Collider signals of brane fluctuations

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Abstract

Assuming that we live on a non rigid brane with TeV-scale tension, the scalar fields that control the coordinates of our brane in the extra dimensions give rise to missing energy signals at high-energy colliders with a characteristic angular and energy spectrum, identical to the one due to graviton emission in 6 extra dimensions. LEP bounds and LHC capabilities are analyzed.

1 Introduction

Branes in large extra dimensions offer a new possible explanation of why gravity is much weaker than gauge interactions \[1\]. Large extra dimensions give clear experimental signals if larger than \(0.03\) mm or if other particles more detectable than gravitons (i.e. ‘sterile neutrinos’ \[2\]) propagate in the extra dimensions. SN1987a poses strong bounds on both possibilities \[3, 4\]. The missing energy carried away by Kaluza Klein (KK) excitations of the gravitons can be detected at LHC if \(\tilde{M}_d \equiv (2\pi)^{(2+\delta)/2}\tilde{M}_d \lesssim (4 \div 7)\) TeV \[5\] (where \(\tilde{M}_d\) is the reduced Planck mass of the \(d = 4 + \delta\) dimensional theory) depending on \(\delta = (6 \div 3)\). Effective operators of dimension 8 are generated by the heaviest KK gravitons \[5\]. It would be nice to have further computable signals of large extra dimensions.

Here we discuss an experimental way of testing if we live on a non rigid brane. If this is the case there are new scalar fields (‘branons’) that control the coordinate position of our brane in the extra dimensions. Branon interactions with particles on the brane are dictated by general considerations \[6, 7, 8\]. It seems reasonable to assume that branons are light and cannot be directly detected, so that they only give rise to missing energy signals. Such signals (for example \(e\bar{e} \to \gamma E/\) at LEP1 and LEP2 and \(pp \to \text{jet}E_T, pp \to \gamma E_T\) at LHC) can be computed in terms of only one parameter \(f\), related to the tension of the brane \(\tau\) and to the number \(\delta\) of branons by \(f = \tau^{1/4}/\delta^{1/8}\). Unfortunately these branon signals have the same energy and angular spectrum as the “super-string signal” produced by KK gravitons propagating in \(d = 6\) extra dimensions\[5\]. The magnitude of the two signals coincide when \(\tau = (\pi\delta/30)^{1/2}M_{10}^4\). In both cases detectable effects appear only at energies so high that the effective Lagrangian used to compute them becomes questionable.

In section 2 we present the effective branon Lagrangian. In section 3 we discuss branon missing energy signals, and comment about the reliability and the relevance of our computation. In section 4 we study their detectability at high energy colliders. Explicit expressions for the relevant cross sections are given in appendix A.

\[\ast\]Like gravitons in 6 extra dimensions, branons produce also an attractive long range force proportional to \(m_1m_2/(rf)^8\) \[7\].
The minimal brane action is [6, 7, 8]

\[ S_{\text{brane}} = \int d^4 x \det e [-\tau + L_{\text{SM}}], \] (2.1)

where \( e^a_\mu \) is the induced vierbein and \( L_{\text{SM}} \) is the covariant Standard Model (SM) Lagrangian. We denote by \( y_i(x) \) \( (i = 1, \ldots, \delta) \) the \( \delta \) coordinates of the point \( x \) of our brane in the \( \delta \) extra dimensions. Even neglecting bulk gravity, \( e \) is non trivial when the brane is warped: \( e^a_\mu = (1 - (\partial^a y_i)(\partial_\mu y_i))^{1/2} \). Expanding at first order in \( \tau \) and rescaling \( Y_i = y_i\sqrt{\tau} \)

\[ S_{\text{brane}} = \int d^4 x \left[-\tau + L_{\text{SM}} + \frac{(\partial_\mu Y_i)(\partial_\nu Y_i)}{2}(\delta_{\mu\nu} + \frac{T^{\text{SM}}_{\mu\nu}}{\tau}) + \cdots \right] \]

one obtains the flat-space SM Lagrangian \( L_{\text{SM}} \), coupled with \( \delta \) canonically normalized scalar fields \( Y_i \) proportionally to the SM energy-momentum tensor \( T^{\text{SM}}_{\mu\nu} = \sum_{\gamma, G} \frac{i}{4}(\bar{\Psi}\gamma_\mu D_\nu\Psi - D_\mu(\bar{\Psi}\gamma_\nu\Psi)) + \sum_{\gamma, G}(F^{a}_{\mu\rho}F^{a}_{\rho\nu} - \frac{\eta_{\mu\nu}}{4}F^{a}_{\rho\sigma}F^{a}_{\rho\sigma}) + \cdots \)

