Elliptic flow in proton-proton collisions at $\sqrt{s} = 7$ TeV

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Abstract. The angular correlations measured in proton-proton collisions at $\sqrt{s} = 7$ TeV are decomposed into contributions from back to back emission and elliptic flow. Modeling the dominant term in the correlation functions as a momentum conservation effect or as an effect of the initial transverse velocity of the source, the remaining elliptic flow component can be estimated. The elliptic flow coefficient extracted from the CMS Collaboration data is 0.04 – 0.08. No additional small-angle, ridge-like correlations are needed to explain the experimental data.

1 Introduction

Proton-proton collisions at the CERN Large Hadron Collider with the energy of 7 TeV are the most violent elementary collisions studied in a laboratory. The high multiplicity of produced particles means that a short-lived, high density system is formed. By analogy to the physics of ultrarelativistic heavy-ion collisions, one could expect new collective phenomena to emerge in the highest multiplicity events. A striking experimental observation has been made by the CMS Collaboration, measuring two-particle correlations $R(\Delta \phi, \Delta \eta)$ in the relative azimuthal angle and the relative pseudorapidity $|\Delta \eta| > 2.0$. The angular correlation function is decomposed into a contribution from global momentum flow and a smaller elliptic component. The dominant structure in the correlations function in the relative azimuthal angle is due to an enhancement of back to back emission. This effect is qualitatively described by Monte Carlo event generators [1]. The experimental data compared to the PYTHIA generator results show two essential differences. For all transverse momentum and multiplicity classes presented, the measured angular correlations show deviations from model calculations. In simulations and in the data, the correlations are enhanced for the emission with $|\Delta \phi| \approx \pi$, but the width and the height of the peak is different. The most important new feature in the measured correlation functions is the appearance of a small-angle enhancement of the distribution. This small-angle structure (ridge) is present in a broad range in the relative pseudorapidity between the particles in the pair. This small-angle enhancement is not due to the usual correlations from the jet fragmentation, and is not reproduced by any Monte Carlo event generator.

We analyze the possibility that the new effect is due to the elliptic flow present besides the dominant back to back angular correlations. For some transverse momentum and multiplicity cuts, the relative importance of the elliptic flow component is big enough to make it explicitly visible in the small-angle region. However, the second harmonic $\cos(2\Delta \phi)$ cannot be unambiguously unraveled in the whole range (0 – $\pi$) from the dominant background in the correlation function.

The importance of the elliptic flow is that it is a signature of the collective expansion of the fireball created in relativistic heavy-ion collisions [4]. It is important to check whether similar phenomena occur in elementary collisions, but the search for the elliptic flow in smaller systems is hindered by statistical fluctuations and non-flow correlations. Several model estimates of the elliptic flow generated in proton-proton collisions have been given [5–13]. Most commonly, one assumes that the elliptic flow is generated during a short hydrodynamic expansion, in a very similar way as in heavy-ion collisions. The most important differences between calculations concern the origin of the azimuthal asymmetry of the initial energy density distribution in the transverse plane. If the presence of an elliptic flow of collective origin could be demonstrated in proton-proton collisions, it would indicate the a strongly interacting fireball has been formed in hadron collisions at the highest energies.

The elliptic flow correlations are subleading. The extraction of the elliptic flow coefficient requires a careful analysis of other effects. In the following, we propose two models for the dominant back to back correlations. The first estimate is based on the momentum conservation ef-
fects, requiring the conservation of the total transverse momentum. The second calculation assumes that particles separated in pseudorapidity are emitted from sources moving (on average) in opposite transverse directions. This picture is natural if the two particles come from two different back to back jets. In a high multiplicity event, and for low transverse momenta of the particles, a simple picture can be used, with two sources moving with opposite transverse velocities.

To reproduce the observed ridge correlations, an additional azimuthal asymmetric component (elliptic flow) is added. The estimated elliptic flow coefficient is \( v_2 = 0.06-0.1 \), for the parameters where the ridge is observed. The elliptic flow interpretation agrees qualitatively with the multiplicity and transverse momentum dependence of the observed effect. The strength of the small-angle two-particle correlations is proportional to \( v_2^2 \), and should increase as \( p_{\perp}^2 \).

