Curvilinear one-dimensional antiferromagnets

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Within the field of curvilinear magnetism, antiferromagnetic materials represent a technologically and fundamentally promising, but completely unexplored topic. Here, we consider curvature effects on static and dynamic magnetism in a classical spin chain possessing nearest-neighbor exchange and dipolar interactions. It is shown that an intrinsically achiral magnet behaves as a biaxial chiral helimagnet because of curvature-induced inhomogeneous Dzyaloshinskii–Moriya interactions and uniaxial anisotropies stemming from exchange. A phase transition between locally homogeneous and periodic states in space curves is found for arbitrarily small curvature and torsion, while flat curves exhibit only a single homogeneous ground state. As fundamental results on linear excitations of the chains, we show how non-zero curvature leads to spin wave mode hybridization.

Introduction—Antiferromagnets (AFMs) are materials with vanishing net moment. Recently, they attracted significant attention with the birth of a novel research field of antiferromagnetic spintronics [1–4] and related fundamental discoveries of staggered spin-orbit torques in AFMs possessing broken inversion symmetry in a local environment [4–6] which is also a source of Dzyaloshinskii–Moriya interaction (DMI) [7–9]. The presence of DMI is peculiar for noncollinear antiferromagnets characterized by weak ferromagnetism and chiral helimagnetism [10]. Furthermore, DMI significantly affects dynamics of solitary excitations including much higher domain wall velocities [11] and absence of gyroforce (Magnus force) for skyrmions [12, 13]. The portfolio of material systems available for these studies is very limited due to the stringent requirement to the magnetic symmetry of AFM. This requirement delivers the progress in AFM spintronics to depend on time consuming material screening and optimization of intrinsic chiral properties of AFMs.

For ferromagnets chiral responses in nanowires and thin films can be tailored by using curvilinear geometries [14–17]. This approach allows to decouple to certain extend the response from intrinsic material properties, opening the attractive possibility to induce chiral response in otherwise conventionally achiral ferromagnets [16, 18, 19]. This family of the geometrical effects is known as curvilinear magnetism, which is well developed for ferromagnetic materials [20–22]. However, no theory of curvilinear antiferromagnetism is available to date.

In this Letter, we put forth a fundamental theory of curvilinear one-dimensional (1D) AFMs. We explore curvature effects in a prototypical AFM system, namely, spin chain where spins are coupled via local exchange and long-range dipolar interactions. The presence of dipolar interaction leads to the stabilization of the effective hard axis anisotropy with the axis direction along the chain. The nearest-neighbor exchange interaction brings about two geometry-induced responses: the intrinsically achiral curvilinear AFM spin chain behaves as a chiral helimagnet possessing an additional biaxial anisotropy. We apply our theory to analyze static and dynamic responses of flat geometrically curved as well as helical AFM spin chains to demonstrate consequences of the coupling between the geometry and AFM order parameter (the Néel vector). We show that generic curvilinear 1D AFM exhibits the full set of Lifshitz invariants whose strength is determined by the local torsion and curvature. Spin chains arranged along space curves with non-zero torsion exhibit a magnetic phase transition from homogeneous to periodic states, which are tunable by controlling geometrical parameters. The appearance of the curvature-induced DMI results in the hybridization of spin wave modes in linear dynamics.

Model of a curvilinear antiferromagnet—We start with a classical spin chain taking into account the antiferromagnetic nearest-neighbor exchange and dipolar interaction. Its static and dynamic properties are determined by the Landau–Lifshitz equation

\[
\frac{d \mathbf{m}_i}{dt} = \frac{1}{\hbar S} \mathbf{m}_i \times \frac{\partial \mathcal{H}}{\partial \mathbf{m}_i},
\]

with the Hamiltonian specific to the collinear intrinsically achiral classical AFM

\[
\mathcal{H} = J S^2 \sum_i \mathbf{m}_i \cdot \mathbf{m}_{i+1} - \frac{\mu}{2} \sum_i \mathbf{m}_i \cdot \mathbf{H}_i^d.
\]

