SUPERSTRING AXION, GAUGINO CONDENSATION AND DISCRETE SYMMETRIES

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Here I present the effect of gaugino condensation and discrete symmetries to the model-independent axion potential, collaborated with Georgi and Nilles. It is shown that with appropriate discrete symmetries the model-independent axion can solve the strong CP problem.

1 Introduction

For a confining gauge group, we have a physical $\theta$ parameter. For QCD, this parameter $\bar{\theta}$ is known to be extremely small,

$$|\bar{\theta}| < 10^{-9}$$

from the upper bound of neutron electric dipole moment. Thus $\bar{\theta}$ is a very small parameter of the standard model. This smallness of the parameter is one of the parameter problems, usually known as the strong CP problem. On the other hand, from instanton calculus, the $\bar{\theta}$-dependence of $V$ is such that it is minimum at $\bar{\theta} = 0$. $V$ is periodic with period of $2\pi$.

If $\bar{\theta}$ is treated as a coupling, then a theory with any $\bar{\theta}$ will become a good theory, but the bound (1) excludes the most regions of $\bar{\theta}$, which is the strong CP problem.

Figure 1: $V[\theta]$ versus $\theta$. In axionic models, $\theta = a/F_a$.

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CP problem. It is most elegantly solved in the axion framework. If \( \theta \) is a dynamical field, i.e. if it appears with the kinetic energy term, then \( \theta \) slides down the hill of \( V[\theta] \), and will eventually settle at \( \theta = 0 \), which is the axion solution.

In axion models, therefore, one identifies \( \theta \) as an axion \( a \),

\[
\tilde{\theta} = \frac{a}{F_a} \tag{2}
\]

Note that the axion \( a \) does not have any potential except that coming from the \( \tilde{\theta}F\tilde{F} \) term. Otherwise, the mechanism does not work. As given in Eq. (2), the axion model introduces a mechanism to introduce the so-called axion decay constant \( F_a \). It can arise from a spontaneous symmetry breaking scale, or from the gravitational scale, or from a compositeness scale. Thus the axion can be attractive to those who emphasize one of these points as the most fundamental aspect of the particle theory.

There are many interesting physical phenomena due to the existence of axion: domain walls, axionic strings, cosmic axion energy density, galaxy formation, stellar energy loss, etc. From these, studies, we can restrict the range of the axion decay constant to \( 10^9 \text{ GeV} < F_a < 10^{12-13} \text{ GeV} \), where the upper bound is slightly extended since heavy particle decays can raise the original bound estimate. Now the cavity experiments are going on to detect the galactic axions corresponding to the upper bound of \( F_a \).

From now on, I will try to discuss the resurrection of the axion in the string theory.

2 Simple Solutions of the Strong CP Problem

There are two examples in which the strong CP problem is automatically solved. One is the axion solution, and the other is a massless u-quark solution. Since our discussion on the model-independent axion solution relies on these aspect, we briefly review the ideas of these two.

2.1 Heavy Quark Axion

The simplest axion model is the heavy quark axion in which only \( \tilde{\theta} \) is introduced in addition to the standard model fields below the axion scale \( F_a \). The Lagrangian is

\[
\mathcal{L} = \sigma \bar{Q}_R Q_L + \text{h.c.} - V
\]

where we suppressed couplings and \( V \) is assumed to respect the PQ symmetry \( U(1)_A \),

\[
Q_L \to e^{-ia}Q_L, \quad Q_R \to e^{ia}Q_R, \quad \sigma \to e^{ia} \sigma, \quad \theta \to \theta - \alpha \tag{3}
\]
For a nonzero VEV $<\sigma> = F_a/\sqrt{2}$, $Q$ obtains a mass at scale $F_a$, and at low energy there remains only the axion $a$. Thus from the kinetic term $D_\mu \sigma^* D^\mu \sigma$, we obtain $(1/2)\partial_\mu a \partial^\mu a$ where $\sigma = ([F_a + \rho]/\sqrt{2}) e^{ia/F_a}$.

Thus, below the scale $F_a$, the light fields are gluons and $a$ (plus the other SM fields). The relevant part of the Lagrangian respecting the symmetry (4) (i.e. with $a \rightarrow a + \alpha F_a$) is

$$L = \frac{1}{2} (\partial_\mu a)^2 + (\text{derivative term of } a) + \left(\theta + \frac{a}{F_a}\right) \frac{1}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \quad (4)$$

Note that we created the needed $F\tilde{F}$ coupling minimally. Usually, $a$ is redefined as $a - \theta F_a$ so that the coefficient of $F\tilde{F}$ is $a/F_a \equiv \bar{\theta}$. Thus, the above effective Lagrangian is seen to be invariant under the symmetry transformation Eq. (4).

