Brane Quintessence

Kei-ichi Maeda
Department of Physics and Advanced Research Institute for Science and Engineering, Waseda University, Shinjuku, Tokyo 169-8555, Japan

We propose a new quintessence scenario in the brane cosmology, assuming that a quintessence field $Q$ is confined in our 3-dimensional brane world. With a potential $V(Q) = \mu^{\alpha+4} Q^{-\alpha}$ ($\alpha \geq 2$), we find that the density parameter of the scalar field decreases as $\Omega_Q \sim a^{-(\alpha-2)/(\alpha+2)}$ in the epoch of quadratic energy density dominance, if $\alpha \leq 6$. This attractor solution is followed by the usual tracking quintessence scenario after a conventional Friedmann universe is recovered. With an equipartition of initial energy density, we find a natural and successful quintessence model for $\alpha \gtrsim 4$.

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One of biggest mysteries in the Universe is a cosmological constant, if it exists\cite{1}. Recent observation of Type Ia Super Novae suggests that the expansion of the Universe is accelerating now\cite{2}. This acceleration could be explained by a cosmological constant with $\Omega_{\Lambda} \sim 0.7$. Although a cosmological constant seems to be preferred from a view point of the age of the Universe and a structure formation in the Universe, one may wonder why the cosmological constant is almost the same order of magnitude as the present mass density of the Universe. It is very difficult to explain such value from a view point of particle physics, which usually predicts much larger value of a vacuum energy. If the observation is confirmed, we will face on a serious problem in fundamental physics. One of the way out would be the so-called “quintessence”\cite{3}, in which a potential of a scalar field plays as a decaying cosmological constant\cite{4,5}. In the quintessence scenario, a scalar field with some specific potential shows an interesting behavior called “tracking” (or “scaling”) in its evolution\cite{6,5}. The energy of a scalar field tracks the radiation energy (or matter energy) for rather long time and then eventually becomes dominant after matter dominant era. In order for this scenario to work well, a scalar field will catch up neither radiation nor matter so early. This naïvely means that the initial energy of a scalar field was very small. In the quintessence models, however, the final value of quintessence energy density is insensitive to the initial conditions because of its attractor property. For example, a potential of $V = \mu^{\alpha+4} Q^{-\alpha}$ will catch up matter density late in the evolution of the Universe for a wide range of initial conditions, if $\mu$ is suitably chosen. However, this suitable choice of $\mu$ may need a finetuning\cite{6}. Some modified models have been proposed to solve this problem\cite{7,8}. Here, in order to resolve this mystery, we propose a scenario based on a brane universe.

Recently, a new type of world view has been proposed, which is called a brane world based on a superstring or M-theory\cite{9,10}. Our 3-dimensional universe is described by a brane in a higher-dimension\cite{11}, and usual matter field and force except for gravity are confined on the brane. Among them, Randall and Sundrum’s second model\cite{12} gives an interesting picture of gravity, i.e. although the extra-dimension is not compact, four-dimensional Newtonian gravity is recovered in five-dimensional anti-de Sitter spacetime (AdS$_5$) in low energy limit. Since gravity in those brane world could be quite different from the 4-dimensional Einstein theory, many authors discussed interesting difference from a conventional cosmological model\cite{13,14}. The quadratic term of energy-momentum appears and may be important in the early stage of the universe, and dark “radiation”, which is constrained by a successful nucleosynthesis, may also exist\cite{14,15}. In particular, the former will change the expansion law of the Universe in the very early stage, then we may expect some important difference in a “quintessence” scenario.

Here, we will show that the quadratic term will indeed change drastically the evolution of a scalar field and its density parameter will decrease in time until the quadratic term becomes unimportant. This provides us a successful and natural scenario for a conventional quintessence model.

