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Study of Crack Closure Effect of Hull Plate under Low Cycle Fatigue

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Abstract: The crack closure phenomenon significantly influences low cycle fatigue (LCF) crack growth. The crack closure theory deems that a crack can grow only when the applied load is greater than the fatigue crack opening and closing loads. The revised crack closure theory proposed in this paper provides a new understanding of crack growth: It is no longer the range of stress intensity factor $\Delta K$ that controls the crack growth rate, but the effective stress intensity factor $\Delta K_{\text{eff}}$. Therefore, it is of great importance to study the crack closure phenomenon of LCF. A combination of experiments and the finite element method (FEM) was used to study the effect of overload on the crack closure effect, and the study was carried out using compact tensile (CT) specimens made of AH32 steel. The FEM was used to obtain the stress changes near the crack tip and the opening displacement changes in the crack trailing area after a single tensile overload, to study the intrinsic mechanism of overload on crack closure, and to obtain the LCF crack opening and closing loads by the nodal displacement method. The effect of overload on crack morphology was observed by using high-magnification electron microscopy in combination with testing.

Keywords: low cycle fatigue; crack closure effect; crack opening displacement; crack tip stress; crack opening/closing load; overload

1. Introduction

Ships are often subjected to alternating loads during navigation [1,2]. The addition of a single tensile overload to a constant amplitude cyclic load is the simplest and most typical form of variable amplitude cyclic load. Numerous tests have shown that appropriate overloads can increase fatigue life. The current explanations for the overload retardation effect mainly include residual stress [3], plasticity-induced crack closure (PICC) [4], irregularity of the crack front [5], and blunting of the crack tip [6]. However, among the above overload mechanisms, only PICC can explain the three stages of tensile overload: (i) the crack transient speed-up stage (this phenomenon is mainly caused by a reduction in crack surface contact); (ii) the increase in crack surface contact produces a retardation phenomenon; (iii) when the crack passes through the overload plastic zone, the crack growth rate returns to the state under constant amplitude load. In 1971, Elber [7,8] first proposed the crack closure theory and introduced the concept of the effective stress intensity factor $\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{op}}$ ($K_{\text{max}}$ as the maximum stress intensity factor and $K_{\text{op}}$ as the stress intensity factor at the crack opening), defining it as the driving force for crack expansion. The crack closure theory considers that a crack can only extend in a fully open state, i.e., the applied load must be between the maximum load and opening load, and the crack will not extend forward between the opening load and the minimum load.

Based on the above, many scholars have studied crack closure behavior [9–13] under overload. Ding [14] used the FEM to numerically simulate standard CT specimens of Ni Q345R steel, and analyzed in detail the crack surface contact behavior under a single
overload. Finally, the crack opening load, crack surface contact load, and crack contact length were found to be the main parameters influencing crack closure. Li [15] performed numerical calculations of fatigue crack opening load for a CT specimen made of A537 material with a single tensile overload, obtained the variation law of crack opening load with crack extension distance, and discussed the influence of different stress ratios on the length of retardation of the crack closure effect under overload. Feng et al. [16] used the finite element method to study the tension load of CT specimens under a single tensile overload and proposed a growth rate model based on crack closure. Fleck [17] simulated plasticity-closing crack closure under plane strain by the finite element method, and he found that the crack closure process changed from continuous crack closure to discontinuous after large incremental steps of crack expansion. The discontinuous crack closure was a phenomenon in which the distant crack surface behind the crack tip made contact first, and crack closure did not occur at the first node after the crack tip. McClung [18] pointed out that continuous crack closure occurs in the plane strain state, but the crack tension stress level in the plane strain state is lower than in the plane stress state. Zao et al. [19] performed respective numerical simulations of CT specimens in the plane stress and plane strain states. In the stable extension stage of fatigue cracks under cyclic tensile loading, no plasticity-inducing crack closure phenomenon was observed in the plane strain state. However, the crack closure phenomenon was obvious in the plane stress state. Lei [20] considered that continuous crack closure behavior is mainly due to the crack trailing area behind the crack tip and crack tip plasticity, while the distal crack closure behavior is not related to the crack trailing area and is mainly due to the passivation of the crack tip under the maximum applied load and strain hardening of the material. Based on the crack closure theory, Newman [21] was the first to propose the Newman closure theory under the plane stress state. This model can successfully characterize crack growth behavior under low stress and low load, but it does not consider the effect of thickness under the plane strain state. Silva [22] established a crack growth rate model based on plasticity-induced closure theory under a negative stress ratio ($R < 0$) and obtained the relationship between crack length and plastic zone size considering an inconsistent stress overload ratio.

