A time-varying wavelet phase extraction method using the wavelet amplitude spectra

Peng Zhang, Yong-shou Dai, Yong-cheng Tan, Hongqian Zhang and Chunxian Wang

ABSTRACT
The vertical resolution of seismic data is greatly affected by time-varying seismic wavelets, and the phase extraction is the key topic for the accurate wavelet extraction. The traditional methods are limited to the piecewise stationary and constant-phase assumptions, so that the extracted results are not satisfactory. In this paper, we try to explore a new time-varying wavelet phase extraction method using the wavelet amplitude information. The propagation and attenuation law of seismic wavelets is analyzed, and the relationship between the amplitude and phase spectra of time-varying wavelets is obtained by deriving the wave equation for a viscoelastic medium. We find that accurately estimating the amplitude spectra at different time points and the initial phase spectrum is necessary to extract accurate time-varying wavelet phase spectra. Based on this conclusion, the wavelet amplitude spectra and initial phase spectrum are estimated based on time–frequency spectral modelling and higher-order cumulants, respectively, so that the phase spectra of time-varying wavelets are estimated by applying the obtained relationship. The numerical simulation and real seismic data processing results demonstrate that the proposed method can improve the accuracy of time-varying wavelet phase extraction compared to the traditional method.

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Time-varying seismic wavelet; phase extraction; amplitude spectra; seismic wave equation

Introduction
Seismic wavelets are one of the most important factors that affect the vertical resolution of seismic data. In real seismic data, seismic wavelets are scattered from and absorbed by the underground medium during propagation, resulting in energy attenuation and causing the phase distortion of the wavelets. Thus, seismic wavelets vary with time, which greatly affects the accuracy of seismic exploration (Margrave, 1998; Margrave & Lamoureux, 2001). However, time-varying wavelet phase spectra are hard to estimate accurately in the current methods, which is an urgent problem to be solved. Some researchers carried out a series of research work for the extraction and correction of time-varying wavelet phase spectra to improve the accuracy of seismic data processing. Since Wang (2002, 2006, 2008) proposed for the first time a stabilized amplitude compensation in the inverse Q filter, the full inverse Q filter which simultaneously preforming amplitude compensation and phase correction has been widely used (Wang & Chen, 2014; Wang, Song, & Yang, 2014; Zhang et al., 2014). But the method depends heavily on the Q, which is often not simple to derive an exact Q-model from the seismic data. One common approach is to divide a non-stationary seismic trace into several segments using multiple overlapped windows. van der Baan (2008, 2012) proposed a time-varying wavelet estimation method by kurtosis maximization, which decomposed a non-stationary seismogram into several overlapping segments of equal length, in which the subsection wavelet was extracted and elongated under the constant-phase assumption. Economou and Vafidis (2010, 2012) compensated for seismic attenuation and correct phase distortion by using piecewise-extracted seismic wavelets. Wang, Gao, and Zhang (2010, 2012) proposed an adaptive molecular decomposition method, which adaptively decomposes a non-stationary seismogram according to the characteristics of the seismogram without artificially determining the length and number of segments, thus improving the segmentation extraction method. van der Baan and Fomel (2009) estimated time-varying wavelet phase spectra by using local kurtosis criterion and constant phase rotation in the subsequent study, which overcomes the piecewise smooth hypothesis, but is still restricted to the constant phase assumption. From what has been discussed above, the existing extraction methods of time-varying wavelet phase spectra are implemented under the assumption of constant phase, while the seismic wavelet is usually mixed-phase in the real
seismic data, so that the estimation results are not accurate.

The propagation law of seismic waves in viscoelastic media is analyzed in this paper to further improve the extraction accuracy of time-varying wavelet phase spectra. Additionally, the relationship between time-varying wavelet amplitude spectra and phase spectra is summarized and derived from the wave equation in a viscoelastic medium, with a focus on post-stack seismic data. Then, the time-varying wavelet phase spectra can be extracted through the following steps. First, the amplitude spectra of time-varying wavelets at each sample point. The theoretical analysis, numerical simulation and real seismic data processing results demonstrate that the extracted time-varying wavelet phase spectra from the proposed method are more accurate than the conventional method.

