Identification of Automotive Seat Rattle Noise Using an Independent Component Analysis-Based Coherence Analysis Technique

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Abstract: This study proposed a contribution evaluation through the independent component analysis (ICA) method. The necessity of applying ICA to the evaluation of contribution was investigated through numerical simulation. Moreover, the estimation of the number of input sources, the labeling of signals, and the restoration of the signal amplitude were considered to perform the ICA-based coherence evaluation. The contribution evaluation was performed using the coherence evaluation method and by applying the established ICA-based coherence evaluation method to the seat rattle noise of the vehicle. According to the result of the evaluation, with the coherence evaluation technique it was difficult to calculate the contribution in identifying noise sources that overlap in both spatially and in frequency, because it was challenging to distinguish between the two measured signals. By contrast, the ICA-based coherence evaluation was able to restore the original source and investigate the contribution.

Keywords: independent component analysis; coherence analysis technique; rattle noise

1. Introduction

As noises caused by internal combustion engines have become less of an issue with the advent of electric vehicles, the sound quality of noise has become an important factor representing vehicle competitiveness. The Buzz, Squeak, and Rattle (BSR) of vehicles are typical affective quality items of Noise, Vibration, and Harshness (NVH), which are noises generated from assembly seams parts or joints parts [1,2]. Studies on vehicle BSR have recently been actively conducted, mainly using experimental methods [3–5]. Previous studies regarding the improvement of BSR have focused mainly on the accurate quantification of noise and the location of noise sources, such as cockpit modules and seats. D. H. seo at el. used the beamforming technique to locate the noise sources [6], identifying interior/exterior BSR noise for the full vehicle. The beamforming method involves directing each microphone constituting the array in a certain direction by giving an appropriate weight and phase delay. When estimating the location of the noise source, this method is used to discover the direction of the largest output by varying the orientation direction. However, although this method can be used to determine the location of a noise source on a 2D plane, it is difficult to determine exactly where the noise is generated if the sources overlap with the parts or are proximal to each other. Another popular solution to identify sound sources is a method of finding noise sources using complex acoustic intensity [7,8]. Although this method has a definite advantage in identifying the noise source, it is difficult to determine the exact location of the noise source from two independent noise sources with similar frequency characteristics and similar generation locations.

Among the methods for identifying the location of the noise source, a coherence analysis method as a signal processing method is used in several areas [9,10]. This method, which is applied to the road
noise, booming noise, and rumble noise of vehicles, is used to identify noise sources. Furthermore, this method is widely used due to the advantage of evaluating the pure contribution of each input by removing the correlation between noise sources when applied to home appliances. The noise source identification method using these contributions assumes that the inputs are independent of each other when the correlation is removed. When the locations of the signal sources are clear and far from each other (namely, the sources are independent and do not affect each other), the coherence analysis method is widely used [11,12]. In particular, the partial coherence function (PCF) is applied when the inputs are highly correlated with each other [13,14]. However, this method is difficult to apply when the locations of occurrence are ambiguous and proximal to each other. This is because the noise generated from the noise source affects all the nearby microphones, and the contribution analysis method cannot be used to accurately evaluate the contribution.

In order to solve this problem, it is necessary to restore an independent original source through the measured signals. The contribution analysis method is used to calculate the contribution to the independent signal output by removing the correlation between the two signals. Thus, it is necessary to apply an algorithm to the contribution analysis method for restoring an independent original signal by using the mixed signals during measurement.

The well-known basic techniques in source separation are principal component analysis (PCA) and independent component analysis (ICA). According to recent studies, independent component analysis (ICA) shows a better performance in signal separation [15,16]. Independent component analysis (ICA) is a method of restoring independent original signals from mixed signals. ICA originates from the field of signal processing, where it has recently attracted a lot of attention. In addition, the literature on ICA has become extremely vast; however, there have been few reports on the identification of sources. It relies on the paradigm that a mixture of sources can be separated, provided that the sources bear different enough statistical properties. In a sense, the sources should have disjoint support sets in some statistical space. Some examples of sound signal separation are given in [17,18], although they are closer to communication signal processing than the issue addressed in the present paper.

