In the real life, there exists limited information or uncertainty in knowledge about failure and repair rates which follow one of the standard distributions as exponential distribution and Weibull distribution with some parameters. We suppose that these parameters are fuzzy which allow one to specify a system design for a "worst-case scenario." In this paper, the fuzzy availability and the fuzzy reliability of a redundant repairable parallel k-out-of-n warm standby system with common-cause failure are evaluated. We assume that the failure time of each operating unit or warm standby unit follows Weibull distribution with two fuzzy parameters and the repair time of any failed unit follows exponential distribution with one fuzzy parameter. Each fuzzy parameter is represented by triangular membership function estimated from statistical data taken from random samples of each unit. Also, we give a numerical example for a fuzzy repairable parallel 3-out-of-5 warm standby system with two active and three warm standby units to get analytically and represent graphically the fuzzy availability and reliability function of this fuzzy system.

1. Introduction

We study parallel k-out-of-n systems. These systems operate (perform their operational function) provided at least k units operate. Such systems occur in many contexts: in process industries, e.g., in desalination plants [1]; in transportation systems, e.g., electric traction motors on commuter trains [2]; in power generation, e.g., secondary-cooling pumping systems [3]; in distribution networks, e.g., electricity supply [4]; Larsen et al. (2017); in manufacturing e.g. soft drinks production [5]; in aerospace navigation systems [6]; and in oil and gas production infrastructure, e.g., in [7] and in the military [8]. Essentially, such systems are designed with additional capacity or redundancy [9] so that operation is resilient [10] to unit failure.

All units may be operating and sharing production load equally. These are described as hot-standby systems [11, 12], e.g., parallel reverse-osmosis trains in a seawater desalination plant [13]. When standby units are held in reserve, we might assume that nonoperating units are not deteriorating. This is so-called cold standby. Otherwise, and more realistically, we can assume that standby units deteriorate more slowly than operating units. This is warm standby [14–16].

Redundancy or standby cannot guarantee system availability. Switching may be imperfect [17], whereby there is a delay between a failure of one unit and the start-up of a standby unit. We consider perfect switching because switching times are typically small relative to unit lifetimes and because switching can be near-instantaneous when standby units are already operating under reduced load.
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( warm standby ). Nonzero repair times for failed units will also tend to limit availability, particularly if repair times are long, e.g., due to unavailability of spare parts [18]. In our model, we assume warm standby and that repair times are nonzero. Finally, common-cause failures can significantly limit availability. Common causes can damage many units simultaneously, e.g., all auxiliary power units failed simultaneously at Fukushima [19]. Common causes are often environmental, e.g., in a maritime engineering, one single iceberg damaged several watertight compartments of the Titanic [20] but need not be [21]. Measures are taken to prevent common-cause failures (e.g., separation of auxiliary systems or specification of different types of auxiliary systems). Indeed, in maritime propulsion in particular many k-out-of-n warm standby system configurations have been proposed [22]. In this paper, we shall consider common-cause failures.

We shall suppose that units in our system are statistically identical and independent. Also, the state of each unit can be monitored and the failed units are repaired simultaneously to a state that is as good as new [23]. The time to failure and the time to repair are random variables with hazard rates that we specify and call the failure and the repair rates, respectively. We assume that common-cause failures occur randomly according to a homogenous Poisson process [24]. In this way, notionally, a common-cause failure is an external shock (e.g., [25]). Finally, and in the principal contribution of this paper, we suppose that there exists limited information about failure rates and repair rates. Thus, not only are the times to failure and the times to repair of units uncertain, modelled in the standard way using reliability distributions [26] but also the parameters of these distributions are uncertain. We suppose that these parameters are fuzzy (Jovan, 2002). Considering both common causes and uncertainty in the knowledge of failure and repair parameters, we do allow one to specify a system design for a “worst-case scenario.” In the literature, there exist works on k-out-of-n parallel systems that consider warm standby [27]; She and Pecht, 1992; [28–34]; and works that consider common-cause failures [35–39] and works that consider fuzzy specification of failure and repair rates [21, 40–45], but none to our knowledge consider all these aspects in one model. Furthermore, we think it is important to consider these aspects simultaneously because these features interact in interesting ways that are important to study. In a sense, warm standby describes an aspect of uncertainty in redundancy because we might have weaker information about reserve or auxiliary units. Common-cause failures add another layer of uncertainty. Also, parameters are often uncertain due to lack of sufficient data [46, 47] or unknown failure-modes. Furthermore, the circumstances we model (k-out-of-n, warm standby, common-cause failures, and parameter uncertainty) arguably provides a better description of reality and a complete framework for mathematical development.

