Revisiting the Supersymmetric Pati-Salam Models from Intersecting D6-branes

Tianjun Li *,1,2,3 Adeel Mansha†,1,2 and Rui Sun‡4

1CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China
2School of Physical Sciences, University of Chinese Academy of Sciences, No.19A Yuquan Road, Beijing 100049, P. R. China
3Institute of Theoretical Physics, Jiangxi Normal University, Nanchang 330022, P. R. China
4Yau Mathematical Sciences Center, Tsinghua University, Haidian District, Beijing 100084, P. R. China

Employing new scanning methods, we revisit the systematic construction of three-family \( N = 1 \) supersymmetric Pati-Salam models in Type IIA orientifolds on \( T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2) \) with intersecting D6-branes. Arising from the stacks of D6-branes with \( U(n) \) gauge symmetries, the Pati-Salam gauge symmetries \( SU(4)_C \times SU(2)_L \times SU(2)_R \) can be broken down to the Standard Model via D-brane splitting as well as D- and F-flatness preserving Higgs mechanism. Also, the hidden sector contains \( USp(n) \) branes, which are parallel with the orientifold planes or their \( \mathbb{Z}_2 \) images. We find that the Type II T-duality in the previous study is not an equivalent relation in Pati-Salam model building if the model is not invariant under \( SU(2)_L \) and \( SU(2)_R \) exchange, and provides a way to obtain new models. We systematically construct the new models with three families, which usually do not have gauge coupling unification at the string scale. We for the first time construct the Pati-Salam models with one wrapping number equal to 5. In particular, we find that these models carry more refined gauge couplings, and thus with more possibility to have approximate gauge coupling unification. With dimension reduction method “LatentSemanticAnalysis”, we show that the three-family \( N = 1 \) supersymmetric Pati-Salam models gather on islands, where we might find interesting models there.
I. INTRODUCTION

The goal of string phenomenology is to construct the $N = 1$ supersymmetric Standard Models (SM) or the SM from string theories. In Type I, Type IIA and Type IIB string theories, D-branes as boundaries of open strings plays an important role in phenomenologically interesting model building [1]. For the open string sectors, conformal field theory provides the consistent constructions of four-dimensional supersymmetric $N = 1$ chiral models with non-Abelian gauge symmetry on Type II orientifolds. Within such framework, we obtain the chiral fermions on the worldvolume of the D-branes which are located at orbifold singularities [2–8], and/or at the intersections of D-branes in the internal space [9], which have a T-dual description in terms of magnetized D-branes [10, 11].

Within the intersecting D6-brane models on Type IIA orientifolds [12–14], many non-supersymmetric three-family Standard-like models and grand unified models have been constructed [12–25]. However, they typically suffer from the large Planck scale corrections at the loop level, or in other words, there exists the gauge hierarchy problem. On the other hand, a large number of the supersymmetric three-family Standard-like models and grand unified models have been constructed as well [26–45], which can solve the above problem. For a review, see Ref. [46].

In Ref. [37], Cvetiˇ c, Liu and one of us (TL) systematically constructed the three-family $N = 1$ supersymmetric Pati-Salam models from Type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with intersecting D6-branes where all the gauge symmetries come from $U(n)$ branes. The Pati-Salam gauge symmetries $SU(4)_C \times SU(2)_L \times SU(2)_R$ can be broken down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_3}$ via D6-brane splittings, and further down to the SM via four-dimensional $N = 1$ supersymmetry preserving Higgs mechanism. Thus, it provides a road to the SM without any additional anomaly-free $U(1)$’s around the electroweak scale. Also, the hidden sector contain $USp(n)$ branes, which are parallel with the orientifold planes or their $\mathbb{Z}_2$ images. These models have at least two confining gauge groups in hidden sector, whose gaugino condensation can in turn trigger supersymmetry breaking and (some) moduli stabilization. In particular, Chen, Mayes, Nanopoulos and one of us (TL) found one of these models with a realistic phenomenology [42, 44], and study its variations as well [43]. Thus, we shall revisit such kind of three-family $N = 1$ supersymmetric Pati-Salam model building in this work.

Moreover, it has been pointed out that there are a few other potentially interesting constructions which might lead to the SM [37]. For example, the possible massless vector-like Higgs fields, which do not arise from a $N = 2$ subsector, can break the Pati-Salam gauge symmetry down to the SM or break the $U(1)_{B-L} \times U(1)_{I_3}$ down to $U(1)_Y$. However, because the large wrapping numbers is required by the increased absolute values of the intersection numbers between $U(4)_C$ stack of D-branes and $U(2)_R$ stack or its orientifold image, it might be very difficult to find such models. Another interesting scenario is to construct the $SU(2)_L$ and/or $SU(2)_R$ gauge symmetries from filler branes, i.e., $SU(2)_{L,R} = USp(2)_{L,R}$. And then the number of the SM Higgs doublet pairs...
might be decreased. However, we do not want to construct the $SU(2)_{L,R}$ gauge symmetries from the splittings of higher rank $USp(N)$ ($N \geq 4$) branes, which would lead to even number of families in general. In such case, the absolute value for one wrapping number of $U(4)$ branes larger than 2 cannot be avoided, which might make the model building very difficult due to the tadpole cancellation conditions. Interestingly, with the better scanning method, one can definitely try to construct these models in the future.

Employing new scanning method, we will further systematically study the three-family $N = 1$ supersymmetric Pati-Salam model building in Type IIA orientifolds on $T^6/\left(Z_2 \times Z_2\right)$ with intersecting D6-branes in which the $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetries arise from $U(n)$ branes. In particular, we construct the new models with large winding numbers as well, and find that the approximate gauge coupling unification can be achieved at the string scale.

The paper is organized as follows. In Section II we briefly review the basic rules for supersymmetric intersecting D6-brane model building on Type IIA $T^6/\left(Z_2 \times Z_2\right)$ orientifolds, the tadpole cancellation conditions, and the conditions for D6-brane configurations which preserve four-dimensional $N = 1$ supersymmetry. Also, we will briefly review the T-duality symmetries and its variations in the supersymmetric model building with intersecting D6-branes.

In Section III we study the supersymmetric D6-brane model building with bigger winding numbers and generic T-duality into consideration. We point out that the Type II T-duality in Ref [37] is not an equivalent relation in Pati-Salam model building if the model is not invariant under $SU(2)_L$ and $SU(2)_R$ exchange, and provides a way to obtain the new model. With this construction, we obtain the supersymmetric D6-brane models with only one $USp$ group in the hidden sector, which have three families of the SM fermions, as well as satisfy the tadpole cancellation conditions and $N = 1$ supersymmetry preserving conditions. Furthermore, we for the first time expand our investigation to the models with large wrapping number, a la 5, and obtain the approximate gauge coupling unification in these models.

In Section IV we discuss the phenomenological consequences of new models in different classes. For each class, we show the full phenomenology table for one representative. As explicit examples, we present the chiral spectra in the open string sector for each class of models. The difference of T-dual model with paralleled third two-torus in spectrum can also be found in this section.

In Section V we perform machine learning method to show in Figure how the Minimal Supersymmetric SM (MSSM)-like models expand in our scanning according to the reduced latent dimension (which reduced from 18 wrapping numbers). We find that the MSSM-like models tend to gather in islands and indicates more chances to find more MSSM-like models in the nearby region of them.

In Section VI we briefly discuss the other potentially interesting setups and conclude. Also, we present the D6-brane configurations and intersection numbers for supersymmetric Pati-Salam models in Appendix.
II. $T^6/(Z_2 \times Z_2)$ ORIENTIFOLDS WITH INTERSECTING D6-BRANES

First, let us briefly review the basic rules to construct the supersymmetric models on Type IIA $T^6/(Z_2 \times Z_2)$ orientifolds with D6-branes intersecting at generic angles, as well as to obtain the massless open string state spectra in Refs. [27, 29]. In Type IIA string theory which is compactified on a $T^6/(Z_2 \times Z_2)$ orientifold, we consider $T^6$ as a six-torus factorized as three two-tori $T^6 = T^2 \times T^2 \times T^2$. The corresponding complex coordinates for the $i$-th two-torus are $z_i, i = 1, 2, 3$, respectively.

The $\theta$ and $\omega$ generators for the orbifold group $Z_2 \times Z_2$, which are respectively associated with the twist vectors $(1/2, -1/2, 0)$ and $(0, 1/2, -1/2)$, act on the complex coordinates $z_i$ as below

$$\theta : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3),$$

$$\omega : (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3).$$

We implement the orientifold projection by gauging the $\Omega R$ symmetry, where $\Omega$ is world-sheet parity, and $R$ acts on the complex coordinates as follows

$$R : (z_1, z_2, z_3) \rightarrow (\bar{z}_1, \bar{z}_2, \bar{z}_3).$$

Therefore, we have four kinds of orientifold 6-planes (O6-planes) respectively for the actions of $\Omega R$, $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$. In order to cancel the RR charges of O6-planes, we introduce stacks of $N_a$ D6-branes wrapping on the factorized three-cycles. Also, there are two kinds of complex structures for a two-torus: rectangular and tilted [13, 27, 29, 44], which are consistent with orientifold projection. The homology classes of the three cycles wrapped by the D6-brane stacks can be expressed in terms of $n^i_a[a_i] + m^i_a[b_i]$ and $n^i_a[a_i'] + m^i_a[b_i]$ for the rectangular and tilted tori respectively, where $[a_i'] = [a_i] + \frac{1}{2}[b_i]$. Thus, a generic one cycle can be labelled as $(n^i_a, l^i_a)$ in both cases, where in terms of the wrapping numbers $l^i_a \equiv m^i_a$ and $2l^i_a \equiv 2m^i_a + n^i_a$ for a rectangular two-torus and a tilted two-torus, respectively. And then $l^i_a - n^i_a$ must be even for a tilted two-torus.

