Observed tidal evolution of Kleopatra’s outer satellite*

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ABSTRACT

Aims. The orbit of the outer satellite Alexhelios of (216) Kleopatra is already constrained by adaptive-optics astrometry obtained with the VLT/SPIREHE instrument. However, there is also a preceding occultation event in 1980 attributed to this satellite. Here, we try to link all observations, spanning 1980–2018, because the nominal orbit exhibits an unexplained shift by +60′ in the true longitude.

Methods. Using both a periodogram analysis and an ℓ = 10 multipole model suitable for the motion of mutually interacting moons about the irregular body, we confirmed that it is not possible to adjust the respective osculating period $P_1$. Instead, we were forced to use a model with tidal dissipation (and increasing orbital periods) to explain the shift. We also analysed light curves spanning 1977–2021, and searched for the expected spin decceleration of Kleopatra.

Results. According to our best-fit model, the observed period rate is $P_2 = (1.8 ± 0.1) \times 10^{-8} \text{d}^{-1}$ and the corresponding time-lag $\Delta t = 42 \text{s}$, for the assumed value of the Love number $k_2 = 0.3$. This is the first detection of tidal evolution for moons orbiting 100 km asteroids. The corresponding dissipation factor $Q$ is comparable with that of other terrestrial bodies, albeit at a higher loading frequency $2\omega_n - \nu$. We also predict a secular evolution of the inner moon, $P_1 = 5.0 \times 10^{-11}$, as well as a spin deceleration of Kleopatra, $P_0 = 1.9 \times 10^{-12}$. In alternative models, with moons captured in the 3:2 mean-motion resonance or more massive moons, the respective values of $\Delta t$ are a factor of between two and three lower. Future astrometric observations using direct imaging or occultations should allow us to distinguish between these models, which is important for our understanding of the internal structure and mechanical properties of (216) Kleopatra.

Key words. minor planets, asteroids: individual: (216) Kleopatra – planets and satellites: individual: I Alexhelios – planets and satellites: dynamical evolution and stability – celestial mechanics – methods: numerical

1. Introduction

It is already known that small (1 km) binary asteroids are driven by radiative torques, tides, or both (e.g. Scheirich et al. 2021). In the case of binaries, the secondary orbital evolution is obtained by measuring the steady decrease or increase in the period of eclipses. However, the primary rotation evolution is not observed.

For large (100 km) asteroids with relatively small satellites, the situation is different. Radiative torques (cryptographically, ‘BYORP’) are considered weak because they scale as (Čuk & Burns 2005):

$$\Gamma L \approx 3.0 \times 10^{-12} \text{s}^{-1} \left( \frac{a_0}{a_{bo}} \right)^2 \left( \frac{\rho}{\rho_0} \right)^{-1} \left( \frac{a_1}{a_{10}} \right)^{-1} \left( \frac{R_1}{R_{20}} \right)^{-1} \frac{P_1}{P_{10}},$$

where $\Gamma$ denotes the torque, $L$ angular momentum, $a_0$ heliocentric semimajor axis, $\rho$ density, $a_1$ binary semimajor axis, $P_1$ orbital period, and $R_2$ secondary radius. The normalisation is given for $a_{bo} = 1 \text{au}$, $\rho_0 = 1750 \text{kg m}^{-3}$, $a_{10} = 2 \text{km}$, $R_{20} = 0.15 \text{km}$, $P_{10} = 20 \text{h}$, and synchronous rotation.

* Based on observations made with ESO Telescopes at the La Silla Paranal Observatory under program 199.C-0074 (PI Vernazza).
event, which was later attributed to the outer moon of Kleopatra, and was designated S/2008 (216) 1, or I Alexhelios (Descamps et al. 2011). The event lasted only 0.9 s, but was observed by two independent observers separated by 0.61 km. Its sky position in the $(u,v)$ plane coincided with the respective orbit of the outer (second) moon.

When we compared this observation with the revised ephemeris of Brož et al. (2021) – constrained by adaptive-optics (AO) datasets, hereafter denoted DESCAMPS, SPHERE2017, and SPHERE2018 – it turned out that the orbit orientation is offset by $-60^\circ$ in the true longitude $\lambda_2$ (black cross, orange line). This offset corresponds to a shorter orbital period $P_2$ in the past.

2. Observed tidal evolution

2.1. Increasing orbital period $P_2$

Naively, we expected that a minor change of the osculating period $P_2$ within the present uncertainty would be sufficient, but it was not. Indeed, the time-span of the AO datasets (2008–2018, or 3780 d) is comparable with the preceding occultation (1980–2008, 10220 d). Moreover, their phase coverage constrains both periods $P_1$, $P_2$.

To demonstrate this clearly, we computed simplified periodograms as follows. We used our previous converged model (Brož et al. 2021) to determine the true longitudes $\lambda_2$ (unfolded) and orbital epochs $E_i$ of all 2008–2018 observations with respect to $T_0 = 2454728.761806$. We then added one point corresponding to the 1980 occultation with the respective epoch $E_i = \lambda_2/(2\pi) = 0.55$. We assumed uncertainties of $\sigma_E = 0.001$, which corresponds to an astrometric uncertainty of about 10 mas. These data were compared with two simplified ephemerides – constant mean period$^1$ (linear epoch):

$$E(t) = \frac{t}{P_2} ,$$

or linear period (quadratic epoch):

$$E(t) = \frac{1}{P_2} \ln \left( 1 + \frac{P_2}{P_2} \right) = \frac{t}{P_2} - \frac{1}{2P_2^2} \dot{P}_2 t^2 .$$

The difference between $E_i$, $E(t)$, expressed as $\chi^2$, is plotted in Fig. 2. It is not possible to fit all epochs $E_i$ with any of the constant periods. The structure of the periodograms is determined by the AO datasets, not by the occultation. On the other hand, a linearly variable period, with a suitable derivative $P_2 = (1.8 \pm 0.1) \times 10^{-8}$ d d$^{-1}$, is satisfactory (and better by two orders of magnitude).

