The structure of the complex plasma boundary

P M Bryant
Centre for Interdisciplinary Plasma Science, Max-Planck Institut für Extraterrestrische Physik, D-85740 Garching, Germany
E-mail: bryant@mpe.mpg.de

New Journal of Physics 6 (2004) 60
Received 22 March 2004
Published 10 June 2004
Online at http://www.njp.org/
doi:10.1088/1367-2630/6/1/060

Abstract. In recent experiments on the International Space Station, a void or dust-free region was often observed in the central region of a complex plasma. A characteristic feature of the void-complex plasma boundary is a region, several inter-particle distances thick, where the complex plasma is compressed. Typically, $\epsilon = 0.6$ is the ratio of mean particle separation at the boundary to the bulk region. A collisionless model is developed, under the assumption of an electrostatic double layer at the boundary. However, the model showed no compression with $\epsilon = 6.2$, and the boundary width approximately the ion absorption length, $\lambda_{i,ab} \approx 1$ cm, due to absorption by the micro-spheres. A sufficient condition for compression is obtained which shows that, if the ion density gradient is sufficiently greater than the electron density gradient, the boundary becomes compressed. The collisionless model was modified by including ion-neutral collisions, of mean free path $\lambda_{in}$, in the complex plasma. It was found that if $\lambda_{in} \ll \lambda_{i,ab}$, the ion density gradient is greatly increased. The degree of compression is significantly reduced to $\epsilon = 1.2$. Despite the boundary not being compressed, the boundary width is in good agreement with experimental observations.
1. Introduction

Under certain conditions, micro-spheres introduced into a plasma can condense into highly organized structures or plasma crystals [1]–[3]. In most experiments, the negatively charged micro-spheres levitate near the plasma-sheath boundary, balanced by gravity and the sheath electric field. However, gravity limits the crystal size [4] and compresses the lower layers [5]. In some experiments, dust particles grown in situ are often confined to the plasma bulk [6]–[8]. Some recent studies have sought to remove the effects of gravity with thermophoresis [9]–[11], or to conduct experiments under micro-gravity [12]. In the plasma bulk, dust dynamics is governed by the electric and ion (neutral) drag forces [13]. The thermophoretic force has recently been shown to be negligible in the bulk due to insignificant temperature gradients. However, in the electrode sheaths, the thermophoretic force becomes comparable to the ion drag force [14]. In these experiments, where the effects of gravity are insignificant, a centimetre sized dust-free region (or void) with a sharp boundary is often observed. In particular, Samsonov and Goree [8] found that, above a critical particle size, the dust cloud developed an instability that eventually formed the void. Optical emission and line ratio imaging revealed that ionization in the void was enhanced, due to a greater electron density and reduced electron temperature. These observations are partly supported by numerical simulations that show greater plasma density [15, 16] but also a higher electron temperature in the void [16]. Observation of the motion of micro-spheres in the void have demonstrated the symmetrical nature and existence of centrifugal forces [11, 12].

Recently, additional structures at the void boundary were observed during the Plasma Krystal Experiment (PKE) on the International Space Station. In a region several inter-particle spacings thick, the micro-sphere density was significantly increased. The compression of the void boundary can also be seen in the results of Thomas et al [11] and Morfill et al [12]. Prior to these works, compression was not observed because the particles could not be resolved individually. In this paper, a model of the void boundary is developed to determine the mechanism for compression.

2. The PKE experiment

Figure 1 shows the void and the surrounding complex plasma observed during the PKE experiment. Reflected laser light is captured by CCD cameras to record the particle positions.
Figure 1. Cross-sectional view of the void and complex plasma in argon at 49 Pa and 0.25 W net power. Ten frames superimposed over 0.4 s reveal the vortices seen on the far right of the figure and the solid region at the centre. Inset: average intensity profile (taken over horizontal position) and particle separation from one frame in the solid region (indicated by the dashed box).

The picture is a superposition of 10 frames taken over 0.4 s. Refer to Nefedov et al [17] for more technical information regarding the experimental arrangement. In this experiment, spherical melamine formaldehyde particles \((a = 3.4 \, \mu m \, \text{radius}, \, \rho = 1.5 \, g \, \text{cm}^{-3})\) were introduced into a 0.25 W (net power) argon plasma at 49 Pa. The void is spheroidal, with a characteristically sharp and compressed boundary. The structure of the void boundary is smoothed out by the superposition of frames and the motion of the micro-spheres. The vortices observed by Morfill et al [12] are clearly seen at the edges of the electrodes. Near the centre of the electrodes, the complex plasma is more solid with the micro-spheres moving about average positions (dashed box).

The inset in figure 1 shows the average intensity profile (taken over horizontal position) and the nearest neighbour distances from one frame in the solid region. In the complex plasma, the intensity profile increases gradually from the sheath boundary, where the mean particle separation is \(0.17 \pm 0.03 \, \text{mm}\). The large spread of particle separations is an indication of short range disorder. Near the void boundary, the intensity rises to a sharp peak (0.15 mm from the boundary), with the average particle separation dropping to \(0.12 \pm 0.03 \, \text{mm}\). The complex plasma is clearly compressed within a boundary layer about 0.5 mm thick (about 4 mean particle separations). The ratio of the mean particle separation (or reciprocal light intensity) at the boundary to the bulk region, gives a measure of the degree of compression, \(\epsilon\). From figure 1, \(\epsilon_s \approx 0.7 \pm 0.3\) from the mean particle separations. This is comparable to \(\epsilon_l \approx 0.5 \pm 0.2\) obtained from the light intensities. This gives an average value of \(\epsilon = 0.6 \pm 0.3\).
3. Discussion of current void theories

