Spin Hall Effect in a Doped Mott Insulator

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The existence of conserved spin Hall currents is shown in a strongly correlated system without involving spin-orbit coupling. The spin Hall conductivity is determined by intrinsic bulk properties, which remains finite even when the charge resistivity diverges in strong magnetic fields at zero temperature. The state in the latter limit corresponds to a spin Hall insulator. Such a system is a doped Mott insulator described by the phase string theory, and the spin Hall effect is predicted to coexist with the Nernst effect to characterize the intrinsic properties of the low-temperature pseudogap phase.

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INTRODUCTION

Recently, the proposals\textsuperscript{1,2} of manipulating spin currents by an electric field via spin Hall effect (SHE) have attracted a lot of attention, in which studies have been mainly focused on non-interacting or weakly interacting semiconductor systems with substantial spin-orbit (SO) coupling. Due to the SO coupling, an applied electric field can induce intrinsic spin Hall currents in a transverse direction, which can be well separated from the dissipative charge currents along the longitudinal direction.

But the intrinsic SHE in an SO system is usually very sensitive to disorder\textsuperscript{4,5} because the spin polarization is tied to momentum and the spin current is not conserved. Without the conservation law, the spin Hall conductivity studied in the linear response theory generally may not be directly connected to physical spin accumulations in such SO systems\textsuperscript{4,5}, and a disorder effect can drastically affect the latter, leading to the debate whether an intrinsic SHE can meaningfully exist in realistic systems.

It is thus a very interesting issue whether an intrinsic SHE can exist in electron systems without the SO coupling such that the conservation law still holds for the spin current and the spin transport becomes well-described. Without the SO coupling, however, the difficulty is how to effectively couple an electric field to a neutral spin current detached from the charge current. In other words, one needs to identify a system in which the spin current is not directly carried by the dissipative charge current, and at the same time the former can be directly driven by the electric field. For a weakly interacting system where elementary excitations are quasiparticles which carry both charge and spin under a Fermi-Liquid description, the spin and charge currents usually cannot be simply separated to find such kind of “dissipationless” spin Hall effect.

On the other hand, in the strongly correlated electron systems related to the high-$T_c$ cuprates, a (partial) separation of spin and charge degrees of freedom\textsuperscript{6} makes it possible to manipulate conserved spin currents which may be distinguishable from ordinary dissipative charge currents. In this paper, we shall make a proposal that “non-dissipative” spin Hall currents indeed exist in the so-called lower pseudogap phase (LPP) of the cuprates under the description of the phase string theory\textsuperscript{7}. To be sure, so far there is no consensus concerning the correct microscopic theory for high-temperature superconductivity. But making a self-consistent theoretical prediction is very meaningful and possibly the only way to subject a particular theory experimentally testable.

A quantitative prediction of the present work is that a conserved neutral spin current, $J_s^a$, along the $i$-th direction within the two-dimensional (2D) plane with the spin polarized in the $\hat{z}$-axis (out of the plane), can be generated by an applied electric field as follows:

$$J_s^a = \sigma_H^s \epsilon_{ij} E_j$$

with $\epsilon_{ij}$ as the 2D antisymmetric tensor and $E_j$ the $j$-th component of the electric field. Here the spin Hall conductivity is given by

$$\sigma_H^s = \frac{\hbar \chi_s}{g \mu_B} \left( \frac{B}{n_v \Phi_0/2} \right)^2$$

which only depends on the intrinsic properties of the system: $\chi_s$ is the uniform spin susceptibility and $n_v$ denotes the density of $s = 1/2$ neutral spin excitations, with the electron $g$-factor $g \simeq 2$, $\mu_B$ the Bohr magneton, and $\Phi_0 = h c/e$ the flux quantum. Note that an external magnetic field $B$ is applied perpendicular to the 2D plane, reducing the spin rotational symmetry of the system to the conservation of the $S^z$ component only, satisfying $\sigma_H^s + \nabla \cdot \mathbf{J}^s = 0$. In contrast, the charge current still remains dissipative and the resistivity may even become divergent at low temperature in a strong perpendicular magnetic field, leading to a spin Hall insulator where $\sigma_H^s$ still remains finite. In this sense we may say that the spin Hall current is dissipationless, following a similar characterization in an anomalous Hall effect system\textsuperscript{8}. 

