COMPACT DIFFERENCES OF COMPOSITION OPERATORS FROM BLOCH SPACE TO BOUNDED HOLOMORPHIC FUNCTION SPACE IN THE POLYDISC

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Abstract. Let \( \varphi \) and \( \psi \) be holomorphic self-maps of the unit polydisc \( U^n \) in the \( n \)-dimensional complex space, and denote by \( C_\varphi \) and \( C_\psi \) the induced composition operators. This paper gives some simple estimates of the essential norm for the difference of composition operators \( C_\varphi - C_\psi \) from Bloch space to bounded holomorphic function space in the unit polydisc. Moreover the compactness of the difference is also characterized.

1. Introduction

The algebra of all holomorphic functions with domain \( \Omega \) will be denoted by \( H(\Omega) \), where \( \Omega \) is a bounded domain in \( \mathbb{C}^n \). Let \( \varphi = (\varphi_1(z), \ldots, \varphi_n(z)) \) and \( \psi(z) = (\psi_1(z), \ldots, \psi_n(z)) \) be holomorphic self-maps of \( \Omega \). The composition operator \( C_\varphi \) induced by \( \varphi \) is defined by
\[
(C_\varphi f)(z) = f(\varphi(z)),
\]
for \( z \) in \( \Omega \) and \( f \in H(\Omega) \).

We recall that the essential norm of a continuous linear operator \( T \) is the distance from \( T \) to the compact operators, that is,
\[
\|T\|_e = \inf\{\|T - K\| : K \text{ is compact}\}.
\]

Notice that \( \|T\|_e = 0 \) if and only if \( T \) is compact, so that estimates on \( \|T\|_e \) lead to conditions for \( T \) to be compact.

During the past few decades much effort has been devoted to the research of such operators on a variety of Banach spaces of holomorphic functions with the goal of explaining the operator-theoretic behavior of \( C_\varphi \), such as compactness and spectra, in terms of the function-theoretic properties of the symbol \( \varphi \). We recommend the interested readers refer to the books by J. H. Shapiro [15] and Cowen and MacCluer [11], which are good sources for information on much of the developments in the theory of composition operators up to the middle of last decade.

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In the past few years, many authors are interested in studying the mapping properties of the difference of two composition operators, i.e., an operator of the form

\[ T = C_\phi - C_\psi. \]

The primary motivation for this has been the desire to understand the topological structure of the whole set of composition operators acting on a given functions. Most papers in this area have focused on the classic reflexive spaces, however, some classical non-reflexive spaces have also been discussed lately in the unit disc in the complex plane. In [9], MacCluer, Ohno and Zhao characterized the compactness of the difference of composition operator on \( H^\infty \) spaces by Poincaré distance. Their work was extended to the setting of weighted composition operators by Hosokawa, Izuchi and Ohno [5]. In [11], Moorhouse characterized the compact difference of composition operators acting on the standard weighted bergman spaces and necessary conditions on a large scale of weighted Dirichlet spaces. Lately, Hosokawa and Ohno [5 and 6] gave a characterization of compact difference on Bloch space in the unit disc. In [20] and [2], Carl and Gorkin et al., independently extended the results to \( H^\infty(B_n) \) spaces, they described compact difference by Carathéodory pseudo-distance on the ball, which is the generalization of Poincaré distance on the disc.

The present paper continues this line of research, gives some simple estimates of the essential norm for the difference of composition operators induced by \( \phi \) and \( \psi \) acting from Bloch space to bounded function space in the unit polydiscs \( U^n \), where \( \phi(z) \) and \( \psi(z) \) be holomorphic self-maps of the unit polydisc in \( n \)-dimensional complex space. As its applications, a characterization of compact difference is given.

2. Notation and background

Throughout this paper, let \( D \) be the unit disc in the complex plane \( \mathbb{C} \), \( U^n \) the unit polydisc in \( n \)-dimensional complex space \( \mathbb{C}^n \), and \( |||z||| = \max\{|z_j|\} \) stand for the sup norm on \( U^n \). For a holomorphic function in \( U^n \), define \( \nabla f(z) = (\frac{\partial f}{\partial z_1}(z), \ldots, \frac{\partial f}{\partial z_n}(z)) \), \( Rf(z) = \langle \nabla f(z), \bar{z} \rangle \).

