Constraining Ultra-light Axions with Galaxy Cluster Number Counts

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Abstract. In this paper we investigate the potential of current and upcoming cosmological surveys to constrain the mass and abundance of ultra-light axion (ULA) cosmologies with galaxy cluster number counts. ULAs, sometimes also referred to as Fuzzy Dark Matter, are well-motivated in many theories beyond the Standard Model and could potentially solve the $\Lambda$CDM small-scale crisis. Galaxy cluster counts provide a robust probe of the formation of structures in the Universe. Their distribution in mass and redshift is strongly sensitive to the underlying linear matter perturbations. In this forecast paper we explore two scenarios, firstly an exclusion limit on axion mass given a no-axion model and secondly constraints on an axion model. With this we obtain lower limits on the ULA mass on the order of $m_a \gtrsim 10^{-24}$ eV. However, this result depends heavily on the mass of the smallest reliably observable clusters for a given survey. Cluster counts, like many other cosmological probes, display an approximate degeneracy in the ULA mass vs. abundance parameter space, which is dependent on the characteristics of the probe. These degeneracies are different for other cosmological probes. Hence galaxy cluster counts might provide a complementary window on the properties of ultra-light axions.

Keywords: galaxy clusters – cluster counts – axions – cosmology of theories beyond the SM
1 Introduction

In the current standard model of cosmology, consisting of a cosmological constant, Λ, and a cold dark matter component, CDM, 26% [1] of the total mass-energy content of the universe consists of cold dark matter. Most commonly it is considered to be a yet undiscovered elementary particle, which is cold in the sense that its velocity dispersion is non-relativistic. This constituent is referred to as dark matter, because so far only its gravitational imprint has been detected. While being remarkably successful on larger scales, ΛCDM simulations show significant discrepancies to observations at galactic scales, known as missing satellite problem (e.g. [2–4]), too-big-to-fail problem (e.g. [5–8]) and cusp-core problem (e.g. [9–13]). Even though these problems might be solved invoking baryonic physics (e.g. [14–31]), this tension of observations with ΛCDM predictions on small scales makes modifications on these scales appealing.

One possible modification is substituting (part of) the CDM with so-called fuzzy dark matter (FDM) [32]. This dark matter variant consists of very light, scalar bosonic fields. These fields have similar properties as axions in QCD and are therefore often called axion-like particles (ALPs) or ultra-light axions (ULAs). Many theories beyond the standard model — most notably many string theories — can naturally produce cosmologically relevant mass densities, i.e. \( \Omega_a \sim \Omega_{\text{dm}} \) of these particles over a wide range of particle masses [33]. For string theories in particular, a single theory often predicts non-thermal production of many different ULA particles (“axiverse” [34]) in the interesting axion mass range \( m_a \sim 10^{-22} \) eV, their number depending on the existence of anti-symmetric tensors in the theory and the topology of the compactified extra dimensions [35–37]. Since we only use mass and energy density as ULA parameters, our approach is theory independent. For referring to the particles, we will use the term ULA throughout, \( m_a \) for the mass of the particle and \( \Omega_a \) for the energy density in axions or \( F = \Omega_a/\Omega_{\text{dm}} \) for the fractional axion energy density with respect to the overall dark matter density \( \Omega_{\text{dm}} \).

For masses around \( m_a \sim 10^{-22} \) eV the deBroglie wavelength \( \lambda_{\text{dB}} \) of the particles becomes of order of 2 kpc [33], i.e. macroscopic, thus preventing formation of density cusps below \( r < \lambda_{\text{dB}} \) at the centre of galaxies and effectively suppressing structures on scales smaller than \( \lambda_{\text{dB}} \) [32, 38, 39]. The effect is similar to, but not degenerate with the thermal free-streaming induced suppression in warm dark matter (WDM) models, like for models with
massive neutrinos. Another key signature of ULAs is their distinct early-time and late-time behaviour: Being scalar fields ULAs obey the Klein-Gordon (KG) equation. Due to the tiny mass \( m_a \ll H \), compared to the Hubble scale \( H \) at early times, the KG equation for the ULA field is over-damped, the field has a slow-roll phase and behaves like dark energy. At late times, when \( m_a \gg H \) the KG equation oscillates rapidly. Here an effective fluid approximation \[40\] is used, in order to cycle-average over the oscillations. The ULA field then behaves like a DM-like particle with a scale dependent sound speed. \[e.g. \ 38, 40–42\]

