FRIGA, A New Approach To Identify Isotopes and Hypernuclei In N-Body Transport Models.

A. Le Fèvre¹, Y. Leifels¹, J. Aichelin², Ch. Hartnack², V. Kireyev³ and E. Bratkovskaya⁴

¹GSI Helmholtzzentrum für Schwerionenforschung, Darmstadt, Germany; ²Subatech Nantes, France; ³JINR Dubna, Russia; ⁴FIAS and ITP Frankfurt, Germany.
E-mail: A.LeFevre@gsi.de

Abstract. We present a new algorithm to identify fragments in computer simulations of relativistic heavy ion collisions. It is based on the simulated annealing technique and can be applied to n-body transport models like the Quantum Molecular Dynamics. This new approach is able to predict isotope yields as well as hyper-nucleus production. In order to illustrate its predicting power, we confront this new method to experimental data, and show the sensitivity on the parameters which govern the cluster formation.

Introduction

In heavy ion reactions at energies between 20 A.MeV and several A.GeV, many clusters are formed. This cluster formation presents a big challenge for transport models in which nucleons are the degrees of freedom which are propagated. Identifying clusters in a transport code which transports nucleons is all but simple and therefore in many approaches the fragment formation is simply omitted. This invalidates the prediction of single particle observables as well, because the cluster formation – and therefore the modification of the single particle spectra due to the fragment formation – depends on the phase space region and, as a consequence, cannot be approximated by a momentum independent scaling factor.

The simplest way to identify clusters is by employing coalescence or a minimum spanning tree procedure. The first needs a multitude of free parameters, whereas the second allows only for an identification at the end of the reaction which excludes any study on the physical origin [1]. In addition, quantum effects, like additional binding energies due to closed shells or pairing energies, are not supplied by the underlying transport theory which is semi-classical.

The principles of the fragment recognition.

If one wants to identify fragments early, while the reaction is still going on, one has to use the momentum as well as the coordinate space informations. An idea how to do this has been launched by Dorso et al. [2]. It has been further developed into the Simulated Annealing Clusterisation Algorithm (SACA) [3] in the late 1990’s and has been successfully applied to understand the measured fragment charge distribution and spectra as well as bimodality [4, 5]. Starting from the positions and momenta of the nucleons at a given time during the reaction, nucleons are combined in all possible ways into fragments or single nucleons applying a simulated annealing technique. Neglecting the interaction among nucleons in different clusters, but taking
into account the interaction among the nucleons in the same fragment, this algorithm identifies that combination of fragments and free nucleons which has the highest binding energy. If applied after the time when the energetic initial collisions are over, this most bound configuration has been proven to be close to the final distribution of fragments identified by the minimum spanning tree method at the end of the reaction[3]. The reason for this is the fact that fragments are not a random collection of nucleons at the end, but an initial-final state correlation. SACA can be applied at any moment during the reaction and allows therefore for a detailed study of the fragment production mechanism.

In SACA, for accounting the interaction in-between nucleons, only the bulk Skyrme interaction supplemented by a Yukawa potential is used – which is also the potential used for the propagation of the nucleons in the QMD transport model. To obtain more realistic fragment observables and to be able to predict observables for isotopes and hyper-nuclei, we employ in our new approach a more realistic interaction and add the secondary decay because the fragments, when identified, have a (moderate) excitation energy. This new approach is dubbed FRIGA ("Fragment Recognition In General Application").

The new features of FRIGA.

In order to predict the absolute multiplicity of the isotope yields, we have added new features to the SACA cluster identification. They include the asymmetry energy, pairing and quantum effects.

For the asymmetry energy, we adopt the parametrisation from IQMD [8], a transport code which we use in the present article for the transport of nucleons. For a proton the single particle energy thus reads:

$$B_{asy} = E_0 \left( \frac{\langle \rho_B \rangle}{\rho_0} \right)^{\gamma-1} \left( \rho_n - \rho_p \right)$$

where $E_0$=23.3 MeV, and $\rho_n$, $\rho_p$, $\rho_B$, $\rho_0$ are the neutron, proton, baryonic and saturation densities, respectively. In the present work, we take $\gamma$=1 ("stiff" asymmetry potential).

