Strong flavour changing effective operator contributions to single top quark production

P.M. Ferreira † R. Santos ‡
Centro de Física Teórica e Computacional, Faculdade de Ciências,
Universidade de Lisboa, Av. Prof. Gama Pinto, 2, 1649-003 Lisboa, Portugal

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Abstract. We study the effects of dimension six effective operators on the production of single top quarks at the LHC. The operator set considered includes terms with effective gluon interactions and four-fermion terms. Analytic expressions for the several partonic cross sections of single top production will be presented, as well as the results of their integration on the parton density functions.

1 Introduction

The top quark [1] is the heaviest particle thus far discovered. Its large mass makes it a natural laboratory to investigate deviations from Standard Model (SM) physics. Recently [2] we undertook a model-independent study of possible new physics effects on the phenomenology of the top quark. To this effect we considered a set of dimension six effective operators and analyzed its impact on observable quantities related to the top quark, such as its width or the cross section for single top quark production at the LHC. This procedure - the use of effective operators of dimension larger than four, the complete list of dimension five and six operators obtained in reference [3] - has been widely used to study the top particle. In refs. [4] the contributions from several dimension five and six operators for top quark physics, both at the Tevatron and the LHC, were studied. The $Wtb$ vertex was studied in great detail by the authors of ref. [5]. Because many proposals for theories that extend the SM (such as two Higgs doublet models or Supersymmetry) have potentially large contributions to flavour changing neutral currents, they have been the subject of many detailed studies, such as those found in [6]. Recent examples concerning single top production in supersymmetric models may be found in [7]. NLO and threshold corrections to flavour changing effective operators involving the top quark were studied in [8]. A different type of study, using four-fermion operators to tackle the issue of $t\bar{t}$ production, was undertaken in [9].

In ref. [2] we chose a particular set of dimension six operators and studied its effects on the properties of the top quark. Our choice was motivated by the following arguments: it included several operators already studied by other authors, albeit not in conjunction; those operators model possible effects arising from several different interesting extensions of the SM; they had
little or no impact on phenomena occurring at energy scales inferior to the LHC’s; and finally, the operators chosen involved flavour changing strong interactions with a single top quark and a gluon. Our philosophy in [2] was also somewhat different from that of most previous works in this field, in that we presented, whenever possible, analytical expressions. Our aim was, and is, to provide our experimental colleagues with formulae they can use directly in their Monte Carlo simulations.

In [2] we studied the simplest physical consequences of the operator set we chose: its contributions to the top’s width; the possibility of direct top production at the LHC; their effects on the production of a single top at the LHC via interference with the SM processes. In this work we will apply these same operators to more complicated processes of single top production, namely via partonic channels such as $gg \rightarrow t\bar{c}$, $gu \rightarrow gt$, $q\bar{q} \rightarrow t\bar{u}$ and others. We will also expand our operator set, considering three different types of four-fermion operators, which have great relevance for eight different processes of single top production. Their interference with the gluonic operators will also be studied.

This paper is structured as follows: in section 2 we will review the effective operator formalism, the criteria behind our choice of operators in ref. [2] and the results therein obtained. We will also introduce the new four-fermion operators and explain the logic behind their choice, and how their presence is demanded by the very equations of motion. In section 3 we present the calculations of the partonic cross sections for the processes $gg \rightarrow t\bar{u}$ and $gu \rightarrow gt$, as well as, after integration in the partonic density functions (pdf’s), their expected values at the LHC. In section 4 we will compute partonic cross sections of the form $qq' \rightarrow tq''$, where the four-fermion operators are now of crucial importance. Finally, in section 5 we will make a general discussion of the results obtained and draw some conclusions.

2 Effective operators and the top quark

The effective operator approach is based on the assumption that, at a given energy scale $\Lambda$, physics effects beyond those predicted by the SM make themselves manifest. We describe this by assuming the lagrangean

$$\mathcal{L} = \mathcal{L}^{SM} + \frac{1}{\Lambda^5} \mathcal{L}^{(5)} + \frac{1}{\Lambda^6} \mathcal{L}^{(6)} + O \left( \frac{1}{\Lambda^7} \right),$$

where $\mathcal{L}^{SM}$ is the SM lagrangean and $\mathcal{L}^{(5)}$ and $\mathcal{L}^{(6)}$ are all of the dimension 5 and 6 operators which, like $\mathcal{L}^{SM}$, are invariant under the gauge symmetries of the SM. The $\mathcal{L}^{(5)}$ terms break baryon and lepton number conservation, and are thus not usually considered. This leaves us with the $\mathcal{L}^{(6)}$ operators, some of which, after spontaneous symmetry breaking, generate dimension five terms. The list of dimension six operators is quite vast [3], therefore some sensible criteria of selection are needed. Underlying all our work is the desire to study a new possible type of physics, flavour changing strong interactions. The first criterion is to choose those $\mathcal{L}^{(6)}$ operators that have no sizeable impact on low energy physics (below the TeV scale, say). Another criterion was to only consider operators with a single top quark, since we will limit our studies to processes of single top production. Finally, we will restrict ourselves to operators with gluons, or four-fermion ones. No effective operators with electroweak gauge bosons will be considered.

