Comments on old and recent experiments of ”stickiness”
of a soft solid to a rough hard surface

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Abstract

The old asperity model of Fuller and Tabor had demonstrated almost 50 years ago surprisingly good correlation with respect to quite a few experiments on the pull-off decay due to roughness of rubber spheres against roughened Perspex plates. We revisit here some features of the Fuller and Tabor model in view of the more recent theories and experiments, finding good correlation can be obtained only at intermediate resolutions, as perhaps in stylus profilometers. In general we confirm the predictions of the Persson & Tosatti and Bearing Area Model of Ciavarella, as stickiness depends largely on the long wavelength content of roughness, and not the fine features. Therefore, multi-instruments measurements should hopefully not be needed.

Keywords:
Adhesion, Fuller and Tabor model, soft matter, roughness, stickiness.

1. Introduction

Although there is no doubt that roughness has a crucial effect on many aspects of tribology, there are very few quantitative models today finding its role in details. This is true for the earlier attempts to describe roughness with ”asperities” (Greenwood and Williamson, 1966) which had only qualitative success in explaining, for example, a linear dependence of real contact area with normal load but not to predict quantitatively tribological significant quantities such as friction coefficient or a wear coefficient, as dependent on roughness parameters such as rms roughness or rms slope or curvature. Significant progress has been made in recent years with more sophisticated
geometrical descriptions of roughness as self-affine fractals, and the attempt to develop "multiscale" models is in progress (Vakis, et al. 2018), but the quantitative predictive capability is still remote in most areas.

Perhaps one of the few exceptions to this general trend in tribology can be found in the mechanics of adhesion, where a very simple and very approximate model by Fuller and Tabor (1975. FT, in the following) seemed to obtain quite accurate quantititative estimate of pull-off for a rubber sphere in adhesive contact with roughened Perspex plates, relative to the case of an almost atomically smooth plate. The original FT paper shows even before any theory is introduced that pull-off force (relative to the smooth sphere case) for rubber spheres against roughened plates decays quite rapidly (at least in a linear scale) with amplitude of roughness (see Fig.1) (no clear dependence on the radius of the rubber sphere was found), but differently for the three rubber elastic moduli used in experiments. Notice that the relative pull-off scale can also be interpreted as a scale of relative surface energy \( \Delta \gamma_{eff}/\Delta \gamma \), where \( \Delta \gamma \) is the smooth sphere case, and \( \Delta \gamma_{eff} \) the "effective" surface energy as reduced by roughness, and both can be related to pull-off loads using the JKR simple formula for pull-off, \( P = \frac{3}{2\pi} R \Delta \gamma \), where \( R \) is sphere radius.

In order to further collapse the curves, a rather crude model was introduced based on the at the time popular model of "asperities", assuming a large number of largely spaced identical ones whose height follows a Gaussian distribution. However, in these models the asperity "radius" is a quantity hard to define (Greenwood & Wu 2001, Afferrante et al., 2018) when resolution is increased or in Persson’s terminology (Persson and Tosatti, 2001), magnification \( \zeta \) is large (\( \zeta = \lambda_L/\lambda_1 \) where \( \lambda_L, \lambda_1 \) are respectively the longest and the smallest wavelength in the roughness). Further, in FT each individual asperity was assumed to follow the JKR (Johnson, Kendall, Roberts, 1975) theory, and, again when using smaller and smaller asperities, the JKR theory should not be adequate since the so-called Tabor parameter (Tabor, 1977) tends to very small values \( \mu = \frac{\sigma_{th}}{E^*} \left( \frac{R_{asp}}{l_a} \right)^{1/3} \to 0 \) where \( \sigma_{th} \) is theoretical strength, \( E^* \) the plane strain elastic modulus, and \( R_{asp} \) is the radius of the asperity. Here, \( l_a = \Delta \gamma/E^* \) is an adhesive representative length scale, being \( \Delta \gamma \) the nominal surface energy of the pair of materials in contact.

