Precise predictions for $t\bar{t}\gamma/t\bar{t}$ cross section ratios at the LHC

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Abstract: With the goal of increasing the precision of NLO QCD predictions for the $pp \rightarrow t\bar{t}\gamma$ process in the di-lepton top quark decay channel we present theoretical predictions for the $R = \sigma_{t\bar{t}\gamma}/\sigma_{t\bar{t}}$ cross section ratio. Results for the latter together with various differential cross section ratios are given for the LHC with the Run II energy of $\sqrt{s} = 13$ TeV. Fully realistic NLO computations for $t\bar{t}$ and $t\bar{t}\gamma$ production are employed. They are based on matrix elements for $e^+\nu_e\mu^-\bar{\nu}_\mu\bar{b}b$ and $e^+\nu_e\mu^-\bar{\nu}_\mu\bar{b}\gamma$ processes and include all resonant and non-resonant diagrams, interferences, and off-shell effects of the top quarks and the $W$ gauge bosons. Various renormalisation and factorisation scale choices and parton density functions are examined to assess their impact on the cross section ratio. Depending on the transverse momentum cut on the hard photon a judicious choice of a dynamical scale allows us to obtain $1\% - 3\%$ percent precision on $R$. Moreover, for differential cross section ratios theoretical uncertainties in the range of $1\% - 6\%$ have been estimated. Until now such high precision predictions have only been reserved for the top quark pair production at NNLO QCD. Thus, $R$ at NLO in QCD represents a very precise observable to be measured at the LHC for example to study the top quark charge asymmetry or to probe the strength and the structure of the $t\bar{t}\gamma$ vertex. The latter can shed some light on possible new physics that can reveal itself only once sufficiently precise theoretical predictions are available.

Keywords: NLO Computations, QCD Phenomenology, Heavy Quark Physics

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1 Introduction

Top quark studies, that are currently driven by the Large Hadron Collider (LHC) experiments, ATLAS and CMS, play a major role in deciphering the fundamental interactions. At the LHC top quarks are mostly produced in pairs through strong interactions, but they can also be created individually in single-top production via electroweak interactions. Thus, depending on the production mode, the top quark allows for different tests of the underlying forces. Both experiments concentrate on the measurements of top quark properties like for example the top quark mass ($m_t$), the top quark width ($\Gamma_t$), the top quark charge ($Q_t$), the total and differential cross sections as well as the top quark spin correlations and the top quark charge asymmetry ($A_{Ct\bar{t}}$) including differential top quark charge asymmetries. High in the LHC program is the determination of the top quark couplings to gauge bosons and the Standard Model (SM) Higgs boson. Due to the large top quark mass various new physics scenarios introduce modifications within the top quark sector. Some examples include heavy new particles decaying into top quark pairs, flavour changing neutral currents, anomalous missing transverse momentum, same-sign top pair production or charged Higgs production. Such new physics models can be tested by precise measurements of top quark pairs, that are abundantly produced at the LHC. Furthermore, top quark production, also with additional $b$- or light jet(s), constitutes dominant irreducible backgrounds to many of the searches for new physics processes. Thus, it is vital to understand the properties and the characteristics of top production and decay mechanisms. The level of precision available on the theory side can have a huge impact on whether we can actually see the effects of new physics.

As a result of the large collision energy at the LHC also more exclusive final states, like for example $t\bar{t}\gamma$, have started to be accessible [1–3]. Even though the cross section for the $t\bar{t}\gamma$ production process at the LHC is much smaller than the cross section for the production of the top quark pair alone, the former can provide key information on the strength and the structure of the top quark coupling to the photon. Thus, it can for example substantially constrain anomalous top quark couplings at the LHC. Regardless
of the applications, whether these are measurements within the SM or outside of this framework, precise theoretical predictions are compulsory to carry out such measurements.

To increase the theoretical precision for $pp \rightarrow t\bar{t}\gamma$ higher order corrections in QCD should be consistently included. Moreover, the most accurate description of top quark decay chains has to be incorporated. Appropriate calculations have recently been made available. Specifically, a complete description of top quark pair production in association with a hard photon in the di-lepton top quark decay channel has been provided in Ref. [4]. The calculations include factorisable and non-factorisable contributions at NLO in QCD, that imply a cross talk between production and decays of top quarks which require going beyond the so-called Narrow Width Approximation (NWA). Specifically, they include double resonant, single resonant and non-resonant Feynman diagrams with respect to the top quark and $W$ gauge boson, interferences among them as well as finite-width effects of $t$ and $W$. With a fairly inclusive selection of cuts on the final states, which are two $b$-jets, a hard photon, two charged leptons and the missing transverse momentum, $p_T^{miss}$, the full $pp$ cross section for a fixed renormalisation and factorisation scale choice receives negative and moderate NLO QCD corrections of 10%. An assessment of the uncertainties of theoretical origin left us with a 14% theoretical error. Inclusion of a kinematic dependent scale, that captures some parts of the unknown higher order effects, has improved the situation yielding positive and small NLO corrections of 2.5%. In this case the theoretical uncertainties resulting from scale variations have been estimated at the level of 6% only. Impact of higher order corrections on differential distributions, however, was much larger. For some observables, incidentally important in searches for new physics, shape distortions of more than 100% have been observed. As expected for specific phase space regions also the theoretical errors have increased substantially. Improvement of the accuracy of theoretical predictions also at the differential level to a few percent level can be obtained by including the next order in the perturbative expansion in $\alpha_s$. However, going beyond NLO even for on-shell $t\bar{t}\gamma$ production seems to be a formidable task at present. Inclusion of non-factorisable QCD contributions at NNLO is simply difficult to imagine. Instead, a ratio of cross sections can be studied since it may be significantly more stable against radiative corrections and scale variations than the cross sections themselves. Moreover, a ratio may reduce other theoretical uncertainties like for example those stemming from parton distribution functions. To this end a process that is under excellent theoretical control must to be employed in the denominator of the ratio. The $t\bar{t}$ production process, albeit in the same decay channel, seems to be the best candidate for the job due to its large cross section and similar behaviour with regard to radiative corrections [5–7]. Consequently, the following cross section ratio

$$ R = \frac{\sigma_{t\bar{t}\gamma}}{\sigma_{t\bar{t}}} $$

represents an interesting quantity to search for deviations from the SM theory at the LHC. Moreover, a set of differential cross section ratios can be constructed, to look for any shape deviations from those predicted within the SM

$$ R_X = \left( \frac{d\sigma_{t\bar{t}\gamma}}{dX} \right) \left( \frac{d\sigma_{t\bar{t}}}{dX} \right)^{-1} $$

(1.2)
Here $X$ stands for the particular observable under consideration, e.g. the invariant mass of two charged leptons, $m_{\ell\ell}$, the invariant mass of two $b$-jets, $m_{b\bar{b}}$, etc. Since for a realistic analysis specific cuts on top quark decay products need to be imposed a reliable description of top quark decays is mandatory for both processes $pp \to t\bar{t}\gamma$ and $pp \to t\bar{t}$. In order to avoid the introduction of additional unnecessary theoretical uncertainties to the construction of the cross section ratio, the same level of accuracy in the modelling of top quark decays must to be employed in the numerator and denominator of $R$ and $R_X$. Besides the modelling of top quark decays, where the incorporation of radiative corrections is mandatory, a proper renormalisation and factorisation scale choice has to be carefully investigated. The scale choice should play an even greater role when various differential cross section ratios are constructed. For the latter phase space regions away from those dominated by double resonant top quark contributions, which are sensitive to non-factorizable QCD corrections, would be probed as well.

The purpose of this paper is twofold. First, we would like to provide a systematic analysis of the two processes $pp \to t\bar{t}\gamma$ and $pp \to t\bar{t}$ in the di-lepton top quark decay channel and extract the most accurate NLO prediction for the total cross section ratio. Such precise theoretical results can be used in comparisons with the LHC data. The second goal of the paper is to examine whether differential cross section ratios have enhanced predictive power for new physics searches, by investigating possible correlations between the two processes in various phase space regions in the quest of reducing theoretical errors. Calculations for both processes will be carried out with the same input parameters, parton distribution functions (PDFs), jet algorithm and the same set of inclusive cuts up to the cuts on the hard photon, which are present only in the case of $t\bar{t}\gamma$ production. Finally, for both processes factorisation and renormalisation scales will be set to a common fixed value, whereas for a dynamical scale choice scales as similar as possible will be selected. Cross section ratios calculated in this way are free of additional and undesired theoretical uncertainties that are introduced when different input parameters are employed in the numerator and the denominator of $R$. The size of such additional theoretical uncertainties, however, must be estimated. In various experimental analyses different Monte Carlo (MC) programs are employed to provide theoretical predictions for $t\bar{t}$ and $t\bar{t}\gamma$ production. Such general purpose MC frameworks are often used by experimental collaborations with a default set up, among others with a different scale choice and parton distribution functions for $pp \to t\bar{t}$ and $pp \to t\bar{t}\gamma$. Thus, in the paper we will quantify the impact of the additional theoretical uncertainties coming from different theoretical inputs. Finally, the stability of the cross section ratio with respect to the transverse momentum cut on the hard photon will be examined. To this end, theoretical predictions for $t\bar{t}\gamma$ production will be evaluated for two different values of the $p_{T,\gamma}$ cut.

