Dynamical brane creation and annihilation via a background flux

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We study the dynamical Myers effect by allowing the fuzzy (or the dynamical dielectric brane) coordinates to be time dependent. We find three novel kinds of the dynamical spherical dielectric branes depending on their respective excess energies. The first represents a dynamical spherical brane carrying a negative excess energy (having a lower bound) with its radius oscillating periodically between two given non-zero values. The second is the one with zero excess energy and whose time dependence can be expressed in terms of a simple function. This particular dynamical spherical configuration represents the dielectric brane creation and/or annihilation like a photon in the presence of a background creating an electron-position pair and then annihilating back to a photon. The third is the one carrying positive excess energy and the radius can also oscillate periodically between two non-zero values but, unlike the first kind, it passes zero twice for each cycle. Each of the above can also be interpreted as the time evolution of a semi-spherical D-brane–anti semi-spherical D-brane system.

dynamical Myers effect, D-branes, string/M-theory

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1 Introduction

The discovery of static SUSY preserving BPS p-branes in string theories helps establish various duality relations among different string theories and the eleven-dimensional supergravity and leads to the unification of these theories to a big not yet established theory called M-theory. This latter theory is non-perturbative in nature and as such our understanding of it is so far still very limited, in spite of much progress made during the last three decades, mainly due to our lack of ability in dealing with non-perturbative phenomena in general. It is fair to say that the current efforts in string/M theory community are largely still in the stage of theoretical data collection for the M-theory. Any thing non-trivial and going beyond the static BPS configurations is in general hard and worth trying. Among these, the studies of D-brane-anti D-brane systems and the related tachyon condensations made by Sen [1, 2] and others stand out. The recent astronomical observation of the accelerating expansion of our universe which leads to a small positive cosmological constant requires also the effort in string/M theory community to study possible dynamical processes which might provide an explanation of this.

We here try to make a small step in this direction by studying the dynamical creation and/or annihilation of a higher dimensional spherical D-brane from lower dimensional seed

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BPS D-branes via a R-R flux in a given background. This is motivated by the following: Myers studied the creation of a static dielectric brane from neutral objects like D0 branes in the presence of 4-form RR flux, the analog of the static dielectric effect in ordinary electromagnetism. Can such a dynamical brane be created and what are its properties and the corresponding physical effect? We know that a photon with sufficient energy can create an electron-positron pair and then annihilate back to a photon but this requires the presence of a background field to conserve the energy-momentum. Can we realize such an analog in the present case? As we will see, we can indeed do so in addition to many other novel physical effects. This work provides not only a dynamical process of the dielectric brane creation and/or annihilation but also a non-trivial concrete realization of the dynamics of a Dp-brane-anti Dp-brane system. This work might also provide a non-trivial concrete realization of the dynamics of a Dp-brane/anti Dp-brane system. This work provides not only a dynamical process of the dielectric brane creation and/or annihilation but also a non-trivial concrete realization of the dynamics of a Dp-brane-anti Dp-brane system. This work might also provide means for viable cosmological model building.

To make the basic ideas behind the current work clear, we in this paper limit our study on generating a spherical D2 brane from N BPS seed D0-branes via a constant R-R four-form flux. In addition to the R-R flux, we will consider Minkowski flat metric which satisfies the bulk equations of motion to the leading order once certain constraints (mentioned later on) are satisfied for large N. Extending the current work to other branes and other backgrounds is straightforward.

This paper is organized as follows. In sect. 2, we give a brief description of the basic setup for the dynamical brane configurations considered in this paper. We discuss in sect. 3 three separate dynamical brane configurations, with each depending on the so-called excess energy and representing a spherical D2 brane, from N seed D0-branes via a constant R-R 4-form flux. We end up this paper with a discussion in sect. 4.

