\[ N = 8 \text{ SCFT and M Theory on } AdS_4 \times \mathbb{RP}^7 \]

Changhyun Ahn

*Rm 27-217, Center for Theoretical Physics, Seoul National University, Seoul 151-742, Korea*

chahn@spin.snu.ac.kr

Hoil Kim

*Topology and Geometry Research Center, Kyungpook National University, Taegu 702-701, Korea*

hikim@gauss.kyungpook.ac.kr

Bum-Hoon Lee and Hyun Seok Yang

*Department of Physics, Sogang University, Seoul 121-742, Korea*

bhl, hsyang@physics4.sogang.ac.kr

(February 28, 2008)

Abstract

We study M theory on \( AdS_4 \times \mathbb{RP}^7 \) corresponding to 3 dimensional \( N = 8 \) superconformal field theory which is the strong coupling limit of 3 dimensional super Yang-Mills theory. For \( SU(N) \) theory, a wrapped D6 brane on \( S^6 \) which is connected to a D2 brane on the boundary of \( AdS_4 \) by \( N \) fundamental strings can be interpreted as baryon vertex in Type IIA string theory. For \( SO(N)/Sp(2N) \) theory, by using the property of (co-)homology of \( \mathbb{RP}^6 \), we classify various wrapping branes. Then we consider strings, domain walls and the baryon vertex in Type IIA string theory and point out a perplexing puzzle on the baryon vertex.
I. INTRODUCTION

In [1] the large $N$ limit of superconformal field theories (SCFT) was described by taking the supergravity limit on anti-de Sitter (AdS) space. The scaling dimensions of operators of SCFT can be obtained from the masses of particles in string/M theory [2]. In particular, $\mathcal{N} = 4$ $SU(N)$ super Yang-Mills theory in 4 dimensions is described by Type IIB string theory on $AdS_5 \times S^5$. This AdS/CFT correspondence was tested by studying the Kaluza-Klein (KK) states of supergravity theory and by comparing them with the chiral primary operators of the SCFT on the boundary [3]. There exist also $\mathcal{N} = 2, 1, 0$ superconformal theories in 4 dimensions which have corresponding supergravity description on orbifolds of $AdS_5 \times S^5$ [4] and the KK spectrum description on the twisted states of $AdS_5$ orbifolds was discussed in [5]. The energy of quark-antiquark pair [6], glueball mass spectrum [7] and the energy of baryon as a function of its size [8,9] were analytically calculated based on this correspondence. The field theory/M theory duality also provides a supergravity description on $AdS_4$ or $AdS_7$ for some superconformal theories in 3 or 6 dimensions, respectively [1]. The maximally supersymmetric theories have been studied in [10,11] and the lower supersymmetric case was also realized on the worldvolume of M theory at orbifold singularities [12].

The gauge group of the boundary theory becomes $SO(N)/Sp(2N)$ [13] by taking appropriate orientifold operations for the string theory on $AdS_5 \times S^5$ (See also [10,14]). By analyzing the discrete torsion for $B$ fields, the possible models of gauge theory are topologically classified and many features of gauge theory are described by various wrapping branes. By generalizing the work of [13] to the case of $AdS_7 \times \mathbb{RP}^4$ where the eleventh dimensional circle is one of $AdS_7$ coordinates, $(0, 2)$ six dimensional SCFT on a circle rather than uncompactified full M theory was described in [15]. For $SU(N)$ $(0, 2)$ theory, a wrapped D4 brane on $S^4$ together with fundamental strings connecting a D4 brane on the boundary of $AdS_7$ with the D4 brane on $S^4$ was interpreted as baryon vertex. By putting $N$ M5 branes in the $\mathbb{R}^5/\mathbb{Z}_2$ orbifold singularity [16], where the $\mathbb{Z}_2$ acts by a reflection of the 5 directions transverse to the M5 branes and also by changing the sign of the 3-form field $C_3$, the large $N$
limit of the $SO(2N)\ (0,2)$ SCFT and $\mathbb{RP}^4$ orientifold after removing the $\mathbb{R}^5/\mathbb{Z}_2$ orbifold singularity were obtained. Then, using the property of (co-)homology of $\mathbb{RP}^i \subset \mathbb{RP}^4$, various wrapping branes and their topological restrictions were discussed.

Recently, Sethi [17] found that $\mathcal{N} = 8 \ Sp(2N)$ and $SO(2N + 1)$ gauge theories in 3 dimensions flow to the same strong coupling fixed point. This was confirmed by turning on discrete torsion of M2 branes on $\mathbb{R}^8/\mathbb{Z}_2$. By evaluating $C_3 \wedge G_4$ over $\mathbb{RP}^7$ where $G_4 = dC_3$ is four-form field in M theory, it was shown that there exists M2 brane charge shift for the two types of orientifold two-plane. This result implies that in IR limit three dimensional $\mathcal{N} = 8$ SYM theories can flow to two distinct strong coupling conformal field theories.

In this paper, we generalize the work of [13,15] to the case of $AdS_4 \times \mathbb{RP}^7$ where the eleventh dimensional circle is one of $\mathbb{RP}^7$ coordinates, as we briefly mentioned this possibility in the previous paper [15]. In section II, it will be shown that for $SU(N)$ theory, a wrapped D6 brane on $S^6$ together with $N$ fundamental strings connecting a D2 brane on the boundary of $AdS_4$ with the D6 brane on $S^6$ can be interpreted as baryon vertex [8,20]. By putting $N$ M2 branes in the $\mathbb{R}^8/\mathbb{Z}_2$ orbifold singularity, where the $\mathbb{Z}_2$ acts by a reflection of the 8 directions transverse to the M2 branes, we will obtain the large $N$ limit of $\mathcal{N} = 8$ SCFT on $\mathbb{RP}^7$ orbifold [13]. Then, using the property of (co-)homology of $\mathbb{RP}^i \subset \mathbb{RP}^6$ (the reason why we consider $\mathbb{RP}^6$ rather than $\mathbb{RP}^7$ is that we are considering with Type IIA string theory approach [14]), we classify various wrapping branes and discuss their topological restrictions.

1 Although the eleven dimensional solution by uplifting the D2 brane solution is not exactly M2 brane solution in general, when the eleventh dimension is compact, by taking M2 branes to be localized in the eight transverse dimensions, they will resemble each other more and more in M theory limit [15,19].

2 We are using Type IIA string theory description throughout this paper and intend to give M theory interpretation to the results according to the relation between Type IIA string theory and M theory. Given a Type IIA p-brane whose worldvolume $X$ is a $(p+1)$-dimensional submanifold in
logical restrictions. In section III, we consider strings, domain walls and the baryon vertex in Type IIA string theory. Finally, in section IV, we will discuss important open problems and comment on the future directions.

