Wear Modeling of Polyethylene Based Polymer for Lubricated Conditions

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Abstract. Generally, the wear test of HDPE or UHMWPE under lubricated conditions needs a long time duration to obtain a detectable wear loss, because of their low wear rates. In this paper, a theoretical model which couples elastohydrodynamic lubrication theory and Reye’s wear model is proposed to predict the long term wear of HDPE under lubricated conditions. Dependency of the wear on parameters including viscosity of lubricants, sliding velocity, roughness, coefficient of friction, and normal load are determined. This model is expected to reduce the time consumption of the long term wear test and may provide a helpful support for the biomedical application of the polyethylene based polymer.

1. Introduction
The wear modeling of polymeric materials under lubricated conditions is more complicated than that of dry sliding conditions, due to the complex nature of lubrication and the various affecting factors of the wear. However, the major cause of the revision of total joint replacements is the wear of the polyethylene component. A healthy natural joint works under effective lubricated condition with the synovial liquid as a lubricant resulting in very low friction (COF ranges from 0.003-0.024) [1, 2]. This value of COF is much lower than the values yielded using any artificial bearing materials in equivalent conditions. Therefore, lubrication is an important factor to estimate the wear of the potential bearing material in artificial joints. In addition, Archard’s law probably is not appropriate to be used in the wear of polymers under lubricated conditions. However, most of the published studies either neglected the lubrication or applied Archard’s law to estimate the wear of UHMWPE component in artificial joints. For example, Wang et al. [3] proposed a wear law for the UHMWPE under lubricated multi-directional sliding conditions without considering the influence of lubrication. In this paper, a wear modeling based on elastohydrodynamic lubrication theory is built up to predict the long term wear for lubrication conditions.

2. Wear modeling under lubricated conditions

2.1. Wear law
The ability to predict wear and lifespan of a material is essential for its tribological application, because the eventual purpose of tribological study is to protect surface against wear. The lubrication itself is a
just a method to decrease wear. Archard’s law is very frequently used in the wear modelling and can be expressed as:

\[
\frac{V}{L} = K_a \frac{N}{H}
\]  

(1)

Where \( V \) is the volume of the worn off material, \( L \) is sliding distance, \( K_a \) is a constant, \( N \) is the normal load, and \( H \) is the hardness of the softer material in the two rubbing bodies. The hardness, \( H \), is usually bundled up into a single coefficient \( K_a \). Both sides of the equation are divided by the contact area, then Equation (2) is:

\[
\frac{h}{L} = K_a p
\]

(2)

Where \( p \) is the contact pressure, and \( h \) is wear depth. The wear process is considered as dynamic and the wear depth change with respect to each distance increment, therefore, the first order differential form of Equation (2) is:

\[
\frac{\Delta h}{\Delta L} = K_a p(\Delta L)
\]

(3)

Here the contact pressure is a function of the sliding distance which is considered as the time in the dynamic process of wear.

However, Archad’s law may not be appropriate for the wear of polymer under lubricated conditions. A new proposed wear law is:

\[
\frac{V}{L} = K \bar{F}
\]

(4)

Where \( K \) is a constant and \( \bar{F} \) is the average friction force. As \( \bar{F} \) can be expressed as a product of the COF (\( \mu \)) and the normal load (\( N \)) based on the definition of COF, Equation (4) is rewritten as:

\[
\frac{V}{L} = K \bar{\mu} N
\]

(5)

Similar with Equation (3), the change of wear depth can be determined as:

\[
\frac{\Delta h}{\Delta L} = K \bar{\mu} p(\Delta L)
\]

(6)

Where \( \bar{\mu} \) refers as the average of COF. Although Equation (3) and (6) seem very similar with only difference of \( \bar{\mu} \) term, they represent two distinct methods. Equation (6) is believed to be appropriate to be used in lubricated conditions, as \( K \) in Equation (6) only represents the material properties. The different lubricants have significant influence on the COF which can be represented by the average COF. The COF under a specific lubricant is easily obtained or even measured in a very short wear test duration. If using Equation (1) in lubricated conditions, even with the same material, a specific \( K_a \) has to be given for a certain lubricated condition. Therefore, in this work, Equation (6) is used to the wear modeling under lubricated condition.

2.2. Wear simulation

From Equation (6), a numerical solution can be derived with a finite difference of the wear depth as following:

\[
h_j - h_{j-1} = K \bar{\mu} p_j(\Delta L) \Delta L_j
\]

(7)
Where \( h_j \) is the wear depth at \( j \)th iteration while \( h_{j-1} \) is the wear depth at the previous iteration. 

