Multivariate Bühlmann-Straub credibility model for claim reserving

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Abstract. One of the approaches that is used for claim reserving in insurance companies is credibility theory, which allows claim reserving by combining claim payment data with other information. In this paper, the Bühlmann-Straub credibility model is used. Furthermore, in general, claim reserving in a company is done by calculating the claim reserve in each line of business (LoB) in the company, then the total claim reserve for the company (aggregate reserve) is obtained by adding up the claim reserve in each LoB. Considering the possibility that there is correlation between the existing LoBs, the value of aggregate reserve can actually be less than the sum of the claim reserve in each of the existing LoB. Therefore, research on the claim reserving then evolves by considering claim payment data from various LoBs in a company, or also called claim reserving in multivariate context. In this paper, a research is conducted on the development of multivariate Bühlmann-Straub credibility model for claim reserving along with estimation for model’s parameters. The model is used to calculate claim reserve for three LoBs of insurance company in United State, based on the data of claim amount during the period of 2008-2017 that was published by Association of Insurance Commissioners of the United State. It appears that the error of multivariate Bühlmann-Straub credibility model is lower than the error of standard Bühlmann-Straub credibility model.

Keywords: Bühlmann-Straub credibility theory, claim payment, line of business, multivariate, run-off triangle

1. Introduction
Insurance is agreement between two parties, insurer (insurance company) and insured (policy holder), which underlie the obligation of the policy holder to pay premium to the insurance company as well as the obligation of the insurance company to pay sum insured to the policy holder in case of loss [1]. To make sure that the insurance company has the ability to pay the sum insured, the company has to prepare some amount of money, namely claim reserve. One of the method than can be used to calculate claim reserve is chain ladder, which assumes that there is development factor of claim payment, and the development factor is used to predict claim payment in the following period [2]. The other method that can be used in claim reserving is credibility theory. Credibility theory is a set of quantitative tools that allows an insurer to perform prospective experience rating (adjustment future premium based on experience) on a risk or group of risks [3]. By using credibility theory, it is possible to calculate the claim reserve by combining claim payment data with other information.
One of the methods in credibility theory is greatest accuracy credibility theory which assumes that every policy holder has a risk parameter that quantifies the risk that is faced by each policy holder. Bühlmann-Straub credibility model is part of greatest accuracy credibility theory that is commonly used. In Bühlmann-Straub credibility model, the predictor of hypothetical mean (average amount of claim payment given the risk parameter of the policy holder) is assumed as a linear function of the claim payment in the past. It is also assumed that the conditional mean of the claim payment is constant for every period, and there is a measuring exposure that differentiates the conditional variance of claim payment that comes from different periods [3].

The products of insurance companies are commonly divided into several lines of business. Furthermore, in general, the calculation of claim reserve in a company is done by calculating the claim reserve in each line of business in the company, then the claim reserve for the company (aggregate reserve) is obtained by adding up the claim reserves in each line of business. Taking into account the possibility of correlation between existing lines of business in insurance companies, the value of aggregate reserve can actually be less than the sum of the claim reserves in each of the existing lines of business [4]. Therefore, research on the calculation of claim reserve then evolves by considering claim payment data from various lines of business in a company, or also called claim reserve calculation in a multivariate context. In this paper, a research is conducted on the development of multivariate Bühlmann-Straub credibility model for claim reserving along with estimation for parameters of the model.

2. Experimental

2.1. Run-Off triangle

For every loss that incurred, there could be delay from the time of loss until the claim payment for the loss. To accommodate the delayed claim payment data, an instrument called run-off triangle is constructed. There are two main components in run-off triangle. The first is accident year, which indicates the year when loss incurred, and the second is development year, which indicates time delay, in years, from the occurrence of loss to the year when claim payment is paid out. In this paper, we assume that the claim payment data is available from \( N \) run-off triangle and every run-off triangle contains claim payment data from one line of business in insurance company. In figure 1, we provide the illustration of data in run-off triangle, where every row in the run-off triangle represents accident year, and every column in the run-off triangle represents development year.

In the run-off triangle shown in figure 1, \( X_{i,j}^{(n)} \) denotes the claim payment for the loss that incurred in accident year \( i, i \in \{0, 1, \ldots, I\} \) and paid out \( j \) year after the loss incurred from the \( n \)-th run-off triangle where \( j \in \{0, 1, \ldots, J\} \) and \( n \in \{1, 2, \ldots, N\} \). For every accident year \( i \), the claim payment that has been paid out is from development year \( 0, 1, \ldots, I - i \). Claim payment that is paid out in development year \( I - i + 1, \ldots, J \) are not observed yet, and the value is predicted.