\[\text{end}\]
Such an SHE with conserved and “non-dissipative” spin currents is present in a novel phase of the doped Mott insulator described by the phase string theory, known as the spontaneous vortex phase, which is characterized by a nonzero electron pairing amplitude $\Delta^0$ without true superconducting (SC) phase coherence. The SC pairing order parameter is given by

$$\Delta^{SC} = \Delta^0 e^{i\Phi^s} \quad (3)$$

where the phase $\Phi^s$ is composed of $\pm 2\pi$ vortices which are disordered in this state. This spontaneous vortex (SV) phase has been previously proposed to describe the LPP in the high-$T_c$ cuprate superconductors, featured by nontrivial Nernst effect. In the same theory, the Nernst signal is interpreted as contributed by the phase string theory in which $\Delta^0 \propto (\psi_m^2 - \overline{\psi}_m^2)$ still remaining finite. So the SV phase exists in a regime $T < T_c$, with $T_c$ as the characteristic temperature for the holon condensation. Generally, at $T < T_c$, no free vortices should appear in the condensate $\psi_h$, except for those $\pm 2\pi$ vortices in $\phi_h$ whose cores are bound to free spinons, which then can be always absorbed into $A^s \rightarrow \tilde{A}^s$ and $\Phi^s \rightarrow \tilde{\Phi}^s$, with

$$\tilde{\Phi}^s(r) = \int d^2r' \text{Im} \ln |z - z'| \left[ n^+(r') - n^-(r') \right] \quad (7)$$

where $n^\pm(r) = \sum \delta(r - r^\pm_l)$, with $r^\pm_l$ denoting the coordinate of the $l$-th spinon, carrying a $2\pi$ vorticity of sign $\pm$. So the sign of the vorticity for a vortex carried by a spinon is not directly associated with the spin index $\sigma$, thanks to the freedom in $2\phi_h$ in $\Delta^0$ or $\nabla \phi_h$ in $J$, which ensures the spin rotational symmetry of the system as previously discussed in Refs. \[8,11\].

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**Ginzburg-Landau description**

We start with a generalized Ginzburg-Landau (GL) description of the SV phase in the phase string theory \[11\]

$$\alpha \psi_h + \beta |\psi_h|^2 \psi_h + \frac{1}{2m_h} (-i \nabla - A^s - A^c)^2 \psi_h = 0 \quad (4)$$

where $\psi_h(r) = \sqrt{\rho_h} e^{i\phi_h(r)}$ describes the holon condensate and the charge current is determined by

$$J = \frac{\rho_h}{m_h} [\nabla \phi_h - A^s - A^c] \quad (5)$$

with $m_h$ as the effective mass. Here $A^c$ is the electromagnetic field, and $A^s$ is the internal gauge field defined by

$$A^s(r) = \frac{1}{2} \int d^2r' \frac{\hat{z} \times (r - r')}{|r - r'|^2} \left[ n^+_h(r') - n^-_h(r') \right] \quad (6)$$

in which $n^\pm_h(r)$ is the spin density. Physically $A^s$ depicts $\pm \pi$ fluxoids bound to spins, as “felt” by the holon condensate in $\tilde{A}^s$, which reflects the basic mutual influence between charge and spin degrees of freedom in the phase string theory \[12\].

The phase $\Phi^s$ in the SC order parameter \[8\] is related to $A^s$ by $A^s = \mp \Phi^s/2$ which gives rise to $\Phi^s(r) = \int d^2r \text{Im} \ln |z - z'| \left[ n^+_h(r') - n^-_h(r') \right]$. The SC phase coherence is realized at low temperatures when all spins are resonating-valence-bond (RVB) paired, leading to the cancellation in $\Phi^s(r) \tilde{\Phi}^s$. Free (unpaired) $s = 1/2$ spinons then give rise to free $\pm 2\pi$ vortices in $\Delta^{SC}$ via $\Phi^s$ and thus destroy the phase coherence. It results in the SV phase with $\Delta^0 \propto (\psi_m^2 - \overline{\psi}_m^2)$ still remaining finite. So the SV phase exists in a regime $T_c < T < T_v$, with $T_v$ as the characteristic temperature for the holon condensation.

