Conditional Reliability in Uncertain Graphs

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Abstract—Network reliability is a well-studied problem that requires to measure the probability that a target node is reachable from a source node in a probabilistic (or uncertain) graph, i.e., a graph where every edge is assigned a probability of existence. Many approaches and problem variants have been considered in the literature, all assuming that edge-existence probabilities are fixed. Nevertheless, in real-world graphs, edge probabilities typically depend on external conditions. In metabolic networks a protein can be converted into another protein with some probability depending on the presence of certain enzymes. In social influence networks the probability that a tweet of some user will be re-tweeted by her followers depends on whether the tweet contains specific hashtags. In transportation networks the probability that a network segment will work properly or not might depend on external conditions such as weather or time of the day.

In this paper we overcome this limitation and focus on conditional reliability, that is assessing reliability when edge-existence probabilities depend on a set of conditions. In particular, we study the problem of determining the $k$ conditions that maximize the reliability between two nodes. We deeply characterize our problem and show that, even employing polynomial-time reliability-estimation methods, it is $\mathsf{NP}$-hard, does not admit any $\mathsf{PTAS}$, and the underlying objective function is non-submodular. We then devise a practical method that targets both accuracy and efficiency. We also study natural generalizations of the problem with multiple source and target nodes. An extensive empirical evaluation on several large, real-life graphs demonstrates effectiveness and scalability of the proposed methods.

Index Terms—Uncertain graphs, Reliability, Conditional probability.

1 Introduction

Uncertain graphs, i.e., graphs whose edges are assigned a probability of existence, have recently attracted a great deal of attention, due to their rich expressiveness and given that uncertainty is inherent in the data in a wide range of applications. Uncertainty may arise due to noisy measurements [2], inference and prediction models [1], or explicit manipulation, e.g., for privacy purposes [8]. A fundamental problem in uncertain graphs is the so-called reliability, which asks to measure the probability that two given (sets of) nodes are reachable [3]. Reliability has been well-studied in the context of device networks, i.e., networks whose nodes are electronic devices and the (physical) links between such devices have a probability of failure [3]. More recently, the attention has been shifted to other types of network that can naturally be represented as uncertain graphs, such as social networks or biological networks [20], [24].

To the best of our knowledge, all reliability queries so far considered have been modeled without taking into account any external factor that could influence the probability of existence of the links in the network. In this paper we overcome this limitation and introduce the notion of conditional reliability, which takes into account that edge probabilities may depend on a set of conditions, rather being fixed. This situation models real-world uncertain graphs. As an example, Figure 1 shows a link $(u, v)$ of a social influence network, i.e., a social graph where the associated probability represents the likelihood that a piece of information (e.g., a tweet) originated by $u$ will be “adopted” (re-tweeted) by her follower $v$. The re-tweeting probability clearly depends on the content of the tweet. In the example $v$ is much more interested in sports than politics. Hence, if the tweet contains the hashtag #NFL, then it will likely be re-tweeted, while if it is about elections (i.e., it contains the hashtag #GetToThePolls), it will be re-tweeted only with a small probability.

![Fig. 1: A link $(u, v)$ of a social influence network, where the associated probability represents the likelihood that a tweet by $u$ will be re-tweeted by her follower $v$. This probability depends on the content of the tweet. In this example if the tweet contains the hashtag #NFL, then it will likely be re-tweeted, while if it is about elections (i.e., it contains the hashtag #GetToThePolls), it will be re-tweeted only with a small probability.](image)

Given an uncertain graph with external-factor-dependent edge probabilities, in this work we study the following problem: Given a source node, a target node, and a small integer $k$, identify a set of $k$ catalysts that maximizes the reliability between $s$ and $t$. This problem arises in many real-world scenarios, such as the ones described next.

Pathway formation in biological networks. To understand metabolic chain reactions in cellular systems, biologists utilize metabolic networks [19], where nodes represent compounds, and an edge between two compounds indicates that a compound can be transformed into another one through a chemical reaction. Reactions are controlled by various enzymes, and each enzyme defines a probability that the underlying reaction will actually take place. Thus, reactions (edges) are assigned various probabilities of ex-

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instance, which depend on the specific enzyme (external factor). In this setting a fundamental question posed by biologists is to identify a limited set of enzymes that will guarantee with high probability that a sequence of chemical reactions will take place to convert an input compound into a target compound. This corresponds to solving an instance of our problem: Given a source compound and a target compound, what is the set of enzymes that maximizes the probability that the source compound will be converted into the target compound via a series of chemical reactions?

**Information cascades.** Studying information cascades in social networks is receiving more and more attention, mainly due to its large applicability in marketing strategies. The influence among users in a social microblogging platform such as Twitter can in fact be exploited in viral marketing campaigns. Such social influence can be modeled as in Figure 1, i.e., by means of a probability that once a user has been “activated” in a cascade, she will influence other users to perform the same action (e.g., re-tweeting). This probability typically depends on the topic and content of the cascading tweet. As an example, Lady Gaga would have more influence on her followers for music and fashion-related tweets, while Barack Obama would have more influence on his followers for politics-based tweets. In this context it is crucial for a marketing company to identify the top-k features that a tweet should possess in order to maximize the information cascade from a set of early adopters (or influential users) to a group of target customers. This corresponds to identifying the top-k conditions that maximize the reliability between two (sets of) nodes in the social graph, i.e., the problem we study in this work.

In summary, identifying the top-k conditions maximizing reliability would be critical in cost-effective planning (e.g., finding the top-k enzymes for creation of specific pathways in a biological network), satisfying medium-specific constraints in information cascade (e.g., Twitter limits the message length to 140 characters), as well as in exploratory data analysis, e.g., computing the top-k conditions maximizing reliability can provide interesting insights in the key relations existing between two nodes or two sets of nodes.

**Case study.** To demonstrate the effectiveness of our novel query over uncertain networks, we show case studies in Table 1 with DBLP collaboration graph. Each edge, which represents a collaboration between two authors, is annotated by a set of keywords (e.g., databases, distributed, learning, crowd, etc.), that are present in the titles of the papers, co-authored by the respective authors. The edge probability for a specific keyword is derived based on its frequency, i.e., number of its occurrences in the titles of the papers co-authored by the two authors (For details, see Section 5). The intuition is that the more the times u and v co-authored on keyword c, the higher the chance (i.e., the probability) that u influences v (and, vice versa) for that keyword. Therefore, keywords correspond to catalysts for information cascade in our problem setting.

We retrieve the top-10 keywords between (1) “Surajit Chaudhuri” (expert in databases) and “Jiawei Han” (expert in data mining), as well as between (2) “Surajit Chaudhuri” and “Anastasia Ailamaki” (expert in database systems). We find that the top-10 keywords for the first query consist of many data mining-based terms, e.g., sequence, mining, knowledge, and statistical (which are due to “Jiawei Han” and his co-authors), together with some database-related terms such as query and SQL. On the contrary, the top-10 keywords between “Surajit Chaudhuri” and “Anastasia Ailamaki” are primarily databases and systems related, such as performance, OLAP, transaction, databases, data, and systems. These results show the usefulness of our query. We present several connecting paths formed by these top-10 keywords in Figure 2 which provide insights about the key relations existing between the respective authors.

**Challenges and contributions.** The problem that we study in this work is a non-trivial one. Computing standard reliability over uncertain graphs is a #P-complete problem. We show that, even assuming polynomial-time sampling methods to estimate conditional reliability (such as RHT-sampling), our problem of computing a set of k catalysts that maximizes conditional reliability between two nodes remains NP-hard. Moreover, our problem turns out to be not easy to approximate, as (i) it does not admit any PTAS, and (ii) the underlying objective function is shown to be non-submodular. Therefore, standard algorithms, such as iterative hill-climbing that greedily maximizes the marginal gain at every iteration, do not provide any approximation guarantees and are expected to have limited performance. Within this view, we devise a novel algorithm that first extracts highly-reliable paths between source and target nodes, and then iteratively selects these paths so as to achieve maximum improvement in reliability while still satisfying the constraint on the number of conditions.

After studying the single-source-single-target query, we focus on generalizations where multiple source and target nodes can be provided as input, thus opening the stage to a wider family of queries and applications. We study two variants of this more general problem: (i) maximizing an aggregate function over pairwise reliability between nodes in source and target sets, and (ii) maximizing the probability that source and target nodes remain all connected.

The main contributions of this paper are as follows:

- We focus for the first time on the notion of conditional reliability in uncertain graphs, which arises when the
input graph has conditional edge-existence probabilities. In particular, we formulate and study the problem of finding a limited set of conditions that maximizes reliability between a source and a target node (Section 2).

- We deeply characterize our problem from a theoretical point of view, showing that it is NP-hard and hard to approximate even when polynomial-time reliability estimation is employed (Section 2).
- We design an algorithm that provides effective (approximated) solutions to our problem, while also looking at efficiency. The proposed method properly selects a number of highly-reliable paths so as to maximize reliability while satisfying the budget on the number of conditions (Section 3).
- We generalize our problem and algorithms to the case of multiple source and target nodes (Section 4).
- We empirically demonstrate effectiveness and efficiency of our methods on real-life graphs, while also detailing three concrete case studies (Section 5).

