Solving Velocity Ambiguity for Pulse Doppler Radar Space Target Measurement

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Abstract. In order to solve the problem of severe velocity ambiguity for pulse Doppler radar high-speed space orbital targets measurement, a new velocity ambiguity resolution method based on the short arc orbit determination technique is presented in this paper. By means of a modified orbit determination model, this method can adopt the original radar measurements with the range-Doppler coupling term directly. It uses the numerical integration method to solve the differential equation of the state transition matrix and corrects the range-Doppler coupling term simultaneously, and then the optimal reference orbit is obtained by using the iterative least squares method. Finally, the velocity ambiguity is determined with the accurate reference velocity which can be calculated from the above reference orbit. The experimental results indicate that the proposed method can estimate the reference velocity accurately and satisfies the performance requirement of velocity ambiguity resolution.

1 Introduction
The range and velocity ambiguities are frequently encountered problems for pulse Doppler radar measurement. Because of the periodicity of transmitting pulses, the waveforms with low pulse repetition frequency (PRF) are ambiguous in velocity [1].

Current common methods for solving the velocity ambiguity in pulse Doppler radar detection are divided into two types. One is to calculate the unambiguous velocity by multiple measurements with multiple PRFs [2]-[5], among which the most representative is the Chinese Remainder Theorem [6]. There are some algorithms based on the Chinese Remainder Theorem, such as one-dimensional set algorithm [7], group algorithm [8] and the method of resolving ambiguity based on the theory of compressed perception [9]; the other is to use the reference velocity which can be obtained by the range differentiation after getting the accurate range measurement [10]. As long as the error between the reference velocity and the actual velocity does not exceed one ambiguity unit, the unambiguous velocity can be obtained correctly. Range-difference algorithm and invariant embedding algorithm[11] are typical algorithms in this type.

The above two conventional methods solve the velocity ambiguity problem from the radar waveform characteristics and the mathematical points, respectively. In the space target observation scenario, the orbits of non-maneuvering or weakly maneuvering space targets have significant feature. Therefore, from the perspective of the observed target, it is also possible to obtain a reference velocity by constructing an optimized trajectory from the original observation data. Based on this idea, a velocity ambiguity resolution method based on short arc orbit determination model is proposed in this paper and its effectiveness is verified.
The rest of the paper is organized as follows. The model and algorithm for solving the velocity ambiguity are introduced in the second section, detailed algorithm steps are also given at the end of this section. A performance comparison and analysis of the short arc orbit determination method and the conventional range-difference method to solve the velocity ambiguity are presented in the third section. The last section concludes the paper.

2 Model and algorithm

2.1 Doppler shift and velocity ambiguity

There often exist errors in the guided trajectory derived from the orbit motion model with inaccurate state vector or orbital elements, and those errors will lead to a Doppler deviation when the echo signal accumulates. We assume \( \lambda \) is the carrier wavelength of the transmitting signal, the velocity deviation can be calculated according to the relation between velocity and Doppler frequency, and then the actual velocity of the target can be obtained by modifying the velocity deviation to the guided velocity:

\[
\Delta V = \Delta f_c / 2 \\
v_m = \hat{v} + \Delta V
\]

where \( \Delta f_c \) and \( \Delta V \) are the Doppler deviation and the velocity deviation respectively, \( \hat{v} \) is the guided velocity and \( v_m \) is the actual velocity of the target.

Due to the periodicity of radar transmitting signal, the Doppler ambiguity will occur when the Doppler deviation is bigger than half of the PRF, which is \( |\Delta f_c| \geq \text{PRF}/2 \), so the actual velocity cannot be got by (2) directly. At this point, the actual velocity of the target should be

\[
v_m = \hat{v} + \Delta V - N v_{\text{PRF}}
\]

where \( v_{\text{PRF}} \) is the velocity corresponding to a PRF, which is \( v_{\text{PRF}} = \lambda \text{PRF}/2 \), and the integer \( N \) is a fold factor which is usually determined by the reference velocity \( v_{\text{ref}} \) in engineering:

\[
N = \text{round}\left( \frac{\Delta V - v_{\text{ref}}}{v_{\text{PRF}}} \right)
\]

(4) requires an accuracy of the reference velocity less than \( v_{\text{ref}} \), and \( v_{\text{ref}} \) is called the Nyquist velocity [12], which is \( |v_{\text{ref}}| \leq v_n = v_{\text{PRF}}/2 = \lambda \text{PRF}/4 = c \text{PRF}/(4 f_c) \), where \( c \) is the speed of light and \( f_c \) is the carrier frequency. It can be seen that the improvement of carrier frequency or the decrease of PRF will require higher reference velocity accuracy.