Here, \( \mathbf{m}_i \) is the unit magnetic moment of \( i \)-th site, \( \hbar \) is the Planck constant, \( S \) is the spin length, \( J > 0 \) is the exchange integral, \( \mu = g \mu_b S \) is the total magnetic moment of one site with \( g \) being Landé factor, and \( \mu_b \) being Bohr magneton. The dipolar field at \( i \)-th site reads...
FIG. 1. (Color online) (a) Schematics of the antiferromagnetic spin chain. Sublattices are shown by magenta and blue arrows, TNB basis is shown by red. The Dzyaloshinskii vector $\mathbf{d}$ lies in the TB plane. Hard and easy anisotropy axes are labeled by $\mathbf{e}_1$ and $\mathbf{e}_3$, respectively. (b) Helix spin chain with radius $R$ and pitch $P$. (c) Diagram of equilibrium states for helix spin chain. Open symbols and triangles correspond to periodic and homogeneous states, respectively, obtained in spin-lattice simulations. Solid curve shows the numerically obtained boundary between states. The dashed line shows the limiting values torsion when the homogeneous state is stable. (d–f) Schematics of the homogeneous and periodic states for one helix pitch. The director $\mathbf{n}$ is shown by green arrow. Spheres illustrate the trajectories of $\mathbf{n}$.

The continuum counterpart of (1) is formulated based on two vector fields, namely the total magnetization $\mathbf{m}(s) = (\mathbf{m}_t + \mathbf{m}_n)/2$ and vector order parameter (or Néel vector) $\mathbf{n}(s) = (\mathbf{m}_t - \mathbf{m}_n)/2$, where fields $\mathbf{m}_{t,n} = \mathbf{m}_{t,n}(s)$ correspond to the two sublattices of the AFM. In the long-wave approximation, the density of Lagrangian $L = \int \mathcal{L} dx$, corresponding to the model (1) reads

$$\mathcal{L} = \frac{M^2}{\gamma_0^2 \Lambda} (\partial_t \mathbf{n})^2 - \mathcal{E}.$$  \hspace{1cm} (2a)

Here the density of effective energy

$$\mathcal{E}_g = A(\mathcal{E}_g + \mathcal{E}_{\text{DM}} + \mathcal{E}_\lambda) + K(\mathbf{n} \cdot \mathbf{e}_t)^2$$

$$\mathcal{E}_g = |\mathbf{n}|^2,$$  \hspace{1cm} (2b)

$$\mathcal{E}_{\text{DM}} = \mathcal{F}_{\alpha\beta}(n_\alpha n_\beta' - n_\beta n_\alpha'),$$

$$\mathcal{E}_\lambda = \mathcal{K}_{\alpha\beta}(n_\alpha n_\beta), \quad \alpha, \beta \in \text{T,N,B}$$

where $M_s = \mu/(2a)$ is the magnetization of one sublattice, $\gamma_0$ is the gyromagnetic ratio, $\Lambda = 2JS^2/\alpha$ is the constant of the uniform exchange, $A = JS^2a/2$ is the exchange stiffness and $K \approx 2.7\mu^2/\alpha^4$ is the anisotropy constant, see Supplementary [23] for details. The model (2) is valid for $|\mathbf{m}| \ll |\mathbf{n}|$ implying that $K \ll \Lambda$ and $\mathbf{n}$ can be considered as a unit vector. Here, the Einstein summation rule is applied and prime means derivative with respect to $s$. The Frenet tensor $\mathcal{F}_{\alpha\beta}$ has four nonzero components $\mathcal{F}_{12} = -\mathcal{F}_{21} = \kappa$ and $\mathcal{F}_{23} = -\mathcal{F}_{32} = \tau$. The characteristic length and time scales are given by the magnetic length $\ell = \sqrt{A/\Lambda}$ and the frequency of the antiferromagnetic resonance $\omega_0 = c/\ell = \gamma_0 \sqrt{AK}/M_s^2$ with $c$ being the characteristic magnon speed. The exchange energy density expands into three terms, with only one, $\mathcal{E}_g$, possessing the form of a regular inhomogeneous exchange in straight spin chains.

