Predictive modeling of surface roughness in the inner thread grinding considering the effects elastic deformation

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Abstract. Surface roughness is an important factor to consider in the field of precision manufacturing. In this study, a model of the surface roughness in inner thread grinding that takes into account the thread helix angle and effect of elastic deflection between the wheel and the workpiece is presented. The surface roughness is analyzed under different spindle speeds, workpiece speeds, grinding depths, and thread helix angles. The proposed model provides a theoretical basis for the optimization of high-precision inner thread grinding.

1. Introduction

Surface roughness is an important indicator of the grinding quality and performance. Therefore, predicting the grinding force is essential to optimizing the grinding process. A number of grinding force prediction models were recently introduced in the literature. Feng et al. [1] developed an empirical model of roughness for steel turning, and the model could evaluate the roughness under certain cutting parameters. Agarwal et al. [2] developed a predictive model of roughness by considering the effect of grains overlapping. The experimental results showed that the model with considering the grains overlapping can improve the accuracy of surface roughness prediction. However, the model ignored the effect of material removal mechanism. Shao et al. [3] established a physics-based model of roughness considering the combined effect of brittle and ductile material removal mechanism. Zhang et al. [4] built surface roughness incorporating the effect of overlapping of triangular-shaped grooves, the predicted surface roughness showed a good agreement with experimental data. Sun et al. [5] developed a predictive model of the surface roughness considering the effects of abrasive grain size, processing parameters and grinding grooves distribution and discussed effects of various processing parameters on roughness. To date, none of the research conducted on the surface roughness in inner thread grinding has considered the effect of elastic deflection between the wheel and the workpiece.

The objective of this study was to develop a surface roughness model based on Wheel-workpiece contact length and the elastic deflection between the wheel and the workpiece in inner thread grinding. Then, the effects of spindle speed, workpiece speed, and grinding depth on the surface roughness were discussed.
2. Modeling of surface roughness in inner thread grinding

In contrast to normal cylindrical grinding processes, an angle between the wheel axis and workpiece axis exists in inner thread grinding, the effective equivalent wheel diameter can be defined as

\[ d_e = \frac{d_s}{1 - \frac{d_s}{d_w} \cos \alpha} \]  

(1)

Where, \( d_s \) is the diameter of the grinding wheel, \( d_w \) is the diameter of the workpiece, and \( \alpha \) is the thread helix angle.

2.1. Wheel-workpiece contact length (\( l_c \))

Contact length (\( l_c \)) is important to both grinding efficiency and workpiece surface integrity. Therefore, accurately calculating the contact length is crucial. As the effect of uneven wheel wear and other random factors, the wheel imbalance will occurs in the actual grinding process. In this paper, suppose the relative vibration between wheel and workpiece is a sine wave, the vibration equation of the grinding wheel center can be expressed as

\[ M \ddot{y} + C \dot{y} + Ky = m e \omega \sin(\omega t) + \varphi \]  

(2)

\[ y(t) = A \sin(\omega t + \varphi) \]  

(3)

Where \( M, K \) and \( C \) are the total mass, rigidity, and damping of the grinding wheel system respectively. \( \omega \) is the angular speed of the grinding wheel, \( e \) is the eccentricity. \( A \) is the amplitude, \( \varphi = \arctan \left( \frac{2 \zeta \lambda}{1 - \lambda^2} \right) \),

\[ \zeta = \frac{C}{2(MK)^{1/2}}, \quad \lambda = \frac{\omega}{\omega_n} \ll 1, \quad \omega_n \] is the natural frequency.

The motion of a single abrasive grain in the x, y and z directions can be expressed as follows

\[ \begin{cases} 
  x = v_i t \\
  y = \frac{d_r}{2}(1 - \cos(\omega_i t)) + A \sin(\omega t + \varphi) \\
  z = \frac{d_r}{2} \sin(\omega_i t) 
\end{cases} \]  

(4)

Where, \( \omega_i \) is the relative angular speed of the grinding wheel to workpiece, \( \omega_i = \frac{2v_i}{d_e} + \frac{2v_w}{d_w}, \quad v_i \) is the feed speed of the workpiece.

Then, the undeformed chip length can be expressed as

\[ l_c = \int_{0}^{\frac{a_p}{v_i}} \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 + \left( \frac{dz}{dt} \right)^2 \]^{1/2} dt

(5)

\[ = \int_{0}^{\frac{a_p}{v_i}} \left( v_i^2 + \left( \frac{\omega_i d_e}{2} \sin(\omega_i t) + \omega A \cos(\omega t + \varphi) \right)^2 \right)^{1/2} dt \]

Where, \( a_p \) is the cutting depth.
2.2. The elastic deflection analysis between the wheel and the workpiece

The elastic deflection analysis between the wheel and the workpiece in inner thread grinding is assumed as a rigid grain impressing into an elastic binder, as shown in Figure.1.

\[
F_b = k\delta
\]

Where, \( \delta \) is the grain deflection, \( \delta = E(h) - E_{true}(h) \), \( E(h) \) is the average undeformed chip thickness of a single grain, \( E(h) = \left( \frac{a_p v_w}{2\phi\eta l_c} \right)^{1/2} \), \( v_s \) and \( v_w \) are the grinding wheel speed and workpiece speed, respectively, \( \phi \) is the effective coefficient of grains overlapping, \( \phi = 0.79 \), \( \eta \) is the number of active grains per unit area of the wheel periphery. \( E_{true}(h) \) is the average undeformed chip thickness of a single grain with deflection. \( k \) is the stiffness, \( k = 2E_p R(1 - \nu^2)^{-1} \), \( E_p \) is the modulus of the binder, \( \nu \) is the poisson ratio of the binder. \( R \) is the radius of the spherical grinding grain.

