot be reduced below about 15%. However, the combination of X-ray, SZ, and weak-lensing data together with modeling of the equilibrium of the hot gas should allow us to reduce the error to around 5%. The planned weak-lensing surveys should make the required imaging data available in the near future. Alternatively, galaxy velocity distributions from optical observations can be used with numerical simulations to determine physical parameters of individual clusters, as was done by Gomez, Hughes, & Birkinshaw (2000) and Roettiger et al. (1997; see the discussion of merging below).

Peculiar velocities of clusters introduce enhanced (approaching cluster) or decreased (receding cluster) SZ measurements because of the kinematic SZ effect (Sunyaev & Zeldovich 1980; for a derivation, see Birkinshaw 1999). In CDM models, cluster peculiar velocities are 400–500 km s$^{-1}$ (Colberg et al. 2000; Ueda, Itoh, & Suto 1993). This would introduce only a few percent error in $D_A$. However, some observations suggest larger peculiar velocities: 1000 km s$^{-1}$ (Bahcall & Soneira 1983; Lauer & Postman 1994). If these large velocities are real, the kinematic SZ effect may cause an error of up to 25% in $D_A$. Fortunately one can separate the static and kinematic SZ effects based on their different frequency dependence, or equivalently, the peculiar velocities can be determined by measuring the crossover frequency of the total SZ effect (see, e.g., Molnar & Birkinshaw 1999). Peculiar velocities also introduce a small bias in the redshift determination of the cluster. If clusters are selected based on their SZ signal, this effect would cause a biased sample of clusters, and a systematic overestimate of $D_A$ would result.

Primordial CMB fluctuations introduce systematic effects in the distance determinations with their positive or negative contributions to the microwave decrement misinterpreted as SZ signal. The amplitude of systematic errors introduced by CMB fluctuations is a strong function of the observation strategy. The CMB fluctuations are reduced at arcminute angular scales compared to their degree-scale amplitudes but still introduce a scatter of about 10% in the distance determinations (Cen 1998). At smaller scales (high $\ell$-values) the power in CMB fluctuations becomes negligible. A further reduction in the level of this error can be achieved using spectral separation of the thermal SZ effect from the primordial fluctuations (and the kinematic SZ effect). Gravitational lensing transfers power from large-scale primordial fluctuations to small-scale fluctuations (Metzetal 1998; Seljak 1996). As a result, if the CMB is not separated from the SZ effect, it would give larger, but symmetric, scatter in $D_A$ or a fractional error of about 8% (Cen 1998). Note, however, that the kinematic SZ effect and primordial CMB fluctuations have the same frequency dependence; thus, they cannot be separated from each other based on their fre-
frequency signature. Fortunately, both effects can be separated out from the static SZ effect simultaneously based on their different frequency dependence.

The errors in the cosmological parameters caused by random modeling errors (asphericity, cluster peculiar velocities, primordial fluctuations, etc.) can be reduced using a properly selected sample of clusters. This sample must avoid selection biases that themselves introduce systematic errors. As was emphasized by Birkinshaw, Hughes, & Arnould (1991), clusters should not be selected based on their SZ or X-ray central brightnesses since such a selection would produce a sample containing an excess of clusters with prolate geometry, high positive peculiar velocity in the LOS, and contamination from negative CMB fluctuations (if the Rayleigh-Jeans frequency band is used) and result in a biased $D_A$. X-ray–selected clusters with a flux limit well above the detection limit might be used as in Mason, Myers, & Readhead (2001) and Jones et al. (2001). Alternatively, if there are multifrequency measurements, the best solution would be to separate the kinematic SZ effect and the primordial fluctuations from the static SZ effect and use the total static SZ effect flux density as a selection criterion.

### 3.2.2. Nonrandom Modeling Errors

**Nonisothermality** is one of the major sources of systematic error in the determination of $D_A$.

The isothermal assumption for the intracluster gas is clearly an approximation. Even if the central region of the cluster is virialized and isothermal, the outer regions will be subject to shocks from merging and gas infalling from filaments. Merging with massive clusters will change the temperature relative to the single-component cluster virialized value, even in the core region. Observations show that the central regions are nearly isothermal, but thermal substructures have also been found in clusters (Sarazin 1988). The effect of temperature variations on the distance estimate depends on the instrument and observing technique used. SZ measurements are insensitive to temperature variations in the cluster if the projected pressure profile is unchanged. Nonspatially resolved X-ray measurements determine emission-weighted temperatures and so are sensitive to the thermal structure of the central region of clusters. Thus, the temperature deduced from X-ray measurements is well suited to comparison with SZ measurements by radio interferometers, which are also most sensitive to the central region and have less response to the outer parts of the cluster (e.g., assuming $2 \sigma$ detection, BIMA is sensitive out to about $3 r_e$, and about 85% of the observable X-ray flux of a typical cluster, using ROSAT PSPC, is within $3 r_e$).

Birkinshaw & Hughes (1994) and Holzapfel et al. (1997) analyzed Abell 2218 and Abell 2163, respectively, assuming an isothermal $\beta$-model and a model with falling temperature with radius. Their results show that $D_A$ may be overestimated by 20%–30% if nonisothermal distributions are assumed to be isothermal. Numerical simulations show similar errors due to nonisothermality; thus, the simulations of Inagaki et al. (1995) lead us to conclude that an overestimate of 25% in $D_A$, mostly because of the overestimated SZ amplitude, may result from assuming isothermality when the temperature is lower in the outer regions. Simulations of merging clusters by Roettiger et al. (1997) obtained a similar result, that an overestimate of 10%–30% may result in $D_A$ from nonisothermality, with the range depending on the projection geometry. Note, however, that these results are based on single-dish measurements; for interferometric observations, these effects are usually smaller.

