Research article

Linear and nonlinear optical properties in spherical quantum dots: Modified Möbius squared potential

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A B S T R A C T

The optical properties of quantum dots (QDs) in modified Möbius squared (MMS) potential are studied. To obtain the energy expressions and the wave functions, we solved the Schrödinger equation by using Nikiforov-Uvarov (NU) method. We investigated the linear, third-order nonlinear and total absorption coefficients (AC) and refractive index changes (RIC) using the density matrix. The numerical results show that the structure parameters and optical intensity have a strong influence on AC and RIC.

1. Introduction

In the past years, the study of low dimensional materials (quantum well, quantum rings, quantum wires and quantum dots) has gained popularity [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Among them, QDs offer a wide research perspective due to confinement in three dimensions. They have shown unique properties which makes them useful for development of novel devices [11, 12]. Therefore, in the field of nonlinear optics, the study of QDs has become very interesting [13, 14, 15].

The electronic and optical properties of QDs have been studied in some potentials such as Rosen-Morse [16], Inversely quadratic Hellmann plus Kratzer [17], Manning-Rosen [18], Woods-Saxon [19, 20], Inversely quadratic Hellmann [21] and many other potentials. Similar studies have discussed wave equations with different potentials [22, 23, 24, 25, 26, 27, 28]. But, optical properties of QDs with MMS potential have not been reported. Consequently, this study becomes necessary.

The aim of this article is to calculate the AC and RIC of spherical QDs with the MMS potential.

2. Theory

In the framework of effective mass approximation, the Schrödinger equation in spherical coordinate for an electron is

\[ -\frac{\hbar^2}{2\mu} \left( \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right) \psi_{nlm}(r, \theta, \phi) + V(r)\psi_{nlm}(r, \theta, \phi) = E_{nl}\psi_{nlm}(r, \theta, \phi) \]

(1)

where \( \mu \) is the effective mass of the electron, \( \hbar \) is the reduced Planck’s constant, \( n \) and \( l \) are the principal and orbital quantum numbers, \( V(r) \) is the potential and \( \psi_{nlm}(r, \theta, \phi) \) is the wave function expressed as,

\[ \psi_{nlm}(r, \theta, \phi) = \frac{R_n(r)}{r} Y_{lm}^m(\theta, \phi). \]

(2)

In this work, the MMS potential is the confinement potential. This potential is expressed by [29]

\[ V(r) = -V_a \left( \frac{A + Be^{-2ar}}{1 - e^{-2ar}} \right)^2, \]

(3)

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where \( V_o \) is the potential depth, \( A \) and \( B \) are constant coefficients, and \( \alpha \) is an adjustable screening parameter, respectively.

Substituting Eqs. (3) and (2) into Eq. (1) gives the radial Schrödinger as

\[
\frac{d^2 R_m(r)}{dr^2} + \frac{2\mu}{\hbar^2} \left[ E_m - \frac{l(l+1)\hbar^2}{2\mu r^2} + V_o \left( \frac{A + Be^{-2\alpha r}}{1 - e^{-2\alpha r}} \right) \right] R_m(r) = 0. \tag{4}
\]

The centrifugal term \( 1/r^2 \) in Eq. (4) is replaced by Greene-Aldrich-type approximation \([30]\)

\[
\frac{1}{r^2} \approx \frac{4\alpha^2 e^{-2\alpha r}}{(1 - e^{-2\alpha r})^2}. \tag{5}
\]

Eq. (5) is similar to the approximation by the authors \([31, 32, 33, 34]\).

Putting Eq. (5) into Eq. (4) with the transformation \( s = e^{-\alpha r} \), we have

\[
\frac{d^2 R_m(s)}{ds^2} + \frac{(1-s)}{s(1-s)} \frac{dR_m(s)}{ds} - \frac{C_1s^2 + C_2s - C_3}{s^2(1-s)^2} R_m(s) = 0, \tag{6}
\]

where

\[
\begin{align*}
C_1 &= \sqrt{-\frac{\mu}{2\alpha^2 \hbar^2}} (E_{nl} + V_0 B^2), \\
C_2 &= \frac{\mu}{\alpha^2 \hbar^2} (ABV_o - E_{nl}) - l(l+1), \\
C_3 &= -\frac{\mu(E_{nl} + V_0 A^2)}{2\alpha^2 \hbar^2}.
\end{align*}
\]