To address the inconsistencies of the \( \Lambda \)CDM model on scales of stellar streams and dwarf galaxies, a mass range of around \( m_a \sim 10^{-22} \) eV is required \[43–48\]. At lower masses relative heights of the acoustic peaks in the cosmic microwave background (CMB) temperature power spectrum \[40\] as well as CMB polarisation \[49\] provide a lower limit of \( m_a \sim 10^{-23} \) eV. A similar constraint is obtained from the UV luminosity function of high redshift galaxies \[50–52\]. Significantly higher masses are ruled out by the existence of an old central globular clusters within the ultra-faint dwarf galaxy Eridanus-II. This places interesting constraints on the ULA DM mass around \( m_a \sim 10^{-20} \) eV \[53, 54\]. ULAs would also impact the orbital decay of compact binaries \[55\] and via black hole superradiance the spin of black holes \[56\]. The existence of spinning super-massive and solar-mass black holes places limits on high ULA masses at \( m_a \sim 10^{-11} \) eV \(-10^{-19} \) eV \[57\], M87* specifically provides a limit around \( 4 \times 10^{-21} \) eV \[58\]. Lyman-\( \alpha \) forest measurements \[59–64\] provide very tight constraints up to \( m_a \sim 10^{-20} \) eV, potentially limiting the capability of ULAs to solve \( \Lambda \)CDM small scale problems, although various astrophysical effects could alleviate this bound \[33, 65\]. Another interesting possibility to constrain axion properties is through observations of the x-ray spectrum of galaxy clusters \[66–68\]. Here, due to the presence of a magnetic field in galaxy clusters, axions and photons interact and leave a distinct imprint on the x-ray spectrum.

One of the main motivations to introduce cold dark matter in the modelling of the Universe is to explain the observed large-scale structure. This was early on investigated with dark matter particles originating from extensions of the standard model of particle physics \[69–71\] leaving imprints on the formation of structures on various astrophysical scales \[72\]. Axions have been considered for a long time with respect to their influence on the structure formation process \[73\]. Since the nature of dark matter plays an important role in structure formation, we can exploit measures of the distribution of the large-scale structure to constrain these models \[74–80\]. One of the most sensitive probes of the process of structure formation in the Universe is the distribution of galaxy clusters in mass and redshift \[81, 82\]. In recent years clusters of galaxies have developed into a state-of-the art cosmological probe \[83–88\]. With the currently flying Spektr-RG space observatory and eRosita instrument \[89\] and the upcoming Euclid mission of the European Space Agency (ESA) \[90, 91\] thousands of clusters will be observed and can provide powerful constraints for cosmological models \[92\].

In this paper we will investigate the potential of galaxy cluster counts to constrain properties of ULAs. The paper is structured as follows: In chapter 2 we review the calculation of cluster number counts from the halo mass function and our implementation of the Monte Carlo analysis. In chapter 3 our results are presented, distinguishing between the two cases whether or not ULAs are detectable with a given survey. We conclude in chapter 4.

## 2 Modelling Galaxy Cluster Counts in the Presence of Ultra-light Axions

To forecast cluster number counts, or to compare observed counts to a given cosmological model, we need to know the distribution of halos for a given mass \( M \) and redshift \( z \). \[93\]
obtained a first estimate by simply calculating the probability of the formation of a halo and assuming that the underlying distribution of the density fluctuations is Gaussian. This approach relies on a critical density threshold. If a linear perturbation is above this threshold the corresponding region starts to collapse and forms a massive halo. This simple estimation was later put on a more sound footing by the excursion set approach [94, 95]. A more realistic treatment, taking into account all subtleties of structure formation, is to exploit cold dark matter N-body simulations [96–98]. This allows calibrating the mass function, where the functional form is still motivated by the analytical estimates. Typically the mass function is given in terms of the multiplicity function

\[ f(\sigma) = A \left( \frac{\sigma}{\sigma_0} \right)^{-a + 1} e^{-c/\sigma^2} \]

(2.1)

where \( \sigma(R,z) \) is the r.m.s. density fluctuation smoothed on a scale \( R \) and at a given redshift \( z \). \( R \) is related to the mass of the halo via the background density of the Universe, i.e. \( M = 4\pi/3\rho_m,0 R^3 \). The mass variance \( \sigma^2(R) = 1/(2\pi^2) \int_0^\infty dk P(k)k^2 W(kR)^2 \) is obtained from the linear matter power-spectrum \( P(k) \) applying a real-space spherical top hat window function \( W(kR) \).

In [98] the multiplicity function is calibrated with multiple detailed simulations and parameterised in the form, \( f(\text{inker}) = A \left( \frac{\sigma}{\sigma_0} \right)^{-a} e^{-c/\sigma^2} \). Here \( a, b \) and \( c \) depend on redshift \( z \) and the chosen value for the over-density in halos \( \Delta \). We use \( \Delta = 200 \) throughout. The formulation in terms of the multiplicity function makes it possible to study the cosmological universality of the mass function. This provides 5% accuracy over a reasonable parameter range assuming a ΛCDM framework.

In order to compute the mass function to predict the number of clusters at a given mass \( M \) and redshift \( z \), we first need to compute matter power spectra for ULA cosmologies. The matter power spectrum is calculated from matter density fluctuations as a function of scale \( k \) and redshift \( z \). For this we need to solve the system of coupled differential equations for the fluctuations of the different components of the cosmological model. An early overview of this is given in [99]. The solution of the hierarchy of Boltzmann equations is computed by Boltzmann solvers such as [100] or [101]. To include the effects of ULAs we need to extend the standard hierarchy of perturbation equations by the ULA component.