In this letter, we shall analyze a cosmological solution based on the Randall-Sundrum model, although the similar result would be obtained in other brane world models. Assuming flat Friedmann-Robertson-Walker spacetime in our brane world, we find the effective Friedmann equation as follows\cite{16,17}:

$$H^2 = \frac{\kappa_4^2}{3} \rho + \frac{\kappa_5^2}{36} \rho^2 + \frac{C}{a^5}$$

(1)

where $\kappa_4^2 = 8\pi G_N$ and $\kappa_5^2$ are 4- and 5-dimensional gravitational constants, respectively, $H = \dot{a}/a$ is the Hubble parameter, and $C$ is a constant, which denotes “dark” radiation\cite{18}. The 4-dimensional Planck mass $m_4 (= \kappa_4^{-1} = 2.4 \times 10^{18}\text{GeV})$ and the 5-dimensional one $m_5 (= \kappa_5^{-2/3})$ will be used in the following discussion. Here the 4-dimensional cosmological constant is set to zero. We also assume that all matter field including a scalar field $Q$ are confined on the brane. As for
the potential, we consider one of typical quintessence-type potential, i.e. $V(Q) = μ^{α+4}Q^{-α}$, although the present mechanism may work for other potentials. It is worth noting that this potential may be naturally derived in some supersymmetric QCD with a fermion condensation. In that case, $α$ is given by the numbers of colors and of flavors [19]. Since the energy density decreases when the universe expands, the quadratic term ($ρ^2$) dominates in the early stage of the universe. The conventional Friedmann universe is recovered after when the quadratic and the linear terms are equal at $ρ = ρ_c = 12κ^2_t/κ^2_s = 12m^6_s/m^4_5$. Since we know well about the behavior of the scalar field in the linear-term dominant stage, which is the conventional cosmological model, we first study the behavior of the scalar field in the quadratic-term dominant stage. In order to study the dynamics of a scalar field, we discuss two cases separately: the radiation dominant era and the scalar-field dominant era.

First we analyze the case with radiation dominance. The Friedmann equation is approximated as $H = \sqrt{κ/3}ρ$, where we assume that the Universe is expanding ($H > 0$). This gives the expansion law of the Universe as $a \propto t^{1/4}$. Then the equation of motion for the scalar field is

$$\ddot{Q} + \frac{3}{4t} \dot{Q} - αμ^{α+4}Q^{-(α+1)} = 0. \quad (2)$$

We find an exact solution for $α < 6$, that is

$$Q \propto t^{4/α} \quad \text{and} \quad ρ_Q \propto t^{-2/α} \propto t^{-8/α}. \quad (3)$$

The density parameter of the scalar field, which we denote as $Ω_Q$, is

$$Ω_Q = \frac{ρ_Q}{ρ_p + ρ _m} \sim \frac{ρ_Q}{ρ_t} \sim β \left( \frac{a}{a_s} \right)^{-2(α-2)/α}, \quad (4)$$

where $β (≤ 1)$ is a constant and $a_s$ is an “initial” scale factor. If $α > 2$, $Ω_Q$ decreases with time, just contrary to a tracking solution. The scalar field energy decreases faster than the radiation energy. This is a new interesting feature for a quintessence because smallness of a quintessence-field energy when the Universe enters the conventional stage could be dynamically obtained. We will show that it is really the case. If $α = 2$, $Ω_Q$ is constant until the linear term becomes dominant. This is the so-called “scaling” solution.

We also find that the above solution [3] is an attractor, and can show that any solutions in the radiation dominant era will eventually converge to this attractor solution [21]. The constant $β$ then denotes $Ω_Q$ when the Universe reaches this solution at $a = a_s$.

One may wonder if the scalar field initially dominates the radiation. If $α < 2$, the potential term will overcome the kinetic term, leading to an inflationary universe as $a \propto \exp[H_0(t-2-α)/2]$, where $H_0$ is a constant determined by $μ$ and $m_5$ [23]. For the case with $α = 2$, we find a power-law solution [21], i.e. $a \propto t^μ$ with $p = \frac{1}{5}(1 + \frac{1}{5}(μ/m_5)^6)$ and $Q = 2\sqrt{2}m^6_5t^{4/3}$. If $p > 1$ (i.e. $μ > 1.85m_5$), we have a power-law inflationary solution, which is an attractor of the dynamical system. While, if $p < 1/4$ (i.e. $μ < 1.26m_5$), this solution is no longer an attractor, leading to the radiation dominant era discussed above. If $α > 2$, we can show that a kinetic term of the scalar field will always become dominant even if we start with a potential dominance [20]. Since $ρ_Q \propto a^{-6}$ in the case of kinetic-term dominance, we find that the radiation will overcome the scalar-field energy and the Universe will evolve into the radiation dominant era discussed above. Since the solution [3] is a unique attractor in the radiation dominant era, any solution in $ρ^2$-dominant stage will approach to this attractor solution, if $2 < α < 6$. Then the evolution of $Ω_Q$ is given by Eq. (4).

