Two-dimensional optimal jetless solutions for hypervelocity impact and shock waves interaction

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Abstract. Two problems with similar hydrodynamic flows are considered. The first one is the problem of the hypervelocity impact of axisymmetric bodies with cavities on colliding surfaces. The second one is also axisymmetric case of interaction of shock wave (SW) propagating along axis and incident under some angle on the boundary of contact with a substance of higher impedance. In this case the incident SW is always oblique in coordinates of contact boundary. Let denote by T the intersection point of the free boundaries of the bodies in the first case, i.e., the point where begins the boundary of contact of two bodies. In the second case, T is the point at which the front of the incident SW reaches the boundary of contact of two media. In both cases, we have a similar hydrodynamic flows in the region behind the point T bounded by SW and the given flow parameters before SW. Managing movement of T by different ways one may get different solutions. Under optimal solution we mean here movement of T is such way that speed of T in coordinates of the correspondent substance is equal to the speed of the SW in the substance behind T during all time of movement in direction to axis of symmetry. For the problems, an assumption is reasonable that these optimal solutions inside a small area with a given finite space size reach the maximum pressure and temperature of the substance at the maximum compression for the given velocities and materials.

1. Introduction
The obtaining of high pressures and temperatures for highly compressed substance or, in other words, the obtaining of extreme matter is one of the key problems for a broad range of scientific and practical tasks. For example, the extremely-high compression of matter is essential to obtain various structures of chemical elements and compounds at high pressures and temperatures, as well as creating the conditions for nuclear reactions, including nuclear fusion reactions. The use of the cumulation phenomenon [1] for high energy densities, which are of great interest in physics, is widely known. When focusing of shock waves in cylindrical and spherical geometry, the energy supply is carried out indirectly through the creation of high pressure on the surface of a spherical or cylindrical body. In the case of high-speed impact, using shape closed to conical, allows to put the energy directly in the vicinity of the shock front.

Ideologically, this work is closest to [2–10] in which a substance filling the conical target is compressed by plane impactor. So in [6], it is shown the possibility of synthesizing diamond in conical targets, and in theoretical and experimental studies [2,3] of nuclear fusion. In [7] the analysis of jetless shock compression of a plate on a wedged-shaped target is presented. In that work the problem is discussed in terms of the rotation of the flow behind oblique shock. It is
Figure 1. (a) Scheme of hypervelocity impact for numerical simulation: 1—axis of symmetry; 2—numerical grid; 3—front of SW; 4—point T; 5—impactor 1; 6—impactor 2; 7—surface of cavity of colliding bodies. (b) Diagram explaining the calculation of speed of T.

noted that there is a strong solution with a high pressure behind the shock wave (SW) front and weak solutions with a lower pressure behind the front of SW. In the problem considered in [7] for the aluminum impactor and a lead target are realized only solutions of week family in both media. In the presented work it will be explained.

Unlike previous works in this paper the shapes of the impactors are not set initially. They are determined from the calculation.

2. Problem formulation
Let us consider scheme of a counter-collision of axisymmetric impactors with cavities shown in figure 1(a). We will refer to this problem as hypervelocity impact problem. Initially, the entire energy of the system is in the kinetic energy of the impactors. Behind T the kinetic energy of the impactors is transferred to the energy of SW. We will consider the development of the process in time. Let us consider changing of the solution in vicinity of T to the next point in time. If the speed of movement of T higher than speed of SW then the pressure behind SW drop down due to deviation of incident flux to SW front from normal direction. If the speed of movement of the T less than SW speed then SW overtakes T goes on the free surface and its pressure fall due to the rarefaction wave coming from the free surface.

Under optimal solution here we mean the case when during whole the time of convergence of the SW to the axis of symmetry the velocity of its front at T exactly equal to the movement velocity of T itself. The shapes of the impactors are called optimal in this case respectively. In this case, condition that incoming flux in coordinates of SW front has normal direction to the surface of the SW on both sides of T is automatically satisfied. This in turn provides maximum pressure behind SW. If we consider the resulting optimal solution as a process of transition from one state to another, it can be represented as a sequence of events where every state on next time have maximum pressure behind SW in point T in compare with all other possibilities. This allow to assume that these optimal solutions achieve maximum pressure and temperature parameters of the substance under maximum compression for the given initial conditions of these problems inside small area with a given finite space size also on stage after reflection of SW from axis of symmetry. The additional condition on the finite space size area is due to the fact that solutions with flows converging to the axis of symmetry can produce infinite pressure in infinitely small areas near the axis. For numerical methods in this case, it is convenient to use the cell size as
Figure 2. (a) Scheme of an oblique shock wave interaction: 1—axis of symmetry; 2—numerical grid; 3—reflected SW; 4—T; 5—passing SW; 6—impactor 1; 7—impactor 2; 8—incident SW; 9—surface of contact of two media. (b) Diagram explaining the calculation of speed of T.

