FORMATION OF AN EVANESCENT PROTO–NEUTRON STAR BINARY AND THE ORIGIN OF PULSAR KICKS

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ABSTRACT

If core collapse leads to the formation of a rapidly rotating, bar-unstable proto–neutron star surrounded by fallback material, then we might expect it to cool and fragment to form a double (proto–)neutron star binary in a supersonate orbit. The lighter star should survive for awhile, until tidal mass loss propels it toward the minimum stable mass of a (proto–)neutron star, whereupon it explodes. Imshennik & Popov have shown that the explosion of the unstable, cold star can result in a large recoil velocity of the remaining neutron star. Here, we consider several factors that mitigate the effect and broaden the range of final recoil speeds, in particular the finite velocity and gravitational deflection of the ejecta, a range of original masses for the low-mass companion and its cooling history, rotational phase averaging of the momentum impulse from noninstantaneous mass loss, and the possibility of a common envelope phase. In spite of these mitigating factors, we argue that this mechanism can still lead to substantial neutron star recoil speeds, close to, or even above, 1000 km s⁻¹.

Subject heading: pulsars: general — stars: neutron — stars: rotation — supernovae: general

1. INTRODUCTION

It is recognized that radio pulsars have peculiar space velocities, between ≈30 and ≈1600 km s⁻¹, significantly greater than those of their progenitor stars. The highest speed ever recorded is from the Guitar Nebula pulsar (Cordes, Romani, & Lundgren 1993; Cordes & Chernoff 1998), while the lowest has been recently measured for B2016+28 by Brisken et al. (2002) from very accurate VLBA pulsar parallaxes. Statistical studies, aimed at inferring the peculiar velocity at birth from the observed speed, give mean three-dimensional velocities of 100–500 km s⁻¹ for the isolated pulsars (Lyne & Lorimer 1994; Lorimer, Bailes, & Harrison 1997; Hansen & Phinney 1997; Cordes & Chernoff 1998; Arzoumanian, Chernoff, & Cordes 2002).

An early explanation for the large space velocities called for recoil in a close binary that becomes unbound at the time of (symmetric) supernova explosion (Blauw 1961; Iben & Tutukov 1996). Now, a number of observations hint at a natal origin of these high space velocities, or at a combination of orbital disruption (when the progenitor lives in a binary) and internal kick reaction. Evidence of a kick at the time of neutron star birth is now found in a variety of systems: in runaway OB associations (Leonard & Dewey 1993), in highly eccentric Be/neutron star binaries (van den Heuvel & Rappaport 1987; Portegies Zwart & Verbunt 1996), in the binary pulsar J0045–7319 (Kaspi et al. 1996; to explain its current spin-orbit configuration), and in double neutron star binaries, such as B1913+16 (Bailes 1988; Weisberg, Romani, & Taylor 1989; Cordes, Wasserman, & Blaskiewicz 1990; Kramer 1998; Wex, Kalogera, & Kramer 2000), where misalignment between the spin and orbital angular momentum axes indicates velocity asymmetry in the last supernova. All these observations support the view that the formation of a neutron star is accompanied by anisotropic explosion. This notion is bolstered by evolutionary studies of binary populations (Dewey & Cordes 1987; Fryer & Kalogera 1997; Fryer, Burrows, & Benz 1998) and by studies (see, e.g., Cordes & Wasserman 1984) on the survival of binary systems into their late evolutionary stages after supernova explosion.

Among the physical processes that have been proposed to account for the kicks are large-scale density asymmetries seeded in the presupernova core (leading to anisotropic shock propagation), asymmetric neutrino emission in the presence of ultrastrong magnetic fields (see Lai, Chernoff, & Kalogera 1997; Fryer, Burrows, & Benz 1998) and by studies (see, e.g., Cordes & Wasserman 1984) on the survival of binary systems into their late evolutionary stages after supernova explosion.

In this paper, we reconsider the idea, first put forward by Imshennik & Popov (1998), that in the collapse of a rotating core, one or more self-gravitating lumps of neutronized matter can form in close orbit around the central nascent neutron star, transfer mass in the short-lived binary, and ultimately explode, causing the remaining, massive neutron star to acquire a substantial kick velocity, as high as the highest observed. The light member explodes as mass transfer drives it below the minimum stable mass for a neutron stars. In the light star, stability is lost upon decompression by the 3-decaying neutrons and nuclear fissions by radioactive neutron-rich nuclei (Colpi, Shapiro, & Teukolsky 1989, 1991; Blinnikov et al. 1990), which deposit energy-driving matter into rapid expansion (Colpi, Shapiro, & Teukolsky 1993; Sumiyoshi et al. 1998). The kick has its origin in the orbital motion of this evanescent superclose binary, which forms in the collapse of a rapidly rotating (isolated) iron core. We study several effects that might modify the magnitude of the kick, such as gravitational bending of the exploding debris, rotational averaging of the momentum impulse, orbit decay, and delayed neutron star cooling.

Formation of a proto–neutron star companion around the main neutron star has never been verified in numerical simulations, because of computational limitations. For this
reason, we elaborate on a study of Bonnell (1994) on the formation of binary/multiple systems in collapsing gas cloud cores and its extension to the stellar core collapse in the aftermath of a supernova explosion (Bonnell & Pringle 1995) to motivate our working hypothesis. Many works have conjectured the formation of such exotic binaries (Ruffini & Wheeler 1971; Clark & Eardley 1977; Blinnikov et al. 1984; Nakamura & Fukugita 1989; Stella & Treves 1987).

