Principles of equilibrium statistical mechanics revisited: The idea of vortex energy

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Introduction

The physics of phenomena is chiefly perceived through the interactions given by the energy function in line with the principles of holonomic mechanics and equilibrium thermodynamics. This brilliantly unifying guideline created by Euler and Lagrange has found ways into all pores of physics, and interpretations have spread out as if the guideline is a genuine universal law of nature. But with no outlined borders of validity, the law is a default belief, a source of circular theories and fallacies.

In this regard, it is worth recalling the forces called circulatory or vortex with all their cumulative impact beyond the energy function concept that can be huge, as known since the 19th century, and the term “dry water” coined by John von Neumann stuck to viscosity-neglect hydrodynamic studies as inadequate, see [1,2,3]. Also since the 19th century, e.g. [4,5], it was exposed in mechanics and other fields the invalidity of the concept due to the reaction forces of ideal non-holonomy, performing no work on the system, as is the case of rigid bodies rolling without slipping on a surface. Recall also a general symmetry argument provoked by the $H$-theorem of Boltzmann and showing the Loschmidt’s fundamental paradox [6] of reversibility on the way to conform the real world with the energy function concept.

Physics nowadays in line with quantum mechanics claimed as more valid than classical mechanics, further spreads the conviction in the genuine energy law with no limits of validity, and quantum theory has a commanding influence on both fundamental and applied research. This common trend has various sophisticated possibilities to fall into the trap of circular theories, which in my judgment, occurs mainly due to playing with concepts of entropy and energy.

The gist of it is both energy and entropy are then conceived as conformed to equilibrium statistical mechanics and thermodynamics, which binds them as duals of Legendre transform. This work aims to break the cycle and show the existence of non-conventional energy stored in systems in equilibrium. Such energy is not a function of system states. It is related to non-Hamiltonian dynamics but at variance with entropy trends and its law of conservation. It leads to rethinking many standard views. The work clarifies my earlier results [7,8] on the idea of energy suggested first in connection with the strong vortex effect of high frequency fields.

The dilemma of energy function concept and the issue of equal footing

The established consistent pattern of the world around us is basically relaxation to recurrent trends of motion. Its stability implies the ubiquity of irreversible forces as the generalized forces whose infinitesimal work depends on the path of system motion rather than just its instant state.

It might seem correct to refer the irreversible forcing to the averaging of irrelevant variables of a conservative many-body system given by a microscopic Hamiltonian and random initial conditions. This cue, however, misleads in the question of both statistical and dynamical (over fast motion) averaging in that there is no way to come to the irreversible behaviors from the formalism of energy functions unless resorting to the inexact reasoning residing in the averaging methods and truncations irreducible to the separation by canonical transformations [7,9].

At the same time, the perception of myriad outer influences, even treated as time-varying Hamiltonian interactions, is inevitably via smoothing which barely complies with the exact separation given by canonical transformations, hence, contributes to the irreversible forcing that can greatly accumulate for long times along with arbitrariness in modeling the trends. This is like having your cake and eating it too. On the one hand, the irreversible forces, unlike reversible, cannot be derived from a Hamiltonian or effective potential. On the other, insofar as the true physics of phenomena is perceived through the interactions given by energy functions, so should be the physics of irreversible phenomena.

This dilemma is inherent to the perception in terms of energy function and brings in fundamental inexactness.
There is no other way to account for the inexactness but to integrate reasoning in such terms with a tentative (statistical) measure of energy blur/relaxation rates. This element pertains to both classical and quantum mechanical descriptions. The uncertainty principle of the latter as related to the postulated discreteness of energy transfer has nothing to do with the dilemma, and the integration in point puts both descriptions on an equal footing.

Many phenomena in radiation, superconductivity and other fields are commonly referred to as indescribable classically, which might be proper within some specific context. As for unconstrained assertions, it contradicts the above argument of equal footing. The same should concern the ideas of quantum computing claimed to be beyond classical physics. Even for such phenomena as extremely deep cooling of matter by high frequency resonance fields, both ways have led to its independent prediction and showed the classical way direct, free of linkage to the uncertainty principle defined by Planck’s constant as for the cooling mechanisms as its limitations, see [10] and our earlier work cited there.

