Deeply Equal-Weighted Subset Portfolios

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Abstract

The high sensitivity of optimized portfolios to estimation errors has prevented their practical application. To mitigate this sensitivity, we propose a new portfolio model called a Deeply Equal-Weighted Subset Portfolio (DEWSP). DEWSP is a subset of top-$N$ ranked assets in an asset universe, the members of which are selected based on the predicted returns from deep learning algorithms and are equally weighted. Herein, we evaluate the performance of DEWSPs of different sizes $N$ in comparison with the performance of other types of portfolios such as optimized portfolios and historically equal-weighed subset portfolios (HEWSPs), which are subsets of top-$N$ ranked assets based on the historical mean returns. We found the following advantages of DEWSPs: First, DEWSPs provides an improvement rate of 0.24% to 5.15% in terms of monthly Sharpe ratio compared to the benchmark, HEWSPs. In addition, DEWSPs are built using a purely data-driven approach rather than relying on the efforts of experts. DEWSPs can also target the relative risk and return to the baseline of the EWP of an asset universe by adjusting the size $N$. Finally, the DEWSP allocation mechanism is transparent and intuitive. These advantages make DEWSP competitive in practice.

1 Introduction

Despite the significant success of deep learning, its application to stock trading remains extremely challenging owing to the volatile movements of stock prices, making it difficult to define the input values and understand how to apply the output values. Machine learning models are built on a training set and are tested on a disjointed test set to prove their generalization capability, and are commonly applied in various applications such as image processing, image recognition, speech recognition, and Internet searches. However, this approach is limited when applied to the financial field owing to the time-evolving properties of the financial markets, for example, structural breaks at occasional time points [1, 2], volatility clustering [3], and time-varying mean returns [4]. Furthermore, the time ordering of financial data prevents the use of cross-validation as a reliable estimate of the ensemble.

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generalization error. As a result, the performance of financial time-series models tends to be extremely sensitive to pre-specified periods, showing the high power of in-sample (IS) prediction and the poor power of out-of-sample (OOS) prediction [5]. This hampers the practical use of portfolio optimization techniques because an optimization is prone to the ‘garbage in, garbage out’ phenomenon, in which biases occur in a portfolio selection unless predictions are adjusted suitably for an estimation error. To mitigate this problem, we built a new model called a Deeply Equal-Weight Subset Portfolios (DEWSPs) that combines deep learning techniques with an equal-weight strategy.

Portfolio theory The mean-variance portfolio (MVP) theory, pioneered by Markowitz (1952) [6], has long been recognized as the cornerstone of modern portfolio theory (MPT). It provides a mathematical framework for determining a set of portfolios with a maximized expected return per unit of risk; in addition, the return and risk of a set are drawn as a line, called an efficient frontier, on a risk-return plane. However, despite the theoretical advances in portfolio models including the MVP and its extensions, their practical use remains problematic owing to the difficulty in estimating reliable expected returns, which critically affect the performance of the portfolio [7]. For example, MVPs are not necessarily well-diversified [8], portfolio optimizers are often “error maximizers” [9], and a mean–variance optimization can produce extreme or non-intuitive weights for some of the assets in the portfolio [10, 11]. Many studies have attempted to apply improved estimation procedures and mitigate the estimation error problem. These include Bayesian methods [12, 13], shrinkage methods [14] [15, 16], a factor structure imposed on the returns [17], and the combination of a tangency portfolio, a risk-free rate, and a global minimum variance portfolio [18].

Equal-weight portfolio (EWP) There is a growing body of evidence showing that the use of simple rules of thumb is more successful than optimization. The most well-known example is the EWP, also called 1/N naive diversification, which is free of parameter uncertainty and has the following properties: It never shorts any assets, it avoids a concentration, and upon a rebalancing of the dates, it sells high and buys low, thus exploiting a possible mean-reversion effect [19]. The strength of an EWP is well known experimentally [7, 12, 20, 21]. DeMiguel et al.’s study [20] is particularly convincing because the authors evaluated 14 models on 7 empirical datasets. They found that none of the 14 models consistently outperform a 1/N EWP. Tu and Zhou (2011) [22] showed the combination of an EWP and more sophisticated modes [6, 12, 17, 18] is a way to improve performance. The importance of the EW approach lies in its simplicity and widespread use. In addition, Bernartzi and Thaler (2001) [23] demonstrated that EW diversification is ingrained in human behavior by finding that a considerable fraction of participants equally distribute their contributions across the available investment opportunities. This implies that investment decisions tend to use intuition to choose a security and do not necessarily rely on sophisticated formal techniques. Investors can execute an EW strategy with a large universe but extremely low transaction costs using equally weighted exchange traded funds (ETFs), for example, Direxion NASDAQ-100 Equal Weighted Index Shares, First Trust Dow 30 Equal Weight ETF, and Goldman Sachs Equal Weight U.S. Large Cap Equity ETF.

