Dark matter in three-Higgs-doublet models with $S_3$ symmetry

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Motivation: check $S_3$-3HDM for dark matter candidates.
Particle physics is best described by the Standard Model. However, it fails to describe dark matter. Possible solution: extend the scalar sector with interplay of symmetries. Motivation: check $S_3$-3HDM for dark matter candidates.

Outline:

- Inert Doublet Model;
- General $S_3$-3HDM;
- Dark matter within $S_3$-3HDM;
Main building block: SU(2) scalar doublet, $h = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$. 
Inert Doublet Model: Generalities

Main building block: SU(2) scalar doublet, \( h = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \).

Bilinear \( h_{ij} = h_i \dagger h_j \) is a singlet under SU(2).
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Bilinear \( h_{ij} = h_i^\dagger h_j \) is a singlet under SU(2).

\[
\mathcal{L}_{\text{2HDM}} = m_{11}^2 h_{11} + m_{22}^2 h_{22} - \left( m_{12}^2 h_{12} + \text{h.c.} \right) \\
+ \frac{1}{2} \lambda_1 h_{11}^2 + \frac{1}{2} \lambda_2 h_{22}^2 + \lambda_3 h_{11} h_{22} + \lambda_4 h_{12} h_{21} \\
+ \left\{ \frac{1}{2} \lambda_5 h_{12}^2 + \lambda_6 h_{11} h_{12} + \lambda_7 h_{22} h_{12} + \text{h.c.} \right\}.
\]
Main building block: SU(2) scalar doublet, $h = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$.

Bilinear $h_{ij} = h_i^\dagger h_j$ is a singlet under SU(2).

$$\mathcal{V}_{\text{IDM}} = m_{11}^2 h_{11} + m_{22}^2 h_{22} - \left( m_{12}^2 h_{12} + h.c. \right)$$
$$+ \frac{1}{2} \lambda_1 h_{11}^2 + \frac{1}{2} \lambda_2 h_{22}^2 + \lambda_3 h_{11} h_{22} + \lambda_4 h_{12} h_{21}$$
$$+ \left\{ \frac{1}{2} \lambda_5 h_{12}^2 + \lambda_6 h_{11} h_{12} + \lambda_7 h_{22} h_{12} + h.c. \right\}.$$
Inert Doublet Model: Profile

LEP excluded

$\bar{f} f^*$

V resonant co-annihilation

$\rightarrow h$

$\rightarrow \{W^+ W^-, ZZ, hh\}$

$\Omega h^2$

$m_{DM}$ [GeV]
Inert Doublet Model: Profile

DM DM → \bar{f}f^*
**Inert Doublet Model: Profile**

- **DM DM → ℓℓ:**
- **V resonant co-annihilation**

![Graph](image)

- $\Omega h^2$ vs. $m_{DM}$ [GeV]
Inert Doublet Model: Profile

LEP excluded

$\Omega h^2$

$V$ resonant co-annihilation

$DM \rightarrow \bar{f}f^*$

$m_{DM}$ [GeV]
Inert Doublet Model: Profile

LEP excluded

DM DM → \bar{f} f^*

V resonant co-annihilation

DM DM → h

\Omega h^2

m_{DM} [GeV]
Inert Doublet Model: Profile

- LEP excluded
- DM → \bar{ff}^*
- V resonant co-annihilation
- DM → h
- DM → \{W^+W^-, ZZ, hh\}

\[ \Omega h^2 \]

\[ m_{DM} \text{ [GeV]} \]
Dark Matter in Inert Doublet Model and Three-Higgs-Doublet Models

**SCALAR DM MASS RANGES**

IDM: [1612.00511], [1809.07712];
IDM2 (one inert doublet): [1911.06477];

3HDM: [1407.7859], [1507.08433], [1712.09598];
CP-3HDM: [1608.01673];
Dark Matter in Inert Doublet Model and Three-Higgs-Doublet Models

**SCALAR DM MASS RANGES**

| Model          | Mass Ranges |
|----------------|-------------|
| IDM            | 50 GeV - 500 GeV |
| IDM2           | 1000 GeV    |
| 3HDM           | 50 GeV - 1000 GeV |
| CP-3HDM        |             |

**References**

IDM: [1612.00511], [1809.07712];
IDM2 (one inert doublet): [1911.06477];
3HDM: [1407.7859], [1507.08433], [1712.09598];
CP-3HDM: [1608.01673];
(Two inert doublets)
The Lagrangian for the Next-to-Heavy-Doublet Model (NHDM) is given by:

\[
\mathcal{L}_{\text{NHDM}} = \sum_{i=1}^{N} (D^\mu h_i)^\dagger (D_\mu h_i) - \mathcal{V}(h_1, \ldots, h_N) - \mathcal{L}_{\text{Yukawa}}.
\]

Where, \(D^\mu h_i\) and \(D_\mu h_i\) represent the kinetic terms, \(\mathcal{V}\) is the scalar potential, and \(\mathcal{L}_{\text{Yukawa}}\) is the Yukawa Lagrangian.
Multi-Higgs-Doublet Models

\[ \mathcal{L}_{\text{NHDM}} = \sum_{i=1}^{N} (D^\mu h_i)\dagger (D_\mu h_i) - \mathcal{V}(h_1, \ldots, h_N) - \mathcal{L}_{\text{Yukawa}}. \]

Real parameters (dependent) of NHDM [1007.1424]:

\[ N_{\text{tot}} = \frac{1}{2} N^2 \left( N^2 + 3 \right) \rightarrow \begin{cases} N = 1 : & N_{\text{tot}} = 2, \\ N = 2 : & N_{\text{tot}} = 14, \\ N = 3 : & N_{\text{tot}} = 54, \\ \ldots \end{cases} \]
Consider an equilateral triangle: \((C)\).
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Possible transformations:

- 2 rotations
- 3 reflections
- Identity
Consider an equilateral triangle: \( \triangle \).