## 2 SINGLE-SOURCE SINGLE-TARGET: PROBLEM STATEMENT

An uncertain graph \( G \) is a quadruple \((V, E, C, P)\), where \( V \) is a set of \( n \) nodes, \( E \subseteq V \times V \) is a set of \( m \) directed edges, and \( C \) is a set of external conditions that influence the edge-existence probabilities. We hereinafter refer to such external conditions as catalysts. \( P : E \times C \to [0, 1] \) is a function that assigns a conditional probability to each edge \( e \in E \) given a specific catalyst \( c \in C \), i.e., \( P(e|c) \) denotes the probability that the edge \( e \) exists given the catalyst \( c \).

The bulk of the literature on uncertain graphs assumes that edge probabilities are independent of one another \([20]\). In this work we make the same assumption. Additionally, we assume that the existence of an edge is determined by an independent process (coin flipping), one per catalyst \( c \), and the ultimate existence of an edge is decided based on the success of at least one of such processes. This assumption naturally holds in various settings. For instance, in a metabolic network with an initial compound and an enzyme, the probability that a target compound would be produced depends only on that specific reaction, and it is independent of other chemical reactions defined in the network. As a result, the global existence probability of an edge \( e \) given a set of catalysts \( C_1 \subseteq C \) can be derived as

\[
P(e|C_1) = 1 - \prod_{c \in C_1} (1 - P(e|c)).
\]

Given a catalysts set \( C_1 \), the uncertain graph \( G \) yields \( 2^m \) deterministic graphs \( G \subseteq \mathcal{G}_{C_1} \), where each \( G \) is a pair \((V, E_G)\), with \( E_G \subseteq E \), and its probability of being observed is:

\[
P(G|C_1) = \prod_{e \in E_G} P(e|C_1) \prod_{e \in E \setminus E_G} (1 - P(e|C_1))
\]

For a source node \( s \in V \) and a target node \( t \in V \), we define conditional reliability \( R((s, t)|C_1) \) as the probability that \( t \) is reachable from \( s \) in \( G \) given catalysts \( C_1 \). Formally, for a possible graph \( G \subseteq \mathcal{G}_{C_1} \), let \( I_G(s, t) \) be an indicator function taking value 1 if there exists a path from \( s \) to \( t \) in \( G \), and 0 otherwise. \( R((s, t)|C_1) \) is computed as follows:

\[
R((s, t)|C_1) = \sum_{G \in \mathcal{G}_{C_1}} [I_G(s, t) \times P(G|C_1)]
\]

The problem that we tackle in this work is introduced next.

### Problem 1 (s-t TOP-k CATALYSTS)

Given an uncertain graph \( G = (V, E, C, P) \), a source node \( s \in V \), a target node \( t \in V \), and a positive integer \( k \), find a catalysts set \( C^* \subseteq C \) of size \( k \) that maximizes the conditional reliability \( R((s, t)|C_1) \) from \( s \) to \( t \):

\[
C^* = \arg\max_{C_1 \subseteq C} R((s, t)|C_1)
\]

subject to \( |C_1| = k \).

Intuitively, the top-k catalysts set \( C^* \) yields multiple high-probability paths from source \( s \) to target \( t \). Any specific path can have edges formed due to different catalysts.

### Theoretical characterization

Problem 1 intrinsically relies on the classical reliability problem, which is \#P-complete \([3]\). As a result, Problem 1 is hard as well.

However, like standard reliability \([4]\) conditional reliability can be estimated in polynomial time via Monte Carlo (MC) sampling, or other sampling methods \([20]\). Thus, the key question is whether our problem remains hard even if polynomial-time conditional-reliability estimation is employed. As formalized next, the answer to this question is positive.

### Theorem 1

Problem 1 is NP-hard even assuming polynomial-time computation for conditional reliability.

### Proof

We prove NP-hardness by a reduction from the MAX k-COVER problem. In MAX k-COVER, we are given a universe \( U \) and a set of \( h \) subsets of \( U \), i.e., \( \mathcal{S} = \{S_1, S_2, \ldots, S_h\} \), where \( S_i \subseteq U \), for all \( i \in [1..h] \). The goal is to find a subset \( S^* \) of \( \mathcal{S} \) of size \( k \) such that the number of elements covered by \( S^* \) is maximized, i.e., so as to maximize \( \sum_{S \in \mathcal{S}} |S| \). Given an instance of MAX k-COVER, we construct in polynomial time an instance of our s-t TOP-k CATALYSTS problem as follows.

We create an uncertain graph \( G \) with a source node \( s \) and a target node \( t \). We add to \( G \) a set of nodes \( u_1, u_2, \ldots, u_Z \), one for each element in \( U \) (\( Z = |U| \)). We connect each of these nodes \( u_i \) to the target node \( t \) with a (directed) edge \((u_i, t)\), and assume that each of such edges \((u_i, t)\) can occur only in the presence of a single catalyst \( c \) with a certain probability \( p < 1 \), i.e., \( \forall i \in [1..Z] : P((u_i, t)|c) = p \) and \( P((u_i, t)|c') = 0, \forall c' \neq c \). Similarly, we put in \( G \) another set of nodes \( x_1, x_2, \ldots, x_Z \) (again one for each element in \( U \)), and connect each of these nodes \( x_i \) to the source node \( s \) with an edge \((s, x_i)\). Each of such edges \((s, x_i)\) can also be present only in the presence of catalyst \( c \), with probability \( P((s, x_i)|c) = p \). Finally, if some element \( u_i \in U \) is covered by at least one of the subsets in \( S \), we add a directed edge \((x_i, u_i)\) in \( G \). For each set \( S_j \in \mathcal{S} \) that covers item \( u_i \), we consider a corresponding catalyst \( c_j \) and set the probability \( P((x_i, u_i)|c_j) = 1 \).

Now, we ask for a solution of s-t TOP-k CATALYSTS on the uncertain graph \( G \) constructed by using \( k + 1 \) catalysts. Note that every solution to our problem necessarily takes catalyst \( c \) because otherwise there would be no way to connect \( s \) to \( t \). Moreover, given that the paths connecting \( s \) to \( t \) are all disjoint and each of them exists
Theorem 2. Problem 1 does not admit any polynomial time, while non-submodularity is shown next with NP-approximation guarantees. Non-supermodularity easily follows for the objective function therein. Thus, standard

Proof 2. See Appendix.

As a further evidence of the difficulty of our problem, it turns out that neither submodularity nor supermodularity holds for the objective function therein. Thus, standard greedy hill-climbing algorithms do not directly come with approximation guarantees. Non-supermodularity easily follows from NP-hardness (as maximizing supermodular set functions under a cardinality constraint is solvable in polynomial time), while non-submodularity is shown next with a counter-example.

Fact 1. The objective function of Problem 1 is not submodular. A set function $f$ is submodular if $f(A \cup \{x\}) - f(A) \geq f(B \cup \{x\}) - f(B)$, for all sets $A \subseteq B$ and all elements $x \notin B$. Look at the example in Figure 3. Let $C_1 = \{c_2\}$, $C_2 = \{c_1, c_2\}$. We find that $R((s, t)|C_1) = 0$, $R((s, t)|C_1 \cup \{c_3\}) = 0$, $R((s, t)|C_2) = 0.3$, and $R((s, t)|C_2 \cup \{c_3\}) = 0.475$. Clearly, submodularity does not hold in this example.

3 Single-source single-target: baselines

In this section we present two simple baseline approaches and discuss their limitations (Sections 3.1 and 3.2). Then, in Section 4.3 we propose a more sophisticated algorithm that aims at overcoming the weaknesses of such baselines.

3.1 Individual top-k baseline

The most immediate approach to our top-k catalysts problem consists of estimating the reliability $R((s, t)|\{c\})$ between the source $s$ and the target $t$ attained by each catalyst $c \in C$ individually, and then outputting the top-k catalysts that achieve the highest individual reliability.

Time complexity. For each catalyst, we can estimate reliability via MC sampling; sample a set of $K$ deterministic graphs from the input uncertain graph, and estimate reliability by summing the (normalized) probabilities of the graphs where target is reachable from source. The time complexity of MC sampling for a single catalyst is $\mathcal{O}(K(n+m))$, where $n$ and $m$ denote the number of nodes and edges in the input uncertain graph, respectively. Hence, the overall time complexity of the individual top-k baseline is $\mathcal{O}(|C| K(n+m) + |C| \log k)$, where the last term is due to top-k search.

Shortcomings. The Individual top-k algorithm suffers from both accuracy and efficiency issues.

- Accuracy: This baseline is unable to capture the contribution of paths containing different catalysts. For example, in Figure 5 the individual reliability attained by each catalyst is 0. Thus, if we are to select the top-2 catalysts, there will be no way to discriminate among catalysts, which will be picked at random. Instead, in reality, the top-2 catalysts set is $\{c_1, c_2\}$.
- Efficiency: To achieve good accuracy, MC sampling typically requires around thousands of samples [24]. Performing such a sampling for each of the $|C|$ catalysts can be quite expensive on large graphs ($|C|$ may be up to the order of thousands as well, see Section 6).