ULAs obey the Klein-Gordon equations of motion for a scalar field \( \phi \) and small perturbations \( \delta \phi \). The perturbed equation in synchronous gauge and Fourier space is given by [40, 102]:

\[ \ddot{\delta \phi} + 2\frac{a}{\dot{a}} \dot{\delta \phi} + \left( m_a^2 a^2 + k^2 \right) \delta \phi = -\frac{1}{2} \dot{\phi} \dot{\zeta}, \]

(2.2)

where \( \zeta \) is the scalar potential of the synchronous gauge [99]. Here \( a \) refers to the scale factor and overdots are derivatives w.r.t. conformal time. Eq. (2.2) can be expressed in terms of the over-density \( \delta_a = \delta \rho_a/\bar{\rho}_a \) and the velocity \( u_a = (1 + w_a)v_a \), which are given in terms of the ULA background density \( \bar{\rho}_a \), the velocity perturbation \( v_a \) and equation of state parameter \( w_a \) [40]:

\[ \dot{\delta}_a = -k u_a - (1 + w_a)\dot{\zeta}/2 - 3\dot{a}/a(1 - w_a)\delta_a - 9\dot{a}/a(1 - c_{ad}^2)u_a/k \]

(2.3)

\[ \dot{u}_a = 2\dot{a}/au_a + k\delta_a + 3\dot{a}/a(w_a - c_{ad}^2)u_a \]

(2.4)

\[ ^1\text{Usually this parameter is called } h. \text{ We changed it to avoid confusion with the reduced Hubble parameter.} \]
with adiabatic sound speed $c_{\text{ad}}^2 = w_a - \frac{\ddot{w}_a}{3\dot{a}/a(1+w_a)}$.

At late times these equations oscillate heavily due to $m_a \gg H$, making an exact solution impossible. Therefore [40] employ a WKB approximation to average over the fast time scale. This effective fluid approximation leads to averaged equations [40]

\begin{align}
\delta_a &= -ku_a - \frac{\dot{\xi}}{2} - 3\dot{a}/a c_a^2 \delta_a - 9\ddot{a}/a c_a^2 u_a/k \quad (2.5) \\
\dot{u}_a &= -\dot{a}/au_a + \frac{c_a^2}{2} k \delta_a + 3\dot{a}/a c_a^2 u_a, \quad (2.6)
\end{align}

with time-averaged ULA sound speed in perturbations $c_a^2 = \frac{k^2}{4n_a^2 a^2}$ [38]. Once we have $\delta_a$, the matter power-spectrum can be calculated via $P(k) \equiv \langle |\delta_a(k)|^2 \rangle$. Due to the scale dependent sound speed of ULAs, structure growth does not evolve equally with redshift on all scales, so the standard, scale-independent growth factor description is no longer valid [103].

There are modifications for the two well-known Boltzmann solvers CAMB [100] and CLASS [101] accounting for light scalar fields as a dark matter candidate: AxionCAMB\(^2\) [40] and CLASS.FreeSF\(^3\) [104] (for a comparison see [41, 105]). In this work we use AxionCAMB.

In AxionCAMB the transition to the effective fluid approximation is made at $m_a = 3H$ which has been shown to reproduce the exact solution reasonably well [41]. For more implementation details, see [40].

In figure 1 we show the total matter power spectra for different ULA cosmologies at redshift $z = 0$. We choose the cosmological parameters to be from the Planck cosmology [1] with fractional energy density in baryons $\Omega_b = 0.0495$ and in dark matter $\Omega_{\text{dm}} = 0.267$, scalar spectral index $n_s = 0.966$, Hubble parameter $H_0 = 100\, \text{h}\,\text{km}\,\text{s}^{-1}\text{Mpc}^{-1}$ with $h = 0.672$, scalar amplitude $\ln (A_s \times 10^{10}) = 3.04$ and optical depth of reionisation $\tau_{\text{re}} = 0.0515$. On the left side we see a very distinct cutoff at a scale $k_J \propto m_a^{1/2} F^{1/4}$ depending on the ULA mass and fraction. This scale is governed by the de Broglie wavelength of the ULA particles,

\(^2\)Publicly available at https://github.com/dgrin1/axionCMB.

\(^3\)Publicly available at https://github.com/lurena-lopez/class.FreeSF.
suppressing structure formation at larger scales with decreasing axion mass. On the left hand side in figure 1 the increase of the cut-off scale (decreasing $k$) with increasing axion mass $m_a$ is very noticeable. However, if we consider the changes with increasing axion fraction $F$, on the right hand side in figure 1, we have to keep in mind that we are using the total matter power spectrum, with $P \propto (1-F)^2P_{dm} + F^2P_a$. Hence the cut-off scale becomes more visible for higher axion fraction. Since the scale itself depends only mildly on the fraction, $k_J \propto F^{1/4}$, the position of the cut-off barely changes with $F$. The models with $m_a = 10^{-18}\text{eV}$ (left) and $F = 0.01$ (right) are indistinguishable from ΛCDM in the given parameter range.

