An electroweak model of SU(3) × U(1) gauge group is studied. From the group theoretical constraint, the symmetry breaking of this model to the standard model occurs at 1.7 TeV or lower. Hence the mass of the new neutral gauge boson is less than 1.7 TeV. The $Y^\pm$ and $Y^{\pm\pm}$ masses are found to be less than half of the $Z_2$ mass. Thus, the decays $Z_2 \rightarrow Y^{++}Y^{--}$ with $Y^{++} \rightarrow 2\ell^+ (\ell = e, \mu, \tau)$ is allowed, providing spectacular signatures at future colliders. From the flavor-changing neutral current processes, the representations of quarks can be uniquely determined. The neutrino-isoscalar scattering experiments are also considered.
I. INTRODUCTION

A model of SU(15), which includes doubly charged gauge bosons ($Y^{±±}$) and their isospin partners ($Y^{±}$), was proposed by Frampton and Lee [1] two years ago. The model conserves baryon number in gauge interactions, thus proton decay is naturally suppressed [2]. The process $e^-e^- \rightarrow \mu^-\mu^-$ would be the best experiment testing the existence of a doubly charge gauged boson. However, the only machine relevant to this process is operated at the center-of-mass energy 1.112 GeV [3]. Nevertheless, there are $u$-channel contributions to the Bhabha scattering $e^+e^- \rightarrow e^+e^-$ leading to the mass lower bound $M_{Y^{++}} > 210$ GeV (95% C.L.) [4] and right-handed current contributions to muon decay leading to the bound $M_{Y^+} > 270$ GeV (90% C.L.) [5].

Motivated by a doubly charged gauge boson, a model of SU(3)$_L \times$ U(1)$_X$ is introduced by Frampton [6] and Pisano et al. [7]. The former author looked for a simple solution which included dileptons $Y^{±±}$; the latter author argued that $Y^{--}$ is necessary in order to avoid unitarity violation for $e^-e^- \rightarrow W^-Y^-$ at high energies. $Y^{±±}$ and $Y^{±}$ are called dileptons because they couple to two leptons, thus they have two units of lepton number. Many other electroweak models [8] of SU(3) × U(1) were suggested some years ago with different choices of particle content. Here, this model has minimal particle content yielding some interesting new physics, such as stringent constraints on the new gauge boson masses.

The anomaly in this model is not cancelled within each generation. However, the representation of one of the quark generations is chosen in such a way that the anomaly is cancelled among 3 generations. Thus the number of generations is a multiple of 3. Note that the third and the first generation quark multiplets were chosen arbitrarily by the authors in Refs. [6] and [7], respectively. In this paper, we show that only the former choice provides a consistent phenomenology.
breaking scales can be as high as the grand unification scale. Here, the breaking of $SU(3)_L \times U(1)_X$ occurs at 1.7 TeV or less (but greater than 250 GeV). Therefore, this model will be discovered or ruled out at the future colliders. We will organize this paper as follows: Section II describes the model; in Secs. III and IV, gauge boson and fermion masses are discussed. Section V investigates flavor-changing neutral current processes; neutrino-hadron scattering is studied in Sec. VI; finally, the conclusions are presented in Sec. VII.

II. DESCRIPTION OF THE MODEL

The simplest anomaly-free solution [6], which includes the standard model, of a gauge symmetry $SU(3)_c \times SU(3)_L \times U(1)_X$ is given as follows:

$$\psi_{1,2,3} = \begin{pmatrix} e & \mu & \tau \\ \nu_e & \nu_{\mu} & \nu_{\tau} \\ e^c & \mu^c & \tau^c \end{pmatrix}, \quad (1, 3^*, 0) \ , \quad (2.1a)$$

$$Q_{1,2} = \begin{pmatrix} u & d \\ c & s \\ D & S \end{pmatrix}, \quad (3, 3, -\frac{1}{3}) \ , \quad (2.1b)$$

$$Q_3 = \begin{pmatrix} t \\ T \end{pmatrix}, \quad (3, 3^*, \frac{2}{3}), \quad (2.1c)$$

$$d^c, s^c, b^c : \quad \frac{1}{3}, \quad (2.1d)$$

$$u^c, c^c, t^c : \quad -\frac{2}{3}, \quad (2.1e)$$

$$D^c, S^c : \quad \frac{4}{3}, \quad (2.1f)$$

$$T^c : \quad -\frac{5}{3}. \quad (2.1g)$$

where $D, S, T$ are new quarks. For a minimal particle content, the anomaly is not cancelled within each generation, but cancelled among three generations by choosing the third quark generation as an $SU(3)_L$ anti-triplet. So far, the choice of the quark generation is arbitrary.
However, as we shall discuss later, the third generation is chosen in order to have consistent phenomenology.

