Coherent imaging of extended objects
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Abstract

When used with coherent light, optical imaging systems are inherently unable to reproduce both the amplitude and the phase of a two-dimensional field distribution. This is because their impulse response function varies slowly from point to point, a property known as non-isoplanatism. For sufficiently small objects, this usually results in a phase distortion and has no impact on the measured intensity. Here, we show that the intensity distribution can also be dramatically distorted when extended objects are imaged. We illustrate the problem using two simple examples: the pinhole camera and the aberration-free thin lens. The effects predicted by our theoretical analysis are also confirmed by experimental observations.

Key words: Coherent Optics, Diffraction, Imaging, Optical Instruments, Lithography, Microscopy, Holography
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1. Introduction

Current technology, especially the ability to manufacture aspherical surfaces, makes it possible to design lenses and mirrors that minimize the most important geometrical aberrations. Such optical elements are nearly ideal instruments obeying the laws of Gaussian optics even for far off-axis points and non-paraxial rays. Self-luminous objects or objects illuminated with incoherent light can be imaged with high fidelity. The quality of the resulting image is only determined by the resolution of the instrument, which is related to its numerical aperture. The instrument itself is said to be diffraction-limited and can be considered as a linear filter for the intensity of light [1,2].

When imaging objects with coherent light, the conditions for accurate image formation are more severe since both the relative amplitudes and the relative phases of the object points have to be mapped to the corresponding image points (up to the resolution capability of the instrument). This only happens if the effect of the optical instrument on the field from a given point source is independent of its position in the object plane, or in other words, if the coherent impulse response [1,2] of the system is space-independent. The instrument then acts as a linear filter for the complex field amplitude. According to the terminology of [1], such an instrument is said to be isoplanatic. In most cases, even aberration-free optical instruments designed to map some planar object to an image plane under incoherent illumination do not meet this condition. Some spatial phase distortion is unavoidably introduced, which, when combined with a finite resolution, severely modifies the intensity distribution of the image. This was first recognized by Dumontet [3], and later by Tichenor and Goodman [4], who showed that a thin lens can only be considered as an isoplanatic system if both the object and the image lie on spherical surfaces $S_o$ and $S_i$, which are tangent to the geometrical-optics object and image planes $O$ and $I$ respectively, and have their center of curvature in the plane of the lens (see Sec. 3).

In practice, an aberration-free optical instrument can be treated as an ideal imaging system with coherent light whenever the spherical surfaces $S_o$ and $S_i$ can be approximated by their tangent planes $O$ and $I$. We emphasise that this is only viable when the object to be imaged is very small and lies close to the optical axis. A weaker imaging condition has been obtained by Tichenor and Goodman, who showed that the non-isoplanatism of a thin lens has a negligible effect on the intensity distribution of the image if the object diameter is smaller than about a quarter of the lens diameter [4]. In this paper, we investigate the effect of non-isoplanatism on coherent image formation when this condition is not met, and explain, both mathematically and physically, the image distortions that are then observed. These distortions may be encountered in many fields of optics where large-sized objects are imaged through powerful

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limited-aperture instruments, as in coherent far-field microscopy, optical lithography [5], holographic data-storage [6], and dipole-trapping of neutral atoms [7]. In particular, arrays of coherently emitting point sources – like individual trapped atoms excited by the same laser beam [8,9] – will be subject to the phenomena discussed here. Effects similar to those that we described in the context of imaging are also expected when lenses are used to perform the spatial Fourier transform of a two-dimensional field distribution, as in holographic dipole-trapping of atoms [10].

2. Non-isoplanatism of a pinhole camera

The problem of non-isoplanatism only originates in the propagation of light. It is not related to refraction or aberration in the imaging system, regardless of its nature. Because the pinhole camera is the simplest imaging instrument, we use it to introduce the main features of non-isoplanatism. The case of the lens, which is more important in practice than the pinhole camera is the simplest imaging instrument, we consider in the following sections. Because the geometry of the lens, which is more important in practice but presents some additional subtleties, will be analysed in more detail in the following sections.

