p-ADIC PATH INTEGRALS FOR QUADRATIC ACTIONS

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Abstract

The Feynman path integral in p-adic quantum mechanics is considered. The probability amplitude $K_p(x'', t''; x', t')$ for one-dimensional systems with quadratic actions is calculated in an exact form, which is the same as that in ordinary quantum mechanics.

1. Introduction

In the last decade years p-adic numbers have been successfully applied to many parts of theoretical and mathematical physics (for a review, see, e.g. Refs. 1-3).

As is known, numerical experimental results belong to the field of rational numbers $\mathbb{Q}$, and $\mathbb{Q}$ is dense in $\mathbb{R}$ and in the field of p-adic numbers $\mathbb{Q}_p$ ($p$ is a prime number). $\mathbb{R}$ and $\mathbb{Q}_p$, for every $p$, exhaust all possible number fields which can be obtained by completing $\mathbb{Q}$.

Any $x \in \mathbb{Q}_p$ can be presented as an expansion

$$x = p^\nu(x_0 + x_1p + x_2p^2 + \cdots), \quad \nu \in \mathbb{Z}, \quad (1.1)$$

where $x_i = 0, 1, \cdots, p - 1$. p-Adic norm of a term $x_i p^{\nu+i}$ in (1.1) is $|x_i p^{\nu+i} |_p = p^{-(\nu+i)}$. Since p-adic norm is the non-archimedean (ultrametric) one, i.e. $|a + b |_p \leq \max\{|a |_p , |b |_p\}$, it means that the canonical expansion (1.1) has $|x |_p = p^{-\nu}$. There is no natural ordering on $\mathbb{Q}_p$, but one can define a linear order in the following way: $x < y$ if $|x |_p < |y |_p$, or when $|x |_p = |y |_p$, there exists such index $m \geq 0$ that digits satisfy $x_0 = y_0$, $x_1 = y_1, \cdots, x_{m-1} = y_{m-1}, x_m < y_m$.

There is an analysis over $\mathbb{Q}_p$ connected with map $\varphi : \mathbb{Q}_p \to \mathbb{Q}_p$, as well as another one related to map $f : \mathbb{Q}_p \to \mathbb{C}$, where $\mathbb{C}$ is the field of complex numbers. We use both of
these analysis. Derivatives\(^4\) of \(\varphi(x)\) are defined as in the real case, but with respect to the p-adic norm. In the case of mapping \(\mathbb{Q}_p \to \mathbb{C}\) there is well-defined integral with the Haar measure\(^2\). In particular, we use the Gauss integral\(^2\)

\[
\int_{\mathbb{Q}_p} \chi_p(\alpha x^2 + \beta x)dx = \lambda_p(\alpha) \mid 2\alpha \mid_p^{-1/2} \chi_p\left(-\frac{\beta^2}{4\alpha}\right), \quad \alpha \neq 0,
\]

where \(\chi_p(a) = \exp(2\pi i \{a\}_p)\) is an additive character and \(\{a\}_p\) is the fractional part of \(a \in \mathbb{Q}_p\), \(\lambda_p(x)\) is an arithmetic function, \(\lambda_p(0) = 1\) and if \(x \in \mathbb{Q}_p^*\) then

\[
\lambda_p(x) = \begin{cases} 
1, & \nu = 2k, \quad p \neq 2, \\
\left(\frac{a}{p}\right), & \nu = 2k + 1, \quad p \equiv 1(\text{mod } 4), \\
i\left(\frac{x_0}{p}\right), & \nu = 2k + 1, \quad p \equiv 3(\text{mod } 4),
\end{cases}
\]

\[
\lambda_2(x) = \begin{cases} 
\frac{1}{\sqrt{2}}[1 + (-1)^{x_1}i], & \nu = 2k, \\
\frac{1}{\sqrt{2}}(-1)^{x_1 + x_2}[1 + (-1)^{x_1}i], & \nu = 2k + 1,
\end{cases}
\]

where \(x\) is given by \((1.1), k \in \mathbb{Z}\), and \(\left(\frac{a}{p}\right)\) is the Legendre symbol. The properties

\[
\lambda_p(0) = 1, \quad \lambda_p(a^2x) = \lambda_p(x), \quad \lambda_p(x)\lambda_p(y) = \lambda_p(x + y)\lambda_p(x^{-1} + y^{-1}), \\
\lambda_p^*(x)\lambda_p(x) = 1
\]

will be used.