SU(3)_L \times U(1)_X will first be broken down to the standard model SU(2)_L \times U(1)_Y by a nonzero vacuum expectation value (VEV) of a triplet scalar \( \langle \Phi \rangle_T = (0, 0, u/\sqrt{2}) \), yielding a massive neutral gauge boson \( (Z') \) and two charged gauge bosons \( (Y^+, Y^{++}) \) as well as new quarks \( (D, S, T) \). The breaking of SU(2)_L \times U(1)_Y to U(1)_{em} can be achieved by \( \langle \Delta \rangle_T = (0, v/\sqrt{2}, 0) \) and \( \langle \Delta' \rangle_T = (v'/\sqrt{2}, 0, 0) \). In order to obtain acceptable masses for charged leptons, a sextet \( \eta \) is necessary. Hence, the required scalar multiplets are summarized as follows:

\[
\Phi = \begin{pmatrix} \phi^{++} \\ \phi^+ \\ \phi^0 \\ \Delta_1^+ \\ \Delta_2^0 \end{pmatrix} : (1, 3, 1), \\
\Delta = \begin{pmatrix} \Delta^0 \\ \Delta_2^- \end{pmatrix} : (1, 3, 0), \\
\Delta' = \begin{pmatrix} \Delta'^- \\ \Delta'-- \end{pmatrix} : (1, 3, -1),
\]

and

\[
\eta = \begin{pmatrix} \eta_1^{++} & \eta_1^+ / \sqrt{2} & \eta_1^0 / \sqrt{2} \\ \eta_1^+ / \sqrt{2} & \eta^0 & \eta_2^- / \sqrt{2} \\ \eta_2^- / \sqrt{2} & \eta_2^- / \sqrt{2} & \eta_{2--} \end{pmatrix} : (1, 6, 0).
\]

**III. GAUGE BOSON MASSES**

To obtain the gauge interactions, let us first define the covariant derivative for triplets
where $\lambda^a (a = 1, \cdots, 8)$ are the SU(3)$_L$ generators, and $\lambda^0 = \sqrt{\frac{2}{3}}$ diagonal(1,1,1) are defined such that $\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$ and $\text{Tr}(\lambda^0 \lambda^0) = 2$. $g$ and $g_X$ are the gauge coupling constants for SU(3)$_L$ and U(1)$_X$ with their gauge bosons $W^a$ and $V$, respectively. The covariant derivative for the sextet is

$$D_\mu \eta^{\alpha\beta} = \partial_\mu \eta^{\alpha\beta} - \frac{ig}{2} W^a \left[ \lambda^{a\beta}_{\alpha\beta} \eta^{\alpha\beta} + \lambda^{a\alpha}_{\alpha\alpha} \eta^{\beta\prime} \right].$$

(3.2)

As the triplet scalar $\Phi$ acquires a VEV, the symmetry SU(3)$_L \times U(1)_X$ breaks down to SU(2)$_L \times U(1)_Y$, where $Y \equiv \sqrt{3}(\lambda^8 + \sqrt{2} X \lambda^9)$ is the hypercharge. The coupling constant of U(1)$_Y$, $g'$ is given by

$$\frac{1}{g'^2} = 3 \left( \frac{1}{g^2} + \frac{2}{g_X^2} \right).$$

(3.3)

Therefore we obtain

$$\frac{g_X^2}{g^2} = \frac{6 \sin^2 \theta_W}{1 - 4 \sin^2 \theta_W}.$$  (3.4)

where $g'/g = \tan \theta_W$. Therefore, $\sin^2 \theta_W$ has to be smaller than 1/4 at the breaking scale. Below this breaking scale, there are three doublets, $(\Delta^+_1, \Delta^0_0), (\Delta^0, \Delta^-)$ and $(\eta^0, \eta^-)$, one triplet $(\eta^{++}_1, \eta^+_1, \eta^0)$, and three singlets $\eta^-_2, \Delta^-_2$ and $\Delta'^-\cdots$ under the standard model. Including all these Higgs multiplets, we obtain a one-loop running of $\sin^2 \theta_W$. Therefore, the upper bound of the SU(3)$_L$ breaking, $u$, can be computed from the equation $\sin^2 \theta_W (u) = 1/4$. Since the result is very sensitive to the value of $\sin^2 \theta_W$ at $M_Z$, we plot in Fig. 1 the breaking scale $u$ as a function of $\sin^2 \theta_W (M_Z)$ for $\alpha^{-1}_{em} = 127.9$ in the $\overline{MS}$ scheme. In particular, for $\sin^2 \theta_W (M_Z) = 0.2333$, we obtain that the breaking scale is less than 1.7 TeV.

