Study of $B^0 \rightarrow \rho^\pm \pi^\mp$ Time-dependent CP Violation at Belle

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Abstract

We present a time-dependent analysis of CP violation in $B^0 \to \rho^\pm \pi^\mp$ decays based on a 140 fb$^{-1}$ data sample collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider. We fully reconstruct one neutral $B$ meson in the $\rho^\pm \pi^\mp$ final state and identify the flavor of the accompanying $B$ meson from its decay products. We obtain the charge asymmetry $A_{\rho\pi}^{CP} = -0.16 \pm 0.10\text{(stat)} \pm 0.02\text{(syst)}$. An unbinned maximum likelihood fit to the proper-time distributions yields $C_{\rho\pi} = 0.25 \pm 0.17\text{(stat)}^{+0.02}_{-0.06}\text{(syst)}$, $\Delta C_{\rho\pi} = 0.38 \pm 0.18\text{(stat)}^{+0.02}_{-0.04}\text{(syst)}$, $S_{\rho\pi} = -0.28 \pm 0.23\text{(stat)}^{+0.10}_{-0.08}\text{(syst)}$, and $\Delta S_{\rho\pi} = -0.30 \pm 0.24\text{(stat)} \pm 0.09\text{(syst)}$. The direct CP violation parameters for $B \to \rho^+\pi^-$ and $B \to \rho^-\pi^+$ decays are $A_{\rho\pi}^{+} = -0.02 \pm 0.16\text{(stat)}^{+0.05}_{-0.02}\text{(syst)}$ and $A_{\rho\pi}^{-} = -0.53 \pm 0.29\text{(stat)}^{+0.09}_{-0.04}\text{(syst)}$.

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In the Standard Model (SM) of elementary particles, $CP$ violation arises from the Kobayashi-Maskawa (KM) phase in the weak-interaction quark-mixing matrix. Recently, the Belle and BaBar collaborations reported results on $CP$ violation via $b \to u\bar{d}d$ transitions in $B \to \pi^+\pi^−$ decays, which are related to the $CP$ violation parameter $\phi_2$. Here we present a study of $B \to \rho^±\pi^±$, which is another $\phi_2$ related decay. Since $B \to \rho^±\pi^±$ is not a $CP$ eigenstate decay, four decay modes with different charge and flavor combinations in the neutral $B$ system must be considered.

In the decay chain $\Upsilon(4S) \to B^0\bar{B}^0 \to (\rho^±\pi^±)f_{\text{tag}}$, one of the $B$ mesons decays at time $t_{\rho\pi}$ to $\rho^±\pi^±$ and the other meson decays at time $t_{\text{tag}}$ to a final state $f_{\text{tag}}$ that distinguishes between $B^0$ and $\bar{B}^0$. The decay rate for $B^0(\bar{B}^0) \to \rho^±\pi^±$ has a time dependence given by

$$\mathcal{P}_{\rho^±\pi^±}(\Delta t) = (1 \pm \mathcal{A}_{CP}^{\rho\pi}) \frac{e^{-|\Delta t/\tau_{B^0}|}}{8\tau_{B^0}} \times \{1 + q \cdot [S_{\rho\pi} \pm \Delta S_{\rho\pi} \sin(\Delta m_{d}\Delta t) - (C_{\rho\pi} \pm \Delta C_{\rho\pi}) \cos(\Delta m_{d}\Delta t)]\},$$

where $\tau_{B^0}$ is the $B^0$ lifetime, $\Delta m_d$ is the mass difference between the two $B^0$ mass eigenstates, $\Delta t = t_{\rho\pi} - t_{\text{tag}}$, and the $b$-flavor charge $q = +1(-1)$ when the tagging $B$ meson is a $B^0(\bar{B}^0)$. The time and flavor integrated charge asymmetry $\mathcal{A}_{CP}^{\rho\pi}$ is defined as

$$\mathcal{A}_{CP}^{\rho\pi} = \frac{N(\rho^+\pi^-) - N(\rho^-\pi^+)}{N(\rho^+\pi^-) + N(\rho^-\pi^+)};$$

