QCD corrections to $H \rightarrow gg$ in FDR

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Abstract

I apply FDR -a recently introduced Four Dimensional Regularization approach to quantum field theories- to compute the NLO QCD corrections to $H \rightarrow gg$ in the large top mass limit. The calculation involves all key ingredients of massless QCD, namely ultraviolet, infrared and collinear divergences, besides $\alpha_S$ renormalization. I show in detail how the correct result emerges in FDR, and discuss the translation rules to dimensional regularization.

Keywords:
QCD, Higgs, Higher order corrections, New NLO techniques

1. Introduction

Many of the difficulties of higher order calculations in quantum field theory (QFT) can be traced back to the treatment, in the framework of dimensional regularization (DR) [1], of the infinities arising in the intermediate steps of the computation. Ultraviolet (UV), infrared (IR) and collinear (CL) divergences are first dimensionally regulated, and then renormalized away -in the UV case- or canceled by combining virtual and real contributions, or reabsorbed in the collinear behavior of the initial state parton densities. In order to attack the problem numerically, it is often necessary to subtract and add back approximations of the IR/CL singular structures. At one loop, several well tested subtraction procedures have been introduced in the last two decades [2, 3, 4, 5, 6, 7]. At two loops and beyond, the situation is more involved, but progress is under way [8, 9, 10, 11, 12, 13].

The first obvious ingredient, which may lead to a significant simplification in the above picture, is a computational procedure in which all parts of the
calculation can be directly treated in four dimensions. As for the virtual contribution, the FDR approach has been recently introduced in reference [14], which allows a subtraction of the UV divergences at the level of the integrand, leaving a four dimensional integration over the loop momenta. In the same work, the use of FDR as an IR/CL regulator in QED is also suggested.

In this paper, I present the first application of the FDR ideas in the context of massless QCD, where the issues related to gauge invariance are much more subtle than in the QED case. I concentrate, in particular, on the calculation of the $\mathcal{O}(\alpha_S)$ gluonic corrections to the $H \to gg$ decay in the $m_{\text{top}} \to \infty$ limit, and re-derive the well known fully inclusive result [15,16]

$$\Gamma(H \to gg) = \Gamma^{(0)}(\alpha_S(M_H^2)) \left[ 1 + \frac{95}{4} \frac{\alpha_S}{\pi} \right],$$

where

$$\Gamma^{(0)}(\alpha_S(M_H^2)) = \frac{G_F \alpha_S^2(M_H^2)}{36 \sqrt{2} \pi^3} M_H^3$$

is the lowest order contribution, with $N_F = 0$ in $\alpha_S(M_H^2)$, since only gluons are considered.

Despite its simplicity, all key ingredients of massless QCD are present in this process, such as the simultaneous occurrence of IR/CL divergences and UV renormalization. The fact that the correct expression is reproduced, shows that FDR is a valid and consistent calculational scheme in massless QFTs, and gives confidence on its potential to simplify multi-leg/loop computations.

The outline of the paper is as follows. Section 2 provides the set-up of the calculation. In Section 3 I review the FDR treatment of the UV divergences and discuss its interplay with the IR and CL infinities. Section 4 presents the FDR computation of the virtual part, while Section 5 deals with the real contribution and its merging with the one-loop piece. The connection between FDR and DR is discussed in Section 6 and the final conclusions are drawn in Section 7.

2. The model for $H \to gg$

The effective interaction of one Higgs field $H$ with two, three and four gluons -mediated by an infinitely heavy top loop- is described by the La-
The corresponding Feynman rules are given in [20], and the diagrams for the decay rate $\Gamma(H \rightarrow gg)$ are drawn in Figure 1. There are five graphs contributing to the

\begin{align*}
\Gamma_V(H \rightarrow gg) + \Gamma_R(H \rightarrow ggg).
\end{align*}

Figure 1: Virtual and real diagrams contributing to $H \rightarrow gg(g)$ at $O(\alpha_S^3)$. The gray blobs in $V_6$ and $V_7$ represent gluon wave function corrections and the dashed line stands for the Higgs field. $R_1(p_i, p_j, p_k)$ corresponds to three diagrams with permuted gluons.

