Antiferroelectric resonance in noncentrosymmetric multi-sublattice magnets

V. N. Kruvoruchko and D. A. Yablonskii

Physicotechnical Institute, Ukrainian SSR Academy of Sciences, Donetsk

(Submitted 19 February 1988)

Zh. Eksp. Teor. Fiz. 94, 268-276 (September 1988)

We predict the phenomenon of antiferroelectric resonance (AFER), in which an AC electric field causes magnetic ions located at noncentrosymmetric positions in a multi-sublattice magnet to undergo magnetic transitions corresponding to exchange collective excitations of the system. We construct a theory of such resonances, and show that in magnets with collinear magnetic structures AFER is caused by relativistic and exchange-relativistic magnetoelectric interactions, while in the noncollinear magnets a significant contribution can also come from the exchange magnetoelectric interaction. We predict an exchange enhancement of the resonance by exchange modes, and discuss the role of AFER in causing resonant enhancement of magnetooptical phenomena. The main results of our theory are illustrated with the four-sublattice rhombohedral antiferromagnets α-Fe₂O₃ and Cr₂O₃ as examples.

I. INTRODUCTION

The traditional method of investigating the resonance properties of magnetically-ordered crystals consists of subjecting them to a high-frequency (HF) magnetic field. Using this method we can reliably identify both acoustic modes (AM) and, in special cases, exchange modes (EM) of the crystal. Excitation of EM with an HF magnetic field is possible only if the former are magnetically-active (i.e., coupled to oscillations of the ferromagnetism vector M of the system). However, the coupling of the EM to M comes about through relativistic and exchange-relativistic interactions, and therefore the intensity of absorption by EM exchange is weakened compared to AM exchange. For non-magnetically-active EMs interaction with a magnetic field is impossible, by virtue of selection rules (for example, in the case of EM which are odd under inversion).

In this paper we show that a natural way to resonantly excite magnetic-system exchange modes which are odd under inversion is to act on them with a HF electric field. We construct a theory which describes the excitation of such modes by an AC electric field. The corresponding resonance will be referred to as antiferroelectric (AFER), because the EM are oscillations of the antiferromagnetism vectors L, .

The general condition for the existence of AFER will be formulated below. Here we only remark that in the majority of cases a sufficient condition for the existence of AFER in noncentrosymmetric crystals is the presence of magnetic ions at the noncentrosymmetric positions. Examples of such magnets are hematite, iron garnets, ferrite spinels, orthoferrites, and other compounds.

II. ANTIFERROMAGNETIC RESONANCE IN HEMATITE

As an example of a noncentrosymmetric crystal with a magnetic structure which is even under inversion, let us discuss the four-sublattice rhombohedral antiferromagnetic α-Fe₂O₃ (hematite). Following Ref. 10, we introduce the following linear combinations of the sublattice magnetizations \( M_i \) (\( i = 1, 2, 3, 4 \)):

\[
\begin{align*}
M_i & = M + M_i + \frac{1}{2} M_{i'} + \frac{1}{2} M_{i''} \\
L_i & = M_\perp - M_i + \frac{1}{2} M_{i'} - \frac{1}{2} M_{i''} \\
L_{i'} & = M_\perp - M_i + \frac{1}{2} M_{i'} - \frac{1}{2} M_{i''} \\
L_{i''} & = M_\perp - M_i + \frac{1}{2} M_{i'} - \frac{1}{2} M_{i''}
\end{align*}
\]

where \( M_\perp \) is the magnitude of the sublattice magnetization. In Table I we show the classification of the vectors \( l \) in Tables II and III relative to the irreducible representations of the magnetic and electric fields.