Here \( D_\mu = \partial_\mu - ig_3 g^a_\mu(x)T^a - ie\gamma_\mu(x)q \), \( F_{\mu\nu} \) are the usual field strength tensors of the photon \( \gamma_\mu \) and gluons \( g^a_\mu \). Finally, \( \cdots \) denotes the remaining \( W, Z \) and higgs contributions.

We now list various different ways in which a non minimal brane action can differ from the minimal one that we consider.

- The collective position of \( n > 1 \) coinciding \( D \)-branes arising in string theory is described by a non minimal set of branon-like fields.
- Effects associated with a non-minimal brane structure (‘fat branes’) have been studied in [9].
- The presence of our brane spontaneously breaks also a part of the \( d \)-dimensional Lorentz symmetry. However there is no need of introducing more Goldstone fields than the ‘branons’ corresponding to the breaking of translational invariance.
- If extra dimensions are flat, when bulk gravity is taken into account branons are eaten by the off-diagonal components of the metric that get a mass of order \( f^2/M_{\text{Pl}} \). We can neglect this mass and, due to the equivalence theorem, describe low-energy interactions in terms of branons. If instead some extra-dimensions are not flat so that a shift in the extra dimensions requires energy, some branons acquire non negligible mass terms [3]. The main effect of such mass terms would be that only branons lighter than \( \sim \sqrt{\tau}/2 \) affect processes at a given center of mass energy \( \sqrt{s} \). For example bounds from SN1987a [6] and LEP1 (see below) do not apply if all branons are heavier than \( M_Z/2 \).

Unless otherwise indicated, we assume that extra dimensions have the simplest shape (a product of \( \delta \) circles) so that there are \( \delta \) massless branons. Since we will study experimental signals in which branons manifest as missing energy (so that the signal is proportional to the number \( \delta \) of branons) for our purposes branon interactions are described by a Lagrangian with only one free parameter \( f \)

\[ L_{\text{int}} = \frac{(\partial_\mu Y)(\partial_\nu Y)}{2f^4}T^{\text{SM}}_{\mu\nu}, \]

where \( Y \) is a single canonically normalized real scalar field and \( f^4 = \tau/\delta^{1/2} \).

### 3 Missing energy signals

We study \( e\bar{e} \to \gamma \bar{E} \) at LEP and \( pp \to \gamma \bar{E}_T, pp \to \text{jet} \bar{E}_T \) at LHC. Explicit expressions for the differential cross sections for all contributing processes and parton subprocesses

\[ e\bar{e} \to \gamma YY, \quad q\bar{q} \to \gamma YY, \quad q\bar{q} \to gYY, \quad qg \to qYY, \quad \bar{q}g \to \bar{q}YY, \quad gg \to gYY \quad (3.1) \]
are listed in the appendix. Here we show that branons produce the same missing energy signals as gravitons propagating in $\delta = 6$ extra dimensions\(^1\). In particular, in both cases the signals grow as the 6th power of the collision energy. The overall normalization of branon and graviton signals is the same if $\tau$ is the branch tension, $M_{10}$ is the reduced Planck mass in 10 dimension, $\tilde{M}_{10} \equiv (2\pi)^{3/4}M_{10}$ is the phenomenological parameter used in \(^2\) and $f^8 = \frac{\tilde{M}_{10}^8}{1920\pi^5}$, \(^3\)

$$\tau^2 = \frac{\pi \delta}{30} M_{10}^8,$$  \hspace{0.5cm} i.e. \hspace{0.5cm} $f^8 = \frac{\tilde{M}_{10}^8}{1920\pi^5},$ \hspace{0.5cm} (3.2)

where $\tau$ is the branch tension,$^3$ $M_{10}$ is the reduced Planck mass in 10 dimension, $\tilde{M}_{10} \equiv (2\pi)^{3/4}M_{10}$ is the phenomenological parameter.

