Spontaneous Emission Near Superconducting Bodies

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In the present paper we study the spontaneous photon emission due to a magnetic spin-flip transition of a two-level atom in the vicinity of a dielectric body like a normal conducting metal or a superconductor. For temperatures below the transition temperature \( T_c \) of a superconductor, the corresponding spin-flip lifetime is boosted by several orders of magnitude as compared to the case of a normal conducting body. Numerical results of an exact formulation are also compared to a previously derived approximate analytical expression for the spin-flip lifetime and we find an excellent agreement. We present results on how the spin-flip lifetime depends on the temperature \( T \) of a superconducting body as well as its thickness \( H \). Finally, we study how non-magnetic impurities as well as possible Eliashberg strong-coupling effects influence the spin-flip rate. It is found that non-magnetic impurities as well as strong-coupling effects have no dramatic impact on the spin-flip lifetime.

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It is well-known that the rate of spontaneous emission of atoms will be modified due to the presence of a dielectric body [1]. In current investigations of atom microtraps this issue is of fundamental importance since such decay processes have a direct bearing on the stability of e.g. atom chips.

In magnetic microtrap experiments, cold atoms are trapped due to the presence of magnetic field gradients created e.g. by current carrying wires [2]. Such microscopic traps provide a powerful tool for the control and manipulation of cold neutral atoms over micrometer distances [3]. Unfortunately, this proximity of the cold atoms to a dielectric body introduces additional decay channels. Most importantly, Johnson-noise currents in the material give rise to electromagnetic field fluctuations. For dielectric bodies at room temperature made of normal conducting metals, these fluctuations may be strong enough to deplete the quantum state of the atom and, hence, expel the atom from the magnetic microtrap [4]. Reducing this disturbance from the surface is therefore strongly desired. In order to achieve this, the use of superconducting dielectric bodies instead of normal conducting metals has been proposed [5]. Some experimental work in this context has been done as well, e.g. by Nierengarten et al. [6], where cold atoms were trapped near a superconducting surface.

In the present article we will consider the spin-flip rate when the electrodynamic properties of the superconducting body are described in terms of either a simple two-fluid model or in terms of the detailed microscopic Mattis-Bardeen [7] and Abrikosov-Gor’kov-Khalatnikov [8] theory of weak-coupling BCS superconductors. In addition, we will also study how non-magnetic impurities, as well as strong coupling effects according to the low-frequency limit of the Eliashberg theory [9], will affect the spontaneous emission rate.

Following Ref. [10] we consider an atom in an initial state |\( i \rangle \) and trapped at position \( \mathbf{r}_A = (0, 0, z) \) in vacuum near a dielectric body. The rate \( \Gamma_B \) of spontaneous and thermally stimulated magnetic spin-flip transition into a final state |\( f \rangle \) is then

\[
\Gamma_B = \frac{\mu_0}{\hbar} \frac{2 (\mu B g s)^2}{\hbar} \sum_{j,k} S_j S_k^* \times \text{Im} [\nabla \times \nabla \times G(\mathbf{r}, \mathbf{r}, \omega)]_{jk} (\pi + 1),
\]

where we have introduced the dimensionless components \( S_j \equiv \langle f | S_j | i \rangle \) of the electron spin operators \( S_j \) with \( j = x, y, z \). Here \( g_S = 2 \) is the gyromagnetic factor of the electron, and \( G(\mathbf{r}, \mathbf{r}, \omega) \) is the dyadic Green tensor of Maxwell’s theory. Eq. (1) follows from a consistent quantum-mechanical treatment of electromagnetic radiation in the presence of an absorbing body [11, 12]. In this theory a local response is assumed, i.e. the characteristic skin depth should be larger than the mean free path of the electric charge carriers of the absorbing body. Thermal excitations of the electromagnetic field modes are accounted for by the factor \((\pi + 1)\), where \( \pi = 1/(e^{\hbar \nu/T} - 1) \) and \( \omega \equiv 2 \pi \nu \) is the angular frequency of the spin-flip transition. Here \( T \) is the temperature of the dielectric body, which is assumed to be in thermal equilibrium with its surroundings. The dyadic Green tensor is the unique solution to the Helmholtz equation

\[
\nabla \times \nabla \times G(\mathbf{r}, \mathbf{r}', \omega) - k^2 e(\mathbf{r}, \mathbf{r}) G(\mathbf{r}, \mathbf{r}', \omega) = \delta(\mathbf{r} - \mathbf{r}') \mathbf{1},
\]

