Statistical dynamics of early creep stages in disordered materials*

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Abstract. When materials are loaded below their short-term strength over extended periods, a slow time-dependent process known as creep deformation takes place. During creep deformation, the structural properties of a material evolve as a function of time. By means of a generic coarse-grained mesoscopic elastoplastic model which envisages deformation as a sequence of stochastically activated discrete events, we study the creep deformation of disordered materials. We find that the structural evolution of the material during creep modifies not only the average material properties but also changes the statistics of those properties. We analyze the emergence of correlations in the strain localization and deformation activity patterns, the variation of the event rate and the evolution of the inter-event time distribution. We find that the event rate follows the Omori law of aftershocks, which is the discrete counterpart of Andrade’s transient creep law, and that the exponent of these laws only depends on the microstructural heterogeneity. Finally, we find during the initial stages of transient creep a transition from Poisson distributed inter-event times towards a non-trivial power law distribution.

1 Introduction

It is well-known [1,2] that when materials are loaded below their short-term strength over extended periods, a slow time-dependent deformation known as creep takes place. Understanding creep remains of general concern in view of the long-term behavior of structural materials [3] and geosystems such as Earth crust faults zones [4]. Creep deformation is characterized by several deformation regimes. Upon application of an external load, an instantaneous deformation takes place which is followed by a regime of decelerating deformation rates known as transient or primary creep. Transient creep is described by the Andrade law [2,5], which relates deformation to the time since application of the external load as \( \epsilon \sim t^m \), with \( m \approx 2/3 \). Nonetheless, the value of \( m \) is not universal and depends on the specific material under study [5]. Alternatively, transient creep is sometimes reported to be logarithmic, \( \epsilon \sim \log(t) \) [1,6]. After the transient regime, if the temperature is high enough, deformation enters the stationary or linear creep regime, \( \epsilon \sim t \) [5,7].

The description of transient creep in terms of the Andrade law has traditionally been associated with a conceptualization of creep in the framework of continuum mechanics, where plastic deformation appears as a smooth and deterministic process. Nonetheless, over the last decades, it has become clear that plasticity on small scales in which individual plastic events can be resolved appears as a stochastic and intermittent phenomenon [8,9]. When individual deformation events can be resolved, the slowing-down of the intermittent activity is described by the Omori law. The Omori law was originally established for earthquakes [10,11] even before the early observations of transient creep which led to the Andrade law [2]. Specifically, it describes the rate of earthquake aftershocks as \( \dot{n} \sim t^{-p} \), where \( \dot{n} \) represents the rate of events and \( p \) is typically close to 1. Omori-like behavior of plastic deformation activity indicators has been observed in a wide diversity of materials such as, e.g., rocks [12], porous materials [13], protein gels [14] or bulk metallic glasses [15].

Given the underlying fluctuating nature of the plastic activity, relating the phenomenological laws of creep to specific microstructural processes has been a major task in understanding creep deformation. In the case of crystals, this has been achieved by means of dislocation theory [16,17], where the elementary deformation events correspond to the activation of dislocation segments and their motion between metastable configurations, and the structural disorder arises from the complex stochastic patterns that are formed by the dislocation system [18,19]. In the case of metallic glasses, the elementary deformation events correspond to shear transformations [20,21]. A shear transformation can be understood as a local atomic rearrangement encompassing around 100 atoms which

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re-arrange in a shear-like fashion in order to accommodate the local shear stress. The structural disorder inherent to a metallic glass is the result of non-equilibrium features frozen in the structure after a liquid melt is quenched into the glassy state.

Similar creep behavior is found across a wide range of materials, such as polycrystalline metals [2], metallic glasses [15,22], rocks [23], paper [24] or granular matter [25]. Such independence of microstructural details suggests a conceptualization of creep phenomena within a framework that does not rely on material-specific microstructural processes. To this end, mesoscale elastoplastic models of disordered materials have been widely employed in recent years to reproduce and analyze intermittent plastic activity from a coarse-grained perspective [26]. In such models, the material is discretized into a grid of mesoscopic elements of size equal to or above the length scale of the elementary deformation events, with statistically distributed elemental properties in order to represent the underlying disordered microstructure.

In our work, we consider a mesoscale elastoplastic model based on the two closely related concepts of local energy barriers and local yield thresholds. An energy barrier separates two stable states within a mesoscopic element, such that the transition between both states can be envisaged as the introduction of a local plastic strain. Under a stress that acts upon the mesoscopic element, the energy landscape becomes tilted and, at a critical stress which defines the local yield threshold, the energy barrier vanishes and the transition becomes spontaneous. The yield threshold thus corresponds to a mesoscopic internal state variable akin to the yield stress of continuum mechanics and characterizes the local resistance to plastic deformation at zero temperature. A barrier transition, either spontaneous because the stress acting on a mesoscopic volume matches the lowest yield threshold within this volume, or by aid of thermal activation, is envisaged as a localized plastic activity within that element that produces a local plastic strain. Interactions between different elements are then described in terms of the continuum concepts of stress and strain, i.e., within a materials mechanics framework. This approach aims at capturing the essentials of plastic flow at the microscale without resolving atomistic details. The concept of elements with local energy barriers/yield thresholds, that interact via long-range stresses, is of broad applicability irrespective of the detailed nature of the underlying microstructure. Advantages of the concept reside in the possibility of directly measuring the yield stress probability distribution by probing mesoscopic regions in molecular dynamics [27,28] or discrete element simulations, and by the connection it makes between plasticity and interface depinning problems [29–32].

Ultimately, the approach relies on the existence of a characteristic scale in the internal energy landscape (a correlation length), which allows us to treat processes on the scale of the correlation length as “local” in a model that uses a spatial resolution slightly larger than the correlation length. The existence of such a correlation length was demonstrated implicitly by Patinet et al. [28] who showed that the yield thresholds obtained by probing small mesoscopic sub-volumes in large-scale molecular dynamics simulations possess a high degree of predictive power regarding the occurrence of structural re-arrangements under remote loading. Explicit studies of spatial correlations of different structural and energetic characteristics were performed by Cubuk et al. [33] who considered the spatial distribution of a machine learned quantity called “softness” which characterizes the propensity to local atomic re-arrangements, and by Wei et al. [34] who consider a whole spectrum of characteristics including the local energy barriers to irreversible atomic displacements as well as the “softness” parameter of Cubuk et al. [33], and find for all of these exponentially decaying correlation functions with a common correlation length that lies, for amorphous solids, on the scale of a few atomic spacings.