To see this, consider for example the branon process $e(p_1)\bar{e}(p_2) \rightarrow \gamma(q)Y(k_1)Y(k_2)$ and the graviton process $e(p_1)\bar{e}(p_2) \rightarrow \gamma(q)G(k)$ (with $k \equiv k_1 + k_2$). We compute the cross section for emitting a photon of energy $y_0 = x\sqrt{s}/2$ and direction $\theta$ with respect to the $e\bar{e}$ center of mass frame. Kinematics fixes $k^2 = s(1 - x)$. The differential cross section for producing a photon of given energy and direction in the two cases can be computed as follows.

- The amplitude for the branon process can be written as $\mathcal{M} = T_{\mu\nu}k_{\mu}k_{\nu}/f^4$ where $T_{\mu\nu} \equiv \langle \gamma|T_{\mu\nu}^{SM}|e\bar{e}\rangle$ is traceless because we only consider processes involving SM particles with negligible mass. Decomposing the three body phase space as $d\phi(e\bar{e} \rightarrow \gamma YY) = d\phi(\gamma)d\phi(YY)$ (see eq. (A.1)) one has

$$d\sigma(e\bar{e} \rightarrow \gamma YY) = \frac{d\phi(\gamma)}{2s} \cdot \int |\mathcal{M}|^2 \frac{d\phi(YY)}{2!} = \frac{x \, dx \, d\cos \theta}{64\pi^2} \cdot \frac{s^2(1 - x)^2}{1920\pi f^8} \cdot T_{\mu\nu} T_{\mu\nu}.$$  

The integration over the phase space of the two branons has been performed using eq. (A.2) and noticing that terms proportional to $k_{\mu}T_{\mu\nu}$ and to $T_{\mu\nu}$ vanish since $T_{\mu\nu}$ is a conserved traceless tensor.

- The amplitude for the graviton process can be written as $\mathcal{M} = T_{\mu\nu}\epsilon_{\mu\nu}/M_{10}^4$ where $\epsilon_{\mu\nu}$ is the polarization tensor of the graviton. Integrating over the orientation of the transverse momentum of the graviton in the 6 extra dimensions, $\int d^4k_T = \pi^3 k^4(d^2k^2)/2$, one obtains

$$d\sigma(e\bar{e} \rightarrow \gamma G) = \frac{d^dk_T}{M_{10}^2} \cdot \frac{d\phi(\gamma)}{2s} \cdot \sum_{G \text{ spin}} |T_{\mu\nu}\epsilon_{\mu\nu}|^2 = \frac{\pi^3 s^3(1 - x)^2}{2M_{10}^8} \cdot \frac{x \, d\cos \theta}{32\pi s} \cdot T_{\mu\nu} T_{\mu\nu}.$$  

The sum over the polarizations of the graviton has been performed using eq. (43) of \(^1\), omitting terms that vanish since $T_{\mu\nu}$ is a conserved traceless tensor.

A comparison of the graviton and branon cross sections gives eq. (3.2). The equivalence (3.2) does not hold for processes involving SM particles with non negligible mass like Higgs and Z decays.

\(^1\)Another missing energy signal \(^1\)[10] with the same energy dependence and with a different angular spectrum can arise in supersymmetric models from gravitino production, if supersymmetry is spontaneously broken at TeV energies.

\(^2\)In string models the tension of one 4-dimensional supersymmetric D brane moving in the simplest 6-dimensional compactifed space (a product of 6 circles with equal radii) is predicted to be $\tau = \tau_1 = \sqrt{\pi} M_{10}$. Therefore, using eq. (1.2), in this toy string model the emission of gravitons give effects 5 times larger than the emission of branons. In such models $n$ coinciding branes have tension $\tau_n = n\tau_1$, a gauge group U(n) localized on them, and $n^2$ branons. This gives the hope of obtaining a realistic gauge group. Branon effects get suppressed by the larger brane tension and enhanced by the larger number of branons. However $n^2 - 1$ of the $n^2$ branons are in the adjoint of the U(n) gauge group. No light scalar gluons or scalar $W_\pm$ bosons exist. These unwanted states become heavy if some force, maybe generated after supersymmetry breaking, keeps the $n$ branes together. The tension of the $n$ bound D-branes becomes $\tau_n = n\tau_1 -$ (some binding energy). In conclusion, the ‘prediction’ for the ratio between branon and graviton rates gets corrected by unknown model dependent order one factors.

