Is the resonance $D(2637)$ really a radial excitation?

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Abstract

We consider various possible identifications of the quantum numbers of the resonance $D(2637)$ recently observed by DELPHI in the $D^*\pi\pi$ channel. We argue that in spite of a good agreement of the measured mass with the quark-model prediction for the radial excitation, a total width as small as $\leq 15$ MeV is hardly compatible with its identification as a radial charm excitation. The $J^P = 2^-, 3^-$ orbitally excited mesons with such a mass could have widths of the observed order of magnitude. However in this case one would expect two neighbouring states with the mass difference of about 30-50 MeV corresponding to the nearly degenerate components of the heavy-meson multiplet with light-quanta angular momentum $j=5/2$, and moreover, according to the quark-model predictions the mass of the orbital excitation should be more than 50 MeV larger than 2637 MeV. Thus we conclude that, at present, we find no fully convincing understanding of the quantum numbers of the observed resonance.

Recently, DELPHI has observed a narrow resonance $D(2637)$ in the $D^*\pi\pi$ channel with a total width of less than the detector resolution: 15 MeV [1]. The mass of the observed resonance turns out to be in perfect agreement with predictions of the quark models [2, 3] for the charm radial excitation $D^{**}(J^P = 1^-)$. This coincidence has lead to a quick identification of the discovered resonance state with the radially excited vector charm meson.

In this letter we reconsider the identification of the quantum numbers of the observed resonance by submitting it to the following criteria:

i the resonance mass should be $\simeq 2637\pm 6$ MeV;

ii the resonance width should be $\leq 15$ MeV;

iii the resonance should have a sizeable branching ratio of the channel $D(2637) \to D^*\pi\pi$ in which it has been observed.

We estimate the decay rate of a radial excitation of the reported mass and find that, although the partial widths are strongly model-dependent, the total rate is conservatively estimated to be significantly larger than about 50 MeV. We thus conclude that the observed width of only $\leq 15$ MeV is hardly compatible with its identification as a radial charm excitation.

An identification of the observed resonance with an orbital charm excitation seems to be more favourable: two mesons with the quantum numbers $J^P = 2^-, 3^-$ (quark orbital momentum $L = 2$) could have a width of the observed order of magnitude. However, theoretical estimates yield a mass for the latter orbitally-excited states approximately $\geq 50$ MeV above the reported value. In addition, in this case, one would expect two neighbouring states with a mass difference of about 30-50 MeV (similar to the mass difference between $D_{s+}(2460)$ and $D_{s+}(2420)$ in the $j=3/2$ positive-parity sector), while only one resonance has been reported. Altogether we conclude that, at the moment, there is no fully convincing understanding of the quantum numbers of the observed resonance.

Our analysis is based on combining the heavy-quark symmetry relations for the transition amplitudes between heavy mesons through the emission of the light hadrons with the quark-model estimates. Namely, we estimate the decay rates of radially and orbitally excited charm resonances into $D\pi, D^*\pi, D\pi\pi$ and $D^*\pi\pi$ assuming the resonance mass of 2637 MeV as measured by DELPHI.

To obtain estimates of the branching ratios of the three-body decays with two pions in the final state we treat them as cascade two-body decays $D(2637) \to (D, D^*)R \to (D, D^*)\pi\pi$ through the intermediate Breit-Wigner resonance with relevant quantum numbers. The status of the $(\pi\pi)_{l=0, I=0}$ channel is not well-defined,
and we varied the corresponding \( \sigma \) resonance mass in the range 400 \( \div \) 800 MeV and the width in the range \( \Gamma(\sigma) \approx 700 \pm 900 \) MeV. The low-energy \( (\pi\pi)_l=1 \) partial-wave is dominated by \( \rho \). Higher partial waves of the \( \pi\pi \) system give negligible contributions.

Table 1 lists the candidate charm states and their allowed decay modes as given by the spin-parity conservation.

