Static Quantities of the W Boson in the MSSM

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Abstract

We systematically analyze the anomalous dipole $\Delta k_\gamma$ and quadrupole $\Delta Q_\gamma$ moments of the W gauge bosons in the context of the minimal supersymmetric standard model as functions of the soft SUSY breaking parameters $A_0, m_0, M_{1/2}$ and the top quark mass. The severe constraints imposed by the radiative breaking mechanism of the electroweak symmetry $SU(2) \times U(1)$ are duly taken into account. The supersymmetric values of $\Delta k_\gamma$ and $\Delta Q_\gamma$ can be largely different, in some cases, from the standard model predictions but of the same order of magnitude for values of $A_0, m_0, M_{1/2} \leq O(1\text{TeV})$. Therefore possible supersymmetric structure can be probed provided the accuracy of measurements for $\Delta k_\gamma, \Delta Q_\gamma$ reaches $10^{-2} - 10^{-3}$ and hence hard to be detected at LEP2. If deviations from the standard model predictions are observed at LEP2, most likely these are not due to an underlying supersymmetric structure. In cases where $M_{1/2} \ll A_0, m_0$, the charginos and neutralinos may give substantial contributions saturating the LEP2 sensitivity limits. This occurs when their masses $m_{\tilde{\chi}}, m_{\tilde{Z}}$ turn out to be both light satisfying $m_{\tilde{\chi}} + m_{\tilde{Z}} \approx M_W$. However these extreme cases are perturbatively untrustworthy and besides unnatural for they occupy a small region in the parameter space.

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One of the most crucial tests of the standard theory of electroweak interactions will be the study of the three gauge boson couplings to be probed in forthcoming experiments in the near or remote future. Although there is little doubt that the nonabelian structure of the standard model is the right framework for describing the electroweak phenomena at low energies, in the vicinity of the electroweak scale, nevertheless we are still lacking a direct experimental verification of it. Such a study will take place at LEP2 as well as in next round experiments at HERA, NLC, LHC, with high accuracy putting bounds on the dipole $\Delta k_\gamma$ and quadrupole $\Delta Q_\gamma$ form factors which are directly related to the anomalous magnetic moment $\mu_W$ and the electric quadrupole moment $Q_W$ of the $W$-boson. The standard model predicts $\Delta k_\gamma = \Delta Q_\gamma = 0$ at the tree level but higher order corrections modify these values by finite amounts that can be tested in the laboratory with high accuracy of the order of $10^{-2}$ to $10^{-3}$. Such measurements can be of vital importance not only for the self consistency of the standard model but also for probing possible structure beyond that of the standard theory signalling the presence of new physics.

The last years there has been a revived interest towards supersymmetric extensions of the standard model. The supersymmetric structure seems to be a necessary ingredient in the efforts towards embedding the standard model in larger schemes unifying all existing forces of nature and is also suggested by precision data on the gauge couplings $\alpha_1, \alpha_2, \alpha_3$ which merge at a unification scale $M_{GUT} \simeq 10^{16}$ GeV, provided the SUSY breaking scale $M_S$ lies in the TeV range. Such relatively low values for the supersymmetry breaking scale $\sim$ TeV may have important consequences for phenomenology. The TeV scale may be the onset of new physics and detection of supersymmetric particles with masses $\lesssim O(M_S)$ might not be out of reach in future experiments. For this we need energies and luminocities that production of new particles is feasible with rates that are accessible to the new machines. Therefore phenomenological study of supersymmetry is of utmost importance. Below the threshold for the production of superpartners of the known particles the only evidence for the existence of supersymmetry will be the study of physical quantities which are affected by the presence of the underlying supersymmetric structure. In this case the supersymmetric particles are not observed in the final states, and hence we talk about virtual SUSY; however they induce radiative corrections to the physical quantities of interest and make them deviate from their standard model values. In order to measure these one needs, at the theoretical level, higher order calculations while experimentally we need high accuracy tests capable of measuring such small differences. In case the experiments point towards the affirmative it will be an evidence for the existence of new physics SUSY being a strong candidate to play that role.

In this paper we study the radiative corrections to the dipole and quadrupole moments of the $W$-bosons in the minimal supersymmetric standard model (MSSM) with soft supersymmetry breaking terms. It is a well known fact that softly broken supersymmetry leads to $SU(2)_L \times U(1)_Y$ symmetry breaking through radiative effects and therefore within the MSSM the elegant ideas of supersymmetry, gauge coupling unification and natural explanation of the hierarchy $M_W / M_{Planck} \simeq 10^{-16}$ can be simultaneously realized. There are numerous papers studying the phenomenology of $\Delta k_\gamma, \Delta Q_\gamma$ both on and off shell in the context of the standard model (SM). Also supersymmetric versions
of the SM have been considered in which SUSY is either exact \[1\] or broken \[11, 12\] by soft terms but there is no systematic, to our knowledge, phenomenological analysis that properly takes into account all the effects of supersymmetry breaking and the constraints imposed by the renormalization group and the radiative breaking of the electroweak symmetry \[10\]. In this work we undertake this problem and discuss the supersymmetric values of the dipole and quadrupole moments as functions of the soft SUSY breaking terms and the top quark mass, taking into account all the constraints imposed by the radiative breaking of the electroweak symmetry. As a sneak preview of our results we state that the supersymmetric values differ substantially from those of the standard model, in some cases, contrary to what has been claimed in the literature \[11\].

The MSSM is described by a lagrangian

\[ \mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft} \]  

where \( \mathcal{L}_{SUSY} \) is its supersymmetric part derived from a superpotential \( \mathcal{W} \) bearing the form \[13\]

\[ \mathcal{W} = (h_U \hat{Q}^i \hat{H}_2^c \hat{U}^c + h_D \hat{Q}^i \hat{H}_1^c \hat{D}^c + h_E \hat{L}^i \hat{H}_1^c \hat{E}^c + \mu \hat{H}_1^i \hat{H}_2^c) \epsilon_{ij}, \quad \epsilon_{12} = +1 \]  

and \( \mathcal{L}_{soft} \) is its supersymmetry breaking part given by

\[ - \mathcal{L}_{soft} = \sum_i m_{\Phi_i}^2 |\Phi_i|^2 + (h_U A_U Q H_2 U^c + h_D A_D Q H_1 D^c + h_E A_L L H_1 E^c + h.c.) \]

\[ + (\mu B H_1 H_2 + h.c.) + \frac{1}{2} \sum_a M_a \bar{\lambda}_a \lambda_a. \]

In Eq. (3) the sum extends over all scalar fields involved and we have suppressed all family indices.

