THE EFFECT OF THE ACTIVE RESISTANCE OF THE PULSE TRANSFORMER WINDINGS ON THE PARAMETERS OF VOLTAGE PULSES GENERATED ON A CAPACITIVE LOAD

Goal. Analysis of the influence of the active resistance of the primary and secondary windings of a pulse transformer on the voltage at the load capacitance based on the developed methodology for the analysis of transients caused by the discharge of the storage capacitance in the primary winding. Methodology. A model for calculating transients is developed using the Laplace transform. Transient modeling is carried out in the MATLAB software package. The results of transient calculations are compared with experimental results. Results. A method for calculating transients in test installations with pulse transformers has been developed, which allows taking into account the effect of power losses in the primary and secondary windings on the voltage at the load capacitance. The calculated relations are obtained, allowing to take into account the influence of the active resistance of the primary and secondary windings of the transformer on the voltage at the load capacitance, the currents in the primary and secondary windings of the transformer, as well as on the voltage drop on the inductance of the primary winding of the transformer. Scientific novelty. A mathematical model is developed for calculating transients in the primary and secondary windings of a pulse transformer, taking into account the influence of the active resistance of the windings when it changes over a wide range of possible values. Practical value. Using the proposed technique, it is possible to determine the parameters of the discharge circuit at which test voltage pulses are formed on the load capacitance without distorting the shape of the pulse front.

References 14, figures. 5.

Key words: pulse transformer, capacitive load, winding active resistance, test voltage pulse, electrical insulation test.

Introduction. The problem of controlling the stability of insulation of electric power equipment in relation to overvoltages due to various causes is usually solved by using pulse voltage generators. Such generators can be developed using the widespread Arkadyev-Marx scheme [1], which, when applied, implies the charge of electric capacitors when they are connected in parallel, followed by discharge when connected in series. An example of the practical application of the mentioned example of the practical application of the mentioned...
method for generating test pulses is given in [2], which describes a generator with stored energy up to 0.48 MJ, for generating voltage pulses with amplitude of up to 3 MV. This approach allows to simulate overvoltage pulses that occur as a result of lightning strikes, as well as switching overvoltages. A detailed description of the metrological equipment used in the practice of forming high-voltage voltage pulses is given in [3]. Although the use of Marx generators allows the generation of voltage pulses with sufficient amplitude levels and time characteristics that are satisfactory for practical purposes, the practical implementation of such schemes leads to certain difficulties, primarily due to the need to use a significant number of arresters [4].

Another widespread approach that is used in the practice of generating high-voltage pulses is based on the implementation of various circuits, which involve amplifying voltage pulses to the required level using pulse transformers. A typical example is the pulse transformer with a magnetic core consisting of 68 ferrite rods described in [5]. In some technical applications, certain advantages can be obtained by using air transformers, since transformers of this type do not require additional demagnetization circuits, which are usually used to ensure maximum magnitude of magnetic flux density in the core [6].

One of the most common problems for high-voltage installations with pulse transformers is the need to determine the voltage at the load capacitance in a wide range of its values. In the case of using pulse transformers with magnetic cores, relatively small values of the open circuit current in some cases allow mathematical analysis of the discharge of the storage capacitor, neglecting the value of the magnetization inductance. The results of mathematical modelling of discharge processes of a storage capacitor on the primary circuit of a pulse transformer with a magnetic core, performed in [4], showed that an increase in the load capacitance leads to a decrease in the voltage across it. In the case of using an air transformer, its analysis is often carried out without taking into account the active resistance of the primary and secondary windings. A detailed analysis of the transient in a pulse transformer, taking into account the influence of energy losses in the primary and secondary windings on the voltage value at the load capacitance, was performed in [8]. However, the solution of a differential equation of the 4\textsuperscript{th} order, which determines the shape of the current in the primary and secondary windings, was obtained in the form in which the existence of only complex conjugate roots of the characteristic equation is implied. These types of roots usually occur in the case of analysis of circuits with a sufficiently high quality factor. Therefore, the scope of the mentioned analysis is limited by the range of problems that occur in the case of rather insignificant losses in the primary and secondary windings. Although such an analysis is sufficient for the vast majority of practical cases, an increase in losses in the primary and secondary windings can lead to other solutions of the characteristic equation. Obviously, such pulse transformers will have degraded technical characteristics compared to transformers with reduced losses. Nevertheless, if it is necessary to generate voltage pulses with certain requirements for the duration of the front and the cutoff of the pulse, for example, when forming voltage pulses in a shape close to aperiodic, circuits with a reduced quality factor may be of some interest. The increase in active resistance allows to reduce or completely eliminate the distortion of voltage pulses, which are caused by oscillatory processes in electrical circuits with high quality factor. Therefore, for some cases, it is preferable to develop a more universal solution that allows to analyze transients in the primary and secondary windings of a pulse transformer for a wider range of power losses in the windings. Such a problem was also considered in [9]; however, the presented solutions, similarly to the results of [8], describe the case of weakly damped oscillations, which usually occur in the case of relatively insignificant losses in the primary and secondary windings. In addition, issues related to determining the voltage across the capacitance of the tested object are not addressed in [9]. The expression for the voltage at the load capacitance in operator form and general form is given in [10]. However, the original of this expression was determined for its simplified form, in which the value of the active resistance of the windings was not taken into account. The solutions given in [11] take into account the influence of the secondary active resistance on the voltage on the capacitance of the secondary circuit, but the analysis was carried out for the case of primary circuit excitation by harmonic voltage. In this paper, attention is focused on the case of primary circuit excitation by discharging the storage capacitance. A detailed analysis of the conditions for obtaining maximum voltages on the capacitance of the secondary circuit without taking into account the influence of the active resistance of the windings of the primary and secondary circuits on the temporal characteristics of the voltage was performed in [12]. There are no publications that take into account the attenuation of the voltage at the load capacitance associated with the parameters of the primary and secondary circuits [13]. The relations given in [13] for the voltage on the electric capacitance of the secondary winding obtained after such an analysis are also based on the consideration of the oscillation voltage on it. Thus, the issues of the formation of test voltage pulses at the load capacitance close to the aperiodic shape are closely related to the results given in [8, 13]. Nevertheless, if it is necessary to form such pulses, the analysis should be carried out for the case of more significant values of the active resistance of the windings, which lead to a different type of roots of the characteristic equation.