Consider the following situation (Fig. 1): Two mutually coherent point-objects, \(P_1\) and \(P_2\), are imaged through a small pinhole (of radius \(a\)) from an object plane \(O\) to an image plane \(I\). We call \((x_{o,k}, y_{o,k})\) the transverse coordinates of the point sources \(P_k\), and \((x_{i,k}, y_{i,k})\) the transverse coordinates of their geometrical images \(P_k'\). For simplicity, we choose \(y_{o,k} = y_{i,k} = 0\) and assume that the point sources emit in phase. We also assume that the pinhole is so small that the resolution is limited by far-field diffraction. The “images” of \(P_1\) and \(P_2\) are then defined by the geometrical image points \(P_1'\) and \(P_2'\). We choose the distance between \(P_1'\) and \(P_2'\) to be equal to the resolution limit, i.e. \(x_{i,1} - x_{i,2} = 0.61\lambda z_i/a\). For any given point on \(I\), the relative phase of the fields originating from \(P_1\) and \(P_2\) is \(\phi = 2\pi(r_1 - r_2)/\lambda\). This relative phase \(\phi\) varies as a function of \(x_{o,m} \equiv (x_{o,1} + x_{o,2})/2\), the mean distance of the point sources from the optical axis. Therefore, the Airy patterns centered on \(P_1'\) and \(P_2'\) will interfere differently, depending on the location of the \(P_1' - P_2'\) pair in the object plane. The phase \(\phi\) is obviously null if \(x_m = 0\), but it reaches the value \(\pi/2\) when \(|x_{o,m}|\) is as small as \(a/2.44\). Since \(a\) is usually about 1 mm for a pinhole camera, \(\phi\) varies extremely rapidly when the two point sources move away from the optical axis.

The preceding example shows that different intensity distributions must be expected from identical object patterns depending on their positions in the object plane. This is due to an incorrect phase mapping between the object and image plane. We refer to this situation as the non-isoplanatism of coherent imaging. It is caused by the space variance of the impulse response function of the camera. Note that any linear optical system shows this behaviour [2].

The complex field amplitudes in the object and image planes, \(U_o(x_o, y_o)\) and \(U_i(x_i, y_i)\), satisfy the integral relation

\[
U_i(x_i, y_i) = \iint h(x_i, y_i|x_o, y_o) U_o(x_o, y_o) \, dx_o \, dy_o.
\]

The impulse response of the camera (in the far-field approximation) is given by

\[
h(x_i, y_i|x_o, y_o) = |M| \exp(i\frac{\pi}{\lambda} (r_1 + r_2)) \times \delta_{\phi}(x_i - M x_o, y_i - M y_o),
\]

where \((x_o, y_o)\) are the coordinates of the point source in the object plane, \((x_i, y_i)\) are the coordinates of the “observation point” in the image plane, \(M \approx -z_i/z_o\) is the geometrical magnification ratio, and

\[
r = \sqrt{x_o^2 + y_o^2 + z_o^2},
\]

\[
s = \sqrt{x_i^2 + y_i^2 + z_i^2},
\]

are the distances from the point source and the observation point respectively to the pinhole center. The point spread function introduced in Eq. (1) is given by

\[
\delta_N(x,y) = |N| \frac{J_1(2\pi N \sqrt{x^2 + y^2})}{\sqrt{x^2 + y^2}} \quad (N \in \mathbb{R}_o),
\]

where \(J_1(x)\) is a Bessel function of first kind. Its square, \(\delta_N^2(x,y)\), is the usual Airy pattern of diffraction-limited optical systems. It is normalized so that \(\lim|N|\to\infty \delta_N(x,y) = \delta(x,y)\) (the Dirac distribution).

The phase factor in Eq. (1) plays an important role. Without it, the impulse response function corresponding to any point \((x_o, y_o)\) lying in the object plane would be the same as the impulse response function of the origin \((0,0)\), but translated to the geometrical image point \((M x_o, M y_o)\). In that case, the system would be space-invariant, or isoplanatic, and the image of a coherent object with field amplitude \(U_o(x_o, y_o)\) could be computed by simply convoluting \(U_o(x_o, y_o)\) with the impulse response function. In other words, the pinhole camera would act as a linear filter that reproduces the object \(U_o(x_o, y_o)\) with a lower resolution. The example analysed in the beginning of this section clearly
shows that this is not the case. Because of the phase factor, the impulse response function is different for different points in the object plane. This creates interference patterns that modify the image more significantly than a simple blur. The image of a field profile $U_o(x_o, y_o)$ is given by

$$U_i(x_i, y_i) = \frac{1}{|M|} \exp[\frac{2\pi}{\lambda z_o} (x_i/M - x_o/M, y_i/M - y_o/M)],$$

where we made use of the fact that

$$\delta\frac{x}{x_o} = \frac{1}{|M|} \exp[\frac{2\pi}{\lambda z_o} (x/M - x_o/M, y/M - y_o/M)].$$

Seen as a function of $(x_i, y_i)$, the Airy pattern has a first-zero full-width $c_o = 1.22\lambda z_o/a$ that represents the region of the image plane most influenced by the field originating from the point source at $(x_o, y_o)$ in the object plane. Seen as a function of $(x_o, y_o)$, its first-zero full-width $c_o = 1.22\lambda z_o/a$ represents the area of the object plane that most contributes to the field at the point $(x_i, y_i)$ in the image plane. In the non-isoplanatic situation described by Eq. (3), important interferences occur when the phase factor $\exp[\frac{2\pi}{\lambda z_o} r]$ cannot be considered as constant over the circular region of area $\pi(c_o/2)^2$ centered on $(x_i/M, y_i/M)$ in the object plane.