![Figure 1. Illustration of Run-Off Triangle](image-url)
In general, we could say that the claim payment data in upper triangular part of run-off triangle is observed, meanwhile the claim payment data in lower triangular part of run-off triangle is not observed yet, and going to be predicted.

The amount that has to be reserved by the company is determined by the sum of the predicted amount of claim payment that will be settled in the future for each accident year in every run-off triangle, that is the claim payment that is settled in the \( I - i + 1, ..., J \) development year for the \( i \)-th accident year. The claim reserve, well known as the outstanding claim, for the \( i \)-th accident year in the \( n \)-th run-off triangle, notated by \( R_i^{(n)} \), is defined as below.

\[
R_i^{(n)} = \sum_{j=i+1}^{J} X_{i,j}^{(n)}
\]

### 2.2. Bühlmann-Straub credibility model

In this section, we will discuss about Bühlmann-Straub credibility Model. Suppose that \( X_1, X_2, ..., X_n \) denote claim amount of a policy holder in the \( j \)-th period where \( j \in \{1, ..., n\} \). In this paper, we will discuss about the random variable \( X_j \) nonparametrically, so that we will not have any assumption about the distribution of \( X_j \). Assume that the policy holder has risk parameter \( \theta \). Suppose that:

\[
\begin{align*}
\mathbb{E}[X_j|\theta] &= \mu(\theta), \quad j = 1, ..., n. \\
\text{Var}[X_j|\theta] &= \frac{\nu(\theta)}{m_j}, \quad j = 1, ..., n.
\end{align*}
\]

where \( m_j \) is measuring exposure that is proportional to risk size in \( j \)-th period. \( \mu(\theta) \) is called hypothetical mean, the average of claim amount of a policyholder with a given value of \( \theta \) and \( \nu(\theta) \) is called process variance, the variance of claim amount of a policyholder with a given value of \( \theta \). Suppose that \( \mu = \mathbb{E}[\mu(\theta)] \) is the expected value of hypothetical mean, \( \nu = \mathbb{E}[\nu(\theta)] \) is the expected value of process variance, and \( \alpha = \text{Var}[\mu(\theta)] \) is the variance of hypothetical mean. The claim amount in the \( n + 1 \) period will be predicted by using the predicted value of hypothetical mean. By assuming that the predictor of the hypothetical mean is in the form of linear function of the past claim payment and by doing some mathematical operation, we obtain the predictor of the hypothetical mean as follow:

\[
\hat{\mu}(\theta) = Z\bar{X} + (1 - Z)\mu
\]

with

\[
\bar{X} = \frac{\sum_{j=1}^{n} m_j X_j}{m}
\]

\[
m = \sum_{j=1}^{n} m_j
\]

\[
Z = \frac{m}{m + \frac{\nu}{\alpha}}
\]

\( Z \) is called Bühlmann-Straub credibility factor.
3. Results and discussion

We provide the flowchart to summarize the step by step process that will be done in the results and discussion section in figure 2. Each of the step in figure 2 will be explained more detail in every subsection in the section of results and discussion.

3.1. Multivariate Bühlmann-Straub credibility model

In this section, we will build the multivariate Bühlmann-Straub credibility model. Suppose that there are \( N \geq 1 \) run-off triangles where each run-off triangle contains claim payment data from a line of business in insurance company. Note that the run-off triangles contain the claim payment data of loss that occurred in accident year \( i, i \in \{0,1,...,I\} \). The claim payment of the loss could be settled in the same year with the year when the loss incurred, one year after the loss incurred, until \( j \) year after the loss incurred, where \( j \in \{0,1,...,J\} \). The delayed time from the occurrence of loss until the settlement of claim is called development year. In run-off triangle, \( I = J \), and in this paper, it is assumed that the \( N \) run-off triangles have the same size. Hereinafter, those claim payment data is called as incremental claim payment. For every accident year \( i \) in the \( n \)-th run-off triangle, the claim payment data that has been observed are the data from development year \( i \), meanwhile the claim payment in development year \( i + 1, \ldots, J \) will be predicted.