For heavy-meson decays, additional constraints are given by the heavy-quark (HQ) symmetry \( [4] \). Namely, in the heavy quark limit the heavy quark spin decouples from other degrees of freedom and remains conserved in hadron transitions. Thus, in strong decays of heavy hadrons, the total angular momentum of the heavy and light degrees of freedom are conserved separately, in addition to the conservation of the parity and total angular momentum. Hence, with respect to strong decays, heavy hadrons can be assigned an additional conserved quantum number, \( j \), which is the total angular momentum of the light degrees of freedom. The consequences of the HQ symmetry for hadron transitions have been worked out by Isgur and Wise \( [3] \). Namely, the HQ symmetry allows one to relate to each other different amplitudes of strong transitions between the states with fixed \( j \) and \( j' \), the latter being the angular momenta of the light degrees of freedom in the initial and final hadronic states, respectively. Table 2 also presents the HQ symmetry allowed transitions in terms of the few independent amplitudes. The \( O(1/m_Q) \) corrections in the effective Hamiltonian yield corrections to these HQ symmetry relations. However, for our order-of-magnitude analysis these corrections are generally unimportant and will be neglected unless explicitly specified.

For the calculation of the independent amplitudes \( (\alpha, \beta, \delta, \text{ and } \xi \text{ in Table 3}) \) one needs a non-perturbative approach. We apply here a naive quark-pair-creation \( (3P_0) \) model \( [3] \) which, in spite of its simplicity, has proven to provide a reasonable quantitative description of the two-body hadronic decays. The model is based on the assumption that the spectator quarks do not change their SU(3) quantum numbers, nor their momenta and spins. The created quark-antiquark pair should be therefore in a \( |\bar{q}q\rangle \rangle \) wave function, \( q \) and \( \bar{q} \) being the quark and antiquark, respectively. Table 3 also shows the HQ symmetry allowed transitions in terms of the few independent amplitudes. The remaining parameters to be fixed are the size of the light-light and heavy-light wave functions. They are given by two radii the light-light \( (R) \) and heavy-light \( (R_D) \). For the ground state these radii are simply \( R^2 = 2/3 <(\vec{r}_q - \vec{r}_q)^2> \).

The heavy meson has to be smaller than the light one. From estimates with different potentials, we assume the radii to satisfy the relation \( R_D^2/R^2 \approx 0.5 \pm 0.7 \) and allowed \( R^2 = 6 \pm 9 \) GeV\(^2\): such values of \( R \) are compatible with previous descriptions of the spectrum and decay rates \( [3] \) and in addition, we check, Table 3, that the \( 3P_0 \) model with these parameters describes correctly the experimentally observed \( D_{2+} \rightarrow (D, D^*)\pi \) decay rates. For the \( D^* \rightarrow D\pi \) transition, Table 3, we also express the decay width in terms of a dimensionless coupling constant defined as follows

\[
\langle D^0(p_2)\pi^+(q)|D^{*+}(p_1)\rangle = g_{D\pi\pi}^\mu q_\mu \epsilon_1^\mu,
\]

where \( \epsilon(\mu) \) is the vector-meson polarization vector, the states being normalized covariantly and where we have omitted the momentum conservation delta function. The \( g_{D\pi\pi}^\mu \) of the \( 3P_0 \) model agrees with the experimental bound although seems to be a bit small compared with other theoretical estimates. Notice however that the \( 3P_0 \) model is non-relativistic and as such does not describe properly the soft pion limit. In the \( D^* \rightarrow D\pi \) decay the pion is produced almost at rest and the model is indeed expected to underestimate the coupling constant as observed. But the model has proven to work rather well for hadronic resonance decays in the usual domain of the emitted pion energies, i.e. \( E_\pi \approx 300-600 \) MeV. Indeed, as we will see later, the model estimates for the decay of a heavier radially-excited \( D' \) are in better agreement with covariant methods.

Having thus fixed the ranges of the basic parameters from the light sector and \( D_{2+} \rightarrow (D, D^*)\pi \) decays, we apply the model to the analysis of the decay of radially and orbitally excited negative-parity states.