Isoplanatism is recovered when one considers the imaging problem from $S_o$ to $S_i$ (see Fig. 1), where $S_o (S_i)$ is a spherical surface of radius $z_o (z_i)$ centered on the pinhole, and having its vertex on $O \ (I)$. The spherical field distributions on $S_o$ and $S_i$ are described by the amplitudes $U_{S_o}(x_o, y_o)$ and $U_{S_i}(x_i, y_i)$ respectively ($z$-coordinates are dependent variables). For slowly varying fields, these are related to the field amplitudes on $O$ and $I$ by a phase factor: $U_{S_i}(x_i, y_i) = U_o(x_o, y_o) \exp[i2\pi(r - z_i)/\lambda]$ and $U_{S_i}(x_i, y_i) = U_i(x_i, y_i) \exp[-i2\pi(z - z_i)/\lambda]$. Inserting these expressions into Eq. (3) and removing the constant phase factor $\exp[i2\pi(z_o + z_i)/\lambda]$, one gets:

$$U_{S_i}(x_i, y_i) = \frac{1}{|M|} \int U_{S_o}(x_o, y_o) \exp\left[\frac{2\pi}{\lambda z_o} (x_i - x_o, y_i - y_o)\right] dx_o \ dy_o,$$

which is a convolution relation, as expected for an isoplanatic system. The physical reason why isoplanatism is recovered when the object lies on the spherical surface $S_o$ is simple to understand: since all the point sources are at the same distance $z_o$ from the pinhole, their point spread functions always interfere constructively.

3. Non-isoplanatism of a thin lens

The scenario exhibited in the example of the pinhole camera occurs in all optical imaging systems. When imaging with lenses or mirrors, the diffraction effects are less dramatic than with a pinhole camera. However, the problem of phase distortion remains, and may sometimes induce unwanted intensity modulation in the image.

![Fig. 2. Scheme of coherent image formation by an aberration-free thin lens.](image)

An aberration-free thin lens is an ideal diffraction-limited optical system that acts locally on the impinging field as a pure phase transparency $T(x, y) = \exp[-i\pi(x^2 + y^2)/(\lambda f)]$, where $f$ is the focal length of the lens. It turns out that the impulse response of a thin lens is given by the same formula as for the pinhole camera — Eq. (1) — with the understanding that $z_o$ is now related to $z_i$ through the lens formula $1/z_o + 1/z_i = 1/f$. To stress the analogy between the pinhole camera and the thin lens systems, we have drawn on Fig. 2 the same information as on Fig. 1, and used the same notation. Note that $2a$ now represents the diameter of the lens, which is usually considerably larger than the aperture of the pinhole camera. Therefore, diffraction effects will be weaker, and non-isoplanatism less pronounced. Apart from this comment, the results and discussion of the last section also apply to the thin lens system. As with the pinhole camera, isoplanatism can be recovered when the thin lens is used to image the spherical surface $S_o$ to $S_i$ [see Eq. (4)]. However, this is less obvious here because, in contrast with the pinhole camera, a lens does not have an infinite field of view. That $S_i$ is the image surface of $S_o$ is therefore questionable (see Sec. 5).

Whether non-isoplanatism leads to interference when a plane emitter is imaged to a plane receptor depends not only on the size of the lens, but also on the size of the object and its position with respect to the optical axis. The condition for avoiding any interferences due to non-isoplanatism has been derived in [4] for one-dimensional imaging. Extended to the two-dimensional case (see Appendix A), it reads:

$$e_o \left(\rho_o^2 + \frac{c_o}{4}\right) \ll \lambda z_o,$$

where $\rho_o^2 = \sqrt{(x_i/M)^2 + (y_i/M)^2}$ is the off-axis distance of the geometrical object point under consideration and $e_o = 1.22\lambda z_o/a$ is the first-zero full width of the Airy pattern in the object plane. The main conclusions are:
- When imaging points very close to the optical axis (\(p^2_0 \ll \varepsilon_0/4\)), interference due to non-isoplanatism does not occur if \(a^2/\lambda z_o \gg 1\). This last condition is always satisfied in practice with lenses. However, for a pinhole camera it fails to be satisfied.
- When imaging off-axis points (\(p^2_0 \gg \varepsilon_0/4\)), interference due to non-isoplanatism does not occur if \(p^2_0 \ll a\), i.e., if the object points are not as far off-axis as the edges of the lens.