Despite these strong approximations, FT experiments suggested quite good correlation with the model (see Fig.1 again, where FT model predictions are shown as solid lines), while varying the elastic modulus of the spheres, their radius, and the rms (CLA) roughness of the plates all of a factor of about
Figure 1: The relative pull-off force in the original Fuller and Tabor experiments decays rapidly with center line average roughness but differently for three different elastic moduli (from Fuller and Tabor, 1975). The shaded areas show experimental measurements and solid lines the FT predictions using their choice of parameters.

10. This is quite remarkable but is surprising in view of the weaknesses of asperity models we know today. Can this be a pure coincidence? We therefore discuss some recent theoretical and experimental findings, commenting also the original FT results.

2. Recent models on adhesive contacts

There has been considerable effort after FT in modelling the role of adhesion in contact mechanics (see a review in Ciavarella et al., 2019). Today, there is some consensus that surfaces are close to true "fractals", i.e. having a cascade of features from the macroscopic down to the possibly atomic scale. With modern instruments, the possibility to measure roughness have increased enormously since the times of Fuller and Tabor, and one is tempted
to use several instruments in overlapping range of wavelengths to attempt to describe roughness down to the Ångström-scale (Dalvi et al. 2019). In this attempt to be more "sophisticated", we have seen in the literature some debate although mostly theoretical and numerical and not enough experimental assessments like the original FT. With the development of large scale numerical simulations, debate was prompted in particular by results obtained by Pastewka & Robbins (2014) and Müser (2016) which seemed to suggest an important role of small scale features of roughness like rms slope or curvatures which for a true fractal become ill-defined quantities. This, in a sense, was a similar problem of asperity models, which had to define a "radius" for asperities. However, there is some degree of consensus today (see Violano et al., 2019, Ciavarella, 2020, Joe et al. 2017, 2018, for theoretical considerations, and Tiwari et al., 2020 for some experimental assessment) that "stickiness" should be for most surfaces a well defined quantity (i.e. independent on small scale features of the surface, like local slopes or curvatures). To show this, keeping the matter as simple as possible, let us consider a pure power law PSD (Power Spectral Density) starting from the longest wavelength in roughness $\lambda_L$, i.e. a PSD $C(q) = Z q^{-2(1+H)}$ for wavevectors $q > q_0 = \frac{2\pi}{\lambda_L}$, and $H$ is the Hurst exponent (equal to $3 - D$ where $D$ is the fractal dimension of the surface). We shall consider typical value for $H = 0.8$ (Persson, 2014) in general. The rms roughness is easily obtained as $h_{\text{rms}} = \sqrt{2\pi Z q_0^{-2H} \left( \frac{\zeta^{-2H-1}}{-2H} \right)} \simeq \sqrt{\frac{\pi Z}{H} q_0^{-H}}$ and so it clearly depends on the long wavelength components and not the detailed measurements, as it is obvious. Let us then consider just the "threshold" of stickiness, i.e. the condition where pull-off between the two solids becomes zero, or anyway many orders of magnitude smaller than the value for smooth surfaces. For this case, very simple results are obtained using Persson & Tosatti (2001, PT in the following) or the BAM model (Ciavarella, 2018) theories (see Ciavarella, 2020), suggesting that adhesion is not destroyed until we reach a very similar threshold on the rms roughness $h_{\text{rms}}$

$$h_{\text{rms}} > \sqrt{0.24l_a\lambda_L} ; \quad \text{Persson-Tosatti} \quad (1)$$

$$h_{\text{rms}} > \sqrt{0.6l_a\lambda_L} ; \quad \text{BAM} \quad (2)$$

The two theories reach (almost) the same conclusion despite starting off from quite different perspectives, and neither of them has any parameter related to rms slopes or curvatures. PT obtain their simple model arguing
with a energy balance between the state of full contact and that of complete loss of contact that the effective energy available at pull-off with a rough interface is:

$$\Delta \gamma_{\text{eff}} = \Delta \gamma - \frac{U_{\text{el}}}{A_0}$$

(3)

where $U_{\text{el}}$ is the elastic strain energy when we squeeze the roughness flat, and $A_0$ is the nominal contact area. This elastic energy $U_{el}(\zeta)$ generally depends on magnification, but converges quite rapidly (therefore eliminating the dependence on the smallest scales in the spectrum, those affecting slopes or curvatures) for the most general case of low fractal dimension, suggesting there is a true "fractal limit" to the adhesive contact problem, in agreement also with other, completely different theories, namely Joe et al. (2017, 2018). BAM, instead, assumes a simplified Maugis-Dugdale force-separation law and a geometric evaluation of the region of attraction which, together with the adhesiveless theory of Persson, gives a full solution to the problem.

