Characterization and Performance of PADME’s Cherenkov-Based Small-Angle Calorimeter

A. Frankenthal*a, J. Alexandera, B. Buonomo*b, E. Capitolo*c, C. Capoccia*c, C. Cesarotti*a,b, R. De Sangro*c, C. Di Giulio*a, F. Ferrarotto*a, L. Foggetta*a, G. Georgiev*d,e, P. Gianotti*e, M. Hunyadi*e, V. Kozhuharov*d,e, A. Krasznahorkay*g, E. Leonardi*d, G. Organtini*c,d, G. Piperno*e, M. Raggi*c,d, C. Rella*d, A. Saputi*e, I. Sarra*e, E. Spiriti*e, C. Tarugi*e, P. Valente*d

*aCornell University, Ithaca, NY 14853, USA
*bHarvard University, Cambridge, MA 02138, USA
*cSapienza Università di Roma, Rome, RM 00185, Italy
*dINFN Sezione di Roma, Rome, RM 00185, Italy
*eINFN Laboratori Nazionali di Frascati, Frascati, RM 00044, Italy
*fUniversity of Sofia, Sofia, 1504, Bulgaria
*gInstitute for Nuclear Research, Hungarian Academy of Sciences, Debrecen, H-4026, Hungary

Abstract

The PADME experiment, at the Laboratori Nazionali di Frascati (LNF), in Italy, will search for invisible decays of the hypothetical dark photon via the process $e^+e^- \rightarrow \gamma A'$, where the $A'$ escapes detection. The dark photon mass range sensitivity in a first phase will be 1 to 24 MeV. We report here on measurement and simulation studies of the performance of the Small-Angle Calorimeter, a component of PADME’s detector dedicated to rejecting 2- and 3-gamma backgrounds. The crucial requirement is a timing resolution of less than 200 ps, which is satisfied by the choice of PbF$_2$ crystals and the newly released Hamamatsu R13478UV photomultiplier tubes (PMTs). We find a timing resolution of 81 ps (with double-peak separation resolution of 1.8 ns) and a single-crystal energy resolution of 5.7%/√E with light yield of 2.07 photo-electrons per MeV, using 100 to 400 MeV electrons at the Beam Test Facility of LNF. We also propose the investigation of a two-PMT solution coupled to a single PbF$_2$ crystal for higher-energy applications, which has potentially attractive features.

Keywords: PbF$_2$ Crystals, Ultra Fast Calorimeter, Dark Photon, Dark Matter, Cherenkov Detectors

1. Introduction

Despite the recent discovery of the Higgs boson at the LHC, solidifying our understanding of the standard model (SM) of particle physics, many lingering questions remain that the SM cannot yet explain. One of the prime examples is the dark matter problem – the fact that we have abundant evidence for its existence yet few clues about its composition.

One proposed solution to the dark matter problem invokes the existence of a dark sector, coupled to the SM sector by its ‘dark photon’, a vector mediator of a new abelian gauge force that kinetically mixes with the SM $U(1)_Y$. The kinetic mixing coupling can be small, allowing the dark photon to evade discovery so far.

The PADME experiment, set to start operation at the INFN Laboratori Nazionali di Frascati (LNF), in Italy, during the summer of 2018, is a novel fixed-target, missing-mass experiment to search for the dark photon via its effective coupling to ordinary SM electromagnetism [1]. The goal is to look for remnants of the interaction $e^+e^- \rightarrow \gamma A'$, where $A'$ is the dark photon.

A beam of 550 MeV positrons is produced and accelerated by a LINAC at LNF’s Double Annular Φ Factory for Nice Experiments (DAΦNE), and driven to one of two Beam Test Facility (BTF) [2] magnetic lines. The beam strikes a low-Z target, and the interaction with atomic electrons produces a photon and a dark photon, after conversion via kinetic mixing. The energy and direction of the outgoing photon are measured with an electromagnetic calorimeter (ECAL). Combined with an accurate knowledge of the positron beam, the missing mass of the system (i.e. the dark photon invariant
Figure 1: Layout of the PADME detector. On the left is the Small-Angle Calorimeter (in red), which sits behind the central hole in the main calorimeter. Other components include an electron and positron veto spectrometer, an active diamond target, and a 0.5 T magnet.

mass) can be inferred.

