Reaction cross sections for proton scattering from stable and unstable nuclei based on a microscopic approach

H. F. Arellano\textsuperscript{1,2,\ast} and M. Girod\textsuperscript{2}

\textsuperscript{1}Department of Physics - FCFM, University of Chile
Av. Blanco Encalada 2008, Santiago, Chile

\textsuperscript{2}Commissariat à l’Energie Atomique,
Département de Physique Théorique et Appliquée,
Service de Physique Nucléaire, Boîte Postale 12,
F-91680 Bruyères-le-Châtel, France

Abstract

Microscopic optical model potential results for reaction cross sections of proton elastic scattering are presented. The applications cover the 10-1000 MeV energy range and consider both stable and unstable nuclei. The study is based on \textit{in-medium} $g$-matrix full-folding optical model approach with the appropriate relativistic kinematic corrections needed for the higher energy applications. The effective interactions are based on realistic $NN$ potentials supplemented with a separable non-Hermitian term to allow optimum agreement with current $NN$ phase-shift analyzes, particularly the inelasticities above pion production threshold. The target ground-state densities are obtained from Hartree-Fock-Bogoliubov calculations based on the finite range, density dependent Gogny force. The evaluated reaction cross sections for proton scattering are compared with measurements and their systematics is analyzed. A simple function of the total cross sections in terms of the atomic mass number is observed at high energies. At low energies, however, discrepancies with the available data are observed, being more pronounced in the lighter systems.

PACS numbers: 25.40.Cm, 25.60.Bx, 24.10.Ht

Keywords: Proton scattering, optical model, total reaction cross-section

\textsuperscript{\ast}Electronic address: arellano@dfi.uchile.cl; URL: \url{http://www.omp-online.cl}
I. INTRODUCTION

Nucleon-nucleus integrated cross sections constitute a key observable in fundamental nuclear research as well as in applications of nucleon-induced reactions. These quantities are of particular importance in nuclear technology such as nuclear transmutation, nuclear waste treatment, safety assessment and medical therapy, among many examples. From a fundamental point of view, their description within global microscopic approaches constitutes a stringent test to the understanding of the underlying physics involved in the interaction of nucleons colliding with a nucleus. Depending on the energy range of the projectile, total cross sections may constitute an important input in the study of diverse phenomena. For instance, the study of $r$-processes in astrophysics require the knowledge of total cross sections at energies near and below a few MeV, whereas spallation applications may well require data up to GeV energies.

In this article we present a study of the systematics exhibited by proton-nucleus ($pA$) reaction cross sections based on the microscopic full-folding (FF) optical model potential approach [1, 2]. The study spans in energy from a few MeV up to 1 GeV, considering various even-even isotopes from carbon up to lead. The calculated reaction cross sections are compared with existing data, whereas results for proton scattering from unstable nuclei are examined as functions of the number of constituents of the target.

Research on proton reaction cross section of $pA$ elastic scattering has received significant attention during recent years. These studies include experimental [3], phenomenological [4, 5] and microscopic [6, 7, 8, 9] analyses. Global analyses in the framework of Glauber theory have also been reported [10]. Recent experimental efforts by Auce and collaborators [3] have been very valuable in reporting new measurements at intermediate proton energies. The phenomenological studies of Refs. [4, 5] represent attempts to provide simple parametrizations of the observed cross sections in terms of geometry and mass distribution of the targets. The microscopic studies reported in Refs. [6, 7, 8] are based on the referred ‘coordinate-space $g$-folding model’ [11], where a local medium-dependent $NN$ effective interaction is folded with the target ground-state mixed density. As a result, a nonlocal optical potential is obtained, where the non locality stems from the inclusion of the exchange term. The results reported here differ from the above $g$-folding model in two aspects. The first of them is that the antisymmetrized $NN$ effective interaction is handled in momentum space.
with no special considerations on an eventual local structure in coordinate space [1]. Thus, we retain the intrinsic off-shell behavior of the interaction. The second difference lies in the representation on the mixed density. Whereas the $g$-folding approach treats explicitly the mixed density extracted from shell-models, the results we report here rely on its Slater representation using only the diagonal elements. An assessment of this approximation has been made within the free $t$ matrix full-folding approach, where it is shown that it affects only slightly the differential scattering observables at momentum transfers ($q$) of $1$ fm$^{-1}$, becoming more visible at $q \geq 2.5$ fm$^{-1}$ (c.f. Fig. 3 of Ref. [12]).

