QUARKS IN THE UNIVERSE

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1. Local change in the vacuum structure: from QED to QCD

Matter in its present form was formed when our Universe emerged from the quark-gluon phase (QGP) at about 30\(\mu\)s into its evolution. To explore this early period in the laboratory, we study highly excited matter formed in relativistic heavy ion collision experiments: heavy nuclei crash into each other, and form compressed and energetically excited nuclear matter, resembling in its key features the stuff which filled the early Universe. In these experiments we further explore the physics of the vacuum structure of strongly interacting gauge theory, Quantum Chromodynamics (QCD). The common beginning for both, heavy ion collisions, and vacuum structure investigations, is the physics of the quantum electrodynamic (QED) vacuum in the presence of the supercritical external field that is formed when two highly charged heavy ions are brought together near to the Coulomb barrier in a considerably lower reaction energy collision.\(^1\)

QED of strong fields research program was initiated by Walter Greiner in Frankfurt many years ago. The 2nd island of super-heavy elements at \(Z = 164\) demanded the understanding of relativistic atomic physics structure, and more specifically, the understanding of what happens when the most tightly bound electrons disappear into the lower Dirac continuum. At the Montreal meeting in late August 1969, shortly following his first study of atomic structure of super-heavy elements,\(^2\) Walter Greiner made the following comments: *Heavy-ion physics is the tool . . . . . . we find in the future elements in the area of \(Z = 164\) . . . with accelerators. . . . this would lead us into new field of quantum electrodynamics . . . of strong fields . . . an unsettled problem and a new and rich field of research. . . . if you come to very high \(Z\)-numbers the 1s-electron levels . . . dives into the lower continuum.* Walter prophecy came true with regard to his theory of high \(Z\)-atoms. As for “future elements in the area of \(Z = 164\),” this remains today good material for science fiction.

* Dedicated to Walter Greiner on occasion of his 70th birthday.
Along with a few other young students I joined this adventure in a new field, the QED of strong fields. One Saturday morning, in the Fall of 1971, the process of positron auto-ionization was finally understood. After a bit more work the charged vacuum was born, the positron carried out the positive charge, while the balancing negative charge was in the vacuum, localized near the source of the supercritical field. An important consequence of all this was the proper understanding of the behavior of electrons and positrons in rapidly changing strong fields, from which the prediction of the shape of the emitted positron spectra emerged.

Several technically challenging experiments followed, yet the detection of the spontaneous positron production in supercritical fields remains an open subject, despite years of diligent work. The QED vacuum kept its secret, buried in the high noise generated by other processes accompanying atomic and nuclear reactions of very heavy atoms. We have not demonstrated that the QED vacuum state can change locally.

We now search for another local vacuum modification, the melting of the structure of the vacuum of strong interactions. Unlike the case of QED, where the local non-perturbative structure is created in the experiment, we aim here to locally dissolve the global non-perturbative, color charge structure, and to locally liberate quarks confined in hadronic particles. In both cases, the QED of strong fields, and the QCD at high temperature, we probe the same principle, that a local change of the vacuum state is possible.

By showing that it is possible to dissolve color confining vacuum structure, we demonstrate that the vacuum state can change locally. We further complete the understanding of the origin of the mass of matter, which is as we believe today, due to the confinement properties inherent in the non-perturbative nature of the true QCD vacuum. We further learn how the QGP energy becomes matter in the process we call hadronization. We thus learn about the matter formation mechanisms in the expanding Universe. There hadronization occurs at about $T_h = 160$ MeV.

The research area of high energy heavy ion collisions emerges as a new field in the '70-s. The first application is at that time the exploration of compressed nuclear matter. Walter Greiner is among the first to propose hydrodynamic description of the evolution of the strongly interacting matter. He proposes an interpretation of some results in terms of shock waves. These could help compress nuclear matter to conditions expected in the interior of the neutron stars.

Our understanding of the hot hadronic matter formed in these reactions expands rapidly. We recognize that already at rather modest heavy ion reaction energies we can encounter deconfinement. By 1982 our theoretical work suggests that QGP is formed in heavy ion collisions and can be observed.