In our analysis we assume universal boundary conditions for the soft masses at a unification scale \( M_{GUT} \approx 10^{16} \text{ GeV} \). The evolution of all couplings as well as all soft masses and the mixing parameters \( \mu, B \) from \( M_{GUT} \) down to energies \( E \) in the vicinity of the electroweak scale is given by their renormalization group equations (RGE) \[6, 13\]. These are known up to two loop order \[14\]. As arbitrary parameters of the model we take the running top quark mass \( m_t(M_Z) \) at the Z-boson mass the angle \( \tan \beta \) defined by \( v_2(M_Z)/v_1(M_Z) \) and the soft SUSY breaking parameters \( A_0, m_0, M_{1/2} \) at \( M_{GUT} \). \( v_{1,2}(M_Z) \) are the vacuum expectation values of the Higgses \( H_{1,2} \) and the values of the trilinear scalar couplings \( A_0 \), common scalar mass \( m_0 \) and common gaugino mass \( M_{1/2} \) are meant at the unification scale \( M_{GUT} \). In this approach, which has been adopted by other authors too \[10, 14\] \( B, \mu \) are not free parameters; in fact their values at \( M_Z \) are determined through the minimizing equations of the scalar potential,

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1. Even in ref.\[11\] the SUSY breaking effects are actually ignored since only the supersymmetric limit of the MSSM is considered in the discussion of the physical results.

2. The physical top quark mass \( M_t \) is defined by \( M_t = m_t(M_t)/(1 + 4\alpha_3(M_t)/3\pi) \) when the one loop corrections are taken into account \[15\].

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\[
\frac{M_Z^2}{2} = \frac{\bar{m}_1^2 - \bar{m}_2^2 \tan^2 \beta}{\tan^2 \beta - 1} \\
\sin 2\beta = -\frac{2B\mu}{\bar{m}_1^2 + \bar{m}_2^2}.
\]  

In Eqs. (4) and (5) the masses appearing are defined by
\[
\bar{m}_{1,2}^2 = m_{1,2}^2 + \frac{\partial \Delta V}{\partial v_{1,2}^2}, \quad m_{1,2}^2 = m_{H_{1,2}}^2 + \mu^2
\]

where \(\Delta V\) accounts for the one loop corrections to the effective potential which should be included in the minimizing equations. If they are not the results are known to depend drastically on the choice of the scale becoming ambiguous and untrustworthy \[18\] \[17\] \[16\]. Eqs. (4) and (5) cannot be analytically solved to obtain the values \(B(M_Z), \mu(M_Z)\) and our numerical routines have to run several times to reach convergence. In solving the RGE’s we have also to properly take into account the appearance of the thresholds of the various particles opened as we approach low energies. We have duly taken care of these effects along the lines of ref \[19\]. We have found that their presence little upsets the picture as long as one loop corrections to the \(\Delta k_\gamma\) and \(\Delta Q_\gamma\) are concerned.

The supersymmetric limit of the model is realized when all soft SUSY breaking terms vanish, \(A_0 = m_0 = M_{1/2} = 0\), the v.e.v.’s \(v_1\) and \(v_2\) become equal and the mixing parameter \(\mu\) vanishes. It is in this limit that particles and their superpartners get a common mass. In that limit the quadrupole moment \((\Delta Q_\gamma)_{\text{SUSY}}\) vanishes but the same does not happen for the dipole moment \((\Delta k_\gamma)_{\text{SUSY}}\). For the latter the difference \(\Delta k_\gamma - (\Delta k_\gamma)_{\text{SUSY}}\) is a measure of the importance of SUSY breaking effects on the dipole moment.

After this introductory remarks concerning the MSSM and an outline of the numerical procedure we shall follow we embark on discussing separately the contributions of the various particles involved to the quantities of interest.

The most general \(W^+W^-\gamma\) vertex consistent with current conservation can be written as
\[
\Gamma_{\mu \alpha \beta} = -ie \left\{ f [2g_{\alpha \beta} \Delta_\mu + 4(g_{\alpha \mu}Q_\beta - g_{\beta \mu}Q_\alpha)] + 2\Delta k_\gamma (g_{\alpha \mu}Q_\beta - g_{\beta \mu}Q_\alpha) + \frac{4\Delta Q_\gamma}{M_W^2} \Delta_\mu (Q_\alpha Q_\beta - \frac{Q^2}{2}g_{\alpha \beta}) \right\} + \ldots
\]

where the W’s are on their mass shell and the ellipsis denote C, P violating terms. The labelling of the momenta and the assignment of Lorentz indices is as shown in Figure 1. To lowest order \(f = 1\), \(\Delta k_\gamma = \Delta Q_\gamma = 0\). The values of the form factors \(\Delta k_\gamma, \Delta Q_\gamma\) at zero momentum transfer are related to the actual magnetic dipole moment \(\mu_W\) and electric quadrupole moment \(Q_W\) by \[4\] \[3\] \[9\]
\[
\mu_W = \frac{e}{2M_W^2} (1 + \kappa_\gamma + \lambda_\gamma), \quad Q_W = -\frac{e}{M_W^2} (\kappa_\gamma - \lambda_\gamma)
\]

where \(\Delta k_\gamma \equiv \kappa_\gamma + \lambda_\gamma - 1\) and \(\Delta Q_\gamma \equiv -2\lambda_\gamma\).
The standard model one loop predictions for the dipole and quadrupole moments have been calculated and can be traced in the literature \[^9\]. The contributions of gauge bosons and matter fermions of the MSSM are identical to those of the SM. These are displayed in Table I in units of \(g^2/16\pi^2\). For the contributions of the fermions we agree with the findings of other authors as far as the isospin \(T_3 = -1/2\) fermions are concerned but we disagree in the sign of the \(T_3 = +1/2\) chiral fermion contributions. This means for instance that “up” and “down” quarks’ contributions to the dipole/quadrupole moments have the same sign despite the fact that they carry opposite electric charges. This is due to the fact that the triangle graph with the up quark coupled to the photon and the corresponding one with the down quark playing that role are crossed; the dipole and quadrupole terms however change sign under crossing, resulting to a same sign contribution, unlike the anomaly which preserves its sign yielding \(\approx \text{Tr}(Q)\). Since this is a rather delicate point which, we think, has been overlooked in previous works we shall discuss it in more detail.