This article is organized as follows: In Section II we outline the general framework upon which we base our study. In Section III we present results for the calculated reaction cross sections for $pA$ elastic scattering at energies between 10 MeV and 1 GeV. Additionally, we examine the systematics exhibited by the calculated reaction cross sections and the total cross sections for neutron-nucleus ($nA$) elastic scattering at energies above 400 MeV. Finally, in Section IV we present a summary and the main conclusions of this work.

II. FRAMEWORK

The microscopic approach we follow is the in-medium FF optical model approach [1], which is a realization of the double convolution of an $NN$ antisymmetrized effective interaction with the target ground-state mixed density. In its formulation, with the use of the Slater representation of the mixed density and the assumption of a weak dependence of the $g$ matrix on one of the momentum integrals, the FF optical potential for $pA$ collisions at beam energy $E$ takes the simplified form

$$U_{pp}(k', k; E) = \int dZ e^{i(k' - k) \cdot Z} \sum_{\alpha=p,n} \rho_{\alpha}(Z) \bar{g}_{p\alpha}[k', k; \rho_A(Z)], \quad (1)$$

where $\rho_{\alpha}$ is the point density of specie $\alpha$, and $\bar{g}_{p\alpha}$ represents an off-shell Fermi-averaged amplitude in the appropriate $p\alpha$ pair and evaluated at the local target density $\rho_A(Z)$. More explicitly, in an infinite nuclear matter model these density-dependent amplitudes are given by

$$\bar{g}_{NN}(k', k; \bar{p}) = \frac{3}{4\pi k^3} \int \Theta(\hat{k} - |P|) g(k_r', k_r; \sqrt{s}; \bar{p}) dP, \quad (2)$$
where \( g(k_r', k_r; \sqrt{s}; \bar{\rho}) \) corresponds to off-shell \( g \) matrix elements for symmetric nuclear matter of density \( \bar{\rho} \). The relative momenta, \( k_r', \) and \( k_r \), depend on the asymptotic momenta \( k', k \) as well as \( P \), the mean momentum of the struck target nucleon. Their general form is \( k_r = Wk - (1 - W)p \), with \( p = P/2 + k - k' \), and \( W = W(E; k', k, P) \). As there is no local prescription to handle the \( g \) matrix, the results we report here retain the intrinsic nonlocalities of the \( NN \) interaction. The \( s \)-invariant specifies the energy at which the interaction is evaluated, which depends on the beam energy \( E \) and kinematics of the interacting pair. Details about the implementation of the Fermi-averaged elements and relativistic kinematics corrections have been discussed in Ref. [2]. With the above considerations we obtain a nonlocal optical potential which is handled exactly within numerical accuracy.

An important element for the realization of the optical potential is the two-body effective interaction, which in our model takes the form of the nuclear-matter \( g \) matrix. This is based on the Brueckner-Bethe-Goldstone model for symmetric nuclear matter and described by the integral equation

\[
g(\omega) = V + V \frac{\hat{Q}}{\omega - \hat{e}_1 - \hat{e}_2} g(\omega),
\]

with \( V \) the free-space two-nucleon potential, \( \hat{e}_1 \) and \( \hat{e}_2 \) the quasiparticle energies, \( \hat{Q} \) the Pauli blocking operator and \( \omega \) the starting energy just above the real axis. In the case of the free \( t \) matrix applications, the Pauli blocking operator is set to unity and the nuclear self-consistent fields vanish. Thus, \( g \to t \), the \( NN \) scattering matrix in free space.

A physically acceptable \( g \) matrix for GeV nucleon energy applications requires a bare \( NN \) potential model consistent with its high energy phenomenology, particularly the presence of complex phase-shifts above pion-production threshold [13]. In our approach this is achieved by supplementing realistic \( NN \) potential models \( (V_R) \) with a separable term of the form [2]

\[
V = V_R + |\xi\rangle \Gamma(E) \langle\xi|.
\]

In this study the form factors \( |\xi\rangle \) are taken as harmonic oscillator states characterized with \( \hbar \omega = 450 \text{ MeV} \). The strength \( \Gamma(E) \) is energy and state dependent, becoming complex in those states where loss of flux is observed. These coefficients are calculated analytically for each state to reproduce the year-2000 \( np \) continuous-energy solution (SP00) of the phase-shift analysis by R. Arndt et al. [13]. With these considerations we are able to account for the absorption stemming from the elementary \( NN \) interactions. Since most realistic bare potentials do not share a common phase-shift data basis, in particular the SP00 solution, we
suppress the separable contribution below 300 MeV. The inclusion of the separable strength is done gradually out to 400 MeV, energy from which the full strength is taken into account.

