Experimental demonstration of complementarity relations between quantum steering criteria

Huan Yang, Zhi-Yong Ding, Xue-Ke Song, Hao Yuan

1School of Physics and Material Science, Anhui University, Hefei 230601, China
2Institutes of Physical Science and Information Technology, Anhui University, Hefei 230601, China
3Department of Experiment and Practical Training Management, West Anhui University, Luan 237012, China
4School of Physics and Electronic Engineering, Fuyang Normal University, Fuyang 236037, China
5Key Laboratory of Functional Materials and Devices for Informatics of Anhui Educational Institutions, Fuyang Normal University, Fuyang 236037, China
6CAS Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei 230026, China

The ability that one system immediately affects another one by using local measurements is regarded as quantum steering, which can be detected by various steering criteria. Recently, Mondal et al. [Phys. Rev. A 98, 052330 (2018)] derived the complementarity relations of coherence steering criteria, and revealed that the quantum steering of system can be observed through the average coherence of subsystem. Here, we experimentally verify the complementarity relations between quantum steering criteria by employing two-photon Bell-like states and three Pauli operators. The results demonstrate that if prepared quantum states can violate two setting coherence steering criteria and turn out to be steerable states, then it cannot violate the complementary settings criteria. Three measurement settings inequality, which establish a complementarity relation between these two coherence steering criteria, always holds in experiment. Besides, we experimentally certify that the strengths of coherence steering criteria dependent on the choice of coherence measure. In comparison with two setting coherence steering criteria based on $l_1$ norm of coherence and relative entropy of coherence, our experimental results show that the steering criterion based on skew information of coherence is more stronger in detecting the steerable of quantum states. Thus, our experimental demonstrations can deepen the understanding of the relation between the quantum steering and quantum coherence.

1. INTRODUCTION

Quantum steering describes a nontrivial trait of quantum world that one subsystem of bipartite systems can instantaneously affects another one by using local measurements [1, 2]. In contrast to the entanglement [3] and Bell nonlocality [4, 5], quantum steering has been attracted extensive attentions in the field of quantum information only in recent years [6]. The detection of quantum steering can be realized through the violations of steering criteria (also called steering inequalities), which can be obtained by using correlations, state assemblages, and full information [7]. There are various steering criteria, including linear and nonlinear steering criteria [8, 9], steering inequality from uncertainty relations [10–13], steering criterion from geometric Bell-like inequality [14], steering inequalities from the semidefinite programs [15], and full information steering inequality [16]. So far, quantum steering embodies vital application values in sub-channel discrimination, resource theory of steering, quantum communication, quantum teleportation, randomness generation, and so on [7]. Also, it has been demonstrated in a series of significant experiments [17–26].

Coherence, which originates from the superposition principle of quantum mechanics, reflects one of fundamental essences in many quantum phenomena [27, 28]. Although the investigations concerning coherence have a long-standing history, however, the rigorous quantification of coherence in the field of quantum information had never established. Until 2014, based on incoherent operations, Baumgratz et al. [29] put forward the general frame of quantifying coherence for quantum states, and this quantification relies on a fixed reference basis. One can measure quantum coherence via $l_1$ norm of coherence [29], relative entropy of coherence [29], robustness of coherence [30, 31], and skew information of coherence [32–34]. Recently, quantum coherence becomes a hot topic in both theory [35–40] and experiment [41–44]. It plays a central part in different fields, such as quantum metrology, quantum thermodynamics, quantum algorithms, and quantum channel discrimination [27].

Noteworthily, it is a new tendency for theoretically exploring the complementarity and trade-off relations among different quantities in recent years. According to these relations, a bound for a quantity can be established via another complementary quantity. The understanding of the quantum state space and information as well as correlation can also deepened through these relations. There are several promising efforts in concerning fields [45–52]. Singh et al. [45] obtained complementarity between maximal coherence and mixedness, and examined the limits imposed by mixedness of a quantum system with respect to quantum coherence. Cheng et al. [46] explored complementarity relations between the coherences of mutually unbiased bases, and derived relations among coherence, purity, and uncertainty. Considering the maximal violations of the Clauser-Horne-Shimony-Holt inequality, the trade-off relations of Bell violations among pairwise qubit systems were investigated by Qin et al. [47], and the relations constrain the distribution of nonlocality among the subsys-
tems. By using the relative entropy of coherence, Sharma et al. [48] presented the trade-off relation between the systems coherence and disturbance induced by a completely positive trace-preserving map. For a multipartite system, the trade-off relations for tripartite nonlocality were established by Zhao et al. [49]. Experimentally, different complementarity and trade-off relations were also tested [53–55]. By employing a photonic qutrit-qubit hybrid system, Zhan et al. [53] experimentally verified contextuality-nonlocality trade-off relation, and the results certified that entanglement is a particular form for fundamental quantum resource. Weston et al. [54] experimentally tested the universally valid complementarity relations satisfied for any joint measurement of two observables. In two noncommuting reference bases, Lv et al. [55] experimentally verified the trade-off relation of quantum coherence, and their results displayed that the lower and upper bounds restrict the sum of quantum coherence under these bases.