II. $SO/SP$ SYM AND BRANES ON $RP^6$

A. The Baryon Vertex in $SU(N)$

Let us consider M theory on $\mathbb{R}^3 \times \mathbb{R}^7 \times S^1$. For $\mathbb{R}^3$, we can take $(x^0, x^1, x^2)$ directions and for $\mathbb{R}^7$, we take $(x^3, x^4, x^5, x^6, x^7, x^8, x^9)$ transverse to M2 branes. The eleventh coordinate $x^{10}$ is compactified on a circle $S^1$ and is a periodic coordinate of period $2\pi$. For small radius of $S^1$, we can regard M theory as Type IIA string theory which can be described in the context of $AdS_4 \times S^7$ where D2 branes are realized by transverse M2 branes. The radial function $\rho = \sqrt{\sum_{i=3}^{10} (x^i)^2}$ of $\mathbb{R}^7 \times S^1$ will be one of the $AdS_4$ coordinates, the other three being the ones in $\mathbb{R}^3$. Since we are using the Type IIA description, we are interested in the orthogonal space to $S^1$, which looks like $\mathbb{R}^3 \times \mathbb{R}^7$ or $AdS_4 \times S^6$ locally.

The $AdS_4 \times S^6$ compactification has $N$ units of six-form flux on $S^6$ as follows:

$$\int_{S^6} \frac{G_6}{2\pi} = N,$$  (II.1)

where $G_6$ is six-form field which is a hodge dual of $G_4 = dC_{IIA}^3$ four-form field. When a D6 brane is wrapped on $S^6$, on the worldvolume of the D6 brane, there exists a $U(1)$ gauge field which can couple $G_6$ by

$$\int_{S^6 \times \mathbb{R}} a \wedge \frac{G_6}{2\pi},$$  (II.2)

$\mathbb{R}^{10}$, we identify $X$ with a submanifold $X' = w \times X$ in $S^4 \times X$, where $w$ is any point in $S^4$. More precisely, an M5-brane should be understood as the bound state of a D4-brane and $n$ D0-branes for every $n$ and a D6-brane as a KK monopole whose transverse space including the eleventh circle is described by Taub-NUT metric.
where $\mathbf{R}$ is a timelike curve in $AdS_4$. From these two relations (II.1) and (II.2), $G_6$ field has the $N$ units of a charge. There should be $-N$ units of a charge somewhere in order to satisfy the vanishing of a charge in a closed universe. This can be done by putting oriented $N$ fundamental strings ending on the D6 brane.

In $SU(N)$ gauge theory, the gauge invariant combination of $N$ quarks should be completely antisymmetric. This antisymmetry of baryon vertex can be understood with the following boundary conditions of fundamental strings connecting D6 brane to the boundary of $AdS_4$ space. The Pauli exclusion principle in this case is geometrically realized by Bachas and Green in [20]. The time zero section of $AdS_4 \times S^6$ is a copy of $\mathbf{R}^3 \times S^6$. We can consider a static D2 brane which has the worldvolume of $S^2 \times R$ where $R$ is a point of $S^6$ and $S^2$ is a large two sphere near infinity in $\mathbf{R}^3$. Then our $N$ strings connect between D2 brane on $S^2 \times R$ with D6 brane on $Q \times S^6$ where $Q$ is a point in a time zero slice of $AdS_4$. The D2 brane and the D6 brane are linked in the $\mathbf{R}^3 \times S^6$, but they do not intersect each other. They have linking number $\pm 1$. The string stretching between linked D-branes has eight mixed Dirichlet-Neumann boundary conditions. Then the string zero point energy is 0 in the Ramond sector as always, and the zero point energy in NS sector is positive, 1/2. Thus true ground state is a Ramond state, whose Fock space generates Clifford algebra. Thus the strings stretching between the boundary (or a D2 brane) and the D6 brane are certainly fermionic strings [20]. So we can interpret the wrapped D6 brane as baryon vertex or antibaryon vertex.

B. The $\mathbb{RP}^6$ Orientifold

It was observed [10] that the large $N$ limit of $\mathcal{N} = 8$ SCFT in 3 dimensions corresponds to $N$ M2 branes coinciding at $\mathbf{R}^8/\mathbb{Z}_2$ orbifold singularity. In order to reduce to the Type IIA
description, let us consider M theory on $\mathbb{R}^3 \times (\mathbb{R}^7 \times S^1)/\mathbb{Z}_2$. The $\mathbb{Z}_2$ acts by sign change on all eight coordinates in $\mathbb{R}^7$ and $S^1$ as follows: $(x^3, \ldots, x^{10}) \rightarrow (-x^3, \ldots, -x^{10})$. This gives two orbifold singularities at $x^{10} = 0$ and $x^{10} = \pi$, each of which locally looks like $\mathbb{R}^8/\mathbb{Z}_2$.

The angular directions in $\mathbb{R}^8/\mathbb{Z}_2$ are identified with $\mathbb{RP}^7$. Consider $N$ parallel M2 branes which are sitting at an orbifold two-plane (O2-plane) which is located at $x^3 = \cdots = x^{10} = 0$. Note that there is another singularity at $x^{10} = \pi$ but we will focus on the theory at the origin which has corresponding interacting superconformal field theory [21]. The space orthogonal to $S^1$ near the singularity at $x^{10} = 0$ locally looks like $\mathbb{R}^3 \times \mathbb{R}^7/\mathbb{Z}_2$ or $AdS_4 \times \mathbb{RP}^6$. For small radius of $S^1$, we can thus regard this as Type IIA orientifold on $\mathbb{R}^7/\mathbb{Z}_2$ which can be described in the context of $AdS_4 \times S^6$ with scale dependent radius. Then the orientifolding operation replaces six sphere $S^6$ around the origin in $\mathbb{R}^7$ with $\mathbb{RP}^6 = S^6/\mathbb{Z}_2$. We will describe how $\mathcal{N} = 8$ SCFT in 3 dimensions can be interpreted as M theory on $AdS_4 \times \mathbb{RP}^7$ where the eleventh dimensional circle is in $\mathbb{RP}^7$ space. This is our main goal in this paper.

