Structural stability and electronic property in K$_2$S under pressure†

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In this work, the structures, phase sequence, and metallic properties of K$_2$S have been systematically explored. We confirm that the P6$_3$/mmc phase is the best possible candidate for the stable structure of K$_2$S at low pressure range. Although the phases of P6$_3$/mmc and Cmcm K$_2$S are semiconductors, two new structures of P6$_3$/mmm and P3m1 emerge with metallic characters at high pressures. The analyses of electronic localization functions reveal that the conductivity mainly comes from the electrons surrounding S atom chains, which supplies a potential way to improve the conductivity of sulfur to enhance the electrode recharge ability and rate capability in alkali sulfide battery under pressure.

Introduction

The storage and conversion of energy continue to be important to society. Batteries, which interconvert chemical and electrical energy, are widely used in industry and for consumer applications, e.g., appliances and laptop computers. The battery energy storage system, e.g. lead-acid battery, solid oxide fuel battery, solar battery, lithium ion battery and so on, are focused and studied for a long time. The lead-acid battery is the most-used battery system with advantages including large current discharge, rechargeable, and low-cost. On the other hand, the disadvantage of lead-acid battery is obvious, for examples, the heavy weight and pollution.1–3 Solid oxide fuel battery is an energy conversion device that can convert chemical energy directly into electricity without pollution, nevertheless, the high operating temperature limits its widely application in our daily life.4–6 Solar battery is excepted to have large-scale applications if the problem of low efficiency is solved.7–9 Lithium ion battery is an attractive battery for the property of portable type, high energy density, and non-pollution, and is widely used for digital products, electric vehicles, household appliances, etc.7–9 The principle of operation lithium ion battery is that Li is removed from an intercalation cathode, for example LiCoO$_2$ on charging and is inserted between sheets of carbon atoms in the graphite negative electrode; discharge reverses this process.10 Though the compound of Li$_2$S and Na$_2$S with high specific energy (energy per unit weight) and energy density (energy per unit volume) are candidates for applications in solid state batteries,11,12 it is confronted with major challenge, i.e., poor electrode recharge ability and limited rate capability owing to the insulating nature of sulfur and the solid reduction products.13

The ground-state properties of alkali metal sulfides of X$_2$S (X = Li, Na, K, Rb), e.g., elastic properties, electronic structure, and optical properties are widely studied.14–17 The calculated band structures show the Li$_2$S, K$_2$S, and Rb$_2$S are indirect bandgap materials, whereas Na$_2$S is a direct gap material.14 At the same pressure, bulk modulus of the alkali metal sulfides show a clear decrease with heavier mass of alkali atoms.15 The similar tendency of decrease with atomic mass of X in X$_2$S has been also reported on the elastic constants under the same pressure. When the pressure is enhanced, except for the linear decrease of the elastic constant of $C_{44}$, the bulk modulus and the elastic constant of $C_{11}$, $C_{12}$ increase in the crystals of Rb$_2$S, K$_2$S and Na$_2$S. Similar trends are also reported in the Li$_2$S except for the elastic constants $C_{44}$. It is reported that $C_{44}$ increases linearly with pressure which is related to the difference in the bonding nature of the bottom of the conduction band.14

K$_2$S has been extensively investigated by theoretical and experimental methods since it was first synthesized by Zintl et al.14 It is reported that K$_2$S can crystallize in the antifluorite structure, i.e. Fm$ar{3}$m at ambient pressure, then transforms to an orthorhombic crystal with the space group of Pnma at about 2.7 GPa.16 Theoretical prediction points out that Pnma crystal can transform to a distorted P6$_3$/mmc at 4.4 GPa.14,15 Nonetheless, the Pnma phase is not observed by experiment to date.19 In this work, four stable phases of P6$_3$/mmc, Cmcm, P6$_3$/mmm and P3m1 have been found at different pressures through ab initio ELocR code.20 The low-pressure phase of Pnma proposed by an experimental analysis19 is found unstable by our theoretical calculations. Based on the reported X-ray diffraction and our theoretical analysis, we update the low-pressure phase with a hexagonal crystal of P6$_3$/mmc. Our calculations reveal the
insulating character of the two phases of $P6_3/mmc$ and $Cmcm$, and the Raman spectra are subsequently analyzed for the experimental convenience. Electronic band structure and the partial density of states show the metallic properties of $P6_3/mmm$ and $P3m1$ at pressures. As shown in the chart of electronic localization functions (ELF), the delocalization of electrons around S atoms in the phases of $P6_3/mmm$ and $P3m1$ denotes the improvement of conductivity of sulfur in Li$_2$S battery under pressure.

