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Computational and theoretical modeling of the transmission dynamics of novel COVID-19 under Mittag-Leffler Power Law

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Abstract In the current article, we studied the novel corona virus (2019-nCoV or COVID-19) which is a threat to the whole world nowadays. We consider a fractional order epidemic model which describes the dynamics of COVID-19 under nonsingular kernel type of fractional derivative. An attempt is made to discuss the existence of the model using the fixed point theorem of Banach and Krasnoselkii's type. We will also discuss the Ulam-Hyers type of stability of the mentioned problem. For semi analytical solution of the problem the Laplace Adomian decomposition method (LADM) is suggested to obtain the required solution. The results are simulated via Matlab by graphs. Also we have compare the simulated results with some reported real data for Commutative class at classical order.

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1. Introduction

It is a real fact that safe and peaceful environment is the important need of every society. But from the start of the world, in each time people on this earth have faced various threats in which outbreaks of different diseases were the most dangerous. The occurrence of the outbreak due to various diseases in history have greatly destroyed human civilization. In many occasions, they have completely changed the life style of people. Therefore in each time researchers and experts have tried to save their societies from such disasters. Proper interpretation of a disease play an important role to remove the threat from the community. Further, implementation of suitable control strategies against the disease transmission have been assumed a big challenge. To handle such type of challenges the approach of mathematical modeling is key in this regard. A number of general and diseases models have been investigated in existing literature which enables us to explore and control the spread of infectious diseases in a better way [1,2].

Here, we remark that in recent time a threatful outbreak which has been originated from China is spreading throughout...
the globe very rapidly. Thousands of people have been faced death due to this disease. Nearly 9 million have got the infection in all over the world [3]. Some infected people have been recovered from the mentioned disease. Different researchers and policy makers are struggling to control the disease from further spreading. One big factor of spreading this disease is immigration of infected people from place to place which effect more people and hence cause spreading this disease. Therefore on international level, almost all the countries of the world have suspended air traffic for some time and also they have announced lock down in cities so that some precautionary measure may be taken to reduce maximum loss of human lives. Also each country in the globe try to reduced unnecessary traveling of people and to reduce the cases of infection in their countries [4]. It is remarkable that such like outbreak in past have caused of million people death around the globe. Since scientists and researchers are trying to investigate cure or vaccine for the aforesaid outbreak so that in future such like pandemic may be controlled. From medical engineering point of view infectious disease can be well understand by using the concept of mathematical model. This concept was started during in 1927. After that various mathematical models have been established for different disease in history. For some famous studies, we refer [5–7] and the cited papers therein.

Keeping in mind the treat of the present pandemic, researchers have investigated the current disease of COVID-19 from some aspects, for detail see [8–12]. On the other hand as mathematical models also can help us in understanding and to construct some strategies, how to control the disease from being spreading and to take precautionary measures. In this regards some models have been very recently studied about the current novel virus disease, see for detail [13]. Since the infection is rapidly transmitting from person to person, therefore in most of the countries, the people are advised to keep social distance from each other. In this regard some authors have investigated the immigration effect on the transmission dynamics of this disease by considering the modified version of predator–prey model [14].

In the present time, one of the interesting area of research is arbitrary order integration and differentiation. The subject was started after a famous conversation between Liebnitz and L’Hospital in the last decay of seventeenth century. Arbitrary order calculus has many applications in different field of science and engineering, such is biological sciences, chemical engineering, dynamical system, electrodynamics, etc., (for detail see [15–17] and reference there in). It can also be used to model phenomena like earthquakes, oscillation, porous media, flow of seepage and as well as in various models of fluid dynamics. Fractional differential equations (DEs) have been studied from many authors for existence and uniqueness of solution [18–23]. For this, different authors used various theories like topological degree theory, “Banach contraction theorem and Leray-Shaudar fixed point theorem”, etc. There has been significant interest for obtaining exact analytical and numerical solutions for the fractional DEs. It is quiet difficult task to find exact solutions to every arbitrary order DEs. So, authors are trying for optimal and numerical solutions corresponding to which they used numerous approaches, available in the literature. Two type of solutions we investigate in applied analysis, one is analytical results for which, we use some analytical techniques, while other is computed through numerical method called numerical solutions. In this regards, a lot of contribution has been made by many researchers for solving fractional DEs using different techniques like perturbation methods, integral transform techniques, spectral techniques, collocation methods, etc [24,25]. Here we will study our considered problem for mathematical analysis using theory of existence and uniqueness, Ulam-Hyers stability along with approximate solution using LADM.

On the other hand the definition of fractional derivative is not unique. In literature there are various approaches to define fractional derivative with different kernels. This fact motivate the researchers to use the operator which is best to the model they considered. In recent time some new type of fractional differential operators with nonsingular kernel have got great attention. Until 2015, all the fractional operators had singular kernels. These singularities make difficulties to model some physical phenomena. To ride of this difficulty, Caputo and Febrizio [26] has introduced a new definition for fractional derivative. Actually it is the extension of the Caputo fractional derivative to more general frame work having nonsingular kernel. This new idea successfully studied many real world phenomenon for example see these [27–37] and the references for further applications and properties of Caputo-Febrizio derivatives. This new approach has attracted many researchers because it has a non-singular kernel. Although, this definition has some problems because of the locality of its kernel. A year later, Atangana and Baleanu (AB) [38,39] introduced another definition of fractional derivative with “Mittag-Leffler” function as a non-local and nonsingular kernel. With this generalized “Mittag-Leffler” function works as a kernel. AB approach makes an excellent memory description and inhold properties for mean-square displacement [40,41]. Further, in AB derivative the “Mittag-Leffler” kernel is nonsingular, which gives information at the start as well as in the final stage of the evolution propagation. In this regard, the mentioned derivative has got significant interest of researchers. Therefore, “Mittag-Leffler” fractional DEs have studied from theoretical as well as numerical aspects. Theoretically, the existence and uniqueness of solutions of “Mittag-Leffler” fractional DEs are in progress. Recently, the AB derivative has been used in modeling various real world phenomena, for example see [42,43]. Further, AB derivative has also used to model various infectious diseases like Ebola virus, dynamics of smoking, Leprosirosis, etc [44–51] in more comprehensive way.