![FIG. 1: (Color online) Vortices of $\pm$ vorticity can be driven by a perpendicular electric field to form a non-dissipative current and a spin current is simultaneously produced because each vortex is bound to an $S^s = \pm 1/2$ spin at the core in the spontaneous vortex phase (see text).](image)
We now consider some important consequences of this generalized GL theory. For a steady current state with \( \partial_t \mathbf{J} = 0 \), we find, in the transverse gauge, the electric field \( \mathbf{E} \equiv -\partial_t \mathbf{A}^e = \partial_t \mathbf{A}^s(r) \) in terms of \( \mathbf{A}^s \). Then by using \( \partial_t \mathbf{A}^s(r) = -\mathbf{z} \times \pi \sum \left[ \mathbf{t}^+_l \delta (\mathbf{r} - \mathbf{r}^+_l) - \mathbf{t}^-_l \delta (\mathbf{r} - \mathbf{r}^-_l) \right] \), the following relation can be established

\[
\mathbf{J}^s = -\frac{1}{\pi} \mathbf{E} \times \mathbf{z} \tag{8}
\]

with \( \mathbf{J}^s(r) = \sum \left[ \mathbf{t}^+_l \delta (\mathbf{r} - \mathbf{r}^+_l) - \mathbf{t}^-_l \delta (\mathbf{r} - \mathbf{r}^-_l) \right] \) depicting the vortex current. The Nernst signal generated by a vortex current flowing down along the temperature gradient \(-\nabla T\) has been shown \([6]\) based on \([5]\), which is a basic feature of the SV phase.

In the following, we focus on the case that \( \mathbf{J}^s \) is driven directly by the electric field, \( \mathbf{E} \), instead of by a temperature gradient \(-\nabla T\), as schematically illustrated in Fig. 1. Obviously, it is non-dissipative according to \([5]\) and since \([5]\) holds, locally individual vortices and antivortices will move in opposite directions with \( \mathbf{t}^+_l = -\mathbf{t}^-_l = \mathbf{v}^s \) in the uniform electric field, additively contributing to the vortex current

\[
\mathbf{J}^s = \left[ n^+(\mathbf{r}) + n^-(\mathbf{r}) \right] \mathbf{v}^s \tag{9}
\]

Here the densities of vortices and antivortices, \( n^\pm(\mathbf{r}) \), is constrained by the condition \( \langle \mathbf{t}_c \cdot \mathbf{r} \cdot \mathbf{J} \rangle = 0 \) for an arbitrary loop \( C \) such that on average

\[
n^+(\mathbf{r}) - n^-(\mathbf{r}) = -\frac{B}{\pi} \tag{10}
\]

based on \([5]\). Namely, the polarization of spinon-vortices and antivortices is determined by the external magnetic field applied perpendicular to the 2D plane. In the superconducting phase, without the presence of spontaneous (thermally excited) vortices, equation \( \langle 10 \rangle \) reduces to \( n^- = \frac{B}{2\pi} \) which represents the flux quantization condition by noting that the flux quantum \( \Phi_0 = 2\pi \) in the units of \( \hbar = c = e = 1 \) and the above GL theory predicts an \( s = 1/2 \) being always trapped at the core of a magnetic vortex \([11]\).

### Spin Hall effect

Now we focus on the spins carried by these spinon-vortices. Define \( n^\sigma_\pm(\mathbf{r}) \) as the spinon-vortex density with a vorticity \( \pm \) and a spin index \( \sigma \). Then \( n^\pm(\mathbf{r}) = \sum \frac{1}{2} n^\pm_\sigma(\mathbf{r}) \), and the spin current carried by spinon-vortices can be expressed as

\[
\mathbf{J}^s = \frac{1}{2} \sum \sigma \left( n^\sigma_\uparrow - n^\sigma_\downarrow \right) \mathbf{v}^s.
\]