Note that every accident year has risk parameter, so it is possible that loss that happen in a accident year is much higher than loss that happen in other accident year. The risk parameter is notated by \( \theta_i \), risk parameter for \( i \)-th accident year. We define \( \mu_i^{(n)} \) as prior information about the total of loss that incurred in the \( i \)-th accident year in the \( n \)-th run-off triangle and \( y_j^{(n)} \) as the proportion of the claim that is being paid in the \( j \)-th development year to prior information about the total of loss that incurred in the \( n \)-th run-off triangle. The expected value of the incremental claim payment is as below:

\[
E[X_{i,j}^{(n)}] = y_j^{(n)} \mu_i^{(n)} = w_{i,j}^{(n)}, \quad i = 0, \ldots, I, \quad j = 0, \ldots, J, \quad n = 1, \ldots, N. \tag{8}
\]
As the claim payment data comes from several different lines of business in insurance company and it is possible that the claim payment in one line of business is much higher than the claim payment in the other line of business, we normalize the claim payment data. We define normalized incremental claim payment as below.

\[ Y_{i,j}^{(n)} = \frac{X_{i,j}^{(n)}}{\mu_{i,j}^{(n)}} = \frac{X_{i,j}^{(n)}}{w_{i,j}^{(n)}} \]  

and the expected value of the normalized incremental claim payment is as below:

\[ E\left[ Y_{i,j}^{(n)} \right] = E\left[ \frac{X_{i,j}^{(n)}}{\mu_{i,j}^{(n)}} \right] = E\left[ \frac{X_{i,j}^{(n)}}{w_{i,j}^{(n)}} \right] = w_{i,j}^{(n)} = 1. \]  

Furthermore, for every accident year \( i, i \in \{0, ..., I\} \), the normalized incremental claim payment data in the \( j \)-th development year that comes from \( N \) run-off triangle is summarized into a vector, \( Y_{ij} = \left( Y_{i,j}^{(1)}, Y_{i,j}^{(2)}, ..., Y_{i,j}^{(N)} \right)^T \) for \( j = 0, ..., J \), so that for every accident year there are \( J + 1 \) vectors where each vector summarize normalized claim payment for each development year. Those \( J + 1 \) vectors for every accident year is summarized into a vector, \( Y_i = \left( Y_{i,0}^T, Y_{i,1}^T, ..., Y_{i,J}^T \right)^T \). Now, we have \( I + 1 \) vectors, \( Y_i \), for every accident year. There are several model assumptions that are applied in this paper, those assumptions are necessary to build the model and estimate the parameters of the model. Those assumption are as below.

- \((\theta_0, Y_0), (\theta_1, Y_1), ..., (\theta_I, Y_I)\) are independent.
- \(\theta_0, ..., \theta_I\) are identically distributed.
- For \( i = 0, ..., I, j = 0, 1, ..., J, Y_{i,j}^{(1)}|\theta_0, ..., Y_{i,j}^{(N)}|\theta_0\) are independent.
- For \( i = 0, ..., I, Y_{i,0}|\theta_0, ..., Y_{i,J}|\theta_i\) are independent.
- \(E\left[ Y_{i,j}^{(n)}|\theta_i \right] = \mu_n(\theta_i), \) for \( i = 0, ..., I, j = 0, ..., J, \)

\[ E[Y_{i,j}|\theta_i] = \begin{pmatrix}
E[Y_{i,j}^{(1)}|\theta_i] \\
E[Y_{i,j}^{(2)}|\theta_i] \\
\vdots \\
E[Y_{i,j}^{(N)}|\theta_i]
\end{pmatrix} = \begin{pmatrix}
\mu_1(\theta_i) \\
\mu_2(\theta_i) \\
\vdots \\
\mu_N(\theta_i)
\end{pmatrix} = \mu(\theta_i) \]  

\[ E[\mu_n(\theta_i)] = E\left[ E\left( Y_{i,j}^{(n)}|\theta_i \right) \right] = 1, n = 1, ..., N. \]  

\(\mu_n(\theta_i)\) is the hypothetical mean of the \( i \)-th accident year in the \( n \)-th run-off triangle. We define \( \tau_n^2 = \text{Var}[\mu_n(\theta_i)] \) as the variance of hypothetical mean for the \( n \)-th run-off triangle and \( \tau_{nm}^2 = \text{Cov}[\mu_n(\theta_i), \mu_m(\theta_i)] \) as the covariance between the hypothetical mean from the \( n \)-th and \( m \)-th run-off triangle.
\( \text{Var}\left[Y_{i,j}^{(n)} \bigg| \Theta_i \right] = \frac{\sigma_i^2(\Theta_i)}{w_{i,j}^{(3)}} \), where \( w_{i,j}^{(3)} = w_{i,j}^{(n)} \mu_i^{(n)} \) for \( n = 1, \ldots, N \).

