Proving Security Goals With Shape Analysis Sentences

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Abstract

The paper that introduced shape analysis sentences presented a method for extracting a sentence in first-order logic that completely characterizes a run of CPSA. Logical deduction can then be used to determine if a security goal is satisfied.

This paper presents a method for importing shape analysis sentences into a proof assistant on top of a detailed theory of strand spaces. The result is a semantically rich environment in which the validity of a security goal can be determined using shape analysis sentences and the foundation on which they are based.
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1 Introduction

A central problem in cryptographic protocol analysis is to determine whether a formula that expresses a security goal about behaviors compatible with a protocol is true. Following [7], a security goal is a quantified implication:

$$\forall \vec{x}. \Phi_0 \supset \bigvee_{1 \leq i \leq n} \exists \vec{y}_i. \Phi_i. \tag{1}$$

The hypothesis $\Phi_0$ is a conjunction of atomic formulas describing regular (honest) behavior. Each disjunct $\Phi_i$ that makes up the conclusion is also a conjunction of atomic formulas. When $\Phi_i$ describes desired behaviors of other regular participants, then the formula is an authentication goal. The goal says that each run of the protocol compatible with $\Phi_0$ will include the regular behavior described by one of the disjuncts. When $n = 0$, the goal’s conclusion is false. In this case, if $\Phi_0$ mentions an unwanted disclosure, Eq. 1 says the disclosure cannot occur, thus a security goal with $n = 0$ expresses a secrecy goal.

Guttman [7] presented a model-theoretic approach to establishing security goals in the context of strand space theory. In that setting, a skeleton describes regular behaviors compatible with a protocol. For skeleton $k$ and formula $\Phi$, he defined $k, \alpha \models \Phi$ to mean that the conjunction of atomic formulas that make up $\Phi$ is satisfied in $k$ with variable assignment $\alpha$.

A realized skeleton is one that includes enough regular behavior to specify all the non-adversarial part of an execution of the protocol. In a realized skeleton, its message transmissions combined with possible adversarial behavior explain every message reception in the skeleton.

In strand space theory, a homomorphism is a structure-preserving map $\delta$ that shows how the behaviors in one skeleton are reflected within another. As skeletons serve as models, homomorphisms preserve satisfaction for conjunctions of atomic formulas.

The Cryptographic Protocol Shapes Analyzer (cpsa) constructs homomorphisms from a skeleton $k_0$ to realized skeletons [14]. If cpsa terminates, it generates a set of realized skeletons $k_i$ and a set of homomorphisms $\delta_i : k_0 \mapsto k_i$. These realized skeletons are all the minimal, essentially different skeletons that are homomorphic images of $k_0$ and are called the shapes of the analysis.

Ramsdell [13] described cpsa’s support for security goals. cpsa includes a tool that extracts a sentence that characterizes a shape analysis. This so
called shape analysis sentence is special in that it encodes everything that can be learned from the shape analysis.

Given a shape analysis sentence, a security goal is achieved if the goal can be deduced from the sentence. CPSA includes a Prolog program that translates shape analysis sentences into Prover9 [11] syntax. Typically, a goal that is a theorem is quickly proved by Prover9.

There is another advantage to this approach. It can be tedious to generate security goals. Realistic ones can be large and complicated. An easy way to create one is to modify a shape analysis sentence. This typically involves deleting parts of the conclusion.

There is a disadvantage to this approach. When a goal cannot be deduced from a shape analysis sentence, one cannot conclude that there is a counterexample. It could be simply that the sentence is not relevant to the security goal. It could also be that a proof of the goal depends on a fact not exposed by a shape analysis sentence. For example, the precedes relation on nodes in a skeleton is transitive, but that fact is not available to Prover9.

This paper describes the method that was used to import shape analysis sentences into the proof assistant PVS [12] on top of a detailed theory of strand spaces specified in PVS. In this environment, if the proof of a security goal depends on the transitivity of the precedes relation, that fact is available as a lemma. Furthermore, if a security goal is false, one can construct a counterexample and use it to prove the security goal is in fact false.

In CPSA, executions of protocols are represented by skeletons. Associated with each skeleton is a free message algebra generated by a finite set of variables. Skeletons are used as models in the original paper on shape analysis sentences.

The PVS strand space theory uses bundles over an initial algebra as its representation of executions of protocols. This allows for a shallow embedding of strand space theory in which algebra variables are replaced by logical variables in PVS. This specification choice alleviates the need to manipulate homomorphisms within PVS. Section 3 contains two descriptions that relate skeletons to bundles.

Motivating Example. The running example used throughout this paper is now presented. An informal version of the example is presented here, and the example with all of the details filled in is in Section 4.
The following simple example protocol is due to Bruno Blanchet [2].

\[ A \rightarrow B : \{\{s\}_{a^{-1}}\}_b \]
\[ B \rightarrow A : \{d\}_s \]

Alice (A) freshly generates symmetric key \( s \), signs the symmetric key with her private uncompromised asymmetric key \( a^{-1} \) and intends to encrypt it with Bob’s (B) uncompromised asymmetric key \( b \). Alice expects to receive data \( d \) encrypted, such that only Alice and Bob have access to it.

The protocol was constructed with a known flaw for expository purposes, and as a result the secret is exposed due to an authentication failure. The protocol does not prevent Alice from using a compromised key \( b' \), so that Mallory (M) and Eve (E) can perform this man-in-the-middle attack:

\[ A \rightarrow M : \{\{s\}_{a^{-1}}\}_b' \]
\[ M \rightarrow B : \{\{s\}_{a^{-1}}\}_b \]
\[ B \rightarrow E : \{d\}_s \]

The protocol fails to provide a means for Bob to ensure the original message was encrypted using his key. The authentication failure is avoided with this variation of the protocol:

\[ A \rightarrow B : \{\{s, b\}_{a^{-1}}\}_b \]
\[ B \rightarrow A : \{d\}_s \] (2)

In strand space theory, a *strand* is a linearly ordered sequence of events \( e_0 \Rightarrow \cdots \Rightarrow e_{n-1} \), and an *event* is either a message transmission \( \bullet \rightarrow \) or a reception \( \bullet \leftarrow \). In CPSA, adversarial behavior is not explicitly represented, so strands always represent regular behavior.

