Turbulent flow over spanwise-varying roughness in a minimal streamwise channel

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Abstract. We report direct numerical simulations in a minimal streamwise domain of turbulent channel flow over spanwise-alternating patches of rough and smooth walls. Despite the minimal streamwise domain overpredicting streamwise-velocity fluctuations and inhibiting the meandering of long turbulent structures, it captures the rotational behaviour of mean secondary flows also observed in other studies with spanwise-varying roughness. To extend the study of spanwise-varying roughness, we prescribe a lateral velocity to the wall roughness to mimic flow over oblique patches of roughness. Far from the wall, long-lived turbulent structures are convected in the direction of the moving roughness, but their speeds are only weakly perturbed from a preferential value of around 40% of the friction velocity. The turbulence-driven secondary flows laterally convect at comparable speeds, but depend on the roughness patch width.

1. Introduction
Turbulent flow over spanwise-varying patches of roughness are associated with secondary (cross-plane) flows in the form of large counter-rotating rolls that appear in the time mean. Although secondary flows are only about 10% of the magnitude of the bulk streamwise flow, they alter the transfer of mean momentum and perturb the spatial homogeneity of the mean velocity field [1, 2]. In addition to effects on the mean flow, secondary flows are also associated with long meandering structures that are phase-locked to the wall’s spanwise heterogeneity [3, 4]. While secondary flows can be studied by deliberately imposing spanwise-varying roughness [5], whether they manifest for any generic surface cannot be reliably predicted (e.g. complex rough surface in [6]). The occurrence of secondary flows has predominantly been studied via the equilibrium budgets of turbulent kinetic energy [7, 8] and streamwise vorticity [9, 8], via optimal growth [10], or via stability of variations [11], but few studies have directly tested the robustness of secondary flows to perturbations in the flow. One such perturbation commonly found in application is the misalignment of the flow direction with the roughness patches, i.e. obliquely varying roughness.

In this paper, we investigate whether secondary flows over spanwise-varying roughness persist or strengthen when we remove the spatial meandering of turbulent structures; we do so by conducting direct numerical simulation (DNS) in a periodic domain with very short streamwise extent, which inhibits the streamwise evolution of turbulent structures [12, 13]. The small domain also alleviates computational costs, permitting a parametric study over the spanwise wavelength and the velocity of a laterally translating roughness geometry. The latter mimics the study of flow over patches of oblique roughness (figure 1a); whereas obliquely varying roughness
consists of a spanwise roughness distribution that moves laterally with fetch, we model it in a minimal streamwise domain by moving the spanwise roughness distribution laterally with time.

2. Method

We numerically solve the continuity and Navier–Stokes equations for a pressure-gradient driven channel: \( \partial_t u_i = 0 \) and \( \partial_t u_i + \partial_j (u_i u_j) = -(1/\rho) \partial_t p + \nu \partial_j \partial_i u_i + \Pi \delta_{i1} + f_i \); here, \( \rho \) is the density, \( p \) is the periodic pressure, \( \nu \) is the kinematic viscosity, \( \Pi (>0) \) is the driving kinematic pressure gradient, and \( f_i \) is the immersed-boundary force that implements the roughness geometry using a volume-of-fluid approach [14]. The streamwise, spanwise, and wall-normal directions are denoted by \{x_1, x_2, x_3\} \equiv \{x, y, z\} with corresponding velocity components \{u_1, u_2, u_3\} \equiv \{u, v, w\}. We refer the reader to Rouhi et al. [15] for details of the present numerical code. The governing equations are spatially discretised on a staggered grid with a fourth-order finite-difference scheme that conserves mass, momentum, and energy. Time marching is achieved using a low-storage third-order Runge–Kutta scheme with only the wall-normal diffusive terms treated implicitly. At the end of each Runge–Kutta substep, the fractional-step method is applied to ensure a divergence-free velocity field. The simulations are run with constant time step and a prescribed constant mass flux that sets the friction Reynolds number \( Re_x \equiv \delta u_r/\nu \) to approximately 1000, where \( \delta \) is the half-channel height defined relative to the mean roughness height (figure 1b) and \( u_r \equiv \sqrt{\tau_0/\rho} \) is the friction velocity based on the mean wall shear stress \( \tau_0 = \rho \Pi \delta \). The domain is periodic in the streamwise and spanwise directions. The bottom \((z = -4k/3)\) and top \((z = 2\delta + 4k/3)\) planes of the domain are free slip \( (\partial_z u = \partial_z v = 0) \) since the solid boundary is set by the immersed-boundary method (figure 1b). The exceptions are the smooth-wall channel simulations (cases HS & HS2Hi), where we prescribe no-slip boundary conditions at the bottom \((z = 0)\) and top \((z = 2\delta)\) of the domain. The computational cells are clustered near the wall by using a hyperbolic-tangent mapping of the wall-normal grid from the roughness crest to the channel centreline; the wall-normal grid is uniform below the roughness crest.