2. LIGHT FRAGMENTS AROUND PROTO–NEUTRON STARS

2.1. The Scenario

Formation of a light companion around a main body implies breaking of spherical and axial symmetry during collapse and following core bounce. During dynamical collapse, unstable bar modes \( m = 2 \) can grow in a fluid (even nonrotating) that might end with fragmentation. However, this is known to occur only if the cloud core contracts almost isothermally, as in the case of star’s formation from unstarred cold gas clouds (Bonnell 1984). Core collapse in Type II supernovae is far from isothermal (it is described by an effective polytropic index \( \gamma \simeq 1.3 \)), so that instabilities of this type do not have time to grow (Lai 2000), and simulations of nonaxisymmetric rotating core collapse confirm this trend (Rampp, Müller, & Ruffert 1998; Centrella et al. 2001). Can fragmentation/fission be excited after core bounce?

Rapid rotation in equilibrium bodies is known to excite nonaxisymmetric dynamical instabilities, and these instabilities might grow in the proto–neutron star core. Interestingly, core-collapse simulations of unstable rotating iron cores (Heger, Langer, & Woosley 2000; Fryer & Heger 2000) or polytropes (Zwerger & Müller 1997) indicate that proto–neutron stars, soon after formation, can rotate differentially above the dynamical stability limit set when the rotational-to-gravitational potential energy ratio \( \frac{\alpha_{\text{cent}}}{W} \) is larger than the value \( \alpha_{\text{cent}} = 0.25-0.26 \) (Saio et al. 2001). Strong nonlinear growth of the dominant barlike deformation \( m = 2 \) is seen in these cores (described as polytropes by Rampp et al. 1998). However, there is no sign of fission into separate condensations. The bar evolves, producing two spiral arms that transport the core’s excess angular momentum outward (see also Shibata, Baumgarte, & Shapiro 2000). This reduces the bar’s angular momentum, so that a single central body is formed. How can a body develop local condensations when bar-unstable?

According to Bonnell’s picture, the evolution of the bar instability is more complex, in reality. If the rapidly spinning proto–neutron star core goes bar-unstable when surrounded by a fallback disk, then matter present in the bar-driven spiral arms interacts with this material. The sweeping of a spiral arm into fallback gas can gather sufficient matter to condense into a fragment of neutronized matter. This occurs because the \( m = 1 \) mode grows during the development of the \( m = 2 \) mode. The \( m = 1 \) mode causes the displacement of the unpinned (free-to-move) core and creates an off-center spiral arm that sweeps up more material on one side than the other during “continuing” accretion. The condensation eventually collapses into a low-mass neutron star.

Detailed simulations confirming or dismissing the occurrence of such instability are still lacking, so further considerations of the lump masses, temperatures, and entropy contents are necessarily speculative.

2.2. Cooling Scenarios and the Minimum Mass

Here we wish to explore the possibility that a light (proto–)neutron star forms around the main central body, lives for a while, and later explodes, imprinting a kick to the neutron star that remains (because of linear momentum conservation) before gravitational waves or hydrodynamical effects can drive it toward the central star. Can a light (proto–)neutron star form from the condensation of material accumulated in the off-centered spiral arm? What limits can be imposed on its mass?

Cooling plays a key role in addressing these questions. Goussard, Haensel, & Zdunik (1998), Strobel, Schaab, & Weigel (1999), and Strobel & Weigel (2001) have shown that the value of the minimum stable mass \( m_{\text{mmc}} \) for a neutron star (located at the turning point in the mass-radius relation of equilibria) is a function of the temperature: it varies from \( \geq 1 \, M_\odot \) at 50–100 ms after core bounce (setting the actual value of the mass of the central neutron star), to \( \sim 0.7 \, M_\odot \) after \( \sim 1 \) s, down to \( \sim 0.3 \, M_\odot \) after 30 s, reaching the value of \( \sim 0.025 \, M_\odot \) (Baym, Pethick, & Sutherland 1971) known for cold, catalyzed matter \( (T < 1 \, \text{MeV}) \) several hundred seconds later. Thus, the lump of nuclear matter gathered in the spiral arm by the instability might become self-bound if its mass is above the minimum corresponding to that particular temperature. Once formed, it is stabilized against expansion by cooling. The actual value of \( m \) is thus determined by the overall dynamics of collapse after core bounce and varies between 0.025 and \( \sim 1 \, M_\odot \). It depends on the time at which the instability sets in, on the amount of fallback material (potential reservoir of matter in the lump), and on the cooling history.

The value of \( m \) and of the mass ratio \( q = m/M \) in the binary (with \( M \) the heavier of the two stars) remains unpredictable to us at this level, so we can depict just three possible scenarios for the formation and evolution of this evanescent binary: case 1, when the instability sets in after few hundreds of milliseconds or a second after core bounce and binary formation occurs, so that \( m \geq 0.7 \, M_\odot \) (implying a preexisting iron core of large mass if the primary is as massive as \( 1.4 \, M_\odot \)); case 2, when a lighter star forms around a main body with \( m = 0.2-0.4 \, M_\odot \) after several tens of seconds from core bounce; and case 3, when cooling is sufficiently advanced that the minimum mass approaches its asymptotic value and the binary can have \( m \leq 0.2 \, M_\odot \).

The magnitude of a natal kick in case 1 is difficult to estimate, and we must wait for realistic simulations of core collapse. Values of \( m/M \) above a given threshold in a binary are known to lead to unstable mass transfer and thus to final coalescence. We note that the evolution in this case might be similar to that described in coalescing neutron star binaries with slightly unequal masses (see Rosswog et al. 2000), where it has been shown that large kicks can be acquired as a consequence of mass loss via a wind. Cases 2 and 3 are explored in this paper. Imshennik & Popov (1998) estimated neutron star recoil speeds of order 1500–2000 km s\(^{-1}\) for scenario 3.

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\(^3\) Large-scale asymmetries imprinted in the iron core prior to collapse can lead to anisotropic explosions that produce kicks, as indicated by Goldreich, Lai, & Sarihling (1996). Whether they lead to fragmentation is unknown.
In cases 2 and 3, one factor that could lower the final neutron star speed is the finite velocity of the ejecta. Colpi et al. (1993) have shown that in the dynamical phase of the explosion, the ejecta can attain speeds varying from 10,000 to \( \sim 50,000 \) km s\(^{-1}\) (the upper bound being related to the value of the binding energy of the star at the minimum mass relative to a dispersed state of iron). These speeds are close to the escape velocity from the binary, and therefore the final kick imparted to the remaining neutron star might be influenced substantially by gravitational deflection of the ejecta and velocity phase averaging during the explosion. A further threat would be a rapid decay of the orbital separation due to unstable mass transfer, which might lead to final coalescence and emission of gravitational waves.

