Description of the ground state of axially deformed nuclei within the Relativistic Hartree-Fock-Bogoliubov model

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Abstract. The Relativistic Hartree-Fock-Bogoliubov model for axially deformed nuclei (RHFBz) is presented. The model involves a phenomenological Lagrangian with density-dependent meson-nucleon couplings in the particle-hole channel and the central part of the Gogny force in the particle-particle channel. The RHFBz equations are solved by expansion in the basis of a deformed harmonic oscillator. Illustrative RHFBz calculations are performed for Neon isotopes.

1. Introduction
Nuclear energy density functionals (EDF) represent a tool of choice for the microscopic description of both static and dynamic properties of nuclei over the whole nuclide chart. They subsum nucleonic short-range in-medium correlations, whereas static long-range correlations are taken into account by allowing a single-determinant state to break the symmetries of the nuclear Hamiltonian [1]. Many structure phenomena in both stable and exotic nuclei have successfully been described by EDFs involving the non-relativistic Gogny and Skyrme [2] effective interactions, as well as relativistic phenomenological Lagrangian densities [3]. The Relativistic Mean Field (RMF) framework [3] is an example of a covariant EDF. The corresponding phenomenological Lagrangians provide a quantitative description of a variety of ground-state data. However, it does not include explicitly the Fock term, but implicitly takes it into account through the fit of model parameters to structure data. A more involved approach, the Relativistic Hartree-Fock (RHF) theory [4], treats the exchange contributions explicitly.
The first RHF models used to predict under-bound nuclei in comparison to experimental data. This problem originated from the lack of a medium dependence in the corresponding effective nucleonic interaction [4]. To overcome this problem, an explicit nucleon-density dependence of the nucleon-meson couplings was included [5]. The resulting improvement brought the current RHF models to a similar level of accuracy as the standard RMF approach for a quantitative description of nuclear structure phenomena [6]. In particular, recent studies by W.H. Long et. al. [5, 6, 7, 8] and H. Liang et. al. [9, 10] have emphasized that, compared to the RMF approach, the explicit treatment of Fock terms can improve the description of nuclear matter and finite nuclei. Moreover, it explicitly includes the tensor contributions to the inter-nucleon interaction generated by the exchange of the \(\pi\) and \(\rho\) mesons. So far, the RHF framework has been limited to the description of spherical nuclei. We consider an extension of this approach to deformed, axially-symmetric nuclei: the Relativistic Hartree-Fock-Bogoliubov model with density-dependent meson-nucleon couplings (RHFBz) [11, 12]. In Sec. 2 the formalism of the RHFBz model is briefly presented. In Sec. 3 we present and discuss applications of the RHFBz model to ground-state properties of neon isotopes. Finally, a short summary and discussion of possible future studies are displayed in Sec. 4.

2. Formalism of the RHFBz model

The RHFBz approach is based on a phenomenological Lagrangian density involving the relevant degrees of freedom for nuclear structure, namely nucleons and mesons:

\[
\mathcal{L} = \bar{\psi} \left\{ i\gamma^{\mu} \partial_{\mu} - M - g_\sigma (\rho_\sigma) \sigma - g_\omega (\rho_\omega) \gamma_\mu \omega_\mu - \frac{f_\pi (\rho_\pi)}{m_\pi} \gamma_\mu \gamma_\rho \partial^{\rho} \pi_\mu \right\} \psi \\
+ \frac{1}{2} \left( \partial_\mu \sigma \partial^{\mu} \sigma - m_\sigma^2 \sigma^2 \right) - \frac{1}{2} \left( \Omega_{\mu\nu} \Omega^{\mu\nu} - m_\omega^2 \omega_\mu \omega_\mu \right) \\
- \frac{1}{2} \left( \tilde{\Omega}_{\mu\nu} \tilde{\Omega}^{\mu\nu} - m_\rho^2 \rho_\mu \rho_\mu \right) \\
+ \frac{1}{2} \left( \partial_\mu \pi \partial^{\mu} \pi - m_\pi^2 \pi^2 \right) - \frac{1}{2} (F_{\mu\nu} F^{\mu\nu}) \right. \tag{1}
\]