The fermionic Lagrangian relevant for the calculation of \(\Delta k_\gamma\) and \(\Delta Q_\gamma\) is

\[
\mathcal{L} = \frac{g}{\sqrt{2}} (W^+_{\mu} \tilde{\Psi}_L T^\mu T_+ \Psi_L + h.c.) + e A_\mu (\bar{\Psi}_L Q \gamma^\mu \Psi_L) \tag{8}
\]

In this \(\Psi_L\) is a column involving all left handed fermions, \(T^\pm = T_1 \pm iT_2\) are weak isospin raising/lowering matrices and \(Q\) is the diagonal electric charge matrix. The graphs we have to calculate are shown in Figure 2, where \(p, p'\) are the momenta carried in by the external \(W\)’s. The Feynman integral of the graph shown in Figure 2b has a momentum dependence which follows from that of Figure 2a under the interchange \(\alpha \leftrightarrow \beta\) and \(p \leftrightarrow p'\) or equivalently \(Q_\mu \leftrightarrow Q_\mu, \Delta_\mu \leftrightarrow -\Delta_\mu\). One is easily convinced of that by explicitly writing down the expressions of the two graphs shown in Figure 2. Therefore one has

\[
\text{Graph}(2a) = \text{Tr}(T_+ T_- Q) V_{\mu\alpha\beta}(Q, \Delta)
\]

\[
\text{Graph}(2b) = \text{Tr}(T_- T_+ Q) V_{\mu\beta\alpha}(Q, -\Delta)
\]

where the tensor structure of \(V_{\mu\alpha\beta}(Q, \Delta)\) including the axial anomaly term is

\[
V_{\mu\alpha\beta}(Q, \Delta) = \alpha_0 e_{\alpha\beta\mu\lambda} \Delta^\lambda + \beta_1 g_{\alpha\beta} \Delta_\mu + \beta_2 (g_{\alpha\mu} Q_\beta - g_{\beta\mu} Q_\alpha) + \beta_3 \Delta_\mu Q_\alpha Q_\beta + \ldots \tag{9}
\]

The first term is the anomaly and \(\beta_{1,2,3}\) are form factors where the contributions to the charge renormalization, dipole and quadrupole moments are read from. The anomaly term does not flip its sign under \(\alpha \leftrightarrow \beta\), \(Q_\mu \leftrightarrow Q_\mu, \Delta_\mu \leftrightarrow -\Delta_\mu\) but the remaining terms do. As a result we get for the sum of the two graphs

\[
\text{Tr}(Q \{T_-, T_+\}) e_{\alpha\beta\mu\lambda} \Delta^\lambda + \text{Tr}(Q [T_+, T_-])(\beta_1 g_{\alpha\beta} \Delta_\mu + \ldots) \tag{10}
\]

where for the sake of the argument we have suppressed the fermion mass dependence entering into \(\beta_{1,2,3}\) by assuming that all fermions have the same mass. In the standard model \(\{T_-, T_+\} = \text{constant} \times 1\) and the anomaly is proportional to \(\text{Tr}(Q)\) which vanishes, a well known result. The second term of Eq.(10) however yields a contribution proportional to \(\text{Tr}(Q T_3)\), when all fermions have the same mass. From this it becomes obvious that “up” and “down” quarks’ contributions to the dipole quadrupole moments
have the same sign since they carry opposite isospin and electric charges. Since the fermion masses are different the graphs of Figure 2 yield, ignoring the anomaly term,

\[ \sum_f (Q_f T^3_f) V^{\alpha\beta\mu}(Q, \Delta, m^2_f, m^2_{f'}) \]

(11)

where \( f, f' \) are left handed fermions belonging to doublets of the weak \( SU(2) \). The explicit expressions for \( \Delta k_\gamma, \Delta Q_\gamma \) of each fermion doublet \( (\tilde{f}_f, \tilde{f}'_f)_L \) are given in the Table I, where they are expressed in terms of the dimensionless ratios \( r_{f,f'} = (m_{f,f'}/M_W)^2 \). The factor \( C_g \) appearing in these formulae is the color factor, one for the leptons and three for the quarks.

The discussion of the sleptons and squarks is complicated by the fact that the superpartners \( \tilde{f}_L, \tilde{f}'_L \) of the known fermions \( f_L, f'_L \) are not mass eigenstates. This is the case for the third family fermions and especially for the stops \( \tilde{t}_L, \tilde{t}'_L \) due to the heaviness of the top quark which results to large \( \tilde{t}_L, \tilde{t}'_L \) mixings. The first two families have small mass and such mixings are not large. In the stop sector the relevant mass matrix squared has the following form,

\[
\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix}
m^2_{\tilde{t}_L} + m^2_{\tilde{t}_L} & \cos(2\beta)\frac{1}{2} - \frac{2}{3}\sin^2\theta_W & m_t(A + \mu\cot\beta) \\
m^2_{\tilde{t}_L} & m^2_t(\mu\cot\beta) + M_Z^2& m^2(\mu\cot\beta) \\
m^2_{\tilde{t}_L} & m^2_t(A + \mu\cot\beta) & m^2_t(A + \mu\cot\beta) + M_Z^2\cos(2\beta)
\end{pmatrix}
\]

