Charmless hadronic $B_c \to VA, AA$ decays in the perturbative QCD approach

Xin Liu* and Zhen-Jun Xiao†

Department of Physics and Institute of Theoretical Physics,
Nanjing Normal University, Nanjing,
Jiangsu 210046, People’s Republic of China

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Abstract

In this work, we calculate the branching ratios (BRs) and the polarization fractions of sixty two charmless two-body $B_c$ meson decays into final states involving one vector and one axial-vector meson ($VA$) or two axial-vector mesons ($AA$) within the framework of perturbative QCD approach systematically, where $A$ is either a $^3P_1$ or $^1P_1$ axial-vector meson. All considered decay channels can only occur through the annihilation topologies in the standard model. Based on the perturbative calculations and phenomenological analysis, we find the following results: (i) the $CP$-averaged BRs of the considered sixty two $B_c$ decays are in the range of $10^{-5}$ to $10^{-9}$; (ii) since the behavior for $^1P_1$ meson is much different from that of $^3P_1$ meson, the BRs of $B_c \to A(^1P_1)(V,A(^1P_1))$ decays are generally larger than that of $B_c \to A(^3P_1)(V,A(^3P_1))$ decays in the perturbative QCD approach; (iii) many considered decays modes, such as $B_c \to a_1(1260)^\pm \omega$, $b_1(1235)\rho$, etc, have sizable BRs within the reach of the LHCb experiments; (iv) the longitudinal polarization fractions of most considered decays are large and play the dominant role; (v) the perturbative QCD predictions for several decays involving mixtures of $^3P_1$ and/or $^1P_1$ mesons are highly sensitive to the values of the mixing angles, which will be tested by the ongoing LHC and forthcoming Super-B experiments; (vi) the $CP$-violating asymmetries of these considered $B_c$ decays are absent in the standard model because only one type tree operator is involved.

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* liuxin.physics@gmail.com
† xiaozhenjun@njnu.edu.cn
I. INTRODUCTION

Unlike the $B_q$ meson with $q = (u, d, s)$, the $B_c$ meson is the only heavy meson embracing two heavy quarks $b$ and $c$ simultaneously. Researchers believe that the $B_c$ physics must be very rich if the statistics reaches high level. With the running of Large Hadron Collider (LHC) experiments, a great number of $B_c$ meson events, about 10% of the total $B$ meson data, will be collected and this will provide a new platform for both theorists and experimentalists to study the perturbative and nonperturbative QCD dynamics, final state interactions, even the new physics scenarios beyond the standard model (SM) [1].

Very recently, we studied the two-body charmless hadronic decays $B_c \rightarrow PP, PV/VP,VV$ and $B_c \rightarrow AP$ (here $P, V$ and $A$ stand for the light pseudo-scalar, vector and axial-vector mesons respectively) [2, 3]. All these decays can only occur via the annihilation type diagrams in the SM. Although the contributions induced by annihilation diagrams are suppressed in the decays of ordinary light $B_q$ ($q = u, d, s$) mesons, they could be large and detected at LHC experiments [4] in $B_c$ meson decays. According to the discussions as given in Ref. [4], the charmless hadronic $B_c$ decays with decay rates at the level of $10^{-6}$ could be measured at the LHC experiments with the accuracy required for the phenomenological analysis, it is therefore believed that they can help the people to understand the annihilation decay mechanism in $B$ physics well. In this paper, we will extend our previous studies of two-body $B_c$ decays to $B_c \rightarrow V A$ and $A A$ modes, which are also pure annihilation type decays, and expected to have rich physics since there are three polarization states involved in these decays.

In this paper, we will calculate the $CP$-averaged branching ratios (BRs) and polarization fractions of the sixty two charmless hadronic $B_c \rightarrow VA, AA$ decays by employing the low energy effective Hamiltonian [5] and the perturbative QCD (pQCD) factorization approach [6–8]. In the pQCD approach, the annihilation type diagrams can be calculated analytically, as have been done for example in Refs. [6, 7, 9–14]. First of all, the size of annihilation contributions is an important issue in the $B$ meson physics, and has been studied extensively, for example, in Refs. [6, 7, 9, 10, 15, 16]. Second, the internal structure of the axial-vector mesons has been one of the hot topics in recent years [17–19]. Although many efforts on both theoretical and experimental sides have been made [20–26] to explore it through the studies for the relevant decay rates, $CP$-violating asymmetries, polarization fractions and form factors, etc., we currently still know little about the nature of the axial-vector mesons. Furthermore, through the polarization studies in the considered $B_c \rightarrow VA, AA$ decays, these channels can shed light on the underlying helicity structure of the decay mechanism.

The paper is organized as follows. In Sec. II we present the formalism and give the essential input quantities, including the operator basis and the mixing angles between $3P_1$ and/or $1P_1$ mesons. The wave functions and distribution amplitudes for $B_c$ and light vector and axial-vector mesons are also given here. Then we perform the perturbative calculations for considered decay channels in Sec. III. The analytic expressions of the decay amplitudes for all considered sixty two $B_c \rightarrow VA, AA$ decay modes are also collected in this section. The numerical results and phenomenological analysis are given in Sec. IV. The main conclusions and a short summary are presented in the last section.

II. INPUT QUANTITIES AND FORMALISM

In the following we shall briefly discuss the mixing of axial-vector mesons and summarize all the input quantities relevant to the present work, such as the operator basis, mixing angles,
wave functions and light-cone distribution amplitudes for light vector and axial-vector mesons. Finally, the formalism of pQCD approach will also be presented briefly.

A. Effective Hamiltonian

For those considered charmless hadronic $B_c$ decays, the related weak effective Hamiltonian $H_{\text{eff}}$ is given by [5]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{cb}^* V_{ud} (C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)) \right] ,$$

with the local four-quark tree operators $O_{1,2}$

$$O_1 = \bar{u}_\beta \gamma^\mu (1 - \gamma_5) D_\alpha \bar{c}_\gamma \gamma^\mu (1 - \gamma_5) b_\alpha ,$$  

$$O_2 = \bar{u}_\beta \gamma^\mu (1 - \gamma_5) D_\beta \bar{c}_\alpha \gamma^\mu (1 - \gamma_5) b_\alpha ,$$

where $V_{cb}, V_{ud}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, "$D$" denotes the light down quark $d$ or $s$ and $C_i(\mu)(i = 1, 2)$ are Wilson coefficients at the renormalization scale $\mu$. For the Wilson coefficients $C_i(\mu)$, we will also use the leading order expressions, although the next-to-leading order calculations already exist in the literature [5]. This is the consistent way to cancel the explicit $\mu$ dependence in the theoretical formulas. For the renormalization group evolution of the Wilson coefficients from higher scale to lower scale, we use the formulas as given in Ref. [2] directly.

B. Mixtures and Mixing Angles

In the quark model, there exist two distinct types of light parity-even axial-vector mesons, namely, $3P_1 (J^{PC} = 1^{++})$ and $1P_1 (J^{PC} = 1^{+-})$. The $3P_1$ nonet consists of $a_1 (1260), f_1 (1285), f_1 (1420)$ and $K_{1A}$ states, while the $1P_1$ nonet has $b_1 (1235), h_1 (1170), h_1 (1380)$ and $K_{1B}$ states. In the SU(3) flavor limit, these mesons can not mix with each other. Because the $s$ quark is heavier than $u, d$ quarks, the physical mass eigenstates $K_1 (1270)$ and $K_1 (1400)$ are not purely $1^3P_1$ or $1^1P_1$ states, but believed to be mixtures of $K_{1A}$ and $K_{1B}$.[1] Analogous to $\eta$ and $\eta'$ system, the flavor-singlet and flavor-octet axial-vector meson can also mix with each other. It is worth mentioning that the mixing angles can be determined by the relevant data, but unfortunately, there is no enough data now for these mesons which leaves the mixing angles basically free parameters.

The physical states $K_1 (1270)$ and $K_1 (1400)$ can be written as the mixtures of the $K_{1A}$ and $K_{1B}$ states:

$$\begin{pmatrix} K_1 (1270) \\ K_1 (1400) \end{pmatrix} = \begin{pmatrix} \sin \theta_K & \cos \theta_K \\ \cos \theta_K & -\sin \theta_K \end{pmatrix} \begin{pmatrix} K_{1A} \\ K_{1B} \end{pmatrix}$$

(3)

The mixing angle $\theta_K$ still not be well determined because of the poor experimental data. In this paper, for simplicity, we will adopt two reference values as those used in Ref. [19]: $\theta_K = \pm 45^\circ$.

[1] For the sake of simplicity, we will adopt the forms $a_1, b_1, K', K''$ to denote the axial-vector mesons $a_1 (1260), b_1 (1235), K_1 (1270), K_1 (1400), f_1 (1285), f_1 (1420), h_1 (1170)$ and $h_1 (1380)$ correspondingly in the following sections, unless otherwise stated. We will also use $K_1, f_1$ and $h_1$ to denote $K_1 (1270)$ and $K_1 (1400), f_1 (1285)$ and $f_1 (1420), and h_1 (1170)$ and $h_1 (1380)$ for convenience unless explicitly otherwise stated.
Analogous to the $\eta$-$\eta'$ mixing in the pseudoscalar sector, the $h_1(1170)$ and $h_1(1380)$ ($1^1P_1$ states) system can be mixed in terms of the pure singlet $|h_1\rangle$ and octet $|h_8\rangle$,

$$
egin{pmatrix}
    h_1(1170) \\
    h_1(1380)
  \end{pmatrix}
  =
  \begin{pmatrix}
    \cos \theta_1 & \sin \theta_1 \\
    -\sin \theta_1 & \cos \theta_1
  \end{pmatrix}
  \begin{pmatrix}
    h_1 \\
    h_8
  \end{pmatrix}
$$

(4)

Likewise, $f_1(1285)$ and $f_1(1420)$ (the $1^3P_1$ states) will mix in the form of

$$
egin{pmatrix}
    f_1(1285) \\
    f_1(1420)
  \end{pmatrix}
  =
  \begin{pmatrix}
    \cos \theta_3 & \sin \theta_3 \\
    -\sin \theta_3 & \cos \theta_3
  \end{pmatrix}
  \begin{pmatrix}
    f_1 \\
    f_8
  \end{pmatrix}
$$

(5)

where the component of $h_1, f_1$ and $h_8, f_8$ can be written as

$$
|h_1\rangle = |f_1\rangle = \frac{1}{\sqrt{3}} (|\bar{q}q\rangle + |\bar{s}s\rangle),
$$

$$
|h_8\rangle = |f_8\rangle = \frac{1}{\sqrt{6}} (|\bar{q}q\rangle - 2|\bar{s}s\rangle),
$$

(6)

where $q = (u, d)$. The values of the mixing angles for $1^1P_1$ and $1^3P_1$ states are chosen as

$$
\theta_1 = 10^\circ \ \text{or} \ 45^\circ; \quad \theta_3 = 38^\circ \ \text{or} \ 50^\circ.
$$

(7)

C. Wave Functions and Distribution Amplitudes

In order to calculate the decay amplitude, we should choose the proper wave functions for the heavy $B_c$, and light vector and axial-vector mesons. For the wave function of $B_c$ meson, we adopt the form (see Ref. [2], and references therein) as,

$$
\Phi_{B_c}(x) = \frac{i}{\sqrt{2N_c}} [(P + m_{B_c})\gamma_5\phi_{B_c}(x)]_{\alpha\beta},
$$

(8)

where the distribution amplitude $\phi_{B_c}$ would be close to $\delta(x - m_c/m_{B_c})$ in the non-relativistic limit because of the fact that $B_c$ meson embraces two heavy quarks. We therefore adopt the non-relativistic approximation form for $\phi_{B_c}$ as

$$
\phi_{B_c}(x) = \frac{f_{B_c}}{2\sqrt{2N_c}} \delta(x - m_c/m_{B_c}),
$$

(9)

where $f_{B_c}$ and $N_c$ are the decay constant of $B_c$ meson and the color number, respectively.

For the wave functions of vector and axial-vector mesons, one longitudinal($L$) and two transverse($T$) polarizations are involved, and can be written as,

$$
\Phi^{L}_V(x) = \frac{1}{\sqrt{2N_c}} \left\{ m_V \xi^{L}_V \phi_V(x) + \xi^{L}_V P\phi_V(x) + m_V \phi^{\ast}_V(x) \right\}_{\alpha\beta},
$$

(10)

$$
\Phi^{T}_V(x) = \frac{1}{\sqrt{2N_c}} \left\{ m_V \xi^{T}_V \phi^{\ast}_V(x) + \xi^{T}_V P\phi^{\ast}_V(x) + m_V i\epsilon^{\mu\rho\sigma} \gamma_5 \gamma^\mu \epsilon^{\ast\nu}_{\ast} n^{\rho} v^{\sigma} \phi^{\ast}_V(x) \right\}_{\alpha\beta},
$$

(11)

$$
\Phi^{L}_A(x) = \frac{1}{\sqrt{2N_c}} \gamma_5 \left\{ m_A \xi^{L}_A \phi_A(x) + \xi^{L}_A P\phi_A(x) + m_A \phi^{\ast}_A(x) \right\}_{\alpha\beta},
$$

(12)

$$
\Phi^{T}_A(x) = \frac{1}{\sqrt{2N_c}} \gamma_5 \left\{ m_A \xi^{T}_A \phi^{\ast}_A(x) + \xi^{T}_A P\phi^{\ast}_A(x) + m_A i\epsilon^{\mu\rho\sigma} \gamma_5 \gamma^\mu \epsilon^{\ast\nu}_{\ast} n^{\rho} v^{\sigma} \phi^{\ast}_A(x) \right\}_{\alpha\beta},
$$

(13)
where $\epsilon_{V(A)}^{L,T}$ denotes the longitudinal and transverse polarization vectors of vector(axial-vector) meson, satisfying $P \cdot \epsilon = 0$ in each polarization, $x$ denotes the momentum fraction carried by quark in the meson, and $n = (1, 0, 0_T)$ and $v = (0, 1, 0_T)$ are dimensionless light-like unit vectors. We here adopt the convention $\epsilon_{0123} = 1$ for the Levi-Civita tensor $\epsilon^{\mu \nu \alpha \beta}$.

The twist-2 distribution amplitudes for the longitudinally and transversely polarized vector meson can be parameterized as:

\[
\phi_V(x) = \frac{3f_V}{\sqrt{2N_c}} x(1-x) \left[ 1 + 3a_{1V}^\parallel (2x-1) + a_{2V}^\parallel \frac{3}{2}(5(2x-1)^2 - 1) \right],
\]

\[
\phi_T^V(x) = \frac{3f_T^V}{\sqrt{2N_c}} x(1-x) \left[ 1 + 3a_{1V}^\perp (2x-1) + a_{2V}^\perp \frac{3}{2}(5(2x-1)^2 - 1) \right],
\]

Here $f_V$ and $f_T^V$ are the decay constants of the vector meson with longitudinal and transverse polarization, respectively.