It is not easy to correct for nonisothermality. The SZ effect is proportional to $(n_e T_e) k_{\text{B} \text{los}}$ and for an accurate calculation of the SZ effect, it is often necessary to have good information about the temperature out to $R_{\text{vir}}$ (about $10 r_e$) or more. It is difficult to carry out spatially resolved spectroscopy at such large radii because of the low X-ray surface brightness of the outer regions of clusters ($S_X \propto n_e^2$). At present, the best evidence from BeppoSAX data extends $T(r)$ measurements to $(0.5–0.75)R_{\text{vir}}$ (Irwin & Bregman 2000; De Grandi & Molendi 1999). The increased sensitivity available with Chandra and XMM should enable us to use spatially resolved spectroscopy to determine the temperature profiles of clusters that both resolve the inner cooling flow region and collect enough photons to measure useful temperatures in the clusters’ outer regions (Schmidt, Allen, & Fabian 2001; Tamura et al. 2001).

Unfortunately, the hydrostatic equilibrium assumption of the $\beta$-model breaks down in the central parts of clusters with high enough density for cooling to be important. In these cases the ICM will radiate via thermal bremsstrahlung and line emission (with the balance depending on the temperature) and develop a pressure gradient and a subsonic inflow, a so-called cooling flow (see review of Fabian 1994). The increased central density at the core of the cluster leads to an increased level of X-ray emission, which is often used as an indicator of the presence of a cooling flow region.

Phenomenological models have been developed and numerical simulations have been performed to study cooling flows (White & Sarazin 1987; Rizza et al. 2000; Fabian 1994 and references therein). Majumdar & Nath (2000) estimated the effect of cooling flows on the determination of the Hubble constant. They found that only at the very center of the cooling flow (within the sonic radius) will the X-ray luminosity drop because of the decreased temperature of the ICM. They show, further, that there is a gradual increase in pressure toward the center of the cooling flow. Outside this region, the approximation of the hydrostatic equilibrium profile remains good, while within there will be an SZ effect excess. Thus, the results of Majumdar & Nath (2000) indicate that one should exclude 80% of the cooling flow region to reduce the error in $D_A$ to below 10%. However, this may be an upper limit on the error in $D_A$ since the calculation ignored the effect of the cooling flow on the structure fitting. Figures 3 and 4 in Reese et al. (2000) suggest that a compensatory error occurs here, and this lowers the error in $D_A$.

A further effect from cooling flows, pointed out by Schlickeiser (1991), is that the build-up of cold gas at the center of cooling flows might lead to significant free-free emission in the radio band, which would reduce the SZ signal. This works in the opposite sense to the pressure effect but provides another reason for excluding the central parts of SZ images of cooling flow clusters from the SZ/X distance-scale analysis.

Excluding cooling flow clusters completely from SZ/X distance-scale studies would be the most complete solution to the problem that they pose. However, most clusters close to hydrostatic equilibrium (where the underlying physics required to model the ICM is most straightforward) possess cooling flows. Modeling cooling flows, and excluding their...
centers, is therefore necessary to build up large samples for Hubble constant work.

High-redshift cooling flow clusters can be recognized by using their X-ray spectra even if we cannot resolve the central region since cooling flow clusters have emission-weighted metallicity 1.8 times higher than noncooling flow clusters (Allen & Fabian 1998). Mohr, Mathiesen, & Evrard (1999) analyzed a nearby cluster of galaxies using ROSAT PSPC data, modeling cooling flow regions where necessary, and found that fitting an isothermal $\beta$-model to cooling flow clusters will underestimate both $r_e$ and $\beta$ and therefore produce a poor fit even outside the cooling flow. They identified cooling flow clusters based on two criteria: (1) nonrandom residuals consistent with a central emission excess and (2) a relaxed cluster with no asphericity or substructure. They concluded that a double $\beta$-model fit gives an unbiased estimate of $D_A$. Cooling flow contamination in the determination of the average Hubble constant using the SZ/X method can be recognized by searching for a dependence of individual Hubble constants (determined for each cluster) on IC gas metallicity.

Clumping in the IC gas is potentially one of the most important systematic effects. It is well known that radio halos, hot bubbles from supernova eruptions, cold condensed gas in galaxies, etc., constitute a level of clumpiness in the ICM. The important question is whether the effect of clumping is important in the SZ/X method of measuring $D_A$.

As a first approximation, clumping will enhance the X-ray surface brightness since $S_X \propto n_e^2$ and does not change $\Delta T_S$ since $\Delta T_S$ is proportional to the LOS-averaged pressure, and clumps should be in pressure equilibrium with their surroundings if they are to be long-lived (assuming no substantial magnetic fields exist). As a consequence, $D_A$ is underestimated. However, the emission-weighted temperature of the cluster measured by X-ray spectroscopy will also decrease. This reduces the systematic effects of clumping to only a few percent error in $D_A$. Birkinshaw et al. (1991) studied the effects of isobaric clumping of the intracluster gas in Abell 665. The fractional error in $D_A$ from clumping is

$$\frac{\delta D_A}{D_A} = \frac{\langle n_e^2 \rangle \langle \Lambda(T_e) \rangle}{\langle n_e^2 \Lambda(T_e) \rangle} - 1,$$

where the angle brackets imply averages over regions larger than the scale of the clumping. For isothermal clumps, $\Lambda_e$ factors out. Then, since $\langle n_e^2 \rangle/\langle n_e \rangle^2$ is always greater than or equal to 1, clumping will always cause an underestimate of $D_A$. Birkinshaw et al. (1991) found $\langle n_e^2 \rangle^2/\langle n_e \rangle^2 < 3$ or so. Holzapple et al. (1997) studied the effects of isobaric clumping on Abell 2163. Assuming cold clumps, they estimate that error from clumping could lead to an overestimate of about 10% in $D_A$.

