Mechanical property analysis of joint on the cylindrical waveform topography surface

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Abstract. This paper separates the regular and periodic morphology features from the random tough components in the typical machining surface, extracts the microscopic morphology features on the machined surface, building several joint surface models with regular surface features. This project is based on the combination of cylindrical waveform topography surface and start with the Hertz contact theory, using the analytical solution method to reconstruct the constitution relation of the cylindrical waveform surface contact.

Key words: Join; topography; cylinder model; contact model.

1. Introduction
The field of mechanical dynamics modeling and dynamics prediction of mechanical devices has always been a difficulty of research. The research shows that the stiffness of the joint surface in the machine tool accounts for 60%~80% of the total stiffness, and the contact damping of the joint surface accounts for more than 90% of the total damping of the machine tool [1]. The early comprehensive research on the joint surface of the machine tool is a Japanese scholar Yoshimura Yunxiao [2]. Yi Dongyi [3] simplified the dynamic characteristics of combined surface, based on the mechanical properties of the joint surface area, and extended to the joint surface of different areas. Since then, Huang Yumei et al. [4] of Xi’an University of Technology proposed the design principle of the experimental device combined with the mechanical parameters of the surface and the treatment method of the influencing factors. Zhang Xueliang et al. [5] of Taiyuan University of Science and Technology improved some of the shortcomings in the experimental study on dynamic mechanical properties of mechanical joints and carried out a large number of experiments. In addition, joint surface identification studies have been carried out by dynamic response of structural members containing bonding surfaces [6]. However, the experimental results provided by these researchers are inconsistent and even contradictory. In the study, the processing methods and surface roughness were used as the characterization parameters of the contact surface, but it is difficult to describe the morphology of the joint surface in detail by only two parameters.
In practice, there is regular and periodic characterization machined surfaces [7][8]. Based on this regularity and periodicity, this paper abstracts a concrete ideal surface, taking the contact surface of the cylindrical waveform and the topography as the research object. Starting with the determination of the morphological joint surface, the mechanical properties of the regular joint surface are studied by analytic solution method. Firstly, the contact mechanical properties of a pair of cylinders are studied, and then the research results are extended to the joint case to explore the mechanical properties of the joint under different contact angles which shown in figure 1.

![Fig 1. The contact model between two cylinders](image)

2. The analysis of contact problem between two indefinite length cylinder in elastic area

2.1. Geometric characteristics of contact between two cylinders in elastic range

For contacting between two cylinders, when two non-conformal solids first contact each other, they contact at a point or along a line, as shown in Figure 2. At the same time, they deform elastically near the initial contact point under small loads. This deformation causes them to contact in a limited area much smaller than their own structural size. Generally, it is considered that the shape of each surface is smooth on macro and micro scales. On macro scales, the first and second derivatives of the surface shape function and surface shape function are incompatible and continuous in the contact area. So two cylindrical surfaces near the origin can be approximated by the following expressions.

\[ z_1 = \frac{1}{2R_1} x_1^2 \]
\[ z_2 = \frac{1}{2R_2} x_2^2 \]  

Where \( R_1 \), \( R_2 \) are principal curvature radius of surface \( z_1 \), \( z_2 \) at Origin.

![Fig 2. Initial point and line contact of two cylinders](image)

For convenience of calculation, the coordinates of surface \( z_1 \), \( z_2 \) are transformed into common coordinate system \((x, y)\) (as shown in Figure 3). The clearance between two surfaces can be obtained by formula \( h = z_1 + z_2 \). After coordinate transformation, \( h \) can be formed as

\[ h = Ax^2 + By^2 + Cxy \]

Where A, B and C are all constants related to the radius of principal curvature and \( \alpha \), \( \beta \) and greater than zero.
When the relationship between A and B satisfies Eq. (3), C = 0 can be obtained.

\begin{equation}
B - A = \frac{1}{2} \left[ \left( \frac{1}{R_1} \right)^2 + \left( \frac{1}{R_2} \right)^2 + 2 \left( \frac{1}{R_1 R_2} \right) \cos 2\phi \right]^{\frac{1}{2}}
\end{equation}

Finally, \( h \) can be formed as

\begin{equation}
h = Ax^2 + By^2 = \frac{1}{2R} x^2 + \frac{1}{2R^2} y^2
\end{equation}

Where \( R' \) and \( R'' \) are defined as relative principal curvature radius.

