The effect of generation of narrow ultrarelativistic beams of positrons (electrons) in the process of resonant photoproduction of pairs on nuclei in a strong electromagnetic field

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Abstract

The generation of narrow beams of high-energy positrons (electrons) in the process of resonant photogeneration of ultrarelativistic electron–positron pairs by high-energy gamma quanta in the field of the nucleus and a strong electromagnetic wave is theoretically predicted. It is shown that if the energy of the initial gamma quanta significantly exceeds the characteristic energy of the process, then ultrarelativistic positrons (electrons) are emitted with energies very close to the energy of gamma quanta. Moreover, the resonant differential cross-section of such processes can exceed the corresponding differential cross-section without an external field by thirteen orders of magnitude. This effect makes it possible to obtain narrow beams of ultrarelativistic positrons (electrons) in strong electromagnetic fields with high probability.

Keywords: high-energy gamma quanta, ultrarelativistic electrons (positrons), nuclei, strong electromagnetic fields

(Some figures may appear in colour only in the online journal)

1. Introduction

Currently, laser systems of high intensities are being intensively created [1–8], as well as sources of high-energy particles [9–12]. This contributes to the intensive development of quantum electrodynamics in strong electromagnetic fields [13–65]. An important place in such processes is occupied by resonant effects (Oleinik resonances [24, 25]) associated with the release of an intermediate particle in an external electromagnetic field onto the mass shell (see, for example, articles [26–38]). It is important to emphasize that the resonant differential cross-sections can significantly exceed the corresponding non-resonant differential cross-sections [29–38].

The transformation of electromagnetic radiation into matter is one of the most interesting phenomena of quantum field theory. There are various methods of obtaining electron–positron pairs in nature [40]. Among them, the processes of Bethe et al [56] and Breit and Wheeler [57] stand out. It is important to note that the Bethe-Heitler process can be modified...
by an external electromagnetic field. As a result, the laser-assisted Bethe-Heitler process can proceed in a resonant manner (see, for example, [30, 32–36]), i.e. it effectively decays into two subprocesses: the external field-stimulated Breit-Wheeler process [23] and the external field-assisted Mott process (scattering of an intermediate electron (positron) on the nucleus in the external field) [58–60].

In a recent paper [36], the resonant photoproduction of pairs (RPPs) on nuclei in strong light fields (resonant laser-assisted Bethe-Heitler process) was studied. At the same time, however, the possibility of generating positrons (electrons) with energies close to the energies of the initial gamma quanta has not been studied. This article will consider this case. At the same time, in contrast to the article [36], the methodology of consideration of this resonant process will be changed (a characteristic quantum parameter of the process \( \eta \) is introduced (see expression (3))).

We will study the resonant process of photogeneration of pairs for high-energy initial gamma quanta, as well as produced electrons and positrons when the basic classical parameter

\[
\eta = \frac{eF \lambda}{mc^2}
\]  

(1)

satisfies the relation

\[
\eta \ll \frac{\hbar \omega_i}{mc^2} \gg 1, \quad \eta \ll \frac{E_i}{mc^2} \gg 1.
\]  

(2)

Here \( e \) and \( m \) are the charge and mass of the electron, and \( F \) and \( \lambda = c/\omega \) are the electric field strength and wavelength, \( \omega \) is the frequency of the wave; \( \omega_i \) is the high-energy initial gamma quantum, \( E_i \approx mc^2 \) is the ultrarelativistic energy of the positron (electron). It is important to note that the value of the classical parameter \( \eta \) should be significantly less than not only the energies of the initial and final particles (in units of electron rest energy), but also the limit value associated with the critical Schwinger field \( (\eta \ll \eta_c) \), where the value \( \eta_c \) corresponds to the critical Schwinger field \( (I \ll I_s \sim 10^{23} \text{ Wcm}^{-2}) \) [61, 62]. So, in the field of optical frequencies, the value of \( \eta_c \approx 10^5 \).