The article is organised as follows. In Section 2 the HELAC-NLO computational framework and input parameters used in our studies are described. In section 3 the normalised differential cross sections for off-shell $t\bar{t}\gamma$ and $t\bar{t}$ production are provided in order to study the correlation of the two processes. The results given there are used to understand how the theoretical errors on the cross section ratios should be estimated. NLO predictions for absolute cross sections are presented in Section 4 together with the theoretical uncertainties
from the scale dependence. Additionally, results with different parton distribution functions are shown in Section 4. They are calculated to estimate the size of theoretical uncertainties that come from the parametrisation of parton distribution functions. In Section 5 we provide results for NLO cross section ratios. Theoretical uncertainties are also discussed there. Theoretical predictions for the differential cross section ratios and their theoretical uncertainties are exhibited and discussed in Section 6. Finally, in Section 7 our conclusions are laid out.

2 Computational Framework and Input Parameters

All the LO and NLO results for $e^+\nu_e \mu^-\bar{\nu}_\mu \bar{b}b\gamma$ and $e^+\nu_e \mu^-\bar{\nu}_\mu \bar{b}b$ production, which are presented in this paper, have been obtained with the help of the HELAC-NLO MC framework [8]. The package comprises HELAC-1LOOP [9] with CUTTOOLS [10] for the virtual corrections and HELAC-DIPOLES [11, 12] for the real emission part. The HELAC-DIPOLES software deals with singularities from soft or collinear parton emissions that are isolated via subtraction methods for NLO QCD calculations. Specifically, the commonly used Catani-Seymour dipole subtraction [11, 13, 14] and the so-called Nagy-Soper subtraction scheme [12] are both implemented in the HELAC-DIPOLES program and used in our simulations. The integration over the phase space has been achieved with the help of KALEU [17]. For unstable top quarks the complex mass scheme is utilised [18, 19]. At the one loop level the appearance of $\Gamma_t \neq 0$ in the propagator requires the evaluation of scalar integrals with complex masses, which is supported by the ONELOOP program [20]. Further details of calculations can be found in our earlier work on $pp \to t\bar{t}$, $pp \to t\bar{t}j$ and $pp \to t\bar{t}\gamma$ [4, 6, 15, 16]. In each case all resonant and non-resonant Feynman diagrams, interferences and finite width effects of the top quark as well as $W$ gauge boson have been included consistently at the NLO QCD level. The methods developed there have been straightforwardly adapted in the current studies and, therefore, do not need a recollection. We refer the interested readers to previously published results. In the calculations of cross sections we employ the following SM parameters

$$G_F = 1.166378 \cdot 10^{-5} \text{ GeV}^{-2},$$
$$m_t = 173.2 \text{ GeV},$$
$$m_W = 80.385 \text{ GeV},$$
$$m_Z = 91.1876 \text{ GeV},$$
$$\Gamma_{t}^{\text{LO}} = 1.47848 \text{ GeV},$$
$$\Gamma_{W} = 2.0988 \text{ GeV},$$
$$\Gamma_{Z} = 2.50782 \text{ GeV},$$
$$\Gamma_{t}^{\text{NLO}} = 1.35159 \text{ GeV}. $$

(2.1)

All other particles including bottom quarks are treated as massless. Since leptonic $W$ gauge boson decays do not receive NLO QCD corrections, to account for some higher order effects the NLO QCD values for the gauge boson widths are used everywhere, i.e. for LO and NLO matrix elements. The electromagnetic coupling $\alpha$ is calculated from the Fermi constant $G_F$ in the $G_\mu$-scheme via

$$\alpha_{G_\mu} = \frac{\sqrt{2}}{\pi} G_F m_W^2 \left( 1 - \frac{m_W^2}{m_Z^2} \right).$$

(2.2)
For the emission of the isolated photon, however, $\alpha_{\text{QED}} = 1/137$ is used instead. The running of the strong coupling constant $\alpha_s$ with two-loop (one-loop) accuracy at NLO (LO) is provided by the LHAPDF interface [21]. The number of active flavours is set to $N_F = 5$, however, contributions induced by the bottom-quark parton density are neglected due to their numerical insignificance. Following recommendations of PDF4LHC [22] for the usage of parton distribution functions (PDFs) suitable for applications at the LHC Run II we employ CT14 [23], which is our default choice, MMHT14 [24] and NNPDF3.0 [25] PDFs. Our calculation, like any fixed-order calculations, contains a residual dependence on the renormalisation ($\mu_R$) and the factorisation scales ($\mu_F$) arising from the truncation of the perturbative expansion in $\alpha_s$. As a consequence, all observables depend on the values of $\mu_R$ and $\mu_F$ that are provided as input parameters. The theoretical uncertainty of the total cross section, associated with neglected higher order terms in the perturbative expansion, can be estimated by varying $\mu_R$ and $\mu_F$ in $\alpha_s$ and in the PDFs. We assume that $\mu_R$ and $\mu_F$ are set to a common value $\mu_R = \mu_F = \mu_0$. However, the scale dependence is evaluated by varying $\mu_R$ and $\mu_F$ independently in the range

$$
\frac{1}{2} \mu_0 \leq \mu_R, \mu_F \leq 2 \mu_0,
$$

with the additional condition

$$
\frac{1}{2} \leq \frac{\mu_R}{\mu_F} \leq 2.
$$

In practice, such restrictions are equivalent to evaluating the following scale variations

$$
\left( \frac{\mu_R}{\mu_0}, \frac{\mu_F}{\mu_0} \right) = \{(2, 1), (0.5, 1), (1, 1), (1, 0.5), (2, 2), (0.5, 0.5)\}.
$$

The final error is estimated from the envelope of the resulting cross sections. For the central value of the scale, $\mu_0$, we consider the fixed scale (the phase-space independent scale choice) $\mu_0 = m_t/2$ and the dynamic scale (the phase-space dependent scale choice) $\mu_0 = H_T/4$. The latter is defined on an event-by-event basis according to

$$
H_T = p_{T,e^+} + p_{T,\mu^-} + p_{T,\mu^+} + p_{T,b_1} + p_{T,b_2},
$$

where $p_{T,\text{miss}}$ denotes missing transverse momentum and $p_{T,b_1}, p_{T,b_2}$ are transverse momenta of the two $b$-jets. In the case of $pp \to t\bar{t}\gamma$ the transverse momentum of the hard photon is also included into the definition of $H_T$. Jets are constructed from final-state partons with pseudo-rapidity $|\eta| < 5$ with the help of the infrared safe anti-$k_T$ jet algorithm [26] with the separation parameter $R = 0.4$. For $e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}$ production exactly two $b$-jets, two charged leptons and missing transverse momentum are required. Additionally, for the $e^+\nu_e\mu^-\bar{\nu}_\mu bb\gamma$ production process an isolated hard photon is requested. The latter is defined with $p_{T,\gamma} > 25$ GeV (our default transverse momentum cut on the hard photon) and $|y_\gamma| < 2.5$ [2, 3]. To examine the stability of our theoretical predictions at NLO in QCD we also present results for the higher value of the $p_{T,\gamma}$ cut, namely for $p_{T,\gamma} > 50$ GeV. To ensure infrared safety we use the photon isolation prescription described in Ref. [27] that is based on a modified cone approach. The photon isolation condition is implemented
in the same way for quarks and gluons. For each parton $i$ we evaluate the distance in the
rapidity-azimuthal angle plane between this parton and the photon, according to
\[ \Delta R_{\gamma i} = \sqrt{\Delta y_{\gamma i}^2 + \Delta \phi_{\gamma i}^2} = \sqrt{(y_{\gamma} - y_i)^2 + (\phi_{\gamma} - \phi_i)^2}. \] (2.7)
We reject the event unless the following condition is fulfilled
\[ \sum_i E_{T,i} \Theta(R - R_{\gamma i}) \leq E_{T,\gamma} \left( \frac{1 - \cos(R)}{1 - \cos(R_{\gamma j})} \right), \] (2.8)
where $R \leq R_{\gamma j} = 0.4$ and $i$ runs over all partons. Moreover, $E_{T,i}$ is the transverse energy
of the parton $i$ and $E_{T,\gamma}$ is the transverse energy of the photon. We apply all other
selection criteria to jets if and only if their separation from the photon exceeds
$R_{\gamma j}$. A jet reconstructed inside the cone size $R_{\gamma j}$ is not subjected to any cuts. All final states have to
fulfil the subsequent selection criteria that mimic as closely as possible the ATLAS and the
CMS detector acceptances [2, 3]
\[ p_{T,\ell} > 30 \text{ GeV} \quad p_{T,b} > 40 \text{ GeV} \quad p_{T,\text{miss}} > 20 \text{ GeV} \]
\[ |y_{\ell}| < 2.5 \quad |y_b| < 2.5 \quad \Delta R_{\ell\gamma} > 0.4 \quad \Delta R_{bb} > 0.4 \quad \Delta R_{\ell\ell} > 0.4. \] (2.9)
We set no restriction on the kinematics of the extra (non $b$-)jet.

