Optimal control of stochastic magnetization dynamics by spin current

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Abstract – Fluctuation-induced stochastic magnetization dynamics plays an important role in spintronics devices. Here we propose that it can be optimally controlled by spin currents to minimize or maximize the Freidlin-Wentzell action functional of the system hence to increase or decrease the probability of the large fluctuations. We apply this method to study the thermally activated magnetization switching problem and to demonstrate the merits of the optimal control strategy.

Introduction. – One of the key issues in the modern magneto-electronics is the reliable control of the magnetization dynamics. In comparison with the conventional Oersted field generated by the electric current, the spin transfer torque (STT) [1,2] acting on the magnet by the spin current has been proven to be a more efficient manipulation method as the magnetic structures are miniaturized towards the nanoscale [3,4]. Various devices based on this principle has been realized, such as the STT-magneto-resistive random access memory, spin-torque oscillators, spin logic, etc. [5] Although the effect of STT on the deterministic magnetization dynamics has been extensively studied, its effect on the magnetization fluctuations, which can induce the stochastic dynamics of the magnetization and have a prominent impact on the performance of the spintronics devices [6–12], has only started to receive attention very recently. It was demonstrated experimentally that spin current may either suppress or enhance the magnetization fluctuations [13], which coincides with the theoretical studies [10,11,14]. Some control strategies have also been proposed [15,16]. However, there is controversy whether the effective damping should be decreased or increased in order to suppress the magnetization fluctuations [15,16], and there is lack of solid theoretical foundation for the choice of control strategies [16]. Therefore, a better understanding of the effect of spin current on the magnetization fluctuation and study of better control strategy are demanded for further applications of the spin current in spintronics devices.

Unlike the control problem for deterministic systems, the main difficulty of the control of stochastic dynamics comes from the random behavior of the systems in the presence of fluctuations. The realization of such kind of control is meaningful only in the statistical sense. However, randomness does not imply that there is no rule for the dynamics. For example, when the noise can be considered to be weak, the large fluctuations, which appear rarely but constitute the most significant part of the stochastic dynamics, will obey the large deviation principle [17,18]. Although the exact instant at which these large fluctuation will appear is unpredictable, their patterns are almost deterministic as long as the noise is small enough. Based on this observation, the general theory to control the large fluctuations has been developed [19–24], and has been applied to several physical systems, such as analogy electronic circuits [20,21,25], chemical kinetics and reaction dynamics [26,27], chaos [28,29], optical trapping particles [30], etc. In this paper, this theory is applied to discuss how to use the spin current to optimally control the thermally activated magnetization dynamics. Especially, we will analyze how the spin current will act on the stochastic magnetization dynamics based on the large deviation principle, and how to optimize the spin current pulses to control the large magnetization fluctuations under certain constraint conditions. The application of the idea to the case of thermally activated magnetization switching in the presence of STT will be demonstrated as an example. The relevance of the present...
study to the practical applications in spintronics industry and possible difficulties will also be discussed.

**General formalism.** For simplicity, we consider the dynamics of the single-domain magnet with magnetization vector \( \mathbf{M} = M_s \mathbf{m} \) and volume \( V \), where \( M_s \) denotes the constant magnetization magnitude and \( \mathbf{m} \) denotes the unit direction vector. The dynamics of the single-domain magnet is driven by the deterministic effective magnetic field given by the micromagnetic energy density, the STT due to the spin current, and the stochastic magnetic field arising from the thermal fluctuation. Explicitly, the effective magnetic field is \( \mathbf{H}_{\text{eff}} = -\nabla_m E(\mathbf{m})/M_s \), with the micromagnetic energy density \( E(\mathbf{m}) \). The spin current \( I^s \) is defined by its amplitude \( a_J \) and spin polarization vector \( \mathbf{P} \), and we will only consider the adiabatic STT [1,2] although the non-adiabatic STT can be important in some cases [31]. The fluctuating magnetic field is assumed as \( \sqrt{\gamma} \mathbf{W} \), where the amplitude \( \epsilon = \frac{2a_J K_B T}{\gamma^2 M_s V} \) is proportional to temperature \( T \) and Gilbert damping coefficient \( \alpha \), and \( \mathbf{W} \) is the Gaussian white-noise process satisfying \( \langle \mathbf{W}(t) \rangle = 0 \) and \( \langle \mathbf{W}(t) \mathbf{W}^\dagger(t') \rangle = \delta_{jj}(t-t') \). Here we have introduced the Boltzmann constant \( K_B \) and the gyromagnetic ratio \( \gamma \). The form of the random magnetic field taken here implies that the fluctuation-dissipation theorem is assumed to be still valid even in the presence of spin current. This should be reasonable because the magnetization dynamics considered here is dominated by the thermal fluctuations and the spin current will be treated perturbatively. The far-from-equilibrium magnetization dynamics driven by strong spin current and the possible deviations of the fluctuation-dissipation theorem are out of the scope of the present study. Then the stochastic LLG equation can be written in the compact form [32]