2.2. Monopole model including tides

Tidal dissipation in Kleopatra is a likely dynamical mechanism explaining the secular evolution of the orbital period $P_2$. To

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$^1$ These mean Keplerian periods are different from osculating periods reported in Brož et al. (2021) by a factor of approximately 1.02246.
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determine the basic parameters of the tides, we used a time-
lag model (Mignard 1979; Neron de Surfuy & Laskar 1997). The
additional acceleration (and torque) was implemented in the
SWIFT integrator (Levison & Duncan 1994) as follows:
\[ f_{\text{tides}} = K_1 [K_2 r' - K_3 r - K_4(r \times \omega + v) + K_5 (K_6 r - K_7 r')], \tag{5} \]
\[ K_1 = \frac{3Gm^* R^5 \Delta \lambda}{(r^2 \Delta \lambda)^3}, \tag{6} \]
\[ K_2 = \frac{5}{r^2} \left[ r' \cdot r(\omega \times r + r' \cdot v) - \frac{1}{2r^2} \cdot v(5(r' \cdot r)^2 - r'^2 r^2) \right], \tag{7} \]
\[ K_3 = r \cdot \omega \times r + r' \cdot v, \tag{8} \]
\[ K_4 = r' \cdot r, \tag{9} \]
\[ K_5 = r - v, \tag{10} \]
\[ K_6 = 5r' \cdot r, \tag{11} \]
\[ K_7 = r'^2, \tag{12} \]
\[ \Gamma = r \times m' f_{\text{tides}}. \tag{13} \]
The classical notation assumes an Earth–Moon–test particle, but
this can be any triple system and any combination of bodies
denoted by indices \((i, j \neq i, k \neq i)\). Ergo, \(m^*\) denotes the mass
of the Moon, \(m'\) the mass of the test particle, \(R\) the radius of the
Earth, \(k_3\) the Love number of the Earth, \(\Delta \lambda\) the time-lag, \(r\) the
vector Earth–Moon (i.e. perturbing body), \(v\) the orbital velocity
of the Moon, \(r'\) the vector Earth–test particle (interacting body),
\(\omega\) the spin rate vector of the Earth, and \(\Gamma\) the torque acting
on the spin of Earth. This general formula is used to compute cross-
tides among all triples. In our case, non-negligible interactions
are expected for Kleopatra–first moon–first moon, Kleopatra–
second moon–second moon; where the tidal dissipation occurs in
Kleopatra itself. Both moons have to be accounted for, because
they contribute to the total torque (spin-down). A simple Euler
integrator is then used to evolve spins, assuming principal-axis
rotation. The time steps were 0.02 d (orbital) and 1 d (spin).

There are three relevant radii of Kleopatra: \(R = 59.6\ \text{km}\)
(volume-equivalent), 69.0 km (surface-equivalent), and 135 km
(maximal). The volume equivalent is commonly used, but if
tidal dissipation happens in surface layers, the surface equiva-
Ient should be preferred. In case of Kleopatra, we decided to use
the maximal radius, because the strongest dissipation is expected at
the ‘extremes’ of the elongated body. Other parameters are the
Love number \(k_2 = 0.305\) (here, we used the same value as for
the Earth), and the moment of inertia \(I = 1.72 \times 10^{28}\ \text{kg m}^2\), as
derived from the ADAM model (Marchis et al. 2021). We vari-
ed only the time-lag and obtained \(\Delta \lambda \simeq 47\ \text{s}\), meaning that the
offset in true longitude is \(\Delta l_2 = -60^\circ\) with respect to the model
without tides, or \(-^0\) with respect to the observation (occul-
tation). The evolution is shown in Fig. 3. It is very smooth because
we included only the monopole for Kleopatra and we overplotted
orbits computed separately, without perturbations.

For comparison, the inner (first) moon should tidally evolve
with \(P_1 = 5.0 \times 10^{-8}\), which is inevitably larger than \(P_2 = 1.8 \times 10^{-8}\)
due to the smaller distance. The accumulated change in the
rotation phase of Kleopatra due to both moons over the entire
time-span of 1980–2018 should then reach 1° (see Sect. 2.4).

\[ \chi^2_{\text{sky}2} = 113 \quad \text{(absolute astrometry)}, \quad \chi^2_{\text{sky}2} = 66 \quad \text{(relative astrometry)}, \quad \chi^2_{\text{sky}2} = 621 \quad \text{(adaptive-optics)}, \text{and the joint metric was given as } \chi^2_{\text{sky}2} + \chi^2_{\text{sky}2} + 0.3 \chi^2_{\text{sky}2}. \]

Fig. 3. Tidal evolution of Kleopatra spin (dashed magenta) and moon
orbits (solid green, blue), computed as the difference in the true lon-
gitudes \(\Delta l_0, \Delta l_1, \Delta l_2\) between dynamical models with and without
tides. The value of the time-lag \(\Delta \lambda = 47\ \text{s}\) corresponds to \(P_2\) in Fig. 2.
The epoch when mean periods coincide was arbitrarily shifted (↔)
towards 2 456 500. Moreover, the mean periods were adjusted (↗) to fit
observations in 2008 and 2018.