The ECAL has a central hole due to the large rate of Bremsstrahlung events, which are sharply peaked at small angles. In order to have a manageable rate in the ECAL and in the scintillating bars acting as veto detectors for Bremsstrahlung events, the current of the positron beam must be limited to \( \approx 20 \text{nA} \), corresponding to \( 2.5 \times 10^4 e^+ \) in 200 ns long LINAC pulses.

The proposed experiment requires excellent background rejection capability. Among the largest sources of background are 2-gamma \( (e^+ e^- \rightarrow \gamma \gamma) \) and 3-gamma \( (e^+ e^- \rightarrow \gamma \gamma \gamma) \) events, where 1 (or 2) photons escape detection via the hole in the ECAL. To mitigate such backgrounds, a very fast Small-Angle Calorimeter (SAC) is placed behind the main ECAL, flush with the central hole. The in-time correlation of photon events in the SAC and ECAL allows the tagging of 2- and 3-gamma events and hence the efficient vetoing of such backgrounds.

In this paper we evaluate the performance of the SAC with a test beam done at LNF, using fast Lead Fluoride crystals (PbF\(_2\)) and the newly developed Hamamatsu R13478UV photomultiplier tube (PMT), optimized for fast response. We demonstrate that this detector meets the requirements for an efficient rejection of 2- and 3-gamma events, namely: (a) a timing resolution less than 200 ps for Cherenkov radiation detection; (b) moderate single-crystal energy resolution better than 10% at close to beam energy; (c) moderate light yield between 0.5 and 2 photo-electrons (p.e.)/MeV; (d) double-peak separation resolution capable of distinguishing several dozen photons in a 200 ns time span; (e) radiation hardness of order 1 Gy per \( 10^{13} \) positrons on target; and (f) acceptance of low-wavelength light due to the Cherenkov spectrum \([3]\). Furthermore, we encourage the investigation of a related (but more expensive) setup which uses two compact ultra-fast PMTs (R9880U-110) coupled to a single PbF\(_2\) crystal in order to provide independent efficiency measurements and timing references, for higher-energy applications.

2. The PADME Small-Angle Calorimeter

Fig. 1 shows the general layout of the PADME detector, including the placement of the SAC behind the main calorimeter. The SAC consists of 25 PbF\(_2\) crystals, each with transverse dimensions \( 30 \times 30 \text{mm}^2 \), and length 140 mm. The total transverse area is therefore \( 150 \times 150 \text{mm}^2 \), slightly larger than the central square hole of the ECAL. The non-interacting beam is diverted to an off-axis beam dump by means of a 0.5 T magnet. The photon rate in the central crystal due to Bremsstrahlung is expected to reach several hundred MHz, depending on beam intensity.

The lateral surfaces of each crystal are wrapped with 50\(\mu\)m thick black Tedlar to minimize optical cross-talk and the back surfaces are coupled to Hamamatsu R13478UV PMTs via UV transparent optical grease, with matching index of refraction for optimal light transmission. The investigation that led to the choices of
3. Crystal/Glass and PMT Choices

Given the requirements for SAC performance outlined in Section 1, two options of radiating material were considered: SF57 (used e.g. in the Large Angle Veto of the NA62 experiment at CERN [4]) and PbF$_2$ (used in the calorimeters of the Muon g-2 experiment at Fermilab [5, 6]). Both materials are suitable Cherenkov radiators due to their high refractive index, allowing for good timing resolution. However, SF57 has two (related) main disadvantages compared to PbF$_2$. The SF57 transparency window cuts off at 450 nm wavelength, whereas the Cherenkov spectrum is peaked at lower wavelengths due to its $1/E_\gamma$ energy dependence. This can be seen in Fig. 2, which shows the measured transparency profiles (measured by the Atomki Lab in Debrecen) for SF57 and PbF$_2$ compared with the Cherenkov spectrum. For this reason, SF57 offers a low light yield: in our preliminary tests, we obtained 0.15 p.e./MeV, which disfavors its use at energies below 1 GeV.\(^1\)

