Analysis of Magnetic Field between Two Different Permeable Membranes with Two Immiscible Fluids

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Abstract: In this paper the couette flow of two immiscible fluid flows passing through a parallel channel with isothermal wall in the presence of magnetic field has been investigated. The lower permeable membrane consists of predetermined thickness with large permeability and the upper membrane consists of unlimited thickness with small permeability. The flow in the lower permeable membrane is explained by using Brinkman equation and the flow in the upper permeable membrane is characterized by Darcy’s law. Here the Navier-Stokes equations are used to govern the flow between the two membranes. The velocity in the presence of magnetic field of the fluids is calculated and the solution is obtained. The effects of different types of parameters like, Darcy number, Reynolds number, Viscosity ratio, the magnetic field, Slip parameter, etc. on the velocity field have been solved graphically, numerically and analyzed quantitatively.

Keywords: Couette flow, immiscible fluid, permeable membrane.

1. Introduction

Many researchers are doing their research work in two phase flow, which is transient two phase flow, separate two phase flow and dispersed two phase flow etc. This two phase flow methods are commonly applied in large power station, turbine factories and also in different areas such as air and water, oil and nature gas, climate system, ground water system and wave on the etc. Vajravelu et al. [1] have concluded that the magnetic effect on unsteady flow of two immiscible conducting fluids between two permeable membranes. Singh et al. [2] founded Couette flow of two immiscible viscous fluids with heat transfer using Brinkman model. Malashetty et al. [3], [4] have determined the MHD flow of two fluid and heat transfer in an inclined channel, and flow in an inclined channel containing porous and fluid layer. Umavathi et al. [5], [6] an oscillatory Hartmann two-fluid flow and an unsteady two-fluid flow of heat transfer in a horizontal channel is found. Anwar Beg et al. [7] studied the rotating non-Darcian porous medium in the parallel plate configuration of transient Couette flow. Sastry et al [8] the velocity fields in various parameter such as mass flow rate, Darcy number and the effect of Reynolds number in the Couette flow of two immiscible fluids between two permeable membrane was investigated.

From the above mentioned studies, the analysis of the magnetic field in the two immiscible fluids between two permeable membranes is investigated. The different types of parameters like, Darcy number, Reynolds number, Viscosity ratio, Mass flow rate, the magnetic field, Slip parameter, etc. on the velocity field solved numerically, graphically and analysed quantitatively.
2. Formulation of the Problem

Consider the horizontal channel of height $2h$ with Couette flow of two immiscible fluids is bounded by two permeable membranes of various permeabilities. The lower permeable membrane has large permeability with predetermined thickness $H$ whereas the upper permeable membrane has small permeability with unlimited thickness. Figure 1 describes the geometry of the flow. Let $k_1$ be the permeability of lower membrane. Let $k_2$ be the permeability of upper membrane. A pressure gradient is $C$, where $C = -\frac{\partial p}{\partial x}$ which plays a role at the mouth of the channel. Let $\mu_1$ and $\rho_1$ be the viscosity and density of the upper fluid. It assumed that, the upper fluid occupies the upper half of channel (i.e., $0 \leq y \leq h$), and this region is called zone I. In the lower fluid, the viscosity $\mu_2$ is greater than $\mu_1$ and the density $\rho_2$ is greater than $\rho_1$ occupies the region $[-(h + H) \leq y \leq 0]$, the lower half of the channel and this is called zone II. The lower permeable membrane of the flow region is called zone III and the pattern of flow is governed by Brinkman equation. The upper permeable membrane of flow region is called zone IV and the pattern of the flow by Darcy’s law.

![Figure 1: Physical Model](image)

The non-dimensional quantities are,

\[
\frac{y}{h}; \frac{u}{U}; \frac{u_1}{U}; \frac{u_2}{U}; \frac{u_3}{U}; u_{b1}; u_{b1}; \frac{Q}{u}; \frac{Q}{u}; D = \frac{h}{Da}; \frac{k_2}{h}; \frac{1}{h}; u_{b1}; u_{b1}; Then \frac{d}{d} \left( \frac{u_{b1}}{Q} \right); M^2; \frac{B^2}{R^2}; \frac{h^2}{1}.
\]

we obtained the different zones:

2.1. The governing equations

are Zone I ($0 \leq \eta \leq 1$)

\[
\frac{d^2 u}{d \eta^2} = 2RS \frac{M^2 u}{0}.
\]

Zone II ($-1 \leq \eta \leq 0$)
\[
\frac{d^2 u}{d^2} - 2R \quad M^2 u \quad 0
\]  
Zone III \((-1 < \eta < 1\)
\[
\frac{d^2 u_3}{d^2} - 3u_3 - M^2 u_3 2R
\]

2.2. Boundary conditions are,
\[
u \quad \frac{du}{d} \quad \frac{Q}{\sqrt{Da}} \quad \frac{Q}{\sqrt{Da}} \quad \text{at} \quad \text{at} \quad \text{at} \quad \text{at} \quad \text{at}.
\]
Where \(Q=2RS Da\):
\[
u \quad \frac{du_1}{d} \quad \frac{du_1}{d} \quad \frac{du_2}{d} \quad \frac{du_2}{d} \quad \frac{du_3}{d} \quad \frac{du_3}{d} \quad \text{at} \quad \text{at} \quad \text{at}.
\]
\[
u \quad 0 \quad \text{at} \quad \text{at}.
\]

2.3. Solutions
The equations (1), (2) and (3) denote the Solutions of momentum, by using the above boundary conditions.

\[
u_1 \quad c_1 \cosh( m) \quad c_2 \sinh( m) \quad \frac{2R S}{m} ; \]
\[
u_2 \quad c_3 \cosh( m) \quad c_4 \sinh( m) \quad \frac{2R}{\ell} ; \quad u_3 \quad c_5 \cosh( z) \quad c_6 \sinh( z) \quad \frac{2R S}{\ell} ; \]

3. Result and Discussions
From figure – 2, the effect of magnetic field for different values of \(M\), the velocity distribution in both region \(\text{I,II}\) found to increases with an increment in magnetic parameter \(M\), then other parameters are fixed. Also observe that the velocity distribution in the both region have a tendency increases with an increases in magnetic parameter.

