Pattern recognition in the ALFALFA.70 and Sloan Digital Sky Surveys: A catalog of \( \sim 500,000 \) HI gas fraction estimates based on artificial neural networks.

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ABSTRACT

The application of artificial neural networks (ANNs) for the estimation of HI gas mass fraction (\( M_{\text{HI}}/M_* \)) is investigated, based on a sample of 13,674 galaxies in the Sloan Digital Sky Survey (SDSS) with HI detections or upper limits from the Arecibo Legacy Fast Arecibo L-band Feed Array (ALFALFA). We show that, for an example set of fixed input parameters (\( g-r \) colour and \( i \)-band surface brightness), a multidimensional quadratic model yields \( M_{\text{HI}}/M_* \) scaling relations with a smaller scatter (0.22 dex) than traditional linear fits (0.32 dex), demonstrating that non-linear methods can lead to an improved performance over traditional approaches. A more extensive ANN analysis is performed using 15 galaxy parameters that capture variation in stellar mass, internal structure, environment and star formation. Of the 15 parameters investigated, we find that \( g-r \) colour, followed by stellar mass surface density, bulge fraction and specific star formation rate have the best connection with \( M_{\text{HI}}/M_* \).

By combining two control parameters, that indicate how well a given galaxy in SDSS is represented by the ALFALFA training set (PR) and the scatter in the training procedure (\( \sigma_{\text{PR}} \)), we develop a strategy for quantifying which SDSS galaxies our ANN can be adequately applied to, and the associated errors in the \( M_{\text{HI}}/M_* \) estimation. In contrast to previous works, our \( M_{\text{HI}}/M_* \) estimation has no systematic trend with galactic parameters such as \( M_* \), \( g-r \) and SFR. We present a catalog of \( M_{\text{HI}}/M_* \) estimates for more than half a million galaxies in the SDSS, of which \( \sim 150,000 \) galaxies have a secure selection parameter with average scatter in the \( M_{\text{HI}}/M_* \) estimation of 0.22 dex.

Key words: galaxies: fundamental parameters - galaxies: evolution - methods: data analysis - methods: statistical - Astronomical data bases: surveys

1 INTRODUCTION

Neutral hydrogen plays an important role in the fuelling pipeline for star formation activity in galaxies. The study of HI can therefore provide insights into some of the main physical processes that drive galaxy evolution. To this end, numerous surveys (see Giovanelli & Haynes 2015 for a recent review) have been conducted with the aim of determining the HI mass (\( M_{\text{HI}} \)) of large numbers of galaxies, such as the Arecibo Dual Beam Survey (ADBS, Rosenberg & Schneider 2000), the HI Parkes All-Sky Survey (HIPASS, Barnes et al. 2001) and the HI Jodrell All-Sky Survey (HIJASS, Lang et al. 2003). Most recently, the Arecibo Legacy Fast Arecibo L-band Feed Array (ALFALFA) survey has provided a wide area, moderate depth (\( \log M_{\text{HI}}/M_\odot > 8.5 \)), blind survey of HI in the local (\( z < 0.06 \)) universe (Giovanelli et al. 2005). Due to the broad inverse correlation between gas fraction and stellar mass, relatively few massive galaxies are included in the ALFALFA detections. To complement the strategy of ALFALFA, the GALEX Arecibo SDSS Survey (GASS) has therefore performed deep 21 cm observations of \( \sim 1000 \) massive (\( \log M_*/M_\odot > 10 \)) galaxies down to a fixed gas fraction limit (Catinella et al. 2010, 2013). Coupled with multi-wavelength data from other large surveys, these large HI surveys have provided unprecedented insight into the connections between global gas properties and environment, nuclear activity, star formation and chemical enrichment (Cortese et al. 2011; Fabello et al. 2011; Bothwell et al. 2013; Lara-Lopez et al. 2013; Hughes et al. 2013).

Despite the tremendous efforts of HI surveys, the measurement of HI masses continues to lag behind optical surveys. For example, in contrast to the \( \sim 5000 \) detections available for the 40 per cent ALFALFA data release (Haynes et al. 2011), the main galaxy sample of the Sloan Digital Sky Survey (SDSS) contains more than two orders of magnitude more galaxies out to \( z \approx 0.2 \) (e.g. Strauss...
et al. 2002). For this reason, there is a long history of efforts that attempt to infer \( M_{\text{HI}} \) from more readily available optical properties (Roberts 1969; Bothun 1984; Roberts & Haynes 1994). In the last decade, such efforts have been able to capitalize on large, homogeneous datasets, often in combination with multi-wavelength data in the UV and IR regimes (Kannappan 2004; Zhang et al. 2009; Catinella et al. 2010, 2012; Toribio et al. 2011; Wang et al. 2011; Li et al. 2012; Denes, Kilborn, & Koribalski 2014; Eckert et al. 2015). These works have used different linear combinations of physical parameters to reduce scatter between the estimated and observed HI gas mass fraction (defined as \( M_{\text{HI}}/M \), throughout this paper). However, despite the availability of extensive physical parameters, and the identification of the ‘best’ parameters for \( M_{\text{HI}}/M \) estimation, the improvement in the accuracy of scaling laws has been small. For example, Zhang et al. (2009) used a linear combination of \( i \)-band surface brightness and \( g-r \) colour that was motivated by the Kennicutt-Schmidt law to achieve a scatter in their \( M_{\text{HI}}/M \), estimator of \( \sim 0.31 \) dex. Catinella et al. (2010) found that the near ultraviolet (NUV) NUV−\( r \) colour was the single best estimator of HI mass, and determined a ‘gas fraction plane’ by combining the stellar mass surface density (\( \mu_* \)), NUV−\( r \) and HI gas mass fraction. However, the scatter in this relation remained at 0.3 dex (see also Catinella et al. 2012). Motivated by the finding that gas fractions are linked to outer disk colours (Wang et al. 2011), Li et al. (2012) add the colour gradient to their linear estimator, applied to ALFALFA and GASS samples, but again the scatter in the HI gas fraction was not improved beyond \( \sim 0.3 \) dex. Moreover, many of these linear scaling relationships still have significant systematics relative to stellar mass, age, colour or concentration indices (e.g., Zhang et al. 2009).

Despite their current limitations, linear scaling relations and the gas fraction plane are useful as indicators of ‘HI normalcy’, which can be used to identify outliers (within a given sample) that are either particularly gas-rich or gas-poor (e.g. Catinella et al. 2012). Similarly, Cortese et al. (2011) demonstrate that the distance from the gas fraction plane is a good alternative to the classic ‘HI deficiency’ parameter (Haynes & Giovanelli 1984). However, extreme caution must be applied in using scaling relations to estimate \( M_{\text{HI}}/M \), due to the inherently biased nature of the samples from which they are (currently) constructed. For example, the application of scaling relations derived from the blind, but shallow, ALFALFA data tends to over-estimate \( M_{\text{HI}}/M \), in deeper, or targeted surveys (e.g., Huang et al. 2012). Problems may persist even when precautions are taken to identify galaxies with broadly similar properties as the calibration sample. For example, Zhang et al. (2009) identify a sample of star-forming galaxies from which they determine their \( M_{\text{HI}}/M \) scaling relation. It might therefore be expected that the distinction between star-forming and quiescent galaxies may help to identify samples of galaxies appropriate for the application of the Zhang et al. (2009) parametrization. However, Huang et al. (2012) show that the Zhang et al. (2009) relation is not a good predictor of \( M_{\text{HI}}/M \) in ALFALFA, despite the domination of star-forming galaxies in the latter. Therefore, whilst linear scalings reveal the typical relationship between \( M_{\text{HI}}/M \), and other optical/NUV/infrared (IR) properties for a given sample, they can not be used as a universal predictor of HI gas mass fraction, particularly in circumstances when galaxies are expected to deviate from the ‘normal’ relation (e.g. Cortese et al. 2011).