The goal of the work is an analysis of the influence of the active resistance of the primary and secondary windings of a pulse transformer on the voltage at the load capacitance based on the developed technique for the analysis of transients caused by the discharge of the storage capacitance in the primary winding.

Analysis of the equivalent circuit of a pulse transformer. Transient analysis is performed for the transformer’s equivalent circuit shown in Fig. 1.
In the equivalent circuit in Fig. 1, $C_1$, $C_2$ represent the capacitances of the capacitor in the primary winding (storage capacitor) and the load capacitance of the test object in the secondary winding, respectively; $R_1$, $R_2$ are the resistances of the primary and secondary circuit, respectively; $M$ is the mutual induction coefficient between the primary and secondary windings; $L_1$, $L_2$ are the inductances of the primary and secondary windings, respectively.

The analysis is carried out under the assumption of an insignificant parasitic capacitance of the primary and secondary windings (see Fig. 1). In resonance mode, the equality holds

$$L_1C_1 = L_2C_2.$$  \hspace{1cm} (1)

In this case, the equivalent circuit (Fig. 1) is actually a Tesla transformer’s equivalent circuit. For the case of negligibly small active resistances of the primary and secondary windings and the previously given equality (1), which determines the relationships between the inductances $L_1$, $L_2$ and capacitances $C_1$, $C_2$, the voltage at the load capacitance $C_2$ can be determined using the following expression [14]:

$$U_2 = \frac{U_1}{2} \sqrt{\frac{C_1}{C_2}} \left[ \cos \left( \frac{\omega_0}{\sqrt{1-k}} \right) - \cos \left( \frac{\omega_0}{\sqrt{1+k}} \right) \right].$$  \hspace{1cm} (2)

where $k$ is the coupling coefficient between primary and secondary windings, $\omega_0$ is the natural frequency of oscillations of the primary and secondary windings [14]:

$$\omega_0 = \frac{1}{\sqrt{L_1C_1}} = \frac{1}{\sqrt{L_2C_2}}.$$  \hspace{1cm} (3)

Applying the Laplace transform to the expressions for the voltage drop across the elements of the equivalent circuit (Fig. 1), we can obtain relations for determining the current in the secondary winding in the operator form:

$$(pL_1 + R_1 + \frac{1}{pC_1})i_1 - pMi_1 = \frac{U_{c1}}{p},$$  \hspace{1cm} (4)

$$(pL_2 + R_2 + \frac{1}{pC_2})i_2 - pMi_2 = 0,$$  \hspace{1cm} (5)

where $u_{c1}$ is the voltage across the storage capacitor at the beginning of the transient.

Taking the expressions (4), (5), we can write the expressions for the current in the secondary winding:

$$i_2(p) = \frac{p^2Mu_{c1}C_2}{ap^4 + bp^3 + cp^2 + dp + 1},$$  \hspace{1cm} (6)

where constants $a$, $b$, $c$, $d$ can be determined using the following expressions:

$$a = L_2C_2L_1C_1 - M^2C_1C_2,$$  \hspace{1cm} (7)

$$b = L_2C_2R_1C_1 + R_2C_2L_1C_1,$$  \hspace{1cm} (8)

$$c = L_2C_2 + R_2C_2R_1C_1 + L_1C_1,$$  \hspace{1cm} (9)

$$d = R_2C_2 + R_1C_1.$$  \hspace{1cm} (10)

In accordance with the usual scheme of applying the Laplace transform, the expression for the dependence of the current in the secondary circuit on time can be written in the general form:

$$i_2(t) = \sum_{n=1}^{4} \frac{N(p_n)}{M'(p_n)} e^{p_n t},$$  \hspace{1cm} (11)

where all $N(p_n)$ are the numerator values in formula (6) at the points that correspond to the roots of the polynomial in the denominator of (6), and all $M'(p_n)$ are the values of the derivative of the polynomial in the denominator of the expression (6) at the points corresponding to the zeros of this denominator.

