Big Bang Nucleosynthesis with an Evolving Radion in the Brane World Scenario

B. Li† and M. -C. Chu‡

Department of Physics, The Chinese University of Hong Kong, Hong Kong SAR, China

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We consider the big bang nucleosynthesis (BBN) in the Brane world scenario, where all matter fields are confined on our 3-brane and the radion of the Brane evolves cosmologically. In the Einstein frame fundamental fermion masses vary and the results of standard BBN (SBBN) are modified. We can thus use the observational primordial element abundances to impose constraints on the possible variations of the radion. The possibility of using the evolving radion to resolve the discrepancies between the Wilkinson Microwave Anisotropy Probe (WMAP) and SBBN values of the baryon-to-photon ratio ($\eta$) is also discussed. The results and constraints presented here are applicable to other models in which fundamental fermion masses vary.

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I. INTRODUCTION

The Brane world scenarios have been attracting much attention during recent years, being motivated by the developments in superstring theory and acting as alternatives to supersymmetric or technicolor models in addressing the gauge hierarchy problem. In such scenarios the matter fields are confined in our world, which is believed to be a 3-dimensional brane embedded in a $1 + 3 + n$-dimensional bulk spacetime, while gravitation fills all the dimensions. In Ref. [1] Arkani-Hamed, Dimopoulos and Dvali (ADD) proposed a model in which the fundamental energy scale can be as low as TeV while the weakness of gravity is due to the large compactification radius of extra dimensions (millimeter in case of two extra dimensions, for example). A different setup was introduced by Randall and Sundrum (RS) [2] later, in which our brane is one (the negative tension one) of the two boundaries of a 5-dimensional anti-de Sitter (AdS$_5$) spacetime and the gravitation is localized around the positive tension brane while observers on our brane only measure a weak tail of it. In some works, e.g., [3, 4, 5], the various matter fields are also allowed to propagate along the extra dimensions and interesting consequences are discovered such as a different unification scale and the localization of zero-mode fermions onto our brane.

Brane model cosmology has also been investigated extensively (see, e.g., [6, 7] for an introduction). In particular, it is found [6] that the gravitational equations on our 3-brane (in a 5-dimensional bulk, for example) appear in the form

\[
(4) \, G_{\mu \nu} = -\Lambda_4 g_{\mu \nu} + 8\pi G_N \tau_{\mu \nu} + \frac{64 \pi^2}{M_{Pl}^2} \tau_{\mu \nu} - E_{\mu \nu}, \tag{1}
\]

where $\tau_{\mu \nu}$ is the energy momentum tensor confined on the brane and $M_{Pl}$ the fundamental (5-dimensional) Planck mass; $\Lambda_4$ is the effective 4-dimensional cosmological constant which is related to the bulk cosmological constant $\Lambda$ and the brane tension $\lambda$ by

\[
\Lambda_4 = \frac{4\pi}{M_{Pl}^2} \left( \Lambda + \frac{4\pi}{3M_{Pl}^2} \lambda^2 \right). \tag{2}
\]

The 4-dimensional Newtonian constant $G_N$ is defined by

\[
G_N = \frac{4\pi \lambda}{3M_{Pl}^2}. \tag{3}
\]

The $\tau_{\mu \nu}$ term in Eq. (1) is a quadratic function of $\tau_{\mu \nu}$ and negligible when the energy density on the brane is much smaller than the brane tension, i.e., $\tau_{\mu \nu}/\lambda \ll 1$, and the last term is a part of the high dimensional Weyl tensor carrying information of the gravitation outside the brane, which acts in 4-dimensional theory as a relativistic energy component – the so-called dark radiation. If the non-conventional field equations hold down to low energies and the dark radiation term is nonzero, then the predictions of the standard cosmological model, such as BBN and CMB (Cosmic Microwave Background), might be modified dramatically. Consequently the observations of primordial element abundances and CMB power spectrum would place stringent constraints on the parameters in Eq. (1) [6, 8, 9].

In Ref. [10] the authors considered the effects of a non-standard cosmic expansion rate on the outputs of BBN; yet there is another source for modifying the SBBN yields in the Brane World scenario – namely the variation(s) of the fundamental constant(s). This is because, although in most extra dimensional models the moduli fields (for instance the radion in the brane model) are believed to have been stabilized before the commencement of the primordial nucleosynthesis, it is not necessary to dismiss the possibility of cosmological evolution of the moduli fields in order to avoid the long range forces and deviations from general relativity (GR) [11]. It is conceivable that there is a cosmological attractor mechanism as in some scalar-tensor theories, driving the model towards the conventional GR [12]; it is also probable that the radion itself is a chameleon field [13], acquiring a large effective

*Email address: bli@phy.cuhk.edu.hk
†Email address: mchu@phy.cuhk.edu.hk
mass via its self-interaction together with the interaction with matter and thus evading the constraints from local gravitational measurements while evolving cosmologically \[14, 15\]. Such a time evolution of the radion field will lead to variations of fundamental constant(s). BBN turns out to impose the most stringent constraints on the possible time evolution of the extra dimensions because it is more sensitive than, say, CMB to the changes of the fundamental constants. Therefore, it is of interest in the present work to find out how the outputs of BBN are influenced by an evolving radion field (See \[16, 17, 18\] for the BBN constraints on other specific models and \[16, 20, 21, 22, 23\] etc. for constraints on the variations of individual fundamental constants such as the fine structure constant and the strong coupling).

