Mode-locking in modeless laser cavity

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Abstract: We demonstrate experimentally that random phase modulation of an erbium-doped fiber ring-laser with an intra-cavity electro-optic phase modulator did not inhibit ultra-short pulse operation (so-called mode-locking). Stable mode-locking was achieved even when the phase modulator was driven with random sequences sufficiently fast and strong to render the cavity modeless, as determined by heterodyne measurements. No significant change in the pulse characteristics was observed. The insensitivity to the random phase modulation is expected, given the lack of phase-sensitive elements in the cavity.

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1. Introduction

Ultrashort pulses from so-called mode-locked (ML) fiber lasers can be created in a variety of laser cavities and operating regimes. The pulse train is often described as a superposition of longitudinal cavity modes with a fixed spacing and phase relation, i.e., phase-locked or mode-locked [1], and “mode-locked laser” (MLL) and “ultrashort pulse laser” are often used synonymously. On the other hand, one can argue that the term “mode-locking” is a misnomer [2], since the intuitive picture of a pulse propagating in the cavity with periodic amplification and outcoupling of energy does not rely on any cavity modes, and the description in terms of phase-locking of modes is far-fetched and can even be misleading for lasers such as those with frequency-shifted feedback [3]. Thus, as an alternative, many ultrashort-pulse lasers are described and modeled predominantly in the time domain, and the pulse formation is described, e.g., in terms of a saturable-absorption effect together with a balance of self-phase modulation and group velocity dispersion [4]. This description does not rely on the existence of cavity modes with a round-trip phase of an integer number of $2\pi$, which suggests that this is not a requirement.

In this work, we present an erbium-doped fiber laser passively mode-locked and with an electro-optic phase modulator (EOPM) spliced in the cavity to investigate the effect of the cavity roundtrip phase. Even when the EOPM imposes random changes of the cavity phase strong enough to make the cavity modeless, pulses were still formed. The spectral and temporal characteristics of the pulses were nearly the same with and without random phase modulation. Optical heterodyning of both the MLL and a similar free-running continuous-wave (CW) laser showed that the power of signal-reference beat notes decreased with increasing amplitude of the random phase sequence, down to nearly zero. The residual low power can be explained by measurement noise and limited trace lengths. This verifies the absence of modes with a round-trip phase of an integer of $2\pi$ and is one of several examples of additional measurements and extensions beyond those presented in our previous two-page conference paper [5].

2. Passively mode-locked erbium-doped fiber laser with randomly driven intra-cavity phase modulator
The experimental setup of the laser is depicted in Fig. 1. Except for the semiconductor saturable absorber mirror (SESAM) arrangement, fiber and fiberized components are used throughout. A 1.8-m-long Er$^{3+}$-doped fiber (EDF) (Fibercore, mode field diameter 5.5 µm, NA 0.22) with peak absorption 37 dB/m at ~1530 nm was pumped by a diode laser centered at 1470 nm with up to 150 mW of power launched through a wavelength division multiplexer (WDM). A second WDM at the other end of the EDF coupled out unabsorbed pump power. A polarizing beam splitter (PBS) introduced polarization-dependent loss (PDL) and ensured that the input to a subsequent EOPM (iXblue, 10 GHz, $V_\pi = 4.4$ V @ 50 kHz) was linearly polarized. The PBS also coupled out the orthogonal polarization from the cavity through port 1 (P1). The outcoupling corresponds to a cavity loss which depends on the polarization state, which will to some degree self-adjust to minimize the loss. The EOPM was driven by a 250-MSa/s arbitrary waveform generator (AWG) with ~4-ns risetime (AFG31052, Tektronix). Except for the PBS and EOPM, all components and fibers were nominally polarization-independent, without significant PDL. The EOPM was followed by a tap coupler with 5% outcoupling through port P2 and a three-port circulator. A SESAM with 14% modulation depth, glued to a gold-plated cylinder, was coupled to the circulator’s intermediate port (#2) by two lenses. Their focal lengths led to a suitable energy fluence on the SESAM. These were estimated to 17 mm (f1, Thorlabs C260 TME-C) and 12 mm (f2, Thorlabs C220 TME-C) at 1560 nm (chromatic dispersion makes the focal lengths slightly longer than the values specified at shorter wavelengths). Two polarization controllers (PCs) controlled the effect of the nonlinear polarization rotation and the polarization-dependent loss in the PBS. The total cavity length was ~16.2 m and the dispersion approximately ~0.26 ps$^2$. In this configuration, the SESAM ensures that pulsing self-starts whereas the nonlinear polarization rotation adds additional pulse shaping [6]. Diagnostics included an optical spectrum analyzer (Ando AQ6315E) to measure optical power spectra. A 20 GHz, 50 GSa/s oscilloscope (Tektronix DSA72004B) captured temporal traces, typically 4 µs long, from biased InGaAs detectors with bandwidth 15 GHz (EOT ET-3500F) and 22 GHz (EOT ET-3600F). The total effective bandwidths can be estimated according to $f_{tot}^2 = f_{osc}^2 + f_{det}^2$ to around 12 GHz and 15 GHz, respectively.
We first ran the MLL without phase modulation. With the two PCs properly adjusted, we obtained stable fundamental mode-locking centered at ~1560 nm with single pulses at 12.7 MHz pulse repetition frequency (PRF) (cavity round-trip time 78.7 ns), for a pump power of ~20 mW. Then, in order to study the impact of the modulation of the roundtrip phase on the pulse formation, the AWG was set to drive the EOPM with a 10 V (peak-to-peak) repeated random sequence of 0.4 ms duration (100,000 points at 250 MSa/s). Sections of the AWG output are shown in Fig. 2(a), over 12 μs, and (b), over 80 ns, i.e., just over one roundtrip. These were measured with the oscilloscope and recalculated to the phase $\phi_{\text{EOPM}}$. 