Mesoscale elastoplastic models of the type used in the present study have successfully reproduced much of the phenomenology associated with plastic deformation of disordered materials under stress or strain driven conditions [29,30,32,35–37]. However, comparatively little attention has been given to creep deformation [38–41]. In this paper, we implement a model that is closely related to the model of creep failure by Castellanos and Zaiser [39], but focus on the short-time transient dynamics and the long-time stationary flow regime. The model introduces stochastic behavior by considering thermally activated events and statistically distributed microstructural properties, which allows us to link the changes in the macroscopic creep response to the structural evolution. The model is able to capture the aging of the structure as well as mechanical rejuvenation as a function of time, external stress, temperature, and microstructural disorder. We study the spatial correlations between plastic events, strain localization, event rate and inter-event waiting time distributions during transient and stationary creep regimes.

2 Model

We coarse grain the microscale description of plastic deformation events and represent the material as a 2D array of \( L \times L \) yielding elements [29,30,32,35–37] of volume \( V_e \) centered at positions \( r_i \). The state of an element \( i \) is represented by (i) a local stress tensor \( \Sigma(r_i) \) which is the superposition of the stresses resulting from external boundary conditions (for creep: temporally constant applied tractions) and internal stresses resulting from the heterogeneity of the plastic strain field, (ii) its accumulated plastic strain \( \epsilon(r_i) \) and (iii) a local yield threshold \( \hat{\Sigma}(r_i) \) which characterizes the internal state of an element. Plastic deformation is assumed to be governed by the yield function

\[
\Phi = \sqrt{(3/2)\Sigma' : \Sigma'} - \hat{\Sigma} = \Sigma_{eq} - \hat{\Sigma}. \tag{1}
\]

Here, \( \Sigma' \) is the deviatoric part of the stress tensor and the equivalent stress \( \Sigma_{eq} \) is defined in such a manner that the model amounts to a stochastic generalization of J2 plasticity (for further generalizations see Budrikis et al. [29]). The yield function depends on the local yield threshold
and the local stress tensor, which is computed from the external boundary conditions and the plastic strain field $\epsilon_i(r_i)$ using Finite Element methodology as detailed in Appendix A.

On microscopic scales below the scale of resolution of our model, microscopic plastic re-arrangements are assumed to result in deformation events which produce on the element scale a tensorial plastic strain increment

$$\Delta \epsilon^p = \dot{\epsilon} \Delta \dot{\epsilon}^p$$  \hspace{1cm} (2)

where $\Delta \epsilon^p$ is chosen such that the local equivalent stress is reduced from $\Sigma_{eq}$ to $\epsilon \Sigma_{eq}$ where $0 \leq \epsilon < 1$. The tensor $\dot{\epsilon}$, which gives the “direction” of the local strain increment is in the spirit of an associated flow rule chosen to maximize energy dissipation by setting

$$\dot{\epsilon}_{ij} = \partial \Phi / \partial \Sigma_{ij}.\hspace{1cm} (3)$$

### 2.1 Rules for local activation of deformation events

The plastic flow law we adopt is characterized by the idea that deformation occurs as a stochastic sequence of local events. These events are activated according to the local values $\Phi(r_i)$ of the yield function. We use the following rules:

- an event is activated instantaneously if $\Phi(r_i) > 0$. We denote this process as mechanical activation.
- if $\Phi(r_i) < 0$, an event is activated with finite rate that depends on temperature $T$ according to

$$\nu(r_i) = \nu_{el} \exp \left( - \frac{E(\Sigma)}{k_B T} \right)$$  \hspace{1cm} (4)

where $\nu_{el}$ is an attempt frequency for event activation within the element volume $V_i$. We approximate the stress dependence of the characteristic activation energy $E$ by a linear dependency on the equivalent stress, $E = E_0 - V_A \Sigma_{eq}$ where $V_A$ is an activation volume. The activation barrier goes to zero if $\Phi = 0$, hence $E_0$ relates to the activation threshold stress via $E_0 = \Sigma V_A$ and we can write the activation rate alternatively as

$$\nu(r_i) = \nu_{el} \exp \left( \frac{\Phi(r_i)}{\Sigma_T} \right)$$  \hspace{1cm} (5)

where the parameter $\Sigma_T = k_B T / V_A$ characterizes the influence of thermal fluctuations on event activation. Besides thermal activation over a barrier, this parameter might also stem from other thermally activated processes that may trigger an event, e.g. the rate of chemical attack in chemically assisted microcracking. Beyond thermal activation, the same parameter might also be interpreted in terms of an effective magnitude of stochastic stress fluctuations of non-thermal origin affecting the region of interest, as long as such fluctuations can be represented by an effective temperature. Irrespective of the physical origin of the fluctuations, we denote this activation process as thermal activation.

- the duration of a deformation event is assumed negligibly small.

In the limit where the activation thresholds $\Sigma$ are spatially uniform and the stress changes $\Delta \epsilon$ are infinitesimally small, our model reduces for $T \to 0$ (no thermal effects) to a standard $J_2$ plasticity model. On the other hand, at low external stress, the model reduces to a viscoplastic creep model where the rate of plastic flow is given by

$$\dot{\epsilon} = \dot{\epsilon}_{el} \Delta \epsilon \exp(\Phi / \Sigma_T).$$

### 2.2 Distribution of yield thresholds

Statistical heterogeneity of the material is represented by considering the local yield thresholds $\Sigma(r_i)$ as random variables. For the initial probability distribution of these thresholds we consider a Weibull distribution with a mean value of $\Sigma_0$ and exponent $k$. The corresponding probability density is

$$P_0(\Sigma) = \frac{k}{\Sigma^*} \left( \frac{\Sigma}{\Sigma^*} \right)^{k-1} \exp \left( - \left( \frac{\Sigma}{\Sigma^*} \right) \right)$$  \hspace{1cm} (6)

where the scale parameter $\Sigma^*$ is chosen to ensure that the distribution has a mean of $\Sigma_0$ for a specific given value $k$. The value of $k$ relates to the width of the threshold distribution and consequently characterizes the degree of disorder of the material – lower values of $k$ correspond to broader distributions, i.e. higher disorder. We note that, in materials mechanics, Weibull distributions often result from some kind of weakest-link behavior [42]. In the present context, however, we merely use equation (6) as a convenient form of a probability distribution with positive support and independently tunable mean value and standard deviation. Its interpretation will be discussed in some more detail in Appendix B.