In § 3 we describe the gravitational bending of the ejecta, while in § 4 we study initial conditions and, subsequently, orbit decay and mass exchange for cases 2 and 3. We then give an estimate of the kick speed including phase-velocity averaging.

3. GRAVITATIONAL BENDING OF THE EXPLODING DEBRIS AND THE FINAL KICK

Consider now the instant at which the light secondary of mass \( m \) explodes, having reached the dynamical instability point \((m \simeq m_{\text{ms}})\) while orbiting the heavier, primary neutron star of mass \( M \). Obtaining the final kick speed imparted to the heavy neutron star is complicated by several factors. First, some of the material ejected from the exploding star can remain bound to the system and eventually accrete onto the remaining star. Second, material that escapes might not be moving at extremely large speeds, even right at ejection, and therefore emerges with a different momentum than it has at the point of explosion. Third, the mass of ejected material can be of the order of 10% of the total mass of the system, and self-gravity could play a role. Fourth, the ejecta are a fluid and, depending on how much time elapses between neutron star formation and the explosion of the low-mass star, move through ambient gas left over from the original supernova explosion. If the ejecta are slowed significantly by the surrounding gas, some material that would be judged unbound upon ejection might actually end up falling back toward the neutron star, possibly to accrete onto it.

Suppose that, relative to the remaining heavy neutron star, the orbital velocity of the exploding star is \( V \) at the instant of explosion \((t = 0)\) and the position of the center of mass of the exploding star is at \( r \). Let us write the final velocity of the remaining neutron star as

\[
V_{\text{kick}} = -\eta \frac{mV}{(M + m)},
\]

where \( V = |V| \). If the ejecta are expelled isotropically, relative to \( r \), and at speeds large compared with \( V \), then we expect \( \eta = |\eta| \simeq 1 \), in which case the remaining neutron star recoils with the maximum possible kick speed,

\[
V_{\text{kick, max}} \simeq \frac{mV}{(m + M)},
\]

as implied by momentum conservation in the center of mass of the binary.

To get a more realistic, but still approximate, idea of the size of \( \eta = V_{\text{kick}}/V_{\text{kick, max}} \), we work to zeroth order in the ejected mass and also treat the trajectories of the ejecta ballistically; this is equivalent to ignoring the third and fourth complications listed above entirely. By working to zeroth order in the mass of the ejecta, we can also assume that all particles are ejected virtually from a single point, \( r \). Let the velocity of a particle relative to the exploding star be \( w \), and assume that \( w \) is distributed isotropically about the position of the exploding star with probability \( P(w) \). Since we work to zeroth order in the mass of the ejecta, we can take the heavy neutron star to remain at a fixed position in the calculation.

The problem that remains is the orbital mechanics for each particle relative to the massive star. The orbits are characterized by the conserved quantities

\[
E = \frac{1}{2}|V + w|^2 - \frac{GM}{r}, \quad J = r \times (V + w), \quad A = -\frac{GMr}{r} + (V + w) \times J,
\]

which are the orbital energy, angular momentum per unit mass, and the Runge-Lenz vector, respectively. We note that

\[
A = |A| = GMO = \sqrt{(GM)^2 + 2EJ^2},
\]

where \( e < 1 \) for bound orbits and \( e > 1 \) for unbound orbits.

To calculate \( \eta \), we need to find the momentum per unit mass carried away by the ejecta that escape to infinite distance. First, we need to identify the initial velocities of particles that escape. For these, we must have \( E > 0 \), and from equation (3) we find the condition

\[
V \cdot w = V_{\text{vu}} > \frac{GM}{r} - \frac{(V^2 + w^2)}{2},
\]

where \( V = |V| \), \( w = |w| \), and \( \mu = V \cdot w/V_{\text{vu}} \). Second, we need to find the mapping between initial and final velocity. To this end, it is useful to define a coordinate system in which \( e_3 = J, \ e_1 = A, \) and \( e_2 = J \times A \), where \( J = J/J \) and \( A = A/A \). The orbit is then confined to the 1-2 plane, and we can read off the final velocity in this system from equation (15.12) in Landau & Lifshitz (1969):

\[
v_{\infty} = \left[-A\sqrt{2E} + (2E/GM)J \times A \right].
\]

We wish to express the final velocity \( v_{\infty} \) in a coordinate system fixed in the binary at the point of explosion. We thus choose, explicitly,

\[
r = r \hat{x} \quad \text{and} \quad V = V \hat{y},
\]

so that the orbital angular momentum of the light star relative to \( M \) just prior to explosion points along \( \hat{z} \). We have explicitly selected the point of explosion to occur at either pericenter or apocenter of the orbit, and for simplicity we specialize to circular orbits below. (Both tidal effects and gravitational radiation will tend to circularize the orbits, but even for eccentric orbits, we expect tidal disruption to be likeliest at pericenter.) In this coordinate system,

\[
J = -\hat{y}w_z + \hat{z}r(V + w_y),
\]

\[
A = \hat{x}[-GM + r(V + w_y)^2 + rw_z^2] + \hat{y}[-rww_z(V + w_y)] + \hat{z}(-rww_zw_z),
\]
The velocity of the ejecta can thus be expressed in terms of the known explosion parameters \( w, V, \) and \( r \) by combining equation (6) with equations (7)-(10). The result is that the total momentum carried off per unit mass of ejecta is

\[
\frac{u}{\eta} = \frac{dW}{dw} P(w) \int d^3 \mathbf{w} P(w) \mathbf{v}_{\infty}(w),
\]

so that \( \eta = u/V \), to zeroth order in \( m/M \). As \( P(w) = P(w) \) is isotropic, we can separate out the integrals,

\[
u = \int d^3 \mathbf{w} P(w) \int d^2 \mathbf{w} P(w) \mathbf{v}_{\infty}(w),
\]

and consider different distributions of ejection speeds separately. The integration only extends over unbound orbits. For \( w > (2GM/r)^{1/2} + V \), all ejecta escape, and for \( w < (2GM/r)^{1/2} - V \), no ejecta can escape.