Mathematical modeling plays an important role to investigate the dynamics of a disease and hence its control particularly in the absence of vaccination or at early stages of the disease. The area devoted to investigate biological models for infectious diseases is warm area of research in recent time. Also, one can look for possible prevention strategies as well. In this regard recently, Lin and his co-authors in [13] have been considered model for COVID-19 with integer order derivative as

\[
\begin{align*}
S' (\theta) &= -\frac{\alpha SF}{N} - \frac{\alpha SI}{N} - \mu S, \\
E' (\theta) &= \frac{\alpha SF}{N} + \frac{\alpha SI}{N} - (\sigma + \mu)E, \\
I' (\theta) &= \sigma E - (\gamma + \mu)I, \\
R' (\theta) &= \gamma I - \mu R, \\
N' (\theta) &= -\mu N, \\
D' (\theta) &= \gamma I - \lambda D, \\
C' (\theta) &= \sigma E,
\end{align*}
\]
where $\beta(\theta) = \beta_0 (1 - \zeta)(1 - \frac{\sigma}{\sigma_{\text{th}}})^\gamma$.

Here, $S$ represent susceptible populations, $E$ represent exposed populations, $I$ represent infectious populations, $R$ represent the removed population (recovered or dead), $N$ represent total populations, $D$ represent mimicking the public perception of risk regarding the number of severe and critical cases and deaths and $C$ representing the number of cumulative cases (both reported and not reported).

The detail of parameters used in model (1) with complete descriptions are given in Table 1. Inspired from the above model of COVID-19 in this work, we consider model (1) under $AB$ fractional derivative in Caputo sense, shortly ($ABC$) for existence theory, Ulam-Hyers’s stability and semi analytical solution. The upper mentioned model of COVID-19 with $ABC$ fractional derivative can be written as

$$
\begin{align*}
ABC D^\theta_0 S(\theta) &= -\frac{\beta_0 SF}{N} - \frac{\beta_0 IS}{N} - \mu S, \\
ABC D^\theta_0 E(\theta) &= \frac{\beta_0 IS}{N} + \frac{\beta_0 SE}{N} - (\sigma + \mu) E, \\
ABC D^\theta_0 I(\theta) &= \sigma E - (\gamma + \mu) I, \\
ABC D^\theta_0 R(\theta) &= \gamma I - \mu R, \\
ABC D^\theta_0 N(\theta) &= -\mu N, \\
ABC D^\theta_0 D(\theta) &= \nu I - \lambda D, \\
ABC D^\theta_0 C(\theta) &= \sigma E.
\end{align*}
$$

where $0 < \theta \leq 1$, $\beta(\theta) = \beta_0 (1 - \zeta)(1 - \frac{\sigma}{\sigma_{\text{th}}})^\gamma$.

We may assume the initial conditions for considered model as,

$$
\begin{align*}
S(0) &= S_0, \\
E(0) &= E_0, \\
I(0) &= I_0, \\
R(0) &= R_0, \\
N(0) &= N_0, \\
D(0) &= D_0, \\
C(0) &= C_0.
\end{align*}
$$

With the help of LADM, we want to handle the proposed problem for semi analytical solution. The concerned techniques was rarely studied for the aforementioned derivative of fractional order. Further from analysis point of view, existence is necessary to be investigated for the considered problem. Therefore, we use the fixed point theory due to Banach and Krassnoselskii’s to establish some sufficient conditions regarding existence and uniqueness of the solution. Since stability is needed in respect of approximate solution, so we attempt on Ulam type of stability for the considered system via nonlinear functional analysis.

2. Fundamental results

Here, we recall some useful definitions and results from fractional calculus and nonlinear analysis.

**Definition 2.1.** [54] $ABC$ fractional integral is define as

$$
\begin{align*}
ABC I^\theta_0 \Psi(\theta) &= \frac{(1 - \vartheta)}{\Gamma(\vartheta)} \Psi(\theta) + \frac{\vartheta}{\Gamma(\vartheta)\Gamma(\vartheta)} \int_0^\theta (\theta - \eta)^{\vartheta - 1} \Psi(\eta) d\eta,
\end{align*}
$$

provided the integral on right converges.

