Physics-guided deep learning for data scarcity

Jinshuai Bai\textsuperscript{a,b,c}, Laith Alzubaidi\textsuperscript{a,b}, Qingxia Wang\textsuperscript{c,d}, Ellen Kuhl\textsuperscript{e}, Mohammed Bennamoun\textsuperscript{f}, Yuantong Gu\textsuperscript{a,b,*}

\textsuperscript{a} School of Mechanical, Medical and Process Engineering, Queensland University of Technology, Brisbane, QLD 4000, Australia

\textsuperscript{b} ARC Industrial Transformation Training Centre - Joint Biomechanics, Queensland University of Technology, Brisbane, QLD 4000, Australia

\textsuperscript{c} School of Civil Engineering, The University of Queensland, Brisbane, QLD 4072, Australia

\textsuperscript{d} Centre for Applied Climate Sciences, University of Southern Queensland, Toowoomba, QLD 4305, Australia

\textsuperscript{e} Department of Mechanical Engineering, Stanford University, Stanford, CA, United States

\textsuperscript{f} Department of Computer Science and Software Engineering, The University of Western Australia, Perth, WA, Australia.

* Corresponding authors.

\textit{E-mail address:} vuantong.gu@qut.edu.au (Y. Gu),

Abstract

Data are the core of deep learning (DL), and the quality of data significantly affects the performance of DL models. However, high-quality and well-annotated databases are hard or even impossible to acquire for use in many applications, such as structural risk estimation and medical diagnosis, which is an essential barrier that blocks the applications of DL in real life. Physics-guided deep learning (PGDL) is a novel type of DL that can integrate physics laws to train neural networks. It can be used for any systems that are controlled or governed by physics laws, such as mechanics, finance and medical applications. It has been shown that, with the additional information provided by physics laws, PGDL achieves great accuracy and generalisation when facing data scarcity. In this review, the details of PGDL are elucidated, and a structured overview of PGDL with respect to data scarcity in various applications is presented, including physics, engineering and medical applications. Moreover, the limitations and opportunities for current PGDL in terms of data scarcity are identified, and the future outlook for PGDL is discussed in depth.
Introduction

Deep learning (DL) has experienced significant development and has become one of the most promising machine learning techniques in recent decades. Great successes in using DL have been witnessed in various fields, including biostructure exploration\(^1\), disease prediction\(^2\) and financial prediction\(^3\). These remarkable successes are an incitant for researchers from different backgrounds and thus stimulate great interest in exploring new possibilities via DL techniques\(^4\). Deep learning provides us with a powerful tool to extract features and unveil the underlying relationships that hide behind raw data\(^5\). Data play an essential role in DL applications, and implementing DL without sufficient data is like ‘making bricks without straw’. Deep learning models with data scarcity can suffer from severe failures in terms of low prediction accuracy, biased training and poor generalisation\(^6\). Therefore, having sufficient high-quality data is essential for any successful DL application and can significantly boost the performance of DL models.

However, obtaining sufficient data for DL is quite challenging for some applications. Despite significant developments and improvements in experimental fields, some of the data remain deeply hidden and are hard to measure directly\(^7\), such as the quantities (such as stress and displacement) and topology inside solid structures or substances\(^8\). In addition, high quality data annotation and curation can be costly\(^9\). In fact, many fields, such as medicine and healthcare, currently possess significant unlabelled data\(^10\). However, changing those unlabelled data to high-quality annotated data requires expertise.

When using DL to deal with some problems, there are physics laws or equations that can describe those problems and have been summarised by previous researchers; however, they seldomly contribute to training DL models\(^11\). For example, early combinations between fluid dynamics and DL focused on using DL models to map the relations between measured data and observations\(^12\). However, the well-known Navier–Stokes equations that are widely used in fluid dynamics are usually excluded from DL models\(^13\). In fact, the Navier–Stokes equations can effectively and efficiently describe most fluid systems and have been studied extensively for many decades.

Recently, a novel type of DL technique – physics-guided deep learning (PGDL) – has been proposed\(^13\). In PGDL, physics laws can be incorporated to train neural networks by tailoring physics-informed loss functions. With physics-informed loss functions, physics laws, equations and even empirical knowledge can be transferred as training data as the remedy for
data scarcity\textsuperscript{14}. Furthermore, those transferred data are naturally annotated. Physics-guided deep learning is applicable to any system that is governed by physics or equations. It has been proven that even with fewer data, PGDL can still exhibit favourable predictions and a good generalisation ability\textsuperscript{15}. A typical example of PGDL is shown in Fig. 1. Hence, PGDL has attracted increasing attention in many applications\textsuperscript{11,16}. In what follows, a brief introduction of PGDL is firstly presented. Then, examples of PGDLs for data scarcity are highlighted, and the limitations and opportunities for PGDL are listed. Finally, the future outlook for PGDL is discussed.

**Fig. 1.** Example of physics-guided deep learning (PGDL) for solving Poisson’s equation. a, Consider a square problem domain where only limited observation data (cross signs) are available on the boundary lines. b, The problem domain is potentially governed by Poisson’s equation (physics laws) with corresponding boundary conditions. c, Predictions from both the data-driven DL model and the PGDL model. The result from the PGDL aligns well with the ground truth solution, while the result from the DL model fails to capture the contour. d, The structure of the PGDL used in this problem. A PGDL model comprises a neural network and a physics-informed loss function. Note that the partial differential terms can be analytically obtained through automatic differentiation\textsuperscript{17}. Poisson’s equation and the corresponding data are embedded in the physics-informed loss function. (The code for this problem is available at https://github.com/JinshuaiBai/PGDL_review.)

**Physics-guided deep learning**

Physics-guided deep learning comprises a neural network and a physics-informed loss function\textsuperscript{13}. In PGDL, the neural network is used to map nonlinear relationships, while the
physics-informed loss function qualifies the performance of the neural network via physics laws. Here, the physics laws can be the conservation laws or empirical mathematical models that potentially govern the change of the quantities within the problem domain (such as the conservation of momentum in mechanics problems and the Black–Scholes equation in finance). In this manner, sample points in the problem domain can be arbitrarily generated and evaluate the performance of neural networks via physics laws. Note that neural network mapping can be the exact solution to the problem only when the residual or error from every sample point in the problem domain vanishes to zero. Thus, optimisers will be applied to modify neural network mapping with the help of the physics-informed loss function until convergence.

Advantages of PGDL over data-driven DL

**Trustworthy data.** Trustworthy data are an essential prerequisite for establishing trustworthy DL. Neural networks can train poorly due to the use of incomplete or biased data. By integrating physics laws, PGDL can provide trustworthy data for training neural networks. Those physics laws and equations offer additional knowledge that can supplement information for incomplete, imperfect or biased training data. Additionally, it is worth noting that the data generated from physics laws are naturally annotated and of high quality, i.e. the residuals of every generated sample should be zero. Thus, PGDL improves the performance of neural networks when facing data scarcity. For example, as presented in Fig. 1, the initial observation data are all located at the boundary of the problem domain. Additional data that follow the governing equation (physics laws) are manually generated in the problem domain. With these additional data, PGDL produces accurate predictions while the data-driven DL fails to capture the features in the in-domain area.