Fig. 2. (a) Time-domain waveform generated from the random sequence and used to drive the EOPM, as recalculated to induced phase difference. (b) Zoom into one pulse roundtrip of (a). (c) Distribution of EOPM phase change in 80 ns (i.e., $\phi_{\text{EOPM}}(t) - \phi_{\text{EOPM}}(t - 80\text{ ns})$) as evaluated every 1 ns for a 4 μs trace and folded into the range [0, 2$\pi$]. (d) Optical spectrum at unpolarized port P2 for 20 mW pump power with (black curve) and without (red curve) phase modulation. They are nearly identical; the top curve shows the difference. (e) Output temporal traces at polarized port P1 with (black curve) and without (red curve) phase modulation. The pulses are temporally offset for clarity. No significant difference is observed. (f) Zoom-in on a single pulse. (g) Pulse traces with random-sequence phase modulation switched on and then off. All curves use either 10 V or 0 V modulation voltage.
induced by the EOPM with $V_x = 4.4$ V. The phase varies within a range of $\pm 1.13$ rad (i.e., over a total range of $2.27\pi$ rad = 7.13 rad). Fig. 2(c) shows the distribution of changes in $\varphi_{\text{EOPM}}(t)$ in one roundtrip time, i.e., $\varphi_{\text{EOPM}}(t) - \varphi_{\text{EOPM}}(t - 80 \text{ ns})$, as folded into the range $[0, 2\pi]$. This was evaluated every 1 ns for a 4-μs trace. The EOPM phase changes in one roundtrip are nearly uniformly distributed over $[0, 2\pi]$. This suggests that the variations are fast and large enough to eliminate any cavity modes as characterized by a roundtrip phase of an integer multiple of $2\pi$.

Optical pulse spectra were measured at port P2 with the OSA at 0.05 nm (~ 6 GHz) resolution, as shown in Fig. 2 (d), with and without phase modulation. We repeated the measurement three times to verify consistency. The characteristics of a ML spectrum with Kelly sidebands are present both with and without phase modulation. Although there is a noticeable change in the spectrum, the change is small. The 3-dB spectral bandwidth was ~5.45 nm (670 GHz), which corresponds to a transform-limited pulse duration of around 0.6 ps. Fig. 2 (e) shows the corresponding pulse trace monitored at P1, where the random-sequence phase modulation produced no discernible change, and Fig. 2 (f) shows the trace of a single pulse. The duration is 290 ps (full-width at half-maximum, FWHM), i.e., some 500 times the transform limit. Under the assumption of a linear chirp, this becomes $670 \text{ GHz}/290 \text{ ps} = 5.2 \text{ GHz/ps} = (192 \text{ ps})^{-1}$. Fig. 2 (g) shows the temporal trace of the MLL captured at P1 as the phase modulation turns on and then off after a period of time. There are no obvious transient effects (e.g., loss of mode-locking or pulse amplitude or period changes). Furthermore, we found that pulsing (mode-locking) self-starts with phase modulation both on and off. We conclude that the single-pulse state of the laser persisted steadily with and without random phase modulation in the cavity, and did not observe any significant difference. Nevertheless, we note that properties such as jitter are likely to degrade, where a phase change of $2\pi$ may change the cavity roundtrip time by one optical cycle (5.2 fs).

