Perturbative Dynamics of Quantum General Relativity

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Abstract

The quantum theory of General Relativity at low energy exists and is of the form called "effective field theory". In this talk I describe the ideas of effective field theory and its application to General Relativity.

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1 Introduction

The conference organizers originally suggested that the title of this talk be: “Gravity and the Quantum: The view from particle physics”. While it is presumptuous for me to claim to speak for all particle physicists, there is in fact a widely held “view” within the particle community that carries an important insight not always appreciated within the gravity community. In this visual analogy, we see clearly a variety of beautiful low-lying hills representing the Standard Model and its applications in known physics. However, there are two sets of clouds on the horizon which ultimately obscure our view. One cloud is located at 1 TeV, just beyond the reach of present accelerators. Beyond this scale, we expect to find the physics which governs electroweak symmetry breaking, with the expectation being that we will uncover new particles and new interactions that change the way that we think of Nature. The other cloud is more ominous, sitting at the Planck scale. Beyond this, we don’t know what to expect, but it likely will be something totally new. Neither the Standard Model nor General Relativity is likely to emerge unchanged beyond this scale. So this “view” recognizes that the scenery that we see (our low energy theory) is likely to change if we are ever able to see beyond the “clouds”.

The insight behind this view suggests a new way of looking at quantum General Relativity. Since we only know that General Relativity is valid at low energy, the key requirement is that the quantum theory can be applied to gravity at present scales. What goes on beyond the Planck scale is a matter of speculation, but gravity and quantum mechanics had better go together at the scales where they are both valid.

The good news is that the quantum theory of General Relativity at low energies exists and is well behaved. It is of the form of a type of field theory called Effective Field Theory [1]. This is true no matter what the ultimate high-energy theory turns out to be. Given all the work that has gone into quantum gravity, I feel that this is a significant result. The development of effective field theory is an important part of the past decade, and anyone who cares about field theory should learn about it. It has become a standard way of calculating within particle physics, and the way of thinking is widely internalized in the younger generation. In fact, it is would be a reasonably common expectation of young theorists that it is possible that the gravitational effective field theory may turn out to be a better quantum theory
than the Standard Model, as the former may extend in validity all the way up to the Planck scale, while the Standard Model will likely be fundamentally modified at 1 TeV.

This talk describes some of the features of the effective field theory of General Relativity \(^2\) \(^3\). The effective field theory completes a program for quantizing General Relativity that goes back to Feynman and De Witt \(^4\), and which has received contributions from many researchers over the years. Earlier work focused on quantization and on the divergence structure at high energy. The contribution of effective field theory is to shift the focus back to the low energy where the theory is valid, and to classify the reliable predictions. The low energy quantum effects are distinct from whatever goes on at high energy. Of course, the effective theory does not answer all the interesting questions that we have about the ultimate theory. However, in principle it answers all those questions that we have a right to know with our present state of knowledge about the content of the theory. I will attempt to be clear about the limits of the effective theory as well as its virtues.

The outcome of this is that we need to stop spreading the falsehood that General Relativity and Quantum Mechanics are incompatible. They go together quite nicely at ordinary energies. Rather, a more correct statement is that we do not yet know the ultimate high energy theory in Nature. This change in view is important for the gravity community to recognize, because it carries the implication that the ultimate theory is likely to be something new, not just a blind continuation of General Relativity beyond the Planck scale.

### 2 Effective Field Theory

First let’s describe effective field theory in general. Once you understand the basic ideas it is easy to see how it applies to gravity. The phrase “effective” carries the connotation of a low energy approximation of a more complete high energy theory. However, the techniques to be described don’t rely at all on the high energy theory. It is perhaps better to focus on a second meaning of “effective”, “effective” \(\sim\) “useful”, which implies that it is the most effective thing to do. This is because the particles and interactions of the effective theory are the useful ones at that energy. An “effective Lagrangian” is a local Lagrangian which describes the low energy interactions. “Effective
field theory” is more than just the use of effective Lagrangians. It implies a specific full field-theoretic treatment, with loops, renormalization etc. The goal is to extract the full quantum effects of the particles and interactions at low energies.