The breaking of the SM to U(1)$_{em}$ can be achieved by $\langle \Delta^0 \rangle = v/\sqrt{2}, \langle \Delta^0 \rangle = v'/\sqrt{2}$ and $\langle \eta^0 \rangle = w/\sqrt{2}$, where $\langle \eta^0 \rangle = 0$ is assumed for lepton number conservation. The charged gauge bosons
\[ Y^+ = (W^6 - iW^7)/\sqrt{2}, \quad (3.5b) \]

and

\[ Y^{++} = (W^4 - iW^5)/\sqrt{2}, \quad (3.5c) \]

acquire masses

\[ M_{W}^2 = \frac{1}{4} g^2 (v^2 + v'^2 + w^2) , \quad (3.6a) \]
\[ M_{Y^+}^2 = \frac{1}{4} g^2 (u^2 + v'^2 + w^2) , \quad (3.6b) \]

and

\[ M_{Y^{++}}^2 = \frac{1}{4} g^2 (u^2 + v'^2 + 4w^2) , \quad (3.6c) \]

respectively. For \( v'^2 = w^2 = 0 \), we have an approximate mass relation, \( M_{Y^\pm}^2 = M_{Y^{\pm\pm}}^2 + M_W^2 \).

Therefore, we would expect \( Y^\pm \) to be heavier than \( Y^{\pm\pm} \).

The mass-squared matrix for the neutral gauge bosons \( \{W^3, W^8, V\} \) is given by

\[
\begin{bmatrix}
\frac{1}{4} g^2 (v^2 + v'^2 + w^2) & -\frac{1}{4\sqrt{3}} g^2 (v^2 - v'^2 + w^2) & -\frac{1}{2\sqrt{6}} gg_X v'^2 \\
-\frac{1}{4\sqrt{3}} g^2 (v^2 - v'^2 + w^2) & \frac{1}{12} g^2 (4u^2 + v^2 + v'^2 + w^2) & -\frac{1}{6\sqrt{2}} gg_X (2u^2 + v'^2) \\
-\frac{1}{2\sqrt{6}} gg_X v'^2 & -\frac{1}{6\sqrt{2}} gg_X (2u^2 + v'^2) & \frac{1}{6} g^2_X (u^2 + v'^2)
\end{bmatrix}
\quad . \quad (3.7)\]

We can easily identify the photon field \( \gamma \) as well as the massive bosons \( Z \) and \( Z' \)

\[ \gamma = + \sin \theta_W W^3 + \cos \theta_W \left( \sqrt{3} \tan \theta_W W^8 + \sqrt{1 - 3 \tan^2 \theta_W} V \right) , \quad (3.8a) \]
\[ Z = + \cos \theta_W W^3 - \sin \theta_W \left( \sqrt{3} \tan \theta_W W^8 + \sqrt{1 - 3 \tan^2 \theta_W} V \right) , \quad (3.8b) \]

and

\[ Z' = - \sqrt{1 - 3 \tan^2 \theta_W} W^8 + \sqrt{3} \tan \theta_W V , \quad (3.8c) \]

where the mass-squared matrix for \( \{Z, Z'\} \) is given by

\[ (M_{Z}^2, M_{Z'}^2) \]
with

\[ M_Z^2 = \frac{1}{4} \frac{g^2}{\cos^2 \theta_W} (v^2 + v'^2 + w^2), \quad (3.10a) \]

\[ M_{Z'}^2 = \frac{1}{3} g^2 \left[ \frac{\cos^2 \theta_W}{1 - 4 \sin^2 \theta_W} u^2 + \frac{1 - 4 \sin^2 \theta_W}{4 \cos^2 \theta_W} (v^2 + v'^2 + w^2) \right. \
\left. + \frac{3 \sin^2 \theta_W}{1 - 4 \sin^2 \theta_W} v'^2 \right], \quad (3.10b) \]

\[ M_{ZZ'}^2 = \frac{1}{4 \sqrt{3}} g^2 \left[ \sqrt{1 - 4 \sin^2 \theta_W} \cos \theta_W (v^2 + w^2) - \left( \frac{1 + 4 \sin^2 \theta_W}{1 - 4 \sin^2 \theta_W} \right) v'^2 \right]. \quad (3.10c) \]