Our issue here is not only that the physics of phenomena can be perceived through any self-sufficient construction and that one can’t see through its wall unless allowing for a dual of the formalism with respect to wider frameworks. It is also that the idea of energy in physics incorporates the energy function concept as its part and that it exceeds the part enormously, as shown below. Being vast it floats over the issues of equal footing and perception dilemma. Let us first outline the existence domain of energy function concept.

The entrainment theorem

Let us think of the energy function concept in terms of generalized thermodynamic potential commonly accepted in the study of phase transitions, transport through barriers and many other things. The generalized potential of a system relaxing in steady conditions to a density distribution \( \rho_{st} \) connects to it by

\[
\rho_{st}(z) = Ne^{-\Phi(z)}, \quad N^{-1} = \int e^{-\Phi} d\Gamma
\]  

(1)

where the integral is over the volume \( \Gamma \) of system phase space variables \( z \) and the reversible motion is on surfaces

\[
\Phi(z) = \text{const.}
\]

(2)

The properties of the system mainly depend then on the local properties of the minima of \( \Phi \). Also, it gives insight from the observed symmetries of a physical system. An analogous approach to systems under high frequency fields is in terms of the picture where the hf field looks fixed or its effect is time-averaged. In all this, Eq. (1) can be viewed as merely redefining the distribution \( \rho_{st} \) in terms of function \( \Phi \), whereas, taking this function as the energy integral of reversible motion provides the physical basis of the theory, but implies rigid constraints.

Commonly, going back to Boltzmann, Onsager, Graham and Haken [11], to mention a few, the constraints are reasoned based on microscopical reversibility. It corresponds to detailed balance of transition probabilities between each pair of system states in equilibrium within the framework of autonomous Fokker-Planck equations treated by means of division of the variables and parameters into odd and even with respect to time reversal, with a reserve on factors like magnetic field. In so doing the logic of time reversal is model-bound, ill-suits unsteady conditions and the reserve rule is imposed as if universal while it can be easily broken, e.g., in nuclear processes and where spin-orbit interactions are a factor, particularly near surfaces, interfaces, dislocations. It makes detailed balance a non-self-maintained concept.

A different approach to outlining the overall domain of exactness in question was suggested in [7] and will be developed here. Its basis is in keeping with invariance under transformations of variables. On doing so the energy integral of reversible motion implies the invariance under univalent transformations \( z \rightarrow Z \), of Jacobian

\[
| \det \{ \partial Z_k(z,t)/\partial z_i \} | = 1
\]

(3)

where \( i,k \) run through all components of \( z \) and \( Z \). \( \Phi(z) \) (1) satisfies this condition, for then not only \( \rho_{st} d\Gamma \) is invariant (being a number) but also \( d\Gamma \). The environment as a fluctuation/dissipation source for the system brings in another invariance. Connecting \( \Phi \) to the system’s energy function implies scaling this function in terms of environmental-noise energy levels. The energy scales set this way must vary proportionally with the energy function in arbitrary moving frames \( Z = Z(z,t) \) to hold \( \Phi \) invariant. Since the energy function changes in moving frames, this constraint can hold only for the systems entrained – carried along on the average at any instant for every system’s degree of freedom with the environment causing irreversible drift and diffusion.

Also account must be taken where the limit of weak background noise poses as a structure peculiarity – transition to modeling of evolution without regard to diffusion. The entrainment constraint then keeps its sense as the weak irreversible-drift limit grasped via the scenarios of motion along the isolated paths of motion in line with d’Alembert-Lagrange variational principle. This principle still allows for the ideal non-holonomic constraints that do not perform work on the system but reduce the number of its degrees of freedom, which violates the desired invariance of \( \Phi(z) \). Hence, the invariance necessitates the domain of entrainment free of that, termed ideal below.