2.2. Contact between two axes intersecting cylinders in elastic range

Under the action of pressure, as shown in Fig. 4 (b), the deformation displacement of the lower cylinder in the contact part is \( \omega_1 \) (rigid body displacement is excluded), and that of the upper cylinder in the contact part is \( \omega_2 \) (rigid body displacement is excluded). The distance between the two points on the Z1 axis and the Z2 axis "far from O" is \( \delta \), which should be greater than the sum of the above-mentioned deformation. It can be seen from Fig. 4 (a) that besides the total deformation of the two cylinders, there is also an original contact gap of \( \delta \). From the geometric relation, \( \delta \) can be formed as

\begin{equation}
\delta = h + \omega_1 + \omega_2
\end{equation}

Eq. (6) can be obtained from equation (6) and the displacement formula of points in the body under the action of distributed pressure on the semi-infinite boundary plane [9].
\[ (k_1 + k_2) \iint \frac{F'}{R} d\xi' d\eta' = \delta - h \quad (6) \]

Where \( k_1 = \frac{1 - \mu_i^2}{\pi E_i}, \quad k_2 = \frac{1 - \mu_i^2}{\pi E_i} \); \( F' d\xi' d\eta' \) is the pressure acting on element \( d\xi' d\eta' \) of the contact part; \( R \) is the distance from any point of the contact surface to the designated point of the contact surface.

When the intersection angle of axes is 0 degrees, that is, when two cylinders with parallel axes contact each other, the contact area is rectangular; the two axes have a certain intersection angle \( \theta \), the contact area \( S \) is elliptical, the long and short axes of the ellipse are \( a \) and \( b \) respectively, and the pressure distribution in the contact surface is a half ellipsoid.

\( P \) is the total pressure exerted on two objects in contact, it is equal to the volume under the semi-ellipsoid. So it can be formed as

\[ P = \iint p dx dy = \frac{2}{3} \pi ab \rho \quad (7) \]

In addition, the elliptical centrifugal rate \( e \) of the contact surface satisfies the following conditions, in which \( K \) and \( E \) are complete elliptical integrals with \( n/m \) modulus and \( e \) is elliptical centrifugal rate. From Eq.(8), the centrifugal rate of the contact area between two cylinders can be calculated.

\[ \frac{B}{A} = 1 - \frac{2(1-e^2)}{e^2} \left[ \frac{K(e)}{E(e)} \right] = \cos \theta \quad (8) \]

Where

\[ K(e) = \int_0^\frac{\pi}{2} \frac{d\phi}{\sqrt{1-e^2 \sin^2 \phi}} \quad E(e) = \int_0^\frac{\pi}{2} \sqrt{1-e^2 \sin^2 \phi} d\phi \quad (9) \]

Therefore, when two cylinders are in contact with each other at a certain intersection angle, the contact area is ellipse and the pressure distribution in the contact area is half ellipsoid. When the ellipse centrifugal rate \( e \) is determined, the length of the half-axis \( a, b \) of the ellipse contact area can be calculated according to the integral results in Eq. (6).

\[ b = \left[ \frac{3E(e)}{2(1-e^2)} \frac{k_1 + k_2}{A + B} \right]^\frac{1}{2} \quad (10) \]

\[ a = b \sqrt{1-e^2} = \sqrt{1-e^2} \left[ \frac{3E(e)}{2(1-e^2)} \frac{k_1 + k_2}{A + B} \right]^\frac{1}{2} \quad (11) \]

Similarly, according to the integral results in equation (6), for the contact between two infinite cylinders in the elastic range, any two points located on the common normal of contact point \( O \) and quite far away from contact point \( O \) are close to each other due to compression. Deformation \( \delta \) of two cylinders can be expressed as

\[ \delta = \frac{2}{3} \frac{K(e)}{\pi a} \frac{k_1 + k_2}{P} \quad (12) \]
For the contact stiffness $A$ between two cylinders, the following formula can be used to calculate the contact stiffness $A$ according to the definition as

$$k(\theta) = \frac{dP}{d\delta}$$  

(13)