The key to the separation of high energy from low is the uncertainty principle. When one is working with external particles at low energy, the effects of virtual heavy particles or high energy intermediate states involve short distances, and hence can be represented by a series of local Lagrangians. A well known example is the Fermi theory of the weak interactions, which is a local effective Lagrangian describing the effect of the exchange of a heavy W boson. This locality is true even for the high energy portion of loop diagrams. An example of the latter is the high energy portion of the fermion self energy, which is equivalent to a mass counterterm in a local Lagrangian. In contrast, effects that are non-local, where the particles propagate long distances, can only come from the low energy part of the theory. The exchange of a massless photon at low energy can never be represented by a local Lagrangian. From this distinction, we know that we can represent the effects of the high energy theory by the most general local effective Lagrangian.

The second key is the energy expansion, which orders the infinite number of terms within this most general Lagrangian in powers of the low energy scale divided by the high energy scale. To any given order in this small parameter, one needs to deal with only a finite number of terms (with coefficients which in general need to be determined from experiment). The lowest order Lagrangian can be used to determine the propagators and low energy vertices, and the rest can be treated as perturbations. When this theory is quantized and used to calculate loops, the usual ultraviolet divergences will share the form of the most general Lagrangian (since they are high energy and hence local) and can be absorbed into the definition of renormalized couplings. There are however leftover effects in the amplitudes from long distance propagation which are distinct from the local Lagrangian and which are the quantum predictions of the low energy theory.

This technique can be used in both renormalizable and non-renormalizable theories, as there is no need to restrict the dimensionality of terms in the Lagrangian. (Note that the terminology is bad: we are able to renormalize non-renormalizable theories!) Renormalizable theories are a particularly compact and predictive class of theories. However, many physical effects require non-renormalizable interactions and these need not destroy the quantum theory.
In fact, a common calculational device is to isolate the relevant interactions only, even if this implies a non-renormalizable theory, and to use the techniques of effective field theory to perform a simpler calculation than if one were to compute using the full theory. This is done in Heavy Quark Effective Theory \[5\] as well as in the theory of electroweak radiative corrections.

The effective field theory which is most similar to general relativity is chiral perturbation theory \[6\], which describes the theory of pions and photons which is the low energy limit of QCD. The theory is highly nonlinear, with a lowest order Lagrangian which can be written with the exponential of the pion fields

\[
\mathcal{L} = \frac{F_\pi^2}{4} \text{Tr} \left( \nabla_\mu U \nabla^\mu U^\dagger \right) + \frac{F_\pi^2 m_\pi^2}{4} \text{Tr} \left( U + U^\dagger \right) + \frac{F_\pi^2 i \tau^i \pi^i(x)}{F_\pi},
\]

with \( \tau^i \) being the SU(2) Pauli matrices and \( F_\pi = 92.3 \text{MeV} \) being a dimensional coupling constant. This shares with general relativity the dimensional coupling, the non-renormalizable nature and the intrinsically nonlinear Lagrangian. This theory has been extensively studied theoretically, to one and two loops, and experimentally. There are processes which clearly reveal the presence of loop diagrams. In my talk, I displayed some of the predictions and experimental tests of chiral perturbation theory, most of which were taken from the book published with my co-authors \[1\]. The point was to illustrate the fact that effective field theory is not just an idea, but is a practical tool that is applied in real-world physics. In a way, chiral perturbation theory is the model for a complete non-renormalizable effective field theory in the same way that QED serves as a model for renormalizable field theories.

3 Overview of the gravitational effective field theory

At low energies, general relativity automatically behaves in the way that we treat effective field theories. This is not a philosophical statement implying that there must be a deeper high energy theory of which general relativity is the low energy approximation. Rather, it is a practical statement.
Whether or not general relativity is truly fundamental, the low energy quantum interactions must behave in a particular way because of the nature of the gravitational couplings, and this way is that of effective field theory.

The Einstein action, the scalar curvature, involves two derivatives on the metric field. Higher powers of the curvature, allowed by general covariance, involve more derivatives and hence the energy expansion has the form of a derivative expansion. The higher powers of the curvature in the most general Lagrangian do not cause problems when treated as low energy perturbations [7]. The Einstein action is in fact readily quantized, using gauge-fixing and ghost fields ala Feynman, DeWitt, Faddeev, Popov [4]. The background field method used by ’tHooft and Veltman [8] is most beautiful in this context because it allows one to retain the symmetries of general relativity in the background field, while still gauge-fixing the quantum fluctuations. The applications of these methods allow the quantization of general relativity in as straight-forward a way as QCD is quantized.