**Radial excitation \( D_{1-} \)**

The decay of the vector radially-excited \( D_{1-} \) into \( D\pi \) and \( D^*\pi \) is governed by the following amplitudes

\[
\langle D^{00}(p_2)\pi^+(q)|D^{*-+}(p_1)\rangle = g_{D^{*-}\pi}\epsilon_1^\mu q_\mu \epsilon_2^\mu
\]

\[
\langle D^{*0}(p_2)\pi^+(q)|D^{*-+}(p_1)\rangle = ig_{D^{*-}\pi}\epsilon_1^\mu \epsilon_2^\mu p_2^\nu \alpha_1^\nu \beta_2^\nu
\]

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with the coupling constants $g_{D^*D\pi}$ and $g_{D^*D\pi}$ to be determined on the basis of a dynamical approach. In the heavy-quark limit the constants are related to each other as follows

$$g_{D^*D\pi} = M_{D^*}, \quad g_{D^*D\pi}. \quad (3)$$

To estimate these coupling constants one can applied various theoretical approaches. For instance, one may use the PCAC definition of the pion field (although the pion is not soft at all in this decay) in which case a complicated problem of calculating the coupling constants of interest is reduced to a relatively simpler one of calculating the $D^* \to (D, D^*)$ transition form factors through the axial-vector current. Namely, one finds

$$g_{VP\pi} = \frac{1}{f_{\pi}}[(MV + MP)A_1(0) + (MV - MP)A_2(0)]. \quad (4)$$

For estimating the meson transition form factors $f$ and $a_+$ we used the relativistic dispersion approach of Ref. [9] adopting wave functions of the ground-state and radially-excited $D$ which provide the values $f_D \simeq 200$ MeV, $f_{D^*} \simeq 240$ MeV and $f_{D'} \leq 400$ MeV.

Actually, one observes a suppression of the form factor $A_1$ in the $D^* \to D$ transition, as compared with the $D^* \to D$ one, because of the orthogonality of the wave functions of the orbitally-excited and the ground states $D_{1^+}$: namely, $A_{D^*D}^{1+}(0) \simeq 0.5$ and $A_{D^*D}^{1+}(0) \simeq 0.2 - 0.3$. In the absence of the second term of (4) the $D^* \to (D, D^*)$ transition would thus be suppressed by a factor 2 to 4 in rate. However such a suppression due to orthogonality in the soft pion limit is expected to be reduced when the momentum recoil (which is large in this case) is taken into account. The transition $D^* \to (D, D^*)$ would only produce a soft pion if the mass of the $D^*$ would be close to that of the $D^{(*)}$, thus canceling the second term in (4). But we are not in such a situation, the second term is not negligible since $A_{D^*D}^{2+}(0) \simeq 0.9 - 2.5$ and one ends up with the relation $g_{D^*D\pi} = (0.5 - 1.5)g_{D^*D\pi}$ where $g_{D^*D\pi} \simeq 15$. Notice that the large uncertainties in $g_{D^*D\pi}$ are connected with a strong sensitivity of the latter to the subtle details of the sign-changing wave function of the radially-excited state. Altogether, and using the HQ symmetry relation (3), we find a rather large range for our estimate: $\Gamma(D^* \to D\pi) = 20 - 200$ MeV and $\Gamma(D^* \to D^*\pi) = 25 - 250$ MeV.

To obtain more precise estimates we also used the $3P_0$ model and found values in the region $g_{D^*D\pi} = 13 \pm 15$ which are compatible with PCAC based estimates. Table 4 presents the $3P_0$ model estimates of the $D^*_{2^+}(2637)$ decay rates.

So for the sum of the decay rates of the channels $\Gamma(D^* \to D\pi) + \Gamma(D^* \to D^*\pi)$ one finds rather uncertain estimates ranging from 40–50 MeV, which is only slightly above the reported width of the DELPHI resonance, to several hundreds MeV, which is far above.

Notice that anyway an important contribution to the total width of the radially-excited $D^*$ is given by the decay channel $D^* \to D^{(*)}\pi\pi$ allowed by the HQ symmetry in the S-wave (Table 4). The $3P_0$ model estimate of its rate is 120-160 MeV. So, conservatively, one cannot expect the total width of the radially excited state $D^*_{2^+}(2637)$ to be less than 50 MeV and presumably it should be much broader. This has the double effect of predicting a large branching ratio for the observed $D^*\pi\pi$ channel, but unhappily also a large total width. Thus we conclude that identification of the resonance $D(2637)$ as a radial excited $J^P = 1^-$ charmed meson is hardly compatible with the reported value of the resonance total width $\leq 15$ MeV.