The roughness elements implemented in the simulations are three-dimensional sinusoids, the surface of which is defined by \( k \sin(2\pi x/\lambda) \sin(2\pi y/\lambda) \). Here, the amplitude and wavelength of the sinusoid are given by \( k/\delta = 0.056 \) and \( \lambda/k = 7.1 \), respectively. Each wavelength of the sinusoid is resolved with \( 24 \times 48 \times 80 \) cells in the streamwise, spanwise, and wall-normal directions, respectively. The computational domain of all the simulations are minimal streamwise domains with streamwise extent \( L_x^+ \approx 400 \), where the superscript ‘+’ indicates scaling by \( \nu \) and \( u_r \). Since \( Re_x \) is approximately 1000, we have \( L_x/\delta \approx 0.4 \). To accommodate an integer number of roughness wavelengths in the streamwise and spanwise extents of the domain, we set \( L_x/\delta = \lambda/\delta \approx 0.398 \) and \( L_y/\delta = 16\lambda/\delta \approx 6.36 \). The top and bottom walls of the channel are mirrored across the channel centreplane for all simulations (figure 1b). The height of the bottom
Table 1. Simulation parameters. The number of cells in the streamwise, spanwise, and wall-normal directions of the domain are given by $N_x$, $N_y$, and $N_z$, respectively. The corresponding grid resolutions are given by $\Delta_x$, $\Delta_y$ and $\Delta_z$, respectively. $T$ is the sampling window duration after initial transients have decayed.

| Case   | $\Lambda/\delta$ | $\theta$ | $U_b^+$ | $V_b^+$ | $Re_\tau$ | $N_x \times N_y \times N_z$ | $\Delta_x^+$ | $\Delta_y^+$ | $\Delta_z^+$ | $Tu_\tau/\delta$ |
|--------|------------------|----------|---------|---------|----------|-----------------------------|---------------|---------------|---------------|------------------|
| HS     | 20.8             | 0°       | 11.5    | 0.00    | 1013     | $24 \times 768 \times 400$ | 15.9          | 7.9           | 0.6–15.2      | 27.1             |
| HS_Hi  | 20.5             | 0°       | 11.3    | 0.00    | 1004     | $24 \times 768 \times 400$ | 15.6          | 8.1           | 0.3–7.7       | 19.7             |
| S08_15 | 0.80             | 0°       | 11.6    | 0.00    | 1039     | $24 \times 768 \times 400$ | 16.5          | 8.2           | 1.3–18.0      | 12.4             |
| S16_15 | 1.59             | 0°       | 11.3    | 0.05    | 996      | $24 \times 768 \times 400$ | 16.5          | 8.3           | 1.3–18.1      | 14.9             |
| S32_15 | 3.18             | 0°       | 10.0    | 0.00    | 1004     | $24 \times 768 \times 400$ | 16.6          | 8.3           | 1.3–18.1      | 12.5             |
| S64_15 | 6.36             | 0°       | 9.2     | 0.00    | 993      | $24 \times 768 \times 400$ | 16.5          | 8.2           | 1.3–18.0      | 12.4             |
| S08_30 | 0.80             | 0°       | 10.2    | 0.00    | 1010     | $24 \times 768 \times 400$ | 16.7          | 8.4           | 1.4–18.3      | 15.2             |
| S16_30 | 1.59             | 0°       | 10.7    | 0.04    | 1030     | $24 \times 768 \times 400$ | 17.0          | 8.5           | 1.4–18.7      | 15.4             |
| S32_30 | 3.18             | 0°       | 11.0    | 0.05    | 996      | $24 \times 768 \times 400$ | 16.5          | 8.3           | 1.4–18.1      | 14.9             |
| S64_30 | 6.36             | 0°       | 9.2     | 0.00    | 973      | $24 \times 768 \times 400$ | 16.1          | 8.1           | 1.3–17.7      | 14.6             |
| S08_60 | 0.80             | 0°       | 9.2     | 0.00    | 986      | $24 \times 768 \times 400$ | 16.3          | 8.2           | 1.3–17.9      | 14.8             |
| S16_60 | 1.59             | 0°       | 9.2     | 0.00    | 993      | $24 \times 768 \times 400$ | 16.6          | 8.3           | 1.4–18.2      | 15.1             |
| S32_60 | 3.18             | 0°       | 9.2     | 0.00    | 1004     | $24 \times 768 \times 400$ | 16.3          | 8.2           | 1.3–17.9      | 14.8             |
| S64_60 | 6.36             | 0°       | 9.2     | 0.00    | 1038     | $24 \times 768 \times 400$ | 17.2          | 8.6           | 1.4–18.9      | 15.6             |

