Cross-Mode Interference Characterization in Cellular Networks with Voronoi Guard Regions

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Abstract—Advances in cellular networks such as device-to-device communications and full-duplex radios, as well as the inherent elimination of intra-cell interference achieved by network-controlled multiple access schemes, motivates the investigation of the cross-mode interference under a guard region corresponding to the Voronoi cell of an access point. By modeling the positions of interfering access points (APs) and user equipments (UEs) as Poisson distributed, analytical expressions for the statistics of the interference generated by either APs or UEs are obtained based on appropriately defined density functions. The considered system model and analysis are general enough to capture many operational scenarios of practical interest, including conventional downlink/uplink transmissions with nearest AP association, as well as transmissions where not both communicating nodes lie within the cell. Analysis provides insights on the level of protection offered by a Voronoi guard region and its dependence on type of interference and receiver position. Various examples demonstrate the validity/accuracy of the analysis in obtaining the system coverage probability for operational scenarios of practical interest.

Index Terms—Stochastic geometry, cellular networks, guard region, D2D communications, full-duplex radios, interference.

I. INTRODUCTION

Characterization of the interference experienced by the receivers of a wireless network is of critical importance for system analysis and design [1]. This is a task that has been performed by engineers throughout the last decades resulting in the successful deployment of wireless networks with the most prominent example being the cellular network. Cellular networks are currently experiencing fundamental changes in their architecture, technology, and operation [2], which have significant impact on the interference footprint. Interference characterization under these new system features is of utmost importance in order to understand their potential merits as well as their ability to co-exist.

Towards increasing the spatial frequency reuse, two of the most prominent techniques/features considered for the future cellular network are device-to-device (D2D) communications [3] and full-duplex (FD) radios [4]. Although promising, application of these techniques introduces additional, cross-mode interference. For example, an uplink transmission is no longer affected only by interfering uplink transmissions but also by interfering D2D and/or downlink transmissions. Although it is reasonable to expect that the current practice of coordinated transmissions within the cell of each access point (AP) as a means to eliminate intra-cell interference will also hold in the future [5], the continually increasing density of APs and user equipment (UEs) suggest that inter-cell interference will be the major limiting factor of D2D- and/or FD-enabled cellular networks, rendering its statistical characterization critical.

A. Previous Work

Stochastic geometry is by now a well accepted framework for analytically modeling interference in large-scale wireless networks [6]. Under this framework, most of the numerous works on D2D-enabled cellular networks (without FD radios) consider the interfering D2D nodes as uniformly distributed on the plane, i.e., there is no spatial coordination of D2D transmissions (see, e.g., [7], [8], [9]). Building on the approach of [10], various works consider the benefits of employing spatially coordinated D2D transmissions where for each link in the system a circular guard region (zone) is established, centered at either the receiver or transmitter, within which no interfering transmissions are performed [11], [12]. This type of guard zone can be viewed as a model for a CSMA-type transmission protocol [1, Sec. 3.7], which implies that D2D links operate as an independent-from-APs tier. However, when the D2D links are network controlled [5], a more natural and easier to establish guard region is the (Voronoi) cell of a coordinating AP. Under a non-regular AP deployment [13], this approach results in a random polygon guard region, which makes the interference characterization a much more challenging task.

Interference characterization for this type of guard region has only been partially investigated in [14], [15], [16] where conventional uplink transmissions with nearest AP association and one active UE per cell are considered. As the interfering point process is distributed as a Voronoi perturbed lattice process (VPLP) [17], [18], which is analytically intractable, it is approximated as a Poisson point process (PPP) of an appropriately defined density. This approach is shown to have reasonable accuracy for uplink interference characterization, which suggests the same approach also for the case when the considered receiver is located at an arbitrary position in the plane (not that of an AP). However, the interferer density functions considered in the works on uplink analysis are heuristically proposed and, as shown in [18], they may change drastically when a receiver position other than that of the coordinating AP is considered. In addition, the uplink analysis does not hold when the interference is generated by APs.

Investigation of interference statistics under a cell guard region is also missing in the (much smaller) literature on FD-enabled cellular networks, which typically assumes no spatial...
coordinated for the UE transmissions (see e.g., [19], [20], [21]).

B. Contributions

This paper considers a stochastic geometry framework for modeling the interference power experienced at an arbitrary receiver position due to transmissions by APs or UEs and under the spatial protection of a Voronoi guard region. The system model is general enough to capture operational scenarios such as conventional downlink and uplink communications with nearest AP association, as well as communications where the receiver location is not guaranteed to lie within the guard region. The latter case has not been previously investigated in the literature and is of particular interest in, e.g., D2D communications where cross-cell links may be established.

For this system model, the interference power at the receiver is statistically characterized in terms of its Laplace transform. The exact Laplace transform is obtained for the case of AP-generated interference, whereas a tighter lower bound is provided for the Laplace transform of the UE-generated interference power. Both Laplace transforms are uniquely determined by appropriately defined equivalent densities of interferers, which also serve as an intuitive quantity for understanding the interference properties. It is shown that the equivalent interferer density and, hence, interference properties, significantly change when the type of interference (AP- or UE-generated) and/or receiver position change. Emphasis is given on the more challenging for analysis UE-generated interference scenario, where various special instances of the system model are investigated in detail, including the standard Planck communication scenario.

Analysis provides insights, among others, on the validity of the heuristic density functions considered previously for the analysis of the conventional uplink scenario, the effectiveness of a Voronoi guard region formed around the receiver or transmitter, and the position within the Voronoi cell where the interference in its close vicinity is minimal. As an application of the Laplace transform expressions, the coverage probability of potentially cross-cell D2D communications, is presented.

