Scalar fields superdense gravitating systems

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Abstract. Solutions for scalar fields superdense gravitating systems of flat, open and closed type obtained in the frame of gauge theories of gravitation are discussed. Properties of these systems in dependence on parameter $\beta$ and initial conditions are analyzed.

KEYWORDS: Gauge theories of gravity, scalar fields, superdense gravitating systems

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1 Introduction

As it was shown at first in [1], generalized cosmological Friedmann equations (GCFE) deduced in the frame of gauge theories of gravity (GTG) [2] besides cosmological solutions lead to solutions for some hypothetical objects — so-called superdense gravitating systems (SGS). SGS have extremely high energy density and their dynamics is essentially non-Newtonian. In considered approximation of homogeneous isotropic space SGS oscillate between minimum and maximum values of the scale factor. Later solutions for such systems in the case of various equations of state for gravitating matter were discussed in [3–5]. Below SGS are investigated for systems including scalar fields.

2 Generalized cosmological Friedmann equations

GCFE for systems including scalar field $\phi$ minimally coupled with gravitation and radiation have the following form [7].

\[
\frac{k}{R^2} Z^2 + \left\{ H \left[ 1 - 2\beta (2V + \dot{\phi}^2) \right] - 3\beta V' \phi \right\}^2 = 8\pi \frac{\rho_r}{3M_p^2} \left[ \rho_r + \frac{1}{2} \dot{\phi}^2 + V - \frac{1}{4} \beta \left( 4V - \dot{\phi}^2 \right)^2 \right] Z, \tag{1}
\]

\[
H^2 \left[ 1 - 2\beta (2V + \dot{\phi}^2) \right] Z
+ H^2 \left\{ \left[ 1 - 4\beta (V - 4\dot{\phi}^2) \right] Z - 18\beta^2 \dot{\phi}^4 \right\}
+ 12\beta H \dot{\phi} V' \left[ 1 - 2\beta (2V + \dot{\phi}^2) \right]
- 3\beta \left[ (V''\dot{\phi}^2 - V') Z + 6\beta \dot{\phi}^2 V' \right]
= 8\pi \frac{3M_p^2}{3M_p^2} \left[ V - \dot{\phi}^2 - \rho_r - \frac{1}{4} \beta (4V - \dot{\phi}^2)^2 \right] Z, \tag{2}
\]

where $R$ is the scale factor, $k = +1, 0, -1$ for closed, flat and open models respectively, $H$ is the Hubble parameter, $\rho_r$ is radiation energy density, $V(\phi)$ is a scalar field potential, $V' = \frac{dV}{d\phi}, V'' = \frac{d^2V}{d\phi^2}, M_p$ is Planckian mass, $Z = 1 - \beta (4V - \dot{\phi}^2)$. (The system of units with $\hbar = c = 1$ is used.) Eqs. (1)–(2) include indefinite

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1 One supposes the interaction between scalar field and radiation is negligibly small.
parameter $\beta$ with inverse dimension of energy density, the value of $|\beta|^{-1}$ determines the scale of extremely high energy densities.

Besides Eqs. (1)–(2) gravitational equations of GTG lead to the following relation for torsion function $S$ and nonmetricity function $Q$ [2, 6]

$$S - \frac{1}{4} Q = \frac{3\beta}{2} \left( H\dot{\phi} + V' \right) \frac{\dot{\phi}}{1 - \beta (4V - \dot{\phi}^2)}. \tag{3}$$

The conservation law in usual form follows from Eqs. (1)–(2), as result we obtain the equation for scalar field

$$\ddot{\phi} + 3H\dot{\phi} = -V' \tag{4}$$

and the integral for radiation $\rho_r R^4 = \text{const.}$

If the condition

$$|\beta (4V - \dot{\phi}^2)| \ll 1 \tag{5}$$

is valid, solutions of GCFE coincide practically with that of general relativity (GR) and the torsion and nonmetricity functions are negligibly small.