\[
\dot{\mathbf{m}} = \mathbf{b}(\mathbf{m}) + \sqrt{\gamma} \sigma(\mathbf{m}) \mathbf{W},
\]

where the vector \( \mathbf{b} \) and the matrix \( \sigma \) are defined as

\[
\mathbf{b}(\mathbf{m}) = \sigma \mathbf{H}_{\text{eff}} - a_J \gamma K_s \mathbf{P}, \quad \sigma(\mathbf{m}) = \gamma' (K_A + aK_S).
\]

Here we have \( \gamma' = \frac{\gamma}{1+\gamma^2} \) and the elements of the anti-symmetric matrix \( K_A \) and symmetric matrix \( K_S \) are [32]

\[
(K_A)_{\mu\nu} = \epsilon_{\mu\nu\rho} m_{\rho}, \quad (K_S)_{\mu\nu} = \delta_{\mu\nu} - m_{\mu} m_{\nu}.
\]

Under the thermal noise, the probability \( P(\mathbf{m}_i \to \mathbf{m}_f) \) that the magnet will fluctuate from the initial state \( \mathbf{m}_i \) to the final state \( \mathbf{m}_f \) is contributed by all the possible stochastic trajectories connecting \( \mathbf{m}_i \) and \( \mathbf{m}_f \). According to the large deviation principle [17,18,32–34], the probability for a given stochastic trajectory \( \{\mathbf{m}(t)\} \) is proportional to \( e^{-S[\mathbf{m}(t),I^s(t)]/\epsilon} \), where \( S[\mathbf{m}(t),I^s(t)] \) is the Freidlin-Wentzell (FW) action functional and is defined for the magnetic system here as [32–34]

\[
S[\mathbf{m}(t),I^s(t)] = \frac{1}{2} \int [\sigma^{-1}(\dot{\mathbf{m}} - \mathbf{b})]^2 dt.
\]

Then the probability \( P(\mathbf{m}_i \to \mathbf{m}_f) \) is written in the path integral form as

\[
P(\mathbf{m}_i \to \mathbf{m}_f) \propto \int \mathcal{D}[\mathbf{m}(t)] e^{-S[\mathbf{m}(t),I^s(t)]/\epsilon}.
\]

As shown here, the probability \( P \) is dependent on the spin current \( I^s \), which can then be optimally chosen to change the probability. Generally, in order to minimizing or maximizing the probability \( P \), the optimal spin current \( I_{\text{opt}}^s \) is obtained by solving the variational problem over the function space of spin current with certain constraint conditions. Mathematically, this is formulated as

\[
\delta\{P(\mathbf{m}_i \to \mathbf{m}_f) + \Lambda \mathcal{F}[I^s(t)]\} = 0,
\]

where we have set the constraint conditions \( \mathcal{F}[I^s(t)] = 0 \) for the spin current, and \( \Lambda \) is the Lagrange multiplier. Equations (2), (3) and (4) are the theoretical foundations for us to optimally control the thermally activated dynamics of the single-domain magnet by spin current \( I^s \). Notice that different constraint conditions can be chosen according to the practical experimental conditions. For example, the polarization direction of the spin current may have to be fixed in the multilayer magnetic structures, which will result in different solution for the variational problem. However, in order to show the full potential of the proposal, here we assume that both the amplitude and the polarization direction of the spin current can be dynamically modulated.

