Decoherence of Topological Qubit in Linear Motions: Decoherence Impedance, Anti-Unruh and Information Backflow

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Abstract. In this work we study the decoherence of topological qubits in linear motions. The topological qubit is made of two spatially-separated Majorana zero modes which are the edge excitations of Kitaev chain [1]. In a previous work [2], it was shown by one of us and his collaborators that the decoherence of topological qubit is exactly solvable, moreover, topological qubit is robust against decoherence in the super-Ohmic environments. We extend the setup of [2] to consider the effect of motions on the decoherence of the topological qubits. Our results show the thermalization as expected by Unruh effect. Besides, we also find the so-called “anti-Unruh” phenomena which shows the rate of decoherence is anti-correlated with the acceleration in short-time scale. Moreover, we modulate the motion patterns of each Majorana modes and find information backflow and the preservation of coherence even with nonzero accelerations. This is the characteristics of the underlying non-Markovian reduced dynamics. We conclude that he topological qubit is in general more robust against decoherence than the usual qubits, and can be take into serious consideration for realistic implementation to have robust quantum computation and communication. This talk is based on our work in [3].

1. Introduction
In the past twenty years, quantum information sciences have achieved tremendous progress both theoretically and experimentally. However, it is still far from reaching out a large-scale quantum computer. The main obstacle is short of realistic robust qubits against quantum decoherence so that it is hard to scale up the quantum resource for computation. One way to break this obstacle is to find the qubits which are topological in nature ensured by the underlying quantum dynamics. One candidate for such a qubit is the edge excitations of the topological insulators and superconductors, for which their robustness is guaranteed dynamically. In [2] we have demonstrated that the topological qubit made of two Majorana zero modes of the Kitaev chain [1] is indeed robust against decoherence in the super-Ohmic environment. The interaction Hamiltonian between the Majorana zero modes and the environment is given by

\[ V := \gamma_1 O_1 + \gamma_2 O_2 \]  \hspace{1cm} (1)

where \( \gamma_{1,2} \) are the Majorana zero modes, i.e., they satisfy the Clifford algebra: \( \gamma_1^2 = \gamma_2^2 = 1 \) and \( \gamma_1 \gamma_2 + \gamma_2 \gamma_1 = 0 \). The operators \( O_i \)'s are the anti-Hermitian environmental operators obeying the locality constraint

\[ \langle O_1(t)O_2(t') \rangle = 0 \]  \hspace{1cm} (2)
where $O_i(t) := e^{iH_E t} O_i e^{-iH_E t}$ with $H_E$ the environmental Hamiltonian.

Moreover, it was shown in [2] that the open-system quantum reduced dynamics of the topological qubit is exactly solvable, and the result is as follows. Given the initial state of the topological qubit

$$\rho_M(t = 0) = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{01} & 1 - \rho_{00} \end{pmatrix},$$

its reduced dynamics has been solved in [2] and takes the simple closed form

$$\rho_M(t) = \begin{pmatrix} \frac{1}{2} + (\rho_{00} - \frac{1}{2}) e^{\tau_1(t) + \tau_2(t)} & e^{\tau_2(t)} \Re \rho_{01} + ie^{\tau_1(t)} \Im \rho_{01} \\ e^{\tau_2(t)} \Re \rho_{01} - ie^{\tau_1(t)} \Im \rho_{01} & \frac{1}{2} - (\rho_{00} - \frac{1}{2}) e^{\tau_1(t) + \tau_2(t)} \end{pmatrix}$$

where the “influence functional” associated with the $i$-th Majorana zero mode is

$$I_i(t) := 2 \int_{t'}^{t} d\tau \int_{t'}^{\tau} d\tau' \overline{G}_{i,sym}(\tau - \tau')$$

with

$$\overline{G}_{i,sym}(t - t') = \frac{1}{2} \left( \langle O_i(t) O_i(t') \rangle + \langle O_i(t') O_i(t) \rangle \right).$$

Here, $\overline{G}_{i,sym}$ is the so-called Majorana-dressed symmetric Green function, which is different from the usual fermionic one, i.e., $G_{i,sym}(t - t') = \frac{1}{2} \left( \langle O_i(t) O_i(t') \rangle - \langle O_i(t') O_i(t) \rangle \right)$. The sign change is due to the Majorana dressing so that time-ordering for $O_i$ changes from the fermionic one to the bosonic one. Based on this reduced density matrix, it is straightforward to see that the quantum dynamics is robust for the super-Ohmic environments, as shown in [2].

