Equity–Premium Puzzle: Evidence From Brazilian Data

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Equity-Premium Puzzle: Evidence From Brazilian Data*

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Abstract

This paper uses 1992:1-2004:2 quarterly data and two different methods (approximation under lognormality and calibration) to evaluate the existence of an equity-premium puzzle in Brazil. In contrast with some previous works in the Brazilian literature, I conclude that the model used by Mehra and Prescott (1985), either with additive or recursive preferences, is not able to satisfactorily rationalize the equity premium observed in the Brazilian data. The second contribution of the paper is calling the attention to the fact that the utility function may not exist if the data (as it is the case with Brazilian time series) implies the existence of states in which high negative rates of consumption growth are attained with relatively high probability.

1 Introduction

It has been now twenty years since the seminal paper by Mehra and Prescott (1985) raised the question of the "Equity-Premium Puzzle".

This is the name economists give to the fact that basic representative-agent models of asset pricing (e.g., Lucas (1978), Breeden (1979) and Mehra and Prescott’s (1985) adaptation of Lucas’ (1978) work1) have not been able

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1To be detailed later in this paper. These models are usually denominated CCAPM (Consumer Asset Pricing Models).
to satisfactorily rationalize the fact that US real returns on stocks have been, between 1889 and 1978, about six percent per year higher than those on T-Bills.

The title Mehra and Prescott used in their (now) famous article generated a curious semantic herd behavior in the profession. What should be just one more case of the universal problem regarding the failure of a certain model to explain a certain data set was named a "puzzle". Following the trend, several other macroeconomically disappointing exercises of statistically unsatisfactory analysis, pertaining to the domain of what is usually dubbed "calibration", were also given the name "puzzle".

Among these is the "risk-free rate puzzle", introduced in the literature by Weil (1989). This puzzle relates to the (low) long-run level of Treasury returns in the United States, when compared to those implied by Mehra and Prescott’s type models.

Consistently with the underlying time frame of their model, Mehra and Prescott used US long-term data. Here, because long-term data is not available for Brazil, I follow Sampaio (2002), Bonomo and Domingues (2002), Alencar (2002) and Issler and Piqueira (2000, 2002) and use short-term data. Relatively to these previous works, this work uses data of a more recent period.

The economic reasoning behind the first two puzzles cited above is as follows. Assets paying higher returns should do it, in equilibrium, according to the basic asset-pricing models previously mentioned, based on the fact that they present a higher covariance with the consumption stream of the representative agent. However, the empirical data for the US has shown that, under the values of the risk-aversion and of the time-discount parameters which are considered to be reasonable, stock returns’s covariance with consumption is not sufficiently greater than that of Treasury Bills, in order to explain the observed difference in their yields. This is what is behind the "equity-premium puzzle".

To explain the high spread between stocks and bonds, one needs values of the risk-aversion parameter that are considered too high. Assuming standard utility functions, in which the risk aversion equals the inverse of the elasticity of substitution, this is to say that agents should highly dislike growth. However, if this were true, the necessary risk-free rates to explain the amount of saving in the US, an economy in which real per-capita consumption growth reaches almost two percent per year, should be much higher than the historical one percent per year observed in the data. This leads to Weil’s "risk-free
rate puzzle" to which I have referred above.

Such puzzles emerge under the assumptions of complete markets, costless asset trading, and that consumer preferences can be represented by the standard utility function used in macroeconomics. Explanations of the puzzles, therefore, can always be made by leaving aside one or more of these three assumptions. There must be, though, consistent support from the empirical evidence in doing so.\(^3\)

A considerable amount of academic research has been developed attempting to solve these puzzles. The explanations include simply denying the existence of a puzzle (see, e.g., Cecchetti and Mark (1990)), or arguments based on: i) recursive utility (Epstein and Zin (1991)); ii) habit formation (e.g., Constantinides (1990), Campbell and Cochrane (1999)); iii) idiosyncratic risk ((Heaton and Lucas 1996, Constantinides and Duffie (1996)); iv) probability of a large drop in consumption (Rietz (1988)); v) borrowing constraints (Davis and Willen (2000)); vi) liquidity premium (Bansal and Coleman (1996)) and; vii) changes in tax rates (McGrattan and Prescott (2001)).

In the present work, the only departure from the assumptions outlined in Mehra and Prescott (1985) that I shall consider regards the modification of the utility function to allow for Kreps-Porteus (1978) preferences, in the line of Epstein and Zin (1991).

In Brazil, some papers concerning, among other subjects, the possible existence of these two puzzles, have been conveniently collected in a book edited by Bonomo (2002). In the second part of this book, three different techniques have been used by several authors in order to assess if the CCAPM models could explain the respective data generated by the Brazilian economy. Among these, Sampaio (2002) follows the basic methodology used by Mehra and Prescott. Bonomo and Domingues (2002) innovate by modeling the consumption series using a Markov-switching model and also by using a Kreps-Porteus (1978) utility function. Alencar (2002) uses the approach of Hansen and Jagannathan (1991), interpreting the equity premium puzzle in terms of the "market price of risk" generated by the data, and constructing volatility bounds. Finally, Issler and Piqueira (2000 and 2002), approach the problem by testing the Euler equations implied by three different models.

A common fact about these works is that they find either that "there is no equity-premium puzzle in Brazil (Issler and Piqueira (2000, p. 233),

\(^3\)For instance, one could abandon the hypothesis of costless trading and argue that the equity premium can be explained by the fact that trading stocks is much more costly than buying Treasury Bills. However, the empirical evidence about this assumption, at least in the US, is not conclusive.
Bonomo and Domingues (2002, p. 116) and Sampaio\(^4\) (2002, p. 99) or that there is a puzzle, but it less intense than the one which emerges with data related to the US (Alencar, p. 152)\(^5\).

When one concentrates solely on the basic assumptions of the model underlying these empirical analyses, such a conclusion of nonexistence of an equity-premium puzzle is intriguing. Indeed, it is hard to argue that Brazil would be any closer to complete markets or costless stock trading than the United States, United Kingdom, Japan, Germany, and France, just to cite some countries where the empirical failure of CCAPM models has been documented\(^6\) (see, e.g., Allais and Nalpas (1999), Iwata (1996), Canova and Nicolo (1995) or Mehra (2003)).

Under this view, the purpose of the present work is reassessing if the Mehra and Prescott’s model, either under the usual CRRA utility or under recursive utility, can be indeed a reasonable model to explain the returns on equities and risk-free bonds in Brazil.