For \( z, w \in D \), the pseudo-hyperbolic distance between \( z \) and \( w \) is defined by

\[ \rho(z, w) = \frac{|z - w|}{1 - \overline{z}w}. \]

It is well known that if \( f : D \to D \) is holomorphic, then

\[ \rho(f(z), f(w)) \leq \rho(z, w) \quad z, w \in D. \]

The Bergman metric on the unit polydiscs is given by

\[ H_z(u, \overline{v}) = \sum_{j=1}^{n} \frac{u_j v_j}{(1 - |z_j|^2)^2}. \]
The Kobayashi distance $k_{U^n}$ of $U^n$ is given by
\[ k_{U^n}(z, w) = \frac{1}{2} \log \frac{1 + |||\phi_z(w)|||}{1 - |||\phi_z(w)|||}, \]
where $\phi_z : U^n \to U^n$ is the automorphism of $U^n$ given by
\[ \phi_z(w) = \left( \frac{w_1 - z_1}{1 - \overline{z}_1 w_1}, \ldots, \frac{w_n - z_n}{1 - \overline{z}_n w_n} \right). \]

Let $H^\infty$ denote the space of bounded holomorphic functions $f$ on the unit polydiscs with the sup norm $|||f|||_\infty = \sup_{z \in U^n} |f(z)|$.

According to [18] and [19], the Bloch space $B$ in $U^n$ consists of those holomorphic functions such that
\[ |||f|||_B = \sup_{z \in U^n} \mathcal{Q} f(z) < \infty \]
where
\[ \mathcal{Q} f(z) = \sup \left\{ \frac{|\nabla f(z) \cdot u|}{H_z(u, \overline{u})} : u \in \mathbb{C}^n - \{0\} \right\}. \]

It is well known that
\[ ||f \circ \phi||_B = ||f||_B \]
for any automorphism $\phi$ of $U^n$, and $B$ is a Banach space under the norm $||f||_1 = |f(0)| + ||f||_B$.

If we put
\[ G_f(z) = \sum_{j=1}^n (1 - |z_j|^2) \left| \frac{\partial f}{\partial z_j}(z) \right| \]
and
\[ ||f|| = |f(0)| + \sup_{z \in U^n} G_f(z). \]

It follows from (3.8) and (3.9) in [18] that
\[ \frac{1}{n} G_f(z) \leq \max_{1 \leq j \leq n} (1 - |z_j|^2) \left| \frac{\partial f}{\partial z_j}(z) \right| \leq Q_f(z) \leq n G_f(z), \]
this implies that $\frac{1}{n} ||f|| \leq ||f||_1 \leq n ||f||$, so $B$ is also a Banach space with the norm $|| \cdot ||$.

**Lemma 1.** Assume $f \in B$, then
\[ |f(z) - f(w)| \leq n^2 ||f|| k_{U^n}(z, w) \]
for any $z, w \in U^n$. 
Proof.

\[ |f(z) - f(0)| = \left| \int_0^1 Rf(tz) \frac{dt}{t} \right| = \left| \sum_{j=1}^{n} \int_0^1 z_j \frac{\partial f}{\partial \zeta_j}(tz) dt \right| \]

\[ \leq \sum_{j=1}^{n} \int_0^1 \frac{|z_j|}{1 - |tz_j|^2} \left| \frac{\partial f}{\partial \zeta_j}(tz) \right| (1 - |tz_j|^2) dt \]

\[ \leq \|f\|_B \sum_{j=1}^{n} \int_0^{||z||} \frac{1}{1 - t^2} dt = \frac{1}{2} \|f\|_B \sum_{j=1}^{n} \log \frac{1 + |z_j|}{1 - |z_j|} \]

\[ \leq n \|f\|_B \frac{1}{2} \log \frac{1 + ||z||}{1 - ||z||}. \]

