The Methodology of Distinguish Between Random and Chaotic Machine Tool Oscillations

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Abstract. Machine tool oscillations are irregular or aperiodic. Most often, these oscillations are chaotic but, in some cases, they can be quasi-periodic or random. The methodology for characterizing oscillations in the first of two steps uses the nonparametric hypothesis tests which the observed oscillations confirmed as irregular. The methodology for the final characterization of oscillations is based on chaos quantifiers. A time series defined as the measured values of oscillations in the time domain is the basis for calculating the quantifiers of chaos. There are four quantifiers of chaos: the Lyapunov exponent, Kolmogorov entropy, fractal dimension and correlation dimension. The correlation dimension and Kolmogorov entropy are important for distinguishing between random and chaotic oscillations. Other quantifiers of chaos are not used for this purpose. The methodology requires a multidisciplinary approach based on combining Nonlinear Dynamics and Probability Theory and Statistics. The methodology can be applied to many oscillating phenomena. Therefore, the paper mainly used the term oscillations, not vibrations, chatter, etc.

1. Introduction

The difficulty of predicting machine tool oscillations was already recognized and described by Frederick Winslow Taylor [1856-1915], published in a scientific paper "On the Art of Cutting Metals" in 1907. Since then, several attempts have been made to explain this phenomenon. The introduction of the regenerative effect and the corresponding mathematical models -Delay Differential Equations (DDEs) resulted a breakthrough in the modelling of machine tool oscillations. After the extensive work of Tlusty, Tobias, Kudinov, Thompson, Werner, Stépán, Insperger, Moon, Kalmár-Nagy, Warmiński, Litak and others, regenerative effect became the most commonly accepted explanation for machine tool oscillations. [1-5]

The time series analysis method is very popular to analyse machine tool oscillations. This method is based on the use of a time-series of experimentally generated data (acceleration, velocity or displacement of oscillation). The time-series analysis method is turning out to be one of the major contributions of nonlinear dynamics. This method is based on the notion that even for a multidimensional dynamical system the time series record of a single variable is often sufficient to determine many of the properties of the full dynamics of the system. The paper used the time series data of a single variable to create a multidimensional embedding space. The use of a single time series to generate a reconstruction space to characterize nonlinear dynamical systems was put on a firm mathematical basis by Floris Takens. The mathematical basis is Takens’ theorem embedding (reconstruction) theorem which is a bridge between the nonlinear dynamics and the time-series
analysis method. The time-series is the basis for calculating the quantifiers of chaos. The quantifiers of chaos can be specified quantitatively whether or not a system's apparently erratic behavior is indeed chaotic. This method was criticized by Moon and Kalmár-Nagy because it contained no physics and several other important reasons. However, in combination with an experimental design stochastic modeling, the time-series analysis method gives good results in the analysis machine tool oscillations.

2. Random and Chaotic Machine Tool Oscillations

Any oscillatory motion of a mechanical system about its equilibrium position is called vibration or mechanical oscillation. The oscillations may be periodic, random or chaotic. Periodic oscillations repeat motion in equal intervals of time. The sufficient and necessary conditions for the existence of chaotic oscillations are [7]: nonlinearity, energy dissipation, external excitation and sensitivity on the initial conditions. Machine tools as dynamic systems meet the above-mentioned conditions so that machine oscillations are usually chaotic. Any chaotic behavior is unpredictable with the possibility of predicting declining over time. An example is the weather forecast.

Machine tool oscillations are self-excited vibrations. The mathematical model of machine tool oscillations is a Retarded Functional Differential Equations - RFDE. The physical basis of this model is the regenerative effect. This effect is related to the cutting force variation due to the wavy workpiece surface cut one revolution ago (in turning) or one tooth ago (in milling). In grinding, based on doubly regenerative theory, the grinding force is proportional to the grinding depth which involves both the wheel and the workpiece regenerations. Retarded Functional Differential Equations - RFDEs are mathematical terminologies. In engineering RFDEs called Delay Differential Equations and abbreviation as DDEs. DDEs are also called time-delay systems. [2, 5, 12]

![Figure 1. The acceleration diagram, complete and on short time](image)

Both the random and the chaotic machine tool oscillations are irregular. Figure 1 shown the acceleration diagram of experimentally measured grinding machine oscillations during cylindrical (transversal) grinding.