Notice that the pull-off force should not depend on elastic modulus for contact of a smooth sphere vs a smooth plane, as the JKR theory predicts pull off $P = 3/2 \pi R \Delta \gamma$, whereas the elastic modulus does enter into play in all the theories via the dependence on the factor $l_a$. This has prevented measurement of adhesion between hard macroscopic bodies until either JKR used soft elastomers in contact with smooth glass surfaces in 1971. FT then introduced some micrometer scale roughness in their 1975 paper, and introduced an asperity model as we shall describe in the next paragraph.

3. The FT paper

The FT asperity model leads to a single parameter encapsulating all parameters in the adhesion problem. Stickiness is virtually destroyed (more precisely pull-off is reduced exponentially by several orders of magnitude, see Ciavarella & Papangelo, 2018) when the ratio of the separation at pull-off in the JKR model for a single asperity, $\delta_c$, is large enough, say about 3

$$\frac{1}{\Delta_c} = \frac{h_{rms}}{\delta_c} = \left(\frac{4}{3}\right)^{5/3} \frac{h_{rms}}{R_{\text{asp}}^{1/3}} \left(\frac{1}{\pi l_a}\right)^{2/3} > 3$$

(4)

We neglect to consider the correction due to increase of surface area which is included in the original paper of Persson-Tosatti (2001) and, in modified form, in that of Dalvi et al. (2019), as we shall discuss further later on.
As we have anticipated, one parameter of the FT dimensionless factor \( \frac{1}{\Delta_c} \) that clearly is very delicate to estimate is the asperity radius \( R_{asp} \). A “common” value is found in FT experiments (Tab.3) for the mean “radius” of the asperities of all “bead-blasted” surfaces despite the change of center line average roughness and density of asperities, having the quite round value \( R_{asp} = 100 \mu m \), whereas it varies a little for the ”abraded surface”, as \( R_{asp} = 150 \mu m \). The fact that the radius of asperities is a very ”difficult” quantity to measure is well known and for any power law tail of PSD, is given as

\[
R_{asp} \approx \frac{2}{h_{rms}} \sim 2 \sqrt{\frac{2-H}{\pi Z}} q_1^{H-2},
\]

where \( q_1 \) the upper truncating wavevector of roughness. The quantitative agreement between FT predictions and their experiments is therefore dependent on the choice of \( q_1 \), and although \( R_{asp} \) appears elevated to a power 1/3 which makes the dependence weaker, in today’s view which makes it possible to measure various decades of roughness, a change of a factor 1000 in \( q_1 \) would mean

\[
\left( \frac{q_1}{q_1} \right)^{(H-2)/3} \approx 1000^{0.8-2/3} = 6 \times 10^{-2}
\]

factor change in \( \frac{1}{\Delta_c} \).

But let us discuss for the moment how the threshold (4) compares with more recent proposals (1, 2). Elaborating eqt.(4) we get after some algebra that

\[
h_{rms} > \alpha^{-1/4} \beta^{3/4} \left( \frac{3}{4} \right)^{5/4} \left( \frac{2-H}{H} \right)^{1/8} (\lambda_L l_a)^{1/2} \zeta^{H/4-1/2}
\]

i.e. typically for \( H = 0.8 \)

\[
h_{rms} \approx \sqrt{2} (\lambda_L l_a)^{1/2} \zeta^{0.3}
\]

Comparing therefore with the other theories (1, 2), we find, perhaps unexpectedly, exactly the same parametric dependence on the \( l_a \) and \( \lambda_L \) quantities, which means the same dependence on elastic modulus, surface energy, and longest wavelength in roughness of Persson-Tosatti and BAM. This is encouraging and may explain already partly the success of the FT model, despite the largely crude origins of the finding. However, we also find a spurious dependence on ”magnification” \( \zeta \), and hence, exact correspondence between FT and PT stickiness thresholds occurs only when

\[
\zeta = \left( \frac{\sqrt{2}}{\sqrt{0.24}} \right)^{1/0.3} \approx 34
\]
i.e. quite low magnifications. The comparison will be further elucidated in the next paragraph.