Another significant part of the binding energy of light isotopes are the shell structure and odd-even effects (pairing). In the conditions of high pressure and temperature where FRIGA is used to determine the pre-fragments, these structure effects are not well known. E. Khan et al. [9] showed that there are some indications that they affect the primary fragments. The authors demonstrate that the pairing vanishes above a nuclear temperature $T_V \approx 0.5 \Delta_{pairing}$ (pairing energy). At normal density the pairing energy tends to be negligible for heavy nuclei, with $\Delta_{pairing} = \frac{12}{\sqrt{A}} MeV$, whereas it is strong for light isotopes, like $^4He$ and $^3He$ with 12 MeV and 6.9
MeV, respectively. In FRIGA, the primary fragments are usually produced slightly below the saturation density (typically around half of it) and quite cold, with $T < 1 - 2 MeV$, and hence below $T_V$. Therefore, one cannot neglect the pairing energy. The same is true for shell effects which produce experimentally a visible enhancement of the fragment yield for closed shell nuclei.

In order to determine the contribution of all structure effects to the binding energy of clusters, we make two hypotheses independent of the density and the average kinetic energy of the fragment environment.

First, the relative ratio of this nuclear structure contribution to the overall binding energy remains unchanged at the moderate temperatures and at the density at which clusters are formed which is not far away from the saturation density.

Introducing among the nucleons, initialised with the right root-mean-square radius, two body interactions, which corresponds in infinite matter to the Skyrme equation of state, the total fragment energy

$$E_B(N, Z) = \langle H \rangle = \langle T \rangle + \langle V \rangle$$

$$= \sum_i \frac{p_i^2}{2m_i} + \sum_{i>j} \int f_i(r, p, t) V(r, r', p, p') f_j(r', p', t) \, dr \, dr' \, dp \, dp' \quad (1)$$

where $f_i$ is the single-particle Wigner density

$$f_i(r, p, t) = \frac{1}{\pi^3 \hbar^2} e^{-\frac{2}{\hbar^2} (r-r_i(t))^2} e^{-\frac{2}{\hbar^2} (p-p_i(t))^2} \quad (2)$$

reproduces very well the nuclear binding energy given by the Weizsäcker mass formula for ground state nuclei, $B_0$ [6]. (Fig.12).

Our second hypothesis is that eq. 2 remains the right description of the binding energy if the nuclei are deformed or excited when the fragments are identified by the FRIGA algorithm.

Taken both assumptions together, we can express the nuclear structure contribution to the binding energy of a deformed cluster with $Z$ protons and $N$ neutrons in the following way:

$$B_{\text{struct}} = E_B(N, Z) \frac{B_{\text{exp}}(Z, N) - B_{\text{BW}}(Z, N)}{B_{\text{BW}}(Z, N)}$$

where $B_{\text{exp}}$ and $B_{\text{BW}}$ are the experimentally measured binding energy (which contains the structure contribution and the sum of the volume and surface terms of the Bethe-Weizsäcker formula). (Hyper-)Isotopes which are not stable at all in nature, are discarded in FRIGA by assigning to them a very repulsive $B_{\text{struct}}$. The total binding energy of a cluster with $N$ and $Z$, which is used in the annealing algorithm, will then be:

$$B = E_B(N, Z) + B_{\text{asy}} + B_{\text{struct}}.$$

in contradistinction to SACA in which only the first term is used.

Fig. 1 shows the influence of the asymmetry energy and of the structure energy on the isotope yield in the reaction $^{124}Xe + ^{112}Sn$ at 100 A.MeV. We display here the results for central collisions ($b < 0.2 \, b_{\text{max}}$). This figure illustrates as well how the various ingredients influence the fragments yield obtained in FRIGA, assuming an early clusterisation at $t=60 \, \text{fm/c}$. From that time on, the size of the pre-fragments does not change anymore. We see that the charge distributions are not strongly modified for the different options, whereas details of the isotopic yield are strongly influenced: the asymmetry energy tends to narrow the distributions towards the valley of stability, whereas the structure effects contribute to restore the natural abundances, particularly strong for the $^4He$ clusters.
Excitation energy and density of the primary fragments.
The pre-fragments, called also “primary” fragments, created in FRIGA, are often produced non
relaxed in shape and density. When turning to their ground state, the shape surface energy is
converted into excitation energy. Using QMD simulations, for beam energies between 50 A.MeV
- 1 A.GeV, FRIGA obtains for central heavy ion collisions a mean excitation energy of the
intermediate mass fragments between 0 and 3 A.MeV, depending on the fragment size and very
similarly to the experimental measurements of [12]. This excitation energy is sufficiently large
that the secondary decay of the pre-fragments causes a significant contribution to the yield of
small clusters. For this reason, we optionally allow in FRIGA the excited cluster to undergo
sequential secondary decays, using the GEMINI algorithm [13].