The range-Doppler coupling of the chirp signal is another problem should be considered when using the short arc orbit determination method to solve the velocity ambiguity [13]. For a typical negative chirp signal, the relation between delay \( t_{\text{delay}} \) and Doppler shift \( f_c \) is

\[
t_{\text{delay}} = -\frac{f_c T_p}{B}
\]

where \( T_p \) is the pulse width. The additional range deviation caused by \( t_{\text{delay}} \) is

\[
\Delta R = t_{\text{delay}} \cdot \frac{c}{2} = -\frac{f_c T_p}{B} \cdot \frac{\lambda f_c}{2} = -\frac{f_c T_p}{B} \cdot V
\]

where \( B \) is the bandwidth of the transmitting signal, \( V \) is the unknown radial velocity of target. The range after the correction of range-Doppler coupling is

\[
R_r = R + \Delta R = R - \frac{f_c T_p}{B} \cdot V
\]

where \( R \) is the radial range before the correction of range-Doppler coupling. Obviously, we should use the \( R_r \) in the short arc orbit determination model, but \( R_r \) needs to be solved by the unknown \( V \). Therefore, in order to obtain a more accurate reference velocity, it is necessary to update \( R_r \) simultaneously during short arc orbit determination.
2.2 Orbit and measurement model

We use \( \mathbf{r}, \mathbf{v} \) to indicate the position and velocity vectors of the target in the Earth Centered Inertial (ECI) Coordinate System respectively; \( \mathbf{r}, \mathbf{v} \) to indicate the position and velocity vectors of the target in the Topocentric Horizon (ENZ) coordinate system respectively. The positional relationship between the ECI coordinate system and the Earth Centered Earth-Fixed (ECEF) Coordinate System is the rotation with respect to the hour angle, and the relationship between the ECEF coordinate system and the ENZ coordinate system is the translation of the origin and the rotation of the axis. Then we have

\[
\mathbf{r}_s = \mathbf{E} \mathbf{U} (\mathbf{r} - \mathbf{r}_{\text{Site-ECI}}) \tag{8}
\]

\[
\dot{\mathbf{r}}_s = \mathbf{E} \mathbf{U} (\dot{\mathbf{v}} - \dot{\mathbf{v}}_{\text{Site-ECI}}) + \mathbf{E} \frac{d\mathbf{U}}{dt} (\mathbf{r} - \mathbf{r}_{\text{Site-ECI}}) \tag{9}
\]

\[
\dot{\mathbf{E}} = \mathbf{R}_e (-\pi/2 + \text{lat}) \cdot \mathbf{R}_e (-\pi/2 - \text{lon}) \tag{10}
\]

\[
\mathbf{U} = \mathbf{R}_e (\theta) \tag{11}
\]

where \( \mathbf{E} \) is the rotation matrix of ECEF coordinate system to ENZ coordinate system, \( \mathbf{U} \) is the rotation matrix of ECI coordinate system to ECEF coordinate system, \( \mathbf{r}_{\text{Site-ECI}} \) is the coordinates of the radar station in the ECI coordinate system, \( \mathbf{v}_{\text{Site-ECI}} \) is the velocity vector of the radar in the ECI coordinate system, \( \text{lat}, \text{lon} \) is the latitude and longitude of the radar station respectively, \( \theta \) is the Greenwich Hour Angle corresponding to the measurement time, \( \mathbf{R}_e (\theta) \) represents the rotation matrix corresponding to the YZ plane around the X axis with rotation angle \( \theta \), \( \mathbf{R}_e (\theta) \) has the same meaning as \( \mathbf{R}_e (\theta) \), but represents the XY plane rotating around the Z axis [14].

