Three-body bound state calculations using three-dimensional low-momentum interaction $V_{\text{low } k}$

M. R. Hadizadeh$^{1,*,†}$

$^1$Institute of Nuclear and Particle Physics, Department of Physics and Astronomy, Ohio University, Athens, OH 45701, USA

$^*$E-mail: hadizadm@ohio.edu

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Three-dimensional Faddeev integral equations are solved for the three-body (3B) bound state problem without using the partial wave form of low-momentum two-body interaction $V_{\text{low } k}$ which is constructed from spin-independent Malfliet–Tjon V (MT-V) potential. The dependence of 3B binding energy on the cutoff momentum of $V_{\text{low } k}$ is investigated for a wide range of $\Lambda$ from 1.0 to 7.0 fm$^{-1}$. The properties of the Faddeev components and 3B wave function are displayed and the effect of the number of grid points for momentum and angle variables on the accuracy and stability of the numerical results is studied by calculation of the expectation value of the total Hamiltonian.

Subject Index D00, D05, D06

1. Introduction

The low-momentum nucleon–nucleon (NN) interaction $V_{\text{low } k}$ is derived in partial wave (PW) representation, based on the strategy of integrating out the high-momentum part of the NN interaction $V_{\text{NN}}$. The properties of the NN system for laboratory energies $E_{\text{lab}} < 350$ MeV (like deuteron binding energy, low-energy phase shifts, and half-on-shell $T$-matrix of bare potential $V_{\text{NN}}$) are all preserved by $V_{\text{low } k}$, which is derived from very different NN potential models (like Paris, CD Bonn, Argonne, Idaho, and the Nijmegen potentials), when it is confined within a cutoff momentum $\Lambda \sim 2.0$ fm$^{-1}$. Different methods are developed in PW representation to derive $V_{\text{low } k}$, such as the renormalization group (RG) [1–3] and the model space techniques like Lee–Suzuki similarity transformations [4,5]. It has been shown that the Lee–Suzuki method is equivalent to the RG method and reproduces the same results, whereas its numerical procedure is less cumbersome in comparison to the solution of the differential equation in the RG method.

On this basis, recently the three-dimensional (3D) form of the low-momentum NN interaction $V_{\text{low } k}$ has been formulated into the model space Lee–Suzuki method as a function of momentum vectors and is calculated for a spin-independent potential model [6] and also for a modern NN interaction [7]. It has been shown that, similar to the PW representation, the 3D form of low-momentum interaction reproduces the same NN observables from bare potential $V_{\text{NN}}$, whereas it avoids the

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highly involved angular momentum algebra occurring for the transition and permutation operators of the PW representation.

In this paper we have employed the spin-independent $V_{\text{low} M}$ constructed from the MT-V potential [8], which acts on all PWs, to calculate the three-body (3B) binding energy in a non-PW scheme by solution of 3D Faddeev integral equations. The momentum cutoff dependence of the binding energy is studied and it is shown that the 3B binding energy is strongly cut off dependent on low cutoff values, similar to what is predicted by PW-based calculations [9,10].

Implementation of the 3D form of low-momentum NN interaction obtained with the similarity renormalization group (SRG) method [11] in NN scattering [12], as well as three- and four-body bound state calculations [13–15] are in progress.

The paper is organized as follows. In Sect. 2 the 3D formalism of $V_{\text{low} M}$ for the spin-independent case is briefly presented and our numerical results for matrix elements of $V_{\text{low} M}$ are given. In Sect. 3 we review the three-dimensional Faddeev integral equations for 3B bound state, and in Sect. 4 our numerical results for 3B binding energies and momentum distribution functions for low-momentum potentials, and for a wide range of cutoff momentums, are given. To test the accuracy of our numerical results the expectation value of the 3B Hamiltonian is calculated and compared with the eigenvalue binding energy. We conclude in Sect. 5.