**Saddle point approximation and weak spin current limiting case.** The probability \( P(\mathbf{m}_i \to \mathbf{m}_f) \) in eq. (3) takes the form of path integral, and its calculation can be simplified in the spirit of saddle point approximation in the zero noise limitation, i.e., \( \epsilon \to 0 \). In this case, the main contribution of \( P \) comes from the special trajectory which is obtained by minimizing the FW action function in eq. (2) and its surrounding trajectories. We name this most probable trajectory as the “optimal path” as in ref. [22] and denote it as \( \{\mathbf{m}_{\text{opt}}(t)\} \). The corresponding minimal value of the FW action functional is the so-called quasi-potential [32] and is denoted as \( V[\mathbf{m}_{\text{opt}},I^s(t)] \), and then the probability reduces to \( P(\mathbf{m}_i \to \mathbf{m}_f) \sim e^{-V[\mathbf{m}_{\text{opt}},I^s(t)]/\epsilon} \). For the thermally activated magnetization switching process, this quasi-potential is equivalent to the energy barrier for the magnet to overcome in the Néel-Brown law, and the optimal control problem is to find the optimal spin current to maximizing (or minimizing) this quasi-potential in order to decrease (or increase) the switching probability.

The formulated optimization problem above can be further simplified for the weak spin current case. For “weak spin current”, we means that the spin current is weak enough that its contribution to the quasi-potential \( V[\mathbf{m}_{\text{opt}},I^s(t)] \) is much smaller than the contributions from the other terms such as the effective fields. However, it should still be strong enough that its contribution to

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the quasi-potential is larger than the noise amplitude \( \epsilon \), hence the probability \( P(\mathbf{m}_i \rightarrow \mathbf{m}_f) \) can be exponentially changed by this weak spin current. In this case, the optimal path \( \{ \mathbf{m}_{\text{opt}} \} \) can be approximated by the trajectory in the absence of the spin current. Thus, the quasi-potential \( V[\mathbf{m}_{\text{opt}}, I^s] \) can be approximated up to the first order of \( a \) as \( V[\mathbf{m}_{\text{opt}}, I^s] = V_0[\mathbf{m}_{\text{opt}}] + \Delta V[\mathbf{m}_{\text{opt}}, I^s] \), where

\[
V_0 = \frac{1}{2} \int_{t_i}^{t_f} |\sigma^{-1}(\mathbf{m}_{\text{opt}} - \mathbf{b}_0)|^2 dt,
\]

\[
\Delta V = \int_{t_i}^{t_f} a \gamma |\sigma^{-1}(\mathbf{m}_{\text{opt}} - \mathbf{b}_0), \sigma^{-1}K_S \mathbf{P}| dt.
\]

For convenience, we have introduced the vector \( \mathbf{b}_0 \equiv \sigma \mathbf{H}_{\text{eff}} \), and notice that \( \sigma \), \( K_S \), \( \mathbf{H}_{\text{eff}} \) are all functions of \( \mathbf{m}_{\text{opt}} \). \( t_i \) and \( t_f \) are the initial and final time of the optimal path respectively. Thus, the presence of a weak spin current \( I^s \) gives the change of the initial quasi-potential \( V_0 \) in the amount of \( \Delta V \), and the probability \( P(\mathbf{m}_i \rightarrow \mathbf{m}_f) \) has been exponentially changed by the factor \( e^{-\Delta V/\epsilon} \). The optimal spin current \( I^s_{\text{opt}} \) is then obtained by the reformulated variational problem

\[
\delta \{ \Delta V[\mathbf{m}_{\text{opt}}(t), I^s(t)] + \lambda F[I^s(t)] \} = 0,
\]

with the reformulated constraint condition \( F[I^s(t)] = 0 \) and the Lagrange multiplier \( \lambda \).

