Production of $\Omega_{scb}$ Baryons in Electron-Positron Collisions

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Abstract

The total and differential cross sections for the production of $\Omega_{scb}$ baryons in electron-positron collisions are calculated at the $Z$-boson pole.

1. Introduction

Baryons involving two or three heavy quarks ($c, b$) have not yet been observed experimentally. Theoretical investigations into the mass spectra of hadrons that contain two or more heavy quarks, the cross sections for their production in various processes, and their lifetimes and decay modes form a rather new line of research in particle physics. For an overview of these investigations, the interested reader is referred to [1]. Calculations presented in the literature that are aimed at determining the production cross sections for baryons featuring two heavy quarks either rely on the fragmentation approach [2], or treat the production of unbound four-quark states in a bounded phase space [3], or consider (as is done in the majority of the most recent studies on this subject) the production of the relevant heavy diquark [4-9]. In particular, the production of doubly heavy baryons in electron-positron collisions was analyzed in [3, 9]. No attention has so far been given to the production of triply heavy baryons.

Investigation of the mechanisms responsible for the production of multiply heavy hadrons is of interest from the theoretical point of view since this provides the possibility of further verifying QCD (more precisely, of obtaining deeper insight into this theory). This involves testing both its perturbative aspect, which is used to describe the simultaneous production...
of a few quark pairs, and QCD-inspired non-perturbative models of bound states. We recall that, even in apparently obvious cases, the results of QCD calculations often unexpectedly prove to be at odds with experimental data, as was, for example, in the hadronic production of $J/\psi$ particles. On the other hand, a derivation of theoretical cross-section estimates is of importance from the point of view of applications to searches for such particles and investigation into their properties. The presence of two or more heavy quarks in a hadron substantially affects the properties of its weak decays. At the same time, the reliability of theoretical predictions is higher for such hadrons, and this makes it possible to test model concepts more thoroughly.

In the present study, we reveal some features of $\Omega_{scb}$ baryon production in electron-positron annihilation. The choice of a process where the initial state is purely leptonic was motivated, on one hand, by the fact that the relevant calculations are simpler here than in the case of hadronic production and, on the other hand, by the fact that the case of leptonic production offers a number of advantages in what is concerned with a possible experimental observation, which include favorable background conditions and a precisely known initial energy.

At the quark level, the subprocess $e^-e^+ \rightarrow \bar{s}\bar{c}\bar{c}\bar{b}b$ which is of order $\alpha^2\alpha_s^4$ in conventional perturbation theory, is associated with the process being considered. In evaluating the square of the relevant matrix element, use is made of the method that was proposed in [10] and which is referred to as the method of orthogonal amplitudes (previously, this method was employed, for example, in [7, 8]). The fusion of the product $s$, $c$, and $b$ quarks into a $\Omega_{scb}$ baryon is described within the standard nonrelativistic approximation [11-13]. A detailed account of the technical facet of our calculations is given in Section 2. The numerical results obtained on the basis of these calculations are discussed in Section 3.

2. Computational procedure

Our calculations are based on considering the partonic process

$$e^-(p_{e^-}) + e^+(p_{e^+}) \rightarrow s(p_1, \xi) + c(p_2, \zeta) + b(p_3, \chi) + \bar{s}(p_4, \xi') + \bar{c}(p_5, \zeta') + \bar{b}(p_6, \chi'),$$

where the parentheses referring to the colliding particles enclose their 4-momenta, while the parentheses referring to the product quarks and antiquarks enclose their 4-momenta and color indices. As usual, we disregard Feynman diagrams featuring the electroweak interaction of the quarks involved. In other words, we consider only those diagrams where the interaction between the quarks is mediated by gluons. For this class of diagrams, Fig. 1 shows nine
basic diagrams in which quark and gluon lines are connected in different ways. Here, six (three) nonequivalent dispositions of the \( ss \), \( cc \), and \( bb \) lines correspond to each of the first seven (last two) diagrams in Fig. 1 - that is, diagrams 1-7 (diagrams 8 and 9). Considering that the annihilation channel may involve either a photon or a \( Z \) boson, we conclude that the total number of diagrams in question is 96.