We have discussed \( \Phi(z) \) (1), but the reasoning holds for any one-to-one function of \( \rho_{st} \). For the systems describable by a time-dependent density distribution \( \rho(z,t) \), the adequacy of energy function formalism also requires the entrainment ideal. The arguments used above for the systems of steady \( \rho_{st}(z) \) become applicable there with univalent transformations of \( \rho(z,t) \) into \( t \)-independent distribution functions.
The converse is also true: the behaviors governed by a dressed Hamiltonian $H(z, t)$ imply the entrainment ideal and the existence of a density distribution $\rho(z, t)$. The velocity function $\dot{z} = \dot{z}(z, t)$ of underlying motion is then constrained by $\dot{z} = [z, H]$ with $[,]$ a Poisson bracket, so the divergence $\text{div} \dot{z} = \text{div}[z, H] = 0$ and $\text{div}(\dot{z} f) = -[H, f]$ for any smooth $f(z, t)$. It implies
\[
\frac{\partial \rho}{\partial t} = [H, \rho] \tag{4}
\]
which determines $\rho(z, t)$ from a given initial distribution and the boundary conditions at $|z| \to \infty$ taken natural ($\rho$ and its derivatives vanish) to preserve the normalization and continuity, for all other constraints are embodied in $H$. In no way does the solution to (4) ceases to exist as unique, non-negative and not normalizable over the phase space of $z$ where $H(z, t)$ governs the behaviors. The entrainment ideal there takes place since the solution turns into a function $\rho(H)$ in the interaction picture where $H$ is $t$-independent. This completes the proof.

Thus, the necessary and sufficient conditions where the energy function concept is duly adequate to the evolution described by distribution functions come down to the entrainment ideal. This theorem lays down the overall domain of desired energy function adequacy. It includes the systems isolated or in thermodynamic equilibrium, as well as entrained in steady or unsteady environments generally of non-uniform temperature or indescribable in temperature terms so long as the diffusion, irreversible drift and ideal nonholonomy can be neglected. Its criterion as an asymptotic limit in the parameter space of modeling is related to a boundary layer and intermittency where the limit trend can be deprived of evidential force in the close vicinity of the ideal like transitions to turbulence for large Reynolds numbers.

**The energy measure formulation**

Let us refine our concepts. The issues of energy and energy function under study relate to the systems that interact with the environment whose influences of short correlation time are accounted for via the notion of entrainment introduced above. The systems are assumed to be describable by a smooth evolution of the density distribution $\rho(z, t)$ of phase space states $z$, a set of continuous variables $z = (x, p)$ with the generalized coordinates $x = (x_1, \ldots, x_n)$ and conjugated moments $p = (p_1, \ldots, p_n)$ of proper $n$ taken so in neglect of the constraints breaking the energy function formalism; $z$ may include sets of normal mode amplitudes of waves in media. The smoothness of $\rho$ will be understood to mean
\[
\frac{\partial \rho}{\partial t} = -\text{div}(v \rho) \tag{5}
\]
with $v \rho$ the $2n$-vector flux of phase fluid at $z, t$. Eq. (5) turns into the evolution equation of $\rho(z, t)$ with $v$ treated as a non-anticipating functional of $\rho(z, t)$ that accounts for all constraints on the phase flows under the boundary conditions taken natural for the components of $z$ set unbound. In neglect of nonlocal and retarded constraints, $v$ is generally a $t$-dependent field divergent in $z$.

At that, while the $x$, $p$ of $\rho(z, t)$ of (5) is a set of phase space variables, the principle of virtual work on the system and the law of energy conservation, which are to be taken as prime as so the material world is perceived, are formulated in terms of isolated paths with $x$ and $p$ the functions of $t$. The notion of integration along the paths, hence, the work along them is ambiguous in conditions of diffusion; the mechanics pertains then solely to the forces of drift, which is to $v$ a vector function $v(z, t)$. As for a general case, we treat any conceivable isolated paths as abstraction of the Cauchy problem of kinetics of $\rho$, so the integrable correspondence between the two descriptions is to imply the principles of continuity and causality. Integrable is in the sense of averaging and truncations in the limit of short correlation time influences as defined in line with the basics of stochastic integral measure extended in [12–14].

The $n$ components $(v_{n+1}, v_{n+2}, \ldots, v_{2n})$ of the actual (from given initial conditions) phase flux at $z$, $t$ act then as the generalized force conjugated to $x = (z_1, \ldots, z_n)$, and the scalar product
\[
v_{n+1}\delta z_1 + v_{n+2}\delta z_2 + \ldots + v_{2n}\delta z_n \tag{6}
\]
represents the virtual work on the system irrespective of whether this sum is reducible to the variation of a scalar function or not. Accordingly, for the generalized coordinates taken in the geometric conditions not involving time explicitly, the density power on the phase fluid comes down to the scalar product
\[
(v_{n+1}v_1 + \ldots v_{2n}v_n)\rho. \tag{7}
\]
In particular, the energy of the system is conserved as long as the integral of this density over the whole phase volume remains zero,
\[
\int(v_{n+1}v_1 + \ldots v_{2n}v_n)\rho d\Gamma = 0. \tag{8}
\]
This criterion itself bears no relation to the entrainment ideal and shows up in both entrained and non-entrained systems and also as under steady constraints (autonomous Eq. (5)) as unsteady.