C. Notation

The origin of the two-dimensional plane $\mathbb{R}^2$ will be denoted as $o$. The Euclidean norm of $x \in \mathbb{R}^2$ is denoted as $|x|$ with operator $|\cdot|$ also used to denote the absolute value of a scalar or the Lebesgue measure (area) of a bounded subset of $\mathbb{R}^2$. The polar form representation of $x \in \mathbb{R}^2$ will be denoted as $(|x|, \angle x)$ or $|x| \angle x$, where $\angle x$ is the principal branch of the angular coordinate of $x$ taking values in $[−\pi, \pi)$. The open ball in $\mathbb{R}^2$, centered at $x \in \mathbb{R}^2$ and of radius $R > 0$, is denoted as $B(x, R) \triangleq \{y \in \mathbb{R}^2 : |y - x| < R\}$, whereas its boundary is denoted as $C(x, R) \triangleq \{y \in \mathbb{R}^2 : |y - x| = R\}$. $F(\cdot)$ is the indicator $(0, 1)$-operator, $P(\cdot)$ is the probability measure, and $E(\cdot)$ is the expectation operator. Functions $\arccos(\cdot) : [−1, 1] \to [0, \pi]$ and $\arcsin(\cdot) : [−1, 1] \to [−\pi/2, \pi/2]$ are the principal branches of the inverse cosine and sine, respectively. The Laplace transform of a random variable $z$ equals $L_z(s) \triangleq E(e^{-sz})$, for all $s \in \mathbb{R}$ for which the expectation exists. A function $f$ defined over some set $S \subset \mathbb{R}^2$ will be referred to as circularly-symmetric with respect to $(w.l.o.g.) z \in S$ if it holds $f(z + x) = f_z(|x|)$, $x \in S$, where $f_z$ is a function defined over $[0, \infty)$.

II. SYSTEM MODEL

A large-scale model of a cellular network with APs and UEs positioned over $\mathbb{R}^2$ is considered. The positions of APs are modeled as a realization of a homogeneous PPP (HPPP) $\Phi_a \subset \mathbb{R}^2$ of density $\lambda_a > 0$ (average number of APs per unit area) and the positions of the UEs are modeled as a realization of another, independent HPPP $\Phi_u \subset \mathbb{R}^2$ of density $\lambda_u > 0$ (average number of UEs per unit area). Within the cellular network there exists a node of interest (either AP or UE), referred to in the following as typical node. Without loss of generality (w.l.o.g.), the typical node will be assumed to be positioned at the origin $o$. The typical node establishes a communication link with another node (AP or UE) arbitrarily positioned on the plane. The receiver position of the link will be denoted as $x_R \in \mathbb{R}^2$ ($x_R = o$ in the case when the typical node is in receive mode).

The receiver experiences interference generated by the other transmitting nodes (APs and UEs) in the network. For simplicity, it will be assumed that all APs and UEs in the system are in transmit mode with a fixed power $P_a > 0$ and $P_u > 0$, respectively. Generalization of the analysis in the case when the transmit powers of the nodes of the same type (APs or UEs) are modeled as independent and identically distributed (i.i.d.) nonnegative random variables is straightforward based on the theory of independently marked PPPs [23].

Let $x^* \triangleq \arg \min_{x \in \Phi_a} |x|$ denote the position of the closest AP to the typical node ($x^* = o$ in the case the typical node is an AP) and let $V^*$ denote its Voronoi cell generated by the Poisson-Voronoi tessellation of the plane from APs, i.e.,

\[
V^* \triangleq \{y \in \mathbb{R}^2 : |y - x^*| < |y - x|, \text{ for all } x \in \Phi_a \setminus \{x^*\}\}.
\]

In order to reduce the interference experienced at the receiver position $x_R$, the AP at $x^*$ coordinates transmissions such that $V^*$ effectively forms a spatial guard region within which no interference is generated. An example of a Voronoi guard region is shown in Fig. [1]. Note that $V^*$ is a random guard region with no guarantee that $x_R$ lies within $V^*$, i.e., it holds $P(x_R \in V^*) < 1$, unless $x_R$ is a point on the line segment joining the origin with $x^*$.

Denoting $I_{x_R,a}$ and $I_{x_R,u}$ the interference power experienced at $x_R$ due to transmissions by APs and UEs that lie outside $V^*$, respectively, the standard interference model is adopted in this paper, namely

\[
I_{x_R,k} \triangleq \sum_{x \in \Phi_k \setminus V^*} P_k g_x |x - x_R|^{-\alpha_k}, k \in \{a, u\},
\]

where $g_x \geq 0$ the channel gain of a transmission generated by a node at $x \in \mathbb{R}^2$, assumed to be an independent of $x$. 
exponentially distributed random variable of mean 1 (Rayleigh fading), and \( \alpha_k > 2 \) is the path loss exponent. Note that the model is general enough so that, by appropriately choosing \( x_R \) and/or \( x^* \), \( I_{x_R,a} \) and \( I_{x_R,u} \) correspond to many instances of (cross-mode) interference experienced in D2D/FD-enabled as well as conventional cellular networks. For example, with \( x_R = 0 \) and \( x^* \neq 0 \), \( I_{x_R,a} \) is the interference power experienced by a standard downlink transmission with nearest AP association \(^{[24]}\), whereas \( I_{x_R,u} \) is the cross-mode interference power generated by UEs in, e.g., D2D mode. Various other cases of practical interest will be discussed in the following sections.

For characterization of the performance of the communication link as well as obtaining insights on the effectiveness of the guard region \( \mathcal{V}^* \) for reducing interference, it is of interest to characterize the statistical properties of the random variables \( I_{x_R,a} \) and \( I_{x_R,u} \). This is the topic of the following section, where the marginal statistics of \( I_{x_R,a} \) and \( I_{x_R,u} \) conditioned on \( x^* \) and treating \( x_R \) as a given, but otherwise free, parameter, are investigated in detail. The respective unconditional interference statistics can be simply obtained by averaging over the distribution of \( x^* \), which corresponds to a uniformly distributed \( x^* \) and a Rayleigh distributed \( |x^*| \) of mean \( 1/(2\sqrt{\alpha_0}) \) \(^{[23]}\). Characterization of the joint distribution of \( I_{x_R,a} \) and \( I_{x_R,u} \) is left for future work.