This article is arranged as following: in Sec. II we discuss how an evolving radion field changes the fundamental constant(s) and thus leads to modified predictions of BBN. We will work in the Einstein frame where \(G_N\) stays unaltered and show that the only one varying fundamental constant in the model is the Higgs vacuum expectation value (VEV) \(v = \langle H \rangle\). The BBN constraints on a changing Higgs VEV have been investigated previously in \[24, 25\] and more recently in \[26\], but ours differs from the previous works in several ways: firstly, we consider some effects ignored in previous works, such as the radiative and Coulomb corrections to the neutron lifetime (and weak interaction rates), their radion dependence and the radion dependence of the \(p(n, D)\gamma\) cross section; secondly, we have done a complete likelihood analysis using the recent compilation on the nuclear rates and uncertainties \[27, 28\] and measurements of the primordial abundances of \(\text{D}, \text{^4He}\) and \(\text{^7Li}\); thirdly, we show that the \(\text{^7Li}\) yields could be changed significantly with a small evolution of the radion, tending to reduce the inconsistency between the SBBN and WMAP-implied values of \(\eta\), the baryon-to-photon ratio \[29\]. Because of the accuracy of the WMAP result, we also use it to constrain the variation of the radion. Our numerical results obtained from a modified BBN code is presented in Sec. III and then Sec IV is devoted to a discussion. Throughout this work we will assume three species of massless neutrinos (or negligible neutrino masses) and zero chemical potentials for neutrinos as in SBBN.

## II. The Influences of an Evolving Radion on BBN

In this section we discuss the influences of an evolving radion on the primordial nucleosynthesis. We first write down the low energy action in the 4-dimensional effective theory and show that only the Higgs VEV (and thus fundamental fermion masses) changes. For simplicity we will concentrate on large flat extra dimensions; the more realistic models with warped extra dimensions are discussed in the literature, e.g., \[12\]. (In \[12\] the authors considered a general class of two brane models, including the RS model as a special case; the time dependence of fermion masses in the Einstein frame and time independence of gauge couplings can also be found there.) Then we briefly discuss the implications on BBN. The derivations in this section closely follow our recent work \[18\].

### A. The Low Energy Effective Action

Let us start from a general \(4 + n\)-dimensional model, with \(n\) being the number of the large extra dimensions. The full line element is given as:

\[
\begin{align*}
\text{d}s^2 &= G_{AB} \text{d}X^A \text{d}X^B = g_{\mu\nu} \text{d}x^\mu \text{d}x^\nu + h_{ab} \text{d}y^a \text{d}y^b,
\end{align*}
\]

(4)

where \(\mu, \nu = 0, 1, 2, 3\) label the four ordinary dimensions, \(a, b = 4, \cdots, 3 + n\) label extra dimensions and \(A, B = 0, 1, 2, \cdots, 3 + n\) describe the whole spacetime. (We shall not consider cross terms such as \(G_{ap}\) in Eq. (4)).

The extra dimensions are assumed to compactly on an orbifold, and their coordinates \(y^a\) take values in the range \([0, 1]\). The quantities \(h_{ab}\) have dimensions of [Length]^2 since \(y^a\) are dimensionless in our choice.

Then the effective 4-dimensional action in the gravitational sector can be obtained by dimensionally reducing Eq. (4) as:

\[
\begin{align*}
S_{\text{Gravity}} &= \frac{1}{\kappa_{4+n}^2} \int d^{4+n}X \sqrt{|G|} R_{4+n}[G] \\
&= \frac{1}{\kappa_4^2} \int d^4x \sqrt{|g|} \left[ R_4[g] - \frac{1}{4} \partial_\mu h^{ab} \partial^\mu h_{ab} - \frac{1}{4} h_{ab} \partial_\mu h_{ab} \cdot h_{cd} \partial^\mu h_{cd} \right],
\end{align*}
\]

(5)

in which \(|g|, |h|\) and \(|G|\) are respectively the determinants of the metrics of the ordinary dimensions, the extra dimensions and the whole spacetime. \(R_4[g]\) and \(R_{4+n}[G]\) are the Ricci scalars of the ordinary 4 and the total \(4+n\) dimensional spacetimes. \(\kappa_4, \kappa_{4+n}\) are related to the 4 and \(4+n\) dimensional Planck masses through \(\kappa_4^2 = 2M_{Pl,4}^2\) and \(\kappa_{4+n}^2 = 2M_{Pl,4+n}^2\), while they themselves are con-
nected by a volume suppression \( \kappa_{4+n}^2 = \kappa_4^2 \cdot V \), \( V \) being a measure of the extra space volume whose present-day value is denoted by \( V_0 \) in Eq. (5) (Note that because of the specified choice of \( V_0 \) and because the higher dimensional quantity \( \kappa_{4+n} \) is treated as a constant, the \( \kappa_4 \) above also takes its currently measured value and is a constant rather than a variable).