\( K \bar{p} j \Delta L_j \) Refers to the incremental wear depth, which is a function of the incremental sliding distance, the contact pressure at the corresponding sliding distance and the average COF. For each increment of \( \Delta L_j \), \( p_j(L) \Delta L_j \) is actually the integral of \( p(x) \) with \( x \). It can be expressed as following:

\[
p_j(L) \Delta L_j = \int_{x_a}^{x_b} p(x) \, dx
\]

Based on simplified Reynolds equation, \( p \) can be calculated by the equation of:

\[
p(x) = \frac{12 \eta U}{h_0^2} \sqrt{\frac{2}{2Rh_0}} P^*(x)
\]

Where \( \eta \) is viscosity of lubricants, \( U \) is the half of the sum velocity of the two rubbing surface, and \( R \) is the radius of the contacting cylinder.

\[
h_0 = 1.66 \left( \frac{\alpha \eta U}{R} \right)^{2/3} R
\]

Where \( \alpha \) is the pressure-viscosity coefficient.

\[
P^*(x) = \frac{1}{2} \left[ \gamma(x) + \frac{\pi}{2} + \frac{\sin 2\gamma(x)}{2} - 1.226 \left\{ \frac{3}{4} \left( \gamma(x) + \frac{\pi}{2} \right) + \frac{\sin 2\gamma(x)}{2} + \frac{\sin 4\gamma(x)}{16} \right\} \right]
\]

Where \( \gamma = \tan^{-1} \left( \frac{x}{\sqrt{2Rh_0}} \right) \).

After the incremental wear depth is known, the incremental wear volume is calculated by multiplying the incremental wear depth with the length (\( l \)) of the cylinder. Therefore, an accumulation of the incremental wear volume for all iterations (\( n \)) can be given as:

\[
V = \sum_{j=1}^{n} \Delta V_j = l \sum_{j=1}^{n} \Delta h_j \Delta L_j = l \sum_{j=1}^{n} (h_j - h_{j-1}) \Delta L_j
\]

Where \( b \) is the radius of the Hertzian contact area.

With the wear depth accumulated in each iteration, the sliding distance is also an accumulation of the incremental sliding distance for all iterations (\( n \)) and is given as:

\[
L = \sum_{j=1}^{n} \Delta L_j
\]

Therefore, the wear depth on a contact surface for a specified sliding distance \( L \) can be estimated. It is noted that Equation (12) is appropriate for the case that the soft material is continuously worn off. Therefore, for the tribology system, the accumulation of the incremental sliding distance can be calculated as following:

\[
L = 2\pi r \sum_{j=1}^{n} j
\]

Where \( r \) is the radius of wear track. In future wear modelling, Equation (14) will be used.

As we know, in each rotation, only a very tiny wear depth is generated resulting in a very small geometric change. For shortening the computation time, the incremental wear depth in Equation (30) can be defined as the incremental wear depth in every \( N \) rotations period, and then Equation (30) can be rewritten as:
\[ h_j - h_{j-1} = NK\theta\mu p_j(\Delta L)\Delta L_j \]  

(15)

Where \( \theta \) is the frequency of a rotation with unit of Hz.

Thereafter, the accumulation of the incremental sliding distance (Equation (14)) can be reformed as:

\[ L = 2\pi r\theta N j (j = 1,2,3, \ldots, n) \]  

(16)

If we let \( N=3600 \), the incremental wear depth \( (h_j - h_{j-1}) \) is actually the wear depth change in an hour. \( n \) is therefore the duration hours of the wear test.

The accumulation of the incremental wear volume for all iterations \( (n) \) can be given as:

\[
V = \sum_{j=1}^{n} \Delta V_j = \begin{cases} 
\sum_{j=1}^{n}(h_j - h_{j-1}) \cdot \left(2\pi r \cdot \frac{h_j}{2h_m}\right) \cdot \left(2R \cdot \frac{h_j}{2h_m}\right) & \text{if } h_j < 2h_m \\
\sum_{j=1}^{n}(h_j - h_{j-1}) \cdot (2\pi r) \cdot (2R) & \text{if } h_j > 2h_m 
\end{cases} 
\]  

(17)

(18)