Inagaki et al. (1995) find from their $\Omega_m = 1$ numerical simulations that $D_A$ will be underestimated by about 15% because of clumping. However, there is evidence that lower matter density models produce less clumpy structures, so that if our universe has $\Omega_m < 1$, Inagaki et al. will have overestimated the effect of clumping.

Self-consistent modeling of small-scale clumping is difficult because many physical processes contribute to its creation and destruction. Gunn & Thomas (1996) used phenomenological multiphase models to study X-ray emission from clumpy IC gas. Their isobaric model implies a fractional error in the X-ray central surface brightness

$$\frac{\delta S_X(0)}{S_X0} \propto \frac{\langle n_e^2 \rangle \langle n_e^{-1} \rangle \alpha^2}{\langle n_e^2 \rangle^{-1} \alpha^2},$$

where $\alpha$ is the emissivity exponent. This would cause about a 10% error in $D_A$.

Nagai, Sulkanen, & Evrard (2000) discussed biases in the Hubble constant determination from a multiphase, spherically symmetric, ICM with isobaric clumping of variance

$$\sigma^2 = \frac{\sigma_x^2}{1 + (r/r_e)^2},$$

where $\sigma_x$ and $\epsilon$ are free parameters describing the strength and radial dependence of the variance, respectively. They assumed a lognormal distribution for the gas density phase distribution, which was motivated by simplicity and the effect of nonlinear gravitational growth of initially Gaussian density fluctuations (Cole, Fisher, & Weinberg 1994). They assumed that the multiphase model has the same emission-weighted temperature and the X-ray emission profile as a fiducial single-phase model. Based on their results, the error in the distance from an incorrect assumption of a single-phase ICM is

$$\frac{\delta D_A}{D_A} = 2 \exp \left[ \frac{(1-\alpha)(2-\alpha)}{4} \sigma_x^2 \right] - 1,$$

where $\alpha$ is again the power-law exponent in the emissivity function. We can conclude that clumping may cause a 5%–20% error in $D_A$.

Spatially resolved X-ray spectroscopy, as can be performed using Chandra and XMM, will help to estimate the clumpiness of the IC gas. Emission lines between 0.5 and 1.5 keV originating in cool regions, such as the Fe L-shell lines and H- and He-like lines from N, O, Ne, Mg, will provide a strong test on multiphase models. Unfortunately, this method will not constrain all types of clumpiness. Hughes & Birkinshaw (1998) and Mason (1999) suggest that since SZ/X Hubble constant determinations are not very different already discussed, ellipticity introduces scatter rather than potential model fit gives an unbiased estimation of $D_A$.
Roettiger et al. (1997) suggest that clusters at the early stages of merging should be excluded from distance determinations. Dynamically active clusters may be recognized from their galaxy velocity distributions. Clusters with dynamical activity will have a large $\beta$ discrepancy ($\Omega_{\text{fit}} \neq \Omega_{\text{spec}}$, see details in Sarazin 1988). Anisotropy in galaxy velocity distribution also signals dynamical activity. More relaxed systems, like merging clusters after they reached quasi-equilibrium, may be modeled and included in the distance determination. Simulations help to analyze individual merging clusters (Roettiger, Burns, & PKinney 1995; Gomez et al. 2000); adjustments to the initial parameters of merging clusters are made until the resulting merged cluster has the observed SZ and X-ray appearance and galaxy velocity distribution.

The $\beta$-model gives divergent masses if not truncated at some finite radius, which is usually taken to be about 10 core radii. This fact is a sign that at large radii, the $\beta$-model cannot be correct. When calculating the distance to a cluster using the isothermal $\beta$-model, we assume an infinite extent, and therefore the *finite extent* of clusters introduces a systematic bias in $D_A$. The SZ effect is more sensitive to outer regions than X-ray bremsstrahlung since it is proportional to $n_e^2$, unlike X-ray bremsstrahlung, which is proportional to $n_e$. In theory, the SZ effect measurements are more suitable for studying the outer regions of clusters and also are subject to more error caused by the finite extent of clusters.

Inagaki et al. (1995) discussed the effect of finite cluster sizes. They find that the ratio between finite truncated and full $\beta$-models is

$$\frac{\delta D_A}{D_A} = \left( \frac{1 - \left\{ B_q(3\beta - 2 - 1, 1/2) \right\} / \left\{ B(3\beta - 2 - 1, 1/2) \right\}}{1 - \left\{ B_q(3\beta - 2 - 1, 1/2) \right\} / \left\{ B(3\beta - 2 - 1, 1/2) \right\}} \right)^2,$$

where $q = 1/(1 + p^2)$, $p = R_{\text{cut}} / r_{\text{core}}$, $B$ and $B_q$ are the beta and incomplete beta functions, respectively, and assuming no change in $\Omega_{\text{fit}}$. The results of Inagaki et al. (1995) suggest that this effect can cause an underestimation of about 10%–20% of $D_A$ for typical parameter values (see also Puy et al. 2000). By contrast, Birkinshaw & Hughes (1994) and Holzapfel et al. (1997), analyzing Abell 2218 and Abell 2163, respectively, find that finite cluster extent contributes only about 6% and 2% overestimate error, respectively, in $D_A$. Observationally, Molnar (2001) attempted to find the outer cutoff of Abell 3571 using RXTE scans across the cluster. Abell 3571 seems to extend well beyond its virial radius; however, this conclusion is not strong because of poor statistics in the data.