Possible transformations:

- 2 rotations
- 3 reflections
- Identity

\( S_3 \) irreducible representation: \( \chi_1 \oplus \chi_1' \oplus \chi_2 \).
Assume an $S_3$ structure $(h_S)_1 \oplus \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}_2$. 
Assume an $S_3$ structure $(h_S)_1 \oplus \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}_2$.

$$V_{3\text{HDM}} = \mu_1^2 [2 \otimes 2]_1 + \mu_0^2 [1 \otimes 1]_1$$

$$+ \lambda_1 ([2 \otimes 2]_1 \otimes [2 \otimes 2]_1) + \lambda_2 ([2 \otimes 2]_{1'} \otimes [2 \otimes 2]_{1'})$$

$$+ \lambda_3 ([2 \otimes 2]_2 \otimes [2 \otimes 2]_2) + \left\{ \lambda_4 ([2 \otimes 2]_2 \otimes [1 \otimes 2]_2) + \text{sym} \right\}$$

$$+ \lambda_5 ([2 \otimes 2]_1 \otimes [1 \otimes 1]_1) + \lambda_6 ([1 \otimes 2]_2 \otimes [2 \otimes 1]_2)$$

$$+ \left\{ \lambda_7 ([1 \otimes 2]_2 \otimes [1 \otimes 2]_2) + \text{sym} \right\} + \lambda_8 ([1 \otimes 1]_1 \otimes [1 \otimes 1]_1).$$
Assume an $S_3$ structure $(h_S)_1 \oplus \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}_2$.

\[ V_{3HDM} = \mu_1^2 (h_{11} + h_{22}) + \mu_0^2 h_{SS} \]
\[ + \lambda_1 (h_{11} + h_{22})^2 + \lambda_2 (h_{12} - h_{21})^2 + \lambda_3 \left[ (h_{11} - h_{22})^2 + (h_{12} + h_{21})^2 \right] \]
\[ + \{ \lambda_4 [h_{S_1} (h_{12} + h_{21}) + h_{S_2} (h_{11} - h_{22})] + \text{h.c.} \} + \lambda_5 [h_{SS} (h_{11} + h_{22})] \]
\[ + \lambda_6 [h_{1S} h_{S_1} + h_{2S} h_{S_2}] + \{ \lambda_7 \left[ h_{S_1}^2 + h_{S_2}^2 \right] + \text{h.c.} \} + \lambda_8 h_{SS}^2. \]
Assume an $S_3$ structure $(h_S)_1 \oplus \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}_2$.

$V_{3\text{HDM}} = \mu_1^2 (h_{11} + h_{22}) + \mu_0^2 h_{SS}$

$$+ \lambda_1 (h_{11} + h_{22})^2 + \lambda_2 (h_{12} - h_{21})^2 + \lambda_3 \left[ (h_{11} - h_{22})^2 + (h_{12} + h_{21})^2 \right]$$

$$+ \{ \lambda_4 [h_{S1} (h_{12} + h_{21}) + h_{S2} (h_{11} - h_{22})] + \text{h.c.} \} + \lambda_5 [h_{SS} (h_{11} + h_{22})]$$

$$+ \lambda_6 [h_{1S} h_{S1} + h_{2S} h_{S2}] + \{ \lambda_7 [h_{S1}^2 + h_{S2}^2] + \text{h.c.} \} + \lambda_8 h_{SS}^2.$$

Symmetries reduce free parameters:

$\text{NHDM} \xrightarrow{3\text{HDM}} (54) \xrightarrow{S_3} 12 \xrightarrow{\text{Re}} 10.$
Assume an $S_3$ structure $(h_S)_1 \oplus \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}_2$.

\[
\mathcal{V}_{3\text{HDM}} = \mu_1^2 (h_{11} + h_{22}) + \mu_0^2 h_{SS} \\
+ \lambda_1 (h_{11} + h_{22})^2 + \lambda_2 (h_{12} - h_{21})^2 + \lambda_3 \left[ (h_{11} - h_{22})^2 + (h_{12} + h_{21})^2 \right] \\
+ \{ \lambda_4 [h_{S1} (h_{12} + h_{21}) + h_{S2} (h_{11} - h_{22})] + \text{h.c.}\} + \lambda_5 [h_{SS} (h_{11} + h_{22})] \\
+ \lambda_6 [h_{1S} h_{S1} + h_{2S} h_{S2}] + \left\{ \lambda_7 \left[ h_{S1}^2 + h_{S2}^2 \right] + \text{h.c.}\right\} + \lambda_8 h_{SS}^2.
\]

Symmetries reduce free parameters:

\[
\text{NHDM} \xrightarrow{3\text{HDM}} (54) \xrightarrow{S_3} 12 \xrightarrow{\text{Re}} 10.
\]

$S_3$-3HDM models were classified in [1601.04654]:

\[
\text{vacuum: } \begin{cases}
11 \text{ real } (w_1, w_2, w_S), \\
17 \text{ complex } (\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S).
\end{cases}
\]
Whenever $w_S \neq 0$ we can construct a trivial Yukawa sector, $\mathcal{L}_Y \sim 1_f \otimes 1_h$:

$$M_u = \frac{1}{\sqrt{2}} \text{diag} \left( y_1^u, y_2^u, y_3^u \right) w_S^*, \quad M_d = \ldots$$
Whenever $w_S \neq 0$ we can construct a trivial Yukawa sector, $\mathcal{L}_Y \sim 1_f \otimes 1_h$:

$$\mathcal{M}_u = \frac{1}{\sqrt{2}} \text{diag} (y_1^u, y_2^u, y_3^u) w_S^*, \quad \mathcal{M}_d = \ldots$$

Fermions can transform non-trivially under $S_3$, $\mathcal{L}_Y \sim (2 + 1)_f \otimes (2 + 1)_h$:

$$2 : (Q_1 Q_2)^T, (u_{1R} u_{2R})^T, (d_{1R} d_{2R})^T \quad \text{and} \quad 1 : Q_3, u_{3R}, d_{3R},$$

$$\mathcal{M}_u = \frac{1}{\sqrt{2}} \begin{pmatrix}
    y_1^u w_S^* + y_2^u w_2^* & y_2^u w_1^* & y_4^u w_1^* \\
    y_2^u w_1^* & y_1^u w_S^* - y_2^u w_2^* & y_4^u w_2^* \\
    y_5^u w_1^* & y_5^u w_2^* & y_3^u w_S^*
\end{pmatrix}, \quad \mathcal{M}_d = \ldots$$
Massless state:

\[ \mathcal{V}(Uh) = \mathcal{V}(h), \]
\[ \langle 0 | (Uh) | 0 \rangle \neq \langle 0 | h | 0 \rangle. \]
Massless state:

\[ \mathcal{V}(Uh) = \mathcal{V}(h), \]
\[
\langle 0 | (Uh) | 0 \rangle \neq \langle 0 | h | 0 \rangle.
\]

