Status of $B$-mixing in the standard model and beyond

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Abstract. The theoretical methods underlying predictions of mixing quantities $\Delta M_q$ and $\Delta \Gamma_q$ are well understood nowadays. Currently the uncertainties are dominated by nonperturbative matrix elements of local $|\Delta B| = 2$ operators of dimension 6 and 7, which require preciser calculations, hopefully with lattice methods in the future. Global fits show good support of the standard model, restricting potential effects of new physics to be small. An improved measurement of $\Delta \Gamma_d$ will be helpful to further analyse the dimuon charge asymmetry as measured by DØ.

1. Introduction

The phenomenon of neutral $B_q$ meson mixing ($q = d, s$) proceeds in the standard model (SM) only at the loop level and provides important constraints in the determination of elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, $V_{tb}V_{tq}^{\ast}$. Moreover, the loop suppression makes it sensitive to new sources of CP and flavour violation. It affects CP violation in all neutral $B_q$ meson decays in the form of mixing-induced CP asymmetries. In the absence of reliable methods for the calculation of strong (CP-conserving) phases of decay amplitudes, mixing thus provides an important additional access to CP violating parameters.

The time evolution of the two flavour eigenstates $B_q$ and $\overline{B}_q$ is described by

$$i \frac{d}{dt} \left( \frac{|B_q(t)|}{|\overline{B}_q(t)|} \right) = \left( M_q^q - \frac{\Gamma_q^q}{2} \right) \left( \frac{|B_q(t)|}{|\overline{B}_q(t)|} \right).$$

Under the assumption of CPT-invariance, the elements of the $2 \times 2$ mass- and decay-matrices, $M_q$ and $\Gamma_q$, fulfill the relations $M_{11}^q = M_{22}^q = M_{B_q}$, $M_{12}^q = (M_{21}^q)^\ast$ and similarly for $\Gamma_q$. The diagonalisation yields the light and heavy mass eigenstates $|B_{L,H}\rangle = p|B\rangle \pm q|\overline{B}\rangle$ and their averaged masses, $M_{B_q}$ and decay widths $\Gamma_{B_q}$. The according differences, $\Delta M_q$ and $\Delta \Gamma_q$, as well as the flavour-specific CP-asymmetries, $a_{sl}^q$

$$\Delta M_q = M_{H}^q - M_{L}^q = 2|M_{12}^q| + \ldots \geq 0,$$

$$\Delta \Gamma_q = \Gamma_{L}^q - \Gamma_{H}^q = 2|\Gamma_{12}^q| \cos(\zeta_q) + \ldots \geq 0,$$

$$a_{sl}^q = \frac{\Gamma(B_q \to f) - \Gamma(\overline{B}_q \to \overline{f})}{\Gamma(B_q \to f) + \Gamma(\overline{B}_q \to \overline{f})} = \frac{\Gamma_{12}^q}{M_{12}^q} \sin(\zeta_q) + \ldots.$$
are determined by the off-diagonal elements $M_{12}^q$ and $\Gamma_{12}^q$ and their relative phase $\zeta_q = \arg(-M_{12}^q/\Gamma_{12}^q)$. Above the dots stand for terms of $O(1/8)\Gamma_{12}^q/M_{12}^q|^{2}\sin^2\zeta_q$, which is smaller than $10^{-6}$ in the SM for both systems $q = d, s$ [1].

The structure of electroweak (EW) interactions in the SM allows for the construction of an effective field theory, in which the off-diagonal element $M_{12}^q$ is given by $|\Delta B| = 2$ operators, whereas $\Gamma_{12}^q$ by $|\Delta B| = 1$ current-current operators, which again can be matched on local $|\Delta B| = 2$ operators by means of heavy quark expansion (HQE).