(12)

which is diagonalized by a matrix \( \mathbf{K}^\dagger \), i.e. \( \mathbf{K}^\dagger \mathcal{M}^2_{\tilde{t}} \mathbf{K}^T = \text{diagonal} \). Since the top mass is large \( \mathbf{K}^\dagger \) deviates substantially from unity. In the same way the sbottom and stau mass matrices get diagonalized by \( \mathbf{K}^b, \mathbf{K}^\tilde{\tau} \). However in that case the corresponding \( \tilde{f}_L, \tilde{f}'_L \) mixings are not substantial. The fermion contributions to \( \Delta k_\gamma, \Delta Q_\gamma \) are presented in the Table I. They are expressed in terms of the diagonalizing matrices discussed previously and the dimensionless ratios \( R_{\tilde{f}_i} = (m_{\tilde{f}_i}/M_W)^2 \), where \( i = 1, 2 \) runs over the mass eigenstates. In the supersymmetric limit the matrices \( \mathbf{K}^\dagger \mathbf{K}^\dagger \) corresponding to the sfermion doublet \( (\tilde{f}_f, \tilde{f}'_f)_L \) are unit matrices and \( m_{\tilde{f}_{1,2}} \) (\( m_{\tilde{f}'_{1,2}} \)) become equal to the fermion mass \( m_f \) (\( m_{f'} \)). In our numerical analysis we have taken only the top quark mass to be nonvanishing and therefore the aforementioned discussion regards only the stops. The inclusion of the bottom and tau masses little affects the quantities \( \Delta k_\gamma, \Delta Q_\gamma \) owing to the fact that the induced mixings in the corresponding sbottom and stau sectors are small. In the Table I we also give the dipole moment in the supersymmetric limit discussed earlier. Notice that in that limit the fermion/sfermion contributions to the quadrupole moment of each family separately cancel against each other.

We turn next to discuss the Higgs sector. In the MSSM we have two doublets of Higgs scalars \( H_1, H_2 \) giving masses to “down” \( (T_3 = -1/2) \) and “up” \( (T_3 = 1/2) \) fermions when they develop nonvanishing v.e.v’s as a result of electroweak symmetry breaking. The relative strength of their v.e.v’s is parametrized by the angle \( \beta \) defined as \( \tan\beta = v_2/v_1 \). These are running parameters depending on the scale, as is the running Z-boson mass \( m_Z^2 = (g^2 + g'^2)(v_1^2 + v_2^2)/2 \). The physical Z-boson mass is defined
as the pole of the Z-boson propagator occurring therefore at the point \( M_Z \) for which
\[ m_Z(M_Z) = M_Z. \]
Experimentally we know that \( M_Z = 91.2 GeV \).

As a result of the electroweak symmetry breaking we have five physical Higgses
\( A, h_0, H_0, H_\pm \) with the following masses,
\[
A (neutral) : m_A^2 = m_1^2 + m_2^2
\]
\[
H_0, h_0 (neutral) : m_{H,h}^2 = \frac{1}{2}\{(m_A^2 + M_Z^2)^2 \pm \sqrt{(m_A^2 + M_Z^2)^2 - 4M_Z^2m_A^2\cos^2(2\beta)}\}
\]
\[
H_\pm (charged) : m_{H_\pm}^2 = m_A^2 + M_W^2
\]
The lightest of the neutral Higgses \( h_0 \) has always a tree level mass smaller than \( M_Z \).
However it is known that radiative corrections due to the top/stops can be large, due
to the heaviness of the top quark \cite{24} and shift its upper limit to values that can exceed
\( M_Z \). In that case \( h_0 \) can escape detection at LEP2. In the supersymmetric limit \( h_0 \)
and the pseudoscalar \( A \) become massless, \( H_\pm \) has a mass \( M_W \) and \( H_0 \) a mass \( M_Z \).

The Higgs contributions to the dipole and quadrupole moments are shown in the
Table I as functions of the dimensionless parameters
\[ R_\alpha = (m_\alpha/M_W)^2, \alpha = h_0, H_0, A, H_\pm \]
and a mixing angle \( \theta \); for \( \sin^2 \theta = 1 \), \( h_0 \) becomes the standard model Higgs scalar \cite{1}.
In the standard model only one neutral Higgs survives the spontaneous electroweak
symmetry breaking having mass \( m_{Higgs} \) whose contributions to \( \Delta k_\gamma, \Delta Q_\gamma \) are expressed
in terms of the ratio \( \delta = (m_{Higgs}/M_W)^2 \); for comparison these are also given in the
Table I along with the Higgs contributions in the supersymmetric limit.

The neutralino and chargino sectors have the most complicated structure and will be
discussed in more detail. Their weak and electromagnetic currents are given by,
\[
J^\mu_+ = \sum_{\alpha,i} \bar{\tilde{\zeta}}_\alpha \gamma^\mu (P_R C_{\alpha i} + P_L C_{\alpha i}^L) \tilde{C}_i
\]
\[
J^\mu_\text{em} = \sum_i \bar{\tilde{C}}_i \gamma^\mu \tilde{C}_i
\]
(13)

where the sums are over neutralino, \( \alpha = 1, 2, 3, 4 \), and/or chargino, \( i = 1, 2 \), indices and
\( P_R, L \) are the right and left handed projection operators \( (1 \pm \gamma_5)/2 \).
\( \tilde{Z}_\alpha, \alpha = 1, 2, 3, 4 \) are four component Majorana spinors
\begin{equation}
\mathcal{M}_N = \begin{pmatrix}
M_1 & 0 & g'v_1/\sqrt{2} & -g'v_2/\sqrt{2} \\
0 & M_2 & -g'v_1/\sqrt{2} & g'v_2/\sqrt{2} \\
g'v_1/\sqrt{2} & -g'v_1/\sqrt{2} & 0 & -\mu \\
-g'v_2/\sqrt{2} & g'v_2/\sqrt{2} & -\mu & 0
\end{pmatrix}
\end{equation}
(14)

while the charginos \( \tilde{C}_i, i = 1, 2 \) are Dirac fermions, mixtures of winos \( \tilde{W}_+ \) and Higgsinos
\( \tilde{H}_+ \) and they are eigenstates of the chargino mass matrix,
\begin{equation}
\mathcal{M}_C = \begin{pmatrix}
M_2 & -g v_2 \\
-g v_1 & \mu
\end{pmatrix}
\end{equation}
(15)

\( \sin^2 \theta = (m_\gamma^2 + M_Z^2 \sin^2(2\beta) - m_A^2)/(m_A^2 - m_\gamma^2) \). No dependence of the Higgs contributions
on \( \sin \theta \) appears in the results cited in ref. \cite{21} ; however we agree with the expressions given by the same
authors in ref. \cite{12} for the limiting cases discussed in that paper.
The right and left handed couplings $C^R_{\alpha i}$ appearing in (13) are given by

$$C^R_{\alpha i} = - \frac{1}{\sqrt{2}} O_{3\alpha} U^*_{i2} - O_{2\alpha} U^*_{i1}$$

$$C^L_{\alpha i} = + \frac{1}{\sqrt{2}} O_{4\alpha} V^*_{i2} - O_{2\alpha} V^*_{i1}$$

(16)

The real orthogonal matrix $O$ diagonalizes $M_N$, and the unitary matrices $U, V$ diagonalize $M_C$, i.e $O^T M_N O = diagonal$ and $U M_C V^\dagger = diagonal$.

