May Gravity reveal Tsunami?

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Abstract The present gravitational wave detectors are reaching lowest metric deviation fields able to detect galactic and extra-galactic gravitational waves, related to Supernova explosions up to Virgo cluster. The same gravitational wave detector are nevertheless almost able to reveal, in principle, near field Newtonian gravitational perturbations due to fast huge mass displacements as the ones occurring during largest Earth-Quake or Tsunami as the last on 26nd December 2004 in Asiatic area. Virgo and Ligo detector are unfortunately recording on high frequencies (above tens Hz) while the signal of the Tsunami lay at much lower range (below 0.1 Hz). Nevertheless prompt gravitational near field deformation by the Tsunami might reach the future LISA threshold sensitivity and frequency windows if such an array is located nearby \( \approx 3000\text{-}10000 \) km distances. Unfortunately the present LISA system should be located at Lagrange point too far away (1.5 million km. far away). We note however that the later continental mass rearrangement and their gravitational field assessment on Earth must induce, for Richter Magnitude 9 Tsunami, a different terrestrial inertia momentum and a different principal rotation axis. In conclusion we remind that gravitational geodetic deviation on new precise satellites (GOCE 2006), assisted by GPS network, might nevertheless reach in the near future the needed threshold and accuracy to reveal Tsunami by their prompt tidal gravity field deviations. An array of such geoid detector with LISA-like satellite on Earth orbits may offer the fastest alarm system.
1 INTRODUCTION

The last lethal Tsunami event corresponds to an energy release equal or greater than $E_{Ts} \simeq 10^{27}\text{erg}$ (Richter Magnitude 9 Tsunami) released in a very short time, shaping the sea into a wide spread Tsunami wave front. These events are well observed by normal seismic detector, whose time response are related by the surface or sound propagation on Earth. On the contrary the huge energy release maybe in principle source of other signals detectable at the faster velocity of light. Here I consider a preliminary model associated with the mass displacement occurring during the largest Tsunami as the last huge one; in order to make the approximation flexible I will calibrate the results following the energy release scale 8.9 Richter scale or $E_{Ts} \simeq 10^{27}\text{erg}$. We foresee that a detectable shrinking of the averaged Earth radius of nearly $\Delta R_\oplus \simeq 3.4 \mu\text{m}$, and a consequent faster Earth spinning by a ratio $\frac{\Delta w_\oplus}{w_\oplus} \simeq -1.08 \cdot 10^{-12}$ had take place and a year duration suffered a shortening of the order of $\Delta t_{\text{year}} \simeq -34 \mu\text{s}$; if all the energy released is absorbed by the terrestrial rotation energy the opposite would occur, $\frac{\Delta w_\oplus}{w_\oplus} \simeq 2.59 \cdot 10^{-10}$, with a longer year length:$\Delta t_{\text{year}} \simeq 8.17 \cdot 10^{-3} \text{s}$; because the terrestrial gravitational energy is nearly 480 times its rotational one, the mutual energy exchange may in general leads to a spin up or a spin down of the day length within $\mp \Delta t_{\text{day}} \simeq 22.4 \cdot 10^{-6}\text{s}$, well within detection.

1.1 Energy output of Largest Earthquake and Tsunami

Let us remind the huge power occurring during largest Tsunami or Earth-Quake, comparable to nearly one and a half million of Hiroshima nuclear explosions or $3.2 \cdot 10^{10}\text{TNT}$. In other words it is like $10^4$ kilometer water cube are falling down from a km height, or an ice asteroid of $0.1 km^3$ volume was hitting the Earth. If this event would be triggered by artificial or exotic means, as discussed below, for example by a ton anti-meteorite annihilation, a million nuclear Hiroshima weapons explosion, a ton mass mini-black-hole evaporation, than its detection would be promptly achieved by its prompt neutrino emission and the warning alarm might be immediately given. However the source of the Tsunami are related to geological elastic (and after all rotational and gravitational) energy release and their prompt associated signal is nearly undetectable out of a tiny near field Newtonian gravitational perturbation.