Let us study the property of $AdS_4 \times \mathbb{RP}^6$ orientifold. Let $x$ be the generator of $H^1(\mathbb{RP}^6, \mathbb{Z}_2)$ which is isomorphic to $\mathbb{Z}_2$, $\Sigma$ be a string worldsheet and $w_1(\Sigma) \in H^1(\Sigma, \mathbb{Z}_2)$ be the obstruction to its orientability. Then we only consider the map $\Phi: \Sigma \rightarrow AdS_4 \times \mathbb{RP}^6$ such that $\Phi^*(x) = w_1(\Sigma)$. Since $\mathbb{Z}_2$ action on $S^6$ is free (no orientifold fixed points), there is no open string sector. In the orientifold the string world sheet need not be orientable and a basic case of an unorientable closed string worldsheet is $\Sigma = \mathbb{RP}^2$, which can be identified with the quotient of the two sphere $S^2$ by the overall sign change. The map $\Phi: \mathbb{RP}^2 \rightarrow \mathbb{RP}^6$ satisfying the constraints $\Phi^*(x) = w_1(\mathbb{RP}^2)$ is the embedding $(x_1, x_2, x_3) \rightarrow (x_1, x_2, x_3, 0, 0, 0, 0, 0)$.

In M theory, there is the Chern-Simons interaction in eleven dimensional supergravity,

3The eleven dimensional spacetime is not a principal $U(1)$-bundle over 10 dimensional spacetime although it can be defined as a $U(1)$-bundle over 10 dimensional spacetime. However, it will still make sense to consider RR $U(1)$ gauge field as one-form with values in the twisted bundle. We thanks to K. Hori for pointing out this.
\[-\frac{1}{24\pi^2} \int C_3 \wedge G_4 \wedge G_4. \] (II.3)

where \( G_4 = dC_3. \) A compactification of M-theory on an eightfold \( X_8 \) receives tadpole contribution for the \( C_3 \) three-form field in one-loop \[22,23\]

\[- \int_{X_8} C_3 \wedge I_8(R), \] (II.4)

where \( J = -\int_{X_8} I_8(R) = \chi/24, \) with \( \chi \) the Euler characteristic of the eightfold \( X_8 \) and \( I_8(R) \) is an eight-form constructed as a quartic polynomial in the curvature. The condition for a consistent M theory compactification on an eightfold is thus

\[ \frac{\chi}{24} - \frac{1}{8\pi^2} \int_{X_8} G_4 \wedge G_4 - n = 0, \] (II.5)

where \( n \) is the number of M2 branes filling the vacuum. It can be deduced from the relation (II.3) the orbifold \( \mathbb{R}^8/\mathbb{Z}_2 \) carries \(-1/8\) units of M2 brane charge \[24,4\].

It was shown by Sethi \[17\] that there exist three O2-planes \[4\]

i) \( SO(2N) \) with \( O2^- \) : M theory on \( \mathbb{R}^3 \times (\mathbb{R}^7 \times S^1)/\mathbb{Z}_2, \)

ii) \( SO(2N + 1) \) with \( \widetilde{O2}^+ \) : M theory on \( \mathbb{R}^3 \times (\mathbb{R}^7 \times S^1)/\mathbb{Z}_2, \)

iii) \( Sp(2N) \) with \( O2^+ \) : M theory on \( \mathbb{R}^3 \times (\mathbb{R}^7 \times S^1)/\mathbb{Z}_2. \) (II.6)

Here \( O2^- \) characterized by \( \mathbb{R}^7/\mathbb{Z}_2 \) has \(-1/4\) unit of D2 brane charge. By promoting to M theory, each fixed point (0 and \( \pi \) on the circle) realized by \( \mathbb{R}^8/\mathbb{Z}_2 \) carries \(-1/8\) unit of M2 brane charge. We should have a single D2 brane stuck on the \( O2^- \) plane to get \( SO(2N + 1) \) gauge group and the orientifold \( \mathbb{R}^7/\mathbb{Z}_2 \) carries \( 3/4 \) units of D2 brane charge. It is denoted \[5\]

\[4\]We count the number of brane charges on \( \mathbb{Z}_2 \) orbifold in the double cover of the \( \mathbb{Z}_2 \) quotient where the charge of a D2 brane is 1.

\[5\]We will also not consider nontrivial holonomy (RR \( U(1) \) Wilson line) around the eleventh circle as in \[17\] since the gauge theory has a smooth strong coupling limit in decompactified M theory limit and the holonomy indeed vanishes due to \( H^2(\mathbb{R}P^6, \mathbb{Z}) = 0. \)
by $O^2_-$ plane splits into two orbifolds $R^8/Z_2$. Apparently the stuck M2 brane also splits into two fluxes, each carrying 1/2 unit of M2 brane charge. Each fixed point $R^8/Z_2$ has thus $3/8 = -1/8 + 1/2$ units of M2 brane charge. As shown by Sethi [17], the M theory realization of the charge shift is given by the term (II.3):

$$-\frac{1}{2} \int_{\mathbb{RP}^7} \frac{C_3}{2\pi} \wedge \frac{G_4}{2\pi} = -\frac{1}{2} \int_{\mathcal{M}} \frac{G_4}{2\pi} \wedge \frac{G_4}{2\pi} = \frac{1}{4},$$

where $G_4/2\pi$ is the torsion class and $\mathbb{RP}^7$ is the boundary of the smooth eightfold $\mathcal{M}$. Finally we can obtain $O^2_+$ plane by the basis choice of the Chan-Paton factors giving gauge group $Sp(2N)$, which has the 1/4 unit of D2 brane charge. In this case, the M theory interpretation on the orbifold singularity is a little different: the singularity at $x^{10} = 0$ has the charge $3/8 = -1/8 + 1/2$ units of M2 brane where the 1/2 (in the double cover $S^7$) charge shift is realized by turning on the discrete torsion as in Eq. (II.7) while the singularity at $\pi$ having $-1/8$ units of M2 brane charge. Thus we wish to classify the O2-planes in terms of distinct fluxes for the four-form $G_4$ on $\mathbb{RP}^7$ at the origin which correspond to distinct strong coupling limits for O2 planes [17]. The relevant cohomology corresponds to the possible choices of discrete torsion and is given by

$$H^4(\mathbb{RP}^7, \mathbb{Z}) = \mathbb{Z}_2.$$  

(II.8)

Consequently, $O^2_-$ plane flows to the case without discrete torsion while $\tilde{O}^2_+ , O^2_+$ flow to the case with discrete torsion.