Moreover, for a stack $a$ of $N_a$ D6-branes along the cycle $(n^i_a, l^i_a)$, we need to introduce their $\Omega R$ images $a'$ stack of $N_a$ D6-branes with wrapping numbers $(n^i_a, -l^i_a)$. The homology three-cycles for a stack $a$ of $N_a$ D6-branes and its orientifold image $a'$ respectively are

$$[\Pi_a] = \prod_{i=1}^3 \left( n^i_a[a_i] + 2^{-\beta_i}l^i_a[b_i] \right), \quad [\Pi_{a'}] = \prod_{i=1}^3 \left( n^i_a[a_i] - 2^{-\beta_i}l^i_a[b_i] \right),$$

where $\beta_i = 0$ or $\beta_i = 1$ for the rectangular or tilted $i$-th two-torus, respectively. The homology three-cycles, which are wrapped by the four O6-planes, are given by

$$\Omega R : [\Pi_{\Omega R}] = 2^3[a_1] \times [a_2] \times [a_3],$$

$$\Omega R\omega : [\Pi_{\Omega R\omega}] = -2^{3-\beta_2-\beta_3}[a_1] \times [b_2] \times [b_3].$$
\[ \Omega R \theta \omega : [\Pi_\Omega R \theta \omega] = -2^{3-\beta_1-\beta_3} [b_1] \times [a_2] \times [b_3], \]  

(6) 

\[ \Omega R \theta : [\Pi_\Omega R] = -2^{3-\beta_1-\beta_2} [b_1] \times [b_2] \times [a_3]. \]  

(7) 

Thus, the intersection numbers can be expressed in terms of wrapping numbers as follows

\[ I_{ab} = [\Pi_a][\Pi_b] = 2^{-k} \prod_{i=1}^{3} (n_a^i l_b^i - n_b^i l_a^i), \]  

(8) 

\[ I_{ab'} = [\Pi_a][\Pi_{b'}] = -2^{-k} \prod_{i=1}^{3} (n_a^i l_{b'}^i + n_{b'}^i l_a^i), \]  

(9) 

\[ I_{aa'} = [\Pi_a][\Pi_{a'}] = -2^{3-k} \prod_{i=1}^{3} (n_a^i l_a^i), \]  

(10) 

\[ I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k} (-l_{a'}^1 l_a^3 + l_a^1 n_{a'}^2 n_a^3 + n_a^1 l_a^2 n_{a'}^3 + n_a^1 n_{a'}^2 l_a^3), \]  

(11) 

where \( k = \beta_1 + \beta_2 + \beta_3 \) is the total number of tilted two-tori, and \([\Pi_{O6}] = [\Pi_{\Omega R}] + [\Pi_{\Omega R \theta \omega}] + [\Pi_{\Omega R \theta}] + [\Pi_{\Omega R \theta \omega}]\) is the sum of four O6-plane homology three-cycles.

The generic massless particle spectrum for intersecting D6-branes at general angles, which is valid for both rectangular and tilted two-tori, can be expressed via the intersection numbers as listed in Table II. In addition, the two main constraints on the four-dimensional \( N = 1 \) supersymmetric model building from Type IIA orientifolds with intersecting D6-branes are: RR tadpole cancellation conditions and \( N = 1 \) supersymmetry preservation in four dimensions, which are given in the following subsections A and B, respectively.

### A. The RR Tadpole Cancellation Conditions

The tadpole cancellation conditions directly lead to the \( SU(N_a)^3 \) cubic non-Abelian anomaly cancellation \cite{15, 16, 27}, while the cancellation of \( U(1) \) mixed gauge and gravitational anomaly or \( [SU(N_a)]^2 U(1) \) gauge anomaly can be achieved by Green-Schwarz mechanism mediated by untwisted RR fields \cite{15, 16, 27}. The D6-branes and orientifold O6-planes, which are the sources of RR fields, are restricted by the Gauss law in a compact space, namely, the sum of the RR charges of D6-branes and O6-planes must be zero due to the conservations of the RR field flux lines. The conditions for RR tadpole cancellations are given by

\[ \sum_a N_a [\Pi_a] + \sum_{a'} N_{a'} [\Pi_{a'}] - 4 [\Pi_{O6}] = 0, \]  

(12) 

where the last terms arise from the O6-planes, which have \(-4\) RR charges in D6-brane charge unit.
TABLE I: General massless particle spectrum for intersecting D6-branes at generic angles. In this table, the representations refer to $U(N_a/2)$, the resulting gauge symmetry because of $Z_2 \times Z_2$ orbifold projection \[27\]. The chiral supermultiplets contain both scalars and fermions in such supersymmetric constructions. And in our convention, the positive intersection numbers give us the left-handed chiral supermultiplets.

| Sector | Representation |
|--------|----------------|
| $aa$   | $U(N_a/2)$ vector multiplet |
|        | 3 adjoint chiral multiplets |
| $ab + ba$ | $I_{ab} (\square, \bar{\square})$ fermions |
| $ab' + b'a$ | $I_{ab'} (\square, \bar{\square})$ fermions |
| $aa' + a'a$ | $\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{a, O6}) \square$ fermions |
|        | $\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{a, O6}) \bar{\square}$ fermions |

For simplicity, we define the following products of wrapping numbers

$$A_a \equiv -n_a^1 n_a^2 n_a^3, \quad B_a \equiv n_a^1 l_a^2 n_a^3, \quad C_a \equiv l_a^1 n_a^2 n_a^3, \quad D_a \equiv l_a^1 l_a^2 n_a^3,$$

$$\bar{A}_a \equiv -l_a^1 l_a^2 n_a^3, \quad \bar{B}_a \equiv l_a^1 n_a^2 n_a^3, \quad \bar{C}_a \equiv n_a^1 l_a^2 n_a^3, \quad \bar{D}_a \equiv n_a^1 n_a^2 l_a^3. \quad (13)$$

To cancel the RR tadpoles, we introduce an arbitrary number of D6-branes wrapping cycles along the orientifold planes, dubbed as “filler branes”, which contribute to the RR tadpole cacellation conditions while trivially satisfy the four-dimensional $N = 1$ supersymmetry conditions. The tadpole conditions then take the form of

$$-2^k N^{(1)} + \sum_a N_a A_a = -2^k N^{(2)} + \sum_a N_a B_a =$$

$$-2^k N^{(3)} + \sum_a N_a C_a = -2^k N^{(4)} + \sum_a N_a D_a = -16, \quad (14)$$

where $2N^{(i)}$ is the number of filler branes wrapping along the $i$-th O6-plane that is given in Table \[II\]. The filler branes, which give us the $USp$ group, carry the same wrapping numbers as one of the O6-planes as shown in Table \[III\]. When the filler branes have non-zero $A$, $B$, $C$ or $D$, we refer to the $USp$ group as the $A$-, $B$-, $C$- or $D$-type $USp$ group, respectively.

B. Conditions for Four-Dimensional $N = 1$ Supersymmetric D6-Brane

In four-dimensional $N = 1$ supersymmetric models, 1/4 supercharges from ten-dimensional Type I T-dual are required to be preserved, namely, these 1/4 supercharges survive the orientation projection of the intersecting D6-branes and the $Z_2 \times Z_2$ orbifold projection on the background manifold. It was shown that the four-dimensional $N = 1$ supersymmetry can be preserved after the orientation projection iff the rotation angle of any D6-brane with respect to the orientifold
TABLE II: The wrapping numbers for four O6-planes.

| Orientifold Action | O6-Plane | \((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) |
|--------------------|----------|------------------------------------------------|
| \(\Omega R\)       | 1        | \((2^{\beta_1}, 0) \times (2^{\beta_2}, 0) \times (2^{\beta_3}, 0)\) |
| \(\Omega R\omega\)  | 2        | \((2^{\beta_1}, 0) \times (0, -2^{\beta_2}) \times (0, 2^{\beta_3})\) |
| \(\Omega R\theta\omega\) | 3       | \((0, -2^{\beta_1}) \times (2^{\beta_2}, 0) \times (0, 2^{\beta_3})\) |
| \(\Omega R\theta\)  | 4        | \((0, -2^{\beta_1}) \times (0, 2^{\beta_2}) \times (2^{\beta_3}, 0)\) |

plane is an element of \(SU(3)\) \([9]\), or in other words, \(\theta_1 + \theta_2 + \theta_3 = 0 \mod 2\pi\), where \(\theta_i\) is the angle between the \(D6\)-brane and orientifold-plane in the \(i\)-th two-torus. Because the \(Z_2 \times Z_2\) orbifold projection will automatically be survived for such \(D6\)-brane configuration, the four-dimensional \(N = 1\) supersymmetry conditions can be written as below \([29]\)

\[x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0,\]

\[A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0, \quad (15)\]

where \(x_A = \lambda, \ x_B = \lambda 2^{\beta_2+\beta_3}/\chi_2\chi_3, \ x_C = \lambda 2^{\beta_1+\beta_3}/\chi_1\chi_3, \ x_D = \lambda 2^{\beta_1+\beta_2}/\chi_1\chi_2, \) where \(\chi_i = R_i^2/R_i^3\) are the complex structure moduli for the \(i\)-th two-torus. And we introduce the positive parameter \(\lambda\) to put all the variables \(A, B, C, D\) on an equal footing. Based on these conditions, we can classify all the possible \(D6\)-brane configurations, which preserve four-dimensional \(N = 1\) supersymmetry, into three types:

1. The filler brane which has the same wrapping numbers as one of the O6-planes in Table II. The gauge symmetry is \(USp\) group. Because one and only one of the wrapping number products \(A, B, C\) and \(D\) has non-zero and negative value, we refer to the corresponding \(USp\) group as the \(A\)-, \(B\)-, \(C\)- or \(D\)-type \(USp\) group as mentioned in the last section.

2. The \(Z\)-type \(D6\)-brane with one zero wrapping number. There are two negative and two zero values in \(A, B, C\) and \(D\).

3. The \(NZ\)-type \(D6\)-brane without zero wrapping number. Among \(A, B, C\) and \(D\), three of them are negative while one of them is positive. Based on which one is positive, we can classify the \(NZ\)-type branes into the \(A\)-, \(B\)-, \(C\)- and \(D\)-type \(NZ\) branes. Each type has two forms of wrapping.
numbers defined as follows

\[ A_1 : (-, -) \times (+, +) \times (+, +), \quad A_2 : (-, +) \times (-, +) \times (-, +); \quad (16) \]

\[ B_1 : (+, -) \times (+, +) \times (+, +), \quad B_2 : (+, +) \times (-, +) \times (-, +); \quad (17) \]

\[ C_1 : (+, +) \times (+, -) \times (+, +), \quad C_2 : (-, +) \times (+, +) \times (-, +); \quad (18) \]

\[ D_1 : (+, +) \times (+, +) \times (+, -), \quad D_2 : (-, +) \times (-, +) \times (+, +). \quad (19) \]

To be convenient, we shall refer the Z-type and NZ-type D6-branes to be U-branes in the following since they carry U(n) gauge symmetry.

C. T-Duality Symmetry and its Variations

In string theory, two theories are equivalent when T-duality can be performed to map one to the other. This also applies to D-brane model building when two models are related by T-duality. For D6-brane configurations, two models are equivalent if their three two-tori as well as their corresponding wrapping numbers for all the D6-branes are correlated by an element of the permutation group \( S_3 \) acting on three two-tori. In addition, two D6-brane configurations are equivalent if their wrapping numbers on two arbitrary two-tori have the same absolute values but opposite sign, while their wrapping numbers on the third two-torus are the same. In this case, we call it as the D6-brane Sign Equivalent Principle. As T-duality is not the key discussion point in our work, we refer to Ref. [37] for the details about how T-dualities and its variants perform in intersecting D6-brane model building technically.

