Faraday effect of light caused by plane gravitational wave

Andrey A. Shoom

Max Planck Institute for Gravitational Physics (Albert Einstein Institute),
Leibniz Universität Hannover, Callinstr. 38, D-30167, Hannover, Germany

Gravitational field can cause a rotation of polarisation plane of light. This phenomenon is known as the gravitational Faraday effect. We study the gravitational Faraday effect of linearly polarised light propagating in the gravitational field of a weak plane gravitational wave (GW) with “+”, “x”, and elliptical polarisation modes. The corresponding gravitational Faraday rotation is proportional to the wave amplitude and to the squared distance traveled by light and it is inversely proportional to the GW’s squared wavelength. The rotation is also maximal if light propagates in the direction perpendicular to the GW propagation, along directions of its polarisation. There is no gravitational Faraday rotation when light and the GW propagate in the same or opposite directions, or it propagates along directions perpendicular to directions of the GW polarisation. Helicity of elliptically polarised GW gives higher-order contribution to the gravitational Faraday rotation.

I. INTRODUCTION

The Faraday effect of light is a magneto-optical phenomenon—rotation of polarisation plane of light propagating in a transparent material in the presence of magnetic field along the light propagation [1]. There is analogous effect, the gravitational Faraday effect of light—rotation of polarisation plane of an electromagnetic wave propagating in a gravitational field of non-zero angular momentum. Such gravitational field can be created by a rotating black hole, whose angular momentum causes “dragging” of inertial frames, which results in rotation of polarisation plane of light and gyroscope precession. These phenomena can be observed in local frames which do not rotate relative to asymptotic rest frame. Study and observation of the gravitational Faraday effect of light can be found in many works [2–19].

It is known that gravitational wave (GW) carries angular momentum. Thus, space-time of a GW can also cause inertial frame dragging, and, as a result, the gravitational Faraday effect. This phenomenon was already studied on the example of linearly polarised electromagnetic shock wave interacting with strong gravitational field of a plane-fronted GW in full nonlinear regime, with different definition of the Faraday rotation angle [20, 21]. Here we consider this effect due to a weak plane gravitational wave in linearised gravity. We study gravitational Faraday rotation caused by a plane GW of “+”, “x”, and elliptical polarisation modes.

This paper is organised as follows. In section II we briefly review the laws of geometric optics in a curved space-time and describe the gravitational Faraday effect of light in the formalism presented in [5] and adopted here to linear polarisation. Section III contains study of the gravitational Faraday effect of light caused by plane gravitational wave. The last section contains discussion of our results. We shall use a system of units with $c = G = 1$ and conventions adopted in the book [22].

II. GRAVITATIONAL FARADAY EFFECT OF LIGHT

When electromagnetic wave (light) propagating in a curved space-time background is highly monochromatic over some space-time regions, we can use an asymptotic short-wave (geometric optics) approximation [22].

This approximation is valid when the reduced wavelength (wavelength/2π) measured in a local Lorentz frame is much less than inhomogeneities in the wave and in the space-time. By using this approximation we consider the electromagnetic wave as a plane-fronted monochromatic wave whose propagation can be described by laws of geometric optics [2].

\[
\begin{align*}
    k^\alpha k_\alpha &= 0, & k^\beta k^\alpha_{\beta} &= 0, \\
    p^\alpha p_\alpha &= 1, & k^\beta p^\alpha_{\beta} &= 0.
\end{align*}
\]

Here \(k^\alpha = dx^\alpha/d\lambda\) is wave vector tangent to a light ray \(\Gamma: x^\alpha = x^\alpha(\lambda)\), where \(\lambda\) is affine parameter of the ray, and \(p^\alpha\) is the linear polarisation vector. These laws imply that light rays are space-time null geodesics and the polarisation vector is orthogonal to the light ray and parallel propagated along it [2].

Our goal is to study propagation of the polarisation vector along spatial trajectory of a light ray. This can be done by an observer-defined local decomposition of space-time into space and time, which is known as space-time threading approach [1]. Consider a family of observers filling a three-dimensional space like a continuous medium (see Fig. 1). World lines of these observers form a congruence of integral curves of the timelike future directed

---

1 Here and in what follows the semicolon stands for the covariant derivative associated with space-time metric \(g_{\alpha\beta}\).

