Symmetric and asymmetric nuclear matter in the relativistic approach at finite temperatures

H. Huber, F. Weber, and M. K. Weigel
Sektion Physik, Universität München
Am Coulombwall 1, D-85748 Garching, Germany
14th August 2018

PACS numbers: 21.65.+f, 21.60.Jz, 24.10.Cn, 97.60.Jd

Abstract
The properties of hot matter are studied in the frame of the relativistic Brueckner-Hartree-Fock theory. The equations are solved self-consistently in the full Dirac space. For the interaction we used the potentials given by Brockmann and Machleidt. The obtained critical temperatures are smaller than in most of the nonrelativistic investigations. We also calculated the thermodynamic properties of hot matter in the relativistic Hartree–Fock approximation, where the force parameters were adjusted to the outcome of the relativistic Brueckner–Hartree–Fock calculations at zero temperature. Here, one obtains higher critical temperatures, which are comparable with other relativistic calculations in the Hartree scheme.
I Introduction

The properties of hot and dense nuclear matter play an essential role in the understanding of high-energy heavy-ion collisions, supernova explosions and proto-neutron stars. For that reason the problem of hot nuclear matter has been studied over the last decades in several investigations, which however were predominantly performed within the nonrelativistic scheme [1], using either effective density dependent interactions [1, 2, 3, 4] or the Brueckner approach [1, 5, 6, 7, 8]. In the relativistic approach investigations of the equations of state for $T \neq 0$ are relatively scarce. The majority of such calculations were performed in the relativistic Hartree approximation (RH), where the extension to finite temperatures is straightforward. Details of this scheme are given, for instance, in Refs. [9]-[15]. More complicated are the relativistic Hartree–Fock– [14, 15] and the Brueckner–Hartree–Fock approximation [16], and the application to finite nuclei [17].

In this contribution we will concentrate on the relativistic Brueckner–Hartree–Fock treatment (RBHF) of symmetric and asymmetric nuclear matter generalizing the formalism as described in Refs. [18, 19] to $T \neq 0$. The RBHF–approach seems to be of special interest, since it is known for $T = 0$ that the resulting EOSs are much stiffer than their nonrelativistic counterparts [20]. To our knowledge such an investigation has been only performed so far by the Groningen group for symmetric nuclear matter [16]. As described in more details in Refs. [16, 18], their method uses a nonunique ansatz for the $T$–matrix in terms of five Fermi invariants, which can lead to ambiguous results for the self-energies (see, e.g., Refs. [21, 22]). In order to avoid this problem we solve, according to the original scheme of the Brooklyn group [23], the RBHF–approximation in the full Dirac space, which is more tedious (for a more detailed discussion, see Ref. [18]). Since the formal structure of the problem is the same as for $T = 0$, where one has to solve three coupled equations, namely the Dyson equation for the one-body Green’s function $G$, the (reduced) Bethe–Salpeter equation for the effective scattering matrix $T$ in matter and the equation for the self–energy $\Sigma$, we will not repeat here the equations. As in Refs. [16, 18, 19] we will restrict ourselves to the incorporation of intermediate positive–energy nucleon states only, where now the Fermi step functions are replaced by the Fermi distribution functions $n_{F}(T)$ at finite temperature $T$ (for details, see Ref. [25]). The Green’s function obeys
for $T \neq 0$ the spectral representation \cite{13, 24}

$$G(p) = \int d\omega A(\vec{p}, \omega) \left\{ \frac{f(\omega)}{p_p - \omega - i\eta} + \frac{f(-\omega)}{p_p - \omega + i\eta} \right\} , \quad \text{(I.1)}$$

with

$$f(\omega) = (e^{\beta\omega} + 1)^{-1} \left\{ T = 0 \right\} \Theta(-\omega) . \quad \text{(I.2)}$$

The formal structure of the spectral function $A(\vec{p}, \omega)$ \cite{18} remains unaltered to the case for $T = 0$.