Thermally activated magnetization switching is a typical example of the stochastic dynamics described above, and the concept of “effective temperature” has been introduced to the Néel-Brown activation law to take account into the effect of spin current on the magnetization switching probability [7-11]. This fact can be easily verified from eqs. (5) and (6) for a weak spin current with constant amplitude, where \( \Delta V \) is now proportional to \( a \) and the magnetization switching probability is formally written as \( P_{\text{switch}} \sim e^{-(1 - a \gamma^2)\mu_0/\epsilon} \). Here, \( a \gamma \) is the critical value for the spin current to switch the magnet, and is given by

\[
a_\gamma = \frac{\int_{t_i}^{t_f} |\sigma^{-1}K_S \mathbf{H}_{\text{eff}}|^2 dt}{\int_{t_i}^{t_f} |\sigma^{-1}K_S \mathbf{H}_{\text{eff}}, \sigma^{-1}K_S \mathbf{P}| dt}.
\]

To get eq. (8), we have used the fact that in the absence of spin current, the optimal path \( \{ \mathbf{m}_{\text{opt}} \} \) is given by the equation [33,34]

\[
\mathbf{m}_{\text{opt}} = \gamma' (K_A - \alpha K_S) \mathbf{H}_{\text{eff}}.
\]

Notice that eq. (9) is not the magnetization dynamics equation. If the spin current is too large to be treated perturbatively, eq. (9) will not hold and the optimal path should be solved numerically [32]. In fact, the detailed analysis on the critical current was performed quite recently based on the same spirit [35]. From eq. (8), it is seen that the value of \( a \gamma \) is dependent on the choice of the spin polarization vector \( \mathbf{P} \) of the spin current. Especially, if \( \mathbf{P} \) is always in anti-parallel with the effective magnetic field \( \mathbf{H}_{\text{eff}} \), then \( a \gamma \) will be negative and the magnetization switching probability \( P \) will be decreased exponentially. From eq. (1), one might regard that the Gilbert damping should be effectively enhanced by the spin current in order to stabilize the magnet. One the other hand, \( \mathbf{P} \) should be parallel with \( \mathbf{H}_{\text{eff}} \) in order to increase \( P \). Thus, our results here support the viewpoint in ref. [16] that the spin current should effectively increase damping in order to suppress the noise in spin valves. Furthermore, eq. (8) gives a rough estimation of the critical spin current magnitude as \( a \gamma \). \( \alpha \) is proportional to the Gilbert damping coefficient and the maximal effective field [10,11]. We emphasize that the results here are valid only for the cases with small noise and weak spin current, which is different from the case discussed in ref. [16] where strong spin current has been exploited for noise control. In order to improve the \( \text{ad hoc} \) approach in ref. [16], one needs to solve the general variational problem (4) to get the optimal spin current for control.

If the spin current amplitude is assumed to be time-dependent, the optimal pulse shape \( a_{\gamma}^{\text{opt}}(t) \) to control the optimal path \( \{ \mathbf{m}_{\text{opt}} \} \) can be obtained from eq. (7). One natural constraint condition would be that the total dissipation energy of the spin current pulse should be constant, i.e., \( \int_{t_i}^{t_f} a_{\gamma}^{\text{opt}}(t) dt = \mathcal{E} \). Then the general solution \( a_{\gamma}^{\text{opt}}(t) \) of eq. (7) is given by

\[
a_{\gamma}^{\text{opt}}(t) = \frac{1}{\lambda} \alpha \gamma^2 \left( \int_{t_i}^{t_f} (\sigma^{-1}K_S \mathbf{H}_{\text{eff}}, \sigma^{-1}K_S \mathbf{P})^2 dt \right)^{1/2},
\]

and the Lagrange multiplier \( \lambda \) is determined from the constraint condition as

\[
\lambda = \pm \alpha \gamma^2 \left( \int_{t_i}^{t_f} (\sigma^{-1}K_S \mathbf{H}_{\text{eff}}, \sigma^{-1}K_S \mathbf{P})^2 dt \right)^{1/2}.
\]