With the matter power-spectrum we can now calculate the halo mass function. In order to calculate the halo mass function we adapt an approach previously exploited in [106]. We should note that simulations for ULA cosmologies using a pseudo-spectral method with - for the first time - subsequent determination of the halo mass function have been presented in a recent paper [107]. The authors only find significant quantitative differences in the power spectrum and halo distribution, compared to ΛCDM, at much lower halo masses as the ones we consider here. Although they consider larger axion masses than our analysis, we do not expect significant changes on the relevant scales for cluster cosmology beyond linear effects. The effects of ULAs on linear scales are well modelled in AxionCAMB. Implementing ULAs firstly requires modelling of the transfer function and growth of structures [32, 44] and secondly the implementation of an adjusted critical linear over-density in order to account for the fact that no structures are formed below the Jeans mass of the ULA cosmology. Note that the Jeans mass $M_J$ for ULA cosmologies is proportional to $m_a^{-3/2}$ [32, 108–113]. Different approaches to achieve this exist [44, 114, 115]. They all have in common that there are no significant effects on the mass function for halo masses above $10^{13}h^{-1}\text{M}_\odot$. So for the purpose of this paper it is sufficient to model correctly the growth and transfer function for the ULA cosmologies and these are obtained by the application of the Boltzmann solver including axions [40]. We can then apply this appropriately calculated matter power spectrum to the ΛCDM mass function [98].

Figure 2 shows the influence of ULA parameters on the halo mass function in the mass range relevant for galaxy clusters. As expected, the influence of ULAs is larger for smaller halo masses with the ULA mass $m_a$ determining the mass range of halos where significant suppression relative to ΛCDM occurs. The ULA abundance, parametrised by $F = \Omega_a/\Omega_{dm}$, determines how strong the suppression is. Since the models $m_a = 10^{-18}\text{eV}$ and $m_a = 10^{-23}\text{eV}$ (figure 2, left) are almost indistinguishable from each other, one can already see that surveys sensitive to cluster masses $>10^{13}h^{-1}\text{M}_\odot$ will possibly not be able to distinguish these ULA masses from ΛCDM cosmology. In general the models with $m_a = 10^{-18}\text{eV}$ (left) and $F = 0.01$ (right) are indistinguishable from ΛCDM in the given mass range.

In order to predict the number of observed clusters we first need to multiply the halo mass function with the observed volume of the survey. For this we need to calculate the co-moving volume element $\frac{\text{d}V}{\text{d}z\text{d}A} = r(z)^2/H(z)$, where $r(z)$ is the co-moving coordinate distance and $H(z)$ the Hubble parameter. Clusters of galaxies are typically detected and observed via either their Sunyaev-Zel’dovich signature in the cosmic microwave background [85, 116–120], their x-ray flux [121–124] or their optical richness [125–128]. Since true masses of galaxy clusters are not directly accessible with these observations, it is required to assume a scaling relation between the observable and the true mass of the cluster. While the general form of the scaling relation can be motivated for x-ray or Sunyaev-Zel’dovic detected clusters by the underlying physics [84, 129–131], the situation is less clear for the relation between the optical richness and the mass [132]. Here we choose to explore the ability of future optical
clusters surveys to constrain ULA parameters, though we expect that our results hold also for other types of observations. Hence, we follow a generic approach to the mass-observable scaling relation (as e.g. in [87, 132–135]):

\[
\ln \frac{M}{M_{\text{norm}}} = a_M + \alpha_M \ln \frac{O}{O_{\text{norm}}},
\]  

(2.7)

where \(M\) is the true underlying mass of the galaxy cluster, \(O\) is the prediction for the observable given \(M\), \(a_M\) and \(\alpha_M\) are scaling relation parameters, and \(O_{\text{norm}}\) and \(M_{\text{norm}}\) are normalisation factors. Note that in this generic forecast exercise we keep the mass-observable relation fixed with redshift. We normalise the scaling relation at \(M_{\text{norm}} = 10^{14} h^{-1} M_\odot\). For our analysis we choose the normalisation by adapting the value from observations of the galaxy cluster richness, with \(O_{\text{norm}} = N_{\text{gal norm}} = 40\) [132]. We further assume that the observed value of the observable \(O_{\text{obs}}\) follows a log-normal distribution around the "true" theoretical prediction for the observable \(O\) [133]:

\[
p(O_{\text{obs}}|O) = \frac{1}{\sqrt{2 \pi \sigma_{\ln O_{\text{obs}}|M}^2}} \exp \left[ -\frac{x^2(O, O_{\text{obs}})}{2 \sigma_{\ln O_{\text{obs}}|M}^2} \right]
\]  