**Computational methods**

The structural searching of K$_2$S has been carried out by *ab initio* ELocR code, and the further confirmation with high accuracy of structural relaxations and electronic localization functions (ELF) are completed by the Vienna *ab initio* simulation package (VASP code) with a plan wave cutoff energy of 1000 eV and Brillouin zone sampling grid with a spacing of $2\pi \times 0.04 \text{ Å}^{-1}$. Total energy calculations are performed using the VASP codes with the Perdew–Burke–Ernzerhof (PBE) parameterization of generalized gradient approximation (GGA) used to treat the exchange–correlation energy. The partial augmented wave (PAW) method is adopted with the PAW potentials where 2s2p and 3s3p4s are treated as valence electrons. The electronic self-consistent calculation is stopped until the energy convergence less than $1 \times 10^{-5}$ eV.

The electronic projected density of states and the electronic band structure are calculated by CASTEP code with the cutoff energy of 720 eV, GGA-PBE exchange–correlation functional, Brillouin zone sampling grid with a spacing of $2\pi \times 0.03 \text{ Å}^{-1}$, and norm-conserving pseudopotentials. The Raman spectra are obtained within the density functional theory (DFT) formalism, and PBE exchange–correlation functional with norm-conserving pseudopotentials in the CASTEP code. The dynamical stability properties phonon calculations are carried out by a finite displacement approach through the PHONOPY code. The supercell are: $2 \times 2 \times 2, 2 \times 2 \times 3, 2 \times 2 \times 3, 3 \times 3 \times 3$ for $P6_3/mmc$, $Cmcm$, $P6_3/mmm$, and $P3m1$, respectively. The force constants are calculated from forces on atoms at atomic finite displacements, and the finite displacement size is 0.01 Å in this work.

**Results and discussion**

We have performed extensive structural searches on K$_2$S compounds at selected pressures of 0, 50, 100, 150, and 200 GPa. Five phases with lowest enthalpy, i.e. $Fm\bar{3}m$, $P6_3/mmc$, $Cmcm$, $P6_3/mmm$, and $P3m1$, are predicted as presented in Fig. 1. According to our enthalpy curves, the K$_2$S can crystallize in a face-centered cubic lattice with the space group of $Fm\bar{3}m$ at ambient pressure, and keep the symmetry up to 3 GPa. The $Fm\bar{3}m$ phase has been synthesized experimentally at ambient pressure and reported by Zintl *et al.* as shown in Fig. 2(a). The unstable phase $Pm\bar{3}m$ is also rebuilt in Fig. 2(b) for the convenience of comparison. Elevating pressure above 3 GPa, a hexagonal phase with $P6_3/mmc$ symmetry emerges and is stable till 88 GPa, it has been reported by experiment. The layer stacking sequence in the c-axis can be denoted by the repeated ABAC stacking. K atoms occupy the crystallographic 2a position with 3m symmetry along a axis in A layers, the other nonequivalent K atoms occupy the crystallographic 2d position with 6m2 symmetry, locating in B and C layers. S atoms occupy the crystallographic 2c position with 6m2 symmetry and locate at C and B positions in B and C layers respectively, as shown in Fig. 2(c). The base-centered orthorhombic phase of $Cmcm$ is found with the lowest enthalpy in the pressure range of 88 GPa to 136 GPa, which is about 0.03 eV lower than another body-centered orthorhombic structure of $Immm$ per K$_2$S at 100 GPa. There are two nonequivalent atoms in the crystal lattice, and thus consists of one K$_2$S unit as shown in Fig. 2(d). K atoms occupy the crystallographic 8g position (site symmetry is ..m) while S atoms occupy the crystallographic 4c position (site symmetry is m2m). Each K is surrounding by four S atoms to form a triangular pyramid geometry, and the distance between S and nearest K is 2.492 Å. After 136 GPa, another hexagonal crystal with $P6_3/mmm$ symmetry takes over $Cmcm$ phase and becomes most competitive on enthalpy up to 163 GPa. K atoms occupy the crystallographic 2d position with 6m2 symmetry, S atoms occupy the crystallographic 1a position with 6/mmm symmetry. There, K atoms form an interesting graphene-like layered structure in the ab plane, and the distance between S and nearest K is 2.590 Å as shown in Fig. 2(e). At high pressure range from 163 GPa to 200 GPa, a trigonal phase of $P3m1$ obtains the minimum enthalpy, which includes two nonequivalent atoms of K and S consisting of layered K–S geometry. The layer stacking sequence in the c-axis can be denoted by the repeated ABA stacking. K atoms occupy the crystallographic 2d position with 3m symmetry along b axis in A layers. S atoms occupy the crystallographic 1b position with 3m symmetry, locating in layers B, as shown in Fig. 2(f). The calculated structural parameters, volume, and Wyckoff positions for each phase are summarized in Table 1.

