Passive Beamforming Design for Reconfigurable Intelligent Surface Enabled Integrated Sensing and Communication

Zhe Xing, Graduate Student Member, IEEE, Rui Wang, Senior Member, IEEE, and Xiaojun Yuan, Senior Member, IEEE

Abstract

To exploit the potential of the reconfigurable intelligent surface (RIS) in supporting the future integrated sensing and communication (ISAC), this paper proposes a novel passive beamforming strategy for the RIS-enabled ISAC (RIS-ISAC) system in consideration of the target size. To this end, the detection probability for target sensing is derived in closed-form based on the illumination power on an approximated scattering surface area (SSA) of the target, and a new concept of ultimate detection resolution (UDR) is defined for the first time to measure the capability of the target detection. Subsequently, an optimization problem is formulated to maximize the signal-to-noise ratio (SNR) at the user-equipment (UE) under a minimum detection probability constraint. To solve this problem, a novel convexification process is performed to convexify the detection probability constraint with matrix operations and a real-valued first-order Taylor approximation. The semidefinite relaxation (SDR) is then adopted to relax the problem. A successive convex approximation (SCA) based algorithm is finally designed to yield a phase-shift solution, followed by a detailed analysis on the problem feasibility condition as well as the algorithm convergence. Our results reveal the inherent trade-offs between the sensing and the communication performances, and between the UDR and the duration of a sensing time slot. In comparison with two existing approaches, the proposed strategy is validated to be superior when detecting targets with practical sizes.

Index Terms

Reconfigurable intelligent surface (RIS), integrated sensing and communication (ISAC), beamforming optimization, detection probability, ultimate detection resolution (UDR).

Z. Xing and R. Wang are with the College of Electronics and Information Engineering, Tongji University, Shanghai 201804, China. R. Wang is also with the Shanghai Institute of Intelligent Science and Technology, Tongji University, Shanghai 201804, China (e-mail: zxing@tongji.edu.cn; ruiwang@tongji.edu.cn).

X. Yuan is with the National Key Laboratory of Science and Technology on Communications, University of Electronic Science and Technology of China, Chengdu, 610000, China (e-mail: xjyuan@uestc.edu.cn).
I. INTRODUCTION

The forthcoming beyond fifth- (B5G) and sixth-generation (6G) mobile communications have been envisioned as pivotal enablers for many innovative applications, such as the autonomous mobility, virtual/augmented reality (VR/AR), digital twin, and human-machine interaction, etc. [1], [2]. Supporting these applications requires a tight cooperation of wireless communication and environmental sensing, which have been concurrently developed with rare coordination and mutual benefit for decades [3], [4]. Recently, owing to their commonalities in regard of signal processing methods, hardware platforms and system architectures, etc. [4], the coexistence and merging of the two individual functionalities have attracted considerable interest, thereby promoting the emergence and development of a novel paradigm shift, termed integrated sensing and communication (ISAC) [5].

The ISAC can be performed with the aid of various key enabling technologies, including the millimeter wave (mmWave), ultra-dense network (UDN) and multiple-input-multiple-output (MIMO) radar. These technologies have been incorporated to boost the communication and sensing capabilities by improving the spectral efficiency and spatial degrees of freedom (DoF), but are still unable to adequately address several critical challenges [6], [7]. For instance, the mmWave is highly susceptible to obstructions on the line-of-sight (LoS) path, and suffers from severe propagation loss in the atmosphere. To compensate for the resultant signal attenuation over the wireless channel, higher transmit power and/or antenna gain are requisite to enhance the emitted signal strength, thereby increasing the energy consumption (EC) to a large extent. In addition, the dense deployment of the ISAC base stations (BSs) with massive MIMO arrays brings about high hardware cost (HC), while further pushing the total EC in the network to an exorbitant level. To tackle these issues, recent attention has been paid to a new burgeoning concept termed reconfigurable intelligent surface (RIS), or intelligent reflecting surface (IRS) [8], which early appeared as a prototype of intelligent wall [9], and was developed by the landmark works [10], [11] three years ago. An RIS is a near-passive reflecting metasurface with many small controllable units, which can be digitally configured to perform passive beamforming by changing the physical properties of the impinging electromagnetic wave (such as the phase-shift), so as to create a reliable virtual LoS link and manipulate the propagation environment intelligently [12], [13]. Since the RIS is generally fabricated with cheap hardware components without energy-consuming radio-frequency (RF) chains [14], it is envisioned as a promising candidate technology in compliant with the notion of green communication in B5G and 6G.

Up to now, plenty of researches have been focusing on the performance analysis and application potential of the RIS in both communication and sensing fields. As for communication, the RIS was
leverage to transfer passive information [15], build index modulation scheme [16] and achieve secure physical-layer transmission [17], [18], owing to the flexibility of the phase-shift adjustment. More importantly, it was revealed that a sufficiently large RIS with \( N \) reflecting units could yield a remarkable signal-to-noise ratio (SNR) gain by \( \mathcal{O}(N^2) \) over the cascaded channel [19]. In light of this, the RIS was also widely employed to combat the unfavorable channel conditions by forming concentrative passive beams toward the desired users, so as to improve the achievable data-rate [6], [20], spectral/energy efficiency [11], outage probability [21], [22] and bit-error-rate (BER) performance [23] of the assisted wireless communication system. As for sensing, some prior works exploited the RIS to perform user localization in combination with the codebook search [24], the phase-shift optimization as well as the parameter estimation [25], [26], and incorporated the RIS into the conventional radar system to assist the target (TG) detection by providing additional reception link for echoes [27]–[29]. According to their results, such meta-localization and meta-radar systems were validated to be able to outperform the traditional ones without the RIS, especially when the reflection arrays were fabricated to be large.

Owing to the benefits brought by the utilization of the RIS, recent progresses have been made to incorporate the RIS into the ISAC system. For instance, Jiang et al. [30], first introduced the RIS to the dual-function radar and communication (DFRC) system with single user-equipment (UE) and single TG, and jointly optimized the reflection matrix and the transmit precoder. The optimization problem was solved by the semidefinite relaxation (SDR) and the majorization–minimization (MM) methods. After that, Song et al. [31], further considered a single-user RIS-ISAC scenario with multiple TGs to be detected, and proposed to maximize the minimum beampattern gain in several sensing directions under the transmit power constraint at the BS and the SNR constraint at the UE. Liu et al. [32], extended the beamforming design to a multi-user RIS-ISAC system, and alternatively optimized the transmit beamformer, receive filter and reflection coefficients by maximizing the sum-rate of the UE under the radar SNR constraint. Wang et al. [33], performed the joint waveform and phase-shift design in an RIS-aided multi-user DFRC system to minimize the multi-user interference (MUI). Tong et al. [34], exploited the RIS to assist the uplink multi-user ISAC by dividing the sensing space into several blocks, and applied the generalized approximate message passing to determine the environmental information. Unlike these works considering the completely passive and continuous RIS, Prasobh Sankar et al. [35], introduced the hybrid RIS to the multi-user ISAC system, where partial elements on the RIS were designed to be active while the others remained passive. Wang et al. [36], considered a more practical scenario where the RIS phase-shifts were discrete, and took the Cramér-Rao bound (CRB) of the direction-of-arrival (DOA) estimation as a sensing performance metric.
In these prior works, heterogeneous RIS-aided beamforming strategies were developed to jointly fulfil the communication and sensing demands. Although these initial attempts made a big step forward in this direction, some challenging problems remained unsolved. First, the TGs to be sensed are mostly treated as points. However, the cross-section area of a TG is generally non-ignorable because it is essentially associated with the scattering capability. Second, under the assumption of point TG, the beamforming design is confined to the conventional end-to-end channel model and abandons much freedom in sensing beampattern adjustment. Such freedom is crucial for further improvement of the TG detection performance in an RIS-ISAC system. Consequently, it is of significance to develop a new RIS-enabled passive beamforming strategy in consideration of the TG size, which motivates our research.

In this paper, we take the TG size into account, and propose to jointly optimize the performances of the UE communication and TG detection via a novel phase-shift design. On the basis of our conference version [1], here we further introduce a new concept for sensing capability measurement and provide extensive analysis on the proposed design. Our contributions are summarized as follows.

• **Derivation of the Detection Probability and Definition of the Ultimate Detection Resolution (UDR):** In this work, a physics-based model is adopted to characterize the practical RIS reflection. Considering the TG size, the scattering surface area (SSA) of the TG is approximated as a smooth surface in accordance with empirical radar cross-section (RCS) measures of practical TGs. Then, based on the illumination power computed integrally over the SSA of the TG in a specific sensing direction, the detection probability of the TG is derived in closed-form. According to the detection probability, a new concept of UDR is mathematically defined for the first time. The UDR has not been reported in any previous works to the best of our knowledge, but is essential for sensing capability measurement by characterizing the minimum size of the detectable TG under a certain system setup and a given detection probability requirement.

• **Passive Beamforming Design for the RIS-ISAC:** An optimization problem is formulated to optimize the phase-shifts of the RIS, by maximizing the SNR at the UE under a minimum detection probability constraint. To solve the non-convex problem, a novel convexification process is proposed to linearise the detection probability constraint by means of a series of matrix manipulations and a real-valued first-order Taylor approximation. Afterwards, the SDR is adopted for problem relaxation, and a successive convex approximation (SCA) based algorithm is designed to iteratively find the phase-shift solution. Finally, the feasibility condition of the convexified problem is analysed in detail and is used to determine the UDR, while the designed algorithm is rigorously proved to be convergent. It is revealed that the optimization problem is guaranteed to
be feasible at every iteration step if the initial point is properly set.

- **Performance Evaluation**: The analytical properties of the proposed beamforming strategy are testified via the simulations. Two important trade-offs, namely, 1) communication vs. sensing trade-off, and 2) UDR vs. sensing-duration trade-off, are numerically investigated. Then, the proposed strategy is compared with two recently developed state-of-the-art approaches which assume point TGs. Simulation results validate that the proposed strategy significantly outperforms the approach which merely considers the BS-RIS-TG link, and is superior to the approach in consideration of the echo link when detecting TGs with practical sizes.

The rest of this paper is organized as follows. Section II describes the system model and the problem formulation. Section III proposes the convexification process and designs the SCA-based algorithm, with detailed analysis on the problem feasibility condition and the algorithm convergence. Section IV carries out the simulations for performance evaluations. Section V draws the final conclusion.

**Notations**: Boldfaces represent the vectors or matrices, while italics symbolize the variables or constants. \((\cdot)^*, (\cdot)^T, (\cdot)^H, (\cdot)^{-1}\) denote the conjugate, transpose, Hermitian and inversion operators, respectively. \(|\cdot|\) and \(\|\cdot\|_2\) denote the modulus and \(\ell_2\) norm. \(\mathbb{E}\{\cdot\}\), \(\text{rank}(\cdot)\) and \(\text{tr}(\cdot)\) are the expectation, the rank and the trace. \(\otimes\) denotes the Kronecker product. \(\int_{S_r} (\cdot) \, dS\) is the surface integral over \(S_r\). \(\text{vec}(\cdot)\) and \(\text{vec}^{-1}(\cdot)\) stand for the vectorize and matrixing operators. \(\Re(\cdot)\) and \(\Im(\cdot)\) represent the real part and imaginary part. \(\langle x, y \rangle\) denotes the inner product of \(x\) and \(y\).

## II. System Model and Problem Statement

As depicted in Fig. 1, we consider an mmWave RIS-ISAC system composed of a BS equipped with \(M\) transmitting antennas and \(M\) receiving antennas arranged in uniform linear arrays (ULAs) with antenna spacing of \(d\), an RIS equipped with \(N = N_x N_y\) reflecting elements in a uniform planar array (UPA) with element spacing of \(d\), a TG to be detected, and a single-antenna communication UE served by the BS. The UE is assumed to be static. An obstruction is assumed to block the LoS path between the BS and the UE/TG, such that the corresponding direct links are unavailable on account of limited and weak non-LoS scattering of the mmWave [7].

In the considered RIS-ISAC system, the BS is responsible for simultaneously communicating with the UE and detecting the TG in the environment. Since the TG near or around the BS can be easily sensed by the direct-link echoes [33], [36], this system is dedicated to the detection of the TG behind the obstacle without the LoS path. In such a scenario, to attain a favourable communication/sensing performance, the RIS is leveraged to establish strong virtual LoS links between the BS and the UE/TG. By properly adjusting the phase shifts, the RIS is expected to produce considerable passive
Fig. 1: The considered mmWave RIS-ISAC system, where the RIS is leveraged to reflect the ISAC signal from the BS to the UE and the TG for communication and sensing purposes. A portion of the signal wave scattered by the TG is then reflected back to the BS for TG detection.

