Equalization of Response Functions of the SK and SNO

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Abstract

We have studied the equalization of the response functions of the SK and SNO for total event rate and event rate for the energy bins. To calculate the response functions of SNO, we have used the latest theoretical values of the cross section of the neutrino-deuteron CC process. By using these new theoretical values, we find that the trigger threshold of the SK at which its response function is equalized to the response function of SNO (at the trigger threshold of 6.75 MeV), is 8.5 MeV. This value is 0.1 MeV smaller than the value calculated using the old theoretical values of the cross section of neutrino-deuteron process. The use of these new theoretical values also produces a small change in the range of the energy bins where the response functions are equalized.

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1 Introduction

Homestake, GALLEX, SAGE, Kamiokande, Super-Kamiokande and SNO [1-6], these are the six solar neutrino experiments for which up to date experimental results are available. Out of these six detectors, first three are radiochemical detectors and can measure only the total event rate produced by the electron neutrinos, whereas other three are water-cherenkov detectors and provide the facility to record the time of the event, energy of the scattered electron and scattering angle. These detectors are sensitive to all flavors of neutrinos. All these detectors have detected solar neutrinos at the rate much smaller than predicted by the Standard Solar Model (SSM) [7]. This discrepancy between SSM and experimental measurements is called solar neutrino puzzle (SNP). The puzzle can be explained in two general ways.

1. Perhaps the solar models do not accurately describe the sun.
2. Perhaps the known theories of neutrino do not accurately describe it.

At present it is difficult to accept (1) because the SSM has been very successful in describing many features of the sun, particularly the latest confirmation of the SSM’s prediction of the velocity of sound at the surface of sun through helioseismological studies [7]. Now it is generally believed that the solution of the puzzle is to come from the particle theory. The most elegant solution, which the particle physics provides, is the ‘neutrino oscillation’. The phenomena in which different flavors of neutrino may oscillate into each other while passing through vacuum or matter [8]. The exact amount of depletion which may be caused by the oscillations, however, depends on the neutrinos squared mass differences and mixing angles. The 'Global analysis of solar neutrino data', in which we fix the values of these parameters so that the difference between the predicted and measured rates is minimum, reveals that there are many possible solutions in the framework of neutrino oscillations [9]. However, these solutions have different goodness-of-fit (g.o.f). The latest analysis shows that the Large Mixing Angle (LMA) solution have the highest g.o.f and hence most probable [9].

There is another mode of analysis in which we try to probe the oscillations from the experimental data, without using the prediction of solar neutrinos flux by the SSM. In this model independent analysis we compare the data in the same or different solar neutrinos experiments and try to find out whether it is consistent with the neutrino oscillations or not [10]. This comparison of the
solar neutrino data obtained from different detectors, however, requires the equalization of their response functions. The equalization of the response functions of two different detectors is non-trivial phenomena and so far it has been possible only for the SK and SNO detectors. In [10,11,12] the authors have proved that it is possible to equalize the response functions of the SK and SNO by changing the trigger threshold or energy bins of one of these detectors. The comparison, after the equalization of the response functions, reveals that the results obtained from the SK and SNO are consistent with the neutrino oscillations[6,10].

In the equalization of these response functions, accurate values of the cross sections of neutrino-electron (ve) and neutrino-deuteron (vd) reactions, occurring in the SK and SNO respectively, are very important. The most accurate calculations of the cross sections of ve process is given in [16]. These calculations include the effect QED and QCD radiative corrections up to one-loop. For neutrino-deuteron reactions, the most successful method used is the standard nuclear physics approach (SNPA) based on one-body impulse approximation terms and two-body exchange-current terms acting on non-relativistic nuclear wave function. The status of the first detailed calculation based on SNPA is given in [17], referred as Kubodera-Nozawa (KN) calculations. The calculated values of the cross section of vd process in [17] have been used in the equalization of response functions of the SK and SNO in [10,11,12]. The work of Kubodera and Nozawa was further improved by using more accurate NN potentials and nucleon weak-interaction form factors [18]. In the literature this improved work is referred as the NSGK calculations. The estimates of NSGK calculations have been further improved by about 1% by updating some of its inputs and by taking into account the results of recent effective-field-theoretical calculation [19].