In this paper, we explore the application of non-linear methods, such as artificial neural networks (ANNs), to the challenge of gas fraction estimation. We have previously demonstrated the application of ANN to various astronomical problems, including pattern recognition for the spectral classification of galaxies (Teimoorinia 2012), fitting applications in order to estimate line fluxes which can be used for Active Galactic Nucleus (AGN) classifications, metal abundances etc. (Teimoorinia & Ellison 2014) and a large public catalog of estimated IR luminosities (Ellison et al. 2016a) that can be used to determined star formation rates for galaxies dominated by AGN (Ellison et al. 2016b). Most recently, we have presented a method for ranking the parameters involved in galaxy quenching, which effectively disentangles the complex multi-variate nature of this physical process (Teimoorinia et al. 2016). We refer the interested reader to those works for details of the general ANN method. The non-linear ANN techniques used in these previous works are readily applied to the estimation of the gas fraction from large input datasets. In this work, we will train and validate the ANN using galaxies drawn from the ALFALFA.70 data release, matched to galaxies in the SDSS DR7. The trained network can then be applied to all galaxies in the DR7 (not covered by ALFALFA.70) in order to predict the gas fraction based on their optical properties.

The layout of the paper is as follows. In Section 2 we describe the samples that are used for training the ANN and the method used for rejecting galaxies that may have their 21 cm fluxes contaminated by the presence of a close companion within the Arecibo beam. The results in the following sections are then presented with respect to three main objectives. First, we will explore whether non-linear methods are able to reduce the scatter in \( M_{\text{HI}}/M \); scaling relations below what has been previously achieved for linear calibrations for a given set of input parameters (Section 3). Specifically, we use the same optical input parameters as Zhang et al. (2009) in order to compare the scatter of their linear calibration with that of the non-linear methods. Although this example uses a limited number of variables for training, it serves as a direct comparison of the performance of linear and non-linear techniques. In Section 3 we demonstrate how differences in HI surveys can influence the application of scaling relations. In Section 4 the input data for the ANN are described and also a performance function, \( R \), is introduced for ranking the weights of input parameters in the fitting procedure. A second performance metric is then used to determine which galaxy variables show the most important physical connection with \( M_{\text{HI}}/M \). The third objective is then to develop the non-linear relationships between \( M_{\text{HI}}/M \), and galactic properties into a procedure that can be robustly used to estimate \( M_{\text{HI}}/M \) for galaxies in the SDSS, the results of which are presented in Section 5. In Section 6 we describe a pattern recognition procedure that determines how similar a given galaxy is to those that have been used in the training set (i.e. detections in ALFALFA). We hence are able to assign an ALFALFA detection probability, \( PR \), to any SDSS galaxy; a robust gas fraction can be best determined for galaxies with high detection probabilities (i.e. the galaxy is similar to the ALFALFA detected galaxies on which the ANN was trained). The final strategy for defining a robust sample for applying the ANN is presented in Section 7. In Section 8 a short summary is presented. A public catalog of the HI gas mass fraction, the control parameters and errors accompanies this paper in electronic format.

2 SAMPLES

2.1 Training Set

At the heart of our ANN approach is a training set based on matches between the SDSS DR7 galaxy catalog and the ALFALFA.70 data
HI gas fraction estimations

2.2 Additional Samples and Ancillary Data

In order to test our $M_{\text{HI}}/M_*$ estimator on independent datasets, we will also make use of several other samples that have been matched with the SDSS. The largest of these is the GASS sample\(^2\) for which we use the 342 detections from the data release 3 (Catinella et al. 2013). Additionally, we use the Cornell catalog (~30 galaxies, Giovanelli et al. 2007) as incorporated into the GASS representative sample and a sample of ~40 post-mergers (PM) presented by Ellison et al. (2015). We have also used 279 galaxies (of 2839, matched with our ANN’s input parameter space) from the Nancay Interstellar Baryons Legacy Extragalactic Survey (Nibles) sample, presented by van Driel et al. (2016). They use a sample in the redshift range $z<0.04$ selected on $z$-band magnitude ($-24 < M_z < -13.5$) as a proxy for stellar mass. Galaxies in the NIBLES sample are at the bright end of the SDSS distribution. All of the above samples have large amounts of ancillary information available, which may be used as input variables for the $M_{\text{HI}}/M_*$ estimation, from the following sources:

- Photometry is taken from SDSS DR7. Structural parameters, such as bulge fractions and galaxy sizes are taken from the reprocessed SDSS images by Simard et al. (2011) and Mendel et al. (2014). The fluxes in different bands ($u,g,r,i,z$) are all corrected for Galactic extinction.
- Stellar masses are taken from Mendel et al. (2014) based on their re-assessment of SDSS photometry.
- Total star formation rates were taken from the MPA/JHU catalogs, which applied a colour dependent aperture correction to account for the light outside of the SDSS fibre (Brinchmann et al. 2004; Salim et al. 2007). Star formation rates are only used for those galaxies classified as ‘star-forming’ by the definition of Kauffmann et al. (2003). Specific SFRs are determined by combining the SFRs described above with the stellar masses from Mendel et al. (2014).
- The halo masses come from the group catalogue of Yang et al. (2007, 2009).
- Local environmental densities are computed as $\Sigma_n = \frac{N_\text{n}}{\pi d_n^2}$, where $d_n$ is the projected distance in Mpc to the $n$th nearest neighbour within $\pm 1000$ km s$^{-1}$. Normalized densities, $\delta_n$, are computed relative to the median $\Sigma_1$ within a redshift slice $\pm 0.01$. In this study we adopt $n = 5$.
- Stellar mass density is defined as $\log(\mu_*) = \log(M_*) - \log(2\pi R_{\text{vir}}^3)$ in which $M_*$ is the stellar mass and $R_{\text{vir}}$ is the radius (in kpc) enclosing 50 per cent of the total Petrosian $i$-band flux.

3 A SIMPLE EXAMPLE OF HI MASS ESTIMATION FROM NON-LINEAR METHODS

Before launching into a complex many-parameter ANN application for $M_{\text{HI}}/M_*$ estimation, we show a simple example of comparing the linear estimation of Zhang et al (2009) to a non-linear method. Zhang et al (2009) used a sample of ~800 galaxies with HI masses in the Hyperleda database that are matched to galaxies in the SDSS to determine a calibration between $M_{\text{HI}}/M_*$ and the $g$- and $r$-band

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\(^1\) This equation is in excellent agreement with Broeils & Rhee (1997).