Regular behavior is constrained by a set of roles that make up the protocol. In this protocol, Alice’s behaviors must be compatible with an initiator role, and Bob’s behaviors follow a responder role.

```
init

\[ \bullet \rightarrow \{\{s\}_{a^{-1}}\}_b \quad \{\{s\}_{a^{-1}}\}_b \rightarrow \bullet \]

\[ \bullet \leftarrow \{d\}_s \quad \{d\}_s \leftarrow \bullet \] (3)
```

The important authentication goal from Bob’s perspective is that if an instance of a responder role runs to completion, there must have been an
instance of the initiator role that transmitted its first message. Furthermore, assuming the symmetric key is freshly generated, and the private keys are uncompromised, the two strands agree on keys used for signing and encryption.

A CPSA analysis of the authentication goal requires two inputs, a specification of the roles that make up the protocol, as in Eq. 3, and a question about runs of the protocol. The question in this case is the hypothesis of Eq. 4, that an instance of the responder role ran to completion. In these diagrams, a strand instantiated from a role is distinguished from a role by placing messages above communication arrows, and $\succ$ is used to assert an event occurred after another.

\[
\text{resp} \vdash \{\{s\}_{a-1}\}_{b} \implies \text{init} \vdash \{\{s\}_{a-1}\}_{b'}
\]

CPSA produces the conclusion in Eq. 4, that an instance of the initiator role must have transmitted its first message, but it does not conclude that the strands agree on the key used for the outer encryption. When CPSA is run using the amended protocol in Eq. 2, the strands agree on the key, and the authentication goal is achieved.

The contribution of this paper is a method of importing security goals and the results of a CPSA analysis into PVS such that proofs about the goals can rely on a detailed theory of strand spaces. The shape analysis sentence associated with this example is presented in Section 4.

**Some Related Work.** This paper is the result of implementing security goals as described by Guttman in [7]. The original motivation for extracting shape analysis sentences rather than following the procedure in [7] was ease of implementation. With shape analysis sentences, most of the work is performed by a post-processing stage, and there were only a few changes made to the core CPSA program. Only later was it realized the sense in which shape analysis sentences completely characterize a shape analysis.

The Scyther tool [3] integrates security goal verification with its core protocol analysis algorithm. Security goals are easy to state as long as they can be expressed using a predefined vocabulary, however, there is no sense in which Scyther goals characterize an analysis.
The Protocol Composition Logic [4] provides a contrasting approach to specifying security goals. It extends strand spaces by adding an operational semantics as a small set of reduction rules, and a run of a protocol is a sequence of reduction steps derived from an initial configuration. The logic is a temporal logic interpreted over runs.

Structure of this Paper. Section 2 describes strand spaces as formalized in pvs, Section 3 reintroduces shape analysis sentences, and Section 4 displays the example above in full detail. Appendix A describes an extension that can be used to prove security goals that involve long-term state.

Notation. A finite sequence is a function from an initial segment of the natural numbers. The length of a sequence $X$ is $|X|$, and sequence $X = (X(0), \ldots, X(n-1))$ for $n = |X|$. If $S$ is a set, then $S^*$ is the set of finite sequences over $S$, and $S^+$ is the non-empty finite sequences over $S$. The prefix of sequence $X$ of length $n$ is $X\mid n$.

2 Strand Spaces

PVS is based on classical, typed higher-order logic. It has dependent types and parameterized theories.

This section describes the PVS definition of strand spaces [15] in a style motivated by the PVS language [12], that is, the presentation attempts to minimize the gap between the actual proofs and this content.

Message Algebra. An order-sorted algebra [6] is a generalization of a many-sorted algebra in which sorts may be partially ordered. The carrier sets associated with ordered sorts are related by the subset relation.

Figure 1 shows the simplification of the CPSA message algebra signature used by the examples in this paper. Sort $\top$ is the sort of all messages. Messages of sort $A$ (asymmetric keys), sort $S$ (symmetric keys), and sort $D$ (data) are called atoms. Messages are atoms or constructed using encryption $\{\cdot\}_{\cdot}$ and pairing $(\cdot, \cdot)$, where the comma operation is right associative and parentheses are omitted when the context permits.

The message algebra $\mathfrak{Z}$ is the initial quotient term algebra over the signature. The canonical representative for each message is the term that contains
no occurrences of the inverse operation \((\cdot)^{-1}\). The set of messages associated with a sort is called its \textit{carrier set}. The set of message algebra atoms is \(\mathcal{B}\).

A message \(t_0\) is \textit{carried by} \(t_1\), written \(t_0 \sqsubseteq t_1\) if \(t_0\) can be extracted from a reception of \(t_1\), assuming plaintext is extractable from encryptions. In other words, \(\sqsubseteq\) is the smallest reflexive, transitive relation such that \(t_0 \sqsubseteq t_0\), \(t_0 \sqsubseteq (t_0, t_1)\), \(t_1 \sqsubseteq (t_0, t_1)\), and \(t_0 \sqsubseteq \{\|t_0\|\}_{t_1}\).

**Strand Spaces.** A run of a protocol is viewed as an exchange of messages by a finite set of local sessions of the protocol. Each local session is called a \textit{strand}. The behavior of a strand, its \textit{trace}, is a finite non-empty sequence of messaging events. An \textit{event} is either a message transmission or a reception. Outbound message \(t \in \mathcal{A}\) is written as \(+t\), and inbound message \(t\) is written as \(-t\). The set of traces over \(\mathcal{A}\) is \(\mathcal{C} = (\pm \mathcal{A})^+\). A message \textit{originates} in trace \(C\) at index \(i\) if it is carried by \(C(i)\), \(C(i)\) is outbound, and it is not carried by any event earlier in the trace.

A \textit{strand space} \(\Theta\) over algebra \(\mathcal{A}\) is a finite non-empty sequence of traces in \(\mathcal{C}\). A strand \(s\) is a member of the domain of \(\Theta\), and its trace is \(\Theta(s)\). An atom \(t\) is \textit{non-originating} in a strand space \(\Theta\), written \(\text{non}(\Theta, t)\), if it originates on no strand.

Message events occur at nodes in a strand space. For each strand \(s\), there is a node for every event in \(\Theta(s)\). The \textit{nodes} of strand space \(\Theta\) are \(\{(s, i) \mid s \in \text{Dom}(\Theta), 0 \leq i < |\Theta(s)|\}\), and the event at a node is \(\text{evt}_{\Theta}(s, i) = \Theta(s)(i)\). A node names an event in a strand space. The relation \(\Rightarrow\) defined by \(\{(s, i) -
1) ⇒ (s, i) | s ∈ Dom(Θ), 1 ≤ i < |Θ(s)|} is called the strand succession relation. An atom t uniquely originates in a strand space Θ at node n, written uniq(Θ, t, n), if it originates in the trace of exactly one strand s at index i, and n = (s, i).