One of the greatest achievements in the use of p-adic numbers in physics is a formulation of p-adic quantum mechanics\(^5,6\). The elements of the corresponding Hilbert space \(L_2(\mathbb{Q}_p)\) are some complex-valued functions of a p-adic argument. Quantization is performed by the Weyl procedure. Instead of the Schrödinger equation, an eigenvalue problem and dynamical evolution of a system are defined by means of an unitary representation of the evolution operator \(U_p(t)\) on \(L_2(\mathbb{Q}_p)\). Like ordinary quantum mechanics the kernel \(K_p(x''; t''; x', t')\) of \(U_p(t)\) is defined as

\[
U_p(t'')\psi_p(x'') = \int_{\mathbb{Q}_p} K_p(x'', t''; x', t')\psi_p(x', t')dx'.
\]

As in the real case, \(K_p(x''t''; x', t')\) is called the quantum-mechanical propagator or the probability amplitude for a quantum particle to go from a space-time point \((x', t')\) to a space-time point \((x'', t'')\). Anyhow, \(K_p(x''; t''; x', t')\) is of central importance in both ordinary and p-adic quantum mechanics.

For one-dimensional systems with quadratic Lagrangians it has been assumed\(^7\)–\(^10\) that

\[
K_p(x'', t''; x', t') = N_p(t'', t')\chi_p\left(-\frac{1}{\hbar}\mathcal{S}(x'', t''; x', t')\right),
\]

\(2\)
where $N_p(t''; t')$ is a normalization factor, $\tilde{S}(x'', t''; x', t')$ is a p-adic classical action quadratic in $x''$ and $x'$, and $\hbar$ is the Planck constant. It has been shown that (1.7) satisfies the group property
\[
\int_{\mathbb{Q}_p} \mathcal{K}_p(x'', t''; x, t) \mathcal{K}_p(x, t; x', t') \, dx = \mathcal{K}_p(x'', t''; x', t'),
\] (1.8)
and $N_p(t''; t')$ has been also calculated in an explicit form for a harmonic oscillator\(^{5,8}\), a free particle\(^{5,8}\), a particle in a constant field\(^8\), a harmonic oscillator with time-dependent frequency\(^9\), a minisuperspace - the de Sitter model of the universe\(^{10}\), as well as for some other minisuperspace cosmological models\(^{11}\) with quadratic actions.

This Letter is devoted to p-adic generalization of the Feynman path integral approach to $\mathcal{K}(x'', t''; x', t')$ in ordinary quantum mechanics. It will be derived in an exact way for one-dimensional systems with quadratic classical actions, and it will be also shown that such $\mathcal{K}_p(x'', t''; x', t')$ has the same form as that in ordinary quantum mechanics.

2. p-Adic Path Integrals

Since 1948, when Feynman published\(^{12}\) his first paper on the path integral, it has been a subject of permanent interest in theoretical physics, and now it presents one of the best approaches to quantum theory. On the recent progress, present status and some future prospects of the path integral can be found Ref. 13.