We suppose the measurement vector at the time \( t \) is \( \mathbf{z}(t) = (R \ \mathbf{A} \ \mathbf{E})^T \), where \( \mathbf{A} \) and \( \mathbf{E} \) are the azimuth and elevation respectively, the relative position and velocity vectors in the ENZ coordinate system corresponding to \( \mathbf{z}(t) \) are \( \mathbf{\rho} = (\rho_e \ \rho_n \ \rho_z) \) and \( \dot{\mathbf{\rho}} = \left( \dot{\rho}_e \ \dot{\rho}_n \ \dot{\rho}_z \right) \) respectively. According to the conversion between the ENZ coordinate system and the polar coordinate system, there is

\[
R_e = R_e \cdot \frac{f_e \cdot T_e \cdot \mathbf{V}}{B} = \frac{\sqrt{\rho_e^2 + \rho_n^2 + \rho_z^2}}{B} \cdot \frac{f_e \cdot T_e \cdot \rho_e + \rho_n + \rho_z \cdot \dot{\rho}_z}{\sqrt{\rho_e^2 + \rho_n^2 + \rho_z^2}}
\]

\[
A = \begin{cases} 
\arccos \frac{\rho_n}{\sqrt{\rho_e^2 + \rho_n^2}}, \rho_e \geq 0 \\
2\pi - \arccos \frac{\rho_n}{\sqrt{\rho_e^2 + \rho_n^2}}, \rho_e < 0
\end{cases}
\]

\[
E = \arctan \frac{\rho_z}{\sqrt{\rho_e^2 + \rho_n^2}}
\]

We suppose the state vector at the time \( t \) in the ECI coordinate system is \( \mathbf{x}(t) = (\mathbf{r} \ \mathbf{v}) \), where \( \mathbf{r} = (r_x \ r_y \ r_z) \) and \( \mathbf{v} = (v_x \ v_y \ v_z) \), then the partial derivative of \( \mathbf{z}(t) \) to \( \mathbf{x}(t) \) can be expressed as

\[
\frac{\partial \mathbf{z}(t)}{\partial \mathbf{x}(t)} = \begin{bmatrix}
\frac{\partial R_e}{\partial r} & \frac{\partial R_e}{\partial v} \\
\frac{\partial A}{\partial r} & \frac{\partial A}{\partial v} \\
\frac{\partial E}{\partial r} & \frac{\partial E}{\partial v}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial \mathbf{E} \mathbf{U} \mathbf{O}_{t,3}}{\partial \mathbf{p}} \\
\frac{\partial \mathbf{A} \mathbf{E} \mathbf{O}_{t,3}}{\partial \mathbf{p}} \\
\frac{\partial \mathbf{E} \mathbf{O}_{t,3}}{\partial \mathbf{p}}
\end{bmatrix}\tag{13}
\]
the \( O_{13} \) in the (13) represents a zero matrix of \( 1 \times 3 \) order. According to (8),(9) the \( \rho \) and \( \hat{\rho} \) can be obtained by the state vector \( x(t) \), and then \( \frac{\partial x(t)}{\partial x(t)} \) can be solved by the (12),(13).

Assuming that \( x(t_i) \) represents the state vector at time \( t_i \), the relation between \( x(t) \) and \( x(t_i) \) can be obtained by the state vector \( \phi(t,t_i) \), and then \( \frac{\partial x(t)}{\partial x(t)} \) can be solved by the (12),(13).

\[
\phi(t,t_i) = \begin{bmatrix} \frac{\partial x(t)}{\partial x(t_i)} \end{bmatrix}_{i=6} 
\]

(14)

In order to adapt to the actual radar observation requirements, only the influence of gravity field on the earth is considered in this paper. The effects of precession, nutation and the earth pole shift are ignored. Since the partial differential equation corresponding to the state transition matrix considering the perturbation term is very complicated, it is often impossible to obtain an analytical solution and must be solved by numerical integration. The variable order variable step method proposed by Shampine & Gordon 1975 [15] is employed in this paper. This method can obtain a numerical solution with satisfactory precision because the order and step size can be changed during the integration process.