\(^2\) Data publicly available at http://wwwmpa.mpa-garching.mpg.de/GASS/data.php
Petrosian apparent magnitudes and \(i\)-band surface brightness, defined as \(\mu_i = m + 2.5 \log(2\pi R_{\text{eff}}^2)\), where \(R_{\text{eff}}\) is the radius (in units of arcsecond) enclosing 50 per cent of the total Petrosian \(i\)-band flux. The calibration presented by Zhang et al. (2009) is:

\[
\log \frac{M_{\text{HI}}}{M_*} = -1.73(g - r) + 0.22\mu_i - 4.08
\]  

(1)

Using Eq. 1 we estimate \(M_{\text{HI}}/M_*\) for the samples described in Section 2: AC1, GASS, PM and Cornell, where we group the latter two samples together for plotting purposes due to their small size. In Fig. 1 we compare the \(M_{\text{HI}}/M_*\) estimated by Zhang et al. (2009)’s calibration (Eq. 1) and the observed values. In each panel, the values in the lower right corner give the mean difference between the estimated and observed \(M_{\text{HI}}/M_*\), and the scatter (\(\sigma\)) in these differences. There are systematic offsets between the observed and estimated \(M_{\text{HI}}/M_*\) values, that vary between the samples. For example, the top panel of Fig. 1 shows that \(M_{\text{HI}}/M_*\) in AC1 (ALFALFA detections) is underestimated, on average, by 0.22 dex by Zhang’s formula. The same seems to be true in the NIBLES data, with the offset occurring in a similar regime (i.e. \(\log M_{\text{HI}}/M_*\) (Obs)\(>\) 0). The offset between AC1 and the Zhang et al. formulation has been previously reported by Huang et al. (2012) who suggest that differences in the methods of calculating the stellar masses may account for a systematic deviation of \(\sim 0.2\) dex.

The top panel of Fig. 1 and explanation by Huang et al. (2012) demonstrate an important caveat for the application of any calibration method – if parameters are not uniformly derived, even ‘perfect’ calibrations will perform poorly. It is therefore vital to apply calibrations to datasets whose parameters have been derived as closely as possible to the original data.

The second plot from the top of Fig. 1 shows that, in contrast to what was seen in AC1, the \(M_{\text{HI}}/M_*\) estimated for the GASS sample using Eq. 1 has a tendency to be mildly over-estimated. Taken together, the top and middle panels therefore imply a total difference between AC1 and GASS estimations \(\sim 0.45\) dex. As described above, Huang et al. (2012) have suggested that this may be due, at least in part, to differences in stellar mass calibrations. In order to test this suggestion, we can re-derive the best fit linear coefficients of \((g - r)\) and \(\mu_i\) in Eq. 1 using AC1 and test this calibration on the GASS data. Since both samples have consistent sources of input parameters and stellar mass measurements, any systematic trend caused by inconsistencies therein should be removed. The AC1-calibrated version of Eq. 1 is then

\[
\log \frac{M_{\text{HI}}}{M_*} = -2.332(g - r) + 0.168\mu_i - 2.528
\]  

(2)

In Fig. 1 we now compare the observed and estimated \(M_{\text{HI}}/M_*\) for the newly derived coefficients in Eq. 2 in the AC1 and GASS samples (blue and red points respectively). The first important result demonstrated by Fig. 2 is that Eq. 2 does not perform well at estimating the data on which it is calibrated (AC1), indicating that AC1 can not be well fit with the parametrization in Eqs. 1 and 2. Application to the GASS sample also results in a strong systematic offset. Importantly, the mean offset between the estimated (using Eq. 2) and the observed GASS data is still \(\sim 0.45\) dex, the same as was inferred between AC1 and GASS using the original coefficients in Eq. 1. This difference can no longer be attributed to differences in inhomogeneous stellar mass estimates or photometry, which were derived identically for the two samples. As we will demonstrate below, the offset is due to the fundamentally different nature of the galaxies in the two samples.

To compare the performance of the linear (Zhang et al. 2009) and non-linear approaches, we follow the methods outlined in Teimoorinia & Ellison (2014), in which non-linear methods were
used to determine Balmer decrements and emission line fluxes. Here, we use the AC1 sample of HI detections from ALFALFA.70, combined with the same parameters used by Zhang et al. (2009) and tested in the above linear fit tests \((g-r) \text{ and } \mu_i\). Therefore, the only change we are making is in the methodology of the fitting, not in the parameters used. We use the Levenberg-Marquardt algorithm (Marquardt 1963) to find the coefficients in the following equation:

\[
f(C, X) = \sum_{i=1}^{N} C_i X_i + \sum_{i=1}^{N} C_i + C'.
\]

In the above equation, \(f(x) = \log M_{HI}/M_\star\), and \(X\) is:

\[
X = \begin{pmatrix} m_t \\ m_r \\ m_i \\ R_{HI} \end{pmatrix}
\]

Here, \(N=4\) and \(X_i\) (\(i=1\) to \(4\)) in which \(m_t\), \(m_r\), \(m_i\) are apparent magnitudes. These are the individual variables that comprise \(g-r\) and \(\mu_i\) in the parametrization of Zhang et al. (2009) in Eqs. 1 and 2. We use AC1 as the training set and determine the 15 coefficients required for the above parametrization.

Fig. 3 shows the estimated vs. observed gas fractions derived from Eqs. 3 which is a representation of the matrix-based solution. The residual tilt in Fig. 2 in the AC1 data (blue points) is now completely removed and scatter is reduced from 0.31 dex in the linear calibration (Eq 2) to 0.22 dex. This demonstrates that although \(g-r\) and \(\mu_i\) can be used for \(M_{HI}/M_\star\), calibration, a more complex representation, such as the matrix form, can provide a better match (due to more complex connections between the parameters). Moreover, this matrix form can also remove the deviation and skew seen in Fig. 2 for AC1, which is obtained using only three (different) coefficients.

Despite the significant improvement in AC1, Fig. 3 shows that no improvement in scatter or systematic offset is seen in the GASS dataset. The persistent systematic offset shown in Fig. 3 for the GASS data could be due to the two samples (AC1 and GASS) representing galaxies of different physical nature, so that even the matrix form of the estimator cannot extrapolate it in a suitable manner. This is demonstrated in Fig. 3 where we plot \(\mu_i\) vs. \(g-r\) for the two samples; whilst there is some overlap, the GASS galaxies are preferentially located at the reddest \(g-r\) colours and lowest values of \(\mu_i\), that is poorly represented in AC1. It is possible to select a subset of the GASS data that are relatively well represented in the AC1 data, for which we might expect the calibration to perform better. In the lower panel of Fig. 4 we again plot the comparison of estimated and observed \(M_{HI}/M_\star\), shown in Fig. 2 but now limiting the GASS data to the range \(\mu_i > 20\) and \(g-r < 0.6\), which is more populated by the AC1 sample. The estimated \(M_{HI}/M_\star\) does not remove all the offsets; however, this estimate is now closer to the observed one for this limited GASS sample. These tests demonstrate that whilst a single ANN model could not be found that is a good representation of all of the galaxies in the combined GASS and ALFALFA samples, it is nonetheless possible to post-facto exclude those galaxies for which the ANN is not suitable. For this reason, our approach for the remainder of the paper is to train our networks only using the ALFALFA sample, and then to develop a set of criteria from which we can assess the robustness of the gas fraction estimate. The cuts used in Fig. 4 represent a rather crude approach, and practical here only because we are dealing with a two-dimensional input parameter space. With greater dimensionality (higher number of input parameters for the calibration), such simple cuts are cumbersome and complex, and ultimately subjective with no quantitative assessment of suitability. Therefore, whilst the general approach of limiting the data suitable for the application of a given calibration is desirable, a more sophisticated method is required. We return to this issue in Section 6.