DEWSP DEWSPs are constructed by incorporating deep learning techniques into an EW
strategy. The building procedure consists of three steps: First, the 1-month ahead return of assets is forecasted using deep learning algorithms. Second, assets are ranked in descending order based on the forecasts. Finally, subset portfolios are constructed with top-$N$ ranked assets that are equally weighted. Our contribution is as follows:

- DEWSP is fully data-driven based on hyperparameter optimization. The entire process is automatic without the views of human experts in building the models, which contributes to reduced costs in terms of portfolio management. We also use public data on the prices and volume, which can be publicly obtained from various Web sites. Thus, DEWSPs are easily reproducible.

- DEWSPs show an increase in their risk and return from the baseline of the EWP with a decrease in the number of assets. This means that it is possible to control the aggressiveness of DEWSPs in terms of their risk-return tradeoff. This mitigates difficulties in understanding the black-box portfolio optimization and in tailoring the risk and return of ranked portfolios based on financial factors (e.g., based on size, value, and leverage).

**Related papers** This study covers stock prediction using deep learning methods and ranked-portfolios. Deep learning models are on the rise, showing impressive results in modeling the complex behavior of financial data. Examples include stock prediction based on long short-term memory (LSTM) networks [24], deep portfolios based on deep autoencoders [25], threshold-based portfolios using recurrent neural networks [26], deep factor models using deep feed-forward networks [27], a time-varying multi-factor model using LSTM networks [28], and an enhancing standard factor model using deep learning [29].

Ranked portfolios are widely used with varying degrees of complexity, and their basic premise is the same: ranking stocks-based on factors such as their value, momentum, quality, size, low risk, and a combination of these factors, and then selecting a particular proportion of the top-ranked stocks to add to the portfolio. These include portfolios ranked in terms of size and book-to-market [30], portfolios ranked on value and momentum factors [31], portfolios ranked on time-series momentum [32], and portfolios ranked on binary classification using returns predicted through deep learning [24].

The remainder of this paper is organized as follows: In Section 2, we describe the data and preprocessing methods applied. In Section 3, we describe the experimental setting and implementation. In Section 4, we provide the experimental results and compare different portfolio models. Finally, some concluding remarks are offered in section 5.

## 2 Data and preprocessing

### 2.1 Universe

Small portfolios are considered for an easier analysis, and are important for several practical reasons [33]: First, it is difficult for small investors to acquire and continuously monitor a large portfolio. Second, large investors need to identify a threshold where the cost exceeds the
benefit of risk reduction from diversification. Third, large portfolios amplify the estimation errors during the optimization process. To select a small but well-diversified universe, we refer to the most commonly applied classification system, i.e., the Global Industry Classification Standard (GICS). The asset universe consists of 22 diversified stocks in Standard and Poor’s 500 index (S&P 500) that belong to 11 different GICS sectors:

- **Energy**: ExxonMobil (XOM) and Chevron (CVX), **Utilities**: Duke Energy (DUK) and Consolidated Edison (ED), **Materials**: Sherwin-Williams (SHW) and DuPont (DD), **Industrials**: Boeing (BA) and Union Pacific (UNP), **Consumer Discretionary**: Amazon (AMZN) and McDonald’s (MCD) **Consumer Staples**: Coca-Cola (KO) and Procter & Gamble (PG) **Healthcare**: United Health Group (UNH) and Johnson & Johnson (JNJ) **Financials**: Berkshire Hathaway (BRK-B) and JPMorgan Chase (JPM) **Information Technology Sector**: Apple (AAPL) and Microsoft (MSFT), **Communication Services**: Facebook (FB) and Alphabet (GOOG), **Real Estate**: American Tower (AMT) and Simon Property Group (SPG).