Combining the above partial derivatives, the relationship between \( z(t) \) and the initial state vector \( x(0) \) is obtained:

\[
\begin{bmatrix} \frac{\partial z(t)}{\partial x(t_i)} \end{bmatrix}_{i=6} = \begin{bmatrix} \frac{\partial z(t_i)}{\partial x(t_i)} \end{bmatrix} \cdot \phi(t,t_i) 
\]

(15)

2.3 Linearization and least squares estimation

We denote an \( n \)-dimensional vector of measurements at times \( t_1, \ldots, t_n \) is

\[
Z = (z_1, \ldots, z_n)^T
\]

(16)

The measurement vector \( z_i(t_i) \) are described by

\[
z_i(t_i) = h_i(t_i, x(t_i)) + \epsilon_i
\]

(17)

where \( i = 1,2,\ldots,n \), \( h_i(t_i, x(t_i)) \) denotes the \( i \)th value of the measurement vector as a function of time \( t_i \) and the state \( x(t_i) \); the quantities \( \epsilon_i \) account for the difference between actual and modelled observations due to measurement errors. The (17) can be simplified to

\[
Z = h(x) + \epsilon
\]

(18)

The (18) is a standard vector least squares model. According to the definition of least squares, for the measurements vector \( Z \), we need to find the state vector \( x^{lsq} \) to minimize the loss function

\[
J(x) = (Z - h(x))^T (Z - h(x))
\]

(19)

The practical solution of (19) is complicated due to the fact that \( h(x) \) is a highly nonlinear function of the unknown vector \( x \). So it is necessary to linearize the problem, the Herrick-Gibbs initial orbit method [16] is employed to obtain an approximation value \( x^{ref} \) of the initial state firstly, and then all quantities are linearized around \( x^{ref} \). The residual vector is approximately given by

\[
Z - h(x) \approx Z - h(x^{ref}) - \frac{\partial h(x)}{\partial x}(x - x^{ref})
\]

(20)

where \( \Delta Z \) denotes the difference between the actual measurements and the predicted measurements derived from the reference orbit, \( \Delta x \) denotes the difference between the initial state and the initial reference state, and \( H \) is the Jacobian matrix denotes the partial derivatives of the predicted
measurements vector derived from the reference orbit model with respect to the state vector \( x \) at time \( t_0 \). Combined with (15), we have

\[
\begin{bmatrix}
\frac{\partial h_1}{\partial x_0} \\
\frac{\partial h_1}{\partial x_1} \\
\vdots \\
\frac{\partial h_n}{\partial x_0} \\
\frac{\partial h_n}{\partial x_1} \\
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial x_0}{\partial x_0} \\
\frac{\partial x_0}{\partial x_1} \\
\vdots \\
\frac{\partial x_n}{\partial x_0} \\
\frac{\partial x_n}{\partial x_1} \\
\end{bmatrix}_{n \times 6} \tag{21}
\]

where \( x_1 \cdots x_n \) denote the state vectors corresponding to the vectors of predicted measurements \( h_1 \cdots h_n \). So the partial derivative matrix \( H \) corresponding to \( n \)-dimensional vector of measurements should be \( 3n \times 6 \) dimensions.

In this way, the orbit determination problem is reduced to the linear least squares problem of finding \( \Delta x_0^{\text{lsq}} \). Consider the weighted matrix \( W \), we have

\[
\Delta x_0^{\text{lsq}} = (H^T W H)^{-1} H^T W \Delta z 
\]

\[
W = \text{diag}(\sigma_1^{-2}, \ldots, \sigma_n^{-2}) 
\]

\[
\sigma = \text{diag}(\sigma_{R_i}, \sigma_{A_i}, \sigma_{E_i}), i = 1, 2, \ldots n. 
\]

where \( \sigma \) denotes the diagonal matrix consisting of the random noise measured at the range, azimuth and elevation of the point \( i \). Because of the approximation of \( x_0^{\text{ref}} \), the value of \( x_0^{\text{lsq}} = x_0^{\text{ref}} + \Delta x_0^{\text{lsq}} \) determined so far is not yet the exact solution of the orbit determination problem. In practical engineering applications, we can let \( x_0^{\text{ref}} = x_0^{\text{lsq}} \) and repeat the same procedure. After several iterations, the reference orbit can basically meet the accuracy requirements.

