EXACT RESULTS IN SOFTLY BROKEN SUPERSYMMETRIC MODELS

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Abstract

We show how recent exact results in supersymmetric theories can be extended to models which include explicit soft supersymmetry breaking terms. We thus derive new exact results for non-supersymmetric models.

1 Introduction

The superpotential in the Wilsonian effective Lagrangian of a supersymmetric (SUSY) theory is holomorphic in the fields and invariant under any gauged or global symmetries of the theory [1, 2]. Recently Seiberg [3] has used these properties to prove non-perturbative non-renormalization theorems in SUSY theories and to explicitly explore the vacuum structure of strongly interacting SUSY gauge theories. One of his important tools is to embed the theory under study in a larger SUSY model in which the couplings of the original theory are treated as chiral superfields (i.e. supersymmetric spurions). The coefficients of the kinetic terms for these chiral ‘coupling superfields’ are later taken to infinity, so that they become constant, non-propagating, background fields. However, the superpotential of this larger theory must still be holomorphic in the coupling fields. By imposing a non-zero vacuum expectation
value (vev) for the lowest, or scalar, component of the coupling fields the embedded theory is recovered.

It is known that adding soft breaking terms to SUSY theories does not destroy the perturbative non-renormalization theorems [4]. That is, soft breaking does not induce any power divergences and leads only to logarithmic corrections to the superpotential. In this paper we show that these non-renormalization theorems continue to be valid non-perturbatively as well. Our method is, similarly, to embed the theory we wish to study in a larger SUSY theory in which again couplings are treated as chiral superfields. However, now we couple each of the coupling superfields to an enlarged Wess-Zumino (WZ) sector which is an O’Raifeartaigh model [5]. The superpotential is again determined exactly by the symmetries of the theory and holomorphy. The O’Raifeartaigh sectors generate non-zero expectation values for the $F$-terms of the coupling fields as well as the scalar components. Then supersymmetry is spontaneously broken and soft SUSY breaking terms are generated in the embedded theory. However, because the larger theory is exactly supersymmetric, with SUSY only broken spontaneously, the holomorphic properties of the superpotential remain intact and lead to exact results.

We will thus derive exact results for the Wess-Zumino model and for SUSY gauge theories in the presence of all possible soft SUSY breaking terms. Among these soft SUSY breaking terms are ones which may be used to give large masses to the gauginos (e.g. gluinos and photino) and the scalar components of matter superfields (e.g. squarks and sleptons). This will allow us to embed QED and QCD in larger SUSY gauge models and then to subsequently decouple all of the superpartners without changing the holomorphic properties of the potential. We anticipate therefore that our methods will be useful in investigating whether Seiberg’s results in SUSY QCD (SQCD) will be preserved in QCD. Work in this direction is in progress.
2 The Wess-Zumino Model

2.1 Non-Renormalization theory: one flavor WZ Model

We begin by reviewing Seiberg’s method in the Wess-Zumino model.

\[ L_0 = \Phi^\dagger \Phi|_D + m\Phi^2|_F + g\Phi^3|_F \]  

(2.1)

where we have neglected the term with a single power of \( \Phi \) since it may be removed by a shift in the field. The model may be embedded in a larger theory where \( m \) and \( g \) are treated as fields:

\[ L = L_0 + \Lambda_m m^\dagger m|_D + \Lambda_g g^\dagger g|_D \]  

(2.2)

where \( \Lambda_m \) and \( \Lambda_g \) are some scales. The Wess-Zumino model is recovered in the limit where the coupling fields \( c = m, g \) are given vacuum expectation values (vevs) in their scalar components \( A_c \) and then \( \Lambda_c \to \infty \). In this limit the kinetic terms of the coupling fields have infinite coefficients. Hence any contribution to the functional integral with field variation is suppressed, and fluctuations of the coupling fields can be neglected. Furthermore, the vev of the scalar fields may be set to any value by appropriate choice of a source term. Once the source is turned off, the vev will remain fixed even if it does not minimize the potential energy. This is easily seen by examining the equation of motion for the scalar components of the coupling fields, which have the form

\[ \partial^2 A_c \sim \frac{1}{\Lambda_c} \frac{\partial U}{\partial A_c}, \]  

(2.3)

where \( U \) is the scalar potential.

