The reduction of \( CO_2 \) emission using the optimal control approach

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Abstract. Indonesia is the country that has the largest tropical forest with a wide variety of biodiversity. Forests are natural resources that have an important role in human life. However, the existence of forest is currently decreasing along with the increasing of industrialization. The depletion of forest area and the expansion of industrialization can cause the increasing of \( CO_2 \) emission. Therefore, it is necessary to control forest resources and industrial development, thus the level of \( CO_2 \) emission can be minimized. In this study, the dynamics of \( CO_2 \) emission, forest area and industrialization are modelled mathematically. In order to reduce the level of \( CO_2 \) emission, we design two control variables in the model which are reforestation and government policy. The optimal control theory using Pontryagin Minimum Principle is applied to find the optimal level of control variables. Furthermore, numerical simulation is given to illustrate the performance of the optimal control variables. Based on the simulation result, the level of \( CO_2 \) emission with controls is less than the level of \( CO_2 \) emission without controls. This result suggests that providing reforestation and government policy can reduce the concentration of \( CO_2 \).

1. Introduction
Indonesia is the country with the largest and the most biodiversity tropical forests in the world. Forests have an important role in providing water, producing oxygen, and preventing global warming. If the use of forest resources is not controlled, it can cause the depletion of forest resources. The cause of the reduced forest resources in Indonesia is the increasing population growth rate which causes the low level of welfare. Therefore, people need a lot of natural resources to improve welfare, such as utilizing forest resources in developing industrial units, using land for agriculture, and so on.

East Kalimantan is the provinces with abundant natural resource potential which is utilized for industrial development. Therefore, East Kalimantan is one of the largest contributors to \( CO_2 \) emissions in Indonesia. The level of greenhouse gas emissions in East Kalimantan in 2014 was 1,594.42 million tons of \( CO_2 \) with 96.19% or 1,533.62 million tons of \( CO_2 \) come from land-based activities in the form of conversion of forest and land areas. Meanwhile, the energy, transportation, and industrial sectors contributed 3.17% or 50.6 million tons of \( CO_2 \), and waste sector contributed 0.64% or 10.24 million tonnes of \( CO_2 \) [1]. Moreover, the rate of deforestation in East Kalimantan is around 60,000 ha/year in forest areas. Most of the forested land becomes bush and open land (65%), and 35% left is for agricultural activities. The forest degradation in the forest area is about 30,000 ha and 84% of the total degradation in East Kalimantan occurs in Limited Production Forests [2].
All deforestation activities which are quite high can cause the depletion of biodiversity and natural disasters such as floods, forest and land fires, and landslides. Therefore, East Kalimantan needs to have the efforts to reduce $CO_2$ emission. Some efforts can be carried out to control the degradation of forest resources, for example reforestation activities and industrial licensing policies. Reforestation is an effort to plant tree species in damaged forest areas to restore forest function. While the industrial licensing policy is a government policy related to the granting of industrial activity permits. One example of an industrial permit is the Industrial Business License. According to the government regulation of Indonesia number 107 [3], Industrial Business License is a permit given to anyone to carry out industrial business activities.

This issue is interesting to be studied and analyzed. The mathematical model can be used to describe the dynamics of $CO_2$ emission, forest, and industrialization. There are some researches regarding this topic. In 2008, Caetano et. al. studied regarding the optimal control for $CO_2$ and greenhouse effect reducing. In that work, the mathematical models are presented to explain the dynamics of $CO_2$ emission, forest area, and GDP (Gross Domestic Product) [4]. The other work is conducted by Dubey at. al. in 2009 who introduced the mathematical model for the depletion of renewable (biological) resources caused by population and industrialization [5]. Refers to that paper, Anggriani at. al proposed the model decreasing forest resources due to industrialization [6].

Furthermore, this research investigates the dynamics of $CO_2$ emission, forest area, and industrialization by referencing the work of [4, 5, 6]. In this study, the optimal controls such as reforestation and government policy on industrial licensing are proposed and implementing in the models. The aim of this work is to investigate the effect of providing the control variables to reduce the concentration of $CO_2$ emission. The method used to solve the optimal control problem is Pontryagin Minimum Principle (PMP). Furthermore, the numerical simulation using the Runge Kutta Forward-Backward Sweep method is given to show the performance of the optimal controls in the system.