2.3. Multipole model including tides
In order to have a complete dynamical model, we also imple-
mented tides (Eqs. (5)–(13)) in Xi tau$^2$ (Brož 2017; Brož et al.
2021), which enabled us to fit all observations. Let us recall that
the model already included multipoles up to the order \(r = 10\) and
mutual moon perturbations, and that our previous best-fit model
(Brož et al. 2021), which included multipoles up to the order \(r = 2\),
had \(\chi^2 = 368^3\).

We proceeded as follows: (i) we unsuccessfully tried to re-conver-
erge periods \(P_1, P_2\) (without tides), but the value of \(\chi^2\) remained too high, at \(\chi^2 = 677\), compared to the number of mea-
surements (reported in Table 1); (ii) we successfully converged
\(P_1, P_2\) together with a non-zero time-lag \(\Delta \lambda\) and obtained \(\chi^2 = 388\); (iii) we verified there is no deeper local minimum in the
surroundings (see Fig. 4); and (iv) we converged all remaining
parameters, with the final \(\chi^2 = 360\) (see Fig. 5). The respective
parameters are presented in Table 1.

Although multipole perturbations (4 km in \(a_2\)) or mutual
perturbations (2 km) are orders of magnitude larger than tides
(1 m yr$^{-1}$ in \(a_0\)), the former are strictly conservative, or periodic,
and the latter are dissipative, or non-periodic. Tides are crucial
to explain the 1980 occultation.

Moreover, the tidal evolution may partially explain the sys-
 tematic errors in our previous fitting of the SPHERES2017 dataset.
When the osculating periods are constant and constrained by
DESCAMPS and SPHERE2018, some offsets (of the order of
10 mas) are required for the intermediate dataset, especially for
the first moon which is more affected by tides. A detailed com-
parison shows that the offsets may be decreased when tides are
included (see Fig. 6). However, tidal evolution cannot explain all
remaining systematic errors (cf. our discussion of astrometry
in Brož et al. 2021).

2.4. Possibly increasing rotation period \(P_0\)
As discussed in Sect. 2.2, if the moons are affected by tides,
then must be the rotation of Kleopatra. If the period is evolving

\[ \text{http://sirrah.troja.mff.cuni.cz/~mira/xitaun/} \]

\[ ^3 \text{More specifically, the individual contributions were } \chi^2_{\text{sky}2} = 113 \quad \text{(absolute astrometry), } \chi^2_{\text{sky}2} = 66 \quad \text{(relative astrometry), } \chi^2_{\text{sky}2} = 621 \quad \text{(adaptive-optics), and the joint metric was given as } \chi^2_{\text{sky}2} + \chi^2_{\text{sky}2} + 0.3 \chi^2_{\text{sky}2}. \]
Table 1. Best-fit models with no tides (left) and including tides (middle), together with realistic uncertainties of the parameters (right).

| var.   | val.       | val.       | Unit     | σ       |
|--------|------------|------------|----------|---------|
| m₁     | 1.492735 × 10⁻¹² | 1.492735 × 10⁻¹² | Mₛ       | 0.16 × 10⁻¹² |
| m₂     | 2 × 10⁻¹⁶   | 2 × 10⁻¹⁶   | Mₛ       | 2 × 10⁻¹⁶  |
| m₃     | 3 × 10⁻¹⁶   | 3 × 10⁻¹⁶   | Mₛ       | 3 × 10⁻¹⁶  |
| P₁     | 1.822359    | 1.822281    | day      | 0.004156 |
| log e₁ | −3.991     | −3.991      | 1        | −3 (i.e. 0.001) |
| i₁     | 70.104     | 70.104 deg  | 1.0      |         |
| Ω₁     | 252.920    | 252.920 deg | 1.0      |         |
| ω₁     | 0.089      | 0.089 deg   | 10.0     |         |
| λ₁     | 59.665     | 59.665 deg  | 1.0      |         |
| P₂     | 2.745820   | 2.745791    | day      | 0.004820 |
| log e₂ | −3.998     | −3.998      | 1        | −3      |
| i₂     | 70.347     | 70.347 deg  | 1.0      |         |
| Ω₂     | 252.954    | 252.954 deg | 1.0      |         |
| ω₂     | 1.601      | 1.601 deg   | 10.0     |         |
| λ₂     | 108.357    | 108.357 deg | 1.0      |         |
| ℓpole  | 72.961     | 72.961 deg  | 1.0      |         |
| hpole  | 19.628     | 19.628 deg  | 1.0      |         |
| Δ₁     | –          | 42.1 s      | 1.0      |         |
| nₖsky  | 68         | 68          |          |         |
| nₖsky₂ | 28         | 28          |          |         |
| nₙao   | 3240       | 3240        |          |         |
| χ₁²    | 617        | 110         |          |         |
| χ₂²    | 66         | 60          |          |         |
| χ₆²    | 621        | 621         |          |         |
| χ₇    | 872        | 360         |          |         |
| χ₈    | 9.07       | 1.62        |          |         |
| χ₉    | 2.35       | 2.14        |          |         |
| χ₁₀    | 0.19       | 0.19        |          |         |