Furthermore, PbF$_2$ is denser and hence more compact, and 10x more radiation-hard. Table 1 contrasts some PbF$_2$ and SF57 properties which are relevant for Cherenkov-based calorimetry. PbF$_2$ has shorter radiation length and smaller Moliere radius compared to SF57. Smaller electromagnetic showers reduce the detector occupancy, thus enhancing its rate capabilities. Hence, PbF$_2$ is the preferred solution for PADME’s requirements.

We also considered two candidates for PMT in the SAC: the ultra-compact Hamamatsu R9880U-110, and the Hamamatsu R13478 (both Q and UV versions). The first option is a compact PMT with only 16 mm in diameter (8 mm sensitive area), and a rise time of 0.57 ns and a transit time of 0.2 ns. The small size of this PMT allows the coupling of two such devices to the back of a single crystal. This could provide several benefits, such as improved light yield and an independent time reference between the two PMTs. Furthermore, having two PMTs enables an efficiency measurement of each one separately.

The second candidate considered was the Hamamatsu R13478, Q and UV versions. This PMT has a diameter of 26 mm (17.3 mm of which is sensitive area) and thus covers a larger area overall (26%). Compared to the R9880U-110, it has a similar rise time of 0.9 ns but a slower transit time of 9.1 ns. These specifications are fast enough for the PADME use case, and the improved light collection efficiency enables only one PMT per crystal.

The disadvantage of the R9880U-110 PMT, however, is that its small dimensions limit single-PMT light collection efficiency. Without employing the two-PMT solution, a single R9880U-110 PMT has a cross-sectional area of only 5.5% of the crystal transverse dimensions. Even coupling two of them provides only 11% of geometric acceptance, and costs twice as much. Given PADME’s relatively low beam energy, this light yield is unacceptably small. Nevertheless, for higher energy applications, the two-PMT setup could be an interesting solution to explore.

Fig. 3 illustrates the key measures of performance underscoring the determination of the optimal PMT. On the left are the quantum efficiency (QE) curves for the different PMTs, as well as the PbF$_2$ transparency profile and the Cherenkov spectrum. The plot on the right shows

\(^1\)However, note that this measurement was also limited by the PMT’s small dimension (model R9880U-110, described below).
the convolution of the PbF$_2$, Cherenkov, and QE curves for each PMT. Due to the drop in PbF$_2$’s transparency around a wavelength of 250 nm, there is no significant difference in performance between the more expensive R13478Q model and the more affordable R13478UV. The compact R9880U-110 actually performs better than the R13478’s by a factor of roughly 30%, but its reduced light acceptance must also be accounted for as mentioned above.

To decide between the R9880U-110 and the R13478UV PMTs, we completed a simple study to understand the signal distribution expected for each one. We considered two configurations: the R13478UV PMT with a tapered voltage divider, with typical gain $G$ of $3.2 \times 10^5$, and the more compact R9880U-110, with a typical gain of $2 \times 10^6$. Both PMTs are assumed to be coupled to a crystal producing ~ 20 Cherenkov photons reaching the photosensors face ($N_{ph}$) for each MeV of deposited energy $E_{dep}$, and to have average 20% QE. To account for differences between the two PMTs, two corrections are added depending on the PMT. The light acceptance due to photocathode size $A_C$ corrects for the different active areas, while $QE_{corr}$ corrects for the different integrated QEs (see Fig. 3). First we calculate the number of photo-electrons (p.e.) produced by each configuration:

$$N_{p.e.} = N_{ph} \times E_{dep} \times QE \times A_C \times QE_{corr}. \quad (1)$$

With these assumptions, we estimate the expected charge distribution from an incident particle based on a Gaussian spread model:

$$Q_{tot} = \text{Gaus}(N_{p.e.}, \sqrt{N_{p.e.}}) \times G \times e. \quad (2)$$

The results are shown in Fig. 4, with the charge distributions and corresponding estimated resolutions. Despite a higher gain, the R9880-110’s small surface area limits the light collection efficiency and hence the charge resolution. The R13478UV PMT offers in this configuration roughly a factor of 2 better resolution.