2 The basic setup

Myers [3] made a remarkable observation a while ago that lower dimensional D-branes can couple to the RR potentials of higher ranks, associated with higher dimensional D-branes, if the non-Abelian effect of these lower dimensional D-branes is taken into consideration. As such, he found that a static charge neutral (but carrying a R-R electric-like dipole moment) spherical D2-brane can be produced from the D0-branes in the presence of a constant R-R 4-form flux background1). We here try to extend Myers and others’ work and find that Myers static spherical D2-brane is a special case of a large class of dynamical spherical D2-branes and there are rich dynamical properties associated with them. Actually there are three kinds of these dynamical branes depending on the excess energies above the original D0-branes for which we now discuss one by one in order.

To make things concrete, let us consider first the matrix theory description of N D0 branes in the presence of a constant RR 4-form flux and a flat bulk spacetime with \( G_{\mu\nu} = \eta_{\mu\nu}, B_{\mu\nu} = 0, \phi = \text{constant} \), following refs. [3, 4]. The corresponding Lagrangian up to the order of \( O(x^2) \) is

\[
L = -T_0N + \frac{T_0 l^2}{2} \text{Tr}(\Phi'^2) + \frac{T_0 l^2}{4} \text{Tr}[\Phi', \Phi]^2 \nonumber + \frac{i l^2 T_0}{3} \text{Tr} \Phi' \Phi' \Phi'^2 F_{a,b}. \tag{1}
\]

Here we choose already the static gauge \( t = \phi^0, \lambda = 2\pi l^2 \) with \( l \) the string length scale, \( T_0 = 2\pi/[g_s(2\pi l)^{p+1}] \) the D-brane tension with \( g_s = e^\phi \) the string coupling. As in ref. [3], we take

\[
F_{a,b} = \begin{cases} -2f \epsilon_{ijk} & \text{for } i, j, k \in \{1, 2, 3\}, \\ 0 & \text{otherwise,} \end{cases} \tag{2}
\]

with \( f \) carrying dimensions of \( l^{-1} \). The non-trivial part of the equations of motion along 1, 2, 3 directions from the above Lagrangian is

\[
\Phi' + [(\Phi', \Phi')^2, \Phi] + 2i f \Phi' \Phi'^2 \epsilon_{ijk} = 0, \tag{3}
\]

where we have made use of eq. (2) for \( F_{a,b} \). As in ref. [3], we take \( \Phi' \) for \( i = 1, 2, 3 \) in the \( N \times N \) irreducible representation of \( SU(2) \) but instead we are seeking a general dynamical charge neutral (but carrying a time dependent RR dipole moment) spherical D2 brane configuration. In other words, we take \( \Phi' = u(t)J^i \) with \( u(t) \) a function of time in general and the \( SU(2) \) generators \( J^i \) satisfying \( [J^i, J^j] = i \epsilon_{ijk}J^k \). Then the above equations are reduced to a single second order non-linear differential equation:

\[
\ddot{u} - 2fu'^2 + 2u^3 = 0, \tag{4}
\]

which can be derived either from the following Lagrangian:

\[
L = -T_0N + \frac{\lambda^2 T_0 N(N^2 - 1)}{8} \left( \dot{u}^2 - u^4 + \frac{4}{3} f u'^2 \right), \tag{5}
\]

or the Hamiltonian

\[
H = T_0N + \frac{\lambda^2 T_0 N(N^2 - 1)}{8} \left( \dot{u}^2 + u^4 - \frac{4}{3} f u'^2 \right). \tag{6}
\]

In having the above, we have set \( \Phi' = u(t)J^i \) for \( i = 1, 2, 3 \) and \( \Phi' \) to be diagonal constant matrices for \( i \neq 1, 2, 3, \) for

1) The magnetic analogue of this was later discussed in ref. [4].
simplicity\textsuperscript{2)}, in the original action (1). We have also made use of
\[
\text{Tr}(f)^2 = \frac{1}{12} N(N^2 - 1),
\]
for \(i = 1, 2, 3\), respectively. We can read from either eqs. (5) or (6) the following effective potential for \(u\) as:
\[
V = \frac{\lambda^2 t_0 N(N^2 - 1)}{8} \left( u^4 - \frac{4}{3} f u^3 \right).
\]

Figure 1 shows the characteristic behavior of the above potential with two extremal points at \(u = 0\) and \(u = f\), respectively, for which the former is in fact an (decreasing) inflection point while the latter gives the minimum of the potential. In addition to \(u = 0\), the other zero point of the potential occurs at \(u = 4 f/3\) as is evident from the expression of the potential.