3 Differential Cross Sections at NLO in QCD

In this Section we present results for differential cross section distributions for both processes:
$pp \rightarrow e^+\nu_e\mu^-\bar{\nu}_{\mu}b\bar{b}\gamma + X$ at $\mathcal{O}(\alpha_s^3\alpha^5)$ and $pp \rightarrow e^+\nu_e\mu^-\bar{\nu}_{\mu}b\bar{b} + X$ at $\mathcal{O}(\alpha_s^3\alpha^4)$.
They are obtained for the LHC Run II energy of $\sqrt{s} = 13$ TeV. For brevity, we will refer to
these reactions as $pp \rightarrow t\bar{t}\gamma$ and $pp \rightarrow t\bar{t}$. To understand similarities and potential differences
between the two production processes, it is helpful to identify the dominant partonic
subprocesses. In both cases the most important production mechanism is via scattering of
two gluons. With our selection of cuts, the $gg$ channel contributes 79% (88%) to the LO
$pp \rightarrow t\bar{t}\gamma$ ($pp \rightarrow t\bar{t}$) cross section while the $q\bar{q} + \bar{q}q$ channels account for 21% (12%).
The dominance of the $gg$ production process in both cases suggests that $pp \rightarrow t\bar{t}\gamma$ and $pp \rightarrow t\bar{t}$ should show similar features in the kinematics of the final states, i.e. two charged leptons,
the missing transverse momentum and two $b$-jets. All differential cross sections that are
presented in the following have been obtained for the CT14 PDF set. For both production
processes we use the kinematic-independent factorisation and renormalisation scales
$\mu_R = \mu_F = \mu_0$ with the central value $\mu_0 = m_t/2$ rather than simply $\mu_0 = m_t$. Even though
the mass of the heaviest particle appearing in the process seems to be a more natural option,
the $\mu_0 = m_t/2$ scale choice is very well motivated by the fact that $pp \rightarrow t\bar{t}$ at the LHC is
dominated by $t$-channel gluon fusion, which favours smaller values of the scale. Additionally,
effects beyond NLO that include soft-gluon resummation for the hadronic cross-section
at next-to-leading logarithmic accuracy are smaller for $\mu_0 = m_t/2$ than for $\mu_0 = m_t$ [28, 29].
as we have explicitly checked with the help of the Top++ program [30]. From the QCD point of view both processes $pp \to t\bar{t}$ and $pp \to t\bar{t}\gamma$ are similar, which motivates our scale choice for $pp \to t\bar{t}\gamma$ as well.

We start with a collection of angular cross section distributions that are given in Figure 1. Specifically, we present the averaged rapidity distribution of the $b$-jet ($y_b$), the distance in the azimuthal angle rapidity plane between two $b$-jets ($\Delta R_{bb}$), the averaged rapidity of the charged lepton ($y_\ell$) as well as the distance in the azimuthal angle rapidity plane between two charged leptons ($\Delta R_{\ell\ell}$) as well. Results for two different values of the transverse momentum cut on the hard photon are shown. The NLO CT14 PDF set is employed and $\mu_R = \mu_F = \mu_0 = m_t/2$ is used.

**Figure 1.** Comparison of the normalised NLO differential cross sections for $pp \to e^+\nu_e, \mu^-\bar{\nu}_\mu, bb\gamma + X$ and $pp \to e^+\nu_e, \mu^-\bar{\nu}_\mu, b\bar{b} + X$ at the LHC with $\sqrt{s} = 13$ TeV. We present: the averaged rapidity of the $b$-jet ($y_b$), the distance in the azimuthal angle rapidity plane between two $b$-jets ($\Delta R_{bb}$), the averaged rapidity of the charged lepton ($y_\ell$) as well as the distance in the azimuthal angle rapidity plane between two charged leptons ($\Delta R_{\ell\ell}$). Results for two different values of the transverse momentum cut on the hard photon are shown. The NLO CT14 PDF set is employed and $\mu_R = \mu_F = \mu_0 = m_t/2$ is used.
Singularities stemming from the collinear $g \to b\bar{b}$ splitting are, however, screened off by the (effective) invariant mass cut of $m_{bb} \gtrsim 16$ GeV. The latter is implied once the $\Delta R_{bb}$ separation between the two $b$-jets of 0.4 is introduced by the jet algorithm together with the requirement of having both $b$-jets with transverse momentum larger than 40 GeV. For the two charged leptons the situation is rather simplified due to the fact that we simulate decays of the weak bosons to different lepton generations only, thus, virtual photon singularities stemming from collinear $\gamma \to \ell^+\ell^-$ decays are avoided. As might be observed in Figure 1 we can not see large shape differences in dimensionless observables, when the emission of the additional hard photon is included. This is in line with our expectation that the $t\bar{t}$ and $t\bar{t}\gamma$ production processes are similar from the QCD point of view. All the kinematical features described above are insensitive to the $p_{T,\gamma}$ cut as can be additionally observed in Figure 1 since results for two cases $p_{T,\gamma} > 25$ GeV and $p_{T,\gamma} > 50$ GeV are plotted.

In the next step, we consider dimensionful observables like for example the averaged transverse momentum of the $b$-jet, the averaged transverse momentum of the charged lepton as well as the invariant mass of the two charged leptons and the two $b$-jets. They are collected in Figure 2. Again shapes of all observables are not affected by the hard photon
Figure 3. Comparison of the normalised NLO differential cross sections for $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}\gamma + X$ and $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b} + X$ at the LHC with $\sqrt{s} = 13$ TeV. The top quark kinematics is shown. Specifically, the invariant mass of the reconstructed $t\bar{t}$ system ($m_{t\bar{t}}$) as well as the averaged transverse momentum ($p_{T,t}$) and rapidity ($y_t$) of the top quark are depicted. Results for two different values of the transverse momentum cut on the hard photon are shown. The NLO CT14 PDF set is employed and $\mu_R = \mu_F = \mu_0 = m_t/2$ is used.

emissions. In the case of $pp \to t\bar{t}\gamma$ all plotted spectra are slightly harder. However, this is a consequence of the additional $p_{T,\gamma}$ cut that effectively sets higher transverse momentum thresholds on the whole $t\bar{t}$ system, thus, consequently on all top quark decay products. Overall, for $t\bar{t}$ and $t\bar{t}\gamma$ production similarities in the jet activity and the way charged leptons are produced could be observed.

Subsequently, we turn our attention to the common underlying $t\bar{t}$ kinematics. To this end, in Figure 3 we depict the invariant mass of the $t\bar{t}$ system as well as the averaged transverse momentum and rapidity of the top quark. We note here, that top quarks are reconstructed from their decay products assuming exact reconstruction of the $W$ gauge boson. Specifically, we have defined $p(t) = p(b) + p(e^+) + p(\nu_e)$ and $p(\bar{t}) = p(\bar{b}) + p(\mu^-) + p(\bar{\nu}_\mu)$, where $b$ and $\bar{b}$ denotes the $b$-jets. We could observe harder spectra for the averaged transverse momentum of the top quark and for the invariant mass of the $t\bar{t}$ system in the case of the $t\bar{t}\gamma$ production process as compared to the corresponding distributions for $t\bar{t}$ production. Since we consider the whole reconstructed top quark system, not only its decay products separately, the higher transverse momentum threshold set by the $p_{T,\gamma}$ cut is more pronounced here. Moreover, for both processes the top quarks are predominantly produced in the central rapidity regions and in the back-to-back configuration.
Table 1. NLO cross sections for \( pp \rightarrow e^+\nu_e\bar{\mu} - \bar{\nu}_\mu b\bar{b} + X \) and \( pp \rightarrow e^+\nu_e\bar{\mu} - \bar{\nu}_\mu b\bar{b}\gamma + X \) at the LHC with \( \sqrt{s} = 13 \) TeV. Also included are theoretical errors as obtained from the scale variation. In the case of \( pp \rightarrow e^+\nu_e\bar{\mu} - \bar{\nu}_\mu b\bar{b}\gamma + X \) results for two different values of the \( p_{T,\gamma} \) cut are given. Various PDF sets are employed.

