Interference-optical methods in mechanics for the multi-parameter description of the stress fields in the vicinity of the crack tip

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Abstract. Digital photoelasticity method is used to analyze experimentally the complete Williams series expansion of the stress and displacement fields in the vicinity of the crack tip in isotropic linear elastic plates under Mixed Mode loading. The distribution of the isochromatic fringe patterns is employed for obtaining the stress field near the crack tip by the use of the complete Williams asymptotic expansion for various classes of the experimental specimens (plates with two collinear cracks under tensile loading and under mixed mode loading conditions). The higher order terms of the Williams series expansion are taken into account and the coefficients of the higher order terms are experimentally obtained. The stress field equation of Williams up to fifty terms in each in mode I and mode II has been considered. The comparison of the experimental results and the calculations performed with finite element analysis has shown the importance and significant advantages of photoelastic observations for the multi-parameter description of the stress field in the neighborhood of the crack tip.

1. Introduction

Defects like crack, sharp notches and inclusions play a critical role in the failure of structural components [1]. These defects reduce the strength of structure and upon loading leads to the growth of crack followed by failure. The presence of crack alters the stress and strain field around the crack tip. In fracture mechanics, the stress intensity factor (SIF) expresses the strength of singular elastic stress field and they also characterize near-tip stress field. SIF is a function of applied load, crack length and geometry of the cracked body. The critical SIF value will decide the propagation of crack and the resulting failure under service load. SIF can be estimated using analytical, numerical, or experimental techniques. However, analytical methods are restricted to simple configurations and boundary conditions. For complex configurations, SIF needs to be extracted using either numerical or experimental method. In this work experimental evaluation of SIF using digital photoelasticity is carried out. In the past researchers have been estimating SIF with whole field non-contact optical methods such as holographic interferometry, Moiré interferometry, electronic-speckle-pattern interferometry, coherent gradient sensing, method of caustics, photoelasticity, and digital image correlation (DIC). Further, contact methods such as resistance strain gauge were also used. Methods like holography and other interferometric techniques are very sensitive to vibration and require a coherent light source and complex experimental setup. Among these experimental techniques, photoelasticity provides rich field data for complex geometry and loading with simple optical setup and specimen preparation. Photoelasticity is an optical, non-contact technique used for whole field stress analysis which provides the information of principal stress difference (isochromatic) and principal stress direction (isoclinic) in the form of fringe contours. In the early days of its
development, quantitative isoclinic and isochromatic data were obtained only along the fringe contours. With the advent of personal computer-based digital image processing systems, automation of photoelastic parameter estimation has now become popular and is often referred as digital photoelasticity [1,2].

The advent of computers coupled with developments in personal computer digital image processing has had a great influence in the development of modern photoelasticity. The term “digital photoelasticity” refers to the automation of the photoelastic data collection and analysis [2]. The classical manual procedure of analysis is usually very tedious and time consuming and requires skilled and experienced personnel. With the advent of digital image processing techniques in photomechanics, digital photoelasticity has become very popular [1-20]. The development of the computer-aided analysis of the experimental data obtained from photoelastic experiment caused rapid growth of the photoelasticity experiments [1-20], especially in fracture mechanics for analysis of the crack tip fields and crack tip parameters. In the past two decades crack-tip mechanics has been studied increasingly using full-field techniques, namely DIC [15] and photoelasticity [2].

Thus, in [3] the experimental technique of photoelasticity has been utilized for calculating bi-material notch stress intensities as well as the coefficients of higher order terms. Employing the equations of multi-parameter stress field allows data collection from a larger zone from the notch tip and makes the data collection from experiments more convenient. Moreover, the effects of higher order terms in the region near the notch tip are taken into account. For the photoelasticity experiments, a laboratory specimen known as the Brazilian disk with a central notch, consisting of Aluminum and Polycarbonate, has been utilized in [3]. Using this specimen, different mode mixities could be easily produced by changing the loading angle. The bi-material notch stress intensities and the first non-singular stress term (called T-stress) were calculated for different test configurations. In order to utilize the advantages of whole-field photoelasticity and minimize the experimental errors, a large number of data points were substituted in the multiparameter stress field equations. Then the resulting system of nonlinear equations was solved by employing an overdeterministic least squares method coupled with the Newton–Raphson algorithm. It has been shown [3] that considering the T-stress term improves, to a large extent, the accuracy of the stress intensities calculated through the photoelasticity technique. Moreover, by reconstructing the isochromatic fringes, the effects of the T-stress term on the shape and size of these fringes around the notch tip were investigated for a 30° notch. The experimental photoelasticity results were also compared with the corresponding values obtained from finite element analysis and a good correlation was observed.