Another important input in our calculations is the target ground-state radial density. These are obtained following self-consistent Hartree-Fock-Bogoliubov calculations with the Gogny force [14, 15]. This interaction contains a central finite-range term, a zero-range spin-orbit contribution and a zero-range density-dependent contribution. This approach has been instrumental for obtaining the spherical radial densities for $^{12}$C and the isotope families $^{12}$O, $^{34-64}$Ca, $^{50-86}$Ni, $^{80-100}$Zr, $^{96-136}$Sn and $^{176-224}$Pb included in this study.

III. APPLICATIONS

A. Scattering from stable nuclei

Following the considerations outlined above, we have calculated the reaction cross sections for proton-nucleus elastic scattering $\sigma_R$ at beam energies ranging from 5 MeV up to 1 GeV. The targets considered are $^{208}$Pb, $^{90}$Zr, $^{60}$Ni, $^{58}$Ni, $^{40}$Ca, $^{16}$O and $^{12}$C, whose proton and neutron root-mean-square (r.m.s.) radii are summarized in Table II. The results for $\sigma_R$ are shown in semi-log scale in Fig. II where the data are taken from Refs. [3] (open squares), [16] (filled circles) and [17] (triangles). The calcium and nickel data have been colored blue and red, respectively, to distinguish them when they overlap. The solid curves represent results from the FF optical model using the nuclear matter $g$ matrix based on the $np$ Argonne V18 reference potential [18], whereas the dashed curves represent results using the free $t$ matrix corresponding to the same bare interaction. As observed, the level of agreement between the calculated cross sections and the data is qualitatively different above 200 MeV from that below 100 MeV. At proton energies below 100 MeV—with the exception of $^{208}$Pb—all FF results overestimate $\sigma_R$, being more pronounced for the $^{12}$C and $^{16}$O targets. In these two cases differences of $\sim$100 mb at 30 MeV are observed, with no proper account of the maxima near 25 MeV exhibited by the data. Instead, the calculated cross sections decrease monotonically in the range 10–300 MeV. This feature is also observed in Ref. [9], where a much closer agreement with the data is reported. The deficiencies of microscopic optical models at low energies have been addressed recently in Ref. [19], where it is suggested that the inclusion of nucleon-phonon couplings may be needed to improve the agreement with
the low energy data.

For the heavier targets ($^{40}$Ca, $^{58,60}$Ni, $^{90}$Zr and $^{208}$Pb) the calculated cross sections follow the same qualitative trend of the data. The discrepancies observed at energies between 20 and 100 MeV can be characterized by a uniform 50-mb overestimate of the calculated cross sections relative to the data. At energies above 200 MeV the agreement with the measured cross sections is considerably improved, with the exception of the 860-MeV measurements of $^{12}$C and $^{208}$Pb, where a clear disagreement is exposed. At this specific energy we have performed FF optical model calculations for the targets considered and our predictions are summarized in Table III. These predictions are consistent with the data from Ref. [17] near and above 1 GeV (open triangles), an indication of the adequacy of our approach at these high energies.

For completeness in this section, in Fig. 2 we compare the measured [20] and calculated $nA$ total cross sections ($\sigma_T$). The calculated results were obtained within the FF optical model approach, and the applications cover the energy range between 5 MeV and 1 GeV. What becomes clear from this figure, in contrast to the description of $\sigma_R$, is that the closest agreement with the data occurs with the lighter self-conjugate targets. Instead, for the isospin asymmetric $^{90}$Zr and $^{208}$Pb systems, the calculated $\sigma_T$ lacks the pronounced oscillating pattern exhibited by the data. Microscopic calculations within the $g$-folding approach have been reported [9] to provide an excellent account for the total cross-section data in the 60-200 MeV energy range. Its extension to 600 MeV has been achieved by means of a simple parametric form [21]. From the prospective of the microscopic FF approach, the account for $\sigma_R$ for isospin asymmetric targets remains a pending issue.