2. Standard model
The prediction of $M_{12}^q$ is well understood in the SM,

$$M_{12}^q = \frac{G_F^2 M_B}{12\pi^2}(V_{tb}V_{tq}^*)^2 m_W^2 S_0(x_l) \eta B_{B_s} f_B^2 f_{B_q}^2.$$  \hspace{1cm} (3)

Contributions of the EW interaction at short-distance scales of the order of the $W$-boson mass, $m_W$, are decoupled and are contained at leading order (LO) in $S_0(x_l = m_t^2/m_W^2)$ [2] depending also on the top-quark mass $m_t$, at next-to-LO (NLO) QCD $\eta$ [3], whereas tiny NLO EW corrections [4] are usually neglected. The accuracy of the determination of CKM elements is strongly limited by the uncertainties of the matrix element of a single local operator \(\langle B_q | (\bar{b}q) V_A (\bar{b}q) V_A | B_L \rangle = 8/3 f_{B_L}^2 B_R^* M_B \) in terms of the $B_q$-meson decay constant, $f_{B_q}$ and the associated bag parameter $B_{B_q}$. Both are nowadays calculated with the help of lattice QCD techniques — see [5] — and the current precision of latest world averages [6] ($N_f = 2 + 1$) are summarised in table 1, together with the arising uncertainty on $\Delta M_q$. Since the small uncertainty of the decay constant is currently dominated by a single lattice calculation [7, 8], it would be reassuring to have confirmations from other lattice groups in the future. Beyond the SM, NP scenarios can give rise to additional $|\Delta B| = 2$ operators. The renormalisation group evolution of their Wilson coefficients from $m_W$ to $m_b$ are known up to NLO $[9, 10]$ in QCD, whereas the according bag parameters are currently only available from the single unquenched lattice ($N_f = 2$) calculation [11] — for quenched results see [12] — and some preliminary ($N_f = 2 + 1$) results are reported in [13].

The calculation of $\Gamma_{12}^q$ is based on HQE [14] giving rise to a double series in $\lambda = \Lambda_{QCD}/m_b \sim 0.15$ and the strong coupling, $\alpha_s$,

$$\Gamma_{12}^q = \lambda^3 \left( \Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \ldots \right) + \lambda^4 \left( \Gamma_4^{(0)} + \ldots \right) + \lambda^5 \left( \Gamma_5^{(0)} + \ldots \right), \quad \hspace{1cm} (4)$$

which is also applied to calculate of the total and partial inclusive decay widths [15]. The individual contributions shown in (4) can be found in [16, 17, 18, 19, 20, 21]. The expansion of $\Delta\Gamma^q$ (and also $\Gamma^q$) shows a convergent behaviour and predictions are in satisfactory agreement with measurements — see figure 1 for the $B_s$ system. Especially lifetime ratios can be predicted quite precisely $\tau_{B_s}/\tau_{B_d} \in [0.996, 1.000]$ [1], since they are free of hadronic uncertainties, and
agree well with the measured value $0.995 \pm 0.006$. Concerning $\Delta \Gamma_q$, current measurements [22] and the according SM predictions [1] are

$$
\begin{align*}
\Delta \Gamma_d|_{\text{Exp}} &= (0.0059 \pm 0.0079) \text{ps}^{-1}, \\
\Delta \Gamma_d|_{\text{SM}} &= (0.0029 \pm 0.0007) \text{ps}^{-1}, \\
\Delta \Gamma_s|_{\text{Exp}} &= (0.091 \pm 0.008) \text{ps}^{-1}, \\
\Delta \Gamma_s|_{\text{SM}} &= (0.087 \pm 0.021) \text{ps}^{-1}.
\end{align*}
$$

(5)

A comprehensive summary of theoretical uncertainties [1] shows that the largest sources are due to matrix elements of local $|\Delta B| = 2$ operators of dimension 6 and 7, about 5% and 17%, respectively. Especially bag factors of dim-7 operators are currently known only with low precision from sum rule calculations [23], but no dedicated lattice studies are pursued at the moment. Further, with most recent values of $f_{B_q}$ the according relative uncertainty is the same as for $\Delta M_q$ given in table 1. Another large source of uncertainty is the factorisation scale dependence on $\mu_b \sim m_b$ with about 8%.

Whereas experimental uncertainties are below theoretical ones for $\Delta \Gamma_s$, $\Delta \Gamma_d$ is not well known yet. A better measurement of the latter can also help to further investigate the deviation of the dimuon charge asymmetry measured by DØ [24]. Although the interpretation of this measurement within the SM has been refined lately [25]

$$
A_{\text{CP}} = C_d a_d^d + C_s a_s^s + C_{\Delta \Gamma_d} \frac{\Delta \Gamma_d}{\Gamma_d} + C_{\Delta \Gamma_s} \frac{\Delta \Gamma_s}{\Gamma_s},
$$

(6)

it still points towards large deviations of about $3.6\sigma$ from the SM predictions of $a_q^q$ and/or $\Delta \Gamma_d$ [24, 26]. The numerical values of the coefficients $C_{d,s,\Delta \Gamma}$ can be extracted from [25, 24] and $C_{\Gamma_s}$ turns out to be negligible.

