Testing Lorentz violation with binary pulsars: constraints on standard model extension *

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Abstract Under the standard model extension (SME) framework, Lorentz invariance is tested in five binary pulsars: PSR J0737–3039, PSR B1534+12, PSR J1756–2251, PSR B1913+16 and PSR B2127+11C. By analyzing the advance of periastron, we obtain the constraints on a dimensionless combination of SME parameters that is sensitive to timing observations. The results imply no evidence for the break of Lorentz invariance at the $10^{-10}$ level, one order of magnitude larger than the previous estimation.

Key words: gravitation — relativity — pulsars: individual (PSR 0737–3039, PSR B1534+12, PSR J1756–2251, PSR B1913+16, PSR B2127+11C)

1 INTRODUCTION

Unification of general relativity (GR) and quantum mechanics is a great challenge in the realm of fundamental physics. Some candidates of a self-consistent quantum theory of gravity emerge from tiny violations of Lorentz symmetry (Kostelecký 2005; Mattingly 2005). To describe observable effects of these violations, effective field theories could be a theoretical framework for tests.

The standard model extension (SME) is one of those effective theories. It includes the Lagrange densities for GR and the standard model for particle physics and allows possible breaking of Lorentz symmetry (Bailey & Kostelecký 2006). The SME parameters $\bar{s}_{\mu\nu}$ control the leading indicators of Lorentz violation for gravitational experiments in the case of the pure-gravity region of the minimal SME. By analyzing archival lunar laser ranging data, Battat et al. (2007) constrained these dimensionless parameters in the range from $10^{-11}$ to $10^{-6}$, which means there is no evidence for Lorentz violation at this level.

However, tighter constraints on $\bar{s}_{\mu\nu}$ would be hard to obtain in the solar system because the gravitational field is weak. Thus, for this purpose, binary pulsars provide a good opportunity. Because of their stronger gravitational fields, for example the relativistic periastron advance in binary pulsars could exceed the corresponding value for Mercury by a factor of $\sim 10^5$, these systems are taken as an ideal and clean test-bed for testing GR, alternative relativistic theories of gravity and modified gravity, such as the works by Bell et al. (1996), Damour & Esposito-Farèse (1996), Kramer et al. (2006), Deng et al. (2009) and Deng (2011).

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Motivated by this advantage of binary pulsars, we will try to test Lorentz invariance under the SME framework with five binary pulsars: PSR J0737–3039, PSR B1534+12, PSR J1756–2251, PSR B1913+16 and PSR B2127+11C. In Section 2, the orbital dynamics of binary pulsars in the SME will be briefed. Observational data will be used to constrain the SME parameters in Section 3. The conclusions and discussions will be presented in Section 4.

2 ORBITAL DYNAMICS OF BINARY PULSARS IN SME

When the pure-gravity region of the minimal SME is considered, it will cause secular evolutions of the orbits of binary pulsars. Since timing observations of binary pulsars can obtain its value very precisely, the periastron advance plays a much more important role in constraining $\bar{s}^{\mu\nu}$ and, with widely used notations in celestial mechanics, it reads as (Bailey & Kostelecký 2006)

$$
\left\langle \frac{d\omega}{dt}\right\rangle_{\text{SME}} = -\frac{n}{\tan i(1-e^2)^{1/2}}\left[\frac{e^2}{e}\bar{s}_{kp}\sin \omega + \frac{(e^2 - e)}{e^2}\bar{s}_{kQ}\cos \omega - \frac{\delta m}{M}\frac{2n\varepsilon}{e}\bar{s}_k\cos \omega \right] - n\frac{(e^2 - 2e)}{2e^4}(\bar{s}_{PP} - \bar{s}_{QQ}) + \frac{\delta m}{M}\frac{2n\varepsilon}{e^3(1-e^2)^{1/2}}\bar{s}_Q,
$$

