A multibody model of a wheel loader with pneumatic boom suspension and proving ground testing

N L Pavlov and D I Dacova
Technical University of Sofia, Faculty of Transport, Department of Combustion Engines, Automotive Engineering and Transport, Technical University of Sofia, 8 Kliment Ohridski Blvd., 1756 Sofia, Bulgaria
e-mail: npavlov@tu-sofia.bg

Abstract. In this paper a multibody dynamic model of a wheeled tractor-front-end loader system with pneumatic boom suspension is created. A front-end loader was designed and manufactured to validate the dynamic model and to conduct proving ground tests. Comparisons of some simulation results with the measured data revealed that the proposed dynamics model can well describe the vibrations of the tested compact wheel loader. In addition, the results of tests for RMS acceleration and spectral densities at different speeds are presented.

1. Introduction
Hitching the mounted type implements to the tractors due to increased their mass and their moments of inertia. These implements have particularly negative effect on increasing of the mass moment of inertia around the lateral axis. This leads to a reduction in the oscillation frequency around the lateral axle (pitch oscillation) and may increase their amplitudes when the tractor is moving. It causes a reduction in ride comfort performance, resulting in a faster occurrence of fatigue in the driver and a reduction in his ability to work. In addition, there are large variations in the dynamic forces of the wheels, sometimes resulting in loss of contact with the road. This reduces the stability, handling and road safety of the tractor. Tractors which are aggregated with a front loader are most commonly used in building construction. The moving of wheeled tractors between construction sites within a city or nearby towns and villages often takes place on their own, without using transport platforms. The movement is carried out at high speed (often the tractor maximum speed), resulting in large vertical and angular low frequency oscillations, exceeding the healthy doses. When the movement is carried out with a load in the bucket, angular oscillations with large amplitudes may result in the loss of part of the load. In order to improve the operation comfort of the wheel loader and reduce material falling off from the bucket, a type of vibration absorber systems are created [1]. Dynamic absorber systems with hydro-pneumatic accumulators, operating as elastic elements and damping in hydraulic cylinders, were initially proposed. These systems are passive and they require a compromise solution when choosing constant spring ratio (mass of the gas is constant) and damping for different loads in the bucket of the loader. In work [2] is noted that only passive oscillation systems are subject to mobile machine series production till then (2004). In order to fully solve the problem at different loads in the bucket, more complex mechatronic systems of valves and pressure regulators in hydraulic cylinders have to be used.

If instead of constant gas weight hydro-pneumatic accumulators are used such with a variable mass of filed gas, the system would be effective in the all loading range in the bucket of the loader. The advantage of the pneumatic system is that, by increasing the load while the volume of the pneumatic
element is constant, the natural frequencies of the load bucket vibrations remain constant and close to the natural frequencies of the pith oscillations of the tractor.

The aim of this work is to create multibody dynamic model and use it to study a compact wheeled tractor equipped with front-end loader on pneumatic suspension. Also to conduct proving ground tests. Therefore should be designed and made special front-end loader. The front-end loader will be attached on compact wheeled tractor with suspended axles. Some of the results from numerical experiment will be compared with the proving ground test results.

2. Multibody model of a wheel loader with pneumatic boom suspension

The model has eight degrees of freedom (8 DOF) and consists of a tractor sprung mass, four unsprung masses and mass of the loader bucket (figure 1). The vertical, pitch and roll motions of the sprung mass are considered. The four unsprung masses are connected to the corners of the tractor body.

\[
q = \begin{bmatrix} z_0 & \varphi_0 & \theta_0 & \dot{\theta}_f & z_{t1} & z_{t2} & z_{t3} & z_{t4} \end{bmatrix}^T.
\]

(1)

To find the equations of motion for the dynamic model, we use the Lagrange method. The kinetic energy of the system is:

\[
T = \frac{1}{2} m_0 \dot{z}_0^2 + \frac{1}{2} I_{0x} \dot{\varphi}_0^2 + \frac{1}{2} I_{0y} \dot{\theta}_0^2 + \frac{1}{2} m_f l_f^2 \dot{\theta}_f^2 + \frac{1}{2} m_{tf} \dot{z}_{t1}^2 + \frac{1}{2} m_{tf} \dot{z}_{t2}^2 + \frac{1}{2} m_{tr} \dot{z}_{t3}^2 + \frac{1}{2} m_{tr} \dot{z}_{t4}^2.
\]

(2)