The matrix element for the process in (1) can be represented in the form

\[
\mathcal{M} = \frac{g^4 g^2}{24 \cos^2 \theta_w (s - M_Z^2 + i M_Z \Gamma_Z)} \varepsilon^{\xi' \xi' \xi' \xi} A_{\xi' \xi' \xi' \xi}^Z - \frac{g^4 e^2}{6 s} \varepsilon^{\xi' \xi' \xi' \xi} A_{\xi' \xi' \xi' \xi}^Z, \tag{2}
\]

where

\[
A_{\xi' \xi' \xi' \xi}^Z = B_{\xi' \xi' \xi' \xi}^{\xi' \xi' \xi' \xi}, \tag{3}
\]

\[
B_{\xi' \xi' \xi' \xi}^{\xi' \xi' \xi' \xi} = \left. \left\{ \left( p_2 + p_1 + p_4 \right)^2 - m_b^2 \right\}^{-1} \left[ (p_{e+} + p_{e-} - p_6)^2 - m_b^2 \right]^{-1} (p_1 + p_4)^{-2} \times \right.
\]

\[
\left. \times (p_1 + p_2 + p_4 + p_6)^{-2} \bar{u}(p_2) T_{\xi' \xi' \xi' \xi}^{\xi' \xi' \xi' \xi} \gamma' \left( \hat{p}_2 + \hat{p}_1 + \hat{p}_4 + m_b \right) \gamma_5 v(-p_5) \times \right.
\]

\[
\left. \times \bar{u}(p_3) T_{\xi' \xi' \xi' \xi}^{\xi' \xi' \xi' \xi} \left( \hat{p}_2 + \hat{p}_1 + \hat{p}_4 + m_b \right) \gamma_5 v(-p_6) + \right.
\]

\[
\left. \left[ (p_5 + p_1 + p_6)^2 - m_b^2 \right]^{-1} \left[ (p_{e+} + p_{e-} - p_3)^2 - m_b^2 \right]^{-1} (p_1 + p_4)^{-2} \times \right.
\]

\[
\left. \times (p_1 + p_2 + p_4 + p_6)^{-2} \bar{u}(p_2) T_{\xi' \xi' \xi' \xi}^{\xi' \xi' \xi' \xi} \gamma_5 \left( \hat{p}_2 + \hat{p}_1 + \hat{p}_4 + m_e \right) \gamma_5 v(-p_5) \times \right.
\]

\[
\left. \times \bar{u}(p_3) T_{\xi' \xi' \xi' \xi}^{\xi' \xi' \xi' \xi} \left( \hat{p}_2 + \hat{p}_1 + \hat{p}_4 + m_e \right) \gamma_5 v(-p_6) + \right.
\]

\[
\left. \left[ (p_5 + p_1 + p_6)^2 - m_b^2 \right]^{-1} \left[ (p_{e+} + p_{e-} - p_3)^2 - m_b^2 \right]^{-1} (p_1 + p_4)^{-2} \times \right.
\]

\[
\left. \times (p_1 + p_2 + p_4 + p_6)^{-2} \bar{u}(p_2) T_{\xi' \xi' \xi' \xi}^{\xi' \xi' \xi' \xi} \gamma_5 \left( \hat{p}_2 + \hat{p}_1 + \hat{p}_4 + m_e \right) \gamma_5 v(-p_5) \times \right.
\]

\[
\left. \times \bar{u}(p_3) T_{\xi' \xi' \xi' \xi}^{\xi' \xi' \xi' \xi} \left( \hat{p}_2 + \hat{p}_1 + \hat{p}_4 + m_e \right) \gamma_5 v(-p_6) + \right.
\]

\[
\left. \left[ (p_2 + p_5 + p_3)^2 - m_b^2 \right]^{-1} \left[ (p_{e+} + p_{e-} - p_6)^2 - m_b^2 \right]^{-1} (p_1 + p_4)^{-2} \times \right.
\]

\[
\left. \times (p_2 + p_5)^{-2} \bar{u}(p_2) T_{\xi' \xi' \xi' \xi}^{\xi' \xi' \xi' \xi} \gamma_5 \left( \hat{p}_2 + \hat{p}_5 + \hat{p}_3 + m_b \right) \times \right.
\]

\[
\left. \gamma_5 \left( \hat{p}_2 + \hat{p}_5 + \hat{p}_3 + m_b \right) \gamma_5 v(-p_6) + \right.
\]

