Calculation of the bottom of vertical cylindrical steel tanks under the action of special loads

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Abstract. The operation of the steel tanks under the action of two types of special loads is considered. The first type of loads represents the overpressure resulting from the ignition of vapors in the reservoir itself or from the temperature effects of a burning adjacent object. The second type of loads is related to hydrodynamic pressure of product on shell during earthquake. New engineering method of calculation of shell-to-bottom joint is proposed. The results can be used in normative rules.

1. Introduction

Shell-to-bottom joint is the most stressed part of a vertical steel storage tank. Numerous publications, which are devoted to analysis of the behavior of this joint, can be roughly divided into four groups. The first research group employs the system of plates and shells. For example, in the paper [1] the bottom joint of the tank is made up from long cylindrical shell and annular ring. Each of the elements are modeling within shell theory. Integration of the equations for these elements constitute the system of seven algebraic equations. In the paper [2] arbitrary configuration rotational shells analysis methods are suggested. Obviously, that the models of such type have the cumbersome mathematical apparatus.

The second research group combines publications, performed by finite-element method (for example, the second part of paper [1]). The finite-element model uses specific numerical data. The presence of large amount of input parameters cause difficulties for reporting of the numeric results in convenient form. In the third group of models, the shell and the bottom are considered as a system of two semi-infinite beams-bands of unit width. Both beams are supported on elastic foundations, which consider spatial rigidity of cylindrical shell and real foundation characteristics [3]. The problem is solved by the force method and provides the simplicity of design ratios. However, the employment of such solution in practice is correct if the shell is supported on earth foundation. Most of the tanks are supported on concrete foundation, what fundamentally alters operation scheme of shell-to-bottom joint. The fourth research group considers only that section of the bottom, which is out of touch with rigid foundation. The bottom is modeled as the beam-band and two unknowns characterize its distortion: the radial dimension of the beam and the angle of rotation of the shell [4], [5], [6].

It must be noted that foregoing publications consider hydrostatic load exerted only at normal operating conditions. In this paper shell-to-bottom joint analysis methodology is suggested for the storage tank, which is subjected to the impact of special loads. The first type of such loads represents internal pressure, caused either by vapors inflammation inside this tank, or by thermal effect from adjacent object, which is on fire (Figure 1a) [7], [8]. The second type of loads is associated with horizontal hydrodynamic product pressure against cylindrical shell walls during earthquake (Figure 1b) [9]. In either case, there is an uplift of the shell and a lifting of some section of the bottom over the foundation. The de-
formation scheme of the lower part of the tank alters completely at that in comparison with deformations at normal operating conditions. Numerous publications are devoted to the analysis of the tank, subjected to the impact of seismic loads [9]. The papers [10], [11], [12] they are interesting because they allow to implement the results in codes and regulations. However, article [10] uses a model of the bottom of the tank, which does not take into account the action of longitudinal forces. Analysis of this assumption is given in [12]. The purpose of this publication is to obtain design ratios ensuring safe operation of tanks under conditions of special loads. The principal difference of the new method is to take into account not only the bending deformations of the bottom, but also the longitudinal deformations of the bottom that occur when the uplift shell above the foundation.

![Diagram of tank bottom under special loads](image)

**Figure 1.** Deformation of the tank bottom under special loads

2. **Shell-to-bottom joint**

Consider the portion of the tank bottom adjacent to the shell. It is in a bending state and can be examined within beam theory [3], [4], [5]. The calculation is made according to the deformed scheme (Figure 1c) and takes into account not only the bending state of the thin-walled bottom, but also the radial tension forces $N_0$ arising in it from the transverse load $q$. This load is determined by the formula:
where \( \rho \) – product density, \( H \) – liquid level, \( t \) - bottom thickness, \( \rho_m \) – specific mass of bottom metal of thickness \( t \), \( p_0 \) – overpressure in the gas space of the tank, \( g \) – acceleration of gravity.