The enlarged model has a global \( U(1) \otimes U(1)_R \) symmetry under which the fields transform as

\[
\begin{array}{c|c|c}
 & U(1) & U(1)_R \\
\hline
\Phi & 1 & 1 \\
m & -2 & 0 \\
g & -3 & -1 \\
\end{array}
\]  

(2.4)

\(^*\)We use the conventions and notation of Wess and Bagger throughout this paper.
Holomorphy and these U(1) symmetries restrict the superpotential of the Wilsonian effective Lagrangian to the form

$$W_{\text{eff}} = m\Phi^2 \ f \left( \frac{g\Phi}{m} \right)$$

(2.5)

where $f$ is any holomorphic function. The effective superpotential receives contributions from perturbative 1PI loop diagrams. It is easy to see that all such diagrams are incompatible with the form of (2.5). For sufficiently small coupling $g$, any remaining non-perturbative contributions to $W_{\text{eff}}$ must approach zero faster than $g^N$ for any $N$. We assume that they do so uniformly for all $\Phi$ – in other words the non-perturbative corrections should be non-singular for all field values at sufficiently weak coupling. This assumption rules out any corrections, e.g., of the form $e^{-1/g}$ since to conform with (2.5) they must actually be of the form $e^{-m/g\Phi}$ and hence are singular for $\Phi$ small and negative. We conclude there are no corrections compatible with the form of (2.5) and therefore the superpotential is not renormalized:

$$W_{\text{eff}} = m\Phi^2 + g\Phi^3 = W_{\text{tree}}.$$  \hspace{1cm} (2.6)

Note that one could also add to the Lagrangian (2.2) a term

$$\delta L = (K^\dagger K \ \Phi^\dagger \Phi)|_D + \Lambda_K K^\dagger K|_D.$$  \hspace{1cm} (2.7)

A non-zero vev of the scalar component of $K$ would change the normalization of the $\Phi$ kinetic term, thus we set it to zero. However a term like (2.7) will be useful to us in what follows. Note that $K$ carries arbitrary $U(1)$ charges. If $K$ were to appear in the superpotential the number of powers of $K$ would be dictated by its $U(1)$ charges. However, the theory must be invariant under the arbitrary assignment of charge, and the only function of $K$ which has this property is the constant function. Hence there is no $K$ dependence in the superpotential. The $D$-terms involving $K$ will of course in general be renormalized by an arbitrary, non-holomorphic function of the fields in the model (wavefunction renormalization).
It is important to note that the relationship between the superpotential and the potential of any model depends on the coefficients of the $D$-terms. This is because the auxiliary fields $F$ must be eliminated in order to obtain the potential as a function of propagating fields only. Since the coefficient functions of the $D$-terms are not holomorphic in the fields, this means that an exact result for the superpotential does not necessarily yield an exact result for the potential.

### 2.2 N flavor WZ model

The arguments of section 2.1 may be extended to an $N$ flavor WZ model with Lagrangian

$$L_0 = \sum_i (K_i^\dagger K_i + 1)\Phi_i^\dagger \Phi_i |_D + \lambda_i \Phi_i |_F + m_{ij} \Phi_i \Phi_j |_F + g_{ijk} \Phi_i \Phi_j \Phi_k |_F, \quad (2.8)$$

where $i, j, k = 1, \ldots, N$. We may again embed the model in a larger Lagrangian in which the couplings are themselves chiral superfields with infinite $D$-terms. The theory now has $N$ global $U(1)$ symmetries in addition to $U(1)_R$. The fields transform as

\[
\begin{array}{ccc}
\Phi_i & U(1)_k & U(1)_R \\
\lambda_i & \delta_{ik} & 1 \\
m_{ij} & -\delta_{ik} & 1 \\
g_{ij} & -(\delta_{ik} + \delta_{jk}) & 0 \\
g_{ij} & -(\delta_{ik} + \delta_{jk} + \delta_{lk}) & -1
\end{array}
\]