Because of reflection symmetry with respect to the binary orbital plane, \( u \) vanishes identically. The kick is thus given in the orbital plane of the preexplosion binary. It is easy to show that when the particles are ejected with equal velocity \( w_0 \), so that \( P(w) = \delta(w - w_0)/(4\pi w_0) \), the integrals yield \( u_e \sim (GM/r)w_0^{-1} \) and \( u_t \sim V \) for very large values of \( w_0 \) (\( \gg V \)), which is consistent with the requirement that at large ejection speeds, the outflow reaches infinity with an average velocity comparable to that of the exploding star. Figure 1 shows \( \eta \) as a function of the dimensionless explosion velocity \( w_0/(GM/r)^{1/2} \). Note that there is no kick (i.e., \( \eta \) vanishes) when \( E = 0 \) (see eq. [5]). This occurs at a critical value of the expansion speed, \( w_0/(GM/r)^{1/2} = \sqrt{2} - V/(GM/r)^{1/2} \), which is \( \approx 0.414 \) for \( V \sim (GM/r)^{1/2} \) to zeroth order in \( m/M \). When \( E \leq 0 \), the mass outflow vanishes identically and the ejecta fall back to the remaining neutron star, transferring all their linear momentum to it.

4. FINAL KICK SPEED FROM THE SHORT-LIVED BINARY

4.1. Masses and Orbital Separations

The final kick speed imparted to the remaining neutron star depends crucially on the evolutionary scenario leading to the formation and disruption of its low-mass companion. The range of possible values of \( m_{in} \), the initial mass of the secondary, depends on the time the fragment forms and how fast it cools. Both of these complications distinguish the problem of “early” formation and decay of an evanescent proto–neutron star binary (case 2) from the problem of “late” decay of a neutron star binary (case 3), originally discussed by Clark & Eardley (1977), Blinnikov et al. (1984), and Imshennik & Popov (1998); the final outcome might differ from the results found by the authors for cold neutron stars.

Figure 2 shows an estimate of the binary separation \( r_{orb} \) at the time of formation (solid line) as a function of \( m_{in} \), which we regard as an independent variable here. To obtain this estimate, we assumed that the angular momentum \( J_{\text{dyn}} \) of the bar-unstable proto-neutron star with \( T_{\text{tot}}/|W| \sim \beta_{\text{dyn}} \) goes entirely into orbital angular momentum of the binary. (Even if the lump forms in corotation, its angular momentum is smaller than the orbital angular momentum of the isolated rotating preexplosion binary.)
momentum of the system, substantially so if the orbital separation is considerably larger than its radius.) As a reference value for $J_{\text{dyn}}$, we adopt $\sim 4 \times 10^{49}$ g cm$^2$ s$^{-1}$, the value given by Sajjo et al. (2001) for differentially rotating compact stars at $\beta_{\text{dyn}}$ (note that this is the minimum $J_{\text{dyn}}$ since the $m = 2$ and $m = 1$ modes can grow at values of $T_{\text{rot}}/|W|$ larger than $\beta_{\text{dyn}}$). As shown in the figure, lighter secondaries can be accommodated on wider orbits, in which the timescale of orbit decay for emission of gravitational waves, $\tau_{\text{GW}} = 5 \xi^{5/4} a/\kappa$ (256$G^3 M_{\text{tot}} M_{\text{mn}}$), is longer ($M_{\text{tot}} = M + m_{\text{mn}}$ is the total mass of the collapsing core).

Along the relation $r_{\text{orb}} = h_{\text{orb}}$ for $J_{\text{orb}} = J_{\text{dyn}}$, the circles show values of $m_{\text{mn}}$ at different cooling times $\tau_{\text{cool}}$, according to the cooling models of Strobel et al. (1999). The dotted lines in the sample plane correspond to loci of fixed values of the gravitational radiation decay timescale for the orbit, $\tau_{\text{GW}}$ (from the top, 1, 30, 100, and 300 s, as labeled in Fig. 2).

Note that gravitational radiation timescales are generally longer than cooling timescales, with the only exception occurring when $m_{\text{mn}} \sim 0.7 M_\odot$. Coalescence of the two stars is likely to occur in this case, with some mass loss that can only be estimated via hydrodynamical simulations. In the opposite case, when $m_{\text{mn}}$ and $r_{\text{orb}}$ are such that $\tau_{\text{GW}}$ is longer than the cooling time $\tau_{\text{cool}}$, the binary survives for a while, and below we study its evolution.

### 4.2. Binary Evolution, Mass Transfer, and Maximum Kicks

In the case of late formation (case 3), a phase of stable mass exchange can set in, driven by gravitational radiation emission, which terminates when the secondary reaches a critical mass, $m_{\text{tid}}$ (Clark & Eardley 1977; Blinnikov et al. 1984; Jaroszewski & Krolak 1992; Bildsten & Cutler 1992; Imshennik & Popov 1998). Subsequently, the low-mass star (which has a core-envelope structure) loses its extended halo, evolving along a sequence of hydrostatic equilibrium while the orbit widens (Blinnikov et al. 1984); the star reaches the minimum stable neutron star mass, whereupon it explodes (Page 1982; Colpi et al. 1989, 1991, 1993; Blinnikov et al. 1990; Sumiyoshi et al. 1998). In the case of early formation of the binary (case 2), we show below that mass transfer is likely to be unstable, and we argue that because of this it might just drive the star to the point of exploding dynamically.