**Definition 2.2.** [54] $ABC$ fractional derivative is define as

$$
\begin{align*}
ABC D^\theta_0 \Psi(\theta) &= \frac{\ABC(\vartheta)}{(1 - \vartheta)} \Psi(\theta) + \frac{\vartheta}{\Gamma(\vartheta)\Gamma(\vartheta)} \int_0^\theta (\theta - \eta)^{\vartheta - 1} \Psi(\eta) d\eta,
\end{align*}
$$

where $0 < \vartheta \leq 1$, $\Psi \in L^1(\alpha, \beta)$ and $\ABC(\vartheta)$ is a normalization constant such that $\ABC(0) = \ABC(1) = 1$. Here, $E_{\alpha, \beta}$ is well known “Mittag-Leffler” function and define as

$$
E_{\alpha, \beta}(\theta) = \sum_{j=0}^{\infty} \frac{\theta^j}{\Gamma(j\vartheta + \beta)},
$$

where two parameter “Mittag-Leffler” function is given as,

$$
E_{\alpha, \beta}(\theta) = \sum_{j=0}^{\infty} \frac{\theta^j}{\Gamma(j\vartheta + \beta)}.
$$

**Lemma 2.3** [54]. Let $\tilde{f}(\theta) \in C([0, \tau])$, then the solution of fractional DE

$$
\begin{align*}
\begin{cases}
ABC D^\theta_0 \Psi(\theta) = \tilde{f}(\theta), & \theta \in [0, \tau], \ 0 < \vartheta \leq 1, \\
\Psi(0) = f_0,
\end{cases}
\end{align*}
$$

is given by

$$
\Psi(\theta) = f_0 + \frac{(1 - \vartheta)}{\ABC(\vartheta)} \tilde{f}(\theta) + \frac{\vartheta}{\ABC(\vartheta)\Gamma(\vartheta)} \int_0^\theta (\theta - \eta)^{\vartheta - 1} \Gamma(\eta) d\eta.
$$

**Definition 2.4.** The Laplace transform of $ABC$ derivative of a function $\Psi(\theta)$ is defined by

$$
\mathcal{L}\left[ABC D^\theta_0 \Psi(\theta)\right] = \frac{\ABC(\vartheta)}{s^{\vartheta}(1 - \vartheta) + \vartheta} \left[ s^\vartheta \mathcal{L}[\Psi(\theta)] - s^{\vartheta - 1} \Psi(0) \right].
$$

Let $X = C([0, \tau])$ be a Banach space with norm define as

$$
\|\Psi\| = \max_{\theta \in [0, \tau]} |\Psi|, \ \text{for all } \Psi \in X.
$$

| Table 1 | Description of the parameters used in model (1). |
|---------|-----------------------------------------------|
| Notation | Parameters description                         |
| $F$     | Number of zoonotic cases                      |
| $\beta_0$ | Transmission rate                    |
| $\zeta$ | z                                             |
| $\kappa$ | Intensity of responds                      |
| $\mu$   | Mean latent period                          |
| $\sigma^{-1}$ | Emigration rate    |
| $\gamma^{-1}$ | Mean infectious period        |
| $\nu$   | Proportion of severe cases                  |
| $\lambda^{-1}$ | Mean duration of public reaction    |
Theorem 2.5. (Krasnosel’skii’s fixedpoint theorem) Let $E \subset X$ be closed, convex non empty subset of $X$ and there exist two operators $F$ and $G$ such that

$$(2) F \Psi + G \Psi \in E \forall \Psi \in E$$

$(2) F$ is contraction and $G$ is compact and continuous.

Then there exist at least one solution $\Psi \in E$ such that $F \Psi + G \Psi = \Psi$.

3. Qualitative analysis of the considered model

Before analyzing any biological model, it is natural to ask whether such dynamical problem really exist or not. This question is guaranteed by the fixed point theory. Here, we will try to use the same theory for the proposed problem (2) being part of this research. Upon computation, the basic reproductive number of the model (2) as given by

$$R = \frac{\beta_0 \sigma}{\gamma (\sigma + \mu)}.$$  

Here we remark that if $R < 1$, then the disease free equilibrium point will asymptotically stable. Further we are interested in qualitative analysis based on fixed point approach and analytical results which we compute through LADM. Regarding the aforesaid need, we fix the right sides of model (2) as

$$\begin{align}
\Psi_1(\theta, S, E, I, R, N, D, C) &= -\frac{\beta_0 SF}{N} - \mu S, \\
\Psi_2(\theta, S, E, I, R, N, D, C) &= \frac{\beta_0 SF}{N} + \frac{\beta_0 E}{N} - (\sigma + \mu)E, \\
\Psi_3(\theta, S, E, I, R, N, D, C) &= \sigma E - (\gamma + \mu)I, \\
\Psi_4(\theta, S, E, I, R, N, D, C) &= \gamma I - \mu R, \\
\Psi_5(\theta, S, E, I, R, N, D, C) &= -\mu N, \\
\Psi_6(\theta, S, E, I, R, N, D, C) &= \nu I - \beta D, \\
\Psi_7(\theta, S, E, I, R, N, D, C) &= \sigma E.
\end{align}$$

With the help of (8), the developed model (2) can be written in the form of

$$\begin{align}
\begin{bmatrix} A \ B \ C \ D \ E \ F \ G \end{bmatrix} \Psi(\theta) &= \Phi(\theta, \Psi(\theta)), \ \theta \in [0, \tau], \ 0 < \theta \leq 1, \\
\Psi(0) &= \Psi_0.
\end{align}$$

In view of Lemma 2.3, Eq. (9) yields

$$\begin{align}
\Psi(\theta) &= \Psi_0(\theta) + \frac{(1 - \theta)}{\Theta(X)} \Phi(\theta, \Psi(\theta)) + \frac{\theta}{\Theta(X)} \overline{\Phi}(\theta) \\
&\times \int_0^{\theta} (\theta - \eta)^{\alpha - 1} \Phi(\eta, \Psi(\eta)) d\eta.
\end{align}$$

Define data dependence results, let $X = C([0, \tau])$ be the Banach space, assume the following hold:

(H1) There exists constants $K, M$, and $q \in [0, 1]$ such that $|\Phi(\theta, \Psi(\theta))| \leq K|\Psi|^q + M$.  

