Non-supersymmetric Orientifolds with D-branes at Angles

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Abstract: We present special classes of orientifold models involving supersymmetry breaking via branes at angles. Type II superstring theories are compactified on a two torus times a four-dimensional orbifold. Combining worldsheet parity with a reflection of half of the compact coordinates leads to D6-branes at angles which are mapped onto each other by the orbifold group, while applying the geometric action only along one coordinate leads to intersecting D8-branes with non-trivial transformation properties under the orbifold group. The models differ in the gauge groups and matter content.

1 Introduction

One of the outstanding problems of identifying string theory as the underlying theory which unifies the four known fundamental forces is the huge variety of different vacua. While early attempts of mimicking the standard model concentrated on constructing four-dimensional $N = 1$ supersymmetric vacua of the weakly coupled heterotic string, the development of orientifold constructions [1] and the concept of D-branes [2] as charged objects supporting the gauge groups led to a renewed interest in compactifications of type II theories. Within these classes of models, non-supersymmetric theories might provide a low energy spectrum consistent with the standard model and at the same time explain the hierarchy between the electroweak and the Planck scale. In contrast to the heterotic case, by compactifying on manifolds of large dimensions transverse to the branes, the string scale could be lowered down to the electroweak scale [3]. One possible way of realizing the supersymmetry breaking in the open string sector of type II orientifolds is to consider torus compactifications with a constant magnetic background flux [4, 5]. This tool also provides a mechanism to obtain chiral fermions and gauge symmetry breaking. In a T-dual picture, the background fluxes translate into relative angles of intersecting branes wrapping lower-dimensional cycles in the compact space [6]. It has already been known for some time that massless chiral fermions are located at the intersection points of two D-branes [7] and the correspondence between magnetic background fluxes and branes wrapping cycles has been established in [8]. The resulting class of orientifold models can be further generalized by including a discrete NSNS-sector B field [9, 10].

Phenomenological issues within the framework of branes at angles have been first addressed in [11] where D4-branes of type II theory were considered. This ansatz has been further exploited in [12]. Searches for the Standard Model in orientifold constructions
with D6-branes have successively been performed for torus compactifications and some orbifold groups \([6, 13, 14]\). In \([15]\) supersymmetric vacua of this kind were found which can provide chiral semi-realistic models.

The ansatz presented here is somewhat different from the previously described ones in the sense that we consider hybrid models of \([6, 10]\) and \([11]\). Our starting point is to take an orientifold on a two torus times a four-dimensional orbifold and allow for non-trivial magnetic background fields only on the two torus. Depending on the geometric action that we choose to combine with the worldsheet parity, we obtain either D6-branes \([16]\) at angles wrapping a 2-cycle on the orbifold in the T-dual picture or D8- branes wrapping the entire volume of the orbifold. The latter case has not been worked out before. Contrarily to all models with D6-branes, this model can be T-dualized to include only D4-branes with the orbifold along the transverse directions admitting a large volume compactification.

2 The Concept of D-branes at angles

The notion of D-branes at angles in orientifold constructions was first developed within the framework of supersymmetric non-chiral models \([17, 18]\). Combining the worldsheet parity \(\Omega\) with a complex conjugation \(R(i)\) of \(i\) internal complex coordinates \(z^j\) leaves \(O(9-i)\)-planes invariant and therefore induces the existence of \(D(9-i)\)-branes to cancel the RR-charges. Introducing in addition a \(Z_N\) rotation \(\Theta : z^j \rightarrow e^{2\pi iv_j}z^j\) with \(\sum_j v_j = 0\) leads to partial supersymmetry breaking in the closed string sector such that we obtain either an \(N = 1\) or \(N = 2\) supergravity multiplet in four dimensions. Placing all D-branes on top of the O-planes leads to local RR charge cancellation and a non-chiral supersymmetric spectrum. The more generic situation is to allow for \(D_a\)-branes with wrapping numbers \((n_a, m_a)\) along the two fundamental cycles \((e_1, e_2)\) of a two torus. In this case, RR charges are cancelled only globally, part of the open spectrum is chiral and generically supersymmetry is broken. The geometric data of the T-dual two torus for vanishing and non-trivial NSNS field \(B^{45} = \frac{b_0}{R_1R_2}\) can be read off from the figure below where the reflection is taken to act on the \(x^5\) direction.

