Holographic superconductor in a deformed four-dimensional STU model

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ABSTRACT: In this paper, we consider deformed STU model in four dimensions including both electric and magnetic charges. Using AdS/CFT we study holographic superconductor and obtain transport properties. We find that presence of magnetic charge is necessary to have maximum electrical conductivity. Also we show that thermal conductivity increases with magnetic charge.

KEYWORDS: Superconductor; Black hole; Holography.
1 Introduction

AdS/CFT correspondence, which relate a conformal field theory (CFT) in d-dimensional and a string theory in (d + 1)-dimensional anti-de Sitter (AdS) space, is a powerful mathematical tool to study strongly correlated systems \cite{1–3}. The extra dimension in the AdS side can be understood as the energy scale of the CFT on the boundary. AdS/CFT correspondence already used to study quark-gluon plasma (QGP) \cite{4, 5}, and it is possible to study nuclear force using AdS/CFT correspondence \cite{6}. An interesting application of AdS/CFT is in condensed matter physics known as AdS/CMT which claim that there is a dual gravitational description of superconductivity \cite{7, 8}. In order to review some important aspects of the theory of holographic superconductors see Refs. \cite{9, 10}.

Superconductivity is a phenomenon that has a history of about 100 years. Superconductivity is a state of a material in which \( \rho = 0 \) (disappearance of resistivity). It occurs only below a certain temperature \( T_C \), below a certain current level \( J_C \), and below a certain magnetic field \( H_C \). Superconductivity was first discovered by Onnes in 1911 for mercury. Later, many researches has been done, theoretically and scientifically. The goal of the scientists was to justify this phenomenon by condensed matter and many-body models. From these kinds of models, we have "Fritz and Heinz" two fluid model (1935), phenomenological theories" (QM) were proposed to explain superconductivity by Ginzburg and Landau and (1950) and cooper’s BCS (Bardeen-Cooper-Schrieffer) pair model \cite{11}. But cooper’s theory can’t justify superconductivity in higher temperatures \cite{12, 13}. On the other side, scientific works in this field are done in the basis of increasing the superconductivity transition temperature. In these days, scientists have reached a transition temperature of about 140K. But reaching higher temperatures is a great challenge for researchers. The present work is...
done on the basis of a computing and analytical gravitation method and discusses the relation between magnetic monopole model and superconductivity. Heavy monopole initially suggested by 't Hooft and Polyakov in the framework of SU(2) gauge theories [14–16]. Our model tries to link the superconductivity to magnetic monopole charge carriers mobility which have major effect on thermal and superconductive properties.

As we know, ordinary superconductors are well described by microscopic theory of superconductivity known as BCS theory [11]. But, unusual superconductors including the pairing mechanism, is not good understood using BCS theory and one need another theoretical model of a strongly coupled system like AdS/CFT correspondence. According to the AdS/CFT dictionary, a black hole and charged scalar field are holographic dual of temperature and condensate of a superconductor respectively [7]. In order to reproduce the superconductor phase diagram, one can consider a black hole with scalar hair at low temperature. STU black hole is an important model with both electric and magnetic charges [17]. The STU model is just some $N = 2$ supergravity [18, 19]. It generally involves 8 charges (4 electric and 4 magnetic). There is complete STU symmetry, so the "4" have "3" charges that are on the same footing, while the last has different couplings. The STU model can be interpreted in string theory by embedding. There are different ways to do this but the simplest, in type IIA on $T^2 \times T^2 \times T^2$, interprets the 8 charges as D0, D2 (3 of these, one for each $T^2$), D4 (3 of these as well) and D6. Special case of STU model in 5 dimensions with three electric charges which admits a chemical potential for the $U(1)^3$ symmetry already used to study QGP which is called STU/QCD correspondence [20–24]. Also $D = 5, N = 2$ STU model considered as dual picture of superfluid [25].