After each deformation event, the local yield threshold of the deforming element is assigned a new value from the probability distribution $P_0(\Sigma)$ in order to represent structural changes. This distribution thus serves a dual role: It represents the initial distribution of thresholds, and at the same time it statistically characterizes the change of threshold that occurs after a barrier crossing. These changes are of a stochastic nature: the renewed yield threshold may be either higher than the previous one, representing a more stable micro-state (in the context of amorphous solids, one might think of reduced local free volume), or the new threshold may be lower (a less stable state of enhanced free volume). In the present simple model, we do not consider any systematic change of the distribution $P_0(\Sigma)$ with time or strain. This allows the system to reach a steady state. A modified rule according to which the parameter $\Sigma^*$ systematically decreases with increasing plastic strain was considered in reference [39] – in physical terms, this might be envisaged as a systematic accumulation of free volume, which leads to
strain softening, strain localization and ultimately to creep failure.

We finally note that the renewal distribution \( P_0(\hat{\Sigma}) \) must be distinguished from the sample distribution of thresholds \( P(\Sigma) \) that characterizes the statistics of local thresholds at a given moment in time. The two distributions are identical only in the beginning, whereas in the course of straining, the distribution of thresholds \( P(\Sigma) \) systematically shifts to larger thresholds, as states with high threshold (difficult activation) have a longer lifetime than those with low threshold (easy activation). The resulting aging processes are discussed in more detail in the following, and also in Appendix B.

### 2.3 Simulation protocol

Simulations are carried out under pure shear conditions by imposing on the free surfaces of the system spatially uniform tractions giving rise to a homogeneous shear stress, in the following denoted as \( \Sigma \), which is kept fixed during a simulation. Deformation proceeds as a sequence of plastic deformation avalanches. A thermally activated event initiates an avalanche in form of a cascade of mechanically triggered events happening on a short time scale which we assume to be negligible in comparison with the inter-event time between different thermally activated events. We avoid thus competition between time scales \([43]\), which is beyond the scope of the present investigation. The thermal events are selected by the Kinetic Monte Carlo Method with the local activation rates given by equation (5); the KMC formalism yields both the location of the thermally activated event and the inter-event time that has elapsed since the last avalanche. After event initiation, we increase the local strain at the activated site \( i \) according to equations (2) and (3). We then re-compute the stress field by evaluating the stress changes \( \Delta \Sigma(\mathbf{r}_i) \) that result everywhere in the system from the local strain increment \( \Delta \epsilon(\mathbf{r}_j) \) in the activated element. To this end, we use a Finite Element framework as described in Appendix A. Note that, because of macroscopic stress equilibrium, \( \sum_j \Delta \Sigma(\mathbf{r}_j) = 0 \) so we may appropriately speak of a stress re-distribution. We then check whether, as a consequence of this stress re-distribution, there are new elements which meet the mechanical activation condition \( \Phi(\mathbf{r}_i) < 0 \). These sites also become activated and yield, leading to further stress changes and possible mechanical activation of further elements. The ensuing avalanche proceeds adiabatically in a series of deformation steps, in each of which one or more elements are mechanically activated and yield, and the stress changes resulting from the strain increments at the activated sites are evaluated in a single FEM step (parallel update). The avalanche terminates once, after the stress update, the mechanical activation condition \( \Phi(\mathbf{r}_i) < 0 \) is no longer fulfilled in any element \([29,35]\). The avalanche size \( S \) is then defined as the total number of activated events between avalanche initiation and termination. After termination of the avalanche, a next Kinetic Monte Carlo step determines the initiation site and initiation time of the next avalanche. We repeat this process until a specified end strain is reached. To obtain statistically representative results, we perform ensemble averages over many realizations of the disorder (typically \( 10^3 \) samples for each set of parameters).

### 2.4 Parameters

In the simulations, stress is measured in units of \( \hat{\Sigma}_0 \), strain in units of \( \hat{\Sigma}_0/E \) (where \( E \) is Young’s modulus) and time in units of \( \mu_s^{-1} \). The model depends on the non-dimensional coupling constant \( 1 - e \) which controls the characteristic fraction of the local stress that is elastically re-distributed after an event. The model relates to particular disordered systems through the values of \( e, \hat{\Sigma}_0 \) and \( k \). Unless otherwise stated, during this work we assume the default values \( e = 0.95, L = 64, \Sigma_T = 0.0075 \) and \( k = 4 \).

Since creep loading requires the application of a constant external load, we need to determine the value of such load before performing creep tests. To this end, we use as reference the critical macroscopic yield stress \( \Sigma_c \) at which the system enters a state of sustained macroscopic plastic flow without the need for thermal activation. The value of \( \Sigma_c \) averaged over many simulations for different system parameters is shown in Figure 1. Specifically, for the default set of parameters given above, we find \( \Sigma_c = 0.668 \). We set a default value for the external stress of \( \Sigma = 0.7 \Sigma_c \) during our creep simulations unless otherwise stated.

### 2.5 General remarks on the modelling approach

In conclusion of our model description, some general remarks regarding the scope of the model and the expected outcomes are in place. In a typical simulation scenario as described in the next section, the sequence of thermal activation events and subsequent avalanches will produce an average strain rate signal that first decelerates during an initial transient (primary creep, stage I). The average creep rate then approaches a constant asymptotic value, where the spatio-temporal stochastic flow process becomes stationary (secondary creep, stage II). In this regime, plastic flow is heterogeneous in space and time.