## 2 Momentum conservation

The correlation function \( R(\Delta \phi) \) is obtained selecting particles separated in pseudorapidity [1]. This procedure eliminates jet-like correlations on the near side. The form of the dominant remaining correlations shows, that the associated particle is emitted preferentially in the opposite direction in the azimuthal angle. This could be a consequence of the momentum conservation in the microscopic particle production mechanism. Such correlations can be very important in small multiplicity events, as in proton-proton collisions [14–17]. In the following we take into account the transverse momentum conservation in the same way as proposed in [14,17]. In an event of multiplicity \( M \), the multiparticle distribution in transverse momenta \( p_{\perp} \) and pseudorapidities \( \eta_i \) is given as

\[
\begin{align*}
\langle f_M(p_{\perp}, \eta_1, \ldots, \eta_M) \rangle &= \int \delta^2(p_{\perp} + \ldots + p_M) f(p_{\perp}, \eta_1) \ldots f(p_M, \eta_M) / \langle f_M(p_{\perp}) \rangle, \\
\langle f(p_{\perp}, \eta) \rangle &= f_{av}(p_{\perp}, \eta) (1 + 2v_2 \cos(2\phi)),
\end{align*}
\]

and is normalized to one. \( M \) is not necessarily the total multiplicity in the event, it is rather the number of particles created in the microscopic process conserving the momentum. \( v_2 \) is the elliptic flow coefficient, that is assumed to have negligible variation over the transverse momentum, pseudorapidity or multiplicity bins where the averages are taken. The two-particle distribution is obtained from Eq. (1) by integrating over the momenta of \( M - 2 \) particles. For large \( M \), small \( v_2 \), and to the order \( 1/N \), one obtains [14,17]

\[
\begin{align*}
\langle f_{2}(p_{\perp}, \eta_1, \eta_2) \rangle &= \langle f(p_{\perp}, \eta_1) f(p_{\perp}, \eta_2) \rangle \left( 1 + 2 \frac{v_2^2}{M} - \frac{p_{\perp}^2}{M(p_{\perp}^2)} \right),
\end{align*}
\]

where the integration is performed over the considered transverse momentum bin and fulfilling the condition on the pseudorapidity separation \( 2.0 < |\eta_1 - \eta_2| < 4.8 \), also the numerator and the denominator in the above expression are averaged over events in a given multiplicity \( N_{trk} \) class. We obtain a simple expression

\[
R(\Delta \phi) = -c_1 \cos(\Delta \phi) + c_2 \cos(2\Delta \phi),
\]

where the coefficients are

\[
c_1 = \frac{(p_{\perp})_B^2 ((N) - 1)}{(p_{\perp}^2)_B N_{eff}}, \quad \frac{1}{N_{eff}} = \frac{1}{M} B
\]

and

\[
c_2 = 2((N) - 1)v_2^2.
\]

To extract the elliptic flow coefficient the average multiplicities \( \langle N \rangle \) in each \( p_{\perp} \) bin are used. These multiplicities are obtained from the integration of the \( p_{\perp} \) distribution of particles produced in proton-proton collisions at \( \sqrt{s} = 7 \) TeV [18]. For minimum bias events the transverse momentum distribution is well described using a Tsallis distribution

\[
\frac{dN}{dp_{\perp} d\eta} = C \frac{p_{\perp}}{E} \left( 1 + \frac{E - m_0}{nT} \right)^{-n}
\]