Results of [2001.01994]:

| Constraints     | Continuous symmetries | # of massless states |
|-----------------|------------------------|----------------------|
| \[ \lambda_4 = 0 \] | O(2)                   | 1                    |
| \[ \cdots + [\lambda_7 = 0] \] | O(2) \otimes U(1)_{h_S} | 2                    |
| \[ \cdots + [\lambda_2 + \lambda_3 = 0] \] | SU(2) | 3 |
|                  | [ O(2) \otimes U(1)_{h_1} \otimes U(1)_{h_2} \otimes U(1)_{h_S} ] | 3 |
### S₃-Symmetric Three-Higgs-Doublet Models: Dark Matter Models

| Vacuum          | vevs            | λ₄   | symmetry          | # massless states | fermions under S₃ |
|-----------------|-----------------|------|-------------------|-------------------|-------------------|
| R-I-1           | (0, 0, wₛ)      | ✓    | S₃, h₁ → −h₁      | none              | trivial           |
| R-I-2a          | (w, 0, 0)       | ✓    | S₂                | none              | non-trivial       |
| R-I-2b,2c       | (w, ±√3w, 0)    | ✓    | S₂                | none              | non-trivial       |
| R-II-1a         | (0, w₂, wₛ)     | ✓    | S₂, h₁ → −h₁      | none              | trivial           |
| R-II-2          | (0, w, 0)       | 0    | h₁ → −h₁, hₛ → −hₛ | 1                 | non-trivial       |
| R-II-3          | (w₁, w₂, 0)     | 0    | hₛ → −hₛ         | 1                 | non-trivial       |
| R-III-s         | (w₁, 0, wₛ)     | 0    | h₂ → −h₂         | 1                 | trivial           |
| C-I-a           | (ˆw₁, ±iˆw₁, 0)  | ✓    | cyclic Z₃         | none              | non-trivial       |
| C-III-a         | (0, ˆw₂eᵢσ₂, ˆwₛ) | ✓    | S₂, h₁ → −h₁      | none              | trivial           |
| C-III-b         | (±iˆw₁, 0, ˆwₛ)  | 0    | h₂ → −h₂         | 1                 | trivial           |
| C-III-c         | (ˆw₁eᵢσ₁, ˆw₂eᵢσ₂, 0) | 0    | hₛ → −hₛ         | 2                 | non-trivial       |
| C-IV-a          | (ˆw₁eᵢσ₁, 0, ˆwₛ)  | 0    | h₂ → −h₂         | 2                 | trivial           |

Possible DM candidates: 3 (exact S₃) and 8 (softly broken S₃) solutions.
### S₃-Symmetric Three-Higgs-Doublet Models: Dark Matter Models

| Vacuum | vevs          | λ₄ | symmetry                  | # massless states | fermions under S₃ |
|--------|---------------|----|---------------------------|-------------------|------------------|
| R-I-1  | (0, 0, wₛ)   | √  | S₃, h₁ ↦ −h₁              | none              | trivial          |
| R-I-2a | (w, 0, 0)     | √  | S₂                        | none              | non-trivial      |
| R-I-2b,2c | (w, ±√3w, 0) | √  | S₂                        | none              | non-trivial      |
| R-II-1a| (0, w₂, wₛ)  | √  | S₂, h₁ ↦ −h₁              | none              | trivial          |
| R-II-2 | (0, w, 0)     | 0  | h₁ ↦ −h₁, hₛ ↦ −hₛ        | 1                 | non-trivial      |
| R-II-3 | (w₁, w₂, 0)  | 0  | hₛ ↦ −hₛ                  | 1                 | non-trivial      |
| R-III-s| (w₁, 0, wₛ)  | 0  | h₂ ↦ −h₂                  | 1                 | trivial          |
| C-I-a  | (̂w₁, ±i ̂w₁, 0) | √  | cyclic ℤ₃                 | none              | non-trivial      |
| C-III-a | (0, ̂w₂eᵢσ₂, ̂wₛ) | √  | S₂, h₁ ↦ −h₁              | none              | trivial          |
| C-III-b| (±i ̂w₁, 0, ̂wₛ) | 0  | h₂ ↦ −h₂                  | 1                 | trivial          |
| C-III-c| (̂w₁eᵢσ₁, ̂w₂eᵢσ₂, 0) | 0  | hₛ ↦ −hₛ                  | 2                 | non-trivial      |
| C-IV-a | (̂w₁eᵢσ₁, 0, ̂wₛ) | 0  | h₂ ↦ −h₂                  | 2                 | trivial          |

Possible DM candidates: 3 (exact S₃) and 8 (softly broken S₃) solutions.
### S₃-Symmetric Three-Higgs-Doublet Models: Dark Matter Models