Where the energy of system is conserved, there its energy measure exists in strict sense. So, the conditions where criterion (8) holds outlines the existence domain of the energy measure. It includes the whole existence domain of the energy measure in the entrainment ideal, which is obviously where $v$ is a $t$-independent divergent-free function of $z$, but can extend fairly far beyond it – however far in principle both the irreversible drift and diffusion terms of $v$ permit and whether they are retarded and $t$-dependent or $t$-independent.

Thereby the energy function concept serves for the energy concept as means, instruments in modeling via kinetic equations and measurements, e.g., yardstick in
calorimetry, but at issue is, as usual, how we interpret the results of measurements and what concept is more consistent and wider applicable without crutches.

The canonic invariance theorem of kinetic operators

For the evolution of $\rho$ modeled by a kinetic equation

$$\frac{\partial \rho}{\partial t} = [H, \rho] + I$$

(9)

where $H = H(z,t)$ is, unlike in Eq. (4), an arbitrary smooth function, we get from (5) for the term $I$

$$I = -div[(v - \dot{z})\rho]$$

(10)

with $\dot{z} = [z, H]$ the local velocity of Hamiltonian phase flows governed by $H$. An important feature of presentation (9) is the canonical invariance of $I$ holds as in as off the entrainment ideal. To prove, note that a canonical (univalent) transformation $z \rightarrow Z$ implies not only the invariance of $\rho$ and Poisson brackets but also the constraint

$$\frac{\partial Z(z,t)}{\partial t} = [Z, H]$$

(11)

with $G$ a scalar function of $z, t$. Herein $\partial Z(z,t)/\partial t$ is the relative velocity of reference frame $Z$ at $z$ and $t$, so the function $G(z,t)$ plays the role of Hamiltonian governing this relative motion. The canonical invariance of $\partial \rho/\partial t - [H, \rho]$ in (9) follows and, hence, of the $I$ term whatever its functional form may be. This formulation generalizes our theorem IV in [7].

In the entrainment ideal, $I$ reduces to a $[H, \rho]$-like Poisson bracket since the evolution is then to be governed by a dressed Hamiltonian. An example is when the $I$ term is modeled as a heat bath – a superposition of Hamiltonian subsystems with randomly distributed initial conditions. Beyond the ideal, however, the entrainment theorem implies that $I$ is not reducible to a $[H, \rho]$-like bracket, hence, both canonical invariants, $[H, \rho]$ and $I$, then cease to be invariant in the process of actual evolution for any choice of $H(z,t)$.

Abstracting the evolution, the state of $\rho$ at any given instant $t = t_i$ can be taken for ideally entrained by fitting. Due to this and since $\rho$ is assumed smooth in $t$, the effect of the irreducibility of $I$ is weak for $t$'s close to $t_i$, so it might seem reasonable to judge about its figure of merit by popular perturbation methods of celestial mechanics, e.g. [15]. But such insight is insufficient. It fails in the long run beyond the ideal entrainment to match the future with the past and so conforms to the trends of $\rho$ in line with a dressed Hamiltonian, which conduces to the belief in this theory beyond its above-established rigid constraints.

Physically, as the canonical transform is equivalent to the imposition of fields given by Hamiltonian $G(z,t)$, the fields superimposed on the system affect directly the conditions of its entrainment, and so the reversibility of the overall evolution of $\rho$ is affected in response. This generally translates into a vortex (in spatial subspace) impact exerted on the system in the picture at “rest”, where the field $G$ looks frozen. Though small at $t \rightarrow t_i$, it tends to accumulate exponentially and is not a nuisance. In particular, this shows up rigorously for the systems in high frequency resonance fields, especially at parametric and combination resonances, as elucidated in [7] and our earlier work cited there. The arising steady states of $\rho$ and behavior near them in the picture at rest were shown to differ radically from that given by the theory of generalized thermodynamic potential.

