Meridional circulation of gas into gaps opened by giant planets in three-dimensional low-viscosity disks

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ABSTRACT

We examine the gas circulation near a gap opened by a giant planet in a protoplanetary disk. We show with high resolution 3D simulations that the gas flows into the gap at high altitude over the mid-plane, at a rate dependent on viscosity. We explain this observation with a simple conceptual model. From this model we derive an estimate of the amount of gas flowing into a gap opened by a planet with Hill radius comparable to the scale-height of a layered disk (i.e. a disk with viscous upper layer and inviscid midplane). Our estimate agrees with modern MRI simulations (Gressel et al., 2013). We conclude that gap opening in a layered disk cannot slow down significantly the runaway gas accretion of Saturn to Jupiter-mass planets.

1. Introduction

Understanding what sets the terminal mass of a giant planet in a runaway gas accretion regime is an open problem in planetary science. Runaway gas accretion is the third stage of the classical core-accretion scenario for the formation of giant planets (Pollack et al., 1996). We remind the reader that in stage I, a solid core of 5-10 Earth masses ($M_{\oplus}$) is formed by planetesimal accretion (or possibly by pebble-accretion; see Lambrechts and Johansen 2012, Morbidelli and Nesvorny, 2012). In stage II, the core starts to capture gas from the protoplanetary disk, forming a primitive atmosphere in hydrostatic equilibrium; the continuous accretion of planetesimals heats the planet and prevents the atmosphere from collapsing. In stage III the combined mass of core and atmosphere becomes large enough (the actual mass-threshold depending on opacity and energy input due to planetesimal bombardment) that the latter cannot remain in hydrostatic equilibrium anymore; thus the
planet enters in an exponential gas-accretion mode, called runaway.

The accretion timescale in the runaway accretion mode is very fast and, once started, can lead to a Jupiter-mass planet in a few $10^4$ y (see for instance the hydro-dynamical simulations in Kley, 1999; D’Angelo et al., 2003; Klahr and Kley, 2006; Ayliffe and Bate, 2009; Tanigawa et al., 2012; Gressel et al., 2013, Szulagyi et al., 2014) in a proto-planetary disk with mass distribution similar to that of the Minimum Mass Solar Nebula model (Weidenschilling, 1977; Hayashi, 1981). This rapid accretion mechanism can explain how giant planets form. On the other hand, there is no obvious reason for this fast accretion to stop. As its timescale is much shorter than the proto-planetary disk life time (a few My Haisch et al., 2001), it is unlikely that the disk disappears just in the middle of this process, raising the question why Jupiter and Saturn and many giant extra-solar planets did not grow beyond Jupiter-mass.

It is well known that giant planets open gaps in the protoplanetary disk around their orbits (Lin and Papaloizou, 1986; Bryden et al., 1999). Thus it is natural to expect that the depletion of gas in the planet’s vicinity slows down the accretion process. Still, all of the hydro-dynamical simulations quoted above that feature the gap-opening process show that the mass-doubling time for a Jupiter-like planet is not longer than $10^5$ years.

However, these simulations may have been hampered by the assumption of a prescribed viscosity throughout the protoplanetary disk, or by significant numerical viscosity in the simulation scheme. It is expected that planets form in dead zones of the protoplanetary disk (Gammie, 1996), where the viscosity is much smaller than in numerical simulations, so that Jovian-mass planets could presumably open much wider and deeper gaps, with consequent inhibition of further growth (e.g. Thommes et al., 2008; Matsumura et al., 2009; Ida and Lin, 2004).