**Generalisation.** By training neural networks through physics laws, PGDL is capable of achieving favourable generalisation properties even with limited observation data. This is because physics laws or equations describe phenomena from a higher perspective than direct observations. For example, using data-driven DL with observation data can be regarded as low-order curve fitting. In contrast, by embedding physics laws, higher-order constraints are added during curve fitting processes. Note that the higher-order derivative fitting algorithms are considered to be more accurate than the lower-order curve fitting algorithms. In this case, satisfying or enforcing physics laws can significantly affect the predictions of neural networks. The strong generalisation property of PGDL with limited data attracts considerable attention.
for extrapolation problems, such as forecasting the evolution of infectious diseases\textsuperscript{20} and predicting the physical phenomenon with initial-boundary conditions\textsuperscript{7,21}.

**Interpretability and uncertainty control.** Meanwhile, the use of physics-informed loss functions improves the interpretability of neural networks, and makes PGDL a way for explainable artificial intelligence (XAI)\textsuperscript{11}. The residual from the physics-informed loss function indicates which physics laws contribute to the poor performance of neural networks. Since PGDL has good interpretability, uncertainty control in PGDL can be easy and straightforward. It is possible to approximate arbitrary probability densities governed by physics laws through PGDL\textsuperscript{14,22}. Furthermore, other uncertainty qualification methods (such as the Bayesian distribution) can also be introduced in PGDL to avoid overfitting with substantial noise\textsuperscript{22}. With a view towards uncertainty control, PGDL has great potential to conduct comprehensive studies of uncertain systems with noisy limited data where repetitive experiments are prohibitive or impossible.

The following section highlights examples of using PGDL for data scarcity, focusing on the areas of physics, engineering and medical applications. More PGDL applications are summarised in Table 1.

**Physics**

The discipline of physics investigates fundamental mechanisms, constituents and behaviours of matter\textsuperscript{24}. Physics knowledge is accumulated and summarised by physics phenomena of the observable universe. However, observing such physics phenomena can be difficult. For example, atomic interactions occur at the microscale, which makes their observation and quantification challenging. Therefore, data scarcity is one of the essential difficulties that hinder DL applications for physics. Fortunately, many principles and laws have been established for various physics systems. Therefore, with these established principles and laws, PGDL can be an ideal choice to address the data scarcity in physics and significantly contribute to physics exploration. Herein, typical PGDL models for data scarcity in physics are presented as examples.

- **Case studies. (Material science)** Physics-guided deep learning has been leveraged to aid in exploring materials at the microscale\textsuperscript{25}. For example, understanding the interaction between atoms can help with modelling microscale phenomena at the atomic level as well as revealing the mechanism of material properties at the macroscale\textsuperscript{26}. However, experimentally quantifying interactions between atoms can be difficult\textsuperscript{27}.
Another way to acquire interaction data is to conduct density functional theory (DFT) calculations, but the extremely expensive computational cost makes it prohibitive to obtain sufficient interaction data for diverse atomic types. Therefore, PGDL is applied to integrate physics constraints (virial tensor) with limited DFT results to train neural networks. It has been proven that frameworks based on PGDL can achieve higher accuracy in predicting the energy and force fields of molecules than data-driven DL models (Fig. 2a). Furthermore, such PDGL frameworks also perform well for extended molecular-level systems that are not provided in training datasets. Another study applied PGDL to establish force fields for different materials with limited force interaction data (Fig. 2b). The force fields predicted by PGDL with additional physics maintain high accuracy outside the training set, where data-driven DL fails to produce large deviations. Currently, PGDL-based frameworks have profoundly decreased the cost of exploring emerging new materials at the microscale, which was previously restricted due to data scarcity, thus opening a new avenue for material exploration.

In addition, PGDL has also been applied to infer the microscale information of materials. For example, a PGDL framework has been introduced to evaluate the microstructure of polycrystalline nickel using ultrasonic surface acoustic wavefield data, offering great potential for the evaluation of non-destructive materials.

- **Case studies. (Biological science)** Additionally, PGDL enables the exploration of biological systems. For example, a PGDL-based framework has been proposed to study the food drying process, which is an essential method of food preservation. With a better understanding of the drying process, the process can be optimised to obtain better quality results and reduce the energy consumption. However, the heterogeneous, hygroscopic and porous microstructures of food materials greatly increase the difficulties for experimental investigations. Note that the drying process is governed by diffusion laws. In this manner, these laws are seamlessly embedded in the physics-informed loss function to train neural networks. As shown in Fig. 2c, although both DL and PGDL can reconstruct cellular moisture concentration for the training dataset, the DL can produce significant errors when predicting the moisture concentration for the testing dataset. In contrast, PGDL can maintain high accuracy for the moisture distribution for the testing dataset, indicating that it is more generalisable than DL with limited training data. In another study, a biologically-informed PDGL model was proposed for biological mechanistic modelling with sparse data.
Fig. 2 Examples of physics-guided deep learning (PGDL) for physics applications. 

a. Comparisons of the performances between PGDL and data-driven DL for predicting different molecular systems. The mean absolute error of the predicted energy (meV) and force (meV/Å) fields with respect to ground truths are listed in the table. The data-driven DL is trained via pure density functional theory (DFT) calculation results, while the PGDL is trained via virial tensor and the DFT result. The table was re-generated from Ref.27.

b. Comparisons of predicted energy between the PGDL and data-driven DL models with respect to the simple cubic (top figure) and diamond cubic (bottom figure) molecular systems. The PGDL model accurately predicts the energy of the systems outside the training set, while data-driven DL exhibits a poor generalisation property for both systems outside the training set. The figures were reproduced from Ref.30.

c. Comparisons of the in-cell moisture distribution predicted by data-driven DL and PGDL. Although both DL and PGDL can achieve high accuracy for the training set (top figures), DL produces large errors compared with PGDL for the testing set (bottom figures). The figures were reproduced from Ref.35.

Engineering

Engineering is an area that has been closely related to DL. Many DL models have been proposed for engineering purposes. However, when collecting training data in engineering, difficulties can arise from various aspects. Despite the significant development of experimental equipment, most experiments are restricted to limited small domains or laboratory settings, and some quantities inside structures or substances are challenging or even impossible to measure even with state-of-the-art equipment. Furthermore, since novel materials with complex constitutive properties keep emerging, it is impossible to obtain a well-developed database that covers all material properties. Hence, data scarcity greatly hinders the application of DL in engineering.
PGDL has alleviated the data scarcity problem of DL in this field. Note that the responses of engineering systems normally satisfy the governing equations of conservation laws and material models, and the governing equations of mechanics have been extensively studied. In this manner, the governing equations can be leveraged as training data via PGDL, and the insufficient representative information due to deficient data can be enhanced by the governing equations.

- **Case studies. (Structural analysis)** In general, only observation data on the boundaries or surfaces of structures are accessible for structural analysis. When DL models are trained only with surface data, they produce large error for structural analysis because they do not generalise well to prediction inside structures. By implementing physics laws for training neural networks, PGDL greatly improves the accuracy of structural analysis by using neural networks\(^{39-41}\). Furthermore, the nature of the material (such as hyperelasticity and elastoplasticity) and internal topology (such as the geometry of voids inside structures) are interpretable via limited responses of the structure under loadings\(^{8,39,42}\) (Fig. 3a).

