Global nuclear structure effects of tensor interaction

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A direct fit of the isoscalar spin-orbit (SO) and both isoscalar and isovector tensor coupling constants to the $f_{5/2} - f_{7/2}$ SO splittings in $^{40}\text{Ca}$, $^{56}\text{Ni}$, and $^{48}\text{Ca}$ nuclei requires a drastic reduction of the isoscalar SO strength and strong attractive tensor coupling constants. The aim of this work is to address further consequences of these strong attractive tensor and weak SO fields on binding energies, nuclear deformability, and high-spin states. In particular, we show that contribution to the nuclear binding energy due to the tensor field shows generic magic structure with the tensorial magic numbers at $N(Z)=14, 32, 56$, or $90$ corresponding to the maximum spin-asymmetries in $1d_{5/2}$, $1f_{7/2} \oplus 2p_{3/2}$, $1g_{9/2} \oplus 2d_{5/2}$ and $1h_{11/2} \oplus 2f_{7/2}$ single-particle configurations and that these numbers are smeared out by pairing correlations and deformation effects. We also examine the consequences of strong attractive tensor fields and weak SO interaction on nuclear stability at the drip lines, in particular close to the tensorial doubly magic nuclei and discuss the possibility of an entirely new tensor-force driven deformation effect.

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I. INTRODUCTION

The primary goal of energy density functional (EDF) methods is to describe ground-state energies of fermion systems, i.e., in nuclear physics applications – masses of nuclei. The existence of an universal functional describing exactly masses of odd, odd-odd and even-even nuclei is warranted by the Hohenberg-Kohn [1] and Kohn-Sham [2] (HKS) theorems. The HKS theorems, however, provide no universal rules underlying the construction of such functional. The complexity of the functional and our lack of knowledge with respect to the in-medium strong interaction that governs the structure of finite nuclei make the situation even more difficult. It does not permit the determination for any ab initio constraints of the nuclear EDF, except for dilute neutron systems [3]. It forces us to use effective functionals with coupling constants fitted directly to the data. Hence, a proper selection of empirical data to be used in the process of constraining parameters of the functional becomes, irrespective of the form of functional, the key issue for the overall good performance of the nuclear density functional theory (DFT) [4].

The typical strategy used to construct the nuclear EDF is to start with either the finite-range Gogny [5] or zero-range Skyrme [6] effective interactions and construct non-local or local functional, respectively, by averaging the interaction within the Hartree-Fock (HF) method. The datasets used to adjust the free parameters of the theory are dominated by bulk nuclear matter data and by nuclear binding energies of selected doubly magic nuclei with much lesser attention paid to the single-particle energies (SPE).

The major reason is the effective mass scaling of the single-particle (SP) level density, $g$, in the vicinity of the Fermi energy, $\varepsilon_F$. In homogeneous nuclear matter the SP level density scales according to the simple rule: $g(\varepsilon_F) \rightarrow \frac{m^*}{m} g(\varepsilon_F)$. In finite nuclei the situation is slightly more intricate mostly due to $r$-dependence of the $m^*(r)$. Several authors [7, 8, 9] analyzed the SP level density scaling arguing that the physical density of SP levels around the Fermi energy can be restituted only after the inclusion of particle-vibration coupling, i.e., by going beyond mean-field (MF). This viewpoint is difficult to reconcile with the effective EDF theories. Indeed, these theories should warrant a proper value of the effective mass through the fit to empirical data and readjust other coupling constants to this value of $m^*$. This should lead to fairly $m^*$ independent predictions, provided that the (spherical) SPE are calculated from the differences between the binding energies in even-even doubly-magic cores and the lowest SP states in odd-$A$ single-particle/hole neighbors. Within the EDF approach, the mean-field or, more precisely, Kohn-Sham SP energies computed in e-e doubly magic core serve only as auxiliary quantities.

A new strategy for fitting the spin-orbit (SO) and tensor parts of the nuclear EDF was recently suggested by our group [10]. Instead of performing large-scale fits to binding energies we proposed a simple and intuitive three-step procedure that can be used to fit the isoscalar strength of the SO interaction as well as the isoscalar and isovector strengths of the tensor interaction. The entire idea is based on the observation that the $f_{7/2} - f_{5/2}$ SO splittings in spin-saturated isoscalar $^{40}\text{Ca}$, spin-unsaturated isoscalar $^{56}\text{Ni}$, and spin-unsaturated isovector $^{48}\text{Ca}$ form distinct pattern that can neither be understood nor reproduced based solely on the conventional SO interaction. Following the general philosophy of the nuclear DFT we compute the $f_{7/2} - f_{5/2}$ SO splittings from the differences between the binding energies of these doubly-magic cores and their odd-$A$ neighbors. However, we use the same functional to calculate both the ground state energies, what is well justified, as well
as the low-lying SP excitations in the odd-A neighbors. How reasonable is the latter assumption it remains to be studied.

The procedure reveals the need for a sizable reduction (from $\sim 20\%$ up to $\sim 35\%$ depending on the parameterization and, in particular, on the value of $m^*$) of the SO strength and at the same time, for much stronger tensor fields as compared to the commonly used values. The new parameterization systematically improves the performance of the functional with respect to SP properties like the SO splittings or the magic gaps but deteriorates the binding energies [10]. The aim of the present work is to address further consequences of strong attractive tensor and weak SO fields on binding energies, time-even and time-odd polarization effects, and the nuclear deformability. The paper is organized as follows. In Sect. II we briefly present the theoretical background of our model. In Sect. III we show that the contribution to the binding energy due to the tensor interaction forms a generic pattern closely resembling that of the shell-correction with the tensorial magic numbers shifted up as compared to the standard magic numbers toward $N(Z)=14, 32, 56,$ or 90. The tensorial magic numbers reflect the maximum spin-asymmetry in $1d_{5/2}, 1f_{7/2} \oplus 2p_{3/2}, 1g_{9/2} \oplus 2d_{5/2},$ and $1h_{11/2} \oplus 2f_{7/2}$ configurations, respectively, in the extreme SP scenario at spherical shape. The tensorial magic number is smeared out by configuration mixing caused by pairing and deformation effects. In Sect. IV we demonstrate that one can construct the EDF capable to reproduce reasonably well both the SO splittings as well as binding energies of the doubly magic spherical nuclei. In Sect. V we discuss the influence of strong tensor fields on time-even and time-odd polarization effects in the $f_{7/2} - f_{5/2}$ SO splittings. In Sect. VI we analyze deformation properties of the new functionals and discuss a possible novel mechanisms related to the onset of nuclear deformation in the presence of strong attractive tensor fields. The paper is concluded in Sect. VII. This analysis complements our preliminary results communicated in two earlier conference publications [11, 12].