We use data from Yahoo Finance from January 1997 to October 2019, which is the common period of data availability. The monthly stock dataset contains five attributes: open price, high price, low price, adjusted close price, and volume (OHLCV). The last of the daily OHLCV datasets per month is used as the raw dataset. For each experiment, we split the data into an in-sample (70%) period and an out-of-sample (30%) period. The in-sample data are divided again into a training dataset (50%) for developing the prediction models and a validation set (50%) for evaluating its predictive ability.

**Technical indicators** A technical analysis is a method for forecasting price movements using past prices and volume and includes a variety of forecasting techniques such as a chart analysis, cycle analysis, and computerized technical trading systems.

A technical analysis has a long history of widespread use by participants in speculative markets [34, 35, 36, 37, 38, 39], and there is a large body of academic evidence demonstrating the usefulness of such analysis, including theoretical support [40] and empirical evidence [41, 42], as well as the role of such analysis in out-of-sample equity premium predictability [43, 44, 45]. We used a full set of 14 technical indicators based on 3 types of popular technical strategies, i.e., the moving average crossover, momentum, and volume rules:

- The time-series momentum indicator, \( \text{MOM}(m) \), is the generation of a buy signal when the price is higher than the historical price. Its validation is supported by the observation that the “trend” effect persists for approximately 1 year and then partially reverses over a longer timeframe. Here, \( \text{MOM}_t(m) \) at time \( t \) is defined as follows:

\[
\text{MOM}_t(m) = \begin{cases} 
1 \text{ (Buy signal)}, & \text{if } P_t \geq P_{t-m} \\
-1 \text{ (Sell signal)}, & \text{otherwise.}
\end{cases}
\] (1)

where \( P_t \) is the index value at time \( t \), and \( m \) is the look-back period. We use \( m = 1, 3, 6, 9 \) and 12, which are respectively labeled as \( \text{MOM}_t(1M) \), \( \text{MOM}_t(3M) \), \( \text{MOM}_t(6M) \), \( \text{MOM}_t(9M) \), and \( \text{MOM}_t(12M) \).
The moving average indicator, MA\((s, l)\), provides a signal for an upward or downward trend. A buy signal is generated when the short-term moving average crosses above the long-term moving average because this represents the beginning of an upward trend. A sell signal is generated when the short-term moving average falls below the long-term moving average because this represents the beginning of a downward trend.

Let us define a simple moving average of the index as follows:

\[
MA_{j,t}^P = \frac{1}{j} \sum_{i=0}^{j-1} P_{t-i} \quad \text{for } j = s \text{ or } l, \tag{2}
\]

where \(s\) and \(l\) are the look-back periods for short and long moving averages. The moving average indicator \(MA_t(s, l)\) is then designed as follows:

\[
MA_t(s, l) = \begin{cases} 
1 \text{ (Buy signal)}, & \text{if } MA_{s,t}^P \geq MA_{l,t}^P \\
-1 \text{ (Sell signal)}, & \text{otherwise.}
\end{cases} \tag{3}
\]

The six moving average indicators are constructed for \(s = 1, 2, \text{ and } 3, \text{ and for } l = 9 \text{ and } 12, \text{ which are symbolized as } MA(1M-9M), MA(1M-12M), MA(2M-9M), MA(2M-12M), MA(3M-9M), \text{ and } MA(3M-12M).

The volume indicator, VOL\((s, l)\), indicates a strong market trend if the recent stock market volume and stock price increase. Let us define the on-balance volume (OBV) as follows:

\[
OBV_t = \sum_{k=1}^{t} VOL_k D_k, \tag{4}
\]

where \(VOL_k\) is a measure of the trading volume (i.e., number of shares traded) during period \(k\), and \(D_k\) is a binary variable:

\[
D_k = \begin{cases} 
1, & \text{if } P_k \geq P_{k-1} \\
-1, & \text{otherwise.}
\end{cases} \tag{5}
\]