| Irreducible representation | Irreducible spin configuration | Polarization of HF electric and magnetic fields | Irreducible spin configuration | Polarization of HF electric and magnetic fields |
|---------------------------|------------------------------|-----------------------------------------------|-----------------------------|-----------------------------------------------|
| \( A_{1g} \)              | \( l_{1g} \)                 | \( (\pm, \pm), (\pm, -\pm) \)                  | \( A_{1g} \)               | \( l_{1g} \)                 | \( (\pm, \pm), (\pm, -\pm) \)                  |
| \( A_{1u} \)              | \( l_{1u} \)                 | \( (\pm, \pm), (\pm, -\pm) \)                  | \( A_{1u} \)               | \( l_{1u} \)                 | \( (\pm, \pm), (\pm, -\pm) \)                  |
| \( E_{g} \)               | \( l_{eg} \)                 | \( (\pm, \pm), (\pm, -\pm) \)                  | \( E_{g} \)                | \( l_{eg} \)                 | \( (\pm, \pm), (\pm, -\pm) \)                  |
| \( E_{u} \)               | \( l_{eu} \)                 | \( (\pm, \pm), (\pm, -\pm) \)                  | \( E_{u} \)                | \( l_{eu} \)                 | \( (\pm, \pm), (\pm, -\pm) \)                  |

1986 Sov. Phys. JETP 67 (9), September 1988 0038-5646/88/091886-05$04 00 © 1989 American Institute of Physics
because they are odd under inversion. At the same time, an oscillation of the vectors

\[ \mathbf{R}(t) \] 

corresponds to oscillations of the vectors

\[ \mathbf{E}(t) \] 

under inversion; these oscillations couple to the external magnetic field, which do not contain relativistic origin which do not contain relativistic origin.

To linear approximation in the spin deviations, we have

\[ \mathbf{P}(t) = -\mathbf{M} \mathbf{E}(t) \] 

Introducing the electric polarization tensor

\[ \mathbf{P}(\omega) = \alpha(\omega) \mathbf{E}(\omega) \] 

we find from Eqs. (4)–(7) and the relations above that the spin part of the HF electric polarization tensor \( \mathbf{P}(\omega) \) in the easy-plane phase with the magnetic field \( \mathbf{H} \) has the following nonzero components:

\[ a_{m}(\omega) = -4 \mathbf{K}_{s}^{T} \left[ (\mathbf{I} + \chi) \mathbf{E}(\omega) \right] (\omega^{2} - \omega_{s}^{2})^{-1} \] 

\[ a_{m}(\omega) = -4 \mathbf{K}_{s}^{T} \left[ (\mathbf{I} + \chi) \mathbf{E}(\omega) \right] (\omega^{2} - \omega_{s}^{2})^{-1} \] 

In Eq. (10) we neglect the term \( \tau \mathbf{m} \) compared to \( \mathbf{K}_{s} \), while in Eq. (13) we neglect \( \tau \mathbf{m} \) compared to \( \mathbf{K}_{s} \).

For the easy-axis phase \( (I_{z} = \pm 1, \mathbf{H}z) \), we have

\[ P_{\mathbf{h}}(t) = \partial \mathbf{R}_{0}(t) \mathbf{m} \] 

\[ P_{\mathbf{h}}(t) = \partial \mathbf{R}_{0}(t) \mathbf{m} \] 

Calculating the oscillation amplitudes of the antiferromagnetic vectors \( \mathbf{h}_{s}(t) \) and \( \mathbf{l}_{s}(t) \) under the action of AC electric field \( \mathbf{E}(t) \) in this phase and substituting them into Eqs. (14), we obtain the following nonzero spin contributions to the electric polarization tensor:

\[ \mathbf{P}(\omega) = \mathbf{P}(\omega) \mathbf{E}(\omega) \] 

\[ \mathbf{P}(\omega) = \mathbf{P}(\omega) \mathbf{E}(\omega) \]
Now, the EM frequency equals
\[ \gamma \omega_{\text{em}} = \left[ \left( H_0^2 + H_0 H_1 + H_1^2 \right) \right]^{-1} \]
(17)

It is clear from Eqs. (10)-(13), (15) and (16) that in the easy-plane and easy-axis phases the residues at the EM poles of the HF electric polarization tensor are enhanced by exchange. The absorption intensity of the electric field at these frequencies is determined by the magnitude of the antiferroelectric constant; in what follows we give a numerical estimate of this constant.