The term $\mathcal{E}_{\text{DM}} = \mathbf{d} \cdot [\mathbf{n} \times \mathbf{n}']$ has the functional form of a DMI allowed in crystals with magnetic symmetry groups $C_n$ and $S_4$ acting on 1D magnetic textures [8]. However, its origin is not the spin-orbit interaction as for the case of intrinsic DMI but the exchange interaction. The vector $\mathbf{d} = d_t \mathbf{e}_t + d_n \mathbf{e}_n$ acts as the Dzyaloshinskii vector with components $d_t = 2A\tau$ and $d_n = 2Ax$. It corresponds to the full set of Lifshitz invariants, allowed in
the 1D magnet. The DMI vector \( \mathbf{d} \) is linear with respect to \( \tau \) and \( \kappa \). This allows strong chiral effects in curvilinear 1D AFMs. The strength of the curvature-induced DMI can be estimated as the relation to the exchange stiffness. For instance, in the case of a Mn-DNA chain bent to the radius of 15 nm, the \( \alpha \)D/A/A \( = 0.036 \) with \( \mathbf{D} \) being constant of non-uniform DMI, where ultrafast motion of AFM domain walls was predicted [11].

In addition to the linear in \( \tau \) and \( \kappa \) DMI terms, the expression for energy \( \mathcal{E} \) contains weaker bilinear terms, representing a curvature-induced anisotropy \( \mathcal{E}_\kappa \), whose coefficients are represented by the tensor \( \mathcal{K}_{\alpha \beta} = \mathcal{F}_{\alpha \gamma} \mathcal{F}_{\beta \gamma} \propto \kappa^2, \tau^2 \kappa \tau \). It contains non-diagonal terms, causing the tilt of \( \mathbf{n} \) within the rectifying surface formed by \( \mathbf{e}_\tau \) and \( \mathbf{e}_s \).

The last term in the effective energy \( \mathcal{E} \) represents the hard-axis anisotropy induced by the dipolar interaction. The anisotropy axis is along the tangential direction \( \mathbf{e}_\tau \). This model is an analogue of the shape anisotropy approach, which means that the effective anisotropy \( \mathcal{K} \) resembles the demagnetization coefficient \( \mathcal{N} \) for the uniform ground state in a straight spin chain. The presence of the two anisotropies (hard axis stemming from the dipolar interaction and easy axis stemming from the exchange interaction) renders a curvilinear AFM spin chain to behave as a biaxial AFM. The directions of the primary hard axis \( \mathbf{e}_1 \) and secondary easy axis \( \mathbf{e}_3 \) are determined by the diagonalization of the tensor of the total anisotropy \( \mathcal{K}_{\alpha \beta} + \delta_{1 \alpha} \delta_{1 \beta} K \) with \( \delta_{1 \alpha} \) being Kronecker symbol, see Fig. 1(a). The axis \( \mathbf{e}_1 \) lies within the rectifying surface. Although the anisotropy induced by the dipolar interaction is the strongest one, it determines the plane, where the Néel vector rotates. The direction of the vector \( \mathbf{n} \) within the easy plane is given by the curvature-induced anisotropy \( \mathcal{E}_\kappa \). Independent of the strength of \( \mathcal{E}_\kappa \), the system has no competing easy axis terms and governs the orientation of the Neel vector even for \( A \kappa^2, A \tau^2 \ll K \). We note that the energy barrier determining the stability of the state is given by the strength of the \( \mathcal{E}_\kappa \).

**Ground state of AFM helix chain**—To illustrate the behavior of curvilinear AFM spin chains, described by (2), we analyze the helix chain as one of the simplest systems with a constant curvature and torsion. The geometry of a helix is characterized by the radius \( R = \kappa / (\kappa^2 + \tau^2) \) and pitch \( P = \frac{2 \pi \tau}{\left( \kappa^2 + \tau^2 \right)} \), see Fig. 1(b). It is convenient to introduce the angular parametrization of the Néel vector \( \mathbf{n} = e_\tau \cos \theta + e_s \sin \theta \cos \phi + e_\phi \sin \theta \sin \phi \) with \( \theta = \theta(s, t) \) and \( \phi = \phi(s, t) \) being polar and azimuthal angles, respectively. Note that in contrast to ferromagnets where the order parameter is a vector, in the case of AFMs, the order parameter \( \mathbf{n} \) is a director. Therefore, states with \((\theta, \phi)\) and \((\pi - \theta, \phi \pm \pi)\) are equivalent. Then, the linear energy density reads

\[
\mathcal{E} = A(\theta' + \kappa \cos \phi)^2 + A [\sin \theta (\phi' + \tau) - \kappa \cos \theta \sin \phi]^2 + K \cos^2 \theta, \quad (3)
\]

As a biaxial chiral helimagnet, helix spin chains support homogeneous and periodic equilibrium states dependent on the strength of the DMI, see Fig. 1(c). For the case of the homogeneous state, which is realized for \( \tau < \gamma_0(\kappa) \approx \kappa \) at \( \kappa \ell \ll 1 \), see Fig. 1(d,e) and Supplementary [23] for details, the orientation of the Néel vector is given by

\[
\theta_{\text{hom}} = \frac{\pi}{2} - \psi, \quad \phi_{\text{hom}} = \frac{\pi}{2}, \quad \psi \approx \ell^2 \kappa \tau \quad (4)
\]

with \( \kappa \ell, |\tau| \ell \ll 1 \).