\[
\left. \left[ (p_2 + p_5 + p_3)^2 - m_b^2 \right]^{-1} \left[ (p_{e+} + p_{e-} - p_6)^2 - m_b^2 \right]^{-1} (p_1 + p_4)^{-2} \times \right.
\]

\[
\left. \times (p_2 + p_5)^{-2} \bar{u}(p_2) T_{\xi' \xi' \xi' \xi}^{\xi' \xi' \xi' \xi} \gamma_5 \left( \hat{p}_2 + \hat{p}_5 + \hat{p}_3 + m_b \right) \times \right.
\]

\[
\left. \gamma_5 \left( \hat{p}_2 + \hat{p}_5 + \hat{p}_3 + m_b \right) \gamma_5 v(-p_6) + \right.
\]

\[
\left. \left[ (p_1 + p_4 + p_6)^2 - m_b^2 \right]^{-1} \left[ (p_{e+} + p_{e-} - p_3)^2 - m_b^2 \right]^{-1} (p_1 + p_4)^{-2} \times \right.
\]

\[
\left. \times (p_2 + p_5)^{-2} \bar{u}(p_2) T_{\xi' \xi' \xi' \xi}^{\xi' \xi' \xi' \xi} \gamma_5 \left( \hat{p}_2 + \hat{p}_5 + \hat{p}_3 + m_b \right) \times \right.
\]

\[
\left. \gamma_5 \left( \hat{p}_2 + \hat{p}_5 + \hat{p}_3 + m_b \right) \gamma_5 v(-p_6) + \right.
\]
\[ \times \gamma^\delta(-\hat{p}_1 - \hat{p}_4 - \hat{p}_6 + m_b)\gamma^\nu(g^b_V - g^b_A\gamma_5)v(-\mathbf{P}_6) - (i/2)[(p_e + p_c - p_6)^2 - m^2_{\bar{c}}]^{-1}(p_1 + p_4)^{-2}(p_2 + p_5)^{-2} \times \]
\[ \times (p_1 + p_2 + p_4 + p_5)^{-2} f^{abc}[(\mathbf{P}_1 - \mathbf{P}_4 + \mathbf{P}_2 + \mathbf{P}_5)g^{\mu\nu} + \]
\[ + (\mathbf{P}_1 - \mathbf{P}_4 + \mathbf{P}_2 + \mathbf{P}_5)g^{\mu\nu} + (2p_4^\mu + 2p_4^\nu)g^{\nu\delta}]\bar{u}(\mathbf{P}_2)T^b_{\xi'\xi\gamma\mu}v(-\mathbf{P}_5) \times \]
\[ \times \bar{u}(\mathbf{P}_3)T^{d'\gamma}_{\chi'\chi\nu}(\hat{p}_{e+} + \hat{p}_{e-} - \hat{p}_3 + m_b)\gamma_\epsilon(g^b_V - g^b_A\gamma_5)v(-\mathbf{P}_6) - (i/2)[(p_e + p_c - p_3)^2 - m^2_{\bar{c}}]^{-1}(p_1 + p_4)^{-2}(p_2 + p_5)^{-2} \times \]
\[ \times (p_1 + p_2 + p_4 + p_5)^{-2} f^{abc}[(\mathbf{P}_1 - \mathbf{P}_4 + \mathbf{P}_2 + \mathbf{P}_5)g^{\mu\nu} + \]
\[ + (\mathbf{P}_1 - \mathbf{P}_4 + \mathbf{P}_2 + \mathbf{P}_5)g^{\mu\nu} + (2p_4^\mu + 2p_4^\nu)g^{\nu\delta}]\bar{u}(\mathbf{P}_2)T^b_{\xi'\xi\gamma\mu}v(-\mathbf{P}_5) \times \]
\[ \times \bar{u}(\mathbf{P}_3)T^{d'\gamma}_{\chi'\chi\nu}(\hat{p}_{e+} + \hat{p}_{e-} - \hat{p}_3 + m_b)\gamma_\epsilon(g^b_V - g^b_A\gamma_5)v(-\mathbf{P}_6)\}	imes \]
\[ \times \bar{u}(\mathbf{P}_1)T^a_{\xi\gamma\mu}v(-\mathbf{P}_4)v(-\mathbf{P}_e+)^\gamma\gamma^e(g^e_V - g^e_A\gamma_5)u(\mathbf{P}_e-), \]