2. The formulation of the optimal control problem
Generally, the formulation of the optimal control problem consists of the mathematical model of the system, the objective function or performance index, the boundary conditions and the physical constraints on the states and/or controls [7].

2.1. The mathematical model
In this study, the concentration of $CO_2$ is denoted by $X$. It is assumed that the level of $CO_2$ increases with the logistic emission growth rate. In addition, the concentration of $CO_2$ can increase because of industrialization existence (expressed by $I$) that can produce carbon dioxide. The carbon dioxide is then discharged to the air and causing the $CO_2$ concentration increased. One thing that can reduce the level of $CO_2$ concentration is the presence of forests (stated by $Z$). This is due to the photosynthetic process in which $CO_2$ and water are absorbed by plants in the subsistence of sunlight to produce food and energy. However, the existence of forests can become increasingly depleted due to the increasing of industrialization. Therefore, it is necessary to have the government policy on industrial licensing and doing reforestation, so that the existence of the forest is sustained and the emission of $CO_2$ can be reduced. In this paper, two aspects of those are considered as the control variables. Furthermore, the dynamics of this process are modeled mathematically using the differential ordinary as shown in Equations (1)-(3).

\[
\frac{dX}{dt} = rX \left(1 - \frac{X}{s}\right) - aZ + h_1I, \quad (1)
\]

\[
\frac{dZ}{dt} = \gamma Z + u_1Z - h_2Z - h_3IZ, \quad (2)
\]

\[
\frac{dl}{dt} = \beta h_3IZ - u_2I. \quad (3)
\]
Subsequently, the Equations (1)-(3) can be expressed as follow

\[ \dot{x}(t) = f(x(t), u(t), t) \]

where

\[ x(t) = (X(t), Z(t), I(t))' \in \mathbb{R}^3. \]

\begin{table}
\centering
\begin{tabular}{l|l|l}
\hline
Parameter & Description & Unit \\
\hline
\hline
r & The growth rate of \( CO_2 \) concentration & year\(^{-1}\) \\
s & The carrying capacity of \( CO_2 \) & ppm \\
\alpha & The level of emission reduction due to forest resources & ppm ha\(^{-1}\) year\(^{-1}\) \\
u_1 & The optimal control of the reforestation & year\(^{-1}\) \\
u_2 & The optimal control of the government policy on industrial licensing & year\(^{-1}\) \\
h_1 & The emission growth rate due to industrialization & ppm unit\(^{-1}\) year\(^{-1}\) \\
h_2 & The natural depletion rate of forest resources & year\(^{-1}\) \\
h_3 & The depletion rate of forest resources due to industrialization & unit\(^{-1}\) year\(^{-1}\) \\
\beta & The industrialization growth rate due to the availability of forest resources & unit ha\(^{-1}\) \\
\gamma & The natural forest growth rate & year\(^{-1}\) \\
\hline
\end{tabular}
\caption{The description of the parameters.}
\end{table}

2.2. The objective function

The purpose of this problem is to obtain the optimal control variables by minimizing the level of \( CO_2 \) emission, the number of industry, and the costs of government efforts to carry out reforestation activities and government policy on industrial licensing. Therefore, we considered the objective function as follows:

\[ \min J = \frac{1}{2} \int_{t_0}^{t_f} \left[ C_1 X^2(t) + C_2 Z^2(t) + C_3 u_1^2(t) + C_4 u_2^2(t) \right] dt \]  \hspace{1cm} (4)

with \( t_0 \) dan \( t_f \) are fixed initial time and fixed final time respectively, while \( C_i > 0 \) for \( i = 1, 2, 3, 4 \) are the weighting parameters.

2.3. The boundary condition

The type of the optimal control problem in this research is fixed final time and free final state. Therefore, we have the boundary condition as follows

\[ x(t_0) = x_0 \text{ and } \lambda^{*}(t_f) = \left( \frac{\partial S}{\partial x} \right)_{x(t_f)} \]

where \( S \) is the Mayer Function of the objective function. Nevertheless, the objective function in Equation (4) does not contain Mayer Function. Accordingly, it is equal to zero.
3. The solution of the optimal control problem

The Pontryagin Minimum Principle (PMP) is used to solve this optimal control problem. The steps of this method are explained in this section [7].