2. 3D representation of $V_{\text{low} M}$ in the model space Lee–Suzuki method

The low-momentum interaction in the model space Lee–Suzuki method, which reproduces the model space components of the wave function from the full-space wave function, is given by:

$$V_{\text{low} M} = PV_{\text{NN}}(P + Q \omega P),$$

where $V_{\text{NN}}$ is the bare two-body (2B) interaction, $P$ and $Q$ are 2B projection operators which project a state onto the model space (low-momentum space) and its complement (high-momentum space), respectively. $\omega$ is an operator which transforms the states of the $P$ space to the states of the $Q$ space. As shown in Ref. [6], the momentum space representation of Eq. (1) by considering the integral form of the projection operators $P$ and $Q$ in a 3D approach reads:

$$V_{\text{low} M}(p', p) = V_{\text{NN}}(p', p) + \int_{\Lambda \leq k < \infty} d^3 k \ V_{\text{NN}}(p', k) \ \omega(k, p),$$

where $p, p'$ are 2B momentum vectors in model space $P$ and $k$ is the 2B momentum in the complement model space $Q$. The matrix elements of $\omega(k, p)$ can be obtained by solution of the following integral equation:

$$\omega(k, p) = \int_{0 \leq p' \leq \Lambda} d^3 p' \ \Psi_{p'}^{\text{NN}}(k) \ \bar{\Psi}_{p'}^{\text{NN}}(p),$$

where $\Psi_{p'}^{\text{NN}}(p)$ and $\Psi_{p'}^{\text{NN}}(k)$ are the wave function components of the $P$ and $Q$ spaces of the full space, respectively. They are given in the form of the half-on-shell (HOS) 2B $T$-matrix by:

$$\Psi_{p'}^{\text{NN}}(k) = \frac{T(k, p', k^2)}{p'^2 - k^2},$$

$$\Psi_{p'}^{\text{NN}}(p) = \delta^3(p - p') + \frac{T(p, p', p'^2)}{p'^2 - p^2 + i\epsilon},$$

2/12
where the HOS 2B $T$-matrix can be obtained from the Lippmann–Schwinger equation in the 3D representation [16]:

$$ T(p', p, p^2) = V_{NN}(p', p) + \int d^3 p'' V_{NN}(p', p'') T(p'', p, p^2). $$  (6)

The $V_{low-k}$ given by the Lee–Suzuki method is non-Hermitian, i.e. $V_{low-k}(p', p) \neq V_{low-k}(p, p')$, and is specifically constructed to preserve the HOS $T$-matrix $T(p', p, p^2)$. This interaction of course preserves the phase shift which is given by the fully-on-shell $T$-matrix $T(p, p, p^2)$.

Details of the numerical solution of the 3D integral equations (2), (3), and (6) by choosing suitable coordinate systems are given in Ref. [6] and are not repeated here. As is shown in this reference, if we choose the Jacobi momentum vector $p$ parallel to the z-axis and the vector $p'$ is in $x$–$z$ plane, we can obtain the matrix elements of the low-momentum interaction $V_{low-k}(p', p, x')$ from solution of the integral equation (2), where $p$ and $p'$ are the magnitudes of the Jacobi momenta and $x'$ is the angle between them.

In this paper, for the numerical solution of the integral equations we have used the spin-independent MT-V potential with the parameters given in Ref. [17]. Figure 1 shows the 2D plots of the angle-averaged low-momentum interaction $V_{low-k}(p', p)$ as a function of the momentum variables $p$ and $p'$ for a range of cutoffs $\Lambda$ from 1.2 to 7.0 fm$^{-1}$. It indicates that the non-Hermiticity of $V_{low-k}(p', p)$ is rather weak and it is a smooth potential for the cutoff momentum $\Lambda$ in the vicinity of 2.0 fm$^{-1}$. The bare MT-V potential is also shown as $\Lambda = \infty$.