| Vacuum | vevs                  | \(\lambda_4\) | symmetry                        | # massless states | fermions under \(S_3\) |
|--------|-----------------------|----------------|---------------------------------|------------------|------------------------|
| R-I-1  | \((0, 0, w_S)\)       | \(\checkmark\) | \(S_3, h_1 \rightarrow -h_1\)  | none             | trivial                |
| R-I-2a | \((w, 0, 0)\)         | \(\checkmark\) | \(S_2\)                         | none             | non-trivial            |
| R-I-2b,2c | \((w, \pm \sqrt{3}w, 0)\) | \(\checkmark\) | \(S_2\)                         | none             | non-trivial            |
| R-II-1a| \((0, w_2, w_S)\)      | \(\checkmark\) | \(S_2, h_1 \rightarrow -h_1\)  | none             | trivial                |
| R-II-2 | \((0, w, 0)\)         | 0               | \(h_1 \rightarrow -h_1, h_S \rightarrow -h_S\) | 1                | non-trivial            |
| R-II-3 | \((w_1, w_2, 0)\)      | 0               | \(h_S \rightarrow -h_S\)       | 1                | non-trivial            |
| R-III-s| \((w_1, 0, w_S)\)      | 0               | \(h_2 \rightarrow -h_2\)       | 1                | trivial                |
| C-I-a  | \((\hat{\omega}_1, \pm i\hat{\omega}_1, 0)\) | \(\checkmark\) | cyclic \(\mathbb{Z}_3\)         | none             | non-trivial            |
| C-III-a| \((0, \hat{\omega}_2 e^{i\sigma_2}, \hat{\omega}_S)\) | \(\checkmark\) | \(S_2, h_1 \rightarrow -h_1\)  | none             | trivial                |
| C-III-b| \((\pm i\hat{\omega}_1, 0, \hat{\omega}_S)\) | 0               | \(h_2 \rightarrow -h_2\)       | 1                | trivial                |
| C-III-c| \((\hat{\omega}_1 e^{i\sigma_1}, \hat{\omega}_2 e^{i\sigma_2}, 0)\) | 0               | \(h_S \rightarrow -h_S\)       | 2                | non-trivial            |
| C-IV-a | \((\hat{\omega}_1 e^{i\sigma_1}, 0, \hat{\omega}_S)\) | 0               | \(h_2 \rightarrow -h_2\)       | 2                | trivial                |

Possible DM candidates: 3 (exact \(S_3\)) and 8 (softly broken \(S_3\)) solutions.
S₃-Symmetric Three-Higgs-Doublet Models: Dark Matter Models

| Vacuum       | vevs                | λ₄ | symmetry       | # massless states | fermions under S₃ |
|--------------|---------------------|----|----------------|------------------|------------------|
| R-I-1        | (0, 0, wₛ)          | ✓  | S₃, h₁ → −h₁  | none             | trivial          |
| R-I-2a       | (w, 0, 0)           | ✓  | S₂             | none             | non-trivial      |
| R-I-2b,2c    | (w, ±√3w, 0)        | ✓  | S₂             | none             | non-trivial      |
| R-II-1a      | (0, w₂, wₛ)         | ✓  | S₂, h₁ → −h₁  | none             | trivial          |
| R-II-2       | (0, w, 0)           |    | h₁ → −h₁, hₛ → −hₛ | 1              | non-trivial      |
| R-II-3       | (w₁, w₂, 0)         |    | hₛ → −hₛ     | 1                | non-trivial      |
| R-III-s      | (w₁, 0, wₛ)         |    | h₂ → −h₂     | 1                | trivial          |
| C-I-a        | (w₁, ±îw₁, 0)      | ✓  | cyclic ℤ₃     | none             | non-trivial      |
| C-III-a      | (0, ̂w₂eᵢσ₂, ̂wₛ)   | ✓  | S₂, h₁ → −h₁ | none             | trivial          |
| C-III-b      | (±îw₁, 0, ̂wₛ)      | ✓  | h₂ → −h₂     | 1                | trivial          |
| C-III-c      | (w₁eᵢσ₁, ̂w₂eᵢσ₂, 0)|  | hₛ → −hₛ     | 2                | non-trivial      |
| C-IV-a       | (w₁eᵢσ₁, 0, ̂wₛ)    |    | h₂ → −h₂     | 2                | trivial          |

Possible DM candidates: 3 (exact S₃) and 8 (softly broken S₃) solutions.
Vacuum: \( \{0, w_2, w_5\} \).
Vacuum: \( \{0, w_2, w_S\} \).

The \( \mathbb{Z}_2 \) symmetry is preserved for: 
\[
\begin{align*}
h_1 &\to -h_1, \\
\{h_2, h_S\} &\to \pm\{h_2, h_S\}.
\end{align*}
\]
Vacuum: $\{0, w_2, w_S\}$.

The $\mathbb{Z}_2$ symmetry is preserved for: $h_1 \rightarrow -h_1$, $\{h_2, h_S\} \rightarrow \pm\{h_2, h_S\}$.

The inert doublet is associated with $h_1$. 
Vacuum: \( \{0, w_2, w_S\} \).

The \( \mathbb{Z}_2 \) symmetry is preserved for: \( h_1 \rightarrow -h_1, \ \{h_2, h_S\} \rightarrow \pm\{h_2, h_S\} \).

The inert doublet is associated with \( h_1 \).

Trivial Yukawa sector \( \mathcal{L}_Y \sim 1_f \otimes 1_h \).
R-II-1a: Physical Spectrum

Vacuum: \{0, w_2, w_S\}.

The \(\mathbb{Z}_2\) symmetry is preserved for: \(h_1 \rightarrow -h_1\), \(\{h_2, h_S\} \rightarrow \pm\{h_2, h_S\}\).

The inert doublet is associated with \(h_1\).

Trivial Yukawa sector \(\mathcal{L}_Y \sim 1_f \otimes 1_h\).

Mass eigenstates:

\[
\begin{align*}
    h_1 &= \begin{pmatrix} h^+ \\ \frac{1}{\sqrt{2}} (\eta + i \chi) \end{pmatrix}, \\
    h_2 &= \begin{pmatrix} \sin \beta \ G^+ - \cos \beta \ H^+ \\ \frac{1}{\sqrt{2}} (\sin \beta \ v + \cos \alpha \ h - \sin \alpha \ H + i (\sin \beta \ G^0 - \cos \beta \ A)) \end{pmatrix}, \\
    h_S &= \begin{pmatrix} \cos \beta \ G^+ + \sin \beta \ H^+ \\ \frac{1}{\sqrt{2}} (\cos \beta \ v + \sin \alpha \ h + \cos \alpha \ H + i (\cos \beta \ G^0 + \sin \beta \ A)) \end{pmatrix}.
\end{align*}
\]
Vacuum: \( \{0, w_2, w_S\} \).