As it was shown in [7], the GCFE allow to build regular inflationary cosmological models with dominating ultrarelativistic matter at a bounce, if the scale of extremely high energy densities is much less than the Planckian one $|\beta|^{-1} \ll 1M_p^4 (\beta < 0)$. Numerical solutions obtained in [7, 8] for closed and flat models show that after inflation ($|\phi| < 1M_p$) during certain time intervals the dynamics of inflationary models has oscillating character, namely scalar field and the Hubble parameter oscillate near values zero. Such situation is typical for SGS. This means, if $|\phi| < 1M_p$ and the relation (5) is not fulfilled, by choosing some initial conditions we can obtain numerical solutions of GCFE for SGS including scalar fields. We have to take into account that by given initial conditions for magnitudes ($\phi, \dot{\phi}, R, \rho_r$) there are two different solutions corresponding to two values of the Hubble parameter, which according to (1) are

$$H_\pm = \frac{3\beta V' \pm \sqrt{D}}{1 - 2\beta (2V + \dot{\phi}^2)}.$$ 

where

$$D = \frac{8\pi}{3M_p^2} \left[ \rho_r + \frac{1}{2} \dot{\phi}^2 + V - \frac{1}{4} \beta \left( 4V - \dot{\phi}^2 \right)^2 \right] Z_0 - \frac{k}{R^2} Z^2 \geq 0.$$ 

Solutions of GCFE corresponding to $H_+$ and $H_-$ were called in [8] as $H_+$-solutions and $H_-$-solutions respectively.

Initial conditions will be chosen at the moment $t = 0$ corresponding to extremum of the scale factor $R$: $R(0) = R_0$ and $H_+(0) = 0$ or $H_-(0) = 0$. Then according to (1) initial values of $R_0$, $\rho_r 0$, $\phi_0$ and $\dot{\phi}_0$ satisfy the following relation

$$\frac{k}{R_0^2} Z_0^2 + 9\beta^2 V''_0 \phi_0^2 = \frac{8\pi}{3M_p^2} \left[ \rho_r 0 + \frac{1}{2} \dot{\phi}_0^2 + V_0 - \frac{1}{4} \beta \left( 4V_0 - \dot{\phi}_0^2 \right)^2 \right] Z_0.$$ 

Eq. (6) determines on the plane $P$ with the axes ($\phi, \dot{\phi}$) curves ($H_0$-curves), in points of which the Hubble parameter vanishes for $H_+$-solutions or $H_-$-solutions. From (2) the time derivative of the Hubble parameter $\dot{H}_0$ at the moment $t = 0$ is

$$\dot{H}_0 = \left\{ \frac{8\pi}{3M_p^2} \left[ V_0 - \dot{\phi}_0^2 - \rho_r 0 - \frac{1}{4} \beta (4V_0 - \dot{\phi}_0^2)^2 \right] + 3\beta \left[ (V''_0 + \dot{\phi}_0^2 - V'^2_0) + 6\beta \dot{\phi}_0^2 V''_0 Z_0^{-1} \right] \right\} \times \left[ 1 - 2\beta (2V_0 + \dot{\phi}_0^2) \right]^{-1}, \tag{7}$$

where $Z_0 = 1 - \beta (4V_0 - \dot{\phi}_0^2)$.

In order to analyze solutions properties near the origin of coordinates on the plane $P$ we will examine $H_0$-curves defined by (6) for some simple scalar field potentials.
3 Extremum $H_0$-curves for simplest scalar field potentials

According to (1) in the case of flat and closed models we have $Z \geq 0$ ($\beta < 0$) and admissible values of scalar field $\phi$ and derivative $\dot{\phi}$ on the plane $P$ are limited by two bounds $L_\pm$ defined by equation

$$\dot{\phi} = \pm (4V + |\beta|^{-1})^{\frac{1}{2}}. \quad (8)$$

$H_0$-curves $B_1$ and $B_2$ for flat models ($k = 0$) are situated on the plane $P$ between bounds $L_+$ and $L_-$ having with them common points (points $K_1$ and $K_2$ in Fig. 1) on the axis $\phi$ (if $V'(0) = 0$). Each of $H_0$-curves contains two parts — extremum curve for $H_+$-solutions ($H_0+$-curves) and extremum curve for $H_-$-solutions ($H_0-$-curves) [8].

The family of $H_0$-curves for closed models is situated on the plane $P$ between curves $B_1$ and $B_2$. The family of $H_0$-curves for open models is situated on the plane $P$ between bounds $L_\pm$ and corresponding $H_0$-curves for flat models.²

