On Spiral Turbulent Flow in an Annular Concentric Channel

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Abstract. The paper presents the results of Newtonian-fluid spiral flow simulation in an annular channel. The simulation parameters were diameters ratio of 0.5; Re = 100 ÷ 10,000 and dimensionless inner cylinder rotation rate χ = 0.2 ÷ 5. The turbulent flow regimes were simulated using URANS, a non-stationary statistical turbulence model. While simulation, the following flow regimes were detected: homogeneous flow without pronounced structures (Re = 100, χ ≤ 3 and Re = 300, χ ≤ 0.5); flow with Görtler-type continuous spiral vortices near inner cylinder (Re = 300, χ = 1 and Re = 1000 ÷ 10000, χ = 0.5); flow with Taylor-type vortices (Re = 100, χ ≥ 3 and Re = 300, χ ≥ 3); flow with small-scale Görtler-type vortices near both channel walls (Re > 1000 and χ ≥ 1). The spiral spin of Görtler-type vortices near the outer cylinder was co-directed with the rotation of the inner cylinder, while the vortices near the inner cylinder spin against the rotation. Formation of large Taylor-type vortex structures led to a decrease in the channel skin friction coefficient faRe. Görtler-type vortices interaction near the channel walls triggered the formation of small-scale structures increasing in the channel resistance as the Reynolds number and rotation speed grew.

1. Introduction

The problem of fluid flow between two cylinders is common for many engineering systems such as centrifuges, rotation viscometers, heat exchangers, sliding-surface bearings, borehole drilling systems and many others. In this problem, the fluid’s axial flow along the channel can be accompanied by the fluid’s rotational motion to produce a spiral flow. In this case, the flow regime and laminar-turbulent transition depend not only on the Reynolds number but also on the cylinder’s rotation rate. Despite the system's apparent simplicity, this hydrodynamic problem is hard to solve, but these solutions are in great demand.

Most of the experimental works addressing the issue are devoted to the questions of stability and visualization of laminar fluid flows [1 - 3]. A sufficient number of spiral flows has been detected so far, but the studies contain almost no data on channel resistance.

Nouri et.al. [4] and Escudier et.al. [5] experimentally studied a turbulent rotational flow in an eccentric annular channel with diameters ratio of 0.5 for a range of Re reaching 26600. The dimensionless velocity in the performed experiments was relatively low and did not exceed 0.86.

Several numerical studies were devoted to using direct numerical simulation (DNS) [6, 7] and large eddy simulation (LES) [8, 9] for investigating a turbulent spiral flow in a concentric annular channel. Their results have become a great validation tool for other numerical studies because LES results are
usually in good match with DNS and experimental ones. In the concerned studies, Re reaches 9000, and the dimensionless rotation rate less 0.86, but none of them contains a systematic analysis of the flow structure and its integral parameters.

The objective of the study presented in this paper was to widen the range of studied annular channel flows, identify their flow regime and analyze their channel drag behavior.

2. Problem statement
To meet the objective, a spiral flow of Newtonian fluid in an annular channel was simulated. In the model, the outer cylinder of diameter $D_o$ rests, while the inner cylinder of diameter $D_i$ rotates with a given rotation rate $\Omega$. The inner/outer diameters relate as $D_i/D_o = 0.5$. The problem accounts for two dimensionless parameters such as Reynolds number $Re = \rho U(D_o - D_i)/\mu$ and rotation rate $\chi = \Omega D_i/2U$. Here, $U$ denotes the mean axial fluid velocity; $\rho$ is the fluid density, and $\mu$ is the viscosity. The modeling was performed for $Re = 100 \div 10,000$ and $\chi = 0.2 \div 5$.

3. Simulation method
3.1. Numerical method
To simulate the unsteady flow, Unsteady Reynolds-Averaged Navier-Stokes (URANS) approach and $k-\omega$ SST Menter's model [10] were used. The numerical algorithm was based on the finite volume discretization on an arbitrary unstructured mesh. The convective terms of the momentum equation were approximated using the high-order Rossi approximation [11]. For temporal integration, the unconditionally stable Crank-Nicolson method of second-order accuracy was used. For purposes of operator splitting, both viscous and convective terms of the momentum equation were considered as implicit. The hydrodynamic equations were solved using the SIMPLE-C algorithm.

3.2. Computational mesh
For modeling of the considered flows, the channel lengths equal to four hydraulic diameters $D_h = D_o - D_i$ under periodic boundary conditions were thought to be sufficient [12]. To obtain a developed flow pattern in our study, a channel of $4.2D_h$ in length was used.

For every calculation performed, the condition $\Delta r^+ \leq 1$ for every mean value was met. The the number of nodes in radial distribution were selected based on a pre-estimated steady-state flow. The number of computational nodes was constant both for tangential ($N_\theta = 100$) and streamwise directions ($N_z = 128$).

The time step was $\Delta t = 0.02D_h/U$, and mean CFL number is less than one for all the computation. On average, it took no more than 200$D_h/U$ seconds for a simulated flow to reach a developed flow. After that, the flow was time-averaged within an interval of 400$D_h/U$ to obtain its statistical characteristics.

3.3. Validation
To validate the URANS results, they were compared against LES modeling [8] for Re=8900, $\chi = 0.43$ and 0.85. In [13], the authors claim that flows with high rotational degree cannot be properly described by turbulent models using eddy viscosity approach. In the flow modeled in our study, unsteady large-scale vortex structures of Taylor/Görtler type got formed that resulted in a significant improvement of the model’s ability to predict the mean flow velocities (figure 1), and fist (figure 2) and second (figure 3) velocity field moments. The URANS computational mesh parameters can be seen in table 1. It is noteworthy that URANS reduced the channel resistance values by 12% if compared to those calculated using LES [8] (table 2).
Figure 1. Distributions of mean axial velocity $W$ normalized by mean axial velocity $U$ and tangential velocity $V$ normalized by rotational velocity of inner cylinder $V_{ω}=ωD/2$, $χ=0.858$.