Note that since the \( s = 1/2 \) spin and the sign of the vorticity for a spinon-vortex are independent of each other, the spin polarizations in the magnetic field should equal for \( \pm \) vortices, i.e., \( \sum_{\sigma} \sigma n^\sigma_\pm = \frac{1}{2} \sum_{\sigma} n^\sigma_\pm \). One then obtains \( \mathbf{J}^s = -\frac{2}{\pi} \frac{\Phi_0}{\eta_s} \mathbf{J}^s \) where \( \langle \mathbf{s}_\sigma \rangle = \frac{1}{2} \sum_{\sigma} \sigma (n^\sigma_\uparrow + n^\sigma_\downarrow) \) and \( n_v \equiv n^+ + n^- \). By using \([5]\) and \( g \mu_B \langle S^z \rangle = \chi_s B \), one finally arrives at \([11]\) and \([2]\) after restoring \( \hbar \). No quantities related to dissipation explicitly appear in \([5]\). Notice that both \( \mathbf{J}^s \) and \( \mathbf{E} \) are invariant under time-reversal, and \( \sigma_H \) is also explicitly unchanged under \( B \to -B \), in contrast to the charge Hall conductance. Furthermore, without the spin-orbit coupling, the spin current \( \mathbf{J}^s \) always remains conserved in the SV phase.

The above spin Hall effect follows directly from the GL equations \([11, 13]\). Note that the non-dissipative relation \([5]\) is independent of the “Coulomb drag” between vortices and antivortices moving in opposite directions, caused by the 2D Coulomb-like (logarithmic) interaction which exists in the generalized GL equations \([5]\). Generally, interactions do not affect \([5]\) so that both the spinon-vortex current and spin current are “protected” in the SV phase by the pairing amplitude \( \Delta \neq 0 \).

Then let us consider the charge current in the SV phase. Due to the presence of free spinon vortices, the SC phase coherence is destroyed and the system gains a finite resistivity due to the vortex motion. Such a charge resistivity has been previously obtained based on the generalized GL theory as follows \([5]\)

\[
\rho = \frac{n_v}{\eta_s} \left( \frac{\Phi_0}{2e} \right)^2 \tag{11}
\]

where \( \eta_s \) denotes the viscosity of spinon vortices. Note that the contribution from quasiparticles is not considered here. In the units of \( \hbar = c = e = 1 \), equation \( \langle 11 \rangle \) can be written in a duality form

\[
\sigma \sigma_{sv} = \frac{1}{\pi^2} \tag{12}
\]

where \( \sigma = 1/\rho \) and spinon conductance \( \sigma_{sv} \equiv \frac{n_v}{\eta_s} \). Thus, a charge current is always dissipative in the SV phase, which is well separated from the spin Hall current which is independent of the viscosity \( \eta_s \) of spinon vortices. In this sense, the latter is considered to be dissipationless.

### Spin Hall insulator

In the SC phase below \( T_c \), vortex-antivortex are bound together (spinon confinement) and no free (unpaired) spinon-vortices present in the bulk. Consequently, \( \sigma_{sv} \) vanishes such that \( \sigma = \infty \) according to \( \langle 12 \rangle \). On the other hands, \( \sigma \) becomes finite when free spinons emerge in the bulk, which destroy the SC phase coherence as discussed before and contribute to a finite \( \sigma_{sv} \). In particular, if a finite density of free spinons is present at low temperatures as stabilized by, say, a strong magnetic field,
then these unpaired bosonic spinon-vortices can experience a Bose condensation such that \( \sigma_{sv} \to \infty \) at \( T \to 0 \). Correspondingly, based on (12), the charge conductance \( \sigma \to 0 \), leading to an insulator as the ground state of the SV phase. By contrast, the spin Hall conductance \( \sigma_{h}^s \) in (2) remains finite as given by (13) below, unaffected by the vanishing longitudinal charge conductivity. Therefore, such a ground state of the SV phase is a spin Hall insulator, presumably stabilized by strong perpendicular magnetic fields.

