Electroweak symmetry breaking by a neutral sector: 
Dynamical relaxation of the little hierarchy problem

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We propose a new dynamical relaxation mechanism of the little hierarchy problem, based on a singlet extension of the minimal supersymmetric standard model (MSSM). In this scenario, the small soft mass parameter of an MSSM singlet is responsible for the electroweak symmetry breaking and the non-zero Higgs vacuum expectation value, whereas the effect of the large soft mass parameter of the Higgs boson, \(-m_{h_u}^2\), is dynamically compensated by a flat direction of the MSSM singlets. The small singlet’s soft mass and the Z boson mass can be protected, even if the stop mass is heavier than 10 or 20 TeV, since the gravity-mediated supersymmetry breaking effects and the relevant Yukawa couplings are relatively small. A “focus point” of the singlet’s soft mass parameter can emerge around the stop mass scale, and so various fine-tuning measures can reduce well below 100. Due to the relatively large gauge-mediated effects, the MSSM superpartners are much heavier than the experimental bounds, and the unwanted flavor changing processes are adequately suppressed.

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One of the long standing problems in theoretical particle physics is the gauge hierarchy problem. It is basically a naturalness problem associated with the relatively small Higgs boson mass and the resulting electroweak (EW) interaction scale much lower than a ultraviolet (UV) cutoff energy scale, below which the standard model (SM) can be valid. For last four decades, the question how the small Higgs boson mass can naturally be maintained against the large quantum corrections without a fine tuning has encouraged many physicists to propose various UV theories embedding the SM just above the EW scale. Thus, a new physics has been expected to be present around the EW scale, by which the counter operators are provided to cancel the quadratic divergences appearing in the radiative corrections to the Higgs mass parameter, and renormalize it. Otherwise, a fine-tuning associated with its renormalization becomes serious.

In particular, introduction of supersymmetry (SUSY) at the EW scale has been accepted as the most promising way to resolve the problem [1,2]. In SUSY theories, the needed counter terms in the Lagrangian are dynamically generated by superpartners at the SUSY scale. In the minimal supersymmetric standard model (MSSM), moreover, the renormalization group (RG) evolutions of the three SM gauge coupling constants turned out to be precisely unified around 10^{16} GeV energy scale, when the superpartners’ contributions to them are included [3,4]. It might be an evidence of the presence of a theory unifying all the SM gauge interactions at that scale, and so the MSSM has been regarded as a guiding model leading to such a grand unified theory (GUT).

In supergravity (SUGRA) models, the EW symmetry is radiatively broken at low energy through the RG effect on the Higgs soft mass parameter \(m_{h_u}^2\) due to the large top quark Yukawa coupling. As seen in the following two extreme conditions of the scalar potential for the two Higgs bosons, \(h_u\) and \(h_d\) in the MSSM [1,4],

\[
|\mu|^2 + \frac{1}{2} M_Z^2 = \frac{m_{h_u}^2 - m_{h_d}^2 \tan^2 \beta}{\tan^2 \beta - 1},
\]

\[
2|B\mu| + M_Z^2 \sin 2\beta = (m_{h_u}^2 - m_{h_d}^2) \tan 2\beta,
\]

non-zero Higgs vacuum expectation values (VEVs) and the Z boson mass \(M_Z^2 = (g_3^2 + g_2^2)((h_u)^2 + (h_d)^2)/2\) are generated with \(\pi/4 < \beta < \pi/2\), when \(m_{h_u}^2\) becomes negative via its RG evolution at low energies. Here \(\mu\) and \(B\mu\) denote the mass of the Higgsinos (superpartners of the Higgs scalars) and its corresponding soft mass parameters (“B-term”) and \(\tan \beta \equiv (h_u)/(h_d)\) is the ratio of the VEVs of the two Higgs doublets. In SUSY models, thus, \(m_{h_u}^2\), \(|\mu|^2\) and \(M_Z^2\) are required to be of a similar size for the naturalness of the Z boson mass. Although the energy scale the LHC probes has been raised higher and higher so far, however, any new physics signal has not be observed yet. It implies that as the UV cutoff scale of the SM becomes higher and higher, the fine-tuning problem for the Higgs mass parameter is being serious more and more. In fact, all the theoretical puzzles raised in the SM still remain unsolved at the moment.

A barometer of the naturalness of the MSSM is the stop (superpartner of the top quark) mass: A too heavy stop mass induces a large value of \(m_{h_u}^2\), which requires a fine-tuning with other parameters in Eq. (1) to get the Z boson mass of 91 GeV. However, the experimental stop (gluino) mass bound has been already exceeded 1 TeV (2 TeV) [5], by which a fine-tuning of sub-percent level seems to be needed already. Moreover, the observed Higgs boson mass, 125 GeV [6] is too heavy as a SUSY Higgs boson mass, because it requires a too heavy stop mass for explaining it. According to the recent theoretical analyses based on three loop calculations, 10–20 TeV stop mass is necessary in the MSSM for explaining the 125 GeV Higgs mass without a quite large stop mixing effect [6]. Accordingly, a fine-tuning of order 10^{-3} or 10^{-4}
seems to be unavoidable, even if there exists SUSY at 10–20 TeV scale. It is called “little hierarchy problem.”

Apart from such a fine-tuning problem, some phenomenological problems were also pointed out in two representative SUSY breaking scenarios [1]. In gravity mediation scenario, where all the scalar fields obtain SUSY breaking soft masses, sizable flavor changing neutral currents (FCNC) are generically admitted. On the other hand, in gauge mediation, where only scalar fields carrying the SM gauge charges acquire the soft masses at the leading order, it is hard to get the $\mu/B\mu$ terms of desirable size, while the flavor problem doesn’t arise.