and where the expression for \( A_{\xi\xi'\xi'\chi'\chi''}^2 \) is obtained from the expression \( A_{\xi\xi'\xi'\chi'\chi''}^2 \) by setting in it \( g^V_V = 1, g^V_A = 0, g^V_t = Q_q, \) and \( g^V_A = 0, \) with \( Q_q \) being the electric charge of the quark \( q (q = s, c, b) \) in units of the electron charge \( e. \) Since the number of nonequivalent Feynman diagrams belonging to type 8 or 9 (Fig. 1) and differing from one another only by a permutation of the \( s\bar{s}, c\bar{c}, \) and \( b\bar{b} \) lines is equal to three and since the quantity \( A_{\xi\xi'\xi'\chi'\chi''}^2 \) from Eq. (3) involves six terms that are obtained from one another by permuting the quantum numbers of \( s\bar{s}, c\bar{c}, \) and \( b\bar{b} \) quark-antiquark pairs, we introduce, in the eighth and the ninth term in expression (4), an additional (in relation to the Feynman formulation) factor of 1/2, whereupon we arrive at the correct results for this quantity and for the matrix element in (2).

Let us consider in more detail the color structure of the matrix element given by Eqs. (2)-(4). Since any baryon is a color-singlet object, the \( scb \) state that is produced in process (1) must be an \( SU(3)_c \) singlet that is contained in the tensor product of three \( SU(3)_c \) triplets. It follows that the \( scb \) state must be fully antisymmetric in the color indices of the quarks; to take this into account, it is necessary to introduce, in the amplitude of the process, the antisymmetric tensor \( \varepsilon^{\xi\xi'\chi'}/\sqrt{6}, \) which is normalized to unity. Since the initial electron-positron state is also a color singlet, the state of three unbound \( \bar{s}, \bar{c} \) and \( \bar{b} \) antiquarks accompanying the product baryon must also be a singlet. We note in passing that the tensor \( \varepsilon^{\xi\xi'T^{a}_{\xi\xi'}T^{b}_{\xi'\chi'}T^{c}_{\chi\chi'}} \) is already fully antisymmetric in its indices \( \xi', \xi' \) and \( \chi' \); therefore, it not necessary is \( \varepsilon^{\xi\xi'\chi'}/\sqrt{6} \) that the projection operator be explicitly present there. The presence of this operator is technically useful, however, since this makes it possible to perform summation over color indices at the amplitude level—that is, prior to squaring the amplitude.
Further, it is straightforward to prove the identity\(^3\)

\[
\varepsilon^{\xi\xi'}\varepsilon^{\zeta\zeta'} f^{abcd} T_a^{\xi \xi'} T_b^{\zeta \zeta'} T_c^{\chi \chi'} = 0,
\]

from which it follows that the contributions of diagrams involving a three-gluon vertex [eighth and ninth term in Eq. (4)] vanish. At the same time, all of the remaining terms in Eqs. (2)-(4) have the same color structure. Summation over color indices yields

\[
\frac{1}{6} \varepsilon^{\xi\xi'}\varepsilon^{\zeta\zeta'} T_a^{\xi \xi'} T_b^{\zeta \zeta'} T_c^{\chi \chi'} = \frac{4}{9}
\]

In performing summation over fermion polarizations (with the aid of the REDUCE system [14] for analytic calculations), we employed the method of orthogonal amplitudes. Briefly, the essence of the method is as follows. Suppose that we have the quantity \(\bar{u}(p') R u(p'')\), where \(u(p')\) and \(u(p'')\) are spinors that obey the Dirac equation, while \(R\) is an operator that is expressed in terms of the matrices and their contractions with 4-vectors. In general, this quantity then admits a linear decomposition in terms of four orthogonal amplitudes

\[
w_1 = \bar{u}(p') u(p''), \quad w_2 = \bar{u}(p') \hat{K} u(p''), \quad w_3 = \bar{u}(p') \hat{Q} u(p''), \quad w_4 = \bar{u}(p') \hat{K} \hat{Q} u(p'').
\]

That two different amplitudes are orthogonal implies the vanishing of the quantity obtained by summing, over the polarizations of the two spinors, the product of one of these amplitudes and the complex conjugate of the other. This is so if the 4-vectors \(K^\mu\) and \(Q^\mu\) are orthogonal to the 4-momenta and \(p'^\mu\) and \(p''^\mu\) to each other - that is \(K_\mu p'^\mu = 0, K_\mu p''^\mu = 0, Q_\mu p'^\mu = 0, Q_\mu p''^\mu = 0,\) and \(K_\mu Q^\mu = 0;\) otherwise, the 4-vectors \(K^\mu\) and \(Q^\mu\) are arbitrary.