The forces that result in the compressed boundary are also important in the formation of the void. Due to a lack of conclusive experimental evidence, a number of explanations have been put forward. Morfill et al. [12] originally proposed that plasma-induced thermophoresis was the dominant force. However, recent numerical modelling of the complex plasma under micro-gravity [14, 15] has shown this force to be insignificant. By taking a thermodynamic approach, Avinash [18] predicted voids to form near the fluid phase boundary. It was assumed that the neutral shadowing force [19] is dominant and attractive. However, some evidence suggests that this force is repulsive, because of the greater particle surface temperature compared with the neutral gas temperature [16, 20]. Samsonov and Goree [8] suggested that the void forms due to dust density fluctuations. This causes the ion drag force to exceed the local electric and repulsive Debye–Hückel forces. This pushes the particles away from the centre of the plasma until equilibrium is reached. This seems to be consistent with experimental observations of the void, numerical simulations of complex plasmas [14, 15], and the motion of micro-spheres in the void [11, 12].

The hypothesis of Samsonov and Goree [8] was used by Goree et al. [21] to develop a one-dimensional model of the void, under micro-gravity conditions. The void was modelled as a low-pressure collisionless plasma, with uniform ionization. From the balance of the plasma electric field and the ion drag force, the void boundary was found to be at the point where the ions attained the local Bohm speed. By including ion-neutral collisions, Tsytovich et al. [22] found the void size to increase with ionization, in agreement with observation [8, 12]. Goree et al. [21] proposed that the sheath was localized in the dust region, forming a dusty sheath. A fluid model of the dusty sheath was developed [23], with ionization and quasi-neutrality neglected. It was found that all properties of the dusty sheath depend only on the Mach number of the void ions. Stability is ensured in permitted zones of Mach numbers, being subsonic and supersonic. For supersonic Mach numbers, compression of the dust structure was only observed at the dust–electrode sheath boundary, contrary to experimental observations. For subsonic Mach numbers, compression was observed at the void–dust boundary. However, a typical boundary layer width of $12\lambda_{Di}^2/a$ is about 40% of the dust structure size (about $30\lambda_{Di}^2/a$), for $a/\lambda_{Di} = 0.5$, where $\lambda_{Di}$ is the ion Debye length. These results are clearly inconsistent with experimental observations where the boundary layer makes up about 10% (see figure 1).

In a recent paper, a numerical model of the complex plasma was developed [15]. The model uses the fluid description of the plasma species and the dust in the steady state. The results show a void region with compression of the dust at the void–complex-plasma boundary. The compression occurs in a region $\sim 4.7$ mm thick, with a density peak occurring 2.8 mm from the boundary. This is significantly different from values measured from figure 1. The compression is due to a potential well in the vicinity of the boundary. Ions flow into the well, compressing the dust on both sides. However, the potential well could be caused by the neglect of double layers at the boundary. Annaratone et al. [24] suggested that double layers, or even triple layers, could exist at the void–complex-plasma boundary. This has been confirmed in a recent numerical simulation by Akdim and Goedheer [16]. Despite this, compression of the boundary was not observed. The absence of small-scale dust structures of the order of several inter-particle distances is not surprising in models where the dust is treated as a fluid.

The model of Annaratone et al. [24] essentially follows that of Prewett and Allen [25], for a biased hot cathode immersed in a collisionless plasma. For a ‘closed’ void, an additional
boundary condition is that the net current across the double layer should be zero. Annaratone et al [24] considered various situations, such as a finite current across the layer (‘open’ voids) and non-zero Maxwell stress. The formation of the double layer requires the electron flux, from the complex plasma, to be approximately at the Langmuir limit. In this case, the collective electric field at the complex plasma boundary is zero. This meets the requirement of (exact) quasi-neutrality for planar boundaries (or sufficiently large voids), since only zero or constant electric fields are permissible. Ion-neutral collisions, ionization, and non-Maxwellian electron distributions are neglected in the model.

4. Collisionless model of the boundary layer

In this paper, the collisionless theory of Annaratone et al [24] is used with a model of the complex plasma boundary layer. The model treats the boundary as a multi-layered charged porous surface into which positive ions, \( n_{iv} \), and electrons, \( n_{ev} \), from the void are being gradually absorbed. This approach is different from existing models [15, 16] in that small-scale dust structures can be modelled. Figure 2 shows a schematic of the complex plasma. The layers are separated by a distance \( \Delta x_l \), where \( l \) denotes the layer number. The micro-spheres are arranged into regular rows and columns in the \( y-z \) plane, with \( L \) layers along the \( x \) axis. The