Clearly, more observations are needed to find out the extent to which the $\beta$-model is correct and how the transition happens from the cluster to the surrounding regions (dominated by filaments according to numerical simulations). A falling temperature profile with radius is a sign of deviations from the $\beta$-model and may provide the best tracer of additional structure of this type. We expect errors from this effect to be reduced greatly when the new spectral information from Chandra and XMM is available.

One of the most important contaminants of the SZ effect is emission from radio point sources. Such emission, especially at the center of the cluster, will decrease the fitted amplitude of the SZ decrement in the Rayleigh-Jeans frequency region and cause an underestimate in the distance of the cluster. Using the sensitivity of the instrument, one can estimate the maximum flux density of unresolved point sources that can contribute to this error: for BIMA and the RT, the systematic errors are in the 10%–15% range. The level of radio source confusion varies with the observing technique used. Multibaseline interferometric observations have the most favorable confusion level. A deep survey, carried out at different frequencies or interferometer baselines, would help to find point sources up to a limit when the error due to unresolved point sources would be negligible relative to other errors. Interferometric measurements also have the advantage over single-dish observations of being able to monitor the brightnesses of (potentially variable) point sources while simultaneously measuring the SZ effect.

As Loeb & Refregier (1997) pointed out, systematic effects in the determination of the baseline for the SZ effect arise from gravitational lensing by the cluster gravitational potential. The brightnesses of point sources are enhanced by lensing, which brings them above the detection threshold, and thus they are removed from the field. This overremoval of point sources from the background lowers the background flux relative to a control field and thus leads to an overestimate of the SZ signal and an underestimate of about 10% of $D_A$. However, this effect is important only at frequencies lower than about 30 GHz, and its presence can be tested (and corrected for) using the model cluster mass distribution that can be obtained by conventional analyses of the X-ray data for each cluster (see also Blain 1998).

Other, probably small, effects contributing to the error budget, which should be checked and treated individually when necessary, are contributions in the radio band from synchrotron emission from resolved halo (and other) sources in clusters, free-free emission from cool gas in spirals, free-free emission from radio halos in the X-ray band (Birkinshaw 1979), and any contribution from the nonthermal SZ effect. As was pointed out by Molnar & Birkinshaw (1999), the Kompaneets equation and its relativistic extensions are equivalent to a single scattering approximation; thus, a small effect will arise from ignoring finite optical depth (multiple scattering). Any nonthermal population of IC electrons would produce a nonthermal SZ effect. There are some theoretical arguments for such a population (Petrosian 2001; Blasi 2001; Colafrancesco 1999; Sarazin 1999), and some observations have detected the expected excess hard X-ray emission (e.g., in the Coma Cluster), but such excess X-ray emission is rare, and hence, the nonthermal electron population is weak (Fusco-Femiano et al. 2001; Maloney & Bland-Hawthorn 2001). Clusters with complex morphology should not be used for distance determinations because of the difficulty in their modeling. If substructure in high-redshift clusters (as in RX J1347–1145; see Komatsu et al. 2001) is common, one should check such individual clusters for complex morphology using a high-resolution instrument.

The *cumulative* systematic effect resulting from all these sources of error is best addressed via numerical simulations. Yoshikawa, Itoh, & Suto (1998) used numerical simulations to estimate the systematic error from fitting $\beta$-models to clusters. They used a spatially flat fiducial model with $\Omega_m = 0.3$, $\Omega_{\Lambda} = 0.7$, and $h = 0.7$ for their simulations. They found that the isothermal $\beta$-model describes clusters well in both SZ and X-ray imaging. They found that even though
the fitted $\beta$-model parameters were different when fitted to SZ or X-ray images (because of nonisothermality, nonasphericity, and clumpiness), the systematic errors in the Hubble constant (and so in distances) are negligible at low redshifts. At high redshift ($z \approx 1$), about a 20% overestimate of $D_A$ might occur, mainly because of nonisothermality and asphericity. Inagaki et al. (1995) find that the cumulative effect of nonisothermality and asphericity leads to about a 10%–20% overestimate of $D_A$ since the overestimate due to nonisothermality is larger than the underestimate due to clumpiness (although in their simulations, $\Omega_m = 1$).

3.3. Summary of the Error Budget

The new generation of satellites will allow us to determine more precisely the temperature profile and spatial structure of the ICM and thus minimize the systematic errors in $D_A$ that result from temperature variations. When necessary, the spatial resolution of these new instruments will allow us to model the gas in each individual cluster beyond the spherical isothermal $\beta$-model. Numerical modeling of individual clusters will help us to derive their physical parameters. The improved spatial resolution will also allow us to study clumpiness, which is another important source of uncertainty. In general, those effects that have a different spectral signature than the static SZ effect should be separated from the SZ effect using multifrequency observations.

Based on our evaluation of the error budget, we estimate that the random error in $D_A$ achievable in the near future using known techniques, assuming that lensing measurements will be used to eliminate errors from cluster asphericity, might be as low as 7%. This estimate is made up by quadrature combination of the errors, with the dominant terms being from $T_e$ (5%) and spatial fitting (5%). A systematic error of 5% might also be obtained, although this would be difficult.

Evolution effects are clearly important limitations in determining cosmological parameters. The advantage of the SZ/X method is that as long as hydrostatic equilibrium and simple geometry hold, it should be reliable even if scaling laws (e.g., mass-temperature) evolve. However, a detailed analysis of evolution effects is out of the scope of our paper. We restrict ourselves to simply studying the effect of an additional systematic error in the $D_A$ determination with a linear gradient with redshift.