### 2.2 Massless u-quark Solution

To see the $\theta$-independence of the effective Lagrangian of QCD below the chiral symmetry breaking scale with the massless u-quark, let us consider the one-flavor QCD first with a mass parameter $m_u$,

$$L = -m_u \bar{u}_R u_L + \text{h.c.}$$

which possess the following hypothetical symmetry

$$u_L \rightarrow e^{i\alpha} u_L, \quad \bar{u}_R \rightarrow e^{i\alpha} \bar{u}_R, \quad m_u \rightarrow e^{-2i\alpha} m_u, \quad \theta \rightarrow \theta + 2\alpha \quad (5)$$

Since $m_u$ is endowed with a transformation even though it is not a symmetry, Eq. (5) is useful to trace the $m_u$ dependence in the effective theory below the quark condensation scale $<\bar{u}u> \propto v^3 e^{i\eta/v}$,

$$\begin{align*}
V &= \frac{1}{4} m_u \lambda_1 \Lambda^2 e^{i\theta} - \frac{1}{4} \lambda_1 \Lambda^2 v^3 e^{i(\eta/v-\theta)} - \frac{1}{2} \lambda_2 m_u v^3 e^{i\eta/v} \\
&\quad + \lambda_3 m_u^2 \Lambda^2 e^{2i\theta} + \lambda_4 e^{i(\eta/v-\theta)} + \cdots + \text{h.c.}
\end{align*}$$

where the strong interaction scale $\Lambda$ is inserted to make the dimension appropriate. Note that, for $m_u = 0$, $\eta - F_a \theta$ can be redefined as a new $\eta$, removing the $\theta$ dependence. Thus $\theta$ is unphysical in a massless $u$-quark theory, solving the strong CP problem.

Since the interactions at the gravitational scale may violate global symmetries, one can consider the following nonrenormalizable interactions for the massless u-quark case,

$$\frac{1}{M_P} \bar{u}u e^{-2i\theta}, \quad \frac{1}{M_P} \bar{u}us e^{-i\theta} \quad (6)$$
where \( \sigma \) is a singlet scalar field. If only the first term is the only allowed nonrenormalizable interaction, \( \theta \) is shifted by a tiny amount, \( 10^{-38} \); thus the massless u-quark solution is still valid. However, if the second term is allowed with nonvanishing VEV of \( \sigma \), the VEV must be bounded to be less than \( 10^4 \) GeV to have phenomenologically allowable \( \theta \), given in Eq. (1). Thus, the massless u-quark idea is not automatically solving the strong CP problem with gravitational interactions.

It is interesting to see how one can obtain the axion mass from the above symmetry argument. At the minimum of the potential, the \( a-\eta \) mass matrix for \( m_u \neq 0 \) is

\[
M^2 = \begin{pmatrix}
\lambda \Lambda v + \lambda' mv, & -\lambda \Lambda v^2 / F_a \\
-\lambda \Lambda v^2 / F_a, & -\frac{F_a \Lambda + \lambda' \Lambda v^2}{F_a}
\end{pmatrix}
\]

Diagonalizing the above mass matrix for \( F_a \gg \) (other mass parameters), we obtain for vacuum at \( \theta = 0 \)

\[
m_a^2 = \frac{m_u \Lambda}{F_a^2} \left( \frac{\lambda \Lambda v^4}{\lambda \Lambda v + \lambda' m_u v} - \Lambda^2 \right), \quad m_\eta^2 = (\lambda \Lambda + \lambda' m) v
\]

which shows the essential features of the axion mass: it is suppressed by \( F_a \), multiplied by \( m_q \), and the rest of condensation parameters. If the above mass squared is negative, we chose a wrong vacuum and choose \( \theta = \pi \) instead as the vacuum.

For a realistic axion mass, however, we consider one family QCD

\[
\mathcal{L} = -m_u \bar{u} u - m_d \bar{d} d
\]

which possesses the fictitious \( U(1)_u \times U(1)_d \) symmetry,

\[
u_L \rightarrow e^{i\alpha} \nu_L, \quad d_L \rightarrow e^{i\beta} d_L, \quad m_u \rightarrow e^{-2i\alpha} m_u,
\]

\[
m_d \rightarrow e^{-2i\beta} m_d, \quad \theta \rightarrow \theta + 2(\alpha + \beta)
\]  

(7)

Following the same procedure, we obtain

\[
m_a = \frac{m_a \pi \alpha \sqrt{Z}}{F_a} \frac{\sqrt{Z}}{1 + Z}
\]  

(8)

where \( Z = m_u / m_d \). The above formula is valid for the KSVZ model. For the PQWW and DFSZ models, one needs extra consideration, for removing the longitudinal component of \( Z^0 \). In the limit of \( F_a \gg \) (other mass parameters), i.e. in the DFSZ model, the above formula is also valid.
3 Superstring Axion

The standard introduction of axion through spontaneous breaking of $U(1)_A$ global symmetry is ad hoc. The reason is that there exist so many ways to introduce the symmetry.