This spin current pulse gives the change of the quasi-potential as \( \Delta V \sim -2\mathcal{E} \). According to eq. (10), the sign of \( \lambda \) is positive (or negative) when the optimal spin polarization vector \( \mathbf{P}_{\text{opt}} \) is parallel (or anti-parallel) with the effective magnetic field \( \mathbf{H}_{\text{eff}} \), which will enhance (or suppress) the probability \( P(\mathbf{m}_i \rightarrow \mathbf{m}_f) \) by the factor \( e^{2\lambda \mathcal{E}/\epsilon} \).

**Numerical simulation.** – Based on the theoretical analysis above, we use the stochastic LLG equation to simulate the optimal control of the thermally activated magnetization switching by weak spin current numerically. Considering a ferromagnetism film with easy axis along z-axis, and demagnetization field direction along x-axis, then the energy density \( E(\mathbf{m}) \) is given as [10,11,16,32]

\[
E(\mathbf{m}) = -\frac{1}{2} H_s M_s m_z^2 + 2\pi M_s^2 m_z^2.
\]

Here, we assume that no external magnetic field is applied, the anisotropic field is \( H_s = 0.05 \) T, and the demagnetization field is \( 4\pi M_s = 1.2 \) T. Besides, we set the Gilbert damping coefficient \( \alpha = 0.03 \), the temperature \( T = 300 \) K,
and the magnet volume \( V = 1500 \text{nm}^3 \). The initial direction of the magnet is assumed as \( \mathbf{m}(0) = (0, 0, 1) \), and the stochastic LLG equation (1) is simulated with the Heun method, where the time step is set as 1 ps. We also got the amplitude of the critical spin current numerically as \( a_c = 0.024 \) at zero temperature. Notice that we have chosen a relatively small magnet otherwise the switching time will be too long to be numerically feasible.

If there is no thermal noise, the magnet will stay at one of the two stable positions \((0, 0, 1)\) and \((0, 0, -1)\). At finite temperature, the thermal fluctuation will cause the magnet to randomly move around one of the two stable positions and then suddenly switch to another stable point. The switching trajectories are randomly distributed. In the zero noise limit, these random switching trajectories will converge to the deterministic switching trajectories according to the saddle point approximation discussed above. Such switching trajectories are the minimum energy paths in the absence of spin current. The energy profile \( E(\mathbf{m}) \) in (12) has two saddle points, i.e., \((0, 1, 0)\) and \((0, -1, 0)\), thus there exist two corresponding optimal paths for the magnet to switch from the initial stable point, say \((0, 0, 1)\), to the other one \((0, 0, -1)\). Each switching path consists of two stages, i.e., (a) the path from \((0, 0, 1)\) to one of the saddle points \((0, \pm 1, 0)\) given by the Eq. (9); (b) the path from the saddle points \((0, \pm 1, 0)\) to \((0, 0, -1)\) given by the deterministic LLG equation without the thermal noise. Notice that there is a fundamental difference between these two stages of the switching path. The first stage from the initial stable point to the saddle point is thermally activated with rare probability, although its path is expected to be the optimal path in the zero noise limit; while the second stage from the saddle point to the final stable point is governed by the LLG dynamics. Then the weak spin current pulse to control the thermally activated magnet switching problem should be applied to the first stage.

In fig. 1(a), we show the optimal path \( \{\mathbf{m}_{\text{opt}}(t)\} \) from \((0, 0, 1)\) to \((0, 1, 0)\), which is obtained from eq. (9). As one can see, there are some oscillations around the stable point before the magnet arrives at the saddle point, and this process takes about 2 ns. This gives the \textit{a priori} prediction of the thermally activated switching path of the magnet through the saddle point \((0, 1, 0)\) in the zero noise limit. Any practical switching path with finite noise will randomly deviate from this optimal path. To illustrate this point, fig. 1(b) demonstrates one random switching trajectory of the magnet simulated by the stochastic LLG equation (1) with no spin current. It is found that the main features of the real switching trajectories are indeed qualitatively captured by the optimal path given in fig. 1(a), although there are some quantitative differences because the noise amplitude is not small enough in our simulations. The simulated switching trajectories are expected to converge to the optimal path predicted in fig. 1(a) when the noise amplitude approaches to zero, but the switching time will be extremely long and makes the numerical simulation impractical. From fig. 1(b), one can also see that the magnet randomly moves around the stable point \((0, 0, 1)\) for a rather long time (about 1839 ns in this sample trajectory) until the switching occurred in less than 1 ns, which suggests that the switching is indeed a large fluctuation and rare event.