Notes. The left model does not fit the October 10 1980 occultation (see Fig. 1); without this observation, its χ² would be 368. Orbital elements of the moons are osculating for the epoch T₀ = 2454728.761806, where m₁ denotes the mass of body 1 (i.e. Kleopatra), m₂ body 2 (first moon), m₃ body 3 (second moon), P₁ the orbital period of the first orbit, e₁ eccentricity, i₁ inclination, Ω₁ longitude of node, ω₁ longitude of pericentre, λ₁ true longitude, etc. of the second orbit; ℓpole ecliptic longitude of Kleopatra’s rotation pole, hpole ecliptic latitude; n numbers of observations (SKY, SKY2, AO), χ² values, χ²_red ≡ χ²/n reduced values. The angular orbital elements are expressed in the standard stellar reference frame. If the orbits lie in the equatorial plane of body 1, they fulfil ℓ = 90° – hpole, Ω = 180° + ℓpole.

Fig. 4. χ² = χ²_red + χ²_res values for a range of osculating periods P₁, P₂ and converged models. All black crosses correspond to local minima of χ²; colours are interpolated. A normal χ² map would be much more irregular. The dotted lines show the periods of the global minimum.

in time, then the value of P₀ = 5.3852824(10) h reported by Marchis et al. (2021) corresponds to the middle of the 1977–2018 time-span. To estimate a realistic uncertainty of this ‘mean’ rotation period, we created 1000 bootstrapped samples of the light-curve data set (random selection of light curves and random selection of points in those light curves) and used them as input for convex light-curve inversion. The data set of Marchis et al. (2021) was supplemented with other observations that are listed in Table A.1, and now consists of 198 light curves covering the interval 1977–2021. This led to the mean rotation period of:

$$P_0 = (5.3852827 ± 0.0000003) \, \text{h}. \quad (14)$$

This improved uncertainty of the rotation period corresponds to uncertainty in Kleopatra’s rotation phase of 1.3° over the interval of 44 yr, which is of the same order as the expected 1° shift estimated in Sect. 2.2.

To check whether or not the predicted deceleration of the main body’s rotation is ‘visible’ in the data, we divided light curves into two sets: the first one covering the interval 1977–1994 and the second one 2002–2021. If the rotation period is changing, we should see some difference in the periods for these two data sets. Similarly, as with the full data set, we created 1000 bootstrapped samples and performed the light curve inversion...
independently for all of them to estimate parameter errors. For the interval 1977–1994, the rotation period was:

\[ P_{0}^{1977−1994} = (5.3852821 \pm 0.0000010) \text{ h} \]  

(15)

and the corresponding phase shift 1.8°. For 2002–2021, the values were:

\[ P_{0}^{2002−2021} = (5.3852822 \pm 0.0000005) \text{ h} \]  

(16)

and 1.0°. The uncertainty intervals are therefore larger (due to the shorter time span) than with the full data set and they overlap, that is, there is no indication that the rotation period is changing. Controversially, the mean period derived from 1977–2021 observations is slightly longer than periods for 1977–1994 and 2002–2021 subsets, while we would expect it to be somewhere in between the two values. This is partly caused by the correlation between the period and the pole direction (which is also optimised for each bootstrapped sample), but we think that the main reason is some small but systematic errors present in some of the light curves.

To test the sensitivity of our approach, we generated an equivalent set of synthetic observations using the non-convex ADAM shape model from Marchis et al. (2021), Hapke's light-scattering model, and two values of \( P_{0} \), 3.2 × 10^{-12} and 1.6 × 10^{-12}. We then treated the synthetic data set as real data and applied the same bootstrap approach to detect possible changes in rotation period. For \( P_{0} = 3.2 \times 10^{-12} \), the effect of changing period was clearly visible as a systematic difference between periods for 1977–1994 and 2002–2021 data. In this way, we checked that the choice of using convex or non-convex models does not affect the results in a systematic way. However, when using \( P_{0} = 1.6 \times 10^{-12} \) and adding 2% random noise to our synthetic light curves (which is a realistic estimate of observational uncertainties), the effect of changing period was no more detectable – both subsets of bootstrapped light curves had statistically the same rotation period.

We also tried to detect a possible evolution of Kleopatra’s rotation period by including \( P_{0} \) as a free parameter in the light curve inversion. In practice, we used the same approach as Kaasalainen et al. (2007) or Durech et al. (2018) when searching for the YORP effect that influences light curves in the same way – rotation period changes linearly over time (more precisely, angular velocity changes linearly over time but the difference is negligible). We used the same bootstrap sample as in the case of fitting light curves with a constant-period model. The results are shown in Fig. 7, where \( P_{0} \) is plotted against \( P_{0} \). There is a strong anticorrelation between these two parameters – positive \( P_{0} \) (deceleration of the rotation) and shorter initial rotation (at the beginning of the observing time interval in 1977) has a similar outcome as negative \( P_{0} \) (acceleration of the rotation) and slower initial rotation. From bootstrap, \( P = (−0.5 \pm 4.2) \times 10^{-12} \), which means that the effect we are searching for, \( P_{0} = 1.9 \times 10^{-12} \), is consistent with the data but cannot be confirmed. Zero
\[ P_0 \] is also compatible with the data. Due to correlation, the marginal uncertainty of \( P_0 \) is 0.00000009 h, which is larger than when assuming \( P_0 = 0 \).