There exists another very fundamental way to introduce the axion. It is in the string theory. Furthermore, the axion must be present in string theory for the automatic strong CP solution in string models. Nevertheless, the route to the axion solution in string models is not always present, and therefore let us first see what are the problems along this line and then present a possible route.

Ten dimensional string models contain massless bosons $G_{MN}$ ($MN$ symmetric), $B_{MN}$ ($MN$ antisymmetric), and $\phi$, where $M, N$ run through indices $0, 1, \cdots, 9$. Our interest here is the antisymmetric tensor field $B_{MN}$ which contains two kinds of axions: model-independent axion (MIA) and model-dependent axions.

The MIA is basically $B_{\mu\nu}$ where $\mu, \nu$ is the 4D indices $0, 1, 2, 3$. The dual of the field strength is defined as the derivative of MIA $a$,

$$\partial^\sigma a \sim \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho}, H_{\mu\nu\rho} \sim \epsilon^{\mu\nu\rho\sigma} \partial^\sigma a$$

The question is why we interpret this as an axion. It is due to Green, Schwarz, and Witten,\[5\] the gauge invariant field strength $H$ of $B$ is $H = dB - \omega^0_{3Y} + \omega^0_{3L}$ with the Yang-Mills Chern-Simmons term $\omega^0_{3Y} = \text{tr}(AF - A^3/3)$ and the Lorentz Chern-Simmons term $\omega^0_{3L} = \text{tr}(\omega R - \omega^3/3)$. These satisfy $d\omega^0_{3Y} = \text{tr} F^2$ and $d\omega^0_{3L} = \text{tr} R^2$. Therefore,

$$dH = -\text{tr} F^2 + \text{tr} R^2$$

Note also that one had to introduce a nontrivial gauge transformation property of $B$. Then gauge anomaly is completely cancelled by introducing the so-called Green-Schwarz term,\[5\]

$$S_{GS} \propto \int (B \text{tr} F^4 + \cdots)$$

which contains the coupling of the form

$$\epsilon_{ijklmnopqr} B_{ij} F^K L F^M N F^O P F^Q R \sim B_{ij} (\epsilon^{\mu\nu\rho\sigma} F^\mu F^\nu F^\rho F^\sigma) \sim \epsilon^{ijklpq}$$

where we note the Minkowski indices $\mu, \nu, \cdots$ and the internal space indices $i, j, \cdots$. The nonvanishing VEV $< F >$ gives $a' F \tilde{F}$ coupling at tree level.
Thus we are tempted to interpret $a'$ as an axion, and it was called model-dependent axion\(^6\). But we have to check that there is no dangerous potential term involving $a'$, and indeed it has been shown that world-sheet instanton contribution

$$i \int_{\Sigma_J} d^2z B_J \omega^I_{ij} (\partial X^i \partial X^j - \overline{\partial X^i \partial X^j}) = 2 a'B_J$$

gives $a'(=B_I)$ dependent superpotential\(^10\), removing $a'$ as a useful degree for relaxing a vacuum angle. In the above equation, $a'$ is the string tension and $\omega^I_{ij}$ represents the topology of the internal space,

$$B = B_{\mu\nu} dx^\mu dx^\nu + B_I \omega^I_{ij} dz^i d\overline{z}^j \quad (12)$$

But $B_{\mu\nu}$ is still good since it does not get a contribution from the stringy world-sheet instanton effect. Eq. (10) implies

$$\Box a = -\frac{1}{M} (\text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} - \text{Tr} R_{\mu\nu} \tilde{R}^{\mu\nu}) \quad (13)$$

obtained from an effective Lagrangian of the form

$$L = \frac{1}{2} (\partial_\mu a)^2 - \frac{a}{16\pi^2 F_a} (\text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} - \cdots) \quad (14)$$

Thus $a$ is the axion (MIa). Any string models have this MIa and its decay constant is of order $10^{16}$ GeV\(^9\). At this point, we comment that there are two serious problems of the superstring axion:

(A) The axion decay constant problem—It is known that the decay constant of MIa is of order $10^{16}$ GeV\(^9\) which is far above the cosmological upper bound of $F_a$. A large $F_a$ can be reconciled with cosmological energy density if a sufficient number of radiation are added below 1 GeV of the universe temperature, but then it is hopeless to detect the cosmic axion by cavity detectors. Therefore, $F_a$ is better to be lowered to around $10^{12}$ GeV.