My conclusions here are contrasting with those obtained by Sampaio (2002), Bonomo and Dominges (2002) and Issler and Piqueira (2000, 2002). I have not been able to explain the Brazilian equity-premium with Mehra and Prescott’s model. On the other hand, I have found no evidence of "an inverted risk-free puzzle", as pointed out by Bonomo and Domingues (2002). The model is able to satisfactorily explain the risk-free rates of interest prevailing in Brazil. Such a contrast in the conclusions can be found in many different grounds, including different data sets, distinct methodologies, and the use of alternative discrete-state modelings of the consumption-growth series.

The remaining of this work proceeds as follows. Sections 2 and 3 present the basic model and the conditions for the existence of an expected utility. Section 4 is used to discuss the new data set from which I derive new results, and to compare it with the data sets used by other authors. Section 5 displays new empirical outcomes, initially in the form of approximations under assumptions of lognormality and, next, as the asset returns implied by the simulations. Given the contrasting evidence regarding the empirical results of this work and of its predecessors, Section 6 offers tentative explanations for such discrepancies. Section 7 recalculates the simulations under

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\(^4\)In the case on non-seasonally adjusted data.

\(^5\)Soriano, though, arrives at this milder conclusion by considering seasonally adjusted data. The remaining works, including the results I report here, were based in nonseasonally adjusted data.

\(^6\)As pointed out by Mehra (2003), the United States, the United Kingdom, Japan, Germany, and France account for more than 85 percent of global equity value.
the assumption of recursive utility. Section 8 concludes.

2 The Basic Model

Mehra and Prescott (1985) have adapted Lucas’s (1978) pure-exchange model by assuming that consumption growth, rather than consumption itself, follows a first-order Markov process. This adaptation aims at capturing possible nonstationarities of the real per-capital consumption series.

I describe the model below using initially an Arrow-Debreu structure with contingent claims all traded at time 0. Next, I assume a Markovian structure of endowments and use the pricing sequence previously obtained to price one-period Arrow securities in a sequential-trading structure.

There are several identical consumers in the economy. A representative consumer maximizes the discounted utility (\( U \)) of consumption (\( c \)):

\[
U(c) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1
\]

or, given the assumption of a finite number of states, and with \( \lambda^t \) standing for the history of states till time \( t \):

\[
U(c) = \sum_{t=0}^{\infty} \sum_{\lambda^t} \beta^t u(c_t(\lambda^t)) \pi^t_0(\lambda^t)
\]

subject to the constraint:

\[
\sum_{t=0}^{\infty} \sum_{\lambda^t} q_t^0(\lambda^t)c_t(\lambda^t) \leq \sum_{t=0}^{\infty} \sum_{\lambda^t} q_t^0(\lambda^t)e_t(\lambda^t)
\]

Above, \( \beta \) stands for the time-discount parameter, \( E_t \) for the expectation conditional on the information available at time \( t \), \( q_t^0 \) for the time-zero price of a security promising to pay one unit of consumption at time \( t \), \( \pi^t_0 \) for the conditional probability (on the information available at time zero) of having the history \( \lambda^t \) and \( e_t \) for the endowment at time \( t \). Equation 2 uses the fact that the linear functional defined in the commodity space of the problem has a dot product representation.\(^8\)

\(^7\)By assumption \( \lim_{c \to 0} u(c) = \infty \) and \( u(c) \) is concave.

\(^8\)Note that with exogenous growth the commodity space cannot be the usual \( L_\infty \). Following Mehra (1988), the commodity space \( L \) here is the normed linear space of infinite
Make \( 1 + g_{t+1} = \frac{e_{t+1}}{e_t} \), the aggregate endowment growth in period \( t + 1 \). Endowments are assumed to be governed by a Markov process. In equilibrium, endowments will also be equal to the real per-capita consumption growth.

Assume that:

\[
u(c) = \frac{e^{1-\gamma} - 1}{1-\gamma}, \quad 0 < \gamma < \infty \tag{3}\]

In the present description of the model, at time \( t_0 \) households trade claims on the time \( t \) consumption good at all nodes \( \lambda^t \). After \( t_0 \), no further trade occurs. There are no enforcement- or information-related incentive problems.

The equilibrium allocations in this economy are equivalent to those of an economy with sequential trading. In particular, the allocations at each point in time depend only on the realized aggregate endowment. It is also assumed that the necessary and sufficient condition for the existence of the utility function given by Proposition 1 below is satisfied.

Under such circumstances, the price/dividend ratio \( Q \) can be easily determined, after a straightforward application of the Kuhn-Tucker theorem (the point of departure of which, in this infinite-dimensional case, is the Hahn-Banach separation theorem).

First, use the auxiliary Lagrangean function to obtain:

\[
u_t^t(\lambda^t) = \beta \frac{u'(c_{t+1}(\lambda^{t+1}))}{u'(c_t(\lambda^t))} - \pi_{t+1}(\lambda^{t+1} \mid \lambda^t) \tag{4}\]

This expression gives the time-\( t \) price of an asset that pays off 1 if history \( \lambda^{t+1} \) comes true, given that \( \lambda^t \) has happened.

Next, (4) will be used to price an ex-dividend one-period ahead stock, at time \( t \).

Because of the Markovian structure that we assume for the endowment process, decisions taken at date \( t \) only depend on the state variables at this date, and not on the whole history till date \( t \) (\( \lambda^t \)). This allows us to work conditionally only on the information set available at time \( t \), and only consider the possible states that can be reached at date \( t + 1 \), instead of having to keep track of all previous history \( \lambda^t \), as in (4).

sequences of vectors \( z \) with the \( t - th \) vector indexed by the event \( e_t = (g_1, g_2, ..., g_t) \). The set of possible period \( t \) events is called \( E_t \), and has cardinality \( n^t \). The norm of an element \( z \in L \) is given by:

\[\| z \| = \sup_{t \in E_t} \max_{e_t \in E_t} \left| \frac{z(e_t)}{y_t} \right|\]

where \( y_t = y_0 g_1 g_2 ... g_t \) is the event contingent maximum output of the firm.
Call the sequential-trading price of an ex-dividend one-period ahead stock \( Q_t \). Then, using (4):

\[
Q_t(\lambda_t) = \sum_{\lambda_{t+1}} \beta^{u'(c_{t+1}(\lambda_{t+1}))} (Q_{t+1}(\lambda_{t+1}) + e_{t+1}(\lambda_{t+1})) \pi_{t+1}(\lambda_{t+1} | \lambda_t) \tag{5}
\]

Dividing both terms by \( e_{t+1} \) and using (3) leads to the following expression for the price/dividend relation \( P_t = Q_t / e_t \):

\[
P_t = \sum_{\lambda_{t+1}} \beta((1 + g_{t+1}(\lambda_{t+1}))^{1-\gamma}) \pi_{t+1}(\lambda_{t+1} | \lambda_t)(P_{t+1}(\lambda_{t+1} + 1) \tag{6}
\]