When criterion (5) is satisfied for any point on the object, the phase factor \(\exp(2\pi i r/\lambda)\) can be moved out of the integral in Eq. (3), giving

\[
U_i(x_i, y_i) = \frac{1}{|M|} e^{i\pi \frac{x_i^2 + y_i^2}{M^2}} \iint U_o(x_o, y_o) \delta_{x, y} \left( \frac{x_M}{M} - x_o, \frac{y_M}{M} - y_o \right) \, dx_o \, dy_o, \tag{6}
\]

where the phase factor comes from the second order approximation of \(s(x_i, y_i) + r(x_i, y_i/M)\) and constant phases have been removed. An intensity detector in the plane \(I\) will record the same image as in the isoplanatic case — Eq. (4). It should however be noted that the impulse response function is still non-isoplanatic because the phase curvature has not been removed. It is important to keep this mind when a phase-sensitive detector (hologram) is used or if further optical processing is applied.

4. Non-isoplanatism and large field imaging

In practical applications of coherent imaging, the assumption is usually made [2] that the field mapping from the object space to the image space is given by Eq. (6) for any lens of a given optical system, i.e., the system is isoplanatic. This assumption is very convenient from a theoretical point of view since it makes the analysis of optical systems much simpler by the use of standard Fourier optics methods. In this section, we show that the slight difference between Eqs. (6) and (3) may lead to strong distortions if the object does not fulfill condition (5). We explain and interpret these effects.

Let’s first point out what is wrong with Eq. (6) from a physical point of view. Consider an object field \(U_o(x_o, y_o)\) that is varying slowly on the length scale \(\varepsilon_o = 1.22\lambda z_o/a\) of the peaked function \(\delta_{x, y}(\lambda z_o)(x/M - x_o, y/M - y_o)\). Then, according to Eq. (6), one can write

\[
|U_i(x_i, y_i)|^2 = \frac{1}{|M|^2} \left| U_o \left( \frac{x_M}{M}, \frac{y_M}{M} \right) \right|^2.
\tag{7}
\]

This equation implies that the energy is conserved: \(\iint |U_i(x_i, y_i)|^2 \, dx_i \, dy_i = \iint |U_o(x_o, y_o)|^2 \, dx_o \, dy_o\). However, independently of how slowly the field varies in space, Eq. (7) cannot hold for far off-axis points when the lens has a limited aperture. Radiation from far off-axis points (like \(P_3\) on Fig. 2) is partially lost, and energy cannot be conserved. For instance, in the case of a plane wave travelling along the optical axis, radiation from the neighborhood of

![Fig. 3. Intensity in the image plane \(I\) when a large square aperture, centered on the optical axis, is imaged with a thin lens. The square aperture is illuminated by a plane wave travelling along the optical axis. The sides of the square are \(b = 9.5\) mm. The different panels correspond to different lens diameters. From left to right and top to bottom, the lens diameter is progressively increased in 2 mm-steps from \(2a = 6\) mm to \(2a = 22\) mm. The relevant parameters are: \(\lambda = 780\) nm, \(f = 12\) mm, and \(x_o = 2\) m. The scale of the figures is expressed in microns. The intensity scale is such that 1 corresponds to the expected uniform intensity in the center of the square in the limit of an infinite lens. P2 will not be transmitted at all. Taking the phase factor \(\exp(2\pi i r/\lambda)\) out of the integral sign in Eq. (3) breaks down the energy balance.

To give a deeper insight of the effect into the phase factor \(\exp(2\pi i r/\lambda)\), we use Eq. (3) instead of Eq. (6) to compute the intensity distribution in the image plane for a slowly varying object field. We now obtain

\[
|U_i(x_i, y_i)|^2 = \frac{1}{|M|^2} \left| U_o \left( \frac{x_M}{M}, \frac{y_M}{M} \right) \right|^2 \times \left| \iint e^{i2\pi (\xi x_i + \eta y_i)} \delta_{\xi, \eta} (\xi, \eta) \, d\xi \, d\eta \right|^2,
\tag{8}
\]

where \(\xi = x/M - x_o\) and \(\eta = y/M - y_o\). The integral in Eq. (8) has the form of a Fourier transform integral. For points \((x_i, y_i)\) which are sufficiently far away from the optical axis, the quadratic phase terms \(\xi^2/2\) and \(\eta^2/2\) can, as a first order approximation, be neglected, and the integral reduces to the Fourier transform of the Airy pattern in the object space. Using

\[
\int e^{i2\pi (x\xi + y\eta)} \, d\xi \, d\eta = \text{circ} \left( \frac{x}{N}, \frac{y}{N} \right),
\tag{9}
\]