Let us apply the method of orthogonal amplitudes to the specific problem at hand. For this purpose, we introduce the quantities

\[
w_{s1} = \bar{u}(p_1) v(-p_4), \quad w_{s2} = \bar{u}(p_1) \hat{K}_s v(-p_4),
\]

\[
w_{s3} = \bar{u}(p_1) \hat{Q}_s v(-p_4), \quad w_{s4} = \bar{u}(p_1) \hat{K}_s \hat{Q}_s v(-p_4),
\]

\[
w_{c1} = \bar{u}(p_2) v(-p_5), \quad w_{c2} = \bar{u}(p_2) \hat{K}_c v(-p_5),
\]

\[
w_{c3} = \bar{u}(p_2) \hat{Q}_c v(-p_5), \quad w_{c4} = \bar{u}(p_2) \hat{K}_c \hat{Q}_c v(-p_5),
\]

\(^3\)Of three indices \(a, b \text{ and } d\) corresponding to nonzero values of the structure constant \(f^{abcd}\) two are always the numbers of Gell-Mann matrices such that they undergo no changes upon transposition, while the remaining index is associated with a Gell-Mann matrix that changes sign upon transposition. By simultaneously replacing the primed indices by unprimed ones and transposing the matrices \(T^a, T^b\) and \(T^d\), we arrive at

\[
\varepsilon^{\xi\xi'}\varepsilon^{\zeta\zeta'} T_a^{\xi \xi'} T_b^{\zeta \zeta'} T_c^{\chi \chi'} = -\varepsilon^{\xi\xi'}\varepsilon^{\zeta\zeta'} T_a^{\xi \xi'} T_b^{\zeta \zeta'} T_c^{\chi \chi'} = -\varepsilon^{\xi\xi'}\varepsilon^{\zeta\zeta'} T_a^{\xi \xi'} T_b^{\zeta \zeta'} T_c^{\chi \chi'},
\]

whence we immediately obtain relation (5).
\[ w_{b1} = \bar{u}(p_3)v(-p_6), \quad w_{b2} = \bar{u}(p_3)\hat{K}_b v(-p_6), \]
\[ w_{b3} = \bar{u}(p_3)\hat{Q}_b v(-p_6), \quad w_{b4} = \bar{u}(p_3)\hat{K}_b \hat{Q}_b v(-p_6), \]
\[ w_{e1} = \bar{v}(-p_{e+})\hat{K}_e u(p_{e-}), \quad w_{e2} = \bar{v}(-p_{e+})\hat{Q}_e u(p_{e-}), \]

where
\[
\begin{align*}
K_s^\mu &= \varepsilon^{\mu\nu\rho\sigma}p_{1\nu}p_{4\rho}a_{s\sigma}, \\
Q_s^\mu &= \varepsilon^{\mu\nu\rho\sigma}p_{1\nu}p_{4\rho}K_{s\sigma}, \\
K_c^\mu &= \varepsilon^{\mu\nu\rho\sigma}p_{2\nu}p_{5\rho}a_{c\sigma}, \\
Q_c^\mu &= \varepsilon^{\mu\nu\rho\sigma}p_{2\nu}p_{5\rho}K_{c\sigma}, \\
K_b^\mu &= \varepsilon^{\mu\nu\rho\sigma}p_{3\nu}p_{6\rho}a_{b\sigma}, \\
Q_b^\mu &= \varepsilon^{\mu\nu\rho\sigma}p_{3\nu}p_{6\rho}K_{b\sigma}, \\
K_e^\mu &= \varepsilon^{\mu\nu\rho\sigma}p_{e+\nu}p_{e-\rho}a_{e\sigma}, \\
Q_e^\mu &= \varepsilon^{\mu\nu\rho\sigma}p_{e+\nu}p_{e-\rho}K_{e\sigma},
\end{align*}
\]

the 4-vectors \(a_{s\sigma}, a_{c\sigma}, a_{b\sigma},\) and \(a_{e\sigma}\) being arbitrary.

