Revealing the Formation of Stellar-mass Black Hole Binaries: The Need for Deci-Hertz Gravitational-wave Observatories

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Abstract

The formation of compact stellar-mass binaries is a difficult, but interesting problem in astrophysics. There are two main formation channels: in the field via binary star evolution, or in dense stellar systems via dynamical interactions. The Laser Interferometer Gravitational-wave Observatory (LIGO) has detected black hole binaries (BHBs) via their gravitational radiation. These detections provide us with information about the physical parameters of the system. It has been claimed that when the Laser Interferometer Space Antenna (LISA) is operating, the joint observation of these binaries with LIGO will allow us to derive the channels that lead to their formation. However, we show that for BHBs in dense stellar systems dynamical interactions could lead to high eccentricities such that a fraction of the relativistic mergers are not audible to LISA. A non-detection by LISA puts a lower limit of about 0.005 on the eccentricity of a BHB entering the LIGO band. On the other hand, a deci-Hertz observatory, like DECIGO or Tian Qin, would significantly enhance the chances of a joint detection and shed light on the formation channels of these binaries.

Key words: black hole physics – gravitational waves – methods: analytical – stars: kinematics and dynamics

1. Introduction

The first Laser Interferometer Gravitational-wave Observatory (LIGO) events, GW150914 and GW151226 (Abbott et al. 2016c, 2016d), are consistent with mergers of General Relativity black holes (BHs). Data analysis reveals that the orbits started at a semimajor axis of $a \sim 10$ Schwarzschild radii ($R_S$) with an eccentricity of $e < 0.1$. The BH masses are about $M_1 \approx 36$ and $M_2 \approx 29 M_\odot$ for GW150914 and $M_1 \approx 14$ and $M_2 \approx 7.5 M_\odot$ for GW151226. The detections can be used to infer more realistic event rates, of about $9-240$ Gpc$^{-3}$ yr$^{-1}$ (Abbott et al. 2016b). This rate agrees with two formation channels (see Abbott et al. 2016a for a review): (i) evolution of a binary of two stars in the field of the host galaxy, where stellar densities are very low, or (ii) via exchange of energy and angular momentum in dense stellar systems, where the densities are high enough for stellar close encounters to be common.

LIGO and other ground-based gravitational-wave (GW) observatories, such as Virgo, are, to some degree, blind with regards to the formation channels of BH binaries (BHBs). Both channels predict populations in the $10^{-3}$-Hz detector band with similar features, i.e., masses larger than the nominal $10 M_\odot$, a mass ratio ($q = M_2/M_1$) of about 1, low spin, and nearly circular orbits (Abbott et al. 2016a; Amaro-Seoane & Chen 2016).

It has been suggested that a joint detection with a spaceborne observatory, such as Laser Interferometer Space Antenna (LISA; Amaro-Seoane et al. 2017), could allow us to study different moments in the evolution of BHBs on their way to coalescence: LISA can detect BHBs when the BHs are still $10^2 - 10^3 R_S$ apart, years to weeks before they enter the LIGO/Virgo band (Miller 2002; Sesana 2016; Seto 2016; Vitale 2016). At such a separation, the orbital eccentricity bears the imprint of the formation channel because (i) BHBs in dense stellar systems form on systematically more eccentric orbits and (ii) the GW radiation at this stage is too weak to circularize the orbits (Miller 2002; Wen 2003; Gültekin et al. 2004, 2006; O’Leary et al. 2006). Therefore, circular binaries typically form in the field, while eccentric ones form through the dynamical channel. Recent studies further predict that those BHBs with an eccentricity of $e > 0.01$ in the LISA band preferentially originate from the dynamical channel (Kyutoku & Seto 2016; Nishizawa et al. 2016, 2017; Breivik et al. 2016; Seto 2016).

In this Letter, we prove that eccentric BHBs originating in dense stellar environments have a large chance to elude the LISA band.

