Technipion contribution to $b \rightarrow s\gamma$

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Abstract

We show that the present limit on the inclusive decay $b \rightarrow s\gamma$ provides strong constraints on Technicolor models. In particular, small values of $F_{\pi}$ and the mass of charged octet and singlet technipions are excluded, assuming the most natural form of the technipion coupling to the ordinary quarks.
The $b \to s\gamma$ decay is very sensitive to the physics beyond the standard model \cite{1}. The one-loop W-exchange diagrams that generate this decay at the lowest order in the standard model are shown in Figure 1. In most extensions of the standard model (such as the two-higgs-doublet models), there are additional contributions to this process coming from charged scalar exchange (replacing the W with the charged scalar(s) in Figure 1). This has been used to obtain lower bounds on the masses of the charged scalars.

Technicolor theories \cite{2} are an attempt to explain electroweak symmetry breaking without elementary scalars. Non-minimal models can contain pseudo-Goldstone bosons (technipions), which arise from the breakdown of global chiral symmetries \cite{3}. Here we consider the contribution of color singlet and color octet, weak isorotriplet technipions to the process $B \to X_s\gamma$.

Randall and Sundrum have studied the short-distance extended technicolor (ETC) \cite{3}, \cite{4} contribution to $b \to s\gamma$, that comes from the single ETC gauge boson exchange connecting purely left-handed doublets (Figure 2) \cite{5}. They show that this contribution is suppressed relative to the standard model (SM) one by $m_t/4\pi v$, where $v = 246\text{GeV}$. We shall see that the leading contributions are long-distance, nonanalytic contributions from technipion exchange, and that they are comparable to that from W-exchange. These contributions will be used obtain bounds on masses of the charged color octet and color singlet technipions.

Consider an ETC model incorporating the one family technicolor model. The exchange of ETC gauge bosons induces interactions of the form

$$\frac{g_{ETC}^2}{M_{ETC}^2} Y_u^{ij} \left( \bar{\psi}_L^i \gamma_\mu T_L^j \bar{U}_R^j \right) + \frac{g_{ETC}^2}{M_{ETC}^2} Y_d^{ij} \left( \bar{\psi}_L^i \gamma_\mu T_L^j \bar{D}_R^j \right) + h.c.$$  \hspace{1cm} (1)

where $g_{ETC}$ is the ETC gauge boson coupling and $M_{ETC}$ the generic gauge boson mass. Here, $T_L$ is the colored techni-doublet while $U_R$, $D_R$ are the right-handed partners; $\psi_L^i$ is the $i$-th generation quark doublet and $u_R^i$, $d_R^i$ are its right-handed partners. The matrices $Y_u(d)$ are model dependent constants that play a role similar to the standard model Yukawa couplings.

The relevant effective ETC interactions of the technipions and the top and bottom quarks (ignoring terms proportional to the CKM matrix element $V_{td}$) are obtained from PCAC arguments using the above Lagrangian. They have the form, respectively,

$$c_1 \frac{\pi_{\text{singlet}}^+}{\sqrt{6}F_\pi} \left( m_t (\bar{t}_R V_{tb} b_L + \bar{t}_R V_{ts} s_L) - m_b (\bar{t}_L V_{tb} b_R + \bar{t}_L V_{ts} s_R) + h.c. + \cdots \right)$$  \hspace{1cm} (2a)
and
\[ c_8 \frac{\pi_{\text{octet}}^+}{F_\pi} (m_t (\bar{t}_R V_{tb} \frac{\lambda^a}{2} - b_L + \bar{t}_R V_{ts} \frac{\lambda^a}{2} s_L) - m_b (\bar{t}_L V_{tb} \frac{\lambda^a}{2} - b_R + \bar{t}_L V_{ts} \frac{\lambda^a}{2} s_R) + \text{h.c.} + \cdots) \quad (2b) \]

for the color singlet and color octet technipions — the only technipions that couple to ordinary (color-triplet) quarks \([6]\). The \(\lambda^a\)'s are the SU(3) generators normalised so that \(\text{tr}(\lambda^a \lambda^b) = 2\delta^{ab}\). The constants \(c_1, c_8\) are model dependent factors of order 1. We shall take \(c_{1,8} = 1\); alternatively, all our results are really functions of \(F_\pi/c_{1,8}\), rather than \(F_\pi\).