2 As it is in magneto-optical Faraday effect, here we consider light with linear polarisation.

3 The threading approach is different from another space-time decomposition known as slicing or the ADM approach. It was originally developed by Möller, Zel’manov, and Cattaneo, and discussed in detail in [23, 24]. It is considered in [23] as well.

4 Only one such curve passes though each event of the space-time.
unit vector field $u^\alpha = u^\alpha(x^\alpha)$, $u^\alpha u_\alpha = -1$.

Each of the observers defines the local frame of reference, a right-handed orthonormal Lorentz frame:

$$\{e_0^\alpha = u^\alpha, \ e_a^\alpha \ ; a = 1, 2, 3\},$$

$$u^\alpha u_\alpha = -1, \ e_a^\alpha u_\alpha = 0, \ e_\alpha^\alpha e_\beta^\beta g_{\alpha\beta} = \delta_{ab}. \quad (3)$$

Here $\delta_{ab}$ is the three-dimensional Kronecker tensor. Let $\Sigma_a$ be three-dimensional local subspace of the tangent space defined at every event on the observer’s world line and orthogonal to $u^\alpha$. A vector from the tangent space can be projected into the subspace $\Sigma_a$ by the projection operator

$$p^\alpha_{\beta} = \delta^\alpha_{\beta} + u^\alpha u_\beta, \quad (4)$$

where $\delta^\alpha_{\beta}$ is the four-dimensional Kronecker tensor. This operator also defines the induced metric on $\Sigma_a$:

$$p_{\alpha\beta} = p^\mu_{\alpha} p^\nu_{\beta} g_{\mu\nu} = g_{\alpha\beta} + u_\alpha u_\beta. \quad (5)$$

Applying the projection operator to $k^\alpha$ we construct the unit spacelike vector $n^\alpha$ that defines spatial direction of a light ray. Accordingly, we have

$$k^\alpha = \omega (u^\alpha + n^\alpha), \quad (6)$$

where $\omega = -k_\alpha u^\alpha$ is angular frequency of light measured by a local observer (see Fig. 1).

Let us now consider polarisation vector. As follows from the geometric optics equations, there is the gauge freedom

$$p^\alpha \rightarrow \tilde{p}^\alpha + \kappa k^\alpha, \quad (7)$$

where $\kappa$ is constant along null ray, such that $k^\alpha \kappa_{\alpha} = 0$. The gauge transformation implies that at any event on the light world line one can add to the polarisation vector a multiple of the wave vector without affecting the propagation laws (2). We fix the gauge so that

$$p^\alpha u_\alpha = 0. \quad (8)$$

Then, according to the second relation in (2) and the expression (4) we have

$$p^\alpha n_\alpha = 0. \quad (9)$$

Thus, the polarisation vector lies in a two-dimensional subspace perpendicular to $u^\alpha$ and $n^\alpha$. If we orient the local frame so that $n^\alpha = e_3^\alpha$, then we can expand the polarisation vector in the local two-dimensional basis $\{e_1^\alpha, e_2^\alpha\}$. To make such an expansion, we note that linear polarisation is a superposition of left-hand and right-hand circular polarisation modes whose polarisation vectors rotate in the anti-clockwise and clockwise directions respectively, when viewed along light ray propagation from a particular point on the ray. As in the magneto-optical Faraday effect, due to the gravitational Faraday effect there is an additive contribution $\pm \phi$ to each mode’s phase $k_\alpha x^\alpha$. This contribution results in the net contribution to the linear polarisation mode’s phase, which causes rotation of the linear polarisation vector in the anti-clockwise direction by $\phi$. Taking this into account we have

$$p^\alpha = \cos \phi e_1^\alpha - \sin \phi e_2^\alpha. \quad (10)$$

We shall call the angle $\phi$ the polarisation phase. It defines orientation of the polarisation vector (see Fig. 2). We would like to calculate how the polarisation phase

\footnote{For details see Eqs. (95) and (101) in [16].}
changes along the null ray. Using the third expression in [2] and the expression (11) we derive

\[ k^\alpha \phi_a = -e_{1\alpha} k^\beta e_2^\alpha ;\beta. \] (11)