4. Persson-Tosatti theory in terms of FT parameter

The PT theory gives naturally not only the threshold but the full decay curve of pull-off (or surface energy), and in particular for the usual case of \( H = 0.8 \), or \( D = 2.2 \), we obtain under the power law PSD simplifying assumption (Ciavarella, 2020)

\[
\frac{\Delta \gamma_{\text{eff}}}{\Delta \gamma} = 1 - 4.2 \frac{h_{\text{rms}}}{l_0 \lambda_L} \tag{8}
\]

For this case, the FT parameter (4) can be reinterpreted as

\[
\frac{1}{\Delta_c} = 1.9 \zeta^{0.4} \left( \frac{h_{\text{rms}}^2}{l_0 \lambda_L} \right)^{2/3} \tag{9}
\]

and therefore the full Persson-Tosatti curve (8) can be written in terms of the FT parameter as

\[
\frac{\Delta \gamma_{\text{eff}}}{\Delta \gamma} = 1 - 1.6 \zeta^{-0.6} \left( \frac{1}{\Delta_c} \right)^{3/2} \tag{10}
\]

Using the FT curve for the pull-off decay as taken from the original FT paper, we can compare it with the PT theory prediction (10) for \( \zeta = 10, 100, 1000 \), in Fig.2. It would seem a reasonable fit is obtained only for quite low "magnification" \( \zeta \approx 10 \), as it seems results are very far already at \( \zeta \approx 100 \). However, since we don’t have detailed measurements of roughness of the original FT paper, we don’t know what an appropriate choice for \( \zeta \) would be in FT case, so it is interesting to compare the two predictions starting from a more recent experimental paper, that of Dalvi et al. (2019), as in the next paragraph.
Figure 2: A comparison of relative pull-off force (i.e. relative surface energy) in the original Fuller and Tabor theory (solid thick line), with the Persson-Tosatti theory (10) at different magnifications $\zeta = 10, 100, 1000$ (colour lines with symbols) with power law PSD roughness. Clearly, the agreement is satisfactory only for very low magnifications $\zeta$.

5. Dalvi et al. experiments

While the literature abounds of measurements of hard small particles adhesion, where the effect of elasticity is very limited, the measurements with soft materials like those in FT are scarce. A very detailed set of experiments, similar in principle to FT, has been discussed recently by Dalvi et al. (2019) reporting soft elastic polydimethylsiloxane (PDMS) hemispheres with elastic modulus ranging from 0.7 to 10 MPa (see Tab.1) in contact with four different polycrystalline diamond substrates measured very accurately with several instruments to cover 7 orders of magnitude in wavelengths (see main roughness parameters in Tab.2). Notice that the rms roughness is at nanometer scale rather than the micrometer scale of FT paper, while the radius of asperities (measured at the full resolution) varies between 0.6nm and 1.8nm, which means values 5 orders of magnitude smaller than those
which FT reported (mostly because the surfaces in FT where only measured with old profilometers, and here at probably at least 4 orders of magnitude larger resolution).

Dalvi et al. (2019) discuss the apparent work of adhesion as defined from the approach curves using a JKR curve fit, rather than the retraction curves (where the pull-off point occurs, and where FT made their measurements), as they find considerable hysteresis and probably dependence on the maximum compressive preload. They find values reported in Tab.1 as mean values for the entire set of measurements. They then apply the Persson & Tosatti (2001) model (3) which however in the full form includes a modification of the surface energy to take into account of the hard rough surface has having a bigger area $A_{true}$ than the nominal one:

$$\Delta \gamma_{eff} = \Delta \gamma \frac{A_{true}}{A_0} - \frac{U_{el}}{A_0}$$

(11)