where the slope parameter \( T = 0.145 \) GeV and \( n = 6.6 \). For our estimate, we assume the same distribution in the whole pseudorapidity range, with the constant \( C \) adjusted to reproduce the total multiplicity in the interval \( |\eta| < 2.4 \). A noticeable increase of the mean transverse momentum with multiplicity is observed [19]. To take this effect into account, we assume that the slope parameter depends on the multiplicity \( N_{trk} \) in the CMS acceptance region \( \langle |\eta| < 2.4, p_{\perp} > 0.4 \) GeV \( T = (0.02 + 0.03\sqrt{N_{trk}}) \) GeV. Such a distribution, when recalculated for the ATLAS acceptance, reproduces the increase of the mean transverse momentum with multiplicity [19]. Using the parameterization \( (9) \) we calculate the mean multiplicities in different \( p_{\perp} \) bins and multiplicity classes \( N_{trk} \). The events used in the analysis have at least two particles in the chosen kinematic
The correlation function \( R(\Delta \phi) \) as function of the relative angle (5), for particle pairs with pseudorapidity separation \( 2.0 < |\Delta \eta| < 4.8 \), for different multiplicity and transverse momentum bins [1]. The solid lines represent the fit \( -c_1 \cos(\Delta \phi) + c_2 \cos(2\Delta \phi) \), the dashed lines represent the \( -c_1 \cos(\Delta \phi) \) term only.

Fig. 1. The correlation function \( R(\Delta \phi) \) as function of the relative angle (5), for particle pairs with pseudorapidity separation \( 2.0 < |\Delta \eta| < 4.8 \), for different multiplicity and transverse momentum bins [1]. The solid lines represent the fit \( -c_1 \cos(\Delta \phi) + c_2 \cos(2\Delta \phi) \), the dashed lines represent the \( -c_1 \cos(\Delta \phi) \) term only.

For small multiplicities we take the mean multiplicity \( \langle N \rangle \) only for events fulfilling the condition that \( N \geq 2 \). The correction is made using a Poisson distribution.

The expression (6) is used to fit the angular correlation function \( R(\Delta \phi) \) measured in proton-proton collisions at \( \sqrt{s} = 7 \) TeV [1]. The quality of the fit is excellent. The angular correlation has two components, the momentum conservation and the elliptic azimuthal asymmetry correlations. These effects account for all the angular structure observed in the experiment for particles emitted in different pseudorapidity regions. Some deviations are visible in the highest momentum bin, especially in the low multiplicity class. This may signal the presence of two-particle correlation not of the assumed form (6). The expansion used in deriving momentum conservation effects breaks down for large \( p_T \) and small \( N_{\text{trk}} \) \((M)\). The coefficient \( c_1 \) \((0.4-0.9)\), denoting the strength of the momentum conservation correlations, is of the order one, as expected from the formula (7).

The extracted \( c_2 \) can be used to calculate the strength of the elliptic flow in the particle azimuthal distributions

\[
v_2 = \sqrt{\frac{c_2}{2\langle N \rangle}}
\]

which is equivalent to the second order cumulant method [20]. In Fig 2 is shown the elliptic flow as function of the transverse momentum bin for the four multiplicity classes considered. The first observation is that the results for low multiplicity events differ from the others. It may indicate that the correlations in low multiplicity events are of different origin than in high multiplicity classes. For the high multiplicity events, the value of the elliptic flow in-
creases from 0.03, in the lowest, to 0.1, in the highest \( p_{\perp} \) bin. This value is too high to be interpreted as entirely due to collective effects. The strong increase with \( p_{\perp} \), and the large absolute value indicate that at least part of the effect is due to the non-flow correlations that cannot be separated. Hydrodynamic model estimates for the elliptic flow coefficient in proton-proton collisions are of the order 0.04-0.06, if viscosity effects reducing the anisotropy are taken into account [5,13].

We have followed the same procedure, using a fit with the first and second harmonics, for the PYTHIA simulated distributions [1]. The resulting elliptic flow coefficient is of the same order as found in the data. This indicates that the non-flow, jet-like correlations, included in PYTHIA have both \( \cos(\Delta \phi) \) and \( \cos(2\Delta \phi) \) components. Of course, the elliptic flow component in PYTHIA is not of a collective origin. On the other hand, the ridge structure is not present in the correlation function from the Monte Carlo generated events. Moreover, since the particle pair is separated in pseudorapidity, most of the short range correlations, coming from jets or resonance decays, are strongly reduced [21]. The remaining non-flow correlations are modifying the angular correlation function for large relative angles, as seen in the PYTHIA simulations. It indicates that the assumed form of the background \( -c_1 \cos(\Delta \phi) \) is oversimplified, and that non-flow higher harmonics are present in the observed data. The small-angle structure can be explained as resulting from the elliptic flow, but the extracted values of the \( v_2 \) coefficient depend on the form of the correlation \( R(\Delta \phi) \) in the whole range 0-\( \pi \), and contain a large contribution of non-flow origin. To estimate the value of the elliptic flow related to the appearance of the ridge-like structure, we fit a simple formula