The gluon operators that survive these criteria are but two, which, in the notation of ref. [3], are written as

$$\mathcal{O}_{uG} = i \frac{\alpha_s}{\Lambda^2} \left( \bar{u}^i_R \lambda^a \gamma^\mu u^j_R \right) G^{a}_\mu\nu.$$
\[ \mathcal{O}_{uG\phi} = \frac{\beta_{ij}}{\Lambda^2} \left( q^i_L \lambda^a \sigma^{\mu\nu} u^j_R \right) \bar{\phi} G^a_{\mu\nu} \]  

(2)

$q_L$ and $u_R$ are spinors (a left quark doublet and up-quark right singlet of $SU(2)$, respectively), $\phi$ is the charge conjugate of the Higgs doublet and $G^a_{\mu\nu}$ is the gluon tensor. $\alpha_{ij}$ and $\beta_{ij}$ are complex dimensionless couplings, the $(i, j)$ being flavour indices. According to our criteria, one of these indices must belong to the third generation. After spontaneous symmetry breaking the neutral component of the field $\phi$ acquires a vev ($\phi_0 \to \phi_0 + v$, with $v = 246/\sqrt{2}$ GeV) and the second of these operators generates a dimension five term. The lagrangean for new physics thus becomes

\[
\mathcal{L} = \alpha_{tu} \mathcal{O}_{tu} + \alpha_{ut} \mathcal{O}_{ut} + \beta_{tu} \mathcal{O}_{tu\phi} + \beta_{ut} \mathcal{O}_{ut\phi} + \text{h.c.}
\]

\[
= \frac{i}{\Lambda^2} \left[ [\alpha_{tu} (\bar{t}_R \lambda^a \gamma^\mu u_R) + \alpha_{ut} (\bar{u}_R \lambda^a \gamma^\mu t_R)] G^{a}_{\mu\nu} + \frac{v}{\Lambda^2} [\beta_{tu} (\bar{t}_L \lambda^a \sigma^{\mu\nu} u_R) + \beta_{ut} (\bar{u}_L \lambda^a \sigma^{\mu\nu} t_R)] G^{a}_{\mu\nu} + \text{h.c.} \right].
\]

(3)

This lagrangean describes new vertices of the form $g \bar{t} u$ ($g \bar{t} \bar{u}$) and $g g \bar{t} u$ ($g g \bar{t} \bar{u}$). We will also consider an analogous lagrangean (with new couplings $\alpha_{tc}, \beta_{ct}, \ldots$) for vertices of the form $g \bar{t} c$ ($g \bar{t} \bar{c}$) and $g g \bar{t} c$ ($g g \bar{t} \bar{c}$). Notice how the operators with $\beta$ couplings correspond to a chromo-magnetic momentum for the $t$ quark. Several extensions of the SM, such as supersymmetry and two Higgs doublet models, may generate contributions to this type of operator [10]. The Feynman rules for these anomalous vertices are shown in figure 11, with quark momenta following the arrows and incoming gluon momenta. The double gluon vertex was not considered in ref. [2] because it was not necessary there but, as we shall shortly see, it is of vital importance for this paper.

In ref. [2] we calculated the effect of these operators on the width of the quark top. They allow for the decay $t \to u g$ ($t \to c g$) (which is also possible in the SM, albeit at higher orders), and the corresponding width is given by

\[
\Gamma(t \to u g) = \frac{m_t^3}{12\pi\Lambda^4} \left\{ m_t^2 |\alpha_{ut} + \alpha_{tu}^*|^2 + 16 v^2 \left( |\beta_{tu}|^2 + |\beta_{ut}|^2 \right) + 8 v m_t \text{Im}[(\alpha_{ut} + \alpha_{tu}^*) \beta_{tu}] \right\}
\]

(4)

and an analogous expression for $\Gamma(t \to c g)$. In this expression, and throughout the entire paper, we will consider all quark masses, except the top’s, equal to zero; the imprecision introduced by this approximation is extremely small, as we verified having performed the full calculations. Direct top production is also possible with these new vertices (meaning, the production of a top quark from partonic reactions such as $g u \to t$ or $g c \to t$), and the corresponding cross section at the LHC is given by

\[
\sigma(p p \to t) = \sum_{q = u, c} \Gamma(t \to q g) \frac{m_t^2}{m_t^2} \int_0^1 \frac{2 m_t}{E_{CM}^2 x_1} f_g(x_1) f_q(m_t^2/(E_{CM}^2 x_1)) \, dx_1.
\]

(5)

In this expression $E_{CM}$ is the proton-proton center-of-mass energy (14 TeV at the LHC) and $f_g$ and $f_q$ are the parton density functions of the gluon and quark, respectively. It is not surprising that this cross section is proportional to the partial widths $\Gamma(t \to q g)$ - after all, the amplitudes for the decay $t \to u g$ or for direct top production via the channel $u g \to t$ are closely related by a time inversion transformation. Numerical results for these quantities were obtained in ref. [2], where we also derived bounds on the values of the $\{\alpha, \beta\}$ couplings.
\[
\frac{\lambda}{\Lambda^2} \left[ \gamma_\mu \gamma_R (\alpha_{ij} p_\nu + \alpha_{ij}^* q_\nu) + v \sigma_{\mu\nu} (\beta_{ij} \gamma_R + \beta_{ij}^* \gamma_L) \right]
\]
\[
(k^\mu g^\nu - k^\nu g^\mu)
\]

\[
\frac{\lambda}{\Lambda^2} \left[ \gamma_\mu \gamma_R (\alpha_{ij} q_\nu + \alpha_{ij}^* p_\nu) + v \sigma_{\mu\nu} (\beta_{ij} \gamma_R + \beta_{ij}^* \gamma_L) \right]
\]
\[
(k^\mu g^\nu - k^\nu g^\mu)
\]

\[
\frac{i g_s}{\Lambda^2} \left[ f_{abc} \left\{ \frac{\gamma_\mu \gamma_R (\gamma_{ij} p_\nu + \gamma_{ij}^* q_\nu)}{2} \right. \right. + 2 v \sigma_{\mu\nu} (\beta_{ij} \gamma_R + \beta_{ij}^* \gamma_L) \bigg] +
\frac{g_s}{\Lambda^2} \left[ (k_{ij} g_{\mu\nu} - k_{ij}^* \gamma_{ij}^\nu) \right. + \left. (k_{2j} g_{\mu\nu} - k_{2j}^* \gamma_{2j}^\nu) \right]
\]

Figure 1: Feynman rules for anomalous gluon vertices.