In this section, we have demonstrated that for a given set of input parameters a non-linear approach can yield a significant improvement in the estimation of \(M_{HI}/M_\star\) over a linear fit to the data. The matrix representation provides a very good option for deter-
Are we want to use a higher dimensional space as a training set then can be correlated with each other in different criterion such as a cut on a physical parameter. Since the parameters samples. This might be done in linear examples using a simple criterion; for example, we use both g- and r-band magnitudes as well as g-r colour. This repetition is not detrimental to the network’s performance, but contributes stability. Photometry and galaxy colour represent some of the raw observables that have been shown to correlate with gas fraction (e.g. Kannappan 2004; Denes et al. 2014; Eckert et al. 2015). These are in turn related to physical parameters such as galactic stellar mass, which has been shown to exhibit a strong anti-correlation with Mstellar/MBH (e.g. Zhang et al. 2009; Catinella et al. 2010; Huang et al. 2012). However, internal galaxy structure, size and the distribution of stellar mass appear to be even more tightly correlated with gas fraction than stellar mass itself (Zhang et al. 2009; Catinella et al. 2010; Toribio et al. 2011; Wang et al. 2016). Furthermore, Brown et al. (2015) have argued that specific star formation rate also modulates Mstellar/MBH, at fixed Mstellar. Environment also appears to play a role in determining gas fraction with gas fractions suppressed in both the cluster (e.g. Chung et al. 2007; Cortese et al. 2011; Denes et al. 2014) and group (e.g. Verdes-Montenegro et al. 2001; Rasmussen et al. 2008; Kilborn et al. 2009; Catinella et al. 2013; Hess & Wilcots 2013; Denes et al. 2016; Odekon et al. 2016) environments. We have included two environmental metrics in our list of training variables, halo mass and δ200, although Brown et al. (2016) have recently shown that it is the former of these that dominates the environmental dependence of Mstellar/MBH.

The full list of 15 parameters used in our work, based on the SDSS imaging and spectroscopic data, is presented in Table 1 which includes photometry, metrics of internal size and structure, star formation and environment. We note that not all the 15 parameters are available for all galaxies. This requirement reduces the number of galaxies in both the GASS and NIBLES samples from their complete data release. In the analysis that follows, the full AC1 and AC3 samples are therefore sometimes restricted further by the lack of input parameter data. If a certain input variable (such as a magnitude in a band) is required for a given training run, galaxies without a robust measurement of that input variable are excluded. Amongst the parameters in Table 1 there are two notable omissions of parameters that may significantly dictate Mstellar/MBH. The first is angular momentum, which has recently been proposed by Obreschkow et al. (2016) to dictate HI gas fractions based on arguments of gas instability in the interstellar medium (see also Huang et al. 2012;
Table 1. Galaxy variables used in training sets.

| Input data | Description          |
|------------|----------------------|
| $M_\star$  | stellar mass         |
| $M_\text{u}$ | $u$ band absolute magnitude |
| $M_\text{g}$ | $g$ band absolute magnitude |
| $M_\text{r}$ | $r$ band absolute magnitude |
| $M_\text{i}$ | $i$ band absolute magnitude |
| $M_\text{z}$ | $z$ band absolute magnitude |
| $g-r$      | observed colour      |
| $\mu_{\text{i}}$ | $i$-band stellar mass density |
| SFR        | star formation rate  |
| sSFR       | specific star formation rate |
| $M_{\text{Halo}}$ | halo mass |
| $\delta_\text{g}$ | local galaxy density |
| rhalf      | half light radius (kpc) in the $r$ band |
| B/T        | bulge-to-total fraction in the $r$ band |
| rdisk      | disk radius (kpc) in the $r$ band |

Maddox et al. (2015). Unfortunately, we do not have metrics of angular momentum available for our sample. We have also not included a NUV colour, proposed by several works (e.g. Cortese et al. 2011; Catinella et al. 2013, Brown et al. 2015) to be the single most important variable in their samples. Requiring NUV photometry reduces our sample size by a factor of more than 4, to only ~1400 galaxies, which was found to be inadequate for ANN training.

4.2 The ANN performance metric, $R$

In this section, we introduce the use of a performance function, as a metric of the quality of the fit between gas fraction and a given variable. This performance metric can be a simple linear regression of the estimated and the observed values or a Spearman’s rank correlation number (see Huang et al. 2012 for more details), but it may also be a more complicated figure of merit (e.g. Teimoorinia et al. 2016). We use the coefficient of determination, $R^2$, which is a measure of goodness of fit and is defined as:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(\text{target} - \hat{y}_{\text{fit}})^2}{\text{Var}(\text{target})}.$$  

In this equation $\hat{y}_{\text{fit}}$ is a linear fit to the target (observed data) and the estimated values (obtained by ANN). Var is the variance and $n$ is the number of objects in the sample. $R$ ranges from 0 to 1 in which $R \sim 0$ indicates that the fit is not significantly better than a model in which $y_{\text{fit}} = \text{constant}$. A value of $R=1$ indicates that the linear equation ($y_{\text{fit}} = ax + b$), where $X$ is the observed $M_{\text{HI}}/M_\star$, predicts 100% of the variance in the target ($y$, in this case the predicted gas fraction). Each parameter listed in Table 1 has a certain contribution to estimating $M_{\text{HI}}$ which can be considered as a weight in the fitting procedure, such that parameters with higher values of $R$ contribute more significantly to the combined estimate of $M_{\text{HI}}/M_\star$.

In Figure 5 we show four different parameters taken from Table 1 as an example of the performance metric functionality. The value of $R$ for each variable is given in the lower right of each panel. Amongst these four examples, it can be seen that stellar mass has the highest value of $R=0.85$, and indeed the scatter between the observed and predicted $M_{\text{HI}}/M_\star$ is relatively small. Although parameters with low $R$, such as B/T (R=0.29) provide little improvement in our predictions of $M_{\text{HI}}/M_\star$, their inclusion with the ensemble of parameters can still provide stability to the network. We also note that in some regimes, some of our parameters may become unreliable, such as $M_{\text{Halo}}$ in the low mass regime. In these cases, variables act simply as random numbers, and in the limit of a large training sample such as ours, such random variables do not decrease the performance of the network. For these reasons, all 15 parameters are used in our final ANN.

In Figure 6 we show the performance numbers for all the 15 input parameters. It should be noted that these performance numbers are only the weights that show the contribution of each single parameter in the fitting procedure (i.e., when we use only a parameter from 15 for fitting) and should not be considered as a ranking in physical importance. In order to extract physical determinism, different statistical methods such as receiver operating characteristics (ROC) (Teimoorinia et al. 2016 and reference therein) can be applied.

