Multifragmentation model for the production of astrophysical strangelets

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Determination of baryon number (or mass) distribution of the strangelets, that may fragment out of the warm and excited strange quark matter ejected in the merger of strange stars in compact binary stellar systems in the Galaxy, is attempted here by using a statistical disassembly model. Finite mass of strange quarks is taken into account in the analysis. Resulting charges of the strangelets and the corresponding Coulomb corrections are also included to get a plausible size distribution of those strangelets as they are produced from binary stellar merger. From this mass distribution, an approximate order of magnitude estimate for the possible flux of strangelets is also attempted by using a simple diffusion model for the propagation of those strangelets in the Galaxy. Such an estimate may be useful in view of the ongoing effort to detect galactic strangelets by the recent satellite-borne experiments.

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I. INTRODUCTION

Strange quark matter (SQM), containing almost equal numbers of up, down and strange quarks enclosed in a MIT bag \cite{1}, may be the true ground state of hadronic matter \cite{2,3}. Finite size effects notwithstanding, small lumps of SQM (or, \textit{strangelets}) can also be more stable than ordinary nuclei \cite{3}. A possible scenario for the formation of strangelets in the Galaxy is the fragmentation of bulk SQMs ejected in the tidal disruptions of strange stars (SSs) in compact binary stellar systems \cite{4}. Simulations of SS mergers \cite{5,6}, in fact, show clumpy structures in SQM ejecta. It is reasonable to assume that further fragmentation and separation of those lumps, as the ejected material approaches its thermodynamic and chemical equilibrium, will ultimately yield a set of strangelets, distributed over a range of mass, that contributes to the primary cosmic rays (PCR) \cite{7,8}. Several other authors have also pointed out the possibility of obtaining strangelets in cosmic rays \cite{8,10}. As an aid to the ongoing efforts to detect strangelets in PCR by PAMELA \cite{9}, AMS-02 \cite{10} and other experiments, we here determine a plausible mass spectrum of strangelets \textit{at their source} by invoking a statistical multifragmentation model (SMM) that is often used in the analysis of fragmentation of hot nuclear matter in various contexts \cite{11,12,13,14}. This paper is, in fact, a continuation of our earlier attempt \cite{7} to find the basic rate of injection of strangelets of various sizes in the Galaxy; in Ref. \cite{7}, we had ignored the quark-masses for the sake of simplicity. While this approximation is reasonable for the \textit{u} and the \textit{d} quarks, the mass \(m_s\) of the \textit{s} quarks is expected to be about 95 ± 5 MeV \cite{15}. Without detailed investigation, one cannot simply rule out the possibility that the surface tension of the strangelets and their electric charge, that are associated with non-zero values of \(m_s\) \cite{3,16}, may make the strangelets unstable so that little or no strangelets may be available in PCR in solar neighborhood. It is also possible that the fragmentation pattern of strangelets and its variation with various physical parameters, that we found in Ref. \cite{7}, would undergo quantitative or even qualitative changes as the effect of finite mass of \textit{s} quarks is taken into account. An examination of these aspects of the fragmentation model presented in Ref. \cite{7} is undertaken in this paper. Fragmentation of color-flavor-locked strange matter (CFL SQM) \cite{17,18} will be examined on another occasion in the near future (see also Ref. \cite{19}). The paper is arranged into the following sections. In Sec. II, we briefly review the disassembly model originally presented in Ref. \cite{7}. Equations governing the thermodynamic equilibrium of a single strangelet are presented in Sec. III. In Sec. IV, we apply the formalism of SMM to find the size distribution of strangelets. Discussion of the results and their observational implications are presented in Sec. V.

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II. THE MULTIFRAGMENTATION MODEL

In this model, it is assumed that the fluctuations (or fractures), that would eventually lead to fragmentation into strangelets, might have naturally occurred in the warm and excited initial bulk SQM tidally ejected in the merger of two SSs. The initial average temperature of this bulk matter might have been high enough during its compression in the merger process to allow for such fluctuations. After it becomes gravitationally unbound from the merged SSs [3, 6], the fractured system undergoes quasistatic evolution during which it tries to minimize its free energy by cooling and expanding, while the initial fractures develop into more or less well-defined clumps of different baryon numbers (or sizes) still interacting among themselves. The system eventually occupies a freeze-out volume in thermodynamic equilibrium at a certain temperature $T$. It is assumed that, in this volume, the strong interactions between the fully developed fragments cease to exist [7]. This freeze-out volume is considered to be larger than the original volume of the initial matter. The equilibrium temperature $T$ at freeze-out is also considered to be lower than the initial temperature of the tidally released SQM. At the outset, it may appear that, to achieve thermodynamic and chemical equilibrium, the strangelet complex at freeze-out must be at a temperature of a few tens of MeV which is of the order of the binding (or breaking) energy of the bulk SQM. The fact that such condition is not a binding one is perhaps exemplified by the nuclear statistical equilibrium (NSE) established in a pre-bounce, collapsing stellar core in the course of its evolution towards a type II supernova (SN II). There, NSE is established at a temperature $\sim 100$ keV [20] whereas the average binding energy of nuclei is $\sim 8$ MeV.

In the following, we adapt SMM to find a plausible mass distribution in the strangelet complex in thermodynamic equilibrium at a certain temperature at freeze-out. In real SS merger events, there may, however, be a distribution of those temperatures in the regions in which strangelets are formed. In the absence of observations or the results of relevant numerical simulations, it is difficult to definitely predict a range of temperatures of those regions. Here, we arbitrarily consider the values of $T$ within a range $(0.001 - 1.0)$ MeV at freeze-out. The lower limit $(\sim 1$ keV) corresponds to the possible lower bound of the temperatures usually associated with the accretion disks of the low mass X-ray binary systems [21]. The upper limit $(\sim 1$ MeV) corresponds to the magnitude of temperatures attained by the materials ejected from the tip of the tidal arms formed sometimes in the simulations [21] of merger between two neutron stars (NSs). At such temperatures at freeze-out, the multiplicity $\omega^i$ of the strangelets of specie ‘$i$’, that is characterized by its baryon number $A^i$, is written as [7, 13]