Two proposed dedicated experimental facilities, the accelerator projects at the LBL (Venus) and GSI (SIS100), do not attract funding in the early eighties. Despite this initial setback the field of nuclear physics moves decisively into this new area. The interest in the QGP research program grows rapidly, both in Europe and USA. The theoretical effort is soon supported by experiment, with a large number of
experimental nuclear physicists entering into research collaborations with particle physicists, and jointly developing full fledged experiments at the particle physics laboratories, at CERN in Geneva, and at BNL in New York.

2. Creation of Matter in Laboratory

In laboratory experiments, there are two primary steps in the particle production from QGP as illustrated in figure 1.

- cooking of the energy content towards QGP $u, d, s$ quarks and $G$ gluon yield (chemical) equilibrium;
- combination of quark content into final state hadrons, in figure 1 the precooked strangeness content combines into $\Omega(s\bar{s}\bar{s})$ and $\Xi^0(ssu)$.

Fig. 1. A schematic illustration by example of the two step particle formation process: within QGP strangeness is produced in gluon fusion $GG \rightarrow s\bar{s}$, and later combined into final state particles.

The hadronic particles emerge in a quark combination process; see arrows in figure 1. The hadronic particle can be surface-produced in this way during the entire history of plasma evolution. However, given the high collective outflow velocity of the matter, driven by the high collisional compression pressure, it is widely believed that the bulk of hadrons emerges at the end of the expansion in the global volume dissociation. At this point in time, across the fireball volume, the temperature decreases to the point at which the deconfined phase cannot continue.
Quarks in the Universe

to exist. This final breakup of the QGP phase formed in the laboratory is, as pion interferometry HBT results show, a very fast process. We thus conclude that the dense matter fireball expands in an explosive manner, and undergoes a fast bulk “hadronization”.

The resulting particle yields are well described by the statistical hadronization model (SHM). SHM originates in the Fermi-hypothesis: strong interactions saturate the quantum particle production matrix elements. Therefore, pursuant to the golden rule of quantum mechanics (attributed to Fermi) the yield of particles is given by the accessible phase space. For a more detailed discussion of SHM and its parameters we refer to the recent review. 14

In the original Fermi model the accessible phase space is considered in terms of the available energy. Today we refer to this method as micro-canonical. Normally, we use the grand canonical approach, which substitutes for the total available fireball energy a temperature-like parameter $T_f$. Even if $T_f$ is reported in context of SHM, that does not mean by necessity that there is an equilibrated gas of hadrons. Particle spectra and abundances may imply different values of $T_f$ if following the hadron formation there is a period in which hadrons interact and reequilibrate. In this case, $T_f$ is called chemical freeze-out (particle formation) temperature, and another parameter $T_t$ appears, the thermal freeze-out temperature. If QGP is the source of hadrons, $T_f$ is closely related to the hadron source temperature $T_h$ of the QGP.

The state of the art of SHM is today more complex than in time of Fermi. To make a quantitative model we must deal with strong interactions among particle, this is done introducing the production of hadron resonances. In addition we consider chemical potentials associated with all conserved quantum numbers. We can either work with baryon number $B$, hyperon charge $Y$ and electrical charge $Q/|e|$, or the net number $N_i$ of each of the three valance quark flavors,

$$N_i = q_i - \overline{q}_i, \quad i = u, d, s$$  \hspace{1cm} (1)

Specifically,

$$B = \frac{1}{3}(N_u + N_d + N_s), \quad Q = \frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s, \quad Y = B + S = B - N_s$$  \hspace{1cm} (2)

In general there are three chemical fugacities $\lambda_i$, or potentials, $\mu_i = T \ln \lambda_i$, with either $i = u, d, s$ or equivalently $i = b, Q, Y$ or any linearly independent combination of the three which must be introduced to be able to satisfy the conservation of these three quantities. One often refers to light quarks by $\lambda_q \equiv \sqrt{\lambda_u \lambda_d}$, $\mu_q = (\mu_u + \mu_d)/2$.

Conservation laws do not tell us anything about actual ‘filling’ of phase space. For example in laboratory experiments, initially there are very few, if any, strange quark pairs present. As the collision reaction progresses, the yield of strangeness grows. This is described by a parameter $\gamma_s(t)$ which expresses how close one is to a yield expected when the system had long time to cook strangeness in the QGP.

