A PAPR Reduction Method Based on the CT Codes

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Abstract. This paper proposes a Peak-to-Average Power Ratio (PAPR) reduction method based on the Chinese Transform (CT) codes intended for the Orthogonal Frequency Division Multiplexing (OFDM) systems. Based on the advantage that remainder is smaller than the corresponding modulus during the CT encoding, the proposed method adopts the strategy of dynamical controlling the dynamic range of the OFDM transmission signal at the front stage. The simulation results show that the proposed method is effective in reducing the PAPR of OFDM system.

1. Introduction

In order to reduce the high Peak-to-Average Power Ratio (PAPR) of Orthogonal Frequency Division Multiplexing (OFDM) signals, many studies have been conducted [1-2]. Based on that whether the original signal suffers from losses caused by distortion or not, the PAPR reduction techniques can be divided into two categories: loss-based PAPR reduction techniques and lossless PAPR reduction techniques. The loss-based PAPR reduction techniques include the clipping algorithm, peak windowing, and companding transformation. Among them, the most popular clipping algorithm (Clipping) achieves high effectiveness in PAPR reduction with a simple algorithm [3-4]. However, when the clipping noise level is high, the signal waveform can be distorted, and the bit error rate can be increased. Therefore, the loss-based PAPR reduction has certain limitations. On the other hand, the lossless PAPR reduction techniques include the coding-based techniques and probabilistic techniques [5-7]. The basic idea of the former is to add the redundant information to limit the set of signal codewords, allowing only the codewords whose peak values are below the threshold to be selected for transmission. This type of techniques has a high capability of error correction, but they are applicable only to the OFDM systems with a large number of sub-carriers. Besides, instead of completely avoiding the high signal peaks performed by the loss-based reduction techniques, the probabilistic techniques aim at lowering the probability of occurrence of peaks without generating the signal distortion, showing high efficiency in reducing the PAPR value. However, using sideband information such as phase factor and branch number requires reliable and error-free signal transmission, reduces the spectrum utilization, and involves high-complexity computing.

Nevertheless, the fountain codes is a new channel coding technique [8], and the CT codes represent the cluster of fountain codes implementation methods based on the Chinese Remainder Theorem (CRT) [9-11]. At the transmitter end, the decimal number of each original packet is first decomposed into the remainders of several prime numbers. Then, the chaotic location scrambling method is used to construct
the transmission packets. When the packets containing the remainder information are received at the receiver end, the corresponding chaotic mapping and decoding algorithm can be used to restore the original data to the specific structural ratio. This paper proposes a new lossless PAPR reduction method based on the CT codes. The main idea is to divide the original signal into a number of the simultaneous inputs. The signal is converted by modular operations into the corresponding number of remainders, which are fed to the corresponding remainder channels for OFDM modulation. The dynamic range of each remainder channel signal is determined by the value of the corresponding remainder. In this way, a flexible and lossless PAPR reduction is achieved.

2. Principal of the papr and ct codes

2.1. Description of PAPR in OFDM Systems

The basic structure of the OFDM transmitter is shown in Fig. 1, where $N$ represents the number of subcarriers in OFDM system, $T$ represents the duration of the OFDM symbol, and $d_i (i = 0, 1, \cdots, N - 1)$ denotes the data symbol allocated to the sub-channel. The signal is sampled at a rate of $T/N$, so $t = KT / N, k = 0, 1, 2, \cdots, N - 1$. After a serial-to-parallel conversion, the data are converted into $N$ parallel data, which are then processed by the Inverse Fast Fourier Transform (IFFT) to produce the OFDM output signal [12]:

$$s_k = s(KT / N) = \sum_{i=0}^{N-1} d_i \exp\left(\frac{2\pi ik}{N}\right) \quad 0 \leq k \leq N - 1$$

Figure 1. Basic structure of the OFDM transmitter.

High peaks may occur due to the superposition of multiple signals with synchronous phase, which is the subject of this study. The ratio of the maximum peak power to the average power is defined by:

$$PAPR = \frac{\max [x(t)]}{E[x(t)]} = 10\log\left(\frac{\max |N_k|}{E[|S_k|]}\right) \quad , \quad k = 0, 1, 2, \cdots, N - 1$$

The PAPR is a random quantity whose performance is usually described by its Complementary Cumulative Distribution Function (CCDF), which is used to calculate the probability of the PAPR exceeding a certain threshold $z$.

$$P\{ PAPR > z \} = 1 - \{ PAPR \leq z \}$$

2.2. The CT Codes

The CT codes encoding process is as follows. First, the original binary information sequence is divided into simple packets. Then, the binary sequence of each simple packet is converted and a modulus to do remainder funding operation is selected. Finally, the coded packets with the moduli and remainders are
formed and transmitted. After receiving a certain number of coded packets, the receiver uses the CRT to decode the data [9]. An example of simple packets encoding using the CRT is presented in Fig.2.