In the paper, fuzzy reliability and availability are calculated using an algorithm based on the a-cut technique [48]. The proposed algorithm is appropriate because it uses simplified fuzzy arithmetic operations [49] and its execution is faster than other methods [50]. The validity of the proposed method is demonstrated in the numerical example that we describe. Our results illustrate the advantages of the algorithm and its application to complicated systems. The structure of the paper is as follows. In Section 2, we describe the system and the model and its assumptions. In Section 3, we obtain the system reliability and availability. Section 4 describes the estimation method for the fuzzy parameters and the method for obtaining the fuzzy reliability and the fuzzy availability. A numerical example is presented in Section 5. Conclusions and directions for future work are given in the final section.

2. The Model Description and Assumptions

We consider a repairable k-out-of-n warm standby system that consists of n units in parallel. Each unit in the system is operating or failed or on warm standby. The system is operating if and only if k units are operating. The set of possible states of the system at any time t is denoted by \( \{S_C, S_F, S_i \mid i = 0, 1, \ldots, n - k \} \) where \( S_C \) is system failure due to a common-cause failure, \( S_F \) is the system failure due to failure of at least \( n - k + 1 \) units, and \( i \) is the number of failed units when the system is operating.

2.1. Assumptions

1. When an operating unit fails, the newest warm standby unit changes its state to operating instantaneously if there exists at least one unit on warm standby. The process by which a unit changes from warm standby to operating is called switching.

2. No matter how long a unit stays in warm standby, it is already switched to be operating with time varying rate \( h_1(t) \).

3. The time varying failure rate of the newest warm standby unit \( h_2(t) \) is less than \( h_1(t) \) in any time \( t \).

4. When an operating unit or warm standby unit fails, it is immediately sent for repair. After repair, the failed unit is as good as new.

5. All failed units are repaired simultaneously.

6. The repair rate for any failed unit is constant and equal \( \mu \).

7. The function of the system might be interrupted due to common cause includes design error, human error in installation, improper maintenance and operation, extreme operating conditions such as vibration, heavy rains, high temperature, humidity, fire, external shocks created by earth quakes, and floods.

8. Common-cause failures arise according to a Poisson process with constant failure rate \( \lambda_c \).

9. Units that fail due to a common-cause are repaired simultaneously and the repair rate of each unit in this case is \( \mu_c \).
(10) Common-cause failure and failure of any single unit are mutually independent which means that the failure of any operating unit \( i; i = 1, 2, \ldots, k \) is independent on the state of the other units.

(11) Switching is perfect: switching of one unit does not change the state of other units; the time to failure of a unit is statistically independent of the switching of another unit.

3. Availability and Reliability Model

Depending on the above assumptions, all possible states of the warm standby repairable parallel \( k \)-out-of-\( n \) system with replacement at each common-cause failure are presented in Table 1 and the state transition diagram is shown in Figure 1.

