Stationary Thermomagnetic Convection of Ferrofluid in an Enclosed Loop

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Abstract: This paper presents an experimental study of thermal convection of ferrofluid inside a closed hydrodynamic loop heated from the side in the presence of a magnetic field applied to the loop section nearby the heater. The pipes of the circuit are blown with a stream of thermostatic air, which ensures a constant heat transfer coefficient on the outer surface of the pipes and an exponential temperature distribution along the circuit. The value of the exponent measured provide information on the integral axial heat flux (Nusselt number). The experiments were conducted with undecane under natural gravitational convection and with a colloidal solution of magnetite in kerosene of moderate concentration under mixed (gravitational and thermomagnetic) convection at the Rayleigh numbers varied in the range $10^3 – 10^4$. It is shown that thermomagnetic convection causes a 4 – 6 – fold increase in heat transfer.

1. Introduction
Thermal convection of ferrofluids has attracted the attention of researchers because it includes two mechanisms that may initiate fluid motion [1-3]. The first mechanism is the buoyancy force related to thermal expansion. It is typical for all fluids. In this case the convective heat transfer is determined by the dimensionless Rayleigh number

$$Ra = \frac{g \beta_1 r_1^3}{\eta \gamma} \Delta T$$

where $g$ is the gravity acceleration, $\beta_1$ is the coefficient of thermal expansion of a fluid, $r_1$ is the characteristic size, $\Delta T$ is the characteristic temperature difference, $\rho$, $\eta$, and $\gamma$ are the density, dynamic viscosity, and the thermal diffusivity of the fluid, correspondently. The second mechanism is unique to magnetic fluids only and is the temperature dependence of magnetization leading to thermomagnetic convection in the nonuniform magnetic field. The gradient of the magnetic field strength plays the role of the gravity. The convective heat transfer is determined in this case by the magnetic Rayleigh number

$$Ra_m = \frac{\mu_0 \beta_2 M H r_1^3}{\eta \gamma} \Delta T$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the magnetic constant, $\beta_2$ is the pyromagnetic coefficient, $M$ stands for the magnetization of the fluid, and $H$ is the strength of the magnetic field in the fluid. Simple estimates of the characteristic Rayleigh numbers (1) and (2) show that with the same geometry of convective cavities and temperature differences, the numerical value of the magnetic Rayleigh number $Ra_m$ can exceed the value of $Ra$ by an order of magnitude or more [3]. According to these estimates, one might hope that heat transfer in cooling devices can be enhanced by using ferrofluid instead of common coolants (water, transformer oil, etc.) and by positioning a permanent magnet (or any other source of a magnetic field) nearby a heater. Although this problem attracted the attention of researchers more than 50 years ago, progress in this direction is poorly represented in the literature.
The majority of studies on the thermal convection in magnetic fluids [4-6] examined the stability of the mechanical equilibrium or a weak supercritical flows in a uniform applied field; the ferrofluid cavity had the shape of a flat layer, cube or sphere. In all these cases, the nonuniformity of the magnetic field in the fluid occurred primarily due to the temperature inhomogeneity, and therefore, the intensity of thermodermagnetic convection was comparable to that of gravitational one. No significant enhancement in heat transfer effect was observed. Besides, under certain conditions, there was a competition between two types of convection [7]. A significant increase in heat transfer was reported in refs. [8-11], in which a ferrofluid was placed in an applied field of a known configuration. It was shown in [11] that “at least 50% increase in heat transfer efficiency is achievable with an appropriate positioning of the magnet”. According to [12], the use of a ferrofluid as a cooling medium for small electric motors can increase their power factor by about 20-25%.