### 2.5. Discussion of the quality factor \( Q \)

Our modelling of tidal evolution indicates the time-lag around \( \Delta t = 42 \text{s} \), with the assumed Love number of \( k_2 = 0.3 \). According to the approximate relation (Efroimsky & Lainey 2007):

\[
Q = \frac{1}{\Delta t 2|\omega - n|} \Rightarrow \Omega_{\text{mean}} \approx 1 \text{s} \quad \text{for} \quad \Omega_{\text{real}} \approx 3 \text{d} \quad \text{and} \quad \Omega_{\text{obs}} \approx 1 \text{d} \text{.}
\]

where \( Q \) denotes the quality factor, \( \omega_n = 2\pi/P_0 \) the spin rate, and \( n \) the mean motion, \( Q = 40 \), or \( Q/k_2 = 131 \). This \( Q \) value is relatively low (i.e. dissipation high), which seems reasonable for (216) Kleopatra – an irregular body close to critical rotation (Marchis et al. 2021). The value of \( k_2 \) cannot realistically be orders-of-magnitude lower, because \( Q \) would be unrealistically low. For comparison, the Earth and Moon have \( Q = 280 \pm 60 \) and \( 38 \pm 4 \), respectively (Konopliv et al. 2013; Lainey 2016), but they correspond to low loading frequencies, \( \xi \equiv 2|\omega - n| \), and the expected dependence \( Q(\xi) \) is positive \( (Q \propto \xi^{0.3} \text{ for } \xi \geq 10^{-2} \text{ rad d}^{-1}; \text{Efroimsky & Lainey 2007}) \). This is demonstrated in Fig. 8.

For uniform bodies, there is a relation between the Love number \( k_2 \) and the material rigidity \( \mu \) (Goldreich & Sari 2009; Eq. (24)):

\[
\mu = \left( \frac{3}{2k_2} - 1 \right) \frac{6}{19} \frac{Gm_1^2}{R^2} \frac{1}{S} \approx \frac{9}{19} \frac{1}{k_2} \frac{Gm_1^2}{R^2} \frac{1}{S} \text{.}
\]

where \( S \) denotes the surface area; the approximation holds for bodies with substantial \( \mu \) (or small \( k_2 \)). Because we know \( Q/k_2 \), we can obtain \( \mu Q = 2.7 \times 10^9 \text{ Pa} \). This is the same order of magnitude as the estimate for 1 km asteroids (Scheirich et al. 2015), but is three orders of magnitude smaller than the value \( \mu Q = 10^{10} \text{ Pa} \) derived for other 100 km asteroids (Marchis et al. 2008a,b). We can also try to express \( \mu = 6.7 \times 10^9 \text{ Pa} \) (from \( k_2 \)), but this is not independently constrained. It seems compatible with loose material, or at least regolith-covered bodies.

There is also a relation to the regolith thickness (Nimmo & Matsuyama 2019; Eq. (6)):

\[
l = \sqrt{\frac{m_3n_2^2R^2}{3m_1Gp f Q}} \text{.}
\]

where \( f = 0.6 \) is the assumed friction coefficient. This gives \( l = 13 \text{ m} \). For non-spherical bodies, there may be significant deviations. In particular, when we used the maximum radius \( R \) and only a part of the surface is at this distance, the regolith needed to explain all the dissipation is probably accordingly thicker.

### 2.6. \( Q \) for orbits in the 3:2 resonance

The orbits of the two moons appear to be very close to the 3:2 mean-motion resonance; the respective critical angle \( \sigma \) does not librate though, because orbits are so perturbed by the multipoles of Kleopatra and eccentricities are too small (Brož et al. 2021). Nevertheless, if they are locked, tides act on both moons at the same time and, inevitably, \( P_2 = 1.5 P_1 \). According to our numerical experiments (using the machinery of Sect. 2.2), the value of \( P_1 \) decreases and \( P_2 \) increases compared to their nominal values. In order to obtain the same offset of \( \Delta t_2 = 60^\circ \), the required values are now \( P_0 = 0.9 \times 10^{-12} \), \( P_1 = 1.2 \times 10^{-8} \), and \( P_2 = 1.8 \times 10^{-8} \). This corresponds to a time-lag of approximately \( \Delta t_1 = 22 \text{s} \).
Consequently, the dissipation factor as well as other derived quantities from Sect. 2.5 are revised as follows: $Q = 76$, $Q/k_2 = 250$, $\mu Q = 5.0 \times 10^3$ Pa, and $l = 9$ m. The assumption of the $3:2$ resonance thus decreases the dissipation rate and puts Kleopatra somewhat closer to the theoretical dependence of $Q(\ell)$ in Fig. 8.

2.7. $Q$ for more massive moons

In an alternative model, moons can be more massive (more dense than Kleopatra), with $m_2 = 4 \times 10^{-16} M_S$ and $m_3 = 9 \times 10^{-16} M_S$ (Brož et al. 2021), and the deformation potential is proportionally larger (Eq. (6)). Again, to obtain $\Delta t_2 = +60^\circ$, $\Delta t_3 = 16$ s is required, together with $P_0 = 3.6 \times 10^{-12}$, $P_1 = 3.1 \times 10^{-8}$, and $P_2 = 1.8 \times 10^{-6}$. The value of $P_0$ is increased substantially, but still not enough to be confirmed (or excluded) by observations. Adjustments of other parameters are as follows: $Q = 100$, $Q/k_2 = 330$, $\mu Q = 8.2 \times 10^3$ Pa, and $l = 13$ m. This puts Kleopatra even closer to the theoretical dependence on Fig. 8 and indicates that mechanical properties of Kleopatra’s material may actually be similar to those of terrestrial bodies.