Motivated by the evidence of superconductive detector for moving magnetic monopoles [26], we would like to investigate effect of magnetic charge on the conductivities. Special case of STU model in five dimensions already considered to study transport properties of superconductor [27]. In that case, various kinds of STU model are important from AdS/CFT point of view and statistical analysis [28], because it is extension of Yang-Mills theory to the case of having chemical potential. Recently, a deformation of the $N = 2, D = 4$ STU model, characterized by a non-homogeneous special Kähler manifold, considered to solve BPS attractor equations and to construct static supersymmetric black holes with radial symmetry [17]. In that case the relevant physical properties of the resulting black hole solution are explained and one can see that it is four charged STU model, three electric and one magnetic charges. So we consider $N = 2, D = 4$ STU model as dual picture of a superconductor and investigate effect of magnetic charge on the transport properties. This paper is organized as follows. In the next section we review holographic superconductor and recall basic equations. Then, in section 3 we consider non-homogeneous STU black hole in four dimensions and write important properties of the model. In section 4 we discuss briefly about the thermodynamics of the model and calculate some useful quantities to study conductivities. In section 5 we study electrical end thermal conductivities and discuss about the effect of magnetic charge on them. Finally in section 6 we give conclusion and summary of results together outlook of future works.
2 Holographic superconductor

It has been shown that a gravitational background may considered as holographically dual picture of a superconductor [7]. Perturbation equation help us to obtain transport coefficients from the general Lagrangian of the form,

\[ L \equiv \frac{\mathcal{L}}{\sqrt{-g}} = R - \frac{1}{4} G_{ij} F_{\mu\nu}^i F^{\mu\nu j} + \cdots , \]  

(2.1)

where \( R \) stands for Ricci scalar and dotes denote scalar field and Chern-Simons terms, and also \( F_{\mu\nu} \) is the field-strength tensor. Induced metric \( G_{ij} \) related to the background metric \( g_{ij} \) with determinant \( g \) and introduced later. In the Ref. [27], electrical and thermal conductivities of R-charged black hole in 4, 5, and 7 dimensions calculated for the general D-dimensional space-time,

\[ ds^2 = g_{tt} dt^2 + g_{rr} dr^2 + g_{xx} d\Omega^2_{D-2} , \]  

(2.2)

where \( d\Omega^2 \) is (D-2)-dimensional space. We will use results of the Ref. [27] to study transport properties of superconductor via a non-homogeneous deformation of R-charged black hole (STU black hole) in four dimensions. Perturbation equation for the gauge field of general case can be write as follow,

\[ \frac{d}{dr} \left( N_i \frac{d}{dr} \phi_i(r) \right) - \omega^2 N_i g_{rr} g^{rt} \phi_i(r) + \sum_{j=1}^{m} M_{ij} \phi_j(r) = 0 , \]  

(2.3)

where

\[ N_i = \sqrt{-g} g^{xx} g^{rr} , \]  

(2.4)

and

\[ M_{ij} = F_{rt}^i \sqrt{-g} g^{xx} g^{rr} G_{jj} F_{rt}^j , \]  

(2.5)

with the condition \( M_{ij} = M_{ji} \). \( \phi_i \) related to the \( x \)-component of the gauge field via,

\[ A_x^i = \frac{\mu^i}{2} \Phi_i(r) e^{i(qz - \omega t)} , \quad \phi_i(r) = \mu^i \Phi_i(r) , \]  

(2.6)

with \( \omega \) and \( q \) represent the frequency and momentum in \( z \) direction, respectively, and \( \mu^i \) denotes the chemical potential. It should be noted that the last term of the equation (2.3) comes from the metric perturbation part which interpreted as interaction between different gauge fields, so it will be vanish for the case of non-interacting fields. Also the second term of the equation (2.3) vanishes for the case of low frequency limit (\( \omega^2 \to 0 \)).

The diagonal and off diagonal components of conductivity given by the following expressions respectively,

\[ \sigma_{ii} = \frac{1}{8\pi G_D} \left[ \sqrt{g_{rr}} \sum_{k=1}^{m} \frac{N_k(r) \phi_k(r, \omega) \phi_k(r, -\omega)}{(\phi_i)^0(\phi_i)^0} \right]_{r=r_+} , \]  

(2.7)

and

\[ \sigma_{ij} = \frac{1}{16\pi G_D} \left[ \sqrt{g_{rr}} \sum_{k=1}^{m} \frac{N_k(r) \phi_k(r, \omega) \phi_k(r, -\omega)}{(\phi_i)^0(\phi_j)^0} \right]_{r=r_+} , \]  

(2.8)
where \( r_+ \) is the horizon radius, and \( G_D \) is \( D \)-dimensional Newtonian constant. We will use above relations to study electrical conductivity of a superconductor dual of deformed STU model in four dimensions.