Vectors in isospin space are denoted by arrows. The Dirac spinor \(\psi\) denotes the nucleon with mass \(M\). \(m_\sigma, m_\omega, m_\rho, m_\pi\) are the masses of the \(\sigma\)-meson, the \(\omega\)-meson, the \(\rho\)-meson and the \(\pi\)-meson, respectively. \(g_\sigma, g_\omega, g_\rho\) and \(f_\pi\) are the meson-nucleon coupling constants. \(A^\mu\) stands for the electromagnetic 4-potential. \(e^2/4\pi = 1/137.036\). The (density-dependent) coupling constants and meson masses are parameters, adjusted to reproduce nuclear matter properties and ground-state properties of finite nuclei. \(\Omega^{\mu\nu}, \tilde{\Omega}^{\mu\nu}, \) and \(F^{\mu\nu}\) are the field tensors of the vector fields \(\omega, \rho,\) and of the photon [4]. A nucleon-density dependence of the meson-nucleon couplings accounts for medium polarisation and three-body correlations [13, 14, 5].

3. Results and discussion

This section presents an application of the RHFBz model in the calculation of ground-state properties of neon isotopes. The RHFBz model is used with the PKO2 and PKO3 effective interactions [15, 16] in the particle-hole channel, and the central part of the Gogny D1S force [17] in the particle-particle channel. The PKO3 effective interaction is related to a covariant EDF including explicitly the pion degree of freedom, which is one actor of the tensor force. On the contrary, the PKO2 effective interaction corresponds to a covariant EDF where the pion degree of freedom is not treated explicitly.

The two-neutron separation energy \(S_{2n} \equiv E_{\text{tot}}(Z,N) - E_{\text{tot}}(Z,N-2)\) of Ne isotopes, calculated with PKO2 and DD-ME2, are compared to data in Fig. 1. In general, the RHFBz
results obtained with the PKO2 parameter set are closer to the experimental two-neutron separation energies. Both PKO2 and DD-ME2 predict $^{32}$Ne to be the last bound isotope.

Figure 1: Two-neutron separation energy in the neon isotopic chain. The relativistic mean-field results: RHFBz with PKO2 [15, 16], and RHB with DD-ME2 [18], are compared to data (Audi-Wapstra [19]).

The evolution of the axial deformation parameter $\beta$ in the neon isotopic chain is illustrated in Fig. 2. In general, the deformation predicted by PKO3 is larger than that calculated with PKO2 and, therefore, closer to the results obtained with the DD-ME2 and Gogny D1S effective interactions. PKO3 predicts an oblate shape for $^{24}$Ne (quasi degenerate in energy with a prolate solution at $\beta = 0.3$) whereas a prolate ground-state shape for this nucleus is obtained with PKO2, DDME2, Gogny D1S (quasi degenerate in energy with an oblate solution at $\beta = -0.15$) and Skyrme SLy4 interactions. Moreover, all these interactions, except PKO3 that predicts a prolate ground-state, give no deformation for $^{26}$Ne and $^{28}$Ne. The comparison between the two PKO3 curves where the pion coupling is switched on and off shows that the prolate shape of $^{26,28}$Ne is driven by the pion.

4. Conclusion

The relativistic Hartree-Fock-Bogoliubov model for axially deformed nuclei (RHFBz) is based on an effective Lagrangian with density-dependent meson-nucleon couplings. In this work RHFBz calculations have been performed for neon isotopes. Results obtained with the RHF effective forces PKO2 and PKO3 have been compared to experimental $S_{2n}$. In addition, ground-state deformation has been shown in comparison with the predictions of the relativistic DD-ME2 effective interaction, as well as with the results calculated with the non-relativistic Gogny D1S and Skyrme SLy4 interactions. The effect of explicitly including the pion field has been investigated for deformation parameters. The inclusion of the tensor $\rho$-nucleon coupling will complete the model and thus enable studies of the role of tensor components of the effective inter-nucleon interaction in the evolution of shell structures in deformed nuclei.
Figure 2: Axial deformation parameter \( \beta \) of Ne nuclei as function of the mass number. The calculated values correspond to the PKO2 and PKO3 [15, 16], DD-ME2 [18], Gogny D1S [17], Skyrme SLy4 [20] and Skyrme SGII [21] effective interactions. PKO3 calculations with \( f_\pi(\rho) \) set to 0 are also shown.

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