![Fig. 1. Critical external stress \( \Sigma_c \) at which macroscopic flow occurs in stress driven loading conditions, as a function of the disorder parameter \( k \); inset: dependency of critical stress on system size.](image-url)
(the temporal dynamics is characterized by avalanches, the spatial dynamics is characterized by transient shear bands), however, statistical homogeneity in space and time is preserved. A simulation without prescribed end strain would therefore continue forever, which is, of course, unphysical: real creep samples undergo a third, accelerating creep stage (tertiary creep, stage III) during which spatial and temporal homogeneity is lost as deformation localizes into a single shear band and accelerates towards a finite-time singularity of the creep rate (a catastrophic avalanche), resulting in sample failure. This raises two questions: (i) why can the present model not capture the processes during tertiary creep and (ii) why do we use it nevertheless?

The phenomenology of tertiary creep is characterized by a macroscopic deformation instability leading to local acceleration of deformation under constant overall load (localization and acceleration). In a material whose internal dynamics reaches a stable steady state, such instability must be related to geometry changes and is therefore contingent on deformation geometry and boundary conditions [44]. By imposing pure shear conditions, we exclude such a geometrical instability. However, many materials (amorphous or not) exhibit tertiary creep and creep failure even when loaded in pure shear mode as e.g. in a torsion test. We are thus dealing with a material rather than a geometrical instability (see [44] for a formal discussion of the distinction between geometrical and material instabilities in plastic deformation).

Condition for a material instability of plasticity in a rate-dependent material is that the macroscopic stress required to sustain a given strain rate is either a decreasing function of strain rate (strain-rate softening) or that it is an increasing function of strain rate but a decreasing function of strain (strain softening) [45]. In the present case, thermally activated overcoming of barriers is facilitated by stress and therefore a higher stress always implies a higher strain rate. Instability must then be of the strain softening type. Such an instability as the prime cause of creep acceleration and strain localization in tertiary creep leading to creep failure was studied in a companion paper [39], where a damage variable was introduced that reduces, on statistical average, the deformation thresholds with increasing local strain. In physical terms, in metallic glasses one may think of free volume accumulation. For a study of tertiary creep and creep failure, which were the exclusive focus of reference [39], we think the inclusion of such a damage or softening variable is indispensable.

In the present study, on the other hand, we focus on primary and secondary creep. Comparison of our results with simulations of the same creep stages using the model of reference [39] shows no significant differences during the early creep stages. We thus conclude that the inclusion of a damage variable (e.g., free volume accumulation) is not needed to describe the transition from primary to secondary creep, or the statistical dynamics during the linear creep stage. As the damage variable and the associated equations and parameters are not relevant to the problem studied here, we use a simplified model without this variable for conceptual clarity. Besides, the fact that the linear creep stage has the characteristics of a stationary stochastic process has the additional benefit that it facilitates statistical sampling.

### 3 Simulation results

#### 3.1 Creep curves

Based on the evolution of the average strain as a function of time (see Fig. 2, top), we observe two deformation regimes. The first regime, referred to as transient or stage-I creep, is characterized by a decelerating strain rate. The second regime, referred to as linear or stage-II creep, is characterized by a constant strain rate. Tertiary creep, which is characterized by an accelerating strain rate prior to failure, is not in the focus of the present work but has been discussed in detail in a parallel study [39].

Immediately after the application of the load at $t = 0$, all elements with a yield threshold lower than the external stress yield in an instantaneous and uncorrelated manner. Consequently, a macroscopic avalanche with a characteristic size of order $\langle S \rangle \sim L^2$ takes place. This avalanche
can be interpreted as the instantaneous creep stage \[5\].

After the instantaneous creep, the average avalanche size quickly drops as activity is mainly due to thermal activation of individual plastic events. This gives rise to the transient regime, during which deformation continues to decelerate. After the transient regime, the deformation enters a stationary regime, where activity is controlled by thermally activated events, with an amount of mechanically triggered sequels that depends on the stress level and diverges as \(\Sigma \to \Sigma_c\) but is typically small away from this critical point.

In the following, we consider different quantities that will help us gain insight into the internal dynamics of systems undergoing creep deformation. We study the evolution of these quantities along the creep process and their dependence on system parameters and deformation conditions.

### 3.2 Evolution of yield thresholds

We first consider the time evolution of the average local yield threshold \(\langle \hat{\Sigma} \rangle\), and the threshold scatter measured by the standard deviation of thresholds std(\(\hat{\Sigma}\)). As shown in Figures 3 (top) and 4 (top), the average threshold first increases from its initial value of \(\langle \hat{\Sigma} \rangle = 1\). This hardening is responsible for the slowing down of creep deformation in the transient regime. Commonly known as statistical hardening, it is the consequence of a survival bias due to the statistically distributed yield thresholds: The lowest threshold elements are, according to the rules of Section 2.1, more likely to yield and the re-assignment of a new threshold leads, on average, to a higher local threshold. The evolution of the average threshold can be interpreted as structural aging, by which thermally activated plastic activity leads to a more stable structure. The value of std(\(\hat{\Sigma}\)) is found to drop until reaching a minimum, after which it grows until attaining its plateau value at the stationary creep regime (see Figs. 3 and 4, bottom).

The existence and location of the minimum depends on temperature (it disappears at high temperatures) but is robust upon variation of the external stress. As statistical hardening proceeds, the thresholds move to the high strength side of the distribution \(6\), which in turn increases the probability that upon activation a higher threshold is replaced by a lower one. Ultimately, a dynamic equilibrium situation is reached where the mean and scatter of the yield thresholds in the sample no longer change (see also Appendix B). Thus, the system enters a stationary creep regime which is characterized by a constant deformation rate, as shown in Figure 2 (bottom).

### 3.3 Spatial activity patterns

#### 3.3.1 Strain localization

During the initial stages of the transient regime, internal stresses are almost absent, and the system behavior is controlled by the externally applied shear stress which is spatially homogeneous. As a result, plastic deformation is activated in a spatially uniform random manner.
Fig. 5. Average yield threshold $\langle \Sigma_{II} \rangle$ in the stationary creep regime for different system parameters.