with appropriate boundary conditions. Here \( k = \omega/c \) is the wavevector in vacuum, \( c \) is the speed of light and \( \mathbf{1} \) the unit dyad. The tensor \( G(\mathbf{r}, \mathbf{r}', \omega) \) contains all relevant information about the geometry of the material and, through the relative electric permittivity \( e(\mathbf{r}, \omega) \), about its dielectric properties. The fluctuation-dissipation theorem is build into this theory [11, 12].
The decay rate $\Gamma_B^0$ of a magnetic spin-flip transition for an atom in free-space is well-known (see e.g. Refs.\([12]\)). This free-space decay rate is $\Gamma_B^0 = \Gamma_B S^2$, where $\Gamma_B = \mu_0 (\mu_B g_s)^2 k^3/(3\pi h)$ and where we have introduced the dimensionless spin factor $S^2 = S_x^2 + S_y^2 + S_z^2$. The free-space lifetime corresponding to this magnetic spin-flip rate is $\tau_B^0 = 1/\Gamma_B^0$. In the present paper we only consider $^{87}$Rb atoms that are initially pumped into the $|5S_{1/2}, F = 2, m_F = 2\rangle \equiv |2, 2\rangle$ state, and assuming the rate-limiting transition $|2, 2\rangle \rightarrow |2, 1\rangle$ in correspondence to recent experiments \([6, 13, 14, 15]\). The spin factor is $S^2 = 2/\pi$ (c.f. Ref.\([10]\)) and the frequency is $\nu = 560$ kHz. The numerical value of the free-space lifetime then is $\tau_B^0 = 1.14 \times 10^{25}$ s.

In the following we will consider a geometry where an atom is trapped at a distance $z$ away from a dielectric slab with thickness $H$. Vacuum is on both sides of the slab, i.e. $\epsilon(r, \omega) = 1$ for any position $r$ outside the body. The slab can be e.g. a superconductor or a normal conducting metal, described by a dielectric function $\epsilon(\omega)$. The total transition rate for magnetic spontaneous emission

$$\Gamma_B = (\Gamma_B^0 + \Gamma_B^{\text{slab}})(\overline{n} + 1), \quad (3)$$

can then be decomposed into a free part and a part purely due to the presence of the slab. The latter contribution for an arbitrary spin orientation is then given by

$$\Gamma_B^{\text{slab}} = 2 \Gamma_B^0 \left( (S_x^2 + S_y^2) I_{\parallel} + S_z^2 I_{\perp} \right), \quad (4)$$

with the atom-spin orientation dependent integrals

$$I_{\parallel} = \frac{3}{16kz} \text{Re} \left\{ \int_0^{2kz} dx e^{ix} \left[ \mathcal{C}_N(x) - \frac{x}{2kz} \mathcal{C}_M(x) \right] \right\} + \int_{kz}^{\infty} dx e^{-x} \frac{1}{x} \left[ \mathcal{C}_N(ix) + \frac{x}{2kz} \mathcal{C}_M(ix) \right] \right\}, \quad (5)$$

$$I_{\perp} = \frac{3}{8kz} \text{Re} \left\{ \int_0^{2kz} dx e^{ix} \left[ 1 - \frac{x}{2kz} \right] \mathcal{C}_M(x) \right\} + \int_{kz}^{\infty} dx e^{-x} \frac{1}{x} \left[ 1 + \frac{x}{2kz} \right] \mathcal{C}_M(ix) \right\}, \quad (6)$$

where the scattering coefficients are given by \([16]\)

$$\mathcal{C}_N(x) = r_p(x) \frac{1 - e^{ix} H/z}{1 - r_p^2(x) e^{ix} H/z}, \quad (7)$$

$$\mathcal{C}_M(x) = r_s(x) \frac{1 - e^{ix} H/z}{1 - r_s^2(x) e^{ix} H/z}. \quad (8)$$

The electromagnetic field polarization dependent Fresnel coefficients are

$$r_p(x) = \frac{\epsilon(\omega) x - \sqrt{(2kz)^2(\epsilon(\omega) - 1) + x^2}}{\epsilon(\omega) x + \sqrt{(2kz)^2(\epsilon(\omega) - 1) + x^2}}, \quad (9)$$

$$r_s(x) = \frac{\epsilon(\omega) x - \sqrt{(2kz)^2(\epsilon(\omega) - 1) + x^2}}{\epsilon(\omega) x + \sqrt{(2kz)^2(\epsilon(\omega) - 1) + x^2}}. \quad (10)$$