Our main concern in this study is to create optimal conditions for the onset of thermodermagnetic convection and to get information on the enhancement of heat transfer by ferrofluid under these conditions. For this purpose, we examined three important factors like nonuniformity applied magnetic field (as, for example, in [9-12]), heat exchanger geometry, and concentration of magnetic phase in ferrofluid. The experiments were carried out using a closed convection loop that is a classical model for studying convective flows. Such a loop was described in the numerous studies on thermal convection (for example, [13-16]), in which the authors explored the stability of the mechanical equilibrium and laminar flow, the chaotic regimes, and the efficiency of mathematical models of cooling systems. We have no intention of considering all the problems mentioned above; our primary purpose here is to study the integral heat flux along the loop in the thermodermagnetic convection mode.

2. Temperature distribution along the cooled section of the loop

Let’s consider a closed convective loop heated from the side over a relatively small area. The most portion of the loop is in air so that heat transfer on its outer surface occurs according to the Newton’s law (figure 1). The density of the heat flux released from the outer surface is written as \( J = \alpha T \), where \( \alpha \) is the heat transfer coefficient (considered constant), and the air temperature is taken as a reference point. We introduce a local coordinate system and arbitrary small element \( \Delta z \) of the pipe as shown in figure 2.

The balance of heat fluxes is recorded in relation to a closed surface that bounds this element and consists of two channel cross sections \( z = z_1 \) and \( z = z_2 \) and the outer surface of the pipe \( r = r_2 \).

The axial heat flux is defined as the sum of molecular fluxes through the cross-sections of the glass pipe and fluid and the convective flux. Thus, we have

\[
Q_1 = -\left[ \pi \lambda_1 r_1^2 \left( \frac{\partial T}{\partial z} \right)_{z=z_1} + \rho C q T_1 \right], \quad z = z_1 \tag{3}
\]

\[
Q_2 = -\left[ \pi \lambda_2 r_2^2 \left( \frac{\partial T}{\partial z} \right)_{z=z_2} + \rho C q T_2 \right], \quad z = z_2 \tag{4}
\]

where \( \lambda_1, \lambda_2 \) are the thermal conductivity coefficients of the fluid and glass, respectively, \( \rho, C \) are the density and specific heat of the fluid, and \( q \) is the convective flow through the cross-sections of the pipe which is related to the fluid velocity \( v(r) \) by the relation:

\[
q = 2\pi \int_0^r v(r) r \, dr = \text{const}.
\]
For an incompressible fluid and non-deformable pipe walls, the integral fluid flow is constant. The radial heat flux can be defined in two ways: in terms of temperature gradients in the glass and in terms of the heat transfer coefficient at the side of the circuit.

\[ Q_3 = -2 \pi \lambda_2 r_2 \frac{\partial T}{\partial r} \Delta z = 2 \pi \alpha T \Delta z \]  

(5)

For steady-state heat transfer

\[ Q_1 - Q_2 - Q_3 = 0. \]  

(6)

Having expanded the temperature and its derivative in (4) into a power series with respect to the small parameter \( \Delta z \), we get

\[ Q_2 - Q_1 = -\left[ \pi \lambda_1 r_1^2 + \pi \lambda_2 (r_2^2 - r_1^2) \right] \frac{\partial^2 T}{\partial z^2} \Delta z + \rho C q \frac{\partial T}{\partial z} \Delta z. \]  

(7)

We recall here that the ratio of molecular heat fluxes in equations (7) and (5) is of the order \( r_2 (r_2 - r_1) / L^2 \), where \( L \) is the characteristic distance along the loop at which temperature disturbances associated with the convection motion decay. As we’ll see later, under intense convection, the value of \( L \) approaches the loop length, \( r_2 (r_2 - r_1) / L^2 \ll 1 \), and therefore the first term in equation (7) can be ignored, resulting in good accuracy. Substituting (5) into the balance equation (6) yields an equation for describing temperature distribution along the loop

\[ \rho C q \frac{\partial T}{\partial z} + 2 \pi \alpha T = 0. \]  

(8)

