Mass of the Electroweak Monopole

Y. M. Cho,1,2 Kyoungtae Kimm,3 and J. H. Yoon4

1Administration Building 310-4, Konkuk University, Seoul 143-701, Korea
2School of Physics and Astronomy, Seoul National University, Seoul 151-747, Korea
3Faculty of Liberal Education, Seoul National University, Seoul 151-747, Korea
4Department of Physics, Konkuk University, Seoul 143-701, Korea

We present two independent methods to estimate the mass of the electroweak monopole. Our result strongly implies the existence of a genuine electroweak monopole of mass around 4 to 10 TeV, which could be detected by MoEDAL at present LHC. We emphasize that the discovery of the the electroweak monopole should be the final test of the standard model.

PACS numbers: PACS Number(s): 14.80.Hv, 11.15.Tk, 12.15.-y, 02.40.+m

The recent “discovery” of Higgs particle at LHC has reconfirmed that the standard model describes the real world [1]. Indeed it has been interpreted that the standard model has passed the “final” test with the discovery. But we emphasize that the final test of the standard model should come from the discovery of the electroweak (“Cho-Maison”) monopole, because the theory predicts the electroweak monopole [2, 3]. It has the monopole topology, and naturally accommodates the Cho-Maison monopole as the electroweak generalization of the Dirac’s monopole [4].

Ever since Dirac predicted the existence of the monopole, the monopole has been an obsession [5–9]. The Abelian monopole has been generalized to the non-Abelian monopoles by Wu and Yang [5, 6] who showed that the pure SU(2) gauge theory allows a point-like monopole, and by ’t Hooft and Polyakov [7, 8] who have constructed a finite energy monopole solution in Georgi-Glashow model as a topological soliton. Moreover, the monopole in grand unification has been constructed by Dokos and Tomaras [9].

But it has been asserted that the Weinberg-Salam model has no topological monopole of physical interest [10]. The basis for this “non-existence theorem” is that the quotient space $SU(2) \times U(1)/U(1)_{\text{em}}$ allows no non-trivial second homotopy which can accommodate the monopole.

This claim, however, is unfounded [2, 3]. This is because the Weinberg-Salam model, with the hypercharge $U(1)_Y$, could be viewed as a gauged $CP^1$ model in which the (normalized) Higgs doublet plays the role of the $CP^1$ field. So, if the standard model is correct, the electroweak monopole must exist. This makes the experimental detection of the electroweak monopole an urgent matter.

Fortunately the latest MoEDAL detector (“The Magnificent Seventh”) at LHC is actively searching for such monopole [11]. To help the experiment discover the monopole, however, we need a theoretical estimate of the monopole mass. The purpose of this Letter is to estimate the mass of the electroweak monopole to be around 4 to 10 TeV.

The importance of the electroweak monopole is twofold. First, it is the straightforward and natural generalization of the Dirac monopole to the electroweak theory which is unavoidable when the electrodynamics is unified to the electroweak theory. This means that the monopole which should exist in the real world is not likely to be the Dirac monopole but this one.

Second, unlike the Dirac monopole which is optional, the electroweak monopole must exist because the standard model has the monopole topology. This means that the final test of the standard model is not the discovery of the Higgs particle, but the confirmation of the electroweak monopole. Indeed the discovery of the electroweak monopole should be regarded as the topological test of the standard model which has never been done before.

The Cho-Maison monopole may be viewed as a hybrid between the Dirac monopole and the ’tHooft-Polyakov monopole, because it has a $U(1)$ point singularity at the center even though the SU(2) part is completely regular. Consequently it carries an infinite energy at the classical level, so that the mass of the monopole can be arbitrary. A priori there is nothing wrong with this, but nevertheless one may wonder whether one can estimate the mass of the electroweak monopole. In the following we provide three independent methods to estimate the monopole mass.