Figure 2. Schematic of the complex plasma, the double layer and the void presheath. The complex plasma is arranged into \( L \) layers, with \( x_l \) the layer position. The Debye–Hückel forces from adjacent layers \( F_{l-1}(\Delta x_{l-1}) \), \( F_{l+1}(\Delta x_l) \) and the local ion drag force, \( F_{id,l} \), determine the layer separation, \( \Delta x_l \). The vertical separation of micro-spheres, \( \Delta x_{\perp} \), is constant for all layers. The micro-spheres are at the normalized floating potential (in eV/\( k_B T_e \) units), \( \eta_{l,l} \). The total normalized potential drop in the presheath, \( \eta_0 \), and in the double layer, \( \eta_{DL} \), are also shown.
complex plasma is considered to be infinite in extent away from the void boundary \((x_l > 0)\) and in the \(y-z\) plane. Effects arising from other boundaries, such as the electrode sheath, are neglected. The micro-sphere separation along the \(y\) and \(z\) axes, \(\Delta x_\perp\), is constant for all layers. Quasi-neutrality is assumed, with the micro-spheres at the normalized floating potential, \(\eta_{f,l} = eV_{f,l}/k_B T_{ev}\), producing local electric fields. The position of each layer is determined by the balance between the repulsive Debye–Hückel forces, \(F_{i+1}(\Delta x_l)\) and \(F_{i-1}(\Delta x_{l-1})\), from adjacent layers, and the ion drag force, \(F_{id,l}\), from the penetrating void ions. Other forces arising from ion and neutral shadowing are neglected. Current theories of ion shadowing are inappropriate because the interaction considered is between two micro-spheres in a uniform electron–ion plasma [19]. In general, the ion (neutral) shadowing force results from anisotropic ion (neutral) bombardment. This is caused by a non-uniform distribution of micro-spheres and background ions ( neutrals). The net force is directed towards the region where the ion (neutral) momentum transfer is lowest.

In the steady state, the force balance equation is simply:

\[
F_{i+1}(\Delta x_l) - F_{i-1}(\Delta x_{l-1}) - F_{id,l} = 0 \quad (1)
\]

with the separation, \(\Delta x_{l-1}\), being obtained from the force balance at the previous layer and \(\Delta x_l\) is the desired solution. Due to the assumption of quasi-neutrality, the collective electric field in the complex plasma is taken to be zero. However, electric fields may exist in the complex plasma near the boundary. In this case, an additional term accounting for the electric force on the micro-spheres should be included.

In figure 2, the electric potential is also shown starting from the void presheath, through the double layer and into the complex plasma. In the void presheath, cold ions are accelerated from rest to the Bohm speed through the normalized potential drop, \(\eta_o\). They are then accelerated through an additional drop of \(\eta_{DL}\) in the double layer, before reaching the void boundary with velocity \(v_{iv}\). Ionization in the complex plasma produces background densities of ions, \(n_{ic}\), and electrons, \(n_{ec,l}\), where the subscript ‘c’ denotes the complex plasma. Electrons are assumed to be in Boltzmann equilibrium with temperatures \(T_{ec}\) and \(T_{ev}\). The equation of quasi-neutrality is then given by

\[
n_{iv,l} + n_{ic} = n_{ev,l} + n_{ec,l} + Q_l N_l, \quad (2)
\]

where \(N_l = [((\Delta x_l + 2\Delta x_\perp)/3)]^{-3}\) is the average micro-sphere density and \(Q_l\) is the surface charge. For vacuum capacitance \((a \ll \lambda_D)\), the surface charge in \(e\) units is given by \(Q_l = 4\pi \epsilon_0 a k_B T_{ev} \eta_{f,l}/e^2\), with \(a\) the micro-sphere radius. Note that \(n_{ic}\) is assumed to be a constant and is determined by boundary conditions.

The floating potential can be obtained by equating the electron and ion fluxes. Near the boundary, the ion flux to a micro-sphere is the sum of the background fluxes and the directed void fluxes. Recently, Allen et al [26] showed that for an isolated micro-sphere in a collisionless Maxwellian plasma, the orbital motion limited (OML) theory is inconsistent for \(T_i \leq T_e\), even for particles of small radius. Charging theories in the case of a complex plasma have not been reported in the literature to the author’s knowledge. In the collisionless limit and for sufficiently small isolated dust particles, the OML theory was found to be a good approximation to the full orbital motion theory [27]. Further, the potential distribution was found to be well approximated by the Debye–Hückel potential (using the linearized plasma Debye length) over most of the sheath. At large distances from the dust particle, the Debye–Hückel approximation underestimates the full solution because of ion-absorption effects. The OML theory extended to flowing plasmas.
where  \( v_{e(i)} = (8k_BT_{e(i)}/\pi n_{e(i)})^{1/2} \) is the electron or ion thermal velocity. The OML radii, \( b_{iv,j} \) and \( b_{nth,j} \), are given by \( a[1 + (2k_BT_{iv}\eta_{j,l}/m_i v_{\perp}^2)]^{1/2} \) and \( a[1 + (\eta_{j,l}T_{iv}/T_{ic})]^{1/2} \), respectively. Here, \( T_{ic} \) is the background ion temperature in the complex plasma, taken to be 0.1 eV. Note that in a dense collisionless complex plasma, both \( b_{iv,j} \) and \( b_{nth,j} \) cannot exceed \( \Delta x_\perp/2 \).

The potential distribution around each micro-sphere is approximated by a Debye–Hückel or Yakawa potential: \( V_I(r) = (Q_i/e/4\pi\epsilon_o r) \exp(-r/\lambda_{D,l}). \) Here, \( \lambda_{D,l} = (\lambda_{c,l}^{-2} + \lambda_{e,l}^{-2})^{-1/2} \) is the local plasma screening length, with \( \lambda_{c,l}^{-2} = e^2 [(n_{iv,l}/T_{iv}) + (n_{ic,l}/T_{ic})]/\epsilon_o b_{iv}, \lambda_{e,l}^{-2} = e^2 [(n_{ev,l}/T_{ev}) + (n_{ec,l}/T_{ev})]/\epsilon_o b_{iv} \) and \( T_{iv} = m_i v_{\perp}^2/2k_B \) the effective ion temperature of the void ions. These assumptions are valid when \( n_{ic} \gg n_{iv,l} \). Distortion of the screening potential due to ion flow [29] can then be neglected. It will be seen in the next section that this condition is satisfied. Distortion of the force due to neighbouring micro-spheres is ignored for simplicity. Considering only the nearest neighbours, the Debye–Hückel force, \( F_{l+1} \), on a micro-sphere in layer \( l \) from 9 micro-spheres in an adjacent layer \( l + 1 \) (see figure 2) is obtained by summing over the force components along the \( x \)-axis. By symmetry, the other components cancel, giving