This term $\frac{A_{true}}{A_0}$ or the surfaces of Dalvi et al. (2019) even considering the full spectrum, since $h'_{rms} \simeq 1$ and using Fig.S7B in the Dalvi et al. (2019) paper, is of the order of 1.3, so it is not a major factor. Also, since (see Tab.1), the curvature is extremely similar for all surfaces, despite the very different rms roughness, the factor $\frac{A_{true}}{A_0}$ corresponds to merely adjusting the "intrinsic" work of adhesion $\Delta \gamma$ which is not unambiguously defined, and indeed Dalvi et al. (2019) take as best-fit from the data (not, as FT, from a ideally "smooth" sphere adhesion test). There is however a further modified form of the PT theory, which reads

$$\Delta \gamma_{eff} = \Delta \gamma \frac{A_{true}}{A_0} - \gamma_1 \left( \frac{A_{true}}{A_0} - 1 \right) - \frac{U_{el}}{A_0}$$

(12)

where $\gamma_1$ is the surface energy of the elastomer alone. Given $\frac{A_{true}}{A_0} \simeq 1.3$ for all surfaces and they take $\gamma_1 = 25mJ/m^2$, this correction corresponds merely to a decrease of all measured values of $\Delta \gamma_{eff}$ of a factor $7.5mJ/m^2$. This is probably why Dalvi et al. (2019) obtain as best fit $\Delta \gamma = 25mJ/m^2$ when using the original PT theory (11), and $\Delta \gamma = 37mJ/m^2$ when applying the modified form (12). Dalvi et al. (2019) argue in favour of their modified PT form (12) based on correlation with experiments. Notice that the polished ultrananocrystalline diamond surface is close to be considered ideally smooth, and this case is what causes the most curious results, since the apparent work of adhesion seems to increase with the elastic modulus (see Tab.1), rather than decreasing as one would expect, and as it occurs for all other cases.
Dalvi et al. (2019) obtain two other values for the “intrinsic” work of adhesion $\Delta \gamma$, $\Delta \gamma = 37.0 \pm 3.7 \text{mJ/m}^2$ from the measured contact radius curve as a function of applied load fitting a JKR curve, and $\Delta \gamma = 46.2 \pm 7.7 \text{mJ/m}^2$ from a measurement of the closed-circuit integral of the force-displacement curve, assuming, as they show, that the energy loss should be proportional to the intrinsic work of adhesion, and the true area at maximum preload (which they measure).

\[
\begin{array}{|c|c|c|c|c|}
\hline
 & E = 0.69 \text{MPa} & E = 1.03 \text{MPa} & E = 1.91 \text{MPa} & E = 10.0 \text{MPa} \\
\hline
\text{PUNCD} & 41 & 42 & 46 & 59 \\
\text{UNCD} & 39 & 42 & 40 & 23 \\
\text{NCD} & 21 & 20 & 17.5 & 8.4 \\
\text{MNCD} & 23.5 & 25 & 17.6 & 4.1 \\
\hline
\end{array}
\]

Tab.1 - Work of adhesion $\Delta \gamma_{eff}$ during approach (mean value) [mJ/m²] in Dalvi et al. (2019) for PDMS hemispheres with various elastic moduli, against different roughened plates. PUNCD, UNCD, NCD, MCD stand for polished ultrananocrystalline diamond, ultrananocrystalline diamond, nanocrystalline diamond, microcrystalline diamond. Notice that $\Delta \gamma_{eff}$ decreases for increasing elastic modulus, except for the PUNCD.

\[
\begin{array}{|c|c|c|}
\hline
 & h_{rms}[\text{nm}] & h_{rms}^p[\text{nm}^{-1}] (*) \\
\hline
\text{PUNCD} & 4.6 & 1.13 \\
\text{UNCD} & 23 & 3.37 \\
\text{NCD} & 121 & 3.19 \\
\text{MNCD} & 127 & 2.83 \\
\hline
\end{array}
\]

Tab.2 - Main roughness parameters measured for the different roughened plates (PUNCD, UNCD, NCD, MCD stand for polished ultrananocrystalline diamond, ultrananocrystalline diamond, nanocrystalline diamond, microcrystalline diamond). Notice how the curvature is extremely similar for all surfaces, despite the very different rms roughness. (*) using the full measured spectrum.