virtual part $\Gamma_V$ -without counting gluon wave function corrections- and four diagrams for the real radiation $\Gamma_R$. In the following, I separately compute, in FDR, the two pieces, showing how IR/CL divergences drop in the sum

\begin{align*}
\Gamma_V(H \rightarrow gg) + \Gamma_R(H \rightarrow ggg).
\end{align*}
3. FDR vs infinities

The FDR subtraction of UV infinities is better illustrated with an explicit example. Consider the one-loop quadratically divergent rank-two tensor

\[ \int d^4q \frac{q_\alpha q_\beta}{D_0 D_1}, \]  

with

\[ D_i = q^2 - d_i, \quad d_i = M_i^2 - p_i^2 - 2(q \cdot p_i), \quad p_0 = 0. \]  

Its UV convergence can be improved by first deforming the propagators by a vanishing amount

\[ D_i \to \bar{D}_i = D_i - \mu^2, \]  

and then by repeatedly using the identity

\[ \frac{1}{D_i} = \frac{1}{\bar{q}^2} \left(1 + \frac{d_i}{\bar{D}_i}\right), \]  

where

\[ \bar{q}^2 = q^2 - \mu^2. \]  

Note that the propagator deformation in eq. (8) is needed to avoid possible infrared divergences in the r.h.s. of eq. (9). The integrand in eq. (6) can then be rewritten as

\[ \frac{q_\alpha q_\beta}{D_0 D_1} = q_\alpha q_\beta \left( \left[ \frac{1}{\bar{q}^4} \right] + \left[ \frac{d_0 + d_1}{\bar{q}^6} \right] + \left[ \frac{d_1^2}{\bar{q}^8 D_1} \right] + \frac{d_0 d_1}{\bar{q}^6 D_1} + \frac{d_0^2}{\bar{q}^4 D_0 D_1} \right), \]  

where the terms in square brackets are UV divergent, but depend only on \( \mu^2 \). The FDR definition of the integral in eq. (6) is obtained by integrating the expansion in eq. (11), after dropping the divergent pieces, and taking the limit \( \mu \to 0 \):

\[ B_{\alpha \beta}(p_1^2, M_0^2, M_1^2) = \int [d^4q] \frac{q_\alpha q_\beta}{D_0 D_1} \equiv \lim_{\mu \to 0} \int d^4q q_\alpha q_\beta \left( \frac{d_1^3}{\bar{q}^8 D_1} + \frac{d_0 d_1}{\bar{q}^6 D_1} + \frac{d_0^2}{\bar{q}^4 D_0 D_1} \right). \]
The r.h.s. of eq. (12) corresponds to a well defined four dimensional integral, in which all UV divergences are explicitly subtracted. Furthermore, IR and CL divergences get also regulated by the propagator deformation. The gauge invariance properties of this definition are discussed in detail in [14] and [21].

In the rest of this Section, I mostly concentrate on CL and IR infinities, and, in particular, on the matching between virtual and real contributions.

A convenient starting point to study the CL singularities is the fully massless limit of eq. (12)

\[ B_{\alpha\beta}(0,0,0) = \lim_{\mu \to 0} \int d^4 q \frac{q_\alpha q_\beta \delta^3}{q^8 D_1} = -8 p_1^\rho p_1^\sigma p_1^\tau \lim_{\mu \to 0} \int d^4 q \frac{q_\alpha q_\beta q_\rho q_\sigma q_\tau}{q^8 D_1} = 0 , \]  

(13)

which vanishes, after tensor decomposition, since \( p_1^2 = 0 \). Analogously, one proves that

\[ B_{0}(0,0,0) = \int [d^4 q] \frac{q_0}{D_0 D_1} = 0 , \]

\[ B(0,0,0) = \int [d^4 q] \frac{1}{D_0 D_1} = 0 . \]  

(14)

Those results coincide with DR -in which scale-less integrals are zero- and are due to a cancellation between two \( \ln(\mu^2) \) of CL and UV origin, respectively.