Hematite is a straightforward example of a multi-sublattice system in which excitation of EM by a magnetic field is impossible by virtue of general selection rules, while for an AC electric field these rules allow such excitations. Let us now turn to a different system—CrO3.

III. ELECTRIC-DIPOLE-ACTIVE VIBRATIONS IN CrO3

The crystal CrO3 possesses a magnetic structure which is odd under inversion. In the magnetic class which includes CrO3, combination involves with the time-reversal operation \( \mathcal{T} \) and this results in a linear magnetoelectrical effect. The ground state is \( A_{1g} \), with \( I_{1g} \). The magnetic properties of CrO3 are described by the potentials (2) and (3), in which it is necessary to make the replacements \( H_1 \rightarrow -H_1, H_2 \rightarrow -H_2, \) and \( H_3 \rightarrow -H_3 \).

In the exchange approximation the acoustic type of oscillations in CrO3 correspond to transverse oscillations of the vectors \( I_1 \) and \( m \), while those of exchange type correspond to transverse oscillation of the vectors \( I_1 \) and \( I_2 \). Allowance for the Dzyaloshinskii interaction of \( H_{1g} \) and \( H_{2g} \) in (2) leads to dynamic coupling of the EM and AM. Without pausing for detailed calculations we will present the final results.

A. Acoustic modes. Accurate to terms of order \( H_{1g}/H_0 \) and \( I_{1g}/H_0 \) \( 1/3 \) inclusively, the frequencies of the AM are equal to
\[ \gamma \omega_{\text{ac}} = \omega_{\text{ac}} \pm \mu_0 H_0. \]
(18)

The nonzero transverse components of the HF magnetic susceptibility tensor have in this frequency interval the form
\[ \chi_{xx} = \left[ \left( H_0^2 + H_0 H_1 + H_1^2 \right) \right]^{-1} \]
(19)

\[ \chi_{xx} = \left[ \left( H_0^2 + H_0 H_1 + H_1^2 \right) \right]^{-1} \]
(20)

\[ \chi_{xx} = \left[ \left( H_0^2 + H_0 H_1 + H_1^2 \right) \right]^{-1} \]
(21)

\[ \chi_{xx} = \left[ \left( H_0^2 + H_0 H_1 + H_1^2 \right) \right]^{-1} \]
(22)

\[ \chi_{xx} = \left[ \left( H_0^2 + H_0 H_1 + H_1^2 \right) \right]^{-1} \]
(23)

\[ \chi_{xx} = \left[ \left( H_0^2 + H_0 H_1 + H_1^2 \right) \right]^{-1} \]
(24)

\[ \chi_{xx} = \left[ \left( H_0^2 + H_0 H_1 + H_1^2 \right) \right]^{-1} \]
(25)

\[ \chi_{xx} = \left[ \left( H_0^2 + H_0 H_1 + H_1^2 \right) \right]^{-1} \]
(26)

\[ \chi_{xx} = \left[ \left( H_0^2 + H_0 H_1 + H_1^2 \right) \right]^{-1} \]
(27)

Since the vectors \( I_1, m \), which are even under inversion, and the vectors \( I_1, I_2 \), which are odd under inversion, particles in the EM and AM oscillations, these EM and AM can be excited both by magnetic and electric fields with \( E, D \) (see Table III). However, from an experimental point of view the important thing is the magnitude of the absorption by the EM and AM. We obtain from (19)–(22) and (24)–(27) the following estimate of the susceptibility near the resonance frequency:
\[ \alpha_{\text{em}} = -\gamma M R (w - \omega_o)^{-1} - \gamma M (H_1/H_0) (w - \omega_o)^{-1}, \]
(28)

\[ \alpha_{\text{am}} = -\gamma M R (w - \omega_o)^{-1} - \gamma M (H_1/H_0) (w - \omega_o)^{-1}. \]
(29)

The intensity of the absorption of an electric field by the EM is \( (H_1/H_0)^{1/3} \) times larger than absorption by AM. For a
magnetic field, the situation is reversed: the intensity of the absorption by AM is \( (H_x/H_y) \) times larger than that due to the EM. It is also clear that for \( R \gg H_x/H_y \), excitation of EM by an electric field is easier than by a magnetic field.