Extending to real applications, PGDL exhibits great potential for complex engineering systems. For example, PGDL has been applied to study hypersonic vehicles with limited training data and knowledge-based engineering\(^{43}\). Compared with data-driven DL, PGDL achieves higher accuracy for maximum take-off weights, especially with limited training data. Furthermore, PGDL improves the efficiency of generating design data generation. PGDL has also contributed to structural topology optimisation. Data is the key to data-driven DL-based topology optimisation\(^{44}\). However, it is impossible to create a complete database that covers every loading and geometry scenario. In this manner, design targets (such as lightweight designs or alleviating stress concentrations) are treated as physics to train neural networks, exploring the ideal designs under different senarios\(^{45,46}\).

- **Case studies. (Fluid mechanics)** Fluid mechanics is another field that benefits from PGDL. The Navier–Stokes equations serve as physics laws in PGDL to produce favourable predictions for fluid flows. For example, Raissi et al.\(^7\) proposed hidden fluid mechanics (HFM) that can infer velocity and pressure fields via local incomplete observation data (Fig. 3b). It is worth noting that before PGDL, DL-based models trained with limited fluid videos or snapshots were mostly confined to the animation and gaming engine fields due to their low accuracy\(^{47}\). Currently, various PGDL-based
frameworks have been invented for hydrodynamics modelling only with initial and boundary conditions\textsuperscript{48-52}. Meanwhile, PGDL has also been applied to fluid-related fields that suffer from data scarcity. For example, wind field information can provide valuable assessments for wind resources and wind turbine control\textsuperscript{53}. Nevertheless, it is difficult to obtain large amount of wind field data due to the limitations of the current sensor technology\textsuperscript{54}. Instead, only sparse local measurements are available. Therefore, with the incorporation of Navier–Stokes equations with sparse data, a PGDL framework was proposed that successfully predicts the spatiotemporal wind field. The study in Ref.\textsuperscript{55} applied a PGDL to reconstruct the surficial velocity and pressure fields of structures under vibration. In that work, acoustic holographic data and the Kirchhoff–Helmholtz equation were implemented to train a convolutional neural network (CNN).

\textbf{Fig. 3} Examples of physics-guided deep learning (PGDL) for engineering. a, Predicting the deformed configurations of the geometries with limited displacement data. The displacement data are only accessible at the measurement points on the boundaries (red crosses). Amid training, the PGDL predictions align well with the ground truths. The figures were reproduced from Ref.\textsuperscript{8}. b, Predicting quantities deeply hidden in fluid flows using
PGDL with limited observations (passive scalar). The limited observation data (bottom-left figure) are generated within the shape of the flower inside the channel (top-left figure). A PGDL framework leverages the limited observations and Navier–Stokes equations to further predict the velocity and pressure fields (right figures) inside the flower-shaped domain. The figures were reproduced from Ref. 7.

**Medical applications**

Deep learning has been widely applied for medical applications, including disease diagnosis and case predictions, and the great success of DL for medical purposes has been witnessed\(^5^6\). Although much of the data in medical applications can now be recorded thanks to the development of medical engineering, most data are raw and not annotated\(^1^0\). The raw data must be annotated by experts, making it both tedious and costly. Furthermore, for some problems, the severe conditions of tissues or microscale situations can present significant barriers to measurement. More importantly, given that each patient has their own unique patient history, DL models trained with limited data can induce severe generalisation problems when applied to predict the behaviour of new patients. These dilemmas can be alleviated by introducing physics laws for training neural networks.

- **Case studies. (Medical diagnosis)** Physics-guided deep learning is now increasingly applied to extract information with limited observation data (such as medical images) and help with diagnosing diseases. For example, artificial intelligence velocimetry (AIV) – a PGDL-based framework – was proposed to characterise the potential risk factors in microaneurysms\(^5^7\) (Fig. 4a). The limited data from 2D medical images of microaneurysms are seamlessly incorporated with fluid mechanics knowledge to train neural networks. In so doing, the well-trained neural networks can accurately approximate the 2D and 3D velocity and pressure fields inside the microaneurysms, thereby offering a better understanding of pathophysiology. In addition, the no-slip flow can be identified with the help of filtered velocity fields, from which it is possible to rebuild the motion and position of the artery wall and extract disease predictors in terms of dissipation of kinetic energy, wall pressure and stresses.

Furthermore, numerous studies have focused on integrating physics laws to investigate the condition of organs using limited magnetic resonance imaging (MRI) or computed tomography imaging (CTI) in clinics\(^5^8-^6^0\). It has been demonstrated that PGDL can significantly improve personalised predictions and treatment planning (Fig. 4b). It is also worth noting that MRI and CTI observations usually suffer from poor resolution, with heavy outliers. Great efforts regarding data reconstruction and de-noise manipulations are necessary to process these images, making it more costly to obtain.
the training data for DL models\textsuperscript{61}. By using PGDL, the inherent uncertainty and error can be identified and quantified with the help of physics laws. Thus, minimising the physics-informed loss function can improve errors and outliers in noisy measurements\textsuperscript{61}.

- **Case studies. (Case predictions/epidemiology)** Additionally, PGDL has contributed to case predictions, which is challenging for data-driven DL models. Physics-guided deep learning integrates the knowledge from epidemiology to provide additional information that guides neural network training\textsuperscript{20}. For example, one study applied a well-known epidemiology model, the susceptible–exposed–infectious–removed (SEIR) model, to the PGDL with the previously reported data for predicting COVID-19 cases\textsuperscript{62}. The proposed PGDL framework accurately forecasted the COVID-19 spread within 15 days. Besides, Bayesian inference can be integrated into PGDL to provide uncertainty qualification for a seasonal endemic infectious disease\textsuperscript{63}. Compared with data-driven DL that only trains neural networks via previously reported data, PGDL achieves better accuracy for predicting future case data with the help of the SEIR model (Fig. 4c). Consequently, even with imperfect and incomplete training data, PGDL can effectively and accurately predict the evolution of new cases and holds great potential for providing insights into more complex infectious diseases\textsuperscript{62}. Accurate epidemiology prediction is vital for evaluating the consequences and efficacy of various measures to contain the spread of infectious illnesses. Policy and decision-making can be rationally influenced by probabilistic projections of a pandemic’s progress.
Fig. 4. Examples of physics-guided deep learning (PGDL) for medical applications. a. Predicting the 3D velocity, pressure and wall shear stress fields of a microaneurysm via artificial intelligence velocimetry (AIV) with limited microfluid images. The figures were reproduced from Ref.57. b, A PGDL framework for a personalised left-ventricular study. Magnetic resonance imaging (MRI) observations and physics laws are used to train the neural network. The MRI observations provide a personalised geometry of the left ventricle, and physics laws govern the quantities in the left ventricle. The personalised left-ventricular conditions in terms of microstructure, tissue properties and systemic parameters are fed as inputs, and the PGDL framework predicts the dynamics and functions of the personalised left-ventricular model. The figures were reproduced from Ref.59. c, Comparisons between DL and PGDL for predicting future COVID-19 cases using previously recorded data. The figures were reproduced from Ref.63.

Limitations and opportunities

Currently, PGDL is in its infancy despite its successful application in various fields. In this section, the limitations of current PGDL are identified, and potential opportunities and solutions are discussed.

Imbalanced loss. Since most PGDL models simply sum up the residuals from physics laws, the differences in magnitude between different loss terms may induce severe bias in training problems.64 Consequently, balancing the weights of different physics-informed loss terms is essential for the effectiveness and efficiency of PGDL. If the weights of different loss terms are not adequately balanced, optimisers may only focus on minimising the loss terms with a relatively larger magnitude while ignoring the others.