2 GRAVITATIONAL NEAR FIELD BY A TSUNAMI

Some geological ideas on geological effects observable by interferometer detector has been already put forward recently. Here, independently, we estimate the adimensional metric displacement by a Tsunami by the following approximation: we assume that a sea water mass $m_w$ is raised by a Tsunami by a height $h$ (its exact value is irrelevant), assuming that

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the energy conversion from the Tsunami $\Delta E_{Tsu}$ to the gravitational energy is comparable (out of a factor $\eta$):

$$m_w = \frac{\Delta E_{Tsu}}{gh} \cdot \eta$$

The consequent adimensional metric deviation $h_{\alpha\beta}$ along Cartesian axis along the unitary vectors due to the tidal mass displacement has a value:

$$h_{\alpha\beta} = \delta g_{\alpha\beta} \simeq 2 \frac{Gm_w}{c^2r} \cdot \eta$$

Therefore the final absolute value (out of an angular projection not considered here) is:

$$h_{\alpha\beta} = 2 \cdot \frac{G \Delta E_{Tsu}}{c^2r^2} \cdot \eta = 1.3 \cdot \eta 10^{-22} \left(\frac{10^9 cm}{r}\right)^2$$

These metric perturbation are within present and future LISA thresholds and frequency ($10^{-2}-10^{-3}$ Hz.). However the energy conversion $\eta$ maybe 1% and the needed threshold is lost. However it is quite surprising that the newtonian tidal field is already in the range of values of present GW antennae. At a nearby distance of 3000 km the detection threshold is reduced to $10^{-21}$ well within detector thresholds.

### 2.1 Terrestrial Gravitational shrinking and angular velocity speed-up

One of the long term consequence of the huge mass displacement inside the Earth is the change of its Inertial Momentum as well as of the terrestrial gravitational field; its readjustment must compensate the energy dispersion of the Tsunami itself. Elementary consequence are derived by angular momentum and energy conservation, once the elastic energy is replenished by the Earth gravitational one:

$$E_{g\oplus} = -\frac{G}{2} \cdot \frac{M_\oplus^2}{R_\oplus}$$

$$\frac{\Delta r}{R_\oplus} = \frac{\Delta E_{Tsu}}{E_{g\oplus}} \simeq -5.43 \cdot 10^{-13} \frac{E_{Tsu}}{10^{27} erg}$$

The consequent average terrestrial radius contraction is:

$$\Delta r \simeq -3.4 \frac{E_{Tsu}}{10^{27} erg} \cdot m$$

The angular momentum conservation imply a faster spin frequency for the Earth: $\Delta w = 2 \cdot \frac{\Delta r}{r}$ and a consequent shortening of the year duration by a well detectable value:

$$\Delta t_{Tsu} \simeq -3.4 \cdot 10^{-5} \cdot \eta s.$$  

A value well within present time measure accuracy.
2.2 Terrestrial Rotational Energy dissipation and angular velocity slow-down

There is also the possibility that part of the energy released has been promptly absorbed by the terrestrial rotational energy $E_{\text{Rot}⊕} = \frac{1}{2} I w^2_{\text{Rot}⊕}$ by a drastic inertial mass redistribution; in this case the terrestrial angular velocity may be slowed down by a larger fraction:

$$\frac{\Delta E_{T \text{su}}}{E_{\text{Rot}⊕}} \simeq 2.59 \cdot 10^{-10} \frac{E_{T \text{su}}}{10^{27} \text{erg}}$$

and our previous estimate reverse on a slow down and a day length increase of opposite sign and nearly 2 order of magnitude larger. This extreme case is of great interest and each year time accumulate increase will be very soon detected: $\Delta t_{T \text{su}} \simeq 8.13 \cdot 10^{-3} \cdot \eta s$, while the corresponding day length will be increased by a time Tsunami delay:

$$\Delta t_{T \text{su}} \simeq 2.24 \cdot 10^{-5} \cdot \eta s.$$

2.3 Rotational-Tsunami Energy Equipartition by Earth’s Gravitational field

In some sense previous estimate are both to be considered as an upper and a lower bound limits, because the energy balance budget (for either Tsunami and Terrestrial rotation) maybe covered by the dominant Gravitational one whose value is nearly 480 times the terrestrial rotational energy.

