1. Introduction

Moving up a roller coaster lift hill can be an exciting experience with views from higher and higher points and anticipation of the accelerations in drops, loops, twists and turns. In this paper, however, we focus on uniform rectilinear motion, where Newton’s first law applies, and the sum of the forces from the ride on the body must exactly compensate the force of gravity.

Smartphones offer a number of ways to illustrate motion. Different sensors offer complementary representations, and the data from the phone are easily accessible through apps, such as Physics Toolbox [1, 2] and Phyphox [3]. For example, using the pressure sensor in an elevator is a way to measure the change in elevation, which can in turn be used to obtain a graph of how velocity varies with time [4]. Staacks et al [3] showed how the elevation and velocity graphs can be combined with with accelerometer data within the Phyphox app, itself, complementing the experience of the body. These examples illustrate how smartphones can be used to bring real-life examples of motion into the classroom.
Section 2 focuses on motion on an inclined plane, with a roller coaster lift hill (figure 1) as a special case, discussed in section 2.1, and illustrated in section 2.2 with data from a smartphone riding on the hand rail of an escalator. The analysis uses a combination of data from accelerometer, gyroscope and pressure sensors, as well as observations and length measurements in an escalator.

Section 2.3 introduces the motion in an LSM (linear synchronous motor) launch with a slight uphill slope as a special case of an inclined plane. The Tracker app [5] is used to support the counter intuitive result that the acceleration is very small during most of the launch.

Finally, section 3 analyses the essentially uniform rectilinear motion with rotation around the heartline.

1.1. Accelerometers, forces and Newton’s second law

Acceleration as second derivative of position may seem abstract, but when connected to force through Newton’s second law, the experience of the body can be added to the more formal representations. With Newton’s second law, $F = ma$, forces and mass enter the descriptions of motion. Since the force of gravity, $mg$, always acts, the total force on an object can be written as a sum $F = mg + X$, where $X$ is the sum of external forces contributing to the acceleration. For free fall, where $X = 0$, the acceleration is given by $a = g$. For rest or non-accelerated motion, $a = 0$ and the sum of external forces must be given by $X = -mg$.

Whether found in more traditional educational equipment [6, 7] or in smartphones, an accelerometer, in spite of its name, does not measure acceleration but instead $X/m = a − g$, or $X/mg = (a − g)/g$, relating $X$ to the force of gravity. This ‘G factor’ is independent of the mass—a consequence of the equivalence between inertial and gravitational mass [8–10]. The data from a 3D accelerometer is expressed in relation to the axes of the sensor—just like the experience of a human body depends on its orientation. Calculating the G factor acting on your body in a ride may feel more relevant than checking if a result like 16.7 kN for the force on a roller coaster car agrees with the answer in the back of the book.

Traditionally, the coordinate axes for biomechanical effects on human bodies are defined with the x axis pointing forward and the z axis up along the spine as in figure 1. For a right-handed coordinate system the y axis must then point to the left. Many phones now have access to gyroscopes, measuring angular velocities around the x, y and z axes, referred to as roll, pitch and yaw. They can
be thought of as doing a cartwheel, a somersault and a pirouette in a sports class. The escalator and heartline roll examples in this paper involve pitch and roll rotations, respectively. (See also [11, 12] and check out the axes on your phone.)

2. Inclined planes: lift hills, escalators and LSM launches

Force and motion on inclined planes are a mandatory part of any introductory physics course, and the teaching typically includes force arrows in the textbook, possibly also blocks, dynamometers, and demonstrations to illustrate that the normal force must be $mg \cos \theta$ and the tangential force $mg \sin \theta$ to compensate for the downward force of gravity, $mg$, to keep the mass at rest or at constant speed (figure 2). However, the forces on an inclined plane can also be illustrated using a 3D accelerometer, e.g. in a smartphone.

2.1. Roller coaster lift hills

A roller coaster lift hill, e.g. as shown in figure 1, is a special case of an inclined plane and an example of uniform rectilinear motion. The sum of the downward force of gravity, and the forces from the train on a rider must thus be zero, as the train moves uphill with constant velocity. Figure 1 shows how the force from the seat, orthogonal to the track, and the force from the backrest, parallel to the track, combine to compensate for the force of gravity in the 30° lift hill.

