COMPARISON THE PERFORMANCE OF ORDINARY KRIGING AND INVERSE DISTANCE WEIGHTING METHODS FOR MAPPING NICKEL LATERITE PROPERTIES

Hendro Purnomo
Mining Engineering, Institut Teknologi Nasional Yogyakarta
Jl Babarsari, Catuutunggal, Depok, Sleman Yogyakarta, Indonesia.
hendro.purnomo.hpu@gmail.com

Abstract

Selection of an optimal interpolation method for predicting ore grade at un-sampled location is an important issue to map the anomaly distribution and resources estimation. Objective of this research were to evaluate the performance of ordinary kriging (OK) and inverse distance weighting (IDW) methods for predicting distribution and potential resources of nickel (Ni) and cobalt (Co) in the limonite and saprolite zone. In this study the ArcGIS 10.2 with Geostatistical Analyst Extensions was used in exploratory data analysis. To choose the variogram model and optimal interpolation method were used root mean square error (RMSE) value obtained from a cross validation procedure. Experimental variograms were fitted with the spherical, exponential and gaussian variogram models. The model with the smallest RMSE value was chosen as the best. In the limonite zone, IDW power of 2 performed best for Ni while OK procedure gave the best results when applied to thickness variable. In the saprolite zone, IDW power of 5 performed best for Ni whereas IDW power of 1 indicated the best result when applied to both Co and thickness. Result of the interpolation indicated that the nickel distribution and additional potential resources in the limonite and saprolite zone still open to the northeast and southwest of the research area.

Keyword: ArcGIS, cross validation, IDW, OK, RMSE
Kriging that requires preliminary modeling step of the variogram before kriging itself is conducted. In this research, as a comparison the IDW and OK procedure were applied to map the distribution of the nickel (Ni) and Cobalt (Co) anomalies in the nickel laterite deposit type.

Objectives of this study were to evaluate the relative performance of the IDW and OK methods in predicting ore distribution which contain Ni and Co higher than cut of grade in the limonite and saprolite zone, and to estimate the potential amount of the Ni and Co resources, based on the root mean square error (RMSE) value.

Figure 1: Location of the research area and map of the distribution of drill holes.

2. Method
2.1 Study area and sampling design

The research area is about 9 km² located in the Konawe regency, South East Sulawesi Province of Indonesia. A total of 28 drill holes with various depths from 7.5 to 64.2m with spacing between each drill hole were 500m. Samples were collected at 1m interval for every hole and those were assayed for Ni and Co. The distribution of drill hole and location of the research area is show in FIGURE 1.

Geologically the area is located in the south east arm of Sulawesi that is widely occupied by ophiolite rocks complex consist of basaltic and ultramafic rocks, with the primary north west – south east trending structure [2] see FIGURE 2. The ultramafic rocks type is known as a potential source of the nickel laterite in this area.

2.2 Descriptive of estimation methods

Statistical analysis was held in two stages. First, the distribution of data was summarized by using basic statistics such as maximum, minimum, mean, median, standard deviation, skewness, kurtosis and coefficient of variation. Second, geostatistical analysis both OK and IDW estimations were processed by using the geostatistical software package ArcGIS 10.2 with Geostatistical Analyst Extensions.
Kriging is a technique of spatial prediction for linear optimum unbiased interpolation with a minimum mean interpolation error [3]. Primary tool in kriging estimation is variogram, which reflects the special relationship between neighboring data observations. The variogram is obtained from the result of fitting between experimental variogram and theoretical model. The most widely used models in mines are spherical, exponential and gaussian [4]. In this study to select a variogram theoretical model is based on the root mean square error (RMSE) value whereas the smallest value was chosen as the best model [5]. The value of experimental variogram can be calculated with the following equation [6]:

\[
y(h) = \frac{1}{2n(h)} \sum_{i=1}^{n(h)} [Z(x_i) - Z(x_i + h)]^2
\]

where:
- \(Z(x_i)\): Sample value at point \(x_i\)
- \(Z(x_i + h)\): Sample value at a point distance \(h\) from point \(x_i\)
- \(y(h)\): The experimental semivariogram value at the distance interval \(h\)
- \(n(h)\): Number of sample pairs separated by a distance \(h\).

2.2.1 Ordinary Kriging (OK)

Ordinary kriging is one of the basic on kriging methods that provides an estimate at unobserved location, based on weighted average of around observed sites within an area [7].