II. THEORY: FROM TWO-BODY SPIN-ORBIT AND TENSOR INTERACTIONS TO ENERGY DENSITY FUNCTIONALS AND MEAN-FIELDS

In our study we explore the local energy density functional $\mathcal{H}(r)$. It is the sum of the kinetic energy and the isoscalar ($t=0$) and isovector ($t=1$) potential energy terms:

$$\mathcal{H}(r) = \frac{\hbar^2}{2m} \rho_0 + \sum_{t=0,1} \left\{ \mathcal{H}_t(r)^{\text{even}} + \mathcal{H}_t(r)^{\text{odd}} \right\},$$

which are conventionally decomposed into parts build of bilinear forms of either only time-even or only time-odd densities, currents and their derivatives:

$$\mathcal{H}_t^{\text{even}} = C_t^0 [\rho_0] \rho_t^2 + C_t^\Delta \rho_0 \Delta \rho_t + C_t^\nabla \rho_t \nabla \cdot \mathbf{J}_t,$$

$$\mathcal{H}_t^{\text{odd}} = C_t^0 [\rho_0] s_t^2 + C_t^\Delta s_t \Delta s_t + C_t^\nabla s_t \cdot \mathbf{T}_t = C_t^0 [\rho_0] \rho_t^2 + C_t^\Delta \rho_0 \Delta \rho_t + C_t^\nabla \rho_t \nabla \cdot \mathbf{J}_t,$$

For the time-even, $\rho_t, \Delta \rho_t,$ and $\Delta \mathbf{J}_t$, and time-odd, $s_t, \mathbf{T}_t,$ and $\mathbf{j}_t$, local densities we follow the convention introduced in Ref. [10] (see also Refs. [14, 15] and references cited therein).

In the present study we focus on the spin-orbit and tensor parts of the EDF:

$$\mathcal{H}_t^{T} = C_t^0 J_t^0 + C_t^1 J_t^1,$$

$$\mathcal{H}_t^{SO} = C_t^{\nabla} \rho_0 \nabla \cdot \mathbf{J}_t + C_t^{\nabla} \rho_1 \nabla \cdot \mathbf{J}_1.$$

These two parts of the EDF are strongly tied together through their mutual and unique contributions to the one-body spin-orbit potential. This relation can be best visualized by decomposing the spin-current tensor density $J_{\mu \nu}$ into scalar $J^{(0)}$, vector $J_\mu$ and symmetric-tensor densities $J_{\mu \nu}^{(2)}$:

$$J_{\mu \nu}^{(0)} = \nabla \cdot J^{(0)} \delta_{\mu \nu} + \frac{1}{2} \varepsilon_{\mu \nu q} J_q + J_{\mu \nu}^{(2)},$$

$$J^2 = \sum_{\mu \nu} J_{\mu \nu}^{(2)} = \frac{1}{2} (J^{(0)})^2 + \frac{1}{2} J^2 + \sum_{\mu \nu} (J_{\mu \nu}^{(2)})^2,$$

and going to the spherical-symmetry (SS) limit where the scalar $J^{(0)}$ and the symmetric-tensor densities $J_{\mu \nu}^{(2)}$ vanish identically. In this limit the spin-current tensor density:

$$J_{\mu \nu}^{\text{SS limit}} = \frac{1}{2} \varepsilon_{\mu \nu q} J_q$$

reduces, therefore, to the spin-orbit vector density having a single radial component, $\mathbf{J}_t = \frac{1}{2} J_t(r)$. The variation of the tensor and SO parts of the EDF over the radial SO densities $J_t(r)$ gives the spherical isoscalar ($t=0$) and isovector ($t=1$) SO MFs,

$$W_t^{SO} = \frac{1}{2r} \left( C_t^0 J_t(r) - C_t^{\nabla} \frac{d \rho_t}{dr} \right) \mathbf{L} \cdot \mathbf{S},$$

which can be easily translated into the neutron ($q=n$) and proton ($q=p$) SO MFs,

$$W_q^{SO} = \frac{1}{4r} \left\{ (C_q^0 - C_1^0) J_0(r) + 2C_1^1 J_q(r) \right\} \frac{d \rho_q}{dr} - \left\{ (C_0^{\nabla} - C_1^{\nabla}) \frac{d \rho_q}{dr} - 2C_1^{\nabla} \frac{d \rho_0}{dr} \right\} \mathbf{L} \cdot \mathbf{S}.$$
and, conventionally, tensor force generates a clear attributes to the EDF in a non-uniform way. Hence, the theoretical need to generalize the EDF (4) by using three independent coupling constants multiplying each of the three terms appearing in Eq. (13):

$$\mathcal{H}^T_i \rightarrow C^{(0)}_i (J^{(0)}_i) + C^{(1)}_i J^2 + C^{(2)}_i \sum_{\mu \nu} (J^{(2)}_{i,\mu \nu})^2. \quad (15)$$

The effects of such an extension, where the new coupling constants need to be adjusted can only be probed in deformed nuclei. Since conventional effective interactions and functionals are rather successful in describing nuclear deformation there is no first-hand motivation for such a generalization. Hence, in the present study, we do not implement this possible extension of the EDF, and we use the unique tensor coupling constants $C^T_i$, as defined in Eq. (2).

The contribution to the time-odd part of the EDF coming from the tensor interaction is:

$$\delta \mathcal{H}^{\text{odd}}_i = B^T_i \left( s_i \cdot T_i - 3 s_i \cdot F_i \right) + B^\Delta_i \left( s_i \cdot \Delta s_i + 3 (\nabla \cdot s_i)^2 \right), \quad (16)$$

where

$$B^\Delta_i = \frac{1}{52} (t_c - t_o) \quad \text{and} \quad B^{\Delta s}_i = -\frac{1}{52} (3t_c + t_o). \quad (17)$$

New terms which appear in the time-odd part of the EDF, namely the $s_i \cdot F_i$ and $\left( \nabla \cdot s_i \right)^2$ will not be considered here mostly due to lack of clear experimental indicators allowing to fit their strength. Extensive discussion linking the Skyrme forces to the tensor component in the EDF including, in particular, the definition of the density $F_i$ can be found in Ref. [10], see also Ref. [15].