The problem with the field theory program comes not at the level of quantization, but in attempting to make meaningful calculations. The dimensionful nature of the gravitational coupling implies that loop diagrams (both the finite and infinite parts) will generate effects at higher orders in the energy expansion [9]. In previous times when we only understood renormalizable field theory, this was a problem because the divergences could not be dealt with by a renormalization of the original Lagrangian. However, in effective field theory, one allows a more general Lagrangian. Since the divergences come from the high energy portion of loop integrals, they will be equivalent to a local term in a Lagrangian. Since the effective Lagrangian allows all terms consistent with the theory, and each term is governed by one or more parameters describing its strength, there is a parameter available corresponding to each divergence. We absorb the high energy effects of the loop diagram into a renormalized parameter, which also contains other unknown effects from the ultimate high energy theory. The one and two loop counterterms for graviton loops are known [8, 10] and, as expected, go into the renormalization of the coefficients in the Lagrangian. However, these are not really predictions of the effective theory. The real action comes at low energy.

How in practice does one separate high energy from low? Fortunately, the calculation takes care of this automatically, although it is important to know what is happening. Again, the main point is that the high energy
effects share the structure of the local Lagrangian, while low energy effects are different. When one completes a calculation, high energy effects will appear in the answer in the same way that the coefficients from the local Lagrangian will. One cannot distinguish these effects from the unknown coefficients. However, low energy effects are anything that has a different structure. Most often the distinction is that of analytic versus non-analytic in momentum space. Analytic expressions can be Taylor expanded in the momentum and therefore have the behavior of an energy expansion, much like the effects of a local Lagrangian ordered in a derivative expansion. However, non-analytic terms can never be confused with the local Lagrangian, and are intrinsically non-local. Typical non-analytic forms are $\sqrt{-q^2}$ and $\ln(-q^2)$. These are always consequences of low energy propagation.

Having provided this brief overview of the way that effective field theory may be applied to general relativity, let me be a bit more explicit about some of these steps.

4 The energy expansion in general relativity

What is the rationale for choosing the gravitational action proportional to $R$ and only $R$? It is not due to any symmetry and, unlike other theories, cannot be argued on the basis of renormalizability. However physically the curvature is small so that in most applications $R$ terms would be yet smaller. This leads to the use of the energy expansion in the gravitational effective field theory.

There are in fact infinitely many terms allowed by general coordinate invariance, i.e.,

$$S = \int d^4x \sqrt{g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \ldots + L_{\text{matter}} \right\}$$

(2)

Here the gravitational Lagrangians have been ordered in a derivative expansion with $\Lambda$ being of order $\partial^0$, $R$ of order $\partial^2$, $R^2$ and $R_{\mu\nu} R^{\mu\nu}$ of order $\partial^4$ etc. Note that in four dimensions we do not need to include a term $R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}$ as the Gauss Bonnet theorem allows this contribution to the action to be written in terms of $R^2$ and $R_{\mu\nu} R^{\mu\nu}$.

The first term in Eq.21, i.e., $\Lambda$, is related to the cosmological constant, $\lambda = -8\pi G \Lambda$. This is a term which in principle should be included, but
cosmology bounds $| \lambda | < 10^{-56} \text{cm}^{-2}$, $| \Lambda | < 10^{-46} \text{GeV}^4$ so that this constant is unimportant at ordinary energies. We then set $\Lambda = 0$ from now on.

In contrast, the $R^2$ terms are able to be shown to be unimportant in a natural way. Let us drop Lorentz indices in order to focus on the important elements, which are the numbers of derivatives. A $R + R^2$ Lagrangian

$$\mathcal{L} = \frac{2}{\kappa^2} R + c R^2$$

has an equation of motion which is of the form

$$\Box h + \kappa^2 c^2 \Box h = 8\pi G T$$

The Greens function for this wave equation has the form

$$G(x) = \int \frac{d^4 q}{(2\pi)^4} \frac{e^{iq \cdot x}}{q^4 + \kappa^2 c q^4}$$

$$= \int \frac{d^4 q}{(2\pi)^4} \left[ \frac{1}{q^2} - \frac{1}{q^2 + 1/\kappa^2 c} \right] e^{-iq \cdot x}$$

The second term appears like a massive scalar, but with the wrong overall sign, and leads to a short-ranged Yukawa potential

$$V(r) = -G m_1 m_2 \left[ \frac{1}{r} - \frac{e^{-r/\sqrt{\kappa^2 c}}}{r} \right].$$

The exact form has been worked out by Stelle [11], who gives the experimental bounds $c_1, c_2 < 10^{74}$. Hence, if $c_i$ were a reasonable number there would be no effect on any observable physics. [Note that if $c \sim 1, \sqrt{\kappa^2 c} \sim 10^{-35} m$]. Basically the curvature is so small that $R^2$ terms are irrelevant at ordinary scales.