In our work we have studied the response functions of the SK and SNO detectors for B8 neutrinos using the most accurate estimate of cross sections of ve and vd process taken from reference [16] and [19] respectively. In section 2 we present the basic definitions of the response functions. In section 3 and 4 we present our results of the comparison of the response functions. As a consequence of the approximate equalization of the response functions, we show in section 5, that in the presence of active neutrino oscillations, how the total event rates and event rates for energy bins of the SK and SNO detectors are related to each other.
2 SK and SNO Response Functions

The solar neutrinos are detected in the SK by the following elastic scattering (ES) process.

\[ v_{e,a} + e^- \rightarrow v_{e,a} + e^- \]  \hspace{1cm} (1)

This process is caused either by the electron neutrino \((v_e)\) or other two active neutrinos \(v_a(a = \mu, \tau)\) with cross sections \(\sigma^e\) and \(\sigma^a\) respectively.

In the SNO, the neutrinos are detected by the following charged-current (CC) and neutral-current (NC) processes.

\[ v_e + d \rightarrow p + p + e^- \] \hspace{1cm} (CC Process) \hspace{1cm} (2)
\[ v_{e,\mu,\tau} + d \rightarrow p + n + e^- \] \hspace{1cm} (NC Process) \hspace{1cm} (3)

The CC process is caused by only electron neutrinos with cross section \(\sigma^e\), whereas as NC process is caused by all three flavors of neutrinos with the same cross section \(\sigma^n\).

All the scattering events in the SK and SNO are detected by detecting the cherenkov radiation emitted by the scattered electron along its direction of motion. This method allows us to record the energy as well as the direction of the scattered electrons. However, due to finite detector resolution the measured energy \((E_e)\) is expected to be scattered around its actual value \((E'_e)\) according to some well-defined distribution function, called resolution of the detector [13-15]. In this way it is possible to define the total event rates by applying some limit on the minimum value of the measured energy, called trigger threshold and event rate for a measured energy range, called energy bin. Normally the trigger threshold and energy bin are taken not less than 5-6 MeV to reduce the background effects. At this higher value of threshold only \(B_8\) or possibly hep neutrinos contribute to the measured events.

The response function for the total event rate (or the rate for an energy bin) is the normalized effective cross section for producing the scattered electron with the measured energy greater than the trigger threshold \(E_{th}\) (or lying in a specific energy bin) [11,12,14]. The response functions (RF) relevant for our work are

\[ \rho_{E_{th}}^{e}(E_v), \text{ the RF of ES } (v_e, e) \text{ for event rate above } E_{th} \]
\[ \rho_{E_{th}}^{a}(E_v), \text{ the RF of ES } (v_a, e) \text{ for event rate above } E_{th} \]
The response functions are defined as following.

$$\rho_{E_{th}}^c(E_v) = \frac{\lambda_B(E_v) \int_{E_{th}}^{E_{max}} dE_e \int_0^{E_v} dE'_e \frac{d\sigma^c(E_e,E'_e)}{dE'_e} R_{SK}(E_e,E'_e)}{\sigma_{E_{th}}^c}$$  (4)

$$\rho_{E_{th}}^a(E_v) = \frac{\lambda_B(E_v) \int_{E_{th}}^{E_{max}} dE_e \int_0^{E_v} dE'_e \frac{d\sigma^a(E_e,E'_e)}{dE'_e} R_{SK}(E_e,E'_e)}{\sigma_{E_{th}}^a}$$  (5)

$$\rho_{E_{th}}^i(E_v) = \frac{\lambda_B(E_v) \int_{E_{th}}^{E_{max}} dE_e \int_{E_{i_{min}}}^{E_{i_{max}}} dE_e \frac{d\sigma^i(E_e,E'_e)}{dE'_e} R_{SK}(E_e,E'_e)}{\sigma_i^e}$$  (6)