4.3 The physical link between galaxy variables and gas fraction

In Teimoorinia et al. (2016) we described a different performance number, namely the area under the curve (AUC) of ROC plots which can be used to link the physical significance of one variable to the value of another. This method was applied by Teimoorinia et al. (2016) to determine which were the most important variables for determining whether or not a galaxy’s SFR was quenched. A full description and worked examples of the AUC parameter can be found in that paper. In brief, the AUC ranges from 0.5 (a random number with no physical import) to 1 (outstanding performance governing the physical link between two variables). In Figure 7 we show the ordered values of AUC for the 15 parameters used in this work. The numerical values of AUC are traditionally associated with qualitative ranks, ranging from ‘outstanding’ (AUC>0.9) to ‘random’ (no physical importance, AUC<0.6), e.g. Hosmer & Lameshow (2000). Figure 7 shows that $g-r$ colour and stellar mass surface density are the two best performing indicators of $M_{\text{HI}}/M_\star$. 
and indeed these parameters have featured widely in the literature (e.g. Zhang et al. 2009; Catinella et al. 2010, 2012; Cortese et al. 2011). Whereas $B/T$ has a low value of $R \sim 0.3$, it has an $AUC = 0.71$, placing it as the 3rd most important parameter for determining $M_\text{HI}/M_*$. Specific SFR also plays a marginally acceptable role in governing $M_\text{HI}/M_*$, but the 11 other parameters in our list have a formally low impact on the gas fraction. This includes parameters that are linked to environment: halo mass and $\delta_5$, indicating that such parameters are not the prime drivers of $M_\text{HI}/M_*$, although they may still contribute at a lower level once higher performing variables have been accounted for. Moreover, although the value of $R$ for stellar mass is high, we can see from Figure 7 that the $AUC$ value associated with $M_*$ is 0.63, and this parameter therefore performs little better than a random variable. Although none of the individual parameters has a particularly high $AUC$, when all 15 are considered together, the $AUC = 0.86$, characterized as an ‘excellent’ (but not ‘outstanding’) indicator.

5 FITTING RESULTS

Having defined our samples (Section 2.1), removed galaxies with contamination (Section 2.1), chosen 15 relevant variables for fitting the target data (Section 4.1 and Table 1), defined their respective weights (Section 4.2) and ranked their relative physical importance (Section 4.3), we now proceed with a Bayesian neural network model (e.g., Ellison et al. 2016a) to fit the 6329 uncontaminated galaxies from AC1. In practice, we train 25 different networks with different initialization conditions and select the 20 best performing networks. Hence, each galaxy has 20 estimations of $M_\text{HI}/M_*$, from which we adopt the mean value. We can also quantify the scatter ($\sigma_{\text{fit}}$) in the estimated $M_\text{HI}/M_*$ (for a given galaxy, and for a given variable) for these best 20 performing networks, which can be used to quantify an uncertainty in the network estimation. That is, if a given galaxy’s $M_\text{HI}/M_*$ is robustly estimated by the 20 networks (for a given variable) $\sigma_{\text{fit}}$ will be small. If the $M_\text{HI}/M_*$ estimation is unstable, $\sigma_{\text{fit}}$ will be large. We have previously used $\sigma_{\text{fit}}$ as a way of identifying which subset of galaxies have robust ANN estimations (see Ellison et al. 2016a, for a more technical discussion).

In Figure 8 we show the fitting results for the 15 parameters for the training sample AC1, by comparing the estimated and observed $M_\text{HI}/M_*$ in the top panel. There is now a systematic difference between the predicted $M_\text{HI}/M_*$ and the observed value, with the latter being on average

![Figure 6](image6.png)

**Figure 6.** The performance number $R$ for all 15 parameters listed in Table 1. The $R$ parameter reflects the amount of scatter present between the observed and predicted gas fraction and hence the relative weight of a given parameter in a fit.

![Figure 7](image7.png)

**Figure 7.** The performance number $AUC$ for all 15 parameters listed in Table 1. In contrast to $R$, the $AUC$ indicates the relative importance of a given parameter for determining $M_\text{HI}/M_*$. and indeed these parameters have featured widely in the literature (e.g. Zhang et al. 2009; Catinella et al. 2010, 2012; Cortese et al. 2011). Whereas $B/T$ has a low value of $R \sim 0.3$, it has an $AUC = 0.71$, placing it as the 3rd most important parameter for determining $M_\text{HI}/M_*$. Specific SFR also plays a marginally acceptable role in governing $M_\text{HI}/M_*$, but the 11 other parameters in our list have a formally low impact on the gas fraction. This includes parameters that are linked to environment: halo mass and $\delta_5$, indicating that such parameters are not the prime drivers of $M_\text{HI}/M_*$, although they may still contribute at a lower level once higher performing variables have been accounted for. Moreover, although the value of $R$ for stellar mass is high, we can see from Figure 7 that the $AUC$ value associated with $M_*$ is 0.63, and this parameter therefore performs little better than a random variable. Although none of the individual parameters has a particularly high $AUC$, when all 15 are considered together, the $AUC = 0.86$, characterized as an ‘excellent’ (but not ‘outstanding’) indicator.

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HI gas fraction estimations

The approach adopted here is to attempt to distinguish which galaxies are represented in AC1, as opposed to those that appear as non-detections in AC3. Artificial neural networks are powerful tools for such pattern recognition problems and they can also provide us with statistical information about a data set in order to categorize the data in a quantitative way. We recall that sample AC1 contains 6837 galaxies detected at 21 cm, all of which have measurements of the 15 parameters listed in Table 1. In order to create a statistical balance within the network, we randomly select 6837 galaxies out of the 23,652 non-detections in AC3. These 13674 galaxies are used as the main training sample for the pattern recognition step of our analysis. The objective of this step is to train a network that can distinguish the 21 cm detections from the non-detections, based solely on the 15 input SDSS parameters. Note that we use all 6837 of AC1 regardless of the potential contamination described in Section 4 since what is important here is whether or not a galaxy is detected, not its exact 21 cm flux.

We use a binary classification for our training set such that a value of 0/1 is initially assigned to all galaxies in AC3 (non-detections) and AC1 (detections), respectively. During the training procedure, the 13674 galaxies of the training set are randomly separated into two sub-samples of training (70%) and validation sets (30%), to avoid any over-fitting problems. Using these input and target values, we train 40 networks and use the average output of the best 20. The network output will be the estimated probability that the input pattern (of SDSS parameters) belongs to one of the two categories. We refer to this probability of detection and non-detection galaxies as the pattern recognition detection metric, PR, which has a value between 0 and 1.

In the ideal case, for the training set of 13674 galaxies, we would expect to see two completely separated groups with PR values of 0 and 1, i.e. HI detections and non-detections that are completely separable in terms of their SDSS properties. However, in practice, the detection pattern exhibits a more continuous behaviour, due to the smoothly varying properties within the galaxy populations (see also Teimeoorinia et al. 2016). In Figure 10 we show the actual (normalized) distribution of the detection pattern for the galaxies in the training set, plotted separately for AC1 (blue line) and AC3 (red line). The figure shows that the detection pattern peaks near 1 and 0 for AC1 and AC3 respectively, demonstrating that the network is largely successful in discriminating the two samples based on their SDSS properties. However, both samples show long tails in their PR distributions, indicating that not all galaxies are correctly classified with the 15 parameters in our training set. In other words, this is not a perfect classification so that some detections in AC1 have low probabilities estimated by the ANN, and some non-detections in AC3 have high estimated values of PR. For a decision boundary of PR=0.5 such that AC1 (PR<0.5) and AC3 (PR>0.5) represent misclassifications, we find more than 80 per cent of galaxies of AC1 sample are correctly classified. The exact choice of PR threshold will depend on the specific application of the data and the requisite combination of purity and accuracy. According to binary classification methods, the level of separation shown in Figure 10 can be considered as ‘successful’. In this method, the AUC = 0.86 and is therefore indicative of a very good classification.