Consider a binary in which Roche lobe overflow is in effect; then, the radius of the low-mass star $R(m)$ overfills the Roche lobe, and this determines the typical orbital separation at the onset of mass transfer,

$$ r = \frac{2.2 R(m)(M + m)^{1/3}}{m^{1/3}}. \tag{13} $$

The dotted lines in Figure 2 give the tidal radius as a function of $m$ for the two different mass-radius relations, describing a cold and a warm neutron star, respectively. A ring/disk of material forms around the primary star at the moment of Roche filling, and accretion begins to the primary. As the disk drains angular momentum from the orbit, there might not be sufficient time to return it to the orbit through disk-donor tidal torques (for details, see Bildsten & Cutler 1992; Lubow & Shu 1975). If $1 - f$ is the fraction of orbital angular momentum stored in the disk, the orbital separation would vary with time (following Bildsten & Cutler 1992) as

$$ \frac{d}{dt} = \frac{2m(J^2 - m^2)}{Mm(M + m)} - \frac{64G^3 m M (M + m)}{5 \xi^4}. \tag{14} $$

Fitting the values of $(J^2/m)$ computed by Hut & Paczynski (1984) for the Roche problem, Bildsten & Cutler (1992) found

$$ f \sim \frac{\frac{S}{3}}{\frac{m}{M}} \frac{1}{3} \frac{3}{2} \frac{2}{5} \frac{2}{5} \frac{2}{5}. \tag{15} $$

During Roche spillover, $m/R(m) \propto (M + m)/r^3$, and the rate of change of $r$ is

$$ \frac{d}{dt} = \frac{\frac{1}{3}}{\frac{1 + a}{a} - b(b/m) - 2(J^2 - m^2)} \frac{M}{M + m} < 0, \tag{16} $$

where $m$ is the mass transfer rate. Using this result in equation (14) for the orbital evolution, we have

$$ \frac{1}{3} \frac{d}{dt} \frac{R(m)}{dM} = \frac{2(J^2 - m^2)}{M(M + m)} < 0, \tag{17} $$

given the fact that $m/m < 0$.

In case 3, if we adopt the mass-radius relationship valid for a cold star (and mass below $1 M_\odot$, as in Jaroszewski & Krolak 1992), $R(m) = R_0 m^2/(m - m_0)^2$, where $R_0 = 7.5$ km, $m_0 = 0.09 M_\odot, b = 0.79$, and $a = 0.83$, we find that the condition for stable mass transfer is

$$ T = \frac{1}{3} + a + (a - b)(m/m_0) - 2(J^2 - m^2) < 0. \tag{19} $$

In general, there is only a finite interval of the mass $m$ for stability, whose boundaries are determined by the two roots of the equation $T = 0$: $m_{\text{tid}}$ (the lower bound) and $m_{\text{tid}}^+$ (the upper bound). When $m_{\text{tid}} < m < m_{\text{tid}}^+$, mass transfer is stable. Outside this interval stability is lost. There might also be no roots, in which case the fate of the light star depends on a number of details that we discuss below.

Adopting a constant value for $f$, and setting $\beta \sim \gamma$, to simplify analysis, we note that one regime in which the necessary inequality fails is at low masses, near $m \simeq m_0$,

$$ \frac{m_{\text{tid}}}{m_0} \simeq 1 + \frac{\beta}{2f - 1/3}. \tag{20} $$

(For $f = 0.4$ and $\beta = \gamma = 0.8, m_{\text{tid}}/m_0 = 2.7$, while if $f = 1, m_{\text{tid}}/m_0 \simeq 1.5$.) The other regime of instability is at masses well above $m_0$, in which case $d\ln R(m)/d\ln m$ is relatively small, and $m = m_{\text{tid}}^+$, with

$$ \frac{m_{\text{tid}}^+}{M} \simeq \frac{1}{2} \sqrt{\frac{4f - 23}{36} - \frac{1}{12}}. \tag{21} $$

(For $f = 0.4, m_{\text{tid}}^+/M \simeq 0.4$, while for $f = 1, m_{\text{tid}}^+/M \simeq 0.8$.) Thus, if there is a disk that stores orbital angular momentum (i.e., $f < 1$), the range of masses for which there can be stable, Roche-filling transfer shrinks considerably, relative
to the case \( f = 1 \) explored in Blinnikov et al. (1984) and Imshennik & Popov (1998).

We have computed the roots of equation (19) numerically for different \( R(m) \). For the Jaroszewski & Krolak (1992) parameterization of \( R(m) \) for cold neutron stars (case 3), we find the following intervals of \((m_{\text{tid}}^{m}, m_{\text{tid}}^{f})\) in units of \( M_{\odot} \): (0.14, 1.08) or (0.14, 1.33) for \( f = 1 \) and \( M = 1.4 \) or 1.7 \( M_{\odot} \); for \( f \) given by equation (15), there are no roots for \( M = 1.4 \) \( M_{\odot} \) and the roots are (0.29, 0.50) for \( M = 1.7 \) \( M_{\odot} \). The stability interval \((m_{\text{tid}}^{m}, m_{\text{tid}}^{f})\) depends sensitively on the logarithmic derivative of \( R(m) \). Note that the root \( m_{\text{tid}}^{m} \) exists because of the special dependence of \( R(m) \) on the parameter \( m_{0} \). Numerical hydrodynamical models for the description of these semidetached tight binaries are necessary to address the problem of mass transfer, and preliminary results points toward the existence of stable phases (M. B. Davies 2002, private communication). Below, we adopt the values of \( m_{\text{tid}}^{m} \) and \( m_{\text{tid}}^{f} \) for a 1.7 \( M_{\odot} \) neutron star.

