Anomalous impurity resonance in graphene

Yu.G. Pogorelov

IFIMUP/Departamento de Física, Universidade do Porto, R. Campo Alegre, 687, Porto, 4169, Portugal

A Green function analysis has been developed for quasiparticle spectrum and localized states of a 2D graphene sheet in presence of different types of substitutional disorder, including vacancies. The anomalous character of impurity effects in this system is demonstrated, compared to those in well known doped semiconductors, and explained in terms of conical singularities in the band spectrum of pure graphene. The criteria for appearance of localized states on clusters of impurity scatterers and for qualitative restructuring of band spectrum are established and a phase diagram in variables “disorder” vs “electron density” is proposed.

PACS numbers: 03.65.Pm; 71.30.+h; 71.55.-i; 81.05.Uw

There is a growing attention to electronic properties of a single carbon layer known as graphene. Its 2D honeycomb lattice defines a peculiar band structure with two nodal points in the Brillouin zone where conical energy surfaces (with zero effective mass) of conduction and valence bands touch each other. This gives rise to electronic dynamics of relativistic Dirac type, extraordinary for condensed matter, and generates such unusual phenomena as half-integer Hall effect and, possibly, the magnetic catalysis of an excitonic gap. For Dirac quasiparticles under realistic perturbation, phenomena as half-integer Hall effect and, possibly, the magnetic catalysis of an excitonic gap are sometimes contradictory and not fully confirmed by the existing experimental data. To this time, the disorder effects in graphene were theoretically studied, searching for weak localization in this 2D electronic system under weak scattering (Born limit) or for strong localization under infinitely strong (unitary limit) perturbation. This work is aimed on a consequent description of restructured electronic spectrum, at arbitrary perturbation strength and in a rather broad range of impurity concentration, and on specifics of this restructuring for Dirac quasiparticles under realistic perturbation, compared to usual quasiparticles with parabolic dispersion.

Let us start from the simplest tight-binding Hamiltonian restricted to nearest neighbor hopping

$$H = t \sum_{n, \delta} a_n^\dagger b_{n+\delta} + h.c.,$$

where $t$ is the hopping amplitude, $a_n$ and $b_{n+\delta}$ are the Fermi operators of (spinless) electrons on sites of type 1 and 2 respectively, and atomic energy on each site is chosen zero. The vectors $\delta$ point to three nearest neighbors of a site (the vectors for a type 1 site being the inverses of those for a type 2 site, see Fig. 1a). Passing from site operators to plane waves: $a_n = N^{-1/2} \sum_k e^{-i k \cdot n} a_k$ and $b_n = N^{-1/2} \sum_k e^{-i k \cdot n} b_k$ ($N$ being the number of cells in the lattice), and then to the eigen-modes

$$\alpha_k = \frac{1}{\sqrt{2}} \left( e^{i \phi_k/2} a_k + e^{i \phi_k/2} b_k \right),$$

$$\beta_k = \frac{1}{\sqrt{2}} \left( e^{i \phi_k/2} b_k - e^{-i \phi_k/2} a_k \right),$$

the Hamiltonian, Eq. 1, is diagonalized:

$$H = \sum_k \varepsilon_k \left( \alpha_k^\dagger \alpha_k - \beta_k^\dagger \beta_k \right),$$

with eigen-energies $\varepsilon_k = 2 t |f_k|$. Here the function $f_k = \sum_{q=0}^\infty e^{i k \cdot q} F_q |f_k|$ vanishes near two isolated points in the Brillouin zone: $K = (0, 2\pi/3a)$ and $-K$ (Fig. 1). Due to the absence of inversion symmetry for the point group $C_3$, this vanishing is linear in small difference $q = k - K$ or $q = k + K$: $f_k \equiv f_q \approx \frac{\sqrt{3}}{2} (q_x - i q_y)$ defining the conical form of isoenergetic surfaces $\varepsilon = \pm \varepsilon_q$ where

$$\varepsilon_q \approx \hbar v_F q$$

with the Fermi velocity $v_F = \sqrt{3} t a / \hbar$. The following analysis of this system is restricted to the low energy physics which is essentially determined by the vicinities of two nodal points $\pm K$. The four relevant long-wave modes, picked up from the eigen-modes, Eq. 2, near each nodal point, can be combined into the Dirac 4-spinor $\psi_q$ whose components (in common notation) are: $\psi^{(+)}_q = \alpha_{q+K}$, $\psi^{(+)}_q = \alpha_{q-K}$, $\psi^{(-)}_q = \beta_{q+K}$, $\psi^{(-)}_q = \beta_{q-K}$. Here, “particles” and “antiparticles” obviously correspond to electrons and holes and the “Dirac spin” indices to the nodal points (while the physical spin indices stay suppressed). The respective Dirac form of the Hamiltonian, Eq. 3 is