$$\omega^i = \frac{V}{(L^i)^3} e^{(\mu^i - F^i)/T}. \quad (1)$$

Here, $V$ is the available volume, i.e. the freeze-out volume minus the volume of the produced fragments. In Eq. (1), $\mu^i = \sum_f \mu_f N^i_f$ is the chemical potential of the $i^{th}$ specie. The quark chemical potential is $\mu_f$ with ‘$f$’ indicating a particular quark-flavor (i.e. $f = u, d, s$). Also, $N^i_f = \left( -\frac{\partial E}{\partial \mu_f} \right)_{\nu^i, T}$ is the number of quarks of the $f^{th}$ flavor in the $i^{th}$ specie of volume $V^i$, their thermodynamic potential is $\Omega^i_f$. For an approximate value of the mass $m^i$ of a strangelet of baryon number $A^i$, we consider the mass-formulae derived in Refs. [16, 22] by using a bulk approximation to the chemical potentials at $T = 0$ MeV. The mass corresponding to $m_s = 95$ MeV is obtained by means of an interpolation between the ones derived in Ref. [22] for different values of $m_s$. The masses corresponding to various bag values are obtained by using the scaling law derived in Ref. [10]. The thermal de-Broglie wavelength of the specie is $L^i = h/\sqrt{2\pi m^i T}$ where $h$ is the Planck’s constant. Here, $F^i = \Omega^i + \mu^i + E^i_C$ stands for the Helmholtz free energy of the $i^{th}$ specie while $\Omega^i$ is its thermodynamic potential and $E^i_C$ is its Coulomb energy. $F^i$ may be rewritten as $F^i = \Omega^{i_{\text{tot}}} + \mu^i$, where, $\Omega^{i_{\text{tot}}} = \Omega^i + E^i_C$. Thus, Eq. (1) can be reframed as

$$\omega^i = \frac{V}{(L^i)^3} e^{-\Omega^{i_{\text{tot}}}/T}. \quad (2)$$

We will use Eq. (2) to determine the multiplicities of various fragments in the strangelet complex after specifying the thermodynamic quantities pertaining to a single strangelet in that complex. We also add that, throughout this work, we choose our units such that $\hbar = c = k_B = 1$, where $c$ is the speed of light and $k_B$ is the Boltzmann constant.

III. THERMODYNAMICS OF A STRANGELET

To describe the thermodynamic equilibrium of the strangelet of a particular specie, we confine ourselves to the multiple reflection expansion method [23] with smoothed density of states as applied to the standard MIT bag model [3, 16, 22]. This method is similar to the liquid drop model used in the theories of nuclear structure [16]. It is known that the model can satisfactorily fit with the overall behavior of the strangelets that are obtained from the mode-filling calculations of the shell
model \cite{24}. The strength of the QCD coupling between the quarks is taken to be zero here; it was argued that the effect of such coupling may be absorbed by a rescaling of the bag constant \cite{2,10}. Strangelets are assumed to be spherical in shape. Radius of a strangelet of the \( i \)th specie is \( R^i = r^i_0(A^i)^{1/3} \), \( r^i_0 \) being its radius parameter. Volume, surface and curvature of a strangelet are denoted as \( V^i = \frac{4}{3} \pi (R^i)^3 \), \( S^i = 4 \pi (R^i)^2 \) and \( C^i = 8 \pi R^i \), respectively. Thermodynamic potential of the \( i \)th specie is written as

\[
\Omega^i = \Omega^i_V V^i + \Omega^i_S S^i + \Omega^i_C C^i + BV^i, \tag{3}
\]

where,

\[
\Omega^i_V = \frac{37}{90} \pi^2 T^4 - \left( \frac{\mu^2_u + \mu^2_d}{2} \right) T^2 - \left( \frac{\mu^2_s + \mu^2_d}{2} \right) T^2 - \left( \frac{\mu^2_s + \mu^2_d}{4 \pi^2} \right) - \left( \frac{\mu^2_s + \mu^2_d}{4 \pi^2} \right) \left[ (1 - \frac{5}{2} \lambda^2_s) \sqrt{1 - \lambda^2_s} + \frac{3}{2} \lambda^4_s \ln \left( \frac{1 + \sqrt{1 - \lambda^2_s}}{\lambda_s} \right) \right] + 2 \pi^2 \left( \frac{T}{\mu_s} \right)^2 \sqrt{1 - \lambda^2_s} + \frac{7}{15} \pi^3 \left( \frac{T}{\mu_s} \right)^4 \left( \frac{1 - \frac{3}{2} \lambda^2_s}{1 - \lambda^2_s} \right)^{3/2},
\]

\[
\Omega^i_S = \frac{3}{4 \pi} \mu^3_s \left[ \left( 1 - \frac{\lambda^2_s}{6} \right) - \frac{\lambda^2_s}{3} (1 - \lambda_s) \right] - \frac{1}{3 \pi} \left\{ \tan^{-1} \left( \frac{\sqrt{1 - \lambda^2_s}}{\lambda_s} \right) + \lambda_s^3 \ln \left( \frac{1 + \sqrt{1 - \lambda^2_s}}{\lambda_s} \right) - 2 \lambda_s \sqrt{1 - \lambda^2_s} \right\} + \frac{\pi^2}{3} \left( \frac{T}{\mu_s} \right)^2 \left\{ \frac{\pi}{2} - \tan^{-1} \left( \frac{\sqrt{1 - \lambda^2_s}}{\lambda_s} \right) \right\} + \frac{7}{180} \pi^3 \left( \frac{T}{\mu_s} \right)^4 \left( \frac{\lambda^2_s}{1 - \lambda^2_s} \right)^{3/2},
\]