This expression defines change of the polarisation phase along the null ray with respect to the basis \( \{ e_1^\alpha, e_2^\alpha \} \). This change depends on propagation of the basis along the null ray. For example, if the basis is propagated arbitrarily, the polarisation phase change can take arbitrary value, regardless the null ray and gravitational field. Thus, in order to find the polarisation phase change due to the gravitational field only, we need to find out how to propagate the basis along the ray. This problem is analogous to measurement of polarisation in the magneto-optical Faraday effect where we have to align properly optic axes of a polariser and an analyser. For example, if light trajectory is a straight line, we orient the polariser and analyser optic axes parallel to one another and perpendicular to the light trajectory. If light trajectory is a plane curve, we orient their optic axes parallel to one another and perpendicular to the trajectory plane everywhere along the curve. And finally, if the light trajectory is not plane, we propagate the analyser axis from the initial point, where it is aligned with the polariser axis, to the final point along the light trajectory in such a way that it does not rotate with respect to the trajectory during infinitesimally small step of propagation. This procedure generalises the straight and plane trajectory cases. Such a propagation is called Fermi-Walker transport and is defined by vanishing Fermi derivative of the unit vector indicating direction of the analyser optical axis along the unit vector of the light trajectory (see equation (12) below) [26]. Strictly speaking, “does not rotate with respect to the trajectory” means that the unit vector undergoes only rotation in the plane spanned by the unit vector of the light trajectory and “acceleration” vector, which defines its rate of change along the curve (see §6.5 in [22]). For Fermi-Walker transport along a timelike curve the acceleration vector acquires its usual meaning.

Returning to the expression (11) we see that it contains derivative of the basis vector along the null ray vector. To define propagation law for the basis along the null ray we shall use the decomposition (6), which allows to express the basis propagation law via its propagations along the vectors \( u^\alpha \) and \( n^\alpha \),

\[ k^\beta e_\alpha ;\beta = \omega (u^\beta e_\alpha ;\beta + n^\beta e_\alpha ;\beta), \] (12)

where \( a = 1, 2 \). Thus, we need to define transport law along the observer’s world line and along the ray spatial trajectory. We already discussed above how the basis should be transported along the ray trajectory. Namely, its spatial Fermi-Walker derivative along the tangent to the trajectory unit vector \( n = n^\alpha \partial_\alpha \) should vanish,

\[ \nabla_n w_\alpha = \nabla_n e_\alpha - w(n, e_\alpha) + n(w, e_\alpha) = 0. \] (13)

Here \( e_\alpha = e_\alpha \partial_\alpha \), \( w = \nabla_n n \) is the mentioned above “acceleration” vector, the nabla \( \nabla \) stands for the covariant derivative associated with the spatial metric \( p_{\alpha \beta} = e_\alpha e_\beta p_{\alpha \beta} \), such that \( \nabla p_{\alpha \beta} = 0 \), and \( (\ldots) \) stands for a scalar product in the metric \( p_{\alpha \beta} \). This covariant derivative is related to the covariant derivative associated with the space-time metric \( g_{\alpha \beta} \) as follows:

\[ (\nabla_n e_\alpha)^\alpha = p^\alpha_\gamma n^\beta e_\alpha ;\beta. \] (14)

Note that the orthogonality condition \( (e_\alpha, n) = 0 \) is preserved by the Fermi-Walker derivative.

To define propagation of the basis \( \{ e_\alpha, a = 1, 2 \} \) along \( u^\alpha \) we discuss first the observers congruence \( u^\alpha(x^\alpha) \). We shall consider observers whose local frames are “tied” to Lorentz frame at asymptotically flat space-time region, which is in turn tied to “fixed stars”. Such observers congruence forms a rigid Cartesian latticework where each observer has fixed coordinate position. In stationary space-times such observers are analogous to Killing observers, that is their 4-velocity is proportional to the space-time’s timelike Killing vector field. In contrast to such observers, inertial (freely falling) observers are “dragged” by the local gravitational field together with null rays. These inertial observers cannot detect local gravitational effects. On the other side, the congruence observers detect the ambient gravitational field. The congruence observers move with acceleration \( a^\alpha = u^\beta u_\alpha ;\beta \) and rotate with angular velocity

\[ \omega^\alpha = \frac{1}{2} \varepsilon^{\alpha \beta \gamma \delta} u_\beta u_\gamma e_\delta, \] (15)

which is also called the vorticity vector of the observers congruence [27]. Here \( \varepsilon^{\alpha \beta \gamma \delta} \) is the Levi-Civita (pseudo) tensor, which for the observer-adapted frame (8) is normalised as

\[ \varepsilon_{\alpha \beta \gamma \delta} u^\alpha e_1^\beta e_2^\gamma e_3^\delta = +1. \] (16)

