Analytic study of properties of holographic \( p \)-wave superconductors

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Abstract

In this paper, we analytically investigate the properties of \( p \)-wave holographic superconductors in \( AdS_4 \)-Schwarzschild background by two approaches, one based on the Sturm-Liouville eigenvalue problem and the other based on the matching of the solutions to the field equations near the horizon and near the asymptotic \( AdS \) region. The relation between the critical temperature and the charge density has been obtained and the dependence of the expectation value of the condensation operator on the temperature has been found. Our results are in very good agreement with the existing numerical results. The critical exponent of the condensation also comes out to be 1/2 which is the universal value in the mean field theory.

1 Introduction

The first successful microscopic theory of superconductivity, BCS theory [1], was formulated over fifty years ago and correctly describes the superconducting phenomenology of a large number of metals and alloys [2]. The basic idea of superconductivity in these weakly coupled systems is the spontaneous breaking at low temperatures of a \( U(1) \) symmetry due to a charged condensate. The condensate is a Cooper pair of electrons bound together by lattice vibrations or phonons. However, it has been appreciated that the understanding of the pairing mechanism leading to the charged condensate is far from reality for materials exhibiting superconductivity at high temperatures (high \( T_c \) cuprates). There are indications that the relevant new physics is strongly coupled and hence requires a new theoretical insight. One such insight comes from the \( AdS/CFT \) correspondence.

The \( AdS/CFT \) correspondence, which has been a powerful tool to deal with strongly coupled systems, is regarded as the most remarkable discovery in string theory. It provides

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an exact correspondence between a gravity theory in a \((d+1)\) dimensional \(AdS\) spacetime and a conformal field theory (CFT) living on its \(d\)-dimensional boundary [3]-[5]. In recent years the \(AdS/CFT\) correspondence has provided some remarkable theoretical insights in order to understand the physics of high \(T_c\) superconductors. The holographic description of s-wave superconductors consists of a AdS black hole and a complex scalar field minimally coupled to an abelian gauge field. The black hole admits scalar hair formation below certain critical temperature \((T_c)\) indicating the onset of a charged scalar condensate in the dual CFTs. The mechanism behind this condensation is the breaking of a local \(U(1)\) symmetry near the event horizon of the black hole [6]-[11].

Examples of superconducting black holes exhibiting \(p\)-wave gap has also been found [12]-[14]. The results here are based on classical solutions to field equations of Einstein-Yang-Mills theory with a negative cosmological constant

\[
S = \frac{1}{2\kappa^2} \int d^4x \left\{ R - \frac{1}{4} (F_{\mu\nu})^2 + \frac{6}{l^2} \right\}
\]

where \(F_{\mu\nu}\) is the field strength of an \(SU(2)\) gauge field. Here the idea is to consider a \(U(1)\) subgroup as the gauge group of electromagnetism and to persuade the gauge bosons, charged under this \(U(1)\), to condense outside the horizon of the black hole.

Till date a number of numerical as well as analytical studies have been performed on both \(s\) and \(p\)-wave holographic superconductor models [11]-[31]. However, analytical study of properties of holographic \(p\)-wave superconductors in \(AdS_4\)-Schwarzschild background has so far been missing in the literature. In this paper, we present two analytical approaches, one based on the Sturm-Liouville (SL) eigenvalue problem [15] and the other based on the matching of the solutions to the field equations near the horizon and near the asymptotic \(AdS\) region [10] to find the relation between the critical temperature and the charge density and to compute the expectation value of the condensation operator for holographic \(p\)-wave superconductors. This eventually helps us to compare the strength of both the methods simultaneously. The critical exponent for the condensation near the critical temperature can also be obtained naturally in our analysis. We compare our analytical results with the numerical results existing in the literature [12]-[14]. It is reassuring to note that all our calculations have been carried out in the probe limit.