Dong et al. attempted to apply ICA to the fields of noise and vibration. They applied ICA to the inverse problem of noise to identify noise sources that overlap in both the spatial and frequency domains. Furthermore, they identified the noise source by applying ICA to a diesel engine, which is a complex structure [19,20]. Moreover, Chang et al. applied ICA to operational transfer path analysis (OTPA) and removed the crosstalk effect of the reference signal through ICA. The method in the study identified a more accurate delivery route than the existing OTPA method [21].

Thus, it is necessary to apply the ICA method to the existing contribution analysis method. The method of restoring independent signals will enable the contribution to be evaluated in situations in which it is difficult to apply the existing coherence analysis method. However, there are several problems to be solved in order to use ICA for coherence analysis. First, the number of input sources should be estimated. Noise is measured at several locations to estimate the noise source, which may not be where the noise is actually generated. To confirm the location, it is necessary to estimate the number of input sources to determine exactly where the noise is generated. Minka Bayesian Model Selection (MIBS) can be used to accurately separate signals only by determining the correct number of sources [22]. Second, the labeling of signals should be performed. When a signal is separated using ICA, it is difficult to determine which signal originated from which location because each signal is not labelled. In order to identify the noise source, it is necessary to determine where the separated signal originated. The rest of the paper is devoted to proposing solutions to these issues.

This study intends to identify the exact location of the seat rattle noise of vehicles. For this purpose, an ICA-based coherence technique was proposed. Section 2 deals with the coherence analysis method, the theory of ICA, and the performed numerical simulations. Section 3 applies the proposed method to the seat rattle noise. Furthermore, it discusses the estimation of the number of input sources, labeling of signals, and size restoration to utilize the ICA method in the coherence evaluation method.
Finally, it identifies the location of the seat rattle noise of the vehicle using an ICA-based contribution analysis method.

Thus, in this paper, the ICA-based coherence technique is proposed by improving the coherence method. In order to apply the ICA to the coherence evaluation, the following problems were solved: the restoration of signal amplitude, the estimation of the number of signals, and the labeling of signals.

2. Theory

2.1. Ordinary and Partial Coherence

The identification of a major noise source is typically difficult because it is challenging to evaluate the contribution between input and output when the inputs are partially coherent. In signal processing, the partial contribution analysis method calculates the contribution by using optimal linear frequency response functions in which the correlation between inputs is removed. This reveals the degree to which the input source directly contributes to the output. In the case of vehicle noise, the expected multiple-input/single-output model for the input/output and transfer function for the expected generation location of major BSR noise is shown in Figure 1.

![Correlated multiple-input/single-output model.](image)

The equation for output with respect to a certain $q$ number of inputs in this multiple-input/single-output spectrum model can be represented as shown in Equation (1) in the frequency domain.

$$S_{yy}(f) = \sum_{j=1}^{q} \sum_{i=1}^{q} H_{ij}^*(f) H_{ij}(f) S_{ij}(f) + S_{mn}(f)$$

(1)

where $H_{ij}(f)$ represents a transfer function that relates the $i$-th input to output $y$, and $S_{ij}(f)$ denotes a cross-power spectrum between inputs $X_i(f)$ and $X_j(f)$. $H_{ij}^*(f)$ is the conjugate complex number of the transfer function. $H_{ij}(f)$ represents a coherence relation between the transfer paths. $S_{mn}(f)$ is expressed as a power spectrum of noise $N(f)$.