Eq. (8) becomes

\[
|U_i(x_i, y_i)|^2 = \frac{1}{|M|^2} \left| U_o \left( \frac{x_M}{M}, \frac{y_M}{M} \right) \right|^2.
\]
\[
\times \text{circ}^2\left(\frac{x_i}{Ma}, \frac{y_i}{Ma}\right),
\]

(10)

In Eqs. (9) and (10),

\[
\text{circ}(x, y) = \begin{cases} 
1 & \text{if } \sqrt{x^2 + y^2} < 1, \\
0 & \text{if } \sqrt{x^2 + y^2} > 1.
\end{cases}
\]

(11)

Eq. (10) shows that the intensity in the image plane exhibits a cut-off. No intensity reaches the image plane at a distance higher than \(Ma\) from the optical axis. This can be understood in the following way: since the field has been assumed to be slowly varying, the diffraction in the propagation from the object plane to the lens is negligible and the limited aperture of the lens has the same effect as a stop of radius \(a\) in the object plane. The function \(\text{circ}(x_i/(Ma), y_i/(Ma))\) is the image of that virtual stop and can be interpreted as the shadow of the lens. This is, however, only a first order approximation, since the quadratic phase terms \(\xi^2/2\) and \(\eta^2/2\) in Eq. (8) have been neglected. The effect of these quadratic phase terms is to create radial intensity oscillations in the image.

In Fig. 3, the preceding discussion is illustrated with an example. A square object of size \(9.5 \times 9.5\, \text{mm}^2\) is imaged using a lens of diameter \(2a\) varying from 6 mm to 22 mm. The object field \(U_o(x_o, y_o)\) is 1 inside the square and zero outside. Fig. 3 shows the intensity distribution in the image plane computed using Eq. (3). With small lenses (\(2a\) up to 10 mm) the clipping predicted by Eq. (10) is observed. One can clearly distinguish the disk (11) that limits the observable part of the object, as well as the intensity ripples due to the quadratic phase terms in (8) which were neglected when deriving (10). Interestingly, the intensity oscillations do not disappear as soon as the lens becomes bigger than the object. For \(2a = 22\, \text{mm}\), some residual modulation still remains. We use realistic parameters in the simulation: the object distance \(z_o\) has been fixed to 2 m (close to infinite-conjugate ratio imaging) and the focal length \(f\) to 12 mm (i.e. the numerical aperture ranges from 0.24 to 0.67). Obviously, energy is lost when imaging large objects through small lenses: For the simulations shown in Fig. 3, the percentage of transmitted energy is, from left to right and top to bottom, 31.1%, 56.1%, 83.4%, 96.6%, 99.2%, 99.8%, and nearly 100% for the last three images.

For quickly varying fields the previous discussion does not hold, but non-isoplanatism still has some effects on imaging. No general conclusions can be drawn in that case. To get some insight, consider the following example.

A large square grid (\(19 \times 19\, \text{mm}^2\)) of mutually coherent point sources is imaged through a lens of diameter \(2a = 8\, \text{mm}\). The imaging conditions are otherwise the same as in Fig. 3. Let’s consider that all the point sources are in phase and discuss the image formation when the spacing \(d\) between the point sources is varied. If the Airy patterns do not overlap (\(d \gg c_o/2\)), no interference takes place and non-isoplanatism has no effect on the intensity distribution in the image plane. If the Airy patterns overlap strongly (\(d \ll c_o/2\)), the object field varies slowly in space, and a fringe pattern similar to the one in Fig. 3 is expected. The interesting case is \(d \approx c_o/2\). Fig. 4 shows the intensity distribution in the image plane when \(d\) ranges from 0.6 to 2.2 times \(c_o/2\). For \(d \leq c_o/2\), a fringe pattern similar to the one in Fig. 3 is seen in the center of the field. However, periodic replicas of this pattern are also observed. For \(d > c_o/2\), the circular fringe patterns intersect each other, but the sparse sampling due to the grid structure of the image makes this structure barely visible (periodicity, however, remains). For clarity, only the central \(30 \times 30\, \text{µm}^2\) region of the image is displayed in the last six panels of Fig. 4. The simulations in Fig. 4 show that the intensity distribution in the image plane exhibits two distinct pseudo-periods (in both the \(x\) and \(y\) directions): the small-scale pseudo-period \(Md\) due to the grid structure of the object and the large-scale pseudo-period \(X\) associated with the ring patterns due to non-isoplanatism. Strictly speaking the image is periodic only if \(X\) is an integer multiple of \(Md\). Hereafter, we use this property to deduce the value of \(X\). The object field is modelled as a two-dimensionnal Dirac comb:

\[
U_o(x_o, y_o) = \sum_{n,m} \delta(x_o - n d, y_o - m d),
\]

where the integers \(n\) and \(m\) run from \(-\infty\) to \(+\infty\). Using
Eq. (3), we then find that

\[ |U_i(x_i, y_i)|^2 = \frac{1}{|M|^2} \times \left| \sum_{n,m} e^{i\phi_{nm}} \delta_{nm} \left( \frac{x_i + X}{M} - nd, \frac{y_i - nd}{M} \right) \right|^2, \quad (12) \]

with \( \phi_{nm} = \frac{\pi}{\lambda z_o} (n^2 + m^2) d^2 \).

