Spin textures, screening and excitations in dirty quantum Hall ferromagnets

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We study quantum Hall ferromagnets in the presence of a random electrostatic impurity potential. Describing these systems with a classical non-linear sigma model and using analytical estimates supported by results from numerical simulations, we examine the nature of the ground state as a function of disorder strength, $\Delta$, and deviation, $\delta \nu$, of the average Landau level filling factor from unity. Screening of an impurity potential requires distortions of the spin configuration, and in the absence of Zeeman coupling there is a disorder-driven, zero-temperature phase transition from a ferromagnet at small $\Delta$ and $|\delta \nu|$ to a spin glass at larger $\Delta$ or $|\delta \nu|$. We examine ground-state response functions and excitations.

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Quantum Hall ferromagnets (QHFMs) are interesting especially as systems in which spin configurations and charge density are closely linked. At small Zeeman energy and for Landau level filling factor $\nu$ close to unity, this link has the celebrated consequence that the charged quasiparticles with the lowest energy are not single electrons but skyrmions. These bound states of a minority-spin electron with one or many spin waves may be viewed classically as topological excitations of an ordered ferromagnet: in this description, the deviation of local charge density from that of a filled and ferromagnetically polarised Landau level is proportional to the topological density of the spin configuration. For a clean QHFM with sufficiently small Zeeman energy, skyrmions or antiskyrmions are introduced at zero temperature on varying the average filling factor from $\nu = 1$ to larger or smaller values and, at non-zero temperature, are generated thermally in pairs together with spin waves. For dirty QHFMs, coupling of an electrostatic impurity potential to the charge density offers an additional mechanism by which spin textures may arise: the consequences of such a coupling are the subject of this paper.

The interplay between disorder and exchange for QHFMs has been examined previously from several different viewpoints. Fogler and Shklovskii, building on earlier discussions, have developed a mean field treatment in different viewpoints. Within this approximation, developed by Efros for the comparable problem in spin-polarised Landau levels when $\nu$ lies near half-integer values, the ground-state charge density at weak disorder is determined by the condition that the Hartree potential should everywhere match the chemical potential: $\mu = V(r) + \int U(r - r')\rho(r')d^2r'$. We are concerned with circumstances in which the resulting density varies smoothly on the scale of the magnetic length, $l_B$, and has fluctuations from $\nu = 1$, $\delta \rho(r) \equiv \rho(r) - (2\pi l_B^2)^{-1}$, which are small: $|\delta \rho(r)| \ll \rho(r)$. Restoring exchange interactions under these conditions results locally in maximal ferromagnetic polarisation of electron spins, with a direction that may vary in space. Denoting this direction by the three-
component unit vector $\vec{S}(r)$, its spatial fluctuations are linked to electron density via

$$\delta \rho(r) = (8\pi)^{-1} \epsilon_{\alpha\beta\gamma} S^\alpha \partial_r S^\beta \partial_r S^\gamma.$$ (1)

An exchange energy is associated with such variations: combining exchange (with interaction constant $J$), impurity and Hartree contributions to the total energy, and choosing for simplicity a short-range interaction $U(r - r') = U_0 \delta(r - r')$, we take as our description of a dirty quantum Hall ferromagnet the configurational energy

$$\mathcal{H} = \int \left( \frac{J}{2} |\nabla \vec{S}(r)|^2 + V(r) \delta \rho(r) + \frac{U_0}{2} [\delta \rho(r)]^2 \right) d^2 r.$$

(2)

Doing so, we neglect quantum fluctuations of $\vec{S}(r)$, as is justified in the semiclassical limit, $|\nabla \vec{S}(r)| \ll l_H^{-1}$, to which we are already restricted. We choose for simplicity $V(r)$ Gaussian distributed with amplitude $\Delta$ and correlation length $\lambda$. (More realistic choices for $U(r)$ and for the distribution of $V(r)$ will be considered elsewhere.)