### III. INTERFERENCE CHARACTERIZATION

Towards obtaining a tractable characterization of the distribution of the (AP or UE) interference power, or, equivalently, its Laplace transform, the following standard result in PPP theory is first recalled \(^{[23]}\).

**Lemma 1.** Let \( I \triangleq \sum_{x \in \Phi} P h_x |x - z|^{-\alpha}, z \in \mathbb{R}^2, P > 0, \) with \( \Phi \) an inhomogeneous PPP of density \( \lambda : \mathbb{R}^2 \to [0, \infty), \) and \( \{h_x\}_{x \in \Phi} \) i.i.d. exponential random variables of mean 1. The Laplace transform of \( I \) equals

\[
\mathcal{L}_I(s) = \exp \left\{ -\int_{\mathbb{R}^2} \lambda(x) g(sP|x - z|^{-\alpha}) dx \right\}, \tag{2}
\]

where \( g(t) \triangleq 1 - \frac{1}{1+t} \) and the second equality holds only in the case of a circularly-symmetric density function w.r.t. point \( z \), i.e., \( \lambda(z + x) = \lambda_z(x) \), for an appropriately defined function \( \lambda_z : [0, \infty) \to [0, \infty) \).

The above result provides a complete statistical characterization in integral form of the interference experienced at a position \( z \in \mathbb{R}^2 \) due to Poisson distributed interferers. Of particular interest is the case of a circularly-symmetric density, which only requires a single integration over the radial coordinate and has previously led to tractable analysis for various wireless network models of interest such as mobile ad-hoc \(^{[1]}\) and downlink cellular \(^{[24]}\). In the following it will be shown that even though the interference model of Sec. II suggests a non circularly-symmetric interference density due to the random shape of the guard region, the Laplace transform formula of \(^{[3]}\) holds exact for \( I_{x_R,a} \) and is a (tight) lower bound for \( I_{x_R,u} \) with appropriately defined circularly-symmetric equivalent density functions.

#### A. AP-Generated Interference

Since, by definition, no other AP lies within \( \mathcal{V}^* \) except the one at \( x^* \), the actual shape of the guard region \( \mathcal{V}^* \) has no effect on AP-generated interference apart from restricting the AP at \( x^* \) to transmit. However, since \( x^* \) is the closest AP position to the origin, the interfering APs, given \( x^* \), are effectively distributed as an inhomogeneous PPP with density \(^{[24]}\)

\[
\lambda_a(x) = \lambda_a \mathbb{I}(x \notin B(o, |x^*|)), x \in \mathbb{R}^2, \tag{4}
\]

i.e., an implicit circular guard zone is introduced around the typical node (see Fig. 1). This observation may be used to determine the Laplace transform of the AP-generated interference experienced at \( x_R \) directly from the formula of \(^{[3]}\). However, the following result shows that the two-dimensional integration can be avoided as the formula of \(^{[3]}\) is also valid in this case with an appropriately defined equivalent density function.

**Proposition 2.** The Laplace transform, \( \mathcal{L}_{I_{x_R,a}}(s, x^*) \triangleq \mathbb{E}(e^{-sI_{x_R,a}} \mid x^*) \), of the AP-generated interference power \( I_{x_R,a} \), conditioned on \( x^* \), equals the right-hand side of \(^{[3]}\) with \( P = P_o, \alpha = \alpha_a \) and \( \lambda_z(r) = \lambda_{z_R,a}(r) \), where

\[
\lambda_{z_R,a}(r) \triangleq \begin{cases} \lambda_a, & r > |x^*| + |x_R| \\ 0, & |x^*| > |x_R|, \\ \lambda_a, & |x^*| < |x_R|, \quad r \leq ||x^*| - |x_R|| \\ \lambda_a/2, & |x^*| = |x_R| \\ \lambda_a (1 - \arccos(d)), & \text{otherwise}, \end{cases} \tag{5}
\]

with \( d \triangleq \frac{r^2 - (|x^*|^2 - |x_R|^2)}{2|x_R|} \), for \( x_R \neq 0 \), and

\[
\lambda_{z_R,a}(r) = \lambda_a \mathbb{I}(r \geq |x^*|), \quad \text{for } x_R = 0. \tag{6}
\]

**Proof:** See Appendix A.

The following remarks can be made:
1) The interference experienced at an arbitrary position \( x_R \neq o \) under the considered guard region scheme is equal in distribution to the interference experienced at the origin due to interferers distributed as an inhomogeneous PPP of density given by (5).

2) Even though derivation of the statistics of \( I_{x_R,a} \) was conditioned on \( x_R \) and \( x^* \), the result depends only on their norms \( |x_R| \) and \( |x^*| \).

3) Function \( \lambda_{x_R,a}(r) \) appearing in Prop. 2 can be considered as an equivalent interfering AP density experienced at \( x_R \). However, \( \lambda_{x_R,a}(r) \) is equal to the actual interfering AP density experienced at \( x_R \) when \( \angle x_R \) is uniformly distributed in \([−π, π]\).

4) \( \lambda_{x_R,a}(r) \) is a decreasing function of \( |x^*| \), corresponding to a (statistically) smaller AP interference power due to an increased guard zone area.

5) For \( x_R = o \), the interference corresponds to the one experienced at the receiver of a downlink cellular transmission with nearest AP association and Prop. 2 coincides with the analysis of [24]. The case of a downlink transmission without nearest AP association, i.e., where the receiver is not guaranteed to lie in the cell of the serving AP is also captured by the analysis by setting \( x^* = o \).

6) The case \( x_R = x^* \) corresponds to the nearest-neighbor transmission scenario considered in [25] where the validity of \( \lambda_{x_R,a}(r) \) in (5) as an equivalent density function for computation of the Laplace transform of the interference power was not observed.