The starting point of our consideration is always the conventional Skyrme-force-inspired functional with coupling constants fixed at the values characteristic for either SkP [17], SLy4 [18], or SkO [19] Skyrme parametrization. The variants of the EDF with tensor and spin-orbit strengths modified along the prescription of Refs. [10, 20] will be marked by an additional subscript $T$: SkP$_T$, SLy4$_T$, and SkO$_T$. In the time-odd sector we test two variants of the functional with the coupling constants fitted to the empirical values of the $s$-wave Landau parameters $g_{0}=0.4, g'_{0}=1.2$ and to the Gogny-force values of the $p$-wave Landau parameters $g_{1}=-0.19, g'_{1}=0.62$:

$$g_{0} = N_{0} (2C^{T}_0 + 2C^{T}_0 \beta \rho^{2/3})^2, \quad g_{1} = -2N_{0}C^{T}_0 \beta \rho^{2/3}, \quad (18)$$

$$g'_{0} = N_{0} (2C^{T}_1 + 2C^{T}_1 \beta \rho^{2/3})^2, \quad g'_{1} = -2N_{0}C^{T}_1 \beta \rho^{2/3}, \quad (19)$$

where $\beta = (3\pi^2/2)^{2/3}$, and $N_{0}^{-1} = \pi^2 \hbar^2 / 2m^* k_F$ is an effective mass-dependent normalization factor. In these variants of the EDF we additionally assume density independence of $C^T_i$ coupling constant, set the spin-surface term $C^{\Delta s}_i = 0$ to zero, and assume gauge-invariant relations $C^T_i = -C^T_i$ and $C^{\Delta s}_i = C^{\Delta s}_i$. Concerning the time-odd tensor coupling constants, $C^T_i$, the following two possibilities will be tested: (i) broken gauge-symmetry scenario with $C^T_i$ fitted to the Landau parameters and (ii) gauge-invariant scenario with $C^T_i = -C^T_i$ determined using the time-even coupling constants $C^T_i$. The variants of the EDF with spin fields defined using the Landau parameters will be labeled either by a subscript $L_S$ or by

References, see e.g. Refs. [13, 15], and will not be repeated here. We provide here spherical formulas [9] and [10] because they are crucial for understanding of our fitting strategy of the tensor and spin-orbit coupling constants which relies on the $f_{7/2} - f_{5/2}$ SO splittings in spherical doubly magic $^{40}\text{Ca}, ^{56}\text{Ni}$ and $^{48}\text{Ca}$ nuclei, see Ref. [10] and discussion below.

The functional of the form [2 - 8] can be obtained by averaging the conventional Skyrme effective interaction, $v_{Sk}(r)$, within the Skyrme-Hartree-Fock (SHF) approximation [13]. In such an approach twenty EDF coupling constants $C_i$ are uniquely expressed by means of ten auxiliary Skyrme-force parameters, see [14]. Hence, the SHF approximation superimposes relatively strong limitations to the EDF. It serves, though, as a reasonable starting point for further studies. The approach to the EDF starting from the effective force indicates also possible ways of generalization of the nuclear EDF. In particular, investigation of the tensor component requires the use of a generalized Skyrme force augmented by a local tensor interaction $v_T(r)$:

$$v(r) = v_{Sk}(r) + v_T(r) \quad (11)$$

where

$$v_T(r) = \frac{1}{2} \left\{ \left[ 3(\vec{\sigma}_1 \cdot \vec{k}) (\vec{\sigma}_2 \cdot \vec{k}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \vec{k}^2 \right] \delta(r) + \frac{1}{2} \left[ 3(\vec{\sigma}_1 \cdot \vec{k}) (\vec{\sigma}_2 \cdot \vec{k}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \vec{k}'^2 \right] \delta(r) \right\} + t_o \left[ 3(\vec{\sigma}_1 \cdot \vec{k}' \delta(r)(\vec{\sigma}_2 \cdot \vec{k}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \vec{k}' \delta(r) \vec{k} \right], \quad (12)$$

and, conventionally, $r = r_1 - r_2$ and $k = -\frac{i}{2} (\nabla_1 - \nabla_2)$ are relative coordinate and momentum while $k'$ is complex conjugation of $k$ acting on the left-hand side. By averaging $v_T(r)$ within the HF approach, one obtains the following contribution to the time-even part of the EDF [13]:

$$\delta \mathcal{H}^{\text{even}}_i = \frac{5}{3} B^T_i (J_i^{(0)})^2 - \frac{2}{3} B^T_i J_i^2 + \frac{1}{3} B_j^T \sum_{\mu \nu} (J_i^{(2)}_{\mu \nu})^2, \quad (13)$$

where

$$B^T_0 = -\frac{2}{5} (t_c + 3t_o) \quad B^T_1 = \frac{1}{5} (t_c - t_o). \quad (14)$$

Note, that there are two independent contributions to the tensor part of the EDF. The Skyrme force contributes, through the exchange term, to the tensor part of the EDF in an uniform manner, i.e., it depends on a unique coupling constant $\delta$. In contrast, the tensor force contributes to the EDF in a non-uniform way. Hence, the tensor force generates a clear theoretical need to generalize the EDF [4] by using three independent coupling constants multiplying each of the three terms appearing in Eq. (17).
a subscript $L_B$ for the gauge-invariant and the gauge-symmetry-violating functionals, respectively.

III. TOPOLOGY OF TENSOR CONTRIBUTION TO THE NUCLEAR BINDING ENERGY

Recent revival of interest in the tensor interaction was triggered by empirical discoveries of strong and systematic changes in the shell structure of neutron-rich oxygen [24], neon [25], sodium [26, 27], magnesium [28], or titanium [29, 30] nuclei including new shell gap opening at $N=32$. These empirical discoveries were successfully interpreted within the nuclear shell-model after introducing the so-called monopole shifts. To account for the data the monopole shifts are: (i) attractive between $j_2^y$ and $j_2^x$ orbitals and (ii) repulsive between $j_2^x$ and $j_2^y$ orbitals where $j_2 = l \pm 1/2$. The physical origin of these monopole shifts was attributed to the tensor interaction [31, 32, 33]. Soon after successful shell-model calculations, the mechanism was confirmed within self-consistent mean-field calculations using finite range Gogny force augmented by the finite-range tensor interaction, see [34]. It was shown in Ref. [34] that, apart from explaining the shell-structure evolution in light exotic nuclei, the tensor interaction was also capable to account for empirical trends in the relative positions of the 1$^{28}$ and 1$^{29}$ in antimony [37] isotopes.

For strictly pragmatic reasons, like technical complexity and lack of firm experimental indicators constraining further the BSF estimate, in many Skyrme parameterizations (including SIII [33], SLy4 [18], SkM* [44], SkO* [19]), the tensor terms are simply disregarded by setting $C_t^I = 0$. Moreover, the $C_t^I$ coupling constants established through fits to, predominantly, bulk nuclear data seem to contradict the BSF estimates. Indeed, the isoscalar tensor coupling constants of such popular forces like SLy5 [18], SkP [17] or Skxc [15] are relatively weak while their isovector coupling constants are positive. These coupling constants lie outside the BSF triangle as shown in Fig. 1.