The value of \(OBV_t\) conceptionally measures both positive and negative volume based on the belief that changes in volume can predict a stock movement. The volume-based indicator is then defined as the difference between the moving averages with an \(s\)-period and an \(l\)-period:

\[
VOL(s, l) = \begin{cases} 
1 \text{ (Buy signal)}, & \text{if } MA_{s,t}^{OBV} \geq MA_{l,t}^{OBV} \\
-1 \text{ (Sell signal)}, & \text{otherwise.}
\end{cases} \tag{6}
\]

Here, \(MA_{j,t}^{OBV} = (1/j) \sum_{i=0}^{j-1} OBV_{t-i}\) is the moving average of \(OBV_t\) for \(j = s \text{ or } l\). The six moving average indicators are constructed for \(s = 1, 2, \text{ and } 3, \text{ and for } l = 9 \text{ and } 12, \text{ which are symbolized as follows: } VOL(1M-9M), VOL(1M-12M), VOL(2M-9M), VOL(1M-12M), VOL(3M-9M) \text{ and } VOL(3M-12M).
3 Frameworks

3.1 Portfolios

For a comparative analysis, we also built three different types of portfolios, which are distinct in terms of their optimization or estimation process. All portfolios are built on the following assumptions: (1) all stocks are infinitely divisible; (2) there are no restrictions on the buying or selling of any selected portfolio; (3) there is no friction (e.g., transaction costs, taxation, commissions, or liquidity); and (4) it is possible to buy and sell stocks at the closing prices at any time \( t \). We adapt a periodic rebalancing strategy in which the investor adjusts the weights in the investor’s portfolio at the close price on the last business day of every month.

List of portfolios considered:

- **DEWSP**: This is a subset of portfolios that consist of the top \( N \)-th ranked assets among all \( N_0 \) assets based on their expected returns.

- **EW whole portfolio (EWWP)**: This is a traditional EWP of all assets \( N_0 \), and can be viewed as a special case of DEWSP when \( N = N_0 \). Because there are no parameter estimations, it serves as the baseline for an evaluation of the risk and return of the DEWPs of different sizes.

- **Historically EW subset portfolios (HEWSPs)**: Like DEWSPs, HEWSPs are top-ranked subset portfolios, although their expected returns are estimated as a historical average over the training and validation (HEWSP-TV) and historical average over the validation (HEWSP-V). This reveals the effect of the return prediction of the DEWSPs.

- **Randomly EW subset portfolios (REWSPs)**: These are subsets of portfolios consisting of \( N \) assets selected randomly, without the use of a ranking method. A comparison between REWSPs and DEWSPs and HEWSPs reveals the effect of the estimated return prediction.

- **Maximum Sharpe ratio portfolios (MSRPs)**: These are complete portfolios that are maximized to achieve the highest Sharpe ratio, and are mathematically defined as follows:

\[
\max_{w_t} w_t^T \mu_t / \sqrt{w_t^T \Sigma_t w_t} \quad \text{s.t.} \quad w_t^T 1 = 1, \quad \text{and} \quad w_{i,t} \geq 0, \forall i, \quad (7)
\]

where \( \mu_t \) is a vector of \( N_0 \) predicted returns, \( w_t = (w_{1,t}, \ldots, w_{N_0,t})^T \) is a vector of portfolio weights, \( \Sigma_t \) is a covariance matrix of the asset returns, \( 1_N = (1, \ldots, 1)^T \) is an \( N \)-dimensional vector, and \( w_t^T \mu_t \) and \( w_t^T \Sigma_t w_t \) are the portfolio return and variance, respectively. Because \( \mu_t \) and \( \Sigma_t \) are unknown in practice, we replace them with \( \hat{\mu}_t \) from deep learning algorithms and \( \hat{\Sigma}_t \) from an in-sample dataset. A comparison with DEWSPs reveals the effect of the estimation error on a portfolio optimization.

- **Minimum variance portfolios (MVPs)**: These are complete portfolios optimized for the lowest volatility, and solve the following constrained minimization problem:

\[
\min_{w} w_t^T \Sigma_t w_t \quad \text{s.t.} \quad w_t^T 1 = 1, \quad \text{and} \quad w_{i,t} \geq 0, \forall i. \quad (8)
\]
A comparison with DEWSPs reveals the effectiveness of optimization under the condition of no estimation errors.