Notice how both the top width (4) and the cross section (5) depend on $\Lambda^{-4}$. There are processes with a $\Lambda^{-2}$ dependence, namely the interference terms between the anomalous operators and the SM diagrams of single top quark production, via the exchange of a W gauge boson - processes like $u \bar{d} \rightarrow t \bar{d}$. They were studied in ref. [2] in detail, and we discovered that, due to a strong CKM suppression, the contributions from the anomalous vertices are extremely small. We will come back to this point later.

Now, the operators that compose the lagrangean (3) are not, in fact, completely independent. If one performs integrations by parts and uses the fermionic equations of motion [3, 11], one obtains the following relations between them:

\[
O^t_{u} = O_{tu} - i \frac{1}{2} \left( \Gamma^t_{u} O^t_{ut\phi} + \Gamma_u O_{tu\phi} \right)
\]
\[
O^t_{u} = O_{tu} - i g_s \bar{t} \gamma_\mu \gamma_R \lambda^a u \sum_i \left( \bar{u}^i \gamma^\mu \gamma_R \lambda_a u^i + \bar{d}^i \gamma^\mu \gamma_R \lambda_a d^i \right)
\]

where $\Gamma_u$ are the Yukawa couplings of the up quark and $g_s$ the strong coupling constant. In the second of these equations we see the appearance of four-fermion terms, indicating that they have to be taken into account in these studies. Equations (6) then tell us that there are two relations between the several operators, which means that we are allowed to set two of the couplings to zero.

A careful analysis of the operators listed in [3] leads us to consider three types of four-fermion operators:
above, an overall factor of $g$ the effects of the several operators, we included, in the definitions of the four-fermion terms studying strong flavour changing effects. For this reason, and for an easier comparison between operators also signals their origin within the strong interaction sector, in line with our aim of considering the singlet operators but, since their spinorial structure is identical to these (lacking only the Gell-Mann matrices) we opted to leave them out. The presence of the $c$ changing interactions with the only the Gell-Mann matrices) we opted to leave them out. The presence of the $\lambda^a$ in these operators also signals their origin within the strong interaction sector, in line with our aim of studying strong flavour changing effects. For this reason, and for an easier comparison between the effects of the several operators, we included, in the definitions of the four-fermion terms above, an overall factor of $g_s$. The relative signs and disposition of quark spinors in the operators of types 2 and 3 are a reflex of their particular structure, emerging as they do from combinations of $SU(2)$ singlets and doublets. In eq. (10) we included a multiplicative factor of “$x$”; as can be seen from that equation, the four-fermion operator in question concerns only the bottom quark - unlike the operator in eq. (11), for which the flavour of the down-type quarks was left free. This means that for processes involving the bottom quark there will be more contributions to the amplitude than for non-bottom quarks. We will reflect this in our results by expressing them in terms of $x$ - the reader will then know that if that particular cross section contribution involves a bottom quark one must set $x = 1$, if not, then $x = 0$.

We emphasize that many possible operators with a single top quark were left out, due to another of our criteria, that low energy physics be not affected. For instance, we could have considered a type 2 operator of the form $(Q_L \lambda^a u_R) (\bar{u}_R \lambda^a Q'_L)$, where $Q_L$ is the quark doublet of the third generation, $Q_L = (t_L, b_L)$ and $Q'_L$ is a quark doublet of another generation. This would produce two terms in the lagrangean, namely

$$ (\bar{t} \lambda^a \gamma_R u) (\bar{u} \lambda^a \gamma_L u') + (\bar{b} \lambda^a \gamma_R u) (\bar{u} \lambda^a \gamma_L d') ,$$

and the second term in this expression has no bearing on top physics. It would only impact bottom physics, for instance, and thus its effects are already immensely constrained by the existing data. Finally, the reader will notice that we considered the same constants $\gamma_1, \gamma_2$ and $\gamma_3$ regardless of the flavour structure of the four-fermion operators. Having distinguished earlier on between $O_{ct}$ and $O_{cc}$, for instance, with the couplings $\alpha_{ct}$ and $\alpha_{cc}$, we should do the same here for consistency. However, that would introduce an enormous number of unconstrained parameters in the calculations, which would constitute a needless complication. We chose this simpler approach.
3 Cross sections for $g g \to t \bar{u}$ and $g u \to g t$

In ref. [2] we considered the contributions from the anomalous gluon operators to the simpler processes of single top production. We now present the results for more elaborate reactions, such as $g g \to t \bar{u}$. There are six Feynman diagrams contributing to this partonic cross section, shown in fig. (2). With the Feynman rules shown in fig. (1) it is a simple, if laborious, task to calculate the cross section $\sigma(g g \to t \bar{u})$. However, one important calculational detail warrants a special mention, given how seldom it is mentioned in the literature. For any process involving a single gluon with polarization $\epsilon_\mu^a(k)$, the calculation of the squared amplitude will involve the sum of the gluon polarizations, $\sum_i \epsilon_\mu^a(k) \epsilon_\nu^b(k)$. The use of the “Feynman trick” from QED is allowed and one may replace this sum by $-\delta^{ab} g_{\mu\nu}$. However, for any process involving two or more gluons, this is no longer possible. Instead, one must use the more complex expression [12]

$$\sum_{\text{spins}} \epsilon_\mu^a \epsilon_\nu^a = \delta^{ab} \left[ -g_{\mu\nu} + \frac{2}{s} \left( p_1\mu p_2\nu + p_1\nu p_2\mu \right) \right] ,$$