\[
\begin{align*}
\begin{array}{c}
\text{Figure 1: } b = 0 \\
\text{Figure 2: } b = 1/2
\end{array}
\end{align*}
\]

The angle \(\pi \varphi\) of a \(D_a\) brane w.r.t. the invariant axis \(x^4\) can be expressed in terms of the wrapping numbers, \(\tan(\pi \varphi) = \frac{(m_a + bn_a)}{n_aR_1R_2}\), which translates into the magnetic background \(F_{a}^{45} = \frac{m_a n'_a}{n_a R_1 R_2}\) plus \(B^{45}\) upon T-duality along the \(x^5\) direction. Two distinct branes \(D_a\) and \(D_b\) support chiral fermions as well as scalars with masses depending on the intersection angle (generically including tachyonic states) at the intersection loci, the multiplicity of states being determined by the intersection number on the fundamental torus, \(I_{ab} = n_a m_b - n_b m_a\). For consistency of the theory, one always has to take into account the mirror images \(D'_a\) under the geometric action \(R(i)\) which are specified by the wrapping numbers \((m'_a, n'_a) = (n_a, -m_a - 2bn_a)\). The open string spectrum consists then of supersymmetric non-chiral fields living on one set of \(N_a\) branes and providing the
gauge group $U(N_a)$ and of non-supersymmetric chiral matter in $(N_a, \overline{N}_b)$ of two distinct intersecting types of branes $D_a$ and $D_b$. The representation of those states which arise from intersections of mirror branes has to be determined by regarding the transformation properties of the mass eigenstates and intersection points w.r.t. the reflection $R_{(i)}$ and the orbifold generator $\Theta$.

The allowed combinations of numbers $N_a$ of identical branes and wrapping numbers $n_a$ along the invariant plane are determined by computing the contributions to the RR-tadpoles of closed and open string 1-loop amplitudes and requiring the total RR charge to vanish. The angles $\pi \varphi$ between intersecting branes enter the Annulus and Möbius strip amplitude through the modified oscillator modding $\alpha_{m-\varphi}$, whereas identical branes contribute Kaluza Klein momenta $P = r/L$ along the brane and ‘windings’ $\alpha W = sR_1R_2/L$ perpendicular to the brane (where $L$ is the length of the wrapped 1-cycle on the torus).

For example, the Annulus amplitude $A = c \int_0^\infty \frac{dt}{t^3} \text{Tr}_{\text{open}} \left( \frac{1}{2} \mathbf{P}_{\text{orb}} \mathbf{P}_{\text{GSO}} (-1)^{s} e^{-2\pi tL_\varphi} \right)$ in the loop channel transforms into scattering of closed strings between two boundary states in the tree channel, $\tilde{A} = \int_0^\infty dl \langle B | e^{-2\pi tlH_{\varphi}} | B \rangle$. When performing this transformation, the role of Kaluza Klein and winding states as well as ‘twist sectors’ and insertions of the orbifold projector are exchanged. There is always a contribution from the trivial part of the orbifold projector. This part determines the untwisted RR charge in the tree channel and thus fixes the net number of branes. In models with D6-branes, the orbifold generator $\Theta$ rotates the positions of branes and therefore does not contribute to the tadpoles. In the tree channel picture, this means that no twisted closed strings couple to the D-branes and O-planes. This is in contrast to the models with D8-branes. In the latter case, $\Theta$ does not affect the positions of branes but acts non-trivially on the Chan-Paton factors fixing the trace of the representation of the orbifold group, $\text{tr} \gamma_k$. In the tree channel, this has to be interpreted as couplings of twisted closed strings to D-branes and O-planes. A model of each kind will be presented in section 3 and 4, respectively.