Our problem is then described in terms of 128 orthogonal amplitudes of the form
\[ w_{ijkl} = w_{s_i}w_{c_j}w_{b_k}w_{e_l}, \quad i, j, k = 1, 2, 3, 4, \quad l = 1, 2. \] (9)

In order to find the coefficients \(c_{ijkl}\) in the expansion of the matrix element (2) in the amplitudes specified by Eq. (9),
\[ M = \sum_{i,j,k} w_{ijkl}c_{ijkl}, \] (10)

we multiply both sides of this equality by the quantity \(w_{ijkl}^*\), sum the result over the polarizations of all fermions, and make use of the orthogonality of the different amplitudes \(w_{ijkl}\). Denoting by \(|w_{ijkl}|^2\) the quantity obtained by summing, over the polarizations of all fermions, the squared modulus of the amplitude \(w_{ijkl}\), we have
\[ c_{ijkl} = \left\{ \sum_{\text{polar.}} Mw_{ijkl}^* \right\}/|w_{ijkl}|^2. \] (11)

For the squared modulus of the relevant matrix element, summation over the polarizations of product particles and averaging over the polarizations of colliding particles is performed by the formula
\[ |M|^2 = \frac{1}{4} \sum_{i,j,k} |c_{ijkl}|^2|w_{ijkl}|^2. \] (12)

We note that we did not include the quantities \(\bar{v}(-p_{e+})u(p_{e-})\) and \(\bar{v}(-p_{e+})\hat{K}_e \hat{Q}_e u(p_{e-})\) in the list of basic orthogonal amplitudes in (7), since the corresponding coefficients in the expansion of the matrix element [Eqs. (2)-(4)] vanish (this is because the traces that arise
in performing summation over the polarizations of massless electrons and positrons involve an odd number of Dirac $\gamma$ matrices).

The question of why it is profitable to employ the method of orthogonal amplitudes is in order here. Upon directly squaring the matrix element specified by Eqs. (2)-(4), we would obtain, with allowance for the equality in (5), 3570 terms, and an individual operation of summation over particle polarizations would correspond to each of these terms. Within the method of orthogonal amplitudes, we compose one REDUCE code for evaluating traces and tensor contractions that corresponds to 84 terms in the quantity $\mathcal{M} w_{1111}^*$, whereupon we apply text editors (for example, joe or gedit) to perform obvious substitutions in this code, thereby obtaining REDUCE codes for evaluating all 128 coefficients $c_{ijkl}$. We note that analytic expressions for 128 coefficients $c_{ijkl}$ occupy 370 Mb.

3. Numerical results

In order to describe a bound state of heavy quarks, we make use of the nonrelativistic approximation [11-13], according to which the relative velocities of the quarks in a heavy hadron are assumed to be low. In the case of S-wave states, these velocities can be set to zero. Accordingly, the velocities of all three quarks in the final state of process (1) are taken to be identical, while the momenta of the quarks are assumed to be proportional to their masses; that is,

$$p_1 = (m_s/M) p, \quad p_2 = (m_c/M) p, \quad p_3 = (m_b/M) p, \quad M = m_s + m_c + m_b.$$  \hspace{1cm} (13)

Concurrently, the six-particle phase space of the final state of process (1) reduces to the four-particle phase space of the process

$$e^- (p_{e^-}) + e^+ (p_{e^+}) \to \Omega_{scb}(p) + \bar{s}(p_4) + \bar{c}(p_5) + \bar{b}(p_6),$$  \hspace{1cm} (14)

while the probability of bound-state formation is controlled by the value of the baryon wave function at the origin of coordinates, the only model parameter in this approach. Eventually, the differential cross section for process (14) assumes the form

$$d\sigma = \frac{(2\pi)^4 |\mathcal{M}|^2}{2s} \frac{|\psi(0)|^2}{M^2} \delta^4(p_{e^-} + p_{e^+} - p_4 - p_5 - p_6 - p) \times \frac{d^3p_4}{(2\pi)^3 2E_4} \frac{d^3p_5}{(2\pi)^3 2E_5} \frac{d^3p_6}{(2\pi)^3 2E_6} \frac{d^3p}{(2\pi)^3 2E}.$$  \hspace{1cm} (15)