Let \( P_i(t); i = 1, 2, \ldots, n - k, P_c(t), \) and \( P_F(t) \) be the probabilities that the system is in the states \( S_i, S_c, \) and \( S_F \) at any time \( t \), respectively. From the state transition diagram, shown in Figure 1, and by elementary and continuity arguments, we can get the set of first-order differential equations governing the stochastic behavior of our system in terms of the rates \( h_1(t), h_2(t), \mu, \mu_c, \) and \( \lambda_c \) as follows:

\[
\begin{align*}
\frac{dP_0(t)}{dt} &= -[kh_1(t) + (n - k)h_2(t) + \lambda_c]P_0(t) + \mu P_1(t) + \mu_c P_c(t), \\
\frac{dP_i(t)}{dt} &= -[kh_1(t) + (n - k - i)h_2(t) + \lambda_c + i\mu] \\
&\quad \cdot P_i(t) + [kh_1(t) + (n - k - i + 1)h_2(t)]P_{i-1}(t) + (i + 1) \\
&\quad \cdot \mu P_{i+1}(t) + \mu_c P_c(t); i = 1, 2, \ldots, n - k - 1, \\
\frac{dP_{n-k}(t)}{dt} &= -[kh_1(t) + \lambda_c + (n - k)\mu] \\
&\quad \cdot P_{n-k}(t) + [kh_1(t) + h_2(t)]P_{n-k-1}(t) + (n - k + 1) \\
&\quad \cdot \mu P_F(t) + \mu_c P_c(t); n - k \geq 2,
\end{align*}
\]

\[
\frac{dP_F(t)}{dt} = -(n - k + 1)\mu P_F(t) + kh_1(t)P_{n-k}(t),
\]

\[
\frac{dP_c(t)}{dt} = 1 - P_F(t) - \sum_{i=0}^{n-k} P_i(t).
\]

The initial conditions for our model if the process is in state “0” at the beginning are given by

\[
P_0(0) = 1 \quad \text{and} \quad P_i(0) = 0; i = 1, 2, \ldots, n - k, F, C.
\]

When the Weibull failure rates of both the operating and stand-by units are given by \( h_1(t) = \gamma_1\beta_1 t^{\gamma_1 - 1} \) and \( h_2(t) = \gamma_2\beta_2 t^{\gamma_2 - 1}, 0 < h_2(t) < h_1(t), \beta_1, \beta_2 > 0 \) and \( \gamma_1, \gamma_2 > 1. \)

Under the specified initial conditions (6), the system of the first-order differential equations (1)–(5) can be solved graphically or by any mathematical method as substitution and elimination to get the transition probabilities \( P_i(t); i = 1, 2, \ldots, n - k, F, C \) then, the system availability at time \( t \) can be evaluated by

\[
A(t) = \sum_{i=0}^{n-k} P_i(t), \ t \geq 0.
\]

To obtain the system reliability function \( R(t) \), the set of failed states in equations (1)–(5) are treated as absorbing states in the initial model (i.e., set all transition repair rates from these states equal to zero by putting \( \mu_c = 0 \) in all equations and \( \mu = 0 \) in equations (3) and (4)). If we consider \( P_i(t) \rightarrow P^*_i(t); i = 0, 1, \ldots, n - k, F, C \) and solve these equations again with the same initial conditions (6) to get \( P^*_i(t) \), the system reliability is given by

\[
R(t) = \sum_{i=0}^{n-k} P^*_i(t), \ t \geq 0.
\]

4. Fuzzy Availability and Reliability

We can extend the applicability of our system by assuming that all failure rates and repair rates in our model have fuzzy parameters. Each fuzzy parameter can be represented by a fuzzy set \( \tilde{A} \). \( \tilde{A} \) is defined on a universe of discourse \( X \) and is denoted by \( \tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) : x \in X \} \) if \( \mu_{\tilde{A}}(x) : X \to [0, 1] \).
[0, 1] is a map called the membership function which determines the degree of membership of any element $x \in X$ in the fuzzy set $\tilde{A}$ and the choice of the membership is dependent on the observer and application. In our model, we suppose each fuzzy parameter has a triangular membership function which is denoted by $\tilde{A} = a, b, c$ and given by

$$
\mu_{\tilde{A}}(x) = \text{triangle}(x; a, b, c) = \begin{cases} 
0, & x < a \\
\frac{x - a}{b - a}, & a \leq x \leq b \\
\frac{c - x}{c - b}, & b \leq x \leq c \\
0, & x > c
\end{cases}
$$

