A New Massive Type IIA Supergravity from Compactification

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ABSTRACT

We consider the most general form for eleven dimensional supersymmetry compatible with on-shell superfields. This allows for the introduction of a conformal Spin(1,10) connection. In eleven dimensional Minkowski space this modification is trivial and can be removed by a field redefinition, however, upon compactification on S¹ it is possible to introduce a non-trivial ‘Wilson line’. The resulting ten dimensional supergravity has massive 1-form and 3-form potentials and a cosmological constant. This theory does not possess a supersymmetric eightbrane soliton but it does admit a supersymmetric non-static cosmological solution.

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1. Introduction

Prior to the advent of D-branes as carriers of RR charge [1] the most useful tools in the study of non-perturbative string theory were supergravities. Indeed even now, next to the newly developed and much studied M(atrix) theory, eleven dimensional supergravity [2] remains one of the few tools available to study M theory.

One problem with our present knowledge of M theory is that there is no eleven dimensional understanding of the type IIA eightbrane. From the supergravity point of view this can be described by a BPS soliton of Romans’ massive type IIA supergravity [3,4]. In this case the cosmological constant is interpreted as the Hodge dual of the eightbrane field strength. There has been some recent work relating the Romans field equations to the consistency conditions for $\kappa$-symmetry of a massive type IIA D-twobrane [5] and the corresponding picture in eleven dimensions [6]. However there is no known way to obtain the Romans theory directly from eleven dimensions. Furthermore there has arisen the possibility of additional type IIA branes [7] for which there is also no eleven dimensional interpretation.

Thus it is of interest to understand the origin of massive supergravities by compactification from eleven dimensions. In fact since the bosonic field content of the (massless) type IIA supergravity contains the $p$-form fields $(\sigma, A_m, B_{mn}, C_{mnp})$ [8] there are potentially three distinct Higg’s mechanisms which could give rise to massess; the vector $A$ could eat the scalar $\sigma$, the 2-form $B$ could eat the vector, or the 3-form $C$ could eat the 2-form. In Romans theory the 2-form eats the vector but this leaves open the possibility that other massive type IIA supergravities can be found. Indeed below we shall verify this and obtain a new massive type IIA supergravity by compactifying eleven dimensional supergravity. In this theory the other two Higg’s mechanisms are both employed. In addition the method we describe appears to be applicable in other dimensions.

Recently it has been argued that there is no possibility for a cosmological constant in eleven dimensions [9] which sheads doubt on the most straightforward
interpretation of the type IIA eightbrane as a wrapped M theory ninebrane (here we mean a BPS soliton of uncompactified eleven dimensional supergravity carrying a 10-form charge). Our construction is consistent with the conclusions of [9] since the cosmological constant is associated with a compact direction and so does not appear in eleven dimensional Minkowski space.

The interpretation of the eightbrane as a wrapped ninebrane suffers from additional problems. In particular the worldsheet effective action for an M theory ninebrane would naively be based on a ten dimensional $N = 1$ supermultiplet with one scalar. However the only suitable supermultiplets also contain states with spins greater than one and have more degrees of freedom than that contained in the $N = 1, D = 10$ Maxwell supermultiplet which describes the other D-branes. One the other hand all the D-branes are related by T-duality and hence they should all carry the same number of degrees of freedom on their worldvolumes. A second problem is the lack of zerobrane/eightbrane type IIA bound states preserving one quarter of the spacetime supersymmetry. If the eightbrane is the double dimensional reduction of a ninebrane then these states should exist as Kaluza-Klein modes on the eightbrane [10]. Thus one expects that the type IIA eightbrane’s M theory origin is like that of the type IIA sixbrane. Its eleven dimensional interpretation is as a Kaluza-Klein monopole which cannot exist in the uncompactified theory. As a result its worldsheet theory only possesses three scalar fields because the “zero mode” associated to the compact Killing direction is massive in the quantum theory (see also [7,11]).