Within the usual framework of statistical thermodynamics of quarks and gluons these parameters enter the quantum Fermi and Bose distributions, for example for
the conserved strangeness flavor we have

\[ s = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\lambda_s^{-1} \gamma_s^{-1} e^{E(p)/T} + 1} \]

\[ \bar{s} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\lambda_s^{-1} \gamma_s^{-1} e^{E(p)/T} + 1} \]

(3)

In a good approximation,

\[ \gamma_s \simeq \frac{s + \bar{s}}{s_{eq} + \bar{s}_{eq}} \]

(4)

and similarly for all other quark flavors. Here the equilibrium distribution arises for \( \gamma_s \to 1 \). One often refers to light quarks by \( \gamma_q \equiv \sqrt{\gamma_u \gamma_d} \).

It is not customary to introduce a chemical potential associated with the fugacity \( \gamma_i \) since in any physical system, as reaction time evolves, \( \gamma_i(t) \to 1 \) maximizing the entropy. Thus the associated chemical potential is not time independent, but rapidly evolves to zero. On the other hand, the conserved quantum number chemical potentials

\[ \mu_B = \frac{3}{2} (\mu_u + \mu_d), \quad \mu_Q = \frac{2}{3} \mu_u - \frac{1}{3} \mu_d - \frac{1}{3} \mu_s, \quad \mu_Y = \mu_B - \mu_s \]

(5)

are normally constant in time. However, in the heavy ion reaction environment the expansion-driven cooling of the system leads to a decreasing value of \( T \). Note that the dimensionless quantity \( \mu_B/T \) is nearly conserved in hydrodynamic expansion of QGP.

3. In search of a new phases of matter

3.1. Phase boundary

The best way to discover a new phase of matter is to find a phase boundary. We recall Gibbs definition of the phase boundary. For the case of chemical equilibrium:

a) The pressure is equal, or else mechanical force would move the boundary in space, or in time, until the pressures in both domains are equal;

b) The temperature is equal, or else radiative processes would transport energy between domains in space until this condition is reached;

c) The chemical potential(s) is(are) equal, or else particle transport across boundary would change the particle number so that this condition is satisfied.

The last condition follows from the first law of thermodynamics.

However, unlike the Gibbs case, we deal here with systems in which particle number is not conserved, so we need to modify this condition to get:

c') The ‘conserved quantum number’ chemical potential(s) is(are) equal, or else particle and antiparticle transport across boundary would change the baryon number, charge etc, so that this condition is satisfied.

If the particle yield equilibrium (chemical equilibrium) cannot be assured, for example when the physical constraints are evolving fast, we speak of chemical non-equilibrium. In this case the phase boundary is defined by micro-canonical properties which have not otherwise been considered:
d) The near-conservation of entropy, and hard-to-form particles across the phase boundary.

The entropy cannot decrease at the phase boundary, but it could increase. However since QGP is an entropy-rich phase undergoing a fast phase transformation, this should occur without significant on the scale of entropy already generated, entropy enhancement. There are several options to accomplish this, e.g. by volume expansion or phase space occupancy. 

In chemical non-equilibrium the Gibbs condition c) and its variant c') have to be applied to both the particle and antiparticle number separately. Using strangeness as example, see Eq. (3), we write:

\[ \gamma_s = T \ln \left( \gamma_s \lambda_s \right) = T \ln \gamma_s + \mu_s \]  
\[ \gamma_{\bar{s}} = T \ln \left( \gamma_s \lambda_s^{-1} \right) = T \ln \gamma_s - \mu_s \]

Note that \( \mu_s^s - \mu_s^{\bar{s}} = 2\mu_s \) is independent of \( \gamma_s \), and assures that net strangeness is conserved. Similarly, \( \mu_s^s + \mu_s^{\bar{s}} = 2T\ln \gamma_s \) is independent of \( \mu_s \) and assures that number of strange quark pairs (up to additive constant and factor two) is conserved.