The transport law for each observer from the congruence is (see, e.g., §13.6 in [22])

\[ u^\beta e_\alpha ;\beta = (u^\alpha a_\beta - a^\alpha u_\beta) e_\beta - \varepsilon_{\alpha \beta \gamma \delta} u^\gamma e_\delta. \] (17)

From (16) with \( e_3^\alpha = n^\alpha \) it follows that

\[ \varepsilon_{\alpha \beta \gamma \delta} u^\alpha e_1^\beta e_2^\gamma = n_\delta. \] (18)

Then, using the expressions above we can compute the right-hand side of the expression (11) and derive

\[ k^\alpha \phi_a = \frac{d\phi}{d\lambda} = \omega_\alpha k^\alpha. \] (19)

This result allows us to find evolution of linear polarisation angle \( \phi \) along a null ray \( \Gamma: x^\alpha = x^\alpha(\lambda) \),

\[ \phi = \int_\Gamma \omega_\alpha k^\alpha d\lambda, \] (20)

which is known as the gravitational Faraday effect of light.
III. PLANE GRAVITATIONAL WAVE

GW’s detections from coalescing compact binaries opened up a new era of observational astronomy and cosmology. Up to date, there are about 90 GW’s detections. Sources of the currently detected GW’s have luminosity distance in the range from about 40 to about 5300 Mpc. Detected GW’s are well modelled by a plane GW solution to the linearised Einstein field equations. In the transverse-traceless (TT) gauge the space-time metric of a monochromatic plane gravitational wave propagating in the z-direction has the following form:

$$ds^2 = -dt^2 + (1 + h_+)dx^2 + 2h_+dxdy + (1 - h_+)dy^2 + dz^2,$$  \hspace{1cm} (21)

where

$$h_+ = \Re\{A_+ e^{-i\Omega(t-z)}\}, \quad h_\times = \Re\{A_\times e^{-i\Omega(t-z)}\}.$$  \hspace{1cm} (22)

Here $A_+$ and $A_\times$ are constant amplitudes of the “+” and “×” polarisation modes and $\Omega$ is their constant frequency.

By the construction, the TT gauge corresponds to a global Lorentz frame $u^\alpha = \delta^\alpha_\tau$, which is inertial; world line of $u^\alpha$ is the space-time (21) timelike geodesic, as well as any timelike world line with fixed spatial coordinates $x^\alpha = \text{const}$. Such geodesics have fixed coordinate separation (see Exercise 35.5 in [22]). According to the construction presented in the previous section, to measure gravitational Faraday effect we have to consider non-inertial observers congruence. Therefore, for the given metric (21) the congruence of inertial observers, $u^\alpha(x^\alpha) = \delta^\alpha_\tau$, $x^\alpha = \text{const}$, is not suitable. The reason is that inertial observers cannot measure local gravitational effects, and in particular, detect gravitational Faraday rotation. The best these observers can do is to measure deviation of their world lines, which is constant in the TT gauge. Solution to this problem is to work in the proper frame of one of such observers. As it is known, local geodesic deviation can indeed be detected in the proper reference frame (see §35.5 in [22]). Here we consider the proper frame of an inertial observer in the Fermi normal coordinates (see p. 332 in [22] and for details [28]):

$$ds^2 = -(1 + R_{\tau ab}x^a x^b)dt^2 - \frac{4}{3} R_{abcd}x^a x^b dx^c dx^d + O((x^a)^2) dx^a dx^b,$$  \hspace{1cm} (23)

where the components of the Riemann tensor $R_{\alpha\beta\gamma\delta}$ are evaluated on the observer’s world line $t = \tau$, $x^\alpha = 0$, where $\tau$ is the proper time, i.e. $R_{\alpha\beta\gamma\delta}$ depends on $t$ only.