![Figure 2. The CT codes encoding of simple packets.](image)

3. **Papr reduction method based on the ct codes**

The PAPR reduction scheme based on the CT codes is shown in Fig. 3. The method involves a number of variable moduli \( \{m_1, m_2, \cdots, m_v\} \), where \( v \) is adjustable, and the data sent by the source are represented by \( \{D_0, D_1, \cdots, D_{N-1}\} \).

![Figure 3. The block diagram of PAPR reduction based on the CT codes.](image)

After modulo operation, the data stream is split into \( v \) remainder channels for parallel transmission, and the remainder systems is expressed by:
Where \( \{a_{m_0, m_1, \ldots, m_{N-1}}\} \) represents the remainder signals of the remainder channels corresponding to moduli \( m_i \). The remainder signals are obtained by the frequency division multiplexing or other division schemes. After the IFFT transformation is performed, according to Eq. (1), the remainder channels \( a_i \) can be expressed by:

\[
s_{i,m_i}(kT/N) = \sum_{i=0}^{N-1} a_{m_i} \exp\left(\frac{2\pi ik}{N}\right), \quad 0 \leq k, i \leq N-1
\] (5)

The transmitter can flexibly choose the number of moduli \( V \) according to the requirements for PAPR reduction, and synthesize and modulate \( v \) parallel remainder channels; thus, performing all functions of the transmitter.

The receiver is mainly composed of the remainder channel receiving module, FFT module, and CT codes decoding module. It performs parallel processing of multiple channels, as shown in Fig. 4. The signal containing the channel noise is sent to the receiver end. The \( v \) remainder channel receiving modules receive the signals of the corresponding remainder channels. Finally, the CT codes decoding module restores the original data.

\[
\begin{align*}
& \{a_{m_0, m_1, \ldots, m_{N-1}}\} \\
& \{a_{m_2, m_1, \ldots, m_{N-1}}\} \\
& \vdots \\
& \{a_{m_v, m_1, \ldots, m_{N-1}}\}
\end{align*}
\] (4)

\[\text{Figure 4. The receiver block diagram.}\]

4. Simulations

4.1. Simulation theory

On the basis of the PAPR given by Eq. (2), the OFDM output signal defined by Eq. (1), and the CT-OFDM output signal presented by Eq. (5), the PAPR of the OFDM signal can be expressed by:
$PAPR_{OFDM} = 10 \log \left( \frac{\max \left| \sum_{i=0}^{N-1} d_i \exp(j \frac{2\pi ik}{N}) \right|^2}{E \left( \sum_{i=0}^{N-1} d_i \exp(j \frac{2\pi ik}{N}) \right)} \right)$ \quad (6)

$\leq 10 \log \left( \frac{\sum_{i=0}^{N-1} d_i \exp(j \frac{2\pi ik}{N})^2}{E \left( \sum_{i=0}^{N-1} d_i \exp(j \frac{2\pi ik}{N}) \right)} \right) (dB)$

On the other hand, the PAPR expression based on the CT codes is given by:

$PAPR_{CT-OFDM} = 10 \log \left( \frac{\max \left| \sum_{i=0}^{N-1} r_{m,v,i} \exp(j \frac{2\pi ik}{N}) \right|^2}{E \left( \sum_{i=0}^{N-1} r_{m,v,i} \exp(j \frac{2\pi ik}{N}) \right)} \right)$ \quad (7)

$\leq 10 \log \left( \frac{\sum_{i=0}^{N-1} r_{m,v,i} \exp(j \frac{2\pi ik}{N})^2}{E \left( \sum_{i=0}^{N-1} r_{m,v,i} \exp(j \frac{2\pi ik}{N}) \right)} \right) (dB)$

In Eq. (9), when the phases of $N$ subcarriers are synchronous, the PAPR reaches the maximum, and the inequality becomes an equation. According to the Central Limit Theorem, when the number of subcarriers is large, the real and imaginary parts of an OFDM signal have a Gaussian distribution, and their envelopes have the Rayleigh distributions given by $R(\sigma \sqrt{0.5\pi}, (2-0.5\pi)\sigma^2)$, with the mean value of $E(X) = \sigma \sqrt{0.5\pi}$, and the variances of $D(X) = (2-0.5\pi)\sigma^2$.