3.2. Non-equilibrium phase boundary in heavy ion reactions

An important difference between \( \gamma_i \) and \( \mu_i \) is that the Gibbs criteria of a smooth functional connection across phase boundary do not apply to \( \gamma_i \). The Gibbs condition, that at the phase boundary transport of particles of given conserved number content must vanish requires in general for continuous \( \mu_i \) a discontinuity in all \( \gamma_i \), since in general the size of the phase space is not the same in the two matter phases considered. For example, conserving entropy in hadronization amounts to hadronization of equilibrated QGP into oversaturated HG:

\[ \gamma_{q_{\text{HG}}} = 1 \rightarrow \gamma_{q_{\text{HG}}} \simeq e^{m_s/2T} \simeq 1.6. \]

Similarly, since strangeness phase space size is about 2.5 times smaller in HG compared to QGP, chemically equilibrated strange QGP implies \( \gamma_{s_{\text{HG}}} \simeq 2.5 \).

We note further physical meaning of the non-equilibrium parameters with regard to hadron yields. Comparing the yield of strange to non-strange hadrons of the same type (e.g. nucleon with a hyperon, kaon with a pion etc.) we are evaluating the ratio \( \gamma_s/\gamma_{u,d} \). Similarly, the relative yield of baryons (\( \propto \gamma_q^3 \)) to mesons (\( \propto \gamma_q^2 \)) is controlled at fixed values of \( \gamma_s/\gamma_q, T \) by \( \gamma_q \). The observed baryon-to-meson ratio in nuclear collisions at RHIC is strongly enhanced compared to the yield seen in pp reactions at the same energy. Thus we know that the mechanism of baryon production in heavy ion reactions (quark combination mechanism) is different from pp reactions. If these yields can be described well by SHM model, we are expecting \( \gamma_q > 1 \).

We have given here highlights of what needs to be considered in setting up the QGP fireball breakup into hadrons within the SHM. More details can be found in the manuals to SHARE suite of programs (Statistical HAdronization with REsonances). Further discussion of the impact of the heavy ion dynamics on phase boundary have also been recently described.
4. Strangeness and the Discovery of QGP

4.1. Measuring QGP degrees of freedom

The QGP at hadronization, in the early Universe as in the laboratory, consists of $u, d, s$ and their anti-quarks $\bar{u}, \bar{d}, \bar{s}$. In laboratory experiments, strangeness formation continues throughout the temporal evolution of the plasma until the break up temperature $T_h$. Because of the coincidence of scales with $T_h \simeq 170 \pm 20$ MeV and $2m_s \simeq 190 \pm 30$ MeV being not very different, the yield of strangeness is a natural probe of QGP.

One way to understand if QGP has been formed is to study the available number of degrees of freedom. This can be accomplished by comparing the strangeness pair yield $N_s$ with entropy $S$. We denote here the yield of strange quarks by $N_s$, which is the same as the yield of strange quark pairs. Both $N_s$ and $S$ are extensive in the volume and thus not subject to dependence on precise collision history. Their ratio is in effect, up to a factor 4, the ratio of strangeness degeneracy $g_s$ to all active degrees of freedom in plasma $g_{QGP}$. The factor 4 allows (in good approximation) for the entropy per particle content in a nearly massless gas:

$$\left| \frac{N_s}{S} \right|_{QGP} = \frac{3,2s \cdot \tilde{\gamma}_s}{(2 + \tilde{\gamma}_s)3c2s + 8c2s} \approx 0.03,$$

Here $\tilde{\gamma}_s < \gamma_s$ allows for the reduction in the effectively acting massless strangeness degrees of freedom due to the strange quark mass, $m_s \neq 0$, and due to under-saturation of the phase space described by factor $\gamma_s$.

The final value,

$$\gamma_s(t_h) \equiv \gamma_s^{QGP}(\sqrt{s_{NN}}, A)$$

reached at time of hadronization $t_h$, is growing with increasing collision energy $\sqrt{s_{NN}}$, and with increasing participant number $A$, i.e. volume $V \propto A$. Thus we expect that as function of these variables, $\gamma_s^{QGP} \to 1$ for a sufficiently large $\sqrt{s_{NN}}, A$. In this limit the analysis of experimental data would yield $N_s/S \to 0.03$.

The measurement of this value for $N_s/S$, which is believed to be preserved in hadronization of QGP, amounts to a measurement of the relative strength of strangeness among all QCD degrees of freedom. An in-depth analysis of the experimental conditions shows that for the most central RHIC collisions we indeed have $N_s/S \to 0.03$.