On the other hand, coefficient of thermal conductivity given by,

\[
\kappa_T = \left( \frac{\epsilon + P}{T} \right)^2 \frac{1}{\sum_{i,j=1}^{m} \rho_i (\frac{G}{G})_{ij}^{-1} \rho_j},
\]

where \( \epsilon \) and \( P \) are the local energy density and pressure respectively. Also, \( \rho_i \) are charge density satisfy the following thermodynamics equations,

\[
\epsilon + P = Ts + \sum_{i=1}^{m} \mu_i \rho_i,
\]

and

\[
d\epsilon = T ds + \sum_{i=1}^{m} \mu_i d\rho_i,
\]

where \( s \) and \( \mu_i \) are entropy and chemical potential respectively.

### 3 Non-homogeneous STU black hole in four dimensions

Non-homogeneous deformation of the STU model of \( N = 2, D = 4 \) gauged supergravity has been studied by the Ref. [17], and we review some important properties of the model which will be useful to study holographic superconductor. The general form of the metric given by,

\[
ds^2 = -U(r)dt^2 + \frac{dr^2 + \psi(r)d\Omega^2_2}{U(r)},
\]

where \( d\Omega^2_2 \) is the two-dimensional metric of surface. Determinant of above line element is,

\[
\sqrt{-g} = \frac{\psi(r)}{U(r)} f_k,
\]

with \( k = 0, \pm 1 \), where \( f_1 = \sin \theta \), \( f_0 = \theta \) and \( f_{-1} = \sinh \theta \), which are corresponding to closed, flat and open universes.

The bosonic Lagrangian given by [17],

\[
L = R - g_{ij} \partial_\mu z^i \partial^\mu z^j - \frac{1}{4} \mathcal{I}_{\Lambda \Sigma} F^\Lambda_\mu F^{\mu \Sigma} + \cdots,
\]

where

\[
\mathcal{I}_{\Lambda \Sigma} = \frac{1}{4} e^{-\kappa} \begin{pmatrix} 1 & 0 \\ 0 & 4g_{ij} \end{pmatrix},
\]

with

\[
g_{ij} = \frac{1}{2(\lambda^1 \lambda^2 \lambda^3 - \frac{4}{3}(\lambda^3)^2)^2} \begin{pmatrix}
(\lambda^2)^2(\lambda^3)^2 & \frac{4}{3}(\lambda^3)^4 & -\frac{2A}{3} \lambda^2(\lambda^3)^3 \\
\frac{4}{3}(\lambda^3)^4 & (\lambda^1)^2(\lambda^3)^2 & -\frac{2A}{3} \lambda^1(\lambda^3)^3 \\
-\frac{2A}{3} \lambda^2(\lambda^3)^3 & -\frac{2A}{3} \lambda^1(\lambda^3)^3 & (\lambda^1)^2(\lambda^2)^2 + \frac{A^2}{3}(\lambda^3)^4
\end{pmatrix},
\]
and
\[ e^{-K} = 8 \left( \lambda^1 \lambda^2 \lambda^3 - \frac{A}{3} \lambda^3 \right), \]  
(3.6)
where \( A \) is deformation parameter which may be fix as unit, so \( A = 0 \) give us ordinary STU model. The superpotential specified by the dyonic Fayet-Iliopoulos (FI) gauging with FI parameters \( (g^A, g_A) \). Also, in this model both magnetic \( (p^A) \) and electric \( (q^A) \) charges are possible. We will consider only the following non-zero charges, \( p^0, q_1, q_2 \) and \( q_3 \) which means that we have one magnetic and three electric charges. This choice requires \( (g_0 p^0 - g^i q_i = -k, \text{where} \ k = 0, \pm 1) \) that some FI parameters vanish, so we have only \( g^1, g^2, g^3 \) and \( g_0 \) which interpreted as gauge coupling constant (all positive) and related to the scalar fields \( \lambda^1, \lambda^2 \) and \( \lambda^3 \) which will mention later. In that case, full black hole solution of the non-homogeneous STU model given by,
\[ \psi(R) = (ar^2 - c)^2, \]  
(3.7)
and
\[ U(r) = \frac{2g_0 g^3 (ar^2 - c)^2}{\lambda^3_{\infty} (ar - g_0 \beta^0 - \frac{g_0 \beta_0}{\lambda^3_{\infty}})^2 \sqrt{(ar + 2g_0 \beta^0)(ar + 2g_0 \beta^0 \beta^3)}}, \]  
(3.8)
where \( a \) and \( c \) are positive constant and
\[ \lambda^3_{\infty} = \sqrt{\frac{g_0 g^3}{g^1 g^2 - \frac{A}{3} (g^3)^2}}, \]  
(3.9)
is asymptotic value of \( \lambda^3 \). Also, \( \beta^0 \) and \( \beta_i \) with \( i = 1, 2, 3 \), are constants [29]. It is found that,
\[ \lambda^1 = \frac{a g^1 (\lambda^3_{\infty})^2 r - g_0 \beta_3 (\frac{a}{g^3} - \frac{A (\lambda^3_{\infty})^2}{g^2}) - \beta_0 (q_3)^2}{\sqrt{(2g_0 \beta^0 + ar)(2g_0 \beta^0 + ar (\lambda^3_{\infty})^2)}}, \]
\[ \lambda^2 = \frac{g^2}{g^3} \lambda^3, \]
\[ \lambda^3 = \lambda^3_{\infty} \sqrt{\frac{ar + 2g_0 \beta^0 (\lambda^3_{\infty})^2}{ar + 2g_0 \beta^0}}, \]  
(3.10)
In that case all quantities described by \( \beta^0, \beta_i, a, c, p^0 \) and \( q_i \).
Black hole horizon given by
\[ r_+ = \sqrt{\frac{c}{a}}. \]  
(3.11)
Now, we can discuss about the perturbation equation (2.3) and obtain \( \phi_i \) with \( i = 0, 1, 2, 3 \) corresponding to one magnetic and three electric charges of black hole. Using the equation
it is easy to find that,