3.2 Greedy baseline

A more advanced baseline consists of greedily selecting the catalyst that brings the maximum marginal gain to the total reliability, until $k$ catalysts have been selected. More precisely, assuming that a set $C_1$ of catalysts has been already computed, in the next iteration this Greedy baseline selects a catalyst $c^*$ such that:

$$c^* = \arg \max_{c \in C \setminus C_1} [R((s, t)|C_1 \cup \{c\}) - R((s, t)|C_1)]$$

Note that, since the top-k catalysts problem is neither submodular nor supermodular, this greedy approach does not achieve any approximation guarantees.

Time complexity. The time complexity of each iteration of the greedy baseline is $\mathcal{O}(|C| K(n+m))$, as we need to estimate the reliability achieved by the addition of each catalyst in order to choose the one maximizing the marginal gain. For a total of $k$ iterations (top-k catalysts are to be reported), the overall complexity is $\mathcal{O}(|C| k K(n+m))$.

Shortcomings. While being more sophisticated than Individual top-k, the Greedy baseline still suffers from both accuracy and efficiency issues.

- Accuracy: Although Greedy partially solves the accuracy issue related to the presence of paths with multiple catalysts, such an issue is still present at least in the initial phases of this second baseline. For example, in Figure 5 the individual reliability attained by each catalyst is 0. Therefore, in the first iteration the Greedy algorithm has no information to properly select a catalyst, thus ending up with a completely random choice. If $c_3$ is selected as a first catalyst, then the second catalyst selected would be $c_1$. Thus, Greedy would output $\{c_1, c_3\}$, while the top-2 catalyst set is $\{c_1, c_2\}$. We refer to this issue as "cold-start" problem.
- Efficiency: MC sampling is performed $|C| k$ times. This is more inefficient than the Individual top-k baseline.
Following we provide the details of each of the two steps.

**Most-reliable Paths**

Given an uncertain graph $G = (V, E, C, P)$, a source node $s \in V$, a target node $t \in V$, positive integers $k$, $r$

**Ensure:** Subset of catalysts $C^* \subseteq C$

1. $P \leftarrow$ Algorithm 1 on input $(G, s, t, r)$
2. $P_1 \leftarrow$ Algorithm 2 on input $(G, s, t, k)$
3. $C^* \leftarrow$ catalysts present on $P_1$

**Algorithm 1 Most-reliable Paths**

**Require:** Uncertain graph $G = (V, E, C, P)$, source node $s \in V$, target node $t \in V$, positive integers $k$, $r$

**Ensure:** Subset of catalysts $C^* \subseteq C$

1. $P \leftarrow$ Algorithm 1 on input $(G, s, t, r)$
2. $P_1 \leftarrow$ Algorithm 2 on input $(G, s, t, k)$
3. $C^* \leftarrow$ catalysts present on $P_1$

**Algorithm 2 Top-$r$ Most Reliable Path Selection**

**Require:** Uncertain graph $G = (V, E, C, P)$, source node $s \in V$, target node $t \in V$, positive integer $r$

**Ensure:** $P$: top-$r$ most reliable paths from $s$ to $t$

1. for all $e \in E$ do
2. let $C(e) = \{c_1, c_2, \ldots, c_i\}$ be the set of all catalysts s.t. $P(e|c_j) > 0$, $\forall j \in [1,i]$.
3. assign probability $P(e|c_j) = P(e|c_j)$
4. assign edge-weight $W(e_j) = -\log P(e_j|c_j)$
5. end for
6. $P \leftarrow$ top-$r$ shortest paths from $s$ to $t$ in the constructed multigraph

**4 Single-source single-target: Proposed method**

Here we describe the method we ultimately propose to provide effective and efficient solutions to the $s$-$t$ top-$k$ Catalysts problem.

The main intuition behind our method directly follows from the shortcomings of the two baselines discussed above. Particularly, both baselines highlight how considering catalysts one at a time is not accurate. This can easily be explained as a single catalyst can bring information that is related only to single edges. Instead, what really matters in computing the reliability between two nodes is the set of paths connecting source to target. This observation finds confirmation in the literature [12].

Motivated by this, we design the proposed method as composed of two main steps. First, we select the top-$r$ paths exhibiting highest reliability from source to target. Second, we iteratively include these paths in the solution so as to maximize the marginal gain in reliability, while still keeping the constraint on total number of catalysts satisfied. Apart from the main advantage due to considering paths instead of single catalysts, designing our algorithm as composed of two separate steps also allows us to achieve high efficiency. Indeed, the first step can be efficiently solved by fast algorithms for finding the top-$r$ shortest paths in a graph, while the second step requires MC sampling to be performed in a significantly reduced version of the original graph.

The outline of the proposed method, which we call Most-reliable Paths, is reported in Algorithm 1. In the following we provide the details of each of the two steps.

**4.1 Step 1: Most-reliable path selection**

The first step of the proposed method consists of finding the top-$r$ most reliable paths from source to target nodes. Given an uncertain graph $G = (V, E, C, P)$, a source node $s \in V$, and a target node $t \in V$, we first convert $G$ into an uncertain, multigraph $G'$ (Algorithm 2). For each edge $e = (u, v) \in E$, let $C(e)$ denote the set of all catalysts $c$ such that $P(e|c) > 0$. Assume $C(e) = \{c_1, c_2, \ldots, c_i\}$. Then, we add $i$ edges $\{e_1, e_2, \ldots, e_i\}$ between $u$ and $v$ in the multigraph $G'$. To each newly constructed edge $e_j$, $j \in [1..i]$, we assign a single catalyst $C(e_j) = c_j$ and set $P(e_j|c_j) = P(e|c_j)$. It can be easily noted that $G$ and $G'$ are equivalent in terms of our problem. The construction of $G'$ only serves the purpose of selecting the top-$r$ most reliable paths from $s$ to $t$ in such a way that, for each intermediate pair of nodes $x, y$ along a path, a single edge (and, thus, a single catalyst) among the many ones possibly created by the $G \rightarrow G'$ transformation is picked up. The reliability of a path is defined as the product of the edge-probabilities along that path.

To ultimately compute the top-$r$ most reliable paths, we further convert the uncertain multigraph $G'$ into an edge-weighted multigraph $G''$ by assigning a weight $-\log(p_e)$ to each edge $e$ with probability $p_e$ of $G'$. This way, the top-$r$ most reliable paths in $G''$ will correspond to the top-$r$ shortest paths in $G''$. To compute the top-$r$ shortest paths in $G''$, we apply the well-established Eppstein’s algorithm [13], 2 which has time complexity $O(|C(m+n\log n+r)|)$.

We point out that the number $r$ of paths to be selected is an input parameter of the algorithm. In general, parameter $r$ constitutes a knob to tradeoff between efficiency and accuracy (larger $r$ leads to higher accuracy and lower efficiency). It can be set by, e.g., observing when inclusion of the top-$(r+1)$-th reliable path does not significantly increase the reliability gained by the top-$r$ paths.

**4.2 Step 2: Iterative path inclusion**

The second step of our most-reliable-path method aims at selecting a proper subset from the top-$r$ most-reliable path set so as to maximize reliability between source and target nodes, while also meeting the constraint on the number of output catalysts. Denoting by $Rel_P(s, t)$ the reliability between $s$ and $t$ in the subgraph induced by a path set $P$, this step formally corresponds to the following problem:

**Problem 2 (Iterative Path Inclusion).** Given set $\mathcal{P}$ of top-$r$ most reliable paths from $s$ to $t$ in multigraph $G'$, find a path set $\mathcal{P}^* \subseteq \mathcal{P}$ such that:

$$\mathcal{P}^* = \arg \max_{\mathcal{P}^* \subseteq \mathcal{P}} \sum_{e \in \mathcal{P}^*} C(e) \ s.t. \ |\cup_{e \in \mathcal{P}^*} C(e)| \leq k \tag{4}$$

The iterative path inclusion problem can be shown to be NP-hard via a reduction from max-$k$ cover. The proof is analogous to the one in Theorem 4, we thus omit it.

**Theorem 3.** Problem 2 is NP-hard.

2 We discuss Eppstein’s algorithm in the Appendix.
Algorithm. We design an efficient greedy algorithm (Algorithm 3) for the iterative path inclusion problem. At each iteration, we add a path $P^*$ to the already computed path set $P_1$ which brings the maximum marginal gain in terms of reliability. While selecting path $P^*$, we also ensure that the total number of catalysts used in the paths $P_1 \cup \{P^*\}$ is no more than $k$. The algorithm terminates either when there is no path left in the top-$r$ most reliable path set $P$, or no more paths can be added without violating the catalyst budget $k$. We report the catalysts present in $P_1$ as our final solution. If the total number of catalysts present in $P_1$ is $k' < k$, additional $k - k'$ catalysts that are not in $P_1$ can be selected with some proper criterion (e.g., frequency on the non-selected paths). Next, we report an example of our Iterative Path Inclusion algorithm in action.