this finite space size and flow parameters in this cell when the maximum flow parameters are achieved.

The second problem will be referred to as the problem of an oblique shock wave interaction. Figure 2(a) shows the optimal oblique shock wave interaction scheme. As in case of considered hypervelocity impact we have divergent shock waves from T. The difference is only that the energy supply here is the energy of incident SW. Figure 3 presents optimal [figure 3(a)] and other oblique shock wave interaction scheme. If the speed of movement of T higher than speed of SW behind T [figure 3(b)], then the pressure behind SW drop down due to increase of incident flux to SW front from normal direction. If the speed of movement of T less than SW speed behind T, figure 3(c), then incident SW de-attaches from contact discontinuity and irregular SW reflection with Mach stem T–T′ appears.

2.1. Calculation of motion T

The solution of hypervelocity impact here is a generalization of solution of the problem of the impact of bodies of the same material [11].

Numerically, the position of the front at each time step is calculated by the Huygens principle by constructing a envelope for perturbations from the nodes of the numerical grid boundary. Figure 4 shows the reconstruction of a fragment of the numerical boundary represented at a new time step in the vicinity T. The boundary of the grid is a sequence of nodes connected by segments of straight lines. The new position of SW front is marked in figure 2, the old one is 1. Let us consider the calculation of new front location in more detail. First, the nodes of the boundary shift with the velocity of the substance before SW front. In figure 4, the circles drawn by a continuous line have centers in the shifted nodes and the radii are proportional to the propagation velocity of the disturbance in the medium. The shifted nodes are not shown in figure 4. But to illustrate the propagation of perturbation in a moving medium, a dotted circle with the same radius vectors drawn at the corresponding nodes of the old boundary is shown. Then the points of tangency of the straight lines simultaneously touching the obtained circles from the outside are determined. For nodes with convex angles new position of the node
Figure 3. Wave structures in the vicinity of point T: (a) with the optimum angle $\alpha$ between the incident SW and the contact surface; (b) with an angle $\beta < \alpha$; (c) with an angle $\gamma > \alpha$; 1—contact discontinuity; 2—reflected SW; 3—passing SW; 4—T; 5—incident SW; 6—contact between two media.

is defined at the circumference midway between the two touch points. Straight lines connecting adjacent nodes of the new border, in this case intersect the circle of perturbation. This is seen in figure 4 and is a reflection of the construction of the border, not the negligence of the sketch. For concave angles, the new node position is the intersection of tangent lines. Node T is a special node at the SW front, which is simultaneously contact point separating two media. By managing the movement of T at each time step, we determine the contact boundary of the two media. The formulas obtained below determine the movement of this node, according to the specified declaration of optimal solution. This is what determines the desired shape of cavity of colliding bodies. For considered two problems these formulas applicable in the sector shown in figure 4.

From consideration of figure 1(b) one may write

$$\begin{align*}
D_1 \sin a_1 &= D_2 \sin a_2, \\
D_1 \cos a_1 + D_2 \cos a_2 &= U,
\end{align*}$$

where indexes 1 and 2 denote correspondent impactor, $D$ is the velocities of SW in coordinates of impactor, $a$ is the angle between direction of counter-strike collision and surfaces of impactor, $U$ is the difference in the velocities of impactors. From (1) we have

$$a_i = \arccos \frac{U^2 + D_i^2 - D_{(3-i)^2}}{2UD_i}, \quad i = 1, 2. \tag{2}$$