Thus, assuming that the load capacitance at the beginning of the transient is not charged, the voltage on it can be found using the following expression:

$$u_{c2}(t) = \frac{1}{C_2} \int_0^t i_2(t) dt = \frac{1}{C_2} \sum_{n=1}^{4} \frac{N(p_n)}{M'(p_n)} \left( e^{p_n t} - 1 \right).$$  \hspace{1cm} (12)

Since the transfer of energy from the primary circuit to the secondary one is made by inductive coupling, the analysis of transients must be performed taking into account the time dependence of the voltage on the inductance of the primary winding. Taking into account relations (4), (5), the expression for the current in the primary circuit can be written in the form:

$$i_1(p) = \frac{u_{c1}(p^2L_2C_2 + pR_2C_2 + 1)}{ap^4 + bp^3 + cp^2 + dp + 1}.$$  \hspace{1cm} (13)

The currents in the primary and secondary circuits are determined by expressions in which the denominator is the same (compare (6) and (13)). This circumstance makes it possible to simplify the simulation of transients caused by the discharge of capacitor $C_1$. Taking into account (13), the expression for the dependence of the current in the primary winding on time can be written in the form:

$$i_1(t) = \sum_{n=1}^{4} \frac{W(p_n)}{M'(p_n)} e^{p_n t},$$  \hspace{1cm} (14)

where $W(p_n)$ represent the values of the numerator from (13) at the points that correspond to the roots of the polynomial from the denominator (6) and (13).

Taking into account the expression (14), the voltage drop across the inductance of the primary winding $L_1$ can be determined using the following expression:

$$u_{L_1}(t) = L_1 \sum_{n=1}^{4} \frac{W(p_n)}{M'(p_n)} p_n e^{p_n t}.$$  \hspace{1cm} (15)

Taking into account (11), (14), as well as (4), (5), the expressions for the voltages on the load capacitance (12) and the inductance of the primary winding (15) can be written in the form:
Expressions (16), (17) can also be used to partially verify the previously given formulas for the voltage at the load capacitance and inductance of the primary winding.

Simulation results of transients in the primary and secondary windings caused by the discharge of the storage capacitor. Figures 2-5 show the results of transient calculations in the described model of the pulse transformer during its operation on the capacitive load. All calculations and measurements were carried out with the mutual induction between the primary and secondary windings \( M = 1.133 \times 10^{-4} \) H. The inductance of the primary winding is taken equal to \( L_1 = 186 \times 10^{-6} \) H. Secondary winding inductance: \( L_2 = 126 \times 10^{-6} \) H. Despite the fact that in practical applications such circuit parameters do not provide certain advantages from the point of view of technical characteristics, they can be used both to verify the described solutions and to determine the general trends of transients in the primary and secondary windings.

It can be seen from the above results that even in the case of the aperiodic discharge mode of the storage capacitor \( C_1 \) (calculation results in Fig. 3), which can be achieved by increasing the primary active resistance, the mathematical simulation does not allow the aperiodic shape of the voltage across the load capacitance to be obtained (results calculation in Fig. 4). In this case, at the aperiodic discharge mode of the storage capacitor, the absence of jumps that distort the shape of the voltage at the load capacity of the test object, which are clearly visible in Fig. 5 is not provided. Elimination of the illustrated distortions can be achieved by increasing the time constant of the \( RC \) circuit in the secondary circuit of the transformer, whose action on the shape of the voltage pulse is seen in Fig. 4.

Conclusions.

1. The described solutions obtained using the Laplace transform can be used if it is necessary to generate voltage pulses on the capacitive load that are close in shape to aperiodic by adjusting the active resistance values of the primary and secondary transformer windings.

2. An increase in the active resistance of the primary winding allows achieving an aperiodic discharge mode of the storage capacitor. However, even with such a discharge mode, it is not possible to provide an aperiodic
shape of voltage at the load capacitance. One of the possible ways to obtain voltage pulses at a capacitive load that is as close as possible to the aperiodic shape is to increase the active resistance of the primary winding to values providing an aperiodic discharge of the storage capacitance, followed by an increase in the active resistance of the secondary winding, to eliminate jumps that distort the shape of the voltage.

3. A negative consequence of the described approach to the formation of voltage pulses is the fact that the elimination of jumps that distort the shape of the voltage due to an increase in the active resistance of the windings is accompanied by an inevitable decrease in the amplitude of the voltage pulse at the capacitive load.

4. Using the above methodology for calculating transients allows controlling the active resistance of the windings in order to eliminate distortion of the front of the voltage pulse, generated at the capacitive load, by high-frequency oscillations.

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