The energy duality

Let us focus on the systems relaxing to stable distributions of their states in stationary conditions. The energy-measure criterion (8) includes then the whole area of reversible-motion criterion (2) but is not confined to it at all, which is indicative of the fact that the conditions of $\rho_{st}$ where the conserved energy is indescribable via a generalized thermodynamic potential are common and may range far. In terms of Eq. (9) we get

$$[H, \rho] + I = 0$$

(12)

where the branch $I$ acts on a par with $[H, \rho]$ in jointly keeping the circulation and transformations of conserved energy both within and beyond the entrainment ideal. As for beyond, it implies the conserved energy irreducible to a function of system states. Accordingly, whereas the conserved energy of motion (chaotic motion including) can be conceived within the ideal as the circulation of the kinetic and potential energies within the framework of dressed Hamiltonian, the conserved energy circulating in the systems beyond the ideal includes or constitutes entirely the energy form complementary to the forms describable by a Hamiltonian, hence, quantizable.

Such energy, which we called integral or vortex energy, is also under no bound to the principles of detailed balance, energy transfer directionality, stability and preference of phases – all that given by the conventional theory of phase transitions, transport through barriers and other phenomena based on the generalized thermodynamic potential. The dualism associated with the complementary energy in point also has nothing to do with the particle-wave duality in quantum mechanics and the concept of energy transfer by energy quanta. It questions the all-physics adequacy of quantum approach. The quantum approach, just as the classical one, to be adequate would imply incorporating the scope of energy - energy function duality.
The vortex energy and the directional Brownian motion

Look first at a particle hopping upon a horizontal reflecting plate. Gravity tends to bring it into contact with the plate and the ambient noise keeps it hopping in stationary conditions. Now let the particle be charged and the field of permanent magnet be applied horizontally. This causes the hopping particle to drift in the direction across the field. The net drift is modified but does not vanish when the reflecting surface is uneven or rolled or forms a box, and it persists as the particle motion state relaxes to a stationary \( \rho_{st} \). The same trend is for a number of interacting charged particles in the presence of reflecting walls. The energy of steady macromotion is then conserved, but it is not describable by an energy function of macromotion states and holds vortex energy. The general theorem shown below makes it evident.

The existence of distribution \( \rho_{st}(z) \) in a stable entrainment ideal in stationary conditions implies, along with relaxation to the ideal, the system’s dressed Hamiltonian \( H(z) \) to exist, be bound below, commute with \( \Phi(z) \) and be a monotonic function of \( \Phi \). Thereat, the vanishing irreversible forcing on the average for every component \( i \) of system variables \( z \) implies according to (9) and (12) the constraints

\[
\left( f_i - d_{ik} \frac{\partial}{\partial z_k} + \ldots \right) \rho_{st}(z) = 0 \quad (13)
\]

where \( f = \{ f_i(z) \} \) is the irreversible drift forces, \( d = \{ d_{ik}(z) \} \) is a symmetric non-negative definite matrix of diffusion and ellipsis stands for the higher order diffusion terms of expansion of \( I \) into a series in \( \partial / \partial z \). As \( I \) is generally an integrodifferential form in \( z \), so is the operator bracket of (13). The constraints of (13) generalize the conditions of detailed balance.

Neglecting the higher order terms in the bracket reduces Eq. (13) to the algebraic fluctuation-dissipation relations

\[
f_i = -(d\Phi/dH)_{ik} \dot{z}_k \quad (14)
\]

with \( \dot{z} = [z, H] \). For the distribution \( \rho_{st} \) of Maxwell-Boltzmann form and general Gibbs form, \( d\Phi/dH = \beta \) is independent of \( H \), which reduces (14) to

\[
f = -\beta \dot{z} = -\beta [z, H]. \quad (15)
\]

\( \beta^{-1} = \Theta \) is the energy scale of absolute temperature whose meaning expounds the known equipartition theorem: for every component of \( z \) (coordinate or momentum) whose contribution to \( H \) reduces to a square term, say, \( k_1(z_j - k_2)^2 \) with \( k_1 > 0 \) and \( k_{1,2} \) independent of \( z_j \) but may depend on other components of \( z \) and \( t \), its mean over the Gibbs statistics comes to \( \langle k_1(z_j - k_2)^2 \rangle = \Theta \).