### 3. 3D Faddeev integral equations for 3B bound state

The bound state of three particles which interact by pairwise forces can be described by the Faddeev equation:

$$ |\psi\rangle \equiv |\psi_{12,3}\rangle = G_0 t P |\psi\rangle, $$  (7)

where $G_0 = (E - H_0)^{-1}$ is a free propagator, $P = P_{12}P_{23} + P_{13}P_{23}$ a permutation operator. $t = V + VG_0 t$ is a 2B $t$-matrix which can be obtained from the Lippmann–Schwinger equation, where $V$ is applied to the low-momentum potential or the original one. The total wave function of the 3B system can be obtained from Faddeev component (7) as:

$$ |\Psi\rangle = (1 + P)|\psi\rangle. $$  (8)

In order to project Eq. (7) in momentum space, we need to define 3B basis states which are composed of two Jacobi momentum vectors $p$ and $q$. The momentum vector $p = \frac{k_2 - k_1}{2}$ is the relative momentum of the pair in the 2B subsystem and $q = \frac{1}{2} (k_1 - \frac{1}{2} (k_2 + k_3))$ is the momentum of the third particle relative to the center of mass of the pair. The 3B basis states $|pq\rangle$ are complete and normalized as:

$$ \int d^3 p \int d^3 q \langle pq | pq \rangle = 1, \quad \langle pq | p' q' \rangle = \delta^3(p - p')\delta^3(q - q'). $$  (9)

The projection of the Faddeev component, Eq. (7), in the basis states of Eq. (9) reads:

$$ \psi(p, q) = \frac{1}{E - \frac{p^2}{m} - \frac{3q^2}{4m}} \int d^3 q' t_s(p, \frac{1}{2} q + q'; \epsilon) \psi(q + \frac{1}{2} q', q'), $$  (10)

where the two-body subsystem energy is defined as $\epsilon = E - \frac{3q^2}{4m}$. The symmetrized 2B $t$-matrix in the kernel of the integral equation is $t_s(p, q; E) = t(p, q; E) + t(-p, q; E)$. The total 3B wave
Fig. 1. Angle-averaged bare and constructed low-momentum interactions calculated from MT-V potential as a function of momentum variables $p$ and $p'$. 

function can be obtained as:

$$\Psi(p, q) = \psi(p, q) + \psi\left( -\frac{1}{2} p - \frac{3}{4} q, \frac{1}{2} q \right) + \psi\left( -\frac{1}{2} p + \frac{3}{4} q, -\frac{1}{2} q \right). \tag{11}$$

Since we have ignored spin–isospin degrees of freedom and we study the three-boson bound state, the Faddeev amplitude $\psi(p, q)$ and consequently the total wave function $\Psi(p, q)$ should be symmetric under the exchange of interacting particles in the 2B subsystem. On the other hand, they are symmetric under the exchange $p$ to $-p$, which can be easily verified from Eqs. (10) and (11). In order to numerically solve the integral equation (10), as shown in Ref. [17], choosing a suitable coordinate system, $q$ parallel to the $z$-axis and $p$ in the $x$–$z$ plane, leads to a three-dimensional integral equation:

$$\psi(p, q, x) = \frac{1}{E - \frac{p^2}{m} - \frac{3q^2}{4m}} \int_0^{\infty} dq' \int_{-1}^{+1} dx' \int_0^{2\pi} d\varphi' t_s(p, \pi', x_{p\pi'}; \epsilon) \psi(\pi, q', x_{\pi q'}), \tag{12}$$
where the angle variables and shifted momentum arguments are given as:

\[ x_{qp} \equiv \hat{q} \cdot \hat{p} = x \]
\[ x_{qq'} \equiv \hat{q} \cdot \hat{q}' = x' \]
\[ x_{pq} \equiv \hat{p} \cdot \hat{q}' = xx' + \sqrt{1 - x^2} \sqrt{1 - x'^2} \cos(q') \]
\[ \tilde{\pi} = \sqrt{\frac{1}{4} q^2 + q'^2 + qq'x'} \]
\[ x_{p\tilde{\pi}} = \frac{1}{2} qx + q'x_{pq} \]
\[ \pi = \sqrt{q^2 + \frac{1}{4} q'^2 + qq'x'} \]
\[ x_{\pi q'} = \frac{q x' + \frac{1}{2} q'}{\pi} \]

(13)

By using the Faddeev amplitude \( \psi(p, q, x) \), and by considering the coordinate system defined in Eq. (13), the three-body wave function \( \Psi(p, q, x) \) can be obtained from Eq. (11).