$$H = \sum_q \varepsilon_q \psi_q^\dagger \gamma_0 \psi_q,$$
to study the effects of localized perturbations on its dynamics, which is our main purpose here. To this end, we adopt the Lifshitz model of impurity perturbation [17], supposing a certain shift $V$ of the on-site energy at random sites of the lattice. This model looks more adequate to the case of rare defects in graphene, than the alternative choice [13] of Anderson model with random perturbations at each lattice cite [14]. We denote $p_1$ the defect sites of type 1 with concentration $c_1$ and $p_2$ those of type 2 with concentration $c_2$ (not necessarily equal to $c_1$), the total impurity concentration being $c_1 + c_2 = c \ll 1$. This perturbation scatters the modes of Eq. [4] accordingly to the Hamiltonian

$$H' = \frac{V}{2N} \sum_{q, q'} \psi^+_q \left\{ e^{-i\varepsilon_q} \hat{U}_1 e^{i\varepsilon_{q'}} \sum_{p_1} e^{i(q' - q)\cdot p_1} + e^{i\varepsilon_q} \hat{U}_2 e^{-i\varepsilon_{q'}} \sum_{p_2} e^{i(q' - q)\cdot p_2} \right\} \psi_{q'},$$

(6)

with $\hat{U}_1, 2 = (\hat{\tau}_0 \mp \hat{\tau}_1) \otimes (\hat{\sigma}_0 \mp \hat{\sigma}_1)$. The equations of motion for perturbed GF matrix $[20]$ with Hamiltonian $H + H'$ can be solved in the T-matrix approximation. For the most important, momentum-diagonal GF, this results in

$$\langle \langle \psi_q | \psi_{q'} \rangle \rangle = \left( \hat{G}_q^0 \right)^{-1} - \hat{\Sigma} \left( \varepsilon \right)$$

where the self-energy matrix is

$$\hat{\Sigma} \left( \varepsilon \right) = \left( c_1 \hat{U}_1 + c_2 \hat{U}_2 \right) \frac{V}{2D(\varepsilon)}.$$  

(7)

Its denominator $D(\varepsilon)$ at an energy $\varepsilon = \varepsilon_{res}$ such that $\text{Re} \, D(\varepsilon_{res}) = 0$. But, accordingly to Eq. [8] this requires a strong enough perturbation: $|V| > V_{cr}$, where the critical value is $V_{cr} \approx 0.947W$. For a numerical check, we adopt the common value of $t \approx 2.5$ eV which leads to $V_{cr} \approx 3.6$ eV. The defects usually discussed in graphene are vacancies [12], for which the “acceptor” perturbation parameter $V$ can be approximated by the 1st ionization potential of carbon [21]: $V \approx I_C \approx 11.3$ eV $\approx 3W$. It results in a low resonance energy: $\varepsilon_{res} \approx 0.048W \approx 0.18$ eV, as shown in Fig. [2]. This can be compared with the two traditional impurities for carbon compounds: acceptors by boron (1st ionization potential $I_B \approx 8.3$ eV) and donors by nitrogen ($I_N \approx 14.5$ eV). The related estimates for impurity perturbation: $V_B \approx I_C - I_B \approx 3$ eV and $V_N \approx I_C - I_N \approx -3.2$ eV, make it less probable that these impurities produce resonances in graphene

where the $4 \times 4$ matrix $\hat{\gamma}_0 = \hat{\tau}_3 \otimes \hat{\sigma}_0$, the tensor product of Pauli matrices $\hat{\tau}_i$, acting on “particle-antiparticle” indices, and $\hat{\sigma}_i$, on “Dirac spin” indices. We describe the dynamics of this system by the (Fourier transformed) two-time Green functions (GF’s) [16], here combined into a $4 \times 4$ matrix $\langle \langle \psi_q | \psi_{q'} \rangle \rangle$. For the unperturbed system, Eq. [4] its exact form is $\langle \langle \psi_q | \psi_{q'} \rangle \rangle = \hat{G}_q^0 \delta_{q, q'}$ with

$$\hat{G}_q^0 = \frac{\varepsilon + \varepsilon_q \hat{\gamma}_0}{\varepsilon^2 - \varepsilon_q^2}. $$