(H2) There exists constants $L > 0$, such that for each $\Psi$, $\Psi \in X$ such that $|\Phi(\theta, \Psi) - \Phi(\theta, \Psi)| \leq L|\Psi| - |\Psi|$.  

Next, we define operator $T : X \to X$ as

$$\begin{align}
T \Psi(\theta) &= F \Psi(\theta) + G \Psi(\theta),
\end{align}$$

such that

$$\begin{align}
\begin{bmatrix} A \ B \ C \ D \ E \ F \ G \end{bmatrix} \Psi(\theta) &= \Phi(\theta, \Psi(\theta)), \\
\Psi(0) &= \Psi_0.
\end{align}$$

Theorem 3.1. Under the assumptions (H1, H2), the considered problem (2) has at least one solution if $(\frac{1 - \theta}{\Theta(X)} L) < 1$.

Proof. First, we show operator $F$ is contraction. Let $\Psi \in B$, where $B = \{\Psi \in X : ||\Psi|| \leq \rho, \rho > 0\}$ is closed convex set, then

$$\begin{align}
||F(\Psi) - F(\bar{\Psi})|| &= \frac{\max_{(\theta, \bar{\theta}) \in [0, \tau]} |\Phi(\theta, \Psi(\theta)) - \Phi(\theta, \bar{\Psi}(\theta))|}{\Theta(X)} \\
&\leq (\frac{1 - \theta}{\Theta(X)} L) ||\Psi - \bar{\Psi}||.
\end{align}$$

Hence $F$ is contraction. Next to prove that $G$ is compact and continuous, for any $\Psi \in B$, then $G$ is continuous as $\Phi$ is continuous, thus

$$\begin{align}
G(\Psi) &= \max_{(\theta, \bar{\theta}) \in [0, \tau]} \frac{\alpha}{\Theta(X)} \int_0^{\theta} (\theta - \eta)^{\alpha - 1} \Phi(\eta, \Psi(\eta)) d\eta, \\
&\leq \frac{\alpha}{\Theta(X)} \int_0^{\theta} (\theta - \eta)^{\alpha - 1} \Phi(\eta, \Psi(\eta)) d\eta, \\
&\leq \frac{\alpha}{\Theta(X)} (K|\Psi|^q + M).
\end{align}$$

Which shows $G$ is bounded. Further, let $\theta_1 > \theta_2 \in [0, \tau]$ such that

$$\begin{align}
||G(\Psi) - G(\bar{\Psi})|| &= \frac{\max_{(\theta, \bar{\theta}) \in [0, \tau]} |\Phi(\theta, \Psi(\theta)) - \Phi(\theta, \bar{\Psi}(\theta))|}{\Theta(X)} \\
&\leq (\frac{1 - \theta}{\Theta(X)} L) ||\Psi - \bar{\Psi}||.
\end{align}$$
Thus, \( G \) is equi-continuous. So, by Arzelà Ascoli theorem \( G \) is compact. Hence, the corresponding problem has at least one solution. \( \square \)

**Uniqueness:**

**Theorem 3.2.** Assume there exist a constant \( R > 0 \) such that

\[
R = \left[ (1 - \phi)L_\Phi + \frac{e^\varepsilon L_\Phi}{ABC(\theta)} \Gamma(\theta) \right] < 1, \tag{15}
\]

then \( T \) has unique fixed point.

**Proof.** Let \( \Psi, \hat{\Psi} \in X \), then

\[
\|T\Psi - T\hat{\Psi}\| \leq \|T\Psi - \hat{T}\Psi\| + \|G\Psi - G\hat{\Psi}\| \\
\leq \|(1 - \phi)L_\Phi \| \|\Phi(\theta, \Psi(\theta)) - \Phi(\theta, \hat{\Psi}(\theta))\| \\
+ \frac{e^\varepsilon L_\Phi}{ABC(\theta)} \max_{[0, \theta]} \int_0^\theta (\theta - \eta)^{\varepsilon-1} \Phi(\eta, \Psi(\eta)) d\eta \\
- \frac{e^\varepsilon L_\Phi}{ABC(\theta)} \max_{[0, \theta]} \int_0^\theta (\theta - \eta)^{\varepsilon-1} \Phi(\eta, \hat{\Psi}(\eta)) d\eta, \\
\leq \left( \frac{1 - \phi}{ABC(\theta)} + \frac{e^\varepsilon L_\Phi}{ABC(\theta)} \right) \|\Psi - \hat{\Psi}\|, \\
= R \|\Psi - \hat{\Psi}\|.
\]

Hence, \( T \) has unique fixed point, by Banach contraction principle. Consequently, problem (1) has unique solution. \( \square \)

4. Ulam-Hyers stability of the considered model

Stability is important aspect of differential equations. Among different form of stability, one of the interesting type is Ulam-Hyers type stability. The stated stability introduced by Ulam [52], further Ulam-Hyers stability was further generalized by Rassias [53], to more general framework known is Ulam-Rassias type stability. For the last few years the stated stability has been studied by many authors, for example see [27–32,38]. So, we also consider the mentioned problem for Ulam-Rassias type stabilties.