The coherence in a typical input/output model is represented using an ordinary coherence function (OCF) that represents correlations between inputs and between the input and output. This can be described as follows:

$$Y^2_{ij}(f) = \frac{|S_{ij}(f)|^2}{S_{ii}(f) S_{jj}(f)} \quad (i, j = 1, 2, \ldots, q; i \neq j)$$

(2)

$$Y^2_{iy}(f) = \frac{|S_{iy}(f)|^2}{S_{ii}(f) S_{yy}(f)} \quad (i, j = 1, 2, \ldots, q)$$

(3)

All of the inputs in the seat have transfer paths that are coupled to each other. In particular, it is difficult to distinguish the major noises in a complex structure, and the noises are coherent due to the close proximity of the noise measurement locations during measurement. For example, because two input sources are located close to each other, there are cases where similar signals are measured from the two microphones used in measuring the noises (Figure 2).
In this case, when the correlation between the input signals is significantly high, it is difficult to evaluate the contribution of the input to the output. When \( q = 2 \) in Equation (1), the output spectrum is given by Equation (4).

\[
S_{yy}(f) = \sum_{i=1}^{2} |H_{iy}(f)|^2 S_{ii} + S_{nn}(f) = |H_{1y}(f)|^2 S_{11} + |H_{2y}(f)|^2 S_{22} + S_{nn}(f)
\] (4)

In this case, the optimal transfer function between these two inputs is shown in Equation (5).

\[
L_{11} = \frac{S_{11}}{S_{11}}, \quad L_{22} = \frac{S_{22}}{S_{22}}
\] (5)

where \( S_{221}(f) \) refers to the input spectrum in which the correlation of the \( S_1 \) was removed from the \( S_2 \).

The residual spectrum between the input and output is expressed using the optimal transfer function, as shown in Equation (6).

\[
S_{221}(f) = S_{22}(f) - |L_{12}(f)|^2 S_{11}(f),
\]

\[
S_{2y1}(f) = S_{2y}(f) - |L_{1y}(f)|^2 S_{12}(f).
\] (6)

The output spectrum for any \( q \) number of inputs can be expressed as shown in Equation (7) using a residual spectrum.

\[
S_{yy}(f) = \sum_{i=1}^{d} |L_{iy}(f)|^2 S_{ii-(i-1)!}(f) + S_{nn}(f)
\] (7)

Equation (8), which is a partial coherence function, can evaluate the contribution of the input to the output by considering the correlation between the inputs.

\[
\gamma_{ij}^2(f) = \frac{|S_{iy-(i-1)!=j}(f)|^2}{S_{ii-(i-1)!=j}(f) S_{yy-(i-1)!=j}(f)} \quad (i, j = 1, 2, \ldots, q)
\] (8)

The multiple coherence function (MCF) is used to determine whether an appropriate input source has been selected, which is shown in Equation (9).

\[
\gamma_{yq!}^2 = 1 - \left[ (1 - \gamma_{1y}^2)(1 - \gamma_{2y}^2) \cdots (1 - \gamma_{qy-(q-1)!}^2) \right] \quad (i, j = 1, 2, \ldots, q)
\] (9)

2.2. Independent Component Analysis

The properties of the multidimensional data can be changed by projecting the data to a certain axis. In this case, the number of axes to be projected can be matched with the number of original data dimensions so that the dimensions of the data created after the projection are the same as those of the data before the projection or smaller to ensure the effect of compression. In particular, ICA is a method...
of transforming the projected values into independent ones by setting the axis in the direction with a high non-Gaussianity to remove the correlation between dimensions.

Let a \( d \) number of independent signal inputs be \( S_1, S_2, S_3, \ldots, S_d \), and the new signals obtained through linear combinations of the inputs be \( x_1, x_2, x_3, \ldots, x_n \). Then, the relationship between the signals can be expressed as shown in Equation (10).

\[
x_i = \sum_{j=1}^{d} a_{ij} s_j \quad (i = 1, 2, \ldots, n)
\]

where \( s_j \) and \( x_i \) are vectors which can be represented as \( s = (s_1, s_2, \cdots, s_d)^T \) and \( x = (x_1, x_2, \cdots, x_d)^T \), respectively, and \( a_{ij} \) is defined as the element of the mixing matrix. Then, Equation (10) can be represented as a vector matrix shown in Equation (11).

\[
x = As
\]

Here, in the state where \( s \) is not given and there is no information about mixing matrix \( A \), source \( s \) and mixing matrix \( A \) are estimated with only \( x \). This process is called blind source separation. Equation (12) can be written as follows to estimate the source:

\[
s = A^{-1} x = Wx
\]

where \( W \) is an inverse matrix of \( A \).