OK prediction at an unsampled location \(\hat{Z}\) is defined by an equation:

\[
\hat{Z} = \sum_{t=1}^{n} \lambda_t Z_t
\]

with the weight \(\lambda_t\) calculated by an equation:

\[
\sum_{t=1}^{n} \lambda_t c(t,l) + \mu = c(t,0), \text{ with } \sum_{t=1}^{n} \lambda_t = 1
\]

where:

Comparison the Performance of Ordinary Krigging and Inverse Distance Weighting Methods (Hendra Purnomo)
2.2.2 Inverse Distance Weighting (IDW)

To calculate a sample weight, IDW assumed that degree of correlations and similarities between neighbors is proportional to the distance between them. The IDW equation that is used in weighting is written below [4]:

$$w_i = \frac{1}{d_i^k} \sum_{i=1}^{n} \frac{1}{d_i^k}$$  \hspace{1cm} (4)

To estimate a predicted point is used equation below:

$$\hat{Z}_0 = \sum_{i=1}^{n} w_i \cdot Z_i$$  \hspace{1cm} (5)

where:
- $Z_0$ : A point where the value should be estimated.
- $w_i$ : A sample weight in point $i$.
- $d_i$ : A distance between point $i$ and a prediction point.
- $k$ : A power parameter.
- $Z_i$ : A sample value in point $i$.

2.2.3 Data transformation and interpolation

The best performance of kriging analysis is on normal distribution data. If the data distribution is not normal then transformations of the data can help to make it appropriately normal [8]. A logarithmic transformation can be considered where the coefficient of skewness is greater than 1 and square root transformation is between 0.5 and 1 [9]. To transform back through exponentiation can be used the equation bellow [10]:

$$\hat{Z}(x_i) = \exp \left[ \hat{\rho}(x_i) + \frac{\hat{\sigma}^2(x_i)}{2} \right]$$  \hspace{1cm} (6)

Where:
- $\hat{\sigma}^2(x_i)$ : The corresponding lognormal kriging variance.
- $\hat{\rho}(x_i)$ : The lognormal kriging estimate.
- $\hat{Z}(x_i)$ : The corresponding back transformed result in the original data domain.

2.2.4 Criteria for comparison

To select the best variogram model between potential models and to compare the accuracy of interpolation method were used parameter of root mean square error (RMSE). The RMSE can be obtained from cross validation technique, and it was calculated with the equation below [11]:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\hat{Z}(x_i) - Z(x_i))^2}$$  \hspace{1cm} (7)

where:
- $\hat{Z}(x_i)$ : The estimation value.
- $Z(x_i)$ : A mean value.
- $n$ : Total number of the estimation.

The prediction is slightly deviate if a root mean square error value is low.
3. Result and Discussion

In this research, thickness of the mineralization layer and geochemical assay data consisted of Ni, Co and MgO were obtained from 28 drill holes. The assay data was then discriminated and composited into two zone namely: limonite and saprolite base on the MgO content, where MgO ≤ 5% implied into limonite zone and MgO > 5% included into saprolite zone. The discrimination result indicated that there were 28 holes contain limonite type and 22 holes contain saprolite type.

Figure 3: Samples classification for (a) limonite Ni, (b) limonite Co, (c) saprolite Ni and (d) saprolite Co.
FIGURE 3 shows the spatial distribution of samples in the limonite and saprolite zones which were classified based on the cutoff grade value. Summary statistic for Ni, Co and thickness obtained from 28 composite data in the limonite zone and 22 data in the saprolite zone are given in TABLE 1. There were two variables, Co in the limonite zone and thickness in the saprolite zone, have coefficient skewness value greater than 1 (1.75 for Co and 1.54 for thickness), therefore the natural logarithm is applied for a kriging analysis. Those were later back-transformed with using equation (6).

To identify the possible spatial structure of different variables, variogram experimental were calculated according to isotropy model with using equation (1). The variogram theoretical models consisted of exponential, spherical and gaussian were fitted to the experimental variogram and the model with the lowest RMSE value was chosen as the best model. The RMSE value was obtained from the cross validation results and it was calculated by using equation (7). TABLE 2 provides a spatial ratio value, RMSE value and different theoretical variogram models as a result of matching with experimental variogram for each variable. In the limonite zone, the exponential model was found as the best variogram model for Ni while the spherical model as the best variogram for both Co and thickness variable. In the saprolite zone, the exponential model was identified as the best variogram for Ni whereas the gaussian model as the best variogram for both Co and thickness variables. The nugget to sill ratios for the best variogram models in the limonite and saprolite zone indicated a ratio value from 35% to 83%.