Machine tool oscillations measurement system was shown in figure 2. The piezoelectric accelerometers were installed on the machine and they were connected to a "Spider" device that registered the voltage change in the accelerometers. "Spider" device was connected to a computer. The computer with the "Catman" software converted the voltage changes to the acceleration values (time series), one monitored the measurement, the type of processed data and saved data in MS Excel [6].
It was obvious that the oscillations were aperiodic or irregular. The qualitative analysis of the diagram cannot determine whether the observed oscillations were random or chaotic. Therefore, it was necessary to introduce a quantitative method.

3. Correlation Dimension

The quantifiers of chaos are often the only method for reliably recognition the types of dynamics. The quantifiers of chaos - the correlation dimension and the Kolmogorov entropy are important for distinguishing the random and chaotic oscillations [6].

Grassberger and Procaccia 1983. introduced a dimension based on the behavior of a correlation sum. This dimension is called the correlation dimension $D_c$. For the scalar time series $y(i), i = 1, 2, \ldots, N$, with $N$ points and time delay $\tau$, the correlation sum is [7, 16]:

$$C(\rho) = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \Theta(\rho - |y_i - y_j|).$$  (1)

The scalar time series $y(i), i = 1, 2, \ldots, N$, is experimentally generated data some of the oscillation’s characteristic or real data set. Figure 1 shown diagram of experimentally generated data the acceleration of the grinding machine oscillations during cylindrical grinding. $N$ points is actually the number of experimentally generated data and in the measurement shown above is about $N \approx 20.000$.

In equation (1) $\Theta(x)$ is the Heaviside step function:
\[ \Theta(x) = \begin{cases} 1 & \text{if } x \geq 0 \\
0 & \text{if } x < 0 \end{cases}. \] (2)

In equation (1) the distance \( \rho \) is assigned value, which the \( \Theta(x) \) compare with the absolute values of the differences \( |y_i - y_j| \), except \( j = i \), which means that it does not compare \( y_i \) with itself. If it is \( \rho \geq |y_i - y_j| \), then it is \( \Theta(x) = 1 \), and if there is \( \rho < |y_i - y_j| \), then there is \( \Theta(x) = 0 \). Any real data set consists of a finite number of points. There is some minimum distance between trajectory points. When \( \rho \) is less than that minimum distance, the correlation sum, equation (1), is equal to 0. When there is \( \rho \) larger than the size of the attractor and all points are within \( \rho \) of each other and \( C(\rho) \) is equal to 1, not depending to \( \rho \)[7].

If there is \( y(i), i = 1, 2, \ldots, N, \) are periodic, then there is \( C(\rho) = \text{const.} \) provided \( 0 < C(\rho) < 1 \), not depending to \( \rho \). The reason for this is the repetition of the values of \( y_i \) and \( y_i \), which causes that the same number of differences \( |y_i - y_j| \) are inside and outside the distance \( \rho \).

The correlation dimension \( D_c \) is then defined to be the number that satisfies \[ C(\rho) = \lim_{\rho \to 0} g^\rho. \] (3)

or, more precisely, after taking logarithms the correlation dimension \( D_c \) is:

\[ D_c = \lim_{\rho \to 0} \frac{\log C(\rho)}{\log \rho}. \] (4)

The correlation dimension \( D_c \) is practically the slope of the regression line logarithmic value of \( C(\rho) \) and \( \rho \). The range of \( D_c \) values is \( 0 \leq D_c \leq 1 \), as the maximum slope angle of the regression line is \( \alpha = 45^\circ \).

The correlation dimension distinguishes between random and chaotic data provided that the number of data \( N \) is large (\( N > 1000 \)). The second condition is that the value of the correlation dimension is not equal to the number of degrees of freedom of the state space (attractor) \( d \). The second condition is met if \( d \gg 1 \) [7]. When the above conditions are met, then the following criterion can be used:

- if the regression line is constant, \( \alpha \approx 0^\circ \Rightarrow D_c \approx 0 \), the oscillations are periodic;
- if the slope of the regression line is \( 0 < \alpha < 45^\circ \Rightarrow 0 \leq D_c \leq 1 \), the oscillations are chaotic;
- if the slope of the regression line is \( \alpha \approx 45^\circ \Rightarrow D_c \approx 1 \), the oscillations are random.

**Figure 3.** The oscillation slope
For practical application, the above-mentioned criterion is too restrictive and there is necessity to introduce the concept of oscillation zones, figure 3.

For experimentally measured acceleration of the grinding machine oscillations (figure 1), using “Matlab”, a regression line was achieved whose equation was

$$\log C(\rho) = 0.73455 \cdot \log \rho - 0.18.$$  

Therefore, the correlation dimension value is $D_c = 0.73455$, which means that the oscillations were chaotic.