2.8. Discussion of the origin

Regarding the origin of the moons, it is interesting to estimate the timescale as the angular momentum over the tidal torque, $L_2/\Gamma_2 = 1.3 \times 10^8$ yr, because this would indicate the moons are very young. The dependence of both tidal and radiative torques, computed for the Kleopatra system according to Eqs. (1) and (2), is shown in Fig. 9. If the initial distance coincided with the last stable orbit (LSO), at about $r_{\text{LSO}} = 280$ km (or $P = 0.8$ d) according to our numerical tests, and the final distance is comparable to half of the Hill sphere, $r_{\text{Hill}} = 33.100$ km, the overall evolution would take over $2 \times 10^9$ yr$^4$. In a broader perspective, this is comparable to the dynamical timescale of the rings of Saturn (Charnoz et al. 2009; although cf. Crida et al. 2019).

The moons are definitely younger than Kleopatra, because a large-scale collisional event would leave observable traces (an asteroid family). The moons may alternatively be related to small-scale craterings, which are much more frequent. Of the three following options, the latter appears the most plausible: (i) a cratering with a direct re-accretion of multi-kilometre moons; (ii) a collisional spin-up of Kleopatra over its critical frequency and mass shedding; (iii) low-speed ejection of material from the surface below the L1 critical point (see Fig. 6 in Marchis et al. 2021) and continuous accretion from ring. This suggested mechanism requires lower kinetic energy of collisions.

However, the long-term evolution could be complicated. If Kleopatra has been close to its rotation limit for a prolonged period of time, many moons have likely been created. This implies there are perhaps more moons within the Hill sphere, as suggested by some Keck images. The most likely distance seems to be about 1500 km, where $\Gamma/L$ is lowest and evolution is slowest. Such a hypothetical third moon would be close to the 3:1 resonance with the second moon and capture is inevitable. Subsequent evolution of eccentricity, which is increased by tides (Goldreich 1963; Correia et al. 2012), would lead to an instability of the moon system and an ejection of one or two moons beyond the Hill sphere. The timescale of evolution is determined by the inner moon. The instability may be delayed by the protective resonant mechanism, or alleviated if the moons have been rotating synchronously (1:1) and dissipating due to higher tidal modes (3:2, 2:1).

3. Conclusions

Astrometric and occultation observations of Kleopatra’s outer moon indicate a secular evolution of its orbital period $P_2 = (1.8 \pm 0.1) \times 10^{-8}$, which is the first such observation in a system of moons orbiting a large (100 km) asteroid. This should be linked to the secular evolution of the rotation period $P_0 = 1.9 \times 10^{-12}$ of (216) Kleopatra itself. The latter value is not excluded by current photometric observations, but their precision (about 1$^\circ$ in phase, or 3 milliseconds in period) is still not sufficient to exclude $P_0 = 0$.

For future observers, we predict a secular evolution of the first moon $P_1 = 5.0 \times 10^{-9}$, which is inevitable when the second moon is driven by tides. If the observed value is found to be different, this could indicate, for example, stronger mutual interactions, different masses of the moons ($m_2, m_3$), or a greater proximity to the 3:2 mean-motion resonance. If the moons are inside the 3:2 resonance, the tides acting on the first moon also act on the second moon, and a lower dissipation in Kleopatra is sufficient to explain the offset in true longitude $\lambda_2$. In more complex rheological models the time-lag $\Delta t$ (or $Q$) also depends on loading frequencies, i.e. $2|\omega - n|$. However, in the Kleopatra triple system, the loading frequencies are perhaps too close ($49.1, 51.4 \text{ rad d}^{-1}$) to measure this dependence directly by means of accurate astrometry.

At the same time, adaptive-optics observations of fast-moving shadows (at higher phase angles) could perhaps be used to better constrain the rotation phase of Kleopatra and detect a possible difference between measured $P_0$ and $P_0'$ inferred from tides (similarly as in the Earth–Moon system; cf. post-glacial rebound). Consequently, ground-based observations with the VLT/SPHERE instrument have the potential to constrain the ‘geophysical’ internal evolution of large asteroids.

There will be another opportunity to observe (216) Kleopatra and its moons in 2022–2024. According to our ephemeris,
Fig. 10. Sky-plane projection of Kleopatra and moon orbits for the Besselian year 2022.80 (October), i.e. one of the epochs when eclipses and transits will be observable. The spacing between points corresponds to 0.02 d. Approximate sizes of the moons are 10 km, corresponding to 5 mas.

transits and eclipses of the moons will occur (e.g. Fig. 10). The intervals when orbital planes cross Kleopatra are as follows:

- 2022.34–2022.41 May 2.32 au
- 2022.80–2022.87 Oct.–Nov. 1.34 au
- 2023.93–2024.05 Dec.–Jan. 1.94 au
- 2024.51–2024.59 July 3.70 au.