**Method**

**Relationship between wavelet amplitude and phase spectra in a viscoelastic medium**

The seismic wave equation for a viscoelastic medium can be expressed in the frequency domain as follows (Wang, 2006; Wang & Chen, 2014):

\[
U(z + \Delta z, \omega) = U(z, \omega) \exp[-j(k(\omega) \Delta z)],
\]

(1)

where \( U \) is the Fourier spectrum of the seismic wave; \( z \) is the propagation distance; \( \Delta z \) is the depth increment; \( \omega \) is the angular frequency, and \( \omega \geq 0 \); \( j \) is the imaginary symbol; and \( k(\omega) \) is the wavenumber. When the seismic wave propagates in inhomogeneous media, \( k(\omega) \) must be a complex value that includes not only an imaginary part, the frequency-dependent attenuation coefficient, but also a real part, the dispersive wavenumber (Wang, 2008):

\[
k(\omega) = \frac{\omega}{v(\omega)} - j\alpha(\omega),
\]

(2)

\[
\alpha(\omega) = \frac{\omega}{2Qv(\omega)},
\]

(3)

where \( \alpha(\omega) \) is the attenuation coefficient, which can be expressed in terms of the angular frequency \( \omega \), phase velocity \( v(\omega) \) and quality factor \( Q \). According to Kolsky’s model, the phase velocity \( v(\omega) \) is given by (Aki & Richards, 1980):

\[
\frac{1}{v(\omega)} = \frac{1}{v_r} \left( 1 - \frac{1}{\pi Q} \ln \left| \frac{\omega}{\omega_r} \right| \right),
\]

(4)

where \( v_r \) and \( \omega_r (\omega_r = 2\pi f_r) \) are reference phase velocity and frequency. Wang (2008) concludes by recommending setting \( f_r \) at a large value (e.g., 500 Hz). After applying equations (2), (3) and (4) to equation (1) and neglecting an argument proportional to \( 1/Q^2 \) (Braga & Moraes, 2013), which is small for the usual range of \( Q \) values used in exploration (say, 20–200 or higher), the frequency-domain expression of the seismic wavelet in the \( z \) direction is as follows:

\[
U(z + \Delta z, \omega) = U(z, \omega) \exp \left[ -\frac{j\omega \Delta z}{v_r} \right.
\]

\[
+ \frac{j\omega \Delta z}{\pi Qv_r} \ln \left| \frac{\omega}{\omega_r} \right| - \frac{\omega \Delta z}{2Qv_r}. \]

(5)

Then, equation (5) can be transformed from the depth domain to the time domain and the travel time is defined by \( \Delta t = \Delta z/v_r \), so that equation (5) can be expressed in the time domain as follows:

\[
U(t + \Delta t, \omega) = U(t, \omega) \exp \left[ -j\omega \Delta t \right.
\]

\[
+ \frac{j\omega \Delta t}{\pi Q} \ln \left| \frac{\omega}{\omega_r} \right| - \frac{\omega \Delta t}{2Q}. \]

(6)

and the amplitude spectrum \( A \) and phase spectrum \( \phi \) of the seismic wavelet in the time domain are given by equations (7) and (8), respectively:

\[
A(t + \Delta t, \omega) = A(t, \omega) \exp(-\frac{\omega \Delta t}{2Q}),
\]

(7)

\[
\phi(z + \Delta z, \omega) = \phi(z, \omega) - \omega \Delta t + \frac{\omega \Delta t}{\pi Q} \ln \left| \frac{\omega}{\omega_r} \right|. \]

(8)

And then, equations (7) and (8) can be written as equations (9) and (10) respectively:

\[
2 \ln A(t + \Delta t, \omega) - 2 \ln A(t, \omega) = -\frac{\omega \Delta t}{Q},
\]

(9)

\[
\frac{\pi}{\ln(\omega/\omega_r)} [\phi(t, \omega) - \phi(t + \Delta t, \omega) - \omega \Delta t] = -\frac{\omega \Delta t}{Q}, \]