\(^3\)The (uncomputable) virtual effects give a simple example of this fact. For example, let us compare $s$-channel exchange of gravitons and branons with transferred squared momentum $s$. Tree-level exchange of virtual gravitons in 6 extra dimensions gives the scattering amplitude \(^1\)

$$\mathcal{M} = ic_G(T_{\mu\nu} T_{\mu\nu}^{SM} - \frac{1}{8} T_{\mu\nu}^{SM} T_{\nu\nu}^{SM}) \cdot \left| c_G = -\frac{1}{M_{10}} \int^A \frac{d^6k_T}{s - k_T^2} \cdot \frac{\pi^3}{2M_{10}^8} \cdot \frac{\Lambda^4}{s^2 \ln(-s)} \cdot \ldots \right|$$  \hspace{0.5cm} (3.3a)

where the integral runs over transverse graviton momentum. One loop exchange of virtual branons gives rise to the scattering amplitude \(^2\)

$$\mathcal{M} = ic_Y(T_{\mu\nu} T_{\mu\nu}^{SM} - \frac{1}{2} T_{\mu\nu}^{SM} T_{\nu\nu}^{SM}) \cdot \left| c_Y = \frac{1}{f^8} \int_0^1 \frac{d^6k_T}{i(2\pi)^4} \cdot \frac{k^4/24}{|k^2 + sx(1 - x)|^2} \cdot \frac{5\Lambda^4 - s^2 \ln(-s)}{240f^8(4\pi)^2} \cdot \ldots \right|$$  \hspace{0.5cm} (3.3b)

where the integral runs over the loop momentum of the branons. In both cases we have cut-off divergent integrals in a naive way and we have shown only terms of order $\Lambda^4$ and $s^2 \ln(-s)$:
Up to which values of $\sqrt{s}/f$ can our computation be trusted? Higher order terms of the expansion in the brane tension become relevant when $\sqrt{s} \gtrsim (3 \div 6)f$ and give additional contributions to the missing energy signal, with 4, 6, 8, ... branons in the final state. As we will see, the missing energy signals emerge from the SM background in a similar range of $\sqrt{s}/f$ values. Furthermore, the Lagrangian (2.1), itself is only a non-renormalizable effective Lagrangian, whose validity is expected to break down at some unknown energy. This unknown energy must be smaller than about $\sqrt{s} \lesssim 4f$, otherwise the 2 body cross section $e\bar{e} \rightarrow YY$ in eq. (2.3) exceeds the unitarity bound $\sigma \lesssim 1/s$.

A more relevant question is: are branon signals a serious candidate for new physics? If $f$ is small (for example $f \sim M_Z$) branons are detectable and kill KK signals [3], including the ones due to gravitons. An optimal solution to the hierarchy problem suggests a sub-TeV value of the effective cutoff $\Lambda_{UV}$ for quadratically divergent corrections to the higgs mass, since in the SM

$$\delta m^2_h \approx \delta m^2_m\text{(top)} \approx +(0.3\Lambda_{UV})^2 \approx m^2_h \quad \text{at} \quad \Lambda_{UV} \approx 400\text{ GeV}.$$  

Even assuming that new physics conserve CP, $B$, $L$, $L_i$, $B_i$, the agreement of precision measurements at the Z pole with SM predictions requires that various dimensions 6 non-renormalizable operators (NRO) must be suppressed by a scale $\Lambda_{NRO}$ larger than $(5 \div 10)$ TeV [4].

One possibility is that all scales of new physics are comparable: if $f \approx M_d \approx \Lambda_{UV} \approx \Lambda_{NRO} > (5 \div 10)$ TeV there is no conflict with precision measurements but extra dimensions alone do not provide a complete solution to the hierarchy problem, since $\delta m^2_{h\text{(top)}} \approx 100m^2_h$. In this case graviton and branon missing energy signals cannot be detected, and only the lightest string (or whatever) states that interact with SM particles could provide a signal at LHC (either as NRO or via direct production).