Solutions. One of the accessible ways of addressing this problem is to apply adaptive learning, i.e. introducing additional hyperparameters to balance loss terms and automatically optimising them during the training process. Thus, the optimisers can pay more attention to the loss terms that are naturally of a low magnitude during training, and thus alleviate the biased training. Examples of adaptive schemes based on physics,65 the Gaussian probabilistic model50 and neural tangent kernel (NTK) theory66 have been proposed. It has been reported that PGDL with adaptive learning can achieve additional accuracy as well as efficiency. Nonetheless, the existing adaptive learning schemes are problem based. A more general and effective adaptive learning scheme still calls for more in-depth research.

Another effective way is to establish specific physics-informed loss functions that naturally balance the loss terms or unify the loss terms into the same unit. For example, the principle of potential energy can be equivalently used as the physics laws for static mechanics problems. In contrast to governing equations with corresponding boundary conditions, the physics-informed loss function based on the energy principle expresses all the loss terms in an energy unit.39,64 Comparisons between the use of partial differential equation (PDE) based and energy-based loss functions suggest that the energy-based loss function can effectively predict
displacement fields and accurately capture local stress concentration, while the PDE-based loss function can fail to regenerate stress concentration due to imbalanced loss terms\textsuperscript{64}.

**Complex physics.** The effectiveness and robustness of PGDL are still limited by challenging and complex physics (such as non-linear equations) even though PGDL has been successfully applied to applications with complex PDEs. These complex equations make training neural networks a non-convex optimization problem. As a consequence, the training is likely to achieve local optima instead of the global optimum.

**Solutions.** The non-convex optimisation problem can be solved with transfer learning (TL). By leveraging neural networks that have been trained based on related problems, the knowledge embedded in the trained neural networks can be used to solve new problems as well as to enhance the efficiency of the training process\textsuperscript{6}. For example, a well-trained neural network for predicting the deflection field of a plate was used for other plate problems with different materials\textsuperscript{67}. The results showed that PGDL with TL could maintain accuracy while significantly decreasing the training cost for new scenarios. However, how to effectively apply TL in PGDL is an important issue. Criteria and standards need to be established to qualify the correlation between the two physical systems.

Domain decomposition is another way of alleviating the influences caused by challenging complex physics. Dividing the problem domain into pieces can decrease the complexity of targets. For example, the study in Ref.\textsuperscript{68} combined PGDL with domain decomposition towards various nonlinear functions, including the nonlinear Helmholtz equation, the advection equation, the diffusion equation and the viscous Burgers’ equation. The study in Ref.\textsuperscript{69} utilised domain decomposition to deal with variational problems. By using multiple neural networks to approximate solutions, it has been proven that PGDL can achieve higher accuracy with smaller neural networks than using a single neural network for global approximation. Furthermore, decomposed PGDL problems are suitable for parallel computing, which can significantly improve the computational efficiency of PGDL applications\textsuperscript{70}. However, domain decomposition inevitably generates new boundaries. Information transformation between different domains via boundaries is also an essential issue that must be considered.

**Extreme conditions.** Numerous complex systems are governed by equations with singularity and discontinuity, such as structural fractures\textsuperscript{71} and multi-degree-of-freedom friction problems\textsuperscript{72}. Physics laws containing extreme conditions (such as singularity and discontinuity) can also be challenging for formulating physics-informed loss functions.
Solutions. To address this problem, existing numerical schemes that simplify extreme physics can be integrated into PGDL. For example, the commonly used phase-field method can be integrated with PGDL to denote the discontinuities in fracture problems.\cite{73,74} Another study incorporated the peridynamics differential operators to deal with sharp gradient elastoplastic deformation.\cite{75} Note that the sharp gradient in elastoplastic problems can induce severe local deformation; therefore, using a single neural network to predict the deformation globally may fail to capture the local phenomenon. The partial differential terms obtained from peridynamics operators offer strong constraints to gradient predictions obtained from automatic differentiation. Coupling PGDL with other numerical schemes significantly enhances the capability of PGDL to cope with extreme conditions, thus broadening its application scope.

Customising specific neural network structures in PGDL can be another solution for extreme conditions. Currently, most PGDL models are based on feedforward neural networks (FNN), and more efforts should be made to explore the possibility of integrating other DL models into PGDL. For example, the study in Ref.\cite{62} implemented a long short-term memory (LSTM) neural network to deal with temporal-dependent predictions. On the other hand, the components inside neural networks, such as the output layer and activation function, can be also tailored to help the neural networks face extreme conditions. Examples can be found in refs\cite{39,48,76}, where the output layers of neural networks were designed to naturally satisfy boundary conditions. It has been demonstrated that PGDL with a tailored output layer can accurately impose boundary conditions for complex geometries.\cite{76} Furthermore, the boundary conditions are no longer required in formulating the physics-informed loss function with the tailored output layers, which simplifies the complexity of training neural networks.

Theoretical support. Finally, despite the widespread attempts to apply PGDL in various fields, rigorous mathematical proof is still required to build confidence in PGDL.

Solutions. To provide rigorous mathematical support for PGDL, general DL theories and techniques can be migrated into PGDL. Since PGDL is a branch of DL, it shares many common features with other DL models. For example, the study in Ref.\cite{66} extended NTK theory to PGDL to study the dynamic training process and convergence rate. Furthermore, in that study, using NTK, the authors proposed a gradient descent training scheme that enhances the efficiency of the PGDL training process. Another study applied the Hölder regularisation to PGDL and provided convergence theory for PGDL.\cite{77} Certainly, theoretical investigations based on the particularity of PGDL are also encouraged. For example, the study in Ref.\cite{78} focused on studying the generalisation error control from the physics-informed loss function. Besides, an
abstract formalism was proposed to separately assess uncertainties from the training error and volume of training data. More importantly, Ref.\textsuperscript{78} demonstrated that PGDL can achieve sufficient accuracy by satisfying some conditions. However, only limited studies regarding the theoretical support have been conducted for PGDL. Investigations regarding PGDL from the theoretical point of view need to be further strengthened.