The mass eigenstate are

\[ Z_1 = \cos \theta \ Z - \sin \theta \ Z', \quad (3.11a) \]

and

\[ Z_2 = \sin \theta \ Z + \cos \theta \ Z', \quad (3.11b) \]

where the mixing angle is given by

\[ \tan^2 \theta = \frac{M_Z^2 - M_{Z_1}^2}{M_{Z_2}^2 - M_Z^2}. \quad (3.12) \]

with \( M_{Z_1}^2 \) and \( M_{Z_2}^2 \) being the masses for \( Z_1 \) and \( Z_2 \). Here, \( Z_1 \) corresponds to the standard model neutral gauge boson and \( Z_2 \) corresponds to the additional neutral gauge boson.

Since \( 1 - 4 \sin^2 \theta_W \approx 0.06 \) and \( v'^2 \ll u^2 \), we can conclude that \( M_{ZZ'}^2 \ll M_{Z'}^2 \). Hence, we obtain

\[ M_{Z_1}^2 = M_Z^2 \left( 1 - \frac{M_{ZZ'}^4}{M_Z^4 M_{Z'}^4} \right), \quad (3.13a) \]

\[ M_{Z_2}^2 = \frac{1}{3} g^2 \frac{\cos^2 \theta_W}{1 - 4 \sin^2 \theta_W} u^2, \quad (3.13b) \]

and

\[ \theta = \frac{M_{ZZ'}^2}{M_{Z'}^2}. \quad (3.13c) \]

In particular, for \( v' = 0 \), the mass of \( Z_1 \) is shifted by a factor of \( 1 - \frac{1}{1 - 4 \sin^2 \theta_W} M_Z^2 \).
From the symmetry breaking hierarchy, $\eta > v, v', w$, we obtain the lower mass bound of $Z_2$

$$M_{Z_2} > \sqrt{\frac{4}{3}} \sqrt{\frac{\cos^2 \theta_W(M_{Z_2})}{1 - 4 \sin^2 \theta_W(M_{Z_2})}} M_{Z_1}$$

$$> 400 \text{ GeV}.$$  

(3.14)

In many extensions of the SM, such as SO(10), E$_6$, L-R models, etc., the masses of the additional neutral gauge bosons are unconstrained in general. From $Z$-$Z'$ mixing [10], the lower limits are typically from 200 GeV to 1000 GeV, depending on the models. They can also be as heavy as the unification scale. This model, on the other hand, predicts $M_{Z'}$ to be within 400 GeV and 1.7 TeV. In addition, the masses of the new charged gauge bosons $Y^+$ and $Y^{++}$ are expected to be

$$M_{Y^+} \simeq M_{Y^{++}} = M_Y \simeq \sqrt{\frac{3}{4}} \sqrt{\frac{1 - 4 \sin^2 \theta_W}{\cos \theta_W}} M_{Z_2}.$$  

(3.15)

which is depicted numerically in Fig. 2. We find that $M_Y$ is always less than 0.5 $M_{Z_2}$. Therefore, we expect that the decays $Z' \rightarrow Y^{++}Y^{--}$ and $Y^{\pm \pm} \rightarrow 2\ell^\pm (\ell = e, \mu, \tau)$ are allowed, leading to spectacular signatures in the future colliders. From the collider experiments [1] and muon decay [2], $M_{Y^{++}}$ and $M_{Y^+}$ are greater than 210 GeV (95% C.L.) and 270 GeV (90% C.L.) respectively. From Fig. 2, we obtain a limit, $M_{Z_2} > 1.3 \text{ TeV}$, for $M_Y > 270 \text{ GeV}$.

### IV. FERMION MASSES

The Yukawa interactions corresponding to the scalar multiplets $\Phi$, $\Delta$, $\Delta'$ and $\eta$ are given as follows

$$-\mathcal{L}(\Phi) = h_{D,S}^{1,2} Q_{1,2}(D^c, S^c) \Phi^* + h_T^3 Q_3 T^c \Phi, \quad (4.1a)$$

$$-\mathcal{L}(\Delta) = h_3^3 Q_3 t^c \Delta + h_{1,2}^{ij} Q_{1,2}(d^c, s^c, b^c) \Delta^* + h^{ij}_{1} \psi_1 \psi_1 \Delta^*, \quad (4.1b)$$

(4.1c)
and

\[-\mathcal{L}(\eta) = y^{ij}_e \psi_i \psi_j \eta, \]

where \(h^{ij}_e\) and \(y^{ij}_e\) are antisymmetric and symmetric matrices in flavor space.