4.2. Strange antibaryons

A promising indicator for the formation of QGP is the anomalous yield of strange antibaryons. Their production occurs through combination of earlier produced quarks and thus anti-strangeness rich QGP is a particularly good source of otherwise more rarely produced strange antibaryons. Enhanced production of $\bar{\Lambda}(\bar{s}\bar{q}\bar{q}), \Xi(\bar{s}\bar{s}\bar{q}), \Omega(\bar{s}\bar{s}\bar{s})$ increasing with the $\bar{s}$ content is the signature of QGP.
The detection of these particles is assisted by their natural radioactive decay patterns, which can be seen tracking secondary charged particles. For example, to observe a $\Xi^-$ we note its decay:

$$\Xi^- (\bar{s} \bar{s} \bar{d}) \rightarrow \Lambda (\bar{u} \bar{d} \bar{s}) \rightarrow \bar{p} + \pi^+ + \pi^-.$$  \hspace{1cm} (11)

The simplest yield ratio to consider is $\Lambda / p$. After cancellation of combinatorial and phase space factors this ratio is determined by relative quark yields available at hadronization. If no QGP were formed one could at best hope for chemical equilibrium yields in the hadron gas matter. In both cases, aside of directly produced $\Lambda, p$, there are decays of resonances. We assume here that these multiply the yields of $\Lambda, p$ by the same factors irrespective if these are originating in QGP or HG. For the purpose of comparing the magnitude of this ratio originating in either QGP or HG, these corrections can be ignored at first.

In a baryon-rich QGP environment the light antiquark $\bar{u}, \bar{d}$ abundances are suppressed by the baryochemical potential, while $\bar{s}$ is suppressed by strange quark mass, and we find:

$$\frac{\Lambda}{\bar{p}} \bigg|_{\text{QGP}} = \frac{N_{\bar{s}} N_\bar{u} N_{\bar{d}}}{N_\bar{s} N_\bar{u} N_{\bar{d}}} \sim \frac{1}{2} \frac{m_s^2}{T_h^2} K_2 (m_s/T_h) e^{(\mu_u - \mu_s)/T_h} = 0.9 e^{(\mu_u - \mu_s)/T_h}, \hspace{1cm} (12)$$

where the last equality follows the currently accepted value $m_s/T_h \simeq 0.7$.

The thermal yield originating in the hadron phase comprises, in place of strange quark mass suppression, the hadron phase space suppression factor:

$$\frac{\Lambda}{\bar{p}} \bigg|_{\text{HG}} = \left( \frac{m_\Lambda}{m_{\bar{p}}} \right)^{3/2} e^{-\frac{m_\Lambda - m_{\bar{p}}}{T_f}} e^{(\mu_u - \mu_s)/T_f} = 1.3 e^{-180 \text{ MeV}/T_f} e^{(\mu_u - \mu_s)/T_f}. \hspace{1cm} (13)$$

For $T_f \simeq 160 \pm 20$ MeV we obtain a significant reduction of the expected relative yield, which is also clearly less than unity. If the chemical equilibrium in HG is not reached we further have a multiplicative factor $\gamma_s/\gamma_q$. It is very hard, indeed impossible, to ever obtain a result that would exceed unity in case of HG-based production.

In SHM fits of ratio $\Lambda / \bar{p}$ the presence of QGP is expressed by the magnitude of $\gamma_s/\gamma_q$, which in order to accommodate the large strangeness content of QGP, can exceed unity. The additional baryons produced by the quark combination mechanism (comparing to HG yield) imply that $\gamma_q > 1$. We further note that the above argument can be repeated for $\Xi^- / \Lambda, \Omega^- / \Xi^+$, easily and exactly in the same way. Thus arises the original prediction that strange antibaryon enhancement grows with the strangeness content.

It is of some interest to see how current AGS and SPS experimental results compare to this initial prediction. The results are shown in figure 2, based on compilation of data and theoretical results by the NA49 collaboration. We see that the central rapidity ratio $\Lambda / \bar{p}$ is well above unity at all available reaction energies. With decreasing reaction energy, this ratio increases. This suggests that the increasing baryon density and its suppressing effect outweigh any reduction of the relative yield due to reduced strangeness abundance.
At the bottom of figure 2, predictions based on the SHM fits to the experimental data made by different groups are shown. For the three highest reaction energies, the discrepancy between theoretical interpretation and experimental data is fully accounted for by the need to correct the experimental ratio for the included weak decays $Ξ \rightarrow \Lambda$, and the fact that the thermal rapidity distribution of $\Lambda$ is narrower than that of $\bar{p}$, which enhances the central rapidity ratio. Despite the large error bars, it is hard to explain the trend at very low reaction energies, considering that we would not expect that QGP is formed below a certain energy threshold. We will need further data to resolve this intriguing trend. The experimental difficulty here is the relatively low yield of all very massive particles, and the decreasing sensitivity of antibaryon detection.

