Evolution of curvature perturbation in generalized gravity theories

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Received 11 March 2009, in final form 1 June 2009
Published 2 July 2009
Online at stacks.iop.org/CQG/26/145016

Abstract
Using the cosmological perturbation theory in terms of the $\delta N$ formalism, we find the simple formulation of the evolution of the curvature perturbation in generalized gravity theories. Compared with the standard gravity theory, a crucial difference appears in the end-boundary of the inflationary stage, which is due to the non-ideal form of the energy–momentum tensor that depends explicitly on the curvature scalar. Recent study shows that ultraviolet-complete quantum theory of gravity (Hořava–Lifshitz gravity) can be approximated by using a generalized gravity action. Our paper may give an important step in understanding the evolution of the curvature perturbation during inflation, where the energy–momentum tensor may not be given by the ideal form due to the corrections from the fundamental theory.

PACS numbers: 98.80.Cq, 98.80.Es, 04.60.Kz

1. Introduction
The inflationary universe scenario is the most successful model for explaining the large-scale structure of the universe. The spectrum of the cosmological perturbations created during inflation is expected to be scale invariant and Gaussian. An inflationary universe is consistent with current observations of the temperature anisotropy of the cosmic microwave background (CMB), except for some anomalies in the spectrum. These anomalies include a small departure from exact scale invariance and a certain non-Gaussian character [1, 2], both of which can help reveal the underlying gravitational and field theoretical model as well as the dynamics of the fields during inflation. There are many models of the mechanism for generating curvature perturbations, which can be categorized either in terms of (A) when and (B) how they are created. Focusing on the question of when perturbations are generated, curvature perturbations can be generated (A-1) at the horizon exit [3], (A-2) by the evolution during inflation [4, 5], (A-3) at the inhomogeneous end of inflation [6, 7], (A-4) just after inflation...
during preheating/reheating [8–10], or (A-5) late after inflation by curvatons [11, 12] or by an inhomogeneous phase transition [13]. In this paper, we consider the curvature perturbations in generalized gravity theories for (A-1), (A-2) and (A-3): the generation at the horizon crossing, evolution during inflation and the correction at the inhomogeneous end of inflation. Focusing on the mechanism of generating curvature perturbations, curvature perturbations can be created by (B-1) perturbations at the boundary between different scaling era [13], (B-2) non-adiabatic evolution [4] or (B-3) the inhomogeneities in the fraction of the contents that have different scaling (curvatons)[11]. The case (B-1) includes inhomogeneous end of inflation [6, 7], inhomogeneous reheating [9] or inhomogeneous phase transition [13]. This paper considers the cases (B-1) and (B-2) for the inflationary stage. Although curvatons (B-3) or the creation of the curvature perturbations after inflation ((A-4) and (A-5)) may lead to a significant result in the generalized gravity theory1, they usually depend on the specific choice of the inflationary model and the fields in the effective action. Therefore, for simplicity of the argument, we do not consider the creation of curvature perturbations after inflation ((A-4) and (A-5)), which includes inhomogeneous reheating/preheating and the curvatons.

Considering the discussions of multi-field inflation, there are at least three ways to reach the same result of the cosmological perturbations. One is the straight calculation of ˙R, which is considered in [15]. The calculation is based on the field equations and the perturbations. Another is to calculate the evolution of the ‘perturbed expansion rate’ [4, 17] using the equations for the energy–momentum and the perturbations, which is considered in this paper. The calculation based on this method is very easy and straight compared with the former one. The last method, which is intuitive but may be rather sketchy, is to calculate first the total number of e-foldings, then calculate δN. This method must be complemented by the study of the evolution during inflationary stage [18].

2. Curvature perturbations in generalized gravity theories

In this paper, the Lagrangian that defines the generalized $f(\phi, R)$ gravity is given by

\[
\mathcal{L} = \frac{1}{2} f(\phi^I, R) + P(X, \phi^I),
\]

(2.1)

where $X$ is defined by $X = -\frac{1}{2} G_{IJ} \partial_\mu \phi^I \partial^\mu \phi^J$. Here $\phi^I$ and $R$ are the scalar fields and the scalar curvature. We consider the separation

\[
P(X, \phi^I) = K(X, \phi^I) - V(\phi^I),
\]

(2.2)

and set $8\pi G = 1$ for simplicity. We define the energy–momentum in the standard way [16]. General coordinate invariance suggests that the energy–momentum tensor follows the conservation law, which is true without using the Einstein field equations.