**B. Scattering from unstable isotopes**

In order to explore the behavior of the integrated cross sections as functions of the mass number, particularly its isotopic asymmetry, we have evaluated $\sigma_R$ and $\sigma_T$ for nucleon-nucleus elastic scattering. These applications include results at 0.4, 0.7 and 1.0 GeV nucleon energies, covering even-even isotopes of oxygen, calcium, nickel, zirconium, tin and lead. In Fig. 3 we show a log-log plot of the calculated $\sigma_R$ (red circles) and $\sigma_T$ (blue circles) at 0.4, 0.7 and 1.0 GeV nucleon energy as functions of A, the number of nucleons of the target.
The straight lines represent the least-square regression of the type

\[ \sigma = \sigma_0 A^p, \]

with \( \sigma_0 \) the reduced cross section and \( p \) the power-law exponent. Both parameters depend on the nucleon energy \( E \). For clarity in the figure, the results for 0.7 GeV and 1.0 GeV have been offset by factors of 10 and 100, respectively.

As observed from the figure, with the exception of \( \sigma_R \) for the oxygen isotopes, the overall trend of the calculated cross sections follows very well the \( A^p \) power law. In the case of the oxygen isotopes at 1.0 GeV, a slight deviation as a function of the asymmetry \( A - 2Z \) is observed. In this particular case the maximum deviation of the calculated \( \sigma_R \) with respect to the \( A^p \) law is bound by 3%. Our estimate is that this manifestation of the isotopic asymmetry is genuine, although a more reliable evaluation would require a better handling of the target mixed density. Such considerations go beyond the scope of the present work.

In Table II we summarize the results from the power law regression of the calculated cross sections. In all cases we include the standard deviation of the obtained reduced cross section and corresponding exponent. Judging by the standard deviation, the results at 1 GeV exhibit the closest agreement with the \( A^p \) behavior, case in which \( \sigma_R \sim A^{0.644} \), and \( \sigma_T \sim A^{0.748} \). Clearly these results differ from the \( A^{2/3} \) geometric law, suggesting the relevance of the hadron dynamics and implicit correlations in the collision phenomena.

A closer comparison between the calculated cross section and the least-square power law fit is shown in Fig. 4, where the solid curves represent the cross sections for nucleon scattering from \(^{16}\text{O} \) at 1 GeV using the corresponding parameters from Table II. The filled circles represent the calculated \( \sigma_R \) and \( \sigma_T \) using the FF approach, where dashed lines are drawn to guide the eye. In this figure it becomes evident the departure from the \( A^p \) behavior when the neutron number varies. Indeed, the calculated cross sections are weaker than those obtained from the power law for increasing neutron excess. Conversely, the oxygen isotopes with fewer neutrons than protons yield cross sections greater than the ones prescribed by the parametrization. These features become more pronounced in \( \sigma_R \) than in \( \sigma_T \). Although we do not have thorough interpretation of this feature, we notice that the elementary total cross sections, \( \sigma_{pp} \) and \( \sigma_{pn} \), grow very rapidly between 500 MeV and 1.3 GeV nucleon laboratory energy. This increase is more pronounced in \( \sigma_{pp} \) than in \( \sigma_{pn} \), where near 1 GeV \( \sigma_{pn} \) is overtaken by \( \sigma_{pp} \). The smaller \( \sigma_{pn} \) relative to \( \sigma_{pp} \) weakens the increase of the total cross
section as the number of neutrons is increased.

IV. SUMMARY AND CONCLUSIONS

We have presented a global study of the reaction cross section for proton scattering from various unstable and stable isotopes at energies between 10 MeV and 1 GeV. The study is based on the microscopic in-medium FF optical model potential. The effective interaction is represented by the nuclear matter $g$ matrix which is obtained from bare $NN$ potentials with complete account for the loss of flux of the interaction above pion production threshold. This feature is implemented with the inclusion of a separable, energy-dependent component added to the reference potential, in this case the Argonne AV18 $NN$ potential model. To this purpose, we have used the SP00 phase-shift analysis available from the George Washington University, Data Analysis Center [13]. The target ground-state densities were obtained from Hartree-Fock-Bogoliubov calculations based on the finite range, density dependent Gogny force.

The calculated $\sigma_R$ for proton scattering from stable nuclei are in reasonable description of the data above 200 MeV, but lack comparable agreement with the data below 100 MeV. These discrepancies become more pronounced in the case of the lightest targets studied, i.e. $^{12}$C and $^{16}$O, but diminish with increasing size. At higher energies the FF optical model based on the described $NN$ interactions are consistent with the measured data. A comparison of the FF optical model approach with the total cross section data for $nA$ scattering shows close agreement in the cases of self-conjugate targets, but clear disagreement for the isospin asymmetric nuclei. Thus far we have been unable to identify a microscopic mechanism able to account for such discrepancy.

