The Lorentz and CPT violating effects on the 
\( Z \rightarrow l^+l^- \) decay

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Abstract

We study the Lorentz and CPT violating effects on the branching ratio, the CPT violating asymmetry and the ratio of the decay width, including only the Lorentz violating effects, to the one obtained in the standard model, for the flavor dependent part of the lepton flavor conserving \( Z \rightarrow l^+l^- (l = e, \mu, \tau) \) decay. The inclusion of the Lorentz and CPT violating effects in the standard model contribution are too small to be detected, since the corresponding coefficients are highly suppressed at the low energy scale.

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1 Introduction

A considerable theoretical work has been done to construct a fundamental theory at higher scales, like the Planck scale, which the standard model (SM) of particle physics is its low energy limit. In such scales, there are hints that the Lorentz and CPT symmetries are broken \cite{1}, in contrast to their conservation in the SM. The string theories \cite{2} and the non-commutative theories \cite{3} are the examples of high energy extensions of the SM. Even if the Lorentz and CPT symmetry violations exist in the extended theories given above, the small violations of these symmetries can appear at the low energy level.

The general Lorentz and CPT violating extension of the SM is obtained in \cite{4,5}. In the extension of the SM the Lorentz and CPT violating effects are carried by the coefficients coming from an underlying theory at the Planck scale. These coefficients can arise from the expectation values in the string theories or some coefficients in the non-commutative field theories \cite{3}. The loop quantum gravity \cite{6}, the space time foam \cite{7} and cosmological scalar fields \cite{8} are the possible sources of the Lorentz violating coefficients. Furthermore, the space-time varying couplings are also associated with the Lorentz violation, and they affect the photon dynamics \cite{9}.

In the literature, there are various studies in which some of the coefficients are probed, by using the experiments \cite{10,11}. The general Lorentz and CPT violating Quantum Electrodynamics (QED) extension has been studied in \cite{12,13} and in \cite{13} the one loop renormalizability of this extension has been shown. In \cite{14} the Lorentz and CPT violating effects on the branching ratio ($BR$) and the CP violating asymmetry $A_{CP}$ for the lepton flavor violating (LFV) interactions $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$, has been analyzed in the model III version of the two Higgs doublet model (2HDM) and the relative effects of new coefficients on these physical parameters have been studied. The Lorentz and CPT violating effects in the Maxwell-Chern-Simons model have been examined in \cite{15,16} and these effects in the non-commutative space time have been analyzed in \cite{17}. In \cite{18}, the theoretical overview of Lorentz and CPT violation has been done; in \cite{19}, the possible signals of Lorentz violation in sensitive clock-based experiments has been investigated and in \cite{20}, the superfield realizations of Lorentz-violating extensions of the Wess-Zumino model were presented. The threshold analysis of ultra-high-energy cosmic rays can also be used for Lorentz and CPT violation searches. The basis for such threshold has been investigated in \cite{21}.

In the present work, we study the Lorentz and CPT violating effects on the $BR$, the CPT violating asymmetry ($A_{CPT}$) and the ratio $R$ of the decay width $\Gamma$, including only the Lorentz
violating effects, to the one obtained in the SM, for the flavor dependent part of the lepton flavor conserving $Z \rightarrow l^+l^- (l = e, \mu, \tau)$ decay. The additional contribution, coming from the Lorentz and CPT violating effects, on the physical parameters we study is too small to be detected, since the corresponding coefficients are highly suppressed at the low energy scale. Our aim is to investigate the relative importance of the coefficients which are responsible for the Lorentz and CPT violating effects on the $BR$ of the decays under consideration. Furthermore, we predict the possible CPT violating asymmetry $A_{CPT}$ which is carried by the limited number of coefficients, $e_\mu$ and $g_{\mu\nu\alpha}$ in the present process. The $A_{CPT}$ is sensitive to the flavor structure of the process, however, it is considerably small, as expected. Finally, we study the ratio $R$ to understand contribution of the Lorentz and CPT violating effects on the flavor structure of the decay and we observe that these effects are too weak to be detected in the present experiments.

The paper is organized as follows: In Section 2, we present the theoretical expression for the decay width $\Gamma$, the $A_{CPT}$ and the ratio $R$, for the lepton flavor conserving $Z \rightarrow l^+l^- (l = e, \mu, \tau)$ decay, in the case that the Lorentz and CPT violating effects are switched on. Section 3 is devoted to discussion and our conclusions.