The effective Ricci curvature term is not canonical in Eq. (5); to make it so, let us take the conformal transformation

\[
g_{\mu\nu} \to e^{2\vartheta} g_{\mu\nu} \tag{6}
\]

and choose the field \( \vartheta \) to satisfy

\[
\frac{\sqrt{|h|}}{V_0} e^{2\vartheta} = 1. \tag{7}
\]

Then we obtain the effective 4-dimensional gravitational action in the Einstein frame:

\[
S_{\text{Gravity}} = \frac{1}{\kappa_4^2} \int d^4 x \sqrt{|g|} \left[ R_4 - \frac{1}{4} \partial_\mu h^{ab} \partial^\mu h_{ab} + \frac{1}{8} h^{ab} \partial_\mu h_{ab} \cdot h^{cd} \partial^\mu h_{cd} \right]. \tag{8}
\]

Next we turn to the matter sector of the effective action, firstly for the scalar fields. The action of a brane scalar field \( \phi \) is given by:

\[
S_\phi = \int d^4 x \sqrt{|g|} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \hat{U}(\phi) \right], \tag{12}
\]

because the \( n \) large extra dimensions are inscient to the field and have been integrated out. Then the same conformal transformation given in Eqs. (6) and (7) transforms Eq. (12) into the following form:

\[
S_\phi = \int d^4 x \sqrt{|g|} \left\{ \frac{1}{2} \exp \left[ -\kappa \sqrt{\frac{n}{n+2}} \right] g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \exp \left[ -2\kappa \sqrt{\frac{n}{n+2}} \right] \hat{U}(\phi) \right\}. \tag{13}
\]

Note that from now on we will use \( \kappa \) instead of \( \kappa_4 \) for simplicity. The kinetic term of the scalar field in Eq. (13) could be made canonical by redefining a new scalar field as

\[
\varphi \equiv \exp \left[ -\frac{1}{2} \kappa \sqrt{\frac{n}{n+2}} \right] \phi, \tag{14}
\]

and the action becomes (up to the higher order derivative term):

\[
S_\varphi = \int d^4 x \sqrt{|g|} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - U(\varphi) \right], \tag{15}
\]

where the potential \( U(\varphi) \) is given by

\[
U(\varphi) = \frac{1}{2} \exp \left[ -\kappa \sqrt{\frac{n}{n+2}} \right] m_\varphi^2 \varphi^2 + \lambda \varphi^4 \tag{16}
\]

provided that the original potential \( \hat{U}(\phi) \) takes the following form:

\[
\hat{U}(\phi) = \frac{1}{2} m_\varphi^2 \varphi^2 + \lambda \varphi^4. \tag{17}
\]

The same technique could be applied to gauge fields, whose action before conformal transformation is given as

\[
S_{\text{Gauge}} = - \int d^4 x \sqrt{|g|} \frac{1}{4 g^2} F^{\mu\nu} F_{\mu\nu}, \tag{18}
\]
where \( \hat{g} \) is the gauge coupling constant (a tilde is used to distinguish it from \( g \), the determinant of the metric) and \( \hat{F}_{\mu\nu} \) are corresponding gauge field strengths. With the conformal transformation Eq. (6) the above action is changed into

\[
S_{\text{Gauge}} = - \int d^4x \sqrt{|\hat{g}|} \frac{1}{4 \hat{g}^2} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu}, \tag{19}
\]

where \( \hat{g} \) is unchanged because \( F_{\mu\nu} = \hat{F}_{\mu\nu} \) have zero conformal weights and are unaltered under conformal transformations.

We can also obtain the effective action for the Dirac fermion field in a similar way. Starting from the brane action:

\[
S_\Psi = \int d^4x \sqrt{|g|} \left\{ i \bar{\Psi} \gamma^\mu D_\mu \Psi - \exp \left[ -2g \sqrt{\frac{n}{n + 2}} \bar{\Psi} \Psi \right] \right\}. \tag{20}
\]

and using the conformal transformation Eq. (6) we get

\[
S_\Psi = \int d^4x \sqrt{|g|} \left\{ \exp \left[ \frac{3\kappa}{2} \sqrt{\frac{n}{n + 2}} \right] i \bar{\Psi} \gamma^\mu D_\mu \Psi - \exp \left[ -2g \sqrt{\frac{n}{n + 2}} \bar{\Psi} \Psi \right] \right\}. \tag{21}
\]

