Controllable drilling parameter optimization for roller cone and polycrystalline diamond bits

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Abstract
Oil well drilling data from 23 oil wells in northern Iraq are analyzed and optimized controllable drilling parameters are found. The most widely used Bourgoyne and Young (BY) penetration rate model have been chosen for roller cone bits, and parameters were extracted to adjust for other bit types. In this regard, the collected data from real drilling operation have initially been averaged in short clusters based on changes in both lithology and bottom hole assemblies. The averaging was performed to overcome the issues related to noisy data negative effect and the lithological homogeneity assumption. Second, the Dmitriy Belozerov modifications for polycrystalline diamond bits compacts have been utilized to correct the model to the bit weight. The drilling formulas were used to calculate other required parameters for the BYM. Third, threshold weight for each cluster was determined through the relationship between bit weight and depth instead of the usual Drill of Test. Fourth, coefficients of the BYM were calculated for each cluster using multilinear regression. Fifth, a new model was developed to find the optimum drill string rotation based on changes in torque and bit diameter with depth. The above-developed approach has been implemented successfully on 23 oil wells field data to find optimum penetration rate, weight on bit and string rotation.

Keywords Bourgoyne and Young model · Clustering · Drilling · Multiple linear regression · Optimization

Introduction
Oil well drilling technique is the process of making a safe vertical or inclined cased hole (Rabia 2002). An experienced team of qualified personnel, drilling program, and powerful drilling rig are the main basic requirements to perform a drilling operation (Azar and Samuel 2007). Reduction in cost and/or time without compromising safety is key topics for optimized drilling performance. Optimization in drilling operations started with the first attempt to drill in 1900 (Eren and Ozbayoglu 2010). The process of optimization was slow in the beginning and became quicker with scientific and technological developments. Despite all advances, however, still, there are challenges in understanding the overall drilling operation process. These challenges pushed researchers to continue their work to identify and model drilling processes and to study further the main parameters that affect oil well operations (Darley 1969; Khodja et al. 2010). From 1920 to date, many parameters have been studied in different models. Bourgoyne and Young (BY) was one of these models used in optimizing the controllable drilling parameters. This model was conducted in 1974 for eight parameters in one equation for optimal rate of penetration (ROP) in roller cone bits (Bourgoyne et al. 1986; Bourgoyne and Young 1974; Caenn et al. 2011; Eckel 1968; Eren and Ozbayoglu 2010). Other researchers used charts, relationship between a set of parameters in their works (Bingham 1965; Gatlin 1960; Maurer 1962; Warren 1987). Computer programs were developed in different areas to find optimal weight on bit (WOB) and rotation per minute (RPM) (Barragan et al. 1997; Bourgoyne and Young 1974; Coelho et al. 2005; Galle and Woods 1963). Lummus, for example, studied the optimization process through analysis of mud hydraulic properties (Lummus 1971). Different mathematical models implemented to optimize WOB, RPM, and bit hydraulics (Bourgoyne et al. 1986; Graham and Muench 1959; Speer 1958). Borehole cleaning, lifting capacity, bit types, and hydraulic parameters have been studied by many
researchers in seeking more optimized operations (Garnier and Van Lingen 1959; Maurer 1962; Naganawa 2012). Many researchers focused on regression analysis and empirical correlation in their works to reduce the drilling time (Duklet and Bates 1980).

Despite all advances still, the need for more optimized operational is crucial. Recently, the BYM has been studied by Eren Tuna (Eren and Ozbayoglu 2010). He studied the real-time optimization of ROP using regression analysis to find the BYM coefficients. Tuna used the data of two wells to optimize the controllable parameters of a third well. In fact, many other researchers considered the BYM as one of the best drilling optimization models because it is based on a statistical analysis of past drilling parameters, like WOB and RPM. The BYM has been successfully implemented in different locations on roller cone bits (Elahifar et al. 2012; Eren and Ozbayoglu 2011; Irawan and Anwar 2014; Miyora et al. 2015). Nevertheless, this model needs substantial adjustments before using it on other types of drilling bits like fixed cutter bits. Recently, Christensen, Inc. and Shell Oil Co. used the corrected BYM for polycrystalline diamond (PDC) bits to optimize the penetration rate (Duklet and Bates 1980). In almost all drilling operations, the operator uses different types of drilling bits (roller cone and fixed cutter bits) in the same well. Thus, using the BYM with corrections expands the applicability of the model. In this paper, field data from the Bazian oil block in northern Iraq are used with the original model in the upper intervals where the roller cone bits are used. A corrected BYM was used in other intervals where the PDC bits were used in the drilling operations. The drilled sections were partitioned into 20 clusters based on bottom hole assemblies (BHA) and the lithology changes. Roller cone bits were mainly used in drilling the upper clusters, while, PDC bits were used in the other clusters. Optimized WOB, RPM, and ROP parameters were predicted for future operations in the same geological setting.

Methodology

In this study, the drilling operations of 23 oil wells were monitored in three different oil blocks in northern Iraq. The three oil blocks were Taq Taq, Bazian, and Miran. The Bn-1 oil well located in the Bazian block was selected to be a key well for this study. This oil well has been monitored closely to collect all the operational data starting from the civil works to the site restoration. Appendix Table 7 shows a part of the field data from the mud logging unit (MLU) that were collected and used in this study. A set of data for each meter from the surface to the final depth at 3715 m has been collected from different sources. The MLU data were selected because it was the most continuous and complete data set.

The collected data of WOB, RPM, ROP, and torque were averaged in short intervals of 10 m to reduce the effects of noisy data and homogeneity assumptions. Then, the collected data have been divided into 20 clusters based on lithology and bottom hole assembly (BHA) changes as shown in Fig. 1. The needed parameters for the BYM were determined from standard drilling formula (DF) calculations. Parameters such as equivalent circulation density (ECD), bottom hole pressure (BHP), Reynold number, pore pressure gradient (Pg) and annular pressure were calculated through the DF. The BYM contains the effect of eight variables of drillability, normal and abnormal compaction, pressure difference, bit weight, string rotation, bit wear, and bit hydraulics. The eight effective operational parameters ($x_1$–$x_8$) of the BYM were calculated through specific equations (Bourgoyne and Young 1974), as in Appendix Table 8. Multiple linear regression techniques were used to compute optimum coefficients ($a_1$–$a_8$), as Appendix Table 6. Based on the relationship between RPM, bit diameter and torque with depth, a new model was derived to calculate optimum RPM. The new model is a function of only two parameters (Torque and Bit diameter). The main methodology steps are illustrated in Fig. 1.