$$\rho_{i_{th}}^c(E_v) = \frac{\lambda_B(E_v) \int_{E_{i_{th}}^{i_{max}}}^{E_{i_{th}}^{i_{min}}} dE_e \int_0^{E_v} dE'_e \frac{d\sigma^c(E_e,E'_e)}{dE'_e} R_{SNO}(E_e,E'_e)}{\sigma_{i_{th}}^c}$$  (7)

$$\rho_{i_{th}}^a(E_v) = \frac{\lambda_B(E_v) \int_{E_{i_{th}}^{i_{max}}}^{E_{i_{th}}^{i_{min}}} dE_e \int_0^{E_v} dE'_e \frac{d\sigma^a(E_e,E'_e)}{dE'_e} R_{SNO}(E_e,E'_e)}{\sigma_{i_{th}}^a}$$  (8)

$$\rho_{i_{th}}^i(E_v) = \frac{\lambda_B(E_v) \int_{E_{i_{th}}^{i_{max}}}^{E_{i_{th}}^{i_{min}}} dE_e \int_{E_{i_{min}}}^{E_{i_{max}}} dE_e \frac{d\sigma^i(E_e,E'_e)}{dE'_e} R_{SNO}(E_e,E'_e)}{\sigma_{i_{th}}^i}$$  (9)

Where the differential cross sections $\frac{d\sigma^c}{dE_e}$ and $\frac{d\sigma^a}{dE_e}$ for ES processes are taken from ref [16] and $\frac{d\sigma^i}{dE_e}$ for CC process is taken from the ref [19]. It is noted that in [10,11,12] the authors have used old theoretical data for CC process [17]. $R_{SK}$ and $R_{SNO}$ are the resolutions functions of the SK and SNO respectively. $\lambda_B(E_v)$ is the normalized energy spectrum of $B^8$ neutrinos. The denominators represent the $B^8$ neutrinos total cross sections for producing an electron with the observed energy greater than the trigger threshold or lying with in the specific energy bin as per the definition of response function. The denominator in each expression is obtained by integrating the numerator over the energy ($E_v$) of $B^8$ neutrinos.
3 Equalization of the Total Response Functions

In this section, it is shown that the following total response functions for the SK and SNO can be equalized to a good approximation by an appropriate choice of the trigger threshold.

\[ \rho_{E_{th}}(E_v) = \rho_{E_{th}}^a(E_v) \]  
(10)

\[ \rho_{E_{th}}'(E_v) \simeq \rho_{E_{th}}^c(E_v) \]  
(11)

Equalization of ES response functions of the SK for \( v_e \) and \( v_{\mu} \) neutrinos is accurately possible where as the ES response function of the SK for \( v_e \) can be approximately equalize to the CC response function of SNO for \( v_e \) by changing the trigger threshold of either of the detectors. In order to compare the response functions, we determine the following integral differences.

\[ \Delta_1 = \int dE_v |\rho_{E_{th}}(E_v) - \rho_{E_{th}}^a(E_v)| \]  
(12)

\[ \Delta_2 = \int dE_v |\rho_{E_{th}}'(E_v) - \rho_{E_{th}}^c(E_v)| \]  
(13)

By requiring the minimization of these integral differences we obtain the best equalization of the response functions. These integral difference are zero if the equations (10) and (11) satisfy exactly. It is found that the first integral difference is zero at all values of \( E_{th} \). In minimizing the second integral difference, we take SNO’s threshold, \( E_{th} = 6.75 \) MeV and obtain the value of \( \Delta_2 \) for different values of SK’s threshold \( E_{th}' \). The results in Figure 1 shows that \( \Delta_2 \) has one minima at \( E_{th}' = 8.5 \) MeV. At this value of SK’s threshold, the response function are best equalized. Figure 2 shows the graphs of these approximately equalized response functions of the SK and SNO. The value of \( E_{th}' \) obtained by the old theoretical data of CC cross section in [10,11], is 8.6 MeV. In the combined SK and SNO data analysis by SNO collaboration in [6], the value of 8.5 MeV was used, which is in agreement with our calculation.