The article [36] shows that the resonant energy of the produced positron (for channel (A)) and electron (for channel (B)) is determined by their outgoing angles, as well as the quantum parameter (see expressions (11) and (13))

\[
\varepsilon_{\eta}(\gamma) = r\varepsilon \eta \geq 1, \quad \varepsilon \eta = \frac{\omega_i}{\omega_\eta}.
\]  

(3)

Here the parameter \( \varepsilon_{\eta}(\gamma) \) is numerically equal to the product of the number of absorbed photons in the external field stimulated Breit-Wheeler process \( (r = 1, 2, 3 \ldots) \) by the quantum parameter \( \varepsilon_\eta \). This parameter is equal to the ratio of the energy of the high-energy initial gamma quantum to the characteristic energy of the process \( \hbar \omega_\eta \) which is determined by the experimental conditions and the laser installation:

\[
\hbar \omega_\eta = \frac{(mc^2)^2}{(\hbar \omega \sin^2(\theta_i/2)} \left(1 + \eta^2\right),
\]  

(4)

Here \( \theta_i \) is the angle between the momentum of the initial gamma quantum and the direction of wave propagation. It can be seen from expression (4) that the value of the characteristic energy \( \hbar \omega_\eta \) is inversely proportional to the photon energy of the wave \( (\hbar \omega) \) and is also directly proportional to the intensity of the wave \( (I \sim \eta^2 \left( \text{Wcm}^{-2} \right))\).

Note that the value of the characteristic energy of the process arises due to the fact that one of the subprocesses is the external field-stimulated Breit-Wheeler process [23]. At the same time, in the case of a weak field, when the classical parameter \( \eta \ll 1 \) and the process with the absorption of one photon of the wave is most likely, the characteristic energy makes sense of the threshold energy \( (\omega_\eta = \omega_\eta(b)) \). For the energies of the initial gamma quanta lower than the threshold energy \( (\omega_i < \omega_\eta(b)) \), the RPP process is suppressed (see, for example, [32–34]). However, for strong fields \( (\eta \gtrsim 1) \), processes with the absorption of one or more photons of the wave may be of the same order. Because of this, the value \( \omega_\eta \) loses the meaning of the threshold energy, since resonant processes with the energy of the initial gamma quanta greater or less than the value of the characteristic energy have a physical meaning [36].

It is important to note that the number of absorbed photons under resonance conditions significantly depends on the range of values of the quantum parameter \( \varepsilon_\eta \) (3) or the relationship between the characteristic energy of the process and the initial energy of the gamma quantum. If \( \varepsilon \eta < 1 \ (\omega_i < \omega_\eta) \), then the number of absorbed photons of the wave in the external field-stimulated Breit-Wheeler process must exceed or be equal to some minimum number of photons of the wave \( r_{\min} \), which is determined by the parameter \( \varepsilon_\eta \) (see the equation (14)). At the same time, the number of absorbed photons of the wave can start with quite large numbers when \( r_{\min} \gg 1 \ (\omega_i \ll \omega_\eta) \) (this is usually true for very strong electromagnetic fields). If it is a quantum parameter \( \varepsilon \eta \gg 1 \ (\omega_i \gtrsim \omega_\eta) \), then the number of absorbed photons of the wave always begins with one photon (see the equation (15)). It is important to emphasize that the number of absorbed photons of the wave significantly affects the magnitude of the resonant differential cross section. For a small number of absorbed photons of the wave \( (r \sim 1) \), the resonant cross section will be significantly larger than for a large number of absorbed photons \( (r \gg 1) \). Because of this, the case when the energy of the initial gamma quanta exceeds the characteristic energy of the process is of undoubted interest. Note that case (14) was studied in detail in [36]. At the same time, case (15) was not considered.

In this paper, within the framework of the ratio (15) we will mainly study consider the case

\[
\omega_i \gg \omega_\eta \quad \left(\varepsilon_{\eta(\gamma)} = r\varepsilon \eta \gg 1, \ r = 1, 2, 3 \ldots \right),
\]  

(5)

when the resonant generation of ultrarelativistic positrons (electrons) takes place with maximum probability and with energies close to the energies of the initial gamma quanta. Note that the cross-channel of this process was considered in the articles [37, 38].

We will use the relativistic system of units: \( \hbar = c = 1 \).
2. Resonant energies of positrons (electrons) in strong fields

Let us consider this process in the field of a plane circularly polarized wave propagating along the \( z \) axis [36]

\[
A(\phi) = \frac{F_0}{\omega} (e_\gamma \cos \phi + \delta \cdot e_\delta \sin \phi), \quad \phi = k x = \omega (t - z),
\]

where \( k = (\omega, k) \) is the wave vector, \( \delta = \pm 1 \) and \( e_\gamma = (0, e_\gamma) \), \( e_\delta = (0, e_\delta) \) are the polarization four-vectors of the wave, particularly \( e_{\gamma,z} = -1 \), \( (e_{\delta,z})^2 = k^2 = 0 \). Note that a choice of the circular polarization of the wave is associated with a simpler representation of special functions (Bessel functions) for the probability of the external field-stimulated Breit-Wheeler process.

