Effect of magnetic field on the rotation curves of spiral galaxies

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Abstract. Discrepancies between expected and observed rotation curves in spiral galaxies are commonly interpreted as evidence for the existence of dark matter. Dark matter can explain the observed flat rotation curves at the outer radii. However, some rotation curves of spiral galaxies exhibit rising features at large distances from the galactic centers that cannot be attributed to dark matter alone. Addition of magnetic field contribution to the rotation curves of spiral galaxies has been proposed in some literature to explain the rising features. In this work we investigated the rotation curves of three spiral galaxies, i.e. NGC 2841, NGC 6946, and M 31, whose profiles of azimuthal magnetic field are available in literature. In general, decomposition scenarios using four components (stellar disk, gas, bulge, and dark matter halo) with magnetic field fit better to the observational data.

1. Introduction

Rotational velocities vary with radius, reflecting mass distribution in spiral galaxies. The luminous component of spiral galaxies consists of three constituents, i.e. disk (stars and gas), bulge, and stellar halo. However, contribution of the stellar halo to the rotation curves of spiral galaxies is negligible. It is well known that the observed rotation curves of spiral galaxies tend to be flat at large distances from the centers of the galaxies, and do not show keplerian declines as expected from the distribution of the luminous components. This suggests that an additional non-luminous component, dubbed as dark matter, is needed to explain the shape of the observed rotation curves.

It was long discussed whether magnetic fields in the disks of spiral galaxies had contribution to the rotation curves. Magnetic fields are detected in many astrophysical objects, from planets to pulsars, as well as to larger structures such as galaxies. Observations of galactic magnetic fields show that the presence of magnetic fields plays important roles in the evolution of galaxies and the formation of large-scale structures. In addition, magnetic fields affect gas dynamics in molecular clouds. Early studies of the influence of magnetic fields on the rotation curves of spiral galaxies were directed towards explaining the flat rotation curves without introducing any kind of dark matter (see for example [1] and [2]). However, [3] argued that magnetic fields could not contribute larger than 10 km/s to the rotational velocities. They derived this value from virial constraints, a larger value of magnetic field in the outer radius of a spiral galaxy would induce a large vertical force that would disperse gas in the galactic plane.

Some spiral galaxies exhibit increasing rotation curves at large distances from the centers. [4] and [5] conducted a study of M 31 and Milky Way and showed that adding only dark matter component without involving magnetic fields in the two galaxies could not explain the rising features of the rotation curves at large distances from the centers of the galaxies. Adding contributions of the magnetic fields to the rotation curve models resulted in better fits to the observational rotation curves of the two galaxies.
In this work we investigated rotation curves of three spiral galaxies, i.e. NGC 2841, NGC 6946, and M 31, whose models of azimuthal magnetic field profiles are available. This paper is organized as follows: in section 2 we describe data and method we used in this investigation, including the mass models of the components; results and analysis is explained in section 3, and we finally conclude our study in section 4.

2. Data and method
We used photometric parameters and rotational velocity data of NGC 2841 and NGC 6946 from Spitzer Photometry and Accurate Rotation Curve (SPARC) [6], while for M 31 we used data from [7] for the photometric parameters of the galaxy and from [8] for the rotational velocity data. SPARC photometric parameters are given in 3.6 micro meter, whereas [7] used R-bands surface brightness profile. Magnetic field models were obtained from [9] for NGC 2841 and from [10] for NGC 6946 and M 31.

We performed three scenarios of rotation curve decompositions in order to investigate the influence of magnetic field. In the first scenario, rotation curves were decomposed into four components (star, gas, bulge, and halo dark matter). In the second scenario, we decomposed the rotation curves into the four components plus a contribution from the magnetic field. In performing fitting procedure, we let the velocity contribution of the magnetic field to vary. The third scenario was similar to the second one, but this time we fixed the contribution of the magnetic field by using available constraints on the radial profiles of the magnetic field strength. We employed Markov Chain Monte Carlo (MCMC) method with Metropolis-Hastings algorithm using Bayesian rules in those decompositions. The models of rotational velocities for the five components including magnetic field are explained below. For NGC 2841 and NGC 6946 we used the mass model suggested by SPARC.