(10)

where \( \ln A(t + \Delta t, \omega) \) and \( \ln A(t, \omega) \) describe the logarithmic amplitude spectra at different time points, and \( \phi(t + \Delta t, \omega) \) and \( \phi(t, \omega) \) are the phase spectra at different time points. We define \( -\omega \Delta t/Q \) as the seismic attenuation factor, and according to equations (9) and (10), the amplitudes and phases are related to ‘attenuation’, a medium property. Thus, if we regard the seismic attenuation factor as a bridge, then the relationship between the
amplitude and phase spectra of time-varying wavelets can be obtained and expressed as follows:

\[ 2 \ln A(t + \Delta t, \omega) - 2 \ln A(t, \omega) = \frac{\pi}{\ln(\omega/\omega_0)} [\varphi(t, \omega) - \varphi(t + \Delta t, \omega) - \omega \Delta t]. \quad (11) \]

where \( A(t + \Delta t, \omega) \) and \( A(t, \omega) \) are the unknown variables that require estimation. Equation (11) indicates that accurately estimating the amplitude spectra \( A(t + \Delta t, \omega) \) and \( A(t, \omega) \) at different sample points and the initial phase spectrum \( \varphi(t, \omega) \) is necessary to extract accurate phase spectra of wavelets at each sample point such that time-varying wavelet phase extraction can be achieved. The derived relationship between wavelet amplitude and phase spectra is based on the Kolsky-Futterman model and can be applied when the value of \( Q \) is constant or piecewise constant with time.

**The extraction method of time-varying wavelet phase spectra**

The non-stationary convolutional model of a seismic trace is often defined as follows (Margrave, 1998; Margrave, Lamoureux, & Henley, 2011; Wang & Chen, 2014):

\[ x(t) = \int_{-\infty}^{\infty} w(t, \tau) r(\tau) d\tau, \quad (12) \]

where \( x(t) \) is the non-stationary seismogram, \( r(t) \) is the reflectivity, and \( w(t, \tau) \) is the function that describes the time-varying wavelet at time \( \tau \), and \( w(t, \tau) \) can be obtained by:

\[ w(t, \tau) = \int_{-\infty}^{\infty} \hat{w}(\omega) \exp \left(-j\omega t - j\frac{\omega \tau}{2Q(\tau)} \right) e^{j\omega \tau} d\omega, \quad (13) \]

where \( \hat{w}(\omega) \) is the Fourier spectrum of stationary seismic wavelet.

First, time-varying wavelet amplitude spectra are estimated by using the time–frequency spectral modelling method, the theory of which was described in the literature of Zhou et al. (2014) and Dai, Wang, Li, Zhang, and Tan (2016), and that is to say \( A(t + \Delta t, \omega) \) and \( A(t, \omega) \) in equation (11) have been extracted.

Then, the initial phase spectrum \( \varphi(t, \omega) \) must be estimated so that the time-varying wavelet phase spectra at each sample point can be extracted with equation (11).

Higher-order cumulant and higher-order spectrum that contain the complete information of the signal are introduced into the mixed-phase wavelet extraction. The initial phase spectrum of the wavelet can be estimated from the bispectrum of the seismogram according to the nature of the higher-order cumulant. Many phase reconstruction methods exist, and we use the Matsuoka-Ulrych algorithm (Matsuoka & Ulrych, 1984) in this paper.

The bispectrum is the Fourier transform of the third-order cumulant of the signal. The third-order cumulants of the seismogram can be expressed as follows:

\[ c_x(t_1, t_2) = E[x(t)x(t + t_1)x(t + t_2)], \quad (14) \]

where the symbol \( E \) represents the mathematical expectation. Then, \( c_x(t_1, t_2) \) is processed by Fourier transformation, and the bispectrum of the seismogram \( B_x(\omega_1, \omega_2) \) can be expressed as follows:

\[ B_x(\omega_1, \omega_2) = \sum_{t_1=-\infty}^{\infty} \sum_{t_2=-\infty}^{\infty} c_x(t_1, t_2) e^{-j(\omega_1 t_1 + \omega_2 t_2)}. \quad (15) \]