For given scalar field potential $V(\phi)$ the behaviour of $H_0$-curves depends on parameters of potential $V$, on parameter $\beta$ and on the model type ($k = 0$, $\pm 1$). Below $H_0$ are analyzed for scalar field potential $V = \frac{1}{4}m^2\phi^2$ ($m = 10^{-6}M_p$) in the case of $\rho_r = 0$. For flat models $H_0$-curves are given in Fig. 1 ($\beta = -1.6 \cdot 10^{12}$). The derivative $\dot{H}_0$ is positive in points of $H_0$-curves if the value of $|\beta| < |\beta_1| \sim 1.8 \cdot 10^{12}$. This means that $H_0$-curves in considered case are bounce curves discussed in [8]. By increasing of the value of $|\beta|$ the points $K_1$ and $K_2$ approach to the origin of coordinates and the derivative $\dot{H}_0$ becomes negative in the neighbourhood of points $K_1$ and $K_2$ (see Fig. 2). In points of $H_0$-curves with $\dot{H}_0 < 0$ the

²In the case of open models $k = -1$ Eq. (6) allows solutions if $Z < 0$. These solutions are not discussed in present paper.
evolution of system corresponds to transition from expansion to compression. The existence of regions on $H_0$-curves with $\dot{H}_0 < 0$ is necessary condition for appearance of oscillating solutions for SGS. The behaviour of $H_0$-curves for open models is similar to that for flat models. The curves $B_1$ and $B_2$ are limiting $H_0$-curves for open models under $R_0 \to \infty$. By decreasing of the value of $R_0$ corresponding $H_0$-curves of open models approach to bounds $L_{\pm}$. All $H_0$-curves of open models contain points $K_1$ and $K_2$, where $H_+ = H_-$. Like to flat models oscillating SGS-solutions of open type can appear by sufficiently large values of $|\beta|$. 

In the case of closed models by certain value of $\beta$ there are three kinds of $H_0$-curves (Fig. 3). For very large values of $R_0 > R_{01}$ $H_0$-curves are closed curves with the center in origin of coordinates (curve (a) in Fig. 3), they correspond to $H_0$-curves of GR. The value of $R_{01}$ depends on $|\beta|$, for example $R_{01} \sim 7.14 \cdot 10^7 M_p^{-1}$ for $|\beta| \sim 1.0 \cdot 10^{14} M_p^{-1}$. In the case of $R_0 < R_{01}$ we have $H_0$-curves (b) and (c) presented in Fig. 3. The Hubble parameter in points of curves (b) is positive (bounce-curves), and it is negative in points of curves (c). So, in points of $H_0$-curves ($\phi_0 \neq 0$, $\dot{\phi}_0 = 0$) the values of $R_0$ and $V_0$ are connected by usual relation of GR $R_0^{-2} = \frac{8\pi}{3M_p^2} V_0$, and the derivative $\dot{H}_0 = \frac{8\pi}{3M_p^2} V_0 - \frac{3\beta V_0''}{1 - 4\beta V_0} > 0$. In points of $H_0$-curves on the axis $\dot{\phi}$ ($\phi_0 = 0$, $\dot{\phi}_0 \neq 0$) we have

$$R_0^{-2} = \frac{4\pi}{3M_p^2} \frac{\dot{\phi}_0^2}{Z_0} \left(1 - \frac{1}{2} \beta \dot{\phi}_0^2\right)$$

and

$$\dot{H}_0 = -\frac{\dot{\phi}_0^2}{1 - 2\beta \dot{\phi}_0^2} \left[\frac{8\pi}{3M_p^2} \left(1 + \frac{1}{4} \beta \dot{\phi}_0^2\right) - 3\beta V_0''\right] < 0.$$ 

The presence on $H_0$-curves of regions with different signs of $H_+$ ($H_-$) leads to appearance of oscillating solutions. Note, that in points of $H_0$-curves lying on coordinates axes we have $H_+ = H_-$. 

The analysis shows that the behaviour of $H_0$ in the case of potential $V_2 = \frac{1}{4}\lambda \phi^4$ ($\lambda = \text{const}$) qualitatively is like to that of considered above potential $V_1 = \frac{1}{2}m^2\phi^2$. 

4 Numerical analysis of solutions for scalar fields SGS

By using various scalar field potentials applying in inflationary cosmology (in particular, $V_1 = \frac{1}{2}m^2\phi^2$, $V_2 = \frac{1}{4}\lambda \phi^4$), numerical analysis of SGS solutions of Eqs. (1), (2), (4) for flat, open and closed type was carried out, solutions properties in dependence on value of parameter $\beta$ and initial conditions were investigated. As result the following conclusions were obtained:

1. SGS-solutions of flat and open type have non-stationary character and are unsta-
2. SGS-solutions of closed type can have quasi-stationary as well as non-stationary regime in dependence on the value of $|\beta(4V_0 - \dot{\phi}_0^2)|$. The value of $\dot{\phi}_0^2$ has to be less than $4V_0$ at several orders to conserve the stable character of discussed solutions. In connection with this the analysis of SGS-solutions will be given below in dependence on the value of $x = 4|\beta V_0|$.