**Conclusions**

Fuelled by the recent development of PGDL, the performances of neural networks in terms of accuracy, reliability and generalisation have been greatly improved via physics laws when facing data scarcity. Physics-guided deep learning has become a powerful technology as a result of these merits and benefits to provide trustworthy data, strong generalisation, interpretability and uncertainty control, which have led to remarkable widespread achievements in diverse fields. Nevertheless, PGDL is still in its infancy, and many challenges in terms of robustness, embedding challenging and complex physics remain to be addressed. In addition, rigorous proof regarding the capability and limitations of PGDL is required. Therefore, via this review, we hope to inspire the further development of PGDL and stimulate future research and applications that will embrace diverse opportunities for PGDL, such as in the fields of physics, engineering and medical application.
| Field               | Specific Area          | Reference | Difficulties faced by data-driven DL                                                                 | Physics laws                                                                                       | Advantages/Comment                                                                                     |
|---------------------|------------------------|-----------|-----------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
| **Physics**         | Material science       | 27,29,31,33,79-83 | 1. Phenomena and mechanisms in microscale systems are hard to observe; 2. The number of features of the system under investigation is extraordinarily high. | Bond-order potential; Virial stress. Departure from the nucleate boiling model; Full-vector Maxwell’s equations. | 1. PGDL is effective for studying microscale systems that are hard to obtain; 2. PGDL can maintain accuracy for extended microscale systems. |
|                     | Nuclear engineering    | 84,85     |                                                                                                     |                                                                                                   |                                                                                                   |
|                     | Chemical engineering   | 86-91     |                                                                                                     | Electrochemical–thermal model; Diffusion equation.                                                  |                                                                                                   |
|                     | Biological science     | 35,36,92-94 |                                                                                                     | Diffusion laws; Constitutive models.                                                                |                                                                                                   |
| **Engineering**     | Structure analysis     | 8,39, 43,46,64,67,71,95-102 | 1. Limited and incomplete data (such as surficial observations) due to the limitations of experimental equipment; 2. Infinite possibilities (such as emerging new materials and different loading scenarios) challenge the generalisation ability of DL models. | Solid mechanics; Constitutive models. Vibration function.                                           | 1. PGDL can extract information that is deeply hidden inside structures and substances; 2. PGDL can perform well in different scenarios that are not included in the training set. |
|                     | Dynamics system control| 63,103    |                                                                                                     |                                                                                                   |                                                                                                   |
|                     | Fluid mechanics        | 7,21,48-52 |                                                                                                     |                                                                                                   |                                                                                                   |
|                     | Environmental science  | 54,104,105 |                                                                                                     |                                                                                                   |                                                                                                   |
| **Medical application** | Diagnosis             | 57,59,61,93,106-109 | 1. Limited observation data with uncertainties (such as low-resolution MRI or CTI); 2. Costly to annotate data; 3. Generalisation for different patients; 4. Extrapolating the predictions based on limited previous data. | Solid and fluid mechanics. Epidemiology.                                                          | 1. PGDL can accurately predict the dynamics and functions of organs via limited MRI or CTI. 2. PGDL can help with identifying uncertainties and errors in MRI or CTI. 3. PGDL trained with physics can take patients’ personalisation into account. 4. PGDL can accurately predict the evolution of infectious cases using previously reported cases. |
|                     | Case prediction         | 20,62,63,110,111 |                                                                                                     |                                                                                                   |                                                                                                   |
Table 2. Challenges and potential solutions of current physics-guided deep learning (PGDL).

| Challenge            | Description                                                                 | Potential solution                                      | Example reference |
|----------------------|-----------------------------------------------------------------------------|---------------------------------------------------------|-------------------|
| Imbalanced loss/data | Scale difference of loss terms induces bias training.                       | Adaptive training.                                       | 50,65,66          |
|                      |                                                                             | Specific loss function.                                  | 39,42,64,69,95    |
| Complex physics      | Training process becomes a non-convex optimisation problem due to complex physics. | Transfer learning.                                       | 67,73,96          |
|                      |                                                                             | Domain decomposition.                                    | 68-70,112         |
| Extreme physics      | Extreme conditions, including singularity and discontinuity, are hard to embed. | Coupling with other numerical schemes.                   | 73-75             |
|                      |                                                                             | Customisation of neural network structures.              | 39,48,62,76       |
| Theoretical support  | Rigorous mathematical analysis regarding the PGDL’s limitations and capabilities is required. | Migration from general DL theories.                      | 66,77,78          |
References

Acknowledgements

Support from the Australian Research Council research grants (IC190100020 and DP200102546) is gratefully acknowledged.

Author contributions

All authors wrote and edited the manuscript.

1 Senior, A. W. et al. Improved protein structure prediction using potentials from deep learning. *Nature* **577**, 706-710, doi:https://doi.org/10.1038/s41586-019-1923-7 (2020).

2 Frazer, J. et al. Disease variant prediction with deep generative models of evolutionary data. *Nature* **599**, 91-95, doi:https://doi.org/10.1038/s41586-021-04043-8 (2021).

3 LeCun, Y., Bengio, Y. & Hinton, G. Deep learning. *Nature* **521**, 436-444, doi:https://doi.org/10.1038/nature14539 (2015).

4 Guo, Y. et al. Deep Learning for 3D Point Clouds: A Survey. *IEEE Trans Pattern Anal Mach Intell* **43**, 4338-4364, doi:10.1109/TPAMI.2020.3005434 (2021).

5 Bengio, Y., Goodfellow, I. & Courville, A. Deep learning. Vol. 1 (MIT press Massachusetts, USA:, 2017).

6 Alzubaidi, L. et al. Review of deep learning: concepts, CNN architectures, challenges, applications, future directions. *Journal of Big Data* **8**, 53, doi:https://doi.org/10.1186/s40537-021-00444-8 (2021).

7 Raissi, M., Yazdani, A. & Karniadakis, G. E. Hidden fluid mechanics: Learning velocity and pressure fields from flow visualizations. *Science* **367**, 1026-1030 (2020).

8 Zhang, E., Dao, M., Karniadakis, G. E. & Suresh, S. Analyses of internal structures and defects in materials using physics-informed neural networks. *Science Advances* **8**, eabk0644, doi:https://doi.org/10.1126/sciadv.abk0644 (2022).

9 Liang, W. et al. Advances, challenges and opportunities in creating data for trustworthy AI. *Nature Machine Intelligence*, doi:https://doi.org/10.1038/s42256-022-00516-1 (2022).

10 Krishnan, R., Rajpurkar, P. & Topol, E. J. Self-supervised learning in medicine and healthcare. *Nat Biomed Eng*, doi:https://doi.org/10.1038/s41551-022-00914-1 (2022).

11 Karniadakis, G. E. et al. Physics-informed machine learning. *Nature Reviews Physics* **3**, 422-440, doi:https://doi.org/10.1038/s42254-021-00314-5 (2021).

12 Brunton, S. L., Noack, B. R. & Koumoutsakos, P. Machine learning for fluid mechanics. *Annual Review of Fluid Mechanics* **52**, 477-508, doi:https://doi.org/10.1146/annurev-fluid-010719-060214 (2020).
Raissi, M., Perdikaris, P. & Karniadakis, G. E. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics* **378**, 686-707, doi:https://doi.org/10.1016/j.jcp.2018.10.045 (2019).

Zhu, Y., Zabaras, N., Koutsourelakis, P.-S. & Perdikaris, P. Physics-constrained deep learning for high-dimensional surrogate modeling and uncertainty quantification without labeled data. *Journal of Computational Physics* **394**, 56-81, doi:https://doi.org/10.1016/j.jcp.2019.05.024 (2019).

Sun, L., Gao, H., Pan, S. & Wang, J.-X. Surrogate modeling for fluid flows based on physics-constrained deep learning without simulation data. *Computer Methods in Applied Mechanics and Engineering* **361**, doi:https://doi.org/10.1016/j.cma.2019.112732 (2020).

Faroughi, S. A. *et al.* Physics-Guided, Physics-Informed, and Physics-Encoded Neural Networks in Scientific Computing. arXiv preprint arXiv:07377 (2022).

Baydin, A. G., Pearlmutter, B. A., Radul, A. A. & Siskind, J. M. Automatic differentiation in machine learning: a survey. *Journal of Machine Learning Research* **18** (2018).

Mishin, Y. Machine-learning interatomic potentials for materials science. *Acta Materialia* **214**, doi:10.1016/j.actamat.2021.116980 (2021).