III. SUPERSYMMETRIC PATI-SALAM MODEL BUILDING

A. Construction of Supersymmetric Pati-Salam Models

To construct the SM or SM-like models from the intersecting D6-brane scenarios, besides the \( U(3)_C \) and \( U(2)_L \) gauge symmetries from stacks of branes, we must have at least two extra \( U(1) \) gauge groups in both supersymmetric and non-supersymmetric models to obtain the correct quantum number for right-handed charged leptons [16, 27–29]. One is the lepton number symmetry \( U(1)_L \), while the other is similar to the third component of right-handed weak isospin \( U(1)_{I_3R} \). And then the hypercharge is given by

\[ Q_Y = Q_{I_3R} + \frac{Q_R - Q_L}{2}, \quad (20) \]

where \( U(1)_R \) is the overall \( U(1) \) of \( U(3)_C \). In general, the \( U(1) \) gauge symmetry, which comes from a non-Abelian symmetry, is anomaly free and then its gauge field is massless. Thus, to forbid the mass term for the \( U(1)_{I_3R} \) gauge field via \( B \wedge F \) couplings, \( U(1)_{I_3R} \) can only arise from
the non-Abelian $U(2)_R$ or $USp$ gauge symmetry. And to obtain an anomaly-free $U(1)_{B-L}$ gauge symmetry, the $U(1)_L$ gauge symmetry should come from non-Abelian group as well. Note that the $U(1)_L$ stack should be parallel to the $U(3)_C$ stack on at least one two-torus, we can obtain it by splitting one $U(4)$ stack of branes into $U(1)_L$ and $U(3)_C$ stacks, which generate the $U(3)_C$ gauge symmetry in the mean time.

If $U(1)_{I_{3R}}$ arises from the stack of D6-branes on top of orientifold $^{[27,28]}$, i.e., from the $USp$ group, there exist at least 8 pairs of SM Higgs doublets, and two extra anomaly free $U(1)$ gauge symmetries in general. These $U(1)$ gauge symmetries could in principle be spontaneously broken via the Higgs mechanism by the scalar components of the chiral superfields whose quantum numbers are the same as the right-handed neutrinos. However, the D-flatness conditions cannot be preserved, and then supersymmetry is broken. Thus, the scale of symmetry breaking should be around the electroweak scale. Moreover, we typically do not have any other candidates, which can preserve the D-flatness and F-flatness conditions, and break these gauge symmetries at an intermediate scale.

Therefore, similar to Ref. $^{[37]}$, we concentrate on the Pati-Salam models in which $U(1)_{I_{3R}}$ arises from the $U(2)_R$ symmetry. Because it is very difficult to find the interesting models with $SU(2)_L$ from the D6-branes on the top of O6-plane $^{[37]}$, we study the supersymmetric $SU(4)_C \times SU(2)_L \times SU(2)_R$ model building from three stacks of D6-branes, which are not on the top of orientifold planes. In our model, we can break the Pati-Salam gauge symmetry down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ via D6-brane splittings, and further down to the SM gauge symmetry via Higgs mechanism with Higgs particles from a $N = 2$ subsector $^{[37]}$. Because we do not have any extra anomaly free $U(1)$ gauge symmetry around the electroweak scale, we solve a generic problem in previous constructions $^{[27,28]}$.

In short, we introduce three stacks of D6-branes, $a$, $b$, $c$ with D6-brane numbers 8, 4, and 4, which respectively give us the gauge symmetries $U(4)_C$, $U(2)_L$ and $U(2)_R$. The gauge anomalies from three $U(1)$s are cancelled by the generalized Green-Schwarz mechanism, and these $U(1)$s gauge fields obtain masses via the linear $B \wedge F$ couplings. Thus, we obtain the Pati-Salam gauge symmetries $SU(4)_C \times SU(2)_L \times SU(2)_R$. Moreover, to have three families of the SM fermions, we require the intersection numbers to satisfy

$$I_{ab} + I_{ab'} = 3,$$

$$I_{ac} = -3,$$

where the conditions $I_{ab} + I_{ab'} = 3$ and $I_{ac} = -3$ give us three generations of the SM fermions, whose quantum numbers under $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetries are $(4,2,1)$ and $(\bar{4},1,2)$. To satisfy the $I_{ac'} = 0$ condition, the stack $a$ D6-branes must be parallel to the orientifold ($\Omega R$) image $c'$ of the $c$ stack of D6-branes along at least one two-torus, where in our model building we choose to be the third two-torus. And then we have Open strings that stretch between the $a$ and $c'$ stacks of D6-branes. When the minimal distance square $Z^2_{ac'}$ (in $1/M_s$ units) between
these two stacks on the third two-torus is small, namely when the minimal length squared of
the stretched string is small, we obtain the light scalars with squared-masses $Z_{(ab')}'^2/(4\pi^2\alpha')$ from
the NS sector, and the light fermions with the same masses from R sector $\{15, 16, 36\}$, which
form four-dimensional $N = 2$ hypermultiplets. Thus, we have $I_{ac}'^{(2)}$ (the intersection numbers for
$a$ and $c'$ stacks on the first two two-tori) vector-like pairs of the chiral superfields with quantum
numbers $\{4, 1, 2\}$ and $\{4, 1, 2\}$. These vector-like particles are the Higgs fields, which can break
the Pati-Salam gauge symmetry down to the SM gauge symmetry, while keep the four-dimensional
$N = 1$ supersymmetry. Especially, they are massless when $Z_{(ac')}'^2 = 0$. Due to the symmetry
transformation $c \leftrightarrow c'$, the model with intersection numbers $I_{ac} = 0$ and $I_{ac'} = -3$ are equivalent
to that with $I_{ac} = -3$ and $I_{ac'} = 0$, so we shall not discuss it here.

To break the Pati-Salam gauge symmetry to the SM, we split the $a$ stack of D6-branes into $a_1$
and $a_2$ stacks respectively with 6 and 2 D6-branes. And then the $U(4)_C$ gauge symmetry is broken
down to $U(3)_C \times U(1)$. The gauge fields and three chiral multiplets in adjoint representation of
$SU(4)_C$ are broken down to the gauge fields and three chiral multiplets in adjoint representations
of $SU(3)_C \times U(1)_{B-L}$ accordingly. Also, we assume that the numbers of symmetric and anti-
symmetric representations for $SU(4)_C$ are $n_a^{(1)}$ and $n_a^{(2)}$, respectively, similar convention for $SU(3)_C,$
$SU(2)_L,$ and $SU(2)_R.$ These chiral multiplets for $SU(4)_C$ are broken down to the $n_a^{(1)}$ and $n_a^{(2)}$ chiral
multiplets in symmetric and anti-symmetric representations for $SU(3)_C,$ and $n_a^{(3)}$ chiral multiplets
in symmetric representation for $U(1)_{B-L}$. Moreover, there exist $I_{a_1a_2'}$ new fields with quantum number $\{3, -1\}$ under $SU(3)_C \times U(1)_{B-L}$ arising from the open strings at the intersections of $a_1$ and $a_2'$ stacks of D6-branes, while the rest of the particle spectrum remains the same. Also, the anomaly free gauge symmetries from $a_1$ and $a_2$ stacks of D6-branes are $SU(3)_C \times U(1)_{B-L}$, the
$SU(4)_C$ subgroup.

To break $U(2)_R$ gauge symmetry, we split the $c$ stack of D6-branes into $c_1$ and $c_2$ stacks, and
each one has two D6-branes. And then the gauge fields and three chiral multiplets in adjoint
representation of $SU(2)_R$ are broken down to the gauge fields and three chiral multiplets in adjoint
representation of $U(1)_{I_{3R}},$ respectively. The $n_c^{(1)}$ chiral multiplets in symmetric representation of
$SU(2)_R$ are broken down to the $n_c^{(1)}$ chiral multiplets in symmetric representation of $U(1)_{I_{3R}},$ while
the $n_c^{(1)}$ chiral multiplets in anti-symmetric representation $SU(2)_R$ will be gone. Also, there are
$I_{c_1c_2'}$ new fields that are neutral under $U(1)_{I_{3R}}$ arising from the open strings at the intersections
of $c_1$ and $c_2'$ stacks of D6-brane, while the rest of the particle spectrum remain the same. The anomaly free gauge symmetry from $c_1$ and $c_2$ stacks of D6-branes becomes $U(1)_{I_{3R}},$ the $SU(2)_R$
Cartan subgroup.

With the above D6-brane splittings, we obtain the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ gauge
symmetry. In order to break it further down to the SM gauge symmetry, we assume the minimal
distance square $Z_{(azc_1')}'^2$ to be small, and thus obtain $I_{azc_1'}^{(2)}$ pairs of chiral multiplets with quantum
numbers $\{1, 1, -1, 1/2\}$ and $\{1, 1, 1, -1/2\}$ under $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}.$ These
vector-like particles can break the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_3R}$ gauge symmetry down to the SM while keep the D- and F-flatness since their quantum numbers are the same as those of the right-handed neutrino and its complex conjugate. In particular, they are massless when $Z^2_{(a2c')_1} = 0$. Therefore, the complete chains for symmetry breaking are

\[
SU(4) \times SU(2)_L \times SU(2)_R \rightarrow a \rightarrow a_1 + a_2^T \quad SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}
\]

\[
c \rightarrow c_1 + c_2 \quad SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}
\]

Higgs Mechanism $\rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$.

For Type IIA orientifolds with intersecting D6-branes, the dynamical supersymmetry breaking has been studied in Ref. [33]. There exist some filler branes carrying $USp$ gauge symmetries that are confining, and then could allow for gaugino condensation, supersymmetry breaking, as well as moduli stabilization.

The gauge kinetic function for a generic stack $x$ of D6-branes is given by [33]

\[
f_x = \frac{1}{4} \left[ n_x^1 n_x^2 n_x^3 S - \left( \sum_{i=1}^3 2^{\beta_j - \beta_k} n_x^i \bar{U}^j \bar{U}^k \right) \right],
\]

where the real parts of dilaton $S$ and moduli $U^i$ respectively are

\[
\text{Re}(S) = \frac{M^3_{11} R^3_{11} R^3_{11}}{2\pi g_s},
\]

\[
\text{Re}(U^i) = \text{Re}(S) \chi_j \chi_k,
\]

where $i \neq j \neq k$, and $g_s$ is the string coupling. So the gauge coupling constant associated with a stack $x$ is

\[
g_{D6_x}^{-2} = |\text{Re}(f_x)|.
\]

In our models, the holomorphic gauge kinetic functions for $SU(4)_C$, $SU(2)_L$ and $SU(2)_R$ are identified with stacks $a$, $b$, and $c$, respectively. The holomorphic gauge kinetic function for $U(1)_Y$ is then a linear combination of these for $SU(4)$ and $SU(2)_R$. As shown in [13, 44], we have

\[
f_Y = \frac{3}{5} (\frac{2}{3} f_a + f_c).
\]

Also, we can express the tree-level MSSM gauge couplings in the form of

\[
g_\alpha^2 = \alpha g_6^2 = \beta g_3^2 = \gamma \left[ \pi e^{\phi_4} \right]
\]

where $g_6^2$, $g_3^2$, and $\frac{5}{3} g_Y^2$ are the strong, weak and hypercharge gauge couplings, respectively, and $\alpha, \beta, \gamma$ are the ratios between them. Moreover, the Kähler potential is given by

\[
K = -\ln(S + \bar{S}) - \sum_{I=1}^3 \ln(U^I + \bar{U}^I).
\]
Three stacks of D6-branes, which carry $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry, generically determine the complex structure moduli $\chi_1, \chi_2$ and $\chi_3$ because of the four-dimensional $N = 1$ supersymmetry conditions. Thus, we only have one independent modulus field. In order to stabilize the modulus, we must have at least two $USp$ groups with negative $\beta$ functions which can be confined and then allow for gaugino condensations \cite{47,49}. In general, the one-loop beta function for the $2N^{(i)}$ filler branes, which are on top of $i$-th O6-plane and carry $USp(N^{(i)})$ group, is given by \cite{37}

$$\beta_i^g = -3\left(\frac{N^{(i)}}{2} + 1\right) + 2|I_{ai}| + |I_{bi}| + |I_{ci}| + 3\left(\frac{N^{(i)}}{2} - 1\right) = -6 + 2|I_{ai}| + |I_{bi}| + |I_{ci}| . \quad (31)$$

If supersymmetry is broken by gaugino condensations, we may need to consider gauge mediation since gravity mediation is much smaller. Thus, the supersymmetry CP problem may be solved as well. Unlike Ref. \cite{37}, we will not require at least two $USp$ gauge group factors with negative $\beta$ functions in our Pati-Salam model building.