(2.9)

The effective superpotential is now constrained to the form

$$W_{eff} = m_{ij} \Phi_i \Phi_j \ f \left( \frac{\lambda_a \Phi_a}{m_{ij} \Phi_i \Phi_j} \ ; \ \frac{g_{abc} \Phi_a \Phi_b \Phi_c}{m_{ij} \Phi_i \Phi_j} \right). \quad (2.10)$$

The superpotential is independent of $K$ since $K$ has arbitrary $U(1)_R$ charge. Using similar arguments as for the one flavor model we see that the superpotential is again not renormalized and is exact even at the non-perturbative level.
2.3 Soft SUSY breaking terms in the WZ model

Now we will generalize the previous analysis to allow for soft breaking of supersymmetry. Consider the following model which is a subclass of the $N$ flavor WZ model discussed in section 2.2

\[ L = (K_0^+ K_0 + 1)\Phi^\dagger \Phi |_D + m_0 \Phi^2 |_F + g_0 \Phi^3 |_F \]
\[ + \Lambda_m (m_i^\dagger m_i |_D + \alpha_m m_0 |_F + \beta_m m_1 m_2 |_F + \gamma_m m_2^2 m_0 |_F) \]
\[ + \Lambda_g (g_i^\dagger g_i |_D + \alpha_g g_0 |_F + \beta_g g_1 g_2 |_F + \gamma_g g_1^2 g_0 |_F) \]
\[ + \Lambda_K (K_i^\dagger K_i |_D + \alpha_K K_0 |_F + \beta_K K_1 K_2 |_F + \gamma_K K_1^2 K_0 |_F) \]
\[ + h.c. \] (2.11)

where $m_i, g_i, K_i$ are superfields with $i = 0, 1, 2$ and the couplings $\alpha, \beta$ and $\gamma$ are complex. Since this model is a specific example of the $N$ flavor WZ model we know that its superpotential is not renormalized. The potentials for $m_i, g_i$ and $K_i$ are simply those of the O’Raifeartaigh model \[3\] and can be minimized by

\[ \langle F_{c_0} \rangle = \alpha_c^* \]
\[ \langle A_{c_0} \rangle \neq 0 \]
\[ \langle A_{c_i} \rangle = 0 \quad i = 1, 2 \] (2.12)

where the subscript $c_i \equiv m_i, g_i, K_i$. The $\langle F \rangle$ vevs spontaneously break SUSY. The potential is flat at $O(\Lambda_c)$ in $A_{m_0}, A_{g_0}$ and $A_{K_0}$ so we may choose these vevs arbitrarily when we take the limit $\Lambda_c \to \infty$. We shall choose these vevs equal to the desired one flavor WZ couplings and also take $\langle A_{K_0} \rangle = 0$.

After taking the limit of all $\Lambda_c$ to infinity, we obtain the following $F$-terms in the effective

\[ \text{\textsuperscript{1}We can actually eliminate all of the } c_i, i = 1, 2 \text{ fields from (2.11) and still have spontaneous SUSY breaking.} \]
theory
\[
\left[ \langle A_{m_0} \rangle \Phi^2 |_F + \langle A_{g_0} \rangle \Phi^3 |_F + h.c. \right] + 2 \text{Re}(\langle F_{m_0} \rangle A^2 + \langle F_{g_0} \rangle A^3) \quad (2.13)
\]

where \( A \) is the scalar component of the chiral field \( \Phi \).

