Configuration mixing in strange tetraquarks $Z_{cs}$

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ABSTRACT

The BESIII Collaboration has observed a candidate for a $c\bar{c}s\bar{u}$ tetraquark $Z_{cs}$ at $(3982.5^{+1.8}_{-2.6} \pm 2.1) \text{ MeV}$ and width $(12.8^{+5.3}_{-4.4} \pm 3.0) \text{ MeV}$, while the LHCb Collaboration has observed a $Z_{cs}$ candidate in the $J/\psi K^-$ channel with mass of $(4003 \pm 6^{+14}_{-14}) \text{ MeV}$ and width $(131 \pm 15 \pm 26) \text{ MeV}$. In this note we examine the possibility that these two states are distinct eigenstates of a mixing process similar to that which gives rise to two axial-vector mesons labeled by the Particle Data Group \cite{PDG} $K_1(1270)$ and $K_1(1400)$. The main point is that on top of a $\bar{c}c$ pair, the $Z_{cs}$ states have the same light quark content as the $K_1$-s. In the compact tetraquark picture this implies several additional states, analogous to members of the $K_1$ nonet. These states have not yet been observed, nor are they required in the molecular approach. Thus experimental discovery or exclusion of these extra states will be a critical test for competing models of exotic mesons with hidden charm.

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I INTRODUCTION

The discovery of mesons with minimal quark configuration $c\bar{c}q\bar{q}'$, where $q$ and $q'$ are light quarks $u, d, s$, was pioneered by the observation \cite{X3872} of the $X(3872)$. The quark configuration of this state includes $c\bar{c}u\bar{u}$, $c\bar{c}$, and some lesser admixtures, while in terms of mesons it is mainly $D^0\bar{D}^{*0}$ charge conjugate (c.c.) The spin, parity, and charge-conjugation eigenvalue of this state is $J^{PC} = 1^{++}$ \cite{PDG}.

Over the course of the nearly twenty subsequent years, partners of this state have emerged. Within the compact tetraquark model this implies the existence of a flavor $SU(3)$ nonet whose neutral members have $J^{PC} = 1^{++}$, as well as candidates for membership in the $c\bar{c}(4020)$.
in a $1^+$ nonet $[^48]$. Candidates for the strange members of these nonets have been identified. The BESIII Collaboration has observed a candidate for a $c\bar{c}s\bar{s}u$ tetraquark $Z_{cs}$ at $(3982.5_{-2.6}^{+1.8} \pm 2.1)\text{ MeV}$ and width $(12.8_{-4.4}^{+5.3} \pm 3.0)\text{ MeV}$, while the LHCb Collaboration has observed a $Z_{cs}$ candidate in the $J/\psi K^-$ channel with mass of $(4003 \pm 6_{-14}^{+4})\text{ MeV}$ and width $(131 \pm 15 \pm 26)\text{ MeV}$.

A number of interpretations of the $Z_{cs}$ candidates have been put forth, reviewed briefly in [14]. Here we propose that within the compact tetraquark model these two states are likely to be distinct eigenstates of a mixing process similar to that which gives rise to two axial-vector strange mesons labeled by the Particle Group $K_{1}(1270)$ and $K_{1}(1400)$. We discuss the system of these two kaons in Sec. II, apply similar methods to the $Z_{cs}$ states in Sec. III, and conclude in Sec. IV.

