Sum Rate Analysis for Massive MIMO Downlink With MRC Beamforming and User Selection

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Abstract. Average allocation of data rate to each user is inefficient since the resource a base station can allocate is limited. Thus, user selection and user scheduling need to be applied into multi-user massive multiple-input multiple-output (MIMO) downlink system. In this paper, we mainly focus on the methods of user selection. First, we establish a downlink system model including transmission model and channel model. Then, two user-rate based user selection algorithms via the signal-to-interference-plus-noise-ratio (SINR) are proposed, where the SINR is generated by MRC beamforming. Finally, simulation results are provided to compare the performance of two proposed algorithms and their fairness towards selected users. In the simulation results, location-based selection algorithm and random selection algorithm are jointly compared. The second proposed algorithm possesses the highest total sum-rate and is the optimal algorithms among the four algorithms.

1. Introduction
Nowadays, 5G has become a popular technique for its high bit rate and higher system capacity in order to accommodate the widespread use of smart devices and the qualitative change in association with the Internet of Things (IoT). However, because of the great number of users waiting for signal communication with a Base Station (BS), the resource a BS can provide to each user will be extremely limited in a multi-users Massive MIMO system. Therefore, it is important to distribute data rate effectively.

In that case, we need to find a solution to select and schedule users to receive the signal from the BS. Those users who have better communication channels or higher data rate should be given more power. To achieve this purpose, sum-rate based user selection are considered. This is related to the beamforming technique in Massive MIMO system [1-3].
When a base station has a large number of antennas, the phase of each received signals by the base station can be adjusted to form a superimposed signal, which usually means each transmitted antennas in this massive MIMO system need lower power to transmit signals. Beamforming is an important technique, by performing different beams points to large number of users, the interference between users and beams can be effectively reduced.

However, as everyone knows, there are no systems with no resources limitation exists in the real world, the massive MIMO system have very limited resources compare to the huge amount of users it need to support, it just do not have enough antennas to handle tens of thousands communication requirements between users, to make the overall performance of this massive MIMO system better, we have to make a choice, we select those users have better communication with the base station(BS) and give them more power [4].

This paper is more focused on the downlink user selection. Which means we are thinking a way to let the BS to think which user it needs to provide more power, pays less attention to those users have worse signal or have noisier channel, by doing the power allocation control, this system can run more efficient and have much better performance.

By saying the word better performance, we need a standard to see what we are talking about exactly, we are doing the sum rate analysis of the massive MIMO system, we conduct quantitative studies on the sum-rate performance of the massive MIMO system has limited antennas.

To achieve higher efficiency, we choose to optimize the plan of user selection, so we focus on downlink situation. First we need to consider about different users in one same channel. Several users exist in one channel will cause interference and higher path loss during signals transition. A method of user selection based on zero-forcing beamforming is provided in [4]. Their method is useful to be an alternative choice of Dirty Paper(DP)-based schemes. For less numeration, we choose this method as part of our project. It proposes a user selection algorithm considering about signal-to-leakage-and-noise-ratio(SLNR) and derived sum-rate expression with contrast low-complexity. Also, simulation results show the advantage of this special method compared with existed algorithm. A two-step method of user selection has been proposed in [5]. First, cut the high-frequent users and low-quality users. Second, use channel estimation to make an order of the other users. A special algorithm used in Massive MIMO and OFDMA system is mentioned in [6], which will maximum the total sum-rate of all sub carriers. In practical situations, there are more users than antenna arrays. Thus, dealing with the numeration of choosing users online becomes a big problem for base station.

Solving the problem of user selection, our method’s purpose is to provide a better power allocation. Assumed the system is working in time division duplex(TDD) mode with beamforming training(BT) method to transmit downlink pilots, it is possible to achieve higher power rate with different power allocation on different users [7]. Then, a deep understanding of this kind of user selection is provided in [8]. It uses machine learning to learn the connection between the positions of user equipments (UEs) and better power allocation. The special thing of this method is that it does not need compute any statistical average, which means higher accuracy because of the base station is able to collect accurate data of UEs. We learn the method of how to compute the distance between base station and UEs from [9]. This method helps our method to reduce a large amount of work need to measure the distance and angles between base station and UEs. With all the algorithms mentioned before, thus, our method could help the process of forming a better power allocation at base station with all numerations finished online. The result of code simulation proves the feasibility of our method.

Based on the result appears after our analysis, we have a more straight forward looking on the system design and performance. We are using two different user selections algorithm compared to the random selection to see which one is better, as the selected number of users increased, the performances of three
algorithms are changing, one could be better when there are less users, but as the number of users increasing, the performance is surpassed by another algorithm.

The rest of this paper is organized as followed. In section II, the system model including a transmission model and a channel model will be presented. Section III proposes two user selection algorithms. And the following section IV and V are the results analysis and conclusion of this paper.

2. System Model

This section introduced the transmission model in massive MIMO downlink and the channel model based on location.