For example

\[ B(p_2^2,0,0) = -i\pi^2 \lim_{\mu \to 0} \int_0^1 dx \left[ \ln(\mu^2 - p_2^2 x(1-x)) - \ln(\mu^2) \right] , \]  

(15)

where the first logarithm develops a CL singularity in the limit \( p_2^2 \to 0 \). Thus, the virtual CL infinities, generated by \( 1 \to 2 \) splittings of massless particles, are naturally regulated by the \( \mu^2 \)-deformed propagators inside the loop, while the external momenta remain massless, as illustrated in Figures 2 (a) and (c). The real counterpart of this procedure is exemplified in Figures 2 (b) and (d), and corresponds to a phase space in which all the would be massless external particles are given a common mass \( \mu \) and the internal ones stay massless. In other words, one has to replace \[ \frac{1}{2(p_i \cdot p_j)} \to \frac{1}{(p_i + p_j)^2} \]  

(16)

\footnote{See Figure 2 (b).}
in any possible singular denominator of the real matrix element squared, integrate over the aforementioned massive phase space and take the limit $\mu \to 0$.

As for the IR divergences, the reasoning follows the same lines. For example, the only IR/CL divergent scalar one-loop three-point function generated
by the cut in Figure 2 (e) is

\[ C(s) = \int [d^4 q] \frac{1}{D_0 D_1 D_2} = \lim_{\mu \to 0} \int d^4 q \frac{1}{D_0 D_1 D_2}, \tag{17} \]

with

\[ M_0^2 = M_1^2 = M_2^2 = p_1^2 = p_2^2 = 0, \quad s = -2(p_1 \cdot p_2). \tag{18} \]

By denoting

\[ \mu_0 = \frac{\mu^2}{s}, \tag{19} \]

one computes

\[ C(s) = \lim_{\mu \to 0} \frac{i\pi^2}{2s} \ln^2 \left( \frac{\sqrt{1 - 4\mu_0} + 1}{\sqrt{1 - 4\mu_0} - 1} \right) = \frac{i\pi^2}{s} \left[ \frac{\ln^2(\mu_0) - \pi^2}{2} + i \pi \ln(\mu_0) \right], \tag{20} \]

which is indeed fully matched by the inclusive real contribution in Figure 2 (f), as will be shown in Section 6.

In the following, I use the described approach to UV/CL/IR infinities to compute \( \Gamma_V^V (H \to gg) \) and \( \Gamma_R^V (H \to ggg) \).

4. The virtual part \( \Gamma_V^V (H \to gg) \)

The calculation is greatly simplified by eqs. (13) and (14). In fact, only diagrams \( V_1 \) and \( V_2 \) in Figure 1 contribute - as in DR- and gluon wave function corrections vanish. One computes

\[ \Gamma_V^V (H \to gg) = -\frac{3}{\pi} \alpha_s \Gamma^{(0)}(\alpha_s) M_H^2 \Re e \left[ \frac{C(M_H^2)}{i\pi^2} \right]. \tag{21} \]

This simple expression is obtained after a standard Passarino-Veltman decomposition, the only subtlety being the FDR treatment of \( \mu^2 \) [14, 21]: for consistency with eq. (8), a \( q^2 \) appearing in the numerator of a diagram should also be deformed

\[ q^2 \to \bar{q}^2, \tag{22} \]

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2The FDR integration corresponds to a normal integration, in this case, because \( C(s) \) is UV finite.
and integrals involving $\mu^2$, such as

$$\tilde{B}(p_1^2, M_0^2, M_1^2) = \int [d^4q] \frac{\mu^2}{D_0 D_1},$$

(23)

require the same integrand expansion as if $\mu^2 = q^2$. For example, from eq. (12),

$$\tilde{B}(p_1^2, M_0^2, M_1^2) = \lim_{\mu \to 0} \int d^4q \mu^2 \left( \frac{d_1^3}{q^8 D_1} + \frac{d_0 d_1}{q^6 D_1} + \frac{d_0^2}{q^4 D_0 D_1} \right)$$

$$= \frac{i\pi^2}{2} \left( M_0^2 + M_1^2 - \frac{p_1^2}{3} \right).$$

(24)