Consider the standard Weinberg-Salam model,

$$\mathcal{L} = -|D_\mu \phi|^2 - \frac{\lambda}{2} (|\phi|^2 - \frac{\mu^2}{\lambda})^2 - \frac{1}{4} F_{\mu\nu}^2 - \frac{1}{4} G_{\mu\nu}^2,$$

$$D_\mu \phi = (\partial_\mu - i \frac{g}{2} \vec{A} \cdot \vec{\tau}) \phi - \frac{g'}{2} B_\mu \phi, \quad (1)$$

where $\phi$ is the Higgs doublet, $F_{\mu\nu}$, $\vec{A}_\mu$ and $G_{\mu\nu}$, $B_\mu$ are
the gauge fields of $SU(2)$ and $U(1)_Y$. Now choose the ansatz in the spherical coordinates $(t, r, \theta, \varphi)$

$$
\phi = \frac{1}{\sqrt{2}} \rho(r) \xi(\theta, \varphi), \quad \xi = i \left( \frac{\sin(\theta/2) e^{-i\varphi}}{-\cos(\theta/2)} \right),
$$

$$
\vec{A}_\mu = \frac{1}{g} A(r) \partial_\mu t \hat{r} + \frac{1}{g} f(r - 1) \hat{r} \times \partial_\mu \hat{r},
$$

$$
\hat{r} = -\xi^\dagger \vec{r},
$$

$$
B_\mu = \frac{1}{g^2} B(r) \partial_\mu t - \frac{1}{g}(1 - \cos \theta) \partial_\mu \varphi.
$$

(2)

The ansatz has an apparent string singularity along the negative z-axis in $\xi$ and $B_\mu$. But they are a pure gauge artifact which can easily be removed making $U(1)_Y$ non-trivial. So the above ansatz describes a most general spherically symmetric ansatz of the electroweak dyon.

But this, of course, requires $U(1)_Y$ to be non-trivial. In other words, we need $U(1)_Y$ to be non-trivial to have the monopole. So the important question is why $U(1)_Y$ must be non-trivial.

To understand this choose the unitary gauge with the gauge transformation

$$
\xi \to U \xi = \begin{pmatrix} 0 \\ 1 \end{pmatrix},
$$

$$
\vec{A}_\mu \to \frac{1}{g} \begin{pmatrix} -f(r)(\sin \varphi \partial_\mu \theta + \sin \theta \cos \varphi \partial_\mu \varphi) \\ f(r)(\cos \varphi \partial_\mu \theta - \sin \theta \sin \varphi \partial_\mu \varphi) \\ A(r) \partial_\mu t - (1 - \cos \theta) \partial_\mu \varphi \end{pmatrix},
$$

(3)

and express the electromagnetic and Z-boson potentials $A_\mu^{(\text{em})}$ and $Z_\mu$ by

$$
\begin{pmatrix} A_\mu^{(\text{em})} \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_w & \sin \theta_w \\ -\sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ A_\mu^3 \end{pmatrix}
$$

$$
= \frac{1}{\sqrt{g^2 + g^2'}} \begin{pmatrix} g & g' \\ -g' & g \\
\end{pmatrix} \begin{pmatrix} B_\mu \\ A_\mu^3 \end{pmatrix},
$$

(4)

where $\theta_w$ is the Weinberg angle. Clearly $A_\mu^3$ has the string singularity along the negative z-axis. This is because the $U(1)$ subgroup of $SU(2)$ is non-trivial. This justifies the string singularity in $B_\mu$, because $A_\mu^3$ already has the singularity. In other words, it is inconsistent (i.e., in violation of self-consistency) to insist $U(1)_Y$ to be trivial. This tells that the standard model must have the monopole.