3.2 Prediction models

3.2.1 Training

A multilayer feedforward neural network (FFNN) was used in this study. We used Tree-structured Parzen Estimator (TPE) approach [46] for automated hyperparameter tuning and Table 1 presents the list of hyperparameters and their values. Each optimization run was initialized with randomly selected points, after which it proceeded sequentially for a total of 50 function evaluations. During one evaluation run, the FFNN was trained over an in-sample training data. The mean squared error (MSE) is calculated on a validation set per function evaluation, early stopping was applied when there is no improvement on the validation accuracy after 10 continuous epochs.

3.2.2 Regularizer

We used two popular regularization methods, i.e., a dropout and batch normalization (BN). A dropout [47] is a simple way to prevent co-adaptation among hidden nodes of a deep feed-forward neural network by dropping out randomly selected hidden nodes. In recent years, BN [48] has replaced a dropout in modern neural network architectures, and uses the distribution of the summed input to a specific neuron over a mini-batch of training cases to compute the mean and variance, which are then used to normalize the summed input to that neuron for each training case. A dropout and BN layers were employed for all hidden layers.

3.3 Evaluation metrics

**Average percent change (APC)** The APC measures the rate of change in a DEWSP return and the volatility as size $N$ increases from $N = 1$ to $N = N_0$ to see the rate of change from the baseline of $N = N_0$ to $N = 1$, and is defined as follows:

$$\text{APC}_x = \frac{1}{N_0 - 1} \sum_{N=1}^{N_0-1} \frac{x^N - x^{N+1}}{x^{N+1}},$$  \hspace{1cm} (9)

where $x$ is $r_t$ or $\sigma_t$.

**Average Sharpe ratio improvement rate (ASRIR)** ASRIR measures the relative improvement of the DEWSPs as compared to the HEWSP benchmark in terms of the Sharpe ratio (SR), and is defined as follows:

$$\text{ASRIR} = \frac{1}{N_0} \sum_{N=1}^{N_0} \frac{x^N_{\text{DEWSP}} - x^N_{\text{HEWSP-TV/T}}}{x^N_{\text{HEWSP-TV/T}}},$$  \hspace{1cm} (10)

where $x$ is the SR of DEWSPs and HEWSPs of the same size $N$. 7
Table 1: List of hyperparameters and range of each hyperparameter.

| Hyperparameter                    | Considered values/functions |
|-----------------------------------|-----------------------------|
| Number of Hidden Layers           | {2, 3}                      |
| Number of Hidden Units            | {2, 4, 8, 16}               |
| Standard deviation                | {0.025, 0.05, 0.075}        |
| Dropout                           | {0.25, 0.5, 0.75}           |
| Batch Size                        | {28, 64, 128}               |
| Optimizer                         | {RMSProp, ADAM, SGD (no momentum)} |
| Activation Function               | Hidden layer: {tanh, ReLU, sigmoid}, Output layer: Linear |
| Learning Rate                     | {0.001}                     |
| Number of Epochs                  | {100}                       |

Number of layers: number of layers of a neural network. Number of hidden units: number of units in the hidden layers of a neural network. Standard deviation: standard deviation of a random normal initializer. Dropout: dropout rates. Bath size: number of samples per batch. Activation: sigmoid function $\sigma(z) = 1/(1 + e^{-z})$, hyperbolic tangent function $\tanh(z) = (e^z - e^{-z})/(e^z + e^{-z})$, and rectified linear unit (ReLU) function $\text{ReLU}(z) = \max(0, z)$. Learning Rate: learning rate of the back-propagation algorithm. The Number of Epochs: number of iterations over all training data. Optimizer: stochastic gradient descent (SGD) [49], RMSProp [50], and ADAM [49]

4 Experiments and Results

We examined the portfolio performance over both IS and OOS periods for three different universes: a total of 22 stocks (Exp. I), with 11 stocks consisting of the first stocks of each sector on the list (Exp. II), and the other 11 stocks (Exp. III). The following observation was made based on the empirical simulation results.

4.1 In-sample performance

- The left side of Figure 1 graphically shows the realized risk and return points of the portfolios on the risk-return plane. Each color represents a different type of portfolio, and different points with the same color represent different sizes. A comparison of DEWSPs and HEWSPs with the REWSPs of a (seemingly) random pattern indicates that the prediction-based ranking assets can be used to construct portfolios with increasing return and volatility as $N$ decreases. In Table 2, APC$_r$s and APC$_\sigma$s indicate quantitative measurements of the increase over Exp. I, II, and III, and APC$_r$/APC$_\sigma$ shows the degree of trade-off between the return and risk.