\[ N_0 = 2f_k \left( \lambda^1 \lambda^2 \lambda^3 - \frac{A}{3} (\lambda^3)^3 \right) U(r), \]

\[ N_1 = \frac{4f_k (\lambda^3)^2 (\lambda^3)^2}{(\lambda^1 \lambda^2 \lambda^3 - \frac{A}{3} (\lambda^3)^3)} U(r), \]

\[ N_2 = \frac{4f_k (\lambda^1)^2 (\lambda^3)^2}{(\lambda^1 \lambda^2 \lambda^3 - \frac{A}{3} (\lambda^3)^3)} U(r), \]

\[ N_3 = \frac{4f_k \left( (\lambda^1)^2 (\lambda^3)^2 + \frac{4}{3} (\lambda^3)^4 \right)}{(\lambda^1 \lambda^2 \lambda^3 - \frac{A}{3} (\lambda^3)^3)} U(r), \]

(3.12)

where \( U(r) \) given by the equation (3.8). Then, non-interacting version of the equation (2.3) reads as,

\[ \frac{d}{dr} \left( N_i \frac{d}{dr} \phi_i(r) \right) + \omega^2 N_i \frac{\phi_i(r)}{U(r)} = 0. \]  

(3.13)

So, we have four different equation to obtain \( \phi_0(r), \phi_1(r), \phi_2(r), \) and \( \phi_3(r) \). In the simplest case of low frequency we can neglect \( \omega^2 \). In that case one can obtain,

\[ \phi_i(r) = C_1 \int N_i^{-1} dr + C_2, \]  

(3.14)

where \( C_1 \) and \( C_2 \) are integration constants.

Numerically, we solve equation (3.13) and obtain behavior of \( \phi_i \) with respect to \( r \) (\( r < r_+ \)) as illustrated by the plots of Fig. 1. We find that presence of all FI parameters are necessary to obtain finite and real value of scalar fields. It is illustrated that scalar fields asymptotically yields to infinity near the black hole horizon (\( r_+ = 1 \) for \( a = c = 1 \)). We have shown that \( \phi_2 \geq \phi_0 \geq \phi_3 > \phi_1 \). It is clear that \( \phi_i \) are increasing function of \( r \) inside black hole. Behavior of scalar fields are in agreement with the results of the Ref. [27] in the region of \( r < r_+ \). We also find that effect of deformation if decreasing value of scalar field. It means that for the bigger \( A \) we have smaller \( \phi_i \).