In ordinary quantum mechanics, Feynman has postulated\(^{12}\) $\mathcal{K}(x'', t''; x', t')$ to be the path integral
\[
\mathcal{K}(x'', t''; x', t') = \int \exp \left( \frac{2\pi i}{\hbar} \mathcal{S}[q] \right) \mathcal{D}q ,
\] (2.1)
where $\mathcal{S}[q] = \int_{t'}^{t''} L(q, \dot{q}, t) \, dt$ is an action and $x'' = q(t'')$, $x' = q(t')$. The integral (2.1) symbolizes an intuitive understanding that a quantum-mechanical particle may propagate from $x'$ to $x''$ using uncountably many paths which connect these two points. Thus the Feynman path integral means a continual summation of single amplitudes $\exp \left( \frac{2\pi i}{\hbar} \mathcal{S}[q] \right)$ over all paths $q(t)$ connecting $x'$ and $x''$. In practical calculations it is the limit of an ordinary multiple integral of $n - 1$ variables $q_i = q(t_i)$ when $n \to \infty$. For the classical action $\tilde{S}(x'', t''; x', t')$ which is polynomial quadratic in $x''$ and $x'$ it has been shown (see, e.g. Ref. 14) that in ordinary quantum mechanics
\[
\mathcal{K}(x'', t''; x', t') = \left( \frac{i}{\hbar} \frac{\partial^2 \tilde{S}}{\partial x'' \partial x'} \right)^{1/2} \exp \left( \frac{2\pi i}{\hbar} \tilde{S}(x'', t''; x', t') \right). \] (2.2)
p-Adic generalization of (2.1) was suggested in Ref. 5 and one can write it in the form

\[ K_p(x''; t''; x', t') = \int \chi_p \left( -\frac{1}{h} S[q] \right) Dq = \int \chi_p \left( -\frac{1}{h} \int_{t'}^{t''} L(q, \dot{q}, t) dt \right) \prod_t dq(t), \quad (2.3) \]

where \( \chi_p(a) \) is p-adic additive character. In (2.3) we regard \( h \in \mathbb{Q} \) and \( q, t \in \mathbb{Q}_p \). In fact, an integral\(^{15,16} \int_{t'}^{t''} L(q, \dot{q}, t) dt \) is a difference of antiderivative of \( L(q, \dot{q}, t) \) in points \( t'' \) and \( t' \). However, \( dq(t) \) is the Haar measure and p-adic path integral is the limit of a multiple Haar integral.

The path integral (2.3) is elaborated, for the first time, for the harmonic oscillator\(^16\). In particular, it was shown that there exists the limit

\[ K_p(x''; t''; x', t') = \lim_{n \to \infty} K_p^{(n)}(x''; t''; x', t') = \lim_{n \to \infty} N_p^{(n)}(t'', t') \int_{\mathbb{Q}_p} \cdots \int_{\mathbb{Q}_p} \right. \]

\[ \times \chi_p \left( -\frac{1}{h} \sum_{i=1}^{n} \bar{S}(q_i, t_i; q_{i-1}, t_{i-1}) \right) dq_1 \cdots dq_{n-1}, \quad (2.4) \]

where \( N_p^{(n)}(t'', t') \) is the corresponding normalization factor for the harmonic oscillator. So obtained \( K_p(x''; t''; x', t') \) is in complete agreement with that earlier found in Refs. 5,8 by the method described in Introduction.

In the similar way the path integral has been recently calculated\(^17\) for a particle in a constant external field. Also it is done\(^18\) for the linear oscillator with time-dependent frequency. The obtained results confirm the form (1.7) and yield the corresponding \( N_p(t'', t') \).

Thus one can conclude that p-adic path integrals calculated for particular physical models give the results which resemble those known in ordinary quantum mechanics. This is the reason to look for a general expression in p-adic case which would be an analogue of (2.2) in the real one. Note that in addition to (1.8), the path integral must also satisfy

\[ \int_{\mathbb{Q}_p} K_p^x(x'', t''; x', t') K_p(z, t''; x', t') dx' = \delta_p(x'' - z), \quad (2.5) \]

\[ K_p(x'', t''; x', t') = \lim_{t'' \to t'} K_p(x'', t''; x', t') = \delta_p(x'' - x'), \quad (2.6) \]

where \( \delta_p(a - b) \) is the p-adic \( \delta \)-function\(^2\).