2. Topological qubits in linear motions

We consider the decoherence patterns of a topological qubit made of two Majorana zero modes in the generic linear and circular motions in the Minkowski spacetime. Based on the formulation of the topological qubit and its reduced dynamics developed in [2], we develop the formalism to study the reduced dynamics for a moving topological qubit by generalizing the one given in [4] for the usual Unruh-DeWitt (UDW) detector. We show that the reduced dynamics is exact without Markov approximation. We can also obtain the exact transition probability of the topological qubit beyond the Markovian approximation.

The key idea of [4] is that the motion of the particle is induced by performing proper coordinate transformations on the evolving Hamiltonians for either UDW detector or the environmental fields. In this work we consider the linear motions of the Majorana zero modes as shown in Fig. 1. As shown in (4), the reduced dynamics is encoded in the influence functionals. In the case of linear motion, we can choose to observe the reduced dynamics in the comoving frame of $k$-th Majorana zero modes (called this frame as “$M_k$"-frame), then the influence functional associated with the $i$-th Majorana zero modes as function of proper time $\tau_k$ is

$$I_i^{M_k}(\tau_k) = -2 \int_{0}^{\tau_k} d\tau'_k \int_{0}^{\tau_k} d\tau''_k \lambda_i(\tau_i(\tau'_k)) \lambda_i(\tau_i(\tau''_k)) \int_{-\infty}^{\infty} d\omega |\omega|^{d - 1} A_i(\omega) e^{-i\omega(t(\tau'_k) - t(\tau''_k))}$$

$$\times \oint d\Omega \exp \left[ i\omega \hat{n} \cdot \left( \vec{x}(\tau'_k) - \vec{x}(\tau''_k) \right) \right]$$

where $\hat{n} := \vec{k}/|\vec{k}|$ is the unit normal to the unit sphere of measure $\oint d\Omega$, $d$ is the space dimension, and $A_i(\omega)$ is the environmental spectral density. On the other hand, we can choose to observe the
Figure 1. A pair of topological qubits with their constituent Majorana zero modes in incoherent linear boost or acceleration. The Majorana zero modes $\gamma_{1,2}$ form a topological qubit of which we study the effect of the motions to the decoherence pattern.

Reduced dynamics in the environmental frame (called “E-frame”), then the influence functional associated with $i$-th Majorana zero mode becomes

$$\mathcal{I}_i^E(t) = -2 \int_0^t dt' \int_0^t dt'' \lambda_i(t') \lambda_i(t'') \frac{d\tau_i}{dt'} \frac{d\tau_i}{dt''} \int_{-\infty}^{\infty} d\omega |\omega|^{d-1} A_i(\omega) e^{-i\omega(t''-t')} \times \oint d\Omega \exp \left[ i\omega \tilde{n} \cdot \left[ \vec{x}_i(t') - \vec{x}_i(t'') \right] \right].$$

In both case, the worldline relations $t = t(\tau_i)$ and $\vec{x} = \vec{x}(\tau_i)$ are implicitly used.

Moreover, we can extract from the reduced dynamics the transition probability of UDW detector as follows:

$$P_{i \rightarrow f} = \lim_{\tau \rightarrow \infty} \langle f | \sum_m \langle m| U(t)|0\rangle \langle i| (0| U^\dagger(t)|m\rangle |f \rangle = \lim_{\tau \rightarrow \infty} \langle f | \text{Tr}_E \rho^D(t) |f \rangle$$

where $\text{Tr}_E \rho^D(t)$ is the reduced density matrix for the (M)UDW detector and note that $ho^D(t = 0) = |0\rangle \langle i| |i\rangle \langle 0|$ so that $\rho^D(t) = U(t) \rho^D(t = 0) U^\dagger(t)$. Note that unlike the usual treatment for the UDW detector, the transition probability derived here is exact, not just up to first order of coupling between probe and environment. We can then compare our results of Unruh effect with the ones obtained by usual treatment.

3. Summary of our results
Despite that the topological qubit is novel and non-local, the effect of the (acceleration) motions to its decoherence patterns may bear some generic features shared by the usual qubit in the similar motions. In some sense, it can be thought as the manifestation of the Unruh effect in the decoherence. Here we list what we found in this work, which we think should be generic:

- Thermalization: We find that the acceleration does cause thermalization as expected by the Unruh effect. This can be seen by the complete decoherence of the accelerating topological qubit in the super-Ohmic environment as this kind of qubits without acceleration are robust against decoherence. However, the influence functional is different from the thermal one. It is worthy of further study to see if the influence functional of the usual qubit in the constant acceleration is the same as the thermal one or not.