It is desirable that scalar field should be well-defined outside horizon with positive value. At the low frequency limit, one can use the equation (3.14) and obtain asymptotic values of the scalar fields for the large \( r \) to find behavior of scalar field as illustrated by the Fig. 2. We obtain plots corresponding to \( A = 1 \) but situation is similar for \( A = -1 \) which used in the Ref. [27]. We will show later that, negative value is suitable for the deformation parameter \( A \) as used in the Ref. [27]. So the boundary value of the scalar field \( (\phi_i)0 \) is a constant (may be zero or one), for example \( (\phi_i)0 \approx 1 \) for the given value of model parameters as Fig. 1 and Fig. 2.

In agreement with the Ref. [27] and coincide with both large and small \( r \) we can propose the following function for the scalar field,

\[ \phi_i(r) = \frac{a_i + b_i r}{c_i + d_i r}, \]  

(3.15)

with \( a_i < 0, b_i < 0, c_i < 0 \) and \( d_i > 0 \). It is indeed corresponding to \( C_2 = 0 \) of the solution (3.14). However we can consider it as general behavior of the scalar field satisfying the differential equation (3.13) for non-interacting case with \( \omega^2 \to 0 \).
Figure 1. Typical behavior of scalar fields in terms of $r$ ($r \leq r_+$) with initial values $\phi(0) = 0$ and $\phi'(0) = 1$. All model parameters set one ($a = c = A = g_0 = g^1 = g^2 = g^3 = \beta^0 = \beta_3 = f_k = 1$). $\omega = 1$ (dashed blue), $\omega = 0.5$ (dotted green), $\omega = 1$ (solid red).

Figure 2. Typical behavior of scalar fields in terms of $r$ ($r \geq r_+$) for $\omega^2 = 0$. All model parameters set one ($a = c = A = g_0 = g^1 = g^2 = g^3 = \beta^0 = \beta_3 = f_k = C_1 = C_2 = 1$). Solid lines denote first order approximation while dashed lines represent second order approximation.
4 Thermodynamics

Hawking temperature of black hole $T = \frac{1}{4\pi} U'(r_+)$ indicated that our black hole is in zero temperature limit. However near the horizon we have infinitesimal temperature which is decreasing function of $r$, and the black hole entropy given by [27],

$$s = \frac{\pi A}{3 p^0},$$

(4.1)

where the black hole horizon area given by,

$$A = 2 \left( -p^0 q_3 \left[ ( -p^0 q_3 )^2 + 12 (p^0)^2 q_1 q_2 \right] + \left[ ( -p^0 q_3 )^2 - 4 (p^0)^2 q_1 q_2 \right]^2 \right),$$

(4.2)

with,

$$p^0 = \frac{ac}{2g_0} - 2g_0 (\beta^0)^2,$$

(4.3)

as magnetic charge and,

$$q_1 = 2(\beta_3)^2 g^2 \left( g^1 g^2 - \frac{A}{3} (g^3)^2 \right) - g^2 \frac{ac}{2(g^1 g^2 - \frac{4}{3}(g^3)^2)},$$

$$q_2 = \frac{1}{2g^2} (\beta_0 g_0 + \beta_3 g^1 g^2 g^3)^2 - g^1 \frac{ac}{2(g^1 g^2 - \frac{4}{3}(g^3)^2)}$$

$$+ \frac{A}{3} \beta_3 \frac{g^3}{g^2} \left( g^3 (g^1 g^2 g^3 - \beta^0 g_0 - \frac{A}{2} \beta_3 g^3) \right),$$

$$q_3 = \frac{g^2}{g^3} q_2 - A \frac{g^3}{g^2} q_1,$$

(4.4)

as electric charges.

In the Fig. 3 we show values of magnetic and electric charges for the case of $a = c = \beta^0$ corresponding to $r_+ = 1$. According to the Fig. 3 (a) it is clear that infinitesimal $g_0$ yields to large positive value for the magnetic charge while large value of $g_0$ yields to large negative value for the magnetic charge. It is clear that magnetic charge vanishes for $g_0 = \pm \sqrt{ac}$. As initial assumption FI parameters all are positive, so positive sign is acceptable and $g_0 = \frac{\sqrt{ac}}{2\beta^0}$ is corresponding to zero magnetic charge. In the case of $a = c = \beta^0$ we have $g_0 = 0.5$ where $p^0 = 0$. In the case of $g_0 = 0.1$ we have $p^0 = 5$.