On the other hand, direct fits to the SP level splittings [10, 38, 40] clearly reveal that drastic changes in the commonly accepted tensor coupling constants are needed in order to accommodate for the SP data. This is visualized in Fig. 1 where the new parameterizations are marked by black triangles and black diamonds, respectively. The fact that new values of $C_t^I$ are still slightly scattered is a consequence of different fitting strategies as well as different starting point parameterizations used by different groups.

Our strategy of fitting the coupling constants of the nuclear EDF, see Ref. [10], differs from the strategies applied by other groups. Unlike the other groups, we fit simultaneously the isoscalar spin-orbit, $C_{tO}^I$, as well as the isoscalar, $C_{tI}^I$, and the isovector, $C_{tI}^J$, tensor coupling constants using a simple three-step method. The entire idea of our procedure is based on the observation that the empirical $1f_{5/2} - 1f_{5/2}$ SO splitting in $^{40}$Ca, $^{56}$Ni, and $^{48}$Ca form a distinct pattern, which cannot be reproduced by using solely the conventional SO interaction. The readjustment of the coupling constants goes as follows: (i) in the first step $C_{tO}^I$ is established in the isoscalar spin-saturated nucleus $^{40}$Ca; (ii) next, the $C_{tI}^J$ coupling constant is readjusted in spin-unsaturated isoscalar nucleus $^{56}$Ni; (iii) finally, the $C_{tI}^J$ coupling constant is readjusted to the spin-unsaturated isovector nucleus $^{48}$Ca.

Our results, see Refs. [10, 11, 20], show that drastic changes in the isoscalar SO strength and the tensor coupling constants are required as compared to the commonly accepted values. In turn, one obtains systematic improvements for such single-particle properties like SO splittings and magic-gap energies. It is also interesting to note that the isoscalar SO and the isovector tensor coupling constants resulting from such a fit are, to large
extent, independent on the parameterization and equal to $C_0^{\mathcal{V}J} \approx -60 \pm 10 \text{ MeV fm}^5$ and $C_0^{IJ} \approx -40 \pm 10 \text{ MeV fm}^5$, respectively. The uncertainties are rough estimates reflecting the sensitivity of the method. The isovector tensor coupling constant, $C_1^{IJ}$, is less certain. It depends on the actual ratio $C_0^{\mathcal{V}J}/C_0^{IJ}$ of the SO coupling constants which, in the adjustment process, was kept fixed to its Skyrme force value. This is due to lack of empirical data in $^{48}$Ni which does not allow for firm independent readjustment of the fourth coupling constant $C_1^{\mathcal{V}J}$.

The influence of the tensor interaction on nuclear SPE and SP level splittings has been analyzed by many authors. It was shown that the tensor interaction leaves unique and robust fingerprints when the SPE and SP level splittings are studied along isotopic or isotonic chains. It appears also that the contribution to the binding energy coming from the tensor interaction, $\delta B_T(N,Z)$, shows several highly interesting and robust topological features. In particular, the contribution $\delta B_T(N,Z)$ shows a generic pattern closely resembling that of a shell-correction. The tensorial magic numbers at $N(Z)=14, 32, 56, 90$ correspond to the maximum spin-asymmetries in $1d_{5/2}^1 \oplus 2p_{3/2}^1$, $1g_{9/2}^1 \oplus 2d_{5/2}^1$ and $1h_{11/2}^1 \oplus 2f_{7/2}^1$ single-particle configurations, respectively, in the extreme SP scenario at spherical shape. The robustness, i.e., model independence of the tensorial magic pattern, results from rather unambiguously established order of single-particle levels which is relatively well reproduced by state-of-the-art nuclear MF models, in particular, in light and medium-mass nuclei. Note, that the tensorial magic numbers are only slightly shifted as compared to classic magic numbers at $N(Z)=8, 20, 28, 50$, and 82.

The tensorial magic pattern is clearly visible in Fig. 2.
Panels a)-c) show the contribution to the binding energy coming from: a) the isovector part of the tensor term, \( \delta E^T_1 = C^J_1 \int d^3 \mathbf{r} J^2_1(\mathbf{r}) \), b) the isoscalar part of the tensor term, \( \delta E^T_0 = C^J_0 \int d^3 \mathbf{r} J^2_0(\mathbf{r}) \), and c) the total tensor contributions to the EDF. These calculations were performed using spherical Hartree-Fock-Bogolyubov (HFB) code HFBRAD [46] with the SLy4\( _T \) functional of Ref. [10] in the particle-hole channel and the volume \( \delta \)-interaction in the pairing channel. This part of Fig. [2] shows several interesting features including:

- Additional smearing of the SP tensorial magic structure due to configuration mixing caused by nuclear pairing. In light and medium-mass nuclei a substantial tensor contributions located in relatively broad regions centered around the SP tensorial magic numbers. In heavier nuclei, where pairing effects are relatively stronger due to larger density of SP levels, the erosion of the SP tensorial magic structure is stronger. The maximum of the tensor contribution is shifted away from the SP magic numbers. The details, however, are strongly model-dependent mostly due to large uncertainties in the positions of the SP levels in heavier nuclei. Indeed, Skyrme models have persisting problems to reproduce absolute positions of the experimental SPE as shown recently in Ref. [17].

- Contribution due to the isovector part of the tensor interaction is much weaker than the isoscalar contribution. This conclusion depends on the tensorial coupling constants.

- The isoscalar tensor interaction creates oscillatory effects in nuclear masses that depend on the degree of spin-unsaturation in a given nucleus. This additional non-uniform \( N \) and \( Z \) dependence may, in particular, obscure conclusions deduced from widely used binding-energy indicators technique.

The second major source of configuration mixing is due to the spontaneous breaking of spherical symmetry inherent to the MF method. The influence of nuclear deformation on the topology of the tensor contributions to the binding energy is illustrated in Fig. [2], i.e.f. The calculations presented in the figure were performed for e-e nuclei with \( N \geq Z \) ranging from \( 6 \leq Z \leq 64 \) using the HFODD code [48]. The same SLy4\( _T \) interaction of Ref. [10] was used in the p-h channel and volume-\( \delta \) interaction in the p-p channel. It is clearly visible that the effect of deformation does not change the topology of the tensor energy contribution but strongly reduces its magnitude. One should stress though that quantitative estimate of the deformation effect is uncertain. The magnitude of the deformation is extremely sensitive to the balance between SO and tensor strengths. This effect will be discussed in detail in Sect. V.