As a slightly technical aside, in an effective field theory we should not treat the $R^2$ terms to all orders, as is done above in the exponential of the Yukawa solution, but only include the first corrections in $\kappa^2 c$. This is because at higher orders in $\kappa^2 c$ we would also be sensitive to yet higher terms in the effective Lagrangian ($R^3, R^4$ etc.) so that we really do not know the full $r \to 0$ behavior. Rather, for $\sqrt{\kappa^2 c}$ small we can note the Yukawa potential becomes a representation of a delta function
The low energy potential then has the form

\[
V(r) = -Gm_1M_2 \left[ \frac{1}{r} + 128\pi^2G(c_1 - c_2)\delta^3(x) \right]
\]

(8)

\( R^2 \) terms in the Lagrangian lead to a very weak and short range modification to the gravitational interaction.

Thus when treated as a classical effective field theory, we can start with the more general Lagrangian, and find that only the effect of the Einstein action, \( R \), is visible in any test of general relativity. We need not make any unnatural restrictions on the Lagrangian to exclude \( R^2 \) and \( R_{\mu\nu}R^{\mu\nu} \) terms.

5 Quantization

There is a beautiful and simple formalism for the quantization of gravity. The most attractive variant combines the covariant quantization pioneered by Feynman and De Witt [4] with the background field method introduced in this context by 't Hooft and Veltman [8]. The quantization of a gauge theory always involves fixing a gauge. This can in principle cause trouble if this procedure then induces divergences which can not be absorbed in the coefficients of the most general Lagrangian which displays the gauge symmetry. The background field method solves this problem because the calculation retains the symmetry under transformations of the background field and therefore the loop expansion will be gauge invariant, retaining the symmetries of general relativity.

Consider the expansion of the metric about a smooth background field \( \bar{g}_{\mu\nu}(x) \),

\[
g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + \kappa h_{\mu\nu}
\]

(9)

Indices are now raised and lowered with \( \bar{g} \). The Lagrangian may be expanded in the quantum field \( h_{\mu\nu} \).

\[
\frac{2}{\kappa^2} \sqrt{\bar{g}} R = \sqrt{\bar{g}} \left\{ \frac{2}{\kappa^2} \bar{R} + \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \cdots \right\}
\]
Here $D_\alpha$ is a covariant derivative with respect to the background field. The total set of terms linear in $h_{\mu\nu}$ (including those from the matter Lagrangian) will vanish if $\bar{g}_{\mu\nu}$ satisfies Einstein’s equation. We are then left with a quadratic Lagrangian plus interaction terms of higher order.

However, the quadratic Lagrangian cannot be quantized without gauge fixing and the associated Feynman-DeWitt-Fadeev-Popov ghost fields. In this case, we would like to impose the harmonic gauge constraint in the background field, and can choose the constraint \[ G^\alpha = 4\sqrt{\bar{g}} \left( D^\nu h_{\mu\nu} - \frac{1}{2} D_\mu h^\lambda_\lambda \right) t^{\nu\alpha} \] (11)

where

$$\eta^{\alpha\beta} t^{\mu\alpha} t^{\nu\beta} = \bar{g}^{\mu\nu}$$

This leads to the gauge fixing Lagrangian \[ \mathcal{L}_{gf} = \sqrt{\bar{g}} \left\{ \left( D^\nu h_{\mu\nu} - \frac{1}{2} D_\mu h^\lambda_\lambda \right) \left( D_\sigma h^{\mu\sigma} - \frac{1}{2} D^\mu h^\sigma_\sigma \right) \right\} \] (13)