In this letter, we will discuss the possibility that the EW phase transition is triggered by a mass parameter of a singlet rather than $m_{h_u}^2$ in the SUSY framework. We will employ both the gravity- and gauge-mediated SUSY breaking scenarios, assuming the gauge-mediated effects dominate over the gravity-mediated ones such that all the super particles carrying the SM gauge quantum numbers are made much heavier than the experimental bounds and unwanted flavor changing processes are sufficiently suppressed. As mentioned above, 10–20 TeV stop mass could explain the observed Higgs mass well. On the other hand, the soft masses of SM singlets remain relatively small in this case, since they are generated only by the gravity mediation. Hence, the scale of the EW symmetry breaking can be much lower than the ordinary MSSM SUSY particles’ mass scale. We could restore the traditional radiative EW symmetry breaking scenario with it.

As a benchmark model, let us consider the following form of a singlet extension of the MSSM in the superpotential:

$$W \supset (\lambda_1 X + \lambda_2 \phi + \mu) h_u h_d + MXY + \frac{\kappa}{2} Y \phi^2. \tag{3}$$

where $X$, $Y$, and $\phi$ denote newly introduced singlet superfields inert under the MSSM gauge interaction. Here $M$ is a mass parameter of order 1–10 TeV, while $\{\lambda_1, \lambda_2, \kappa\}$ are dimensionless coupling constants. As seen in the first two terms of Eq. (3), the MSSM $\mu$ term is promoted to the trilinear couplings among $X$, $\phi$, and the MSSM Higgs fields apart from the bare $\mu$ term. In Eq. (3), we ignored the existence of e.g. $\phi Y$ term, assuming its dimensionful SUSY coupling is small enough, because it is not crucial in our analysis. Nonetheless, this superpotential does not admit any accidental symmetry. As will be seen below, instead, a flat direction is found in the SUSY limit in this model. Eq. (3) could be a remnant of the $U(1)_{\text{PQ}}$ breaking mechanism at an intermediate scale [2].

As a UV model, one can consider, for instance,

$$W_{\text{UV}} \supset \Psi (y_1 X + y_2 X_2) Z + y_3 \Psi^c Z \phi + (y_4 X_1 + y_5 X_2) h_u h_d + \frac{(\Psi^c)^2}{M_P} (y_6 X_1 + y_7 X_2) Y + \frac{\kappa}{2} Y \phi^2, \tag{4}$$

where $M_P$ denotes the reduced Planck mass ($\approx 2.4 \times 10^{18} \text{GeV}$), and two $X$s, i.e., $\{X_1, X_2\}$, and the spurion fields $\{\Psi, \Psi^c\}$ breaking the $U(1)_{\text{PQ}}$ are introduced. The global $U(1)_{\text{PQ}}$ charge assignment is presented in Table I. We suppose that the scalar components of $\{\Psi, \Psi^c\}$ develop non-zero VEVs at an intermediate scale inside the “axion window” [5], say, of order $10^{11} \text{GeV}$. By non-zero VEVs of the scalar components of $\{\Psi, \Psi^c\}$, the $U(1)_{\text{PQ}}$ is completely broken, and $Z$ and one combination of $X_{1,2}$ $[= (y_1 X_1 + y_2 X_2)/\sqrt{y_1^2 + y_2^2} \equiv X_H]$ become superheavy. Integrating out such heavy superfields leaves Eq. (4), in which $X$ is identified with the light mode of $X_{1,2}$ orthogonal to $X_H$, and $\lambda_2$ is proportional to $y_2 (\Psi^c)/\langle \Psi \rangle$. The mass term of $X$ and $Y$ in Eq. (3), is generated from the non-renormalizable term in Eq. (4). A mass term of $\phi$ and $Y$ is also induced. However, it turns out to leave intact the existence of the flat direction: It just deform it. We will ignore the term just for simplicity. In a similar way, the bare $\mu$ term in Eq. (3) can also be generated with more spurion fields.

The resulting scalar potential with Eq. (3) is given by

$$V \supset |\lambda_1 X + \lambda_2 \phi + \mu|^2 |H|^2 + \left| \frac{\kappa}{2} \phi^2 + M \right|^2 + m_X^2 |X|^2 + m_Y^2 |Y|^2 + m_\phi^2 |\phi|^2 + \left\{ (A_1 X + A_2 \phi + B Y h_u h_d + M b X Y + \frac{\kappa}{2} a Y \phi^2 + \text{h.c.}) \right\},$$

where $|H|^2 \equiv |h_u|^2 + |h_d|^2$, and $\{m_X^2, m_Y^2, a, b, A_1\}$ are soft SUSY breaking parameters. Here we assume a hierarchy between $M$ and such soft parameters of the MSSM singlets, $|M|^2 \gg |m_X^2, m_Y^2, a, b, A_1|^2$, $|b|^2, |B|^2$. In addition, we will regard these MSSM singlets’ soft parameters as being relatively suppressed also than the ordinary MSSM soft (squared mass) parameters. It can be realized if the gravity-mediated SUSY breaking effects are relatively suppressed than the gauge-mediated ones. Here we note that a flat direction, $\kappa \phi^2/2 + M X = 0$ with $\langle h_u \rangle = \langle h_d \rangle = \langle Y \rangle = 0$ exists in the SUSY limit, since the Higgs gets a VEV only by a soft mass parameter as will be seen below. Accordingly, the VEVs of $\phi$ and $X$ can be arbitrarily large in this limit. The flat direction is lifted only by small soft parameters. From Eqs. (3) and (5), the effective $\mu$ and $B\mu$ parameters read as follows:

$$\mu_{\text{eff}} = \lambda_1 \langle X \rangle + \lambda_2 \langle \phi \rangle + \mu, \tag{6}$$

$$B\mu_{\text{eff}} = (\lambda_1 M^* + \lambda_2 k^* \langle \phi \rangle^*) \langle Y \rangle^* + A_1 \langle X \rangle + A_2 \langle \phi \rangle + B \mu,$$

which replace $\mu$ and $B\mu$ in Eqs. (1) and (2) in our case.