(5)

The notable distinction of graphene from genuine relativistic systems of quantum field theory is the possibility

FIG. 1: 2D lattice structure of a graphene sheet. a) Rhombic primitive cell (shadowed) with two non-equivalent positions for carbon atoms, 1 (open circles) and 2 (solid circles), and elementary translation vectors $a_1$ and $a_2$ of length $a = 2.46\rho\AA$. b) Rhombic Brillouin zone (shadowed) with two non-equivalent nodal points, $K$ (open) and $-K$ (solid), and vectors of reciprocal lattice $b_1$ and $b_2$ of length $4\pi/(a\sqrt{3})$. 

The notation $\langle \langle \psi_q | \psi_{q'} \rangle \rangle = \hat{G}_q^0 \delta_{q, q'}$ with

$$\hat{G}_q^0 = \frac{\varepsilon + \varepsilon_q \hat{\gamma}_0}{\varepsilon^2 - \varepsilon_q^2}. $$

(5)

The notable distinction of graphene from genuine relativistic systems of quantum field theory is the possibility
matrix, through a spinor rotation:  

$$V \equiv \tau$$  

ceptor impurities at the choice of perturbation parameter  

$$\hat{U}$$  

is symmetric in these indices. Thus the impurity scattering in this case perturbs only two symmetric modes, whose dispersion is described by the equations

$$\varepsilon_{1,2}(q) - \text{Re}\Sigma(\varepsilon_{1,2}(q)) \pm \varepsilon_q = 0. \quad (11)$$

Using the explicit function \( g(\varepsilon) \), Eq. 8 and the linear law \( \varepsilon_q \), Eq. 3 we obtain the dispersion curves \( \varepsilon_{1,2}(q) \) as shown in Fig. 2 (the picture for “donor” perturbation \( V < 0 \) will be simply inverted, \( \varepsilon \rightarrow -\varepsilon \), by the particle-antiparticle symmetry).

If Eqs. 14 were valid down to \( q \rightarrow 0 \) (though in fact they are not), they would describe the shift of nodal point energy from zero to a finite value \( \varepsilon_{1,2}(0) \equiv \varepsilon_0 = \text{Re} \Sigma(\varepsilon_0) \). This value grows with the defect concentration as

$$\varepsilon_0 \approx \begin{cases} cV, & c \ll c_0, \\ \varepsilon_{\text{res}}(1 - c_0/c), & c \gg c_0, \end{cases} \quad (12)$$

where the characteristic concentration \( c_0 \sim (\varepsilon_{\text{res}}/W)^2 \) defines, as will be seen below, the threshold for qualitative restructuring of the spectrum. For \( c > c_0 \), there appears a certain interval of negative dispersion in the valence band (a Z-like feature in Fig. 2). It resembles the known situation near resonances in parabolic bands [20], however in this case negative dispersion appears rather far from the resonance \( \varepsilon_{\text{res}} \) (which lies within the conductance band). And the restructured conductance and valence bands are both shifted towards this resonance, not repelled from it (as commonly for doped semiconductors).

To clarify the physical origin of this anomalous behavior, it is instructive to compare it with the well studied case of shallow donor levels below a 2D parabolic conductance band  

$$E_k = \hbar^2 k^2/2m \quad (25).$$

The respective quasiparticle energy formally coincides with the above expression for \( \Sigma(\varepsilon) \) but including the lattice sum \( g(\varepsilon) = N^{-1} \sum_k (\varepsilon - E_k)^{-1} \approx \text{Re}(E_m/\varepsilon) \) and a weak attractive perturbation \( 0 < -V < E_m = 2\hbar^2/ma^2 \). Its expansion near the localized donor level \( \varepsilon_{\text{loc}} \approx -E_m e^{-E_m/|V|} \) reads: \( \Sigma(\varepsilon) \approx cV|\varepsilon_{\text{loc}}|/(\varepsilon - \varepsilon_{\text{loc}}) \), so that the perturbed band energies \( \varepsilon = E_k + \Sigma(\varepsilon) \) are repelled from this level. In fact, this repulsion is due to the simultaneous action of attractive perturbation \( V \) in the numerator and denominator of \( \Sigma(\varepsilon) \) (so it remains also true for repulsive perturbation, \( V < 0 \), by shallow acceptors). In contrary, such expansion for the actual case of linear band dispersion reads: \( \Sigma(\varepsilon) \approx -cV|\varepsilon_{\text{res}}|/(\varepsilon - \varepsilon_{\text{res}}) \), like if the repulsive perturbation \( V > 0 \) in the denominator of \( \Sigma(\varepsilon) \) turns to be attractive in its numerator (or vice versa for donors), resulting in the overall attraction of the bands to the impurity resonance.