The wall $z_w$ is described by the equation

$$z_w(x, y, t) = \begin{cases} 0 \text{ (i.e. smooth)} & \text{if } \lfloor 2(y-v_r t)/\Lambda \rfloor \equiv 0 \mod 2, \\ k \sin (2\pi x/\lambda) \sin (2\pi (y-v_r t)/\lambda) & \text{otherwise.} \end{cases}$$

(2.1)

Here, $\lfloor \cdot \rfloor$ is the floor function, $\Lambda$ the wavelength of the spanwise heterogeneity, and $v_r$ a prescribed spanwise velocity of the wall geometry. We choose $\Lambda/\delta$ to sweep across the values (0.80, 1.59, 3.18, 6.36) so as to cover the range $\Lambda/\delta = O(1)$, in which secondary flows are expected to be strongest [1, 16]. Details of the simulation parameters can be found in Table 1.

The first set of simulations consists of a stationary roughness geometry, i.e. $v_r = 0$ in (2.1). In this case, the volume-of-fluid fields which define the wall geometry $z_w$ are independent of time and thus only need to be computed once for each simulation. The second set of simulations consists of a roughness geometry that translates at a constant spanwise velocity $v_r = U_b \tan \theta$, where $U_b$ is the prescribed bulk velocity and $\theta$ is an effective oblique angle. We can remove the cost of computing the volume-of-fluid at every time step by running the simulation in a reference frame $S$ in which the wall geometry appears stationary. Let us denote the Cartesian coordinate system of $S$ as $\{x, y, z\}$ with corresponding velocity field components $\mathcal{U}_i$. The transformation of the velocity field to the laboratory reference frame from $S$ is given by $u_i(x, y, z) = \mathcal{U}_i(\xi \equiv x, \eta \equiv y - v_r t \mod L_y, \zeta \equiv z) + \delta_{2}v_r$. In $S$, the wall geometry $z_w$ is defined analogously to (2.1) with $v_r = 0$; the no-slip boundary condition at the wall is given by $\mathcal{U}_i(\xi, \eta, \zeta = z_w) = -\delta_{2}v_r$, which translates to zero skin velocity in the laboratory reference
frame. Hereafter, we only report velocity fields in the laboratory reference frame. Since the moving wall geometry affects the channel’s mass flow rate, the prescribed $U_b$ for the simulations are chosen through trial and error so that the steady state $Re_\tau$ is approximately 1000.