In order to find \( X \), we require that \( |U_i(x_i, y_i)|^2 = |U_i(x_i + X, y_i)|^2 \) when \( X \) is a multiple of \( Md \). Using eq. (12), we have

\[ |U_i(x_i + X, y_i)|^2 = \frac{1}{|M|^2} \times \left| \sum_{n,m} e^{i\phi_{nm}} \delta_{nm} \left( \frac{x_i + X}{M} - nd, \frac{y_i - nd}{M} \right) \right|^2. \]

The argument \( (x_i + X)/M - nd \), can be rewritten as \( x_i/M - n'd \) with \( n' = n - X/(Md) \in \mathbb{Z} \). Replacing the sum over \( n \) by a sum over the values of \( n' \), we obtain

\[ |U_i(x_i + X, y_i)|^2 = \frac{1}{|M|^2} \times \left| \sum_{n',m} e^{i\phi_{n'm}} \delta_{n'm} \left( \frac{x_i}{M} - n'd, \frac{y_i - md}{M} \right) \right|^2. \]

where

\[ \phi_{n'm} = \frac{\pi}{\lambda z_o} \left[ n'^2 + 2n' \frac{X}{Md} + m^2 \right] d^2. \]

One can note that \( \phi_{n'm} \) and \( \phi_{nm} \) just differ by a factor \( n' \times 2\pi \), and therefore \( |U_i(x_i, y_i)|^2 = |U_i(x_i + X, y_i)|^2 \), if \( X \) is given by

\[ X = M \times \frac{2\lambda}{d}, \quad (13) \]

When \( X \) is not a multiple of \( Md \), formula (13) is still valid if \( X \) is understood as the large-scale pseudo-period. However, the notion of “large-scale pseudo-period” only holds if \( X/(Md) \approx a e_0/d^2 \gg 1 \) (\( e_0 = 1.22 \lambda z_o/a \)). Since \( a/d \) is usually a large number, this condition can still be valid even if \( d \) strongly exceeds the distance \( e_0/2 \) corresponding to the Rayleigh criterion. The periodic patterns displayed in Fig. 4 prove that the field in the image plane corresponding to the Airy pattern of a given point in the grid influences the entire image on a length scale much longer than the usually considered “Airy pattern diameter” \( e_0 \). Readers familiar with signal processing will notice that the reason why periodicity appears here is the same one that makes the Fourier spectrum of a sampled signal (Dirac comb) periodic. We however stress that the periodicity that is described here appears in the image plane and not in the Fourier plane of the object field. This is a peculiarity of non-isoplanatic imaging. There is also an intuitive way of understanding the non-isoplanatic effect displayed in Fig. 4. We image a two-dimensional grating. Since the lens sits far from the grating, it transmits only plane waves with discrete inclination angles \((\alpha_n, \beta_m) = \lambda (n\alpha/d, m\beta/d)\). Because the lens has a finite diameter, each plane wave \((\alpha_n, \beta_m)\) projects a shadow that is centered on different positions \((x_i^n, y_i^m) = (z_i \alpha_n, z_i \beta_m)\) with a periodicity given by \( X \) in Eq. (13). The image of the grid can only be observed properly when all the plane waves originating from the object overlap and interfere in the image plane.

In many applications of coherent optics (like optical lithography), the object is made of lines instead of single points or filled surfaces. Lines are objects on which the field varies slowly in one direction (the direction tangent to the line) and rapidly in the orthogonal one. Therefore, one can expect that the effects of non-isoplanatic will be intermediate between the two previous cases. Fig. 5 helps to understand how lines are imaged through a non-isoplanatic optical system. The imaging conditions are the same as in Fig. 3, except that the lens diameter is fixed: \( 2a = 6 \text{ mm} \). Panel (a) shows the image of three squared contours. The outer square is exactly the contour of the filled square imaged in Fig. 3. The comparison with the upper-left panel of Fig. 3 shows that the major part of the contour is now visible; only the corners of the square are clipped. Closer examination shows that, for a straight line, only a segment of length \( 2a \) in the object plane \((2Ma \text{ in the image plane})\) is transmitted. The part of the line that is clipped corresponds to the orthogonal projection of the shadow of the lens on the straight line. This can be easily understood by analysing the propagation of the cylindrical waves emitted by straight lines through the spherical lens. The smaller square in panel (a) is transmitted because its side is shorter than the diameter of the lens. The intermediate square is at the limit of the cut-off. As shown in the panel (b) of Fig. 5, circles are never clipped, whatever
their radii, because the orthogonal projection of the lens disk on the circle is the circle itself.