(2.8)

with

\[
x(O, O_{\text{obs}}) = \frac{\ln O_{\text{obs}} - \ln O(M)}{\sqrt{2 \sigma_{\ln O_{\text{obs}}|M}^2}}.
\]  

(2.9)

Here \(\sigma_{\ln O_{\text{obs}}|M}\) is the scatter in \(\ln O_{\text{obs}}\) for fixed \(M\) and we have used \(\sigma_{\ln O_{\text{obs}}|M} = \sigma_{\ln O_{\text{obs}}|O}\).
We can predict the number of galaxy clusters in bins of the observable \([O_{i}^{\text{obs}}, O_{i+1}^{\text{obs}}]\) and redshift \([z_{j}, z_{j+1}]\) with [134]:

\[
N_{ij} = \Delta \Omega \int_{z_{j}}^{z_{j+1}} dz \frac{dV}{d\Omega} \int_{-\infty}^{\infty} d\ln O \frac{\alpha_{M}}{O_{i}^{\text{obs}}} \frac{dn}{d\ln O} \frac{1}{2} (\text{erfc}(x_{i}) - \text{erfc}(x_{i+1})),
\]

(2.10)

using \(x_{i} = x(O, O_{i}^{\text{obs}})\), the sky coverage of the survey \(\Delta \Omega\) and having expressed the halo mass function in terms of the observable via eq. (2.7). We would like to emphasize that the choice of parameterization of the scaling relation is quite general. As pointed out below we work with mass limits instead of observable limits in the analysis presented here. This allows the results to be transferable to other survey set-ups, like for example SZ or x-ray observations. The crucial parameter however is the scatter in the mass observable relation. This parameter defines how many clusters scatter into the mass-bin from neighbouring bins or the observational limits. Hence the uncertainty in the scattering parameter will be degenerate with cosmological parameters, as pointed out for the case of dark energy in [133].

Throughout our analysis we assume survey parameters comparable to next generation cosmological surveys such as the Euclid mission of the European Space Agency ESA [92]. Specifically we assume a sky coverage of \(\Delta \Omega = 10^{4}\) deg\(^{2}\) and a redshift range \(0.1 < z < 0.6\) with five equally sized bins in redshift \(z\) and in the mass \(\log_{10} M\), resulting in a total of 25 bins of galaxy cluster counts. In order to understand the influence of the limiting mass of a survey, we explore the lower bound from values at \(10^{13} h^{-1} M_{\odot}\) to more realistic values of \(5 \times 10^{14} h^{-1} M_{\odot}\). Since for any realistic cosmology there are no clusters of masses above \(5 \times 10^{15} h^{-1} M_{\odot}\) expected, we set the upper integration bound to this value. Figure 3 shows the predicted number counts for different ULA parameters. For the standard cosmological parameters we use Planck 2018 values [1], as listed above. In order to explore the influence of the ULA parameters, in figure 3 we set the observable-mass relation parameters to the no bias, ideal scaling and no scatter values, i.e. \(a_{M} = 0\), \(\alpha_{M} = 1\) and \(\sigma_{\ln O_{\text{obs}}|M} \approx 0\).

Since cluster number counts are a binned version of the halo mass function convolved with the purely geometrical volume element, the dependence on the ULA parameters for number counts follows the trend observed in the halo mass function as shown in figure 3. Here we can already see that the lowest mass bins are crucial for the analysis. Firstly, because this is where ULAs produce the biggest relative differences in NCs and secondly, because the highest number of absolute counts also has the biggest impact on the value of the likelihood. In redshift we see an increasing number of clusters due to a growing co-moving volume. The suppression of the halo mass function only becomes dominant at much higher redshifts. \(m_{a} = 10^{-18} eV\) in the top and \(F = 0.01\) in the bottom panels are indistinguishable from a \(\Lambda\)CDM model.

3 Modelling the Likelihood and Forecasting Constraints on Ultra-light Axion Parameters

3.1 Sampling of the Likelihood

The dominant error of galaxy number counts follows a Poisson statistics, hence we need to compute the Poissonian log-likelihood (compare e.g. [85])

\[
\sum_{k} \ln P(N_{\text{obs}}^{k}|\theta) = \sum_{k} N_{\text{obs}}^{k} \ln N_{\text{th}}^{k} - N_{\text{th}}^{k} + \text{const.},
\]

(3.1)
with $\theta$ a vector of cosmological parameters of interest. The sum is over all observational bins, which for our analysis is a grid of 5 bins each in cluster mass and redshift. Note that here we ignore the contribution due to sample covariance [136]. Of course any precision cosmological analysis should include this term. However, the sample covariance is proportional to the square of the number density of observed clusters. Since we will concentrate our forecast on observational programs which cover a quarter of the sky, we do not expect this contribution to make a big difference for our analysis, even in the low mass bins.