Again, since the Langrangian we started with in (2.11) was exactly supersymmetric, and of the form (2.8), the terms in (2.13) are not renormalized – they are \textit{exact}. While (2.13) appears to violate SUSY explicitly, albeit softly, we actually obtained it as the limit of a model in which SUSY is only spontaneously broken. Hence the persistence of the exact result.

The \( D \)-term in the bare Lagrangian now generates a soft SUSY breaking term
\[
\langle F_{K_0}^* F_{K_0} \rangle A^* A \ . \quad (2.14)
\]

While we cannot specify the exact renormalized \( D \)-term, it is highly plausible that if \( \langle F_{K_0}^* F_{K_0} \rangle \) is large enough this breaking term will persist in the effective theory.

We have induced the following soft SUSY breaking terms (writing \( A = a + ib \)):
\[
\begin{align*}
\langle F_{K_0}^* F_{K_0} \rangle & (a^2 + b^2) \\
\text{Re}\langle F_{M_0} \rangle & (a^2 - b^2) \\
\text{Im}\langle F_{M_0} \rangle & 2ab \\
\text{Re}\langle F_{g_0} \rangle & (a^3 - 3ab^2) \\
\text{Im}\langle F_{g_0} \rangle & (b^3 - 3a^2b) .
\end{align*}
\quad (2.15)
\]

We note that these exhaust the types of soft SUSY breaking terms which are consistent with the perturbative non-renormalization result \cite{4}. Each term in (2.13) is exact, with the exception of the first, which arises from a \( D \)-term.

\footnote{In \cite{4} the imaginary terms were not obtained because only real couplings were considered.}
Allowing vevs of the $F$ components of the coupling constants can lead to additional $D$-terms which are holomorphic in $\phi$ and hence mimic terms in the softly broken WZ model. Specifically, terms of the form

$$ f(g^\dagger g)(g\phi m^\dagger)^l |_D $$

with $l \geq 1$ will appear in the effective Lagrangian. These terms are consistent with the $U(1)$ symmetries and lead to $F$-terms in $\phi$ when $\langle F_{g,m} \rangle \neq 0$. To arrive at the form (2.16), we have eliminated terms which are singular as $m$ or $\phi \to 0$, or which can become singular at weak coupling (e.g. $e^{-m/g\phi}$). The terms remaining in (2.16) can be seen to receive contributions within perturbation theory from specific supergraphs, and may also receive non-perturbative contributions of the form $e^{-1/(g^\dagger g)}$. Thus we cannot exactly determine the functions $f(g^\dagger g)$. However, simple power counting tells us that the term linear in $\phi$ has at worst logarithmically divergent contributions and that higher order terms have finite contributions. Therefore these additional terms do not spoil the non-renormalization results of [3].

3 SUSY QCD

Soft SUSY breaking terms may also be introduced into SQCD. The SQCD Lagrangian is

$$ L = \frac{1}{4} (\tau W^{a\dot{a}} W_{a\dot{a}} |_F + \tau^{\dagger} W_{a\dot{a}} W^{a\dot{a}} |_F) 
+ (1 + K^{\dagger} K) Q^{\dagger} e^V Q |_D 
+ (1 + \tilde{K}^{\dagger} \tilde{K}) \bar{\tilde{Q}}^{\dagger} e^{-V} \tilde{Q} |_D 
+ m \tilde{Q} Q |_F + m^{\dagger} Q^{\dagger} \tilde{\tilde{Q}}^{\dagger} |_F. $$

(3.1)