However, the above referred dispersion curves are only reliable when the respective band states are well defined, which can be checked by the Ioffe-Regel-Mott (IRM) criterion [24] that the quasiparticle lifetime is long enough.
state with energy $\varepsilon$: $|\psi_z\rangle = \sum_n (\psi_n^{(1)} a_n^\dagger + \psi_n^{(2)} b_n) |0\rangle$, whose amplitudes on type 1 and 2 sites $\psi_n^{(1,2)}$ are found from the Schrödinger equation, $(H + H' - \varepsilon)|\psi_z\rangle = 0$. The solutions having central symmetry with respect to the impurity site $\mathbf{p}$:

$$
\psi_n^{(1)} = \psi_p^{(1)} \frac{V}{N} \sum_{\mathbf{q}} \frac{\varepsilon \hat{G}^{\mathbf{q},\mathbf{p}}(p,n)}{\varepsilon^2 - \varepsilon_q^2} \\
\psi_n^{(2)} = \psi_p^{(2)} \frac{V}{N} \sum_{\mathbf{q}} \frac{\varepsilon q \hat{G}^{\mathbf{q},\mathbf{p}}(p,n)}{\varepsilon^2 - \varepsilon_q^2} \\
\psi_n^{(1,2)} = \psi_p^{(1,2)} \frac{V}{N} \sum_{\mathbf{q}} \frac{\varepsilon q \hat{G}^{\mathbf{q},\mathbf{p}}(p,n)}{\varepsilon^2 - \varepsilon_q^2}.
$$

are proportional to the locator Green functions. Their long distance asymptotics (at $|\mathbf{n} - \mathbf{p}| \gg a$) is:

$$
\psi_n^{(1,2)} \sim \left(\frac{a}{|\mathbf{n} - \mathbf{p}|}\right)^{3/2} \cos \left(\frac{|\mathbf{n} - \mathbf{p}|}{r(\varepsilon)}\right),
$$

with the characteristic length $r(\varepsilon) = (Wa)/(\sqrt{\varepsilon}) \gg a$. In particular, the value $r_0 = r(\varepsilon_{res})$ defines the length scale of local perturbation of quasiparticle spectrum near a defect. So, a qualitative restructuring of this spectrum should happen if local perturbations begin to overlap, at characteristic concentration of defects: $c_0 \sim a^2/(\pi r_0^2) = (\varepsilon_{res}/W)^2$. For the considered vacancy model, this value is $\sim 0.2\%$. In fact, when the concentration $c$ reaches this level, important changes occur in the spectrum characteristics calculated from the GF, Eq. 10.

For instance, the quasiparticle density of states (DOS) is $\rho(\varepsilon) = (\pi N)^{-1} \text{Im} \text{Tr} \sum_{\mathbf{n}} \langle \psi_{\mathbf{n}} | \hat{G} | \psi_{\mathbf{n}} \rangle$. At $c \ll c_0$, it is close to the unperturbed DOS $\rho_0(\varepsilon) = (\pi N)^{-1} \text{Im} \text{Tr} \sum_{\mathbf{q}} \hat{G}^2_{\mathbf{q}} = |\varepsilon|/W^2$, but at $c \sim c_0$ a hump appears in $\rho(\varepsilon)$ near the resonance $\varepsilon_{res}$ and then progressively increases at $c > c_0$ (Fig. 4).

Next, the Fermi energy $\varepsilon_F$ is obtained in function of $c$ from the condition $\int_{-\infty}^{\varepsilon_F} \rho(\varepsilon) d\varepsilon = (1 - c)/2$, and (in the given model) it displays a rapid initial growth at $c \ll c_0$ (inset in Fig. 4), entering the conductance band and so realizing an anomalous n-conductance at nominal p-doping. But, before $c$ approaches $c_0$, the level $\varepsilon_F$ makes a sharp downturn and crosses the monotonically growing nodal energy $\varepsilon_0$, thus restoring a usual type of conductance.