2. Inaudible Black Hole Binaries

Non-circular BH binaries have two distinct properties. First, the eccentricities will damp the characteristic amplitude ($h_c$) of each GW harmonic. In Figure 1, we depict two sources similar to GW150914 but originating from two distinct channels, i.e., with two different initial eccentricities. In the low-eccentricity case, the $n = 2$ harmonic predominates, and it is strong enough to be jointly detected by LISA and LIGO/Virgo. In the (very) eccentric case, however, the amplitudes of the harmonics are orders of magnitude below the noise level of LISA, so that a joint detection is ruled out. When the eccentricity has been significantly damped, about one hour before the merger, the dominant harmonic starts to converge to the $n = 2$ one and, later, upon entering the LIGO band, becomes indistinguishable from that in the circular case. Therefore, the imprint about the formation channel is lost. Second, increasing the eccentricity shifts the peak of the relative power of the GW harmonics toward higher frequencies (see Figure 3 of Peters & Mathews 1963). Hence, more eccentric orbits emit their maximum power at frequencies farther away from LISA. More precisely, when $e = 0$, all the GW power is radiated through the $n = 2$ harmonic, so that the GWs...
3. Dense Stellar Environments

BHBs such as the one we have used for our last example completely miss the LISA/TJ band. Eccentric binaries typically originate from dense stellar systems such as globular clusters (GCs) and nuclear star clusters (NSCs), as shown by a number of authors in a number of publications (Miller 2002; Wen 2003; Gültekin et al. 2004, 2006; O’Leary et al. 2006; Breivik et al. 2016; Nishizawa et al. 2016, 2017). In these systems, BHs diffuse toward the center via a process called mass segregation. To model it, we adopt a Plummer model, and we assume that the mean stellar density is \( \rho_0 = 5 \times 10^5 M_\odot \, \text{pc}^{-3} \) and the one-dimensional velocity dispersion is \( \sigma_v = 15 \, \text{km s}^{-1} \). These values correspond to a typical GC with a final mass of \( M_{\text{GC}} \approx 10^6 M_\odot \) and a half-mass radius of \( R_h \approx 0.5 \, \text{pc} \). We note, however, that the main conclusions derived in this work do not significantly change for an NSC.

The two driving and competing mechanisms in the evolution of any BHB in the center of the cluster are (i) interaction with other stars, “interlopers,” which come in at a rate of \( \Gamma \sim 2 \pi G \rho_0 a (M_1/M_2) / \sigma_v \), with \( M_\star = 10 M_\odot \) the mean mass of the interlopers because the cluster has gone through mass segregation, and (ii) gravitational radiation, which shrinks the orbital semimajor axis at a rate of

\[
a_{\text{gw}} = -\frac{8}{5} c \, \epsilon_0^2 \left(1 + \frac{q}{2} \right) \left(1 + \frac{73}{24} \epsilon^2 + \frac{37}{96} \epsilon^4 \right) \quad (1)
\]

(Peters 1964). We can readily separate the phase space in two distinct regimes according to these two competing processes by

\[\text{Figure 1. Characteristic amplitude} \, h_c \, \text{of the first four harmonics (indicated with numbers) emitted by a BHB with masses} \, M_1 = M_2 = 30 M_\odot \, \text{and at a luminosity distance of} \, D = 500 \, \text{Mpc}. \, \text{The amplitude is calculated as described}\, \text{in}\, \text{Barack \& Cutler (2004) and the orbital evolution as in Peters (1964). We display a BHB starting at a semimajor axis of} \, a_0 = 0.1 \, \text{au} \, \text{and initially with two very different eccentricities, so as to illustrate the main idea of this article:} \, (i) \, a_0 = 0.05 \, (\text{thin colored lines}), \, (\text{ii}) \, \text{an extreme case,} \, a_0 = 0.999 \, (\text{thick colored lines}). \, \text{Along the harmonics we mark several particular moments with dots, where the labels show the time before the coalescence of the binary and the corresponding orbital eccentricities. The two black solid curves depict the noise curves (} \sqrt{S_n(f)} \text{) for LISA and LIGO in its advanced configuration. Although we have chosen a very high eccentricity for the second case in this example, we note that lower eccentricities can also be inaudible to LISA (see the discussion).}