This paper is organized as follows. In section 2, we provide the basic holographic set up for the \(p\)-wave superconductors, considering the background of a Schwarzschild-\(AdS_4\) spacetime. In section 3, ignoring the back reaction of the dynamical matter field on the space time metric, we compute the critical temperature as well as the temperature dependence of the condensate using the SL eigenvalue problem. In section 4, we present a simple analytical method based on matching the solutions to the field equations near the horizon and near the asymptotic \(AdS\) region to find the relation between the critical temperature and the charge density and to compute the expectation value of the condensation operator and compare our results with the SL method and the numerical results. We conclude finally in section 5.

## 2 Basic set up for \(p\)-wave superconductors

Our construction of the holographic \(p\)-wave superconductor is based on the fixed background of Schwarzschild-\(AdS\) space time. The metric of a planar Schwarzschild-\(AdS\) black
hole reads
\[ ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(dx^2 + dy^2) \] (2)

with
\[ f(r) = r^2 - \frac{r_+^3}{r} \] (3)
in units in which the AdS radius is unity, i.e. \( l = 1 \). The Hawking temperature is related to the horizon radius \((r_+)\) and is given by
\[ T = \frac{3r_+}{4\pi} \] (4)

In order to study the holographic \( p \)-wave superconductors in the probe limit, we need to introduce an \( SU(2) \) Yang-Mills action in the bulk theory. The Lagrangian density of this is given by
\[ \mathcal{L} = -\frac{1}{4} F_{a\mu\nu} F^{a\mu\nu} \] (5)

where
\[ F_{a\mu\nu} = \partial_{\mu} A_{\nu}^a - \partial_{\nu} A_{\mu}^a + \epsilon^{abc} A_{\mu}^b A_{\nu}^c \] (6)
is the Yang-Mills field strength, \((a, b, c) = (1, 2, 3)\) are the indices of the generators of \( SU(2) \) algebra. \( A_{\mu}^a \) are the components of the mixed valued gauge field \( A = A_{\mu}^a \tau^a dx^\mu \), where \( \tau^a \) are the \( SU(2) \) generators satisfying the usual commutation relations.

We now choose the following ansatz for the gauge field \([12, 13]\)
\[ A = \phi(r) \tau^3 dt + \psi(r) \tau^1 dx \] (7)

In the above ansatz, the \( U(1) \) subgroup of \( SU(2) \) generated by \( \tau^3 \) is identified to be the electromagnetic \( U(1) \) \([12, 13]\). The gauge boson with non-zero component \( \psi(r) \) along \( x \)-direction is charged under \( A_3^a = \phi(r) \). According to the AdS/CFT dictionary, \( \phi(r) \) is dual to the chemical potential in the boundary field theory while \( \psi(r) \) is dual to the \( x \)-component of some charged vector operator \( J \). The condensation of \( \psi(r) \) spontaneously breaks the \( U(1) \) gauge symmetry and triggers the phenomena of superconductivity on the boundary field theory.

The equations of motion for the fields \( \phi(r) \) and \( \psi(r) \) computed on this ansatz read
\[ \phi''(r) + \frac{2}{r} \phi'(r) - \frac{\psi^2(r)}{r^2 f} \phi(r) = 0 \] (8)
\[ \psi''(r) + \frac{f'}{f} \psi'(r) + \frac{\phi^2(r)}{f^2} \psi(r) = 0 \] (9)

where prime denotes derivative with respect to \( r \). In order to solve the non-linear equations \([8]\) and \([9]\), we need to seek the boundary condition for \( \phi(r) \) and \( \psi(r) \) near the black hole horizon \( r \sim r_+ \) and at the spatial infinity \( r \to \infty \). At the horizon, we require \( \phi(r_+) = 0 \) for
the $U(1)$ gauge field to have a finite norm and $\psi(r_+)$ should be finite. Near the boundary of the bulk, we have

$$\phi(r) = \mu - \frac{\rho}{r},$$

$$\psi(r) = \frac{\psi^{(1)}}{r}.$$  \hspace{1cm} (10)

$\mu$ and $\rho$ are dual to the chemical potential and charge density of the boundary CFT and $\psi^{(1)}$ is dual to the expectation value of the condensation operator $J$ at the boundary. Under the change of coordinates $z = \frac{r_+}{r}$, the field equations become

$$\phi''(z) - \frac{\psi^2(z)}{r_+^2 (1 - z^3)} \phi(z) = 0$$

$$\psi''(z) - \frac{3z^2}{1 - z^3} \psi'(z) + \frac{\phi^2(z)}{r_+^2 (1 - z^3)^2} \psi(z) = 0$$

where prime now denotes derivative with respect to $z$. These equations are to be solved in the interval $(0, 1)$, where $z = 1$ is the horizon and $z = 0$ is the boundary. The boundary condition $\phi(r_+) = 0$ now becomes $\phi(z = 1) = 0$.