Similar to the static case discussed in ref. [3], we expect that eq. (4) can also be derived from the worldvolume action of a single spherical dynamical D2 brane with a worldvolume \(U(1)\) flux describing \(N\) D0 branes \(F_{\phi \theta} = N \sin \theta/2\) moving in a flat Minkowski background with the coordinates \(x^i\) for \(i = 1, 2, 3\) expressed in spherical polar ones as:
\[
d\tilde{s}^2 = -dr^2 + dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + \sum_{i=4}^9 (dx^i)^2.
\]

In doing so, we also take the static gauge \(a^\theta = t, \sigma^1 = \theta, \sigma^2 = \phi\) and take the radial coordinate \(r\) to be time dependent and \(x^i (i = 4, \cdots, 9)\) to be constant. We further need to take the motion to be non-relativistic, i.e., \(r \ll 1\) and \(r^2 \ll \lambda N/2\) which can indeed be satisfied consistently as we will see later on. Notice also now that \(F_{\text{reih}} = -2 f r^2 \sin \theta \to C_{\phi \theta} = 2 f r^2 \sin \theta/3\). With all these, the equation of motion for \(r\) can indeed be reduced to the one for \(u(t)\) above if we identify \(r(t) = \lambda(t) u(t)/2\). So \(r \ll \sqrt{\lambda N/2}\) implies that \(u(t) \ll \sqrt{\lambda N}/2\) which further implies \(f \ll O(1/ \sqrt{\lambda N})\) once the solution for \(u(t)\) is considered.

Let us comment on a few things before we proceed. In general, our chosen flat background with a constant RR 4-form flux does not satisfy the bulk equations of motion as mentioned in ref. [3]. The equivalence of the above two descriptions puts constraints on both \(u(t)\) and the parameter \(f\). In particular, for large \(N\), both \(u(t)\) and \(f\) are tiny but the size of the spherical D2 brane can still be large since we need only \(r \ll \sqrt{\lambda N/2}\) as pointed out in ref. [3]. The smallness of \(u(t)\) makes our ignoring higher order terms in the matrix action justified while that of \(f\) implies that the flat background satisfies the bulk equations of motion to the leading order since the metric correction due to the presence of the 4-form is now \(\delta h_{\mu \nu} \ll 1\) as discussed in ref. [4]. We therefore should take large \(N\) to validate our following discussion.

### 3 The dynamical brane configurations

Our focus now is eq. (4). Integrating it once, we have
\[
\dot{u}^2 = C + \frac{4}{3} f u^3 - u^4,
\]
where the integration constant \(C\) is actually the so-called reduced excess energy \(\Delta E_t \equiv 8 (H - T_0 N) (\lambda^2 T_0 N (N^2 - 1)) = u^2 + u^4 - 4 f u^3/3\) of the system as can be seen from the Hamiltonian (6) and is no less than \(-f^2/3\) from the above equation given that \(\dot{u}^2(t) \geq 0\) and the reduced potential \(V_t \equiv 8 V/ (\lambda^2 T_0 N (N^2 - 1)) = u^4 - 4 f u^3/3\) has a minimum value of \(-f^2/3\) at \(u = f\). Note that \(u = 0\) gives the extremes of the \(u(t)\) with respect to time \(t\). Except for \(C = -f^2/3\) corresponding to Myers static solution with \(u = 0, \ddot{u} = 0 = u = f\), and \(C = 0\) corresponding to a degenerate root \(u(t) = 0\) and the other \(u(t) = 4 f/3\), we have in general four roots of the quartic equation obtained by setting \(\dot{u} = 0\) in eq. (10) and two of them denoted as \(a\) and \(b\) with \(a > b\) are real and the other two are complex conjugate to each other denoted as \(\bar{a}\) and \(\bar{b}\). These roots can be expressed in terms of \(f\) and \(C\) as:
\[
a = \frac{f}{3} \left[ 1 + y^{1/2} + \sqrt{3 + 2 y^{-1/2} - y} \right],
b = \frac{f}{3} \left[ 1 + y^{1/2} - \sqrt{3 + 2 y^{-1/2} - y} \right],
c = -i \frac{f}{3} \left[ 1 - y^{1/2} - \sqrt{y + 2 y^{-1/2} - 3} \right],
\]
where we have \(y = 1 + 3 x^{1/3} (1 - \sqrt{1 - x})^{1/3} + (1 + \sqrt{1 - x})^{1/3} / 2 / -f^2/3 \leq C < 0\) with \(0 < x \equiv -3 C f^2/4 \leq 1\), and \(y = 1 - 3 x^{1/3} (1 - \sqrt{1 + x})^{1/3} + (1 + \sqrt{1 + x})^{1/3} / 2 \) for \(C > 0\) with \(x \equiv 3 C f^2/4 \geq 0\). From eq. (10), we then have
\[
\ddot{u}^2(t) = -(u - a)(u - b)(u - c)(u - \bar{c}) \geq 0,
\]
giving \(b \leq u(t) \leq a\).
We then have the general solution in terms of Jacobian elliptic function $cn(t)$ as:

$$u(t) = \frac{aB + Ab - (aB - Ab) cn\left(\frac{t - t_0}{2}\right)}{(A + B) + (A - B) cn\left(\frac{t - t_0}{2}\right)}, \quad (12)$$

where $A^2 = (a - b_1)^2 + a_1^2$ and $B^2 = (b - b_1)^2 + a_1^2$ with $a_1^2 = -(c + \bar{c})^2/4, b_1 = (c + \bar{c})/2, g = 1/\sqrt{AB}$. In addition to Myers’ static solution, we have from the above the other three kinds of time dependent solutions, depending on the value of the integration constant $C$. We now come to discuss each of them in order.

### 3.1 Case 1: $-f^4/3 \leq C < 0$

Except for $C = -f^4/3$ giving Myers’ static solution $u = f$ the D2-anti D2 ground state, the general solution in this case cannot be expressed in terms of elementary functions and one has to use Jacobian elliptic function $cn[(t - t_0)/g]$ with $1/g = 2f(2y^2 + 1/\sqrt{3y^2})^{1/4}/3$. Since the $cn(x)$ is a periodic function like $\cos x$, taking values also between $-1$ and $1$ with its standard period (the corresponding $k^2 = 1/2 + 3(1 - y)/[4(2y^2 + 1/\sqrt{3y^2})^{1/2}]$), therefore $u(t)$ is also a periodic oscillating between the roots $b$ and $a$ as given in eq. (11) with respect to its expected equilibrium point at $u = f$ which can be determined, for example, by the requirement of maximum speed from eq. (10) or the minimum of the potential (8). Note that since both $a$ and $b$ are positive for the present case due to $1 < y \leq 4$, $u(t)$ is also positive which implies that the radius of the spherical D2 brane oscillates periodically between $\lambda N b/2$ and $\lambda N a/2$ with its equilibrium radius $r = \lambda N f/2$. According to ref. [5], a spherical dielectric D2-brane can be viewed as a semi-spherical D2-brane-anti semi-spherical D2-brane system and as such one would naively expect the appearance of a tachyon mode. However, the equilibrium position $r = \lambda N f/2$ of this brane-anti brane occurs at the potential minimum of the N D0 system at $u = f$ and this indicates that the D2 brane should be stable if the back-reaction as well as the radiation is ignored. In other words, we should not expect a tachyon mode to appear in this case (or the would-be tachyon mode becomes a stable one). We will discuss this more later on.