In this paper, independent components were found using the FastICA algorithm proposed by Hyvarinen and Oja [23]. The FastICA method uses a fixed-point algorithm based on the theory that finding independent components with the central limit theorem is the same as finding the most non-Gaussian direction [24]. The central limit theorem is the theory that the sum of independent random variables becomes closer to a Gaussian distribution than each independent component. Thus, matrix \( A \) is found so that the estimated source is non-Gaussian.

2.3. Numerical Simulation

Numerical simulations were performed for the case where noise sources with similar frequency bands overlap, or are located nearby, showing a high correlation with each other. The assumption was made that there are three sources, two of which have similar frequency bands and physical locations, resulting in a high correlation. Thus, it was assumed that the two sources are mixed with each other during measurement and are indistinguishable (Figure 3). A simulation was performed based on these numerical assumptions, as shown in the following equation.

\[
s_1 = \sin(2\pi f_1 t) + \sin(2\pi f_2 t) \quad (f_1 = 9 \text{ Hz}, \ f_2 = 23 \text{ Hz})
\]

\[
s_2 = \sin(2\pi f_3 t) + \sin(2\pi f_4 t) \quad (f_3 = 9 \text{ Hz}, \ f_4 = 11 \text{ Hz})
\]

\[
s_3 = \cos(2\pi f_5 t) + \cos(2\pi f_6 t) \quad (f_5 = 14 \text{ Hz}, \ f_6 = 17 \text{ Hz})
\]

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 
\end{bmatrix} = \begin{bmatrix}
0.85 & 0.81 & 0.01 \\
0.81 & 0.83 & 0.02 \\
0.01 & 0.015 & 0.8 
\end{bmatrix} \begin{bmatrix}
s_1 \\
s_2 \\
s_3 
\end{bmatrix}
\]

\[
y = x_1 + x_2 + x_3
\]

where \( s \) refers to the source and \( x \) refers to the measured signal.
Thus, PCF was performed to identify noise sources at 9, 11, and 23 Hz, and the results are shown in Figure 6.

For this numerical model, OCF was performed for the measured signals nearby, the measured signals through matrix $A$, and the output (Figure 5a,b). Signal $x_3$ contributes to the output at 14 and 17 Hz (Figure 5c). Thus, PCF was performed to identify noise sources at 9, 11, and 23 Hz, and the results are shown in Figure 6.

The frequencies of $s_1$ and $s_2$ are the same at 9 Hz, and $s_1$ and $s_2$ are mixed and measured through matrix $A$. In other words, because these two signals are physically overlapped or are located nearby, the measured signals $x_1$ and $x_2$ have similar characteristics to those of the frequency band. For this numerical model, OCF was performed for the measured signals $x_1$, $x_2$, and $x_3$ and the output $y$ (Figure 5).

Figure 4 shows the results of the frequency analysis performed for each measured signal $x$. The frequencies of $s_1$ and $s_2$ are the same at 9 Hz, and $s_1$ and $s_2$ are mixed and measured through matrix $A$. In other words, because these two signals are physically overlapped or are located nearby, the measured signals $x_1$ and $x_2$ have similar characteristics to those of the frequency band. For this numerical model, OCF was performed for the measured signals $x_1$, $x_2$, and $x_3$ and the output $y$ (Figure 5). The measured $x_1$ and $x_2$ signals, which are highly correlated with each other, show the same contribution to the output (Figure 5a,b). Signal $x_3$ contributes to the output at 14 and 17 Hz (Figure 5c). Thus, PCF was performed to identify noise sources at 9, 11, and 23 Hz, and the results are shown in Figure 6.
Figure 4. Frequency response of $x_1$, $x_2$, and $x_3$. The frequencies of $s_1$ and $s_2$ are the same at 9 Hz, and $s_1$ and $s_2$ are mixed and measured through matrix $A$. In other words, because these two signals are physically overlapped or are located nearby, the measured signals $x_1$ and $x_2$ have similar characteristics to those of the frequency band. For this numerical model, OCF was performed for the measured signals $x_1$, $x_2$, and $x_3$ and the output $y$ (Figure 5).