It is easily seen that the \( \rho_{st} \) taken a Gibbs rules out persistent currents since for any \( \langle \dot{z}_i \rangle \), a function of \( z_i \) averaged over the phase subspace off \( z_i \), one gets on integrating by parts

\[
\langle \dot{z}_i \rangle = N \int \langle z_i, H \rangle e^{-\beta H} (d\Gamma/dz_i) = 0 \quad (16)
\]

by virtue of natural boundary conditions for \( \Phi(z) \). The theorem \( \langle \dot{z}_i \rangle = 0 \) holds not only for Gibbs but for any arbitrary statistics of \( \rho_{st} \), a function of \( z \). The proof ensues from \( [z, H] = [z, \Phi H] \) with \( H' = dH/d\Phi > 0 \), for the sign of every \( [z, \Phi] \) is implied so for stability.

These results show no place for a stable macromotion state in stationary conditions within the framework of generalized thermodynamic potential. Such states are thus a Litmus test of conserved vortex energy. A distinctive feature of the phenomenon is robustness as the stability of macromotion state is to be asymptotic, with relaxation a factor and with reversion in response to weak perturbations. It extends the paradigm of Brownian motion caused by eternal chaos as non-directional to that of directional motions caused by eternal chaos.

While any system at a certain standing can be taken via fitting as ideally entrained, governed by an energy function of its states, the theories of transition from these under a shift of parameters to a stable macromotion beg a question whenever the emerging macromotion state is again treated as a state given by an energy function. The macromotion is then attributed to spontaneous symmetry breaking, topological defects and what-not, which is problematic as it implies the conditions (13) to be somehow miraculously restored. Anyhow, in the end one faces the above theorem banning a stable macromotion within this beaten path down-the-line. To claim the phenomenon as just quantum is not sufficient, for as in classics this needs consistently applied principles to account for the transition to a stable stored energy of vortex form.

In contrast to the essence of pattern formation as a process that makes the Cauchy problem of kinetic equation (9) and its quasi-static \( (\partial / \partial t \rightarrow 0 \), not just \( \partial / \partial t \rightarrow 0 \) limit (12) the corner stone of the theory of energy, as we do, the theory of phase transitions in question makes, in fact, the boundary value problem of Liouville type kinetic equations the corner stone. This results in the geometrization beauty of kinetics but rules out the formation intrinsic to a stable non-entrained state, hence, the macromotion and vortex energy.

Thermodynamic laws in the light of vortex energy

Let us look into equilibrium thermodynamics. It proceeds from the existence of internal energy \( E \) of system as a function of external parameters \( a = \{ a_k \} \) and temperature \( \Theta \) so that the differential \( dE \) in space \( (a, \Theta) \)

\[
dE = \frac{\partial E}{\partial \Theta} d\Theta + \frac{\partial E}{\partial a_k} da_k = dQ + dW \quad (17)
\]

expresses the first law by introducing the heat transfer \( Q \) as the difference between the internal energy and the
work on the system $W$ defined for any processes as purely mechanical, for $\Theta$ fixed. For the processes to proceed the parameters are assumed to vary in time, but slowly - in the quasi-static limit $d(a, \Theta)/dt \to 0$. Whereas $Q$ and $W$ may freely depend on the path chosen in $(a, \Theta)$ with $\delta Q$ and $\delta W$ not bound to be exact differentials, Eq. (17) implies for any cyclic process

$$\oint \delta Q = - \oint \delta W.$$  \hspace{1cm} (18)

Therein lays the principle of equivalence between the work and heat. Being for any path in $(a, \Theta)$, it means two separate relations of detailed energy balance, Eq. (18) for the work of irreversible forcing and Eq. (17) with $\delta Q + \delta W$ replaced by their reversible part for the reversible forcing. Treating both as a projection of the separating principle between the balances of reversible and irreversible forcing we formulated in the paragraph with Eq. (13) shows the first law as the law of energy conservation bound to energy function concept for the case and, since equilibrium we formulated in the paragraph with Eq. (13) shows the second relation shows $\Theta$ as the Helmholtz free energy, a function comprising the work of forces $E$ and poses the energy $E$ and forces $A_k = -\partial E/\partial a_k$ as the averages

$$E = \int H e^{(\psi-H)/\Theta} d\Gamma,$$  \hspace{1cm} (19)