To make the kinetic part of the fermion action Eq. (21) canonical, we rescale the field as

\[
\psi = \exp \left[ -\frac{3\kappa}{4} \sqrt{\frac{n}{n + 2}} \right] \Psi, \tag{22}
\]

and then using the conformality of the coupling of massless Weyl fermions, we rewrite Eq. (21) as:

\[
S_\psi = \int d^4x \sqrt{|g|} \left\{ i \bar{\psi} \gamma^\mu D_\mu \psi - m(\sigma) \bar{\psi} \psi \right\}, \tag{23}
\]

in which

\[
m(\sigma) = \exp \left[ -\frac{\kappa}{2} \sqrt{\frac{n}{n + 2}} \right] \bar{\psi} \psi. \tag{24}
\]

Note that our results above are equal to those of [30] when there are no universal extra dimensions (i.e., \( n = 0 \) in their model).

It is apparent from Eq. (24) that if the radion \( \sigma \) evolves, then the fermion masses also vary. As in the standard model, we assume that the fundamental fermion masses are generated by the Higgs mechanism and that the Higgs potential takes the form as Eq. (17). We find from Eq. (16) that the Higgs VEV \( \langle v \rangle \), which is obtained by simply minimizing the Higgs potential, has the same radion dependence as the fermion masses (Eq. (24)) so that the Yukawa couplings are independent of radion. Recalling also that the gauge couplings would not depend on radion (Eq. (19)), which is because of, as pointed above, the conformal invariance of the gauge kinetic term and this is true in general scalar tensor gravity theories [16]. So here we indeed encounter a model in which the only varying fundamental constant is \( v \). On the other hand, if the gauge fields exist in the full dimensions as suggested by the universal extra dimensional scenario [5], then the above conclusion does not hold anymore and the gauge couplings will be dependent on the radion field [16].

**B. BBN with a Varying Higgs VEV**

The primordial nucleosynthesis with a varying Higgs VEV has been discussed previously in [24, 25, 26], and here we shall only briefly identify the places where effects of Higgs VEV enter. We also omit the introduction to the BBN theory and observation and refer these to existing literatures, e.g., [31, 32, 33, 34, 35, 36, 37].

A different \( v \) from its standard value will modify BBN mainly in two aspects, the weak interactions and the nuclear reactions. Consider firstly the weak interactions

\[
\nu_e + n \leftrightarrow p + e^-, \quad e^+ + n \leftrightarrow p + \bar{\nu}_e, \quad n \leftrightarrow p + e^- + \nu_e,
\]

which interconvert the neutrons and protons. Roughly speaking, they are important for BBN because they determine the neutron density at the beginning of BBN, and, because nearly all the neutrons are incorporated into \(^4\text{He}\) at last, they are crucial for the final \(^4\text{He}\) output (for more detailed discussions see e.g., [22]). The rates of these \( n \leftrightarrow p \) interactions could be well described as

\[
\Gamma(n \rightarrow p) = A \int_1^\infty \frac{e^2}{1 + \exp(-\epsilon z_n)} \cdot \frac{(e - q)^2(e^2 - 1)^{1/2}}{1 + \exp(-\epsilon z_n)} \cdot \frac{(e + q)^2(e^2 - 1)^{1/2}}{1 + \exp(\epsilon z_n)} \cdot \frac{d\epsilon}{1 + \exp(\epsilon z_n)} + A \int_1^\infty \frac{d\epsilon}{1 + \exp(\epsilon z_n)} \cdot \frac{d\epsilon}{1 + \exp(-\epsilon z_n)} \cdot \frac{(e - q)^2(e^2 - 1)^{1/2}}{1 + \exp(-\epsilon z_n)} \cdot \frac{(e + q)^2(e^2 - 1)^{1/2}}{1 + \exp(\epsilon z_n)}.
\tag{25}
\]
\[
\Gamma(p \rightarrow n) = A \int_1^\infty d\epsilon \frac{\epsilon(\epsilon - q)^2(\epsilon^2 - 1)^{1/2}}{[1 + \exp(\epsilon z_\nu)] \{1 + \exp[\epsilon(q - z_\nu)]\}} + A \int_1^\infty d\epsilon \frac{\epsilon(\epsilon + q)^2(\epsilon^2 - 1)^{1/2}}{[1 + \exp(-\epsilon z_\nu)] \{1 + \exp[\epsilon + q(z_\nu)]\}}.
\]

(26)

using the Born approximation, where we have defined dimensionless quantities \( q = m_{np}/m_\epsilon, \epsilon = E_\epsilon/m_\epsilon \) and \( z_\nu = m_\nu/T_\nu \) and \( z_\epsilon = m_\epsilon/T_\epsilon \) with \( m_\epsilon \) being the electron mass, \( m_{np} \) the neutron-proton mass difference and \( T_\nu, T_\epsilon \) the temperature of the neutrinos (the electromagnetic plasma). Here \( A \) is a normalization factor determined by the requirement that at zero temperature \( \Gamma(n \rightarrow p + e^- + \bar{\nu}_e) = \tau_n^{-1} \) with \( \tau_n \) the neutron lifetime, which means that:

\[
A = \tau_n^{-1} \lambda(q)^{-1} \propto G_F^2 m_\epsilon^5,
\]