We have also investigated the systematics of the total cross sections as a function of the mass number for various isotope families. The results for both $\sigma_R$ and $\sigma_T$ for proton and neutron scattering, respectively, suggest an $A^p$ power-law accurate within 3%. At nucleon energies between 0.7 and 1.0 GeV the exponents for $\sigma_R$ and $\sigma_T$ are close to 2/3 and 3/4, respectively. Some slight deviation from this law is observed when the number of neutrons departs from that for the most stable isotope. This feature becomes more pronounced in the case of $\sigma_R$ at 1 GeV for $^{16}$O, but weakens for heavier targets and at lower energies.

The work reported here constitutes a global assessment of the microscopic momentum-
space FF optical model approach on its account for proton reaction and neutron total cross section, covering nearly three orders of magnitude in energy. Similar studies have been reported within alternative theoretical approaches, such as Glauber theory \[10\], the *in-medium* $g$-folding approach and global Dirac phenomenology \[7\]. Although some differences occur in the quality of the description of the data, particularly with respect to the $g$-folding approach, it remains difficult to identify the sources of such differences. Indeed, in Ref. \[11\] the effective interaction is represented in coordinate space as an expansion of Yukawa form factors of various ranges and complex energy dependent strength, in the form [c.f. Eqs. (7.1) and (7.2) of Ref. \[11\]]

$$t_{\text{eff}}^{ST}(r, \omega) = \sum_i \langle \theta_i \rangle \ t_{\text{eff}}^{(i)ST}(r, \omega),$$

with

$$t_{\text{eff}}^{(i)ST}(r, \omega) = \sum_{j=1}^{n_i} S_j^{(i)}(\omega) e^{-\mu^{(i)r}/r}.$$

The calculations reported here do not make any consideration regarding the coordinate space structure of the $g$ matrix. Actually, they are taken directly from the solution of the Brueckner-Bethe-Goldstone equation. Additionally, the representation of the mixed density may be critical in the case of the light systems. In this regard, the $g$-folding approach is more detailed by making explicit use of the full mixed density based on shell models, a treatment which is pending in the FF approach.

From a broader prospective, the realization of the FF optical model relies on a weak dependence on the struck nucleon momentum. This assumption is crucial to reduce the number of multidimensional integrals by three, making computationally feasible the evaluation of optical potentials in the FF and $g$-folding approaches. However, the conditions under which this assumption becomes most (or least) adequate have not been investigated. Other aspects such as charge-symmetry of the bare $NN$ interaction and asymmetric nuclear matter effects may also need to be examined.

**Acknowledgments**

The authors are indebted to Prof. H. V. von Geramb for providing the separable strength needed for the high energy applications of this work. They also thank J.-P. Delaroche for careful and critical reading of the manuscript. H.F.A. acknowledges partial funding provided
by FONDECYT under grant 1040938.

[1] H. F. Arellano, F. A. Brieva, and W. G. Love, Phys. Rev. C 52, 301-315 (1995).
[2] H. F. Arellano and H. V. von Geramb, Phys. Rev. C 66, 024602 (2002)
[3] A. Auce, A. Ingemarsson, R. Johansson, M. Lantz, G. Tibell, R. F. Carlson, M. J. Shachno,
    A. A. Cowley, G. C. Hillhouse, N. M. Jacobs, J. A. Stander, J. J. van Zyl, S. V. Förtsch, J.
    J. Lawrie, F. D. Smit, and G. F. Steyn, Phys. Rev. C 71, 064606 (2005).
[4] A. Kohama, K. Iida, and K. Oyamatsu, Phys. Rev. C 72, 024602 (2005)
[5] A. Ingemarsson and M. Lantz, Phys. Rev. C 72, 064615 (2005)
[6] K. Amos, W. A. Richter, S. Karataglidis, and B. A. Brown, Phys. Rev. Lett. 96, 032503
    (2006).
[7] P. K. Deb, B. C. Clark, S. Hama, K. Amos, S. Karataglidis, and E. D. Cooper, Phys. Rev. C
    72, 014608 (2005)
[8] P. K. Deb and K. Amos, Phys. Rev. C 67, 067602 (2003).
[9] P. K. Deb, K. Amos, S. Karataglidis, M. B. Chadwick, and D. G. Madland, Phys. Rev. Lett.
    86, 3248 (2001).
[10] A. de Vismes, P. Roussel-Chomaz, and F. Carstoiu, Phys. Rev. C 62, 064612 (2000).
[11] K. Amos, P. J. Dortmans, H. V. von Geramb, S. Karataglidis, and J. Raynal, Adv. in Nucl.
    Phys. 25, 275 (2000).
[12] H. F. Arellano, F. A. Brieva, and W. G. Love, Phys. Rev. C 42, 652 (1990).
[13] R. A. Arndt, I. I. Strakovsky, and R. L. Workman, Phys. Rev. C 62, 034005 (2000).
    Also, CNS DAC [SAID], Physics Department, The George Washington University,
    http://gwdac.phys.gwu.edu/.
[14] J. Dechargé and D. Gogny, Phys. Rev. C 21, 1568-1593 (1980).
[15] J. -F. Berger, M. Girod, and G. Gogny, Comput. Phys. Commun. 63, 365 (1991).
[16] R. F. Carlson, Atomic Data and Nucl. Data Tables 63, 93-116 (1996).
[17] F. S. Dietrich, E. P. Hartouni, S. C. Johnson, G. J. Schmid, R. Soltz, W. P. Abfalterer, R. C.
    Haight, L. S. Waters, A. L. Hanson, R. W. Finlay, and G. S. Blanpied, J. Nucl. Sci. Technol.,
    Supplement 2, 269 (2002).
[18] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C 51, 38 (1995).
[19] M. Dupuis, S. Karataglidis, E. Bauge, J. P. Delaroche, and D. Gogny, Phys. Rev. C 73, 014605 (2006).

