EM algorithm applied for estimating non-stationary region boundaries using electrical impedance tomography

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Abstract. EIT has been used for the dynamic estimation of organ boundaries. One specific application in this context is the estimation of lung boundaries during pulmonary circulation. This would help track the size and shape of lungs of the patients suffering from diseases like pulmonary edema and acute respiratory failure (ARF). The dynamic boundary estimation of the lungs can also be utilized to set and control the air volume and pressure delivered to the patients during artificial ventilation. In this paper, the expectation-maximization (EM) algorithm is used as an inverse algorithm to estimate the non-stationary lung boundary. The uncertainties caused in Kalman-type filters due to inaccurate selection of model parameters are overcome using EM algorithm. Numerical experiments using chest shaped geometry are carried out with proposed method and the performance is compared with extended Kalman filter (EKF). Results show superior performance of EM in estimation of the lung boundary.

1. Introduction

In many clinical and research studies long term ventilation has been applied and investigated. Accurate non-invasive method is useful for apnea detection in infants and adults, ventilation in intensive care units and lung monitoring for respiratory complexes [1]. EIT has been applied as an imaging technique to measure the resistivity changes for monitoring apnea and edema [2]. During the respiratory cycle, the air flows in and out of the lungs resulting in resistivity change. If the resistivity distribution inside the lung, thorax and the other tissues are assumed to be known a priori then the unknown information can be size and shape of lung. The volume changes in the lung can be monitored and thus can provide valuable information when applying artificial ventilation. The boundaries are assumed to be smooth, therefore, can be represented using Fourier series [3]. The Fourier coefficients determine the size and shape of the boundary to be estimated. The fast moving boundary can be estimated using dynamic estimation methods.

In dynamic estimation, the inverse problem is treated as a state estimation problem and the time varying boundary coefficients are the state variables to be estimated [4, 5]. For the application of
Kalman-type estimators like EKF, UKF, the model parameters (dynamics of the evolution, the initial states, and the noise covariance of process and measurement models) have to be predefined [5]. In real situations, the dynamics of the evolution and noise matrices are complex and it is difficult to model them in a prior form. In situations, when there is uncertainty in determining the model parameters, the estimation performance of the Kalman-type filters is affected. Therefore, in this study, we apply EM as an inverse algorithm to reduce model uncertainties in estimating the closed boundaries [6].

2. Forward problem: boundary representation

Let us assume that the outer boundary of the body, that is, \( \partial \Omega \) is known. If phase boundaries \( C_k \in \Omega \) of the objects are sufficiently smooth, they can be approximated as a linear combination of known functions ([3, 5]),

\[
C_k(s) = \left( \begin{array}{c} x_1(s) \\ y_1(s) \end{array} \right) = \sum_{i=1}^{N_\gamma} \gamma_i \theta_i^x(s) + \sum_{j=1}^{N_\theta} \theta_j(s) \theta_j^y(s)
\]

where \( \theta_i^x(s) \) and \( \theta_j^y(s) \) are periodic differentiable basis functions, \( P \) is the number of distinct regions and \( N_\theta \) is the number of basis functions. In this paper, we express the phase boundaries as Fourier series in two-dimensional coordinates with respect to parameter \( s \), i.e., basis functions are of the form,

\[
\theta_i^x(s) = \sin(2\pi \frac{\omega_1}{2} s), \quad \omega_1 = 2, 4, 6, \ldots, N_\theta - 1
\]

\[
\theta_j^y(s) = \cos(2\pi \frac{\omega_1 - 1}{2} s), \quad \omega_1 = 1, 3, 5, \ldots, N_\theta,
\]

where \( s \in [0,1] \) and \( \alpha \) denotes either \( x \) or \( y \). Furthermore, using (1), the boundary coefficients are identified with the vector \( \gamma \in \mathbb{R}^{N_\theta-1} \), that is,

\[
\gamma = (\gamma_1^x, \ldots, \gamma_{N_\theta-1}^x, \gamma_1^y, \ldots, \gamma_{N_\theta-1}^y, \gamma_1^z, \ldots, \gamma_{N_\theta-1}^z)^T
\]