Figure 5. Ordinary coherence between input and output: (a) $\gamma_{y_1}$, (b) $\gamma_{y_2}$, (c) $\gamma_{y_3}$.

The results of the PCF showed that the measured signal $x_2$ had no contribution to the output (Figure 6b). This is because the ordering of $x_1$ is high in the process of calculating the mutual contribution of $x_1$ and $x_2$. Thus, it is difficult to accurately evaluate the contribution of coherence functions because the signals of the noise source are mixed, showing similar contributions, and it is challenging to show the contribution of the actual noise source to the output because the PCF calculates the mutual contribution based on this situation.

Thus, to solve this problem, it is necessary to restore the signal using ICA and evaluate the contribution through OCF based on the obtained signals. Because the signals are independent of each other when the signals are restored using ICA, there is no need to remove the correlation. Thus, it is possible to evaluate the contribution through OCF alone without using PCF.

First, a frequency analysis was performed by restoring the noise measured for three input sources using ICA (Figure 7).
The measured $x_1$ and $x_2$ signals, which are highly correlated with each other, show the same contribution to the output (Figure 5a,b). Signal $x_3$ contributes to the output at 14 and 17 Hz (Figure 5c). Thus, PCF was performed to identify noise sources at 9, 11, and 23 Hz, and the results are shown in Figure 6.

Figure 6. Partial coherence between input and output: (a) $\gamma_{1y}^2$, (b) $\gamma_{2y}^2$, (c) $\gamma_{3y}^2$.

The results of the PCF showed that the measured signal $x_2$ had no contribution to the output (Figure 6b). This is because the ordering of $x_1$ is high in the process of calculating the mutual contribution of $x_1$ and $x_2$. Thus, it is difficult to accurately evaluate the contribution of coherence functions because the signals of the noise source are mixed, showing similar contributions, and it is challenging to show the contribution of the actual noise source to the output because the PCF calculates the mutual contribution based on this situation.

Thus, to solve this problem, it is necessary to restore the signal using ICA and evaluate the contribution through OCF based on the obtained signals. Because the signals are independent of each other when the signals are restored using ICA, there is no need to remove the correlation. Thus, it is possible to evaluate the contribution through OCF alone without using PCF.

First, a frequency analysis was performed by restoring the noise measured for three input sources using ICA (Figure 7).

Figure 7. Frequency response of $s_1$, $s_2$, and $s_3$.

It can be seen that the restored signal showed its major frequency characteristics to be nearly identical to those of the original signal. The contribution of the restored signal to the output signal was evaluated using OCF (Figure 8).

Figure 8. Ordinary coherence based on ICA: (a) $\gamma_{1y}^2$, (b) $\gamma_{2y}^2$, (c) $\gamma_{3y}^2$.

The evaluation results were found to be consistent with those set in the simulation. $s_1$ and $s_2$ showed the contribution to the output at 9 Hz. Furthermore, the results showed that $s_1$ made a contribution to the output at 23 Hz, $s_2$ contributed at 11 Hz, and $s_3$ made a contribution at 14 and...
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![Ordinary coherence based on ICA](image)

Figure 8. Ordinary coherence based on ICA: (a) \( \gamma_{1y}^2 \), (b) \( \gamma_{2y}^2 \), (c) \( \gamma_{3y}^2 \).

The evaluation results were found to be consistent with those set in the simulation. \( s_1 \) and \( s_2 \) showed the contribution to the output at 9 Hz. Furthermore, the results showed that \( s_1 \) made a contribution to the output at 23 Hz, \( s_2 \) contributed at 11 Hz, and \( s_3 \) made a contribution at 14 and 17 Hz. The results showed the same results as the previous set in the simulation. Thus, it would be useful to utilize the contribution evaluation using ICA under the condition where signals that are physically overlapped or located nearby with overlapping frequency bands show a high correlation with each other.

