IN THE MODIFIED PERTURBATIVE APPROACH

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We investigate the transverse momentum effects in $\gamma\gamma \rightarrow \pi^+\pi^-$ at moderately large total center of mass energy $\sqrt{s}$ in the modified perturbative approach. The calculation of the differential cross section using different approximations shows that the inclusion of $k_\perp$ in the quark and gluon propagators cannot lead to a considerable improvement of the theoretical prediction.

1 Introduction

For a long time it has been a matter of debate whether or not exclusive reactions are perturbatively dominated at moderate momentum transfer. Calculations of various processes, the pion form factor being the most prominent, in the standard perturbative approach could not settle this question in a satisfactory manner. The validity of this approach at scales of order of a few GeV has been questioned in Refs. 2, 3, where large endpoint contribution to form factors have been found. These large endpoint contributions render perturbative calculations inconsistent, in the sense that observables receive their bulk contributions in domains where the strong coupling is rather large, thus destroying the applicability of perturbation theory. A more sophisticated investigation resulting in the modified perturbative approach including transverse momenta and Sudakov corrections lead to a self-consistent perturbative treatment of meson and nucleon elastic form factors. However, their results have clearly shown that the pion and proton form factors are not perturbatively dominated. Recent calculation of soft, overlap contribution to pion and proton form factors successfully describe the data at moderate momentum transfer.

Since the cross section of $\gamma\gamma \rightarrow \pi^+\pi^-$ has only been obtained in the standard perturbative approach, i.e. neglecting transverse momenta and Sudakov

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corrections, a new, improved investigation within the modified approach has been necessary. The existing LO prediction and NLO prediction are far below the existing data. Moreover, these authors have used a rather large value for the pion form factor as a phenomenological input, which comes out much smaller in a self-consistent perturbative calculation. The object of this work is to provide such an improved perturbative analysis of the process.

2 The scattering amplitude

Since the calculation of the transition amplitude for $\gamma\gamma \rightarrow \pi^+\pi^-$ in the modified approach is completely analogous to that of the pion form factor in Ref. 6, we shall only very briefly introduce the objects appearing in the hard scattering formula.

The factorized amplitude for two gamma annihilation into pion pairs is given by a six dimensional phase space integral:

$$M_{\gamma\gamma \rightarrow \pi^+\pi^-} = \int dx dy \int \frac{d^2 b_\perp}{2\pi^2} \frac{d^2 b'_\perp}{2\pi^2} \hat{\Psi}(x, b_\perp) \hat{T}_H(x, y, b_\perp, b'_\perp; s, t) \times \hat{\Psi}(y, b'_\perp) e^{-S}, \quad (1)$$

where $s$ and $t$ are the usual Mandelstam variables. $\hat{\Psi}$ denotes the Fourier transform of the light-cone wave function of the pion’s valence Fock state and $\hat{T}_H$ is the Fourier transform of the hard scattering amplitude to be calculated perturbatively. The fractions $x, y$ describe how the quarks share their parents’ pion momenta (cf. Fig. 1) and $b_\perp$ and $b'_\perp$ are the Fourier conjugated variables with respect to the intrinsic transverse momenta $k_\perp$ and $k'_\perp$. The factor $e^{-S}$ is the Sudakov form factor, which accounts for gluonic radiative corrections.

Our ansatz for the pion wave function in transverse distance space reads

$$\hat{\Psi}(x, b_\perp) = \frac{f_\pi}{2\sqrt{6}} \phi_\pi(x, \mu_F) 4\pi \exp \left[ -\frac{x(1-x)}{4a_\pi^2} b_\perp^2 \right], \quad (2)$$

where $f_\pi$ is the pion decay constant, $a_\pi$ is the transverse size parameter and $\mu_F$ is the factorization scale, which is of the order of $\sqrt{s}$. The functional form of the wave function meets various theoretical constraints and the parameters are fixed from normalization and certain pion decay processes respectively. There is now broad agreement as to the distribution amplitude $\phi_\pi(x, \mu_F)$ being close to its asymptotic form $\phi_\pi^{\text{AS}}(x) = 6x(1-x)$, which we shall employ in this work.

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\[^b\text{We implicitly assume that the running coupling and the Sudakov factor are the same in the spacelike and timelike regions. We avoid further discussion of these delicate questions, which are beyond the scope of the present work.}\]
Figure 1: The four basic diagrams for the $\gamma\gamma \rightarrow \pi\pi$ amplitude in the HSP. We use the common notation $\bar{x} = 1 - x$.