In [4] it is noted that the V-notches are most possible case for initiation of cracks in structure elements. The specifications of cracks on the tip of the notch will be influenced via opening angle, tip radius and depth of V-notch. In [4] the effects of V-notch’s opening angle on stress intensity factor and T-stress of crack on the notch has been investigated. The experiment has been done in different opening angles and various crack length in mode I loading using photoelasticity method. The results illustrate that while angle increases in constant crack’s length, stress intensity factor (SIF) and T-stress will decrease. Beside, the effect of V-notch angle in short crack is more than long crack. These V-notch affects are negligible by increasing the length of crack, and the crack’s behavior can be considered as a single-edge crack specimen

Guaglianon [5] et al considered the multi-parameter description of the crack tip in elastic materials. The elastic stress field around a crack tip is fully defined through multiparameter equations. A code and program were implemented to evaluate the characteristic parameters of the stress field around a crack tip by photoelastic analysis. The possibility to change the number of parameters makes it possible to adapt the study to different cases, increasing the extension of the analyzed area in order to have a correct modeling of the photoelastic fringes. The performed experimental tests allow emphasizing the importance of using multiparameter equations in the study of the stress field around the crack tip.

Inspired by the Brazilian disk geometry the authors of [6] examine the utility of an edge cracked semicircular disk (ECSD) specimen for rapid assessment of fracture toughness of brittle materials using compressive loading. It is desirable to optimize the geometry towards a constant form factor for evaluating SIF $K_I$. In this investigation photoelastic and finite element results for $K_I$ evaluation
highlight the effect of loading modeled using a Hertzian. A Hertzian loading subtending $4^\circ$ at the center leads to a surprisingly constant form factor of 1.36. This special case is further analyzed by applying uniform pressure over a chord for facilitating testing.

Lei et al [7] applied photoelasticity method for study of structural imperfection of ZnGeP$_2$ crystal. The stresses, related to rows and accumulations of dislocations were revealed by photoelastic method for ZnGeP$_2$ crystals, grown by Vertical Bridgman method. In [8] it is noted once again that the photoelastic technique has seen some renewed interest in past few years with digital images and image processing new methods becoming readily available. However, further research is needed to improve the precision, the accuracy and the automation of photoelastic technique. The aim of this research work is to get new numerical equations for the phase-shifting method in digital photoelasticity using a plane polariscope. The model was developed to plane polariscope because of the simplicity and low cost of this equipment. To develop the phase shift and respective intensity equations only the analyzer is rotated. A ring under diametral compression is used for the experimental validation. From these intensity equations, the equations for isoclinic and isochromatic parameters are deduced by applying a new numerical technique. This approach can be used to calculate the isoclinic and isochromatic parameters using any number of images. Several analysis are performed with different number of photographic images. The results showed errors reduce when more phase-stepped images are utilized. Hence, one concludes that the uncertainties in results due to effects of errors on photoelastic images can be reduced with a larger amount of phase-stepped images.

In [9] the recent advances in digital photoelasticity have made it possible to use it conveniently for the stress analysis of articles and components made of glass. Depending on the application, the retardation levels to be measured range from a few nanometres to several thousand nanometres, which necessitates different techniques and associated equipments. This paper reviews the recent advances in the photoelasticity of glass with a focus on the techniques/methods developed in the last decade.