and

\[
\Omega^i_C = \frac{19}{36} T^2 + \left( \frac{\mu^2_u + \mu^2_d}{8 \pi^2} \right) + \frac{\mu^2_s}{8 \pi^2} \left[ \frac{\pi}{2} - \tan^{-1} \left( \frac{\sqrt{1 - \lambda^2_s}}{\lambda_s} \right) \right] + \left( \frac{\lambda^2_s}{\pi} \right) \left( \frac{T}{\mu_s} \right)^2 \left\{ \frac{\pi}{2} - \tan^{-1} \left( \frac{\sqrt{1 - \lambda^2_s}}{\lambda_s} \right) \right\} + \lambda_s^3 \left\{ \pi + \ln \left( \frac{1 + \sqrt{1 - \lambda^2_s}}{\lambda_s} \right) \right\} - \frac{3 \pi^2}{2} \lambda_s - \left( \frac{2 \pi^2}{3} \right) \left( \frac{T}{\mu_s} \right)^2 \frac{1}{\sqrt{1 - \lambda^2_s}} - \frac{7}{60} \pi^3 \left( \frac{T}{\mu_s} \right)^4 \left( \frac{\lambda^2_s (1 + \lambda^2_s)}{(1 - \lambda^2_s)^{5/2}} \right).
\]

Here, \( \Omega^i_V \), \( \Omega^i_S \) and \( \Omega^i_C \) are the thermodynamic potential densities associated with the volume, surface and curvature of the strangelets. In Eqs. (4), \( \lambda_s = \frac{m_s}{\mu_s} \) and \( B \) is the bag pressure. In case of the assumption \( m_s \to 0 \) MeV, we obtain \( \lambda_s \to 0 \) and \( \mu_u = \mu_d = \mu_s = \mu \). Eqs. (3) and (4) then reduce to the thermodynamic potential of the strangelets in the limit of massless quarks \cite{2,10}. For \( T = 0 \) MeV, we recover the thermodynamic potential of the cold strangelets with massive \( s \)-quarks \cite{10} from Eqs. (3) and (4).

As \( s \)-quarks are massive, the number of those quarks is less than the numbers of quarks of other (viz. \( u \)- and \( d \)-) flavors in the strangelets. This gives a positive charge number \( Z^i \) to a strangelet of the \( i \)th specie. Debye screening modulates this charge distribution in the strangelet. Electric field within the strangelet does not allow its charge density to be uniform; positive charge density migrates towards the surface of a large (i.e. \( R^i \gg \lambda_D \); \( \lambda_D \) is the Debye screening length) strangelet so that the charge is confined within a thickness \( \sim \lambda_D \) from the surface. The interior of that strangelet is, therefore, charge-neutral \cite{27}. Such charge-neutrality does not necessarily require the existence of electrons inside the strangelets. It may occur entirely due to the adjustment of chemical potentials of the quarks of different flavors among themselves as they move freely inside
the strangelets \cite{24, 26}. Typically, $\lambda_D \approx 5$ fm; it is almost independent of the mass of the $s$-quarks \cite{25}. On the other hand, the de-Broglie wavelength of a quark is $\lambda_f \sim \frac{2\hbar}{m_f} \sim 4$ fm by assuming $\mu_f \sim 300$ MeV \cite{27}. As $\lambda_D \sim \lambda_f$, the particular expression of the local quark chemical potential $\mu_f$ of a strangelet that was derived in Ref. \cite{25} as containing a part having functional dependence on the radial position coordinate $r$ of the quark within the strangelet, as measured from the center of that strangelet, does not apply in this case unless $R^i \gtrsim (3 - 5)\lambda_D$ \cite{27}. Moreover, as we assume the strangelet complex to be in thermodynamic and chemical equilibrium at freeze-out, the effective quark chemical potential of all flavors of quarks must be the same in all the strangelets throughout the fragmenting complex at equilibrium. In this paper, we ignore the chemical potential of the electrons. This possibly is justified as the Compton wavelength of the electron is larger than the size of the strangelets of mass $A^i \ll 10^7$ so that the electrons reside mostly outside the strangelets \cite{9, 10}. The effect of those electrons is, however, considered while calculating the external pressure on the strangelets. Keeping these issues in mind, we have taken an average $\bar{\mu}$ of the maximum (in the case $R^i \ll \lambda_D$) and the minimum (in the case $R^i \gg \lambda_D$) values of the local quark chemical potential $\mu_f$, as obtained in Ref. \cite{25}, to arrive at the relation $\mu_f = \bar{\mu} - q_f \frac{\Delta \bar{\mu}}{2}$. We consider $\bar{\mu}$ to be an approximate value of the effective quark chemical potential of the strangelet cluster at freeze-out; $q_f$ is the fractional charge of the quark of the $f$th flavor in the units of $e$. Here, $\Delta \bar{\mu} \approx \frac{\rho_e^*}{4\bar{\mu}}$ for $m_e^* \ll \bar{\mu}^2$ \cite{24, 25} as is the condition applicable in this case in which $m_e = 95$ MeV and $\bar{\mu} \sim 300$ MeV. The averaging procedure mentioned above replaces the position dependent term in the expression of $\mu_f$ by the effective quark chemical potential $\bar{\mu}$ of the fragmenting system that is independent of the flavor and the position of any quark in a particular strangelet. This approach is perhaps justified by the fact that the liquid drop model of the strangelets, that we employ in this paper, is concerned only about the average properties of the strangelets unlike the shell model that examines the internal structure of the strangelets. The total charge (in the units of $e$) and the Coulomb energy of a strangelet are the properties approximately obtained by integrating over the radial coordinate $r$ of that strangelet; they are considerably less sensitive to the nature of the variation of $\mu_f$ with $r$. We, therefore, adopt the expressions for these quantities as given in Ref. \cite{25}:

$$Z^i \approx \frac{1}{\alpha} R^i \Delta \bar{\mu} \left[ 1 - \frac{\tanh(R^i/\lambda_D)}{R^i/\lambda_D} \right]$$

and

$$E_C^i \approx \frac{(\Delta \bar{\mu})^2}{2\alpha} R^i \left[ 1 - \frac{3\tanh(R^i/\lambda_D)}{2(R^i/\lambda_D)} \right] \left( \frac{\cosh(R^i/\lambda_D)}{\cosh(\lambda_D/\lambda_D)} \right)^2$$

with $\alpha$ being the fine structure constant. Here, we consider an approximate estimate (see below) of the Debye shielding length $\lambda_D$ in the strangelets even though it is strictly valid only in the case of cold ($T = 0$ MeV) strangelets. In the above, our approach to consider the average quark chemical potential but to retain the integrated values for the charge and the Coulomb energy of a strangelet is analogous in spirit to the one considered in Ref. \cite{28} while calculating the threshold value $Z^2/A$ for spontaneous fission. Recall that Bohr and Wheeler used the classical expressions for the Coulomb energy and the surface energy, two integrated quantities, to demonstrate that this threshold is approximately equal to 40, in fairly good agreement with experimental data.