To summarise this part, let us repeat that as anticipated both \( \bar{t}t\gamma \) and \( \bar{t}t \) production processes are highly correlated. This fact will be exploited in the next section when the theoretical error for the \( \bar{t}t\gamma \) and \( \bar{t}t \) cross section ratio will be estimated. Additionally, conclusions drawn here are independent of the \( p_{T,\gamma} \) cut. Furthermore, they are not modified when the dynamical scale choice (\( \mu_R = \mu_F = \mu_0 = H_T/4 \)) is used instead for both processes or when different PDF sets (MMHT14 or NNPDF3.0) are employed.

4 Absolute Cross Sections at NLO in QCD

In this Section we present predictions for \( pp \rightarrow e^+\nu_e\bar{\mu} - \bar{\nu}_\mu b\bar{b} \) and \( pp \rightarrow e^+\nu_e\bar{\mu} - \bar{\nu}_\mu b\bar{b}\gamma \) at the LHC with \( \sqrt{s} = 13 \) TeV. NLO QCD cross sections are shown in Table 1 together with their theoretical errors from the scale dependence. Results are presented for the following two values of the transverse momentum cut on the hard photon \( p_{T,\gamma} > 25 \) GeV and \( p_{T,\gamma} > 50 \) GeV. The default CT14 PDF set is employed together with two additional PDF sets, namely MMHT14 and NNPDF3.0. Moreover, the following two scale choices, \( \mu_0 = m_t/2 \) and \( \mu_0 = H_T/4 \), are studied. In the first step we examine results that we have obtained for the CT14 PDF set. Looking at the total cross sections, which are mostly influenced by final state production relatively close to the \( \bar{t}t \) threshold, both scale choices are in equally good shape since the results agree well within the corresponding theoretical errors. However, the size of the theoretical uncertainties, especially in the case of \( \bar{t}t\gamma \) production, does depend on the scale choice. The latter finding tells us that the absolute cross sections for \( \bar{t}t \) and \( \bar{t}t\gamma \) production in the di-lepton top quark decay channel with the selection of cuts that we have imposed are not as inclusive observables as one would expect. Specifically, for \( \mu_0 = m_t/2 \) the NLO theoretical uncertainties for the \( pp \rightarrow \bar{t}t\gamma \) process are of the order of 14% for \( p_{T,\gamma} > 25 \) GeV and 17% for \( p_{T,\gamma} > 50 \) GeV. In the case of \( \bar{t}t \) production theoretical
uncertainties of the order of 9% have been estimated. For the dynamical scale choice in each case the theoretical uncertainties are well below 10%. Specifically, our judicious dynamical scale choice has allowed us to obtain 7% for $t\bar{t}$ production and 6% for $t\bar{t}\gamma$ production with $p_{T,\gamma} > 25$ GeV. In the latter case an increase of the transverse momentum cut to 50 GeV has resulted in the smaller theoretical error of 4%. These facts suggest that the proposed dynamical scale efficiently describes the multi-scale kinematics of the process.

Before discussing results for other PDF sets let us remind the reader in this place that in the case of on-shell $t\bar{t}$ and $t\bar{t}\gamma$ production for stable top quarks the size of the theoretical error as obtained from the scale dependence is not substantially reduced when the dynamical scale choice is used instead of the fixed one, of course as long as this scale is properly selected. To better outline this conclusion, we show the NLO QCD results for on-shell $t\bar{t}$ and $t\bar{t}\gamma$ production at the LHC, that we denote with a special index “(on-shell)” to distinguish them from the results with top quark and $W$ gauge boson decays and off-shell effects included. Results are generated with the same input parameters and the value of the $p_T,\gamma$ cut as given in Section 2. Additionally, we present these results for the following two scale choices $\mu_R = \mu_F = \mu_0 = m_t/2$ and $\mu_R = \mu_F = \mu_0 = E_T/4$. The dynamical scale choice, which is defined as

\[ E_T = \sqrt{p_T^2(t) + m_t^2} + \sqrt{p_T^2(\bar{t}) + m_t^2}, \]  

(4.1)

is similar to our previous choice $\mu_0 = H_T/4$. For obvious reasons the latter can not be applied for the on-shell $t\bar{t}$ and $t\bar{t}\gamma$ production. Once more in the case of $pp \to t\bar{t}\gamma$ the transverse momentum of the hard photon, $p_{T,\gamma}$, has been added to the definition of $E_T$.

Our results for top quark pair production can be summed up as

\[
\begin{align*}
\sigma_{t\bar{t}}^{\text{NLO (on-shell)}}(\mu_0 = m_t/2, \text{CT14}) &= 797.07^{+65.88}_{-82.41} \text{ pb}, \\
\sigma_{t\bar{t}}^{\text{NLO (on-shell)}}(\mu_0 = E_T/4, \text{CT14}) &= 770.11^{+74.61}_{-83.92} \text{ pb}.
\end{align*}
\]

(4.3)

For $pp \to t\bar{t}\gamma$, on the other hand, we have obtained

\[
\begin{align*}
\sigma_{t\bar{t}\gamma}^{\text{NLO (on-shell)}}(\mu_0 = m_t/2, \text{CT14}, p_{T,\gamma} > 25 \text{ GeV}) &= 2.035^{+0.137}_{-0.211} \text{ pb}, \\
\sigma_{t\bar{t}\gamma}^{\text{NLO (on-shell)}}(\mu_0 = E_T/4, \text{CT14}, p_{T,\gamma} > 25 \text{ GeV}) &= 1.901^{+0.209}_{-0.227} \text{ pb}.
\end{align*}
\]

(4.4)

The theoretical uncertainties for the on-shell $t\bar{t}$ and $t\bar{t}\gamma$ production process are at the level of 10% - 11% for $pp \to t\bar{t}$ and 10% - 12% for $pp \to t\bar{t}\gamma$.

\footnote{For example for $\mu_R = \mu_F = \mu_0$ set to $\mu_0 = E_T/4$ we have obtained the following results for $e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}$ and $e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{\nu}_\gamma$ production

\[
\begin{align*}
\sigma_{e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}}^{\text{NLO}}(\mu_0 = E_T/4, \text{CT14}) &= 1628.4^{+19.7}_{-69.9} \text{ fb}, \\
\sigma_{e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{\nu}_\gamma}^{\text{NLO}}(\mu_0 = E_T/4, \text{CT14}) &= 7.524^{+10.106}_{-3.93} \text{ fb}.
\end{align*}
\]

(4.2)

In the latter case the $p_{T,\gamma} > 25$ GeV cut has been applied on the hard photon.}
The theoretical uncertainties as obtained from the scale dependence of the studied cross sections are, however, not the only source of systematic uncertainties. Another source of theoretical uncertainties is associated with the parameterisation of PDFs. Thus, we have given in Table 1 NLO results for two additional PDF sets MMHT14 and NNPDF3.0 for $\mu_0 = m_t/2$. In this way the various theoretical assumptions that enter into parameterisation of the PDFs, which are difficult to quantify within a given scheme, are assessed. When comparing CT14 results for $\sigma_{t\bar{t}\gamma}^{NLO}$ and $\sigma_{t\bar{t}}^{NLO}$ with the corresponding numbers for MMHT14 and NNPDF3.0 we observe that the PDF uncertainties for NLO cross sections are of the order of 1% for MMHT14 for both production processes. In the case of the NNPDF3.0 set they are at the level of 4%. Taken very conservatively as the maximum of MMHT14 and NNPDF3.0 results PDF uncertainties for $t\bar{t}$ and $t\bar{t}\gamma$ are estimated to be of the order of 4%. Consequently, they are below the theoretical uncertainties due to scale dependence, which remain the dominant source of the theoretical systematics.

