The understanding of dependence structure measurement:
evidence from natural rubber imports of ASEAN

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Abstract. This paper is proposed to focus on the time series data study and rare structures of
natural rubber, which cannot precise imply in regular non-linear estimation. The Maximum
entropy bootstrapping estimator (MEboot), Stochastic dominance, the AR-GARCH model and
Copula models were applied to research the outcome of natural rubber demand in ASEAN. The
results illustrate which countries were the dominator who leads rubber importing trend in
ASEAN region and also beneficial for develop policies concerned by the natural rubber
production development and promotion.

1. Introduction
Since 1986, ASEAN has become the world largest rubber consuming area as its population and
economic growth, with 57 percent in account of global demand in natural rubber, leads ASEAN region
be the most powerful natural rubber in the world [1]. This research paper offers a rare structure and
highlight on natural rubber in the time series data, which cannot indicate by regular nonlinear
estimation. The varieties detection in the patterns of dependent structural and the discovery the
relation between market leader and followers regarding to the demand on natural rubber are the main
purpose of this research paper.

The main issue of mislead in nonlinear regressive lead to an uninterruptable in the financial indexes.
From this circumstance, when the data of the imports in natural rubber is the time series data, the
estimation of error term can be utilised if data is not identify as the concrete connection between
independent variables and dependent variables. Hence, the corrected error terms were demonstrated by
this study. It can be productively used for investigating the structural form by testing via Maximum
entropy bootstrapping approach (MEboot). The decision-making below the uncertainty and risk is
verified by the stochastic dominance (SD) [2]. The comparison of national rubber consumption
performance regard to the value of import rubber was applied in this research paper.

The paper is organized into sections. The reviews of literature are represented in section 2. The
methodology including the AR-GARCH model, Maximum entropy bootstrapping estimator (MEboot),
Stochastic Dominance (SD) and the Copula models are described in section 3. Section 4 represents
empirical results discuss of the dominator that leads the rubber-importing trend in ASEAN region base
decision on Copula approach, and the suggestions and conclusions are clarified in section 5.
2. Methodology

2.1. The AR-GARCH model
Ferenstein and Gasowski [3] analysed two sets of stock prices data, which could specify error distributions by the AR-GARCH model. The Autoregressive (AR) and GARCH are formed by AR-GRARCH that basically estimated by MLE or Maximum Likelihood estimator. The Autoregressive method set up an understanding at time t, which is a linear combination between some error term and the previous understanding (p). Let \( p = 0 \), \( X_t = \omega_t \), with no AR term.

The GARCH approach is an expansion of the ARCH approach. Let \( \alpha_\omega > 0 \), \( \alpha_\nu > 0 \), \( \beta_\nu > 0 \) and \( \alpha_\nu + \beta_\nu < 1 \) to generalize ARCH or GARCH approach [4]. As can be shown by equation of AR-GARCH as below,

\[
X_t + \sigma_t^2 = \sum_{j=1}^{p} \phi_j X_{t-j} + \omega_t + \gamma V_t + \alpha \mu_{t-1}^2 + \beta \sigma_{(t-1)}^2
\]  

(1)

2.2. The maximum entropy bootstrapping (MEboot)
Vinod [5] stated that maximum entropy is a forceful computing for abstains from all needless distribution assumptions. As Bartirmo [6] investigated the maximum entropy method by maximizing information entropies for getting the distribution of fluctuated stock prices. This computing evaluation pursue a given constraints and finite sets of n circumstance with probabilities \( q_i \) defined as a prior evaluation form with latest information \( i; \sum_i q_i a_{ki} = 0 \) or \( \sum_i q_i a_{ki} \geq 0 \), where \( a_{ki} \) and \( b_{ki} \) be known numbers. Thus, the model of entropy maximization is shown as follow,

\[
H(q) = \sum_i q_i \log(q_i) - \log(n)
\]  

(2)

The uniqueness, invariant, and subset independence in minimizing function, \( H(q) \), is equal to the ordinary that can be get by maximizing entropy [7].