Escalators are more easily accessible than roller coaster lift hills, and offer good practice in understanding sensor data. The accelerometer graphs (figure 3) from a smartphone riding on the hand rail of an escalator (figure 4) look very similar to the lift-hill part of accelerometer graphs from rides in traditional roller coasters. Most escalators, in fact, have the same slope as the lift hill of Liseberghanan (figure 1). The experience of the rider is, of course, very different: in an escalator you stand up, just as if you were on the ground, whereas in the Liseberghanan lift hill your body is tilted backwards by 30°. Similarly, a smartphone resting on the escalator hand rail is rotated by 30° when the escalator moves upward.

2.2. Escalators

Escalators connect different levels in stores, stations and amusement parks, such as in figure 4. Figure 3 shows data from the accelerometer, gyro and pressure sensors of a smartphone. (How can the different graphs in figure 3 tell whether the motion is uphill or downhill?)

As an escalator moves upward (or downward) after the first transition steps, the motion is an example of Newton’s first law, where the force from the escalator on a person riding it must compensate for the force of gravity. This holds also for a smartphone placed on the hand rail. However, whereas the force on a person points straight up from the step, the smartphone axes are aligned with the slope. The data in figure 3 agree well with an angle $\theta \approx 30°$ as obtained also from the measurement of the length and height of an escalator step (figure 4). For this angle the normal force on the phone (defined as the $z$ axis in the graphs) is expected to be $mg \cos \theta = mg\sqrt{3}/2$, while the force along the forward direction of motion (defined as the $x$ axis in the graphs) is given by $\pm mg \sin \theta = \pm mg/2$. A third way of estimating the angle $\theta$ is to use the relation $\theta(t) - \theta(t_0) = \int_{t_0}^{t} \omega(t') dt'$ and perform a numeric integration of the data for the angular velocity, $\omega(t)$. An approximation is obtained by counting rectangles over the graph in
In real life, checking your answer is unlikely to be in the form of checking the back of the book, but rather by checking consistencies between different ways of obtaining the result. The escalator data offer many additional consistency checks: the pressure difference over the longest escalator shown in figure 3 is $\Delta p \approx 2$ hPa. For an air density $\rho \approx 1.3 \text{ kg m}^{-3}$ this gives $\Delta h \approx 16$ m. A manual count in a non-moving escalator found 87 steps with 20 cm height, but less for a few steps at either end. Approximately 80 steps times 20 cm/step also gives $\Delta h \approx 16$ m. The graph shows that the uphill ride in the longest elevator takes about 64 s, corresponding to a vertical velocity component of $0.25 \text{ m s}^{-1}$. The dimensions shown in figure 4 give a slope of $30^\circ$. For long elevators, $30^\circ$ is the international standard slope, and $0.5 \text{ m s}^{-1}$ is a common speed, which is found to give optimal capacity for a continuous flow of people. This corresponds to a $0.25 \text{ m s}^{-1}$ vertical velocity component, consistent with the measurements presented here.

2.3. A roller coaster launch

Traditional lift hills with chains are replaced by launch technologies in many newer roller coasters. Hydraulic launch coasters typically give the trains a couple of seconds of forward acceleration of 1 g or more (as studied, e.g. in [13]) along a nearly horizontal track. The energy for the launch can also be provided with LSM (linear synchronised motor) technology, where electromagnetic interactions are used to propel the train. Figure 6 shows the linear part of the track that forms the second LSM launch of the Helix roller coaster. The $9.7^\circ$ uphill slope is less steep than traditional lift hills, but makes it possible to add mechanical energy to the train in a shorter distance than if the launch had been horizontal. Towards the later part of the launch, the train is found to move uphill with nearly constant velocity.

2.3.1. Accelerometer data. Figure 7 shows the 'vertical' component (i.e. the '$z$' component, orthogonal to the track) of the accelerometer data for the Helix roller coaster, collected using a wireless dynamic sensor system (WDSS) [6]. The parts with essentially uniform rectilinear
Smartphones and Newton’s first law in escalators and roller coasters

motion are shaded. During linear parts of the track, this force orthogonal to the track should be $mg \cos \theta \approx 0.99 \ mg$ independent of the acceleration, as illustrated in figure 2. The graph in figure 7 shows, indeed, a vertical force very close to $mg$ for both of the launches, as expected for the small angles.