As SU(3)_L \times U(1)_X breaks down to SU(2)_L \times U(1)_Y, D, S, T acquire masses which are expected to be less than 10 TeV. As \(\Delta, \Delta'\) and \(\eta\) break the SU(2)_L \times U(1)_Y to U(1)_{em}, all the usual fermions acquire masses. We have redefined the SU(3)_L singlets in such a way that \(m_t = \frac{1}{\sqrt{2}} h^3_t v\) and \(m_b = \frac{1}{\sqrt{2}} h^3_b v'\). Therefore, we expect that \(v' \ll v\) as we assumed in the previous section. The mass matrix linking the left-handed to right-handed quarks is given by

\[
\begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \sqrt{2} m_b
\end{pmatrix}
\]

The mass matrix of the up-sector has the same form. It would be natural to assume that \(m_b\) is much bigger than the other elements after the redefinition. Therefore, the mixing hierarchy will be \(D^L_{sb} = m_s/m_b\) and \(D^L_{db} = m_d/m_b\) for the left-handed sector, whereas \(D^R_{sb} = (m^2_s/m_b^2)\) and \(D^R_{db} = (m^2_d/m_b^2)\) for the right-handed sector. Hence we obtain the CKM matrix elements \(V_{cb} \simeq m_s/m_b\) and \(V_{ub} \simeq m_d/m_b + D^L_{uc}(m_s/m_b) \simeq D^L_{uc}(m_s/m_b).\) Thus \(V_{ub}/V_{cb} \simeq 0.1\) [11], implies \(D^L_{uc} \simeq 0.1.\)

For the lepton sector, the charged lepton mass matrix is \(h^{ij}_e v/\sqrt{2} + y^{ij}_e w/\sqrt{2}\). Without the \(\eta\), the matrix is symmetric, yielding an unacceptable relationship, namely \(m_\mu = m_\tau\). In addition, we have assumed \(\langle \eta' \rangle = 0\) so that neutrinos remain massless and there will be residual lepton number conservation. In general, if \(\langle \eta' \rangle \neq 0\), heavy SU(3)_L \times U(1)_X singlet neutrinos are required for a see-saw mechanism in order to obtain realistic masses for the light neutrinos. Nevertheless, assuming \(\langle \eta' \rangle = 0\) will not affect our discussion in this paper.
V. FLAVOR-CHANGING NEUTRAL CURRENT PROCESSES

In this model, the interactions of $Z'$ discriminate among quark generations. Since $M_{Z'}$ is expected to be smaller than 1.7 TeV, flavor-changing neutral current processes induced by $Z'$ would be important tests for this model. As explained in Sec. II, the choice of an anti-triplet quark multiplet is arbitrary; here we first choose the third generation. To define the convention properly, we explicitly write out all the fermions and neutral gauge bosons ($A$, $Z$ and $Z'$) as follows:

$$\mathcal{L}(A) = Q_f e A^\mu \bar{f} \gamma_\mu f,$$

$$\mathcal{L}(Z) = \frac{g}{\cos \theta_W} Z^\mu \bar{f} \gamma_\mu (g_V(f) + g_A(f) \gamma_5) f,$$  \hspace{1cm} (5.1)
with $g_V(f) = \frac{1}{2} T_f - Q_f \sin^2 \theta_W$ and $g_A(f) = -\frac{1}{2} T_f$, and

$$\mathcal{L}(Z') = \frac{g}{\cos \theta_W} Z'^\mu \bar{f} \gamma_\mu (a_f + b_f \gamma_5) f,$$  \hspace{1cm} (5.2)

with

$$a_\nu = -b_\nu = \frac{1}{4\sqrt{3}} \sqrt{1-4\sin^2 \theta_W},$$ (5.4a)

$$a_{e,\mu,\tau} = 3b_{e,\mu,\tau} = 3a_\nu,$$ (5.4b)

$$a_{u,c} = \frac{1}{4\sqrt{3}} \frac{-1+6\sin^2 \theta_W}{\sqrt{1-4\sin^2 \theta_W}}, \quad b_{u,c} = \frac{1}{4\sqrt{3}} \frac{1+2\sin^2 \theta_W}{\sqrt{1-4\sin^2 \theta_W}},$$ (5.4c)