A further difference in comparison with Ref. \cite{16} is that we take the momentum dependence of the self–energies into account. Since the pole of the quasi–particle propagators occurs for $T \neq 0$ in the integration domain of the intermediate states, one obtains, in principle, complex effective scattering matrix elements and self–energies. It was checked in Ref. \cite{16} that the imaginary part of $\Sigma$ turned out to be small. Therefore we neglect also $\text{Im} \, \Sigma$ in the calculations. For the one–boson–exchange interaction we used the modern potentials constructed by Brockmann and Machleidt \cite{26}. We select for the presentation the so-called potential $B$, which gives the best results for the EOS ($E/A = -15.73$ MeV; $\rho_0 = 0.172$ fm$^{-3}$; $K = 249$ MeV; $J = 32.8$ MeV) at zero temperature in RBHF–calculations (see Refs. \cite{18, 19}, for the potential $A$ the outcome is similar, see Ref. \cite{23}). For the sake of comparison we also treated the RHF–approximation, where we adjusted the force parameters to the outcome for the EOS for symmetric and asymmetric nuclear matter at $T = 0$ \cite{13, 25}. The RHF–approximation has in comparison with the RH–approximation the advantage to resemble in its formal structure more to the RBHF–approximation with the benefit of a much simpler numerical treatment than in the RBHF case.

For finite temperatures one needs for the determination of the pressure the free energy per baryon, defined as

$$f = u - Ts , \quad \text{(I.3)}$$

with the entropy per baryon:

$$s = -\frac{2}{\rho h^3} \sum_{\tau} \int d^3p \left[ n(\vec{p}) \ln n(\vec{p}) + (1 - n(\vec{p})) \ln (1 - n(\vec{p})) \right] . \quad \text{(I.4)}$$

The pressure is given by:

$$P = \rho \sum_{\tau} \rho \frac{\partial f}{\partial \rho} \bigg|_{T, \rho = \tau} . \quad \text{(I.5)}$$
II Results and discussion

Common to almost all nonrelativistic treatments are EOSs typical in shape to the standard Van de Waals behavior. The value for the critical temperature depends strongly on the choice of the forces (and approximations). The bandwidth reaches from 14 to 22 MeV, with a critical density of about 1/3 of the saturation density $\rho_0$. In a recent calculation of asymmetric matter for $T \neq 0$ a critical temperature of 20.8 MeV ($\rho_c = 0.39 \rho_0$) was obtained for symmetric matter, which decreased to 8.0 MeV ($\rho_c = 0.24 \rho_0$) for the limiting asymmetry of $\delta = 0.75$. In the relativistic treatment within the framework of the relativistic Hartree approximation the Van der Waals behavior is still present, but in general one obtains lower critical temperatures (for instance, $T_c = 14.4$ MeV, $\rho_c = 0.31 \rho_0$ [12], $T_c \approx 14$ MeV [13]).

Of great theoretical interest is the microscopic RBHF–treatment for two reasons: Firstly, as mentioned before, the RBHF–EOSs are much stiffer than their nonrelativistic counterparts [20]. Secondly, according to Refs. [1, 16] the typical Van der Waals behavior may be questionable, and one obtains a rather low critical temperature, $T_c \approx 8 - 9$ MeV [1] or 12 MeV ($\rho_c \approx 0.6 \rho_0$) [16], depending on the approximative treatment of the self–energy (see, also the discussion in Ref. [1]). Due to these reasons we recalculated the RBHF–approximation in the full Dirac space as described in Refs. [18, 25] with the Brockmann–Machleidt potentials [26], which give better results for the saturation properties [18]. As discussed before we treated also the RHF–approximation, where the force parameters are adjusted to the outcome of the RBHF–treatment at $T = 0$ [19, 25].

In Figs. 1 and 2 we present first on a larger scale the EOSs for different asymmetries and temperatures, computed for RBHF and RHF, respectively. On this scale the energy per nucleon differs not very much in both approximations. This holds also for the pressure (see Fig. 3), free energy and the entropy (see Fig. 4). As expected the EOSs seem to be stiffer as in non-relativistic Brueckner calculations (cf., for instance, with Ref. [7]). The entropy behavior is similar to Ref. [7] and agrees with the experimental situation (see, e.g., Fig. 6 of Ref. [7]). Interesting is the EOS in the lower density domain, where one expects a phase transition. The situation is depicted for symmetric matter in Figs. 5 and 6, from which we deduce a critical temperature of 10.4 MeV (RBHF) and 15.2 MeV (RHF), respectively. (For asymmetric matter, see Ref. [23].) The RBHF value is in accordance with Ref. [16] and with one non-relativistic BHF–calculation [4]. The RHF result for $T_c$ is higher and agrees
more or less with the result of Ref. [12], where one uses a smaller equilibrium density. The critical temperatures and pressures for different asymmetries in the RBHF–treatment are given in Fig. 7 (for RHF, see Ref. [25]). The critical densities are shown in Fig. 8. For symmetric matter they are slightly above 1/3 of the saturation density but smaller than in Ref. [10].