(27)

where

\[
\lambda(q) = \int_1^q d\epsilon \frac{\epsilon(\epsilon - q)^2(\epsilon^2 - 1)^{1/2}}{[1 + \exp(\epsilon z_\nu)] \{1 + \exp[\epsilon(q - z_\nu)]\}}.
\]

(28)

From Eqs. (25)-(28) we can see that \( v \) determines the weak rates through \( G_F, m_\epsilon \) and \( m_{np} \) respectively. The dependences of \( G_F \) and \( m_\epsilon \) on \( v \) can be simply parameterized as

\[
G_F \propto \frac{1}{v^2}, \quad m_\epsilon \propto v.
\]

(29)

(30)

The neutron-proton mass difference \( m_{np} \) is a sum of the electromagnetic contribution \( \sim -0.76 \text{ MeV} \) and the \( u-d \) quark mass difference \( \sim 2.053 \text{ MeV} \), for which the former is unchanged in the present model because the (electromagnetic and strong) gauge couplings are radion independent while the latter is proportional to the Higgs VEV. So \( m_{np} \) could be described as

\[
m_{np} = 2.053 \rho \text{ MeV} - 0.76 \text{ MeV},
\]

(31)

in which

\[
\rho \equiv \frac{v_{\text{BBN}}}{v_{\text{NOW}}}
\]

(32)

is the ratio between the Higgs VEVs at the BBN era and at present. \( m_{np} \) is important also because it determines the equilibrium neutron-to-proton ratio through

\[
\frac{n_n}{n_p} = \exp \left[ -\frac{m_{np}}{T} \right].
\]

(33)

Although the Born approximation in Eqs. (25) and (26) captures the essential features, various corrections are needed for more accurate estimations (see e.g., [37, 38, 39, 40]). For example, the accuracy of the theoretical value of \( \tau_n \) from Born approximation is \( \sim 7\% \) while including the zero-temperature and Coulomb corrections reduces it to be \( \sim 0.1\% \). These two corrections are the most important ones and the radiative correction itself depends on \( m_{np} \) (and thus on \( \rho \)); in our calculation we numerically integrate Eqs. (25) and (26) and include these corrections explicitly. We have neglected other corrections for simplicity and as the accuracy is adequate for our purpose here (see [37, 38, 39, 40] for discussions on this issue).

Next we turn to the nuclear reaction sector. The change of \( v \) will modify the pion mass \( m_\pi \), which is related to the light quark mass \( m_q \)

\[
m_\pi \propto m_q^n \propto \rho^5.
\]

(34)

According to recent investigations of the dependence of the nuclear potential on the pion mass [42, 43], the deuteron binding energy \( B_d \) shows a strong decrease when \( m_\pi \) increases; although the relationships between \( B_d \) and \( m_\pi \) as obtained in [42, 43] have large uncertainties, they could be well approximated by

\[
B_d = B_{d,0} \left( r + 1 - \frac{r m_\pi}{m_{\pi,0}} \right)
\]

(35)

within the small range of \( m_\pi \) (or \( \rho \)) we are working with [26, 41], in which \( B_{d,0} \) and \( m_{\pi,0} \) denote their present-day values and the central value of \( r \) ranges from 6 [42] to 10 [43]. In the following we shall work with \( r = 10 \) and 6 in parallel and show that the resulting qualitative features are the same.

The deuteron binding energy plays an important role in BBN because it determines whether a significant amount of D could be produced (via the reaction \( p(n, D)\gamma \)), which then leads to the synthesis of heavier elements. Because of the huge number of photons (recall that \( \eta \sim 10^{-10} \)), the inverse process \( D + n \rightarrow p + d + \gamma \) could be active down to low energies (\( \sim 0.08 \text{ MeV} \)) and sensitive to the variations in \( B_d \) [22, 44]. Besides, the forward cross section of \( p(n, D)\gamma \) will also depend on \( B_d \) [15]: to estimate this dependence, we adopt the cross section formula calculated using the effective field theory without pion [46, 47]. The authors of [46, 47] gave an expression of the \( p(n, D)\gamma \) cross section in terms of several parameters in which the deuteron binding energy \( B_d \) has by far the largest leverage. In our modified BBN code we average the cross section given in [46] over the thermal distribution of the particles numerically and obtain the reaction rate as a function of the temperature [48]; we have checked that the results obtained in this way are essentially identical to those obtained using the fitted \( p(n, D)\gamma \) rates of [32, 45].
FIG. 1: The primordial abundances of $^4\text{He}$, $^D$ and $^7\text{Li}$ as a function of the baryon-to-photon ratio $\eta$, for various values of $\rho$ as indicated by the legend in the lower panel and $r = 10$. Also shown are the observational $1\sigma$ ranges of the abundances of $^4\text{He}$ from [35] (grey region) and [59] (dark grey region), $^D$ from [60] (grey region), $^7\text{Li}$ from [61] (grey region), as well as the baryon-to-photon ratio implied by WMAP [29], $\eta_{\text{WMAP}} = (6.14 \pm 0.25) \times 10^{-10}$ (the vertical lines).