For the rest of this paper we shall pursue a construction of supergravity where the origin of the mass comes from topological effects. We shall see that the massive supergravity we obtain does admit a natural BPS state but in contrast to the other supergravities it is not static. As we mentioned above a cosmological constant is generally associated with an eightbrane but here the resulting solution appears to represent some kind of dynamical instability of the compactification.
2. Weyl Superspace

In this paper underlined quantities refer to tangent space indices, letters from the beginning of the alphabet denote eleven dimensional indices and those from the middle ten dimensional indices. Hatted variables refer to eleven dimensional fields and we use the “mostly plus” signature.

In the superspace formulation of eleven dimensional supergravity the field equations are entirely determined by solving constraints on superspace [12]. The starting point is to introduce a spin connection on superspace taking values in Spin(1,10). However it is possible to allow for the introduction of a conformal spin connection [13]. The solution to the constraints then proceeds as before provided that the conformal curvature vanishes.

We write the $CSpin(1,10)$ connection as

\[ \hat{\Upsilon}_a = \frac{1}{4} \hat{\Omega}^{bc} \Gamma_{bc} + 2k_a. \]  

(2.1)

The condition that the conformal part of the curvature vanishes is then simply that $dk = 0$. The Lorentz part may be further written as

\[ \hat{\Omega}^{ab}_a = \hat{\omega}^{ab} + 2(\hat{e}^a_k k_b - \hat{e}^b_k k_a), \]  

(2.2)

where $\hat{\omega}$ is the Levi-Civita connection. The additional terms in (2.2) are needed to ensure that the torsion of the connection $\hat{\Upsilon}$ vanishes. The bosonic degrees of freedom are carried by a vielbein $\hat{e}^b_a$ and anti-symmetric 3-form gauge potential $\hat{B}_{abc}$ with Weyl weight 2 and field strength $\hat{H}_4 = \hat{D}\hat{B}_3$. The field equations are [13]

\[ \hat{R}_{ab} - \frac{1}{2} \hat{g}_{ab} \hat{R} = -\frac{1}{48} \left( 4 \hat{H}_{aced} \hat{H}_b^{cd} - \frac{1}{2} \hat{g}_{ab} \hat{H}^2 \right), \]  

(2.3)

\[ \hat{D}^a \hat{H}_{abcd} = \frac{1}{36} \frac{1}{48} \epsilon_{bcd\ldots f\ldots} \hat{H}^e\ldots \hat{H}^f\ldots. \]

Note that the curvatures and covariant derivatives appearing in the eleven dimensional expressions are those of the conformal spin connection $\hat{\Upsilon}$. As it is written
equation (2.3) only considers the bosonic fields. However precisely the same equation is true if we interpret the fields as being eleven dimensional superfields. The equations then possess the supersymmetry

\[
\delta \hat{e}_a^b = -i \epsilon \Gamma^b \psi_a,
\]
\[
\delta \hat{B}_{abc} = -3i \epsilon \Gamma_{[ab} \psi_{c]} ,
\]
\[
\delta \hat{\psi}_a = \hat{D}_a \epsilon + \frac{1}{36} \left( \epsilon \Gamma^{bcd} \hat{H}_{abcd} + \frac{1}{8} \epsilon \Gamma_{abcde} \hat{H}^{bcde} \right).
\]

We note here also the both the massless type IIA supergravity and Romans theory can be given a superspace formulation [14].

3. Massive Electrodynamics and Topology

Before discussing the compactification of eleven dimensional supergravity with the addition conformal connection it is instructive to first consider the analogous case for electrodynamics. To this end suppose we have the action

\[
S = \frac{1}{4} \int_{\mathcal{M}} d^dx \sqrt{-g} \hat{F}_{MN} \hat{F}^{MN} ,
\]

defined over a d dimensional manifold \( \mathcal{M} \) with a metric \( g_{MN} \) which we will assume to be non-dynamical and \( M, N = 0, \ldots, d - 1 \). In standard electrodynamics one introduces the exterior derivative \( d \) and defines \( \hat{F} = d \hat{A} \) as the 2-form field strength of a \( U(1) \) connection \( A \). However, if we introduce a 1-form \( k \) then we could also define another derivative \( D = d + k \) and curvature \( \hat{F} = D \hat{A} \). By taking \( k \) to be closed we maintain \( D^2 = 0 \) and as a result the action (3.1) is invariant under the transformation

\[
\hat{A}_M \rightarrow \hat{A}_M + D_M \lambda .
\]