One should however mention in this context that in general BHF–calculations are facing numerical convergence problems for small densities [7] and may even not be applicable [27]. For that reasons the conclusions drawn from the BHF–approximation for low densities (and temperatures) should be considered with some caution and a phenomenological description with adjustable parameters, for instance, with Skyrme forces, might be a suitable alternative in this sensitive domain. Unfortunately the critical temperature can not be extracted directly from experiment and additional models and assumptions are needed to obtain it from hot nuclei produced in heavy ion collisions [1]. However more recent experiments imply that a lower critical temperature might be possible [28].

As a final point we would like to mention that the symmetry energy remains – as expected – a monotonic increasing function of the density in the case for finite temperatures (see Fig. 9), so that the composition of proto-neutron stars should show, as in cold neutron stars for relativistic EOSs, the tendency to lower asymmetries with increasing density [3, 14]: contrary to some nonrelativistic EOSs with a non-monotonic symmetry energy [29].

III Conclusions

In conclusion, we have performed a calculation of hot symmetric and asymmetric nuclear matter within the relativistic Brueckner–Hartree–Fock scheme using modern OBE–interactions constructed by Brockmann and Machleidt. It turned out that the critical temperatures are smaller than it is the case for the majority of nonrelativistic treatments. We have additionally treated the relativistic Hartree–Fock approximation at $T \neq 0$, where the Lagrangian parameters were adjusted to the outcome of the RBHF–treatment for $T = 0$. Here the critical temperature is in the range of other relativistic treatments performed in the Hartree scheme.

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Figure captions

Fig. 1: Energy per nucleon for different asymmetries \( \delta = \frac{\rho_n - \rho_p}{\rho} \) and temperatures as function of the density in the RBHF–approximation (Brockmann–Machleidt potential B).
Fig. 2: Energy per nucleon for different asymmetries and temperatures in the RHF–approximation.
Fig. 3: Density dependence of the pressure for different asymmetries and temperatures in the RBHF–approximation. (In the RHF–approximation the pressure increases less for higher temperatures).
Fig. 4: Entropy per baryon for nuclear matter versus density at different temperatures (for the experimental comparison, see Fig. 6 of Ref. [7]).
Fig. 5: Pressure as a function of baryon density for nuclear matter ($\delta = 0$) at different temperatures in the RBHF–approximation.
Fig. 6: Pressure versus baryon density for nuclear matter at different temperatures in the RHF–approximation.
Fig. 7: Critical temperature and pressure as function of the asymmetry (RBHF; $T_c$ and $P_c$ are smaller than in the RHF).
Fig. 8: Critical density as function of the asymmetry.
Fig. 9: Symmetry energy as function of the baryon density at different temperatures (RBHF).
The graph shows the entropy per nucleon as a function of density ($\rho$) for different temperatures ($T$). The curves are labeled as follows:

- **RBHF**
- **RHF**

The temperatures considered are 5 MeV, 10 MeV, 15 MeV, and 20 MeV. The graph indicates the behavior of entropy per nucleon with respect to density at these temperatures, with the RBHF and RHF methods showing distinct trends.
\begin{align*}
\rho \text{ (fm}^{-3}\text{)} & & P \text{ (MeV/fm}^3\text{)} \\
T = 17.5 \text{ MeV} & & \\
T = 15 \text{ MeV} & & \\
T = 12.5 \text{ MeV} & & \\
T = 10.4 \text{ MeV} & & \\
T = 7.5 \text{ MeV} & & \\
T = 5 \text{ MeV} & & \\
T = 2.5 \text{ MeV} & & \\
T = 0 \text{ MeV} & & 
\end{align*}
$e_{\text{sym}}(\rho)$ (MeV) vs $\rho$ (fm$^{-3}$)

- Solid line: $T = 0$ MeV
- Dashed line: $T = 10$ MeV
- Dotted line: $T = 20$ MeV