The periodic state can be stabilized in systems possessing torsion \( \tau > \gamma_0(\kappa) \). In the periodic state, the Néel vector is almost uniform in the plane perpendicular to the helix axis. Hence, it is modulated in the local reference frame, see Fig. 1(f,g). The emergence of the periodic state is a consequence of the exchange-induced DMI, \( \mathcal{E}_{\text{DMI}} \), with the main contribution given by the torsion-related term \( \mathcal{E}_\tau \). When the curvature is much smaller than the torsion, the state can be described as the Dzyaloshinskii spiral [26]

\[
\theta_{\text{per}} = \frac{\pi}{2}, \quad \phi_{\text{per}} = -\tau s, \quad (5)
\]

The boundary between the homogeneous and periodic states \( \tau_0(\kappa) \) is plotted by the solid red line in the Fig. 1(c).

It is instructive to compare the results above with ferromagnetic spin chains, where dipolar interactions induce easy axis anisotropy. In contrast to ferromagnetic helices, the phase transition between periodic and homogeneous states in AFMs has no threshold in curvature [15]. Hence, the transition to the periodic state in the case of AFM helical chains can be observed for very small curvature radii. This is a consequence of the specificity of the curvilinear AFM systems where the stability of the state is given by the weak easy axis anisotropy stemming from the exchange interaction.

**Ground state of AFM flat chain**—Flat curves are characterized by \( \tau = 0 \) (the torsion-related DMI \( d_\tau \) is absent) and curvature \( \kappa \) with alternating sign in (3). In this case, the easy axis \( \mathbf{e}_1 \) is along the \( \mathbf{e}_s \), which is oriented perpendicular to the plane of the flat curve. The hard axis \( \mathbf{e}_3 \) is tangential to the curve and is oriented along \( \mathbf{e}_\tau \). The energy density (3) is a positively defined quadratic form. It gives the only ground state \( \mathcal{E}_0 \) with \( \theta_0 = \phi_0 = \pi / 2 \), which is analogous to the homogeneous state discussed for the case of helix. The periodic state is absent in the case of flat AFM spin chains because the spiralizing direction of the curvature-induced DMI \( d_\kappa \) coincides with the hard axis \( \mathbf{e}_\tau \) of the dipole-induced anisotropy.

The curvature-induced easy axis anisotropy oriented perpendicular to the plane can contribute to the model
Hamiltonian, which is used for the description of Cr-based molecular wheels with nearest-neighbor exchange and easy axis perpendicular to the wheel plane [27, 28].

Linear dynamics—To illustrate linear excitations in a curved chain we start from the flat ring with the ground state \( \mathbf{n} = \mathbf{e}_a \). The Euler–Lagrange equations for the Lagrangian (2) are linearized by \( \theta(s,t) = \theta_0 + \varphi(s,t) \) and \( \phi(s,t) = \phi_0 + \varphi(s,t)/\sin\theta_0 \). Here, \( \varphi(s,t) \) and \( \varphi(s,t) \) are small deviations from the equilibrium state. The corresponding equations read

\[
\begin{align*}
\kappa^2 \phi'' - \partial_t \phi &= K_1 \phi + c D_2 \varphi', \\
\kappa^2 \varphi'' - \partial_t \varphi &= K_2 \varphi - c D_2 \phi',
\end{align*}
\]

where \( K_{1,2} \) and \( D_2 \) are functions of curvature, playing a role of the renormalized effective anisotropy and DMI coefficients, respectively and periodic boundary conditions are implied, see Supplementary [23]. For the large curvature radius \( K_1 \approx \omega_0^2 + c^2 \kappa^2 \), \( K_2 \approx c^2 \kappa^2 \) and \( D_2 = 2 c \kappa \).