\( B_x(\omega_1, \omega_2) \) can also be described by equation (16) according to the relationship among the seismogram, the reflection coefficient sequence and the seismic wavelet:

\[ B_x(\omega_1, \omega_2) = B_r(\omega_1, \omega_2) B_w(\omega_1, \omega_2), \quad (16) \]

where \( B_r(\omega_1, \omega_2) \) and \( B_w(\omega_1, \omega_2) \) are the bispectra of the reflection coefficient sequence and the seismic wavelet, respectively. According to the literature (Matsuoka & Ulrych, 1984), \( B_r(\omega_1, \omega_2) = E[r^3(t)] = C \neq 0 \), so equation (16) can also be expressed as follows:

\[ |B_x(\omega_1, \omega_2)| \exp[i\psi_x(\omega_1, \omega_2)] = C \cdot |W(\omega_1)W(\omega_2)W(-\omega_1 + \omega_2)|, \quad (17) \]

where \( |B_x(\omega_1, \omega_2)| \) and \( \psi_x(\omega_1, \omega_2) \) are the amplitude and phase spectra of the bispectrum of the seismogram, respectively, and \( W(\omega) = |W(\omega)| \exp(i\varphi(\omega)) \), where \( W(\omega) \) and \( |W(\omega)| \) and \( \varphi(\omega) \) are the frequency spectrum, the amplitude spectrum and the phase spectrum of the seismic wavelet, respectively. Thus, the relationship between the phase spectrum of the bispectrum of the seismogram and the phase spectrum of the seismic wavelet can be expressed as follows:

\[ \psi(\omega_1, \omega_2) = \varphi(\omega_1) + \varphi(\omega_2) - \varphi(\omega_1 + \omega_2). \quad (18) \]

By using equation (18), the initial phase spectrum of the wavelet can be estimated from the bispectrum of the seismogram.

The estimated amplitude spectra and initial phase spectrum are input into equation (11) such that the phase spectra of the time-varying wavelets can be extracted and merged at each sample point. And then, to improve the resolution of the seismogram, the phase correction can be implemented using the estimated phase spectra:

\[ X_0(t, \omega) = X(t, \omega) \cdot e^{-j\varphi_0(t, \omega)}, \quad (19) \]

where \( \varphi_0(t, \omega) \) is the extracted phase spectra of the time-varying wavelets, and \( X(t, \omega) \) and \( X_0(t, \omega) \) are the time–frequency spectra of the seismogram before and after the phase correction respectively.
Results

Numerical simulation and analysis of the results

The original mixed-phase wavelet that was used in this experiment can be expressed with the following autoregressive–moving-average (ARMA) formula:

\[
x(t) - 3.45x(t - 1) + 5.03x(t - 2) - 3.49x(t - 3) + 1.05x(t - 4) = r(t) - 0.8r(t - 1) + 0.6r(t - 2) - 1.2r(t - 3).
\]

(20)

The equivalent of this formula in the z domain is

\[
W(z) = \frac{1 - 0.8z^{-1} + 0.6z^{-2} - 1.2z^{-3}}{1 - 3.45z^{-1} + 5.03z^{-2} - 3.49z^{-3} + 1.05z^{-4}}.
\]

(21)

Figure 1(a) shows the original mixed-phase wavelet, and Figure 1(b) presents the corresponding reflectivity sequence, which satisfies the assumptions of independence and identical distribution (IID) and follows a Bernoulli-Gaussian distribution. The sampling interval is 1 ms. Figure 1(c) shows a non-stationary seismogram that was synthesized with this time-varying wavelet and

![Figure 1.](image1.png)

![Figure 2.](image2.png)
Figure 3. Results of time-varying wavelet phase spectra extraction. (a) Theoretical results (111, 381, 619, 892 ms); (b) extracted results using the kurtosis maximization method (111, 381, 619, 892 ms); (c) extracted results using the proposed method (111, 381, 619, 892 ms).

reflectivity sequence based on the non-stationary convolutional model in equation (12).