(1)

where $M = m_1 + m_2$, $\delta m = m_2 - m_1$ ($m_2 > m_1$) and $e = 1 - (1-e^2)^{1/2}$. In this expression, the coefficients $\bar{s}_k$ and $\bar{s}_Q$ for Lorentz violation with subscripts $P$, $Q$ and $k$ are projections of $\bar{s}^{\mu\nu}$ along the unit vectors $P$, $Q$ and $k$ respectively. The unit vector $k$ is perpendicular to the orbital plane of the binary pulsar, $P$ points from the focus to the periastron, and $Q = k \times P$. By definitions given by (Bailey & Kostelecký 2006),

$$
\bar{s}_k \equiv \bar{s}^{0j}k^j, \quad \bar{s}_Q \equiv \bar{s}^{0j}Q^j, \quad \bar{s}_{kp} \equiv \bar{s}^{ij}k^i P^j, \quad \bar{s}_{kQ} \equiv \bar{s}^{ij}k^i Q^j, \quad \bar{s}_{QP} \equiv \bar{s}^{ij}P^i Q^j, \quad \bar{s}_{QQ} \equiv \bar{s}^{ij}Q^i Q^j.
$$

However, according to Equation (1), it is easy to see that the measurement of $\bar{s}_\omega$ is sensitive to a combination of $\bar{s}^{\mu\nu}$ instead of its individual components. Bailey & Kostelecký (2006) defined the combination as

$$
\bar{s}_\omega \equiv \bar{s}_{kp}\sin \omega + (1-e^2)^{1/2}\bar{s}_{kQ}\cos \omega - \frac{\delta m}{M}\frac{2n\varepsilon}{e}\bar{s}_k\cos \omega
$$

$$
+ \tan i\frac{(1-e^2)^{1/2}(e^2-2e)}{2e^2\varepsilon}(\bar{s}_{PP} - \bar{s}_{QQ}) + \frac{m}{M}\frac{2n\varepsilon}{e}\tan i\frac{(e^2 - e)}{e}\bar{s}_Q,
$$

(2)

and crudely estimated its value at the level of $10^{-11}$.

Together with the contribution from GR, the total secular periastron advance of a binary pulsar system is

$$
\dot{s} = 3\frac{P_b}{2\pi}(-5/3)^{-\frac{5}{3}}\left(\frac{GM}{c^3}\right)^{2/3}(1-e^2)^{-1} - \frac{n\varepsilon}{\tan i(1-e^2)^{1/2}}\bar{s}_\omega
$$

$$
= 3\frac{P_b}{2\pi}(-5/3)^{-\frac{5}{3}}T_{\odot}^{-2/3}\left(\frac{M}{M_{\odot}}\right)^{2/3}(1-e^2)^{-1} - \frac{2\pi\varepsilon s}{P_b(1-e^2)^{1/2}c^2(1-s^2)^{1/2}} \bar{s}_\omega,
$$

(3)

where $T_{\odot} \equiv GM_{\odot}/c^3 = 4.925490947 \mu s$ and

$$
s = x\left(\frac{P_b}{2\pi}\right)^{-2/3}T_{\odot}^{-1/3}M^{2/3}m_2^{-1}.
$$

(4)

The quantity $x$ in Equation (4) is the projected semi-major axis, which is usually given by the timing observations, but, in some cases, $x$ can be measured directly so that there is no need to evaluate it from this equation. In this work, Equation (3) will be taken to find the constraints on $\bar{s}_\omega$ with timing measurements of binary pulsars.
3 OBSERVATIONAL CONSTRAINTS

Long-term timing observations can determine the geometrical and physical parameters of binary pulsars very well. Among them, PSR J0737–3039 (Kramer et al. 2006), PSR B1534+12 (Stairs et al. 2002), PSR J1756–2251 (Faulkner et al. 2005), PSR B1913+16 (Weisberg et al. 2010) and PSR B2127+11C (Jacoby et al. 2006) are good samples for gravitational tests. Some of their timing parameters are listed in Table 1. In terms of the estimated uncertainties given in parentheses after $\dot{\omega}$, the pool of data is divided into two groups: Group I, where all the binary pulsars are taken; and Group II, including PSR B1913+16, PSR B1534+12 and PSR B2127+11C, which have the smallest uncertainties.