Li, T. E., Nitzan, A. & Subotnik, J. E. Energy-efficient pathway for selectively exciting solute molecules to high vibrational states via solvent vibration-polariton pumping. *Nat Commun* **13**, 4203, doi:http://doi.org/10.1038/s41467-022-31703-8 (2022).
27 Zhang, L., Han, J., Wang, H., Car, R. & E, W. Deep Potential Molecular Dynamics: A Scalable Model with the Accuracy of Quantum Mechanics. *Phys Rev Lett* **120**, 143001, doi:[https://doi.org/10.1103/PhysRevLett.120.143001](https://doi.org/10.1103/PhysRevLett.120.143001) (2018).

28 Unke, O. T. *et al.* Machine Learning Force Fields. *Chem Rev* **121**, 10142-10186, doi:[https://doi.org/10.1021/acs.chemrev.0c01111](https://doi.org/10.1021/acs.chemrev.0c01111) (2021).

29 Wang, H., Zhang, L., Han, J. & E, W. DeePMD-kit: A deep learning package for many-body potential energy representation and molecular dynamics. *Computer Physics Communications* **228**, 178-184, doi:[https://doi.org/10.1016/j.cpc.2018.03.016](https://doi.org/10.1016/j.cpc.2018.03.016) (2018).

30 Pun, G. P. P., Batra, R., Ramprasad, R. & Mishin, Y. Physically informed artificial neural networks for atomistic modeling of materials. *Nat Commun* **10**, 2339, doi:[https://doi.org/10.1038/s41467-019-10343-5](https://doi.org/10.1038/s41467-019-10343-5) (2019).

31 Galib, M. & Limmer, D. T. Reactive uptake of N2O5 by atmospheric aerosol is dominated by interfacial processes. *Science* **371**, 921-925, doi:[https://doi.org/10.1126/science.abd7716](https://doi.org/10.1126/science.abd7716) (2021).

32 Pun, G. P. P., Yamakov, V., Hickman, J., Glaessgen, E. H. & Mishin, Y. Development of a general-purpose machine-learning interatomic potential for aluminum by the physically informed neural network method. *Physical Review Materials* **4**, doi:10.1103/PhysRevMaterials.4.113807 (2020).

33 Shukla, K., Jagtap, A. D., Blackshire, J. L., Sparkman, D. & Em Karniadakis, G. A Physics-Informed Neural Network for Quantifying the Microstructural Properties of Polycrystalline Nickel Using Ultrasound Data: A promising approach for solving inverse problems. *IEEE Signal Processing Magazine* **39**, 68-77, doi:[https://doi.org/10.1109/msp.2021.3118904](https://doi.org/10.1109/msp.2021.3118904) (2022).

34 Shukla, K., Di Leoni, P. C., Blackshire, J., Sparkman, D. & Karniadakis, G. E. Physics-Informed Neural Network for Ultrasound Nondestructive Quantification of Surface Breaking Cracks. *Journal of Nondestructive Evaluation* **39**, doi:[https://doi.org/10.1007/s11831-021-00953-0](https://doi.org/10.1007/s11831-021-00953-0) (2020).

35 C. P. Batuwatta-Gamage *et al.* A physics-informed neural network-based surrogate framework to predict moisture concentration and shrinkage of a plant cell during drying. *Journal of Food Engineering*, doi:[https://doi.org/10.1016/j.jfoodeng.2022.111137](https://doi.org/10.1016/j.jfoodeng.2022.111137) (2022).

36 Lagergren, J. H., Nardini, J. T., Baker, R. E., Simpson, M. J. & Flores, K. B. Biologically-informed neural networks guide mechanistic modeling from sparse experimental data. *PLoS Comput Biol* **16**, e1008462, doi:[https://doi.org/10.1371/journal.pcbi.1008462](https://doi.org/10.1371/journal.pcbi.1008462) (2020).

37 Viana, F. A. C. & Subramaniyan, A. K. A Survey of Bayesian Calibration and Physics-informed Neural Networks in Scientific Modeling. *Archives of Computational Methods in Engineering* **28**, 3801-3830, doi:[https://doi.org/10.1007/s11831-021-09539-0](https://doi.org/10.1007/s11831-021-09539-0) (2021).

38 Nti, I. K., Adekoya, A. F., Weyori, B. A. & Nyarko-Boateng, O. Applications of artificial intelligence in engineering and manufacturing: a systematic review. *Journal of Intelligent Manufacturing* **33**, 1581-1601, doi:[https://doi.org/10.1007/s10845-021-01771-6](https://doi.org/10.1007/s10845-021-01771-6) (2021).

39 Samaniego, E. *et al.* An energy approach to the solution of partial differential equations in computational mechanics via machine learning: Concepts, implementation and
applications. Computer Methods in Applied Mechanics and Engineering 362, doi:https://doi.org/10.1016/j.cma.2019.112790 (2020).

40 Rao, C., Sun, H. & Liu, Y. Physics-Informed Deep Learning for Computational Elastodynamics without Labeled Data. Journal of Engineering Mechanics 147, doi:https://doi.org/10.1061/(asce)em.1943-7889.0001947 (2021).

41 Abueidda, D. W., Lu, Q. & Koric, S. Meshless physics-informed deep learning method for three-dimensional solid mechanics. International Journal for Numerical Methods in Engineering, doi:https://doi.org/10.1002/nme.6828 (2021).

42 Nguyen-Thanh, V. M., Zhuang, X. & Rabczuk, T. A deep energy method for finite deformation hyperelasticity. European Journal of Mechanics - A/Solids 80, doi:https://doi.org/10.1016/j.euromechsol.2019.103874 (2020).

43 Chen, D., Li, Y., Guo, J. & Li, Y. Estimation of hypersonic vehicle weight using Physics-Informed neural network supported by knowledge based engineering. Expert Systems with Applications 195, doi:https://doi.org/10.1016/j.eswa.2022.116609 (2022).

44 Zhou, Y. et al. A new data-driven topology optimization framework for structural optimization. Computers & Structures 239, doi:https://doi.org/10.1016/j.compstruc.2020.106310 (2020).

45 Zhang, Z. et al. TONR: An exploration for a novel way combining neural network with topology optimization. Computer Methods in Applied Mechanics and Engineering 386, doi:https://doi.org/10.1016/j.cma.2021.114083 (2021).

46 Chandrasekhar, A. & Suresh, K. TOuNN: Topology Optimization using Neural Networks. Structural and Multidisciplinary Optimization 63, 1135-1149, doi:https://doi.org/10.1007/s00158-020-02748-4 (2020).

47 Bai, J. et al. A data-driven smoothed particle hydrodynamics method for fluids. Engineering Analysis with Boundary Elements 132, 12-32, doi:https://doi.org/10.1016/j.enganabound.2021.06.029 (2021).

48 Wessels, H., Weissenfels, C. & Wriggers, P. The neural particle method – An updated Lagrangian physics informed neural network for computational fluid dynamics. Computer Methods in Applied Mechanics and Engineering 368, doi:https://doi.org/10.1016/j.cma.2020.113127 (2020).

49 Jin, X., Cai, S., Li, H. & Karniadakis, G. E. NSFnets (Navier-Stokes flow nets): Physics-informed neural networks for the incompressible Navier-Stokes equations. Journal of Computational Physics 426, doi:https://doi.org/10.1016/j.jcp.2020.109951 (2021).

50 Xiang, Z., Peng, W., Zheng, X., Zhao, X. & Yao, W. Self-adaptive loss balanced Physics-informed neural networks for the incompressible Navier-Stokes equations. arXiv preprint arXiv:2104.06217 (2021).