The neutralino and chargino contributions to the dipole and quadrupole moments are displayed in the Table I. For $\Delta Q_\gamma$, we are in complete agreement with the findings of ref. [11] but for $\Delta k_\gamma$ we disagree in the first term of the equation for $\Delta k_\gamma$ appearing in that table. However our result in the limit of vanishing right handed coupling, $C^R_{\alpha i} = 0$, receives a form which up to group factors is exactly the same as that of a massive fermion family as it should. This consists a check of its correctness. The sign$(m, m_a)$ in the second term of $\Delta k_\gamma$ takes care of the fact that we have not committed ourselves to a particular sign convention for the chargino and neutralino mass eigenvalues $m_i$ and $m_a$. The prefactors $F_{\alpha i}$ and $G_{\alpha i}$ appearing in the expressions for $\Delta k_\gamma$ and $\Delta Q_\gamma$ are defined as

$$F_{\alpha i} = |C^R_{\alpha i}|^2 + |C^L_{\alpha i}|^2 , \quad G_{\alpha i} = (C^L_{\alpha i} C^R_{\alpha i}^* + h.c)$$

(17)

In the supersymmetric limit two of the neutralinos states, namely the photino $\tilde{\gamma} = (\cos \theta_W) \tilde{B} + (\sin \theta_W) \tilde{W}_3$ and the axino $\tilde{\alpha} = \frac{1}{\sqrt{2}} (\tilde{\chi}_1 + \tilde{\chi}_2)$, have vanishing mass [1]. The first combines with the photon and the second with the scalars $h_0, A$ to form supersymmetric multiplets of zero mass. The other two linear combinations $\tilde{\alpha}^T = \frac{1}{\sqrt{2}} (-\tilde{\chi}_1 + \tilde{\chi}_2)$, orthogonal to $\tilde{\alpha}$, and the “Zino” $\tilde{Z} = -(\sin \theta_W) \tilde{B} + (\cos \theta_W) \tilde{W}_3$, which is orthogonal to the photino state, have both mass $M_Z$. These two form a new “Zino”, $\tilde{\zeta}$, which is a Dirac fermion describing thus four physical states of mass $M_Z$. This combines with the heavy neutral $H_0$ and the $Z$-boson to forming a multiplet having mass $M_Z$. Also in that limit we have two charginos, Dirac fermions of mass $M_W$, which along with the charged Higgses $H^\pm$ and the $W$-bosons belong to supermultiplets of mass $M_W$. This is how supersymmetry manifests itself in the SUSY limit discussed previously. In that limit the contributions of fermions and bosons of each multiplet to $\Delta Q_\gamma$ cancel each other as can be seen from Table I. Such a cancellation however does not hold for the dipole moment as is well known [10].

We pass now to discuss our physical results. As we have already mentioned in the beginning $\tan \beta(M_Z)$ and $m_t(M_Z)$ are considered as free parameters; given these the top Yukawa coupling at $M_Z$ is known and so is its value at the Unification scale $M_{GUT} \simeq 10^{16}$ GeV. At all scales we have imposed the perturbative requirement $h_t^2/(4\pi) \leq \mathcal{O}(1)$. Therefore given $m_t(M_Z)$, $\tan \beta$ is forced to a minimum value due to the presence of an infrared fixed point of the Yukawa coupling [22]. In our analysis we have limited ourselves to vanishing values for the bottom and tau Yukawa couplings. This excludes large values of $\tan \beta (\geq 15)$; accepting nonvanishing bottom and Yukawa couplings produces only minor changes to the one loop dipole and quadrupole moments. Then starting with

$^4\tilde{\chi}_{1,2}$ are phase rotated Higgsino fields $\tilde{\chi}_{1,2} = i \tilde{H}_{1,2}$. 

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\(A_0, m_0, M_{1/2}\) at \(M_{\text{GUT}}\) we run the RGE’s of all parameters involved except those of \(B\) and \(\mu\) whose values \(B(M_Z), \mu(M_Z)\) are determined through the minimization conditions \((4), (5)\). Their values at any other scale are found by running their corresponding RGE’s using their near decoupling from the rest of the renormalization group equations. For their determination we properly take into account the radiative corrections to the effective potential as we have already discussed.

For a given pair of \(\tan \beta(M_Z), m_t(M_Z)\) the parameter space \(A_0, m_0, M_{1/2}\) can be divided into three main regions,

i) \(A_0, m_0, M_{1/2}\) comparable: 
This includes also the dilaton dominated SUSY breaking mechanism with vanishing cosmological constant \([24]\).

ii) \(M_{1/2} \gg A_0, m_0\) : 
In this case the gaugino mass is the dominant source of SUSY breaking. This includes also the no-scale models (see second ref. in \([13]\)) which in the physically interesting cases favor values \(A_0 = m_0 = 0\).

iii) \(A_0, m_0 \gg M_{1/2}\) : 
This includes the light gluino case.

We have explored the entire parameter space for values of the top mass ranging from 130 to 180 GeV and in Table II we present sample results covering the previously discussed cases. In every case appearing in that table the dominant SUSY breaking terms have been taken equal to 300 GeV. One observes that the MSSM values for the quantities of interest, and especially for \(\Delta k_\gamma\), in some particular cases can be largely different from those of the SM but of the same order of magnitude indicating that possible supersymmetric structure is hard to be detected at LEP2. This behaviour characterizes the most of the parameter space provided that the typical SUSY breaking scale is in the range \(\mathcal{O}(100\text{GeV} - 17\text{eV})\). The distinction between SM and MSSM predictions can be therefore made detectable once the accuracy reaches the level of \(\mathcal{O}(10^{-2} - 3)\).

In general the contributions of each sector separately to dipole and quadrupole moments, in units of \(g^2/16\pi^2\), are as follows:

The matter fermions contributions to \(\Delta k_\gamma\) are of order \(\mathcal{O}(1)\) and negative while those to \(\Delta Q_\gamma\) are of the same order of magnitude but positive.