For the special case $H = \infty$, the integrals in Eqs.\((5)\) and \((6)\) are simply a convenient re-writing of Eqs.\((8)-(12)\) in Ref.\([17]\). Note that $I_{\parallel} \approx 2I_{\parallel}$ provided $kz \ll 1$. Throughout this article, we use the same spin-orientation as in Refs.\([3, 15]\), i.e. $S_y^2 = S_z^2$ and $S_x = 0$.

![FIG. 1: $\tau_B$ of a trapped atom near a superconducting film as a function of the temperature $T/T_c$. The solid as well as the dashed-dotted line correspond to the two-fluid mode and the Gorter-Casimir temperature dependence. We use $\lambda_1(0) = 35$ nm and $\delta(T_c) \approx 150$ µm, corresponding to niobium. The critical temperature is $T_c = 8.31$ K \([21]\). For $T/T_c \geq 1$ we put $\sigma_2(T) \approx 0$ but $\sigma_1(T) = 2/\omega \mu_0 \delta(T_c)^2$. The dashed line corresponds to a film made of gold described by the dielectric function given by Eq.\((15)\). The uppermost graph (dotted line) shows to the lifetime $\tau_B^0/(\bar{n} + 1)$, i.e. the free-space lifetime with $\tau_B^0 = 1.114 \times 10^{25}$ s.

As the total current density responds linearly and locally to the electric field, the dielectric function can be written

$$\epsilon(\omega) = 1 - \frac{\sigma_2(T)}{\epsilon_0 \omega} + i \frac{\sigma_1(T)}{\epsilon_0 \omega}. \quad (11)$$

Here $\sigma(T) \equiv \sigma_1(T) + i\sigma_2(T)$ is the complex optical conductivity. We may now parameterize this complex conductivity in terms of the London penetration length $\lambda_L(T) \equiv \sqrt{1/\omega \mu_0 \sigma_2(T)}$ and the skin depth $\delta(T) \equiv \sqrt{2/\omega \mu_0 \sigma_1(T)}$. In this case, the dielectric function is $\epsilon(\omega) = 1 - 1/k^2 \lambda_L^2(T) + i/2k^2 \delta^2(T)$. If, in addition, we consider a non-zero and sufficiently small frequency in the range $0 < \omega \ll \omega_0 \equiv 2\Delta(0)/h$, where $\Delta(0)$ is the energy gap of the superconductor at zero temperature, the current density may be described in terms of a two-fluid model \([19]\). The London penetration length is $\lambda_L(T) = \lambda_L(0)/\sqrt{n_n(T)/n_0}$ and the skin depth is $\delta(T) = \delta(T_c)/\sqrt{n_n(T)/n_0}$. Here the electron density in the superconducting and normal state are $n_n(T)$ and $n_n(T)$, respectively, such that $n_n(T) + n_n(T) = n_0$ and $n_n(0) = n_n(T \geq T_c) = n_0$ \([19]\). A convenient summary...
of the two-fluid model is expressed by the relations
\[ \sigma_1(T) = \sigma_n \sqrt{\frac{n_n(T)}{n_0}}, \quad \sigma_2(T) = \sigma_L \sqrt{\frac{n_s(T)}{n_0}}, \] (12)
where \( \sigma_n \equiv \sigma_1(T_c) \) and \( \sigma_L \equiv 1/\omega_0 \lambda_L^2(0) \). Considering, in particular, the Gorter-Casimir temperature dependence \[20\] for the current densities, the electron density in the normal state is \( n_n(T)/n_0 = (T/T_c)^4 \). For niobium we use \( \delta(T_c) = \sqrt{2}/\omega_0 \sigma_0 \approx 150 \mu m \) as \( \sigma_0 = 2 \times 10^7 \text{(Sm)}^{-1} \) and \( \lambda_L(0) = 35 \text{ nm} \) according to Refs.\[21\]. In passing, we remark that the value of \( \sigma_n \) as obtained in Ref.\[22\] is two orders of magnitude larger than the corresponding value inferred from the data presented in Refs.\[21\].