For the other reactions which are of importance to the $^D$ yields, such as $^D(D,D^3\text{He})$, $^D(D,p)^3\text{H}$, $D(p,\gamma)^3\text{He}$, $D(3\text{H},n)^4\text{He}$ and $D(3\text{He},p)^4\text{He}$, their cross sections may also be dependent on $m_\pi$ or $B_d$ but we have no similar effective field theory calculations on them; so we ignore them in the present article. If we simply take these cross sections to be related to the size of the deuteron radius, i.e., $\sigma \propto 1/B_d$, as in Ref. [22], then in the interested range of $\eta$ and $\rho$ (See FIG. 5 below) we find modifications to the final outputs of $^4\text{He}$ and $^7\text{Li}$ by $< \sim 6\%$, $< \sim 7 \times 10^{-4}$ and $< \sim 3\%$ respectively, all well lying within the corresponding $1\sigma$ observational uncertainties ($\sim 16\%$ for $^4\text{He}$, $\sim 0.84\%$ for $^4\text{He}$ and $\sim 21\%$ for $^7\text{Li}$, see Eqs. (36)-(38) below). In the present work we do not adopt the $1/B_d$ parametrization of these cross sections due to the lack of more explicit expressions; rather we treat the above estimations as possible errors introduced by neglecting their $m_\pi$ or $B_d$ dependences and emphasize that further related theoretical calculations are needed to reduce these errors.

Note that the variation of the Higgs VEV probably will also cause modifications in the binding energies of other nuclei and thus change the corresponding reverse reaction rates. However there are again no explicit calculations of these modifications. Fortunately these effects are small because the abundances of these heavier nuclei fall far below their nuclear statistical equilibrium (NSE) values long before the beginning of BBN and the reverse reactions are effectively switched off. For example, according to our constraints on $\rho$, i.e., $\rho \sim 1.01$ (see below), the deuteron binding energy $B_d$ will be decreased by $\sim 5\%$. Let us suppose that the other binding energies have variations of similar magnitude, since different nuclei may appear in the left-hand and right-hand sides of a reaction (e.g., $^3\text{He}$ and $^3\text{H}$ in $^3\text{He}(n,p)^3\text{H}$) and we do not know whether the variations of their binding energies correlate or anti-correlate. We thus arrived at a conservative estimation of $\sim 10\%$ for the Q-values of these reactions. If the Q-values of all the reactions relevant to BBN are changed by $10\%$, then the abundances of $^D$, $^4\text{He}$ and $^7\text{Li}$
FIG. 3: Upper panel: individual contours of D (dotted), $^4$He (dashed) and $^7$Li (dot-dashed) at 68% confidence level. Lower panel: joint contours of $D^+ + ^4He + ^7Li$ at 68% (dark grey region) and 95% (grey region) confidence levels; the best-fitting parameters $(\eta, \rho) \simeq (6.28 \times 10^{-10}, 1.009)$ are denoted by the white circle and the case of SBBN by the horizontal dash-dotted line. In both panels the range of $\eta_{\text{WMAP}}$ is represented by vertical solid lines. Here $r$ is taken to be 10.

would be varied by no more than $\sim 0.1\%$, $\sim 5 \times 10^{-5}$ and $\sim 0.5\%$ respectively, in the range of $\eta$ of interest.

There is also a modification to the cosmic expansion rate in the present model. As the Newtonian constant $G_N$ is unchanged in the Einstein frame, this modification originates from the variation of energy density compared with SBBN — due to the modified electron and positron masses as described in [18, 26]. We include this effect in the calculation although it is small compared with others.

III. NUMERICAL RESULTS

We have incorporated all the effects discussed in the previous section into the standard BBN code by Kawano [51] and used it to obtain the numerical results presented in this section. The nuclear reaction rates are taken from the NACRE compilation [27] (for the rates not given in [27] we adopt those of [52]) and their uncertainties from the work of Cyburt, Fields and Olive [28]. For the present neutron lifetime we use the value suggested by the Particle Data Group [52], $\tau_{\text{ex}}^{\text{ee}} = 885.7 \pm 0.8$ s.