**Bundles.** The pair Υ = (Θ, →) is a bundle if it defines a directed acyclic graph, where the vertices are the nodes of Θ, and an edge represents communication (→) or strand succession (⇒) in Θ. For communication, if n₀ → n₁, then there is a message t such that Θ₀ (n₀) = +t and Θ₀ (n₁) = −t. For each reception node n₁, there is a unique transmission node n₀ with n₀ → n₁.

Each acyclic graph has a transitive irreflexive relation \( \prec \) on its vertices. The relation specifies the causal ordering of nodes in a bundle. A transitive irreflexive binary relation is also called a strict order.

**Runs of Protocols.** In a run of a protocol, the behavior of each strand is constrained by a role in a protocol. Adversarial strands are constrained by roles as are non-adversarial strands. A role is a set of role items of the form \( r(C, N, U) \), where \( C ∈ C, N ∈ P(Β)⁺, U ∈ P(Β)⁺ \), and the lengths of \( C, N, \) and \( U \) agree. The trace of the role item is \( C \), its non-origination assumptions are \( N \), and its unique origination assumptions are \( U \). A strand is an instance of a role item in a strand space, written \( inst(Θ, s, r(C, N, U)) \), if for \( h = |Θ(s)| \),

1. \( h \leq |C| \),
2. \( C | h = Θ(s) \),
3. for all \( i < h, t ∈ N(i) \) implies non(Θ, t), and
4. for all \( i < h, t ∈ U(i) \) implies uniq(Θ, t, (s, i)).

A protocol is a set of roles. A bundle \( Υ = (Θ, →) \) is a run of protocol \( P \) if there is a role assignment \( rl : Dom(Θ) → P \) such that for each \( s ∈ Dom(Θ) \), there exists \( r(C, N, U) ∈ rl(s) \) such that \( inst(Θ, s, r(C, N, U)) \). Let \( R_P \) be the set of bundles that are runs of protocol \( P \).

The description of roles differs from most presentations. Role origination assumptions usually are specified by a set of atoms, instead of a sequence of sets of atoms. The PVS theory follows the technique used in the CPSA implementation. A sequence is used so as to make explicit the length of the
create \((t \in \mathcal{B}) = \langle +t \rangle \)

pair \((t_0 : \top, t_1 : \top) = \langle -t_0, -t_1, +(t_0, t_1) \rangle \)

sep \((t_0 : \top, t_1 : \top) = \langle -(t_0, t_1), +t_0, +t_1 \rangle \)

enc \((t : \top, k : A|S) = \langle -t, -k, +\|t\|_k \rangle \)

dec \((t : \top, k : A|S) = \langle -\|t\|_k, -k^{-1}, +t \rangle \)

Figure 2: Adversary Traces

instance of a role at which each origination assumption applies. Furthermore, roles are normally described as templates to be copied and refined, rather than as sets of role items. This difference will be addressed in the next section.

Adversary Model. The traces of the roles that constrain adversarial behavior are in Figure 2. For the encryption related traces, \(k : A|S\) asserts that \(k : A\) or \(k : S\). There are no origination assumptions in the adversary’s roles.

The parameter of the create role is restricted to atoms. In fact, the defining characteristic of an atom is it denotes the set of messages the adversary can create out of thin air modulo origination assumptions.

3 Importing Protocol Analyses

Unlike the pvs theories, cpsa does not use bundles as its representation of runs of a protocol. Instead, it uses abstract interpretation to discuss sets of bundles using an object called a skeleton.

Skeletons. Skeletons and bundles share the same signature, but their algebras differ. Rather than using the initial algebra, each skeleton has a free algebra generated from a finite set of variables. Subscripting is used to indicate when a free algebra is in use. Thus, if \(X\) is a set of variables along with their sorts, then \(\Theta_X\) is a strand space over the free algebra generated by \(X, \mathcal{A}_X\).

The treatment of roles is slightly different in cpsa. The pvs theories define a role as a set of role items as described earlier. In cpsa, a role is a template that is instantiated to produce the equivalent of a role item via an algebra homomorphism \(\sigma\). Thus for cpsa role \(r = r(C_X, N_X, U_X)\), the related
role item-like object is \( r(\sigma \circ C_X, \sigma \circ N_X, \sigma \circ U_X) \), which by abuse of notation, we write as \( \sigma(r) \). A PVS role is *template inspired* by \( r = r(C_X, N_X, U_X) \) if it is of the form \( \{ \sigma(r) \mid \sigma \in \mathcal{A}_X \rightarrow \mathcal{A} \} \).

Associated with each skeleton is protocol \( P \) as a set of roles in template form, and a strand space \( \Theta_X \). In cpsa syntax, the trace and role associated with a strand is specified by an *instance*. An instance is of the form \( i(r, h, \sigma) \), where \( r \in P \) is a role, \( h \) specifies the length of a trace instantiated from the role, and \( \sigma \) specifies how to instantiate the variables in the role to obtain the trace. Thus the trace in \( C_X \) associated with \( i(r(C_Y, U_Y, N_Y), h, \sigma) \) is \( \sigma \circ C_Y \upharpoonright h \), the prefix of length \( h \) that results from applying \( \sigma \) to \( C_Y \), where \( \sigma \) is a homomorphism from \( \mathcal{A}_Y \) to \( \mathcal{A}_X \).

A *skeleton* has the form \( k(P, I_X, \prec, N_X, U_X) \), where \( P \) is the protocol, \( I_X \) is an instance map, \( \prec \) is a strict node ordering, \( N_X \) is a set of atoms assumed to be non-originating, and \( U_X \) is a set of atoms assumed to be uniquely originating. The instance map \( I_X \) is a finite non-empty sequence of instances, where the range of the homomorphism associated with each instance is \( \mathcal{A}_X \).

The strand space associated with a skeleton is defined by its instance map. When \( I_X(s) = i(r(C_Y, U_Y, N_Y), h, \sigma) \), trace \( \Theta_X(s) = \sigma \circ C_Y \upharpoonright h \). We write \( k(P, I_X, \prec, N_X, U_X) \) as \( k_X(P, I, \prec, N, U) \) in what follows.