Example 1. In Figure 4(a) we show the top-3 reliable paths from node $s$ to node $t$, i.e., $P_1 = e_1e_2$, $P_2 = e_3e_4$, and $P_3 = e_5e_6$. Assume that there is a budget constraint of 3 catalysts. In the first iteration we select the path $P_1$ since it has the highest reliability compared to the two other paths. In the second iteration $P_1$ and $P_2$ together have higher reliability than $P_2$ and $P_3$. However, the former combination requires 4 catalysts, thus violating the constraint. Hence, we select $P_2$ and $P_3$. After that, the algorithm terminates as no more path can be included without violating the constraint on catalysts.

Approximation guarantee. The Iterative Path Inclusion algorithm achieves approximation guarantee under some assumptions. If the top-$r$ most reliable paths are node-disjoint (except at source and target nodes), Iterative Path Inclusion exhibits an approximation ratio proportional to $r$.

Theorem 4. The Iterative Path Inclusion algorithm, under the assumption that the top-$r$ most reliable paths are node-disjoint, achieves an approximation factor of:

$$\frac{1}{k_{rel}} \left(1 - \left(\frac{K_C - k_{rel}}{K_C}\right)^{k_C}\right),$$

where

$$K_C = \max_{P_1 \subseteq P} \{|P_1| : |C(P_1)| \leq k\}$$

and

$$k_C = \min_{P_1 \subseteq P} \{|P_1| : |C(P_1)| \leq k\}$$

and

$$k_{rel} = 1 - \min_{P \subseteq P} \frac{\text{Rel}_{P}(s, t) - \text{Rel}_{P \setminus \{P\}}(s, t)}{\text{Rel}_P(s, t)}.$$

Proof 3. See Appendix.

In the above approximation-guarantee result, $K_C$ and $k_C$, respectively, denote maximum and minimum size of the maximal feasible path set that can be derived from $P$, $k_{rel}$ denotes the curvature of our optimization function, which can be shown to be submodular when paths in $P$ are node-disjoint (see Appendix). Hence, in this case $k_{rel} \in (0, 1)$. Assuming that $P$ contains at least one path having less than $k$ catalysts, then in the worst case the approximation ratio is $\geq \frac{1}{k_{rel}} \geq \frac{1}{r}$ (where $r$ is the total number of paths in the top-$r$ path set $P$). In other words, the approximation ratio is guaranteed to be at least $\frac{1}{r}$.

Time complexity. Let us denote by $n'$ and $m'$ the number of nodes and edges, respectively, in the subgraph induced by the top-$r$ most reliable path set $P$. At each iteration, our iterative path selection algorithm performs MC sampling over the subgraph induced by the selected paths. The number of iterations is at most $r$. Thus, if $K$ is the number of samples used in each MC sampling, the iterative path selection algorithm takes $O(r^2K(n' + m'))$ time. Including the time due to the first step of selecting the top-$r$ most reliable paths, we get that the overall time complexity of the proposed Most-reliable Paths algorithm is $O((|C|m + n \log n + r^2K(n' + m'))$. We point out that the subgraph induced by the top-$r$ most reliable paths is typically much smaller than the input graph $G$. Thus, our Most-reliable Paths method is expected to be much more efficient than the two baselines introduced earlier. Experiments in Section 6 confirm this claim.

5 Multiple Sources and Targets

Real-world queries often involve sets of source and/or target nodes, instead of a single source-target pair. As an example, the topic-aware information cascade problem [4], [6] asks for a set of early adopters who maximally influence a given set of target customers. Motivated by this, in the following we discuss problems and algorithms for the case where multiple source nodes can be provided as input. Such a generalization opens the stage to various formulations of the problem. Here we focus on two variants: (1) maximizing an aggregate function over all possible source-target pairs (Section 5.1), and (2) maximizing connectivity among all query nodes (Section 5.2). Note that our first problem formulation has a notion of “clique” connectivity, as it applies an aggregate function over all pairs of query nodes.

5.1 Maximizing aggregate functions

We formulate our problem as follows.

Problem 3 (top-$k$ catalysts w/ aggregate). Given an uncertain graph $G = (V, E, C, P)$, a source set $S \subseteq V$, a target set $T \subseteq V$, and a positive integer $k$, find a catalyst set $C^*$ of size $k$ that maximizes an aggregate function $F$ over conditional reliability of all source-target pairs:

$$C^* = \arg \max_{C \subseteq C} \max_{(s, t) \in S \times T} F((s, t)|C)$$

subject to $|C| = k$. (9)
Being a generalization of the $s$-$t$ Top-$k$ Catalysts problem, Problem 3 can easily be recognized as NP-hard. In this work we consider three commonly-used aggregate functions: average, maximum, and minimum. These aggregates give rise to three variants of Problem 3 which we refer to TOP-$k$ Catalysts Avg, TOP-$k$ Catalysts Max, and TOP-$k$ Catalysts Min, respectively:

- **Average.** Find the top-$k$ catalysts such that the average reliability over all $\langle s, t \rangle$ pairs is maximized. This is equivalent to maximization of the sum of reliability over all $\langle s, t \rangle$ pairs. This problem occurs, e.g., in the topic-aware information cascade scenario when the campaigner wants to maximize the spread of information to the entire target group.

- **Maximum.** Find the top-$k$ catalysts such that the reliability of the $\langle s, t \rangle$ pair with the highest reliability is maximized. In the topic-aware information cascade problem this is equivalent to the scenario that each early adopter is campaigning a different product of the same campaigner. The campaigner wants at least one target user to be aware about one of her products (e.g., each target user might be a celebrity user in Twitter). Therefore, the campaigner would be willing to maximize the spread of information from at least one early adopter to at least one target user.

- **Minimum.** Find the top-$k$ catalysts such that the reliability of the $\langle s, t \rangle$ pair having the lowest reliability is maximized. In the topic-aware information cascade setting this is equivalent to the problem that each early adopter is campaigning a different product of the same campaigner, and the campaigner wants to maximize the minimum spread of her campaign from any of the early adopters to any of her target users. This is motivated, in reality, because only a small percentage of the users who have heard about one target user to be aware about one of her products (e.g., each target user might be a celebrity user in Twitter). Therefore, the campaigner would be willing to maximize the spread of information from at least one early adopter to at least one target user.

In the following we describe the algorithm we propose for each of the aforementioned aggregate functions. Extending the baselines presented in Sections 3.1 and 3.2 to the multiple-query-nodes case is instead straightforward (regardless of the aggregate function). We thus omit the details.

Unless otherwise specified, below we assume that $S \cap T = \emptyset$, that is source and target sets are non-overlapping. We will discuss case by case how our algorithms can (easily) handle the case when $S$ and $T$ overlap.

### 5.1.1 Algorithm for average reliability

We first identify the top-$r$ most reliable paths for each $\langle s, t \rangle$ pair (Algorithm 4). Then, we add two dummy nodes $s'$ and $t'$ to the graph (Figure 5). We add directed edges (i) from $s'$ to every source node $s \in S$, and (ii) from every target node $t \in T$ to $t'$. Each of these newly included edges has probability 1 regardless of any catalysts, i.e., including these edges in our solution does not increase the total number of catalysts used. Next, we apply the Iterative Path Inclusion algorithm (Algorithm 5) to add paths from $s'$ to $t'$, while maintaining the budget $k$ on total number of catalysts satisfied. Catalysts present in the selected paths are reported as a solution to the TOP-$k$ Catalysts Avg problem.

#### Time complexity

The time required to find the reliable paths for all $\langle s, t \rangle$ pairs is $O(|S||T| (m + n \log n + r))$, as we apply Eppstein’s algorithm for $|S||T|$ times. Next, the time complexity of the iterative path inclusion phase is $O\left((|S||T|^2 (n' + m') K)\right)$, where $n'$ and $m'$ are the number of nodes and edges of the subgraph induced by the reliable paths, and $K$ is the number of samples used in each MC sampling.

### 5.1.2 Algorithm for maximum reliability

For each $\langle s, t \rangle$ pair, we identify the top-$r$ most reliable paths, and then apply the Iterative Path Inclusion algorithm to find the top-$k$ catalysts. Then, we select the $\langle s, t \rangle$ pair which attains the maximum reliability. We report the corresponding top-$k$ catalysts as the solution to the $s$-$t$ Top-$k$ Catalysts problem.

#### Time complexity

The time required to find the reliable paths for all $\langle s, t \rangle$ pairs is $O(|S||T| (m + n \log n + r))$. Similarly, the time complexity of the iterative path inclusion phase is $O\left(|S||T| r^2 (n' + m') K\right)$. There is an additional cost $O(|S||T|)$ to find the $\langle s, t \rangle$ pair with the maximum reliability, which is, however, dominated by the time spent in path inclusion.