Recently, Mondal et al. [56] obtained the complementarity relations between coherence steering criteria by employing different quantifications of quantum coherence. This work established a connection between two valuable quantum resources in quantum information task, i.e., quantum steering and quantum coherence. However, the test of the complementarity relations in experiment is still lacking. The concerning investigation may further deepen our understanding of the relation between the quantum steering and quantum coherence in practice. Also, it can demonstrate a new method to detect quantum steering in experiment, namely, witness quantum steering in experiment, which is used to convince or fool Bob via $\rho_A^3$. The $\rho_{AB}$ is steerable state if and only if the joint probabilities of measurement outcomes (Alice performs the measurement of $A$ on her subsystem and obtains the outcome $a \in \{0,1\}$) cannot be described by employing a local hidden variable-local hidden state (LHV-LHS) model [6].

In 2018, Mondal et al. [56] proposed the coherence steering criteria, which can help us to observe the quantum steering of a two-qubit state via the quantum coherence of subsystem. Consider two-qubit states $\rho_{AB} = (I \otimes I + r \cdot \sigma \otimes I + I \otimes s \cdot \sigma + \sum_{i,j} t_{ij} \sigma_i \otimes \sigma_j) / 4$ and three Pauli operators $\{\sigma_i\}$ for a complete set of mutually unbiased bases (MUBs), where $r = \text{tr}(\rho_{AB} \sigma \otimes I)$, $s = \text{tr}(\rho_{AB} I \otimes \sigma)$, and $t_{ij} = \text{tr}(\rho_{AB} \sigma_i \otimes \sigma_j)$. Assuming that Alice implements a projective measurement on her system by using the eigenbasis of $\sigma_i$ and the corresponding outcome is $a \in \{0,1\}$. The corresponding probability is $p(\rho_{B|M_1^a}) = \text{Tr}[M_k^a I \rho_{AB}]$, and measurement operator is $M_k^a = [I + (-1)^a \sigma_i] / 2$. Similarly, Alice can measure her system by employing another measurement operator $M_k^a = [I + (-1)^a \sigma_k] / 2$, and $k \neq i, k \in \{x, y, z\}$. For each projective measurement implemented by Alice, Bob can measure the coherence of his conditional state $\rho_{B|M_1^a}$ under the eigenbasis of each Pauli operator. According to the number of Pauli operators chosen by Bob, the coherence steering criteria can be divided into one measurement setting (or one setting) and two measurement setting (or two setting), respectively [56]. Explicitly, the probability superposition of coherences of $\rho_{B|M_1^a}$ can be defined as

$$S_{\ell}^B(\rho_{AB}) = \sum_{i,a} p(\rho_{B|M_1^a}) C_{\ell+i}^q(\rho_{B|M_1^a}),$$

where $\ell \in \{0,1,2\}$, $C_{\ell+i}^q(\rho)$ represents different coherence measures under the eigenbasis of Pauli operator $\sigma_i$, including $l_1$ norm of coherence ($q = l_1$ and $C_{\ell+i}^{l_1}(\rho) = \sum_{i \neq j} ||\langle k_i | \rho | k_j \rangle||$), relative entropy of coherence ($q = \text{REC}$ and $C_{\ell+i}^{\text{REC}}(\rho) = S(\rho^{\text{diag}}) - S(\rho)$), and skew information of coherence ($q = \text{SIC}$ and $C_{\ell+i}^{\text{SIC}}(\rho) = -\text{Tr}[(\sqrt{\rho} \cdot \sigma_i^2)^2] / 2$). $\{k_i, k_j\}$ denote the eigenvectors of $\sigma_i$, $S(x)$ is the von Neumann entropy, and $\rho^{\text{diag}} = \sum_i |k_i \rangle \langle k_i | \rho | k_i \rangle |k_i \rangle$. If Bob measures the coherence only in one Pauli operator $\sigma_i$, which is same as each projective measurement chosen by Alice (i.e., $\ell = 0$), and then the one setting coherence steering criteria are given by

$$S_{\ell}^B(\rho_{AB}) = \sum_{i,a} p(\rho_{B|M_1^a}) C_{\ell+i}^q(\rho_{B|M_1^a}) \leq \varepsilon^q.$$  