The gauge bosons contributions to \(\Delta k_\gamma\) is of the same order as that of the fermions but opposite in sign. However those to the quadrupole moment \(\Delta Q_\gamma\) are almost an order of magnitude smaller.

The supersymmetric Higgses yield \(\Delta k_\gamma \sim \mathcal{O}(1)\), \(\Delta Q_\gamma \sim \mathcal{O}(10^{-2})\) which little deviate from the SM predictions provided the standard model Higgs boson has a mass in the range \(\simeq 50 - 100\text{GeV}\). Actually in the MSSM with radiative electroweak breaking one of the Higgses, namely \(h_0\), turns out to have a mass in the aforementioned range; this is predominantly the standard model Higgs. The rest have large masses, of the order of the supersymmetry breaking scale, giving therefore negligible contributions.

The squark and slepton sector is the one yielding the smallest contributions. These are of order \(\simeq \mathcal{O}(10^{-2})\) or less having therefore negligible effect on both \(\Delta k_\gamma\) and \(\Delta Q_\gamma\).

It remains to discuss the effects of the charginos and neutralinos to the dipole and quadrupole moments. In general this sector gives \(\simeq \mathcal{O}(10^{-2})\) to both \(\Delta k_\gamma\) and \(\Delta Q_\gamma\). However in some cases and for positive values of the parameter \(\mu\), \(\Delta k_\gamma\) can be substan-
tially larger, \( \simeq \mathcal{O}(1) \). This occurs only when \( M_{1/2} \ll m_0, A_0 \), case (iii), and the solution for \( \mu \) as given from the Eq. (4) and (5) happens to allow for light chargino and neutralino states with masses \( m_{\tilde{\chi}}, m_{\tilde{\nu}} \) such that \( m_{\tilde{\chi}} + m_{\tilde{\nu}} \approx M_W \). In such cases the values of \( \Delta k_\gamma, \Delta Q_\gamma \) are enhanced and a structure is observed \( [4] \) (see Fig 3). However even for such relatively large contributions of this sector we can not have values approaching the sensitivity limits of LEP2. Only in a very limited region of the parameter space and when accidentally \( m_{\tilde{\chi}} + m_{\tilde{\nu}} \) turns out to be almost equal to W - boson mass, the chargino and neutralino contributions can be very large saturating the sensitivity limits of LEP2. We disregard such large contributions since they are probably outside the validity of the perturbation expansion. Even if it were not for that reason these cases are unnatural since they occupy a very small portion of the available parameter space which is further reduced if the lower experimental bounds imposed on the chargino mass are strictly observed.

The results presented in Table II are representative of the cases discussed previously. To simplify the discussion the dominant SUSY breaking parameters have been assumed equal. Notice the large chargino/neutralino contribution to \( \Delta k_\gamma \) and \( \Delta Q_\gamma \) in the case where \( M_{1/2} = 80 \text{GeV}, m_0, A_0 = 300 \text{GeV} \). Smaller values for \( M_{1/2} \) can lead to even larger contributions resulting to moments \( \Delta k_\gamma, \Delta Q_\gamma \) which may approach the sensitivity limits of LEP2. However the lower experimental bound put on the chargino mass (\( \geq 45 \text{GeV} \)) constraints the situation a great deal not allowing for arbitrarily low \( M_{1/2} \) values. In the same table the supersymmetric values \( \Delta k_\gamma^{\text{SUSY}}, \Delta Q_\gamma^{\text{SUSY}} \) are also shown. For the later we know that \( \Delta Q_\gamma^{\text{SUSY}} = 0 \). Its vanishing merely serves as a check of the correctness of the calculations. \( \Delta k_\gamma^{\text{SUSY}} \) is nonvanishing however receiving the value \( 1.273(g^2/16\pi^2) \) for \( m_t = 160 \text{GeV} \). For a comparison of our results we also display the standard model predictions for Higgs masses \( m_{Higgs} = 50, 100 \) and \( 300 \text{GeV} \).

In order to examine the behaviour of the dipole and quadrupole moments with varying the supersymmetry breaking scale we plot in Figure 3 \( \Delta k_\gamma, \Delta Q_\gamma \) as functions of \( m_0, A_0 \) for the physically interesting case \( M_{1/2} \ll m_0, A_0 \). To compare with the previously discussed cases we have taken \( M_{1/2} = 80 \text{GeV} \) and \( m_0 = A_0 \) ranging from \( 200 \text{GeV} \) to \( 1 \text{TeV} \) for both \( \mu > 0 \) and \( \mu < 0 \). Also in order to see how sensitive are our results to the top quark mass we plot \( \Delta k_\gamma, \Delta Q_\gamma \) for \( m_t = 140 \text{GeV} \) and \( m_t = 160 \text{GeV} \). The two cases yield almost identical results showing in a clear manner that are insensitive to the choice of the top quark mass. They differ appreciably when we are close to values of \( m_0, A_0 \) for which a dip in \( \Delta k_\gamma, \Delta Q_\gamma \) is developed, occuring when \( m_t = 160 \text{GeV} \) and \( \mu > 0 \), due to the large neutralino and chargino contributions discussed previously. For comparison in the same figure we have drawn the standard model predictions for a Higgs mass equal to \( 100 \text{GeV} \) and \( m_t = 160 \text{GeV} \). The dependence of the quantities of interest on the value of the parameter \( \tan \beta \) is rather smooth. In Figure 4 we display \( \Delta k_\gamma, \Delta Q_\gamma \) as functions of \( \tan \beta \) for values ranging from \( 2 \) to \( 10 \). No strong dependence on \( \tan \beta \) is observed either although for \( \mu > 0 \) the dipole moment is appreciably larger for small values of \( \tan \beta \). The cases shown correspond to \( m_0 = A_0 = 300 \text{GeV} \), \( M_{1/2} = 80 \text{GeV} \) and \( m_t = 160 \text{GeV} \). However these are representative of the more general situation.