Well profile

The drilling operation in Bn-1 oil well was spudded on October 2009 and finished in 256 days after penetrating the Triassic rocks at a total depth of 3715 m (Korea National Oil Corporation, KNOC 2009). The final casing setting depth was with the 7-in. liner at 3640 m (Korean National Oil Corporation, KNOC 2009). Figure 2 shows penetrated formations, casing size and setting depth for all the drilled sections from the surface with 30-in. conductor pipe ended to the 6-in. casing for and Open Hole Section at 3715 m. The geological stratigraphy showed alternation of different geological formations from the surface to the final depth (Darwesh 2014). Figure 3 shows 20 different geological formations starting from Eocene–Pila Spi formation to Triassic–Kura Chine formation. The actual drilling time spent on all the operating activities was more than the planned time due to drilling problems and shortage in the offset data (Darwesh 2014).

The surface section is characterized by hard, highly fractured and abrasive formations which cause excessive vibrations of the drill string and a total loss in the drilling fluid. Swelling and packing off were the problems in the intermediate section. Problems of tight spots and bridges were encountered also in this section. The production and liner...
section showed a good drillability but with frequent loss of circulation and problems of fishing and sidetracking.

The rate of penetration (ROP) model

The BYM (1974) is a linear relationship between the penetration rate and a set of eight effective parameters $x_1$–$x_8$. This model was based on the statistical synthesis of the collected data from the previously drilled wells. This model considers the following eight effects:

- Formation strength
- Formation depth
- Formation compaction
- The pressure differential across the bottom of the hole
- Bit diameter and weight on bit
- Rotary speed
- Bit wear
- Bit hydraulics

First, the model was introduced for roller cone bits, as shown in Eq. 1 (Adam et al. 1991).

$$\frac{df}{dt} = e^{a_1 + \sum_{j=2}^{8} a_j x_j}$$

(1)

where $\frac{df}{dt}$ is the rate of penetration (ft./h), $a_j$ are the coefficients of the model and $x_j$ the eight drilling parameters and explained below.

Effect of formation strength

This effect is represented by coefficient $a_1$ in the BYM and it is inversely proportional to the natural logarithm of the square of the drillability. The less value for this coefficient means less ROP. The coefficient also includes the effects of parameters not mathematically modeled, such as the effect of drilled cuttings in terms of size, type, quantity or any other effects (Bourgoyne and Young 1974). Other factors such as drilling fluid properties, solid content, the efficiency of the equipment/material, crew experience, and service companies' efficiency could be included under this function or as separate coefficients in future considerations. Equation 2 defines the formation of strength related effects with the same units of ROP.

$$f_1 = e^{\epsilon_1}$$

(2)
where $f_1$ is the rate of penetration (ft./h) and the $a_1$ is a dimensionless coefficient for the formation strength.

**Effect of formation compaction**

Equations 3 and 4 define the effect of normal and abnormal formation compaction. These effects are represented by coefficients $a_2$ and $a_3$.

$$f_2 = e^{a_2 x_2}$$  \hspace{1cm} (3)

$$f_3 = e^{a_3 x_3}$$  \hspace{1cm} (4)

where $f_2$ and $f_3$ are the rate of penetration (ft./h) and the $a_2$ and $a_3$ are dimensionless coefficients for the effect of normal and abnormal formation compaction.

The normal compaction parameter $x_2$ assumes an exponential decrease in ROP with depth, while the abnormal compaction effect on penetration rate $x_3$ assumes an exponential increase in ROP with the pore pressure gradient (Bourgoyne and Young 1974). Equations 5 and 6 show the effect of formation compaction in BYM.

$$x_2 = 10000 - D$$  \hspace{1cm} (5)

$$x_3 = D^{0.69}(g_p - 12.5)$$  \hspace{1cm} (6)

where $g_p$ is the pore pressure gradient in units of ppg, $D$ is depth in ft. and the numbers 10,000 and 12.5 have been chosen to normalize the effect of depth and pore pressure.
Effect of bottom hole pressure differential

Coefficient $a_4$ in Eq. 7 represents the effect of pressure differential on ROP. The penetration rate will reduce with the decrease in the bottom hole pressure difference. Whenever the pressure differential between mud hydrostatic pressure and the formation pressure is zero, this effect will be equal to 1 (Bourgoyne and Young 1974).

$$f_4 = e^{a_4 x_4}$$

$$x_4 = D(g_p - \text{ECD})$$

where $f_4$ is the rate of penetration (ft./h) and the $a_4$ is a dimensionless coefficient for the effect of bottom hole pressure differential, ECD is equivalent circulating mud density at the bottom hole in units of ppg.

Effect of bit diameter and weight on bit, ($w/d$)

The bit weight and bit diameter are considered to have a direct effect on the ROP. The constant $a_5$ in Eq. 9 represents the effect of WOB and bit diameter on the penetration rate. The parameter $x_5$ assumes that the ROP is directly proportional to $\left(\frac{w}{d}\right)^{a_5}$ (Bourgoyne and Young 1974).

$$f_5 = e^{a_5 x_5}$$

$$x_5 = \ln \left[ \frac{\left(\frac{w}{d}\right) - \left(\frac{w}{d}\right)^2}{4 - \left(\frac{w}{d}\right)^2} \right]$$

where $f_5$ is the rate of penetration (ft./h) and the $a_5$ is a dimensionless coefficient for the effect of Bit Diameter and Weight on Bit, $w/d$ is the WOB per inch of bit diameter in
units of 1000 lb/in. \((w/d)\), is the threshold WOB per inch of bit diameter. The number 4 is the normalized weight on bit in units of 1000 lb per bit diameter in an inch.

Fix cutter bits like polycrystalline diamond bit (PDC) was used widely in the drilling of Bn-1 and many other wells. This type of bit showed high performance in drilling soft-shale formations. Adding some special design features in recent years made this type of bits more effective in drilling medium-to-hard formations also (Belozerov 2015).

