General Property of Neutrino Mass Matrix and CP Violation*

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It is found that the atmospheric neutrino mixing angle of \( \theta_{\text{atm}} \) is determined to be \( \tan \theta_{\text{atm}} = \text{Im}(B)/\text{Im}(C) \) for \( B = M_{\nu e}M_{\nu \mu} \) and \( C = M_{\nu e}M_{\nu \tau} \), where \( M_{ij} \) is the \( ij \) element of \( M_{\nu} \) with \( M_{\nu} \) as a complex symmetric neutrino mass matrix in the \((\nu_e, \nu_\mu, \nu_\tau)\)-basis. Another mixing angle, \( \theta_{13} \), defined as \( U_{e3} = \sin \theta_{13} e^{-i\delta} \) is subject to the condition: \( \tan 2\theta_{13} \propto |\sin \theta_{\text{atm}} B + \cos \theta_{\text{atm}} C| \) and the CP-violating Dirac phase of \( \delta \) is identical to the phase of \( \sin \theta_{\text{atm}} B^* + \cos \theta_{\text{atm}} C^* \). The smallest value of \( |\sin \theta_{13}| \) is achieved at \( \tan \theta_{\text{atm}} = -\text{Re}(C)/\text{Re}(B) \) that yields the maximal CP-violation and that implies \( C = -\kappa B^* \) for the maximal atmospheric neutrino mixing of \( \tan \theta_{\text{atm}} = \kappa = \pm 1 \). The generic smallness of \( |\sin \theta_{13}| \) can be ascribed to the tiny violation of the electron number conservation.

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I. INTRODUCTION

Properties of neutrino oscillations have been extensively studied in various experiments \[1, 2\] since the confirmation of the atmospheric neutrino oscillations by the SuperKamiokande collaborations in 1998 \[3\]. It is next expected that leptonic CP-violation can be observed in neutrino-related reactions \[4\] via CP-violating phases in the neutrino parameterization of Majorana neutrinos of the three flavors, \( \nu \). The recent experimental data \[2, 7\] show that the presence of tiny masses for neutrinos implied by Eq.(5) is understood by the seesaw mechanism \[8, 9\] and by the radiative mechanism \[10, 11\]. It is also stressed that the oscillations show 1) a hierarchy of \( \Delta m^2_{\text{atm}} \gg \Delta m^2_\odot \) and 2)
\sin^2 2\theta_{\text{atm}, \odot} = \mathcal{O}(1) \text{ while } \sin^2 \theta_{13} \ll 1. \text{ Among various theoretical proposals to find clues behind the mystery, there are theoretical ideas based on 1) a conservation of the } L_e - L_\mu - L_\tau (\equiv L') \text{ number \cite{10} and 2) a } \mu-\tau \text{ permutation symmetry \cite{11, 12, 13, 14, 15}. The hierarchy of } \Delta m_{20}^2 \gg \Delta m_{32}^2 \text{ is due to the ideal situation with } \Delta m_{32}^2 = 0 \text{ and } \Delta m_{20}^2 \neq 0, \text{ which are accounted by the } L'\text{-conservation, while the maximal mixing of } \sin^2 2\theta_{\text{atm}} = 1 \text{ arises from the } \mu-\tau \text{ permutation symmetry. However, to discuss how to depart from these ideal cases is physically important. Furthermore, if CP-violating phases are included, the possible form of the neutrino mass matrix is not fully understood \cite{16}. We would like to discuss it to clarify its general properties and implications on neutrino physics.}

II. NEUTRINO MASS MATRIX

The neutrino mass terms are described by

\[ -\mathcal{L}_{\text{mass}} = \frac{1}{2} (\nu_e, \nu_\mu, \nu_\tau)^T M_\nu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} + \text{h.c.}, \]

where \( M_\nu \) is a complex symmetric mass matrix. It is understood that the charged leptons and neutrinos are rotated, if necessary, to give diagonal charged-current interactions and to define \( \nu_e, \nu_\mu \) and \( \nu_\tau \). This flavor neutrino mass matrix can be diagonalized by \( U_{\text{PMNS}} \) to give

\[ U_{\text{PMNS}}^T M_\nu U_{\text{PMNS}} = M_\nu^{\text{diag}} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \]