The aim of the present study is to found coefficients of the multiparameter asymptotic expansion of the stress field in the vicinity of the tips of two collinear cracks in the isotropic linear elastic plate using the photoelasticity method. The main idea of the paper is to keep the higher-order terms of the Williams series expansion of the stress filed, to reveal and evaluate the effect of the higher-order terms of the asymptotic expansion. The motivation of this study is twofold. First, we would like to compare theoretical results obtain in [10,11] with experimental data. According to [10,11] the higher-order terms of the complete asymptotic expansion of the crack-tip stress field can play a significant role. The more distance from the crack tip the more terms it is necessary to keep in the Williams asymptotic expansion. Thus it follows the second reason when the photoelastic data is processing the number of terms of the asymptotic expansion can not be chosen arbitrary. It depends on the distance from the crack tip. Therefore, the distance from the crack tip should be taken into account when we suppose the structure of the solution in the vicinity of the crack tip.

Note that the coefficients of higher-order terms in the Williams series expansions were computed by different approaches in numerous studies [11-15].

In [12] digital photoelasticity technique is used to estimate the crack tip fracture parameters for different crack configurations. Conventionally, only isochromatic data surrounding the crack tip is used for SIF estimation, but with the advent of digital photoelasticity, pixel-wise availability of both isoclinic and isochromatic data could be exploited for SIF estimation in a novel way. A linear least square approach is proposed to estimate the mixed-mode crack tip fracture parameters by solving the multi-parameter stress field equation. The stress intensity factor (SIF) is extracted from the estimated crack parameters. The isochromatic and isoclinic data around the crack tip is estimated using the ten step phase shifting technique. The experimental SIF values estimated using the proposed method are compared with analytical/finite element analysis (FEA) results, and are found to be in good agreement.

In [12] the method of photoelasticity is used to study the effects of first non-singular stress term on isochromatic fringe patterns around the tip of a mode I sharp V-notch. Notches are divided into two categories: notches with opening angles a) less than 45°, and b) between 45° and 152°. First, utilizing the mathematical relations of the isochromatic fringes, the effects of the first non-singular stress term
on the shape and size of the fringes are studied theoretically. Good correlation between the analytical and experimental results is observed.

Harilal et al [13] an experimental study is carried out to estimate the mixed-mode stress intensity factors (SIF) for different cracked specimen configurations using digital image correlation (DIC) technique. For the estimation of mixed-mode SIF's using DIC, a new algorithm is proposed for the extraction of crack tip location and coefficients in the multi-parameter displacement field equations. From those estimated coefficients, SIF could be extracted. The experimental results have been compared with the analytical values and they are found to be in good agreement, there by confirming the accuracy of the algorithm being proposed.

The coefficients of higher-order terms in the Williams series expansions were computed using the DIC method which is a noncontact full-field optical technique [15]. First, the fundamental concepts of DIC method were described and then, this method was proposed to obtain the higher-order terms of the Williams expansion for a CT specimen under pure mode I loading. The displacement field around the crack tip in the CT specimen was determined by the DIC approach. The displacements were utilized in order to obtain the coefficients of Williams expansion. Then, these coefficients were also calculated by using the FE method from the displacement field in the vicinity of the crack tip. The values of stress intensity factor and T-stress obtained from the DIC and FE techniques were compared with the results of previous researches. The efficiency and accuracy of the DIC technique in determining the coefficients of higher order terms in the Williams expansion were demonstrated for the CT specimen. As it is noted in [11] the accuracy of SIF estimate could be improved by improving the accuracy of the isoclinic parameter estimate using the white light photoelasticity thereby eliminating the isochromatic–isoclinic interaction noise. An advanced experimental technique for determination of the stress intensity factor (SIF) and the T-stress is developed in [18] and carefully verified. The approach employs optical interferometric measurements of local deformation response to small crack length increment. Narrow notches are used for crack modeling. Initial experimental data represent inplane displacement component values measured by electronic speckle-pattern interferometry in the vicinity of the crack tip. Determination of the first four coefficients of Williams’ series is the main feature of the developed technique. Relationships for transition from measured in-plane displacement components to required fracture mechanics parameters are presented. Availability of high-quality interference fringe patterns, which are free from rigid-body motion, serves as a reliable indicator of real strain state near the crack tip. Experimental verification of the proposed method is performed for non-symmetrical and symmetrical crack in thin rectangular plates subjected to uniaxial tension. The distributions of SIF and T-stress values for cracks of different length in residual stress fields near electronically welded joints of thin plates are presented as an example of practical implementing.