From the phenomenological point of view, we want to emphasize that Type II T-duality in Ref. \cite{37} is not an equivalent relation in Pati-Salam model building if the model is not invariant under $SU(2)_L$ and $SU(2)_R$ exchange. Under the Type II T-duality, the transformations of the wrapping numbers for any stacks of D6-branes in the model are

$$n_x^i \to -n_x^i, \; l_x^i \to l_x^i, \; n_x^j \leftrightarrow l_x^j, \; n_x^k \leftrightarrow l_x^k, \quad (32)$$

where $i \neq j \neq k$, as well as $x$ runs over all D6-branes in the model. In particular, it is easy to show that all the intersection numbers will change signs.

For a three-family supersymmetric Pati-Salam model, we obtain a corresponding new three-family supersymmetric Pati-Salam models by exchanging $b$-stacks and $c$-stacks of D6-branes

$$b \leftrightarrow c . \quad (33)$$

Especially, the quantum numbers for $SU(2)_L$ and $SU(2)_R$ in the particle spectrum, as well as the $SU(2)_L$ and $SU(2)_R$ gauge couplings at string scale will be interchanged due to the $b$-stacks and $c$-stacks exchange $b \leftrightarrow c$. Therefore, from the phenomenological point of view, this is not an equivalent relation if the particle content is not invariant under $SU(2)_L$ and $SU(2)_R$ exchange or if $SU(2)_L \times SU(2)_R$ gauge couplings are not unified at the string scale. To be more concise, this is not an equivalent relation if the Pati-Salam model is not invariant under $SU(2)_L$ and $SU(2)_R$ exchange.

Moreover, there is a variation of type II T-duality \cite{37}. Under it, the transformations of the wrapping numbers for any stacks of D6-branes in the model are

$$l_x^1 \to -l_x^1, \; l_x^2 \to -l_x^2, \; l_x^3 \to -l_x^3, \; b \leftrightarrow c , \quad (34)$$

where $x$ runs over all D6-branes in the model.
B. Scanning of Supersymmetric Pati-Salam Models

We shall search for the new Pati-Salam models with basic properties in the previous subsection. Similar to Ref. [37], we introduce three stacks of D6-branes, a, b, and c with number of D6-branes 8, 4, and 4, respectively. The corresponding gauge symmetries are $U(4)_C$, $U(2)_L$ and $U(2)_R$. Unlike the strategy in Ref. [37], we do not restrict ourselves with that at least two $USp$ groups in the hidden sector have negative $\beta$ functions, and instead we do a broader scanning without any constraint on the hidden sector.

In general, if all three two-tori are not tilted, we can not obtain the particle spectra with odd generations of the SM fermions. Thus, we have three kinds of scenarios: one tilted two-torus, two tilted two-tori, and three tilted two-tori. As pointed out in Ref. [37], the model buildings with two and three tilted two-tori either do not have three families or violate the RR tadpole cancellation conditions. And our scanning confirms this observation. Therefore, here we concentrate on the new scanning with only one tilted torus. We choose the third two-torus to be tilted and study the new inequivalent Pati-Salam models in the following. In our broader scanning, we obtain several classes of new models.

The first class of models including Models IX, X, XI, and XII has only one $USp$ group, in which Models IX and X, as well as Models XI and XII are T-dual to each other. Especially, the Models XI and XII do not have the colored chiral exotic particles. The Higgs particles in Models IX and XI arise from $N = 2$ subsectors at the intersections of b- and c-stacks of D6-branes, while the Higgs particles in Models X and XII arise from $N = 2$ subsectors at the intersections of b- and c'-stacks of D6-branes. There exist four and eight exotic Higgs-like particles in Models IX and X as well as Models XI and XII respectively.

The second class of models has two $USp$ groups, and the representative models are Models XIII and XIV, which are T-dual to each other. The Higgs particles in Model XIII arise from $N = 2$ subsector at the intersections of b- and c-stacks of D6-branes, while the Higgs particles in Models XIV arise from $N = 2$ subsectors at the intersections of b- and c'-stacks of D6-branes. Because all these $USp$ groups have negative $\beta$ functions, we may stabilize the modulus and break the supersymmetry via gaugino condensations.

The third class of models has more than two $USp$ groups, and the representative models are Models XV, XVI, XVII, and XVIII. There exist at least two $USp$ groups in these models, which have negative $\beta$ functions. So we may stabilize the modulus and break the supersymmetry via gaugino condensations as well. Model XVIII which have four confining $USp(N)$ gauge groups and can considered as T-dual of Model I-Z-10 in [37], is the only model in our current scan which has exact gauge coupling unification at the string scale.

The fourth class of models has the absolute value of at least one wrapping number equal to 5, and the representative models are Models XX and XXI. This kind of models has not been found in the previous search [37]. Interestingly, we observe that the MSSM gauge coupling values are
more refined, and there exists the approximate gauge coupling unification. Also, Model [XXI] has
two $USp$ groups with negative β functions. While in Model [XX] there is only one $USp$ group with
negative β function, so we might need to stabilize the modulus with different mechanism.

IV. PRELIMINARY PHENOMENOLOGICAL STUDIES

In this section, we shall discuss the phenomenological features of our models. We start with
Models [IX] and [X] which are constructed with one $USp$ group. The gauge symmetry is $U(4) \times $ $U(2)_L \times U(2)_R \times USp(4)$, while the β function of $USp(4)$ group is zero. So we cannot break supersymmetry via gaugino condensation, and then need the other mechanism for supersymmetry
breaking. Also, how to decouple the exotic particles, which are charged under $USp(4)$, is an
interesting question since $USp(4)$ is not confined. In Models [XI] and [XII] the gauge symmetry
is $U(4) \times U(2)_L \times U(2)_R \times USp(4)$ as well. The β function of $USp(4)$ group is negative, so we
can break supersymmetry via gaugino condensation, and decouple the exotic particles. In all these
models, we need to address the modulus stabilization issue as well, which is generic for the models
with one $USp$ group.

| Field | $B - L$ |
|-------|---------|
| $Q_{LL}$ | 0 |
| $Q_{LR}$ | 1 |
| $H'$ | -1 |

The Models [XIII] and [XIV] are with two confining gauge groups in hidden sector. They are T-dual to each other in the same manner as for the one orientifold plane class of models, and they can be obtained from Model I-Z-2 in Ref. [37] via generic Type II duality. The full spectrum of Model [XIV] is shown in Table V. Because we do not have $SU(2)_L \times SU(2)_R$ gauge coupling unification at
the string scale, the Models XIII and XIV are not equivalent to the Model I-Z-2 in Ref. [37] from phenomenological point of view. At the string scale, we have $SU(4) \times SU(2)_{R} \times SU(2)_{L} \times USp(4)$ unification in Models XIII and XIV while $SU(4)_{C} \times SU(2)_{L}$ gauge coupling unification in Model I-Z-2 of [37]. Recalling the definition of hypercharge gauge coupling, we find that the unification in Models XIII and XIV, while its corresponding model, which is constructed via generic Type II duality, has $SU(3)_{C} \times U(1)_{Y}$ gauge coupling unification, and vice versa. Similarly, the Model XVII has $SU(3)_{C} \times SU(2)_{L}$ gauge coupling unification while its corresponding model, which is constructed via generic Type II duality, has $SU(3)_{C} \times U(1)_{Y}$ gauge coupling unification. Thus, the generic Type II duality provides a new way to construct the new models. For Model XV with approximate $SU(3)_{C} \times U(1)_{Y}$ gauge coupling unification, we obtain Model XVI with approximate $SU(3)_{C} \times SU(2)_{L}$ gauge coupling unification via generic Type II T-duality. However, we should note that this construction is not simply swapping the b-stack and c-stack of D6-branes, but usually the non-trivial Type II T-dualities are performed. For the examples of performing b-stack and c-stack of D6-branes swapping under Eq. (34), we show Models XXII, XXIII and XXIV respectively from Models XIX, XX and XXI via Type II duality, in which the SM gauge couplings are shifted resulting from such D6-brane swapping.

As we mentioned, the $USp(N)$ gauge groups with negative beta functions in hidden sector have a potential to be confining, and thus the non-perturbative effective superpotential can be generated via gaugino condensations. The ground state, which is determined by the minimization of this supergravity potential, can stabilize the dilaton and complex structure toroidal moduli, and breaks supersymmetry in some cases. For the models with two confining $USp(N)$ gauge groups, a general analysis of the non-perturbative superpotential with tree-level gauge couplings can be performed, and it was shown that there can exist extrema with the stabilizations of dilaton and

| Χ         | $SU(4) \times SU(2)_{L} \times SU(2)_{R} \times USp(4)$ | $Q_{4}$ | $Q_{2L}$ | $Q_{2R}$ | $Q_{em}$ | $B - L$ | Field       |
|-----------|-----------------------------------------------------|--------|----------|----------|----------|--------|-------------|
| $ab'$     | $3 \times (4, 2, 1, 1)$                             | 1      | 1        | 0        | $-\frac{1}{3}, \frac{2}{3}, -1, 0$ | $\frac{2}{3}, -1$ | $Q_{L, LL}$ |
| $ac$      | $3 \times (1, 1, 2, 1)$                             | -1     | 0        | 1        | $\frac{1}{3}, -\frac{2}{3}, 1, 0$ | $-\frac{1}{3}, 1$ | $Q_{R, LR}$ |
| $bc$      | $4 \times (1, 2, 1, 1)$                             | 0      | 1        | -1       | $1, 0, 0, -1$ | 0      | $H$         |
| $a2$      | $2 \times (1, 1, 1, 4)$                             | -1     | 0        | 0        | $-\frac{1}{3}, \frac{2}{3}$ | 0      |             |
| $b2$      | $1 \times (1, 1, 1, 4)$                             | 0      | -1       | 0        | $\frac{1}{3}, -\frac{2}{3}$ | 0      |             |
| $a2'$     | $1 \times (1, 1, 1, 4)$                             | 0      | 0        | -1       | $\frac{1}{3}, 1$ | $-\frac{2}{3}, 2$ |             |
| $a'$      | $1 \times (10, 1, 1, 1)$                            | 2      | 0        | 0        | $\frac{1}{3}, -\frac{2}{3}, -1$ | $\frac{2}{3}, -2$ |             |
| $c$       | $1 \times (5, 1, 1, 1)$                             | -2     | 0        | 0        | $-\frac{1}{3}, 1$ | $-\frac{2}{3}, 2$ |             |
| $b$       | $2 \times (1, 3, 1, 1)$                             | 0      | 2        | 0        | 0        | 0      |             |
| $c$       | $2 \times (1, 1, 1, 1)$                             | 0      | -2       | 0        | 0        | 0      |             |
| $c$       | $6 \times (1, 1, 1, 1)$                             | 0      | 0        | -2       | 0        | 0      |             |
TABLE V: The chiral spectrum in the open string sector for Model XIV