### 5.1.3 Algorithm for minimum reliability

We start again by finding the top-$r$ most reliable paths for each $\langle s, t \rangle$ pair. Instead, applying the Iterative Path Inclusion algorithm in this case is more subtle. Specifically, if there are many $\langle s, t \rangle$ pairs and a limited budget $k$ of catalysts, spending too many catalysts for a single $\langle s, t \rangle$ pair might prevent us from finding a path for another pair. This way there will be pairs with conditional reliability very low, thus the solution to the Top-$k$ Catalysts Min problem, i.e., the pair exhibiting minimum conditional reliability, would be quite poor. To mitigate this issue, we consider an additional
step where we find a minimum catalyst set before applying the Iterative Path Inclusion algorithm. The subsequent steps remain instead identical, regardless of whether $S$ and $T$ overlap or not.

Finding minimum catalyst set. The objective of this step is to select a minimum set of catalysts which ensure that at least one path for every $⟨s, t⟩$ pair exists. This step corresponds to the following problem.

\textbf{Problem 4 (Minimum Catalyst Set).} Given a source set $S$, a target set $T$, a set of paths $P$, an uncertain graph $\mathcal{G} = (V, E, C, P)$ induced by $P$, find the smallest catalyst set $C^* \subseteq C$ such that the conditional reliability $R((s, t)|C_1)$ for each $(s, t)$ pair is larger than zero:

$$\begin{align*}
C^* &= \arg\min_{C_1 \subseteq C} |C_1| \\
&\text{subject to } R((s, t)|C_1) > 0, \forall (s, t) \in S \times T. (10)
\end{align*}$$

\textbf{Theorem 5.} Problem 4 is NP-hard.

\textbf{Proof 4.} NP-hardness can easily be verified by noticing that the SET COVER problem can be reduced to a specific instance of MINIMUM CATALYST SET where each path in $P$ can be formed using a single catalyst. In this case, in fact, we ask for the minimum number of catalysts required to cover at least one path of every $(s, t)$ pair, which exactly corresponds to what SET COVER asks for.

Minimum Catalyst Set Finding algorithm. We design an algorithm to provide approximated solutions to MINIMUM CATALYST SET which consists of four steps:

- \textbf{Step 1.} Mark all $(s, t)$ pairs as disconnected.
- \textbf{Step 2.} For all disconnected $(s, t)$ pairs, find a path $P$ that connects one of such $(s, t)$ pairs, while adding the minimum number of new catalysts to the set of already selected catalysts.
- \textbf{Step 3.} Mark that $(s, t)$ pair as connected. Include the catalysts in path $P$ to the set of selected catalysts.
- \textbf{Step 4.} If there is at least one disconnected $(s, t)$ pair, go to step 2.

We report the set of selected catalysts as our minimum set. If the size of this minimum set is more than $k$, we perform an additional step. From the selected set $C^*$, if a subset $C'$ can be removed, a path can still be found for all connected $(s, t)$ pairs with the remaining catalysts in $C^* \setminus C'$, then $C'$ is removed from $C^*$. We illustrate our algorithm with an example below.

\textbf{Example 2.} As shown in Figure 6 let us assume that the source set is $S = \{s_1, s_2\}$, and the target set is $T = \{t_1, t_2\}$. The figure illustrate the top-2 most reliable paths for each source-target pair. Assume there is a budget $k = 3$ on the number of output catalysts. We now apply our Minimum Catalyst Set finding algorithm. First, we select the catalyst $c_3$, since this catalyst is sufficient to have an edge for the $(s_1, t_1)$ pair. Then, we select the catalyst $c_4$ because $\{c_3, c_4\}$ together add an edge for the $(s_2, t_1)$ pair. Next, we consider the catalyst $c_1$ in order to have an edge for the pair $(s_1, t_2)$. At this point, we have already saturated the budget of 3 catalysts: $\{c_1, c_3, c_4\}$, but we are yet to add an edge for the $(s_2, t_2)$ pair. Thus, we delete $c_4$ from the selected catalysts set, because this still allows a path for three previously connected source-target pairs. Finally, we add catalyst $c_2$ to the set. The final catalysts set $\{c_1, c_2, c_3\}$ allows a path for all source-target pairs.

\textbf{Time complexity.} In each iteration, we find the path with the smallest number of new catalysts, which requires $O(|S||T|(n' + m'))$ time. Then, we also remove the redundant catalysts, which requires another $O(kr|S||T|(n' + m'))$ time. Since there can be at most $|S||T|$ iterations, overall time complexity of our Minimum Catalyst Set finding algorithm is $O(kr|S|^2|T|^2(n' + m'))$.

Intuitively, the minimum set finding step ensures that, given large enough budget on catalysts, there will be at least one path for all $(s, t)$ pairs. Therefore, the objective function of the TOP-$k$ CATALYSTS MIN problem will be guaranteed to be larger than zero. If our budget has not been exhausted yet and more catalysts can be added, we next apply the Iterative Path Inclusion algorithm as follows. At each iteration, we find the $(s, t)$ pair exhibiting minimum conditional reliability. We then add a path that maintains the catalysts budget, while also maximizing the marginal gain in reliability for that $(s, t)$ pair. The algorithm terminates when no more paths can be added without exceeding the catalysts budget, or all top-$r$ paths for all $(s, t)$ pairs have been selected.

\subsection*{5.2 Maximizing connectivity}

In the second variant of the $s$-$t$ TOP-$k$ CATALYSTS problem applied to multiple query nodes, we do not distinguish between source and target nodes. All query nodes are considered as peers: the objective of this CONNECTIVITY TOP-$k$ CATALYSTS problem is to find a set of top-$k$ catalysts which maximize the probability that all query nodes are connected in the subgraph induced by edges containing those catalysts. An example of application of this problem is finding a suitable topic list of a thematic scientific event among researchers. The event would be successful not only if the invitees are experts for those topics, but also if they can find connections (e.g., paths formed due to collaborations in the DBLP network) with each other based on those topics.

We formally define our problem below.

\textbf{Problem 5 (Connectivity TOP-$k$ Catalysts).} Given an uncertain graph $\mathcal{G} = (V, E, C, P)$, a set of query nodes
In the above statement $J_C(Q)$ is an indicator function over a possible deterministic graph $G \subseteq \overline{G(C_1)}$ taking value 1 if nodes in $Q$ are all connected in $G$, and 0 otherwise. For simplicity, in directed graphs we consider a weak notion of connectivity, i.e., connectivity disregarding edge-directions. The extension to strong connectivity is straightforward.

**Algorithm.** The CONNECTIVITY TOP-$k$ CATALYSTS problem is a generalization of the $s$-$t$ TOP-$k$ CATALYSTS basic problem (Problem [1]). Thus, it can be immediately be recognized as NP-hard.

To provide high quality approximation, we design an algorithm whose outline is similar in spirit to the Most-reliable Paths algorithm proposed for $s$-$t$ TOP-$k$ CATALYSTS. As a main difference, however, since our goal in CONNECTIVITY TOP-$k$ CATALYSTS is to maximize connectivity among a set of peer nodes, we ask for the top-$r$ minimum Steiner trees as a first step of the algorithm (rather than top-$r$ most reliable paths between a single source-target pair). A Steiner tree for a set $Q$ of nodes in a weighted graph is a tree that spans all nodes of $Q$. A minimum Steiner tree is a Steiner tree whose sum of edge-weights is the minimum. We first apply the technique proposed in [13] to find the top-$r$ minimum Steiner trees from an equivalent edge-weighted, multi-graph $G''$. We recall that $G''$ can be obtained from the input uncertain graph $G$ by following Algorithm [2]. Next, we iteratively include the Steiner trees in our solution so as to maximize the marginal gain in the probability that nodes in $Q$ are connected, while not exceeding the budget on catalysts.

**Time Complexity.** The complexity to find the top-$r$ minimum Steiner trees is $O\left(\binom{\binom{|Q|}{2} \cdot n + 2^{Q(|Q| + \log n)} n + e}{2} \right)$ [13]. As for Iterative Path Inclusion, the complexity of our iterative tree inclusion method is $O(r^2 (|n + m|) K)$, where, we recall, $K$ is the number of samples used in each MC sampling, and $n'$ and $m'$ are the number of nodes and edges in the subgraph induced by the top-$r$ minimum Steiner trees, respectively.

## 6 Experimental evaluation

We report empirical results to show accuracy and efficiency of the proposed methods. We also provide case studies to demonstrate the applicability of the top-$k$ catalysts identified by our methods. We report sensitivity analysis by varying all main parameters involved in the problem, e.g., the number of catalysts, reliable paths, and query nodes, as well as distance between source and target nodes. The code is implemented in C++ and experiments are performed on a single core of a 100GB, 2.26GHz Xeon server.

### 6.1 Experimental setup

**Datasets.** We use three real-world uncertain graphs. DBLP (http://dblp.uni-trier.de/xml). We use this well-known collaboration network, downloaded on August 31, 2016, to showcase a real-world case study. Each node represents an author, and an edge denotes co-authorship. Each edge is defined by a set of keywords, that are present within the title of the papers, co-authored by the respective authors. We selected 347 distinct keywords from all paper titles, e.g., databases, distributed, learning, crowd, verification, etc, based on frequency and how well they represent various sub-areas of computer science. We count occurrences of a specific keyword in the titles of the papers co-authored by any two authors. Edge probabilities are derived from an exponential cdf of mean $\mu = 5$ to this count [20]; hence, if a keyword $c$ appeared $t$ times in the titles of the papers co-authored by the authors $u$ and $v$, the corresponding probability is $p((u, v)|c)) = 1 - \exp^{-t/5}$. The intuition, as we stated earlier, is that the more the times $u$ and $v$ co-authored on keyword $c$, the higher the chance (i.e., the probability) that $u$ influences $v$ (and, vice versa) for that keyword. Therefore, keywords correspond to catalysts for information cascade in our problem setting.