\[\text{Figure 2. Different detectors’ bands for a binary of} \, M_1 = M_2 = 30 M_\odot. \, \text{We have considered four types of detectors:} \, (i) \, \text{a ground-based interferometer like LIGO and Virgo (pink stripe), with the minimum and maximum observable frequencies} \, (f_c, f_o) \sim (10, 10^3) \, \text{Hz (e.g., Abbott et al. 2016d)}, \, (ii) \, \text{a space-borne solar-orbit interferometer such as the DECIGO-hertz Interferometer Gravitational-wave Observatory (DECIGO; blue) with} \, (f_c, f_o) \sim (0.1, 10) \, \text{Hz (Kawamura et al. 2011)}, \, (iii) \, \text{a geocentric space observatory like the Tian Qian project (TQ; orange) with} \, (f_c, f_o) \sim (10^{-2}, 0.3) \, \text{Hz (Luo et al. 2016)}, \, \text{and} \, (iv) \, \text{another solar-orbit interferometer but with a million-kilometer baseline, like LISA or Tai Ji (TJ; cyan), which operates at milli-Hz,} \, (f_c, f_o) \sim (10^{-3}, 0.1) \, \text{Hz (Gong et al. 2015; Amaro-Seoane et al. 2017}). \, \text{The upper, horizontal limit in the color stripes corresponds to an orbital period of one week for LIGO/Virgo/DECIGO, one month for TQ, and one year for LISA/TJ, as imposed by the restrictions in the search of the different data streams. The green solid lines show the evolutionary tracks of a binary evolving only due to GW emission, in the approximation of Keplerian ellipses (Peters 1964). The dashed, black lines are isochrones displaying the time to relativistic merger in the same approximation (see the text), provided that the evolution is driven only by GWs. The thick gray stripe displays the last stable orbit, below which the two BHs will merge within one orbital period. We also display with red stars the positions of the eccentric BHB in Figure 1 at different stages, to illustrate the process.}

\[\text{have a single frequency of} \, 2/P, \, \text{where} \, P = 2\pi (GM_{12}/a^3)^{-1/2} \, \text{is the orbital period and} \, M_{12} = M_1 + M_2. \, \text{On the other hand, when} \, e \ll 1, \, \text{the} \, n = 2.16(1 - e)^{-3/2} \, \text{harmonic becomes predominant (Farmer \& Phinney 2003), so most GW power is radiated at a frequency of} \, f_{\text{peak}} = 2.16(1 - e)^{-3/2}/P^{1/2}. \, \text{To better see the consequences of the second property, in Figure 2 we display the} \, a = (1 - e) \, \text{plane for a BHB. The boundaries of the stripes have been estimated by looking at the minimum and maximum frequencies audible by the detectors,} \, f_1 \, \text{and} \, f_2, \, \text{and letting} \, f_1 < f_{\text{peak}} < f_2, \, \text{with} \, f_{\text{peak}} \text{defined before. If a BHB is evolving only due to GW emission, it will evolve parallel to the green lines. These tracks are parallel to the stripes because as long as} \, e \ll 1, \, \text{the pericenter distance,} \, r_p = a (1 - e), \, \text{is almost constant during the evolution (Peters 1964), and a constant} \, r_p \, \text{corresponds to a constant} \, f_{\text{peak}} \text{. Because of this parallelism, a BHB cannot evolve into the band of a GW detector if it initially lies below the detector stripe.} \, \text{Hence, we can see that some binaries will fully miss the LISA/TJ range. A good example is the eccentric BHB we chose for Figure 1. A detector operating at higher frequencies, such as TQ or DECIGO, can, however, cover the relevant part of the phase space, so that a joint search is possible. These detectors could alert LIGO/Virgo decades to hours before an event is triggered, as one can read from the isochrones of Figure 2.}\]
Figure 3. Phase-space structure of a BHB with $M_1 = M_2 = 30 M_\odot$. The top right box fences in the birthplace of 95% of a thermal distribution of primordial binaries, i.e., those binaries formed not dynamically but via binary stellar evolution. In this box, but limited within the radii $a_{ij}$ and $a_{ej}$, the hard and ejection radii, which end at the boundary of the dynamical region because of the absence of interlopers, we also find the vast majority of binaries formed dynamically, i.e., 95% of their thermal distribution. The colored, dashed lines depict the birthplaces of BHBs formed via three different processes, which we explain in the main text. The green lines display the evolutionary tracks of a BHB entering the LIGO/Virgo band at two different eccentricities, $e = 0.1$ (lower) and $e = 5 \times 10^{-3}$ (upper). The first LIGO detections have an eccentricity $e \lesssim 0.1$, meaning that they have formed between the lower green line and the upper thick, black line.