From these considerations we conclude the larger Hamamatsu R13478UV PMT, coupled to PbF$_2$ crystals, is the best solution for PADME’s requirements. This has acceptable timing resolution while providing sufficient...
light yield for the low beam energies in the experiment, at a reasonable cost. For a higher-energy experiment, we recommend an investigation of the two-PMT solution (R98880U-110), since there are potentially significant benefits in timing resolution and independent efficiency measurement and calibration.

4. Optical Monte Carlo Simulation

To establish our crystal specifications and to study different aspects of the light collection efficiency, timing and energy resolution, we developed a detailed optical Monte Carlo (MC) simulation using the Geant4 framework [10]. The primary goal was to investigate differences in the light yield and photon arrival times as a function of crystal length and find the optimal length. Manufacturing limitations restrict the crystal’s length to 18 cm.

The simulation consists of a PbF$_2$ crystal, with transverse dimensions 30 × 30 mm$^2$, and a variable length in the range 10 to 20 cm. It can be wrapped with either white millipore or black tedlar materials. The crystal is coupled to a thin layer of grease, modeled as Epoxy EJ-500 (radius 12.7 mm, thickness 1 mm), and then to a sensitive detector layer which models the PMT. The QE of the R13478UV PMT is implemented as a function of energy according to its datasheet [8]. Other optical properties are also implemented as functions of energy, such as the absorption length from PbF$_2$ transparency [7], and the refractive index via a parametrized dispersion formula [11]. These were calculated for optical photons with energy ranging from 1.6 to 5.0 eV in steps of 0.02 eV.

A single energetic photon (energy: 200 MeV) is fired at a distance of 1 mm from the crystal’s front surface, which produces a few thousand optical photons after showering inside the crystal. The simulation then tracks each individual optical photon, until it either reaches the sensitive area corresponding to the PMT, or gets lost along the way due to different physical processes. We run 100 events per crystal length.

The arrival time distribution of the photons for each length considered is displayed in Fig. 5. Two peaks can be identified in each curve: a narrow, high peak which represents the arrivals of most photons, and a lower, broader peak, which corresponds to photons that underwent back-scattering inside the crystal. A shift and broadening of the narrow peak with increasing length is clearly visible.

The arrival profiles were fitted to a convolution of a Landau (narrow peak) and a Gaussian (broader peak) distribution, and the sigma of the Landau was taken as indicative of the timing resolution of the detector in MC. The choice of a Landau fit was empirically driven and not based on the underlying physical process. We don’t account for effects from the PMT itself, but those should be of second order and so this procedure at least allows the relative comparison of different crystal lengths. The MC timing resolution is plotted in Fig. 6. There is a roughly 16% effect in the resolution when going from 14 cm to 18 cm.

Fig. 6 also shows the light yield as a function of crystal length. The light yield is determined from a convolution of the PMT’s QE with the energy distribution of arriving photons. The light yield decreases with increasing length due to a higher chance of Cherenkov photons getting lost (absorbed or escaping) while traveling towards the PMT. In particular, there is a drop of about 14% in collected light between lengths of 18 cm and 14 cm. The light yield for a 14 cm crystal, according to the simulation, is around 1.7 p.e./MeV, to be compared with test beam data.