II MIXING OF STRANGE KAONS

We label quark-antiquark mesons with the notation $^{2S+1}L_J$, where $S = 0, 1$ is the total quark spin, $L = S, P, D, \ldots$ stands for orbital angular momentum $0, 1, 2, \ldots$, and $J = 0, 1, 2, \ldots$ denotes total angular momentum. The lowest-level quark-antiquark mesons consist of

$$S\text{-waves: } J^{PC} = 0^{-+} = {}^1S_0, \ 1^{--} = {}^3S_1;$$

$$P\text{-waves: } J^{PC} = 0^{++} = {}^3P_0, \ 1^{++} = {}^3P_1, \ 1^{+-} = {}^1P_1, \ 2^{++} = {}^3P_2.$$ (2)

Here $P = \pm 1$ stands for parity eigenvalue, while $C = \pm 1$ denotes eigenvalue under charge conjugation, which is only a good quantum number for a $qq'$ pair if $q' = q$. More generally we shall use the basis labels $(A, B)$ for $(^{3}P_1, {^1}P_1)$, respectively.

The assignment of some of the observed light mesons to the above quantum numbers is given in Table 15.2 of Ref. [15]. The two lightest nonets of $J^P = 1^+$ mesons are also listed in Table I.

| $^{2S+1}L_J$ | $J^{PC}$ | $I = 1$ | $I = 1/2$ | $I = 0$ | $I = 0$ |
|-------------|----------|---------|-----------|---------|---------|
| $^{3}P_1$   | $1^{++}$ | $a_1(1260)$ | $K_{1A}^\dagger$ | $f_1(1420)$ | $f_1(1285)$ |
| $^{1}P_1$   | $1^{+-}$ | $b_1(1235)$ | $K_{1B}^\dagger$ | $h_1(1415)$ | $h_1(1170)$ |

$^\dagger K_{1}(1270)$ and $K_{1}(1400)$ are mixtures of $K_{1A,B}$ (see text).

Nonstrange states, in the limit of isospin conservation, possess a definite $G$-parity, defined as

$$G \equiv C \exp(i\pi I_2)$$

which reduces to $G = C(-1)^I$ for the neutral member of an isospin multiplet with isospin $I$. However, because the strange quark is heavier than the up and down quarks, the corresponding symmetry based on the $U$-spin and $V$-spin [16] subgroups of $SU(3)_F$ does not

$^8$Strictly speaking, $1^{++}, 1^{+-}$ denotes the neutral member of a nonet in which the light quark-antiquark pair is in a $^{3}P_1, {^1}P_1$ state. We shall label these basis states $(A, B)$.
3982.5 and 4003 MeV are the analogues of $K^0_3$.

The light-quark degrees of freedom are linear combinations of a $c\bar{c}$ pair.

This scheme is close to a superposition of Solutions 1 and 2 of Ref. [5], in which

$\phi$ mixing between mass eigenstates and basis states

exist (though an analogue of $G$-parity can be formally defined in the $SU(3)_F$ symmetry limit [17]).

Evidence for two resonant states with $J^P = 1^+$ (axial vector kaons) in the $K\pi\pi$ spectrum was first noted in Ref. [18]. The mass eigenstates arise from mixing between $K_{1A} = 3P_1$ and $K_{1B} = 1P_1$, for instance as a result of sharing $K^*$(892) and $K\rho$ decay modes [19, 20]. They may be written as [19] (Ref. [21] uses a different phase convention)

$$|K_1(1270)\rangle = |K_{1A}\rangle \cos \phi - |K_{1B}\rangle \sin \phi ,$$

$$|K_1(1400)\rangle = |K_{1A}\rangle \sin \phi + |K_{1B}\rangle \cos \phi .$$

Both $K_1(1270)$ and $K_1(1400)$ can couple to $K^*(892)\pi$ and $K\rho$, in $S$ and $D$ waves. Ref. [21] quotes parameters of Ref. [22] for these decays as shown in Table II while current PDG values including those obtained from nondiffractive axial kaon production are summarized in Table III. The analysis of Ref. [19] favored the range $10^0 < \phi < 35^0$. For that range, the mixing between mass eigenstates and basis states $|K_{1A,B}\rangle$ is such that $K_1(1270)$ couples mainly to $K\rho$ in $S$ wave, with suppressed $S$ wave coupling to $K^*\pi$ despite greater phase space, while $K_1(1400)$ couples mainly to $K^*\pi$ in $S$ wave. This pattern is qualitatively similar to that in an updated version [20, 21]. Mixing of states with similar quantum numbers such that different eigenstates have different decay modes is familiar from many other examples in hadron spectroscopy [23–25].