If instead $f \approx M_d \approx \Lambda_{UV} < 1$ TeV extra dimensions really solve the hierarchy problem, graviton and branon missing energy signals can be detected, but we do not understand why precision measurements agree with SM predictions. To know if this is a motivated candidate for new physics would require a predictive theory of quantum gravity. String brane models with realistic gauge groups [15] could give some useful hint.

4 Signals at colliders

Since branons give missing energy signals similar to the gravitino and graviton ones studied in [14] [3] we facilitate a comparison by following these papers for what concerns tentative experimental cuts and relative backgrounds.

4.1 LEP1

The precision measurements done mostly at LEP1 set a lower bound on $f$ since branons lighter than $M_Z/2$ contribute to the width of the $Z$. Estimating their contribution as $\Gamma(Z \rightarrow fYY)/\Gamma(Z \rightarrow ff) \approx (M_Z/f)^8/[12(4\pi)^4]$, we obtain the bound $f \gtrsim M_Z/2$. Since branon effects strongly increase with energy, it is not surprising that this bound is stronger than the astrophysical bound from the 1987a supernova, $f \gtrsim 10$ GeV [8].

4.2 LEP2

For the same reason bounds on $e\bar{e} \rightarrow \gamma E$ at LEP2 with $\sqrt{s} = 200$ GeV induce a stronger bound on $f$ than LEP1. Assuming an integrated luminosity $L = 4 \cdot 500$ pb$^{-1}$, and performing the same cuts on the photon energy and direction as in [8], the branon signal exceeds the discovery cross section $\sigma_{\text{discovery}} = 5\sqrt{\sigma_{\text{bekn}}/L} = 0.17$ pb when $f < 100$ GeV. With optimized cuts, LEP collaborations have recently performed a search for missing energy

- The $s^2\ln(-s)$ terms are fixed by the low energy effective theory. We see that the equivalence (3.2) holds for such terms, but only if the energy-momentum tensor is traceless. These terms generate an attractive $1/r^8$ force between non relativistic particles [1] (in the case of gravitons this force of course is 10-dimensional gravity).
- The region of integration with $k^2, k^2 \gg s$ gives rise to effective dimension-8 operators. In both cases the coefficients of the operators cannot be computed from the low energy effective Lagrangian, because given by divergent integrals over transverse graviton momentum and over loop branon momentum, respectively.

Such operators could be of experimental interest. Searches for virtual graviton effects have been performed by the L3 collaboration [7] at LEP2 and by the D0 collaboration [3] at Fermilab. Using eqs. (2.3a,b) and assuming some value for the two $\Lambda$, the resulting ‘bound’ on $M_{10}$ could be directly converted into a ‘bound’ on $f$, since $T^{SM}_{\mu\nu}$ of the colliding particles is negligible.

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graviton signals, obtaining bounds on $\tilde{M}_d$. Using the equivalence (3.2) between branon and graviton signals, the OPAL bound, $\tilde{M}_d > 530$ GeV at 95% confidence level [16], can be rescaled to $f > 100$ GeV.

### 4.3 Electron colliders

To explore larger values of $f$ one needs colliders with higher center of mass energy. In fig. 1a we show, as function of $f$, the total branon $e\bar{e} \to \gamma E$ cross section at an $e\bar{e}$ collider with $\sqrt{s} = 500$ GeV integrated over $E_{T\gamma} > 60$ GeV and $E_{\gamma} < 160$ GeV. $E_{\gamma}$ is the photon energy, $E_{T\gamma} \equiv E_{\gamma} \sin \theta$ is the transverse photon energy and $E$ is the missing energy, carried away by the branons. The cut on $E_{\gamma}$ suppresses the $e\bar{e} \to \gamma Z \to \gamma \nu \bar{\nu}$ SM background. The dot-dashed line represents the background arising from the SM process $e\bar{e} \to \gamma \nu \bar{\nu}$, computed with the program COMPHEP [17]. We see that the signal emerges from the SM background when $0.3\sqrt{s} \gtrsim f$. At such energies the effective Lagrangian approximation becomes questionable. A polarization of the electron beams would reduce the background without affecting the signal [19]. Depending on the acceleration technique, it seems possible to obtain a 80% polarization [18]. In fig. 1b we show the missing energy cross section, integrated over $E_{\gamma} < 450$ GeV and $E_{T\gamma} > E_{T\gamma}^{\text{min}}$ as function of $E_{T\gamma}^{\text{min}}$ for $f = 160$ GeV and for $f = 180$ GeV. The dot-dashed line represents the SM background.

