Effect of applied electric field and grating spacing on performance of ring resonator

Ruchi Singh* R A Yadav and D P Singh
Department of Physics, Banaras Hindu University, Varanasi-221005, India,
*E-mail: ruchisingh8bhu@gmail.com

Abstract. In this paper we analyze the effect of grating spacing on relative oscillation intensity and two-beam coupling gain of a ring resonator in presence of an applied electric field. The photorefractive crystal used in the ring resonator is KNSBN. It is found that the relative oscillation intensity increases exponentially with grating spacing in presence as well as in absence of the applied electric field, and approaches to a saturation state after some grating spacing. The increase is however slower under applied field condition, hence, the saturation state is reached at higher value of the grating spacing. The two-beam coupling gain decreases with grating spacing for both conditions on electric field. The decrease is however less abrupt in presence of applied electric field. It is found that the saturation state occurs for the grating spacing over 2 to 4 µm.

1. Introduction
Over the last four decades the photorefractive materials and devices have been a very active field of research because of their enormous applications in telecommunication and sensing. The ring resonator using a photorefractive crystal is an important device because of its rich dynamic behavior and pattern formation characteristics. Several interaction geometries in photorefractive crystal have been studied both theoretically and experimentally [1-4]. The ring resonator geometry consists of a photorefractive crystal as an active nonlinear medium with three partially reflecting mirrors $M_1$, $M_2$, and $M_3$ as shown in figure 1. Mostly the ring resonators with BaTiO$_3$ crystal have been studied [5-7]. In this work we studied the two-beam coupling performance of a ring resonator with potassium-sodium-strantium-barium niobate (KNSBN) crystal in the resonator cavity. The crystal is pumped by an external light beam. The inhomogeneities and impurities in the crystal produce beam fanning. The scattered light in the direction of signal beam is reflected by the cavity mirrors back into the crystal where amplification occurs by the process of two-wave mixing. The oscillation beam builds up if the two-wave mixing gain is above a threshold value that is when the gain exceeds losses including the absorption in the crystal. High powers can be achieved even with moderate values of two-beam coupling coefficient provided the losses are small. The scattered light away from the signal path will be lost. In the photorefractive ring configuration, the light propagation inside the cavity should be unidirectional as the two-wave mixing is directional determined by the crystal symmetry, alignment and charge transport properties [5]. The two beam
coupling gain and the relative oscillation intensity are the two most important parameters that characterize the performances of a ring resonator. In this work we analyze the effect of grating spacing on the oscillation intensity and the two-beam coupling gain both in absence and in presence of an applied electric field. It is seen that an efficient resonator can be realized even with moderate coupling constant if the absorption coefficient is low. These conditions are fulfilled by KNSBN. A He-Ne laser has used as a pump beam.

![Figure 1. Schematic diagram of ring resonator.](image)

2. Theory
2.1. Two beam coupling gain
The two-beam coupling gain is determined by amplification of the signal beam in two-wave mixing. The amplification occurs under conditions that (i) the two-beam coupling overcomes passive losses inside the resonator, and (ii) the phase shift of the oscillating wave in making one round-trip in the resonator is an integral multiple of $2\pi$ [5]. That is,

$$GR \geq 1$$  \hspace{1cm} (1)  

$$\Delta \psi_c + \int k_g ds = 2N\pi$$  \hspace{1cm} (2)  

where $G$ denotes parametric two beam coupling gain, $R$ represents the product of reflectivities of mirrors $M_1$, $M_2$ and $M_3$, $\Delta \psi_c$ is the additional phase shift due to two beam coupling, $k_g$ is the grating wave vector, and the integration in the Eq.(2) is over a trip around in the resonator cavity. The parametric two-beam coupling gain can be defined as, and is given by [5].
where \( m \) is the ratio of intensities of the two waves at the input, \( l \) is the crystal thickness, \( \alpha l \) is the absorption strength, \( \gamma_0 l \) is the coupling strength, \( \omega = \omega_{\text{pump}} - \omega_{\text{signal}} \) is frequency detuning, and \( \tau \) is the response time (a function of grating spacing) given as [5, Eq.3.6-34]

