Algorithm for non-parametric modeling of the cutting process of dense snow formations with snow plow blade

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Abstract. The paper presents an algorithm for non-parametric modeling of the process of cutting dense snow formations with snow plow blade. The following parameters are used as parameters involved in the model: cutting force components, width, cutting angles and installation of snow plow blade, depth of cut and physicomechanical properties of dense snow formations (density from 400 to 500 kg/m$^3$). The initial data for the construction of the model were the results of experiments conducted on a laboratory model of snow plow blade for snow removal equipment. Optimization of the non-parametric regression model was carried out using evolutionary strategies, and the result was improved by local descent methods. The model is necessary to predict the course of the cutting process in a wide range of changes in the parameters in the process of studying and using it on full-scale real snow removal equipment.

1. Introduction

The paper deals with the task of building a model of a system of interrelated cutting parameters of dense snow formations with snow plow blade. As parameters involved in the model, are used: cutting force components, width, cutting angles and installation of snow plow blade, depth of cut and physicomechanical properties of dense snow formations (density from 400 to 500 kg/m$^3$). The model is necessary to predict the course of the cutting process in a wide range of changes in the parameters in the process of studying and using it on full-scale real snow removal equipment.

The initial data for the construction of the model were the results of experiments conducted on a laboratory model of snow plow blade made on a scale of 1:10. In the course of the experiments, measurements were made of all the previously listed interrelated parameters.

The complexity, the multifactorial nature of the cutting process, the absence of its analytical models necessitate the use of approaches to the construction of models based solely on measurement data. Among others, such approaches include those proposed in the framework of the theory and methodology of non-parametric regression estimation [1, 2].

Optimization of the non-parametric modeling algorithm was carried out using evolutionary strategies, and the result was improved by local descent methods [3].

In constructing the model, a non-parametric method of statistical data processing was used, which is based on estimates proposed by Rosenblatt and generalized by Parzen [1–3], and the method for solving extremal tasks – evolutionary strategies that are stochastic optimization methods based on the theory of
evolution and adaptation. The method is based on the theory of the «cultivation» of the best solution in the space of alternatives [2].

2. Development of non-parametric modeling algorithm

The experimental data are a sample \((\overline{x}_i, y_i), i = 1, 2 \ldots 240\) in some ball \(B(c) = \{x: \|x - c\| \leq r\}\) of the space \(R^6\).

\[
\text{where, 6 is the dimension of the extremum search task space;}
\]
\[
e - \text{ball center;}
\]
\[
r - \text{ball radius.}
\]

Input data:
\[
x_1 - \text{the angle of installation of the snow plow blade;}
\]
\[
x_2 - \text{values of the cutting angle of the snow plow blade;}
\]
\[
x_3 - \text{cutting depth value.}
\]

Output data:
\[
y_1 - \text{the values of the horizontal component of the cutting force;}
\]

According to the available sample and non-parametric Rosenblat – Parzen regression equation:
\[
f(\overline{x}; a) = \frac{\sum_{i=1}^{240} y_i \Phi(x_i'; a', \overline{a})}{\sum_{i=1}^{240} y_i \Phi(x_i'; a', \overline{a})};
\]

where, \(\overline{a}\) – vector of blur parameters;
\[
\overline{x}_i - \text{measurement vector of model factors;}
\]
\[
\overline{y}_i - \text{selective values of the system output - component of the cutting effort;}
\]
\[
\Phi - \text{bell-shaped function.}
\]

You can find the relationship between the input variables and the resulting variables. For this, for each input variable it is necessary to find the parameters of blur. Finding the blur parameters leads to an optimization task [4]:

\[
I(\overline{a}) = \frac{\sum_{i=1}^{240} (y_i - f(x_i))}{f(x; a)} \rightarrow \min_{\overline{a} \cdot R};
\]

\[
I(\overline{a}) = \sum_{i=1}^{240} (x_i - f(x_i, \overline{a}))^2 \rightarrow \min_{\overline{a} \cdot R};
\]

where, \(I(\overline{a})\) objective function defined in some area of space \(R^6\);