As the fluid flow through the channel cross-section is constant, the solution to equation (6) is the decaying component

\[ T = T_0 e^{-\kappa z}, \quad \kappa = -\frac{d}{dz} \ln(T) = \frac{2 \pi \alpha}{\rho C q}, \]  

(9)

where the factor \( \kappa \) provides information on the intensity of convective motion. Now we introduce the Nusselt number as the ratio of the convective flux along the loop to the molecular flux without taking into account the heat flux along the glass pipe

\[ Nu = \frac{\rho C q T}{-\pi \lambda f_1 \frac{dT}{dz}} = \frac{2 r_2 \alpha}{\lambda f_2 \kappa^2} \propto \alpha \kappa^{-2}. \]  

(10)

The Nusselt number, determined according to (10), makes it possible to identify the components of the heat flow associated with the liquid and independent of the pipe material. Taking into account the
heat flux through the glass pipe under experimental conditions leads to a doubling of the denominator
in formula (10), but there is no qualitative change in the curve \( Nu = Nu(Ra) \). Equation (10) shows that,
up to coefficients independent of the temperature and fluid velocity, the Nusselt number can be obtained
from the damping coefficient of temperature perturbations along the loop – \( Nu \propto \kappa^2 \), which gives the
desired dependence \( Nu = Nu(Ra) \).

3. Details of the experiment

The oval convective loop made of the glass tube with a circular cross section with an inner radius
\( r_1 = 2.5 \text{ mm} \) and an outer radius \( r_2 = 3.5 \text{ mm} \) was located in the vertical plane (figure 1). The total length
of the loop was \( L = 35 \text{ mm} \). The fluid (in the vertical section) was heated from the side. A schematic
representation of the heater is shown in figure 3. The heating coil was made up by winding the 0.09 mm
nickrome wire and filled with epoxy resin. The inner diameter of the coil was equal to the inner diameter
of the glass pipe. The electrical resistance of the coil at room temperature was 338 Ohms. The heater
was powered from the stabilized DC power supply HY3003-D2. The GDM-8246 multimeter was used
as a voltage monitoring device. The rest of the loop was cooled by an air flow coming from the radiator
regulated by the MLW UT-8 water thermostat. The blade fan mounted behind the radiator promoted air
mixing in the chamber (figure 4). The use of a stabilized current source HY3003-D2 (by the second
channel) ensured a constant fan speed and a constant heat transfer coefficient on the surface of the cooled
loop.

The non-uniform magnetic field by strength up to 24 kA/m was generated by a permanent magnet
equipped with a ferrite magnetic core and pole pieces with a cross-section of 10\( \times \)10 mm. The field
covered the loop section about 30 mm long, in the lower part of which the electric heater was positioned.
figure 5 shows how the horizontal component of the magnetic field depends on the vertical coordinate
\( z \) on the pipe axis. Measurements were made using a SH1-8 magnetometer.

For temperature measurements, we used several miniature copper–constantan thermocouples with a
copper conductor 0.1 mm in diameter. To improve thermal contact with the loop walls and to reduce an
error associated with heat removal along the copper conductor, the hot junctions of the thermocouples
were soldered to the thin copper plates 3 mm wide which were glued to the outer surface of the glass
pipe. The cold junctions of the thermocouples were submerged into a Dewar vessel with melting ice
cribuses. The Rayleigh number was calculated in terms of the temperature difference \( \Delta T \) between the
points located 1 cm below and above the heater; the temperature difference was measured with a
differential thermocouple. In addition, a cooling air temperature was measured.
The test experiments on gravitational convection of a non-magnetic fluid were performed with undecane, and the experiments on thermomagnetic convection – with ferrofluid based on kerosene and colloidal magnetite, obtained by diluting the base sample. The base sample with the density of 1.57 g/cm³ was synthesized by the standard chemical condensation method \[17\] and diluted with kerosene to the density \( \rho = 1.13 \text{ g/cm}^3 \). The magnetization curve for the ferrofluid was determined by the sweep method, in which differential magnetic susceptibility \( \chi \) of the fluid is directly measured and the magnetization curve is found by numerical integration as in references \[18\]:

\[
M = \int_{0}^{H} \chi(H') \, dH'
\]