$$a_t = \frac{1}{4\sqrt{3}} \frac{1+4\sin^2 \theta_W}{\sqrt{1-4\sin^2 \theta_W}}, \quad b_t = -\frac{1}{4\sqrt{3}} \sqrt{1-4\sin^2 \theta_W},$$ (5.4d)

$$a_{d,s} = -\frac{1}{4\sqrt{3}} \frac{1}{\sqrt{1-4\sin^2 \theta_W}}, \quad b_{d,s} = \frac{1}{4\sqrt{3}} \sqrt{1-4\sin^2 \theta_W},$$ (5.4e)

$$a_b = \frac{1}{4\sqrt{3}} \frac{1-2\sin^2 \theta_W}{\sqrt{1-4\sin^2 \theta_W}}, \quad b_b = -\frac{1}{4\sqrt{3}} \frac{1+2\sin^2 \theta_W}{\sqrt{1-4\sin^2 \theta_W}},$$ (5.4f)

$$a_{D,S} = \frac{1}{2\sqrt{3}} \frac{1-9\sin^2 \theta_W}{\sqrt{1-4\sin^2 \theta_W}}, \quad b_{D,S} = -\frac{1}{2\sqrt{3}} \frac{1-\sin^2 \theta_W}{\sqrt{1-4\sin^2 \theta_W}}.$$ (5.4g)
Q_f = -1, 2/3, -1/3, -4/3 and 5/3 for f = (e, μ, τ), (u, c, t), (d, s, b), (D, S) and T, respectively. The weak isospin for fermion f, T_f, is defined as 1/2, -1/2 and 0 for (ν_e, ν_µ, ν_τ, u, c, t), (e, μ, τ, d, s, b) and (D, S, T), respectively. Thus the couplings of D, S and T to the Z boson are vector-like. Since the third generation transforms differently, their couplings to Z' differ from those for the first and second generations, leading to the flavor-changing neutral currents (FCNCs). In particular, the FCNC in the down sector is given by

$$L_{FCNC} = \frac{g}{\cos \theta_W} \left[ -\sin \theta Z_1 + \cos \theta Z_2 \right] \left\{ b \gamma_5 \mu \delta_L \left( \frac{1 - \gamma_5}{2} \right) b \right\}, \quad (5.5)$$

where

$$\delta_L = (a_b - a_d) + (b_b - b_d) = \frac{1 - \sin^2 \theta_W}{\sqrt{3} \sqrt{1 - 4 \sin^2 \theta_W}}. \quad (5.6)$$

There is no FCNC for the right-handed currents as the right-handed fermions transform identically. The B^0–\overline{B}^0 mixing will be calculated to be

$$\Delta M_{B^0} \simeq \frac{4 \pi \alpha}{3 \sin^2 \theta_W \cos^2 \theta_W} \left( \frac{m_d}{m_b} \right)^2 \frac{\delta_L}{\sqrt{M_{Z_1}^2 + \sin^2 \theta \rho}} B_B f_B^2 M_B, \quad (5.7)$$

where f_B and B_B are the decay constant and bag factor of a B-meson. Taking \( \sin^2 \theta_W = 0.2333 \), \( \sqrt{B_B f_B} = 160 \text{ MeV} \), \( m_b = M_B = 5.3 \text{ GeV} \), \( m_d > 5 \text{ MeV} \) and \( \Delta M_B < 3 \times 10^{-13} \text{ GeV} \), we obtain \( \sin \theta < 0.2 \) and \( M_{Z'} > 180 \text{ GeV} \). Because of the suppression factor \( (m_d/m_b)^2 \), the Z' mass and the mixing angle sin \( \theta \) are not stringently constrained. For K^0–\overline{K}^0 mixing, there will be an additional suppression factor \( (m_s/m_b)^2 \) which leads to a negligible contribution.

b – s transitions can also be induced by the Yukawa interactions

$$\left[ \sqrt{2} \frac{m_b}{v} \Delta^0 - \sqrt{2} \frac{m_b}{v'} \Delta^{\alpha} \right] D_{sb} \bar{T} \left( \frac{1 - \gamma_5}{2} \right) s. \quad (5.8)$$

Assuming \( v' > 10 \text{ GeV} \) and \( m_{\Delta^0} > 250 \text{ GeV} \) the contribution to \( \Delta M_{B^0} \) is less than \( 3 \times 10^{-13} \text{ GeV} \). The contribution from the first term is even smaller as \( v \simeq 250 \text{ GeV} \).