4. Numerical results

4.1. 3B binding energy and wave function

We have solved the 3D Faddeev integral equation (10) for bare and low-momentum interactions constructed from the MT-V potential. For a numerical solution of this integral equation we have first discretized the continuous momentum and angle variables. To this end we have used Gauss–Legendre quadrature with 40 mesh points for all momentum and angle variables. For discretization of the momentum variables \( p \) and \( q \) we need to determine their corresponding momentum cutoffs \( p_{\text{max}} \) and \( q_{\text{max}} \). In order to avoid extrapolation on the symmetrized two-body \( t \)-matrix \( t_s(p, \tilde{\pi}, x_{p\tilde{\pi}}; \epsilon) \) and Faddeev component \( \psi(\pi, q', x_{\pi q'}) \) for the solution of integral equation (12), the following condition should be satisfied:

\[ \pi_{\text{max}} = \pi_{\text{max}} = 1.5 q_{\text{max}} \leq p_{\text{max}} \]

(14)

whereas we have considered the same momentum cutoff for \( q \) and \( q' \), i.e. \( q_{\text{max}} \). A linear mapping \( \tilde{\pi}_2(1 + x) \) is used for discretization of the Jacobi momenta \( p \) and \( q \), where the \( x \) are the roots of the Gauss–Legendre polynomial. In the calculations with bare potential, we have used \( c = 20.0 \text{ fm}^{-1} \) and \( c = 7.0 \text{ fm}^{-1} \) for Jacobi momenta \( p \) and \( q \), respectively. These cutoffs satisfy the condition \( p_{\text{max}} \geq 1.5 q_{\text{max}} \). Since the low-momentum potential \( V_{\text{low}}k \), which is dependent on the relative two-body momenta \( p \) and \( p' \), is confined in a low-momentum space with a cutoff \( \Lambda \), consequently the symmetrized two-body \( t \)-matrix, which appears in the kernel of the Faddeev integral equation, is also defined in a momentum interval from 0 to \( \Lambda \). So, for 3B calculations with low-momentum potentials, \( c = \Lambda \) is used for both \( p \) and \( q \) Jacobi momenta. Of course, for this case, in each step of iteration, we equate the value of the Faddeev component \( \psi(\pi, q', x_{\pi q'}) \) for \( \pi_{\text{max}} > \Lambda \) to zero. The integral equation is solved by a Lanczos-type technique, which is based on iteration (see Appendix C2 of Ref. [18]). The iteration of the integral in Eq. (10) requires a very large number of two-dimensional interpolations on the Faddeev component and 2B \( t \)-matrix; to this end we have used cubic Hermite splines to achieve high computational accuracy and speed. Usually 7 to 10 iterations is enough to reach convergence in the solution of integral equation.
Table 1. Three-body binding energy for bare and low-momentum interaction for MT-V potential.

| $\Lambda$ [fm$^{-1}$] | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 |
|-----------------------|-----|-----|-----|-----|-----|-----|
| $E_3$ [MeV]           | -6.146 | -6.568 | -6.917 | -7.190 | -7.395 | -7.544 |
| $\Lambda$ [fm$^{-1}$] | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 | 2.1 |
| $E_4$ [MeV]           | -7.646 | -7.713 | -7.754 | -7.777 | -7.788 | -7.792 |
| $\Lambda$ [fm$^{-1}$] | 2.5 | 3.0 | 4.0 | 5.0 | 6.0 | 7.0 |
| $E_5$ [MeV]           | -7.783 | -7.766 | -7.744 | -7.736 | -7.737 | -7.738 |
| $V_{NN}$              | -7.738 |        |        |        |        |        |

Fig. 2. The dependence of the three-body binding energy $E_3$ on the cutoff of low-momentum potential $\Lambda$.