In order to describe the response of a material to an externally applied stress, the single-crystal elastic constants of...
The structures of K$_2$S. (a) The Fm$ar{3}$m phase at ambient pressure, (b) the unstable Pmma phase at 4.4 GPa, (c) the P6$_3$/mmc phase at 10 GPa, (d) the Cmcm phase at 100 GPa, (e) the P6/mmm phase at 150 GPa, (f) the P3m1 phase at 200 GPa. Purple and yellow atoms are K and S, respectively.

Table 1 Structural information of K$_2$S phases

| Space group | Pressure (GPa) | Volume ($\text{Å}^3$/K$_2$S) | Lattice parameter (Å) | Atom (Wyckoff) | Atomic coordinates |
|-------------|---------------|-----------------------------|------------------------|----------------|--------------------|
| P6$_3$/mmc   | 10            | 66.86821                    | $a = b = 4.9652$       | S(2c)          | X: 0.33333, Y: 0.66667, Z: 0.25000 |
|             |               |                             | $c = 6.2634$           | K(2a)          |                                 |
|             |               |                             |                        | K(2d)          |                                 |
| Cmcm         | 100           | 37.7420125                  | $a = 7.7793$           | S(4c)          | X: 0.00000, Y: -0.70000, Z: -0.25000 |
|             |               |                             | $b = 4.4456$           | K(8g)          |                                 |
|             |               |                             | $c = 4.3653$           | S(1a)          | X: 0.00000, Y: 0.00000, Z: 0.00000 |
| P6/mmm       | 150           | 32.81625                    | $a = b = 3.9710$       | S(1b)          | X: 1.00000, Y: 1.00000, Z: -0.50000 |
|             |               |                             | $c = 2.3994$           | S(1b)          |                                 |
| P3m1         | 200           | 29.64067                    | $a = b = 3.9022$       | S(1b)          | X: 2.33333, Y: 1.66666, Z: -0.90932 |
|             |               |                             | $c = 2.2477$           | S(1b)          |                                 |