2.1. Disk mass model
Stellar disk is described to be exponential [11], the surface mass distribution follows the equation below,

$$\Sigma = \Sigma_{d0} \exp\left(-\frac{R}{R_d}\right),$$

(1)

where $\Sigma_{d0}$ is the central density of the stellar disk and $R_d$ is a radial scale factor. The contribution of the exponential disk to the rotational velocity is [11]

$$v_d^2(R) = \pi G \Sigma_{d0} \frac{R}{R_d} \left[ I_0 \left(\frac{R}{2R_d}\right) K_0 \left(\frac{R}{2R_d}\right) \right] - I_1 \left(\frac{R}{2R_d}\right) K_1 \left(\frac{R}{2R_d}\right),$$

(2)

where $I_0$, $K_0$, $I_1$, $K_1$ are the modified Bessel functions of the first and second kinds and $\Sigma_{d0}$ is related to the central surface brightness of the disk $I_{d0}$ as $\Sigma_{d0} = \frac{M_d}{L_d} I_{d0}$, with $\frac{M_d}{L_d}$ is the mass to light ratio of the disk.

2.2. Bulge mass model
We determined contribution of the bulge component to the rotational velocity from

$$v_b(R) = \left[ \frac{4\pi G}{R} \int_0^R \rho_b(r)^2 dr \right]^{1/2},$$

(3)

where $\rho_b$ is the bulge density,

$$\rho_b(R) = \rho_b(R_e) \left[ 1 + \left(\frac{R}{R_e}\right)^2 \right]^2,$$

(4)

and $R_e$ is the effective radius.

2.3 Mass model of halo dark matter
The contribution of halo dark matter is modeled as isothermal halo as follows,
\[ v_h(R) = \left[ \frac{4\pi G}{R} \int_0^R \rho_h(r)r^2 dr \right]^{1/2}, \]

where \( \rho_h \) denotes halo density at scale radius,

\[ \rho_h(R) = \rho_h(R_c) \left[ 1 + \left( \frac{r}{R_c} \right)^2 \right]^{-2}, \]

here \( R_c \) is a radial scale factor.

### 2.4 Contribution of magnetic field to the rotation curve

To determine contribution of magnetic fields to the rotation curves of spiral galaxies, we first consider the equation of fluid motion [12] as follows,

\[ \rho \frac{\partial \vec{v}_0}{\partial t} + \rho \vec{v}_0 \cdot \nabla \vec{v}_0 + \nabla P = n\vec{F} + \frac{1}{4\pi} \vec{B} \cdot \nabla \vec{B} - \nabla \left( \frac{\rho v^2}{2\pi} \right), \]

where \( \rho \) is the gas density, \( \vec{v}_0 \) is the velocity of the fluid, \( P \) is the pressure, \( \vec{F} \) is the total force due to gravity and \( \vec{B} \) is the magnetic field.

Assuming an axisymmetric and pure rotation, \( \vec{v}_0 = v_{0r}, v_{0\phi}, v_{0z} = (0, \theta, 0) \), we get equation of fluid motion in the radial direction (in cylindrical coordinate)

\[ \rho \left( \frac{\partial \Phi(r)}{\partial r} + \frac{\theta^2}{r} \right) - \frac{dp}{dr} - F_r^{mag} = 0, \]

where \( \Phi(r) \) is the gravitational potential, \( F_r^{mag} \) is the radial component of the magnetic force, and \( P \) is the pressure of the fluid. The radial component of the magnetic field can be written as

\[ F_r^{mag} = \frac{1}{4\pi} \left( \frac{B_{\phi}^2}{r} + \frac{1}{2} \frac{dB_{\phi}^2}{dr} \right), \]

where \( B_{\phi} \) is the magnetic field in the tangential direction and \( r \) is the galactocentric distance. Assuming that pressure gradient is negligible [13], we get the contribution of magnetic field to the rotational velocity

\[ v_{mag}^2 = \frac{r}{4\pi} \left( \frac{B_{\phi}^2}{r} + \frac{1}{2} \frac{dB_{\phi}^2}{dr} \right), \]

where,

\[ B_{\phi}(r) = \frac{B_1}{1 + r_1/r}, \]

\( B_1 \) is the amplitude of \( B_{\phi} \) and \( r_1 \) is the characteristic scale of the magnetic field.