In [19] the common definitions for mode I and mode II are evaluated and improved. For this purpose, the in-plane linear elastic stress field around the crack tip is written as a set of infinite series expansions. Mode I and mode II fields are classically defined as symmetric and anti-symmetric parts of these expansions, respectively. There is also a constant term called “T-stress” in these expansions; parallel to the crack line and independent of the distance from the crack tip. Previous definitions assume that Ts-tress exists only in pure mode I or combined mode I and mode II conditions. Based on these definitions, T-stress always vanishes in pure mode II. However, the published results of several analytical and experimental researches indicate that the constant stress term can exist in mode II stress field, as well. In this paper, some examples are presented which indicate the presence and importance of T-stress in pure mode II conditions. Then, the classical definition for mode I and mode II is modified to make it consistent with the results presented in the literature [20-36].

Thus, the photoelasticity techniques have been extensively used for experimentally determining the state of stress in actual mechanical components. Nowadays digital image processing and new image analysis algorithms become very important [37, 38].

In this research, digital photoelasticity was employed to assess the singular and higher-order coefficients of the Williams series expansion for the stress field in the vicinity of the crack tip. To utilize the advantages of the whole – field photoelasticity and minimize the experimental errors, the overdeterministic method [18] has been used. The experimental equipment is shown in Fig. 1. The aim of this paper is to obtain the coefficients of the higher-order terms in the Williams expansion and to
estimate the influence of these terms on the stress field description taking into account as many as possible terms in the asymptotic presentation of the crack tip fields.

2. Elastic stress field in the vicinity of the crack tip

2.1. M. Williams asymptotic series expansion

In the development of fracture mechanics, Williams made a major breakthrough in the analysis of asymptotic stress field at the vicinity of the crack tip in isotropic linear elastic plane media. With the well-known eigenfunction expansion method it is possible to establish the separable variable nature of the solution and to obtain asymptotic expressions for the stress field in a plane medium with a traction-free crack submitted to mode I, mode II and mixed-mode (mode I and mode II) loading conditions:

\[ \sigma_m(r,\theta) = \sum_{m=1}^{2} \sum_{k=-\infty}^{\infty} a_{ik}^{m} r^{k/2-1} f_{m,j}^{(k)}(\theta), \]

with index \( m \) associated to the fracture mode; \( a_{ik}^{m} \) amplitude coefficients related to the geometric configuration, load and mode; \( f_{m,j}^{(k)}(\theta) \) angular functions depending on stress component and mode.

Analytical expressions for angular eigenfunctions \( f_{m,j}^{(k)}(\theta) \) are available [3,4,35,36]:

\[ f_{1,11}^{(k)}(\theta) = (k/2) \left[ 2 + k/2 + (-1)^k \right] \cos(k/2 - 1)\theta - (k/2 - 1) \cos(k/2 - 3)\theta, \]
\[ f_{1,12}^{(k)}(\theta) = (k/2) \left[ 2 - k/2 - (-1)^k \right] \cos(k/2 - 1)\theta + (k/2 - 1) \cos(k/2 - 3)\theta, \]
\[ f_{1,21}^{(k)}(\theta) = -(k/2) \left[ 2 + k/2 - (-1)^k \right] \sin(k/2 - 1)\theta + (k/2 - 1) \sin(k/2 - 3)\theta, \]
\[ f_{1,22}^{(k)}(\theta) = -(k/2) \left[ 2 - k/2 + (-1)^k \right] \sin(k/2 - 1)\theta - (k/2 - 1) \sin(k/2 - 3)\theta, \]
\[ f_{2,11}^{(k)}(\theta) = -(k/2) \left[ 2 + k/2 - (-1)^k \right] \sin(k/2 - 1)\theta - (k/2 - 1) \sin(k/2 - 3)\theta, \]
\[ f_{2,12}^{(k)}(\theta) = (k/2) \left[ 2 - k/2 + (-1)^k \right] \cos(k/2 - 1)\theta + (k/2 - 1) \cos(k/2 - 3)\theta, \]
\[ f_{2,22}^{(k)}(\theta) = -(k/2) \left[ 2 - k/2 + (-1)^k \right] \cos(k/2 - 1)\theta - (k/2 - 1) \cos(k/2 - 3)\theta. \]