(9)

where $a$ is the lower value, $b$ is the medium value, and $c$ is the upper value. Also, we can get $\alpha$-cut interval of $\tilde{A}$ as follows:

$$
\tilde{A}[\alpha] = [A_L(\alpha), A_U(\alpha)]
= [a + \alpha(b - a), c - \alpha(c - b)] \forall \alpha \in [0, 1].
$$

By using statistical data taken from random sample of each unit, we can obtain $(1-\alpha)100\%$ confidence intervals of any unknown parameter at different values of significance level $\alpha; 0 < \alpha < 1$. These confidence intervals can be converted directly to triangular fuzzy parameters [51].

4.1. Estimation of Weibull Distribution Parameters $\gamma$ and $\beta$.
Suppose $t_1, t_2, \ldots, t_n$ is a random sample from operating active or warm standby units with Weibull failure time. We can use the likelihood function $L(\beta, \gamma)$ to calculate $\hat{\beta}$ and $\hat{\gamma}$ which are the point estimations (average) of the unknown parameters $\beta$ and $\gamma$ (e.g., [52, 53]). Also, we approximate $(1-\alpha)100\%$ confidence interval of $\hat{\beta}$ and $\hat{\gamma}$ as follows:
following procedures:

4.2. Estimation of Exponential Distribution Parameters $\mu$, $\mu_c$, or $\lambda_c$. Given a random sample $x_1, x_2, \ldots, x_m$ of size $m$ from a unit having exponentially distributed failure/repair time, the $(1-\alpha)100\%$ confidence interval of the unknown parameter $\phi = (\mu, \mu_c, \lambda_c)$ of the exponential distribution is [54]

$$
\hat{\phi} - \frac{X_{\alpha/2, (m-2)}}{2} \leq \hat{\phi} = (\hat{\mu}, \hat{\mu}_c, \hat{\lambda}_c) \leq \hat{\phi} + \frac{X_{\alpha/2, (m-2)}}{2}, 0 \leq \alpha \leq 1,
$$

where $\hat{\phi}$ is the point estimation (average) of the unknown parameters $\phi = (\mu, \mu_c, \lambda_c)$ and $\sum_{j=1}^{m} x_j$ is the total test time of our random sample.

4.3. System Availability and Reliability with Fuzzy Failure and Repair Rates. The fuzzy availability and reliability of our model can be computed by the $a$-cut technique with the following procedures:

1. From any active operating unit with fuzzy Weibull hazard failure rate $\tilde{h}_1(t) = \tilde{y}_1 \tilde{p}_1 t^{-\tilde{\alpha}_1}$, take a random sample with size $n_1$ at fixed values of the parameters $\tilde{p}_1$, $\tilde{y}_1$; $\tilde{p}_1 > 0$, $\tilde{y}_1 > 1$; then, calculate their confidence intervals by expression (11).

2. From any warm standby unit with fuzzy Weibull hazard failure rate $\tilde{h}_2(t) = \tilde{y}_2 \tilde{p}_2 t^{-\tilde{\alpha}_2}$, $0 < \tilde{h}_2(t) < \tilde{h}_1(t)$, take a random sample with size $n_2$ at fixed values of the parameters $\tilde{p}_2$, $\tilde{y}_2$; $\tilde{p}_2 > 0$, $\tilde{y}_2 > 1$; then, calculate their confidence intervals as expression (11).

3. From any failed unit with fuzzy constant repair rate $\tilde{\mu}$, take a random sample with size $m$ at fixed value of the parameter $\tilde{\mu}$; then, calculate the confidence interval of $\mu$ as expression (12).

4. When common-cause failure occurs, take two other samples to get the confidence intervals of the constant common-cause failure rate $\tilde{\lambda}_c$, and repair rate $\tilde{\mu}_c$ as expression (12).