In evaluating the cross sections in question, we employed codes for integration that enter as ingredients into the CompHEP package [15]. As a necessary test, we first of all made sure
that numerical values of the cross sections are identical for different choices of the 4-vectors $a_{s\sigma}, a_{c\sigma}, a_{b\sigma},$ and $a_{e\sigma}$, which are involved in the construction of the basic amplitudes. Having proven this, we prescribed ten iterations for the cross sections, each involving 100 000 steps of a Monte Carlo sampling of the integrand. The error in evaluating the total cross sections was 2.0-2.5 %, while the error in the differential cross sections was about 10 %, on average.

Among theoretical uncertainties that affect cross-section values, the choice of renormalization scale in the running coupling constant for strong interaction, the values of the baryon wave function at the origin, and numerical values of the quark masses are of greatest importance. In what is concerned with the quark masses, the results of the calculations are the most sensitive to the choice of value for the lightest quark (strange one), because, for some gluon propagators, the minimal values of the denominators are $4m_s^2$. To illustrate this dependence, we everywhere present the results obtained at two values of the strange-quark mass, $m_s = 300$ and 500 MeV. The remaining parameters were set to the following values: $m_c = 1500$ MeV, $m_b = 4800$ MeV, $\alpha = \alpha(M_Z) = 1/128.0$, $\alpha_s = \alpha_s(M_Z/2) = 0.134$, and $\sin^2\theta_W = \sin^2\theta_W(M_Z) = 0.2240$; the numerical value of the wave function for the spin-3/2 $\Omega_{scb}$ baryon at zero relative coordinates of its quarks was borrowed from [16]:

$$|\psi(0)|^2 = 0.90 \cdot 10^{-3} \text{ GeV}^6. \quad (16)$$

We note that the change in the characteristic energy scale $\alpha_s(\mu)$ in the running coupling constant $\alpha_s(\mu)$ from $\mu = M_Z/2$ to $\mu = M_Z$ leads to a change in the calculated cross sections by a common factor of $[\alpha_s(M_Z)/\alpha_s(M_Z/2)]^4 = 0.665$.

For electron-positron collisions at $\sqrt{s} = 91.2$ GeV the table presents the values of the total cross sections $\sigma_{\text{tot}}$ and the forward-backward asymmetry at the $Z$-boson pole. This asymmetry is defined as

$$A_{FB} = (\sigma_F - \sigma_B)/(\sigma_F + \sigma_B), \quad (17)$$

where $\sigma_F(\sigma_B)$ is the cross section for the production of $\Omega_{scb}$ baryons traveling in the forward (backward) direction with respect to the direction of the electron momentum.

**Table 1.** Features of $\Omega_{scb}$ baryon production in electron-positron collisions at the $Z$-boson pole

| $m_s$, MeV | $\sigma_{\text{tot}}$, fb | $A_{FB}$ |
|------------|----------------|--------|
| 300        | 0.0534±0.0014  | 0.162±0.024 |
| 500        | 0.0153±0.0004  | 0.158±0.016 |

Figure 2 displays the transverse-momentum ($p_T$) and rapidity ($Y$) distributions of $\Omega_{scb}$ baryons at the strange-quark-mass values of $m_s = 300$ and 500 MeV. For both values of the mass $m_s$, the differential cross sections $d\sigma/dp_T$ peak at $p_T$ values approximately equal
to one-fourth of the total energy of colliding particles, while the quantities \( d\sigma/dY \) peak at small positive values of the rapidity \( Y \).