51 Bai, J. et al. A general Neural Particle Method for hydrodynamics modeling. Computer Methods in Applied Mechanics and Engineering 393, doi:https://doi.org/10.1016/j.cma.2022.114740 (2022).

52 Mao, Z., Jagtap, A. D. & Karniadakis, G. E. Physics-informed neural networks for high-speed flows. Computer Methods in Applied Mechanics and Engineering 360, doi:https://doi.org/10.1016/j.cma.2019.112789 (2020).
Howland, M. F. & Dabiri, J. O. Wind Farm Modeling with Interpretable Physics-Informed Machine Learning. *Energies* **12**, doi:https://doi.org/10.3390/en12142716 (2019).

Zhang, J. & Zhao, X. Spatiotemporal wind field prediction based on physics-informed deep learning and LIDAR measurements. *Applied Energy* **288**, doi:https://doi.org/10.1016/j.apenergy.2021.116641 (2021).

Olivieri, M., Pezzoli, M., Antonacci, F. & Sarti, A. A Physics-Informed Neural Network Approach for Nearfield Acoustic Holography. *Sensors (Basel)* **21**, doi:https://doi.org/10.3390/s21237834 (2021).

Yu, K. H., Beam, A. L. & Kohane, I. S. Artificial intelligence in healthcare. *Nat Biomed Eng* **2**, 719-731, doi:https://doi.org/10.1038/s41551-018-0305-z (2018).

Kissas, G. *et al*. Machine learning in cardiovascular flows modeling: Predicting arterial blood pressure from non-invasive 4D flow MRI data using physics-informed neural networks. *Computer Methods in Applied Mechanics and Engineering* **358**, doi:https://doi.org/10.1016/j.cma.2019.112623 (2020).

van Herten, R. L., Chiribiri, A., Breeuwer, M., Veta, M. & Scannell, C. M. Physics-informed neural networks for myocardial perfusion MRI quantification. *arXiv preprint arXiv:.12844* (2020).

Bhouri, M. A. *et al*. COVID-19 dynamics across the US: A deep learning study of human mobility and social behavior. *Computer Methods in Applied Mechanics and Engineering* **382**, doi:https://doi.org/10.1016/j.cma.2021.113891 (2021).

Linka, K. *et al*. Bayesian Physics Informed Neural Networks for real-world nonlinear dynamical systems. *Computer Methods in Applied Mechanics and Engineering*, doi:https://doi.org/10.1016/j.cma.2022.115346 (2022).

Li, W., Bazant, M. Z. & Zhu, J. A physics-guided neural network framework for elastic plates: Comparison of governing equations-based and energy-based approaches. *Computer Methods in Applied Mechanics and Engineering* **383**, doi:https://doi.org/10.1016/j.cma.2021.113933 (2021).

Wang, S., Yu, X. & Perdikaris, P. When and why PINNs fail to train: A neural tangent kernel perspective. *Journal of Computational Physics*, doi:https://doi.org/10.1016/j.jcp.2021.110768 (2021).
Zhuang, X., Guo, H., Alajlan, N., Zhu, H. & Rabczuk, T. Deep autoencoder based energy method for the bending, vibration, and buckling analysis of Kirchhoff plates with transfer learning. *European Journal of Mechanics - A/Solids* 87, doi:https://doi.org/10.1016/j.euromechsol.2021.104225 (2021).

Dong, S. & Li, Z. Local extreme learning machines and domain decomposition for solving linear and nonlinear partial differential equations. *Computer Methods in Applied Mechanics and Engineering* 387, doi:https://doi.org/10.1016/j.cma.2021.114129 (2021).

Kharazmi, E., Zhang, Z. & Karniadakis, G. E. M. hp-VPINNs: Variational physics-informed neural networks with domain decomposition. *Computer Methods in Applied Mechanics and Engineering* 374, doi:https://doi.org/10.1016/j.cma.2020.113547 (2021).

Shukla, K., Jagtap, A. D. & Karniadakis, G. E. Parallel physics-informed neural networks via domain decomposition. *Journal of Computational Physics* 447, doi:https://doi.org/10.1016/j.jcp.2021.110683 (2021).

Goswami, S., Anitescu, C., Chakraborty, S. & Rabczuk, T. Transfer learning enhanced physics informed neural network for phase-field modeling of fracture. *Theoretical and Applied Fracture Mechanics* 106, doi:https://doi.org/10.1016/j.tafmec.2019.102447 (2020).

Goswami, S., Anitescu, C. & Rabczuk, T. Adaptive fourth-order phase field analysis using deep energy minimization. *Theoretical and Applied Fracture Mechanics* 107, doi:https://doi.org/10.1016/j.tafmec.2020.102527 (2020).

Haghighat, E., Bekar, A. C., Madenci, E. & Juanes, R. A nonlocal physics-informed deep learning framework using the peridynamic differential operator. *Computer Methods in Applied Mechanics and Engineering* 385, doi:https://doi.org/10.1016/j.cma.2021.114012 (2021).

Sukumar, N. & Srivastava, A. Exact imposition of boundary conditions with distance functions in physics-informed deep neural networks. *Computer Methods in Applied Mechanics and Engineering*, doi:https://doi.org/10.1016/j.cma.2021.114333 (2021).

Shin, Y., Darbon, J. & Karniadakis, G. E. On the Convergence of Physics Informed Neural Networks for Linear Second-Order Elliptic and Parabolic Type PDEs. *Communications in Computational Physics* 28, 2042-2074, doi:https://doi.org/10.4208/cicp.OA-2020-0193 (2020).

Mishra, S. & Molinaro, R. Estimates on the generalization error of physics-informed neural networks for approximating PDEs. *IMA Journal of Numerical Analysis*, doi:https://doi.org/10.1093/imanum/drab093 (2022).
Chen, Y., Lu, L., Karniadakis, G. E. & Dal Negro, L. Physics-informed neural networks for inverse problems in nano-optics and metamaterials. *Opt Express* **28**, 11618-11633, doi:https://doi.org/10.1364/OE.384875 (2020).

Ghaderi, A., Morovati, V. & Dargazany, R. A Physics-Informed Assembly of Feed-Forward Neural Network Engines to Predict Inelasticity in Cross-Linked Polymers. *Polymers (Basel)** **12**, doi:https://doi.org/10.3390/polym12112628 (2020).

Zhou, K., Sun, H., Enos, R., Zhang, D. & Tang, J. Harnessing deep learning for physics-informed prediction of composite strength with microstructural uncertainties. *Computational Materials Science* **197**, doi:https://doi.org/10.1016/j.commatsci.2021.110663 (2021).

Ghaderi, A., Morovati, V., Chen, Y. & Dargazany, R. A physics-informed multi-agents model to predict thermo-oxidative/hydrolytic aging of elastomers. *International Journal of Mechanical Sciences* **223**, doi:https://doi.org/10.1016/j.ijmecsci.2022.107236 (2022).

Amini Niaki, S., Haghighat, E., Campbell, T., Poursartip, A. & Vaziri, R. Physics-informed neural network for modelling the thermochemical curing process of composite-tool systems during manufacture. *Computer Methods in Applied Mechanics and Engineering* **384**, doi:https://doi.org/10.1016/j.cma.2021.113959 (2021).