Figure 1. Combined region of $\Delta \Gamma_s$ vs. $\Gamma_s$ from different channels. The $(B_s \rightarrow \text{CP-odd})$ decays allow to measure directly $\Gamma_s^H$, whereas $(B_s \rightarrow \text{CP-even})$ decays $\Gamma_s^L$. Also shown the SM prediction of $\Delta \Gamma_s$ [1]. For comparison, a “prediction” of $\Gamma_s = (0.660 \pm 0.004) \text{ps}^{-1}$ might be obtained when using the prediction of $\tau_{B_s}/\tau_{B_d}$ [1], and the current measured world average of $\tau_{B_d} = (1.519 \pm 0.005) \text{ps}$ [22, 26].

3. Beyond the standard model

Effects of NP can be model-independently parametrised with two complex-valued parameters $\Delta_q$ and $\tilde{\Delta}_q$ as

$$
\begin{align*}
M_{12}' &= M_{12}^{\text{SM}} \Delta_q, \\
\Gamma_{12}' &= \Gamma_{12}^{\text{SM}} \tilde{\Delta}_q, \\
\Delta_q &= |\Delta_q| e^{i\phi_{\Delta_q}}, \\
\tilde{\Delta}_q &= |\tilde{\Delta}_q| e^{i\phi_{\tilde{\Delta}_q}}.
\end{align*}
$$

(7)
Figure 2. The allowed regions of the NP parameters $\Delta_d$ (left) and $\Delta_s$ (right), as determined in a global CKM-fit [27].

The global fits of the CKM elements in the framework of the SM take into account also $\Delta M_q$. Beyond the SM, recent constraints on the scenario of NP only in $M_{s12}$, i.e. $\Delta_d \neq 0$, $\Delta_s \neq 0$ and also permitting NP independently in the $K\bar{K}$-system, are shown in figure 2. A $1.5\,\sigma$ deviation is obtained for the 2-dimensional SM hypothesis $\Delta_d = 1$ and $0.0\,\sigma$ for $\Delta_s = 1$. The SM point $\Delta_d = \Delta_s = 1$ is disfavoured by $1\,\sigma$, compared to the $3.6\,\sigma$ in the year 2010 [28]. The highest pull value of $3.4\,\sigma$ arises for $A_{CP} = C_d a_{sl}^d + C_s a_{sl}^s$, i.e. setting $C_{\Gamma} = 0$. Large deviations from the SM are by now excluded, whereas at $1\,\sigma$ CL deviations of $O(40\%)$ and $O(20\%)$ are still allowed

$$|\Delta_d| = 0.81^{+0.27}_{-0.10}, \quad \phi_d^\Delta = (-7.9^{+5.0}_{-2.3})^\circ,$$

$$|\Delta_s| = 0.97^{+0.20}_{-0.08}, \quad \phi_s^\Delta = (-0.3^{+5.1}_{-5.3})^\circ.$$ (8)

An important role in this analysis plays the LHCb measurement of mixing-induced CP asymmetry $S(B \to J/\psi\phi)$ [29, 30], which does not permit an explanation of the DØ result for $A_{CP}$ in terms of NP in $M_{s12}$. Instead, from the best fit point of $M_{s12}^0$ follows the prediction $a_{sl}^d = (-2.46^{+0.63}_{-0.45}) \cdot 10^{-3}$, which is enhanced by a factor of almost 8 over the SM prediction [1]. Improved determinations of $a_{sl}^d$ at LHCb and Belle II are needed to further elucidate this issue. In a more general NP scenario, also $\Gamma_{12}^d$ might be modified. Whereas $\Gamma_{12}^d$ is determined in the SM by Cabibbo-favoured tree-level exchange $b \to c\bar{c}s$ that gives rise to a large inclusive decay rate $Br(b \to c\bar{c}s) = (23.7 \pm 1.3\%)$, $\Gamma_{12}^d$ is dominated by $b \to c\bar{c}d$, which leads only to small $Br(b \to c\bar{c}d) = (1.31 \pm 0.07\%)$ [31]. Therefore larger deviations from the SM prediction in $\Gamma_{12}^d$ are at present less constrained, compared to $\Gamma_{12}^s$ [32, 33]. Modifications of $\Gamma_{12}^d$ affect both $\Delta \Gamma_d$ and $a_{sl}^d$, being presently not well measured and thus potentially alleviating current tensions in $A_{CP}$.