We solve Eq. (6) by using the NU method \([35, 36, 37, 38, 39, 40]\). Given the equation:

\[
\frac{d^2\psi(s)}{ds^2} + \frac{\alpha_1 - \alpha_2 \alpha_5}{s(1-s)} \frac{d\psi(s)}{ds} + \frac{-\xi_1 s^2 + \xi_2 s - \xi_3}{s^2(1-s)^2} \psi(s) = 0. \tag{8}
\]

Using the proposed method in Ref. [36], we have solutions of Eq. (8) as

\[
\psi(s) = N_{nl} s^{\alpha_1}(1 - \alpha_3 s)^{-\alpha_1/(\alpha_3/\alpha_1)} P_n^{(\alpha_1 - 1, \alpha_3 - 1)}(1 - 2\alpha_3 s), \tag{9}
\]

where \( P_n^{(\alpha_1, \alpha_3)} \) are Jacobi polynomials, and

\[
(\alpha_2 - \alpha_3)n + \alpha_2 n^2 - (2n + 1)\alpha_5 + (2n + 1)(\sqrt{\alpha_5} + \alpha_3 \sqrt{\alpha_5}) + \alpha_7 + 2\alpha_2 \alpha_8 + 2 \sqrt{\alpha_5} \alpha_9 = 0, \tag{10}
\]

with

\[
\begin{align*}
\alpha_4 &= \frac{1}{2}(1 - \alpha_1), \\
\alpha_5 &= \frac{1}{2}(\alpha_2 - 2\alpha_1), \\
\alpha_6 &= \alpha_2^2 + \xi_1, \\
\alpha_7 &= 2\alpha_3 \alpha_5 - \xi_2, \\
\alpha_8 &= \alpha_2^2 + \xi_3, \alpha_9 &= \alpha_1 \alpha_7 + \alpha_2^2 \alpha_3 + \alpha_7, \alpha_{10} &= \alpha_1 + 2\alpha_3 + 2 \sqrt{\alpha_5}, \\
\alpha_{11} &= \alpha_1 - 2\alpha_3 + 2 \left( \sqrt{\alpha_5} \alpha_3 + \alpha_3 \sqrt{\alpha_5} \right), \\
\alpha_{12} &= \alpha_2 + \sqrt{\alpha_5} \alpha_3, \alpha_{13} &= \alpha_5 - \left( \sqrt{\alpha_5} + \alpha_3 \sqrt{\alpha_5} \right).
\end{align*} \tag{11}
\]

Now, comparing Eq. (8) with Eq. (6) we deduce

\[
\begin{align*}
\alpha_1 &= 1, \quad \alpha_2 = 1, \quad \alpha_3 = 1, \quad \xi_1 = \xi_3^2, \quad \xi_2 = C_2, \quad \xi_3 = C_3. \tag{12}
\end{align*}
\]

We also obtain the constants from Eq. (11) as

\[
\begin{align*}
\alpha_4 &= 0, \quad \alpha_5 = -\frac{1}{2}, \quad \alpha_6 = C_1^2 + \frac{1}{4} \alpha_7 = -C_2, \quad \alpha_8 = C_3, \\
\alpha_9 &= -\frac{\mu V_0 (A + B)^2}{2\alpha^2 \hbar^2} + l(l+1) + \frac{1}{4}, \quad \alpha_{10} = 1 + 2 \sqrt{C_3}, \\
\alpha_{11} &= 2 \left( 1 + \sqrt{C_3} \right) + \sqrt{(2l+1)^2 - \frac{2\mu V_0 (A + B)^2}{a^2 \hbar^2}}, \\
\alpha_{12} &= \sqrt{C_3} \alpha_{13} = -\frac{1}{2} \left( 1 + \sqrt{(2l+1)^2 - \frac{2\mu V_0 (A + B)^2}{a^2 \hbar^2}} \right) - \sqrt{C_3}. \tag{13}
\end{align*}
\]

Using Eqs. (13), (12) and (10), we derive the energy as

\[
E_{nl} = -V_o A^2 - \frac{2\alpha^2 \hbar^2}{\mu} \left[ \frac{n(n+1)}{2} + \frac{l(l+1)}{2} + \left( n + \frac{1}{2} \right) \sqrt{(2l+1)^2 - \frac{2\mu V_0 (A + B)^2}{a^2 \hbar^2}} - \frac{\mu V_0 A (A + B)}{a^2 \hbar^2} \right]^2. \tag{14}
\]

Eq. (14) is similar to Eq. (22) of Ref. [29], if we make \( k = l \).