Its average power is given by:

$E(X^2) = D(X) + [E(X)]^2 = (2-0.5\pi)\sigma^2 + 0.5\pi\sigma^2 = 2\sigma^2$ \quad (8)

By substituting Eq. (8) into Eqs. (6) and (7), respectively, it can be seen that for a specific Rayleigh process, the denominators of the PAPR of both OFDM and CT-OFDM are both constant. Then, the numerators of the expressions should be compared, i.e., $\sum_{i=0}^{N-1} d_i \exp(j \frac{2\pi ik}{N})$ and $\sum_{i=0}^{N-1} r_{m,v,i} \exp(j \frac{2\pi ik}{N})$. Since the remainder $r_{m,v,i}$ of the number $d_i$ is always smaller than the original number, multiplying both of them with the phase factor $\exp(j \frac{2\pi ik}{N})$ and summing the obtained products result in $\sum_{i=0}^{N-1} r_{m,v,i} \exp(j \frac{2\pi ik}{N}) < \sum_{i=0}^{N-1} d_i \exp(j \frac{2\pi ik}{N})$. Therefore, it can be concluded that by utilizing the decomposable modulo operation, the proposed method can achieve a successful PAPR reduction.
4.2. Simulation results and analysis

In Fig. 5, the curves CTm1 - PAPR, CTm2 - PAPR, and CTm3 - PAPR represent the PAPR performances of the signals of the remainder channels corresponding to the moduli \( m_1, m_2, \) and \( m_3 \), respectively, and the curve Original-PAPR represents the peak-to-average performance of the conventional OFDM system.

In the simulation, the following moduli were used \( m_1 = 323, \quad m_2 = 251, \quad m_3 = 161 \). The number of sub-carriers was 1,024 for the QPSK input signal, and the number of input symbols was 1,000. The CCDF graph was used to compare the PAPR reduction performances.

Figure 5. The PAPR performances of the CT-OFDM.

In Fig.5, it can be seen that the branch \( m_1 \) determined the PAPR performance of the CT-OFDM method. At the CCDF of 0.1, the PAPR performance of the CT-OFDM method was 1 dB better than that of the conventional OFDM system. Moreover, the CT-OFDM converted the original signal to smaller remainders. As the remainders were smaller than the original figure by default, the dynamic range of the signal was controlled at the front stage, which means the PAPR of OFDM was suppressed.

5. Conclusion

In this paper, a new PAPR reduction method based on the CT codes is presented. The proposed method can dynamically adjust the input signal at the front stage according to the specific PAPR requirements of the OFDM system to ensure that the peak value of the output signal changes within a relatively small range. Moreover, this method can effectively suppress the peak-to-average ratio to meet the requirement for a limited dynamic range of the amplifier avoiding the signal distortion. The simulation results showed that the proposed method was of efficiency.

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References

[1] Jiang Tao, Wu Yiyian. An overview: Peak-to-average power ratio reduction techniques for OFDM signals [J]. IEEE Transactions on Broadcasting, 2008, 54 (2): 257-268.
[2] Chen Houshou, Chung Kuochen. A low complexity PTS technique using minimal trellis in OFDM systems [J]. IEEE Transactions on Vehicular Technology, 2018, 67 (1): 817-821.
[3] Gong Lin, Yang Shuhui, Chen Yinchen. Research on the reduction of PAPR for OFDM signals by companding and clipping method [C] //Proceedings of the IEEE WiCOM2010. Chengdu, China: IEEE, 2011: 1-4.
[4] Kelvin A, Cagri T, Bamidele A, et al. A new approach to iterative clipping and filtering PAPR reduction scheme for OFDM systems [J]. IEEE Access, 2017, PP (99): 1-11.

[5] Juwono F. H, Gunawan D. The effectiveness of using source coding to reduce PAPR in OFDM system [C] //Proceedings of the IEEE WiCOM2010. Chengdu, China: IEEE, 2010: 1-3.

[6] Fathy S. A, El-Mahallawy M. S, Hagras E. A. A. SLM technique based on particle swarm optimization algorithm for PAPR reduction in wavelet-OFDM systems [C] //32nd National Radio Science Conference (NRSC). 2015:163-170.

[7] Kang S. L, Young J. C, Jun Y. W, et al. Low-complexity PTS schemes using OFDM signal rotation and preexclusion of phase rotating vectors [J]. IET Communications, 2016, 10 (5): 540-547.

[8] Shokrollahi A. Raptor codes [J]. IEEE Trans on Inform Theory, 2006, 52 (6): 2551-2567.

[9] Huang Cheng, Yi Benshun, Gan Liangcai, et al. Fountain Codes Based on Modulo and Chaos [J]. Journal of Beijing University of Posts and Telecommunications, 2010, 33 (3): 121-125.

[10] Xiao Hanshen, Huang Yufeng Ye Yu. Robustness in Chinese Remainder Theorem for Multiple Numbers and Remainder Coding [J]. IEEE Transactions on Signal Processing, 2018, 66 (16): 4347-4361.

[11] Chang C. C, Lee J S. Robust t-out-of-n oblivious transfer mechanism based on CRT [J]. Journal of Network and Computer Applications, 2009, 32 (1): 226-235.

[12] Yao Yi, Hu Jianhao, and Ma Shang. Residue Number System Based OFDM Method for Lossless PAPR Reduction [J]. Journal of University of Electronic Science and Technology of Chin, 2013, 42 (5): 667-671.