\[
R(\Delta \phi) = b + c_2 \cos(\Delta \phi)
\]

in the range \( \Delta \phi \leq \frac{\pi}{3} \). The results for the multiplicity class \( 90 \leq N_{\text{trk}} < 110 \) is shown as the lower points in Fig. 2.

The momentum resolution of the correlation data is not sufficient to look for a hydrodynamic origin of the effect. Future data with more statistics would allow to extract a detailed behavior in the soft momentum region \( p_{\perp} < 2.0 \) GeV. The charge independence of the effect found in [1] is consistent with a collective flow origin of the correlations. The multiplicity dependence of the second harmonic coefficient in the correlation function \( R(\Delta \phi) \) \( (c_2 \propto \langle N \rangle) \) indicates that the underlying correlations \( (v_2) \) are of a non-flow origin. On the contrary, the approximate multiplicity independence of the coefficient \( c_1 \) shows that the underlying correlations decrease linearly with the size of the system, which suggests a non-flow origin of the effect.

Let us comment on the first harmonic term in the correlations function \( R(\Delta \phi) \). The formula (8) allows to estimate the value of effective multiplicity \( N_{\text{eff}} \) of the process for which the momentum conservation occurs. We find values between 10, in low \( p_{\perp} \) bins, and 80, in high momentum and high multiplicity bins. Such values are small compared to the multiplicity in the range \( |\eta| < 2.4 \), and even more so with respect to the total multiplicity in the event. This, and the fact that higher harmonics in the angular correlation function are important shows that the observed dominant correlations have a more complicated form than \( \cos(\Delta \phi) \), especially in high \( p_{\perp} \) bins.

### 3 Transverse source velocity

In this section we analyze the angular correlations assuming a different model for the dominant back to back peak. In the first collision between partons a hard momentum transfer occurs. Subsequent particle production is determined by the initial transverse momentum exchange. We use a simple estimate for the angular correlation resulting from such a mechanism. Particles separated in pseudorapidity originate predominantly from initial partons with opposite transverse momenta. Let us assume that in the rest frame of the source connected to a given fragmenting parton, massless particles are emitted isotropically. If the source is moving with velocity \( \beta_\perp \) in the direction \( \phi = \pi \) the final angular distribution takes the form

\[
\frac{dN}{d\phi} \propto \frac{\sqrt{1 - \beta_\perp^2}}{1 + \beta_\perp \cos(\phi)}.
\]

The source from which the other particle in the pair originates, moves on the average with velocity \( -\beta_\perp \) (Fig. 3). If the transverse momentum and pseudorapidity dependence factorizes from the angular one, we obtain for the angular correlation of the pair

\[
R(\Delta \phi) = (\langle N \rangle - 1) \left( \frac{2\sqrt{1 - \beta_\perp^2}}{2 - \beta_\perp^2 + \beta_\perp \cos(\Delta \phi)} - 1 \right) \tag{13}
\]

The above correlation function is peaked for back to back emission and contains the first, second, and higher harmonic components. In this respect it is different from the

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**Fig. 3.** Schematic picture of the source created in a proton-proton collision. At each rapidity it has a transverse velocity. The source is azimuthally deformed and its collective expansion would lead to a nonzero elliptic flow.
simple $-c_1 \cos(\Delta \phi)$ expression resulting from the momentum conservation effects.