**Definition 4.1.** The Eq. (9) is Ulam-Hyers stable if for \( \epsilon > 0 \) and let \( \Psi \in X \) be any solution of inequality

\[
\|ABC^\varepsilon \Phi(\theta) - \Phi(\theta, \Psi(\theta))\| \leq \epsilon, \quad \theta \in [0, \tau]. \tag{16}
\]

there exist unique solution \( \Psi \) of Eq. (9) with \( C_\theta > 0 \) such that

\[
\|\Psi - \Psi\| \leq C_\theta \epsilon, \quad \theta \in [0, \tau]. \tag{17}
\]

**Definition 4.2.** Further, if there exist \( \phi \in C([0, \tau], R) \) with \( \phi(0) = 0 \), for any solution \( \Psi \) of Eq. (16) and \( \Psi \) be unique solution of (9) such that

\[
\|\Psi - \Psi\| \leq \phi(\epsilon), \tag{18}
\]

then Eq. (9) is generalized Ulam-Hyers stable.

**Remark 4.3.** If there exist \( \varsigma(\theta) \in C([0, \tau], R) \), then \( \Psi \in X \) satisfies inequality (16) if

(i) \( |\varsigma(\theta)| \leq \epsilon, \forall \theta \in [0, \tau] \),

(ii) \( ^{ABC}D_{1\theta}^\varepsilon \Phi(\theta) = \Phi(\theta, \Psi(\theta)) + \varsigma(\theta), \forall \theta \in [0, \tau] \).

Consider the corresponding perturb Eq. of problem (9) as

\[
\begin{align*}
\begin{cases}
ABC^\varepsilon \Phi(\theta) = \Phi(\theta, \Psi(\theta)) + \varsigma(\theta), \\
\Psi(0) = \Psi_0.
\end{cases}
\end{align*}
\]

(19)

For further analysis, we need the following relation.

**Lemma 4.4.** The following result hold for perturb Eq. (19).

\[
|\Psi(\theta) - T\Psi(\theta)| \leq \left[ \frac{1 - \phi}{ABC(\theta)} + \frac{e^\varepsilon L_\Phi}{ABC(\theta)} \right] \epsilon. \tag{19}
\]

**Proof.** Using Lemma 2.3, the solution of perturb problem (19) is given by

\[
\Psi(\theta) = \Psi_0 + ABC^\varepsilon \Phi(\theta, \Psi(\theta)) + ABC^\varepsilon \varsigma(\theta)
\]

Now using Eq. (14), we have

\[
|\Psi(\theta) - T\Psi(\theta)| \leq \left[ \frac{1 - \phi}{ABC(\theta)} |\varsigma(\theta)| + \frac{e^\varepsilon L_\Phi}{ABC(\theta)} |\int_0^\theta (\theta - \eta)^{\varepsilon-1} |\varsigma(\eta)| d\eta| \right] + \left[ \frac{1 - \phi}{ABC(\theta)} + \frac{e^\varepsilon L_\Phi}{ABC(\theta)} \right] \epsilon. \tag{19}
\]

**Theorem 4.5.** Under the Lemma 4.4, the solution of the considered problem (9) is Ulam-Hyers stable and also generalized-Ulam-Hyers stable if \( \left[ \frac{1 - \phi}{ABC(\theta)} + \frac{e^\varepsilon L_\Phi}{ABC(\theta)} \right] < 1 \).

**Proof.** Let \( \Psi \in X \) be any solution and \( \hat{\Psi} \in X \) be unique solution of Eq. (9), then

\[
|\Psi(\theta) - \hat{\Psi}(\theta)| = |\Psi(\theta) - T\Psi(\theta)| \\
\leq |\Psi(\theta) - T\Psi(\theta)| + |T\Psi(\theta) - T\hat{\Psi}(\theta)| \\
\leq \left[ \frac{1 - \phi}{ABC(\theta)} + \frac{e^\varepsilon L_\Phi}{ABC(\theta)} \right] \epsilon + \left[ \frac{1 - \phi}{ABC(\theta)} + \frac{e^\varepsilon L_\Phi}{ABC(\theta)} \right] |\Psi - \hat{\Psi}| \\
\leq \left[ 1 - \frac{1 - \phi}{ABC(\theta)} - \frac{e^\varepsilon L_\Phi}{ABC(\theta)} \right] \epsilon.
\]

Hence, problem (9) is Ulam-Hyers stable. Consequently it is generalized Ulam-Hyers stable. \( \square \)

**Definition 4.6.** The Eq. (9) is Ulam-Hyers-Rassias stable for \( \phi \in C([0, \tau], R) \), if for \( \epsilon > 0 \) and let \( \Psi \in X \) be any solution of inequality

\[
\|ABC^\varepsilon \Phi(\theta) - \Phi(\theta, \Psi(\theta))\| \leq \phi(\theta) \epsilon, \quad \theta \in [0, \tau].
\]

there exist unique solution \( \Psi \) of Eq. (9) with \( C_\theta > 0 \) such that

\[
\|\Psi - \Psi\| \leq C_\theta \phi(\theta) \epsilon, \forall \theta \in [0, \tau]. \tag{21}
\]
Definition 4.7. Further, for \( \phi \in C([0, \tau], R) \) if there exist \( C_{q, \phi} \) and for \( \epsilon > 0 \), let \( \Psi \) be any solution of inequality (20) and \( \Psi \) be unique solution of Eq. (9) such that
\[
||\Psi - \Psi|| \leq C_{q, \phi} \phi(\theta), \forall \theta \in [0, \tau],
\]
then Eq. (9) is generalized Ulam-Hyers-Rassias stable.