On the other hand, if the first or second generation is chosen to be an anti-triplet of \( SU(3)_L \), the K^0–\overline{K}^0 mixing is unsuppressed. As a result, \( M_{Z'} \) has to be greater than 40 TeV.
This is in contradiction with the analyses in Sec. III. Therefore, in order that this model be viable, the third generation should be chosen to be the SU(3)$_L$ anti-triplet. In addition, due to the mixing hierarchy, the new contributions to $B^0_s\bar{B}^0_s$ mixing would be important.

VI. NEUTRINO ISOSCALAR SCATTERING

Models with additional neutral gauge bosons, such as E$_6$ and L-R models, have been intensively investigated [10]. In particular, the mass bounds at 90\% C.L. for $Z_2$ from neutrino-hadron scattering for E$_6$ (χ-model) and L-R models are 555 GeV and 795 GeV [12], respectively. The corresponding mixing angles are bound to be less than $6 \times 10^{-3}$ and $5 \times 10^{-3}$. Here, the coupling strength of $Z'$ to quarks in this model is stronger than that of leptons; neutrino-quark scattering would be the most important process among the precision measurements.

In the low-energy limit, the four-fermion interactions are given by

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}} (J_{\mu} J^\mu - 2\delta \xi J_{\mu} J'^{\mu} + \xi J'_{\mu} J'^{\mu}) ,$$

(6.1)

where

$$J_{\mu} = \bar{f} \gamma_{\mu} (g_f^V + g_A^f \gamma_5) f , \quad J'_{\mu} = \bar{f} \gamma_{\mu} (a^f + b^f \gamma_5) f$$

(6.2)

and

$$\delta = \frac{M_{Z'Z'}}{M_Z^2}, \quad \xi = \frac{M_Z^2}{M_{Z'}^2}, \quad \rho = 1 - \delta^2 \xi .$$

(6.3)

The above expressions, which can be obtained by taking the inverse of Eq. (3.9), are exact without any approximation. Hence, the ratio of neutral to charged current cross sections,
\[ \mathcal{R}_\nu = \left[ (\epsilon^u_L)^2 + (\epsilon^d_L)^2 \right] + \left[ (\epsilon^u_R)^2 + (\epsilon^d_R)^2 \right] r, \]  

where

\[ \epsilon^u_L = \frac{1}{\bar{\rho}} \left[ \rho_{\nu N} \left( \frac{1}{2} - \frac{2}{3} \kappa_{\nu N} \sin^2 \theta_W + \lambda^u_L \right) - \delta \xi \frac{\sqrt{1 - 4 \sin^2 \theta_W}}{4 \sqrt{3}} \left( \frac{1}{1 - 4 \sin^2 \theta_W} + \frac{4}{3} \sin^2 \theta_W \right) - \frac{1}{12} \xi (1 - 4 \sin^2 \theta_W) \right], \]  

(6.5a)

\[ \epsilon^u_R = \frac{1}{\bar{\rho}} \left[ \rho_{\nu N} \left( -\frac{2}{3} \kappa_{\nu N} \sin^2 \theta_W + \lambda^u_R \right) - \delta \xi \frac{\sqrt{1 - 4 \sin^2 \theta_W}}{4 \sqrt{3}} \left( \frac{8 \sin^2 \theta_W}{1 - 4 \sin^2 \theta_W} - \frac{4}{3} \sin^2 \theta_W \right) + \frac{1}{3} \xi \sin^2 \theta_W \right], \]  

(6.5b)

\[ \epsilon^d_L = \frac{1}{\bar{\rho}} \left[ \rho_{\nu N} \left( -\frac{1}{2} + \frac{1}{3} \kappa_{\nu N} \sin^2 \theta_W + \lambda^d_L \right) + \delta \xi \frac{\sqrt{1 - 4 \sin^2 \theta_W}}{4 \sqrt{3}} \left( 2 + \frac{1}{1 - 4 \sin^2 \theta_W} - \frac{2}{3} \sin^2 \theta_W \right) - \frac{1}{12} \xi (1 - 2 \sin^2 \theta_W) \right], \]  