4. CONSTRAINTS ON COSMOLOGICAL PARAMETERS

As discussed earlier, in the near future SZ surveys will discover hundreds of high- and low-redshift clusters, and we can expect Chandra and XMM observations of hundreds of clusters. However, accurate angular diameter measurements require long SZ and X-ray integrations, and therefore we do not expect all discovered clusters to have accurate SZ/X distance measurements. If the SZ/X method is to be used to measure cosmological parameters, we should select a sample of clusters that minimizes the systematic errors ($\% 3$) while not requiring excessive observing time. As pointed out in Huterer & Turner (2001), the ideal distribution of clusters in redshift space would be a superposition of $P$ delta functions in redshift, where $P$ is the number of cosmological parameters to be determined. In practice, however, in any narrow redshift band we will not have enough clusters to achieve good statistics, and objects at $z > 1$ would need excessive integration times to reach high individual accuracies. A detailed study to select the optimum redshift distribution of clusters, taking into account the differing numbers of suitable clusters at different redshifts and the detailed characteristics of the possible instruments and observational strategies, is beyond the scope of our paper and more properly devolves on the groups proposing to construct such instruments. Here we do a simpler problem, by assuming that distances with similar accuracies are available for a set of clusters uniformly distributed in redshift space between $z = 0.01$ and $z = 1$. This choice covers almost half of the redshift interval most sensitive to the cosmological constant ($z = 0.5$–1.8) but excludes the more distant objects for which excessive integration times would be needed.

We carried out simulations to estimate the constraints from the angular diameter distance-redshift function on the parameter space defined by $\Omega_m$, $\Omega_\Lambda$, and $h$ and also by $\Omega_m$, $\Omega_\Lambda$, $w$, and $h$ (assuming a spatially flat geometry $\Omega_\Lambda = 1 - \Omega_m$). We simulated clusters using a spatially flat fiducial CDM model with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and $h = 0.65$. We use the $\chi^2$ statistic to evaluate errors from our simulations since we are dealing with large errors, 50% in cosmological parameters, and therefore the assumptions leading to the Fisher matrix formalism are not satisfied. We choose realizations that have best-fit values close to those of the input fiducial model and offset by the values of $\Delta \chi^2 = 3.53$, $8.02$, and $14.2$ appropriate for the number of fitted parameters to find the Gaussian $1$, $2$, and $3 \sigma$ error surfaces (defined by probability levels of $68\%$, $95.4\%$, and $99.73\%$, respectively).

As a first realization, we assumed a sample of 500 clusters uniformly distributed in redshift (so that $250$ lie at $z > 0.5$) with $4\%$ random error in $D_A$, which might be achieved if systematic errors can be tightly controlled (we assume that lensing measurements will be used to eliminate errors from cluster shape). We use these results to demonstrate the ultimate constraints on cosmological parameters that might be achievable from the SZ/X method. In Figures 2a, 2b, and 2c we show the results of fitting cosmological parameters to the observationally determined angular diameter distance function (1, 2, and 3 $\sigma$ concentric ellipses [solid lines] assuming three independent parameters, corresponding to $\Delta \chi^2$ of $3.53$, $8.02$, and $14.2$, appropriate for three fitted parameters). Two-dimensional projections of the three-dimensional surface of 3 $\sigma$ constraints are shown using dotted lines. In the $\Omega_m$-$\Omega_\Lambda$ plane (Fig. 2a) the error ellipses are elongated roughly along the line $2\Omega_m - \Omega_\Lambda = \text{constant}$. This is similar to the constraint obtained from the SNe Ia experiment, which measures cosmological parameters via the predicted apparent magnitude distribution and uses different redshift limits. Very low redshift measurements of clusters lead to error ellipses, which are elongated along the line $\Omega_m \Omega_\Lambda = \text{constant}$. As higher redshift objects are added, this elongated feature rotates counterclockwise. From Figure 2c, which shows constraints in the $h$-$\Omega_\Lambda$ plane, we can conclude that with the assumed accuracy, the cosmological constant is not well constrained. However, $h$ is well measured even without additional information. In general, the dispersion of measurements (1, 2, and 3 $\sigma$ contours) is determined by random and not systematic errors. Systematic errors introduce only bias, shifting the mean of the measurements away from the expected value in the parameter space while preserving their dispersion. A $\pm 3\%$ systematic error in the $D_A$ will not affect the results in the $\Omega_m$-$\Omega_\Lambda$ plane but simply shift the error ellipses we obtained from random errors...
up or down along the $h$-axis by 0.02 (3% of the fiducial value of $h$) since the amplitude of $D_A$ is set by the Hubble constant (Figs. 2b and 2c; 3σ solid ellipses above and below the 1, 2, and 3σ concentric ellipses corresponding to random errors [solid lines]). With the assumed random and systematic errors, the determination of $h$ becomes limited by systematic errors. We also carried out simulations assuming a systematic error with a gradient in redshift growing from 0% at $z = 0$ to ±3% at $z = 1$ in addition to the assumed 4% random error to study the effect of evolution. (Figs. 2b and 2c; short-dashed and dot-dashed 3σ lines). The error ellipses from random errors will be shifted in the $\Omega_m$-$\Omega_{\Lambda}$ plane (Fig. 2a). From Figures 2b and 2c we can conclude that the Hubble constant will not be affected by this type of systematic error, which follows from the fact that the Hubble constant is constrained by the low-redshift regime where the assumed systematic errors are small. $\Omega_m$ and $\Omega_{\Lambda}$ are strongly affected since their relation is determined by redshift between 0.5
and 1.8 (see Fig. 1), where the systematic errors become larger; thus, $\Omega_m$ and $\Omega_A$ become limited by this type of systematic error. An indication of systematic errors in $D_A$ from evolution might be found using an $\Omega_m$-$h$ plot (Fig. 2).