\[
F_{l+1} = \frac{Q_i Q_{l+1} e^2 \Delta x_\perp}{4\pi\epsilon_o} \sum_{n=0}^{\infty} \kappa_n \exp\left(-\frac{r_n}{\lambda_{D,l+1}}\right) \left(\frac{1}{r_n^{\frac{3}{2}}} + \frac{1}{\lambda_{D,l+1}}\right),
\]

where \( r_n = (n\Delta x_\perp^2 + \Delta x_\perp^2)^{1/2} \) is the inter-particle distance. The parameter \( \kappa_n \) has the following values: \( n = 0, \kappa = 1; n > 0, \kappa = 4 \). The force \( F_{l-1} \) exerted by the previous layer \( l - 1 \) is obtained in a similar manner.

The ion drag force arises from the total momentum transferred to the micro-sphere from direct ion impact and from scattering in the potential well. Using classical scattering [30] and OML theory for the ions, Barnes et al [13] obtained an approximate expression for the drag force:

\[
F_{id,l} = n_{iv,l} v_{\perp,l}^2 m_i \pi (b_{iv,l}^2 + 4b_{iv,l}^2 \Lambda_i),
\]

where \( b_{iv,l} = Q_i e^2/4\pi\epsilon_o m_i v_{\perp,l}^2 \) is the classical distance of closest approach, \( b_{iv,l} \) is the OML impact parameter and \( \Lambda_i = \ln[(b_{a,l}^2 + b_{b,l}^2)/(b_{a,l}^2 + b_{b,l}^2)]^{1/2} \) is the Coulomb logarithm. Barnes et al [13] defined the maximum scattering radius \( b_{in,l} \) to be the screening distance \( \lambda_{D,l} \). Recently, Khrapak et al [31] have pointed out that this definition does not account for large-angle scattering. They suggested a new definition with \( b_{m,l} = \lambda_{D,l}(1 + 2\beta_l)^{1/2} \), where \( \beta_l = b_{in,l}/\lambda_{D,l} \). However, this is only applicable when \( \beta_l < 5 \) and \( \Delta x_\perp/2 > b_{m,l} \). In this paper both of these conditions are easily satisfied and the new definition of Khrapak is used.

As the void ions and electrons penetrate the complex plasma they are continuously absorbed by the micro-spheres. The effective absorption area for ions is \( \pi b_{iv,l}^2 \) and for electrons it is \( 4\pi a^2 \exp(-\eta_{i,l}) \). The ion flux, after passing through a micro-sphere layer of surface density \( \sigma = 1/\Delta x_\perp^2 \), is given by \( \Gamma_{iv,l+1} = \Gamma_{iv,l} (1 - \pi \sigma b_{iv,l}^2) \). Similarly, for the void electrons, \( \Gamma_{ev,l+1} = \Gamma_{ev,l} (1 - 4\pi \sigma a^2 \exp(-\eta_{i,l})) \). The reduction in the transmitted ion flux due to scattering was found to be insignificant, resulting in slightly larger floating potentials and layer separations.
By taking the condition of zero Maxwell stress and zero net current across the double layer, Annaratone et al [24] obtained $\eta_{DL} = 1.007$, $J_0 = 0.049$ and $\eta_0 = 0.577$ (for definitions see figure 2). Here, $\eta_{DL} = eV_{DL}/k_B T_{ev}$ is the normalized potential drop, $J_0 = j_0 \sqrt{m_e/e}$ and $n_{DL} = n_{DL}/(2\eta_{DL})^{3/2}$ is the normalized electron current density from the complex plasma with $n_{DL}$ the electron density at the presheath edge. The fluxes at the first layer, $\Gamma_{iv,0} = n_{iv,0} v_{iw}$ and $\Gamma_{ev,0} = n_{ev,0} v_{eth}/4$, can then be obtained using the conservation of ion flux, the void plasma density, $n_p$, and the electron thermal velocity. The electron density at the boundary is given by $n_{ec,0} = 4j_{ec}/e\gamma^{1/2}v_{eth}$.

An iterative procedure was used to determine the first layer separation. From an initial estimate of $\Delta x_0$, the ion density $n_i$ and floating potential $\eta_{fl,0}$ were obtained by simultaneously solving equations (2) and (3). The local Debye length, $\lambda_{D,0}$, the ion drag force, $F_{id,0}$, and the ion and electron fluxes at the next layer are calculated. Note that in a collisionless complex plasma, the ion flux is conserved between layers. Equations (2) and (3) are used to determine $\eta_{i,1}$ and $n_{ed,1}$ to give $\lambda_{D,1}$ and $Q_1$. The Debye–Hückel force, $F_1$, acting on the first layer is then calculated from equation (4). From force balance (equation (1)) the layer separation is obtained and compared with $\Delta x_0$. This process is repeated until a self-consistent solution is obtained. The force balance equation is then solved up to $L$ layers. For a sufficiently large $L$, and due to quasi-neutrality, the quantities $\eta_{fl,l}$, $n_{ec,l}$ and $\Delta x_l$ become constant. Note that for $l > 0$ it is necessary to calculate $F_{l-1} (\Delta x_{l-1})$.

