Linear and nonlinear development of controlled disturbances in the supersonic boundary layer on a swept wing at Mach 2.5

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Abstract. Experimental data on the linear and nonlinear wave train development in 3D supersonic boundary layer over a 45° swept-wing at Mach number 2.5 are presented. Travelling artificial disturbances were introduced in the boundary layer by periodical glow discharge at frequencies 10 and 20 kHz. The spatial-temporal and spectral-wave characteristics of the wave train of unstable disturbances in the linear region are obtained. It is shown that the additional peaks in β'-spectra arise for both subharmonic and fundamental frequencies. The experiments indicate the presence of subharmonic resonance mechanism in 3D boundary layer at Mach number 2.5.

1. Introduction
The problem of the transition to turbulence in swept-wing boundary layers has been the focus of considerable attention by researchers for more than 30 years [1, 2]. High practical relevance is directly related to airplane wings. However, a variety of factors involved in the process of laminar–turbulent transition pose many difficulties in its study. The fundamental property of a swept-wing boundary layer is the presence of crossflow, which leads to the appearance of new instabilities in comparison with a two-dimensional boundary layer. The modern theoretical concept of the transition mechanisms in a three-dimensional (3D) boundary layer gives four basic types of flow instabilities: (1) attachment line boundary layer instability on the swept leading edge; (2) stationary crossflow instability; (3) crossflow instability to the travelling disturbances; and (4) possible instability of Tollmien–Schlichting waves. Their relative role in the laminar-turbulent transition strongly depends on environmental conditions, such as level of free stream disturbances, surface quality and Mach number.

Most of the theoretical and experimental results on stability and transition of the 3D boundary layers are obtained for subsonic flow. In such flows there are two main scenarios for transition [1]. One such scenario is connected with the domination of stationary disturbances (crossflow vortices), and can occur if the free-stream turbulence level is small, i.e., the stationary disturbances are larger, relative to the initial amplitude of the non-stationary disturbances [3]. In this case, stationary vortices, created as a result of 3D roughness near the leading edge, are linear developed and eventually saturate to become subjected to high-frequency secondary instability with further laminar-flow breakdown [4, 5]. The other scenario for transition is the nonlinear interaction between travelling modes and stationary vortices, which can be observed in natural conditions when the level of unsteady freestream disturbance is high. Deyhle and Bippes [6] pointed out that, for turbulence levels Tu > 0.2% and smooth surfaces, travelling
modes begin to dominate. Observations of travelling instability modes were also made under controlled disturbance conditions when wave packets were generated by a localized pulse [7]. We should note that this is in fact the only paper by Wasserman and Kloker [8] that scrutinizes the crossflow breakdown by travelling modes in detail by using direct numerical simulation (DNS).

Supersonic boundary layer is far more complex than incompressible boundary layer. Major differences between incompressible and compressible flows can also be expected both for the linear and nonlinear regimes. Several studies have been devoted to the evolution of travelling disturbances in 3D supersonic boundary layer and it was suggested that the transition is more likely to be dominated by them [9, 10]. To excite controlled travelling disturbances in a supersonic boundary layer, a glow discharge technique was developed at ITAM SB RAS [11]. Such technique allowed to scrutinize the linear development of a wave train on a swept wing with 45°-sweep angle at Mach number M = 2.0 and obtain the main stability characteristics for disturbances with frequencies 10 and 20 kHz [12, 13]. Moreover, at ITAM SB RAS the technique of pulsed excitation of a wave packet with a broad disturbance spectrum is developed [14]. We can expect that Yatskikh et. al. will also provide the detailed wave analysis for a wide range of frequencies in 3D supersonic boundary layer.

Later stages of the transition are characterized by nonlinearity of the disturbance flow field. The early nonlinear transition regime for the 2D boundary layer at moderate supersonic Mach numbers has been studied extensively by Kosinov and co-workers [15]. They have focused on an ‘asymmetric subharmonic resonance’ breakdown mechanism. The viability of this kind of resonance in 2D compressible boundary layer was confirmed theoretically [16]. Principal possibility of occurrence of resonant triplets in swept-wing boundary layers is known over a long period of time [17]. However, to date, very few experimental investigations of the subharmonic resonance in 3D supersonic boundary layer have been carried out. The goal of the present study is to perform the controlled disturbance experiments in the swept-wing supersonic boundary layer at M = 2.5 and to obtain the wave characteristics of the most unstable disturbances in the region of their linear and weakly nonlinear development.