**Orbital excitations $D_{2^-,3^-}$**

To proceed with these states, we assume the HQ symmetry at the level of the transition amplitudes but use the physical masses to compute the relevant phase-space factors. Namely, we calculate the transition amplitudes of the modes $D_{2^-,j=5/2} \to D^{(*)}\pi$ and $D_{3^-,j=5/2} \to D^{(*)}\pi\pi$ and determine all other related amplitudes through the HQ symmetry relations listed in Table 4.

Notice that the transition of $D_{2^-,j=5/2}$ into the four positive parity states $D_{2^+,j=1/2,3/2,5/2} \to D^{(*)}\pi\pi$, is allowed, in the heavy-quark limit, only in the D-wave, since the latter positive parity states with $j = 1/2$ or $j = 3/2$ cannot be produced with an S-wave pion from a $j = 5/2$. A rough estimate convinced us that, due to the small final momenta, these decays will be even more suppressed than the other $D^{(*)}\pi\pi$ channels to be discussed later.

Taking into account the phase-space factors yields the results listed in Tables 3 and 4. Some comments on the presented numbers are in order. It can be seen from Table 4 that the decay into $D^{(*)}\pi\pi_{j=1}$ can go

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We are indebted to Damir Becirevic and Alain Le Yaouanc for attracting our attention on this suppression which they find even stronger in a related approach [11] based on the Dirac equation.
through a P-wave between $\pi\pi$ and the charmed meson. Compared to the F-wave for $D^{(*)}\pi$ and the D-wave for $D^{(*)}(\pi\pi)_{J=0}$, this decay channel has to be considered even though only a small part of the $\rho$-meson Breit-Wigner tail is included in the phase space. The $D^{(*)}(\pi\pi)_{J=1}$ is suppressed by one order of magnitude as compared to the $D^{(*)}(\pi\pi)_{J=0}$ due to the centrifugal barrier suppression.

The physical $D_{3^-}$ state is dominantly the $D_{3^-j=5/2}$ one, and one can safely estimate $\Gamma(D_{3^-}) \simeq \Gamma(D_{3^-j=5/2})$, since a possible small admixture of the narrow $D_{3^-j=7/2}$ state does not practically change its width. For the $D_{2^-}$ state however this is not the case: one might expect a sizable increase of the width of the physical $D_{2^-}$ with respect to the width of the $D_{2^-j=5/2}$. For example, even a small admixture of the $D_{2^-j=3/2}$ to the dominant $D_{2^-j=5/2}$ may increase the total width of the physical $D_{2^-}$ meson, since one expects $\Gamma(D_{2^-j=3/2}) \gg \Gamma(D_{2^-j=5/2})$ due to the $P$-wave decay mode $D_{2^-j=3/2} \to D^*\pi$ allowed by the HQ symmetry.

A similar situation has been observed in the positive-parity $D_{j=3/2}$ multiplet: namely, the experimental ratio of the decay rate of $D_{1^+}(2420)$ which is dominantly $D_{1^+,j=3/2}$ and the decay rate of $D_{2^+}(2460)$ which is practically the rate of a pure $D_{2^+,j=3/2}$ is:

$$\frac{\Gamma(D_{1^+}(2420))}{\Gamma(D_{2^+}(2460))} = 0.71$$

that is about twice larger than the leading-order HQ symmetry estimate

$$\frac{\Gamma(D_{1^+,j=3/2})}{\Gamma(D_{2^+,j=3/2})} = 0.3.$$  

The latter value is obtained by assuming the leading-order HQ symmetry relations between the amplitudes and taking the physical masses of the corresponding states for the calculation of the relevant phase-space factors.

This discrepancy can be solved by invoking the $1/m_c$ corrections e.g. by assuming a small admixture of a broad $D_{1^+,j=1/2}$ to a narrow $D_{1^+,j=3/2}$ in the physical $D_{1^+}(2420)$ as proposed in Ref. [1]. Another possibility (see [12] and refs therein) is to have a rather strong increase in the decay rate of a pure $D_{1^+,j=3/2}$ due to the $1/m_c$ corrections to the effective Hamiltonian (A combination of these two variants is of course also possible). Anyway the $D_{j=3/2}$ positive-parity sector prompts that the net effect of the $1/m_c$ corrections is a doubling of the ratio $\Gamma(D_{j=1/2,j})/\Gamma(D_{j=1/2,j})$.