Oleinik resonances occur when an intermediate electron (positron) in the electromagnetic wave field enters the mass shell [24, 25, 36]. Because of this, for channels A and B, we get (see figure 1):

\[
\tilde{q}_-^2 = m^2, \quad \tilde{q}_+ = k_1 - \tilde{p}_\pm + rk.
\]

Here \( \tilde{q}_- \) and \( \tilde{q}_+ \) are the 4-quasimomenta of intermediate electron (for channel (A)) and intermediate positron (for channel (B)), \( m \) is the effective mass of the electron (positron) in the field of a circularly polarized wave [23, 36]

\[
\tilde{p}_\pm = p_\pm + \eta^2 \frac{m^2}{2(kp_\pm)} k, \quad \tilde{q}_+ = q_\mp + \eta^2 \frac{m^2}{2(kq_\mp)} k,
\]

\[ j = i, f. \]

\[ \tilde{p}_\pm^2 = m^2, \quad m_\mp = m \sqrt{1 + \eta^2}. \]

In expressions (7) and (8) \( k = (\omega, k) \) is the 4-momentum of the external field photon, \( k_1 = (\omega_1, k_1) \) is the 4-momentum of the initial gamma quantum, \( p_\pm = (E_\pm, p_\pm) \) is the 4-momentum of the positron (electron). Such behavior is caused by the quasi-discrete energy spectrum of fermion propagating within the plane electromagnetic wave. Due to that fact, one may interpretate it as the reduction of the second order process into the two successive second order processes in fine structure constant: the process of scattering of an intermediate electron (positron) on the nucleus in the wave field and the external field–stimulated Breit-Wheeler process (see figure 1).

In this paper, we examine the case of high-energy energies of the initial gamma quantum (2). Moreover, we confine ourselves with the configuration, where all produced ultrarelativistic particles propagate within the narrow cone with the initial gamma quantum direction. Additionally, we demand that the directions of initial gamma quantum and external wave propagation do not coincide, otherwise resonances are impossible [29, 30, 36]:

\[
\theta_{\pm} = \angle (k_1, p_\pm) \ll 1, \quad \theta_{\mp} = \angle (p_\pm, p_\mp) \ll 1,
\]

\[
\theta_1 = \angle (k_1, k) \sim 1, \quad \theta_\pm = \angle (k_1, p_\pm) \sim 1. \quad (10)
\]

In this paper, we will consider the energies of the initial gamma quantum \( \omega \lesssim 10^3 \text{GeV} \) and also in a wide range of photon energies of an electromagnetic wave \( (1 \text{eV} \lesssim \omega \lesssim 10^4 \text{eV}) \). At the same time, we will consider the intensities of the electromagnetic wave significantly less than the critical intensities of the Schwinger \( (I \ll I_s \sim 10^{26} \text{Wcm}^{-2}) \).

We determine the resonant energy of the positron \( E_{\eta^+(r)} \) (for channel (A), see figure 1(A)) and electron \( E_{\eta^-(r)} \) (for channel (B), see figure 1(B)). We take into account the relations (10) in the resonant condition (7). After simple calculations, we get [36]

\[
x_{\eta(r)} = \frac{\varepsilon_{\eta(r)} - \delta_{\eta}^2}{2(\varepsilon_{\eta(r)} + \delta_{\eta}^2)}, \quad j = \pm. \quad (11)
\]

Here it is indicated:

\[
x_{\eta(r)} = \frac{F_{\eta^\pm(r)}}{\omega_j}, \quad \delta_{\eta} = \frac{\omega_1 \theta_{\pm}}{2m_\mp}. \quad (12)
\]

It can be seen from the expression (11) that there are restrictions on the values of the quantum parameter \( \varepsilon_{\eta(r)} \) and the outgoing angles of the positron (electron):

\[
\varepsilon_{\eta(r)} = r \varepsilon_{\eta} \gtrsim 1, \quad \delta_{\eta}^2 \lesssim \delta_{\max}^2 = \varepsilon_{\eta(r)} - 1. \quad (13)
\]