5. After substituting in equations (1)–(5) by the crisp confidence intervals of the failure and repair rates obtained at distinct values of $\alpha$: $\alpha \in [0, 1]$, solve them mathematically by using Maple program under the initial conditions (6) to obtain the transition probabilities $P_i(t); i = 1, 2, \ldots, n - k, F, C$; then, calculate the crisp intervals of the fuzzy availability function $\breve{A}(t)$ at time $t$ corresponding to the chosen values of $\alpha$ by using relation (7).

6. Repeat the previous step to obtain the fuzzy reliability function $\breve{R}(t)$ corresponding to the chosen values of $\alpha$ by using relation (8) after putting $\mu_c = 0$ in equations (1)–(5) and $\mu = 0$ in (3) and (4).

5. Numerical Example

Suppose a power generator consists of three parallel equipment groups placed in close proximity by operating independently. Because of the requirement for more output of electric power, another two equipment groups are added in parallel. Our power generator satisfies power supply requirement if and only if at least three of the five equipment groups are in operation. The new two equipment groups are arranged to be in standby, but the old three equipment groups have priority to operate. By using a perfect switching mechanism, we can replace any failed equipment group by a standby one instantaneously and repair it. Suppose the failure rates of both the operating and stand by components are $\breve{h}_1(t) = \breve{y}_1 \breve{p}_1 t^{-\breve{\alpha}_1}$ and $\breve{h}_2(t) = \breve{y}_2 \breve{p}_2 t^{-\breve{\alpha}_2}$, $0 < \breve{h}_2(t) < \breve{h}_1(t)$ and the repair rate is $\breve{\mu}$. High surrounding temperature may affect suddenly on our system and cause complete failure with fuzzy failure rate $\tilde{\lambda}_c$ and then, it is repaired with fuzzy repair rate $\tilde{\mu}_c$.

Our system represents parallel repairable 3-out-of-5 warm standby system with common-cause failure. The first-order differential equations of our system in terms of the fuzzy rates $\breve{h}_1(t)$, $\breve{h}_2(t)$, $\breve{\mu}$, $\breve{\mu}_c$, and $\tilde{\lambda}_c$ are
Table 3: The intervals for $\tilde{y}_1$, $\tilde{y}_2$, $\tilde{p}_1$, $\tilde{p}_2$, $\tilde{h}_1(t)$, $\tilde{h}_2(t)$, $\tilde{\lambda}$, $\tilde{\lambda}_C$, and $\tilde{\mu}_C$ corresponding to $\alpha$-cut = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.