The last inequality follows by the fact the map \( t \to \log \frac{1 + t}{1 - t} \) is strictly increasing on \([0,1)\). Setting \( z = \phi_w(z) \), it follows that

\[ |f(\phi_w(z)) - f(0)| \leq n \|f\|_B \frac{1}{2} \log \frac{1 + ||\phi_w(z)||}{1 - ||\phi_w(z)||} \]

\[ = n \|f \circ \phi_w\|_B \frac{1}{2} \log \frac{1 + ||\phi_w(z)||}{1 - ||\phi_w(z)||}. \]

That is,

\[ |f \circ \phi_w(z)) - f \circ \phi_w(w)| \leq n \|f \circ \phi_w\|_B \frac{1}{2} \log \frac{1 + ||\phi_w(z)||}{1 - ||\phi_w(z)||}. \]

Replacing \( f \circ \phi_w \) by \( f \circ \phi_w \circ \phi_w^{-1} \),

\[ |f(z) - f(w)| \leq n \|f\|_B \frac{1}{2} \log \frac{1 + ||\phi_w(z)||}{1 - ||\phi_w(z)||} \leq n^2 \|f\|_B \|k_U^n(z, w). \]

This completes the proof of the lemma.

Lemma 2. Suppose \( f \in B \), for fixed \( 0 < \delta < 1 \), let \( G = \{z \in U^n : ||z|| \leq \delta\} \). Then

\[ \lim_{r \to 1} \sup_{||f|| \leq 1} \sup_{z \in G} |f(z) - f(rz)| = 0. \]
Proof

\[
\sup_{z \in G}|f(z) - f(rz)| = \sup_{z \in G}\left|\sum_{j=1}^{n}(f(rz_1, rz_2, \cdots, rz_{j-1}, z_j, \cdots, z_n) - f(rz_1, rz_2, \cdots, rz_j, z_{j+1}, \cdots, z_n))\right|
\]

\[
\leq \sup_{z \in G} \sum_{j=1}^{n} \left|\int_{r}^{1} z_j \frac{\partial f}{\partial z_j}(rz_1, rz_{j-1}, tz_j, z_{j+1}, \cdots, z_n)dt\right|
\]

\[
\leq (1 - r)n \sup_{z \in G} \left|\frac{\partial f}{\partial z_j}(z)\right| \leq (1 - r)n \sup_{z \in G} \left|\frac{\partial f}{\partial z_j}(z)\right| \left(1 - |z_j|^2\right) \frac{1}{1 - |z_j|^2}
\]

\[
\leq (1 - r)n \|f\| \sup_{z \in G} \frac{1}{1 - \|z\|^2}
\]

\[
\leq \frac{(1 - r)n \|f\|}{1 - \delta^2}.
\]

The lemma follows as \( r \to 1 \).

3. Main theorem

Theorem. For \( \delta > 0 \), write \( F_{\delta} = \{z \in U^n : \max\{||\varphi(z)||, ||\psi(z)||\} \leq 1 - \delta\} \). Suppose \( \varphi, \psi : U^n \to U^n \) and \( C_{\varphi} - C_{\psi} : B \to H^\infty \) is bounded, then

\[
\frac{1}{4} \lim_{\delta \to 0} \sup_{z \in E_{\delta}} \|\phi_{\varphi}\psi(z)\| \leq \|C_{\varphi} - C_{\psi}\|_e \leq 2n^2 \lim_{\delta \to 0} \sup_{z \in E_{\delta}} k_{U^n}(\varphi(z), \psi(z))
\]

where \( E_{\delta} = U^n - F_{\delta} \).

Proof. We consider the upper estimate first. For fixed \( 0 < r < 1 \), it easy to check both \( C_{r\varphi} \) and \( C_{r\psi} \) are compact operators. Therefore,

\[
\|C_{\varphi} - C_{\psi}\|_e \leq \|C_{\varphi} - C_{r\varphi} + C_{r\psi}\|
\]