where $N(\rho^+\pi^-)$ and $N(\rho^-\pi^+)$ are the sum of the yields for $B^0$ and $\bar{B}^0$ decays to $\rho^+\pi^-$ and $\rho^-\pi^+$, respectively. The mixing-induced $CP$ violation parameter $S_{\rho\pi}$ is related to $\phi_2$ and $C_{\rho\pi}$ is the flavor-dependent direct $CP$ violation parameter. The asymmetry between the decay rates, $\Gamma(B^0 \to \rho^+\pi^-) + \Gamma(\bar{B}^0 \to \rho^-\pi^+)$ and $\Gamma(B^0 \to \rho^-\pi^+) + \Gamma(\bar{B}^0 \to \rho^+\pi^-)$, is described by $\Delta C_{\rho\pi}$, while the strong phase difference between the amplitudes contributing to $B^0 \to \rho\pi$ decays is described by $\Delta S_{\rho\pi}$. These parameters are related to $\phi_2$ as $S_{\rho\pi} \pm \Delta S_{\rho\pi} = \sqrt{1 - (C_{\rho\pi} \pm \Delta C_{\rho\pi})^2} \sin(2\phi_{2\text{eff}} \pm \delta)$, where $2\phi_{2\text{eff}} = \arg[(q/p)(\Delta_{\rho\pi}/A_{\rho\pi})]$ and $\delta = \arg[A_{\rho\pi}/A_{\rho\pi}^\dagger]$, $\arg[q/p]$ is the $B^0$-$\bar{B}^0$ mixing phase. The terms $A_{\rho\pi}^\dagger$ and $A_{\rho\pi}$ denote the transition amplitudes for the processes $B^0(\bar{B}^0) \to \rho^+\pi^- + B^0(\bar{B}^0) \to \rho^-\pi^+$, respectively. The angles $\phi_{2\text{eff}}$ are equal to $\phi_2$ if there is no penguin contribution. The effect of direct $CP$ violation can also be expressed in terms of another set of parameters, $A_{\rho\pi}^{\pm\mp}$ and $A_{\rho\pi}^{\mp+}$:

$$A_{\rho\pi}^{\pm\mp} = \frac{N(B^0 \to \rho^\mp\pi^\pm) - N(\bar{B}^0 \to \rho^\pm\pi^\mp)}{N(B^0 \to \rho^+\pi^-) + N(\bar{B}^0 \to \rho^-\pi^+)}$$

$$= \pm \mathcal{A}_{CP}^{\rho\pi} \pm C_{\rho\pi} \pm \mathcal{A}_{CP}^{\rho\pi} \cdot \Delta C_{\rho\pi}.$$
at the KEKB asymmetric-energy $e^+e^-$ collider, which collides 8.0 GeV $e^-$ and 3.5 GeV $e^+$ beams. The $\Upsilon(4S)$ is produced with a Lorentz boost of $\beta\gamma = 0.425$ nearly along the electron beamline. Since the $B^0$ and $\bar{B}^0$ mesons are approximately at rest in the $\Upsilon(4S)$ center-of-mass system (CM), $\Delta t$ can be determined from $\Delta z$, the displacement in $z$ between the $\rho^\pm\pi^\mp$ and $f_{\text{tag}}$ decay vertices: $\Delta t \simeq (z_{\rho^\pm\pi^\mp} - z_{f_{\text{tag}}})/\beta\gamma c$. The $z$ axis is anti-parallel to the positron beam.

The Belle detector is a large-angle general purpose spectrometer that consists of a silicon vertex detector (SVD), a central drift chamber (CDC), an array of aerogel threshold Čerenkov counters (ACC), time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of CsI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux return located outside of the coil is instrumented to detect $K^0_L$ mesons and identify muons.