The PDC bit breaks the rock through shearing, and they do not have any moving or rotating parts as in roller cone bits. PDC bits consist of many fixed blades that are integral with rotation as a single unit of the drilling string (Belozerov 2015). Figure 4 shows the main differences between the roller cone and PDC bits.

Adjustments were applied on the \(x_5\) to suit PDC bits before running the regression analysis, and both bit weight and bit diameter were adjusted. Belozerov (2015) adjustments were selected to adjust the effect of bit weight parameter. Dmitriy Belozerov adjustments were based on the relationship between mechanical and critical weights on the PDC bit, as seen in Eqs. 11, 12 and 13 to find \(x_5\).

\[
x_5 = \ln \left[ \frac{C_r \cdot \left( \frac{w}{d} \right) - 0.942 \cdot \Delta P_b \cdot \left( \frac{d-1}{d} \right)}{WOB_c} \right]
\]

\[
x_5 = \ln \left[ \frac{C_r \cdot WOB_a - 0.942 \cdot \Delta P_b \cdot \left( \frac{d-1}{d} \right)}{WOB_c} \right]
\]

\[
\Delta P_b = \frac{q^2 \cdot \rho}{12031 \cdot A_n^2}
\]

where \(C_r\) is the dimensionless weight split between the 12¼” drill bit and 13½” under-reamer; in case of no under-reamer it will equal to 1. \(WOB_c = 800\) lbs/in. is the chosen weight to normalize WOB value for PDC bits. The coefficient 0.942 was high, probably due to hydraulic friction losses, and therefore, it was reduced by 0.02 in this paper. \(WOB_a\) is the measured weight on the surface in l bf. \(A_n\) is the nozzle total flow area in a square inch and \(q\) is the mud pump flow in units of mph.

The minimum needed WOB to start the drilling process (threshold weight) was found through the relationship between the ROP and the WOB as in Fig. 5. This force can be determined on-site by drill-off tests for each formation during the drilling operation by plotting the ROP as a function of WOB per bit diameter and then extrapolating back to a zero-drilling rate (Bourgoyne et al. 1986). In Fig. 5 before point (a) there is no record of ROP with the applied WOB, but then a linear relationship appears between the points (a, b and c). Subsequently, the relationship is nonlinear from (c) to the (d). After point (d), any increase in the WOB will not give an increase in the ROP (24). Thus, point (a) is the threshold value of that specific formation and drilling bit.

In this study, the threshold weight was determined through the plotted relationship between WOB and the depth in each cluster separately. Figure 6 shows the relationship between the applied weight and the depth in the first cluster (Pila Spi formation—Eocene). The minimum weight of 0.7

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**Fig. 4** Main features of PDC and roller cone bits used in Bn-1 (Karadzhova 2014)

**Fig. 5** Weight on bit and rate of penetration relationship (Ahmed et al. 2018)
ton was recorded for the mentioned formation. Later, this weight was converted to \((\text{weight}/\text{drilling bit diameter})\) as shown in Table 1.

The same procedure was performed to determine other threshold weight for all other clusters.

**Effect of rotary speed** \(N\)

The effect of the rotary speed \((N)\) described by the coefficient \(a_6\) and the parameter \(x_6\). It is assumed that the ROP is directly proportional to the rotation speed of the bit. Based on the Dmitriy Belozerov model (Belozerov 2015), the normalizing value to equalize the rotary speed function differs between roller cone bits and PDC bits. ROP usually increases linearly with the rotary speed at low values of rotary speed, but this effect will change at high rotation speeds into a nonlinear relationship (Bourgoyne and Young 1974). At higher values of rotary speed, the ROP will decrease due to an increase in drilled cutting and a decrease in lifting capacity (Warren 1987). This effect is modeled in Eqs. 14, 15 and 16 below for roller cone and PDC bits (Adam et al. 1991) (Belozerov 2015):

\[
 f_6 = e^{a_6 x_6} 
\]  

(14)

**Table 1**: Bn-1 clustering intervals and threshold weight

| Cluster no. | Lithology no | Formation       | Interval (ft.) | Hole diameter (in.) | \((w/d)_i\) (1000 lb/in.) | Clustering base         |
|-------------|--------------|-----------------|----------------|---------------------|---------------------------|-------------------------|
| 1           | 1            | Pila Spi        | 32–450         | 26                  | 0.0577                    | Geological Fm.          |
| 2           | 2            | Gercus          | 450–808        | Surface section     | 0.131                     | Geological Fm.          |
| 3           | 3            | Khurmala        | 808–1082       | 0.585               |                           | Geological Fm.          |
| 4           | 4            | Sinjar          | 1082–1444      | 0.681               |                           | Geological Fm.          |
| 5           | 5            | Kolosh          | 1444–1496      | 0.592               |                           | Bit change              |
| 6           |              |                 | 1496–1575      | 0.427               |                           | Bit change              |
| 7           | 6            | Aliji           | 1575–2326      | 17.5                | 0.257                     | Bit size change         |
| 8           | 7            | Upper Shiranish | 2326–3445      | Intermediate section| 0.20                      | Bit change              |
| 9           | 8            | Lower Shiranish | 3445–3930      | 0.137               |                           | Bit change              |
| 10          |              |                 | 3930–5489      | 12.25               | 0.196                     | Bit and Fm. change      |
| 11          | 1            | Production      | 5489–6033      | 0.359               |                           | Bit change              |
| 12          |              | section         | 6033–6375      | 0.433               |                           | Bit size change         |
| 13          |              | Liner section   | 6375–6670      | 8.5                 | 0.18                      | Bit change              |
| 14          |              |                 | 6670–7477      | Liner section       | 0.259                     | Geological Fm.          |
| 15          | 9            | Kometan         | 67475–7795     | 67475–7795          | 0.376                     | Geological Fm.          |
| 16          | 10           | Qamchuqa        | 7795–8179      | 0.047               |                           | Bit change              |
| 17          |              |                 | 8179–10,253    | 0.047               |                           | Bit change              |
| 18          |              |                 | 10,253–11,552  | 0.047               |                           | Bit change              |
| 19          | 11           | Qamchuqa/Sarmord| 11,552–11,926  | 0.067               |                           | Bit size change         |
| 20          |              |                 | 11,926–12,480  | Open hole           | 0.050                     | TVD                     |

**Fig. 6** Weight on bit versus depth for the first cluster (Pila Spi) formation
\[ x_6 = \ln \left( \frac{N}{100} \right) \quad \text{For roller cone bits} \quad (15) \]
\[ x_6 = \ln \left( \frac{N}{160} \right) \quad \text{For PDC bits} \quad (16) \]

where \( f_8 \) is the rate of penetration (ft./h), \( N \) = rotations per minute (RPM), numbers 100 and 160 are normalization factors for roller cone and PDC bits.