Our aim in the following is to understand the zero-temperature phase diagram of the model defined by Eq. (2), and to characterise its ground states via their response functions and excitations. As an example of a disordered electron system, it is unusual in that there is an exchange gap for single-particle excitations, even if the ground-state spin configuration $\vec{S}(r)$ is not ferromagnetically ordered, so that the only low-energy excitations involve collective spin modes. As an example of a ferromagnet with quenched disorder, the system is also unusual in several ways. First, the coupling to disorder leaves spin-rotation symmetry intact but breaks time-reversal symmetry, in contrast to random magnetic fields, which break both symmetries, and to random exchange interactions, which leave both symmetries intact. Second, because of the same coupling, the spin system responds to applied electric fields: we calculate the wavevector-dependent dielectric susceptibility, comparing with behaviour found in more conventional disordered electron systems. Third, the link between spin and charge also endows spinwaves with an electric dipole moment: we calculate the spin-wave contribution to the optical conductivity in disordered spin states, complementing Green’s results for the polarised ferromagnet with impurities.

We start with a simple discussion of the phase diagram. The model is characterised by two energy scales, $J$ and $\Delta$, and two length scales, $\lambda$ and $L_H \equiv (U_0/J)^{1/2}$. The last of these, which we call the Hartree length, plays an important role in what follows. Its significance can be made clear by comparing, for a skyrmion of fixed shape and radius $R$ in a clean system, the contributions to total energy from exchange and from Hartree interactions, of order $J$ and $U_0/R^2$ respectively: exchange dominates on length scales large compared with $L_H$, while Hartree interactions dominate at smaller distances. We use the limit $L_H \gg \lambda$ as a source of simplifications in analytical estimates, but take $L_H \sim \lambda$ in numerical simulations.

Examine first the ground state for $\langle \delta \rho \rangle = 0$ as a function of $\Delta/J$. At weak disorder ($\Delta \lesssim J$) it can be pictured as a dilute glass of skyrmions and antiskyrmions, discussed in Ref. [6]. This reflects the existence of a threshold $\lambda/J$, defined by $|V(r)| = 4\pi J$: in most parts of the system (roughly, those where $|V(r)|$ is below threshold) ferromagnetic order is essentially unaffected by the impurity potential [7]. More precisely, for any $\vec{S}(r)$ one has $|\nabla \vec{S}(r)|^2 \geq 8\pi |\delta \rho(r)|$ and hence $\mathcal{H} \geq \int [4\pi J |\delta \rho(r)| + V(r) |\delta \rho(r)|] d^2 r$, which for $V(r)$ below threshold everywhere implies that the ground state is the perfectly aligned ferromagnet with $\delta \rho(r) = 0$. At stronger disorder ($\Delta \gg J$), by contrast, screening is almost perfect at short distances and $\delta \rho(r) \simeq -V(r)/U_0$. Corrections to such screening arise at and beyond the scale $L_H$, where exchange is important. We can summarise the effect of exchange by dividing the system into regions of area $L_H^2$, finding for each such area the integral $Q \equiv -\int V(r)/U_0 d^2 r$, and adjusting the total screening charge within every region to the integer value closest to $Q$. We argue that these integers are predominantly zero in a ferromagnetic phase, and predominantly non-zero in a phase without ferromagnetic order. To see this, consider a well-ordered ferromagnetic phase, in which $S(r)$ as a function of $r$ has small fluctuations around a global direction of magnetisation. In this case, the net topological charge in each region has magnitude much less than one. Conversely, in a phase without such order, unit topological charge accumulates in a region of linear size given by the ferromagnetic correlation length. The phase boundary for the ferromagnet is therefore located by setting $(Q^2)^{1/2} \sim 1$. Since each area of size $\lambda^2$ contributes to $Q$ a charge of magnitude $\lambda^2 \Delta / U_0$ with random sign, $(Q^2)^{1/2} \sim \lambda L_H \Delta / U_0$ yielding $\Delta_c \sim U_0/(\lambda L_H) = J(L_H/\lambda)$. For $\Delta > \Delta_c$ the ground state has no ferromagnetic order; because within a classical zero-temperature description spins are frozen, we identify this phase as a spin glass.