The entropy of the strangelet of the $i$th species is $S^i = -(\frac{\partial E^i}{\partial T})_{T, \mu_i}$. Thus, the total entropy of the strangelet may be written as

$$E^i = TS^i + \mu^i + \Omega_{\text{tot}}$$

$$= TS^i + \mu^i + \Omega + E_C^i.$$ (7)

For thermodynamic equilibrium, the strangelet fragments, in addition to being in chemical equilibrium are also in mechanical equilibrium, i.e. $P_{\text{ext}}^i = -(\frac{\partial E^i}{\partial V})_{T, \mu_i}$, where, $P_{\text{ext}}^i$ is the external pressure on a strangelet of the $i$th species. This pressure is exerted by the electrons maintaining an overall charge-neutrality of the fragments. This is calculated in the Wigner-Seitz (WS) approximation \cite{29} in which a single strangelet of the $i$th species having charge $Z^i$ is surrounded by a neutralizing sphere of relativistic electrons with number density $n_e^i = Z^i/V_{\text{cell}}^i$; $V_{\text{cell}}^i$ being the volume of the particular WS-cell. We choose $V_{\text{cell}}^i = \frac{A^i}{\sum_i A^i \omega^i} V$ satisfying the constraint $\sum_i \omega^i V_{\text{cell}}^i = V$. The expression for the external pressure of the electrons on the strangelet is written as

$$P_{\text{ext}}^i = (3\pi^2)^{1/3} \left[ \frac{(n_e^i)^{1/3}}{4} \right] + \left( \frac{\pi^2}{6} \right) \left[ \frac{T^2}{(3\pi^2)^{1/3}} \right] (n_e^i)^{2/3}$$

$$- \left( \frac{3}{10} \right) \left( \frac{4\pi}{3} \right)^{1/3} (Z^i)^{2/3} (n_e^i)^{4/3}$$

$$- \left( \frac{1}{6} \right) \left( \frac{324}{175} \right) \left( \frac{4}{9\pi} \right)^{2/3} (3\pi^2)^{1/3} (Z^i)^{4/3} \alpha^2 (n_e^i)^{4/3}.$$ (8)

In Eq. (8), the first two terms on the right hand side represent the pressure of a degenerate Fermi gas of non-interacting, relativistic electrons, the third term represents the classical coulomb energy of the strangelet in
the field of uniformly distributed electrons and the fourth term represents the Thomas-Fermi deviations from uniform charge distribution of relativistic electrons. The terms representing exchange energy and spin-spin electrons are here neglected as they are found to be much smaller in comparison with the above terms. With the definitions given in Eqs. (4), (5), (6) and (8), the condition for the mechanical equilibrium mentioned above ultimately yields an expression for the total thermodynamic potential $\Omega^i_{\text{tot}}$ of the strangelet at thermodynamic equilibrium at freeze-out. This expression is

$$\Omega^i_{\text{tot}} = (-\Omega^i_{\nu} - B)\mathcal{V} \left( \frac{\Omega^i C + 3E^i_C - \Delta E^i_C - 3P^i_{\text{ext}}}{2gS^i + \Omega^i C + \Delta E^i_C} \right).$$

(9)

where,

$$\Delta E^i_C \approx \frac{(\Delta \hat{\mu})^2}{2\alpha} R^i \left[ 1 - \cosh^{-2} \left( \frac{R^i}{\lambda_D} \right) \left\{ 1 + \left( \frac{R^i}{\lambda_D} \right) \tanh \left( \frac{R^i}{\lambda_D} \right) \right\} \right].$$

(10)

In the following, we will use Eqs. (9) and (10) to evaluate the multiplicities of strangelets of the $i^{th}$ specie as defined in Eq. (2). In doing so, we require an additional relation

$$N^i_u = A^i + Z^i,$$

(11)

that is obtained from the definition of the baryon number and the charge of a strangelet. Eq. (11) may be rewritten in the form of a complicated transcendental equation involving the radius parameter $r^i_n$ that is solved iteratively to obtain the radius parameter of a particular specie of the strangelets that corresponds to each trial value of $\hat{\mu}$.

### IV. MASS SPECTRA OF STRANGELETS

Eqs. (2)-(6),(8), (9), (10) and (11) with the added condition for the conservation of the initial baryon number $A_b$, namely

$$A_b = \sum_i A^i \omega^i,$$

(12)