In Fig. 1 we plot the abundances of the light nuclei $^4$He, D and $^7$Li with a modified Higgs VEV ($v$), which is characterized by $\rho$, in the case of $r = 10$. It is apparent that if $v$ is larger at the BBN era ($\rho > 1$), then the deuterium output increases while the $^4$He and $^7$Li (for $^7$Li we only consider the larger-$\eta$-case implied by the WMAP result [29]) yields decrease compared with SBBN. The behavior of the $^4$He output is a consequence of several effects: firstly, a $\rho$-value larger than 1 leads to a larger $m_{\text{ap}}$ (from Eq. (31)) and a smaller neutron density at the beginning of the nucleosynthesis (from Eq. (33)), thus finally to a smaller $^4$He output; secondly, the weak interaction rates will be smaller than those in SBBN, leading to increased final $^4$He abundance. Thirdly, the deuteron binding energy $B_d$ becomes smaller (from Eq. (35)) than in SBBN so that the nucleosynthesis will commence at a later time and become less efficient, producing less $^4$He and $^7$Li while leaving more D unprocessed (there is an extra decrease in the forward rate of $p(n, D)_\gamma$, whose influence is small compared with the effect of commencing BBN later). The sum of these effects is a smaller $^4$He abundance (see Fig. 1 of [26] for a more explicit comparison among these effects). For comparison we also plot in Fig. 2 the case of $r = 6$, where the effect of $B_d$ be-
comes weaker than \( r = 10 \) but the essential features are the same.

The results in Figs. 1 and 2 suggest that the discrepancy between the SBBN theory and WMAP observations is reduced if \( \rho \) is greater than 1. It is well known that adopting the WMAP-implied value of \( \eta \), the SBBN can reproduce the observed D abundance, but more \( ^4 \text{He} \) and \( ^7 \text{Li} \) than their observational abundances. One possibility is that this discrepancy is due to some systematic errors in the \( ^4 \text{He} \) and \( ^7 \text{Li} \) observations. However, it may also originate from some new physics, such as a varying fine structure constant \( \alpha \), an altered deuteron binding energy \( E_\text{d} \), more than three relativistic (effective) neutrino species \( \nu \), a lepton asymmetry \( \eta \), or some combination of them. Our results show that we might add another candidate, a varying Higgs VEV, to this list. To see this point more quantitatively, and to derive a constraint on the possible evolution of the radion, next we will give the likelihood analysis of this model. For this purpose, we choose to use the linear propagation approach proposed by Fiorentini et al., and then generalized by Cuoco et al., to estimate the error matrix. The observational abundances and uncertainties of \( ^4 \text{He}, ^\text{D} \) and \( ^7 \text{Li} \) are taken from Luridiana et al., Kirkman et al., and Bonifacio et al., respectively.

\[
Y_P^{\text{obs}} = 0.2389 \pm 0.0020; \quad \alpha \eta_0 = 2.78_{-0.38}^{+0.44} \times 10^{-5}; \quad \beta_\rho = 2.19_{-0.38}^{+0.46} \times 10^{-10}
\]

(Note that the \( ^4 \text{He} \) abundance is quantified by its mass fraction.) These observational results are also shown in Figs. 1 and 2 as grey regions, and in Fig. 1 we also show the earlier wider range of \( ^4 \text{He} \) given by Olive, Steigman and Walker.

by the dark grey region as a comparison, even though we will not use it in our likelihood analysis.

The results for \( r = 10 \) are plotted in the upper panel of Fig. 3, in which the joint constraints from \( \text{D}^+ + ^4 \text{He} + ^7 \text{Li} \) at 68% and 95% C.L. are shown. Our best-fitting point in the parameter space (the white circle) is at \( \eta \sim 1.009 \) and \( \rho \sim 6.28 \times 10^{-10} \). This central value of \( \eta \) is slightly larger than that of \( \eta_{\text{WMAP}} = 6.14_{-0.25}^{+0.25} \times 10^{-10} \). To constrain the SBBN (horizontal line) is only allowed marginally at 95% C.L. and should be further excluded at the same level with \( \eta_{\text{WMAP}} \) taken into account. A similar situation is presented for \( r = 6 \) (Fig. 4), while the details are a little different: here the best-fitting point \( \sim (6.67 \times 10^{-11}, 1.02) \) lies slightly outside the 1\(\sigma \) range of \( \eta_{\text{WMAP}} \) and SBBN is excluded at the 95% C.L. using the observational primordial abundances only. In any case we see an improvement compared with SBBN if \( \rho \) is appropriately chosen (note also that, if the 1/\( B_d \) scaling of the cross sections discussed in Sec. II B is a good approximation, then \( \rho > 1 \) will also increases the rates of these reactions by reducing \( B_d \), making the destructions of D more efficient; this will lead to less D yields and shift the D and joint contours leftward).