(12)

where $p_1$ and $p_2$ are the 4-momenta of the incoming particles (even if they are not gluons). The reason for this more complicated structure is the non-abelian nature of the theory. The extra terms in eq. (12) arise from the need to introduce Fadeev-Popov ghosts in the quantification of QCD. Failure to use the full structure of eq. (12) will result in a break of unitarity and negative cross sections.

For this process, then, the full calculation yields

$$\frac{d\sigma(g g \to t \bar{u})}{dt} = -\frac{g_s^2}{4 m_t^2} \frac{F_{gg}}{u t s^3 (s + t)^2 (s + u)^2} \Gamma(t \to u g) ,$$

(13)

where

$$F_{gg} = 4 s^2 t (s + t)^3 \left( s^2 + 2 s t + 2 t^2 \right) + s (s + t)^2 \left( 4 s^4 + 11 s^3 t + 48 s^2 t^2 + 52 s t^3 + 18 t^4 \right) u$$
$$+ 2 (s + t) \left( 10 s^5 + 27 s^4 t + 69 s^3 t^2 + 90 s^2 t^3 + 45 s t^4 + 9 t^5 \right) u^2$$
$$+ (s + t) \left( 44 s^4 + 115 s^3 t + 203 s^2 t^2 + 162 s t^3 + 36 t^4 \right) u^3$$
$$+ 2 \left( 26 s^4 + 85 s^3 t + 135 s^2 t^2 + 99 s t^3 + 27 t^4 \right) u^4 + 4 \left( 2 s + t \right) \left( 4 s^2 + 9 s t + 9 t^2 \right) u^5$$
$$+ 4 \left( s + 2 t \right) \left( 4 s^3 + 9 s^2 t + 9 s t^2 + t^3 \right) u^6$$
\[ + 2 \left( 4s^2 + 9st + 9t^2 \right) u^6 \]  

Remarkably, as in the case of the cross section for direct top production, this result is proportional to the width \( \Gamma(t \rightarrow ug) \).

For the process \( gu \rightarrow gt \) the procedure is very similar. We also have six diagrams, shown in fig. 3. The cross section for this process is clearly related to that of the previous one by

\[ \frac{d\sigma(gu \rightarrow gt)}{dt} = \frac{g_s^2}{24m_t^2} \frac{F_{gu}}{u s (s+u)^2 (t+u)^2} \Gamma(t \rightarrow ug), \]

where now we have

\[ F_{gu} = 16s^8 + s^7 \left( 241t + 72u \right) + 4s^6 \left( 305t^2 + 222t + 34u^2 \right) + s^5 \left( 3003t^3 + 3750t^2u + 1367tu^2 + 136u^3 \right) + 4t \left( t + u \right)^3 \left( 4t^4 + 6t^3u + 4t^2u^2 - u^4 \right) + 2s^4 \left( 1999t^4 + 3675t^2u + 2295t^2u^2 + 591tu^3 + 34u^4 \right) + s^3 \left( 3003t^5 + 7350t^4u + 6733t^3u^2 + 5010t^2u^3 + 636tu^4 + 4u^5 \right) + 2s^2 \left( t + u \right) \left( 610t^5 + 1265t^4u + 1030t^3u^2 + 475t^2u^3 + 104tu^4 - 6u^5 \right) + s(t+u)^2 \left( 241t^5 + 406t^4u + 314t^3u^2 + 148t^2u^3 + 26tu^4 - 4u^5 \right) \]

and once again we obtain a result proportional to \( \Gamma(t \rightarrow ug) \). For the processes \( gg \rightarrow t\bar{c} \) and \( gc \rightarrow gt \) we obtain expressions analogous to (13) and (15), with \( \Gamma(t \rightarrow ug) \) replaced by \( \Gamma(t \rightarrow cg) \).

If we assume that the branching ratio \( BR(t \rightarrow bW) \) is approximately 100% and use \( \Gamma(t \rightarrow bW) = 1.42 |V_{tb}|^2 \) GeV (a value which includes QCD corrections) \( [11][13] \), we may express the partial widths of eqs. (13) and (15) as \( \Gamma(t \rightarrow qg) = 1.42 |V_{tb}|^2 BR(t \rightarrow qg) \). In terms of these branching ratios, and using the CTEQ6M structure functions \( [14] \) to perform the integration in the pdf’s, we obtain, for the total cross sections, the following results (expressed in picobarns):

\[ \sigma(pp \rightarrow gg \rightarrow t\bar{q}) = \left[ 0.5BR(t \rightarrow ug) + 0.5BR(t \rightarrow cg) \right] |V_{tb}|^2 10^4 \]

1We used a factorization scale equal to the mass of the quark top, that being the characteristic scale of these reactions. This choice of \( \mu_F \) produces smaller cross section values than, saying, choosing it equal to the partonic center-of-mass energy \( [15] \).
Table 1: Coefficients of $BR(t \rightarrow ug)$ (in picobarn) in equations (17) for several values of the $p_T$ cut.