### 3.2 Case 2: $C = 0$

This is the case for which the total excess energy is zero. In addition to the trivial solution $u(t) = 0$, we have the following non-trivial one:

$$u(t) = \frac{12f}{9 + 4f^2(t - t_0)^2}, \quad (13)$$

with $t_0$ the integration constant. We can interpret this solution in two ways depending on whether we take our initial time $t = -\infty$ or $t = t_0$. If we take $-\infty < t < \infty$, then at initial and final stages, $u(t) = 0, \dot{u}(t) = 0,$ and $\ddot{u}(t) = 0$, therefore we have only N D0 branes at the two ends while in-between we have the creation of the dynamical spherical D2-brane. This is the analog of a photon creating an electron-positron pair and then annihilating back to a photon but the current process takes an infinity amount of time and the number of D0 branes is large. Interpreting the spherical D2 as a semi-spherical D2-anti semi-spherical D2 pair and with the 4-form flux switching on, we create this pair whose size increases with time up to $t = t_0$ where the size reaches its maximum and then decreases to zero at $t \to \infty$. If we choose our initial time at $t = t_0$, then the pair starts from the largest size and ends with a zero one, becoming N D0 branes. Again, we do not expect the appearance of unstable tachyon mode since neither the extremal $u = 0$ nor the extremal $u = f$ point of the potential $V(u)$ given in eq. (8) gives rise to any tachyon mode. We will elaborate more on this later on.

### 3.3 Case 3: $C > 0$

This is the case for which the excess energy is positive. The solution now for $u(t)$ has exactly the same form as in case (1) as given in eq. (12) and once again we need to use the Jacobian elliptic function. But in this case, the variable $y$ is different as given below eq. (11). The other difference is that now the parameter $b < 0$ due to $0 < y < 1$. Since $u(t)$ oscillates periodically between $b$ and $a$ ($a > f$), so we expect that $u(t)$ goes through zero twice for each cycle. In other words, assuming initial zero size for the spherical D2 branes (or just N D0 branes) the radius of the D2 brane grows with time, for example, passing through the equilibrium radius at $\lambda N f/2$ to reach the maximal size $\lambda N a/2$, then shrink to zero size again after passing through the equilibrium radius one more time, then grows toward the size $\lambda N |b|/2$ and finally shrinks back to zero size to finish one cycle. One can also interpret this process as the time evolution of a semi-spherical D2-brane-anti semi-spherical D2 brane with positive excess energy, but once again one does not expect the appearance of unstable tachyon mode for the same reason as given before. We would like to point out here about the connection of the present solution or the one in case (1) to the one in case (2). Actually, if we take $C \to 0$ from the solution either in case (1) or in case (3), it reduces to either the trivial $u(t) = 0$ or the non-trivial one given by eq. (13) in case (2). For the trivial $u(t) = 0$, the Jacobian elliptic function $cn[(t - t_0)/g] = 1 - (t - t_0)^2/(2g^2)$ while for the non-trivial one $cn[(t - t_0)/g] = -1 + (t - t_0)^2/(2g^2)$, noting for either case $g \to \infty$ as $C \to 0$ with all the other parameters in the solution
also taking the corresponding proper limits.

4 Discussion and conclusion

In each of the above three cases, the excess energy is conserved because we take the system under consideration basically as an isolated one (we ignore radiation and back reaction). Because of this, the system in case (1) has less total energy than that of N D0 branes and therefore the spherical D2 brane cannot shrink its size to zero (or the semi-spherical D2-anti semi-spherical D2 pair cannot annihilate each other back to N D0 branes) to set down to N D0 branes. Case (2) has zero excess energy which is just the right amount of excess energy for the final state of N BPS D0 branes. Therefore, we must expect that the final state consists of N D0 branes as the solution indicates. In case (3), we have a positive excess energy above N BPS D0 branes which has no way to go given our assumption of no interaction with surroundings. Now the spherical D2 brane has enough energy to shrink its size to zero (or the D-brane-anti D-brane pair has enough energy to annihilate each other) but it cannot stay there because of the positive excess energy. Given our assumption, the dynamical D2 brane in each case as well as the N D0 branes in case (2) has no way to give away its or their excess energy above the ground state to settle down to the true Myers’ static spherical D2 or the D2-anti D2 ground state.