The expected equity return conditional on the information available in time \( t \) is given by:

\[
1 + r_{t+1} = \sum_{\lambda_{t+1}} Q_{t+1}(\lambda_{t+1}) + e_{t+1}(\lambda_{t+1}) \pi_{t+1}(\lambda_{t+1} | \lambda_t) / Q_t
\]

Making

\[
\frac{Q_{t+1} + e_{t+1}}{Q_t} = \frac{e_{t+1}}{Q_t} \left(1 + \frac{Q_{t+1}}{e_{t+1}}\right)
\]

one gets, after multiplying and dividing the above expression by \( e_t \):

\[
1 + r_{t+1} = \sum_{\lambda_{t+1}} (1 + g_{t+1}(\lambda_{t+1})) \pi_{t+1}(\lambda_{t+1} | \lambda_t) 1 + P_{t+1}(\lambda_{t+1}) / P_t \tag{7}
\]

To obtain the risk-free rate note that (5) allows us to write, for \( m = r \) or \( m = r_f \) (\( r_f \) standing for the risk-free rate):

\[
1 = \sum_{\lambda_{t+1}} \beta^{u'(c_{t+1}(\lambda_{t+1}))} (1 + m_{t+1}) \pi_{t+1}(\lambda_{t+1} | \lambda_t) \tag{8}
\]

From which we get (also conditional on the information set available at time \( t \), and noticing that by definition \( r_{f,t} \) does not depend on the realization of \( \lambda_{t+1} \)):

\[
1 + r_{f,t+1} = \frac{1}{\beta \sum_{\lambda_{t+1}} (1 + g_{t+1}(\lambda_{t+1}))^{-\gamma} \pi_{t+1}(\lambda_{t+1} | \lambda_t)} \tag{9}
\]
3 Existence of the Expected Utility

As pointed out by Mehra and Prescott (1985, p. 151), the introduction of nonstationarity in Lucas’s (1978) model requires, among other things, verifying if the conditions under which the expected utility exists are satisfied. The Proposition below, the proof of which Mehra and Prescott (1985, p. 151) refer the reader to Mehra and Prescott (1984), provides a direct way to check it.

**Proposition 1** (Mehra and Prescott (1984)): Suppose the preferences of the representative consumer are ordered over random paths by (1) and (3). Denote by $W$ the transition matrix of the (ergodic) Markov chain associated with $g_{t+1} \in \{g_1, g_2, ..., g_N\}, g_i \geq 0$ for all $i$ and all $t$ and suppose that $e_0 > 0$. Then, a necessary and sufficient condition for expected utility to exist is that the matrix $A$ with elements $a_{i,j} = \beta W_{i,j} (1 + g_j)^{1-\gamma}$ obeys:

$$\lim_{m \to \infty} A^m = 0$$  \hspace{1cm} (10)

**Proof.** See Mehra and Prescott (1984) or Mehra (1988).

Note that the condition (10) is equivalent to the matrix $A$ having eigenvalues all of which lie within the unit circle in the complex plan.

An intuition of this result is easily obtained from the analysis of the deterministic case, in which, trivially, $EU(c) < \infty$ if and only if $\beta(1+g)^{1-\gamma} < 1$. In the stochastic case, one has to consider all states attained by $g$. In particular, note that when $\gamma$ is very high (in an attempt of the researcher to obtain high equity premia with the model) one can easily have $\beta(1+g)^{1-\gamma} > 1$ for values of $g$ slightly below zero. This can pose a problem for the fulfillment of condition (10).

Condition (10) is also sufficient to guarantee a unique positive solution to the $n$ by $n$ system of linear equations (when $n$ states are considered) given by (6).

4 Data

The Brazilian data used in this work has a quarterly frequency and ranges from 1992:1 to 2004:2. This period was chosen because 1992:1 is the point after which the official quarterly consumption series derived from the quarterly GDP series calculated by FIBGE (Fundação Instituto Brasileiro de Geografia e Estatística) and by the IPEA (Instituto Brasileiro de Pesquisa Econômica Aplicada) is available. The consumption series is provided by Macrodados.
Population data are from the FIBGE. For the two first quarters of 1994 population data were extrapolated under the assumption that the rate for 2004 was the same as the one for 2003. Within each year the series was quarterly interpolated assuming a log-linear growth.

The price index used to deflate consumption and to obtain real interest rates was the INPC (Índice Nacional de Preços ao Consumidor) provided by the FIBGE. The Selic (Sistema Especial de Liquidação e Custódia) interbank rate was used to generate the "risk-free" rate, and the return of the IBOVESPA index to generate the equity rate (a claim to the stochastic endowment). Both were deflated by the inflation rate calculated using the INPC. The source of the Selic and of the FGV-100 was the databank of the IBRE (Instituto Brasileiro de Economia da Fundação Getulio Vargas).

Table 1 summarizes the data used by different authors, including Prescott and Mehra for the US and Sampaio (2002), Domingues and Rodrigues (2002), Issler and Piqueira (2000, 2002), Alencar (2002) and Cysne (this work) for Brazil.

Table 1 - Data (%) on Per.Cap. Cons. Growth (g) and Real Returns

| Range/Per. | av(g) | st(g) | av(r) | st(r) | av(rf) | st(rf) | e.p. |
|------------|-------|-------|-------|-------|--------|--------|-----|
| M./P. (US) | 1889-78 A | 1.83 | 3.57 | 6.98 | 16.54 | 0.80 | 5.67 | 6.18 |
| Sampaio    | 1980-98 Q | 2.02 | 7.20 | 29.13 | 29.30 | 7.82 | 9.70 | 21.31 |
| B./D.      | 1986-98 Q | 0.80 | 6.80 | 24.21 | 31.12 | 13.96 | 6.28 | 10.25 |
| I./P.      | 1975-94 A | -    | -    | -    | -    | -    | -    | 29.06 |
| Alencar    | 1980-98 Q | 1.61 | 2.2  | 28.65 | 29.1  | 7.40 | 5.70 | 21.25 |
| Cysne      | 1992-04 Q | 3.12 | 4.80 | 31.33 | 24.89 | 15.41 | 4.82 | 15.92 |

av=average, st=st. dev., r = equity rate, rf = risk-free rate, e.p.=equity premium
Q=Quarterly, A=annual, Per=Periodicity, M./P = Mehra and Prescott (Table 1, p. 147) B./D. = Bonomo and Domingues (Tables 1 and 2, p. 106) I./P. = Issler and Piqueira (p. 234 of I./P. (2000)). Data for Alencar comes from Table 2, p. 128 and for Sampaio from Table 1, p.194.

