MHD free convective flow past a vertical plate

Nor Raihan Mohamad Asimoni¹, Nurul Farahain Mohammad¹, Abdul Rahman Mohd Kasim² and Sharidan Shafie³

¹Department of Computational and Theoretical Sciences, Kulliyyah of Science, International Islamic University, 25200 Kuantan, Pahang, Malaysia
²Faculty of Industrial Sciences & Technology, Universiti Malaysia Pahang, 26300 Gambang, Kuantan, Pahang, Malaysia
³Department of Mathematics, Faculty of Science, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia

E-mail: raihan.asimon@gmail.com, farahain@iium.edu.my, rahmanmohd@ump.edu.my, sharidan@utm.my

Abstract. The free convective flow in incompressible viscous fluid past a vertical plate is studied under the presence of magnetic field. The flow is considered along the vertical plate at \( x \)– axis in upward direction and \( y \)– axis is taken normal to it. The governing equations are written in vector form. Afterwards, the equations are solved numerically using finite element method with automated solution techniques. Later, the effects of magnetic field strength to the velocity and temperature of the fluid are obtained. It is found that for heated plate, the velocity and the temperature of the fluid decreases when the magnetic field strength increases. Meanwhile for cooled plate, the velocity decreases but the temperature increases when the magnetic field strength increases.

1. Introduction

Heat transfer has close relationship with the fluid flows. It compromises of three types which are convection, conduction and radiation. The transfer of heat through fluids occurs in convention caused by molecular motion. There are three types of convective flow in viscous fluid due to broad range of applications such as forced convection [1], free convection [2] and mixed convection [3].

The effect of magnetohydrodynamics (MHD) on the convective flow has been widely studied at vertical plate. Soundalgekar et al. [4] solved MHD free convective flow at vertical plate using Laplace-transform technique. They mentioned different results for the heated and cooled plate. They also found that increasing magnetic parameter increases the velocity in the heated plate while decrease the velocity in the cooled plate. Nevertheless, if the plate is cooled in a great low temperature, the velocity increases. Raptis & Singh [5] focused on the accelerated vertical plate and used same method which is Laplace-transform technique. They stated that skin friction decreases due to the effect of magnetic parameter.

Besides, Helmy [6] concentrated on the unsteady state at porous plate with perturbation technique. He concluded that increasing magnetic parameter affects velocity and temperature profile to decrease. Furthermore, Palani & Srikanth [7] also investigated the unsteady state with mass transfer using finite difference method. However, different result is found compared to [6] where increasing magnetic parameter will increase the temperature but decrease the velocity.
In addition, several papers started using automated solution technique which is FEniCS package ([8],[9]). FEniCS is an open-source project developed by a group of scientists and software developers with a goal to solve partial differential equation using finite element method efficiently.

Motivated by above researches, this paper focuses on MHD free convective flow of viscous incompressible electrically conducting fluid past a vertical plate and is solved using automated solution technique. The effects of the magnetic field and the other parameters governing the problem are discussed.

2. Mathematical Formulation
Consider steady two-dimensional MHD free convective flow of incompressible viscous fluid past a finite vertical plate in the presence of transverse magnetic field. The flow is considered along the \( x \)-axis which is taken along the vertical plate in the upward direction, and the \( y \)-axis is taken normal to it.

\[
\begin{align*}
\nabla \cdot \mathbf{u} &= 0 \\
\rho (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\nabla p + \mu \nabla^2 \mathbf{u} - \sigma \mathbf{B}_0^2 \mathbf{u} + \rho \mathbf{g} \\
\rho C_p (\mathbf{u} \cdot \nabla T) &= c \nabla^2 T
\end{align*}
\]

Figure 1. Flow configuration with domain \( \Omega = [0,1] \times [0,1] \).