In practice, the system under consideration has couplings with the bulk and with other branes even though these could be weak for large N and \( \mathcal{N} f^2 \ll 1 \). We therefore have radiation and/or back-reaction for such system. Note that \( u = 0 \), corresponding to the true ground state of N D0 branes in the absence of the RR 4-form flux, is now an inflection point while Myers’ blown-up static spherical D2 brane or the D2-anti D2 is the true ground state which has \( u = f \) and occurs at the potential minimum. Therefore all the dynamical motion considered in this paper can be viewed one way or the other as the one oscillating around the equilibrium position \( u = f \) or the D2-anti D2 ground state. This implies that all the dynamical D2 branes as well as the D0 branes will eventually settle down to the static spherical D2 or the D2-anti D2 ground state after the underlying system radiates away their excess energy above the ground state via interactions with other modes. This may happen via a chain of processes such as from case (3) to case (2), then to case (1) and finally to the ground state or directly from case (3) to case (1), then finally to the ground state and so on, depending on where we start with and how the energy is radiated. Here we must assume that the interactions involved must be weak and the ground state itself is not altered in the radiation process.

Given what has been said, for large N and small flux \( \mathcal{N} f^2 \ll 1 \), the interactions of the underlying system with all other possible modes should be weak enough and the system itself is therefore almost isolated. We then expect that each kind of dynamical solutions discussed in this paper should live with a rather long lifetime. This is further supported by the absence of unstable tachyon mode. At first look, this is rather strange since each of the dynamical D2 is actually a D2-anti D2 system and in general we should expect the appearance of a tachyon mode. However, at either extremal \( u = 0 \) or extremal \( u = f \) point of the potential (8), we do not have a tachyon mode. The former corresponds to an inflection point of the potential, indicating that the solution \( u = 0 \), though not giving rise to tachyon modes, will not stay there even with a small positive perturbation and will eventually settle down to the potential minimum at \( u = f \). This can be demonstrated\(^3\) using the reduced potential introduced after eq. (10):

\[
V_i = u^4 - \frac{4}{3} fu^3. \tag{14}
\]

From this, it is clear that the corresponding reduced mass squared evaluated at \( u = 0 \) as \( m^2 = \frac{d^3V_i}{du^2}|_{u=0} = 0 \), i.e., vanishing rather than \( < 0 \), therefore free from a tachyon mode. However, a perturbation \( \delta u \) around the solution \( u = 0 \) gives the leading term to this potential as:

\[
V_i = -\frac{4}{3} f(\delta u)^3, \tag{15}
\]

which becomes positive when \( \delta u < 0 \), so the solution \( u = 0 \) has a tendency to stay where it is, therefore stable. However, the potential becomes negative when \( \delta u > 0 \), the static solution \( u = 0 \) will start to move towards \( u = f \) once we have such a positive perturbation, indicating the non-tachyonic instability of the potential at the reflection point \( u = 0 \). This is also clear from the potential profile given in Figure 1. The latter solution \( u = f \) at the potential minimum corresponds to the true ground state, giving rise to Myers’ static stable spherical D2 or the D2-anti D2. As discussed in refs. [2, 6-8], whether a brane-anti brane system is stable (or a tachyon mode appears) depends on certain parameter(s) associated with the system. Before switching on the 4-form RR flux, the N D0 branes are BPS stable one. After switching on, the flux only lowers the potential minimum from zero to a negative one and the new ground state of the system under consideration becomes more stable than the original N D0 one even though it is a D2-anti D2. This very fact indicates also the absence of unstable tachyon mode following the line in refs. [2, 6-8].

\(^3\) We thank one of anonymous referees for raising the instability issue related to the potential reflection point in the early version, therefore leading to the discussion given around eqs. (14) and (15).
Since the ground state is stable, we do not expect that the dynamical $D_2$, viewed as excited one in each case, has an issue with the tachyonic instability.

If we replace the 4-form RR flux by the flux $H_{ijk}$ as pointed out in ref. [3], we will end up with the same action, therefore the same conclusion can be reached.

A different time dependent $D_2$ brane solution is discussed in ref. [9]. For other related work, see refs. [10-18].

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