3. Estimation of Fourier coefficients using EM algorithm

EM algorithm is a kind of maximum likelihood estimator which estimates the state variables and model parameters by minimizing the likelihood function using E and M step. In E step, the hidden variables ( \( \gamma \) ) are estimated with the assumed model parameters ( \( \theta = [\mu, IT, F, Q, R] \)). In M step, using estimated \( \gamma \), model parameters \( \theta \) are estimated such that they increase the log-likelihood function. The EM algorithm is formulated for boundary estimation based on Kalman smoother. In E step, the expectation of the hidden variables is obtained using RTS Kalman smoother using \( n \) observations. The computed hidden variables are used to estimate the model parameters in the M step as follows [6],

\[
F = BA^{-1}
\]

\[
Q = n^{-1}(C - BA^{-1}B^T)
\]

\[
R = n^{-1}\sum_{k=1}^{n} J_k^T P_{in} J_k + (V_k - J_k \gamma_{in})(V_k - J_k \gamma_{in})^T
\]

\[
\mu(r+1) = \gamma_{in} \cdot \prod(r+1) = p_{in},
\]

where

\[
A = \sum_{k=1}^{n} (P_{i-k, in} + \gamma_{i-k, in} \gamma_{i-k, in}^T)
\]

\[
B = \sum_{k=1}^{n} (P_{i+k, in} + \gamma_{i+k, in} \gamma_{i+k, in}^T)
\]

\[
C = \sum_{k=1}^{n} (P_{i+n, in} + \gamma_{i+n, in} \gamma_{i+n, in}^T)
\]
4. Results and discussion

In order to test the EM algorithm, a 16-electrode chest shaped mesh with size and shape as that of an average human being is chosen. The conductivities of the thorax and the other organs inside chest are assumed to be known *a priori* figure 1(a) [1]. Opposite method is used as a current injection method and to simulate the dynamic scenario it is assumed that lung boundary changes after every single current injection. Different meshes with 5370 elements and 5220 elements are used in forward and inverse solver to avoid inverse crime. Four frames of data are considered in which the first two frames is for breathing or inhaling where the lung expands and two frames is for breathing out where the lung contracts. To represent the true dynamics of expansion and contraction, first-order kinematic model is used and the resulting scenario is shown in figure 1(b). In inverse computation, it is assumed that we do not have the prior knowledge of the evolution therefore random-walk model is used. To simulate with that of real conditions, random noise of zero-mean Gaussian noise having STD 1% of the value of the corresponding voltage is added to the computed voltage.

![Figure 1](image1.png)

**Figure 1.** Chest shaped mesh used in numerical results (a) conductivity profile (b) generated scenario.

![Figure 2](image2.png)

**Figure 2.** Numerical results for estimated lung boundary with 1% noise. Solid line represents the true boundary, dotted line is with EM algorithm and dashed line is using EKF.
Reconstructed lung boundary for 1% relative noise is shown in figure 2. Alternative images are displayed in the reconstructed boundary. From figure 2, it can be noticed that EM has better estimation of the lung boundary as compared to EKF. The parameters used in the simulation with EKF and EM are given in Table 1. EM was able to track the fast changes of expansion and contraction of lung in one respiration cycle. The superior performance of EM is due to the estimation of model parameters (initial states, state evolution matrix and noise covariance matrix). The transition period is less as compared to EKF. For quantitative comparison, RMSE for boundary coefficients is computed and is shown in figure 3. It can be seen that EM has lower RMSE values as compared to EKF in all the iterations.

Table 1. Initial settings of model parameters used in EM and EKF.

| E | Q | R | P_{00} |
|---|---|---|--------|
| I_M ∈ R^{M×M} | 0.5I_M | 500I_M | 0.5I_M |

5. Conclusions

This paper presents an application of 2D-EIT using EM algorithm to estimate non-stationary lung boundary during respiratory cycle. The boundary of lungs and other organs are assumed to be smooth and therefore are represented using truncated Fourier series. It is assumed that the conductivities of thorax and other organs inside chest are constant and are known a priori. In respiratory cycle, it is assumed that the boundaries of heart and backbone are known and the unknown boundary is that of lung. The lung expands and contracts during the respiratory cycle and these changes are estimated using EM algorithm. The EM algorithm is formulated for boundary estimation in EIT and the model parameters along with the boundary coefficients are estimated. Numerical experiments are carried out to test the performance of the proposed method. Through the results, it is found that EM has better estimation of lung boundaries and thus can be used as an imaging tool for artificial ventilation.

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