The Gegenbauer moments have been studied extensively in the literatures [29, 30], here we adopt the following values from the recent updates [31–33]:

\[
a_{1K^*}^{\parallel} = 0.03 \pm 0.02, a_{2K^*}^{\parallel} = 0.11 \pm 0.09, a_{2\rho}^{\parallel} = a_{2\phi}^{\parallel} = 0.15 \pm 0.07, a_{2\omega}^{\parallel} = 0.18 \pm 0.08 ;
\]

\[
a_{1K^*}^{\perp} = 0.04 \pm 0.03, a_{2K^*}^{\perp} = 0.10 \pm 0.08, a_{2\rho}^{\perp} = a_{2\phi}^{\perp} = 0.14 \pm 0.06, a_{2\omega}^{\perp} = 0.14 \pm 0.07 .
\]

The asymptotic forms of the twist-3 distribution amplitudes $\phi_V^{t,s}$ and $\phi_V^{v,a}$ are [10]:

\[
\phi_V^t(x) = \frac{3f_V^T}{2\sqrt{2N_c}} (2x - 1)^2, \quad \phi_V^s(x) = -\frac{3f_V^T}{2\sqrt{2N_c}} (2x - 1),
\]

\[
\phi_V^v(x) = \frac{3f_V}{8\sqrt{2N_c}} (1 + (2x - 1)^2), \quad \phi_V^a(x) = -\frac{3f_V}{4\sqrt{2N_c}} (2x - 1).
\]

The twist-2 distribution amplitudes for the longitudinally and transversely polarized axial-vector $^3P_1$ and $^1P_1$ mesons can be parameterized as [10, 24]:

\[
\phi_A(x) = \frac{3f_A}{\sqrt{2N_c}} x(1-x) \left[ a_{0A}^\parallel + 3a_{1A}^\parallel (2x-1) + a_{2A}^\parallel \frac{3}{2}(5(2x-1)^2 - 1) \right],
\]

\[
\phi_T^A(x) = \frac{3f_A}{\sqrt{2N_c}} x(1-x) \left[ a_{0A}^\perp + 3a_{1A}^\perp (2x-1) + a_{2A}^\perp \frac{3}{2}(5(2x-1)^2 - 1) \right],
\]

Here, the definition of these distribution amplitudes $\phi_A(x)$ and $\phi_T^A(x)$ satisfy the following relations:

\[
\int_0^1 \phi_{0^3P_1}^A(x) = \frac{f_{0^3P_1}}{2\sqrt{2N_c}}, \quad \int_0^1 \phi_{0^1P_1}^A(x) = \frac{f_{0^1P_1}}{2\sqrt{2N_c}} ;
\]

\[
\int_0^1 \phi_{1^3P_1}^A(x) = \frac{f_{1^3P_1}}{2\sqrt{2N_c}}, \quad \int_0^1 \phi_{1^1P_1}^A(x) = \frac{f_{1^1P_1}}{2\sqrt{2N_c}}.
\]

where $a_{0^{3/1}P_1} = 1$ and $a_{0^{3/1}P_1}^\perp = 1$ have been used.

As for twist-3 distribution amplitudes for axial-vector meson, we use the following form [24]:

\[
\phi_A^t(x) = \frac{3f_A}{2\sqrt{2N_c}} \left\{ a_{0A}^\parallel (2x-1)^2 + \frac{1}{2} a_{1A}^\perp (2x-1)(3(2x-1)^2 - 1) \right\},
\]

\[
\phi_A^s(x) = \frac{3f_A}{2\sqrt{2N_c}} \frac{d}{dx} \left\{ x(1-x)(a_{0A}^\parallel + a_{1A}^\perp (2x-1)) \right\}.
\]
\[
\phi_A^+(x) = \frac{3f}{4\sqrt{2}N_c} \left\{ \frac{1}{2} a_{0A}^a (1 + (2x - 1)^2) + a_{1A}^a (2x - 1)^3 \right\},
\]
\[
\phi_A^0(x) = \frac{3f}{4\sqrt{2}N_c} \frac{d}{dx} \left\{ x(1 - x)(a_{0A}^a + a_{1A}^a (2x - 1)) \right\}.
\]

where \( f \) is the decay constant. It should be noted that in the above distribution amplitudes of strange axial-vector mesons \( K_{1A} \) and \( K_{1B} \), \( x \) stands for the momentum fraction carrying by the \( s \) quark.

The Gegenbauer moments have been studied extensively in the literatures (see Ref. [19] and references therein), here we adopt the following values:

\[
\begin{align*}
  a_{a_1}^a &= -0.02 \pm 0.02; & a_{1a_1}^a &= -1.04 \pm 0.34; & a_{1b_1}^a &= -1.95 \pm 0.35; \\
  a_{2f_1}^a &= -0.04 \pm 0.03; & a_{1f_1}^a &= -1.06 \pm 0.36; & a_{1h_1}^a &= -2.00 \pm 0.35; \\
  a_{2f_8}^a &= -0.07 \pm 0.04; & a_{1f_8}^a &= -1.11 \pm 0.31; & a_{1h_8}^a &= -1.95 \pm 0.35; \\
  a_{1K_{1A}}^a &= 0.00 \pm 0.26; & a_{2K_{1A}}^a &= -0.05 \pm 0.03; & a_{0K_{1A}}^a &= 0.08 \pm 0.09; \\
  a_{1K_{1A}}^a &= -1.08 \pm 0.48; & a_{0K_{1B}}^a &= 0.14 \pm 0.15; & a_{1K_{1B}}^a &= -1.95 \pm 0.45; \\
  a_{2K_{1B}}^a &= 0.02 \pm 0.10; & a_{1K_{1B}}^a &= 0.17 \pm 0.22.
\end{align*}
\]

(27)

D. Formalism of pQCD approach

Since the \( b \) quark is rather heavy, we work in the frame with the \( B_c \) meson at rest, i.e., with the \( B_c \) meson momentum \( P_1 = (m_{B_c}/\sqrt{2})(1, 1, 0_T) \) in the light-cone coordinates. For the non-leptonic charmless \( B_c \to M_2 M_3 \) decays, assuming that the \( M_2 (M_3) \) meson moves in the plus (minus) \( z \) direction carrying the momentum \( P_2 (P_3) \) and the polarization vector \( \epsilon_2 (\epsilon_3) \). Then the two final state meson momenta can be written as

\[
P_2 = \frac{m_{B_c}}{\sqrt{2}} (1 - r_3^2, r_2^2, 0_T), \quad P_3 = \frac{m_{B_c}}{\sqrt{2}} (r_3^2, 1 - r_2^2, 0_T),
\]

respectively, where \( r_2 = m_2/m_{B_c} \), \( r_3 = m_3/m_{B_c} \) with \( m_2 = m_{M_2} \) and \( m_3 = m_{M_3} \). The longitudinal polarization vectors, \( \epsilon_2^L \) and \( \epsilon_3^L \), can be given by

\[
\epsilon_2^L = \frac{m_{B_c}}{\sqrt{2}m_2} (1 - r_2^2, -r_2^2, 0_T), \quad \epsilon_3^L = \frac{m_{B_c}}{\sqrt{2}m_3} (-r_3^2, 1 - r_2^2, 0_T).
\]

(29)

And the transverse ones are parameterized as \( \epsilon_2^T = (0, 0, 1_T) \), and \( \epsilon_3^T = (0, 0, 1_T) \). Putting the (light) quark momenta in \( B_c, M_2 \) and \( M_3 \) mesons as \( k_1, k_2, \) and \( k_3 \), respectively, we can choose

\[
k_1 = (x_1 P_1^+, 0, k_{1T}), \quad k_2 = (x_2 P_2^+, 0, k_{2T}), \quad k_3 = (0, x_3 P_3^-, k_{3T}).
\]

(30)

Then, for \( B_c \to M_2 M_3 \) decays, the integration over \( k_{1T}, k_{2T} \), and \( k_{3T} \) will conceptually lead to the decay amplitudes in the pQCD approach,

\[
\mathcal{A}(B_c \to M_2 M_3) \sim \int dx_1 dx_2 dx_3 db_1 db_2 db_3 db_3 \\
\cdot \text{Tr} \left[ C(t) \Phi_{B_c}(x_1, b_1) \Phi_{M_2}(x_2, b_2) \Phi_{M_3}(x_3, b_3) H(x_i, b_i, t) S_i(x_i) e^{-S(t)} \right].
\]

(31)

\footnote{For the sake of simplicity, in the following, we will use \( M_2 \) and \( M_3 \) to denote the final state mesons respectively, unless otherwise stated.}
where $b_i$ is the conjugate space coordinate of $k_i T$, and $t$ is the largest energy scale in function $H(x_i, b_i, t)$. The large logarithms $\ln(m_W/t)$ are included in the Wilson coefficients $C(t)$. The large double logarithms ($\ln^2 x_i$) are summed by the threshold resummation [34], and they lead to $S_t(x_i)$ which smears the endpoint singularities on $x_i$. The last term, $e^{-S_t(t)}$, is the Sudakov form factor which suppresses the soft dynamics effectively [35]. Thus it makes the perturbative calculation of the hard part $H$ applicable at intermediate scale, i.e., $m_{Bc}$ scale. We will calculate analytically the function $H(x_i, b_i, t)$ for the considered decays at leading order in $\alpha_s$ expansion and give the convoluted amplitudes in next section.

III. PERTURBATIVE CALCULATIONS IN pQCD APPROACH

There are three kinds of polarizations of a vector or axial-vector meson, namely, longitudinal ($L$), normal ($N$), and transverse ($T$). Similar to the pure annihilation type $B_c \to V V$ decays [2], the amplitudes for a $B_c$ meson decaying into one vector and one axial-vector meson or two axial-vector mesons are also characterized by the polarization states of these vector and axial-vector mesons. In terms of helicities, the decay amplitudes $\mathcal{M}^{(\sigma)}$ for $B_c \to M_2(P_2, \epsilon^*_2) M_3(P_3, \epsilon^*_3)$ decays can be generally described by

$$\mathcal{M}^{(\sigma)} = \epsilon^*_2(\sigma) \epsilon^*_3(\sigma) \left[ a \, g^{\mu\nu} + \frac{b}{m_2 m_3} P_1^\mu P_1^\nu + i \frac{c}{m_2 m_3} \epsilon^{\mu\alpha\beta} P_2^\alpha P_3^\beta \right],$$

$$\equiv m_{Bc}^2 \mathcal{M}_L + m_{Bc}^2 \mathcal{M}_N \epsilon_2^*(\sigma = T) \cdot \epsilon_3^*(\sigma = T) + i M_T \epsilon_2^*(\sigma = T) \epsilon_3^*(\sigma = T) P_2^\mu P_3^\rho,$$

(32)

where the superscript $\sigma$ denotes the helicity states of one vector and one axial-vector meson or two axial-vector mesons with $L(T)$ standing for the longitudinal (transverse) component. And the definitions of the amplitudes $\mathcal{M}_i (i = L, N, T)$ in terms of the Lorentz-invariant amplitudes $a$, $b$ and $c$ are

$$m_{Bc}^2 \mathcal{M}_L = a \, \epsilon_2^*(L) \cdot \epsilon_3^*(L) + \frac{b}{m_2 m_3} \epsilon_2^*(L) \cdot \epsilon_3^*(L) \cdot P_2,$$

$$m_{Bc}^2 \mathcal{M}_N = a,$$

$$m_{Bc}^2 \mathcal{M}_T = \frac{c}{r_2 r_3}.$$

(33)

We therefore will evaluate the helicity amplitudes $\mathcal{M}_L, \mathcal{M}_N, \mathcal{M}_T$ based on the pQCD factorization approach, respectively.

In the following we will present analytically the factorization formulas for sixty two charmless hadronic $B_c \to AV/VA, AA$ decays. From the effective Hamiltonian (1), there are four types of diagrams contributing to these considered decays as illustrated in Fig. 1 with single $(V-A)(V-A)$ currents. From the first two diagrams (a) and (b) in Fig. 1 by perturbative QCD calculations,
we can obtain the Feynman decay amplitudes for factorizable annihilation contributions for $B_c \to AV, VA, AA$ as the following sequence,

\[
F^L_{fa}(AV) = -8\pi C_F m^2_{B_c} \left[ \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 b_4 \times \left\{ \right. \\
\times \left\{ \left[ x_2 \phi_A(x_2) \phi_V(x_3) - 2 r^2_3 r^2_3 \left( (x_2 + 1) \phi_A^*(x_2) + (x_2 - 1) \phi_A^*(x_2) \right) \phi_V^*(x_3) \right] \\
\phi_V^*(x_3) \right\} E_f a(t_a) h_f a(1 - x_2, x_2, b_3, b_2) + E_f a(t_b) h_f a(x_2, 1, x_2, b_2, b_3) \\
\times \left[ (x_2 - 1) \phi_A(x_2) \phi_V(x_3) - 2 r^2_3 r^2_3 \phi_A^*(x_2) \left( (x_2 - 1) \phi_V^*(x_3) - x_2 \phi_V^*(x_3) \right) \right] \right\} , \quad (34)
\]

\[
F^N_{fa}(AV) = -8\pi C_F m^2_{B_c} \left[ \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 b_4 r^2_3 r^2_3 \phi_V^*(x_3) \\
\times \left\{ h_f a(1 - x_2, x_2, b_3, b_2) E_f a(t_a) \left[ (x_2 + 1) \phi_A^*(x_2) \phi_V^*(x_3) + \phi_A(x_2) \phi_V(x_3) \right] \\
\times (x_2 - 1) \phi_A^*(x_2) \phi_V^*(x_3) \right\} E_f a(t_b) h_f a(x_2, 1, x_2, b_2, b_3) \\
\times \left[ (x_2 - 1) \phi_A(x_2) \phi_V(x_3) - 2 r^2_3 r^2_3 \phi_A^*(x_2) \left( (x_2 - 1) \phi_V^*(x_3) - x_2 \phi_V^*(x_3) \right) \right] \right\} , \quad (35)
\]

\[
F^T_{fa}(AV) = -8\pi C_F m^2_{B_c} \left[ \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 b_4 r^2_3 r^2_3 \phi_V^*(x_3) \\
\times \left\{ h_f a(1 - x_2, x_2, b_3, b_2) E_f a(t_a) \left[ (x_2 + 1) \phi_A^*(x_2) \phi_V^*(x_3) + \phi_A(x_2) \phi_V(x_3) \right] \\
\times (x_2 - 1) \phi_A^*(x_2) \phi_V^*(x_3) \right\} E_f a(t_b) h_f a(x_2, 1, x_2, b_2, b_3) \\
\times \left[ (x_2 - 1) \phi_V(x_2) \phi_A(x_3) + 2 r^2_3 r^2_3 \phi_V^*(x_2) \left( (x_2 - 1) \phi_A^*(x_3) - x_2 \phi_A^*(x_3) \right) \right] \right\} , \quad (36)
\]

\[
F^L_{fa}(VA) = -8\pi C_F m^2_{B_c} \left[ \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 b_4 \\
\times \left\{ x_2 \phi_V(x_2) \phi_A(x_3) + 2 r^2_3 r^2_3 \left( (x_2 + 1) \phi_V^*(x_2) + (x_2 - 1) \phi_V^*(x_2) \right) \phi_A^*(x_3) \\
\times \phi_A^*(x_3) \right\} E_f a(t_a) h_f a(1 - x_2, x_2, b_3, b_2) + E_f a(t_b) h_f a(x_2, 1, x_2, b_2, b_3) \\
\times \left[ (x_2 - 1) \phi_V(x_2) \phi_A(x_3) + 2 r^2_3 r^2_3 \phi_V^*(x_2) \left( (x_2 - 1) \phi_A^*(x_3) - x_2 \phi_A^*(x_3) \right) \right] \right\} , \quad (37)
\]

\[
F^N_{fa}(VA) = -8\pi C_F m^2_{B_c} \left[ \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 b_4 r^2_3 r^2_3 \phi_V^*(x_3) \\
\times \left\{ h_f a(1 - x_2, x_2, b_3, b_2) E_f a(t_a) \left[ (x_2 + 1) \phi_V^*(x_2) \phi_A^*(x_3) + \phi_V(x_2) \phi_A^*(x_3) \right] \\
\times (x_2 - 1) \phi_V^*(x_2) \phi_A^*(x_3) \right\} E_f a(t_b) h_f a(x_2, 1, x_2, b_2, b_3) \\
\times \left[ (x_2 - 1) \phi_V(x_2) \phi_A(x_3) + 2 r^2_3 r^2_3 \phi_V^*(x_2) \left( (x_2 - 1) \phi_A^*(x_3) - x_2 \phi_A^*(x_3) \right) \right] \right\} , \quad (38)
\]