It is important to note that the resonant energy of the positron and electron is determined by the corresponding outgoing angle (ultrarelativistic parameter \( \delta_{\eta}^2 \) (12)), as well as the quantum parameter \( \varepsilon_{\eta(r)} \) (3). Note that the resonant energy spectrum (11) is essentially discrete, since each value of the number of absorbed laser photons corresponds to its resonant energy: \( r \to E_{\eta^\pm(r)} \) (11). Note that the first relation in expression (13), depending on the value of the quantum parameter \( \varepsilon_{\eta} \), can be represented as a condition for the number of the wave absorbed photons required for the resonant process:

\[
r \geq r_{\min} = \frac{1}{\varepsilon_{\eta}}, \quad \text{if} \quad \varepsilon_{\eta} < 1 \quad (\omega_i < \omega_\eta), \quad (14)
\]
the characteristic energy of the process decreases. The resonant process of photogeneration of pairs for optical laser frequencies in the region (14) was studied in detail in the article [36].

Note that the choice of specific numbers for wave intensities (see expression (16)) is connected with the choice of specific values of wave frequencies and parameter values $\eta$.

Here, within the framework of the relation (15), we will study the case very high energies of the initial gamma quanta (5). Because of this, for not very large outgoing angles, the resonant energies of the positron and electron (11) will take the form:

$$
x_{\eta_{\pm}(r)} \approx 1 - \left(1 + 4\delta^2_{\eta_{\pm}} / 4\varepsilon_{\eta(r)}^2\right) \approx 1, \left(\delta^2_{\eta_{\pm}} \ll \varepsilon_{\eta(r)}, \varepsilon_{\eta(r)} \gg 1\right).
$$

(17)

Here we have taken into account the maximum energy of the positron (electron). From here it can be seen that one of the possible values of the positron (electron) energies is close to the energy of the initial gamma quantum.

3. Maximum resonant cross section of the RPP in a strong field

It is important to note that the resonant energy of the electron–positron pair is determined for channel (A) by the positron outgoing angle, and for channel (B) by the electron outgoing angle. In addition, channels A and B of the resonant process do not interfere with each other. Because of this, the resonant differential cross section for channel (A) can be integrated at the electron outgoing angles, and for channel (B) at the positron outgoing angles. In the article [36], a general relativistic expression was obtained for the resonant differential cross-section of the RPP process in the field of a strong electromagnetic wave with intensities up to $10^{27}$ W cm$^{-2}$. Integrations were carried out on the energies of the electron (positron), as well as the outgoing angles of the electron (for channel (A)) or positron (for channel (B)), on which the resonant energy of the positron (channel (A)) or electron (channel (B)) does not depend. Moreover, the integration at the outgoing angles was carried out in a special kinematic region, in which small relativistic corrections of the order $(m_e / \omega)^2 \ll 1$ in the momentum transmitted to the nucleus are taken into account [32–38, 55]. It is this integration that leads to the appearance of a large order parameter $(\omega / m_e)^2 \gg 1$ in the resonant differential cross-section [32–38]. Also assume that the flux of initial gamma quanta is directed opposite to the direction of propagation of the electromagnetic wave. Taking this into account, the resonant differential cross section of the RPP with simultaneous registration of the energy and outgoing angles of the positron (sign ‘+’ in equation (18), channel (A)) or electron (sign ‘–’ in equation (18), channel (B)) can be represented as follows:

$$
R_{\eta_{\pm}(r)}^{\text{max}} = \frac{d\sigma_{\eta_{\pm}(r)}^{\text{max}}}{d\varepsilon_{\eta_{\pm}(r)} d\delta^2_{\eta_{\pm}}} = \left(Z^2 \alpha e^2 \right) c_{\eta_{\pm}} H_{\eta_{\pm}(r)}.
$$

(18)
Here $\alpha$ is the fine structure constant, $Z$ is the charge of the nucleus, $r_0$ is the classical radius of the electron, $H_{\eta \pm}(r)$ are functions that determine the energy spectrum and angular distribution of a positron or electron:

$$H_{\eta \pm}(r) = \frac{x_{\eta \pm}(r)}{\rho_{\eta \pm}(r)} \left(1 - x_{\eta \pm}(r)\right)^3 P(u_{\eta \pm}(r), \varepsilon_{\eta}(r)), \quad (19)$$

$$\rho_{\eta \pm}(r) = x_{\eta \pm}(r) \delta_{\eta \pm}^2 + \frac{1}{4(1 + \eta^2)}, \quad (20)$$

and the magnitude of the $c_{\eta}$ coefficient is determined by the small transmitted momenta of the order $(m_e/\omega)^2 \ll 1$, as well as the resonance width

$$c_{\eta} = 2 \left(\frac{2\pi \omega_L}{\alpha m_e K(\varepsilon_{\eta})}\right)^2 \lesssim 10^6 \left(\frac{\omega_L}{m_e}\right)^2 \gg 1. \quad (21)$$