| Field | $Q_{4}$ | $Q_{2L}$ | $Q_{2R}$ | $Q_{cm}$ | $B - L$ | $SU(4) \times SU(2)_{L} \times SU(2)_{R} \times USp(4)$ |
|-------|---------|---------|---------|---------|---------|---------------------------------|
| $ab$  | 3       | 0       | -1      | $0, 0, 1$ | $1$     | $Q_{L, L_{L}}$                      |
| $ac'$ | 3       | 1       | -1      | $0, 0, 1$ | $-1$    | $Q_{R, L_{R}}$                      |
| $bc$  | 0       | 1       | 0       | $0, 0, 1$ | 0       | $H$                               |
| $a3$  | 1       | 0       | 0       | $0, 0, 1$ | 0       | $H$                               |
| $a4$  | 1       | 0       | 1       | $0, 0, 1$ | 0       | $H$                               |
| $b3$  | 2       | 0       | 0       | $0, 0, 1$ | 0       | $H$                               |
| $b4$  | 1       | 0       | 0       | $0, 0, 1$ | 0       | $H$                               |
| $c4$  | 1       | 0       | 1       | $0, 0, 1$ | 0       | $H$                               |
| $b$   | 2       | 0       | 0       | $0, 0, 1$ | 0       | $H$                               |
| $c$   | 2       | 0       | 0       | $0, 0, 1$ | 0       | $H$                               |

TABLE VI: The chiral spectrum in the open string sector for Model XXI

| Field | $Q_{4}$ | $Q_{2L}$ | $Q_{2R}$ | $Q_{cm}$ | $B - L$ | $SU(4) \times SU(2)_{L} \times SU(2)_{R} \times USp(4)$ |
|-------|---------|---------|---------|---------|---------|---------------------------------|
| $ab'$ | 3       | 1       | 0       | $0, 0, 1$ | $1$     | $Q_{L, L_{L}}$                      |
| $ac$  | 3       | 0       | 1       | $0, 0, 1$ | $-1$    | $Q_{R, L_{R}}$                      |
| $bc$  | 1       | 0       | 1       | $0, 0, 1$ | 0       | $H$                               |
| $a2$  | 1       | 0       | 0       | $0, 0, 1$ | 0       | $H$                               |
| $a4$  | 1       | 0       | 1       | $0, 0, 1$ | 0       | $H$                               |
| $b2$  | 1       | 0       | 0       | $0, 0, 1$ | 0       | $H$                               |
| $b3$  | 2       | 0       | 0       | $0, 0, 1$ | 0       | $H$                               |
| $b4$  | 2       | 0       | 0       | $0, 0, 1$ | 0       | $H$                               |
| $c3$  | 5       | 0       | 1       | $0, 0, 1$ | 0       | $H$                               |
| $b$   | 8       | 0       | 0       | $0, 0, 1$ | 0       | $H$                               |
| $c$   | 3       | 0       | 1       | $0, 0, 1$ | 0       | $H$                               |
| $c$   | 3       | 0       | 1       | $0, 0, 1$ | 0       | $H$                               |

complex structure moduli. However, these extrema might be saddle points and thus do not break supersymmetry. Interestingly, if the models have three or four confining $USp(N)$ gauge groups, the non-perturbative superpotential allows for the moduli stabilization and supersymmetry breaking at the stable extremum in general.

Among our representative models, two Models (IX and X) carry one $USp(N)$ gauge group with zero beta function, three Models (XI, XII, and XX) have one confining $USp(N)$ gauge group with negative beta function, six Models (XIII, XIV, XV, XVI, XVII, and XXI) carry two confining
USp(N) gauge groups with negative beta functions, and one Model XVIII have four confining USp(N) gauge groups and considered as T-dual of Model I-Z-10 in [37]. Therefore, for the latter seven models, there may exist the stable extrema with moduli stabilization and supersymmetry breaking due to gaugino condensations, which are very interesting from the phenomenological points of view. However, as pointed out in Ref. [33], the cosmological constants at these extrema are likely to be negative and close to the string scale, and thus the gaugino condensations in these models might not address the cosmological constant problem.

All the models contain the exotic particles that are charged under the hidden gauge groups. The strong coupling dynamics in hidden sector at certain intermediate scale might provide a mechanism for all these particles to form bound states or composite particles, which are compatible with anomaly cancellation conditions. And then similar to the quark condensation in QCD, these particles would be only charged under the SM gauge symmetry [30]. The USp groups have two kinds of neutral bound states in general. The first one is the pseudo inner product of two fundamental representations that is generated by decomposing the rank two anti-symmetric representation, and is general for USp groups. In some sense, this is the reminiscent of a meson that is the inner product of one pair of fundamental and anti-fundamental representations of SU(3)_C in QCD. The second one is the rank 2N anti-symmetric representation of USp(2N) group for N ≥ 2, which is an USp(2N) singlet and somewhat similar to a baryon, as a rank three anti-symmetric representation of SU(3)_C in QCD. Our models, which contain the second kind of neutral bound states, are Models XI and XII with confining USp(4) group, as well as Models XIII and XIV with confining groups USp(4) × USp(4) in the hidden sector. We note that Model XIII and XIV are T-dual to each other, and are constructed by their b-stack and c-stack of D6-branes swapped from Model I-Z-2 in [37] with proper T-duality transformations. For N = 1, these two kinds are the same.

Now we take Models XII and XIV as examples to show explicitly the new composite states. In Model XII, we present the confined particle spectrum in Table VII. Because it has one confining gauge group USp(4) with two charged intersections. Therefore, besides self-confinement, the mixed-confinement between different intersections is also possible, which yields the chiral supermultiplets (1, 2, 2, 1). In Model XIV the confined particle spectra are given in Table VIII. It has two confining gauge groups USp(4) and USp(4) both with two charged intersections. Besides the self-confinement, the mixed-confinement between different intersections yields the chiral supermultiplets (7, 2, 1, 1, 1), (4, 2, 1, 1, 1), (1, 2, 2, 1, 1), and (4, 1, 2, 1, 1). Note that when there is only one charged intersection, we do not have mixed-confinement, and only the tensor representations are yielded from self-confinement. Moreover, it is easy to check from the spectrum that no new anomaly is introduced to the remaining gauge symmetry, so this model is still anomaly free.

This kind of self-confinement and mixed-confinement between different intersections also applies to the other models except for the Models IX and X. In these two models, we do not have asymptotical free gauge symmetries in the hidden sector, so the states charged under these symmetries cannot be confined. Because the anomaly cancellations for the confined particle spectra
TABLE VII: The composite particle spectrum for Model XII formed due to the strong forces in hidden sector.

| Model XII | Confining Force | Intersection | Exotic Particle Spectrum | Confined Particle Spectrum |
|-----------|-----------------|--------------|--------------------------|---------------------------|
|           | $USp(4)_2$      | $b_2$        | $1 \times (1,2,1,\overline{4})$ | $1 \times (1,2^2,1,1), 3 \times (1,2,\overline{2},1)$ |
|           | $c_2$           |              | $3 \times (1,1,\overline{2},4)$ | $6 \times (1,1,\overline{2}^2,1)$ |

TABLE VIII: The composite particle spectrum for Model XIV formed due to the strong forces in hidden sector.

| Model XIV | Confining Force | Intersection | Exotic Particle Spectrum | Confined Particle Spectrum |
|-----------|-----------------|--------------|--------------------------|---------------------------|
|           | $USp(4)_3$      | $a_3$        | $1 \times (\overline{4},1,1,4,1)$ | $1 \times (\overline{4}^2,1,1,1,1), 2 \times (\overline{4},2,1,1,1)$ |
|           | $b_3$           |              | $2 \times (1,2,1,\overline{4},1)$ | $4 \times (1,2^2,1,1,1)$ |
|           | $USp(4)_4$      | $a_4$        | $1 \times (4,1,1,1,\overline{4})$ | $1 \times (4^2,1,1,1,1), 1 \times (4,\overline{2},1,1,1)$ |
|           | $b_4$           |              | $1 \times (1,\overline{2},1,1,4)$ | $1 \times (1,\overline{2}^2,1,1,1), 1 \times (1,\overline{2},2,1,1)$ |
|           | $c_4$           |              | $1 \times (1,1,2,1,\overline{4})$ | $1 \times (1,1,2^2,1,1), 1 \times (4,1,2,1,1)$ |

are not automatically guaranteed, one extra field associated with composite states may be needed to satisfy t’ Hooft anomaly matching condition. To avoid the unnecessary complications, we only consider relatively simple examples here.

V. MACHINE LEARNING AND MODEL BUILDING

We perform the machine learning methods in Figure 1 to visually show how the models with three families of the SM fermions expand in the whole random scanning data according to their wrapping numbers. We observe that with dimension reduction method “LatentSemanticAnalysis”, the scanning data expand in the blue region where the models with three families are marked with red triangular. It is obvious that the total scanning expands with pattern according to the first and second latent dimensions (which reduced from 18 wrapping numbers) as shown in Figure 1. The fact, where the MSSM-like models gather on islands, indicates that there will be more chances to construct the new MSSM-like models there. However, the dense observation also contains the contributions from the T-dual MSSM-like models there are constructed in the nearby region. With linear algorithm, this behaviour, which the MSSM-like models tend to gather, can be confirmed as well.
Pati-Salam model building if the model is not invariant under \( SU(2) \) can be broken down to the generic Type II T-duality transformation. Thus, the MSSM-like models from Type IIA orientifolds on triangles. It turns out that the MSSM-like models populate in several separated islands.