From Eq. (9), we have

\[
R_{nl}(s) = N_{nl} s^{\alpha_1}(1 - s)^{\alpha_2/(\alpha_3)} P_n^{(\alpha_1 - 1, \alpha_3 - 1)}(1 - 2s), \tag{15}
\]
with $s = e^{-2ar}$,

$$R_n(r) = N_m e^{-2ar} \sqrt{C_1} \left(1 - e^{-2ar}\right)^{1/2} P_n^{2(1+1)} \left(1 - 2e^{-2ar}\right),$$  \hfill (16)

where,

$$x_i = \sqrt{\left(2l + 1\right)^2 - \frac{2\mu V_s(A + B)^2}{\alpha^2 \hbar^2}}.$$  \hfill (17)

The $N_m$ normalization constant can be obtained using [3]

$$\int \psi_{\text{in}}(r, \theta, \phi) \psi_{\text{in}}^*(r, \theta, \phi) d\Omega = 1,$$  \hfill (18)

where $d\Omega = r^2 dr \sin \theta d\theta d\phi$.

### 3. Optical absorption coefficients and refractive index changes

We apply the density matrix to derive the AC and RIC [16, 18]. The system under study is excited by an electromagnetic field of frequency $\omega$ such as

$$E(t) = E_0 \cos(\omega t) = E e^{i\omega t} + E^* e^{-i\omega t}.$$  \hfill (19)

We obtain the time evolution of the matrix elements of one-electron density operator $\rho$ as [3, 16, 18, 41]

$$\frac{d\rho}{dt} = \frac{i}{\hbar} \left[ H_0 - e \varepsilon \left(\omega(t), \rho\right) \right] - \Gamma \left(\rho - \rho^{(0)}\right).$$  \hfill (20)

where $H_0$ is the system Hamiltonian without electromagnetic field $E(t)$, $e$ is the electronic charge. The sign[], is the quantum mechanical commutator, $\rho^{(0)}$ is the unperturbed density matrix operator and $\Gamma$ is a damping operator due to the collision processes [2]. We assume that $\Gamma$ is a diagonal matrix whose elements are equal to the inverse of the relaxation time $\tau$ [16, 18]. Eq. (20) can be solved using an iterative method [16, 18, 41]:

$$\rho(t) = \sum_{n=0}^\infty \rho^{(n)}(t),$$  \hfill (21)

with

$$\frac{d\rho^{(n+1)}}{dt} = \frac{i}{\hbar} \left[ [H_0, \rho^{(n+1)}]_{ij} - i\hbar \Gamma_{ij}^{(n+1)} \right] - \frac{i}{\hbar} \left[ e\varepsilon_n, \rho^{(n)} \right]_{ij} E(t).$$  \hfill (22)

The electronic polarization $P(t)$ due to the electric field is given as [41]

$$P(t) = \varepsilon_0 \chi(\omega) \varepsilon e^{i\omega t} + \varepsilon_0 \chi(-\omega) \varepsilon^* e^{-i\omega t} = \left\{ \frac{1}{V} \right\} \text{Tr}(\rho M),$$  \hfill (23)

where $M$ is the dipole operator, $\chi(\omega)$ is the susceptibility, $V$ is the volume, $\varepsilon_0$ is the permittivity of vacuum and $\text{Tr}$ (trace) denotes the summation over the matrix diagonal elements.