3 Sturm-Liouville method

3.1 Relation between critical temperature and charge density

With the above set up in place, we now move on to investigate the relation between the critical temperature and the charge density. At the critical temperature $T_c$, $\psi = 0$, so the field equation (12) reduces to

$$\phi''(z) = 0.$$  \hspace{1cm} (14)

With the boundary condition (10), the solution of this equation reads

$$\phi(z) = \lambda r_{+(c)} (1 - z)$$

$$\lambda = \frac{\rho}{r_+^{(c)}}.$$  \hspace{1cm} (15)

Using the above solution, we find that as $T \to T_c$, the equation for the field $\psi$ approaches the limit

$$- \psi''(z) + \frac{3z^2}{1 - z^3} \psi'(z) = \frac{\lambda^2}{(1 + z + z^2)^2} \psi(z).$$  \hspace{1cm} (17)

Near the boundary, we define $\psi(z) = \frac{(J)}{\sqrt{2r_+}} z F(z)$  \hspace{1cm} (18)
where $F(0) = 1$. Substituting this form of $\psi(z)$ in eq. (17), we obtain

$$- F''(z) + \left( \frac{3z^2}{1 - z^3} - \frac{2}{z} \right) F'(z) + \frac{3z}{1 - z^3} F(z) = \frac{\lambda^2}{(1 + z + z^2)^2} F(z)$$ (19)

to be solved subject to the boundary condition $F'(0) = 0.$

The above equation can be put in the Sturm-Liouville form

$$\frac{d}{dz} \left\{ p(z) F'(z) \right\} - q(z) F(z) + \lambda r(z) F(z) = 0$$ (20)

with

$$p(z) = z^2(1 - z^3), \quad q(z) = 3z^3, \quad r(z) = \frac{z^2(1 - z)}{1 + z + z^2}. \quad (21)$$

With the above identification, we now write down the eigenvalue $\lambda^2$ which minimizes the expression

$$\lambda^2 = \frac{\int_0^1 dz \left\{ p(z)[F'(z)]^2 + q(z)[F(z)]^2 \right\}}{\int_0^1 dz \ r(z)[F(z)]^2}
= \frac{\int_0^1 dz \left\{ z^2(1 - z^3)[F'(z)]^2 + 3z^3[F(z)]^2 \right\}}{\int_0^1 dz \ \frac{z^2(1-z)}{1+z+z^2}[F(z)]^2}.$$ (22)

To estimate it, we use the following trial function

$$F = F_\alpha(z) \equiv 1 - \alpha z^2$$ (23)

which satisfies the conditions $F(0) = 1$ and $F'(0) = 0.$

Hence, we obtain

$$\lambda^2_\alpha = \frac{60(\alpha - \frac{3}{4} - \frac{27\alpha^2}{40})}{(-130 + 21\alpha - 10\sqrt{3}\pi\alpha + 30(\alpha + 4)\ln 3)\alpha + 10(-9 + \sqrt{3}\pi + 3\ln 3)} \quad (24)$$

which attains its minimum at $\alpha \approx 0.5078.$ The critical temperature therefore reads

$$T_c = \frac{3}{4\pi} r_{+}(c) = \frac{3}{4\pi} \sqrt{\frac{\rho}{\lambda_{\alpha=0.5078}}} \approx 0.1239\sqrt{\rho}$$ (25)

which is in very good agreement with the exact $T_c = 0.125\sqrt{\rho} \quad [12].$

### 3.2 Critical exponent and condensation values

In this section, we shall compute the condensation values of the condensation operator $J$ in the boundary field theory.

