1. Introduction

In order to study any system, it is necessary to model it. The purpose of modeling is to obtain relevant conclusions of the original, based on model study. In the case of automotive systems, the model will be a mathematical one and describes their operation under the conditions or absence of disturbances imposed by the engineer. Mathematical model is theoretically elaborated and finalized on the basis of the data obtained from the experimental tests [5,6].

More and more restrictive pollution rules, traffic safety requirements and the need for effective diagnosis have led to the emergence and proliferation of electronic control systems onboard modern cars. All of these systems have reference databases containing records of the parameters concerned. At the vehicle level there is a large amount of data that needs to be processed in an efficient way. Thus, the need to use statistical methods and models arises [3,4].

Statistical models are obtained on the basis of large amounts of data. To achieve the goal, we needed to develop a testing program of vehicles dynamics. It aimed to capture a large range of operating regimes. There were used 14th same type cars with gasoline engines and different mileage. Vehicles have traveled over a year on runways like mosaic tiles, asphalt or pavement and in various weather conditions, snow, sleet, rain or sunny and warm weather. Driving style varied between normal and sport ones. Some of the engine parameters like power, consumption and torque were measured on test bench, but most data were recorded of on board sensors using a specialized tester with a reading frequency of 10 values per second. The result of the experimental research program is a set of 64th tests. A test contains 16 time-series of the measured parameters. Each time-series has 256 values. Data were saved in a database and became the reference for the modeling [1,7,8].
2. Model used

"Determining the best model in the set, guided by the data is the System Identification method." It is used parametric models having vectors as arguments. It is well known that the model of the vector $\xi$ is noted $M(\xi)$. If the mathematical model of the process is "parameterized" by the vector $\xi$, the problem of the identification resides in determining or assessing the model’s parameters on the basis of the experimental data of the input and output variables of the system (parameters identification). In other words, the analytical expression of the mathematical model is known, but the values of the coefficients (parameters) are not known. Model validation is done using different criteria. One of these is the one based on the least squares method. Thus, the objective function that is minimized is:

$$V(\theta, z) = \frac{1}{n}\sum_{i=1}^{n}[y(t) - \hat{y}(i\theta)]^2,$$

where $n$ is the number of values of the experimental series data, $y(t)$ is the experimental series, $\hat{y}(i\theta)$ is estimated series, obtained by identification and $z$ is an expression $z(t)=[y(t)u(t)]$ that encompasses $y(t)$ as output size and $u(t)$ as input size. Both series are vectors with discrete values like experimental data are.

The general form of the model is:

$$A(q)y(t) = B(q)u(t - nk) + C(q)e(t)$$

where $y(t)$ is the system’s output, $x(t)$ is the system’s input, $e(t)$ is the noise and $t$ is the independent variable (actually the time, usually given in discrete domain). The number of the delaying elements along the system’s input-output chain is $nk$. The polynomials featuring the model are given by:

$$A(q) = 1 + a_1q^{-1} + a_2q^{-2} + \cdots + a_{na}q^{-na}$$

$$B(q) = b_1 + b_2q^{-1} + b_3q^{-2} + \cdots + b_{nb}q^{-nb+1}$$

$$C(q) = 1 + c_1q^{-1} + c_2q^{-2} + \cdots + c_{nc}q^{-nc}$$

$$D(q) = 1 + d_1q^{-1} + d_2q^{-2} + \cdots + d_{nd}q^{-nd}$$

$$F(q) = 1 + f_1q^{-1} + f_2q^{-2} + \cdots + f_{nf}q^{-nf}$$

where $q$ is known as the delay operator and is given like:

$$q^{-i}u(t) = u(t - i),$$

while $na$, $nb$, $nc$, $nd$ and $nf$ are the polynomial’s orders. Model described by equation (1) is a linear prediction if its polynomial coefficients are constant.

Particular models have been developed from the generalized model. Each of these is intended to model certain categories of processes. The vehicle taken as a whole can be considered an automatic system. The most proven model for automated systems is ARMAX (AutoRegressive–Moving-Average with eXogenous inputs). Its mathematical equation is:

$$A(q)y(t) = B(q)u(t - nk) + C(q)e(t)$$

and it is featured by the following conditions:

$\begin{align*}
\{ & nd = nf = 0 \\
D(q) &= F(q) = 1
\end{align*}$

The major advantage is the freedom in describing the properties of the disturbance term. It adds flexibility to that by describing the equation error as a moving average of white noise. This gives the model:

$$y(t) + a_1y(t - 1) + \cdots + a_{na}y(t - na) = b_1u(t - 1) + \cdots + b_{nb}u(t - nb) + e(t) + c_1e(t - 1) + \cdots + c_{ne}e(t - nc)$$

(11)
3. Modelling vehicles dynamics
The mostly used checking method is the “predicted value method“. This is a simulation by which a process is reproduced in identical conditions by a certain number of times. Calculations require:
- the experimental series of input and output sizes;
- a mathematical model which is in this case ARMAX;
- the prediction horizon \( k \) that determines how many times the process is reproduced. In this study it is equal with 1. [9,10]

One model studied was a SISO one with vehicle speed as output and engine’s load-\( \xi \) as input \( V = f(\xi) \). Figure 1 presents the prediction and residual of test I20n.