In the Fig. 3 (b) we draw $q_1$ as a function of deformation parameter $A$ because $q_1$ is not depend on $g_0$. Minimum value of $q_1$ obtained for $A \approx 1.5$ (positive charge) and $A \approx 4.5$ (negative charge). In the case of $A = -1$ we have $q_1 = 3$ and $A = -2$ we have $q_1 \approx 3.5$. For the negative $A$ we have positive $q_1$ as well as $0 \leq A < 3$, while for the $A > 3$ we have negative $q_1$.

Again in the Fig. 3 (c) we draw $q_2$ in terms of $g_0$ and see that $g_0 < 0.3$ yields to negative electric charge while for the case of $g_0 > 0.3$ we have positive $q_2$ which is increasing function of $g_0$.

In the Fig. 3 (d) we can see that $q_3$ is totally positive for positive $g_0$ and it is increasing function of $g_0$. In the case of $g_0 = 0.1$ we have $q_3 = 6.9$. 

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Figure 3. Magnetic and electric charges in terms of $g_0$ for $a = c = g^1 = g^2 = g^3 = \beta^0 = \beta_3 = 1$ and $A = -2$.

Figure 4. Typical behavior of entropy in terms of $g_0$ for $\omega^2 = 0, a = c = g^1 = g^2 = g^3 = \beta^0 = \beta_3 = 1$. $A = -2$ Solid, $A = -3$ dotted, $A = -4$ dashed.

In the Fig. 4 we draw entropy (4.1) and see that it is decreasing function of $g_0$ and yields to constant value for large $g_0$. For the selected value of model parameter we should choose negative $A$ to have positive entropy. Because the black hole is in zero temperature, hence specific heat is also zero so black hole is in the stable phase. Therefore, we investigate thermodynamics quantities near the horizon. Temperature of the black hole in terms of $r$ drawn in the Fig. 5. We can see that black hole temperature is zero at $r = r_+ = 1$ as expected and it is decreasing function near the horizon. For the small
Figure 5. Typical behavior of temperature in terms of $r$ for $\omega^2 = 0, a = c = g^1 = g^2 = g^3 = \beta^0 = \beta_3 = 1, A = -2$. $g_0 = 0.1$ blue dot, $g_0 = 0.2$ cyan dash dot, $g_0 = 0.5$ (zero magnetic charge) red solid, $g_0 = 0.8$ green dash, $g_0 = 1$ orange long dash.

value of $g_0$ (large positive value of magnetic charge), the black hole temperature is totally decreasing function of $r$. For the larger value of $g_0$, the black hole temperature increases first to a maximum value, then decreases to zero at the black hole horizon. Zero magnetic charge case represented by solid red line of the Fig. 5.

5 Conductivities

In this section, as main purpose of the paper, we try to obtain electrical and thermal conductivities. In order to obtain electrical conductivity we assume low frequency limit ($\omega^2 \to 0$) and use the metric (3.1) and scalar field (3.15) in the relations (2.7) and (2.8).

5.1 Electrical conductivity

Assuming $(\phi_i)^0 = (\phi_j)^0$ give us similar diagonal and off diagonal conductivity components, hence $\sigma = \sigma_{ii} = \sigma_{ij}$. In the unit of $8\pi G = 1$ and $m = 1$ we can obtain near horizon behavior of electrical conductivity components, graphically, as illustrated by plots of the Fig. 6 (a). It is clear that electrical conductivity increases with $r$, hence decreased with temperature. It means that we have high conductivity at low temperature as expected. Also, in the Fig. 6 (b) we can find effect of magnetic charge on the electrical conductivity near the horizon. In the case of $A = -2$, it has been shown that maximum conductivity given by $g_0 \approx 0.35$ which means $p^0 \approx 0.5, q_1 \approx 3.5, q_2 \approx 0.1$ and $q_3 \approx 7.4$ (see Fig. 3). It means that, in order to have maximum conductivity, presence of magnetic charge is necessary. However, decreasing or increasing magnetic charge decreases value of electrical conductivity. It means that there is a critical magnetic charge where the maximum of conductivity exist. However, deformation parameter is also important quantity in conductivity. In the Fig. 7 (a) we draw electrical conductivity for small negative deformation parameter and find that superconductivity happen for the large positive value of magnetic charge. It means
that for the infinitesimal value of deformation parameter, superconductivity is due to large value of magnetic charge. It means that superconductivity enhanced near horizon due to the magnetic charge for the case of ordinary STU model \( (A = 0) \). In this case \( (A = 0) \) we find special cases of superconductivity (see Fig. 7 (b)) with \( g_0 \approx 0.0036 \) where \( q_2 = q_3 \approx 1 \) \( (q_1 = 2.5) \) and \( p_0 \approx 137 \). In that case \( q_2 \) and \( q_3 \) interpreted as electron charge so value of magnetic charge is about 137 electron charge.