Fig. 2: Illustration of the joint sensing and communication scheme over the time line. The sensing and communication tasks can be jointly performed as described in Fig. 2. Specifically, we consider one complete ISAC round as an example, which is evenly divided into $D$ transient time slots, with each time slot lasting for a time duration of $T_0$. In one time slot, the RIS updates its phase-shift variables to simultaneously generate passive beamforming gains toward the direction $\Psi_U$ for UE communication, and toward a sensing direction $\Psi_S$ for TG detection. Here, $\Psi_U$ is fixed owing to the assumption of motionless UE, whereas $\Psi_S$ remains unchanged within one time slot but varies from time slot to time slot to scan over the neighbourhood. A sensing beam is assigned to each $\Psi_S$ with a time duration of $T_0$. As such, through the environmental scanning in the entire ISAC round,
the direction in which the TG appears can be determined. It is noteworthy that passive beamforming
needs to be optimized in each time slot, so as to change the sensing beam toward different \( \Psi_S \). In
fact, it is sufficient to focus on the beamforming optimization in one time slot based on a specific \( \Psi_S \)
and \( \Psi_U \), while those for other time slots can be done in a similar way. Therefore, we will consider
a specific \( \Psi_S \) and \( \Psi_U \), followed by an elaboration of the communication and sensing performance
metrics, as well as the problem formulation.

A. Communication Performance Metric

We first describe the UE communication performance metric. Let \( P_{tx} \) and \( x(t) \) denote the transmitting
power and the transmitted ISAC waveform, respectively, with \( t \) being the time index. According to
\[30\], \[32\], \( x(t) \) can be designed as \( x(t) = w_{tx} x(t) \), where \( w_{tx} \in \mathbb{C}^{M \times 1} \) is the overall unit-norm
transmit beamformer satisfying \( \| w_{tx} \|_2 = 1 \), and \( x(t) \) denotes the tailored ISAC signal \[30\], satisfying
\( \mathbb{E}\{|x(t)|^2\} = 1 \), which is applied to deliver useful communication information from the BS to the UE
while enabling the TG detection in the sensing direction. Based on \( P_{tx} \) and \( x(t) \), the signal received
by the UE is expressed as

\[
y_u(t) = \sqrt{P_{tx} G_{tx} G_{rx,u}} \mathbf{h}_{RU}^H \Omega(\Psi_R, \Psi_U) \mathbf{H}_{BR} x(t) + n_u(t),
\]

where \( G_{tx} \) and \( G_{rx,u} \) are the transmitting antenna gain at the BS and the receiving antenna gain at the
UE, respectively. For instance, we have \( G_{tx} = G_{rx,u} = 1 \) if the antennas are isotropic \[29\]. \( n_u(t) \) is the
complex zero-mean additive white Gaussian noise at the UE side with variance of \( \sigma_{n,u}^2 \); \( \Psi_R = (\theta_R, \varphi_R) \)
and \( \Psi_U = (\theta_U, \varphi_U) \) represent the incident and reflective directions, with \( \theta_R \) and \( \varphi_R \) being the elevation
and azimuth angle-of-arrivals (AOAs) at the RIS, and \( \theta_U \) and \( \varphi_U \) being the elevation and azimuth angle-of-departures (AODs) at the RIS toward the UE; \( \Omega(\Psi_R, \Psi_U) \) is the diagonal RIS response matrix with
respect to \( \Psi_R \) and \( \Psi_U \); \( \mathbf{H}_{BR} \) and \( \mathbf{h}_{RU} \) denote the BS-RIS and RIS-UE channels, modelled as \[1]\n
\[
\mathbf{H}_{BR} = \rho_{BR}^2 \delta(t - \tau_{BR}) e^{j2\pi f_{BR}t} a(\theta_R, \varphi_R) a^H(\theta_B, \varphi_B),
\]

\[
\mathbf{h}_{RU} = \rho_{RU}^\frac{1}{2} \delta(t - \tau_{RU}) e^{j2\pi f_{RU}t} a(\theta_U, \varphi_U),
\]

where \( \rho_{BR} \) and \( \rho_{RU} \) denote the free-space path loss coefficients of the BS-RIS link and the RIS-UE
link, given by

\[
\rho_{BR} = \left( \frac{\lambda}{4\pi d_{BR}} \right)^2, \quad \rho_{RU} = \left( \frac{\lambda}{4\pi d_{RU}} \right)^2,
\]

\[1\] We assume that the RIS is appropriately deployed, so that the LoS paths exist in the BS-RIS and RIS-UE channels. Measurement
campaigns \[39\], \[40\] have revealed that the power of the mmWave LoS path is much stronger (around 13 dB higher) than the sum power
of the mmWave NLoS paths. As such, we consider that \( \mathbf{H}_{BR} \) and \( \mathbf{h}_{RU} \) are LoS-dominated channels, by following the prior works \[7\], \[37\], \[38\].
with $\lambda$ being the signal wavelength, and $d_{BR}$ or $d_{RU}$ being the distance between the BS and the RIS or between the RIS and the UE; $\delta(\cdot)$ is the impulse function; $\tau_{BR}$ and $\tau_{RU}$ are the time delays of the BS-RIS and the RIS-UE links; $f_{BR}$ and $f_{RU}$ represent the Doppler frequencies; $\theta_B$ and $\varphi_B$ are the elevation and azimuth AODs at the BS toward the RIS; $a(\theta_B, \varphi_B)$, $a(\theta_R, \varphi_R)$ and $a(\theta_U, \varphi_U)$ are the steering vectors, given by

$$a(\theta_B, \varphi_B) = \left(1, e^{j\frac{2\pi d}{\lambda} \sin \theta_B \sin \varphi_B}, \ldots, e^{j\frac{2\pi d}{\lambda} (M-1) \sin \theta_B \sin \varphi_B}\right)^T,$$

$$a(\theta_R, \varphi_R) = \left(1, e^{j\frac{2\pi d}{\lambda} \sin \theta_R \cos \varphi_R}, \ldots, e^{j\frac{2\pi d}{\lambda} (N_u-1) \sin \theta_R \cos \varphi_R}\right)^T \otimes \left(1, e^{j\frac{2\pi d}{\lambda} \sin \theta_R \sin \varphi_R}, \ldots, e^{j\frac{2\pi d}{\lambda} (N_u-1) \sin \theta_R \sin \varphi_R}\right)^T,$$

$$a(\theta_U, \varphi_U) = \left(1, e^{j\frac{2\pi d}{\lambda} \sin \theta_U \cos \varphi_U}, \ldots, e^{j\frac{2\pi d}{\lambda} (N_u-1) \sin \theta_U \cos \varphi_U}\right)^T \otimes \left(1, e^{j\frac{2\pi d}{\lambda} \sin \theta_U \sin \varphi_U}, \ldots, e^{j\frac{2\pi d}{\lambda} (N_u-1) \sin \theta_U \sin \varphi_U}\right)^T.$$

In this paper, we assume that the locations of the BS, the RIS and the UE are known in advance, such that the channel state information (CSI) of $H_{BR}$ in (2) and $h_{RU}$ in (3) can be obtained from the location information [7], [41]. Besides, we use the physics-based model in [42], [43] to characterize the practical RIS reflection, where $\Omega(\Psi_R, \Psi_U)$ is modelled as [42]

$$\Omega(\Psi_R, \Psi_U) = \frac{\sqrt{4\pi}}{\lambda} g_{uc}(\Psi_R, \Psi_U) \text{diag}(\omega),$$

where $\omega = (e^{j\beta_1}, e^{j\beta_2}, \ldots, e^{j\beta_N})^T$ is the unit-modulus adjustable phase-shift vector with $\beta_i$ for $i = 1, 2, \ldots, N$ being the phase-shift variables, and $g_{uc}(\Psi_R, \Psi_U)$ is the inherent unit-cell response factor. Note that an expression of $g_{uc}(\Psi_R, \Psi_U)$ is derived in [42, Eq. (16)] based on the physical reflection properties. Interested readers may refer to [42] for more details.

For the signal model in (1), the SNR at the UE, defined by

$$\text{SNR}_{\text{UE}} = \frac{P_{tx} G_{ts} G_{ru,u}}{\sigma_u^2} |\mathbf{h}_{RU}^H \Omega(\Psi_R, \Psi_U) \mathbf{H}_{BR} \mathbf{w}_{tx}|^2,$$

is employed as the communication performance metric.

**B. Sensing Performance Metric**

The TG detection is performed based on the echo power depending on the TG size. To facilitate the analysis, we consider detecting a possible TG in the sensing direction of $\Psi_S = (\theta_S, \varphi_S)$ at a distance of $r$ away from the RIS, as shown in Fig. [3] Due to the randomness and irregularity of practical TGs, the SSA of the TG is approximated as a smooth surface, in accordance with the empirical measures of the radar cross-sections (RCSs) of practical TGs [Table I, 48]. The SSA, denoted by $S(\Delta\theta, \Delta\varphi)$, has an elevation angle-spread of $\Delta\theta$ and an azimuth angle-spread of $\Delta\varphi$ from the view of the RIS. By analysing the illumination power on the SSA, we will derive the detection probability of the TG.
in closed-form. Based on the expression of the detection probability, we will also introduce a new concept of UDR for detection capability measurements.

To begin with, we analyze the signal power in each transmission hop. When the RIS is reflecting the signal from the BS to an arbitrary direction \( \Psi = (\theta, \varphi) \), the reflective radiation power pattern toward \( \Psi = (\theta, \varphi) \) is expressed as

\[
P_{R}(\theta, \varphi) = \frac{P_{tx}G_{tx}\lambda^{2}}{4\pi d_{BR}^{2}} \left| \frac{\sqrt{4\pi}}{\lambda} g_{uc}(\Psi_{R}, \Psi) a^{\dagger}(\theta, \varphi) \text{diag}(\omega) H_{BR} w_{tx} \right|^{2}.
\]

Then, at the distance of \( r \) away from the RIS, the reflective radiation power density is given by \( \frac{P_{R}(\theta, \varphi)}{4\pi r^{2}} \), and the illumination power on a differential surface area (DSA) of \( dA \) is given by \( \frac{P_{R}(\theta, \varphi) E[\delta_{S}]}{4\pi r^{2}} dA \).

Let \( E[\delta_{S}] \) denote the average loss of the first-order scattering. It is reported in [44] that \( E[\delta_{S}] \) is around \(-10 \text{ dB} \) with root-mean-square deviation of 4 dB in practice. Then, the differential signal power scattered by the DSA is given by \( dP_{S}(\theta, \varphi, r) = \frac{P_{R}(\theta, \varphi)}{4\pi r^{2}} E[\delta_{S}] dA \), according to which the corresponding differential echo power received by the BS is derived as

\[
dP_{rx}(\theta, \varphi, r) = dP_{S}(\theta, \varphi, r) G_{rx} \left( \frac{\lambda}{4\pi r} \right)^{2} \left( \frac{\lambda}{4\pi d_{BR}} \right)^{2} \left| g_{uc}(\Psi, \Psi_{R}) w_{rx}^{H} H_{BR}^{H} \text{diag}(\omega) a(\theta, \varphi) \right|^{2}.
\]

(8)

where \( G_{rx} \) denotes the receiving antenna gain of the BS, and \( w_{rx} \) is the overall unit-norm receive beamformer at the BS satisfying \( \| w_{rx} \|_{2} = 1 \).