Since \( M_\nu \) is not Hermitian, one has to deal with the complexity due to the existence of all three phases, one Dirac phase of \( \delta \) \cite{11} and two Majorana phases of \( \rho \) and \( \sigma \) \cite{20}. As a simpler choice, we use a Hermitian matrix \cite{21, 22}:

\[ M = M_\nu^\dagger M_\nu, \]

\[ U_{\text{PMNS}}^\dagger M U_{\text{PMNS}} = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}, \]

(8)

to examine the structure of \( M_\nu \) so that two Majorana phases in \( K \) become irrelevant. We parameterize \( M_\nu \) by

\[ M_\nu = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}, \]

(9)

leading to

\[ M = \begin{pmatrix} A & B & C \\ B^* & D & E \\ C^* & E^* & F \end{pmatrix}, \]

(10)

where

\[ A = |a|^2 + |b|^2 + |c|^2, \quad B = a^* b + b^* d + c^* e, \quad C = a^* c + b^* e + c^* f, \]

\[ D = |b|^2 + |d|^2 + |e|^2, \quad E = b^* c + d^* e + e^* f, \quad F = |c|^2 + |e|^2 + |f|^2. \]

(11)

Note that \( B, C \) and \( E \) are complex.

To examine the possible form of \( M_\nu \) compatible with the observed data of Eqs\( (10) \) and \( (11) \), we have directly performed the computation of Eq.\( (10) \) and have found the following constraints:

\[ c_{12}\Delta_1 - s_{12} [s_{13}^* (c_{23}B^* - s_{23}C^*) + c_{13}\Delta_2] = 0, \]

\[ s_{12}\Delta_1 + c_{12} [s_{13}^* (c_{23}B^* - s_{23}C^*) + c_{13}\Delta_2] = 0, \]

\[ c_{12} (s_{12}\Delta_1 + c_{12} [s_{13}^* (c_{23}B^* - s_{23}C^*) - s_{13}\Delta_2]) - s_{12} (c_{12}\Delta_2 + c_{12} [s_{13}^* (c_{23}B^* - s_{23}C^*) - s_{13}\Delta_2]) = 0, \]

\[ \Delta_1 = c_{13}s_{13}^* (A - \lambda_3) + c_{13}^2 (s_{23}B + c_{23}C) - s_{13}^2 (s_{23}B^* + c_{23}C^*), \quad \Delta_2 = c_{23}^2 E - s_{23}E^* + s_{23}c_{23} (D - F). \]

(14)
and the diagonalized masses:
\[
\begin{align*}
m_1^2 &= c_{12}^2 \lambda_1 + s_{12}^2 \lambda_2 - 2 c_{12} s_{12} X, \\
m_2^2 &= s_{12}^2 \lambda_1 + c_{12}^2 \lambda_2 + 2 c_{12} s_{12} X, \\
m_3^2 &= c_{13}^2 \lambda_3 + s_{13}^2 A + c_{13} \left[ \tilde{s}_{13} (s_{23} B + c_{23} C) + \tilde{s}_{13}^* (s_{23} B^* + c_{23} C^*) \right],
\end{align*}
\]

and
\[
\begin{align*}
\lambda_1 &= c_{13}^2 A - c_{13} \left[ \tilde{s}_{13} (s_{23} B + c_{23} C) + \tilde{s}_{13}^* (s_{23} B^* + c_{23} C^*) \right] + s_{13}^2 \lambda_3, \\
\lambda_2 &= c_{23}^2 D + s_{23}^2 F - 2 s_{23} c_{23} \text{Re} (E), \\
\lambda_3 &= s_{23}^2 D + c_{23}^2 F + s_{23} c_{23} \text{Re} (E), \\
2X &= c_{13} \left( c_{23} B - s_{23} C \right) - \tilde{s}_{13}^* \Delta_2^* + c_{13} \left( c_{23} B^* - s_{23} C^* \right) - \tilde{s}_{13} \Delta_2,
\end{align*}
\]