$$A_k = \int (-\partial H/\partial a_k) e^{(\psi-H)/\Theta} d\Gamma$$  \hspace{1cm} (20)

with

$$\psi = -\Theta \ln N, \quad N = \int e^{-H/\Theta} d\Gamma$$  \hspace{1cm} (21)

and the Hamiltonian $H$ assumed a function of $z$ and slowly varying parameters $a$ but not $\Theta$. It follows

$$A_k = -\partial \psi/\partial a_k, \quad E = \psi - \Theta \partial \psi/\partial \Theta, \quad S = -\partial \psi/\partial \Theta.$$  \hspace{1cm} (22)

The first relation shows $\psi(a, \Theta)$ as the Helmholtz free energy, a function comprising the work of forces $A = \{A_k\}$, so the second shows $\Theta \partial \psi/\partial \Theta$ as the binding energy function; and the problem of energy and forces at equilibrium is determined by a single function $\psi$ of system states. $S$ represents the entropy function $\int (\delta Q/\Theta)$ introduced in pure thermodynamics by postulating the existence of the integrating multiplier of $\delta Q$ with $1/\Theta$, so the constraint on function $S(a, \Theta)$ to be maximal at thermodynamic equilibrium implies the direction of relaxation only to such ideal. Gibbsian concept makes more sense in physics and shows entropy in (22) as not a self-sustained notion for that matter and that the first and second laws do not extend to the vortex energy and its trends.

The latter assertion is to be common to any extensions of entropy function as within the first law (17) as for more general entrainment ideal conditions. Indeed, the entropy function and the generalized potential must always commute since the ideal entrainment holds where this potential for the system is its energy integral. The violation of entropy conservation law would mean that the entropy is not a function of parameters entering in the potential for the case. This is also so in stationary conditions where the law of energy conservation holds beyond the entrainment ideal, for the opposite would then mean the existence of the energy integral of the system. As to the conservation law of entropy in conditions where the energy of system is not conserved, the entropy cannot be related to the system energy, for such notion does not exist then, which means the entropy conservation is out of physical perception.

Of various entropy functions linked to the conservation law $\oint \delta Q/\Theta = 0$ for slow cyclic processes, only Gibbs statistics assigns to the pure thermodynamics the meaning given by the equipartition theorem. But at that, only a small area of Gibbs statistics domain fits the thermodynamics, as particularly evident from the paragraph with Eqs. (13)–(16). It implies $H$ to be bound from below and the additivity postulate to limit its long-ranged interactions, and the interactions and parameters entering into $H$ should not depend on $\Theta$ and statistical factors - to preserve the very separating principle between the balances of reversible and irreversible forcing and avoid ambiguity in its definitions.

In this light, the known Landau theorem [16], that a closed system of interacting parts in thermal equilibrium admits only uniform translation and rotation as a whole, referred to as the outright ban on classical routes to persistent currents, should not be treated so. The proof [16] proceeds from the system’s entropy $S$ taken in the form of a sum $\sum S_i$ where each summand $S_i$ is a function of the difference $E_i - P_{i}^2 / 2m_i$ between the total and kinetic energy only of part $i$. The statistics of $\rho_{\text{st}}$ is not specified, but the additivity assumption is very restrictive. Also, once the entropy function is taken even in the moments $P_i$’s of system parts, so the distribution of $\rho_{\text{st}}$ is, which automatically rules out persistent currents. But the general conditions of outright ban do not rely on the parity in point, as seen from Eq. (16) and the theorem below it.

We now make a comment on the theory of matter stability, its element based on Gibbsian thermodynamics for Coulomb systems. By the rigorous theory, see [17,18], and the mean-field theories going back to Debye the screening of long-range Coulomb potential $1/r$ between moving charges by the charges of opposite sign in matter makes the potential short-ranged, so the free energy per unit volume is bound below and tends to a finite limit as the system volume increases. But all
that presumes Gibbsian thermodynamics. The sufficient conditions would include the stability with respect to wider possibilities of energy conservation, for the screening arises due to the diffusion and relaxation of gradient of charge-particle density under field perturbations. Within the domain of generalized thermodynamic potential the sufficiency reduces to criterion (13), whereas beyond, the vortex energy emerges, the energy function concept loses force, so the energy integral transits into the energy functional (8) and the stability criterion (13) into that where \( f \) comprises all drift forces, which is accessible for measurements.