The linear $\chi(1)$ and the third-order nonlinear $\chi(3)$ susceptibility coefficients are derived from Eqs. (22) and (23) as [21]

$$\varepsilon_0 \chi(1)(\omega) = \frac{\sigma_e |M_{21}|^2}{E_{21} - \hbar \omega - i\hbar \Gamma_{12}}.$$  \hfill (24)

and

$$\varepsilon_0 \chi(3)(\omega) = -\frac{\sigma_e |M_{21}|^2 |E|^2}{E_{21} - \hbar \omega - i\hbar \Gamma_{12}} \left[ \frac{4 |M_{21}|^2}{(E_{21} - \hbar \omega)^2 + (\hbar \Gamma_{12})^2} + \frac{(M_{22} - M_{11})^2}{(E_{22} - \hbar \omega - i\hbar \Gamma_{12})^2} \right].$$  \hfill (25)

where $M_{ij} = \langle \psi_{n', m'} | e\varepsilon | \psi_{n, m} \rangle$ is the matrix elements of the electric dipole moment, $E_{ij} = E_i - E_j$ is the energy difference between two states, $\hbar \omega$ is the photon energy and $\sigma_e$ is the carrier density. The matrix elements of electric dipole moment can be known using [3, 21]

$$M_{ij} = e \int \psi_{n', m'}^* (r, \theta, \phi) \psi_{n, m} (r, \theta, \phi) r^2 dr \sin \theta d\theta d\phi,$$  \hfill (26)

where $z = r \cos \theta$ and the wave function gives

$$\psi_{\text{in}}(r, \theta, \phi) = N_m e^{-2ar} \sqrt{C_1} r^{-1} \left(1 - e^{-2ar}\right)^{1/2} P_n^{2(1+1)} \left(1 - 2e^{-2ar}\right) Y_m^m(\theta, \phi).$$  \hfill (27)

For spherical symmetric systems, $M_{22} = 0 \Rightarrow M_{11} = M_{22} = 0$. Therefore, Eq. (25) becomes [3, 21]

$$\varepsilon_0 \chi(3)(\omega) = -\frac{\sigma_e |M_{21}|^2 |E|^2}{E_{21} - \hbar \omega - i\hbar \Gamma_{12}} \left[ \frac{1}{(E_{21} - \hbar \omega)^2 + (\hbar \Gamma_{12})^2} \right].$$  \hfill (28)

The susceptibility $\chi(\omega)$ determines the AC $\alpha(\omega)$ via the following expression [16, 18, 41]:

$$\alpha(\omega) = \alpha \sqrt{\frac{\hbar \Gamma_{12}}{x}} \Im \left[ \varepsilon_0 \chi(\omega) \right],$$  \hfill (29)
where the permittivity is $\varepsilon = n_r^2 \varepsilon_0$ with $n_r$ as the refractive index. The linear and third-order nonlinear AC is expressed as [43, 44]

$$a^{(1)}(\omega, I) = \omega \sqrt{\frac{\mu_0}{\varepsilon}} \frac{\sigma_i \hbar \Gamma_{12} |M_{21}|^2}{(E_{21} - \hbar \omega)^2 + (\hbar \Gamma_{12})^2},$$

(30)

and

$$a^{(3)}(\omega, I) = -\omega \sqrt{\frac{\mu_0}{\varepsilon}} \frac{\sigma_i \hbar \Gamma_{12} |M_{21}|^2}{(E_{21} - \hbar \omega)^2 - (\hbar \Gamma_{12})^2} \times$$

$$\times \left\{ 4 |M_{21}|^4 - \left[ |M_{22} - M_{11}|^2 \left[ 3E_{21}^2 - 4E_{21}\hbar \omega + \hbar^2 (\omega^2 - \Gamma_{12}^2) \right] \right] \right\}.$$

(31)

For $M_{ii} = 0$, Eq. (31) reduces to [3]

$$a^{(3)}(\omega, I) = -\omega \sqrt{\frac{\mu_0}{\varepsilon}} \frac{\sigma_i \hbar \Gamma_{12} |M_{21}|^4}{(E_{21} - \hbar \omega)^2 + 2(\hbar \Gamma_{12})^2}.$$

(32)

The total AC gives [45]

$$a(\omega, I) = a^{(1)}(\omega) + a^{(3)}(\omega, I).$$

(33)

The refractive index has a relationship with the susceptibility [41]

$$\frac{\Delta n(\omega)}{n_r} = \text{Re} \left( \frac{\chi(\omega)}{2n_r^2} \right).$$

(34)