So far, although many scholars have studied the effect of overload on the crack closure effect, few have systematically analyzed the intrinsic mechanism of crack closure due to overload. In this paper, CT specimens of AH32 steel were used to study the effect of overload on the crack closure effect by combining finite element and testing methods. Firstly, ABAQUS finite element method (FEM) software was used to study the effect of overload on the crack closure effect by combining finite element and testing methods. Secondly, ABAQUS finite element method (FEM) software was used to obtain the stress change near the crack tip and crack opening displacement change in the crack trailing area after a single tensile overload, and to systematically analyze the intrinsic mechanism of the crack closure effect under overload. Based on the above analysis, the influence length of overload on crack closure and residual stress generated by overload as well as the crack opening and closing loads were obtained. The crack morphology was observed by using a high-magnification electron microscope during the test, and the change in crack morphology after overload was analyzed.

2. Low Cycle Fatigue Crack Closure Finite Element Simulation

In this paper, ABAQUS FEM software was selected to numerically simulate the marine AH32 steel CT specimens. The geometry of the CT specimen is shown in Figure 1, and the pre-crack was 2 mm. Considering the symmetry, only half of the model was needed. To take into account the Bauschinger effect and the anisotropic hardening properties of the material when cyclically loaded, the Chaboche combined intrinsic model was used, and model parameters were obtained by uniaxial cyclic tensile testing of AH32 steel as shown in Table 1. The CPS4R planar stress four-node reduced integration cell was used for meshing. Since the elastic-plastic stress-strain field in the area near the crack tip has a large gradient, in order to better simulate the fatigue crack extension and obtain a more accurate stress-strain field at the crack tip, the mesh refinement technique was used in the area near the crack tip, and the mesh size of the refinement area was 0.05 mm, as
shown in Figure 2. A dynamic crack growth approach was used, where periodic release nodes allowed the crack to extend gradually, and boundary conditions were simulated by constraints and contacts. To simulate the phenomenon of crack closure on crack surfaces, contacts were set on both the upper and lower surfaces of fatigue cracks, and rigid lines were defined at the parting interface so that the upper and lower surfaces cannot cross each other. The rigid line and the crack surface in contact with it were respectively defined as the master contact and slave contact surface, which were smooth and frictionless. The nodes on the contact surface were initially bonded together, and later the crack was extended by releasing the nodes periodically with the application of cyclic loads.

![Figure 1. CT specimen geometry (mm).](image)

Table 1. Parameters of the Chaboche combined principal structure model.

| Parameter | Value |
|-----------|-------|
| $E$       | 206 GPa |
| $\nu$     | 0.3   |
| $Q$       | 72 MPa |
| $k$       | 8 MPa  |
| $C_1$     | 314,310 |
| $C_2$     | 28,071  |
| $C_3$     | 1950   |
| $\alpha_1$ | 800   |
| $\alpha_2$ | 321   |
| $\alpha_3$ | 0     |

![Figure 2. Finite element model and refined mesh.](image)

In the simulation calculations, the maximum applied load $P_{\text{max}} = 35$ kN, the stress ratio $R = 0.1$, and a node was released every two cycles. Single tensile overload peaks were applied when the crack length extended to 3.55 mm with overload ratios of 1.2 and 1.3, as shown in Table 2. The finite element fixed increment method was used to obtain the stress change near the crack tip and the change in opening displacement in the crack trailing area after a single tensile overload to study the intrinsic mechanism of overload on crack closure.

### Table 2. Test conditions of CT specimens.

| No. | Stress Ratio (R) | $a$/mm | $R_{OL}$ | $P_{\text{max}}$/kN | Life/Cycle |
|-----|-----------------|--------|----------|---------------------|------------|
| CT01| 0.1             | 2      | /        | 35                  | 3855       |
| CT02| 0.1             | 2      | 1.2      | 35                  | 4055       |
| CT03| 0.1             | 2      | 1.3      | 35                  | 4273       |
2.1. Crack Closure Process Analysis

The stress ratio $R = 0.1$, the maximum applied load was 35 kN, and the crack closure process curve was calculated by ABAQUS FEM using fixed increments to derive the crack trailing area after the crack tip in a block cycle. A preliminary analysis of the crack closure process was performed as shown in Figure 3.