\[
\text{Cov} \left( Y_{i,j}, Y_{i,j}^{(n)} \big| \Theta_i \right) = \begin{pmatrix}
\frac{\sigma_i^2(\Theta_i)}{w_{i,j}^{(3)}} & 0 & \ldots & 0 \\
0 & \frac{\sigma_i^2(\Theta_i)}{w_{i,j}^{(2)}} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \frac{\sigma_i^2(\Theta_i)}{w_{i,j}^{(N)}}
\end{pmatrix}
\]

\( \sigma_i^2(\Theta_i) \) is the process variance for the \( i \)-th accident year in the \( n \)-th run-off triangle and \( \sigma_i^2 = E[\sigma_i^2(\Theta_i)] \) is the expected value of process variance for the \( n \)-th run-off triangle.

Please note that the term of risk parameters is a theoretical term that not are not possible to be measured in real life case. Therefore, when we use the model to predict claim reserve in real insurance industry, it is not necessary to check the model assumptions.

For every accident year \( i, i \in \{0, \ldots, I\} \) in every run-off triangle \( n, n \in \{1, \ldots, N\} \), the normalized incremental claim payment that has been settled in \( I - i + 1 \) development year, development year \( 0, 1, \ldots, I - i \), is summarized into a scale, that is the weighted average of the normalized incremental claim payment in the \( i \)-th accident year in the \( n \)-th run-off triangle. We define the weighted average, as below.

\[
K_i^{(n)} = \sum_{j=0}^{I-i} w_{i,j}^{(n)} y_{i,j}^{(n)}
\]

where

\[
w_{i}^{(n)} = \sum_{j=0}^{I-i} w_{i,j}^{(n)}
\]

For every accident year \( i, i \in \{0,1,\ldots,I\} \), the weighted average from the \( N \) run-off triangle is summarized into a vector, as below.

\[
K_i = \begin{pmatrix}
K_i^{(1)} \\
K_i^{(2)} \\
\vdots \\
K_i^{(N)}
\end{pmatrix}
\]

Through some mathematical process, the multivariate Bühlmann-Straub credibility model to predict hypothetical mean for each accident year in every run-off triangle by giving some weight to \( K_i^{(n)} \) is obtained as below.

\[
\mu(\Theta_i)^{cred} = A_i K_i + (I - A_i) 1
\]

where

\[
A_i = T \cdot \left( T + W_i^{-2} S W_i^{-2} \right)^{-1}
\]
\[
T = \begin{pmatrix}
\tau_1^2 & \tau_{12} & \ldots & \tau_{1N} \\
\tau_{21} & \tau_2^2 & \ldots & \tau_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\tau_{N1} & \tau_{N2} & \ldots & \tau_N^2
\end{pmatrix}, \quad S = \begin{pmatrix}
\sigma_1^2 & 0 & \ldots & 0 \\
0 & \sigma_2^2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \sigma_N^2
\end{pmatrix}
\]

(19)

\[
W_i^{-1} = \begin{pmatrix}
\frac{1}{2} \left( \frac{1}{2} \mu_i^{(1)} \sum_{j=0}^{i-1} y_j^{(1)} \right) & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & \frac{1}{2} \left( \frac{1}{2} \mu_i^{(N)} \sum_{j=0}^{i-1} y_j^{(N)} \right)
\end{pmatrix}
\]

(20)

\[
K_i = \begin{pmatrix}
K_i^{(1)} \\
K_i^{(2)} \\
\vdots \\
K_i^{(N)}
\end{pmatrix}, \quad I = \begin{pmatrix}
1 & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & 1
\end{pmatrix}, \quad 1 = \begin{pmatrix}
1 \\
1 \\
\vdots \\
1
\end{pmatrix}
\]

(21)

\( S \) matrix in equation 19 is the matrix that contains expected value of process variance from each run-off triangle in the main diagonal and \( T \) matrix in equation 19 is the matrix that contains covariance between hypothetical mean of the run-off triangles.