Proposal structures are calculated and analyzed, as shown in Table 2. The matrix of elastic constants $C_{ij}$ must be positive definite,$^{39}$ it is worth noting that negative values are not prohibited for $C_{ij}$. For hexagonal structure P6$_3$/mmc and P6/mmm, there are five independent elastic constants ($C_{11}$, $C_{12}$, $C_{13}$, $C_{33}$, and $C_{44}$), they all satisfy the inequality $C_{44} > 0$, $C_{11} > |C_{12}|$, $C_{11} + 2C_{12}C_{33} > 2C_{11}^2$, it confirms that P6$_3$/mmc and P6/mmm are mechanically stable. As to orthorhombic structure Cmcm, independent elastic constants $C_{22}$, $C_{33}$, $C_{44}$, $C_{55}$, $C_{66}$, $C_{12}$, $C_{13}$, and $C_{34}$ meet the inequality $C_{11} > 0$, $C_{22} > 0$, $C_{33} > 0$, $C_{12} + C_{33} + 2[C_{12} + C_{13} + C_{23}] > 0$, $(C_{11} - C_{22} - 2C_{13}) > 0$, $C_{44} > 0$, $C_{55} > 0$, $C_{66} > 0$, $(C_{22} + C_{33} - 2C_{23}) > 0$, which conforms mechanical stability of Cmcm at corresponding pressure range. P3m1 belongs to trigonal system, the independent elastic constants also satisfy the inequality $C_{44} > 0$, $C_{11} > |C_{12}|$, $(C_{11} + 2C_{12})C_{33} > 2C_{11}^2$, indicating the mechanical stability of P3m1. So, the proposal structures all satisfy Born–Huang criterion,$^{39}$ representing the stability of the mechanical property.

The mechanically and thermodynamically stability of proposal phases have been discussed, and they are all stable at relevant pressure range. Furthermore, the phonon band structure and partial phonon density of states (PHDOS) for atoms with nonequivalent Wyckoff positions are calculated to judge its dynamical stability as shown in Fig. 3. The absence of imaginary frequency modes in the entire Brillouin zone indicates dynamic stability of the structures. From the PHDOS on nonequivalent atoms in P6$_3$/mmc crystal at 50 GPa as plotted in Fig. 3(a), we can see clearly that the low frequency modes mainly
dominated by the K atoms occupying the crystallographic 2a position, but the other nonequivalent K atoms occupying the crystallographic 2d position contribute more than K atoms with 2a position at high frequency modes. In the Cmcm phase, S atoms mainly contribute to the frequency upon 14 THz while the low frequency below 14 THz mainly comes from the vibrations of K atoms at 100 GPa, see the Fig. 3(b). At higher pressure range from 150 GPa to 200 GPa, there are no distinguishing characteristics in P6/mmm and P3m1 K2S except K atoms contribute more than S atoms as a whole because the number of K atoms is twice the number of S atoms in K2S crystals as shown in Fig. 3 (c) and (d).

There is a discussion needed with the proposal phase in low pressure range. Experiment proposes a crystal with the space group of Pnma at 4.4 GPa, which is not consistent with the structure of P63/mmc proposed in this work. We rebuild Pnma phase according to the space group and atomic coordinates given by the experiment report, and calculate the phonon dispersion and elastic constants. The matrix of elastic constants \( C_{ij} \) are as shown in ESI,† which contains \( C_{11} < 0, C_{55} < 0, C_{66} < 0, C_{11} + C_{22} - 2C_{12} < 0, C_{11} + C_{33} - 2C_{13} < 0 \). For the orthorhombic structure, elastic constants do not satisfy Born–Huang criterion,† representing the instability of the mechanical property. Phonon band structure is added in ESI."
Fig. 4  (a) Calculated difference X-ray power diffraction for P6_3/mmc and Pmma which is rebuilt though data from experiment at 4.4 GPa, (b) Raman active modes of P6_3/mmc phase, (c) experiment X-ray power diffraction for Pmma at 4.4 GPa, (d) Raman active modes of Cmcm phase.

Fig. 5  The electronic band structure and PDOS of K2S. (a) The electronic band structure of P6_3/mmc, (b) the electronic band structure and PDOS of P6/mmmm, (c) the electronic band structure of Cmcm, (d) the electronic band structure and PDOS of P3m1.
close peaks at 337 cm\(^{-1}\) and 341 cm\(^{-1}\) combine the most intense peak.