A truncated parabola was used as a bell-shaped function:
\[
\Phi(t, a') = \left\{0.75(1 - \frac{a'}{a})^2\right\} \text{ если } \frac{a}{a'} \leq 1; \]

Due to the peculiarities of the adopted model structure, we will use the sliding exam method, which involves checking the adequacy of the model by the discrepancy at a particular sample point, provided that
\[
I(\vec{a}) = \sum_{i=1}^{240} \sum_{j=1, j \neq i}^{240} \left( y_j - f(x_j, \vec{a}) |_{x,y}^{(x_{ij}, y_{ij})} \right)^2 \to \min_a ,
\]

where, \( f(x_j,\vec{a}) |_{x,y}^{(x_{ij}, y_{ij})} \) – model point is not involved in the construction \( (x_{ij}, y_{ij}) \).

Thus, the modeling task is reduced to an optimization task on the space of real numbers. The objective function is non-analytic and, in general, multi-extremal. To find the extremum, it was proposed to use an evolutionary search method.

To solve the unconditional extremal task, a hybrid algorithm was used consisting of the following sequence of actions [5, 6]:

- at stage 1 we select, depending on how this is required by the type of selection, individuals of applicants.
- at stage 2, according to a predetermined probability of crossing, we randomly select individuals from candidates for crossing, leaving the rest unchanged.
- at the 3rd stage, a mutation operation occurs for all individuals.
- at the 4th stage, we determine the suitability function for newly derived individuals.
- at the 5th stage, the coordinate of the vector of the current solution is randomly selected and the direction of improvement is sought.
- at the 6th stage, we check the stopping condition, that is, whether the desired suitability (or the value of the objective function) or the maximum number of generations is reached. If the stop condition is not fulfilled, we form a population of individuals obtained after mutation [2].

Every chromosome \( H_i, i = 1, N \) represented by a set of object parameters and a set of strategic parameters [2, 5], i.e

\[
H_i = \langle op_i, sp_i \rangle ; \quad \text{ (6)}
\]

\[
op_i = (o_{i}^{1}, \ldots, o_{q}^{i}) ; \quad \text{ (7)}
\]

\[
sp_i = (s_{i}^{1}, \ldots, s_{q}^{i}) , \quad \text{ (8)}
\]

where, \( o_{j}^{i} \in R, s_{j}^{i} \in R, i = 1, N, j = 1, q \). Thus, each individual \( I \) has its chromosome and the corresponding value of the suitability function:

\[
I_i = \langle H_i, fit(op_i) \rangle , i = 1, N \quad \text{ (9)}
\]

We define the mutation operation. Mutation in evolutionary algorithms can be represented as:

\[
H_{\text{mutation}} = \langle op_{\text{mutation}}, sp_{\text{mutation}} \rangle ; \quad \text{ (10)}
\]

\[
op_{\text{mutation}} = (o_{1} + \vec{N}(0,s_{1}), \ldots, o_{q} + \vec{N}(0,s_{q})) ; \quad \text{ (11)}
\]

\[
sp_{\text{mutation}} = (s_{1} \cdot A_{1}, \ldots, s_{q} \cdot A_{q}) , \quad \text{ (12)}
\]
where, $\tilde{N}(m,\sigma^2)$ – normally distributed one-dimensional random variable with mathematical expectation $m$ and variance $\sigma^2$;

$A_i, i = 1, q$ – random variable such that $P(A_i = \alpha) = 0.5$ – in the classic statement, $P(A_i = \alpha) = p$ – in a modified statement. Value $\alpha$ and $p$ are set by the researcher or

$H_{\text{mutation}} = < op_{\text{mutation}}, sp_{\text{mutation}} >$; \hfill (13)

