Mathematical models of magnetic circuits of a magnetomodulation DC converter

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Abstract. Monitoring of the operation mode of autonomous power supplies with magnetically modulating DC converters and system control, differential sensitivity, improve accuracy and reliability and monitor its static characteristics in the form of a line graph.

1. Introduction
Currently, a large number of current converters (dc converter) are known and this creates certain difficulties in choosing the required type, specific design of these converters. In this regard, it is advisable to classify dc converter, which will reveal their fundamental and design features. Some are devoted to the classifications of high AC current transducers, while others are devoted to the classifications of high DC current transducers. Currently, there is no generalized and detailed classification of direct current sensors.

2. Energy efficiency
Currently existing dc converter are subdivided according to the method of their inclusion in the measured circuit into two large groups: 1) contact dc converter, based on the measurement of the voltage drop across the resistor included in the circuit of the measured current; b) non-contact dc converter, based on the measurement of the magnetic field created by the measured current.

DC converters of the first group contain, as a rule, a certain set of elements - a shunt, modulator, transformer, demodulator, low-pass filter, in a large number of standard versions of such DC converter. These elements are only supplemented by new ones, contributing to some improvement in their characteristics. Therefore, in the following, DC converter of only the second group will be described.

DC magnetomodulation converters are most widely used to measure direct current, in which the parameters that change under the action of an external magnetic field are modulated. By the type of modulated parameter, the following four groups of active DC magnetomodulation converters are distinguished; and -converters.

An example α - the transducer is a rotating frame in a magnetic field; S- converter – circuit, fixed on the faces of the piezoelectric crystal; N- converter – device, in which the ferromagnetic rotor rotates relative to the ferromagnetic stator.

The well-known DC magnetomodulation converters has a significant drawback: with fast inrush currents of the monitored circuit, the voltage at the DC magnetomodulation converters output in this case carries false information about the direction of the converted current.

In order to eliminate this drawback, by memorizing the direction of the current in case of current surges in the controlled circuit for the time required for the voltage discriminator to operate at the output...
with a frequency detector (figure 1) [1]. Such a device is realized by adding to the DC magnetomodulation converters one that responds to the current derivative $\frac{dI_x}{dt}$ and two waiting multivibrators [2]. With current surges, one of the waiting multivibrators, depending on the direction of the current, forms a pulse, the duration of which is determined by the ratio:

$$T_p > T_{fd},$$

where $T_p, T_{fd}$ – respectively the duration of the pulse and the transient in (frequency detector).

In this case, the operational amplifier A2 is saturated with a voltage of the desired polarity for the time $T_p$. If at the end of the pulse the current $I_x$ continues to be greater than the current $I_0$, then the output voltage (frequency detector) through the (voltage discriminator) maintains the corresponding switch K1 or K2 in the open state and amplifier A2 remains in a saturated state until the current $I_x$ becomes less than current $I_0$.

**Figure 1.** DC magnetomodulation converters with memory of the current direction.

If at the end of the pulse the current $I_x$ continues to be greater than the current $I_0$, then the output voltage (frequency detector) through the (voltage discriminator) maintains the corresponding switch K1 or K2 in the open state and amplifier A2 remains in a saturated state until the current $I_x$ becomes less than current $I_0$.

The magnetic circuit of the converted current DC magnetomodulation converters (figure 2) is a circuit with distributed magnetic and lumped electrical parameters. [3]

The distributed parameters include the linear values of the magnetic resistance of the ferromagnetic rings ($Z_{\mu n}$) and the magnetic capacity of the air gap between the coaxial ferromagnetic rings ($C_{\mu n} = const$) per unit angle of the ferromagnetic rings. The lumped parameter includes ampere-turns ($F_x$) created by the converted current $I_x$. 
For a theoretical study of the main characteristics of DC magnetomodulation converters, it is necessary to develop mathematical models of their magnetic circuits, i.e. find analytical expressions for magnetic flux and magnetic voltage as a function of the length of the magnetic circuit.