2 The $Z \rightarrow l^+l^- (l = e, \mu, \tau)$ decay with the addition of the Lorentz and CPT violating effects

In this section, we study the Lorentz and CPT violating effects on the $BR$, the CPT asymmetry and the ratio $R$ for the leptonic $Z$ decay. In the SM, this process is allowed at tree level and the $BR$ is weakly sensitive to the lepton flavor. The insertion of the Lorentz and CPT violating effects in the tree level brings new contribution and its size is regulated by the magnitudes of the new coefficients coming from the tiny Lorentz and CPT violation. The Lorentz and CPT violating lagrangian in four space-time dimensions responsible for the decay of $Z$ boson to the lepton pair reads [4]

$$L = \frac{i}{2}(\bar{\psi}_L \Gamma^\mu D_\mu \psi_L + \bar{\psi}_R \Gamma^\mu D_\mu \psi_R)$$

(1)

where

$$\Gamma^\mu = \gamma^\mu + \Gamma_1^\mu,$$

$$\Gamma_1^\mu = c^\alpha \gamma_\alpha + d^\alpha \gamma_5 \gamma_\alpha + e^\mu + i f^\mu \gamma_5 + \frac{1}{2} g^{\lambda\nu\mu} \sigma_{\lambda\nu}.$$  

(2)

Here the coefficients $c_\alpha$, $d_\alpha$, $e_\mu$, $f_\mu$ and $g_{\lambda\nu\mu}$ are responsible for the Lorentz violation. Even if the U(1) charge symmetry and renormalizability does not exclude the part of lagrangian
including the coefficients $e_\mu$, $f_\mu$ and $g_{\lambda\nu\mu}$, they are not compatible with the electroweak structure of the SM extension. However, the possible nonrenormalizable higher dimensional operators respecting the electroweak symmetry and including Higgs field with vacuum expectation value can create these highly suppressed terms (see [4] for details). In our analysis we also take these terms into consideration since they are sources of the CPT violation ([13]).

Now, we would like to present the additional vertex due to the Lorentz and CPT violating effects for the $Z \to l^+l^-$ decay:

$$V_{LorVio} = -\frac{i Q_l e}{s_W c_W} e^{\alpha\mu} \gamma_\alpha + d^{\alpha\mu} \gamma_\alpha + e^\mu + i f^\mu \gamma_5 + \frac{1}{2} g^{\lambda\nu\mu} \sigma_{\lambda\nu} (c_L L + c_R R),$$

(3)

where $L(R) = \frac{i}{2}(1 \pm \gamma_5)$, $c_L = a_{\gamma} + s^2_{W}$, $c_R = s^2_{W}$ and $Q_l = -1$. Our aim is to calculate the decay width of the $Z \to l^+l^-$ process including the Lorentz violating effects. It is known that the invariant phase-space elements in the presence of Lorentz violation are modified [16]. In the conventional case where there are no Lorentz violating effects, the well known expression for decay width in the $Z$ boson rest frame reads

$$d\Gamma = \frac{(2\pi)^4}{6m_Z} \delta^{(4)}(p_Z - q_1 - q_2) \frac{d^3 q_1}{(2\pi)^3 2 E_1} \frac{d^3 q_2}{(2\pi)^3 2 E_2} |M|^2 (p_Z, q_1, q_2)$$

(4)

with the four momentum vector of $Z$ boson (lepton, anti-lepton) $p_Z$ ($q_1$, $q_2$), and the matrix element $M$ for the process $Z \to l^+l^-$. The inclusion of the new Lorentz violating parameters changes the lepton dispersion relation and an additional part in the phase space element is switched on. The variational procedure generates the Dirac equation

$$(\gamma^\mu q_\mu - m + \Gamma_1^\mu q_\mu) \psi = 0.$$  

(5)