At last, the width $\Gamma$ of the energy interval near $\varepsilon_0$ (hatched in the inset in Fig. 4), where the IRM criterion ceases to hold, is as narrow as $\sim \varepsilon_0 c/(c_0 \ln(1/c_0))$ at $c \ll c_0$ but suddenly expands at $c \sim c_0$ to the whole range from $\varepsilon_0$ to $\varepsilon_{res}$, and then grows as $\sim \sqrt{c}/\ln(1/c)$ at $c \gg c$. This energy interval is filled by localized states, however their localization is realized as a rule on certain clusters of defects, rather than on single defects.

FIG. 3: Check for the IRM criterion at different concentrations of defects. a) Below the critical concentration, $c = 0.1\% < c_0$, all the states are band-like except a very narrow vicinity of the nodal point energy $\varepsilon_0$ (shown under a great magnification in the inset), where the states are localized. b) At approaching the critical concentration, $c = 0.2\% \approx c_0$, the area of localized states rapidly expands on the whole interval between $\varepsilon_0$ and $\varepsilon_{res}$. c) At overcritical concentration, $c = 1\% \gg c_0$, the area of localized states of width $\Gamma$ extends beyond the resonance level $\varepsilon_{res}$.

A closer insight on the localization process can be obtained, beginning from the case of a single impurity on site $\mathbf{p}$ (say, of type 1) and constructing a quasiparticle.
On each side of this interval, the localized states are separated from the band-like states by a Mott’s mobility edge, defining a metal-insulator transition when the Fermi energy crosses this edge.

It is of interest to compare this type of spectrum restructuring and the quadratic law $c_0 \sim (\epsilon_{\text{res}}/W)^2$ with the known examples for non-relativistic spectra. Thus, the low-frequency acoustic resonance $\omega_{\text{res}} \sim \omega_D \sqrt{M/M'} \ll \omega_D$ by impurities with mass $M' \gg M$ in a crystal with atomic mass $M$ and Debye frequency $\omega_D$ gives rise to splitting of phononic spectrum near $\omega_{\text{res}}$ with opening of a quasi-gap (seen as a dip in DOS) of width $\sim c\omega_D^2/\omega_{\text{res}}$, at surpassing the characteristic impurity concentration $c_0 \sim (\omega_{\text{res}}/\omega_D)^3$ (the cubic law) [22]. This dip corresponds to repulsion of band levels from the impurity resonance (in contrast to the hump in DOS and attraction of band levels to the resonance expected in graphene). Another example is the donor level $\epsilon_{\text{loc}}$ near a parabolic conductance band [22], which rapidly expands and merges with this band when the donor concentration exceeds $c_{\text{loc}} \sim \epsilon_{\text{loc}}/E_m$ (linear law), in the 2D case, or $c_{\text{loc}} \sim (\epsilon_{\text{loc}}/E_m)^{3/2}$, in the 3D case. The 3/2 law also defines the characteristic concentration $c_0 \sim (\omega_{\text{res}}/J)^{3/2}$ of weakly coupled ($J' \ll J$) impurity spins in a Heisenberg ferromagnet when the magnon spectrum splits near the resonance frequency $\omega_{\text{res}} \sim J' \ll J$, and the cubic law $c_0 \sim (\omega_{\text{res}}/J)^3$ defines such effect in an aniferromagnet [24]. Hence the case of defects in graphene differs from all those, even at seemingly identical linear dispersion (as for phonons and antiferromagnons).

Observable effects related to the restructured electronic spectrum of graphene with defects need a detailed consideration which shall be done elsewhere. Here we only outline a general framework for this study, through the phase diagram in concentration of (vacancy) defects $c$ vs. electron density $n$ (the latter can be independently tuned, e.g., by the external bias voltage or by adding non-resonant donor and acceptor impurities). Accordingly to the concentration dependence of characteristic energy parameters in the inset in Fig. 4, this diagram (Fig. 5) presents three different phase regions: the “inversely doped” metallic phase $M_1$ with $\epsilon_F > \epsilon_0$, the insulating phase $I$ with $\epsilon_F$ inside the domain $\Gamma$ of localized states, and the “normally doped” metallic phase $M_2$ with $\epsilon_F < \epsilon_0$. Within the latter phase, a specific region of inverse, negative dispersion $N$ can be distinguished, where the Fermi carriers should behave like holes by their electric charge but like electrons by their velocity, the latter turning extremely high near the inversion line between $M_2$ and $N$. This peculiar dynamics can produce even more extraordinary anomalies under applied magnetic field (which effect is already anomalous in the pure graphene). The latter factor and also interaction between electrons in presence of defects need a more involved treatment. All the mentioned features can be potentially used for applications in a specific “relativistic” electronics, based on properly doped graphene, and undoubtedly deserve further attention.