(see Fig. 2, patterns). As deformation proceeds, the internal stress field created by previous plastic events induces elastic couplings within the system, which leads to self-organization of the deformation activity and the emergence of strain patterns in the form of non-permanent, mutually perpendicular shear bands. The orientation of the bands is the result of the loading mode: our external load induces a pure shear state with principal axes oriented along $\pm \pi/4$ with respect to the horizontal axis in Figures 2 and 8. The plastic events follow this orientation, and, as a consequence, positive stress redistribution favoring the formation of bands takes place along the directions 0 and $\pi/2$ with respect to the horizontal axis. However, the ensuing strain localization is only transient: As the creep strain increases during the stationary creep regime, the plastic strain pattern arising from the superposition of strain bands remains statistically homogeneous.

3.3.2 Event correlation

To quantitatively study the spatial activity, we consider the Pearson correlation coefficient $\rho$ between the locations of the triggering events of two consecutive avalanches. Specifically, we consider the projected locations on the horizontal axis, $x$, and $x'$. As discussed by [39], care must be taken in case of symmetry breaking induced by, e.g., permanent strain localization in the form of a macroscopic shear band. However, since we study the stationary creep regime without the presence of any softening mechanism, the activity is symmetric with respect to the interchange of $x$ and $y$ directions. We define thus the coefficient

$$\rho(\epsilon) = \frac{\langle xx'\rangle_{\epsilon} - \langle x\rangle^2_{\epsilon}}{\sigma(x,\epsilon)^2}$$

(7)

where $\langle \cdot \rangle_{\epsilon}$ denotes an average over a narrow strain window centered at $\epsilon$ and $\sigma(\cdot,\epsilon)$ is the standard deviation of event locations within that strain window. The evolution of the correlation coefficient $\rho$ with strain is shown in Figure 2 (bottom). The initial lack of elastic coupling is reflected by near-zero correlation coefficient $\rho_{II}$, while on the approach to the stationary regime the coefficient saturates at a value $\rho_{II}$.

The dependence of $\rho_{II}$ on different system parameters is shown in Figure 7. Since spatial correlations are the result of the internal stress field induced by the plastic events, a higher (spatially homogeneous) external stress reduces the effects of such internal stress field and leads to a drop of spatial correlation. Similarly, by increasing temperature we promote the stochastic yielding of elements, leading to reduced spatial correlation. The drop as the system size increases stems from the fact that consecutive events are more likely to occur far away from each other in bigger systems. We observe that, as the disorder increases, the correlation increases. This behavior can be understood by looking at the strain patterns obtained at the same value of strain with different values of $k$. As shown in Figure 8, higher disorder (lower $k$) promotes activity localization in small clusters that are uniformly scattered across the system. In the case of lower disorder, plastic activity gives rise to comparatively longer-wavelength features, which leads to a lower correlation coefficient.

3.4 Event rate

The event rate $\dot{n}$ is defined as the number of avalanches per unit time. Since avalanches are considered as instantaneous events and are triggered by thermally activated events, $\dot{n}$ matches the rate of thermal activation.
3.4.1 Transient regime

The event rate is found to decrease after the application of the external load as \( \dot{n} \sim t^{-p} \) with \( p \approx 0.89 \) (see Fig. 9, left). This relation is known as Omori law and describes the decay of activity with time after a large perturbation occurs in the system. It is commonly used in geophysics to model the sequence of aftershocks after a main shock \([11,49]\) and has been found in materials failure tests after a partial rupture of the sample \([13]\). In our case, the perturbation corresponds to the sudden application of the external load at \( t = 0 \). The drop of event rate can be explained as the result of the growth of the yield thresholds (Fig. 2, bottom) which leads to lower activation rates in equation (5).

We have studied the parameter dependence of the Omori exponent \( p \). We find that, within error bars, the value of \( p \) does not vary with the value of the applied external stress, the temperature or the system size. However, we find a systematic evolution of the value of \( p \) upon variation of the Weibull exponent \( k \) (see Fig. 9, right). The exponent \( p \) decreases upon increasing the threshold disorder (i.e., lower \( k \)), and seems to converge towards \( p = 1 \) for \( k = 0 \). Since avalanches have, during our simulations, a constant characteristic size \( \langle S \rangle \), the cumulative deformation during the creep process can be obtained by integrating with respect to time the event rate, \( \epsilon \sim \int \dot{n} dt \).

Consequently, we recover the Andrade law \( \epsilon \sim t^{m} \), with an exponent \( m = 1 - p \), in the case of highly ordered systems of high \( k \). On the other hand, as disorder increases and \( p \to 1 \), we recover logarithmic creep \( \epsilon \sim \log(t) \).

3.4.2 Stationary regime

When the creep process enters the stationary regime, the average threshold becomes constant (Sect. 3.2). Since the element activation rates are related to the thresholds through equations (5) and (1), the activity in the stationary regime \( \dot{n}_H \) becomes eventually constant. The impact of different simulation parameters on \( \dot{n}_H \) is shown in Figure 10. The exponential dependence on applied stress and inverse temperature are direct results of the underlying thermal activation process as given by equation (5), i.e., our model is in accordance with viscoplastic creep models. The event rate scales with system size as \( L^2 \), which is the consequence of considering thermal activation a Poisson process in which \( L^2 \) elements with the same average activation rate (in the stationary regime) attempt to yield simultaneously. Increasing the disorder is found to strongly reduce the event rate in the stationary regime, which is explained by the the higher saturation values of the local strength (Fig. 5) and again illustrates the paradigm “more disordered is stronger”.

3.5 Inter-event time distributions

The inter-event time between two thermally activated events is computed by the Kinetic Monte Carlo method. Thus, thermal activation is envisaged as a Poisson process in which the mesoscopic elements attempt to yield independently. However, the distribution of activation barriers \( \Phi \) depend on internal stresses and therefore change due to mutual interactions between events occurring at different times. Therefore, the probability density \( P(\Delta t) \) is expected to reflect the different degrees of activity correlation found along the creep curve.