Let combined this estimation with the “bootstrap” that is the computing statistical method regarding the relation between populations and samples. The function of bootstrap percentile is calculated, let \( q_i \) be motif statistic: \( q \rightarrow \mathbb{R} \). Assumed \( q_1, \ldots, q_n \) be identically independence distribution (i.i.d), and let \( p(q|I) \) be the density of choices [8]. Accordingly, setting \( y = f(q_i) \) for each I, then, defined the bootstrap percentile as follow,

\[
P_{BS}(f, q) = \frac{|\{y|y<y\}|}{n}
\]  

(3)

The study by Chaiboonsri and Chaitip [9] claimed that algorithms evaluated by the maximum entropy bootstrapping could originate the ensemble of global variation time. Moreover, Vinod and Lopez-de-Lacalle [10] mentioned the MEboot that can be a possible instrument to calculate more precisely in the linear model.

2.3. Stochastic dominance (SD)
The stochastic dominant method is ground on risk-averse preferences in an axiomatic method [11], which occurred in the main theory [12] and was later expansive to normal distributions [13][14].

Stochastic dominance rules have served as a foundation in the literature on risk theory [15][16]. They are the consensus rules to rank distributions for all individuals in specific sets. For instance, FSD or the first-degree stochastic dominance corresponds to all non-satiable decision makers. If individuals are not only non-satiable but also risk-averse, then SSD or the second-degree stochastic dominance can be applied to rank distributions for all of these individuals. Let a random variable be \( X \) with the probability \( P_X \). The right continuous cumulative distribution function itself defined by the first performance function \( F_X^1 \):

Some text.

\[
F_X^1(\eta) = F_X(\eta) = P[X \leq \eta \} \text{ for } \eta \in R.
\]  

\[
F_X^2(\eta) = \int_{-\infty}^{\eta} F_X(\xi) d\xi \text{ for all } \eta \in R,
\]  

(4)

(5)

The corresponding strict dominance relation \( \succ_{FSD} \) and \( \succ_{SSD} \) are defined by the standard rule \( X \succ Y \iff X \succeq Y \text{ and } Y \nprec X \). Meaning that \( X \) dominates \( Y \) under the FSD rules \( X \succ_{FSD} Y \), if
The Efficient under second-degree stochastic dominance rules is called by a feasible portfolio $X \in Q$ when there is no $Y \in Q$ such that $Y \succ_{SSD} X$.

The Outcome – Risk (O-R) diagram can justify partial harmony of the MAD method with the SSD rules, since $\delta_X = F_X(\mu_X)$ is a well-established geometrical characteristic of the diagram. However, the Gini’s mean difference cannot be placed within the O-R diagram. Therefore, Levy and Kroll [18] considered the quantile model of stochastic dominance. Ogryczak and Ruszczyński [19] represented the comprehensive discussion of a stochastic dominance and its relation to the downside risk measures.

### 2.4. The C-Vine copula approach

The Gaussian copula models were employed in this research paper to analyze the dominator that leads the rubber-importing trend in ASEAN. Gruber and Czado [20] reviewed using regular vine copulas by the multivariate dependence model, the general outlines of stepwise tree-by-tree strategies were discussed the contemporary approaches for model selection. Regarding Sklar’s concept [21] express that we can explain every multivariate cumulative distribution function as follow,

$$F(x_1, x_2, ..., x_p) = P(X_1 \leq x_1, X_2 \leq x_2, ..., X_p \leq x_p)$$

(7)

Any $(x_1, x_2, ..., x_p)$ in equation (7) show joint cumulative distribution function (CDF) in random variables, let defined $(F_j(X))$ a marginal distributions of those random variables by:

$$F_j(x) = P(X_j \leq x), j = 1, 2, ..., p$$

(8)

![Figure 1. The structure of a market leader and market followers divided by a C-vine copula [9].](image)

The random copula models are observed by tail dependent structure, for more details in mathematical form, see Brechmann and Schepsmeier [22]. Figure 1 represents pair-copula type of dependence structure between 5 random variables by the C-vine trees copula. In addition, the C-vine copula was adopted in this research paper to measure the dependence between countries in order to identifying market leader and market followers in natural rubber demand between 2009 – 2018.