(6.5c)

\[ \epsilon^d_R = \frac{1}{\bar{\rho}} \left[ \rho_{\nu N} \left( \frac{1}{3} \kappa_{\nu N} \sin^2 \theta_W + \lambda^d_R \right) + \delta \xi \frac{\sqrt{1 - 4 \sin^2 \theta_W}}{4 \sqrt{3}} \left( \frac{4 \sin^2 \theta_W}{1 - 4 \sin^2 \theta_W} - \frac{2}{3} \sin^2 \theta_W \right) - \frac{1}{6} \xi \sin^2 \theta_W \right], \]  

(6.5d)

is expected to impose the most stringent constraint on the parameters \( \delta \) and \( \xi \). \( r \) in Eq. (6.4) is the ratio of antineutrino to neutrino scattering cross sections. \( \lambda \)'s, \( \rho_{\nu N} - 1 \) and \( \kappa_{\nu N} - 1 \) are the electroweak radiative corrections. Assuming that they are dominated by the oblique corrections, we then obtain the approximate expressions \( \rho_{\nu N} \simeq 1 + \frac{3G_F}{8\sqrt{2}\pi^2} m_t^2 \) and \( K_{\nu N} \simeq 1 \) for the \( \overline{\text{MS}} \) renormalization scheme \cite{11,13}. Using the most precisely measured values for \( \mathcal{R}_\nu \) obtained by the CDHS \cite{14} and CHARM \cite{15} collaborations, we plot

\[ \chi^2 = \left( \frac{\mathcal{R}_\nu (\text{CDHS}) - \mathcal{R}_V}{\sigma(\text{CDHS})} \right)^2 + \left( \frac{\mathcal{R}_\nu (\text{CHARM}) - \mathcal{R}_V}{\sigma(\text{CHARM})} \right)^2 \]  

(6.6)

for \( \chi^2 = 0.5 \) and 2 in Fig. 3, where we take the global fit for the top-quark mass \( m_t = 124 \text{ GeV} \). In this paper, we do not consider a comprehensive analysis of this additional neutral gauge boson.
parameter $M_{Z'}^2 (\simeq M_{Z_2}^2)$ is not stringently constrained. For example, for $|\delta| < 0.5$, $M_{Z'}$ (or $M_{Z_2}$) can be anywhere in the allowed region.

**VII. CONCLUSION**

In this paper, we have considered an electroweak theory of SU(3)×U(1) which introduces three new quarks $D, S$ and $T$ with charges $-4/3, -4/3$ and $5/3$. All the lepton generations transform identically under this gauge symmetry; whereas one of the quark generations, which is an SU(3) anti-triplet, transforms differently from the other two. An SU(3) triplet Higgs scalar, $\Phi$, is required to breaking the symmetry into the standard model. Two SU(3) triplets, $\Delta$ and $\Delta'$, are responsible for breaking the standard model as well as providing masses for the usual fermions. To obtain realistic lepton masses, a sextet, $\eta$, is needed. From the flavor changing neutral current processes, the third generation quark is chosen to be the SU(3) anti-triplet.

By matching the coupling constants at the symmetry breaking scale, we find that $\sin^2 \theta_W$ should be less than $1/4$, leading to a breaking scale under 1.7 TeV for $\sin^2 \theta_W (M_{Z_1}) = 0.2333$. From the symmetry breaking hierarchy, namely $1.7 \text{ TeV} > u > v, v', w$, the mass of the additional neutral gauge boson $Z_2$ ranges from 400 GeV to 1.7 TeV. In addition, the mass of the new charged gauge bosons $Y^\pm$ and $Y^{\pm\pm}$ is less than a half of $M_{Z_2}$. Therefore the decay $Z_2 \rightarrow Y^{++} Y^{--}$ with $Y^{\pm\pm} \rightarrow 2 \ell^{\pm}$ provides unique signatures in the future colliders.

From the muon decay experiments, $M_Y$ is found to be at least 270 GeV at 90% confidence level. Hence, we obtain narrow windows for $M_Y$ and $M_{Z_2}$, $270 \text{ GeV} < M_Y < 330 \text{ GeV}$ and $1.3 \text{ TeV} < M_{Z_2} < 1.7 \text{ GeV}$. Therefore, this model can be either discovered or ruled out at the future colliders.
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FIGURES

FIG. 1. the breaking scale $u$ as a function of $\sin^2 \theta_W(M_Z)$

FIG. 2. the new charged gauge boson mass $M_Y$ as a function of $M_{Z'}$ for $\sin^2 \theta_W(M_Z) = 0.2333 \pm 0.0016$

FIG. 3. contour plot of $\chi^2 = 0.5$(solid line) and 2(dotted line) as a function of $\delta$ and $\xi$