3. Identification of Automotive Seat Rattle Noise

3.1. Experimental Setup and Data Acquisition

A microphone was installed at the height of the driver’s ear to measure the frequency characteristics of the rattle noise (Figure 9a). Because rattle noise exhibits the characteristics of a relatively low-frequency band, the measurement frequency band was selected as 0–1500 Hz. The methods of generating rattle noise can be largely classified into vehicle tests and exciter tests. Because various noises occur in the case of the vehicle test, including engine noise, road noise, and exhaust noise during driving, as well as foreign noises from the dashboard or doors, it is difficult to evaluate the
rattle noise from a seat. Furthermore, this test is disadvantageous in terms of low reproducibility due to the influence of the driver, the road surface, and the test environment. Thus, tests using an exciter are mainly performed. Vibrator tests can be divided into tests with an electronic exciter and tests with a hydraulic exciter. The electronic exciter has a low noise level output, while a hydraulic exciter has a high operating noise, but it has the advantage in that noises occur that are similar to the actual vehicle driving conditions due to the simultaneous excitation of several axes. Thus, in this study, excitation was performed using a hydraulic exciter with white noise.

Figure 9b shows the frequency characteristics of noise at the driver’s ear level when rattle noise occurs. The high-level frequency band of 10–800 Hz indicated that this band corresponded to a rattle noise band. The highest noise level was determined at 58 Hz. The peak of pure tone near 400 Hz was confirmed to be a component due to the hydraulic system of the exciter. Thus, the major frequency of rattle noise was determined at 58 Hz.

3.2. ICA-Based Coherence Analysis Technique

In order to apply the ICA to the contribution evaluation, the signal must first be restored using ICA. Typically, the following matters should be considered to restore the signal:
1. Restoration of signal amplitude;
2. Estimation of the number of signals;
3. Labeling of signals.

The rest of this paper describes techniques for evaluating the contribution under these three considerations.
3.2.1. Restoration of Signal Amplitude

When the signal is restored using ICA, the amplitude of the signal is not restored. Thus, in order to apply ICA to other signal-processing techniques, it is necessary to devise a method to restore the amplitude of the signal. However, general contribution evaluation is a method of calculating the correlation between two different signals in the frequency band. In other words, the coherence is obtained by calculating whether the frequencies of the two signals are correlated with each other rather than considering the amplitudes of the frequency response of two signals. The general contribution function and the partial coherence method are commonly used methods. The general contribution function is applied when the signals are not correlated with each other, while the partial coherence technique is used when the signals are correlated. Although both techniques are related to calculating the correlation of signals, the restoration of signal amplitude may not be considered in evaluating the contribution using ICA.

3.2.2. Estimation of Number of Signals

Typically, the relationship between the input (excitation) and output (response) in a linear system is expressed by the following equation:

\[ Y_i = \sum_j H_{ij} X_j \]  

(18)

where \( Y_i \) refers to the output spectrum at point \( i \), \( X_j \) refers to the input spectrum at point \( j \), and \( H_{ij} \) refers to the frequency response function between the degrees of freedom \( i \) and \( j \).

In order to estimate the input by measuring the output in such a linear system, an inverse transform technique is typically used. Inverse transform technology has been applied to many fields. An error occurs in a complex system when the inverse transform technique is applied, and such errors are called ill-condition or ill-posed problems.

The process of finding the solutions to these problems can be represented by the following linear equation and linear least squares equation:

\[ Ax = b \]  

(19)

\[ \min_x \|Ax - b\|_2 \]  

(20)

Here, if the ratio of the maximum and minimum values is large among the singular values of \( A \), the ill condition appears, which suggests that the solution responds highly sensitively to small changes. Thus, in this input restoration problem, unnecessary matrix terms may result in very large errors. In this respect, the estimation of the number of signals is crucial in restoring signals. In this paper, the number of signals is estimated using Bayesian information criterion, which is a method widely used to estimate the number of input signals. Minka proposed a method to estimate the dimensionality of data using Bayesian model selection, which is called Minka Bayesian Model Selection (MIBS), expressed in the following equation:

\[ \text{MIBS}(n) \approx p_n \left( \prod_{j=1}^{n} \lambda_j \right) \frac{N}{\sigma^2} N^{(m-n)} |A_n|^{-\frac{1}{2}} (2\pi)^{\frac{dn+n}{2}} N^{-n/2} \]  