7) In [26], the Laplace transform of the interference power experienced at \( x_R \in \mathbb{R}^2 \) due to a Poisson hole process, i.e., with interferers distributed as an HPPP over \( \mathbb{R}^2 \) except in the area covered by randomly positioned disks (holes), was considered. The holes were assumed to not include \( x_R \) and a lower bound for the Laplace transform was obtained by considering only a single hole [26, Lemma 5], which coincides with the result of Prop. 2. This is not surprising as the positions of the APs, conditioned on \( x^* \), are essentially distributed a single-hole Poisson process. Note that Prop. 2 generalizes [26, Lemma 5] by allowing \( x_R \) to be covered by the hole and considers a different proof methodology.

Figure 2 shows the normalized density function \( \lambda_{x_R,a}(r)/\lambda_a \) for various values of \( |x_R| \) and assuming that \( |x^*| = 1/(2\sqrt{\lambda_a}) \).

Recollecting that in the absence of guard region, the interfering AP density would be constant over all \( \mathbb{R}^2 \) and equal to \( \lambda_a \), it can be seen that the presence of a guard region results in reducing the interferer density in certain intervals of the radial coordinate \( r \), depending on the value of \( |x_R| \). In particular, when \( |x_R| < |x^*| \), it is guaranteed that no APs exist within a radius \( |x^*| - |x_R| > 0 \) from \( x_R \). In contrast, when \( |x_R| > |x| \) there is no protection from APs in the close vicinity of \( x_R \).

The case \( |x_R| = |x^*| \) is particularly interesting since it corresponds to the case when the receiver of interest is the AP at \( x^* \). For \( x^* \neq o \), corresponding to an uplink transmission by the typical node to its nearest AP, it can be easily shown that it holds

\[
\lambda_{x_R,a}(r) = \lambda_a \left( \frac{1}{2} + \frac{r}{2\pi|x^*|} \right) + O(r^3), r \to 0, \tag{7}
\]

i.e., the guard region results in the serving AP experiencing about half of the total interfering APs density in its close vicinity. Figure 3 shows the normalized \( \lambda_{x_R,a}(r) \) for various values of \( |x^*| \), where its linear asymptotic behavior as well as the advantage of a larger \( |x^*| \) are clearly visible.

### B. UE-Generated Interference

The positions of interfering UEs, conditioned on the realization of \( \Phi_a \), are distributed as an inhomogeneous PPP of density

\[
\tilde{\lambda}_u(x|\Phi_a) = \lambda_u(x \notin \mathcal{R}^*), x \in \mathbb{R}^2. \tag{8}
\]

Although the expression for \( \tilde{\lambda}_u \) is very similar to the that of the density \( \lambda_a \) of interfering APs given in (4), the random
Corollary 4. The PCA function is lower bounded as
\[
p_c(x | x^*) \geq e^{-\lambda_u \pi |x-x^*|^2}, \quad x \in \mathbb{R}^2,
\]
with equality if and only if \(x^* = o\).

Remark: The right-hand side of (9) equals the probability that \(x\) belongs to the Voronoi cell of the AP positioned at \(x^*\) when the latter is not conditioned on being the closest AP to the origin or any other point in \(\mathbb{R}^2\).

Figure 4 depicts \(p_c(x | x^*)\) for the case where \(|x^*| = 1/\sqrt{\lambda_u}\) (behavior is similar for other values of \(|x^*| > 0\)). Note that, unless \(x^* = o\), \(p_c(x | x^*)\) is not circularly symmetric w.r.t. any point in \(\mathbb{R}^2\), with its form suggesting that points isotropically distributed in the vicinity of \(x^*\) are more probable to lie within \(\mathcal{V}^*\) than points isotropically distributed in the vicinity of \(o\).

The lower bound \(e^{-\lambda_u \pi |x-x^*|^2}, \quad x \in \mathbb{R}^2\), is also shown in Fig. 4, clearly indicating the probabilistic expansion effect of the Voronoi cell of an AP, when the latter is conditioned on being the closest to the origin. This cell expansion effect is also demonstrated in Fig. 5 where the conditional average cell area, equal to

\[
\mathbb{E}(|\mathcal{V}^*| | x^*) = \mathbb{E} \left( \int_{\mathbb{R}^2} I(x \in \mathcal{V}^*) dx | x^* \right)
\]

is plotted as a function of the distance \(|x^*|\), with the integral of (10) evaluated numerically using Lemma 3. Simulation results are also depicted serving as a verification of the validity of Lemma 3. It can be seen that the average area of the guard region increases with \(|x^*|\), which implies a corresponding increase of the average number of UEs that lie within the region. However, as will be discussed in the following, even though resulting in more UEs silenced on average, an increasing \(|x^*|\) does not necessarily imply improved protection from UE interference, depending on the receiver position.

The interference statistics of the UE-generated interference power are given in the following result.

Proposition 5. The Laplace transform \(\mathcal{L}_{e^{-sI_{x^u}}}(s | x^*) \triangleq \mathbb{E}(e^{-sI_{x^u}} | x^*)\) of the UE-generated interference power \(I_{x^u}\) conditioned on \(x^*\), is lower bounded as the right-hand side of (3) with \(P = P_u, \alpha = \alpha_u\) and \(\lambda_x(r) = \lambda_{x,u}(r)\), where

\[
\lambda_{x,u}(r) \triangleq \lambda_u \left( 1 - \frac{1}{2\pi} \int_0^{2\pi} p_c((r, \theta) + x_R | x^*) d\theta \right),
\]

with \(p_c(x | x^*)\) as given in Lemma 3.
bound is tight in the sense that it corresponds to the exact equivalent density when \( x^* = o \).

**Proposition 6.** The equivalent density \( \lambda_{x_R,u}(r) \) of Prop. 5 is upper bounded as

\[
\lambda_{x_R,u}(r) \leq \lambda_u \left[ 1 - e^{-\lambda_o \pi (|x^*|^2 + |x_R|^2 + r^2)} I_0(\lambda_a 2\pi r) \right],
\]

where \( \bar{x} \triangleq \max(|x^*|, |x_R|) \) and \( I_0(\cdot) \) denotes the zero-order modified Bessel function of the first kind. Equality holds only when \( x^* = o \).

