Meliorated Weighted Least Square Method with Variable Parameters for Digital Filter Design

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Abstract. In published weighted least square method, the weighting matrix has no connection with the targeted filter, which limits the application of this method. To overcome this drawback, this article presents a novel meliorated weighted least square method with variable parameters for digital filter design. In the new method, the targeted filter is used to form the weighting matrix with two parameters. Through the parameters, the attenuation of stop band of the filter is able to be controlled. Simulations of the new method show that the new method is able to effectively create high performance digital filter.

1. Introduction

Least square method has been deeply investigated for several decades. Multiple variants of least square method have been developed. Weighted least square method and projection approach are the main representatives of these variants. All these least square related methods have been widely applied in many fields. A novel least square method was proposed in [1]. Though the new least square method presented in [1] was able to adjust the attenuation of the designed filter by varying a parameter, the meaning of the parameter was not provided. And in [1], the procedure obtaining the new method had no connection with the weighted least square method. In [2], projection approach was used to create a multi-beamformer. Based on weighted least square method, an eigenvalue decomposition approach for antenna array pattern synthesis was presented in [3]. In [4], weighted least square method was applied to array pattern synthesis. In [5], alternating adaptive projection approach was applied to array synthesis. Weighted alternating adaptive projection approach was applied to array synthesis in [6]. In [7], weighted least square method was applied to addressing electrohydrodynamic problems.

Digital filter has been broadly used in many domains. In line with the specifically applied situation, many novel digital filter design approaches have been presented. In [8], linearly constrained pole optimization was applied to designing recursive parallel form digital filters. In [9], high speed all pass transformation based variable digital filters were designed and implemented. In [10], an adjustable weighting vector of weighted least square approach was used to improve the attenuation of the designed digital filter.

The least square method’s advantage is that it is easy to use. Hence, a lot of published articles were dedicated to develop new meliorated least square approach to further improve its performance. In all aforementioned methods [2-10] related to weighted least square method, the weighting matrix has no connection with the targeted filter, which limits the application of this method. To overcome this drawback, based on [1], this article presents a novel meliorated weighted least square method with...
variable parameters for digital filter design. In the new method, the targeted filter is used to form the weighting matrix with two parameters. Through the parameters, the attenuation of stop band of the filter is able to be controlled. Simulations of the new method show that the new method is able to effectively create high performance digital filter.

The rest of this article is organized as follows: the new meliorated weighted least square approach is presented in Section 2; some examples are taken to simulate the new method in Section 3; a conclusion is drawn in Section 4.

2. The New Method
Let \( N \) be an odd integer, the amplitude-frequency response \( H_\delta(\omega) \) of an \( N \)-order Type 1 linear finite impulse response (FIR) filter can be written as

\[
H_\delta(\omega) = \sum_{n=0}^{K} h(n)\cos(n\omega)
\]  

(1)

With \( h(n) \) being the filter’s time sequences, \( K = (N-1)/2 \), and \( \omega \) is the digital angle frequency.

The targeted filter is set as \( H_d(\omega) \). And set \( \omega \) in a periodical interval \([0, \pi]\), and let the discrete values of \( \omega \) be \( \omega_0, \omega_1, \omega_2... \omega_{179}, \omega_{180} \) in turn. Then let

\[
Q = \begin{bmatrix}
1 & \cos(\omega_0) & \ldots & \cos(K\omega_0) \\
1 & \cos(\omega_1) & \ldots & \cos(K\omega_1) \\
\vdots & \vdots & \ddots & \vdots \\
1 & \cos(\omega_{180}) & \ldots & \cos(K\omega_{180})
\end{bmatrix},
\]  

(2)

\[
h = [h(0) \ h(1) \ \ldots \ h(K)]^T,
\]

(3)

\[
f = a = [H_d(\omega_0) \ H_d(\omega_1) \ \ldots \ H_d(\omega_{180})]^T,
\]

(4)

Where superscript \( T \) represents the operation of transpose.