3. Investigation of the impact of intra-cavity random phase modulation on laser modes by heterodyne detection

We used optical heterodyning to reach the MHz-level spectral resolution required to study the modal behavior with intra-cavity random-sequence phase-modulation. If a laser can be described in terms of cavity modes, its rapidly varying field $E(t)$ can be written as $E(t) = \sum C_k e^{i2\nu_k t}/2$, where $C_k$ is the amplitude and $\nu_k$ is the frequency of mode $k$. Ideally, the parameters are time-independent but may vary slowly in time. Furthermore, $E(t)$ can naturally and conveniently be taken to be real-valued. Then, $C_k^* = C_{-k}$, where the star denotes complex conjugate. Because of dispersion, the frequencies of the cavity modes are not necessarily exactly equidistant. However, in case of a conventional MLL, $\nu_k = k \Delta \nu \pm \nu_0$, where the frequency spacing $\Delta \nu$ is the same for all modes (and equal to the PRF) and $\nu_0$ is the carrier envelope offset [2], with different sign for positive and negative frequencies. In both these cases, the mode spectrum is discrete, and all power is in the modes.

Experimentally, the output of the laser-under-test was mixed with that of a stabilized single-frequency (SF) polarized reference laser (IDPHOTONICS CoBrite-DX4) to down-convert the spectrum to the radio-frequency (RF) domain. Following square-law photodetection, the signal becomes $V(t) = C_{\text{ref}}^2/2 + E^2(t) + C_{\text{ref}} \sum C_k(e^{2it\Delta \nu} + e^{-2it\Delta \nu})/4$. Here, the reference field has been expanded as $E_{\text{ref}}(t) = C_{\text{ref}}(e^{2it\nu_0} + e^{-2it\nu_0})/2$ and the laser field as $E(t) = \sum C_k e^{i2\nu_k t}/2$ in the third term of $V(t)$ (but not in the second term). Furthermore, some terms oscillating with zero mean at twice the optical frequency are neglected to simplify the expression. The signal $V(t)$ is proportional to the instantaneous power, and thus, in the absence of the reference laser, to the instantaneous power of the laser-under-test (proportional to $E^2(t)$ as averaged over the optical cycle). The signal was measured with the oscilloscope and its spectrum $\hat{V}(f)$ obtained as a Fourier transform (FT) [7]. $\hat{V}(f)$ includes the laser’s
intensity FT (IFT) \( \hat{E}^2(f) \). This is the spectrum conventionally measured with a RF spectrum analyzer (without mixing with a reference laser), and is centered at zero frequency (DC) with single-sided spectral width equal to the laser linewidth or less. It is independent of the optical phase, and is therefore less interesting for us. Instead, our primary interest is the phase-dependent beating of different spectral components of the signal laser with the reference laser. Its Fourier transform becomes \( C_{\text{ref}} [\hat{E}(f - v_{\text{ref}}) + \hat{E}(f + v_{\text{ref}})]/2 \), where \( \hat{E} \) is the Fourier transform of the optical wave \( E \) (and \( |\hat{E}|^2 \) corresponds to the optical power spectrum). Thus, each of the terms has a linewidth equal to the optical linewidth of the test laser and provided that \( v_{\text{ref}} \) is tuned to lie outside the optical spectrum, the two terms will be well separated into a positive-frequency and negative-frequency branch. It is also possible to tune \( v_{\text{ref}} \) so that the IFT is spectrally separated. Over a spectral range chosen to include only one of the positive-frequency and negative-frequency branch and exclude the IFT, \( |\hat{V}(f)|^2 \) then becomes the optical power spectrum as down-converted to the RF. Any modes are now resolved, and their power can be evaluated. Note also that the RF spectra we plot are \( |\hat{V}(f)|^2 \) and may thus include the square of the IFT rather than the IFT itself.