2.3. Stiffness of cylindrical waveform joint surface

When calculating the contact static stiffness of two cylinders and the contact static stiffness of cylindrical waveform, the number of contact points in the circular contact area is needed, as shown in Figure 5. Assuming that the intersection angle of the axes is $\theta$, when the rotation angle between the two cylinders is $\theta$, the distance between the central points of the two cylinders is $d = 2R / \sin \theta$. Ignoring the boundary conditions of the circular region, the number of contact points in the region can be calculated as.

$$N = \frac{\pi D^2 \sin \theta}{4R^2}$$  

(14)

Where $D$ denotes the diameter of a circular region,$R$ denotes the radius of a single cylinder and $\theta$ denotes the angle at which a single cylinder begins to rotate parallel to its axis.

(a) Angle $\theta_1$  
(b) Angle $\theta_2$

Fig 5. Number of contact points

Total number of contact points $N$ can be expressed as

$$N = \frac{\pi^2 D^2 \cdot \sin \theta}{16L \cdot R\alpha^2}$$  

(15)

So the pressure $P_n$ of the interface of the cylindrical waveform can be obtained.

$$P_n = \frac{NP}{\pi \left(\frac{D}{2}\right)^2} = \frac{4NP}{\pi D^2}$$  

(16)

The contact stiffness $K$ of the cylindrical surface micro-surface bonding surface contact can be formed as

$$K = k(\theta) \cdot N = \frac{dP}{d\delta} \cdot N$$  

(17)

3. Study on mechanical properties of bonded surface contact

The actual machined surface topography is affected by the main parameters such as processing mode, cutting parameters and tool characteristics. There are both regularity and random factors [8]. However,
these characteristics are not fully utilized in the study of mechanical properties of the joint surface. According to the regular solution method of cylindrical waveform micro-topography theory, the variation law of mechanical properties of joint surface under the influence of different contact angles of axes is explored.

3.1. Effect of axis crossing angle on deformation
For the contact problem of cylindrical waveform micro-morphology of gray cast iron HT250, assuming that L is a length in a period where L=4R, the average Ra of the calculated profile is 3.2μ, Poisson's ratio is 0.26, and the elastic modulus is 1.1e8Pa. The effect of external loading P on the deformation and normal contact stiffness when the intersection angles of axes are 30 degree, 45 degree, 60 degree and 90 degree, as shown in Fig. 6

![Graph](image)

(a) The Relation between Contact Stress and Deformation

![Graph](image)

(b) The Relation between Contact Stress and Contact Stiffness

**Fig 6.** Relation between force and displacement at special angle

From the variation trend of each angle in Fig. 6 (a), for gray cast iron HT250, when the cross angle of axes is equal to 30°, 45°, 60° and 90° respectively, the external loading load increases gradually, and
the deformation increases obviously at the initial contact, and then the growth rate slows down. Under the same stress condition, the overall change of deformation is \( \delta_{90^\circ} > \delta_{60^\circ} > \delta_{45^\circ} > \delta_{30^\circ} \). The reason is that the change rate of contact area is different due to the different cross angles. The smaller the angle is, the faster the contact area will increase. For example, when the minimum cross angle is 0°, the contact area of two infinite semi-cylinders is nearly infinite, so the same load is applied when the same load is applied. From an angle, the amount of deformation will naturally be much smaller. The overall change of contact stiffness, as shown in Fig. 6 (b), \( K_{30^\circ} > K_{45^\circ} > K_{60^\circ} > K_{90^\circ} \). The reason for the change is the same as that of deformation. Because the change rate of contact area caused by different contact angles is different, the greater the contact area is, the greater the stiffness corresponding to the same material. Therefore, the normal contact stiffness will gradually decrease with the increase of cross angle.

![Image](image_url)

**Fig 7.** The relationship between deformation and normal contact stiffness

The change of deformation and normal contact stiffness is directly related to the intersection angle of the axes of the two contact cylinders. Similarly, the change process between deformation and normal contact stiffness has some regularity. From Fig. 7, it can be seen that the normal stiffness decreases with the increase of the cross-angle when the deformation is constant; while the stiffness remains unchanged, the deformation increases with the increase of the angle; if the cross-angle is constant, the stiffness is positively correlated with the deformation, but the change rate of the stiffness decreases gradually. In a word, the normal contact stiffness decreases and the deformation increases with the increase of the cross angle. There is a negative correlation between the two, which proves the reliability of the conclusion.