We now consider congruence of observers $x^\alpha = \text{const}$ in the vicinity of the proper frame (24). In accordance with (23), 4-velocity of the congruence computed up to second order in $x^\alpha$ is

$$u^\alpha(x^\alpha) \approx \left(1 - \frac{1}{2} R_{\tau ab}x^a x^b\right) \delta^\alpha_\tau,$$  \hspace{1cm} (24)

and its covariant form reads

$$u^\alpha(x^\alpha) \approx -\left(1 + \frac{1}{2} R_{\tau ab}x^a x^b\right) \delta^\alpha_\tau - \frac{2}{3} R_{\tau abcd}x^a x^b \delta^\alpha_\tau.$$  \hspace{1cm} (25)

The corresponding vorticity [15] of the congruence is

$$\omega^\alpha \approx \varepsilon^{\alpha b c} R_{b c d x} x^d \delta^\tau_\alpha,$$  \hspace{1cm} (26)

where we used the Bianchi identity $R_{(bcd)} = 0$. To compute the polarisation phase [20] we need to know vector $k^\alpha$ of a null geodesic of the space-time (23). The space-time Christoffel coefficients are of first order in $x^\alpha$. Therefore, the polarisation phase consistent with the metric (23) should be computed up to first order in $x^\alpha$. This implies that $k^\alpha$ should be computed in flat (Minkowski) metric, that is null geodesics passing through the proper frame origin at $\lambda = 0$ are

$$x^\alpha(\lambda) \approx k^0_0 \lambda, \quad k^0_0 = (\omega_0, k^0_0) = \text{const},$$  \hspace{1cm} (27)

where $\omega_0$ is light frequency and $k^0_0$ is three-dimensional wave vector, such that $|k|^2 = \omega^2_0$. Then, the polarisation phase can be computed as follows:

$$\phi = -\int_0^\lambda \varepsilon_{a b c} R_{b c d t} [t(\lambda)] x^d(\lambda) k^0_d d\lambda,$$  \hspace{1cm} (28)

where we used the Minkowski metric approximation $\varepsilon_{a b c} \approx -\varepsilon_{a b c}$, where $\varepsilon_{abc}$ is the three-dimensional Levi-Civita (pseudo) tensor.

To compute the polarisation phase we need to know the Riemann tensor components. In the linearised theory of gravity, where $g_{\alpha\beta} = h_{\alpha\beta} + h_{\alpha\beta}$, with $|h_{\alpha\beta}| \ll 1$, we have (see, e.g. p. 438 in [22]):

$$R_{\alpha\beta\gamma\delta} = \frac{1}{2} (h_{\alpha\delta,\beta\gamma} + h_{\beta\gamma,\alpha\delta} - h_{\gamma\delta,\alpha\beta} - h_{\delta\alpha,\beta\gamma}).$$  \hspace{1cm} (29)

The chosen proper frame (24) coincides with the TT frame (21) to first order in $h_{ab}$. Therefore, the Riemann tensor components of the TT frame are the same to first order in $h_{ab}$ as these of the proper frame. Using accordingly the expressions (29) and (22) we derive

$$R_{txxx} = -R_{tyzy} = -\frac{1}{2} h_{+,tz}, \quad R_{ttxy} = R_{tzyz} = \frac{1}{2} h_{\times, tz}.$$  \hspace{1cm} (30)

These components are evaluated at $x^\alpha = 0$, in accordance with the proper frame construction (23). The proper frame quadratic approximation (23) is valid if $|x^\alpha|$ is sufficiently less than the reduced GW wavelength, $\lambda_{\text{GW}}$.

In what follows, we first consider “+” and “×” polarisation modes separately, and then discuss their superposition—elliptically polarised modes.

A. “+” polarisation

We can now compute the polarisation phase (23) due to the plane GW with “+” polarisation. Using the expressions above and defining direction of light ray $k^0_0$ in
the proper frame by its spherical angular coordinates $\varphi_0$ and $\theta_0$ we derive
\[ \phi_+(t) = -\frac{A_+}{2} \sin^2 \theta_0 \sin(2\varphi_0) [\Omega t \sin(\Omega t) + \cos(\Omega t) - 1]. \] (31)

Here $t$ is the time measured by proper observer. Let time $t$ corresponds to the proper distance $\ell = ct$ ($c$ is the speed of light) of the light propagation from the proper frame origin, then according to the quadratic approximation in [23], we should have $\Omega t = \ell / \kappa_{\text{GW}} \ll 1$. In this case, the expression (31) can be approximated as follows:
\[ \phi_+(\ell) \big|_{\ell \ll \kappa_{\text{GW}}} \approx -\frac{A_+}{4} \frac{\ell^2}{\kappa_{\text{GW}}} \sin^2 \theta_0 \sin(2\varphi_0). \] (32)