Here, $q \in \{l_1, \text{REC}, \text{SIC}\}$, and $\varepsilon^q \in \{\sqrt{6}, 2.23, 2\}$ represent corresponding upper bounds for different coherence measures. The criteria cannot be violated by the state with the
LHS model. If Bob measures the coherence in the eigenbasis of another two Pauli operators \(\sigma_j\) and \(\sigma_k\) (\(j \neq k \neq i\)) (corresponding to \(\ell = 1, 2\)), which are different from the one chosen by Alice’s measurement, there exists the two setting coherence steering criteria

\[
\frac{1}{2} S^B_{12}(\rho_{AB}) = \frac{1}{2} (S^B_1(\rho_{AB}) + S^B_2(\rho_{AB})) \leq \varepsilon^2. \tag{3}
\]

Any state with the LHS model obeys these steering criteria. The violation of the criteria means that the \(\rho_{AB}\) is steerable. If Bob measures the coherence under the eigenbasis of three Pauli operators after each projective measurement performed by Alice on her subsystem, the inequality of three measurement settings is

\[
\frac{1}{3} S^B_{012}(\rho_{AB}) = \frac{1}{3} (S^B_0(\rho_{AB}) + S^B_1(\rho_{AB}) + S^B_2(\rho_{AB})) \leq \varepsilon^3. \tag{4}
\]

This inequality cannot be used to detect the quantum steering of \(\rho_{AB}\) due to the fact that the inequality is satisfied by all two-qubit states. In reality, the Eq. (4) describes a complementarity relation between one setting coherence steering criteria and two setting coherence steering criteria, that is,

\[
\frac{1}{3} S^B_{012}(\rho_{AB}) = \frac{1}{3} (S^B_0(\rho_{AB}) + S^B_1(\rho_{AB})) \leq \varepsilon^3. \tag{5}
\]

The results manifest that if one criterion between Eqs. (2) and (3) is violated, and then the other one as a compensation can never be violated.

### III. Experimental Demonstrations and Results

In the process of our experimental implementation, we choose two-photon Bell-like states as test states. The polarized photons are encoded as qubits. The horizontally and vertically polarized states are described by using \(|H\rangle\) and \(|V\rangle\), respectively. Hence, two-photon Bell-like states are

\[
\rho_{AB} = |\phi_{AB}\rangle \langle \phi_{AB}| \tag{6}
\]

with

\[
|\phi_{AB}\rangle = \cos \theta |H H\rangle + \sin \theta |V V\rangle. \tag{7}
\]

**TABLE I. The settings of wave plates for realizing different PMOs on the photon of Alice in the module (b).**

| Settings | \(M_0^w\) | \(M_1^w\) | \(M_0^b\) | \(M_1^b\) | \(M_0^f\) | \(M_1^f\) |
|----------|--------|--------|--------|--------|--------|--------|
| \(w_1\)  | HWP    | HWP    | QWP    | QWP    | HWP    | HWP    |
| \(\theta_1\) | 22.5°  | -22.5° | 45°    | -45°   | 0°     | 45°    |
| \(w_2\)  | HWP    | HWP    | QWP    | QWP    | HWP    | HWP    |
| \(\theta_2\) | 22.5°  | -22.5° | -45°   | 45°    | 0°     | 45°    |

Figure 1 provides the schematic diagram of all-optical experiment setup which is used to realize the verification of the complementarity relations. The setup contains three modules. The yellow area is the module (a) to prepare test states. To be explicit, high-power continuous pumped beam (the power is 130mW and the wavelength is 405nm) passes through the polarization beam splitter (PBS). The state of pumped beam transforms into horizontally polarized state \(|H\rangle\). This light beam first passes through the half-wave plate (HWP), and then is focused on two type-I \(\beta\)-barium borate; QWP, quarter-wave plate; IF: interference filter; SPD: single photon detector. \(w_1\) and \(w_2\): the types of wave plates; \(\theta_1\) and \(\theta_2\): the angles of optical axes of wave plates.
in Table I. For each Bell-like state prepared in experiment, the postmeasurement states are expressed by $\rho_{M_1^0B}, \rho_{M_2^0B}, \rho_{M_3^0B}, \rho_{M_4^0B}, \rho_{M_5^0B}, \rho_{M_6^0B}$, and $\rho_{M_7^0B}$. The pink area in Fig. 1 indicates the module (c) of the quantum state tomography [59, 60], which is used to attain density matrices of quantum states.