Consider next the ground state at fixed $\Delta < \Delta_c$, as a function of $\langle \delta \rho \rangle$. Charge is introduced into the system for $\langle \delta \rho \rangle > 0$ as skyrmions of size $R$. In the presence of Hartree energy alone, $R$ is divergent, but fluctuations of an impurity potential establish an optimal size, by generating random potential wells. For $R \gg \lambda$ these have an average depth $\Delta(\lambda/R)$ (the case $R \lesssim \lambda$ is treated in Ref. [8]). The contributions to the skyrmion energy from these two sources are of order $U_0/R^2$ and $-\Delta \lambda / R$, respectively; minimising their sum, we find $R \sim U_0/(\Delta \lambda) = L_H(\Delta_c/\Delta)$. (Note that, since $R > L_H$ for $\Delta < \Delta_c$, screening as discussed in the previous paragraph does not alter this argument.) We ex-
pect ferromagnetic order to persist with increasing \( \langle \delta \rho \rangle \) until such skyrmions overlap, so the phase boundary lies at \( \langle \delta \rho \rangle_c \sim L^{-2}(\Delta/\Delta_c)^2 \). In this way we arrive at the schematic phase diagram shown in the inset to Fig. 1.

In order to test these arguments, we have studied ground states of a lattice model numerically. In outline, we replace Eq. 3 with a Heisenberg model on a square lattice which has nearest-neighbour ferromagnetic interactions of strength \( J \) and a topological charge defined on each elementary plaquette. These charges contribute to the total energy by a local Hartree interaction with coefficient \( U_0 \), and by their interaction with plaquette potentials, independently and uniformly distributed in \([-\Delta, \Delta] \). Thus \( \lambda = 1 \) in units of the lattice spacing. We find low-energy states using a simulated annealing procedure similar to that of Ref. 12, and can identify states that are equivalent under a global rotation by comparing total energies and also charge distributions. Using system sizes up to \( 56^2 \) spins and annealing with \( 10^6 \) Monte Carlo steps per spin, for a given disorder realisation in successive runs we repeatedly reach states from a small set: we take lowest of these in energy to be the ground state. In the ground state for each disorder realisation we calculate the site-averaged magnetisation, \( M = |\langle \vec{S}(r) \rangle| \). We also determine: the ground-state susceptibility \( \chi \), from the response to a Zeeman field applied in the direction of the magnetisation; the ground-state spin stiffness \( \rho_S \), from the energy cost of long-wavelength twists imposed on the spin configuration; and the wavevector-dependent compressibility, from the linear response of \( \delta \rho(r) \) to a periodic potential added to \( V(r) \). Further details of our methods will be described elsewhere 11.

Representative results are shown in Fig. 2. Taking \( \langle \delta \rho \rangle = 0 \) and \( U_0/J = 2 \), we present \( M \), \( \chi \) and \( \rho_S \) as a function of disorder strength, \( \Delta/J \), using \( 40^2 \) spins and an average over three disorder realisations. Three regimes are evident from the behaviour of \( M \). For \( \Delta/J < 1 \), the ground state is a fully-polarised ferromagnet, because we use a bounded disorder distribution. For \( 1 < \Delta/J \lesssim 2.5 \), the ground state is a partially polarised ferromagnet (though with an almost saturated magnetisation for \( 1 < \Delta/J \lesssim 1.8 \)). And for \( 2.5 \lesssim \Delta/J \), the ground state is, we argue, a spin glass, with a small non-zero \( M \) arising as a finite-size effect. In support of this interpretation, a large peak in \( \chi \) indicates a phase transition at \( \Delta/J \approx 2.5 \). Moreover, the distinction between phases is illustrated by their spin stiffness, specified in full by a \( 3 \times 3 \) symmetric tensor. We plot the eigenvalues of this in Fig. 2. For the fully-polarised ferromagnet, rotations about the magnetisation direction do not alter the spin configuration, and so one eigenvalue is zero while the other two are degenerate. For the partially-polarised ferromagnet, all three eigenvalues are non-zero, with two remaining degenerate. And at \( 2.5 \lesssim \Delta/J \), magnetic isotropy and finite spin stiffness in the spin glass are illustrated by the fact that all three eigenvalues are approximately degenerate and non-zero.