The final result follows by inserting eq. (20) into (21)

$$\Gamma_V(H \to gg) = \frac{3\alpha_S}{2\pi} \Gamma^{(0)}(\alpha_S) \left( \pi^2 - \ln^3 \frac{M_H^2}{\mu^2} \right).$$

(25)

5. The real radiation $\Gamma_R(H \to ggg)$ and the fully inclusive result

The unpolarized matrix element squared, derived from the real emission diagrams in Figure 1, reads

$$|M|^2 = 192 \pi \alpha_S A^2 \left[ \frac{s_{12}^3}{s_{12}s_{13}} + \frac{s_{13}^3}{s_{12}s_{23}} + \frac{s_{12}^3}{s_{13}s_{23}} + \frac{2(s_{13}^2 + s_{23}^2)}{s_{12}} + \frac{3s_{13}s_{23}}{s_{13}} \right.$$

$$+ \frac{2(s_{12}^2 + s_{23}^2)}{s_{13}} + \frac{3s_{12}s_{23}}{s_{23}} + \frac{2(s_{12}^2 + s_{13}^2)}{s_{23}} + \frac{3s_{12}s_{13}}{s_{23}}$$

$$+ 6(s_{12} + s_{13} + s_{23}) \left. \right], \quad (26)$$

where $s_{ij} = (p_i + p_j)^2$. This expression is obtained from the massless result with the replacement $2(p_i \cdot p_j) \to s_{ij}$, in accordance with eq. (16). As described in Section 3, in order to match the virtual IR/CL singularities, $|M|^2$ should be integrated over a massive three-gluon phase space with $p_i^2 = \mu^2$, which can be parametrized as

$$\int d\Phi_3 = \frac{\pi^2}{4s} \int ds_{12} ds_{13} ds_{23} \delta(s - s_{12} - s_{13} - s_{23} + 3\mu^2),$$

(27)
where $\sqrt{s}$ is the Higgs mass. It is convenient to introduce the dimensionless variables

$$
x = \frac{s_{12}}{s} - \mu_0, \quad y = \frac{s_{13}}{s} - \mu_0, \quad z = \frac{s_{23}}{s} - \mu_0,
$$

with $\mu_0$ given in eq. (19), in terms of which, by using the condition

$$
x + y + z = 1,
$$

all IR/CL divergent bremsstrahlung integrals can be reduced to the following ones

$$
I(s) = \int_R dx dy \frac{1}{(x+\mu_0)(y+\mu_0)}, \quad J_p(s) = \int_R dx dy \frac{x^p}{(y+\mu_0)} \quad (p \geq 0),
$$

where the integration region reads, in the fully inclusive case,

$$
\int_R dx dy \equiv \int_{3\mu_0}^{1/2} dx \int_{y_-}^{y_+} dy,
$$

with

$$
y_\pm = \frac{1}{4(x+\mu_0)} [(1-\mu_0)^2 - (R_0 \mp R_1)^2] - \mu_0,
\quad R_0 = \sqrt{(x-\mu_0)^2 - 4\mu_0^2}, \quad R_1 = \sqrt{(1-x)^2 - 4\mu_0}.
$$

Thus

$$
\Gamma_R(H \to ggg) = 3\frac{\alpha_S}{\pi} \Gamma^{(0)}(\alpha_S) \left( \frac{1}{4} + I(M_R^2) - \frac{3}{2} J_0(M_R^2) - J_2(M_R^2) \right).
$$