Based on the differential echo power in (8), the total echo power scattered from \( S(\Delta_{\theta}, \Delta_{\varphi}) \) and
where \( P_{tx}(\theta_S, \varphi_S, r, \Delta_\theta, \Delta_\varphi) \) is the probability density function of the received echo at the BS. The probability density function is given by

\[
P_{tx}(\theta_S, \varphi_S, r, \Delta_\theta, \Delta_\varphi) = \iint_{S(\Delta_\theta, \Delta_\varphi)} dP_{tx}(\theta, \varphi, r)
\]

where \( S(\Delta_\theta, \Delta_\varphi) \) is the region of interest. TheReceived echo is sampled over the duration of \( T_0 \) with a frequency of \( f_s \), and is dedicated to the following hypothesis test:

\[
\mathcal{H}_0 : \ y_{rx}(m) = n_{rx}(m), \quad m = 0, 1, 2, \ldots, T_0 f_s - 1,
\]

\[
\mathcal{H}_1 : \ y_{rx}(m) = E_{rx}(m) + n_{rx}(m), \quad m = 0, 1, 2, \ldots, T_0 f_s - 1,
\]

where \( m \) is the discrete time index; \( y_{rx}(m) \) is the discrete noise-disturbed echo received by the BS; \( E_{rx}(m) \) is the effective signal part with an average power of \( P_{rx}(\theta_S, \varphi_S, r, \Delta_\theta, \Delta_\varphi) \); \( n_{rx}(m) \) is the zero-mean additive noise at the BS side with variance of \( \sigma_n^2 \). Hypothesis \( \mathcal{H}_0 \) denotes the absence of \( E_{rx}(m) \), whereas hypothesis \( \mathcal{H}_1 \) represents the opposite. Based on \( \mathcal{H}_0, \mathcal{H}_1 \) and a predetermined threshold \( \eta \), the decision falls into hypothesis \( \mathcal{H}_1 \) if the received signal strength exceeds \( \eta \); otherwise, the decision falls into hypothesis \( \mathcal{H}_0 \). In accordance with the Neyman-Pearson criterion, the detection probability is given by [45, Eq. (3.8)]

\[
\mathbb{P}_d(\theta_S, \varphi_S, r, \Delta_\theta, \Delta_\varphi) = Q \left( Q^{-1}(\mathbb{P}_f) - \frac{T_0 f_s P_{rx}(\theta_S, \varphi_S, r, \Delta_\theta, \Delta_\varphi)}{\sigma_n} \right),
\]

where \( Q(\cdot) \) and \( Q^{-1}(\cdot) \) are the Q-function and the inverse Q-function, respectively; \( \mathbb{P}_f = Q \left( \frac{\eta}{\sqrt{\sigma_n^2/(T_0 f_s)}} \right) \) represents the false alarm rate.

According to (11), we note that under a certain system setup (i.e. given \( P_{tx}, H_{BR}, h_{RU}, \) etc.), the detection probability is positively related to \( S(\Delta_\theta, \Delta_\varphi) \) associated with the TG size, owing to the integral operator \( \iint_{S(\Delta_\theta, \Delta_\varphi)} (\cdot) \). If the detection probability is required to satisfy \( \mathbb{P}_d(\theta_S, \varphi_S, r, \Delta_\theta, \Delta_\varphi) \geq \gamma_{p(\min)} \) with \( \gamma_{p(\min)} \in (0, 1) \) being a constant, the TG size should be larger than a certain minimum. Based on the above analysis, we provide the following definition.

**Definition 1.** Under a certain system setup, a TG at the distance of \( r \) in the sensing direction of \( \Psi_S = \)
\((\theta_S, \varphi_S)\) is called to be “\(\gamma_{p_d(\text{min})}\)-detectable”, if its SSA, i.e. \(S(\Delta\theta, \Delta\varphi)\), can make \(P_d(\theta_S, \varphi_S, r, \Delta\theta, \Delta\varphi) \geq \gamma_{p_d(\text{min})}\) hold. Then, the SSA of the smallest \(\gamma_{p_d(\text{min})}\)-detectable TG is defined as the ultimate detection resolution (UDR), mathematically given by

\[
\text{UDR}(\Delta\theta, \Delta\varphi, \gamma_{p_d(\text{min})}) \triangleq \min_{P_d(\theta_S, \varphi_S, r, \Delta\theta, \Delta\varphi) \geq \gamma_{p_d(\text{min})}} \{S(\Delta\theta, \Delta\varphi)\}.
\]  

(12)

The UDR defined in (12) characterizes the capability of the TG detection from the perspective of the size of the detectable TG. In Section III, we will show that the UDR can be explicitly determined via the feasibility condition of the optimization problem.

C. Problem Statement

Based on the performance metrics in (6) and (11), our objective is to improve the UE communication performance as much as possible, while ensuring the minimum detection probability requirement. The overall optimization problem is then formulated as

\[
\begin{align*}
(P1) : \quad & \text{maximize} & \quad \text{SNR}_{\text{UE}}, \\
& \text{subject to} & \quad \| [\omega]_\ell \| = 1, \ell = 1, 2, \ldots, N, \quad (13b) \\
& & \quad \| w_{\text{tx}} \|_2 = \| w_{\text{rx}} \|_2 = 1, \quad (13c) \\
& & \quad P_d(\theta_S, \varphi_S, r, \Delta\theta, \Delta\varphi) \geq \gamma_{p_d(\text{min})}, \quad (13d)
\end{align*}
\]

where constraint (13b) comes from the unit-modulus property of the adjustable phase-shift variables; constraint (13c) comes from the unit-norm transmit and receive beamformers at the BS; constraint (13d) means that the detection probability is guaranteed to be not lower than a threshold \(\gamma_{p_d(\text{min})}\). As (P1) is non-convex, we will propose a novel convexification process and develop an SCA-based algorithm to solve (P1) in the next section.

III. Beamforming Optimization

This section is dedicated to solving (P1) by optimizing \(w_{\text{tx}}\), \(w_{\text{rx}}\) and, particularly, \(\omega\). Note that under the assumption of the LoS-dominated BS-RIS channel, the solutions of \(w_{\text{tx}}\) and \(w_{\text{rx}}\) can be readily attained in closed-form as [7]

\[
w_{\text{tx}} = w_{\text{rx}} = \frac{1}{\sqrt{M}} a(\theta_B, \varphi_B),
\]

(14)

which aims to achieve the utmost beam power concentration toward the RIS.
Since the optimal $w_{tx}$ and $w_{rx}$ have been settled, we now recast (P1) as

$$\text{(P2):} \quad \max_{\omega} \quad \text{SNR}_{\text{UE}}, \tag{15a}$$

subject to

$$\|\omega\|_1 = 1, \ell = 1, 2, \ldots, N; \tag{15b}$$

$$p_d(\theta_S, \phi_S, r, \Delta_\theta, \Delta_\phi) \geq \gamma_{p_d}^{(\min)}, \tag{15c}$$

which is non-convex as well with respect to the optimization variable $\omega$. To solve (P2), we first propose to convexify constraint (15c) into a linear constraint with matrix operations and a real-valued first-order Taylor approximation. Then, we use the SDR to relax the problem and design an SCA-based algorithm to acquire the phase-shift solution. Finally, we provide essential analysis on the problem feasibility condition, the tightness of the SDR, and the algorithm convergence.

A. Convexification of Constraint (15c)

To convexify constraint (15c), we rewrite $p_d(\theta_S, \phi_S, r, \Delta_\theta, \Delta_\phi) \geq \gamma_{p_d}^{(\min)}$ as

$$Q \left( Q^{-1}(P_f) - \frac{\sqrt{T_0}f_s P_{rx}(\theta_S, \phi_S, r, \Delta_\theta, \Delta_\phi)}{\sigma_n} \right) \geq \gamma_{p_d}^{(\min)}, \tag{16}$$

yielding

$$P_{rx}(\theta_S, \phi_S, r, \Delta_\theta, \Delta_\phi) \geq \frac{\sigma_n^2}{T_0 f_s} \left( Q^{-1}(P_f) - Q^{-1}(\gamma_{p_d}^{(\min)}) \right)^2. \tag{17}$$

Note that in (9), $P_{rx}(\theta_S, \phi_S, r, \Delta_\theta, \Delta_\phi)$ includes a constant term $\frac{P_{rx} G_{tx} G_{rx} \lambda^2 \mathbb{E}\{\delta_s\}}{1024 \pi^5 \lambda^2 d_{BR}^4}$, which is independent of $\omega$, $\theta$ and $\phi$. Hence, by defining

$$f(\omega) = \int_{\Theta_S}^{\Theta_S + \frac{\Delta_\theta}{2}} \int_{\Theta_S - \frac{\Delta_\theta}{2}}^{\Theta_S} [g_{uc}(\Psi_R, \Psi)a^H(\theta, \phi) \text{diag}(\omega) H_{BR} w_{tx}]^2 [g_{uc}(\Psi, \Psi_R) w_{rx}^H H_{BR}^H \text{diag}(\omega) a(\theta, \phi)]^2 \sin \theta \, d\theta \, d\varphi,$$

$$g = \left( \frac{P_{tx} G_{tx} G_{rx} \lambda^2 \mathbb{E}\{\delta_s\}}{1024 \pi^5 \lambda^2 d_{BR}^4} \right)^{-1} \frac{\sigma_n^2}{T_0 f_s} \left( Q^{-1}(P_f) - Q^{-1}(\gamma_{p_d}^{(\min)}) \right)^2,$$

the inequality in (17) can be equivalently written as

$$f(\omega) \geq g. \tag{18}$$

Here, we define two auxiliary matrices with respect to $\theta$ and $\phi$, i.e. $A(\theta, \phi)$ and $B(\theta, \phi)$, as

$$A(\theta, \phi) = |g_{uc}(\Psi_R, \Psi)|^2 \left[ \text{diag}\{a^H(\theta, \phi)\} H_{BR} w_{tx} w_{tx}^H H_{BR}^H \text{diag}\{a(\theta, \phi)\} \right]^T, \tag{19}$$

$$B(\theta, \phi) = |g_{uc}(\Psi, \Psi_R)|^2 \text{diag}\{a^H(\theta, \phi)\} H_{BR} w_{rx} w_{rx}^H H_{BR}^H \text{diag}\{a(\theta, \phi)\} \sin \theta. \tag{20}$$

Then, we have

$$|g_{uc}(\Psi_R, \Psi)a^H(\theta, \phi) \text{diag}(\omega) H_{BR} w_{tx}|^2 = \omega^H A(\theta, \phi) \omega, \tag{21}$$

$$|g_{uc}(\Psi, \Psi_R) w_{rx}^H H_{BR}^H \text{diag}(\omega) a(\theta, \phi)|^2 \sin \theta = \omega^H B(\theta, \phi) \omega. \tag{22}$$
which simplifies $f(\omega)$ into

$$f(\omega) = \int_{\varphi_S - \Delta\varphi}^{\varphi_S + \Delta\varphi} \int_{\theta_S - \Delta\theta}^{\theta_S + \Delta\theta} \omega^H A(\theta, \varphi) \omega \omega^H B(\theta, \varphi) \omega \, d\theta \, d\varphi. \quad (23)$$

In (23), $f(\omega)$ is composed of a product of double quadratic forms involved in an integral, which is still highly non-convex and intractable, and motivates us to derive the following Proposition 1 to convexify $f(\omega)$.

**Proposition 1.** By introducing a positive semidefinite rank-one matrix $Q = \omega \omega^H$, $f(\omega)$ can be transformed into

$$f(\omega) = \|v(Q)\|^2, \quad (24)$$

where $v(Q) \in \mathbb{C}^{N^2 \times 1}$ is a vector with respect to $Q$, given by $v(Q) = [\text{tr}(S_1 Q), \text{tr}(S_2 Q), \ldots, \text{tr}(S_{N^2} Q)]^T$, in which $S_i$ for $i = 1, 2, \ldots, N^2$ are specified in (49) in Appendix A, and are independent of $Q$.

**Proof.** The proof is given in Appendix A.

According to (24), constraint (15c) can be recast as

$$\|v(Q)\|_2 \geq \sqrt{\mathcal{G}}. \quad (25)$$

In addition, using the definition of $Q = \omega \omega^H$, the objective function of (P2) and the constraint (15b) can be, respectively, converted into

$$\text{SNR}_{\text{UE}} = \text{tr} (\Xi Q), \quad (26)$$

$$\text{tr} (E_{\ell} Q) = 1, \ell = 1, 2, \ldots, N, \quad (27)$$

where $\Xi = \frac{4\pi P_d G_t G_r \sigma_w}{\lambda^2} |g_{uc}(\Psi_R, \Psi_U)|^2 \text{diag} (w_{tx}^H H_{BR}^H) h_{RU} h_{RU}^H \text{diag} (H_{BR} w_{tx})$, and $E_{\ell}$ is a selecting matrix with each element satisfying

$$[E_{\ell}]_{m,n} = \begin{cases} 1, & m = n = \ell, \\ 0, & \text{otherwise.} \end{cases} \quad (28)$$

Combining (25), (26) and (27), Problem (P2) is reformulated as

(P3) : maximize $\quad \text{tr} (\Xi Q), \quad (29a)$

subject to $\quad \text{tr} (E_{\ell} Q) = 1, \ell = 1, 2, \ldots, N, \quad (29b)$

$$\|v(Q)\|_2 \geq \sqrt{\mathcal{G}}, \quad (29c)$$

$$\text{rank}(Q) = 1. \quad (29d)$$
Remark 1. Since $tr(\cdot)$ is linear and $\| \cdot \|_2$ is convex, $\| v(Q) \|_2$ is convex with respect to $Q$. However, $\| v(Q) \|_2 \geq \sqrt{G}$ is a non-convex constraint, due to the inequality of “convex”$ \geq$“constant”. Besides, $\text{rank}(Q) = 1$ is non-convex as well. Therefore, Problem (P3) is still non-convex and cannot be readily or efficiently solved.

