Charge sensitivity of superconducting single-electron transistor

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Abstract

It is shown that the noise-limited charge sensitivity of a single-electron transistor using superconductors (of either SISIS or NISIN type) operating near the threshold of quasiparticle tunneling, can be considerably higher than that of a similar transistor made of normal metals or semiconductors. The reason is that the superconducting energy gap, in contrast to the Coulomb blockade, is not smeared by the finite temperature. We discuss also the increase of the maximum operation temperature due to superconductivity and a new peak-like feature on the $I-V$ curve of SISIS structures.
Electron transport in the systems of small-capacitance tunnel junctions shows a variety of single-electron effects [1]. The simplest and most thoroughly studied circuit revealing these effects is the so-called Single Electron Transistor [2] (SET) which consists of two tunnel junctions connected in series. At low temperatures \( T \ll e^2/C_\Sigma \), \( C_\Sigma = C_1 + C_2 \) where \( C_1 \) and \( C_2 \) are the junction capacitances) the current through this structure depends on the background charge \( Q_0 \) of the central electrode (the dependence is periodical with a period equal to the electron charge \( e \)). Hence, controlling \( Q_0 \) (for example, by a capacitive gate) it is possible to control the current \( I \) through the circuit. The possibility to use the SET as a highly-sensitive electrometer has been confirmed in numerous experiments.

The most developed technology of the SET fabrication uses the overlapping narrow aluminum films with a typical junction capacitance about few times \( 10^{-16} \) F (see, e.g. Refs. [3–5]). Consequently, the operation temperature is typically less than 1 K, and the electrodes are in the superconducting state unless the superconductivity is intentionally suppressed by the magnetic field. It has been noticed [3,4,6] that the superconductivity of electrodes improves the performance of the SET (operating near the threshold of quasiparticle tunneling) as an electrometer in comparison with the normal-state operation. However, we are not aware of quantitative theoretical analysis of this issue, which will be the subject of the present paper.

There are two major characteristics of the SET operation as an electrometer. The first one is the amplitude of the output signal modulation for \( Q_0 \) variations larger than \( e \). It was found experimentally [4] that the use of superconducting electrodes increases the modulation amplitude of current \( I \) (for fixed bias voltage \( V \)), especially at temperatures comparable to \( e^2/C_\Sigma \), thus increasing the maximum temperature. The theoretical results of the present paper confirm this statement for both \textit{NISIN} and \textit{SISIS} structures.

The other, even more important characteristic of the SET operation is the noise-limited sensitivity (ability to detect variations of \( Q_0 \) much smaller than \( e \)). The best achieved sensitivity so far (by the normal state SET) is \( 7 \times 10^{-5} e/\sqrt{Hz} \) at 10Hz [5]. In the present-day technology this figure is limited by 1/f noise which is most likely caused by random
trapping-escape processes in nearby impurities. So, in some sense, the sensitivity does not depend much on the parameters of the SET, but rather on the purity of the sample. It is unlikely that superconductivity of electrodes can significantly affect these processes. Hence, the present-day sensitivities of superconducting and normal SETs with similar parameters should not differ much for reasonably low temperatures when both SETs show sufficient modulation amplitude.

However, with the technology improvement one can expect the reduction of the noise due to impurities. Then the charge sensitivity of the SET would achieve the limit determined by the intrinsic noise $[7, 8]$ of the device caused by random electron jumps through tunnel junctions (this “white” noise has been recently measured in experiment $[9]$). Though the theory of the “classical” thermal/shot intrinsic noise of the SET is applicable to the general case of one-particle tunneling (normal metals, semiconductors, quasiparticle current in superconductors, etc.), most numerical results in Refs. $[7]$ and $[8]$ as well as in a number of subsequent papers on this subject (see, e.g. Refs. $[10–13]$) were obtained only for SETs made of normal metals. (Recently some generalization was done $[14]$ to include the possibility of two-particle tunneling which can be important in the superconducting case. Let us also mention Ref. $[6]$ in which the noise in $NISIN$ SET was briefly considered.)