| Cut in $p_T$ (GeV) | 1  | 5  | 10 | 15 | 20 |
|-------------------|----|----|----|----|----|
| $g u \rightarrow g t$ | 33.4 | 29.3 | 12.0 | 8.2 | 6.4 |
| $g g \rightarrow \bar{q} t$ | 1.0 | 0.7 | 0.6 | 0.5 | 0.4 |

These are to be compared with the results obtained in [2] for the direct top cross section,

$$\sigma(pp \rightarrow gg \rightarrow \bar{q} t) = |V_{tb}|^2 10^4$$

(17)

$$\sigma(pp \rightarrow gq \rightarrow t) = [10.5 BR(t \rightarrow u g) + 1.6 BR(t \rightarrow c g)] |V_{tb}|^2 10^4$$

(18)

The larger values of the coefficients affecting the up-quark branching ratios in eqs. (17) and (18) derive from the fact that the pdf for that quark is larger than the charm’s. The numerical integration has an error of less than one percent. Except for the direct top channel, all of these cross sections (as well as the four-fermion results we will soon present) are integrated with a cut on the transverse momentum ($p_T$) of the light parton in the final state of 15 GeV. This is to remove the collinear and soft singularities in the gluon-quark subprocesses to render finite partonic cross sections, for a finite $p_T$ cut eliminates both of those divergences in two-to-two scattering processes. In a realistic analysis including backgrounds, a higher $p_T$ cut might well be needed, to suppress background rates in order to observe the signal events. That study, however, is beyond the scope of this work. Observe how the direct channel cross section is larger than the others. Notice, however, that due to the kinematics of that channel, no $p_T$ cut was applied. When imposing such a cut on the decay products of the top quark produced in the direct channel, the corresponding cross section will certainly be reduced.

We expect that a cut in $p_T$ should reduce the cross section for the gluon-quark channel in a more severe way than the gluon-gluon one. This is due to the fact that the $p_T$ cut eliminates most of the soft gluons in the gluon-top final state, thus placing us further away from a region where the cross section would be larger due to infrared divergencies. In table 1 we show the value of the coefficient multiplying the branching ration $BR(t \rightarrow ug)$ in equations (17), for the gluon-gluon and gluon-quark channels, for several values of the $p_T$ cut. As expected, the reduction of the gluon-quark cross section is much more severe than that of the gluon-gluon channel. Nevertheless, a somewhat surprising feature of these results is the larger values obtained for the channels $gq \rightarrow gt$, compared to the double gluon channel, $gg \rightarrow \bar{q} t$. This runs contrary to the conventional wisdom that the gluon-gluon channel ought to be the most important at the LHC, and stems from the fact that at the large energy scales expected at the LHC the quark content of the proton becomes larger. It also derives from a different factor on the average of the initial colours: the double gluon channel requires a colour average factor of 1/64, whereas the gluon-quark channel corresponds to a larger factor of 1/24.

It is quite remarkable that these cross sections are all proportional to the branching ratios
for rare decays of the top. These are possible even within the SM, at higher orders. For instance, one expects the SM value of $BR(t \rightarrow c g)$ to be of about $10^{-12}$ [10], $BR(t \rightarrow u g)$ two orders of magnitude smaller. What this means is that, if whatever new physics lies beyond the SM has no sizeable impact on the flavour changing decays of the top quark, so that its branching ratios are not substantially different from their SM values, then one does not expect any excess of single top production at the LHC through these channels. On the other hand, if an excess of single top production is observed, even a small one, the expressions (17) and (18) tell us that $BR(t \rightarrow c g)$ and $BR(t \rightarrow u g)$ will have to be very different from their SM values. In fact, in models with two Higgs doublets or supersymmetry, one expects the branching ratios $BR(t \rightarrow c g)$ and $BR(t \rightarrow u g)$ to increase immensely [10], in some models becoming as large as $\sim 10^{-4}$. If that is the case, eqs. (17) and (18) predict a significant increase in the cross section for single top production at the LHC. This cross section is therefore a very sensitive observable to probe for new physics.

### 4 Four-fermion channels

Other possible processes of single top production involve quark-quark (or quark-antiquark) scattering. There are in fact eight such possible reactions, which we list in table 2. Notice that in this table we included only processes where there is a “single” flavour violation - in other words, though processes like $s \bar{d} \rightarrow t \bar{u}$ are a priori possible from the four-fermion operators we considered, we will not study them here. In fact, this is consistent with our choice of gluonic operators, as such processes are not possible with the vertices in fig. [1]. The resulting cross sections now have contributions from both the gluonic operators [3] and from the four fermion operators described earlier. The Feynman diagrams for the process $u u \rightarrow t u$ are shown in figure [4], but in what concerns the four-fermion contributions, there is a subtlety that must be mentioned: depending on the process considered, each four-fermion operator may contribute twice to the squared amplitude. For instance, for the process just mentioned, there is an operator of Type 1 that surely contributes: $\gamma_{u1} (t \lambda^a \gamma^\mu \gamma_R u) (\bar{u} \lambda^a \gamma_\mu \gamma_R u)$. When deducing the Feynman
Figure 4: Feynman diagrams for $qq \rightarrow qt$. The four-fermion graph can generate both “t-channel” and “u-channel” contributions.

rule corresponding to this term, we conclude that it gives us a “t-channel” (when the first $u$ spinor in the operator corresponds to the first incoming momentum) and a “u-channel” (when the first incoming momentum is attributed to the second $u$ spinor), both contributions to the amplitude differing by a minus sign. In fig. [1] we represent only one four-fermion graph for simplicity. Notice that for the process $uc \rightarrow tc$ the $u$-channels we just considered (both from four-fermion operators and gluonic ones) are not present. When antiquarks are present, such as