Following our procedure in the WZ model we promote $\tau$, $m$, $K$ and $\tilde{K}$ to the status of chiral superfields and then couple each field to an O’Raifeartaigh model allowing the $F$ and scalar components of each field to acquire a non-zero vev. After fixing the coupling constant fields we obtain the SQCD Lagrangian plus the SUSY breaking terms.
\[ \Delta L = \langle F_K^*F_K \rangle \mid A_Q \mid^2 + \langle \tilde{F}_K^*\tilde{F}_K \rangle \mid \tilde{A}_Q \mid^2 + 2\text{Re}(\langle F_m \rangle \tilde{A}_Q A_Q) \\
+ \frac{1}{4} \langle F_\tau \rangle \lambda^\alpha \lambda_\alpha + \frac{1}{4} \langle F_\tau^* \rangle \bar{\lambda}_\dot{\alpha} \bar{\lambda}_{\dot{\alpha}}. \] (3.2)

Writing the squark fields as \((a_Q + ib_Q)\) we explicitly induce the soft SUSY breaking parameters

\[ \langle F_K^*F_K \rangle (a_Q^2 + b_Q^2) \]
\[ \langle \tilde{F}_K^*\tilde{F}_K \rangle (\tilde{a}_Q^2 + \tilde{b}_Q^2) \]
\[ \langle F_\tau \rangle \lambda^\alpha \lambda_\alpha + \langle F_\tau^* \rangle \bar{\lambda}_{\dot{\alpha}} \bar{\lambda}_{\dot{\alpha}} \] (3.3)

which have the effect of giving masses to the gaugino and squark fields.

The model possesses an anomaly free global \(SU(N_f)_L \otimes SU(N_f)_R \otimes U(1)_V \otimes U(1)_R\) symmetry where the fields transform under the \(U(1)_R\) group as

\[ \begin{array}{ccc}
W & 1 \\
\tau & 0 \\
Q & (N_f - N_c)/N_f \\
\tilde{Q} & (N_f - N_c)/N_f \\
m & 2N_c/N_f \\
\lambda & \text{arbitrary}
\end{array} \] (3.4)

As a simple example, consider the case of \(N_c > N_f\). The effective superpotential (or more precisely, the \(F\)-term part of \(L_{\text{eff}}\)) is therefore determined by the symmetries to be of the form

\[ W_{\text{eff}} = f \left( \tau, \frac{\det(\tilde{Q}Q)^{1/(N_f - N_c)}}{W^\alpha W_\alpha}, m^{N_f(N_c - N_f)/N_c} \det(\tilde{Q}Q) \right) W^\beta W_\beta + \text{h.c.} \] (3.5)

(If \(N_f \geq N_c\) then objects other than those in (3.5) can appear \([7]\).) Again the superpotential is not dependent on \(K\) since it has arbitrary charge. The contribution to the potential at lowest order in a derivative expansion is the term reported in \([3]\):

\[ \Lambda^{(3N_c-N_f)/(N_c-N_f)} \frac{\det(\tilde{Q}Q)^{1/(N_c-N_f)}}{\det(\tilde{Q}Q)^{1/(N_c-N_f)}}. \] (3.6)
The dimensional factor $\Lambda$ must be proportional to $\Lambda_{SQCD}$, which is the only scale in the problem other than $m$, which has $R$ charge. $\Lambda_{SQCD}$ can be related to the gauge coupling constant: $\Lambda_{SQCD} \sim \mu \exp(-\langle A_\tau \rangle / b)$, where $b$ is determined by the $\beta$-function. The general function of $\tau$ which appears in $f_{\alpha}$ is then determined to be $\exp(-\tau/b)$, and can be straightforwardly evaluated when $\langle F_\tau \rangle \neq 0$. The functional form of the superpotential is unchanged when we allow the $F$-terms of $M$, $K$ and $\tau$ to have non-zero vevs and therefore we expect many of Seiberg’s results in SQCD to apply in theories with soft SUSY breaking terms.

As in the case with the WZ model, additional terms which are holomorphic in the physical (non-coupling constant) fields can arise from soft breaking. In particular, the following $D$-terms are allowed by the symmetries (we consider the massless case $m = 0$):

$$f (\tau, \tau^\dagger, \det(\tilde{Q}Q)^{1/(N_c-N_f)} W^\alpha W_\alpha) \big|_D. \quad (3.7)$$

When $\langle f_\tau \rangle \neq 0$, this leads to additional superpotential terms. Terms of the form (3.7) are not generated within perturbation theory but may receive nonperturbative contributions. They are qualitatively similar to terms already present in (3.5) which couple the quark and gauge superfields.

