STABLY-INTERIOR POINTS AND THE SEMICONTINUITY OF THE AUTOMORPHISM GROUP

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1. Introduction

It is a familiar fact of everyday life that a symmetric object can become less symmetric by the tiniest of changes, but an object that lacks a certain symmetry has to undergo a change of a definite size to acquire that symmetry. A tire that is even a little out of (round) balance makes the ride bumpy. And to re-balance it will require some definite amount of counter-weighting. In mathematical terminology, one might say that the amount of symmetry an object has is semicontinuous.

This idea can be made precise and valid in many contexts. In Lie group theory, it gave rise to the by-now classical result of Montgomery and Samelson [11]:

If $G$ is a Lie group and $H$ is a compact subgroup, then there is a neighborhood $U$ of $H$ in $G$ such that every subgroup $K$ contained in $U$ is isomorphic to a subgroup of $H$.

Situations where the possible symmetries of the objects are not constrained a priori to belong to a fixed Lie group lead to additional subtleties, however. This paper is about such a situation as it arises in complex analysis. We shall be concerned with the general question: “Given a domain $\Omega_0$ in $\mathbb{C}^n$, to what extent is it true that nearby domains $\Omega$ have no more symmetry than $\Omega_0$ itself?” The natural concept of symmetry here is that of the automorphism group, that is, the group of biholomorphic maps of the domain to itself. Thus one is led to ask:

If $\Omega$ is close to $\Omega_0$, is the automorphism group $\text{Aut}(\Omega_0)$ isomorphic to a subgroup of $\Omega_0$?

The question becomes precise only with the specification of which types of domains are considered and what “closeness” is taken to mean.

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We shall be considering bounded domains; for these the automorphism group is always a Lie group so subgroup should mean injective Lie group homomorphism. The domains will have at least some degree of boundary smoothness and closeness will involve closeness in some $C^k$ topology. We now turn to detailed statements on this.

For a subset $K$ of $\mathbb{R}^N$ which is the closure in $\mathbb{R}^N$ of its own interior, the $C^k$-norm of a complex-valued function $f$, smooth on an open set $U$ containing $K$ is given by

$$\|f\|_{C^k(K)} = \sup_{p \in K} \left| \frac{\partial^{i_1 + \cdots + i_N} f}{\partial x_1^{i_1} \cdots \partial x_N^{i_N}}(p) \right|.$$ 

For a mapping $F = (f_1, \ldots, f_m): U \to \mathbb{C}^m$, its $C^k$-norm on $K$ will then be defined by

$$\|F\|_{C^k(K)} = \sup \{\|f_j\|_{C^k(K)} : j = 1, \ldots, m\}.$$ 

This gives rise to the following

**Definition 1.1.** The bounded domains $\Omega_1$ and $\Omega_2$ in $\mathbb{C}^n$ are said to be $\epsilon$-close in the $C^k$-sense, if there is a $C^k$ diffeomorphism $F: cl(\Omega_1) \to cl(\Omega_2)$ between their closures satisfying

$$\|F - I\|_{C^k(cl(\Omega_1))} < \epsilon,$$

where $I$ represents the identity map of $\mathbb{C}^n$ and $\mathbb{C}^n$ is identified with $\mathbb{R}^{2n}$ as usual. The topology on a collection of domains following this construction is called the $C^k$-topology.

The automorphism group of a domain $\Omega$ in $\mathbb{C}^n$ is defined by

$$\text{Aut}(\Omega) := \{f : \Omega \to \Omega \mid \text{bijective, holomorphic}\}.$$ 

Equipped with the topology of uniform convergence on compact subsets of $\Omega$, $\text{Aut}(\Omega)$ is a topological group under the law of composition, and when $\Omega$ is bounded, $\text{Aut}(\Omega)$ is a Lie group (cf. [5]). The semicontinuity phenomenon for the automorphism groups discussed in generality above now becomes in precise terms the following: when a sequence of bounded domains $\Omega_j$ in $\mathbb{C}^n$ converges to another domain $\Omega_0$ in $C^k$-sense, there should be an integer $N$ such that, for every $j > N$, there exists an injective Lie group homomorphism $\psi_j: \text{Aut}(\Omega_j) \to \text{Aut}(\Omega_0)$.