On this issue, it is worth stressing that there is quite of a confusion on the role of
viscosity in gap opening. In a 2D disk isothermal model, Crida et al. (2006) showed that
the width and the depth of a gap saturates in the limit of vanishing viscosity. This paper
has been challenged recently by Duffell and MacFadyen (2013) and Fung et al. (2013), still
for 2D disks. For a massive planet, the results of Duffell and MacFadyen actually agree
with those of Crida et al., because the former group also finds that the gap has a saturated
depth and width in the limit of null viscosity and they demonstrate that this result is not
due to numerical viscosity. The actual disagreement is on the ability of small planets to
open gaps in disks much thicker than their Hill sphere. Fung et al. address this case just
by extrapolation of formulæ obtained for massive planets in viscous disks, so it is not very
compelling. We believe that gap opening by small planets in Duffel and MacFadyen is due
to the use of an adiabatic equation of state $P = E_{\text{int}}(\gamma - 1)$ which, despite adopting a
value for the parameter $\gamma$ very close to unity, is not equivalent to the isothermal case (see
Paardekooper and Mellema, 2008). The issue, however, deserves further scrutiny.

This controversy is nevertheless quite academic, because real disks have a 3 dimensional
structure. Thus, in this Note we discuss gap opening in 3D disks and we focus on giant
planets that are massive enough to undergo runaway gas accretion, i.e. Jupiter-mass bodies.
In Sect. 2 we present the structure of the gaps and the gas circulation in their vicinity,
using three-dimensional isothermal simulations. We also interpret the results with a simple
intuitive model. From this model, we derive in Section 3 an estimate for the flow of gas
into a gap in a layered disk (i.e. a disk that is viscous on the surface and “dead” near
the mid-plane), that is in agreement with the numerical results of Gressel et al. (2013).
Conclusions and discussion of the implications for terminal mass problem of giant planet
are reported in Sect. 4.
2. Gaps in 3D disks

In the framework of a study on planet accretion (Szulagyi et al., 2014), we conducted 3D simulations of a Jupiter-mass planet embedded in an isothermal disk with scale height of 5% and \(\alpha\)-prescription of the viscosity (Shakura and Sunyaev, 1973). We adopted \(\alpha = 4 \times 10^{-3}\) (viscous simulation) and \(\alpha = 0\) (inviscid simulation). The latter simulation was conducted with two different resolutions, to change the effective numerical viscosity of the simulation code. The technical parameters of the simulations are described in detail in Szulagyi et al. (2014) and the main ones are briefly reported in the caption of Fig. 1.

In Szulagyi et al. (2014) we reported that the flux of material into the gap (and therefore the accretion rate of the planet) decreases by a factor of 2 when the resolution is increased by a factor of 2 (and hence the numerical viscosity is halved). This suggested that the gap properties do not saturate in the limit of small viscosity (prescribed or numeric), or at least not in the range of resolution that we have been able to attain. This observation motivated a deeper investigation, which is the object of the present work.

Fig. 1 gives a clear demonstration of how the gap profile opened by a Jupiter-mass planet in 3D simulations changes with prescribed and numerical viscosity (i.e. with resolution). Each curve shows the radial profile of the surface density, averaged over the azimuth. The surface density has been computed by integrating the volume density over the vertical direction. Clearly, here no convergence is achieved with resolution.

We explain this fact with the following simple model. Consider first the gas in the mid-plane. Its dynamics has to be similar to that in a 2D disk model, and therefore the planet opens a gap with a given profile independent of viscosity in the small viscosity limit. In the vertical direction, the disk has to be in hydrostatic equilibrium. This implies that
Fig. 1.— Gap profiles in the simulations by Szulagyi et al. (2014). The perturbed column density distribution is normalized by the original column density. Here “Viscous” means and \( \alpha \) parameter of 0.004 in the Shakura and Sunyaev (1973) viscosity prescription and “inviscid” means \( \alpha = 0 \); “Low resolution” means 628x208x15 cells for the directions of azimuth, radius, co-latitude, respectively, but with mesh refinement in the vicinity of the planet (we use 7 levels of mesh refinements, starting from a box centered on the planet with half-sizes \( 0.27 \times 0.27 \times 0.12 \) in code units resolved in \( 112 \times 112 \times 24 \) cells, and doubling the resolution at each level); “High resolution” means twice the number of cells in each direction at each level and still \( \alpha = 0 \). Notice that the gap becomes deeper with decreasing viscosity and increasing resolution, i.e. decreasing the effective viscosity (prescribed or numeric). The simulation has been performed with the code JUPITER written by F. Masset.
the volume density $\rho$ scales with $z$ (the distance from the mid-plane) as

