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**Title:** Advanced Multi-Mode Phase Retrieval for Dispersion Scan

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Multi-Mode Root Preserving Ptychographic Phase Retrieval Algorithm for Dispersion Scan Supplemental Information

In this supplementary document we analyze how various pulse train instabilities affect the retrieval of a pulse from the d-scan trace using the single-mode algorithm. Retrievals from simulated traces with simulated instabilities are shown.

1. PULSE TRAIN INSTABILITIES AND THE COHERENT ARTIFACT

In most scanning dispersion scan (d-scan) measurements, thousands of pulses in a train are measured and are implicitly assumed to be identical to each other. Generally, this is a valid assumption as the pulse train is reasonably stable. However, when the pulse train is unstable, the instabilities are encoded into the trace and the retrieval algorithm faces the impossible task of trying to recover a single pulse from a trace containing information from thousands of pulses.

For some measurement techniques (e.g. autocorrelation, FROG, SPIDER) retrievals can exhibit a coherent artifact: the retrieved pulse from an unstable pulse train is not representative of the average pulse in the train [1–3]. The effect of pulse train instabilities and the coherent artifact has been extensively studied for frequency resolved optical gating (FROG) [3, 4], and recently has been explored for other d-scan retrieval algorithms [5–7]. While d-scan has been shown to not exhibit a misleading coherent artifact [6], it is still instructive to understand how pulse instabilities affect the d-scan trace and how our algorithm behaves in the presence of pulse train instabilities.

We have identified four common sources of instabilities in laser pulse trains that we will test the algorithm on: shot-to-shot energy fluctuations, phase fluctuations, pointing instabilities within the laser, and unstable gain pulling during amplification. Figure S1 shows the results of four separate simulations, each exhibiting one of these instabilities. The leftmost column shows the simulated traces for each instability source. The middle-left column shows the retrieved traces. The middle-right column shows the ideal simulated pulse (blue) and retrieved pulse (red-dashed). The rightmost column displays the ideal spectrum (blue), retrieved spectrum (red-dashed), ideal spectral phase (purple), and retrieved spectral phase (yellow-dashed). Here, ideal refers to the initial pulse or spectrum before the fluctuations were applied.

Shot-to-shot energy fluctuations are extremely common in all laser systems, especially lower repetition rate systems. Figures S1(a) through S1(d) show the results of a simulation where each column of the trace was generated from a pulse with identical phase and spectral content, but whose overall energy was randomly sampled from a normal distribution with a standard deviation of 25%. The random energy fluctuations can easily be seen in the simulated trace, but they are noticeably absent in the retrieved trace. The retrieval algorithm behaves extremely favorably under such conditions as the retrieved pulse, spectrum, and spectral phase are essentially identical to the ideal pulse, spectrum and spectral phase. The NRMSE for the retrieval in Fig. S1(a) - S1(d) was $\delta = 0.0215$.

Spectral phase fluctuations are also somewhat common in modern laser systems for various reasons. They can arise from pointing instabilities in the laser, or running the laser in unstable operating regimes. Figures S1(e) through S1(h) show a simulation where the spectral amplitude of the pulse was identical for each column of the trace but the spectral phase of each column was the sum of the the simulated phase, $\phi_S$, and a random phase $\phi_R$ which is given by:

$$\phi_R(\omega) = R_2\phi_2(\omega - \omega_0)^2 + R_3\phi_3(\omega - \omega_0)^3 + R_4\phi_4(\omega - \omega_0)^4 + \sin(R_S(\omega - \omega_0))$$  \hspace{1cm} (S1)

Here, $\phi_2 = 100fs^2$, $\phi_3 = 1000fs^3$, $\phi_4 = 10000fs^4$, $R_2$ through $R_4$ represent a number randomly sampled from a normal distribution with a mean of 0 and standard deviation of 1, and $R_S$ represents a number randomly sampled from a normal distribution with a mean of 0 and standard deviation of 10. Each $R_S$ was randomly sampled for each column of the trace. We find that the algorithm again behaves favorably under such strong fluctuations. Minimal deviations in the...
Fig. S1. Simulated pulse retrievals on traces with different sources of error. The four rows correspond to the following sources of error: energy fluctuations in the pulse train, phase fluctuations in the pulse train, shot-to-shot pointing instabilities into the pulse compressor, and unstable gain pulling in the amplifier.

The reconstructed spectrum and spectral phase are seen only near the very edges of the bandwidth. The pulse structure is accurately reconstructed with only minimal errors in the width of the side lobes. The NRMSE for the retrieval in Fig. S1(h) was \( \delta = 0.0165 \).

One extremely common source of error in laser systems is pointing instabilities within the laser itself. This can slightly alter the spectral phase content of each pulse in the train due to the different paths each pulse in the train will take through the laser system. This is especially problematic with regards to grating d-scan as each pulse can enter the pulse compressor at a different incident angle, which would lead to a different grating phase being applied to each pulse. Figures S1(i) through S1(l) show a simulated retrieval where the incident angle of the beam onto the compressor gratings for each column of the trace was randomly sampled from a normal distribution centered around \( \theta_i = 34.05^\circ \), with a standard deviation of half a degree. The retrieval algorithm assumed a Littrow incident angle onto the gratings for the entire trace. In this case, the simulated trace does not seem to exhibit as many visible fluctuations as in the previous cases and the retrieved trace matches it extremely well. Similarly to the previous example, the retrieved spectrum and spectral phase deviate from the simulated spectrum and spectral phase very slightly near the edges of the bandwidth. Additionally, slight errors in the retrieved pulse structure can be seen in the side lobes of the temporal pulse. The NRMSE for the retrieval in Fig. S1(i) - S1(l) was \( \delta = 0.0027 \).
Lastly, we will discuss the effects of random gain pulling on the d-scan trace. Such a phenomena is common in chirped pulse amplification systems where the amplifier is operated in an unstable regime and the long wavelength side of the chirped seed pulse experiences a disproportionate amount of gain relative to the short wavelength side randomly from pulse to pulse. This effectively randomly pulls the spectrum of the amplified pulse towards the red side of the spectrum. To simulate gain pulling, the spectral field for each column of the trace was multiplied by a weighting function to pull the spectrum towards longer wavelengths:

$$\tilde{E}_k(\omega) = \tilde{E}_0(\omega)(1 + \text{erf}(-R_k(\omega - \omega_0)))$$  \hspace{1cm} (S2)

Here, $\tilde{E}_k(\omega)$ is the spectral field of the $k$th column, $\tilde{E}_0(\omega)$ is the unperturbed spectral field, $R_k$ is a number randomly sampled from a normal distribution with a mean of 1 fs and a standard deviation of 5 fs. Figures S1(m) through S1(p) show the results of a simulated pulse retrieval on a trace that exhibits random gain pulling in each column of the trace. The two primary effects this has are 1) the retrieved spectrum is essentially the average of all the spectra used to generate the trace, and 2) the temporal pulse is the average of all the pulses in the trace. The NRMSE for the retrieval in Fig. S1(m) - S1(p) was $\delta = 0.0093$.

In effect, we see that the retrieval algorithm tends to retrieve essentially the average pulse in the pulse train when many unique pulses are present. The cases presented above are exaggerated examples of real world phenomena that are present in most modern laser systems. In reality, combinations of all these effects are present in real-world data, but as can be seen from the above examples, the algorithm performs well under such conditions and does not exhibit a misleading coherent artifact similar to how other d-scan algorithms perform under such scenarios [6]. While the results shown here are for the single mode algorithm, the multi-mode algorithm performs similarly under such conditions.

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