**Proof:** See Appendix C.

An immediate observation of the above result is that, given \( x_R \), the equivalent interfering UE density decreases for any \( x^* \neq o \) compared to the case \( x^* = o \). This behavior is expected due to the cell expansion effect described before. However, as the bound of (13) is independent on \( \angle x_R \), it does not offer clear information on how the interfering UE density changes when different values of \( x^* \neq o \) are considered for a given \( x_R \).

In order to obtain further insights on the UE-generated interference properties, \( \lambda_{x_R,u}(r) \) is investigated in detail in the following section for certain special instances of \( x_R \) and/or \( x^* \), which are of particular interest in cellular networks.

**IV. Analysis of Special Cases of UE-Generated Interference**

A. \( x_R = x^* \)

This case corresponds to an uplink cellular transmission with nearest AP association, no intra-cell interference, and interference generated by UEs outside \( V^* \) operating in uplink and/or D2D mode. For this case, a tighter upper bound than the one in Prop. 6 is available for \( \lambda_{x^*,u}(r) \), which, interestingly, is independent of \( |x^*| \), in the practical case when \( x^* \neq o \).

**Lemma 7.** For \( x_R = x^* \), the equivalent density \( \lambda_{x_R,u}(r) = \lambda_{x^*,u}(r) \) of Prop. 5 is upper bounded as

\[
\lambda_{x^*,u}(r) \leq \lambda_u \left( 1 - e^{-\lambda_o \pi r^2} \right),
\]

with equality if and only if \( x^* = o \).

**Proof:** Follows by replacing the term \( p_c((r, \theta) + x_R | x^*) = p_c((r, \theta) + x^* | x^*) \) appearing in (11) with its bound given in Cor. 4 which evaluates to \( e^{-\lambda_o \pi r^2} \).

The bound of (14) indicates that \( \lambda_{x^*,u}(r) \) tends to zero at least as fast as \( O(r^2) \) for \( r \to 0 \), irrespective of \( x^* \).

The following exact statement on the asymptotic behavior of \( \lambda_{x^*,u}(r) \) shows that \( \lambda_{x^*,u}(r) \sim \alpha r^2, r \to 0 \), with the value of \( \alpha \) independent of \( |x^*| \) when \( x^* \neq o \).

**Proposition 8.** For \( x_R = x^* \), the equivalent density \( \lambda_{x_R,u}(r) = \lambda_{x^*,u}(r) \) of Prop. 5 equals

\[
\lambda_{x^*,u}(r) = \lambda_u \lambda_o b \pi r^2 + O(r^3), r \to 0,
\]

with \( b = 1 \) for \( x^* = o \) and \( b = 1/2 \) for \( x^* \neq o \).
Another approach is to consider a piecewise-linear curve, which, in addition to having a simple closed form, results in a closed form approximate expression for the bound of $L_{I,x^*}(s \mid x^*)$ as given below (details are straightforward and omitted).

**Lemma 9.** With the piecewise-linear approximation

$$\lambda_{x^*,u}(r) \approx \lambda_u \min(\delta r, 1), \ r \geq 0,$$

for some $\delta > 0$, the lower bound for $L_{I,x^*,u}(s \mid x^*)$ equals

$$\exp\left\{ -\lambda_u \pi \left[ C(P_u s)^{2/\alpha} - \frac{1}{2\delta} \left( \frac{2}{3} \tilde{F}(3) - \tilde{F}(2) \right) \right] \right\}$$

where $C \triangleq \frac{1}{\sin(2\pi/\alpha)}$ and $\tilde{F}(x)$ with $2F_1(\cdot)$ denoting the hypergeometric function.

A piecewise-linear approximation as in [16] was heuristically proposed in [15] for a slightly more general system model than the one considered here. A closed form formula for $\delta$ was provided, only justified as obtained by means of curve fitting without providing any details on the procedure. For the system model considered in this paper, this formula results in $\delta \approx 0.82687 \sqrt{\lambda_u}$ and the corresponding piecewise-linear approximation of the equivalent density is also depicted in Fig. 6. It can be seen that this approximation is essentially an attempt to capture the average behavior of $\lambda_{x^*,u}(r)$ over $|x^*|$, thus explaining its good performance reported in [15]. However, it is not clear why this particular value of $\delta$ is more preferable than any other (slightly) different value. A more rigorous approach for the selection of $\delta$ is to consider the tightest piecewise-linear upper bound for $\lambda_{x^*,u}(r)$, which leads to a lower (looser) lower bound for $L_{I,x^*,u}(s \mid x^*)$. This bound corresponds to a value of $\delta \approx 1.13118 \sqrt{\lambda_u}$, found by numerical optimization, and is also shown in Fig. 6.

### B. $x^* = o$

This case corresponds to the typical node being an AP, i.e., a typical AP. Note that, in contrast to the cases when $x^* \neq o$, the Voronoi cell of the typical AP is not conditioned to cover any point in $\mathbb{R}^2$, therefore, no cell expansion effect is observed and, for $x_R \neq o$, it is possible that the receiver lies outside the guard region. This is a different operational scenario from the common downlink with nearest AP association [24] where the intended receiver by default lies within the Voronoi cell of the serving AP. It may arise, for example, in case of D2D communications, where both communicating UEs receive data (e.g., control information) from the same AP.

The equivalent interfering UE density is given in Prop. 8 and is depicted in Fig. 7 for various values of $|x_R|$. Note that the case $x_R = o$ corresponds to the typical AP in receive mode, whereas for $x_R \neq o$ the typical AP transmits. It can be seen that increasing $|x_R|$ effectively results in reduced protection in the close vicinity of the receiver since the latter is more probable to lie near the edge or even outside the Voronoi guard region.
\( \lambda(x^*, u) \) with nearest AP association, whereas it decreases with its close vicinity as AP, experiencing an increased equivalent interferer density in is not as well protected from interfering UEs as its nearest.