What is the fate of the binary, then? In case 3, if the two roots \( m_{\text{tid}}^{m} \) and \( m_{\text{tid}}^{f} \) exist, and if the initial mass of the low-mass star is \( m_{\text{tid}}^{m} < m_{\text{in}} < m_{\text{tid}}^{f} ( \ll M ) \), the binary might first go through a stage of evolution via gravitational radiation without mass exchange, then undergo a phase of evolution with mass exchange at a rate determined by the amount of angular momentum loss due to gravitational radiation, and finally become unstable when \( m_{\text{tid}}^{m} \) is reached (Blinnikov et al. 1984; Imshennik & Popov 1998). If, on the other hand, \( m_{\text{tid}}^{m} \leq m_{\text{in}} \leq m_{\text{tid}}^{f} \), then the binary opens as a consequence of gravitational radiation without mass loss, until \( m_{\text{in}} \) fills its Roche lobe, whereupon it becomes unstable. If we define \( m_{\text{stab}} \) to be the stellar mass at the onset of instability, then \( m_{\text{stab}} = m_{\text{tid}}^{m} \) if \( m_{\text{in}} > m_{\text{tid}}^{f} \) and \( m_{\text{stab}} = m_{\text{in}} \) if \( m_{\text{mmn}} \leq m_{\text{in}} \leq m_{\text{tid}}^{f} \).

What happens once \( m = m_{\text{tid}}^{m} \) sets in has been described by Blinnikov et al. (1984). As mentioned above, they argue that although the equilibrium radius of a low-mass neutron star can become larger than the Roche radius, the vast majority of its mass can still be enclosed within it (99% is contained inside the inner 38 km or so). The stripping of the stellar mass envelope occurs slowly enough that the companion evolves through a series of nearly equilibrium states until \( m_{\text{mmn}} \) is attained, while the orbit widens.

More accurately, equation (14), in the absence of gravitational wave losses, leads to a change in orbital radius \( r \) as a function of mass loss equal to

\[
\frac{d \ln r}{d \ln m} = -2f + \frac{2fm}{M + m} + \frac{2m^2}{M(M + m)},
\]

which can be integrated to yield, when the mass is \( m \),

\[
r \sim \left( \frac{m_{\text{in}}}{m} \right)^{2f} \exp \left[ \frac{2(1 - f)(m_{\text{in}} - m)}{M + m} \right],
\]

neglecting changes in \( M \). In this approximation, the orbit widens by a factor \( \sim (m_{\text{stab}}/m_{\text{mmn}})^{2f} \) as the mass decreases from \( m_{\text{stab}} \) toward \( m_{\text{mmn}} \).

The orbital radius at the point of explosion is somewhat larger than the radius of Roche spillover and is, accordingly,

\[
r_{\text{mmn}} \sim \frac{2.2R(m_{\text{stab}})M^{1/3}m_{\text{stab}}^{2f-1/3}}{m_{\text{mmn}}^2},
\]

the orbital speed of the low-mass neutron star when it explodes is

\[
V \sim \sqrt{\frac{2GM}{r_{\text{mmn}}}} \approx \frac{0.68G^{1/2}M^{1/3}m_{\text{stab}}}{m_{\text{mmn}}^{(f-1)/6}[R(m_{\text{stab}})]^{1/2}},
\]

implying a maximum kick speed

\[
V_{\text{kick, max}} \approx \frac{m_{\text{mmn}}}{(m_{\text{mmn}} + M)} \frac{M}{V} \approx \frac{0.68G^{1/2}m_{\text{mmn}}^{2f-1}}{m_{\text{stab}}^{(f-1)/6}[R(m_{\text{stab}})]^{1/2}} M^{-2/3}
\]

that scales as \( M^{-2/3} \), because of the dependence of \( V \) on \( r_{\text{mmn}} \). This is a direct consequence of the hypothesis of Roche lobe mass transfer; the system has lost memory of the initial orbital separation. Also note that in \( V_{\text{kick, max}} \) we have \( m_{\text{mmn}} \) as numerator, even when \( m_{\text{stab}} = m_{\text{in}} \). When \( m_{\text{stab}} = m_{\text{in}} \approx 0.29 \ M_{\odot} \) (or when \( m_{\text{stab}} = m_{\text{mmn}} \approx 0.0925 \ M_{\odot} \)), the maximum kick is \( V_{\text{kick, max}} \approx 936 \text{ km s}^{-1} \) (800 km s\(^{-1}\)), and \( f \sim 0.4 \).

Concerning case 2, warmer proto–neutron stars seem to satisfy a smoother mass-radius relation. A fit to Strobel et al. (1999) data gives \( R(m) \propto m^{-6/5} \), which would lead to a stability interval \((m_{\text{mmn}}, m_{\text{tid}}) \) only for \( f \) close to unity (note that there is only a single root in this case). Since disk-donor torqueing can act on a timescale longer than the lifetime of the binary, it is likely that mass transfer is always unstable under these circumstances, and as soon as the light star overfills its Roche lobe at \( m_{\text{stab}} \), unstable mass transfer begins and there might be time for a phase of common envelope evolution. In this context, we wish to argue that the star is driven almost immediately toward explosion. The cooling time, \( \tau_{\text{cool}} \approx 50 \text{ s} \) (case 2; see Fig. 2), exceeds the typical time for mass transfer, \( \tau_{\text{mass}} \approx m_{\text{mmn}}/\rho cs_{s}R_{s}^2 \approx 0.2 \text{ s} \) (\( \rho \approx 10^{11} \text{ g cm}^{-3} \) denotes the density, close to neutron drip, of the mass losing star in its envelope, \( R \approx 50 \text{ km} \) the size of the Roche lobe, and \( cs_{s} \) the sound speed in units of \( 10^{10} \text{ km s}^{-1} \)). In this case, the mass-losing star encounters the instability point \( m_{\text{mmn}}(T) \) before being stabilized by cooling. We speculate that the core of the light star explodes on its own dynamical time \( \sim 0.001 \text{ s} \) before coalescence of the two stars in completed over an orbital period of \( P_{\text{orb}} = \frac{2\pi (r_{\text{mmn}}/GM)^{1/2} \approx 0.03 \text{ s} \). Common evolution is known to be accompanied by ejection of part of the envelope enshrouding the system, and this mass loss might provide an additional thrust to the merged object. Only hydrodynamical simulations can give credit to this scenario, and we expect a continuous transition in the physics when moving from case 3 (cold, late-type mode) to case 1 (hot, hydrodynamical mode), passing through case 2 (warm, early-time mode). If we were to compute \( V_{\text{kick, max}} \) by assuming that the warm star explodes right at the time it fills the Roche lobe, we would obtain nominal speeds close to 10,000 km s\(^{-1}\). The actual value of the kick velocity is computed below, combing gravitational bending with orbital phase averaging of the momentum impulse.