In Figure 11 we compare the physical parameter space of galaxies in AC1 (left panels) and AC3 (right panels), where points are colour coded by PR. Figure 11 shows that the distribution of \( g-r \) colours, stellar masses, B/T and SFRs are quite different between the two samples, as expected based on the ALFALFA survey design. For example, most of the detections are located on the star forming main sequence and have blue colours, whereas non-detections additionally populate the region of quiescent galaxies with red colours and high bulge fractions. However, the distribution of PR between AC1 and AC3 is qualitatively similar. For example, the highest values of PR in AC3 are seen for galaxies on the main sequence (bottom right panel) and blue colours (top and middle right panels). The PR values are typically lower than seen for AC1, as expected given that, in reality, the AC3 galaxies are indeed not detected. It should be noted that in the pattern recognition procedure we do not use any direct connection between the observed \( M_{\text{HI}}/M_* \) values and the 15 parameters. PR is better interpreted as a
connection between the best parameters and the nature of the survey. The closer the resemblance of a given galaxy with those in the original survey, the smaller the error in the estimation.

We now apply the pattern recognition networks to all 561,585 galaxies in SDSS for which all 15 of our parameters are measured. The resultant distribution of PR values is shown in Figure 12. In the lower panel of Figure 12 we show the number of galaxies remaining in the sample as a function of progressively more stringent PR cut.

In Section 5 we described how the scatter amongst the 20 best trained networks yields an uncertainty on the ANN estimation, \(\sigma_{fit}\); we now compute \(\sigma_{fit}\) for the networks trained with our 15 parameters. For reference, in the top panel of Figure 13 we show the distribution of \(\sigma_{fit}\) obtained for AC1. The lower panel shows \(\sigma_{fit}\) for the 561,585 galaxies with the 15 available parameters from SDSS. As can be seen from the top panel, log \(\sigma_{fit}\) spans the interval \(-2.5\) to \(-1\) for AC1. On the other hand, when we apply the networks to the SDSS, the distribution (in the lower panel) exhibits a more extended distribution, notably with a tail to higher values. This indicates (not surprisingly) that some galaxies in the SDSS have an estimation of \(M_{HI}/M_*\) that is not as stable as in the training set. From a pattern recognition point of view the two distributions can be considered as two distinguishable groups with considerable overlap. A cut at log \(\sigma_{fit}=-1\) removes \(\sim 135,000\) galaxies from the SDSS sample, indicating that \(\sim 1/3\) of galaxies in SDSS have a
We now have at our disposal two parameters that can be used to select a sample of SDSS galaxies for which robust $M_{\text{HI}}/M_*$ estimations can be made, based on their similarity to the training set (PR) and the uncertainty in the networks ($\sigma_{\text{fit}}$). Both of these parameters are included in our public catalog. As a demonstration of how judicious cuts in these parameters can reduce the scatter between the estimated and observed $M_{\text{HI}}/M_*$, in Fig. 14 we show results for the GASS (left panels) and PM & Cornell samples (right). The top panels show all galaxies in the sample, and the lower panels show the galaxies remaining after we restrict $\text{PR} > 0.5$ and $\sigma_{\text{fit}} < 0.1$. These cuts can be viewed as nominal threshold values for the SDSS catalog, given the natural decision boundary at PR=0.5 and

higher uncertainty in their $M_{\text{HI}}/M_*$ estimation than in the training set.

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the distribution of $\sigma_{\text{fit}}$ in the training set (as shown in the top panel of Figure 13). However, different cuts may be required for surveys with different properties. For example, for the GASS sample, imposing these default cuts does reduce the scatter from 0.33 to 0.24 dex, although a small offset still persists, indicating that even more stringent thresholds may be required. The PM & Cornell sample is small and shows some outliers; excluding the single most significant outlier reduces the scatter of the full sample from 0.35 to 0.29 dex. Furthermore, placing a PR > 0.5 and $\sigma_{\text{fit}} < 0.1$ cut on the PM & Cornell sample further reduction in scatter to 0.24 dex (lower right panel of Figure 13). One approach for a practical application of $\sigma_{\text{fit}}$ and PR is to make cuts on each of these parameters, as we have done above, with choices that are best suited to the science application in hand. However, an alternative approach is to combine $\sigma_{\text{fit}}$ and PR into a single parameter, which we explore in the next section.

7 A COMBINATION OF PR AND $\sigma_{\text{fit}}$ AND A DETERMINATION OF FINAL ERROR

In Fig. 10 we find that ~25% of non-detections (i.e., AC3) are predicted to be detections (i.e., false positive); so a cut only on PR is not the best way to distinguish and separate more secure estimations. On the other hand, we have introduced a control parameter taken only from the detections in the fitting step: $\sigma_{\text{fit}}$. We want to
go a step further to introduce a more secure control parameter: a combination of PR and $\sigma_{\text{fit}}$. To do this we notice that in Figure 13, log $\sigma_{\text{fit}}$ is extended in the interval $-2.5 - 0$, a range broader than AC1. The SDSS distribution can be normalized to AC1 via a simple ‘inverse’ mapping process, by setting the minimum and maximum values of the two distributions to 0 and 1, where 1 is the ‘best’ value, representing the minimum scatter in the distribution. We call this normalized distribution $\sigma_{\text{fitN}}$. Figure 15 shows $\sigma_{\text{fitN}}$ for both SDSS (lower panel) and AC1 (upper panel).

We now have two ‘quality control’ parameters that are distinct indicators of the $M_{\text{HI}}/M_*$ estimation: $\sigma_{\text{fitN}}$ and PR, where both have values between 0 and 1. The best estimations will be when both $\sigma_{\text{fitN}}$ and PR are large (i.e. tend towards 1). Moreover, these two parameters can now be trivially combined ($\sigma_{\text{fitN}} \times \text{PR}$), and then normalized to define a single variable, $C_{\text{fgas}}$, that represents the confidence of the $M_{\text{HI}}/M_*$ estimation. This confidence metric also has a value between 0 and 1, where the higher the number, the more secure the $M_{\text{HI}}/M_*$ estimation will be. This is demonstrated in Figure 16 where we plot $\sigma_{\text{fitN}}$ vs PR colour coded by $C_{\text{fgas}}$ for the SDSS sample. The area above the black solid line shows $\sim 150000$ galaxies with $C_{\text{fgas}}>0.5$.

In Fig. 17 we show how the $C_{\text{fgas}}$ metric performs on one of our validation samples, namely the GASS sample. In the top panel, we show the estimated vs. observed $M_{\text{HI}}/M_*$, colour coded by $C_{\text{fgas}}$. The familiar over-estimate of $M_{\text{HI}}/M_*$ by the ANN at low gas fractions is seen. However, the colour coding by $C_{\text{fgas}}$ shows that the greatest offsets correspond to the lowest values of $C_{\text{fgas}}$. This trend is emphasized in the lower panel by plotting the difference between the estimated and observed $M_{\text{HI}}/M_*$ as a function of the observed value. In the inset panel, we show the GASS points with $C_{\text{fgas}}>0.5$; the scatter is reduced to 0.22 dex and with a small systematic error, which can be eliminated by increasing the cut in $C_{\text{fgas}}$ to $\sim 0.7$. For other surveys in our combined sample, a threshold of $C_{\text{fgas}}>0.5$ provides a good nominal selection of robust $M_{\text{HI}}/M_*$ determinations, and is our recommended default value.

