We have used the light-cone formulation of Chiral-Quark Soliton Model to estimate the width of the lightest pentaquark $\Theta^+$. We have found that the effect of nonzero momentum transfer is important and reduces drastically the width to about 0.43 MeV. This means that this effect is a piece of the small width puzzle of exotic baryons.

1 Introduction

Chiral-Quark Soliton Model ($\chi$QSM) has been recently formulated in the infinite momentum frame (IMF) \cite{1,2}. This provides a new approach for extracting predictions out of the model. The light-cone formulation is attractive in many ways. For example light-cone wave functions are particularly well suited to compute matrix elements of operators. One can even choose to work in a specific frame where the annoying part of currents, i.e. pair creation and annihilation part, does not contribute. On the top of that it is in principle also easy to compute parton distributions once light-cone wave functions are known.

The technique has already been used to study vector and axial charges of the nucleon and $\Theta^+$ pentaquark width up to the 5-quark component \cite{2,3}. It has been shown that relativistic effects are non-negligible. For example they explain the reduction of the naïve quark model value $\frac{2}{3}$ for the nucleon axial charge $g_A^{(3)}$ down to a value close to 1.257 observed in beta decays.

The existence of an exotic antidecuplet is still under debate. Even though most of the latest experiments suggest that it does not exist, no definitive answer can be given \cite{4}. Theoretically pentaquarks are expected to have narrow width. In their seminal paper \cite{5}, Diakonov, Petrov, Polyakov have given an upper bound $\Gamma_{\Theta^+} \leq 15$ MeV. In a previous paper using the present technique \cite{3} we have obtained $\Gamma_{\Theta^+} \sim 2$ MeV. On the experimental side, if $\Theta^+$ does exist, its width should be $\Gamma_{\Theta^+} = 0.36 \pm 0.11$ MeV \cite{6}. Such a small value is below experimental resolution and does not contradict any other experimental result on $\Gamma_{\Theta^+}$. More conservative phenomenological estimations give only upper bounds of 1-5 MeV \cite{7}. Using the so-called “model-independent approach” to $\chi$QSM \cite{8} it has been shown that the model may be consistent with $\Gamma_{\Theta^+} < 1$ MeV and even the experimental value. Our estimation \cite{3} is one order of magnitude higher than the DIANA result. This can be related to the fact that we did not take into account the difference of masses between nucleon and $\Theta^+$. The axial matrix element was then evaluated at zero momentum transfer. That is the reason that motivated this study.

In this paper we present our results for nonzero momentum transfer. We show that the effect is far from being negligible and thus is probably part of the explanation for the small width of pentaquarks. While in a previous work we have considered relativistic corrections to quark wavefunction, in this study we limit ourselves to the nonrelativistic case so that the computations remain tractable in a reasonable amount of time. In section 2 we show how we have considered nonzero momentum transfer in this study. In section 3 we explain how to compute the matrix elements within the $\chi$QSM in IMF. Then we give the results obtained and the numerical values of relevant integrals in section 4.
2 Nonzero momentum transfer

Were nucleon and $\Theta^+$ degenerate in mass there would be no momentum transfer. Let us consider a $\Theta^+$ pentaquark with 4-momentum $P$ decaying into a nucleon and a kaon with 4-momenta $P'$ and $q$ respectively. We consider the $z$-direction as the pentaquark momentum one and all the particles on mass-shell

$$P = \left(\sqrt{P_z^2 + M^2}, \vec{0}, P_z\right),$$

(1)

$$P' = \left(\sqrt{X^2 P_z^2 + q_{\perp}^2 + M'^2}, -q_{\perp}, XP_z\right),$$

(2)

$$q = \left(\sqrt{(1 - X)^2 P_z^2 + q_{\perp}^2 + m^2}, q_{\perp}, (1 - X)P_z\right)$$