2. Experimental setup and data processing

The experiments were conducted in the T-325 low noise supersonic wind tunnel of ITAM SB RAS at M = 2.5 and unit Reynolds number \( \text{Re}_1 = 5 \times 10^6 \text{ m}^{-1} \). A swept-wing model was used in the experiments. The model had a sweep angle of 45° and a slightly blunted leading edge of radius 0.2 mm. The swept wing had a 2.6% lens-shape airfoil at the upper side and flat surface at the bottom side with maximum thickness of 12 mm. The curvature radius of the upper surface of the model was approximately 4000 mm. A sketch of the experimental model and the coordinate systems are shown in figure 1; dimensions are given in mm. The average roughness of the upper surface of the model was no more than 1 micron. The model was fixed in the central plane of the test section at approximately a zero angle of attack with an alignment error of 0.06°. A source of localized artificial disturbances was built in the model, similar to that described by Kosinov et al. [11]. Controlled pulsations had been generated using high-frequency glow discharge in a chamber and were injected into the boundary layer through an aperture 0.4 mm in diameter in the upper surface of the model. The source aperture was located at \( x_0 = 56.6 \pm 0.3 \text{ mm} \) from the leading edge on the center line of the model symmetry. A sine-wave generator and amplifier with transformer output were used to produce high voltage at a frequency of 10 kHz. The actuator worked at a frequency of 20 kHz, thereby enabling artificial perturbations to be excited at frequencies of 10 and 20 kHz in the boundary layer.

The wind tunnel had an automated measuring system that allowed for the determination of the flow parameters in “real time”. The flow disturbances were measured by a constant-temperature anemometer (CTA). A tungsten hotwire of 10 \( \mu \text{m} \) in diameter and 1.6 mm in length was used. The axis of the hotwire was parallel to the \( z \)-axis. The probe was moved in the \( x \), \( y \), \( z \)-directions with the help of traversing gears with which the wind tunnel was equipped. The measurement accuracy was about 0.1 mm in the \( x \)- and \( z \)-directions and about 0.01 mm in the \( y \)-direction. Hotwire measurements were performed at a fluctuation maximum in the boundary layer and at \( y = \text{const} \) when the probe moved parallel to the leading
edge. The fluctuation maximum was determined at \( z' = 0 \). Because the overheat ratio was taken to be about 0.8, the measured disturbances of up to 95% consisted of mass flow fluctuations.

The pulsation measurements were synchronized with a disturbance source and sine wave generator of a special device in a computer-automated measurement and control crate. The AC and DC signals from the CTA were written to the PC using a 12-bit analog-to-digital converter with sampling rate 750 kHz and by DC voltmeter, respectively. Four time traces of 65536 points in length were measured and recorded in each space position of the hotwire. This was sufficient to detect even very small amplitudes of the controlled pulsations after averaging. To analyze the evolution of natural fluctuations, the full time traces were considered.

\[ m(x', z', t) = \frac{e(x', z', t)}{Q \cdot E(x', z')} \]

Figure 1. A sketch of the swept wing and coordinate systems used; (a) – side view; (b) – plan view; (1) marks the disturbance source.

The obtained fluctuation time traces normalized by the mean voltage CTA output at each spatial measurement point were transformed to the mass flow fluctuations according to the relation \( m(x', z', t) = e(x', z', t)/(Q \cdot E(x', z')) \). Here \( e(x', z', t) \) is the fluctuation signal, \( E(x', z') \) the mean voltage from CTA output, and \( Q \) the probe sensitivity to the mass flow fluctuations. Note that the probe measures the \( x \)-component of the mass flow fluctuation. The frequency-wave disturbance spectra at a fixed \( x' \)-coordinate were determined using the discrete Fourier transform in the form:

\[
A_{jj'} = \sqrt{\text{Re}^2(m_{jj'}) + \text{Im}^2(m_{jj'})},
\]

\[
\Phi_{jj'} = \arctan(-\text{Im}(m_{jj'})/\text{Re}(m_{jj'})),
\]

The streamwise wavenumber \( \alpha \) is defined as

\[
\alpha = \frac{\Delta \Phi}{\Delta x},
\]

where \( \Delta x \) is the distance between the measured sections in the \( x \)-direction.