Considering Eqs. (24)–(25) and (34), the linear and third-order nonlinear RIC gives [41, 42]

$$\frac{\Delta n^{(1)}(\omega)}{n_r} = \frac{\sigma_i |M_{21}|^2}{2n_r^2 \varepsilon_0} \frac{E_{21} - \hbar \omega}{(E_{21} - l\hbar \omega)^2 + (\hbar \Gamma_{12})^2},$$

(35)

and

$$\frac{\Delta n^{(3)}(\omega)}{n_r} = -\frac{\sigma_i |M_{21}|^4}{4n_r^2 \varepsilon_0} \frac{\Gamma_{12} \mu_0 e I}{[(E_{21} - \hbar \omega)^2 + (\hbar \Gamma_{12})^2] \times}$$

$$\times \left\{ 4(E_{21} - \hbar \omega) |M_{21}|^2 - \left( \frac{M_{22} - M_{11}}{(E_{21})^2 + (\hbar \Gamma_{12})^2} \right)^2 \times$$

$$\left[ E_{21} - \hbar \omega(E_{21} - \hbar \omega - (\hbar \Gamma_{12})^2) - (\hbar \Gamma_{12})^2 (2E_{21} - \hbar \omega) \right] \right\}.$$  

(36)

For $M_{ii} = 0 \Rightarrow M_{11} = M_{22} = 0$, Eq. (36) is

$$\Delta n^{(3)}(\omega) = -\frac{\mu_0 e I}{n_r^2 \varepsilon_0} \frac{\sigma_i |M_{21}|^4}{(E_{21} - \hbar \omega)^2 + (\hbar \Gamma_{12})^2}.$$  

(37)

where $\mu_0$ is the vacuum permeability, $c$ is the speed of light and $I = 2\varepsilon_0 n_r e |E|^2$ is the optical intensity. Eqs. (35) and (37) gives the total RIC as

$$\frac{\Delta n(\omega)}{n_r} = \frac{\Delta n^{(1)}(\omega)}{n_r} + \frac{\Delta n^{(3)}(\omega)}{n_r}.$$  

(38)

4. Results and discussions

In this part, we selected GaAs for our computation. We used the parameters: $\mu = 0.067 m_s$, $\sigma_i = 5 \times 10^{22} m^{-3}$, $n_r = 3.2$, and the radius as $R = 10$ nm [18]. Therefore, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m is the vacuum permittivity, $\mu_0 = 4\pi \times 10^{-7}$ H/m is the vacuum permeability, and $\Gamma_{12} = 1/\tau_{12}$, where $\tau_{12} = 0.14$ ps. Other fixed parameters are $A = 1$ and $B = -1.5$. Here, we note that the ground and excited states were chosen as $n = 0$, $l = 0$ and $n = 0$, $l = 1$. We perform our calculations in atomic units ($h = m_s = e = 1$).

In Fig. 1, we plot the MMS potential versus the distance $r$ for value of the screening parameter $a$. Fig. 2 shows the variation of energy against the screening parameter for different quantum numbers $n$ and $l$. We can see that by increasing the screening parameter, the energy decreases gradually. In Fig. 3, we plotted the variation of energy against the barrier height for different values of the quantum number $n$. It is noted that as the barrier height increases, the energy decreases linearly.

In order to study the effect of the confinement barrier slope $\gamma = 1/\alpha$ on AC, we plot the linear, third-order nonlinear and total AC against the energy $h \omega$ with optical intensity $I = 2.10^{10}$ W/m$^2$ and barrier height $V_0 = 112.23$ meV for various barrier slopes in Fig. 4. It is seen that increasing the barrier slope $\gamma$, the peaks of linear, third-order nonlinear as well as total AC shift to the higher energies. This fact is due to the increasing energy difference between the ground and excited states ($E_{21}$). The increasing energy difference results in the total AC increases. These findings are in agreement with reports in [20].

In Fig. 5, we plot the linear, third-order nonlinear and total AC against the incident photon energy $h \omega$ for different $V_0$ with barrier slope $\gamma = 3$ nm and incident optical intensity $I = 2.10^{10}$ W/m$^2$. We can observe that with increasing $V_0$, the resonant peak positions of AC move to higher energies (blue shift) in agreement with the literature [20]. As it was observed in Fig. 4, where the increasing barrier slope results in the energy difference increases, similarly, the increasing barrier height leads to energy difference between the ground and excited states, which causes the shift of the peaks to higher energies. In addition, with increasing $V_0$ the peaks become increasingly larger and sharper as a result of the overlaps of the wave functions [19].
Fig. 1. Variation of the modified Möbius potential $V(r)$ with $r$ for various values of the screening parameter $\alpha$. The barrier height is $V_0 = 112.23$ meV.