$op_{\text{mutation}} = (o_i + \tilde{N}(0,s_i),...,o_q + \tilde{N}(0,s_q))$; \hfill (14)

$sp_{\text{mutation}} = \left(|s_i + \tilde{N}(0,1)|,...,|s_q + \tilde{N}(0,1)|\right)$ \hfill (15)

With discrete crossing, a descendant randomly inherits a gene from one of its parents:

$H_{\text{crossover}}(i, j) = < op_{\text{crossover}}, sp_{\text{crossover}} >$; \hfill (16)

$P(o_k = o^i_k) = P(o_k = o^j_k) = 0.5$; \hfill (17)

$P(s_k = s^i_k) = P(s_k = s^j_k) = 0.5$. \hfill (18)

The suitability function is calculated by the formula

$fit(op) = \frac{1}{1 + Q(o_1,...,o_q)}$ \hfill (19)

As a local optimization procedure, we used the method of random coordinatewise descent, the essence of which is that the coordinate of the vector of the current solution is chosen randomly, then the search for improvement direction is performed.

Let be $x \in \mathbb{R}^n$ – current solution. The scheme algorithm of the algorithm is as follows:

- choose the coordinate $i \in N$ by which the search will be performed:

  $p(i = j) = \frac{1}{n}, j = 1, n$;

- set the search step $\alpha \in \mathbb{R}^+$;

- randomly select the direction of the search $p(s = -1) = p(s = 1) = 0.5$;

- we carry $N$ out steps towards the direction found:

  $y_j = x_j, j = 1, n, y_i = x_i + s \cdot \alpha$;

- if a $Q(y) < Q(x)$ (to solve the minimization task), then $x = y$, otherwise repeat the previous step for $s = -s$. In case none of the search directions has given any improvements, choose a new coordinate. The coordinate is selected $k$ once, so the random coordinate search is determined by the following parameters:

  $k$ – number of choice of descent coordinates; $N$ – number of steps in a given direction; $\alpha$ – step size.

The scheme of the random coordinate descent algorithm is shown in figure 1.
Local descent carries the ratio of individuals who exercise and individuals who remain unchanged after mutation.

The selection process at each stage of the stochastic descent algorithm is determined from the principle of maximum entropy [5, 6]. In other words, when we do not know anything about the search space, we choose a random coordinate and take a step in a random direction.

In this paper, a non-parametric mathematical model is presented as an application program developed in the MATLAB environment.

The assessment of the reliability of the mathematical model was carried out by comparing the calculated values of the horizontal component of the cutting effort obtained using the non-parametric modeling algorithm with the snow plow blade model obtained in experimental studies of the cutting process of dense snow formations. When comparing the results, there may be discrepancies, the reasons for which are:

- assumptions adopted in the calculation method;
- measurement errors and errors in determining the physical and mechanical properties of dense snow formations;
- to eliminate the influence of the drag prism volume on the cutting resistance, it is assumed that all resistances are fixed for the moment when the drag prism has formed, but the cutting process of dense snow formations continues, i.e. when the most power consumption is required [7].

A graphical interpretation of the results is presented in figure 2-4.

Figure 1. Algorithm of coordinate descent.

Figure 2. Horizontal component: the data obtained using the mathematical model are marked in red, the data obtained experimentally in blue.

Figure 3. Side component: the data obtained using the mathematical model are marked in red, the data obtained experimentally in blue.
3. Conclusion

Analyzing these dependencies, it was found that the experimental and calculated values of the cutting effort components coincide with sufficient accuracy (the relative error of the horizontal component is not more than 9%, the lateral one is not more than 4%, the vertical is not more than 10%), which confirms the accuracy of the developed non-parametric model. The non-parametric evolutionary model takes into account the influence on the components of the cutting effort of such parameters as width, cutting angles and installation of the working body, depth of cut, and physical and mechanical properties of dense snow formations with a density of 400 to 500 kg / m³. The application of the proposed method can be extended for dense snow formations of various density, hardness and existing snow plow blade.

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