3. Results

3.1. Natural development of disturbances

Before considering controlled disturbances, firstly we need to determine the regions of linear and nonlinear development of natural disturbances. Downstream distributions of the mean voltage \( E \) and
RMS mass flow pulsations $<m>$ are presented in figure 2a. Note that the longitudinal coordinate $x$ is measured from the disturbance source. The measurements were performed at constant mass flow that corresponds to the maximum in the disturbance amplitude profile. The region where a monotonic increase of $<m>$ is started is usually associated with the finishing of the linear stage of the laminar-turbulent transition and the onset of the nonlinear evolution of disturbances. From this data, it can be seen that the transition onset occurs at $x \approx 50$ mm (i.e., at a distance of around 106 mm from the leading edge of the model or up to Reynolds number $Re_x = Re_1 \times (x + 56.6 \text{ mm}) \approx 0.53 \times 10^6$). Figure 2b shows the downstream evolution of the amplitude spectrum of the natural mass-flow fluctuations. The first three lines belong to the linear pulsation growth. Here the pulsation amplitude amplification occurs within the frequency range $f = 4$–$30$ kHz. In the region of nonlinear disturbance growth ($x > 50$ mm), both the low- and high-frequency disturbances with frequencies up to $f = 200$ kHz have rapid growth. At the last measured point ($x = 129$ mm), the spectrum is close to turbulent flow.

Hence, from this data, it can be concluded that the source of the controlled disturbances is located in the linear regime of the natural fluctuation development and, additionally, that the fundamental (20 kHz) and subharmonic (10 kHz) frequencies fall within the frequency range for most unstable natural disturbances.

![Figure 2.](image)

**Figure 2.** (a) Evolution of the mean voltage $E$ and RMS mass flow fluctuations $<m>$ downstream. (b) Amplitude spectrum of the natural disturbances.

### 3.2. Linear development of disturbances

In this section the controlled disturbance development was measured in three different sections at a fixed coordinate $x'$ by means of displacing the anemometer probe along the $z'$ coordinate, that is parallel to the leading edge of the model at $x = 40$, 50 and 60 mm. The measurements were made in the layer of maximum fluctuations, that corresponds to the condition $y/\delta \approx 0.6$, where $\delta$ is the boundary layer thickness.

The amplitude distributions of perturbations for both frequencies in the spanwise direction are shown in figure 3. One can see that in some places inside the wave packet the amplitude grows downstream, whereas the main amplitude maximum is slowly displaced in the positive direction of the $z'$-coordinate. Recall that the asymmetry of the amplitude distributions along the spanwise $z'$-coordinate, relative to the position of the discharge ($z' = 0$ mm), is associated with the presence of crossflow on the swept wing. The wave train is spread along the leading edge of the model in the direction opposite to the crossflow that is in agreement with previously obtained results for Mach number 2.0 [12, 13]. Note, that the amplitude for disturbances with frequency 10 kHz is even higher than that for disturbances with frequency 20 kHz.

Figure 4 shows the amplitude $\beta'$-spectra of the controlled disturbances at frequencies 10 and 20 kHz. As distinct from the flat plate case, the swept-wing $\beta'$-spectra are asymmetric. The amplitude growth is observed in the wide range of the spanwise wavenumbers, but the amplitude maximum is located at
$\beta' = 0.65 \text{ rad/mm for } 10\text{ kHz and } \beta' = 0.9 \text{ rad/mm for } 20\text{ kHz. These values correspond to the linear peaks in downstream wave train development. We should note that along with the linear peaks there is a number of additional peaks. Their existence is not explained by the linear theory and be discussed in the section 3.3.}$

**Figure 3.** Amplitude distributions of disturbances in the spanwise direction. (a) $f = 10\text{ kHz}$ and (b) $f = 20\text{ kHz}$.