The lifetime \( \tau_B \equiv 1/T_B \) for spontaneous emission as a function of \( T \) is shown in Fig.\[1\] for \( H = 0.9 \mu m \) (solid line). We confirm the observation in Ref.\[18\] that for \( T < T_c \) for the normal state is\[20\] for the current densities, the electron density in the normal state is \( n_n(T)/n_0 = (T/T_c)^4 \). For niobium we use \( \delta(T_c) = \sqrt{2}/\omega_0 \sigma_0 \approx 150 \mu m \) as \( \sigma_0 = 2 \times 10^7 \text{ (Sm)}^{-1} \) and \( \lambda_L(0) = 35 \text{ nm} \) according to Refs.\[21\]. In passing, we remark that the value of \( \sigma_n \) as obtained in Ref.\[22\] is two orders of magnitude larger than the corresponding value inferred from the data presented in Refs.\[21\].

For \( T > T_c \), we have to resort to numerical investigations. In contrast to the traditional Drude model, more realistic descriptions of a normal conducting metal in terms of a permittivity include a significant real contribution to the dielectric function in addition to an imaginary part. One such description is discussed in Ref.\[23\], where
\begin{equation}
\bar{\epsilon}(\omega, T) = 1 - \frac{\omega^2}{\omega^2 + \nu(T)^2} + i \frac{\nu(T)^2 \omega^2}{(\omega^2 + \nu(T)^2)^2},
\end{equation}

and \( \omega \nu(T) = 0.0847(T/\theta)^5 \int_{\theta/T}^{T} dx x^5 e^x/(e^x - 1)^2 \text{ eV} \) using a Bloch-Gr"uneisen approximation. Here \( \theta = 175 \text{ K} \) for gold. The plasma frequency is \( \bar{\omega}_D = 9 \text{ eV} \). For temperatures \( T \approx 0.25T_c \), we observe that Eq.\[15\] leads to \( \sigma_1(T) \approx \sigma_2(T) \), and that for lower temperatures \( \sigma_2(T) \) will be the dominant contribution to the conductivity. For temperatures \( T/T_c \approx T_c \), in the use of Eq.\[15\] we can set \( \sigma_2(T) \approx 0 \) when calculating the lifetime. For a bulk material of gold this leads to almost two orders of magnitude longer lifetime as compared to niobium since for gold \( \delta(T_c) \approx 1 \mu m \), using the parameters corresponding to Fig.\[1\]. This finding is in accordance with Eq.\[14\]. As seen from Fig.\[1\] for a thin film and for \( T/T_c \geq 1 \) we find the opposite and remarkable result, i.e. a decrease in conductivity can lead to a larger lifetime.

A much more detailed and often used description of the electrodynamic properties of superconductors than the simple two-fluid model was developed by Mattis-Bardeen\[7\], and independently by Abrikosov-Gor’kov-Khalatnikov\[8\], based on the weak-coupling BCS theory of superconductors. In the clean limit, i.e. \( l \gg \xi_0 \), where \( l \) is the electron mean free path and \( \xi_0 \) is the coherence length of a pure material, the complex conductivity, normalized to \( \sigma_n \equiv \sigma_1(T_c) \), can be expressed in the form\[24\]
\[ \frac{\sigma(T)}{\sigma_n} = \frac{\sigma(T)}{\bar{\epsilon}(T) - \nu} \frac{dx}{\nu} \text{ tanh} \left( \frac{x_0 + h\nu}{2k_BT} \right) g(x) \]
\[ = \int_{0}^{\frac{\omega_D}{\sqrt{\bar{x}^2 + \Delta^2(0)}}} \frac{dx}{\sqrt{\bar{x}^2 + \Delta^2(0)}} \text{ tanh} \left[ \frac{\sqrt{\bar{x}^2 + \Delta^2(0)}}{2k_BT} \right], \]
(16)
where \( g(x) \) is the Debye frequency and \( \Delta(0) = 3.53k_BT_c/2 \). For niobium, the Debye frequency is \( \omega_D = 25 \text{ meV} \). According to a theorem of Anderson\[25\], the presence of non-magnetic impurities, which we only consider in the present paper, will not modify the superconducting energy gap as given by Eq.\[17\]. The complex conductivity will, however, in general be modified due to the presence of such impurities.