Then, (1) is able to be re-written as

\[
Qh = f.
\]

The weighted least square solution of (5) is \([10]\)

\[
h_{WLS} = (Q^DQ)^{-1}Q^Df
\]

(6)

With superscript \( H \) representing the operation of conjugate transpose, and \( D_l = \text{diag} v \) with \( \text{diag} \) denoting the operation of forming a diagonal matrix using the vector \( v \)'s elements. In \([10]\), the vector \( v \)'s elements are able to be arbitrarily specified but have no connection with the targeted filter vector \( a \), which is inconvenient for application. To overcome the shortcoming and further improve the performance of the method, let \( l \) and \( m \) being integer variables and let

\[
f = a^{l+m} = [H_{d}^{l+m}(\omega_0) \ H_{d}^{l+m}(\omega_1) \ \ldots \ H_{d}^{l+m}(\omega_{180})]^T,
\]

(7)

\[
D_l = D^l,
\]

(8)

\[
D = \text{diag} a = \text{diag} [H_d(\omega_0) \ H_d(\omega_1) \ \ldots \ H_d(\omega_{180})]^T.
\]

(9)

Then, we can obtain the meliorated weighted least square solution from (6) as

\[
h_{WLS} = (Q^DQ)^{-1}Q^D a^{l+m} = (Q^DQ)^{-1}Q^D a^m.
\]

(10)

It is apparent that the following equation is valid
\[ a^e = D^e e, \]  
\[ (11) \]

Where \( e \) is a column vector whose all elements is 1. Therefore, we can obtain another form of (10) as

\[ h_{\text{MWLS}} = (Q^{lH} D^{mH})^{-1} Q^{lH} D^{mH} e. \]  
\[ (12) \]

We call the new method described as (12) the meliorated weighted least square (MWLS) method. In (12), when \( m=0 \), one form can be obtained as

\[ h_{\text{MWLS}} = (Q^{lH} D^{mH})^{-1} Q^{lH} e, \]  
\[ (13) \]

And, when \( l=0 \), another form can also be obtained as

\[ h_{\text{LS}} = (Q^{lH} Q^{mH})^{-1} Q^{lH} e. \]  
\[ (14) \]

It is obvious that (14) is the least square solution of (5) with the targeted filter vector being \( a^e \), and that (13) is similar with the conclusion equation (16) in [1]. So either the least square method described as (14) or the method in [1] is a special case of MWLS method described as (12).

3. Simulations
Matrix Laboratory (MATLAB) is used to simulate the method and demonstrate its performance.

Let the discrete values of the digital angle frequency \( \omega \) in \([0, \pi]\) be equal to \(0, \pi/180, 2\pi/180... 179\pi/180, \pi\) in turn. We use three examples to illustrate the advantage of the new method.

Firstly, a band-stop filter is designed using (12) and (14) and the targeted filter is set as

\[ H_d(\omega) = \begin{cases} 10^{-2} & 0 < \omega < \omega_p \\ 1 & \text{other} \end{cases} \]  
\[ (15) \]

With \( N=71 \), \( \omega_p=1/3\pi \), and \( \omega_s=2/3\pi \). The designed filters’ amplitude-frequency responses are shown in Figure 1.

As shown in Figure 1, the \( x \)-axis represents \( \omega \) in rad/s, and the \( y \)-axis represents the designed filter’s amplitude in dB. In Figure 1, “target” refers to the targeted filter’s graph; “ls” refers to the designed filter’s graph using (14) when \( m=1 \); and other three captions refer to the designed filter’s graphs using (12) with \( l=1, m=1; l=2, m=1; \) and \( l=3, m=1 \) respectively.

![Figure 1. The designed band-stop filters’ amplitude-frequency responses.](image)

From Figure 1, it can be seen that the least attenuation of the curve of \( \text{ls} \) is -11.1 dB while the targeted filter’s attenuation is -20 dB. However, the designed filter’s attenuation using (12) with \( l=1, m=1; l=2, m=1; \) and \( l=3, m=1 \) are -17.4 dB, -28.2 dB, and -39.3 dB sequentially at the expense of the
width of the transitional band being slightly rising with the increasing attenuation. The outcomes demonstrate that the new method’s performance is far better than the least square method.