**B. “×” polarisation**

The polarisation phase evolution due to the plane GW with “×” polarisation is
\[ \phi_\times(t) = \frac{A_\times}{2} \sin^2 \theta_0 \cos(2\varphi_0) [\Omega t \sin(\Omega t) + \cos(\Omega t) - 1]. \] (33)

According to the quadratic approximation in [23], for $t$ corresponding to the distance $\ell$ of the light propagation from the proper frame origin, we should have $\ell \ll \kappa_{\text{GW}}$. In this case, the expression (33) can be approximated as follows:
\[ \phi_\times(\ell) \big|_{\ell \ll \kappa_{\text{GW}}} \approx -\frac{A_\times}{4} \frac{\ell^2}{\kappa_{\text{GW}}} \sin^2 \theta_0 \cos(2\varphi_0). \] (34)

**C. Elliptical polarisation**

Elliptical polarisation mode can be presented as a superposition of the “+” and “×” polarisation modes of different amplitudes,
\[ h_\sigma = \Re \{A_+ + i A_\times \sigma^\epsilon e^{-i\Omega(t-z)} \}. \] (35)

Here $\sigma = \pm 1$ and $+(-)$ sign stands for the right-(left-)handed polarisation when viewed from the source along the wave propagation. The polarisation phase evolution due to the elliptically polarised plane GW is
\[ \phi_\sigma(t) = -\frac{A_+}{2} \sin^2 \theta_0 \sin(2\varphi_0) [\Omega t \sin(\Omega t) + \cos(\Omega t) - 1] + \frac{\sigma A_\times}{2} \sin^2 \theta_0 \cos(2\varphi_0) [\sin(\Omega t) - \Omega t \cos(\Omega t)]. \] (36)

As in the cases above, according to the quadratic approximation in [23], we should have $\ell \ll \kappa_{\text{GW}}$. Then the expression (36) can be approximated as follows:
\[ \phi_\sigma(\ell) \big|_{\ell \ll \kappa_{\text{GW}}} \approx -\frac{A_+}{4} \frac{\ell^2}{\kappa_{\text{GW}}} \sin^2 \theta_0 \sin(2\varphi_0) + \frac{\sigma A_\times}{6} \frac{\ell^3}{\kappa_{\text{GW}}} \sin^2 \theta_0 \cos(2\varphi_0). \] (37)

We see that the helicity $\sigma$ contribution to the gravitational Faraday rotation is of higher order.

**IV. DISCUSSION**

Here we studied the gravitational Faraday effect of light due to a weak plane gravitational wave with “+”, “×”, and elliptical polarisation modes. We computed the polarisation phase in the proper frame, which is valid for $\ell \ll \kappa_{\text{GW}}$, where $\ell$ is distance from the proper frame origin. The derived expressions show that the gravitational Faraday rotation is proportional to the GW amplitude. It is maximal for electromagnetic wave (light) propagating in the direction perpendicular to the GW propagation $\theta_0 = \pi/2$, along directions of its polarisation: $\phi_0 = 0, \pi/2$ for “×” polarisation mode, and $\phi_0 = \pi/4, 3\pi/4$ for “+” and elliptical polarisation modes. Within the order of approximation, there is no gravitational Faraday rotation when the light and GW propagate in the same directions, or/and the light propagates along directions perpendicular to directions of the GW polarisation. Considering GW of elliptical polarisation we also found that the GW helicity $\sigma = \pm 1$ contribution to the gravitational Faraday rotation is of higher order in $\ell / \kappa_{\text{GW}}$. It would be interesting to analyse the actual amount of the expected polarisation rotation for observed GW, which we may consider elsewhere.