At the classical level, phenomenological features of models with intersecting branes can be described in terms of the geometric quantities on the compact space: the gauge coupling constant pertaining to a set of $D_a$-branes is related to the length $L_a$ of the 1-cycle on the two torus which the branes wrap, $\frac{1}{g_a^2} \sim \frac{N_a}{L_a^2}$, leading to a gauge hierarchy. The trilinear coupling of e.g. two fermions $F_L^i, F_R^j$ and a scalar $H^k$ is exponentially suppressed by the area of the worldsheet spanned among the three intersecting branes involved, $Y_{ijk} \sim \exp (-A_{ijk})$, producing a hierarchy of Yukawa couplings. The Planck scale obtained from dimensional reduction depends on the compact volume, $M_P \sim \sqrt{\frac{N_a N_b}{\lambda_s}}$. In models with D6-branes, this relation fixes the string and the Planck scale to be of the same order. Models containing only D8 branes admit, however, a dual description in terms of D4-branes transverse to the orbifold. This suggests that large volume compactifications can be used to lower the string scale down to the electroweak scale.

3 A Model with D6-branes

The model with D6-branes that we present here consists of an orientifold of IIA theory on $R^{1,3} \times T^2 \times (T^2)^2 / Z_3$ where we choose the coordinates $x^{0,\ldots,3}$ along $R^{1,3}$ and $x^{4,\ldots,9}$ along $(T^2)^3$. The reflection $R_{(3)}$ inverts $x^{5,7,9}$ and the orbifold group generator $\Theta$ acts on the second and third torus only. Thus, we obtain the $N = 2$ supergravity multiplet from the closed string sector. In this model, $\Theta$ rotates the position of the D6-branes and we are left with only an untwisted tadpole condition as discussed in section 3, namely $\sum_a n_a N_a = 4$. 
In order to construct models which do not contain any anti-branes, we have to impose $n_a > 0$. So as to get chiral fermions, at least two kinds of intersecting branes are needed. The gauge group $SU(3) \times U(1)$ can be engineered by choosing the numbers of identical branes and wrapping numbers as follows,

\[
N_1 = 3, \quad (n_1, m_1) = (1, 1),
\]
\[
N_2 = 1, \quad (n_2, m_2) = (1, 2).
\]

By taking the NSNS B-field background to be trivial, i.e. $b = 0$, we obtain the spectrum listed in table 1 where we have only given the charge of the anomaly free $U(1)$ combination $Q_{\text{non-an.}} = Q_1 - 3/2Q_2$. The other combination acquires a mass by the generalized Green-Schwarz mechanism involving couplings to the untwisted RR forms, e.g. $\int_{R^1,3} C^{(2)} \wedge F_a$.

In this model, each intersection also accommodates a tachyonic state. The existence of such a scalar state is typical for intersecting brane world models on tori. One possibility of avoiding some tachyons is to project them out by the orbifold group $Z_2$ instead of $Z_3$ as discussed in [16]. Another approach of projecting some tachyons out is discussed in the following section.

### Chiral spectrum for Model I

| Sector | $SU(3) \times U(1)_{\text{non-an.}}$ | mult. |
|--------|---------------------------------|-------|
| 11'    | $(3)_{2}$                       | 4     |
| 12     | $(3)_{-5/2}$                    | 2     |
| 12'    | $(3)_{-1/2}$                    | 6     |

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## 4 A Model with D8-branes

The model presented in this section is based on a similar construction to the one reviewed in section 3. Again, we consider a IIA orientifold on $R^{1,3} \times T^2 \times (T^2)^2 / Z_3$ where the orbifold group acts non-trivially only on the second and third torus. The difference w.r.t. the previous case is that we choose the orientifold projection to be $\Omega R_{(1)}$ where the reflection $R_{(1)}$ acts on $x^5$ only. The resulting model requires D8-branes for tadpole cancellation. For the supersymmetric non-chiral set-up, the decompactification limit on the two torus of the T-dual theory is given by the $R^{1,5} \times T^4 / Z_3$ model in [13]. The orbifold group acts non-trivially on the Chan-Paton factors of open strings, and a stack of $D8_a$-branes of identical position is decomposed into its different $Z_3$ eigenvalues $\alpha^i$, i.e. $N_a = \sum_{i=0}^{2} N_a^i$. Also twisted closed strings couple to D8-branes and O8-planes. Untwisted and twisted RR-charges have to be cancelled separately,

\[
\sum_a n_a N_a = 16,
\]
\[
\sum_a n_a \left( N_a^0 - \frac{N_a^1 + N_a^2}{2} \right) = 4 \quad \text{and} \quad N_a^1 = N_a^2.
\]