3. Results and discussion

3.1. Smooth wall

To further alleviate computational costs associated with the parameter sweep, most simulations have spatial resolutions $\Delta_x^+ \approx 16$ and $\Delta_y^+ \approx 8$ (table 1), which are coarser than similar channel simulations [17, 13]. We thus perform a grid-sensitivity study for the smooth-wall channel. We first run an additional simulation where the spatial resolution is refined in each direction, and then compare the coarse and fine simulations’ profiles of the mean velocity (figure 2a) and of the streamwise-velocity variance $u'u'$ (figure 2b), where the overline presently denotes the $xyt$-mean and the prime denotes the fluctuation from this mean. The streamwise minimal domain is known to have a stronger wake in the mean velocity profile and to overpredict $u'u'$ relative to conventional DNS results [13]. The coarser simulation’s mean velocity profile (figure 2a) agrees with the finer resolution’s profile. The variance $u'u'$ is slightly underpredicted in the coarser simulation (figure 2b), but is perhaps not due to spatial resolution but statistical convergence. The spanwise spectral density of $u'u'$ is slightly underpredicted at the near-wall peak for the coarser simulation (figure 2d), whereas wide scales far from the wall have a slightly different peak (figure 2c). The jaggedness of the spectra and the width of the error bars, especially at wide scales, hint to the lack of statistical convergence in the present DNS; for comparison, we presently use windows of $10–30$ large-eddy turnover times ($\delta/u_\tau$) to collect statistics whereas Abe et al. [13] used 400. Keeping in mind the order of statistical and resolution uncertainties here, we turn our focus to secondary flows over spanwise-heterogeneous roughness.

3.2. Stationary rough wall

In the cases where the roughness geometry is stationary, the spanwise heterogeneity is expected to give rise to large counter-rotating rolls [16, 1]. We compare the present DNS results with the results of a boundary layer experiment with an analogous setup, but with sandpaper instead of sinusoids tiled at the rough patches [4]. Since Wangsawijaya et al. [4] could not directly
measure \( u_\tau \), we presently make an estimate for \( u_\tau \) so that results are comparable to the present study. By performing a composite fit of the velocity profile for a homogeneous smooth (rough) wall to the log-law and a wake profile, we can estimate the corresponding friction velocity \( u_\tau,S \) (\( u_\tau,R \)). We then assume the wall shear stresses in the spanwise-heterogeneous roughness cases are equal to the average of the wall shear stresses in the homogeneous smooth and homogeneous rough wall cases, i.e. \( u'^2 = (u'^2_{r,S} + u'^2_{r,R})/2 \). This gives a final \( Re_\tau \) of approximately 3000, where \( \delta \) is taken as the 99\% boundary-layer thickness. In figure 3, we compare the experimental results with the present results in terms of the mean cross-plane velocity and the mean swirl strength \( \lambda_{ci} \). Presently, we compute \( \lambda_{ci} \) using the mean cross-plane velocity field that is box-filtered (width 0.2\( \delta \)) to smooth fluctuations due to lack of statistical convergence. Although secondary flows in both the present simulations and experiments have the same rotational sense, i.e. downwards over the roughness and upwards over the smooth wall (e.g. [5, 8, 1]), the present DNSs capture a weaker intensity in \( \lambda_{ci} \) compared to the experiments. The DNSs also capture peaks in \( \lambda_{ci} \) (solid blue box) at the rough-smooth transition next to the roughness crest that are not seen in the experiments. This observed difference is likely due to the difference in roughness geometry: sinusoids with height 0.056\( \delta \) presently and P36 grit sandpaper with average particle size approximately 0.007\( \delta \) in the experiments. We also compare the cross-plane integrated mean swirl strength \( I_{\lambda_{ci}} \) and cross-plane integrated streamwise-velocity variance \( I_{u'u'} \) conditioned on secondary flow locations; explicitly, they are defined in [4] as:

\[
I_{\lambda_{ci}} = \frac{1}{D} \int_0^D \int_{y_{sc} - D/4}^{y_{sc} + D/4} \lambda_{ci} \, dy \, dz,
\]

and

\[
I_{u'u'} = \frac{2}{D^2} \int_0^D \int_{y_{sc} - D/4}^{y_{sc} + D/4} u'w' \, dy \, dz,
\]