Fig. [3] shows again the total contribution to the binding energy calculated using the spherical HFB model. The map is overlaid with the map of known, according to Ref. [42], nuclei. The aim of the figure is to illus-
trate mass-regions where enhanced tensor effects, and in turn perhaps new physics, may be expected on the neutron-rich and proton-rich side. On the neutron-rich side the regions of interest, i.e., those which are or can be accessible experimentally in the nearest future, include: $Z \approx 14$ and $N \approx 32$, $Z \approx 32$ and $N \approx 56$, and $Z \approx 56$, $N \approx 90$. In particular, recent measurements of exotic $^{40}_{20}\text{Mg}_{28}$ and $^{42}_{13}\text{Al}_{29}$ by Bauman et al. [49] (see also the discussion in Ref. [50]) approach closely the first of the above mentioned mass regions. However, whether or not extra binding due to the strong attractive tensor interaction gives rise to stabilization of these nuclei and nuclei around them remains to be studied. Mean-field calculations using conventional Skyrme forces predict these nuclei to be bound [51].

IV. ENERGY DENSITY FUNCTIONAL FITTED TO THE SINGLE-PARTICLE SPIN-ORBIT SPLITTINGS AND TO THE TOTAL BINDING ENERGIES OF SPHERICAL NUCLEI

The topology of the tensor contribution to the total binding energy is, as discussed above, a generic feature related to shell-structure and the degree of spin-saturation. The quantitative features including the total magnitude and the isovector to isoscalar ratio of the tensor contributions depend, however, upon the actual values of the tensorial coupling constants. The two strategies of fitting effective forces, namely the conventional one based on the large-scale fit to the binding energies and the one based on the fit of $C^J_t$ and $C^{\Sigma J}_t$ coupling constants directly to the SO splittings seem to yield contradicting results. This is clearly visible in in Fig. 4 where the ranges of the $C^J_t$ strengths deduced using these two methods simply exclude each other.

A direct fit to the SO splittings leads to drastic changes in the isoscalar SO strength and the tensor coupling constants as compared to the commonly accepted values. In particular, the pronounced reduction of the SO strength which varies from 20\% for large effective mass, $m^*$, forces to even 35\% for low ($m^* \approx 0.7$) forces imperils the agreement with observed binding energies. This effect cannot be solely compensated by strong attractive tensor fields. Hence, further readjustments of the other coupling constants of the EDF are necessary to assure good performance on masses.

In Ref. [10] we have demonstrated that a considerable improvement in the quality of mass fits can be achieved be relatively small readjustments of the EDF coupling constants. For the purpose of this work we have performed similar calculations using multi-dimensional minimization technique but starting from the SkP$_T$ force. In both cases tiny modifications (of the order of a fraction of a percent) in the coupling constants clearly improve the quality of the mass fit as compared to the SLy4$_T$ and SkO$_T$ forces but still do not provide the quality of the original SLy4 and SkO parameterizations.

Inherent to the multi-dimensional minimization technique is a merit function being a subject of minimization. In our calculations the merit function is constructed out of relative deviations from measured masses of selected spherical doubly magic nuclei. It appears that the merit function in the multi-dimensional space spanned by the EDF coupling constants varying in the minimization process is very steep for some specific directions and extremely flat in others. It implies that the entire minimization problem is not well defined. We will visualize this by taking as a starting point the SkO parameterization.

By reducing the SO strength by 15\% corresponding to $C^J_0 \approx -65.6$ MeV fm$^2$ and $C^J_0 \approx 84.5$ MeV fm$^5$ and taking $C^\Sigma_0 \approx -44.1$ MeV fm$^5$ and $C^J_0 \approx -91.6$ MeV fm$^5$ we create a modified version of the SkO$_T$ parameterization of Ref. [10]. This parameterization, dubbed SkO$_{T_1}$, has slightly stronger SO term and more attractive tensor fields as compared to the SkO$_T$. These changes aim to improve mass performance of the SkO$_T$. Direct calculations show, see Fig. 4, that the SkO$_{T_1}$ functional reproduces masses at a similar level of accuracy as one of the most popular SLy forces. The calculations illustrated in Fig. 4 were performed using the code HFODD with 20 spherical shells. We were forced to use the HFODD code because the spherical HBFRAD code has no two-body center-of-mass correction implemented.

The performance of the SkO$_{T_1}$ force can be further improved in many different ways. One example, dubbed SkO$_{T_{1a}}$, is illustrated in Fig. 4. This force was obtained by readjusting isoscalar and isovector central fields in the following way: $C^\Sigma_0 \rightarrow 1.00015C^\Sigma_0$ and $C^J_0 \rightarrow 0.99C^J_0$. As a result, standard deviation drops from $\sigma(\text{SkO}_{T_1}) \approx 1.663$ MeV to $\sigma(\text{SkO}_{T_{1a}}) \approx 1.475$ MeV where, for compar-
ison, $\sigma(\text{SLy4}) \approx 1.879\text{MeV}$. Similar improvements can be made by, for example, readjusting the density dependent term.

A reasonable performance of the SkO$_T$ or SkO$_{T'}$ functionals with respect to the binding energies of spherical doubly-magic nuclei is of great interest. It may help to resolve the conflict concerning the tensor and SO coupling constants resulting from (local) fits to the SO splittings and the SPE [10, 47] preferring strong tensor and weak SO terms on one hand and from large-scale fits to the binding energies [16] pointing toward weak tensor and stronger SO terms on the other hand. Indeed, our present result indicates that one should explore in large-scale fits functionals having non-conventional isovector spin-orbit functionals with respect to the binding energies of spherical doubly-magic nuclei.

In all variants of the calculations the odd-even core in the presence of strong attractive coupling constants as compared to the SkO functional can still reproduce the empirical $1f_{5/2}$ splitting data. Note also, that the time-odd polarization effects are indeed large, reaching a few hundred keV.

Open circles illustrate the results of our calculations using the SkO parameterization. Open and filled diamonds represent calculations using SkO$_T$ functional. In all variants of the calculations the $C^T_{2\Delta}$ strength was set to zero to assure convergence. Different panels represent different variants of the calculations concerning treatment of the time-odd sector. Panels a) and e) show bare, unpolarized SO splittings deduced directly from the SP spectra calculated in the doubly magic $^{40}$Ca, $^{48}$Ca, and $^{56}$Ni nuclei. Results presented in the other panels are calculated from the binding energies in the doubly magic cores and their one-particle(hole) odd-$A$ neighbors following the prescription given in Ref. [10]. In the calculations the odd-$A$ binding energies correspond to fully aligned ($I_{sp}$) $= j$ states at oblate, for one-particle, and prolate, for one-hole, nuclei, respectively. Unlike in our previous study [10], the present calculations include polarization effects exerted by the odd particle or hole on the even-even core in the presence of strong attractive tensor fields.