$$\rho(z) = \rho(0) \exp \left( -\frac{z^2}{2H^2} \right),$$

(1)

where $\rho(0)$ is the mid-plane density and $H$ is the scale-height of the disk. Therefore, at equilibrium the radial density profile of the gap has to be the same at all altitudes. However, the planet cannot sustain the same profile at high altitude as on the mid-plane, because its gravitational potential:

$$\Phi(d, z) = \frac{G M_p}{\sqrt{d^2 + z^2}}$$

(2)

(where $G$ is the gravitational constant, $M_p$ is the planet’s mass, $d$ is the planet-fluid element distance projected on the mid-plane) weakens with increasing $z$. Please notice that $z$ plays the role here of the smoothing parameter (usually denoted $\epsilon$) used in 2D simulations, and it is well known that the depth of the gap decreases with increasing smoothing length (see Fig. 2).\(^1\)

Away from the mid-plane, the gas tends to refill the gap by viscous diffusion because it is not sufficiently repelled by the planetary torque. As soon as the gas penetrates into the gap, however, it has to fall towards the mid-plane because the relationship (1) is no longer satisfied and has to be restored. Therefore, there is always more gas near the mid-plane in the gap than it would in the ideal case where vertical motion is disabled. This excess of gas is partially accreted by the planet and partially repelled away from the planet orbit into the disk, like in the gap opening process. Outside of the gap, then, the relationship (1) is also not fulfilled, because there is an excess of gas near the mid-plane, coming from within the

\(^1\)This shows that neglecting the $z$-dependence of the potential in the numerical simulations, as done in Zhu et al. (2013), changes the dynamics of the gas at a qualitative level and can enable small-mass planets to sustain partial gaps.
Fig. 2.— Perturbations of the azimuthally averaged surface density profile in a 2D disk, after 5000 orbits in the vicinity of a Jupiter mass planet on fixed orbit. The size of the cells is $\delta r = 0.0167$ and $\delta \theta = 0.0193$. We have checked that this resolution is enough (doubling the number of cells doesn’t affect the shape of the gap. The viscosity is $\nu = 10^{-6.5}r_p^2\Omega_p$ (independent of radius). Different colors show results obtained with different values of the smoothing length $\epsilon$ of the planetary potential. This shows the crucial role of the smoothing parameter which, in a 3D disk, is played by the vertical distance from the midplane.
gap, and a deficit of gas at high altitude, which flowed into the gap. So, the gas has to move towards the surface of the disk to restore the hydrostatic vertical equilibrium profile (1).

In conclusion, there has to be a 4-step meridional circulation in the gas dynamics: (1) the gas flows into the gap at the top layer of the disk; (2) then it falls towards the disk’s mid-plane; (3) the planet keeps the gap open by accreting part of this gas and by pushing back into the disk the gas that flowed outside of the Hill sphere (4) the gas expelled from the gap near the mid-plane rises back to the disk’s surface. If no gas were permanently trapped in the vicinity of the planet (i.e. no planet growth), this meridional circulation would basically be a closed loop. Instead, planet accretion makes the flow at step (3) smaller than that at step (1).

We can observe this meridional loop in the numerical simulations. Fig.3 shows the volume density of the disk in $r, \phi$ coordinates and the arrows show the mass transport, both averaged over the azimuth. Kley et al. (2001), Ayliffe and Bate (2009), Machida et al. (2010), Tanigawa et al. (2012), Gressel et al. (2013) and Szulagyi et al. (2014) already showed that gas flows into the gap near the surface of the disk, but Fig.3 is the first clear portrait of the meridional circulation explained above.