where \( y_{sc} \) is the spanwise location of the secondary-flow cell centre and \( D \) is the diameter of the secondary-flow cell, both of which are visually estimated based on coherent regions of \( \lambda_{ci} \). Unlike in the grid-sensitivity study, here the overline denotes the \( xt \)-mean and \( u' \) hence denotes the fluctuation from the \( xt \)-mean. The integration areas are depicted in figure 3 as the red dashed box for \( I_{\lambda_{ci}} \) and the grey-shaded region for \( I_{u'u'} \); for a smooth wall (\( \Lambda/\delta = 0 \)), the integration area is just the entire cross plane. Figure 4 compares these integrated quantities between the present study and the experiment. The trend of \( I_{\lambda_{ci}} \) with increasing \( \Lambda/\delta \) is reasonably similar between the experimental and simulation results; \( I_{\lambda_{ci}} \) increases with \( \Lambda/\delta \) up to a certain spanwise wavelength, approximately 1.2 for the experiments and 0.8 for the simulations. The main difference is the wider peak of \( I_{\lambda_{ci}} \) in the experiments compared to the simulations. In terms of \( I_{u'u'} \), the experimental and DNS results differ more. Firstly, the trend with increasing \( \Lambda/\delta \) is reversed; the experimental results peak in \( I_{u'u'} \) at \( \lambda/\delta \approx 1.5 \), whereas the DNS results reach a minimum in \( I_{u'u'} \) at approximately the same spanwise
wavelength. Secondly, the magnitudes of \( I_{uw} \) appear to differ by a factor more than 3 between the studies. The minimal streamwise domain is expected to only overpredict \( I_{uw} \) by a factor of approximately 2 for a smooth wall based on figure 2 (b), suggesting that part of the observed disparity in \( I_{uw} \) between the experimental and present results is perhaps due to the inaccurate estimation of \( u_\tau \) used to scale the experimental results. The disparity in trends could also be due to differences between internal and external flows. For example, since the smooth and rough patches in spanwise-varying roughness experience different wall shear stresses, secondary flow patterns may reasonably depend on this contrast of wall shear stresses. In the limit of infinite \( \Lambda/\delta \), the plane-averaged wall shear stresses over the smooth and rough parts of the channel wall are dictated by the driving pressure gradient and will hence approach equal values, whereas those for a boundary layer will in general approach a non-unity ratio.

The minimal streamwise domain, despite truncating the streamwise evolution of turbulent structures, can reproduce the mean secondary flow pattern, but not the intensity thereof, induced by spanwise-heterogeneous roughness. Though Wangsawijaya et al. [4] observed that spanwise-heterogeneous roughness is associated with meandering structures with non-negligible streamwise evolution, the appearance of secondary flows in the minimal domain suggests that long streamwise evolution of turbulent structures, despite strengthening the streamwise-velocity fluctuations, may not be essential for the main features of secondary flows to manifest.

3.3. Model for oblique roughness: temporal roughness in minimal streamwise domain

Modelling oblique roughness, we consider the effect of a laterally translating roughness geometry, which we simply refer to as ‘the moving roughness’. As there is no clear relation between an actual oblique angle and the roughness velocity, \( v_r \) in (2.1), we define an effective oblique angle \( \theta \) using an arbitrary relation \( v_r = U_b \tan \theta \). The velocity of the fluid at the wall surface is still 0, i.e. \( v_r \) only moves the geometry. The moving roughness induces a bulk spanwise velocity \( \bar{V}_y \) in table 1, but there is no net drag in the spanwise direction. Although not the focus of this paper, we find that the moving wall increases \( U_b \) relative to the stationary wall cases for \( \Lambda/\delta \gtrsim 3.2 \).

Intuitively, one might expect the moving roughness to laterally convect the flow in its direction of movement, and this is indeed what we observe. However, the velocity at which the flow is laterally convected by the roughness is not \( v_r \), but a much lower velocity. Figure 5 shows the time evolution of the streamwise-velocity fluctuation 0.88 from the wall at a fixed \( x \) location of the domain for the case where \( \Lambda/\delta = 3.2 \) and \( \theta = 30^\circ \). The high- and low-velocity structures indeed travel in the \( y \)-direction, but a crude estimate would yield a velocity of only 0.4\( u_\tau \) (corresponding to 2.3° inclination in figure 5), much smaller than the prescribed \( v_r \approx 5.8u_\tau \) (30° inclination).