equating their associated timescales: $\tau_{\text{int}} = 1/\Gamma$ and $\tau_{\text{gw}} = (1/4)(a_\ast /a_{\text{gw}})$, which defines the threshold shown as the thick, black line in Figure 3. The reason for the 1/4 factor is given in Peters (1964). Below the curve, BHBs will evolve due to GW emission. Above it, close encounters with interlopers are the main driving mechanism, so that BHBs can be scattered in both directions in angular momentum in a random-walk fashion. The scattering in energy is less significant but also present (see, e.g., Alexander 2017).

4. Possible Ways of Forming Relativistic BHBs

Different mechanisms have been proposed in the literature to form a BHB that eventually might end up emitting detectable GWs.

(1) Primordial binaries: in stellar dynamics, this term refers to binaries already present in the cluster that form via stellar evolution. Population synthesis models predict that these binaries populate the area of phase space displayed as the gray thick-dashed box of Figure 3 (from Belczynski et al. 2004). A large fraction of them lie above the demarcation line, implying that interaction with interlopers is needed to drive them into the LISA/TJ band.

(2) Dynamics: (2.1) close encounters of multiple single, i.e., initially not bound, objects also form BHBs (Kulkarni et al. 1993; Sigurdsson & Hernquist 1993). Their formation follows a thermal distribution in $e$ (Antognini & Thompson 2016), like primordial binaries, but the distribution of $a$ is better constrained: When the binding energy of the binary, $E_b = GM_1M_2/(2a)$, becomes smaller than the mean kinetic energy of the interlopers, $E_k = 3M_1\sigma_{\ast}^2/2$, the binary ionizes. The threshold condition $E_b = E_k$ can be expressed in terms of a “hard radius,” $a_h = GM_1M_2/(3M_1\sigma_{\ast}^2)$. These “hard” binaries heat up the system, meaning that they deliver energy to the rest of the stars interacting with them: Binaries with $a < a_h$ impart on average an energy of $\Delta E \approx kG\mu M_a/a$ to each interloper, where $\mu$ is the reduced mass of the binary and $k$ is about 0.4 when $M_1 \approx M_2 \approx M_\odot$ (Heggie 1975). The interloper hence is re-injected into the stellar system with a higher velocity because of the extra energy, $v \sim (3\sigma_{\ast}^2 + 2kG\mu/a)^{1/2}$, and the center of mass of the BHB recoils at a velocity of $v_b \sim M_1v/(M_1 + M_2)$. Occasionally, the BHB will lose the system if this velocity exceeds the escape velocity of the GC, $v_{\text{esc}} = \sqrt{2GM_\odot/r_b}$ (Rodriguez et al. 2016). The threshold for this to happen is defined by the condition $v_b = v_{\text{esc}}$, i.e., the binary must have a semimajor axis smaller than the “ejection radius,” $a_{ej}$. Therefore, all of these BHBs are confined in $a_h < a < a_{ej}$ of Figure 3. Because of their thermal distribution, 95% of them have $e < 0.975$. Therefore, they populate an even smaller area than those primordial binaries.