1. Determine Pontryagin function, \( \mathcal{H} \)

Based on the dynamics model on Equation (1)-(3) and the objective function on Equation (4), we can consider the Pontryagin Function as follows

\[
\mathcal{H}(x(t), u(t), \lambda(t), t) = V(x(t), u(t), t) + \lambda'(t)f(x(t), u(t), t)
\]

\[
= \frac{1}{2}(C_1X^2(t) + C_2I^2(t) + C_3u_1^2(t) + C_4u_2^2(t))
\]

\[+ \lambda_1 \left(rX \left(1 - \frac{X}{S}\right) - \alpha Z - h_1I\right)\]

\[+ \lambda_2 (\gamma Z + u_1Z - h_2Z - h_3IZ)\]

\[+ \lambda_3 (\beta h_3I - u_2I)\].

2. Minimize \( \mathcal{H} \) with respect to \( u \)

The optimal control equation can be obtained by minimizing \( \mathcal{H} \) with respect to each control variable, \( u_1 \) and \( u_2 \).

- Minimizing \( \mathcal{H} \) for \( u_1 \)

\[
\frac{\partial \mathcal{H}}{\partial u_1} = 0,
\]

\[
C_3u_1 + \lambda_2Z = 0,
\]

\[
u_1 = -\frac{\lambda_2Z}{C_3}.
\]

- Minimizing \( \mathcal{H} \) for \( u_2 \)

\[
\frac{\partial \mathcal{H}}{\partial u_2} = 0,
\]

\[
C_4u_2 - \lambda_3I = 0,
\]

\[
u_2 = \frac{\lambda_3I}{C_4}.
\]

In this case, the control variables, \( u_1 \) and \( u_2 \) are the proportion. Therefore the value of \( u_1 \) and \( u_2 \) are constrained as \( 0 \leq (u_1, u_2) \leq 1 \). Hence, we find the optimal control of \( u_1 \) and \( u_2 \) as

\[
u_1^* = \min \left(1, \max \left(0, -\frac{\lambda_2Z}{C_3}\right)\right),
\]

\[
u_2^* = \min \left(1, \max \left(0, \frac{\lambda_3I}{C_4}\right)\right).
\]

3. Determine state and costate variables

State and costate equation can be obtained by \( \dot{x} = \frac{\partial \mathcal{H}}{\partial \lambda} \) and \( \dot{\lambda} = -\frac{\partial \mathcal{H}}{\partial x} \) respectively. Following the formula, we can find:

- State equation

\[
\dot{X} = rX \left(1 - \frac{X}{S}\right) - \alpha Z + h_1I
\]

\[
\dot{Z} = \gamma Z + u_1Z - h_2Z - h_3IZ
\]

\[
\dot{I} = \beta h_3I - u_2I.
\]
Costate equation

\[
\begin{align*}
\dot{\lambda}_1 &= -C_1 X - \lambda_1 r + \frac{2rX}{s} \lambda_1 \\
\dot{\lambda}_2 &= \lambda_1 \alpha - \lambda_2 \gamma - \lambda_2 u_1 + \lambda_2 h_2 + \lambda_3 h_3 l - \lambda_3 \beta h_3 l \\
\dot{\lambda}_3 &= -C_2 l - \lambda_1 h_1 + \lambda_2 h_3 Z - \lambda_3 \beta h_3 Z + \lambda_3 u_2.
\end{align*}
\]

Subsequently, by substituting the optimal control, \(u_1^*\) and \(u_2^*\) into the state and costate equation, we can obtain the optimal state and costate equation. Finally, the differential equation systems of state and costate are solved numerically with the initial condition \(x_0 = (X_0, Z_0, I_0)\) and transversality condition as follows

\[
\lambda^*(t_f) = \left( \frac{\partial S}{\partial x} \right)_{t_f},
\]

\[
\lambda^*(t_f) = \left( \frac{\partial}{\partial x} \right)_{0, t_f},
\]

\[
\lambda^*(t_f) = 0.
\]

In other word, we can decide that \(\lambda_i^*(t_f) = 0\) for \(i = 1, 2, 3\).