### 3. Results and analysis

For the decomposition of NGC 2841 and NGC 6946 we used the rotational velocities of the stellar disk (hereafter is referred to as "disks") and the bulge components for the value of \( M/L = 1 \) \( M_\odot/L_\odot \) obtained from the galactic mass model according to [6] which is also found in the SPARC website (http://astroweb.cwru.edu/SPARC/). The actual \( M/L \) values of the two components that best fit the observational rotation curves were then determined through a fitting process. Since neutral gas does not contribute in generating luminosity at 3.6 micro meters, the rotational velocity of the gas was taken directly from the contribution of gas velocity according to [6] given on the SPARC website. For M 31, the rotational velocity for each component is obtained by using Equation (1), (3), and (5). The free parameters for NGC 2841 and NGC 6946 are mass-luminosity ratio of the disk \( (M/L)_{disk} \) and of the bulge \( (M/L)_{bulge} \), radial scale factor \( R_c \) of halo dark matter, and halo density at scale radius \( \rho_h(R_c) \).
Meanwhile, for M 31 the free parameters are mass-luminosity of the disk $(M/L)_{d,ak}$, the density of the bulge $\rho_b(R_e)$, radial scale factor $R_c$ of halo dark matter, and halo density at scale radius $\rho_h(R_c)$. The results of the decompositions are shown in Figure 1.

Figure 1. Best fits for decompositions of the rotation curves in the first scenario (luminous + dark matter). The blue dots represent observational rotation curves and the associated error bars. The black lines are the best fits of the rotation curves. The yellow, red, blue, and green lines represent contributions of the disk, bulge, gas, and halo dark matter components respectively. The reduced chi-square of the plots for NGC 2841, M 31 and NGC 6946 are 1.3840, 1.1481, and 1.4656 respectively.

In the second scenario, we decomposed the rotation curves into contributions of luminous components (stars, gas, and bulge), halo dark matter and magnetic fields. We used the results of the first scenario for disk and bulge contributions and varied only dark matter and magnetic fields contributions by using Equation (5) and (10) to get the best fits to the observational rotation curves. Therefore, in the second scenario we had four free parameters, i.e. $R_c$, $\rho_h(R_c)$, amplitude of the magnetic field $B_1$ and a characteristic scale of the magnetic field $r_2$.

Using multiwavelength polarized radio observations in the region between 6 and 14 kpc for M 31, [14] obtained that the value of the strength of azimuthal magnetic field at radius 14 kpc is about 4.6 $\mu G$ [4]. By substituting this into Equation (11), we find a relation between $B_1$ and $r_2$. We therefore can rewrite the Equation (11) for M 31 into

$$B_\phi(r) = \frac{4.6r_2 + 64.4}{r_1 + r}$$

(12)

where $r$ is in kpc and $B_\phi(r)$ is in $\mu G$. The free parameters in the decomposition of M 31 are $R_c$, $\rho_h(R_c)$, and $r_1$. The decomposition results are shown in Figure 2.
Figure 2. Best fits for the decomposition of rotation curves in the second scenario (luminous + dark matter + varying magnetic field). The purple lines are contributions of the magnetic fields. All other lines and symbol are the same as in Figure 1. The reduced chi-square of the plots for NGC 2841, M 31 and NGC 6946 are 1.1176, 1.0261, and 1.4660 respectively.

In the third scenario, we decomposed the rotation curves into luminous components, dark matter halo and the magnetic field components as in the second scenario. However, this time we fixed the contribution of the magnetic fields by using constraints for $B_9$ and $r_9$ obtained from profiles modelled in literature, i.e. from [9] for NGC 2841, and [10] for M 31 and NGC 6946. The radial profile of azimuthal magnetic field of NGC 2841 for example is shown in Figure 3. Since the profile is available only as graphic in the literature, we first retrieved the numerical data from it and fitted this profile using Equation (11) with $B_1$ and $r_1$ as free parameters.
Figure 3. Best fit for the radial profile of azimuthal magnetic field of NGC 2841. The yellow dot is the model adapted from [9] and the orange line is the best fit of the radial profile following Equation (11).