Therefore it is probable that $\Delta t_{T \text{su}} \simeq \mp 22.4 \cdot 10^{-6} \frac{E_{T \text{su}}}{10^{27} \text{erg}} \cdot \eta s$ a day, while a terrestrial average radius variation $\Delta r \simeq -3.4 \cdot \eta \mu \cdot m$; in case of a more probable combined Earth Gravitational energy flow in equipartition both in Tsunami and in the Rotation terrestrial energy the estimated averaged Earth radius shrinking would be twice as large: $\Delta r \simeq -6.8 \cdot \eta \mu \cdot m \frac{E_{T \text{su}}}{10^{27} \text{erg}}$ and the terrestrial spin will be speed up by a few or ten microsecond a day, depending on the exact mass redistribution. The nutation of the bending angle of the Earth spin axis (due to the Earth inertial mass change) might as large as the same factor $\simeq 2.6 \cdot 2\pi \cdot 10^{-10} \frac{E_{T \text{su}}}{10^{27} \text{erg}}$ leading to a prompt terrestrial axis pole displacement as long as a cm. size; however being the whole Earth nutation axis trajectory around the pole already a few tens a meter a year, this miss-alignment might be difficult to be disentangled, while the day period increase (or decrease) might be probed also by its steady grow and by the accurate atomic and astronomical timing technique.

2.4 Tidal fields and Geoid detections by GOCE satellite

In analogy with the near field estimate the gravitation acceleration deviation is:

$$\delta g \simeq 2 \cdot \frac{G \Delta E_{T \text{su}}}{gr^3}$$
Gravitational tidal waves to reveal Tsunami

\[
\delta g \simeq 3.56 \cdot 10^{-9} \left( \frac{3 \cdot 10^3 \text{km}}{r} \right)^3 \text{cm s}^{-2}
\]

just at the edge (3 nGal) of present interferometer thresholds for LISA like detectors. Let us mention the very recent proposal of GOCE, a satellite able to track at best its keplerian trajectory up to highest resolution by its GPS system. The exact geodetic track of GOCE allows to reveal a geoid deformation below a centimeter length. The Tsunami tides were comparable to 1.8 cm size. Therefore future GOCE detector (a novel satellite able to better trace geoid perturbations) will be able to reveal largest Tsunami by their gravity signals, see ref[4]. A very exciting novel use of near field gravity detector.

3 CONCLUSIONS

While the recent Tsunami ejected by normal geological ways (P and S waves) its huge and lethal message and negligible amount of energy is radiated (as G.W.) at wave zone, a very tiny prompt signal took place in its near field gravity zone: its prompt tide perturbation might be already detectable in principle by future LISA (if operating at its nominal threshold and within $10^4$ km distance and around the Earth). Unfortunately LIGO and VIRGO antennae while being in the same metric threshold range, are tuned at much higher frequencies (tens-thousands Hz). Naturally the geological waves did over-excite all the terrestrial antennas, but we are dealing here with a immediate gravitational perturbation at light of flight velocity connection. Other kind of Tsunami or Earth-Quake events of artificial origin (Appendix C) or by exotic source (Appendix B) considered in this paper could be promptly observed. The gravity detection as a prompt event may become the fastest alarm system for earliest warn alarm: GOCE satellite[4] (like existing GRACE experiment) might become in a near (2006) future such a first gravity detection system. Anyway the consequences of such a huge event is recorded and imprinted within
Fig. 2 Geoid deviation estimated from the last Tsunami (on Dec. 26 2004) in Asia, see ref.4.

Fig. 3 Geoid deviation accuracy at different horizontal resolution for present and future GOCE satellite experiment, see ref.4.

our day time length with an increase of time a day $\Delta t_{Tsu} \simeq \pm 2.24 \cdot 10^{-5} \cdot \eta_s \frac{E_{Tsu}}{10^{27} \text{erg}}$.