(B) The hidden sector problem—It is a general belief here that a hidden sector confining force, e.g. $SU(N)$, is needed for supersymmetry breaking at $10^{12} \sim 10^{13}$ GeV. If so, MIa gets mass also due to the $a_{MI} F' \tilde{F}'$ coupling where $F'$ is the field strength of the hidden sector confining gauge field, and we expect $m_a \simeq \Lambda^2_h/F_a$ which is obviously too large to bring down MIa to low energy scale for the solution of the strong CP problem. For example, the axion gets potential both from the hidden sector scale $\Lambda_h$ and the QCD scale $\Lambda_{QCD}$ (if there is no matter) in the following way,

$$-\Lambda_{QCD}^4 \cos(\frac{a_{MI}}{F_a} \theta^0) - \Lambda_h^4 \cos(\frac{a_{MI}}{F_a} \theta_h^0)$$
where we added two terms with independent phases $\theta^0, \theta^0_h$ which arise at the string scale when CP is broken. Because $\Lambda_h \gg \Lambda_{QCD}$, the vacuum chooses $a_{MI} + \theta^0_h = 0$, i.e. $< a_{MI} > \approx -\theta^0_h F_a$, implying $\theta \approx \theta^0 - \theta^0_h$ which is not zero in general. If we want to settle both $\theta_h$ and $\bar{\theta}$ at zero, then we need two independent axions. However, it is known that only MIa is available at string induced low energy physics. Therefore, we say that there is the hidden sector problem in the MIa phenomenology.

The above two problems have to be resolved if the string theory render an acceptable low energy standard model and also if the axion solution has a profound root in the fundamental theory of the universe. It turns out that it is very difficult to achieve. Only in a limited case, it may be possible to find a possible route toward a solution.

4 Anomalous $U(1)$

In anomalous $U(1)$ gauge models\footnote{With the massless gaugino with only renormalizable gaugino self interactions, there is an $R$ symmetry.}, the Green-Schwarz term contains

$$\epsilon_{MNPQRSTU} B^{MN} \cdot \text{Tr} F^{OP} \cdot < F^{QR} > < F^{ST} > < F^{UV} > \quad (15)$$

which introduces the coupling $M_c (\partial^\mu a_{MI}) A_\mu$. Namely, the MIa becomes the longitudinal degree of freedom of $A_\mu$. Thus, the $U(1)_A$ gauge boson becomes massive and $a_{MI}$ is removed at low energy. Below the scale $M_c$, then there exists a global symmetry\footnote{With the massless gaugino with only renormalizable gaugino self interactions, there is an $R$ symmetry.}.

Superstring models also need an extra hidden sector confining force. Then, even if the MIa is present, it obtains a dominant contribution to the mass from the hidden sector instanton effects. One can make the contribution to the MIa mass absent if there is a massless hidden colored fermion. It is similar to the massless u-quark solution of the strong CP problem. The first obvious choice is the theory of a massless hidden sector gaugino without a hidden matter. But the hidden sector gaugino is NOT massless. Nevertheless, the final hidden sector gaugino mass is not introduced by hand, but it arises from the condensation of the hidden sector gauginos, thus the contribution to the MIa potential is absent in this limit. However, the string (or gravitational) theory does not preserve any global symmetry, thus there must be interactions violating the $R$ symmetry\footnote{With the massless gaugino with only renormalizable gaugino self interactions, there is an $R$ symmetry.}. It is similar to the interactions (6) in massless u-quark model.