Since the $SO(N)/Sp(2N)$ gauge theories for large $N$ can be distinguished by the sign of the string $\mathbb{RP}^2$ diagram, this can be classified by the discrete torsion of the $B_{NS}$ field [13]. Note that the orbifolding in $R^8/Z_2$ does not act on $C_3$ but it acts on $x_{10}$. Since the $B_{\mu\nu}$ field of Type IIA string theory corresponds in M theory to $C_{\mu\nu10}$, $\mathbb{Z}_2$ action in ten dimensions flips the sign of $B_{NS}$. This means that a cohomology class $[H_{NS}]$ takes values in a twisted integer coefficient $\tilde{\mathbb{Z}}$ where the twisting is determined by an orientation bundle. The relevant cohomology groups measuring the topological types of the fields $B_{NS}$ is given by

$$H^3(\mathbb{RP}^6, \tilde{\mathbb{Z}}) \approx \mathbb{Z}_2.$$  

(II.9)
Thus the relevant cohomology groups measuring the topological types for the O2 planes are given by

\[ H^3(\mathbb{RP}^6, \tilde{\mathbb{Z}}) \approx \mathbb{Z}_2, \quad H^4(\mathbb{RP}^7, \mathbb{Z}) \approx \mathbb{Z}_2. \]  

(II.10)

If we denote the values of the cohomologies \( H^3(\mathbb{RP}^6, \tilde{\mathbb{Z}}) \) and \( H^4(\mathbb{RP}^7, \mathbb{Z}) \) as \((\alpha, \beta)\) respectively, we get the topological classification of the three models:

\[ O2^- : (\alpha, \beta) = (0, 0), \quad \tilde{O}2^+ : (\alpha, \beta) = (0, 1), \quad O2^+ : (\alpha, \beta) = (1, 1). \]

(II.11)

Note that the topological type of \( O2^+ \) at \( x^{10} = \pi \) is \((\alpha, \beta) = (1, 0)\).

C. Various Wrapped Branes

Now we consider the possibilities of brane wrapping on \( \mathbb{RP}^6 \) in the Type IIA string theory. We first recall that fundamental strings, D4 branes and D6 branes in Type IIA string theory are M2 branes, M5 branes wrapped around the circle \( S^1 \) and KK monopoles (Taub-NUT space) in M theory, respectively \([25]\). The wrappings of string and NS5 brane are classified by the twisted homology \( H_i(\mathbb{RP}^6, \tilde{\mathbb{Z}}) \) for wrapped on an \( i \)-cycle in \( \mathbb{RP}^6 \) along the line of \([13,15]\):

(i) unwrapped string, giving a onebrane in \( AdS_4 \).

The wrapping modes that would give zero-branes are not possible since \( H_1(\mathbb{RP}^6, \tilde{\mathbb{Z}}) = 0 \).

The unwrapped NS5 brane is not possible since it does not fit in \( AdS_4 \).

(ii) wrapped on a two-cycle, to give a threebrane in \( AdS_4 \), classified by \( H_2(\mathbb{RP}^6, \tilde{\mathbb{Z}}) = \mathbb{Z}_2 \).

(iii) wrapped on a four-cycle, to give a onebrane in \( AdS_4 \), classified by \( H_4(\mathbb{RP}^6, \tilde{\mathbb{Z}}) = \mathbb{Z}_2 \).

The wrappings of D2 and D4 brane are classified by the ordinary (untwisted) homology \( H_i(\mathbb{RP}^6, \mathbb{Z}) \) for wrapped on an \( i \)-cycle in \( \mathbb{RP}^6 \) since the twobrane charge is even under the orientifolding operation (it comes from the fact that, in M theory, \( \mathbb{Z}_2 \) action does not act on the three-form \( C_3 \) and the D2 brane is a transverse M2 brane) and the fourbrane is dual to
the twobrane.

(iv) unwrapped D2 brane, giving a twobrane in $AdS_4$,

(v) wrapped on a one-cycle, to give a onebrane in $AdS_4$, classified by $H_1(R\mathbb{P}^6, \mathbb{Z}) = \mathbb{Z}_2$,

The unwrapped D4 brane is not possible.

(vi) wrapped on a one-cycle, to give a threebrane in $AdS_4$, classified by $H_1(R\mathbb{P}^6, \mathbb{Z}) = \mathbb{Z}_2$,

(vi) wrapped on a three-cycle, to give a onebrane in $AdS_4$, classified by $H_3(R\mathbb{P}^6, \mathbb{Z}) = \mathbb{Z}_2$.

The $k$ units of KK momentum mode around the eleventh circle $S^1$ can be identified as $k$ D0-branes which is charged BPS particles. In the three dimensional gauge theory context \cite{21}, a new scalar modulus appears as the dual of a photon, the magnetic scalar photon, coming from dualizing the vector in three dimensions. This corresponds to the expectation value of the localized eleven dimensional coordinate of a D2 brane. Then the effect of D0 branes exchange between D2 branes can be captured by instanton effects in 3-dimensional SYM and renders it $SO(8)$ invariant \cite{19}. The wrapping D6 brane are classified by the twisted homology $H_i(R\mathbb{P}^6, \tilde{\mathbb{Z}})$ for wrapped on an $i$-cycle in $R\mathbb{P}^6$ since the six brane is dual to zerobrane. It is obvious it is not possible to have a wrapped D6-brane on $R\mathbb{P}^i \subset R\mathbb{P}^6$ for $i \leq 2$.

---

6 Note that this situation is different from the case of O4 plane considered in \cite{15} and similar phenomena will also appear in the D0 and D6 system.

7 Note that the D0 brane charge is odd under the $\mathbb{Z}_2$ action since $A_{\mu}^{RR} = G_{\mu 10}$ and the $\mathbb{Z}_2$ flips the orientation of $S^1$ and thus the RR $U(1)$ gauge field should be considered as twisted one-form.

8 We should mention that this reasoning seems to cause a perplexing problem and we have no definite solution although we suggest some possible solutions in Sec.III. In treating the wrappings of D6 branes, we may need a proper $U(1)$ fibration of $R\mathbb{P}^7$, which is taken as a $U(1)$ bundle over $C\mathbb{P}^3$ with $R\mathbb{P}^1$ fibers, since the D6 branes are magnetically charged under the $U(1)$ gauge field. Unfortunately, this Hopf fibration does not give a correct R-symmetry and expected spectrums \cite{26}.
(viii) wrapped on a four-cycle, to give a twobrane in \( AdS_4 \), classified by \( H_4(\mathbb{RP}^6, \mathbb{Z}) = \mathbb{Z}_2 \),

(ix) wrapped on a six-cycle, to give a zerobrane in \( AdS_4 \), classified by \( H_6(\mathbb{RP}^6, \mathbb{Z}) = \mathbb{Z} \).