The couplings of the charged technipions to the quarks are similar to that of the charged Higgs coupling to the quarks in two Higgs doublet model of type I in which both the up- and down-type quarks get mass from Yukawa couplings to the same Higgs doublet \([7]\). In particular, it is important to observe the relative negative sign between the terms proportional to \(m_t\) and on \(m_b\). This sign is model-dependent. For instance, the charged technipions in a two-doublet technicolor model (with \(SU(4) \times SU(4)\) chiral symmetry) couple to ordinary quarks without the relative sign difference if one of the doublets is used for giving mass to the up-type quarks and the other to the down type quarks (if the same doublet is used, one reproduces Eq. 3). We shall only study the case when the relative sign is negative as that is most conservative for the lower bounds on technipion masses.

We are interested in the technipion contribution to the coefficient \(C_7\) of the operator known as \(\hat{O}_7\) in the notation of \([8]\)
\[ \hat{O}_7 = \left(\frac{4G_F}{\sqrt{2}}\right) V^*_{ts} V_{tb} \left(\frac{e}{16\pi^2}\right) m_b (\bar{\tau}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \quad (3) \]

where \(F_{\mu\nu}\) is the electromagnetic field strength. The relevant Feynman diagrams are obtained by replacing the W with the charged technipions in Figure 1. The virtual top quark contribution is the most important one since the technipion coupling to quarks is expected to be proportional to the quark mass.

Comparing with the results of \([9]\) \([10]\) for the two Higgs-doublet models, the technipion contributions to \(C_7\) are found to be
\[ C_7(\pi_{\text{singlet}}) = + \frac{\sqrt{2}}{4G_F F_\pi^2} \left(\frac{1}{6}\right) \left( B(x) - \frac{A(x)}{6} \right) \quad (4a) \]
and
\[ C_7(\pi_{\text{octet}}) = + \frac{\sqrt{2}}{4G_F F_\pi^2} \left(\frac{4}{3}\right) \left( B(x) - \frac{A(x)}{6} \right) \quad (4b) \]
where \( x = (m_t^2/M_{\pi_T}^2) \) and

\[
A(x) = x \left( \frac{\frac{5}{2}x^2 + \frac{5}{12}x - \frac{7}{12}}{(x - 1)^3} - \frac{\left(\frac{3}{2}x^2 - x\right) \ln x}{(x - 1)^4} \right) \tag{5}
\]

\[
B(x) = \frac{x}{2} \left( \frac{\frac{5}{6}x - \frac{7}{3}}{(x - 1)^2} - \frac{(x - \frac{2}{3}) \ln x}{(x - 1)^3} \right) \tag{6}
\]

For comparison, the W-exchange contribution to \( C_7 \) that is present in SM as well as technicolor models is \((-A(x)/2)\) [11] if QCD corrections are ignored. The sign of the technipion contribution is positive and tends to decrease the overall magnitude of \( C_7 \) when added to the SM contribution.

There are many theoretical and experimental uncertainties present in the prediction for \( \text{BR}[B \to X_s \gamma] \) [12]. The two most important sources of uncertainties are the uncertainty in \( \alpha_s \) [13] and next-to-leading-order QCD effects [12]. We incorporate the uncertainty in \( \alpha_s(M_z) \) by obtaining results for \( \alpha_s(M_z) = 0.11 \) and \( \alpha_s(M_z) = 0.13 \). By varying the renormalization scale \( \mu \) by a factor of 2 in both directions around 5 GeV we estimate the size of the next-to-leading QCD corrections — a 25% effect on \( C_7 \) [12].