The potential energy is:

\[
\Pi = \frac{1}{2} k_f (\theta_f l_1 - \theta_0 l_2)^2 + \frac{1}{2} k_f (z_0 - \varphi_0 \frac{d}{2} - \theta_0 a - z_{t1})^2 + \frac{1}{2} k_f (z_0 + \varphi_0 \frac{t}{2} - \theta_0 a - z_{t2})^2 + \frac{1}{2} k_r (z_0 + \varphi_0 \frac{d}{2} + \theta_0 b - z_{t3})^2 + \frac{1}{2} k_r (z_0 - \varphi_0 \frac{t}{2} + \theta_0 b - z_{t4})^2 + \frac{1}{2} k_{tf} (z_{t1} - h_1)^2 + \frac{1}{2} k_{tf} (z_{t2} - h_2)^2 + \frac{1}{2} k_{tr} (z_{t3} - h_3)^2 + \frac{1}{2} k_{tr} (z_{t4} - h_4)^2.
\]

(3)

And the Rayleigh dissipation function is:
Applying Lagrange method

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial \Pi}{\partial \dot{q}_i} = H_i \quad i = 1, 2 \ldots 8,
\]

provides us with following equations of motion:

**Differential equation of vertical motion of tractor body:**

\[
m_0 \ddot{z}_0 + (2k_f + 2k_r)z_0 + (2k_f b - 2k_f a)\theta_0 - k_f z_{t1} - k_r z_{t2} - k_r z_{t3} - k_r z_{t4} + (2c_f + 2c_r)\dot{z}_0 + (2c_r b - 2c_p a)\dot{\theta}_0 - c_f \dot{z}_{t1} - c_f \dot{z}_{t2} - c_r \dot{z}_{t3} - c_r \dot{z}_{t4} = 0.
\]

**Differential equation of roll motion of tractor body:**

\[
J_{0x} \ddot{\theta}_0 + \left( 2k_f \frac{d^2}{4} + 2k_r \frac{d^2}{4} \right) \phi_0 + k_f \frac{d}{2} \dot{z}_{t1} - k_f \frac{d}{2} \dot{z}_{t2} - k_r \frac{d}{2} \dot{z}_{t3} + k_r \frac{d}{2} \dot{z}_{t4} + \left( 2c_f \frac{d^2}{4} + 2c_r \frac{d^2}{4} \right) \phi_0 + c_f \frac{d}{2} \dot{z}_{t1} - c_f \frac{d}{2} \dot{z}_{t2} - c_r \frac{d}{2} \dot{z}_{t3} + c_r \frac{d}{2} \dot{z}_{t4} = 0.
\]

**Differential equation of pitch motion of tractor body:**

\[
J_{0y} \ddot{\theta}_0 + (2k_r b - 2k_f a)z_0 + (2k_f l_1^2 + 2k_f a^2 + 2k_r b^2)\theta_0 - 2k_f l_1 z_{t1} - 2k_f l_2 \theta_{f1} + k_f a z_{t1} + k_f b z_{t3} - k_r b z_{t4} + (2c_r b - 2c_f a)\dot{z}_0 + \left( 2c_f l_1 + 2c_f a^2 + 2c_r b^2 \right) \dot{\theta}_0 - 2c_f l_1 \dot{z}_{t1} + c_f a \dot{z}_{t1} + c_f a \dot{z}_{t2} = 0.
\]

**Differential equation of angular motion of loader boom:**

\[
m_{fi l_1^2} \ddot{\theta}_{f1} - 2k_f l_1 l_2 \theta_0 + 2k_f l_1^2 \theta_{f1} - 2c_f l_1 l_2 \dot{\theta}_0 + 2c_f l_1 l_2 \dot{\theta}_{f1} = 0.
\]

**Differential equation of vertical motion of left-side front tire:**

\[
m_{tf} \ddot{z}_{t1} - k_f z_0 + k_f \frac{d}{2} \phi_0 + k_f a \theta_0 + (k_f + k_{tf})z_{t1} - c_f \dot{z}_0 + c_f \frac{d}{2} \phi_0 + c_f a \dot{\theta}_0 + (c_f + c_{tf}) \dot{z}_{t1} = k_{tf} h_1 + c_{tf} \dot{h}_1.
\]

**Differential equation of vertical motion of right-side front tire:**

\[
m_{tf} \ddot{z}_{t2} - k_f z_0 + k_f \frac{d}{2} \phi_0 + k_f a \theta_0 + (k_f + k_{tf})z_{t2} - c_f \dot{z}_0 - c_f \frac{d}{2} \phi_0 + c_f a \dot{\theta}_0 + (c_f + c_{tf}) \dot{z}_{t2} = k_{tf} h_2 + c_{tf} \dot{h}_2.
\]