We want to analyse the ability of future surveys to constrain the ULA parameters. For
| Type         | Symbol       | Definition                        | Fiducial Value | Prior             |
|--------------|--------------|-----------------------------------|----------------|-------------------|
| ULAs         | $\log_{10}(m_a[eV])$ | Fiducial value of ULA mass         | various        | various           |
|              | $F$          | Fraction of DM in ULAs             | various        | various           |
| Cosmology    | $\Omega_{dm}$ | DM energy density                 | 0.267          | Planck            |
|              | $\ln(A_s \times 10^{10})$ | Amplitude of primordial perturb.  | 3.04           | Planck            |
|              | $n_s$        | Scalar spectral index             | 0.966          | Planck            |
| obs.-mass    | $\alpha_M$   | Scaling Exponent                  | 1.06 ± 0.11    | Gauss             |
| distrib.     | $\alpha_M$   | Scaling Offset                    | 0.75 ± 0.1     | Gauss             |
|              | $\sigma_{ln O_{obs}|M}$ | Scaling Scatter                  | 0.45 ± 0.1     | Gauss             |

Table 1. Parameters for posterior analysis. *Planck* denotes Gaussian priors obtained from Planck 2018 [1] covariance matrix. Fiducial values for cosmological parameters also come from [1], the observable-mass distribution parameters from [132]. The prior in ULA mass is logarithmically flat but needs to be shifted for various values for $M_{min}$ to not sample too much of the unconstrained parameter space.

In order to efficiently sample the posterior likelihood we employ a Monte-Carlo Markov Chain (MCMC) algorithm. Since we do not have prior knowledge of the prior probability for ULA parameters we use uninformative flat priors as shown in table 1. For the standard cosmological parameters we use the Planck covariances from the 2018 data release [1] as priors. For the numerically demanding probability analysis we use the dark matter content $\Omega_{dm}$, the scalar primordial perturbation amplitude $A_s$ and the spectral index $n_s$ as free parameters. All other cosmological parameters are fixed. They are the parameters, which most affect the matter power spectrum and hence the halo mass function. Note that we fix the Planck covariance to the $\Lambda$CDM case. For a detailed investigation with real data, of course a joint analysis between the two probes should be performed. Including the ULA parameters would most likely widen the prior range from Planck. Given that we also fix parameters, like the baryon contents of the universe, we expect this approximation to be sufficient for the forecast analysis presented here. Priors on observable-mass distribution parameters have been taken from the analysis by Rozo et al. [132] (their table 4). An overview for all the parameters and their priors used in our analysis can be found in table 1.

The analysis is conducted using the affine-invariant code *emcee* [137], convergence is tested using the indicator from [138].

### 3.2 Lower Limits on the Axion Mass

We would first like to address the question up to which limit galaxy cluster counts can exclude an axion cosmology given a $\Lambda$CDM fiducial cosmology. Hence we perform a likelihood analysis with fiducial values on the standard cosmological parameters from [1] and an axion mass an order of magnitude above the expected exclusion limit. For relatively large particle masses ULA cosmologies cannot be distinguished from $\Lambda$CDM anymore as figure 4 shows. The posterior for ULA mass is not Gaussian, instead it resembles a logistic function, running into the upper prior bound of the axion mass $m_a$. This makes sense, since large axion mass cosmologies are indistinguishable from a $\Lambda$CDM model. Lower axion masses are ruled out for a fiducial $\Lambda$CDM model, due to their large impact on the halo mass function at the mass scales probed by galaxy clusters. For large abundances $F$ the differences in the posterior
Figure 4. Posterior for ULA parameters indistinguishable from ΛCDM ($\log_{10}(m_a[\text{eV}]) = -23.2, F = 0.1$). Dashed grey lines denote fiducial parameter values, the dashed red line is the 95% exclusion limit of the marginalised 1D density for $\log_{10}(m_a[\text{eV}])$. Minimum cluster mass observable $10^{14} h^{-1} M_{\odot}$.

Different shades of blue from dark to light denote 68%, 95% and 99% posterior contours respectively.

primarily arise due to number count differences in the lowest-mass bins of the survey. Smaller $F$ have less impact on the level of suppression, therefore smaller ULA masses are allowed for lower abundance values $F$. Above this exclusion threshold the posterior is relatively flat for the ULA mass. The preference for small ULA abundances shown in the posterior distribution of $F$ is caused by a larger unconstrained region in $m_a$ for small $F$.

The dark matter density and scalar spectral index do not portray significant degeneracy with ULA parameters. Only $A_s$ experiences a non-significant shift to slightly smaller values when the lower bound on axion mass is approached.