Fig. 2 is analogous to fig. 1, but with a higher collider energy: $\sqrt{s} = 1$ TeV. The cuts have been accordingly modified to $E_{\gamma} < 450$ GeV and $E_{T\gamma} > 100$ GeV.

### 4.4 LHC

Next, we study the discovery potential of the approved LHC $pp$ collider with $\sqrt{s} = 14$ TeV. The most promising signal is $pp \to \text{jet} E_T$ (mostly produced by $qg \to qYY$). A less promising signal is $pp \to \gamma E_T$ (produced in $q\bar{q}$ collisions). The total signal cross sections for these two processes are plotted as function of $f$ and compared with SM backgrounds in fig. 3a and 3b (continuous lines). The cuts on the transverse energy $E_T$ and on the pseudorapidity $\eta$ of the jet (fig. 3a) and of the photon (fig. 3b) are specified in the caption. The dashed lines show the signal produced including only the contribution from collisions with partonic center of mass energy $\sqrt{s} < 2\pi f$. As discussed in the previous section, the discrepancy between the continuous and dashed lines indicates the breakdown of our effective Lagrangian approximation. There is no good reason for choosing $2\pi f$ rather than
another comparable number. The trustable signal is rapidly reduced below the background if the cut on $\hat{s}$ is lowered.

Almost all the signal is produced by partons that carry (20% $\div$ 60%) of the proton momentum. We have used the partonic distribution functions of \cite{19}. With a different choice of partonic distributions, the signal can vary by few 10%.

Before LHC, a $p\bar{p} \to \text{jet}$ $E_T$ signal can be searched at the upgraded Tevatron collider. However a positive evidence can emerge from the SM background only if $f$ is just above the range excluded by LEP2.

5 Conclusions

If we really live on a non rigid brane in $\delta$ flat extra dimensions, the scalar fields that control the coordinates of our brane in the extra dimensions have low energy interactions with SM particles suppressed by the tension $\tau$ of the brane and described by a predictive effective non-renormalizable Lagrangian. We have computed the missing energy signals produced by these ‘branon’ fields and studied their detectability at high energy colliders. These signals depend only on one parameter $f \equiv \tau^{1/4}/\delta^{1/8}$. LEP2 gives the strongest bound on it: $f > 100$ GeV. Branons produce a jet + missing energy signal detectable at LHC if $f \lesssim 900$ GeV. However, the angular and energy spectrum of the missing energy produced by brane fluctuations in any number of extra dimensions is identical to the missing energy signal produced by graviton emission in $\delta = 6$ extra dimensions. The two signals have the same magnitude if the brane tension $\tau$ and the 10-dimensional reduced Planck mass are related by $\tau = \sqrt{\pi \delta/30} M_{10}^2$. In particular, in both cases the signal rises as the 6th power of the collision center of mass energy: therefore the energy at which it becomes larger than the SM background is close to the energy at which the effective Lagrangian approximation becomes questionable.

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Figure 3: The continuous lines show the total branon cross section for (3a) $pp \rightarrow \text{jet} + \not{E_T}$ with $E_{T\text{jet}} > 1 \text{ TeV}$, $|\eta_{\text{jet}}| < 3$ and (3b) $pp \rightarrow \gamma + \not{E_T}$ with $E_{T\gamma} > 0.4 \text{ TeV}$, $|\eta_\gamma| < 2.5$ at LHC with $s = (14 \text{ TeV})^2$ as function of $f$. The dashed line include only the contribution from partonic collisions with $\sqrt{s} < 2\pi f$. The horizontal dot-dashed line represents the SM background.

Figure 4: The Feynman graphs that contribute to $e\bar{e} \rightarrow \gamma Y Y$.