This topic has deep roots and has been investigated in various contexts over a long period of time ([12], [11], [4], [7], [8], [6]). However, previous work has been almost exclusively about strongly pseudoconvex domains (with $k \geq 2$).
In this paper, this type of result will be obtained in the more general context of pseudoconvex domains of finite type in the sense of D’Angelo.

The main specific result is the following:

**Theorem 1.2.** Let $\Omega_0$ be a bounded pseudoconvex domain in $\mathbb{C}^2$ with its boundary $\mathcal{C}^\infty$ and of finite type in the sense of D’Angelo [2] and with $\text{Aut}(\Omega_0)$ compact. If a sequence $\{\Omega_j\}$ of bounded pseudoconvex domains in $\mathbb{C}^2$ converges to $\Omega_0$ in $\mathcal{C}^\infty$-topology, then there is an integer $N > 0$ such that, for every $j > N$, there exists an injective Lie group homomorphism $\psi_j: \text{Aut}(\Omega_j) \to \text{Aut}(\Omega_0)$.

This is a semicontinuity result in the same sense as those of [7], [8] and [6], but strong pseudoconvexity is no longer required as a hypothesis. The result is however, restricted to domains in $\mathbb{C}^2$.

The method we develop here is both simpler and more general than the methods used previously for strongly pseudoconvex domains. In particular this method gives a more concise and intuitive proof even in the strongly pseudoconvex case. And furthermore, the method applies also to the case of convex domains with only $C^1$ smooth boundary. This will be demonstrated in a later section.

2. **Stably-interior points and the semicontinuity theorem for domains with finite-type boundary**

Let $\mathcal{D}(n)$ be the collection of bounded pseudoconvex domains in $\mathbb{C}^n$ with $\mathcal{C}^\infty$-smooth boundary equipped with the $\mathcal{C}^\infty$-topology described above.

**Definition 2.1.** Let $\mathcal{S} := \{\Omega_j : j = 1, 2, \ldots\}$ be a sequence in $\mathcal{D}(n)$ converging to $\Omega_0 \in \mathcal{D}$ in the $\mathcal{C}^\infty$-topology. Then a point $p \in \Omega_0$ is said to be stably-interior if there exists $N > 0$ and $\delta$ such that, for every $j > N$, $p \in \Omega_j$ and $\text{dist}(\varphi_j(p), \mathbb{C}^n - \Omega_j) > \delta$ for every $\varphi_j \in \text{Aut}(\Omega_j)$.

We now prove the existence of such stably-interior points which will play a crucial role in establishing the semicontinuity theorems.

**Theorem 2.2.** If $\Omega_0 \in \mathcal{D}(2)$ is such that its automorphism group $\text{Aut}(\Omega_0)$ is compact, and that its boundary is of finite type in the sense of D’Angelo then, for any sequence in $\mathcal{S} := \{\Omega_j : j = 1, 2, \ldots\}$ in $\mathcal{D}(2)$ converging to $\Omega_0$ with respect to the $\mathcal{C}^\infty$ topology, there exists a stably-interior point in $\Omega_0$.

For the sake of smooth exposition, recall first the concept of the tangent cone. If $\Omega$ is a domain in $\mathbb{C}^n$ with a boundary point $q$, then
the tangent cone to $\Omega$ at $q$ is defined to be

$$T_q(\Omega) := \text{cl} \left( \bigcap_{r > 0} \{ \lambda (p - q) \mid \lambda \in \mathbb{C}, p \in \Omega, \|p - q\| < r \} \right),$$

where the notation $\text{cl}(A)$ represents the closure of $A$ in $\mathbb{C}^n$.

Now we present the following lemma, an important step toward the proof of Theorem 2.2.

**Lemma 2.3.** If a point $p_0 \in \Omega_0$ is not stably-interior for the sequence $S := \{\Omega_j : j = 1, 2, \ldots\}$, $\Omega_j \in \mathcal{D}(n)$, converging to $\Omega_0 \in \mathcal{D}(n)$ in $C^\infty$-topology with its automorphism compact and its boundary of finite D’Angelo type, then there exists another sequence $S' := \{\Omega_j' : j = 1, 2, \ldots\}$ in $\mathcal{D}(n)$ converging to $\Omega_0' \in \mathcal{D}(n)$ in $C^\infty$-topology and a sequence $p_j' \in K$ for some compact subset $K$ of $\Omega' \cap \bigcap_{j=1}^\infty \Omega_j'$ satisfying:

1. $\Omega_0'$ and $\Omega_j'$ are $\mathbb{C}$-affine linearly biholomorphic to $\Omega_0$ and $\Omega_j$ respectively, for every $j = 1, 2, \ldots$.
2. $0 = (0, \ldots, 0) \in \partial \Omega_0' \cap \bigcap_{j=1}^\infty \partial \Omega_j'$.
3. $T_0(\Omega_0') = T_0(\Omega_j') = \{(z_1, \ldots, z_n) \in \mathbb{C}^n : \text{Re } z_1 \geq 0\}$, and
4. $\exists \psi_j' \in \text{Aut}(\Omega_j'), j = 1, 2, \ldots$, such that $\psi_j(p_j') = (\epsilon_j, 0)$ with $\epsilon_j > 0$ and $\lim_{j \to \infty} \epsilon_j = 0$.

**Proof.** Assume that $p_0 \in \Omega_0$ is not a stably-interior point for the sequence $\{\Omega_j : j = 1, 2, \ldots\}$ which converges to $\Omega_0$.

Since this point is not stably-interior, there exists a sequence $\{\varphi_j \in \text{Aut}(\Omega_j) : j = 1, 2, \ldots\}$ such that $\lim_{j \to \infty} \text{dist}(\varphi_j(p), \mathbb{C}^n - \Omega_j) = 0$. Choosing a subsequence, we may assume that

$$\lim_{j \to \infty} \varphi_j(p) = p^*.$$

One can easily check that $p^* \in \partial \Omega_0$.

Choose a unitary map $U$ of $\mathbb{C}^n$ such that the complex rigid motion $\tilde{U}$ defined by $\tilde{U}(z) := U(z - p^*)$ satisfies

$$T_0(\tilde{U}(\Omega_0)) = \tilde{U}(T_{p^*}(\Omega_0)) = \{(z_1, \ldots, z_n) \in \mathbb{C}^n : \text{Re } z \geq 0\}.$$

Now, for each $j$ sufficiently large, let $p_j^* \in \partial \tilde{U}(\Omega_j)$ be the point satisfying

$$\|\tilde{U} \circ \varphi_j(p) - p_j^*\| = \inf_{q \in \partial \Omega_j} \|\tilde{U} \circ \varphi_j(p) - q\|.$$

Then take a unitary map $U_j$ of $\mathbb{C}^n$ such that the complex rigid motion $\tilde{U}_j$ defined by $\tilde{U}_j(z) = U_j(z - p_j^*)$ satisfies

1. $\tilde{U}_j(T_{p_j^*}(\tilde{U}(\Omega_j))) = \{(z, w) \in \mathbb{C}^2 : \text{Re } z \geq 0\}$, and
2. $\tilde{U}_j \circ \tilde{U} \circ \varphi_j(p) = (\epsilon_j, 0)$ where $\epsilon_j = \|\tilde{U} \circ \varphi_j(p) - p_j^*\|$. 

Notice that $\lim_{j \to \infty} \epsilon_j = 0$ and $\lim_{j \to \infty} \tilde{U}_j = I$.

Hence it suffices to set:
\[
\begin{align*}
\Omega'_0 &:= \tilde{U}(\Omega_0), \\
\Omega'_j &:= \tilde{U}_j \circ \tilde{U}(\Omega_j), \\
\psi_j &:= (\tilde{U}_j \circ \tilde{U}) \circ \varphi_j \circ (\tilde{U}_j \circ \tilde{U})^{-1}, \text{ and} \\
p'_j &:= \tilde{U}_j \circ \tilde{U}(p_0).
\end{align*}
\]

The properties (0)–(3) follow immediately, and the proof of Lemma 2.3 is now complete. \hfill \Box

**Proof of Theorem 2.2:** Recall that we are concerned here with complex dimension 2 only.

Suppose the contrary, that there is no stably-interior point. Then by Lemma 2.3, we may assume without loss of generality that $\{\Omega_j\}$ is a sequence convergent to $\Omega_0$ in $C^\infty$-topology satisfying the properties:

1. $0 = (0, 0) \in \partial \Omega_0 \cap \bigcap_{j=1}^\infty \partial \Omega_j$.
2. $\mathcal{T}_0(\Omega_0) = \mathcal{T}_0(\Omega_j) = \{(z, w) \in \mathbb{C}^2 : \text{Re} \ z \geq 0\}$, and
3. there exist a sequence $p_j \in \Omega_0 \cap \bigcap_{j=1}^\infty \Omega_j$ with $\lim_{j \to \infty} p_j = p_0 \in \Omega_0 \cap \bigcap_{j=1}^\infty \Omega_j$ and a sequence $\varphi_j \in \text{Aut} (\Omega_j)$, $j = 1, 2, \ldots$, with $\varphi_j(p_j) = (\epsilon_j, 0)$, $\epsilon_j > 0$ and $\lim_{j \to \infty} \epsilon_j = 0$.

Thanks to Theorem 11 of p. 149 of [3], there exists $N > 0$ such that, for every $j > N$, the boundary of $\Omega_j$ is of finite type at the origin, bounded by the D'Angelo finite type, which we set to be $2m$, of $\Omega_0$ at the origin. This implies in particular that, for each $j$, the local defining inequality for $\Omega_j$ near the origin 0 can be written as
\[
\text{Re} \ z > H_j(w) + E_j(w, \text{Im} \ z),
\]
where the following two conditions hold:

(A) $H_j(w) = \sum_{2 \leq k + \ell \leq 2m} A_{j,k,\ell}w^k \bar{w}^\ell$, a homogeneous subharmonic polynomial of degree $2m$, and

(B) $E_j(w, \text{Im} \ z) = o(|w|^{2m} + |\text{Im} \ z|)$.

On the other hand, $\Omega$ near the origin is defined by
\[
\text{Re} \ z > H(w) + E(w, \text{Im} \ z),
\]
where:

(A) $H(w) = \sum_{k + \ell = 2m} A_{k,\ell}w^k \bar{w}^\ell$, a subharmonic polynomial, and

(B) $E(w, \text{Im} \ z) = o(|w|^{2m} + |\text{Im} \ z|)$. 

Moreover, \( \lim_{j \to \infty} H_j = H \), and \( \lim_{j \to \infty} E_j = E \), on any ball of positive radius centered at the origin.

Now we shall apply the scaling method, in which we follow the arguments developed by Berteloot in [1]. We first introduce the finite dimensional vector space \( V_{2m} \) of the polynomials in \( w \) and \( \bar{w} \) of degree not greater than \( 2m \). The norm \( \| \Phi(w) \| \) for polynomial \( \Phi(w) = \sum_{j+k \leq 2m} c_{jk} w^j \bar{w}^k \) of degree \( \leq 2m \) is defined to be

\[
\| \Phi(w) \| = \max \{ |c_{jk}| : \Phi(w) = \sum_{j+k \leq 2m} c_{jk} w^j \bar{w}^k \}.
\]

Then, for each \( j \), consider \( \delta_j > 0 \) such that

\[
1 = \left\| \frac{1}{\epsilon_j} H_j(\delta_j w) \right\|.
\]

Such \( \delta_j \) exists and satisfies that \( \delta_j 2^m \lesssim \epsilon_j \). Then one may choose a subsequence so that the sequence \( \frac{1}{\epsilon_j} H_j(\delta_j w) \) converges in the norm just introduced. Let

\[
H_\infty(w) = \lim_{j \to \infty} \frac{1}{\epsilon_j} H_j(\delta_j w).
\]

Let \( L_j(z, w) := (z/\epsilon_j, w/\delta_j) \). According to [1] (Proposition 2.2, p. 623), by choosing a subsequence again if necessary, the sequence \( L_j \circ \varphi_j : \Omega_j \to \mathbb{C}^2 \) converges, uniformly on compact subsets of \( \Omega_0 \), to a holomorphic mapping

\[
\sigma : \Omega_0 \to M_\infty
\]

where \( M_\infty = \{ (z, w) \in \mathbb{C}^2 : \text{Re } z > H_\infty(w) \} \).

Note that, choosing a subsequence again if necessary, the sequence of inverse maps of \( L_j \circ \varphi_j \) also converges uniformly on compact subsets of \( M_\infty \) as \( \Omega_j \)'s are contained in a bounded neighborhood of \( \Omega_0 \). Altogether it follows that the map \( \sigma : \Omega_0 \to M_\infty \) is a biholomorphism.