**Differential equation of vertical motion of right-side rear tire:**

\[
m_{tf} \ddot{z}_{t3} - k_r z_0 - k_r \frac{d}{2} \phi_0 - k_r b \theta_0 + (k_r + k_{tr})z_{t3} - c_r \dot{z}_0 - c_r \frac{d}{2} \phi_0 - c_r b \dot{\theta}_0 + (c_r + c_{tr}) \dot{z}_{t3} = k_{tr} h_3 + c_{tr} \dot{h}_3.
\]
Differential equation of vertical motion of left-side rear tire:

\[
m_r \ddot{z}_{t4} - k_r z_0 + k_r \frac{d}{2} \varphi_0 - k_r b \theta_0 + (k_r + k_{tr}) \dot{z}_{t4} - c_r \ddot{z}_0 + c_r \frac{d}{2} \dot{\varphi}_0 \]

\[
- c_r b \theta_0 + (c_r + c_{tr}) \dot{z}_{t4} = k_{tr} h_4 + c_{tr} \dot{h}_4. \tag{13}
\]

The equations of motion may be written in a matrix form:

\[
[M] \ddot{q} + [C] \dot{q} + [K] q = [K_t] H + [C_t] \dot{H}, \tag{14}
\]

where \([M], [K] \) and \([C]\) are mass matrix, elastic matrix and dissipative matrix, respectively. \([K_t]\) is an 8×8 matrix with the following nonzero elements \([K_t]_{5,5} = [K_t]_{6,6} = k_{tf} \) and \([K_t]_{7,7} = [K_t]_{8,8} = k_{tr}.\)

The matrix \([C_t]\) has also 8×8 dimension and nonzero elements \([C_t]_{5,5} = [C_t]_{6,6} = c_{tf} \) and \([C_t]_{7,7} = [C_t]_{8,8} = c_{tr}.\)

\[
H = [0 \ 0 \ 0 \ 0 \ h_1 \ h_2 \ h_3 \ h_4]^T \tag{15}
\]

A sinusoidal function is used to describe the excitations caused by road surface \([3].\) Thus the functions to the all four tires are approximated by:

\[
h_1 = A \sin(\omega t); \tag{16}
\]

\[
h_2 = A \sin(\omega (t + a_1)); \tag{17}
\]

\[
h_3 = A \sin(\omega (t + a_2)); \tag{18}
\]

\[
h_4 = A \sin(\omega (t + a_1 + a_2)), \tag{19}
\]

where \(A\) and \(\omega\) are the amplitude and the frequency of the sinusoidal road disturbance, respectively. The parameter \(a_1\) indicates the time delay between the kinematic excitation function to two front tires, or to two rear tires, respectively. On the other hand, \(a_2 = \frac{l}{\nu}\) indicates the time delay between the kinematic excitation function to front-right and rear-right tires, where \(l\) is the wheelbase and \(\nu\) is the movement speed.

The natural frequency of the dynamical system is found by using the undamped and free vibration equations of motion:

\[
[M] \ddot{q} + [K] q = 0. \tag{20}
\]

The equations are presented in Cauchy normal form:

\[
\dot{y} + Ly = 0; \tag{21}
\]

\[
y = [y_1 \ y_2 \ \ldots \ \ y_{16}]^T = [q_1 \ q_2 \ \ldots \ \ q_8 \ \dot{q}_1 \ \ldots \ \dot{q}_8]^T; \tag{22}
\]

\[
L = \begin{bmatrix}
[M]^{-1} [O] & [M]^{-1} [K] \\
[I] & [O]
\end{bmatrix}, \tag{23}
\]

where \([I]\) is an identity matrix and \([O]\) is a zero matrix.

For vibration in case of kinematic disturbance:

\[
\dot{y} + Ly = Y; \tag{24}
\]

\[
L = \begin{bmatrix}
[M]^{-1} [C] & [M]^{-1} [K] \\
-[I] & [O]
\end{bmatrix}; \tag{25}
\]

\[
Y = \begin{bmatrix}
[M]^{-1} H(t) \\
[O]
\end{bmatrix}. \tag{26}
\]
In case of pneumatic suspension when the load is applied to an automatically leveling air spring, air is fed into the air spring until the original boom level is restored. This causes the mass of the air in the spring to increase. In order to keep the suspension at its original height, however, the volume of air in the spring must remain the same. The final air pressure in the spring required for a load increase lies on the polytropic line. The spring thus undergoes a linear, load-dependent change in stiffness:

$$k_{fl} = \frac{nA_e^2p}{V_0},$$  \hspace{1cm} (27)

where $k_{fl}$ is an air spring stiffness in static equilibrium ($n = 1$); $n$ – polytropic index ($1 < n < 1.4$), where $n = 1$ for a static (isothermal) spring event and $n = 1.4$ for a dynamic (adiabatic) spring event; $p$ – the absolute pressure is the sum of the ambient pressure and the internal pressure within the spring; $A_e$ – air spring effective area; $V_0$ – air spring volume in static equilibrium.