\[
F^T_{fa}(VA) = -8\pi C_F m^2_{B_c} \left[ \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 b_4 r^2_3 r^2_3 \phi_V^*(x_3) \\
\times \left\{ h_f a(1 - x_2, x_2, b_3, b_2) E_f a(t_a) \left[ (x_2 + 1) \phi_V^*(x_2) \phi_A^*(x_3) + \phi_V(x_2) \phi_A^*(x_3) \right] \\
\times (x_2 - 1) \phi_V^*(x_2) \phi_A^*(x_3) \right\} E_f a(t_b) h_f a(x_2, 1, x_2, b_2, b_3) \\
\times \left[ (x_2 - 1) \phi_V(x_2) \phi_A(x_3) + 2 r^2_3 r^2_3 \phi_V^*(x_2) \left( (x_2 - 1) \phi_A^*(x_3) - x_2 \phi_A^*(x_3) \right) \right] \right\} , \quad (39)
\]
\[ F_{f_a}^{L}(AA) = 8\pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \]
\[ \times \left\{ (x_2 \phi_2(x_2) \phi_3(x_3)) + 2r_2^A r_3^A ((x_2 + 1) \phi_2^s(x_2) + (x_2 - 1) \phi_2^d(x_2)) \right. \]
\[ \left. \times \phi_3^1(x_3) E_{f_a}(t_a) h_{f_a}(1 - x_3, x_2, b_3, b_2) + E_{f_a}(t_b) h_{f_a}(x_2, 1 - x_3, b_2, b_3) \right. \]
\[ \left. \times [(x_3 - 1) \phi_2(x_2) \phi_3^0(x_3) + 2r_2^A r_3^A \phi_2^s(x_2) ((x_3 - 2) \phi_3^0(x_3) - x_3 \phi_3^1(x_3))] \right\}, \tag{40} \]
\[ F_{f_a}^{N}(AA) = 8\pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 r_2^A r_3^A \]
\[ \times \left\{ h_{f_a}(1 - x_3, x_2, b_3, b_2) E_{f_a}(t_a) [(x_2 + 1) \phi_2^0(x_2) \phi_3^0(x_3) + \phi_2^e(x_2) \phi_3^e(x_3)] \right. \]
\[ + (x_2 - 1) (\phi_2^0(x_2) \phi_3^0(x_3) + \phi_2^e(x_2) \phi_3^e(x_3))] \]
\[ \left. + [(x_3 - 2) (\phi_2^0(x_2) \phi_3^0(x_3) + \phi_2^e(x_2) \phi_3^e(x_3))] - x_3 (\phi_2^0(x_2) \phi_3^0(x_3) + \phi_2^e(x_2) \phi_3^e(x_3))] \right\} \]
\[ \times E_{f_a}(t_b) h_{f_a}(x_2, 1 - x_3, b_2, b_3) \}, \tag{41} \]
\[ F_{f_a}^{T}(AA) = 16\pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 r_2^A r_3^A \]
\[ \times \left\{ h_{f_a}(1 - x_3, x_2, b_3, b_2) E_{f_a}(t_a) [(x_2 + 1) (\phi_2^0(x_2) \phi_3^0(x_3) + \phi_2^e(x_2) \phi_3^e(x_3))] \right. \]
\[ + (x_2 - 1) (\phi_2^0(x_2) \phi_3^0(x_3) + \phi_2^e(x_2) \phi_3^e(x_3))] \]
\[ \left. + [(x_3 - 2) (\phi_2^0(x_2) \phi_3^0(x_3) + \phi_2^e(x_2) \phi_3^e(x_3))] - x_3 (\phi_2^0(x_2) \phi_3^0(x_3) + \phi_2^e(x_2) \phi_3^e(x_3))] \right\} \]
\[ \times E_{f_a}(t_b) h_{f_a}(x_2, 1 - x_3, b_2, b_3) \}, \tag{42} \]

where the superscripts \( V \) and \( A \) in the formulas express the types of mesons involved in the considered decays, the subscripts \( f_a \) and \( na \) (to be shown below) are the abbreviations of factorizable annihilation and nonfactorizable annihilation respectively, and \( C_F = 4/3 \) is a color factor. Moreover, the terms proportional to \( r_{2(3)}^2 \) can not change the results significantly and they have been neglected safely because the values of \( r_{2(3)}^2 \) are numerically small: \( r_{2(3)}^2 < 5\% \).

For the function \( h_{f_a} \), the scales \( t_i \), and \( E_{f_a}(t) \), we use the expressions as given in Appendix B of Ref. [2].

For the nonfactorizable diagrams (c) and (d) in Fig. 7 all three meson wave functions are involved. The integration of \( b_3 \) can be performed using \( \delta \) function \( \delta(b_3 - b_2) \), leaving only integration of \( b_1 \) and \( b_2 \). The corresponding decay amplitudes are

\[ M_{na}^{L}(AV) = -\frac{16\sqrt{6}}{3} \pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \]
\[ \times \left\{ E_{na}(t_c) [(r_c - x_3 + 1) \phi_A(x_2) \phi_V(x_3)) - r_2^A r_3^V (\phi_A^s(x_2))((3r_c + x_2 - x_3 + 1) \phi^V(x_3))] \right. \]
\[ \times \phi_1^V(x_3) - (r_c - x_2 - x_3 + 1) \phi_1^V(x_3)) + \phi_A^d(x_2)((r_c - x_2 - x_3 + 1) \phi_1^V(x_3)) \]
\[ + (r_c - x_2 + x_3 - 1) \phi_1^V(x_3)))] h_{na}^c(x_2, x_3, b_1, b_2) - h_{na}^d(x_2, x_3, b_1, b_2)E_{na}(t_d) \]
\[ \times [(r_b + r_c + x_2 - 1) \phi_A(x_2) \phi_V(x_3)) - r_2^A r_3^V (\phi_A^s(x_2))((4r_b + r_c + x_2 - x_3 - 1) \phi_1^V(x_3)) \]
\[ \times \phi_1^V(x_3) - (r_c + x_2 + x_3 - 1) \phi_1^V(x_3)) + \phi_A^d(x_2)((r_c + x_3 + 1) \phi_1^V(x_3)) - (r_c + x_2 - x_3 - 1) \phi_1^V(x_3))] \right\}, \tag{43} \]

\[ M_{na}^{N}(AV) = -\frac{32\sqrt{6}}{3} \pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 r_2^A r_3^V \]
\[ \times \left\{ r_c [\phi_A^0(x_2) \phi_V(x_3)) + \phi_A^e(x_2) \phi_1^V(x_3)) E_{na}(t_c) h_{na}^c(x_2, x_3, b_1, b_2) \right. \]
\[ - r_b [\phi_A^0(x_2) \phi_A^0(x_3)) + \phi_A^e(x_2) \phi_1^V(x_3)) E_{na}(t_d) h_{na}^d(x_2, x_3, b_1, b_2)] \right\}, \tag{44} \]
\[
M_{na}^L(AV) = -\frac{16\sqrt{6}}{3} \pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 dB_1 B_2 dB_2 r^A_2 r^A_3 V
\times \{ r_c [\phi^6_2(x_2)\phi^6_2(x_3) + \phi^6_2(x_2)\phi^6_2(x_3)] E_{na}(t_c) h^c_{na}(x_2, x_3, b_1, b_2)
- r_b [\phi^6_2(x_2)\phi^6_2(x_3) + \phi^6_2(x_2)\phi^6_2(x_3)] E_{na}(t_d) h^d_{na}(x_2, x_3, b_1, b_2) \} .
\]

\[
M_{na}^N(VA) = -\frac{32\sqrt{6}}{3} \pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 dB_1 B_2 dB_2 r^A_2 r^A_3
\times \{ r_c [\phi^A_2(x_2)\phi^A_2(x_3) + \phi^A_2(x_2)\phi^A_2(x_3)] E_{na}(t_c) h^c_{na}(x_2, x_3, b_1, b_2)
- r_b [\phi^A_2(x_2)\phi^A_2(x_3) + \phi^A_2(x_2)\phi^A_2(x_3)] E_{na}(t_d) h^d_{na}(x_2, x_3, b_1, b_2) \} .
\]

\[
M_{na}^T(VA) = -\frac{64\sqrt{6}}{3} \pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 dB_1 B_2 dB_2 r^A_2 r^A_3
\times \{ r_c [\phi^A_2(x_2)\phi^A_2(x_3) + \phi^A_2(x_2)\phi^A_2(x_3)] E_{na}(t_c) h^c_{na}(x_2, x_3, b_1, b_2)
- r_b [\phi^A_2(x_2)\phi^A_2(x_3) + \phi^A_2(x_2)\phi^A_2(x_3)] E_{na}(t_d) h^d_{na}(x_2, x_3, b_1, b_2) \} .
\]

\[
M_{na}^L(AA) = \frac{16\sqrt{6}}{3} \pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 dB_1 B_2 dB_2
\times \{ E_{na}(t_c) [(r_c - x_2 + 1)\phi_2(x_2)\phi_2(x_3) + r^A_2 r^A_3 (\phi_2^6(x_2)((3r_c + x_2 - x_3 + 1)
\times \phi_A^6(x_3) - (r_c - x_2 - x_3 + 1)\phi_A^6(x_3)))] E_{na}(t_c) h^c_{na}(x_2, x_3, b_1, b_2)
- E_{na}(t_d) h^d_{na}(x_2, x_3, b_1, b_2) \} .
\]

where $r_b = m_b/m_{B_c}$, $r_c = m_c/m_{B_c}$, and $r_b + r_c \approx 1$ for $B_c$ meson.
There are three kinds of polarizations in these $B_c \to VA, AA$ decays, namely, longitudinal ($L$), normal ($N$) and transverse ($T$). The decay amplitudes are classified accordingly, with $H = L, N, T$. From the effective Hamiltonian [1], based on Eqs. (34–31), we can combine all contributions to these considered decays and obtain the total decay amplitude generally as,

$$
\mathcal{M}^H(B_c \to M_2 M_3) = V_{cb} V_{ud} \left\{ f_{B_c} f_{M_2 M_3} a_1 + M_{M_2 M_3} C_1 \right\},
$$

where $a_1 = C_1/3 + C_2$. Then we can write down the total decay amplitudes for sixty two charmless two-body nonleptonic $B_c$ meson decays into final states involving one vector and one axial-vector meson ($VA$) or two axial-vector mesons($AA$) one by one.

1. $B_c \to VA/AV$ decay modes

   (i) For $\Delta S = 0$ processes,

   $$
   \sqrt{2} \mathcal{M}^H(B_c \to \rho^+ a_1^0) = V_{cb} V_{ud} \left\{ \left[ f_{B_c} F_{\rho a_1^0} + M_{\rho a_1^0 C_1} \right] \\
   - \left[ f_{B_c} F_{\rho a_1^0} + M_{\rho a_1^0 C_1} \right] \right\},
   $$

   (53)

   $$
   \sqrt{2} \mathcal{M}^H(B_c \to a_1^+ \rho^0) = -\sqrt{2} \mathcal{M}^H(B_c \to a_1^+ \rho^0) = V_{cb} V_{ud} \left\{ \left[ f_{B_c} F_{a_1^+ \rho^0} + M_{a_1^+ \rho^0 C_1} \right] \\
   - \left[ f_{B_c} F_{a_1^+ \rho^0} + M_{a_1^+ \rho^0 C_1} \right] \right\},
   $$

   (54)

   $$
   \sqrt{2} \mathcal{M}^H(B_c \to a_1^+ \omega) = V_{cb} V_{ud} \left\{ \left[ f_{B_c} F_{a_1^+ \omega} + M_{a_1^+ \omega C_1} \right] \\
   + \left[ f_{B_c} F_{a_1^+ \omega} + M_{a_1^+ \omega C_1} \right] \right\},
   $$

   (55)

   $$
   \sqrt{2} \mathcal{M}^H(B_c \to \rho^+ b_1^0) = V_{cb} V_{ud} \left\{ \left[ f_{B_c} F_{\rho b_1^0} + M_{\rho b_1^0 C_1} \right] \\
   - \left[ f_{B_c} F_{\rho b_1^0} + M_{\rho b_1^0 C_1} \right] \right\},
   $$

   (56)

   $$
   \sqrt{2} \mathcal{M}^H(B_c \to b_1^+ \rho^0) = -\sqrt{2} \mathcal{M}^H(B_c \to b_1^+ \rho^0) = V_{cb} V_{ud} \left\{ \left[ f_{B_c} F_{b_1^+ \rho^0} + M_{b_1^+ \rho^0 C_1} \right] \\
   - \left[ f_{B_c} F_{b_1^+ \rho^0} + M_{b_1^+ \rho^0 C_1} \right] \right\},
   $$

   (57)

   $$
   \sqrt{2} \mathcal{M}^H(B_c \to b_1^+ \omega) = V_{cb} V_{ud} \left\{ \left[ f_{B_c} F_{b_1^+ \omega} + M_{b_1^+ \omega C_1} \right] \\
   + \left[ f_{B_c} F_{b_1^+ \omega} + M_{b_1^+ \omega C_1} \right] \right\},
   $$

   (58)

   $$
   \mathcal{M}^H(B_c \to \rho^+ f') = V_{cb} V_{ud} \left\{ \frac{\cos \theta_3}{\sqrt{3}} \left[ f_{B_c} (F_{\rho f'_1} + F_{\rho f'_2}) a_1 \\
   + (M_{\rho f'_1} + M_{\rho f'_2}) C_1 \right] - \frac{\sin \theta_3}{\sqrt{6}} \left[ f_{B_c} \\
   (F_{\rho f'_{g_1}} + F_{\rho f'_{g_2}}) a_1 + (M_{\rho f'_{g_1}} + M_{\rho f'_{g_2}}) C_1 \right] \right\},
   $$

   (59)
\[ \mathcal{M}^H(B_c \to \rho^+ f'') = V_{cb}^* V_{ud} \left\{ -\sin \theta_3 \left[ f_{Bc}(F_{fa;H}^{\rho f_1} + F_{fa;H}^{f_2}) a_1 \right. \right. \\
+ (M_{fa;H}^{\rho f_1} + M_{fa;H}^{f_2}) C_1 \left. \right] + \frac{\cos \theta_3}{\sqrt{6}} \left. \left[ f_{Bc} \right. \right. \\
\left. \left. \cdot (F_{fa;H}^{\rho f_2} + F_{fa;H}^{f_2}) a_1 + (M_{fa;H}^{\rho f_2} + M_{fa;H}^{f_2}) C_1 \right] \right\}, \]  

(60)

\[ \mathcal{M}^H(B_c \to \rho^+ h') = V_{cb}^* V_{ud} \left\{ \cos \theta_1 \left[ f_{Bc}(F_{fa;H}^{\rho h_1} + F_{fa;H}^{h_2}) a_1 \right. \right. \\
+ (M_{fa;H}^{\rho h_1} + M_{fa;H}^{h_2}) C_1 \left. \right] + \frac{\sin \theta_1}{\sqrt{6}} \left. \left[ f_{Bc} \right. \right. \\
\left. \left. \cdot (F_{fa;H}^{\rho h_2} + F_{fa;H}^{h_2}) a_1 + (M_{fa;H}^{\rho h_2} + M_{fa;H}^{h_2}) C_1 \right] \right\}, \]  

(61)

\[ \mathcal{M}^H(B_c \to \rho^+ h'') = V_{cb}^* V_{ud} \left\{ -\sin \theta_1 \left[ f_{Bc}(F_{fa;H}^{\rho h_1} + F_{fa;H}^{h_2}) a_1 \right. \right. \\
+ (M_{fa;H}^{\rho h_1} + M_{fa;H}^{h_2}) C_1 \left. \right] + \frac{\cos \theta_1}{\sqrt{6}} \left. \left[ f_{Bc} \right. \right. \\
\left. \left. \cdot (F_{fa;H}^{\rho h_2} + F_{fa;H}^{h_2}) a_1 + (M_{fa;H}^{\rho h_2} + M_{fa;H}^{h_2}) C_1 \right] \right\}, \]  

(62)

\[ \mathcal{M}^H(B_c \to K^{*0} K'^+) = V_{cb}^* V_{ud} \left\{ \sin \theta_K \left[ f_{Bc}(F_{fa;H}^{K^{*0} K'^+}) a_1 + M_{fa;H}^{K^{*0} K'^+} C_1 \right] \right. \\
+ \cos \theta_K \left. \left[ f_{Bc}(F_{fa;H}^{K^{*0} K'^+}) a_1 + M_{fa;H}^{K^{*0} K'^+} C_1 \right] \right\}, \]  