As the universe expands and the energy density decreases below $ρ_c$, we find the conventional Friedmann universe, in which many authors studied quintessence models [3] [4] [5] [21]. During the radiation or matter dominant era, when the Universe evolves as $a \propto t^{1/2}$ or $t^{3/4}$, the tracking attractor solution in the present model was found, giving the evolution of $Ω_Q$ as

$$Ω_Q \sim ρ_Q/ρ_c \propto a^{5/6} \quad \text{(radiation dominant)}$$

$$Ω_Q \sim ρ_Q/ρ_m \propto a^{-3/2} \quad \text{(matter dominant)}. \quad (5)$$

In both radiation dominant and matter dominant cases, the energy density of the scalar field decreases slower than that of radiation or matter fluid, and will eventually overcome those, resulting in an accelerating Universe.

From [3] and [4], the present value of $Ω_Q$ is estimated as

$$Ω_{Q,0} = β \left( \frac{a_c}{a_s} \right)^{-2(α-2)/α} \left( \frac{a_{eq}}{a_c} \right)^{4/α} \left( \frac{a_0}{a_{eq}} \right)^{2/α} \left( \frac{T_c}{T_{eq}} \right)^{4/α} \left( \frac{T_0}{T_{eq}} \right)^{-4/α}, \quad (6)$$

where $a_c$, $a_{eq}$, and $a_0$ are scale factors at the time when the quadratic term of energy density drops just below the linear term, when radiation density becomes equal to matter density, and the present time, and $T_c$, $T_{eq}$, and $T_0$ are corresponding temperatures of the Universe, respectively. In the present brane quintessence model, $Ω_Q$ first decreases during the quadratically-energy dominant stage, and the scalar field energy could be very small when the conventional cosmology is recovered. This may explain naturally why the scalar field energy was so small in the early stage of the Universe in the conventional quintessence scenario.

Now we discuss about constraints on parameters to find a successful quintessence scenario. Since we do not know about initial condition, we set our initial time when
the attractor solution is reached \((a = a_s)\). In this case, we have only one unknown parameter \(\beta\) (the value of \(\Omega_Q\) at \(a = a_s\)). Although we do not know the value of \(\beta\), it should be smaller than unity because the attractor solution appears only in the radiation dominant era. If an equipartition for the energy of each particle is assumed at \(a_s\), we expect \(\beta \sim 1/g\), where \(g\) is a degree of freedom of particles. \(\beta\) could be smaller because the kinetic energy of the scalar field might be dominant before the attractor solution.

Setting \(T_0 = 2.73\) K and \(T_{eq} \sim 10^4\) K, and assuming \(\Omega_{Q,0} \sim 0.7\) and equipartition at \(a_s\) (i.e. \(\beta = 1/g = 0.01\)), we find a relation between \(T_s\) and \(T_c\), which is shown in Fig. 1 by a solid line for each \(\alpha\).

One stringent constraint in any cosmological models is nucleosynthesis. It must take place in the conventional radiation dominant era to explain the present amount of light elements. Therefore, \(T_s\) must be higher than the temperature at nucleosynthesis, \(T_{NS} \sim 1\) MeV. This constraint implies \(m_5 > 1.6 \times 10^4 (g/100)^{1/6} (T_{NS}/1\text{MeV})^{2/3}\) GeV, which is included in Fig. 1. The r.h.s. of the vertical line is the allowed region. If the second Randall-Sundrum model turns out to be a fundamental theory, in order to recover the Newtonian force above 1 mm scale, the 5-dimensional Planck mass is constrained as \(m_5 \geq 10^8\) GeV \([2]\), which is satisfied in the r.h.s. region of the dotted vertical line in Fig. 1.