BioMine (https://www.cs.helsinki.fi/group/biomine). This is the database of the BIOMINE project. The graph is constructed from biological articles [24]. Nodes represent biological concepts such as genes, proteins, etc., and edges denote real-world phenomena between two nodes, e.g., a gene “codes” for a protein. Edge probabilities quantify the existence of a phenomenon between the two endpoints of that edge. In our setting these phenomena correspond to catalysts.

Freebase (http://www.freebase.com). This is a knowledge graph, where nodes are named entities (e.g., Google) or abstract concepts (e.g., Asian people), while edges represent relationships among those entities (Jerry Yang is the “founder” of Yahoo!). Relationships corresponds to catalysts. We use the probabilistic version of the graph as derived in [10].

**Query Selection.** For each set of experiments we select 500 different queries. If we do not impose any distance constraint between source and destination, both source and destination are picked uniformly at random. When we would like to maintain a maximum pairwise distance $d$ from source to destination, we first select the source uniformly at random. Then, out of all nodes that are within $d$-hops from it, one node is selected uniformly at random as the destination. All reported results are averaged over 500 such queries.

**Competing methods.** We compare the proposed Most-reliable Paths method (Algorithm [1]) to the two baselines, Individual top-$k$ and Greedy, discussed in Sections [3.1][3.2]. For the sake of brevity, in the remainder of this section we refer to the proposed method as Rel-Path, and to the Individual top-$k$ baseline as Ind-$k$.

**Reliability estimation.** Both the proposed method and the baselines need a subroutine that estimates conditional reliability for given source node(s), target node(s), and number

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3 We describe Ding et al.’s algorithm [13] in the Appendix.
of catalysts. To this end, we employ the well-established Monte Carlo-sampling method. In particular, to improve efficiency, we combine MC sampling with a breadth first search from the source node (set) \( [20] \), meaning that the coin for establishing if an edge should be included in the current sample is flipped only upon request. This avoids to flip coins for edges in parts of the graph that are not reached with the current breadth first search, thus increasing the chance of early termination. In the experiments we found that MC sampling converges at around \( K = 1000 \) samples in our datasets. This is roughly the same number observed in the literature \([20], [23]\) for these datasets. Hence, we set \( K = 1000 \) in all sets of experiments.

### 6.2 Case study

In addition to the two case studies demonstrated in Introduction, we focus on another case study in Table 4 using the DBLP dataset, and setting top-\( k = 5 \). As earlier, the goal is to demonstrate that our Rel-Path is able to identify the important keywords that can explain the connectivity between two authors. To this end, we run our Rel-Path and the two baselines, with authors Arnab Nandi (source) and Hanghang Tong (target), who are well-known researchers in databases and data mining, respectively.

There is no path with only one keyword between authors Arnab Nandi and Hanghang Tong in the dataset. Hence, Ind-\( k \) reports the top-5 keywords uniformly at random, and indeed the majority of them are not very relevant, which can be observed from the lower reliability achieved by these keywords. Greedy also suffers from the cold-start problem, thus its top-5 keywords are not always relevant. For example, the keyword ‘language’ is not very relevant to explain the connectivity between the two authors. On the other hand, the top-5 keywords identified by Rel-Path are quite interesting – ‘systems’ and ‘query’ refer to the expertise of “Arnab Nandi”, whereas ‘graph’ denotes the expertise of “Hanghang Tong”. The other two keywords, ‘xml’ and ‘data’ appear due to their intermediate connections, in particular, “H. V. Jagadish” and “Wenfei Fan”. In fact, by following these top-5 keywords, we identified two important connections between the query authors as follows: (1) Arnab Nandi \( \rightarrow \) H. V. Jagadish \( \rightarrow \) Christos Faloutsos \( \rightarrow \) Hanghang Tong, and (2) Arnab Nandi \( \rightarrow \) Philip Bohannon \( \rightarrow \) Wenfei Fan \( \rightarrow \) Shuai Ma \( \rightarrow \) Hanghang Tong. We note that the reliability achieved by these top-5 keywords significantly outperforms that obtained from the other two baselines. This case study demonstrates the effectiveness and applicability of our proposed technique.

### 6.3 Single-source single-target

#### Experiments over different datasets

In Table 4 we show conditional reliability and running time of all competitors for top-5 output catalysts. For our Rel-Path, we use top-20 most reliable paths with Freebase and BioMine and top-50 most reliable paths over DBLP, as we observe that increasing the number of paths beyond 20 (Freebase and BioMine) and 50 (DBLP) does not significantly increase the quality in respective datasets. Results with varying the number of most reliable paths will be reported shortly.

Conditional reliability illustrates the quality of the top-\( k \) catalysts found: the higher the reliability, the better the quality. The proposed Rel-Path achieves the best quality results on all our datasets.

Concerning running time, we observe that Rel-Path is 2-3 orders of magnitude faster than Ind-\( k \), and 3-4 orders faster than Greedy. This confirms that performing MC sampling on a significantly reduced version of the input graph leads to significant benefits in terms of efficiency, without affecting accuracy. Surprisingly, Greedy is orders of magnitude slower than Ind-\( k \). The reason is the following. Although only a factor \( k \) separates Ind-\( k \) from Greedy based on our complexity analysis, what happens in practice is that Ind-\( k \) benefits from MC-sampling’s early termination much more than Greedy, as Ind-\( k \) considers each catalyst individually, while Greedy considers catalyst sets. One may also note that the running times over BioMine is higher than that over Freebase. Although Freebase has more number of nodes and edges, the graph is sparse compared to BioMine. Therefore, a breadth first search in BioMine often traverses more nodes, thus increasing its processing time.

#### Varying number of catalysts

We show results with varying the number \( k \) of output catalysts in Table 5. Similar trends have been observed in all datasets, thus, for brevity, we only report results on BioMine. As expected, conditional reliability and running time increase with more catalysts. Our Rel-Path remains more accurate and faster than both baselines for all \( k \).

#### Varying distance from source to target

Table 6 reports on results with varying the distance from source to target query nodes. Keeping fixed the number of output catalysts, as expected, the reliability achieved by all three methods decreases with larger distance from source to target. However, we observe that the reliability drops sharply for Ind-\( k \). This is because with increasing distance it becomes less likely that there would be a path due to only one catalyst from source to target. We also note that the reliability decreases more in Greedy than in the proposed Rel-Path. This is due to the cold-start problem of Greedy: It is more likely for Greedy to make mistaken choices in the initial steps if source to target are connected by longer paths.

#### Varying number of most reliable paths

We also test Rel-Path for different values of the number \( r \) of most reliable paths discovered in the first step of the algorithm. We report

| Source    | Target     | Top-5 Keywords | Greedy Rel-Path | Ind-k Rel-Path | Conditional Reliability | Running time (sec) |
|-----------|------------|----------------|----------------|---------------|------------------------|-------------------|
| Arnab Nandi | Hanghang Tong | manifold language systems | | | 0.17 | 1.38 |
| | | spark data query data | | | 0.59 | 1220 |
| | | data databases data | | | 0.60 | 2290 |
| | | topic query xml | | | 0.62 | 2290 |
| | | visual learning graph | | | 0.63 | 2303 |

| dataset | Ind-k Rel-Path | Greedy Rel-Path | Conditional Reliability | Running time (sec) |
|---------|----------------|----------------|------------------------|-------------------|
| Freebase | 0.15 | 0.15 | 0.17 | 1.38 |
| BioMine | 0.18 | 0.43 | 0.59 | 1220 |
| DBLP | 0.11 | 0.26 | 0.28 | 85.97 |

| dataset | Ind-k Rel-Path | Greedy Rel-Path | Conditional Reliability | Running time (sec) |
|---------|----------------|----------------|------------------------|-------------------|
| Freebase | 0.15 | 0.15 | 0.17 | 1.38 |
| BioMine | 0.18 | 0.43 | 0.59 | 1220 |
| DBLP | 0.11 | 0.26 | 0.28 | 85.97 |

| dataset | Ind-k Rel-Path | Greedy Rel-Path | Conditional Reliability | Running time (sec) |
|---------|----------------|----------------|------------------------|-------------------|
| Freebase | 0.15 | 0.15 | 0.17 | 1.38 |
| BioMine | 0.18 | 0.43 | 0.59 | 1220 |
| DBLP | 0.11 | 0.26 | 0.28 | 85.97 |

| dataset | Ind-k Rel-Path | Greedy Rel-Path | Conditional Reliability | Running time (sec) |
|---------|----------------|----------------|------------------------|-------------------|
| Freebase | 0.15 | 0.15 | 0.17 | 1.38 |
| BioMine | 0.18 | 0.43 | 0.59 | 1220 |
| DBLP | 0.11 | 0.26 | 0.28 | 85.97 |
TABLE 6: Reliability and efficiency with varying source-to-target distance. Single source-target pair, BioMine dataset, top-5 catalysts.

