Modeling of Timing Jitter for Single Photon Avalanche Diodes

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Keywords: Single photon avalanche diode, Timing jitter, Diffusion tail, Gaussian distribution.

Abstract. Timing jitter is an important indication reflecting on the performance of single photon avalanche diodes (SPADs). In this paper, an analytical model is established to predict the temporal response characteristic of SPADs based on the physical progress of photon absorbing and avalanche triggering. The diffusion and drift of carriers in the neutral layer are considered by solving the continuity equation for exponential tail modeling. A Gaussian distribution is used to model the peak response corresponding to photon absorption in avalanche region. The model key physical parameters such as electric field and avalanche triggering probability come from TCAD simulation, which improves the model accuracy. This analytical model is useful to predict the temporal response for given SPAD device structure.

Introduction

In the past twenty years, silicon-based single photon avalanche diodes (SPADs) have gained great interest in photon counting and timing applications due to fast temporal response, high resolution, low cost, etc. [1]. We have known that the timing jitter performance is a dominant factor in photon timing application such 3D imaging, which plays an important role in resolution and speed of SPAD imagers. The timing jitter is attributed to two individual contributions, including photon absorption and induced avalanche triggering in avalanche and neutral regions, respectively. The peak of temporal response is due to the photon absorption in the avalanche region, depending on different absorption positions and the randomness of collision ionization. The photo-generated carriers in the neutral region enter the high field region and subsequently trigger avalanche phenomenon, which is known as diffusion tail of temporal response. The peak of timing jitter and the tail amplitude have a strong influence on timing performance. Therefore, it is necessary to establish an analytical model to predict the time jitter characteristic in the device design stage. Previous models have revealed that the timing jitter with peak and tail features by physics and number simulation. However, Monte Carlo simulation is time-consuming. Additionally, there is a lack of an analytical expression to directly calculate the temporal response of SPADs.

In this paper, we established an analytical model to predict the temporal response of SPADs, mainly focusing on the tail of timing jitter. By solving continuity equation, we obtain the carriers temporal evolution in the neutral region, further deriving the analytical expression of temporal response with exponential tail. TCAD simulation was performed to acquire the accurate model parameters of electric field and avalanche triggering probability, enabling the improvement of modeling. The presented simple model is suitable for the temporal response prediction for the specified device structure.

Device Structure and Principle

In this work, we take a typical P+/NWELL device structure for example to establish the timing jitter model. The SPAD device structure and the schematic diagram of photons absorbing in different regions are shown in Fig. 1(a) and (b), respectively. In this device, P+ and N-well form the depleted
layer as the main detection area to absorb photons. To prevent the device from premature breakdown, STI and P-guard ring area are placed on the edge.

![Diagram](image)

**Figure 1.** (a) Typical P+/NWELL structure; (b) Schematic diagram of photons absorbing in three regions.

Due to the high electric field, photon-generated electron-hole pairs rapidly accelerate and ionize in the depletion region, triggering avalanche events. Differently, if a photon impinging on the neutral region is absorbed, the minority carrier is generated and randomly drifts in a weak electric field. In the shallow P+ neutral region, the minority carriers (electrons) which are not capable to reach the boundary of the depleted region will eventually recombine. Similarly, in the deep N-well neutral region, when moving toward the substrate, the minority carriers (holes) will thermalize [2]. Otherwise if the minority carriers in two neutral regions approach to the edge of the depleted region, a strong electric field will allow them to accelerate and there is a probability of triggering an avalanche. Ultimately, a photon that is absorbed directly by the substrate far away from the depletion region cannot be detected.

**Modeling of Timing Jitter**

We adopt a mathematical and physical modeling method to evaluate the temporal response and the photon detection efficiency (PDE) of SPAD. The temporal response curve is divided into two parts, i.e., Gaussian distribution in the depleted region and an exponential tail in the neutral layer as

$$
PDE(t) = G(\mu, \sigma) + P_{\text{neu}}(t).$$  \hspace{1cm} (1)

**Gaussian Distribution in the Depleted Region**

The photons hitting onto the depleted layer are absorbed directly, resulting in electron-hole pairs. In a strong electric field, the electrons and holes drift in different directions, and both can trigger avalanche breakdown [3]. The following equation can be used to calculate the peak value of Gaussian distribution:

$$P_{\text{Dep}} = \int_0^{W_2} \alpha e^{-\alpha x} P_{\text{pair}}(x) dx,$$

where $\alpha$ is the absorption coefficient, $W_2$ is the thickness of the depleted layer and $P_{\text{pair}}(x)$ is the avalanche triggering efficiency. The absorption coefficient is a function of wavelength at a certain temperature which is dependent on the material and depth of the device.

Actually, the temporal response curve of the depleted region can be fitted with Gaussian distribution:

$$G(\mu, \sigma) = P_{\text{Dep}} e^{-\frac{(t-\mu)^2}{2\sigma^2}},$$  \hspace{1cm} (3)

where $\mu$ is the average avalanche time obtained in the experiment which depends on the thickness, about 10ps/µm, of the depleted layer [4], and $\sigma$ is the standard deviation determined by full-width at half-maximum $H_m$ [5]:
\[ \sigma = \frac{H_m}{2\sqrt{2\ln 2}}. \]  

**Exponential Tail in the Neutral Region**

As for the temporal response in the neutral layer, both the shallow P+ layer and the deep N-well layer, we calculate the photon detection probability density by

\[ P_{\text{neu}}(t) = H_f P_h(x_D) + E_f P_e(-x_S), \]  

where \( H_f \) is the hole flux density, \( E_f \) is the electric flux density, \( P_h(x_D) \) and \( P_e(-x_S) \), are respectively the hole triggering efficiency at \( x_D \) and the electron triggering efficiency at \( -x_S \).