In the present paper we apply the theory of Refs. $[7]$ and $[8]$ to the cases of capacitively coupled superconducting $SISIS$ and $NISIN$ SETs (the analysis of a resistively coupled SET can be done in a similar way - see Ref. $[7]$). We show that the noise-limited sensitivity of a SET-electrometer can be considerably improved by the use of superconducting electrodes.

We consider only the quasiparticle tunneling, neglecting the Josephson current, resonant tunneling of Cooper pairs, Andreev reflection, and cotunneling. This assumption is appropriate when the Josephson coupling is negligible and the normal state resistances $R_1$ and $R_2$ of tunnel junctions are well above the resistance quantum $R_Q = \pi \hbar / 2e^2$. We use the “orthodox” theory $[1,2]$ of the SET and the BCS theory $[15]$ for the calculation of the tunneling rates.

Figure 1 shows the $I – V$ curves at different temperatures for (a) the normal metal
NININ case, (b) NISIN case (which is equivalent to SINIS case), and (c)–(d) SISIS case. SETs with \( C_1 = C_2 \) and \( R_1 = R_2 = R_S/2 \) are chosen, and we neglect the gate capacitance \( C_g \) because it can always be formally distributed between \( C_1 \) and \( C_2 \) (see, e.g., Ref. [16]). Three curves in each set represent \( Q_0 = 0, e/4, \) and \( e/2 \), respectively. Temperature increase decreases the superconducting energy gap \( \Delta(T) \) (which is assumed to be equal in all S-electrodes) leading to the noticeable shift to the left of the positions of the current jumps in Figs. 1(c) and 1(d). The pure BCS theory would lead to the abrupt jumps of the current in SISIS case. To take into account the unavoidable smoothing of the jumps in reality, we assume additionally the inhomogeneous broadening of \( \Delta(0) \) with Gaussian distribution characterized by the dispersion \( w_0 \). This phenomenological parameter is chosen as \( w_0 = 0.05\Delta(0) \) in Figs. 1(c) and 1(d) (for finite temperatures \( w(T) = w_0[\Delta(T)/\Delta(0) - (T/\Delta(0))(d\Delta(T)/dT)] \) was used).

One can see that in the normal metal case the current \( I \) can be considerably modulated \((I_{max}/I_{min} \gtrsim 2) \) by \( Q_0 \) (\( V \) is fixed) only at \( T \lesssim 0.15e^2/C_S \), while at \( T = 0.3e^2/C_S \) the modulation is already negligible, \((I_{max} - I_{min})/I_{max} \approx 5\% \). Notice that the maximum relative modulation is achieved at small voltages and does not depend on ratios \( C_1/C_2 \) and \( R_1/R_2 \).

NISIN transistor with \( \Delta(0) = 0.5e^2/C_S \) shows considerable modulation crudely up to \( T \approx 0.2e^2/C_S \), while SISIS transistors with \( \Delta(0) = 0.5e^2/C_S \) and \( \Delta(0) = 2.0e^2/C_S \) operate well almost up to the critical temperature \( T_c \) \((T_c/(e^2/C_S) = 0.28 \) and 1.14 respectively\). The case \( \Delta(0) = 0.5e^2/C_S \) corresponds to the typical present-day experimental situation with aluminum junctions and \( C_S \approx 0.4 \) fF (see, e.g. Ref. [4]). Comparison of Figs. 1(c) and 1(d) shows that the increase of \( \Delta(0) \) provides further improvement of the transistor performance at high temperatures. Using Fig. 1(d) one can predict the operation of the niobium-based SET with \( C_S \approx 0.2 \) fF (current state-of-the-art for aluminum junctions) at temperatures up to 7 K.