(3)

where $M$, $M'$ and $m$ are respectively the $\Theta^+$, nucleon and kaon masses and $X$ the fraction of the total longitudinal momentum kept by the nucleon. In the Infinite Momentum Frame (IMF) $P_z \to \infty$ the energy conservation law yields the following condition

$$M^2 = \frac{M'^2 + q_{\perp}^2}{X} + \frac{m^2 + q_{\perp}^2}{1 - X}.\quad (4)$$

The nucleon is described as a superposition of $3+2n$-quark Fock components with $n = 0, 1, 2, \ldots$ while a pentaquark has $n = 1, 2, \ldots$. The momenta of the individual quarks have to sum up to the total momentum of the baryon they belong to

$$\sum_{i=1}^{3+2n} \vec{p}_i = \vec{P}.\quad (5)$$

We introduce $z_i = p_{iz}/P_z$ the fraction of the total longitudinal momentum carried by quark $i$. The two other components of the momentum are collectively called $\vec{p}_{i\perp}$. This means that

$$\sum_{i=1}^{3+2n} \vec{p}_{i\perp} = \vec{P}_{\perp} \quad \text{and} \quad \sum_{i=1}^{3+2n} z_i = 1.\quad (6)$$

Using eq. (6) one concludes that

$$\sum_{i=1}^{3+2n} \vec{p}_{i\perp} = \vec{0}, \quad \sum_{i=1}^{3+2n} z_i = 1 \quad \text{and} \quad \sum_{i=1}^{3+2n} \vec{p}_{i\perp}' = -\vec{q}_{\perp}, \quad \sum_{i=1}^{3+2n} z_i' = 1$$

(7)

where the unprimed variables refer to pentaquark and primed ones to nucleon. The current strikes only one quark line, say $j_0$, so one obtains the following relations

$$\vec{p}_{j\perp} = \vec{p}_{j\perp}', \quad \vec{p}_{j_0\perp} = \vec{p}_{j_0\perp} + \vec{q}_{\perp},$$

(8)

$$z_j = Xz_j', \quad z_0 = Xz_0' + (1 - X)$$

(9)

where $j \neq j_0$ refers to the non-struck quarks. Since in the IMF all the quarks are moving in the same direction as the baryon they belong to, one expects $z_i, z_i' \in [0, 1]$. In fact, from eq. (8) one can see that in order for the pentaquark to decay any internal configuration is not allowed and depends on the fraction of total momentum carried by the nucleon

$$z_j \in [0, X] \quad \text{and} \quad z_{j_0} \in [1 - X, 1].$$

(10)

Any value of $X$ is also not allowed because of the energy constraint (1). In this study we have considered the cases of a massless $m = 0$ and a massive $m = 495$ MeV kaon and have used $M = 1530$ and $M' = 938$ MeV yielding to $X \in [0.376, 1]$ and to $X \in [0.468, 0.803]$ respectively. In the equal mass limit $M = M'$ with masses kaon $m = 0$ one has $X = 1$ and $\vec{q}_{\perp} = 0$ as it should be.

\footnote{We work in the Drell frame $q^+ = 0$ where quark-antiquark creation and annihilation are absent. The momentum transfer can then only be transverse.}
3 Nonzero momentum transfer integrals

It was shown in [2] that four integrals were needed to compute the axial charge of the $\Theta \rightarrow K\pi$ decay. If one considers nonzero (transverse) momentum transfer then one has to compute five integrals. They can be written in the general form

$$K_I = \frac{M^2}{2\pi} \int d\chi \frac{d^3P'}{(2\pi)^3} \Phi_0(P', X) G_I(P', X)$$

(11)

where $\Phi_0$ is a valence probability distribution, $G_I$ is a quark-antiquark probability distribution and $I = \pi\pi, \sigma\sigma, 3\pi, 2\pi$. These integrals are regularized by means of Pauli-Villars procedure. In order to keep them tractable in a reasonable amount of time, we used the nonrelativistic form of the valence probability distribution

$$\Phi_0(P', X) = \int d\chi\int d^2p'_{1,2,3} \frac{d^2p'_{1,2,3}}{(2\pi)^6} \delta(p'_{1}/M + z'_1 + z'_2 + z'_3 - 1)(2\pi)^2 \delta^2(P'_{\perp} + p'_{\perp} + q_{\perp})$$

(12)

where $P'_{\perp} = (z'_1 + z'_2)M = Z'M$ and $P'_{\perp} = p'_{\perp} + p'_{\perp}$. More details about the expressions used can be found in [2] [3].