5 Cross Section Ratios at NLO in QCD

In the following, we study the cross section ratios. Our main goal here is to verify whether even further improvement in the accuracy of theoretical predictions can be obtained. More precisely we would like to see if theoretical uncertainties below 10% can be obtained for the fixed scale choice. On the other hand, in the case of the dynamical scale choice, that has been adopted for these studies, we would like to determine whether a few percent precision, i.e. comparable accuracy to that of NNLO calculations for $t\bar{t}$ production [31, 32], might be achieved. To this end results for $R = \sigma_{t\bar{t}\gamma}/\sigma_{t\bar{t}}$ cross sections ratio for the $p_{T,\gamma}$ cut of $p_{T,\gamma} > 25$ GeV and $p_{T,\gamma} > 50$ GeV are provided. They are constructed with the help of the absolute cross sections that are collected in Table 1. The theoretical error for the cross section ratio is estimated by calculating

$$R = \frac{\sigma_{t\bar{t}\gamma}^{NLO}(\mu_1)}{\sigma_{t\bar{t}}^{NLO}(\mu_2)},$$

(5.1)

where $\mu_1 = \mu_2 = \mu_0$ and due to correlation of $pp \rightarrow t\bar{t}\gamma$ and $pp \rightarrow t\bar{t}$ only the following combinations are considered, see e.g. Ref. [33]

$$\left(\mu_1, \mu_2\right) = \{(2,2), (0.5,0.5)\}.$$  

(5.2)

For $p_{T,\gamma} > 25$ GeV we have obtained the following results for $R$ at NLO in QCD

$$R(\mu_0 = m_t/2, CT14, p_{T,\gamma} > 25 \text{ GeV}) = (4.56 \pm 0.25) \cdot 10^{-3} (5\%),$$

$$R(\mu_0 = H_T/4, CT14, p_{T,\gamma} > 25 \text{ GeV}) = (4.62 \pm 0.06) \cdot 10^{-3} (1\%),$$

(5.3)

while for $p_{T,\gamma} > 50$ GeV our findings can be summarised as follows

$$R(\mu_0 = m_t/2, CT14, p_{T,\gamma} > 50 \text{ GeV}) = (1.89 \pm 0.16) \cdot 10^{-3} (8\%),$$

$$R(\mu_0 = H_T/4, CT14, p_{T,\gamma} > 50 \text{ GeV}) = (1.93 \pm 0.06) \cdot 10^{-3} (3\%).$$

(5.4)
The observed change in the value of $R$ for the scale variation is truly asymmetric. Thus, theoretical errors, which are quoted as well, are taken very conservatively as a maximum of these two results. Ratio results for our default $p_{T,\gamma}$ cut of 25 GeV for the two different scale choices are in perfect agreement within theoretical errors that are provided. This outcome is not affected by a higher value of the $p_{T,\gamma}$ cut, albeit, the absolute value of the ratio is smaller in the latter case. We notice that for $\mu_0 = m_t/2$ theoretical uncertainties from the scale variation are, indeed, below 10%, i.e. they are at the level of 5% and 8% respectively for the $p_{T,\gamma}$ cut of 25 GeV and 50 GeV. For $\mu_0 = H_T/4$, however, theoretical errors are substantially reduced down to 1% and 3%. Such precision is comparable to the precision one would rather expect from NNLO QCD results for top quark pair production. Thus, the ratio of $t\bar{t}\gamma$ and $t\bar{t}$ cross sections represents a very precise observable to be used at the LHC. One of the possible applications might be the measurement of the strength and the structure of the $t\bar{t}-\gamma$ vertex in $t\bar{t}\gamma$ production. The latter could shed some light on possible new physics that can reveal itself only once sufficiently precise theoretical predictions are available.

Once again, if on-shell $t\bar{t}$ and $t\bar{t}\gamma$ production is employed to construct the cross section ratio

$$
R^{\text{on-shell}} = \frac{\sigma_{t\bar{t}\gamma}^{\text{NLO (on-shell)}}}{\sigma_{t\bar{t}}^{\text{NLO (on-shell)}}},
$$

no substantial reduction in the theoretical uncertainties could be observed when replacing $\mu_R = \mu_F = \mu_0 = m_t/2$ with $\mu_R = \mu_F = \mu_0 = E_T/4$. Indeed, we can write

$$
R^{\text{on-shell}} (\mu_0 = m_t/2, CT14, p_{T,\gamma} > 25 \text{ GeV}) = (2.55 \pm 0.04) \cdot 10^{-3} (2\%),
$$

$$
R^{\text{on-shell}} (\mu_0 = E_T/4, CT14, p_{T,\gamma} > 25 \text{ GeV}) = (2.47 \pm 0.03) \cdot 10^{-3} (1%).
$$

It is worth mentioning at this point that, the theoretical error for $R^{\text{on-shell}}$ as calculated from the scale dependence is at the 2% level already for the fixed scale choice. From the experimental point of view, however, measurements in the phase space regions defined by the specific selection cuts that simulate as closely as possible detector response are more appropriate, simply because such measurements do not introduce additional and unnecessary uncertainties due to model-dependent extrapolations to parton level $t$ and $\bar{t}$ objects and to phase-space regions outside the detector sensitivity. Having on-shell results at hand we can also study the impact of top quark decays on the cross section ratio. We note that the central value of $R^{\text{on-shell}}$ is smaller by a factor of 1.8 when comparing to $R$. The cuts on the final state decay products in conjunction with hard photon emission from $b$-jets and charged leptons modify the ratio substantially. Since the set of selection cuts is different in both cases there is no particular reason why one would expect $R^{\text{on-shell}}$ and $R$ to be equal.

To assess the PDF uncertainties, we have recalculated the $R$ observable for two different PDF sets, namely MMHT14 and NNPDF3.0 with $\mu_0 = m_t/2$. For $p_{T,\gamma} > 25 \text{ GeV}$ theoretical
predictions for $\mathcal{R}$ are given by

$$
\mathcal{R} (\mu_0 = m_t/2, \text{MMHT14}, p_{T,\gamma} > 25 \text{ GeV}) = (4.54 \pm 0.26) \cdot 10^{-3} (6\%),
$$

$$
\mathcal{R} (\mu_0 = m_t/2, \text{NNPDF3.0}, p_{T,\gamma} > 25 \text{ GeV}) = (4.55 \pm 0.26) \cdot 10^{-3} (6\%),
$$

while for $p_{T,\gamma} > 50 \text{ GeV}$ we have found instead

$$
\mathcal{R} (\mu_0 = m_t/2, \text{MMHT14}, p_{T,\gamma} > 50 \text{ GeV}) = (1.87 \pm 0.17) \cdot 10^{-3} (9\%),
$$

$$
\mathcal{R} (\mu_0 = m_t/2, \text{NNPDF3.0}, p_{T,\gamma} > 50 \text{ GeV}) = (1.88 \pm 0.17) \cdot 10^{-3} (9\%).
$$

For the scale choice $\mu_0 = H_T/4$ we have estimated the size of the PDF uncertainties to be $\pm 0.02$ independently of the $p_{T,\gamma}$ cut. Thus, they are below 0.5% for $p_{T,\gamma} > 25 \text{ GeV}$ and of the order of 1% for $p_{T,\gamma} > 50 \text{ GeV}$. We can summarise our best NLO QCD predictions for the $\mathcal{R}$ observable at the LHC with $\sqrt{s} = 13 \text{ TeV}$ for $\mu_0 = H_T/4$ as

$$
\mathcal{R} (\mu_0 = H_T/4, \text{CT14}, p_{T,\gamma} > 25 \text{ GeV}) = (4.62 \pm 0.06 \text{ [scales]} \pm 0.02 \text{ [PDFs]}) \cdot 10^{-3}
$$

$$
\mathcal{R} (\mu_0 = H_T/4, \text{CT14}, p_{T,\gamma} > 50 \text{ GeV}) = (1.93 \pm 0.06 \text{ [scales]} \pm 0.02 \text{ [PDFs]}) \cdot 10^{-3},
$$

where we have included theoretical errors both from the scale dependence and from the PDFs. The former being a factor of 3 larger than the latter. Also for the $\mathcal{R}$ observable the dominant source of theoretical systematics is associated with the scale dependence.