2.4 The steps of algorithm

Step 1: Obtaining the approximate initial reference state \( x_0^{\text{ref}} \) using the Herrick-Gibbs method based on the measurements vector \( Z \).

Step 2: Obtaining \( x_1, \ldots, x_n \) by propagating \( x_0^{\text{ref}} \) to times \( t_1, \ldots, t_n \), and then obtaining the vectors \( h_1, \ldots, h_n \).

Step 3: Solving the partial derivative matrix of \( h_1, \ldots, h_n \) with respect to \( x_1, \ldots, x_n \) by (13), solving the state transition matrix by the numerical integration method, and then obtaining the Jacobian matrix \( H \).

Step 4: Solving the \( \Delta x_0^{\text{lsq}} \) by the least squares method.

Step 5: Judging that the \( \Delta x_0^{\text{lsq}} \) whether or not satisfies the requirement of solving the velocity ambiguity, if satisfied, turns to step 6 otherwise let \( x_0^{\text{ref}} = x_0^{\text{lsq}} \) and repeats step 2 to step 5.

Step 6: Solving the reference velocity \( v_{\text{ref}} \) by coordinate transformation, and then solving the velocity ambiguity.

3 The experimental results

3.1 The simulation

In order to verify the effectiveness of the short arc orbit determination method proposed above, we choose to compare it with the range-difference method. The principle of the range-difference method is to obtain the reference velocity of the intermediate point by the range-difference between the front and back points. Assume that the radial range and radial velocity of the target at three consecutive times \( t_1, t_2, t_3 \) are \( \{r_i, v_i\}, i = 1, 2, 3 \), since the original measurements include the range-Doppler coupling term, the reference velocity at time \( t_2 \) obtained by the range-difference method is
In this simulation, we use KOMPSAT 5 satellite (NORAD ID: 39227) as the target, and the working parameters of the radar are shown in Table 1. The theoretical guidance is extrapolated by the SGP4 (Simplified General Perturbations 4) model using the TLE (Two Line Element) of KOMPSAT 5, which is considered a precise orbit in this simulation. The measurements are constructed by adding Gaussian white noise and other terms to the theoretical guidance. Finally, the range measurement includes a theoretical range term, a noise term with a standard deviation of 5 m, and the range-Doppler coupling term. The azimuth and elevation measurements contain respective theoretical term and noise term with a standard deviation of 0.14 mrad. The velocity measurement includes the theoretical velocity term, the noise term with a standard deviation of 0.01 m/s, and the ambiguous velocity term \( \Delta v \).

\[
 v_{\text{ref}} = \frac{(r_i - \frac{f_i}{B} \cdot T) - (r_j - \frac{f_j}{B} \cdot T)}{t_j - t_i}
\]

(25)

Table 1. Working parameters of radar

| Waveform 1 | Waveform 2 | Waveform 3 |
|------------|------------|------------|
| Carrier frequency | Signal bandwidth | Duration | Pulse width | Pulse repetition time |
| 8.5 GHz | 10 MHz | 66.7 s | 1 ms | 4 ms |
| 8.5 GHz | 10 MHz | 66.7 s | 2 ms | 7 ms |
| 8.5 GHz | 10 MHz | 66.7 s | 4 ms | 14 ms |

Figure 1. The errors before ambiguity resolution.

Figure 1(a) and Figure 1(b) describe the errors of the measurements and theoretical values of the range and velocity before velocity ambiguity resolution respectively. It can be seen from Figure 1 that the error of range measurement reaches several kilometers and the velocity measurement has an error of more than ten meters per second. In addition, we can see the range error in Figure 1(a) has a jump at the waveform switching point, which is due to the existence of the range-Doppler coupling term, and the velocity error in Figure 1(b) shows the velocity measurement is obviously ambiguous.