It is possible to characterize cosmological perturbations using the line element with linear scalar perturbations of a Friedman–Lemaître–Robertson–Walker (FLRW) background, which can be written as

\[
ds^2 = -(1 + 2A) dt^2 + 2a^2(t) \left[ \nabla_i B \right] dt dx^i + a^2(t) \left[ (1 - 2\psi) \gamma_{ij} + 2 \nabla_i \nabla_j E \right] dx^i dx^j.
\]

(2.3)

In FLRW spacetime, which is spatially homogeneous and isotropic, from the variation of the metric $g_{\mu\nu}$, we get the gravitational field equation and the energy–momentum tensor. Assuming a simple model of two-field inflation with a standard kinetic term $K = X$, we obtain an energy density $\rho$ and pressure $p$ given by [16]

---

1 See [14], for an example with scalar–tensor gravity.
\[
\rho = \frac{1}{F} \left[ X + \frac{RF - f}{2} + V - 3H F \right],
\]
\[
p = \frac{1}{F} \left[ X - \frac{RF - f}{2} - V + 2HF - F \right],
\] (2.4)

where a dot (\(\dot{}\)) above a symbol denotes a derivative with respect to time. In this paper, we consider the adiabatic inflation field \(\phi\) and the so-called entropy field \(s\) for two-field inflation. Using the adiabatic inflation field \(\phi\), \(X\) is redefined as \(X = \frac{1}{2} \dot{\phi}^2\) in FLRW spacetime. \(F\) is defined as \(F = f_R \equiv \partial f / \partial R\). Here \(\dot{F}\) cannot be neglected even in the slow-rolling phase\(^2\). We thus find

\[
\rho + p = \frac{1}{F} [2X - HF + \dot{F}].
\] (2.5)

Our interest is in the evolution of the curvature perturbation \(\psi\) on the constant-time hypersurfaces of the FLRW background. Introducing a normalized 4-vector field \(n^\mu\) orthogonal to the constant-time hypersurfaces, the expansion of the spatial hypersurfaces with respect to the proper time \(d\tau \equiv (1 + A) dt\) is given by

\[
\theta \equiv n^\mu \partial_\mu = 3H(1 - A) - 3\dot{\psi} + \nabla^2 \sigma,
\] (2.6)

where \(\sigma\) is the scalar describing the shear, which is defined by \(\sigma \equiv \dot{E} - B\). A useful definition of the expansion rate with respect to the coordinate time is \([17]\)

\[
\tilde{\theta} = (1 + A)\theta = 3H - 3\dot{\psi} + \nabla^2 \sigma,
\] (2.7)

which can be used to define the perturbed expansion \(\delta\tilde{\theta} \equiv \tilde{\theta} - 3H\). Without using the gravitational field equations, the energy conservation equation \(n^\nu T_{\mu \nu} = 0\) for the first-order density perturbations can be evaluated. The equation is given by \([17]\)

\[
\dot{\delta}\rho = -3H(\delta\rho + \delta p) + (\rho + p)[3\dot{\psi} - \nabla^2 (\sigma + v + B)],
\] (2.8)

which gives the equation of \(\dot{\psi}\) in terms of the local conservation of energy–momentum\(^3\). The gauge-invariant combinations for the curvature perturbation are usually defined by

\[
\zeta = -\psi - H \frac{\delta\rho}{\rho},
\]
\[
R = \psi - H \frac{\delta q}{\rho + p},
\] (2.9)

where \(\delta q\) is the momentum perturbation satisfying

\[
\epsilon_m \equiv \delta\rho - 3H\delta q.
\] (2.10)

Here \(\epsilon_m\) is the perturbation of the comoving density, satisfying the evolution equation

\[
\epsilon_m = -\frac{1}{4\pi G} \frac{k^2}{a^2} \Psi,
\] (2.11)

where \(\Psi\) is related to the shear perturbation, which is assumed to be finite. The continuity equation \(\dot{\rho} = -3H(\rho + p)\), which can be used to relate these definitions, does not depend on the gravitational equations. The following useful expression can be straightforwardly obtained:

\(^2\) A useful example is given by \(F = \phi^2\), which leads to \(\dot{F} = 2\dot{\phi}^2 + 2\ddot{\phi}\phi \simeq 4X\) for slow-rolling approximation \(\dot{\phi} \simeq 0\).