According to the similar arguments done in Type IIB description [13] and Type IIA description [15], one can derive a topological restriction on the brane wrappings on \( \mathbb{RP}^6 \) just described. In particular, since the topological restriction coming from the holonomy of the connection \( A_{RR} \) on the \( U(1) \)-bundle would be not considered for the reason explained in the previous footnote, it is sufficient only to consider the discrete torsion \( \theta_{NS} \) of the field \( B_{NS} \). We will show that the description of brane wrappings on \( \mathbb{RP}^6 \) is consistent with the topological restriction coming from the RR discrete torsion \( \theta_{RR} \), which should vanish in our case.

In the case (ii), there is no restriction on wrapping of NS5-branes on \( \mathbb{RP}^2 \subset \mathbb{RP}^6 \), to make a threebrane in \( AdS_4 \), since \( H^2(\mathbb{RP}^2, \mathbb{Z}) = \mathbb{Z} \). In the case (iii), the NS5 brane can be wrapped on \( \mathbb{RP}^4 \), to make a string in \( AdS_4 \), only if \( \theta_{RR} = 0 \), since \( H^2(\mathbb{RP}^4, \mathbb{Z}) = 0 \).

Similarly, in the cases (v) and (vi), there is no restriction on wrapping of D2 and D4 branes on \( \mathbb{RP}^1 \subset \mathbb{RP}^6 \), to make a string and a threebrane in \( AdS_4 \) respectively, since in this case \( \mathbb{RP}^2 \) cannot even be deformed into the D2 or D4 brane. In the case (vii), the D4 brane can be wrapped on \( \mathbb{RP}^3 \), to make a string in \( AdS_4 \), only if \( \theta_{NS} \neq 0 \), since \( H^2(\mathbb{RP}^3, \mathbb{Z}) = \mathbb{Z}_2 \).

Similarly, in the case (viii), the D6 brane can be wrapped on \( \mathbb{RP}^4 \subset \mathbb{RP}^6 \), to make a twobrane in \( AdS_4 \), only if \( \theta_{NS} = 0 \), since \( H^2(\mathbb{RP}^4, \mathbb{Z}) = 0 \). Finally, in the case (ix), the D6 brane can be wrapped on \( \mathbb{RP}^6 \), to make a particle in \( AdS_4 \), only if \( \theta_{NS} = 0 \), since \( H^2(\mathbb{RP}^6, \mathbb{Z}) = 0 \).

\footnote{Note that there can be a topological obstruction to the brane wrapping since the second Stieffel-Whitney class \( w_2(\mathbb{RP}^6) \) does not vanish, which is first observed and speculated to need to cancel worldsheet global anomalies in [13] and is intended to give a K-theory interpretation in [27]. We will consider it in Sec.III.}
III. GAUGE THEORY AND BRANES ON RP$^6$

A. Strings

Let us consider “solitonic” strings in $AdS_4$ coming from wrapped NS5 brane on $RP^4$ which is the case of $(iii)$. According to the topological restriction, $\theta_{RR} = 0$, the solitonic string is possible for both orthogonal and symplectic group. The solitonic strings can annihilate in pairs due to $H_4(RP^6, \mathbb{Z}) = \mathbb{Z}_2$. Let the NS5-brane worldvolume coordinates be $(x^0, x^4, x^5, x^7, x^8, x^9)$ directions and be specified by $x^1 = x^2 = x^3 = x^6 = 0$. Then the solitonic string worldsheet in $AdS_4$ is parametrized by the radial function $\rho$ of $RP^7$ and $x^0$ and is at $x^1 = x^2 = x^3 = x^6 = 0$. The tension of this string is proportional to the NS5-brane tension, of order $1/\lambda^2$ and stretched to infinity in the radial direction of $AdS_4$. It is not obvious that the solitonic string can carry an external spinor charge of $SO(N)$ as the fat string in $AdS_5$ \cite{13} and it is not clear how to interpret this in the boundary SCFT.

According to previous analysis, we have strings in $AdS_4$ arising from wrapping D2 brane on $RP^1 \subset RP^6$. We call this “fat” strings since it’s property is quite similar to $AdS_5$ “fat” string in \cite{13}, which is different from usual fundamental and D strings. Fat string can annihilate in pairs due to the fact that they are classified by $H_1(RP^6, \mathbb{Z}) = \mathbb{Z}_2$. Their tension is proportional to D2-brane tension and is of order $1/\lambda$. Let us consider D2 brane whose worldvolume is specified by $x^1 = x^2 = x^5 = \cdots = x^9 = 0$ with arbitrary values of $x^0$ and of $x^3, x^4$. From the $AdS_4 \times S^6$ point of view, this D2 brane is wrapped on an $RP^1 \subset RP^6$ and looks like a fat string on $AdS_4$. The $RP^1$ is the subspace of $RP^6$ with $x^5 = \cdots = x^9 = 0$. Then the fat string worldsheet in $AdS_4$ is parametrized by $\rho$ and $x^0$ and is at $x^1 = x^2 = x^5 = \cdots = x^9 = 0$. Since D2 brane wrapped on an $RP^1 \subset RP^6$ meet the D2 brane in $AdS_4$ at $x^1 = \cdots = x^9 = 0$, the ground state of the 2-2 string has zero energy \cite{28}. The ground states of these strings give $N$ fermionic zero modes in the spinor representation of $SO(N)$. The D2 brane regarded as a string in $AdS_4$ has an end point at $\rho = 0$ and this end point lies on the boundary of $AdS_4$ at which there are external spinor charges.
Let us consider two identical adjacent D2 branes whose worldvolumes are of the form $C \times \mathbb{RP}^1$ and $C' \times \mathbb{RP}^1$ at given time where $C$ and $C'$ are two parallel paths in $AdS_4$. We use the same $\mathbb{RP}^1$ for each D2 brane. For example, if $\mathbb{RP}^6$ is obtained by projectivizing a copy of $\mathbb{R}^7$ with coordinates $x_3, \ldots, x_9$, then $\mathbb{RP}^1$ can be defined as the subspace $x_5 = \cdots = x_9 = 0$. We give same orientation to $C \times \mathbb{RP}^1$ and $C' \times \mathbb{RP}^1$ by using that $C$ and $C'$ are parallel and the two $\mathbb{RP}^1$'s are identical. As done in [13], we consider $C$ and $C'$ are semi-infinite paths terminating at points $P$ and $P'$ respectively in order to describe D2 brane annihilation. We pick a two dimensional manifold $Z$ in spacetime whose boundary is the union of $P \times \mathbb{RP}^1$ and $P' \times \mathbb{RP}^1$. $Z$ should have an untwisted orientation which agrees on the boundary with those of $C \times \mathbb{RP}^1$ and $C' \times \mathbb{RP}^1$ since the orientation of the D2 brane worldvolume $X$ is defined in terms of ordinary tangent bundle $TX$.