From figure – 3, for various values of \(R\), we determined that the velocity increases in the increment of Reynolds number increases in the Region I is higher than that at the Region II.

From figure – 4 for different values of \(Da\), velocity increases with the increment of Reynolds number. Also observe that the velocity distribution in the both region have equally increases with an increases in Darcy number.

From Figure - 5 the effect of the slip parameter \(\alpha\) on the Mass flow rate is increasing when velocity increases in region I decrease in region II.

4. Conclusion
Finally it found that the velocity is in increasing with the increment different types of parameters like, Darcy number, Reynolds number, Viscosity ratio, Mass flow rate, the angle of magnetic field, Slip parameter, etc. on the velocity fields are solved numerically, graphically and analysed quantitatively.
5. Appendix-I

\[ z \sqrt{m^2}; \]
\[ f \quad 2R_s; \]
\[ m^2 \]
\[ Q \quad 2R s d a; \]
\[ m \quad g_1 \cosh m \quad m \quad da \quad \sqrt{\sinh m} ; \]
\[ g_2 \sinh m \quad m \quad da \quad \sqrt{\cosh m}; \]
\[ g_2 \cosh m \quad 2 \quad g_2 \cosh \]
\[ m \quad g_1 \sinh m ; \]
\[ Q \quad c_1 \quad \sinh m \quad 2R_s \quad \sinh m \quad \frac{f g_2}{2} ; \]
\[ f_2^m \quad f_2 \quad f_2 \]
\[ Q \quad c_2 \quad \cosh m \quad \frac{2R_s}{2} \quad \cosh m \quad \frac{f g_2}{2} ; \]
\[ f_2^m \quad f_2 \quad f_2 \]
\[ Q \quad c_3 \quad \sinh m \quad \frac{2R_s}{2} \quad \sinh m \quad \frac{f g_2}{2} \quad \frac{2R_s}{2} \quad \frac{2R_s}{2} \quad \frac{2R_s}{2}; \]
\[ f_2^m \quad f_2 \quad f_2 \quad \frac{m}{f_2} \quad m \quad m \]
\[ g_4 \quad m \quad \cosh m \quad \sinh m ; \]
\[ g_6 z \quad \cosh z \quad \sinh z ; \]
\[ g_8 \quad \sinh z \quad 1; \]
\[ d \quad g_3 \quad \sinh m \quad \frac{1}{f_2} ; \]
\[ d \quad g_4 \quad \cosh m \quad \frac{1}{f_2} ; \]
\[ d \quad g_6 g_7 \quad g_8 g_5 ; \]
\[ d \quad g_5 g_8 \quad g_7 g_6 ; \]
\[ c_5 \quad d_1 \quad Qg_8 \quad \frac{2R_s g_8}{2} \quad \frac{d_2 Qg_8}{2} \quad \frac{2R_s g_8}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_8}{2} \quad \frac{f_1 g_2 g_3 g_8}{2} \quad \frac{2R_s g_8}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_8}{2} \quad \frac{f_1 g_2 g_3 g_8}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_8}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_8}{2} ; \]
\[ d_4 \quad m d \quad d_4 \quad s m d_4 \quad f_1 \quad m \quad f_2 \quad g_2 g_4 g_6 \quad \frac{2R_s g_8}{2} \quad \frac{g_2 g_4 g_6}{2} \quad \frac{2R_s g_8}{2} \quad \frac{f_1 g_2 g_3 g_8}{2} \quad \frac{2R_s g_8}{2} \quad \frac{f_1 g_2 g_3 g_8}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_8}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_8}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_8}{2} ; \]
\[ c_6 \quad d_1 \quad Qg_7 \quad \frac{2R_s g_7}{2} \quad \frac{d_2 Qg_7}{2} \quad \frac{2R_s g_7}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_7}{2} \quad \frac{f_1 g_2 g_3 g_8}{2} \quad \frac{2R_s g_7}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_7}{2} \quad \frac{f_1 g_2 g_3 g_8}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_7}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_7}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_7}{2} ; \]
\[ d_3 \quad m d \quad d_3 \quad s m \quad d_3 \quad f_1 \quad m \quad f_2 \quad g_2 g_4 g_6 \quad \frac{2R_s g_7}{2} \quad \frac{g_2 g_4 g_6}{2} \quad \frac{2R_s g_7}{2} \quad \frac{f_1 g_2 g_3 g_8}{2} \quad \frac{2R_s g_7}{2} \quad \frac{f_1 g_2 g_3 g_8}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_7}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_7}{2} \quad \frac{1}{f_1 g_2 g_3 g_8} \quad \frac{2R_s g_7}{2} ; \]
Figure – 4

Figure – 5
6. References

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