To reconstruct $B^0 \to \rho^\pm\pi^\mp$ candidates, we combine pairs of oppositely charged tracks with $\pi^0$ candidates. Each charged track is required to have transverse momenta greater than 100 MeV/$c$ in the laboratory frame. Charged tracks are identified as pions by combining information from the ACC, CDC and TOF. Electron-like tracks are rejected. The $\gamma$ energies for $\pi^0$ candidates are required to be greater than 50 MeV if the photon is detected in the barrel ECL (32° $< \theta < 129^\circ$); otherwise, the energy is required to be larger than 100 MeV, where $\theta$ denotes the polar angle with respect to the $z$-axis. The $\pi^0$ candidates are selected from $\gamma\gamma$ pairs in invariant masses in the range 0.118 GeV/$c^2 < M_{\gamma\gamma} < 0.150$ GeV/$c^2$, and momentum larger than 200 MeV/$c$ in the laboratory frame. In addition, we require $|\cos \theta_{\text{dec}}^\rho| < 0.95$, where $\theta_{\text{dec}}^\rho$ is defined as the angle between the photon flight direction and the boost direction from the laboratory system in the $\pi^0$ rest frame, and we require the $\chi^2$ of the $\pi^0$ mass-constrained fit to be less than 50.

$B$ meson candidates are reconstructed using the beam-energy constrained mass $M_{bc} \equiv \sqrt{E_{\text{beam}}^2 - P_B^2}$ and the energy difference $\Delta E \equiv E_B - E_{\text{beam}}$. The variables $E_B$ and $P_B$ are the reconstructed energy and momentum of the $B$ candidate in the CM frame, and $E_{\text{beam}}$ is the CM beam energy. The $B$ candidates in the region with $M_{bc} > 5.2$ GeV/$c^2$ and $-0.3$ GeV $< \Delta E < 0.2$ GeV are selected. The signal region is defined as $M_{bc} > 5.27$ GeV/$c^2$ and $-0.10$ GeV $< \Delta E < 0.08$ GeV. The $B \to \rho^\pm\pi^\mp$ candidates are formed from 3-body $B \to \pi^+\pi^-\pi^0$ decays with a $\pi^\pm\pi^0$ invariant mass in the range 0.57 GeV/$c^2 < M_{\pi^\pm\pi^0} < 0.97$ GeV/$c^2$ and $\rho$ helicity $|\cos \theta_{\text{hel}}^\rho| > 0.5$, where $\theta_{\text{hel}}^\rho$ is defined as the angle between the charged pion direction and the $B^0$ direction in the $\rho$ rest frame. To avoid the region where the $\rho^+\pi^-$ and $\rho^-\pi^+$ contributions interfere, we exclude candidates with both $M_{\pi^+\pi^-}$ and $M_{\pi^-\pi^0}$ smaller than 1.22 GeV/$c^2$. Candidates with $M_{\pi^+\pi^-} < 0.97$ GeV/$c^2$ are removed to avoid the region where the $\rho^+\pi^-$ or $\rho^-\pi^+$ bands overlap with $\rho^0\pi^0$.

To suppress the dominant $e^+e^- \to qq$ continuum background ($q = u, d, s, c$), we form the likelihood ratio $\mathcal{R} = \mathcal{L}_s / (\mathcal{L}_s + \mathcal{L}_{\text{bkg}})$, where $\mathcal{L}_s$ and $\mathcal{L}_{\text{bkg}}$ are likelihood functions for signal and continuum events, respectively. We use a Fisher discriminant based on five modified Fox-Wolfram moments, and the CM flight direction of the $B$ ($\theta_B$) with respect to the $z$-axis to form the likelihood function. The signal likelihood $\mathcal{L}_s$ is determined from a GEANT-based Monte Carlo (MC), and $\mathcal{L}_{\text{bkg}}$ is based on $M_{bc}$ sideband data, $M_{bc} < 5.26$ GeV/$c^2$. The continuum background is reduced by requiring $\mathcal{R}$ to be greater than 0.8. If there is more than one candidate in an event, we select the candidate with the smallest sum of the $\chi^2$ for the $\pi^+\pi^-$ vertex fit and the $\pi^0$ mass-constrained fit.