The \( \mathbb{Z}_2 \) symmetry is preserved for: \( h_1 \rightarrow -h_1 \), \( \{h_2, h_S\} \rightarrow \pm\{h_2, h_S\} \).

The inert doublet is associated with \( h_1 \).

Trivial Yukawa sector \( \mathcal{L}_Y \sim 1_f \otimes 1_h \).

Mass eigenstates:

\[
\begin{align*}
h_1 &= \begin{pmatrix} h^+ \\ \frac{1}{\sqrt{2}} (\eta + i\chi) \end{pmatrix}, \\
h_2 &= \begin{pmatrix} \sin \beta \ G^+ - \cos \beta \ H^+ \\ \frac{1}{\sqrt{2}} (\sin \beta \ v + \cos \alpha \ h - \sin \alpha \ H + i \ (\sin \beta \ G^0 - \cos \beta \ A)) \end{pmatrix}, \\
h_S &= \begin{pmatrix} \cos \beta \ G^+ + \sin \beta \ H^+ \\ \frac{1}{\sqrt{2}} (\cos \beta \ v + \sin \alpha \ h + \cos \alpha \ H + i \ (\cos \beta \ G^0 + \sin \beta \ A)) \end{pmatrix}.
\end{align*}
\]

Inert, physical states: \( \{h^\pm, \eta, \chi\} \).
Vacuum: $\{0, w_2, w_S\}$.

The $\mathbb{Z}_2$ symmetry is preserved for: $h_1 \rightarrow -h_1$, \(\{h_2, h_S\} \rightarrow \pm \{h_2, h_S\}\).

The inert doublet is associated with $h_1$.

Trivial Yukawa sector $\mathcal{L}_Y \sim 1_f \otimes 1_h$.

Mass eigenstates:

\[
h_1 = \begin{pmatrix} h^+ \\ \frac{1}{\sqrt{2}} (\eta + i\chi) \end{pmatrix},
\]

\[
h_2 = \begin{pmatrix} \sin \beta \, G^+ - \cos \beta \, H^+ \\ \frac{1}{\sqrt{2}} (\sin \beta \, v + \cos \alpha \, h - \sin \alpha \, H + i (\sin \beta \, G^0 - \cos \beta \, A)) \end{pmatrix},
\]

\[
h_S = \begin{pmatrix} \cos \beta \, G^+ + \sin \beta \, H^+ \\ \frac{1}{\sqrt{2}} (\cos \beta \, v + \sin \alpha \, h + \cos \alpha \, H + i (\cos \beta \, G^0 + \sin \beta \, A)) \end{pmatrix}.
\]

Inert, physical states: $\{h^\pm, \eta, \chi\}$. Two possible DM candidates: $\{\eta, \chi\}$. 
Introduction

R-II-1a: Physical Spectrum

Vacuum: \( \{0, w_2, w_S\} \).

The \( \mathbb{Z}_2 \) symmetry is preserved for: \( h_1 \rightarrow -h_1, \) \( \{h_2, h_S\} \rightarrow \pm \{h_2, h_S\} \).

The inert doublet is associated with \( h_1 \).

Trivial Yukawa sector \( \mathcal{L}_Y \sim 1_f \otimes 1_h \).

Mass eigenstates:

\[
\begin{align*}
    h_1 &= \begin{pmatrix} h^+ \\ \frac{1}{\sqrt{2}} (\eta + i \chi) \end{pmatrix}, \\
    h_2 &= \begin{pmatrix} \sin \beta G^+ - \cos \beta H^+ \\ \frac{1}{\sqrt{2}} \left( \sin \beta v + \cos \alpha h - \sin \alpha H + i \left( \sin \beta G^0 - \cos \beta A \right) \right) \end{pmatrix}, \\
    h_S &= \begin{pmatrix} \cos \beta G^+ + \sin \beta H^+ \\ \frac{1}{\sqrt{2}} \left( \cos \beta v + \sin \alpha h + \cos \alpha H + i \left( \cos \beta G^0 + \sin \beta A \right) \right) \end{pmatrix}.
\end{align*}
\]

Inert, physical states: \( \{h^\pm, \eta, \chi\} \). Two possible DM candidates: \( \{\eta, \chi\} \).

Active, physical states: \( \{H^\pm, h - H, A\} \).
The model is analysed using the following input (6 masses + 2 angles):

- Mass of the SM-like Higgs is fixed at \( m_h = 125.25 \) GeV;
- The Higgs basis rotation angle \( \beta \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \)
  and the \( h-H \) diagonalisation angle \( \alpha \in [0, \pi] \);
- The charged scalar masses \( m_{\varphi^\pm} \in [0.07, 1] \) TeV;
- The inert sector masses \( m_{\varphi_i} \in [0, 1] \) TeV.
  Either \( \eta \) or \( \chi \) could be a DM candidate, whichever is lighter;
- The active sector masses \( \{m_H, m_A\} \in [m_h, 1 \) TeV].
The model is analysed using the following input (6 masses + 2 angles):

- Mass of the SM-like Higgs is fixed at $m_h = 125.25$ GeV;
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- The charged scalar masses $m_{\varphi^\pm_i} \in [0.07, 1]$ TeV;
- The inert sector masses $m_{\varphi_i} \in [0, 1]$ TeV.
  Either $\eta$ or $\chi$ could be a DM candidate, whichever is lighter;
- The active sector masses $\{m_H, m_A\} \in [m_h, 1$ TeV$]$

Both theoretical and experimental constraints, at 3-$\sigma$, are evaluated:

- Cut 1: perturbativity, stability, unitarity checks, LEP constraints;
- Cut 2: SM-like gauge and Yukawa sector, electroweak precision observables and $B$ physics;
- Cut 3: $h \rightarrow \{\text{invisible, } \gamma\gamma]\$ decays, $\Omega_{\text{CDM}} h^2$, direct searches;
Trilinear and quartic couplings are not tuneable!

\[ g(\mathbf{XXh}) = g(\mathbf{XXhh}) = 1 \]

\[ v \]

\[ \Omega h^2 \]

\[ m_h^2 + 2m_X^2 \]

R-II-1a: Relic Density

\[ \eta < \chi \]

\[ \chi < \eta \]
Trilinear and quartic couplings are not tuneable!