The normal grinding force acting on the grain can be expressed as

\[
F_g = H_w A_{contact} = H_w \pi R^2 \sin^2 \theta
= H_w \pi R^2 \left[1 - \left(1 - \frac{E_{true}(h)}{R} \right)^2 \right]
\approx 2H_w \pi RE_{true}(h)
\]

Where, \( H_w \) is the workpiece hardness.

Since the internal force of binder is equal to the normal grinding force acting on the grain, \( F_b = F_g \), \( E_{true}(h) \) can be expressed as

\[
E_{true}(h) = \frac{E(h)}{1 + \frac{2H_w \pi R}{k}} = \frac{E(h)}{1 + \pi(1 - \nu^2) \frac{H_w}{E_b}}
\]

2.3. Establishment of surface roughness model

The surface roughness Ra can be written as
\[ R_s = \frac{1}{l} \int_{0}^{l} |y - y_{cl}| dl \]  

(9)

Where, \( y_{cl} \) is the distance of the centerline, \( y_{cl} = \beta \left( \frac{\pi}{2} \right)^{1/2} \), \( y \) is the random profile height.

The grooves can be divided into two types, either above or below the centre line \( y_{cl} \), the total expected value of surface roughness considered overlapping factor can be expressed as

\[ E(R_s) = P_1 E(R_{a1}) + P_2 E(R_{a2}) \]  

(10)

According to Rayleigh probability distribution, the probability \( p_1 \), \( p_2 \) of the undeformed chip thickness smaller and bigger than the centerline position, \( y_{cl} \), can be calculated as

\[
\begin{align*}
    p_1 &= \int_{0}^{y_{cl}} f(h) dh = 1 - e^{-\frac{y_{cl}^2}{2} \beta^2} = 1 - e^{-\frac{\pi}{4}} \quad h < y_{cl} \\
    p_2 &= 1 - p_1 = e^{-\frac{\pi}{4}} \quad h > y_{cl}
\end{align*}
\]  

(11)

\[
\begin{align*}
    E(R_{a1}) &= E(\frac{A_1}{2h_1}) = y_{cl} - \left( \frac{\pi}{4} \right) E(h_1) = 0.656 \beta \\
    E(R_{a2}) &= E(\frac{A_{upper} + A_{lower}}{2h_2}) = 0.362 \beta
\end{align*}
\]  

(12)

The total expected value of surface roughness can be given as

\[ E(R_s) = 0.522 \beta = 0.417 E_{true}(h) = \frac{0.417}{1 + \pi(1 - \nu^2)} \left( \frac{a_p v_w}{2 \phi \eta v_c l_c} \right)^{1/2} \]  

(13)

3. Analysis of the model

The workpiece hardness \( H_w \) is HRC55. An inner thread \( M 42 \times 3 \) was chosen, and the helix angle of the inner thread was \( \alpha = 2^\circ 48' \). The grinding wheel was composed of metal-bonded diamond abrasives with diameter \( d_s = 30mm \), the modulus of the binder is \( E_b = 882 GPa \), the poisson ratio of the binder is \( \nu = 0.2 \). The effects of spindle speed, workpiece speed, grinding depth and the thread helix angle on the surface roughness are presented in Figure 2.

![Figure 2](image-url)

(a) \( v_w = 5m/s\ a_p = 0.05mm \)  
(b) \( a_p = 0.05mm \)  
(c) \( v_w = 18.84m/s \)

**Figure 2.** Effects of spindle speed, workpiece speed, grinding depth and the thread helix angle on the surface roughness
As illustrated in Figure. 2 (a), higher workpiece speeds contribute to an improved surface roughness. Moreover, the higher the thread helix angle, the lower the surface roughness. The surface roughness varies from 0.4344 to 0.3372 as the thread helix angle varies from 0 to $\pi/6$, when $v_w = 400mm/min$, $v_s = 5m/s$, and $a_p = 0.05mm$. The surface roughness is reduced by 22.4%, therefore, the thread helix angle should be taken into consideration to ensure an accurate prediction of the surface roughness in inner thread grinding. The surface roughness decreases as the spindle speed increases, as shown in Figure.2 (b). Furthermore, the larger the grinding depth, the higher the surface roughness under the same workpiece speed, as illustrated in Figure. 2 (c).

4. Conclusions
In this study, a model of surface roughness based on the effective equivalent wheel diameter and the elastic deflection between wheel and workpiece was presented for inner thread grinding. A higher workpiece speed and grinding depth, along with a lower spindle speed and a lower thread helix angle contribute to an improved surface roughness. The results suggest that surface roughness due to the thread helix angle and the elastic deflection between wheel and workpiece should not be ignored.

5. References
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