Another interesting feature of the primary clusters in FRIGA is their internal density. Although
the medium is close to \( \rho_0 \), at the stage of the collision when the primary cluster
formation is stabilised, just after the colliding system begins to separate, the fragments predicted
by FRIGA are produced quite dilute, typically around \( \rho \sim \rho_0/2 \) for intermediate mass fragments,
and around \( \rho \sim \rho_0/5 \) for the light \( Z < 3 \) isotopes. This is explained by the fact that the dense
clusters are disfavoured, because they would contain nucleons which are moving against each
other. In this case the nucleons have a too high relative momenta to form a cluster. Therefore, in
the FRIGA approach, fragment formation tests only the low density behaviour of the potentials,
which are contributing to the binding energy.

Another application of FRIGA: the hypernucleus formation.
An hyper-nucleus is a nucleus which contains
at least one hyperon \((\Lambda(uds), ...)\) in addition
to nucleons. Extending FRIGA to the strange sector requires the knowledge of the \( \Lambda N \)
potential. In this first study, we consider the strange quark as inert and use \( V_{\Lambda N} = \frac{4}{9} V_{nN} \) for
protons as well as for neutrons. Similarly, we
consider the case of multiple strange nuclei as
well, in which more than one hyperon is part
of the fragment. There, the coupling of 2 \( \Lambda \)'s contributes with the potential \( V_{\Lambda \Lambda} = (\frac{4}{9})^2 V_{nN} \).
In the present approach we neglect \( B_{asy} \) for
the hyperons, and take the contribution of the
core nucleus (partner of the hyperons) as if it
were decoupled from the hyperon. Since the
pairing and shell contributions in the binding
energy are not yet well known for hyper-nuclei,
we neglect the \( B_{struct} \) contribution.

Using these modifications of the potentials,
FRIGA produces hyper-nuclei with the same
procedures as non strange fragments. In the
underlying QMD-like programs, which
propagate the hadrons, \( \Lambda \)'s are produced in
different reactions: \( K + N \rightarrow \Lambda + \pi, \pi + n \rightarrow \Lambda + K^+, \pi^- + p \rightarrow \Lambda + K_0, p + p \rightarrow \Lambda + X \). Their abundance, position and momentum
distributions are strongly influenced by the
reaction kinematics, the nuclear equation of state and the in-medium properties of the \( K^+ \)
(kaon potential, etc.) which are implemented in the transport model [14].

Figure 2. Predictions of FRIGA from PHSD [7]
simulations of Au+Au collisions at 11.45 A.GeV
incident energy, \( b=6 \) fm. It shows the multiplicity
per event as a function of the rapidity in the centre
of the collision scaled to the projectile rapidity, for
various (hyper-)clusters (\( A_\Lambda \) are hyper-clusters with
\( A \geq 4 \)) and clusterisation times. \( t_{pass} = 7.5 \) fm/c
is the passing time of the projectile and target in
central collisions. Like in the following figures, the
shaded areas depict the statistical uncertainties.
Due to their composition, the yields of hyper-nuclei are produced when a cluster in coordinate and momentum space absorbs a hyperon. In heavy ions collisions at relativistic energies, the hyperon distributions are strongly peaked around the mid-rapidity region whereas the large fragments have rapidities close to the beam or target rapidity. The closer the rapidity of the hyperon approaches – by production or by subsequent collisions – the target/beam rapidity, the larger is the probability that it can be absorbed by one of this larger clusters. Heavy hyper-nuclei are therefore observed not far away from beam/target rapidity. At the same time hyperons can also form with other nucleons light clusters at mid-rapidity. There, the probability decreases with the cluster size because it is increasingly difficult to form large cluster out of a gas of nucleons. Whereas the large clusters in the beam/target rapidity regime can be identified quite early, the light clusters at mid-rapidity are formed later and many of them decay due to the interactions with the surrounding nucleons which form a gas of a large temperature as compared to the cluster binding energy. This is illustrated in Fig. 2.

As seen previously, in Fig. 1, the ingredients of the cluster binding energy influence the light isotope yields in FRIGA. The same is observed for hypernuclei. Adopting the factor $\frac{2}{3}$ in $V_{AN}$ has a strong effect, decreasing on the average the hypernucleus yields by around 20 percent. The asymmetry energy in the cluster can have a similar effect, depending on the isotope (Z,N) asymmetry.