Repeating such calculations for \( \langle \delta \rho \rangle > 0 \), we determine the phase diagram shown in the main panel of Fig. 2, which is qualitatively similar to that in the inset, even though the simulations are for \( (U_0/J \lambda^2) = 2 \), while our theoretical arguments apply for \( (U_0/J \lambda^2) \gg 1 \). The maximum density range over which disorder may stabilise the ferromagnet, \( |\langle \delta \rho \rangle| \lesssim 10^{-3} \) in units of charge per plaquette, is strikingly narrow.

We now turn to the dielectric response of the partially polarised ferromagnet and spin glass, characterised at zero frequency by the wavevector-dependent dielectric susceptibility \( \chi_c(q) \) or by the compressibility \( \kappa(q) \), with
\[ \kappa(q) = q^2 \chi_e(q) \epsilon_0 / e^2. \] To fix definitions, take \( V(r) \rightarrow V(r) + V_1 \cos(q \cdot r) \) in Eq. (3): the ground-state electron density changes according to \( \delta \rho(r) \rightarrow \delta \rho(r) + \delta \rho_1(r) \) and the disorder-averaged linear response is \( \langle \delta \rho_1(r) \rangle = -\kappa(q)V_1 \cos(q \cdot r) \). We can apply to \( \kappa(q) \) the approach used in our discussion of the phase diagram: the Hartree length \( L_H \) again plays an important role, within our model problem in which Coulomb interactions are omitted. For \( q \gg L_H^{-1} \), exchange may be neglected and \( \kappa(q) \propto U_0^{-1} \). More generally, we anticipate the scaling form \( \kappa(q) = U_0^{-1} f(L_H q) \), with \( f(x) \) constant at large \( x \). For \( q \ll L_H^{-1} \) we expect that exchange dominates and hence that \( \kappa(q) \) should be independent of \( U_0 \) in this regime. In turn this implies for the scaling function \( f(x) \propto x^2 \) at small \( x \), so that \( \kappa(q) \propto (qL_H)^2 U_0^{-1} \equiv qJ^2 \) for \( q \ll L_H^{-1} \). To summarise, the response is that of a metal (\( \kappa(q) \) constant) at large wavevectors, and that of an insulator (\( \chi_e(q) \) constant) at small wavevectors [13].

We have tested these arguments using the simulation methods outlined above. Results for \( \kappa(q) \) are displayed in Fig. 3, where we compare behaviour in systems with \( J = 1 \) and \( U_0 = 1 \) or \( U_0 = 2 \), combining for each case data from lattices of size 40\(^2\) and 56\(^2\) in order to maximise wavevector resolution. The distinction is clear between a \( U_0 \)-independent \( \kappa(q) \), quadratic in \( q \), at small \( q \), and a \( q \)-independent \( \kappa(q) \), varying roughly as \( U_0^{-1} \), at large \( q \).

Finally, we consider optical conductivity \( \sigma(\omega) \) at frequency \( \omega \). A spin glass in the absence of Zeeman energy is expected [13] to support three degenerate Goldstone modes. Their dispersion is linear at small frequencies with speed \( c = (\rho_0 / \chi)^{1/2} \). In a QHFM they have an electric dipole moment, which arises because they propagate in a non-collinear ground state. Their contribution to \( \sigma(\omega) \) is determined by a product of the mean square dipole moment with the density of states. We find [13]

\[
\sigma(\omega) \sim \frac{e^2}{\hbar} \left( \frac{\omega \xi}{c} \right)^2 \frac{\hbar \omega}{J}
\]

where \( \xi \) is the spin correlation length. Variable-range hopping would mask this contribution to \( \sigma(\omega) \) but should be absent if spins locally have maximal polarisation throughout the sample.

In summary, we have shown how the competition between screening and exchange determines the ground-state properties of QHFM, in the presence of a smoothly-varying impurity potential.

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