### 4.3. Velocity Phase Averaging and Kick Speeds

Our estimate of \( V_{\text{kick}} \) for cases 3 and 2 using the expression for \( \eta \) would be not complete without considering that explosion does happen noninstantaneously. The star explodes on a timescale comparable to the dynamical time-
scale, and this can be close to the time of revolution in the binary. In case 2 in particular, this effect might be severe.

The final kick speed will be diminished if the explosion is not instantaneous to a good approximation. The orbital frequency when the companion explodes is

$$\omega_{\text{mmnc}} = \left( \frac{G M}{r_{\text{stab}}^{3/2}} \right)^{1/2} \frac{0.314 G^{1/2} m_{\text{stab}}^2}{m_{\text{stab}}^{3/2} \left(R(m_{\text{stab}})\right)^{3/2}},$$

which ranges from $\omega_{\text{mmnc}} \simeq 17$ s$^{-1}$ for $m_{\text{stab}} = m_{\text{mmnc}}$ to $\omega_{\text{mmnc}} \simeq 360$ s$^{-1}$ for $m_{\text{stab}} = m_{\text{stab}} = \frac{1}{2} m_0 \simeq 0.29 M_\odot$ and $M = 1.7 M_\odot$ (case 3), and $185$ s$^{-1}$ for case 2 with $m_{\text{mmnc}} = 0.3 M_\odot$. At the time of explosion, the light star overfills its Roche radius, $R_{\text{R,exp}} \sim R(m_{\text{stab}})$. Matter ejected from the unstable star at a velocity $w_o$ crosses it in a time $R_{\text{R,exp}}/w_o$, and we can take $R_{\text{R,exp}}/w_o$ as an estimate of the duration of the explosion, $\tau_{\text{exp}}$.

The explosion is almost instantaneous as long as the dimensionless combination

$$\frac{\omega_{\text{mmnc}} R_{\text{R,exp}}}{w_o} \ll 1.$$  

For $m_{\text{stab}} \simeq 0.0952 M_\odot$, the dimensionless ratio is $2800/w_o$, km s$^{-1} \sim 0.1$ for $m_{\text{stab}} \simeq m_{\text{mmnc}}$ (case 3), and $9000/w_o$, km s$^{-1} \sim 0.3$ for $m_{\text{stab}} = m_{\text{mmnc}} = 0.3$ in case 2, taking $w_o \sim 30,000$ km s$^{-1}$ as an example.

To be more quantitative, suppose that the star loses mass at a rate $\dot{m}(t) = m(t)$ once it becomes unstable, with

$$\int_0^\infty dt \Gamma(t) = 1. \quad \text{(29)}$$

Consider circular orbits only, and assume that although mass loss can extend over many orbital periods, the orbit of the exploding star remains unaltered during the mass loss. (The latter approximation should be valid if mass loss is rapid enough, to zeroth order in the decreasing secondary mass.) Adopt a fixed coordinate system defined at $t = 0$, when mass loss begins, so that $x = r(0)/r$ and $y = V(0)/V$.

The momentum per unit ejected mass is now

$$u = \int_0^\infty dt \Gamma(t) u_x \cos \omega t + y \sin \omega t]$$
$$+ u_y(-x \sin \omega t + y \cos \omega t)$$
$$= u_x \dot{e}_x \Gamma + u_y \dot{e}_y \Gamma \Gamma,$$

where $u_x = (u_x + i u_y)/\sqrt{2}$, $u_y = (u_x - i u_y)/\sqrt{2}$, and

$$\dot{e}_x \equiv \int_0^\infty \frac{dt}{\Gamma(t)} e^{i \omega t}.$$  

Here, $(u_x, u_y)$ are the same as were computed in $\S$ 3 for instantaneous mass loss ($t = 0$). When the finite duration of the mass loss is accounted for, the efficiency of the recoil is diminished from $\eta$ (as computed in $\S$ 3 for instantaneous mass loss) to $\eta \Gamma$. For example, if mass is lost at a constant rate, then $\Gamma(t) = \tau_{\text{exp}}^{-1} \Theta(\tau_{\text{exp}} - t)$, and we find

$$\Gamma = \frac{\left| \sin(\omega_{\text{mmnc}} \tau_{\text{exp}})/2 \right|}{\omega_{\text{mmnc}} \tau_{\text{exp}}/2}; \quad \text{(32)}$$

if instead $\Gamma(t) = \tau_{\text{exp}}^{-1} \exp(-t/\tau_{\text{exp}})$, then

$$\Gamma = \frac{1}{\sqrt{1 + \left(\omega_{\text{mmnc}} \tau_{\text{exp}}\right)^2}}.$$  

The correction due to phase averaging, $\Gamma_{\omega}$, is only modest (and somewhat model-dependent) as long as $\omega_{\text{mmnc}} \tau_{\text{exp}} \ll 1$, but for long explosions, the efficiency is diminished by a factor of order $\left(\omega_{\text{mmnc}} \tau_{\text{exp}}\right)^{-1}$, in general.