All data regarding consumption growth, except the enters for Alencar, who only reports seasonally adjusted averages, are non-seasonally adjusted. Standard deviations are calculated under the periodicity of the data shown in column 2. All averages are expressed in annual terms using compounded rates. The equity premia are also annual and correspond simply to the difference between the respective equity and risk-free rates. All Brazilian authors
calculate the mean return on equities using the IBOVESPA (Índice da Bolsa de Valores do Estado de São Paulo).

In Table 1, concerning the Brazilian economy, this work (last row in Table 1) presents the highest value of the per capita consumption growth. This is consistent with the fact that after the Real Plan there were high current account deficits, fostering consumption temporarily. The standard deviation of 0.048 (of consumption growth) is higher than the one reported by Mehra and Prescott (1982) for the US time series (0.036). But lower than the ones reported by Sampaio (0.072) and by Bonomo and Domingues (0.068), for Brazil. This reflects the fact that these previous studies using Brazilian data included a larger share of the period in which inflation was higher. Relatively to the data I present here, they do not include the 1998-2004 low-inflation period.

This work also presents the highest risk-free rate (15.41%), a consequence of the fact that after the Real Plan high interest rates were used to control the residual inflationary pressures. Last, note that the equity premium calculated here (15.92%) is between the one calculated by Bonomo and Domingues (10.25%) and those calculated by Issler and Piqueira (29.06%) and by Sampaio (21.31%).

Figure 1 below presents the data in a way close to the one Mehra and Prescott (1985, Table 1) and Kocherlakota (1996, Table 1) did in their respective works:

**Insert Figure 1 Here**

The numbers in the first column of Figure 1 correspond to the averages presented in the last row of Table 1. However, in Table 1 they are expressed in an annual and percentage basis, whereas Figure 1 presents them in a quarterly and decimal basis. The quarterly standard deviations reported in the last row of Table 1 can also be obtained from Figure 1.

Note that, as it happens with the data reported in Mehra and Prescott (1985), the sample covariance of consumption growth and the return on equity (0.00208) is higher than the sample covariance on consumption growth and the return on the risk-free asset (−0.00032)\(^9\). This implies that the model we have seen in the last section does account for the fact that the return on equity (which pays more when consumption is higher) is observed to command a certain premium over the return on the risk-free rate. We shall see, though, that although qualitatively right, the CCAPM model is not able to provide a satisfactory quantitative rationalization for the equity premium found in the Brazilian data.

\(^9\)In the population, by definition, this value is equal to zero.
5 Empirical Results

5.1 Preliminary Analysis Under Lognormality

Before calibrating a general-equilibrium model as in Mehra and Prescott (1985), I investigate the existence an equity premium puzzle assuming lognormality of returns.

The expressions for asset returns under lognormality are well known in the literature (see, e.g. Hansen and Singleton (1983) or Ljungqvist and Sargent (2002)), being included here just for completeness.

Suppose that consumption growth, the equity return (r) and the risk-free return follow the equations:

\[ 1 + k_{t+1} = (1 + \tilde{k}) \exp(\varepsilon_{k,t+1} - \sigma_k^2/2), \quad \text{for } k = g, \; k = r \; \text{or} \; k = r_f \]  

(11)

with \{\varepsilon_{g,t+1}, \varepsilon_{r,t+1}, \varepsilon_{r_f,t+1}\} jointly normally distributed with zero means and variances \{\sigma_g^2, \sigma_r^2, \sigma_{r_f}^2\}. By using the fact that the moment generating function of a standard normal variable (\(E(\exp(\tau Z))\)) is equal to \(\exp(\tau^2/2)\), one easily concludes that the expected value of \(1 + k_{t+1}\) in (11) is \(\tilde{k}\). This allows for an economic interpretation of (11) that is used below.

From equations (5) and (3), applying the law of iterated expectations:

\[ 1 = \beta E[(1 + g_{t+1})^{-\gamma}(1 + m_{t+1})], \quad \text{for } m = r \; \text{or} \; m = r_f \]  

(12)

Using (11) in (12), for \(m = r \) or \(m = r_f\):

\[ 1 = (1 + \tilde{g})^{-\gamma}(1 + \tilde{m})\beta E\left\{\exp\left[(\varepsilon_{m,t+1} - \sigma_m^2/2) - \gamma(\varepsilon_{g,t+1} - \sigma_g^2/2)\right]\right\} \]  

(13)

Since the variable between brackets is a normal variable with mean \(-\sigma_m^2/2 + \gamma\sigma_g^2/2\) and variance \(\sigma_m^2 + \gamma^2\sigma_g^2 + 2\gamma\text{cov}(\varepsilon_m, \varepsilon_g)\), using again the result above for the moment generating function of a standard normal variable (for \(m = r \) or \(m = r_f\)):

\[ 1 = (1 + \tilde{g})^{-\gamma}(1 + \tilde{m})\beta \left[\exp((1 + \gamma)\gamma\sigma_g^2/2 - \gamma\text{cov}(\varepsilon_g, \varepsilon_m))\right] \]  

(14)

Taking logarithms on both members of (14):

\[ \log(1 + \tilde{m}) = -\log\beta + \gamma \log(1 + \tilde{g}) - \gamma(1 - \gamma)\frac{\sigma_g^2}{2} - \gamma\text{cov}(\varepsilon_g, \varepsilon_m) \]  

(15)

Making for \(m = r \) and \(m = r_f \) above and and subtracting the resulting expressions:

\[ \log (1+r) - \log (1+r_f) = \gamma(\text{cov}(\varepsilon_g, \varepsilon_r) - \text{cov}(\varepsilon_g, \varepsilon_{r_f})) \]  

(16)
Since \( \log(1 + x) \simeq x \) for low (absolute) values of \( x \), (16) allows us to use the sample values of \( \text{cov}(\varepsilon_g, \varepsilon_r) \), \( \text{cov}(\varepsilon_g, \varepsilon_{rf}) \), and the means of \( r \) and \( r_{rf} \), in order to obtain the implied risk-aversion parameter \( \gamma \). Approximating these covariances with the values of Figure 1, we get \( \gamma = 14.15 \), or directly from the values of Figure 2, \( \gamma = 15.8 \). One can also use the fact that the populational value of \( \text{cov}(\varepsilon_g, \varepsilon_{rf}) \) is equal to zero, to obtain even higher values for \( \gamma \) (16.36 and 19.12, respectively). In any case, it is clear from the data the existence of an equity-premium puzzle (since \( \gamma > 10 \) is not acceptable as a reasonable range for the parameter \( \gamma \)).

**Insert Figure 2 Here**

### 5.2 Existence and Calibration

Since the calculations above relied on the assumption of lognormality, the conclusions may have resulted from this hypothesis, rather than from the empirical failure of the underlying model. In this subsection I abandon the hypothesis of lognormality and re-investigate the existence of an equity-premium puzzle under another set of assumptions.