3. General Solution for Quadratic Actions

We will derive now a general solution of the p-adic Feynman path integral (2.3) for systems with Lagrangians \( L(q, \dot{q}, t) \), quadratic in \( q \) and \( \dot{q} \), and classical actions \( \bar{S}(x'', t''; x', t') \), quadratic in \( x'' \) and \( x' \).
Classical mechanics in p-adic case has the same analytic form as in the real one (see Refs. 15,16). An analytic solution of the Euler-Lagrange equation $\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = 0$ gives a p-adic classical path $\bar{q}(t)$. The corresponding classical action is

$$S[\bar{q}] = \bar{S}(x'', t''; x', t') = \int_{t'}^{t''} L(\bar{q}, \dot{\bar{q}}, t) dt, \quad x'' = \bar{q}(t''), \; x' = \bar{q}(t'). \quad (3.1)$$

A possible quantum path $q = q(t)$ can be presented as $q(t) = \bar{q}(t) + y(t)$ with conditions $y(t') = y(t'') = 0$. Since $\delta S[\bar{q}] = 0$, we can write for quadratic actions (Lagrangians) an expansion

$$S[q] = S[\bar{q} + y] = S[\bar{q}] + \frac{1}{2!} \delta^2 S[\bar{q}] = S[\bar{q}] + \frac{1}{2} \int_{t'}^{t''} \left( y \frac{\partial \bar{\varphi}}{\partial q} + \dot{y} \frac{\partial \bar{\varphi}}{\partial \dot{q}} \right)^{(2)} L(q, \dot{q}, t) dt. \quad (3.2)$$

Replacing $S[q]$ by (3.2) and $Dq$ by $Dy$ in (2.3) we have

$$K_p(x''; x', t') = \chi_p \left( -\frac{1}{h} S[\bar{q}] \right) \int_{t'}^{t''} \chi_p \left( -\frac{1}{2h} \int_{t'}^{t''} \left( y \frac{\partial \bar{\varphi}}{\partial q} + \dot{y} \frac{\partial \bar{\varphi}}{\partial \dot{q}} \right)^{(2)} L dt \right) \prod_t dy(t). \quad (3.3)$$

Since the remained path integral in (3.3) does not depend on $x'$ and $x''$ (see, e.g. Ref. 19) it follows that $K_p(x''; x', t')$ has the form (1.7).

Let us use the unitary condition (2.5) to get further information on $N_p(t'', t')$. After substitution (1.7) in (2.5) it becomes

$$| N_p(t'', t') |^2 = \chi_p \left[ \frac{1}{h} \bar{S}(x'', t''; x', t') - \frac{1}{h} \bar{S}(x'', x'; z, t'') \right] dx' = \delta_p(x'' - z), \quad (3.4)$$

where $| \cdot |_\infty$ denotes the usual absolute value. As a consequence of quadratic dependence on $x'', x'$ and $z$ one has

$$\bar{S}(x'', t''; x', t') - \bar{S}(x', t''; x', t') = (x'' - z) \frac{\partial}{\partial z} \bar{S}(0, t'', 0, t')$$
$$+ \frac{1}{2!} [(x'')^2 - z^2] \frac{\partial^2}{\partial z^2} \bar{S}(0, t'', 0, t') + (x'' - z)x' \frac{\partial^2}{\partial x'' \partial x'} \bar{S}(0, t'', 0, t'). \quad (3.5)$$

Replacing (3.5) in (3.4) we obtain

$$| N_p(t'', t') |^2 \chi_p \left[ \frac{1}{h} (x'' - z) \frac{\partial \bar{S}}{\partial z} + \frac{1}{2h} [(x'')^2 - z^2] \frac{\partial^2 \bar{S}}{\partial z^2} \right]$$
$$\times \delta_p \left( (x'' - z) \frac{1}{h} \frac{\partial^2 \bar{S}}{\partial x'' \partial x'} \right) = \delta_p(x'' - z). \quad (3.6)$$
Since \( \chi_p \) and \( \delta_p \)-function depend on \( x'' - z \) one can take \( \chi_p(\cdots) = 1 \) and (3.6) leads to

\[
| N_p(t'', t')|_\infty = \left| \frac{1}{h} \frac{\partial^2}{\partial x'' \partial x'} \bar{S}(x'', t''; x', t') \right|^{1/2}.
\] (3.7)