4. Kolmogorov Entropy

The Kolmogorov entropy ($K$-entropy) is important quantifier of chaos which describes the degree of chaoticity of the systems. It gives the rate of information loss about a position of the phase point on the attractor. The values of the $K$-entropy belong to the set of real positive numbers, $K \in \mathbb{R}^+$ or $0 \leq K < \infty$. Depending on the value of the $K$-entropy for the characterization of oscillations, where a well-known criterion can be used also [17]:

- if there is $K = 0$, then the oscillations are periodic and there is no loss of information;
- if there is $0 \leq K < \infty$, then the oscillations are chaotic and information loss is exponential;
- if there is $K \to \infty$ then the oscillations are random and information loss is instantaneous.

Obviously, there is a correlation between the criteria based on the correlation dimension and the $K$-entropy. One of the criteria is sufficient to characterize the oscillations. Criteria based on the correlation dimension is more practical to apply. The reason is the possible large values of the $K$-entropy which can even take on infinite values. However, it is practical to use both of these criteria, due to mutual complementarity and control.

Time-series are also the basis for calculating the $K$-entropy. To calculate the $K$-entropy, it is necessary to calculate the generalized correlation sum of the vector time series firstly [7]:

$$C_q^{(d)}(\rho) = \left( \frac{1}{N_d} \sum_i \left[ \frac{1}{N_d - 1} \sum_{j,j \neq i} \Theta(\rho - \rho_{ij}) \right]^{q-1} \right)^{\frac{1}{q-1}},$$  (5)

where there are as follows:

- $\rho_{ij}$ - a difference between two of the embedding space $d$-dimensional "vectors" $\vec{y}_i$, which were determined from a scalar time serie as the “Euclidean length” or the "maximum coordinate difference”;
- $q$ - the order of the generalized correlation sum;
- $d$ - the dimension of the vectors $\vec{y}_i$.

The generalized correlation sum is related to the $K$-entropy by the equation ($q = 2$):

$$C_2^{(d)}(\rho) = g \rho^{D_c} e^{-dK}.$$  (6)

Taking the natural logarithm of both sides of the previous equation gives the following:

$$\ln C_2^{(d)}(\rho) \approx D_c \ln \rho - d \cdot K,$$  (7)

where the unimportant $\ln(g)$ term ($g$ is a constant) has been dropped. From equation (7), it is visible how to find $K$: with $\rho$ fixed $- \ln C_2^{(d)}(\rho)$ was plotted as a function of the embedding dimension $d$. The slope of that curve should be $-K$ [18].
For experimentally measured the acceleration of the grinding machine oscillations (figure 1), using "Matlab", a regression line whose equation is $-\ln C_2^{(d)}(\rho) = 0,3918 \cdot d + 0,39$ was achieved, figure 4. Therefore, the K-entropy value is $K= 0,3918$ which also means that the oscillations are chaotic and the information loss is exponential.

$$-\ln C_2^{(d)}(\rho) = 0,3918 \cdot d + 0,39$$

Figure 4. The K-entropy calculation for $\rho = 0,3$

The values of $K$ that distinguish chaotic and random oscillations are a bit messy, especially for $K \rightarrow \infty$. It is therefore necessary to redefine and explain this criterion. Thus, if the attractor of the dynamical system (machine tool) is $d$-dimensional ($d$ is also the number of degrees of freedom of the state space) and if $K > 0$, then it claims that the oscillations are chaotic. Otherwise, if the value of $K$ is a relatively large number and if $K \gg d$, then the oscillations are random. It was based upon the claim from Pesin’s equation according to which the K-entropy was equal to the sum of the positive Lyapunov exponents [19].

5. Non-Parametric Hypothesis Test
A statistical test is a formal technique that relies on the theoretical probability distribution for reaching the conclusion concerning the accepting of the hypothesis. These hypothetical testing related to differences were classified as parametric and nonparametric tests. The parametric test is one which has information about the population parameter. Non-parametric test, as its name tells, is statistical test without parameters [20].