For constant velocity, the forward force (the $x$ component) from the back of the seat would be $mg \cos \theta \approx 0.17 \ mg$ for the $\theta = 9.7^\circ$ uphill slope of the second launch. This value has been marked with a solid green line in figure 8. The accelerometer data are larger, dropping from 0.50 mg to 0.31 mg over 2.32 s. The data thus indicate an acceleration that drops from 0.33 g to 0.14 g (with a ‘jerk’ [11] of $-0.08 \ g \ s^{-1}$).

However, extracting the tangential force from measured data requires an accurate orientation of the coordinate axes, since the accelerometer data are dominated by the force orthogonal to the track. Devices carried on the body are usually not sufficiently accurately oriented to give a

Figure 4. Escalator at Liseberg viewed from inside and outside, and a detail of an escalator step. Measurement on site shows $h \approx 20 \ cm$ and $L \approx 40 \ cm$.

Figure 5. Detail of the angular velocity data from one of the graphs in figure 3. Integration over the angular velocity gives the change in angle from horizontal to the slope of the escalator. Each rectangle corresponds to an angular change of 0.01 radian $\approx 0.57^\circ$. 

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Figure 6. The second launch leading into the inverted top hat of the Helix roller coaster at Liseberg. The 69.5 m long linear part of the track has an uphill slope of $\theta = 9.7^\circ$.

Figure 7. An accelerometer graph for the Helix roller coaster together with elevation data, showing the component vertical to the seat. The shaded areas mark parts with essentially uniform rectilinear motion during the two LSM-launches as well as the heartline roll, discussed in section 3.
Figure 8. A detail of the accelerometer data for the second launch in Helix, shown in figure 6. The red graph shows the force orthogonal to the track and the green graph shows the component in the direction of motion, in units of mg. The blue graph shows the lateral component, which would be zero for a properly aligned sensor. The solid red line corresponds to \( \cos \theta \approx 0.99 \), giving the size of the normal force for \( \theta = 9.7^\circ \). The solid green line marks \( \sin \theta \approx 0.17 \), giving the size of the force required to compensate for the tangential component of the force of gravity, maintain constant speed. The dashed green line is the least squares fit to the measured data. The dashed black line shows the least squares fit for the total G factor, which would be exactly 1 without acceleration.

Figure 9. Screenshots of a movie (30 fps) of the second launch of the Helix roller coaster analysed using the Tracker software [5]. The distance between the wheel axes is 2.50 m, giving a total length of 12.50 m between the first and last axis, and an estimated distance \( L \approx 13.5 \) m between the front and back of the train, as viewed in the video. The three rows of screenshots were aligned from stationary features, indicating small camera movements.
reliable measure of any deviations from the value \( mg \sin \theta \). The orientation of the device relative to the body may also shift slightly during the ride, causing small deviations even after an attempted reorientation of the coordinate axes in the data collected.

The size of total force from the train on the rider for a forward acceleration \( a \) in a \( \theta = 9.7^\circ \) uphill slope (see figure 2) is given by

\[
m \sqrt{(g + a \sin \theta)^2 + (a \cos \theta)^2}
\]

The measured values for the total force should not depend on the orientation of the device. Figure 8 includes a least squares fit to the values the total force (falling from 1.12 mg to 1.09 mg), corresponding to accelerations up the slope in the range of 0.36 g to 0.30 g.

However, due to large cancellations involved in subtracting numbers close to 1, these values can be expected to be even more uncertain than the values based on the forward (x) acceleration discussed above.

2.3.2. Video analysis of motion. Another way to detect changes in speed as the train moves up the launch section of the track is to use a movie clip. Figure 9 shows a section of the launch analysed with the Tracker software [5], following the motion of the front and back of the train, frame by frame. The train length, as visible in the video, is estimated to be 13.5 m.

In the beginning of the visible part of the track, the back of the train passes a certain spot about 20 frames (\( \approx 0.67 \) s) later than the front, corresponding to a speed of 20.2 m s\(^{-1}\). The front of the train reaches the uphill part of the track about 1.7 s after emerging from the left in the video. Towards the end of the linear part, the back lags the front by about 18 frames = 0.6 s, corresponding to a speed of about 22.5 m s\(^{-1}\). The acceleration for this part of the track can then be estimated to 0.14 g. The
whole launch distance is 69.5 m, which is travelled in about 3s. Thus, the first 1.3 s of the launch, where the measured acceleration was larger, were not part of the video analysis.