| $\alpha$-cut | $[\tilde{y}_1^L, \tilde{y}_1^U]$ | $[\tilde{p}_1^L, \tilde{p}_1^U]$ | $[\tilde{h}_1^L(t), \tilde{h}_1^U(t)]$ | $[\tilde{y}_2^L, \tilde{y}_2^U]$ | $[\tilde{p}_2^L, \tilde{p}_2^U]$ | $[\tilde{h}_2^L(t), \tilde{h}_2^U(t)]$ | $[\tilde{\lambda}^L, \tilde{\lambda}^U]$ | $[\tilde{\lambda}_C^L, \tilde{\lambda}_C^U]$ | $[\tilde{\mu}_C^L, \tilde{\mu}_C^U]$ |
|--------------|-----------------|-----------------|-------------------------------|-----------------|-----------------|-------------------------------|-----------------|-----------------|-----------------|
| 0.1          | [1.173, 1.951]  | [1.015, 1.569]  | [1.190^{0.17}, 3.060^{0.95}] | [1.247, 1.887]  | [0.87, 1.404]   | [1.09^{0.25}, 2.65^{0.89}]   | [0.04, 0.115]   | [0.11, 0.042]   | [0.044, 0.106]  |
| 0.2          | [1.258, 1.865]  | [1.075, 1.508]  | [1.35^{0.20}, 2.81^{0.87}]   | [1.317, 1.817]  | [0.929, 1.345]  | [1.22^{0.53}, 2.44^{0.82}]   | [0.043, 0.111]  | [0.013, 0.041]  | [0.048, 0.103]  |
| 0.3          | [1.317, 1.806]  | [1.117, 1.466]  | [1.47^{0.31}, 2.65^{0.81}]   | [1.366, 1.768]  | [0.969, 1.305]  | [1.32^{0.37}, 2.31^{0.77}]   | [0.047, 0.106]  | [0.014, 0.039]  | [0.051, 0.099]  |
| 0.4          | [1.362, 1.762]  | [1.149, 1.434]  | [1.57^{0.36}, 2.53^{0.76}]   | [1.403, 1.731]  | [1.1273]        | [1.46^{0.34}, 2.21^{0.71}]   | [0.051, 0.101]  | [0.016, 0.037]  | [0.054, 0.095]  |
| 0.5          | [1.402, 1.721]  | [1.178, 1.405]  | [1.65^{0.4}, 2.42^{0.72}]    | [1.436, 1.698]  | [1.027, 1.246]  | [1.48^{0.44}, 2.12^{0.69}]   | [0.055, 0.097]  | [0.017, 0.035]  | [0.057, 0.092]  |
| 0.6          | [1.438, 1.686]  | [1.203, 1.380]  | [1.73^{0.44}, 2.33^{0.69}]   | [1.465, 1.669]  | [1.051, 1.222]  | [1.54^{0.47}, 2.04^{0.67}]   | [0.058, 0.092]  | [0.018, 0.032]  | [0.061, 0.088]  |
| 0.7          | [1.471, 1.653]  | [1.227, 1.357]  | [1.80^{0.47}, 2.24^{0.65}]   | [1.492, 1.642]  | [1.075, 1.199]  | [1.66^{0.49}, 2.09^{0.59}]   | [0.062, 0.087]  | [0.020, 0.030]  | [0.064, 0.085]  |
| 0.8          | [1.502, 1.622]  | [1.249, 1.335]  | [1.88^{0.5}, 2.17^{0.63}]    | [1.518, 1.617]  | [1.096, 1.178]  | [1.66^{0.52}, 1.99^{0.62}]   | [0.066, 0.082]  | [0.021, 0.028]  | [0.067, 0.081]  |
| 0.9          | [1.532, 1.591]  | [1.271, 1.313]  | [1.95^{0.53}, 2.09^{0.59}]   | [1.543, 1.591]  | [1.117, 1.157]  | [1.72^{0.54}, 1.84^{0.59}]   | [0.069, 0.078]  | [0.023, 0.026]  | [0.071, 0.077]  |
| 1.0          | 1.562           | 1.292           | 2.018^{0.562}                 | 1.567           | 1.137           | 1.782^{0.567}                 | 0.073           | 0.024           | 0.074           |
Table 4: The statistical information used to get $\tilde{\mu}, \tilde{\lambda} C$, and $\tilde{\mu}_C$.

|          | $\mu$ | $\lambda$ C | $\mu$ C | $\sum m j = 1 x_j$ |
|----------|-------|-------------|--------|-------------------|
| $\tilde{\mu}$ | 12    | 10          | 7      | 110               |
| $\tilde{\lambda}$ C | 420   |             |        |                   |
| $\tilde{\mu}_C$ | 96    |             |        |                   |

Figure 2: The fuzzy availability function versus the time at $\alpha - cut = 0.1, 0.4, 0.7, 1$.  

Figure 3: The fuzzy reliability function versus the time at $\alpha - cut = 0.1, 0.4, 0.7, 1$.  

Figure 4: The fuzzy availability at $t = 0.3$.  