\[
\frac{g(XXh)}{v} \bigg|_{SM} = g(XXhh) \bigg|_{SM} = \frac{1}{v^2} \left[ m_h^2 + 2m_X^2 \right].
\]
All constraints satisfied:

- $m_\eta < m_\chi$: no overlap in all parameters;
- $m_\eta > m_\chi$: overlap in all parameters for $m_\chi \in [52.5, 89]$ GeV;
R-II-1a: Cut 3

Cut 1: Unitarity < $16\pi$

Cut 2: 3-$\sigma$

Cut 2: 2-$\sigma$

Cut 3
R-II-1a: Cut 3

Cut 1: Unitarity $< 16\pi$

Cut 2: 3 - $\sigma$

Cut 2: 2 - $\sigma$

Cut 3
R-II-1a: Cut 3

Cut 1: Unitarity $< 16\pi$

Cut 2: $3 - \sigma$

Cut 2: $2 - \sigma$

Cut 3
R-II-1a: Cut 3

SCALAR DM MASS RANGES

| 50 GeV | 100 GeV | 200 GeV | 500 GeV | 1000 GeV |
|-------|--------|---------|--------|---------|
| $Z_2$ | IDM    |         |        |         |
| $Z_2$ | IDM2   | 3HDM    |        |         |
| $CP$  | 3HDM   |         |        |         |
| $\chi$| R-II-1a|         |        |         |
Conclusions

- Multi-Higgs-doublet models are phenomenology rich and can accommodate a dark matter candidate;
- Possible DM candidates were identified within $S_3$-3HDM;
- Analysed the R-II-1a model, and found a viable dark matter region $[52.5, 89]$ GeV;

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We can generate the following $S_3$ structures:

1: $[2 \otimes 2]_1$, $[1 \otimes 1]_1$, $[1' \otimes 1']_1$;

1': $[2 \otimes 2]_1'$, $[1 \otimes 1']_1'$, $[1' \otimes 1]_1'$;

2: $[2 \otimes 2]_2$, $[1 \otimes 2]_2$, $[2 \otimes 1]_2$, $[1' \otimes 2]_2$, $[2 \otimes 1']_2$;

Products:

\[
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix}
_2 \otimes \begin{pmatrix}
y_1 \\
y_2
\end{pmatrix}
_2 = (x_1 y_1 + x_2 y_2)_1 + (x_1 y_2 - x_2 y_1)_1' + \begin{pmatrix}
x_1 y_2 + x_2 y_1 \\
x_1 y_1 - x_2 y_2
\end{pmatrix}_2,
\]

\[
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix}
_2 \otimes (y')_1' = \begin{pmatrix}
-x_2 y' \\
x_1 y'
\end{pmatrix}_2,
\]

\[
(x')_1' \otimes (y')_1' = (x'y')_1.
\]

$\nu_{3\text{HDM}} = \mu_1^2 [2 \otimes 2]_1 + \mu_0^2 [1 \otimes 1]_1$

\[+ \lambda_1 \left( [2 \otimes 2]_1 \otimes [2 \otimes 2]_1 \right) + \lambda_2 \left( [2 \otimes 2]_1' \otimes [2 \otimes 2]_1' \right) + \lambda_3 \left( [2 \otimes 2]_2 \otimes [2 \otimes 2]_2 \right) + \lambda_4 \left\{ \left( [2 \otimes 2]_2 \otimes [1 \otimes 2]_2 \right) + \sym \right\} + \lambda_5 \left( [2 \otimes 2]_1 \otimes [1 \otimes 1]_1 \right) + \lambda_6 \left( [1 \otimes 2]_2 \otimes [2 \otimes 1]_2 \right) + \lambda_7 \left\{ \left( [1 \otimes 2]_2 \otimes [1 \otimes 2]_2 \right) + \sym \right\} + \lambda_8 \left( [1 \otimes 1]_1 \otimes [1 \otimes 1]_1 \right).\]
Appendix

R-II-1a masses:

\[ m_{h^+}^2 = -2\lambda_3 w_s^2 + \frac{5}{2} \lambda_4 w_2 w_s - \frac{1}{2} (\lambda_6 + 2\lambda_7) w_s^2, \]

\[ m_{H^+}^2 = \frac{\nu^2}{2w_s} \left[ \lambda_4 w_2 - (\lambda_6 + 2\lambda_7) w_s \right], \]

\[ m_n^2 = \frac{9}{2} \lambda_4 w_2 w_s, \]

\[ m_{\chi}^2 = -2(\lambda_2 + \lambda_3) w_2^2 + \frac{5}{2} \lambda_4 w_2 w_s - 2\lambda_7 w_s^2, \]

\[ m_A^2 = \frac{\nu^2}{2w_s} (\lambda_4 w_2 - 4\lambda_7 w_s), \]

\[ m_h^2 = \frac{1}{4w_s^2} \left[ 4 (\lambda_1 + \lambda_3) w_2^2 w_s^2 + \lambda_4 w_2 w_s \left( w_2^2 - 3w_s^2 \right) + 4\lambda_8 w_s^4 - w_s \Delta \right], \]

\[ m_H^2 = \frac{1}{4w_s^2} \left[ 4 (\lambda_1 + \lambda_3) w_2^2 w_s^2 + \lambda_4 w_2 w_s \left( w_2^2 - 3w_s^2 \right) + 4\lambda_8 w_s^4 + w_s \Delta \right], \]

where

\[ \Delta^2 = 16 (\lambda_1 + \lambda_3)^2 w_2^4 w_s^2 - 8 (\lambda_1 + \lambda_3) w_2^2 w_s \left[ \lambda_4 \left( w_2^3 + 3w_2 w_s^2 \right) + 4\lambda_8 w_s^3 \right] \]

\[ + 16\lambda_a^2 w_2^4 w_s^4 - 48\lambda_4 \lambda_a w_2^3 w_s^3 + \lambda_4^2 \left( w_2^6 + 42w_2^4 w_s^2 + 9w_2^2 w_s^4 \right) \]

\[ + 8\lambda_4 \lambda_8 w_2 w_s^3 \left( w_2^2 + 3w_s^2 \right) + 16\lambda_8^2 w_s^6. \]
Appendix