![Figure 3. Schematic diagram of the preliminary analysis of the crack closure process. (a) Crack closure process in a block cycle; (b) crack closure increasing process; (c) crack closure weakening process.](image)

As shown in Figure 3, when the applied load is unloaded to A1, the displacement of individual nodes appears to be equal to 0, at which point the crack begins to close. The first one that starts to show crack closure is the node closest to the crack tip, and the farther the node is away from the crack tip, the larger the node’s opening displacement is. In the figure, it can be seen that A1 represents the applied load corresponding to the time when the fatigue crack is just starting to close, i.e., the low cycle fatigue crack closure load, which is expressed by $P_{cl}$. As the applied load decreases, the crack closure length gradually increases, and when unloaded to the minimum load, the crack closure length is the largest, which is 0.3 mm. During the cyclic applied load loading phase, the crack closure effect gradually decreases as the applied load increases, and the nodes closed in front gradually open up. When the applied load reaches A2, the previously closed nodes will all open and the crack closure phenomenon will disappear, and the corresponding applied load at this time is the crack opening load, indicated by $P_{op}$. This is a preliminary analysis of the crack closure process, and the following will introduce in detail the stress changes near the crack tip and the changes in the opening displacement in the crack trailing area to study the intrinsic mechanism of overload on crack closure.
2.2. Stress Analysis Near Crack Tip after Overload

Figure 4 shows the variation of stress near the crack tip after applying a single tensile overload with stress ratio $R = 0.1$ and maximum applied load of 35 kN. Figure 4a,b shows that after the overload peak action, unloading to the minimum load moment, each new crack tip enters a reverse yielding state due to the stress concentration, and the reverse yielding becomes more and more obvious with the increase in overload ratio. Figure 4c,d shows that as the crack advances away from the overload position, the effect of residual stresses generated by overload gradually decreases and reaches the stress level of constant amplitude loading. At an overload ratio of 1.2, the residual stress generated by overload completely disappears at a crack length of 5.3 mm; at an overload ratio of 1.3, the residual stress generated by overload completely disappears at a crack length of 5.7 mm. From the above finite element results, it can be seen that the crack tip reaches the reverse yield state after unloading due to the stress concentration. The size of the residual stress caused by the stress concentration cannot be ignored. The higher the overload ratio is, the greater the residual stress generated by the overload, which affects the crack length.

![Figure 4](image-url)

**Figure 4.** Crack tip stress distribution after overload. (a) $a = 3.55$ mm; (b) $a = 3.6$ mm; (c) $a = 4.55$ mm; (d) $a = 5.0$ mm.

2.3. Analysis of Crack Opening Displacement

Figure 5 shows the variation of crack opening displacement after applying a single tensile overload with stress ratio $R = 0.1$ and maximum applied load of 35 kN. From Figure 5a, it can be seen that the crack opening displacement is greater with the increase in overload ratio at the moment of applied overload peak, and the closure phenomenon at the moment of overload is not present. The curves of crack opening displacement in Figure 5a,b are generally similar, which indicates that the new crack tip after overload is in a completely closed state after unloading of the current tensile load, and residual plastic deformation does not appear. This is mainly because after overloading the crack tip
forms a large positive plastic deformation, thus leading to the formation of a large residual compressive stress at the crack tip. When the fatigue crack applied load is in the unloading phase, the stress at the crack tip is the result of the combined effect of current applied load and residual stress, thus leading to a new crack tip that does not produce residual plastic deformation. From Figure 5c, the crack closure lengths are 1.65 mm and 2.05 mm when the overload ratios are respectively 1.2 and 1.3. The crack closure length is the distance from the newly created crack tip to the overload point. The crack closure length is 0.3 mm under normal amplitude, and overload causes a substantial increase in crack closure length compared with the normal amplitude. This is primarily because the crack closure effect after overload is due to the combined effect of residual compressive stresses generated by the overload in the crack wake area and the residual compressive stresses generated at the new crack tip. A high residual compressive stress can lead to continuous crack closure consistently from the overload position to the new crack tip position for some distance after overload. From Figure 5d, it can be seen that the crack presents a discontinuous crack closure phenomenon, and crack surface contact consists of two parts: the crack surface contact in the overload region and near the crack tip. No crack closure is detected in a certain section between the crack tip and the overload position. This is mainly because the residual compressive stress from overload decreases as the distance of the crack from the overload point increases, thus leading to the occurrence of discontinuous closure. It can be seen from the above analysis that the higher the overload ratio, the stronger the influence of overload on the length of crack closure.

![Graphs showing crack opening displacement change after overload.](image)

**Figure 5.** Crack opening displacement change after overload. (a) $a = 3.55$ mm; (b) $a = 3.6$ mm; (c) $a = 4.55$ mm; (d) $a = 5$ mm.