IV. PHYSICAL MECHANISM OF AFER

The physical mechanism which gives rise to the spin Hamiltonian is the same as the one discussed previously in constructing a theory of the magnetoelectric effect and of electric effects in magnetic resonance, and a theory of absorption and scattering of light in magnetically-ordered crystals.

A direct indication of the possibility of experimental observation of AFER is provided by experiments in which an antiferromagnetic transition between magnetic levels of paramagnetic ions in noncentrosymmetric sites, i.e., transitions from a state \((L_m, m)\) to states \((L_m \pm 1, m)\) where \(L\) is the orbital and \(m\) the magnetic quantum number. Such transitions were observed in Refs. 19-20 (see also Ref. 22 and citations therein). As we have shown here, in centrosymmetric crystals which have high concentrations of magnetic ions and magnetic structures which are even under inversion (e.g., of FeO\(_2\) type) one consequence of these transitions will be magnetic excitations of exchange type.

The physics of AFER combines the physical mechanisms of electric-dipole paramagnetic resonance associated with impurity magnetic ions in noncentrosymmetric positions\(^{43,44} \) and of absorption and scattering of light in systems with a high concentration of magnetic ions.\(^{45,46} \) In systems with magnetic ions occupying centers of inversion, the electric dipole activity of the magnetic modes can be due to, e.g., the additional effect of a constant electric field.\(^{21,22} \) We emphasize that in previously-studied magnets,\(^{13,43} \) without centers of inversion the effects investigated were due to coupling of electric and magnetic subsystems; in contrast, according to the theory constructed in this paper, AFER in noncentrosymmetric crystals is caused by direct excitation of exchange-type magnetic oscillations by an AC electric field.

We shall determine the antiferroelectric constants for \( \text{Fe}_3\text{O}_4 \), from experiments on the shift of the paramagnetic-resonance lines for \( \text{Fe}^{3+} \) ions.\(^{22} \) The latter give magnetic ions in noncentrosymmetric positions\(^{13,14} \) and of absorption and scattering of light in systems with a high concentration of magnetic ions.\(^{45,46} \) In systems with magnetic ions occupying centers of inversion, the electric dipole activity of the magnetic modes can be due to, e.g., the additional effect of a constant electric field.\(^{21,22} \) We emphasize that in previously-studied magnets,\(^{13,43} \) without centers of inversion the effects investigated were due to coupling of electric and magnetic subsystems; in contrast, according to the theory constructed in this paper, AFER in noncentrosymmetric crystals is caused by direct excitation of exchange-type magnetic oscillations by an AC electric field.

We shall determine the antiferroelectric constants for \( \text{Fe}_3\text{O}_4 \), from experiments on the shift of the paramagnetic-resonance lines for \( \text{Fe}^{3+} \) ions.\(^{22} \) The latter give an estimate of \( 10^{-12} \) for the single-ion spin-Hamiltonian constant. In systems with a high concentration of magnetic ions, contributions to the magnetoelectric effects come also from ion-ion interactions (in particular from exchange and relativistic-exchange interactions), which in individual cases increase the value of the constant by an order of magnitude\(^{23} \). Therefore in \( \text{Fe}_3\text{O}_4 \) (apparently) \( R \sim 10^{-11} \). The value of \( R \) is also an order of magnitude larger.

It should also be noted that the contribution of invariants of exchange origin is proportional to the magnetization (see Eqs. (12) and (13)); therefore, in the canted phases their contribution can significantly exceed the contribution from the exchange-relativistic invariants.