With \cite{2} we can express the Lagrangian \cite{1} in terms of the physical fields

$$
\mathcal{L} = -\frac{1}{4}(\partial_\mu \rho)^2 - \frac{\lambda}{8}(\rho^2 - 2\rho^2 -\rho^2) - \frac{1}{4} F_{\mu \nu}^{(\text{em})^2} - \frac{1}{4} Z_{\mu \nu}^2
$$

$$
-\frac{1}{2} |(D_\mu^{(\text{em})} W_\mu - D_\nu^{(\text{em})} W_\mu)|^2 + i\epsilon g(Z_\mu W_\nu - Z_\nu W_\mu)^2
$$

$$
-\frac{g^2}{8} + \frac{g^2}{4} \rho^2 Z_\mu^2 - \frac{g^2}{4} \rho^2 W_\mu^2 + i\epsilon g Z_{\mu \nu} W_\mu W_\nu
$$

$$
+ i\epsilon F_{\mu \nu}^{(\text{em})} W_\mu W_\nu + \frac{g^2}{4} (W_\mu W_\nu - W_\nu W_\mu)^2,
$$

(5)

where $\rho$ and $W_\mu$ are the Higgs and $W$-boson, $D_\mu^{(\text{em})} = \partial_\mu + i\epsilon A_\mu^{(\text{em})}$, and $e = gg'/\sqrt{g^2 + g^2'}$ is the electric charge.

Moreover, the ansatz \cite{2} becomes

$$
\rho = \rho(r), \quad W_\mu = \frac{i f(r)}{g} e^{-i\varphi} (\partial_\mu \theta + i \sin \theta \partial_\mu \varphi),
$$

$$
A_\mu^{(\text{em})} = e \left( \frac{A(r)}{g^2} + \frac{B(r)}{g'^2} \right) \partial_\mu t - \frac{1}{e} (1 - \cos \theta) \partial_\mu \varphi,
$$

$$
Z_\mu = \frac{e}{gg'} (A(r) - B(r)) \partial_\mu t.
$$

(6)

With this we have the equations of motion

$$
\ddot{r} - \frac{f^2 - 1}{r^2} \dot{r} = \left( \frac{g^2}{4} \rho^2 - A^2 \right) f,
$$

$$
\ddot{\varphi} - \frac{2}{r} \dot{\varphi} - \frac{f^2}{2r^2} \rho = -\frac{1}{4} (B - A)^2 \rho + \lambda (\rho^2 - \frac{\mu^2}{2}) \rho,
$$

$$
\ddot{A} + \frac{2}{r} \dot{A} - \frac{2f^2}{r^2} A = g^2 \left( A - B \right) \rho^2,
$$

$$
\ddot{B} + \frac{2}{r} \dot{B} = g^2 \left( B - A \right) \rho^2,
$$

(7)

which has a singular monopole solution

$$
\dot{f} = 0, \quad \rho = \rho_0 = \sqrt{2\mu^2/\lambda},
$$

$$
A_\mu^{(\text{em})} = -\frac{1}{e} (1 - \cos \theta) \partial_\mu \varphi, \quad Z_\mu = 0.
$$

(8)

Choosing the boundary condition

$$
\rho(0) = 0, \quad f(0) = 1, \quad A(0) = 0, \quad B(0) = b_0,
$$

$$
\rho(\infty) = \rho_0, \quad f(\infty) = 0, \quad A(\infty) = B(\infty) = A_0,
$$

(9)

we obtain the Cho-Maison dyon \cite{2}.

We can also have the anti-monopole or in general anti- dyon solution, the charge conjugate state of the dyon, which has the magnetic charge $q_m = -4\pi/e$ with the following ansatz

$$
\phi' = \rho(r) \xi', \quad \xi' = -i \left( \frac{\sin(\theta/2) e^{-i\varphi}}{-\cos(\theta/2)} \right),
$$

$$
\vec{A}_\mu' = -\frac{1}{g} A(r) \partial_\mu t \hat{r} + \frac{1}{g} f(r - 1) \hat{r} \times \partial_\mu \hat{r},
$$

$$
\hat{r}' = -\xi'^\dagger \vec{r}',
$$

$$
B_\mu' = -\frac{1}{g'} B(r) \partial_\mu t + \frac{1}{g'} (1 - \cos \theta) \partial_\mu \varphi.
$$

(10)