VI. DISCUSSIONS AND CONCLUSIONS

We revisited the systematic construction of the three-family \( N = 1 \) supersymmetric Pati-Salam models from Type IIA orientifolds on \( T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2) \) with intersecting D6-branes, where the \( SU(4)_C \times SU(2)_L \times SU(2)_R \) gauge symmetries arise from the stacks of D6-branes with \( U(n) \) gauge symmetries. We found that the Type II T-duality in Ref. 37 is not an equivalent relation in Pati-Salam model building if the model is not invariant under \( SU(2)_L \) and \( SU(2)_R \) exchange, and provides a way to obtain the new model. Unlike the previous studies, we did not require at least two confining \( USp \) groups. Also, we scanned the wrapping numbers up to 5, and obtained more interesting models with approximate gauge coupling unification. The Pati-Salam gauge symmetry can be broken down to the \( SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{13R} \) via D6-brane splittings, and further down to the SM gauge symmetry via the D- and F-flatness preserving Higgs mechanism in which Higgs fields are the massless open string states from a specific \( N = 2 \) subsector. Moreover, Models \[ \text{XIII} \] and \[ \text{XIV} \] are T-dual to each other and can be obtained from Model I-Z-2 in 37 via generic Type II T-duality transformation. Thus, the \( SU(3)_C \times SU(2)_L \) gauge coupling unification in Model I-Z-2 is shifted to \( SU(3)_C \times U(1)_Y \) gauge coupling unification in Models \[ \text{XIII} \] and \[ \text{XIV} \] and vice versa. Also, Model \[ \text{XV} \] with b-stack and c-stack of D6-branes swapped leads to Model \[ \text{XVI} \] with \( SU(4) \) and \( SU(2)_L \) gauge couplings being closer to unification at the string scale.
Furthermore, we obtained the models with one $USp(4)$ group, and the new confine particle spectrum in Model [XI] shown in Table [VII]. The composite particle spectrum of Model [XIV] which is formed due to two confined $USp(4)$ groups in hidden sector, is given in Table [VIII] where supersymmetry breaking via a “race-track” scenario is still possible.

Last but not least, we found interesting models with one wrapping number equal to 5, which was not found before. For these models, especially Model [XXI] it is obvious that the gauge couplings are in a much more refined form because of the complicity of the intersections due to a large wrapping number. It is also obvious that the approximate gauge coupling unification is achieved because of the refined gauge couplings. Based on these observations, we conjecture that it is worthwhile to search for the Pati-Salam models with broader scanning and large wrapping numbers. And in the machine learning and model building part, we observed visually that the MSSM-like models tend to gather in islands and indicates more chances to find more MSSM-like models in their nearby region.

Acknowledgments

This research was supported by the Projects 11847612 and 11875062 supported by the National Natural Science Foundation of China, the Key Research Program of Frontier Science, CAS, the National Thousand Young Talents Program of China, the China Postdoctoral Science Foundation Grant 2018M631436, and the LMU Munich’s Institutional Strategy LMUexcellent within the framework of the German Excellence Initiative. We would like to thank Jie Ren and Zheng Sun for discussing and participating for the early part of the work, thank Xiaoyong Chu, Andreas Deser, Jiahua Tian, and Yinan Wang for useful discussions. RS would also like to thank Babak Haghighat for his support, and acknowledges Ludwig Maximilian University of Munich, Max Planck Institute for Physics, the Abdus Salam International Centre for Theoretical Physics (ICTP) for their hospitalities where part of this work was carried out.

[1] J. Polchinski and E. Witten, Nucl. Phys. B 460, 525 (1996).
[2] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y. S. Stanev, Phys. Lett. B 385, 96 (1996).
[3] M. Berkooz and R.G. Leigh, Nucl. Phys. B 483, 187 (1997).
[4] G. Shiu and S. H. Tye, Phys. Rev. D 58, 106007 (1998).
[5] J. Lykken, E. Poppitz and S. P. Trivedi, Nucl. Phys. B 543, 105 (1999).
[6] M. Cvetić, M. Plümmacher and J. Wang, JHEP 0004, 004 (2000); M. Cvetić, A. M. Uranga and J. Wang, Nucl. Phys. B 595, 63 (2001).
[7] G. Aldazabal, A. Font, L. E. Ibáñez and G. Violero, Nucl. Phys. B 536, 29 (1998); G. Aldazabal, L. E. Ibáñez, F. Quevedo and A. M. Uranga, JHEP 0008, 002 (2000).
[8] M. Klein and R. Rabadan, JHEP 0010, 049 (2000).
[9] M. Berkooz, M. R. Douglas and R. G. Leigh, Nucl. Phys. B 480 (1996) 265.
[10] C. Bachas, hep-th/9503030.
[11] J. F. G. Cascales and A. M. Uranga, hep-th/0311250.
[12] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, JHEP 0010 (2000) 006.
[13] R. Blumenhagen, B. Körs and D. Lüst, JHEP 0102 (2001) 030.
[14] G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadán and A. M. Uranga, JHEP 0102, 047 (2001).
[15] G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadán and A. M. Uranga, J. Math. Phys. 42, 3103 (2001).
[16] L. E. Ibáñez, F. Marchesano and R. Rabadán, JHEP 0111, 002 (2001).
[17] C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, Phys. Lett. B 489 (2000) 223.
[18] S. Förste, G. Honecker and R. Schreyer, Nucl. Phys. B 593 (2001) 127; JHEP 0106 (2001) 004.
[19] R. Blumenhagen, B. Körs and D. Lüst, T. Ott, Nucl. Phys. B616 (2001) 3.
[20] D. Cremades, L. E. Ibáñez and F. Marchesano, Nucl. Phys. B 643, 93 (2002).
[21] D. Cremades, L. E. Ibáñez and F. Marchesano, JHEP 0102, 009(2002).
[22] D. Cremades, L. E. Ibáñez and F. Marchesano, JHEP 0207, 022(2002).
[23] D. Bailin, G. V. Kranios, and A. Love, Phys. Lett. B 530, 202 (2002); Phys. Lett. B 547, 43 (2002);
     Phys. Lett. B 553, 79 (2003); JHEP 0302, 052 (2003).
[24] J. R. Ellis, P. Kanti and D. V. Nanopoulos, Nucl. Phys. B 647, 235 (2002).
[25] C. Kokorelis, JHEP 0209, 029 (2002); JHEP 0208, 036 (2002); Nucl. Phys. B 677, 115 (2004); JHEP
     0211, 027 (2002); hep-th/0210200.
[26] M. Cvetič, G. Shiu and A. M. Uranga, Phys. Rev. Lett. 87, 201801 (2001).
[27] M. Cvetič, G. Shiu and A. M. Uranga, Nucl. Phys. B 615, 3 (2001).
[28] M. Cvetič and I. Papadimitriou, Phys. Rev. D 67, 126006 (2003).
[29] M. Cvetic, I. Papadimitriou and G. Shiu, Nucl. Phys. B 659, 193 (2003) Erratum: [Nucl. Phys. B 696,
     298 (2004)].
[30] M. Cvetič, P. Langacker and G. Shiu, Phys. Rev. D 66, 066004 (2002).
[31] M. Cvetič, P. Langacker and G. Shiu, Nucl. Phys. B 642, 139 (2002).
[32] M. Cvetič and I. Papadimitriou, Phys. Rev. D 68, 046001 (2003).
[33] M. Cvetič, P. Langacker and J. Wang, Phys. Rev. D 68, 046002 (2003).
[34] R. Blumenhagen, L. Gorlich and T. Ott, JHEP 0301, 021 (2003).
[35] G. Honecker, Nucl. Phys. B 666, 175 (2003).
[36] T. Li and T. Liu, Phys. Lett. B 573, 193 (2003).
[37] M. Cvetic, T. Li and T. Liu, Nucl. Phys. B 698, 163 (2004).
[38] M. Cvetic, P. Langacker, T. Li and T. Liu, Nucl. Phys. B 709, 241 (2005).
[39] C.-M. Chen, G. V. Kraniotis, V. E. Mayes, D. V. Nanopoulos and J. W. Walker, Phys. Lett. B 611,
     156 (2005).
[40] C.-M. Chen, G. V. Kraniotis, V. E. Mayes, D. V. Nanopoulos and J. W. Walker, Phys. Lett. B 625,
     96 (2005).
[41] C. M. Chen, T. Li and D. V. Nanopoulos, Nucl. Phys. B 732, 224 (2006).
[42] C. M. Chen, T. Li, V. E. Mayes and D. V. Nanopoulos, Phys. Lett. B 665, 267 (2008).
[43] C. M. Chen, T. Li, V. E. Mayes and D. V. Nanopoulos, J. Phys. G 35, 095008 (2008).
[44] C. M. Chen, T. Li, V. E. Mayes and D. V. Nanopoulos, Phys. Rev. D 77, 125023 (2008).
[45] C. M. Chen, T. Li, V. E. Mayes and D. V. Nanopoulos, Phys. Rev. D 78, 105015 (2008).
[46] R. Blumenhagen, M. Cvetic, P. Langacker and G. Shiu, Ann. Rev. Nucl. Part. Sci. 55, 71 (2005).
Appendix: Supersymmetric Pati-Salam Models

In this Appendix, we tabulate 12 representative models obtained from our broader scanning method. In the first column for each table, we denote the \(U(4), U(2)_{L},\) and \(U(2)_{R}\) stacks of D6-branes as \(a, b,\) and \(c\) stacks, respectively. We also employ 1, 2, 3, and 4 stacks to represent the filler branes respectively along \(\Omega R, \Omega R\omega, \Omega R\theta\omega,\) and \(\Omega R\theta\) orientifold planes, which result in the \(USp(N)\) gauge symmetries. In the second column, \(N\) is the number of D6-branes in each stack. Moreover, we present the wrapping numbers of the various D6-branes in the third column and specify the third set of wrapping numbers for the tilted two-torus.

In the remaining right columns, we give the intersection numbers between various stacks, where \(b'\) and \(c'\) denote the \(\Omega R\) images of \(b\) and \(c\), respectively. In addition, we present the relation among the moduli parameters imposed by the four-dimensional \(N = 1\) supersymmetry conditions, and the one-loop \(\beta\) functions \((\beta_i^H)\) for the hidden sector gauge symmetries. In particular, we also give the MSSM gauge couplings in the caption of each model, and thus it is easier to check the gauge coupling unification. However, we do not require at least two confining hidden gauge sectors which are needed to realize the moduli stabilization and supersymmetry breaking via gaugino condensation.

TABLE IX: D6-brane configurations and intersection numbers in Model IX and its MSSM gauge coupling relation is \(g_a^2 = \frac{1}{2} g_b^2 = \frac{65}{11} (\frac{5}{3} g_Y^2) = \frac{16\sqrt{5}}{15} \pi e^{\phi^4}.

| Model IX | \(U(4) \times U(2)_{L} \times U(2)_{R} \times USp(4)\) |
|----------|------------------------------------------------|
| stack    | \((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)\) | \(b\) | \(b'\) | \(c\) | \(c'\) |
| \(a\)    | 8 | \((1, -2) \times (0, 1) \times (1, -1)\) | 1 | -1 | 3 | 0 | -3 | 0 | -2 |
| \(b\)    | 4 | \((0, 1) \times (-3, -1) \times (1, 1)\) | 2 | -2 | - | 0 | -4 | 1 |
| \(c\)    | 4 | \((2, -1) \times (-1, 1) \times (-1, -1)\) | -2 | -6 | - | - | - | -1 |
| 1        | 4 | \((1, 0) \times (1, 0) \times (2, 0)\) | \(X_A = 3X_B = \frac{1}{2}X_C = \frac{3}{8}X_D\) | \(\beta_i^H = 0;\) | \(\chi_1 = \frac{1}{4};\) | \(\chi_2 = \frac{3}{2};\) | \(\chi_3 = 4\) |
The gauge coupling relation is $g_a^2 = \frac{1}{2} g_b^2 = \frac{65}{18} (\frac{5}{3} g_Y^2) = \frac{16\sqrt{3}}{15} \pi e^{\phi^4}$.