Away from (but close to) the critical temperature $T_c,$ the field equation [12] for $\phi$ becomes (using eq. [18])

$$\phi''(z) = 2J^2 r_+^2 B(z) \phi(z)$$ (26)

$$B(z) = \frac{z^2 F^2(z)}{2r_+^2(1 - z^3)}$$
where the parameter $\langle J \rangle^2/r_+^2$ is small. Now we expand $\phi(z)$ in the small parameter $\langle J \rangle^2/r_+^2$ as

$$\frac{\Phi}{r_+} = \lambda (1 - z) + \frac{\langle J \rangle^2}{r_+^3} \chi(z) + \ldots \quad (27)$$

From eq(s) $[26, 27]$, we obtain the equation for the correction $\chi(z)$ near the critical temperature

$$\chi''(z) = \lambda r_+ (1 - z) B(z) \quad (28)$$

with $\chi(1) = 0 = \chi'(1)$. Integrating both sides of the above equation between $z = 0$ to $z = 1$, we obtain

$$\chi'(0) = -\lambda r_+ \int_0^1 (1 - z) B(z) dz. \quad (29)$$

Now from eq(s) $[10, 27]$, we have

$$\frac{\mu}{r_+} - \frac{\rho}{r_+^2} z = \lambda (1 - z) + \frac{\langle J \rangle^2}{r_+^3} \chi(z)$$

$$= \lambda (1 - z) + \frac{\langle J \rangle^2}{r_+^3} (\chi(0) + z \chi'(0) + \ldots) \quad (30)$$

where in the second line we have expanded $\chi(z)$ about $z = 0$. Comparing the coefficient of $z$ on both sides of this equation, we obtain

$$\frac{\rho}{r_+^2} = \lambda - \frac{\langle J \rangle^2}{r_+^3} \chi'(0). \quad (31)$$

Substituting $\chi'(0)$ from eq.$[29]$ in the above equation, we get

$$\frac{\rho}{r_+^2} = \lambda \left\{ 1 + \frac{\langle J \rangle^2}{r_+^3} \int_0^1 (1 - z) B(z) dz \right\}. \quad (32)$$

Finally using eq(s) $[4, 16]$ in this equation, we get the following expression for $\langle J \rangle$

$$\langle J \rangle = \gamma T_c^2 \sqrt{1 - \frac{T}{T_c}} \quad (33)$$

where

$$\gamma = \frac{2(4\pi/3)^2}{\sqrt{A}} \quad (34)$$

$$A = \int_0^1 \frac{z^2 F^2(z)}{1 + z + z^2} dz.$$  

Computing $A$ with $\alpha = 0.5078$, we get $\gamma \approx 123.4$ which is close to the exact numerical result $\gamma \approx 104.8$ $[14]$. The critical exponent of the expectation value of the condensation operator also comes out to be $1/2$ which is the universal value in the mean field theory.
4 Critical temperature and condensation values by matching method

In this section, we shall present a simple analytic treatment of superconductivity and compare our results with those obtained from the SL approach discussed above. The method involves finding an approximate solution around both $z = 1$ and $z = 0$ using Taylor series expansion and then connecting these solutions between $z = 1$ and $z = 0$.

4.1 Solution near the horizon: $z = 1$

Near $z = 1$, we expand $\phi(z)$ and $\psi(z)$ as

$$
\phi(z) = \phi(1) - \phi'(1)(1 - z) + \frac{1}{2} \phi''(1)(1 - z)^2 + ... \\
\approx -\phi'(1)(1 - z) + \frac{1}{2} \phi''(1)(1 - z)^2
$$

(35)

$$
\psi(z) = \psi(1) - \psi'(1)(1 - z) + \frac{1}{2} \psi''(1)(1 - z)^2 + ... \\
\approx \psi(1) + \frac{1}{2} \psi''(1)(1 - z)^2
$$

(36)

where we have used the boundary condition $\phi(1) = 0$ in the second line of eq.(35) and $\psi'(1) = 0$ (which readily follows by multiplying eq.(13) by $(1 - z^3)^2$) and then setting $z = 1$ and using $f(1) = 0$ and $\phi(1) = 0$ in the second line of eq.(36). We also set $\phi'(1) < 0$ and $\psi(1) > 0$ for the positivity of $\phi(z)$ and $\psi(z)$.