I assume that real per-capita consumption growth \( (g) \) follows a linear stochastic difference equation driven by a Gaussian disturbance, and that it can be reasonably modeled by a stationary \( AR(1) \) process:

\[
g_t = bg_{t-1} + \varepsilon_t \tag{17}
\]

where \( \varepsilon \) is a normal random variable with mean \( a \).

Using (17) and the value of \( b \) estimated from the sample data, one concludes that \( \varepsilon_t \) has an average equal to 1.0016 and a standard deviation of 0.0480. Based on these data, I generate an approximation of the initial problem using ten different states for \( g \) and a transition matrix \( T \) of the approximating discrete-state Markov chain.

The minimum and maximum values of the series of consumption growth are 0.9065 and 1.1016, respectively. In the discrete-state approximation, such values are, respectively, 0.91 and 1.10\(^{10} \). The states and the stationary probabilities are presented in Figure 3.

**Insert Figure 3 Here**

\(^{10}\text{The Markov chain implied by the data and its discrete-state approximation have also been compared regarding the implied first-order correlation coefficient and the implied standard deviation of the series in order to get the best possible approximation.} \)
• Checking the Existence of an Expected Utility

Before starting the simulations, it is crucial to check if, under the parameter values used in the problem, the necessary and sufficient conditions for the existence of an expected utility function are satisfied. This is done with the help of Proposition 1. The calculations using beta, the transition matrix and the ten states of consumption growth show that Proposition 1 is satisfied for the values of the gammas used in the simulations (from 2 to 6) but not necessarily for \( \gamma > 7 \) (when \( \beta = 0.995 \)) or \( \gamma > 9 \) (when \( \beta = 0.9740 \)).

The fact that the series of real per capita consumption growth used here has a higher average and a lower standard deviation than the ones used by Sampaio (2002) and by Bonomo and Domingues (2002) plays an important role in the existence of the expected utility (an inspection of condition (10) makes this point clear).

• Calibration

The calibration I carry out in this section has the advantage, when compared with those reported by Sampaio, and by Bonomo and Domingues, of approximating the consumption series with ten states, instead of only two.

The equity premium implied by the model is generated in the following way. First, the continuous state Markov chain is approximated by a ten-state Markov chain, as detailed above. Second, the transition matrix is used to simulate a time series of length 15000 for the states assumed by the Markov chain. Third, for each state, different values of the parameters \((\beta, \gamma)\) and (6) are used to calculate the price/dividend ratios. Fourth, such price-dividend ratios are used to calculate, also for each state, the return on equity, the risk-free rate and the equity premium. In this step I used (7), (9), and the fact that the equity premium is defined as \( r - r_f \). Fifth, the 15000 states assumed by the Markov chain are employed to generate time series of \( r, r_f \) and the equity premium. Sixth, I take the averages of each one of these series to obtain the equity return, the risk free rate and the equity premium generated by the model\(^{11}\).

The results of these steps are presented in Figure 4.

\(^{11}\)Since the Markov chain under consideration is ergodic, the state space is compact and the transition function defined by an \( n \) by \( n \) stochastic matrix defines a stable conditional operator (sometimes dubbed as an operator with the "Feller property"), step 6 is a.e.-equivalent, by the strong law of large numbers, to obtaining the averages of each variable using the stationary probabilities implied by the respective Markov chain (see, e.g., theorem 14.7 in Stokey, Lucas and Prescott (1989, p. 425)).
Note in Figure 4 that both the equity return and the risky-free rate decrease for high values of gamma. Under deterministic growth, interest rates vary positively with the risk aversion parameter when consumption-growth rates are strictly positive (equation (15) with \( m = r_f \) makes this point clear). When the variance of consumption growth is not zero, though, this rate tends to become a negative function of \( \gamma \) when this parameter assumes higher values (see equation (15) once more). As one can observe from Figure 3, states 1 to 5 considered in the simulations are characterized by negative rates of growth of consumption. Moreover, \( \sigma_g^2 > 0 \) here. For these reasons, one can get risk-free rates decreasing with the value of gamma.

From Table 1 or from Figure 1, the quarterly equity return for the 1992:01-2004:2 period should run around 7.052\%, and the risk-free return around 3.648\%, generating a quarterly equity premium around 3.40\%. From Figure 4, the maximum equity premium generated by the model happens for \( \beta = 0.974 \) and \( \gamma = 6 \), and is equal to 1.34\%. This result confirms the preliminary calculations carried out under the assumption of lognormality, concluding that there is an equity-premium puzzle in Brazil.

In section 7 I check if this puzzle remains under more elaborate utility functions.

When \( \beta = 0.974 \), the quarterly risk-free rate which emerges from the sample is generated for values of the relative risk-aversion parameter between two and four. Since these are considered to be reasonable parameters for \( \gamma \), I find no risk-free-rate puzzle as the one identified by Weil (1989) for the United States. Neither an "inverted risk-free rate puzzle", as pointed out by Bonomo and Domingues (2002).

### 6 Comments on Previous Works

In section 5 I have found evidence of the existence of an equity-premium puzzle in the Brazilian data, both under a lognormal approximation and in the calibration exercise using Brazilian 1992:1-2004:2 quarterly data.

In contrast with this result, Sampaio (2002, p. 99), using nonseasonally adjusted quarterly data ranging from 1980:1 to 1998:2, finds that the first moments of Brazilian data can be obtained using Mehra and Prescott’s model for values of the risk aversion parameter (\( \gamma \)) and of the time-discount parameter (\( \beta \)) equal to, respectively, 6.1 and 0.91 (quarterly). Sampaio (2002) concludes that, once these parameters are accepted as valid, there is no equity-premium puzzle in Brazil.
Also in contrast with the results obtained here, Bonomo (2002, p. 116) states that "there is no equity premium puzzle in Brazil", whereas Issler and Piqueira (2000) argue that "there is no equity premium puzzle for Brazil due to the great variation observed for the equity premium" (page 233)\textsuperscript{12}.

In the next paragraphs I analyze some of these works in order to understand the difference in the conclusions.

- Sampaio (2002) and of Bonomo and Rodrigues (2002):

A first point to note is that the data set used here differs from the data set used in these works. Not only are the time periods different, but also the consumption series. I use official quarterly IPEA/FIBGE/Macrodados’s consumption data, including non-durables and durables, whereas Soriano, Sampaio (2002) and Bonomo and Rodrigues (2002) (see page 81 in Bonomo (2002)) use a private consumption series produced by Soriano, which excludes durables.

Below I use the data sets provided by Sampaio (2002) and by Bonomo and Domingues (2002) to show that the results reported by these authors do not survive under the lognormal approximation.