Since Eq. (12) gives
\[
\Delta_1 = 0, \quad \tilde{s}_{13}^* \left( c_{23} B^* - s_{23} C^* \right) + c_{13} \Delta_2 = 0,
\]
we obtain that
\[
\tan 2 \theta_{13} = \left| \frac{s_{23} B + c_{23} C}{\lambda_3 - A} \right|, \quad X = \frac{c_{23} \text{Re} (B) - s_{23} \text{Re} (C)}{c_{13}},
\]

and \( \delta \) used in \( \tilde{s}_{13} \) is determined by the phase of \( s_{23} B + c_{23} C \) to be:
\[
s_{23} B + c_{23} C = \left| s_{23} B + c_{23} C \right| e^{-i \delta},
\]
provided that \( s_{23} B + c_{23} C \neq 0 \). By using Eq. (13) with Eq. (17) for \( \Delta_2 \), we find that
\[
\tan \theta_{23} = \frac{\text{Im} (B)}{\text{Im} (C)},
\]
from the constraint on the imaginary part:
\[
c_{23} B - s_{23} C = c_{23} B^* - s_{23} C^*,
\]
and
\[
\tan 2 \theta_{12} = \frac{X}{\lambda_2 - \lambda_1},
\]
from the constraint on the real part. As a result, \( \delta \) is expressed as:
\[
\tan \delta = -\frac{1}{s_{23} \left( \text{Re} (B) + c_{23} \text{Re} (C) \right)}.
\]

Furthermore, considering the relations of \( \Delta_2 \) in Eqs. (14) and (17), we also obtain that
\[
\text{Im} (E) = s_{13} X \sin \delta, \\
\cos 2 \theta_{23} \text{Re} (E) = \frac{\sin 2 \theta_{23}}{2} (F - D) - s_{13} X \cos \delta.
\]

From these relations, the mass parameters are further converted to give
\[
\begin{align*}
m_1^2 &= \frac{\lambda_1 + \lambda_2}{2} - \frac{X}{\sin 2 \theta_{12}}, \\
m_2^2 &= \frac{\lambda_1 + \lambda_2}{2} + \frac{X}{\sin 2 \theta_{12}}, \\
m_3^2 &= \frac{c_{13}^2 \lambda_3 - s_{13}^2 A}{c_{13}^2 - s_{13}^2},
\end{align*}
\]

where
\[
\lambda_1 = \frac{c_{13}^2 A - s_{13}^2 \lambda_3}{c_{13}^2 - s_{13}^2},
\]

and
\[
\Delta m_\odot^2 = 2 \frac{c_{23} \text{Re} (B) - s_{23} \text{Re} (C)}{c_{13} \sin 2 \theta_{12}},
\]
which is the useful relation.

If other parameterizations of the CP-violating Dirac phase in $U_{PMNS}$ are employed, different relations will be derived. However, this difference can be absorbed in the redefinition of the masses. For example, results from $U_{PMNS}$ of the Kobayashi-Maskawa type (with $e^{-i\delta}$ as a CP-violating phase) can be generated by the replacement of $B$ and $C$ by $Be^{-i\delta}$ and $Ce^{-i\delta}$ in the same $M$ as Eq. (10) with the standard $U_{PMNS}$ of Eq. (2). Especially, in Eq. (19), $s_{23}B + c_{23}C = |s_{23}B + c_{23}C|e^{-i\delta}$ becomes $s_{23}Be^{-i\delta} + c_{23}Ce^{-i\delta} = |s_{23}B + c_{23}C|e^{-i\delta}$, leading to $s_{23}B + c_{23}C = |s_{23}B + c_{23}C| (= \text{real})$. Therefore, no CP-violating phase is induced by Eq. (19). Instead, the CP-violating phase is induced by $e_{23}B - s_{23}C = |e_{23}B - s_{23}C|e^{i\delta}$ derived as a solution of $(s_{23}B - c_{23}C)e^{-i\delta} = (s_{23}B^* - c_{23}C^*)e^{i\delta}$ from Eq. (21) with the appropriate replacement of $B$ and $C$. Since $s_{23}B + c_{23}C = \text{real}$, the relation of $\tan \theta_{23}$ becomes $\tan \theta_{23} = -\text{Im}(C)/\text{Im}(B)$ instead of Eq. (20). In this article, we only show the results by using the standard $U_{PMNS}$.