![Image](image_url)

**Fig 8.** Relations between arbitrary angles and deformations
In the case of increasing cross angle, the conclusion that the deformation will increase correspondingly is deduced from several special angles. Figure 8 shows the variation trend of the deformation at any angle $(0 \leq \pi / 2)$. Under the condition of constant external load and constant change of axis intersection angle, for gray cast iron HT250, the axis intersection angle increases continuously when two cylinders contact, and the deformation is also increasing. The increasing trend will be more obvious at the initial contact, and the increasing trend will be slower at the later stage, and the sensitivity of deformation to axis intersection angle changes will decrease.

![Figure 8](image)

(a) Local magnification

**Fig 9.** The relationship between angle and contact stiffness

In Fig. 9 (a), the contact stiffness is inversely proportional to the change of the intersection angle of the axis. When the initial contact angle is small, the stiffness is larger. With the increase of the angle, the contact stiffness decreases rapidly, and tends to be stable at $\theta=10^\circ$. Because the contact area of two infinite cylinders will become infinite when the intersection angle is close to 0° or small, the stiffness will be much greater than that of other angles. Infinite cylindrical contact is an ideal case. When the cross angle of the cylinder becomes larger, the influence of the length on the contact will be weakened. Therefore, the stiffness change after intercepting the cross angle of $10^\circ$, as shown in Figure 9 (b), will be closer to the actual contact process of the two cylinders. The intersection angle of axes has a
significant effect on the contact stiffness. The maximum difference of stiffness can be 6 to 7 times with the change of angle in Fig. 9 (b).

3.2. Stiffness of Cylindrical Waveform Microstructure
For the contact problem of cylindrical waveform micro-morphology of gray cast iron HT250, assuming that L is a period length, then L = 4R, the diameter of the circular contact area is D = 1cm, the average Ra of the calculated profile is 3.2um, Poisson's ratio \( \mu = 0.26 \), and the elastic modulus E = 1.1e8Pa. According to the calculation of the number of contact points, the magnitude of load \( P \) in multi-wave contact is the product of the number of contact points and the load in single-wave contact, it is \( P_c = P_s \times N \). While the surface pressure can be expressed as \( P_n = (P_s \times N) \times S \). Therefore, the influence of the cross angle of axes at 30°, 45° and 60° on the deformation as shown in Fig. 10.

![Graph](image)

(a) Surface pressure and deformation

![Graph](image)

(b) Contact stress and stiffness

Fig 10. Relation between Pressure and Deformation under Special Inclusion Angle
Under the same loading conditions, the overall variation of deformation is $\delta_{90^\circ} > \delta_{60^\circ} > \delta_{45^\circ} > \delta_{30^\circ}$ as shown in Fig. 10 (a), so the deformation will increase with the increase of the angle, which is consistent with the results of single-wave contact between two cylinders.

From the variation trend of each angle in Fig. 10 (b), when the intersection angle of axes is equal to $30^\circ, 45^\circ, 60^\circ$ and $90^\circ$ respectively, the load at contact increases gradually, and the stiffness increases obviously at the initial contact, then the growth rate slows down. The difference between single-wave contact and two cylinders is that under the same load, the stiffness of the two joints is much greater than that of single-wave contact, but the variation law is the same.

4. Conclusion

(1) The contact stiffness and deformation of cylindrical waveform micro-surface interface will increase with the increase of contact load.

(2) For the contact of two cylinders in the elastic range, under the same loading conditions and with the increasing angle, the deformation is also increasing. The increasing trend will be more obvious in the initial contact, but slower in the later contact, and the sensitivity of the deformation to the change of the intersection angle of the axis decreases.

(3) The contact angle is negatively correlated with the normal stiffness. The fundamental reason is that the change rate of the contact area is different due to the different contact angle. So the stiffness of the same material joint increases with the increase of the contact area, so the normal contact stiffness decreases with the increase of the cross angle. Moreover, the intersection angle of axes has a significant effect on the contact stiffness, and the maximum difference of stiffness can be multiplied by the change of the angle.

(4) The cylindrical contact stiffness can be effectively improved by reducing the roughness of the contact surface and the cross angle of the contact.

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