4 Equalization of the Response Functions for Energy Bins

In this section, it is shown that the following response functions of the SK and SNO for the energy bins can be equalized to a good approximation by an appropriate choice of the range of energy
Equalization of the ES response functions of SK for $\nu_e$ and $\nu_a$ neutrino is accurately possible for the same energy bins where as the CC response function of SNO for $\nu_e$ can be approximately equalized to ES response function of SK for $\nu_e$ for different energy bins. In order to compare these response functions, we determine the following integral differences.

$$\Delta_1 = \int dE_{\nu_e} |\rho_{i}^e(E_{\nu_e}) - \rho_{i}^a(E_{\nu_e})|$$ (16)

$$\Delta_2 = \int dE_{\nu_e} |\rho_{i}^e(E_{\nu_e}) - \rho_{i}^c(E_{\nu_e})|$$ (17)

By requiring the minimization of these integral differences, we obtain the best equalization of the response functions for energy bins. It is found that the first integral difference is zero for all energy bins of SK. In minimizing the second integral difference, we take 13 different energy bins of the SK for which the experimental data is available and obtain the values of $\Delta_2$ for different energy bins of SNO. For each energy bin of SK, we find the energy bin of SNO for which $\Delta_2$ is minimum. The results are summarized in the Table 1. Figure 3 and 4 shows the graphs of these approximately equalized response functions for energy bins of the SK and SNO.

## 5 Relation between SK and SNO Event Rates

After equalizing the total and energy bin response functions of the SK and SNO, it is possible to relate the corresponding event rates [10,11,12]. First we relate the total event rate defined above the trigger threshold. These total event rate, assuming no oscillation, are given by.

$$R_{SK}^0 = \phi \sigma_{E_{th}}^e$$ (18)

$$R_{SNO}^0 = \phi \sigma_{E_{th}}^c$$ (19)

where $\phi = 5.15 \times 10^6 cm^{-2}s^{-1}$, is the SSM’s predicted total flux of B$^8$ neutrinos and $\sigma_{E_{th}}^e, \sigma_{E_{th}}^c$ are total effective cross section for observing the event above the trigger threshold of SK and SNO.
respectively. In the presence of neutrino oscillations, which are described by a survival probability function $P_{ee}(E_v)$, the expressions for the total event rates of the SK and SNO are given by

$$R_{SK} = \phi[\sigma^e_{E_{th}} < P_{ee} >^e_{E_{th}} + \sigma^a_{E_{th}}(1 - < P_{ee} >^a_{E_{th}})]$$

(20)

$$R_{SNO} = \phi[\sigma^c_{E_{th}} < P_{ee} >^c_{E_{th}}]$$

(21)

where the terms $< P_{ee} >^x_{E_{th}} \ (x = e, a, c)$ represent the average survival probability weighted by the response functions $\rho^e_{E_{th}}, \rho^a_{E_{th}}$ and $\rho^c_{E_{th}}$. Now if these response functions are equal, as they are by the equations 14 and 15, then it implies the equalization of these three response function’s weighted survival probabilities.

$$< P_{ee} >^e_{E_{th}} = < P_{ee} >^a_{E_{th}} \approx < P_{ee} >^c_{E_{th}} \equiv < P_{ee} >$$

(22)

The event rates normalized to un-oscillated rates are now given by

$$r_{SK} \equiv \frac{R_{SK}}{R_{SK}^0} = [< P_{ee} > + \frac{\sigma^a_{E_{th}}}{\sigma^e_{E_{th}}}(1 - < P_{ee} >)]$$

(23)

$$r_{SNO} \equiv \frac{R_{SNO}}{R_{SNO}^0} = < P_{ee} >$$

(24)

Eliminating $< P_{ee} >$ in equations 23 and 24, we get the following relation between normalized event rates of the SK and SNO.

$$r_{SK} = r_{SNO}(1 - \frac{\sigma^a_{E_{th}}}{\sigma^e_{E_{th}}}) + \frac{\sigma^a_{E_{th}}}{\sigma^e_{E_{th}}}$$

(25)

The value of the ratio $\frac{\sigma^a_{E_{th}}}{\sigma^e_{E_{th}}}$ is calculated to be 0.1518.