Because the gauge constraint contains a free Lorentz index, the ghost field will carry a Lorentz label, i.e., they will be fermionic vector fields. After a bit of work the ghost Lagrangian is found to be

$$\mathcal{L}_{gh} = \sqrt{\bar{g}} \eta^{\mu\nu} \left[ D_\lambda D^\lambda \bar{g}_{\mu\nu} - R_{\mu\nu} \right] \eta^\nu$$

(14)

The full quantum action is then of the form

$$S = \int d^4s \sqrt{\bar{g}} \left\{ \frac{2}{\kappa^2} \bar{R} - \frac{1}{2} h_{\alpha\beta} D^{\alpha\beta,\gamma\delta} h_{\gamma\delta} \right. $$

$$+ \left. \eta^{\mu\nu} \left\{ D_\lambda D^\lambda \bar{g}_{\mu\nu} - \bar{R}_{\mu\nu} \right\} \eta^\nu + \mathcal{O}(h^3) \right\} \] (15)
6 Renormalization

The one loop divergences of gravity have been studied in two slightly different methods. One involves direct calculation of the Feynman diagrams with a particular choice of gauge and definition of the quantum gravitational field \[12\]. The background field method, with a slightly different gauge constraint, allows one to calculate in a single step the divergences in graphs with arbitrary numbers of external lines and also produces a result which is explicitly generally covariant \[8\]. In the latter technique one expands about a background spacetime $\bar{g}_{\mu\nu}$, fixes the gauge as we described above and collects all the terms quadratic in the quantum field $h_{\mu\nu}$ and the ghost fields. For the graviton field we have

$$Z[\bar{g}] = \int [dh_{\mu\nu}] \exp \left\{ i \int d^4x \sqrt{g} \left\{ \frac{2}{\kappa^2} \bar{R} + h_{\mu\nu} D^{\mu\nu\alpha\beta} h_{\alpha\beta} \right\} \right\}$$

where $D^{\mu\nu\alpha\beta}$ is a differential operator made up of derivatives as well as factors of the background curvature. The short distance divergences of this object can be calculated by standard techniques once a regularization scheme is chosen. Dimensional regularization is the preferred scheme because it does not interfere with the invariances of general relativity. First calculated in this scheme by 't Hooft and Veltman \[8\], the divergent term at one-loop due to graviton and ghost loops is described by a Lagrangian

$$\mathcal{L}^{(\text{div})}_{1}\text{loop} = \frac{1}{8\pi^2\epsilon} \left\{ \frac{1}{120} \bar{R}^2 + \frac{7}{20} \bar{R}_\mu^\nu \bar{R}_\mu^\nu \right\}$$

(17)

with $\epsilon = 4 - d$. Matter fields of different spins will also provide additional contributions with different linear combinations of $R^2$ and $R^\mu_\nu R^\mu_\nu$ at one loop.

The fact that the divergences is not proportional to the original Einstein action is an indication that the theory is of the non-renormalizable type. Despite the name, however, it is easy to renormalize the theory at any given order. At one loop we identify renormalized parameters
\[ c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon} \]
\[ c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon} \]  

(18)

which will absorb the divergence due to graviton loops. Alternate but equivalent expressions would be used in the presence of matter loops.

A few comments on this result are useful. One often hears that pure gravity is one loop finite. This is because the lowest order equation of motion for pure gravity is \( R_{\mu\nu} = 0 \) so that the \( \mathcal{O}(R^2) \) terms in the Lagrangian vanish for all solutions to the Einstein equation. However in the presence of matter (even classical matter) this is no longer true and the graviton loops yield divergent effects which must be renormalized as described above. At two loops, there is a divergence in pure gravity which remains even after the equations of motion have been used [10].

\[ L_{\text{2loop}}^{(\text{div})} = \frac{209\kappa^2}{2880(16\pi^2)} \frac{1}{\epsilon} R_{\gamma\delta}^{\alpha\beta} R_{\eta\sigma}^{\gamma\delta} R_{\alpha\beta}^{\eta\sigma} \]  

(19)

For our purposes, this latter result also serves to illustrate the nature of the loop expansion. Higher order loops invariably involve more powers of \( \kappa \) which by dimensional analysis implies more powers of the curvature or of derivatives in the corresponding Lagrangian (i.e., one loop implies \( R^2 \) terms, 2 loops imply \( R^3 \) etc.). The two loop divergence would be renormalized by absorbing the effect into a renormalized value of a coupling constant in the \( \mathcal{O}(R^3) \) Lagrangian.