and a small modification on $E_i$ in the phase space element is obtained. In our case, the corresponding dispersion relation is a complicated function of the various Lorentz violating parameters (see [3] for example). In addition to this, the crowd of Lorentz violating parameters causes large number of fixed directions and makes the angular integrations complicated, since the amplitude has a functional dependence of these angular variables. Finally, spin sums in the final state are not trivial in the case of Lorentz violating effects since the phase factors depend on the outgoing lepton polarizations (see [22] for details). Therefore, in the present work, we do
not take into account these tiny additional effects and use the conventional expression for the decay width in the $Z$ boson rest frame (see eq. (11)). With the inclusion of the Lorentz violating effects in the matrix element, the Lorentz violating part of the decay width $\Gamma(Z \to l^+ l^-)$ is obtained as

$$
\Gamma_{\text{LorVio}} = \frac{e^2 Q_l^2}{48 \pi m_Z^2 c_W^2 s_W^2} s_l \left\{ c_{00} \left( 2 m_l^2 - m_Z^2 \left( 1 - 4 s_W^2 + 8 s_W^4 \right) \right) + d_{00} \left( 2 m_l^2 - m_Z^2 \left( 1 - 4 s_W^2 \right) + \frac{1}{2} g m_Z m_l \left( 1 - 4 s_W^2 \right) \right) \right\}.
$$

(6)

Here $s_l = \sqrt{m_l^2 - m_Z^2}$, the parameters $c_{00}$ and $d_{00}$ are the zeroth components of the coefficients $c_{\alpha \beta}$ and $d_{\alpha \beta}$ and the last term $g = \epsilon_{ijk} g^{ijk}$, where $i, j, k = 1, 2, 3$. Notice that we take only the additional part of the decay width which is linear in the Lorentz violating coefficients.

The eq. (6) shows that $\Gamma_{\text{LorVio}}$ depends on the CPT even ($c_{00}$ and $d_{00}$) and the CPT odd $g$ coefficients. Using $\Gamma_{\text{LorVio}}$ it is easy to calculate the Lorentz violating part of the $BR(BR_{\text{LorVio}})$ as:

$$
BR_{\text{LorVio}} = \frac{\Gamma_{\text{LorVio}}}{\Gamma_Z},
$$

(7)

where the $\Gamma_Z$ is the total decay width of the $Z$ boson and its numerical value is $\Gamma_Z = 2.490 \ (\text{GeV})$.

The coefficient $g$ switches on the CPT asymmetry and it reads

$$
A_{\text{CP}} = \frac{(1 - 4 s_W^2) m_l g}{D},
$$

(8)

where

$$
D = 2 m_Z \left[ \left( 1 - 4 s_W^2 \left( 1 - 2 s_W^2 \right) \right) - \frac{m_l^2}{m_Z^2} \left( 1 + 8 s_W^2 \left( 1 - 2 s_W^2 \right) \right) + \left( 2 - 4 s_W^2 \left( 1 - 2 s_W^2 \right) \right) \right] c_{00} + \left( 1 - 4 s_W^2 \right) \left( 2 \frac{m_l^2}{m_Z^2} - 1 \right) d_{00}.
$$

(9)

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2The modified Dirac equation for the outgoing lepton in the present case is $(\gamma^\mu q_\mu - m + \Gamma_{\text{Lor}} q_\mu) \psi = 0$. Now, we assume that all the Lorentz violating coefficients, except $c_{00}, d_{00}, e^0, f^0$ and $g^{ijk}$, are vanishing. Furthermore, $g^{ijk}$ is small compared to other coefficients. After some algebra, the dispersion relation is obtained as $(q^2 - m_l^2 + 2 m_l E e^0)^2 - 4 m_l^2 E^2 (s^0)^2 = 0$, where $(s^0)^2 = (c^0)^2 - (d^0)^2 - (f^0)^2$ and the energy eigenvalues read $E_{\pm}^n \simeq -m_l (e^0 + (-1)^n s^0) \pm \sqrt{q^2 + m_l^2 (1 + (e^0 + s^0)^2)}$ where $n = 1$ or 2. Following the integration over the anti lepton four momentum $q_1$, the phase factor $1/E_1$ is replaced by

$$
1/ \left( -m_l (e^0 + s^0) + \sqrt{q^2 + m_l^2 \left( 1 + (e^0 + s^0)^2 \right)} \right) \simeq \left( 1 + m_l (e^0 + s^0) / \sqrt{q^2 + m_l^2} \right) / \sqrt{q^2 + m_l^2}. \text{ We expect that the factor } 1/ \sqrt{q_1^2 + m_l^2} \text{ in the additional part further suppresses the Lorentz violating effects in the phase factor, after the kinematical integration over the lepton four momentum } q_1.
$$
This equation shows that the $A_{CPT}$ depends on the flavor part of the decay under consideration and becomes larger for the heavier lepton pair decay.