Here the $K(\varepsilon_{\eta})$ function is determined by the resonant width (the full probability of the external field-stimulated Compton effect) and has the form [23, 36]:

$$K(\varepsilon_{\eta}) = \sum_{r=1}^{\infty} K_r(\varepsilon_{\eta}), \quad K_r(\varepsilon_{\eta}) = \int_{0}^{\varepsilon_{\eta}} \frac{du}{(1 + u)^2} K(u, \varepsilon_{\eta}(r)).$$

$$K(\varepsilon_{\eta}) = -4J_2^2(\gamma_{\eta}(r)) + \eta^2 \left(2 + \frac{u^2}{1 + u}\right) \left(J_{r+1}^2 + J_{r-1}^2 - 2J_r^2\right), \quad (22)$$

$$\gamma_{\eta}(r) = 2r - \frac{\eta}{\sqrt{1 + \eta^2}} \sqrt{\frac{u}{\varepsilon_{\eta}(r)} \left(1 - \frac{u}{\varepsilon_{\eta}(r)}\right)}.$$ \quad (24)

In expression (19) the $P(u_{\eta \pm}(r), \varepsilon_{\eta}(r))$ functions are determined by the probability (per unit of time) of the external field-stimulated Breit-Wheeler process [23, 36]:

$$P(u_{\eta \pm}(r), \varepsilon_{\eta}(r)) = J_r^2(\gamma_{\eta \pm}(r)) + \eta^2 \left(2u_{\eta \pm}(r) - 1\right) \times \left[\left(\frac{r^2}{\gamma_{\eta \pm}(r)} - 1\right) J_r^2 - \frac{1}{4}(J_{r+1} - J_{r-1})^2\right].$$ \quad (25)

Here, the relativistically invariant parameter $u_{\eta \pm}(r)$ and the arguments of the Bessel functions $\gamma_{\eta \pm}(r)$ have the form:

$$u_{\eta \pm}(r) \approx \frac{1}{4x_{\eta \pm}(r) \left(1 - x_{\eta \pm}(r)\right)}, \quad (26)$$

$$\gamma_{\eta \pm}(r) = 2r - \frac{\eta}{\sqrt{1 + \eta^2}} \sqrt{\frac{u_{\eta \pm}(r)}{\varepsilon_{\eta}(r)} \left(1 - \frac{u_{\eta \pm}(r)}{\varepsilon_{\eta}(r)}\right)} \times \frac{x_{\eta \pm}(r) \delta_{\eta \pm}}{\varepsilon_{\eta}(r)}, \quad (27)$$

$$\delta_{\eta \pm}(r) = \frac{u_{\eta \pm}(r)}{\varepsilon_{\eta}(r)} \left(1 - u_{\eta \pm}(r)/\varepsilon_{\eta}(r)\right), \quad (28)$$

$$\delta_{\eta \pm}(r) = \frac{1}{\sqrt{1 + \eta^2}} \sqrt{\frac{u_{\eta \pm}(r)}{\varepsilon_{\eta}(r)} \left(1 - \frac{u_{\eta \pm}(r)}{\varepsilon_{\eta}(r)}\right)} \times \frac{x_{\eta \pm}(r) \delta_{\eta \pm}}{\varepsilon_{\eta}(r)}.$$ \quad (29)

Note that the right part of the expression (27) for the argument of the Bessel functions is obtained by taking into account the relations (11) and (26).