The flavor of the accompanying $B$ meson is identified from the decay products not associated with the reconstructed $B^0 \to \rho^\pm\pi^\mp$ decay. We use the same method as used for
FIG. 1: $\Delta E$ (top) and $M_{bc}$ (bottom) projections for the result of the 2-D unbinned likelihood fit. The plots on the left are the results for the $\rho^+\pi^-$ candidates, while those on the right show the results for the $\rho^-\pi^+$ candidates.

the Belle sin $2\phi_1$ measurement \cite{8, 9}. Two parameters $q$ and $r$ are used to describe the flavor tagging information. The parameter $q$ is defined in Eq. 1 and the parameter $r$ is a MC-determined quality factor that ranges from $r = 0$ for no flavor discrimination to $r = 1$ for unambiguous flavor assignment. It is used only to sort data into six $r$ intervals. The wrong tag fractions for $B^0$ and $B^0$ are obtained from $B \to D^{*}\ell\nu$, $D^*\pi$, $D^*\rho$, and $D\pi$ data for the six $r$ intervals.

The vertex reconstruction algorithm is the same as that used for the sin $2\phi_1$ analysis \cite{8}. The vertex positions for $\rho^\pm\pi^\mp$ and $f_{\text{tag}}$ decays are reconstructed from charged tracks with associated SVD hits and an interaction point constraint. The vertex for $f_{\text{tag}}$ is determined from all well-reconstructed tracks excluding the tracks from the $B^0 \to \rho^\pm\pi^\mp$ decay and $K^0_S$ candidates.

Figure 1 shows the $\Delta E$ ($M_{bc}$) distribution in the $M_{bc}$ ($\Delta E$) signal region for $B^0 \to \rho^\pm\pi^\mp$ candidates after flavor tagging and vertex reconstruction. The $\rho^\pm\pi^\mp$ signal yields are extracted from an unbinned maximum-likelihood fit to the two-dimensional ($M_{bc}, \Delta E$) distribution. The backgrounds are categorized as continuum $q\bar{q}$, $b \to c$ transitions ($B\bar{B}$),
FIG. 2: Signal yields as functions of (left) $M_{\pi\pi}$ and (right) $\cos\theta^0_{\text{hel}}$ in data. The histograms show the results of $B \to \rho^{\pm}\pi^\mp$ MC simulation with areas normalized to the total signal yield.

$B \to \rho K$, and rare charmless decays other than $B \to \rho K$ (rare $B$). The distributions for $\rho\pi$, $B\bar{B}$, $\rho K$, and rare $B$ events are obtained from MC.

The $\rho\pi$ signal PDF contains two components: signal events reconstructed with the correct charge ($P_{\rho\pi}$) and those with incorrect charge ($P_{\text{wc}}^{\rho\pi}$). The fraction of events with incorrect charge in the signal region due to combinations that include a random $\pi^0$ is estimated to be 2.7% from MC and is fixed in the fit. The signal PDF shape is modeled by a smoothed histogram. The $\Delta E$ distributions for $B \to \rho\pi$ signal are parameterized separately for $\pi^0$ momentum below and above 1.2 GeV/$c$ in the laboratory frame. The $\Delta E$ widths for $\rho\pi$ and $\rho K$ are calibrated from $D^{*0} \to D^{0}[K^−π^+]π^0$ data. The $B^+ \to D^{*0}[K^−π^+π^0]π^+$ mode is used to calibrate the $\Delta E$ and $M_{bc}$ peak positions. The $M_{bc}$ and $\Delta E$ distributions for the continuum $q\bar{q}$ are parametrized by an ARGUS background function \[10\] and a linear function, respectively. The contributions from $B \to \rho K$ (with $B = (9.0 \pm 1.6) \times 10^{-6}$ \[11\]) and rare $B$ decays are fixed in the fit, while the yields for $B \to \rho\pi$ signal, $B\bar{B}$ and continuum backgrounds, and the shape parameters for continuum are floated. We obtain $483 \pm 46$ $B \to \rho^{\pm}\pi^\mp$ events, and obtain a time and flavor integrated charge asymmetry $A_{\text{CP}}^{\rho\pi} = -0.16 \pm 0.10$ (stat). The estimated yields for $B \to \rho\pi$, $B \to \rho K$, $q\bar{q}$, $B\bar{B}$ and rare $B$ in the signal region are 328.7, 11.2, 833.0, 23.3 and 18.8, respectively. We remove the requirements on $M_{\pi^\pm\pi^0}$ and $\cos\theta^0_{\text{hel}}$ and examine these distributions to verify that the signals reconstructed as $B \to \pi^+\pi^-\pi^0$ are from the two-body decay $B \to \rho\pi$. Figure 2 shows the signal yields in bins of $M_{\pi^\pm\pi^0}$ and $\cos\theta^0_{\text{hel}}$ for data.