3.1 Struck valence quark integrals $I = \pi\pi, \sigma\sigma, 3\pi$

If the struck quark is a valence one then one can choose $j_0 = 3$ in (8). The quark-antiquark probability distributions are

$$G_{\pi\pi}(P', X) = \theta(P'_z)P'_z\Pi(P)\Pi(P') \int_0^1 dy \int d^2Q_{\perp} \frac{Q_{\perp}^2 + M^2}{Z'Z},$$

(13)

$$G_{\sigma\sigma}(P', X) = \theta(P'_z)P'_z\Sigma(P)\Sigma(P') \int_0^1 dy \int d^2Q_{\perp} \frac{Q_{\perp}^2 + M^2(2y - 1)^2}{Z'Z},$$

(14)

$$G_{3\pi}(P', X) = \theta(P'_z)P'_zP'_z\Pi(P)\Pi(P') \int_0^1 dy \int d^2Q_{\perp} \frac{Q_{\perp}^2 + M^2}{Z'Z},$$

(15)

where $Z = Q_{\perp}^2 + M^2 + y(1 - y)P^2$ and $Z' = Q_{\perp}^2 + M^2 + y(1 - y)P'^2$. The internal quark-antiquark pair variables are defined as $y = y'Z' / Z$ and $Q_{\perp} = yP_{\perp} - (1 - y)P_{\perp}$. One naturally recovers the zero momentum transfer case by setting $X = 1$ and thus $q_{\perp} = 0$.

3.2 Struck quark/antiquark integrals $I = \sigma3, 3\sigma$

If the struck quark is the quark (antiquark) of the pair one has $j_0 = 4$ ($j_0 = 5$) in (8). Let us consider that the current strikes the antiquark. The quark-antiquark probability distributions are then

$$G_{\sigma3}(P', X) = \theta(P'_z)P'_z\Sigma(P)\Pi(P') \int_0^1 dy \int d^2Q_{\perp} \frac{Q_{\perp}^2 - M^2V_z + (1 - y)(Q_{\perp} \cdot V_{\perp} + 2M^2Z')}{DZ'},$$

(16)

$$G_{3\sigma}(P', X) = \theta(P'_z)P'_zP'_z\Pi(P)\Sigma(P') \int_0^1 dy \int d^2Q_{\perp} \frac{Q_{\perp}^2 - M^2(2y - 1)V_z + (1 - y)Q_{\perp} \cdot V_{\perp}}{DZ'},$$

(17)

where

$$D = (1 - y)\frac{V_z}{Z'}(M^2X^2Z'^2 + P_{\perp}^2) + V_z(Q_{\perp}^2 + M^2) + (1 - y)\frac{1}{Z'}V_{\perp}^2 + \frac{2}{Z'}(1 - y)(V_zP_{\perp} + Z'Q_{\perp}) \cdot V_{\perp}.$$  

(18)

For convenience we have introduced the variables $V_z = Z' + \frac{V_z}{Z'}$, $V_z = yZ' + \frac{V_z}{Z'}$, and $V_{\perp} = Z'q_{\perp} - \frac{1 - y}{Z'}P_{\perp}$. Once more one naturally recovers the zero momentum transfer case $K_{3\sigma} = K_{1\sigma}$ by setting $X = 1$ and thus $q_{\perp} = 0$.