Finally, we study the ratio $R = \frac{\Gamma_{\text{LorViol}}}{\Gamma_{\text{SM}}}$

$$R = \frac{4 m_l c_{00} + (1 - 4 s_W^2) (4 d_{00} m_l + g m_Z)}{2 m_l (1 + 8 s_W^2 (1 - 2 s_W^2))}$$

(10)

where $\Gamma_{\text{LorViol}}$ ($\Gamma_{\text{SM}}$) is the flavor dependent part of the decay width including only the Lorentz violating (the SM without Lorentz violating) effects. This ratio is sensitive to the lepton mass and it is dominant for the light lepton pair decay.

3 Discussion

The SM is invariant under the Lorentz and CPT transformations, however, the small violations of Lorentz and CPT symmetry, possibly coming from an underlying theory at the Planck scale, can arise in the extensions of the SM. In this section, we analyze the Lorentz and CPT violating effects on the $BR$ and the $A_{CPT}$ for the $Z \rightarrow l^+ l^-$ ($l = e, \mu, \tau$) decays, in the SM extension. Furthermore, we study the ratio $R = \frac{\Gamma_{\text{LorViol}}}{\Gamma_{\text{SM}}}$ to understand contribution of the Lorentz and CPT violating effects on the flavor structure of the decay. It is well known that these effects are tiny to be observed, however, it would be interesting to see the relative behaviors of different coefficients, which are responsible for the violation of the Lorentz and CPT symmetry.

The natural suppression scale for these coefficients can be taken as the ratio of the light one $m_l$ to the one of the order of the Planck mass. Therefore, the coefficients which carry the Lorentz and CPT violating effects are roughly in the range of $10^{-23} - 10^{-17}$. Here the first (second) number represent the electron mass $m_e$ (m_{EW} \sim 250 GeV) scale. We take the numerical values of the coefficients $|d|, |c|, |e|, |g|$ at the order of the magnitude of $10^{-23} - 10^{-17}$.

In Fig. 1 (2), we present the magnitude of the coefficient dependence of the Lorentz violating part of the $BR$ ($BR_{\text{LorViol}}$) for the decay $Z \rightarrow \tau^+ \tau^-$ ($\mu^+ \mu^-$). Here solid (dashed, small dashed) line represents the dependence to the coefficient $c_{00}$ ($d_{00}, g$), in the case that the other coefficients have the same numerical value $10^{-20}$. Notice that, in the figures, the parameter $\chi$ denotes to the size of $c_{00}, d_{00}$ and $g$ for different lines. It is observed that the $BR$ is more sensitive to the coefficient $c_{00}$ compared to the others and the contribution of the new effects to the $BR$ is at the order of the magnitude of $10^{-19}$ for the large values of the coefficient $c_{00} \sim 10^{-17}$. With the increasing values of the coefficient $g$ the $BR$ decreases for $Z \rightarrow \tau^+ \tau^-$ decay and this effect is weak for the light lepton pair case, namely $Z \rightarrow \mu^+ \mu^-$ decay. Notice that the Lorentz violating
coefficient dependence of the $BR_{LorVio}$ for the decay $Z \to e^+e^-$ is almost the same as the one for the decay $Z \to \mu^+\mu^-$. 

Now, we analyze the CPT violating asymmetry $A_{CPT}$ for the decays under consideration. The coefficient $g$ and the lepton flavor are responsible for this violation as seen in the eq. (8). Fig. 3 is devoted to the magnitude of the coefficient dependence of the $A_{CPT}$. Here solid (dashed, small dashed) line represents the $A_{CPT}$ for the decay $Z \to \tau^+\tau^- (\mu^+\mu^-, e^+e^-)$. The $A_{CPT}$ is sensitive to the lepton flavor and it is at the order of the magnitude of $10^{-20}$ ($10^{-21}$, $10^{-23}$) for the large values of the coefficient $g \sim 10^{-17}$, for the decay $Z \to \tau^+\tau^- (\mu^+\mu^-, e^+e^-)$. The $A_{CPT}$ enhances with the increasing lepton mass.