In addition to using $C_{\text{fgas}}$ for imposing robustness thresholds on the $M_{\text{HI}}/M_*$ estimations, we can also use the scatter in the observed vs. estimated gas fractions as an indicator of the likely uncertainty therein. To do this, first we construct a combined sample of all the validation sets (633 galaxies) shown in the left panels of Fig. 13. Then, in order to make a compromise and also to avoid any bias, we add 633 galaxies randomly selected from AC1 to our combined validation set. In the left panel of Fig. 18 we show the overall distribution of $C_{\text{fgas}}$ in the combined sample (N=1266) and in the right panel the $M_{\text{HI}}/M_*$ comparison. Again, it can be seen that points that have deviant estimations of $M_{\text{HI}}/M_*$ have low values of $C_{\text{fgas}}$.  

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**Figure 15.** The distribution of $\sigma_{\text{fit}}$ after the application of an inverse mapping process in which the distributions of $\sigma_{\text{fit}}$ are normalized the minimum and maximum values of the distribution of Figure 13, with values of 0 and 1 respectively. Top panel: AC1. Bottom panel: SDSS.

**Figure 16.** PR vs $\sigma_{\text{fitN}}$ colour coded by $C_{\text{fgas}}$, defined as $\sigma_{\text{fit}} \times \text{PR}$. The most reliable $M_{\text{HI}}/M_*$ estimations can be made for galaxies in which both PR and $\sigma_{\text{fitN}}$ are large. The area above the black solid line shows $\sim 150000$ galaxies with $C_{\text{fgas}}>0.5$.

**Figure 17.** Top panel: the estimated vs observed gas fraction colour coded by $C_{\text{fgas}}$ for the GASS sample. The inset show points for $C_{\text{fgas}}>0.5$. Bottom panel: The difference between estimated and observed $M_{\text{HI}}/M_*$ vs the observed value, again colour coded by $C_{\text{fgas}}$. 

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Now, we can obtain a relationship between the scatter of the new sample and $C_{\text{fgas}}$. Since $M_{\text{HI}}/M_*$ estimations are more reliable for higher $C_{\text{fgas}}$, we use data only in region $0.33 < C_{\text{fgas}} < 1$. In this way, we avoid using small $C_{\text{fgas}}$ and also, at the same time, we have adequate data points to fit a function to them and extrapolate it to smaller $C_{\text{fgas}}$. The upper plot of Fig. 19 shows these data points (in red circles) and also a polynomial function fitted to them. Here, we show the scatter in the $M_{\text{HI}}/M_*$ estimation for bins in $C_{\text{fgas}}$ ($C_{\text{fgas}} \pm 0.1$, the value on the x-axis). We can see that, for example, the fitted function shows a scatter of ~0.8 dex for $C_{\text{fgas}} = 0$, which is obtained by an extrapolation. In the lower plot of this figure we use the data points (in blue rectangles) in the same regions to obtain mean offset and again fit the similar function to the points. The average estimated offset (by the function) for $C_{\text{fgas}} < 0.5$ is 0.23 (median 0.22) and the scatter in this region is 0.38. This is well matched with the data from sample GASS, which is the dominant sample in this region. There is no significant average offset for $C_{\text{fgas}} > 0.5$ and the mean scatter in this region is 0.22 dex. The recommended values to use are $0.5 < C_{\text{fgas}} \leq 1$ or $0.09 < \sigma_{\text{fgas}} < 0.29$.

The resulting distribution of $M_{\text{HI}}/M_*$ uncertainties for the SDSS sample is shown in the top panel of Fig. 20 and both $\sigma_{\text{fgas}}$ and $C_{\text{fgas}}$ are provided in our online catalog. The distribution of uncertainties has a narrow peak at small values, corresponding to galaxies with a high $C_{\text{fgas}}$ value, and for which $M_{\text{HI}}/M_*$ is expected to be well estimated. In the lower panel, we plot the star forming main sequence, colour coded by $\sigma_{\text{fgas}}$. The bimodal distribution of $\sigma_{\text{fgas}}$ seen in the top panel of Fig. 20 is clearly present in the distribution of SFR as a function of stellar mass. The ANN can make robust estimations for most galaxies on the main sequence, but performs poorly for passive galaxies, as expected from the distribution of PR shown in the lower panels of Fig. 11. As we have seen repeatedly in this paper, this is due to the nature of the AC1 training sample, whose galaxies are mostly star-forming.

In Table 2 we show the first 10 entries of the ANN estimated $M_{\text{HI}}/M_*$ along with all the control parameters. The full catalog is available in the online material that accompanies this paper.

### 7.1 A final caveat on the derivation of $M_{\text{HI}}$ from $M_{\text{HI}}/M_*$

In this paper we have concentrated on the estimation of $M_{\text{HI}}/M_*$, rather than $M_{\text{HI}}$, although the latter can be derived from the former since we have stellar masses available. In Figure 21 we show the estimated value of $M_{\text{HI}}$ vs. the observed one, colour coded by the number of galaxies in each point. A mild skewness is seen in this plot, whereby low $M_{\text{HI}}$ galaxies have their gas masses slightly over-estimated and vice versa. This effect persists even after the application of $C_{\text{fgas}}$ thresholds. Moreover, we have found that this effect persists if the network is re-trained with $M_{\text{HI}}$ as the target. This check includes a complete re-assessment of the most relevant parameters for $M_{\text{HI}}$ estimation (indeed, the parameter rankings for the prediction of $M_{\text{HI}}$ are quite different to those for $M_{\text{HI}}/M_*$). As was previously shown in the top panel of Figure 8 no such skewness exists in the estimation of $M_{\text{HI}}/M_*$. The skewness in the $M_{\text{HI}}$ estimates is again due to the nature of the training sample, the large observed scatter in $M_{\text{HI}}$ at a given stellar mass and the shallow na-
ture of the survey. We therefore caution that whilst we have demonstrated the robustness of our $M_{\text{HI}}/M_*$ estimates, there may be errors of several tenths of a dex in attempts to convert these to gas masses. It is worth noting that previous efforts to calibrate the gas content of galaxies have similarly used $M_{\text{HI}}/M_*$, rather than $M_{\text{HI}}$.

8 SUMMARY

In this paper we have presented a novel method to estimate HI gas mass fraction and the associated uncertainties based on the patterns found in our data sets, using machine learning methods. The ALFALFA survey is used as our main training sample, and we check our model estimations with a range of validation sets, comprised of the GASS and Cornell surveys and a small sample of post-merger galaxies. We have shown that, for a given set of input parameters, non-linear methods can significantly reduce the scatter in the estimation of $M_{\text{HI}}/M_*$, compared with traditional linear fits. Specifically, we demonstrate that using only the $g - r$ colour and $i$-band surface brightness, a matrix-based model can reduce the scatter in a linear model from 0.32 dex to 0.22 dex.

In order to extend our models to include more galactic parameters, we assess the correlations between $M_{\text{HI}}/M_*$ and 15 parameters derived from the SDSS imaging and spectroscopic datasets. Two performance parameters are presented: R, which represents the relative weights of the fit in minimizing scatter and AUC, which quantifies the physical relevance of a given galaxy parameter to its gas fraction. The AUC ranking shows that $g - r$ colour and stellar mass surface density both strongly govern a galaxy’s gas fraction, with specific SFR and B/T having a further marginal relevance.