\footnote{In those cases the integrations over the Feynman parameter are of the form \( \int_0^1 f(t)dt/[(t-\alpha)^2 + \epsilon^2] \) with \( 0 < \alpha < 1 \) and \( \epsilon \) small.}
Concerning our numerical results a last comment is in order. Scanning the parameter space we have intentionally ignored cases giving large contributions due to the presence of Landau singularities of our one loop expressions. These are encountered when the masses involved take values for which the integrands \( f(t) \) develop double poles within the integration region \([0,1]\) of the Feynman parameter \( t \). We are aware of the fact that near a Landau singularity the dipole and quadrupole moments can become sizeable but these cases should be considered with a “grain of salt” not corresponding to an actual physical situation. Our ignorance how to treat these singularities forces us to keep a rather conservative view leaving aside values of the physical masses for which we are in the vicinity of a Landau singularity. These result to large and untrustworthy values for the dipole and quadrupole moments which cannot be handled perturbatively.

The supersymmetric dipole and quadrupole moments though different, in general, from the corresponding standard model quantities are of the same order of magnitude in almost the entire parameter space \( m_0, A_0 \) and \( M_{1/2} \). The MSSM values of these quantities are not sensitive to either top mass or \( \tan \beta \) as long as the former lies in the physical region \( 140 \text{GeV} < m_t < 180 \text{GeV} \). For \( M_{1/2} \) small, as compared to \( m_0, A_0 \), the chargino and neutralino sector may give substantial contributions resulting to \( \Delta k_\gamma, \Delta Q_\gamma \) approaching the sensitivity limits of LEP2. However these cases should be considered with some caution for they may not be allowed within the perturbative regime; in addition these cases occupy a tiny region in the parameter space, being therefore unnatural, and hence to be disregarded on these grounds. Our conclusion is that deviations from the standard model predictions for the dipole and quadrupole moments due to supersymmetry are hard to be observed at LEP2. If such deviations are observed most likely these are not due to an underlying supersymmetric structure.

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Table Captions

**Table I**: Radiative corrections to the dipole $\Delta k_\gamma$ and quadrupole moments $\Delta Q_\gamma$ in units of $g^2/(16\pi^2)$. The symbols are explained in the main text.

**Table II**: The dipole and quadrupole moments, in units of $g^2/(16\pi^2)$, for the input values $m_t(M_Z)$, $\tan\beta(M_Z)$, $A_0$, $m_0$ and $M_{1/2}$ shown in the table. For comparison we display the standard model predictions for Higgs masses 50, 100 and 300 GeV; the corresponding moments in the supersymmetric limit are also shown.

Figure Captions

**Figure 1**: The $W_W+\gamma$ vertex. Lorentz indices and momentum assignments are as shown in the figure.

**Figure 2**: Triangle graph (a) and its crossed (b) contributing to the dipole and quadrupole moments. $Q, T_+, T_-$ denote electric charge and isospin raising and lowering operators respectively.

**Figure 3**(a) Dipole (solid line) and Quadrupole (dashed line) moments, in units of $g^2/(16\pi^2)$, as functions of $m_0$, $A_0$ for $M_{1/2} = 80 GeV$. The cases shown are for $m_t = 140 GeV$ (a) and $m_t = 160 GeV$ (b). For convenience we have taken $m_0 = A_0$. The sign of the parameter $\mu$ is positive. The horizontal lines are the standard model predictions for $m_{Higgs} = 100 GeV$ and $m_t = 160 GeV$.

(b) Same as in (a) for negative sign of the parameter $\mu$.

**Figure 4**: Dipole (solid line) and Quadrupole (dashed line) moments, in units of $g^2/(16\pi^2)$, as functions of $\tan\beta$ for $\mu > 0$ (a) and $\mu < 0$ (b). In both cases $A_0 = m_0 = 300 GeV$, $M_{1/2} = 80 GeV$ and $m_t = 160 GeV$. 

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TABLE I

Corrections to the Dipole ($\Delta k_\gamma$) and Quadrupole ($\Delta Q_\gamma$) moments
(in units of $g^2/(16\pi^2)$)

Gauge bosons

| $\gamma$ | $\Delta k_\gamma = \frac{20}{3}\sin^2\theta_W$ | $\Delta Q_\gamma = \frac{4}{9}\sin^2\theta_W$ |
|---|---|---|
| $Z$ | $\Delta k_\gamma = \frac{20}{3R} - \frac{5}{6} + \frac{1}{2} \int_0^1 dt \frac{t^4 + 10t^3 - 36t^2 + 32t - 16}{t^2 + R(1-t)}$ | $\Delta Q_\gamma = (\frac{8}{3R} + \frac{1}{3}) \int_0^1 dt \frac{t^3(1-t)}{t^2 + R(1-t)}$ |

Leptons, Quarks: \( \begin{pmatrix} f \\ f' \end{pmatrix} \) \( SU(2) \) doublets

| $\Delta k_\gamma = \frac{C_g Q_{f'}}{2} \int_0^1 dt \frac{t^4 + (r_{f'} - r_f - 1)t^3 + (2r_{f'} - r_f)t^2}{t^2 + (r_{f'} - r_f - 1)t + r_f} - [f \leftrightarrow f']$ | \( (r_{f,f'} \equiv (m_{f,f'}/M_W)^2) \)
|---|---|
| $\Delta Q_\gamma = \frac{2C_g Q_{f'}}{3} \int_0^1 dt \frac{t^3(1-t)}{t^2 + (r_{f'} - r_f - 1)t + r_f} - [f \leftrightarrow f']$ | \( (r_{f,f'} \equiv (m_{f,f'}/M_W)^2) \)

Sleptons, Squarks: \( \begin{pmatrix} \tilde{f} \\ \tilde{f}' \end{pmatrix} \) \( SU(2) \) doublets

| $\Delta k_\gamma = -C_g Q_{f'} \sum_{i,j=1}^2 (K_{\tilde{f},\tilde{f}'}^j K_{\tilde{f},\tilde{f}'}^i) \int_0^1 dt \frac{t^2(t-1)(2t-1 + R_{\tilde{f},\tilde{f}} - R_{\tilde{f},\tilde{f}})}{t^2 + (R_{\tilde{f},\tilde{f}} - R_{\tilde{f},\tilde{f}} - 1)t + R_{\tilde{f},\tilde{f}}} - [f \leftrightarrow f']$ | \( (\tilde{m}_{f,f'}/M_W)^2 \); \( \tilde{m}_{f,\tilde{f},\tilde{f}} \) are sfermion masses. |
|---|---|
| $\Delta Q_\gamma = -\frac{2C_g Q_{f'}}{3} \sum_{i,j=1}^2 (K_{\tilde{f},\tilde{f}'}^j K_{\tilde{f},\tilde{f}'}^i) \int_0^1 dt \frac{t^3(1-t)}{t^2 + (R_{\tilde{f},\tilde{f}} - R_{\tilde{f},\tilde{f}} - 1)t + R_{\tilde{f},\tilde{f}}} - [f \leftrightarrow f']$ | \( (\tilde{m}_{f,f'}/M_W)^2 \); \( \tilde{m}_{f,\tilde{f},\tilde{f}} \) are sfermion masses. |