We can more rigorously estimate the lateral convection velocity of the turbulent structures using the spatio-temporal spectrum \( \Psi_{uu}(\kappa_y, \omega) \), where its relation to \( \bar{w}'u' \) (defined as in \( I_{w'u'} \)) is given by \( \bar{w}'u' = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Psi_{uu} \, d\kappa_y \, d\omega \), \( \kappa_y \) is the spanwise wavenumber, and \( \omega \) is the angular frequency. Explicitly, we compute the spectrum

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Psi_{uu} \, d\kappa_y \, d\omega \]

Figure 4. Comparison of integrated (a) mean swirl strength and (b) streamwise-velocity variance, both conditioned on the secondary flow locations. ——, present results; – – –, experimental results [4].
Figure 5. Time evolution of streamwise-velocity fluctuation at fixed \(x\) and \(z/\delta = 0.8\). Case: \(\Lambda/\delta = 3.2\), \(\theta = 30^\circ\). The velocity of the roughness corresponds to the red line with 30\(^\circ\) inclination.

as \(\Psi_{uu} = \langle \hat{u}\hat{u}^* \rangle / (\Delta_{\kappa_y} \Delta_\omega)\), where \(\hat{u}\) is the discrete Fourier transform of \(u'\) given by
\[
\hat{u}(\kappa_y, \omega) = (1/(M_t N_y)) \sum_t \sum_y W(t) u'(y, t) \exp\{-i(\kappa_y y - \omega t)\}; \langle \cdot \rangle\) denotes streamwise averaging and ensemble averaging using 6 windows with 50\% overlap; \(M_t = 100\) is the number of time instants per window; \(W(t) = \sqrt{2/3}[1 - \cos(2\pi t/(M_t \Delta_t))]\) is the normalized Hanning window; \(t = (0, \ldots, M_t - 1) \Delta_t\); \(y = (0, \ldots, N_y - 1) \Delta_y\); \(\kappa_y = (-N_y/2, \ldots, N_y/2 - 1) \Delta_{\kappa_y}\); \(\omega = (-M_t/2, \ldots, M_t/2 - 1) \Delta_\omega\); \(\Delta_t\) is the interval between time instants; \(\Delta_{\kappa_y} \equiv 2\pi/(N_y \Delta_y)\); \(\Delta_\omega \equiv 2\pi/(M_t \Delta_t)\); and \(\hat{u}^*\) is the complex conjugate of \(\hat{u}\). For brevity, we only present quadrants I and II of the spatio-temporal spectra for \(\Lambda/\delta = 3.2\) in figure 6; the discussion to follow applies equally for other cases of \(\Lambda/\delta\). The intersection of the quadrants represents both -0.1 and 0.1 on the horizontal axis due to the logarithmic scale.

We find that the moving roughness has little effect on the small-scale near-wall turbulence, but energises longer-lived turbulent structures both close to and far from the wall and imparts a directional bias. Starting from the smooth wall and stationary roughness cases (\(\theta = 0^\circ\)), figure 6 shows that \(\Psi_{uu}\) is unbiased between the two quadrants and is concentrated near the wall (e.g. \(z/\delta = 0.1\)). Near the wall, the spectra consists of two peaks: the first is the inner peak, characterised by short time scales (blue boxes) representative of the small-scale and short-lived near-wall turbulence (e.g. figure 2 of [18]); the second is the outer peak, characterised by long time scales (magenta boxes) and convects in either spanwise direction at speeds around 0.4\(u_r\) (diagonal black lines). As we look further away from the wall (e.g. \(z/\delta = 0.8\)), the inner peak decays quickly whereas the outer peak persists, albeit weakly. As \(v_r\) (and hence \(\theta\) increases, the inner peak appears mostly unaffected throughout the flow as it still attenuates quickly with wall distance. In contrast, the outer peak becomes increasingly biased to the positive quadrant of the spectrum and becomes more energised away from the wall. In other words, whereas the moving roughness has little effect on the short-scaled inner peak, which is typically associated with wall-detached turbulent structures (e.g. [19]), it ‘drags’ the outer peak, which is typically associated with wall-attached structures. Not only is the outer peak more energised for increasing \(\theta\), the range of velocities at which it laterally convects also increases slightly; we see that this peak is mostly slower than 0.4\(u_r\) for \(\theta = 0^\circ\), but is mostly faster than 0.4\(u_r\) for \(\theta = 30^\circ\). An intuitive reason for this slight change in speed is that these turbulent scales have an intrinsic lateral convection speed of around 0.4\(u_r\), and that the moving roughness is only strong enough to bias the convection direction and slightly perturb this intrinsic speed.