Some possible NP scenarios include violations of CKM unitarity [34, 33], large effects in almost unconstrained channels like $b \to \tau\bar{d}$ or modifications of the current-current operators $b \to c\bar{c}d, c\bar{u}d, u\bar{u}d$ [33]. The latter two scenarios have been investigated in a model-independent fashion based on $|\Delta B| = 1$ dim-6 operators [33]. Concerning $b \to \tau\bar{d}$, model-independent constraints on the complete set of dim-6 operators $O_i \sim [d\Gamma_A b][\overline{\tau\Gamma_B} \tau]$ ($\Gamma_A = P_A, \gamma_\mu P_A, \sigma_{\mu\nu} P_A$
with $P_A = (1 \pm \gamma_5)/2$ have been derived on the according Wilson coefficients. In the case of scalar Wilson coefficients direct bounds from $Br(B_d \rightarrow \tau\tau)$ cannot preclude enhancements of about 60% of $\Delta\Gamma_d$. Only indirect constraints from $Br(B \rightarrow X_d\gamma)$ (circle-shaped), $a_{d}^{s}$ (inner star-shaped) and dim-8 contributions to $\sin(2\beta)$ (outer star-shaped) [33]. Contours in $\Delta\Gamma_d/\Delta\Gamma_{d}^{SM}$ [right].

4. Conclusion

The mixing of neutral $B_q$ mesons ($q = d, s$) plays an important role in CP violating phenomena, tests of the picture of quark mixing in the standard model (SM) and search for new physics (NP). Currently the largest uncertainties in predictions of $\Delta M_q$ and $\Delta\Gamma_q$ are due to hadronic parameters: the decay constants $f_{B_q}$ and bag factors of various $|\Delta B| = 2$ operators. There is encouraging progress in lattice predictions concerning $\Delta M_q$ in the standard model, dominated by $f_{B_q}$ and a single bag factor. However, NP searches require the knowledge of the bag factors of all $|\Delta B| = 2$ operators of dimension 6, which receives currently much less attention. Moreover, $\Delta\Gamma_q$ also depends strongly on bag factors of $|\Delta B| = 2$ operators of dimension 7 that cause at present the largest uncertainties, but for which so far exist only sum rule estimates. Recent global CKM fits that account for NP in $M_{B_d}^0$ only, exclude now larger deviations then $\mathcal{O}(20\%)$ and $\mathcal{O}(40\%)$ in the $B_s$ and $B_d$ system from the SM, respectively. Only the DØ measurement of the like-sign dilepton charge asymmetry has a pull value of about 3$\sigma$. This tension could be caused by non-standard contributions to $\Gamma_{B_d}^{SM}$ for which at present, large NP contributions cannot be excluded. A direct measurement of $\Delta\Gamma_{d}$, more precise data on the individual semi-leptonic CP asymmetries $a_{d}^{s,d}$, but also improved experimental determinations of rare and radiative $b \rightarrow d$ decays would shed light on this issue and should be pursued with

![Figure 3. The allowed regions for the NP contributions of the colour-allowed current-current $b \rightarrow \tau\tau$ operator [left] from $Br(B \rightarrow X_d\gamma)$ (circle-shaped), $a_{d}^{s}$ (inner star-shaped) and dim-8 contributions to $\sin(2\beta)$ (outer star-shaped) [33]. Contours in $\Delta\Gamma_d/\Delta\Gamma_{d}^{SM}$ [right].](image-url)
vigour in the future.