In conclusion, the effects of local perturbation by various types of impurities, including vacancies, in single layer graphene were studied through Green function techniques. Possibility of low energy resonance near nodal points in the relativistic electronic spectrum was indicated for the case of vacancies and the conditions for qualitative restructuring of the quasiparticle spectrum.
were established. A significant distinction of this restructuring for relativistic electrons, compared to the known disordered systems, is demonstrated and the phase diagram in variables “disorder vs electron density” is proposed, indicating possible practical applications of such system.

I would like to thank V.A. Miransky, J.P. Carbotte and S.G. Sharapov for very useful discussions of disorder effects in graphene, and Department of Applied Mathematics of University of Western Ontario for kind hospitality during my sabbatical visit there, when this work was done. The support from Portuguese Fundação para a Ciência e a Tecnologia is gratefully acknowledged.

[1] K.S. Novoselov, A.K. Geim, S.V. Morozov, D. Jaing, Y. Zhang, S.V. Dubonos, I.V. Grigorieva, and A.A. Firsov, Science 306, 666 (2004).
[2] P.R. Wallace, Phys. Rev. 77, 622 (1947).
[3] G. Semenoff, Phys. Rev. Lett. 53, 2449 (1984).
[4] K.S. Novoselov, A.K. Geim, S.V. Morozov, D. Jaing, M.I. Katsnelson, I.V. Grigorieva, S.V. Dubonos, and A.A. Firsov, Nature 438, 197 (2005).
[5] Y. Zhang, Y.-W. Tan, H.L. Störmer, and P. Kim, Nature 440, 201 (2005).
[6] V.P. Gusynin and S.G. Sharapov, Phys. Rev. Lett. 95, 146801 (2005); N.M.R. Peres, F. Guinea, and A.H. Castro Neto, cond-mat/0512091.
[7] V.P. Gusynin, V.A. Miransky, I.A. Shovkovy, Phys. Rev. Lett. 73, 3499 (1994); E.V. Gorbar, V.P. Gusynin, V.A. Miransky, I.A. Shovkovy, Phys. Rev. B 66, 045108 (2002).
[8] D.V. Khveshchenko, Phys. Rev. Lett. 87, 206401 (2001); 87, 246802 (2001).
[9] Y. Kopelevich, P. Esquinazi, J.H.S. Torres, and S. Moehlecke, J. Low Temp. Phys. 119, 691 (2000).
[10] B.I. Shklovskii and A.L. Efros, Electronic properties of doped semiconductors, Springer (1984).
[11] P.A. Lee, Phys. Rev. Lett. 71, 1887 (1993).
[12] W.A. Atkinson, P.J. Hirschfeld, and A.H. McDonald, Phys. Rev. Lett. 85, 3922 (2001).
[13] H. Suzuura and T. Ando, Phys. Rev. Lett. 89, 266603 (2002).
[14] D.V. Khveshchenko, cond-mat/0002398.
[15] N.M.R. Peres, F. Guinea, A.H. Castro Neto, cond-mat/0512091.
[16] V.L. Bonch-Bruevich and S.V. Tyablikov, The Green function method in statistical mechanics. Amsterdam: North-Holland, 1962.
[17] I.M. Lifshitz, Sov. Phys. JETP 17, 1159 (1963).
[18] D.N. Sheng, L. Sheng, Z.Y. Weng, cond-mat/0602190.
[19] P.W. Anderson, Phys. Rev. 109, 1492 (1958).
[20] M.A. Ivanov, V.M. Loktev, Yu.G. Pogorelov, Phys. Rep. 157, 209 (1987).
[21] H. O. Pierson, Handbook of Carbon, Graphite, Diamond, and Fullerences: Properties, Processing, and Applications. New York: Noyes Data. (1993).
[22] M.A. Ivanov, V.M. Loktev, Yu.G. Pogorelov, Yu.V. Skripnik, Sov. J. Low. Temp. Phys. 17, 377 (1991).
[23] A.F. Ioffe and A.R. Regel, Progr. Semicond. 4, 26 (1960); N.F. Mott, Adv. Phys. 16, 149 (1967).
[24] Yu.M. Kagan, Ya.A. Iosilevskii, JETP 15, 182 (1962).
[25] M.A. Ivanov, Sov. Phys. Solid State 12, 1508 (1971).