In terms of the physical fields the ansatz is expressed by

$$
W_\mu' = \frac{i f(r)}{g} \sqrt{2} e^{-i\varphi} (\partial_\mu \theta - i \sin \theta \partial_\mu \varphi) = -W_\mu^*,
$$

$$
A_\mu^{(\text{em})} = -e \left( \frac{1}{g^2} A(r) + \frac{1}{g'^2} B(r) \right) \partial_\mu t
$$

$$
+ \frac{1}{e} (1 - \cos \theta) \partial_\mu \varphi,
$$

$$
Z_\mu' = -\frac{e}{gg'} (A(r) - B(r)) \partial_\mu t = -Z_\mu.
$$

(11)
This clearly shows that the electric and magnetic charges of the ansatz (10) are the opposite to the dyon, which confirms that the ansatz indeed describes the anti-dyon. Notice that the ansatz is basically the complex conjugation of the dyon ansatz. With this we obtain exactly the same equation of motion for the anti-dyon.

The electroweak dyon has two remarkable features. First, unlike the Dirac monopole it has the magnetic charge $4\pi/e$ (not $2\pi/e$). Second, with a non-trivial dressing of weak bosons it looks similar to the Julia-Zee dyon $\mathcal{S}$. But, unlike the Julia-Zee dyon, it has the point singularity at the origin $[2]$.

The point singularity of the Cho-Maison monopole makes it difficult to estimate the mass classically. In the following we predict the monopole mass to be around $10^3$ TeV and present three ways, the dimensional argument, the scaling argument, and the ultraviolet regularization, to back up this.

A. Dimensional Argument

To estimate the order of the monopole mass, it is important to realize that (roughly speaking) the monopole mass comes from the Higgs mechanism which generates the W-boson mass. To see this we first consider the 'tHooft-Polyakov monopole. Let $\tilde{\Phi}$ and $\tilde{A}_\mu$ be the Higgs triplet and the gauge potential, and express the monopole ansatz by

$$\tilde{\Phi} = \rho \hat{r}, \quad \tilde{A}_\mu = \tilde{C}_\mu + \tilde{W}_\mu, \quad \tilde{C}_\mu = -\frac{1}{g} \hat{r} \times \partial_\mu \hat{r}, \quad \tilde{W}_\mu = -f \tilde{C}_\mu,$$  

where $\tilde{C}_\mu$ is the Wu-Yang monopole potential. Notice that, except the overall amplitude $f$, the W-boson part of the ansatz is given by the Wu-Yang potential.

With this we have

$$|D_\mu \tilde{\Phi}|^2 = \frac{1}{2} (\partial_\mu \rho)^2 + \frac{1}{4} g^2 \rho^2 f^2 (\tilde{C}_\mu)^2. \quad (13)$$

This (with $f \approx 1$) tells that the monopole acquires mass through the Higgs mechanism, except that $\tilde{C}_\mu$ contains the extra factor $1/g$.

Similar mechanism works for the Weinberg-Salam model. Indeed with the ansatz (2) we have (with $A = B = 0$)

$$|D_\mu \phi|^2 = \frac{1}{2} (\partial_\mu \rho)^2 + \frac{1}{2} \rho^2 |D_\mu \xi|^2 \quad = \frac{1}{2} (\partial_\mu \rho)^2 + \frac{1}{8} g^2 \rho^2 f^2 (\tilde{C}_\mu)^2.$$  

So we could say that the Higgs mechanism is responsible for the mass of the electroweak monopole (more precisely the SU(2) part of it).

With this understanding, we can use the dimensional argument to predict the monopole energy. Since the monopole mass term in the Lagrangian contributes to the monopole energy we expect

$$E \simeq C \times \frac{4\pi}{e^2} M_W, \quad C \simeq 1.$$  

This implies that the monopole mass should be about $1/\alpha$ times bigger than the electroweak scale, around $10^3$ TeV. Now we have to know how to estimate $C$, and we discuss two ways to do so.