In the SUSY limit \( K_{\tilde{f},\tilde{f}'} \) become unit matrices and \( R_{\tilde{f},\tilde{f}} = r_f \), \( R_{\tilde{f},\tilde{f}} = r_{f'} \)

(continued)
TABLE I  (continued)

Higgses : (H\(\pm\), H\(0\), h\(0\), A)

a) broken SUSY

\(A\) : \(\Delta k_\gamma = D_2(R_A, R_+)\), \(\Delta Q_\gamma = Q(R_A, R_+\))

\(h_0\) : \(\Delta k_\gamma = \sin^2 \theta D_1(R_h) + \cos^2 \theta D_2(R_h, R_+)\)

\(\Delta Q_\gamma = \sin^2 \theta Q(R_h, 1) + \cos^2 \theta Q(R_h, R_+)\)

\(H_0\) : As in \(h_0\) with \(R_h \rightarrow R_H\) and \(\sin^2 \theta \rightleftharpoons \cos^2 \theta\)

\(R_a \equiv (m_a/M_W)^2\) \(a = h_0, H_0, A, H_\pm\)

b) exact SUSY

\(A, h_0\) : \(\Delta k_\gamma = -\frac{1}{6} + \frac{11}{6}, \quad \Delta Q_\gamma = \frac{1}{18}, \frac{1}{18}\)

\(H_0\) : \(\Delta k_\gamma = \frac{1}{6} + \frac{1}{2} \int_0^1 dt \frac{t^4 - 2t^3}{t^2 + R(1-t)}\), \(\Delta Q_\gamma = \frac{1}{3} \int_0^1 dt \frac{t^3(1-t)}{t^2 + R(1-t)}\)

\(R = (M_Z/M_W)^2\)

c) Standard Model Higgs contribution

\(\Delta k_\gamma = D_1(\delta)\), \(\Delta Q_\gamma = Q(\delta, 1)\) \(\left(\delta = (m_{Higgs}/M_W)^2\right)\)

\[D_1(r) \equiv \frac{1}{2} \int_0^1 dt \frac{2t^4 + (-2 - r)t^3 + (4 + r)t^2}{t^2 + r(1-t)}\]

\[D_2(r, R) \equiv \frac{1}{2} \int_0^1 dt \frac{2t^4 + (-3 - r + R)t^3 + (1 + r - R)t^2}{t^2 + (-1 + r - R)t + r}\]

\[Q(r, R) \equiv \frac{1}{3} \int_0^1 dt \frac{t^3(1-t)}{t^2 + (-1 - r + R)t + r}\]
**TABLE I**  (continued)

Neutralinos ($\tilde{Z}_\alpha$, $\alpha = 1...4$)  Charginos ($\tilde{C}_i$, $i = 1, 2$)

\[ \Delta k_\gamma = -\sum_{i,\alpha} F_{\alpha i} \int_0^1 dt \frac{t^4 + (R_\alpha - R_i - 1)t^3 + (2R_i - R_\alpha)t^2}{t^2 + (R_i - R_\alpha - 1)t + R_\alpha} \]
\[ + \sum_{i,\alpha} \text{sign}(m_i m_\alpha) G_{\alpha i} \sqrt{R_\alpha R_i} \int_0^1 dt \frac{4t^2 - 2t}{t^2 + (R_i - R_\alpha - 1)t + R_\alpha} \]
\[ \Delta Q_\gamma = -\frac{4}{3} \sum_{i,\alpha} F_{\alpha i} \int_0^1 dt \frac{t^3(1-t)}{t^2 + (R_i - R_\alpha - 1)t + R_\alpha}, \]
\[ (R_{\alpha,i} \equiv (m_{\alpha,i}/M_W)^2) \]

**b) exact SUSY**

\[ \bar{\gamma} : \quad \Delta k_\gamma = -\frac{8}{3} \sin^2 \theta_W, \quad \Delta Q_\gamma = -\frac{4}{9} \sin^2 \theta_W \]
\[ \bar{\alpha} : \quad \Delta k_\gamma = -\frac{2}{3}, \quad \Delta Q_\gamma = -\frac{1}{9} \]
\[ \bar{\zeta} : \quad \Delta k_\gamma = \frac{1}{6} - \frac{8}{3R} - \int_0^1 \frac{t^4 + 3t^3 - 15t^2 + 12t - 4}{t^2 + R(1-t)} \]
\[ \Delta Q_\gamma = -\left(\frac{2}{3} + \frac{8}{3R}\right) \int_0^1 \frac{t^3(1-t)}{t^2 + R(1-t)} \]
\[ (R = (M_Z/M_W)^2) \]
TABLE II

$m_t = 160, \tan \beta = 2$

$A_0, m_0, M_{1/2} : 300, 300, 300 \ [0, 0, 300] \ (300, 300, 80)$

| $\Delta k_\gamma$ | $\Delta Q_\gamma$ |
|-------------------|-------------------|
| $\mu > 0$         | $\mu < 0$         |
| $q, l$            | -1.973            |
| $W, \gamma, Z$    | 1.179             |
| $h_0, H_{\pm, 0}, A$ | .945 [.945] (.946) |
| $\bar{q}, \bar{l}$ | -.004 [-.024] [.009] |
| $Z, C$            | .013 [.015] (.697) |
| Total             | .159 [.143] (.859) |
| SUSY limit        | $\Delta k_\gamma^{SUSY} = 1.237$ |
| Standard Model    | $\Delta k_\gamma^{SM} = .188, -.106, -.449$ |
|                   | $\Delta Q_\gamma^{SM} = 2.186, 2.174, 2.161$ |
Figure 1

Figure 2
Figure 3

Figure 4
This figure "fig1-1.png" is available in "png" format from:

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