Table II: Masses and decay properties of axial vector kaons as measured by the ACCMOR Collaboration [22] in diffractive production and quoted in Ref. [21]. Small $D$-wave contribution to $K_1(1270) \to K\rho$ neglected.

| $K_1$ | $M$, MeV | $\Gamma_{K_1}$, MeV | $\mathcal{B}(K^*\pi)_S$ | $\mathcal{B}(K^*\pi)_D$ | $\mathcal{B}(K\rho)_S$ |
|-------|----------|-------------------|------------------|-----------------|-----------------|
| $K_1(1270)$ | 1270 ± 7 | 90 ± 8 | 0.13 ± 0.03 | 0.07 ± 0.006 | 0.39 ± 0.04 |
| $K_1(1400)$ | 1410 ± 25 | 165 ± 35 | 0.87 ± 0.05 | 0.03 ± 0.005 | 0.05 ± 0.04 |

Table III: Masses and decay properties of axial vector kaons as summarized in Ref. [1], including nondiffractive production. Small $D$-wave contribution to $K_1(1270) \to K\rho$ neglected.

| $K_1$ | $M$, MeV | $\Gamma_{K_1}$, MeV | $\mathcal{B}(K^*\pi)_S$ | $\mathcal{B}(K^*\pi)_D$ | $\mathcal{B}(K\rho)_S$ |
|-------|----------|-------------------|------------------|-----------------|-----------------|
| $K_1(1270)$ | 1253 ± 7 | 90 ± 20 | 0.08 ± 0.025 † | 0.08 ± 0.025 † | 0.42 ± 0.06 |
| $K_1(1400)$ | 1403 ± 7 | 174 ± 13 | 0.90 ± 0.06 | 0.04 ± 0.01 | 0.03 ± 0.03 |

†Neglecting error in $\mathcal{B}(K^*\pi)_D/\mathcal{B}(K^*\pi)_S = (1.0 \pm 0.7)$%

III  MIXING OF $Z_{cs}$ STATES

Within the compact tetraquark model it is natural to expect that the $Z_{cs}$ candidates at 3982.5 and 4003 MeV are the analogues of $K_1(1270)$ and $K_1(1400)$ with an extra $c\bar{c}$ pair. The light-quark degrees of freedom are linear combinations of a $^3P_1$ and a $^1P_1$ $q\bar{q}'$ pair. This scheme is close to a superposition of Solutions 1 and 2 of Ref. [5], in which $Z_{cs}(3982.5)$
and $Z_{cs}(4003)$ are distinct states of $^{2S+1}P_J$: $^{3,1}P_1$ for Solution 1 and $^{1,3}P_1$ for Solution 2. In our solution, it is the the unmixed strange $^3P_1$ state which should have a mass $M_A$ which is the average of the nonstrange $X(3872)$ and the $c\bar{c}s\bar{s}$ state called $X(4140)$:

$$M_A = (4146.8 + 3871.65)/2 \text{ MeV} = 4009 \text{ MeV},$$

where we have used the latest PDG masses and the identification by Lebed and Polosa [8] of the $X(4140)$ as a $1^{++}$ $c\bar{c}s\bar{s}$ state. In addition to the $X(3872)$, assumed isosinglet, there should exist a nearby nonstrange isotriplet, whose charged members await detection. Strictly speaking, if we regard the $X(3872)$ as mainly $c\bar{c}u\bar{u}$, thanks to its proximity to the $D^0\bar{D}^{*0} + \text{c.c.}$ threshold, there should also exist a state which is mainly $c\bar{c}d\bar{d}$, presumably not far from the $D^+\bar{D}^{*-}$ threshold, so the estimate [4] is uncertain by at least several MeV. One should stress that while the existence of these additional states is a must in the compact tetraquark picture, in the molecular approach they are not necessary and in some cases downright unlikely.