### 2.4. Crack Opening/Closing Loading

Figure 6 shows a schematic diagram of crack opening and closing loads under different overload ratios. After a single tensile overload, the tension load and closure load of the
LCF crack immediately drop to zero, which indicates that crack closure is not present at this time. Then, the crack opening and closing loads increase rapidly. At an overload ratio of 1.2, the proportion of the maximum opening and closing loads to the maximum applied load reaches 48.91% and 39.81%, respectively, which represents an increase of 12.52% and 12.2% compared with normal amplitude cyclic loading. At an overload ratio of 1.3, the proportion of maximum opening and closing loads to the maximum applied load reach 54.39% and 46%, respectively. Compared with that under normal amplitude cyclic loading, the increase in proportion is 18% and 18.39%, respectively. As the crack grows away from where the overload is applied, the fatigue crack opening and closing loads gradually return to the crack opening and closing loads under normal amplitude loading. Defining the position from the applied overload to the return to normal amplitude state as the influence length of overload on opening and closing loads, it can be seen from Figure 6 that the influence length of overload on crack opening and closing loads is 1.65 mm at an overload ratio of 1.2, and the influence range of overload on crack opening and closing loads is 2.05 mm at an overload ratio of 1.3. It can be seen from the above analysis that the higher the overload ratio, the larger the area of influence after crack overload for crack opening and closing loads.

**Figure 6.** Crack opening and closing loads under different overload ratios. (a) LCF crack opening load; (b) LCF crack closure load.

Table 3 shows the effect of overload ratio on different cracking parameters. It can be seen from Table 3 that when the overload ratio is 1.2, the residual stress influence length is 1.75 mm, and the influence length of overload on the crack closure effect is 1.65 mm. When the overload ratio is 1.3, the residual stress influence length is 2.15 mm, and the influence length of overload on the crack closure effect is 2.05 mm. The effect of overload on the length of opening and closing loads is the same as that on crack closure; the residual stresses from overload are slightly more extensive than the effect of overload on crack closure.

**Table 3.** Effect of overload ratio on different cracking parameters.

| Cracking Parameters                  | Overload Ratio of 1.2 | Overload Ratio of 1.3 |
|--------------------------------------|-----------------------|-----------------------|
| Residual stress influence length     | 1.75 mm               | 2.15 mm               |
| Crack closure influence length       | 1.65 mm               | 2.05 mm               |
| Opening/closing load influence length| 1.65 mm               | 2.05 mm               |

3. Experimental Study of Low Cycle Fatigue Crack Closure

The low cycle fatigue crack closure test of CT specimens was carried out at room temperature using the MTS322 electro-hydraulic servo fatigue testing machine as shown in Figure 7, with closed-loop control and data acquisition using the TestStar control system. The test was performed by stress cyclic loading with a sine wave loading frequency of 1 Hz. When tensile overload was applied, the loading frequency was reduced to 0.25 Hz to obtain...
enough data for a detailed description of the effect of overload on crack closure as the crack reached the overload point. The number of cycles between crack measurements before and after application of overload was kept as small as possible, thus increasing the number of test data points. An extensometer was used to measure the specimen flexibility and its variation during the test so as to observe the crack closure effect.

![Image of test setup](image-url)

**Figure 7.** Schematic diagram of LCF crack closure test device for CT specimens.

### 3.1. Principle of Low Cycle Fatigue Crack Closure Test

The crack opening load was determined by using the compliance method. The LCF crack opening load was determined by measuring the relationship between crack mouth opening displacement \( V \) and applied load \( P \), as shown in Figure 8. The crack in the straight-line AB section is in a completely open state, and the slope of this straight line is smaller and more flexible; part of the crack in the straight-line CD section is in a closed state, and the slope of this straight line is greater and less flexible. The BC segment of the arc represents the fatigue crack in a state of gradual opening. Point B is the turning point of fatigue crack extension, and the applied load corresponding to point B is the fatigue crack opening load \( P_{\text{op}} \).

![Diagram of opening load determination](image-url)

**Figure 8.** Schematic diagram of opening load determination.
3.2. Low Cycle Fatigue Crack Opening Load

By using the above LCF crack closure approach, the LCF crack opening load curve was obtained for stress ratio $R = 0.1$ and overload ratios of 1.2 and 1.3, as shown in Figure 9. It can be seen from Figure 9 that the crack opening load test value after overload rapidly decreases to 0 at the crack length of 3.55 mm, then rapidly becomes higher than the opening load value under normal amplitude load, and then gradually reaches a stable value. The higher the overload ratio, the higher the maximum value of the tension load resulting from overload, and the greater the effect of overload on the length of crack closure, which is consistent with the conclusions obtained in the previous finite element calculations. Meanwhile, the increase in LCF crack opening load under overload can explain the retardation of crack extension observed in the tests due to overload. However, the experimental values are lower than the FEM calculated values, and the error range between the experimental values of specimens CT02 and CT03 and the FEM calculated opening loads is 8.2–9.2%.