3.2. Multivariate Bühlmann-Straub credibility model for claim reserving

The prediction of the hypothetical mean \( (\mu_i(\theta_i)) \) that have been obtained before is used to predict outstanding claim \( (R_i^{(n)}) \) for \( i = 1, \ldots, I, \ n = 1, \ldots, N \). The formula of outstanding claim is as below.

\[
R_i^{(n)} = \sum_{j=1-i+1}^{j} X_{i,j}^{(n)} = \sum_{j=1-i+1}^{j} \mu_i^{(n)} Y_j^{(n)} L_j^{(n)}
\]

(22)

The unknown value of \( Y_j^{(n)} \) for \( i = 1, \ldots, I, j = I - i + 1, \ldots, J, j = 1, \ldots, N \) is predicted by the predictor of hypothetical mean that have been obtained before, so that we get the new formula for outstanding claim, as below.

\[
R_i^{(n), cred} = \sum_{j=I-i+1}^{j} \mu_i^{(n)} Y_j^{(n)} \mu_n(\Theta_i) \quad \mu_n(\Theta_i) \quad \mu_n(\Theta_i) \quad \mu_n(\Theta_i) \quad \mu_n(\Theta_i) \quad \mu_n(\Theta_i)
\]

(23)
If the formula for outstanding claim in the \( i \)-th accident year from every run-off triangle in equation 23 are written in the form of vectors, we get the equation below.

\[
R_i^{\text{cred}} = \begin{pmatrix}
\mu_1^{(1)} & 0 & \ldots & 0 \\
0 & \mu_1^{(2)} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & \mu_i^{(m)}
\end{pmatrix}
\begin{pmatrix}
\gamma_1^{(1)} \\
\gamma_2^{(2)} \\
\vdots \\
\gamma_j^{(m)} \\
\vdots \\
\gamma_{j-I}^{(m)}
\end{pmatrix}
\begin{pmatrix}
\mu_2^{(1)}^{\text{cred}} \\
\mu_2^{(2)}^{\text{cred}} \\
\vdots \\
\mu_n^{(m)}^{\text{cred}}
\end{pmatrix}
\]  
(24)

### 3.3. Estimation of the parameters

The estimation of the parameters in the model are as below:

\[
\hat{\gamma}_j^{(n)} = \frac{\sum_{i=0}^{j-i} \hat{w}_{i,j}^{(n)}}{\sum_{i=0}^{j-i} \hat{w}_{i,j}^{(n)}} \text{, for } j = 0, \ldots, f, \ n = 1, \ldots, N.
\]
(25)

\[
\sigma_n^2 = \frac{1}{T} \sum_{i=0}^{T-i} \sum_{i=0}^{I-i} \left( \hat{w}_{i,j}^{(n)} - \hat{K}_i^{(n)} \right)^2
\text{, for } n = 1, \ldots, N
\]
(26)

where

\[
\hat{w}_{i,j}^{(n)} = \hat{y}_j^{(n)} \cdot \hat{\mu}_i^{(n)}
\]
(27)

\[
\hat{w}_i^{(n)} = \sum_{j=0}^{I-i} \hat{w}_{i,j}^{(n)}
\]
(28)

\[
\hat{K}_i^{(n)} = \sum_{j=0}^{I-i} \hat{w}_{i,j}^{(n)} \hat{y}_j^{(n)}
\]
(29)

\[
\hat{\tau}_n = \left[ \hat{w}^{(n)} - (\hat{w}^{(n)})^{-1} \sum_{i=0}^{I-i} \hat{w}_i^{(n)} \right]^{-1} \left\{ \sum_{i=0}^{I-i} \hat{w}_i^{(n)} \left( \hat{K}_i^{(n)} - \hat{K}_i^{(n)} \right)^2 - 1 \sigma_n^2 \right\}
\text{, for } n = 1, \ldots, N.
\]
(30)

\[
\hat{\tau}_{nm} = \frac{\hat{\tau}_{nm}^{**} + \hat{\tau}_{nm}^{**}}{2}
\text{, for } n = 1, \ldots, N, \ m = 1, \ldots, N, \ n \neq m.
\]
(31)

where

\[
\hat{\tau}_{nm}^{**} = \left[ \hat{w}^{(n)} - (\hat{w}^{(n)})^{-1} \sum_{i=0}^{I-i} \hat{w}_i^{(n)} \right]^{-1} \left\{ \sum_{i=0}^{I-i} \hat{w}_i^{(n)} \left( \hat{K}_i^{(n)} - \hat{K}_i^{(n)} \right) \left( \hat{K}_i^{(n)} - \hat{K}_i^{(n)} \right) \right\}
\]
(32)
3.4. Claim reserving calculation for several lines of business in insurance company by using Multivariate Bühlmann-Straub credibility model

After building multivariate Bühlmann-Straub credibility model for claim reserving, we will use the model to predict claim reserve for three lines of business in a company. The data is published by National Association of Insurance Commissioners of the United State. The data that is used in this paper is claim payment of the loss that incurred in 2008-2017 in three line of business, for instance private passenger auto liability/medical (line of business-1), commercial auto/truck liability/medical (line of business-2), and medical professional liability (line of business-3). In this case, 2008 is the 0-th accident year, 2009 is the 1-st accident year, and so on until 2017 is the 9-th accident year. We provide the claim payment data for those three lines of business (that is represented by run-off triangle) for each line of business in table 1, table 2 and table 3.