We shall attempt to apply the FT model, knowing that we expect the choice of the mean asperity radii $R_{asp}$ to be critical. Given the results in the previous paragraph, we anticipate the radii obtained with the full spectrum measured down to the atomic scale is too small. For example we may reduce
the upper wavevector cutoff, which in the original data is \( q_1 = 1.6 \times 10^{10} m^{-1} \), by three orders of magnitude to \( q'_1 \simeq 10^7 m^{-1} \), approximately the limit of stylus profilometer investigations (see Fig.1 of the Dalvi et al. (2019) paper). This implies one needs to multiply the values obtained from Tab.2 as \( R = 2/h''_{rms} \sim q'_1^{-2} \) by a factor \( \beta \simeq \left( \frac{10^7}{1.6 \times 10^{10}} \right)^{0.8-2} \simeq 7000 \). We shall take the value \( \Delta \gamma = 37 mJ/m^2 \) for our initial elaborations, as it seems this is the value Dalvi et al. (2019) give with most confidence. This results in a correlation between experiments and theory for the old FT model which looks very similar to that obtained by Dalvi et al. (2019). To test this more in general, we computed the correlation \( R^2 \) as a function of this multiplicative factor \( \beta \) in Fig.3 and find that the correlation is better than the original PT model for \( \beta \simeq 10^2 - 10^5 \) (Dalvi et al report in this case \( R^2 \simeq 0.29 \) even considering for PT a different fit for \( \Delta \gamma \), namely \( \Delta \gamma = 25 mJ/m^2 \)) and similar than the correlation factor \( R^2 \simeq 0.67 \) they obtained with their modified form of the PT criterion, in a still quite extensive range of \( \beta \). However, correlation becomes very poor \( (R^2 \simeq -2 \text{ for } \beta = 1 \text{ i.e. with the radius as measured at the Angstrom scale, since FT predicts stickiness to be destroyed with the very small radii}) \). Hence, the real troubles seem to start in using FT when measuring roughness with very high resolutions of AFM or TEM as reported by Dalvi et al. (2019). As a further test, we use the other estimate \( \Delta \gamma = 46.2 mJ/m^2 \) obtained by Dalvi et al. (2019) with a different method, and repeat the calculation in Fig.4, obtaining an even better correlation for the FT model in a certain range of choice of the asperity radii. One example of the Dalvi data plotted in the FT representative plot is Fig.5, where we have assumed the case of \( \Delta \gamma = 46.2 mJ/m^2 \text{ and } \beta = 500 \) for which one of the best correlations is found. Notice that the main reason for discrepancy between theory and data are the two points for the rougher surfaces in the case of harder rubbers, for which there is some persistence of stickiness not expected in the FT theory, or the PT theory.

With this we don’t want to suggest that FT parameter is a better choice than PT or Dalvi’s modified form in general, as we are convinced that asperity models cannot be regarded as accurate today, and we prefer a model which is not sensitive to the measuring instrument. Indeed, it is not acceptable that the correlation with experiments becomes extremely poor outside the range we indicated. However, particularly in view of the fact that today there is consensus that stickiness depends mostly on the longest wavelengths in the power spectrum, it is reasonable to assume that the asperity model
captures reasonably well the physics at those scales. This, at least, seems to explain why FT seemed to obtain a good correlation of their theory with experiments, at their time.

Notice that, due to the large hysteresis, there is an additional complication in modelling retraction curves, and there is no attempt by Dalvi et al. (2019) to really estimate pull-off points or effective surface energy, which can be largely greater than that upon loading. This is probably due to the nanoscopic scale of roughness, and the additional complication in modelling retraction curves shows that the problem is still not entirely understood.

Figure 3: $R^2$ values obtained in correlating the apparent surface energy $\Delta \gamma_{eff}$ obtained in the Dalvi experimental data with the Fuller and Tabor theory as a function of the multiplicative factor of the mean asperity radius $\beta$, as compared with the $R^2$ reported in Dalvi’s paper for the Persson-Tosatti and Persson-Tosatti modified criteria, which we consider independent on $\beta$. Here, we assume $\Delta \gamma = 37\, mJ/m^2$. 