We introduce several parameters that permit the assessment of how accurately $M_{\text{HI}}/M_*$ is estimated for a given galaxy in the SDSS. The scatter in the estimation of $M_{\text{HI}}/M_*$ is determined from 20 individually trained networks ($\sigma_{\text{fit}}$) and provides an indication of variation in the fitting process. The inverse normalized version of $\sigma_{\text{fit}}$ ($\sigma_{\text{fitN}}$) has a value from 0 to 1, where higher values indicate a smaller scatter. We also use a pattern recognition technique to identify the similarity of a given galaxy to the ALFALFA detections used in our ANN training set, PR. Again, this parameter has a value between 0 and 1, where higher values indicate a greater similarity to ALFALFA detections, and hence, a higher likelihood that the network can provide an appropriate estimation of $M_{\text{HI}}/M_*$. PR and $\sigma_{\text{fitN}}$ can by multiplied together to yield a single parameter, $C_{\text{fgas}}$, whose value again ranges from 0 to 1, where higher values indicate more robust estimations. $C_{\text{fgas}}$ can also be used to determine an uncertainty associated with a given $M_{\text{HI}}/M_*$ estimation. We demonstrate how the application of these various parameters to the validation sets effectively removes outliers from the ANN estimations. A $C_{\text{fgas}}$ cut of 0.5 yields ~150,000 galaxies with $M_{\text{HI}}/M_*$ estimations from our ANN approach, a factor of ~20 greater than the number of firm 21 cm detections in the 70 per cent data release of ALFALFA. All of the quality control parameters accompany the $M_{\text{HI}}/M_*$ estimations in the online table.

Our catalog of predicted gas fractions offers several advantages over previous attempts to calibrate gas fractions. For example, we have shown that there is no systematic error in our $M_{\text{HI}}/M_*$ estimates with various galaxy properties, and the scatter in our estimates is lower than has been previously achieved. Perhaps most importantly, we have provided a quantitative prescription for assessing the robustness of our estimates, on a galaxy-by-galaxy basis. Our ANN $M_{\text{HI}}/M_*$ estimates also potentially offer advantages over even direct Arecibo observations. The large size of the Arecibo beam (3.5×3.8 arcminutes) is not able to distinguish contributions from multiple galaxies with close separations. Indeed, as shown in Figs. 8 and 9, whilst our predictions reproduce well the gas fractions in the ‘clean’ training sample, galaxies with close companions have observed gas fractions that are typically 0.14 dex above the ANN predictions. Finally, our ANN-based gas fractions extend the number of galaxies in the nearby universe with robust $M_{\text{HI}}/M_*$ estimates by more than an order of magnitude. The growth in sample size yields predictions for large numbers of galaxies in parameter space beyond current samples. For example, most HI surveys are currently limited to $z < 0.06$, whereas our $M_{\text{HI}}/M_*$ predictions extend to $z \sim 0.2$. Indeed, our catalog contains ~61,000 robust ($C_{\text{fgas}} > 0.5$) $M_{\text{HI}}/M_*$ predictions at $z > 0.1$. Figure 22 shows the distribution of gas fractions for the GASS, HiGHz (Catinella & Cortese 2006, whereas our $M_{\text{HI}}/M_*$ estimates are lower than has been previously achieved. Perhaps most importantly, we have provided a quantitative prescription for assessing the robustness of our estimates, on a galaxy-by-galaxy basis. Our ANN $M_{\text{HI}}/M_*$ estimates also potentially offer advantages over even direct Arecibo observations. The large size of the Arecibo beam (3.5×3.8 arcminutes) is not able to distinguish contributions from multiple galaxies with close separations. Indeed, as shown in Figs. 8 and 9, whilst our predictions reproduce well the gas fractions in the ‘clean’ training sample, galaxies with close companions have observed gas fractions that are typically 0.14 dex above the ANN predictions. Finally, our ANN-based gas fractions extend the number of galaxies in the nearby universe with robust $M_{\text{HI}}/M_*$ estimates by more than an order of magnitude. The growth in sample size yields predictions for large numbers of galaxies in parameter space beyond current samples. For example, most HI surveys are currently limited to $z < 0.06$, whereas our $M_{\text{HI}}/M_*$ predictions extend to $z \sim 0.2$. Indeed, our catalog contains ~61,000 robust ($C_{\text{fgas}} > 0.5$) $M_{\text{HI}}/M_*$ predictions at $z > 0.1$. Figure 22 shows the distribution of gas fractions for the GASS, HiGHz (Catinella & Cortese 2006).
Table 2. Catalog of ANN estimated $M_{\rm HI}$/$M_\ast$ along with all the control parameters described in this paper. The first four columns (ID, RA, Dec and $z$) are taken from the SDSS database. The first 10 entries are shown here; the full catalog is available in the online material that accompanies this paper.

| ID (SDSS) | RA     | Dec     | $z$     | $\log M_\ast$ | $\log M_{\rm HI}/M_\ast$ | $C_{\rm gas}$ | PR     | $\sigma_{\rm fitN}$ | $\sigma_{\rm fit}$ | $\sigma_{\rm fit,\text{gas}}$ |
|-----------|--------|---------|---------|---------------|---------------------------|---------------|-------|----------------------|----------------------|------------------------|
| 587736871457719496 | 237.4554943 | 34.90548896 | 0.1670 | 11.4110 | -0.9456 | 0.1178 | 0.1972 | 0.5249 | 0.1006 | 0.6169 |
| 588018091616174223 | 237.54070295 | 34.70075261 | 0.0701 | 9.9120 | -0.2466 | 0.3310 | 0.4111 | 0.7075 | 0.0269 | 0.3913 |
| 587736871457850591 | 237.6627645 | 34.78976388 | 0.1511 | 10.7487 | -0.9023 | 0.3493 | 0.4945 | 0.6207 | 0.0503 | 0.3778 |
| 587736871457850627 | 237.691242 | 34.70155096 | 0.0751 | 10.5140 | -0.5599 | 0.4970 | 0.6302 | 0.6929 | 0.0298 | 0.2922 |
| 587739130812694794 | 225.2513585 | 29.37813267 | 0.0793 | 10.6881 | -0.8689 | 0.3917 | 0.4506 | 0.7637 | 0.0179 | 0.3491 |
| 587736781457916109 | 237.8366547 | 34.56556243 | 0.0751 | 10.5140 | -0.8689 | 0.3917 | 0.4506 | 0.7637 | 0.0179 | 0.3491 |
| 587736781457850627 | 237.6627645 | 34.78976388 | 0.1511 | 10.7487 | -0.9023 | 0.3493 | 0.4945 | 0.6207 | 0.0503 | 0.3778 |
| 587736781457719496 | 237.4554943 | 34.90548896 | 0.1670 | 11.4110 | -0.9456 | 0.1178 | 0.1972 | 0.5249 | 0.1006 | 0.6169 |

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2015) and ALFALFA.70 samples (green, red and grey points respectively, compared with the ANN predictions (blue contours and points, where the latter show individual galaxies with counts below the lowest contour). It can be seen that the ANN predicts a significant population of high stellar mass ($\log M_\ast > 11$) galaxies with gas fractions $\log M_{\rm HI}/M_\ast > -1$. Such galaxies are largely absent from current samples and are predicted to contain some of the highest HI masses in the local universe. Follow-up observations of these high stellar mass, high HI mass galaxies to confirm our ANN predictions would be of great interest.
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