allow us to evaluate the mass (or size) distribution of the strangelets in the fragmenting complex after we self-consistently solve the system of equations for the value of the effective quark chemical potential $\hat{\mu}$ at thermodynamic equilibrium at freeze-out. The available volume $\mathcal{V}$ is a free parameter in Eq. (2). In all the numerical results displayed in the following, we choose $\mathcal{V} = 5V_b = 5(\frac{4\pi}{3}r^3_b A_b)$, even though we examined the variation of multiplicities due to the variation of $\mathcal{V}$ within a range $\mathcal{V} = (2-9)V_b$, usually considered in nuclear fragmentation models. With increase in the available volume, the average fragment size generally decreases after fragmentation, but we have checked that in the temperature range we explore, either the basic fragmentation pattern or the average fragment size does not change drastically even if the available volume is increased much beyond the range mentioned. Here, $V_b$ is the volume of the initial bulk matter with $r_b$ being its bulk radius parameter. For an approximate estimate of $V_b$, we consider the value of the bulk radius parameter at zero temperature which is $r_b \approx \left( \frac{m_b}{\pi n_b} \right)^{1/3}$ with the baryon number density $n_b \approx \frac{1}{3} \left[ \frac{m_b}{\pi} + \frac{m^3_b}{\alpha^2} (1 - \lambda_{\nu}^2)^{3/2} \right]$, where $\lambda_{\nu} = \frac{m_{\nu}}{m_b}$. The value of the quark chemical potential $\mu_b$ of the bulk SQM is approximated as one third of the parameterized form of its energy ($E_b$) per baryon (i.e. $\mu_b = \frac{1}{3}(E_b/A_b)$) at $T = 0$ MeV \cite{10, 22}, after the substitution of the appropriate value of $A_b$. The possible error involved in the above approximations, while determining $V_b$, is absorbed in the free parameter $\mathcal{V}$. For an approximate expression of the Debye shielding length $\lambda_D$, that is required for estimating $Z^i$ and $E^i_C$ from Eqs. (5) and (6), we assume $\lambda_D^{-2} \approx \frac{8\pi}{3} \alpha (\frac{d\mu}{d\mu_b})$ \cite{23}. 

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[10] A. definition

[22] B. assumption
For the baryon number of the initial bulk matter, we choose \( A_b = 1.2 \times 10^{53} \); this corresponds to a population averaged tidally released mass \( \sim 10^{-4} M_\odot \) per binary SS merger obtained in the simulations with a bag value that corresponds to \( B^{1/4} \approx 145 \text{ MeV} \). This value of \( B \) represents its lower bound determined by the fact that the energy per baryon of two-flavored quark matter must be higher than the one of \( ^{56}\text{Fe} \) \[16, 31\], i.e. \((E/A)_{u,d} > 930 \text{ MeV}\). Considering the limited accuracy of the MIT bag model, we may, as well, consider \( B^{1/4} = 145 \text{ MeV} \) as the most favorable choice of the bag constant for which the ordinary nuclei can decay into their strange quark phases only on a timescale longer than the age of the universe \[32\]. In Ref. \[7\], we considered this value of the bag constant to find the basic size distribution of the strangelet fragments; the standard bag value was taken to be represented by \( B^{1/4} \approx 145 \text{ MeV} \) in Ref. \[33\]. In the model calculations of unpaired SQM, the bag value is, however, a bounded parameter that may be varied within the range \( 145 \text{ MeV} \leq B^{1/4} \leq 160 \text{ MeV} \) with its upper limit approximately corresponding to the limit of absolute stability of bulk SQM at zero pressure and zero temperature \[6, 16, 33, 34\]. We have examined the consequence of this variation of bag value in this paper.

Before we present the numerical results, we would like to add that, as in the case of nuclear disassembly models, the derived size distribution of the fragments is sensitive to channel selection (i.e. the selection of baryon numbers of the fragments) in this case also. In Ref. \[7\], such channels were selected somewhat arbitrarily. In this paper, we instead consider all available positive integer values for \( A^i \) of the fragment species to arrive at the number of fragments (i.e. the multiplicity) pertaining to each specie. While selecting those channels, we also take the charge numbers of strangelets into account. For this, we round off the real values obtained from Eq. (5) to their nearest positive integers. The lower cutoff in the baryon number of a strangelet with \( m_s = 95 \text{ MeV} \) is chosen so that the corresponding charge number becomes \( Z^i \approx 1 \) after rounding off.

Figs. 1(a,b) compare the multiplicities of strangelets in two cases, namely \( m_s = 0 \text{ MeV} \) and \( m_s = 95 \text{ MeV} \), for a fixed bag value at a specific temperature at freeze-out. From these figures, it is apparent that the effect of \( m_s \neq 0 \text{ MeV} \) on the multiplicity distribution is not simply equivalent to an enhanced Boltzmann suppression as seems to have been recently suggested in Ref. \[37\]. The actual distribution arises due to a complex interplay of several factors. While the distribution for \( m_s = 0 \text{ MeV} \) starts from \( A^i = 1 \), the one for \( m_s = 95 \text{ MeV} \) starts from \( A^i \approx 11 \) that corresponds to \( Z^i \approx 1 \). Both the distributions have their peaks at \( A^i \approx 37 \) (see Fig. 1(b), an enlarged view of Fig. 1(a) in a smaller mass range). The lower cutoff at \( A^i \approx 11 \), along with the constraint imposed by Eq. (12), force the multiplicities for \( m_s = 95 \text{ MeV} \) to decrease more slowly.
with increasing $A' (> 37)$ in comparison with the ones for $m_s = 0$ MeV. We may also interpret the nature of distribution with $m_s = 95$ MeV in the following way. Apart from giving rise to a lower cutoff at $A' \approx 11$, incorporation of finite $m_s$ also leads to the suppression of lighter fragments and copious production of heavier fragments. This is due to the surface term that depends on finite mass of $s$-quarks and vanishes in the limit of massless quarks. It is known that the surface term represents the energy required for creating the surface whereas the curvature term represents the energy required for bending it. As a consequence of the additional surface term, the total (surface + curvature) requirement of energy for $m_s \neq 0$ MeV is more than the energy required for curvature alone in the case $m_s = 0$ MeV. More energy is, therefore, required to produce small fragments out of the bulk SQM with massive $s$-quarks. This has to be supplied from the limited reserve of thermal energy of the fragmenting system at a fixed temperature. In statistical multifragmentation, an increase in the total (surface + curvature) requirement of energy to form small strangelets results in copious production of larger fragments at the cost of smaller fragments in a way such that the total baryon number is conserved. The converse leads to an enhanced production of lighter fragments at the cost of heavier fragments. These features of multifragmentation appear consistently in our results both in Ref. [7] and in this paper. These features of the disassembly model are independent of whether we consider massless or massive quarks as should become more apparent from the following discussions. Our preliminary calculations presented in Ref. [10] suggest that this nature of fragmentation is also independent of the choice of CFL or unpaired strangelets.