### 7 Quantum Predictions in An Effective Theory

At this stage it is important to be clear about the nature of the quantum predictions in an effective theory. The divergences described in the last section come out of loop diagrams, but they are not predictions of the effective theory. They are due to the high energy portions of the loop integration, and we do not even pretend that this portion is reliable. We expect the real divergences (if any) to be different. However the divergences do not in any
case enter into any physical consequences, as they absorbed into the renormalized parameters. The couplings which appear in the effective Lagrangian are also not predictions of the effective theory. They parameterize our ignorance and must emerge from an ultimate high energy theory or be measured experimentally. However there are quantum effects which are due to low energy portion of the theory, and which the effective theory can predict. These come because the effective theory is using the correct degrees of freedom and the right vertices at low energy. It is these low energy effects which are the quantum predictions of the effective field theory.

It may at first seem difficult to identify which components of a calculation correspond to low energy, but in practice it is straightforward. The effective field theory calculational technique automatically separates the low energy observables. The local effective Lagrangian will generate contributions to some set of processes, which will be parameterized by a set of coefficients. If, in the calculation of the loop corrections, one encounters contributions which have the same form as those from the local Lagrangian, these cannot be distinguished from high energy effects. In the comparison of different reactions, such effects play no role, since we do not know ahead of time the value of the coefficients in $\mathcal{L}$. We must measure these constants or form linear combinations of observables which are independent of them. Only loop contributions which have a different structure from the local Lagrangian can make a difference in the predictions of reactions. Since the effective Lagrangian accounts for the most general high energy effects, anything with a different structure must come from low energy.

A particular class of low energy corrections stand out as the most important. These are the nonlocal effects. In momentum space the nonlocality is manifest by a nonanalytic behavior. Nonanalytic terms are clearly distinct from the effects of the local Lagrangian, which always give results which involve powers of the momentum.

8 Examples

A conceptually simple (although calculationally difficult) example is graviton-graviton scattering. This has been calculated to one-loop in an impressive paper by Dunbar and Norridge [13] using string based methods. Because the reaction involves only the pure gravity sector, and $R_{\mu\nu} = 0$ is the lowest order
equation of motion, the result is independent of any of the four-derivative terms that can occur in the Lagrangian \((R^2 \text{ or } R_{\mu \nu} R^{\mu \nu})\). Thus the result is independent of any unknown coefficient to one loop order. Their result for the scattering of positive helicity gravitons is

\[
\mathcal{A}(++ \to ++) = 8\pi G \frac{s^4}{stu} \left\{ 1 
+ \frac{G}{\pi} \left( t \ln\left(\frac{-u}{\delta}\right) \ln\left(\frac{-s}{\delta}\right) + u \ln\left(\frac{-t}{\delta}\right) \ln\left(\frac{-s}{\delta}\right) + s \ln\left(\frac{-t}{\delta}\right) \ln\left(\frac{-u}{\delta}\right) \right) 
+ \ln\left(\frac{t}{u}\right) \frac{tu(t-u)}{60s^6} \left(341(t^4 + u^4) + 1609(t^3u + u^3t) + 2566t^2u^2\right) 
+ \left( \ln^2\left(\frac{t}{u}\right) + \pi^2 \right) \frac{tu(t+2u)(u+2t)}{2s^7} \left(2t^4 + 2u^4 + 2t^3u + 2u^3t - t^2u^2\right) 
+ \frac{tu}{360s^5} \left(1922(t^4 + u^4) + 9143(t^3u + u^3t) + 14622t^2u^2\right) \right\} \right. (20)
\]

where \(s = (p_1 + p_2)^2\), \(t = (p_1 - p_3)^2\), \(u = (p_1 - p_4)^2\), \((s + t + u = 0)\) and where I have used \(\delta\) as an infrared cutoff \([14]\). One sees the non-analytic terms in the logarithms. Also one sees the nature of the energy expansion in the graviton sector - it is an expansion in \(GE^2\) where \(E\) is a typical energy in the problem. I consider this result to be very beautiful. It is a low energy theorem of quantum gravity. The graviton scattering amplitude must behave in this specific fashion no matter what the ultimate high energy theory is and no matter what the massive particles of the theory are. This is a rigorous prediction of quantum gravity.