For calculating the magnetic circuits of converters of electrical and non-electrical quantities with distributed parameters, in which there are no moving parts, the most effective is the classical method of drawing up and solving differential equations [3].

The magnetic system of which is shown in figure 3 is produced by the classical method of composing and solving differential equations.

In order to simplify the analysis, we neglect the side scattering fluxes and assume that the ring closed cores, as well as the ferromagnetic bridges, diametrically connecting the coaxially located ring closed cores, are identical.
3. Results and discussions

These assumptions introduce minor inaccuracies in the calculations, however, they greatly simplify the analysis of the circuit under consideration.

We will take into account the nonlinearity of the average magnetization curve in the first approximation using the average value of the specific magnetic resistance \( \rho_{\mu \text{mid}} \), defined as

\[
\rho_{\mu \text{mid}} = \rho_{\mu \text{min}} - \frac{\rho_{\mu \text{min}} - \rho_{\mu \text{max}}}{2} = \rho_{\mu \text{min}} + \frac{\rho_{\mu \text{max}}}{2},
\]

(2)

Where \( \rho_{\mu \text{min}} \) and \( \rho_{\mu \text{max}} \) - respectively, the minimum and maximum values of the specific magnetic resistance corresponding to the lower and upper limits of the change in the converted current.

Let us compose differential equations for the magnetic flux in closed annular ferromagnetic magnetic circuits and the magnetic voltage between them on the basis of Kirchhoff’s laws, created by the concentrated magnetic driving force \( F_x \), for an elementary section of the magnetic circuit \( d\alpha \) (figures 3 and 4):

\[
\dot{Q}_\mu(\alpha) - U_\mu(\alpha)C_{\mu\nu}d\alpha - Q_\mu(\alpha) - dQ_\mu(\alpha) = 0
\]

or

\[
\frac{dQ_\mu(\alpha)}{d\alpha} = -U_\mu(\alpha)C_{\mu\nu}.
\]

(3)

\[-U_\mu(\alpha) + Z_{\mu 1}Q_\mu(\alpha)d\alpha + U_\mu(\alpha) + dU_\mu(\alpha) + Z_{\mu 2}Q_\mu(\alpha)d\alpha = 0
\]

or

\[
\frac{dQ_\mu(\alpha)}{d\alpha} = -(Z_{\mu 1} + Z_{\mu 2})Q_\mu(\alpha).
\]

(4)

Where \( Q_\mu(\alpha), U_\mu(\alpha) \) - respectively, the magnetic flux in the ring cores and the magnetic voltage between them; \( Z_{\mu 1} = \frac{1}{\mu_0 b h_1} = Z_{\mu 2} = \frac{1}{\mu_0 b h_2} \) - linear values of the magnetic resistance of ring ferromagnetic cores 1 and 2 per unit angle of the magnetic circuit; \( C_{\mu\nu} = \mu_0 b \) - linear value of the magnetic capacitance of the gap between coaxially located ferromagnetic cores 1 and 2. The geometrical dimensions of cores 1 and 2 are shown in figure 3.

Differentiating (4) and substituting (3) into it, we obtain the following differential equation [4]:

\[
\frac{d^2U_\mu(\alpha)}{d\alpha^2} = (Z_{\mu 1} + Z_{\mu 2})C_{\mu\nu}Q_\mu(\alpha).
\]

(5)

The general solution of the differential equation (5) has the following form [4, p.279]:

\[
U_\mu(\alpha) = A_1 e^{\gamma\alpha} + A_2 e^{-\gamma\alpha},
\]

(6)
$$\gamma = \sqrt{2Z_{\mu}c_{\mu}}$$ – the coefficient of propagation of the magnetic flux along the length of the magnetic circuit; $A_1$ and $A_2$ are the constants of integration.