Zhao, X., Salko, R. K. & Shirvan, K. Improved departure from nucleate boiling prediction in rod bundles using a physics-informed machine learning-aided framework. *Nuclear Engineering and Design* **374**, doi:https://doi.org/10.1016/j.nucengdes.2021.111084 (2021).

Chen, Y. & Dal Negro, L. Physics-informed neural networks for imaging and parameter retrieval of photonic nanostructures from near-field data. *APL Photonics* **7**, doi:https://doi.org/10.1063/5.0072969 (2022).

Li, W. *et al.* Physics-informed neural networks for electrode-level state estimation in lithium-ion batteries. *Journal of Power Sources* **506**, doi:https://doi.org/10.1016/j.jpowsour.2021.230034 (2021).

Ji, W., Qiu, W., Shi, Z., Pan, S. & Deng, S. Stiff-PINN: Physics-Informed Neural Network for Stiff Chemical Kinetics. *J Phys Chem A* **125**, 8098-8106, doi:https://doi.org/10.1021/acs.jpca.1c05102 (2021).

Chen, H., Katelhon, E. & Compton, R. G. Predicting Voltammetry Using Physics-Informed Neural Networks. *J Phys Chem Lett* **13**, 536-543, doi:https://doi.org/10.1021/acs.jpclett.1c04054 (2022).

Ngo, S. I. & Lim, Y.-I. Solution and Parameter Identification of a Fixed-Bed Reactor Model for Catalytic CO2 Methanation Using Physics-Informed Neural Networks. *Catalysts* **11**, doi:https://doi.org/10.3390/catal11111304 (2021).

De Florio, M., Schiassi, E. & Furfaro, R. Physics-informed neural networks and functional interpolation for stiff chemical kinetics. *Chaos* **32**, 063107, doi:https://doi.org/10.1063/5.0086649 (2022).

Tian, J., Xiong, R., Lu, J., Chen, C. & Shen, W. Battery state-of-charge estimation amid dynamic usage with physics-informed deep learning. *Energy Storage Materials* **50**, 718-729, doi:10.1016/j.ensm.2022.06.007 (2022).
Liu, M., Liang, L. & Sun, W. A generic physics-informed neural network-based constitutive model for soft biological tissues. *Computer Methods in Applied Mechanics and Engineering* **372**, doi:https://doi.org/10.1016/j.cma.2020.113402 (2020).

Li, W. & Lee, K.-M. Physics informed neural network for parameter identification and boundary force estimation of compliant and biomechanical systems. *International Journal of Intelligent Robotics and Applications* **5**, 313-325, doi:https://doi.org/10.1007/s41315-021-00196-x (2021).

Yin, M., Zheng, X., Humphrey, J. D. & Em Karniadakis, G. Non-invasive Inference of Thrombus Material Properties with Physics-Informed Neural Networks. *Comput Methods Appl Mech Eng* **375**, doi:https://doi.org/10.1016/j.cma.2020.113603 (2021).

Nguyen-Thanh, V. M., Anitescu, C., Alajlan, N., Rabczuk, T. & Zhuang, X. Parametric deep energy approach for elasticity accounting for strain gradient effects. *Computer Methods in Applied Mechanics and Engineering* **386**, doi:https://doi.org/10.1016/j.cma.2021.114096 (2021).

Haghighat, E., Raissi, M., Moure, A., Gomez, H. & Juanes, R. A physics-informed deep learning framework for inversion and surrogate modeling in solid mechanics. *Computer Methods in Applied Mechanics and Engineering* **379**, doi:https://doi.org/10.1016/j.cma.2021.113741 (2021).

Haghighat, E., Amini, D. & Juanes, R. Physics-informed neural network simulation of multiphase poroelasticity using stress-split sequential training. *arXiv preprint arXiv:2110.03049*, doi:https://doi.org/10.48550/arXiv.2110.03049 (2021).

Fuhg, J. N. & Bouklas, N. The mixed Deep Energy Method for resolving concentration features in finite strain hyperelasticity. *Journal of Computational Physics*, doi:https://doi.org/10.1016/j.jcp.2021.110839 (2021).

Guo, H., Zhuang, X. & Rabczuk, T. A deep collocation method for the bending analysis of Kirchhoff plate. *Computers, Materials & Continua* **59**, 433--456, doi:https://doi.org/10.32604/cmc.2019.06660 (2019).

Henkes, A., Wessels, H. & Mahnken, R. Physics informed neural networks for continuum micromechanics. *Computer Methods in Applied Mechanics and Engineering* **393**, doi:https://doi.org/10.1016/j.cma.2022.114790 (2022).

Jiang, J. et al. Physics-informed deep neural network enabled discovery of size-dependent deformation mechanisms in nanostructures. *International Journal of Solids and Structures* **236-237**, doi:https://doi.org/10.1016/j.ijsolstr.2021.111320 (2022).

Yin, M., Zhang, E., Yu, Y. & Karniadakis, G. E. Interfacing finite elements with deep neural operators for fast multiscale modeling of mechanics problems. *Computer Methods in Applied Mechanics and Engineering*, doi:https://doi.org/10.1016/j.cma.2022.115027 (2022).

Zhai, H. & Sands, T. Comparison of Deep Learning and Deterministic Algorithms for Control Modeling. *Sensors* **22**, doi:https://doi.org/10.3390/s22176362 (2022).

Waheed, U. b., Haghighat, E., Alkhalifah, T., Song, C. & Hao, Q. PINNeik: Eikonal solution using physics-informed neural networks. *Computers & Geosciences* **155**, doi:https://doi.org/10.1016/j.cageo.2021.104833 (2021).

Bottero, L. et al. Physics-informed machine learning simulator for wildfire propagation. *arXiv preprint arXiv:06825* (2020).
Rutkowski, D. R., Roldan-Alzate, A. & Johnson, K. M. Enhancement of cerebrovascular 4D flow MRI velocity fields using machine learning and computational fluid dynamics simulation data. *Sci Rep* **11**, 10240, doi: [https://doi.org/10.1038/s41598-021-89636-z](https://doi.org/10.1038/s41598-021-89636-z) (2021).

Kaandorp, M. P. T. *et al.* Improved unsupervised physics-informed deep learning for intravoxel incoherent motion modeling and evaluation in pancreatic cancer patients. *Magn Reson Med* **86**, 2250-2265, doi: [https://doi.org/10.1002/mrm.28852](https://doi.org/10.1002/mrm.28852) (2021).

Vardhan, M. & Randles, A. Application of physics-based flow models in cardiovascular medicine: Current practices and challenges. *Biophysics Reviews* **2**, 011302 (2021).

Herrera, C. R. *et al.* Physics-informed neural networks to learn cardiac fiber orientation from multiple electroanatomical maps. *arXiv preprint arXiv:2212.362* (2022).

Schiassi, E., De Florio, M., D’Ambrosio, A., Mortari, D. & Furfaro, R. Physics-Informed Neural Networks and Functional Interpolation for Data-Driven Parameters Discovery of Epidemiological Compartmental Models. *Mathematics* **9**, doi: [https://doi.org/10.3390/math9172069](https://doi.org/10.3390/math9172069) (2021).

Rodríguez, A., Cui, J., Ramakrishnan, N., Adhikari, B. & Prakash, B. A. EINNs: Epidemiologically-Informed Neural Networks. *arXiv preprint arXiv:2210.446* (2022).

Jagtap, A. D. & Karniadakis, G. E. in *AAAI Spring Symposium: MLPS*. 