We would like to note here that a meaningful theoretical error on $\mathcal{R}$ coming from the scale variation can be calculated for the first time only at NLO in QCD. At LO theoretical predictions for $\mathcal{R}$ for the fixed scale choice and for the CT14 PDF set with $p_{T,\gamma} > 25 \text{ GeV}$ are given by

$$
\mathcal{R} (\mu_0 = m_t/2, \text{CT14}, p_{T,\gamma} > 25 \text{ GeV}) = (4.94 \pm 0.08) \cdot 10^{-3} (2\%).
$$

The scale variation of $\mathcal{R}$ at LO is much smaller than at NLO. In the latter case we have obtained 5% instead. Since at LO we generate $pp \rightarrow t\bar{t}\gamma$ at $\mathcal{O}(\alpha_s^3)$ and $pp \rightarrow t\bar{t}$ at $\mathcal{O}(\alpha_s^4)$ we have the same order in $\alpha_s$ for both production processes and the dependence on $\alpha_s(\mu_R)$ cancels out in the cross section ratio. The only source of the scale dependence comes from variations in PDFs. The latter, however, also largely cancels out in the cross section ratio. The dependence on $\mu_R$ is introduced for the first time at NLO due to the virtual corrections. Specifically, different one loop structures in both processes give us a handle on $\alpha_s(\mu_R)$. Thus, the LO error is truly underestimated and only the NLO theoretical error should be considered as reliable.

In the next step, we would like to study the effect of various settings in the numerator and denominator of the $\mathcal{R}$ observable on the cross section ratio. In many experimental studies various MC programs are employed usually with the default scale choice implemented in a given program. Thus, one should assess the size of the additional theoretical uncertainties due to the mismatch to see if they are substantial or can be simply ignored. To this end for the CT14 PDF set we calculate cross section ratios assuming different scale choices in the
numerator ($\mu_1$) and in the denominator ($\mu_2$) of the $R$ observable. Specifically, we set $\mu_1$ to the fixed scale choice $m_t/2$ and $\mu_2$ to the dynamical scale choice $H_T/4$ and vice versa. With the $p_{T,\gamma} > 25$ GeV cut we have obtained the following results at NLO in QCD

$$R \left( \frac{\mu_1 = m_t/2}{\mu_2 = H_T/4}, CT14, p_{T,\gamma} > 25 \text{ GeV} \right) = (4.59 \pm 0.33) \cdot 10^{-3} (7\%),$$

$$R \left( \frac{\mu_1 = H_T/4}{\mu_2 = m_t/2}, CT14, p_{T,\gamma} > 25 \text{ GeV} \right) = (4.60 \pm 0.14) \cdot 10^{-3} (3\%).$$

(5.11)

In the case of $p_{T,\gamma} > 50$ GeV cut our NLO QCD findings can be summarised as

$$R \left( \frac{\mu_1 = m_t/2}{\mu_2 = H_T/4}, CT14, p_{T,\gamma} > 50 \text{ GeV} \right) = (1.90 \pm 0.19) \cdot 10^{-3} (10\%),$$

$$R \left( \frac{\mu_1 = H_T/4}{\mu_2 = m_t/2}, CT14, p_{T,\gamma} > 50 \text{ GeV} \right) = (1.92 \pm 0.09) \cdot 10^{-3} (5\%).$$

(5.12)

Even though in each case the central value of the cross section ratio has not been changed, we observe an increase of the theoretical error due to scale dependence. For the $p_{T,\gamma}$ cut of 25 GeV (50 GeV) the following increase of the relative error can be quoted: for $\mu_1 = m_t/2$ the rise from 5% (8%) to 7% (10%) and for $\mu_1 = H_T/4$ from 1% (3%) to 3% (5%). Therefore, in order to have $t\bar{t}\gamma$ production under excellent theoretical control the same scale choice should be employed for the generation of both processes $pp \to t\bar{t}\gamma$ and $pp \to t\bar{t}$. We can also study the impact of using various PDF sets for the $R$ observable. In that case our NLO QCD predictions for $p_{T,\gamma} > 25$ GeV are given by

$$R \left( \mu_0 = m_t/2, \frac{CT14}{MMHT14}, p_{T,\gamma} > 25 \text{ GeV} \right) = (4.50 \pm 0.23) \cdot 10^{-3} (5\%),$$

$$R \left( \mu_0 = m_t/2, \frac{MMHT14}{CT14}, p_{T,\gamma} > 25 \text{ GeV} \right) = (4.60 \pm 0.28) \cdot 10^{-3} (6\%).$$

(5.13)

$$R \left( \mu_0 = m_t/2, \frac{CT14}{NNPDF3.0}, p_{T,\gamma} > 25 \text{ GeV} \right) = (4.39 \pm 0.23) \cdot 10^{-3} (5\%),$$

$$R \left( \mu_0 = m_t/2, \frac{NNPDF3.0}{CT14}, p_{T,\gamma} > 25 \text{ GeV} \right) = (4.74 \pm 0.28) \cdot 10^{-3} (6\%).$$

For $p_{T,\gamma} > 50$ GeV have obtained instead

$$R \left( \mu_0 = m_t/2, \frac{CT14}{MMHT14}, p_{T,\gamma} > 50 \text{ GeV} \right) = (1.87 \pm 0.15) \cdot 10^{-3} (8\%),$$

$$R \left( \mu_0 = m_t/2, \frac{MMHT14}{CT14}, p_{T,\gamma} > 50 \text{ GeV} \right) = (1.90 \pm 0.17) \cdot 10^{-3} (9\%).$$

(5.14)

$$R \left( \mu_0 = m_t/2, \frac{CT14}{NNPDF3.0}, p_{T,\gamma} > 50 \text{ GeV} \right) = (1.82 \pm 0.15) \cdot 10^{-3} (8\%),$$

$$R \left( \mu_0 = m_t/2, \frac{NNPDF3.0}{CT14}, p_{T,\gamma} > 50 \text{ GeV} \right) = (1.96 \pm 0.18) \cdot 10^{-3} (9\%).$$
Although the choice of various PDF sets in the cross section ratio is not theoretically very well motivated it does not affect the estimation of the theoretical errors. Overall, unlike the theoretical uncertainties due to the different scale choice, additional undesired PDF uncertainties are negligible. Let us note here, that the issue of choosing the same value of $\mu_0$ for both production processes is going to play a crucial role when various differential cross section ratios will be constructed.

To summarise this part of the paper, we discuss the stability of the cross section ratio against higher order corrections. To this end we also comment on the size of NLO QCD corrections to the absolute $t\bar{t}\gamma$ cross section at the LHC. With $\mu_R = \mu_F = \mu_0$ set to $\mu_0 = m_t/2$ and for the CT14 PDF set the full $pp \rightarrow t\bar{t}\gamma$ cross section receives negative and moderate NLO corrections of 10% (13%) for the $p_T,\gamma$ cut of 25 GeV (50 GeV). The NLO QCD corrections to cross section ratio $R$ are similar. Specifically, they are also negative and of the order of 8% and 11% depending on the $p_T,\gamma$ cut. For the dynamical scale choice $\mu_0 = H_T/2$ the NLO QCD corrections to the absolute $pp \rightarrow t\bar{t}\gamma$ cross section are positive and small of the order of 2% (5%) for $p_T,\gamma > 25$ GeV ($p_T,\gamma > 50$ GeV). The size of NLO QCD corrections to the $R$ observable as evaluated with $\mu_0 = H_T/4$ is the same. Thus, the cross section ratio is very stable and behaves similarly as the absolute $t\bar{t}\gamma$ cross section when higher order effects are incorporated.