Cluster counts provide additional constraining power to the scaling relation parameters.
a_M and α_M. The marginalised 1-σ limits shrink from ±0.1 and ±0.11 to ±0.03 and ±0.02 respectively. A significant degeneracy between the two parameters and ULAs is visible, for small ULA masses a_M is shifted to lower, α_M to higher values. This negatively impacts the constraining capability of the probe, making tight priors on these two parameters desirable. We performed an analysis with fixed scaling relation parameters. In this case the exclusion on the axion mass changes from \( \log_{10}(m_a[eV]) = -24.29 \) to \( \log_{10}(m_a[eV]) = -24.23 \), corresponding to a 15% increase of the mass limit. We want to stress that this is of course an unrealistic scenario but serves as a limiting case what in the best circumstances is achievable. This could be achieved by complementary observations of the galaxy cluster masses, for example with gravitational lensing observations. We also investigated the case of doubling the mass limits between \( m \) and \( \text{min} \), which translates to our findings for exclusion limits for different minimal mass \( M_{\text{min}} \) of the observed clusters. Table 2 summarises \( \log_{10}(m_a[eV]) \) exclusion limits for different minimal mass \( M_{\text{min}} \) of the observed clusters. Our analysis indicates that \( M_{\text{min}} \) is by far the most important survey parameter when it comes to the ability to constrain axion mass with a certain data-set. An increase in the surface angle of the survey \( \Delta \Omega \) increases the total number of observed clusters and should therefore have a small positive impact on the constraints for real surveys. The same is true for reaching higher redshifts. Since fractional differences of different axion models do not have a significant redshift dependency however (compare figure 3), this effect is subdominant compared to being able to observe clusters with lower masses. The left panels of figure 3 show that this is where relative differences between different axion cosmologies are biggest, which translates to our findings for exclusion limits of \( m_a \) in table 2.

To investigate the influence of the redshift range of the survey on the constraints, we additionally conducted an analysis with \( 0.1 < z < 1.0 \) and \( M_{\text{min}} = 1 \times 10^{14} h^{-1} M_\odot \). We increased the number of redshift bins from five to nine, all other parameters remaining equal. This increases our 95% exclusion limit on \( \log_{10}(m_a[eV]) \) from \(-24.29\) to \(-24.21\), which is a factor 1.2 increase in sensitivity in non-logarithmic units. Two effects explain this slightly more constraining exclusion limit: Increasing the redshift range to higher redshifts increases the number of clusters. Firstly this increased number of clusters reduces the Poisson error of the likelihood and hence improves the constraint. In addition ULA effects are slightly more pronounced at higher redshifts, as can be inferred from figure 3, leading to better constraints for a larger redshift range.

| \( M_{\text{min}}[h^{-1} M_\odot] \) | 95% exclusion for \( \log_{10}(m_a[eV]) \) |
|-----------------------------|-----------------|
| \( 5 \times 10^{14} \)     | -24.09          |
| \( 1 \times 10^{14} \)     | -24.29          |
| \( 5 \times 10^{14} \)     | -24.98          |

Table 2. \( \log_{10}(m_a[eV]) \) exclusion limits for different minimal mass \( M_{\text{min}} \) of the observed clusters.
3.3 Constraining the Axion Mass

Instead of asking which exclusion limit we can obtain given specific survey parameters, we may also wonder how a possible detection might unfurl and be impacted by parameters of the survey. To answer this question we adopt a fiducial model of $F = 0.4$, $\log_{10} m_a [eV] = -24.3$. Although this specific model is already under strain from other probes it is interesting to see if such a model could be "detected" with galaxy cluster counts as a cosmological probe.

ULAs with these parameters could possibly be distinguished from $\Lambda$CDM by cluster surveys with minimal observed clusters masses $M_{\text{min}} \lesssim 1 \times 10^{14} h^{-1} M_\odot$. For $M_{\text{min}}$ values slightly below this limit, it is not surprising to find a significantly degenerate posterior. In section 2 we showed that the effect from the abundance $F$ and ULA mass $m_a$ on the matter power spectrum or the halo mass function look similar. In the limit where ULAs are only barely detectable this similarity naturally leads to a degeneracy in the $m_a - F$ parameter plane as figure 5 shows. For reduced minimal observed cluster masses $M_{\text{min}}$ the joint posterior in $m_a$ and $F$ has a smaller volume and is less steep. One can understand this with the effect of the ULA parameters on the halo mass function in figure 2. Let us consider two ULA models, which look degenerate for a survey with a limiting mass of $M_{\text{min}} = 7 \times 10^{13} h^{-1} M_\odot$, e.g. a fiducial model with $F_1 = 0.4$ and $\log_{10} m_a,1 [eV] = -24.3$ and a second model with $F_2 = 0.8$ and $\log_{10} m_a,2 [eV] = -24.1$. For a different survey with a smaller limiting mass of $M_{\text{min}} = 2 \times 10^{13} h^{-1} M_\odot$, these two models are not degenerate anymore. The second survey has an additional mass bin with $2 \times 10^{13} h^{-1} M_\odot \leq M_{\text{min}} \leq 7 \times 10^{13} h^{-1} M_\odot$. Here the fiducial model is highly suppressed in comparison to the second model. Therefore lowering the abundance $F$ of the second model will result in a better match with the fiducial model and hence the posterior will be less steep in the $\log_{10} m_a - F$ parameter space. The reason for this behaviour essentially is a result of the different effect of the axion mass $m_a$ and the axion fraction $F$ on the total matter power spectrum as discussed in section 2. Reducing the limiting mass $M_{\text{min}}$ of the survey for the same axion mass range is increasing the constraining power of the survey and therefore the posterior volume is smaller. Of course we would like to stress that it is very challenging to use "clusters" in a meaningful way down to such low masses, but we demonstrate here that this could reveal new physics.