These resonant differential cross sections (18) were studied in detail for the case when the energy of the initial gamma quanta did not exceed the characteristic energy of the process (14). However, the most interesting case when the energy of the initial gamma quanta exceeds the characteristic energy of the process (15) has not been studied. Here we consider the maximum resonant differential cross section (18) for the case when the energy of the initial gamma quanta exceeds the characteristic energy of the process (15). It is also of interest to consider the case (5) when the energy of the initial gamma quanta significantly exceeds the characteristic energy of the process $(\omega_i \gg \omega_f)$. In this case, the resonant energy of the positron or electron is close to the energy of the initial gamma quanta (see relation (17)). Given this, after simple transformations, the maximum resonant differential cross sections of the RPP (18)–(26) is significantly simplified and takes the form

$$R_{\eta \pm}(r) = \frac{1}{2} \delta_{\eta \pm}(r) \left[\begin{array}{c} \delta_{\eta \pm}^2 + \frac{1}{4(1 + \eta^2)} \end{array}\right]^{-2} \times P(\delta_{\eta \pm}^2, \varepsilon_{\eta}(r)), r \gg 1. \quad (29)$$

Here the $\Phi_{\eta \pm}(r)$ are functions that determine the spectral-angular distribution of the resonant SB cross-section for channels (A) and (B):

$$\Phi_{\eta \pm}(r) = \left(1 + 4\delta_{\eta \pm}^2\right)^3 \left[\begin{array}{c} \delta_{\eta \pm}^2 + \frac{1}{4(1 + \eta^2)} \end{array}\right]^{-2} \times P(\delta_{\eta \pm}^2, \varepsilon_{\eta}(r)), r \gg 1. \quad (29)$$

and $b_{\eta}$—the coefficient, which is determined by the parameters of the laser installation

$$b_{\eta} = \frac{1}{2\varepsilon_{\eta}} \left(\frac{\pi \omega_L}{2 \alpha m_e K(\varepsilon_{\eta})}\right)^2 \lesssim 10^4 \frac{1}{\varepsilon_{\eta}} \left(\frac{\omega_f}{m_e}\right)^2. \quad (30)$$

In expression (29) the function (25) takes the form:

$$P(\delta_{\eta \pm}^2, \varepsilon_{\eta}(r)) = J_r^2(\gamma_{\eta \pm}(r)) + \eta^2 \left[\begin{array}{c} \frac{\delta_{\eta \pm}^2}{1 + 4\delta_{\eta \pm}^2} - 1 \end{array}\right] \times \left[\begin{array}{c} \frac{r^2}{\gamma_{\eta \pm}(r)} - 1 \end{array}\right] J_r^2 + \frac{1}{4}(J_{r+1} - J_{r-1})^2, \quad (31)$$

$$\gamma_{\eta \pm}(r) = 4r - \frac{\eta}{\sqrt{1 + \eta^2}} \frac{\delta_{\eta \pm}}{\sqrt{1 + \eta^2}(1 + 4\delta_{\eta \pm}^2)}.$$ \quad (32)
order of magnitude \( d\sigma_0 \sim (Z^2\alpha r_0^2)\, d\delta \). Therefore, functions \( R_{\eta\varepsilon}(r) \) \((18)\) and \((28)\) with simultaneous registration of the outgoing angle and the energy of the positron (electron) actually determine the resonant differential cross section in units of the usual differential cross section without an external field.

### 4. Main results

Let the flux of initial gamma quanta propagate towards an external electromagnetic wave \((\theta_i = \pi)\). Let’s choose the energy of the initial gamma quanta \( \omega_i = 60\text{GeV} \). Then, for characteristic energies \( \omega_{\eta} \) \((16)\), the quantum parameter \( \varepsilon_{\eta} \) \((3)\) takes the corresponding values: \( \varepsilon_{\eta} = 0.115; 1.145; 11.453; 45.812; 229.057 \). Here, the first case corresponds to the optical frequencies of the laser \((\omega = 1\text{eV}, I = 1.863 \times 10^{18}\text{ Wcm}^{-2})\) and meets the condition \((14)\) when \( r \gg r_{\text{min}} \). The remaining cases for the x-ray frequencies of the external wave meet the condition \((15)\) when \( r \geq 1 \left( \omega \gg \omega_{\eta} \right) \). Moreover, the last three cases meet the condition \( \varepsilon_{\eta} \gg 1 \left( \omega \gg \omega_{\eta} \right) \) (see expression \((17)\)).

Note that when plotting the resonant differential cross-section \((18)\) and \((28)\), the maximum energy of the positron (electron) was selected (the ‘+’ sign before the square root in expression \((11)\)). It is these positron (electron) energies that make the main contribution to the resonant differential cross section.