The CP violation parameters are obtained from an unbinned maximum-likelihood fit to the observed proper-time distribution for the $B \to \rho\pi$ candidates in the ($M_{bc}, \Delta E$) signal region. The likelihood function describing the proper-time distribution is

$$L = \prod_{i=1}^{N} \left\{ f_{\rho\pi} P_{\rho\pi}(\Delta t_i) + f_{\text{wc}}^{\rho\pi} P_{\rho\pi}^{\text{wc}}(\Delta t_i) + f_{\rho K} P_{\rho K}(\Delta t_i) + f_{q\bar{q}} P_{q\bar{q}}(\Delta t_i) + f_{B\bar{B}} P_{B\bar{B}}(\Delta t_i) + f_{\text{rare}B} P_{\text{rare}B}(\Delta t_i) \right\},$$

where the weighting functions $f_m$ ($m = \rho\pi$, $\rho K$, $q\bar{q}$, $B\bar{B}$, and rare $B$) are determined on
an event-by-event basis as functions of $M_{bc}$ and $\Delta E$ for each flavor tagging $\tau$ interval and $\pi^0$ momentum range in the laboratory system. The time-dependent probability density functions ($\Delta t$ PDFs) $P_{\rho\pi}(\Delta t_i)$ for $B \to \rho \pi$ and $P_{\rho K}(\Delta t_i)$ for $B \to \rho K$ are obtained from the true PDFs convolved with the $\Delta t$ resolution function used in the sin$2\phi_1$ measurement. The true PDF for $B \to \rho t$ is given by Eq.1 modified to incorporate the effect of incorrect flavor tagging. The PDF for $B \to \rho \pi$ signal reconstructed with incorrect charge, $P_{\rho \pi}^{wC}(\Delta t_i)$, is given by $P_{\rho \pi}(\Delta t_i)$. For $B \to \rho K$, $C = S = \Delta S = 0$, $\Delta C = -1$, and $\mathcal{A}^{\rho K} = 0$ is assumed. The resolution function consists of the detector resolution, the shift in vertex position due to secondary tracks originating from charmed particle decays, and smearing due to the approximation $\Delta t \simeq (z_{t-\pi^+} - z_{\text{tag}})/\beta \gamma c$. The $\Delta t$ PDFs for other backgrounds are all parameterized as $P_j = (1 - f_j)\delta(\Delta t - \mu_j) + f_j\exp(-|\Delta t - \mu_j|/\tau_j)$ convolved with $R_j (j = q\bar{q}, B\bar{B}$ and rare $B$), where $f_j$ is the fraction of the background with effective lifetime $\tau_j$. The resolution-like function $R_j$ for background is given by two Gaussians. The parameters of the $\Delta t$ PDF for $q\bar{q}$ background are obtained from a fit to sideband data (5.2 GeV/$c^2 < M_{bc} < 5.26$ GeV/$c^2$ and $\Delta E > -0.15$ GeV). The parameters of the $\Delta t$ PDFs for $B\bar{B}$ and rare $B$ are obtained from a fit to MC.

The maximum likelihood fit to the 1,215 $\rho \pi$ candidates gives $C_{\rho\pi} = 0.25 \pm 0.17^{+0.02}_{-0.06}$, $\Delta C_{\rho\pi} = 0.38 \pm 0.18^{+0.02}_{-0.04}$, $S_{\rho\pi} = -0.28 \pm 0.23^{+0.10}_{-0.08}$ and $\Delta S_{\rho\pi} = -0.30 \pm 0.24 \pm 0.09$, where the first (second) errors are statistical (systematic). The correlation between $C_{\rho\pi}$ and $\Delta C_{\rho\pi}$ is 0.271 and that between $S_{\rho\pi}$ and $\Delta S_{\rho\pi}$ is 0.284, while correlations between other variables are smaller. The data and fit result are shown in Fig. 3.