A long cooled solenoid with two galvanically isolated coaxial coils was used as the source of magnetic field. The direct current was passed through one of the coils, and the weak alternating current of infra-low frequency 0.1 Hz was passed through the second. The frequency was sufficiently low to ignore the relaxation processes in the ferrofluid. The design of the experimental setup made it possible to measure the amplitudes of small magnetization oscillations and the field strength, the ratio of which gave the desired value of the differential susceptibility. The magnetization curve for the ferrofluid is given in figure 6. The particle size distribution was determined via the magnetogranulometric analysis by the method described in paper \[18\]. The basic parameters at temperature 291 K (at which the magnetization curve was measured), are: initial susceptibility \( \chi = 3.15 \), saturation magnetization \( M_\infty = 29.1 \text{ kA/m} \), the average diameter of the magnetic core of particles \( \langle x \rangle = 9.8 \text{ nm} \).

The viscosity of the ferrofluid was measured at 29.6 °C using the Brookfield rotary viscometer. The thermal conductivity of the magnetic fluid was calculated using the formula proposed in reference \[19\]

\[
\lambda = \lambda_b \left[ 1 - \frac{3(\lambda_b - \lambda_m) \varphi_s}{2 \lambda_b + \lambda_m + (\lambda_b - \lambda_m) \varphi_s} \right]
\]

where \( \lambda_b = 0.11 \text{ W/(m·°C)} \) and \( \lambda_m = 5.3 \text{ W/(m·°C)} \) are the thermal conductivities of kerosene and magnetite, respectively, and \( \varphi_s \) is the volume fraction of crystalline magnetite in the colloidal solution. A specific heat capacity of the magnetite phase is calculated according to the formula reflecting its additivity.

\[
C = \rho_m \varphi_s C_m / \rho + (1 - \varphi_s) \rho_b C_b / \rho
\]

where \( \rho \) is the density of the magnetic fluid, \( \rho_b = 0.78 \text{ g/cm}^3 \) and \( \rho_m = 5.2 \text{ g/cm}^3 \) are the densities of kerosene and magnetite, respectively, and \( C_b = 2.0 \times 10^3 \text{ J/(kg·°C)} \) and \( C_m = 5.9 \times 10^2 \text{ J/(kg·°C)} \) respectively.
their specific heat capacities. The coefficient of thermal expansion $\beta_1$ of the ferrofluid is calculated ignoring that of the magnetite. So, we can write

$$\beta_1 = \rho_b \beta_b \left(1 - \varphi_s\right)/\rho$$

where $\beta_b = 0.96 \times 10^{-3} \text{K}^{-1}$ is the coefficient of thermal expansion of kerosene. The physical properties of undecane (density $\rho$, dynamic viscosity $\eta$, thermal expansion coefficient $\beta$, thermal conductivity $\lambda$ and thermal diffusivity $\gamma$), taken from [20], and ferrofluid are given in table 1.

**Table 1.** The physical properties of undecane and ferrofluid (297 K)

|          | $\rho$ (g/cm$^3$) | $\eta$ (Pa·s) | $\beta$ (K$^{-1}$) | $\lambda$ (W/m·°C) | $\gamma$ (m$^2$/s) |
|----------|--------------------|---------------|--------------------|--------------------|--------------------|
| Undecane | 0.74               | 1.0·10$^{-3}$ | 1.0·10$^{-3}$      | 0.13               | 8.0·10$^{-8}$      |
| Ferrofluid | 1.13              | 2.6·10$^{-3}$ | 0.61·10$^{-3}$     | 0.14               | 8.2·10$^{-8}$     |

It should be noted here that thermomagnetic convection is very sensitive to the concentration of colloidal particles in a ferrofluid. In the limit of low particle concentrations, the magnetic Rayleigh number (2) is small due to low magnetization and, in the limit of high particle concentrations, it is small because of the high viscosity of the solution. At the volume fraction of colloidal particles $\varphi_s \approx 0.60 - 0.65$, the ferrofluid completely loses its fluidity [21, 22] and convective motion becomes impossible. The best conditions for thermomagnetic convection are achieved at moderate particle concentrations in a low-viscosity dispersion medium. In this sense, the parameters of the ferrofluid used in this study are close to optimal.