Accounting for uncertainties arising from the train length estimate and in the time resolution provided by the frame rate, the final speed from the video could be written as \( v = (22.5 \pm 1.5) \text{ m s}^{-1} \). The specified speed after the launch of 23.5 m s\(^{-1}\). The average acceleration estimate is based on a change from 20 to 18 frames for the train to pass. An uncertainty of half a frame thus corresponds to 25% uncertainty in the acceleration, giving \( a = (0.14 \pm 0.04) \text{ g} \). The data from the video analysis are consistent with constant speed towards the end of the launch.

2.3.3. Comparison with technical specifications. The 24 stators that form the second launch can provide sufficient energy to get the train over the top even if, for some reason, it reaches the launch with very low (or zero) speed. The maximum acceleration possible is 0.9 g. However, during normal runs much less acceleration is needed, and when the train has reached the specified launch speed of 23.5 m s\(^{-1}\), the rest of the launch only adds energy to keep this speed. The accelerometer data, collected in May 2014, about a week after the opening, indicated an acceleration dropping from 0.33 g to 0.14 g during the launch, although it should be noted that these data were quite noisy. The video, recorded late in the season a few years later, indicated a speed that was instead constant towards the end.
3. Heartline rolls—uniform rectilinear motion with rotation

In the third shaded area of the graphs in figure 7, the vertical component varies significantly more. This is the ‘heartline roll’, where the body moves in a nearly straight line while rotating along an axis close to the heart. The track and train move around the heartline, leading to a less than obvious track shape, shown in figures 10 and 11. Throughout the motion, the train exerts an upward force $-mg$ to compensate for the force of gravity. The experience of the rider is of course very different depending on the orientation of the body. When the body is rotated an angle 90° around the heartline (the $x$ axis) the force from the ride is purely sideways (commonly referred to as the $y$ direction or a ‘lateral’ force.) When the angle is larger than 90°, you experience ‘negative G’s’, with a force primarily between the restraint and your body, which may separate from the seat.

Figure 12 shows the data from a 3D accelerometer moving with a rider in the Helix heartline roll, together with the theoretical values $\sin \omega t$ and $\cos \omega t$ overlaid. Another example is shown in [13].

In the case of purely linear motion without rotation, integrating accelerometer data twice is a way to obtain elevation or distance, as used, for example, in [4, 13–15]. The accelerometer graphs for the uniform rectilinear motion in the heartline roll illustrate why accelerometer data alone cannot be used to extract the change in position. The forces from the train on the rider vary as approximately $mg \cos \omega t$ in the $z$ direction and $mg \sin \omega t$ in the $y$ direction as illustrated in figure 12. Similar graphs can also be obtained by rotation of a smartphone around one axis, as shown in figure 13. These accelerometer data could also be obtained for a very different motion without rotation.

4. Discussion

All examples presented in this work afford a number of different representations of the motion, as well as examples of different ways to obtain results that can be compared to each other. The examples can also be used to remind students that the term ‘normal’ in ‘normal force’ does not refer to $-mg$, which students may have been led to believe from insufficient variation in introductory examples [16]. The examples show how theoretical idealisations can give meaningful comparisons with authentic data, and that the end of the book is not the only source of check on your work.

The escalator and heartline roll examples reflect that humans are not point particles, and that the experience depends on the orientation of the body, which can be captured by smartphone data and their dependence on device orientation. Both examples can also be used to demonstrate that rotation data are needed to give a full description of motion in space.

Sensor data can provide links between mathematical definitions, graphical representations, and the experience of the body in authentic situations. Offering a wider variation of applications can support students’ conceptual learning [17]. This paper aims to inspire teachers to expand their repertoire of examples of Newton’s first law and uniform rectilinear motion.

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ORCID iDs

Ann-Marie Pendrill https://orcid.org/0000-0002-1405-6561

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Ann-Marie Pendrill is a senior professor at Lund university and has been the director of the Swedish National Resource Centre for Physics Education. She has used amusement park for physics teaching since 1995, rain or shine.