(63)

\[ \mathcal{M}^H(B_c \to K^{*0} K'^+) = V_{cb}^* V_{ud} \left\{ \cos \theta_K \left[ f_{Bc}(F_{fa;H}^{K^{*0} K'^+}) a_1 + M_{fa;H}^{K^{*0} K'^+} C_1 \right] \right. \\
- \sin \theta_K \left. \left[ f_{Bc}(F_{fa;H}^{K^{*0} K'^+}) a_1 + M_{fa;H}^{K^{*0} K'^+} C_1 \right] \right\}, \]  

(64)

\[ \mathcal{M}^H(B_c \to K^{0} K'^+) = V_{cb}^* V_{ud} \left\{ \sin \theta_K \left[ f_{Bc}(F_{fa;H}^{K^{0} K'^+}) a_1 + M_{fa;H}^{K^{0} K'^+} C_1 \right] \right. \\
+ \cos \theta_K \left. \left[ f_{Bc}(F_{fa;H}^{K^{0} K'^+}) a_1 + M_{fa;H}^{K^{0} K'^+} C_1 \right] \right\}, \]  

(65)

\[ \mathcal{M}^H(B_c \to K^{0} K'^+) = V_{cb}^* V_{ud} \left\{ \cos \theta_K \left[ f_{Bc}(F_{fa;H}^{K^{0} K'^+}) a_1 + M_{fa;H}^{K^{0} K'^+} C_1 \right] \right. \\
- \sin \theta_K \left. \left[ f_{Bc}(F_{fa;H}^{K^{0} K'^+}) a_1 + M_{fa;H}^{K^{0} K'^+} C_1 \right] \right\}. \]  

(66)

(ii) For \( \Delta S = 1 \) processes,

\[ \mathcal{M}^H(B_c \to K^{*0} a^+_1) = \sqrt{2} \mathcal{M}^H(B_c \to K^{*+} a^+_1) \]

\[ = V_{cb}^* V_{us} \left\{ f_{Bc}(F_{fa;H}^{K^{*0} a^+_1}) a_1 + M_{fa;H}^{K^{*0} a^+_1} C_1 \right\}, \]  

(67)

\[ \mathcal{M}^H(B_c \to K^{*0} b^+_1) = \sqrt{2} \mathcal{M}^H(B_c \to K^{*+} b^+_1) \]

\[ = V_{cb}^* V_{us} \left\{ f_{Bc}(F_{fa;H}^{K^{*0} b^+_1}) a_1 + M_{fa;H}^{K^{*0} b^+_1} C_1 \right\}, \]  

(68)
\[ \mathcal{M}^H(B_c \to K^{*0} \rho^+) = \sqrt{2}\mathcal{M}^H(B_c \to K^{+} \rho^0) \]
\[ = V^* c V_u \left\{ \sin \theta_K \left[ f_{B_c} F_{f_a;H}^{K_1^0\rho^0} a_1 + M_{na;H}^{K_1^0\rho^0} C_1 \right] \\
+ \cos \theta_K \left[ f_{B_c} F_{f_a;H}^{K_1^0\rho^0} a_1 + M_{na;H}^{K_1^0\rho^0} C_1 \right] \right\}, \tag{69} \]

\[ \mathcal{M}^H(B_c \to K^{*0} \rho^+) = \sqrt{2}\mathcal{M}^H(B_c \to K^{*0} \rho^0) \]
\[ = V^* c V_u \left\{ \cos \theta_K \left[ f_{B_c} F_{f_a;H}^{K_1^0\rho^0} a_1 + M_{na;H}^{K_1^0\rho^0} C_1 \right] \\
- \sin \theta_K \left[ f_{B_c} F_{f_a;H}^{K_1^0\rho^0} a_1 + M_{na;H}^{K_1^0\rho^0} C_1 \right] \right\}, \tag{70} \]

\[ \sqrt{2}\mathcal{M}^H(B_c \to K^+ \omega) = V^* c V_u \left\{ \sin \theta_K \left[ f_{B_c} F_{f_a;H}^{K_1^0\omega^0} a_1 + M_{na;H}^{K_1^0\omega^0} C_1 \right] \\
+ \cos \theta_K \left[ f_{B_c} F_{f_a;H}^{K_1^0\omega^0} a_1 + M_{na;H}^{K_1^0\omega^0} C_1 \right] \right\}, \tag{71} \]

\[ \sqrt{2}\mathcal{M}^H(B_c \to K^{*+} \omega) = V^* c V_u \left\{ \cos \theta_K \left[ f_{B_c} F_{f_a;H}^{K_1^0\omega^0} a_1 + M_{na;H}^{K_1^0\omega^0} C_1 \right] \\
- \sin \theta_K \left[ f_{B_c} F_{f_a;H}^{K_1^0\omega^0} a_1 + M_{na;H}^{K_1^0\omega^0} C_1 \right] \right\}, \tag{72} \]

\[ \mathcal{M}^H(B_c \to K^{*+} f') = V^* c V_u \left\{ \cos \theta_3 \left[ f_{B_c} (F_{f_a;H}^{K^{*+}f''}) a_1 \\
+ (M_{na;H}^{K^{*+}f''} + M_{na;H}^{f''}) C_1 \right] \right\}, \tag{73} \]

\[ \mathcal{M}^H(B_c \to K^{*+} f'') = V^* c V_u \left\{ \cos \theta_3 \left[ f_{B_c} (F_{f_a;H}^{K^{*+}f''}) a_1 \\
+ (M_{na;H}^{K^{*+}f''} + M_{na;H}^{f''}) C_1 \right] \right\}, \tag{74} \]

\[ \mathcal{M}^H(B_c \to K^{*+} h') = V^* c V_u \left\{ \cos \theta_1 \left[ f_{B_c} (F_{f_a;H}^{K^{*+}h''}) a_1 \\
+ (M_{na;H}^{K^{*+}h''} + M_{na;H}^{h''}) C_1 \right] \right\}, \tag{75} \]

\[ \mathcal{M}^H(B_c \to K^{*+} h'') = V^* c V_u \left\{ \cos \theta_1 \left[ f_{B_c} (F_{f_a;H}^{K^{*+}h''}) a_1 \\
+ (M_{na;H}^{K^{*+}h''} + M_{na;H}^{h''}) C_1 \right] \right\}, \tag{76} \]
\[ M^H(B_c \to \phi K^{'+}) = V_{cb}^* V_{us} \left\{ \sin \theta_K \left[ f_{B_c} F_{fa:H}^{\phi K^0} a_1 + M_{n:a:H}^{\phi K^0} C_1 \right] \\
+ \cos \theta_K \left[ f_{B_c} F_{fa:H}^{\phi K^0} a_1 + M_{n:a:H}^{\phi K^0} C_1 \right] \right\}, \]  
\[ M^H(B_c \to \phi K^{''+}) = V_{cb}^* V_{us} \left\{ \cos \theta_K \left[ f_{B_c} F_{fa:H}^{\phi K^0} a_1 + M_{n:a:H}^{\phi K^0} C_1 \right] \\
- \sin \theta_K \left[ f_{B_c} F_{fa:H}^{\phi K^0} a_1 + M_{n:a:H}^{\phi K^0} C_1 \right] \right\}. \]  

2. \( B_c \to AA \) decay modes

(i) For \( \Delta S = 0 \) processes,

\[ \sqrt{2} M^H(B_c \to a_1^+ a_1^0) = V_{cb}^* V_{ud} \left\{ f_{B_c} (F_{fa:H}^{a_1^0 a_1^0} - F_{fa:H}^{a_1^0 a_1^+}) a_1 \\
+(M_{n:a:H}^{a_1^0 a_1^0} - M_{n:a:H}^{a_1^0 a_1^+}) C_1 \right\}, \]  
\[ \sqrt{2} M^H(B_c \to b_1^+ b_1^0) = V_{cb}^* V_{ud} \left\{ f_{B_c} (F_{fa:H}^{b_1^0 b_1^0} - F_{fa:H}^{b_1^0 b_1^+}) a_1 \\
+(M_{n:a:H}^{b_1^0 b_1^0} - M_{n:a:H}^{b_1^0 b_1^+}) C_1 \right\}, \]  
\[ \sqrt{2} M^H(B_c \to a_1^+ b_1^0) = V_{cb}^* V_{ud} \left\{ f_{B_c} (F_{fa:H}^{a_1^0 b_1^0} - F_{fa:H}^{a_1^0 b_1^+}) a_1 \\
+(M_{n:a:H}^{a_1^0 b_1^0} - M_{n:a:H}^{a_1^0 b_1^+}) C_1 \right\}, \]  
\[ \sqrt{2} M^H(B_c \to b_1^+ a_1^0) = V_{cb}^* V_{ud} \left\{ f_{B_c} (F_{fa:H}^{b_1^0 a_1^0} - F_{fa:H}^{b_1^0 a_1^+}) a_1 \\
+(M_{n:a:H}^{b_1^0 a_1^0} - M_{n:a:H}^{b_1^0 a_1^+}) C_1 \right\}. \]  

\[ M^H(B_c \to a_1^+ f^0) = V_{cb}^* V_{ud} \left\{ \frac{\cos \theta_3}{\sqrt{3}} [f_{B_c}(F_{fa:H}^{a_1^+ f_1^0} + F_{fa:H}^{f_1^0 a_1^+}) a_1 \\
+(M_{n:a:H}^{a_1^+ f_1^0} + M_{n:a:H}^{f_1^0 a_1^+} C_1] + \frac{\sin \theta_3}{\sqrt{6}} [f_{B_c} \\
\cdot (F_{fa:H}^{a_1^+ f_1^0} + F_{fa:H}^{f_1^0 a_1^+}) a_1 + (M_{n:a:H}^{a_1^+ f_1^0} + M_{n:a:H}^{f_1^0 a_1^+} C_1] \right\}, \]  
\[ M^H(B_c \to a_1^+ f^{''0}) = V_{cb}^* V_{ud} \left\{ \frac{-\sin \theta_3}{\sqrt{3}} [f_{B_c}(F_{fa:H}^{a_1^+ f_1^0} + F_{fa:H}^{f_1^0 a_1^+}) a_1 \\
+(M_{n:a:H}^{a_1^+ f_1^0} + M_{n:a:H}^{f_1^0 a_1^+} C_1] + \frac{\cos \theta_3}{\sqrt{6}} [f_{B_c} \\
\cdot (F_{fa:H}^{a_1^+ f_1^0} + F_{fa:H}^{f_1^0 a_1^+}) a_1 + (M_{n:a:H}^{a_1^+ f_1^0} + M_{n:a:H}^{f_1^0 a_1^+} C_1] \right\}. \]
\[ \mathcal{M}^H(B_c \rightarrow b_1^+ f') = V_{cb} V_{ud} \left\{ \frac{\cos \theta_3}{\sqrt{3}} [f_{B_c}(F^{b_1^+ f_1^u}_{fa:H} + F^{b_1^+ f_2^u}_{fa:H}) a_1 \\
+ (M^{b_1^+ f_1^u}_{na:H} + M^{b_1^+ f_2^u}_{na:H}) C_1] + \frac{\sin \theta_3}{\sqrt{6}} [f_{B_c} \\
\cdot (F^{b_1^+ f_1^u}_{fa:H} + F^{f_1^u b_1^+}_{fa:H}) a_1 + (M^{b_1^+ f_1^u}_{na:H} + M^{b_1^+ f_2^u}_{na:H}) C_1] \right\} , \] (85)

\[ \mathcal{M}^H(B_c \rightarrow b_1^+ f'') = V_{cb} V_{ud} \left\{ \frac{-\sin \theta_3}{\sqrt{3}} [f_{B_c}(F^{b_1^+ f_1^u}_{fa:H} + F^{b_1^+ f_2^u}_{fa:H}) a_1 \\
+ (M^{b_1^+ f_1^u}_{na:H} + M^{b_1^+ f_2^u}_{na:H}) C_1] + \frac{\cos \theta_3}{\sqrt{6}} [f_{B_c} \\
\cdot (F^{b_1^+ f_1^u}_{fa:H} + F^{f_1^u b_1^+}_{fa:H}) a_1 + (M^{b_1^+ f_1^u}_{na:H} + M^{b_1^+ f_2^u}_{na:H}) C_1] \right\} , \] (86)

\[ \mathcal{M}^H(B_c \rightarrow a_1^+ h') = V_{cb} V_{ud} \left\{ \frac{\cos \theta_3}{\sqrt{3}} [f_{B_c}(F^{a_1^+ h_1^u}_{fa:H} + F^{h_1^u a_1^+}_{fa:H}) a_1 \\
+ (M^{a_1^+ h_1^u}_{na:H} + M^{h_1^u a_1^+}_{na:H}) C_1] + \frac{\sin \theta_3}{\sqrt{6}} [f_{B_c} \\
\cdot (F^{a_1^+ h_1^u}_{fa:H} + F^{h_1^u a_1^+}_{fa:H}) a_1 + (M^{a_1^+ h_1^u}_{na:H} + M^{h_1^u a_1^+}_{na:H}) C_1] \right\} , \] (87)

\[ \mathcal{M}^H(B_c \rightarrow a_1^+ h'') = V_{cb} V_{ud} \left\{ \frac{-\sin \theta_1}{\sqrt{3}} [f_{B_c}(F^{a_1^+ h_1^u}_{fa:H} + F^{h_1^u a_1^+}_{fa:H}) a_1 \\
+ (M^{a_1^+ h_1^u}_{na:H} + M^{h_1^u a_1^+}_{na:H}) C_1] + \frac{\cos \theta_1}{\sqrt{6}} [f_{B_c} \\
\cdot (F^{a_1^+ h_1^u}_{fa:H} + F^{h_1^u a_1^+}_{fa:H}) a_1 + (M^{a_1^+ h_1^u}_{na:H} + M^{h_1^u a_1^+}_{na:H}) C_1] \right\} , \] (88)

\[ \mathcal{M}^H(B_c \rightarrow b_1^+ h') = V_{cb} V_{ud} \left\{ \frac{\cos \theta_1}{\sqrt{3}} [f_{B_c}(F^{b_1^+ h_1^u}_{fa:H} + F^{h_1^u b_1^+}_{fa:H}) a_1 \\
+ (M^{h_1^u b_1^+}_{na:H} + M^{b_1^+ h_1^u}_{na:H}) C_1] + \frac{\sin \theta_1}{\sqrt{6}} [f_{B_c} \\
\cdot (F^{b_1^+ h_1^u}_{fa:H} + F^{h_1^u b_1^+}_{fa:H}) a_1 + (M^{h_1^u b_1^+}_{na:H} + M^{b_1^+ h_1^u}_{na:H}) C_1] \right\} , \] (89)

\[ \mathcal{M}^H(B_c \rightarrow b_1^+ h'') = V_{cb} V_{ud} \left\{ \frac{-\sin \theta_1}{\sqrt{3}} [f_{B_c}(F^{b_1^+ h_1^u}_{fa:H} + F^{h_1^u b_1^+}_{fa:H}) a_1 \\
+ (M^{h_1^u b_1^+}_{na:H} + M^{b_1^+ h_1^u}_{na:H}) C_1] + \frac{\cos \theta_1}{\sqrt{6}} [f_{B_c} \\
\cdot (F^{b_1^+ h_1^u}_{fa:H} + F^{h_1^u b_1^+}_{fa:H}) a_1 + (M^{h_1^u b_1^+}_{na:H} + M^{b_1^+ h_1^u}_{na:H}) C_1] \right\} , \] (90)
\[ \mathcal{M}^H(\bar{B}_c \to K^0 K^{+}) = V_{cb}^* V_{ud} \left\{ -\sin^2 \theta_K (f_{B_c} F_{fa;H}^{K_{1A}^0} K_{1A} a_1 + M_{na;H}^{K_{1A}} K_{1A} C_1) \\
+ \cos \theta_K \sin \theta_K (f_{B_c} F_{fa;H}^{K_{1A}^0} K_{1B} a_1 + M_{na;H}^{K_{1A}} K_{1B} C_1) \\
+ \cos \theta_K \sin \theta_K (f_{B_c} F_{fa;H}^{K_{1B}^0} K_{1A} a_1 + M_{na;H}^{K_{1B}} K_{1A} C_1) \\
+ \cos^2 \theta_K (f_{B_c} F_{fa;H}^{K_{1B}^0} K_{1B} a_1 + M_{na;H}^{K_{1B}} K_{1B} C_1) \right\}, \] (91)