If the source is emitting particles in an azimuthally asymmetric way in its rest frame

$$1 + 2v_2 \cos(\phi),$$

(14)

its distribution in the center of mass frame is

$$f_{\beta}(\phi) \propto \sqrt{1 - \beta^2_\perp} \frac{(1 + 2v_2 ((\beta_\perp + \cos(\phi))^2 - (1 - \beta^2_\perp)\sin(\phi))}{1 + \beta_\perp \cos(\phi)}. \quad (15)$$

The correlation function in the relative angle of the pair is obtained integrating over the angle of one of the particles in the pair

$$R(\Delta \phi) = (\langle N \rangle - 1) \left( \int \frac{d\phi}{2\pi} f_{\beta}(\phi)f_{-\beta}(\phi + \Delta \phi) - 1 \right). \quad (16)$$

The elliptic asymmetry in the particle distribution has the same orientation as the momentum exchange (source velocity). This means, that both the hard scattering, that defines the reaction plane, and the generated elliptic flow plane have the same orientation. If the generation of the elliptic flow is a collective expansion from fluctuating initial conditions [10–13], the orientation of the reaction plane from the hard scattering and from the elliptic flow is different. In that case, the angular distribution in the rest frame would be $1 + 2v_2 \cos(\phi - \Delta \Psi)$. An average should be taken over the difference of the two angles $\Delta \Psi$.

The results are similar, with slightly larger $v_2$, as in the scenario using the same orientation of the boost and of the elliptic flow (14).

The formula (16) is fitted to the measured correlation function $R(\Delta \phi)$, adjusting two parameters, the elliptic flow coefficient $v_2$, and the boost velocity $\beta_\perp$. The quality of the fit is very good (Fig. 4). In Fig. 5 is shown the resulting elliptic flow coefficient $v_2$. It is smaller than in the scenario with momentum conservation effects, studied in the previous section. The extracted elliptic flow increases with the transverse momentum $p_\perp$ and depends weakly on the multiplicity. In the $p_\perp$ and multiplicity bins where the ridge is observed, no significant rise of $v_2$ is seen. The ridge appears because the correlations from the transverse boost of the source are smaller. In high multiplicity samples the boost velocity is much smaller than in the two small multiplicity classes (Fig. 6). If such transverse boost velocity is present in the particle emitting source, it could mimic the transverse flow in the $p_\perp$ spectra of particles.

Our model of background back to back correlations from the transverse boost is very simple. We take a single average boost velocity and use massless particles. Going beyond these approximations, would require a specific
model of the particle emission. The qualitative form of the back to back correlation would be similar, but the extracted elliptic flow coefficient could depend on the form of the dominant angular correlations.

4 Conclusions

We show that the angular correlations between particles emitted in proton-proton collisions at $\sqrt{s} = 7$ TeV can be decomposed into two components. The dominant, back to back angular correlations have a kinematic origin. Modeling them with momentum conservation effects results in a first harmonic component in the angular distribution. An- other scenario assumes a transverse boost of the source, leading to back to back correlations as well, but of a different form. The remaining correlations, including the small-angle ridge structure, are modeled using an elliptic flow contribution. The corresponding elliptic flow coefficient $v_2$ is estimated for the first time in proton-proton collisions.

The strength of the elliptic flow depends on the assumed model for the dominant background, that has to be subtracted. In the momentum conservation scenario the elliptic flow coefficient is $0.07-0.1$ in the bins where the ridge is observed, whereas in the transverse boost estimate it is $0.06-0.08$. The most conservative estimate is obtained using a fit of the second harmonic only in the small-angle region of the ridge (lower points in Fig. 2), with the result $v_2 = 0.04-0.06$. There are several arguments showing that the measured elliptic flow contains a collective component. It is approximately multiplicity independent. The associated structure extends over a long range in pseudorapidity. The two particles used to define the second cumulant are separated in pseudorapidity, reducing non-flow effects. It is tempting to interpret the observed azimuthally asymmetric flow as a result of some strong rescattering or a hydrodynamic expansion. It remains as a challenge to provide an alternative microscopic explanation, as the available Monte Carlo generators do not reproduce the observed structures. For the event generators, it would require that such azimuthal correlations appear early, between most of the particles in the event. We note that there is no additional same-side, ridge-like structure in the correlation function. All the observed angular correlations are perfectly well accounted for using a combination of kinematic back to back correlations and of the elliptic flow.

Finally, we stress again the importance of the possible first observation of the elliptic flow in elementary collisions. It would mean that the short-lived multiparticle system created in the collision is very strongly interacting and some degree of collectivity appears.

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