3. Numerical simulation and proving ground testing results

The numerical experiments were performed with MATLAB with the parameters given in table 1.

The natural frequencies with given in table 1 parameters and mass of the bucket – 100 kg and pressure in the boom air springs – 2 bar:

- 1.30 Hz – tractor vertical vibration ($z_0$);
- 1.20 Hz – tractor roll vibration ($\phi_0$);
- 0.75 Hz – tractor pitch vibration ($\theta_0$);
- 1.70 Hz – front loader boom angular vibration ($\theta_{fl}$);
- 7.70 Hz – wheel vertical vibration ($z_{t1}$ ÷ $z_{t4}$).

| Table 1. Numerical values of the system parameters. |
|---------------------------------|-----------------|--------------|-------------|
| Parameter                      | Symbol | Value | Unit |
| Sprung mass                    | $m_0$   | 1000  | kg    |
| Roll axis moment of inertia    | $J_{0x}$| 550   | kgm² |
| Pitch axis moment of inertia   | $J_{0y}$| 1100  | kgm² |
| Mass of the bucket             | $m_{fl}$| var.  | kg    |
| Unsprung mass (front/rear)     | $m_{tf}, m_{tr}$| 60 | kg    |
| Suspension spring stiffness (front/rear) | $k_{f}, k_{r}$ | 20000 | N/m   |
| Boom suspension spring stiffness | $k_{fl}$ | var. | N/m   |
| Tire spring stiffness (front/rear) | $k_{tf}, k_{tr}$ | 120000 | N/m   |
| Damping coefficient of suspension (front/rear) | $c_{f}, c_{r}$ | 1500 | Ns/m  |
| Damping coefficient of boom suspension | $c_{fl}$ | 800  | Ns/m  |
| Damping coefficient of tire (front/rear) | $c_{tf}, c_{tr}$ | 100 | Ns/m  |
| Distance between front axle and tractor mass centre | $a$ | 0.77 | m     |
| Distance between rear axle and tractor mass centre | $b$ | 0.43 | m     |
| Distance between boom joint and boom suspension | $l_3$ | 1.07 | m     |
| Distance between boom joint and tractor mass centre | $l_2$ | 0.33 | m     |
| Boom length                    | $l_{fl}$| 0.9   | m     |
| Axle track                     | $d$    | 1.33  | m     |
| Wheelbase                      | $l$    | 1.2   | m     |

The acceleration data collection was performed by conducting two types of tests – on single obstacle (speed bump or breakers) which is often used when testing cars [4] and on special track (figure 2).
Some test results and their comparison with the results from numerical simulations are shown in figure 3 and 4. There is good comparability in the magnitudes of accelerations but the additional harmonic components are not taken into account in the simulations.

**Figure 3.** Measured (a) and simulated (b) vertical acceleration of the sprung mass. Speed is 5.4 km/h, wave length is 0.15 m, unevenness amplitude 2 cm. Boom suspension air spring pressure is 2 bar.

**Figure 4.** Measured (a) and simulated (b) vertical acceleration of the sprung mass. Speed is 8.3 km/h, wave length is 0.15 m, unevenness amplitude 2 cm. Boom suspension air spring pressure is 2 bar.

Root mean square (RMS) acceleration is the most common index for the evaluation of the vibration intensity. RMS provides a time averaged value that can be considered as a good metric of the discomfort level for steady state vibrations [5].
\[ \text{RMS} = \left[ \frac{1}{t_D} \int_0^{t_D} a_w^2(t) \, dt \right]^{\frac{1}{2}}, \]  

(28)

where \( t_D \) is the duration of the measurement, \( a_w \) is the frequency weighted acceleration in m/s\(^2\) for translational vibration or rad/s\(^2\) for angular vibration [6].