\[ \mathcal{M}^H(\bar{B}_c \to K^{+} K^{+}) = V_{cb}^* V_{ud} \left\{ -\cos \theta_K \sin \theta_K (f_{B_c} F_{fa;H}^{K_{1A}^0} K_{1A} a_1 + M_{na;H}^{K_{1A}} K_{1A} C_1) \\
+ \sin^2 \theta_K (f_{B_c} F_{fa;H}^{K_{1A}^0} K_{1B} a_1 + M_{na;H}^{K_{1A}} K_{1B} C_1) \\
+ \cos^2 \theta_K (f_{B_c} F_{fa;H}^{K_{1B}^0} K_{1A} a_1 + M_{na;H}^{K_{1B}} K_{1A} C_1) \\
- \cos \theta_K \sin \theta_K (f_{B_c} F_{fa;H}^{K_{1B}^0} K_{1B} a_1 + M_{na;H}^{K_{1B}} K_{1B} C_1) \right\}, \] (92)

\[ \mathcal{M}^H(\bar{B}_c \to K^{0} K^{+}) = V_{cb}^* V_{ud} \left\{ \cos \theta_K \sin \theta_K (f_{B_c} F_{fa;H}^{K_{1A}^0} K_{1A} a_1 + M_{na;H}^{K_{1A}} K_{1A} C_1) \\
+ \cos \theta_K \sin \theta_K (f_{B_c} F_{fa;H}^{K_{1A}^0} K_{1B} a_1 + M_{na;H}^{K_{1A}} K_{1B} C_1) \\
+ \sin \theta_K (f_{B_c} F_{fa;H}^{K_{1B}^0} K_{1A} a_1 + M_{na;H}^{K_{1B}} K_{1A} C_1) \\
+ \cos \theta_K \sin \theta_K (f_{B_c} F_{fa;H}^{K_{1B}^0} K_{1B} a_1 + M_{na;H}^{K_{1B}} K_{1B} C_1) \right\}, \] (93)

\[ \mathcal{M}^H(\bar{B}_c \to K^{0} K^{0}) = V_{cb}^* V_{ud} \left\{ \cos^2 \theta_K (f_{B_c} F_{fa;H}^{K_{1A}^0} K_{1A} a_1 + M_{na;H}^{K_{1A}} K_{1A} C_1) \\
- \cos \theta_K \sin \theta_K (f_{B_c} F_{fa;H}^{K_{1A}^0} K_{1B} a_1 + M_{na;H}^{K_{1A}} K_{1B} C_1) \\
+ \cos \theta_K \sin \theta_K (f_{B_c} F_{fa;H}^{K_{1B}^0} K_{1A} a_1 + M_{na;H}^{K_{1B}} K_{1A} C_1) \\
- \sin^2 \theta_K (f_{B_c} F_{fa;H}^{K_{1B}^0} K_{1B} a_1 + M_{na;H}^{K_{1B}} K_{1B} C_1) \right\}. \] (94)

(ii) For \( \Delta S = 1 \) processes,

\[ \mathcal{M}^H(\bar{B}_c \to K^{0} a_1^+ \bar{a}_1^+) = \sqrt{2} \mathcal{M}^H(\bar{B}_c \to K^{+} a_1^0) \]

\[ = V_{cb}^* V_{us} \left\{ \sin \theta_K [f_{B_c} F_{fa;H}^{K_{1A}^0} a_1 + M_{na;H}^{K_{1A}^0} a_1^+] \\
+ \cos \theta_K [f_{B_c} F_{fa;H}^{K_{1B}^0} a_1 + M_{na;H}^{K_{1B}^0} a_1^+] \right\}, \] (95)

\[ \mathcal{M}^H(\bar{B}_c \to K^{0} a_1^+) = \sqrt{2} \mathcal{M}^H(\bar{B}_c \to K^{+} a_1^0) \]

\[ = V_{cb}^* V_{us} \left\{ \cos \theta_K [f_{B_c} F_{fa;H}^{K_{1A}^0} a_1 + M_{na;H}^{K_{1A}^0} a_1^+] \\
- \sin \theta_K [f_{B_c} F_{fa;H}^{K_{1B}^0} a_1 + M_{na;H}^{K_{1B}^0} a_1^+] \right\}, \] (96)
\[ \mathcal{M}^H(B_c \to K'^0b_1^+) = \sqrt{2}\mathcal{M}^H(B_c \to K'^+b_1^0) \]
\[ = V_{cb}^*V_{us}\left\{ \sin\theta_K[f_{B_c}F_{fa;H}^{K'^0_{1A}b_1^+}a_1 + M_{na;H}^{K_{1A}b_1^+}C_1] \\
+ \cos\theta_K[f_{B_c}F_{fa;H}^{K'^0_{1B}b_1^+}a_1 + M_{na;H}^{K_{1B}b_1^+}C_1] \right\}, \quad (97) \]

\[ \mathcal{M}^H(B_c \to K'^+b_1^+) = \sqrt{2}\mathcal{M}^H(B_c \to K'^+b_1^0) \]
\[ = V_{cb}^*V_{us}\left\{ \cos\theta_K[f_{B_c}F_{fa;H}^{K'^0_{1A}b_1^+}a_1 + M_{na;H}^{K_{1A}b_1^+}C_1] \\
- \sin\theta_K[f_{B_c}F_{fa;H}^{K'^0_{1B}b_1^+}a_1 + M_{na;H}^{K_{1B}b_1^+}C_1] \right\}, \quad (98) \]

\[ \mathcal{M}^H(B_c \to K'^+f') = V_{cb}^*V_{us}\left\{ \cos\frac{\theta_3}{\sqrt{3}}\frac{\sin\theta_K}{\sqrt{6}}[f_{B_c}(F_{fa;H}^{K_{1A}f_{1A}^u} + F_{fa;H}^{f_{1A}K_{1A}})a_1 \\
+ (M_{na;H}^{K_{1A}f_{1A}^u} + M_{na;H}^{f_{1A}K_{1A}})C_1] + \frac{\sin\theta_3}{\sqrt{6}}\frac{\sin\theta_K}{f_{B_c}}[f_{B_c} \\
\cdot (F_{fa;H}^{K_{1A}f_{1A}^u} - 2F_{fa;H}^{f_{1A}K_{1A}})a_1 + (M_{na;H}^{K_{1A}f_{1A}^u} - 2M_{na;H}^{f_{1A}K_{1A}})C_1] + \cos\theta_K(\sin\frac{\theta_3}{\sqrt{6}}[f_{B_c} \\
\cdot (F_{fa;H}^{K_{1B}f_{1B}^u} + F_{fa;H}^{f_{1B}K_{1B}})a_1 + (M_{na;H}^{K_{1B}f_{1B}^u} + M_{na;H}^{f_{1B}K_{1B}})C_1] \right\}, \quad (99) \]

\[ \mathcal{M}^H(B_c \to K'^+f'') = V_{cb}^*V_{us}\left\{ -\sin\frac{\theta_3}{\sqrt{3}}\frac{\sin\theta_K}{\sqrt{6}}[f_{B_c}(F_{fa;H}^{K_{1A}f_{1A}^u} + F_{fa;H}^{f_{1A}K_{1A}})a_1 \\
+ (M_{na;H}^{K_{1A}f_{1A}^u} + M_{na;H}^{f_{1A}K_{1A}})C_1] + \cos\frac{\theta_3}{\sqrt{6}}\frac{\sin\theta_K}{f_{B_c}}[f_{B_c} \\
\cdot (F_{fa;H}^{K_{1A}f_{1A}^u} - 2F_{fa;H}^{f_{1A}K_{1A}})a_1 + (M_{na;H}^{K_{1A}f_{1A}^u} - 2M_{na;H}^{f_{1A}K_{1A}})C_1] - \cos\theta_K(\sin\frac{\theta_3}{\sqrt{6}}[f_{B_c} \\
\cdot (F_{fa;H}^{K_{1B}f_{1B}^u} + F_{fa;H}^{f_{1B}K_{1B}})a_1 + (M_{na;H}^{K_{1B}f_{1B}^u} + M_{na;H}^{f_{1B}K_{1B}})C_1] \right\}, \quad (100) \]
\[ M^H(B_c \rightarrow K'' f') = V_{cb} V_{us} \left\{ \frac{\cos \theta_3 \cos \theta_K}{\sqrt{3}} [f_{B_c}(F_{fa;H}^{K_1 A f} + F_{fa;H}^{f K_1 A}) a_1 + (M_{k_1 A f}^{K_1 A} + M_{k_1 A f}^{f K_1 A}) C_1] + \frac{\cos \theta_K \sin \theta_3}{\sqrt{6}} [f_{B_c}(F_{fa;H}^{K_1 A f} + F_{fa;H}^{f K_1 A}) a_1 - (M_{k_1 A f}^{K_1 A} + M_{k_1 A f}^{f K_1 A}) C_1] \right\}, \quad (101) \]

\[ M^H(B_c \rightarrow K'' f'') = V_{cb} V_{us} \left\{ -\frac{\cos \theta_K \sin \theta_3}{\sqrt{3}} [f_{B_c}(F_{fa;H}^{K_1 A f} + F_{fa;H}^{f K_1 A}) a_1 + (M_{k_1 A f}^{K_1 A} + M_{k_1 A f}^{f K_1 A}) C_1] + \frac{\cos \theta_3 \cos \theta_K}{\sqrt{6}} [f_{B_c}(F_{fa;H}^{K_1 A f} + F_{fa;H}^{f K_1 A}) a_1 - (M_{k_1 A f}^{K_1 A} + M_{k_1 A f}^{f K_1 A}) C_1] \right\}, \quad (102) \]

\[ M^H(B_c \rightarrow K''+ h') = V_{cb} V_{us} \left\{ \frac{\cos \theta_3 \sin \theta_K}{\sqrt{3}} [f_{B_c}(F_{fa;H}^{K_1 A h} + F_{fa;H}^{h K_1 A}) a_1 + (M_{k_1 A h}^{K_1 A} + M_{k_1 A h}^{h K_1 A}) C_1] + \frac{\sin \theta_3 \sin \theta_K}{\sqrt{6}} [f_{B_c}(F_{fa;H}^{K_1 A h} + F_{fa;H}^{h K_1 A}) a_1 - (M_{k_1 A h}^{K_1 A} + M_{k_1 A h}^{h K_1 A}) C_1] \right\}, \quad (103) \]
\[ \mathcal{M}^H(B_c \to K^{'+} h'') = V_{cb} V_{us} \left\{ \frac{-\sin \theta_3 \sin \theta_K}{\sqrt{3}} [f_B (F_{fa:H}^{K_1A h''_1} + F_{fa:H}^{h_2 K_1A}) a_1 \\
+ (M_{na:H}^{K_1A h''_1} + M_{na:H}^{h_2 K_1A}) C_1] + \frac{\cos \theta_3 \sin \theta_K}{\sqrt{6}} [f_B (F_{fa:H}^{K_1B h''_1} + F_{fa:H}^{h_2 K_1B}) a_1 \\
- C_1] \right\}, \tag{106} \]

\[ \mathcal{M}^H(B_c \to K^{''+} h'') = V_{cb} V_{us} \left\{ \frac{\cos \theta_3 \cos \theta_K}{\sqrt{3}} [f_B (F_{fa:H}^{K_1A h''_1} + F_{fa:H}^{h_2 K_1A}) a_1 \\
+ (M_{na:H}^{K_1A h''_1} + M_{na:H}^{h_2 K_1A}) C_1] + \frac{\cos \theta_3 \cos \theta_K}{\sqrt{6}} [f_B (F_{fa:H}^{K_1B h''_1} + F_{fa:H}^{h_2 K_1B}) a_1 \\
+ C_1] \right\}, \tag{107} \]

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will calculate numerically the BRs and polarization fractions for those considered sixty two \( B_c \to VA/AV, AA \) decay modes. First of all, the central values of the input parameters to be used are given in the following,
Masses (GeV):

\[
\begin{align*}
    m_W &= 80.41; & m_{B_c} &= 6.286; & m_b &= 4.8; & m_c &= 1.5; \\
    m_\phi &= 1.02; & m_{K^*} &= 0.892; & m_\rho &= 0.770; & m_\omega &= 0.782; \\
    m_{a_1} &= 1.23; & m_{K_{1A}} &= 1.32; & m_{f_1} &= 1.28; & m_{f_s} &= 1.28; \\
    m_{b_1} &= 1.21; & m_{K_{1B}} &= 1.34; & m_{h_1} &= 1.23; & m_{h_8} &= 1.37; \\
\end{align*}
\]  

(107)

Decay constants (GeV):

\[
\begin{align*}
    f_\phi &= 0.231; & f_\phi^T &= 0.200; & f_{K^*} &= 0.217; & f_{K^*}^T &= 0.185; \\
    f_\rho &= 0.209; & f_\rho^T &= 0.165; & f_\omega &= 0.195; & f_\omega^T &= 0.145; \\
    f_{a_1} &= 0.238; & f_{K_{1A}} &= 0.250; & f_{f_1} &= 0.245; & f_{f_s} &= 0.239; \\
    f_{b_1} &= 0.180; & f_{K_{1B}} &= 0.190; & f_{h_1} &= 0.180; & f_{h_8} &= 0.190; \\
    f_{B_c} &= 0.489; \\
\end{align*}
\]  

(108)

QCD scale and \( B_c \) meson lifetime:

\[
\Lambda_{\overline{\text{MS}}}^{(f=4)} = 0.250 \text{ GeV}, \quad \tau_{B_c^+} = 0.46 \text{ ps}.  
\]  

(109)

For the CKM matrix elements, here we adopt the Wolfenstein parametrization for the CKM matrix, and take \( A = 0.814 \) and \( \lambda = 0.2257, \tilde{\rho} = 0.135 \) and \( \tilde{\eta} = 0.349 \) \[25\]. In numerical calculations, central values of input parameters will be used implicitly unless otherwise stated.

For these considered \( B_c \to M_2 M_3 \) decays, the decay rate can be written explicitly as,

\[
\Gamma = \frac{G_F^2 |P_c|}{16\pi m_{B_c}^3} \sum_{\sigma=L,T} \mathcal{M}^{(\sigma)\dagger} \mathcal{M}^{(\sigma)}  
\]  

(110)

where \( |P_c| \equiv |P_{2z}| = |P_{3z}| \) is the momentum of either of the outgoing vector or axial-vector mesons.

Based on the helicity amplitudes \[33\], we can define the transversity amplitudes,

\[
\mathcal{A}_L = -\xi m_{B_c}^2 \mathcal{M}_L, \quad \mathcal{A}_\parallel = \xi \sqrt{2} m_{B_c}^2 \mathcal{M}_N, \quad \mathcal{A}_\perp = \xi m_{B_c}^2 \sqrt{2(r^2 - 1)} \mathcal{M}_T.  
\]  

(111)

for the longitudinal, parallel, and perpendicular polarizations, respectively, with the normalization factor \( \xi = \sqrt{G_F^2 |P_c|/(16\pi m_{B_c}^3)} \) and the ratio \( r = P_2 \cdot P_3/(m_2 m_3) \). These amplitudes satisfy the relation,

\[
|\mathcal{A}_L|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2 = 1  
\]  

(112)

following the summation in Eq. \[110\].

The polarization fractions \( f_L, f_\parallel \) and \( f_\perp \) can be defined as,

\[
f_L(\parallel, \perp) = \frac{|\mathcal{A}_L(\parallel, \perp)|^2}{|\mathcal{A}_L|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2},  
\]  

(113)

With the analytic formulas for the complete decay amplitudes as given explicitly in Eqs. \[33\] \[106\], by employing the input parameters and Eq.\[110\], we calculate and then present the pQCD predictions for the CP-averaged BRs and longitudinal polarization fractions (LPFs) of the considered decays with errors in Tables \[110\]. The dominant errors come from the uncertainties of charm quark mass \( m_c = 1.5 \pm 0.15 \) GeV and the combined Gegenbauer moments \( a_i \) of the relevant meson distribution amplitudes, respectively.