Table 1 contains the claim payment data for private passenger auto liability/medical (line of business-1). For the loss that incurred in the 0-th accident year (2008), the claim payment that is settled in the same year is 64,425,248,000 Dollar, the claim payment that is settled one year after the loss incurred (in 2009) is 64,615,344,000 Dollar and so on until the claim payment that is settled 9 year after the loss incurred (in 2017) is 63,008,624,000 Dollar. The rest of the amount in table 1 as well as the amount in table 2 and table 3 can be read in similar way.

| Accident year | 0     | 1     | 2     | 3     | 4     |
|---------------|-------|-------|-------|-------|-------|
| 0             | 64,425,248 | 64,615,344 | 63,947,967 | 63,451,655 | 63,160,339 |
| 1             | 69,122,182 | 68,215,524 | 67,031,132 | 66,766,830 | 66,824,823 |
| 2             | 70,045,322 | 69,088,325 | 68,294,302 | 68,180,620 | 68,033,348 |
| 3             | 70,761,191 | 69,515,068 | 69,481,139 | 69,413,636 | 69,310,522 |
| 4             | 71,876,094 | 71,240,181 | 71,131,997 | 71,036,946 | 70,970,627 |
| 5             | 73,578,060 | 73,473,509 | 73,570,478 | 73,594,911 | 73,404,339 |
| 6             | 76,907,569 | 77,308,605 | 77,605,035 | 77,605,132 |  |
| 7             | 83,208,096 | 85,116,899 | 85,655,885 |  |
| 8             | 91,056,234 | 91,875,579 |  |
| 9             | 94,631,893 |  |  |  |  |

### Table 1. Run-off triangle from line of business-1 (Thousand dollar)
### Table 1 (continued). Run-off triangle from line of business-1 (Thousand dollar)

| Accident year | Development year | 5     | 6     | 7     | 8     | 9     |
|---------------|------------------|-------|-------|-------|-------|-------|
| 0             | 63,067,370       | 63,039,216 | 63,042,580 | 62,995,003 | 63,008,624 |
| 1             | 66,824,823       | 66,851,999 | 66,787,318 | 66,766,210 |
| 2             | 68,032,646       | 67,965,237 | 67,924,450 |
| 3             | 69,212,679       | 69,240,554 |
| 4             | 70,924,547       |           |           |       |       |       |
| 5             |                   |           |           |       |       |       |
| 6             |                   |           |           |       |       |       |
| 7             |                   |           |           |       |       |       |
| 8             |                   |           |           |       |       |       |
| 9             |                   |           |           |       |       |       |

### Table 2. Run-off triangle from line of business-2 (Thousand dollar)

| Accident year | Development year | 0     | 1     | 2     | 3     | 4     |
|---------------|------------------|-------|-------|-------|-------|-------|
| 0             | 11,300,476       | 11,130,239 | 11,118,936 | 11,058,916 | 11,065,022 |
| 1             | 10,575,756       | 10,200,690 | 10,188,622 | 10,146,042 | 10,152,073 |
| 2             | 10,441,691       | 10,474,983 | 10,654,795 | 10,788,071 | 10,917,577 |
| 3             | 10,635,850       | 11,071,102 | 11,352,879 | 11,524,280 | 11,743,920 |
| 4             | 10,982,527       | 11,330,569 | 11,589,953 | 11,930,071 | 12,077,099 |
| 5             | 11,671,371       | 11,967,753 | 12,571,700 | 12,968,855 | 13,113,621 |
| 6             | 12,361,680       | 12,957,744 | 13,565,408 | 13,920,912 |
| 7             | 13,405,557       | 14,177,961 | 14,776,655 |
| 8             | 14,535,568       | 15,184,659 |
| 9             | 15,554,045       |           |           |       |       |       |
| 5             |                   |           |           |       |       |       |
| 6             |                   |           |           |       |       |       |
| 7             |                   |           |           |       |       |       |
| 8             |                   |           |           |       |       |       |
| 9             |                   |           |           |       |       |       |
Table 3. Run-off triangle from line of business-3 (Thousand dollar)