(21)

Here,

\[ p_n = 2^n \prod_{i=1}^{n} \Gamma \left( \frac{m-i+1}{2} \right) \pi^{-(m-i+1)/2} \]  

(22)
were the major noise source, two microphones (B&K TYPE 4192) were installed at the joints of the headrest stays (Figure 10c). MIBS was applied to estimate how many noise sources were present. Figure 11 shows the application results of MIBS. The relative value was confirmed to converge when the estimated number of sources was two or more, which suggests that there were two major sources of rattle noise. Because the results of the noise source visualization confirmed that the headrest stays were the major noise source, two microphones (B&K TYPE 4192) were installed at the joints of the headrest stays (Figure 10c).

$$|A_n| = \prod_{i=1}^{n} \prod_{j=i+1}^{m} (\hat{\lambda}_i^{-1} - \hat{\lambda}_j^{-1})(\lambda_i - \lambda_j)N$$

(23)

$$\sigma_n^2 = \left( \sum_{j=m-n+1}^{m} \lambda_j \right) / m - n = 1$$

(24)

$$d_n = mn - n(n + 1)/2$$

(25)

where $\lambda$ is the inherent value of the covariance matrix of signal $x$, $m$ refers to the number of measured signals, $n$ refers to the number of original signals, and $N$ refers to the length of the data.

To estimate the number of rattle noises, a 48-channel microphone array (Array-Ring 48-75, Gfal, AcSoft, Bedford, UK) was used (Figure 10a). The results showed that noises occurred mainly from the headrest stays (Figure 10b). Figure 11 shows the application results of MIBS. The relative value was confirmed to converge when the estimated number of sources was two or more, which suggests that there were two major sources of rattle noise. Because the results of the noise source visualization confirmed that the headrest stays were the major noise source, two microphones (B&K TYPE 4192) were installed at the joints of the headrest stays (Figure 10c).

![Figure 10](image1.png)

**Figure 10.** Experimental setup for the localization of rattle noise: (a) experimental setup, (b) result of the rattle noise localization, (c) microphone installation for ICA.

![Figure 11](image2.png)

**Figure 11.** Estimation of the number of sources using MIBS.

3.2.3. Labeling of Sources

The labeling of sources, the next consideration, can be easily solved during the application of the contribution technique. Typically, the microphone is placed in close proximity to the noise source in evaluating the contribution. The characteristics of the noise source are most likely reflected in the nearest microphone. Thus, it is possible to label the restored signals by calculating the correlation between the restored signals and the microphones. Figure 12 shows an example of each of the three sources, the microphone positions, and the restored signals.
Because Mic 1 is close to Source 1, the signal measured at Mic 1 is dominant in the characteristics of Source 1. Similarly, because Mic 2 is close to Source 2, the signal measured at Mic 2 is dominant in the characteristics of Source 2. Thus, among the signals restored using the signals of Mics 1 and 2, a signal with a high correlation with Mic 1 can be estimated as Source 1, and a signal with a high correlation with Mic 2 can be estimated as Source 2.

3.3. Results

The OCF results indicated that the signals measured at Mics 1 and 2 (Figure 10c) were highly correlated with each other, and thus their contributions to the output were similar (Figure 13a,b). Thus, it was difficult to determine which signal contributed more to the output. For this reason, the partial contribution of the output for each signal was calculated using the partial coherence function, which can calculate the pure contribution of each input (Figure 14).

![Figure 12. Strategy of labeling the sources.](image1)

![Figure 13. Ordinary coherence: (a) \(\gamma_{1y}^2\), (b) \(\gamma_{2y}^2\).](image2)
The partial coherence results show that the contribution of $S_2$ to the output is low at 58 Hz (Figure 14b). This can be seen as an incorrect result, because the number of noise sources was estimated as two in the previous section. Therefore, when the noise sources physically overlap and the frequency bands are similar, it is necessary to restore the noise source using ICA and perform a contribution evaluation.