**Homomorphisms.** Let \( k_0 = k_X(P, I_0, \prec_0, N_0, U_0) \) and \( k_1 = k_Y(P, I_1, \prec_1, N_1, U_1) \) be skeletons, and let \( \Theta_0 \) and \( \Theta_1 \) be the strand spaces associated with \( I_0 \) and \( I_1 \). There is a skeleton homomorphism \( (\phi, \sigma): k_0 \mapsto k_1 \) if \( \phi \) and \( \sigma \) are maps with the following properties:

1. \( \phi \) maps strands of \( k_0 \) into those of \( k_1 \), and nodes as \( \phi((s, i)) = (\phi(s), i) \), that is \( \phi \) is in \( \text{Dom}(\Theta_0) \rightarrow \text{Dom}(\Theta_1) \);
2. \( \sigma \in \mathcal{A}_X \rightarrow \mathcal{A}_Y \) is a message algebra homomorphism;
3. \( n \in \text{nodes}(\Theta_0) \) implies \( \sigma(\text{evt}_{\Theta_0}(n)) = \text{evt}_{\Theta_1}(\phi(n)) \);
4. \( n_0 \prec_0 n_1 \) implies \( \phi(n_0) \prec_1 \phi(n_1) \);
5. \( \sigma(N_0) \subseteq N_1 \);
6. \( t \in U_0 \) implies \( \sigma(t) \in U_1 \) and \( \phi(O_{k_0}(t)) = O_{k_1}(\sigma(t)) \);
where $O_k(t)$ is the node of the event at which $t$ originates. Property 6 says the node at which an atom uniquely originates is preserved by homomorphisms.

The definition of a skeleton homomorphism can be extended so that a bundle can be in the range. In this case, the range of the message algebra homomorphism is the initial algebra $\mathfrak{A}$. Property 5 and 6 require small tweaks: for non-origination, $t \in N_0$ implies $\text{non}(\Theta_1, \sigma(t))$, and for unique origination, $t \in U_0$ implies $\text{uniq}(\Theta_1, \sigma(t), \phi(O_k_0(t)))$. Notice that a homomorphism between skeletons preserves the protocol. For the case of a bundle in the range, we require that it be a run of the protocol of the skeleton. Let $pt(k)$ be $P$, the protocol of $k$, so that the final condition can be written as $\Upsilon \in R_{pt(k)}$. The bundles associated with skeleton $k$ are $\{\Upsilon \mid \exists \delta. \delta : k \mapsto \Upsilon\}$.

When given a point-of-view skeleton $k_0$, if CPSA terminates, it produces a shape analysis of the form $\delta_i : k_0 \mapsto k_i$. The skeletons $k_i$ are the shapes of this protocol analysis, and they specify all of the non-adversarial behavior associated with a run compatible with the point-of-view skeleton. The shape analysis is complete if for all $\Upsilon$ and $\delta$, $\delta : k_0 \mapsto \Upsilon$ iff $\exists i, \delta' : k_i \mapsto \Upsilon$. See [10] for a proof of CPSA’s completeness.

**Shape Analysis Sentences.** The results of a shape analysis are imported into PVS by translating the analysis into a sentence that is asserted as an axiom in PVS, justified by the fact that the shape analysis is complete. The translation is similar to the one appearing in [13], however this one is superior due to the foundation provided by the bundle-based strand space theory presented earlier. Much of the translation is simply valid by definition. Pay particular attention to the translation of instances.

We define $K_\Upsilon(k) = (Y, \Phi)$, where $\Phi$ is $k$’s skeleton formula, and $Y$ is the formula’s set of variables along with their sorts. Let $k = k_X(P, I, \prec, N, U)$. The set $Y$ is $X$ augmented with a fresh variable $z_s$ for each strand $s \in \text{Dom}(I)$. In formulas, $z_s$ ranges over $\text{Dom}(\Theta)$, where $\Theta$ is the strand space of $\Upsilon$. The formula $\Phi$ is a conjunction of atomic formulas composed as follows.

- For each $s \in \text{Dom}(I)$, assert $\text{htin}(\Theta, z_s, h, \sigma(r))$, where $I(s) = i(r, h, \sigma)$, and $\text{htin}(\Theta, z_s, h, r) = h \leq |\Theta(s)| \land \text{inst}(\Theta, z_s, r)$.
- For each $(s, i) \prec (s', i')$, assert $(z_s, i) \prec_{\Upsilon} (z_{s'}, i')$.
- For each $t \in N$, assert $\text{non}(\Theta, t)$.
For each $t \in U$, assert $\text{uniq}(\Theta, t, (z_s, i))$, where $(s, i) = \mathcal{O}_k(t)$.

When $\mathcal{K}_\Sigma(k) = (X, \Phi)$, the predicate $\Sigma_k = \lambda \forall \mathcal{Y}. \mathcal{Y} \in \mathcal{R}_{pt(k)} \land \exists X. \Phi$ is closed. (In what follows, $X$ will refer to the set of algebra variables augmented with strand variables.) The bundle $\mathcal{Y}$ is a pair $(\Theta, \rightarrow)$, so the strand space $\Theta$ is the first element of the pair, and $\preceq_\mathcal{Y}$ is derived from the communication edges $\rightarrow$ and the strand succession edges in $\Theta$.

The formula describing a skeleton is order-sorted. A truth assignment that tells one how to interpret each skeleton formula must account for this fact. As such, the domain of discourse for interpretation $I(\mathcal{Y})$ contains the carrier set for each sort in the initial message algebra. Additionally, for $\mathcal{Y} = (\Theta, \rightarrow)$, the domain of discourse includes the set $\text{Dom}(\Theta)$, used to interpret strand variables $z_s$. The interpretation of predicates and function symbols follows the case of a many-sorted algebra [5, Section 4.3]. See [6, Section 4] for a description of the reduction of an order-sorted algebra to a many-sorted algebra.

**Theorem 1.** Let $\mathcal{K}_\Sigma(k) = (X, \Phi)$ and $\Sigma_k = \lambda \forall \mathcal{Y}. \mathcal{Y} \in \mathcal{R}_{pt(k)} \land \exists X. \Phi$. For all bundles $\mathcal{Y}$, $\Sigma_k(\mathcal{Y})$ iff there is a homomorphism from $k$ to $\mathcal{Y}$, i.e.

$$\Sigma_k(\mathcal{Y}) \iff \exists \delta. \delta : k \mapsto \mathcal{Y}.$$ 

Thus $\{ \mathcal{Y} \mid \Sigma_k(\mathcal{Y}) \}$ is another way to specify the bundles associated with skeleton $k$.

The intuition behind this proof is the observation that there is an intimate relationship between the homomorphism and the variable assignment used to interpret existentially quantified variables.