- We also found ASRIRs of 21.15, 27.04, and 13.09% for Exp. I, II, and III, respectively, indicating the superiority of DEWSP during the IS period.

- MVP, as expected, achieves the least volatility of 0.99, and MSRP achieves the highest Sharpe ratio of 0.65 ($\mu = 0.026$ and $\sigma = 0.040$), which outperform those of the
Figure 1: Realized risk vs. return of six different types of portfolios for the in-sample (left) and out-of-sample (right) experiments. The dotted lines specify the maximum SR estimate.

Table 2: Performance evaluation results of DEWSPs over in-sample period.

| Metrics (%)          | Exp. I | Exp. II | Exp. III |
|----------------------|--------|---------|----------|
| APC_r                | 6.11   | 10.06   | 11.23    |
| APC_σ               | 5.07   | 8.86    | 10.39    |
| APC_r/APC_σ         | 1.20   | 1.13    | 1.08     |
| ASRIR (w.r.t. HEWSP-TV) | 21.15  | 27.04   | 13.09    |
| ASRIR (w.r.t. HEWSP-T) | 19.12  | 21.93   | 12.24    |

DEWSPs. This suggests that the prediction quality over the IS period is sufficient, allowing a benefit from the optimization process.

4.2 Out-of-sample performance

- The DEWSPs are built using 1-month ahead predicted returns from the trained model, and the HEWSPs are built using the average historical return over the in-sample period.

- The computation results are summarized on the right side of Figure 1 and in Table 3. As with the IS experiment, the return and volatility of the DEWSPs and HEWSPs still show an increasing pattern with the positive APC values. This allows us to tailor the portfolio’s return and risk for investment purposes.

Table 3: Performance evaluation results of DEWSPs over the out-of-sample period.

| Metrics (%)          | Exp. I | Exp. II | Exp. III |
|----------------------|--------|---------|----------|
| APC_r                | 3.24   | 6.74    | 7.45     |
| APC_σ               | 4.42   | 6.64    | 8.86     |
| APC_r/APC_σ         | 0.73   | 1.01    | 0.84     |
| ASRIR (w.r.t. HEWSP-TV) | 3.91   | 5.15    | 0.24     |
| ASRIR (w.r.t. HEWSP-T) | 3.30   | 3.91    | 0.92     |
• The ASRIRs ranged from 0.24 to 5.15% indicate that the DEWSPs outperform the historical models in terms of the monthly SR. The values are small compared to those of the in-sample ASRIRs, but indicate promising results. First, we can beat the HEWSP benchmark, and second, we can tailor the return and volatility of the portfolios relative to the baseline of the EWWP.

• Although the MVP without a parameter estimation still gives the least volatility at 0.295%, the MSRP has a monthly SR of 0.44% ($\mu = 0.014$ and $\sigma = 0.032$), which is lower than that of the DEWSPs and HEWSPs, indicating that the prediction quality is insufficient for the purposes of portfolio optimization.

5 Conclusion

Despite the significant success of machine learning in numerous fields, stock prediction is still severely limited owing to its seasonal, non-stationary, and unpredictable nature. Consequently, portfolio models are inevitably exposed to the risk of estimation errors, which hinders their performance.

To cope with such risk, we have proposed a new DEWSP model by incorporating deep-learning-based predictions into the framework of the EW strategy. We empirically demonstrated that DEWSPs can be used to target the levels of portfolio return and risk relative to the baseline of the EWWPs by adjusting the number of assets, and that its mechanism is clear in terms of the risk-return trade-off. We also showed that DEWSPs are superior to HEWSPs in terms of the SR and that the mean-variance optimization amplifies the estimation error dramatically, which results in a substantially worse Sharpe ratio. To summarize, DEWSPs are attractive from an implementation perspective, i.e., the use of public stock data, a transparent mechanism based on a risk-return trade-off, automatic hyperparameter optimization, the existence of a baseline of the EWP and a benchmark of the HEWSP, the capability of building portfolios using small numbers of assets (with expandability to large assets), and a simple incorporation of deep learning algorithms into the portfolio scheme.

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