Remark 4.8. If there exist \( \xi(\theta) \in C([0, \tau]; R) \), then \( \Psi \in X \) satisfies the inequality (20) if
(i) \( ||\xi(\theta)|| \leq c\phi(\theta), \forall \theta \in [0, \tau] \),
(ii) \( ABC D_{\phi}^0 \Psi(\theta) = \Phi(\theta, \Psi(\theta)) + \xi(\theta), \forall \theta \in [0, \tau] \).

Lemma 4.9. The following result hold for perturb Eq. (19),
\[
||\Psi(\theta) - T\Psi(\theta)|| \leq \left[ 1 - \frac{\theta}{ABC(\theta)} \right] |\frac{\tau^\theta}{ABC(\theta)\Gamma(\theta)} + \frac{\phi(\theta)}{ABC(\theta)}|\phi(\theta)\epsilon.
\]
Proof. The proof is same as Lemma 4.4 so, we left for the reader. □

Theorem 4.10. Under the Lemma 4.9, the solution of the considered problem (9) is Ulam-Hyers-Rassias stable and also generalized-Ulam-Hyers-Rassias stable if
\[
\frac{(1-\theta)\tau^\theta}{ABC(\theta)} + \frac{\phi(\theta)}{ABC(\theta)} < 1.
\]
Proof. Let \( \Psi \in X \) be any solution and \( \Psi \in X \) be unique solution of Eq. (9), then
\[
||\Psi(\theta) - \Psi(\theta)|| = ||\Psi(\theta) - T\Psi(\theta)|| \\
\leq ||\Psi(\theta) - T\Psi(\theta)|| + ||T\Psi(\theta) - T\Psi(\theta)|| \\
\leq \left[ 1 - \frac{\theta}{ABC(\theta)} \right] |\frac{\tau^\theta}{ABC(\theta)\Gamma(\theta)} + \frac{\phi(\theta)}{ABC(\theta)}|\phi(\theta)\epsilon \\
\leq \left[ 1 - \frac{\theta}{ABC(\theta)} \right] |\frac{\tau^\theta}{ABC(\theta)\Gamma(\theta)} + \frac{\phi(\theta)}{ABC(\theta)}|\phi(\theta)\epsilon.
\]
Hence, problem (9) is Ulam-Hyers-Rassias stable. Consequently it is generalized Ulam-Hyers-Rassias stable. □

5. Algorithm construction for general solution of the model

In this section, we will compute a general series type of solution for the proposed model under ABC derivatives. Taking Laplace transform of both sides of each equation in model (2) and using the initial values, we get
\[
\begin{align*}
\mathcal{L}[S(\theta)] &= \frac{S_0}{\tau} + \frac{\phi(1-\theta)\phi(\theta)}{\tau^\theta ABC(\theta)} \mathcal{L}\left[ \frac{R S F}{N} - \frac{\mu S}{N} \right], \\
\mathcal{L}[E(\theta)] &= \frac{E_0}{\tau} + \frac{\phi(1-\theta)\phi(\theta)}{\tau^\theta ABC(\theta)} \mathcal{L}\left[ \frac{R S F}{N} + \frac{\mu E}{N} - (\sigma + \mu)E \right], \\
\mathcal{L}[I(\theta)] &= \frac{I_0}{\tau} + \frac{\phi(1-\theta)\phi(\theta)}{\tau^\theta ABC(\theta)} \mathcal{L}\left[ \sigma E - (\gamma + \mu)I \right], \\
\mathcal{L}[R(\theta)] &= \frac{R_0}{\tau} + \frac{\phi(1-\theta)\phi(\theta)}{\tau^\theta ABC(\theta)} \mathcal{L}\left[ \gamma I - \mu R \right], \\
\mathcal{L}[N(\theta)] &= \frac{N_0}{\tau} + \frac{\phi(1-\theta)\phi(\theta)}{\tau^\theta ABC(\theta)} \mathcal{L}\left[ N - \sigma N \right], \\
\mathcal{L}[D(\theta)] &= \frac{D_0}{\tau} + \frac{\phi(1-\theta)\phi(\theta)}{\tau^\theta ABC(\theta)} \mathcal{L}\left[ \gamma I - \lambda D \right], \\
\mathcal{L}[C(\theta)] &= \frac{C_0}{\tau} + \frac{\phi(1-\theta)\phi(\theta)}{\tau^\theta ABC(\theta)} \mathcal{L}\left[ \sigma E \right].
\end{align*}
\]
and so on. For \( n \geq 0 \), we write the generalized terms as:

\[
\mathcal{L}[S_{n+1}(\theta)] = \Omega \mathcal{L}\left[\frac{-\theta N S_0(\theta)}{N_0(\theta)} + \frac{\theta I_0(\theta)}{N_0(\theta)} - \mu S_n(\theta)\right], \\
\mathcal{L}[E_{n+1}(\theta)] = \Omega \mathcal{L}\left[\frac{N S_0(\theta)}{N_0(\theta)} + \frac{\theta (S_0(\theta) - \gamma) I_0(\theta)}{N_0(\theta)} - (\sigma + \mu) E_n(\theta)\right], \\
\mathcal{L}[R_{n+1}(\theta)] = \Omega \mathcal{L}[\theta I_0(\theta) - \mu R_n(\theta)], \\
\mathcal{L}[N_{n+1}(\theta)] = \Omega \mathcal{L}[-\mu N_n(\theta)], \\
\mathcal{L}[D_{n+1}(\theta)] = \Omega \mathcal{L}[\gamma I_0(\theta) - \lambda D_n(\theta)], \\
\mathcal{L}[C_{n+1}(\theta)] = \Omega \mathcal{L}[\sigma E_n(\theta)].
\]