**Proof.** Consider the backward implication first. We are given $k = k(P, I, \prec, N, U)$, $\delta = (\phi, \sigma)$, and $\mathcal{Y} = (\Theta, \rightarrow)$ such that $\delta : k \mapsto \mathcal{Y}$. To interpret formula $\Phi$, construct the variable assignment $\alpha$ as follows. For each strand variable $z_s$, $\alpha(z_s) = \phi(s)$. Each algebra variable $x$ has a corresponding logical variable, so $\alpha(x) = \sigma(x)$.

The interpretation $I(\mathcal{Y})$ satisfies $\Phi$ with $\alpha$ if each conjunct does so. For some $s \in \text{Dom}(I)$, consider the atomic formula $\text{htin}(\Theta, z_s, h, \sigma'(r))$, where $I(s) = i(r, h, \sigma')$. Its interpretation is $\text{htin}(\Theta, \alpha(z_s), h, \alpha(\sigma'(r)))$ which is $\text{htin}(\Theta, \phi(s), h, \sigma(\sigma'(r)))$. By definition, $\text{htin}(\Theta, \phi(s), h, \sigma(\sigma'(r))) = h \leq |\Theta(\phi(s))| \land \text{inst}(\Theta, \phi(s), \sigma(\sigma'(r)))$. By Property 1 in the definition of a homomorphism, the length of strand $\phi(s)$ must be greater than or equal
to \( h \). Let \( r = r(C_Y, U_Y, N_Y) \). Recall that \( \text{inst}(\Theta, \phi(s), \sigma'(r)) \) implies that \( \sigma \circ \sigma' \circ C_y \mid h' = \Theta(\phi(s)) \), where \( h' = |\Theta(\phi(s))| \), which is true by Property 3.

For the \( \prec_Y \) predicate, Property 4 in the definition of homomorphism applies, for non, it’s the tweak of Property 5, and for uniq, it’s the tweak of Property 6.

Now consider the forward implication in the theorem. In this case, we are given the variable assignment \( \alpha \) such that \( \mathcal{I}(\Upsilon) \) satisfies \( \Phi \) with \( \alpha \) and must construct the corresponding homomorphism. For each strand variable \( z_s \), \( \phi(s) = \alpha(z_s) \). Each algebra variable \( x \) has a corresponding logical variable, so \( \sigma(x) = \alpha(x) \).

With this definition of \( \delta \), we show that \( \delta : k \mapsto \Upsilon \). Substitution \( \sigma \) is a message algebra homomorphism, thus demonstrating Property 2.

For all \( s \in \text{Dom}(I) \), assume \( \mathcal{I}(\Upsilon) \) satisfies \( \text{htin}(\Theta, z_s, h, \sigma'(r)) \) with \( \alpha \), where \( I(s) = i(r, h, \sigma') \). Therefore, \( \text{htin}(\Theta, \alpha(z_s), h, \alpha(\sigma'(r))) \) is true, and so is \( \text{htin}(\Theta, \phi(s), h, \sigma(\sigma'(r))) \) and by definition \( h \leq |\Theta(\phi(s))| \) and \( \text{inst}(\Theta, \phi(s), \sigma(\sigma'(r))) \). The height restriction \( h \leq |\Theta(\phi(s))| \) ensures \( \phi \) maps correctly as prescribed in Property 1. Consider node \( n = (s, i) \) in \( k \). The event in \( k \) at \( n \) is \( \sigma'(C_Y(i)) \) where \( r = r(C_Y, U_Y, N_Y) \). The \( \text{inst} \) assertion implies that event \( \text{evt}_\Theta(\phi(n)) \) is \( \sigma'(C_Y(i)) \), thus demonstrating Property 3.

Property 4, 5, and 6 are straightforward.

In what follows, a sentence that universally quantifies a bundle, as in \( \forall \Upsilon. \Phi \), is true if for all \( \Upsilon \), \( \mathcal{I}(\Upsilon) \) models \( \Phi \). Define \( \models_{\mathcal{I}(\Upsilon)} \Phi \) to mean \( \mathcal{I}(\Upsilon) \) models \( \Phi \), and \( \models_{\mathcal{I}(\Upsilon)} \Phi \) with \( \alpha \) to mean \( \mathcal{I}(\Upsilon) \) satisfies \( \Phi \) with variable assignment \( \alpha \).

Given a set of homomorphisms \( \delta_i : k_0 \mapsto k_i \), its shape analysis sentence \( \mathcal{S}(\delta_i : k_0 \mapsto k_i) \) is

\[
\forall \Upsilon \in \mathcal{R}_{pt(k_0)}, X_0. \Phi_0 \iff \bigvee_i \exists X_i. \Delta_i \land \Phi_i,
\]

(5)

where \( \mathcal{K}_\Upsilon(k_0) = (X_0, \Phi_0) \). The same procedure produces \( X_i \) and \( \Phi_i \) for shape \( k_i \) with one proviso—the variables in \( X_i \) that also occur in \( X_0 \) must be renamed to avoid trouble while encoding the structure preserving maps \( \delta_i \).

The structure preserving maps \( \delta_i = (\phi_i, \sigma_i) \) are encoded in \( \Delta_i \) by a conjunction of equalities. Map \( \sigma_i \) is coded as equalities between a message algebra variable in the domain of \( \sigma_i \) and the term it maps to. Map \( \phi_i \) is coded as equalities between strand variables in \( \Phi_0 \) and strand variables in \( \Phi_i \). Let \( Z_0 \) be the sequence of strand variables freshly generated for \( k_0 \), and \( Z_i \) be the
The relationship between skeletons and bundles is not as tidy as previously described. \textsc{cpsa} supports something called listener strands that do not appear in bundles. A listener strand in a skeleton is an artificial strand used to assert that some message is available to the adversary. A

![Homomorphism Diagram](image)

**Figure 3: Homomorphism Diagram**

Theorem 2. If $\delta_i : k_0 \mapsto k_i$ is a complete shape analysis then $S(\delta_i : k_0 \mapsto k_i)$ is true.

Proof. We show for all bundles $\Upsilon \in \mathcal{R}_{pt(k_0)}$, $\models_{I(\Upsilon)} \forall x_0. \Phi_0 \iff \bigvee_i \exists x_i. \Delta_i \land \Phi_i$, which reduces to showing $\models_{I(\Upsilon)} \Phi_0 \iff \bigvee_i \exists x_i. \Delta_i \land \Phi_i$ with $\alpha$ for all variable assignments $\alpha$ for $x_0$. Take cases on the truth of $\models_{I(\Upsilon)} \Phi_0$ with $\alpha$.