The holomorphic automorphism group of \( M_\infty \) contains a non-compact subgroup \( \{ \psi_t(z, w) = (z + it, w) : t \in \mathbb{R} \} \). Thus we have reached a contradiction to the assumption that \( \text{Aut}(\Omega_0) \) was compact. \( \square \)

We are now ready to prove Theorem 1.2.

**Proof of Theorem 1.2.** We first assert that there exists \( N > 0 \) such that \( \text{Aut}(\Omega_j) \) is compact for every \( j > N \). We shall prove this by contradiction. Assume the contrary that \( \text{Aut}(\Omega_j) \) is non-compact for every \( j \). Let \( p_0 \in \Omega_0 \) be the stably-interior point the existence of which was established above. Then taking a subsequence we may arrange
that \( p_0 \in \Omega_j \) for every \( j \). Since \( \text{Aut}(\Omega_j) \) is noncompact, there exists \( \psi_j \in \text{Aut}(\Omega_j) \) such that
\[
\|\psi_j(p_0) - q_j\| < \frac{1}{j},
\]
for some \( q_j \in \partial\Omega_j \). (Here we are using the familiar fact that, if \( \text{Aut}(\Omega) \) is noncompact, then the \( \text{Aut}(\Omega) \)-orbit of each point \( p \in \Omega \) is noncompact and hence contains a boundary point of \( \Omega \) in its closure, cf. [9].) Choosing a subsequence, we may assume that \( \lim_{j \to \infty} q_j = q_0 \). Then it is clear that \( q_0 \in \partial\Omega_0 \). Therefore \( p_0 \) is not a stably-interior point. This contradiction proves our assertion.

The remainder of the proof follows the pattern discussed in [6], Section 1. It turns out that in the context of the existence of a stably interior point and the convergence uniformly (together with all derivatives) on compact subsets of a subsequence of every sequence \( \{\varphi_k\} \), \( \varphi_k \in \text{Aut}(\Omega_k) \) \( \forall k \), to an element of \( \text{Aut}(\Omega_0) \), there is always an isomorphism of \( \text{Aut}(\Omega_k) \) to a subgroup of \( \text{Aut}(\Omega_0) \) for all \( k \) sufficiently large.

This actually holds not just for automorphism groups but for any compact group actions: it is a general result on group actions not depending on specific properties of holomorphic functions. While this is presented in detail in [5], we shall outline the arguments here:

Start with a \( C^\infty \) exhaustion function on \( \Omega_0 \), that is, a \( C^\infty \) function \( \rho: \Omega_0 \to \mathbb{R} \) with \( \rho^{-1}((-\infty, \alpha]) \) compact for every real number \( \alpha \). By averaging over the action of the compact group \( \text{Aut}(\Omega_0) \), one can (and we shall) take \( \rho \) to be invariant under the action of \( \text{Aut}(\Omega_0) \). Now fix a number \( A \) such that \( \rho^{-1}((-\infty, A]) \) is nonempty and such that \( A \) is not a critical value for \( \rho \). Then \( \rho^{-1}((-\infty, A]) \) is a smooth compact submanifold-with-boundary of \( \Omega_0 \), in particular a closed subset of \( \Omega_0 \) that is the closure of its nonempty interior. Call this closed set \( C \) for convenience.

Now for all \( k \) large enough, every element of \( \text{Aut}(\Omega_k) \) maps \( \rho^{-1}((-\infty, A+1]) \) into a subset \( \rho^{-1}((-\infty, A+2]) \). This follows from the convergence hypothesis and the fact that \( \rho^{-1}((-\infty, A+2]) \) is a compact subset of \( \Omega_0 \). Thus it makes sense, for each large \( k \), to average the function \( \rho \) on \( \rho^{-1}((-\infty, A+1]) \) with respect to the action of \( \text{Aut}(\Omega_k) \) to produce a function on \( \rho^{-1}((-\infty, A+1]) \). Moreover, because of the convergence hypothesis of the group elements, this averaged function, call it \( \hat{\rho}_k \), which is invariant under \( \text{Aut}(\Omega_k) \), will be close in the \( C^\infty \) sense to the original function \( \rho \) (which was invariant under the action of \( \text{Aut}(\Omega_0) \)). In particular, the value \( A \) will be noncritical for each of
the functions \( \hat{\rho}_k \) when \( k \) is large enough. Moreover, the compact sub-
manifold with boundary is \( C^\infty \) close to the compact submanifold with boundary.