By using the concept of a fragmentation function, we can represent our numerical results in a simpler analytic form that is convenient for phenomenological applications. It is natural to break down the entire set of diagrams considered here into three groups that correspond to the fragmentation of \( b \), \( c \), and \( s \) quarks (in accordance with the flavor of quarks that are produced at the \( \gamma/Z \) vertex). Here, the fragmentation of \( b \) quarks plays a dominant role, whence it follows that, to a high precision, we can approximate the differential cross section as (see, for example, [4] and the discussion on the treatment of experimental data on electron-positron annihilation in [17, 18])

\[
d\sigma/dz = \frac{\sigma_{bb} \cdot D_{b \to \Omega_{scb}}(z)}{D_{b \to \Omega_{scb}}(z)},
\]

(18)

where \( \sigma_{bb} \) is the total cross section for the process \( e^-e^+ \to b\bar{b} \), \( D_{b \to \Omega_{scb}}(z) \) is the function that describes the fragmentation of a \( b \) quark into an \( \Omega_{scb} \) baryon, and the variable \( z \) is expressed in terms of the energy \( E \) of the final hadron and its longitudinal momentum \( p_\parallel \) as

\[
z = \frac{(E + p_\parallel)}{(E + p_\parallel)_{\text{max}}}.
\]

(19)

For reasons of practical convenience, the variable \( z \) is often replaced by the variable \( x_p = p/p_{\text{max}} \) [17-20], which is close to it, or by \( x_E = E/E_{\text{max}} \) [21]. The distinction between these definitions vanishes in the limit of ultrahigh energies, but it can be sizable under actual conditions.

Experimental results obtained for electron-positron annihilation are usually contrasted against the Peterson fragmentation function [22]

\[
D(z) \sim \frac{1}{z[1 - (1/z) - \varepsilon/(1 - z)]^2},
\]

(20)

where \( \varepsilon \) is a phenomenological parameter.

If one disregards the aforementioned small asymmetry in the angular distribution of \( \Omega_{scb} \) baryons and sets \( z \approx x_E \), the relation between the differential distribution of the cross section with respect to the transverse momentum of the product baryon and the fragmentation function assumes the form

\[
\frac{d\sigma}{dp_T} = \frac{4\sigma_{bb}p_T}{s} \int_{2\sqrt{(p_T^2 + M^2)/s}}^{1} \frac{D_{b \to \Omega_{scb}}(z)dz}{\sqrt{[z^2 - 4M^2/s][z^2 - 4(M^2 + p_T^2)/s]}},
\]

(21)

But if the variable \( x_p \) is used instead of \( z \), it is necessary to set \( M = 0 \) in relation (21).
The conclusions drawn from a comparison of relation (21) with our numerical results are as follows: if \( z \simeq x_E \), the parameter values of \( \varepsilon = 0.098 \pm 0.012 \) and \( 0.132 \pm 0.018 \) correspond to the strange-quark masses of \( m_s = 300 \) and 500 MeV, respectively; if \( z \simeq x_p \), the corresponding parameter values are \( \varepsilon = 0.108 \pm 0.016 \) and \( 0.147 \pm 0.022 \).

For the sake of comparison, we present values of the parameter \( \varepsilon \) in the Peterson fragmentation function (20) that were obtained in experiments where electron-positron annihilation was explored at \( \sqrt{s} = 10 \) GeV: \( \varepsilon = 0.23^{+0.09}_{-0.06} \) for \( c \)-quark fragmentation into \( \Lambda_c \) [17]; \( \varepsilon = 0.29 \pm 0.06 \) for \( c \)-quark fragmentation into \( \Sigma_c \) [19]; and \( \varepsilon = 0.24 \pm 0.08 \) and \( \varepsilon = 0.23^{+0.09}_{-0.06} \) for \( c \)-quark fragmentation into \( \Xi_c \) according to the results obtained in [18] and [20], respectively.

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Fig. 1. Basic Feynman diagrams for the process $e^+ + e^- \rightarrow s + c + b + \bar{s} + \bar{c} + \bar{b}$. 
Fig. 2. Differential distributions of the cross section for Ω_{scb} baryon production in electron-positron collisions at the Z-boson pole with respect to the transverse momentum $p_T$ (top left, bottom left) and the rapidity $Y$ (top right, bottom right) at the strange-quark-mass values of $m_s = 300$ MeV (top left, top right), and $m_s = 500$ MeV (bottom left, bottom right). Points represent the results of our Monte-Carlo calculations. The solid curves correspond to the calculations by formula (21) with the Peterson fragmentation function whose parameter takes the value of $\varepsilon = 0.098$ for $m_s = 300$ MeV and the value of $\varepsilon = 0.132$ for $m_s = 500$ MeV.