| Superfields | $X_{1,2}$ | $Y$ | $Z$ | $\phi$ | $\Psi$ | $\Psi^c$ |
|------------|---------|-----|-----|-------|-------|-------|
| $U(1)_{\text{PQ}}$ | $-1$ | $4/3$ | $5/6$ | $-2/3$ | $1/6$ | $-1/6$ |

TABLE I. Extra MSSM Singlet Superfields charged under $U(1)_{\text{PQ}}$. The ordinary MSSM superfields including the two Higgs doublets should carry proper $U(1)_{\text{PQ}}$ charges.
From Eq. (5) the extreme conditions for $X$, $Y$, and $\phi$ are derived as follows:

$$
\begin{align*}
M_X^2 X + M^* b^* Y^* &= -\frac{\kappa}{2} M^* \phi^2 - (\lambda_2 \phi + \mu) \lambda_1^* |H|^2 - \frac{\kappa \phi}{2} M^* b^* h^* h^d,
M_Y^2 Y^* + M b X &= -\frac{\kappa}{2} a \phi^2 - \frac{\lambda_3}{2} M^2 + \frac{\lambda_1}{2} X + \mu \lambda_2^* h^* h^d + \frac{\lambda_3}{2} X + \mu \lambda_2^* h^* h^d,
(\kappa |Y|^2 + |\lambda_2 H|^2 + m^2_{\phi}) \phi + \frac{(\kappa \phi - \lambda_3 X + \mu \lambda_2^* h^* h^d)}{2},
+ (\lambda_1 X + \mu) \lambda_2^* H^2 + A h^* h^d + (\lambda_2 h^* h^d + \kappa \phi^2) Y^* &= 0.
\end{align*}
$$

For brevity, here, we introduced $M_X^2$ and $M_Y^2$ as $M_X^2 = |\lambda_1 H|^2 + m^2_{\phi} + |M|^2 \approx |\kappa \phi|^2 + m^2_{\phi} + |M|^2$, respectively. Together with Eqs. (1) and (2), thus, we should solve the five coupled equations in total. We should first note that in a large limit of $\phi$ and $X$ (i.e., $|A_2|, |\kappa Y|$) with $\kappa \phi^2/2 + M X \approx 0$, the VEV of $H$, $H$, is constrained to roughly be of order $m_{\phi}/\lambda_2$, $\kappa Y$, or $\alpha \kappa Y$ from the third equation. We will see it more clearly below.

For a large enough $M$ and $\phi$, the solutions of $X$ and $Y$ to the first two equations in the above can approximately be expressed in terms of $\phi$ and $H$:

$$
X \approx -\frac{-\kappa \phi^2}{2 M_X^2} M^* \left[ 1 - \frac{(a - b) b^*}{M_Y^2} \right],
Y^* \approx -\frac{-\kappa \phi^2}{2 M_X^2} (a - b) - \frac{(\lambda_1 M + \lambda_2 \kappa \phi) h^* h^d}{M_Y^2}.
$$

Then the flat direction, $(\kappa/2) \phi^2 + M X \approx 0$ is lifted to $(\kappa/2) \phi^2 + M X \approx (\kappa \phi^2/2 M_Y^2)|\lambda_1 H|^2 + m^2_{\phi} + (\kappa^2/2 M_Y^2)(a - b) b^* - (2 M/\kappa \phi)(\lambda_2 \phi + \mu) \lambda_1^* |H|^2)$. Plugging them into Eq. (5), the quartic terms of $\phi$ and the Higgs scalar as $(\kappa \phi^2/2 M_Y^2)|\lambda_1 H|^2 \approx 0$ are induced. They are helpful for raising the Higgs boson mass.

Inserting the above expressions into the third equation, we get the equation for $\phi$ or $T_c$ ($\equiv \kappa \phi/M$):

$$
\frac{T_c^2}{2} (|\lambda_1 H|^2 + m^2_{\phi}) - T_c \left[ \lambda_2 + \frac{\mu}{\lambda_1} \right] \lambda_1^* |H|^2 - T_c^2 \lambda_2^2 \lambda_1 |H|^2 + \frac{\mu}{\phi} \lambda_1^* |H|^2 + \frac{(\lambda_2 H^2 + m^2_{\phi})}{2 T_c^2} \frac{T_c^2}{2} + \frac{2}{4} |(\lambda_2^* H^2 + m^2_{\phi}) - (a - b)|^2,
$$

$$
|H|^2 \approx -\frac{m_{\phi}^2}{2} \phi \frac{(m_{\phi}^2 - |a - b|^2) T_c^2}{2 \lambda_2 + \frac{\mu}{\lambda_1} \lambda_1^* |H|^2} - \frac{\lambda_2 - \lambda_2^* T_c}{2},
$$

unless $\langle \phi \rangle = 0$. Here we set $f_T = 1 + |T_c|^2/2)/(1 + |T_c|^2/2), which drops from 1 to 0 as $|T_c|$ increases. We note here that $H$ can develop a nonzero VEV, when $m_{\phi}^2$ (and/or $m_{\phi}^2$) becomes negative for a positive denominator in Eq. (5). Otherwise, $\langle \phi \rangle$ should be zero. Then, $|H|$ should also vanish particularly for $|\mu|^2 \gtrless -m_{\phi}^2$. We will see later that $\langle \phi \rangle = \langle H \rangle = 0$ is a saddle (stable) point when $m_{\phi}^2$ is negative (positive).