From eq.(12), we compute the coefficient of the third term in eq.(35)

$$
\phi''(1) = \lim_{z \to 1} \frac{\psi^2(z)\phi(z)}{r_+^2 (1 - z^3)} = -\frac{\psi^2(1)\phi'(1)}{3r_+^2} 
$$

(37)

Similarly, from eq.(13), we compute the coefficient of the third term in eq.(36)

$$
\psi''(1) = \lim_{z \to 1} \left\{ \frac{3z^2}{1 - z^3} \psi'(z) - \frac{\phi^2(z)\psi(z)}{r_+^2(1 - z^3)^2} \right\} \\
= -\psi''(1) - \frac{\phi^2(1)\psi(1)}{9r_+^2}
$$

(38)

which gives

$$
\psi''(1) = -\frac{\phi^2(1)\psi(1)}{18r_+^2}. 
$$

(39)

Setting $\phi'(1) = -\beta$ and $\psi(1) = \alpha$ and substituting eq.(37) in eq.(35) and eq.(39) in eq.(36), we finally get the solutions for $\phi(z)$ and $\psi(z)$ near $z = 1$

$$
\phi(z) \approx \beta(1 - z) + \frac{\alpha^2\beta}{6r_+^2}(1 - z)^2 \\
\psi(z) \approx \alpha - \frac{\alpha \beta^2}{36}(1 - z)^2 ; \quad \bar{\beta} = \frac{\beta}{r_+}. 
$$

(40)

(41)
4.2 Solution near the asymptotic AdS region: $z = 0$

Near $z = 0$, we expand $\phi(z)$ and $\psi(z)$ using the asymptotic solutions (10) and (11) as

$$\phi(z) = \mu - \frac{\rho}{r_+} z + \frac{1}{2}\phi''(0) z^2 + ...$$  \hspace{1cm} (42)

$$\psi(z) = \frac{\psi^{(1)}}{r_+} z + ...$$  \hspace{1cm} (43)

Now the second derivative of $\phi(z)$ evaluated at $z = 0$ is given by

$$\phi''(0) = \lim_{z \to 0} \frac{\psi^2(z) \phi(z)}{r_+^2 (1 - z^3)} = 0.$$  \hspace{1cm} (44)

Hence we have

$$\phi(z) = \mu - \frac{\rho}{r_+} z$$  \hspace{1cm} (45)

$$\psi(z) = \frac{\psi^{(1)}}{r_+} z = \frac{\langle J \rangle}{\sqrt{2r_+}} z.$$  \hspace{1cm} (46)

4.3 Matching and phase transition

We now connect the solutions (40), (41) with (45) and (46) respectively. To do this, we take the derivative of the eq.(40) with respect to $z$ and compare with the derivative of eq.(45) with respect to $z$, which finally yields,

$$\frac{\rho}{r_+} = \beta + \frac{\alpha^2}{3r_+^2} (1 - z).$$  \hspace{1cm} (47)

Rearranging the above equation and using eq.(4) to write $r_+$ in terms of $T$, we get

$$\alpha = \frac{4\pi}{3} \frac{6}{(1 - z)} T_c \left(1 - \frac{T}{T_c}\right)^{1/2}$$  \hspace{1cm} (48)

where the critical temperature $T_c$ is given by

$$T_c = \frac{3}{4\pi} \frac{1}{\sqrt{\beta}} \rho^{1/2}.$$  \hspace{1cm} (49)

Now comparing eq(s) (41) and (46), we get

$$\frac{\langle J \rangle}{\sqrt{2r_+}} z = \alpha - \frac{\alpha^3}{36} (1 - z)^2.$$  \hspace{1cm} (50)

Differentiating the above equation with respect to $z$, we obtain

$$\frac{\langle J \rangle}{\sqrt{2r_+}} = \frac{\alpha^3}{18} (1 - z).$$  \hspace{1cm} (51)
Dividing eq.(50) by eq.(51) and rearranging, we get
\[ \tilde{\beta}^2 = \frac{36}{1 - z^2}. \] (52)