To further convexify $\| v(Q) \|_2 \geq \sqrt{G}$, we propose to linearise $\| v(Q) \|_2$ by a real-valued first-order Taylor approximation. Let $\mathcal{T}_{z_k}(h(z))$, given by
\[
\mathcal{T}_{z_k}(h(z)) = h(z_k) + \left. \frac{\partial h(z)}{\partial z} \right|_{z=z_k} (z-z_k),
\]
(30)
denote the first-order Taylor approximation for a function $h(z)$ around a given point $z_k$, where $z_k$ can be, e.g. a local optimum obtained in the $k$-th iteration in a successive optimization process. Then, using the definition in (30), the first-order Taylor approximation of $\| v(Q) \|_2$ around $v(Q_k)$ is denoted by
\[
\mathcal{T}_{v(Q_k)}(\| v(Q) \|_2) = \| v(Q_k) \|_2 + \left. \frac{\partial \| v(Q) \|_2}{\partial v(Q)} \right|_{v(Q)=v(Q_k)} [v(Q) - v(Q_k)].
\]
(31)

It is remarkable that because $\| v(Q) \|_2$ is convex, $\mathcal{T}_{v(Q_k)}(\| v(Q) \|_2)$ is a lower-bound of $\| v(Q) \|_2$ around $\forall v(Q_k)$, i.e. $\| v(Q) \|_2 \geq \mathcal{T}_{v(Q_k)}(\| v(Q) \|_2)$, which implies that $\| v(Q) \|_2 \geq \sqrt{G}$ is guaranteed if $\mathcal{T}_{v(Q_k)}(\| v(Q) \|_2) \geq \sqrt{G}$ holds. Thus, $\mathcal{T}_{v(Q_k)}(\| v(Q) \|_2)$ can be used as a surrogate function of $\| v(Q) \|_2$. To derive $\mathcal{T}_{v(Q_k)}(\| v(Q) \|_2)$, we should first derive $\left. \frac{\partial \| v(Q) \|_2}{\partial v(Q)} \right|_{v(Q)=v(Q_k)}$. Nevertheless, we find that the inner product $\left. \frac{\partial \| v(Q) \|_2}{\partial v(Q)} \right|_{v(Q)=v(Q_k)} [v(Q) - v(Q_k)]$ is generally a complex value, making $\mathcal{T}_{v(Q_k)}(\| v(Q) \|_2) \geq \sqrt{G}$ invalid (i.e. “complex value”$ \geq$“real value”) during the optimization process. Fortunately, we note that the optimization for a complex variable can be intrinsically treated as the optimization for its real and imaginary parts. As such, we propose to transform $v(Q)$ by combining its real and imaginary parts into a new vector, so as to find an equivalent real representation of $\mathcal{T}_{v(Q_k)}(\| v(Q) \|_2)$.

Proposition 2. A real representation of $\mathcal{T}_{v(Q_k)}(\| v(Q) \|_2)$ is derived as
\[
\mathcal{T}_{\tilde{v}(Q_k)}(\| \tilde{v}(Q) \|_2) = tr(\tilde{\mathcal{Y}}(Q_k)Q),
\]
(32)
where $\tilde{v}(Q)$ is a real-value vector expressed as (53) in Appendix B. $\tilde{\mathcal{Y}}(Q_k)$ is a matrix with respect to $Q_k$ but is independent of $Q$, given in (58) in Appendix B.

Proof. The proof is given in Appendix B. \qed

Note that in accordance with the proof, $\mathcal{T}_{v(Q_k)}(\| v(Q) \|_2) \geq \sqrt{G}$ is equivalent to $\mathcal{T}_{\tilde{v}(Q_k)}(\| \tilde{v}(Q) \|_2) \geq$
\[ \sqrt{G}, \text{ such that } \mathcal{T}_{v(k)}(\|v(Q)\|_2) \text{ can be used as a surrogate function of } \|v(Q)\|_2. \] Consequently, the constraint \( \|v(Q)\|_2 \geq \sqrt{G} \) can be convexified as \[ \text{tr} (\Upsilon(Q_k)Q) \geq \sqrt{G}. \tag{33} \]

**B. Problem Relaxation**

Based on (33), Problem (P3) is transformed into

\[
(P4) : \quad \text{maximize} \quad \text{tr} (\Xi Q), \tag{34a}
\]

subject to \( \text{tr} (E_\ell Q) = 1, \ell = 1, 2, ..., N, \tag{34b} \)

\[ \text{tr} (\Upsilon(Q_k)Q) \geq \sqrt{G}, \tag{34c} \]

\[ \text{rank}(Q) = 1. \tag{34d} \]

To address the non-convex (34d), we apply the SDR to relax the rank-one constraint, and transform (P4) into

\[
(P5) : \quad \text{maximize} \quad \text{tr} (\Xi Q), \tag{35a}
\]

subject to (34b), (34c),

\[ \text{rank}(Q) = 1. \tag{35b} \]

which is now a semidefinite programming (SDP) problem and can be solved by existing methods, e.g. the interior-point method. Due to the omission of \( \text{rank}(Q) = 1 \), the solution of (P5), denoted by \( Q^{(\star)} \), is not always rank-one. If the SDR yields \( \text{rank}(Q^{(\star)}) = 1 \), we can perform eigenvalue decomposition for \( Q^{(\star)} \) to acquire the optimal \( \omega \). Otherwise, we need to apply some convex relaxation techniques for phase-only beamforming, e.g. the SDP concave-convex procedure (SDP-CCP) [46], to iteratively acquire an approximate rank-one solution before conducting the eigenvalue decomposition. In fact, according to our previous investigations [47], the SDR can possibly yield \( \text{rank}(Q^{(\star)}) = 1 \) when certain conditions hold, as illustrated in the following remark.

**Remark 2.** According to [47], we have \( \text{rank}(Q^{(\star)}) = 1 \) if the conditions below are simultaneously satisfied:

1) \( \text{rank}(\Xi) = 1 \) and the elements in the eigenvector of \( \Xi \) are non-zero.

2) The solution satisfies \( \text{tr} (\Upsilon(Q_k)Q^{(\star)}) > \sqrt{G} \).

Based on the structure of \( \Xi \), we have \( \text{rank}(\Xi) = \text{rank}(\text{diag}(w_{tx}^H H_{BR}^H h_{RL}; h_{RL}^H H_{BR} w_{tx})) = 1 \). Owing to the non-zero elements in the BS-RIS and RIS-UE channels, \( H_{BR} w_{tx} \) and \( h_{RL} \) do not contain zero-elements in general if \( w_{tx} \neq 0 \). Hence, we conclude that condition 1) is satisfied in the vast
majority of cases. However, there is no guarantee to make condition 2) hold after (P5) is solved. Indeed, in most cases, when the AODs of the RIS toward the UE and the TG are different, only the minimum detection probability is maintained while the SNR\_\text{UE} is maximized. This is because our problem is to concentrate the reflective beam power toward the UE as much as possible to strengthen the UE communication link. For this reason, we have $\text{tr} \left( \mathbf{Y}(\mathbf{Q}_k)\mathbf{Q}^\star \right) \approx \sqrt{G}$ in general. Therefore, $\text{rank}(\mathbf{Q}^\star) = 1$ is not strictly guaranteed in our problem, such that the approximation methods (e.g. the SDP-CCP) are sometimes still necessary to deal with the relaxation in practical applications.

It is worth mentioning that we aim to acquire the phase-shift solution of (P3) by solving (P4) using either the SDR or the SDP-CCP. However, given a certain point $\mathbf{Q}_k$, the solution of (P4) is only a local optimum around $\mathbf{Q}_k$ instead of being the true optimum of (P3). Therefore, we design an SCA-based algorithm to iteratively solve (P4). In the following, we will prove that the objective value of (P4) increases monotonically during the iteration process, so that the solution of (P4) can eventually approach that of (P3) when the algorithm converges.

C. SCA-based Algorithm Design

The developed SCA-based algorithm is detailed in Algorithm 1, which can be summarized as the following 3 steps:

- **Step 1**: Initialize the parameters including the initial point $\mathbf{Q}_1$, iteration index $k$ and the terminating condition $\epsilon > 0$.

- **Step 2**: Successively solve (P4) with the aid of the SDR or SDP-CCP, until the gap between the objective values in adjacent two iterations is smaller than $\epsilon$.

- **Step 3**: Obtain and output the final solution.

It should be noted that Algorithm 1 can be successfully executed only when (P4) has feasible solutions and the iteration process is convergent. As such, we will analyse the problem feasibility condition and the algorithm convergence, as detailed below.

D. Problem Feasibility Condition and Algorithm Convergence

Owing to the constant-modulus constraint of the original phase-shift variables, the value of $\text{tr} \left( \mathbf{Y}(\mathbf{Q}_k)\mathbf{Q} \right)$ is upper-bounded, which implies that given $\forall \mathbf{Q}_k$, there exists a constant $u \geq 0$ such that $\text{tr} \left( \mathbf{Y}(\mathbf{Q}_k)\mathbf{Q} \right) \leq u$. Note that if $u \leq \sqrt{G}$, Problem (P4) is infeasible because we cannot find a $\mathbf{Q}$ making constraint (34) hold, which further means that we cannot derive a reflective beamformer fulfilling the minimum detection probability requirement. The feasibility of (P4) is primarily determined by $\mathbf{Q}_k$ in each
iteration and the value of $\sqrt{G}$ depending on multiple factors, such as $P_{tx}$, $\gamma_{p_{d}(\text{min})}$, etc. However, the point $Q_k$ varies from iteration to iteration, such that the feasibility of $(P4)$ needs to be analysed for all iteration steps. In what follows, we rigorously prove that if Problem $(P3)$ is feasible, Problem $(P4)$ is feasible for $\forall k \geq 1$ during the entire iteration process.

**Proposition 3.** When Problem $(P3)$ is feasible, we have that Problem $(P4)$ is feasible at every iteration step, if the initial point $Q_1$ is chosen from the feasible region of $(P3)$.

**Proof.** If $(P3)$ is feasible, there should exist a $Q$ such that $\|v(Q)\|_2 \geq \sqrt{G}$ holds. Then, because: 1) $\|v(Q)\|_2 = \|\tilde{v}(Q)\|_2$, 2) $\|\tilde{v}(Q)\|_2$ is convex, and 3) $tr(\Upsilon(Q_k)Q)$ is the first-order Taylor approximation of $\|\tilde{v}(Q)\|_2$, there should be $\|v(Q)\|_2 = \|\tilde{v}(Q)\|_2 \geq tr(\Upsilon(Q_k)Q)$ and $\|v(Q_k)\|_2 = \|\tilde{v}(Q_k)\|_2 = tr(\Upsilon(Q_k)Q_k)$ for $\forall Q_k$.

Considering $k = 1$, we can choose $Q_1$ from the feasible region of $(P3)$, such that we have

$$\|v(Q_1)\|_2 = tr(\Upsilon(Q_1)Q_1) \geq \sqrt{G}, \quad (36)$$

indicating that there exists at least one $Q$ (e.g. $Q = Q_1$) making constraint $(34b)$ hold at $k = 1$. Thus, we can conclude that $(P4)$ is feasible at $k = 1$ when the initial point $Q_1$ is chosen from the feasible region of $(P3)$.