The comfort criterion is assessed accordingly to the following guidelines [6]:

**Table 2. Comfort/discomfort ranges.**

| RMS acceleration, m/s\(^2\) | Reaction          |
|------------------------------|-------------------|
| Less than 0.315              | not uncomfortable |
| Between 0.315 and 0.63       | a little uncomfortable |
| Between 0.5 and 1.0          | fairly uncomfortable |
| Between 0.8 and 1.6          | uncomfortable      |
| Between 1.25 and 2.5         | very uncomfortable |
| Greater than 2               | extremely uncomfortable |

Note that the ranges overlap. It comes from the subjective perception of comfort. The assessment of the analysis should be from the down top on the table, for example a value of \( \text{RMS} = 1.3 \) would be assessed as “very uncomfortable” and not as “uncomfortable” [7]. Results for measured RMS vertical acceleration and spectral densities are shown below in figure 5 and figure 6 respectively.

**Figure 5.** RMS acceleration on tractor seat on average engine speed (a) and maximum engine speed (b), wave length is 0.15 m, unevenness amplitude 2 cm. Boom suspension air spring pressure is 2 bar.

**Figure 6.** Spectral density of acceleration on tractor seat on average engine speed and tractor speed 5.4 km/h (a) and maximum engine speed and tractor speed 14 km/h (b), wave length is 0.15 m, unevenness amplitude 2 cm. Boom suspension air spring pressure is 2 bar.
In figure 5 (a), the average engine speed is achieved by placing the manual throttle lever in the middle position, and in figure 5 (b) in the end position. Different speeds are achieved by shifting different gears. Figure 5 shows that in high engine speeds there are high RMS values at the same driving speed. Higher vibration intensity is a characteristic feature of engines with a small number of cylinders [8, 9]. The obtained results are close to the results presented in work [10]. Figure 6 shows that as the tractor speed and engine speed increases, both the maximum value of the spectral density and the overall level at different frequencies increase.

4. Conclusion
In this study an 8-DOF spatial multibody dynamic model of a wheeled tractor-front-end loader system was presented. A front-end loader on pneumatic suspension was designed and manufactured to validate the dynamic model. Comparisons of some simulation results with the measured data revealed that the proposed dynamics model can well describe the vibrations of the tested compact wheel loader. Simulations were performed on deterministic road surfaces without taking into account the engine disturbance. Therefore, no additional harmonic components are observed in the simulations. Tests show that at high movement speeds with a high crankshaft rotational speed, the engine contributes to additional vibrational interferences. The presented results for RMS acceleration values can be compared with those in the standards and to assess the comfort under different driving conditions. Future development of the present work will address the modelling and adding to the model of the tractor engine force disturbance, as well as a stochastic road excitation.

References
[1] Wang Z 2012 The simulation analysis and modeling of the coupling vibration absorber system for the wheel loader Applied Mechanics and Materials 121-126 pp 2358–62
[2] Rahmfeld R and Ivantysynova M 2004 An overview about active oscillation damping of mobile machine structure Int. J. of Fluid Power 5(2) pp 5–24
[3] Zhu Q and Ishitobi M 2006 Chaotic vibration of a nonlinear full-vehicle model Int. J. of Solids and Structures 43(3-4) pp 747–59
[4] Georgiev Z and Kunchev L 2019 Study of the stresses in the front suspension components of a carpassing over speed breakers IOP Conf. Ser.: Mater. Sci. Eng. 664 012012
[5] Gobbi M, Mastinu G, Pennati M and Previati G 2015 Farm tractor ride comfort assessment The Dynamics of Vehicles on Roads and Tracks Proc. of the 24th Symposium of the International Association for Vehicle System Dynamics (Graz, Austria) pp 125–36
[6] ISO 2631-1 1997 International Organization for Standardization, Mechanical vibration and shock–evaluation of human exposure to whole body vibration – part 1: General requirements
[7] Donegan M 2015 How do i measure whole body vibration? Prosig Noise & Vibration Blog Available on: http://blog.prosig.com/2015/03/20/how-do-i-measure-whole-body-vibration/ Accessed on: 08 May 2020
[8] Dimitrov E, Gigov B, Pantchev S, Michaylov P and Peychev M 2018 A study of hydrogen fuel impact on compression ignition engine performance MATEC Web Conf. 234 03001
[9] Pavlov N, Sokolov E, Dodov M and Stoyanov S 2017 Study of the wheel loader vibration with a developed multibody dynamic model MATEC Web Conf. 133 02007
[10] Melemez K and Tunay M 2010 An ergonomic evaluation on whole-body vibration of loading tractors in Turkish forestry FORMEC (Padova, Italy July 11-14)