In order to visualize the role of the tensor interaction, in particular in the time-odd sector, we performed three different variants of the calculations. Figures b) and f) include only time-even (mass and deformation) polarization effect. These results were obtained by setting all time-odd coupling constants to zero. Panels c) and g) illustrate the effect of the time-odd fields. These results include both the time-even and time-odd polarizations. In these calculations we use gauge invariant functional with the $C^T_I = -C^T_{I'}$ tensor coupling constants fitted to the SO splittings. All other coupling constants in this run, except $C^{B\Delta} = 0$, are equal to the values given by the SkO parameterization. Note that the splittings calculated in this way match almost perfectly the empirical data. Note also, that the time-odd polarization effects are indeed large, reaching a few hundred keV.

Figures d) and h) show two variants of the calculations with spin fields readjusted to match empirical Landau parameters, namely the SkO$_{T'L_S}$ and the SkO$_{T'L_B}$ (see Sect. 11). The SkO$_{T'L_S}$ variant is labeled by open diamonds. It corresponds to fully gauge invariant functional. In this variant we use the $C^T_I = -C^T_{I'}$ tensor coupling constants fitted to the SO splittings and the spin-field coupling constants readjusted to the s-wave Landau parameters according to Eqs. [15–19].

In the SkO$_{T'L_B}$ variant, which is labeled by filled diamonds, we readjust first the $C^{T}_{J}$ coupling constants to the Gogny values of the p-wave Landau parameters $g_1 = -0.19$ and $g'_1 = 1.2$. This leads to gauge-symmetry violating functional with the tensorial coupling constant $C^C_0 \approx -44.1\text{MeV fm}^{5}$ and $C^C_I \approx -91.6\text{MeV fm}^{5}$ in the time-even channel and $C^{T}_{0} \approx 9.2\text{MeV fm}^{5}$ and $C^{T}_{I} \approx -29.9\text{MeV fm}^{5}$ in the time-odd channel which are used subsequently to calculate spin-fields coupling constants. Note, that the SO splittings are quite sensitive to the way the functional is set up in the time-odd sector. It means that the entire concept of fitting the time-odd coupling constants to the Landau parameters in the presence of the strong tensor terms must be reconsidered. In particular, the p-wave parameters deduced from the Gogny force $g_1 = -0.19$ and $g'_1 = 1.2$ lead to the $C^C_I$ coupling constants which are completely inconsistent with the time-even values $C^C_I$ deduced from the SO splittings. In turn, the SkO$_{T'L_S}$ and SkO$_{T'L_B}$ have entirely different spin fields with coupling constants equal $C^C_0 \approx 426.4\text{MeV fm}^{5}$ and $C^C_I \approx 48.6\text{MeV fm}^{5}$ and $C^T_0 \approx 18.0\text{MeV fm}^{5}$ and $C^T_I \approx 155.9\text{MeV fm}^{5}$, respectively.

VI. EFFECT OF TENSOR FIELD ON NUCLEAR DEFORMATION

In the previous section we have shown that one can construct the EDF capable to reproduce reasonably well binding energies of the spherical doubly magic nuclei and, simultaneously, account for the SO splittings of the $1f_{7/2} - 1f_{5/2}$ SP levels. By inspecting closer the results presented in Fig. 2 one observes very strong time-even (mass and deformation) polarization effect on the calculated SO splitting in $^{56}$Ni – compare Fig. 2a and Fig. 2b.
FIG. 5: SO splittings between the neutron $\nu 1f_{5/2}$ and $\nu 1f_{7/2}$ (left part) and the proton $\pi 1f_{5/2}$ and $\pi 1f_{7/2}$ (right part) orbitals in $^{40}$Ca, $^{48}$Ca, and $^{56}$Ni nuclei. Filled (open) circles mark empirical and theoretical splittings calculated using the conventional SkO force, respectively. Open and filled diamonds indicate calculations performed using the SkO$^{T'}$ functional. Panels a) and e) show bare, unpolarized SO splittings deduced directly from the SP spectra calculated in the doubly magic $^{40}$Ca, $^{48}$Ca, and $^{56}$Ni nuclei. Parts b) and f) include only time-even (mass and deformation) polarization effect. Panels c) and g) include both the time-even and time-odd polarization effects. Finally, figures d) and h) show two variants of the calculations with spin fields readjusted to match empirical Landau parameters, namely the SkO$^{T'\ L_S}$ (open diamonds) and the SkO$^{T'\ L_B}$ (filled diamonds) functionals.

The aim of this section is to show that deformation properties indeed depend sensitively on the balance between the SO and tensor fields. We will discuss two contrasting cases. First, we will consider an example of the yrast superdeformed (SD) band in $^{56}$Ni [54]. The band is formed by promoting two protons and two neutrons from the $1f_{7/2}$ to the $1f_{5/2}$ orbital or, in terms of more appropriate from mean-field point of view asymptotic Nilsson model quantum numbers, from $[303]7/2$ to $[321]3/2$ Nilsson orbital as illustrated schematically in Fig. [5].

In the spherical ground state of $^{56}$Ni the entire $1f_{7/2}$ is fully occupied while the $1f_{5/2}$ is empty. This creates large spin-asymmetry and, in turn, large contribution to the ground state due to the tensor field. By promoting four-particles from the $1f_{7/2}$ to the $1f_{5/2}$ orbit one creates the SD state which has reduced spin-asymmetry as compared to the ground state. The reduced tensor field shifts the SD state up in energy with respect to the ground state. The subsequent reduction of the SO potential shifts the $1f_{5/2}$ orbit and, in turn, the entire SD band down in energy. This compensating mechanism is illustrated in Fig. [5]a,c,d. The figures show the results of the self-consistent quadrupole-constrained HF calculations for the ground state (GS) and the SD configurations. Since it is impossible to go diabatically through the GS–SD configuration crossing region the self-consistent
results for the GS and SD configurations were, for the sake of simplicity and clarity, extrapolated diabatically through this region. This does not affect the physics discussed below.

Fig. 6b shows the calculations performed using the conventional SkO force. These calculations predict the $0^+$ SD state to be excited by $\sim 4$ MeV with respect to the GS what agrees quite well with the empirical estimate [54]. Readjustment of the tensor coupling constant in the SkO to the value characteristic for the SkO$_T$, functional (this functional is called SkO$_{TX}$) shifts the position of the $0^+$ SD state by 4 MeV up in energy. This intermediate step is illustrated in Fig. 6c. The change in the tensor field on the passage from the GS to the SD minimum is shown in Fig. 6e. Finally, Fig. 6d shows the result obtained using full SkO$_{TX}$ functional. The effects of the reduced SO and strong attractive tensor fields almost cancel each other restoring the position of the $0^+$ SD state close to its empirical (and close to the SkO) value.