We should note also that unlike in the MSSM the size of $|H|^2$ is basically of order $m_{\phi, \phi}^2/|\lambda_1 \lambda_2|$, rather than $O(m_{\phi, \phi}^2)$ for almost whole range of $T_c$, only if $M$ and $\phi$ are large enough. In this model, therefore, their smallness is responsible for the smallness of the Higgs VEV and eventually the Z boson mass. As mentioned above, their smallness could be protected for relatively low scale of the gravity-mediated SUSY breaking, namely, $F_{\text{grav}}/(\sqrt{3} M_p) \ll F_F/(16\pi^2 M_\Lambda)$. Here $F_{\text{grav}}$ and $F_F$ denote the SUSY breaking sources in a hidden sector whose effects are mediated to the observable sector through the gravity and the SM gauge interactions, respectively, and $M_\Lambda$ stands for the messenger scale.

Accordingly, the extreme condition of the Higgs fields, Eq. (1) should be met by the modulus-like field $\phi$: $\langle \phi \rangle$ and $(\langle X \rangle)$ should compensate the large value of $-m_{\phi}^2$ in Eq. (3). As a result, the Higgsino mass $\mu_{\text{eff}}$ is necessarily large in this model, of order $m_{\phi, \phi}^2/|\lambda_1 \lambda_2|$. It is a salient feature of this model, distinguished from other SUSY models pursuing the naturalness [9, 10], or even the split SUSY model [11]. While $\mu_{\text{eff}}$ is quite large, $(|\mu_{\text{eff}}|^2 + m_{\phi, \phi}^2)$ is just of order $M_Z^2$, particularly for large $\tan \beta$. Hence, the EW breaking conditions [1, 4, 11] are easily satisfied.

In terms of $T_c$, $\mu_{\text{eff}}$ in Eq. (9) is presented as

$$
\mu_{\text{eff}} \approx \frac{MT_c}{\kappa} \left( \lambda_2 - \frac{1}{2} \lambda_1 T_c \right) + \mu.
$$

Particularly, if $|M \lambda_2/\kappa|^2$ is much larger (smaller) than $-m_{\phi}^2$, then $T_c$ should dynamically be adjusted to a small (large) value, fulfilling Eq. (11). Although the bare $\mu$ is larger than $-m_{\phi}^2$, it can still be true for $|M \lambda_2/\kappa| > |\mu|$. Note that in this case $|B_{\mu_{\text{eff}}}|$ would be much larger than $|B| = |\mu|$. Actually, the size of $T_c$ depends on the given SUSY parameters such as $M$, $\mu$, $\kappa$, $\lambda_1 \lambda_2$, etc. For $|T_c| \approx 1 \ (|T_c| > 1)$, $H^2$ should decrease the coefficient of the quadratic (quartic) term of $\phi$ in the effective potential, i.e., $|\lambda_3 H|^2 + m_{\phi}^2 \approx 0 \ (|\lambda_1 H|^2 + m_{\phi}^2 \approx 0)$, $|T_c| \approx 1 \ (|T_c| > 1)$, thus, the Higgs VEV is simply given by $\sqrt{-m_{\phi}^2/|\lambda_3|^2}(\sqrt{-m_{\phi}^2/|\lambda_1|^2})$ in Eq. (6).

Although $m_{\phi, \phi}^2$ remain light enough at the messenger scale where the gauge mediation effects come in the observable sector, however, they possibly become much heavier through their RG evolutions below the messenger scale. It is because the singlets, $X$ and $\phi$ are coupled to the MSSM Higgs fields in Eq. (3), and $m_{\phi, \phi}^2$ become quite heavy below the messenger scale by the gauge mediation effects of SUSY breaking. To keep the smallness of $m_{\phi, \phi}^2$, therefore, the coupling constants $\lambda_1 \lambda_2$ need to be small enough and/or the messenger scale to be low enough. With a small enough $\lambda_2 (\ll \lambda_1 \lesssim 1)$, e.g., we can get a sufficiently small $m_{\phi}^2$, and so we will attempt to explain the small Higgs VEV with $m_{\phi}^2$ in Eq. (6). Even if $m_{\phi}^2$ is quite sizable, its contribution to Eq. (6) can still
be suppressed by an extremely small $T_\xi$ i.e. by a quite large value of $M$. Moreover, one can introduce other sizable couplings between $X$ and another heavy singlet sector such that $m_X^2$ is small at low energies, with leaving almost intact $m_\phi^2$ and the MSSM soft parameters. Thus, we will assume the first term is dominant in Eq. 5.

A relatively small value of $\lambda^2$ can make a “focus point” of $m_\phi^2$ emerge around the stop mass scale. The two figures in FIG. 1 show the RG evolutions of $m_\phi^2$ under its various trial values at the GUT scale [$t = \log(Q/\text{GeV}) \approx 37$] with $\lambda_1^2 = 0.5$, $\lambda_2^2 = 8 \times 10^{-3}$, and $\tan \beta = 40$. As seen in the both figures, two focus points of $m_\phi^2$ appear. The left vertical dotted lines indicate the 20 TeV stop mass scale, and the right vertical dotted lines correspond to the messenger scales of 25 TeV and 10$^9$ GeV, respectively. In the first figure, the two dotted lines are almost overlapped because of the similarity of the stop mass and messenger scales. In the both cases, we set all the soft scalar masses being universal ($\equiv m_0$) and all the “A-term” being the same as $m_0$ with the relatively heavier unified gaugino mass, $M_{1/2} = 54$ $m_0$ at the GUT scale. It can be realized in “no-scale” SUGRA models [10]. To keep the gauge coupling unification, here, we assumed the messenger fields compose one pair of $\{5, \bar{5}\}$ of SU(5), which are all decoupled below the messenger scale.