[20] R. W. Finlay, W. P. Abfalterer, G. Fink, E. Monte, T. Adami, P. W. Lisowski, G. L. Morgan, and R. C. Haight, Phys. Rev. C 47, 237 (1993).

[21] P. K. Deb, K. Amos and S. Karataglidis, Phys. Rev. C 70, 057601 (2004).
FIG. 1: (Color online) Measured \cite{3, 16, 17} and calculated reaction cross section based on full-folding optical model potential using the $g$ matrix (solid curves) and the free $t$ matrix (dashed curves).

TABLE I: Root-mean-square radii of the point proton ($R_p$) and neutron ($R_n$) densities used in this study.

| Nucleus | $^{208}$Pb | $^{90}$Zr | $^{60}$Ni | $^{58}$Ni | $^{40}$Ca | $^{16}$O | $^{12}$C |
|---------|-----------|-----------|-----------|-----------|-----------|---------|---------|
| $R_p$ [fm] | 5.437 | 4.210 | 3.717 | 3.695 | 3.405 | 2.676 | 2.419 |
| $R_n$ [fm] | 5.573 | 4.265 | 3.738 | 3.683 | 3.365 | 2.656 | 2.402 |
FIG. 2: The measured and calculated total cross section based on the in-medium FF optical model potential.

TABLE II: Results for the $A^p$ power law regression of the calculated cross sections as a function of the nucleon energy.

| $E$ [GeV] | $\sigma_0$ [mb] | $p$       | $\sigma_0$ [mb] | $p$      |
|----------|-----------------|-----------|-----------------|----------|
| 0.4      | 42.0 ± 0.1      | 0.692 ± 0.003 | 40.9 ± 1.1      | 0.789 ± 0.005        |
| 0.7      | 57.0 ± 0.8      | 0.652 ± 0.002 | 61.4 ± 0.8      | 0.746 ± 0.002        |
| 1.0      | 59.8 ± 0.1      | 0.644 ± 0.002  | 63.0 ± 0.1      | 0.747 ± 0.001        |
FIG. 3: (Color online) Calculated $\sigma_R$ and $\sigma_T$ for nucleon elastic scattering from O, Ca, Ni, Zr, Sn and Pb even-even isotopes. The lower-, middle- and upper-pair curves correspond to 0.4, 0.7 and 1.0 GeV nucleon energy, respectively.
FIG. 4: The calculated $\sigma_R$ and $\sigma_T$ (filled circles) *versus* the $A^p$ power law regression (solid curves) for 1-GeV nucleon scattering from oxygen isotopes.

| Target | Measured $\sigma_R$ [mb] | Predicted $\sigma_R$ [mb] |
|--------|--------------------------|---------------------------|
| $^{208}$Pb | $1680 \pm 40$ | 1852 |
| $^{90}$Zr | — | 1079 |
| $^{60}$Ni | — | 823 |
| $^{58}$Ni | — | 804 |
| $^{40}$Ca | — | 647 |
| $^{16}$O | — | 357 |
| $^{12}$C | $209 \pm 22$ | 284 |