In order to illustrate the predicting power of the FRIGA algorithm, we confront it to experimental observations of light hypernuclei produced in the spectator region in collisions of the light system $^6$Li + $^{12}$C at 2 A.GeV incident energy, measured by the HyPHI Collaboration at the SIS synchrotron of GSI Darmstadt. The data presented here are taken from [15]. Fig. 3 compares the IQMD-FRIGA predictions for the rapidity distributions of $^3_1H$ and $^4_1H$ with the experiment. The best agreement in the experimentally resolved rapidity region (close to the projectile spectator, $y/y_{beam} > 0.7$) has been obtained while excluding the most central collisions (taking $b>3$ fm). This procedure is a very basic approach to the simulate the effect of

![Figure 3](image1.png)

**Figure 3.** Predictions of FRIGA (clustering at $2t_{pass}$, binding energy excluding $B_{asy}$) from IQMD simulations of $^6$Li + $^{12}$C collisions at 2 A.GeV incident energy, $b > 3$ fm compared to the HyPHI experimental data. The results of the model calculations are not filtered for the experimental acceptance. It shows the multiplicity per event per unit of rapidity, as a function of the rapidity in the centre of the collision scaled to the projectile rapidity, for all clusters (in proton-like weighting), $^\Lambda s$, $^3_1H$ and $^4_1H$. Markers are experimental data.

![Figure 4](image2.png)

**Figure 4.** Same as Fig. 3, showing the transverse momentum multiplicity distributions in the projectile spectator region ($y_0 > 0.9$).
the complex experimental trigger. The chosen rapidity region has the highest hadronic yield and
contains still the tail of the $\Lambda$ distribution, as predicted by IQMD-FRIGA. At these rapidities,
the experiment has measured a yield ratio $Y(\Lambda H)/Y(\Lambda H) = 1.4\pm0.8$, with which IQMD-FRIGA
agrees within the experimental uncertainty with $1.3\pm0.2$. Including the asymmetry contribution
$B_{asy}$ in the cluster binding energy in FRIGA, we obtain a yield ratio of $1.9 \pm 0.4$ which is still
within this uncertainty. Therefore, at this level, the role of the asymmetry energy is difficult to
determine. Fig. 4 shows that the transverse momentum distributions in the spectator region agree
as well, in the slopes and the absolute yield. Just a slight discrepancy has to be noticed with
a shift of 0.1 GeV/c in $p_\perp$. Here, we present the results without $B_{asy}$ contribution, but the
conclusion is similar when including it.

Conclusion
We present here the first step towards an understanding of the production of isotopic yields and
hypernuclei in heavy ion reactions. Our clusterisation algorithm FRIGA, an improved version of
the SACA approach, which includes pairing and asymmetry energies as well as other structure
effects is able to describe more precisely the nuclear binding energy and allows for realistic
predictions of absolute (hyper-)isotope yields. We have seen that the asymmetry and pairing
potentials can have a strong influence on both, the yields and momentum anisotropies for the
(hyper-)isotopes. According to this model, the nucleons which form fragments have initially a
fairly low density. They contract and form finally slightly excited fragments which may undergo
secondary decays. Therefore, the fragment formation is sensitive to the density dependence of
the asymmetry energy and the pairing energy. However, fragments test this dependence only
for densities below the saturation density.

[1] P.B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nucl. Phys. A 619 (1997) 379-390.
[2] C. O. Dorso and J. Randrup, Phys. Lett. B 301, 328 (1993).
[3] R. K. Puri and J. Aichelin, J. Comput. Phys. 162, 245 (2000).
[4] K. Zbiri, A. Le Fèvre, J. Aichelin et al., Phys. Rev. C 75 (2007) 034612.
[5] A. Le Fèvre et al., Phys. Rev. C 80 (2009) 044615.
[6] J. Aichelin, Phys. Reports 202, 233 (1991).
[7] W. Cassing, E.L. Bratkovskaya, Nucl. Phys. A 831 (2009) 2.
[8] Ch. Hartnack et al., Eur. Phys. J. A 1 (1998) 151.
[9] E. Khan, Nguyen Van Giai, N. Sandulescu, Nucl. Phys. A 789 (2007) 94.
[10] E. Khan, M. Grasso and J. Margueron, Phys. Rev. C 80 (2009) 044328.
[11] A. Le Fèvre et al., Nucl. Phys. A 735 (2004) 219.
[12] S. Hudan et al. (INDRA collaboration), Phys. Rev. C 67 (2003) 064613.
[13] R.J. Charity et al., Nucl. Phys. A 476 (1988) 516. [14]
[14] C. Hartnack, H. Oeschler, Y. Leifels, E. L. Bratkovskaya and J. Aichelin, Phys. Rept. 510 (2012) 119
[arXiv:1106.2083 [nucl-th]].
[15] Ch. Rappold et al., Phys. Lett. B 747 (2015) 129.