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3.3. **Nonlinear development of disturbances**

Before trying to identify the nonlinear mechanisms in the experiment, the resonance conditions must be given. Generally, this condition for a three-wave triad is a phase synchronization of all three instability waves [18, 19]:

$$f_1 = f_2 + f_3, \quad \beta'_1 = \beta'_2 + \beta'_3, \quad \alpha'_{\beta_1} = \alpha'_{\beta_2} + \alpha'_{\beta_3}. \tag{5}$$

For the subharmonic resonance mechanism the conditions (5) go with $f_1 = 20\text{ kHz}$ and $f_2 = f_3 = 10\text{ kHz}$. Figure 5 shows the amplitude $\beta'$-spectra of the controlled disturbances at frequencies 10 and 20 kHz for the sections $x = 60, 70$ and $80\text{ mm.}$ As previously obtained in 3.2, the greatest amplitude growth is observed at $\beta' = 0.75 \text{ rad/mm for } 10\text{ kHz and } \beta' = 0.89 \text{ rad/mm for } 20\text{ kHz that corresponds to the linear development of disturbances. In addition to the main linear peaks we can see that a second maximum forms near } \beta' \approx 0.13 \text{ rad/mm for } 10\text{ kHz. Its appearance, as previously mentioned, can not be explained by the linear theory, since it is a result of nonlinear wave interactions.}$

To explain the appearance of additional peaks in amplitude $\beta'$-spectra at 10 kHz the estimation of dispersion relation for wave numbers $\alpha(\beta)$ (see formula (4)) and the verification of subharmonic...
resonance condition (5) were made. Figure 6 shows the streamwise wavenumber $\alpha_r$ as a function of the spanwise wavenumber $\beta$ for both frequencies 10 and 20 kHz. Resonance conditions for wave numbers were chosen in accordance to maxima in amplitude $\beta'$-spectra for linear waves. In our case wave triplets can be find at $\beta' = 0.14; 0.75 \text{ rad/mm}$ (or $\beta = 0.03; 0.89 \text{ rad/mm}$) at frequency 10 kHz and $\beta' = 0.89 \text{ rad/mm}$ (or $\beta = 0.9 \text{ rad/mm}$) at 20 kHz. Graphically the subharmonic resonance is presented in figure 6.

![Figure 5. Amplitude $\beta'$-spectra of disturbances. (a) $f = 10$ kHz and (b) $f = 20$ kHz.](image)

![Figure 6. Dispersion relation and wave triplet.](image)

Regarding other additional peaks in $\beta'$-spectra for both frequencies (at $\beta' \approx -1 \text{ rad/mm}$ for 10 kHz and $\beta' \approx -0.5 \text{ rad/mm}$ for 20 kHz), it can be caused by another nonlinear mechanism, for example the oblique breakdown mechanism [18] where the resonance triplet contains the steady wave. As due to a clean and polished surface of the model we can obtain the experimental data regarding only the travelling modes. The confirmation the existence of oblique breakdown mechanism in 3D supersonic boundary layer needs to introduce the controlled modulation of the base flow that is beyond this study.

4. Conclusion

An experimental study of nonlinear interactions of controlled disturbances in supersonic boundary layer on smooth swept wing at Mach 2.5 was carried out. The region of linear and nonlinear stage of natural disturbance evolution was defined. The amplitude distributions of the controlled disturbances are shown to be spatially asymmetric which is attributable to the presence of a crossflow in the three-dimensional boundary layer. It is found that for the linear development of disturbances the amplitude maxima are located at $\beta' = 0.65 \text{ rad/mm}$ for 10 kHz and $\beta' = 0.9 \text{ rad/mm}$ for 20 kHz. Downstream
nonlinearity of the controlled disturbance evolution was detected and appearance of additional peaks in $\beta'$-spectra at subharmonic frequency can be explained by subharmonic resonance. It was experimentally demonstrated nonlinear amplification of stable in the linear sense disturbances in a wide range of $\beta'$ wave numbers in the 3D supersonic boundary layer.

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