Naturally the worst records are imprinted in the tragic human losses.

4 APPENDIX A: GRAVITATIONAL WAVE EMISSION BY TSUNAMI

This energy output is apparently a huge and detectable source of gravitational waves. Indeed the approximate G.W. output proportional to the quadrupole third derivative square maybe written as:

$$\frac{dE_{GW}}{dt} \simeq \frac{32}{5} \frac{G}{c^5} M^2 \cdot \eta_s \cdot \omega^6 = 1.7 \cdot 10^{-10} \text{erg} \cdot s^{-1} \left(\frac{300s}{t_{Tsun}}\right)^2$$
where \( t_{T,\text{sun}} \approx 3 \text{min.} \) is a minimal characteristic Tsunami duration time. This gravitational energy power is indeed one of largest available on Earth and it is within LIGO (VIRGO-LISA) frequency range. Its output is above \( 10^{12} \) times larger than the largest mechanical artificial GW source as a rotating iron cylinder of 20m length and 1 m diameter at fastest rotation angular velocity \( (28 \text{ rad/s}) \). However even such a power flux at a distance of nearly twice the Earth radius, \( 10^9 \text{ cm.} \), is still many order (more than two decades) of magnitude below the LIGO or LISA detection threshold. If this huge energy was been delivered by other means (nuclear underground explosion) it was already, as discussed below, possible to detect, not by its gravity, but by its prompt neutrino emission.

5 APPENDIX B: ANTI-METEORITE ANNIHILATION SOURCE OF A TSUNAMI

If an (hypothetical and very un-probable), one-ton, anti-meteorite annihilate on Earth its anti-nucleon annihilation will release its ( Richter Magnitude 9 Tsunami) energy, mostly in 5 charged and neutral 400\( \text{MeV} \) pions (for each nucleon pair annihilation) whose later decay in flight into \( \mu^\pm \) and later in \( \nu_e,\bar{\nu}_e,\nu_\mu,\bar{\nu}_\mu \) will inject a prompt (hundreds MeV) neutrino burst easily detectable by present Super-Kamiokande detector. Their signal will be observable (by more than thousands events) much better than a galactic supernova and with a quite accurate arrival direction. The energy release will produce a wide Tsunami as well a huge atmospheric thunder. Its gravitational wave emission power will be shorter (than previous 3 minute Tsunami time-scale) and the consequent gravitational wave out-put will be larger by four order of magnitude, but still undetectable by present LIGO and VIRGO detectors. However the explosion itself will occur mostly in air and the explosive destruction will propagate anyway at velocity of light to the observable Earth leading to a prompt un-defensible fire shock-wave. Such an energy \( 10^{27} \text{erg} \) output will shine on air within a few seconds at a luminosity (even observed at far horizons within a thousand km distances) comparable at least to ten thousands sun luminosity. The absence of such an events on sun (appearing as a mini-flare) in the past imply a very low probability of such large anti-meteorite existence. Very similar result will take place for a ton mini-black hole evaporation underground, whose neutrino signal would be even harder and very directional.

6 APPENDIX C: UNDERGROUND NUCLEAR EXPLOSION SOURCE OF A TSUNAMI?

In case of a \( 10^{27} \text{ erg} \) underground nuclear explosion the large (MeV) neutrino outflow number \( 10^{33} \) will be also detectable as a prompt neutrino signal by SK, but at a very low energy threshold and without any angular arrival information. On the contrary the
intensity (hundreds events) in a short bomb time (micro-seconds) will be extreme. The
time correlation between different detectors as SNO, SK (and others future larger neu-
trino detectors as UNO) on Earth may offer a very precise time arrival triggering and
distance triangulation of the nuclear explosion location and output calibration. Naturally
we hope that such an experiment will never take place. Anyway the neutrino signal will
be comparable or exceed any expected galactic supernova event [2]

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