Figure 3 shows the dependences of the maximum resonant differential cross section \((18)\) on the square of the positron (electron) outgoing angle for a fixed number of absorbed photons of the wave in the optical frequency range \((\omega = 1\text{eV}, I = 1.863 \times 10^{18}\text{ Wcm}^{-2})\) under conditions \((14)\) when \( r_{\text{min}} = 9 \). It is important to note that in this case, the maximum value of the resonant differential cross section takes place at the number of absorbed photons \( r = 13 \) and is the value \( R_{\eta\varepsilon}(r_{\text{max}}) \approx 10^{13} (Z^2\alpha r_0^2) \). At the same time, the resonant energies of the positron and electron are approximately equal to half the energy of the initial gamma quanta (see table 1).

Figure 4 shows the dependences of the maximum resonant differential cross section \((18)\) on the square of the positron (electron) outgoing angle for a fixed number of absorbed photons of the wave \( r = 1,2,3 \) in the x-ray frequency range \((\omega = 10\text{keV}, I = 1.863 \times 10^{20}\text{ Wcm}^{-2})\) under conditions \((15)\). At the same time, the energy of the initial gamma quanta slightly exceeds the characteristic energy of the process \((\varepsilon_{\eta} = 1.145)\). It is important to note that in this case, the maximum value of the resonant differential cross section occurs when one photon of the wave is absorbed at \( \delta_{\eta\varepsilon} = 0 \) and is of the order of magnitude \( R_{\eta\varepsilon}(r_{\text{max}}) \approx 10^{15} (Z^2\alpha r_0^2) \). With an increase in the number of absorbed photons of the wave, the peak of the maximum value of the resonant differential cross section shifts towards large outgoing angles of the positron (electron). At the same time, the resonant differential cross section decreases quite quickly. Thus, the ratio of the resonant cross sections for three and one absorbed photons of the wave has the order of magnitude: \( R_{\eta\varepsilon}(3)/R_{\eta\varepsilon}(1) \approx 10^{-2} \). Note also that the resonant energy of a positron (for channel (A)) or an electron (for channel (B)) increases from 40.685 GeV for \( r = 1 \) to 52.727 GeV for \( r = 3 \) with an increase in the number of absorbed photons of the wave (see table 2).

Figures 5–7 show the dependences of the maximum resonant differential cross-section \((18)\) and \((28)\) on the square of the positron (electron) outgoing angle for a fixed number of absorbed photons of the wave \( r = 1,2,3 \) for x-ray frequencies \( \omega = 0.1\text{keV}, 1\text{keV}, 10\text{keV} \) and corresponding wave intensities (see expression \((16)\)). These graphs are constructed under conditions when the energy of the initial gamma quanta significantly exceeds the characteristic energy of the process: \( \varepsilon_{\eta} \approx 11.45, 45.81, 229.06 \) (see the ratios \((15)\)–\((17)\)).

Table 3–5 show the outgoing angles, the resonant energies of the positron (electron), as well as the values of the resonant differential cross sections corresponding to the maxima of the distributions in graphs 5–7. From these figures and tables it can be seen that the maximum value of the resonant differential cross section is realized with one absorbed photon at \( \delta_{\eta\varepsilon} = 0 \). With an increase in the number of absorbed photons, as well as the intensity of the wave, the value of the maximum resonant differential cross section decreases.
So, for the wave intensities \( I = 1.863 \times 10^{22}, \ 7.452 \times 10^{24}, \ 1.675 \times 10^{27} \ \text{Wcm}^{-2} \), as well as for the number of absorbed photons \( r = 1 \) and \( r = 2 \) the value of the maximum resonant differential cross section, respectively, is equal to \( E_{\eta \pm(1)}^{\max} \approx 8.75 \times 10^{12}, \ 1.56 \times 10^{11}, \ 3.03 \times 10^{9} \ (Z\alpha r_{\pm}^{2}) \) and \( E_{\eta \pm(2)}^{\max} \approx 2.95 \times 10^{11}, \ 3.66 \times 10^{9}, \ 4.73 \times 10^{7} \ (Z\alpha r_{\pm}^{2}) \) (see tables 3–5). It is very important to note that with an increase in parameter \( \varepsilon_\eta \) (3), the value of the resonant energy of the positron (for channel (A)) or electron (for channel (B)) all it tends closer to the energy of the initial gamma quanta \( E_i = 60\ \text{GeV} \) (17). So, for the wave intensities \( I = 1.863 \times 10^{22}, \ 7.452 \times 10^{24}, \ 1.675 \times 10^{27} \ \text{Wcm}^{-2} \), as well as for the number of absorbed photons \( r = 1 \) and \( r = 2 \) the magnitude of the resonant energy of the positron (electron) at the maximum of the distribution of the resonant differential cross section (see figures 5–7), respectively, is equal to \( E_{\eta \pm(1)} \approx 58.660, \ 59.671, \ 59.934\ \text{GeV} \) and \( E_{\eta \pm(2)} \approx 59.173, \ 59.820, \ 59.965\ \text{GeV} \) (see tables 3–5).