With no further constraint condition except constant energy, the optimal spin current pulse \( I_{\text{opt}}^* (t) \) to enhance the probability of the optimal switching path \( \{\mathbf{m}_{\text{opt}}(t)\} \) is obtained from eqs. (10) and (11) with the spin polarization direction \( \mathbf{P} \) paralleling with the effective magnetic field \( \mathbf{H}_{\text{eff}} \). The results are shown in fig. 1(c) and (d). The opposite direction of \( \mathbf{P} \) should be chosen if one needs to suppress the magnet switching. Besides, the optimal spin current pulse \( I_{\text{opt}}^* (t) \) to control another optimal switching path \( \{\mathbf{m}_{\text{opt}}(t)\} \) through the saddle point \((0, -1, 0)\) is also obtained but not shown here. The shape of \( a_{J_{\text{opt}}}^* (t) \) shows that the best control strategy is to apply the spin current when the magnet has deviated from the stable point significantly. For the energy profile (12) considered here, \( \mathbf{H}_{\text{eff}} \) has no \( y \)-component, and its \( x \)-component can be quite large even for a small \( \mathbf{m}_{\text{opt}}^x \) due to the large demagnetization field.

As a test, we calculated the mean switching time (MST) of the magnet under different control strategies, where \( 10^5 \) random trajectories are generated for each case. First, the MST without spin current is about 562 ns. Then a constant spin current with \( a_J = 0.05a_c \) will increase the MST to be about 917 ns if \( \mathbf{P} = (0, 0, -1) \), and decrease the MST to be about 351 ns if \( \mathbf{P} = (0, 0, 1) \). For the optimal control, the spin current pulse is not applied until it is
judged that a possible switching is going to occur. The analysis of the optimal path in fig. 1(a) shows that there will be some oscillations before the switching occurs. If the stochastic motion of the magnet shows part of such pattern, one can judge that the switching is going to occur and then should apply the spin current pulse to either decrease or increase its probability. During the simulations, the judgement criteria is set so that the z-component of the magnetization $m_z$ reaches the local minimum value $m_c$ in the optimal paths, and the sign of $m_y$ is used to distinguish the two optimal paths. We have chosen $m_c = 0.89, 0.81, 0.65$ separately, and the initial points of the corresponding spin current pulses are denoted as 1, 2, 3 in fig. 1(c). The amplitudes of $a^{opt}_m(t)$ are given such that each pulse will contain the same energy as a 2 ns constant pulse with $a_J = 0.05a_c$. Thus, the number of pulses used directly reflects the energy consumption during the control process.

The resulting MSTs $\tau$ and mean pulse numbers $n$ are shown in table 1, for both the cases to suppress and enhance the switching probability. Unfortunately, we found that the optimal pulses did not increase the MSTs $\tau_{e}$ when $m_c = 0.89$ and 0.65. For $m_c = 0.81$, the situation is better since the MST $\tau_{e}$ has been increased to 740 ns with 263 pulses, but it still has no obvious advantage compared with the constant control current case. The reason of the failure is that the noise amplitude is not small enough to fulfill the saddle point approximation and the practical switching trajectories $m(t)$ can deviate from the optimal path significantly so that the control pulse will in fact increase their probabilities. To eliminate this effect, we have set the spin polarization direction as $P = (0, 0, -1)$, and the results indeed become much better, as shown by $\tau_{e}$ and $n_{e}$ in table 1. Especially when $m_c = 0.65$, the MST $\tau_{e}$ is 911 ns which is close to the constant spin current case, but the number of pulses $n_{e}$ has been greatly decreased to 58. This shows that the optimal control theory is helpful to find the way to save energy even when the noise amplitude is not small enough to fulfill the theory.