6 Differential Cross Section Ratios at NLO in QCD

In the following we present results for differential cross section ratios defined according to

$$R_X = \left( \frac{d\sigma^{NLO}_{t\bar{t}\gamma}}{dX} (\mu_1) \right) / \left( \frac{d\sigma^{NLO}_{t\bar{t}\gamma}}{dX} (\mu_2) \right)$$

(6.1)

where $X$ stands for the observable that is under scrutiny. In Figure 4 we present differential cross section distributions as a function of the invariant mass of two $b$-jets. Thus, in that case we have $X = m_{bb}$ and $R_{m_{bb}}$. The upper plots show absolute NLO predictions for the $e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}\gamma + X$ production process at the LHC at the centre-of-mass energy of $\sqrt{s} = 13$ TeV. Results are given for $\mu_R = \mu_F = \mu_0$, where $\mu_0 = m_t/2$ or $\mu_0 = H_T/4$, for the CT14 PDF set and for two different values of the $p_T,\gamma$ cut, i.e. $p_T,\gamma > 25$ GeV and $p_T,\gamma > 50$ GeV. Also provided are corresponding uncertainty bands resulting from scale variations. The lower panels display differential cross section ratios together with their uncertainty bands. In the first case, that is presented in the middle panel, we have employed $\mu_1 = \mu_2 = \mu_0$, where $\mu_0 = m_t/2$ or $\mu_0 = H_T/4$. In the second case, which is shown in the bottom panel, $\mu_1 \neq \mu_2$ has been assumed. For that case two options are investigated, $(\mu_1 = m_t/2)/ (\mu_2 = H_T/4)$ and $(\mu_1 = H_T/4)/ (\mu_2 = m_t/2)$. For the fixed scale choice for both values of the $p_T,\gamma$ cut we have observed that the theoretical error due to scale dependence for the absolute differential cross section is in the range of 40%–45% towards the end of the $m_{bb}$ spectrum. On the other hand, for $\mu_0 = H_T/4$ theoretical uncertainties up to only 6%–7% have been estimated in the same region. The situation is substantially changed when the cross section ratio, $R_{m_{bb}}$, is studied instead. In the case of $\mu_1 = \mu_2 = \mu_0 = m_t/2$ a reduction almost by a factor of 2 can be noticed. Indeed, theoretical uncertainties of the order of 20% are
Figure 4. Differential cross section distributions as a function of the invariant mass of two b-jets for the $pp \rightarrow e^+\nu_e\mu^−\bar{\nu}_\mu b\bar{b}\gamma + X$ process at the LHC run II with $\sqrt{s} = 13$ TeV. The upper plots show absolute NLO predictions for $p_{T,\gamma} > 25$ GeV (left panel) and $p_{T,\gamma} > 50$ GeV (right panel) together with the corresponding uncertainty bands resulting from scale variations. Renormalisation and factorisation scales are set to the common value $\mu_R = \mu_F = \mu_0$ where $\mu_0 = m_t/2$ and $H_T/4$. The CT14 PDF set is employed. The lower panels display differential cross section ratios together with their uncertainty bands. In the first case (middle panel), the same fixed (dynamical) scale choice is employed in the numerator and denominator of the cross section ratio. In the second case (bottom panel), different scale choices in the numerator and in the denominator have been assumed.

estimated at the end of the $m_{bb}$ spectrum. This finding is also independent of the $p_{T,\gamma}$ cut. For $\mu_1 = \mu_2 = \mu_0 = H_T/4$, however, one can acquire theoretical uncertainties of the order of 1% – 2% in the whole plotted range. This shows that the differential cross section ratio is also a very precise observable to be studied together with the total cross section ratio at the LHC to constrain physics beyond the SM for example via constraining anomalous top quark couplings. It can also be used to extract the electric charge of the top quark or the top quark charge asymmetry with a very high precision. When the scales $\mu_1$ and $\mu_2$ are chosen independently, however, the size of theoretical uncertainties has increased dramatically up to 30% – 40%, as can be clearly seen in Figure 4. Thus, choosing the same $\mu_0$ in the numerator and denominator of $R_{m_{bb}}$ is essential for building a high precision observable, otherwise the theoretical errors are drastically overestimated.

Similar conclusions can be drawn for differential cross section distribution as a function of the invariant mass of two charged leptons. This observable is plotted in Figure 5 again for
Figure 5. Differential cross section distributions as a function of the invariant mass of two charged leptons for the $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}\gamma + X$ process at the LHC run II with $\sqrt{s} = 13$ TeV. The upper plots show absolute NLO predictions for $p_{T,\gamma} > 25$ GeV (left panel) and $p_{T,\gamma} > 50$ GeV (right panel) together with the corresponding uncertainty bands resulting from scale variations. Renormalisation and factorisation scales are set to the common value $\mu_R = \mu_F = \mu_0$ where $\mu_0 = m_t/2$ and $H_T/4$. The CT14 PDF set is employed. The lower panels display differential cross section ratios together with their uncertainty bands. In the first case (middle panel), the same fixed (dynamical) scale choice is employed in the numerator and the denominator of the cross section ratio. In the second case (bottom panel), different scale choices in the numerator and in the denominator have been assumed.

$p_{T,\gamma} > 25$ GeV and $p_{T,\gamma} > 50$ GeV. The advantage of this observable in comparison to the invariant mass of two $b$-jets lies, however, in the fact that measurements of lepton kinematic observables are particularly precise at the LHC due to the excellent lepton energy resolution of the ATLAS and CMS detectors. Moreover, the reconstruction of the top quarks is not required to construct $m_{t\bar{t}}$. For $m_{b\bar{b}}$, on the other hand, good $b$-jet tagging efficiency and low light jet misstag rate is mandatory. For the cross section ratio, $R_{m_{t\bar{t}}}$, with the dynamical scale choice $\mu_1 = \mu_2 = H_T/4$ (the fixed scale choice $\mu_1 = \mu_2 = m_t/2$) the theoretical uncertainties of the order of $1\% - 4\%$ ($20\% - 25\%$) have been estimated. These should be compared to uncertainties up to $10\%$ ($50\%$) for the absolute differential cross section. Again, our findings mildly depend on the $p_{T,\gamma}$ cut. When different scales are applied to the numerator and the denominator of $R_{m_{t\bar{t}}}$ theoretical uncertainties in the tail of the distribution have increased up to $35\% - 40\%$ for $\mu_1 = m_t/2$ and $\mu_2 = H_T/4$ whereas in the case of $\mu_1 = H_T/4$ and $\mu_2 = m_t/2$ they are in the range of $50\% - 60\%$. 

\[\text{LHC13, } t\bar{t}_\gamma, \text{ CT14} \]

\[p_{T,\gamma} \geq 25 \]

\[\frac{d\sigma/dm_{ll}}{[fb/GeV]} \]

\[\mu_0 = m_t/2 \]

\[\mu_0 = H_T/4 \]

\[\text{LHC13, } t\bar{t}_\gamma, \text{ CT14} \]

\[p_{T,\gamma} \geq 50 \]

\[\frac{d\sigma/dm_{ll}}{[fb/GeV]} \]

\[\mu_0 = m_t/2 \]

\[\mu_0 = H_T/4 \]

\[\text{LHC13, } t\bar{t}_\gamma, \text{ CT14} \]

\[p_{T,\gamma} \geq 25 \]

\[\frac{d\sigma/dm_{ll}}{[fb/GeV]} \]

\[\mu_0 = m_t/2 \]

\[\mu_0 = H_T/4 \]

\[\text{LHC13, } t\bar{t}_\gamma, \text{ CT14} \]

\[p_{T,\gamma} \geq 50 \]

\[\frac{d\sigma/dm_{ll}}{[fb/GeV]} \]

\[\mu_0 = m_t/2 \]

\[\mu_0 = H_T/4 \]
In Figure 6 the differential cross section distribution as a function of the difference in azimuthal angle between the two charged leptons, $\Delta \phi_{ll} = |\phi_{l1} - \phi_{l2}|$, is presented. The $\Delta \phi_{ll}$ observable is also measured very precisely at the LHC by both the ATLAS and CMS collaborations. It can be used for example to construct the leptonic charge asymmetry, $A^\ell\ell$, which is sensitive to signals of numerous beyond the SM scenarios, where among others new heavy states might be produced. In general, angular distributions of charged leptons are of huge importance since they reflect spin correlations of the top quark pair and can be employed to probe the $CP$ numbers of such new states. For the fixed (dynamical) scale choice theoretical uncertainties for $\Delta \phi_{ll}$ in the region given by $\Delta \phi_{ll} \gtrsim 2.5$ are of the order of $40\% - 50\%$ ($15\% - 20\%$) depending on the transverse momentum cut on the hard photon. When the cross section ratio $R_{\Delta \phi_{ll}}$ is investigated instead theoretical errors of $20\% - 30\%$ for $\mu_0 = m_t/2$ and $2\% - 3\%$ for $\mu_0 = H_T/4$ can be estimated in that region. Overall, for $\Delta \phi_{ll} \in (0, \pi)$ with $\mu_0 = m_t/2$ or with $\mu_0 = H_T/4$ theoretical uncertainties for $R_{\Delta \phi_{ll}}$ are in the range $1\% - 30\%$, $1\% - 6\%$ respectively. As in the previous cases when $\mu_1 \neq \mu_2$ is
set instead substantial overestimation of the theoretical uncertainties can be observed for \( \mathcal{R}_{\Delta \phi lt} \). For example for \( \Delta \phi lt \gtrsim 2.5 \) an increase from 20\% – 30\% up to 30\% – 40\% has been procured once \( \mu_1 = m_t/2 \) and \( \mu_2 = H_T/4 \) have been assumed, while for \( \mu_1 = H_T/4 \) and \( \mu_2 = m_t/2 \) we have obtained a change from 2\% – 3\% to 15\%.