The general form of \( N_p(t'', t') \) in (1.7) is

\[
N_p(t'', t') = |N_p(t'', t')|_\infty A_p(t'', t') ,
\] (3.8)

where \( A_p(t'', t') \) is a complex-valued function of \( t', t'' \in Q_p \) and \( |A_p(t'', t')|_\infty = 1 \). To determine \( A_p(t'', t') \) we use the property (1.8) and the Taylor expansions of quadratic actions, e.g.

\[
\bar{S}(x'', t''; x', t') = \bar{S}(0, t''; 0, t') + \left( x'' \frac{\partial}{\partial x''} + x' \frac{\partial}{\partial x'} \right) \bar{S}(0, t''; 0, t') + \frac{1}{2!} \left( x'' \frac{\partial}{\partial x''} + x' \frac{\partial}{\partial x'} \right)^2 \bar{S}(0, t''; 0, t') .
\] (3.9)

The integration in (1.8) is performed by applying the Gauss integral (1.2). To satisfy (1.8), we have the following necessary and sufficient conditions:

\[
A_p(t'', t) A_p(t, t') \lambda_p(\alpha) = A_p(t'', t') ,
\] (3.10)

\[
\left| \frac{1}{h} \frac{\partial^2}{\partial x'' \partial x} \right|^{1/2} \left| \frac{1}{h} \frac{\partial^2}{\partial x \partial x'} \right|^{1/2} |2\alpha|^{-1/2} = \left| \frac{1}{h} \frac{\partial^2}{\partial x'' \partial x'} \right|^{1/2} ,
\] (3.11)

where

\[
\alpha = -\frac{1}{2h} \frac{\partial^2}{\partial x^2} \bar{S}(x'', t''; x, t) - \frac{1}{2h} \frac{\partial^2}{\partial x^2} \bar{S}(x, t; x', t') .
\] (3.12)

Note that there is also another condition induced by an equality between characters, but it does not affect (3.10) and (3.11), and therefore its consideration will be omitted here.

To satisfy (3.11) and to derive an expression for \( A_p(t'', t') \) in (3.10) one has to adopt some \( p \)-adic relations for derivatives of the classical action. It is natural to start with equation (3.11) whose all ingredients are determined. It can be rewritten in the form

\[
\left| \frac{-h \left( \frac{\partial^2}{\partial x^2} \bar{S}(x'', t''; x, t) \right) + \frac{\partial^2}{\partial x^2} \bar{S}(x, t; x', t') }{\frac{\partial^2}{\partial x'' \partial x} \bar{S}(x'', t''; x, t) \frac{\partial^2}{\partial x \partial x'} \bar{S}(x, t; x', t') } \right|_{p}^{-1/2}
\] (3.13)

\[
= \left| h \left( \frac{\partial^2}{\partial x'' \partial x'} \bar{S}(x'', t''; x', t') \right)^{-1} \right|_{p}^{-1/2} .
\]
The relations
\[
\frac{\partial^2}{\partial x^2} \bar{S}(x'', t''; x, t) + \frac{\partial^2}{\partial x'^2} \bar{S}(x, t' ; x', t')
= -u \left( \frac{\partial^2}{\partial x'' \partial x'} \bar{S}(x'', t''; x, t) + \frac{\partial^2}{\partial x \partial x'} \bar{S}(x, t' ; x', t') \right),
\]
\[\tag{3.14}\]
\[
\left( \frac{\partial^2}{\partial x'' \partial x} \bar{S}(x'', t''; x, t) \right)^{-1} + \left( \frac{\partial^2}{\partial x \partial x'} \bar{S}(x, t' ; x', t') \right)^{-1}
= v \left( \frac{\partial^2}{\partial x'' \partial x'} \bar{S}(x'', t''; x', t') \right)^{-1},
\]
\[\tag{3.15}\]
where \(u\) and \(v\) have for any particular \(p\) expansions:
\[
u = 1 + u_1 p + u_2 p^2 + u_3 p^3 + \cdots, \tag{3.16}\]
\[
v = 1 + v_1 p + v_2 p^2 + v_3 p^3 + \cdots, \tag{3.17}\]
satisfy condition (3.11). If \(u_1 = u_2 = v_1 = v_2 = 0\) for \(p = 2\) then by virtue of (1.3) and (1.4)
\[
\lambda_p(u x) = \lambda_p(v x) = \lambda_p(x) \tag{3.18}
\]
for every \(p\). Using condition (3.10), relations (3.14), (3.15), and properties (1.5), (3.18) of the \(\lambda_p\) function it follows
\[
A_p(t'', t') = \lambda_p \left( -\frac{1}{2h} \frac{\partial^2}{\partial x'' \partial x'} \bar{S}(x'', t''; x', t') \right). \tag{3.19}\]