First, we investigated the modes of a randomly cavity-phase-modulated continuous-wave (CW) laser. Its relatively narrow spectrum makes it simpler to investigate than a MLL, since the power per mode is higher and since it is relatively simple to tune \( v_{\text{ref}} \) to spectrally separate the IFT from the signal-reference beating. The experimental setup is shown in Fig. 3. Compared to the MLL, there is now an isolator in place of the SESAM and circulator. Other components remained the same. The outputs from the polarized output port 1 (power 2 mW) and the reference laser (power 40 mW) were combined by a 50:50 polarization-maintaining coupler. Fig. 4 shows the resulting spectra \( |\hat{V}(f)|^2 \) when the EOPM was driven by a random sequence of different amplitudes. For this, 4-\( \mu \)s-long traces (\( \sim 49.2 \) roundtrips) were captured by the 15-GHz detector and oscilloscope at 50-GHz sampling rate and then Fourier-transformed. The peak-to-peak amplitudes (i.e., the total voltage spans) were 0 V, 4 V, 6 V, 8 V, and 10 V, corresponding to phase ranges of 0.91\( \pi \) rad, 1.36\( \pi \) rad, 1.81\( \pi \) rad, and 2.27\( \pi \) rad. We attribute the small shift in the signal-reference beat spectrum seen in Fig. 5 (a) - (b) to small wavelength drifts in the lasers. Spectra without reference laser (“signal” or “IFT”), i.e., the pure IFT without any signal-reference beating, are also shown. The IFT extends out to around \( \pm 1 \) GHz, which is narrower than the spectral width of the signal-reference beating. This indicates there are significant phase variations, which broaden the optical spectrum but
leaves the IFT unaffected. (In the extreme, the IFT is a single line for pure phase modulation.) For the signal-reference beating, the reference was offset by \( \sim 7 \text{ GHz} \) to keep it separated from the IFT and still within the measurement bandwidth. In addition, the higher reference power reduces the IFT relative to the signal-reference beat spectrum. The high-resolution spectra of Fig. 4 (c) – (g) show that the beat notes gradually fade for increasing modulation voltages. We also note that the cavity dispersion is small over linewidths this small (e.g., \( -0.26 \text{ ps}^2 \times (2\pi \times 10 \text{ GHz})^2 = -1.02 \text{ mrad} \) for 10-GHz linewidth). This is small compared to the \( 2\pi \) mode spacing, so no deviation from a constant mode-spacing is observed.

![Diagram](image)

**Fig. 4.** Spectra \( f(f) \) of the CW laser with different levels of random phase modulation for (a) the full bandwidth and (b) zoomed in on the positive branch of the signal-reference beat.
spectra and on the IFT spectrum. Note the change in amplitude between plots. (c)–(g) Further enlargements to frequency ranges of about 7.5–7.6 GHz (around the middle of the signal-reference beat spectrum) for increasing random-modulation level as indicated. The amplitude scale is the same in all cases. (h) Enlarged IFT captured without reference laser.

Fig. 5. (a) – (e) Cumulative power of the CW laser as integrated from 5 GHz normalized and shown between 7.5 and 7.6 GHz for different phase modulation levels as indicated. (f) Fractional power as integrated from 5 GHz to 10 GHz in signal-reference beat notes vs. random phase modulation voltage level.

We next evaluated the fraction of power in the beat peaks. First of all, to visualize this fraction, we integrated the spectra \( |\hat{V}(f)|^2 \) in Fig. 4 in the range 5 GHz – 10 GHz, within which the IFT lines are too small to be observed and most of the signal-reference beat power resides. Fig. 5 (a) – (e) shows the cumulative power, as integrated spectrally from 5 GHz and enlarged to cover a frequency range of 0.1 GHz. The beat peaks create steps in the curves, and the fractional power is given by their sum relative to the total power. The fractional power, and thus the size of the steps, becomes smaller as the phase modulation voltage increases, and increasingly difficult to identify and evaluate. Therefore, to calculate the power in modes even when the background is comparable to the modal peaks, we ensemble-averaged the spectrum into one mode-spacing as follows.

\[
S_1(f) = \sum |\hat{V}(f + k\Delta v)|^2
\]  

(1)

The ensemble-averaged spectrum \( S_1(f) \) is evaluated for frequencies \( f \) varying over one mode spacing \( \Delta v \) (from \( f_0 \) to \( f_0 + \Delta v \), where \( f_0 \) can be chosen arbitrarily). The sum then extends over a chosen frequency range of \( |\hat{V}|^2 \) (5 GHz to 10 GHz in this case). Random voltage fluctuations in the spectral trace from mode-spacing to mode-spacing sum up to a relatively constant background, whereas any power in modes is periodic in the spectrum and thus sum up to a spectral peak. The power in the modes was then evaluated as the power in the highest peak in \( S_1 \) relative to the total spectrally integrated power. This is conceptually simple, but requires that the mode spacing is accurately determined so that the error in \( k\Delta v \) is small.
compared to $\Delta v$. The mode spacing was determined to about 13.617 MHz. See Appendix 1 for further details.