We use the range-difference method and the short arc orbit determination method to solve the reference velocity respectively, and then obtain the unambiguous velocity by the reference velocity, and finally, the more accurate range measurement can be obtained. Figure 2(a) and Figure 2(b) show the range error obtained by the range-difference method and the short arc orbit determination method respectively. Figure 2(c) and Figure 2(d) show the velocity error obtained by the range-difference method and the short arc orbit determination method respectively. We can see from Figure 2(a) and Figure 2(c) that the range error obtained by the range-difference method does not exceed 70 m and the velocity error does not exceed 23 m/s. According to the formula shown in the previous...
section \( v_N = cPRF / (4f_c) \), the Nyquist velocities corresponding to waveform 1, 2, and 3 are 2.2 m/s, 1.26 m/s, and 0.63 m/s, respectively. Therefore, although the range-difference method can improve the range measurement accuracy, it cannot solve the velocity ambiguity problem. It can be seen from Figure 2(b) and Figure 2(d) that the range error is no more than 18 m and the velocity error is no more than 0.04 m/s, thence the velocity ambiguity had been completely solved by the short arc orbit determination method.

![Figure 2](image)

**Figure 2.** The measurements and errors after ambiguity resolution.

### 3.2 The observation data

We used the observation data of SENTINEL 1B satellite (NORAD ID: 41456) from a radar station as the measurements. The observation start time is 18:53:22 on July 21, 2018 (Beijing time), and the total data length is 200 seconds. The actual orbit is generated by the public precise ephemeris of SENTINEL 1B. Table 2 shows the waveforms used in the observation and the Nyquist velocity corresponding to the waveforms.

**Table 2.** The waveforms and the corresponding Nyquist velocities

| Carrier frequency | Pulse width | Pulse repetition time | Nyquist velocity |
|-------------------|-------------|-----------------------|------------------|
| 8.5 GHz           | 4 ms        | 14 ms                 | 0.63 m/s         |
|                   | 3.7 ms      | 13 ms                 | 0.68 m/s         |
|                   | 3.5 ms      | 12 ms                 | 0.73 m/s         |

Figure 3 and Figure 4 are similar to Figure 1 and Figure 2 respectively. Except that the measurements are obtained by observation rather than simulation. From Figure 3 we can see that the
range error has a jump at the waveform switching point, and the velocity error shows the velocity measurement is obviously ambiguous.

Figure 3. The measurements and errors before ambiguity resolution.

Figure 4 shows the range error and velocity error obtained by the range-difference method and the short arc orbit determination method respectively. It can be seen from the Figure 4(a) and Figure 4(b) that the range error obtained by the range-difference method does not exceed 57 m, and the range error obtained by the short arc orbit determination method does not exceed 7 m. From the Figure 4(c) and
Figure 4(d), we can see that the velocity error obtained by the range-difference method does not exceed 18 m/s, and the velocity error obtained by the short arc orbit method can reach 0.06 m/s. It can be seen from the Table 2 that the smallest Nyquist velocity is 0.63 m/s of the waveform with a period 14 millisecond. We can see that the range-difference method cannot solve the velocity ambiguity problem at this time and the short arc orbit determination method can be well solved.

4 Conclusion
In this paper, the application of short arc orbit determination technique to the velocity ambiguity problem of the ground-based pulse Doppler radar space target detection is studied. By analysing the relationship between radar measurements and target state vector, as well as considering the range-Doppler coupling effect, an improved short arc orbit determination model is established, and then the iterative least squares method is used to estimate a more accurate reference velocity, resulting in the unambiguous velocity. The experimental results show that the short arc orbit determination method can be applied directly to the original radar observation data with range-Doppler coupling term, especially when observing the space targets with a high carrier frequency and low PRF radar, and its performance of velocity ambiguity resolution is better than the conventional range-difference method. Generally, the proposed method can improve the velocity measurement accuracy from several meters per second to a few centimeters per second. Using the estimated unambiguous velocity to eliminate the range-Doppler coupling term, the range measurement accuracy can also be increased by several times.

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