2.1. Transmission model

In this paper, we consider a single cell multi-user downlink system. The single cell system model is a Massive MIMO system with beamforming technology. Consider our main application area is IoT (internet of things), generally a great number of different devices and users should be taken into account. However, the resources that a BS (Base Station) can provide to users are limited. The BS need to make a selection and schedule (not presented in this paper) to serve the users.

A downlink user selection and a scheduling are executed in the BS by choosing the users possessing the highest data rate. And the BS will send signal to the selected users in sequence. Then, the overall UE (User Experience) can be better.

We assume the range of BS coverage to be 2000 meters, users are randomly distributed in the circle like Fig.1.

The pass loss of the channel depends on the distance between users and the BS. In this paper, the pass loss coefficient is set to be 3.4. The wavelength is set to be 108 mm as is like in 5G communication network.

The received signal of each user at BS is described as followed:

$$y_{D,k}(n) = h_k \left( \sum_{l=1}^{K} \sqrt{P_{D,l}} x_{D,l}(n) + w_k(n) \right).$$

(1)

In this formula, $x_{D,l}(n)$ is the unit power data symbol transmitted by UE-$l$ in downlink, $P_{D,l}$ is the allocated transmit power by BS for UE-$l$. 

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![Fig 1. Single cell transmission model](image-url)
2.2. Notation

Table 1. Symbols explanation

| Symbol | Explanation |
|--------|-------------|
| $h_k^*$ | Complex conjugate transpose of $h_k$ |
| M     | Total number of antennas of BS |
| U     | Total user set |
| K     | Number of all users |
| rho   | Path loss coefficient |
| $\sigma_w^2$ | Noise factor |
| N     | Number of selected users |

Table 1 shows the explanation of the notations and symbols used in this paper; these notations will appear in the equations in the following sections.

2.3. Channel model

The channel model is built based on the description in [10].

An example of channel is depicted in Fig. 2. $h_k$ is the channel vector between UE-k and BS antenna array. It is given as

$$ h_k = [h_{k,1}, h_{k,2}, ..., h_{k,M}]^T $$

where $\theta_k$ is the angle from the UE-k to the BS. It is a random value from 0 to $2\pi$. And $\beta_k$ is the path loss between UE-k and BS. It is given as

$$ \beta_k = \frac{1}{\sqrt{2}d_k^{-\rho}} $$

where $d_k$ is the distance between the UE-k and BS. It is a random value from 200 to 2000. And $\rho$ is the path-loss exponent.

A MIMO channel matrix $H$ with size $M\times K$ between $K$ UEs and BS antenna is given as

$$ H = [h_1, h_2, ..., h_K]^T. $$
Therefore, we can determine the location of the user based on the angle and distance information, and then indicate its channel.

3. User Rate Calculation for MRC Beamforming Scheme and User Selection Schemes

In this section, an analytical user-rate expression of MRC Beamforming is derived. This expression enables user selection with the direct user-rate criterion. Then, two user-rate-based user selection schemes is proposed for the massive MIMO downlink.

3.1. MRC Beamforming Scheme

The pre-coding schemes for downlink transmission is MRC (Maximal Ratio Combining) beamforming scheme which aims at maximizing the gain towards the selected user.

According to [10], the beamformer $v_k$ satisfies the following conditions

$$\text{Maximum} \quad |h_k^*v_k|^2$$

$$\|v_k\| = 1$$

Thus, the beamforming vector can be expressed as:

$$v_k = \frac{h_k}{\|h_k\|}$$

This method is to maximize the gain of the wanted signal without taking care of the inter-user interference. The inter-user interference can be found in the equation of SINR:

$$\gamma_{D,k} = \frac{P_{D,k}|h_k|^2}{\frac{1}{h_k^*\sum_{i=1}^K P_{D,i}v_i^*v_i} + \sigma^2}$$

where $\sigma^2$ is the noise power assumed to be fixed and $P_{D,k}$ is the transmit power for User k with the total power constraint $\sum_{i=1}^K P_{D,k} \leq P_D$. In this paper, we assume the power allocation among the users will be equal. That is, $P_{D,k} = P_D/K$.

The ergodic rate of User k is given by [11]

$$R_k = E\left\{\log_2 \left(1 + \frac{\gamma_{D,k}}{\sigma^2}\right)\right\}.$$  (9)

and the ergodic sum-rate is

$$R_{\text{sum}} = \sum_k R_k.$$  (10)

3.2. User Selection Schemes

In this subsection, two user selection algorithms are proposed. The first one is User-Rate Based User Selection Addition Algorithm. The second one is User-Rate Based User Selection Subtraction Algorithm.

Both algorithms select users with higher data rate.

In Algorithm 1, the users with higher data rate will be selected one by one to form the user set. And a new beamforming will be executed after the whole selection to derive the real user-rate set which eliminate the interference of unselected users.