The other complete example of this style of calculation is the long distance quantum correction to the gravitational interaction of two masses \([3, 15]\). This is accomplished by calculating the vertex and vacuum polarization corrections to the interaction of two heavy masses. In addition to the classical corrections \([16]\), one obtains the true quantum correction

\[
V_{1pr}(r) = -\frac{Gm_1 m_2}{r} \left[ 1 - \frac{135 + 2N_\nu}{30\pi^2} \frac{Gh}{r^2c^3} + \ldots \right] \quad (21)
\]

for a specific definition of the potential. Note that the result is finite and independent of any parameters. This is easy to understand once one appreciates the structure of effective field theory. The divergences that occur in the loop diagrams all go into the renormalization of the coefficients in the
local Lagrangian, as we displayed above. Since these terms in the Lagrangian yield only delta-function modifications to the potential, they cannot modify any power-law correction that survives to large distance. Only the propagation of massless fields can generate the nonanalytic behavior that yields power-law corrections in coordinate space. Since the low energy couplings of massless particles are determined by Einstein’s theory, these effects are rigorously calculable.

Note that this calculation is the first to provide a quantitative answer to the question as to whether the effective gravitational coupling increases or decreases at short distance due to quantum effects. While there is some arbitrariness in what one defines to be $G_{eff}$, it must be a universal property (this eliminates from consideration the Post-Newtonian classical correction which depends on the external masses) and must represent a general property of the theory. The diagrams involved in the above potential are the same ones that go into the definition of the running coupling in QED and QCD and the quantum corrections are independent of the external masses. If one uses this gravitational interaction to define a running coupling one finds

$$G_{eff}(r) = G \left[ 1 - \frac{135 + 2N_\nu}{30\pi^2} \frac{Gh}{r^2 c^3} \right]$$

(22)

The quantum corrections decrease the strength of gravity at short distance, in agreement with handwaving expectations. (In pure gravity without photons or massless neutrinos, the factor $135 + 2N_\nu$ is replaced by 127.) An alternate definition including the diagrams calculated in [15] has a slightly different number, but the same qualitative conclusion. The power-law running, instead of the usual logarithm, is a consequence of the dimensionful gravitational coupling.

These two results do not exhaust the predictions of the effective field theory of gravity. In principle, any low energy gravitational process can be calculated [17]. The two examples above have been particularly nice in that they did not depend on any unknown coefficients from the general Lagrangian. However it is not a failure of the approach if one of these coefficients appears in a particular set of amplitudes. One simply treats it as a coupling constant, measuring it in one process (in principle) and using the result in the remaining amplitudes. The leftover structures aside from this coefficient are the low energy quantum predictions.
9 Limitations and the high energy regime

The effective field theory techniques can be applied at low energies and small curvatures. The techniques fail when the energy/curvature reaches the Planck scale. There is no known method to extend such a theory to higher energies. Indeed, even if such a technique were found, the result would likely be wrong. In all known effective theories, new degrees of freedom and new interactions come into play at high energies, and to simply try to extend the low energy theory to all scales is the wrong thing to do [18]. One needs a new enlarged theory at high energy. However, many attempts to quantize general relativity ignore this distinction and appear misguided from our experience with other effective field theories. While admittedly we cannot be completely sure of the high energy fate of gravity, the structure of the theory itself hints very strongly that new interactions are needed for a healthy high energy theory. It is likely that, if one is concerned with only pure general relativity, the effective field theory is the full quantum content of the theory.

10 Summary

The quantum theory of general relativity at low energy has turned out to be of the form that we call effective field theories. The result is a beautiful theory that incorporates general coordinate invariance in a simple way, and which has a known methodology for extracting predictions. The theory fits well with the other ingredients of the Standard Model. It is common, but wrong, to imply that general relativity differs for the other interactions because it has no known quantum theory. As we have seen, the quantum theory exists at those scales where General Relativity is reliably thought to apply.

Many of the most interesting questions that we ask of quantum gravity cannot be answered by the effective field theory. This is a warning that these questions require knowledge of physics beyond the Planck scale. Since physics is an experimental science, thoughts about what goes on at such a high scale may remain merely speculation for many years. However, it is at least reassuring that the ideas of quantum field theory can successfully be applied to General Relativity at the energy scales that we know about.
References

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