If the current stroke the quark we would in fact obtain the same quark-antiquark probability distributions since $P'$ and $X$ do not change under a permutation of indices 4 and 5. If we had started with the quark struck we would have ended up with the same expression after a change of variables $y \rightarrow (1 - y)$ and $Q_{\perp} \rightarrow -Q_{\perp}$ corresponding indeed to a permutation of indices 4 and 5.$^2$

$^2$The nonzero momentum transfer breaks the symmetry of the quark-antiquark pair leading to $K_{1\sigma} \neq K_{\sigma3}$. 

3
4 Numerical results

In the evaluation of the scalar integrals we have used the quark mass $M = 345 \text{ MeV}$, the Pauli-Villars mass $M_{PV} = 556.8 \text{ MeV}$ for the regularization and the baryon mass $M = 1207 \text{ MeV}$ as it follows for the “classical” mass in the mean field approximation [9].

The numerical evaluation for $m = 0$ yields

$$K_{\pi\pi} = 0.02198, \quad K_{\sigma\sigma} = 0.00883, \quad K_{33} = 0.01216, \quad K_{3\sigma} = 0.01105, \quad K_{\sigma3} = 0.01066. \quad (19)$$

The numerical evaluation for $m = 495 \text{ MeV}$ yields

$$K_{\pi\pi} = 0.01645, \quad K_{\sigma\sigma} = 0.00630, \quad K_{33} = 0.00907, \quad K_{3\sigma} = 0.00799, \quad K_{\sigma3} = 0.00683. \quad (20)$$

From these integrals one can estimate the width of $\Theta^+$ width as explained in [2]. In table 1 we compare the results for $\Theta^+$ width with zero and nonzero momentum transfer. Not surprisingly one can see that nonzero momentum transfer reduces the width. Indeed one can expect the integrals to be smaller because all the configurations for the pentaquark to decay are not allowed. Of course one should not trust the number of Table 1 as they are because up to now we did not estimate the theoretical errors. Nevertheless we hope that they give some kind of order of magnitude. Note that it is in rather good agreement with the experimental extraction.

| $q_{\perp}$ | $M = M', \ m = 0$ | $M \neq M', \ m = 0$ | $M \neq M', \ m \neq 0$ |
|-------------|------------------|------------------|------------------|
| $g_{A}(\Theta \rightarrow K\pi)$ | 0.202 | 0.063 | 0.042 |
| $g_{oKN}$ | 2.230 | 0.697 | 0.467 |
| $\Gamma_{\Theta} \ (\text{MeV})$ | 4.427 | 0.432 | 0.194 |

5 Conclusion

The question of the pentaquark is a very interesting one since its existence or non-existence would should light on many aspects and problems of baryon physics and low-energy QCD. One of the most interesting question is its width. While there is no definitive theoretical answer, experiments seem to suggest a very small value (<1 MeV) if it does exist. It is thus imperative to see if one can obtain such a small value within a model and try to understand the reason.

Using $\chi$QSM formulated in the Infinite Momentum Frame (IMF) we were able to give an estimation $\approx 2 \text{ MeV}$. Such a small values was attributed to the fact in IMF the current does not create nor annihilate quark-antiquark pairs. So the pentaquark can only be connected the 5-quark Fock component of the nucleon, which is small compared the the 3-quark Fock component.

In this paper we tried to take into account the fact that $\Theta^+$ and $N$ have different masses and thus that the current has nonzero (transverse) momentum. We have obtained $\Gamma_{\Theta^+} \approx 0.43 \text{ MeV}$ which is of the same order of magnitude than the experimental width $\Gamma_{\Theta^+} = 0.36 \pm 0.11 \text{ MeV}$. We have shown that this can be understood by the fact that a nonzero momentum transfer reduces the number of possible configurations for the pentaquark to decay. Although our value cannot be fully trusted (due to unknown theoretical errors) it is an indication that nonzero momentum transfer is partly responsible for such a small width.

Acknowledgements

The author is grateful to RUB TP2 for its kind hospitality, to D. Diakonov for suggesting this work and to M. Polyakov for his careful reading and comments. The author is also indebted to J. Cugnon whose absence would not have permitted the present work to be done. This work has been supported by the National Funds of Scientific Research, Belgium.
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