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References
[1] Lenz A and Nierste U 2011 (Preprint 1102.4274)
[2] Inami T and Lim C 1981 Prog.Theor.Phys. 65 297
[3] Buras A J, Jamin M and Weisz P H 1990 Nucl.Phys. B347 491–536
[4] Gambino P, Kwiatkowski A and Pott N 1999 Nucl.Phys. B544 532–556 (Preprint hep-ph/9810400)
[5] Jüttner A (Preprint this proceedings)
[6] Aoki S, Aoki Y, Bernard C, Blum T, Colangelo G et al. 2013 (Preprint 1310.8555)
[7] McNeile C, Davies C, Follana E, Hornbostel K and Lepage G P 2012 Phys.Rev. D85 031503 (Preprint 1110.4510)
[8] Na H, Monahan C J, Davies C T, Horgan R, Lepage G P et al. 2012 Phys.Rev. D86 034506 (Preprint 1202.4914)
[9] Buras A J, Misiak M and Urban J 2000 Nucl.Phys. B586 397–426 (Preprint hep-ph/0005183)
[10] Buras A J, Jager S and Urban J 2001 Nucl.Phys. B605 600–624 (Preprint hep-ph/0102316)
[11] Carrasco N et al. (ETM Collaboration) 2014 JHEP 1403 016 (Preprint 1308.1581)
[12] Becirevic D, Gimenez V, Martinelli G, Papinutto M and Reyes J 2002 JHEP 0204 025 (Preprint hep-lat/0110091)
[13] Chang C, Bernard C, Bouchard C, El-Khadra A, Freeland E et al. 2013 (Preprint 1311.6820)
[14] Shifman M A and Voloshin M 1985 Sov.J.Nucl.Phys. 41 120
[15] Bigi I I, Uraltsev N and Vainshtein A 1992 Phys.Lett. B293 430–436 (Preprint hep-ph/9207214)
[16] Beneke M, Buchalla G and Dunietz I 1996 Phys.Rev. D54 4419–4431 (Preprint hep-ph/9605259)
[17] Beneke M, Buchalla G, Greub C, Lenz A and Nierste U 1999 Phys.Lett. B459 631–640 (Preprint hep-ph/9808385)
[18] Beneke M, Buchalla G, Lenz A and Nierste U 2003 Phys.Lett. B576 173–183 (Preprint hep-ph/0307344)
[19] Ciuchini M, Franco E, Lubicz V, Mescia F and Tarantino C 2003 JHEP 0308 031 (Preprint hep-ph/0308029)
[20] Lenz A and Nierste U 2007 JHEP 0706 072 (Preprint hep-ph/0612167)
[21] Badin A, Gabbiani F and Petrov A A 2007 Phys.Lett. B653 230–240 (Preprint 0707.0294)
[22] Amhis Y et al. (Heavy Flavor Averaging Group) 2012 (Preprint 1207.1158)
[23] Mannel R, Pecjak B and Pivovarov A 2007 (Preprint hep-ph/0703244)
[24] Abazov V M et al. (D0 Collaboration) 2014 Phys.Rev. D89 012002 (Preprint 1310.0447)
[25] Borissov G and Hoe_neisen B 2013 Phys.Rev. D87 074020 (Preprint 1303.0175)
[26] Bertram I (Preprint this proceedings)
[27] Lenz A, Nierste U, Charles J, Descotes-Genon S, Lacker H et al. 2012 Phys.Rev. D86 033008 (Preprint 1203.0328)
[28] Lenz A, Nierste U, Charles J, Descotes-Genon S, Jantsch A et al. 2011 Phys.Rev. D83 036004 (Preprint 1008.1593)
[29] Aaij R et al. (LHCb collaboration) 2013 Phys.Rev. D87 112010 (Preprint 1304.2600)
[30] DeBruyn K (Preprint this proceedings)
[31] Klimov F, Lenz A and Rauh T 2013 Nucl.Phys. B876 31–54 (Preprint 1305.5390)
[32] Bobeth C and Haisch U 2013 Acta Phys.Polon. B44 127–176 (Preprint 1109.1826)
[33] Bobeth C, Haisch U, Lenz A, Pecjak B and Tetlalmatzi-Xolocotzi G 2014 JHEP 1406 040 (Preprint 1404.2531)
[34] Botella F, Branco G, Nebot M and Sanchez A 2014 (Preprint 1402.1181)