Although we do not know the “initial” temperature \(T_s\), if the 5-dimensional spacetime is fundamental and gravity is unified at the energy scale \(m_5\), we expect \(T_s \lesssim m_5\). Even if the 5-dimensional theory is effective, \(T_s \lesssim m_5\) may be required to justify the present 5-dimensional analysis. This condition with the equipartition at \(a_s\) gives a constraint on the scale of the potential as \(\mu \gtrsim (0.2 - 0.3) m_5\). For reference, we have inserted three lines of \(T_s = 0.1 m_5\) (lower dotted line), \(m_5\) (solid line), and \(10 m_5\) (upper dotted line) in Fig. 1. If \(\alpha \gtrsim 4\), we find a successful quintessence scenario with natural conditions. On the other hand, if \(\alpha < 4\), either \(\beta\) should be much smaller than the value expected from equipartition, or \(T_s \gg m_5\), then we may need a fine-tuned or unnatural initial condition.

In Fig. 2, we show one example of the time variation of \(\Omega_Q\) for the case of \(\alpha = 5\) with \(\beta = 0.01\) and \(T_s = m_5\). For these parameters, we find that \(T_c \sim 280\) MeV \((\gg T_{NS})\), and \(T_s(= m_5) \sim 8.6 \times 10^6\) GeV.

FIG. 2. Time variation of \(\Omega_Q\) in terms of a scale factor \(a\), which present value is normalized to unity, for the model with \(V = \mu^3 Q^{-5}\). We set \(\beta = 0.01\) and \(T_s = m_5\). The energy of the scalar field drops faster than that of radiation in the quadratic-energy dominant stage. After the conventional Friedmann universe is recovered, the scalar field with very small energy density tracks radiation and matter fluid.

If \(\alpha \gtrsim 6\), the above solution \(\square\) is no longer an attractor. We can show that the kinetic term always dominates in the quadratic-term dominant stage. Then \(\Omega_Q\) drops as \(a^{-2}\) until the conventional cosmology is recovered at \(a_c\). However, we know that the kinetic term drops much faster than the potential term after \(a = a_c\) and the tracking solution eventually will be reached. Hence, we could approximate \(\Omega_Q\) at \(a = a_c\) by the potential term, i.e \(\Omega_Q \sim \beta (a_c/a)^{4-\alpha}\). Using this estimation, we find the relation between \(T_s\) and \(T_c\) to obtain \(\Omega_{Q,0} \sim 0.7\). Those results are also included in Fig. 1.

In the brane universe, “dark” radiation \(C(>0)\) may exist. If this term dominates, the present scenario may not work because it evolves as \(a^{-4}\) just as radiation. The preliminary analysis shows that the quadratic energy of the scalar field will drops as the same as the present model, but the linear term will increases because the expansion law of the Universe is \(a \propto t^{1/2}\). It will depend on the initial condition whether we find a successful model or not.

In this letter, we have discussed a quintessence model in the context of a brane world scenario. We show that the energy of a scalar field decreases faster than the radiation when the quadratic energy density is dominant. As a result, when we recover a conventional cosmology,
the density parameter of a scalar field is enough small for a scalar field to dominate just at present epoch. If our 3-dimensional brane universe starts at $T_a \sim m_5$, $\alpha \geq 4$ is required to find a natural quintessence scenario. We should note that the present model may not solve the coincidence problem, but will give a natural explanation of smallness of a scalar field energy in the early stage of the conventional cosmology. It is also important to point out that we could predict when the Universe is getting into an acceleration phase, if we know the values of fundamental parameters such as $m_5$ and $\mu$.

The present model prefers rather large value of $\alpha$ ($\alpha \gtrapprox 4$), but recent observation may force to the constraint of $\alpha \lesssim 2$. If this is the case, we have to look for other type of quintessence potential. For example, an exponential potential could be another candidate. The results for such a study will be published elsewhere.

The essential point in the present scenario is that the dynamics of the scalar field is completely modified in the quadratic energy dominant stage. Although we assume the Randall-Sundrum model in the present analysis, the quadratic energy term will usually appear in any brane universe models, then we expect the similar effect, which may provide us a successful quintessence. Since the dilaton field will also appear in a superstring or M-theory, it will also change the dynamics of the Universe, which requires further analysis.

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