For proof of Prop. 8. \( \lambda \) is downlink cellular transmission with nearest AP association, |\( \lambda \)| where it decreases with |\( \lambda \)| as \( r \rightarrow 0 \) when \( x^* \neq o \), and with a rate that is also proportional to (instead of independent of) \( |x^*| \). This implies that the typical node is not as well protected from interfering UEs as its nearest AP, experiencing an increased equivalent interferer density in its close vicinity as \( |x^*| \) increases. This can be understood by noting that with increasing \( |x^*| \), the origin, although contained in a guard region of (statistically) increasing area, is more probable to lie near the cell edge (Fig. 1 demonstrates such a case.)

The strong dependence of \( \lambda_o(u) \) on \( |x^*| \) observed for \( r \rightarrow 0 \) also holds for all values of \( r \). This is shown in Fig. 8 where the numerically evaluated \( \lambda_o(u) \) is depicted for the same values of \( |x^*| \) as those considered in Fig. 6. It can be seen that \( \lambda_o(u) \) increases with \( |x^*| \) for all \( r \) up to approximately 1 – 1.5 times the average value of \( |x^*| \) (equal to 1/(2\( \sqrt{\lambda_o} \))), whereas it decreases with \( |x^*| \) for greater \( r \).

This behavior of \( \lambda_o(u) \), especially for \( r \rightarrow 0 \), does not permit the use of single function to be used a reasonable approximation or bound for \( \lambda_o(u) \), irrespective of \( |x^*| \), as was done for the case for \( \lambda_o(r) \). Noting that the upper bound of \( \lambda(x^*, u) \) has the same value whenever \( |x^*| \) or \( |x_R| \) are zero, it follows that the curves shown in Fig. 7 with \( |x_R| \) replaced by \( |x^*| \) are upper bounds for the corresponding curves of Fig. 8. Unfortunately, it can be seen that the bound is very loose for small \( r \) and/or large \( |x^*| \).

D. For \( x_R \in (0, |x^*|] \), \( x^* = o \)

The previous cases demonstrate how different the properties of the UE-generated interference become when different receiver positions are considered. This observation motivates the question of which receiver position is best protected from UE interference given the positions \( o \) and \( x^* \neq o \) of the typical node and nearest AP, respectively. This question is relevant in, e.g., relay-aided cellular networks where the communication between a UE and its nearest AP is aided by another node (e.g., an inactive UE) [28]. Although the performance of a relay-aided communication depends on multiple factors including the distances among the nodes involved and considered transmission scheme [29], a reasonable choice for the relay position is the one experiencing less interference.

By symmetry of the system geometry, this position should lie on the line segment joining the origin with \( x^* \) (inclusive). However, as can be seen from Figs. 6 and 8 the shape of the equivalent interferer density does not allow for a natural ordering of the different values of \( x_R \). For example, \( \lambda(x^*, u) \) is smaller than \( \lambda_o(u) \) for small \( r \), whereas the converse holds for large \( r \). Noting that the receiver performance is mostly affected by nearby interference [1], a natural criterion to order the interfering UE densities for varying \( x_R \) is their asymptotic behavior as \( r \rightarrow 0 \). For \( |x_R| = |x^*| \) and \( |x_R| = o \), this is given in Props. 8 and 10 respectively. For all other positions the asymptotic density is given by the following result.

**Proposition 11.** For \( |x_R| \in (0, |x^*|], \angle x_R = 0 \), and \( x^* \neq o \), the equivalent density function \( \lambda(x_R, u) \) of Prop. 5 equals

\[ \lambda_o(u) = \frac{\lambda_o \sqrt{\lambda_o}}{\pi |x^*| r + \mathcal{O}(r^2), r \rightarrow 0.} \]
$$\lambda_{x_R,a}(r) = \lambda_u \lambda_a \frac{8}{\pi^2} \frac{(|x^*|/|x_R|)^2}{|x^*| - |x_R|} r^3 + \mathcal{O}(r^4), r \to 0. \quad (17)$$

It follows that the optimal receiver position must have $|x_R| \in (0, |x^*|)$ since, in this case, the density scales as $\mathcal{O}(r^3)$ instead of $\mathcal{O}(r)$ and $\mathcal{O}(r^2)$ when $|x_R| = 0$ and $|x_R| = |x^*|$, respectively. Its value can be easily obtained by minimization of the expression in (17) w.r.t. $|x_R|$. 

**Corollary 12.** The receiver position that experiences the smallest equivalent UE interferer density in its close vicinity lies on the line segment joining the origin to $x^*$ and is at distance $|x^*|/\sqrt{2}$ from the origin. The corresponding equivalent density equals

$$\lambda_{x_R,a}(r) = \lambda_u \lambda_a \frac{8}{\pi^2} r^3 + \mathcal{O}(r^4), r \to 0. \quad (17)$$

**V. NUMERICAL EXAMPLES**

In this section numerical examples will be given demonstrating the performance of various operational scenarios in cellular networks that can be modeled as special cases of the analysis presented in the previous sections. The system performance metric that will be considered is $L_{L_{R,u}}(\rho^{\alpha_u} \theta |x^*|)$, with $k \in \{a, u\}$ depending on whether AP or UE interference is considered, and $\rho > 0$, $\theta > 0$ given variables. Note that this metric corresponds to the coverage probability that the signal-to-interference ratio (SIR) at $x_R$ is greater than $\theta$, when the distance between transmitter and receiver is $r$, the direct link experiences a path loss $\alpha_k$ and Rayleigh fading, and the transmit power is equal to $P_k$. For simplicity and w.l.o.g., all the following examples correspond to $\lambda_u = 1$ and $\alpha_a = \alpha_u = 4$.

**A. D2D Link Not Necessarily Contained in a Single Cell**

In this example, UE-generated interference is considered at a receiver position $x_R \neq 0$ with $\angle x_R$ uniformly distributed in $[-\pi, \pi)$ and $\rho = |x_R|$. This case models a D2D link where the typical node is a UE that directly transmits to a receiver isotropically distributed at a distance $|x_R| > 0$. A guard region $\mathcal{V}^*$ is established by the closest AP to the typical node, however, it is not guaranteed to include $x_R$, i.e., the D2D nodes may not be contained within the same cell, which is a common scenario that arises in practice, referred to as cross-cell D2D communication [22].