**Effect of tooth wear**

The coefficient \( a_7 \) represents the effect of tooth wear on penetration rate. ROP decreases with the increase in bit teeth wear. Reductions in ROP for tungsten carbide insert (TCI) roller cone bits and PDC bits are not as severe as for milled tooth (MT) roller bits (Gatlin 1960). During the use of TCI and PDC bits, the ROP does not vary significantly with tooth wear. Thus, the tooth wear exponent for TCI roller bits \( a_7 x_7 \) in Eq. 17 becomes zero and this means that \( e^{a_7 x_7} \) becomes equal to 1 (Bourgoyne and Young 1974). In the drilling of Bn-1, the TCI and PDC were dominantly used below 500 m and the effect of tooth wear \( (h_l) \) has set to \((1/8)\) as an alternative of zero. The value of \( 1/8 \) was selected to justify the worse dull case below the casing shoe, with a linear increase from zero to 8/8 to 1/8 (Belozerov 2015). The linear increase of wear in TCI and PDC bits is preferred compared to the nonlinear relation as in MT bits.

\[ f_7 = e^{a_7 x_7} \quad (17) \]
\[ x_7 = -h_l \quad (18) \]

where \( f_7 \) is the rate of penetration (ft./h), \( h_l \) is the fractional tooth height that has been worn away over 8.

**Effect of bit hydraulics**

The coefficient \( a_8 \) in Eq. 19 represents the coefficient for the effect of the bit hydraulics on ROP. The effect of the bit hydraulics \( x_8 \) assumes that the ROP is proportional to a Reynolds number or jet impact force (Bourgoyne et al. 1986).

\[ f_8 = e^{a_8 x_8} \quad (19) \]

The Reynolds number gives the effect of the drilling bit jetting action at the bottom of the hole, which promotes better cleaning of bits teeth. Through Eqs. 20 and 21, this effect can be determined and be normalized to 2.0 hp/in. chosen for the hydraulic horsepower.

\[ x_8 = \frac{H p / A_b}{2} \quad (20) \]

\[ \text{PH} = \frac{\Delta P_b / A_b}{1714} \quad (21) \]

where \( f_8 \) is the rate of penetration (ft./h), \( A_b = \) bit area square inch, \( \Delta P_b = \) bit pressure drops [psi] and \( H p = \) Hydraulic horsepower [hp].

The previous Eqs. 2–21 are used to calculate \( x_1 \) to \( x_8 \), and the coefficients \( a_1 \) to \( a_8 \) from multiple regression and thereby the optimum WOB and RPM can be determined.

**Clustering and threshold weight determination**

Based on the BHA and lithology changes, the whole penetrated depth was partitioned into 20 clusters, as shown in Table 1. The total of 11 geological formations was recorded from the surface to the final depth. In the upper part of the well, four geological formations have been drilled with the same bit but they are treated as individual clusters. In some geological formations, different drilling bits were used and each interval is treated as one cluster.

Instability problems such as shales swelling, cavings, tight spots, and eventual loss of circulations were encountered during the drilling operation (Darwesh 2014). These problems caused an increase in drilled cuttings and therefore increased pore pressure and annular density (Guo and Liu 2011). Equation 22 was used to calculate the ECD that was increased due to the weight of drill cuttings in the annulus (Skalle 2011). This equation takes into account the frictional loss due to the circulation of the drilling fluid (Aadnoy 2011; Lyons and Plisga 2011).

\[ \text{ECD} = \frac{P_{an}}{0.052 h} + M_w \quad (22) \]

where ECD is equivalent circulating density in units of ppg, \( P_{an} \) is Annulus frictional pressure loss in units of psi, \( h \) is depth in ft and \( M_w \) is mud weight in ppg.

A linear regression method which contains more than one variable is called a multiple regression (MLR) (Montgomery and Runger 2010). MLR was conducted for each cluster to calculate \( a_1 \) to \( a_8 \) for the BYM and the results attached and shown in Appendix Table 6. Based on the chosen rig, personal and bottom hole assemblies (BHA) selection, the optimum weights WOB have been calculated using Eq. 23.

\[ \frac{w}{d} \mid \text{opt} = \frac{a_5 H_1 \left( \frac{w}{z} \right)_m - \left( \frac{w}{d} \right)_s}{a_6 H_1 - a_8} \quad (23) \]

where \( a_5 \) and \( a_6 \) can be obtained from the regression analyses. Table 2 contains parameters of the bit wear model as described by (BY references), where \((w/d)_m \) is the maximum weight on bit diameter and \( H_1, H_2 \) and \( H_3 \) are coefficients.
provided by the International Association of Drilling Contractors (IADC). All the results were in an acceptable range (Bourgoyne et al. 1986; Eren and Ozbayoglu 2010) between the recorded minimum and the maximum value of WOB. For example, the required values for the coefficients and variables in the first cluster drilled with bit size 17½ in., TCI type 5 (Pila Spi formation) were $a_5 = 0.08576$, $a_6 = -17.76926$, $(w/d)_m = 3$, $(w/d)_t = 0.0920$ and $H_1 = 1.5$. The same principle was applied to find the optimum WOB in other clusters, as shown in Appendix Table 8.