3. Quasi-stationary regime for SGS-solutions of closed type takes place, if the value $x \sim 1$ and changes in certain interval $x_{\min} \leq x \leq x_{\max}$ depending on the value of $\beta$ and the potential form. So, in the case of potential $V_1$ we have $1.5 \leq x \leq 4.5$ at $|\beta| = 1.0 \cdot 10^{14}$. By increasing of $|\beta|$ the values of $x_{\min}$ and $x_{\max}$ decrease. For example, in the case of $V_1$ and $|\beta| = 1.0 \cdot 10^{20}$ we have $0.12 \leq x \leq 2.0$. The frequency of $\phi$-oscillations and also frequency and amplitude of $H$-oscillations do not depend practically on the value of $\beta$ for given form of scalar field potential in the case of quasi-stationary regime. The realization of quasi-stationary regime means, that the sign of term $3H\dot{\phi}$ in Eq. (4) changes during $\phi$-oscillations period, during one half period this term plays the role of damping force and during the second half period it plays the role of accelerating force. As result, the frequency of $H$-oscillations is two times greater than the frequency of $\phi$-oscillations. In accordance with (3) torsion (nonmetricity) function oscillates with the same frequency as the Hubble parameter$^4$. The evolution of magnitudes $\phi$, $H$, $R$ and the energy density of scalar field $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ is presented in Fig. 5. Note, that scalar field energy of closed SGS changes with the time also.

4. By decreasing of the value of $x_{\min}$ ($x < x_{\min}$) oscillating SGS-solutions of closed type become unstable.

5. If $x \gg 1$ oscillating SGS-solutions of closed type are essentially non-stationary for limited time intervals (Fig. 6). In the case of large time scale oscillations have quasi-modulated character.

6. In accordance with the structure of GCFE the presence of radiation has essential influence on dynamics of SGS in the case of quasi-stationary regime $x \sim 1$ destroying this regime, and such influence is essentially less in the case of non-stationary regime $x \gg 1$.

In conclusion note, that oscillating classical solutions for scalar fields are unstable with respect to quantum processes of transformation of scalar fields into elementary particles and hence such solutions can have physical sense only for limited time intervals.

References

[1] A. V. Minkevich, Dokl. Akad. Nauk BSSR. 30, 311 (1986).

[2] A. V. Minkevich, Vestsi Akad. Nauk BSSR. Ser. fiz.-mat. nauk, no. 2, 87 (1980); Phys. Lett. A80, 232 (1980).

[3] A. V. Minkevich, N. H. Chuong, Vestsi Akad. Nauk BSSR. Ser. fiz.-mat. nauk, no. 5, 63 (1989).

All numerical solutions in Fig. 4–6 are given in the case of potential $V_1 = \frac{1}{2}m^2\phi^2$ ($m = 1.0 \cdot 10^{-6}M_p$) by using the system of units with $\hbar = c = M_p = 1$.

$^4$Quasi-stationary oscillating solutions for closed models were discussed in [9] with purpose to build non-singular cosmological model. From our point of view satisfactory cosmological model can not be built by using solutions for scalar fields SGS.
Figure 4: SGS-solution of flat type ($\beta = -1.0 \cdot 10^{20}$, $\phi_0 = 5.25 \cdot 10^{-5}$ and $\dot{\phi}_0 = 7.09 \cdot 10^{-12}$).

[4] A. V. Minkevich, I. M. Nemenman, *Class. Quantum Grav.* **12**, 1259 (1995).

[5] A. V. Minkevich, A. S. Garkun, *Gravitation & Cosmology* **5**, 115 (1999).

[6] A. V. Minkevich and A. S. Garkun, *Class. Quantum Grav.* **17**, 3045 (2000).

[7] A. V. Minkevich, gr-qc/0303022

[8] A. V. Minkevich, gr-qc/0307026

[9] G. V. Vereshchagin, astro-ph/0308107

Figure 5: SGS-solution of closed type in quasistationary regime ($\beta = -1.0 \cdot 10^{14}$, $\phi_0 = 0.1$ and $\dot{\phi}_0 = 0$).
Figure 6: SGS-solution of closed type in non-stationary regime ($\beta = -1.0 \cdot 10^{16}$, $\phi_0 = 0.1 \cdot 10^{-5}$ and $\dot{\phi}_0 = 0.$)