Knowing that the streamwise-velocity fluctuations laterally convect at a speed much slower than \(v_r\), we thus posit that the secondary flows ought to laterally convect, if they exist in the first place, with velocity \(V_c\) in the direction of the moving roughness. The velocity field conditioned on a lateral convection velocity \(V_c\) is computed as
\[
u_{i|V_c}(x, y, z) = \frac{1}{N_t} \sum_t u_i(x, y - V_c t \mod L_y, z, t),
\]
where \(N_t\) is the total number of time instants for a simulated case. Having conditioned the
Figure 6. Premultiplied spatio-temporal spectra of streamwise-velocity fluctuations for $\Lambda/\delta = 3.2$ (black horizontal dash) show that a moving roughness biases structures moving in the same direction. The left-most column is for a smooth wall case. The red and black diagonal lines indicate constant lateral velocity $v_r$ and $0.4u_\tau$, respectively. Here, $\tau \equiv 2\pi/\omega$ is the time period. The blue and magenta boxes highlight the spectral region associated with short-lived ($\tau \lesssim 0.3\delta/u_\tau$) and long-lived ($\tau \gtrsim 0.7\delta/u_\tau$) scales, respectively.

velocity fields on a sweep over $V_c$, we then attempt to quantify the strength of secondary flows. Firstly, we define the mean kinetic energy of the secondary flows as

$$K \equiv \bar{V}^2|_{V_c} + \bar{W}^2|_{V_c},$$

where $\bar{V} \equiv V - \langle V \rangle_y, \bar{W} \equiv W - \langle W \rangle_y$, and $\langle \cdot \rangle_y$ denotes averaging in $y$. We then calculate the cross-plane average of $K$ as

$$I_K(V_c) = \frac{1}{0.9L_y\delta} \int_{-\delta}^\delta \int_{-0.15}^{0.15} \int_0^{L_y} \bar{V}^2|_{V_c} + \bar{W}^2|_{V_c} \, dy \, dz,$$

(3.2)

where the lower limit for the wall-normal integration is chosen as 0.1δ to avoid the direct influence of the roughness geometry in the roughness sublayer. For a given $\Lambda/\delta$ and $\theta$, figure 7(a) shows that $I_K$ reaches a local maximum for some $V_c$, presumably the lateral convection velocity of secondary flows. Unsurprisingly, the peaks in $I_K$ for stationary roughness ($\theta = 0^\circ$) occur around $V_c = 0$, implying secondary flows are phase locked to the roughness. For $\theta = 30^\circ$, the peaks in $I_K$ occur in the range 0.2$u_\tau$ to 0.55$u_\tau$, i.e. secondary flows are convected in the direction, but not at the speed, of the moving roughness. The peaks in $I_K$ for $\theta = 15^\circ$ also tend towards positive $V_c$, but to a lesser extent than for $\theta = 30^\circ$. In figure 7(d) we plot the lateral convection velocity for the secondary flows (inferred from the peaks in $I_K$) against the effective oblique angle $\theta$. While there is a trend of faster convection velocities for increasing $\theta$, its dependence on $\Lambda/\delta$ is unclear. Since $I_K$ only quantifies the strength of the in-plane flow, we require an alternative quantity to capture the rotational sense and the spatial extent of secondary flows. We thus attempt to identify secondary flows by separately integrating the positive and negative mean swirl strengths in the cross plane. Explicitly, we calculate

$$I_{\lambda^{>}_c}(V_c) = \frac{1}{A_{\lambda^{>}_c}} \int_{-0.15}^{0.15} \int_0^{L_y} \lambda^{>}_c|_{V_c} \, dy \, dz,$$

$$I_{\lambda^{<}_c}(V_c) = \frac{1}{A_{\lambda^{<}_c}} \int_{-0.15}^{0.15} \int_0^{L_y} -\lambda^{<}_c|_{V_c} \, dy \, dz,$$