\(^3\) The above equation is written for an ideal fluid; however, in the generalized gravity, the fluid is in general not of the ideal form. This does not affect the first-order analysis of the current work, since the non-ideal corrections to this equation are either of second order or contain spatial gradients which can be neglected on large scales. We appreciate a reviewer of the journal who sent us the argument related to the second-order perturbations in generalized gravity.
From the definition of $\zeta$, and using equation (2.8) to eliminate $\dot{\psi}$, a simple equation for the evolution of $\zeta$ is given by

\[
\dot{\zeta} \simeq -\frac{\delta \dot{\rho} + 3H(\delta \rho + \delta p)}{3(\rho + p)} + \frac{d}{dt} \left[ \frac{H \delta \rho}{3H(\rho + p)} \right] = -\frac{H}{\rho + p} \delta \dot{\rho} \rho + \delta \rho \delta p_{\text{nad}},
\]

(2.13)

where $\nabla^2(\sigma + \nu + B)$, which should vanish at large scales, is neglected in the first line. To compare the above result with the $\delta N$ formalism [19], it is useful to define the $\delta N$ formalism in terms of the perturbed expansion rate $\delta \theta$:

\[
\zeta = \frac{1}{3} \int_{t_{\text{ini}}}^{t} \delta \theta \, dt = \delta N,
\]

(2.14)

where a hypersurface with a flat slicing at $t_{\text{ini}}$ and uniform density at $t$ is chosen. Therefore, the evolution equation for the $\delta N$ formalism is given by

\[
\frac{d}{dt} \delta N \equiv \left. \frac{1}{3} \delta \theta \right|_{\delta \rho = 0} \simeq -\dot{\psi} \bigg|_{\delta \rho = 0} = \dot{\zeta} + \frac{d}{dt} \left( \frac{H \delta \rho}{\rho} \right) \bigg|_{\delta \rho = 0} = -\frac{H}{\rho + p} \delta \dot{\rho} \rho + \delta \rho \bigg|_{\delta \rho = 0}.
\]

(2.15)

Equations (2.13) and (2.15) are consistent as far as the $\delta N$ formalism is defined for a uniform density slice at $t$. Our goal in this paper is to find the evolution equation for the curvature perturbation $\zeta$, which can be defined in terms of the $\delta N$ formalism;

\[
\dot{\zeta} \equiv -\dot{\psi} \bigg|_{\delta \rho = 0} \simeq -H \delta \dot{\rho} + \delta p \bigg|_{\delta \rho = 0},
\]

(2.16)

where $\dot{\psi}$ is obtained from equation (2.8) on uniform density hypersurfaces ($\delta \dot{\rho} = 0$). $\delta \dot{\rho}$ in the last equation is for the later convenience. In usual inflation for the standard-gravity theory, $\dot{\zeta}_N$ is related to the perturbation of the adiabatic inflation velocity using the relation $\delta(\rho + P) \sim \delta(\dot{\phi}^2)$ [4]. This result is convincing as the origin of the inhomogeneities of the time elapsed during inflation ($\delta t$) is generally given by the inhomogeneities of the 'length' ($\delta \phi$) and the 'velocity' ($\dot{\phi}$). This simple result is not true for generalized gravity theories, as we will show in the following. An intuitive argument would be useful for the argument. In the generalized gravity theory, the gravitational constant may not be a homogeneous constant, which may source the inhomogeneities of the expansion rate. The situation is similar to the scalar–tensor gravity theory considered in [4, 15, 16].

\[4\] The perturbation of the kinetic term sources the creation of the curvature perturbations at the 'bend' in the trajectory of multi-field inflation [4, 18].
The perturbations of the energy density and the pressure are given by [16]

\[
\delta\rho \simeq \frac{1}{F}\left[\delta X + \frac{F\delta R + R\delta F - f_\phi \delta \phi - f_s \delta s}{2}
\right.
\]
\[
+ V_\phi \delta \phi + V_s \delta s - 3H\delta [\dot{F}] - \dot{F}\delta \theta \right] - \frac{\delta F}{F}\rho
\]
\[
= \frac{1}{F}\left[\delta X + \frac{(R - 2\rho)\delta F - f_\phi \delta \phi - f_s \delta s}{2}
\right.
\]
\[
+ V_\phi \delta \phi + V_s \delta s - 3H\delta [\dot{F}] - \dot{F}\delta \theta \right],
\]
\[
(2.17)
\]