Finally, one computes, up to terms which vanish in the limit $\mu_0 \to 0$,

$$
I(s) = \frac{\ln^2(\mu_0) - \pi^2}{2},
$$

and

$$
J_p(s) = -\frac{1}{p+1} \ln(\mu_0) + \int_0^1 dx x^p [\ln(x) + 2 \ln(1-x)]
$$

$$
= -\frac{1}{p+1} \ln(\mu_0) - \frac{1}{p+1} \left[ \frac{1}{p+1} + 2 \sum_{n=1}^{p+1} \frac{1}{n} \right] \quad (p \geq 0),
$$
so that
\[
\Gamma_R(H \to ggg) = \frac{3\alpha_S}{2\pi} \Gamma^{(0)}(\alpha_S) \left( \ln^2 \frac{M_H^2}{\mu^2} - \pi^2 + \frac{73}{6} - \frac{11}{3} \ln \frac{M_H^2}{\mu^2} \right). \tag{36}
\]

Summing this to eq. (25), and adding the finite renormalization term in eq. (4), one obtains
\[
\Gamma(H \to gg) = \Gamma^{(0)}(\alpha_S) \left[ 1 + \frac{\alpha_S}{\pi} \left( \frac{95}{4} - \frac{11}{2} \ln \frac{M_H^2}{\mu^2} \right) \right]. \tag{37}
\]

All CL/IR \( \ln(\mu^2) \) and \( \ln^2(\mu^2) \) cancel in eq. (37), so that the remaining \( \mu \) is directly interpreted as the renormalization scale. This is a typical procedure in FDR: since the UV infinities are subtracted from the very beginning, the unphysical left over \( \mu \) dependence is eliminated, at the perturbative level one is working, by a finite renormalization, which fixes the bare parameters in terms of observables [23]. This is obtained, in the case at hand, by simply replacing \( \Gamma^{(0)}(\alpha_S) \to \Gamma^{(0)}(\alpha_S(\mu^2)) \) \(^3\) in eq. (37). Then, the logarithm is reabsorbed in the gluonic running of the strong coupling constant
\[
\alpha_S(M_H^2) = \frac{\alpha_S(\mu^2)}{1 + \frac{\alpha_S}{2\pi} \frac{11}{2} \ln \frac{M_H^2}{\mu^2}}, \tag{38}
\]
and eq. (1) follows.

6. FDR vs DR

In this section, I discuss the transition rules between FDR and DR. This is particularly important in QCD, where NLO calculations have to be matched with the runnings of \( \alpha_S \) and parton densities, conventionally derived in DR. I consider UV, CL and IR divergences in turn, showing the equivalence of FDR with the Dimensional Reduction [24] version of DR, widely used in supersymmetric theories.

I start by establishing the connection between the \( 1/\epsilon \) DR regulator and the \( \ln(\mu^2) \) appearing in FDR. As for the UV infinities, it is sufficient to

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\(^3\) \( \alpha_S(\mu^2) \) has to be computed in the \( \overline{\text{MS}} \) scheme, as explained in the next Section.
compare the FDR and DR variants of any divergent integral. For instance, the DR counterpart of eq. (15) (with $p^2 \neq 0$) reads

$$\int d^n q \frac{1}{q^2(q + p)^2} = i\pi^2 \int_0^1 dx \left[ \Delta - \ln(-p^2 x(1 - x)) \right], \quad (39)$$

where

$$n = 4 + \epsilon \quad \text{and} \quad \Delta = -\frac{2}{\epsilon} - \gamma_E - \ln \pi. \quad (40)$$

Thus, DR and FDR UV regulators are linked through the simple $\overline{\text{MS}}$ replacement

$$\Delta \rightarrow \ln(\mu^2). \quad (41)$$

CL virtual singularities follow the same pattern, as can be inferred from the exact UV/CL cancellation in eqs. (13) and (14). As a consistency check, the DR version of $J_p$ reads

$$J_p^{\text{DR}}(s) = \frac{(\pi s)^{\frac{1}{2}}}{\Gamma(1 + \frac{1}{2})} \int dx \, dy \, dz \, x^{p} \frac{\delta(1 - x - y - z)(xyz)^{\frac{1}{2}}}{y}$$

$$= -\frac{1}{p + 1}(\Delta - \ln(s)) - \frac{1}{p + 1} \left[ \frac{1}{p + 1} + 2 \sum_{n=1}^{p+1} \frac{1}{n} \right], \quad (42)$$

which indeed coincides with eq. (35) if $\Delta = \ln(\mu^2)$.