Table X: D6-brane configurations and intersection numbers in Model X and its MSSM gauge coupling relation is $g_a^2 = \frac{1}{2} g_b^2 = \frac{65}{18} (\frac{5}{3} g_Y^2) = \frac{16\sqrt{3}}{15} \pi e^{\phi^4}$.

| Model X | $U(4) \times U(2)_L \times U(2)_R \times USp(4)$ |
|---------|--------------------------------------------------|
| stack  | $N ((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $b'$ | $c$ | $c'$ |
| $a$    | $8 (-1, 2) \times (-1, 0) \times (1, 1)$     | 1  | -1  | 0  | 3  | -3  | 0  | -2 |
| $b$    | $4 (0, 1) \times (-1, -3) \times (1, 1)$     | -2 | 2   | -  | 4  | 0   | -1 |
| $c$    | $4 (2, -1) \times (-1, -1) \times (-1, 1)$  | -2 | -6  | -  | -  | -   | -1 |
| $2$    | $4 (1, 0) \times (0, -1) \times (0, 2)$     | $X_A = \frac{1}{2} X_B = \frac{1}{3} X_C = \frac{4}{5} X_D$ |

Table XI: D6-brane configurations and intersection numbers in Model XI and its MSSM gauge coupling relation is $g_a^2 = \frac{1}{2} g_b^2 = \frac{10}{13} (\frac{5}{3} g_Y^2) = \frac{16\sqrt{3}}{15} \pi e^{\phi^4}$.

| Model XI | $U(4) \times U(2)_L \times U(2)_R \times USp(4)$ |
|---------|--------------------------------------------------|
| stack  | $N ((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $b'$ | $c$ | $c'$ |
| $a$    | $8 (1, -1) \times (0, -1) \times (-1, 1)$     | 0  | 0   | 3  | 0  | -3  | 0  | 0  |
| $b$    | $4 (0, -1) \times (3, 2) \times (1, 1)$     | 1  | -1  | -  | 0  | 8   | -3 |
| $c$    | $4 (-1, -1) \times (-1, 0, -1) \times (-1, 1)$ | -3 | 3   | -  | -  | -   | -1 |
| $2$    | $4 (1, 0) \times (0, -1) \times (0, 2)$     | $X_A = \frac{1}{2} X_B = X_C = 6 X_D$ |

Table XII: D6-brane configurations and intersection numbers in Model XII and its MSSM gauge coupling relation is $g_a^2 = \frac{1}{2} g_b^2 = \frac{10}{13} (\frac{5}{3} g_Y^2) = \frac{16\sqrt{3}}{15} \pi e^{\phi^4}$.

| Model XII | $U(4) \times U(2)_L \times U(2)_R \times USp(4)$ |
|---------|--------------------------------------------------|
| stack  | $N ((n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $b$ | $b'$ | $c$ | $c'$ |
| $a$    | $8 (-1, 0) \times (1, -1) \times (-1, 1)$     | 0  | 0   | 3  | 0  | 0   | 0  |
| $b$    | $4 (0, 1) \times (-1, 4) \times (1, 1)$     | 3  | -3  | -  | -8 | 0   | 1  |
| $c$    | $4 (2, -3) \times (1, 0) \times (1, 1)$     | 1  | -1  | -  | -  | -   | -3 |
| $2$    | $4 (1, 0) \times (0, -1) \times (0, 2)$     | $X_A = \frac{1}{2} X_B = 2 X_C = \frac{4}{5} X_D$ |

$\beta_2^g = -2; \chi_1 = 2, \chi_2 = 3, \chi_3 = 1$
TABLE XIII: D6-brane configurations and intersection numbers in Model XIII and its MSSM gauge coupling relation is $g_a^2 = \frac{5}{3} g_b^2 = \frac{5}{3} g_Y^2 = \frac{4\sqrt{5}}{3} \pi e^{\phi_4}$.

| Model XIII | $U(4) \times U(2)_L \times U(2)_R \times USp(4)^2$ |
|------------|-------------------------------------------------|
| stack | $N \times (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $\eta$ | $n$ | $b$ | $b'$ | $c$ | $c'$ | 1 | 2 |
| a | 8 | (0, -1) × (1, 1) × (1, 1) | 0 | 0 | 3 | -3 | 0 | 1 | -1 |
| b | 4 | (1, 1) × (2, -1) × (1, 1) | -2 | -6 | - | 0 | 4 | -1 | 2 |
| c | 4 | (-3, 1) × (0, -1) × (1, 1) | -2 | 2 | - | - | -1 | 0 |
| 1 | 4 | (1, 0) × (1, 0) × (2, 0) | $\beta_1^a = -2, \beta_2^a = -2; \quad \chi_1 = 3, \chi_2 = 1, \chi_3 = 2$ |
| 2 | 4 | (1, 0) × (0, -1) × (0, 2) | $X_A = X_B = 3X_C = 3X_D$ |

TABLE XIV: D6-brane configurations and intersection numbers in Model XIV and its MSSM gauge coupling relation is $g_a^2 = \frac{5}{3} g_b^2 = \frac{5}{3} g_Y^2 = \frac{4\sqrt{5}}{3} \pi e^{\phi_4}$.

| Model XIV | $U(4) \times U(2)_L \times U(2)_R \times USp(4)^2$ |
|------------|-------------------------------------------------|
| stack | $N \times (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $\eta$ | $n$ | $b$ | $b'$ | $c$ | $c'$ | 3 | 4 |
| a | 8 | (-1, 0) × (1, -1) × (-1, 1) | 0 | 0 | 3 | 0 | -3 | -1 | 1 |
| b | 4 | (-1, 1) × (1, 2) × (-1, 1) | -2 | -6 | - | 4 | 0 | 2 | -1 |
| c | 4 | (1, -3) × (-1, 0) × (-1, -1) | 2 | -2 | - | - | - | 0 | 1 |
| 3 | 4 | (0, -1) × (1, 0) × (0, 2) | $X_A = X_B = \frac{1}{3}X_C = \frac{1}{3}X_D$ |
| 4 | 4 | (0, -1) × (0, 1) × (2, 0) | $\beta_1^a = -2, \beta_2^a = -2; \quad \chi_1 = \frac{1}{3}, \chi_2 = 1, \chi_3 = 2$ |

TABLE XV: D6-brane configurations and intersection numbers in Model XV and its MSSM gauge coupling relation is $g_a^2 = \frac{2}{3} g_b^2 = \frac{80}{77} (\frac{5}{3} g_Y^2) = \frac{16\sqrt{5}}{15} \pi e^{\phi_4}$.

| Model XV | $U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(4) \times USp(4)$ |
|------------|-------------------------------------------------|
| stack | $N \times (n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $\eta$ | $n$ | $b$ | $b'$ | $c$ | $c'$ | 1 | 2 | 4 |
| a | 8 | (-1, 1) × (-1, 0) × (1, 1) | 0 | 0 | 3 | -3 | 0 | -1 | 1 |
| b | 4 | (1, 4) × (0, -1) × (1, 1) | -3 | 3 | - | 2 | 4 | 0 | 0 |
| c | 4 | (-1, -2) × (-1, -1) × (1, 1) | 0 | 8 | - | - | -2 | 2 | -1 |
| 1 | 2 | (1, 0) × (1, 0) × (2, 0) | $X_A = \frac{1}{4}X_B = \frac{1}{4}X_C = \frac{1}{4}X_D$ |
| 2 | 4 | (1, 0) × (-1, 0) × (0, 2) | $\beta_1^a = 0, \beta_2^a = -2, \beta_4^a = -3$ |
| 3 | 4 | (0, -1) × (0, 1) × (2, 0) | $\chi_1 = \frac{1}{2}, \chi_2 = \frac{1}{3}, \chi_3 = 1$ |
TABLE XVI: D6-brane configurations and intersection numbers in Model [XVI] and its MSSM gauge coupling relation is $g_a^2 = \frac{16}{15} g_b^2 = \frac{10}{13} \left( \frac{5}{3} g_Y^2 \right) = \frac{16\sqrt{6}}{15} \pi e^{\phi_4}$.

| Model [XVI] | $U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(4) \times USp(4)$ | stack | $N$ | $(n^i, l^i) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $c'$ | $1$ | $2$ | $3$ | $4$ |
|-------------|-------------------------------------------------|-------|-----|---------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| a           | 8                                               | (1, -1) \times (0, 1) \times (1, -1) | 0   | 0   | 3   | -3  | 0   | -1  | 1   |
| b           | 4                                               | (-2, 1) \times (1, -1) \times (1, -1) | 0   | 8   | -   | -2  | 0   | -1  | 2   |
| c           | 4                                               | (-4, 1) \times (1, 0) \times (-1, -1) | -3  | 3   | -   | -   | -   | 0   | 4   |
| 1           | 4                                               | (1, 0) \times (1, 0) \times (2, 0)   | $X_A = \frac{2}{3} X_B = X_C = 6 X_D$ |
| 2           | 4                                               | (0, -1) \times (1, 0) \times (0, 2)  | $\beta_1^g = -3, \beta_2^g = -2, \beta_4^g = 0$ |
| 3           | 4                                               | (0, -1) \times (0, 1) \times (2, 0)  | $\chi_1 = 2, \chi_2 = 3, \chi_3 = 1$ |

TABLE XVII: D6-brane configurations and intersection numbers in Model [XVII] and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = \frac{10}{13} \left( \frac{5}{3} g_Y^2 \right) = \frac{16\sqrt{3}}{3} \pi e^{\phi_4}$.

| Model [XVII] | $U(4) \times U(2)_L \times U(2)_R \times USp(2) \times USp(4) \times USp(2)$ | stack | $N$ | $(n^i, l^i) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $c'$ | $1$ | $2$ | $3$ | $4$ |
|-------------|-------------------------------------------------|-------|-----|---------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| a           | 8                                               | (1, 0) \times (1, -1) \times (1, 1) | 0   | 0   | 3   | 0   | -3  | 0   | -1  | 1   |
| b           | 4                                               | (1, 3) \times (-1, 2) \times (-1, -1) | -6  | -18 | -   | -9  | 0   | -6  | 3   | -1  |
| c           | 4                                               | (-2, 3) \times (0, 1) \times (1, 1)  | 5   | -5  | -   | -   | -3  | 0   | -2  | 0   |
| 1           | 2                                               | (1, 0) \times (1, 0) \times (2, 0)   | $X_A = 2 X_B = \frac{2}{3} X_C = \frac{2}{3} X_D$ |
| 2           | 4                                               | (0, 1) \times (0, -1) \times (0, 2)  | $\beta_1^g = 3, \beta_2^g = -3, \beta_4^g = 0, \beta_5^g = -3$ |
| 3           | 2                                               | (0, -1) \times (1, 0) \times (0, 2)  | $\chi_1 = \sqrt{3}, \chi_2 = \sqrt{3}, \chi_3 = 2 \sqrt{3}$ |
| 4           | 2                                               | (0, -1) \times (0, 1) \times (2, 0)  | |

TABLE XVIII: D6-brane configurations and intersection numbers in Model [XVIII] and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = \frac{5}{3} g_Y^2 = \frac{4\sqrt{6}}{3} \pi e^{\phi_4}$.