In order to analyze the electronic properties of phases \(P6_3/mmc\), \(Cmcm\), \(P6/mmm\), and \(P3m1\), the electronic band structure and the partial density of states are further explored in Fig. 5. Band gap is discovered in the \(P6_3/mmc\) phase at 50 GPa, revealing the nonmetallic character which coincides with previously report.\(^{34}\) The electronic band structure of \(Cmcm\) phase shows the semiconductor character, as displayed in Fig. 5(c). The band gap decreases with pressure from \(P6_3/mmc\) to \(Cmcm\) by contrast to the values of gaps at 50 GPa and 100 GPa in Fig. 5, respectively. Elevating the pressure to 150 GPa, three bands marked with magenta, green, and blue crossing over the Fermi level contribute large total electronic density distribution, and revel the strong metallic character of \(P6/mmm\), as displayed in Fig. 5(b). Three bands intersecting the Fermi level, the \(P3m1\) phase also behaves as a metal, and the bands colored by blue and magenta closing to Fermi level are found to contribute more to the density of states (DOS) of Fermi level, as displayed in Fig. 5(d). From the partial density of states (PDOS) in Fig. 5(b) and (d), the metallic character of \(P6/mmm\) and \(P3m1\) is confirmed by the high level of total electronic density distribution at Fermi level. The majority of occupied states come from \(K(p)\) state, whereas the contribution from \(K(s)\), \(S(s)\) and \(S(p)\) to the states around the Fermi level is quite small.

The electronic localization functions (ELF) is derived from an earlier idea of Lennard-Jones\(^{33}\) and first reported by A. D. Becke and K. E. Edgecombe.\(^{32}\) It has been calculated to characterize the degree of electron localization in the proposal \(K_2S\) crystals. ELF as defined runs from 0 to 1, the values of ELF equaling to 1.0 and 0.5 reflect the extremely strong localization and homogeneous electrons distribution, respectively.\(^{33,34}\) Three-dimensional electron location function (3D ELF) of \(P6_3/mmc\) and \(P3m1\) show free-electron channels with isosurface value of 0.5 (ELF = 0.5) around S atoms, which reveals that the conductivity comes from the contributions of the electrons around the S atoms as presented in Fig. 6(a) and (c). Furthermore, the two-dimensional electron location function (2D ELF) confirm the character of conductivity in two crystals, where connected regions with the ELF equaling to 0.5 are surrounding S atoms as shown in Fig. 6(b) and (d). Potassium is an alkali element and always described as a typical free-electron-like metal at ambient pressure; on the contrary, sulfur is often described as a good insulator at the same condition. Nevertheless, free electrons assemble around only S atom chains demonstrating the conductivity, and not K atoms under high pressure. At ambient pressure, \(K_2S\) is crystallized in face-centered cubic antifluorite structure with \(Fm\bar{3}m\) symmetry which is an ionic crystal\(^{32}\) and performed insulator characters. Nevertheless, the two phases of \(P6/mmm\) and \(P3m1\) exhibit metallic features as discussed above, which implies the available improvement of the conductivity of sulfur to enhance the electrode recharge ability and rate capability in alkali sulfide battery under pressure.

**Conclusion**

In summary, we have systematically explored phase sequence of \(K_2S\) at pressures from ambient pressure to 200 GPa using the \textit{ab initio} ELocR, and stable structures of \(Fm\bar{3}m\), \(P6_3/mmc\), \(Cmcm\), \(P6/mmm\), and \(P3m1\) have been explored thoroughly. Based on enthalpies, phonon band structure, elastic constants, and X-ray power diffraction, we confirm that the reported \(Pmmm\) structure is unstable and can distort into \(P6_3/mmc\) by our calculations. Moreover, we also simulate the Raman spectroscopies of \(P6_3/mmc\) and \(Cmcm\) phases for the convenience of the further studies of experiments. The two phases of \(P6_3/mmc\) and \(Cmcm\) \(K_2S\) are semiconductors under pressure, \(P6/mmm\) and \(P3m1\) emerge with the nature of metallic characters at higher pressures. The analyses of ELF show that the conductivity comes from the electrons surrounding S atom chains, which implies the potential improvement of the conductivity of sulfur to enhance the electrode recharge ability and rate capability in alkali sulfide battery under pressure.

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20. A code for crystal structural prediction and analysis is developed by our group. Some details have been provided in ESI†