In Figures 3a, 3b, and 3c we show similar constraints on $\Omega_m$, $w$, and $h$ from the SZ/X method using the same fiducial CDM model and 500 clusters with a random error of 4% in $D_A$ as before (1, 2, and 3 $\sigma$ concentric ellipses [solid lines]). The constraints form a banana-shaped region elongated in the $\Omega_m$-$w$ plane (Fig. 3a). Systematic errors in the $D_A$ will not affect the results in the $\Omega_m$-$w$ plane but simply shift the ellipses from random errors up or down along the $h$-axis (Figs. 3b and 3c; 3$\sigma$ solid ellipses above and under the 1, 2, and 3 $\sigma$ concentric ellipses of random errors [solid lines]). Again, with the assumed 4% random and $\pm 3\%$ systematic errors, the $h$ determination is going to be limited by systematic errors. We also carried out simulations adding a systematic error with a gradient in redshift growing from 0% at $z = 0$ to $\pm 3\%$ at $z = 1$ to the assumed 4% random error (Figs. 3b and 3c; short-dashed and dot-dashed 3 $\sigma$ lines). Again, we find that $\Omega_m$ and $w$ are strongly affected; their determination is limited by the assumed systematic errors. As before, the Hubble constant determination is not affected by this type of systematic error. Also, an indication of sys-
tematic errors in $D_A$ from evolution might be found using a $w-h$ plot (Fig. 3c).

From Figure 2c, which shows constraints in the $\Omega_m-h$ plane, we can conclude that with the assumed accuracy, the cosmological constant is not well constrained. From Figure 3a we can see that $w$ is also poorly constrained by the SZ/X method. Clearly, whichever set of parameters is to be estimated, other cosmological measurements are needed to constrain these parameters further.

One of the most promising other experiments to measure cosmological parameters is based on CMB fluctuations. We estimated how well we can determine cosmological parameters if we add constraints from CMB experiment, which we would expect to be particularly useful since the strongest dependency in this experiment is on the space curvature (the total average density in the universe). As an illustration of the power of combining these techniques, we used the position of the first Doppler peak in the angular power spectrum ($\Omega_m h^2 = \text{constant}$) as observables from primordial fluctuation studies (see, e.g., Zaldarriaga et al. 1997).

The position of the first Doppler peak can be expressed as

$$l_{\text{peak}} = k_{\text{peak}} r(z_*) ,$$

where $k_{\text{peak}}$ and $r(z_*)$ are the first peak in $k$-space and the effective distance to the last scattering surface at $z_*$,

$$r(z_*) = \frac{1}{\sqrt{K}} \mathcal{S} \left\{ \sqrt{K} [\eta(0) - \eta(z_*)] \right\} ,$$

where $K$ is the curvature, $\mathcal{S}$ is the sin, sinh, and identity function for models with negative, positive, and flat space-time, and $\eta$ is the conformal time. In $k$-space in a CDM model,

$$k_{\text{peak}} \approx c_1 + c_2 w_m + c_3 w_m^2 + c_4 w_h + c_5 w_h^2 + c_6 w_m w_h ,$$

where $w_m = \Omega_m h^2$, $w_h = \Omega_h h^2$, and the coefficients are $c_1 = 0.0112$, $c_2 = 0.0441$, $c_3 = -0.043$, $c_4 = -0.0496$, $c_5 = 2.65$, and $c_6 = 0.162$ (White 1998). We used the approximation of Hu & Sugiyama (1996) for the redshift of the last scattering surface

$$z_* = 1048(1 + 0.0012 w_m^{-0.738})(1 + g_1^{(m)}) ,$$

where

$$g_1 = \frac{0.0783 w_m^{-0.238}}{1 + 39.5 w_m^{0.763}}$$

and

$$g_2 = \frac{0.560}{1 + 21.1 w_h^{1.81}} .$$

In Figures 2a, 2b, and 2c we overplot the 3 $\sigma$ error region based on the position of the first Doppler peak in the CMB power spectrum (long-dashed lines). This region is roughly aligned along the line $\Omega_m + \Omega_\Lambda = \text{constant}$ because of the strong dependence of the position of the first Doppler peak on the total space curvature. For demonstration purposes, we choose $l_{\text{peak}} = 245 \pm 10$, where we assume a 3 $\sigma$ error range of $\Delta l = 10$. As expected, constraints on cosmological parameters from SZ/X distance-redshift relation and the position of the first peak in the CMB fluctuations are highly complementary. For our assumed set of clusters, uniformly sampled in $z < 1$, these two constraints are nearly orthogonal to each other in the $\Omega_m-\Omega_\Lambda$ plane (Fig. 2a). The constraints are also complementary in the $h-\Omega_m$ and $h-\Omega_\Lambda$ planes (Figs. 2b and 2c). Geometrically, we have narrow banana-shaped constraints from the SZ/X method elongated in the $\Omega_m-\Omega_\Lambda$ plane and narrow sheetlike constraints $h = \text{constant}$ from the position of the first peak of the CMB fluctuation spectrum. The intercept of these constraints gives us stringent constraints on the following cosmological parameters: $\Omega_m$, $\Omega_\Lambda$, and $h$ can be determined to $\pm 0.08$, $\pm 0.1$, and $\pm 0.015$, respectively (3 $\sigma$, assuming 4% random error in $D_A$). Note that while the effect on $\Omega_m$ and $\Omega_\Lambda$ from reasonable redshift-independent systematic errors is negligible, $h$ would be dominated by systematic errors: a $\pm 3\%$ systematic error would result in a $\pm 0.02$ change in $h$. A systematic error that grows from 0% to $\pm 3\%$ at $z = 1$ would cause an additional $0.05$, $0.05$, and $0.005$ error in $\Omega_m$, $\Omega_\Lambda$, and $h$, respectively. Constraints from the SZ/X method are also orthogonal to those from cluster evolution (compare constraints on the $\Omega_m-\Omega_\Lambda$ plane; our Fig. 2a and Fig. 1 of Holder et al. 2001). In Figures 2a, 2b, and 2c we also overplot constraints from $\Omega_m h^2 = \text{constant}$ assuming a 10% error in its determination from CMB experiments (triple-dot–dashed lines), as suggested by studies based on the characteristics of the MAP experiment (Zaldarriaga et al. 1997). These constraints seem to be less useful in this parameter space than those from the position of the first Doppler peak.