5.1. Finding empirical and theoretical distribution
The first step is to find an empirical and theoretical distribution. The Weibull theoretical distribution is to be used, because the Weibull distribution is related to a number of other theoretical distributions: the exponential, hyperexponential, normal and Rayleigh one. The procedure is as follows (figure 5) [21]:

a) some value is to be added to all data so that they are all positive (the diagram is to be translated into the first quadrant of the coordinate system, this is possible because the data values at this stage are not important but only their randomness), figure 5- columns B and C;
b) sort the data by values from smaller to bigger (this is possible because the time of data creation is not important, but also their randomness), figure 5- column C;
c) find the empirical distribution by the equation (figure 5 - column D):

$$F_x(i) = \frac{i - 0,3}{n + 0,4},$$

where there are: $i$- ordinal number of the single data and $n$- total number of data (1000 in the example).
d) the shape parameter $\beta$ and the scale parameter $\eta$ in the Weibull distribution:

$$F(z) = 1 - e^{-\left(\frac{z}{\eta}\right)^\beta},$$

(9)

was found by the method of least squares (figure 6):

$$a = \frac{n \sum_{i=1}^{n} (x_i y_i) - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} y_i}{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2},$$

(10)

$$b = \frac{\sum_{i=1}^{n} x_i^2 \cdot \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} (x_i y_i)}{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2},$$

(11)

and the parameters in the Weibull distribution were estimated by the equations (Figure 6):

$$\beta = a,$$

(12)

$$\eta = e^{-a/b}.$$

(13)

At the end of this section, the values of the cumulative Weibull distribution were inserted for the calculated parameter values and the independent variable $z$, figure 5 - column D. A theoretical distribution with that was found.

Figure 5. Finding empirical and theoretical distribution in MS Excel
5.2. Fitting empirical to theoretical probability distribution

In this chapter it was determined how well theoretical Weibull distribution (figure 5 - column I) fitted to the empirical distribution, (figure 5 - column D). Regarding the example (figure 5), it was actually tested how well the theoretical oscillations fitted to the empirical (experimentally measured) oscillations. In this purpose, non-parametric statistical hypothesis tests were used. First, the hypotheses for performing the test was checked. The two types of hypotheses are null and alternate. The null hypothesis ($H_0$) is a statement that indicates no difference exists between an empirical and theoretical distribution. The alternate hypothesis ($H_A$), also called a research hypothesis, is the statement that predicts a difference between an empirical and theoretical distribution. There are several popular types of non-parametric hypothesis tests used in research nowadays: chi-square $\chi^2$ test, Friedman one, Romanovsky, Kolmogorov-Smirnov one and others. Also, for a goodness of fit test the coefficient of determination between an empirical and theoretical distribution can be used. [15, 20, 24]

In this paper the null hypothesis ($H_0$) is - machine tool oscillations were random. The alternate hypothesis ($H_A$) is - machine tool oscillations were not random, i.e. the oscillations were chaotic or regular. It was assumed for the moment that the null hypothesis was true. To test the null hypothesis, chi-square $\chi^2$ test and the correlation coefficient are to be used.

Second, the values of empirical (figure 5 - column D) and theoretical distribution were specified (Figure 5 - column I).

Third, the statistic test was computed. In 1900, Karl Pearson [1857-1936] defined the chi-squared $\chi^2$ statistic [20]:

$$\chi^2 = \frac{(f_1 - f_{1t})^2}{f_{1t}} + \frac{(f_2 - f_{2t})^2}{f_{2t}} + \ldots + \frac{(f_r - f_{rt})^2}{f_{rt}} = \sum_{i=1}^{r} \frac{(f_i - f_{it})^2}{f_{it}},$$

(14)

where there are the following:

- $f_i$ - frequency of the $i^{th}$ class of the empirical distribution;
- $f_{it}$ - frequency of the $i^{th}$ class of the theoretical distribution;
- $r$ - number of classes.
It was recommended that the theoretical frequencies be greater than 5. If $f_{i} < 5$, adding those classes to the adjacenting ones. It was also recommended that the frequency values be $f_{i} > 10$.

Fourth, the value of the chi-square distribution was determined $\chi^{2(k)}_{\alpha}$, where $k$ was the number of degrees of freedom. The $k$ was calculated by the equation [22]:

$$k = r - l - 1,$$

where there are as follows:

- $r$ - number of classes (from equation (14));
- $l$ - number of the parameters in the theoretical distribution. $l = 2$ in the Weibull distribution, from equation (9).

The significance level $\alpha$ is the probability of rejecting the null hypothesis [23]:

$$P(\chi^{2} > \chi^{2(k)}_{\alpha}) = \alpha.$$

The common alpha values $\alpha = 0.05$ and $\alpha = 0.01$ are based on tradition.