5. The Early Universe

In order to address the physics of the early Universe using the results we obtain in the laboratory, we must extrapolate the properties of QGP to the conditions pre-
vailing in the early Universe. The experimental environment we expect to create in relativistic heavy ion collisions differs from what we know about the early Universe in several ways, which our extrapolation must bridge. In addition, we have to face the unknown physical properties of dark matter and dark energy. We discuss these three issues before closing this presentation.

5.1. The matter-mirror matter symmetry

This symmetry is best described by the entropy $S$ content per nett baryon $B$, that is, the excess of baryon, over antibaryon, number. Experiments at RHIC reach $S/B|_{\text{RHIC}} = 300–400$. However, our Universe is much more symmetric at the time of hadronization. The study of the photon content of the Universe leads to:

$$\eta \equiv \frac{B}{N_\gamma} = (6.1 \pm 0.15) \times 10^{-10}. \quad (14)$$

Allowing for the $e^+e^-$ annihilation reheating in the late stage the entropy content of the visible Universe is

$$\left. \frac{S}{B} \right|_{\text{Universe}} = \frac{8.0}{\eta} = (1.3 \pm 0.1) \times 10^{10}. \quad (15)$$

Using $S/B$ as a measure, the early Universe has been $3 \cdot 10^7$ more matter symmetric compared to RHIC, assuming here that possible decay of dark matter has not significantly diluted the $S/B$ ratio (see below).

Another way to understand this difference is to look at the baryochemical potential $\mu_B$, a quantity which controls baryon and antibaryon density. In the RHIC experiments, we find $\mu_B = 24 \text{MeV}$, while in the early Universe this number is $1.1 \text{eV}$. The corresponding ratio is similar to what we noted above, $2 \cdot 10^7$. At LHC we expect to measure $\mu_B \simeq 1–2 \text{MeV}$, reducing the difference with the early Universe to a mere “million”. On the other hand, this particular difference is not very relevant, current evaluation of the phase structure of matter in this domain of $\mu_B$ suggests that the properties of QCD matter change smoothly.

$\mu_B$ is a very tiny fraction of the temperature. At hadronization in the early Universe the $\mu_B/T$ ratio fixes the magnitude of the baryon density,

$$\frac{\mu_B}{3T} \simeq 2 \cdot 10^{-9} \simeq \frac{B}{S} \times \frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}}. \quad (16)$$

The frequently asked question, is what is the origin of this very small value of baryochemical potential, or equivalently net baryon number. The absence of antimatter in our neighborhood forces us to think about baryo-genesis. The orthodox point of view is that this small value was established well ahead of the temperature of interest here, in terms of a baryo-genesis mechanisms which occur well above QCD phase transition scale of energy.

We look for baryo-genesis at high energy since we did not find any trace of it in large laboratory experiments carried out at $T = 1/30 \text{eV}$, nor did we see anomalous
baryon number non-conserving effects in elementary collisions. On the other hand, the smallness of the required asymmetry creates a tantalizing opportunity to seek out other baryo-genesis effects at lower temperatures. In our opinion, it is not possible to exclude that baryon number violating effects occur as late as the era of QGP hadronization.

