GRAVITATION AND ELECTROMAGNETISM IN THEORY OF A UNIFIED FOUR-VECTOR FIELD

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A four-vector field in flat space-time, satisfying a gauge-invariant set of second-order differential equations, is considered as a unified field. The model variational principle corresponds to the general covariance idea and gives rise to nonlinear Born-Infeld electrodynamics. Thus the four-vector field is considered as an electromagnetic potential. It is suggested that space-localized (particle) solutions of the nonlinear field model correspond to material particles. Electromagnetic and gravitational interactions between field particles appear naturally when a many-particle solution is investigated with the help of a perturbation method. The electromagnetic interaction appears in the first order in the small field of distant particles. In the second order, there is an effective Riemannian space induced by the field of distant particles. This Riemannian space can be connected with gravitation.

1. Introduction

The problem of unification of all interactions of material particles is one of the most important problems in modern theoretical physics. Specifically, unification for the two known long-range interactions, viz., electromagnetism and gravitation, can be considered as a priority problem in theoretical investigation of nature.

The approach to this problem presented here is connected with a consistent application of the idea of nonlinear local unified field theory.

Spatially, localized solutions in this theory correspond to solitary material particles. Such solutions are said to be solitary or solitonic. Also the term “soliton” is used in this case. These solutions can be designated particle solutions. But the many-particle world configuration corresponds to a complicated many-particle (many-soliton) solution. The many-particle solution contains the appropriate particle solutions in the following sense.

Each particle solution has at least ten free parameters for space-time rotation and shift. Because of the nonlinearity, a sum of particle solutions is not a solution for the model. But we can consider the free parameters of particle solutions to be weakly time-dependent. This method is well known in the theory of nonlinear dynamics. A sum of particle solutions with time-dependent free parameters can be considered as an initial approximation to a many-particle solution. The time dependence of the free parameters of the particle solutions corresponds to the interaction between the particles.

This method, applied to a suitable model, must give electromagnetic and gravitational interactions for the case when the interacting particles are sufficiently distant from each other.

For the first time, this approach to the problem of unification of the gravitational and electromagnetic interactions appeared in the context of some nonlinear electrodynamics model [1]. Later on, the approach was developed for another nonlinear electrodynamics model (Born-Infeld) [2].

A distinguishing characteristic of this approach is that the gravitational interaction must appear through an effective Riemannian space for propagation of particle under consideration. This effective Riemannian space is induced by the electromagnetic field of distant particles.

2. Model field and equations

As mentioned above, the model field is considered to be electromagnetic, satisfying nonlinear equations. But the model to be considered is an unusual electrodynamics not only because the appropriate equations are nonlinear. This model does not contain the postulated trajectory equation for charged particle in external electromagnetic field, i.e., the electromagnetic interaction. This interaction appears naturally as a manifestation of the nonlinearity of the model.

Thus the model field can be called simply an antisymmetric space-time tensor field instead of the electromagnetic one. More precisely, the model field is the space-time vector field (four-vector potential) because it is this field that is varied in the model variational principle.

The variational principle considered here is similar to the one proposed by A.S. Eddington in the context of general relativity ideas [3] and investigated by A. Einstein in the context of his unified field theory [4]. Afterwards, M. Born and L. Infeld used it in the context of nonlinear electrodynamics [5, 6]. The variational prin-
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As we see, the induced metric $\tilde{g}^{\mu\nu}$ includes the energy-momentum tensor $T^{\mu\nu}$ defined in (6).

A calculation of the determinant for the effective metric (8) in Cartesian coordinates of flat space-time gives

$$\det \tilde{g}^{\mu\nu} = -1 \, .$$

Using (6), (9), we have the following condition for the effective metric in Cartesian coordinates $x^\mu$:

$$\frac{\partial \sqrt{|\tilde{g}|} \tilde{g}^{\mu\nu}}{\partial x^\rho} = 0 \, .$$

These equations differ from the corresponding equations of usual linear electrodynamics only by the substitution $g^{\mu\nu} \rightarrow \tilde{g}^{\mu\nu}$.

3. Electromagnetic interaction

The electromagnetic interaction appears in this model as an electromagnetic force acting on a massive charged particle (2) and a moment of force acting on a particle with an electric or magnetic dipole moment and spin (13). The appropriate dynamical equations follow from integral conservation laws for the field energy-momentum and angular momentum, respectively (for details see (2) (14)). These obtained equations, characterizing the electromagnetic interaction, have the corresponding classical form.

In this case, the mass of a particle appears as the full field energy of the appropriate particle solution in a proper coordinate system. The spin appears as the full angular momentum of the electromagnetic field for the particle solution in the proper coordinate system. There exist static electromagnetic field configurations with spin (2) (15).

The force and the moment of force contain the electromagnetic field of distant particles in the first power. Thus we can say that the electromagnetic interaction appear in the first order in the small field of distant particles.
4. Gravitational interaction

An explanation of the gravitational interaction in the scope of this model based on the effective Riemannian space with the metric $\tilde{g}^{\mu\nu}$ induced by the electromagnetic field. According to the general method stated in the Introduction, distant particles modify the propagation conditions for particle under consideration by means of this effective Riemannian space induced by the field of distant particles. The effective metric includes the electromagnetic field components in even powers. Thus we can say that the gravitational interaction appear in the second order in the small field of the distant particles.

The cause of the gravitational interaction in this approach is the energy density of the distant particle field. But to have the real behaviour of the gravitational potential, i.e. $1/r$, we must take into account a quick-oscillating part of the distant particle field with an electromagnetic wave background. In this case, an averaging can give the necessary behaviour of the energy density, $1/r$ (for some details see [10, 17]).

The gravitational constant in this approach is proportional to an amplitude of the wave background. But this amplitude can have different values for different regions of space. Consequently, the gravitational constant can be varied in this approach. In particular, the so-called dark matter effect can have this origin.

5. Conclusion

Thus the present approach, based on a consistent application of the idea of a nonlinear local unified four-vector field, can really unify electromagnetism and gravitation.

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