### 5. Results

At present, no experimental results are available to indicate typical values for the plasma densities and temperatures in the void and complex plasma. The parameter space was then investigated by changing the parameters individually over a wide range. In all the cases considered, no compression of the complex plasma boundary was observed. It was observed that the compression ratio, $\epsilon = \Delta x_0/\Delta x_1$, and the layer separation decreased with increasing $n_p$ and decreasing $\Delta x_1$. The following parameters were used for an argon plasma: $T_{ev} = 3$ eV, $\gamma = 2$, $n_p = 1 \times 10^{16}$ m$^{-3}$ and $T_{ic} = 0.1$ eV. From the double layer parameters the boundary densities are: $n_{iv,0} = 3.72 \times 10^{15}$ m$^{-3}$, $n_{ev,0} = 2.05 \times 10^{15}$ m$^{-3}$ and $n_{ec,0} = 1.39 \times 10^{15}$ m$^{-3}$. From the PKE experiments, $\alpha = 3.4 \mu$m and $\Delta x_1 = 0.1$ mm. Solving for $\Delta x_0$, $\eta_{fl,0}$ and $n_{ic}$ using the iterative procedure outlined above gives: $\Delta x_0 = 6.82 \times 10^{-2}$ mm, $\eta_{fl,0} = 2.11$ and $n_{ic} = 2.07 \times 10^{16}$ m$^{-3}$ with $\lambda_{D,0} = 1.63 \times 10^{-5}$ m. The background ion density, $n_{ic}$, is slightly greater than the void plasma density due to the double layer. Note that $n_{ic} > n_{iv,0}$ so that distortion of the Debye–Hückel potential due to ion flow [29] can be neglected. The layer separations and normalized floating potentials are plotted against distance from the boundary in figure 3. The layer positions are indicated by symbols. Also shown are curves for the collisional case, these are discussed in the next section. The results show that the boundary is highly uncompressed with $\epsilon = 6.22$. However, the layer separation at $x = 0$ is in approximate agreement with the average particle separation obtained from figure 1, even in the presence of ion-neutral collisions. After about 550 layers, the complex plasma becomes uniform, giving a boundary thickness of 8 mm. Here, the layer separations are significantly smaller than the measured values of about 0.15 mm (see figure 1). The floating potential is shown to decrease with distance, approaching $\eta_{fl} \approx 1$ at larger distances. This is smaller than the floating potential of an isolated particle with $\eta_{fl} = 2.8$ for $T_i/T_e = 0.03$ [26].
Figure 3. Variation of layer separation, $\Delta x_l$, and normalized floating potential, $\eta_{f,l}$, with distance, $x$, from the void boundary. Layer positions are shown by the symbols on the curves. The collisional curves result from the modified collisionless model, where only collisions in the complex plasma are considered.

Figure 4 shows the ion and electron densities at each layer in the complex plasma. The background electron density, $n_{ec,l}$, initially decreases and then increases in order to maintain quasi-neutrality. Both $n_{ev,l}$ and $n_{iw,l}$ decrease on similar length scales determined by losses due to absorption. The characteristic distance for absorption can be estimated using $\lambda_{e(i),ab} = 1/N_l \sigma_{e(i),ab}$. Here, $\sigma_{e,ab} = 4\pi a^2 \exp(-\eta_{f,l})$ and $\sigma_{i,ab} = \pi b_{iv,l}^2$ with $N_l$, $b_{iv,l}$ and $\eta_{f,l}$ averaged over all layers. For $N_l = 2.2 \times 10^{12} \text{m}^{-3}$, $\eta_{f,l} = 1.6$ and $b_{iv,l} = 4.7 \mu \text{m}$ then $\lambda_{i,ab} = 6.6 \text{mm}$ and $\lambda_{e,ab} = 15.5 \text{mm}$. The greater electron absorption length is a consequence of the smaller absorption area. The ion absorption length is in good agreement with the boundary thickness of 8 mm. At larger distances $n_{iv,l} \ll n_{ic}$ and $n_{ev,l} \ll n_{ec,l}$ and the layer separation and floating potential become constant. Note that the inclusion of ionization and energy balance is beyond the scope of this paper. Since $n_{ic} > n_{iv,l}$, as determined by the boundary conditions, the floating potential decreases primarily due to the decrease in the total electron density.

The conditions sufficient for compression can be obtained from quasi-neutrality and the charging equation. Differentiation of equation (2) gives:

$$\frac{\Delta n_{iv,l}}{\Delta x} = \frac{\Delta (n_{ec,l} + n_{ev,l})}{\Delta x} + Q_l \frac{\Delta N_i}{\Delta x} + N_i \frac{\Delta Q_l}{\Delta x},$$

where $\Delta / \Delta x$ denotes a finite differential over two adjacent layers. From differentiation of equation (3), the gradient of floating potential, $\Delta \eta_{f,l} / \Delta x$ (and $\Delta Q_l / \Delta x$), is given by:

$$\frac{\Delta \eta_{f,l}}{\Delta x} = -\frac{1}{a^2 \Gamma^*} \left( \frac{\Delta n_{iv,l}}{\Delta x} - \omega \frac{\Delta n_{ct,l}}{\Delta x} \right),$$

where $\Gamma^*$ and $\omega$ are defined in the text.
Figure 4. Densities of void ion, $n_{iv,l}$, electron, $n_{ev,l}$, and background electron, $n_{ec,l}$, plotted against distance from the void boundary, $x$. The curves are for the collisionless case.