Then, for $k > 1$, we have $tr(\Upsilon(Q_k)Q_k) = \|v(Q_k)\|_2 = \|\tilde{v}(Q_k)\|_2 \geq tr(\Upsilon(Q_{k-1})Q_k)$ because with a given $Q_{k-1}$, there is $\|\tilde{v}(Q)\|_2 \geq tr(\Upsilon(Q_{k-1})Q)$ for $\forall Q \succeq 0$. If $(P4)$ is feasible at iteration $k-1$, we can obtain the solution $Q_k$ around $Q_{k-1}$ by solving $(P4)$, and obtain $tr(\Upsilon(Q_{k-1})Q_k) \geq \sqrt{G}$.
This yields
\[ tr(\mathbf{Y}(Q_k)Q_k) \geq tr(\mathbf{Y}(Q_{k-1})Q_k) \geq \sqrt{\mathcal{G}}, \] (37)
indicating that there exists at least one \( Q \) (e.g. \( Q = Q_k \)) making constraint (34c) hold at iteration \( k \). Thus, we can conclude that (P4) is feasible at iteration \( k \) if it is feasible at iteration \( k - 1 \).

Consequently, combining the above ratiocinations, we complete this proof. \( \square \)

Proposition 3 demonstrates that the feasibility of (P4) depends on the feasibility of (P3). The condition for (P3) to be feasible is given in the following remark.

**Remark 3.** Problem (P3) is feasible if \( \|v(Q^{(\dagger)})\|_2 \geq \sqrt{\mathcal{G}} \) holds, where \( Q^{(\dagger)} \) is the solution of the following optimization problem [3]:

\[ Q^{(\dagger)} = \arg\max_{Q \succeq 0} \|v(Q)\|_2, \]
subject to \( tr(\mathbf{E}_\ell Q) = 1, \ell = 1, 2, ..., N, \ rank(Q) = 1. \) (38)

This is because under the premise of \( \|v(Q^{(\dagger)})\|_2 \geq \sqrt{\mathcal{G}}, \) we can find a \( Q \) satisfying constraint (29c).

Based on \( \|v(Q^{(\dagger)})\|_2 \geq \sqrt{\mathcal{G}}, \) we further obtain
\[ P_{tx} \geq \|v(Q^{(\dagger)})\|_2^{-2} \left( \frac{G_{tx} G_{rx} \lambda^2 \mathbb{E}\{d_S\}}{1024\pi^2 r^2 d_{BR}^2} \right)^{-1} \frac{\sigma_n^2}{T_0 f_s} \left( Q^{-1}(\mathbb{P}_f) - Q^{-1}(\gamma_d^{(\min)}) \right)^2 = P_{tx,\min}, \] (39)
implying that the transmit power should be higher than a threshold if (P3) is feasible. Moreover, if \( P_{tx} \) is given, \( \|v(Q^{(\dagger)})\|_2 \) is positively related to \( \Delta_\theta \) and \( \Delta_\varphi \), because \( \Delta_\theta \) and \( \Delta_\varphi \) affect the integral inside \( \|v(Q^{(\dagger)})\|_2 \). Therewith, by defining \( \|v(Q^{(\dagger)})\|_2 \) as a function of \( \Delta_\theta \) and \( \Delta_\varphi \), i.e. \( \mathcal{F}(\Delta_\theta, \Delta_\varphi) = \|v(Q^{(\dagger)})\|_2 \), we can determine \( \text{UDR}(\Delta_\theta, \Delta_\varphi, \gamma_d^{(\min)}) \) introduced in Section II in accordance with
\[ \text{UDR}(\Delta_\theta, \Delta_\varphi, \gamma_d^{(\min)}) \in \{S(\Delta_\theta, \Delta_\varphi) \mid \mathcal{F}(\Delta_\theta, \Delta_\varphi) = \sqrt{\mathcal{G}}\}. \] (40)
The above (40) means that \( \text{UDR}(\Delta_\theta, \Delta_\varphi, \gamma_d^{(\min)}) \) belongs to a class of \( S(\Delta_\theta, \Delta_\varphi) \), whose corresponding \( \Delta_\theta \) and \( \Delta_\varphi \) make \( \mathcal{F}(\Delta_\theta, \Delta_\varphi) = \sqrt{\mathcal{G}} \) hold. On this basis, if \( \Delta_\theta \) and \( \Delta_\varphi \) are further scaled down, there will be \( \mathcal{F}(\Delta_\theta, \Delta_\varphi) < \sqrt{\mathcal{G}} \) such that (P3) becomes infeasible. As a result, (40) characterizes the SSA of the smallest \( \gamma_d^{(\min)} \)-detectable TG.

**Remark 4.** If \( \|v(Q^{(\dagger)})\|_2 \geq \sqrt{\mathcal{G}} \) is satisfied, we can choose \( Q^{(\dagger)} \) as the initial point of Algorithm 1, i.e. \( Q_1 = Q^{(\dagger)} \), such that the feasibility of (P4) can be ensured during the entire iteration process according to Proposition 3 and Remark 3.

\footnote{Note that Problem (38) is non-convex, which aims to maximize a convex objective. To solve Problem (38), one can also apply SCA to linearise \( \|v(Q)\|_2 \) and iteratively find the optimal \( Q^{(\dagger)} \). This is similar to what has been previously performed in Algorithm 1.}
Finally, we analyse the convergence of Algorithm 1. The details are given as follows.

**Proposition 4.** The Algorithm 1 is convergent.

**Proof.** According to the proof of Proposition 3, we have $tr (\Upsilon Q_k Q_k^\dagger) = \|v(Q_k)\|_2 \geq tr (\Upsilon (Q_{k-1}) Q_k)$ for $\forall k \geq 1$. Since $Q_k$ is the optimal solution of (P4) around point $Q_{k-1}$, there is $tr (\Upsilon (Q_{k-1}) Q_k) \geq \sqrt{G}$. As a result, we have $tr (\Upsilon Q_k Q_k^\dagger) \geq \sqrt{G}$, implying that $Q_k$ is a possible solution of (P4) around point $Q_k$.

If $Q_k$ is the optimal solution around point $Q_k$, we have $Q_{k+1} = Q_k$, resulting in $tr (\Xi Q_{k+1}) = tr (\Xi Q_k)$. Otherwise, there must exist another solution $Q_{k+1}$ that is better than $Q_k$, such that $tr (\Xi Q_{k+1}) > tr (\Xi Q_k)$ holds owing to the maximization of the objective. Consequently, we have $tr (\Xi Q_{k+1}) \geq tr (\Xi Q_k)$ for $\forall k \geq 1$, implying that the objective value is monotonically increasing during the iteration process. Additionally, the objective function $tr (\Xi Q)$ in (P4) is upper-bounded owing to the original unit-modulus phase-shift constraint. Thus, we conclude that Algorithm 1 is convergent.

**IV. SIMULATION RESULTS**

This section carries out the simulations to testify the analytical properties of the proposed algorithm, and to evaluate the performance of our beamforming strategy through the comparisons with two recently developed benchmarks.

**A. Parameter Configurations**

In the simulations, the positions of the BS, the RIS, the UE and the TG are (48, 48, 110), (50, 50, 100), (20, 70, 150) and (49, 50, 103) in meters. The sensing direction $\Psi_S$ is considered to be pointing at the center of the TG. The elevation and azimuth angle-spreads of the SSA satisfy $\Delta \theta = \Delta \phi$. The columns and rows of the RIS reflectors satisfy $N_x = N_y$. In addition, the noise powers are $\sigma_{n,u}^2 = -80$ dBm and $\sigma_n^2 = -50$ dBm; the carrier frequency is $f_c = 30$ GHz, resulting in the signal wavelength of $\lambda = 10$ mm; the antenna gains of the BS and the UE are $G_{tx} = G_{rx} = 2$ and $G_{rx,u} = 1$; the average scattering loss is $E[\delta_S] = -10$ dB; the antenna/element spacing is half-wavelength; the terminating condition of Algorithm 1 is $\epsilon = 0.0002$; the duration of the sensing time slot is $T_0 = 0.2$ s; the sampling frequency at the BS is $f_s = 1$ kHz; the decision threshold is $\eta = 10^{-3.8}/\sqrt{T_0 f_s}$; the Doppler frequencies are $f_{BR} = f_{RU} = 0$ Hz in consideration of a static scenario. The other parameters not listed above will vary for diverse observations.

The proposed algorithm is programmed by MATLAB and executed on a personal computer with Intel Core i7 and 32 GB RAM. The optimization problem is solved by CVX Toolbox with Sedumi
When computing the integral, we apply the trapezoidal method to approximate the integral value with an accuracy of 100 divisions over the variable range.

**B. Feasibility Condition and Convergence Behaviour**

In this part, we numerically investigate the feasibility condition of the optimization problem and the convergence behaviour of Algorithm 1. First, we simulate the feasibility condition by presenting the required minimum transmit power $P_{tx,\text{min}}$ derived in (39). Fig. 4 (a) depicts $P_{tx,\text{min}}$ with respect to $\gamma_{d}(\text{min})$. It is demonstrated that when $\gamma_{d}(\text{min})$ increases from 60% to 95%, $P_{tx,\text{min}}$ concomitantly grows. This implies that if higher minimum detection probability is to be maintained, more transmit power is correspondingly required. In particular, to ensure a desired minimum detection probability of $\gamma_{d}(\text{min}) = 90\%$, the total transmit power should be guaranteed beyond 72.5 dBm at $N = 49$ and $M = 512$. Nevertheless, 72.5 dBm transmit power may be exorbitant indeed for a practical mmWave BS. To address this issue, Fig. 4 (b) further plots $P_{tx,\text{min}}$ by varying $N$. The results indicate that increasing the number of the reflecting elements can reduce the $P_{tx,\text{min}}$ prominently, on account of higher passive beamforming gain generated by larger RIS toward both the sensing direction and the UE. From this perspective, adding more reflectors to the RIS can be a viable approach to conserve energy without posing much performance loss.

![Fig. 4: The required minimum transmit power $P_{tx,\text{min}}$ with respect to: (a) the minimum detection probability $\gamma_{d}(\text{min})$, and (b) the number of the RIS reflectors $N$.](image)

Subsequently, we testify Proposition 4 which proves the algorithm convergence. In specific, we examine the convergence behaviour in the presence of $Q_{1} = Q_{1}^{(1)}$ and several random $Q_{1}$. Fig. 5 depicts
the convergence curves showing the optimized SNR at the UE (i.e. the objective value) pertaining to 1 ∼ 100 iterations. From Fig. 5, we observe that the SNR monotonically increases to an implicit upper limit during the iteration process, which validates the correctness of Proposition 4. Besides, the configuration of $Q_1$ considerably influences the convergence rate of Algorithm 1. Particularly, $Q_1$ corresponding to lower initial objective value results in slower convergence. However, it is worth noting that after sufficient times of iterations, all the curves shown in the figure can eventually converge to a same level. Among these curves, the convergence curve corresponding to $Q_1 = Q^{(1)}$ approaches significantly faster to the optimum (needing about 40 iterations) than the other counterparts with random $Q_1$. This phenomenon indicates that the computational cost of the optimization can be potentially reduced to a large extent, if the initial point is properly designed.

![Convergence Behaviour](image1.png)

**Fig. 5:** Convergence behaviour of Algorithm 1 at $M = 512$.

![Communication vs. Sensing Trade-off](image2.png)

**Fig. 6:** The SNR at the UE with respect to $\gamma_{P_{\text{d(min)}}}$ at $M = 512$.

### C. Communication vs. Sensing Trade-off

According to the overall optimization problem, an inherent trade-off exists between the communication and sensing performances, due to the limited system power and beamforming gain. We identify this trade-off by presenting the joint variations of the SNR at the UE and the minimum detection probability in Fig. 6.

Fig. 6 shows that: 1) the increase of the minimum detection probability degrades the SNR at the UE; 2) the SNR at the UE deteriorates moderately in the presence of high $P_{tx}$ and large $N$, but more seriously when $P_{tx}$ or $N$ decreases. In particular, if $P_{tx} = 79$ dBm and $N = 36$, the SNR at the UE drops from 37.5 dB to 31 dB or so, resulting in a decline of around 6.5 dB when the...
minimum detection probability grows from 60% to 95%. These phenomena occur because when $\Psi_S$ deviates from $\Psi_U$, the optimal passive beamforming gains toward the UE and the sensing direction are generally competing paradoxes, i.e. one’s enhancement requires another’s compromise. Therefore, a balance between sensing and communication performances deserves to be pursued. In addition, we observe that the detection probability can be improved at a cost of only marginal SNR loss with large $N$ (see $N = 49$ for example). This signifies that the enlargement of the RIS can effectively relax the contradiction between the two performance metrics.