Just as important are the constraints imposed on particle systems due to enclosure needed for their confinement as it may not comply without vortex energy. This is so for our example of particle hopping in a box and plausible in phenomena where surfaces, interfaces, dislocations are a factor. Besides nonholonomy the nonconcavity of conservative field Hamiltonian may emerge. It might concern, e.g., superconducting topological insulators commonly treated in quantum terms. Recall also the electron fluid instability suggested by Vlasov [19] by analogy with the physics of capillary waves going back to Stokes and Rayleigh [20] – the attraction of surface particles to the bulk of fluid contributes to the negative potential energy of ripple wave motion, so such states can evolve into a steady ripple that transports mass and charges. Obviously for the phenomenon to exist as robust, held long compared with relaxation time in conditions of vanishing work on the system and scattering, it implies stored vortex energy for stabilization.

**Concluding remarks**

The presented idea of energy as a collective concept of interacting systems departs from the traditional insight.

The central element of departure is the law of conservation of energy, where the energy we proceed from characterizes the ability to produce work defined by d’Alembert principle, not merely its surrogat given by Hamiltonian of systems. Also, while the stored energy is a measure to be given through the evolution of distribution function of system states, taking the energy function concept for granted implies substituting the Cauchy problem of equations governing the evolution by a boundary value problem. The departure is thus from the physics of basically predetermined world to that of real, diverse world where nothing happens by itself but depends on circumstances.

A stumbling block on the path to this diversity is that conserved energy being a measure born in mechanics, not kinetics, is tied up to the notion of characteristics which is applicable only to a very particular type of kinetic equations. The integrable correspondence between the two descriptions of evolution we came to based on the principles of continuity and causality gets over that. The correspondence follows the line of how the measurements of kinetics are perceived and appears completely consistent with the canonical invariance feature we have formulated here for kinetics itself.

These principles, together with the fact of existence of stable matter in stationary conditions, imply the existence and ubiquity of stored vortex energy as complementary to that prescribed by the energy function concept. The presented extension to this concept of common use for interpretations and predictions is like extension from integer numbers to all reals but deeper since it is on functional level. It implies the stable self-sustained motion states in equilibrium out of generalized potential concept. Essentially, the extended notion of stored energy is intrinsic of non-vanishing drift and diffusion. The cumulative effect on equilibrium state, its stability and fluctuations can be huge.

The stored vortex energy being not a function of system states is integral, not quantized, and appears to be under no bound to the trends of conventional equilibrium statistical mechanics and first and second laws of thermodynamics as the steady states are determined by non-selfadjoint operators characteristic of indecomposability.

The law of conservation of entropy is then at variance with the law of conservation of energy, and taking the energy as prime makes the entropy concept unacceptable.

So the entropy argument is not suited for the trends of vortex energy and its existence domain; the stability criterion (13) with \( f \) comprising all drift forces is then of importance as accessible for measurements.

The existence domain of stable matter may extend or shrink not bound to changes of energy function at all – the notion of energy function of system states and their dressed option loses sense as the vortex energy emerges. We meet with roughly the same vortex energy circulation in equilibrium in the systems under high frequency fields in the picture where the hf field is frozen, see [7]. So a vast additional range of objects and phenomena has a bearing on the matter. All that questions the all-physics adequacy of pure quantum approach as it has for the classical energy function concept and its formulation in relativity physics. In particular, it stands to reason that the vortex energy has a bearing on black holes, dark energy and dark matter.

An important result to note is our theorem that rules out any stable macromotion in stationary conditions as soon as the distribution function \( \rho_{\text{st}}(z) \) of system states, of any statistics, is treated within the framework of generalized thermodynamic potential. It makes persistent currents a Litmus test of vortex energy, imposes constraints on the traditional theory of phase transitions, extends the paradigm of Brownian motion caused by eternal chaos as non-directional to that of directional, and gives a natural solution to the fundamental Loschmidt’s paradox.

The revealed vortex form of energy and common misconceptions of conventional energy concept bring forth the necessity to change the whole paradigm of physics based on energy and entropy conservation laws that is reminiscent of flat Earth myths. The change extends the horizons of search for new forms of energy, matter and
macromotion and is important for applied research.

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