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Figure 4: R^2 values as in Fig.3, but now we assume Δγ = 46.2mJ/m^2.

6. Tiwari et al. experiments

Another recent paper on the subject was proposed by Tiwari et al. (2020). They measure this time PMMA spheres against rubber flats, where the PMMA spheres are sandblasted, and the process is shown to modify mainly the long wavelength roughness of the PSD, so while the rms-roughness amplitude changes from a very low value for the smooth surface to 0.78μm and 1.73μm for the two ”rough” ones, the rms-slope which is dependent mainly on the short wavelength roughness, is nearly the same (0.18 and 0.22, respectively and notice that with these values, the $\frac{A_{rms}}{A_0}$ correction of the theory is negligible). In terms of roughness, the cases analyzed are closer to the original microroughness scale in FT paper than to the nanometer scale of the Dalvi et al. Also, in the paper the fits occur all in the retraction curve, contrary to the Dalvi et al. The paper contains no attempt to measure roughness down to nanometer scale, and the main result of the paper is an experimental proof that the long components of roughness kills adhesion, and not the small
Figure 5: Decay of $\Delta \gamma_{\text{eff}}/\Delta \gamma$ for experiments and FT prediction (solid line) assuming $\beta = 500$ and $\Delta \gamma = 46.2 m J/m^2$. The symbols represent PUNCD (red crosses), UNCD (solid green circles), NCD (open purple circles), MCD (x),

wavelength content, as the second surface having $h_{\text{rms}} = 1.73 \mu m$ shows virtually no pull-off. This was a result already suggested from various theories (see Ciavarella 2020) as a result of the scientific debate we have discussed.

It may be instructive to do a quick calculation, since the full PT theory seems to involve integration processes over the full PSD curve which requires in general a digitalization. If we apply the PT theory as simplified for a pure power law PSD as in eqt.12 of Ciavarella (2020), assuming constants indicated in the Tiwari, et al. (2020) paper $\Delta \gamma = 0.2 J/m^2$, $E = 2.3 MPa$, $\nu = 0.5$, $\lambda_L = 0.38 mm$ (the JKR radius of the circular contact region at the point of snap-off for the PDMS surface 1), and taking $H = 0.8$, the stickiness threshold (Ciavarella, 2020) is

$$h_{\text{rms}} > \sqrt{\frac{\Delta \gamma}{E^*} \frac{2H - 1}{\pi H} \lambda_L} = \sqrt{\frac{0.2}{2.3 \times 10^6 / (1 - 0.5^2) 0.38 \times 10^{-3} 2 \times 0.8 - 1} \pi \times 0.8} = 2.4 \mu m$$
which gives the correct order of magnitude although not the exact value since it should be a rms-roughness between the 0.78µm and 1.73µm for surface 1 and 2. Given there are only two experimental points, it is not possible to compare with the FT theory, apart from the general considerations in the first paragraph.

7. Conclusions

We have shown that part of the apparent quantitative excellent correlation between theory and experiment of the Fuller and Tabor asperity model paper was due to a correct functional dependences on the problem’s parameters (mainly, elastic modulus), but part of it was certainly a coincidence in measuring the asperity radius with low resolution measurements. In fact, even in more recent experiments, the FT model would possibly give still good correlation if one uses simple measurement techniques like stylus profilometers. It has taken nearly 50 years after the original experiments of Fuller and Tabor, to have finally a clearer understanding of the issue of "scale-dependency" of the roughness measurement in the model, although we still do not have a complete picture of adhesion of soft solids against rough surface, since the problem is complicated, as for example the Dalvi et al experiments suggest in the difference between the loading and unloading curve behaviour. As recent experiments have clarified that stickiness depends on the long wavelengths of spectrum, perhaps this explains the relative success of the Persson-Tosatti’s theory which in the end is simpler than the Fuller and Tabor one. Complex multi-instrumental measurements of roughness over many decades of wavelengths should not generally be required.

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