In particular, the counter collision of bodies of the same material $D_1 = D_2 = D$ and respectively

$$a = \arccos \frac{U}{2D}. \tag{3}$$

Calculation of the velocity of T in case optimal oblique shock wave interaction is based on the fact that in this formulation, the most far along the contact boundary propagate disturbances form T figure 2(b). The projection of the velocity of motion T in the direction of movement of incident SW is equal to the velocity of incident SW. At the same time, the movement velocity of T is the maximum velocity of the disturbance propagation along the boundary of media. The
Figure 4. Calculations of the new position of the front based on the Huygens principle: 1—the broken line of the border at the previous time step; 2—the broken line of the border at the new time step; 3—contact discontinuity; 4—the sector for calculation of new position $T$ by formulas (2) and (4); 5—$T$.

perturbation velocity consists of the perturbation velocity in unmoved medium and the velocity of the medium. The second media is undisturbed. Therefore, we can write the formula

$$
\alpha = \begin{cases} 
\arccos \frac{D_0}{D_2} & \text{for } D_2 \geq C_1, \\
\arccos \frac{D_0}{C_1} & \text{for } D_2 < C_1,
\end{cases}
$$

(4)

where $C_1^2 = D_1^2 + D_0^2 - (D_0 - U_1)^2$, $U_1$ is the mass velocity behind incident SW, $D_0$ is the velocity of incident SW, $D_1$ is the velocity of reflected SW in medium 1, $D_2$ is the velocity of passed SW in medium 2.

2.2. Numerical simulation strategy

As stated above, the concept of optimal flow leads to a simple formulation for numerical solution, using explicit SW and contact discontinuity tracking method on moving grids. In this case, every impactor has the separate numerical region figure 1(a). They are limited by moving boundary with the front of SW conditions, rigid wall boundary conditions at the top and contact discontinuity between the numerical regions. Before the SW front the unperturbed substance moves at a constant set velocity $U_1$ for left side and $U_2$ for right side. The numerical region limited thus continually increases in size as the proliferation of SW. The code automatically increases the number of cells of the numerical grids to maintain a given spatial resolution. We start the calculations with the grid, which consists of several cells only. When the SW reaches the axis of symmetry, the boundary conditions are changed to the boundary conditions of rigid wall. From calculation we get trajectory of $T$ in coordinates $X, Y$ as function of time $\tau$:

$$
T(X, Y) = \{X(\tau), Y(\tau)\}.
$$

(5)
For hypervelocity impact, the initial distance from each point on the surface of cavity to the trajectory of T is the linear function of time, and the shape of every cavity \( S_i \) is easily restored from the calculation:

\[
S_i(X,Y) = \{X(\tau) - U_i\tau, Y(\tau)\},
\]

where \( U_i \) is mass velocity of every impactor.

Thus, one calculation immediately gives the solution for optimal geometries and calculates optimal geometry itself.

2.3. Mathematical model of medium motion and numerical method

The motion of the medium is described by the equations of Euler hydrodynamics for compressible inviscid media, or equations of elastic-plastic motion model of ideal plasticity as described by Mises. The equations of motion of the medium are closed by equations of state SESAME [12].

The numerical method uses the basis of Godunov’s method for a curved quadrangular movable numerical meshes with explicit discontinuities tracking (contact discontinuities, SW). Unlike the original, the method includes developments in the field of construction and optimization of numerical grids [13].

3. An example of the calculation

It was attempt to calculate optimal solution and optimal shapes for the problem considered in [7] for the aluminum impactor and a lead target colliding with velocity \( U = 4 \) km/s. However, the problem has not optimal solution for these materials with these velocities. For equation (2), there is a restriction

\[
|\left( U_i^2 + D_i^2 - D_{(3-i)}^2 \right) | < 2UD_i, \quad i = 1, 2.
\]

For the problem, we have \( U = 4, D_1 \approx 10, D_2 \approx 3 \). So for lead target \( (i = 2) \), condition (7) is not true and the angle \( a_2 \) can not be defined by (2) and we can not get strong solution for this case. Analyzing (7) one may conclude that the \( D_i \) of interacting media should be close. That is why aluminum was replaced by gold. Calculation was performed for impact of lead \( (i = 1) \) with gold \( (i = 2) \) with a speed of \( U = 4 \) km/s; for cavities with an initial radius \( R_0 = 0.3 \) cm, in the axisymmetric formulation according scheme shown in figure 1(a). It is convenient to use the laboratory coordinate system with zero total pulse of interacting impactors. For determination of \( U_1, U_2 \) in the laboratory coordinates, we use the system of equations

\[
\begin{align*}
\rho_1U_1 + \rho_2U_2 &= 0, \\
U_1 - U_2 &= U,
\end{align*}
\]

where \( \rho_i, U_i \) are density and velocity of correspondent impactor. The calculation is started from 50 cells for grid of gold impactor and 50 cells for grid of lead impactor. At the finish of the calculation, the number of cells increases to 4000 cells for every grid.