In the dirty limit where \( l \ll \xi_0 \), the complex conductivity has been examined within the framework of the microscopic BCS theory (see e.g. Ref.\[30\]). In this case, the complex conductivity, now normalized to \( \sigma_L \), can conveniently be written in the form
\[ \frac{\sigma(T)}{\sigma_L} = \frac{\sigma(T)}{\bar{\epsilon}(T) - \nu} \frac{dx}{\nu} \text{ tanh} \left( \frac{x_0 + h\nu}{2k_BT} \right) \]
\[ \times \left( \frac{g(x) + 1}{u_2 - u_1 + ih/\tau} - \frac{g(x) - 1}{u_2 + u_1 - ih/\tau} \right) \]
\[ = \int_{\theta/T}^{T} dx \frac{\tau_B}{2k_BT} \text{ tanh} \left( \frac{x_0 + h\nu}{2k_BT} \right) \]
\[ \times \left( \frac{g(x) + 1}{u_2 - u_1 + ih/\tau} - \frac{g(x) - 1}{u_2 + u_1 + ih/\tau} \right). \]
(18)
Here we choose \( \tau \) such that \( \hbar /\tau \Delta(0) = \pi \xi_0 / l = 13.61 \), corresponding to the experimental coherence length \( \xi_0 = 39 \text{ nm} \) and the mean free path \( l(T \simeq 9K) = 9 \text{ nm} \). The normalization constant is \( \sigma_L = 1.85 \times 10^{14} (\Omega \text{m})^{-1} \) corresponding to \( \lambda_L(0) = 35 \mu\text{m} \) for niobium [21].

As the temperature decreases below \( T_c \), Cooper pairs will be created. Despite a very small fraction of Cooper pair for temperatures just below \( T_c \), the imaginary part of the conductivity, on the other hand, is more decreases exponentially fast. As seen in Fig. 2, the imaginary part of the conductivity, \( \sigma \), is, however, approximatively twenty percent larger than \( \sigma_1(T) \) as obtained from Eq. (10). This difference has, nevertheless a small effect on the lifetime \( \tau_B \). Hence, computing \( \tau_B \) using Eqs. (16) or (18) for the complex conductivity, we realize that the presence of non-magnetic impurities have no dramatic impact on the lifetime for spontaneous emission (see Fig. 3). A comparison of the values of \( \tau_B \) as obtained using the two-fluid model for \( H = \infty \) as presented in Fig. 1 and the corresponding result as shown in Fig. 3 shows, for our set of physical parameters, that the two-fluid model overestimates \( \tau_B \) with three order of magnitude.

For finite values of the lifetime \( \tau \) and for non-magnetic impurities we can also investigate the validity of the two-fluid model approximation in terms of the lifetime \( \tau_B \) for spontaneous emission processes. As we now will see, there are large deviations between the microscopic theory and the two-fluid model approximation, in particular for small temperatures. According to Abrikosov and Gor‘kov (for an excellent account see e.g. Ref. [29] and references cited therein), the density of superconducting electrons is given by

\[
\frac{n_s(T)}{n_0} \approx \frac{\pi \tau}{\hbar} \Delta(T) \tanh \left( \frac{\Delta(T)}{2k_B T} \right),
\]

provided that \( \tau \Delta(0)/\hbar \ll 1 \). We can now compute the dielectric function Eq. (11) using Eq. (12). We find that \( \sigma_2(T)/\sigma_L \) obtained in this way agrees well the corresponding quantity obtained from Eq. (18). There is, however, a considerable discrepancy between the two-fluid expression for \( \sigma_1(T)/\sigma_L \) and the corresponding expressions obtained from the microscopic theory as given by Eq. (18). The numerical results for the lifetime in this case are illustrated in the dashed-dotted lines in Fig. 3 and in Fig. 1.
Since we are considering low frequencies $0 < \omega < \omega_F \equiv 2\Delta(0)/\hbar$, strong coupling effects can now be estimated by making use of the low-frequency limit of the Eliashberg theory \[9\] and its relation to the BCS theory (see e.g. Ref. \[31\]). The so-called mass-renormalization factor $Z_N$, which in general is both frequency and temperature dependent, is then replaced by its zero-temperature limit, which in general is both frequency and temperature dependent rescaled by $\frac{\tau}{\sigma}$ of magnetic impurities $\sigma_0$. Using the strong-coupling expressions for the optical conductivity in a suitable form as e.g. given in Ref. \[24\], we then find that the complex conductivity $\sigma(T)/\sigma_0$ is rescaled by $\sigma_0 \rightarrow \sigma_0/Z_N$ with the lifetime of non-magnetic impurities rescaled by $\tau \rightarrow \tau/Z_N$. The change in the lifetime for spontaneous emission can then e.g. be inferred from the relation Eq. (13), and we find only a minor decrease of $\tau$ by the numerical factor $1/\sqrt{Z_N} \approx 0.69$, which also agrees well with more precise numerical evaluations.