The amplitude spectra of time-varying wavelets are extracted at different time points from the seismogram in Figure 1(c) by applying the time–frequency spectral modelling method. Figure 2 compares the extracted amplitude spectra to the theoretical amplitude spectra at 111, 381, 619 and 892 ms. As shown in Figure 2, the extracted results are consistent with the theoretical values.

A 200-ms time window is used in Figure 1(c), and the initial phase spectrum \( \phi(t, \omega) \) is extracted in the window based on the higher-order cumulants method. Then, the estimated \( \hat{\phi}(t, \omega) \) and the extracted amplitude spectra are input into equation (11) so that the time-varying wavelet phase spectra at each sample point are extracted. Figure 3 shows the extraction results of time-varying wavelet phase spectra, and the extraction errors are shown in Figure 4. Figures 3 and 4 indicate that the accuracy of the extracted wavelet phase spectra from the proposed method is higher than that from the conventional kurtosis maximization method, and the simulation results are shown in Figures 5 and 6, and we can see that the time-varying seismic wavelets extracted by the proposed method is closer to the theoretical wavelets.

Then, the non-stationary seismogram shown in Figure 1(c) is processed respectively by using the time-varying wavelet phase spectra extracted by the kurtosis maximization method and the proposed method, and the results are shown in Figure 7. By comparing the seismic data in the rectangles in Figure 7(b–d), we can see that the seismogram after phase correction based on the proposed method is closer to zero-phase than that based on the conventional method, and it is proved that our method is effective and accurate, and is of great significance to improve the resolution of seismic data.

Analysis of real seismic data

A post-stack seismic section of an oil field is researched at a sampling interval of 1 ms. To verify the effectiveness and accuracy of the proposed method, reflection coefficient sequence inversion and multi-trace deconvolution are conducted respectively, and the results are shown in Figures 8 and 9. From Figures 8 and 9, we
Figure 4. Errors of the extracted time-varying wavelet phase spectra. (a) extraction errors of the kurtosis maximization method (111, 381, 619, 892 ms); (b) extraction errors of the proposed method (111, 381, 619, 892 ms).

Figure 5. Time-varying wavelet extraction results. (a) Theoretical wavelets (111, 381, 619, 892 ms); (b) extracted wavelets using the conventional method (111, 381, 619, 892 ms); (c) extracted wavelet using the proposed method (111, 381, 619, 892 ms).
Figure 6. Errors of the extracted time-varying wavelets. (a) extraction errors of the conventional method (111, 381, 619, 892 ms); (b) extraction errors of the proposed method (111, 381, 619, 892 ms).

Figure 7. Results after non-stationary phase correction. (a) Reflection coefficient sequence; (b) original non-stationary seismogram in Figure 1(c); (c) seismogram after phase correction by using the kurtosis maximization method; (d) seismogram after phase correction by using the proposed method.
can see that compared with the conventional method, the inverted reflection coefficient sequence using the proposed method is closer to that from the well log, and the resolution of the seismic data after deconvolution with the proposed method is higher than that after deconvolution with the conventional kurtosis maximization method. Thus, the proposed time-varying wavelet phase extraction method can be effectively applied for real seismic data processing.

**Conclusions**

The propagation and attenuation law of seismic wavelets was analyzed in this paper, and the relationship between amplitude and phase spectra of time-varying wavelets was obtained by deriving the wave equation for viscoelastic media. Then, a time-varying wavelet phase extraction method that utilizes the obtained relationship was proposed. The following conclusions can be drawn based on
theoretical analysis, numerical simulation and real seismic data processing:

(1) Accurately estimating the amplitude spectra at different sample points and the initial phase spectrum is necessary to accurately extract the phase spectra of time-varying wavelets.

(2) The proposed time-varying wavelet phase extraction method using the wavelet amplitude spectra, is more effective and feasible than the conventional kurtosis maximization method to improve the resolution of the seismic data, which proves that the proposed method has important practical application value.

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