We ignore uncertainties in \( m_c/m_b \) [14], the top quark mass [15], the ratio of CKM factors \( |V_{ts}^* V_{tb}|^2/|V_{cb}|^2 \), the experimental determination of \( \text{BR}[B \to X_c e \overline{\nu_c}] \) [16] and the spectator model approximation. These effects are expected to give about a 15% change in our calculation of the result. This has a negligible impact on the lower bounds on \( M_{\pi_T} \) (as a function of \( F_\pi \)), as the technipion contribution to \( C_7 \) is opposite in sign to the SM contribution. We use the central values of these other parameters: \( m_t = 175 \text{ GeV} \) [13], \( z = m_c/m_b = 0.316 \) [14] and \( \text{BR}[B \to X_c e \overline{\nu_c}] = 10.7\% \) [16]. We take the central value of 0.95 for \( |V_{ts}^* V_{tb}|^2/|V_{cb}|^2 \) in our computation of the standard model’s leading logarithmic contribution [9], [17]. The limits on the branching ratio (each at 95% CL) of \( B \to X_s \gamma \) are \( 1.0 \times 10^{-4} < \text{BR}[B \to X_s \gamma] < 4.2 \times 10^{-4} \) [18], which correspond to \( 0.18 \leq |C_7| \leq 0.38 \).

The results obtained for the color octet and color singlet cases are shown in Figs 3 and 4. The standard model contribution from W exchange is added to the technipion contributions and the sum is compared with the experimental limits in order to obtain the bounds on the masses of the technipions. The lower bound is larger in the case of the color octet because of the larger group theory factor. The next-to-leading order contributions do not affect the lower bounds on technipion masses significantly (again because the technipion contribution to \( C_7 \) interferes destructively with the SM contribution). However, they do affect the excluded region that comes from the experimental lower bound on \( \text{BR}[B \to X_s \gamma] \).
(the shaded region in between). This is because the technipion contribution is positive
definite and there is a lower limit on $|C_7|$. Consequently, there is a certain region which
is disallowed as $|C_7|$ (which includes the W and technipion contributions) would then be
smaller than the experimental lower bound 0.18. This region is thus sensitive to the next-
to-leading order contributions. We have shown the results from the two extreme cases that
could result from the uncertainty in $\alpha_s(M_Z)$ and the choice of appropriate $\mu$.

A one-family technicolor model has contributions from both the color octet and color
singlet technipions. Both contributions are positive definite and hence only serve to in-
crease the lower bounds on technipion mass for a given $F_\pi$. In Fig. 5 we have plotted
the excluded (shaded) regions in the $M_{\pi_{octet}} - M_{\pi_{singlet}}$ plane in a one-family technicolor
model with $F_\pi = 125$ and $c = 1$. The unshaded region in between is the allowed region of
physical interest. For instance, for a color singlet technipion of mass of about 100 GeV, the
octet mass has to be between 200 and 350 GeV. As above, the shaded region in between
is due to the experimental lower bound on $|C_7(\mu)|$.

Our conclusions depend on the relative negative sign in Eq. (2); $C(\pi_T)$ would be
negative definite if there were no relative sign difference. This is because the $B(x)$ piece
(which is the contribution from the ‘cross term’ in Eq. (2)) dominates. In a more general
technicolor model with more technipions, it is apparent that the lower limits on the
technipion masses would be higher than here if all the relevant technipions couple as in
Eq. (2), i.e., with a relative negative sign. If all of them coupled with a relative positive
sign, the situation is even worse, since then the contribution to $C_7$ would be negative,
like the standard model result. However, the constraint would be relaxed (relative to the
above) if some of the technipions coupled to quarks with a relative negative sign and the
others with a relative positive sign, so that there were some cancellation.

In the Topcolor assisted Technicolor model proposed by Hill [19], top-pions ($\pi_T$) can
contribute to $b \rightarrow s \gamma$. The relevant term in the interaction lagrangian is

$$ \frac{m_t}{f_\pi} \pi_T + \left[ (\bar{T}_R S_L D_{Lbs} + \bar{\tau}_R b_L U^\tau_{Rtc}) + h.c. \right] $$  (7)

where $U_{L,R}$ and $D_{L,R}$ are the field redefinition matrices for the up-type (down-type) quarks.
The top-pions couple only to the third generation weak doublet. By assigning different
quantum numbers to the first two generations and the third generation, the third generation
is distinguished from the other two. Then the $U_{L,R}$ and $D_{L,R}$ cannot be completely
absorbed into the CKM matrix, as in the standard model or the ETC interaction assumed
in Eq. 1. As the elements of $D_L$ is not measurable (only the CKM matrix $V = U_L^\dagger D_L$ is), no definite bounds on the top-pion mass can be obtained (see [19]).