\[
\begin{align*}
\text{Figure 6. Typical behavior of electrical conductivity for } & \omega^2 = 0, a = c = g^1 = g^2 = g^3 = \beta^0 = \\
& \beta_3 = f_k = 1, \text{ and } A = -2. (a) \text{ in terms of } r \text{ with } g_0 = 0.1 \text{ blue dot, } g_0 = 0.2 \text{ cyan dash dot, } g_0 = 0.5 \text{ (zero magnetic charge) red solid, } g_0 = 0.8 \text{ green dash, } g_0 = 1 \text{ orange long dash.} (b) \text{ in terms of } g_0.
\end{align*}
\]

\[
\begin{align*}
\text{Figure 7. Typical behavior of electrical conductivity for } & \omega^2 = 0, a = c = g^1 = g^2 = g^3 = \beta^0 = \\
& \beta_3 = f_k = 1, \text{ (a) in terms of } r \text{ with } A = -0.01 \text{ and } g_0 = 0.001 \text{ blue dot, } g_0 = 0.01 \text{ cyan dash dot, } g_0 = 0.1 \text{ green dash, } g_0 = 0.5 \text{ (zero magnetic charge) red solid. } r = 1 \text{ denote location of the black hole horizon.} (b) \text{ in terms of } g_0 \text{ with } A = 0 \text{ and } r = 1.
\end{align*}
\]

### 5.2 Thermal conductivity

In order to obtain thermal conductivity using the equation (2.9) we use the fact that chemical potential of STU black hole is proportional to the temperature \( \mu \propto T \) [30], also we assume that \( iT \approx \omega \), because \( T \ll 1 \) and \( \omega^2 \ll 1 \). In that case we can rewrite thermal
conductivity as follow,

\[ \kappa_T = \frac{s^2 + \left( \sum_{i=0}^{3} \rho_i \right)^2}{\sum_{i,j=0}^{3} \rho_i \left( G_{ij}(\omega) \right)^{-1} \rho_j}, \]  

(5.1)

where the equation (2.10) is used. In the Fig. 8 we draw thermal conductivity in terms of \( r \) (Fig. 8 (a)) and in terms of \( g_0 \) (Fig. 8 (b)). We find that deformation parameter \( A \) should be negative to have real thermal conductivity. From the Fig. 8 (a) we can see that thermal conductivity is proportional to the temperature. It means that increasing \( r \) (decreasing temperature) reduces value of the thermal conductivity as expected. Effect of magnetic charge on the thermal conductivity is strange. In both cases of large positive and small positive magnetic charge there is no thermal conductivity. On the other hand thermal conductivity increased dramatically for the case of large negative magnetic charge.

**Figure 8.** Thermal conductivity in terms of \( r \) (a) and \( g_0 \) (b) for \( \omega^2 = 0, a = c = g^1 = g^2 = g^3 = \beta^0 = \beta_3 = f_k = 1, A = -2 \). (a) \( g_0 = 0.6 \) red solid, \( g_0 = 0.4 \) blue dot, \( g_0 = 0.3 \) green dash, \( g_0 = 0.1 \) orange dash dot. (b) Near horizon behavior (\( r \approx 1 \)).

6 Conclusion

In this paper, we considered non-homogeneous deformation of STU model in four dimensions including three electric and one magnetic charges and use fluid/gravity correspondence to study holographic superconductor. We obtained transport quantities like electrical and thermal conductivities and study the effect of magnetic charge on them. We find that presence of magnetic charge is necessary to have high electrical and thermal conductivities. Small positive value of magnetic charge is enough to have maximum of electrical conductivity while we need large negative value of magnetic charge to have high thermal conductivity. Using the numerical analysis we found critical value of the magnetic charge where there is maximum of the electrical conductivity. We have shown that ordinary STU model \((A = 0)\) is better model to describe holographic superconductor.

In order to obtain thermal conductivity we used ordinary entropy given by (4.1), while it is interesting to calculate thermal conductivity with logarithmic corrected entropy [31–37] and study thermal fluctuations which is corresponding to \( N^{-1} \) corrections in AdS/CFT.
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