Based on the numerical results as given in Tables \[110\] some remarks are in order:
Among the considered sixty two pure annihilation $B_c \to AV/VA, AA$ decays, the pQCD predictions for the CP-averaged BRs of those $\Delta S = 0$ processes are generally much larger than those of $\Delta S = 1$ channels (one of the two final state mesons is a strange meson), the main reason is the enhancement of the large CKM factor $|V_{ud}/V_{us}|^2 \approx 19$ for those $\Delta S = 0$ decays as expected in general. Maybe there exists no such large differences for certain decays, which is just because the enhancement arising from the CKM factor is partially cancelled by the difference between the magnitude of individual decay amplitude.

There is no $CP$ violation for all these sixty two decays within the SM, since there is only one kind of tree operator involved in the decay amplitude of all considered $B_c$ decays, which can be seen directly from Eq. (52).

For the ten $B_c \to (a_1, b_1)V$ decays, the pQCD predictions of the BRs and LPFs for both $\Delta S = 0$ and $\Delta S = 1$ processes are listed in the Table I. As argued in Ref. [4], the LHCb experiments could observe the BRs of annihilation $B_c$ meson decays at the level of $10^{-6}$, the decays $B_c \to a_1\omega, b_1\rho$ will thus be detected at LHC because they are just within its reach. As for the polarization, all these ten decays are governed by the longitudinal contributions. The LPFs are around 95% within the theoretical errors except for $B_c \to a_1^\pm \omega, a_1 K^*$ modes ($\sim 85\%$), and for $B_c \to a_1\rho$ channels, $f_L(B_c \to a_1\rho) \sim 1$, which will be tested in the LHCb experiments.

Since the behavior of $b_1$ meson is contrary to that of $a_1$ meson, one can find that $Br(B_c \to b_1\rho) > Br(B_c \to a_1\rho)$ as given in Table I and the ratio of the corresponding BRs for $B_c \to a_1\rho$ and $B_c \to b_1\rho$ is that

\[
\frac{Br(B_c \to b_1^+\rho^0)}{Br(B_c \to a_1^+\rho^0)} = \frac{Br(B_c \to b_1^0\rho^+)}{Br(B_c \to a_1^0\rho^+)} \approx 13.2,
\]

Similarly, for $B_c \to a_1 K^*, b_1 K^*$ decays, the BRs of the latter modes are larger than that of the former ones and

\[
\frac{Br(B_c \to b_1^+ K^{*0})}{Br(B_c \to a_1^+ K^{*0})} = \frac{Br(B_c \to b_1^0 K^{*+})}{Br(B_c \to a_1^0 K^{*+})} \approx 5.5.
\]

The above two ratios exhibit the annihilation decay pattern consistent with those as shown in Ref. [23].

| $\Delta S = 0$ | Decay modes | BRs (10^{-7}) | LPFs (%) | $\Delta S = 0$ | Decay modes | BRs (10^{-6}) | LPFs (%) |
|-----------------|-------------|---------------|----------|-----------------|-------------|---------------|----------|
| $B_c \to \rho^+ a_1^0$ | 7.5^{+0.1}_{-0.2}(m_c) + 3.0 | 99.2^{+0.3}_{-0.4} | | $B_c \to \rho^+ b_1^0$ | 9.9^{+4.7}_{-3.9}(m_c) + 5.6 | 93.5^{+2.3}_{-2.4} |
| $B_c \to a_1^{+\rho}$ | 7.5^{+0.1}_{-0.2}(m_c) + 3.0 | 99.2^{+0.3}_{-0.4} | | $B_c \to b_1^{+\rho}$ | 9.9^{+4.5}_{-4.0}(m_c) + 5.7 | 93.8^{+2.5}_{-2.4} |
| $B_c \to a_1^+\omega$ | 20.2^{+3.6}_{-2.8}(m_c) + 7.8 | 84.7^{+5.0}_{-4.4} | | $B_c \to b_1^+\omega$ | 0.6^{+0.5}_{-0.4}(m_c) + 0.3 | 96.3^{+2.3}_{-2.5} |

| $\Delta S = 1$ | Decay modes | BRs (10^{-8}) | LPFs (%) | $\Delta S = 1$ | Decay modes | BRs (10^{-7}) | LPFs (%) |
|-----------------|-------------|---------------|----------|-----------------|-------------|---------------|----------|
| $B_c \to a_1^1 K^{*0}$ | 6.5^{+0.2}_{-0.0}(m_c) + 3.5 | 83.6^{+3.3}_{-3.2} | | $B_c \to b_1^1 K^{*0}$ | 3.6^{+1.1}_{-0.8}(m_c) + 1.8 | 91.2^{+2.5}_{-2.4} |
| $B_c \to K^{*+} a_1^0$ | 3.3^{+0.1}_{-0.0}(m_c) + 1.7 | 83.6^{+3.3}_{-3.2} | | $B_c \to K^{*+} b_1^0$ | 1.8^{+0.5}_{-0.4}(m_c) + 0.9 | 91.2^{+2.5}_{-2.4} |
Since the contributions from $u\bar{u}$ and $d\bar{d}$ components cancel each other exactly and result in the zero BRs for $B_c \to a_1^+ a_1^0$ and $B_c \to b_1^+ b_1^0$. Any other nonzero data for these two channels may indicate the effects of exotic new physics. While for $B_c \to a_1^+ b_1^0$ and $B_c \to b_1^+ a_1^0$, as expected from the analytic expressions, Eqs. (81)(82), due to the same component of $u\bar{u} - d\bar{d}$ involved in both axial-vector $a_1^0$ and $b_1^0$ mesons at the quark level, the pQCD predictions for the BRs and LPFs as given in Table III show the identical results as they should be,

$$Br(B_c \to a_1^+ b_1^0) = Br(B_c \to b_1^+ a_1^0) \approx 2.2 \times 10^{-5},$$

$$f_L(B_c \to a_1^+ b_1^0) = f_L(B_c \to b_1^+ a_1^0) \approx 92\% .$$

where the large BRs ($\sim 10^{-5}$) are within the reach of the LHCb experiments [4] and could be detected at LHC.

Since the $3P_1$ meson behaves like the vector meson and $f_{a_1} \sim f_{\rho}$ from Eq. (108), the pQCD predictions of BRs exhibit the good consistency generally for $B_c \to a_1^+ \omega$ and $B_c \to \rho^+ \omega$, $B_c \to a_1^+ K^0$ and $B_c \to \rho^+ K^0$, $B_c \to a_1^+ b_1^0(a_1^0 b_1^+)$ and $B_c \to \rho^+ b_1^0(\rho^0 b_1^+)$ decays, respectively, within the theoretical errors as roughly estimated. As for the polarizations, which can well manifest the helicity structure for the corresponding modes, the LPFs present the different features from the decay rates except for $B_c \to a_1^0 b_1^0(a_1^0 b_1^+)$ and $B_c \to \rho^+ b_1^0(\rho^0 b_1^+)$ decays. From Table II the LPFs for $B_c \to a_1^+ \omega$ and $B_c \to a_1^+ K^0$ can be read straightforward as: $f_L(B_c \to a_1^+ \omega) = (84.7^{+5.0}_{-4.4})\%$ and $f_L(B_c \to a_1^+ K^0) = (83.6^{+5.3}_{-7.5})\%$. As given in Ref. [2], $f_L(B_c \to \rho^+ \omega) = (92.9^{+2.0}_{-1.7})\%$ and $f_L(B_c \to \rho^+ K^0) = (94.9^{+2.0}_{-1.4})\%$, where the various errors as specified have been added in quadrature. The above results and discussions would be tested with high precision by the relevant experiments operated.

### Table II: Same as Table III but for $B_c \to (a_1, b_1)(a_1, b_1)$ decays.

| Decay modes | BRs (10^{-5}) | LPFs (%) | Decay modes | BRs (10^{-5}) | LPFs (%) |
|-------------|---------------|----------|-------------|---------------|----------|
| $B_c \to a_1^+ a_1^0$ | 0.0 | $\theta_3 = 38^\circ$ | $B_c \to b_1^+ b_1^0$ | 0.0 | $\theta_3 = 38^\circ$ |
| $B_c \to a_1^+ b_1^0$ | $2.2^{+0.6}_{-0.5}(m_c)^{+1.1}_{-0.3}(a_1)$ | 92.4^{+1.9}_{-2.8} | $B_c \to b_1^+ a_1^0$ | $2.2^{+0.6}_{-0.5}(m_c)^{+1.1}_{-0.3}(a_1)$ | 91.8^{+2.0}_{-2.6} |

### Table III: Same as Table II but for $B_c \to (\rho, K^+)(f_1(1285), f_1(1420))$ decays.

| Decay modes | BRs (10^{-5}) | LPFs (%) | Decay modes | BRs (10^{-5}) | LPFs (%) |
|-------------|---------------|----------|-------------|---------------|----------|
| $B_c \to \rho^+ f_1(1285)$ | $2.1^{-0.3}_{+0.4}(m_c)^{-0.3}_{+0.2}(a_1)$ | 82.6^{+5.4}_{-3.8} | $B_c \to K^+ f_1(1285)$ | $1.9^{+0.4}_{-0.0}(m_c)^{+0.7}_{-0.3}(a_1)$ | 82.3^{+3.8}_{-3.8} |
| $B_c \to \rho^+ f_1(1420)$ | $0.4^{+0.0}_{-0.1}(m_c)^{+0.5}_{-0.2}(a_1)$ | 88.7^{+11.6}_{-9.2} | $B_c \to K^+ f_1(1420)$ | $2.2^{+0.2}_{-0.0}(m_c)^{+0.2}_{-0.1}(a_1)$ | 85.7^{+8.6}_{-8.6} |

$^a$Here, the factor 10 is specifically used for the BRs. The following ones have the same meaning.

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at the ongoing LHC and forthcoming Super-B to identify the helicity structure even decay mechanism in these considered channels.

- In Table III from the pQCD predictions of the BRs and LPFs for $B_c \rightarrow \rho^+(f_1(1285), f_1(1420)) (\Delta S = 0)$ and $B_c \rightarrow K^{*+}(f_1(1285), f_1(1420)) (\Delta S = 1)$ decays, one can observe that the BRs of $B_c \rightarrow \rho^+(f_1(1420), K^{*+} f_1(1285))$ are more sensitive than those of $B_c \rightarrow \rho^+ f_1(1285), K^{*+} f_1(1420)$ to the mixing angle $\theta_3$.

$$\frac{Br(B_c \rightarrow \rho^+ f_1(1285))}{Br(B_c \rightarrow \rho^+ f_1(1420))} |_{\theta_3=50^\circ} \approx 5.5; \quad \text{(117)}$$

$$\frac{Br(B_c \rightarrow K^{*+} f_1(1285))}{Br(B_c \rightarrow K^{*+} f_1(1420))} |_{\theta_3=50^\circ} \approx 4.0; \quad \text{(118)}$$

These two relations, Eqs. (117,118), can be understood as that the interferences between $B_c \rightarrow \rho^+ f_1(1285), B_c \rightarrow K^{*+} f_1(1420)$ become highly constructive/destructive with the mixing angle $\theta_3$ changing from 38° to 50°. Moreover,

$$\frac{Br(B_c \rightarrow \rho^+ f_1(1285))}{Br(B_c \rightarrow \rho^+ f_1(1420))} \approx \begin{cases} 
52.5 & \text{for } \theta_3 = 38^\circ \\
8.6 & \text{for } \theta_3 = 50^\circ 
\end{cases}; \quad \text{(119)}$$

$$\frac{Br(B_c \rightarrow K^{*+} f_1(1285))}{Br(B_c \rightarrow K^{*+} f_1(1420))} \approx \begin{cases} 
6.9 & \text{for } \theta_3 = 38^\circ \\
30.0 & \text{for } \theta_3 = 50^\circ 
\end{cases}; \quad \text{(120)}$$

From the decay amplitudes as given in Eqs. 59-74, the above two relations can be understood as follows: (a) for $B_c \rightarrow \rho^+(f_1(1285), f_1(1420))$ decays, the mixing coefficients for the former decay are $\cos \theta_3$ and $\sin \theta_3$, while for the latter one are $-\sin \theta_3$ and $\cos \theta_3$. For the common component $q\bar{q}$, it is found that the contributions from $B_c \rightarrow \rho^+ f_1$ and $B_c \rightarrow \rho^+ f_8$ interfere constructively/destructively for $B_c \rightarrow \rho^+ f_1(1285)(B_c \rightarrow \rho^+ f_1(1420));$ (b) for $B_c \rightarrow K^{*+}(f_1(1285), f_1(1420))$ channels, the mixing parameters remain unchanged, however, a new part of contribution from $s\bar{s}$ component involved in both $f_1$ and $f_8$ with different signs results in the construction/destruction to $B_c \rightarrow K^{*+} f_1(1420)(B_c \rightarrow K^{*+} f_1(1285))$. Additionally, the LPFs for these decays are stable to the mixing angle and play the dominant role except for $f_L(B_c \rightarrow K^{*+} f_1(1285))$, whose value change from 61.0% at $\theta_3 = 38^\circ$ to 33.7% at $\theta_3 = 50^\circ$, which will be confronted with the relevant experiments in the future.

### Table IV: Same as Table II but for $B_c \rightarrow (\rho, K^*) (h_1(1170), h_1(1380))$ decays.

| $\Delta S = 0$ | Decay modes | BRs $(10^{-7})$ | LPFs (%) | $\theta_1 = 10^\circ$ | BRs $(10^{-7})$ | LPFs (%) | $\theta_1 = 45^\circ$ |
|----------------|-------------|----------------|----------|----------------------|----------------|----------|----------------------|
| $B_c \rightarrow \rho^+ h_1(1170)$ | 6.4$^{+3.0}_{-2.4}(m_c)-2.2(a_i)$ | 96.4$^{+7.7}_{-7.0}$ | 8.6$^{+1.7}_{-1.4}(m_c)+0.7(a_i)$ | 96.3$^{+7.4}_{-6.2}$ | 6.9$^{+1.7}_{-1.4}(m_c)-0.6(a_i)$ | 98.0$^{+1.7}_{-2.3}$ |
| $B_c \rightarrow \rho^+ h_1(1380)$ | 2.5$^{+2.6}_{-1.8}(m_c)+2.9(a_i)$ | 96.3$^{+2.1}_{-2.6}$ | 0.3$^{+0.5}_{-0.2}(m_c)+0.6(a_i)$ | 98.0$^{+1.7}_{-2.3}$ |

| $\Delta S = 1$ | Decay modes | BRs $(10^{-7})$ | LPFs (%) | $\theta_1 = 10^\circ$ | BRs $(10^{-7})$ | LPFs (%) | $\theta_1 = 45^\circ$ |
|----------------|-------------|----------------|----------|----------------------|----------------|----------|----------------------|
| $B_c \rightarrow K^{*+} h_1(1170)$ | 0.3$^{+0.1}_{-0.0}(m_c)+0.9(a_i)$ | 82.3$^{+6.2}_{-6.5}$ | 2.1$^{+0.6}_{-0.5}(m_c)+1.1(a_i)$ | 89.5$^{+3.4}_{-3.2}$ | 6.9$^{+1.7}_{-1.4}(m_c)-0.8(a_i)$ | 94.0$^{+1.8}_{-1.4}$ |
| $B_c \rightarrow K^{*+} h_1(1380)$ | 4.8$^{+2.2}_{-1.9}(m_c)+2.4(a_i)$ | 93.2$^{+2.2}_{-2.3}$ | 2.9$^{+1.7}_{-1.3}(m_c)+1.4(a_i)$ | 96.9$^{+1.8}_{-1.4}$ |
• The pQCD predictions for $B_c \rightarrow (\rho^+ , K^{*+})h_1$ decays, as given in Table IV, can be explained in a similar way as for $B_c \rightarrow (\rho^+ , K^{*+})f_1$.

