Comment on “Nonadiabatic Conditional Geometric Phase Shift with NMR”

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Realizing quantum computation by means of geometric origin (adiabatic cyclic Berry phase) is now receiving considerable attention due to its intrinsic tolerance to noise. In a recent letter, Wang and Keiji [1] explored the nonadiabatic implementation of the geometrical quantum phase shift in a nuclear magnetic resonance (NMR) system that previously proposed in Ref. [2,3]. However, it should be further clarified that a parallel extension of the adiabatic scenario, i.e., the multi-loop operation sequence of Eq. (11) in Ref. [1], cannot realize such a goal, even if the resonant case is concerned [4].

Differing from the adiabatic scheme, to keep up the eigenstate of $H_0$ with the speedily rotating fields, two opposite vertical fields are needed as performing the twice opposite cyclic evolutions, i.e., the $(\zeta^+)$ and $(\zeta^-)$ in the notation of Ref. [1], respectively. It is not difficult to show that, for a fixed recurrent initial state, these two cyclic evolutions induce the same total phase (the Lewis-Riesenfeld phase [4]): $\phi_C = \phi_{\bar{C}}$. Consequently, one can verify that the four-loop operation sequence, Eq. (11) of Ref. [1], provides nothing but an exactly identical total phase for the four computational bases $\{|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle\}$. It is to say that such an operation sequence, though appearing to be a natural extension of the adiabatic version, in fact is of no use for geometrical quantum computation.

The origin causing the failure of such an attempt can be revealed in a more profound manner. The key idea to obtain conditional geometric phases in the previous propositions in an NMR system [2,3] uses the fact that the cyclic adiabatic evolutions in opposite directions induce the same dynamical phase and negative geometric phases; so that in the whole procedure the dynamical phases accumulated for different bases are identical which can thus be eliminated as a global phase and the only retained geometric phases are different for the four bases which implies the conditional geometric shift. Now consider the nonadiabatic extension of such a version. The characterized time-dependent Hamiltonian system, i.e., the spin-half nucleus in a rotating magnetic field, has an invariant, $I(t)$:

$$\frac{dI(t)}{dt} = \frac{\partial I(t)}{\partial t} - i[I(t), H(t)] = 0,$$

and the eigenvectors of $I(t)$ form the recurrent solutions of the system. In detail, $I(t)$ can be calculated conveniently from the algebraic dynamical method [1]. For the case of a constantly rotating magnetic field, it is given by

$$I(t) = H(t) - \frac{\omega}{2} n \cdot \sigma,$$

where $n$ stands for the rotation direction and $\omega$ the rotation magnitude. According to algebraic dynamics [1,2,3], the invariant $I(t)$ is related to the total phase of the wave function and the fraction of it [i.e., the second term of Eq. (3)] indicates a gauge potential which is related to the nonadiabatic Berry phase. Note that the two opposite cyclic Hamiltonians that satisfy $H_C(t) = H_{\bar{C}}(T - t)$ induce different instantaneous gauge potentials and thus lead to different invariants: $I_C(t) \neq I_{\bar{C}}(T - t)$. In comparison with the adiabatic situation, the deviation caused here is twofold: the recurrent solutions of the two opposite nonadiabatic evolutions become different, and as a result, the dynamical phases, defined in terms of the expected value of Hamiltonian over the eigenstate of the invariant $I(t)$, induced by the two opposite processes are also different in general. In a sense, the latter deviation is fatal for geometrical realization of quantum computation. The four-loop operation sequence proposed by Wang and Keiji does arrive at the same recurrent instantaneous states for the evolutions $(\zeta^+)$ and $(\zeta^-)$, since the scheme uses two different Hamiltonians that correspond to an identical invariant (hence they induce the same total phase as was pointed out in the previous paragraph). However, the dynamical phases induced by the two evolutions are different and thus they cannot be removed via the proposed scheme [4].

Note added: This work was finished in January 2002 and submitted to PRL. Very recently, we noticed that the same misunderstanding occurred in the paper by Zhu etc. [Phys. Rev. Lett. 89, 097902 (2002)]. That is, the authors neglected the fact that the recurrent solutions of the opposite nonadiabatic evolutions are distinctly different, and the dynamical phases induced accordingly are also different and cannot be removed via the current schemes. The assertion that their scheme can implement nonadiabatic geometrical quantum computation thus is invalid.
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[4] X.-B. Wang and M. Keiji, e-print quant-ph/0101038. In this amended manuscript, the authors believed that the resonant case of the the multi-loop scheme can realize the geometric quantum phase shift. However, we will show that it is not the case.
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[8] For a resonant case, the recurrent states are in the vertical of the rotating axis and the dynamical phases induced by the two opposite cyclic evolutions happen to be identical. However, the geometric phase induced now becomes exactly zero in view that the gauge potential is antiparallel to the rotating axis. One can also verify that this is a natural consequence in accord with the result described in the second paragraph.