5. Isoplanatic imaging through a thin lens

As with the pinhole camera, isoplanatic imaging with a thin lens is possible when the object lies on the spherical surface $S_o$ and the image is observed on the spherical surface $S_i$.

The technique for projecting a plane object onto the spherical surface $S_o$ has been proposed and demonstrated in [4] for 1D objects. It consists in placing a convergent thin lens with focal distance $z_o$ immediately before (or after) the object plane. This lens introduces a spherical phase delay in the object field: $U_o \rightarrow U'_o = U_o \times \exp[-i2\pi r/\lambda]$. This extra phase factor compensates the phase accumulated during the propagation from $O$ to $S_o$. Therefore, the field on $S_o$ is $U_o(x_o, y_o)$ ($z_o$ is now a dependent variable). Similarly, on the image side, a thin lens of focal length $z_i$ placed just after (or before) the image plane can be used to project the image from $S_i$ onto $I$. From a broader point of view, any spherical field distribution in the object space of a centered paraxial optical system can be imaged onto a spherical surface of any curvature using only lenses; some of them will be imaging lenses, while others will play the role of phase-correction transparencies. This is the basis of the so-called metaxial optics theory formulated by Bonnet [11–13].

Though first proposed for lens imaging, the technique described above is questionable in that case, while fully legitimate for a pinhole camera. The difference between these two imaging systems is that the pinhole camera has an infinite depth of field (the distances $z_o$ and $z_i$ between the pinhole, an object point and its image can be chosen arbitrarily), while for the lens, $z_o$ and $z_i$ are related by the lens law. Therefore, referring to Fig. 2, simple Gaussian Optics arguments suggest that if the point $P_2$ is translated horizontally from the object plane $O$ to the surface $S_o$, its image $P'_2$, instead of moving towards $S_i$, should move away from the lens. This argument relies on the Gaussian approximation that the point $P'_2$ is initially in the “image plane” $I$. In reality, because of the field curvature aberration due to the lens, the stigmatic image of $P_2$ is closer to the lens than the surface $S_i$ itself ($P'_2$ lies on the so-called Petzval surface). Bringing $P_2$ on $S_o$ will place its image $P'_2$ exactly on $S_i$ [14]. Consequently, Eq. (4) is exact, while Eq. (3) is only valid in the context of Gaussian approximation. If the lens is still diffraction-limited in this regime (no point-aberrations), the isoplanatic imaging geometry provides a nearly perfect transfer of the coherent field from the object to the image space. Only the resolution is reduced because of the finite size of the lens.

It should be noted that this approach only works for sufficiently slowly varying fields, because the diffraction from $O$ to $S_o$ (and $S_i$ to $I$) has to be negligible for the amplitude of the fields on $O$ and $S_o$ ($S_i$ and $I$) to be the same.

6. Experimental investigation

The theoretical discussion of Secs. 4 and 5 relies on two strong approximations: the paraxial approximation and the thin lens approximation. One may wonder whether our analysis is robust enough to be applied to systems containing powerful lenses, which are usually thick and have a high numerical aperture. The following experiment shows that the previous discussion is also valid for these systems.

The setup is shown in Fig. 6. The test lens $L_1$ is an aspheric lens having a focal length $f = 8$ mm and a diameter $2a = 8$ mm (LightPath 352240). This lens is used to image the surface of a spatial light modulator (SLM), a 1024 x 768 micromirror array, that modulates the amplitude of the reflected beam. Using the SLM, we can generate arbitrary patterns. The resolution is set by the size of the micromirrors ($13 \times 13$ μm²). The object plane is 60 cm away from the lens. We use a double-lens system ($L_3$, $L_4$) to magnify the image produced, and project it onto a CCD camera. $L_3$ is a diffraction-limited aspheric lens and $L_4$ a long-focal achromatic doublet. The numerical aperture of this double-lens system is large enough to prevent any possible clipping or diffraction during the magnification process. The SLM is either illuminated with a plane wave or a spherical wave converging on $L_1$ ($\lambda = 780$ nm). In the first case, the imaging system is non-isoplanatic. As explained in Sec. 5, it becomes isoplanatic when a spherical-wave illumination is used. The spherical wave is obtained from the impinging plane wave by inserting the additional lens $L_2$ (75-cm focal length, achromatic doublet) in front of the SLM. We use this setup to verify the theoretical predictions of Sec. 4.