Constraints from the location of the first peak in the CMB fluctuations are not so useful when considering the alternative set of parameters $w$, $\Omega_m$, and $h$. The $l \approx 210$–240 constraint leads to surfaces that lie almost parallel to the ellipsoids derived from the SZ/X method (compare our Fig. 3a and Fig. 4 of White 1998). In Figures 3a, 3b, and 3c we overplot constraints from $\Omega_m h^2 = \text{constant}$, again assuming a 10% error in its determination (triple-dot–dashed lines). Combining constraints from the SZ/X method and those from $\Omega_m h^2 = \text{constant}$ (based on CMB fluctuation analysis), we obtain stringent constraints on $w$ and $h$ ($\leq 0.2$ and $\pm 0.015$, respectively; 3 $\sigma$). Note that the effect on $w$ from redshift-independent systematic errors is negligible and that, as before, $h$ is dominated by systematic errors: a $\pm 3\%$ systematic error would result in a $\pm 0.02$ change in $h$. A systematic error that grows from 0% to $\pm 3\%$ at $z = 1$ would cause an additional $0.1$ in $w$. From Figure 3b ($\Omega_m$–$h$ plane), we see that a gradient in the systematic error in redshift might be recognized using the $\Omega_m h^2 = \text{constant}$ constraint. Also, constraints from the shape of the power spectrum of CMB fluctuations (as will be achieved by MAP and Planck) are nearly orthogonal to those from the SZ/X method (compare our Fig. 3a and Fig. 13 of Huterer & Turner 2001). Comparing our Figure 3a to Figures 8 and 9 of Haiman et al. (2001), we can see that constraints on the $\Omega_m$–$w$ plane from distance function and cluster evolution are complementary. Also, constraints on $w$ from the SZ/X method combined with cluster abundance would put stringent constraints on $w$.

Since the banana-shaped constraints from the SZ/X method in the parameter spaces defined by $(\Omega_m, \Omega_\Lambda, h)$ or $(\Omega_m, w, h)$ are nearly orthogonal to constraints from cluster evolution, a strategy for determining cosmological constants based only on clusters is likely to be successful. It seems to be possible to choose the parameters of the set of
clusters used in this work such that constraints from the SZ/X method and cluster evolution can be made orthogonal and thus optimized for separating cosmological parameters. Clusters can provide a powerful, independent test for cosmological parameters.

Note that our constraints in the $\Omega_m$-$w$ plane are curved toward the $w$-axis (Fig. 3a; solid and dotted lines), while the constraints of Huterer & Turner (2001) from SNe Ia (which is basically a constraint from distance-redshift function; see their Fig. 23) show no sign of curvature. This is due to the fact that they used the Fisher matrix formalism, and we evaluated the $\chi^2$ statistic directly. Our results show that the likelihood function is strongly non-Gaussian on the $\Omega_m$-$w$ plane.

As a second realization, we carried out simulations with 70 clusters (35 high-redshift clusters; $z > 0.5$), a number likely to be observed in the next few years, and estimated how well we can constrain cosmological parameters. We assumed the same fiducial cosmological model as before: $\Omega_m = 0.3$, $\Omega_b = 0.7$, and $h = 0.65$. For this simulation, however, we assumed a random error of 7% in the angular diameter distance, which might be achievable in the near future (we assume that lensing measurements will be used to eliminate errors from cluster shape).

In Figures 4a, 4b, and 4c we show the resulting 1, 2, and 3 $\sigma$ constraints on the parameter space defined by ($\Omega_m$, $\Omega_b$, $h$), corresponding to $\Delta \chi^2$ of 3.53, 8.02, and 14.2, respectively, appropriate for three fitted parameters (solid lines). Long-dashed lines show the constraints from the position of the first Doppler peak. As before, we assumed $l_{\text{peak}} = 245 \pm 10$ (3 $\sigma$ range). We also overplot constraints from $\Omega_m h^2$ = constant assuming a 10% error in its determination from CMB experiments (triple-dot-dashed lines). From these figures we conclude that by using as few as 35 high-redshift (and 35 low-redshift) clusters, with a random error of 7% in $D_A$, we can constrain $\Omega_m$, $\Omega_b$, and $h$ within $\pm 0.2$, $\pm 0.2$, and $\pm 0.04$ (3 $\sigma$ errors), respectively. Note that the effect of redshift-independent systematic errors in $D_A$ on $\Omega_m$ and $\Omega_b$, occurring in practice, is negligible, but a $\pm 5\%$ systematic error would result in an additional $\pm 0.035$ error in $h$. Assuming that we know our redshift-independent systematic error is less than 15% at the 3 $\sigma$ level, with a $7\%$ random error combined in quadrature, we would be able to determine $h$ with an error of $\pm 0.11$. A systematic error that grows from 0% to $\pm 5\%$ at $z = 1$ would cause an additional $\pm 0.1$, $\pm 0.1$, and $\pm 0.01$ error in $\Omega_m$, $\Omega_b$, and $h$, respectively (Fig. 4; short-dashed and dotted-dashed lines). This means that as few as 70 clusters, with the errors likely to be achieved in the next few years, would be sufficient to exclude models with zero cosmological constant with high significance. These results would be independent of the SNe Ia data; thus, they would provide a robust check to the supernova results.