B. Scaling Argument

Suppose that the quantum correction removes the singularity at the origin. In this case we can use the Derrick’s scaling argument to estimate the monopole mass, because the regularized solution should be stable under the rescaling of its field configuration. So consider the monopole configuration (with $A = B = 0$) and let

$$K_\phi = \int |D_\mu \phi|^2 d^3x, \quad V_\phi = \frac{\lambda}{2} \int (|\phi|^2 - \frac{\mu^2}{\lambda})^2 d^3x, \quad K_A = \frac{1}{4} \int F_{ij}^2 d^3x, \quad K_B = \frac{1}{4} \int B_{ij}^2 d^3x.$$  

Notice that $K_B$ makes the monopole energy infinite.

Now, under the scale transformation $\vec{x} \rightarrow \lambda \vec{x}$, we have

$$K_\phi \rightarrow \lambda K_\phi, \quad K_A \rightarrow \lambda K_A, \quad K_B \rightarrow \lambda K_B, \quad V_\phi \rightarrow \lambda^3 V_\phi.$$  

So we have the following condition for the regularized monopole configuration

$$K_A + K_B = K_\phi + 3V_\phi.$$  

Numerically we have

$$K_A \simeq 0.1852 \times \frac{4\pi}{e^2} M_W, \quad K_\phi \simeq 0.1577 \times \frac{4\pi}{e^2} M_W,$$

$$V_\phi \simeq 0.0011 \times \frac{4\pi}{e^2} M_W,$$  

so that from (18) we have

$$K_B \simeq 0.0058 \times \frac{4\pi}{e^2} M_W.$$  

From this we can estimate the energy of the monopole

$$E \simeq 0.3498 \times \frac{4\pi}{e^2} M_W \simeq 3.85 \text{ TeV}.$$  

This strongly back up the dimensional argument. But here we have assumed the existence of a regularized monopole solution. Now we show how the quantum correction could regularize the singularity at the origin.

C. Ultra-violet Regularization
Notice that [5] describes the “bare” theory which should change to an “effective” theory after the quantum correction which changes the coupling constants to the scale dependent running couplings. To see how this quantum correction can make the monopole energy finite, let us consider the following effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = -|D_\mu \phi|^2 - \frac{\lambda}{2} \left( \rho^2 - \frac{\mu^2}{\alpha} \right)^2 - \frac{1}{4} F_{\mu \nu}^2 - \frac{1}{4} \epsilon (|\phi|^2) G_{\mu \nu}^2.$$  \hspace{1cm} (22)

This type of effective Lagrangian has been used in nonlinear electrodynamics and cosmology, and naturally appears in higher-dimensional unified theory [12–14].

Clearly $\epsilon$ effectively modifies $g'$ of the $U(1)_Y$ gauge coupling to $g'/\sqrt{\epsilon}$, but the Lagrangian still retains the $SU(2) \times U(1)_Y$ gauge symmetry. Moreover, when $\epsilon \to 1$ asymptotically, it reproduces the standard model.

From (22) we have

$$\begin{align*}
\ddot{\rho} + \frac{2}{r} \dot{\rho} - \frac{f^2}{r^2} \rho &= \frac{1}{4} (A - B)^2 \rho + \frac{\lambda}{2} (\rho^2 - \rho_0^2) \rho \\
&\quad + \frac{\epsilon}{\epsilon^*} \left( \frac{1}{r^2} - \ddot{B}^2 \right) \rho,
\end{align*}$$

$$\ddot{f} - \frac{f^2 - 1}{f^2} \dot{f} = \left( \frac{g^2}{4} \rho^2 - A^2 \right) f,$$

$$\ddot{A} + \frac{2}{r} \dot{A} - 2 \frac{f^2}{r^2} A = \frac{g^2}{4} \rho^2 (A - B),$$

$$\ddot{B} + 2 \left( \frac{1}{r} + \frac{\epsilon}{\epsilon^*} \rho \right) \dot{B} = - \frac{g^2}{4 \epsilon} \rho^2 (A - B).$$  \hspace{1cm} (23)

This confirms that effectively $\epsilon$ changes the $g'$ to the “running” coupling $g' = g'/\sqrt{\epsilon}$. So, by making $g'$ infinite at the origin, we can regularize the Cho-Maison monopole.