• The numerical pQCD results for $\Delta S = 0 \ B_c \rightarrow (a_1^+, b_1^+) (f_1(1285), f_1(1420))$ decays, as given in Table V, can be commented in order: (a) the BRs of these modes depend weakly on the mixing angle $\theta_3$ except for $B_c \rightarrow a_1^+ f_1(1420)$; (b) these 4 considered decays are governed by the longitudinal contributions for both $\theta_3 = 38^\circ$ and $\theta_3 = 50^\circ$; (c) as mentioned in the text above, $1^3P_1$ meson behaves close to the vector meson, the phenomenology of $B_c \rightarrow a_1^+ (f_1(1285), f_1(1420))$ can therefore be understood as that of $B_c \rightarrow \rho^+ (f_1(1285), f_1(1420))$; (d) the mixing factors $\cos \theta_3$ and $\sin \theta_3$ make the interference between $B_c \rightarrow (a_1^+, b_1^+) f_1$ and $B_c \rightarrow (a_1^+, b_1^+) f_8$ constructive (destructive) to $B_c \rightarrow (a_1^+, b_1^+) f_1(1285) (B_c \rightarrow (a_1^+, b_1^+) f_1(1420))$.

• From the predictions for $B_c \rightarrow (a_1^+, b_1^+) (h_1(1170), h_1(1380))$ presented in Table VI, some discussions could be addressed as: (a) since the behavior of $1^3P_1$ meson is different even contrary to that of $1^3P_1$ meson, a surprisingly large branching ratio for $B_c \rightarrow b_1^+ h_1(1700) \ (\sim 10^{-4})$ with the constructive effects induced by the interference between $B_c \rightarrow b_1^+ h_1$ and $B_c \rightarrow b_1^+ h_8$ is produced, which will be tested stringently by the forthcoming relevant LHC experiments; (b) once the large BRs are verified by the measurements, the mixing angle $\theta_1$ could be well determined to improve the precision of the perturbative calculations; (c) except for $B_c \rightarrow b_1^+ h_1(1170)$ decay, the rest three channels are sensitive significantly to the mixing angle $\theta_1$; (d) similar to $B_c \rightarrow (a_1^+, b_1^+) (f_1(1285), f_1(1420))$ decays, the longitudinal components play the dominant role for these 4 channels.

• For the $\Delta S = 0 \ B_c \rightarrow \overline{K}^0 \ K_1^+$ and $B_c \rightarrow \overline{K}_1 K^{*+}$ decays, one can see from Table VII that the BRs are large in the range of $10^{-6} \sim 10^{-5}$, which can be detected at the ongoing LHC and forthcoming Super B experiments. Moreover, the corresponding ratios of the BRs for these considered channels

$$\frac{Br(B_c \rightarrow \overline{K}^0 \ K_1(1270)^+)}{Br(B_c \rightarrow \overline{K}^0 \ K_1(1400)^+)} = \frac{Br(B_c \rightarrow \overline{K}_1(1270)^0 K^{*+})}{Br(B_c \rightarrow \overline{K}_1(1400)^0 K^{*+})} \approx 1.7, \quad (121)$$

for $\theta_K = 45^\circ$, while

$$\frac{Br(B_c \rightarrow \overline{K}^0 \ K_1(1270)^+)}{Br(B_c \rightarrow \overline{K}^0 \ K_1(1400)^+)} = \frac{Br(B_c \rightarrow \overline{K}_1(1270)^0 K^{*+})}{Br(B_c \rightarrow \overline{K}_1(1400)^0 K^{*+})} \approx \frac{1}{1.7}, \quad (122)$$

\[\text{TABLE V: Same as Table IV but for } B_c \rightarrow (a_1^+, b_1^+) (f_1(1285), f_1(1420)) \text{ decays.}\]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\Delta S = 0 & \text{Decay modes} & BRs (10^{-6}) & LPFs (%) & \theta_3 = 38^\circ & \theta_3 = 50^\circ \\
\hline
\text{B}_c \rightarrow a_1(1260)^+ f_1(1285) & 6.5^{+1.0}_{-0.9} (m_1)^{+0.9}_{-1.0} (a_i) & 83.6^{+2.4}_{-4.1} & 6.1^{+2.4}_{-0.9} (m_1)^{-0.9}_{+1.0} (a_i) & 84.0^{+2.4}_{-4.0} \\
\text{B}_c \rightarrow a_1(1260)^+ f_1(1420) \times 10 & 0.3^{+0.1}_{-0.1} (m_1)^{+0.7}_{-0.3} (a_i) & 56.8^{+13.2}_{-56.8} & 3.9^{+0.7}_{-0.7} (m_1)^{-0.1}_{+1.3} (a_i) & 78.5^{+7.4}_{-13.9} \\
\hline
\Delta S = 0 & \text{Decay modes} & BRs (10^{-7}) & LPFs (%) & \theta_3 = 38^\circ & \theta_K = 50^\circ \\
\hline
\text{B}_c \rightarrow b_1(1235)^+ f_1(1285) & 2.8^{+4.1}_{-0.5} (m_1)^{+1.8}_{-0.9} (a_i) & 65.2^{+28.3}_{-16.4} & 3.0^{+4.4}_{-0.8} (m_1)^{-0.9}_{+1.5} (a_i) & 68.7^{+21.7}_{-14.6} \\
\text{B}_c \rightarrow b_1(1235)^+ f_1(1420) & 1.4^{+0.2}_{-0.1} (m_1)^{+0.7}_{-0.5} (a_i) & 100.0 \pm 0.0 & 1.2^{+0.4}_{-0.3} (m_1)^{-0.8}_{+1.1} (a_i) & 100.0^{+0.0}_{-0.8} \\
\hline
\end{array}
\]
TABLE VI: Same as Table II but for $B_c \to (a_1^+, b_1^+)(h_1(1170), h_1(1380))$ decays.

| $\Delta S = 0$ | $\theta_K = 10^\circ$ | $\theta_K = 45^\circ$ |
|-----------------|----------------------|----------------------|
| Decay modes     | BRs (10^{-6}) | LPFs (%) | BRs (10^{-6}) | LPFs (%) |
| $B_c \to a_1(1260)^+ h_1(1170)$ | 1.3^{+0.3}_{-0.5}(m_c)^{-0.5}_{+0.4}(a_i) | 68.3^{+2.0}_{-0.8} | 0.7^{+0.8}_{-0.4}(m_c)^{-0.3}_{+0.2}(a_i) | 73.1^{+7.4}_{-28.8} |
| $B_c \to a_1(1260)^+ h_1(1380) \times 10$ | 1.1^{+0.7}_{-0.9}(m_c)^{+1.3}_{-0.5}(a_i) | 68.8^{+23.2}_{-11.5} | 6.8^{+0.2}_{-1.1}(m_c)^{+2.3}_{-2.2}(a_i) | 100.0^{+0.0}_{-1.2} |

| $\Delta S = 0$ | $\theta_K = 10^\circ$ | $\theta_K = 45^\circ$ |
|-----------------|----------------------|----------------------|
| Decay modes     | BRs (10^{-5}) | LPFs (%) | BRs (10^{-5}) | LPFs (%) |
| $B_c \to b_1(1235)^+ h_1(1170)$ | 8.1^{+3.6}_{-2.8}(m_c)^{+3.9}_{-3.4}(a_i) | 96.4^{+1.0}_{-1.6} | 10.3^{+2.0}_{-3.3}(m_c)^{+3.7}_{-3.8}(a_i) | 96.4^{+0.9}_{-1.4} |
| $B_c \to b_1(1235)^+ h_1(1380)$ | 2.5^{+0.5}_{-0.7}(m_c)^{+1.4}_{-1.2}(a_i) | 100.0^{+0.0}_{-1.6} | 0.3^{+0.2}_{-0.1}(m_c)^{+0.5}_{-0.3}(a_i) | 100.0^{+0.0}_{-0.0} |

for $\theta_K = -45^\circ$, which indicate that one could determine the size and sign of the mixing angle $\theta_K$ after enough $B_c$ events become available at the LHC experiments and then improve the precision of the theoretical predictions. In terms of polarization, the longitudinal contributions play the dominated role for both $\theta_K = 45^\circ$ and $\theta_K = -45^\circ$ in $B_c \to K^*_1 K^{*+}$ modes. In the $B_c \to K^{*0}(K_1(1270)^+, K_1(1400)^+)$ decays, the transverse components govern the former channel for $\theta_K = -45^\circ$ while dominate the latter one for $\theta_K = 45^\circ$. These results will be tested by the relevant measurements in the future.

- Form the numerical results for $B_c \to K^+_1(\rho, \omega)$, the $\Delta S = 1$ processes, as displayed in the

TABLE VII: Same as Table II but for $B_c \to (K_1(1270), K_1(1400)) (\rho, K^*, \omega, \phi)$ decays.

| $\Delta S = 0$ | $\theta_K = 45^\circ$ | $\theta_K = -45^\circ$ |
|-----------------|----------------------|----------------------|
| Decay modes     | BRs (10^{-6}) | LPFs (%) | BRs (10^{-6}) | LPFs (%) |
| $B_c \to K_1(1270)^0 \rho^+$ | 3.1^{+1.4}_{-1.1}(m_c)^{+2.6}_{-1.7}(a_i) | 89.5^{+5.5}_{-9.3} | 4.0^{+1.0}_{-1.2}(m_c)^{+2.1}_{-2.2}(a_i) | 99.1^{+0.8}_{-3.6} |
| $B_c \to K_1(1400)^0 \rho^+$ | 4.0^{+0.9}_{-0.8}(m_c)^{+1.2}_{-1.4}(a_i) | 99.1^{+3.4}_{-3.4} | 3.0^{+1.4}_{-1.0}(m_c)^{+2.8}_{-2.5}(a_i) | 89.7^{+5.6}_{-9.3} |
| $B_c \to K_1(1270)^+ \rho^0$ | 1.5^{+0.4}_{-0.5}(m_c)^{+1.3}_{-0.7}(a_i) | 89.5^{+5.9}_{-9.3} | 2.0^{+0.6}_{-0.5}(m_c)^{+1.1}_{-1.0}(a_i) | 99.1^{+0.8}_{-3.6} |
| $B_c \to K_1(1400)^+ \rho^0$ | 2.0^{+0.5}_{-0.6}(m_c)^{+1.0}_{-1.1}(a_i) | 99.1^{+3.4}_{-3.4} | 1.5^{+0.7}_{-0.5}(m_c)^{+1.4}_{-1.0}(a_i) | 89.7^{+5.6}_{-9.3} |
| $B_c \to K_1(1270)^+ \omega$ | 1.4^{+0.5}_{-0.4}(m_c)^{+1.2}_{-0.7}(a_i) | 89.8^{+5.4}_{-8.7} | 1.7^{+0.5}_{-0.6}(m_c)^{+0.9}_{-0.8}(a_i) | 99.1^{+3.9}_{-3.9} |
| $B_c \to K_1(1400)^+ \omega$ | 1.7^{+0.5}_{-0.6}(m_c)^{+0.9}_{-0.9}(a_i) | 99.1^{+3.8}_{-3.8} | 1.4^{+0.6}_{-0.5}(m_c)^{+0.9}_{-0.7}(a_i) | 89.7^{+5.5}_{-8.8} |
| $B_c \to K_1(1270)^+ \phi$ | 1.9^{+0.3}_{-0.5}(m_c)^{+1.1}_{-1.0}(a_i) | 95.9^{+2.7}_{-10.9} | 1.5^{+0.4}_{-0.3}(m_c)^{+1.2}_{-0.8}(a_i) | 29.9^{+30.8}_{-27.3} |
| $B_c \to K_1(1400)^+ \phi$ | 1.4^{+0.3}_{-0.5}(m_c)^{+1.1}_{-1.0}(a_i) | 95.9^{+2.7}_{-10.9} | 1.9^{+0.1}_{-0.3}(m_c)^{+1.3}_{-1.4}(a_i) | 95.2^{+10.8}_{-10.8} |
within errors for both $\theta_K = 45^\circ$ and $\theta_K = -45^\circ$, where the longitudinal components contribute to these considered decays dominantly. The pattern of these decays shown in Eqs. (123-126) can be understood as follows: only the same component $u\bar{u}$ in both of $\rho^0$ and $\omega$ mesons contributes to these physical observables, where the differences mainly arise from the different decay constants. Furthermore, by comparison with $B_c \to K^*(0)^0 K_1^+$ and $B_c \to \overline{K}_1^0 K^*$ decays, one can find that the pQCD predictions of $B_c \to K_1^+(\rho, \omega)$ show the weak dependence on the value of the mixing angle $\theta_K$, which will also be tested by the LHC measurements.

- For $B_c \to K_1^+ \phi$ decays, it is interesting to note that the decay rates, as listed in Table VII, are close to each other within the theoretical uncertainties, however, the LPFs

| Table VII | One can straightforwardly observe that |
| --- | --- |
| $\text{Br}(B_c \to K_1(1270)^+ \omega) \sim \text{Br}(B_c \to K_1(1270)^+ \rho^0)$ | $= \frac{1}{2} \text{Br}(B_c \to K_1(1270)^0 \rho^+)$; (123) |
| $\text{Br}(B_c \to K_1(1400)^+ \omega) \sim \text{Br}(B_c \to K_1(1400)^+ \rho^0)$ | $= \frac{1}{2} \text{Br}(B_c \to K_1(1400)^0 \rho^+)$; (124) |
| $f_L(B_c \to K_1(1270)^+ \omega) \sim f_L(B_c \to K_1(1270)^+ \rho^0)$ | $= f_L(B_c \to K_1(1270)^0 \rho^+)$; (125) |
| $f_L(B_c \to K_1(1400)^+ \omega) \sim f_L(B_c \to K_1(1400)^+ \rho^0)$ | $= f_L(B_c \to K_1(1400)^0 \rho^+)$ . (126) |

TABLE VIII: Same as Table VII but for $B_c \to (K_1(1270), K_1(1400))(a_1, b_1, K_1(1270), K_1(1400))$ decays.
show us the dramatically different features: the former is dominated by the longitudinal
components (~ 95%) while the latter governed by the transverse ones (~ 30%) when
θ_K = 45°. The main reason is that the interferences induced by B_c → K_{14}^+ and
B_c → K_{15}^+φ are constructive (destructive) to B_c → K_1(1400)^+φ(B_c → K_1(1270)^+φ) in
the two transverse polarizations, meanwhile, these interferences in the longitudinal
polarization contribute to these considered two decays oppositely. When θ_K = -45°,
the situation is quite the contrary. The decay mechanism and helicity structure for B_c → K_1^+φ
decays will be tested by the LHCb and Super-B experiments.

- For the ΔS = 0 processes, B_c → K_1 K_1^+ modes, as presented in Table VIII, it is of interest
to notice that the BRs for all these four considered decays are in the order of 10^{-5},
which are within the reach of B_c experiments at LHC greatly as discussed in Ref. [4].
These numerical results also present the strong dependance on the mixing angle θ_K,
which will also be tested by the relevant experiments in the near future. The longitudinal
polarization fractions are around 95% ~ 100% within theoretical errors except for B_c →
K_1(1400)^0 K_1(1400)^+ (∼ 73%) at θ_K = 45° or B_c → K_1(1270)^0 K_1(1270)^+ (∼ 72%) at
θ_K = -45° and play the dominant role.

- As mentioned in the text above, although the suppressed CKM factor V_{us} ∼ 0.22 is involved
in the decay amplitudes (see Eqs. (95-97, 99-102)) for the ΔS = 1 B_c → K_1(a_1, b_1)
decays, the pQCD predictions of the BRs for B_c → K_1b_1 are in the order of 10^{-6} and
larger than that for B_c → K_1a_1 because the ^1P_1 meson behaves differently even contrarily
to the ^3P_1 meson, whose behavior is close to that of the vector meson. For the polarization
fractions, all of these eight channels are governed by the longitudinal contributions
evidently. From the numerical results as given in Table VII one can also find that the
pQCD predictions of B_c → K_1a_1(B_c → K_1b_1) are much (less) sensitive to the mixing angle
θ_K.