[1] L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics, Volume 8: Electrodynamics of Continuous Media* (Pergamon Press, Ltd., Headington Hill Hall, Oxford, England, 1960) p. 335.
[2] G. V. Skrotskii, “On the Influence of Gravity on the Light Propagation,” Soviet Phys. Doklady 2, 226 (1957), [Akademia Nauk SSR, Doklady, 114, 73, 1957].
[3] J. Plebanski, “Electromagnetic Waves in Gravitational Fields,” Phys. Rev. 118, 1396 (1959).
[4] B. B. Godfrey, “Mach’s Principle, the Kerr Metric, and Black-Hole Physics,” Phys. Rev. D 1, 2721 (1970).
[5] S. F. Pineault and R. C. Roeder, “Applications of Geometrical Optics to the Kerr Metric. I. Analytical Results,” Astrophys. J. 212, 541 (1977).
[6] P. A. Connors and R. F. Stark, “Observable gravitational effects on polarised radiation coming near a black hole,” Nature (London) 269, 128 (1977).
[7] P. A. Connors, T. Piran, and R. F. Stark, “Polarization features of X-ray radiation emitted near black holes,”
8 F. Fayos and J. Llosa, “Gravitational Effects on the Polarization Plane,” General Relativity and Gravitation 14, 865 (1982).
9 H. Ishihara, M. Takahashi, and A. Tomimatsu, “Gravitational Faraday rotation induced by a Kerr black hole,” Phys. Rev. D 38, 472 (1988).
10 P. Carini, L. L. Feng, and R. Ruffini M. Li, “Phase evolution of the photon in Kerr spacetime,” Phys. Rev. D 46, 5407 (1992).
11 M. Nouri-Zonoz, “Gravitoelectromagnetic approach to the gravitational Faraday rotation in stationary spacetimes,” Phys. Rev. D 60, 024013 (1999).
12 M. Sereno, “Gravitational Faraday rotation in a weak gravitational field,” Phys. Rev. D 69, 087501 (2004).
13 M. Sereno, “Detecting gravitomagnetism with rotation of polarization by a gravitational lens,” Mon. Not. R. Astron. Soc. 356, 381 (2005).
14 A. Brodutch, T. F. Demarie, and D. R. Terno, “Photon polarization and geometric phase in general relativity,” Phys. Rev. D 84, 104043 (2011).
15 T. Ghosh and A. K. Sen, “The Effect of Gravitation on the Polarization State of a Light ray,” Astrophys. J. 833, 82 (2016).
16 Valeri P. Frolov and Andrey A. Shoom, “Spinoptics in a stationary spacetime,” Phys. Rev. D 84, 044026 (2011), arXiv:1105.5629 [gr-qc]
17 Valeri P. Frolov and Andrey A. Shoom, “Scattering of circularly polarized light by a rotating black hole,” Phys. Rev. D 86, 024010 (2012), arXiv:1205.4179 [gr-qc]
18 Andrey A. Shoom, “Gravitational Faraday and spin-Hall effects of light,” Phys. Rev. D 104, 084007 (2021), arXiv:2006.10077 [gr-qc]
19 Valeri P. Frolov, “Maxwell equations in a curved spacetime: Spin optics approximation,” Phys. Rev. D 102, 084013 (2020), arXiv:2007.03743 [gr-qc]
20 Mustafa Halilsoy and Ozay Gurtug, “Search for gravitational waves through the electromagnetic Faraday rotation,” Phys. Rev. D 75, 124021 (2007), arXiv:gr-qc/0612107
21 A. Al-Badawi, “The exact behavior of electromagnetic Faraday rotation in crossing a gravitational sandwich wave in general relativity,” Phys. Lett. B 683, 50–54 (2010)
22 C. W. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation (W. H. Freeman and Co., San Francisco, 1973).
23 R. T. Jantzen, P. Carini, and D. Bini, “The Many faces of gravitoelectromagnetism,” Annals Phys. 215, 1 (1992).
24 P. Carini and R. T. Jantzen, Gravitoelectromagnetism and the single gyro, in Relativistic Gravitational Experiments in Space, edited by M. Demianski and C. W. F. Everitt (World Scientific Publishing Co. Pte. Ltd., 1993) p. 135.
25 L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields, Course of Theoretical Physics Volume 2 (Pergamon Press, Ltd., Headington Hill Hall, Oxford, England, 1975).
26 J. L. Synge, “Optical observations in general relativity,” Seminario Mat. e. Fis. di Milano 30, 271 (1960).
27 S. W. Hawking and G. F. R. Ellis, The large scale structure of space-time (Cambridge University Press, New York, 1999).
28 F. K. Manasse and C. W. Misner, “Fermi Normal Coordinates and Some Basic Concepts in Differential Geometry,” J. Math. Phys. 4, 735 (1963).