From the optical MC simulation, it is evident that shorter crystals are better, both for optimal light yield and timing resolution. Increasing the length causes more Cherenkov photons to be lost along the way. At the same time, in a longer crystal photons have longer travel paths. Consequently, there is more opportunity for scattering, which broadens the arrival time profile of the photons, worsening the resolution. Nevertheless, there is a limit where the crystal length is too short to develop a complete Cherenkov shower. From [7], about 50% of Cherenkov photons from a 3 GeV shower are produced within the first 60 mm of a PbF$_2$ crystal, and about 80% are produced within the first 100 mm. This makes 140 mm a safe length to ensure we can harness most of the Cherenkov photons in a 500 MeV shower.
Finally, the simulation reveals that increasing the crystal length leads to broader signals and longer tails, affecting the double-peak separation capability, which is a crucial parameter in the PADME SAC performance.

5. Test Beam Setup

The chosen crystal and PMT options were subjected to a test beam at the LNF’s BTF, in order to characterize the SAC response and measure the energy and timing resolution, including the double-peak separation capability. We obtained PbF$_2$ crystals on loan from Brookhaven National Laboratory for this test, with dimensions $30 \times 30 \times 140 \text{ mm}^3$.

The BTF at LNF is part of the DAΦNE accelerator complex. A LINAC provides bunches of $\sim 10^{10}$ electrons or $\sim 10^9$ positrons with energy up to 750 MeV or 550 MeV respectively. The LINAC has to switch between electron and positron modes on a regular interval, in order to top off the main DAΦNE rings, and in this case the energy is fixed to $E_0 = 510$ MeV for both charges. There are 50 bunches per second exiting the LINAC, which are shared between the BTF and the main ring under normal operation. In this regime, the BTF gets 38 bunches/s in electron mode, and 18 bunches/s in positron mode. Each mode lasts 180 s, and then a 90-second switch mode is activated. During the switch mode, there are 60 s with no beam at all and then 30 s where BTF gets the entire 49 bunches/s (one per second is used for beam-energy monitoring). The cycle then repeats.

The BTF setup also allows the tuning of the beam intensity delivered to the experimental hall, from $\sim 10^{10}$ particles/bunch, down to a single particle/bunch, by intercepting the primary beam with a $2\chi_0$ target. The resulting secondary particles can be further filtered to allow tuning of the beam energy from $E_0$ down to few tens of MeV. The beam spot and position can be adjusted by means of quadrupoles, dipoles and correctors in the BTF line, and is monitored in real-time by silicon pixel hybrid detectors (FITPIX [12]) with active area $14 \times 14 \text{ mm}^2$ and 55 µm pitch.

A schematic of the detector test setup is shown in Fig. 7. A R13478UV PMT (voltage: 1600 V) was coupled to a PbF$_2$ crystal using optical grease and then connected to a 12-bit, 5 GSPS, 1024-sample digitizer (model CAEN V1742) for data acquisition. A plastic scintillator coupled to two small ‘finger’ PMTs (model R9880U-110) provided a reference signal for comparison with the PbF$_2$ one.

A picture of the PMT and crystal setup is shown in Fig. 8.

For the studies presented here, the beam energy was varied between 100 and 400 MeV in steps of 100 MeV, and the average number of particles per pulse delivered to the BTF experimental hall was set to $\sim 1$ for electrons. The beam was centered on the crystal and the spot was kept within a standard deviation of 3 mm (at 400 MeV) and 5 mm (at 100 MeV) in the transverse plane.
5.1. Single-crystal Monte Carlo simulation

Since only one PbF$_2$ crystal was available for testing, we implemented a dedicated Monte Carlo (MC) simulation based on the Geant4 framework to characterize the average lateral energy leakage and to provide a correction factor to the light yield measurement.

The simulation consists of a single PbF$_2$ crystal with the same dimensions of the one used at BTF and a beam of photons with similar energy and multiplicity as the test beam. The primary goal was to estimate how much of the incident energy escapes the crystal laterally, contributing to an overall decrease in energy collection. This is a purely geometrical correction which can be used to re-scale the light yield obtained from the test beam data. For example, for 100 MeV incident photons, the deposited energy is only 82%. Note, however, that this procedure can only correct for the average energy leakage, and does not account for experimental fluctuations in that quantity. For this reason, the energy resolution quoted here is only an upper limit to the actual one achievable by the PADME SAC, and should be understood to be a single-crystal energy resolution.