We again note that, due to the unknown renormalization of the $D$-term, the potential is not exactly determined, even when the superpotential is. In the limit where the theory is weakly coupled, the renormalization of the $D$-terms can be computed within perturbation theory, and hence the potential determined (albeit not exactly) from the superpotential. However, if the model is strongly coupled the coefficient functions of the $D$-terms could deviate considerably from their classical values and hence drastically alter the relation between the potential and the superpotential. Thus it does not appear that any exact results can be obtained regarding the potential, despite the dramatic results concerning the superpotential.

Modulo the caveats of the preceding paragraph, it is expected [6] that SQCD with $m = 0$ has no vacuum for $N_c > N_f$. As a simple example of the effect of soft-breaking, consider
allowing

$$\langle F^*_K F_K \rangle = \langle \tilde{F}^*_{\tilde{K}} \tilde{F}_{\tilde{K}} \rangle \neq 0,$$

(3.8)

with no other soft-breaking in the model. The mass terms generated for the squark fields (see (3.3)) then prevent the runaway behavior of the exactly supersymmetric model, and allow it to have a vacuum. Allowing gaugino masses through $$\langle F_{\tau} \rangle \neq 0$$ leads to the following additional term in the potential

$$- \frac{\langle F_{\tau} \rangle}{b} \frac{\Lambda^{(3N_c-N_f)/(N_c-N_f)}}{\det(A_{\tilde{Q}}A_{\tilde{Q}})^{1/(N_c-N_f)}},$$

(3.9)

which does not by itself alter the runaway behavior.

4 Discussion

We have demonstrated that the exact results of [3] which rely on holomorphicity properties of the Wilsonian superpotential can be generalized to models in which supersymmetry is broken softly. This yields a large new class of exact results for non-supersymmetric models.

As a particularly interesting application of our results, vacuum expectation values of $$F_K$$ and $$F_{\tau}$$ can be used to give masses to the scalar components of matter chiral superfields (e.g. squarks, selectrons) and gauginos (gluinos, photino) respectively. This means that we can recover QCD and QED as softly broken versions of their supersymmetric counterparts SQCD and SQED, with exact results intact. Unfortunately, many of the exact results are useful primarily to determine the vacuum structure of the scalar sector of the model, which is precisely what is decoupled in the limit that $$\langle F_K \rangle$$ is taken to infinity. However, the coefficient of $$W^\alpha W_\alpha |_F$$, which is related to the $$\beta$$-function, is also holomorphic and does not decouple.

There are supersymmetric results used in Seiberg’s analysis in [4] that are not left intact by the spontaneous breaking of SUSY. In particular, correlators of lowest components of
chiral superfields

\[ G(x_1, \ldots, x_n) = \langle 0 | T(A(x_1) \ldots A(x_n)) | 0 \rangle \]  

are no longer necessarily position independent or holomorphic. These properties rely on the assumption that the vacuum is supersymmetric, and hence annihilated by the SUSY symmetry generators \( Q_\alpha, \bar{Q}_{\dot{\alpha}} \). This clearly no longer holds in a theory in which SUSY is spontaneously broken. The modification of the supersymmetric Ward-Takahashi identities used to derive the properties of (4.1) (see [2], and references therein) is likely to lead to modifications of some of the results in [7]. We leave this and other issues involving applications of our results to future investigation.

Acknowledgements

The authors would like to thank P. Hernandez, G. Moore, S. Selipsky and R. Sundrum for useful comments or discussions, and N. Seiberg for some important comments about \( D \)-terms. This work was supported under DOE contract DE-AC02-ERU3075.

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