Hence the obtained value of the $D_{2^-j=5/2}$ width should be considered as a lower bound of the physical $D_{2^-}$ width. Still, from the comparison with the $j=3/2$ sector, we expect the physical $D_{2^-}$ width not to exceed the HQ estimate of the $D_{2^-j=5/2}$ state by much more than a factor 2.

Similarly, the subleading $1/m_c$ effects can influence also the rate of the transition $D_{2^-j=5/2} \to D^{**}_{j=1/2,3/2}\pi$. A rough estimate based on the eqs. (13-18), where an S-wave $O(1/m_c)$ decay leads to an increase of about 10 MeV of the width, and considering that the phase space is smaller in the decay of a $D(2634)$ into positive parity resonances than in the decay of the latter into the ground state, we are not too worried. Still, a closer scrutiny of these $O(1/m_c)$ effects would be welcome.

Finally, we conclude that the orbitally excited $D_{2^-}$ and $D_{3^-}$ charm resonances with the mass in the region of $2640$ MeV can have the width of the order reported by DELPHI. However, an identification of the resonance $D(2637)$ with an orbital excitation in the charm system is not straightforward since the reported mass seems to be significantly smaller than the theoretical expectations.

For example, the Godfrey-Isgur (GI) model [13] which describes with a good accuracy nearly all known mesons predicts the mass of the $D_{3^-}$ to be $2830$ MeV. Taking into account that for the $D_{2^+}$ state the GI model gives $2500$ MeV which is $40$ MeV heavier than the observed value of $2460$ MeV, we can expect for the $D_{3^-}$ state the mass $\leq 2800$ MeV. From the typical mass-splitting between the states with neighbouring $j=2$ and $j=3$ in the GI model, one could expect the $D_{2^-}$ mass near $2750$ MeV.

The quark-gluon string model [13] has predicted the masses of the charm resonances $M(D_{2^-}) = 2660 \pm 70$ MeV and $M(D_{3^-}) = 2760 \pm 70$ MeV. The partial rates of the latter were estimated to be $\Gamma(D_{3^-} \to D\pi) = 1.3 \pm 2.0$ MeV and $\Gamma(D_{3^-} \to D^*\pi) = 3.5 \div 7.0$ MeV yielding the total width of $D_{3^-}$ in the necessary range. On the other hand a large mass splitting between the $2^-$ and $3^-$ charm states signals that the $D_{2^-}$ in the quark-gluon string model contains a big admixture of the lighter state $D_{2^-j=3/2}$. So, for the mass of the pure $D_{2^-j=5/2}$ state one would expect a higher value.

Altogether, from the above theoretical analyses we could expect the mass of the $D_{2^-j=5/2}$ in the region of $2650 \div 2750$ MeV which is only marginally compatible with the reported resonance mass of $2637$ MeV.

The branching ratios of the $D^*\pi\pi$ channels in table 5 and 6 are in the range of a few percent, which may seem a little small for these channels to have been observed. Still some non-resonant $\pi\pi$ contributions, which are difficult to estimate, will add up.
Summing up, if the resonance $D(2637)$ with the width of $\leq 15$ MeV is confirmed by further analyses (at the moment CLEO and OPAL do not see it \[1\]) then the theoretical understanding of its quantum numbers is not clear: in spite of the coincidence of the observed mass with the predicted mass of the radial charm excitation $D_{1^{-}}$, its interpretation as a radial excitation is completely ruled out by the small observed width.

On the other hand, although an identification of the state with the $D_{2^{-}}$ orbital excitation seems to be appropriate from the viewpoint of the total width, its mass seems to be too low compared with the quark-model theoretical estimates. Its $D^{*}\pi\pi$ branching ratio of a few percent is a little small. More importantly in the case of the orbital excitations in this mass region, one would expect two neighbouring resonances with the width of order several MeV each and with the mass difference of about $30-50$ MeV, corresponding to the $D_{2^{-}}$ and $D_{3^{-}}$ states. The published plots do not show any sign of a neighbouring $3^{-}$ resonance.

Thus we conclude that the experimental confirmation of this resonance would put forward a challenge of its proper theoretical understanding, unless a neighbouring slightly heavier resonance was found.