Substituting this expression in eq.(49) gives
\[ T_c = \kappa \rho^{1/2} \] (53)
where
\[ \kappa = \frac{3}{4\pi} \frac{(1 - z^2)^{1/4}}{\sqrt{6}}. \] (54)

Similarly, substituting the above expression for \( \tilde{\beta}^2 \) in any one of the eq(s)(50) or (51) and using eq.(48), we obtain
\[ \langle J \rangle = \gamma T T_c \left(1 - \frac{T}{T_c}\right)^{1/2} \]
\[ \approx \gamma T_c^2 \left(1 - \frac{T}{T_c}\right)^{1/2} \] (55)
where
\[ \gamma = \left(\frac{4\pi}{3}\right)^{2} \sqrt{\frac{48}{(1 - z)(1 + z)^2}}. \] (56)

The above expressions for the critical temperature \( T_c \) and the expectation value of the condensation operator \( \langle J \rangle \) depends on the value of \( z \) we choose. Note that \( \gamma \) diverges for \( z = 1 \) and therefore it indicates that the matching of the solutions near the horizon and near the asymptotic \( AdS \) region should be done for \( z < 1 \).

In the table below (table 1), we list the values of \( \kappa \) and \( \gamma \) for some values of \( z \) in the interval \((0, 1)\).

| \( z \) | \( \kappa \) | \( \gamma \) |
|---|---|---|
| 0.1 | 0.0972 | 116.5 |
| 0.2 | 0.0965 | 113.3 |
| 0.3 | 0.0952 | 111.8 |
| 0.4 | 0.0933 | 112.1 |
| 0.5 | 0.0907 | 114.6 |
| 0.6 | 0.0872 | 120.1 |

We now compare (in the table below (table 2)) our analytical values for \( \kappa \) and \( \gamma \) obtained by the SL and matching approach with the existing numerical results in the literature \[12 \] [14]. In order to do that we choose the value of \( \kappa_{\text{matching}} \) and \( \gamma_{\text{matching}} \) corresponding to \( z = 0.3 \).

Both from table 1 and table 2, we observe that for \( z = 0.1 \), \( \kappa_{\text{matching}} \) is closest to both \( \kappa_{\text{SL}} \) and \( \kappa_{\text{numerical}} \). However, on the other hand, \( \gamma_{\text{matching}} \) is always found to be closer to
Table 2: A comparison of the analytical (SL and matching methods) and numerical results for the critical temperature and the expectation value of the condensation operator

| $\kappa_{SL}$ | $\kappa_{\text{matching}}$ | $\kappa_{\text{numerical}}$ | $\gamma_{SL}$ | $\gamma_{\text{matching}}$ | $\gamma_{\text{numerical}}$ |
|---------------|-----------------------------|-----------------------------|----------------|---------------------------|-----------------------------|
| 0.1239        | 0.0952                      | 0.125                      | 123.4         | 111.8                     | 104.8                      |

$\gamma_{\text{numerical}}$ compared to $\gamma_{SL}$. For $z = 0.3$, we observe that $\gamma_{\text{matching}}$ is closest to $\gamma_{\text{numerical}}$. Interestingly this does not seem to change the value of $\kappa_{\text{matching}}$ does not change much from the value at $z = 0.1$. As we increase $z$, both $\kappa_{\text{matching}}$ and $\gamma_{\text{matching}}$ starts to deviate away from the corresponding numerical values. As $z \to 1$, the analytical value for $\gamma$ obtained from the matching method diverges which clearly indicates that the matching should be done closer to $z = 0$.

5 Conclusions

In this paper, we perform analytic computation of holographic $p$-wave superconductors in $AdS_4$-Schwarzschild background by two methods, one based on the Sturm-Liouville eigenvalue problem and the other based on the matching of the solutions to the field equations near the horizon and near the asymptotic $AdS$ region. The relation between the critical temperature and the charge density has been obtained and the dependence of the expectation value of the condensation operator on the temperature has been worked out. Our results are in very good agreement with the existing numerical results [12, 14]. The critical exponent of the condensation also comes out to be 1/2 which is the universal value in the mean field theory.

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