When true, by the proof of Theorem 1, $\alpha$ specifies the homomorphism $\delta'_0 : k_0 \mapsto \Upsilon$. Because the shape analysis is complete, for some $i$, $\delta'_i : k_i \mapsto \Upsilon$. By Theorem 1, $\models_{I(\Upsilon)} \exists x_i. \Phi_i$ and therefore $\models_{I(\Upsilon)} \Phi_i$ with $\alpha_i$, where $\alpha_i$ is the variable assignment derived from $\delta'_i$. Let $\alpha \oplus \alpha_i$ be the union of the mappings in $\alpha$ and $\alpha_i$ (the domains of $\alpha$ and $\alpha_i$ are disjoint). The proof of this case is complete when we show $\models_{I(\Upsilon)} \Delta_i$ with $\alpha \oplus \alpha_i$. Recall that $\delta_i : k_0 \mapsto k_i$ and let $\delta_i = (\phi_i, \sigma_i)$. See Figure 3 and note that $\delta'_0 = \delta'_i \circ \delta_i$. For each variable $x$ in the domain of $\sigma_i$, $\Delta_i$ contains the equation $x = \sigma_i(x)$. Its interpretation is $\alpha(x) = \alpha_i(\sigma_i(x))$. In other words, $\sigma'_0(x) = \sigma'_i(\sigma_i(x))$, because $\sigma'_0 = \sigma'_i \circ \sigma_i$. For each strand $j$ in $k_0$, $\Delta_i$ contains the equation $Z_0(j) = Z_i(\phi_i(j))$. Its interpretation is $\alpha(Z_0(j)) = \alpha_i(Z_i(\phi_i(j)))$. In other words, $\phi'_0(j) = \phi'_i(\phi_i(j))$, because $\phi'_0 = \phi'_i \circ \phi_i$.

When $\not\models_{I(\Upsilon)} \Phi_0$ with $\alpha$, there is no homomorphism of the form $\delta'_0 : k_0 \mapsto \Upsilon$. Suppose for some $i$, $\delta'_i : k_i \mapsto \Upsilon$. Then $\delta'_0 = \delta'_i \circ \delta_i$ is a contradiction, so, for all $i$, $\delta'_i : k_i \not\mapsto \Upsilon$. By Theorem 1, $\not\models_{I(\Upsilon)} \exists x_i. \Phi_i$ and therefore $\not\models_{I(\Upsilon)} \exists x_i. \Phi_i$ with $\alpha$ implying there is no disjunct on the R.H.S. that is true.  

Listeners. The relationship between skeletons and bundles is not as tidy as previously described. \textsc{cpsa} supports something called listener strands that do not appear in bundles. A listener strand in a skeleton is an artificial strand used to assert that some message is available to the adversary. A
listener strand has length two, and the second event is the transmission of the message received by the first event.

When translating a listener strand into a bundle, one simply asserts the existence of a node in the bundle that transmits the strand’s message, and that node inherits the node orderings associated with the nodes of the listener strand.

The definition of a homomorphism into a bundle requires adjustment to allow for the disappearance of listener strands. In particular, the definition of a homomorphism must use the roles in instances to identify listener strands.

4 Detailed Example

The simple example protocol is now revisited.

\[
A \rightarrow B : \{\{s\}_{a^{-1}}\}_{b}
\]

\[
B \rightarrow A : \{d\}_{s}
\]

Symmetric key \(s\) is freshly generated, asymmetric keys \(a^{-1}\) and \(b^{-1}\) are uncompromised, and the goal of the protocol is to keep data \(d\) secret. The PVS description of the protocol in Eq. 3, has an initiator and a responder role. The role items are:

\[
\text{init}(a, b : A, s : S, d : D) = r(\langle +\{\{s\}_{a^{-1}}\}_{b}, -\{d\}_{s}, \langle \emptyset, \emptyset \rangle, \langle \{s\}, \emptyset \rangle \rangle)
\]

\[
\text{resp}(a, b : A, s : S, d : D) = r(\langle -\{\{s\}_{a^{-1}}\}_{b}, +\{d\}_{s}, \langle \emptyset, \emptyset \rangle, \langle \emptyset, \emptyset \rangle \rangle)
\]

The \text{init} role is \(\{ r \mid \exists a, b : A, s : S, d : D. r = \text{init}(a, b, s, d) \}\) and the \text{resp} role is analogous. This rendition of each role ensures it is template inspired.

In this protocol, the unique origination assumption is specified in the \text{init} role, while the two non-origination assumptions are specified in skeletons.

The protocol was constructed with a known flaw for expository purposes, and as a result the secret is exposed due to an authentication failure. The desired authentication goal is:

\[
\forall (\Theta, \rightarrow) \in R_{pt(k_0)}, a, b : A, s : S, d : D, z \in Dom(\Theta).
\]
\[
\text{htin}(\Theta, z, 2, \text{resp}(a, b, s, d)) \land \text{non}(\Theta, a^{-1}) \land \text{non}(\Theta, b^{-1})
\]
\[
\exists a_0 : A, s_0 : S, d_0 : D, z_0 \in Dom(\Theta). \text{htin}(\Theta, z_0, 1, \text{init}(a_0, b, s_0, d_0))
\]

that is, when the responder (\text{B}) runs to completion, there is an initiator (\text{A}) that is using \(b\) for the encryption of its initial message.
\[
k_0 = k_X \{ \text{init}(a_0, b_0, s_0, d_0), \text{resp}(a_1, b_1, s_1, d_1) \},
\langle i(\text{resp}, 2, \{ a_1 \mapsto a, b_1 \mapsto b, s_1 \mapsto s, d_1 \mapsto d \}),
\emptyset,
\{ a^{-1}, b^{-1} \},
\emptyset \}
\]

where \( X = a, b : A, s : S, d : D \)

\[
k_1 = k_Y \{ \text{init}(a_0, b_0, s_0, d_0), \text{resp}(a_1, b_1, s_1, d_1) \},
\langle i(\text{resp}, 2, \{ a_1 \mapsto a, b_1 \mapsto b, s_1 \mapsto s, d_1 \mapsto d \}),
i(\text{init}, 1, \{ a_0 \mapsto a, b_0 \mapsto b', s_0 \mapsto s, d_0 \mapsto d' \})\}
\{ (1, 0) \prec (0, 0) \},
\{ a^{-1}, b^{-1} \},
\{ s \} \}
\]

where \( Y = a, b, b' : A, s : S, d, d' : D \)

\[
\delta_1 = (\langle 0 \rangle, \{ a \mapsto a, b \mapsto b, s \mapsto s, d \mapsto d \})
\]