Similarly, the case (vii) also gives fat string in $AdS_4$ by wrapping D4 brane on $\mathbb{RP}^3 \subset \mathbb{RP}^6$ where they are classified by $H_3(\mathbb{RP}^6, Z) = \mathbb{Z}_2$. The D4 brane whose worldvolume is specified by $x^1 = x^2 = x^7 = x^8 = x^9 = 0$ with arbitrary values of $x^0$ and of $x^3, x^4, x^5, x^6$. From the $AdS_4$ point of view, this D4 brane is wrapped on an $\mathbb{RP}^3 \subset \mathbb{RP}^6$ and looks like a fat string on $AdS_4$. The $\mathbb{RP}^3$ is the subspace of $\mathbb{RP}^6$ with $x^7 = x^8 = x^9 = 0$. Then the fat string worldsheet in $AdS_4$ is parametrized by $\rho$ and $x^0$ and is at $x^1 = x^2 = x^7 = x^8 = x^9 = 0$. Since D4 brane wrapped on an $\mathbb{RP}^3 \subset \mathbb{RP}^6$ meet the D2 brane in $AdS_4$ at $x^1 = \cdots = x^9 = 0$, the ground state of the 2-4 string has zero energy. The ground states of these strings also give $N$ fermionic zero modes in the spinor representation of $SO(N)$. The D4 brane regarded as a string in $AdS_4$ has an end point at $\rho = 0$ and this end point lies on the boundary of $AdS_4$ at which there are external spinor charges. We can also consider two identical adjacent D4 branes whose worldvolumes are of the form $C \times \mathbb{RP}^3$ and $C' \times \mathbb{RP}^3$ at given time where $C$ and $C'$ are two parallel paths in $AdS_4$. According to the similar procedure, we can pick a four dimensional manifold $Z$ in spacetime whose boundary is the union of $P \times \mathbb{RP}^3$ and $P' \times \mathbb{RP}^3$. $Z$ should also have an twisted orientation for the similar reason to the D2 brane case which agrees on the boundary with those of $C \times \mathbb{RP}^3$ and $C' \times \mathbb{RP}^3$.
B. Domain Walls

Let us consider the objects in $AdS_4 \times S^6$ and $AdS_4 \times \mathbb{RP}^6$ that look like two-branes in the four noncompact dimensions of $AdS_4$. Since the $AdS_4$ has three spatial dimensions, the two-brane could potentially behave as a domain wall, with the string theory vacuum “jumping” as one crosses the two-brane. In $AdS_4 \times S^6$ and $AdS_4 \times \mathbb{RP}^6$, the only such objects are the Type IIA D2 brane in the case $(iv)$ and the Type IIA D6 brane in the case $(viii)$ in section IIC. Note that, in the case of $AdS_4 \times S^6$, the D6 brane can not wrap on a four-cycle in $S^6$ since $H_4(S^6, \mathbb{Z}) = 0$, so does not give rise to a domain wall in $SU(N)$ gauge theory.

Since the two-brane is the electric source of the four-form field $G_4$ in Type IIA theory, the integrated four-form flux over $S^6$ or $\mathbb{RP}^6$ jumps by one unit when one crosses the two-brane. This means that the gauge group of the boundary conformal field theory can change, for example, from $SU(N)$ on one side to $SU(N \pm 1)$ on the other side for $AdS_4 \times S^6$. For $AdS_4 \times \mathbb{RP}^6$, if one is crossing the D2 brane, it changes from $SO(N)$ to $SO(N \pm 2)$ or from $Sp(N/2)$ to $Sp(N/2 \pm 1)$ since the D2 brane charge changes by two units on double cover.

The similar situation also occurs in the case of the D6 brane wrapped on $\mathbb{RP}^4 \subset \mathbb{RP}^6$ to make a two-brane. Let $P$ and $Q$ be points on opposite sides of the two-brane. Let $X$ be the three-manifold $X = T \times \mathbb{RP}^2$, with $T$ a path from $P$ and $Q$, intersecting the two-brane once. Since a generic $\mathbb{RP}^4$ and $\mathbb{RP}^2$ in $\mathbb{RP}^6$ have one point of intersection, $X$ generically intersects the D6 brane at one point. The boundary of $X$ is the union of the two-manifolds $P \times \mathbb{RP}^2$ and $Q \times \mathbb{RP}^2$. Because the sixbrane is a magnetic source for the $A_{RR}$ field and intersects $X$ in one point, the total magnetic charge of the $A_{RR}$ field on the boundary of $X$ jumps in crossing the two-brane. Although it cause no change of the gauge group in the boundary theory, this may have an important effect on instanton corrections in field theory.
C. The Baryon Vertex in $SO(N)/Sp(2N)$

The baryon vertex in $SU(N)$ was obtained by wrapping a D6 brane over $S^6$. By analogy, one expects that the baryon vertex in $SO(N)$ or $Sp(2N)$ will consist of a D6 brane wrapped on $RP^6$. If we are considering $SO(2k)$ gauge theory, there are $k$ units of six-form flux on $RP^6$ when the D6 brane wraps once on $RP^6$. But there is no gauge invariant combination, in $SO(2k)$ gauge theory, of $k$ external quarks to obtain a “baryon vertex”. The baryon vertex of $SO(2k)$ gauge theory should couple $2k$ external quarks, not $k$ of them.

Let $\Phi$ be the map of D6 brane worldvolume $X$ to $AdS_4 \times RP^6$. We must impose the condition that the $B_{NS}$ fields should be topologically trivial when pulled back to $X$ as implied in deriving the topological restrictions on the brane wrapping in section II. If we choose the D6 brane topology as $S^6$, the topological triviality of the field $B_{NS}$ is automatically obeyed since $H^3(S^6, \mathbb{Z}) = 0$, the AdS baryon vertex thus exists regardless of the gauge group of the boundary theory. Then the map $\Phi : S^6 \to RP^6$ gives the degree two map, in other words, $S^6$ wraps twice around $RP^6$. Thus we can obtain the correct baryon vertex coupling $2k$ quarks.