With interference generated from other UEs in D2D and/or uplink mode, a lower bound for $L_{L_{R,u}}(\rho^{\alpha_u} \theta |x^*|)$ can be obtained using Prop. 5 with an equivalent density function $\lambda_{[x_R],a}(r)$ as given in (17). Figure 9 shows this lower bound for $\lambda_u = \lambda_a$ (Fig. 9a) and $\lambda_u = 10 \lambda_a$ (Fig. 9b). The position $x^*$ is assumed to be isotropically distributed with $|x^*| = 1/(2 \sqrt{\lambda_a})$ and various values of the ratio $|x_R|/|x^*|$ are considered. For each case, the exact value of $L_{L_{R,u}}(\rho^{\alpha_u} \theta |x^*|)$, obtained via Monte Carlo simulation, is also shown. It can be seen that the quality of the analytical lower bound depends on $\lambda_a$. For $\lambda_u = \lambda_a$, it is very close to the exact coverage probability, whereas for $\lambda_u = 10 \lambda_a$, it is reasonably tight and is able to capture the behavior of the exact coverage probability over varying $|x_R|$. In both cases, a performance degradation is observed with increasing $|x_R|$, due to both increasing path loss of the direct link as well as reduced interference protection by $\mathcal{V}^*$.

**B. AP-to-D2D-Receiver Link**

In this example, AP-generated interference is considered at a receiver position $x_R \neq 0$ with $\angle x_R$ uniformly distributed in $[-\pi, \pi)$ and $\rho = |x_R|$. This case is similar to the previous, however, considering AP interference and the receiver at $x_R$ receiving data from the AP node at $x^*$. This case models the scenario where the typical node establishes a D2D connection with a node at $x_R$, and the AP at $x^*$ sends data to $x_R$ via a dedicated AP link (e.g., for control purposes or for implementing a cooperative transmission scheme). Note that, in contrast to the previous case, the link distance $\rho$
is random and equal to $\rho = |x^*| + |x_R| - 2|x^*||x_R|\cos\psi$, with $\psi$ uniformly distributed in $[-\pi, \pi]$. The exact coverage probability of this link can be obtained using Prop. 2 followed by a numerically computed expectation over $\psi$.

Figure 10 shows $\mathbb{E}(\mathcal{L}_{x^*,u}(\rho^{a*}\theta|x^*))$ for an isotropically distributed $x^*$ with $|x^*| = 1/(2\sqrt{\lambda_a})$, and for various values of the ratio $|x_R|/|x^*|$. Note that the case $|x_R| = 0$ corresponds to the standard downlink transmission model with nearest AP association. Monte Carlo evaluation of the coverage probability (not shown here) perfectly matches the analytical curves. It can be seen that increasing $|x_R|$ reduces the coverage probability for small SIR but increases it for high SIR. This is in direct proportion to the behavior of the equivalent interferer density $\lambda_{x^*,u}(r)$ with increasing $|x_R|$ shown in Fig. 2.

### C. Uplink with Nearest AP Association

In this example, AP-generated interference is considered at a receiver position $x_R = x^*$ with $\rho = |x^*|$ and $\lambda_u = \lambda_a$. This case can be viewed as an approximation of the conventional uplink transmission scenario with nearest AP association and one active UE per cell. Note that the actual interferer process in the uplink scenario is distributed as a VPLP, which is difficult to analytically characterize. Figure 11 shows the lower bound of $\mathcal{L}_{x^*,u}(\rho^{a*}\theta|x^*)$ obtained by Prop. 5 as well as the looser, but more easily computable, lower bound obtained using the closed form expression given in Lemma 7 with $\delta = 1.13118\sqrt{\lambda_a}$ (resulting in the tightest bound possible with a piecewise-linear equivalent density function). The coverage probability is computed for an AP at a distance $|x^*| = c/(2\sqrt{\lambda_a})$ from the typical node, with $c = 1/2, 1, 2$, roughly corresponding to a small, average, and large uplink distance (performance is independent of $\angle x^*$). In addition, the coverage probability obtained by Monte Carlo simulation under both the PPP and VPLP interferer positions models are also shown.

As was the case in Fig. 2a Prop. 5 provides a very tight lower bound for the actual coverage probability (under the PPP model for the interfering UE positions). The bound of Lemma 7, although looser, is nevertheless a reasonable approximation of the performance, especially for smaller values of $|x^*|$. Compared to the VPLP model, the PPP model provides an optimistic performance prediction that is, however, reasonable tight, especially for large $|x^*|$. This observation motivates its usage as a tractable approximation of the actual interference statistics as was also reported in [15], [16]. Interestingly, the bound of Lemma 7 happens to provide an even better approximation for the VPLP performance for $|x^*|$ close to or smaller than $1/(2\sqrt{\lambda_a})$.

### D. Effect of Guard Region on UE-Generated Interference Protection

In order to see the effectiveness of a Voronoi guard region for enhancing link quality under UE-generated interference, the performance under the following operational cases is examined.

- Case A (no guard region is imposed): This results in the standard transmission model under an HPPP of interferers positions of density $\lambda_u$. The exact coverage probability is well known (see, e.g., [1, Eq. 3.29]).
- Case B (transmitter imposes a Voronoi guard region): This case corresponds to $x^* = o, x_R \neq o$.
- Case C (receiver imposes a Voronoi guard region): This case corresponds to $x_R = x^* = o$.