Figure 11 shows the evolution of \( P(\Delta t) \) during the initial stages of the transient regime. Initially, the probability distribution is of exponential shape, suggesting uncorrelated activity. This is in line with the initial lack of spatial correlation of plastic activity described in Section 3.3.2. As deformation proceeds, the average threshold grows (Fig. 3), and the system becomes thus more stable. Consequently, the average time for a thermal fluctuation to
occurs which can overcome an energy barrier increases, which is reflected in the shift of the distribution cut-off toward bigger values. Still within the transient regime, the probability distribution becomes dominated by a power-law regime with an exponent close to $-1$, incompatible with Poissonian statistics. This happens in parallel with the growth of spatial correlations after the application of the load (Fig. 2, bottom). During the stationary creep regime, the distribution of inter-event times becomes stationary and retains its power law characteristics.

We note that, as shown by Castellanos and Zaiser [39], during the third creep stage, i.e. during the approach to failure, the inter-event time distribution undergoes exactly the reverse evolution – it changes from a power law with exponent close to $-1$ into an exponential distribution, and this evolution is concomitant with a loss of spatial correlation between subsequent events.

4 Discussion and conclusions

As described in Section 3.2, the transient creep regime is associated with hardening, reflected in the growth of the average yield threshold, which in turn is the result of the exhaustion of low-threshold elements. The saturation value of the average yield threshold in stationary creep regime is found to depend on system parameters (Fig. 5).

At low temperatures, low threshold elements are typically chosen for yielding. The renewed threshold is thus likely to be significantly higher than the previous one, which enhances statistical hardening. On the other hand, a higher temperature allows for more stable elements to be chosen for yielding, reducing thus the effects of statistical hardening. The stationary threshold is determined, at a certain temperature, by the dynamical equilibrium achieved between both processes, namely the selection of low-threshold elements which become stronger after yielding and the selection of strong elements which become softer after yielding. Increasing the external stress has an effect similar to that of increasing temperature. This can be understood by noting that the stress lowers the activation barriers in a global fashion. Consequently, at a higher external stress, strong elements are chosen for yielding more often than they would at low external stress. This has the effect of shifting the dynamical equilibrium of thresholds toward lower values. By increasing the threshold disorder (i.e., reducing $k$), the stationary average yield threshold is increased, which is a direct consequence of fattening the high-strength tail of the threshold probability distribution.

We find that the decrease of activity during the transient creep regime follows an Omori-like law with an exponent $p$ which is generally close to 1, as is experimentally well-known for a wide range of materials, and recently established in the specific case of metallic glasses [15]. Specifically, we observe that $p$ does not depend on the external stress, temperature or system size but varies systematically with the degree of disorder. The exponent seems to converge towards $p = 1$ at high disorder, while the value decreases with the disorder. As pointed out in Section 3.4.1, the variation of $p$ with macrostructural disorder might explain the variability observed in the exponent of the Andrade law of transient creep or the existence of logarithmic creep. Moreover, the exclusive dependence of the exponent on the disorder of the yield thresholds might allow for quantitative estimation of the disorder at the mesoscopic scale. The disorder at the mesoscale is a crucial parameter in the modeling of the plastic activity of disordered materials [29,31,37,50], which is on the other hand difficult to establish experimentally. It is therefore of interest to establish links between the macroscopically observable response and a quantitative measure of structural heterogeneity.

The evolution of the inter-event time distribution as deformation proceeds (Fig. 11) is characterized by a transition from an initially exponential shape to a power-law-like distribution with an exponent of approximately $-1$. This value is in agreement with experimental findings on creep deformation of bulk metallic glasses [15,22]. The transition from uncorrelated to correlated activity suggested by the distributions of inter-event times is supported by the correlation coefficient of spatial activity (Sect. 3.3.2). Interestingly, Krisponeit et al. [22] found that the distribution of inter-event times in metallic glasses evolves with time, from a distribution initially dominated by a cut-off towards a power-law-like distribution with an exponent of approximately $-0.8 \pm 0.1$, which agrees well with our observations.

In conclusion, we have presented a mesoscale elastoplastic model which can reproduce the time evolution of the intermittent plastic activity of disordered materials under creep conditions. We find that the structural evolution as a function of time is concomitant with a variation of the spatial correlation of events, with the emergence of strain localization patterns, and with a variation of the event rate and the inter-event times distribution. In summary, our approach allows us to study the creep dynamics from a holistic perspective, establishing contact between diverse approaches which are frequently considered separately, the macroscopic smooth flow response, the microscopic stochastic intermittent response, and the evolution of the structural properties as expressed by the distribution of local activation thresholds.
Author contribution statement

D.F.C. wrote the simulation code, performed simulations and analyzed the resulting data. M.Z. contributed analytical considerations and assisted in interpreting the simulation results. Both authors jointly wrote the manuscript.

Appendix A: Numerical method for stress computation

For calculating the stress field associated with the externally imposed boundary tractions and the heterogeneous plastic strain field resulting from stochastic shear transformation activation, we use a Finite Element framework. In finite-element elasticity, one solves the static balance equation of linear momentum in the body domain $\Omega \subset \mathbb{R}^2$,

$$\nabla \cdot \Sigma(r) + b(r) = 0, \quad r \in \Omega. \quad (A.1)$$

This equation is considered in conjunction with the constitutive equation relating stress and strain in linear elasticity, $\Sigma = C : \epsilon(r)$, and the geometrical relation between strain and displacement fields

$$\epsilon(r) = \frac{1}{2} (\nabla u + \nabla u^T), \quad r \in \Omega \quad (A.2)$$

which, for small strains, defines the strain as the symmetrized gradient of the displacement field $u$. Solution of equation (A.1) thus yields a displacement field $u$ from which the strain is then evaluated using its geometrical definition, and stress is obtained from the constitutive equation (Hooke’s law).