Now \( \hat{F} \) is by definition a closed 2-form (with respect to \( D \)) on \( \mathcal{M} \) and the field equation of (3.1) states that \( \hat{F} \) is also co-closed, so that if \( \mathcal{M} \) is compact the classical solutions correspond to elements of the cohomology \( H^2_D(\mathcal{M}) \) of \( D \).
If \( k = m \psi \) is exact (for example if \( \mathcal{M} \) is simply connected), then we may write \( D = e^{-m\psi} \cdot d \cdot e^{m\psi} \) and it follows that \( H^*_D(\mathcal{M}) \) is isomorphic to the De Rham cohomology group \( H^*_{DR}(\mathcal{M}) \). Furthermore by simply redefining \( \hat{A}_M \rightarrow e^{-m\psi} \hat{A}_M \) (i.e. \( A \) has Weyl weight one), \( \hat{g}_{MN} \rightarrow e^{-\psi} \hat{g}_{MN} \) we would obtain the action for standard electrodynamics (on a conformally equivalent manifold).

Let us now suppose that \( \mathcal{M} \) is not simply connected and that \( k \) is not exact. In this case the above field redefinition is no longer globally valid. In particular let us choose coordinates in which \( k_y = m, k_\mu = 0 \) with \( \mu = 0, \ldots, d - 2 \) and compactify the theory along \( y \). As is usual we define \( A_\mu = \hat{A}_\mu \) and \( \phi = \hat{A}_y \) and take them to be independent of \( y \). The equations of motion are now (we assume here that \( \mathcal{M} \) is flat for simplicity)

\[
\partial^\mu F_{\mu\nu} - m \partial_\nu \phi + m^2 A_\nu = 0 , \\
\partial^2 \phi - m \partial^\mu A_\mu = 0 .
\] (3.3)

It is easy to see that the second equation in (3.3) is the integrability condition for the first. This apparent loss of a degree of freedom arises because the symmetry (3.2) reduces to

\[
A_\mu \rightarrow A_\mu + \partial_\mu \lambda , \quad \phi \rightarrow \phi + m \lambda .
\] (3.4)

Thus we may gauge away \( \phi \) and arrive at the equations of massive electrodynamics

\[
\partial^2 A_\mu + m^2 A_\mu = 0 .
\] (3.5)

Note however that the photon field is tachyonic, indicating that this compactification is somehow unstable.
4. Dimensional Reduction to Ten Dimensions

Let us now return to the dimensional reduction of eleven dimensional supergravity [8]. As in the previous section if \( k \) is exact, ie \( k = m \psi \), then it is possible to redefine the fields \( \hat{e}_b^a, \hat{\psi}_a, \hat{B}_3 \) so as to absorb \( \psi \) and we arrive at the standard equations of eleven dimensional supergravity. Let us now consider the case that the eleven dimensional space has the topology \( M^{10} \times S^1 \). If \( y \) is a coordinate for the \( S^1 \) then we may let the conformal part of the \( CSpin(1,10) \) connection take the form

\[
k = m dy .
\]  

(4.1)

To compactify we will simply make the usual ansatz that the fields are independent of \( y \). For the purposes of this paper we shall ignore the fermions in ten dimensions. They may be obtained from the eleven dimensional superspace formulation and are guaranteed to provide a supersymmetric completion of the theory. In addition we will only consider compactifications with \( \hat{H}_{abcd} = 0 \). If we make the standard string-frame reduction

\[
\hat{e}_a^b = \begin{pmatrix} e^{-\sigma/3} e_m^n & A_m e^{2\sigma/3} \\ 0 & e^{2\sigma/3} \end{pmatrix},
\]  