The governing equations for this problem are:

\[
\begin{align*}
\nabla \cdot \mathbf{u} &= 0 \\
\rho (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\nabla p + \mu \nabla^2 \mathbf{u} - \sigma \mathbf{B}_0^2 \mathbf{u} + \rho \mathbf{g} \\
\rho C_p (\mathbf{u} \cdot \nabla T) &= c \nabla^2 T
\end{align*}
\]

where \( \mathbf{u} = (u,v,0) \) is the velocity of the fluid flow, \( \rho \) is the density of the fluid, \( \nabla p \) represents pressure difference due to the ‘pumping’ action of the flow, \( \mu \) is the dynamic viscosity, \( \sigma \) is the electrical conductivity of the fluid, \( \mathbf{B}_0 \) is the magnetic field strength, \( \mathbf{g} \) is a vector consist of \( g = (g_x,0) \) which is gravity acceleration applied to the flow in \( x \)-direction where \( g_x = 9.81 \text{ m/s}^2 \), \( C_p \) is the heat capacitance of the fluid, \( c \) is the thermal conductivity of the fluid and \( T \) is the fluid temperature.

In order to obtain automated solution using FEniCS package in Python programming [10], we need to express the partial derivatives equation (PDE) in variational form. The equations (1) – (3) are formulated in mixed variational forms. First, the equations are multiplied by the test functions and integrated over the domain \( \Omega = [0,1] \times [0,1] \). We have \( \mathbf{u}, p \) and \( T \) as functions such that \( (\mathbf{u}, p, T) \in W \) and \( v, q \) and \( s \) as test functions such that \( (v, q, s) \in W \). The space should be mixed function space \( W = V \times Q \times Q \) where \( v \in V \) and \( (q,s) \in Q \). So, weak forms are obtained as:
\[ f_1 = \int (\nabla \cdot \mathbf{u}) q \, dx = 0 \quad (4) \]
\[ f_2 = \rho \int (\mathbf{u} \cdot \nabla) \mathbf{u} \cdot \mathbf{v} \, dx + \int \nabla \cdot \mathbf{v} \, dx - \mu \int (\nabla^2 \mathbf{u}) \mathbf{v} \, dx + \sigma B_0^2 \int \mathbf{u} \cdot \mathbf{v} \, dx - \rho \int \mathbf{g} \cdot \mathbf{v} \, dx = 0 \quad (5) \]
\[ f_3 = \rho C_p \int (\mathbf{u} \cdot \nabla T) s \, dx - c \int (\nabla^2 T) s \, dx = 0 \quad (6) \]

However, we need to keep the smallest derivatives of the functions. In Equation (5) and (6), we have second order derivatives of \( \mathbf{u} \) and \( T \). Thus, by applying integration by parts and Gauss divergence theorem, it can expressed as

\[ f_2 = \rho \int (\mathbf{u} \cdot \nabla) \mathbf{u} \cdot \mathbf{v} \, dx + \int \nabla \cdot \mathbf{v} \, dx - \mu \int \nabla \mathbf{u} \cdot \nabla \mathbf{v} \, dx + \sigma B_0^2 \int \mathbf{u} \cdot \mathbf{v} \, dx - \rho \int \mathbf{g} \cdot \mathbf{v} \, dx = 0 \quad (7) \]
\[ f_3 = \rho C_p \int (\mathbf{u} \cdot \nabla T) s \, dx - c \int \nabla T \cdot \nabla s \, dx = 0 \quad (8) \]

Finally, we need to combine the equation (4), (7) and (8) such that

\[ F \equiv f_1 + f_2 + f_3 = 0 \quad (9) \]

which is called a fully coupled approach [8].

3. Results and Discussion

After reformulate the governing equations (1) – (3) into variational forms (4), (7) and (8) as FEniCS is based on the finite element method, these equations are written in the Python program and solved numerically using FEniCS package which is imported from DOLFIN [11]. Finally, the solution is plotted using ParaView.

The investigation of the effects of magnetic field on the free convective flow past a vertical plate is presented. Here, we consider incompressible viscous fluid which is water at room temperature 293.15K with properties as in table 1.

**Table 1.** Properties of the water at room temperature.

| Property                      | Value |
|-------------------------------|-------|
| Density (g/m³)                | 0.997 |
| Dynamic Viscosity (g/ms)      | 8.91  |
| Electrical Conductivity (µS/m) | 5.501 |
| Heat Capacitance (J/gK)       | 4.184 |
| Thermal Conductivity (W/mK)   | 0.591 |

Besides, we demonstrated the velocity and temperature of the fluid using two different plates which are heated at 353.15K and cooled at 273.15K and four different magnetic field strength such as \( B_0 = 0, 1, 2, 3 \).