Superconductivity improves the SET performance at relatively high temperatures because, in contrast to the Coulomb blockade, the superconducting energy gap is not smeared
by the finite temperature. In the normal metal case the $I-V$ curve has a cusp at the Coulomb blockade threshold $V_t = \min_{i,n}\{V_{i,n} \mid V_{i,n} > 0\}$, where

$$V_{i,n} = \frac{e}{C_i} \left( \frac{1}{2} + (-1)^i(n + \frac{Q_0}{e}) \right),$$

and this cusp is rounded within the voltage interval proportional to the temperature. In $SISIS$ case the jump of the $I-V$ curve at $V_t$ which is shifted due to the energy gap, $V_t = \min_{i,n}\{V_{i,n} + 2\Delta(T)C_\Sigma/eC_i \mid V_t > 4\Delta(T)\}$, remains sharp even at $T \sim \Delta(T)$, and the subthreshold current increase is only proportional to $\exp(-T/\Delta(T))$. This explains why $SISIS$ transistor shows considerable dependence on $Q_0$ for the temperatures almost up to $T_c$ even if $T \gtrsim e^2/C_\Sigma$. In $NISIN$ case the I-V curve in the vicinity of $V_t = \min_{i,n}\{V_{i,n} + \Delta(T)C_\Sigma/eC_i \mid V_t > 2\Delta(T)\}$ is rounded by the finite temperature, that makes $NISIN$ transistor worse than $SISIS$ transistor, however, it is still better than usual $NININ$ transistor.

Let us briefly discuss the origin of small peaks of the current at moderate temperatures visible in Figs. 1(c) and 1(d) ($SISIS$ case) at voltages close to the middle of the subthreshold region. The position of a peak satisfy Eq. (1) and corresponds to zero energy gain $W$ for a particular tunneling process (hence, it coincides with the position of one of the $I-V$ cusps in the corresponding $NININ$ SET). In this case the singularities in the density of states of two electrodes match, leading to increase of tunneling of thermally excited quasiparticles. Hence, the origin of peaks is similar to that of well-known peaks [15] at $V = (\Delta_1(T) - \Delta_2(T))/e$ in the single junction made of superconductors with different gaps $\Delta_1(T)$ and $\Delta_2(T)$. In our case energy gaps are the same but the Coulomb blockade provides the relative shift of the singularities in the density of states. The analysis of the master equation [1,2] shows that the singularity-matching peak can be significantly high only within the voltage range $2\Delta(T) < V < 2\Delta(T) + e/C_\Sigma$. Hence, not more than two closely located peaks from the set (1) can be well pronounced on the $I-V$ curve. In the case $\Delta(T) \gtrsim e^2/C_\Sigma$ the peak position is close to the center of the subthreshold region, and, hence, close to the peak due to the Josephson-plus-quasiparticle cycle [17].
Peculiarities of another type (jumps of the current) also exist at \( V < V_t \) at finite temperatures. Their positions satisfy equation \( V = 2\Delta(T)C_\Sigma/eC_i + V_{i,n} \) and correspond to the energy gain \( W = 2\Delta(T) \) for a particular tunneling process. The jump height decreases with the decrease of voltage and vanishes at \( V < 2\Delta(T)/e \). These jumps of current are not well-noticeable in Figs. 1(c) and 1(d).