Choosing $\epsilon = (\rho/\rho_0)^8$, we have the finite energy dyon solution shown in Fig. [1]. It is really remarkable that the regularized solution looks very much like the Cho-Maison dyon, except that for the finite energy dyon solution $Z(0)$ becomes zero. Moreover, with $A = B = 0$, we can estimate the monopole energy

$$E \simeq 0.65 \times \frac{4\pi}{e^2} M_W \simeq 7.19 \text{ TeV}. \hspace{1cm} (24)$$

This tells two things. First, a quantum correction could easily make the electroweak monopole mass finite. Second, this strongly supports our estimate of the monopole mass based on the scaling argument.

We can think of another way to regularize the Cho-Maison monopole. Suppose the quantum correction modifies [5] by

$$\delta \mathcal{L} = i e \alpha F_{\mu \nu}^{(\text{em})} W_\mu W_\nu + \beta \frac{g^2}{4} (W_\mu W_\nu - \tilde{W}_\mu \tilde{W}_\nu)^2,$$  \hspace{1cm} (25)

where $\alpha$ and $\beta$ are the scale dependent parameters which vanish asymptotically and modify the theory only at short distance.

To find the finite energy dyon, however, we may treat $\alpha$ and $\beta$ as constants because asymptotically the boundary condition makes them irrelevant [15]. In this case the finite energy condition requires

$$1 + \alpha = \frac{1}{f(0)^2} \frac{g^2}{e^2}, \quad 1 + \beta = \frac{1}{f(0)^4} \frac{g^2}{e^2}. \hspace{1cm} (26)$$

With this we can find a finite energy dyon solution with the boundary condition [1] but without the condition $f(0) = 1$. For instance, when $f(0) = g/e$ (with $\alpha = 0$) we have the solution shown in Fig. [2]. Notice that asymptotically boundary condition makes the solution converge to the Cho-Maison dyon.

We can calculate the monopole energy in terms of $f(0)$. For example we have

$$E(f(0) = 1) \simeq 0.61 \times \frac{4\pi}{e^2} M_W \simeq 6.73 \text{ TeV},$$

$$E(f(0) = g/e) \simeq 1.27 \times \frac{4\pi}{e^2} M_W \simeq 13.95 \text{ TeV}. \hspace{1cm} (27)$$

In general, we can plot the monopole energy as a function of $f(0)$, which is shown in Fig. [3]. Again this assures that a simple quantum cotrection could regularize
the Cho-Maison monopole, and strongly supports the prediction of the monopole mass based on the scaling argument.

Of course (22) or (25) may not describe the true quantum correction, so that the finite energy solutions can only be viewed as approximate solutions of the standard model. But this is not the point. Our point here is to show that a small quantum correction, without new interaction, can regularize the Cho-Maison dyon. As Fig. 1 and Fig. 2 demonstrate, they describe an excellent approximation of the Cho-Maison dyon from which we can estimate the mass of the electroweak monopole.

We close with the following remarks:

1. In electrodynamics $U(1)$ can either be trivial or non-trivial. But in the standard model it is natural to expect $U(1)_{\text{em}}$ to be non-trivial. As we have emphasized, there is no reason why $U(1)_Y$ has to be trivial since the $U(1)$ subgroup of $SU(2)$ is already non-trivial. This assures $U(1)_{\text{em}}$ made of the linear combination of them to be non-trivial. This means that, unlike the Dirac monopole in electrodynamics which is optional, the standard model must have the monopole.

2. The unit magnetic charge of the electroweak monopole must be $4\pi/e$. This again is because $U(1)_{\text{em}}$ is made of the linear combination of $U(1)_Y$ and $U(1)$ subgroup of $SU(2)$, which makes the period of $U(1)_{\text{em}}$ $4\pi$.