Thus, under conditions (17), when the energy of gamma quanta significantly exceeds the characteristic energy of the process, it is very likely to obtain narrowly directed streams of positrons (electrons) with energies close to the energy of the initial gamma quanta.

Note that we are considering sufficiently large energies of the initial gamma quanta \( \omega_i = 60\ \text{GeV} \). The choice of large energies of gamma quanta is related to the considered limiting case of energies when \( \omega_i > \omega_{\eta i} \) and \( \omega_i \gg \omega_{\eta i} \). At the same time, the characteristic energy of the process for the considered frequencies and wave intensities is quite large (see expression (16)). Currently, obtaining gamma-ray beams of
The dependence of the maximum resonant differential cross-section on the square of the outgoing angle of the positron (for channel A) or electron (for channel B) for a fixed number of absorbed photons of the wave in the conditions (15) and (17). The curves are constructed for the maximum energy of the positron (electron) in the corresponding mathematical problem. The solution of the resonant problem in the field of a plane monochromatic wave nevertheless allows solving a number of important problems. First, to identify the main physical parameters of the problem (the characteristic energy of the process $\omega_\eta$ (4), the quantum parameters $\varepsilon_\eta$ and $\varepsilon_\eta(r)$ (3)), which determine the resonant energy of the electron–positron pair, as well as the magnitude of the resonant differential cross section. Secondly, it allowed us to obtain analytical expressions for the resonant differential scattering cross section. Note that all this is very important for the subsequent numerical analysis of the corresponding process in an inhomogeneous electromagnetic field.

5. Conclusions

The study of the possibility of generating narrow ultrarelativistic beams of positrons (electrons) in the process of resonant RPP on nuclei in a strong electromagnetic field showed:

- In this process there is a characteristic quantum parameter $\varepsilon_\eta$, the value of which significantly affects the resonant differential cross section and the energy of the electron–positron pair. This parameter $\varepsilon_\eta$ is equal to the ratio of the energy of the initial gamma quanta ($\omega_\eta$) to the characteristic energy of the process ($\omega_\eta(r)$). The characteristic energy is determined by the parameters of the laser installation: frequency, intensity, as well as the direction of propagation of the electromagnetic wave relative to the momentum of the initial gamma quanta (3) and (4).
• So, if the energy of the initial gamma quanta is less than the characteristic energy (ω γ < ω α), then ε α ≪ 1. In this case, there is a minimum number of photons (r ≫ r_{\min} = [ε^{-1}_α]), starting from which photons are absorbed by the wave in the external field-stimulated Breit-Wheeler process. In strong fields r_{\min} ≫ 1, the resonance process also proceeds with the absorption of a very large number of photons of the wave.

• If the energy of the initial gamma quanta is equal to or exceeds the characteristic energy of the process (ω γ ≳ ω α), then the quantum parameter ε α ≳ 1. In this case, the resonant process takes place for the number of absorbed photons of the wave r ≳ 11. Note that the probability of processes with the absorption of a small number of photons of the wave (r ≳ 1) significantly exceeds the corresponding probability with the absorption of a large number of photons of the wave (r ≫ 1).

• If the energy of the initial gamma quanta significantly exceeds the characteristic energy of the process (ε γ ≫ 1), then the resonant energy of the positron (for channel (A)) or electron (for channel (B)) will be close to the energy of gamma quanta (see relation (17)). Under these conditions, narrow streams of high-energy positrons (electrons) are generated with a very high probability. So, for the number of absorbed photons r = 1 and r = 2 the value of the maximum resonant differential cross section, respectively, is equal to R_{\max,n(1)} ≈ 8.75 × 10^{12}, 1.56 × 10^{11}, 3.03 × 10^{10} (Z^2\alpha r_0^2) and R_{\max,n(2)} ≈ 2.95 × 10^{11}, 3.66 × 10^{10}, 4.73 × 10^{9} (Z^2\alpha r_0^2) (see tables 3–5).

The obtained results can be used to create narrow beams of high-energy positrons (electrons) on modern laser modern installations. In addition, these results can explain the high-energy fluxes of positrons near pulsars and magnetars.

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