Comparison of polynomial and nonlinear models on description of pepper growth

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ABSTRACT: Pepper (Capsicum sp.) is important for the Brazilian agribusiness, serving as raw material for the food, pharmaceutical and cosmetic industries. The adequate evaluation of its plants growth may help in understanding the causes of crops yield variation, with it being able to be studied by regression models, which help to adequate the management with the different phenological phases. This study aimed to compare the fit of linear Polynomial model and the Logistic and Gompertz nonlinear models in the description of pepper plants growth from the Doce cultivar. Estimates were obtained by the Gauss-Newton method, with the quality of fitted models compared by graphical analysis and evaluators: adjusted coefficient of determination ($R^2_{adj}$), Residual Standard Deviation (RSD) and the corrected Akaike Information Criterion ($AICc$). Residues were normal, independent and homocedastic at 5% level of significance. All models properly described the height of the Doce cultivar. The Logistic model was the most adequate according to the fitting evaluators, having higher value of $R^2_{adj}$ and lower RSD and $AICc$ values.

Key words: doce cultivar; growth rates; sigmoid curve

Comparação dos modelos polinomial e não lineares na descrição do crescimento de pimenta

RESUMO: A pimenta (Capsicum sp.) é importante para o agronegócio brasileiro, como fonte de matéria-prima para as indústrias alimentar, farmacêutica e cosmética. A avaliação do crescimento adequado das suas plantas pode auxiliar no entendimento das causas de variação de produtividade da cultura e pode ser efetuada por meio de modelos de regressão, que ajudam a adequar o manejo com as diferentes fases fenológicas. O objetivo deste trabalho foi comparar o ajuste dos modelos linear Polinomial e não lineares Logístico e Gompertz na descrição do crescimento de plantas da cultivar Doce. As estimativas foram feitas utilizando o método de Gauss-Newton, a comparação da qualidade de ajuste dos modelos foi feita por meio de análise gráfica e dos avaliadores: coeficiente de determinação ajustado ($R^2_{adj}$), Desvio Padrão Residual (DPR) e o Critério de Informação de Akaike corrigido ($AICc$). Os resíduos foram normais, independentes e homocedásticos ao nível de 5% de significância. Todos os modelos descreveram adequadamente a altura da cultivar Doce. O modelo Logístico foi o mais adequado conforme os avaliadores de ajuste, apresentou maior valor de $R^2_{adj}$, menores DPR e $AICc$.

Palavras-chave: cultivar doce; taxas de crescimento; curva sigmoide
Introduction

Pepper (Capsicum sp.) is a vegetable from the Solanaceae family, originating from the tropical regions of the Americas, whose production is led by India, followed by China, Thailand, Ethiopia and Indonesia. It is a socioeconomic important crop, serving as a source of jobs and income, as raw material in the pharmaceutical, cosmetic and food industries, as well as being used as an ornamental plant. Pepper consumption is made in form of naturally preserved, sauces, spices, jellies, paprika and in natura. The crop is produced in all regions of Brazil, mostly by small farmers, with the states of Minas Gerais, Goiás, São Paulo, Ceará and Rio Grande do Sul as the largest producers (Santana et al., 2017; Rossato et al., 2018).

According to Finger & Pereira (2016), peppers produced worldwide belong to the Capsicum genus, of which five are more known and produced: C. annuum, C. baccatum, C. chinense, C. frutescens, and C. pubescens. This crop cultivars have three growth stages: vegetative, reproductive (flowering and fruiting) and fruit maturation. (Marinho et al., 2018). They have two growth types, determined and undetermined. Determined cultivars complete the vegetative phase before the beginning of the reproductive one, while the undetermined continue to grow after the first flowering-fruiting, producing new branches and fruit of different ripening stages until the end of the crop cycle, allowing for many harvests and higher yields with the plant growth (Vieira et al., 2014; Carmo et al., 2018).