In extreme sections of bottom with unknown width \( l \): the lifting force \( q_s \), the bending moment \( M_A \), shear forces \( Q_0 \), \( Q_A \). Further calculation is performed for a beam-band of unit width, whose deformations are subject to the following obvious dependencies:

\[
\frac{d^2w}{dx^2} = \frac{M_x}{D} \quad M_x = M_0 + Q_0x - 0.5qx^2 + N_0(w - w_0)
\]

where \( w, M_x \) – accordingly, the vertical displacement and the moment in the section with the \( x \) coordinate; \( w_0, M_0, Q_0 \) – initial parameters representing respectively deflection, bending moment and shear force at \( x=0 \).

The complete mathematical formulation of the problem has the form:

\[
\frac{d^2w}{d\chi^2} - v^2w = -v^2w_0 + (M_0l^2 + Q_0l^3\chi - 0.5ql^4\chi^2)/D
\]

for \( \chi=0 \): \( w_0 = 0 \) \( \phi_0 = 0 \) \( M_0 = 0 \) for \( \chi=1 \): \( M_1(1) = -M_A \)

where \( D \) - cylindrical stiffness of the bottom.

Here and hereafter the symbols and dependencies are used:

\[
\nu^2 = \frac{N_0 l^2}{D} \quad \chi = \frac{x}{l} \quad \phi = \frac{1}{l} \frac{dw}{d\chi} \quad M_s = \frac{D}{l} \frac{d\phi}{d\chi} \quad Q_s = \frac{1}{l} \frac{dM_s}{d\chi}
\]

The integration of equation (3) leads to the following result:

\[
w = w_0 + C_1 e^{\nu(x-1)} + C_2 e^{-\nu x} - [M_0l^2 e^{\nu(x-1)} + Q_0l^3 \nu - ql^4 (0.5\nu^2 + v^2)](Dv^2)^{-1}
\]

where \( C_1, C_2 \) - arbitrary constants to be defined.

Insert equation (6) into the first two boundary conditions (4) and come to the final expression for \( w(x) \). We consistently differentiate the obtained relation and taking into account the third condition (4) we write:

\[
w = \{Q_0(l/\nu)^3[sh(\nu\chi) - \nu\nu] \cdot q(l/\nu)^3[1 - ch(\nu\chi) + 0.5\nu^2\chi^2]\}D^{-1}
\]

\[
\phi = \{Q_0(l/\nu)^3[ch(\nu\chi) - 1] \cdot q(l/\nu)^3[\nu\nu - sh(\nu\chi)]\}D^{-1}
\]

\[
M_s = Q_0(l/\nu)sh(\nu\chi) \cdot q(l/\nu)^3[1 - ch(\nu\chi)]
\]

\[
Q_s = Q_0 ch(\nu\chi) - q(l/\nu)sh(\nu\chi)
\]

We use the equilibrium condition of the system of forces:

\[
Q_0 = ql + Q_A
\]

Taking into account the equality (8), the last dependence (7) at \( \chi=1 \) takes the form:

\[
Q_A = ql f_1(\nu)
\]

Substitute the third relation (7) and equality (8), (9) in the last condition (4) and we get:

\[
M_A = ql^2 f_2(\nu)
\]

In formulas (9), (10) symbols are introduced:
\[ f_1(v) = \frac{vcch(v) - sh(v)}{v[1 - ch(v)]} \]
\[ f_2(v) = \frac{2ch(v) - 2 - v sh(v)}{v^2[1 - ch(v)]} \]  
(11)

Analysis of the graphs \( f_1(v), f_2(v) \) in the real range of the parameter \( 0 < v < 2.5 \) shows (Figures 2a, 2b) that with an error of not more than 6% they can be approximated by the constants \( f_1 = -0.7, f_2 = 0.16 \). Then expressions (9), (10) take the form:

\[ Q_A = -0.7ql \]
\[ M_A = 0.16ql^2 \]  
(12)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Results of calculation of the bottom}
\end{figure}

Assign a condition as a criterion of the bearing capacity of the bottom:

\[ M_A \leq 0.25R_y t^2 \]  
(13)

where \( R_y \) – the yield strength divided by the importance factor of the structure.