Since $\sin^2 \theta_{13} < 0.048$ is reported, let us choose $\sin \theta_{13} = 0$ and no CP-violation phase is induced by Eq. (19) because of $s_{23}B + c_{23}C = 0$ in Eq. (18), leading to real $B$ and $C$ from Eq. (21). The mixing angle of $\theta_{23}$ is determined by

$$\tan \theta_{23} = \frac{C}{B},$$

which should be compared with Eq. (20) for the complex $B$ and $C$. We obtain that $\text{Im}(E) = 0$ and

$$2E \cos 2\theta_{23} = (F - D) \sin 2\theta_{23},$$

from Eq. (24) with $t_{13} = 0$. The solar neutrino mixing angle of $\theta_{12}$ is given by

$$\tan 2\theta_{12} = \frac{2B}{c_{23}(\lambda_2 - \lambda_1)}.$$

The observed atmospheric neutrino mixing is close to the maximal one. We, then, restrict ourselves to the case with $\tan \theta_{23} = \pm 1 \equiv \kappa$ and $\cos \theta_{23} > 0$. The relation in Eq. (20) becomes

$$\text{Im}(C) = \kappa \text{Im}(B),$$

and the constraint on $E$ becomes

$$\kappa (F - D) = \sqrt{2} t_{13} \cos \delta \left( \text{Re}(B) - \kappa \text{Re}(C) \right), \quad \sqrt{2} \text{Im}(E) = t_{13} \sin \delta \left( \text{Re}(B) - \kappa \text{Re}(C) \right).$$

If no CP-violation exists, $\delta = 0$ is required. Namely, it demands $\text{Im}(B) = 0$ from Eq. (20), which, in turn, gives $\tan \theta_{23} = 0$ if $\text{Im}(C) \neq 0$. To get around $\tan \theta_{23} = 0$, $\text{Im}(C) = 0$ should be imposed and Eq. (31) disappears. As a result, $B$, $C$ and $E$ turn out to be all real. The mixing angle of $\theta_{23}$ is determined by $\kappa$ derived from Eq. (32) with $\delta = 0$. Therefore, in order to ensure the appearance of the maximal atmospheric neutrino mixing, we have to be careful to impose the constraint:

- $\text{Im}(C) = \kappa \text{Im}(B)$ for the presence of CP-violation,
- $\kappa (F - D) = \sqrt{2} t_{13} (B - \kappa C)$ with real $B$ and $C$ for the absence of CP-violation.

It should be noted that, for the absence of CP-violation, the approximate equality of $F \sim D$ is often assumed and it results in the familiar relation of $\tan \theta_{23} = \kappa \sim -C/B$ for $|t_{13}| \sim 0$. Another solution with $\tan \theta_{23} = \kappa \sim B/C$ seems appropriate because it may yield $\sin^2 2\theta_{12} \sim 0$ in Eq. (22).

### III. EXAMPLES

To see how the present analysis works, let us examine the specific example of $M_\nu$ with $c = -\kappa b^*$ given by

$$M_\nu = \begin{pmatrix} a & b & -\kappa b^* \\ b & d & e \\ -\kappa b^* & e & d^* \end{pmatrix},$$

whose physical consequence has been discussed in Ref. [24]. From $M_\nu^T M_\nu$, we obtain that

$$A = |a|^2 + 2|b|^2, \quad B = (a^* - \kappa e) b + b^* d, \quad C = (-\kappa a^* + e) b^* - \kappa b d^*, \quad D = F = |b|^2 + |d|^2 + |e|^2, \quad E = -\kappa b^* b^* + 2d^* \text{Re}(e).$$