Comparing the two figures, we see that the focus points of $m_\phi^2$ appear at the almost same energy scale, regardless of the messenger scales [10]. As a result, $m_\phi^2$s remain almost the same value at low energies, regardless of various trial values of $m_\phi^2$s at the GUT scale. In fact, larger (smaller) values of $\lambda_2^2$ and $M_{1/2}$ push the focus points to lower (higher) energy scales. Below the stop decoupling scale, the low energy value of $m_\phi^2$ can be estimated using the Coleman-Weinberg potential [12], where $m_{\phi, u,d}^2$ and $\mu_{\text{eff}}$ dominantly affect $m_\phi^2$ via the $\lambda_2$ coupling. Even if they are not well-focused around the stop mass scale, however, the focusing of $m_\phi^2$ would not much be destroyed with a small enough $\lambda_2^2$.

In TABLE II we list various fine-tuning measures $\Delta X$ ($\equiv |\delta \log M_Z^2/\delta \log X|$) for the two cases, in which the stop squared mass $m_t$ is $\left[\equiv \log(Q/\text{GeV})\right]$ under the various trial universal squared soft mass $m_{0\phi}$s at the GUT scale. The right vertical dotted line indicates the messenger scale of $\Lambda_M = 25$ TeV ($10^6$ GeV) in the Left (Right) figure, and the left vertical lines correspond to the stop mass scale of $m_t = 20$ TeV in the both figures. In the left figures, the two dotted lines are almost overlapped. The other parameter choices in the both figures are the same as those of Case II.

![FIG. 1. RG evolutions of $m_\phi^2$ with $t = \log(Q/\text{GeV})$] under the various trial universal squared soft mass $m_{0\phi}$s at the GUT scale. The right vertical dotted line indicates the messenger scale of $\Lambda_M = 25$ TeV ($10^6$ GeV) in the Left (Right) figure, and the left vertical lines correspond to the stop mass scale of $m_t = 20$ TeV in the both figures. In the left figures, the two dotted lines are almost overlapped. The other parameter choices in the both figures are the same as those of Case II.

| Case I | $\lambda_2^2 = 5 \cdot 10^{-4}$ | $\lambda_1^2 = 0.5$ | $\Lambda_M = 15$ TeV | $\tan \beta = 10$ | $m_0^2 = (10$ TeV$)^2$ | $\Delta m_0^2 = 19.1$ | $\Delta m_{\phi^2} = 97.9$ |
|-------|-------------------------------|------------------|-------------------|-----------------|------------------|----------------|------------------|
| Case II | $\lambda_2^2 = 8 \cdot 10^{-3}$ | $\lambda_1^2 = 0.5$ | $\Lambda_M = 25$ TeV | $\tan \beta = 40$ | $m_0^2 = (20$ TeV$)^2$ | $\Delta m_0^2 = 28.6$ | $\Delta m_{\phi^2} = 56.5$ |
|-------|-------------------------------|------------------|-------------------|-----------------|------------------|----------------|------------------|
| $\Delta m_0^2$ | 19.1 | $\Delta m_{\phi^2}$ | 97.9 |
| $\Delta m_{1/2}$ | 83.2 | $\Delta m_{1/2}$ | 28.6 |
| $\Delta m_{\phi^2}$ | 59.7 | $\Delta m_{\phi^2}$ | 56.5 |
| $\Delta_{GM}$ | 37.1 | $\Delta_{GM}$ | 155.5 |
| $\Delta_{\phi M}$ | 6.0 | $\Delta_{\phi M}$ | 21.3 |

TABLE II. Fine-tuning measures for the various input parameters defined in the text for the two different cases.
The three gaugino masses in Case I and II are quite heavy: \( (M_G, M_W, M_B) \approx (11.8 \text{ TeV}, 4.8 \text{ TeV}, 2.7 \text{ TeV}) \) and \( (22.2 \text{ TeV}, 9.1 \text{ TeV}, 5.2 \text{ TeV}) \), respectively, at low energy. Also the Higgsino mass, \( \mu_{\text{eff}} \) is basically heavy, of order \(-m_{h_{\text{L}}}^2 \) in this model. In Case I and II, it is 2.5 and 2.3 TeV, respectively. The lightest mass eigenstate of the singlet fermions comes mainly from the fermionic components of \( \phi \) and \( X \). Its mass turns out to be about \( \kappa (Y)/(1 + T_2^2) \approx -\frac{1}{2} T_2^2 (a^* - b^*) \), of order sub-GeV or lighter. It can play the role of dark matter [13].

Now let us discuss the physical masses of the singlet scalars and their mixing angles with the SM Higgs boson. In this scenario, a light singlet scalar is essential for compensating the large contribution of \(-m_{h_{\text{L}}}^2 \) to Eq. (1). Since this mechanism works through the \( \mu_{\text{eff}} \) couplings in Eq. (6), a large mixing between the light scalar and the SM Higgs boson might be expected in this class of models. Such a large mixing would induce sizable invisible decay of the Higgs boson. An important reason to introduce the several scalar fields is for avoiding it.

Neglecting the \( a \) and \( b \) parameters for simple analysis, the squared mass matrix for the scalar fields in this model \( (\approx M_S^2) \) takes the following form:

\[
\begin{pmatrix}
    m_{H_1}^2 & \lambda_2 H \mu_{\text{eff}} \\
    \lambda_2 H \mu_{\text{eff}} & m_{\phi}^2 + |\kappa \phi|^2 + \lambda_1 H \mu_{\text{eff}} \\
    \lambda_1 H \mu_{\text{eff}} & \lambda_1 H \mu_{\text{eff}} + \kappa \phi M \end{pmatrix}
\]

in the basis of \( \{H, \phi, X\} \). Here \( m_{H_1}^2 \) collectively denotes the mass parameters in the MSSM Higgs sector. Since \( \langle Y \rangle \) is relatively small while \( Y \) is quite heavy, we ignored \( Y \) here. For the solutions of Eqs. (7) and (8) to be stable, all the eigenvalues of the above mass matrix, \( \{m_{H_1}^2, M_S^2, M_S^2\} \) must be positive definite around our solution. Since the sign of \( m_{H_1}^2 \) is flipped to be negative at low energies, the origin \( \langle \phi \rangle = \langle H \rangle = 0 \) is made unstable, whereas the solution obtained above with non-zero VEVs becomes a stable point. Including the radiative corrections as well as the extreme conditions, Eqs. (1) and (2), the smallest eigenvalue \( m_{H_1}^2 \) could be identified with the observed Higgs mass for \( |m_{H_1}^2|, |\lambda_1 H|, |m_{\phi}^2| \ll |M_S|^2, |\kappa \phi|^2 \). On the other hand, the largest eigenvalue would approximately be \( |M_S|^2 \) or \( |\kappa \phi|^2 \), depending on the solution of \( T_1 \).