6. Charge Reconstruction

The results presented in this paper are based on data taken in July of 2017 at the LNF BTF. Electron beams with average multiplicity of 1 particle/pulse and energies of 100, 200, 300 and 400 MeV impinged on the detector setup. Data acquisition triggered on accelerator signals, and 1024 samples at 5 GSPS (i.e. 0.2 ns/sample) were collected per trigger [13]. Since PbF$_2$ has a fast Cherenkov decay time of less than a few nanoseconds, the signal is centered within a small window of the waveform.

The total energy deposited in the crystal was reconstructed in three steps. First a run-level pedestal was calculated by averaging the ADC counts of each 1000-sample empty event in the run (i.e. only noise, no signal peaks). The last 24 samples of each event were not used. The average ADC counts for all empty events form a Gaussian distribution, the mean of which is taken as the pedestal for that run. The sigma in turn informs the noise level, which was around 1.1 pC for all runs. This is the noise considering all 1000 samples, but for a roughly 50-sample signal window the average noise is scaled down to order 0.1 pC.

After subtracting the pedestal from each ADC count, the integrated charge was calculated by identifying all the signal peaks in a given event, and integrating the area underneath each peak. The peak boundaries were set via a simple threshold (ADC - pedestal/4096 > 0.005). A sample digitized trace, with two electron peaks and thresholds identified, is shown in Fig. 9.

The choice of threshold was made to mitigate the after-pulse ringing which can be seen in the figure. This ringing continues with approximately constant magnitude for about 40 ns after the end of a pulse. Immediately following each pulse, the signal shoots below the pedestal-subtracted zero level, which motivates the low threshold in order to exclude such ringing effects from the charge estimate.

Finally, the distribution of integrated charges was plotted. A representative example, for 300 MeV electrons, is shown in Fig. 10. The peaks were fit to a sum of CrystalBall (CB) functions [14] in order to extract the mean and sigma.

7. Detector Performance

To estimate the single-crystal performance, we measured linearity and energy resolution, using only single-electron peak events, at different incident electron energies. We also determined the timing resolution by using the auxiliary finger PMTs as reference, and performed a data-driven study of our estimated double-peak separation capability. These studies are discussed below.

7.1. Linearity and light yield

Fig. 11 shows the collected charge as a function of deposited energy. Each data point is the mean of the corresponding CB fit. The fit is performed only on single-electron peaks (labeled ‘Peak 1’ in Fig. 10), though we
Figure 10: Integrated charge distribution for 300 MeV electrons. The first 3 peaks were fit with a sum of Crystal-Ball functions for extraction of the means and sigmas.

also plot data points corresponding to multiple-electron peaks for completeness (‘Peak 2’ and ‘Peak 3’). Since a typical single-electron pulse has a short duration and given the sampling resolution of 0.2 ns, only rarely do two or more electron pulses exactly overlap in time, and so multiple-electron events are not appropriate indicators of the charge linearity or energy resolution.

The plot in Fig. 11 is a function of the actual deposited energy on the crystal, and not of incident beam energy. This is done to account for the fact that a single finite-sized crystal does not provide full energy containment. The correction used is shown in Fig. 12, displaying the actual energy deposited on a single 30 × 30 × 140 mm³ crystal as a function of incident energy. This correction was determined using a dedicated Geant4 MC simulation of a high-energy photon impinging on a single crystal, as described in Section 5.1.

From the slope of the solid fit line in Fig. 11, we find a light yield of 2.07 p.e./MeV. Note that the linearity is valid only for the single-pulse regime, which is what we are interested in. For double-electron peaks or higher, the ringing after-pulse is not adequately captured by our pulse-area calculation if the two peaks are close enough in time. This overestimates the total charge and introduces a bias in the linearity.

In Section 4, we estimated the light yield for a 14 cm-long crystal to be around 1.7 p.e./MeV, based on an optical MC simulation with 200 MeV incident photons. This number matches the uncorrected light yield from the data, which is 1.67 p.e./MeV. The uncorrected number assumes the full energy deposit on the crystal.