(3.3)

where $\lambda^{>}_c$ is the mean swirl strength computed with $\bar{V}|_{V_c}$ and $\bar{W}|_{V_c}$ that are box-filtered with width 0.2δ to smooth-out small roll modes, its superscript $>$ indicates conditioning on
positive swirl events, $A_{\lambda_{\alpha}^+}$ is the plan area where the computed swirl strength is positive, and the corresponding terms with superscript < refer to negatively signed swirl events. Figures 7(b,c) show that the peaks in $I_{\lambda_{\alpha}^+}$ and $I_{\lambda_{\alpha}^-}$ occur at approximately the same location as peaks in $I_K$. This agreement suggests the energy measured by $I_K$ is contributed mostly by the large counter-rotating modes. As validation that these integrated quantities indeed identify secondary flows, figure 8 compares the mean swirl strengths conditioned on lateral convection velocities where $I_K$ is maximum and where $I_K$ is small. As expected, large secondary flows are visible in the mean flow when $I_K$ is maximum. Having used multiple quantities to identify the presence of secondary flows, we deduce that the moving roughness convects the secondary flows laterally without destroying them, but the convection speed, like that of the turbulent fluctuations, is substantially lower than the prescribed velocity of the moving roughness. Equivalently, the oblique angle of secondary flows is much smaller than that of the roughness.

4. Conclusion
We reported DNS in a minimal streamwise domain of turbulent flow over stationary and laterally translating spanwise-heterogeneous roughness, the latter of which mimics flow over oblique patches of roughness. When the roughness is stationary, we found that the minimal streamwise domain is able to capture the mean secondary flow behaviour when compared to experimental results of an analogous study [4]. Since the minimal streamwise domain can only accommodate structures up to $0.4\delta$ long, the similarity between experimental and DNS results suggests that the streamwise evolution of turbulent structures may not be essential to the development of mean secondary flows even though they lead to stronger streamwise-velocity fluctuations. When we prescribed a constant lateral velocity to the wall geometry, we found that turbulent structures were convected in the direction of the perturbation. The near-wall short-lived turbulence is largely unaffected by the moving roughness, but turbulent structures with longer lifetimes become biased in the direction of the moving roughness an are energised
Figure 8. The mean swirl strength of secondary flows only emerge when the flow is (a) conditioned on their convection velocities, and do not appear when the flow is (b) averaged in the laboratory reference frame.

far from the wall. The intrinsic lateral convection velocity of these structures is around $0.4u_τ$ and is only weakly affected by the speed of the roughness, which is an order of magnitude faster. Just as turbulent structures convect laterally, secondary flows are also found to convect laterally at comparable speeds. While the convection velocity of secondary flows increases with the speed of the roughness velocity, its dependence on the heterogeneity wavelength $Λ/δ$ remains unclear. Whether these observations in a minimal streamwise domain are seen in flows over oblique roughness would require large-domain simulations or experiments.

Acknowledgments

This work is supported by the Australian Research Council Discovery Projects scheme (DP160102279) and is funded in part by the Coturb program of the European Research Council. The present simulations were run on the supercomputers Magnus and Raijin, respectively maintained by Pawsey Supercomputing Centre and National Computational Infrastructure, using resources granted through the National Computational Merit Allocation Scheme. Mr. Xie is supported by the Australian Government Research Training Program Scholarship. We thank Ms. Wangsawijaya for sharing the experimental data. We thank Dr. Lozano-Durán for his valuable comments on our manuscript. We finally thank our hosts Dr. Kwon and Prof. Jiménez for their insights and support at the Fourth Madrid Turbulent Workshop.

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