Our numerical results for the 3B binding energy for a wide range of cutoffs $\Lambda$, from 1.0 to 7.0 fm$^{-1}$ are given in Table 1. The binding energies are obtained in such a way that Eq. (10) is fulfilled with a relative accuracy of $10^{-6}$ at each set point $(p, q, x)$. In Fig. 2, we have shown the binding energy results as a function of cutoff $\Lambda$. Obviously, the 3B binding energy is strongly cutoff-dependent for small values of cutoffs, whereas for values larger than 5.0 fm$^{-1}$ it is almost cutoff-independent and leads to the bare potential binding energy $-7.74$ MeV. This cutoff dependency is quite reasonable, because 2B interaction has a cutoff $\Lambda$ (P-space of QCD) and therefore has corresponding three- and higher-body forces. Consequently, if we omit the many-body forces, the observables will be cutoff-dependent. By using $V_{low}$, all low-energy 2B observables are cutoff-independent, and therefore, by varying the cutoff $\Lambda$ in our 3B calculations, we can evaluate the effects of the omitted 3B forces. Our results show that the cutoff $\Lambda$ variations from 1.0 to 7.0 fm$^{-1}$ lead to approximately $1.6$ MeV variations for the 3B binding energy, which gives an estimate of the 3B forces’ contribution. In order to reproduce the bare solution and achieve the cutoff-independent results for low-momentum cutoffs, we need to consider the 3B forces induced by a unitary transformation of the Hamiltonian [19].

In Figs. 3 and 4, we have shown the Faddeev components $\psi(p, q, x)$ and total wave functions $\Psi(p, q, x)$ for cutoffs $\Lambda = 1.2, 1.5, 1.7, 1.9, 2.1, 3.0, 5.0, 7.0$ fm$^{-1}$ for fixed angle $x = +1$ on a logarithmic scale on the z-axis. These figures clearly show that when the cutoff $\Lambda$ is decreased to small values, the effect of hard core interaction disappears. This is a consequence of integrating out the short-distance physics, which presents a repulsive core, in the construction of the low-momentum interaction.
Fig. 3. The magnitude of the Faddeev component $\psi(p, q, x)$ for $x = +1$ calculated from the MT-V low-momentum potential for $\Lambda = 1.2$–7.0 fm$^{-1}$. 
Fig. 4. The magnitude of the 3B bound state wave function $\Psi(p, q, x)$ for $x = +1$ calculated from the MT-V low-momentum potential for $\Lambda = 1.2$–7.0 fm$^{-1}$.
Fig. 5. The momentum distribution function $n(p)$ calculated from the MT-V bare and low-momentum potential for $\Lambda = 1.2\sim 7.0\text{ fm}^{-1}$. 
Fig. 6. The momentum distribution function $n(q)$ calculated from the MT-V bare and low-momentum potential for $\Lambda = 1.2$–7.0 fm$^{-1}$.
Table 2. The expectation values $\langle H_0 \rangle$, $\langle V \rangle$, and $\langle H \rangle$ calculated for low-momentum interaction constructed from the MT-V potential for $\Lambda = 2.1$ fm$^{-1}$.

| $N_{\text{jac}}$ | $N_{\text{sph}}$ | $\langle H_0 \rangle$ [MeV] | $\langle V \rangle$ [MeV] | $\langle H \rangle$ [MeV] | $E_3$ [MeV] |
|----------------|----------------|-----------------|----------------|----------------|-------------|
| 20             | 20             | 24.778          | -32.575        | -7.797         | -7.795      |
| 30             | 20             | 24.786          | -32.587        | -7.801         | -7.793      |
| 30             | 30             | 24.786          | -32.587        | -7.801         | -7.793      |
| 40             | 40             | 24.788          | -32.589        | -7.801         | -7.792      |