where

$$\frac{\Delta n_{eT,l}^*}{\Delta x} = \sqrt{\gamma} \exp\left(-\frac{\eta_{f,l}}{\gamma}\right) \frac{\Delta n_{ec,l}}{\Delta x} + \exp\left(-\eta_{f,l}\right) \frac{\Delta n_{ev,l}}{\Delta x}$$

and

$$\Gamma^* = v_{eth} n_e^* + \frac{n_{iw,l} v_{iw} T_{ev}}{T_{iw}} + \frac{n_{ic} v_{ih} T_{ev}}{T_{ic}}$$

with $n_e^* = n_{ev,l} \exp(-\eta_{f,l}) + \gamma^{-1/2} n_{ec,l} \exp(-\eta_{f,l}/\gamma)$. Here, $\omega = v_{eth} a^2 \sqrt{v_{iw,l}} \approx 120$ (for $v_{iw} = [2 k_B T_{ev}(\eta_o + \eta_{DL})/m_i]^{1/2}$ with $a \approx 0.7 b_{iw,l}$). Several conditions for compression can be obtained by considering the magnitude and sign of the density gradients in equations (6) and (7). Condition (A): if $|\Delta n_{iw,l}/\Delta x| > \omega |\Delta n_{eT,l}^*/\Delta x|$ then $\Delta \eta_{f,l}/\Delta x > 0$ with $|\Delta n_{iw,l}/\Delta x| \gg |\Delta (n_{ec,l} + n_{ev,l})/\Delta x|$. From equation (6), $\Delta N_{l}/\Delta x < -(N_{l}/Q_l) \Delta Q_{l}/\Delta x$ and the boundary is compressed. If $|\Delta n_{iw,l}/\Delta x| < \omega |\Delta n_{eT,l}^*/\Delta x|$ then two further possibilities exist. The first requires $\Delta n_{eT,l}^*/\Delta x > 0$ so that $\Delta \eta_{f,l}/\Delta x > 0$. In the parameter space investigated, this condition was never satisfied and is not considered further here. The second possibility, condition (B), requires $\Delta n_{eT,l}^*/\Delta x < 0$ so that $\Delta \eta_{f,l}/\Delta x < 0$. In addition, if $|\Delta n_{iw,l}/\Delta x| > |\Delta (n_{ev,l} + n_{ec,l})/\Delta x|$ then $\Delta N_{l}/\Delta x \lesssim 0$ and compression is possible. Finally, compression may also be possible if $|\Delta n_{iw,l}/\Delta x| < |\Delta (n_{ev,l} + n_{ec,l})/\Delta x|$. This seems unlikely due to the smaller electron absorption area and is not considered further here. In the collisionless case, conditions (A) and (B) were never satisfied.

Figure 5 shows the forces acting on each layer. The ion drag force, $F_{id,l}$, follows the void ion density, decreasing with distance from the boundary. The Debye–Hückel forces, $F_{i+1}$ and
Figure 5. Debye–Hückel forces, $F_{l+1}$, $F_{l-1}$, and the ion drag force, $F_{id,l}$, plotted against distance from the void boundary, $x$, in the collisionless case. The ion drag force decreases on the ion absorption length scale, $\lambda_{iv,ab} \approx 1$ cm. Over the same distance, the Debye–Hückel forces increase rapidly, dominating the force balance within the first few layers.

$F_{l-1}$, both increase rapidly with distance, dominating the force balance within a few layers. At the first layer the upstream force, $F_{id,0}$, is balanced only by the downstream force, $F_{l+1}$, with $F_{l-1} = 0$. However, at the second layer the combined upstream force is almost double due to the gradual absorption of the void ions. The decrease of the floating potential reduces the downstream Debye–Hückel force strength, resulting in smaller layer separations. Consequently, the upstream Debye–Hückel force strength increases until a uniform complex plasma is attained.

6. The effect of ion-neutral collisions

The condition for compression requires a sufficiently large ion density gradient. One possibility is ion-neutral collisions of the void ions. If $\lambda_{in} \ll \lambda_{i,ab}$ then $dn_{iv}(x)/dx = -n_{iv}(x = 0) \exp(-x/\lambda_{in})/\lambda_{in}$. The smaller the mean free path, the greater the ion density gradient. Since $\lambda_{in} \ll \lambda_{e,ab}$ then $dn_{ev}(x)/dx \approx 0$ reducing $\Delta n_{T,1}^e/\Delta x$ in equation (7). In addition, the rapid decrease of the ion drag force reduces the increase in the combined upstream force, resulting in larger layer separations. The boundary thickness is then determined by the ion-neutral mean free path. For charge-exchange collisions at 50 Pa and the Bohm speed $\lambda_{in} \approx 0.1$ mm [32]. These collisions produce a flux of fast neutrals at the next layer, after accounting for neutral–neutral collisions and fast neutrals from the void. This produces an additional upstream drag force on the dust grains. To properly account for the neutral drag, a collisional void presheath model.
including the double layer should be used. This is beyond the scope of this paper and is left for future work. The collisionless model is then modified to take into account ion-neutral collisions in the complex plasma.