The Bose condensation of spinons implies the existence of some sort of antiferromagnetic ordering [12]. Experimentally, both a magnetic-field-induced magnetic ordering [13, 14] and insulating behavior [15] have been observed in the pseudogap regime of the underdoped cuprates. By taking the lattice constant \( a \) comparable to the experimental values in [12], which is comparable to the experimental values in the cuprates. The nontrivial Nernst effect [8] and diamagnetism [10], extending over a wide range of temperature, with \( T_v \) as large as several times of \( T_c \) in underdoping, have been also observed in these cuprate materials, strongly suggesting the presence of 2D spontaneous (cheap) vortices [8, 16, 17] as the physical origin. As a unique prediction of the phase string theory, cheap vortices must have \( s = 1/2 \) spins located at the vortex cores due to the Mott physics which prohibits double occupancy of electrons: a site is either occupied by a hole or by an \( s = 1/2 \) spin. Consequently, a dissipationless spin Hall conductance is naturally obtained when those free vortices are either thermally excited or magnetic-field induced in the SV phase.

Let us finally examine the magnitude of the spin Hall conductance \( \sigma^s_{h} \) given in (2). Generally, \( n_v \Phi_0 \leq B \) according to (10) (the equality holds if the vortices are fully polarized by the magnetic field) and \( \sigma^s_{h} \leq \sigma^0_{h} = h \chi_s / g u_B \). So \( \sigma^s_{h} \) decides an upper bound for \( \sigma^s_{h} \). At a temperature slightly above \( T_c \), a typical \( \chi_s \) can be estimated \( \sim 1.1 \rho_o^{2} / a^{2} \times \text{states/eV} \) in the phase string theory [12], which is comparable to the experimental values in the cuprates. By taking the lattice constant \( a \approx 3.8 \AA \), we then obtain \( \sigma^s_{h} \sim 0.55 (h \mu_o / a^2) \times \text{states/eV} = 0.14 e \). Of course, \( \sigma^s_{h} \) is generally reduced from \( \sigma^0_{h} \) with the increase of the thermally (spontaneously) excited vortices above \( T_c \). On the other hand, at low temperatures where the thermally excited spinon-vortices are negligible, and all vortices are nucleated by the applied magnetic field, one has \( n_v = n^* = B / \Phi_0 \) according to (10). Furthermore, the spins of the vortices are totally polarized by \( \langle S^z \rangle = - \frac{1}{2} n_v^{-} = - \frac{1}{2} n_v \) if the temperature is sufficiently low. Then in this limit one finds

\[
\sigma^s_{h} = \frac{1}{2\pi} e
\]

which approaches a universal number as all spins are in RVB paired except for those associated with the vortices nucleated by the magnetic field [11]. Note that \( \sigma^s_{h} = 0 \) if the SC phase coherence is realized when those vortices are pinned spatially, where \( \partial_t J \neq 0 \) unless the electric field \( E = 0 \) in the bulk. Namely (13) is valid only in the vortex flow regime at low temperature.

**Mutual Chern-Simons theory description**

So far the SHE in the SV phase has been discussed based on the generalized GL equations (4) and (5) of the phase string theory. In the following we briefly discuss how this is a self-consistent result ensured by the underlying mutual duality structure of the microscopic theory.

The mutual-Chern-Simons effective description of the phase string theory is given by an effective Lagrangian

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_h + \mathcal{L}_s + \mathcal{L}_{\text{CS}}
\]

with the charge part

\[
\mathcal{L}_h = \hbar \left\{ i \partial_t - A^0_\mu - A^s_\mu - \frac{1}{2m_h} \left( -i \nabla - A^s - A^c \right)^2 \right\} h
\]

which describes that the charge +\( e \) holon field \( h \) couples to an external electromagnetic field \( A^\mu_\nu \) \((\mu = 0, x, y)\) and an internal U(1) gauge field \( A^s_\mu \). The spin part \( \mathcal{L}_s \) describes the neutral spinon field couples to another U(1) gauge field \( A^\mu_\mu \), whose detailed form (15) is not important here. Here both \( A^s_\mu \) and \( A^h_\mu \) can be regarded as “free” U(1) gauge fields, which are “entangled” by the mutual-Chern-Simons term

\[
\mathcal{L}_{\text{CS}} = \frac{1}{\pi} e^{\nu\lambda} A^s_\nu \partial_\lambda A^h_\lambda
\]

Note that the topological constraint on \( A^s \) in (6) only emerges after the temporal component \( A^s_0 \) is integrated out in the partition function determined by \( \mathcal{L}_{\text{eff}} \). The time-reversal, parity, and global spin rotational symmetries have been shown to be retained in \( \mathcal{L}_{\text{eff}} \) at \( A^s_0 = 0 \), and the global phase diagram, including antiferromagnetic phase, SC phase, pseudogap and SV phases, has been discussed within such a unified description (15).