Second, also using the data reported by these authors, I show that, under the Mehra and Prescott’s methodology, the maximum possible range between states of consumption growth that these works could use falls short of the range displayed in their data. This is a condition implied by (10), on the existence of the expected utility.

**Redoing the Calculations Under Lognormality**

In order to redo the approximate calculations under lognormality, one needs the equity premia, as well as the covariance between consumption growth and the returns on equities and on the risk-free asset. Such data are not directly available, since Sampaio (2002) and Bonomo and Rodrigues (2002) do not report the sample covariances.

However, since these authors use the same real consumption series as Alencar (see Bonomo (2002), p. 81), and since the series of nominal returns are the same (IBOVESPA and Selic), I use the Table of variance/covariance for nonseasonally adjusted data published in Alencar (2002, p. 157. Table 9).

From this table, the difference between the covariance of consumption and the risky rate, and the covariance of consumption and the nonrisky rate, using quarterly data, is equal to 0.00004. Sampaio (2002, p. 96. Table 2)

\textsuperscript{12}Alencar (2002) cannot be used as a direct source for comparison, since this author uses seasonally adjusted data in almost all of his calculations.
reports a quarterly premium of 0.047 whereas Bonomo and Domingues (2002, p. 106, Table 1) report 0.02247. Using (16), such values imply a risk-aversion parameter (\( \gamma \)) equal to, respectively, 1175.0 (!) and 561.75. In contrast with the results obtained by these authors, this implies again, as I have found in this work, that there is an equity-premium puzzle in Brazil\(^{14}\).

**Maximum Allowable Range Between States Implied by the Existence of the Expected Utility**

Neither Sampaio nor Bonomo and Rodrigues report for which range of the parameter values they use one could assure the existence of the expected utility function.

This condition is particularly important in the Brazilian case, because of the relatively lower average\(^{15}\) and higher standard deviation\(^{16}\) of the real per-capita consumption growth (when compared to the US). These facts imply assigning relatively high probabilities to states in which real per-capita consumption decreases sharply, with \( \beta(1 + g)^{1-\gamma} \) assuming values that can be too high for the expected utility to exist.

**Sampaio (2002)**

Using the parameter values and the number of states (equal to 2) reported by Sampaio (2002) I carry out next some calculations with this purpose. Using the parameter values of Table 1 in Sampaio (2002, p. 94) of average consumption growth equal to 0.005, standard deviation of consumption growth 0.072 and first-order autocorrelation \(-0.128\), a implied 2-state Markov chain can be very conservatively (concerning the validity of Proposition 1)\(^{17}\) approximated with the states, stationary probabilities and Transition Matrix given by Figure 5.

\(^{13}\)Using a covariance with the risk-free return equal to zero does not improve the results significantly. The new numbers for gamma are now 671.4 and 321.0, respectively.

\(^{14}\)One could argue that the approximation under lognormality would not be appropriate, but this reasoning would not explain such high values of the risk-aversion parameter.

\(^{15}\)Except in Sampaio (2002).

\(^{16}\)The data reported by Alencar (2002) cannot serve as a basis of comparison here because this author uses seasonally adjusted data.

\(^{17}\)The states that emerge from the matrix of Figure 5, after the due adjustment in the y-coordinate are made, are contained in the range shown in Figure 1 of Sampaio (2002, p. 94). Using more than 1.4 standard deviations (I have used 1.4 here) in the construction of the approximate state space reduces the chances that the condition of Proposition 1 is satisfied, thereby the reported conservativeness of the present calculations.
Note in Figure 5 that the two states of the approximation of the rate of growth of the endowments, 0.88260 and 1.12740, both lie within the range displayed in Figure 1 of Sampaio (2002, p. 94) (by the eye, the sample range in Figure 1 of Sampaio (2002) goes from 0.85 to 1.15). Moreover, the weighed mean of the approximating states, using the stationary probabilities, exactly coincides with 1.005, the sample value of the average rate of growth of consumption reported by this author.

Increasing the range of the approximation would be supported by the data but would make things worse in terms of the existence of an expected utility. The number used here, therefore, are conservative.

Figure 5 also shows the values of the matrix A in Proposition 1, for $\gamma = -6.1$ and $\beta = 0.91$, which are found by Sampaio as those for which there is no premium puzzle. The maximum absolute value of the eigenvalues of this matrix is equal to 1.0547, implying that condition (10) is not satisfied.

An alternative calculation, probably closer to the way how this author may have worked, is inverting the procedure and obtaining the maximum possible range for the rates of consumption growth that still allow for the existence of an expected utility. The results are presented in Figure 6. The new lower and upper bounds for the rate of consumption growth, respectively, 0.906 and 1.104, are farther away from the respective end points of the sample values displayed in Figure 1 in Sampaio (2002).

Bonomo and Domingues

Bonomo and Rodrigues (2002, p. 111) argue that they have been able to reproduce the equity premium modelling consumption with Markov switching using with a value of the parameters of time discount equal to 0.95 per quarter and a value of the risk aversion parameter of 3.23.

Here I use the same data but model consumption growth differently from these authors, using the procedure described in section 5 (though with only two states, as Bonomo and Domingues (2002) did), and check if their results would be robust under this alternative modeling\textsuperscript{18}.

\textsuperscript{18}Bonomo and Domingues approximate the logarithm of the consumption growth series ($log(1+g)$) using a Markov-switching model, rather than a discrete-state Markov chain of the relative change ($1+g$), as I do here. For this reason, the evaluation of the existence of an expected utility function that I carry out here using Bonomo and Domingues’s data does not directly apply to these authors’s framework. It is only descriptive of a case in which the Mehra and Prescott could possibly have been used.
The values of the real per-capita consumption series are displayed in Table 2 of Bonomo and Rodrigues (2002, p. 106). The average and standard deviation of the real per capita consumption growth are equal to 0.002 and 0.068, respectively. The first order correlation coefficient is \(-0.075\).

One can easily see from Figure 1 in Bonomo and Rodrigues (2002, p. 111) that 1.11 is not a upper bound and 0.89 is not a lower bound of \(g = c_{t+1}/c_t\). Acting conservatively, in terms of Proposition 1 (see footnote 17), in a next step I construct states, a transition matrix and stationary probabilities using the same procedure as I did in the case of Sampaio (2002), above. The result is displayed in Figure 7.

Please Insert Figure 7 Here

Note that the mean of the states is exactly 1.002, as reported by Bonomo, for the variable \(g\). As it happened with the data of Sampaio, the maximum absolute value of the eigenvalues of the matrix of Proposition 1 is 1.0974, showing that for such an amplitude between states the expected utility does not exist.

As above, a more interesting calculation, using the data reported by Bonomo and Domingues, would be obtaining the maximum allowable range for the rates of consumption growth that would allow the existence of an expected utility. The results of such a calculation are shown in Figure 8.