In collisional conditions the void ion flux at the next layer can be written as \( \Gamma_{iv,l+1} = \Gamma_{iv,l}(1 - \pi b_{iv,l}^2) \exp(-\Delta x_l/\lambda_{in}) \). The collisionless ion drag and OML theory is valid when \( \lambda_{in} \gg b_{m,l} \gg \lambda_{D,l} \) and \( \lambda_{in,th} \gg b_{th,l} \gg \lambda_{D,l} \). Here, \( \lambda_{in,th} \) is the mean free path for the background ions. However, it has been noted by several authors [33, 34] that the use of OML theory in weakly collisional conditions is inappropriate for micron-sized particles. As the pressure increases, the floating potential decreases from the OML value and then increases in the hydrodynamic limit. In a neon plasma the minimum occurs at around 100 Pa [33], in other gases the minimum may occur at different pressures. Although a simple approximation of the floating potential exists in the hydrodynamic limit, none exists for the lower pressures. Due to the lack of a suitable alternative, the simpler OML theory is used. The exact dependance of the ion drag force on collisionality is still unknown, but it is expected to decrease. This is because collisions destroy orbital motion, increasing the ion absorption by the micro-sphere. Consequently, the momentum transfer due to scattering decreases. The momentum transfer due to ion impact becomes more isotropic, with collisions also occurring downstream of the dust particle. With increasing collisionality, the net momentum transferred to the dust particle decreases.

Figure 3 compares the layer separations and normalized floating potential with the fully collisionless theory. In both cases, the same model parameters are used in addition to \( \lambda_{in} = 0.1 \) mm. For the collisional case and far from the boundary, the layer separations are larger giving \( \epsilon = 1.29 \). This is significantly smaller than the collisionless value of 6.22. The boundary thickness is reduced from 8 to 0.4 mm (≈7 layers) and is comparable to \( \lambda_{in} \). This is in good agreement with the experimentally measured thickness of 0.5 mm. The normalized floating potential follows closely the collisionless curve up to 0.4 mm. At positions greater than this, the floating potential maintained an approximately constant value of \( 1.6k_BT_{ev}/e \). Reducing the mean free path decreases the compression ratio, but values less than 1.1 were not obtained. Variations of \( n_{iv,l} \), \( n_{ev,l} \) and \( n_{ec,l} \) with distance from the boundary are shown in figure 6. As expected, the void ion density (and ion drag) rapidly decreases. The void electron density is approximately constant over 0.4 mm. The dashed curve in figure 6 is the curve \( n_{iv,l} = n_{iv,0} \exp(-x_l/\lambda_{in}) \) and coincides with the model values accurately. The collision frequency is then much greater than the micro-sphere absorption rate. The Debye–Hückel force increases as in the collisionless case, but reaches a constant value of \( 1.9 \times 10^{-12} \) N at 0.4 mm. This is less than the values obtained in the collisionless case (figure 5), suggesting that collisions act to reduce the inter-particle forces in the complex plasma. It is found that even though the ion density gradient is greater than the electron density gradient and that condition (A) is satisfied, no compression is observed. This suggests that condition (A) must be satisfied for compression of the boundary. This was not possible partly because of the relatively large background electron density gradient (due to constant \( n_{id} \)), and partly due to the large multiplying factor \( \omega \) (≈120) in equation (7). The large value is due to \( v_{th}/v_{iv} \approx 250 \) with \( a^2/b_{iv,l}^2 \approx 0.5 \). In weakly collisional conditions, where the floating potential decreases with collisionality [33], the ion current to the micro-sphere increases. This could be caused by collisions far from the micro-sphere, promoting the radial motion of ions. The effective collection area, \( b_{iv,l}^2 \), would then be larger, reducing \( \omega \) and making condition (A) easier to fulfil.

In the hydrodynamic limit [33], the floating potential increases with collisionality. This causes the ion current to the micro-sphere and the collection area to decrease [34].
Figure 6. Densities of void ion, $n_{iv,l}$, electron, $n_{ev,l}$, and background electron, $n_{ec,l}$, are plotted against distance from the void boundary, $x$, in the collisional case. Since the ion-neutral mean free path, $\lambda_{in}$, is much less than the ion absorption length, $\lambda_{iv,ab}$, the void ion density decreases as $n_{iv,l}(x_l) = n_{iv,l=0} \exp(-x_l/\lambda_{in})$ (dashed line). Over this distance, $n_{ev,l}$ is approximately constant.

7. Conclusions

A collisionless model of the complex plasma boundary using the numerical results of Annaratone et al [24] was developed. The presence of the double layer requires the electron density near the boundary of the complex plasma to be sufficiently large. Owing to quasi-neutrality, the ion density in the complex plasma becomes comparable to the central void plasma density. The model showed no compression of the boundary layer, with the layer separation at the boundary $\sim 6$ times larger than the separation far from the boundary (i.e. $\epsilon = 6$). Boundary widths were approximately of the order of the ion absorption length ($\approx 1$ cm), due to absorption by the micro-spheres. From quasi-neutrality and the charging equation, several conditions sufficient for compression were obtained. This shows that in the collisionless case, compression was not observed because the ion density gradient was too low compared with the electron density gradient. This causes the floating potential to decrease from the boundary.

One possible mechanism to increase the ion density gradient is ion-neutral collisions of the void ions. If the ion-neutral mean free path is much less than the ion (electron) absorption length, the void ion density (and ion drag force) decreases rapidly over several collision mean free paths. Consequently, the void electron density gradient is significantly reduced. Boundary layer widths were found to be 0.4 mm and in good agreement with experimentally measured values (typically 0.5 mm). The boundary widths are then several ion-neutral mean free paths. Despite this, the collisional model showed no compression of the boundary. The ion density gradient was too small. This is partly due to the use of collisionless OML theory which results in
a smaller collection area of the micro-spheres. Collisions are expected to increase the collection area, making compression more probable. However, the compression ratio, $\epsilon$, and boundary thickness were significantly reduced. The boundary layer separation was approximately equal to the layer separation far from the boundary with $\epsilon \approx 1.3$. This is much closer to the experimentally measured value of $\epsilon \approx 0.6$. These results indicate the important role of ion-neutral collisions in the boundary layer. However, the additional neutral drag force arising from charge-exchange collisions, not accounted in this model, could modify these results. Further work is then required to understand the role of collisions in determining the structure of the void boundary.