One can then show that \( \mathcal{L}_{\text{CS}} \) will generally result in the following equation of motion

\[
J^s = \frac{1}{2\pi} E^s \times \hat{z} , \quad J = \frac{1}{\pi} E^h \times \hat{z}
\]

where \( J^s = 1/2 \delta \mathcal{L}_s / \delta A^h \) and \( J \equiv \delta \mathcal{L}_h / \delta A^s \), with \( E^s = - \partial_t A^s - \nabla A^s_0 \) and \( E^h = - \partial_t A^h - \nabla A^h_0 \). Thus, a spin current can be generated by a perpendicular “electric field” \( E^s \) and the charge current by \( E^h \) according to (16). The spin-current conservation here is due to the U(1) gauge invariance associated with \( A^h_\mu \).

In particular, consider the SV phase defined by the Bose condensation of holons with \( \langle h(r) \rangle = \psi_h = \sqrt{n_h} e^{i \phi_h(r)} \). According to \( \mathcal{L}_h \), \( E \) (via \( A^s_\mu \)) would accelerate the condensate unless it balanced by \( E^s \) (via \( A^s_\mu \)), satisfying \( E^s + E = - \partial_t \nabla \psi_h + \nabla \partial_t \psi_h \) where the right-hand-side is contributed by the vortices in the phase of \( \psi_h \). By incorporating the latter into \( J^s \) which gives rise
to $J^v$, similar to the previous discussion, one then reproduces $\mathbf{8}$.
Therefore, the non-dissipative spin currents are quite robust so long as $\psi_h$ and thus the pairing amplitude $\Delta^0$ remains finite, which defines the SV phase. On the other hand, vacancy impurities like the zinc substitution in the cuprates may greatly reduce the spin currents by very effectively pinning down the free spinon-vortices. Similarly the Nernst effect is expected to be suppressed by the same reason.

Another interesting property of $\mathbf{10}$ is that a charge current flowing through the sample will generate an “electric filed” $E^h$, which acts on the spinon part via $L_s$ and thus provides a means of “spin pump”. It may be manipulated to design a “spin battery” in such a system $\mathbf{21}$. Furthermore, define $J^v = \sigma_{sv} E^h$ and $J = \sigma E$. Then by simply using the relations in $\mathbf{8}$ and $\mathbf{10}$, one easily finds the interesting duality relation $\mathbf{12}$ between the charge conductance $\sigma$ and spinon conductance $\sigma_{sv}$, previously obtained in the generalized GL theory description $\mathbf{11}$.

**CONCLUSION**

In the paper, the existence of a conserved dissipationless spin Hall current is predicted in the low-temperature pseudogap regime of a doped Mott insulator, known as the spontaneous vortex phase, based on the phase string description. Such a spin Hall current is concomitant with the Nernst signal, but is dissipationless whereas the latter is not. A sizable spin Hall conductivity is obtained which only depends on the intrinsic bulk properties as well as the external magnetic field. Furthermore, the spin Hall conductivity remains finite even if the longitudinal charge resistivity diverges when the pseudogap state is stabilized by strong magnetic fields in the ground state, leading to a spin Hall insulator. Note that the spin Hall insulator here is caused by strongly correlated effect (a quantum vortex liquid) which has nothing directly to do with that discussed in the SO systems where a band gap is present.

We emphasize that the spin Hall current is protected here by the electron pairing amplitude. The SHE constitutes a sharp and very unique prediction for the lower pseudogap phase which can be tested in the future experiment. We point out that the present experimental techniques $\mathbf{21}$ allow one to detect only spin accumulations that the spin currents deposit at interfaces and boundaries, which become detectable with the help of the strong SO coupling $\mathbf{21} \mathbf{22}$. Thus in order to observe the SHE in the high-$T_c$ materials, a natural method is to make a finite-width strip of the relevant materials been sandwiched by semiconductors of strong SO coupling and determine the spin accumulations at the interfaces by the conventional techniques $\mathbf{21}$.