By using the method of weighted least squares, the parameter $\bar{s}_\omega$ is estimated (see Table 2). The estimation made by Group I is $\bar{s}_\omega = (-1.24 \pm 0.54) \times 10^{-10}$ and Group II gives $\bar{s}_\omega = (-1.42 \pm 0.75) \times 10^{-10}$. For comparison, Bailey & Kostelecký (2006) proposed that the attainable experimental sensitivity of $\bar{s}_\omega$ is $10^{-11}$, which is 10 times less than the results we obtain.

4 CONCLUSIONS AND DISCUSSION

In this work, we test Lorentz violation with five binary pulsars under the framework of SME. It is found that $\bar{s}_\omega$, which is a dimensionless combination of SME parameters, is on the order of $10^{-10}$, whether all five systems are taken or only the top three systems with the smallest estimated uncertainties of periastron advances are used. This value, one order of magnitude greater than the estimation by Bailey & Kostelecký (2006), implies there is no evidence for the break of Lorentz invariance at the $10^{-10}$ level.

Nevertheless, as mentioned by Bailey & Kostelecký (2006), the secular evolution of the eccentricity of binary pulsars should be included in the analysis. Its contribution is (Bailey & Kostelecký 2006)

$$\left\langle \frac{de}{dt} \right\rangle = \frac{1}{c^3} n (1 - e^2)^{1/2} (c^2 - 2\varepsilon) \bar{s}_e,$$

where

$$\bar{s}_e = \bar{s}_{PQ} - \frac{\delta m}{M} \frac{2nae\varepsilon}{c^2 - 2\varepsilon} \bar{s}_P. \quad (6)$$

Here $\bar{s}_e$ is a combination of coefficients in $\bar{s}^{\mu\nu}$ and is sensitive to observations. However, there is a lack of timing observations on binary pulsars covering a long enough time so that rare observations could show the secular change of $e$. Even though a few numbers could be derived from these
data, their uncertainties are rather larger than those of periastron advances. Timing observations can usually only set the upper bounds, such as \(|\dot{e}| < 1.9 \times 10^{-14} \text{ s}^{-1}\) for PSR B1913+16 (Taylor & Weisberg 1989) and \(|\dot{e}| < 3 \times 10^{-15} \text{ s}^{-1}\) for PSR B1534+12 (Stairs et al. 2002). Hence, we assume that, at least in the current stage, the constraints made by \(\dot{e}\) might be looser and the resulting upper bound is \(|\bar{s}_e| < 3 \times 10^{-10}\). Although it is consistent with the values of \(\bar{s}_\omega\) we obtain, the exact value of \(\bar{s}_e\) remains unknown. Therefore, unless timing observations could provide much more definitive results about \(\dot{e}\), the secular changes of eccentricity would not impose a tight constraint on \(\bar{s}^{\mu\nu}\) or combinations of \(\bar{s}^{\mu\nu}\).

Another issue for future work is to constrain the components of \(\bar{s}^{\mu\nu}\) directly with binary pulsars. However, the choice of a reference frame affects the values of these components so that a certain reference frame must be specified first and the projections of \(\bar{s}^{\mu\nu}\) will be along its standard unit basis vectors. For example, for comparing the constraints due to binary pulsars and lunar laser ranging, \(\bar{s}^{\mu\nu}\) has to be projected along the same triad of vectors. It means that the unit vectors \(P, Q\) and \(k\) (see Sect. 2) have to be decomposed in terms of these vectors, which requires information about the geometry of the orbit of the binary pulsars, such as the orbital elements \(\Omega\) and \(\omega\). Unfortunately, timing observations are not sensitive to those two elements. This makes the components of \(\bar{s}^{\mu\nu}\) hard to directly access for now and demonstrates the advantages and availability of \(\bar{s}_\omega\).

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