The induced term of \( \left(2\pi r \cdot \frac{h_j}{2h_m}\right) \) in Equation (18) refers to the real worn-off distance. As the wear track has a length of \( 2\pi r \), there are \( M \) number of asperities along this wear track, where \( M \) is:

\[ M = \frac{2\pi r}{\lambda} \]  

(19)

The \( \lambda \) is the wavelength of an asperity. Then the real worn off distance for each asperity at the jth iteration, \( d_j \) shown in Figure 1, can be given as:

\[ d_j = \frac{h_j}{2h_m} \cdot \lambda \]  

(20)

Where \( h_m \) is the average height of the asperities. Therefore, the real worn-off distance \( (s_j) \) along the wear track at the jth iteration is:

\[ s_j = Md_j = \frac{2\pi r}{\lambda} \cdot \frac{h_j - 2h_m}{2h_m} = 2\pi r \cdot \frac{h_j}{2h_m} \]  

(21)

\[ \text{Figure 1. Scheme of the geometry of asperities.} \]

The flow chart for the wear simulation is shown in Figure 2.
2.3. Experiments
To validate the wear modeling for the line contact problem, another pin-on-disk (a stainless steel cylinder sliding against a HDPE disk) wear test is also performed under 95% glycerol/water lubricated conditions as shown in Figure 3. The stainless steel cylinder with length of $4.76 \pm 0.002\text{mm}$ has a circular cross-section with a radius of $2.38\pm 0.001\text{mm}$. The radius of wear track is $5\text{ mm}$. The mass loss of the polymer disk is measured in every 2 hours. The COF is monitored continuously during the wear test. In addition, the normal load, rotation frequency and average initial asperity height is varied for different samples in Table 1. COFs of HDPE under various operating parameters are measured in a 1-hour wear tests and also listed in Table 1.

![Figure 2. Wear simulation flow chart.](image)

![Figure 3. Scheme of a line-contact pin-on-disk wear test.](image)
2.4. Results and discussion

As shown in Figure 4, the experimental mass loss measurements are compared to the simulated results. The parameters in both experiments and simulation are set up as Set No. 1 as shown in Table 1. Both experimental and simulated curves have similar parabolic shape. The simulated results have a good agreement with the test data. At the first stage (about 0-24 hours), the mass loss slowly increases with time due to the wear of the asperities. After around 24 hours, the mass loss of HDPE steeply increases and the curve in this second stage becomes a basic straight line with a constant slope. The transition time from the first stage to the second stable stage represents the moment at which the asperity is entirely worn off. The first stage difference of the mass loss between the experimental and simulated curves is small. However, after around 24 hours, the difference becomes larger and larger due to the different slope of the straight lines in the second stage. This may be due to our ideal assumption that after the asperities are worn off, the work surface of the polymer will become prefect smooth surface.

![Figure 4. Comparison of experimental wear measurements with simulation results (Parameters were set as Set No.1 as shown in Table 1).](image)

### Table 1. The wear simulation data set-ups.

| Simulation data set No. | Lubricants       | Viscosity of lubricants, η (Pa.s) | Rotation frequency, θ (Hz) | Normal load, N (N) | Average asperity height of polymer disk, ℎ₀ (µm) | Average COF, μ |
|------------------------|------------------|-----------------------------------|---------------------------|-------------------|-----------------------------------------------|---------------|
| 1                      | 50% Glycerol     | 0.00496                           | 1                         | 50                | 0.24                                          | 0.042         |
| 2                      | 50% Glycerol     | 0.00496                           | 2                         | 50                | 0.24                                          | 0.043         |
| 3                      | 50% Glycerol     | 0.00496                           | 1                         | 100               | 0.24                                          | 0.030         |
| 4                      | 50% Glycerol     | 0.00496                           | 1                         | 50                | 0.50                                          | 0.042         |
| 5                      | 95% Glycerol     | 0.36440                           | 1                         | 50                | 0.24                                          | 0.031         |
| 6                      | Water            | 0.00089                           | 1                         | 50                | 0.24                                          | 0.072         |

3. Conclusion

In this work, a wear model under lubricated conditions is developed with incorporation of elastohydrodynamic lubrication theory and Reye’s wear model. The wear simulation shows that the wear of HDPE is increased with the increasing of the sliding speed and normal load, but decreased with the increasing of the asperity height and lubricant’s viscosity. The wear model can be used to predict...
the long term wear of polyethylene based polymer under lubricated condition and may reduce the time consumption of the wear test.

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