The above squared mass matrix can be diagonalized into \( \mathcal{M}_3 \cdot \mathcal{O}_3 \cdot (m_{H_1}^2, M_S^2, M_S^2) \cdot \mathcal{O}_3 \) using the \( 3 \times 3 \) orthogonal mixing matrix given by

\[
\mathcal{O}_3 = \begin{pmatrix}
    c_1 c_2 & -s_1 & -c_1 s_2 \\
    c_2 s_1 - s_2 s_3 & c_1 c_3 & -c_3 s_1 s_2 - c_2 s_3 \\
    c_3 s_2 + c_2 s_1 s_3 & c_1 s_3 & -c_2 c_3 - s_1 s_2 s_3
\end{pmatrix}
\]

(12)

where \( c_1, c_2, c_3 \) and \( s_1, s_2, s_3 \) mean \( \cos \theta_{1,2,3} \) and \( \sin \theta_{1,2,3} \), respectively. Since the mixing angles between the MSSM Higgs sector and other neutral scalars should phenomenologically be suppressed [13], we need to show \( \{s_{1,2} \approx \varepsilon_{1,2} \approx 0.1, |c_{1,2} | \approx 1 \} \). \( \mathcal{O}_3 \cdot \mathcal{M}_3 \cdot \mathcal{O}_3 \) yields a symmetric matrix \( M_S^2 \approx (\mathcal{M}_3^2) \) with the following elements:

\[
\begin{align*}
M_{S1}^2 & \approx M_S^2 \varepsilon_2^2 + M_S^2 \varepsilon_1^2 + m_1^2, \\
M_{S2}^2 & \approx M_S^2 \varepsilon_2 \sin \theta + M_S^2 \varepsilon_1 \cos \theta - m_1^2, \\
M_{S3}^2 & \approx M_S^2 \varepsilon_2 \cos \theta - M_S^2 \varepsilon_1 \sin \theta - m_1^2, \\
M_{S23}^2 & \approx \Delta M_{S2}^2 (1 - \varepsilon_2^2) \sin \theta \cos \theta, \\
M_{S22}^2 & \approx \Delta M_{S2}^2 \sin^2 \theta + M_S^2 - \{\Delta M_{S2}^2 \sin^2 \theta + \Delta m_{1}^2 \} \varepsilon_1^2, \\
M_{S33}^2 & \approx \Delta M_{S2}^2 \cos^2 \theta + M_S^2 - \{\Delta M_{S2}^2 \cos^2 \theta + \Delta m_{1}^2 \} \varepsilon_2^2 \nonumber - \Delta M_{S2}^2 \sin^2 \theta \varepsilon_1 \varepsilon_2,
\end{align*}
\]

where \( \theta \) means the mixing angle \( \theta_3 \). For simpler expressions, here we introduced the new parameters defined as

\[
\begin{align*}
\varepsilon_{2,1} & \equiv \varepsilon_{2,1} \cos \theta \pm \varepsilon_{1,2} \sin \theta, \\
\varepsilon^2 & \equiv \frac{1}{2} (\varepsilon_1^2 + \varepsilon_2^2) + \tan \theta \varepsilon_1 \varepsilon_2 + \frac{2\Delta m_{1}^2 \varepsilon_1 \varepsilon_2}{\Delta M_{S2}^2 \sin 2 \theta}, \\
\Delta M_{S2}^2 & \equiv M_S^2 - M_2^2, \quad \text{and} \quad \Delta m_{1}^2 \equiv M_S^2 - m_1^2.
\end{align*}
\]

Since \( M_{S1}^2 \) is identified with the squared mass matrix \( M_S^2 \) obtained above, \( \{M, \kappa \phi, m_1^2, m_2^2, m_3^2, \lambda_1 H, \lambda_2 H, \mu_{\text{eff}} \} \) in \( M_S^2 \) can be expressed in terms of the mass eigenvalues and the mixing angles, \( \{m_{1,2}^2, M_2^2, M_3^2; \varepsilon_1, \varepsilon_2, \theta_3 \} \) by comparing their matrix elements, and using the two equations, Eqs. (11) and (8), by which \( \kappa \phi / M = T_2 \) and \( |\lambda_2 H|^2 \) are related to \( \mu_{\text{eff}} \) and \(-m_{h_{\text{L}}}^2 \), respectively.