The fractional power is plotted in Fig. 5 (f). The power dropped from 95.1% to 13.9% when the random modulation level increased from 0 to 10 V (i.e., $2.27\pi$ rad). Simulations indicated that the residual power may be an artefact of the limited trace length and noise. It is also worth noting that we expect the signal-reference beat notes will diminish in the same or a similar way if the signal is instead phase-modulated outside the cavity.

Fig. 6. Experimental setup for heterodyning of the mode-locked laser

We next heterodyned the mode-locked laser using output port 2 and the configuration shown in Fig. 6. The port-2 output power was $\sim 39$ $\mu$W (pump power $\sim 35$ mW). Since the MLL linewidth and IFT far exceeds our RF measurement bandwidth, two tunable filters (Alnair-Labs BVF200 and Alnair-Labs BVF200CL), each with 3-dB bandwidth of 0.086 nm, were cascaded to select a 0.064-nm (7.87-GHz) (FWHM) central slice of the spectrum at around 1560 nm. This restricts the IFT and beating frequency range. The bandwidth corresponds to a fraction of $\sim 0.86\%$ of the bandwidth of the MLL. At the output of each filter, an erbium-doped fiber amplifier (EDFA) boosted the signal power, which became $\sim 0.795$ $\mu$W (average) at the output of the second filter. Then, a PC adjusted the polarization state of the signal before it was launched into the 10% port of a 90:10 polarization-maintaining coupler to mix it with the reference laser. The resulting beating was measured with a 22-GHz detector. The power of the amplified signal after the combiner was $\sim 2$ $\mu$W. The power of the reference laser after the combiner was $\sim 4.5$ mW to keep it below the power limitation of the photodetector. Assuming a linear chirp of 5.2 GHz/ps in the unfiltered pulses and a linewidth of 7.87 GHz, the pulse duration can be estimated according to $[(0.4 / 7.87 \text{ GHz})^2 + (7.87 \text{ GHz} / 5.2 \text{ GHz/ps})^2]^{1/2} = 50$ ps. Since the first square is three orders of magnitude larger than the second, we expect that the filtered pulses are practically transform-limited, despite the strong chirp of the unfiltered pulses.
Temporal traces of the MLL (without reference laser) and the beating with the reference laser are shown in Fig. 7 (a), and Fig. 7 (b) shows FT spectra $|\hat{V}(f)|^2$ of the temporal traces. The spikes, e.g., at $\sim$12 GHz, are detection artefacts and were removed before further processing of the spectra. The reference laser was offset by $\sim$10 GHz from the center of the filtered spectrum of the MLL. This results in well-separated positive and negative branches of the signal-reference beat spectrum. The linewidth of each branch is determined by the filters to $\sim$8 GHz. Without reference, $|\hat{V}(f)|^2$ has a central lobe (corresponding to the IFT) with width comparable to the width of the beat lobes with reference. Outside the IFT lobe, there is significant noise which even grows to secondary peaks at around $\pm$18 GHz.

In the heterodyned temporal traces in Fig. 7(a), the lasers interfere either constructively or destructively, depending on the phase between signal and reference. Without intra-cavity phase modulation, the phase at the pulse peaks drifts at a constant rate relative to the