- Decoherence impedance: By studying the decoherence patterns we find some novel feature, it seems that the topological qubit is resisting the enforced decoherence due to the sudden change of the motion status such as acceleration or boost. We called this tendency the
Figure 2. Decoherence patterns and transition probability $P_{0\rightarrow 1}$ in the $M$-frame of a circularly moving MUDW detector of constant angular velocity $\Omega$ in the super-Ohmic environments with the switching function of time duration scales: $\sigma = 0.1$ (red) and $\sigma = 2$ (blue). Left: Decoherence patterns for $\Omega = 0.7$ (solid), $\Omega = 0.9$ (dashed) and $\Omega = 0.95$ (dotted). Right: $P_{0\rightarrow 1}$ versus $\Omega$. This figure shows that the decoherence impedance and “anti-Unruh” imply each other.

*Decoherence impedance*, namely, the initial rate of decoherence is anti-correlated with the size of change, such as acceleration. In the late time, this impedance effect will be taken over by thermalization so that in the mean time the overtaking of the decoherence rates happens. As a simple explicit example, it is shown in the left panel of Fig.2 for circularly moving detector.

- **Anti-Unruh**: As shown in (10), in our formalism we can obtain the full transition probability of the topological qubit as the UDW detector, which can be used to characterize Unruh effect.

$$P_{0\rightarrow 1} = \lim_{t \to \infty} \frac{1}{2} \left( 1 - e^{\mathcal{I}_1(t) + \mathcal{I}_2(t)} \right) \quad (10)$$

where $\mathcal{I}_i(t)$ is the influence functional.

Follow the scheme in [5] by decoupling the UDW detector from the environmental field after finite time duration, we can find from the transition probability the aforementioned “anti-Unruh” phenomenon for the accelerating topological qubit in both linear and circular motions. Our result is exact without Markov approximation as done in [5] and thus justifies the “anti-Unruh” phenomenon is generic. An explicit example is shown in the right panel of Fig. 2 for circularly moving detector.

In fact, both the decoherence patterns ($\propto e^{\mathcal{I}_i(t)}$) and the transition probability are all dictated by the influence functional. We would expect the decoherence impedance and “anti-Unruh” phenomenon imply each other as both are the short-time non-equilibrium effects. This is indeed what we observe in this work.

- **Information backflow and time modulation**: As our reduced dynamics is exact without Markov approximation, we shall expect some non-Markovian behaviors characterized by the information flow. We thus apply the time modulations to the coupling constant and the acceleration to see if some modulations can invoke information backflow. We find that not all modulations will bring out information backflow but some does and sometimes even ensures the robustness against decoherence. We have done for the amplitude modulation

$$a(\tau) = \begin{cases} C, & \text{if } \tau_1 < \tau < \tau_2, \\ -C, & \text{if } \tau_2 < \tau < 2\tau_2 - \tau_1, \\ 0, & \text{otherwise.} \end{cases} \quad (11)$$
Figure 3. Decoherence patterns in the $M$-frame of an accelerating topological qubit in the super-Ohmic environment with the amplitude modulation (AM), i.e., Eq. (11) and frequency modulation (FM), i.e., Eq. (12) of the acceleration. Left: AM with $C = 1$ (solid red), $C = 5$ (dotted blue) and $C = 10$ (dashed black) as $\tau_1 = 0.3$ and $\tau_2 = 0.5$. Right: FM with $\omega_G = 10$ (solid red), $\omega_G = 50$ (dotted blue) and $\omega_G = 1$ (dashed black) as $\alpha = 10$.

and the frequency modulation

$$a(\tau) = a \cos \omega_G \tau. \quad (12)$$

The results are shown in Fig. 3.

Moreover, by really exploiting the non-local feature of the topological qubit, we consider the frame dependence issues when both Majorana zero modes move incoherently. We indeed find some novel feature: some incoherent accelerations will instead preserve the coherence of the topological qubit so that it will not be thermalized by the accelerations. This is a phenomenon peculiar for the topological qubit as its constituents can move differently.

The exact solvability of the reduced dynamics even with the nontrivial motions of the topological qubit and the results presented in this paper demonstrate the power of marrying topological order with relativistic quantum information. It is interesting to see if some of the “generic” features aforementioned will also occur for the usual qubit. The answer to this question will definitely shed new light on our understanding of relativistic quantum information.

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