For quantitative estimates of the antiferroelectric interaction constants in \( \text{Cr}_2\text{O}_3 \), we make use of the results of experimental and theoretical studies of the magnetoelectric effect. In this compound \( \text{Cr}_2\text{O}_3 \), the latter give for the parameters of the spin Hamiltonian the values\(^{15,16} \):

\[
\Pi \sim 5 \times 10^{-13}, \quad R \sim 5 \times 10^{-14}.
\]

Taking into account that \( H_x/H_y \sim 10^{-3} \) in \( \text{Cr}_2\text{O}_3 \), the conditions for observation of EM based on the absorption of an electric field can be more favorable than those based on absorption of a magnetic field.

V. LINEAR RESONANT MAGNETOELECTRIC EFFECTS

Let us formulate the general conditions for electric-dipole activity of the magnetic oscillations.

In the general case the EM and AM are of electric-dipole type if the dynamic compounds of the antiferromagnetism and ferromagnetism vectors which correspond to them transform according to the irreducible representations of the unitary subgroup of the system's magnetic symmetry group (i.e., they transform just like the compounds of the electric polarization vector \( \text{P} \)).

Let us investigate in more detail the general conditions for the existence of AFER in magnetic structures whose ground state in the exchange approximation is collinear and satisfies the conditions of the Turov classification\(^{47} \) (see also Ref. 39).

In centrosymmetric crystals it is necessary to distinguish between structures which are even and odd under inversion.

1. The structures \( I^+ \) (i.e., a system whose structure is even under inversion contains magnetic ions which are not located at inversion centers, then in addition to the basic even antiferromagnetism vector \( \text{I}_e \), there exists at least one other antiferromagnetism vector \( \text{I} \) which is odd under inversion. In the thermodynamic potential of such a magnet it is possible to have invariants of the form

\[
K_{ab}E_iL_{ai}, \quad a, b, c = x, y, z
\]

In a static electric field, Eq. (28) leads to the appearance of \( I_e - E_0 \). In analogy with the magnetoelectric effect this phenomenon can be called antiferroelectric (AEEF).

In an AC electric field \( E \sim \exp(i\omega t) \), the relation (28) causes oscillation of the vector \( I_e \) and for \( \omega = \omega_0 \), excitation of EM (\( \omega_0 \) is the exchange frequency).

2. For structures \( I^-(\pm) \) odd under inversion, it is necessary to distinguish between two cases according to parity relative to a translation \( t \).

Systems with \( I^-(-) \) and \( I^+(+) \) pertain to antiferromagnets whose thermodynamic potentials include an invariant of the form

\[
K_{aibc}H_iM_{bi},
\]

where \( M \) is the magnetic moment of the system. If in this case there also exists an antiferromagnetic vector \( \text{I} \) which is odd under inversion, then it is also possible to have an invariant of the form (28). This invariant gives rise to the presence of AFER and AEEF, while (29) makes possible excitation of AM by an electric field, i.e., magnetoelectric resonance.

1. In systems with \( I^-(-) \) (the magnetic unit cell is larger than the crystallographic unit cell) the invariant (29) is forbidden and there is no static magnetoelectric effect. However, an invariant of the form (28) is possible, where now the antiferromagnetic vector \( \text{I} \) is odd under transmutation and even under inversion. In this system there will be both AEEF and AFER.
IV. CONCLUSION

We shall dwell in somewhat greater detail on the possibility of experimentally observing antiferroelectric resonance with EM and AM.

Apparently, low-dimensionality magnets are most convenient from an experimental point of view for observing EM by resonant methods. In such systems, because of the presence of weak exchange interactions, the EM and AM frequencies are comparable as a rule. Examples of low-dimensionality magnet in which it is possible to observe AFER are the eight-sublattice antiferromagnets $\alpha$-Fe$_2$O$_3$ and Cr$_2$O$_3$ located in the infrared wavelength band. Therefore the features of experimental observation of AFER in these compounds are close to those in optical experiments. If the dimensions of the sample are comparable with or smaller than the wavelength of the AC electric field, then the theory developed above is directly applicable.