Equality (13) means that in the design section formed a plastic hinge. After substituting the second
formula (12) into condition (13) we obtain:

\[ l = 1.25 t \sqrt{R_y / q} \]  \hspace{1cm} (14)

In this case, the lifting force \( q_s \), equal to the shear force in the bottom \( Q_A \) (Figure 1c), is determined by the relations (12), (14). As a result we get the following expression:

\[ q_s = 0.875 t \sqrt{q R_y} \]  \hspace{1cm} (15)

We further apply the formula (15) to investigate the limit state of the tank in the case of each of the previously specified special loads.

To calculate the allowable overpressure in the tank \( p_0 \) from the condition of the strength of the bottom, we use the obvious equality:

\[ 2 \pi r q_s + G = \pi r^2 p_0 \]  \hspace{1cm} (16)

where \( r \) – tank radius, \( G \) - the weight of the structures and equipment of the tank, with the exception of the bottom.

From the ratio (16), taking into account the expressions (1), (15) it follows:

\[ p_0 = 1.75 \delta \sqrt{\left( \rho g H + \rho_t g t + p_0 \right) R_y + \frac{G}{\pi r^2}} \]  \hspace{1cm} (17)

where indicated \( \delta = t/r \).

Expressing the required value from the dependence (17), we finally get:

\[ p_0 = \frac{G}{\pi r^2} + 1.53 R_y \delta^2 + \left( \frac{G}{\pi r^2} + 1.53 R_y \delta^2 \right)^2 - \left( \frac{G}{\pi r^2} \right)^2 + 3.06 R_y \delta^2 (\rho H + \rho_t t) \]  \hspace{1cm} (18)

The graphic dependence of the pressure \( p_0 \) on the liquid level \( H \) is shown in Figure 2c. The results are obtained with the following initial data: \( r=11.4 \) m, \( H=12 \) m, \( t=7 \) mm, \( \rho=10 \) kN/m\(^3\), \( \rho_t=78.5 \) kN/m\(^3\), \( G=911 \) kN. Curves 1, 2 correspond to the values of the yield strength \( R_y=295.2 \) MPa and \( R_y=223.8 \) MPa. It is found that the lowest pressure obtained at \( H=0 \) is 3.39 kPa for curve 1 and 3.22 kPa for curve 2. These values are 41.1% and 34.0% higher than normal operating pressure \( p_1=2.4 \) kPa.

We pass to the second type special burden. The main parameter in an earthquake is the overturning moment \( M_t \). This value integrates takes into account the inertial and hydrodynamic loads on the tank body and is determined by the methods presented in the articles [10], [11]. One of the options for the development of an emergency situation is the case when the tank shell is uplift above the foundation (Figure 1b). This type of deformation corresponds to the following inequality [10], [12]:

\[ \frac{M_t}{r^2 (q_s + G / 2 \pi r)} \leq 2 \pi \]  \hspace{1cm} (19)

If condition (19) is met, anchors are not required [10], [12]. However, the bottom may experience significant deformation in the area located on the side of the earthquake epicenter. We apply for this case in the formula (19) equality (15). As a result we have:

\[ M_t \leq 1.75 \pi r^2 t \sqrt{\left( \rho g H + \rho_t g t + p_0 \right) R_y + rG} \]  \hspace{1cm} (20)

The inequality (16) provides the bearing capacity of the bottom according to the criterion (13) and is one of the main conditions of seismic resistance of the tank.
3. Conclusion

Formula (18) allows not only to determine the maximum permissible excess emergency pressure, but also to assign the thickness of the bottom to ensure the strength of the lower part of the tank. In addition, formula (18) can be used when assigning the performance of emergency overpressure relief valves. Inequality (20) allows us to assign the thickness of the bottom of the tank in an earthquake. The specified formulas have the form convenient for use in normative ruler.

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