In Algorithm 2, the users with lower data rate will be selected and move out the user set. Beamforming will be executed each time a user is selected to eliminate the interference of unselected users.
The difference between these two algorithms is whether to do beamforming while selection or to do beamforming after the selection. The selected user-rate results of these two algorithms are what we are truly interested in so as to enable the users to derive the highest user-rate.

**Algorithm 1** User Rate Based User Selection Addition Algorithm

1) **Initialization:**
   • Set $n = 0$ and $S_0 = \emptyset$.

2) **While** $n < N$:
   • Increase $n$ by 1.
   • Find the user with index $s_n$, such that
     \[
     s_n = \arg \max_{u \in U \setminus S_{n-1}} \mathcal{R}_{MRC}(S_{n-1} \cup \{u\})
     \]  
     \[
     S_n = S_{n-1} \cup \{s_n\}
     \]

3) **Re-beamforming:** $v_k \rightarrow v_n$

4) **New user-rate set:** $\mathcal{R}_{MRC}$ generation based on $v_n$

5) **Sum-rate:**
    \[
    R_{sum} = \sum_{i=1}^{N} R_n
    \]

**Algorithm 2** User Rate Based User Selection Subtraction Algorithm

1) **Initialization:**
   • Set $n=0$ and $S_0 = \emptyset$.

2) **While** $n < K-N$:
   • Increase $n$ by 1.
   • Find a user $s_n$, such that
     \[
     s_n = \arg \min_{u \in U \setminus S_{n-1}} \mathcal{R}_{MRC}(S_{n-1} \cup \{u\})
     \]
     \[
     S_n = \mathcal{C}_U\{s_n\}
     \]
   • Re-beamforming: $v_k \rightarrow v_n$
   • Derive new user-rate set $\mathcal{R}_{MRC}$ based on $v_n$.

3) **Sum-rate:**
    \[
    R_{sum} = \sum_{i=1}^{N} R_n
    \]

**4. Result**
In this paper, we got two pictures through MATLAB. In each picture, we evaluate the two algorithms mentioned in section III. The first one is the user-rate based user selection addition algorithm, which will run beamforming after the whole selection. The second one is the user-rate based user selection subtraction algorithm, which will run beamforming while the selection. Fig. 3 visually shows the average sum of the data rates of 10 users from 50 users selected by these two algorithms, and in order to show their fairness, we introduce Fig. 4 to show the average data rate of individual users selected out.
In order to make a comparison with our algorithm and show its efficiency, now there are two new user selection methods.

The first method is to randomly select users. For example, we have 50 users. This algorithm does not consider the relationship between users and BS antennas, and selects 10 users completely at random. The purpose of this group is mainly to make a basic comparison.

According to [12], the second method is to select users based on their channel. Unlike the previous screening method based on user data rate, this method is based on the user's channel to filter, and our channel is defined based on location information. Therefore, this algorithm also has a lot to do with location.

We will screen out users with higher channel quality. For example, the algorithm found the kth user, $s_k$:

$$s_k = \arg\max h_k h_k^*$$  \hspace{1cm} (17)

Then select users based on higher $s_k$, so that we can pick the highest channel quality ten users. After beamforming, we will show the rates of these users.

![Fig. 3. Average sum-rate of users selected by four algorithms](image)

Fig 3. Average sum-rate of users selected by four algorithms

Fig.3 compares the average sum-rates of 10 users selected by four different algorithms. On this figure, we can clearly draw the conclusions that the both algorithm1 and algorithm2 can select users with higher user rates than the random selection.

At the same time, although the data rates of users selected by channel are higher than those of randomly selected users, they are much lower than those of users selected by algorithm1 and algorithm2.

More importantly, User-Rate Based User Selection Subtraction Algorithm can select users with higher user rates than the User-Rate Based User Selection Addition Algorithm. This is a good demonstration of the efficiency of our algorithm.
Fig. 4 illustrates the general fairness of three algorithms. We averaged the data rate of users selected by each algorithm. On this figure, it can be clearly shown that the data rate of each user is not much different. It can be seen from the first point of Algorithm 2 that although the user data rate selected by Algorithm 2 is higher, it is not as stable as Algorithm 1. This result proves that the choice of Algorithm 1 is fair.

5. Conclusion
In this paper, the computer simulation proposes a new channel model and user selection scheme for Massive MIMO Downlink with MRC Beamforming and User Selection. The algorithm is aimed at maximizing the sum rate of served users, which improves user experience in getting better Internet quality. This paper analyzes and examines the performance of all proposed solutions by comparing the data rate of selected users.

The results show that the user selection algorithm based on user rate is better than the user selection algorithm based on channel and the random selection algorithm. Moreover, User-Rate Based User Selection Subtraction Algorithm is better than the User-Rate Based User Selection Addition Algorithm. Finally, through averaging the data rate of individual user selected by three algorithms, the fairness of Algorithm 1 has been proved. In the future work, user selection in complicate environment such as multi-cell systems and user fairness should be further studied.

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