According to Pedó et al. (2013b), the growth of the pepper plants depends on the cultivar and the growing conditions, reaching up to between 0.3 m and 3 m. Generally, undetermined cultivars are larger than those with a relatively short cycle, and are very demanding regarding cultural management due to the higher risk of lodging. According to Zanon et al. (2015), in crops with different growth habits, there is some overlapping of the vegetative and flowering phases, lasting longer in indeterminate cultivars due to the continuous growth of their plants. The longer this overlap period is, the greater will be the competition for nutrients, water and mineral salts between the two phases.

Zanon et al. (2015) consider that the longer period of overlapping between the vegetative and reproductive phase in indeterminate cultivars may give them greater ability in adapting to different growing seasons and cultivation conditions. Cultivars of this type have the ability to quickly recover from short periods of water stress and high temperatures, which gives them the preference of many producers, particularly in cold regions, with short summer period. These facts reinforce the need for further studies to understand the development of undetermined cultivars through appropriate techniques capable of determining the duration and overlapping of the phenological phases from the crops.

In general, in agronomic studies the growth analysis technique has been employed, performed through regression models. Several regression models can be found in the literature studying the growth of plants and other living beings, with the use of simple, polynomial and multiple regression more common, as can be observed in the studies of Pedó et al. (2013a) and Pedó et al. (2013b), both with the pepper crop. According to Mischan & Pinho (2014), the growth of living beings shows a distinct behavior, starting slow, passing to an exponential phase and tending to stabilize at the end. For this reason, many authors like Pereira et al. (2016), Ribeiro et al. (2018a) and Ribeiro et al. (2018b) recommend nonlinear models for growth description, as they are asymptotic and do not reach a maximum point as it happens with polynomial models. These models have the advantage of having smaller number of parameters, generally with biological interpretation, besides facilitating the estimation of daily growth rates, with the maximum rate occurring on the day of the model curve inflection (Gurgel et al., 2011). According to Lúcio et al. (2015) and Sari et al. (2019), through the inflection point (IP) it is possible to determine the phenological phases of the crop and its duration.

Estimate of parameters in nonlinear models is usually done by minimizing the sum of squares of the residuals, obtaining a system of normal equations (SNE) that requires the usage of iterative methods for its solution. The most commonly used iterative method in the literature is the Gauss-Newton (Prado et al., 2013; Fernandes et al., 2017). In linear models, the parameter estimate can also be done by the least squares method directly, because its SNE has explicit solution. Therefore, the present study aimed to compare the fittings of the linear Polynomial model and the nonlinear Logistic and Gompertz models in the description of the height growth of pepper from the Doce cultivar.

Materials and Methods

Height data of the pepper plants, Doce cultivar, were taken from an experiment that can be seen in more detail in Pedó et al. (2013a). In general, this experiment conducted in a greenhouse of the “arch pampean” model and the analyzes were held at the Plant Physiology Laboratory of the Federal University of Pelotas, in a region of temperate climate with well-distributed rains and hot summer, classified as the Cfa type by the Köppen classification.

Doce cultivar has undetermined growth and was sown on 11/02/2010, in expanded polystyrene trays with 128 cells, each one containing commercial H. Decker® as substrate. The trays were irrigated by using a floating system, keeping it at 50 mm high. When the plants had five leaves, on 12/13/2010, they were transplanted to beds of 5.0 x 1.20 m containing planosol-type soil and covered by a low-density black polyethylene film with 0.25 x 0.80 m spacing. Soil fertility was corrected according to the technical recommendations for the crop, and by drip-irrigation method the plants were irrigated after being transplanted, during 4 h d⁻¹ within a 48 h interval. Plants were collected from the fourteenth day after transplantation (DAT), and at regular intervals of fourteen days after transplantation until the end of the cultivation
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...indicated by Gurgel et al. (2011): were fit as well, taking into account the parameterization from Mischan & Pinho (2014), the parameters from these model do not possess practical or biological interpretation.