Now taking inverse Laplace transform to Eqs. (27)-(30) we get

\[
\begin{align*}
S_0(\theta) &= S_0, \\
E_0(\theta) &= E_0, \\
I_0(\theta) &= I_0, \\
R_0(\theta) &= R_0, \\
N_0(\theta) &= N_0, \\
D_0(\theta) &= D_0, \\
C_0(\theta) &= C_0.
\end{align*}
\]

and so on. On the same fashion, by assigning proper values to parameters and initial terms, we get the series solution as

\[
\begin{align*}
S(\theta) &= S_0(\theta) + S_1(\theta) + S_2(\theta) + S_3(\theta) + \ldots, \\
E(\theta) &= E_0(\theta) + E_1(\theta) + E_2(\theta) + E_3(\theta) + \ldots, \\
I(\theta) &= I_0(\theta) + I_1(\theta) + I_2(\theta) + I_3(\theta) + \ldots, \\
R(\theta) &= R_0(\theta) + R_1(\theta) + R_2(\theta) + R_3(\theta) + \ldots, \\
N(\theta) &= N_0(\theta) + N_1(\theta) + N_2(\theta) + N_3(\theta) + \ldots, \\
D(\theta) &= D_0(\theta) + D_1(\theta) + D_2(\theta) + D_3(\theta) + \ldots, \\
C(\theta) &= C_0(\theta) + C_1(\theta) + C_2(\theta) + C_3(\theta) + \ldots.
\end{align*}
\]

6. Convergence analysis

The above series solution is in form of series, which is rapidly convergent series and converges uniformly to the exact solution.

**Theorem 6.1.** Let \( X \) be a Banach spaces and \( T : X \rightarrow X \) be a contractive nonlinear operator such that for all \( \Psi, \Psi \in X \), \( \| T(\Psi) - T(\Psi') \| \leq \delta \| \Psi - \Psi' \|, 0 < \delta < 1 \). Then \( T \) has at most one fixed point \( \Psi \) such that \( T\Psi = \Psi \), where \( \Psi = (S(\theta), E(\theta), I(\theta), R(\theta), N(\theta), D(\theta), C(\theta)) \). Moreover the series given in (34) can be written as by applying Adomian decomposition method

\[
\Psi_n = T\Psi_{n-1}, \quad \Psi_{n-1} = \sum_{j=0}^{n-1} \Psi_j, \quad n = 1, 2, 3, \ldots,
\]

and assume that \( \Psi \in B(\Psi) \) where \( B(\Psi) = \{ \Psi \in X : \| \Psi - \Psi' \| < \epsilon \} \), then we have

\[(A_1) \quad \Psi_n \in B(\Psi),
(A_2) \lim_{n \rightarrow \infty} \Psi_n = \Psi.
\]

**Proof.**

- In view of mathematical induction for \( n = 1 \), we have

\[
\| T\Psi_0 - \Psi \| \leq \delta \| \Psi_0 - \Psi \|.
\]

Let the result is true for \( n - 1 \), then

\[
\| T\Psi_{n-1} - \Psi \| \leq \delta^{n-1} \| \Psi_0 - \Psi \|.
\]

We have,

\[
\| \Psi_n - \Psi \| \leq \| T(\Psi_{n-1}) - T(\Psi) \| \leq \delta \| \Psi_{n-1} - \Psi \| \leq \delta^n \| \Psi_0 - \Psi \|.
\]

Hence,

\[
\| \Psi_n - \Psi \| \leq \delta^n \| \Psi_0 - \Psi \| \leq \delta^n \epsilon < \epsilon,
\]

which implies that, \( \Psi_n \in B(\Psi) \)

- Since \( \| \Psi_n - \Psi \| \leq \delta^n \| \Psi_0 - \Psi \| \rightarrow 0 \) as \( n \rightarrow \infty \). Also norm is continuous, which implies \( \lim_{n \rightarrow \infty} \Psi_n = \Psi. \)
7. Numerical results

For the interpretation of numerical results, we have taken the following values about Wuhan China during the month of January last ten days [13] as.

7.1. First choice

In Figs. 1–7, we plot the solution for the given choices of values as in Table 2 taking first values for $F = 0$, $\mu = 0$/day, $\alpha = 0$/day and $\beta_0 = 0.5944$/day. We graph the solutions in series form as in Eq. (35) for initial ten terms via using Matlab-16.

7.2. Second choice

Corresponding to the given values of the parameters, we present the solution through graphs which indicate that at different fractional order the decreasing and increasing dynamics of various compartment occur at various rate with the small fractional order the considered rate of fall and up is slightly high as compared to larger order. This why the smaller order become first stable as compared to the larger order as shown in Figs. 1–7. On the other hand in the absence of zoonotic cases reported and without Government action the dynamics of total population is straight as no changes is seen at different fractional order while involving the other factor may cause the change in dynamics.