Figure 4: Shape Analysis for the Simple Example Protocol

\[
\forall (\Theta, \rightarrow) \in R_{pt(k_0)}, a_0, b_0 : A, s_0 : S, d_0 : D, z_0 \in Dom(\Theta).
htin(\Theta, z_0, 2, \text{resp}(a_0, b_0, s_0, d_0)) \land \text{non}(\Theta, a_0^{-1}) \land \text{non}(\Theta, b_0^{-1})
\iff
\exists a_1, b_1, s_1 : S, d_1, d_2 : D, z_1, z_2 \in Dom(\Theta).
z_0 = z_1 \land a_0 = a_1 \land b_0 = b_1 \land s_0 = s_1 \land d_0 = d_1 \land
htin(\Theta, z_1, 2, \text{resp}(a_1, b_1, s_1, d_1)) \land
htin(\Theta, z_2, 1, \text{init}(a_1, b_2, s_1, d_2)) \land
(z_2, 0) \prec (\Theta, \rightarrow) (z_1, 0) \land \text{uniq}(\Theta, s_1, (z_2, 0)) \land
\text{non}(\Theta, a_1^{-1}) \land \text{non}(\Theta, b_1^{-1})
\]

Figure 5: Shape Analysis Sentence for the Simple Example Protocol
To investigate this goal, we ask CPSA to find out what other regular behaviors must occur when a responder runs to completion by giving CPSA skeleton \( k_0 \) in Figure 4. CPSA produces shape \( k_1 \) that shows that an initiator must run, but it need not use the same key to encrypt its first message. The shape analysis sentence for this scenario is displayed in Figure 5. Needless to say, the authentication goal cannot be deduced from this sentence due to the man-in-the-middle attack discussed earlier. However, one can prove the security goal is false by constructing a bundle that contains the man-in-the-middle attack specified with the help of adversarial stands, and using it as a counterexample to the security goal. If one repeats the analysis using the protocol in Eq. 2, the generated shape analysis sentence can be used to deduce the authentication goal.

5 Discussion

Theorems 1 and 2 correspond to theorems with the same numbers in [13]. There are several key differences between the two works. Higher-order logic is used for shape analysis sentences here, but [13] follows the first-order logic, model theoretic approach set out in [7]. A first-order formulation of this version of shape analysis sentences is straightforward, but would obscure their use in PVS.

The second difference is this work uses bundles over initial algebras for models, whereas the previous works use skeletons over free algebras. The shallow embedding of strand space theory in PVS motivates this choice.

Finally, this work faithfully captures the semantics of the roles of the protocol being analyzed via the height-instance predicate \( htin \), which is defined using roles as sets of role items. In previous works, a role origination assumption was ignored.

6 Conclusion

This paper presented a method for importing security goals and the results of a CPSA analysis into PVS such that proofs about the goals can rely on a detailed theory of strand spaces. The method uses a shallow embedding of the theory within PVS. To enable the embedding, the concept of roles as sets of role items was introduced. As a result, there is no need to explicitly represent
substitutions, homomorphisms, and skeletons within PVS to prove security goals. Instead, shape analysis sentences perform the task of transporting results from CPSA into PVS.

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A Role Annotations

There is a simple extension to the strand space theory in Section 2 that allows the ability to annotate an event in a role with an object of any type $\Sigma$. In
practice, few events in a role need annotation, so for type $\Sigma$, events are associated with the type $\text{lift}(\Sigma)$. A lifted type has two constructors and one accessor, so $x \in \text{lift}(\Sigma)$ implies that $x = \bot$ or $x = \uparrow y$ for some $y : \Sigma$. If $x = \uparrow y$ then $y = \downarrow x$.

Annotations were added by modifying the definition of a role item to be of the form $r(C, N, U, A)$, where $C$, $N$, and $U$ are as before, $A \in \text{lift}(\Sigma)^+$, and the length of $A$ is the same as the length of $C$. Let role assignment $rl$ demonstrate that bundle $\Upsilon$ is a run of some protocol. Node $n = (s, i)$ in $\Upsilon = (\Theta, \rightarrow)$ is annotated with $a \in \Sigma$, written $\text{anno}(\Upsilon, rl, n, a)$ if

$$\exists r(C, N, U, A) \in rl(s). \quad \text{inst}(\Theta, s, r(C, N, U, A)) \land A(i) = \uparrow a$$

The set of annotated nodes is

$$\text{anode}((\Theta, \rightarrow), rl) = \{n \in \text{nodes}(\Theta) \mid \exists a : \Sigma. \text{anno}((\Theta, \rightarrow), rl, n, a)\}$$

The annotations can be used to enrich the specification of security goals. For example, annotations can be used to combine trust management theories with cryptographic protocols [9]. In this use case, events are annotated with formulas from a trust management logic. A formula on an outbound event is a guarantee and the sender must show the formula is true before sending the message. A formula on an inbound event is an assumption that can be used by the receiver to deduce future guarantees. The bundle-based strand space theory can be used to ensure that whenever a receiver relies on a formula, another principle has previously guaranteed it.

Role annotations can also be used to reason about state-based protocols. The state in the protocol is modeled as a set of states and a transition relation $\tau$. An infinite sequence of states $\pi$ is a path if $\forall i \in \mathbb{N}. (\pi(i), \pi(i+1)) \in \tau$. To use role annotations to reason about state, events in roles are annotated with subsets of the transition relation, that is $\Sigma = \mathcal{P}(\tau)$. The art to making effective use of a state agnostic protocol analyzer is to modify the message-passing part of the protocol so that a representation of state is threaded through an execution via receive-send pairs of strand succession nodes, where the transmitting node is annotated with a set of transitions consistent with the threaded state.

A bundle $\Upsilon$ is compatible [8, Def. 11] with a state-based role assignment $rl$ if there exists $\ell \in \mathbb{N}$, $f \in \text{anode}(\Upsilon, rl) \rightarrow \{0, 1, \ldots, \ell - 1\}$, and $\pi \in \text{path}$ such that
1. $f$ is bijective,

2. $\forall n_0, n_1 \in \text{anode}(\Upsilon, rl). n_0 \prec n_1 \iff f(n_0) < f(n_1)$, and

3. $\forall n \in \text{anode}(\Upsilon, rl), a \in \mathcal{P}(\tau)$.
   