![Figure 9. Fatigue crack opening curves under different overload ratios.](image)

When considering the crack closure effect, the crack closure coefficient $U$ is usually used to characterize the size of crack closure. The crack closure factor $U$ is defined as follows.

$$U = \frac{\Delta K_{eff}}{\Delta K} = \frac{K_{max} - K_{op}}{K_{max} - K_{min}} = \frac{P_{max} - P_{op}}{P_{max} - P_{min}} = 1 - \frac{P_{op}}{P_{max}} = \frac{1 - R}{1 - R}$$

where $\Delta K$ is the range of stress intensity factor, $\Delta K_{eff}$ is the range of effective stress intensity factor, $P_{max}$ is the maximum load, $P_{op}$ is the opening load, $U$ is the parameter characterizing the crack closure, and when $U = 1$, the crack is in the fully open state.

Figure 10 shows a comparison of the crack closure parameter $U$ values for different overload ratios. It can be seen from Figure 10 that the higher the overload ratio is, the lower the post-overload crack closure parameter $U$ is and the more obvious the crack closure effect is. After overloading, the crack closure parameter $U = 1$ at a crack length of 3.55 mm, and then it decreases to the lowest value at a crack length of 3.8 mm. Furthermore, the experimental $U$ value is greater than the value calculated by the finite element method, indicating that the experimental crack closure effect is weaker than the finite element numerical simulation of the crack closure effect.
Comparison of the finite element and test results shows that the results of LCF crack opening load calculated by the finite element method are slightly higher than those obtained by test measurement, and the error is controlled within 9.7%. The crack closure parameter finite element results are slightly lower than the test, with the error controlled within 6%. The crack closure effect of the finite element method is more obvious than that of the test. The reason for this can be explained as follows. (1) The finite element calculation is only an ideal open crack extension model, without considering the effect of crack tip blunting, but in the actual crack extension process, the crack blunting phenomenon exists. In the LCF crack propagation experiment using CT specimens, the residual compressive stress at the crack tip and passivation jointly affect the crack closure effect, and these two factors are mutually governing and interacting. (2) When the distance of the extensometer from the crack tip is farther, its sensitivity to crack closure will be slightly lower, resulting in a low value of the test measured open load.

3.3. Crack Tip Morphology Analysis

Figure 11 shows the crack morphology before and after overload of a single tensile overload test. Figure 11a shows the crack morphology at the end of normal amplitude loading before the overload is applied, from which a crack closure phenomenon can be seen. Figure 11b shows the crack morphology when the overload peak is applied; the crack opening displacement behind the crack tip increases significantly after overload, and the crack tip bifurcates, forming two new crack tips. Figure 11c shows the crack tip morphology when unloading after applying the overload peak, from which it can be seen that there is no crack closure after unloading the applied load to the minimum load after a single tensile overload, and the crack extension appears to accelerate at this time. Figure 11d shows the crack tip morphology in the short distance range after the application of overload. Compared with Figure 11a, the crack closure length after overload leads to a crack closure length greater than that at the constant amplitude, and the crack extension reverts to the original extension direction to continue forward after the appearance of deflection.

By analyzing Figure 11, it can be seen that: (1) there is residual compressive stress in the plastic trailing region, so the crack will show obvious crack closure when unloaded to the minimum load; (2) applying a single tensile overload will lead to stress redistribution in the plastic trailing region, making the crack tip not show crack closure even under the minimum load after applying an overload; (3) a single tensile overload will lead to a greater residual compressive stress in the region near the overload location, thus resulting in a crack closure length greater than that under normal amplitude loading.
Figure 11. Overload crack tip morphology. (a) Before overload; (b) at overload; (c) unloading minimum load after overload; (d) some distance after overload.

4. Conclusions

This paper investigates the effect of overload on the fatigue crack closure effect through experiments and the finite element method, and the following conclusions are obtained.

1. Overload causes fatigue crack opening and closing loads to first rapidly decrease to zero, then rapidly increase to become greater than the fatigue crack opening and closing loads under normal amplitude loading, and finally gradually return to the level under normal amplitude loading.

2. The higher the overload ratio, the greater the maximum opening and closing loads of fatigue cracks in the overload affected area, the larger the overload affected area, and the more obvious the crack closing effect.

3. The finite element closure effect is slightly greater than the experimentally measured crack closure effect. The reason for this is that on the one hand, the finite element method does not consider the effect of crack tip blunting, and on the other hand, sensitivity to crack closure will be slightly lower because the distance of the extensometer from the crack tip is farther.

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