In the present elastic-plastic calculation, there are no body forces $b$ but shear transformations introduce a plastic (stress free) eigenstrain $\epsilon^p(r)$ into the material. This fact does not change the geometrical meaning of the strain field, equation (A.2). However, only the elastic part of the strain is associated with a stress, hence the constitutive equation modifies to

$$\Sigma(r) = C : \epsilon^el(r) = C : (\epsilon(r) - \epsilon^p(r)). \quad (A.3)$$

After inserting into the momentum balance equation, it is evident that the plastic strain formally appears in the form of an internal body force:

$$\nabla \cdot \Sigma(r) - \nabla \cdot C : \epsilon^p(r) = 0, \quad r \in \Omega \quad (A.4)$$

For a given plastic strain field, the stress calculation is thus tantamount to a pure elasticity calculation that determines a stress field from the corresponding internal body force field under the prescribed boundary conditions. Note that, because of the linearity of all equations, the solution of the elastic problem fulfills a superposition principle, i.e., the solution for a plastic strain field $\epsilon^p(r) = \sum \epsilon^p_i(r)$ can be obtained by summation of the solutions for the individual $\epsilon^p_i$. Similarly, the solution for the problem with applied boundary tractions and plastic strain field can be obtained by adding the solution of the boundary value problem (no plastic strain) and the solution of the eigenstrain problem with free boundaries. We therefore proceed in an incremental manner: First we calculate the stresses induced everywhere in the system by the applied boundary tractions. Then, we proceed incrementally: In each update step $k$ where a set of sites $S_k$ becomes activated, we determine the corresponding plastic strain increment $\epsilon^p(r) = \sum_{i \in S_k} \Delta \epsilon^p_i(r_k)$ and use this in a Finite Element step to determine the corresponding stress update, which is then added to the previously existing stress field.

In the Finite Element Calculations equation (A.4) is cast into its so-called weak form, which considers stress equilibrium in terms of a weighted average over a finite domain rather than in every point of the continuum. The weak form of (A.4) in terms of the solution displacement field $u(r)$ reads

$$\int_{\Omega} (\nabla_S w) \cdot C : \nabla_S u d\Omega + \int_{\Omega} (\nabla_S w) \cdot C : \epsilon^p d\Omega - \int_{\Gamma} w^T \cdot t dl = 0, \quad (A.5)$$

where $w$ is a vector of weight functions that are, in principle, arbitrary. In the FE calculation, however, these weight functions are defined in a specific manner: We consider a square-shaped domain $\Omega$ and partition it into a 2D regular lattice of square elements $\Omega_i$, $i = 1, 2, \ldots, L^2$ with 4 corner nodes. Such discretization serves both as support for the definition of the local activation rules and discrete fields given in Section 2, and as a mesh over which (A.5) can be discretized and numerically solved by means of the FE method. To this end, we consider an approximate local solution vector $d_i$ which contains the displacements $u$ in the nodes $j$ in the element domain $\Omega_i$. Local discrete solution is interpolated to any location $r \in \Omega_i$ of the element domain using the so-called shape functions $N^{(i)}(r)$ which are 0 outside $\Omega_i$, 1 on their respective supporting local nodes $j$, and 0 on the other nodes of element $i$. The function representation of the solution vector $u(r), r \in \Omega_i$ is then $u(r) = \sum_j u(r_j)N^{(ij)}$. Inserting this function representation into the weak form of the momentum balance equation, and using as weight functions the shape functions, produces a linear system of equations for the nodal displacements. The reader is referred to e.g. [51,52] for details.

Since by means of the shape functions we can obtain the value of the displacement at any point $r \in \Omega$, the components $e_{ij}^{\text{FEM}}$ of the strain tensor can be computed through the gradients of the displacement, $e_{ij}^{\text{FEM}} = (\partial_i u_j + \partial_j u_i)/2$. Furthermore, in order to smoothen the numerically approximated solution and reduce undesired mesh-alignment effects, we convolute $e_{ij}^{\text{FEM}}$ with a Gaussian kernel of width $\sigma = 0.45$, in units of element linear length. This effectively amounts to a nearest-neighbor weighted average with Gaussian weights, which proves sufficient to make our results invariant upon rotation of the stress axis by 45 degrees. The stress can then be straightforwardly obtained as $\Sigma(r) = C : (e_{ij}^{\text{FEM}}(r) - \epsilon^p(r))$. In order to associate a single stress value $\Sigma(r_i)$ to each of our
mesoscopic elements, we average the stress tensor over each element domain $\Omega_i$,

$$\Sigma(r_i) \equiv \frac{1}{|\Omega_i|} \int_{\Omega_i} \Sigma(r) dr. \quad (A.6)$$

The values $\Sigma(r_i), i = 1, 2, \ldots, L^2$ can be readily used in the rules for plastic activation given in Section 2.1. Similarly, when the plastic field is updated in element $i$ as defined by (2), a homogeneous contribution within $\Omega_i$ is added to the local plastic strain.

The specific boundary conditions used in this work consist of a set of spatially homogeneous tractions on the surfaces of the system (“Neumann” BCs) in such a way that a pure shear state with principal axes $\pm \pi/4$ is induced. To this end, on the top surface a traction $t = (t_1, 0)$ is applied, on the bottom surface $t = (-t, 0)$, on the left surface $t = (0, -t)$ and on the right surface $t = (0, +t)$. The modulus $t$ of the traction is kept constant during a simulation in order to perform creep loading tests. The displacement of the surfaces is not constrained so they can freely deform. For this set of boundary conditions, the externally imposed stress takes the spatially constant value $\Sigma = t[e_x \otimes e_y + e_y \otimes e_x]$.

A corner of the system is fixed to avoid rigid body translations. We use first-order (linear) shape functions. We note that the discretization of the material is performed only once, i.e., we do not re-mesh. This approach is in contrast with traditional FEM-based approaches for describing plastic deformation. In such approaches, the mesh discretizing the system can be rebuilt several times during the deformation process in order to account for large deformations, rotations or material advection. However, the strains reached in room temperature deformation of BMGs are of the order of a few percents only. Such small deformation allows us to consider a single material discretization and to avoid re-meshing. On the other hand, avoiding re-meshing yields a higher computational efficiency, which is of the utmost importance since we aim at performing statistical analysis over a big ensemble of simulated samples.