(4.2)

we find the following equations of motion for the bosonic fields

\[
R_{mn} - \frac{1}{2} g_{mn} R = - \frac{1}{2} \left( F_{mp} F_n^p - \frac{1}{4} g_{mn} F^2 \right) + 2 \left( D_m D_n \sigma - g_{mn} D^2 \sigma + g_{mn} (D\sigma)^2 \right)
\]

\[
+ 18 m \left( D_{(m} A_{n)} - g_{mn} D^p A_p \right) + 36 m^2 \left( A_m A_n + 4 g_{mn} A^2 \right)
\]

\[
+ 12 m A_{(m} \partial_{n)} \sigma + 30 m g_{mn} A^m \partial_m \sigma + 144 m^2 g_{mn} e^{-2\sigma},
\]

\[
D^n F_{mn} = 18 m A_n F_m^m + 72 m^2 e^{-2\sigma} A_m - 24 m e^{-2\sigma} \partial_m \sigma,
\]

\[
6 D^2 \sigma - 8 (D\sigma)^2 = R + \frac{1}{4} F^2 + 360 m^2 e^{-2\sigma} + 288 m^2 A^2 + 96 m A^m \partial_m \sigma
\]

\[
- 36 m D^m A_m,
\]  

(4.3)

where \( F_{mn} = \partial_m A_n - \partial_n A_m \). In the above equations all curvatures and covari-
ant derivatives are with respect to the ten dimensional Levi-Civita connection $\omega$. Clearly if $m = 0$ these equations reduce to the standard equations of type IIA supergravity [8]. Furthermore one can check that these equations are self-consistent in the sense the the intergrability condition for the second equation in (4.3) is implied by the other two equations.

The form of the equations of motion is peculiar and it is not to difficult to see that they cannot be obtained from a Lorentz invariant Lagrangian. Therefore the theory cannot be turned into Romans supergravity by any field redefinition. It is also clear from (4.3) that the $U(1)$ symmetry of the gauge field $A_m$ has been affected by the mass terms. However, a modified non-compact symmetry still exists and the fields equations are invariant under the transformations

$$A_m \rightarrow A_m - \partial_m \lambda ,$$

$$\sigma \rightarrow \sigma - 3m\lambda ,$$

$$g_{mn} \rightarrow e^{-6m\lambda} g_{mn} .$$

(4.4)

Thus for $m \neq 0$ the dilaton $\sigma$ is eaten by the vector field $A_m$ which has become massive. If we had not discarded the antisymmetric tensor field in the compactification then there would also be a 2-form gauge symmetry because $\hat{B}_3$ always appears in the equations of motion through its field strength $\hat{H}_4 = \hat{D}\hat{B}_3$. Hence $\hat{B}_3$ is defined only up to transformations $\delta\hat{B}_3 = \hat{D}\Lambda_2$. In ten dimensions where we may write $\hat{B}_3 = C_3 + B_2 \wedge dy$ this symmetry becomes

$$C_3 \rightarrow C_3 + d\Lambda_2 ,$$

$$B_2 \rightarrow B_2 + d\Lambda_1 + 6m\Lambda_2 .$$

(4.5)

Therefore if $m \neq 0$ the 3-form gauge field eats the 2-form and also becomes massive, analogously to the way that we arrived at massive electrodynamics in the previous section.
5. A Euclidean Eightbrane

In this section we will look for solutions to the field equations which preserve half the supersymmetry. If we look for purely bosonic solutions then we may just consider the supersymmetry transformation (written in eleven dimensional form for simplicity)

\[\delta \hat{\psi}_a = \partial_a \epsilon + \frac{1}{4} \hat{\omega}_a^{bc} \Gamma_{bc} - m \hat{\epsilon}_a^b \hat{\epsilon}^{cy} \Gamma_{bc} \epsilon + 2m \hat{\epsilon}_a^y \epsilon + \frac{1}{36} \left( \epsilon \Gamma^{bcd} \hat{H}_{abcd} + \frac{1}{8} \epsilon \Gamma_{abcde} \hat{H}^{bcde} \right). \tag{5.1}\]