One may even doubt that baryo-genesis is required at all, and instead the baryonic Universe we see around us could be simply a large fluctuation, with other domains containing the missing antibaryons. In this context it is helpful to consider the magnitude of fluctuations in baryon number density. Applying the usual formulas of grand canonical statistical physics and recalling that since in quark phase where $T \gg \mu_B$

$$\overline{B} \propto \mu_B T^2$$

(17)

Here we indicate the grand canonical average by the over-line. We find the thermal fluctuation in baryon number in a given volume with mean baryon number $\overline{B}$:

$$\overline{B}^2 - \overline{B}^2 = T \frac{\partial \overline{B}}{\partial \mu_B} = \frac{T \mu_B}{\mu_B} \overline{B}$$

(18)

The normalized probability distribution of finding baryon number $B$ where $\overline{B}$ would be expected is:

$$P(B) = \frac{1}{2\pi} \frac{1}{(T/\mu_B)\overline{B}} e^{-\frac{(B-\overline{B})^2}{2(T/\mu_B)}}$$

(19)

Considering that $\mu_B / 3T \simeq 10^{-9}$, strong fluctuations in baryon number occurs in volumes in which $\overline{B} < 10^9$, and thus considering Eq. (15), comprising up to about $10^{19}$ particles. One should note that the fluctuation we have considered here is purely thermal and excludes dynamical and non-equilibrium effects which arise, e.g. in hadronization of QGP.

5.2. Time constants

The typical life span of QGP formed in the laboratory is determined by the comparison of the nuclear size $R = 6$ fm to the speed of expansion, which is about $v = 0.6c$. Thus $\tau \simeq 6/0.6 \simeq 3 \times 10^{-23}$ s. This amount of time will not allow the equilibration of particles that interact only by weak, or electromagnetic interactions. In this short time even some strong interaction components in the QGP will not fully equilibrate. For example, computations of strangeness yield equilibration in the deconfined QGP show that in currently available experimental conditions only the most central RHIC collisions at maximum energy come to about 90% chemical equilibration. The study of strangeness chemical equilibration is thus of considerable interest and helps us understand the QGP reaction time. All heavier flavors $c, b, t$ are, like the weak and EM particles, practically decoupled; their abundance is established in first most energetic parton collisions.
The natural time constant in the early Universe is much longer, since the expansion velocity is the Hubble constant, at the time of hadronization. This follows from the two equations which govern the dynamics of the homogeneous Universe adiabatic expansion. One of these corresponds exactly to the heavy ion situation; it describes the entropy conservation in the Universe expansion:

$$\frac{de}{\epsilon + P} = \frac{-3dR}{R},$$

(20)

where $\epsilon$ is the energy density of the gravitating matter, $P$ its pressure, and $R$ the radial Universe size/scale. The expansion of the QGP phase can be described by a similar expression, which allows for different dynamics of longitudinal and transverse expansion.

The other, dynamical, equation is the Friedman equation which one obtains for the Robertson-Walker Universe using the energy-momentum tensor of dust matter in the Universe:

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\epsilon + \frac{\Lambda}{3}$$

(21)

Here $G$ is the gravitational constant, $R$ is the size scale of the Universe, $\Lambda$ is the cosmological constant. $k$ is the curvature index. For $k = +1$, the Universe is closed (analogous to a sphere in 3d) and for $k = -1$, it is open. A flat Universe with $k = 0$ (analogous to a sheet in 3d) is favored by observational cosmology. The reader is invited to consult reviews and reference updates of this rapidly evolving field, such as found e.g. in the PDG-biannual review volumes.

Inspecting the form of Eq. (21) we see that a natural time constant of Universe expansion can be introduced:

$$\tau_U \simeq \sqrt{\frac{3c^2}{8\pi G\epsilon}} = 32\mu s \sqrt{\frac{\epsilon_0}{\epsilon}}, \quad \epsilon_0 = 1 \text{ GeV/fm}^3.$$  

(22)

The QCD energy scale $\epsilon_0 = 1 \text{ GeV/fm}^3$ here used is not the total energy density in the Universe at the time of hadronization. Aside of Quarks and gluons which comprise about 30 degrees of freedom of the visible matter energy density near to the hadronization condition there are furthermore 14.25 other visible matter degrees of freedom (electrons, muons, 3 (left-handed) neutrinos and antineutrinos and photons). And there is dark matter.

5.3. Dark matter at time of hadronization

Potentially relevant is the influence of dark matter on the dynamics of the QCD phase transition. Today, dark matter energy density (24% of all) is about 6 times as large as that of visible matter (4.2%); the balance (72 ± 5%) is the so called dark ‘energy’. The experimental evidence strongly favors dark energy in the form proposed by Einstein, i.e. a gravity repulsive $\Lambda$-term. If this is the case, the presence of dark energy does not influence the quark Universe, since the energy density due
to the Einstein $\Lambda$-term does not change with time, but has been always of the magnitude we see today. However, the visible energy scales with $R^3$, or $R^4$, the faster scaling applies to radiation dominated era of the Universe. Since $R$ describes the growth of the Universe from hadronization era to present, without doubt the density of visible matter dominates the Einstein-like dark energy at hadronization by an astronomical number of orders of magnitude.