Figure 2. Distributions of normalized turbulent kinetic energy $k/U^2$, $χ=0.858$.

Figure 3. Distributions of normalized Reynolds shear stress $〈uw⟩/U^2$, $χ=0.858$.

Table 1. Mesh parameters for URANS.

| $χ$  | 0.429 | 0.858 |
|------|--------|--------|
| $(N_r, N_θ, N_z)$ | (71,100,128) | (71,100,128) |
| $(Δr_r, (RΔθ)^r, Δz^r)$ | (1,21,21) | (1,25,25) |
| CFL | 0.57±0.2 | 0.58±0.22 |
| $ΔtU/D_h$ | 0.02 | 0.02 |

Table 2. Skin friction coefficients $f_a$.

| $χ$  | 0.429 | 0.858 |
|------|--------|--------|
| URANS 0.00925945 | 0.01055305 |
| LES by Chung [8] 0.009861 | 0.012048 |

4. Results and discussion

4.1. Flow structure

The fluid thrown away from the inner cylinder’s surface by a centrifugal force forms the pairs of bidirectional vortex cores wound on the cylinder, their visualization on isosurfaces $s_2$ is shown in figure 4. The isosurfaces are colored by the flow vorticity components $Ω = 1/2$rot($V$) to show the direction fluid particles rotate in a corresponding vortex. Here, $V$ is the flow velocity vector.
Figure 4. Vortices structures visualized by isosurfaces $\lambda_2$ and colored to mark the direction fluid particles rotate.

Figure 5. Vortices structures for $Re = 1000, \chi = 5$. Visualized by isosurfaces $\lambda_2 = -5$. Colored by sign of vorticity $\Omega_x$ (red is positive, blue - negative).
At rotation rate $\chi = 0.2$, the vortex structures did not form, thus criterion $\lambda_2$ remained positive for the whole computation domain and the flow became laminar or turbulent depending on the Reynolds number.

At $Re=100$, $\chi < 3$ and $Re=300$, $\chi < 1$, the vortex structures did not form either, and the flow remained laminar and homogenous. Increasing the rotation rate ($\chi =3\div 5$ and $Re=100$) resulted in formation of closed vortex cores known as Taylor-type toroidal ones (figure 4). Somewhat distorted Taylor-type vortices continued to form at higher $Re = 300$ and $\chi = 5$.

Görtler-type spiral vortices were observed near the inner cylinder at $\chi =1$, $Re = 300$ and $\chi =0.5$, $Re = 1000\div 10,000$ (figure 4). It is noteworthy that they rotated in the direction opposite to that of the inner cylinder (figure 4).

Near the surface of the resting outer cylinder, the flow also contained the Görtler-type spiral vortices, and the direction of their twist is similar to the inner cylinder rotation. A typical example of such a flow can be seen in figure 5, which marks the inner and outer vortices and direction of fluid particles rotation in a corresponding vortex. Interaction of large-scale spiral inner and outer vortices led to their breakup followed by formation of the smaller structures that were observed at $Re = 300$, $\chi =2\div 3$ and $Re \geq 1000$, $\chi >1$.

4.2. Friction factor

At $Re = 100$, the flow remained laminar for all the rotation rates, and the channel resistance had a good match with the analytical solution $faRe = 24$ (figures 6 and 7). In this case, formation of the Taylor-type vortices (figure 4) had no effect on the channel drag.

![Figure 6. Fanning friction factor dependency on Re for different rotations $\chi$.](image1)

![Figure 7. Fanning friction factor dependency on rotations $\chi$ for different Re.](image2)

At constant rotation rate, the channel resistance grew together with increasing $Re$, and the dependency on the rotation rate was not monotonous (figure 6).

At $Re=300$, the spiral vortices also had no pronounced effect on the channel resistance ($\chi \leq 1$), while the formation of small-scale vortex structures increased the resistance ($\chi = 2\div 3$). Further increase of the rotation rate led to formation of the large-scale toroidal vortices but reduced the channel resistance almost down to its analytical value ($\chi = 5$). In other words, within a certain range of $Re$ between 100 and 1000, a non-monotonous channel resistance dependency on rotation rate was observed.

Further increase of both $Re \geq 1000$ and $\chi$ resulted in a monotonous growth of the channel resistance (figures 6 and 7) mostly due to formation and breakup of the large-scale vortices (figures 4 and 5).

5. Conclusions

The performed numerical experiments have demonstrated that the large-scale vortex structures did not form at low rotation rates ($\chi <0.5$) and the flow remained homogeneous.
At Re=100, the flow remained laminar for all the considered rotation rate values, while the analytical dependence $faRe=24$ described the channel resistance with a good degree of accuracy despite formation of the Taylor-type toroid vortices.

Increasing the Reynolds number to Re=300 resulted in formation of the paired Görtler-type spiral vortices near the inner cylinder, while the channel resistance remained close to that determined analytically. Formation of vortex structures near the external cylinder and their interaction with inner-cylinder vortices increased the channel resistance a bit. Further increase of the rotation rate suppressed the small-scale vortex structures and deformed the Taylor-type toroid vortices to bring the channel resistance back closer to the analytical solution $faRe=24$.

Further increase of the Reynolds number (Re=1000÷10,000) and rotation rate led to monotonous channel resistance growth. At low rotation rates ($\chi<1$), the paired Görtler-type spiral vortices could be observed around the inner cylinder, and at ($\chi\geq1$) – around the outer cylinder, both resulting in formation of the small-scale vortex structures.

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