Finally, we study the ratio $R = \frac{\Gamma_{LorVio}^{\text{f lavor}}}{\Gamma_{SM}^{\text{f lavor}}}$ and we present the magnitude of the coefficient dependence of the ratio $R$ for the decay $Z \to \tau^+\tau^- (\mu^+\mu^-, e^+e^-)$ in Fig. 4 (5, 6). Here solid (dashed, small dashed) line represents the dependence to the coefficient $c_{00}$ ($d_{00}$, $g$), in the case that the other coefficients have the same numerical value $10^{-20}$. The ratio $R$ is more sensitive to the coefficients $c_{00}$ and $g$ compared to the coefficient $d_{00}$ and it is at the order of the magnitude of $10^{-17}$ for the large values of the coefficient $c_{00}$ ($g$) $\sim 10^{-17}$, for the decay $Z \to \tau^+\tau^-$. For the $Z \to \mu^+\mu^-$ decay, R is more sensitive to the coefficient $g$ and it can reach the values of $10^{-16}$. In the case of $Z \to e^+e^-$ decay, the sensitivity of $R$ to the coefficients $c_{00}$ and $d_{00}$ is weak, however, it enhances up to the values of $10^{-14}$.

At this stage we would like to summarize our results:

We analyze the Lorentz and CPT violating effects on the $BR$, $A_{CPT}$ and the ratio $R$ and we study the relative behaviors of different coefficients by taking their numerical values at the order of the magnitude of $10^{-20} - 10^{-17}:

- The contribution of the Lorentz and CPT violating part to the $BR$ of the decays $Z \to l^+l^-$ ($l = e, \mu, \tau$) is at most at the order of the magnitude of $10^{-19}$ for the large values of the coefficients $c_{00}$, $d_{00}$ and $g$ and these numbers are too small to be detected.

- We predict the numerical value of $A_{CPT}$ at the order of $10^{-20}$ ($10^{-21}$, $10^{-23}$) for the large values of the coefficient $g \sim 10^{-17}$ for the decay $Z \to \tau^+\tau^- (\mu^+\mu^-, e^+e^-)$. This physical parameter is driven by the coefficient $g$ and the lepton flavor. It enhances with the increasing values of lepton mass.

- We study the ratio $R = \frac{\Gamma_{LorVio}^{\text{f lavor}}}{\Gamma_{SM}^{\text{f lavor}}}$ and we observe that its sensitivity to the coefficient $g$ ($c_{00}$, $d_{00}$) increases (decreases) with the decreasing values of the lepton mass. It is at the order of the magnitude of $10^{-17}$ ($10^{-16}$, $10^{-14}$) for $g \sim 10^{-17}$, for the decay $Z \to \tau^+\tau^-$. 


\[ Z \rightarrow \mu^+ \mu^-, \ Z \rightarrow e^+ e^- \].

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Figure 1: The magnitude of the coefficient dependence of $BR_{LorVio}$ for the decay $Z \to \tau^+\tau^-$. Here solid (dashed, small dashed) line represents the dependence to the coefficient $c_{00}$ ($d_{00}$, $g$), in the case that the other coefficients have the same numerical value $10^{-20}$. 
Figure 2: The same as Fig. but for the decay $Z \to \mu^+\mu^-$. 

Figure 3: The magnitude of the coefficient dependence of the $A_{CPT}$ for the decay $Z \to l^+l^-$, $(l = e, \mu, \tau)$ Here solid (dashed, small dashed) line represents the $A_{CPT}$ for the decay $Z \to \tau^+\tau^-$ ($\mu^+\mu^-, e^+e^-$).
Figure 4: The magnitude of the coefficient dependence of the ratio \( R \) for the decay \( Z \rightarrow \tau^+ \tau^- \). Here solid (dashed, small dashed) line represents the dependence to the coefficient \( c_{00} \ (d_{00}, g) \), in the case that the other coefficients have the same numerical value \( 10^{-20} \).

Figure 5: The same as Fig. 4 but for the decay \( Z \rightarrow \mu^+ \mu^- \).
Figure 6: The same as Fig. 4 but for the decay $Z \rightarrow e^+ e^-$. 