**A Computation of the cross sections**

When computing cross sections for processes like $e(p_1)\bar{e}(p_2) \rightarrow \gamma(q)Y(k_1)Y(k_2)$ it is useful to decompose the three body phase space as

$$d\phi^{(3)}(P \rightarrow q + k_1 + k_2) = \frac{2q_0(2\pi)^3}{2q_0(2\pi)^3}d\phi^{(2)}(k \rightarrow k_1 + k_2), \quad P \equiv p_1 + p_2, \quad k \equiv k_1 + k_2. \quad (A.1)$$

Integrals over the two body phase space of the couple of massless branons can be easily performed using

$$\int d\phi^{(2)} k_1\mu k_1\nu k_2\rho k_2\sigma = \frac{k_\mu k_\nu k_\rho k_\sigma}{240\pi} + \frac{k^4}{1920\pi}(\eta_{\mu\nu}\eta_{\rho\sigma} + \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}) + \frac{k^2}{320\pi}(k_\mu k_\rho\eta_{\eta\sigma} + k_\rho k_\sigma\eta_{\mu\nu}) + \frac{k^2}{480\pi}(k_\mu k_\rho\eta_{\mu\sigma} + k_\rho k_\sigma\eta_{\mu\nu} + k_\mu k_\sigma\eta_{\mu\rho} + k_\rho k_\sigma\eta_{\mu\nu}). \quad (A.2)$$

We now list the differential cross sections for all processes (E.1). All cross sections are averaged over spins of initial particles and summed over spins of final particles.

1. The $e\bar{e} \rightarrow \gamma YY$ (or $\mu\bar{\mu} \rightarrow \gamma YY$) cross section is

$$\frac{d\sigma(e\bar{e} \rightarrow \gamma YY)}{dx \, d\cos \theta} = \sigma_0 \frac{\alpha_{\text{em}}}{4\pi} F_{e\bar{e}}, \quad \text{where} \quad \sigma_0 \equiv \sigma(e\bar{e} \rightarrow YY) = \frac{s^3}{30720\pi f^8}. \quad (A.3)$$
The function $F_{ee}$ is given at the end of the appendix in terms of the photon energy $E_\gamma = x\sqrt{s}/2$ and of the photon direction with respect to beam axis in the center of mass frame. The relevant Feynman diagrams are shown in fig. 3.

2. The $q\bar{q} \to \gamma YY$ and $q\bar{q} \to gYY$ cross sections are

$$\frac{d\sigma(q\bar{q} \to \gamma YY)}{dt \, d(k^2)} = \sigma_0 q^2_\gamma \omega_m \frac{1}{12\pi} F_{q\bar{q}}, \quad \frac{d\sigma(q\bar{q} \to GYY)}{dt \, d(k^2)} = \sigma_0 \frac{4}{3} \frac{1}{12\pi} F_{q\bar{q}}$$

where $q_\gamma$ is the electric charge of the colliding quark. The function $F_{q\bar{q}}$ (related in a simple way to $F_{ee}$) and all functions relative to parton processes are given at the end of the appendix in terms of the invariants $s = (p_1 + p_2)^2$, $t = (p_1 - q)^2$, $k^2 = (k_1 + k_2)^2$ and $u = (p_2 - q)^2 = k^2 - s - t$.

3. The cross sections involving gluons $g$ in the initial state are given by

$$\frac{d\sigma(qg \to qY Y)}{dt \, d(k^2)} = \frac{d\sigma(qg \to \bar{q}gY)}{dt \, d(k^2)} = \frac{\alpha_3}{4\pi} F_{gg}, \quad \frac{d\sigma(gg \to Y Y)}{dt \, d(k^2)} = \frac{\alpha_3}{4\pi} F_{gg}$$

The $F$ functions are

$$F_{ee} = 4(1 - x)^2 \left[ \frac{x^3}{2} (3 - 3x + 2x^2) - \frac{x^3}{2} \sin^2 \theta + \frac{(1 - x)(1 + (1 - x)^2)}{x \sin^2 \theta} \right]$$

(A.4a)

$$F_{q\bar{q}} = 2k^4(k^4 + s^2 - 2k^2t + 2t(s + t))(k^2(s + 4t - 4t(s + t))/s^4tu)$$

(A.4b)

$$F_{gg} = -k^4(2k^4 + s^2 + t^2 - 2k^2(s + t))(k^4 + 4st - k^2(s + t))/(s^4tu)$$

(A.4c)

$$F_{gg} = 6k^4(k^8 - 2k^6(s + t) + 3k^4(s^2 + t^2) + (s^2 + st + t^2)^2 - 2k^2(s^3 + t^3))/(s^6tu)$$

(A.4d)

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