| Model [XVIII] | $U(4) \times U(2)_L \times U(2)_R \times USp(2)^2$ | stack | $N$ | $(n^i, l^i) \times (n^2, l^2) \times (n^3, l^3)$ | $a$ | $b$ | $c$ | $c'$ | $1$ | $2$ | $3$ | $4$ |
|-------------|-------------------------------------------------|-------|-----|---------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| a           | 8                                               | (1, 0) \times (1, -1) \times (1, 1) | 0   | 0   | 3   | 0   | -3  | 0   | -1  | 1   |
| b           | 4                                               | (-1, -3) \times (0, -1) \times (-1, -1) | -2  | 2   | -   | 0   | 3   | 0   | -1  | 0   |
| c           | 4                                               | (1, -3) \times (-1, 0) \times (-1, -1) | 2   | -2  | -   | -   | -3  | 0   | 1   |
| 1           | 2                                               | (1, 0) \times (1, 0) \times (2, 0)   | $3 X_A = 3 X_B = X_C = X_D$ |
| 2           | 2                                               | (1, 0) \times (0, -1) \times (0, 2)  | $\beta_1^g = -3, \beta_2^g = -3, \beta_4^g = -3, \beta_5^g = -3$ |
| 3           | 2                                               | (0, -1) \times (1, 0) \times (0, 2)  | $\chi_1 = \frac{2}{3}, \chi_2 = 1, \chi_3 = 2$ |
| 4           | 2                                               | (0, -1) \times (0, 1) \times (2, 0)  | |
TABLE XIX: D6-brane configurations and intersection numbers in Model [XIX] and its MSSM gauge coupling relation is $g_a^2 = \frac{5}{6} g_b^2 = \frac{11}{3} \left( \frac{5}{4} g_Y^2 \right) = \frac{8 \times 2 \sqrt{3}}{\pi} \pi e^{\phi_1}$.

| Model XIX | $U(4) \times U(2)_L \times U(2)_R \times USp(2)$ |
|-----------|-----------------------------------------------|
| stack     | $N \ (n^3, l^3) \times (n^2, l^2) \times (n^3, l^3)$ | $n$ | $b$ | $b'$ | $c$ | $c'$ |
| $a$       | $(0, 1) \times (1, 1) \times (-1, -1)$          | 0  | 0  | 3  | 0  | -3  |
| $b$       | $(1, 0) \times (5, -2) \times (1, 1)$          | -3 | 3  | -  | 0  | 8   |
| $c$       | $(-1, -1) \times (1, 2) \times (1, 1)$         | -2 | -  | -  | -  | -   |
| $3$       | $(0, -1) \times (1, 0) \times (0, 2)$          | $\beta_3^0 = -2$; $\chi_1 = \sqrt{3}$, $\chi_2 = \sqrt{5/2}$, $\chi_3 = \sqrt{5/2}$ |

TABLE XX: D6-brane configurations and intersection numbers in Model [XX] and its MSSM gauge coupling relation is $g_a^2 = \frac{5}{6} g_b^2 = \frac{35}{13} \left( \frac{5}{4} g_Y^2 \right) = \frac{8 \times 2 \sqrt{3}}{27} \pi e^{\phi_1}$.

| Model [XX] | $U(4) \times U(2)_L \times U(2)_R \times USp(2)^2$ |
|------------|-----------------------------------------------|
| stack      | $N \ (n^3, l^3) \times (n^2, l^2) \times (n^3, l^3)$ | $n$ | $b$ | $b'$ | $c$ | $c'$ |
| $a$        | $(1, -1) \times (1, 1) \times (1, -1)$          | 0  | -4 | 0  | -3 | 0    |
| $b$        | $(-2, 5) \times (-1, 0) \times (1, 1)$          | 3  | -  | -  | 0  | -1  |
| $c$        | $(-1, -2) \times (0, -1) \times (-1, -1)$      | -1 | 1  | -  | -  | 0    |
| $2$        | $(1, 0) \times (0, -1) \times (0, 2)$          | $X_A = \frac{6}{7} X_B = \frac{2}{5} X_C = \frac{3}{5} X_D$; $\beta_2^0 = 1$, $\beta_3^0 = -3$ |
| $3$        | $(0, -1) \times (1, 0) \times (0, 2)$          | $\chi_1 = \sqrt{5}$, $\chi_2 = \sqrt{3}$, $\chi_3 = \sqrt{5}$ |

TABLE XXI: D6-brane configurations and intersection numbers in Model [XXI] and its MSSM gauge coupling relation is $g_a^2 = \frac{71}{63} g_b^2 = \frac{50}{47} \left( \frac{5}{4} g_Y^2 \right) = \frac{16 \times 2 \sqrt{3}}{21} \pi e^{\phi_1}$.

| Model [XXI] | $U(4) \times U(2)_L \times U(2)_R \times USp(2)^3$ |
|-------------|-----------------------------------------------|
| stack       | $N \ (n^3, l^3) \times (n^2, l^2) \times (n^3, l^3)$ | $n$ | $b$ | $b'$ | $c$ | $c'$ |
| $a$         | $(-1, 1) \times (-1, 0) \times (1, 1)$          | 0  | 0  | 0  | -3 | 0    |
| $b$         | $(2, -1) \times (1, 1) \times (1, 1)$          | 0  | 8  | -  | -1 | 0    |
| $c$         | $(5, -2) \times (0, 1) \times (1, -1)$         | -3 | 3  | -  | -  | 0    |
| $2$         | $(1, 0) \times (0, -1) \times (0, 2)$          | $X_A = \frac{6}{7} X_B = \frac{2}{5} X_C = \frac{3}{5} X_D$; $\beta_2^0 = -3$, $\beta_3^0 = 1$, $\beta_4^0 = -2$ |
| $3$         | $(0, -1) \times (1, 0) \times (0, 2)$          | $\chi_1 = \sqrt{3}$, $\chi_2 = \sqrt{2}$, $\chi_3 = \sqrt{10}$ |
| $4$         | $(0, -1) \times (0, 1) \times (2, 0)$          | $\chi_1 = \sqrt{3}$, $\chi_2 = \sqrt{2}$, $\chi_3 = \sqrt{10}$ |
TABLE XXII: D6-brane configurations and intersection numbers in Model XXII and its MSSM gauge coupling relation is $g_2^2 = \frac{11}{4} g_6^2 = \frac{25}{28} \left( \frac{5}{8} g_Y^2 \right) = \frac{8 \times 21/4 \times 5/4}{\pi^2} \pi e^{\phi_4}$.

| Model XXII | $U(4) \times U(2)_L \times U(2)_R \times USp(2)$ |
|------------|--------------------------------------------|
| stack | N | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $\beta$ | $b'$ | c | c' | 2 | 3 |
| a | 8 | $(0, -1) \times (1, -1) \times (-1, 1)$ | 0 | 0 | 3 | 0 | -3 | 0 |
| b | 4 | $(1, 0) \times (5, 2) \times (1, -1)$ | 2 | 6 | 0 | 0 | 0 | -8 | 2 |
| c | 3 | $(0, -1) \times (1, 0) \times (0, 2)$ | 3 | -3 | - | - | - | - | 2 |
| $X_A = X_B = \frac{12}{7} X_C = 6 X_D$ |
| $\beta_0^2 = -2$; $\chi_1 = \frac{\sqrt{6}}{\sqrt{5}}$, $\chi_2 = \frac{\sqrt{3}}{2}$, $\chi_3 = \frac{2 \sqrt{3}}{ \sqrt{5}}$ |

TABLE XXIII: D6-brane configurations and intersection numbers in Model XXIII and its MSSM gauge coupling relation is $g_2^2 = \frac{7}{6} g_6^2 = \frac{25}{28} \left( \frac{5}{8} g_Y^2 \right) = \frac{8 \times 5/4 \sqrt{2}}{\pi^2} \pi e^{\phi_4}$.

| Model XXIII | $U(4) \times U(2)_L \times U(2)_R \times USp(2)^2$ |
|------------|-----------------------------------------------|
| stack | N | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $\beta$ | $b'$ | c | c' | 2 | 3 |
| a | 8 | $(1, 1) \times (1, -1) \times (1, 1)$ | 0 | 4 | 3 | 0 | -3 | 0 | 1 |
| b | 4 | $(1, 0) \times (0, 1) \times (-1, 1)$ | 1 | -1 | - | 0 | 1 | 0 | 1 |
| c | 4 | $(2, -5) \times (1, 0) \times (1, -1)$ | -3 | 3 | - | - | - | - | 5 | 0 |
| 2 | 2 | $(1, 0) \times (0, -1) \times (0, 2)$ | $X_A = \frac{2}{7} X_B = \frac{2}{7} X_C = \frac{2}{7} X_D$ |
| 3 | 2 | $(0, -1) \times (1, 0) \times (0, 2)$ | $\beta_2^2 = 1$, $\beta_3^2 = -3$ |
| $\chi_1 = \frac{\sqrt{2}}{\sqrt{5}}$, $\chi_2 = \frac{\sqrt{3}}{2}$, $\chi_3 = \sqrt{5}$ |

TABLE XXIV: D6-brane configurations and intersection numbers in Model XXIV and its MSSM gauge coupling relation is $g_2^2 = \frac{10}{4} g_6^2 = \frac{35}{33} \left( \frac{5}{7} g_Y^2 \right) = \frac{16 \times 21/4 \times 5/4}{\pi^2} \pi e^{\phi_4}$.

| Model XXIV | $U(4) \times U(2)_L \times U(2)_R \times USp(2)^3$ |
|------------|-----------------------------------------------|
| stack | N | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ | $\beta$ | $b'$ | c | c' | 2 | 3 | 4 |
| a | 8 | $(-1, -1) \times (-1, 0) \times (1, -1)$ | 0 | 0 | 0 | 3 | -3 | 0 | 1 | 0 | -1 |
| b | 4 | $(5, 2) \times (0, -1) \times (1, 1)$ | 3 | -3 | - | - | 1 | 0 | 0 | -5 | 0 |
| c | 4 | $(2, 1) \times (1, -1) \times (1, -1)$ | 0 | -8 | - | - | - | 1 | -2 | -2 | 0 |
| 2 | 2 | $(1, 0) \times (0, -1) \times (0, 2)$ | $X_A = \frac{2}{7} X_B = \frac{2}{7} X_C = \frac{2}{7} X_D$ |
| 3 | 2 | $(0, -1) \times (1, 0) \times (0, 2)$ | $\beta_2^2 = 3$, $\beta_3^2 = 1$, $\beta_4^2 = -2$ |
| $\chi_1 = \frac{\sqrt{5}}{2}$, $\chi_2 = \frac{\sqrt{3}}{2}$, $\chi_3 = \sqrt{10}$ |

### Notes on the Tables
- TABLE XXII: The stack configurations and intersection numbers are organized in a table format, with columns for stack, intersection numbers, MSSM gauge coupling relations, and physical parameters.
- TABLE XXIII: Similar to XXII, but with different quantities and parameters.
- TABLE XXIV: Further extension of the table format, incorporating additional parameters and relations.