Figure 3 shows the kick velocity $V_{\text{kick}}$ of the neutron star that remains (of mass $1.7 M_\odot$); but for $1.4 M_\odot$, scale the velocity with $M^{-2/3}$ as a function of the speed of the ejecta $w_o$ for cases 3 and 2. The solid curves refer to case 3, when $f$ is given by the Roche value (eq. [15]) and $m_{\text{stab}}$ is equal to $m_{\text{stab}} = 0.29 M_\odot$ (lower curve before crossover occurring at $w_o \sim 40,000$ km s$^{-1}$) and $m_{\text{mmnc}}$ (upper curve before crossover). The dotted curve is for $m_{\text{stab}} = m_{\text{stab}} = 0.14 M_\odot$. The dash-dotted line, for case 2, is computed starting the explosion during common envelops, when $m_{\text{mmnc}} \sim 0.3 M_\odot$ and the orbit separation is $P_{\text{orb}}/R_{\text{cross}} = P_{\text{orb}}(V/r)$ times less than the separation at the moment of filling the Roche lobe (eq. [13]), to mimic orbit decay during common envelops. We have included the phase-averaged orbit correction as given in equation (33).

Very high kicks come from such complex hydrodynamical processes, which alternative models have not been able to produce.

5. DISCUSSION

In this paper, we have elaborated on suggestions that substantial neutron star recoil can result from the explosion of a low-mass neutron star formed in the aftermath of rotating core collapse (Imshennik & Popov 1998). A noteworthy feature of this process is that the final kick speed is determined by nuclear physics. The details about the formation mechanism and hydrodynamical effects produce, in reality, a wide-spread range in velocities, as we see.

We have extended earlier work by estimating the effect of the finite speed of the material ejected in the explosion, including both gravitational deflection and phase averaging.
and, indirectly, taking into account cooling effects. We argue that a range of neutron star recoil speeds could arise from disruption scenarios with different companion masses. The recoil speed resulting from the dissolution of an evanescent binary system can be around 1000 km s$^{-1}$, explaining the larger values deduced observationally. It is remarkable that nuclear physics implies a value in the observational range at all, once such a binary system is presumed to form, a concordance that lends some support to the idea.

The kicks that result from this mechanism are confined to the orbital plane of the evanescent neutron star binary. We expect the spin angular momentum vector of the remnant neutron star to be nearly, if not perfectly, aligned with the orbital angular momentum of the binary and, in turn, aligned with the angular momentum vector of the collapsing iron core. Near-alignment would be consistent with the requirements imposed by the observation of geodetic precession of B1913+16, where the kick is constrained to lie very nearly in the plane of progenitor binary, which was most likely perpendicular to the spins of the spun-up neutron star (i.e., B1913+16) and its preexplosion companion star (Wex et al. 2000). In contrast, however, X-ray observations of the Vela pulsar have revealed a jet parallel to its proper motion (Pavlov et al. 2001; Helfand, Gotthelf, & Halpern 2001), and it has been argued that the proper motions of both Vela and the Crab pulsars are closely aligned with their spin axes (Lai et al. 2001). The kick mechanism studied here would not be able to account for parallel spin and velocity. However, we note that the proper motions of both Vela and the Crab correspond to transverse speeds of 70–141 and 171 km s$^{-1}$, respectively, using reasonable estimates of the distances to the pulsars (Lai et al. 2001). For the alignment to be real, the space velocities of these two systems must lie in the plane of the sky. The inferred speeds of these two pulsars are then considerably smaller than the characteristic speed arising from explosion of a low-mass, tidally disrupted companion. We propose that the formation of Vela and the Crab did not produce an evanescent binary, and therefore some other mechanism was responsible for their spin-aligned kicks (such as those explored by Lai et al. 2001). We suggest that the larger kick component is due to formation and disruption of an evanescent binary, that it is perpendicular to the spin axis, and that the smaller kick component is associated with other, less vigorous kicks that tend to align with the rotation axis, perhaps because of phase-averaging (see, e.g., Spruit & Phinney 1998; Lai et al. 2001). A superposition of these two classes of kicks would also be consistent with the requirement of nearly, but not precisely, spin-perpendicular kicks to account for the observation of geodetic precession in B1913+16 (Wex et al. 2000). If this idea is correct, then one expects that lower velocity neutron stars should have their space velocities predominantly along their spin axes, and high-velocity neutron stars should have space velocities predominantly perpendicular to their spins, giving origin to two independent (low- and high-velocity) distributions. A contamination of low-velocity stars (belonging to the low-velocity tail of the high-velocity distribution) would come from those explosions in the evanescent binary in which light-bending and rotational averaging have been more important. This could give rise to a bimodal distribution of velocities, as is inferred from the observations (Arzoumanian et al. 2001).

This scenario predicts a clear signature in the neutrino emission, in the aftermath of core collapse. Two neutrino bursts, occurring several seconds or minutes after core bounce, i.e., after the main neutrino burst, should signal (1) Kelvin contraction of the condensation gathered by the instability and (2) explosion (on the dynamical timescale) of the light neutron star. Neutrino emission between the two bursts should also come from material that accretes onto the neutron star that remains. Heavy r-process elements, debris of the explosion of the light star, should also be ejected and found deep in the supernova expanding shells.

Finally, we note that the same phenomenon could occur if the formation of a light neutron star accompanies rotating core collapse to a black hole. In that case, we find that the recoil velocity of the black hole will be proportional to $M_{\text{bh}}^{-\frac{1}{3}}$, which is not the simple $M_{\text{bh}}^{-\frac{2}{3}}$ scaling one would expect for a kick mechanism that ejects the same amount of momentum irrespective of whether a supernova leaves behind a neutron star or a black hole. The higher kick implied by this scaling could be compensated for by stronger relativistic effects of gravitational bending and unstable mass transfer, which can lead to a much lower escape probability of the exploding debris.

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4 During revision of this manuscript we learned of a paper by Davies et al. (2002), in which a connection between gamma-ray bursts and pulsar kicks is made, where kicks result from recoil by a short-lived binary.

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