Now, this same kind of construction can be extended to produce Rie-
mannian metrics \( g_0 \) on \( C \) and \( g_k \) on the set \( \hat{\rho}_k^{-1}((-\infty, A]) \), for each large
\( k \), with the Riemannian metrics smooth up to and including the bound-
daries and invariant under the actions of Aut (\( \Omega_0 \)) and Aut (\( \Omega_k \)), respectively. Moreover, it is possible to choose the Riemannian metrics \( g_0 \) and
\( g_k \) in such a way that the boundaries of the respective submanifolds-
with-boundary admits one-sided normal tubular neighborhoods in these
metrics: that is, that the product metric

\[
(g_0 \text{ restricted to the boundary } \partial C \text{ of } C) \times (dt^2 \text{ on } [0, \epsilon])
\]

is isometric to the metric \( g_0 \) on \( C \), for some \( \epsilon > 0 \) sufficiently small, in
the \( \epsilon \)-neighborhood of \( \partial C \) in \( C \). (And similarly for \( g_k \) and the compact
submanifold-with-boundary \( \hat{\rho}_k^{-1}((-\infty, A]) \).

One can then take the metric doubles of the smooth manifolds with
boundary and then the situation is exactly that of [4]. And the actions
of Aut (\( \Omega_0 \)) and Aut (\( \Omega_k \)) extend as isometric actions on these met-
ric doubles. The isometry groups of doubles of \( \hat{\rho}_k^{-1}((-\infty, A]) \) will be
isomorphic to a subgroup of the isometry group of the metric double
of \( C \) (which is a compact Lie group) when \( k \) is sufficiently large (cf.
[4]). And this isomorphism is a Lie group isomorphism. It now follows
by the classical theorem of Montgomery and Samelson [Op cit.] and
by the uniform convergence on compact subsets that Aut (\( \Omega_k \)) for \( k \)
large is isomorphic to a subgroup of Aut (\( \Omega_0 \)). (The reader is invited
to consult [6].) \( \square \)

Remark 2.4. The reason why Theorem 1.2 is proved only for complex
dimension two is the convergence problem of the scaling method, used
in the proof of Theorem 2.2; the difficulty of extending to higher di-
mensions is precisely there. When the scaling method can be shown
to be convergent, there is no need for any dimension restriction, as we
shall see in the next section.

3. The cases of Convex domains with \( C^1 \) boundary and
strongly pseudoconvex domains

The arguments established in the preceding section can be modified
to yield the following:

**Theorem 3.1.** Denote by \( \mathcal{E}(n) \) the collection of bounded convex do-
main in \( \mathbb{C}^n \) for \( n > 1 \) with compact automorphism group and with \( C^1 \)
boundary. If \( \{ \Omega_j \}_j \) is a sequence in \( \mathcal{E}(n) \) converging to \( \Omega_0 \in \mathcal{E}(n) \), then
there exists $N > 0$ such that every $j > N$ admits an injective Lie group homomorphism $h_j: \text{Aut}(\Omega_j) \to \text{Aut}(\Omega_0)$.

The proof is almost identical with that for Theorem 1.2, except that one uses here the convergence theorem of Kim-Krantz [10] (Theorem 4.2.1, p. 1295-1296) for the scaling sequence in this case.

It is worth noting that if the type is 2, i.e., the domains are strongly pseudoconvex, the $C^2$ convergence of domains is enough to use the same arguments, in any complex dimension $n > 1$, to establish the following version of the theorem of Greene-Krantz:

**Theorem 3.2** (Greene-Krantz, [8]). If $\{\Omega_j\}$ is a sequence of bounded strongly pseudoconvex domains in $\mathbb{C}^n$, $n > 1$, with $C^2$ boundary converging to a bounded domain $\Omega_0$ in $\mathbb{C}^n$ with $C^2$ smooth strongly pseudoconvex boundary and if $\text{Aut}(\Omega_0)$ is compact, then there exists $N > 0$ such that for every $j > N$ there is an injective Lie group homomorphism $h_j: \text{Aut}(\Omega_j) \to \text{Aut}(\Omega_0)$.

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