4.2. Momentum probability densities

In order to simplify our analysis of the symmetrized 3B wave function $\Psi(p, q, x)$, which is calculated from Eq. (11) after obtaining the Faddeev component $\psi(p, q, x)$ from Eq. (12), and to obtain an insight into how the momentum is shared among the Jacobi coordinates, we have calculated the momentum probability densities, which are defined as:

$$n(u_i) = 2\pi u_i^2 \int_0^\infty du_j u_j^2 \int_{-1}^{+1} dx \Psi^2(p, q, x), \quad 4\pi \int_0^\infty n(u_i) du_i = 1, \quad (15)$$

where $(i, j) = (1, 2)$, $(2, 1)$, $u_1$, and $u_2$ stand for Jacobi momenta $p$ and $q$, respectively. The momentum probability densities $n(p)$ and $n(q)$ calculated for low-momentum potentials for different values of cutoff $\Lambda$ are presented in Figs. 5 and 6 and compared with corresponding results obtained from the bare potential. The momentum probability density $n(p)$ has a dip around 2 fm$^{-1}$, which has been shifted down for $\Lambda$ less than 2.0 fm$^{-1}$ and for larger values it has been moved up. Similar behavior can be seen in the momentum probability density $n(q)$, but the shift is not as visible as for $n(p)$.

4.3. Expectation values

In order to test the accuracy of our numerical solution for the Faddeev integral equation (12), and also the calculation of the total wave function (11), we have calculated the expectation value of the 3B Hamiltonian $\langle \Psi|H|\Psi \rangle \equiv \langle H \rangle$ and compared with the calculated binding energy. The explicit form of the expression $\langle H \rangle$ which should be obtained from expectation values of the free Hamiltonian $\langle H_0 \rangle$ as well as potential $\langle V \rangle$ are given in Ref. [17]. Our numerical results for $\langle H_0 \rangle$, $\langle V \rangle$, and consequently $\langle H \rangle$ calculated for low-momentum interaction with $\Lambda = 2.1$ fm$^{-1}$ are given in Table 2. We have done this test of numerical accuracy for different numbers of mesh points for Jacobi momenta $N_{\text{jac}}$ and angle variables $N_{\text{sph}}$, from 20 to 40. The comparison between the expectation value of the 3B Hamiltonian $\langle H \rangle$ and the eigenvalue energy $E_3$ shows that our results are in good agreement. However a better agreement could be reached if we considered a larger number of mesh points in our calculations.

5. Summary

In this paper, we have performed 3B Faddeev calculations by employing a low-momentum potential $V_{\text{low}}$, which is derived in a non-PW representation and by using the Lee–Suzuki similarity transformation method. The main purpose of this work is to test the 3D form of low-momentum interaction in three-body bound state calculations. The motivation for using the 3D approach is to avoid truncation problems and the necessity of complicated recoupling algebra that accompanies PW-based calculations; instead, the equations and amplitudes are formulated in the 3D approach, directly as functions of momentum vector variables. We have studied the dependence of three-body binding energy on low-momentum cutoff $\Lambda$, which separates the Hilbert space into a low-momentum and a high-momentum...
part, and have compared the results with those obtained by using bare spin-independent Malfliet–Tjon potential. The stability of our numerical results for low-momentum interaction and 3B binding energy has been studied with the calculation of the expectation value of the total Hamiltonian. The agreement between binding energy results from low-momentum interaction and those obtained using the bare potential indicates that the 3D form of low-momentum interaction is suitable to be applied in few-body calculations in a non-PW representation.

After this first successful application of the 3D form of $V_{\text{low } k}$, there is now motivation to pursue further few-body bound and scattering calculations with the non-PW form of low-momentum potential, and we predict that the incorporation of three-body force will be less cumbersome in a 3D approach.

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