The advantage of the optimal control strategy is fully verified for the case to increase the switching probability. In table 1, we found that the MST $\tau_{e}$ is decreased to 28 ns with only 25 pulses for $m_c = 0.89$. If $m_c = 0.65$, the MST $\tau_{e} = 66$ ns is larger, but merely 4 pulses in average is used here. For comparison, we also list the results $\tau_{e}$ and $n_{e}$ if $P$ is set as $(0, 0, 1)$ for the spin current pulses, which are only a little better than the constant spin current case. Obviously, the optimal control strategy successfully increased the switching probability with greatly reduced energy consumption.

**Discussions.** Finally, we discuss some possible difficulties which need to be overcome in order to apply the above proposal to practical spintronics industry, and also we shall discuss the further generalization of the theory to more complicated cases. First, dynamical modulating the polarization direction of the spin current is a rather difficult task, especially in the conventional magnetic multilayer structures where the spin current is generated by the pinned ferromagnetism layer. This might be overcome by exploiting other techniques to generate time-dependent spin current, such as the spin Hall effect manipulated by the electric field [36–38], or spin current created by optical methods [39–42]. Furthermore, if this difficulty cannot be overcome in near future, the optimal control theory here can still be applied by adding more constraint conditions such as fixed polarization direction. Another challenge is the feedback mechanism during the control which will make the device and circuit design complex. Thus, there always exists the balance between control efficiency and circuit complexity in industry applications. Considering the wide applications of feedback control in the integrated circuit in semiconductor industry, there is no reason for which such a design is forbidden in principle, although the technique difficulties do exist. However, one can always simplify the device design at the cost of reducing the control efficiency. Specially, if the feedback mechanism is not allowed at all, the principle here is still useful. In this case, the instantaneous status of the magnet is unknown due to the lack of feedback, then the spin current pulse as the control signal should always be applied in order to change the quasi-potential $\Delta V$ in (6). The form of the spin current can be assumed to be constant or periodic, and the variational procedure will give the optimal parameters such as the amplitude, polarization direction, or frequency of the spin current, etc. This will provide a more practical realization of the optimal control considering the current technique bottlenecks.

By far, we have only considered the single-domain magnet to illustrate the basic idea of optimal control. We expected that this principle can be generalized to the multi-domain magnetization case similarly. The reason is because the key step of the optimal control is to find the optimal path of large fluctuations, such as the switching path between two stable configurations. This has in fact been achieved in some multi-domain magnetization structures [43]. Besides, we have neglected the noise carried by the spin current which can be used to accelerate the reversal of the magnet [44]. Including this additional noise source implies that the noise amplitude $\epsilon$ in eq. (1) has been effectively increased, which then increases the

| $m_c$ | $\tau_{e}/n_{e}$ | $\tau_{e}/n_{e}$ | $\tau_{e}/n_{e}$ | $\tau_{e}/n_{e}$ |
|-------|-----------------|-----------------|-----------------|-----------------|
| 0.89  | 515/428         | 1000/835        | 28/25           | 309/263         |
| 0.81  | 740/263         | 1036/376        | 32/13           | 305/113         |
| 0.65  | 516/32          | 911/58          | 66/4            | 344/22          |
switching probability according to (3). For the weak spin current case, this current noise is less important and ignored.

In conclusion, we have discussed how to apply the spin current to optimally control the stochastic magnetization dynamics based on the large deviation principle, and shown the merits of the proposed control strategy by applying it to the thermally activated magnetization switching problem. The probabilities of the large magnetization fluctuations, which are the dominant part of the stochastic magnetization dynamics, are controlled by changing the corresponding quasi-potential with optimized spin current pulse. We have also discussed the possible technique difficulties to be overcome in practical industrial applications and further generalization of the theory.

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