R-II-1a gauge couplings:

\[
\mathcal{L}_{VVH} = \left[ \frac{g}{2 \cos \theta_W} m_Z Z_{\mu} Z^{\mu} + g m_W W_{\mu}^{+} W_{\mu}^{-} \right] \left[ \sin(\alpha + \beta) h + \cos(\alpha + \beta) H \right] ,
\]

\[
\mathcal{L}_{VHH} = - \left[ \frac{g}{2 \cos \theta_W} Z^{\mu} \left[ \eta \partial^{\mu}_\nu X - \cos(\alpha + \beta) h \partial^{\mu}_\nu A + \sin(\alpha + \beta) H \partial^{\mu}_\nu A \right] \right.
\]

\[
- \frac{g}{2} \left\{ i W_{\mu}^{+} \left[ i h - \partial^{\mu}_\nu \chi + h - \partial^{\mu}_\nu \eta - \cos(\alpha + \beta) H - \partial^{\mu}_\nu h \right.ight.
\]

\[
+ \sin(\alpha + \beta) H - \partial^{\mu}_\nu H + i H - \partial^{\mu}_\nu A \left. \right] + \text{h.c.} \right\}
\]

\[
+ \left[ i e A^{\mu} + \frac{ig \cos(2 \theta_W)}{2 \cos \theta_W} Z^{\mu} \right] \left( h^{+} \partial^{\mu}_\nu h^{-} + H^{+} \partial^{\mu}_\nu H^{-} \right) ,
\]

\[
\mathcal{L}_{VVHH} = \left[ \frac{g^2}{8 \cos^2 \theta_W} Z_{\mu} Z^{\mu} + \frac{g^2}{4} W_{\mu}^{+} W_{\mu}^{-} \right] \left( \eta^2 + \chi^2 + h^2 + H^2 + A^2 \right)
\]

\[
+ \left\{ \left[ \frac{eg}{2} A^{\mu} W_{\mu}^{+} - \frac{g^2 \sin^2 \theta_W}{2 \cos \theta_W} Z^{\mu} W_{\mu}^{+} \right] \left[ \eta h^{-} + i \chi h^{-} - \cos(\alpha + \beta) h H^{-} \right.ight.
\]

\[
+ \sin(\alpha + \beta) H H^{-} + i A H^{-} \right] + \text{h.c.} \right\}
\]

\[
+ \left[ e^2 A^{\mu} A^{\mu} + eg \frac{\cos(2 \theta_W)}{\cos \theta_W} A^{\mu} Z^{\mu} + \frac{g^2}{4} \frac{\cos^2(2 \theta_W)}{\cos^2 \theta_W} Z^{\mu} Z^{\mu} + \frac{g^2}{2} W_{\mu}^{+} W_{\mu}^{-} \right] \left( h^{-} h^{+} + H^{+} H^{-} \right) .
\]
R-II-1a fermionic couplings:

\[ g(h \bar{f} f) = -i \frac{m_f}{v} \frac{\sin \alpha}{\cos \beta}, \quad g(H \bar{f} f) = -i \frac{m_f}{v} \frac{\cos \alpha}{\cos \beta}, \]
\[ g(A \bar{u} u) = -\gamma_5 \frac{m_u}{v} \tan \beta, \quad g(A \bar{d} d) = \gamma_5 \frac{m_d}{v} \tan \beta, \]

and for the leptonic sector, the Dirac mass terms would lead to similar relations.

\[ g(H^+ \bar{u}_i d_j) = i \frac{\sqrt{2}}{v} \tan \beta [P_L m_u - P_R m_d] (V_{CKM})_{ij}, \]
\[ g(H^- \bar{d}_i u_j) = i \frac{\sqrt{2}}{v} \tan \beta [P_R m_u - P_L m_d] (V_{CKM}^\dagger)_{ji}, \]
\[ g(H^+ \bar{\nu} l) = -i \frac{\sqrt{2} m_l}{v} \tan \beta P_R, \]
\[ g(H^- \bar{l} \nu) = -i \frac{\sqrt{2} m_l}{v} \tan \beta P_L. \]
We adopt 3-σ bounds from PDG [2021]:

\[ \kappa_{VV}^2 \equiv |\sin(\alpha + \beta)|^2 \in \{1.19 \pm 3 \sigma\}, \text{ which comes from } h_{SM} W^+ W^-, \]

\[ \kappa_{ff}^2 \equiv \left| \frac{\sin \alpha}{\cos \beta} \right|^2 \in \{1.04 \pm 3 \sigma\}, \text{ which comes from } h_{SM} \bar{b}b. \]

We impose the same sign for these two couplings not to spoil the interference required for \( h_{SM} \to \gamma\gamma \).
Appendix

We adopt the experimental value, \( \text{Br} (\bar{B} \rightarrow X(s)\gamma) \times 10^4 = 3.32 \pm 0.15 \) [PDG 2021] and impose an \((n = 3)\)-\(\sigma\) tolerance, together with an additional 10 per cent computational uncertainty,

\[
\text{Br} (\bar{B} \rightarrow X(s)\gamma) \times 10^4 = 3.32 \pm \sqrt{(3.32 \times 0.1)^2 + (0.15 n)^2}.
\]

The acceptable region, corresponding to the 3-\(\sigma\) bound, is [2.76; 3.88].
Appendix

Scatter plots of additional contributions to the di-photon decay amplitudes, normalised to the SM value, expressed in per cent:

![Scatter plots](image)

Two-gluon versus di-photon Higgs-like particle branching ratios, normalised to the SM value:

![Branching ratios](image)
### Appendix

| Parameter | BP 1   | BP 2   | BP 3   | BP 4   | BP 5   | BP 6   | BP 7   | BP 8   | BP 9   |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| DM ($\chi$) mass [GeV] | 52.6   | 56.1   | 59.6   | 63.02  | 65.7   | 70.3   | 75.0   | 82.2   | 88.6   |
| $\eta$ mass [GeV] | 62.7   | 203.8  | 270.4  | 169.3  | 150.5  | 157.7  | 202.8  | 127.8  | 210.7  |
| $h^+$ mass [GeV] | 115.4  | 167.4  | 273.6  | 188.6  | 214.1  | 170.5  | 232.0  | 151.8  | 243.0  |
| $H^+$ mass [GeV] | 192.6  | 369.5  | 367.4  | 246.6  | 265.5  | 405.8  | 319.8  | 410.6  | 311.9  |
| $H^-$ mass [GeV] | 263.9  | 349.3  | 332.9  | 276.3  | 298.2  | 402.0  | 368.5  | 405.2  | 317.6  |
| $A$ mass [GeV] | 179.2  | 208.0  | 190.7  | 173.9  | 205.2  | 255.3  | 253.1  | 330.0  | 247.0  |
| $\beta/\pi$ | 0.162  | -0.204 | -0.201 | -0.165 | 0.163  | 0.220  | 0.203  | -0.218 | 0.183  |
| $\alpha/\pi$ | 0.252  | 0.763  | 0.765  | 0.752  | 0.254  | 0.225  | 0.239  | 0.769  | 0.238  |
| $\sigma_S$ [10^{-11} pb] | 0.029  | 1.456  | 4.928  | 0.176  | 5.326  | 1.341  | 2.711  | 8.553  | 4.491  |
| $\eta \rightarrow \chi qq$ [%] | 63.27  | 54.38  | 54.35  | 53.95  |  |  |  |  |  |
| $\eta \rightarrow \chi b\bar{b}$ [%] | 0.48   | 14.80  | 14.85  | 13.90  |  |  |  |  |  |
| $\eta \rightarrow \chi\nu\bar{\nu}$ [%] | 24.62  | 20.48  | 20.46  | 20.72  |  |  |  |  |  |
| $\eta \rightarrow \chi l\bar{l}$ [%] | 11.61  | 10.33  | 10.33  | 11.42  |  |  |  |  |  |
| $\eta \rightarrow \chi Z$ [%] | 99.98  | 53.09  | 100    | 100    | 100    | 100    | 100    | 100    | 100    |
| $\eta \rightarrow \chi A$ [%] | 46.91  |  |  |  |  |  |  |  |  |
| $h^+ \rightarrow \chi W^+$ [%] | 100    | 100    | 99.98  | 99.89  | 99.99  | 99.99  | 99.99  | 99.99  | 99.99  |
| $h^+ \rightarrow \eta q\bar{q}$ [%] | 20.18  | 0.30   |  |  |  |  |  |  |  |
| $h^+ \rightarrow \eta l\bar{l}$ [%] | 9.88   | 0.16   |  |  |  |  |  |  |  |
| $h^+ \rightarrow \chi qq$ [%] | 46.94  | 66.82  |  |  |  |  |  |  |  |
| $h^+ \rightarrow \chi l\bar{l}$ [%] | 22.99  | 32.71  |  |  |  |  |  |  |  |
| $H^+ \rightarrow t\bar{b}$ [%] | 9.07   | 43.69  | 58.23  | 95.06  | 95.78  | 30.95  | 96.25  | 31.54  | 93.59  |
| $H^+ \rightarrow AW^+$ [%] | 20.56  | 35.74  | 0.29   | 0.06   | 8.66   | 0.05   | 0.05   | 0.05   | 0.05   |
| $H^+ \rightarrow HW^+$ [%] | 1.94   | 2.67   | 4.46   | 4.00   | 1.23   | 2.86   | 1.15   | 6.20   |  |  |
| $H^+ \rightarrow h^+\eta$ [%] | 85.9   | 43.74  | 61.68  |  |  |  |  |  |  |
| $H^+ \rightarrow h^+\chi$ [%] | 5.0    | 33.74  | 3.26   | 15.36  | 0.68   | 5.53   |  |  |  |
| $H \rightarrow \chi\chi$ [%] | 0.15   | 0.03   | 0.07   | 0.87   | 15.03  | 11.34  | 7.63   | 63.75  |  |  |
| $H \rightarrow \eta\eta$ [%] | 89.9   | 24.89  | 25.31  |  |  |  |  |  |  |
| $H \rightarrow hh$ [%] | 3.07   | 2.64   | 9.40   | 34.59  | 33.53  | 1.33   | 13.43  | 0.88   | 14.72  |
| $H \rightarrow A\bar{Z}$ [%] | 0.09   | 13.55  | 70.93  | 13.91  | 2.87   | 7.61   | 22.78  | 0.07   |  |  |
| $H \rightarrow W^+W^-$ [%] | 4.06   | 3.13   | 10.40  | 34.98  | 33.35  | 1.89   | 16.32  | 1.26   | 14.70  |
| $H \rightarrow ZZ$ [%] | 1.75   | 1.43   | 4.77   | 15.29  | 14.82  | 0.88   | 7.55   | 0.59   | 6.62   |
| $H \rightarrow h^+h^-$ [%] | 0.8    | 78.59  |  |  | 52.94  | 56.33  |  |  |  |
| $H \rightarrow q\bar{q}$ [%] | 0.02   | 4.40   | 0.32   | 0.34   | 10.43  | 28.52  | 8.00   | 0.12   |  |  |
| $A \rightarrow \eta\chi$ [%] | 99.97  | 99.32  | 99.01  |  |  |  |  |  |  |
| $A \rightarrow b\bar{b}$ [%] | 0.02   | 79.78  | 84.15  | 84.63  | 75.28  | 0.07   | 8.84   | 0.42   | 4.76   |
| $A \rightarrow q\bar{q}$ [%] | 3.56   | 3.75   | 3.77   | 3.36   | 0.39   | 0.21   |  |  |  |
| $A \rightarrow \tau^+\tau^-$ [%] | 9.85   | 10.19  | 10.00  | 9.24   | 1.13   | 0.61   |  |  |  |
| $A \rightarrow hZ$ [%] | 6.81   | 1.87   | 1.55   | 12.08  | 0.6    | 89.63  | 0.96   | 94.42  |  |  |
Introduction Inert Doublet Model Three-Higgs-Doublet Models R-II-1a

Appendix