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Table 1: Decay modes of possible candidate states allowed by spin-parity conservation. Modes allowed also by the HQ symmetry, i.e. corresponding also to a separate conservation of the total angular momentum of the light degrees of freedom, are listed in bold and HQ symmetry relations for the corresponding amplitude squared from [8] are given (without phase-space factors included).

|      | $D_{1^{-},j=1/2}$ | $D_{2^{-},j=5/2}$ | $D_{3^{-},j=5/2}$ |
|------|------------------|------------------|------------------|
| $D\pi$ | $L=1[\frac{\alpha^2}{4}]$ | $-$ | $L=3[\frac{\alpha^2}{2}]$ |
| $D^*\pi$ | $L=1[\frac{\alpha^2}{4}]$ | $L=1.3[\alpha^2]$ | $L=3[\frac{\alpha^2}{2}]$ |

$D^{**}\pi$:  
- $D_{0^+,j=1/2}^*$: $L=2$  
- $D_{1^+,j=1/2}^*$: $L=0.2$  
- $D_{1^+,j=3/2}^*$: $L=0.2$  
- $D_{2^+,j=3/2}^*$: $L=2$  
- $D(\pi\pi)_{l=0}$: $L=2[\frac{\delta^2}{4}]$  
- $D^*(\pi\pi)_{l=0}$: $L=0.2$  
- $D(\pi\pi)_{l=1}$: $L=1$  
- $D^*(\pi\pi)_{l=1}$: $L=1.3$  

Table 2: $D^* \rightarrow D\pi$ transition.

| $g_{D^*D\pi}$ | Exp. [7] | $^{3}P_0$ model | Other estimates (see refs in [8]) |
|----------------|----------|-----------------|-----------------------------------|
| $\Gamma(D^* \rightarrow D\pi)$ | $< 21$ | $7 \pm 1$ | $7 \pm 21$ |
| $\Gamma(D^* \rightarrow D\pi)$ | $< 90$ KeV | $7 \pm 10$ KeV | $7 \pm 90$ KeV |

Table 3: Decay rate of the $D_{2^+} \rightarrow (D, D^*)\pi$ transition.

| $\Gamma(D\pi)/\Gamma(D^*\pi)$ | Exp. [7] | $^{3}P_0$ model |
|--------------------------------|----------|-----------------|
| $\Gamma_{tot}$ | $2.3 \pm 0.9$ | $2.6$ |
| $\Gamma_{tot}$ | $23 \pm 5$ MeV | $11 \sim 22$ MeV |

Table 4: Decay rates of the $D_{1^-} \rightarrow (D, D^*)\pi$ transition in the $^{3}P_0$ model.

- $D'(1^-) \rightarrow D\pi$: 150–220 MeV  
- $D'(1^-) \rightarrow D^*\pi$: 200–300 MeV  
- $D'(1^-) \rightarrow D^*\pi\pi$: 120–160 MeV

Table 5: Branching ratios of the $D_{2^-,j=5/2}$ decays in the $^{3}P_0$ model.

| $Br(D\pi)$ | $-$ |
| $Br(D^*\pi)$ | $0.65 \sim 0.7$ |
| $Br(D(\pi\pi)_{l=0})$ | $< 0.01$ |
| $Br(D(\pi\pi)_{l=1})$ | $0.2 \sim 0.3$ |
| $Br(D^*(\pi\pi)_{l=0})$ | $< 0.003$ |
| $Br(D^*(\pi\pi)_{l=1})$ | $0.02 \sim 0.03$ |
| $\Gamma_{tot}$ | $6 \sim 14$ MeV |

Table 6: Branching ratios of the $D_{3^-,j=5/2}$ in the $^{3}P_0$ model.

| $Br(D\pi)$ | $0.65 \sim 0.74$ |
| $Br(D^*\pi)$ | $0.23 \sim 0.27$ |
| $Br(D(\pi\pi)_{l=0})$ | $-$ |
| $Br(D(\pi\pi)_{l=1})$ | $< 0.001$ |
| $Br(D^*(\pi\pi)_{l=0})$ | $< 0.003$ |
| $Br(D^*(\pi\pi)_{l=1})$ | $0.03 \sim 0.06$ |
| $\Gamma_{tot}$ | $8 \sim 22$ MeV |