The multi-parameter fracture mechanics concept consists in the idea that the crack tip stress field is described by means of the Williams expansion (1). Characteristics of the fracture problems such as the geometry of the specimen and intensity of the load influence neither radial nor angular functions in Eq. 1 [21]. All the variety of fracture mechanics problems is therefore taken into account in the sole sequence of coefficients \( a_{ik}^{m} \). Terms with higher orders have been proven to be influential in some circumstances [10,11,16, 21 -36]. In the present paper the experimental technique of photoelasticity was employed to investigate the effects of the higher order terms on both the shape and the size of the near crack tip isochromatic fringes for the wide range of specimens.

2.2. Photoelastic determination of coefficients of the higher order terms in the Williams series expansion

Based on the classical concepts of photoelasticity, the locus of an isochromatic fringe is expressed by the stress optic law

\[ 2\tau_m = N f / h, \]

where \( \tau_m \) is the maximum in-plane shear stress, \( N \) and \( f \) represent the fringe order and the material stress-fringe value, respectively, \( h \) is the specimen thickness. On the other hand, the relation between the maximum shear stress \( \tau_m \) and the Cartesian components is

\[ \tau_m^2 = (\sigma_{11} - \sigma_{22})^2 / 4 + \sigma_{12}^2. \]

The maximum in-plane shear stress around the crack tip can be considering \( K \) terms of Mode I and \( M \) terms of Mode II expansion from Eq. 1. By substituting the truncated series expansion of Eq. 1 (with \( K \) terms of Mode I and \( M \) terms of Mode II) into Eq. 4 the mathematical equation for a fringe developed around the crack tip can be written as
Eq. 5 provides $K$ and $M$ terms in Mode I and Mode II expansions, respectively to represent the near crack tip stress field. Further the method of evaluation of mixed-mode stress field parameters proposed in [17] is used. The method is called as an over-deterministic method for calculating the stress intensity factor as well as the coefficients of the higher-order terms in the Williams series expansions in cracked bodies.

The experimental setup used is shown in Figure 1. A set of cracked specimens was used to calculate the stress intensity factor and the coefficients of the higher-order terms of the complete Williams series expansion for the stress field in the vicinity of the crack tip. These specimens are the plate with two collinear crack of the equal length, the plates with two collinear cracks of different lengths, the plates with two inclined collinear cracks of the equal and different lengths (Figures 2-6). All the specimens in this work were made by casting of polycarbonate. Circular and semi-circular shapes were machined from the sheet to get the test specimens. Material properties of the photoelastic material are Young’s modulus $E = 3 GPa$, Poisson’s ration $\nu = 0.3$ and the material fringe constant $f = 18.38 Pa m fringe$.

2.3. Java code for digital processing interference – optic experimental data

In the present study a Java application for advanced determination of the fracture characteristics of the elastic materials is introduced and used. For automatic operating with experimental data obtained from photoelastic experiments a Java programme code has been written. The program developed allows us to find and collect automatically coordinates of points belong to isochromatic fringes and save the number of isochromatic fringe. Figure 2 shows the isochromatic fringe pattern of circular disk under diametral compression. The green lines in Figure 2 show the skeleton of the isochromatic fringe pattern used in processing experimental data. Figures 3-6 show the isochromatic fringe pattern in the cracked specimens. The green lines shown in Figures 3-6 demonstrate the collected points of isochromatic fringes. The programme code allows us to save the experimental data to a file. Thus, the file collects coordinates of points belong to isochromatic fringes and the fringe order. The file is a source for other programme determining the fracture parameters, namely, the coefficients of the higher order terms in the Williams asymptotic series expansion. The algorithm developed was tested using the simplest specimens, for instance, circular disk under a pair of concentred forces. The specimen and the isochromatic fringes are shown in Figure 2. One can see that the Java application reconstructs the isochromatic fringe pattern in specimens quite well even for the complex isochromatic fringe patterns.