Adaptive-optics and possibly also precise photometric observations could help to constrain the sizes and albedos of the moons. This is also true for stellar occultations (see Appendix B). Regarding hypothetical moons separated by 1500 km or more, where radiative torques should be dominant, a deeper survey with the next-generation AO instruments like VLT/ERIS or Gemini/GPI2 would be useful.

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References

Alton, K. B. 2009, Minor Planet Bull., 36, 69
Brož, M. 2017, ApJS, 230, 19
Brož, M., Marchis, F., Jordà, L., et al. 2021, A&A, 653, A56
Charnoz, S., Dones, L., Esposito, L. W., Estrada, P. R., & Hedman, M. M. 2009, Saturn After Cassini-Huygens, Origin and Evolution of Saturn’s Ring System, eds. M. K. Dougherty, L. W. Esposito, & S. M. Krimigis (Berlin: Springer), 537
Correia, A. C. M., Bous, G., & Laskar, J. 2012, ApJ, 744, L23
Crida, A., Charnoz, S., Hou, H.-W., & Dones, L. 2019, Nat. Astron., 3, 967
Čuk, M., & Burns, J. A. 2005, Icarus, 176, 418
de Pater, I., & Lissauer, J. J. 2010, Planetary Sciences (USA: NASA)
Descamps, P., Marchis, F., Berthier, J., et al. 2011, Icarus, 211, 1022
Šurech, J., Vokrouhlický, D., Pravec, P., et al. 2018, A&A, 609, A86
Erofeyu, M., & Lainey, V. 2007, J. Geophys. Res. Planets, 112, E12003
Goldreich, P. 1963, MNRAS, 126, 257
Goldreich, P., & Sari, R. 2009, ApJ, 691, 54
Hirabayashi, M., & Scheeres, D. J. 2014, ApJ, 780, 160
Kasaalainen, M., Šurech, J., Warner, B. D., Krugly, Y. N., & Gafotynuk, N. M. 2007, Nature, 446, 420
Konopliv, A. S., Park, R. S., Yuan, D.-N., et al. 2013, J. Geophys. Res. Planets, 118, 1415
Lainey, V. 2016, Celest. Mech. Dyn. Astron., 126, 145
Levison, H. F., & Duncan, M. J. 1994, Icarus, 108, 18
Marchis, F., Descamps, P., Baek, M., et al. 2008a, Icarus, 196, 97
Marchis, F., Descamps, P., Berthier, J., et al. 2008b, Icarus, 195, 295
Marchis, F., Jordà, L., Vernazza, P., et al. 2021, A&A, 653, A57
Mignard, F. 1979, Moon Planets, 20, 301
Morabito, L. A., Sýnnott, S. P., Kupferman, P. N., & Collins, S. A. 1979, Science, 204, 972
Neron de Sury, J., & Laskar, J. 1997, A&A, 318, 975
Nimmo, F., & Matsuyama, I. 2019, Icarus, 321, 715
Ostro, S. J., Hudson, R. S., Nolan, M. C., et al. 2000, Science, 288, 836
Pál, A., Szakáts, R., Kiss, C., et al. 2020, ApJS, 247, 26
Peale, S. J., Cassen, P., & Reynolds, R. T. 1979, Science, 203, 892
Rosenblatt, P. 2011, A&ARv, 19, 44
Scheirich, P., Pravec, P., Jacobson, S. A., et al. 2015, Icarus, 245, 56
Scheirich, P., Pravec, P., Kušnirák, P., et al. 2021, Icarus, 360, 114321
Shepard, M. K., Timerson, B., Scheeres, D. J., et al. 2018, Icarus, 311, 197
### Appendix A: List of new light curves

Observational circumstances of new light curves are provided in Table A.1.