Figs. 2(a,b) display the size distributions of strangelets with massive $s$-quarks for a fixed bag value $B^{1/4} = 145$ MeV at three different temperatures, namely $T = 1$ keV, $T = 10$ keV and $T = 1$ MeV, at freeze-out. The variation of size distribution with changing temperature is in qualitative agreement with the one obtained in Ref. [7] in the case of massless quarks. Suppression of heavier fragments and enhanced production of lighter fragments with increasing temperature are noted for $m_s = 0$ MeV and also for $m_s \neq 0$ MeV. The peaks of the distributions are separately plotted in Fig. 2(b) for a limited range of baryon numbers for the sake of clarity. The peak shifts to lower values of $A'$ with increasing temperature and becomes sharper. This peak, however, disappears at $T = 1$ MeV so that we have monotonically decreasing multiplicity from $A' \approx 11$ onwards. Such absence of the peak of the distribution function at high temperatures is commonplace in the results obtained in the case of nuclear fragmentation [12-14].

It is known that the surface free energies and the curvature energies of both the baryonic (i.e. nuclei) and the quasi-baryonic (i.e. SQM) fragments decrease with increasing temperature [37,38]. This, in turn, implies that the total requirement of (surface + curvature) energy to produce small strangelets out of the initial bulk matter is reduced at higher temperature. This reduced requirement of energy is easily met by a larger reserve of thermal energy of the fragmenting complex. This, along with the
condition for the conservation of baryon number, ensure copious production of lighter fragments and suppressed production of heavier fragments with increasing temperature. Such pattern of decreasing fragment-sizes with increasing temperature is in consonance with the standard results of nuclear fragmentation models [12, 14, 20]. Recent discussion on fragmentation in Ref. [35] finds an opposite tendency in the variation of size distribution of CFL strangelets with changing temperature. The authors of Ref. [35] seem to attribute this behavior of the fragmentation derived by them to the finite mass of s-quarks combined with the color-superconductivity of the strangelets. It is relevant here to add that an earlier exploratory work [19] of ours found that the changes in the frequency distribution of CFL strangelets having massless quarks with changing temperature are in qualitative agreement with Figs. 2(a,b).

In Fig. 3, we examine the influence of bag values on the fragmentation pattern of strangelets for $m_s = 95$ MeV at a fixed temperature at freeze-out. With other parameters remaining the same, an enhanced bag value increases the effective quark chemical potential in the fragmentation complex that, in turn, increases the surface and curvature energies of the strangelet fragments. This obviously enhances the energy requirement for the formation of lighter fragments, as a consequence of which the production of heavier fragments at the cost of the lighter ones is preferred. Small variation of the fragmentation pattern with the variation of bag value in Fig. 3 may also be interpreted in terms of an increase in the quark chemical potential $\mu_b$ of the initial bulk SQM due to an increase in the value of the bag parameter $B$. The resulting increase in the baryon number density of the initial bulk matter [16] would favor larger fragments in agreement with the standard results of nuclear fragmentation models [20]. In Fig. 3, the lower cutoff (corresponding to $Z^i \approx 1$) in the baryon number of the distribution changes from $A^i \approx 11$ for $B^{1/4} = 145$ MeV to $A^i \approx 13$ for $B^{1/4} = 160$ MeV. Corresponding baryon numbers, at which the peaks of the distributions are obtained, are about 37 and 39, respectively, in the two cases.

The comparison displayed in Fig. 3 is simply a convenient way of demonstrating the effect of bag values on the fragmentation pattern of bulk SQM in which we keep the initial baryon number $A_b$ fixed for both the bag values. Preliminary simulations [8] of SS mergers, however, found no mass ejection in the case $B^{1/4} \approx 160$ MeV that is due to the resulting compactness of the merging SSs. We may, therefore, assume that the ejected mass, averaged over the possible galactic population of coalescing SSs, would, at the most, be just below the limit of mass resolution $\sim 10^{-5}M_\odot$ [8, 6] of those simulations in this particular case. We have checked that the shape of the fragmentation pattern corresponding to a particular bag value remains almost invariant for such reduced mass of the initial bulk matter except that each of the multiplicities goes down by one order of magnitude approximately. Such scaling makes it convenient to estimate the possible fluxes of strangelets in PCR that correspond to various mass distributions of strangelets injected in the Galaxy for various bag values.

To probe the influence of bag values on the stability of strangelet fragments, we examine the energy per baryon $(E^i/A^i)$ of the fragment species as a function of its baryon number $A^i$. This is performed for four different values of the bag parameter, namely $B^{1/4} = 145$ MeV, $B^{1/4} = 155$ MeV, $B^{1/4} = 158$ MeV and $B^{1/4} = 160$ MeV respectively, at different temperatures at freeze-out. Fig. 4 displays the results at the upper bound $T = 1$ MeV of the temperatures considered in this paper. The stability of strangelets is known to decrease at higher temperatures [39]. The condition of mechanical equilibrium of the strangelets under the pressure exerted by external electrons is taken into account in the results of the calculations displayed in Fig. 4. These results are expected to differ from the ones in Ref. [39] as the external pressure of electrons was not considered there. In Fig. 4, we, however, indicate the value of the quark chemical potential $(\mu_0)$ against each bag value with zero external pressure $(P_{\text{ext}} = 0)$ on the strangelets for convenient comparison. The results in Fig. 4 also differ in detail from the ones in Ref. [7] in which the mass of the $s$-quarks and the resulting charges of the strangelets were ignored. In Fig. 4, we find that all the strangelets having $A^i \geq 11$ are stable relative to $^{56}$Fe nucleus for $B^{1/4} = 145$ MeV.

![Graph](image-url)
FIG. 4: (color online) Variation of energy per baryon ($E/A$) against changing baryon number ($A$) of the strangelet fragments with four different values of the bag parameter at a specific temperature $T = 1$ MeV at freeze-out. Here, the horizontal lines indicate the energy per baryon of $^{56}$Fe nuclei, protons and $\Lambda^0$-hyperons, respectively. These lines are included in the diagram to display the stability of the strangelets with respect to the above particles. Available volume is taken as $V = 5V_b$. Here, $A_b = 1.2 \times 10^{53}$.