For bulk samples of $\alpha$-Fe$_2$O$_3$ and Cr$_2$O$_3$, the equations of motion for the vectors $\mathbf{K}$ (1) must be considered jointly with the Maxwell equations. In this case, the exchange spin modes and the electromagnetic oscillations are found to be coupled, which leads to their mutual restructuring. A detailed analysis of this question is outside the framework of the present communication; therefore we will only pause briefly to treat a specific case.

For examples, let us consider the dispersion relation of coupled right-handed-polarized electromagnetic waves and EM $\omega_{\alpha\beta}(17)$ in hematite has the form

$$\omega^2 - \omega_0^2 \left( R^2 (\omega_0^2 - H_0^2) + R^2 (\omega_0^2 - H_0^2) \right) = 0.$$  

Here $\omega$ is the velocity of light while the parameter $\omega_0$ which determines the value of the coupling of the branches is $\omega_0 = 4\omega M_0^2 R^2 (\omega_0^2 - H_0^2)$. $\alpha_n$ is the refractive index of the medium without allowance for the EM contribution. The dispersion of the EM in this frequency band is not significant and we disregard it. For left-handed-polarized waves the dispersion relation is obtained from (30) by the replacements $\omega_{\alpha\beta} \leftrightarrow \omega_{\beta\alpha}, R \rightarrow -R$.

As is clear from (19)-(22) and (24)-(27), when magnetic oscillations can be excited by an electromagnetic field it is possible to have resonance singularities both in the components of the magnetic susceptibility tensor and in the components of the electric polarization tensor. It is necessary to take this circumstance into account both in development of a theory of propagation of electromagnetic waves and in analysis of the experimental data. In particular, near the exchange resonances it is possible to resonantly enhance such magnetooptic effects as Faraday rotation, the Cotton-Mouton effect, and the Kerr effect. It should also be kept in mind that, in addition to the linear processes we have investigated here, the Hamiltonian (3) also contributes to two-magnon absorption, to paramagnetic instability, and to other nonlinear phenomena, even in an approximation linear in the electric field.

The authors wish to express their gratitude to V. G. Bar'yakhtar, V. V. Eremenko, A. I. Zyazgin, and A. A. Stepanov for discussions of this work.