![Figure 1. Prediction of model \( V = f(\xi) \) for test I20n](image)

Although, the linear dependence between the two vectors is reduced, the correlation coefficient being \( \xi \_V \_f = 0.203 \), the error is low, 1.4%. [11] This is calculated by the formula (12):

\[
\text{Error CV} = \frac{CV(y)-CV(ypred)}{CV(y)} \times 100 \ \% ,
\]

(12)

\( y(t) \) is the experimental series, \( y_{pred}(t) \) is the prediction result and \( CV \) the coefficient of variation. The equation with differences obtained by applying the identification method is for I20n test:

\[
V(k)=0.101V(k-1)+0.867V(k-2)+0.0592 Q(k-1)-0.00173 \xi(k-2)-e(k)+0.84e(k-1),
\]

(13)

where \( k \) is the number of value in the series. Other models were engine torque \( M_e \) or hourly consumption \( C_h \) as outputs and engine load \( \xi \) as input. The equations with differences for each of the two modeling for I20n test are:

\[
M_e (k)=1.01M_e (k-1)+0.0126M_e(k-2)+2.18 \xi (k-1)-2.19 \xi (k-2)+e(k)+0.71e(k-1)
\]

(14)

\[
C_h (k)=0.876C_h(k-1)+0.007C_h (k-2)+0.130 \xi (k-1)-0.14 \xi (k-2)+e(k)+0.432e(k-1)
\]

(15)

The correlation coefficients are 0.992 and 0.872 and the errors are 0.71% and 0.42%.

4. Statistical models of vehicles dynamics'
Statistical models are derived from statistical data and statistical tools. In this case, the equation of each model is obtained using the average values of coefficients obtained by modeling the 64 tests. Taking into account the \( C_h = f(\xi) \) dependence, the following results were obtained:
Figure 2. Prediction of $C_h=f(\xi)$ model with average values of coefficients for test I20n

Figure 3. Coefficients values of all 64 predictions of $C_h=f(\xi)$ model

Figure 4. Errors of modeling $C_h=f(\xi)$ with average values of coefficients
Figure 4 shows a lower error in the case of test I20n for model generated with average coefficients. This is not true for all tests, but errors are lower than 3%, which is a reasonable limit of accuracy. Average coefficients values of this model are presented in the figure 3. Also, there are figured all coefficients values.

The equations with differences with average coefficients for the three models are:

\[ C_h(k) = 0.968C_h(k-1) + 0.0082 C_h(k-2) + 0.126 \xi(k-1) - 0.12 \xi(k-2) + e(k) + 0.31e(k-1) \]  \hspace{1cm} (16)

\[ V(k) = 0.899V(k-1) + 0.087V(k-2) + 0.0117 \xi(k-1) - 0.00975 \xi(k-2) + e(k) - 0.136e(k-1) \]  \hspace{1cm} (17)

\[ M_e(k) = 0.969M_e(k-1) + 0.00659M_e(k-2) + 2.07 \xi(k-1) - 2.03 \xi(k-2) + e(k) - 0.2131e(k-1) \]  \hspace{1cm} (18)

The modeling process can be developed with MISO (Multiple Inputs - Single Output). Engine angular speed \( n \) is added as a second input. New models are: \( V = f(\xi, n) \), \( M_e = f(\xi, n) \) and \( C_h = f(\xi, n) \).
\[ C_h(k) = 0.977C_h(k-1) + 0.00173C_h(k-2) + \cdots + 0.126\xi(k-1) - 0.122\xi(k-2) + 0.0009n(k-1) - 0.0009n(k-2) + e(k) + 0.308e(k-1) \]  \hspace{1cm} (19)

\[ V(k) = 0.802V(k-1) + 0.164V(k-2) + \cdots + 0.0103\xi(k-1) + 0.00303\xi(k-2) + 0.00372n(k-1) - 0.00325n(k-2) + e(k) - 0.0832e(k-1) \]  \hspace{1cm} (20)

\[ M_r(k) = 0.958M_r(k-1) + 0.000918M_r(k-2) + \cdots + 2.07\xi(k-1) - 2.\xi(k-2) - 0.00347n(k-1) + 0.00373n(k-2) + e(k) + 0.18e(k-1) \]  \hspace{1cm} (21)

**Figure 7.** Errors of modeling \( V = f(\xi, n) \) with average values of coefficients

Maximum error of all two inputs MISO models and test is lower than 5%. This demonstrates that the models are well-designed and can be used successfully in analyzing the operation of gasoline powered vehicles.

5. **Conclusions**

Modelling vehicle dynamics using single or multiple linear prediction has good results. It is a useful, helpful and replicable process, which can be integrated in a multitude of experiments/designs. Using average values of coefficients produces models with very good accuracy; the error finding is less than 5%. However, it requires a great volume of accurate experimental data. Using parameters with significant correlation between them creates accurate models. The more factorial parameters are involved, the higher the accuracy of the model we get. Another observation is that the accuracy of the models created, and that of the data collected was directly proportional with the number of tests run.

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