| Accident year | Development year |
|---------------|------------------|
|               | 0                | 1                | 2                | 3                | 4                |
| 0             | 1,919,577        | 1,901,349        | 1,879,668        | 1,828,405        | 1,747,551        |
| 1             | 1,905,384        | 1,855,721        | 1,808,926        | 1,756,678        | 1,696,750        |
| 2             | 1,905,062        | 1,825,058        | 1,807,572        | 1,768,620        | 1,679,909        |
| 3             | 1,991,771        | 1,936,864        | 1,865,927        | 1,779,858        | 1,669,464        |
| 4             | 2,004,277        | 1,947,175        | 1,882,231        | 1,761,241        | 1,746,515        |
| 5             | 1,987,695        | 1,937,882        | 1,881,504        | 1,859,786        | 1,810,324        |
| 6             | 1,972,180        | 1,942,112        | 1,883,439        | 1,883,069        |                  |
| 7             | 1,903,049        | 1,875,697        | 1,859,773        |                  |                  |
| 8             | 1,903,881        | 1,864,325        |                  |                  |                  |
| 9             | 1,987,695        | 1,937,882        |                  |                  |                  |
| 5             | 1,670,821        | 1,603,610        | 1,552,529        | 1,531,182        | 1,512,496        |
| 6             | 1,596,004        | 1,546,077        | 1,511,317        | 1,480,210        |                  |
| 7             | 1,614,731        | 1,603,427        | 1,505,510        |                  |                  |
| 8             | 1,621,925        | 1,561,730        |                  |                  |                  |
| 9             | 1,674,067        |                  |                  |                  |                  |

We also provide the prior information about total of loss that incurred during that period, that is assumed to be obtained from actuary opinion, in table 4.

As we can see in table 4, the prior information about total of loss that incurred in 0-th accident year (2008) for line-of business-1, line of business-2, and line of business-3 respectively are 635,000,000,000 Dollar, 111,000,000,000 Dollar, and 17,100,000,000 Dollar. The rest of the row in table 4 can be seen in similar way. By using Python Programming Language in Enthought Canopy (Version 1.7.4.3348, 2012), we get the estimator of the parameters in the model. After getting the estimators of the parameters in model, we predict the hypothetical mean for accident year \( i \in \{1, 2, \ldots, 9\} \) in every run-off triangle \( n, n \in \{1, 2, 3\} \), as shown in table 5.

In table 5, it is shown that the predicted value of hypothetical mean (expected value of the normalized incremental claim payment, given the risk parameter of the accident year) of the 1-st accident year in the line of business-1, line of business-2, and line of business-3 respectively are 0.999667, 0.998508, and 1.001117. The rest of the value in table 5 can be seen in similar way. After having the predicted value of the hypothetical mean, we predict the outstanding claim for accident year \( i, i \in \{1, 2, \ldots, 9\} \) in every run-off triangle \( n, n \in \{1, 2, 3\} \), as shown in table 6.
Table 4. Prior information about total of loss that incurred (Dollar)

| Accident year | Run-off triangle |
|---------------|------------------|
|               | 1                | 2                | 3                |
| 0             | 635,000,000,000  | 111,000,000,000 | 17,100,000,000   |
| 1             | 673,000,000,000  | 102,000,000,000 | 16,600,000,000   |
| 2             | 683,000,000,000  | 107,000,000,000 | 16,700,000,000   |
| 3             | 694,000,000,000  | 114,000,000,000 | 17,000,000,000   |
| 4             | 709,000,000,000  | 117,000,000,000 | 17,300,000,000   |
| 5             | 731,000,000,000  | 125,000,000,000 | 17,500,000,000   |
| 6             | 767,000,000,000  | 133,000,000,000 | 17,400,000,000   |
| 7             | 838,000,000,000  | 144,000,000,000 | 16,800,000,000   |
| 8             | 905,000,000,000  | 153,000,000,000 | 16,700,000,000   |
| 9             | 936,000,000,000  | 162,000,000,000 | 16,300,000,000   |