An ETC model in which the third generation is distinguished from the other two would also involve experimentally undetermined parameters (in place of the CKM matrix elements), and hence be less severely constrained than the type of models studied here. In the case when ETC is used in conjunction with topcolor, the bounds would be considerably weaker for the technipions, because the technipion coupling is proportional to the ETC generated part of the top quark mass. In topcolor assisted technicolor models, the ETC contribution to the top quark mass $\epsilon m_t$ is small, i.e., $\epsilon \ll 1$.

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References

[1] J. L. Hewett, SLAC-PUB-6521
[2] S. Weinberg, Phys. Rev. D19 (1979) 1277; L. Susskind, Phys. Rev. D20 (1979) 2619
[3] E. Eichten and K. D. Lane, Phys. Lett. 90B (1980) 125
[4] S. Dimopoulos and L. Susskind, Nucl. Phys. B155 (1979) 237
[5] L. Randall and R. Sundrum, Phys. Lett. B312 (1993) 148
[6] J. Ellis et al., Nucl. Phys. B182 (1981) 529
[7] H. E. Haber, G. L. Kane, and T. Sterlings, Nucl. Phys. B161 (1979) 493; L. Hall and M. Wise, Nucl. Phys. B187 (1981) 397
[8] B. Grinstein, R. Springer, M. B. Wise, Nucl. Phys. B339 (1990) 269
[9] B. Grinstein and M. B. Wise, Phys. Lett. B201 (1988) 274
[10] W. S. Hou and R. Willey, Phys. Lett. B202 (1988) 591
[11] T. Inami and C. S. Lim, Progr. Theor. Phys. 65 (1981) 297
[12] A. J. Buras, M. Misiak, M. M"unz and S. Pokorski, Nucl. Phys. B424 (1994) 374
[13] P. Langacker and N. Polonsky, Phys. Rev. D47 (1993) 4028
[14] R. R"uckl, MPI-Ph/36/89
[15] F. Abe et al. (CDF Collaboration), Phys. Rev. D50 (1994) 2966; Phys. Rev. Lett. 73 (1994) 225; S. Abachi et al. (D0 Collaboration), FERMILAB-PUB-95-028-E
[16] Review of Particle Properties, Phys. Rev. D45 1994
[17] R. Grigjanis, P. J. O’Donnell, M. Sutherland and H. Navelet, Phys. Lett. B213 (1988) 355, Phys. Lett. B286 (1992) 413; G. Cella, G. Curci, G. Ricciardi and A. Vicere, Phys. Lett. B248 (1990) 181; M. Misiak, Phys. Lett. B269 (1991) 161; M. Misiak, Nucl. Phys. B393 23; K. Adel and Y. P. Yao, Modern Physics Letters A8 (1993) 1679; M. Ciuchini et al, Phys. Lett. 316 (1993) 127
[18] M.S. Alam et al (CLEO Collaboration), Phys. Rev. Lett. 74 2885 (1995)
[19] C. T. Hill, Phys. Lett. B345(1995) 483
Figure Captions

[1] Feynman diagrams that determine the one-loop $b \to s\gamma$ decay amplitude. The loop with virtual top quark dominates.

[2] The ETC gauge boson exchange contribution to the $b \to s\gamma$ considered by Randall and Sundrum.

[3] The excluded regions (shaded) in the $F_\pi-M_\pi$ plane for the color singlet technipion. Figures 1(a) and 1(b) correspond to the $[\alpha_s(M_z), \mu \text{ in GeV}]$ values of [0.13, 2.5] and [0.11, 10] respectively.

[4] The excluded regions (shaded) in the $F_\pi-M_\pi$ plane for the color octet technipion. Figures 2(a) and 2(b) correspond to the $[\alpha_s(M_z), \mu \text{ in GeV}]$ values of [0.13, 2.5] and [0.11, 10] respectively.

[5] The excluded regions (shaded) in the $M_{\pi_{\text{octet}}}-M_{\pi_{\text{singlet}}}$ plane for $F_\pi = 125 \text{ GeV}$. We take $\alpha_s(M_z) = 0.11$ and $\mu = 10 \text{ GeV}$. 

Figure 5
Figure 1
Figure 2
Figure 3(a)