In Figures 5a, 5b, and 5c we show the 1, 2, and 3 $\sigma$ constraints on the parameter space defined by ($\Omega_m$, $w$, $h$), with the $\Omega_b = 1 - \Omega_m$ constraint (solid lines). We overplot constraints from $\Omega_m h^2$ = constant assuming a 10% error as before (triple-dot-dashed lines). From Figure 5a we see that we will be able to determine the equation-of-state parameter $w$ with an accuracy of 0.45 (3 $\sigma$), even with reasonable systematic errors in $D_A$; with additional information from CMB experiments or SZ effect number counts. A systematic error that grows to $\pm 5\%$ at $z = 1$ would cause an additional error of 0.2 in $w$ (Fig. 5; short-dashed and dot-dashed lines).

5. CONCLUSION

We have estimated the accuracy achievable in the determination of cosmological parameters using the SZ/X method of distance determination. This method uses a sample of clusters to map the distance-redshift relation, which is a sensitive probe of cosmological parameters. The advantages of this well-known method are (1) unlike other distance determination methods, it depends only on the geometry of the universe and its average densities, (2) it is a physical method, based on relatively simple gravitational virialization of clusters (as opposed to complicated physics and chemistry involved in galaxy formation and supernova explosions), and (3) a number of clusters are available for observation, and thus systematic effects can be reduced by using many clusters or selecting clusters appropriately to reduce systematics. The necessary data should be available within the next few years.

The SZ/X method, like other cosmological tests, can constrain well only some combination of some cosmological parameters. We have shown that constraints on ($\Omega_m$, $\Omega_b$, $h$) from the $D_A(z)$ function measured in $z = 0$–1 are nearly orthogonal to constraints from the first peak of the CMB fluctuations and also to constraints from cluster evolution. Constraints on ($\Omega_m$, $w$, $h$) from the $D_A(z)$ function are complementary to those from cluster evolution (compare our Fig. 2 to Figs. 8 and 9 from Haiman et al. 2001). In general, $D_A$ provides constraints on cosmological parameters similar to those from the SNe Ia method. Constraints from cluster evolution [the source counts $N_S(z)$ and $N_X(z)$] are similar to constraints from the full CMB fluctuation spectrum, as will be measured by MAP and Planck.

This result suggests that cosmological tests using only clusters of galaxies can be an important independent check on the values of cosmological parameters measured by other techniques. Cluster-based methods have systematic errors, which are unrelated to those of other methods of measuring cosmological parameters (e.g., from CMB fluctuations and SNe Ia). Furthermore, when the set of cosmological parameters is further extended with additional components of density or structure parameters, joint analyses using multiple cosmological tests will be essential to remove the parameter degeneracy exhibited by any one test.

We demonstrated the effect of systematic errors on the determination of cosmological parameters. If random errors can be kept at a few percent level, systematic errors of similar magnitude will dominate the error in the Hubble constant. $\Omega_m$, $\Omega_b$, and $w$ are not affected by redshift-independent systematic errors. We also showed that a systematic error with a linear gradient in redshift will not affect Hubble constant determinations but causes systematic shifts in the estimates of $\Omega_m$, $\Omega_b$, and $w$.

We showed that in the near future, even with only 35 high-redshift (and 35 low-redshift) clusters, the SZ/X method, combined with the position of the first peak in the power spectrum of CMB fluctuations, will provide enough accuracy to exclude $\Omega_b = 0$ models with high confidence even with constant and redshift-dependent systematic errors (see Fig. 4a). Also, the SZ/X method, combined with the $\Omega_m h^2$ = constant constraint, or cluster abundance, will allow us to determine the equation-of-state parameter $w$ to within 0.45 (3 $\sigma$; Fig. 5a), even with usual (redshift-independent) systematic errors.
This idealized discussion of the SZ/X method and its errors in the determination of cosmological parameters could be improved by using the detailed characteristics of specific instruments and observing strategies. Departures from the assumptions made here could cause increases or decreases in the level of error in the derived parameters, with the largest changes likely for different redshift samplings.

We conclude that the determination of the angular diameter distance-redshift function using the SZ effect and X-ray thermal bremsstrahlung emission from clusters of galaxies can be used with confidence to constrain cosmological parameters. In general, clusters of galaxies alone can be used to constrain cosmological parameters independently from other methods. Clusters lead to parameter limits that are competitive with using other techniques.

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Fig. 4.—Same as Fig. 2, but with only 70 clusters (with 35 at $z > 0.5$) assuming a 7% random error in the angular diameter distances. We omit the redshift-independent systematic error ellipses since they simply correspond to the same shifts on the $h$-axis seen in Fig. 2 but retain the redshift-dependent systematic error ellipses corresponding to a 5% gradient in the error in $D_A$. 

Fig. 4a

Fig. 4b

Fig. 4c
Fig. 5.—Same as Fig. 3, but with only 70 clusters (with 35 at $z > 0.5$) assuming a 7% random error in the angular diameter distances. We omit the redshift-independent systematic error ellipses since they simply correspond to the same shifts on the $b$-axis seen in Fig. 2 but retain the redshift-dependent systematic error ellipses corresponding to a 5% gradient in the error in $D_A$.

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