There is no need to apply the pure contribution evaluation method when evaluating the contribution using ICA. This is because, when the signals are restored using ICA, the signals are independent of each other. The existing pure contribution evaluation method is a method of calculating the pure contribution to the output of signals independent of each other by removing the correlation. Therefore, if the signal is restored using ICA, the contribution to the output can be calculated using OCF. This method has the advantage of not requiring the user to know the important order of sources in advance in the existing partial contribution evaluation method.

The results of restoring noise using ICA and performing OCF are shown in Figure 15. It can be seen that the top and bottom of the headrest show the highest contribution at 58 Hz. In particular, it can be seen that the lower part of the headrest ($S_1$) is the most important source of the seat rattle noise.

Furthermore, the MCF was calculated to confirm whether the input source was properly selected for the output (Figure 16). If the multiple contribution is 0.5 or less at the frequency of interest, this suggests that the correlation between the inputs has a significant influence on the output. Thus, it is necessary to devise measures for the transfer function between the corresponding input terminals. However, because a contribution of more than 0.7 was observed, it confirmed that an appropriate input source was selected and that the major input source was found to be the upper/lower part ($S_1$ and $S_2$) of the headrest.

To confirm this, an accelerometer was attached to the headrest, and modal testing of the single module of the seat was performed (Figure 17). The transfer function for each of the three axes is shown in Figure 17.
Figure 14. Partial coherence: (a) $\gamma_{\alpha\beta}^{\omega\lambda}$, (b) $\gamma_{\lambda\omega}^{\beta\alpha}$.

The partial coherence results show that the contribution of S2 to the output is low at 58 Hz (Figure 14b). This can be seen as an incorrect result, because the number of noise sources was estimated as two in the previous section. Therefore, when the noise sources physically overlap and the frequency bands are similar, it is necessary to restore the noise source using ICA and perform a contribution evaluation. There is no need to apply the pure contribution evaluation method when evaluating the contribution using ICA. This is because, when the signals are restored using ICA, the signals are independent of each other. The existing pure contribution evaluation method is a method of calculating the pure contribution to the output of signals independent of each other by removing the correlation. Therefore, if the signal is restored using ICA, the contribution to the output can be calculated using OCF. This method has the advantage of not requiring the user to know the important order of sources in advance in the existing partial contribution evaluation method.

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Figure 15. ICA-based ordinary coherence: (a) $\gamma_{1y}^2$, (b) $\gamma_{2y}^2$.

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Figure 16. Multiple coherence function.

Figure 17. Frequency response at the head-rest stay (a) Experimental setup for modal testing: (b) x-direction, (c) y-direction, (d) z-direction.
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4. Conclusions

In this study, a contribution evaluation was performed on vehicle seat rattle noise using an ICA-based coherence technique. To perform this analysis, a hydraulic exciter was used to generate seat rattle noise. The number of noise sources at the main frequency of the seat rattle was estimated using MIBS. The contribution evaluation of the vehicle seat rattle noise was performed using the established ICA-based coherence evaluation method.

The existing contribution evaluation method was used to calculate the coherence of the input with respect to the output by removing the correlation between the input sources. However, this method is difficult to apply when the input sources are not independent from each other or when the order of the input sources is not known in advance due to a high correlation between the input sources. In particular, when the frequencies of different input sources are similar and their locations are proximal, it is challenging to determine the designated noise sources and contributions. However, the contribution evaluation method can be utilized as a method of restoring an independent noise source using the ICA method in this case. The conclusions of this study are as follows:

1. There were two major sources of seat rattle noise—the upper and lower part of the headrest stay were the main noise sources
2. Seat rattle noise was generated due to the resonant frequency in the x-direction of the seat.
3. The contribution of the two noise sources generated in close proximity to each other could be evaluated through the contribution evaluation method and the ICA method.

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