| distance (# hops) | conditional reliability | running time (sec) |
|-------------------|-------------------------|--------------------|
|                   | Ind-k | Greedy | Rel-Path | Ind-k | Greedy | Rel-Path |
| 2                 | 0.45  | 0.75   | 0.83     | 346   | 9798   | 4.90     |
| 4                 | 0.08  | 0.38   | 0.64     | 406   | 23140  | 5.37     |
| 6                 | 0.02  | 0.17   | 0.30     | 548   | 29135  | 5.58     |

TABLE 7: Reliability and efficiency with varying number \( r \) of most reliable paths in the proposed Rel-Path. Single source-target pair, BioMine dataset.

| \( r \) | conditional reliability | running time (sec) |
|--------|-------------------------|--------------------|
| 1      | 2.29                    | 10                 |
| 2      | 0.29                    | 4.26               |
| 3      | 0.31                    | 4.30               |
| 4      | 0.31                    | 4.30               |
| 5      | 0.31                    | 4.31               |
| 10     | 0.32                    | 4.31               |
| 15     | 0.32                    | 4.37               |
| 20     | 0.33                    | 5.26               |
| 30     | 0.33                    | 5.29               |
| 50     | 0.33                    | 5.38               |
| 100    | 0.33                    | 5.70               |

Fig. 7: Reliability and efficiency for multiple source-target pairs: Freebase, top-5 catalysts, aggregate function = minimum.

Fig. 8: Reliability and efficiency for multiple source-target pairs: BioMine, top-5 catalysts, aggregate function = minimum.

Fig. 9: Reliability and efficiency for multiple source-target pairs: Freebase, top-5 catalysts, aggregate function = maximum.

Fig. 10: Reliability and efficiency for multiple source-target pairs: DBLP, top-10 catalysts, aggregate function = average.

We find that with more source-target pairs, the minimum reliability achieved decreases (Figures 7(a) and 8(a)). This can be explained as follows. As we keep the number of top-\( k \) catalysts fixed at \( k = 5 \), with more source and target nodes, the likelihood of getting one source-target pair with small reliability attained by those top-\( k \) catalysts increases.

Different aggregate functions and datasets. We demonstrate how our aggregate functions perform over Freebase and DBLP, in Figures 9 and 10 respectively. Due to common trends, we only show Maximum over Freebase and Average over DBLP. We find that Rel-Path results in better reliability compared to Greedy over all experiments. Their difference minimizes in both datasets with more source-target pairs, which is due to the fact that we keep the number of top-\( k \) catalysts fixed at \( k = 5 \) (for Freebase) and at \( k = 10 \) (for DBLP). As before, Rel-Path is at least four to five orders of magnitude faster than Greedy in all scenarios. In particular, Greedy requires about \( 10^5 \) seconds to answer a single query, which makes almost infeasible to apply this baseline technique in any real-world online application.

It is interesting to note that with more source and target pairs, the maximum reliability increases (Figure 9), but the average reliability decreases (Figure 10). This is expected since with more source-target pairs, the chance of getting one pair with higher reliability also increases, thereby improving the maximum reliability. On the other hand, as we consider more source-target pairs while keeping the total number of catalysts same, the average reliability naturally decreases.

Varying distance from source set to target set. We vary the distance from source to target nodes as follows. We first select one node uniformly at random in the graph, and then select our source and target sets from the \( h \)-hop neighborhood of that node. By considering \( h = 1, 2, 3, \) and \( 4 \), we ensure that the maximum pairwise distance from any source to target is bounded by \( 2, 4, 6, \) and \( 8 \)-hops, respectively. In Figure 11 we show our reliability and efficiency results over BioMine and with Average aggregate function. With larger distance, the reliability achieved by all three methods decreases, and the running time increases. However, we notice that even with maximum pairwise distance \( 8 \), Rel-Path is five orders of magnitude faster than Greedy.

6.4 Multiple-sources multiple-targets

Aggregate functions. We perform experiments to evaluate the reliability and efficiency of our methods that maximize an aggregate function over conditional reliabilities from multiple source-target pairs. We consider Minimum aggregate function, and vary the number of source and target nodes from 2 to 5. In these experiments, we fix the maximum distance between any source-target pair as 2. We also ensure that the same node is not included both in the source and target sets.

We compare the performance of our algorithms over Freebase (Figure 2) and BioMine (Figure 3). Similar to queries with single source-target pairs, Rel-Path outperforms Ind-\( k \) and Greedy both in terms of efficiency and conditional reliability. Particularly, due to the presence of multiple source-target pairs, running time differences scale up, and Rel-Path is at least four orders of magnitude faster than the baselines. These results in Table 2. We observe that increasing the number of paths, the reliability initially increases, while saturating at a certain value \( r \). We observe that in all our experiments and over all datasets, reliability saturates at around \( r = 20 \) paths. This behavior is expected since the subsequent paths have very small reliability. Hence, including them does not significantly increase the quality of the solution found so far.
Connectivity maximization. We illustrate the performance of our algorithms that maximize connectivity (defined in Section 5.2) across multiple query nodes. For these experiments, we select 4 query nodes with maximum pairwise distance between any two nodes fixed at 2. We compare the connectivity attained by top-5 catalysts in Table 8. It can be observed that Greedy and Rel-Path perform equally well in Freebase, whereas Rel-Path results in higher connectivity over BioMine and DBLP. We further analyze the top-20 Steiner trees retrieved in BioMine, and find that each of these Steiner trees require 3~5 distinct catalysts. Therefore, in this dataset, Greedy makes more mistakes at initial stages. Because of the complexity of the top-20 Steiner tree finding algorithm, Rel-Path requires more running time in these experiments. However, Rel-Path is still significantly faster than the other two baselines over all our datasets.

7 RELATED WORK

To the best of our knowledge, the notion of conditional reliability and the problem of finding the top-k catalysts that we study in this work are novel. Thus, no direct competitors exist in the literature. In the following, we provide a short overview of relevant work in neighborhood areas.

Conditional reliability. Reliability is a classic problem studied in systems and device networks [3]. The term “conditional reliability” of a system has been used in relation to the notion of time. It has been employed to denote the probability that a system will be reliable for the next $t_2$ time instants assuming that it was reliable for the past $t_1$ instants. In this paper we use the same term “conditional reliability” with a completely different and novel meaning, i.e., to define reliability when edge-existence probabilities in an uncertain graph are conditioned on external factors.

Reliability queries in uncertain graphs. Reliability has been recently studied in the context of large social and biological networks. Due to its #P-completeness [5], efficient sampling, pruning, and indexing methods have been considered [20], [21]. None of those works, however, consider edge-existence probabilities conditioned on external factors.

Constrained reachability queries. Mendelson and Wood show that finding all simple paths in a (deterministic) graph matching a regular expression is NP-hard [22]. There are some query languages which support regular expression queries only in some restricted form, e.g., GraphQL, SoQL, GLEEN, XPATH, and SPARQL. Fan et al. [16] study a special case of regular expressions that can be solved in quadratic time. Edge-label constrained reachability and distance queries have been studied in [9], [19].

Label-constrained reachability queries have been also considered in the context of uncertain graphs [10]. However, in that work the goal was to estimate the reliability between two nodes under the constraint that paths connecting the two nodes contain only some admissible labels. Thus, the input graph still has fixed edge probabilities that do not vary based on external conditions. As a result, label-constrained reachability differs from conditional reliability introduced in this work, and, more importantly, our problem of finding the top-k external conditions is not addressed in those works.

Explaining relationships between entity pairs. Several works aim at identifying the best subgraphs/paths in a graph to describe how some input entities are connected [15], [17], [24]. Sun et al. propose PathSIM [26] to find entities that are connected by similar relationship patterns. However, all these works consider deterministic graphs. The semantics behind the notion of connectivity in uncertain graphs is different.

Topic-aware influence maximization. The classical problem of influence maximization has been recently considered in a topic-aware fashion [7], [11]. Although the input to that problem is similar to the input considered in this work (an uncertain graph where edge probabilities depend on some conditions), topic-aware influence maximization solves a different problem, i.e., finding a set of nodes that maximize the spread of information for a certain topic set. Topic-aware influence maximization can however benefit from the solutions provided by our top-k catalysts problem, e.g., in the case where topics are not known in advance.

A preliminary version of our work was published as a short paper in [21]. Our present work contains a lot of new and significant material: For the single-source single-target top-k catalyst problem, we included important theoretical characterization, algorithm sketches, case studies, and experimental evaluation. We also introduced the multiple-source multiple-target top-k catalyst problem under two scenarios: maximizing an aggregate function and maximizing connectivity, and empirically demonstrated the effectiveness and efficiency of our proposed algorithms.