Notice that steps 2 to 4 of the meridional circulation occur on a dynamical timescale (i.e. independent of viscosity), because they are related to pressure and planet’s gravity. Step 1, instead, occurs on a viscous timescale. If the viscosity is small, the viscous timescale controls the entire timescale of the meridional circulation. This explains why the flow into the gap and the accretion rate onto the planet scale linearly with effective viscosity (prescribed or numeric) as found in Szulagyi et al. (2014). Moreover, because step 1 and step 3 occur on independent timescales, the depth of a gap also increases with decreasing viscosity, as observed. The relationship, though, is not necessarily linear. Part of the gas falling toward the mid-plane in the gap ends in the horseshoe libration region and has to
Fig. 3.— Volume density (in $\frac{g}{cm^3}$) of the disk plotted in $r, \phi$ coordinates, integrated over the azimuth. The arrows show the mass transport $\rho \vec{v}$, also averaged over the azimuth. This simulation has been done with the 3D version of FARGO developed in Lega et al. (2013). The viscosity here is $\nu = 10^{-5} r_p^2 \Omega_p$ (independent of radius) and the resolution is 760, 1152, 96 in radius, azimuth and co-latitude respectively, with radius ranging from $r_{min} = 0.3$ to $r_{max} = 4.2$ (in units of the orbital radius of the planet) and co-latitude from 1.42 to 1.72. The Jupiter mass planet is held fixed at $r_p = 1, \phi = \pi/2$ (midplane). The planet is not allowed to accrete (no mass sink is implemented) so that, by viscosity, most of the gas that falls onto the circum-planetary disk eventually flows outside of the planet’s Hill sphere. Consequently, the meridional circulation shown in the figure has practically reached a steady state as a closed loop.
diffuse to the separatrices of said region (on a viscous timescale) before being removed. If all gas followed this fate, the inflow and the removal would both happen on the same viscous timescale and therefore the gap’s depth would not change with viscosity. If instead all the gas fell on the separatrices of the horseshoe region, only the inflow would depend on viscosity and therefore the gap’s depth would be inversely proportional to the viscosity. Reality is in between the two. In fact, in Fig. 1 the gap’s depth increases by a factor $\sim 1.5$ as the resolution is doubled (e.g. the numerical viscosity is halved).

3. Implications on planet’s accretion from the flow of gas into the gap

The results of the previous section seem to suggest that the flow of gas into the gap and the planet’s accretion rate have to vanish with viscosity. But remember that what governs the meridional circulation of gas in the gap’s vicinity is the viscous timescale near the disk’s surface. Protoplanetary disks cannot be fully inviscid: stars are observed to accrete mass, which implies that angular momentum has to be transported at least in part of the disk. A popular view is that of a layered disk, i.e. a disk which is viscous near the surface and inviscid near the mid-plane, due to ionization at high altitude and turbulence driven by the magneto rotational instability (MRI; Gammie, 1996). This view may have problems (see Turner et al. 2013 for a review), so that other mechanisms (e.g. the baroclynic instability; Klahr, 2004) may be at work. Possibly these alternative mechanism can make the entire protoplanetary disk viscous, but at the very least, a viscous layer has to exist near the surface of the disk, as in the MRI view.

We can now make a simple estimate of the gas-flow into a gap, as follows. It is well known that the radial velocity of the gas in a steady state viscous disk is:

$$v_r = \frac{1}{\Sigma j} \frac{d}{dr} [j \nu \Sigma]$$  \hspace{1cm} (3)
where $\nu$ is the viscosity, $j = \sqrt{GM_\odot r}$ is the specific angular momentum and $\Sigma$ is the surface density of the disk, which is not a constant slope any more, because the profile changed due to the opened gap. From now on we set the units so that the mass of the star $M_\odot$ and the gravitational constant $G$ are both equal to 1, so that we substitute $j$ with $\sqrt{r}$.