Now let us consider the noise-limited sensitivity of the SET. The minimum detectable charge for the given bandwidth \( \Delta f \) is \( \delta Q_0 = (S_I\Delta f)^{1/2}/(\partial I/\partial Q_0) \) where the spectral density \( S_I \) of the current noise is taken in the low frequency limit. The ultimate low-temperature \( (T \ll e^2/C_\Sigma) \) sensitivity in the \textit{NININ} case is \( [7,8] \min \delta Q_0 \approx 2.7C_\Sigma(R_{\min T\Delta f})^{1/2}, R_{\min} = \min\{R_1, R_2\} \). This result can be somewhat improved in the \textit{NISIN} SET (with the same resistances) operating near the threshold \( V_t \) of quasiparticle tunneling. At low temperatures, \( T \ll \min\{e^2/C_\Sigma, \Delta(T)\} \), and for \( V \) close to nondegenerate \( V_t \), we can use approximation \( S_I \approx 2eI, I \approx I_{0,i}((V-V_t)C_1C_2/C_\Sigma), \) where \( I_{0,i}(v) = (1/eR_i)[T\Delta(T)/2]^{1/2}\int_0^\infty dy/\sqrt{y}/[1+\exp(y+(\Delta-ev)/T))]^{-1} \) is the “seed” \( I-V \) curve of \( i \)-th junction. Then the ultimate sensitivity is given by equation

\[
\min \delta Q_0 = C_\Sigma(2e\Delta f)^{1/2}\min_v\{\sqrt{I_0(v)}/(dI_0/dv)\},
\]

and finally we get the result

\[
\min \delta Q_0 \approx 2.6C_\Sigma(R_{\min T\Delta f})^{1/2}[T/\Delta(T)]^{1/4}
\]

which is better than \textit{NININ} sensitivity when \( T < \Delta(T) \). The main reason for the improvement is the increase \([3,4,6]\) of the transfer coefficient \( \partial I/\partial Q_0 \approx (e/C_i)(\partial I/\partial V) \), because the differential resistance \( R_d \) of the “seed” \( I-V \) curve near the onset of quasiparticle tunneling is less than \( R_i \). Notice that the “orthodox” theory used here is valid only if \( R_d \gtrsim R_Q \) because the cotunneling processes \([18,6]\) impose the lower bound for \( (\partial I/\partial V)^{-1} \) on the order of \( R_Q \) \([19]\). For relatively high temperatures the ratio of minimum \( \delta Q_0 \) in \textit{NISIN} and \textit{NININ} cases is larger than \([\Delta(T)/T]^{1/4} \) (e.g., compare the dashed lines in Fig. 2) because \textit{NININ} sensitivity starts to deviate up from the low-temperature approximation at smaller \( T \) than \textit{NISIN} sensitivity.
The improvement of the ultimate sensitivity is more significant in \textit{SISIS} SET. For pure BCS model the “orthodox” theory gives infinite derivative $\partial I/\partial Q_0$ at $V = V_t$ even for finite temperature leading to $\delta Q_0 \to 0$. Hence, the “orthodox” ultimate sensitivity depends on the imperfection of the current jump which is described in our model by the energy gap spread $w_0$ ($w_0 \ll \min\{\Delta(T), e^2/C_\Sigma\}$).

Figure 2 shows $\delta Q_0$ together with current $I$ and ratio $S_I/2eI$, as functions of the voltage for the symmetric \textit{SISIS} SET with parameters $\Delta(0) = 0.5e^2/C_\Sigma$, $w_0 = 0.05\Delta(0)$, $T = 0.1e^2/C_\Sigma$, and $Q_0 = 0.25e$ (numerical calculations are done using the method described in Refs. [7] and [8]). Dashed lines show $\delta Q_0$ for similar \textit{NININ} and \textit{NISIN} SETs. One can see that the sensitivity of \textit{SISIS} SET is much better than for \textit{NININ} and \textit{NISIN} cases within a relatively narrow voltage range which corresponds to the jump of current.