Logistic (Eq. 2) and Gompertz (Eq. 3) non-linear models were fit as well, taking into account the parameterization indicated by Gurgel et al. (2011):

\[ Y_i = \alpha_0 + \frac{(\alpha_1 - \alpha_0)}{1 + \exp(k(\beta - x_i))} + \epsilon_i \]

(2)

\[ Y_i = \alpha_0 + (\alpha_1 - \alpha_0) \exp(-\exp(k(\beta - x_i))) + \epsilon_i \]

(3)

in which \( Y_i \) (height in mm) is the dependent variable, \( x_i \) is the independent variable (age, in days after the transplant) and \( \beta \) are the model parameters, with \( i = 0, 1, 2 \) and 3. According to Ribeiro et al. (2018a), the parameters from these models do not possess practical or biological interpretation.

Results and Discussion

Polynomial model estimates were significant at 5% significance by the t test, except \( \beta_3 \) that it was not significant. As the fitting with the exclusion of the non-significant parameter \( \beta_3 \) produced a model with an even worse performance, so, the analysis was held with the complete model. In Logistic and Gompertz models, all estimates were significant at 1%, except for parameter \( k \) of the Gompertz model (p<0.05). This fact indicates that there is less probability (p<0.01) that the parameters of the Logistic model are equal to zero when...
compared to the Polynomial and Gompertz models. Thus, the inferences to be made based on the Logistic model are more acceptable, as displayed in Table 1.

The nonlinear Logistic model obtained estimates of 772.86 mm of upper horizontal asymptote and inflection point of approximately 54 DAT. The estimated lower asymptote was approximately 160.00 mm, which corresponds to the estimated mean plant height on the day of transplantation. Table 1 displays the values of the Shapiro-Wilk (SW), Breusch-Pagan (BP), and Durbin-Watson (DW) tests. Figure 1 shows the graphical analysis of the residuals of the Polynomial, Logistic and Gompertz models fitted for plant height of pepper from Doce cultivar. Results indicate that the residues are independent and identically distributed following a normal distribution with zero mean and constant variance.

Still in Table 1, it is observed that the values of the parameters bias of the Logistic and Gompertz models were higher than the value of 0.01, except the $k$ parameter. Thus, these parameters are responsible for the nonlinearity of the models. In general, the Logistic model has lower bias values in its parameters.

Results of this study corroborates with Pedó et al. (2013a), since they were close to the maximum height observed by the authors, which was of 780 mm in the Doce cultivar. Costa et al. (2015) evaluated the height growth of 40 accessions of peppers (Capsicum spp.) from different provinces of the Amazonas
state during 180 days and found that 60% of accessions grew, on average, 460 to 650 mm in height, 5% between 660 and 850 mm, and 12.5% over 850 mm. These results show that pepper plants have the ability to increase height growth the longer the growing cycle is, which may be explained by the fact that some accessions have indeterminate growth, such as the Doce cultivar.

It is worth emphasizing that height as a measure of growth is a variable affected by several factors. Sowing at a time outside the most appropriate period for cultivation may affect plant size. Height is also related to the plant population, to the used cultivar and the edaphoclimatic conditions.

Results of the fit quality evaluators indicated that the three models fit properly with the analyzed data. The intrinsic curvature measurement was less than 0.5 in both Logistic and Gompertz models, that is, it was not significant. On the other hand, parametric curvature values were higher than 0.5 in both models. Both measurements values were slightly lower for the Logistic model, which indicates that it is the most appropriate one. The significance of the parametric curvature measurement indicates that there is a slight deviation from linearity in both models, and the reliability of the estimates obtained by the Gompertz model is the most compromised. The Logistic model also obtained a higher value of $R^2_{adj}$ (Eq. 3), lower $RSD$ (Eq. 4), and $AICc$ (Eq. 5), according to Table 2. Thus, this model is the most suitable for the height growth data of pepper plants from Doce cultivar.