4. Simulation and result

The state and costate equations are nonlinear. Hence, in this research, the numerical method using Runge-Kutta Forward Backward Sweep [8, 9] is used to solve the differential equation systems for state and costate. In this simulation, the initial value of state variable is \(x_0 = (340, 116, 20)\) and the value of parameters is given in the Table 2.

| Parameter | Value | Ref. | Parameter | Value | Ref. |
|-----------|-------|------|-----------|-------|------|
| \(r\)     | 0.15  | [4]  | \(h_3\)   | 0.02  | (Assumed) |
| \(s\)     | 700   | [4]  | \(C_1\)   | 0.02  | (Assumed) |
| \(\alpha\) | 0.06  | (Assumed) | \(C_2\)   | 0.04  | (Assumed) |
| \(h_1\)   | 0.8   | (Assumed) | \(C_3\)   | 0.1   | (Assumed) |
| \(h_2\)   | 1     | [5]  | \(C_4\)   | 0.5   | (Assumed) |
| \(\beta\) | 0.1   | (Assumed) | \(\gamma\) | 0.07  | (Assumed) |

Using the initial value and the parameters in Table 2, we obtained the simulation results presented in Figure 1. Based on the Figure 1(a), the concentration of \(CO_2\) emission increased among 25 years. It is shown that without the optimal controls, the dynamic of \(CO_2\) emission increased significantly and reached 922 ppm at the final time. However, with the optimal controls, the concentration of \(CO_2\) emission at the final time is smaller, which is 683 ppm. It shows that reforestation and the government policy on industrial licensing can provide the effect on minimizing the concentration of \(CO_2\).

Furthermore, from Figure 1(b) we can see that the area of forest decreased over the time. Nevertheless, the forest area with the optimal controls is bigger than without the optimal controls. In addition, Figure 1(c) shows the dynamic of industrialization. Based on that figure, the density of industrialization without optimal controls increased over 2.5 years and then decreased slightly to the level of 64 in the end of the time. Meanwhile, when there are the reforestation efforts and government policy, the density of industrialization declined significantly over the time. This result illustrates that government policy on industrial licensing can control the dynamics of industrialization development, as the aim of providing the control in government policy. The performances of control variables,
reforestation and government policy on industrial licensing, are presented in Figure 1(d) and 1(e) successively.

![Graphs showing concentration of CO₂ emission, forest area, industrialization density, reforestation control, and government policy control.](image)

**Figure 1.** The dynamics of (a) the concentration of CO₂ emission, (b) the forest area, (c) the density of industrialization, (d) the optimal control of reforestation, (e) the optimal control of government policy on industrial licensing.
Accordingly, the simulation results show that the concentration of $CO_2$ and the density of industrialization can be minimized by providing the optimal controls. Reforestation and government policy can be the alternatives or the recommendations for reducing the level of $CO_2$ emission.

5. Conclusion
This research related to the optimal control model for reducing $CO_2$ emission provides the following conclusions:
1. The optimal control model for reducing $CO_2$ emission can be modified by providing the control variables, such as reforestation ($u_1$) and government policy on industrial licensing ($u_2$). The model can be expressed in the system of differential equations,

$$\frac{dX}{dt} = rX \left(1 - \frac{X}{s}\right) - \alpha Z + h_1 I,$$

$$\frac{dZ}{dt} = \gamma Z + u_1 Z - h_2 Z - h_3 I Z,$$

$$\frac{dI}{dt} = \beta h_3 Z I - u_2 I.$$

2. This research used the Pontryagin Minimum Principle (PMP) method and obtained the optimal controls ($u^*$) as follows

$$u_1^* = \min \left(1, \max \left(0, -\frac{\lambda_2 Z}{C_3}\right)\right),$$

$$u_2^* = \min \left(1, \max \left(0, \frac{\lambda_3 I}{C_4}\right)\right).$$

3. The efforts of providing reforestation and government policy can be the alternatives to reduce the concentration of $CO_2$ emission and minimize the density of industrialization optimally.

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