Finding optimum string rotation $N$

A nonlinear Eq. 31 resulted from the relationship of the RPM, Torque and Bit diameters trend lines with the depth in calculating optimal RPM as shown in Fig. 7. We assume that the trend lines described by Eqs. 24, 25 and 26 with respect to depth $D$ for RPM, Torque and Bit diameters represent their optimum values.

$$\text{RPM}_{\text{opt}} = 574.53D_0^{-0.351}$$  \hspace{1cm} (24)

The coefficients in Eqs. 24, 25 and 26 are based on the actual collected data and we seek to find the optimum RPM as a function of Torque and Bit diameter. The depth values in Eqs. 25 and 26 are given by

$$D_1 = \frac{10.119 - Tq_{\text{opt}}}{0.0009}$$  \hspace{1cm} (27)

$$D_2 = \left( \frac{Bd_{\text{opt}}}{225.56} \right)^{-2.46}$$  \hspace{1cm} (28)

Connecting Eqs. 24, 27 and 28 by using the depth conversion $D_0 = (D_1 + D_2)/2$ gives

$$\text{RPM}_{\text{opt}} = 574.53D_0^{-0.351}$$  \hspace{1cm} (29)

and

$$\text{RPM}_{\text{opt}} = 574.53\left[5621.6 + 555.55Tq + (0.0022167Bd)^{-2.46}\right]^{-0.351}$$  \hspace{1cm} (30)

The averaged depth $D_0$ represents a choice of adding the same weight to the selectable bit diameter and the Torque parameter which mainly is controlled by the drilled formation. Equation 31 can be approximated further by fitting a trend line by trial and error to get a relation between optimum RPM with torque $Tq$ and bit diameter $Bd$:

$$\text{RPM} = Tq + Bd^{1.4}$$  \hspace{1cm} (31)

The new equation is easy to implement and contains only two variables, torque and bit diameter to the power of 1.4. Equation 31 matched all the drilled sections in 23 drilled wells in the 3 different oil blocks.

### Table 2

Tooth wears parameters recommended by Bourgoyne and Young (Bourgoyne et al. 1986)

| Bit class | $H_1$ | $H_2$ | $H_3$ | $(w/d)_m$ |
|-----------|-------|-------|-------|------------|
| 1-1 to 1-2 | 1.9   | 7.0   | 1.0   | 7.0        |
| 1-3 to 1-4 | 1.8   | 6.0   | 0.8   | 8.0        |
| 2-1 to 2-2 | 1.8   | 5.0   | 0.6   | 8.5        |
| 2 to 3     | 1.76  | 4.0   | 0.48  | 9.0        |
| 3 to 1     | 1.7   | 3.0   | 0.36  | 10.0       |
| 3 to 2     | 1.65  | 2.0   | 0.26  | 10.0       |
| 3 to 3     | 1.6   | 2.0   | 0.20  | 10.0       |
| 4 to 1     | 1.50  | 2.0   | 0.18  | 10.0       |
| Insert     | 1.5   | 1.0   | 0.02  | See Table 3 below |

### Table 3

Maximum recommended WOB by Estes (Eren and Ozbayoglu 2010)

| Bit size inch | Bit class MT | Bit class TCI |
|---------------|--------------|---------------|
| 5 | 6 | 7 | 8 | 9 |
| 1-1 | 1-2 | 1-3 | 1-4 | 2-1 to 2-2 | 2-3 | 3 | 4 |
| 1-1 | 1-2 | 1-3 | 1-4 | 2-1 to 2-2 | 2-3 | 3 | 4 |
| 6 1/8 | 5.6 | 6 | 6.6 | 6.9 | 7.9 |
| 6 1/4 | 5.7 | 6.1 | 6.7 | 7 | 7.1 | 7.2 | 8.5 |
| 7 7/8 | 6 | 6.2 | 6.6 | 7 | 7.5 | 7.6 | 8.7 | 9.4 |
| 8 3/4 | 6.2 | 6.5 | 6.8 | 7.2 | 7.8 | 8 | 9.5 | 10 |
| 9 7/8 | 6.5 | 6.7 | 7.1 | 7 | 7.6 | 7.7 | 8.9 |
| 10 5/8 | 6 | 6.4 | 7 | 8.8 |
| 12 1/4 | 5.9 | 6.1 | 6.7 | 7.3 | 7.4 | 8.5 |
| 14 3/4 | 5.3 | 5.8 | 6.3 | 7 | 7.4 |
| 17 1/2 | 5 | 5.7 | 7 | 3 | 4.2 | 4.2 | 4.8 | 3.8 |

$Tq_{\text{opt}} = -0.0009D_1 + 10.119$  \hspace{1cm} (25)

$Bd_{\text{opt}} = 225.56D_2^{-0.406}$  \hspace{1cm} (26)

The coefficients in Eqs. 24, 25 and 26 are based on the actual collected data and we seek to find the optimum RPM as a function of Torque and Bit diameter. The depth values in Eqs. 25 and 26 are given by

$D_1 = \frac{10.119 - Tq_{\text{opt}}}{0.0009}$  \hspace{1cm} (27)

$D_2 = \left( \frac{Bd_{\text{opt}}}{225.56} \right)^{-2.46}$  \hspace{1cm} (28)

Connecting Eqs. 24, 27 and 28 by using the depth conversion $D_0 = (D_1 + D_2)/2$ gives

$\text{RPM}_{\text{opt}} = 574.53D_0^{-0.351}$  \hspace{1cm} (29)

and

$\text{RPM}_{\text{opt}} = 574.53\left[5621.6 + 555.55Tq + (0.0022167Bd)^{-2.46}\right]^{-0.351}$  \hspace{1cm} (30)
Results and discussion

The ROP optimization leads to the cost reduction, together with elimination of hole problems. It has been reported that drilling optimization should be based on the accumulated and statistically processed empirical data rather than working with implicit relations. All drilling parameters effecting ROP are not fully comprehended and there are difficulties to gather them in one model. For that reason, accurate mathematical model for ROP process has not so far been achieved. Many optimizations models have been considered and lead to reductions in drilling cost and decreasing operations problems. The BYM is one of the most and widely used model that has been implemented successfully among those models. BYM is one of the most important drilling optimization models because it is based on statistical synthesis of the past drilling parameters. This model is considered to be the most suitable method in drilling optimization, and it is considered as one of the complete drilling models in use of the industry for roller-cone type of bits. Through the BYM and the collected data, the optimum WOB, \( N \) and ROP were estimated.