\[
\tau = \tau_{di} \left[ \frac{(1 + \frac{E_d}{E_\mu})^2 + \frac{E_0^2}{E_\mu^2}}{1 + \frac{E_d}{E_q} + \frac{E_0 E_q}{E_\mu E_q}} \right]
\]

where \( \tau_{di} = \varepsilon_r \varepsilon_0 / e \mu n_0 \) is the dielectric relaxation time in which \( \varepsilon_r \) is the relative dielectric constant, \( \varepsilon_0 \) is the permittivity in vacuum, \( \mu \) is the mobility and \( n_0 = \tau_R s I_0 N_D \) is the average electron density. The other quantities in are: \( E_d = k_B T g / e \) the diffusion field, \( E_\mu = \gamma N_A / \mu k_g \) the drift field, \( E_q = e N_A / e_0 \varepsilon_0 k_g \) is the limiting space charge field, \( \tau_R = 1 / \gamma_r N_A \) is the recombination life time, \( s \) is the photo-ionization cross section, \( I_0 \) is the incident light intensity, \( N_D \) is the donor density, \( N_A \) is the acceptor density, \( \gamma_r \) is the recombination constant, \( k_g = 2\pi / \Lambda \) is the grating wave vector with grating spacing \( \Lambda \) and \( k_B T \) is the product of Boltzmann constant \( k_B \) and temperature \( T \).

### 2.2. The relative oscillation intensity

The ring resonator oscillations start from the noise generated by scattering and quantum fluctuations. In photorefractive crystals the scattering contributions dominate. Initially, there would be a little amount of light scattered in the direction of ring resonator. The two-beam coupling process amplifies the scattered light at frequencies differing from the pump beam by \( \omega (\approx 30 \text{ Hz}) \). As the intensity of this light in the resonator cavity builds up the parameter \( m \) decreases. The buildup of oscillation intensity leads to gain saturation. The relative oscillation intensity \( I_r \), which is same as the input intensity ratio \( m \), given by [7]

\[
I_r = \frac{1 - R \exp(-\alpha l)}{R \exp(-\alpha l) - \exp\left(-\frac{\gamma_0 l}{1 + (\omega \tau)^2}\right)}
\]
3. Results and discussion
We have used the Eqs. (3) and (5) together with the Eq. (4) for calculating the two beam coupling gain and relative oscillation intensity with pump beam intensity $I_p = 0.5 \text{ W/cm}^2$. The ring resonator crystal is KNSBN having thickness $l = 5 \text{ mm}$ and absorption coefficient of $\alpha = 10 \text{ cm}^{-1}$. Figure 2 shows the variation of gain with grating spacing in presence and in absence of the applied electric field. It can be seen that in presence of applied electric field the gain is enhanced, and it decreases less abruptly with grating spacing when compared with no external field condition.

![Figure 2. Variation of gain with grating spacing in presence and in absence of the applied electric field.](image)

Figure 3 shows variation of the relative oscillation intensity with the grating spacing. It is seen that the relative oscillation intensity increases exponentially with grating spacing both in absence as well as in presence of the applied electric field. Saturation occurs for the value of grating spacing of 4 µm in presence of applied field compared to 2 µm under no field condition. It is also clear from the figure that the relative oscillation intensity decreases in the presence of applied electric field for all the values of grating spacing. It means that in the presence of applied electric field the required pump beam intensity for the working of the ring resonator is smaller as compared to that required in the absence of applied electric field.

4. Conclusion
It is concluded that an externally applied electric field improves the performance of the ring resonator for all values of grating spacing. It is also found that the performance of the ring resonator is better for smaller values of grating spacing.
5. References

[1] Feinberg J P, Heiman D, Tanguay A R, Hellwarth R 1980 *J. Appl. Phys.* **51** 1297–1305.

[2] Madsen C K and Zhao J H 1999 *Wiley-Interscience Publications*, New-York.

[3] Maurya M K, Yadav T K, Singh Ruchi, Yadav R A and Singh D P 2010 *Opt. Comm.* **283** 2416-2424.

[4] Solymar L, Webb D J, Grunnet-Jepsen A 1996 *Oxford*.

[5] Yeh P 1993 *New York* John Wiley.

[6] Vahala K J 2003 *Nature* **424**, 839.

[7] Maurya M K, Yadav R A 2010 *Opt. Laser Technol.* **42** 883–93.