4. Results and discussion

The purpose of the test experiments was to check the homogeneity of the heat transfer coefficient along the loop and to assess the validity of the exponential temperature distribution (9). The experiments were carried out at an air temperature of 27.7 ± 0.2 °C. A stationary temperature distribution was achieved after several tens of minutes after turning on the heater. The thermocouple readings did not change within ±0.2 °C for 40 minutes, indicating a steady state. Figure 7 shows typical examples of temperature distribution on a logarithmic scale. It can be seen that formula (9) is in good agreement with the experimental data, therefore it can be used to calculate the Nusselt number reduced to the heat transfer coefficient on the side surface of the pipe according to formula (10).

The results of measurements of convective heat fluxes along the loop are shown in figure 8. The abscissa represents the Rayleigh thermal number, which is determined from the inner radius of the pipe and the temperature difference $\Delta T$ between the points below and above the heater. The Rayleigh number was changed by varying the heater power in the range from 0.19 W to 0.66 W. The Nusselt number was calculated using the formula (10). Its value, normalized to the heat transfer coefficient $a$ on the outer surface of the pipe is plotted along the ordinate

$$\frac{Nu}{a} = \frac{2r_2}{\lambda_1 \eta_1 K} = \left[\frac{\Delta \ln(T)}{\Delta T}\right]^{-2}.$$  

It is seen that the results obtained in experiments with undecane and a ferrofluid in zero field are in good agreement with each other (points on curve 1). The only difference is that the lower viscosity undecane provides (ceteris paribus) higher Rayleigh numbers. In this case, the heat flux along the circuit is associated with ordinary heat convection. When the magnetic field is turned on, thermomagnetic convection arises, and the convective heat flux increases by 4 - 6 times (curve 2 in figure 8). In our opinion, this is a reasonable result, which reflects the ratio of the magnetic Rayleigh number to the thermal Rayleigh number. Indeed, according to formulas (1) and (2), we can write

$$Ra_m/Ra = \mu_b \beta_b M \nabla H / g \beta_1 \rho.$$  

Under the conditions of the experiments, $\beta_2 / \beta_1 = 2; M \approx 20 \text{kA/m}; \nabla H \approx 2 \times 10^6 \text{A/m}^2$ such that the ratio of the magnetic Rayleigh number to the thermal Rayleigh number $Ra_m/Ra \approx 6 - 8$, which coincides in an order of magnitude with the heat transfer coefficient due to thermomagnetic convection.
5. Conclusion

So, we have investigated the thermal convection of the ferrofluid in the closed hydrodynamic loop heated from the side in the presence of the magnetic field applied to the loop section nearby the heater. The pipes of the circuit are blown with the stream of thermostatic air, which ensures a constant heat transfer coefficient on the outer surface of the pipes and exponential temperature distribution along the circuit. The exponential factor measured in the experiments gave information on the integral heat flux (Nusselt number). The experiments were carried out with undecane under gravitational convection and with the colloidal solution of magnetite in kerosene of the moderate concentration under mixed (gravitational and thermomagnetic) convection at the Rayleigh numbers varied in the range $Ra = 10^3 - 10^4$. It has been established that thermomagnetic convection causes a 4-6-fold increase in the heat flux along the circuit. Detection of such significant increase in the heat flux associated with thermomagnetic convection is the main achievement of this research.

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