Computer network kept the data flow directly from the different data source to the MLU on site. The multiple regression technique is used to linearly model the relationship between the dependent ROP and the eight independent parameters. Through the analysis of the drilling parameters in each cluster, a relation between the drilling parameters and ROP trend or depth were determined separately also. Many researchers worked on BYM in the past decades. Based on the scope and directions of the researches on BYM, it is possible to divide the researchers into three groups:

First group applied BYM as it is together with data limitation in different locations looking to make it suitable for specific area neglecting difference in conditions comparing to the original model condition. As a result, it has less contribution to model development (Al-Betairi et al. 1988; Bataee et al. 2010; Irawan and Anwar 2014; Kutas et al. 2015; Seifabad and Ehteshami 2013). The second group used to apply the BYM with different mathematical methods to eliminate and overcome data limitation constrain (Bahari and Baradaran Seyed 2007; Bahari et al. 2011; Nejati and Vosoughi). Some methods proved its success and others show its limitations. (Genetic a logarithm method is best of alternative method as it gives realistic and within arrange coefficients.) This paper will be under the work of third group. The third group has the best contributions to develop the BYM through considering additional drilling parameters. Modifying and reducing some drilling parameters together with high accuracy made the BYM applicable in a wider range (Bourgoyne et al. 1986; Eren and Ozbayoglu 2011; Osgouei and Ozbayoglu 2007).
Regardless of dividing the researchers, it is observed that all researches looking for alternative solution to overcome data limitation and neglecting to look for rooms of improvement for multiple regression techniques. Most of researches applications used only tri-cone bits as original proposed model and there is a lack of verifications as it is significantly depending on statistical hypothesis tests together with absence of numerical simulation applications.

In this paper, the original BYM was used in the upperpart of the well down to 470 m where the roller cone bit was used in the drilling operation without any need to weight and bit adjustment. The same model was used for the lower part after adjusting the model for the weight and bit type. The model has been successfully implemented on each of 20 clusters. Working on a small cluster was helpful in overcoming the effect heterogeny and the averaging process to eliminate the effect of noisy data. Some of the coefficients

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**Table 5** Proposed drilling sections in more harmonic situations

| Formations    | Lithology Interval (m) | Sections drilled in Bn-1 | Cluster no. | Proposed drilling sections |
|---------------|------------------------|--------------------------|-------------|---------------------------|
| Pila Spi      | 9.2–137.5              | Surface                  | 1, 2, 3 and 4 | Surface                   |
| Gercus        | 137.5–246              |                          |             |                           |
| Khurmala      | 246–330                |                          |             |                           |
| Sinjar        | 330–440                |                          |             |                           |
| Kolosh        | 440–456, 456–710       | Intermediate             | 5, 6,7,8, 9, and 10 | Intermediate |
| Aaliji        | 710–1050, 1050–1673    | Production               |             |                           |
| Upper Shiranish | 1673–1839, 1839–1943  | Liner                    | 11,12, 13 and 14 | Production |
| Lower Shiranish | 1943–2279              |                          |             |                           |
| Kometan       | 2279–2376, 2379–3635   | Liner                    | 15,16,17,18,19, and 20 | Liner      |
| Qamchuqa/Sarmord | 2376–3635            |                          |             |                           |
| Qamchuqa/Sarmord | 3635–3795            |                          |             |                           |

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**Fig. 7** Relationship between variables RPM, WOB, and bit diameter with depth

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showed negative values in some specific intervals. Those negative values are clear indications that the effect of those parameters was insignificant.

Then, the original model was implemented directly to find the optimal WOB, while the adjusted model was used for clusters below 470 m. The new model (Eq. 31) was used to find the optimal RPM for the entire drilled well. This new model was implemented successfully on 23 drilled wells in the three oil blocks (Bazian, Taq Taq and Miran). The recorded and the predicted optimal values of ROP are shown in Fig. 8 and clearly indicate that the ROP has been optimized.

Optimized values that resulted from the work will help in performing drilling operation in the future. "Appendix 1" lists all the optimum predicted values of WOB, RPM, and ROP. The effects of all eight coefficients $a_1$ to $a_8$ are shown in Fig. 9 for the total drilled depth in Bn-1.

The effects of coefficients $a_1$ to $a_8$ have been summarized in Table 4 in order to provide a qualitative description and relation to lithologies. For example, from Fig. 9 and Table 4, it is observed that the drillability $a_1$ in cluster 10 was low compared to cluster 11 to indicate the hardness of Qamchuqa formation. The effect of compaction was high in cluster 10 compared to cluster 11 due to the different pressure regime between these two clusters. The effect of pressure differences was high in clusters 7 and 8 and this was due to the use of high-density fluid that was used to control the swelling and other hole problems caused by shale in clusters 7 and 8. Upper clusters 1, 2, 3, 4 and lower clusters 10 and 11 drilled were with underbalanced drilling fluid due to presence of fractures and cavities. In general, there was a good control on parameters RPM, teeth wear and hydraulic effects ($a_6$, $a_7$, and $a_8$) as they were positive range in most cases.

The efficiency of the clustering method is more visible through the drawing of the relationship between the depth and log (ROP) as shown in Fig. 8. The optimized ROP values are more stable compared with the original ROP in black color. After the completion of Bn-1, and based on the lithology similarity, we can observe that:

1. There is a high similarity between clusters 1, 2, 3, 4 and it will be more productive if they are drilled as a surface section as shown in Table 5. It will be easier if the clusters 5 to 10 are drilled as the intermediate section due to lithology similarity.
2. Production section can extend down to the top of the Kometan formation.

**Conclusions**

The new model to calculate the RPM is much easier than the complex models used in previous studies.

Optimization is a continuous process and the operation will be improved with the increase in the number of drilled in the specific area.

The BYM can be implemented successfully for PDC bits like roller cone bits after adjustments. A new procedure successfully introduced to find the threshold weight through instead of drill of test.