Figure 5: Feynman diagrams for $q\bar{q} \rightarrow \bar{q}t$. The four-fermion graph can generate $s$, $t$ and $u$ channel contributions.

in the process $u\bar{u} \rightarrow t\bar{u}$, gluonic $s$-channels are also present, as we see in fig. [5]. Again, the four-fermion operators may have several distinct contributions to the amplitudes, namely an $s$ channel and a $t$ one. One way of thinking of these different amplitude contributions is reading the four-fermion operators in terms of interacting currents. Depending on the positioning of the fermion spinors within the operator, we will obtain different currents. For instance, if we number the $u$ spinors in the process just mentioned such that $u_1 \bar{u}_2 \rightarrow t \bar{u}_3$ and look at, for instance, the type 1 four-fermion operators, we see that there are two possibilities for the disposition of

Figure 6: Interpretation of the four-fermion terms contributing to the process $u_1 \bar{u}_2 \rightarrow t \bar{u}_3$ in terms of currents; notice the analog of a $t$-channel and an $s$ one.

the $u$ spinors, namely $(\bar{t} \gamma^\mu \gamma_R u_1) (\bar{u}_2 \gamma^\mu \gamma_R u_3)$ and $(\bar{t} \gamma^\mu \gamma_R u_3) (\bar{u}_2 \gamma^\mu \gamma_R u_1)$, corresponding, in terms of “currents”, to the left and right diagrams of figure [6], respectively - a
t-channel and an s-channel. A careful inspection of the Feynman rules obtained from each of these two terms would lead to the conclusion there is a relative minus sign between both of their contributions.

The expressions we obtain are very elaborate, as there are many interference terms between the several operators. Let us first consider the terms of the squared amplitude that are only proportional to the gluonic couplings. For process (1) in table 2 the expression we obtain is

\[ |T^{(1)}_{\alpha,\beta}|^2 = -\frac{2}{27t} \left\{ F_1 |\alpha_{ct}|^2 + F_2 |\alpha_{tc}|^2 + 16v^2 F_3 \left( |\beta_{ct}|^2 + |\beta_{tc}|^2 \right) + 2F_4 \Re(\alpha_{ct} \alpha_{tc}) \right. \]
\[ + 16mtv \left[ F_5 \Im(\alpha_{ct} \beta_{tc}) - F_6 \Im(\alpha_{tc} \beta_{tc}) \right] \right\} \frac{1}{\Lambda^4} \quad (19) \]

where we have

\[ F_1 = 3m_t^6 t - 6m_t^4 t^2 + 6m_t^2 t^2 u - 3m_t^6 u + 12m_t^4 t u + 5m_t^2 t^2 u - 11t^3 u \]
\[ + 6m_t^4 u^2 + 2m_t^4 u^2 - 16t^2 u^2 + 6m_t^2 u^3 - 11t u^3 \]
\[ F_2 = 3m_t^6 t - 6m_t^4 t^2 + 6m_t^2 t^2 u - 3m_t^6 u - 20m_t^4 t u + 25m_t^2 t^2 u - 11t^3 u \]
\[ - 6m_t^4 u^2 + 2m_t^4 u^2 - 16t^2 u^2 + 6m_t^2 u^3 - 11t u^3 \]
\[ F_3 = 3m_t^6 t - 6m_t^4 t^2 + 6m_t^2 t^2 u - 3m_t^6 u - 4m_t^4 t u + 4t^2 u - 6m_t^2 u^2 + 4t u^2 + 6u^3 \]
\[ F_4 = 3m_t^6 t - 6m_t^4 t^2 + 6m_t^2 t^2 u - 3m_t^6 u - 4m_t^4 t u - 7m_t^2 t^2 u + 11t^3 u \]
\[ - 6m_t^4 u^2 - 7m_t^2 u^2 + 16t^2 u^2 + 6m_t^2 u^3 + 11t u^3 \]
\[ F_5 = 3m_t^6 t - 6m_t^4 t^2 + 6m_t^2 t^2 u - 3m_t^6 u + 4m_t^4 t u - t^2 u - 6m_t^2 u^2 - t u^2 + 6u^3 \]
\[ F_6 = 3(t + u) \left( m_t^4 - 2m_t^2 t + 2t^2 - 2m_t^2 u + t u + 2u^2 \right) \quad (20) \]

The expressions we would obtain for process (3) may be obtained from those of process (1), with the Mandelstam variable replacements \( t \rightarrow s \) and \( u \rightarrow t \). For process (2) the expressions are a lot simpler:

\[ |T^{(2)}_{\alpha,\beta}|^2 = -\frac{2}{9t} \left\{ \frac{2}{9t} (s + u) (4t m_t^2 + s^2 + u^2) |\alpha_{ct}|^2 + \frac{2}{9t} (s + u) (s^2 + u^2) |\alpha_{tc}|^2 \right. \]
\[ + \frac{32v^2}{9t} \left[ (s + u) m_t^2 - 2su \right] \left( |\beta_{ct}|^2 + |\beta_{tc}|^2 \right) \right. \]
\[ + \frac{4}{9t} (t + m_t^2) (s^2 + u^2) \Re(\alpha_{ct} \alpha_{tc}) \]
\[ + \frac{16mtv}{9t} \left[ \left( m_t^4 - t^2 - 2s u \right) \Im(\alpha_{ct} \beta_{tc}) - (s^2 + u^2) \Im(\alpha_{tc} \beta_{tc}) \right] \right\} \frac{1}{\Lambda^4} \quad (21) \]

Processes \{(5), (7), (8)\} have identical expressions to these for process (2); processes \{(4), (6)\} have expressions very similar to those of eq. (21), with the substitution \( t \leftrightarrow s \).