Now let us turn to verify the complementarity relations between quantum steering criteria in experiment. The experimental measurement probabilities $p(\rho_{B|M_i^0})$ in Eqs. (2)-(4) are obtained by virtue of coincidence counts [61], and the corresponding coherence $C_\rho(\rho_{B|M_i^0})$ are calculated according to the density matrices of 66 postmeasurement states reconstructed via quantum state tomography. Thus, the experimental results of $S^B_0(\rho_{AB}), S^B_{12}(\rho_{AB}), S^{REC}_0(\rho_{AB})$ and $S^{SIC}_0(\rho_{AB})$, respectively, can be attained in different coherence measures. In detail, Fig. 2 and Table II depict the results based on $l_1$ norm of coherence ($l_1C$). Figure. 3 and Table III provide the results based on relative entropy of coherence (REC). The results based on skew information of coherence (SIC) are depicted in Fig. 4 and Table IV. It is worthwhile to note that some of the error bars are too short to exhibit in Fig. 2–4. For all figures, the green squares, purple squares, and red squares denote the experimental results of $S^{REC}_0(\rho_{AB}), S^{SIC}_0(\rho_{AB})$, and $S^{SIC}_0(\rho_{AB})$, respectively. The corresponding theoretical results are displayed by means of different colored curves, and the black dotted lines are the upper bounds $ε^{REC} = 2.23$.

![Fig. 2](image1.png)

**FIG. 2.** Experimental results of the $l_1$ norm of coherence. The red squares indicate the experimental results of $S^{REC}_{012}(\rho_{AB})/3$. The purple squares denote the experimental results of $S^{REC}_{12}(\rho_{AB})/2$ and the green squares represent the experimental results of $S^{REC}_0(\rho_{AB})$, respectively. The corresponding theoretical predictions are represented by using solid lines with different colors. The black dashed line represents the upper bound $ε^{REC} = √6$.

![Fig. 3](image2.png)

**FIG. 3.** Experimental results of the relative entropy of coherence. The red squares indicate the experimental results of $S^{REC}_{012}(\rho_{AB})/3$. The purple squares denote the experimental results of $S^{REC}_{12}(\rho_{AB})/2$ and the green squares represent the experimental results of $S^{REC}_0(\rho_{AB})$, respectively. The corresponding theoretical predictions are represented by employing solid lines with different colors. The black dashed line represents the upper bound $ε^{REC} = 2.23$.

![Fig. 4](image3.png)

**FIG. 4.** Experimental results of skew information of coherence. The red squares indicate the experimental results of $S^{SIC}_{012}(\rho_{AB})/3$. The purple squares denote the experimental results of $S^{SIC}_{12}(\rho_{AB})/2$ and the green squares represent the experimental results of $S^{SIC}_0(\rho_{AB})$, respectively. The corresponding theoretical predictions are represented by using solid lines with different colors. The black dashed line represents the upper bound $ε^{SIC} = 2$.