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{(a) Temporal traces of optically filtered MLL without heterodyning and intra-cavity phase modulation and with heterodyning and phase modulation voltages of 0 V and 9 V as indicated. (b) Fourier transform $|\hat{F}(f)|^2$ of heterodyned traces for modulation voltages from 0 V to 9 V. (c)-(e) Zoomed-in examples on a single pulse of the mixed trace with 0 V modulation showing varying interference pattern depending on the relative phase between the MLL and reference laser. (f) As (c) - (e) but without reference laser. (g)-(k) The enlarged spectra in frequency scale at about 6.793-6.855 GHz for modulation voltages from 0 V to 9 V. The signal-reference beat notes are marked in red. The vertical scale is the same in all plots.}
\end{figure}
reference phase, leading to a regular pattern of increasing and decreasing peak voltage in Fig. 7(a). On the other hand, with 9 V of random modulation, the interference at the peaks is constructive or destructive to varying degrees in a random pattern. Fig. 7(f) shows a single pulse without reference laser. The duration is 43 ps (FWHM). This is in good agreement with the estimated transform-limited duration, and shows that the measurement bandwidth is adequate for the filtered pulses. Fig. 7(c)-(e) show examples of heterodyned single pulses. Note that each of the heterodyned pulses dips below the baseline level set by the power of the reference level, indicating destructive interference. Thus, our measurement bandwidth suffices for capturing both constructive and destructive sections of the beating. The positions of the maxima and minima are consistent with a beat frequency of 10 GHz. Note also that the phase of the EOPM can be considered constant during the pulse, since the 4-ns risetime of the driver is nearly two orders of magnitude longer than the filtered pulses.

It is also possible to evaluate the peak power from the beating. The highest peaks reach ~15.42 times the reference level. From this, we can roughly estimate the peak power to be between 1.2 and 1.6 times the reference power (i.e., between 5.4 – 7.2 mW), where noise and fluctuations in the traces limits the accuracy of the estimate. Given a duty cycle of 43 ps × 12.3 MHz = 5.3×10^{-4}, the average detected signal power becomes 2.9 – 3.8 μW, in fair agreement with the directly measured power of ~2 μW. We note also that perfect cancellation to 0 V should occur occasionally, when the instantaneous pulse power matches the reference power but is in antiphase. This is not seen in our temporal traces. Possible reasons are polarization mismatch, broadband background (e.g., amplified spontaneous emission, ASE), and the limited detection bandwidth.

We next evaluate the fraction of power that can be assigned to modes. The procedure is similar to that for the CW laser in Fig. 4, although it is more difficult because noise is higher and the IFT lines extend into the spectral region of the signal-reference beating. First of all, we reduced the noise by selecting a low-noise range of the spectrum, 6.793 GHz to 12.22 GHz, and subtracting the baseline noise (measured without signal) from |\hat{V}(f)|. Also, to make it easier to visually identify the positions in the signal-reference beat notes even when the background is comparable to the modal peaks, we ensemble-averaged the spectrum in sections of five mode spacings as follows.

\[ S_j(f) = \sum_k \left| \hat{V}(f - 5k\Delta\nu) \right|^2 \]  

The sum is over the low-noise range of the signal-reference beat spectrum with \( f \) varying by five mode spacings from 6.793 GHz to 6.855 GHz (as for \( S_j \), the origin of \( f \) is arbitrary in the sense that any offset is compensated for by the summation index \( k \)). As for the CW laser, this requires that we first calculate \( \Delta\nu \) precisely, but this is now straightforward thanks to the distinct pulses of a MLL. The mapping into five mode segments instead of a single mode segment may lead to meaningful differences between the five segments. Other segmentations and frequency spans were investigated, too. Although not discussed here, this proved useful for the development of algorithms. Fig. 7 (g) – (k) show the enlarged spectra in frequency scale at about 6.793-6.855 GHz for intra-cavity random phase modulation voltages between 0 V and 9 V. With increased level of random phase modulation, the amplitude of signal-reference beat notes decreased, as for the CW laser in Fig. 4. Small IFT lines can be seen, too. We tentatively attribute the differences between mode segments to the spectral sampling, where the mode-spacing was not an integer number of spectral samples.

Also as for the CW laser, we calculated \( S_j \) and from that, the fractional power in the signal-reference beat peaks, after we removed the IFT lines. Although their contribution may be small enough to be neglected (e.g., Fig. 7(i)), the IFT lines can still interfere with the identification of the modal peaks at high modulation (e.g., Fig. 7(k)) if they are not removed. There are several options for removing them. The signal-reference beat notes and the IFT lines of the MLL have exactly the same frequency spacing (\( \Delta\nu \approx 12.3 \text{ MHz} \)), but are
generally offset from each other, since the signal-reference beat frequencies depend on the reference frequency. This can be tuned to ensure the two sets of lines do not overlap. By contrast, the IFT lines lie very precisely on the comb \( v_m = m \Delta v \). This is how we identified them, and subsequently removed them as described in Appendix 1. Another way to identify the IFT lines is to switch off the reference laser, leaving only the IFT in \( \hat{V}(f) \). Also, if the measurement bandwidth and intermediate frequency (between the signal and reference) are sufficient, one can select a spectral region without IFT lines. This was possible for us, but limited the spectral range available for the evaluation and was therefore expected to lead to less accurate results.