Additionally figure 5 displays slight multi-modality for the abundance $F \sim 1$. This is not a boundary effect but arises due to small oscillations in the matter power spectrum. The amplitude and position of these oscillations in $k$ space depend on ULA parameters (compare figure 6). These oscillations translate to percent level differences over a wide mass range in the halo mass function, which can not be resolved by typical mass bins of cluster counts. Therefore multi-modalities in the posterior arise, which depend on both ULA as well as survey parameters. Oscillation-induced multi-modalities should be a generic problem of ULA probes from the large scale structure, if not observational effects smear out the differences in the posterior. Further investigation of this effect for different large scale structure probes would be desirable.

4 Conclusion

In this analysis we forecast constraints from galaxy cluster number counts on ultra-light axion cosmologies for upcoming surveys. We have used AxionCAMB to calculate the matter power spectrum and the Tinker halo mass function to estimate the halo mass function including ULAs as dark matter. We have forecasted galaxy cluster number counts in mass and
Figure 5. 68% and 95% joint posterior contours on the ULA abundance $F$ and ULA mass $m_a$ for changing shape of different minimum observed galaxy cluster mass. Dashed lines denote the fiducial model.

Figure 6. The relative difference in the power spectrum $P_k$ and the halo mass function, at $z = 0$, for different models along the degeneracy line for a limiting mass of $M_{\text{min}} = 5 \times 10^{13} h^{-1} M_\odot$. The reference model is given by $\log_{10}(m_a[\text{eV}]) = -24.3$ and $F = 0.4$. The relatively big, short oscillations in the power spectrum, $P_k$, lead to longer oscillations in the halo mass function at the 1% level.
redshift bins using a standard approach, which involved modelling of scaling relation parameters and scatter in the observable. This way we could predict cluster number counts for different standard cosmological and ULA parameters. Our Monte Carlo analysis involves the eight-dimensional parameter space \( \{ m_a, \Omega_a/\Omega_{dm}, \Omega_{dm}, A_s, n_s, a_M, \sigma_{ln} O_{obs}|M \} \). We chose various fiducial values for ULA parameters to investigate which region in the \( m_a - \Omega_a/\Omega_{dm} \) parameter space can be constrained given specific survey parameters. We saw a slight increase in constraining power including higher redshifts, the most important survey parameter to obtain competitive constraints however turned out to be the limiting mass of the surveys \( M_{\text{min}} \). Varying this parameter we obtained 95% exclusion limits for the ULA mass, typically in the region of \( m_a = 10^{-25} - 10^{-24} \text{eV} \). This of course makes sense, since the largest effects of axions on the power spectrum are on smaller scales or smaller masses.

Investigating the posterior for ULAs models which are detectable by a given survey, we find an anisotropic, curved shape. Its slope depends sensitively on the limiting mass, \( M_{\text{min}} \), of the survey and multimodalities from ULA induced oscillations in the matter power spectrum occur, which may or may not be smoothed out by observational effects for a specific probe.

A natural next step is to extended our analysis to more realistic cluster observables and to obtain constraints from existing observational campaigns such as the Dark Energy Survey [141] and Planck [1].

To obtain more accurate constraints for cosmological probes involving the halo mass function, calibration of the halo mass function with respect to ULA simulations might be necessary. First steps in this direction have been taken (e.g. [142], [107]) during the development of this work. However, we do not expect that this is a major source of uncertainty for galaxy clusters as a cosmological probe. The dominant error, in particular for halo masses below \( 10^{13} h^{-1} M_\odot \), will be the calibration of the mass observable relation and much focus will be given to this task in the future. Here the important quantity is the scatter of the mass-observable relation. The better the scatter is known the larger will be the ability to constrain cosmological parameters and also the axion quantities.

With Galaxy cluster counts alone, it is difficult to improve upon existing constraints from CMB observables or the Lyman-\( \alpha \) forest. Cluster counts suffer - similarly to all other cosmological probes for ULAs - under an approximate degeneracy between ULA mass and abundancy. While this degeneracy line looks similar in all cases, its precise form does not only depend on the probe used but also on controllable survey parameters. In case of an ULA detection in the parameter range around \( m_a \sim 10^{-22} \text{eV} \), different probes could be combined to alleviate this degeneracy. Here cluster counts could provide an important contribution in particular if they can be extended to smaller halo masses.

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