Noisy data and homogeneity assumptions eliminated through the averaging and clustering. Controllable parameters WOB, RPM, and ROP have been optimized for future operations.
Fig. 9  Range values of coefficient constant $a_1$ to $a_8$
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Glossary

| Symbol | Unit | Description |
|--------|------|-------------|
| \(a_1\) | Dimension less | Formation strength parameter coefficient |
| \(a_2\) | Dimension less | Normal compaction coefficient |
| \(a_3\) | Dimension less | Under-compaction coefficient |
| \(a_4\) | Dimension less | Pressure differential coefficient |
| \(a_5\) | Dimension less | Bit weight coefficient |
| \(a_6\) | Dimension less | Rotary speed coefficient |
| \(a_7\) | Dimension less | Tooth wear coefficient |
| \(a_8\) | Dimension less | Hydraulic coefficient |
| \(C_b\) | USD | Bit cost |
| \(C_r\) | USD | Daily rig rate |
| \(D\) | [L], ft (m) | Depth of borehole |
| \(d_b\) | [L], in (mm) | Diameter of the bit |
| \(d_n\) | [L], in (mm) | Bit nozzle diameter |
| \(f_1\) | ft/h | Rate of penetration considering formation strength function |
| \(f_2\) | ft/h | Rate of penetration considering formation normal compaction function |
| \(f_3\) | ft/h | Rate of penetration considering formation compaction function |
| \(f_4\) | ft/h | Rate of penetration considering pressure differential of hole bottom function |
| \(g_p\) | [M/L^3], ppg (sg) | Pore pressure gradient of the formation |
| \(h\) | Dimension less | Bit tooth dullness, fractional tooth height worn away |
| \(H_1, H_2\) | Dimension less | Constants for tooth geometry of bit types |
| \(N\) | [T-1], rpm (–) | Rotary speed |
| \(Q, q\) | [L^3/T], gpm (l/m) | Volumetric flow rate |
| \(ROP\) | [L/T], ft/h (m/h) | Rate of penetration |
| \(t_b\) | [T], h | Bit drilling time |
| \(t_c\) | [T], h | Drill pipe connection time |
| \(t_t\) | [T], h | Round trip time |
| \(WOB\) | [ML/T^2], 1000 lbf (N) | Weight on bit |
| \(w/d\) | [M/T^2], 1000 lbf/in. (N/m) | Weight on bit per inch of bit diameter |
| \((w/d)_{max}\) | [M/T^2], 1000 lbf/in. (N/m) | Bit weight per diameter where teeth fails instantaneously |
| \((w/d)_t\) | [M/T^2], 1000 lbf/in. (N/m) | Threshold bit weight at which the bit starts to drill |
| \(\rho\) | [M/L^3], ppg (kg/m^3) | Drilling fluid’s density |
| \(ECD\) | [M/L^3], ppg (sg) | Equivalent circulating mud density at the hole bottom |
| \(\mu\) | [M/LT], cp (Pa s) | Apparent viscosity at 10,000 s^-1 |
| BYM | Dimension less | Bourgoyne and Young |
| IADC | Dimension less | International association of drilling contractors |
| MD | [L], ft (m) | Measured depth |
| MW | [M/L^3], ppg (sg) | Mud weight (density) |
| RPM | [T^-1], rpm (–) | Revolution per minute |
| SPP | [M/(LT^2)], psi (Pa) | Standpipe pressure |
| TD | [L], ft (m) | Total depth |
| TVD | [L], ft (m) | True vertical depth |

Appendix 1

See Table 6.
Appendix 2: Data processing steps

Tables 7 and 8 show examples of the two processing steps involved. The first step involves averaging in 10-meter intervals for noise removal. The second step is the calculations of the eight parameters of the model.
Table 7  Data collection, preparation and 10 m averaging