Following removal of the IFT lines, we calculated the cumulative power for different modulation voltages. Fig. 8 (a) – (e) shows this for the mapping \( S_5(f) \) over five mode-spacings. Note that this mapping does not change the relative power in the signal-reference beat notes. Fig. 8(f) shows that the fraction of power in the beat notes dropped from \( \sim 83.34\% \) to \( \sim 20.59\% \) as the random modulation increased from 0 to 2.05\( \pi \) rad (9 V).

![Normalized cumulative power of the MLL as evaluated from \( S_5 \) for different levels of random-sequence phase modulation voltages as indicated. (f) Fractional power as integrated from 6.793 GHz to 12.22 GHz in signal-reference beat notes vs. random phase modulation voltage level.](image)

According to Fig. 8, the fraction of power in the MLL’s beat notes is significantly less than 100\% even in the absence of phase modulation. Simulations show this may be at least partly explained by the level of noise present in the temporal trace. We simulated the heterodyne detection process for a pulse train without random phase modulation, with and without voltage noise, and then processed the resulting trace as we did the experimental traces. The voltage noise had a normal distribution with zero mean and without correlation from sample to sample (i.e., it is “white”). The average power of the MLL and the RMS level of the voltage noise was set to 37\% and 1.07\% of the reference-laser level, respectively. These parameters are similar to the experimental conditions. Fig. 9 shows the simulated heterodyne temporal trace and spectra \( |\hat{V}(f)|^2 \) without and with added voltage noise. The
fraction of power in modes was evaluated as for the experimental traces, and became 99.53% without voltage noise and 88.4% respectively.

Fig. 9. Simulated heterodyned temporal trace without phase modulation without (a) and with (b) added voltage noise. (c), (d) Corresponding Fourier-transformed spectra $|\mathcal{F}(f)|^2$ shown over four mode spacings.

Simulations also show that the apparent residual power in modes at high phase modulations may at least partly be explained by the limited number of pulses in the trace. Fig. 10 shows sections of heterodyne spectra approximately four mode-spacings wide with 0 ((a), (c)) and $2\pi$ ((b), (d)) random phase modulation for trace-lengths of 49 pulses ((a), (b)) and 49,000 pulses ((c), (d)). At $2\pi$ modulation, the simulated power fraction in modes became 0.18 for the 49-pulse realization, relative to the 0 rad case. This dropped to $1.503 \times 10^{-4}$ for the 49,000-pulse realization.

Fig. 10. (a),(b) Simulated heterodyne spectra over four mode spacings of 49-pulse trace at the random phase modulation level of (a) 0 rad and (b) $2\pi$ rad and of 49,000-pulse trace at the random phase modulation level of 0 rad and $2\pi$ rad.

4. Conclusion

In conclusion, we have demonstrated an ultra-short-pulse laser in which phase modulation of the cavity with a random sequence precluded the existence of conventional cavity modes with
roundtrip phase a multiple of $2\pi$. Experimentally we used an erbium-doped fiber ring-laser with an intra-cavity electro-optic phase modulator and optical heterodyne detection to verify the suppression of signal-reference phase beat notes associated with such modes. No significant change of basic laser characteristics was observed when the phase modulation was introduced. The results are expected since there is no element in the cavity that is sensitive to the absolute phase or pulse-to-pulse phase variations, and confirm that traditional phase-locked cavity modes are not required for a ultrashort-pulse laser. The cavity modes were also suppressed in a CW laser.

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**Appendix 1.**

We here describe the procedure we used to identify and remove IFT lines and evaluate the fractional power in the signal-reference phase beat notes. The starting point is 4-μs-long temporal traces captured by the oscilloscope (200,000 points with 20-ps spacing). The traces were Fourier-transformed to yield $|\hat{V}(f)|^2$ with 250-kHz resolution. The number of spectral points per mode becomes $\approx 12.3$ MHz / 250 kHz = 49.2.