The \( M_{S2}^2 \) in Eq. (13) actually contains the SM and heavy Higgs fields. When \(|\lambda_{1,2} H|^2 \sin 2 \beta \cos 2 \beta \) is relatively smaller than other elements of \( M_S^2 \), the mixing angle between the SM and heavy Higgs is suppressed [15]. Moreover, a relatively small \( A_{1,2} H \) decouples the heavy Higgs from the singlet sectors. In this case, \( m_{1,2}^2 \) in \( M_S^2 \) [or \( M_{S1}^2 \) in Eq. (13)] can be regarded as the physical SM Higgs mass. Including the quartic contributions, then, the light Higgs boson mass would approximately be

\[
M_{2}^2 \cos^2 2 \beta + \frac{|\kappa \phi|^2 |\lambda_{1,2}^2 |^2}{|M_S|^2} |H|^2 + \Delta m_{H}^2, \quad (15)
\]

where the second term corresponds to the tree level contribution of the singlets to the Higgs mass, and the third term indicates the radiative correction. Then, the eigenvalue \( m_1^2 \) in Eq. (13) should reproduce the measured Higgs mass \(|M_{1,2,3}^2 (125 \text{ GeV})^2| \). As is well-known, however, the first term in Eq. (15), the tree level mass is too small to explain it. As seen in Eq. (13), moreover, \( M_{3}^2 \varepsilon_1^2 \) and \( M_{2}^2 \varepsilon_2^2 \) make negative contributions to the observed Higgs mass. Although there are many mechanisms to raise the Higgs mass [16], just for simplicity, in this letter we will restrict our discussion to the cases that they are comparable to the second term of Eq. (15), i.e.

\[
\left| \frac{|\kappa \phi|^2 |\lambda_{1,2}^2 |^2}{|M_S|^2} |H|^2 \right| \approx M_{2}^2 \varepsilon_2^2 + M_{2}^2 \varepsilon_1^2.
\]

(16)
Once we get somehow the stop mass of 10–20 TeV, thus, we will regard the measured Higgs boson mass as being explained by the radiative correction $\Delta m_H^2$ at three-loop level as in the MSSM [1].

The identifications of (1, 2) and (1, 3) components of $M_2^2$ and $M_3^2$ give

$$\lambda_2 H_{\mu eff} \approx M_2^2 \varepsilon_2 \sin \theta + M_2^2 \varepsilon_1 \cos \theta, \quad \lambda_1 H_{\mu eff} \approx M_2^2 \varepsilon_2 \cos \theta - M_2^2 \varepsilon_1 \sin \theta,$$

respectively. Thus, a quite large $\mu_{eff}$ determined from Eq. (1) would result in a quite large mass eigenvalue $M_2^2$ or $M_3^2$. For $|\mu_{eff}| = 1.5–2.5$ TeV ($\sim -M_2^2$) and $\lambda_1 \approx 0.7$, thus, $|\lambda_1 H_{\mu eff}|$ is about $(430$ GeV$)^2$–$(550$ GeV$)^2$. In fact, $-M_2^2$ falls in such a range, when the stop mass is about 10–20 TeV at the messenger scales of 15–25 TeV. Assuming $|M_2^2 \varepsilon_2| \gg |M_2^2 \varepsilon_1|$ and $|\sin \theta| \sim 1 \gg |\cos \theta|$, $M_2^2 = 4.3–5.5$ TeV with $\varepsilon_1 \sim 10^{-2}$ can meet Eq. (17).

Similarly, e.g., $(M_2, M_3) \sim (5$ TeV, $500$ GeV) and $(\varepsilon_1, \varepsilon_2) \sim (10^{-2}, 10^{-1})$ can fit also Eq. (10) as well as Eq. (17) for $|\lambda_2/\lambda_1| \ll 1$ and $|\kappa \phi^2/M^2| \sim 1$. The ratio of the above two equations yields

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_1 \cos \theta - \varepsilon_2 \sin \theta}{\varepsilon_2 \cos \theta + \varepsilon_1 \sin \theta} = \frac{M_2^2 \lambda_2 - \tan \theta}{M_2^2 1 + \frac{\lambda_2}{\lambda_1} \tan \theta},$$

which is also a useful expression.

Writing $\kappa \phi M$ and $\lambda_1 \lambda_2/|H|^2$ with the mixing angle as

$$\kappa \phi M = \pm M_2^2 \sin \theta \cos \theta, \quad \lambda_1 \lambda_2/|H|^2 = \pm \delta M^2 \sin \theta \cos \theta,$$

the new mass parameters, $M_2^2$ and $\delta M^2$ are related to the mass eigenvalues and mixing angles as

$$M_2^2 + \delta M^2 \approx \pm \Delta M_{24}^2/2(1 - \epsilon^2),$$

from the identification of the (2, 3) components of $M_2^2$ and $M_{23}^2$. For $\kappa \phi M \gg \lambda_1 \lambda_2/|H|^2$, thus, $M_2^2$ is relatively much larger than $\delta M^2$, $\kappa \phi M/\lambda_1 \lambda_2/|H|^2 = M_2^2/\delta M^2 \gg 1$.

Let us parametrize $\kappa \phi M$ and $\lambda_1/\lambda_2$ using the angle variables, $\zeta$ and $\xi$:

$$\frac{\kappa \phi M}{M} = \tan \zeta, \quad \frac{\lambda_2}{\lambda_1} = \tan \xi,$$

where $\tan \zeta (= T_\zeta)$ is determined by Eqs. (1) and (11). Then, $|\kappa \phi|^2$ and $|M|^2$ are expressed as $\pm M_2^2 \sin \theta \cos \theta$ and $\pm M_2^2 \sin \theta \cos \theta \cot \zeta$, respectively, while $|\lambda_2/\lambda_1 H|^2$ is given by $\pm \delta M^2 \sin \theta \cos \theta \tan \xi (\cot \xi)$ (cot $\xi$). As a result, $m_\phi^2$ and $m_X^2$ are identified as

$$m_\phi^2 \approx m_2^2 - \Delta m_{24}^2 \epsilon_1^2 + \Delta M_{24}^2 (\epsilon^2 - \epsilon_1^2) \sin^2 \theta$$

$$+ M_2^2 \sin \theta \cos \theta (\tan \theta - \tan \zeta),$$

$$\pm M_2^2 \sin \theta \cos \theta (\tan \theta - \tan \zeta),$$

$$m_X^2 \approx m_2^2 - \Delta m_{21}^2 \epsilon_1^2$$

$$+ \Delta M_{21}^2 \left\{ \left( \epsilon^2 - \epsilon_1^2 \right) \cos^2 \theta - \epsilon_1 \epsilon_2 \sin 2 \theta \right\}$$

$$\pm M_2^2 \sin \theta \cos \theta (\cot \theta - \cot \zeta),$$

$$\pm M_2^2 \sin \theta \cos \theta (\cot \theta - \cot \zeta)$$

from the (2, 2) and (3, 3) components of $M_2^2$ and $M_3^2$.