### Appendix B: Aging dynamics in the absence of elastic interactions

To illustrate the stochastic dynamics implicit in our model and provide a clearer understanding of the underlying assumptions, we discuss here the case of a hypothetical system without elastic interactions between elements. In this case, activation is always thermal, and occurs at a rate

$$\nu(\hat{\Sigma}, \Sigma_{eq}) = \nu_0 \exp \left( \frac{\Sigma_{eq} - \hat{\Sigma}}{\Sigma_T} \right) \quad (B.1)$$

where $\Sigma_{eq}$ is the externally imposed stress only, which is constant in space and time. Threshold renewal under stress implies that all thresholds must be above $\Sigma_{eq}$, hence, new thresholds are taken from the truncated distribution

$$P_0(\hat{\Sigma}|\Sigma_{eq}) = \left\{ \begin{array}{ll} N \left( \frac{\hat{\Sigma}}{\Sigma_{eq}} \right)^{k-1} \exp \left( - \left( \frac{\hat{\Sigma}}{\Sigma_{eq}} \right)^k \right) & \text{if } \hat{\Sigma} > \Sigma_{eq} \\ 0 & \text{if } \hat{\Sigma} \leq \Sigma_{eq} \end{array} \right. \quad (B.2)$$

where $N$ normalizes the truncated distribution. This leads to the following evolution equation for the sample distribution of thresholds, $P(\Sigma)$:

$$\partial_t P(\hat{\Sigma}) = -\nu(\hat{\Sigma}, \Sigma_{eq}) P(\hat{\Sigma}) + \langle \nu \rangle P_0(\hat{\Sigma}|\Sigma_{eq}) \quad (B.3)$$

where $\langle \nu \rangle = \int \nu(\hat{\Sigma}, \Sigma_{eq}) P(\hat{\Sigma}) d\hat{\Sigma}$. The loss term on the right hand side is the rate at which elements of local threshold $\hat{\Sigma}$ get activated and change their threshold, whereas the generation term represents the rate at which elements of any threshold are activated and assigned the renewed threshold value $\Sigma_{eq}$.

We consider equation (B.3) first in the case of zero stress. In this case, its steady-state solution is given by

$$P_\infty(\hat{\Sigma}) = \frac{\langle \nu \rangle}{\nu(\Sigma)} P_0(\hat{\Sigma}) = N' P_0(\hat{\Sigma}) \exp \left( \frac{\hat{\Sigma}}{\Sigma_T} \right). \quad (B.4)$$

In the limit of high temperatures where $\Sigma_T \gg \Sigma^*$, this solution reduces to $P_\infty(\hat{\Sigma}) = P_0(\hat{\Sigma})$. In other words, our choice of $P_0(\hat{\Sigma})$ as the initial distribution in our creep simulation is tantamount to considering the situation immediately after a rapid quench from high temperatures. If we are now at a temperature where $\Sigma_T$ is no longer large in comparison with the typical barrier height $\Sigma^*$, then the system is, according to equation (B.3), bound to undergo an aging process where the distribution shifts to larger thresholds (more stable states) until it reaches the asymptotic distribution given by equation (B.4) that characterizes the fully relaxed system. We note that the idea that a rapidly quenched system exhibits a different threshold distribution from a slowly quenched one, in the sense that thresholds after rapid quenching are lower on average, is consistent with molecular dynamics simulations, see Patinet et al. [28].

Next, we investigate how an external stress changes this behavior. In that case, we need to replace $P_0(\hat{\Sigma})$ by the truncated distribution $P_0(\hat{\Sigma}|\Sigma_{eq})$, and the rate $\nu(\hat{\Sigma})$ by $\nu(\hat{\Sigma}, \Sigma_{eq}) = \nu(\hat{\Sigma}) \exp(\Sigma_{eq}/\Sigma_T)$. The effect of an external stress is, thus, twofold: First, any states that are mechanically unstable are removed from the system. Second, the aging process is accelerated, as all rates are multiplied by a factor of $\exp(\Sigma_{eq}/\Sigma_T)$ which may be large even for moderately low temperatures and moderately high applied stresses. For illustration, with an activation volume of $(1\text{ nm})^3$ and a temperature of $300\text{ K}$, we find a “thermal” stress $\Sigma_T \approx 4\text{ MPa}$. If we now apply a stress of $100\text{ MPa}$, all rates characterizing the aging process increase by a factor of $\exp(25) \approx 7 \times 10^{10}$.

Finally, we look into the changes of the mean activation rate that occur just after loading. From equation (B.3) we
can derive an equation for the mean activation rate:
\[ \partial_t \langle \nu \rangle = - \langle \nu^2 \rangle + \bar{v}_0 \langle \nu \rangle \] (B.5)
where
\[ \langle \nu^2 \rangle = \int [\nu(\Sigma, \Sigma_{eq})]^2 P(\Sigma) d\Sigma \]
\[ \bar{v}_0 = \int \nu(\Sigma, \Sigma_{eq}) P_0(\Sigma | \Sigma_{eq}) d\Sigma. \] (B.6)

We now evaluate the integrals under the assumption that \( \Sigma_T \) is small compared to typical barrier heights (see the above numerical example), and in the limit of high disorder. In that case, we can approximately evaluate the integrals by replacing \( P(\Sigma) \) with its value \( P(\Sigma_{eq}) \) in the vicinity of the lowest possible threshold. We obtain \( \langle \nu^2 \rangle = 2 \langle \nu \rangle^2 / |\Sigma_T P(\Sigma_{eq})| \). The mean activation rate then fulfills the approximate differential equation
\[ \partial_t \langle \nu \rangle = - \frac{2 \langle \nu \rangle^2}{\Sigma_T P(\Sigma_{eq})} + \bar{v}_0 \langle \nu \rangle. \] (B.7)

Initially, the first term on the right-hand side of this equation exceeds the second term by a factor of the order of \( \Sigma^*/\Sigma_T \gg 1 \). Hence, \( \partial_t \langle \nu \rangle \propto -\langle \nu \rangle^2 \) and thus \( \langle \nu \rangle \propto t^{-1} \). This is indeed the behavior observed in our simulations during the very first stage of creep, where events are still isolated and far apart such that elastic interactions between events are of minor importance.

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