Figure 11: Detector linearity as a function of beam energy. Only the single-electron peaks are fit. The residuals are also plotted, demonstrating good linearity up to at least 400 MeV. A bias in the linearity is introduced by multiple-electron peaks due to the nature of our threshold setting and the after-pulse ringing, which can be mismeasured if the two peaks are close enough together. This is not a problem since we are only interested in the single-electron peaks for resolution purposes. The axis on the right assumes a gain of 8 × 10⁵ for a PMT operating voltage of 1600 V [8].

7.2. Energy Resolution

The energy resolution was calculated as the fitted sigma over mean of the single-electron peaks in the charge distribution. Fig. 13 shows the energy resolution as a function of energy. The data were fitted according to the expression:

\[ \frac{\sigma_E}{E} = \sqrt{\frac{p_0^2}{E} + \frac{p_1^2}{E/\text{GeV}}} \]  

(3)

with \( p_0 \) and \( p_1 \) the constant and the intrinsic resolution, respectively. The fit yields \( p_0 = (6.0 ± 3.3) \% \) and \( p_1 = (5.7 ± 0.9) \% \). We find that the energy resolution is better than 20% above 100 MeV, which is sufficient for PADME’s SAC requirements.

7.3. Timing Resolution

The timing resolution of the PbF₂ + R13478UV PMT setup was determined with help from the scintillator bar and the finger PMTs. First a cut was imposed on the integrated charge to select events with only one electron in them. For each selected event, the rising edge of the electron pulse was fit to a straight line, using as endpoints the 20% and 80% heights of the pulse amplitude. The location of the fit at 50% of the amplitude was then taken as the reference time. This procedure was done for all three channels: PbF₂ and the two finger PMTs. The distribution of time differences between each channel was plotted. Two examples can be seen in Fig. 14.

The timing resolution was determined by extracting the sigmas of a Gaussian fit to each such distribution.
From the two-finger distributions, the finger resolution was determined to be 174 ps. Then, from the other distributions, the PbF$_2$ resolution was extracted by inserting the calculated finger resolution and summing in quadrature. The resulting resolutions for all channels and runs are shown in Fig. 15. The PbF$_2$ resolution was found to be 81 ps, which comfortably meets the 200 ps timing requirement for a successful SAC performance.

7.4. Double-Peak Separation Resolution

In addition to the timing resolution, we measured the double-peak separation capability of the setup, using a purely data-driven method. From the charge distribution, a cut was imposed to select only single-electron events. The traces of selected events were then randomly added in pairs, with an artificially introduced time separation between single-electron peaks $\tau$. By varying $\tau$ we simulated increasingly overlapping pulses. For each value of $\tau$, the peak separation was measured by means of the ratio between the height of the separation trough ($h$), and the height of the smaller of the two peaks ($H$). This ratio, $r = h/H$, characterizes the degree of separability between the two peaks. A value closer to 1, for example, implies the two peaks are very close together and distinguishing them is more challenging. On the other hand, a ratio closer to 0 means the peaks are far apart and identifying them is straightforward. A sample trace showing the relevant quantities defined above can be seen in Fig. 16.

Fig. 17 shows the distribution of ratios $r$ for a 300 MeV beam. The black dots correspond to the 90-th percentile of the ratio distribution for each $\tau$. Assuming a minimum separation capability of $r < 0.95$, then roughly 90% of peaks with 1.8 ns separation can be distinguished. As a conservative estimate we take this to be the double-peak separation resolution.

With a more sophisticated algorithm (for example, template fitting), it is likely that this capability can be improved further. Even when there is no local minimum (i.e. the peaks are too close together), in which case this algorithm fails, template fitting might still be able to identify a broader shoulder as a second peak. Nevertheless, this separation resolution already meets the PADME tagging requirements.

8. Conclusions

We have characterized the performance of a single-crystal prototype of the Small-Angle Calorimeter in the
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