3. Since the monopole singularity comes from $B_\mu$, we can regularize it embedding the hypercharge $U(1)$ to $SU(2)_Y$. This, of course, will make the monopole mass heavier because this embedding adds an intermediate scale $M_Y$ (somewhere between the electroweak and grand unification scales) \cite{15}. Such embedding could naturally arise in the left-right symmetric grand unification models, in particular in the $SO(10)$ grand unification.

Certainly the existence of the electroweak monopole of mass around 4 to 10 TeV has important implications. First, this explains why the search for the monopole so far has been unsuccessful. Moreover, this implies that the recent upgrading of LHC could allow the MoEDAL to detect it, because the LHC now might have reached the monopole-antimonopole pair production threshold. But most importantly, it tells that the final test of the standard model should be the discovery of the electroweak monopole, not the Higgs particle. Indeed this should be regarded as the topological test of the standard model, which has never been done before.

But if the monopole mass becomes above 7 TeV, the 14 TeV LHC can not produce it. In this case we may need the “cosmic” MoEDAL to detect it, or the 100 TeV LHC. A detailed discussion of our work will be published in a separate paper \cite{15}.

Acknowledgments

The work is supported in part by the National Research Foundation of Korea funded by the Ministry of Education, Science, and Technology (2012-002-134).

\begin{thebibliography}{99}
  
  [1] G. Aad et al. (ATLAS Collaboration), Phys. Lett. B716, 1 (2012); S. Chatrchyan et al. (CMS Collaboration), Phys. Lett. B716, 30 (2012); T. Aaltonen et al. (CDF and D0 Collaborations), Phys. Rev. Lett. 109, 071804 (2012).
  
  [2] Y. M. Cho and D. Maison, Phys. Lett. B391, 360 (1997); W. S. Bae and Y. M. Cho, JKPS 46, 791 (2005).
  
  [3] Yisong Yang, Proc. Roy. Soc. A454, 155 (1998). See also Yisong Yang, \textit{Solitons in Field Theory and Nonlinear Analysis} (Springer Monographs in Mathematics), p. 322 (Springer-Verlag) 2001.
  
  [4] P.A.M. Dirac, Phys. Rev. 74, 817 (1948).
  
  [5] T.T. Wu and C.N. Yang, in \textit{Properties of Matter under Unusual Conditions}, edited by H. Mark and S. Fernbach (Interscience, New York) 1969; Phys. Rev. D12, 3845 (1975).
  
  [6] Y.M. Cho, Phys. Rev. Lett. 44, 1115 (1980); Phys. Lett. B115, 125 (1982).
  
  [7] G. ’tHooft, Nucl. Phys. B79, 276 (1974); A. Polyakov, JETP Lett. 20, 194 (1974).
  
  [8] B. Julia and A. Zee, Phys. Rev. D11, 2227 (1975); M. Prasad and C. Sommerfield, Phys. Rev. Lett. 35, 760 (1975).
  
  [9] C. P. Dokos and T. N. Tomaras, Phys. Rev. D21, 2940 (1980).
  
  [10] T. Vachaspati and M. Barriola, Phys. Rev. Lett. 69, 1867 (1992); M. Barriola, T. Vachaspati, and M. Bucher, Phys. Rev. D50, 2819 (1994).
  
  [11] B. Acharya et al. [MoEDAL Collaboration], Int. J. Mod. Phys. A29, 1430050 (2014); Y. M. Cho and J. Pinfold, Snowmass Whitepaper, arXiv: hep-ph/1307.8390.
  
  [12] Y.M. Cho, Phys. Rev. D35, R2628 (1987); Y.M. Cho, Phys. Lett. B199, 358 (1987).
  
  [13] Y.M. Cho, Phys. Rev. Lett. 68, 3133 (1992); Y.M. Cho and J.H. Yoon, Phys. Rev. D47, 3465 (1993).
  
  [14] E. Babichev, Phys. Rev. D74, 085004 (2006).
  
  [15] Y. M. Cho, Kyounge S. Kimm, and J. H. Yoon, to be published.
\end{thebibliography}