(2.2) Binary–single interactions: initially, we have a hard BHB that interacts with a single object in a chaotic way. During the interaction, the interloper might excite the eccentricity of the inner binary to such high values that the binary is on an almost head-on-collision orbit, to soon merge and emit a detectable burst of GWs (Gültekin et al. 2006; Samsing et al. 2014; Amaro-Seoane & Chen 2016). This happens only if $\tau_{\text{gw}}$ is shorter than the period of the captured interloper $P_{\text{int}}$. The event rate for BHBs has not been calculated for this scenario but earlier calculations for neutron-star binaries find it to be $1 \text{Gpc}^{-3} \text{yr}^{-1}$ (Samsing et al. 2014). We derive now the eccenticities of these BHBs: suppose the semimajor axis of a BHB changes from $a$ (with, of course, $a_{ej} < a < a_h$) to $a'$ and $e$ to $e'$ during the three-body interaction, and the final orbit of the interloper around the center of mass of the BHB has a semimajor axis of $a_{\text{int}}$. Energy conservation results in the following relations: $a' > a$ and $a_{\text{int}} \approx 2a/(1 - a/a')$ (also see Samsing et al. 2014), where we neglect the initial energy of the interloper because the BHB is assumed to be hard. Then, using a conservative criterion for a successful inspiral, $\tau_{\text{gw}}(a', e') = P_{\text{int}}(a_{\text{int}})$, we derive $e'$ for the BHB, which allows us to confine the range of eccenticities as the dashed blue curve of Figure 3.

(3) Hierarchical triple: this is similar to the previous configuration, but now we only consider $1 < a_{\text{int}}/a < 1.5$ because this requires that $a_{\text{int}} > 6a$, in which case the configuration is stable (Mardling & Aarseth 2001). The stability leads to a secular, periodic evolution of the orbital eccentricity of the inner BHB, which is known as the Lidov–Kozai resonance (Kozai 1962; Lidov 1962). At a critical eccentricity, the inner BHB will decouple via GW emission from the interaction with the tertiary and merge, and the merger rate has been estimated to be $0.3–5 \text{Gpc}^{-3} \text{yr}^{-1}$ (Antonini et al. 2014, 2016; Kipp et al. 2016). We follow the scheme of Antonini et al. (2014) of isolated hierarchical triples but impose four additional requirements that are fundamental for a realistic estimation of the threshold eccentricity in our work. (a) The BHB has $a_{\text{ej}} < a < a_h$; (b) The third body orbiting the BHB has a mass of $M_{\text{int}} = 10 M_\odot$ because of mass segregation and an eccentricity of $e_{\text{int}} = 2/3$, which corresponds to the mean
of a thermal distribution (Antognini & Thompson 2016). (c) The outer binary, i.e., the third object and the inner BHB, is also hard, so that \( a_{\text{int}} < GM_2/(3\sigma v^2) \). (d) The pericenter distance of the outer binary, \( a_{\text{int}}(1 - e_{\text{int}}) \), should meet the criterion for a stable triple (Equation (90) in Mardling & Aarseth 2001). These conditions delimit the range of eccentricities as shown by the dashed orange lines in Figure 3. To reach this region, the inner binary should first cross the LISA band (also see Wen 2003; Antonini & Perets 2012; Antonini et al. 2017), but the signal may be too weak for LISA because of the suppression of harmonics as Figure 1 has shown.