| $θ$  | $S^{REC}_{012}(\rho_{AB})/3$ | $S^{REC}_{12}(\rho_{AB})/2$ | $S^{REC}_0(\rho_{AB})$ |
|------|-----------------------------|-----------------------------|------------------------|
| 0°   | 2.0062±0.0195               | 2.0017±0.0200               | 2.0150±0.0186          |
| 10°  | 2.1915±0.0307               | 2.3376±0.0303               | 1.8993±0.0317          |
| 20°  | 2.2750±0.0197               | 2.6373±0.0171               | 1.5504±0.0250          |
| 30°  | 2.2474±0.0171               | 2.8617±0.0149               | 1.0188±0.0215          |
| 40°  | 2.1085±0.0389               | 2.9810±0.0447               | 0.3636±0.0274          |
| 45°  | 2.0095±0.0474               | 2.9954±0.0465               | 0.0378±0.0493          |
| 50°  | 2.1060±0.0356               | 2.9808±0.0389               | 0.3562±0.0289          |
| 60°  | 2.2453±0.0150               | 2.8628±0.0117               | 1.0102±0.0216          |
| 70°  | 2.2740±0.0138               | 2.6423±0.0113               | 1.5372±0.0189          |
| 80°  | 2.1892±0.0207               | 2.3396±0.0214               | 1.8884±0.0194          |
| 90°  | 2.0018±0.0153               | 2.0008±0.0156               | 2.0038±0.0148          |
From Figs. 2-4 and Tables II-IV, one can find that seven prepared Bell-like states ($\theta = 20^\circ, 30^\circ, 40^\circ, 45^\circ, 50^\circ, 60^\circ, 70^\circ$) can violate two setting coherence steering criterion based on different coherence measures (i.e., $L_1$ C, REC, and SIC). The results demonstrate that these seven prepared Bell-like states are steerable states. It also deserves emphasizing that two prepared Bell-like states with $\theta = 10^\circ$ and $\theta = 80^\circ$ (labeled by 1 and 2 in Figs. 2-4, respectively) can violate two setting coherence steering criterion based on skew information of coherence. However, these two states cannot violate the criteria based on $l_1$ norm of coherence and relative entropy of coherence. The experimental results verify that the quantum steering of these two states can only be detected by the two setting coherence steering criterion from skew information of coherence, and cannot be captured through the ones from $l_1$ norm of coherence and relative entropy of coherence. In order to further certify the results, we perform steering inequality tests on these two states by using steering inequality from general entropic uncertainty relation (SIGEUR), which is an effective tool to detect quantum steering [12, 25, 26]. The SIGEUR is written as $(n - 1)^{-1} \sum \{ 1 - \sum_{a b} \frac{\langle x(a) \rangle^n}{\langle x(a) \rangle^n - 1} \} \geq C_B(n) \geq C_B(n) = \ln_2 [\ln(2d-m-1)] / [\ln(2d-m-1)]$ for $n \in [0, 2]$. Here, $\ln_2(x) = x^{1/n} - 1 / (1 - n)$, and $\rho_{ab}^{(i)}$ ($i = x, y, z$ and $a, b \in \{0, 1\}$) represents the probability of outcome $(a, b)$ for a set of measurements $A_i \otimes B_j$ implemented both on the photons of Alice and Bob. $p_a^{(i)}$ is the probability of marginal outcome for measurement $A_i$ of Alice. $d$ is the dimension of system, and $m$ is number of MUBs. In our experiment, $d = 2$, $m = 3$, and we choose $n = 2$ due to that the SIGEUR is the strongest one in this case [12, 25, 26]. Hence, the lower bound $C_B(n) = 1$. In technology, we remove the module (b) in Fig. 2, and use module (c) to achieve the six PMOs performed on both Alice's and Bob's photon, as illustrated in Table V. Thereby, the steering inequality test can be implemented on prepared Bell-like states with $\theta = 10^\circ$ and $\theta = 80^\circ$. One can conveniently calculate $p_{ab}^{(i)}$ and $p_a^{(i)}$ according to the coincidence counts in experiment. The experimental results are plotted in Fig. 5. It is demonstrated that the experimental left hand sides of SIGEUR (LHS-SIGEUR) for prepared Bell-like states with $\theta = 10^\circ$ and $\theta = 80^\circ$ are equal to $0.8869 \pm 0.0049$ and $0.8876 \pm 0.0043$, respectively. These results violate the SIGEUR, and further certify that the prepared Bell-like states with $\theta = 10^\circ$ and $\theta = 80^\circ$ are steerable states. It means that two setting coherence steering criterion based on skew information of coherence can indeed detect more steerable states than the ones based on $l_1$ norm of coherence and relative entropy of coherence.

### IV. CONCLUSIONS

In this work, we experimentally demonstrate the complementarity relations between quantum steering criteria by employing prepared Bell-like states with high fidelity and three Pauli operators. The experimental results are in accordance with the theoretical curves very well, and one can reveal the steerability of system by detecting the average coherence of subsystem. Whatever coherence measure is used, three mea-
measurement settings inequality is always obeyed by all prepared Bell-like states in experiment. Meanwhile, the experimental $S^B_0(\rho_{AB})$ are anticorrelated with the $S^B_{1/2}(\rho_{AB})/2$. If the prepared Bell-like states violate two setting coherence steering criteria, then the states cannot violate one setting coherence steering criteria. Furthermore, The strengths of coherence steering criteria rely on the choice of coherence measure. In comparison with two setting coherence steering criteria based on $l_1$ norm of coherence and relative entropy of coherence, two setting coherence steering criterion based on skew information of coherence is more effective in witnessing steerable states. Our experimental results may deepen the understanding of the connection between the quantum steering and quantum coherence.

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