Thus we introduce an anomalous $U(1)$. The hidden sector gluino is massless in supersymmetric cases. With supersymmetry breaking, the gluino will obtain mass eventually; but this case is different from nonzero mass u-quark
case since the hidden sector gluino obtains mass by the hidden sector gluino
condensation. So if the R-symmetry is not explicitly broken, there is no con-
tribution to the potential of MIA from the hidden sector. However, the grav-
itational interactions violate the global symmetry. For example, with $SU(N)$
hidden gauge group, we expect

$$V_{\text{eff}} = \frac{1}{2} \bar{\lambda} \lambda \lambda + \epsilon_2 \bar{\lambda} \lambda \lambda \lambda \lambda + (\lambda \lambda)^N e^{-i \theta} + \text{h.c.} + \cdots$$  \hspace{1cm} (16)

Endowing the following fictitious symmetry, $\lambda \rightarrow e^{i \alpha} \lambda$, $\theta_h \rightarrow \theta_h + 2l(G)\alpha$ (where $l(SU(N)) = N$), and $\epsilon_2 \rightarrow e^{-2i \alpha} \epsilon_2$, the effective potential contains

$$\epsilon_2 \left( \frac{v}{M_p} \right)^5 v^4 e^{i \eta/v} + \left( \frac{v}{\Lambda_h} \right)^{3N} \Lambda_h^4 e^{i(\eta N/v - \theta_h)}$$

Note that, if $\epsilon_2 = 0$, then we do not have a $\theta_h$ dependence. The $\cdots$ in Eq. (16) contains higher order terms beyond the $\epsilon_2$ term; but if $U(1)_R$ were exact then there would be no $\theta_h$ dependence of $V_{\text{eff}}$.

\begin{center}
\begin{tikzpicture}
\draw (-2,0) -- (2,0);
\draw (0,0) circle (0.1);
\node at (-2,0) {QCD};
\node at (2,0) {hidden sector};
\end{tikzpicture}
\end{center}

**Discrete Symmetry** $Z_N \subset U(1)_R$

But the above figure cannot be realized since $U(1)_R$ is not exact. Therefore, the best we can hope is that only a discrete subgroup of $U(1)_R$ is exact. Starting from a massless hidden sector gluino, an obvious choice is a subgroup $Z_N$ of $U(1)_R$. Depending on the compactification schemes, only a limited class of terms are allowed in the superpotential. One such example is $W \sim T^{18n+12}$ $n = \text{(integer)}$ in some orbifold compactification where $T$ is the twisted sector fields. Then a possible unbroken discrete subgroup is $Z_3 n$.

It is straightforward to estimate the hidden-sector contribution to the MIA potential with an unbroken $Z_N$ which is shown below,

Table 1. The $\theta_h$ dependence of potential in GeV$^4$ units for $\Lambda_h = 10^{12} - 10^{13}$ GeV

| $Z_N$ | $\epsilon_n$ | $V$ |
|-------|-------------|-----|
| $N=2$ | $n=2$       | $\sim 10^{14} - 10^{25}$ |
| $N=3$ | $n=4$       | $\sim 10^{-29} - 10^{-8}$ |
| $N=4$ | $n=3$       | $\sim 10^{-8} - 10^{10}$  |
| $N=5$ | $n=6$       | $\sim 10^{-50} - 10^{-26}$ |
Thus, for $Z_N$ with $N = 3, 5, 6, \cdots$ the hidden sector contribution to the MIa potential is negligible and MIa acts as the invisible axion of the standard model. Indeed, there are possibilities of realizing these $Z_N$ subgroups in string models.

5 CONCLUSION

The $\bar{\theta}$ parameter problem in the standard model must be resolved in an ultimate theory if it exists. The axion solution is the most attractive one. Usually, it is believed that the gravitational interaction breaks all global symmetries, and the basis for the axion is shaken. The same comment applies to a massless quark solution. However, the string models contain the MIa which can be a good candidate for the nonlinearly realized global symmetry. Nevertheless, we must pass through the hurdle of the hidden sector physics.

The MIa gets contributions to its potential from QCD and the hidden sector as

\[ QCD \]
\[ \text{SU}(N)_h \]

But with an anomalous $U(1)$ and $U(1)_R$ symmetries it gets as

\[ QCD \]
\[ \text{SU}(N)_h \]

Gravitational interaction may break $U(1)_R$ and we expect the

\[ QCD \]
\[ \text{SU}(N)_h \]

Even if the $U(1)_R$ symmetry is broken, the hidden sector contribution is not as large as $\Lambda^4_h$, because the hidden sector gluino obtains mass through the gluino
condensation itself. This is a different point from the massive u-quark case where u-quark mass is given as a parameter in QCD.

But if $Z_{3,5,\cdots}$ subgroup of $U(1)_R$ is unbroken the hidden sector contribution is negligible

$$\begin{array}{cc}
\text{QCD} & \text{SU(N)$_h$} \\
\includegraphics[width=0.3\textwidth]{qcd-su-h.png} & \includegraphics[width=0.2\textwidth]{su-h.png}
\end{array}$$

Thus superstring models can include the invisible axion needed for the strong CP solution.

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