The hard scattering amplitude $T_H$ at Born level is given by a total of 20 diagrams, which can be grouped together into four basic graphs displayed in Fig. 1. The remaining diagrams are obtained by various particle permutations. Since we are only interested in the leading order behaviour in $k^2_{\perp}/s$ we neglect all transverse momenta in the numerator of $T_H$. We need the Fourier transform of the hard amplitude which means we have to handle the product of two quark propagators and one gluon propagator. It is rather desirable to find an analytical expression of the Fourier transformed hard scattering amplitude in order not to lose too much numerical precision due to high dimensional numerical integration. This is a somewhat awkward task because each quark propagator generally may depend quadratically and linearly on $k_{\perp}$ or $k'_{\perp}$ whereas the gluon propagators always depend on both $k_{\perp}$ and $k'_{\perp}$. This means that the Fourier integral in general does not factorize with respect to $k_{\perp}$ and $k'_{\perp}$. Due to this circumstance one has to resort to certain approximations and at the same time make sure that important contributions from integration regions where quark and gluon propagators can go on-shell are not missed.

At detailed numerical investigation has shown that the dominant effect of transverse momenta in the quark propagators is a considerable suppression at moderate values of $s$. This means that we can find an estimate from above of the cross section by using the collinear limit of the fermion propagators and keeping transverse momenta in the gluon propagators. The Fourier transfor-
Figure 2: The differential cross section for $\gamma \gamma \rightarrow \pi^+ \pi^-$ (scaled with $s^3$) at a center-of-mass scattering angle $\Theta_{c.m.} = 90^\circ$ using various approximations ($z = \cos \Theta_{c.m.}$). The solid line shows the collinear case using a running coupling frozen at $\mu = 1$ GeV. The dot-dashed line shows the calculation where we have neglected transverse momenta in the quark propagators while keeping them in the gluon propagators. The dashed line represents the case where we have also taken into account transverse momenta in quark propagators.

In order to make a more realistic approximation one should keep as many of the transverse momentum terms as possible, such that an analytical evaluation of the Fourier transform of $T_H$ is still possible. In particular, one may neglect $k_\perp$ corrections of quark propagators in integration regions where the quark virtuality is spacelike, since in these regions the transverse corrections lead to a mild suppression of roughly 10%. A similar reasoning applies to gluon propagators.

3 Numerical results

Results for the differential cross section are shown in Fig. 2. We show both the approximations discussed above. For comparison we also show the collinear case where we have used a running coupling frozen at $\mu = 1$ GeV. In Fig. 3 we show our results for the integrated cross section. We have refrained from the calculation of $\sigma_{tot}(\gamma \gamma \rightarrow \pi^+ \pi^-)$ in the case where we keep the transverse momenta in the fermion propagators, since, as we have discussed above, it cannot exceed the estimate using collinear fermion propagators.
We would like to emphasize that we have avoided to express the collinear amplitude in terms of the pion form factor. Consequently, we have not explicitly assumed any phenomenological value of the pion form factor as has been done in Refs. 1 and 11. These authors have approximated the pion form factor by $F_\pi(s) = 0.4 \text{ GeV}^2/s$, a value which is far too large from a perturbative point of view, since in the perturbative approach one obtains a much smaller value. Such a large value as cited above can only be obtained by using a frozen coupling constant of the order of one over the whole integration region, a proceeding which is surely not consistent with the application of perturbation theory. In fact, it is true that in order to explain the existing data of the $\gamma\gamma \rightarrow \pi^+\pi^-$ cross section one would need a rather large value for the pion form factor. A fit to these data even yields a value of $F_\pi(s) \approx 0.55 \text{ GeV}^2/s$. However, such a large value is in no way justified by a perturbative calculation.

4 Concluding remarks

Our results show that even an improved perturbative analysis, i.e. taking into account transverse momentum corrections as well as Sudakov effects, does not lead to a considerable enhancement of the theoretical prediction of the $\gamma\gamma \rightarrow \pi^+\pi^-$ cross section. Considering the fact that the collinear approximation of the cross section can be written in terms of the timelike pion form factor, one
might be tempted to expect an enhancement of even a factor four in the new calculation of the cross section, since the ratio of the timelike and spacelike form factor is of the order of two, and the cross section depends quadratically on the form factor. However, it seems that in the collinear case the dependence of the cross section on the form factor is pure coincidence because if one includes transverse momenta in the calculation it is not possible to express the cross section in terms of the form factor, the reason being that in the process under consideration the structure of quark and gluon propagators is generally quite different from the one appearing in the form factor.

In view of this discussion, it is desirable to find a mechanism which accounts for nonperturbative contributions to two photon annihilation into meson pairs. We hope to provide further insights by future investigations in this direction.

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