Substituting (6) into (4), we get:

$$Q_\mu(\alpha) = -\frac{\gamma}{2Z_{\mu}c_{\mu}}A_1 e^{\gamma \alpha} + \frac{\gamma}{2Z_{\mu}c_{\mu}}A_2 e^{-\gamma \alpha}$$ \hspace{1cm} (7)

Integration constants $A_1$ and $A_2$ are determined from the following boundary conditions [4, c.169]:

$$U_\mu(\alpha)|_{\alpha=0} = F_x - Q_\mu(\alpha)|_{\alpha=0}Z_{\mu 0}$$

$$U_\mu(\alpha)|_{\alpha=\alpha_M} = -\left[F_x - Q_\mu(\alpha)|_{\alpha=\alpha_M}Z_{\mu 0}\right]$$ \hspace{1cm} (8)

Where $Z_{\mu 0}$ - the magnetic resistance of the diametrical jumper.

It should be noted that the signs of the values of magnetic voltages between ring cores 1 and 2 on both sides of the magnetic neutral $M - M'$ (for $Z_{\mu 01} = Z_{\mu 02} = Z_{\mu 0}$ the magnetic neutral coincides with the geometric neutral 0 – 0’) are opposite (see figure 3).

Therefore, on the right-hand side of the second equation of system (8), the magnetic voltage $U_\mu(\alpha)$ enters with the "−" sign.

Substituting in (6) and (7) the values of the magnetic stress and magnetic flux corresponding to the boundary conditions, we obtain the following system of algebraic equations:

$$\left\{\begin{array}{l}
\left(1 - \frac{\gamma Z_{\mu 0}}{2Z_{\mu\|}}\right)A_1 + \left(1 + \frac{\gamma Z_{\mu 0}}{2Z_{\mu\|}}\right)A_2 = F_x \\
\left(1 + \frac{\gamma Z_{\mu 0}}{2Z_{\mu\|}}\right)e^{\gamma \alpha_M}A_1 + \left(1 - \frac{\gamma Z_{\mu 0}}{2Z_{\mu\|}}\right)e^{-\gamma \alpha_M}A_2 = -F_x,
\end{array}\right\}$$ \hspace{1cm} (9)

Solving the system of equations (9) for $A_1$ and $A_2$, we get:

$$A_1 = -\frac{F_x}{2\Delta}e^{-\gamma \alpha_M} + \frac{F_x}{2\Delta} \frac{\gamma Z_{\mu 0}}{2Z_{\mu\|}} e^{-\gamma \alpha_M} - \frac{F_x}{2\Delta} \frac{\gamma Z_{\mu 0}}{2Z_{\mu\|}},$$

$$A_2 = \frac{F_x}{2\Delta} \frac{\gamma Z_{\mu 0}}{2Z_{\mu\|}} + \frac{F_x}{2\Delta} \frac{\gamma Z_{\mu 0}}{2Z_{\mu\|}} e^{\gamma \alpha_M},$$ \hspace{1cm} (10)

Where $\Delta = \left(1 + \frac{\gamma Z_{\mu 0}^2}{4Z_{\mu\|}^2} \sinh(y(\alpha_M))\right) + \frac{\gamma Z_{\mu 0}}{Z_{\mu\|}} \cosh(y(\alpha_M)).$

Substituting the found values $A_1$ and $A_2$ in expressions (6) and (7), we get:

$$U_\mu(\alpha) = \frac{F_x}{\Delta} \left[\sinh(\gamma(\alpha_M - \alpha)) - \sinh(\gamma \alpha)\right] + \frac{F_x}{\Delta} \frac{\gamma Z_{\mu 0}}{2Z_{\mu\|}} \left\{\sinh(\gamma(\alpha_M - \alpha)) + \sinh(\gamma \alpha)\right\},$$ \hspace{1cm} (12)

$$Q_\mu(\alpha) = \frac{\gamma F_x}{\Delta} \left\{\sinh(\gamma(\alpha_M - \alpha)) + \sinh(\gamma \alpha)\right\} + \frac{\gamma Z_{\mu 0}}{2Z_{\mu\|}} \left\{\sinh(\gamma(\alpha_M - \alpha)) + \sinh(\gamma \alpha)\right\}.$$ \hspace{1cm} (13)