Once the photon is absorbed in deep N-well neutral region, the generated minority carriers (holes) drift and diffuse follow continuity equation:

\[ \frac{\partial P}{\partial t} = D_p \frac{\partial^2 P}{\partial x^2} - \mu_p E \frac{\partial P}{\partial x} - \mu_p \frac{\Delta P}{\tau_p} + g, \]

where \( \mu_p \) is the hole mobility, \( E \) is the electric field, \( D_p \) is the hole diffusion coefficient and \( \tau_p \) is hole life-time. The term represents the change in the number of holes caused by external factors per unit time and unit volume. In effect, Eq. 6 can be solved numerically with proper boundary conditions.

First of all, the weak electric field in the neutral layer displays a uniform distribution because of relatively low doping concentration. By TCAD simulation, we can obtain the electric field in the deep neutral layer.

Therefore, the Eq. 6 can be simplified as

\[ \frac{\partial \Delta P}{\partial t} = D_p \frac{\partial^2 \Delta P}{\partial x^2} - \mu_p E \frac{\partial \Delta P}{\partial x} - \frac{\Delta P}{\tau_p}, \]

Then the boundary condition \( B_c \) is determined by

\[ B_c = N_p, \]

with \( N_p \) the number of the hole generated after absorbing photons on unit area.

Finally, the solution as a function of time \( t \) and distance \( L \) of this equation is expressed as

\[ \Delta p = \frac{N_p}{\sqrt{4\pi t D_p}} \exp \left[ -\frac{L^2 + t}{4tD_p} \right]. \]

Once the holes concentration is calculated for the deep neutral layer, the hole current density can be obtained by

\[ J(t) = q \mu \left( pE - \frac{kT}{q} \frac{dp}{dx} \right). \]

At last, we are capable to calculate the holes flux density \( H_f(t) \) as

\[ H_f(t) = \frac{J(t)}{q}. \]

On the contrary, the shallow neutral layer is the p-type region where the minority carrier electrons are generated. The thickness of this layer is thin, but the doping concentration is high. When photons impinge on the active area, few can be captured by this layer. Besides, the high recombination rate of electrons plays a decisive role. Therefore, \( H_f P_e(-x_S) \) can be neglected and \( P_{\text{neu}}(t) \) strongly depends on another term \( H_f P_h(x_D) \) as

\[ P_{\text{neu}}(t) = \left( \mu_p E + \frac{x}{2t} \right) \frac{N_p}{\sqrt{4\pi tD_p}} \exp \left[ -\frac{(L-\mu_p E t)^2}{4tD_p} - \frac{t}{\tau_p} \right] P_h(x_D). \]

**Results and Discussion**

Geiger mode TCAD simulation was performed to obtain the avalanche triggering efficiency and electric field distribution by Silvaco Atlas, considering the device structure and manufacturing pro-
cess. Fig. 2(a) shows the electric field profile in deep neutral layer. An almost invariable low electric field of about 700 V/cm is found in the deep neutral layer. Fig. 2(b) illustrates the avalanche triggering probability distribution in depletion region. It can be seen that the avalanche triggering probability on the deep N-well layer boundary is about 0.3. Thanks to the TCAD simulation, we can obtain accurate avalanche triggering probability and electric field to calculate the diffusion tail and peak of the timing jitter.

![Electric Field Profile in Deep Neutral Layer](image)

**Figure 2.** (a) Simulated electric field distribution in deep neutral layer; (b) simulated avalanche triggering probability distribution in depletion region.

Fig. 3 depicts the temporal response, a decay exponential tail in the deep N-well neutral region. When the holes drift and diffuse in a longer distance such as 0.3 μm, the tail represents a slower decay trend, as shown in Fig.3 (a). In addition, the stronger electric field in the depletion region due to larger excess bias voltage results into the faster drift speed of minority carriers, which is responsible for the less response time, as seen in Fig.3(b).

![Temporal Response in Deep N-well Neutral Layer](image)

**Figure 3.** Temporal response in the deep N-well neutral layer: (a) different drift and diffusion distances; (b) different excess bias voltages.

Eventually, according to the above two parts of the discussion, the timing jitter curve is fitted including Gaussian peak plus an exponential tail seen in Fig. 4. The former Gaussian distribution is obtained with μ of 30ps and H_m of 35ps. The latter exponential tail is found at 1V of excess bias voltage and 2μm of drift and diffusion distance.

![Timing Jitter Curve](image)
Summary
In this paper, we established a mathematical and physical model of timing jitter aiming at studying the important characteristics of SPAD. By evaluating the time response in the depleted region and the neutral region, the analytical expression of timing jitter was derived. By means of Geiger mode TCAD simulation, we can obtain accurate model parameter. Finally, the temporal time of SPAD is calculated. The presented model is useful to predict and analyze the timing jitter performance for given SPAD device structure.

Acknowledgement
This work is sponsored by the National Natural Science Foundation of China (No. 61571235) and QingLan Project.

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