The systematic error in $\mathcal{A}_{CP}^{\rho \pi}$ includes a possible background asymmetry ($\pm 0.010$) and charge asymmetry in the tracking ($\pm 0.012$). The charge dependence of tracking efficiency is studied using $D^0 \to K^-\pi^+$ decays from inclusive $D^{*+} \to D^0\pi^+$ and selecting the momentum region corresponding to $B^0 \to \rho^+\pi^-$ decays. The systematic errors for time-dependent measurements include the uncertainties in the vertex reconstruction, background fraction, background $\Delta t$ PDF, wrong-tag fractions, $\rho \pi$ and $\rho K$ $\Delta t$ resolution functions, physics parameters ($\tau_B, \Delta m_d$, $\mathcal{A}_{\rho K}$, $\mathcal{A}_{\rho\pi}$) and fitting bias. The fitting bias is estimated from MC pseudo-experiments. All other systematic uncertainties are obtained by varying parameters within their errors and repeating the fit. The dominant source of systematic error is the vertex reconstruction ($^{+0.012}_{-0.055}$ for $C_{\rho\pi}$, $^{+0.011}_{-0.038}$ for $\Delta C_{\rho\pi}$, $^{+0.094}_{-0.073}$ for $S_{\rho\pi}$, and $^{+0.089}_{-0.092}$ for $\Delta S_{\rho\pi}$).

We perform various consistency checks. We examine the stability of the results as the $R$ selection criterion is varied and the asymmetry of the $\Delta t$ distributions for events in the sideband region. No significant variation or asymmetry is observed. We measure the $B^0$ lifetime with the $B^0 \to \rho^+\pi^-$ candidates and find $\tau_{B^0} = 1.56^{+0.13}_{-0.12}$ ps, which is consistent with the world average value $^{12}$.

The extraction of $\phi_2$ from measurements of time-dependent CP violation parameters in $B \to \rho^+\pi^-$ decays has been studied in several theoretical approaches $^{14,15}$. A Grossman-Quinn type bound $^{16}$ based on isospin (SU(2) symmetry) does not significantly limit the penguin diagram contribution due to the large branching fraction for $B^0 \to \rho^0\pi^0$ $^{17}$. Since the number of measurable quantities (six including $B(B^0 \to \rho^+\pi^-)$) are not sufficient to completely describe the amplitudes for $B^0 \to \rho^+\pi^-$ decay (8 free parameters), either specific models or additional assumptions are involved, such as QCD factorization $^{14}$ or SU(3) flavor symmetry $^{1}$. A recent approach assuming broken flavor-SU(3) implies $\phi_2 = (102 \pm 11 \pm 15)^\circ$ using our results $^{15}$. The first error is experimental while the second is the uncertainty due to SU(3) breaking effects.

In summary, using $152 \times 10^6$ $B\bar{B}$ pairs, we have measured CP violation parameters
for $B^0 \rightarrow \rho^\pm \pi^\mp$ decays. We obtain $A^\rho_{CP} = -0.16 \pm 0.10 \pm 0.02$, $C_{\rho\pi} = 0.25 \pm 0.17_{-0.06}^{+0.02}$, $\Delta C_{\rho\pi} = 0.38 \pm 0.18_{-0.04}^{+0.02}$, $S_{\rho\pi} = -0.28 \pm 0.23_{-0.08}^{+0.10}$ and $\Delta S_{\rho\pi} = -0.30 \pm 0.24 \pm 0.09$. These give the direct $CP$ violation parameters $A^\pm_{\rho\pi} = -0.02 \pm 0.16_{-0.02}^{+0.02}$ and $A^{+\pm}_{\rho\pi} = -0.53 \pm 0.29_{-0.04}^{+0.09}$. These results are consistent with a previous measurement $^{[13]}$. We find no significant mixing-induced or direct $CP$ violation in $B^0 \rightarrow \rho^\pm \pi^\mp$.

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