3.1. Velocity of the Fluid

We have plotted velocity of the fluid in figure 2 for heated plate and figure 3 for cooled plate.
Figure 2. Velocity of the fluid past a vertical heated plate when (a) \( B_0 = 0 \), (b) \( B_0 = 1 \), (c) \( B_0 = 2 \) and (d) \( B_0 = 3 \).

Figure 3. Velocity of the fluid past a vertical cooled plate when (a) \( B_0 = 0 \), (b) \( B_0 = 1 \), (c) \( B_0 = 2 \) and (d) \( B_0 = 3 \).

Based on figure 2 and figure 3, velocity decreases as magnetic field strength increases for both cases which are heated and cooled plate. Although the plate has different temperature, this situation does not affect the velocity field.
3.2. Temperature of the Fluid
We have plotted temperature of the fluid in figure 4 for heated plate and figure 5 for cooled plate.

**Figure 4.** Temperature of the fluid past a vertical heated plate when (a) $B_0 = 0$, (b) $B_0 = 1$, (c) $B_0 = 2$ and (d) $B_0 = 3$.

**Figure 5.** Temperature of the fluid past a vertical cooled plate when (a) $B_0 = 0$, (b) $B_0 = 1$, (c) $B_0 = 2$ and (d) $B_0 = 3$. 
Comparing figure 4 and figure 5, it is clear that the temperature of the fluid decreases for heated plate and increases for cooled plate when magnetic field strength increases. Since the fluid at room temperature at first, it can be seen that when $B_0 = 0$ which means no magnetic field is applied, the temperature of the fluid increases for heated plate and decreases for cooled plate.

4. Conclusion
In this paper, the problem of MHD free convection flow past a vertical plate is investigated. It can be concluded that when magnetic field strength increases, the velocity of the fluid decreases for both cases. In addition, the temperature of the fluid increases for cooled plate and decreases for heated plate when magnetic field strength increases.

Acknowledgements
The authors would like to thank the Ministry of Higher Education (MOHE) for supporting this work through Research Acculturation Grant Scheme (RAGS15-067-0130).

References
[1] Beckerman C and Viskanta R 1987 Forced convection boundary layer flow and heat transfer along a flat plate embedded in a porous medium Int J Heat Mass Transf. 30(7) 1547–987
[2] Pop I and Soundalgekar V M 1980 Free convection flow past an accelerated vertical infinite plate ZAMM-Journal of Applied Mathematics and Mechanics 60(3) 167–168
[3] Acrivos A 1966 On the combined effect of forced and free convection heat transfer in laminar boundary layer flows Chemical Engineering Science 21(4) 343–352
[4] Soundalgekar V M, Gupta S K and Aranake R N 1979 Free convection effects on MHD stokes problem for a vertical plate Nuclear Engineering and Design 51(3) 403–407
[5] Raptis A and Singh A K 1983 MHD free convection flow past an accelerated vertical plate International Communications in Heat and Mass Transfer 10(4) 313–321
[6] Helmy K A 1998 MHD unsteady free convection flow past a vertical porous plate ZAMM-Journal of Applied Mathematics and Mechanics 78(4) 255–270
[7] Palani G and Srikanth U 2009 MHD flow past a semi-infinite vertical plate with mass transfer Nonlinear Anal.: Modell. Control 14(3) 345–356
[8] Zhang C, Zarrouk S J and Archer R 2016 A mixed finite element solver for natural convection in porous media using automated solution techniques Comput Geosci. 96 181–192
[9] Mortensen M, Langangen H P and Wells G N 2011 A FEniCS-based programming framework for modeling turbulent flow by the reynolds-averaged navier-stokes equations. Adv Water Resour. 34(9) 1082–1101
[10] Logg A, Mardal K A and Wells G N 2012 Automated solution of differential equations by the finite element method: The FEniCS book (Vol. 84) Springer Science & Business Media
[11] Logg A and Wells G N 2010 DOLFIN: Automated finite element computing. ACM Transactions on Mathematical Software (TOMS) 37(2) 20