Note that the fourth term appearing in \(\delta \hat{\psi}_a\) does not come with gamma matrix as it does in standard cosmological supergravities such as Romans. Let us look for an eightbrane solution with nine dimensional Poincare symmetry and a single spacelike transverse space with the fields \(\hat{H}_{abcd}\) set to zero. This immediately runs into problems since the second and third terms appearing in \(\delta \hat{\psi}_a\) contain \(\Gamma_{bc}\) which, for spacelike indices, has imaginary eigenvalues while the fourth term always has real eigenvalues. This mis-match comes about because we have identified the compact \(U(1)\) symmetry of diffeomophisms of the circle with noncompact scale transformations. At the level of the field equations there is no difficulty to doing this, since they have the same Lie algebra, but it is reflected by the reality of \(m\) in the supersymmetry transformations (5.1). Thus there is no eightbrane BPS soliton. However, this suggests that we look for BPS solutions which are time dependent.

We therefore will start with the ansatz

\[ds_{10}^2 = -e^{2g} dt^2 + e^{2f} d\mathbf{x} \cdot d\mathbf{x}, \tag{5.2}\]

\[F_{mn} = 0,\]

with \(f, g, \sigma\) functions of time \(t\) only. Demanding that this solve the field equations gives

\[(\dot{f} - \dot{\sigma})e^{\sigma - g} = \pm 2m, \tag{5.3}\]

\[A_m = \frac{1}{3m} \partial_m \sigma, \]

where a dot denotes differentiation with respect to \(t\). If we now ask that the ansatz
(5.2) preserves some supersymmetries we find that

\[ \dot{\sigma} e^{\sigma - g} = \pm 14m, \]
\[ \dot{f} e^{\sigma - g} = \pm 12m, \] (5.4)

and the remaining supersymmetries are generated by

\[ \epsilon = e^{-3\sigma/4} \epsilon_0, \]
\[ \epsilon_0 = \pm \Gamma_0 \Gamma_y \epsilon_0. \] (5.5)

Clearly (5.4) implies that the field equation (5.3) is solved. Note that by a redefinition of \( t \) we are free to choose \( g \) to be any non-singular function. Let content ourselves here with \( g = \sigma \). We then find the solutions

\[ \sigma = \sigma_0 \pm 12mt, \]
\[ f = f_0 \pm 14mt. \] (5.6)

This solution is similar to an eightbrane solution except that it has nine dimensional Euclidean symmetry and the transverse space is timelike. At \( t = 0 \) this solution is simply flat \( \mathbb{R}^9 \). However, as time passes the radius of the compact direction increases or decreases depending on the choice of sign in (5.6). For the plus sign the spacetime decompactifies back up to eleven dimensions but for the minus sign it compactifies even further. We may lift the “Euclidean eightbrane” solution to eleven dimensions where it takes the form

\[ ds_{11}^2 = e^{4\sigma/3} (-dt^2 + dy'^2) + e^{5\sigma/3} d\mathbf{x} \cdot d\mathbf{x}, \] (5.7)

where \( y' = y \pm 4t \).
6. Conclusions

In this paper we have investigated the generation of mass in supergravity through compactification on a circle and we have found a massive supergravity in ten dimensions which is not that obtained by Romans. The resulting theory has a number of unusual features, one of which is that it has no action. In addition the compactification ansatz used in this construction relates the internal compact symmetry of the circle to the non-compact dilation symmetry. This is only required at the Lie algebra level, where the two algebras are isomorphic, but it appears to be responsible for the wrong sign for $m^2$ and the time dependence of the supergravity BPS solution.

Can we provide the supergravity obtained here with a natural interpretation? One interpretation of $p$-branes is that they interpolate between different supersymmetric vacua of a theory [15]. Even though the BPS state constructed here is not a $p$-brane we may try to give it a similar interpretation. As $t \to -\infty$ (we assume the $-$ sign without loss of generality) the radius of the eleventh dimension blows up and they theory decompactifies. However, at late times the radius shrinks to zero. Thus this solution appears to be interpolating between uncompactified M theory and weakly coupled type IIA string theory. In the flat space example in section three the vector field in the lower dimension was tachyonic, indicating that the ground state is unstable. The fact that the solution (5.2) is time dependent suggests that the ten dimensional theory is in fact unstable against decompactification back up to eleven dimensions. Or, conversely, in a time reversed picture the theory is unstable against further compactification.
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