Acknowledgments

This work was supported by the Max-Planck Institut für Extraterrestrische Physik and the Bundesministerium für Bildung und Forschung through the Deutsche Zentrum für Luft und Raumfahrt e.V. (DLR) under the Förderkennzeichen 50WM9852. Helpful comments on the manuscript by Dr Konopka, Dr Annaratone, Professor Morfill and the referees are also gratefully acknowledged. In particular, the author would like to thank Professor Morfill for his continued support and help in times of difficulty.

Note added in proof. A recent paper by Ivlev et al [35] shows the ion drag force to be increased by ion-neutral collisions for $\Lambda = (T_i/T_e \eta_1) \lambda_D/a \gg 1$ and $\lambda_D > \lambda_{in}$. This is caused by a local increase in the ion density downstream of the dust grain due to enhanced ion focusing. An additional electric field is induced increasing the ion drag. For the parameters used in this paper, $\Lambda \approx 0.05$ and $\lambda_D/\lambda_{in} \approx 0.2$ so the results of Ivlev et al [35] are not applicable.

References

[1] Thomas H, Morfill G E, Demmel V, Goree J, Feuerbacher B and Möhlmann D 1994 Phys. Rev. Lett. 73 652
[2] Chu J and Lin I 1994 Phys. Rev. Lett. 72 4009
[3] Hayashi Y and Tachibana K 1994 Japan. J. Appl. Phys. Part 2 33 L476
[4] Morfill G E and Thomas H 1996 J. Vac. Sci. Technol. A 14 490
[5] Zuzic M, Ivlev A V, Goree J, Morfill G E, Thomas H M, Rothermel H, Konopka U, Sütterlin R and Goldbeck D D 2000 Phys. Rev. Lett. 85 4064
[6] Jellum G M and Graves D B 1990 Appl. Phys. Lett. 57 2077
[7] Dorier J L, Hollenstein Ch and Howling A A 1995 J. Vac. Sci. Technol. A 13 918
[8] Samsonov D and Goree J 1999 Phys. Rev. E 59 1047
[9] Jellum G M, Daugherty J E and Graves D B 1991 J. Appl. Phys. 69 6923
[10] Rothermel H, Hagl T, Morfill G E, Thoma M H and Thomas H M 2002 Phys. Rev. E 89 175001
[11] Thomas E, Annaratone B M, Morfill G E and Rothermel H 2002 Phys. Rev. E 66 016405
[12] Morfill G E, Thomas H M, Konopka U, Rothermel H, Zuzic M, Ivlev A and Goree J 1999 Phys. Rev. Lett. 83 1598
[13] Barnes M S, Keller J H, Forster J C, O’Neill J A and Coultas D K 1992 Phys. Rev. Lett. 68 313
[14] Akdim M R and Goedheer W J 2002 Phys. Rev. E 65 015401
[15] Gozadinos G, Ivlev A V and Boeuf J P 2003 New J. Phys. 5 32
[16] Akdim M R and Goedheer W J 2003 Phys. Rev. E 67 066407
[17] Nefedov A P et al 2003 New J. Phys. 5 33
[18] Avinash K 2001 Phys. Plasmas 8 2601
[19] Khrapak S A, Ivlev A V and Morfill G 2001 Phys. Rev. E 64 046403
[20] Daugherty J E and Graves D B 1993 J. Vac. Sci. Technol. A 11 1126
[21] Goree J, Morfill G E, Tsytovich V N and Vladimirov S V 1999 Phys. Rev. E 59 7055
[22] Tsytovich V N, Vladimirov S V, Morfill G E and Goree J 2001 Phys. Rev. E 63 056609
[23] Tsytovich V N, Vladimirov S V and Benkadda S 1999 Phys. Plasmas 6 2972
[24] Annaratone B M et al 2002 Phys. Rev. E 66 056411
[25] Prewett P D and Allen J E 1976 Proc. R. Soc. A 348 435
[26] Allen J E, Annaratone B M and de Angelis U 2000 J. Plasma Phys. 63 299
[27] Kennedy R V and Allen J E 2003 J. Plasma Phys. 69 485
[28] Whipple E C 1981 Rep. Prog. Phys. 44 1197
[29] Lapenta G 2000 Phys. Rev. E 62 1175
[30] Lieberman M A and Lichtenberg A J 1994 Principles of Plasma Discharges and Materials Processing (New York: Wiley)
[31] Khrapak S A, Ivlev A V, Morfill G E and Thomas H M 2002 Phys. Rev. E 66 046414
[32] Sheldon J W 1962 Phys. Rev. Lett. 8 64
[33] Zobnin A V, Nefedov A P, Sinel’shchikov V A and Fortov V E 2000 J. Exp. Theor. Phys. 91 483
[34] Bryant P 2003 J. Phys. D: Appl. Phys. 36 2859
[35] Ivlev A V, Khrapak S A, Zhdanov S K, Morfill G E and Joyce G 2004 Phys. Rev. Lett. 92 205007

New Journal of Physics 6 (2004) 60 (http://www.njp.org/)