(34)
To meet the maximal atmospheric neutrino mixing, one simple choice of mass terms is to assume that $a$ and $e$ are real so that $C = -\kappa B^*$, leading to $\text{Re}(C) = -\kappa \text{Re}(B)$ and $\text{Im}(C) = \kappa \text{Im}(B)$. Therefore, $\tan \theta_{23} = \kappa$ is recovered. From Eq. (32), we have

$$\tan \delta = -\frac{2\text{Im}(B)}{\text{Re}(B) + \kappa\text{Re}(C)} = \pm \infty,$$

leading to $|\delta| = \pi/2$, which indicates the maximal CP-violation. From Eq. (32), we also have $F = D$ by $\cos \delta = 0$ consistent with Eq. (44) as expected and $\text{Im}(E) = \pm \sqrt{2}|t_{13}\text{Re}(B)|$ as an additional constraint, where $\pm$ depends on the sign of $\delta$. Other mixing angles are computed to be:

$$\tan 2\theta_{12} = 2\sqrt{2}\frac{\text{Re}(B)}{c_{13}(D - \kappa \text{Re}(E) - \lambda_1)}, \quad \tan 2\theta_{13} = 2\sqrt{2}\frac{\text{Im}(B)}{D + \kappa \text{Re}(E) - A}.$$

Another example is $M_\nu$ with a $\mu$-$\tau$ permutation symmetry, which suggests that $c = \kappa b$ and $d = f$ giving

$$M_\nu = \begin{pmatrix} a & b & \kappa b \\ b & d & e \\ \kappa b & e & d \end{pmatrix}.$$  

From this matrix,

$$A = |a|^2 + 2|b|^2, \quad B = a^* b + b^* d + \kappa b^* e, \quad C = \kappa a^* b + b^* e + \kappa b^* d,$$

$$D = F = |b|^2 + |d|^2 + |e|^2, \quad E = \kappa|b|^2 + d^* e + e^* d,$$

are obtained. Since $C = \kappa B$ and $X = 0$, we find that $\tan \theta_{23} = \kappa$ and $\tan 2\theta_{12} = 0$ unless $\lambda_1 = \lambda_2$. If $\lambda_1 = \lambda_2$, $\theta_{12}$ is not fixed and the masses of $m_{1,2}$ turn out to satisfy $m_1^2 = m_2^2$ and $\Delta m_2^2 = 0$. The mixing angle of $\theta_{13}$ and the CP-violating phase of $\delta$ are determined to be:

$$\tan 2\theta_{13} = 2\sqrt{2}\frac{|B|}{D + \kappa E - A}, \quad \tan \delta = -\frac{\text{Im}(B)}{\text{Re}(B)}.$$

Since $\sin^2 2\theta_{12} = 4/(4 + x^2)$ with $x = (\lambda_2 - \lambda_1)/X$, we may require that $x \sim 1$:

$$\lambda_2 - \lambda_1 \sim X(= (B - \kappa C)/c_{13}),$$

(40)

that gives $\sin^2 2\theta_{12} \sim 0.8$ after the $\mu$-$\tau$ symmetry is slightly at least broken by $\text{Re}(B) \neq \text{Re}(C)$, which also induces $\Delta m_2^2 \neq 0$.

Finally, let us examine a typical texture among mass matrices with two texture zeros, which is given by

$$M_\nu = \begin{pmatrix} 0 & b & 0 \\ b & d & e (= \kappa d) \\ 0 & e (= \kappa d) & f \end{pmatrix},$$

(41)

where the similar matrix with $b = 0$ and $c \neq 0$ can be treated in the same way, and it yields

$$A = |b|^2, \quad B = b^* d, \quad C = b^* e,$$

$$D = |b|^2 + |d|^2 + |e|^2, \quad E = d^* e + e^* f, \quad F = |e|^2 + |f|^2.$$

(42)

The simplest way to get the maximal atmospheric neutrino mixing is to require that $e = \kappa d$, leading to $C = \kappa B$ and $\tan \theta_{23} = \kappa$. Then, the same relations for $\tan \theta_{13}$ and $\tan \delta$ as those in the $\mu$-$\tau$ symmetric case are also satisfied and, accordingly, $\theta_{12}$ is left undetermined. Additional constraints are given by $\arg(d) = \arg(f)$ from $\text{Im}(E) = 0$ and $|f|^2 = |b|^2 + |d|^2$ from $F = D$ imposed by Eq. (32).