D. UDR vs. Sensing-duration Trade-off

Note that in (40), a trade-off between the UDR and $T_0$ is implied when $P_{tx}$ is given. In specific, $\mathcal{F}(\Delta \theta, \Delta \phi) = \sqrt{G}$ in (40) can be equivalently written as

$$T_0 = \left( \frac{P_{tx} G_{tx} G_{rx} \lambda^2 E\{\delta_S\}}{1024 \pi^3 r^2 d_{BR}^4} \right)^{-1} \frac{\sigma_n^2}{\mathcal{F}^2(\Delta \theta, \Delta \phi) f_s} \left( Q^{-1}(P_f) - Q^{-1}(\gamma_{p_d(min)}) \right)^2,$$

indicating that $T_0$ is inversely proportional to $\mathcal{F}^2(\Delta \theta, \Delta \phi)$. If the angle-spread $\Delta \theta$ or $\Delta \phi$ is scaled down, the value of $\mathcal{F}^2(\Delta \theta, \Delta \phi)$ will be reduced on account of the decrease of the integral value inside. Then, $T_0$ will be prolonged to support the detections for smaller TGs.

![UDR vs. Sensing-duration Trade-off](image_url)

Fig. 7: $T_0$ with respect to $\Delta \theta$ and $\Delta \phi$ in the UDR($\Delta \theta, \Delta \phi, 90\%$), at $M = 512$, $N = 36, 49$ and $P_{tx} = 78, 80$ dBm.

Considering $\Delta \theta = \Delta \phi$ for simplicity, we show this trade-off in Fig. 7 by depicting $T_0$ with respect to $\Delta \theta$ and $\Delta \phi$ in the UDR($\Delta \theta, \Delta \phi, 90\%$) with a minimum detection probability requirement of $\gamma_{p_d(min)} = 90\%$. We observe that longer $T_0$ is required for detecting the TG with smaller $\Delta \theta$ and $\Delta \phi$. The rationale behind is that according to (11), the detection probability is proportional to the echo energy-to-noise ratio (ENR) at the BS, denoted by ENR = $\frac{T_0 f_s P_{rx}(\theta, \phi, r, \Delta \theta, \Delta \phi)}{\sigma_n^2}$. As smaller $\Delta \theta$ and $\Delta \phi$ result in lower $P_{rx}(\theta, \phi, r, \Delta \theta, \Delta \phi)$, $T_0$ should be prolonged to compensate for the reduction of
$P_{tx}(\theta_S, \varphi_S, r, \Delta_\theta, \Delta_\varphi)$ in order to maintain the ENR. Besides, Fig. 7 also indicates that if $T_0$ remains, smaller TG can be detected with a minimum detection probability of 90% when $P_{tx}$ and $N$ grow. In this regard, we conclude that increasing the transmit power or the number of RIS reflectors can enhance the capability of the TG detection.

### E. Performance Comparisons with Existing Benchmarks

Since the proposed beamforming strategy considers the SSA of the TG, we finally compare our method with two recently developed benchmarks which assume point TGs:

**Benchmark 1 (B1):** The approach of maximizing the beamforming gain toward the sensing direction $\Psi_S$ without the consideration of the echoes, while satisfying the minimum SNR requirement of the UE. Similar ideas were adopted in some existing works, such as [31]. This approach can be achieved by solving the following Problem (B1):

\[
(B1) : \max_{\omega, w_{tx}} \left| g_{uc}(\Psi_R, \Psi_S) a^H(\theta_S, \varphi_S) \text{diag}(\omega) H_{BR} w_{tx} \right|^2 \tag{41a}
\]

subject to
\[
|\omega_\ell| = 1, \ell = 1, 2, ..., N, \tag{41b}
\]
\[
\|w_{tx}\|_2 = 1, \tag{41c}
\]
\[
\text{SNR}_{UE} \geq \gamma_{\text{SNR}}. \tag{41d}
\]

**Benchmark 2 (B2):** The approach of maximizing the beamforming gain over the cascaded BS-RIS-TG-RIS-BS link in consideration of the echoes, which however, treats the TGs as points. Similar ideas were adopted in e.g. [30]. This approach can be achieved by solving the following Problem (B2):

\[
(B2) : \max_{w_{tx}, w_{rx}, \omega} \left| g_{uc}(\Psi_R, \Psi_S) g_{uc}(\Psi_S, \Psi_R) w_{rx}^H H_{BR}^H \text{diag}(\omega) a(\theta_S, \varphi_S) a^H(\theta_S, \varphi_S) \text{diag}(\omega) H_{BR} w_{tx} \right|^2 \tag{42a}
\]

subject to
\[
|\omega_\ell| = 1, \ell = 1, 2, ..., N, \tag{42b}
\]
\[
\|w_{tx}\|_2 = \|w_{rx}\|_2 = 1, \tag{42c}
\]
\[
\text{SNR}_{UE} \geq \gamma_{\text{SNR}}. \tag{42d}
\]

Here, Problem (B1) can be readily convexified by the SDR, while Problem (B2) can be solved by the SCA after the objective function is convexified via the processes introduced in Section III-A. To compare the performances, we conduct the following steps:

- **Step 1:** Settle the optimal transmit and receive beamformers according to (14).
- **Step 2:** Execute the proposed Algorithm 1 under certain configurations of $\gamma_{p_d^{(\min)}}$, $\Delta_\theta$ and $\Delta_\varphi$ to acquire the phase-shift solution.
• Step 3: Use the phase-shift solution obtained in Step 2 to compute the SNR at the UE and the detection probability for \( S(\Delta_\theta, \Delta_\varphi) \). The computed SNR and detection probability are denoted by \( \text{SNR}_{\text{UE, (proposed)}} \) and \( P_{d, (\text{proposed})} \), respectively.

• Step 4: Set the \( \gamma_{\text{SNR}} \) in B1 and B2 as \( \gamma_{\text{SNR}} = \text{SNR}_{\text{UE, (proposed)}} \), and perform B1 and B2 to acquire their phase-shift solutions by solving (B1) and (B2).

• Step 5: Use the phase-shift solutions obtained in Step 4 to compute the detection probabilities of B1 and B2 for \( S(\Delta_\theta, \Delta_\varphi) \), denoted by \( P_{d, (\text{B1})} \) and \( P_{d, (\text{B2})} \), respectively.

• Step 6: Compare \( P_{d, (\text{B1})} \), \( P_{d, (\text{B2})} \) with \( P_{d, (\text{proposed})} \).

Table I lists the detection probabilities of the proposed algorithm and B1 with \( \gamma_{\text{SNR}}(\text{min}) = 90\% \) and different transmit powers. The results illustrate that our proposed beamforming strategy significantly outperforms B1 in terms of the detection performance, under the premise of the same SNRs at the UE. This is because B1 simply considers the BS-RIS-TG link, without providing additional passive beamforming gain for the TG-RIS-BS echo link.

| Transmit Power | SNR at the UE | Received Signal Power at the BS | Detection Probability |
|---------------|---------------|---------------------------------|-----------------------|
| Proposed      | 85.5000 dBm   | 29.8057 dB                      | \( P_{d, (\text{proposed})} = 93.26\% \) |
| Benchmark-1   | 85.5000 dBm   | 29.8057 dB                      | \( P_{d, (\text{B1})} = 53.73\% \) |
| Proposed      | 86.0000 dBm   | 32.2692 dB                      | \( P_{d, (\text{proposed})} = 92.92\% \) |
| Benchmark-1   | 86.0000 dBm   | 32.2692 dB                      | \( P_{d, (\text{B1})} = 39.27\% \) |
| Proposed      | 86.5000 dBm   | 34.0240 dB                      | \( P_{d, (\text{proposed})} = 92.64\% \) |
| Benchmark-1   | 86.5000 dBm   | 34.0240 dB                      | \( P_{d, (\text{B1})} = 28.52\% \) |
| Proposed      | 87.0000 dBm   | 35.4269 dB                      | \( P_{d, (\text{proposed})} = 92.45\% \) |
| Benchmark-1   | 87.0000 dBm   | 35.4269 dB                      | \( P_{d, (\text{B1})} = 21.09\% \) |
| Proposed      | 87.5000 dBm   | 36.6209 dB                      | \( P_{d, (\text{proposed})} = 92.34\% \) |
| Benchmark-1   | 87.5000 dBm   | 36.6209 dB                      | \( P_{d, (\text{B1})} = 16.14\% \) |
| Proposed      | 88.0000 dBm   | 37.6780 dB                      | \( P_{d, (\text{proposed})} = 92.27\% \) |
| Benchmark-1   | 88.0000 dBm   | 37.6780 dB                      | \( P_{d, (\text{B1})} = 12.92\% \) |

Table II and Table III list the detection probabilities of the proposed algorithm and B2 with \( \gamma_{\text{SNR}}(\text{min}) = 90\% \) and different transmit powers, in the presence of \( \Delta_\varphi = \Delta_\theta = \frac{\pi}{16} \) and \( \Delta_\varphi = \Delta_\theta = \frac{\pi}{4} \), respectively.

From Table II we observe that the detection probabilities of B2 are very close to those of the proposed algorithm when \( \Delta_\varphi = \Delta_\theta = \frac{\pi}{16} \). Nevertheless, when \( \Delta_\theta \) and \( \Delta_\varphi \) increase from \( \frac{\pi}{16} \) to \( \frac{\pi}{4} \), the performance gap emerges as shown in Table III. In light of this phenomenon, we speculate that our proposed strategy could be more advantageous in comparison with B2 when detecting large possible TGs, which motivates us to conduct an additional comprehensive comparison by varying \( \Delta_\theta \) and \( \Delta_\varphi \).

Fig. 8 shows the detection probabilities with respect to \( \Delta_\theta \) and \( \Delta_\varphi \). We observe that as \( \Delta_\theta \) and \( \Delta_\varphi \) increase, the detection probability of the proposed strategy remains at approximately 90\%, whereas
TABLE II: Performance comparisons with B2 at $N = 25$, $M = 512$, with $T_0 = 0.2$ s and $\Delta \phi = \Delta \theta = \frac{\pi}{16}$.

| Transmit Power | SNR at the UE | Received Signal Power at the BS | Detection Probability |
|----------------|---------------|---------------------------------|-----------------------|
| Proposed       | 89.0000 dBm   | 40.5457 dB                      | $P_{d,(\text{proposed})} = 90.11\%$ |
| Benchmark-2    | 89.0000 dBm   | 40.5457 dB                      | $P_{d,(\text{B2})} = 89.36\%$   |
| Proposed       | 90.0000 dBm   | 43.0503 dB                      | $P_{d,(\text{proposed})} = 89.94\%$ |
| Benchmark-2    | 90.0000 dBm   | 43.0503 dB                      | $P_{d,(\text{B2})} = 89.47\%$   |
| Proposed       | 92.0000 dBm   | 46.7348 dB                      | $P_{d,(\text{proposed})} = 90.26\%$ |
| Benchmark-2    | 92.0000 dBm   | 46.7348 dB                      | $P_{d,(\text{B2})} = 89.63\%$   |

TABLE III: Performance comparisons with B2 at $N = 25$, $M = 512$, with $T_0 = 0.2$ s and $\Delta \phi = \Delta \theta = \frac{\pi}{4}$.

| Transmit Power | SNR at the UE | Received Signal Power at the BS | Detection Probability |
|----------------|---------------|---------------------------------|-----------------------|
| Proposed       | 78.0000 dBm   | 25.8846 dB                      | $P_{d,(\text{proposed})} = 90.75\%$ |
| Benchmark-2    | 78.0000 dBm   | 25.8846 dB                      | $P_{d,(\text{B2})} = 77.20\%$   |
| Proposed       | 82.0000 dBm   | 36.6913 dB                      | $P_{d,(\text{proposed})} = 90.38\%$ |
| Benchmark-2    | 82.0000 dBm   | 36.6913 dB                      | $P_{d,(\text{B2})} = 80.15\%$   |
| Proposed       | 85.0000 dBm   | 40.9910 dB                      | $P_{d,(\text{proposed})} = 91.51\%$ |
| Benchmark-2    | 85.0000 dBm   | 40.9910 dB                      | $P_{d,(\text{B2})} = 81.88\%$   |

that of B2 descends monotonically from 90% to around 65%. This is because when detecting a large possible TG, B2 aims to maximize the beamforming gain toward a point, such that the sensing beam generated by B2 may only cover a portion of the SSA. By contrast, the sensing beam generated by the proposed strategy is optimized in consideration of the TG size, such that it can illuminate the entire SSA of the TG as much as possible.