Finally we obtained the kernel of an evolution operator
\[
K_p(x'', t''; x', t') = \lambda_p \left( -\frac{1}{2h} \frac{\partial^2 \bar{S}}{\partial x'' \partial x'} \right) \left| \frac{1}{h} \frac{\partial^2 \bar{S}}{\partial x'' \partial x'} \right|^{1/2} \chi_p \left( -\frac{1}{h} \bar{S}(x'', t''; x', t') \right) \tag{3.20}\]
as the solution of the p-adic Feynman path integral for quadratic actions.

One can easily verify (3.14) and (3.15) for a free particle, a particle in a constant field and a harmonic oscillator.

4. Concluding Remarks

As has been mentioned earlier the Feynman path integral for quadratic actions in the real case has an exact solution (2.2). It is worth noting that (2.2) can be rewritten in the form
\[
K_\infty(x'', t''; x', t') = \lambda_\infty \left( -\frac{1}{2h} \frac{\partial^2 \bar{S}}{\partial x'' \partial x'} \right) \left| \frac{1}{h} \frac{\partial^2 \bar{S}}{\partial x'' \partial x'} \right|^{1/2} \chi_\infty \left( -\frac{1}{h} \bar{S}(x'', t''; x', t') \right), \tag{4.1}\]
where index $\infty$ denotes the real case and

$$\lambda_{\infty}(0) = 1, \quad \lambda_{\infty}(a) = \frac{1}{\sqrt{2}}(1 - i \text{ sign } a), \quad a \in \mathbb{R}^*$$  \hspace{1cm} (4.2)

is also an arithmetic function with the same properties (1.5). An additive character on $\mathbb{R}$ is defined as $\chi_{\infty}(x) = \exp(-2\pi ix)$.

Let $v$ be an index which characterizes real and any of p-adic cases, i.e. $v \in \{\infty, 2, 3, 5, \cdots\}$. Then the Feynman path integral for a quantum-mechanical amplitude with quadratic classical actions can be written in ordinary and p-adic quantum mechanics in the same compact form

$$K_v(x'', t''; x', t') = \lambda_v \left( -\frac{1}{2h} \frac{\partial^2 \bar{S}}{\partial x'' \partial x'} \right) \left| \frac{1}{h} \frac{\partial^2 \bar{S}}{\partial x'' \partial x'} \right|^{1/2} \chi_v \left( \frac{1}{h} \bar{S}(x'', t''; x', t') \right).$$  \hspace{1cm} (4.3)

Note that $K_v(x'', t''; x', t')$ is a function of $t'' - t'$ if $L = L(q, \dot{q})$.

The expression (4.3) exhibits a generic aspects of quantum particle propagation in metric and ultrametric spaces. It once again underlines the fundamental role of the Feynman path integral method in foundation of quantum theory.

The obtained result (4.3) is also a solid basis for further elaboration of Adelic quantum mechanics (which unifies ordinary and p-adic ones), for an approximate (semiclassical) computation of the p-adic path integrals in the case of non-quadratic Lagrangians, and for a generalization to multi-dimensional systems.

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