Now for any \( 0 < \delta < 1 \),

\[
\|C_{\varphi} - C_{\psi} - C_{r\varphi} + C_{r\psi}\|
\]

\[
= \sup_{\|f\| \leq 1} \|(C_{\varphi} - C_{\psi} - C_{r\varphi} + C_{r\psi})f\|_\infty
\]

\[
\leq \sup_{\|f\| \leq 1} \|f(\varphi(z)) - f(r\varphi(z)) + f(r\psi(z)) - f(\psi(z))\|
\]

\[
+ \sup_{\|f\| \leq 1} \|f(\varphi(z)) - f(\psi(z)) - f(r\varphi(z)) + f(r\psi(z))\|
\]

From Lemma 2, we can choose \( r \) sufficiently close to 1 such that the first term of the right hand side is less than any given \( \epsilon \), and denote the second
term by $I$. Using Lemma 1, it follows that

\[ I \leq \sup_{\|f\| \leq 1} \sup_{z \in E_\delta} (|f(\varphi(z)) - f(\psi(z))| + |f(r\varphi(z)) - f(r\psi(z))|) \]

\[ \leq n^2 \sup_{\|f\| \leq 1} \sup_{z \in E_\delta} (k_{U^n}(\varphi(z), \psi(z)) + k_{U^n}(r\varphi(z), r\psi(z))) \]

\[ \leq 2n^2 \sup_{z \in E_\delta} k_{U^n}(\varphi(z), \psi(z)) \]

the last inequality follows by $k_{U^n}(r\varphi(z), r\psi(z))) \leq k_{U^n}(\varphi(z), \psi(z))$. First let $r \to 1$ and then $\delta \to 0$, the upper estimate follows.

Now we turn to the lower estimate. Setting

\[ E_\delta^l = \{ z \in U^n : \max(|\varphi_l(z)|, |\psi_l(z)|) > 1 - \delta \}. \]

It is easy to see that $E_\delta = \bigcup_{l=1}^n E_\delta^l$. For fixed $l(1 \leq l \leq n)$, define

\[ a_l = \lim_{\delta \to 0} \sup_{z \in E_\delta^l} \frac{|\varphi_l(z) - \psi_l(z)|}{1 - \varphi_l(z)\psi_l(z)}. \]

If we put $\delta_m = \frac{1}{m}$, then $\delta_m \to 0$ as $m \to \infty$.

If $||\varphi_l||_{\infty} = 1$ or $||\psi_l||_{\infty} = 1$, then for enough large $m$ with $E_{\delta_m}^l \neq \emptyset$, so there exists $z^m \in E_{\delta_m}^l$ such that $\lim_{m \to \infty} \frac{|\varphi_l(z^m) - \psi_l(z^m)|}{|1 - \varphi_l(z^m)\psi_l(z^m)|} = a_l$. Since $z^m \in E_{\delta_m}^l$ implies that $|\varphi_l(z^m)| > 1 - \delta_m$ or $|\psi_l(z^m)| > 1 - \delta_m$, without loss of generality we assume $|\varphi_l(z^m)| \to 1$. Setting

\[ f_m(z) = \frac{1 - |\varphi_l(z^m)|}{1 - \varphi_l(z^m)z_l}. \]

A little calculation shows that $\{f_m\}$ converges to zero uniformly on compact subsets of $U^n$ as $m \to \infty$ and $\|f_m\| \leq 2$ for any $m = 1, 2, \ldots$. So the compactness of $K$ implies that $\|Kf_m\| \to 0$ whenever $m \to \infty$, it follows
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that