Please Insert Figure 8 Here

The new lower and upper bounds for the rate of consumption growth \((1 + g)\), respectively, of 0.936 and 1.068, are now farther away from the extremes of the sample values displayed in Figure 1 in Bonomo and Rodrigues (2002, p. 111)\(^{19}\).

- Issler and Piqueira (2000, 2002)

The main purpose of Issler and Piqueira (2000, 2002) is not discussing the equity-premium puzzle (which is done only in the last pages), but to estimate structural parameters for the Brazilian economy for different types of utility functions. In the spirit of Hansen and Singleton (1982), these authors have tested a set of necessary conditions implied by the basic model and have not

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\(^{19}\)Both in Sampaio (2002) and in Bonomo and Domingues (2002), satisfying Proposition 1 turns out to be more likely when one considers more than two states. However, both these authors only considered approximations with two states.
been able to reject it, thereby their conclusion about the nonexistence of an equity premium puzzle in Brazil.

The best way to contrast the evidence provided by Issler and Piqueira with the results found here would be through an empirical re-evaluation of the restrictions implied by the basic CCAPM model using the same data set I used here and the same methodology used by these authors. This is a good suggestion for future research in the area.

7 Empirical Results With Recursive Utility

A restriction imposed by the class of preferences we have used so far is that the same parameter ($\gamma$) assumes simultaneously the role of the coefficient of risk aversion and of the reciprocal of the elasticity of intertemporal substitution. This implies that aversion to uncertainty among states of nature is necessarily related to aversion to consumption variation over time. Epstein and Zin (1991) use Kreps-Porteus (1978) preferences, a class of preferences that avoids this problem. Their recursive utility can be stated as:

$$V_t = ((1 - \delta)c_t^{1-\rho} + \delta(\Phi_t V_{t+1})^{1-\rho})^{\frac{1}{1-\rho}}, \quad \rho \neq 1, \quad 0 < \delta < 1$$

where:

$$\Phi_t V_{t+1} = E_t(V_{t+1})^{\frac{1}{1-\alpha}}$$

Proceeding as before, the expected price/equity ratio is now given by:

$$P_t^e = E_t(\delta(c_{t+1})^{1-\rho}(\frac{V_{t+1}}{\Phi_t V_{t+1}})^{\rho-\alpha}(1 + P_{t+1}^e))$$

Making $W_t = V_t/e_t$ one has:

$$P_t = E_t(\delta(g_{t+1})^{1-\rho}(\frac{g_{t+1}W_{t+1}}{\Phi_t(g_{t+1}W_{t+1})})^{\rho-\alpha}(1 + P_{t+1}^e))$$

Note that this formula is equivalent to (6), except for the presence of the term $(\frac{g_{t+1}W_{t+1}}{\Phi_t(g_{t+1}W_{t+1})})^{\rho-\alpha}$ on the right member and for the fact that $\rho$ is replacing $\gamma$ and $\delta$ is replacing $\beta$. The risk-free rate now reads:

$$r_{f_t} = \frac{1}{E_t(\delta(g_{t+1})^{1-\rho}(\frac{g_{t+1}W_{t+1}}{\Phi_t(g_{t+1}W_{t+1})})^{\rho-\alpha})}$$

The equity rate is determined as before, using the new value of the price-dividend ratio. $\rho$ stands for the inverse of the elasticity of substitution, the same as gamma in the previous section. The risk aversion parameter here
is now given by $\alpha$, thereby being completely independent of the elasticity of substitution $1/\rho$. $\delta/(1 - \delta)$ can be interpreted as the new time-discount parameter (the beta of the previous section).

I redo the previous calculations for $\rho = 2$ and $\alpha = 5, 6$, in order to compare with the previous case. The results are shown in Figure 9.

![Please Insert Figure 9 Here](image)

As it is clear from the last row of Figure 9, using recursive utility does not modify the fact that Mehra and Prescott’s-like models are not able to satisfactorily rationalize the equity premium found in the Brazilian data. On the other hand, the risk-free rate that is generated by the model with $\rho = 2$ and $\alpha = 5$ is basically the same as the one observed in the sample. Again, there is no risk-free rate puzzle, as the one reported by Weil (1989) for the U.S., or "inverted risk-free rate puzzle", as the one reported by Bonomo and Domingues (2002) for Brazil.

### 8 Conclusions

In this paper I have used 1992:1-2004:2 Brazilian quarterly data to evaluate the existence of an equity-premium puzzle. In contrast with some previous works in the Brazilian literature, I have concluded that the model used by Mehra and Prescott (1985), either with additive or recursive preferences, is not able to satisfactorily rationalize the equity premium observed in the Brazilian data.

Such a conclusion was obtained both under assumptions of lognormality or under simulations based on a discrete state approximation of the Markov process implied by the data. The model is able, though, to produce risk-free interest rates in agreement with those observed in Brazil.

An advantage of the modeling of consumption growth I have performed here over those of Sampaio (2002) and of Bonomo and Domingues (2002) is that I have used ten states, instead of just two.

In order to understand why my conclusions differed from those obtained by Sampaio and by Bonomo and Domingues, I have used the parameter values reported by these authors and tried to reproduce their conclusions under the assumption of lognormality of the consumption growth and of the returns on the assets. This implied the attainment of risk-aversion parameters completely outside the usually allowable range (1175.0 and 561.75).
I have also derived the maximum allowable range between states that these authors could use to model consumption growth under the Mehra and Prescott methodology. The results have shown a range very inferior to the one displayed by their data.

This last conclusion points out that the utility function may not exist (under the Mehra and Prescott methodology) when one uses a small number of states to approximate a consumption-growth series with relatively high standard deviation (as it happens with Brazilian data) and tries to rationalize the equity-premium puzzle using values of the risk-aversion parameter that lie within the usually allowable range (0 to 10), but are otherwise too high.

Finally, I have redone the calibrations using Kreps-Porteus (1978) preferences and have maintained my previous conclusion that the usual CCAPM models cannot satisfactorily rationalize the equity premium implicit in the Brazilian data, even though they are able to generate the risk-free interest rates found in Brazil.