   $\text{anno}(\Upsilon, rl, n, a) \supset (\pi(f(n)), \pi(f(n) + 1)) \in a$.

This definition ties together the state and message-passing worlds and allows for the verification of state sensitive security goals. An in-depth paper describing this technique by Dan Dougherty, Joshua Guttman, Paul Rowe, and this author is forthcoming.

This appendix ends with a simple example of a stateful protocol called the Award Card Protocol (ACP) created by Joshua Guttman and the author. The state in this protocol is a card with some boxes. When the card is issued, no box is checked. Each time a buyer purchases an item, the cashier checks one box. The buyer may redeem the card when all boxes are checked. It is assumed that a buyer possesses no more than one card at any time.

For simplicity, suppose every card has just one box and there are two interactions with cashiers. Annotated nodes can be used to prove the two interactions are totally ordered and there must have been a new card issued between the cashier interactions. A sketch of the proof follows. The model of state is described first, next the protocol roles, then the method by which the lemma in the state model is imported into the strand space world, and finally, the use of a shape analysis sentence to finish the proof of the security goal.

The model of state is not restricted to a card with one box. Let $bx$ be the number of boxes on a card. Each state $s \in \upsilon$ is the number of unchecked boxes. The transition relation is $\tau = \{(s_0, s_1) \mid s_0 = s_1 + 1 \lor s_1 = bx\}$, that is one box can be checked, or a new card can be issued when one is redeemed or lost. The following lemma can be proved by induction.

**Lemma 1** (Check or Issue).

$$\forall \pi \in \text{path}, i, k \in \mathbb{N}.
   
   i \leq k \supset
   
   \pi(i) \geq \pi(k) \lor
   
   \exists j \in \mathbb{N}. i < j \land j \leq k \land \pi(j) = bx$$

In words, either a card has less checked boxes than a predecessor or there must have been a new card transition in between.
The Award Card Protocol requires an addition to the signature in Figure 1—an infinite set of constants $g_i$ of sort $\top$ called tags. This protocol uses four tags, $\text{zero} = g_0$, $\text{one} = g_1$, $\text{buy} = g_2$, and $\text{new} = g_3$.

There are three roles in the ACP, a new card issuer, a cashier, and a buyer. The trace of each role is displayed in Figure 6.

An interaction between a cashier and a buyer is authenticated using a Needham-Schroeder-Lowe inspired message pattern. Ignore the third and fourth event in the cashier role to see the pattern.

The remainder of the events in the roles encode the state, most using the encoding produced by the injective function $g(s) = g_s$. The third and fourth event in the cashier role encode a box checking transition. The first and second event in the new card role encode a new card transition, where the first event is a dummy value due to the special form of a new card transition.

In general, state encoding message events are inbound followed by outbound event pairs. The outbound event of the pair is annotated. If $i$ is the index of the outbound event of trace $C$, then it is annotated with $\{(s_0, s_1) \mid g(s_0) = h(C(i - 1)) \land g(s_1) = h(C(i))\}$, where $h$ extracts the portion of the message from an event that encodes the state. In the special case of events of the form of a new card transition, the outbound event is annotated with $\{(s_0, s_1) \mid g(s_1) = h(C(i))\}$.

When analyzing the ACP, CPSA has no means by which to enforce the linear ordering of state encoding nodes in bundles, and it may produce a
Lemma 2 (Bridge).

$$\forall \Upsilon, rl. \text{compatible}(\Upsilon, rl) \supset$$

$$\forall n_0, n_1 \in \text{anode}(\Upsilon, rl), a_0, a_1 \in \mathcal{P}(\tau), s_0, s_1 \in v.$$  

$$\text{anno}(\Upsilon, rl, n_0, a_0) \land \text{anno}(\Upsilon, rl, n_1, a_1) \land n_0 \prec n_1 \land$$  

$$a_0 \subseteq \{(s_2, s_3) \mid s_3 = s_0\} \land a_1 \subseteq \{(s_2, s_3) \mid s_2 = s_1\} \supset$$

$$s_0 \geq s_1 \lor$$

$$\exists n \in \text{anode}(\Upsilon, rl).$$

$$\text{anno}(\Upsilon, rl, n, \{(s_2, s_3) \mid s_3 = bx\}) \land$$

$$n_0 \prec n \land n \prec n_1$$

shape analysis sentence that is incompatible with our notion of state. To verify state aware security goals, we will restrict our attention to the bundles that are compatible with the role assignment implied by the role definitions. Because function $f$ in the definition of compatibility is a bijection, annotated nodes in compatible bundles must be linearly ordered.

The compatible bundle assumption allows one to infer the existence of nodes that are not revealed by $\text{cpsa}$. In the case of the $\text{ACP}$, this is done by importing the Check or Issue Lemma into the strand space world by proving the Bridge Lemma (Lemma 2). The proof of the Bridge Lemma makes use of every part of the definition of compatibility.

The implication in the Check or Issue Lemma corresponds to the second implication in the Bridge Lemma. The correspondence of the conclusions of each implication is straightforward, however, the hypothesis of the Bridge Lemma is much more complicated than the one in the Check or Issue Lemma. Yet all it is saying is that the beginning and ending states over the range of the path are $s_0$ and $s_1$, where as in the Check or Issue Lemma, those states are simply referred to by $\pi(i)$ and $\pi(k)$.

Dear reader, at this point I promised to describe the use of a shape analysis sentence to complete the proof of the security goal. I fibbed. This example is so simple and contrived, there is no need to run $\text{cpsa}$ at all! The fact that when there are two interactions with cashiers, there must have been a new card issued between the cashier interactions follows from the point-of-view skeleton one would use to analyze this security goal. In this respect, this is a very unusual example.

The above procedure for verifying security goals of protocols with state has been successfully applied to the Envelope Protocol [1]. In this case, two
shape analysis sentences are required to prove the most interesting security goal. The PVS proof is detailed and involved, and relies on fundamental properties of bundles.

For example, it was shown in PVS that if node $n_0$ is before some transmission node $n_2$, then either the nodes are on the same strand or there is a reception node $n_1$ before $n_2$ on the same strand, such that $n_0$ is before $n_1$. The compatibility assumption implies a total ordering among transmission nodes with annotations. The above lemma is used to infer the correct ordering of nodes that receive state encoding messages. The lemma is also used in the proof of the ACP security goal.

The proof of the Envelope Protocol security goal will be described in the forthcoming paper mentioned earlier.
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