1. G. Bar'yakhtar, I. M. Vitinskii, and D. A. Yablonski, Zh. Eksp. Teor. Fiz. 76, 1381 (1979) [Sov. Phys. JETP 49, 703 (1979)].
2. A. A. Yablonski, Author's Abstract of Doctoral Thesis in Physics and Mathematical Sciences, Dnepropetrovsk, 1980.
3. A. A. Stepanov, M. I. Kosen, and A. I. Zyazgin, Prikl. Fiz. 9, 764 (1983).
4. V. V. Eremenko, V. M. Naumenko, Yu. G. Passkeushi, and V. V. Polikan, Zh. Eksp. Teor. Fiz. 80, 97 (1981) [JETP Lett. 33, 112 (1981)].
5. V. G. Bar'yakhtar, V. V. Eremenko, V. M. Naumenko et al., Zh. Eksp. Teor. Fiz. 88, 1382 (1985) [Sov. Phys. JETP 61, 823 (1985)].
6. A. I. Zyazgin, M. I. Kosen, V. N. Kruvoruchko, A. A. Stepanov, and D. A. Yablonski, Zh. Eksp. Teor. Fiz. 80, 2296 (1981) [Sov. Phys. JETP 62, 1234 (1981)].
7. R. J. Jeneik, Phys. Rev. 126, 265 (1962).
8. D. A. Yablonski and V. N. Kruvoruchko, Preprint DonFTN 86-1 (130), Donetsk, 1985.
9. E. A. Turov, Physical Properties of Magnetically-Ordered Crystals [in Russian], Moscow, USSR Acad. Sci. Press, 1963.
10. I. E. Dzyaloshinski, Zh. Eksp. Teor. Fiz. 32, 1546 (1957) [Sov. Phys. JETP 5, 1259 (1957)]; A. S. Borovik-Romanov, Lectures on Low-Temperature Magneton, Nauka, Moscow, 1981.
11. L. D. Landau and E. M. Lifshitz, Quantum Mechanics, Nonrelativistic Theory, Pergamon, 1975.
12. D. C. Herber, J. Phys. C2, 1614 (1969).
13. O. Nigai, L. Bonavito, and T. Tsuchik, J. Phys. C5, 1226 (1972).
14. L. Kuznetso and V. Yu. Yushankhai, JINR Preprint P17-9932, Dubna, 1986.
15. E. I. Sushkova and T. Sh. Shirman, Phys. Status Solidi 47, 241 (1972).
16. L. Dzyaloshinski, Zh. Eksp. Teor. Fiz. 37, 881 (1959) [Sov. Phys. JETP 10, 629 (1960)].
17. H. Chrennenes, Phys. Rev. 180, 499 (1969).
18. A. B. Rotnin, Usp. Fiz. Nauk 105, 677 (1971) [Sov. Phys. Usp. 14, 766 (1972)].
19. A. B. Rotnin, Fiz. Tverd. Tela (Leningrad) 5, 151 (1963) [Sov. Phys. Solid State 5, 107 (1963)].
20. V. V. Drachman, Fiz. Tverd. Tela (Leningrad) 7, 948 (1965) [Sov. Phys. Solid State 7, 764 (1965)].
21. T. Moriya, J. Phys. Soc. Jpn. 23, 499 (1967).
22. V. V. Eremenko, Introduction to Optical Spectroscopy of Magnets [in Russian], Kiev, Nauk. Dumka, 1975.
23. I. Ozhogin and V. L. Safarov, J. Magnetism and Magn. Mater. 31, 675 (1983).
24. V. G. Bar'yakhtar and I. E. Chup, Fiz. Tverd. Tela (Leningrad) 11, 324 (1969) [Sov. Phys. Solid State 11, 2628 (1970)].
25. A. G. Smelnitski and I. E. Chup, Fiz. Tverd. Tela (Leningrad) 13, 415 (1972) [Sov. Phys. Usp. 35, 675 (1962)].
26. V. A. Kiselev and L. N. Davydov, Prikl. Fiz. 13, 380 (1971) [JETP Lett. 13, 271 (1971)].
27. L. N. Davydov and Z. A. Svetlik, Usp. Fiz. Zh. 18, 1346 (1966).
28. D. E. Tilley and J. Scott, Phys. Rev. B25, 3521 (1982).
29. I. M. Vitinskii and N. M. Lavrenovski, Zh. Eksp. Teor. Fiz. 12, 1913 (1966) [Sov. J. Low Temp. Phys. 12, 672 (1966)].
30. K. I. Bichurin, V. G. Koslot, E. S. Kovalenko, et al., Fiz. Tverd. Tela (Leningrad) 10, 259 (1968) [Sov. Phys. Solid State 10, 196 (1968)].
31. R. Hornreich and S. Strrikman, Phys. Rev. 164, 506 (1967).
32. T. G. Rado, Int. J. Magnetism 6, 121 (1974).
33. S. Four, Phys. Rev. 130, 183 (1963).
34. T. G. Rado, Theory of Magnetic Excitons [in Russian], Kiev, Nauk. Dumka, 1976.
35. V. V. Eremenko and N. A. Yablonski, Zh. Eksp. Teor. Fiz. 9, 851 (1982) [Sov. J. Low Temp. Phys. 8, 479 (1982)].
36. A. V. Bozhch, H. Swain, A. C. Phath, and W. J. de Jonge, J. Phys. C20, 2107 (1987).