Cases B and C correspond to the analysis considered in Sec. V. B. Assuming the same link distance $\rho$ for all cases, Fig. 12 shows the analytically obtained coverage probability (exact for Case A, lower bound for Cases B and C), assuming $\lambda_u = \lambda_a$. It can be seen that imposing a Voronoi guard region (Cases B and C) is always beneficial, as expected. However, for large link distances, Case B provides only marginal gain as the receiver is very likely to be located at the edge or even outside the guard region. In contrast, the receiver is always guaranteed to be protected under case C, resulting in the best performance and significant gains for large link distances.
VI. CONCLUSION

This paper considered the analytical characterization of cross-mode inter-cell interference experienced in future cellular networks. By employing a stochastic geometry framework, tractable expressions for the interference statistics were obtained that are exact in the case of AP-generated interference and serve as a tight lower bound in the case of UE-generated interference. These expressions are based on appropriately defined equivalent interferer densities, which provide an intuitive quantity for obtaining insights on how the interference properties change according to the type of interference and receiver position. The considered system model and analysis are general enough to capture many operational scenarios of cellular networks, including conventional downlink/uplink transmissions between UEs that do not necessarily lie in the same cell. The analytical expressions of this paper can be used for sophisticated design of critical system aspects such as mode selection and resource allocation towards getting the most out D2D- and/or FD-enabled cellular communications.

APPENDIX A

PROOF OF PROPOSITION 2

Let \( \Phi_a \triangleq \Phi_a \setminus \{ x^* \} \) denote the interfering AP point process. Given \( x^* , \Phi_a \) is an inhomogeneous PPP of density \( \lambda_a(x) , x \in \mathbb{R}^2 \), defined in \( \text{(4)} \). Density \( \lambda_a(x) \) is circularly symmetric w.r.t. \( x_R = 0 \), which, using Lemma \( \text{1} \) directly leads to the Laplace transform expression of \( \text{(3)} \) with radial density as in \( \text{(6)} \). Considering the case \( x_R \neq 0 \), it directly follows from Lemma \( \text{1} \) and \( \text{(2)} \) that

\[
\mathcal{L}_{I_{x_R,a}}(s | x^*) = \exp \left\{ - \int_{\mathbb{R}^2} \lambda_a(x + x_R) g(sP_a | x - x_R |^{-\alpha_a}) dx \right\}
\]

by a change of integration variable. By switching to polar coordinates for the integration, the right-hand side of \( \text{(3)} \) results with \( P = P_a , \alpha = \alpha_a \) and \( \lambda_{x_R,a}(r) = \lambda_{x_R,a}(r) \), where

\[
\lambda_{x_R,a}(r) = \frac{1}{2\pi} \int_0^{2\pi} \lambda_a((r, \theta) + x_R) d\theta
\]

The last equality follows by noting that the integral does not depend on the value of \( L_{x_R} \). Evaluation of the final integral is essentially the evaluation of an angle (see Fig. 13), which can be easily obtained by elementary Euclidean geometry, resulting in the expression of \( \text{(5)} \).

APPENDIX B

PROOF OF PROPOSITION 5

It holds

\[
\mathcal{L}_{I_{x_R,a}}(s | x^*) = E \left( E \left( e^{-sL_{x_R,a}} | \Phi_a \right) | x^* \right)
\]

\[
\geq \exp \left\{ - \int_{\mathbb{R}^2} \lambda_a(x + x_R | \Phi_a) g(sP_a | x - x_R |^{-\alpha_a}) dx \right\}
\]

where \( \lambda_{x_R,a}(r) = \lambda_a(x + x_R | \Phi_a) g(sP_a | x - x_R |^{-\alpha_a}) \)

by a change of integration variable. By switching to polar coordinates for the integration, the right-hand side of \( \text{(3)} \) results with \( P = P_a , \alpha = \alpha_a \) and \( \lambda_{x_R,a}(r) = \lambda_{x_R,a}(r) \), where

\[
\lambda_{x_R,a}(r) = \frac{1}{2\pi} \int_0^{2\pi} \lambda_a((r, \theta) + x_R) d\theta
\]

The last equality follows by noting that the integral does not depend on the value of \( L_{x_R} \). Evaluation of the final integral is essentially the evaluation of an angle (see Fig. 13), which can be easily obtained by elementary Euclidean geometry, resulting in the expression of \( \text{(5)} \).

APPENDIX C

PROOF OF PROPOSITION 6

Replacing the PCA function in \( \text{(11)} \) with its lower bound as per Cor. \( \text{4} \) results in the equivalent density bound

\[
\lambda_{x_R,a}(r)
\]
\[
\begin{align*}
\lambda_u & \leq \lambda_u \left( 1 - \frac{1}{2\pi} \int_0^{2\pi} e^{-\lambda_u \pi |(r, \theta) + x_R - x^*|^2} d\theta \right) \\
& \leq \lambda_u \left( 1 - \frac{e^{-\lambda_u \pi |x^*|^2}}{2\pi} \int_0^{2\pi} e^{-\lambda_u \pi |(r, \theta) + x_R|^2} d\theta \right) \\
& \leq \lambda_u \left( 1 - \frac{e^{-\lambda_u \pi |x^*|^2}}{2\pi} \int_0^{2\pi} e^{-\lambda_u \pi |(r, \theta) + (|x_R|, 0)|^2} d\theta \right) \\
& \leq \lambda_u \left( 1 - \frac{e^{-\lambda_u \pi (|x^*|^2 + |x_R|^2 + r^2)}}{2\pi} \int_0^{2\pi} e^{\lambda_u \pi 2r |x_R| \cos \theta} d\theta \right),
\end{align*}
\]

where (a) follows by application of the triangle inequality, (b) by noting that the integral is independent of $x_R$ and (c) since $|(r, \theta) + (|x_R|, 0)|^2 = r^2 + |x_R|^2 - 2r|x_R| \cos(\pi - \theta) \text{ (cosine law)}$. The integral in the last equation is equal to $2\pi I_0(\lambda_u 2\pi |x_R|^2)$. By exchanging the roles $x^*$ and $x_R$ in (a) and following the same reasoning, another upper bound of the form of (c) results, with the only difference that the integrand has $|x^*|$ instead of $|x_R|$ and evaluates to $2\pi I_0(\lambda_u 2\pi |x^*|^2)$. Considering the minimum of these two bounds leads to the expression of $[13]$.