For $B^{1/4} = 155$ MeV, the compulsion of having integral charge number forces the constraint $A_i \geq 12$ on the strangelets. Here, the strangelets with their baryon numbers lying in the range $12 \leq A^i \leq 23$ are metastable; these strangelets are unstable against nucleons but more stable than the $\Lambda$ particles with their energies per baryon lying in the range $939$ MeV $\leq E_i^i/A^i < 1116$ MeV. Strangelets with $B^{1/4} = 155$ MeV are stable with respect to nucleons (i.e. $930$ MeV $\leq E_i^i/A^i \leq 938$ MeV) for their baryon numbers in the range $24 \leq A^i \leq 40$. Strangelets with such a bag value are absolutely stable (i.e. $E_i^i/A^i < 930$ MeV) for $A^i \geq 41$. In contrast, the strangelets with their bag values given by $B^{1/4} = 158$ MeV are stable with respect to nucleons for their baryon numbers satisfying $A^i \geq 95$. These strangelets are stable compared to $^{56}$Fe nuclei for $A^i \geq 400$. For $B^{1/4} = 160$ MeV, on the other hand, all the strangelets with $A^i \geq 13$ are metastable; their $E_i^i/A^i$ are greater than those of the nucleons but much smaller than the one for the $\Lambda$ particles. The actual value of the bag parameter is still an unsettled issue. For the purpose of this paper, we consider bag values in the range $145$ MeV $\leq B^{1/4} \leq 158$ MeV with the corresponding minimum values of the baryon number lying in a range $11 \leq A^i \leq 95$ to make it sure that the selected strangelets are stable relative to nucleons. We assume those strangelets to be the possibly available ones in PCR in solar neighborhood with the necessary caution that such a selection criterion may be somewhat restrictive. This is because of the fact that the liquid-drop model of the strangelets, that have been employed in this paper, is only an approximation. More rigorous but tedious shell model calculations are found to have stabilizing effect on the strangelets [16]. Lowering the temperature also pushes the lower bound of the window of stability towards smaller baryon numbers. Most importantly, all the bag values that place the energy per baryon of the strangelets in the vicinity of that of nuclear matter cannot, perhaps, be discarded. The precise values of $E_i^i/A^i$ of the strangelets, i.e. whether they lie marginally above the nucleon mass or below the energy per nucleon in $^{56}$Fe, is a matter that involves only $\sim 1\%$ deviation in the numerical calculations [32]. Such deviation may be insignificant in view of the uncertainties in the accuracy of the results derived from the MIT bag model [32]. Keeping these factors in mind, we may perhaps extend the upper bound of the plausible bag values to $B^{1/4} \sim 160$ MeV. For a freeze-out temperature $\sim 10$ keV, for example, the energies per baryon of large ($A^i \sim 600$) strangelets with such a bag value are already too close to nucleon mass, whereas, the size distribution of strangelets extends to $A^i \sim 4500$ in this particular case. Presence of a small number of such large strangelets with bag values $B^{1/4} \sim 160$ MeV in PCR cannot, therefore, be ruled out at the outset. It is also relevant here to add that the large ($A^i \gtrsim \text{few} \times 100$) strangelets considered here have their surface tension $\sim 5 - 10$ MeV/fm$^2$, that easily exceeds the critical surface tension $\sim 0.1$ MeV/fm$^2$ required for their stability against fragmentation (or fission) instability proposed in Ref. [27] in the particular case $m_s \sim 100$ MeV.

V. DISCUSSION

If strange matter hypothesis (SMH) [2] is valid, all compact stars are likely to be the strange stars and the debris of collisions between those stars in the Galaxy may be a major source of strangelets in PCR [4, 16]. In Ref. [7], we attempted to find a rate of galactic injection of those strangelets of various baryon numbers by using the simplifying limit of massless quarks in the analysis. The basic nature of fragmentation was described in that paper. In this paper, we have improved upon that earlier work by incorporating the effects of finite mass of strange quarks as well as a wider range of permissible bag values. Both in Ref. [7] and in this paper, we restrict ourselves to the scenario of strangelet production from SS mergers. Other possibilities, such as the fragmentation of SQM ejected by shock waves during SNe II explosions [10], have been left out of consideration. We, however, believe that the
plausible mass distribution of strangelets, that we find here, may be relevant for those cases as well.

After determining the size distribution of strangelets at their source, we revise our earlier estimate of the flux of those strangelets in the vicinity of the Sun by employing a simple diffusion approximation for their propagation in the Galaxy. Assuming a rate \( \sim 10^{-5} \text{ yr}^{-1} \) Galaxy\(^{-1} \) of SS mergers and assuming the strangelets to spread homogeneously in a galactic halo of radius \( \sim 10 \text{ kpc} \) in their galactic confinement time, an approximate magnitude of the intensity of strangelets of the \( i \)-th specie in solar neighborhood was written in Ref. \( 7 \) as

\[
I(A^i) \sim 5 \times 10^{-48} \omega^i \text{ particles m}^{-2} \text{ sr}^{-1} \text{ yr}^{-1}
\]  

(13)