| Depth (m) | Depth (ft) | WOB (tons) | WOB (AV 10 m) | RPM (AV 10) | Torque (AV 10) | Flow (AV 10) | Flow (AV 10) | Mw in (ppg) | Dh (in.) | Bit no. type | Veloc-ity (AV 10) | Pan ECD | tfb (psi) | HP Ab | Av. bit area (sq.in) | 1000 lb/in. (w/d) | 1000 lb/in. (h) |
|-----------|------------|------------|----------------|-------------|---------------|--------------|--------------|--------------|-----------|-------------|----------------|---------|-----------|-------|---------------------|-----------------|-----------------|
| 10        | 32.81      | 2.6        | 5.7            | 6.6         | 90            | 98           | 8.9          | 29.0         | 142.0     | 12.8        | 396.2         | 365.3   | 8.5       | 26.00 | 1-TCI              | 8.5003          | 0.003           |
| 11        | 36.09      | 3.3        | 7.3            | 6.7         | 78            | 101          | 82.2         | 269.6        | 139.5     | 8.6         | 338.2         | 364.8   | 8.5       | 26.00 | 1-TCI              | 8.5002          | 0.004           |
| 12        | 39.37      | 2.8        | 6.2            | 6.3         | 82            | 102          | 91.6         | 300.3        | 112.6     | 7.3         | 396.2         | 371.6   | 8.5       | 26.00 | 1-TCI              | 8.5003          | 0.005           |
| 13        | 42.65      | 2.0        | 4.4            | 6.3         | 88            | 104          | 65.3         | 214.0        | 82.7      | 10.6        | 396.2         | 365.8   | 8.5       | 26.00 | 1-TCI              | 8.5003          | 0.006           |
| 14        | 45.93      | 2.7        | 6.0            | 7.0         | 91            | 106          | 25.0         | 82.0         | 61.5      | 10.5        | 396.2         | 422.8   | 8.5       | 26.00 | 1-TCI              | 8.5002          | 0.004           |
| 15        | 49.21      | 2.0        | 4.3            | 7.2         | 110           | 109          | 10.5         | 34.3         | 112.6     | 7.3         | 396.2         | 365.8   | 8.5       | 26.00 | 1-TCI              | 8.5002          | 0.005           |
| 16        | 52.49      | 3.0        | 6.6            | 7.1         | 111           | 109          | 80.0         | 262.4        | 50.2      | 10.7        | 396.2         | 478.1   | 8.5       | 26.00 | 1-TCI              | 8.5002          | 0.006           |
| 17        | 55.77      | 3.1        | 6.7            | 7.0         | 111           | 109          | 49.2         | 161.3        | 24.1      | 16.5        | 396.2         | 478.1   | 8.5       | 26.00 | 1-TCI              | 8.5002          | 0.007           |
| 18        | 59.06      | 3.8        | 6.8            | 6.8         | 112           | 109          | 17.4         | 57.0         | 10.5      | 25.1        | 333.4         | 621.4   | 8.5       | 26.00 | 1-TCI              | 8.5002          | 0.006           |
| 19        | 62.34      | 1.7        | 3.7            | 6.0         | 111           | 109          | 2.9          | 9.6          | 7.2       | 3.8         | 396.2         | 662.2   | 8.5       | 26.00 | 1-TCI              | 8.5003          | 0.008           |
| 20        | 65.62      | 3.1        | 6.8            | 6.3         | 112           | 109          | 1.3          | 4.3          | 7.2       | 6.0         | 391.4         | 693.9   | 8.5       | 26.00 | 1-TCI              | 8.5003          | 0.009           |
| 21        | 68.90      | 1.6        | 3.5            | 5.9         | 91            | 110          | 0.3          | 1.1          | 7.1       | 7.6         | 405.9         | 723.0   | 8.5       | 26.00 | 1-TCI              | 8.5003          | 0.010           |
| 22        | 72.18      | 3.0        | 6.5            | 6.3         | 101           | 113          | 0.2          | 0.8          | 8.5       | 9.3         | 338.2         | 745.8   | 8.5       | 26.00 | 1-TCI              | 8.5002          | 0.007           |
| 23        | 75.46      | 4.8        | 10.6           | 5.9         | 113           | 114          | 0.7          | 2.3          | 9.2       | 6.9         | 966.4         | 757.0   | 8.5       | 26.00 | 1-TCI              | 8.5015          | 0.006           |
| 24        | 78.74      | 4.0        | 8.8            | 5.4         | 114           | 114          | 0.4          | 1.2          | 10.3      | 19.8        | 891.3         | 708.7   | 8.5       | 26.00 | 1-TCI              | 8.5013          | 0.005           |
| 25        | 82.02      | 1.4        | 3.1            | 4.7         | 113           | 115          | 0.56         | 1.8          | 11.4      | 11.0        | 918.1         | 664.2   | 8.5       | 26.00 | 1-TCI              | 8.5014          | 0.008           |
| 26        | 85.30      | 2.4        | 5.3            | 4.6         | 114           | 115          | 0.55         | 1.8          | 12.4      | 5.0         | 918.1         | 609.9   | 8.5       | 26.00 | 1-TCI              | 8.5014          | 0.006           |
| 27        | 88.58      | 2.2        | 4.8            | 4.3         | 112           | 115          | 7.47         | 24.5         | 133       | 26.0        | 655.0         | 545.0   | 8.5       | 26.00 | 1-TCI              | 8.5007          | 0.003           |
| 28        | 91.86      | 3.1        | 6.7            | 4.0         | 114           | 116          | 7.47         | 24.5         | 120       | 20.6        | 740.9         | 507.4   | 8.5       | 26.00 | 1-TCI              | 8.5009          | 0.004           |
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| Depth (m) | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | LN ROP (ft/h) |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|---------------|
| 10       | 1.0   | 12,467| −44   | −0.01 | −3.19 | −0.02 | −0.0003| −2.1460| 5.0            |
| 11       | 1.0   | 12,464| −47   | −0.01 | −3.16 | 0.01  | −0.0006| −2.3068| 4.9            |
| 12       | 1.0   | 12,461| −50   | −0.01 | −3.26 | 0.02  | −0.0009| −2.1119| 4.7            |
| 13       | 1.0   | 12,457| −53   | −0.01 | −3.25 | 0.04  | −0.0012| −2.1433| 4.4            |
| 14       | 1.0   | 12,454| −56   | −0.01 | −3.10 | 0.06  | −0.0015| −2.0117| 4.1            |
| 15       | 1.0   | 12,451| −59   | −0.01 | −3.04 | 0.08  | −0.0018| −1.7949| 4.0            |
| 16       | 1.0   | 12,448| −62   | −0.01 | −3.07 | 0.09  | −0.0021| −1.4778| 3.9            |
| 17       | 1.0   | 12,444| −64   | −0.01 | −3.10 | 0.09  | −0.0024| −1.2394| 3.2            |
| 18       | 1.0   | 12,441| −67   | −0.01 | −3.14 | 0.09  | −0.0027| −1.2559| 2.3            |
| 19       | 1.0   | 12,438| −69   | −0.02 | −3.34 | 0.09  | −0.0030| −0.9563| 2.0            |
| 20       | 1.0   | 12,434| −72   | −0.02 | −3.25 | 0.10  | −0.0033| −0.8748| 2.0            |
| 21       | 1.0   | 12,431| −74   | −0.02 | −3.37 | 0.10  | −0.0036| −0.7564| 2.0            |
| 22       | 1.0   | 12,428| −77   | −0.01 | −3.25 | 0.12  | −0.0039| −0.8767| 2.1            |
| 23       | 1.0   | 12,425| −79   | −0.12 | −3.38 | 0.13  | −0.0042| 0.2033 | 2.2            |
| 24       | 1.0   | 12,421| −81   | −0.10 | −3.51 | 0.13  | −0.0045| −0.0096| 2.3            |
| 25       | 1.0   | 12,418| −84   | −0.11 | −3.78 | 0.14  | −0.0048| −0.1098| 2.4            |
| 26       | 1.0   | 12,415| −86   | −0.12 | −3.84 | 0.14  | −0.0051| −0.2802| 2.5            |
| 27       | 1.0   | 12,411| −88   | −0.06 | −3.99 | 0.14  | −0.0054| −0.8431| 2.6            |
| 28       | 1.0   | 12,408| −90   | −0.08 | −4.14 | 0.14  | −0.0057| −0.8628| 2.5            |
| 29       | 1.0   | 12,405| −93   | −0.08 | −4.28 | 0.14  | −0.0060| −1.0309| 2.4            |
| 30       | 1.0   | 12,402| −95   | −0.08 | −4.70 | 0.14  | −0.0063| −1.2199| 2.4            |
| 31       | 1.0   | 12,398| −97   | −0.07 | −4.63 | 0.14  | −0.0066| −1.4551| 2.5            |
| 32       | 1.0   | 12,395| −103  | −0.03 | −5.42 | 0.14  | −0.0069| −1.9654| 2.4            |
| 33       | 1.0   | 12,392| −106  | −0.04 | −5.34 | 0.14  | −0.0072| −1.9686| 2.4            |
| 34       | 1.0   | 12,388| −108  | −0.04 | −7.69 | 0.14  | −0.0075| −2.1708| 2.3            |
| 35       | 1.0   | 12,385| −110  | −0.03 | −7.10 | 0.14  | −0.0078| −2.4438| 2.3            |
| 36       | 1.0   | 12,382| −112  | −0.01 | −6.64 | 0.14  | −0.0081| −2.8336| 2.2            |
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