| Zone   | Variable | CV  | Mean | Min  | Max  | Standard deviation | Skewness | Kurtosis | No of observation |
|--------|----------|-----|------|------|------|-------------------|----------|----------|-----------------|
| Limonite | Ni       | 0.29| 0.97 | 0.27 | 1.35 | 0.29              | -0.74    | 2.61      | 28              |
|        | Co       | 0.49| 0.11 | 0.03 | 0.32 | 0.06              | 1.75     | 7.16      | 28              |
|        | Thickness| 0.57| 11.68| 0.50 | 30.40| 6.65              | 0.67     | 3.39      | 28              |
| Saprolite | Ni      | 0.34| 1.28 | 0.54 | 2.10 | 0.44              | 0.14     | 1.79      | 22              |
|        | Co       | 0.43| 0.05 | 0.02 | 0.10 | 0.02              | 0.76     | 2.92      | 22              |
|        | Thickness| 1.26| 15.85| 0.6  | 64.2 | 19.90             | 1.54     | 3.88      | 22              |

FIGURE 4 shows the best fitted variogram models in the limonite and saprolite zone were chosen. Parameters from the best variogram models were then used in the estimation procedure by method of OK. In this study, estimation procedure by IDW also utilizes variogram and isotropic parameter as well. The IDW predictions were exercised by varying number of power, from 1 to 5, and used a number of the
closest neighboring points with ranging from 2 to 10 that was equal with the parameter was used in the OK estimation procedure.

Result of the RMSE value according to OK and IDW power of 1, 2, 3, 4 and 5 were given in TABLE 3 for the limonite zone and TABLE 4 for the saprolite zone. The result indicated that the interpolation methods with the lowest RMSE values were IDW power of 2 for both Ni and Co, and OK for thickness in the limonite zone, while in the saprolite zone the lowest RMSE values were IDW power of 5 for Ni and IDW power of 1 for both Co and thickness.

Figure 4: Best fitted variograms for laterite properties (a) Ni in the limonite zone, (b) Co in the limonite zone, (c) Ni in the saprolite zone, (d) Co in the saprolite zone, (e) Thickness in the limonite zone and (d) Thickness in the saprolite zone
Table 3: Result of the RMSE value according to OK and IDW powers of 1-5 for limonite zone

| Zone | Variable | Spatial ratio (Nugget/ sill) (%) | Interpolation Method   | RMSE     |
|------|----------|---------------------------------|------------------------|----------|
| Ni   |          | 35                              | OK-Exponential         | 0.307633 |
|      |          |                                 | IDW 1                  | 0.289358 |
|      |          |                                 | IDW 2                  | 0.289314 |
|      |          |                                 | IDW 3                  | 0.289987 |
|      |          |                                 | IDW 4                  | 0.291986 |
|      |          |                                 | IDW 5                  | 0.294786 |
| Co   |          | 94                              | OK-Spherical           | 0.056599 |
|      |          |                                 | IDW 1                  | 0.056512 |
|      |          |                                 | IDW 2                  | 0.056213 |
|      |          |                                 | IDW 3                  | 0.056315 |
|      |          |                                 | IDW 4                  | 0.056630 |
|      |          |                                 | IDW 5                  | 0.057034 |
| Thickness | | 38 | OK-Spherical | 6.527668 |
|      |          |                                 | IDW 1                  | 6.852846 |
|      |          |                                 | IDW 2                  | 6.785693 |
|      |          |                                 | IDW 3                  | 6.740409 |
|      |          |                                 | IDW 4                  | 6.710901 |
|      |          |                                 | IDW 5                  | 6.691917 |