Figure 5 shows numerical grids painting in palette connecting with sound speed for two points in time, illustrating the dynamics of the process. Figure 5(a) corresponds to the time when SW reaches about half of initial cavity size. One may see that numerical grids for both regions have close colors, so they have close sound speed and close SW front velocity. Figure 5(b) corresponds to the time when SW reaches the axis of symmetry.

The dependence of maximum pressure and correspondent density on time for gold part is shown in figure 6. The peak is achieved in the moment of reflection of SW from the axis of symmetry, when the pressure reach 230 Mbar, with cell size of 3 \( \mu \)m. Figure 6(b) shows fine structure of the peak with two peaks of pressure and density oscillations. This pressure is much higher then pressure for plane collision \( \approx 1.6 \) Mbar.

The calculated dependence of the coordinate of point T upon time gives information on the shape of the cavity. Since the hydrodynamics equations allow simultaneous linear transformation
Figure 5. Numerical grids for hypervelocity impact drown with palette for sound speed: (a) at moment 205 ns; (b) after the moment when SW reaches the axis of symmetry, at 405 ns.

Figure 6. The dependence of the maximum pressure and density in gold at impact velocity of 4 km/s with lead for the initial radius of the cavity 0.3 cm: (a) full-time; (b) 3 ns time fragment in the region of maximum compression. The red line corresponds to the pressure, blue line is density.

of space and time, the optimal cavity shape remains unchanged for these materials and this collision velocity for all body sizes. Calculated $S_i$ determined by equation (6) are given in figure 7 together with trajectory of T in dimensionless coordinates normalized to the initial radius of the cavity $R_0$.

At the time when T reaches approximately the middle of the initial cavity size, the calculated angles between the direction of the incident flow and the surface of the front are shown in figure 8(a). The angles are calculated at the middle of every cell of numerical grid along the front separately for lead and gold projectile. They are shown in figure 8(a) by markers and connected by lines. Fist point corresponds to cell on upper boundary with rigid wall boundary condition. Last point corresponds to cell connected with T. One may see that the most deviation from $90^\circ$ at the end of cell sequence, that is near by T. It is explained by high gradients in the
solution near this point and fast angle rotation. That is seen in figure 8(b). As result we have bigger error in the numerical estimation of the angle. One may say that within the error range along all SW there is one angle equal to 90°. There is no need to build a shock polar to conclude that we have solution of strong family in terms of turn of flow behind oblique SW and more over we always have maximum pressure behind SW at top point of shock polar for the incident flow. An interesting fact is that by controlling the movement of one point T we get a strong solution in the entire region of the shock wave propagation.

4. Conclusions

It is shown that, for a high speed counter-collision, there is a unique special solution, which is referred to herein as optimal when the shape of the cavity in the impacting bodies is such that at a given impact velocity, the SW during the whole time of impact, until the exit on symmetry axis, reaches the free surface of the colliding bodies, but do not cross the free surface. In other words, we always have one point in cross section of the free surface reached by SW but no more. Thus, the shape of the cavity computed uniquely and it is fixed for a given material and impact velocity. In the planar case of collision there is a fixed optimum angle between colliding bodies.

This optimal solution is a sequence of events in time, in which every next time we have a maximum pressure behind SW at point T compared to all other possible solutions during the entire time of impact, up to the exit to the axis of symmetry.

Thus, there is a reason to assume that the optimal solution inside a small area with a given finite space size reaches the maximum pressure at the maximum compression of the substance.
for these materials and speeds also for the times after shock-wave reflection from the axis of symmetry.

An efficient method of calculation of both the optimal shapes of impactors and the parameters of the cumulative impact by one calculation based on numerical simulation in moving grids with tracking of contact discontinuities and shock waves is presented.

The solution has restriction for different materials of colliding bodies but it always exist for bodies made of the same material.

It is shown that similar approach may be used for other tasks with shock wave interaction. It was considered problem of oblique shock wave interaction with the contact of two media. The formula for calculation of optimal shape of the contact is presented.

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