Assuming that $\nu = \alpha H^2 \Omega$ as in the usual $\alpha$-model (Shakura and Sunyaev, 1973), where $\Omega$ is the orbital frequency of the gas, one has: $\nu = \alpha (H/r)^2 j$. Thus, the right-hand side of eq. (3) becomes the sum of two terms. Assuming that $H/r$ is independent of $r$, we get

$$v_r = \alpha \left( \frac{H}{r} \right)^2 \left( 1 + \frac{d \ln \Sigma}{d \ln r} \right) r \Omega .$$

(4)

According to Crida et al. (2006) (see formula 14 in that paper, with input from formulæ 11 and 13) in the limit of vanishing viscosity and for a planet with Hill radius $R_h \sim H$, the surface density slope is maximum at a distance $\Delta = 2.5 R_h$ and is:

$$\frac{d(\log \Sigma)}{dr} = \frac{1.3}{r_p(H/r)} ,$$

(5)

where $r_p$ is the orbital radius of the planet. As we explained in the previous section, the same slope is present at every altitude because of the hydrostatic equilibrium relationship in (1). At high altitude, therefore, this is the slope that drives the viscous flow, given that the planet can not sustain the gap there, due to its reduced potential. From (5) the second term of the right-hand side of (4) is inversely proportional to $H/r$ and consequently it dominates over the first term. Thus, we drop the first term from the formula, getting

$$v_r \sim \alpha (H/r)r \Omega .$$

(6)

The flow in the gap is then

$$\dot{M} = 2\pi rv_r 4\Sigma_a$$

(7)
where $\Sigma_a$ is the column density of the active layer of the disk, typically 10g/cm$^2$ in the MRI case, if ionization is due to X-ray penetrating into the disk (Igea and Glassgold, 1999). Notice that factor of 4 multiplying $\Sigma_a$, due to the fact that there are two surface layers in a disk and two sides of the gap.

Let’s now take formulæ (7) and (6) and apply them to a planet with $R_h \sim H$ at 3.5 AU. Assuming $\alpha = 3 \times 10^{-3}$, a typical value for the active layer of a MRI disk (Gressel et al., 2011) and $H/r = 0.05$, formula (6) gives $v_r = 7.9 \times 10^9$ cm/y and $\dot{M} = 10^{26}$ g/y, i.e. $1.7 \times 10^{-2} M_\oplus/y$. If half of this flux is accreted by the central planet, this value matches the accretion rate of a Saturn-mass planet observed in the MRI simulations of Gressel et al. (2011).

4. Conclusions

In this paper we have analyzed the dynamics of gas near gaps, opened by planets in three dimensional proto-planetary disks, with particular emphasis on the small viscosity limit.

We observe a flow of gas into the gap at high altitude in the disk, at a rate dependent on effective viscosity (prescribed or numeric). We have explained this result with a simple analytic model, that assumes that the disk is in vertical hydrostatic equilibrium at all radii. Thus the radial profile of the gas volume density is the same at every altitude in the disk. However, the planet’s potential weakens with altitude $z$, so that the planet can not sustain the same gap on the mid-plane and at the surface of the disk. Consequently, gas flows into the gap, at a viscous rate, at high altitude. Only planets with Hill radius $R_h >> H$ can sustain the same gap at all altitudes $z < H$, but this occurs only for very massive planets (typically many Jupiter masses).
Assuming that proto-planetary disks are layered, as in the MRI description, we have derived from our model a simple formula for the amount of gas flowing into a gap opened by a planet with $R_h \sim H$, which is in good agreement with the MRI simulations of Gressel et al. (2013). The mass flux is large, corresponding to a doubling time for the mass of Jupiter of about 50,000 y.

We conclude that gap opening can not be the answer to the terminal mass problem of giant planets, described in the introduction. A different mechanism is needed in order to slow down the gas accretion of planets of Saturn to Jupiter mass. The role of the circum-planetary disk is a promising one, as shown in Rivier et al. (2012) and Szulagyi et al. (2014), although this needs to be explored further with more realistic simulations.

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