![Fig. 8: The detection probabilities with respect to $\Delta \theta$ and $\Delta \phi$, when $M = 512$, $T_0 = 0.2$ s and $\gamma_{\text{min}} = 90\%$.](image)

V. CONCLUSIONS AND PROSPECTS

In our work, the RIS was leveraged to assist an mmWave ISAC system, and a novel passive beamforming strategy was proposed to jointly optimize the communication and sensing performances.
To this end, the detection probability was derived based on the illumination power on an approximated SSA of the TG, and the UDR was defined to measure the capability of the TG detection. Then, an optimization problem was formulated to maximize the SNR at the UE under a minimum detection probability constraint. To solve the non-convex problem, a new convexification process was performed to convexify the detection probability constraint with a series of matrix manipulations and a real-valued first-order Taylor approximation, and the SDR was adopted for problem relaxation. Afterwards, an SCA-based algorithm was designed to iteratively find an approximate phase-shift solution, followed by the analysis on the problem feasibility condition and the algorithm convergence. Results confirmed that our proposed strategy was superior to the investigated benchmarks when detecting TGs with practical sizes, so that it would be promising in supporting the RIS-ISAC in B5G and 6G scenarios. In the future, the proposed strategy in this paper is worth to be extended to a general multi-user case with multiple TGs to be simultaneously detected.

APPENDIX A
PROOF OF PROPOSITION 1

We prove by first considering the transformation of $\omega^H A(\theta, \varphi) \omega \omega^H B(\theta, \varphi) \omega$ inside the integral in $f(\omega)$. In specific, by defining $Q = \omega \omega^H$, we have

$$\omega^H A(\theta, \varphi) \omega \omega^H B(\theta, \varphi) \omega = tr \{ B(\theta, \varphi) Q A(\theta, \varphi) Q \} = [\text{vec}(Q^T)]^T [A^T(\theta, \varphi) \otimes B(\theta, \varphi)] \text{vec}(Q).$$

Since $[\text{vec}(Q^T)]^T = [\text{vec}(\omega^* \omega^T)]^T = (\omega \otimes \omega^*)^T = \omega^T \otimes \omega^H$ and $\text{vec}(Q) = \text{vec}(\omega \omega^H) = \omega^* \otimes \omega$ hold, we have

$$[\text{vec}(Q^T)]^T [A^T(\theta, \varphi) \otimes B(\theta, \varphi)] \text{vec}(Q) = (\omega^T \otimes \omega^H) [A^T(\theta, \varphi) \otimes B(\theta, \varphi)] (\omega^* \otimes \omega).$$

(43)

In order to use (43) to simplify $f(\omega)$, we define two auxiliary matrices, i.e. $C(\theta, \varphi)$ and $K$, as

$$C(\theta, \varphi) = A^T(\theta, \varphi) \otimes B(\theta, \varphi),$$

(44)

$$K = \int_{\varphi_S - \Delta \varphi}^{\varphi_S + \Delta \varphi} \int_{\theta_S - \Delta \theta}^{\theta_S + \Delta \theta} C(\theta, \varphi) \, d\theta \, d\varphi.$$  

(45)

Then, $f(\omega)$ can be recast as

$$f(\omega) = (\omega^T \otimes \omega^H) K (\omega^* \otimes \omega).$$

(46)

It is noted that because: 1) $C(\theta, \varphi) \succeq 0$ for $\forall \varphi \in [\varphi_S - \frac{\Delta \varphi}{2}, \varphi_S + \frac{\Delta \varphi}{2}]$ and $\forall \theta \in [\theta_S - \frac{\Delta \theta}{2}, \theta_S + \frac{\Delta \theta}{2}]$, and 2) $C^H(\theta, \varphi) = A^*(\theta, \varphi) \otimes B^H(\theta, \varphi) = A^T(\theta, \varphi) \otimes B(\theta, \varphi) = C(\theta, \varphi)$, we have $K \succeq 0$ and $K^H = K$, implying that $K$ is a positive semidefinite Hermitian matrix. Therefore, $K$ can be decomposed into
$K = U\Lambda U^H$ via the eigenvalue decomposition, where $U$ is a unitary matrix, and $\Lambda$ is a diagonal eigenvalue matrix, whose diagonal elements are non-negative.

Substituting $K = U\Lambda U^H$ into (46), we obtain

$$f(\omega) = (\omega^T \otimes \omega^H) \ U\Lambda U^H (\omega^* \otimes \omega) = (\omega^T \otimes \omega^H) \ U\Lambda^{\frac{1}{2}} \ ||(\omega^T \otimes \omega^H) \ U\Lambda^{\frac{1}{2}}||^2_2.$$ (47)

By rewriting $U\Lambda^{\frac{1}{2}}$ as $U\Lambda^{\frac{1}{2}} = [\sigma_1, \sigma_2, \cdots, \sigma_{N^2}]$, we have

$$f(\omega) = \| (\omega^T \otimes \omega^H) [\sigma_1, \sigma_2, \cdots, \sigma_{N^2}] \|^2_2$$

$$= \| [ (\omega^T \otimes \omega^H) \sigma_1, (\omega^T \otimes \omega^H) \sigma_2, \cdots, (\omega^T \otimes \omega^H) \sigma_{N^2} ] \|^2_2$$

$$= \| [ \omega^H (\text{vec}^{-1}(\sigma_1)) \omega, \omega^H (\text{vec}^{-1}(\sigma_2)) \omega, \cdots, \omega^H (\text{vec}^{-1}(\sigma_{N^2})) \omega ] \|^2_2$$

$$= \| \text{tr} \{ (\text{vec}^{-1}(\sigma_1)) \omega \omega^H \} , \text{tr} \{ (\text{vec}^{-1}(\sigma_2)) \omega \omega^H \} , \cdots, \text{tr} \{ (\text{vec}^{-1}(\sigma_{N^2})) \omega \omega^H \} \|_2^2$$

$$= \| \text{tr} \{ (\text{vec}^{-1}(\sigma_1)) Q \} , \text{tr} \{ (\text{vec}^{-1}(\sigma_2)) Q \} , \cdots, \text{tr} \{ (\text{vec}^{-1}(\sigma_{N^2})) Q \} \|_2^2,$$ (48)

where $\sigma_i$ is the $i$-th column of $U\Lambda^{\frac{1}{2}}$. Finally, by denoting $\text{vec}^{-1}(\sigma_i)$ as

$$S_i = \text{vec}^{-1}(\sigma_i), \quad \text{for} \quad i = 1, 2, \ldots, N^2,$$ (49)

we complete the proof of Proposition 1.

APPENDIX B

PROOF OF PROPOSITION 2

Here, we aim to derive the real representation of $T_{v(Q_k)}(\|v(Q)\|_2)$. We rewrite the vector $v(Q)$ as

$$v(Q) = v_R(Q) + jv_I(Q),$$ (50)

where $v_R(Q)$ and $v_I(Q)$ are, respectively, the real part and imaginary part of $v(Q)$, given by

$$v_R(Q) = [\Re\{\text{tr}(S_1 Q)\}, \Re\{\text{tr}(S_2 Q)\}, \cdots, \Re\{\text{tr}(S_{N^2} Q)\}]^T$$

$$= \left[ \frac{1}{2} \text{tr} \left( (S_1 + S_1^H)Q \right), \frac{1}{2} \text{tr} \left( (S_2 + S_2^H)Q \right), \cdots, \frac{1}{2} \text{tr} \left( (S_{N^2} + S_{N^2}^H)Q \right) \right]^T,$$ (51)

$$v_I(Q) = [\Im\{\text{tr}(S_1 Q)\}, \Im\{\text{tr}(S_2 Q)\}, \cdots, \Im\{\text{tr}(S_{N^2} Q)\}]^T$$

$$= \left[ -\frac{j}{2} \text{tr} \left( (S_1 - S_1^H)Q \right), -\frac{j}{2} \text{tr} \left( (S_2 - S_2^H)Q \right), \cdots, -\frac{j}{2} \text{tr} \left( (S_{N^2} - S_{N^2}^H)Q \right) \right]^T.$$ (52)

Then, we combine $v_R(Q)$ and $v_I(Q)$ into a new vector $\tilde{v}(Q)$, formed by

$$\tilde{v}(Q) = \begin{bmatrix} v_R(Q) \\ v_I(Q) \end{bmatrix} \in \mathbb{R}^{2N^2 \times 1}.$$ (53)
Hence, according to (50) and (53), we obtain
\[ \| \tilde{v}(Q) \|_2 = \| v(Q) \|_2 = \sqrt{v^T_R(Q)v_R(Q) + v^T_1(Q)v_1(Q)}, \] (54)
implying that \( \| v(Q) \|_2 \geq \sqrt{\mathcal{G}} \) and \( \| \tilde{v}(Q) \|_2 \geq \sqrt{\mathcal{G}} \) are equivalent. Note that according to (53), \( \tilde{v}(Q) \) is a real-value vector, such that the partial derivatives of \( \| \tilde{v}(Q) \|_2 \) with respect to \( v_R(Q) \) and \( v_1(Q) \) are completely real. After some manipulations, we obtain
\[ \frac{\partial \| \tilde{v}(Q) \|_2}{\partial v_R(Q)} = \frac{v_R(Q)}{\| \tilde{v}(Q) \|_2}, \quad \frac{\partial \| \tilde{v}(Q) \|_2}{\partial v_1(Q)} = \frac{v_1(Q)}{\| \tilde{v}(Q) \|_2}. \] (55)
Then, we have
\[ \frac{\partial \| \tilde{v}(Q) \|_2}{\partial \tilde{v}(Q)} \bigg|_{\tilde{v}(Q) = \tilde{v}(Q_k)} = \left[ \frac{\partial \| \tilde{v}(Q) \|_2}{\partial v_R(Q)} \right]_{\tilde{v}(Q) = \tilde{v}(Q_k)} \tilde{v}(Q) - \tilde{v}(Q_k) \right] \]
\[ = \| \tilde{v}(Q_k) \|_2 \left( \frac{\tilde{v}(Q)}{\| \tilde{v}(Q_k) \|_2} \right)^T \left[ \tilde{v}(Q) - \tilde{v}(Q_k) \right] \]
\[ = \| \tilde{v}(Q_k) \|_2^{-1} \tilde{v}^T(Q_k) \tilde{v}(Q) \]
\[ = \| \tilde{v}(Q_k) \|_2^{-1} \sum_{i=1}^{N^2} tr \left( \frac{1}{2} (S_i + S_i^H)Q_k \right) tr \left( \frac{1}{2} (S_i + S_i^H)Q \right) \]
\[ + \| \tilde{v}(Q_k) \|_2^{-1} \sum_{i=1}^{N^2} tr \left( -\frac{j}{2} (S_i - S_i^H)Q_k \right) tr \left( -\frac{j}{2} (S_i - S_i^H)Q \right) \]
\[ = \sum_{i=1}^{N^2} tr \left( \| \tilde{v}(Q_k) \|_2^{-1} tr \left( \frac{1}{4} (S_i + S_i^H)Q_k \right) (S_i + S_i^H)Q \right) \]
\[ + \sum_{i=1}^{N^2} tr \left( \| \tilde{v}(Q_k) \|_2^{-1} tr \left( -\frac{1}{4} (S_i - S_i^H)Q_k \right) (S_i - S_i^H)Q \right) \]
\[ = tr \left( \Upsilon(Q_k)Q \right), \] (57)
where \( \Upsilon(Q_k) \) is given by
\[ \Upsilon(Q_k) = 4\| \tilde{v}(Q_k) \|_2^{-1} \left\{ \sum_{i=1}^{N^2} tr \left( (S_i + S_i^H)Q_k \right) (S_i + S_i^H)Q \right\} - \sum_{i=1}^{N^2} tr \left( (S_i - S_i^H)Q_k \right) (S_i - S_i^H)Q \right\}. \] (58)
Because \( \| \tilde{v}(Q_k) \|_2 \), \( \frac{\partial \| \tilde{v}(Q) \|_2}{\partial \tilde{v}(Q)} \bigg|_{\tilde{v}(Q) = \tilde{v}(Q_k)} \) and \( [\tilde{v}(Q) - \tilde{v}(Q_k)] \) are all real, \( \mathcal{T}_{\tilde{v}(Q_k)}(\| \tilde{v}(Q) \|_2) \) is real as well. In addition, because \( \| v(Q) \|_2 \geq \sqrt{\mathcal{G}} \) and \( \| \tilde{v}(Q) \|_2 \geq \sqrt{\mathcal{G}} \) are equivalent, \( \mathcal{T}_{v(Q_k)}(\| v(Q) \|_2) \geq \sqrt{\mathcal{G}} \) and \( \mathcal{T}_{\tilde{v}(Q_k)}(\| \tilde{v}(Q) \|_2) \geq \sqrt{\mathcal{G}} \) are also equivalent. Consequently, we prove that \( \mathcal{T}_{\tilde{v}(Q_k)}(\| \tilde{v}(Q) \|_2) \) is a real representation of \( \mathcal{T}_{v(Q_k)}(\| v(Q) \|_2) \).
REFERENCES

[1] Z. Xing, R. Wang and X. Yuan, "Passive beamforming design for RIS enabled integrated sensing and communication," in Proc. IEEE Global Communications Conference, Rio de Janeiro, Brazil, Dec. 2022, pp. 1-6. (under review)

[2] X. You, et al., "Towards 6G wireless communication networks: Vision, enabling technologies, and new paradigm shifts," SCIENCE CHINA Information Sciences, vol. 64, no. 1, pp. 1-74, Jan. 2021.