We transform (8) and (9), using the formulas of hyperbolic trigonometry and introducing some notation, we get:

$$U_\mu(\alpha^*) = \frac{F_x}{\Delta} \left[\cosh(\frac{1}{2} \beta_3) + \beta_3 K_0 \sinh(\frac{1}{2} \beta_3) \cosh\left(\frac{1}{2} \alpha^* - \alpha\right)\right]$$ \hspace{1cm} (14)

$$Q_\mu(\alpha^*) = \frac{\beta_3 F_x}{Z\mu\|\alpha M} \left[\cosh\left(\frac{1}{2} \beta_3\right) + \beta_3 K_0 \sinh\left(\frac{1}{2} \beta_3\right) \cosh\left(\frac{1}{2} \alpha^* - \alpha\right)\right]$$ \hspace{1cm} (15)

here $\lambda \alpha_M = \beta_3$, $K_0 = \frac{Z_{\mu 0}}{2Z_{\mu\|} \alpha M}$, $\alpha^* = \frac{\alpha - \alpha}{\alpha M}$, where $\beta_3$ – attenuation coefficient of the magnetic flux along the magnetic circuit.

To facilitate the analysis of these expressions, we turn to relative units:
Here $U_\mu(0)$ and $Q_\mu\left(\frac{1}{2}\right)$ maximum values respectively $U_\mu(\alpha^*)$ и $Q_\mu(\alpha^*)$.

In figure 5 and figure 6 shows the dependence curves $U_\mu(\alpha^*) = f(\alpha^*)$ and $Q_\mu(\alpha^*) = f(\alpha^*)$ for different values of $\beta_3$.

An analysis of expressions (16), as well as the curves constructed on their basis, shows that with an increase in the attenuation coefficient of the magnetic flux along the magnetic circuit $\beta_3$, the degree of nonlinearity of the change in the magnetic voltage and the variability of the magnetic flux along the magnetic circuit increase.

![Figure 5. Curves of dependence $U_\mu = f(\alpha^*)$ at different values of $\beta_3$.](image1)

![Figure 6. Curves of dependence $Q_\mu = f(\alpha^*)$ at different values of $\beta_3$.](image2)

The magnetic field strength of the current $I_x$ in the middle circular magnetic circuit is defined as:

$$H_x = \frac{1}{\alpha_m} Z_{\mu n1} \int_0^{\alpha_m} Q_\mu(\alpha) d\alpha = \frac{f_\mu}{\alpha_m^4 \left( \frac{\gamma Z_{\mu n}}{Z_{\mu n}} c h(\gamma \alpha_m) - 1 + sh(\gamma \alpha_m) \right)}.$$

(17)
Expressions (15), (16) and (17) are mathematical models of DC magnetomodulation converters magnetic circuits, taking into account the distributed nature of the magnetic resistance of the ring cores and the magnetic capacitance between these coaxially located cores.

It is shown that the magnetic voltage along the magnetic circuit is distributed nonlinearly and changes its sign at the point of magnetic neutral, and the magnetic flux is variable and has a minimum value at the point of magnetic neutral, and with an increase in the attenuation coefficient of the magnetic flux, the degree of nonlinearity of the magnetic voltage distribution and the variability of the magnetic flux along the length the magnetic circuit increases.

4. Conclusion
Thus, generalizing it can be said that the PT, controlling the charging currents of the AB in the APS, the following requirements are imposed:

- sensitivity to the direction of current in the controlled circuit;
- high differential sensitivity;
- high measurement accuracy;
- high linearity of static characteristics in the current range $I_{in} \pm \Delta I$
- maintain the conversion ability in the area of high currents;
- high reliability;
- low power consumption;
- small weight and dimensions.

Thus, ppts used in the systems of control and regulation of the charging current of the buffer AB in the APS must meet the above requirements.

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