Finally, the $\ln^2(\mu^2)$ terms -generated by overlapping IR/CL singularities- drop, together with the full constant part, when adding virtual and fully inclusive real contributions, which can be traced back to the following relation

$$\Re \left[ \frac{C(s)}{i\pi^2} \right] = \frac{1}{s} I(s) \quad (43)$$

between eqs. (20) and (34). An easy calculation shows that the same happens in DR. In fact

$$\Re \left[ \frac{1}{i\pi^2} \int d^n q \frac{1}{q^2(q + p_1)^2(q + p_2)^2} \right] = \frac{1}{s}(\pi s)^{\frac{1}{2}} \Gamma \left(1 - \frac{\epsilon}{2}\right) \left[ \frac{4}{\epsilon^2} - \frac{2}{3}\pi^2 \right], \quad (44)$$
where $p_1^2 = p_2^2 = 0$ and $s = -2(p_1 \cdot p_2)$, and

$$ I^{\text{DR}}(s) = \frac{(\pi s)^{\frac{3}{2}}}{\Gamma(1 + \frac{\epsilon}{2})} \int dx \, dy \, dz \frac{1}{xy} \delta(1 - x - y - z)(xyz)^{\frac{\epsilon}{2}} $$

$$ = (\pi s)^{\frac{3}{2}} \Gamma \left(1 - \frac{\epsilon}{2}\right) \left[\frac{4}{\epsilon^2} - \frac{2}{3} \pi^2\right]. $$

(45)

In summary, eq. (41) is the only needed relation between the two regulators. However, an important difference between DR and FDR follows on from self contractions of metric tensors coming from the Feynman rules. In DR $g_{\alpha\beta}g^{\alpha\beta} = n$, while $g_{\alpha\beta}g^{\alpha\beta} = 4$ in FDR. This, together with eq. (41), and the FDR treatment of $\mu^2$ discussed in section 4 makes explicit the equivalence between FDR and Dimensional Reduction in the $\overline{\text{MS}}$ scheme. Having established this, all the well known transition rules between Dimensional Reduction and DR [25, 26] can be directly applied to FDR. In the case of eq. (37), it turns out that the expression is the same in both Dimensional Reduction (or FDR) and DR. Therefore the correct strong coupling constant to be used is the customary $\alpha_S(\mu^2)$ in the $\overline{\text{MS}}$ scheme, proving that the FDR result coincides with eq. (1).

7. Conclusions

I presented a FDR calculation of the gluonic QCD corrections to $H \rightarrow gg$ in the large top effective theory, demonstrating that ultraviolet, collinear and infrared divergences can be simultaneously and successfully regulated in four dimensions. I proved the equivalence, at the one-loop level, of Dimensional Reduction and FDR, making the latter approach attractive also in supersymmetric calculations, where the fermionic and bosonic sectors must share the same number of degrees of freedom.

The advantage of directly working in the four dimensional Minkowsky space is expected to lead to considerable simplifications in higher order QFT computations, especially in connection with numerical techniques. This issue, together with the extension of FDR to more loops, is currently under study.

Acknowledgments

I thank Fabio Maltoni for pointing me out $H \rightarrow gg$ as a possible case study of FDR applied to massless QCD. This work was performed in the
framework of the ERC grant 291377, “LHCtheory: Theoretical predictions and analyses of LHC physics: advancing the precision frontier”. I also thank the support of the MICINN project FPA2011-22398 (LHC@NLO) and the Junta de Andalucia project P10-FQM-6552.

References

[1] G. ’t Hooft, M. Veltman, Regularization and Renormalization of Gauge Fields, Nucl.Phys. B44 (1972) 189–213.