Finally, if such a SHE does exist in the cuprate materials, it may have some important potential applications in spintronics devices due to its sizable effect, conserved and non-dissipative nature, as well as the relatively wide region of the lower pseudogap phase. Compared to the SHE proposed $\mathbf{1}$ $\mathbf{2}$ in the semiconductor systems with SO couplings, however, we would like also to point out a drawback in the present system, namely, a perpendicular magnetic field is always needed here, which may hinder practical applications and should be further carefully investigated.

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**References**

[1] S. Murakami, N. Nagaosa, and S.C. Zhang, Science 301, 1348 (2003); Phys. Rev. B 69, 235206 (2004).
[2] J. Sinova, D. Culcer, Q. Niu, A.A. Sinitsyn, T. Jungwirth, and A. MacDonald, Phys. Rev. Lett. 92, 126603 (2004).
[3] J. Inoue, G.E. Bauer, and L.W. Molenkamp, Phys. Rev. B 70, 041303 (2004); E. G. Mishchenko, A. Shytov, and B. Halperin, Phys. Rev. Lett. 93, 226602 (2004); S. Murakami, Phys. Rev. B69, 241202 (2004).
[4] D.N. Sheng, L. Sheng, Z.Y. Weng, and F.D.M. Haldane, cond-mat/0504218.
[5] E. I. Rashba, Phys. Rev. B 70, 161201 (2004).
[6] P.W. Anderson, Science, 235, 1196 (1987).
[7] W. L. Lee, S. Watauchi, V. L. Miller, R. J. Cava, N. P. Ong, Science 303, 1647 (2004).
[8] Z. Y. Weng and V. N. Muthukumar, Phys. Rev. B 66, 094509 (2002).
[9] Z.A. Xu, N.P. Ong, Y. Wang, T. Kakihana, and S. Uchida, Nature (London) 406, 486 (2000); Y. Wang, Phys. Rev. B 64, 224519 (2001).
[10] P. W. Anderson, cond-mat/0504453.
[11] V. N. Muthukumar and Z. Y. Weng, Phys. Rev. B 65, 174511 (2002).
[12] Z. Y. Weng, D.N. Sheng, and C.S. Ting, Phys. Rev. Lett. 80, 5401 (1998) and references therein.
[13] B. Khaykovich, Y. S. Lee, R. W. Erwin, S.-H. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R. J. Birgeneau, Phys. Rev. B 66, 014528 (2002).
[14] B Lake, H.M. Ronnow, N.B. Christensen, G. Aeppli, K. Lefmann, D.F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T.E. Mason, Nature 415, 6869 (2002).
[15] Y. Ando, G. S. Boebinger, and A. Passner, T. Kimura and K. Kishio, Phys. Rev. Lett. 52, 184511 (2002).
[16] Y. Wang, L. Li, M.J. Naughton, G. Gu, and N.P. Ong, to be published.
[17] H. H. Wen, Z. Y. Liu, Z. A. Xu, Z. Y. Weng, F. Zhou, Z. X. Zhao, Europhys. Lett. 63, 583 (2003).
[18] S. P. Kou, X. L. Qi, and Z. Y. Weng, Phys. Rev. B 71, 235102 (2005).
[19] X. L. Qi and Z. Y. Weng, Phys. Rev. B 71, 184507 (2005).
[20] Q. F. Sun, H. Guo, and J. Wang, Phys. Rev. Lett. 90, 255301 (2003); P. Sharma and C. Chamon, Phys. Rev. Lett. 87, 096401 (2001).
[21] Y.K. Kato, R.C. Myers, A.C. Gossard, and D.D. Awschalom, Science, 306, 1910 (2004); J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. 94, 047204 (2005).

[22] B.K. Nikoli, S. Souma, L. P. Zarbo, and J. Sinova, Phys. Rev. Lett. 95, 046601 (2005); M. Onoda, N. Nagaosa, cond-mat/0505436