\[
\delta p \simeq \frac{1}{F}\left[\delta X - \frac{(R + 2\rho)\delta F - f_\phi \delta \phi - f_s \delta s}{2}
\right.
\]
\[
- V_\phi \delta \phi - V_s \delta s + 2H\delta [\dot{F}] + \frac{2}{3}\dot{F}\delta \theta + \delta [\dot{F}] \right],
\]
\[
(2.18)
\]

where \(\delta X \equiv \dot{\phi}(\delta \dot{\phi} - \dot{\phi}A)\) and \(\delta [\dot{F}] \equiv \delta \dot{F} - \dot{F}A\) are introduced. Here \(\delta \theta\) denotes the perturbation of the expansion scalar \(\theta\), which follows the perturbed equation given by [16]

\[
H\delta \theta = -\frac{1}{2} \frac{k^2}{a^2} \psi,
\]
\[
(2.20)
\]

which suggests that \(\delta \theta\) is negligible at large scales on uniform density hypersurfaces. From the above equations we find

\[
\delta(\rho + p) = \frac{1}{F}\left[2\delta X - (\rho + p)\delta F - H\delta [\dot{F}] - \frac{1}{3}\dot{F}\delta \theta + \delta [\dot{F}] \right].
\]
\[
(2.21)
\]

For the momentum perturbation it is found that [16]

\[
\delta q = \frac{1}{F}[ -\phi \delta \phi - \delta [\dot{F}] + H\delta F].
\]
\[
(2.22)
\]

Considering the evolution of \(\epsilon_m\) in the standard gravity theory, where \(F = 1\) is assumed, it is found that \(\delta q\) vanishes at large scales on uniform density hypersurfaces. In terms of the \(\delta N\) formalism, \(\delta q \simeq 0\) leads to a natural condition \(\phi \simeq \) const at large scales, which can be interpreted as a flat boundary \((\delta \phi e = 0)\) at the end of inflation appearing in the standard inflationary scenario. On the other hand, in generalized gravity theories the uniform density hypersurfaces does not always lead to a flat \((\delta \phi = 0)\) boundary at \(t\). This result shows that in contrast to the standard gravity theory the generation of the curvature perturbation at the end of inflation, which is usually evaluated using the \(\delta N\) formalism [6], is not trivial in generalized gravity theories\(^5\). Namely, considering the curvature perturbation defined by \(R\), the \(\delta N\) formula can lead to the boundary perturbations \(\delta N_{mi}\) and \(\delta N_e\) defined by

\[
\delta N_{mi} = \frac{H}{\rho + p} \frac{\delta q_{mi}}{\rho + p},
\]
\[
\delta N_e = -H \frac{\delta q_e}{\rho + p}.
\]
\[
(2.23)
\]

\(^5\) See also the discussions in [16].
gravity theories, the flat boundary at $\phi = \phi_c$ does not always lead to $\delta q_c = 0$, but rather to the perturbation $\delta q_c \simeq \frac{1}{2} \{ [\delta F] + H \delta F \}$, which leads to

$$\delta N_c = -\frac{H}{\rho + p F} \frac{1}{F} [\delta F] + H \delta F]_{\phi=0} \neq 0. \quad (2.24)$$

Here, $\delta N_c$ is defined for the expansion between a hypersurface with a uniform density slicing ($\delta q = 0$) and uniform field ($\delta \phi = 0$) at $t = t_c$. The above correction does not appear in the standard gravity theory. The perturbation $\delta N_{ini}$ on the initial boundary is given by

$$\delta N_{ini} = -\frac{H}{\rho + p} \frac{1}{F} [\delta \phi - \delta [F] + H \delta F]_{\phi=0}. \quad (2.25)$$

For two-field inflation at first-order expansion, $\delta F$ is given by $\delta F \simeq F_0 \delta \phi + F_s \delta s$, where $\phi$ and $s$ denote the adiabatic component of the multi-field inflation and the entropy field. On the uniform-field ($\delta \phi = 0$) hypersurfaces, the expansion is given by $\delta F \simeq F_s \delta s$. Assuming that in the slow-rolling phase, the rate of variation of cosmological quantities is much smaller than unity, the sum of the boundary perturbations leads to

$$\delta N_{ini} + \delta N_c \simeq \frac{H}{\rho + p} \frac{1}{F} \left[ -\phi - \frac{\partial [F]}{\partial \phi} + H F_0 \right] \delta \phi, \quad (2.26)$$

where only the perturbations caused by the adiabatic field $\phi$ can survive because of the uniform field condition at the end ($t = t_c$) boundary.