![Figure 7. The chi-square $\chi^{2}$ distribution and significance level $\alpha$ [23]](image)

![Figure 8. Finding the value of $\chi^{2(k)}_{\alpha}$ in MS Excel](image)
For the significance level \( \alpha (\alpha = 0.05 \text{ or } \alpha = 0.01) \) and the number of degrees of freedom \( k \), the value of \( \chi^2 \) was found in MS Excel, figure 8.

If there is \( \chi^2 > \chi^2_{\alpha} \), then the probability, equation (16), is very low and the null hypothesis was rejected. There is a significant difference between the empirical and theoretical distribution.

If there is \( \chi^2 \leq \chi^2_{\alpha} \), then the probability, equation (16), is high and the null hypothesis was accepted. There is not a significant difference between the empirical and theoretical distribution.

In the example, it was determined how well the theoretical Weibull distribution (figure 5 - column I) fitted to the empirical distribution, (figure 5 - column D), for randomly selected 1000 experimentally generated data (total number of data \( N \approx 20,000 \)), figure 1. These 1000 data were classified in 10 classes and determined the frequencies classes. Next, the chi-squared \( \chi^2 \) statistic was computed by the equation (14), figure 9.

\[
\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i},
\]

where there were:
- \( O_i \) - value of the empirical distribution for \( i \)th data;
- \( E_i \) - value of the theoretical distribution for \( i \)th data;
- \( \overline{E} \) - the average value of the empirical distribution.

The coefficient of determination calculated \( R^2 \) by the equation [24]:

\[
R^2 = 1 - \frac{\sum_{i=1}^{N}(F_{ij} - \overline{F}e)^2}{\sum_{j=1}^{N}(F_{oj} - \overline{F}e)^2},
\]

Figure 9. Hypothesis testing in MS Excel
$R^2$ is a statistic that gives the information about the goodness of fit of an empirical and theoretical distribution. $R^2$ normally ranges from 0 to 1. $R^2 = 1$ indicates that the theoretical distribution perfectly fits to the empirical distribution, while $R^2 = 0$ indicates that the calculation fails to accurately model the data at all. The correlation coefficient $R^2 = 0.98575$ was calculated in MS Excel, figure 6. The obtained value of $R^2$ confirmed that the theoretical Weibull distribution fitted very well to the empirical distribution. Recursively, $R^2 = 0.98575$ suggests that 98.57% of the empirical distribution is predicted by the theoretical distribution.

In chapters 3. and 4. $D_c = 0.73455$ and $K = 0.3918$ were calculated, using identical data as in chapter 5. According to $D_c$ and $K$, the oscillations are chaotic, which is not consistent with the accepted null hypothesis and the correlation coefficient. This contradiction was checked for several times and have similar results were obtained.

It can be concluded that a non-parametric hypothesis test does not distinguish well between chaotic and random vibrations. Anyway, the nonparametric hypothesis test only confirms that the oscillations are irregular. The final confirmation can be given based upon the quantifiers of chaos.

6. Conclusion
The above statement can be represented by the algorithm for distinguishing between random and chaotic oscillations, figure 10.

![Algorithm Diagram](image-url)

Figure 10. The algorithm for distinguishing between random and chaotic oscillations
The time series is experimentally generated oscillations data and the input data for distinguishing between random and chaotic oscillations. The methodology for distinguishing between random and chaotic oscillations has two steps. In the first step the nonparametric hypothesis tests are to be used, which the observed oscillations confirm as irregular. The nonparametric hypothesis test determines how well theoretical distribution fits to the empirical distribution. The chi-square test is a very popular type of nonparametric test. The correlation coefficient between the empirical and theoretical distributions can also be used for a goodness of fit.

In the second step, the quantifiers of chaos are to be used. The quantifiers of chaos are often the only method for reliable recognition of the types of dynamics. The methodology for the final characterization of oscillations was based on chaos quantifiers. The quantifiers of chaos - the correlation dimension and the Kolmogorov entropy are crucial for distinguishing between random and chaotic oscillations. The calculated value of the correlation dimension or the Kolmogorov entropy determined exactly whether the oscillations were chaotic or random.

The methodology can be applied to many oscillating phenomena, not only on mechanical oscillations, e.g. temperature oscillations. Therefore, in this paper the term oscillations was mainly used, not vibrations, chatter, etc. In combination with an experimental design stochastic modeling, this methodology gives good results in distinguishing between random and chaotic oscillations.

The methodology requires a multidisciplinary approach based on combining Nonlinear Dynamics and Probability Theory to the Statistics and the successful co-operation among mathematicians, physicists, engineers and experts in various different fields in which irregular oscillations are studied.

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