The radial profile of azimuthal magnetic field of NGC 6946 given in [10] cannot be fitted with a single curve following Equation 11. We therefore divided the profile into two ranges of radius, i.e. the profile for \( r \leq 4 \ kpc \) and for \( r \geq 4 \ kpc \). The value \( B_1 \) and \( r_1 \) obtained for each galaxy is given in Table 1. Having determined the value of \( B_1 \) and \( r_1 \) we could determine and fix the velocity contributions of magnetic fields to the total rotation curves. The free parameters in the decomposition are therefore only \( R_c \) and \( \rho_h(R_c) \). Table 1 and Figure 4 show the results.
Figure 4. Best fit for the decomposition of rotation curves in the third scenario (luminous + dark matter + fixed magnetic field). All lines and symbol are the same as in Figure 2. The reduced chi-square of the plots for NGC 2841, M 31 and NGC 6946 are 1.3780, 1.0261, and 3.4040 respectively.

Table 1. The fitting parameters of the third scenario.

| Parameter | NGC 2841     | NGC 6946     | M 31        |
|-----------|--------------|--------------|-------------|
| $\left(\frac{M}{L}\right)_{\text{disk}}$ in $\left(\frac{M}{L}\right)_0$ | $0.938 \pm 0.006$ | $0.678 \pm 0.0019$ | $1.034 \pm 0.026$ |
| $\left(\frac{M}{L}\right)_{\text{bulge}}$ in $\left(\frac{M}{L}\right)_0$ | $1.057 \pm 0.00843$ | $0.761 \pm 0.0049$ | $0.215 \pm 0.001^a$ |
| $\rho_\text{b}(R_c)$ in $\left(\frac{M}{L}\right)_0$ | $0.046 \pm 0.00007$ | $0.054 \pm 0.00024$ | $0.090 \pm 0.00009$ |
| $R_c$ in (kpc) | $5.251 \pm 0.009$ | $2.734 \pm 0.010$ | $2.453 \pm 0.012$ |
| $B_1$ in ($\mu G$) | $477.000 \pm 0.250$ | $407.60 \pm 3.890$ ($r < 4$ kpc); $437.70 \pm 0.198$ ($r > 4$ kpc) | $994.70 \pm 0.670$ |
| $r_1$ in (kpc) | $0.170 \pm 0.0014$ | $0.452 \pm 0.007$ ($r < 4$ kpc); $0.571 \pm 0.000$ ($r > 4$ kpc) | $0.092 \pm 0.0005$ |

$^a$ For M 31 $\rho_\text{b}(R_c)$ was used as a free parameter

Among the three scenarios discussed above, the second scenario provided the best reduced chi-square value. This result is of course predictable, because freeing the magnetic field component in the fitting means that the magnetic field is allowed to have any value. For the second scenario, the rotational velocities of NGC 2841, NGC 6946, and M 31 due to magnetic fields at the outer radii are 93.27 km/s, 0.025 km/s, and 75.2 km/s, respectively. Meanwhile, for the third scenario, the rotational velocities are 7.54 km/s, 8.51 km/s, and 2.12 km/s respectively. [15] studied the influence of magnetic fields on the Milky Way galaxy using a similar method to our second scenario in this work. The value of rotational velocity of Milky Way due to magnetic field at the outer radius obtained by [15] is about 9.5 km/s, while [5] obtained a value of about 180 km/s using a similar method with our third scenario. However, according to [3], the rotational velocity due to magnetic field could not be larger than 10 km/s, otherwise the magnetic field would induce a too large vertical force that can disperse gas in the galactic plane.

4. Conclusions

We have performed decomposition of the rotation curves of NGC 2841, M 31 and NGC 6946 following 3 different scenarios, with and without contributions from magnetic fields. Obviously magnetic field alone cannot held responsible for the shape of the rotation curves at large radii, due to stability issue. Dark matter survives as the dominant component in the galaxies under study. However, the rising features in the rotation curves at large radii observed in several spiral galaxies cannot be attributed to...
dark matter alone, but well explained by the additional contribution of magnetic fields in our third scenario that does not suffer from stability issue mentioned by [3].

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