Finally, in Figure 7 the differential cross section as a function of the transverse momentum of the hardest charged lepton is shown. This observable is also sensitive to effects of possible new physics beyond the SM [34]. Among others, it can be used to test exotic physics scenarios where top like quarks with the electric charge of \( Q_1 = -4/3 \) might be produced. For the absolute \( t\bar{t}\gamma \) cross section theoretical uncertainties are up to 30\% – 45\% for \( \mu_0 = m_t/2 \) and up to 8\% for \( \mu_0 = H_T/4 \). Once the cross section ratio, \( \mathcal{R}_{p_T,\ell_1} \), is investigated theoretical uncertainties have been substantially reduced down to 20\% – 30\% for the fixed scale choice and to 4\% – 5\% for the phase-space dependent scale choice. If we assume different scales in \( \mathcal{R}_{p_T,\ell_1} \), i.e. \( \mu_1 = m_t/2 \) and \( \mu_2 = H_T/4 \), theoretical errors comparable to
these quoted for the absolute $t\bar{t}\gamma$ cross sections with $\mu_0 = m_t/2$ have been evaluated. On the other hand, setting $\mu_1 = H_T/4$ and $\mu_2 = m_t/2$, has resulted in theoretical uncertainties for $R_{pT,\ell}$ maximally up to 15%.

To summarise this part of the paper we found that, the theoretical uncertainties due to scale dependence of the order of 1% - 6% have been obtained for the studied cross section ratios if $\mu_1 = \mu_2 = \mu_0 = H_T/4$ has been employed to construct $R_X$, where $X$ stands for $X = m_{bb}, m_{\ell\ell}, \Delta\phi_{\ell\ell}, pT,\ell_1$.

7 Conclusions

The purpose of the paper is to obtain more precise theoretical predictions for $t\bar{t}\gamma$ production in the di-lepton top quark decay channel for the LHC Run II energy of $\sqrt{s} = 13$ TeV without the need of including terms beyond NLO in the perturbation expansion in $\alpha_s$. To this end cross section ratios $R = \sigma^{\text{NLO}}_{pp\rightarrow t\bar{t}\gamma}(\mu_1)/\sigma^{\text{NLO}}_{pp\rightarrow t\bar{t}}(\mu_2)$ have been studied. Fully realistic NLO computations for $t\bar{t}$ and $t\ell\gamma$ production have been employed. They are based on LO and NLO matrix elements for $e^+\nu_e\mu^−\bar{\nu}_\mu b\bar{b}$ and $e^+\nu_e\mu^−\bar{\nu}_\mu b\bar{b}\gamma$ production processes that include all resonant and non-resonant top quark and $W$ gauge boson Feynman diagrams, interferences, and off-shell effects of $t$ and $W$. Various renormalisation and factorisation scale choices and parton density functions have been examined to assess their impact on the cross section ratio. Our best NLO QCD predictions for the $R$ observable have been obtained for $\mu_1 = \mu_2 = \mu_0 = H_T/4$ and can be summarised as follows

$$R(\mu_0 = H_T/4, CT14, p_{T,\gamma} > 25 \text{ GeV}) = (4.62 \pm 0.06 \text{ [scales]} \pm 0.02 \text{ [PDFs]}) \cdot 10^{-3}$$

$$R(\mu_0 = H_T/4, CT14, p_{T,\gamma} > 50 \text{ GeV}) = (1.93 \pm 0.06 \text{ [scales]} \pm 0.02 \text{ [PDFs]}) \cdot 10^{-3}. \quad (7.1)$$

The theoretical uncertainties due to scale dependence have been estimated to be of the order of 1% for $p_{T,\gamma} > 25$ GeV and 3% for $p_{T,\gamma} > 50$ GeV. The theoretical uncertainties due to various PDF parameterisations, on the other hand, are 0.5% and 1% respectively. Such small theoretical uncertainties are normally available only in the case of top quark pair production at NNLO in QCD. Thus, the cross section ratio has proven to be a very precise observable that should be measured at the LHC. There are many possible applications, including, but not limited to, precise measurements of the top quark charge as well as searches for new physics effects that can reveal themselves only when a few percent precision on the theory side is available. For the fixed scale choice, which is still commonly used in experimental analyses, our finding for $\mu_1 = \mu_2 = \mu_0 = m_t/2$ are given by

$$R(\mu_0 = m_t/2, CT14, p_{T,\gamma} > 25 \text{ GeV}) = (4.56 \pm 0.25 \text{ [scales]} \pm 0.02 \text{ [PDFs]}) \cdot 10^{-3},$$

$$R(\mu_0 = m_t/2, CT14, p_{T,\gamma} > 50 \text{ GeV}) = (1.89 \pm 0.16 \text{ [scales]} \pm 0.02 \text{ [PDFs]}) \cdot 10^{-3}. \quad (7.2)$$

Also in this case theoretical errors due to scale dependence of the order of a few percent, 5% for $p_{T,\gamma} > 25$ GeV and 8% for $p_{T,\gamma} > 50$ GeV, have been estimated. We have also shown that such high precision can only be obtained if $\mu_1$ and $\mu_2$ are set to a common scale. Otherwise, theoretical uncertainties from the scale dependence are overestimated.
We have argued on the similarity of the two processes, that using the same scale for both is well justified.

Subsequently, we have turned our attention to differential cross section distributions. Four observables have been presented at the differential level for the $t\bar{t}\gamma$ production process at the LHC. Specifically, we have shown the invariant mass of two $b$-jets ($m_{bb}$), the invariant mass of two charged leptons ($m_{\ell\ell}$), the difference in azimuthal angle between two charged leptons ($\Delta\phi_{\ell\ell}$) and the transverse momentum of the hardest charged lepton ($p_{T,\ell_1}$). Afterwards, we have calculated differential cross section ratios for these observables according to

$$R_X = \left(\frac{d\sigma^{\text{NLO}}_{t\bar{t}\gamma}(\mu_1)}{dX}\right)\left(\frac{d\sigma^{\text{NLO}}_{t\bar{t}}(\mu_2)}{dX}\right)^{-1}, \quad (7.3)$$

where $X = m_{bb}, m_{\ell\ell}, \Delta\phi_{\ell\ell}$ and $p_{T,\ell_1}$. A clear conclusion could be drawn from our considerations. For observables that we have presented, which are also important for beyond the SM physics searches, the most precise predictions for $R_X$ have been obtained for $\mu_1 = \mu_2 = \mu_0$. Especially interesting conclusions have been reached for the case of the dynamical scale choice, i.e. $\mu_0 = H_T/4$. For all observables that have been investigated, theoretical uncertainties in the range of $1\%-6\%$ have been estimated. These findings are independent of the transverse momentum cut on the isolated hard photon. Such precise theoretical predictions at the differential level should be now employed to indirectly search for new physics at the LHC. When different scale choices $\mu_1 \neq \mu_2$ for $R_X$ have been assumed instead, theoretical uncertainties have been dramatically overestimated. Thus, care must be taken to ensure that $\mu_1$ and $\mu_2$ are as similar as possible when building the $R_X$ observables to be used in experimental studies. Based on our studies we advocate for the $H_T$ based scale choice for $\mu_1$ and $\mu_2$ in $R_X$. Definitely, mixing dynamical and fixed scales in $t\bar{t}\gamma$ and $t\bar{t}$ production introduces additional and unnecessary theoretical uncertainties that should be avoided.

As a further matter let us note here that, from the experimental point of view measurements in the fiducial phase space, which is the phase space defined by the specific selection cuts that simulate detector response as closely as possible, are the most appropriate for new physics searches in the top quark sector and for precision measurements of top quark properties within the SM theory of particle physics. The reason being that such measurements do not introduce additional and unnecessary systematic uncertainties due to model-dependent extrapolations to parton level $t$ and $\bar{t}$ objects and to phase-space regions outside the detector sensitivity. Therefore, our theoretical predictions for observables like $R_{m_\ell\ell}$, $R_{\Delta\phi_{\ell\ell}}$ and $R_{p_{T,\ell_1}}$, which should be marginally affected by parton shower effects, might be directly compared with experimental data at the fiducial level that are collected at the LHC by the ATLAS and CMS experimental collaborations.

On the technical side let us mention that all our results have been generated with the help of the Helac-NLO MC framework. The final results are available (upon request) as Ntuple files [35]. In detail, they are stored in the form of modified Les Houches [36] and ROOT event files [37] that might be directly used for experimental studies at the LHC.
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