Table 5. Prediction of hypothetical mean

| Accident year | Run-off triangle |
|---------------|------------------|
|               | 1                | 2                | 3                |
| 0             | 0.999667         | 0.998508         | 1.001117         |
| 1             | 1.000469         | 1.003894         | 0.997877         |
| 2             | 0.999473         | 0.999201         | 0.997847         |
| 3             | 0.999973         | 0.997247         | 0.997478         |
| 4             | 0.999961         | 0.999924         | 0.999551         |
| 5             | 1.000733         | 1.002082         | 1.002055         |
| 6             | 1.000591         | 0.999096         | 1.002305         |
| 7             | 0.999817         | 1.001858         | 0.999103         |
| 8             | 0.999784         | 1.002301         | 0.998712         |
| 9             | 0.999667         | 0.998508         | 1.001117         |

From table 6, we can see that for the loss that incurred in 2009 (accident year 1), line of business-1 has to provide claim reserve for the amount of 66,756,994,700 Dollar, line of business-2 has to provide claim reserve for the amount of 10,103,304,000 Dollar, and line of business-3 has to provide claim reserve for the amount of 1,469,911,690 Dollar. The other lines in table 6 can be interpreted in similar way. In table 4, we will show the error of the prediction of claim reserve for each of line of business by using multivariate Bühlmann-Straub credibility model and by using standard Bühlmann-Straub credibility model.

From table 7, we can see that the error of prediction by using multivariate Bühlmann-Straub credibility model is lower than the error of prediction by using standard Bühlmann-Straub credibility model.
Table 6. Prediction of outstanding claim (Dollar)

| Accident year | Run-off triangle |
|---------------|------------------|
|               | 1                | 2                | 3                |
| 0             | 66,756,994,700   | 10,103,304,000   | 1,469,911,690    |
| 1             | 135,592,579,000 | 21,297,455,900  | 2,963,102,980    |
| 2             | 206,533,900,000 | 33,970,566,600  | 4,557,737,230    |
| 3             | 281,631,009,000 | 46,572,815,400  | 6,249,613,040    |
| 4             | 363,175,360,000 | 62,585,580,600  | 8,023,816,440    |
| 5             | 457,952,486,000 | 80,352,160,300  | 9,763,792,330    |
| 6             | 584,313,619,000 | 101,381,080,000 | 11,208,750,500   |
| 7             | 721,686,303,000 | 123,427,123,000 | 12,925,278,300   |
| 8             | 840,942,457,000 | 146,673,656,000 | 14,427,519,800   |
| 9             | 66,756,994,700  | 10,103,304,000  | 1,469,911,690    |

Table 7. Error of prediction for each line of business

| Line of business | Multivariate | Univariate |
|-----------------|--------------|------------|
| 1               | 0.7899 %     | 0.7954 %   |
| 2               | 2.9286 %     | 2.9438 %   |
| 3               | 0.8239 %     | 0.8726 %   |

4. Conclusion

The multivariate Bühlmann-Straub credibility model has been built for claim reserving by considering the claim payment data from several lines of business in a company. There are four parameters in the model, namely $\gamma_j$, $\sigma_j^2$, $\tau_n^2$, and $\tau_{nm}$, that are estimated nonparametrically. The model is used to calculate claim reserve for three LoBs of insurance company in United State, based on the data of claim payment for the loss that incurred during the period of 2008-2017 that was published by Association of Insurance Commissioners of the United State. It appears that the error of prediction by using multivariate Bühlmann-Straub credibility model is lower than the error of prediction by using standard Bühlmann-Straub credibility model.

References

[1] Presiden Republik Indonesia 2014 Undang-Undang Republik Indonesia Nomor 40 Tahun 2014 Tentang Usaha Perasuransian (Indonesia: UU) available at https://www.ojk.go.id/id/kanal/iknb/regulasi/asuransi/undang-undang/Documents/Pages/Undang-Undang-Nomor-40-Tahun-2014-Tentang-Perasuransian/UU%20Nomor%2040%20Tahun%202014.pdf

[2] Wuthrich M V 2018 Neural Networks Applied to Chain-Ladder Reserving available at http://dx.doi.org/10.2139/ssrn.2966126
[3] Klugman S A, Panjer H H and Willmot G E 2012 *Loss Models: From Data to Decision* 4th edition (New York: John Wiley & Sons, Inc.) pp 357-430

[4] Bjarnason T and Sjogren M 2014 *Insurance Loss Reserving* Master thesis (Lund: Mathematical Statistics, Lund University)

[5] Happ S, Maier R and Merz M 2014 *Variance* 8 23-42

[6] Bühlmann H and Gisler A 2005 *A Course in Credibility Theory and Its Application* (Berlin: Springer) pp 167-89