\[
\left( \frac{Nf}{2h} \right)^2 = (\sigma_{11} - \sigma_{22})^2 / 4 + \sigma_{12}^2 = K \sum_{k=1}^{K} a_k^2 r^{k/2-1} f_{11}^{(k)}(\theta) + \\
+ \sum_{k=1}^{K} a_k^2 r^{k/2-1} f_{211}^{(k)}(\theta) - \sum_{k=1}^{K} a_k^2 r^{k/2-1} f_{122}^{(k)}(\theta) - \sum_{k=1}^{K} a_k^2 r^{k/2-1} f_{222}^{(k)}(\theta) \right) / 4 + \\
+ \left( \sum_{k=1}^{K} a_k^2 r^{k/2-1} f_{112}^{(k)}(\theta) + \sum_{k=1}^{M} a_k^2 r^{k/2-1} f_{212}^{(k)}(\theta) \right)^2
\]
shown in Figures 4 and 6. Here these cracked specimens with inclined cracks are used to model mixed mode loading conditions.

![Figure 2. Isochromatic fringe pattern in a disk and the results of selecting points belonging to isochromatic fringes: skeleton of the isochromatic fringes.](image)

![Figure 3. The isochromatic fringes in a plate with two cracks (left) and the result of processing the experimental data: skeleton of the isochromatic fringes (right).](image)

![Figure 4. The isochromatic fringes in a plate with two inclined cracks (left) and the result of processing the experimental data: skeleton of the isochromatic fringes (right).](image)

The results of calculations obtained by the over-deterministic method are presented in Table 1 where the coefficients of the Williams asymptotic expansion for the stress field in the vicinity of the crack tip in the plate with one inclined crack at an angle 45° are given.
Table 1. Coefficients of the Williams expansion for a plate with the central inclined crack.

| $\tilde{a}^1_{l,m+1}$ | $\tilde{a}^2_{l,m+1}$ |
|-------------------------|-------------------------|
| $\tilde{a}^1_1 = 0.7808688cm^{-1/2}$ | $\tilde{a}^2_1 = -0.7808688cm^{-1/2}$ |
| $\tilde{a}^1_2 = -0.245522$ | $\tilde{a}^2_2 = 0$ |
| $\tilde{a}^1_3 = 0.2380697cm^{-3/2}$ | $\tilde{a}^2_3 = -0.2380697cm^{-3/2}$ |
| $\tilde{a}^1_4 = -0.0362911cm^{-5/2}$ | $\tilde{a}^2_4 = 0.0362912cm^{-5/2}$ |
| $\tilde{a}^1_5 = 0.0110643cm^{-7/2}$ | $\tilde{a}^2_5 = -0.0110643cm^{-7/2}$ |
| $\tilde{a}^1_6 = -0.0042166cm^{-9/2}$ | $\tilde{a}^2_6 = 0.0042166cm^{-9/2}$ |
| $\tilde{a}^1_7 = -0.0017990cm^{-11/2}$ | $\tilde{a}^2_7 = 0.0017990cm^{-11/2}$ |
| $\tilde{a}^1_8 = -0.0008233cm^{-13/2}$ | $\tilde{a}^2_8 = 0.0008233cm^{-13/2}$ |
| $\tilde{a}^1_9 = 0.0003943cm^{-15/2}$ | $\tilde{a}^2_9 = -0.0003943cm^{-15/2}$ |
| $\tilde{a}^1_{10} = -0.0001955cm^{-17/2}$ | $\tilde{a}^2_{10} = 0.0001955cm^{-17/2}$ |
| $\tilde{a}^1_{11} = 0.0000999cm^{-19/2}$ | $\tilde{a}^2_{11} = -0.0000999cm^{-19/2}$ |

3. Finite element evaluation of fracture mechanic parameters

Along with the photoelasticity method the finite element study for a wide range of cracked specimens has been performed. The typical configurations are shown in Figure 7. The cracked semicircular bend specimen is chosen for conducting mixed-mode fracture experiments. The broad mixed-mode loading configuration from pure I to II can be achieved by changing the crack inclination angle.
Stress intensity factors and T-stress are determined in normalized forms by the following general equations

\[
K_i = \frac{P}{(2Rt)\sqrt{\pi a}}Y_i(a/R,S/R,\alpha), \quad i = I, II \quad T = \frac{P}{(2Rt)}T^*(a/R,S/R,\alpha)
\]  