8 CONCLUSIONS

In this paper we formulated and investigated a novel problem of identifying the top-k catalysts that maximize the reliability from source to target nodes in an uncertain graph. We proposed a method based on iterative reliable-path inclusion. Our experiments show that the proposed method achieves higher quality and significantly higher efficiency compared to simpler baselines. In future, we plan to consider more complex relationships between an edge and the catalysts, as well as other problems from the perspective of top-k catalysts, such as nearest neighbors and influence maximization.

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Appendix

Proof of Theorem 2. A problem is said to admit a Polynomial Time Approximation Scheme (PTAS) if the problem admits a polynomial-time constant-factor approximation algorithm for every constant $\beta \in (0, 1)$. We prove the theorem by showing that there exists at least one value of $\beta$ such that, if a $\beta$-approximation algorithm for $s$-$t$ TOP-$k$ CATALYSTS exists, then we can solve the well-known SET COVER problem in polynomial time. Being SET COVER NP-hard, clearly this can happen only if $P = \text{NP}$.

In SET COVER we are given a universe $U$, and a set of $h$ subsets of $U$, i.e., $\mathcal{S} = \{S_1, S_2, \ldots, S_h\}$, where $S_i \subseteq U$, for all $i \in [1, h]$. The decision version of SET COVER asks the following question: given $k$, is there any a solution with no more than $k$ sets that cover the whole universe? Given an instance of SET COVER, we construct in polynomial time an instance of our $s$-$t$ TOP-$k$ CATALYSTS problem in the same way as in Theorem 1. On this instance, if $k$ sets suffice to cover the whole universe in the original instance of SET COVER, the optimal solution $C^*$ would have reliability at most \(1 - (1 - p^2)^{2}Z\), where $Z = |U|$ (because at most $Z$ disjoint paths from $s$ to $t$ would be produced, each with existence probability $p^2$). On the other hand, if no $k$ sets cover the whole universe, $C^*$ would have reliability at most \(1 - (1 - p^2)^{2}Z - 1\) (because at least one of the disjoint paths would be discarded).

Approx would yield a solution $C_2$ such that $R((s,t)|C_2) \geq \beta R((s,t)|C^*)$. Now, consider the inequality $1 - (1 - p^2)^{2}Z - 1 < \beta \left(1 - \left(1 - p^2\right)^2\right)$. If this inequality has solution for some values of $\beta$ and $p$, then by simply running Approx on the instance of top-$k$ catalysts constructed this way and checking the reliability of the solution returned by Approx one can answer SET COVER in polynomial time: a solution to SET COVER exists iff the solution given by Approx has reliability $\geq \beta \left(1 - \left(1 - p^2\right)^2\right)$. Thus, to prove the theorem we need to show that a solution to that inequality exists.

To this end, consider the real-valued function $f(p, Z) = \frac{1 - (1 - p^2)^{2}Z - 1}{1 - (1 - p^2)^2}$. Our inequality has a solution iff $\beta > f(p, Z)$. It is easy to see that $f(p, Z) < 1$, for all $Z \geq 1$ and $p > 0$. This means that there will always be a value of $\beta \in (0, 1)$ and $p$ for which $\beta > f(p, Z)$ is satisfied, regardless of $Z$. This in turns means that there exists at least one value of $\beta$ such that the inequality $1 - (1 - p^2)^{2}Z - 1 < \beta \left(1 - \left(1 - p^2\right)^2\right)$ has solution, and, based on the above argument, such that no $\beta$-approximation algorithm for Problem 2 can exist.

Proof of Theorem 4. If both our objective function and the constraint were proved to be submodular, our iterative path inclusion problem (Problem 2) would become an instance of the Sub-modular Cost Sub-modular Knapsack (SCSK) problem [10], and the approximation result in Theorem 4 would easily follow from [10]. In the following we show that indeed both our objective function (Lemma 1) and our constraints (Lemma 2) are submodular, thus also proving Theorem 4.

Lemma 1. The constraint of the iterative path inclusion problem, i.e., total number of catalysts on edges of the included paths is submodular with respect to inclusion of paths.

Proof 5. Consider two path sets $P_1, P_2$ from $s$ to $t$ such that $P_2 \supseteq P_1$. Also, we assume a path $P$ from $s$ to $t$, where $P \notin P_2$. There can be two distinct cases: (a) $P$ has no common catalyst with the paths in $P_2 \setminus P_1$. (b) $P$ has at least one common catalyst with the paths in $P_2 \setminus P_1$. In the first case,

$$\left| \bigcup_{e \in P_1 \cup \{P\}} C(e) \right| - \left| \bigcup_{e \in P_2} C(e) \right| \leq \left| \bigcup_{e \in P_2} C(e) \right| - \left| \bigcup_{e \in P_2} C(e) \right|$$

In the second case,

$$\left| \bigcup_{e \in P_1 \cup \{P\}} C(e) \right| - \left| \bigcup_{e \in P_2} C(e) \right| < \left| \bigcup_{e \in P_2} C(e) \right| - \left| \bigcup_{e \in P_2} C(e) \right|$$

Hence, the result follows.

Lemma 2. If the top-$r$ most reliable paths are node-disjoint (except at source and target nodes), then the objective
function of the iterative path selection problem (Problem 2), i.e., $\text{Rel}_{P_1}(s, t)$ is sub-modular with respect to inclusion of paths.

**Proof 6.** Assume $P_1, P_2 \subseteq \mathcal{P}$, such that $P_1 \subseteq P_2$. Also consider a path $P \in \mathcal{P}$ and $P \notin P_2$. Let us denote by $\text{Rel}_{P_1}(s, t) = p_1$, $\text{Rel}_{P_2 \cup \{P\}}(s, t) = p_1 + \delta$, and $\text{Rel}_{P_2}(s, t) = p_2$. Due to our assumption that the top-$r$ most reliable paths in $\mathcal{P}$ are node-disjoint except at the source and the target nodes, we have: $\text{Rel}_{P_2}(s, t) = 1 - (1 - p_1)(1 - p_2)$, and $\text{Rel}_{P_2 \cup \{P\}}(s, t) = 1 - (1 - p_1 - \delta)(1 - p_2)$. Hence, $\text{Rel}_{P_2 \cup \{P\}}(s, t) - \text{Rel}_{P_2}(s, t) = (1 - p_2)\delta$. This is smaller than or equal to $\delta$, which was our marginal gain for including the path $P$ in the set $P_1$. Therefore, our objective function is sub-modular.

**Finding Top-$r$ Shortest Paths.** We provide a brief description of Eppstein’s algorithm [14] to find the top-$k$ shortest paths in an edge-weighted graph $G$. It first computes a shortest path tree $T$ that consists of shortest paths from all nodes in $G$ to the target node $t$. The shortest path tree for the graph in Figure 12(a) is given in Figure 12(b), where only solid edges constitute the tree. We also show the shortest path distance to $t$. The remaining edges of the graph, that are not part of the shortest path tree, are called sidetracks, and these are marked as dashed edges in Figure 12(b). Given an edge $(u, v)$ in $G$, we define a sideCost for that edge as follows. sideCost$(u, v) = d(v, t) + \text{weight}(u, v) - d(u, t)$. Here, $d(u, t)$ denotes the distance of $u$ to $t$, that is already shown on each node in Figure 12(b). Intuitively, sideCost of an edge $e$ measures how much the distance to $t$ increases by being sidetracked along $e$ instead of taking a shortest path to $t$. For all sidetracks, we show the sideCost in Figure 12(b). Clearly, if an edge is on the shortest path tree, its sideCost is 0. With the help of shortest path tree, Eppstein’s algorithm constructs another heap data structure, called the heap tree, where all paths from $s$ to $t$ are stored. By using this heap tree, it identifies the top-$k$ shortest paths from $s$ to $t$.

$t(v, P)$

$(a) \text{Tree Grow (}T_g\text{)}$

$(b) \text{Tree Merge (}T_m\text{)}$

Fig. 13: Recursive algorithm for top-$r$ minimum Steiner trees

**Finding Minimum-$r$ Steiner Trees.** We provide a high level overview of finding the minimum-$r$ Steiner trees in an edge-weighted graph [13]. Let $Q \subseteq V$ be the set of terminal nodes. Assume $P \subseteq Q$, and let us denote by $T(v, P)$ the smallest subtree rooted at $v$ and containing the terminal nodes in $P$.

Clearly, if $P = \{v\}$, the cost of $T(v, P)$ is 0. Next, one can define the following recursions.

\[
T(v, P) = \min \{T_g(v, P), T_m(v, P)\}
\]

\[
T_g(v, P) = \min_{u \in \text{neighbor}(v)} \{T(v, P) \cup \{u, v\}\}
\]

\[
T_m(v, P_1 \cup P_2) = \min_{P_1 \cup P_2 = \emptyset} \{T(v, P_1) \cup T(v, P_2)\}
\]

$T_g$ and $T_m$ defines tree-growth and tree-merge operations, respectively, and shown in Figure 13. As a special case of Equation 14, if $v$ itself is a terminal node, then the left side of the equation would be $T_g(v, P \cup \{v\})$. The algorithm starts with each terminal node as a tree, and extends it via tree-growth and tree-merge operations, until the minimum-$r$ Steiner trees are found.

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