Creation of perturbations at the boundaries (and during inflation) depends on the definition of the hypersurfaces for the $\delta N$ formalism, while the sum for the inflationary period is independent. $\delta N_c \neq 0$ appears because the standard definition of the $\delta N$ formalism, which has been used in this paper, is for flat hypersurfaces at $t_{ini}$ and uniform density at $t$. In contrast to the above argument, which is for boundary perturbations related to the $\delta N$ formalism defined by the uniform density hypersurfaces at $t$, it would be possible to define the $\delta N$ formalism for flat hypersurfaces at $t_{ini}$ and a uniform ‘field’ ($\phi = \text{const}$) at $t$. In the latter case, no perturbations are generated at the end of inflation (except for a specific case in which the inhomogeneous end of inflation is sourced by the entropy field).

To show the equivalence between the $\delta N$ formalism defined by different hypersurfaces, it would be useful (although it might be trivial for an expert) to consider an explicit calculation of the time integral. Considering a separation

$$\frac{d}{dt} \delta N = -\frac{H}{\rho + p} \delta p_{nad} + \frac{d}{dt} \left( H \frac{\delta \rho}{\dot{\rho}} \right), \quad (2.27)$$

the time integral of the last term vanishes if the $\delta N$ formalism is defined for uniform density hypersurfaces. However, it does not vanish for uniform ‘field’ hypersurfaces and gives

$$\int_{t_{ini}}^{t_c} \left[ \frac{d}{dt} \left( H \frac{\delta \rho}{\dot{\rho}} \right) \right] dt = \left( H \frac{\delta \rho}{\dot{\rho}} \right)_{t=t_c} - \left( H \frac{\delta \rho}{\dot{\rho}} \right)_{t=t_{ini}}$$

$$\simeq \left( H \frac{\delta q |_{\phi=0}}{\rho + p} \right)_{t=t_c} - \left( H \frac{\delta q |_{\phi=0}}{\rho + p} \right)_{t=t_{ini}}, \quad (2.28)$$

where $\delta q$ is defined for uniform ‘field’ hypersurfaces. The sign of $\delta N$ defined for the expansion from uniform density hypersurfaces to uniform ‘field’ hypersurfaces is opposite to that defined for the expansion from uniform ‘field’ hypersurfaces to uniform density hypersurfaces. Obviously, the sum of the perturbations $\delta N \equiv \delta N_{ini} + \delta N_c + \int N \, dr$ gives the identical result for different choices of the hypersurfaces. Equations for the boundary

6 This simple assumption is not true if there is a sharp bend in the trajectory.
perturbations and the evolution in terms of the \( \delta N \) formalism depend on the definition of the hypersurfaces, while the sum of these quantities gives identical result.

The explicit form of the evolution equation can be evaluated with the help of field equations. For two-field inflation with the standard kinetic term \( K = X \) and \( G_{IJ} = \delta_{IJ} \), the equation of motion for the adiabatic inflation field is given by

\[
\ddot{\phi} + 3H \dot{\phi} - \frac{f_\phi}{2} + V_\phi = 0.
\]

(2.29)

From the trace of the gravitational field equation, it is found that [16]

\[
\ddot{F} + 3H \dot{F} + \frac{1}{2} [2X - RF + 2f - 4V] = 0
\]

(2.30)

\[
R = \rho - 3p.
\]

(2.31)

As \( \epsilon_m \) decays at large scales as \( \propto k^2/a^2 \), the comoving energy density on uniform density hypersurfaces leads to

\[
\epsilon_m \simeq \frac{1}{F} \left[ \delta X - \frac{3}{2} (\rho + p) \delta F - \frac{f_\delta}{2} \delta s + V_\delta \delta s - \ddot{\phi} \delta \phi \right] \simeq 0,
\]

(2.32)

which gives the equation for \( \delta X \):

\[
\delta X \simeq \frac{3}{2} (\rho + p) \delta F + \frac{f_\delta}{2} \delta s - V_\delta \delta s + \ddot{\phi} \delta \phi.
\]