In $Sp(k)$ gauge theory, a baryon vertex can decay to $k$ mesons [29]. Thus, one may expect no topological stability for the $AdS_4$ baryon vertex when $\theta_{NS} \neq 0$. However, since the stability of baryon in $SO(k)/Sp(k)$ gauge theory actually can be encoded by the homology, $H_6(RP^6, \mathbb{Z}) = \mathbb{Z}$, which implies the topological stability of a baryonic charge, we get a contradiction with the field theory result. In addition, if the baryon vertex decays via compact seven-manifold $X$ contractable to $RP^6$ through similar mechanism discussed in [13], we also meet the following problem. In previous section, we showed that there is a

\[\begin{equation}
\int_{RP^6} G_6^2 \neq 0,
\end{equation}\]

In fact, one cannot properly define the twobrane charges since, in this case, the flux integral over $RP^6$, $\int_{RP^6} \frac{G_6^2}{2\pi}$, is not well defined because $RP^6$ is an unorientable manifold and $G_6$ is an ordinary (untwisted) six-form field. However, if one would consider the flux integral over $S^6$, the double covering of $RP^6$, the twobrane charge can be well defined.
topological restriction on the D6 brane wrapping on $\mathbb{RP}^6$, which is only possible in $SO(N)$ gauge theory, which is definitely the wrong answer. But, we should consider the possibility on an existence of nontrivial torsion class of the $B_{NS}$ fields due to the topology of the D6 brane worldvolume $X$, which is denoted as $W \in H^3(X, \mathbb{Z})$ [13]. Then this means that the correct global restriction is not that $i^*([H_{NS}]) = 0$ but rather that

$$i^*([H_{NS}]) = W,$$

(III.1)

where $i$ is the inclusion of $X$ in spacetime and $[H_{NS}]$ the characteristic class of the $B_{NS}$ field. A possible $W$ can be determined by using the “connecting homomorphism” in an exact sequence of cohomology groups from the second Stiefel-Whitney class $w_2(X) \in H^2(X, \mathbb{Z}_2)$, which means that the proper global restriction is $i^*([H_{NS}]) = W$, i.e., $\theta_{NS} \neq 0$. If the baryon vertex decays via compact seven-manifold $X$, we will find that $W \neq 0$ since $w_2(X) \neq 0$. Consequently, the brane decay via compact sevenfold $X$ is possible only in $Sp(k)$ gauge theory. Hence, the latter problem can be avoided.

In $SO(N)$ gauge theory where $N$ is even or odd, super Yang-Mills theory with 16 supercharges actually has $O(N)$ symmetry, not just $SO(N)$. The generator $\tau$ of the quotient $O(N)/SO(N) = \mathbb{Z}_2$ behaves as a global symmetry. Since the baryon is odd under $\tau$, it cannot decay to mesons which is even under $\tau$. If there is a Pfaffian-like state which is odd under $\tau$, the possible decay channel may be mesons plus a Pfaffian as in [13]. However, in our case, there is no definite candidate being role of the Pfaffian. It is well-known [29] that two baryons in $SO(N)$ gauge theory can annihilate into $N$ mesons. This is also consistent with the fact that, in $O(N)$, a product of two epsilon symbols can be rewritten as a sum of products of $N$ Kronecker deltas. Their annihilation can be realized by the similar (and more simple) process to the pair annihilation of two fat strings discussed in [13]. That is, two identical D6 branes, whose worldvolumes are of the form $C \times S^6$ and $C' \times S^6$ where $C$ and $C'$ are timelike path, collide at any time where $C$ and $C'$ coincide and the 2N strings consisting baryon vertices are pairwise connected and produce N strings, N mesons, attached to the D2 brane on the boundary.
These results imply that the decay of baryon in $SO(N)/Sp(N)$ gauge theory should belong to the element of $H_6(Y, \mathbb{Z}) = \mathbb{Z}_2$. However, if $Y = \mathbb{RP}^6$, the baryon vertex is stable since $H_6(\mathbb{RP}^6, \mathbb{Z}) = \mathbb{Z}$ and thus we meet a perplexing puzzle. Unfortunately, we have no definite answer about this problem, so we would speculate only some possible resolutions. First, recall that the D6-brane is the electric-magnetic dual of the D0-brane, so it is magnetically charged with respect to D0-brane gauge field. In the transverse space to the D6-brane, this object can be represented by Taub-NUT metric which is asymptotically flat and the transverse space looks near infinity like a nontrivial $S^1$ bundle over $\mathbb{R}^3$ \cite{25}. Thus the Kaluza-Klein direction, $S^1$, is not independent of the other directions but is combined with them in a smooth and topologically nontrivial way. Second, recall that, in going from M-theory to Type IIA, the sixbrane core will be singular, so it has to be careful to treat wrapping of such singular object and there may be a subtlety to manipulate (co-)homology involving D6 brane wrappings. Finally, the recent analysis shows \cite{27} that a wrapped Type IIA brane is classified by K-theory in ten dimensional spacetime with $S^1$ completion, more subtle than Type IIB or Type I case. It will be interesting to check that the proper consideration of the above (and another) factors will provide a solution on the puzzle.

IV. DISCUSSION

To summarize, for $SU(N)$ theory, we interpreted the baryon vertex as a wrapped D6 brane in $S^6$ connected by $N$ fundamental strings ending on a D2 brane on $AdS_4$ boundary. When we go $SO(N)/Sp(2N)$ theory, $\mathbb{R}^8/\mathbb{Z}_2$ orbifold singularity was crucial to understand M theory realization of three types of O2 plane. We constructed the possible brane wrappings on $\mathbb{RP}^6$ in Type IIA string theory and determined their topological restrictions in each case. According to this classification, it was possible to interpret various wrapping branes on $\mathbb{RP}^6$ in terms of strings, domain walls and the baryon vertex where the topological properties on $\mathbb{RP}^6$ are used and pointed out that there is a perplexing puzzle on the baryon vertex.

It was pointed out by Witten \cite{30} that the infrared limit of SYM on $\mathbb{R}^3$ related to $\mathcal{N} = 8$
SCFT is quite subtle and there may be surprising features in the topology in the infrared, such as conservation laws that hold in the infrared but not exactly. It may become important in the limit to consider the nonperturbative instanton corrections in SYM theory, which is essential to recover $SO(8)$ invariance or eleven dimensional Lorentz invariance. In our situation, the baryon vertex is constructed by a wrapped D6-branes on $S^6$ or $RP^6$. But this state will be obscure in the limit since D6-branes are not well-defined in the full M-theory limit. It will be thus interesting to study the infrared limit of three-dimensional SYM more closely.

It was observed by Sethi that M-theory realization of O2 planes gives an amusing interpretation on the shift of membrane charge by the discrete torsion. This interesting phenomenon may be more clearly understood in terms of the method applied to O4 planes by Gimon where, applying the T-S-T transformations, the O4 planes are related to O3 planes which is more well understood. By applying the same strategy, one can relate the O2 planes to O3 planes by the T-S-T transformations where T-duality is taken along the one direction of the O3 plane. We hope this work will be accomplished near future and provide a new understanding on O2 planes.