The lifetime for spontaneous emission exhibits a minimum with respect to variation of the thickness $H$ of the superconducting film. This fact is illustrated in Fig. 5. Below the minimum at $H_{min} \approx 0.1 \mu m$, a decrease of the thickness $H$ leads to an increase of lifetime in proportion to $H^{-1}$. This happens despite the growth in polarization noise because the region generating the noise is becoming thinner as it is limited by $H$ and not $\lambda_L(T)$. Eventually, the lifetime reaches the free-space lifetime is $\tau_B^0$ as $H$ tends to zero. On the other hand, for large $H$, i.e. $H \gg \delta(T)$, the lifetime is constant with respect to $H$, giving the same result as for an infinite thick slab. In the region between, i.e. $\lambda_L(T) \approx H \approx \delta(T)$, the lifetime is proportional to $H$. Numerical studies show that a non-zero $\sigma_2(T)$ is important for a well pronounced minimum of $\tau_B$ as a function of $H$.

Some experimental work has been done using a superconducting body, e.g. Nirrengarten et al. \[8\]. Here cold atoms were trapped near a superconducting surface. At the distance of $440 \mu m$ from the chip surface, the trap lifetime reaches $115 s$ at low atomic densities and with a temperature $40 \mu K$ of the chip. We believe the vast discrepancy between this experimental value and our theoretical calculations must rely on effects that we have not taken into account in our analysis. The use of a thin superconducting film may lead to the presence vortex motion and pinning effects in (see e.g. Refs. \[32, 33\]). The presence of vortices will in general modify the dielectric properties of the dielectric body. If we, as an example, consider a vortex system in the liquid phase in a finite slab geometry, one expects a strongly temperature dependent $\sigma_1(T)$ with a peak value $\sigma_1(T) \approx 1.3 \times 10^7 / H^2 [\mu m] \nu [kHz] \Omega m \[22\]$. Close to this peak $\sigma_1(T) \approx \sigma_2(T)$, and for $\nu \approx 560 kHz$ we find a lifetime for spontaneous emission two orders of magnitude larger than a film made out of gold with the same geometry. It is an interesting possibility that spontaneous emission processes close to thin superconducting films could be used for an experimental study of the physics of vortex condensation. This possibility has also been noticed in a related consideration, which has appeared during the preparation of the present work \[34\]. There are also fabrication issues concerning the Nb-O chemistry \[35\] which may have an influence on the lifetime for spontaneous emission.

To summarize, we have studied the rate for spontaneous photon emission, due to a magnetic spin-flip transition, of a two-level atom in the vicinity of a normal...
conducting metal or a superconductor. Our results confirms the conclusion in Ref. [18], namely that the corresponding magnetic spin-flip lifetime will be boosted by several orders of magnitude by replacing a normal conducting film with a superconducting body. This conclusion holds when describing the electromagnetic properties of the superconductivity in terms of a simple two-fluid model as well as in terms of a more detailed and precise microscopic Mattis-Bardeen and Abrikosov-Gor’kov-Khalatnikov theory. For the set of physical parameters as used in Ref. [18] it so happens, more or less by chance, that the two-fluid model results agree well with the results from the microscopic BCS theory. We have, however, seen that even though the two-fluid model gives a qualitatively correct physical picture for spontaneous photon emission, it, nevertheless, leads to large quantitative deviations when compared to a detailed microscopic treatment. We therefore have to resort to the microscopic Mattis-Bardeen $^7$ and Abrikosov-Gor’kov-Khalatnikov $^8$ theory in order to obtain precise predictions. We have also show that non-magnetic impurities as well as strong-coupling effects have no dramatic impact on the rate for spontaneous photon emission. Vortex condensation in thin superconducting films may, however, be of great importance. Finally, we stress the close relation between the spin-flip rate for spontaneous emission and the complex conductivity, which indicates a new method to study the dynamical properties of a superconductor or a normal conducting metal. In such a context the parameter dependence for a bulk material as given by Eq. (13) may be useful.

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