Fig. 7 shows the pictures recorded by the CCD camera when we image squares of different sizes through the 8-mm diameter aspheric lens $L_1$. Let’s first consider the case when the SLM is illuminated by a parallel beam (non-isoplanatic imaging, left column of Fig. 7). For a square side larger than 8 mm, the clipping effect described in Sec. 4 is clearly observed. Circular fringes similar to those of Fig. 3 are also seen in the image plane. For a square side smaller that 8 mm, the region of non-zero intensity is limited by the size of the square. However, intensity modulation due to non-isoplanatism is still noticeable for a square side as small as 2.7 mm. In the case of the 1.4 x 1.4 mm² square, non-isoplanatism has a negligible effect. Note that, in the simulations of Fig. 3, the lens size was varied while the object size was kept constant. Here, the lens is always the same.

![Fig. 6. Experimental setup: $L_1$, aspheric lens ($f = 8$ mm); $L_2$, achromatic doublet ($f = 750$ mm); $L_3$, aspheric lens ($f = 20$ mm); and $L_4$, achromatic doublet ($f = 500$ mm); SLM, spatial light modulator; CCD, coupled-charge camera; O, object plane; I, image plane.](image-url)
Fig. 7. Images of different-sized squares (the side \( b \) ranges from 1.4 mm to 9.6 mm) recorded using the setup in Fig. 6. The object plane is either illuminated with a parallel beam (non-isoplanatic case, left column) or with a spherical one converging on \( L_1 \) (isoplanatic case, right column).

but the square size is varied instead. For this reason, the ring pattern is the same for every picture in the left column of Fig. 7. When the SLM is illuminated with a spherical wave converging on \( L_1 \) (isoplanatic imaging, right column of Fig. 7) no clipping effect occurs and there is, in principle, no limit to the size of the objects that the system can image (the slight variations in intensity are due to amplitude inhomogeneities in the illumination beam).

In a second experiment, we imaged rectangular grids of points illuminated by a plane wave (non-isoplanatic illumination), a situation that we described theoretically in Sec. 4. Fig. 8 shows the recorded intensity distribution in the image plane for a grid with a period \( d = 109 \, \mu m \), which corresponds to 1.5 times the Rayleigh criterion separation \( (e_o/2) \). We distinguish one central ring pattern, similar to the one shown in Fig. 7 and four replicas intersecting it. The intensity distribution is very similar to the simulations displayed in Fig. 4. The distance between the centers of the ring patterns is 57 \( \mu m \). It is in very good agreement with the value \( X = 58 \, \mu m \) calculated using formula (13). Since the experimental point-spread function is slightly broader than the theoretical Airy pattern, the overlap of the rings is clearly visible for \( d = 1.5 \, e_o/2 \), while it is not easy to observe in Fig. 4. This is because of the insufficient sampling caused by the sparsity of the imaged grid.

The experiments that we performed show how non-isoplanatism affects image formation for slowly and fast varying fields. These effects, which can be easily confused with aberrations or misalignment, are always present when imaging extended objects with small lenses.

7. Conclusion

Even diffraction-limited imaging systems distort the phase of the processed fields. This is of no relevance when working with incoherent light, but has a tremendous ef-
fect on coherent imaging. In combination with Fraunhofer diffraction from the finite instrumental aperture, the phase distortion that arises leads to a severe degradation of the field amplitude in the image plane. We analyzed this phenomenon for the pinhole camera and the thin lens. We showed that substantially different effects arise depending on whether the field varies slowly or rapidly on the length scale of an Airy pattern. However, the degradation of the field amplitude can be overcome, or at least minimized. The main aspects of our analysis have been confirmed experimentally with a powerful thick aspheric lens, demonstrating that the phenomena that we describe are also present beyond the paraxial and thin lens approximations.

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Appendix A. Derivation of condition (5)

The condition for avoiding non-isoplanatism effects is that the phase $2\pi r/\lambda$ in Eq. (3) varies by less than $\pi$ when the point corresponding to coordinates $(x_o, y_o)$ explores the Airy pattern $\delta\varphi(x_i/M - x_o, y_i/M - y_o)$ in the object plane. To express this condition mathematically, we rotate the axes to get $y_i = 0$. From the center of the Airy pattern at $x_o = x_i/M$, the phase $2\pi r/\lambda$ changes most rapidly when $x_o$ moves radially off-axis. The phase change between the center and the most off-axis first zero of the Airy pattern is $\delta\varphi = 2\pi/\lambda(r_1 - r_0)$, where $r_0 \approx r_o(1 + (x_i/M)^2/(2z_o^2))$ and $r_1 \approx r_o(1 + (x_i/M + e_o/2)^2/(2z_o^2))$ in paraxial approximation. Asking that $\delta\varphi \ll \pi$ leads to Eq. (5).

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