The dispersion law can be written using the substitution of plane waves \( \varphi(s,t) = \varphi_k \cos(\kappa N s - \Omega t + \delta_k) \) and \( \phi(s,t) = \phi_k \sin(\kappa N s - \Omega t + \delta_k) \), where \( \partial_k \) and \( \varphi_k \) are small amplitudes, \( N \in \mathbb{Z}_+ \) is the quantum number, \( \Omega \) is frequency and \( \delta_k \) is phase. The spectrum reads

\[
\Omega^2 = \frac{K_1 + K_2}{2} + c^2 \kappa^2 N^2 + \frac{q}{2} \sqrt{(K_1 - K_2)^2 + 4D_2^2 c^2 \kappa^2 N^2}.
\]

The signs of \( q = \pm 1 \) correspond to the oscillations within the normal and rectifying surfaces (BT and BN modes, respectively, analogous to \( zx \) and \( zy \) modes in flat systems [29]), see Fig. 2(a). The value of \( N \) determines the type of oscillations of the Néel vector, i.e. \( N = 0 \) corresponds to homogeneous oscillations, \( N = 1 \) gives uniform rotation etc. The presence of \( D_2 \neq 0 \) results in hybridization of NB and TB modes.

The spin waves on the background of the homogeneous state of helix spin chain are described by the same equations of motion (6) and the dispersion (7) with the replacement of \( \kappa N \) to the wave number \( k \) and \( K_1 \approx \omega_0^2 + c^2(\kappa^2 - \tau^2) \), see Fig. 2(b,c). The torsion does not enter into the mode hybridization within the second order of perturbation because the direction of the spin wave propagation coincides with the spiraling direction of \( d_0 \).

It is instructive to compare (7) with straight 1D AFMs. There are two linearly polarized magnon branches \( \Omega_{1,2}^2(k) = \Omega_{1,2}^2 + c^2 k^2 \) with perpendicular planes of oscillation, \( \Omega_1 = \omega_0 \) and \( \Omega_2 = 0 \) [29]. For the case of small curvature, torsion and wave vector, \( \Omega_{1,2} = \sqrt{K_{1,2}} \), which corresponds to the different signs of \( q \) in (7). In other words, curvature increases the gap for the first branch and provides an additional gap for the second branch.

Note, that there is a critical value of torsion \( \tau_c(k) \) which results in the instability of the homogeneous state and is determined by the condition \( \Omega(k)_{|q=-1} = 0 \) and \( \partial_k \Omega(k)_{|q=-1} = 0 \) with \( c k_c = \sqrt{D_2^2 - (K_1 - K_2)^2/(2D_2)} \), see dashed line in Fig. 1(c).

Conclusions—Analyzing the intrinsically achiral curvilinear spin chain lying on a given curve we show that it behaves as a biaxial chiral helimagnet. The primary hard axis anisotropy is mainly governed by dipolar interaction, while the secondary easy axis appears as the exchange-induced effect which has no other competitors. AFM

![Image](https://via.placeholder.com/150)

**FIG. 2.** (Color online) (a) Lowest eigenfrequencies of linear excitations in the AFM ring as a function of the curvature. Solid and dashed lines correspond to the case \( q = +1 \) and \( q = -1 \) in (7), respectively. Symbols correspond to the simulation data. (b) Standing waves in the plane ring for different quantum numbers \( N \). Schematics shows oscillation type for the given branch. Inset below shows profiles of components of the Néel vector \( \mathbf{n} \). (c) Dispersion curves (7) for helices (homogeneous state). Background color shows intensity of spectral harmonics for \( \kappa \ell = 0.6 \) and \( \tau \ell = 0.1 \). (d) Helix geometries, corresponding to the dispersion curves in (c).
space curves possess the full set of Lifshitz invariants, given by two Dzyaloshinskii constants within the frame of anisotropic nonlinear σ-model for the Néel vector \( \mathbf{n} \). They result in helimagnetic phase transition for space curves, while plane curves have as the only ground state \( \mathbf{n} \) perpendicular to the plane. The curvature-induced chiral interaction results in the hybridization of magnon modes in the chain. The symmetry and possible strength of geometry-induced DMI opens perspectives for applications in antiferromagnetic spin-orbitronics, e.g., for ultrafast chiral domain wall dynamics [11, 25], or gradients of material parameters [30] which can be tuned via functional dependence of curvature and torsion on coordinate.

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