Secondly, a band-pass filter is designed using (12) and (14) and the targeted filter is set as

$$H_d(\omega) = \begin{cases} 1 & \text{if } \omega_p < \omega < \omega_s \\ 10^{-2} & \text{other} \end{cases} \quad (16)$$

With $N=71$, $\omega_p=1/3\pi$, and $\omega_s=2/3\pi$. The designed filters’ amplitude-frequency responses are shown in Figure 2.

![Figure 2. The designed band-pass filters’ amplitude-frequency responses.](image)

As shown in Figure 2, the x-axis represents $\omega$ in rad/s, and the y-axis represents the designed filter’s amplitude in dB. In Figure 2, “target” refers to the targeted filter’s graph; “ls” refers to the designed filter’s graph using (14) when $m=2$; and other three captions refer to the designed filter’s graphs using (12) with $l=1, m=2$; $l=2, m=2$; and $l=3, m=2$ respectively.

From Figure 2, it can be seen that the least attenuation of the curve of ls is -11.5 dB while the targeted filter’s attenuation is -20 dB. However, the designed filter’s attenuation using (12) with $l=1, m=2$; $l=2, m=2$; $l=3, m=2$ are -17.6 dB, -29.5 dB, and -38.5 dB sequentially at the expense of the width of the transitional band being slightly rising with the increasing attenuation. The outcomes demonstrate that the new method’s performance is far better than the least square method again.

Finally, a notch filter is designed using (12) and (14) and the targeted filter is set as

$$H_d(\omega) = \begin{cases} 10^{-2} & \text{if } \omega_p1 < \omega < \omega_s1, \omega_p2 < \omega < \omega_s2 \\ 1 & \text{other} \end{cases} \quad (17)$$

With $N=71$, $\omega_{p1}=1/9\pi$, $\omega_{s1}=1/8\pi$, and $\omega_{p2}=2/3\pi$, $\omega_{s2}=123/180\pi$. The designed filters’ amplitude-frequency responses are shown in Figure 3.

As shown in Figure 3, the x-axis represents $\omega$ in rad/s, and the y-axis represents the designed filter’s amplitude in dB. In Figure 3, “target” refers to the targeted filter’s graph; “ls” refers to the designed filter’s graph using (14) when $m=2$; and other three captions refer to the designed filter’s graphs using (12) with $l=1, m=2$; $l=2, m=2$; and $l=3, m=2$ respectively.

From Figure 3, it can be seen that the least attenuation of the curve of ls is -4.5 dB in the first notch, and -6.1 dB in the second notch, while the targeted filter’s attenuation is -20 dB. However, the designed filter’s attenuation using (12) with $l=1, m=2$; $l=2, m=2$; $l=3, m=2$ are -16.1 dB, -23.2 dB, and -42.5 dB sequentially in the first notch, and -15 dB, -27.2 dB, and -42.3 dB sequentially in the second notch at the expense of the width of the transitional band of both notches being slightly rising with the
increasing attenuation. The results demonstrate that the new method’s performance is far better than the least square method again.

Figure 3. The designed notch filters’ amplitude-frequency responses.

The aforementioned results demonstrate that, when used to design filter, (12) is far better than (14), which is caused by the fact that the term $D^l$ in (12) is the weighting matrix. Because $D = \text{diag}(a)$ and $a$ is the normalized targeted filter vector, the $(i,i)^{th}$ element of $D^l$ is the weighting number of the $i^{th}$ element of $a$ with $i$ being an integer variable. It is apparent that in stop band the weighting numbers are far bigger than 1 and in non-stop band the weighting numbers are 1. And along with the rising $l$, in stop band the weighting numbers are increasing but in non-stop band the weighting numbers still remain 1. On the other hand, it can be seen that the factually targeted filter is $d^{l=m}$ in MWLS method, so, in stop band, the attenuation is increasing with the increasing $l$.

4. Conclusion
A new meliorated weighted least square approach for digital filter design is presented. In the presented method, the targeted filter is used to form the weighting matrix with two parameters. Through the parameters, the attenuation of stop band of the filter is able to be controlled. Simulations of the new method show that the new method is able to effectively create high performance digital filter.

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