(4) Gravitational braking: there is a small probability that two single BHs come to such a close distance that GW radiation dissipates a significant amount of the orbital energy, leaving the two BHs gravitationally bound (Kocsis et al. 2006; O’Leary et al. 2009; Lee et al. 2010; Hong & Lee 2015 and references therein). For GCs, and using optimistic assumptions, these binaries contribute an event rate of 0.06–20 Gpc\(^{-3}\) yr\(^{-1}\) in the LIGO band (Lee et al. 2010; Antonini et al. 2016), while in NSCs it has been estimated to range between 0.005–1 Gpc\(^{-3}\) yr\(^{-1}\) (Tsang 2013; Hong & Lee 2015). The boundaries in Figure 3 for BHBs formed via this mechanism can be calculated using the formulae of O’Leary et al. (2009). For that, we choose an initial relative velocity \( v \) in the range \( \sigma_v < v < 3\sigma_v \) and an initial impact parameter \( b \) in the range \( 0.3b_{\text{max}} < b < 0.99b_{\text{max}} \) to account for the majority of the encounters because the encounter probability is proportional to \( b^2 \), and \( b_{\text{max}} \) is the maximum impact parameter that leads to a bound binary. The first LIGO detections, had they been originated via this mechanism, should originate from the red area above the green line.

5. Discussions and Conclusions

A joint detection of BHBs with LIGO/Virgo and LISA/TJ would be desirable because of the science payback. In this Letter, we show that for GCs, the actual number of BHBs to be coincidently detected is very uncertain. As Figure 2 shows, LISA/TJ is already deaf to mildly eccentric BHBs: for example, a BHB at milli-Hertz orbital frequencies starting at \( a \sim 10^{-3} \) au and \( 0.7 \lesssim e \lesssim 0.9 \) will also be missed by LISA/TJ, but will later be detectable by LIGO/Virgo.

BHBs can form via the five mechanisms that we discussed in the list of possible formations. This allows us to pinpoint the regions in phase space that produce BHBs that eventually will merge via gravitational radiation. The total area of these five regions is a small subset of phase space. It is erroneous to assume that all binaries born in this subset are jointly detectable by LIGO/Virgo and LISA/TJ.

Only a subset of that subset of phase space will lead to successful joint detections. This sub-subset depends on the masses of the BHBs. We can see this in Figures 3 and 4. While in the first figure the hierarchical triple gets into the LISA/TJ band, it does not in the second one.

On the other hand, up to 95% of primordial and dynamical binaries (1 and 2.1 in the list of possible formations) are produced in the box delimited by gray dashed lines. In that box, and in principle, the BHBs can lead to sources jointly detectable by LIGO/Virgo and LISA/TJ. However, exceptions might occur if a scatter results in a BHB jumping toward high eccentricities. This probability has not been fully addressed. It requires dedicated numerical scattering experiments with relativistic corrections (e.g., Amaro-Seoane & Chen 2016), as well as a proper star-cluster model to screen out BHBs that can decouple from the stellar dynamics (e.g., our model as presented in Figures 3 and 4).

We have shown that mergers in GCs produced by the mechanisms (2.2), (3), and (4) are inaudible to LISA. The event rates corresponding to these mergers have been largely discussed in the literature, but are uncertain, due to questionable parameters, such as the cosmic density of GCs and the number of BHs in them. Nevertheless, it has been estimated that the rate could be as large as 2 Gpc\(^{-3}\) yr\(^{-1}\) (rate of BH–BH collisions derived in Lee et al. 2010). More inaudible BHBs could possibly come from the field (isolated triples; Antonini et al. 2017; Silsbee & Tremaine 2017) or NSCs (gravitational braking, Kocsis et al. 2006; Miller & Lauburg 2009; O’Leary et al. 2009; Tsang 2013; Addison et al. 2015; Hong & Lee 2015; and massive BH as a perturber, Antonini & Perets 2012; VanLandingham et al. 2016). So the total event rate could be higher (e.g., \( \lesssim 10^5 \) Gpc\(^{-3}\) yr\(^{-1}\); VanLandingham et al. 2016).

Therefore, future multi-band GW astronomy should prepare for LIGO/Virgo BHBs that do not have LISA/TJ counterparts. A non-detection by LISA/TJ is also useful in constraining astrophysics: it puts a lower limit on the eccentricities of the LIGO/Virgo sources, which according to Figures 3 and 4 is about 0.005.

A deci-Hz detector, by covering the gap in frequencies between LISA/TJ and LIGO/Virgo, would drastically enhance the number of jointly detectable binaries.

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