Similarly by applying the equalization of the response functions of the SK and SNO for energy bins, we can obtain the similar relation between the SK and SNO event rates for energy bins.

$$r^i_{SK} = r^i_{SNO}(1 - \frac{\sigma^a_i}{\sigma^e_i}) + \frac{\sigma^a_i}{\sigma^e_i}$$

(26)

where $r^i_{SK}$ and $r^i_{SNO}$ are the normalized event rates of $ith$ energy bin of SK and SNO respectively and $\sigma_i^{e,a}$ are the effective cross sections for producing the event in the $ith$ energy bin by $v_e$ and $v_a$ neutrinos. The values of these cross sections are given in Table 1.
6 Conclusion

In this work, we have shown that the SK and SNO response functions of ES and CC processes can be approximately equalized with the new theoretical values of the cross sections of neutrino-deuteron CC process. Trigger threshold of the SK at which its response function is best equalized with the response function of SNO is found to be 8.5 MeV. This value is 0.1 MeV smaller than the values obtained with the old values of the cross sections of neutrino-deuteron CC process. The new values of the CC cross sections also produce a small change in the energy bins of the SNO for which the response functions for the energy bins are best equalized.

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| $i$ | Energy Bin SK, MeV | Energy Bin SNO, MeV | $\Delta \times 100$ | $\sigma^e$ $(10^{-46} \text{ cm}^2)$ | $\sigma^a$ $(10^{-46} \text{ cm}^2)$ | $\sigma^c$ $(10^{-42} \text{ cm}^2)$ |
|-----|-------------------|-------------------|-----------------|-----------------|-----------------|-----------------|
| 1   | [8.0, 8.5]        | [5.10, 9.90]      | 6.85            | 14.158          | 2.187           | 0.805           |
| 2   | [8.5, 9.0]        | [5.60, 10.35]     | 5.12            | 11.827          | 1.815           | 0.770           |
| 3   | [9.0, 9.5]        | [6.10, 10.75]     | 3.74            | 9.705           | 1.482           | 0.713           |
| 4   | [9.5, 10.0]       | [6.60, 11.25]     | 2.27            | 7.812           | 1.187           | 0.650           |
| 5   | [10.0, 10.5]      | [7.10, 11.70]     | 1.93            | 6.161           | 0.932           | 0.573           |
| 6   | [10.5, 11.0]      | [7.60, 12.20]     | 1.58            | 4.749           | 0.716           | 0.491           |
| 7   | [11.0, 11.5]      | [8.05, 12.70]     | 1.42            | 3.574           | 0.537           | 0.417           |
| 8   | [11.5, 12.0]      | [8.50, 13.30]     | 1.46            | 2.619           | 0.393           | 0.345           |
| 9   | [12.0, 12.5]      | [9.00, 14.20]     | 1.96            | 1.866           | 0.279           | 0.269           |
| 10  | [12.5, 13.0]      | [9.45, 15.00]     | 2.40            | 1.290           | 0.193           | 0.207           |
| 11  | [13.0, 13.5]      | [9.90, 18.10]     | 2.95            | 0.863           | 0.129           | 0.155           |
| 12  | [13.5, 14.0]      | [10.35, 18.85]    | 3.45            | 0.558           | 0.083           | 0.111           |
| 13  | [14.0, 20]        | [11.30, 19.50]    | 7.75            | 0.812           | 0.120           | 0.483           |

Table 1: Energy bins of SK (1st column), energy bins of SNO (2nd column) where the response functions are equalizee, minimum integral difference of response functions (3rd column) and calculated cross sections of ES and CC processes (rest of the columns)
Figure 1: The value of square integral difference of total response functions at different value of trigger threshold of SK
Figure 2: Best equalization of total response functions of SK (continuous line) and SNO (dotted line)
Figure 3: Equalized response functions of SK (continuous curve) and SNO (dotted curve) for odd energy bins
Figure 4: Equalized response functions of SK (continuous curve) and SNO (dotted curve) for even energy bins