The situation with dark matter is different. We do not know how dark matter density extrapolates back in time to the QGP hadronization era. We note that it is not possible that dark matter was a negligible energy component at time of QGP hadronization, since it is credited with the seeding of the (visible) matter fluctuations, which ultimately were the cause of fast stellar and galactic structure development in the Universe. Thus one would be tempted to believe that dark matter was the dominant gravitational component in the early Universe, also at the time of hadronization. The question is, if the time constants were accelerated by 1, 5, 10, or even 15 or more orders of magnitude, in which case the early Universe may have more resembled the fast exploding QGP created in heavy ion collisions.

We recall that the visible matter energy content has been converted into the background radiation and has been consumed by the Universe expansion — this is recognized by the large entropy per baryon ratio. What we see today is a tiny $10^{-9} – 10^{-10}$ fraction of what was originally the visible energy content of the Universe. Thus both dark and visible energy content of the Universe may have changed considerable since the quark-hadron epoch of the Universe.

We close this discussion with a few examples of how astro-particle models of dark matter, and how these may extrapolate back in time to hadronization epoch:

(i) So called ‘warm’ dark matter candidate could be e.g. a relatively light sterile neutrino with mass of $m_{\nu_s} < 15\text{keV}$.[24] Any such dark matter particle would have frozen out from the dense matter long before hadronization. If we assume that their decay/annihilation is not material on time scale of $30\mu s$, we are considering an upper limit of what the influence of such dark matter may be on the hadronizing Universe.

We note that the Universe expansion ‘cools’ the momentum $\vec{p}$ of dark matter $\vec{p} \rightarrow \vec{p}/R$, while the energy density in the Universe is that of a radiation dominated Universe. Hence, as long as the ambient temperature $T \gg m_{\nu_s}$ is higher than the warm dark matter particles, their density in the Universe is practically the same as that of a thermally coupled particle. Therefore, at the time of QGP hadronization, such dark matter would be at most (i.e. if it does not decay or annihilate) another gravitating effectively massless particle, contributing several (1, 2, ?) degrees of freedom to the total count of about 45. Thus a sterile neutrino, or other warm dark matter would have marginal influence on dynamics of expanding quark-hadron Universe.

(ii) Dark matter particles in mass range of $m_d \simeq 1$–few MeV could annihilate into $e^+e^-$ pairs.[25] Like warm dark matter, at the time of QGP hadronization, this
type of matter would still be relativistic. The abundance can be expected to be in chemical equilibrium with its annihilation products at the temperature well above the formation threshold $e^+e^- \rightarrow 2m_d$. We would have one (for the scalar model studied in depth\textsuperscript{[23]} or at most several, additional degrees of freedom, and again negligible impact on dynamics of the hadronizing Universe.

(iii) Moving up in mass by a few orders in magnitude, we note that dark matter could predominantly consist of relatively very heavy, as measured on current particle mass scale, (\textit{e.g.} super-symmetric) quasi-stable particles with $m_d \gg T_h$. Such particles would make for cold dark matter at the time of QGP hadronization. Being cold at that time, as well as now, and stable on scale of the Hubble-time, they must have already frozen out, and could not appreciably feed their energy into the expansion of the Universe in the period following hadronization. The universe then and now would be filled with a dust of heavy invisible matter. This type of dark matter is the most studied model, and decay rate limits and annihilation rate limits are known for different types of particles\textsuperscript{[23]}

The worrisome thought is that in principle, it is possible that very massive dark matter decays or annihilates at a scale which would deplete its number by much more than a factor $10^{10}$ during the Universe expansion, and yet it would still be the dominant matter form in the present day Universe. It suffices to think of a family of dark matter particles with the lightest one being quasi-stable and contributing today to energy balance in the Universe. All we need to alter the picture of Universe expansion is that the heavier particles are stable on scale of $30\mu s$, or even much shorter, since the dark matter component shortens the natural time constant, see Eq.\textsuperscript{[22]}

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