The second example is shown in Fig. 7. The figure illustrates potential energy curves in $^{80}$Zr calculated using the HFB model with volume-delta interaction, $V\delta(r)$, of the strength $V=-190$ MeV in the particle-particle channel. The three curves represent the SkO, SkO$_{TX}$, and SkO$_T$ functionals in the particle-hole channel. All curves are normalized to the spherical minimum in order to facilitate further discussion.

Unlike the $^{56}$Ni, the $^{80}$Zr is spin-saturated in the spherical minimum. Building up deformation is associated in this nucleus with partial occupation of the $1g_{9/2}$ sub-shell. It leads to increasing spin asymmetry and, in turn, to extra attraction due to the tensor terms. This effect is clearly visible for the SkO$_{TX}$ functional, see Fig. 7. The mechanism is strong enough to create a well deformed minimum. In this case, reduction of the SO strength shifts the $1g_{9/2}$ sub-shell up with respect to the negative parity $fp$ levels. In turn, the well deformed minimum is also lifted up in energy ending up slightly higher as compared to the SkO prediction.

These two examples show that the SkO$_T$ functional have deformation properties quite similar to the conventional Skyrme functionals at least in isoscalar $N\approx Z$ nuclei. The situation is slightly more intricate in the isovector channel due to the uncertainties of the $C_{ij}^{\pi J}$ strength and, in turn, in the $C_{ij}^{\pi J}$ coupling constant. Nevertheless, the two cases analyzed above clearly show that:

- Nuclear deformation properties strongly depend on the balance between tensor and spin-orbit terms
- Detailed and systematic studies of nuclear deformation in the presence of strong tensor fields open
We explicitly construct the functional, dubbed SkO$_{T'}$, which is able to reproduce simultaneously the $f_{5/2} - f_{7/2}$ SO splittings in $^{40}\text{Ca}$, $^{56}\text{Ni}$, and $^{48}\text{Ca}$ nuclei and the binding energies of spherical nuclei. In fact, one can construct many parameterizations reproducing these data in a more or less equivalent manner. This is due to the fact that multi-dimensional merit-functions which are minimized in the fitting process are flat in certain directions and the entire minimization procedure is not well-defined. Reasonable performance on the binding energies of the SkO$_{T'}$ functional which is characterized by its non-conventional isovector SO term is very interesting by itself. Indeed, this result may open up a possibility to bridge the $C^J$ and $C^N_J$ coupling constants resulting from (local) fits to the SPE with the values resulting from global large-scale fits to the binding energies by exploring non-standard local functionals.

Using the SkO$_{T'}$ functional we analyze polarization effects exerted by one particle and one hole on the values of the $f_{5/2} - f_{7/2}$ SO splittings. We show that the polarization effects are large and very sensitive to the way the functional is set up in the time-odd channel. Fits to the Landau parameters are uncertain due to rather poorly known $p$-wave Landau parameters. In particular, the use of the $p$-wave Landau parameters deduced from the Gogny force, which is advocated in Ref. [23], leads to strong gauge-symmetry violating effects and, in turn, large differences in the $C^s$ coupling constants between the gauge-invariant SkO$_{T'L_S}$ functional and the gauge-symmetry violating functional SkO$_{T'L_B}$.

We also demonstrate that deformation properties in atomic nuclei can be easily and strongly modified in the presence of strong tensor fields and that these properties are extremely sensitive to the balance between the tensor and the SO coupling constants. We show that, in the particular case of the SkO$_{T'}$ functional, the tensor effects are almost perfectly compensated, at least in the isoscalar channel, by the reduced SO potential. We explicitly construct the functional, dubbed SkO$_{T'}$, which is characterized by its magic structure with the tensorial magic numbers at $N(Z)=14$, 32, 56, or 90 corresponding to the maximum spin-asymmetries in $1h_{5/2}$, $1f_{7/2} \oplus 2p_{3/2}$, $1g_{9/2} \oplus 2d_{5/2}$ and $1h_{11/2} \oplus 2f_{7/2}$ single-particle configurations and that these numbers are smeared out by pairing correlations and deformation effects.

Among others, we show that contribution to the nuclear binding energy due to the tensor field shows generic magic structure with the tensorial magic numbers at $N(Z)=14$, 32, 56, or 90 corresponding to the maximum spin-asymmetries in $1h_{5/2}$, $1f_{7/2} \oplus 2p_{3/2}$, $1g_{9/2} \oplus 2d_{5/2}$ and $1h_{11/2} \oplus 2f_{7/2}$ single-particle configurations and that these numbers are smeared out by pairing correlations and deformation effects.

The direct fit of the isoscalar spin-orbit and both isoscalar and isovector tensor coupling constants to the $f_{5/2} - f_{7/2}$ SO splittings in $^{40}\text{Ca}$, $^{56}\text{Ni}$, and $^{48}\text{Ca}$ requires (i) a drastic reduction of the isoscalar SO strength and (ii) strong attractive tensor coupling constants [11]. In this work we address the global nuclear structure consequences of this novel fitting strategy of the nuclear EDF. Among others, we show that contribution to the nuclear binding energy due to the tensor field shows generic magic structure with the tensorial magic numbers at $N(Z)=14$, 32, 56, or 90 corresponding to the maximum spin-asymmetries in $1h_{5/2}$, $1f_{7/2} \oplus 2p_{3/2}$, $1g_{9/2} \oplus 2d_{5/2}$ and $1h_{11/2} \oplus 2f_{7/2}$ single-particle configurations and that these numbers are smeared out by pairing correlations and deformation effects.

VII. SUMMARY AND CONCLUSIONS

There is a large, and so far completely unexplored, potential to modify deformation properties by extending the tensor term from the uniform form used here to the non-uniform form [19].

- Possible modifications of collective rotational motion, which is inherently related to the spontaneous breaking of spherical symmetry within the EDF formalism, opens up yet another almost completely unexplored area in studying tensor fields.

We also demonstrate that deformation properties in atomic nuclei can be easily and strongly modified in the presence of strong tensor fields and that these properties are extremely sensitive to the balance between the tensor and the SO coupling constants. We show that, in the particular case of the SkO$_{T'}$ functional, the tensor effects are almost perfectly compensated, at least in the isoscalar channel, by the reduced SO potential. We suggest that the role of the tensor interaction, in particular in the time-odd channel, can be studied through dynamical effects induced by fast nuclear rotation. For this purpose one needs to select and use nuclear states representing, as close as possible, an unperturbed single-particle motion in order to suppress other effects or correlations which may obscure the analysis. The examples include superdeformed bands which are known to be very well described using simple one-body cranking approximation, see Ref. [55] and references therein, or terminating states [20].