[2] S. Frixione, Z. Kunszt, A. Signer, Three jet cross-sections to next-to-leading order, Nucl.Phys. B467 (1996) 399–442.

[3] S. Catani, M. Seymour, A General algorithm for calculating jet cross-sections in NLO QCD, Nucl.Phys. B485 (1997) 291–419.

[4] D. A. Kosower, Antenna factorization of gauge theory amplitudes, Phys.Rev. D57 (1998) 5410–5416.

[5] J. M. Campbell, M. Cullen, E. N. Glover, Four jet event shapes in electron - positron annihilation, Eur.Phys.J. C9 (1999) 245–265.

[6] S. Catani, S. Dittmaier, M. H. Seymour, Z. Trocsanyi, The Dipole formalism for next-to-leading order QCD calculations with massive partons, Nucl.Phys. B627 (2002) 189–265.

[7] Z. Nagy, D. E. Soper, General subtraction method for numerical calculation of one loop QCD matrix elements, JHEP 0309 (2003) 055.

[8] T. Binoth, G. Heinrich, An Automatized algorithm to compute infrared divergent multiloop integrals, Nucl.Phys. B585 (2000) 741–759.

[9] C. Anastasiou, K. Melnikov, F. Petriello, A New method for real radiation at NNLO, Phys.Rev. D69 (2004) 076010.

[10] T. Binoth, G. Heinrich, Numerical evaluation of phase space integrals by sector decomposition, Nucl.Phys. B693 (2004) 134–148.

[11] A. Gehrmann-De Ridder, T. Gehrmann, E. N. Glover, Antenna subtraction at NNLO, JHEP 0509 (2005) 056.
[12] S. Catani, M. Grazzini, An NNLO subtraction formalism in hadron collisions and its application to Higgs boson production at the LHC, Phys.Rev.Lett. 98 (2007) 222002.

[13] M. Czakon, A novel subtraction scheme for double-real radiation at NNLO, Phys.Lett. B693 (2010) 259–268.

[14] R. Pittau, A four-dimensional approach to quantum field theories, JHEP 1211 (2012) 151.

[15] A. Djouadi, M. Spira, P. Zerwas, Production of Higgs bosons in proton colliders: QCD corrections, Phys.Lett. B264 (1991) 440–446.

[16] S. Larin, T. van Ritbergen, J. Vermaseren, The Large top quark mass expansion for Higgs boson decays into bottom quarks and into gluons, Phys.Lett. B362 (1995) 134–140.

[17] M. A. Shifman, A. Vainshtein, M. Voloshin, V. I. Zakharov, Low-Energy Theorems for Higgs Boson Couplings to Photons, Sov.J.Nucl.Phys. 30 (1979) 711–716.

[18] S. Dawson, R. Kauffman, Higgs boson plus multi-jet rates at the SSC, Phys.Rev.Lett. 68 (1992) 2273–2276.

[19] F. Maltoni, Basics of QCD for the LHC: pp → H + X as a case study (2012) 1–27.

[20] R. P. Kauffman, S. V. Desai, D. Risal, Production of a Higgs boson plus two jets in hadronic collisions, Phys.Rev. D55 (1997) 4005–4015.

[21] A. Donati, R. Pittau, Gauge invariance at work in FDR: H → gamma gamma JHEP04 (2013) 167.

[22] G. Passarino, M. Veltman, One Loop Corrections for e+ e- Annihilation Into mu+ mu- in the Weinberg Model, Nucl.Phys. B160 (1979) 151.

[23] R. Pittau, On the predictivity of the non-renormalizable quantum field theories (2013).

[24] W. Siegel, Supersymmetric Dimensional Regularization via Dimensional Reduction, Phys.Lett. B84 (1979) 193.
[25] A. Signer, Helicity method for next-to-leading order corrections in QCD, Ph.D. Thesis (1995).

[26] R. Pittau, Primary Feynman rules to calculate the epsilon-dimensional integrand of any 1-loop amplitude, JHEP 1202 (2012) 029.