In table 3, we present the squared amplitude terms involving only the four-fermion couplings. As is obvious from the definitions of the Type 3 operators (eqs. (9) and (10)) they always mix down and up quarks, thus they have no contribution whatsoever to the processes that involve only up quarks (processes (1) to (5)). Also, due to the chiral structure of the several four-fermion operators, there are no interference terms between them.

To complete the expressions for the squared amplitudes we lack only the interference terms between the gluonic operators and the four-fermion ones. We present the results for the squared amplitudes in tables 4 and 5. Notice the absence of any terms proportional to \( \beta_{ct} \) or \( \gamma_{us} \), a consequence of the particular left-right structures associated with those couplings. The equations (10) and (21), and the expressions presented in tables 3 - 5 refer to the squared amplitudes.
of the several quark-quark processes. Gathering the several multiplicative factors, the differential cross section is given by
\[
\frac{d\sigma}{dt} = \frac{\alpha_s}{144 s^2} |T|^2 ,
\]
with $|T|^2$ the total squared amplitude for each process and where we have included a factor of 1/4 (average on initial spins) and 1/9 (average on initial colours). The overall factor of $\alpha_s$ derives from the fact that all squared amplitudes are proportional to $g_s^2$.

5 Results for the integrated cross sections

We can now gather all the results obtained in this paper and in ref. [2] for the cross sections of single top production. In terms of the couplings, the direct channel, eq. (18), gives us
\[
\sigma_{g u \rightarrow t} = \left\{ 321 \left| \alpha_{ut} + \alpha_{tu}^* \right|^2 + 5080 \left( |\beta_{tu}|^2 + |\beta_{ut}|^2 \right) + 2556 \text{Im} \left[ (\alpha_{ut} + \alpha_{tu}^*) \beta_{tu} \right] \right\} \frac{1}{\Lambda^4} \text{ pb} ,
\]
for the partonic channel $g u \rightarrow t$. For the gluon-gluon and gluon-quark channels, we have, from eqs. (17),
\[
\begin{align*}
\sigma_{g g \rightarrow t u} &= \left\{ 14 \left| \alpha_{ut} + \alpha_{tu}^* \right|^2 + 221 \left( |\beta_{tu}|^2 + |\beta_{ut}|^2 \right) + 111 \text{Im} \left[ (\alpha_{ut} + \alpha_{tu}^*) \beta_{tu} \right] \right\} \frac{1}{\Lambda^3} \text{ pb} \\
\sigma_{g u \rightarrow gt} &= \left\{ 250 \left| \alpha_{ut} + \alpha_{tu}^* \right|^2 + 3952 \left( |\beta_{tu}|^2 + |\beta_{ut}|^2 \right) + 1988 \text{Im} \left[ (\alpha_{ut} + \alpha_{tu}^*) \beta_{tu} \right] \right\} \frac{1}{\Lambda^4} \text{ pb} . 
\end{align*}
\]
Finally, the four-fermion processes can all be gathered (after integration on the parton density functions, as before) in a single expression,}

$$\sigma_{4F}^{(u)} = \left[ 171 |\alpha_{ut}|^2 + 179 |\alpha_{tu}|^2 - 176 \text{Re}(\alpha_{ut} \alpha_{tu}) + 331 \text{Im}(\alpha_{ut} \beta_{tu}) - 362 \text{Im}(\alpha_{tu} \beta_{tu}^*) 
+ 689 \left( |\beta_{tu}|^2 + |\beta_{ut}|^2 \right) + 177 \text{Re}(\alpha_{ut} \gamma_{u1}) - 185 \text{Re}(\alpha_{tu} \gamma_{u1}^*) - 16 \text{Im}(\beta_{tu} \gamma_{u1}^*) 
- 17 \text{Re}(\alpha_{ut} \gamma_{u2}) + 17 \text{Re}(\alpha_{tu} \gamma_{u2}^*) + 0.1 \text{Im}(\beta_{tu} \gamma_{u2}^*) 
+ 525 |\gamma_{u1}|^2 + 94 |\gamma_{u2}|^2 + 88 |\gamma_{u3}|^2 \right] \frac{1}{\Lambda^4} \text{ pb}. \quad (25)$$

For the channels proceeding through the charm quark, we have analogous expressions, with different numeric values in most cases due to different parton content inside the proton. We find

| Process | $\text{Re}(\alpha_{ct} \gamma_{u1})/\Lambda^4$ | $\text{Re}(\alpha_{tc} \gamma_{u1}^*)/\Lambda^4$ | $\text{Im}(\beta_{tc} \gamma_{u1}^*)/\Lambda^4$ |
|---------|-----------------|-----------------|-----------------|
| (1)     | 0               | 0               | 0               |
| (2)     | $-\frac{32}{27} st\left(2 m_t^2 - s\right)$ | $-\frac{32}{27} s^2$ | $-\frac{128}{27} s m_t v$ |
| (3)     | 0               | 0               | 0               |
| (4)     | $-\frac{64}{27} u\left(2 m_t^2 - u\right)$ | $-\frac{64}{27} u^2$ | $-\frac{256}{27} m_t v u$ |
| (5)     | $-\frac{64}{27} u\left(2 m_t^2 - u\right)$ | $-\frac{64}{27} u^2$ | $-\frac{256}{27} m_t v u$ |
| (6)     | $-\frac{16}{9} u\left(2 m_t^2 - u\right)$ | $-\frac{16}{9} u^2$ | $-\frac{64}{9} m_t v u$ |
| (7)     | $-\frac{16}{9} s\left(2 m_t^2 - s\right)$ | $-\frac{16}{9} s^2$ | $-\frac{64}{9} m_t v s$ |
| (8)     | $-\frac{16}{9} u\left(2 m_t^2 - u\right)$ | $-\frac{16}{9} u^2$ | $-\frac{64}{9} m_t v u$ |