The generic gauge group is therefore $\prod_a \prod_{i=0}^{2} U(N_a^i)$ and chiral fermions are labeled by additional indices, $(N_a^i, N_b^j)$. Including $\Omega R_{(1)}$-invariant branes requires some modifications.
In table 2 we list the chiral part of the spectrum obtained from

\[
N_A^1 = 3, \quad (n_A, m_A) = (2, -1), \\
N_B^0 = 2, \quad (n_B, m_B) = (4, -1), \\
N_C^1 = 1, \quad (n_C, m_C) = (1, 0), \\
\]

and \( b = 1/2 \). The gauge group is \( SU(3) \times SU(2) \times U(1)^3 \) with the anomaly free \( U(1) \) charges \( Q_A, Q_Y = Q_A + Q_C - Q_B \) and \( \tilde{Q} = Q_B + Q_C + Q_C \). The generalized Green-Schwarz mechanism now involves couplings to the twisted closed fields, \( \int R^1, 3 \text{tr} (\gamma_a \lambda^a) C^{(2)}_k \wedge F_{a,i} \), which descend from the self-dual four-form in ten dimensions integrated over a vanishing supersymmetric two-cycle on the orbifold, \( C^{(2)}_k = \int \Omega^{10} C^{(4)} \) (see e.g. [20]).

| Sector \( AB\alpha \) | mult. | rep. of \( SU(3) \times SU(2) \) | \( Q_1 \) | \( Q_2 \) | \( Q_3 \) | \( Q_4 \) | \( Q_5 \) | \( Q_Y \) | \( Q \) |
|------------------------|-------|-------------------------------|------|------|------|------|------|------|------|
| \( \alpha^2 \) | 2 | (3, 2) | 0 | 0 | -1 | -1 | -1/3 | -1 | -1/4 |
| \( AC\alpha \) | 2 | (3, 2) | 0 | 0 | 1 | 1 | 1/3 | 1 | -1/4 |
| \( \alpha^1 \) | 1 | (3, 1) | 1 | 0 | 0 | -1 | 2/3 | 1 | -1 |
| \( \alpha^2 \) | 1 | (3, 1) | 0 | 1 | 0 | 1 | -2/3 | 1 | -1 |
| \( BC\alpha \) | 1 | (1, 2) | -1 | 0 | 0 | -1 | 4/3 | -1 | -1 |
| \( \alpha^1 \) | 1 | (1, 2) | 0 | -1 | 0 | 1 | -2/3 | 1 | -1 |
| \( BC'\alpha \) | 3 | (1, 2) | -1 | 0 | -1 | 0 | -1 | -5/4 | -1 |
| \( \alpha^2 \) | 3 | (1, 2) | 0 | -1 | 0 | 1 | -2/3 | 1 | -1 |
| \( BB'\alpha \) | 4 | (1, 1) | 0 | 0 | 2 | 0 | 0 | 1/2 |
| \( \alpha^0 \) | 6 | (1, 1) + (1, 3) | 0 | 0 | 2 | 0 | 0 | 1/2 |
| \( CC'\alpha \) | 2 | (1, 1) | 1 | 1 | 0 | 0 | 0 | 2 |

Chiral matter is accompanied by tachyonic states in the same representation \((N^i_a, N^j_b)\) only for \( i = j \). Thus, only the \( AC\alpha, BB'\alpha \) and \( CC'\alpha \) sectors contribute.

There are many more possibilities to solve the tadpole conditions. In particular, three generation models can be constructed. Some tachyons will always remain in the spectrum due to the presence of mirror branes, but possibly they can serve to trigger a non-standard Higgs mechanism. Furthermore, in the D8-brane models blow-up modes of the orbifold contribute to NSNS tadpoles and might play a role in stabilizing non-supersymmetric models. The work on these tasks is still in progress.

**Acknowledgments**

It is a pleasure to thank Stefan Förste and Ralph Schreyer for collaboration on a large part of the work presented here as well as for discussions on the ongoing work.

This work is supported by the European Commission RTN program HPRN-CT-2000-00131 and the DFG Schwerpunktprogramm (1096).

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