1. Determination of mode-spacing $\Delta \nu$ with high accuracy, so that $\nu_m = m \Delta \nu$ is accurate to within a small fraction of the mode-spacing for the full range of modes and frequencies used in the evaluation. The mode spacing was around $12 - 13$ MHz and the frequency range considered was around 5 GHz (e.g., 6.793-12.22 GHz, span 5.427 GHz for the MLL).

For the MLL, the mode-spacing is equal to the PRF. We determined this from the position of the first and last pulse of each analyzed trace. These are separated by nearly 200,000 points. The pulse trace will be distorted by the beating with the reference laser (see Fig. 8 (c) – (e)) and we estimate the error in the peak position to be comparable to the pulse duration, or around ±40 ps for each peak. Thus, the
relative error limit becomes $2 \times 10^{-5}$, so 109 kHz over 5.427 GHz. This is within 0.088% of the mode spacing and smaller than the spectral resolution.

The periodicity is more difficult to determine for the CW laser, since the variations are much less distinct. Therefore, we calculated the autocorrelation

$$C(\tau) = \int V(t)W(t)\bar{V}(t+\tau)W(t+\tau)dt,$$

where $V(t)$ is the voltage trace and $W(t)$ is a Hamming window function. This produced distinct peaks in a comb with spacing equal to the roundtrip frequency. The peaks faded gradually for larger $\tau$, but could be identified up to more than 2 $\mu$s for a 4-$\mu$s trace. From those, the periodicity is easily evaluated. Whereas dispersion is expected to cause the mode spacing to vary in the CW laser, significant dispersion would also cause the autocorrelation peaks to become less distinct, and this was not observed. This further supports that dispersion is not significant, and that the mode-spacing can be evaluated as the inverse of the periodicity also for the CW laser.

The window function eliminated a second, interleaved, comb, which made it more difficult to determine the peak spacing. Since a standard Hamming window produced adequate results, we did not evaluate alternative windows, which may well have been able to preserve the amplitude of the first comb to larger values of $\tau$ whilst still suppressing the second comb.

The precise determination of the periodicity is important and non-trivial. Although autocorrelation is well-established and proved adequate, there are several more sophisticated and well-established algorithms for precise determination of periodicity with traces that can be more challenging than ours, and which may be superior for our traces. See, e.g., [8] for a review directed towards astronomy.

2. A trace was measured without signal and Fourier-transformed to obtain a noise spectrum $\hat{V}_{\text{noise}}(f)$. The noise spectrum was averaged over 100 MHz and then subtracted from the spectra $\hat{V}(f)$ measured with the signal.

3. The spectra were inspected to identify a region (e.g., 6.793-12.22 GHz) which included most of the signal-reference beat spectrum while exhibiting low IFT and noise.

4. The spectrum was remapped into a single mode-spacing $S_1(f)$ and summed as in Eq. 1. This reduces the noise and facilitates visual inspection of the spectral characteristics of a large number of modes and visual and numerical identification of IFT and signal-reference beat peaks. For this, $\hat{V}(f)$ is segmented and the segments are summed. The offsets $k \Delta \nu$ (i.e., the segment boundaries) are rounded to the closest sample point so $|\hat{V}(f)|^2$ does not have to be resampled. Since the mode-spacing is generally not an integer number of points (e.g., ~49.2 points), the segment lengths vary (e.g., between 49 and 50 points). The extra point in the longer traces is discarded.

5. The position of the IFT line within $S_1(f)$ is determined and the line removed. (In $|\hat{V}(f)|^2$, the lines are at $k \Delta \nu$.)

6. Excluding the IFT line, the highest peak is identified in $S_1(f)$. This is assumed to correspond to the signal-reference beat peak. We checked that it is separated from the IFT line by at least five samples. If not, it may be necessary to select or measure another trace.

7. The average background level in $S_1(f)$ is calculated, excluding the five points closest to each of the IFT and signal-reference beat peak.
8. The IFT is removed by replacing the five samples closest to the IFT line with the average value.
9. The power in the signal-reference beat peak is evaluated as the power in the five samples above the average background level.
10. The total power is evaluated as the sum of the samples in $S_1(f)$ plus the background level in the discarded point (e.g., $49.2 - 49 = 0.2$ times the background level).
11. The fractional power is evaluated as the power in the signal-reference beat peak relative to the total power.