#### Table A.1. New optical disk-integrated lightcurves of (216) Kleopatra used in this work.

| N  | Epoch      | \(N_p\) | \(\Delta\) (AU) | \(r\) (AU) | \(\varphi\) (°) | Filter | Observers/Reference |
|----|------------|---------|-----------------|-----------|----------------|--------|---------------------|
| 1  | 2002-05-15.0 | 31      | 2.45            | 3.45      | 20             | C      | Christophe Demeautis   |
| 2  | 2002-05-15.9 | 17      | 2.45            | 3.45      | 22             | C      | Christophe Demeautis   |
| 3  | 2002-05-17.0 | 35      | 2.45            | 3.45      | 25             | C      | Christophe Demeautis   |
| 4  | 2003-07-19.0 | 39      | 1.68            | 2.64      | 8.9            | C      | Claudine Rinner        |
| 5  | 2004-12-14.1 | 122     | 1.59            | 2.41      | 15.6           | C      | Horacio Correia        |
| 6  | 2004-12-20.1 | 315     | 1.56            | 2.43      | 13.7           | C      | Horacio Correia        |
| 7  | 2010-04-09.9 | 18      | 2.39            | 3.01      | 16.9           | C      | Yassine Damerdji, Jean-Pierre Troncin  
Jean Surej, Philippe Bendjoya  
Davide Ricci, Raoul Behrend  
Thierry De Gouvenain, Mugane Diet  
Mathias Marconi, Jean-By Gros  
Christophe Giordano, Jean-Christophe Flesch  
Ivan Belokogne, Andrei Belokogne  
Axel Bazi |
| 8  | 2010-04-09.9 | 5       | 2.39            | 3.01      | 16.9           | C      | Yassine Damerdji, Jean-Pierre Troncin  
Jean Surej, Philippe Bendjoya  
Davide Ricci, Raoul Behrend  
Thierry De Gouvenain, Mugane Diet  
Mathias Marconi, Jean-By Gros  
Christophe Giordano, Jean-Christophe Flesch  
Ivan Belokogne, Andrei Belokogne  
Axel Bazi |
| 9  | 2010-04-09.9 | 6       | 2.39            | 3.01      | 16.9           | C      | Yassine Damerdji, Jean-Pierre Troncin  
Jean Surej, Philippe Bendjoya  
Davide Ricci, Raoul Behrend  
Thierry De Gouvenain, Mugane Diet  
Mathias Marconi, Jean-By Gros  
Christophe Giordano, Jean-Christophe Flesch  
Ivan Belokogne, Andrei Belokogne  
Axel Bazi |
| 10 | 2010-04-26.9 | 72      | 2.64            | 3.04      | 18.7           | C      | Jacques Montier, Serge Heterier, Raoul Behrend  
Jacques Montier, Jean-Pierre Previt |
| 11 | 2010-05-22.9 | 37      | 3.04            | 3.10      | 18.9           | C      | Jacques Montier, Jean-Pierre Previt |
| 12 | 2015-01-25.1 | 203     | 2.44            | 3.10      | 15.3           | C      | Georg Pehler, Alfons Gabel |
| 13 | 2015-01-29.1 | 128     | 2.40            | 3.11      | 14.5           | C      | Georg Pehler, Alfons Gabel |
| 14 | 2015-02-19.0 | 343     | 2.25            | 3.15      | 9.1            | C      | Pierre Antonini        |
| 15 | 2015-02-19.0 | 67      | 2.25            | 3.15      | 9.0            | C      | Matthieu Conjet        |
| 16 | 2015-02-19.1 | 387     | 2.25            | 3.15      | 9.0            | C      | Rene Roy              |
| 17 | 2015-02-23.0 | 183     | 2.24            | 3.16      | 7.9            | C      | Federico Manzini       |
| 18 | 2015-03-06.0 | 310     | 2.21            | 3.18      | 4.9            | C      | Nicolas Esseiva, Raoul Behrend |
| 19 | 2017-07-16.0 | 154     | 1.72            | 2.68      | 8.6            | C      | Nicolas Esseiva, Raoul Behrend |
| 20 | 2017-07-16.0 | 9       | 1.72            | 2.68      | 8.6            | C      | Nicolas Esseiva, Raoul Behrend |
| 21 | 2017-07-16.0 | 9       | 1.72            | 2.68      | 8.6            | C      | Nicolas Esseiva, Raoul Behrend |
| 22 | 2017-8-30.3  | 74      | 1.75            | 2.56      | 16.2           | I      | Kevin Alton, Alton (2009) |
| 23 | 2017-8-31.3  | 115     | 1.75            | 2.56      | 16.5           | I      | Kevin Alton, Alton (2009) |
| 24 | 2017-8-6.2   | 106     | 1.67            | 2.62      | 9.4            | I      | Kevin Alton, Alton (2009) |
| 25 | 2017-9-10.1  | 120     | 1.81            | 2.53      | 19.0           | I      | Kevin Alton, Alton (2009) |
| 26 | 2017-9-11.1  | 124     | 1.82            | 2.53      | 19.2           | I      | Kevin Alton, Alton (2009) |
| 27 | 2017-9-5.1   | 113     | 1.78            | 2.55      | 17.8           | I      | Kevin Alton, Alton (2009) |
| 28 | 2017-9-7.3   | 83      | 1.79            | 2.54      | 18.3           | I      | Kevin Alton, Alton (2009) |
| 29 | 2019-1-4.2   | 889     | 1.49            | 2.43      | 8.7            | R      | Stéphane Fauvaud       |
| 30 | 2019.1-2019.1| 543     | 1.50            | 2.45      | 8.2            | V      | TESS, Pál et al. (2020) |
| 31 | 2021-04-12.1 | 49      | 2.68            | 3.37      | 13.8           | C      | David Augustin, Raoul Behrend |
| 32 | 2021-04-19.3 | 98      | 2.59            | 3.37      | 12.4           | C      | David Augustin, Raoul Behrend |

**Notes.** For each lightcurve, the table gives the epoch, the number of individual measurements \(N_p\), asteroid’s distances to the Earth \(\Delta\) and the Sun \(r\), phase angle \(\varphi\), photometric filter and the observer(s). Majority of the data is from the Courbes de rotation d’astéroïdes et de comètes database (CdR, [http://obswww.unige.ch/~behrend/page_cou.html](http://obswww.unige.ch/~behrend/page_cou.html)), maintained by Raoul Behrend at Observatoire de Genève.
Appendix B: Predictions for stellar occultations 2022–2026

Predictions of the positions of Kleopatra’s moons for expected stellar occultations 2022–2026 are plotted in Fig. B.1.

Fig. B.1. Predictions of the positions of Kleopatra’s moons in the (u, v) plane for the beginning time of expected stellar occultations in 2022–2026. Our ephemerides including tides (+) and without tides (×) are plotted for comparison. The projected orbital velocity (arrow) and the occultation chord (dashed line) are also indicated. If one of the alternative models is valid (Sects. 2.6, 2.7), the position of the inner moon will be somewhere in between. If chords intersecting Kleopatra are close to the inner moon, the event is very promising (see bold dates).