### 3. Data description

The illustrative of natural rubber import statistics represents in Table 1, five ASEAN regions (Thailand, Philippines, Malaysia, Singapore, and Vietnam) collected in 2009 – 2018 by CEIC data.
Table 1. The illustrative statistic of ASEAN natural rubber demand, 2009-2018.

|        | Vietnam | Thailand | Singapore | Philippines | Malaysia |
|--------|---------|----------|-----------|-------------|----------|
| Mean   | 62.15841| 105.8770 | 74.64723  | 28.07774    | 73.89888 |
| Median | 58.74852| 106.8200 | 72.98994  | 25.20848    | 72.17300 |
| Maximum| 103.8907| 188.3800 | 119.7759  | 56.93180    | 140.6700 |
| Minimum| 14.64985| 30.50000 | 43.35695  | 9.254666    | 43.41800 |
| Std.Dev.| 18.88472| 27.71273 | 12.36457  | 11.67306    | 18.45900 |
| Kurtosis| 2.669398| 3.885010 | 4.094932  | 2.605855    | 3.705372 |
| Jaeque-Bera | 2.254539 | 6.209652 | 12.09046  | 9.128027    | 15.69052 |
| Probability | 0.323916 | 0.044832 | 0.002369  | 0.010420    | 0.000392 |
| Sum     | 7148.217| 12175.86 | 8584.431  | 3228.940    | 8498.371 |
| Sum Sq. Dev. | 40656.12 | 87551.48 | 17428.61  | 15533.69    | 38843.76 |
| Observations | 115      | 115      | 115       | 115         | 115      |

Source: Author’s calculation

The plots of ASEAN rubber imports value during 2009-2018 are not normally distributed as shown in Figure 2. The main indicator for importing natural rubber in ASEAN seems to be Thailand, Malaysia, Singapore, Vietnam, and Philippines respectively.

4. Empirical Results

4.1. The stochastic dominance analysis

This paper applied the Stochastic Dominance – based MEboot particulars of the covariance explained in Table 2; the empirical outcome portray that distribution of Vietnam based on MEboot method dominates the relevant parts in Thailand to both FSD and SSD as shown in Figure 3 and Figure 4. Consequently, this result reveals that the directions of natural rubber importing in ASEAN, Vietnam and Thailand are the best representatives of all five ASEAN nations. Meaning that ASEAN organization should acknowledge that Thailand and Vietnam play crucial signs to promote the natural rubber circumstances across ASEAN.
Table 2. The covariance (Long-run covariance, Barlett Kernel and Newey-West).

|          | Thailand | Malaysia | Vietnam | Philippines | Singapore |
|----------|----------|----------|---------|-------------|-----------|
| Thailand | 110.0768 | -23.49265| 2.163360| 29.67308    | 26.98563  |
| Malaysia | -23.49265| 162.6638 | 8.475151| -5.802298   | -5.965908 |
| Vietnam  | 2.163360 | 8.475151 | 157.1839| 17.64059    | 46.73867  |
| Philippines | 29.67308 | -5.802298| 17.64059| 100.7343    | 3.861986  |
| Singapore| 26.98563 | -5.965908| 46.73867| 3.861986    | 53.93147  |

Source: Author’s calculation

4.2. The C-Vine copula approach

After testing MEboot via the error term from AR-GARCH, graphically estimated the C-vine copula trees. The main market of natural rubber demand region was employed by the Gaussian copula. The C-vine copula results strongly confirm that rubber-importing values in Vietnam are dominated and lead the rubber-importing trend in ASEAN region. The structural dependence obviously displays that positive parallel trends are relied on Vietnam rubber imports. Therefore, it can be concluded that rubber imports in Vietnam are the dominated sign for making policies regarding to rubber volumes flowed among ASEAN countries.
5. Conclusions

According to the technical limitation of copula model package with only five samples to be processed. The estimate residuals of time series data in natural rubber demand of top five rubber’s demand countries in ASEAN (Thailand, Philippines, Malaysia, Singapore, and Vietnam) were accomplished in this research paper. The MEboot in the AR-GARCH method was employed for seeking the most precisely estimation. Then, the stochastic dominance method (SD) can indicate that Vietnam and Thailand are the best representatives of all ASEAN five nations.

In addition, The C-vine copula based Gaussian distribution represented in Figure 5 was graphically to investigate the directions of the market flow among them. The result strongly confirms that rubber-importing values in Vietnam dominates and leads the rubber-importing trend in ASEAN region. The structural dependence obviously displays that the positive parallel trends are relied on Vietnam rubber imports.

Therefore, it can be concluded that rubber imports in Vietnam are the dominated sign for making policies regarding to rubber volumes flowed among ASEAN countries. This can strongly confirm that applying mathematics in econometrics has not only been important for seeking precise forecasting in agriculture areas, but it can be efficiently adapted to predictive studies in many academic subjects.

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