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Figure 1: Sample averages and covariance matrix of the equity rate (1+r) the risk-free rate (1+rf) and real per-capita consumption growth (1+g).

|            | Mean | 1+r  | 1+rf | 1+rf | 1+g  |
|------------|------|------|------|------|------|
| 1+r        | 1.07052 | 0.06193 | -0.00330 | 0.00208 |
| 1+rf       | 1.03648 | -0.00330 | 0.00232 | -0.00032 |
| 1+g        | 1.00770 | 0.00208 | -0.00032 | 0.00230 |

Figure 2: Covariance Matrix Using Logarithms

|            | log(1+r) | log(1+rf) | log(1+g) |
|------------|----------|-----------|----------|
| log(1+r)   | 0.05647  | -0.00322  | 0.00178  |
| log(1+rf)  | -0.00322 | 0.00270   | -0.00037 |
| log(1+g)   | 0.00178  | -0.00037  | 0.00230  |

Figure 3: Discrete-State Markov Approximation

| States | S.P. | States |
|--------|------|--------|
| 1      | 0.03772 | 0.91177 |
| 2      | 0.05349 | 0.94659 |
| 3      | 0.09582 | 0.95441 |
| 4      | 0.14133 | 0.97572 |
| 5      | 0.17164 | 0.99704 |
| 6      | 0.17164 | 1.01836 |
| 7      | 0.14133 | 1.03967 |
| 8      | 0.09582 | 1.06099 |
| 9      | 0.05349 | 1.08281 |
| 10     | 0.03772 | 1.10362 |
Figure 4: Results of the Simulation of the Model Based on the Parameters Derived From the 1992:1 -2004:2 Sample.

|                | 2   | 3   | 4   | 5   | 6   |
|----------------|-----|-----|-----|-----|-----|
| Risky Return  | .995| 1.0183 | 1.0218 | 1.0230 | 1.0221 | 1.0189 |
|                | .9740 | 1.0403 | 1.0436 | 1.0451 | 1.0441 | 1.0409 |

|                | 2   | 3   | 4   | 5   | 6   |
|----------------|-----|-----|-----|-----|-----|
| Risky-Free Rate| 2   | 3   | 4   | 5   | 6   |
|                | .995 | 1.0140 | 1.0152 | 1.0143 | 1.0112 | 1.0059 |
|                | .9740 | 1.0358 | 1.0371 | 1.0362 | 1.0330 | 1.0276 |

|                | 2   | 3   | 4   | 5   | 6   |
|----------------|-----|-----|-----|-----|-----|
| Equity Premia  | .995 | 0.0043 | 0.0055 | 0.0087 | 0.0193 | 0.0130 |
|                | .9740 | 0.0044 | 0.0067 | 0.0089 | 0.0111 | 0.0133 |
Approximation of Stat. Prob. and States for g Using the Data of Sampaio (2002)

| S.P. | States |
|------|--------|
| 1    | 0.50000 0.88260 |
| 2    | 0.50000 1.12740 |

Approximate 2-State Transition Matrix Using the Data of Sampaio (2002)

|     | 1     | 2     |
|-----|-------|-------|
| 1   | 0.41317 | 0.58683 |
| 2   | 0.58683 | 0.41317 |

Matrix of Proposition 1 Calculated Using Data the of Sampaio (2002)

|     | 1     | 2     |
|-----|-------|-------|
| -1  | 0.71084 | 0.28971 |
| -2  | 1.00000 | 0.00000 |

Eigenvalue of Matrix A of Proposition 1 Calculated Using Data the of Sampaio (2002)

|     | 1     |
|-----|-------|
| -1  | 1.05467 |

Figure 5: Evaluation of the Existence of the Expected Utility in Sampaio (2002). (Eigenvalue Higher Than One Implies Nonexistence of Expected Utility).
| S.P. | States |
|------|--------|
| 1    | 0.50000 | 0.90600 |
| 2    | 0.50000 | 1.10400 |

Constrained Approximate 2-State Transition Matrix Using the Data of Sampaio (2002)

|     | 1     | 2     |
|-----|-------|-------|
| 1   | 0.42957 | 0.57043 |
| 2   | 0.57043 | 0.42957 |

Constrained Matrix of Proposition 1 Calculated Using Data the of Sampaio (2002)

|     | 1     | 2     |
|-----|-------|-------|
| --1→| 0.64673 | 0.31326 |
| --2→| 0.85879 | 0.23601 |

Eigenvalue of the Constrained Matrix A of Prop. 1 Calc. Using Data the of Sampaio (2002)

|     | 1     |
|-----|-------|
| --1→| 0.99993 |

Figure 6: Calculation of the Range Under Which The Existence of Expected Utility in Sampaio (2002) Can be Insured
Figure 7: Evaluation of the Existence of the Expected Utility Using the Data Reported By Bonomo and Domingues (2002). (Eigenvalue Higher Than One Implies Nonexistence of Expected Utility).

Approximation of Stat. Prob. and States for g in Bon/Dom (2002)

| S.P. | States   |
|------|----------|
| 1    | 0.50000  |
| 2    | 0.30000  |

Approximate Transition Matrix in Bon/Dom (2002)

|   | 1  | 2       |
|---|---|---------|
| 1 | 0.45211 | 0.54789 |
| 2 | 0.34789 | 0.65211 |

Matrix of Proposition 1 Calculated Using Data of Bon/Dom(2002)

|   | 1       | 2       |
|---|---------|---------|
|--1--| 0.78406 | 0.21594 |
|--2--| 0.52619 | 0.47381 |

Eigenvalue of Matrix A of Proposition 1 Calculated Using Data of Bon/Dom (2002)

|--1--| 1.09738 |

Figure 7: Evaluation of the Existence of the Expected Utility Using the Data Reported By Bonomo and Rodrigues (2002). (Eigenvalue Higher Than One Implies Nonexistence of Expected Utility).

Constrained Approximation of Stat. Prob. and States for g in Bon/Dom (2002)

| S.P. | States   |
|------|----------|
| 1    | 0.50000  |
| 2    | 0.30000  |

Constrained Approximate Transition Matrix in Bon/Dom (2002)

|   | 1  | 2       |
|---|---|---------|
| 1 | 0.47092 | 0.52908 |
| 2 | 0.52908 | 0.47092 |

Constrained Matrix of Proposition 1 Calculated Using Data of Bon/Dom(2002)

|   | 1       | 2       |
|---|---------|---------|
|--1--| 0.62671 | 0.37329 |
|--2--| 0.70411 | 0.29589 |

Eigenvalue of the Constrained Matrix A of Prop. 1 Calc. Using Data of Bon/Dom (2002)

|--1--| 0.99923 |

Figure 8: Calculation of the Range Under Which The Existence of Expected Utility in Bonomo and Rodrigues (2002) Could Be Insured
| Rec. Utility, Risky Return For Rho=2 and Alpha = 5, 8 |  
| --- |  
| --1-->|  
| 1.03688 | 1.03346 |

| Rec. Utility, Risky-Free Rate For Rho=2 and Alpha = 5, 8 |  
| --- |  
| --1-->|  
| 1.02604 | 1.01652 |

| Rec. Utility, Equity Premia For Rho=2 and Alpha = 5, 8 |  
| --- |  
| --1-->|  
| 0.01084 | 0.01694 |

Figure 9: Results With Recursive Utility
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