Klebanov and Witten found $AdS_5 \times T^{1,1}$ model which is an example of holographic theory on a compact manifold which is not locally $S^5$ and the corresponding quantum field theory cannot be obtained from the projection of maximal $\mathcal{N} = 4$ theory. See also recent papers. It is well known that there exist various types of seven dimensional compact Einstein manifold $X_7$ which is not locally $S^7$. It would be interesting to study whether one can find wrapping branes over cycles of $X_7$ and discuss their field theory interpretation.

**Note added:** After this work has been finished, we found the reference which treats related subject.

**Acknowledgments**
We thank O. Aharony, K. Hori and E. Witten for email correspondence. CA thanks K. Oh and R. Tatar for discussions on relating subjects. This work is supported (in part) by the Korea Science and Engineering Foundation (KOSEF) through the Center for Theoretical Physics (CTP) at Seoul National University. BHL and HSY are also partially supported by the Korean Ministry of Education (BSRI-98-2414) and HK is supported by TGRC-KOSEF. We thank Asia Pacific Center for Theoretical Physics (APCTP) for hospitality where this work has been done.
REFERENCES

[1] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231.

[2] S. S. Gubser, I. R. Klebanov, A. M. Polyakov, Phys. Lett. B428 (1998) 105; E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253.

[3] Y. Oz and J. Terning, hep-th/9803167.

[4] S. Kachru and E. Silverstein, Phys. Rev. Lett. 80 (1998) 4855; A. Lawrence, N. Nekrasov and C. Vafa, hep-th/9803015.

[5] S. Gukov, hep-th/9806180.

[6] S.-J. Rey and J. Yee, hep-th/9803001; J. Maldacena, Phys. Rev. Lett. 80 (1998) 4859; D. J. Gross and H. Ooguri, Phys. Rev. D58 (1998) 106002.

[7] C. Csáki, H. Ooguri, Y. Oz and J. Terning, hep-th/9806021; R. de Mello Koch, A. Jevicki, M. Mihailescu, and J. P. Nunes, Phys. Rev. D58 (1998) 105009.

[8] A. Brandhuber, N. Itzhaki, J. Sonnenschein and S. Yankielowicz, J. High Energy Phys. 07 (1998) 020.

[9] Y. Imamura, hep-th/9806162.

[10] O. Aharony, Y. Oz and Z. Yin, Phys. Lett. B430 (1998) 87.

[11] R. G. Leigh and M. Rozali, Phys. Lett. B431 (1998) 311; S. Minwalla, J. High Energy Phys. 10 (1998) 002; E. Halyo, J. High Energy Phys. 04 (1998) 011; J. Gomis, hep-th/9803119.

[12] S. Ferrara, A. Kehagias, H. Partouche and A. Zaffaroni, Phys. Lett. B431 (1998) 42; M. Berkooz, hep-th/9802193; R. Entin and J. Gomis, Phys. Rev. D58 (1998) 105008; C. Ahn, K. Oh and R. Tatar, hep-th/9806041; C. Ahn, K. Oh and R. Tatar, hep-th/9804093, to appear in Phys. Lett. B.
[13] E. Witten, J. High Energy Phys. **07** (1998) 006.

[14] Z. Kakushadze, Nucl. Phys. **B529** (1998) 157; Phys. Rev. **D58** (1998) 106003; A. Fayyazuddin and M. Spalinski, [hep-th/9805096](http://arxiv.org/abs/hep-th/9805096); O. Aharony, A. Fayyazuddin and J. Maldacena, J. High Energy Phys. **07** (1998) 013; C. Ahn, K. Oh and R. Tatar, [hep-th/9808143](http://arxiv.org/abs/hep-th/9808143); S. S. Gubser and I. R. Klebanov, [hep-th/9808073](http://arxiv.org/abs/hep-th/9808073).

[15] C. Ahn, H. Kim and H. S. Yang, [hep-th/9808182](http://arxiv.org/abs/hep-th/9808182).

[16] K. Hori, [hep-th/9805141](http://arxiv.org/abs/hep-th/9805141).

[17] S. Sethi, [hep-th/9809162](http://arxiv.org/abs/hep-th/9809162).

[18] N. Itzhaki, J. M. Maldacena, J. Sonnenschein and S. Yankielowicz, Phys. Rev. **D58** (1998) 046004.

[19] J. Polchinski and P. Pouliot, Phys. Rev. **D56** (1997) 6601; E. Keski-Vakkuri and P. Kraus, [hep-th/9804067](http://arxiv.org/abs/hep-th/9804067); S. Paban, S. Sethi and M. Stern, [hep-th/9808119](http://arxiv.org/abs/hep-th/9808119); S. Hyun, Y. Kiem and H. Shin, [hep-th/9808183](http://arxiv.org/abs/hep-th/9808183).

[20] C. P. Bachas and M. B. Green, J. High Energy Phys. **01** (1998) 015.

[21] N. Seiberg, [hep-th/9705117](http://arxiv.org/abs/hep-th/9705117).

[22] K. Becker and M. Becker, Nucl. Phys. **B477** (1996) 155.

[23] S. Sethi, C. Vafa and E. Witten, Nucl. Phys. **B480** (1996) 213; E. Witten, J. Geom. Phys. **22** (1997) 1.

[24] K. Dasgupta, D. P. Jatkar and S. Mukhi, Nucl. Phys. **B523** (1998) 465.

[25] P. K. Townsend, Nucl. Phys. **B350** (1995) 184.

[26] E. Halyo, [hep-th/9803193](http://arxiv.org/abs/hep-th/9803193).

[27] E. Witten, [hep-th/9810188](http://arxiv.org/abs/hep-th/9810188).
[28] M. Berkooz, M. R. Douglas and R. G. Leigh, Nucl. Phys. \textbf{B480} (1996) 265.

[29] E. Witten, Nucl. Phys. \textbf{B223} (1983) 433.

[30] E. Witten, private communication.

[31] E. G. Gimon, \texttt{hep-th/9806226}.

[32] I. R. Klebanov and E. Witten, \texttt{hep-th/9807080}.

[33] D. R. Morrison and M. R. Plesser, \texttt{hep-th/9810201}; K. Oh and R. Tatar, \texttt{hep-th/9810244}; C. P. Boyer and K. Galicki, \texttt{hep-th/9810250}.

[34] M. Berkooz and A. Kapustin, \texttt{hep-th/9810257}.