The nonlinear Logistic model has the advantage of presenting estimates with practical or biological interpretation, as it can be observed in Lúcio et al. (2016) in the growth study of the mean weight of the cherry tomato fruit, and in Bem et al. (2017) in the description of morphological characters of *Crotalaria juncea* L., unlikely the Polynomial linear model, which has no interpretation.

Archontoulis & Miguez (2015) state that nonlinear models are best suited for the growth study of linear measurements (height, length, diameter, etc.) obtained from intact plants (non-destructive evaluations). Since, besides their parameters being interpretable, they also present a superior asymptote that indicates the growth stabilization when compared to the polynomial models that present the maximum growth point, followed by a decrease. The authors also consider that nonlinear models have predictions that tend to be more robust than linear models.

According to Mischan & Pinho (2014), in some cases, when comparing the Polynomial model with the nonlinear one, such as the Logistic case, better evaluators of the first model can be obtained. However, the authors also stated that the Polynomial model is inadequate for the growth study of linear measurements in intact plants, because it does not stabilize the end of growth, considering that it is always irreversible.

Figure 2 shows the graphical fit of the linear Polynomial, Logistic and Gompertz models. It can be graphically seen that the models fit properly with the observed data, and the Polynomial model can better capture the growth onset while the Gompertz tends to overestimate the height. At the end of growth, the Logistic model best describes plant height, stabilizing at around 108 DAT. The Polynomial model shows a maximum at the 86 DAT and then begins to decrease, similar to that observed by Pedó et al. (2013a).

The curve of the first derivative of the Polynomial model showed maximum HGR at approximately 53 DAT, with this as its PI, when the plants had a mean height of 458.77 mm. Based on the estimates of Table 1, in the Logistic model, the estimated IP occurred at 54 DAT (Figure 3), and the mean height was 468.00 mm (Figure 2).

In the IP, estimated rates for plant height growth were 10.51; 19.62 and 18.92 mm d$^{-1}$ by the Polynomial, Logistic and Gompertz models, respectively. At 97 DAT the estimated HGR by the Polynomial model is zero, indicating the point of maximum growth from which it begins to decay. Visual analysis of the best model, the Logistic one, indicates that the

Table 2. Measurements of the intrinsic curvature ($c'$) and the parametric effect ($c^0$), fit quality evaluators ($R^2_{adj}$, $RSD$ and $AICc$) of the Polynomial, Logistic and Gompertz models, fitted to the height data of pepper from Doce cultivar (mm), sown on 11/02/2010 in a greenhouse of the “arch pampean” model from the Federal University of Pelotas.
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Doce cultivar had three growth phases, with the vegetative phase lasting up to approximately 40 DAT, when the estimated growth rate was 12.03 mm d\(^{-1}\). The second phase (flowering-fruiting) ranged from 40 to 70 DAT, and at approximately 80 DAT, the growth begins to slow down until reaching maximum HGR. However, at the third phase of growth, the maturation, occurred between 70 and 112 DAT, around 98 DAT estimates of HGR began to stabilize from 0.91 mm d\(^{-1}\) to 0.22 mm d\(^{-1}\) at 112 DAT.

These results corroborate with Marinho et al. (2018), with the authors considering that the pepper plants have three growth phases: vegetative, flowering-fruiting and fruit maturation. The authors consider that the duration of the growth stages of pepper crop depends on the species, the type of cultivar as well as the edaphoclimatic conditions.

Analyzing the duration of the phenological phases, it is observed that there was overlapping of vegetative growth and maturation in approximately 72 days. These results are close to those obtained by Costa et al. (2015), when analyzing the growth of 40 accessions of pepper, having observed vegetative growth up to 60 DAT, flowering between 61 to 90 DAT, while the fruiting occurred between 60 to 180 DAT coinciding with the fruit maturation. It is verified that the authors registered vegetative and reproductive phase overlapping in approximately 120 days, a difference of 24 days in relation to the results of this study, which is a result of the difference in the cultivation cycles (112 and 180 days).

Conclusions

The Polynomial, Logistic and Gompertz models adequately describe the analyzed data.
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