(2.33)

At large scales on uniform density hypersurfaces, we thus find

\[
\delta \rho + \delta p \simeq \frac{1}{F} [2\delta X - (\rho + p) \delta F - H \delta [\dot{F}] + \delta [\ddot{F}]]
\]

\[
\simeq \frac{1}{F} [2(\rho + p) \delta F + f_\delta \delta s - 2V_\delta \delta s - H \delta [\dot{F}] + \delta [\ddot{F}]].
\]

(2.34)

A useful expression for the evolution of the curvature perturbation obtained from the above equation is given by

\[
\zeta_N \simeq -2H \frac{\delta F}{F} + \frac{H}{F(\rho + p)} [(f_\delta - 2V_\delta) \delta s - H \delta [\dot{F}] + \delta [\ddot{F}]].
\]

(2.35)

Here \( \delta F \) must be evaluated on uniform density hypersurfaces, which (in contrast to the standard gravity theory) does not lead to \( \delta \phi = 0 \) hypersurfaces, as we discussed for the boundary perturbations. For the standard gravitational theory with \( F = 1 \), this equation gives the well-known result [20]

\[
\zeta_N \simeq 2H \frac{V_\delta}{\phi^2} \delta s,
\]

(2.36)

which agrees with the evolution of the curvature perturbation during inflation in standard gravity theory.

The perturbed equation from equation (2.30) is given by

\[
\delta [\dot{F}] + 3F \delta \theta + 3H \delta [\ddot{F}] \simeq -\frac{1}{2} [2\delta X - RF + F \delta R - 4\delta V],
\]

(2.37)

which shows that \( \delta [\dot{F}] \) can be expressed by using \( \delta [\dddot{F}] \) and other quantities. Here a reasonable approximation would be \( F_{\delta R} \ll F_{\delta \phi} \), which makes it possible to calculate \( \delta [\dot{F}] \) in terms of equation (2.33) for specific models of \( f(\phi, R) \) gravity.
3. Conclusion and discussion

In this paper, the curvature perturbations from the boundaries and the evolution of the curvature perturbations are considered for generalized gravity theories in terms of the $\delta N$ formalism. Our result shows that the creation of the curvature perturbation may be significant during the evolution. The perturbation $\delta F$ can be interpreted as spatial inhomogeneities in the gravitational constant $\delta G$. The second-order perturbations with respect to the field perturbations can be used to estimate the order of the magnitude of the non-Gaussian character of the cosmological perturbations.

In this paper, we considered the generalized gravity that is expressed by $f(\phi^I, R)$. This model is suitable for our present study, as it gives a very clear and intuitive result. However, it would be useful to make some comments for a typical string corrections that may not appear as $f(\phi^I, R)$. For example, the effective action may appear with the following additional corrections in the action [16]:

$$L_{\text{add}} = \frac{1}{2} \xi(\phi) R_{G,B}^2,$$

where $R_{G,B}^2 = R^{abcd} R_{abcd} - 4 R^{ab} R_{ab} + R^2$. Using the explicit form of the energy–momentum tensor given in [16], it is indeed possible to show the explicit formula of the perturbed expansion rate after a lengthy calculation.

Moreover, recent study shows that ultraviolet-complete quantum theory of gravity (Hořava–Lifshitz gravity [21]) may be given by a (more) generalized gravity action. In the generalized gravity theory considered in this paper, a crucial difference appears in the end-boundary of the inflationary stage, which is due to the non-ideal form of the energy–momentum tensor that depends explicitly on the curvature scalar. Our paper may give an important step in understanding the discrepancy between the ultraviolet-complete theory and the conventional gravity in terms of the evolution of the curvature perturbation during inflation, where the energy–momentum tensor may not be given by the ideal form due to the ultraviolet corrections from the fundamental theory.

Acknowledgments

We would like to thank K Shima for encouragement, and our colleagues at Tokyo University for their kind hospitality.

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7 Our equations are not exact in the second-order expansion. They can be used to estimate the order of magnitude of the non-Gaussian character, only when there is no significant source other than the trivial field perturbations.

8 The evolution of the curvature perturbations in terms of the perturbed expansion rate and the $\delta N$ formalism (the formulation used in this paper) is discussed in [22] for the effective action with generalized kinetic terms. DBI action in string theory is a typical example of this kind.
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