In some cases we may be interested in how \( \rho \) alone is allowed to change, and for this purpose it proves to be convenient to use the constraint on \( \rho \) from the WMAP measurement (the \( \eta_{\text{WMAP}} \) given above) by virtue of its high accuracy. We shall construct a likelihood function of \( \rho \) only as:

\[
L(\rho) \propto \int_{-\infty}^{\infty} \exp \left[-\frac{(\eta - \eta_0)^2}{2\sigma_\eta^2}\right] \exp \left[-\frac{\chi^2(\eta, \rho)}{2}\right] d\eta,
\]

in which \( \eta_0 \) and \( \sigma_\eta \) are respectively the central value and 1\(\sigma \) range for \( \eta \) given by WMAP; \( \chi(\eta, \rho) \) is a function of both \( \eta \) and \( \rho \) calculated using the same method as above (for \( \text{D}^+ + ^4 \text{He} + ^7 \text{Li} \)). The results are shown in Fig. 5, where we give the cases for both \( r = 10 \) and \( r = 6 \) and we have found that the 95% C.L. ranges for \( \rho \) in these two cases are \( \rho = 1.0089 \pm 0.0012 \) and \( \rho = 1.0182 \pm 0.0024 \), corresponding respectively to changes in the extra space volume \( V \) as:

\[
1.52 \times 10^{-2} \lesssim \frac{\Delta V}{V_0} \lesssim 1.99 \times 10^{-2}
\]
where $\Delta V = V_0 - V_{BBN}$ is the difference between the extra space volumes at present and at the BBN era. The allowed variation of $\rho$ (or equally the Higgs VEV) we find here is comparable to the value quoted in $[26]$, but the full likelihood analysis using the $\eta_{WMAP}$ constraint and the new observational abundances of $D$, $^4$He and $^7$Li presented here strongly disfavor $\rho \leq 1$. Furthermore, as seen from Fig. 5, the ranges for $\rho$ with $r = 10$ and $r = 6$ do not agree with each other at the 95% C. L., even though in both cases the qualitative features are the same. This simply reflects the demand of a more precise understanding of how the deuteron binding energy depends on the light quark masses.

If no constraint on $\eta$ is used, we obtain looser constraints on $\rho$, namely $\rho = 1.0091 \pm 0.0071$ for $r = 10$ and $\rho = 1.0198 \pm 0.0140$ for $r = 6$ (again at the 95% C. L.), which are comparable to the results above. In this case a variation of the Higgs VEV as large as $\sim 3.4\%$ compared with its present value is still allowed at the BBN era (redshift $z \sim 10^9 - 10^{10}$).

IV. DISCUSSIONS AND CONCLUSIONS

In summary, we have considered in this article the possible implications of a cosmologically evolving Brane moduli field on the outputs of the primordial nucleosynthesis. We begin with the discussions on how the fundamental constants are modified if the radion field of the Brane varies and how these modifications would affect the BBN yields. Some effects not considered previously are included in this procedure. Then we present a likelihood analysis using the recent compilation of the various nuclear reaction rates $[27]$ and observational abundances of the light nuclei $D$, $^4$He and $^7$Li $[59, 60, 61]$. The WMAP constraint on $\eta$ $[28]$ is also adopted in determining the constraint on the quantity $\rho$, which characterizes the variation of the Higgs VEV (and thus the evolution of the radion field in the present scenario). We find that the BBN yields could be changed in such a way that the discrepancy between SBBN and $\eta_{WMAP}$ might be reduced, provided that the Higgs VEV was slightly larger at the BBN era. This conclusion is robust within the present theoretical uncertainty of $r$ ($6 \sim 10$), and the errors introduced by neglecting the $\rho$-dependences of other nuclear reactions are estimated to well fall within the $1\sigma$ D, $^4$He and $^7$Li observational uncertainties. However, further developments in the understanding of these topics might help constrain $\rho$ more accurately. The constraints we obtain in this work are also applicable to other models in which the Higgs VEV (and fundamental fermion masses) varies cosmologically, and could be used to constrain parameters in such models.

It would be also interesting to consider other implications of such an evolving radion field. One example is its influences on the CMB; this was discussed in Refs. $[26, 62]$, and the authors found that these influences were mainly through the variation of the electron mass $m_e$ (see $[62]$ for more details). This might provide another constraint on the radion field, but it is much looser than the one we obtain and at a different cosmic era ($z \sim 10^3$). Another potential implication of interest is the possible existence of di-proton or di-neutron. This is because the bindings of the di-nucleon systems are mainly contributed by the long range pion force which depends on pion mass $m_\pi$. If $m_\pi$ was smaller at the time of BBN, then the bindings of di-proton/di-neutron would be enhanced, and if the di-proton becomes bound, it will open a rapid channel for the hydrogen fusion $[63]$ and thus be catastrophic to the stellar lifetimes. The conditions for the di-proton and di-neutron binding were studied in $[64, 65]$. In the present work, we see that $\rho > 1$ is preferred by our BBN analysis, which means that $m_{\pi,BBN} > m_{\pi,\text{NOW}}$ and the di-nucleon systems would be kept unbound during the BBN era. As the last example of possible implications, let us consider the stability of $^5$He which, if becoming stable, would fill the mass-5 gap in the BBN nuclear reaction chain, drastically enhancing the production of $^7$Li.

In $[23]$ the authors investigated this question and found that $\delta(m_q/\Lambda_{QCD})/(m_q/\Lambda_{QCD}) \gtrsim 0.1$ was required to prevent $^4$He from becoming stable, which is obviously satisfied by our constrained results.

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