(6)

where \(T^*\) is the normalized T-stress and \(Y_i\) and \(Y_n\) are the geometry factors corresponding to each fracture mode. Numerical and experimental works are performed on the mixed-mode I/II brittle fracture by edge cracked semicircular bend specimens. In the test numerical program fracture test were conducted at crack inclination angles of 0°, 10°, 15°, 20°, 30°, 45°, 49°, 50°, 80°. The stress intensity factors for Mode I and Mode II loading and T-stresses are obtained numerically in FEM package SIMULIA Abaqus. The results of calculations are shown in Table 2 and Figure 9. Figure 9 shows the dependences of the shape factors of mode I and II calculated in accordance with Eq. (6).

To validate estimations performed by means of the photoelasticity experiments several FE tests for the same test configurations were performed. The results of FE calculations are shown in Figure 10 (left). The corresponding photoelasticity experiment is shown in Figure 10 (right). One can see that the Mises equivalent stress obtained experimentally and numerically coincide.

**Figure 8.** Detailed of finite element mesh pattern used for the simulation of the semicircular bend specimen.

**Table 2.** Normalized stress intensity factors and T-stress.

| The crack inclination angle \(\alpha\) | \(Y_i\)  | \(Y_n\)  | \(T^*\)  |
|------------------------------------|---------|---------|---------|
| 0                                  | 39.67   | 0.00    | -4      |
| 5                                  | 39.39   | -2.21   | -3      |
| 10                                 | 38.48   | -4.33   | -2      |
| 15                                 | 36.99   | -6.25   | 1       |
| 20                                 | 35.05   | -7.91   | 4       |
| 25                                 | 32.74   | -9.25   | 7       |
| 30                                 | 30.15   | -10.26  | 11      |
| 35                                 | 27.43   | -10.93  | 14      |
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4. Conclusions
The paper introduces a Java application programmed for the advanced determination of the fracture characteristics: coefficients of the Williams series expansion (WE) for the stress field in the vicinity of the crack tip. The tool allows us to collect experimental points from the photoelasticity tests on the cracked specimens. An automatic routine implemented as a Java application permits to determine the values of coefficients of higher order terms of the WE that describe crack-tip fields. These values are calculated using the over-deterministic method which is also applied to the results of the finite element analysis of some mode I and mixed mode test geometry. Thus, the Java application provides an analytical reconstruction of the crack-tip stress field by means of the truncated WE and enables detailed analysis of the crack-tip stress field approximation. The developed procedures simplify the analysis of the description of mechanical fields at a greater distance from the crack tip considerably. The presented study is focused on the optimization of the selection of FE nodal results entering the over-deterministic technique used to determine the values of coefficients of the higher order terms of the WE.

| Angle (°) | Γ1 | Γ2 |
|----------|----|----|
| 40       | 24.61 | -11.27 |
| 45       | 21.81 | -11.31 |
| 50       | 19.08 | -11.08 |
| 55       | 16.47 | -10.62 |
| 60       | 13.97 | -9.96  |
| 65       | 11.61 | -9.12  |
| 70       | 9.39  | -8.10  |
| 75       | 7.29  | -6.90  |
| 80       | 5.25  | -5.46  |
| 85       | 3.19  | -3.68  |
| 89       | 0.94  | -1.75  |

Figure 9. Geometrical factors for various crack inclination angles in the semicircular bend specimen.

Figure 10. Comparison of results obtained by the finite element method and the photoelasticity method.
The results of FEM modeling are compared with the experimental results obtained by the photoelasticity method. The experimental approach based on the photoelasticity method allows us to observe the von Mises equivalent stress distribution in the whole specimen and confirm the FEM studies performed. The comparison shows that in the specimen the pure mode II loading can’t be realized. Analysis of numerical studies and the isochromatic fringe patterns allows us to conclude that in the semicircular disc the mixed mode loadings are realized for all angles from 0° to 80°.

5. References

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