[3] J. A. Zhang, et al., "Enabling joint communication and radar sensing in mobile networks - A survey," IEEE Communications Surveys & Tutorials, vol. 24, no. 1, pp. 306-345, Firstquarter 2022.

[4] J. A. Zhang, et al., "An overview of signal processing techniques for joint communication and radar sensing," IEEE Journal of Selected Topics in Signal Processing, vol. 15, no. 6, pp. 1295-1315, Nov. 2021.

[5] F. Liu, et al., "Integrated sensing and communications: Towards dual-functional wireless networks for 6G and beyond," IEEE Journal on Selected Areas in Communications, Early Access, Mar. 2022, DOI: 10.1109/JSAC.2022.3156632.

[6] Z. Xing, R. Wang, J. Wu and E. Liu, "Achievable rate analysis and phase shift optimization on intelligent reflecting surface with hardware impairments," IEEE Transactions on Wireless Communications, vol. 20, no. 9, pp. 5514-5530, Sept. 2021.

[7] Z. Xing, et al., "Location-aware beamforming design for reconfigurable intelligent surface aided communication system," in Proc. IEEE/CIC International Conference on Communications in China (ICCC), Xiamen, China, Jul. 2021, pp. 201-206.

[8] C. Liaskos, et al., "A new wireless communication paradigm through software-controlled metasurfaces," IEEE Communications Magazine, vol. 56, no. 9, pp. 162-169, Sept. 2018.

[9] L. Subrt and P. Pechac, “Intelligent walls as autonomous parts of smart indoor environments,” IET Communications, vol. 6, no. 8, pp. 1004-1010, May 2012.

[10] Q. Wu and R. Zhang, “Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming,” IEEE Transactions on Wireless Communications, vol. 18, no. 11, pp. 5394-5409, Nov. 2019.

[11] C. Huang, et al., “Reconfigurable intelligent surfaces for energy efficiency in wireless communication,” IEEE Transactions on Wireless Communications, vol. 18, no. 8, pp. 4157-4170, Aug. 2019.

[12] S. Gong, et al., “Towards smart wireless communications via intelligent reflecting surfaces: A contemporary survey,” IEEE Communications Surveys & Tutorials, vol. 22, no. 4, pp. 2283-2314, Fourth quarter 2020.

[13] Q. Wu and R. Zhang, “Towards smart and reconfigurable environment: Intelligent reflecting surface aided wireless network,” IEEE Communications Magazine, vol. 58, no. 1, pp. 106-112, Jan. 2020.

[14] L. Dai, et al., “Reconfigurable intelligent surface-based wireless communications: Antenna design, prototyping, and experimental results,” IEEE Access, vol. 8, pp. 45913-45923, Mar. 2020.

[15] W. Yan, X. Yuan and X. Kuai, “Passive beamforming and information transfer via large intelligent surface,” IEEE Wireless Communications Letters, vol. 9, no. 4, pp. 533-537, Apr. 2020.

[16] E. Basar, “Reconfigurable intelligent surface-based index modulation: A new beyond MIMO paradigm for 6G,” IEEE Transactions on Communications, vol. 68, no. 5, pp. 3187–3196, Feb. 2020.

[17] M. Cui, G. Zhang and R. Zhang, “Secure wireless communication via intelligent reflecting surface,” IEEE Wireless Communications Letters, vol. 8, no. 5, pp. 1410-1414, Oct. 2019.

[18] L. Dong and H.-M. Wang, “Enhancing secure MIMO transmission via intelligent reflecting surface,” IEEE Transactions on Wireless Communications, vol. 19, no. 11, pp. 7543-7556, Nov. 2020.

[19] Q. Wu and R. Zhang, "Beamforming optimization for wireless network aided by intelligent reflecting surface with discrete phase shifts," IEEE Transactions on Communications, vol. 68, no. 3, pp. 1838-1851, Mar. 2020.

[20] C. Huang, A. Zappone, M. Debbah and C. Yuen, "Achievable rate maximization by passive intelligent mirrors," in Proc. IEEE International Conference on Acoustics, Speech and Signal Processing, Calgary, AB, Canada, Apr. 2018, pp. 3714-3718.

[21] C. Guo, Y. Cui, F. Yang and L. Ding, "Outage probability analysis and minimization in intelligent reflecting surface-assisted MISO systems," IEEE Communications Letters, vol. 24, no. 7, pp. 1563-1567, Jul. 2020.

[22] B. Lu, R. Wang and Y. Liu, "Outage probability of intelligent reflecting surface assisted full duplex two-way communications," IEEE Communications Letters, vol. 26, no. 2, pp. 286-290, Feb. 2022.
[23] J. Li, R. Wang and E. Liu, "Passive beamforming design for IRS communication system with few-bit ADCs," in Proc. 4th International Conference on Information Communication and Signal Processing (ICICSP), Shanghai, China, Sept. 2021, pp. 1-6.
[24] J. He, et al., "Adaptive beamforming design for mmwave RIS-Aided joint localization and communication," in Proc. IEEE Wireless Communications and Networking Conference Workshops, Seoul, Korea (South), Apr. 2020, pp. 1-6.
[25] A. Elzanaty, A. Guerra, F. Guidi and M.-S. Alouini, "Reconfigurable intelligent surfaces for localization: Position and orientation error bounds," IEEE Transactions on Signal Processing, vol. 69, pp. 5386-5402, Oct. 2021.
[26] H. Zhang, et al., "MetaLocalization: Reconfigurable intelligent surface aided multi-user wireless indoor localization," IEEE Transactions on Wireless Communications, vol. 20, no. 12, pp. 7743-7757, Dec. 2021.
[27] S. Buzzi, E. Grossi, M. Lops and L. Venturino, "Radar target detection aided by reconfigurable intelligent surfaces," IEEE Signal Processing Letters, vol. 28, pp. 1315-1319, Jun. 2021.
[28] A. Aubry, A. D. Maio and M. Rosamilia, "Reconfigurable intelligent surfaces for N-LOS radar surveillance," IEEE Transactions on Vehicular Technology, vol. 70, no. 10, pp. 10735-10749, Oct. 2021.
[29] H. Zhang, et al., "MetaRadar: Multi-target detection for reconfigurable intelligent surface aided radar systems," IEEE Transactions on Wireless Communications, Early Access, Mar. 2022, DOI: 10.1109/TWC.2022.3153792.
[30] Z.-M. Jiang, et al., "Intelligent reflecting surface aided dual-function radar and communication system," IEEE Systems Journal, vol. 16, no. 1, pp. 475-486, Feb. 2021.
[31] X. Song, et al., "Joint transmit and reflective beamforming for IRS-assisted integrated sensing and communication," arXiv:2111.13511v1, Nov. 2021, [Online]. Available: https://arxiv.org/abs/2111.13511.
[32] R. Liu, M. Li and A. L. Swindlehurst, "Joint beamforming and reflection design for RIS-assisted ISAC systems," arXiv:2203.00265v1, Mar. 2022, [Online]. Available: https://arxiv.org/abs/2203.00265.
[33] X. Wang, Z. Fei, Z. Zheng and J. Guo, "Joint waveform design and passive beamforming for RIS-assisted dual-functional radar-communication system," IEEE Transactions on Vehicular Technology, vol. 70, no. 5, pp. 5131-5136, May. 2021.
[34] X. Tong, Z. Zhang, J. Wang, C. Huang and M. Debbah, "Joint multi-user communication and sensing exploiting both signal and environment sparsity," IEEE Journal of Selected Topics in Signal Processing, vol. 15, no. 6, pp. 1409-1422, Nov. 2021.
[35] R.S. Prasobh Sankar and S. P. Chepuri, "Beamforming in hybrid RIS assisted integrated sensing and communication systems," arXiv:2203.05902v1, Mar. 2022, [Online]. Available: https://arxiv.org/abs/2203.05902.
[36] X. Wang, et al., "Joint waveform and discrete phase shift design for RIS-assisted integrated sensing and communication system under Cramer-Rao bound constraint," IEEE Transactions on Vehicular Technology, vol. 71, no. 1, pp. 1004-1009, Jan. 2022.
[37] P. Wang, J. Fang, X. Yuan, Z. Chen and H. Li, "Intelligent reflecting surface-assisted millimeter wave communications: Joint active and passive precoding design," IEEE Transactions on Vehicular Technology, vol. 69, no. 12, pp. 14960-14973, Dec. 2020.
[38] K. Zhi, et al., "Uplink achievable rate of intelligent reflecting surface-aided millimeter-wave communications with low-resolution ADC and phase noise," IEEE Wireless Communications Letters, vol. 10, no. 3, pp. 654-658, Mar. 2021.
[39] M. Najafi, V. Jamali, R. Schober and H. V. Poor, "Physics-based modeling and scalable optimization of large intelligent reflecting surfaces," IEEE Transactions on Communications, vol. 68, no. 12, pp. 7948-7962, Dec. 2020.
[40] M. K. Samimi, G. R. MacCartney, S. Sun and T. S. Rappaport, "28 GHz millimeter-wave ultrawideband small-scale fading models in wireless channels," in Proc. IEEE 83rd Vehicular Technology Conference, Nanjing, China, May 2016, pp. 1-6.
[41] Z. Muhi-Elddeen, L. Ivriissimitzis and M. Al-Nuaimi, "Modelling and measurements of millimetre wavelength propagation in urban environments," IET Microwaves Antennas & Propagation, vol. 4, no. 9, pp. 1300-1309, Sept. 2010.
[42] X. Hu, C. Zhong, Y. Zhang, X. Chen and Z. Zhang, "Location information aided multiple intelligent reflecting surface systems," IEEE Transactions on Communications, vol. 68, no. 12, pp. 7948-7962, Dec. 2020.
[43] M. Shahmansoori, et al., "Position and orientation estimation through millimeter-wave MIMO in 5G systems," IEEE Transactions on Wireless Communications, vol. 17, no. 3, pp. 1822-1835, Mar. 2018.
[44] S. M. Kay, "Fundamentals of statistical signal processing: Volume II: Detection theory," Englewood Cliffs, NJ: Prentice-Hall, 1993.
[46] M. Á. Vázquez, L. Blanco and A. I. Pérez-Neira, ”Spectrum sharing backhaul satellite-terrestrial systems via analog beamforming,” IEEE Journal of Selected Topics in Signal Processing, vol. 12, no. 2, pp. 270-281, May 2018.

[47] Z. Xing, R. Wang, X. Yuan and J. Wu, ”Location information assisted beamforming design for reconfigurable intelligent surface aided communication systems,” arXiv: 2110.08980v2, Feb. 2022, [Online]. Available: https://arxiv.org/abs/2110.08980.

[48] M. C. Rezende, et al., ”Radar cross section measurements (8-12 GHz) of magnetic and dielectric microwave absorbing thin sheets,” Revista de Fisica Aplicada e Instrumentacao, vol. 15, no. 1, pp. 24-29, Dec. 2002.