In contrast to \textit{NININ} and \textit{NISIN} cases, the approximation $S_I \simeq 2eI$ is not accurate in the vicinity of $V_t$ for \textit{SISIS} SET even at low temperatures (see Fig. 2) because the relatively large tunneling rate in the junction determining $V_t$, is comparable to the tunneling rate in the other junction. This approximation is valid only if $T \ll \Delta(T) \ll e^2/C_\Sigma$, and would lead to inaccuracy typically about 10% for the analytical calculation of $\min \delta Q_0$ if $T \ll \Delta(T) \sim e^2/C_\Sigma$. Nevertheless, it can be used as a crude estimate. Using Eq. (2) and smoothed by $w_0$ low-temperature ($T \ll \Delta(T)$) “seed” $I - V$ curve for SIS junction [15] we get

$$\min \delta Q_0 \simeq 1.8C_\Sigma \left( R_{\min} \Delta f w_0^2/\Delta(T) \right)^{1/2}.$$  \hspace{1cm} (4)

Notice that the numerical factor depends on the particular model describing the shape of the current jump. Comparing Eq. (4) with the result for \textit{NININ} SET, we see that the temperature $T$ is replaced in \textit{SISIS} case by $w_0^2/\Delta(T)$. Hence, the ultimate sensitivity is better in \textit{SISIS} SET (resistances are the same) with sufficiently narrow width of the current jump, $w_0 < (T\Delta(T))^{1/2}$.

In the case of very sharp “seed” $I - V$ curve, $w_0 \lesssim \Delta(T)R_Q/R_i$, the slope of the jump of the SET $I - V$ curve is determined by cotunneling [18] and it cannot be sharper than crudely
\(R_Q^{-1}\) [19]. Then \(\min \delta Q_0\) is on the order of \(C_\Sigma (\Delta f \Delta(T) R_Q^2 / R)^{1/2}\) (we assume \(\Delta(T) \gtrsim e^2 / C_\Sigma\), \(R_1 = R_2\), and the ultimate sensitivity is better than for NININ SET if \(T \gtrsim \Delta(T)(R_Q/R)^2\).

The sensitivity of such an ideal SISIS SET is even better than the “quantum” \((T = 0)\) sensitivity of a symmetric \((R_1 = R_2)\) NININ SET operating at \(V_t \sim e / C_\Sigma\) (in that case [7] \(\min \delta Q \sim (hC_\Sigma \Delta f)^{1/2}\), if \(R/R_Q \gtrsim \Delta(T) C_\Sigma / e^2\). However, notice that the quantum-noise-limited \(\min \delta Q\) of a NININ SET can be made arbitrary small using small \(V_t\) (and large ratio \(R/R_Q\)) [7] or large ratio \(R_1/R_2\) [8]; hence, in this sense the use of superconducting electrodes cannot improve further the ultimate sensitivity.

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FIGURES

FIG. 1. $I - V$ curves for (a) $NININ$, (b) $NISIN$ (or $SINIS$), and (c)–(d) $SISIS$ SETs for three values of $Q_0$ (0, $e/4$, and $e/4$) and several temperatures $T$. The curves for different $T$ are offset vertically for clarity. Notice that the modulation by $Q_0$ survives up to higher $T$ in the superconducting transistors.

FIG. 2. The minimum detectable charge $\delta Q_0$, the current $I$, and the ratio $S_i/2eI$ as functions of the bias voltage $V$ for $SISIS$ SET. Dashed lines show $\delta Q_0$ for $NININ$ and $NISIN$ SETs. The best sensitivity is achieved in $SISIS$ case.
Figs. 1a, b
\[
\Delta(0) = 0.5e^2/C_\Sigma \quad (c) \\
TC_\Sigma/e^2 = 0.25
\]

\[
\Delta(0) = 2.0e^2/C_\Sigma \quad (d) \\
TC_\Sigma/e^2 = 1
\]

Figs. 1c, d
\[
\frac{\Delta Q_0}{e} = \frac{e (R_\Sigma C_\Delta f)}{1/2}, \quad \frac{I R_\Sigma C_\Sigma}{e}, \quad \frac{S_1/2eI}{e R_\Sigma C_\Sigma} \\
\Delta (0) = 0.5e^2/C_\Sigma, \quad T=0.1e^2/C_\Sigma, \quad w_0=0.05\Delta (0)
\]

Fig. 2