Classification of spatial dependence for Ni, Co and thickness of both saprolite and limonite zone were evaluated by the ratio between nugget and sill value. For a ratio >75% indicated weak spatial dependence, a ratio of 25-75% indicated moderate spatial dependence and a ratio of <25% indicated strong spatial dependence [12]. In this study, as shows in TABLE 2, the best fitted semivariogram analysis for all variables indicated a ratio of nugget to sill equal to 35-82% which was classified as medium to weak spatial dependence. This result of the classification was probably because of inappropriate data to form an ideal variogram in the study area, as a literature suggests the minimum required some 100-150 data to achieve a stable variogram [8].
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The interpolated maps of Ni and Co in the limonite and saprolite zone by using the methods with the lowest RMSE value are presented in FIGURE 5. In FIGURE 5a present an interpolation of nickel in the limonite zone with using IDW power of 2. The map shows that the areas with red color indicate the distribution of limonite zone with Ni content > 1%, otherwise area with blue color exhibits limonite zone with Ni grade ≤ 1%. If it is assumed the cutoff grade value is 1% Ni, then the red color areas will be as the Ni anomaly areas. As shows in the map, the most Ni anomaly distribute in the middle and south part of the area with North east – South west trending. Base on the nickel distribution in this area that the next stage of exploration with the limonite ore target can be undertaken within the anomaly (red color) area and to get additional resources can be extent to the north east and south west of the research area.

FIGURE 5b shows the interpolation of cobalt with using IDW power of 2 procedures, it is reveals that the areas with the brown color represent the distribution of limonite zone with Co grade > 0.1% as an
anomaly area and the grey color indicate limonite zone with Co grade ≤ 0.1%. The map shows that there are two blocks of anomaly areas, in the east and west part of the research area with the trend direction to the Northwest – Southeast.

FIGURE 5c indicates the interpolation of nickel in the saprolite zone by using IDW power of 5 method. The map shows that the areas with red color indicate the saprolite zone with Ni content > 1.2% as an anomaly area, while the blue color area represent the saprolite zone with Ni grade ≤ 1.2%. The Ni anomaly distributed in the south and middle of the study area with trend direction to the north east – south west, therefore the next advance exploration stage with the target of saprolite ore may be done within the anomaly (red color) area. FIGURE 5d shows the interpolation of cobalt with using IDW power of 1 procedure, the area with brown color indicate the distribution of saprolite zone with Co grade > 0.05% and the grey color represent saprolite zone with Co content ≤ 0.1%.

Estimation of nickel and cobalt resources were calculated based on the two dimensional model. The tonnage of the resources was obtained from the result of the multiplication of the volume and ore density of each zone, while the volume was obtained from the result of between thickness of each zone by square of the drill hole grid (500m x 500 m). In this research was assumed the density of limonite ore was 1.6 ton/m³ and saprolite ore was 1.5 ton/m³ with cutoff grade was 1% Ni for the limonite ore and 1.2% for the saprolite ore, while cutoff grade of Co was 0.1% for limonite ore and 0.05% for saprolite ore. Base on the best performance interpolation methods, the amount of resources in the limonite zone was calculated with using IDW power of 2 procedure for both Ni and Co, and OK technique for thickness, while resources in the saprolite zone was calculated according to IDW power of 5 for Ni and IDW power of 1 for both Co and thickness. Resource estimation in the limonite zone indicated 868623.53 ton of nickel and 94534.93 ton of cobalt, while in the saprolite zone was 688972.14 ton of nickel and 44843.29 ton of cobalt. Additional potential nickel resources in the limonite and saprolite zone can be extended to the Northeast and Southwest of the research area (see FIGURE 5a and 5c). Result of the Ni and Co resources estimation was presented in TABLE 5.

| Zone   | Ore tonnage | Average grade | Metal tonnage |
|--------|-------------|---------------|---------------|
| Limonite | Ni | 83808000 | 1.04 % Ni | 868623.53 |
|        | Co | 66574806 | 0.14 % Co | 94534.93 |
| Saprolite | Ni | 40638285 | 1.42 % Ni | 688972.14 |
|        | Co | 74800785 | 0.06 % Co | 44843.29 |

4. Conclusion

Base on the RMSE value the comparison result of the two applied interpolation methods indicated that IDW procedure was the most suitable methods for estimation and mapping spatial distribution of Ni and Co in this study area. It is probably because the numbers of our dataset are inappropriate to form a stable variogram. The results showed, in the limonite zone, IDW power of 2 performed best for Ni and Co while OK procedure gave the best results when applied to thickness variable. In the saprolite zone, IDW power of 5 performed best for Ni whereas IDW power of 1 indicated the best result when applied to both Co and thickness.

Result of the interpolation revealed the nickel distributions in the limonite and saprolite zones are still open to the northeast and to the southwest. Resource estimation in the limonite zone indicated 868623.53 ton of nickel and 94534.93 ton of cobalt, while in the saprolite zone was 688972.14 ton of nickel and 44843.29 ton of cobalt. Additional potential nickel resources in the limonite and saprolite zone can be extended to the Northeast and Southwest of the research area.

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