In particular, $X(3872)$ mass is extremely close, within 0.1 MeV of the $D^0\bar{D}^{*0}$ threshold [26]. One pion exchange likely plays a significant role in $D^0\bar{D}^{*0}$ binding, converting a $D^0\bar{D}^{*0}$ system into a $D^{*0}D^0$, which of course has the same mass. On the other hand, in the putative charged partner of $X(3872)$ one pion exchange would have to convert a $D^0D^{*+}$, with threshold at 3875.1 MeV to a $D^{*0}D^+$, with threshold at 3876.5 MeV, a shift of 1.4 MeV, much too large in comparison with the dynamics apparently responsible for $X(3872)$ binding.

In view of these conflicting expectations from the compact tetraquark and molecular pictures, experimental search for the extra states is of very high priority. It can discriminate between these competing models of exotic mesons with hidden charm. A discovery would provide strong support for the compact tetraquark picture, while definite experimental exclusion would be a very strong argument in favor of molecular models.

In what follows we shall refer to nonets by the light-quark spin-parity $(2S+1)L_J$ of their unmixed members, so we will speak of the $^3P_1$ and $^1P_1$ nonets. Regarding decay modes of the $^3P_1$ nonet, those of the $X(3872)$ and $X(4140)$ are not affected by $K_1$-like mixing, because the light quark content of these states is $\bar{u}u$ and $\bar{s}s$, respectively. On the other hand, we propose that as result of the mixing of the $^1P_1$ and $^3P_1$ multiplets $Z_{cs}(3982.5)$ is the eigenstate of $c\bar{c}s\bar{s}$ mixing which couples to $D_s^-D^{*0} + D_s^{*-}D^0$, while $Z_{cs}(4003)$ is the eigenstate which couples to $J/\psi K^-$.

Information about the $^1P_1$ nonet is more sparse. The only firm candidate is the nonstrange isotriplet $Z_c(3900)$, whose latest mass listed by the PDG is $(3887.1\pm 2.6) \text{ MeV}$. The $c\bar{c}s\bar{s}$ candidate with $J^{PC} = 1^{+-}$ is unknown at present. That prevents us from estimating the singly strange member of the nonet with mass $M_B$ from the data. If all members of the $^1P_1$ nonet are $M(Z_c(3900)) - M(X(3872))) \sim 15 - 20 \text{ MeV}$ above those of the $^3P_1$ nonet, we predict a $C = -1$ $c\bar{c}s\bar{s}$ candidate around 4160 MeV. (See also [4][8].) It should be able to decay to $D_s^-D^{*0} + D_s^{*-}D^0$, whose threshold is 4080 MeV.

## IV CONCLUSIONS

We have proposed that the two strange tetraquarks recently observed by the BESIII and LHCb Collaborations, $Z_{cs}(3982.5)$ and $Z_{cs}(4003)$, are mass eigenstates in which the singly strange light quark degrees of freedom with $^{2S+1}L_J = ^{3,1}P_1$ undergo mixing such that the lighter state couples predominantly to $D_s^-D^{*0} + D_s^{*-}D^0$, while the heavier one couples
mainly to the mode in which it was observed, $J/\psi K^+$. This mixing is similar to that giving rise to the axial vector kaons $K_1(1270)$ and $K_1(1400)$, which has been known for more than fifty years. The scheme envisions both states as having $J^P = 1^+$. It predicts a $^1P_1$ state of negative $C$ around 4160 MeV, decaying to $D_s^- D_s^+ + D_s^- D_s^+$ The compact tetraquark scheme discussed here involves complete nonets; in a molecular picture key nonet members (such as the charged partners of the $X(3872)$) may be absent.

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