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[1] P. Hohenberg and W. Kohn, Phys. Rev. 136, B864 (1964); M. Levy, Proc. Nat. Acad. Sci. 76, 6062 (1979).
[2] W. Kohn and L.J. Sham, Phys. Rev. 140, A1133 (1965).
[3] A. Bulgac, Phys. Rev. A 76, 040502(R) (2007).
[4] P. Klüpfel, P.-G. Reinhard, T.J. Burvenich, and J.A. Maruhn, Phys. Rev. C 79, 034310 (2009).
[5] D. Gogny, Nucl. Phys. 237, 399 (1975).
[6] T.H.R. Skyrme, Phil. Mag. 1 (1956) 1043; Nucl. Phys. 9 (1959) 615.
[7] I. Hamamoto, Phys. Lett. 61B (1976) 343.
[8] V. Bernard and N. Van Giai, Nucl. Phys. 348, 75 (1980).
[9] E. Litvinova and P. Ring, Phys. Rev. C 73, 044328 (2006).
[10] M. Zalewski, J. Dobaczewski, W. Satula, and T.R. Werner, Phys. Rev. C 77, 024316 (2008).
[11] M. Zalewski, W. Satula, J. Dobaczewski, P. Olbratowski, M. Rafalski, T.R. Werner, and R.A. Wyss, Phys. Rev. Jour. 42, 577 (2009).
[12] W. Satula, M. Zalewski, J. Dobaczewski, P. Olbratowski, M. Rafalski, T.R. Werner, and R.A. Wyss, Int. J. Mod. Phys. E18, 808 (2009).
[13] Y.M. Engel, D.M. Brink, K. Goewe, S.J. Krieger, and D. Vautherin, Nucl. Phys. A 249, 215 (1975).
[14] M. Bender, P.-H. Heenen, and P.-G. Reinhard, Rev. Mod. Phys. 75, 121 (2003).
[15] E. Perlińska, S.G. Rochoziński, J. Dobaczewski, and W. Nazarewicz, Phys. Rev. C 69, 014316 (2004).
[16] T. Lesinski, M. Bender, K. Bennaceur, T. Duguet, and J. Meyer, Phys. Rev. C 76, 014312 (2007).
[17] J. Dobaczewski, H. Flocard, and J. Treiner, Nucl. Phys. A422, 103 (1984).
[18] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A627 (1997) 710; A635 (1998) 231.
[19] P.-G. Reinhard, D.J. Dean, W. Nazarewicz, J. Dobaczewski, J.A. Maruhn, and M.R. Strayer, Phys. Rev. C60, 014316 (1999).
[20] W. Satula, R.A. Wyss and M. Zalewski, Phys. Rev. C78, 011302(R) (2008).
[21] A.N. Ostrowski et al. Z. Phys. A343, 489 (1992).
[22] M. Bender, J. Dobaczewski, J. Engel, and W. Nazarewicz, Phys. Rev. C65, 054322 (2002).
[23] H. Zdzuńczuk, W. Satula, and R. Wyss, Phys. Rev. C71, 024305 (2005); Int. J. Mod. Phys. E14, 451 (2005).
[24] E. Becheva et al., Phys. Rev. Lett. 96, 012501 (2006).
[25] M. Belleguic et al., Phys. Rev. C72, 054316 (2005).
[26] Y. Utsuno et al., Phys. Rev. C70, 044307 (2004).
[27] V. Tripathi et al., Phys. Rev. Lett. 94, 162501 (2005).
[28] G. Neyens et al., Phys. Rev. Lett. 94, 022501 (2005).
[29] B. Fornal et al., Phys. Rev. C70, 064304 (2004).
[30] D.-C. Dinca et al., Phys. Rev. C71, 041302 (2005).
[31] T. Otsuka, R. Fujimoto, Y. Utsuno, B.A. Brown, M. Honma, and T. Mizusaki, Phys. Rev. Lett. 87, 082502 (2001).
[32] T. Otsuka, T. Suzuki, R. Fujimoto, H. Grawe, and Y. Akaishi, Phys. Rev. Lett. 95, 232502 (2005).
[33] M. Honma, T. Otsuka, B.A. Brown, and T. Mizusaki, Eur. Phys. Jour. 25, s01, 499 (2005).
[34] T. Otsuka, T. Matsuo, and D. Abe, Phys. Rev. Lett. 97, 162501 (2006).
[35] M. Sawicka et al., Eur. Phys. J. A 20, 109 (2003).
[36] A. Korgul, private communication.
[37] J.P. Schiffer et al., Phys. Rev. Lett. 92, 162501 (2004).
[38] B.A. Brown, T. Duguet, T. Otsuka, D. Abe, and T. Suzuki, Phys. Rev. C 74, 064303(R) (2006).
[39] G. Colò, H. Sagawa, S. Fracasso, and P.F. Bortignon, Phys. Lett. B 646, 227 (2007).
[40] D.M. Brink and Fl. Stancu, Phys. Rev. C75, 064311 (2007).
[41] D. Brink, Fl. Stancu, and H. Flocard, Phys. Lett. B68, 108 (1977).
[42] G. Audi and A.H. Wapstra, Nucl. Phys. A595, 409 (1995); Nucl. Phys. A565, 1 (1993).
[43] M. Beiner, H. Flocard, N. Van Giai, and P. Quentin, Nucl. Phys. A238, 29 (1975).
[44] J. Bartel, P. Quentin, M. Brack, C. Guet, and H.B. Häkansson, Nucl. Phys. A386, 79 (1982).
[45] B.A. Brown, Phys. Rev. C58, 220 (1998).
[46] K. Bennaceur and J. Dobaczewski, Comput. Phys. Commun. 168, 96 (2005).
[47] M. Kortelainen, J. Dobaczewski, K. Mizuyama, and J. Toivanen, Phys. Rev. C 77, 064307 (2008).
[48] J. Dobaczewski, J. Dudek, and P. Olbratowski, Comput. Phys. Comm. 158 (2004) 158; HFODD User’s Guide nucl-th/0501008.
[49] T. Baumann et al., Nature 449, 1022 (2007).
[50] P.-H. Heenen, Nature 449, 992 (2007).
[51] W. Nazarewicz and M. Stoitsov, private communication.
[52] A. Oros, Ph.D. thesis, University of Köln, 1996.
[53] N. Schwierz, I. Wiedenhover, and A. Volya, arXiv:0709.3525.
[54] D. Rudolph et al., Phys. Rev. Lett. 82 (1999) 3763.
[55] W. Satula and R. Wyss, Rep. Prog. Phys. 68, 131 (2005).