\[ \|C_\varphi - C_\psi - K\| \geq \frac{1}{2} \limsup_{m \to \infty} \| (C_\varphi - C_\psi) f_m \|_\infty \]
\[ \geq \frac{1}{2} \limsup_{m \to \infty} (\| (C_\varphi - C_\psi) f_m \|_\infty - \| K f_m \|_\infty) \]
\[ = \frac{1}{2} \limsup_{m \to \infty} \| (C_\varphi - C_\psi) f_m \|_\infty \]
\[ \geq \frac{1}{2} \limsup_{m \to \infty} \sup_{z \in U} | f_m(\varphi(z)) - f_m(\psi(z)) | \]
\[ \geq \frac{1}{2} \limsup_{m \to \infty} \| f_m(\varphi(z^m)) - f_m(\psi(z^m)) \|_\infty \]
\[ = \frac{1}{2} \limsup_{m \to \infty} (1 - | \varphi_l(z^m) |) \frac{1}{1 - \varphi_l(z^m) \psi_l(z^m)} - \frac{1}{1 - \varphi_l(z^m) \psi_l(z^m)} \]
\[ = \frac{1}{2} \limsup_{m \to \infty} | \varphi_l(z^m) | \frac{\varphi_l(z^m) - \psi_l(z^m)}{1 - \varphi_l(z^m) \psi_l(z^m)} \]
\[ = \frac{1}{4} \limsup_{m \to \infty} \left| \frac{\varphi_l(z^m) - \psi_l(z^m)}{1 - \varphi_l(z^m) \psi_l(z^m)} \right| \]
\[ = \frac{1}{4} a_l = \frac{1}{4} \limsup_{\delta \to 0} \left| \frac{\varphi_l(z) - \psi_l(z)}{1 - \varphi_l(z) \psi_l(z)} \right| \]

If both \( \| \varphi_l \|_\infty < 1 \) and \( \| \psi_l \|_\infty < 1 \), in this condition, when \( \delta \) is small enough, \( E^l_\delta \) is empty, without loss of generality, we may assume that

\[ \limsup_{\delta \to 0} \left| \frac{\varphi_l(z) - \psi_l(z)}{1 - \varphi_l(z) \psi_l(z)} \right| = 0. \]

Now for each \( l = 1, 2, \cdots, n \), we define

\[ b_l = \limsup_{\delta \to 0} \left| \frac{\varphi_l(z) - \psi_l(z)}{1 - \varphi_l(z) \psi_l(z)} \right| \]

For any \( \epsilon > 0 \), there exists a \( \delta_0 \) with \( 0 < \delta_0 < 1 \), such that

\[ \left| \frac{\varphi_l(z) - \psi_l(z)}{1 - \varphi_l(z) \psi_l(z)} \right| > b_l - \epsilon \]
whenever \( z \in E_{\delta_0} \) and \( l = 1, 2, \cdots, n \). Since \( z \in E_{\delta_0}^l \) implies that \( z \in E_{\delta_l} \), by the argument above we have

\[
\|C_\varphi - C_\psi - K\| \geq \frac{1}{4} \max_{1 \leq l \leq n} \limsup_{\delta \to 0} \sup_{z \in E_\delta^l} \left| \frac{\varphi_l(z) - \psi_l(z)}{1 - \varphi_l(z)\psi_l(z)} \right|
\]

\[
\geq \frac{1}{4} \max_{1 \leq l \leq n} (b_l - \epsilon)
\]

\[
= \frac{1}{4} \limsup_{\delta \to 0} \max_{1 \leq l \leq n} \left| \frac{\varphi_l(z) - \psi_l(z)}{1 - \varphi_l(z)\psi_l(z)} \right| - \frac{\epsilon}{4}
\]

\[
= \frac{1}{4} \limsup_{\delta \to 0} \sup_{z \in E_\delta} \left| \phi_{\varphi(z)}(\psi(z)) \right| - \frac{\epsilon}{4}.
\]

Now the conclusion follows by letting \( \epsilon \to 0 \).

**Corollary** Suppose \( C_\varphi - C_\psi : B \to H^\infty \) is bounded, then \( C_\varphi - C_\psi \) is compact if and only if

\[
\lim_{\delta \to 0} \sup_{z \in E_\delta} \left| \phi_{\varphi(z)}(\psi(z)) \right| = 0.
\]

**Proof.** It follows from main theorem that the necessity is obvious. By the strictly increment of \( \log \frac{1}{1-t} \) on \([0, 1)\), \( \lim_{\delta \to 0} \sup_{z \in E_\delta} \left| \phi_{\varphi(z)}(\psi(z)) \right| = 0 \) implies that \( \lim_{\delta \to 0} k_{U^n}(\varphi(z), \psi(z)) = 0 \), it follows from the main theorem that \( \|C_\varphi - C_\psi\|_e = 0 \), so \( C_\varphi - C_\psi \) is compact, the proof of this corollary is finished.

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