In Figs. 8–14, we plot the solution for the given choices of values as in Table 2 taking the values as $F = 10$, $\mu = 0.0205$/day, $\alpha = 0.5239$ and $\beta_0 = 1.68$/day. We graph the solutions in series form as in Eq. (35) for initial ten terms via using Matlab-16.

From Figs. 8–14, the fall and up in different compartment population is shown against various fractional order which provide global dynamical behaviors of every class. Since the Government action is involved in this case and zoonotic cases are reported which are 10 produce significant effect in the decay of total population which shows that either people are immigrating or dying or get rid from the infection which is straight decay at different fractional order. The decay in total population is faster at smaller fractional order as order increasing the rate going on slowing as shown in Fig. 12. The dynamical behaviors given in Figs. 8–14 are similar as in previous case provided in Figs. 1–7. Hence we
Fig. 3  Fractional dynamics of Infected class $I(t)$ at different fractional order.

Fig. 4  Fractional dynamics of Recovered class $R(t)$ at different fractional order.

Fig. 5  Fractional dynamics of Total class $N(t)$ at different fractional order.
Fig. 6 Fractional dynamics of Mimicking class $D(t)$ at different fractional order.

Fig. 7 Fractional dynamics of Cumulative class $C(t)$ at different fractional order.

### Table 2 The physical interpretation of the parameters and numerical values.

| Parameters          | physical description                        | Numerical value                |
|---------------------|---------------------------------------------|--------------------------------|
| $F$                 | Number of zoonotic cases                    | 0, 10                          |
| $\beta_0$           | Transmission rate                           | 0.5944, 1.68/day               |
| $\alpha$            | Governmental action strength                | 0, 0.4239                      |
| $\kappa$            | Intensity of responds                       | 1117.3                         |
| $\mu$               | Emigration rate                             | 0, 0.0205/day                  |
| $\sigma^{-1}$       | Mean latent period                          | 3                              |
| $\gamma^{-1}$       | Mean infectious period                      | 5                              |
| $\lambda^{-1}$      | Mean duration of public reaction            | 11.2/day                       |
| $v$                 | Proportion of severe cases                  | 0.2                            |
| $N_0$               | Initial population size                     | 14 millions                    |
| $S_0$               | Initial susceptible population              | 0.9 $N_0$                      |
| $E_0$               | Initial exposed population                  | 10 millions                    |
| $I_0$               | Initial infection population                | 5 millions                     |
| $R_0$               | Initial recovered population                | 0 millions                     |
| $D_0$               | Initial mimicking people                   | 0.19 millions                  |
| $C_0$               | Initial cumulative cases (both reported and | 3.33 millions                  |
|                     | not reported assumed)                       |                                |
Fig. 8  Fractional dynamics of Susceptible class $S(t)$ at different fractional order.

Fig. 9  Fractional dynamics of Exposed class $E(t)$ at different fractional order.

Fig. 10  Fractional dynamics of Infected class $I(t)$ at different fractional order.
Fig. 11 Fractional dynamics of Recovered class $R(t)$ at different fractional order.

Fig. 12 Fractional dynamics of Total class $N(t)$ at different fractional order.

Fig. 13 Fractional dynamics of Mimicking class $D(t)$ at different fractional order.
concluded that fractional derivative is more capable to describe the complex dynamics of the novel corona virus–19 disease which has attacked on the whole globe in present time. Next we compare through the proposed model the real values of Cumulative class and simulated results for seventy days for the Wuhan city being reported in [55] as.

Cumulative cases

\[
\begin{align*}
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
\end{align*}
\]

Fig. 14 Fractional dynamics of Cumulative class \(C(t)\) at different fractional order.

\[
\begin{align*}
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
\end{align*}
\]

Fig. 15 Comparison between real data and simulated data for Cumulative class \(C(t)\) at classical order.

\[
\begin{align*}
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
\end{align*}
\]

8. Conclusion

We have comprehensively studied a novel corona virus-19 model from theoretical as well as numerical perspective. We have investigated the proposed model under ABC-nonsingular kernel type derivative. First of all we have guaranteed the existence of the considered model under the said derivative by using the fixed point theory. By nonlinear analysis we have also proved the Hyers-Ulam stability concepts for

\[
\begin{align*}
= [16, 14, 18, 27, 35, 38, 44, 60, 80, 131, 151, 259, 467, 688, 776, 1776, 1776, 1460, 1739, 1984, 2101, 2590, 2827, 3233, 3892, 3697, 3387, 3653, 2984, 2473, 2022, 1820, 1998, 1506, 1278, 2051, 1772, 1891, 399, 894, 397, 650, 415, 518, 412, 439, 441, 435, 579, 206, 130, 120, 143, 146, 102, 46, 45, 23, 31, 26, 11, 18, 28, 29, 40, 41].
\end{align*}
\]
the said model. After that by a powerful analytical method due to Laplace transform and decomposition method we have established semi-analytical solutions for the model under consideration. Also we have taken some real data and plotted the solutions against different fractional order. The fractional order dynamics is global in nature in excellently explain the dynamics of the model under consideration. Smaller the order, faster is the spreading or decay process of different compartments and vice versa. Hence fractional order derivative of ABC can also be used as powerful tool to describe many infectious disease through models. Also immigration of zoonotic recordation produce proper impact on the dynamics of the transmission of disease. We have also compared our considered model through the proposed method for the given reported data.

Authors contribution
Both authors have equal contribution in this work.

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Data availability statement
It is not applicable in this work.

Declaration of Competing Interest
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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