Fig. 2. Variation of the energy $E_{nl}$ with the screening parameter $\alpha$ for various quantum numbers $n$ and $l$. The barrier height is $V_0 = 112.23$ meV.

Fig. 3. Variation of the energy $E_{nl}$ with the barrier height $V_0$ for various quantum number $n$. The screening parameter is set to be $\alpha = 0.02$.

Fig. 6 shows the total changes in the AC as a function of $h\omega$ for different incident optical intensities as $0.1 \times 10^{10}$, $2 \times 10^{10}$, $3 \times 10^{10}$ and $4 \times 10^{10}$ W/m$^2$ with the confinement barrier slope $\gamma = 3$ nm and $V_0 = 112.23$ meV. We note that the amplitude of the total AC is decreased with increasing the optical intensity. This is due to the fact that the linear AC is $I$ independent while the third-order nonlinear absorption coefficient is linear in $I$ which reduces the total AC. It begins to saturate at optical intensity value of $3 \times 10^{10}$ W/m$^2$. The optical intensity has no effect on the resonance peak position of total AC. These results are in agreement with the literature [3, 20].

Fig. 7 shows the results for the linear, third-order nonlinear and total RIC as a function of the incident photon energy for values of the barrier slope. The barrier height and incident optical intensity are set to be $V_0 = 112.23$ meV and $I = 2 \cdot 10^{10}$ W/m$^2$. We can observe that with increasing the barrier slope $\gamma$, the RIC increases higher energies.

We show the linear, third-order nonlinear and total RIC as a function of incident photon energy for different values of the barrier height $V_0$ in Fig. 8. The barrier slope and incident optical intensity are $\gamma = 3$ nm and $I = 2 \cdot 10^{10}$ W/m$^2$. It can be seen that with increasing the energy barrier $V_0$, the amplitude of refractive index changes decrease and their position shift to the higher energies.
Fig. 4. Linear (dashed line), third-order nonlinear (dotted line) and total absorption (solid line) coefficients as a function of incident photon energy $\hbar \omega$ for three different values of confinement barrier slope $\gamma$.

Fig. 5. Linear (dashed line), third-order nonlinear (dotted line) and total absorption (solid line) coefficients as a function of incident photon energy $\hbar \omega$ for three different values of barrier height $V_0$.

Fig. 6. Total absorption coefficient as a function of photon energy $\hbar \omega$ for five different values of optical intensity $I$.

Fig. 9 shows the total RIC as a function of the incident photon energy for different values of the incident optical intensity $I$. The barrier height and barrier slope are fixed as $V_0 = 112.23$ nm and $\gamma = 3$ nm. By changing the incident optical intensity, the total RIC significantly varies in vicinity of the resonant frequency. The total RIC gives rise to two peaks when the incident optical intensity attains a critical value. These findings are in agreement with reports [3].

5. Conclusion

In this work, we studied the linear, third-order nonlinear, and total AC and RIC for spherical quantum dots. We considered the MMS potential as the confinement system. To obtain the eigenvalues and wave functions, we solved the Schrödinger equation using the NU method. The optical
properties are obtained using the density matrix method. The results show that by changing the structural parameters, the peak positions of the AC and RIC can be shifted to higher energies. It should be known that the present approach can be applied to other quantum systems. Therefore, we can say that the NU method is a reliable method of solving solvable potentials. Finally, we hope the results obtained in the present study will contribute to a better understanding of the optical properties of quantum dots and other nanostructures.

**Declarations**

**Author contribution statement**

C.P. Onyenegecha: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

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**Data availability statement**

Data will be made available on request.
Fig. 8. (a) The linear, (b) third-order nonlinear and (c) total refractive index changes as a function of photon energy $\hbar\omega$ for three different values of barrier height $V_0$.

Fig. 9. The total refractive index changes as a function of incident photon energy $\hbar\omega$ for five different values of the incident optical intensity $I$. 
Declaration of interest's statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper

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