with \( \omega^i \) being the multiplicity given in Eq. (2). The approximation (13) provides only a tentative order of magnitude estimate. Important issue of the acceleration of strangelets in astrophysical shock waves has been left out of consideration in this approximation. The diffusion coefficient of galactic strangelets has not been derived from rigorous calculations while making such estimate. This expression also ignores the possibility of interaction of strangelets with the particles in the interstellar medium. Moreover, it does not take the effects of geomagnetic field and solar modulation into consideration. In the particular case \( B^{1/4} \approx 145 \text{ MeV} \), simulations find a population averaged tidally released mass \( \sim 10^{-4} M_{\odot} \) per SS merger so that a summation of the estimate (13) over the values of \( \omega^i \), obtained from the results displayed in Fig. 2, yields an integrated intensity of all of the \( A^i \geq 11 \) stable strangelets. The values of this integrated intensity lie in the range \( \sim 3 \times (10^3 - 10^4) \) particles m\(^{-2} \) sr\(^{-1} \) yr\(^{-1} \) depending on the formation temperature of the strangelets. Changing the available volume in the prescribed range \( (V = (2 - 9)V_\odot) \) alters the fragmentation pattern somewhat, but does not affect appreciably the integrated flux of those strangelets. Increasing the bag value to \( B^{1/4} \approx 155 \text{ MeV} \), may, however, have an appreciable effect. In that case, we are required to reduce the average tidally released mass per stellar merger to any value within a range \( (10^{-5} - 10^{-4}) \) \( M_{\odot} \) to comply with the results of the recent simulations. By using the scaling argument given in Sec. IV, the integrated intensity of \( A^i \geq 24 \) stable strangelets is thus found to lie in a range \( \sim 3 \times 10^3 - 7 \times 10^3 \) particles m\(^{-2} \) sr\(^{-1} \) yr\(^{-1} \). Similarly, a bag value corresponding to \( B^{1/4} \approx 158 \text{ MeV} \) and an average tidally released mass \( \sim 10^{-5} M_{\odot} \) yield an integrated intensity with its values within the range \( \sim 10^{-2} \) - \( 2 \times 10^2 \) particles m\(^{-2} \) sr\(^{-1} \) yr\(^{-1} \) for various temperatures at freeze-out. Most of these estimates are within the range of detectability of the currently launched AMS-02 experiment with its limit of resolution being \( \sim 1 \) particle m\(^{-2} \) sr\(^{-1} \) yr\(^{-1} \).

The above estimates predict measurable flux of the stable, unpaired strangelets within a restricted range of values \( 145 \text{ MeV} < B^{1/4} < 158 \text{ MeV} \) of their uncertain bag parameter. Such a range of bag values was earlier considered in Refs. \( 5, 6, 31, 43 \) while examining the structure and stability of the SSs. The upper bound of this range may possibly be extended to \( B^{1/4} \sim 160 \text{ MeV} \) in view of the uncertainties involved in the model calculations. Such a limited window of stability of the unpaired strangelets in the parameter space possibly led the authors of Ref. \( 35 \) to conclude that the CFL strangelets, instead of the unpaired strangelets, should alone have the possibility of being detected in PCR due to their absolute stability over wider range of parameter values. Accordingly, these authors tried to find the size distribution of CFL strangelets by adapting a nuclear liquid-gas phase transition model \( 14 \). Recently, these authors have reported an inconsistency in their results \( 35 \). Here, we wish to point out that the statement regarding the supposed stability of CFL strangelets is required to be appropriately qualified. For example, the value of the pairing energy gap (i.e. the gap parameter \( \Delta \)) of CFL SQM is uncertain \( 18 \), thus increasing the number of uncertain parameters in the calculations. Moreover, if we assume a “not unreasonable” \( \Delta = 100 \text{ MeV} \) for the gap parameter at zero temperature, the lower bound of bag values for the absolute stability of CFL SQM at \( T = 0 \text{ MeV} \) increases to \( B^{1/4} > 156 \text{ MeV} \) to avoid spontaneous nuclear decay to a two-flavor color superconducting (2SC) state \( 18 \). The results in Ref. \( 18 \) further suggest that, for bag values as large as \( B^{1/4} \sim 180 \text{ MeV} \), seemingly unphysical gap parameters, much in excess of 100 MeV, may be required for the stability of CFL strangelets. To make the case worse, the value of \( \Delta \) decreases appreciably with increasing temperature \( 17 \), so that, a very large value of \( \Delta \) (corresponding to \( T = 0 \text{ MeV} \)) may be required for the absolute stability of CFL strangelets at temperatures typically considered in Ref. \( 35 \), particularly for large bag values. Thus, CFL strangelets have their own limited window of stability like the one obtained in the case of unpaired strangelets.

Concern may also be raised over the possible astrophysical sources of CFL strangelets. Hydrodynamic simulations of the combustion of NS into a quark star \( 10 \) have shown that no CFL SQM may be ejected outside the surface of the star; the conversion front stops before it reaches the stellar surface. On the other hand, the present scenario of tidally released quark matter in SS merger cannot, perhaps, be simply extended to CFL strangelets. This is due to the recent argument \( 47 \) against the possibility of the observed cold compact stars...
being the bare CFL stars (CFLSs). To circumvent this problem, Ouyed et al. [48] have invoked a scenario of collisions of the hot and young CFLSs with their NS companions in the compact binary stellar systems of the Galaxy that may produce CFL strangelets. These authors find an estimate (∼10^2 − 10^4 particles m^-2 sr^-1 yr^-1) for the flux of CFL strangelets in solar neighborhood that is similar to the fluxes obtained in our calculations. The detailed derivation of such fluxes are, however, unavailable in Ref. [48]. An extrapolation of our above estimates corresponding to B^{1/4} ∼ 156 MeV, after the substitution of the possible lower bound ∼ 10^-7 yr^-1 [48] of the rate of CFLS-NS collisions in the Galaxy with an assumed tidally released CFL mass ∼ 10^-5 M⊙ in each of such collisions in approximation (13), would bring down the integrated intensity of CFL strangelets in solar neighborhood to ∼ (1−10) particles m^-2 sr^-1 yr^-1. Extrapolation of the results of the recent simulations of SS merger [5, 6] further suggests that, due to the supposedly compact nature of the CFLSs, there is a possibility that almost the entire product of such CFLS-NS merger may collapse into black-hole before the tidal forces have sufficient time to eject appreciable CFL mass out of the gravitational influence of the combined system. With the existing sensitivity of the AMS-02 detector, we would hardly expect to detect CFL strangelets in PCR in that case.

The ultimate vindication of SMH would perhaps depend on the detection of either unpaired or CFL or both the types of strangelets in PCR. In this paper, we have examined the rate of injection of the plausible size distribution of unpaired strangelets in the Galaxy. A separate study of the possible mass distribution of CFL strangelets at their source as well as an examination of the sophisticated galactic propagation models for both the types of strangelets are required to arrive at a definite prediction of strangelet flux in PCR for AMS-02 and other potential experiments.

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