IV. DISCUSSIONS

In summary, if CP-violation is present, using the complex symmetric mass matrix given by

$$M_\nu = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix},$$

(43)
we have found the important and simple relation:

$$\tan \theta_{23} = \frac{\text{Im}(B)}{\text{Im}(C)},$$

(44)

for $B = a^*b + b^*d + c^*e$ and $C = a^*c + b^*e + c^*f$. Any models with CP-violation should respect $|\text{Im}(B)| \sim |\text{Im}(C)|$ to explain the observed result of $\sin^2 2\theta_{23} \sim 1$. Furthermore, the CP-violating Dirac phase is induced by the phase of $s_{23}B + c_{23}C$. The physically interesting quantity of $\theta_{13}$ is determined to be:

$$\tan 2\theta_{13} = \frac{2|s_{23}B + c_{23}C|}{s_{23}^2D + c_{23}^2F + 2s_{23}c_{23}\text{Re}(E) - A},$$

(45)

for $A = |a|^2 + |b|^2 + |c|^2$, $D = |b|^2 + |d|^2 + |e|^2$, $E = b^*c + d^*e + e^*f$ and $F = |c|^2 + |e|^2 + |f|^2$. Other results include

$$\Delta m^2_\odot = \frac{2X}{\sin 2\theta_{12}}, \quad \tan 2\theta_{12} = \frac{2X}{\lambda_2 - \lambda_1},$$

(46)

where

$$X = \frac{c_{23}\text{Re}(B) - s_{23}\text{Re}(C)}{\cos \theta_{13}}.$$

(47)

In order to understand the origin of $\sin^2 2\theta_{13} \ll 1$, it is instructive to notice that the mass terms are grouped into three categories according to the electron number $L_e$. [24, 27]. Namely, $a$ has $L_e = 2$, $b$ and $c$ have $L_e = 1$ and $d$, $e$ and $f$ have $L_e = 0$. If the mass terms with $L_e \neq 0$ are created by perturbative interactions with $|L_e| = 1$, we may assume that $|a| \ll |b, c| \ll |d, e, f|$, leading to $|A| \ll |B, C| \ll |D, E, F|$. Then, this hierarchy ensures the appearance of $|\tan 2\theta_{13}| \ll 1$ due to the tiny violation of the $L_e$-conservation and at the same time may allow $\Delta m^2_\odot \gg \Delta m^2_\odot$ to arise if $\Delta m^2_{\text{atm}} \gtrsim \mathcal{O}(|D, E, F|)$ since $\Delta m^2_\odot = \mathcal{O}(|\text{Re}(B, C)|)$ [27].

The smallest $|\tan 2\theta_{13}|$ can be achieved at $\tan 2\theta_{23} = \text{Re}(C)/\text{Re}(B)$ to yield $|\text{Im}(B)|/s_{23}|$, pointing to $\tan \delta = \infty$ from Eq. (28) and showing the maximal CP-violation. Considering $\tan \theta_{23} = \text{Im}(B)/\text{Im}(C)$, we observe that $B$ and $C$ are consistently related to be, for the maximal atmospheric neutrino mixing with $\tan \theta_{23} = \kappa(= \pm 1)$,

$$C = -\kappa B^*,$$

(48)

which is the case of Eq. (38) and also corresponds to the mass matrix in Ref. [22]. The same result of the maximal CP-violation is obtained for $U_{PMNS}$ of the Kobayashi-Maskawa type if $\tan \theta_{23} = \text{Re}(B)/\text{Re}(C)$ is chosen and $C = \kappa B^*$ becomes a consistent relation.

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