- For the ΔS = 1 B_c → K_1^+(f_1(1285), f_1(1420)) decays, from the pQCD predictions
presented in Table IX one can see that the contributions to the BRs for these four
decays come from the overlap of various parts of B_c → K_{14}^+ f_1, K_{15}^+ f_1, K_{14}^+ f_8, and
K_{15}^+ f_8, which have been given in Eqs. (99-102). Combining with four mixing parameters
\cos θ_K, \sin θ_K, \cos θ_3 and \sin θ_3, these interferences result in the equivalent BRs for

| ΔS = 1 Decays | θ_K = 45° | θ_K = −45° |
|--------------|-----------|-------------|
| μ_s + μ_u | BRs (10^{-7}) | LPFs (%) | BRs (10^{-7}) | LPFs (%) |
| B_c → K_1(1270)^+ f_1(1285) | 1.4_{±0.9}^{+2.7} (μ_s + μ_u)_{±0.7} (μ_s + μ_u)_{±0.7} (μ_s + μ_u)_{±0.7} (μ_s + μ_u)_{±0.7} | 65.1_{±28.4}^{+28.4} | 1.6_{±0.3}^{+2.7} (μ_s + μ_u)_{±0.3} (μ_s + μ_u)_{±0.3} (μ_s + μ_u)_{±0.3} (μ_s + μ_u)_{±0.3} | 96.7_{±22.8}^{+22.8} |
| B_c → K_1(1400)^+ f_1(1285) | 1.7_{±0.4}^{+2.7} (μ_s + μ_u)_{±0.4} (μ_s + μ_u)_{±0.4} (μ_s + μ_u)_{±0.4} (μ_s + μ_u)_{±0.4} | 69.1_{±19.6}^{+19.6} | 1.5_{±0.5}^{+2.7} (μ_s + μ_u)_{±0.5} (μ_s + μ_u)_{±0.5} (μ_s + μ_u)_{±0.5} (μ_s + μ_u)_{±0.5} | 92.1_{±22.8}^{+22.8} |
| B_c → K_1(1270)^+ f_1(1420) | 1.5_{±0.2}^{+1.2} (μ_s + μ_u)_{±0.2} (μ_s + μ_u)_{±0.2} (μ_s + μ_u)_{±0.2} (μ_s + μ_u)_{±0.2} | 96.7_{±11.5}^{+11.5} | 1.4_{±0.8}^{+1.8} (μ_s + μ_u)_{±0.8} (μ_s + μ_u)_{±0.8} (μ_s + μ_u)_{±0.8} (μ_s + μ_u)_{±0.8} | 65.5_{±27.2}^{+27.2} |
| B_c → K_1(1400)^+ f_1(1420) | 1.5_{±0.3}^{+1.6} (μ_s + μ_u)_{±0.3} (μ_s + μ_u)_{±0.3} (μ_s + μ_u)_{±0.3} (μ_s + μ_u)_{±0.3} | 92.1_{±12.8}^{+12.8} | 1.7_{±0.4}^{+2.2} (μ_s + μ_u)_{±0.4} (μ_s + μ_u)_{±0.4} (μ_s + μ_u)_{±0.4} (μ_s + μ_u)_{±0.4} | 69.5_{±21.9}^{+21.9} |

TABLE IX: Same as Table II but for B_c → (K_1(1270)^+ , K_1(1400)^+) (f_1(1285), f_1(1420)) decays with
θ_3 = 38° (1st entry) and θ_3 = 50° (2nd entry).
$B_c \to K_1(1270)^+ f_1(1285)$ and $B_c \to K_1(1400)^+ f_1(1285)$ decays, and suppressed one for $B_c \to K_1(1270)^+ f_1(1420)$ while enhanced one for $B_c \to K_1(1400)^+ f_1(1420)$. Moreover, (a) the BRs for $B_c \to K_1^+ f_1(1420) / B_c \to K_1^0 f_1(1285)$ depend strongly (weakly) on $\theta_K$ for both $\theta_3 = 38^\circ$ and $\theta_3 = 50^\circ$; (b) the BRs for $B_c \to K_1(1270)^+ f_1$ are more sensitive than that for $B_c \to K_1(1400)^+ f_1$ to $\theta_3$ when $\theta_K = 45^\circ$, while the situation is quite the contrary when $\theta_K = -45^\circ$; (c) the longitudinal contributions play an important role in all these considered channels.

- Based on Eq. (3), apart from an overall sign, the physical states $K_1(1270)$ and $K_1(1400)$ can go one into another with changing the mixing angle $\theta_K$ from $45^\circ$ to $-45^\circ$ and vice versa,

$$|K_1(1270)\rangle_{\theta_K=45^\circ} = |K_1(1400)\rangle_{\theta_K=-45^\circ},$$
$$|K_1(1400)\rangle_{\theta_K=45^\circ} = -|K_1(1270)\rangle_{\theta_K=-45^\circ}. \quad (127)$$

which further results in the decay amplitudes of $B_c \to K_1(V, A)$ (Here, $A$ is a nonstrange axial-vector meson) as follows:

$$\mathcal{A}(B_c \to K_1(1270)(V, A))_{\theta_K=45^\circ} = \mathcal{A}(B_c \to K_1(1400)(V, A))_{\theta_K=-45^\circ}, \quad (128)$$
$$\mathcal{A}(B_c \to K_1(1400)(V, A))_{\theta_K=45^\circ} = -\mathcal{A}(B_c \to K_1(1270)(V, A))_{\theta_K=-45^\circ}. \quad (129)$$

These two relations, i.e., Eqs. (128) and (129), can be manifested by the analytic formulas for $B_c \to K_1(V, A)$ decays as shown in Eqs. (63, 66), (69, 72), (77, 78) and (95, 106). The pQCD predictions for these considered $B_c \to K_1(V, A)$ decays as listed in the second and third columns of Tables VII, VIII, IX and X also display the phenomenologies induced by the same pattern.

- For $B_c \to \overline{K}_1^0 K_1^+$ decays, however, it is not the case as shown in Eqs. (128) and (129). According to the relation shown in Eq. (127), there are some simple relations between the decay amplitudes as given in Eqs. (91, 94) for $B_c \to K_1^0 K_1^+$ decays:

$$\mathcal{A}(B_c \to \overline{K}_1(1270) K_1(1270))_{\theta_K=45^\circ} = -\mathcal{A}(B_c \to \overline{K}_1(1400) K_1(1400))_{\theta_K=-45^\circ}, \quad (130)$$
$$\mathcal{A}(B_c \to \overline{K}_1(1400) K_1(1400))_{\theta_K=45^\circ} = -\mathcal{A}(B_c \to \overline{K}_1(1270) K_1(1270))_{\theta_K=-45^\circ}, \quad (131)$$
$$\mathcal{A}(B_c \to \overline{K}_1(1270) K_1(1400))_{\theta_K=45^\circ} = \mathcal{A}(B_c \to \overline{K}_1(1400) K_1(1270))_{\theta_K=-45^\circ}, \quad (132)$$
$$\mathcal{A}(B_c \to \overline{K}_1(1400) K_1(1270))_{\theta_K=45^\circ} = \mathcal{A}(B_c \to \overline{K}_1(1270) K_1(1400))_{\theta_K=-45^\circ}. \quad (133)$$

| $\Delta S = 1$ | $\theta_K = 45^\circ$ | $\theta_K = -45^\circ$ |
|----------------|-----------------|-----------------|
| Decay modes | BRs (10$^{-6}$) | LPFs (%) | BRs (10$^{-6}$) | LPFs (%) |
| $B_c \to K_1(1270)^+ h_1(1170)$ | 1.4$^{+0.4}_{-0.6} (m_c)^{+1.3}_{-0.8} (a_i)$ | 94.5$^{+2.3}_{-3.9}$ | 1.6$^{+0.7}_{-0.5} (m_c)^{+1.0}_{-1.0} (a_i)$ | 98.5$^{+0.6}_{-0.9}$ |
| | 0.6$^{+0.2}_{-0.3} (m_c)^{+0.3}_{-0.4} (a_i)$ | 87.9$^{+6.5}_{-14.6}$ | 0.2$^{+0.2}_{-0.0} (m_c)^{+0.3}_{-0.0} (a_i)$ | 92.9$^{+7.5}_{-15.1}$ |
| $B_c \to K_1(1400)^+ h_1(1170)$ | 1.6$^{+0.7}_{-0.2} (m_c)^{+1.0}_{-1.1} (a_i)$ | 98.5$^{+0.6}_{-0.9}$ | 1.4$^{+0.6}_{-0.4} (m_c)^{+1.2}_{-0.9} (a_i)$ | 94.6$^{+2.3}_{-3.9}$ |
| | 0.2$^{+0.2}_{-0.0} (m_c)^{+0.3}_{-0.0} (a_i)$ | 93.0$^{+7.3}_{-14.8}$ | 0.5$^{+0.4}_{-0.3} (m_c)^{+0.6}_{-0.2} (a_i)$ | 88.1$^{+6.4}_{-14.4}$ |
| $B_c \to K_1(1270)^+ h_1(1380)$ | 0.9$^{+0.0}_{-0.2} (m_c)^{-0.3}_{-0.1} (a_i)$ | 98.5$^{+0.3}_{-1.4}$ | 1.5$^{+0.4}_{-0.3} (m_c)^{-0.7}_{-0.1} (a_i)$ | 98.6$^{+0.3}_{-1.4}$ |
| | 1.8$^{+0.1}_{-0.1} (m_c)^{+1.1}_{-0.1} (a_i)$ | 98.6$^{+0.8}_{-1.7}$ | 2.8$^{+1.2}_{-0.8} (m_c)^{+1.7}_{-1.3} (a_i)$ | 94.3$^{+1.7}_{-2.9}$ |
| $B_c \to K_1(1400)^+ h_1(1380)$ | 1.5$^{+0.4}_{-0.1} (m_c)^{+0.8}_{-0.7} (a_i)$ | 98.9$^{+2.8}_{-3.9}$ | 0.9$^{+0.3}_{-0.1} (m_c)^{+0.8}_{-0.4} (a_i)$ | 98.5$^{+0.9}_{-1.3}$ |
| | 2.8$^{+1.1}_{-0.8} (m_c)^{+1.6}_{-1.3} (a_i)$ | 94.4$^{+1.7}_{-2.7}$ | 1.7$^{+0.6}_{-0.3} (m_c)^{+1.4}_{-0.7} (a_i)$ | 98.6$^{+0.9}_{-1.1}$ |

TABLE X: Same as Table [11] but for $B_c \to (K_1(1270)^+, K_1(1400)^+)(h_1(1170), h_1(1380))$ decays with $\theta_1 = 10^\circ$ (1st entry) and $\theta_1 = 45^\circ$ (2nd entry).
Of course, the above four relations, i.e., Eqs. (130–133), can also be extracted from the pQCD predictions of BRs for $B_c \rightarrow \overline{K}_1 K^+_1$ decays as presented in Table VII apart from an overall sign.

- At the first sight, it appears that the numerical results for $B_c \rightarrow (f_1, h_1)(V, A)$ (Here, $A$ is either a $^3P_1$ or $^1P_1$ nonstrange axial-vector meson) decays are determined by the mixing angles $\theta_3$ and $\theta_1$, respectively, however, based on Ref. [19], whose values will be eventually determined from $\theta_K$ in $K_{1A}-K_{1B}$ mixing system. Experimentally, it is thus very important to measure the channels precisely involving $K_{1}(1270)$ and/or $K_{1}(1400)$ to determine both of sign and size of the mixing angle $\theta_K$ and reduce the uncertainties of theoretical predictions greatly.

- The pQCD predictions for the $CP$-averaged branching ratios of considered $B_c$ decays vary in the range of $10^{-5}$ to $10^{-9}$. Since the LHC experiment can measure the $B_c$ decays with a branching ratio at $10^{-6}$ level [4], our pQCD predictions for the branching ratios of $B_c \rightarrow a_1^+ \omega$, $b_1 \rho$, $\overline{K}^0(K_1(1270)^+, K_1(1400)^+)$, $(\overline{K}_1(1270)^0, K_1(1400)^0)K^{*+}$, $\rho^+ f_1(1285)$, $a_1^+ b_1^0$, $a_1^+ f_1(1285)$, $a_1^+ h_1(1170)$, $b_1^+ h_1$, $(\overline{K}_1(1270)^0, K_1(1400)^0)K^{*+}$ and $K^{*+} h_1$ decays could be tested in the ongoing LHC experiments.

- It is worth stressing that the theoretical predictions in the pQCD approach still have large theoretical errors induced by the still large uncertainties of many input parameters, e.g. Gegenbauer moments $a_i$. Any progress in reducing the error of input parameters, such as the Gegenbauer moments $a_i$ and the charm quark mass $m_c$, will help us to improve the precision of the pQCD predictions.

V. SUMMARY

In summary, we studied the sixty two charmless hadronic $B_c \rightarrow V A, AA$ decays by employing the pQCD factorization approach based on the $k_T$ factorization theorem systematically. These considered decay channels can only occur via the annihilation type diagrams in the SM and they will provide an important platform for testing the magnitude and decay mechanism of the annihilation contributions and understanding the helicity structure of these considered channels and the content of the axial-vector mesons. Furthermore, these decay modes might also reveal the existence of exotic new physics scenario or nonperturbative QCD effects.

The pQCD predictions for the $CP$-averaged branching ratios and longitudinal polarization fractions are displayed in Tables IX. From our perturbative evaluations and phenomenological analysis, we found the following results:

- The pQCD predictions for the branching ratios vary in the range of $10^{-5}$ to $10^{-9}$. There are many charmless $B_c \rightarrow V A, AA$ decays with sizable branching ratios: $B_c \rightarrow a_1^+ \omega$, $b_1 \rho$, $\overline{K}^0(K_1(1270)^+, K_1(1400)^+)$, $(\overline{K}_1(1270)^0, K_1(1400)^0)K^{*+}$, $\rho^+ f_1(1285)$, $a_1^+ b_1^0$, $b_1^+ a_1^0$, $a_1^+ f_1(1285)$, $a_1^+ h_1(1170)$, $b_1^+ (h_1(1170), h_1(1380))$, $(\overline{K}_1(1270)^0, K_1(1400)^0)K^{*+}$, $b_1^+(h_1(1170), h_1(1380))$, which are with a decay rate at $10^{-6}$ or larger and could be measured at the LHC experiment.

- For $B_c \rightarrow V A, AA$ decays, the branching ratios of $\Delta S = 0$ processes are generally much larger than those of $\Delta S = 1$ ones. Such differences are mainly induced by the CKM factors involved: $V_{ud} \sim 1$ for the former decays while $V_{us} \sim 0.22$ for the latter ones.
• In general, since the behavior for $^1P_1$ meson is much different from that for $^3P_1$ meson, the branching ratios of the pure annihilation $B_c \rightarrow A(^1P_1)(V,A(^1P_1))$ are larger than that of $B_c \rightarrow A(^3P_1)(V,A(^3P_1))$, which can be confronted with the LHC and Super-B experiments.

• The longitudinal contributions play a dominant role in the most of these considered pure annihilation $B_c \rightarrow VA, AA$ decays, which will be tested by the ongoing LHC and forthcoming Super-B experiments in the near future.

• The pQCD predictions for several decays involving mixtures of $^3P_1$ and/or $^1P_1$ mesons are rather sensitive to the values of the mixing angles, which will be tested by the relevant experiments in the future.

• Because only tree operators are involved, the $CP$-violating asymmetries for these considered $B_c$ decays are absent naturally.

• The pQCD predictions still have large theoretical uncertainties, mainly induced by the uncertainties of the Gegenbauer moments $a_i$ in the meson distribution amplitudes. By reducing these uncertainties dramatically, one can improve the precision of the theoretical predictions effectively.

• We here calculated the branching ratios and polarization fractions of the pure annihilation $B_c \rightarrow VA, AA$ decays by employing the pQCD approach. We do not consider the possible long-distance contributions, such as the rescattering effects, although they should be present, and they may be large and affect the theoretical predictions. It is beyond the scope of this work.

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