A($t$)

$R(t)$

$\mu_A(X)$
By the statistical data taken from random samples of operating component and stand by component (shown in Table 3), we obtain number of (1-α)100% confidence intervals for $\tilde{Y}_1$, $\tilde{Y}_2$, $\tilde{Y}_3$, $\tilde{h}_1(t)$, and $d\tilde{h}_2(t)$ at different values of significance level $\alpha$ as shown in Table 3. Then, we produce the triangular-shaped membership function of the fuzzy parameters $\tilde{Y}_1$, $\tilde{Y}_2$, $\tilde{Y}_3$, $\tilde{h}_2$, $\tilde{h}_3$, $\tilde{h}_4(t)$ by placing these confidence intervals one on top of the other as follows:

$$\tilde{Y}_1 = 1.2583, 1.5619, 1.8655,$$
$$\tilde{Y}_1 = 1.0754, 1.2917, 1.5081,$$
$$\tilde{Y}_2 = 1.2474, 1.5671, 1.8869,$$
$$\tilde{Y}_2 = 0.8704, 1.1369, 1.4035.$$

Also, by using the samples’ statistical data shown in Table 4 and relation (12), we obtain number of (1-α)100% confidence intervals for $\tilde{F}_1$, $\tilde{F}_2$, $\tilde{F}_3$, $\tilde{F}_4$, $\tilde{f}_5$, and $d\tilde{f}_6(t)$ at different values of $\alpha$ as shown in Table 3 and the triangular-shaped membership function are given by as follows:

$$\tilde{F}_1 = 0.0397, 0.073, 0.1153,$$
$$\tilde{F}_1 = 0.0114, 0.024, 0.0429,$$
$$\tilde{F}_2 = 0.04427, 0.0737, 0.10637.$$

We solve our model’s equations (13)–(17) numerically under the initial conditions $P_0(0) = 1$, $P_i(0) = 0; i = 1, 2, F$, $C$ at arbitrary values of $\alpha$ to obtain the system fuzzy availability at time $t$ by relation (7) and represent it graphically as shown in Figure 2. Let $\tilde{P}_c = 0$ in equations (13)–(17) and $\tilde{P}_c = 0$ in (15) and (16); then, solve them again under the same initial conditions to get the system fuzzy reliability at time $t$ by relation (8) and graph it as Figure 3. Also, we can graph the fuzzy availability and reliability of our system at any instant value of time (take $t = 0.3$) as shown in Figures 4 and 5.

6. Discussion

The classical approach based on the probability theory became inappropriate for analyzing the system performance due to different types of errors or lack of sufficient data. For this reason, we have presented a simple and faster algorithm for analyzing the performance of fuzzy repairable parallel warm standy k-out-of-n model in the presence of perfect switching and common-cause failure. We have used the concept of $\alpha$-cut to find the fuzzy reliability/availability of this system when the failure time of any active/warm standby unit follows Weibull distribution and the repair time follows exponential distribution with fuzzy parameters represented by triangular membership functions estimated by using sample data and confidence interval concept. A real application example for a fuzzy 3-out-of-5 warm standby system with a common-cause failure is given to demonstrate the introduced analysis procedures to get the system fuzzy
availability and reliability functions with graphs. This method can be only used when the failure time and the repair time follow standard distributions with uncertain parameters accounted by using the fuzzy approach. The work that we have carried out has clear implications for practice because the knowledge about causes and effects of failures is usually described with large uncertainty content in various systems. Our work considers all the system parameters’ are fuzzy numbers and obtains two performance characteristics (availability and reliability) of the system in the fuzzy form. This will be helpful for analyzing some real systems such as electric systems, aerospace navigation systems, and military systems which may have some errors results in many problems. As a future extension to this work, we can apply the proposed method for analyzing other complex repairable and nonrepairable systems such as linear and circular consecutive k-out-of-n with hot, cold, or warm standby having different types of fuzzy time varying failure and repair rates [55–68].

**Notations**

- \( h_i(t); i = 1, 2: \) Time varying failure rate
- \( \mu: \) Constant repair rate
- \( \lambda, \mu_i: \) Common-cause failure and repair rates
- \( \gamma, \beta: \) Shape and scale parameters of Weibull distribution
- \( P_i(t): \) Probability that system is in the state \( S_i; i = 1, 2, \ldots, n - k \) at any time \( t \)
- \( P_C(t): \) Probability that the system is in the common-cause failure state
- \( P_F(t): \) Probability that all the warm units fail
- \( A(t): \) System availability
- \( R(t): \) System reliability.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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