As discussed above, the Higgs VEVs, $(h_u)$ and $(h_d)$ vanish in the SUSY limit, where all the soft mass parameters disappear. Even in the SUSY limit, however, $\phi$ and $X$ can still develop large VEVs: The flat direction along $\kappa \phi^2/2 + M X = 0$ becomes alive in this limit as seen in Eq. (5). Then, the $2 \times 2$ block-diagonal part in $M_2^2$,

$$|\kappa \phi M| = |M|^2 \left( \begin{array}{cc} \tan^2 \zeta & \tan \zeta \\ \tan \zeta & 1 \end{array} \right)$$

is diagonalized to $\operatorname{diag}(M_2^2, M_3^2) \approx \operatorname{diag}(0, |M|^2)$ or $\operatorname{diag}(|M|^2, 0)$ for $|\zeta| \ll 1$ with the mixing angles $\theta = \zeta$ or $\theta = \frac{\pi}{2} + \zeta$, respectively (and $\epsilon_{1,2} = 0$). We are more interested in the second case. Even when all the soft SUSY breaking terms in $M_3^2$ turned on, however, the results would be perturbed just slightly, since the soft parameters and the Higgs VEVs are relatively small.

Around $\theta = \frac{\pi}{2} + \zeta$, i.e., when $\theta = \frac{\pi}{2} + \theta_0 (|\delta \theta|, |\theta| \ll 1$, we have $|\sin \theta| \approx 1 - (\delta \theta)^2/2, \cos \theta \approx -\delta \theta, M_2^2 \gg M_3^2$, and so Eqs. (21) and (22) are approximated as

$$m_\phi^2 \approx M_3^2 - M_2^2 \left[ \frac{\epsilon_1^2 + \epsilon_2^2}{2} + (\epsilon_1 \epsilon_2 + \zeta - \delta \theta) \delta \theta \right],$$

$$m_X^2 \approx M_2^2 \left[ 1 - 2 \epsilon_1 \epsilon_2 \delta \theta - \epsilon_2^2 - (1 - \delta \theta) \frac{\delta \theta}{\zeta} - \cos \theta \delta \theta \right],$$

where we set $\eta \equiv M_3^2/M_2^2$ and $\cos \theta \delta \theta \equiv (\delta \theta)^2[1 - (2 \delta \theta/\zeta + \zeta/\delta \theta)/3]$. For $m_\phi^2 = m_X^2 = 0$ and $\epsilon_{1,2} = 0$, thus, $\delta \theta$ and $M_3^2$ are determined to $\delta \theta = \zeta$ and $M_3^2 = 0$ as expected. On the other hand, for non-zero but small enough $m_\phi^2$, $m_X^2$, and $\epsilon_{1,2}$, the values of $\delta \theta$ and $M_3^2$ become relaxed as follows:

$$\delta \theta \approx \frac{\zeta \left( 1 - \epsilon_2^2 \right)}{1 - \eta + 2 \epsilon_1 \epsilon_2 \zeta} \quad \text{and} \quad M_3^2 \approx M_2^2 \left[ \frac{\epsilon_1^2 + \epsilon_2^2}{2} + \epsilon_1 \epsilon_2 \zeta \right].$$

For a given $\zeta$ and $\lambda_2/\lambda_1$, hence, $\epsilon_1/\epsilon_2$ can be determined from Eq. (18). If $\delta \theta \approx \zeta \sim \mathcal{O}(10^{-1})$, $(\lambda_2/\lambda_1) \tan \theta \lesssim \mathcal{O}(1)$, and $\epsilon_2 \sim \mathcal{O}(10^{-2})$ as considered above, $\epsilon_1$ and $M_2^2/M_3^2$ should be of order $10^{-1}$ and $10^{-2}$, respectively. Although we take an extremely small value of $\zeta$, $\epsilon_1$ and $M_2^2/M_3^2$ still stays around those values for $\epsilon_2 \sim \mathcal{O}(10^{-2})$ and $\lambda_2/\lambda_1 \sim \mathcal{O}(10^{-1} - 10^{-2})$, unless $\lambda_2/\lambda_1$ is much smaller as well. Since $M_2$ is approximately given by $M$, thus, a large enough SUSY parameter $M$ (in $|m_\phi^2|$, $|m_X^2| \ll -m_h^2 < |M|^2 < m_\phi^2$) suppresses $\epsilon_2$ by Eq. (17) as well as $\zeta$ by Eq. (11). On the other hand, a larger (smaller) value of $\mu_{eff}$ makes $\epsilon_2$ larger (smaller).

In conclusion, we have proposed a scenario where the EW symmetry is broken by a negative soft squared mass of an MSSM singlet scalar. We have employed both the gravity- and gauge-mediated SUSY breaking scenarios, assuming the latter effects dominate over the former ones. As a result, the MSSM SUSY particles can be much heavier than the experimental bounds and the FCNC phenomena are adequately suppressed. On the other hand,
the naturalness associated with the EW symmetry breaking can be maintained. By introducing several singlets, a flat direction is admitted in the SUSY limit, and a large mixing between the Higgs boson and the singlet sector can be avoided, $|\epsilon_1,2| \ll 1$. The large effect of $-m_{h_u}^2$ on the Higgs VEV in the MSSM can be compensated dynamically by the flat direction, while the small curvature of the flat direction is compensated by the Higgs boson.

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