Table 4: Interference terms between gluonic and Type 1 four-fermion operators for single top production processes.
\[
\text{Table 5: Interference terms between gluonic and Type 2 four-fermion operators for single top production processes.}
\]

\[
\begin{align*}
\text{Process} & & \frac{\text{Re}(\alpha_{ct} \gamma^*_{u_2})}{\Lambda^4} & & \frac{\text{Re}(\alpha_{tc} \gamma^*_{u_2})}{\Lambda^4} & & \frac{\text{Im}(\beta_{tc} \gamma^*_{u_2})}{\Lambda^4} \\
(1) & & \frac{8}{27} (m_t^2 + s) (t - u) & & \frac{8}{27} (t^2 - u^2) & & \frac{32 m_t v}{27} (t - u) \\
(2) & & - \frac{8}{27} u (2 m_t^2 - u) & & - \frac{8}{27} u^2 & & - \frac{32 m_t v}{27} u \\
(3) & & \frac{8}{27} (m_t^2 + u) (t - s) & & \frac{8}{27} (t^2 - s^2) & & \frac{32 m_t v}{27} (t - s) \\
(4) & & \frac{8}{27} t (2 m_t^2 - t) & & \frac{8}{27} t^2 & & \frac{32 m_t v}{27} t \\
(5) & & \frac{8}{27} s (2 m_t^2 - s) & & \frac{8}{27} s^2 & & \frac{32 m_t v}{27} s \\
(6) & & \frac{8}{27} t (2 m_t^2 - t) & & \frac{8}{27} t^2 & & \frac{32 m_t v}{27} t \\
(7) & & \frac{8}{27} u (2 m_t^2 - u) & & \frac{8}{27} u^2 & & \frac{32 m_t v}{27} u \\
(8) & & \frac{8}{27} s (2 m_t^2 - s) & & \frac{8}{27} s^2 & & \frac{32 m_t v}{27} s \\
\end{align*}
\]

Within the four-fermion cross sections, eqs. (25) and (26), are the results for production of a bottom quark alongside the top, through the processes \(u b \rightarrow t b\) and \(u \bar{b} \rightarrow t \bar{b}\) (and analogous processes for the \(c\) quark). They are given by

\[
\sigma_{t+b}^{(u)} = \left[ 8 |\alpha_{ut}|^2 + 9 |\alpha_{tu}|^2 - 2 \text{Re}(\alpha_{ut} \alpha_{tu}) + 28 \text{Im}(\alpha_{ut} \beta_{tu}) - 32 \text{Im}(\alpha_{tu} \beta_{tu}^*) \right] \frac{1}{\Lambda^4} \text{ pb}.
\]

\[
\sigma_{t+b}^{(c)} = \left[ 0.4 |\alpha_{ct}|^2 + 0.6 |\alpha_{tc}|^2 + 0.2 \text{Re}(\alpha_{ct} \alpha_{tc}) + 2 \text{Im}(\alpha_{ct} \beta_{tc}) - 3 \text{Im}(\alpha_{tc} \beta_{tc}^*) \right] \frac{1}{\Lambda^4} \text{ pb}
\]
where the interference terms between the \{\alpha, \beta\} and the \gamma were left out because they were too small when compared with the remaining terms.

Finally, by changing the pdf integrations and using the second vertex in figure (11), we can also obtain the cross sections for anti-top production. We obtain

\[
\sigma_{g \bar{u} \to \bar{t}} = \left\{ \begin{array}{l}
83 |\alpha_{ut} + \alpha_{tc}^*|^2 + 1312 \left( |\beta_{tu}|^2 + |\beta_{ut}|^2 \right) + 660 \text{Im} \{ (\alpha_{ut} + \alpha_{tu}^*) \beta_{tu} \} \frac{1}{\Lambda^4} \text{ pb} \\
14 |\alpha_{at} + \alpha_{ct}^*|^2 + 221 \left( |\beta_{tu}|^2 + |\beta_{ut}|^2 \right) + 111 \text{Im} \{ (\alpha_{at} + \alpha_{ct}^*) \beta_{tu} \} \frac{1}{\Lambda^4} \text{ pb} \\
45 |\alpha_{ut} + \alpha_{tu}^*|^2 + 711 \left( |\beta_{tu}|^2 + |\beta_{ut}|^2 \right) + 358 \text{Im} \{ (\alpha_{ut} + \alpha_{tu}^*) \beta_{tu} \} \frac{1}{\Lambda^4} \text{ pb} \\
32 |\alpha_{at}|^2 + 32 |\alpha_{ct}^*|^2 - 19 \text{Re} (\alpha_{at} \alpha_{tu}) + 90 \text{Im} (\alpha_{at} \beta_{tu}) - 90 \text{Im} (\alpha_{ct} \beta_{tu}) \\
+ 178 \left( |\beta_{tu}|^2 + |\beta_{ut}|^2 \right) - 21 \text{Re} (\alpha_{at} \gamma_{u1}) + 21 \text{Re} (\alpha_{ct} \gamma_{u1}^*) + \text{Im} (\beta_{tu} \gamma_{u1}) \\
+ 3 \text{Re} (\alpha_{at} \gamma_{u2}) - 1 \text{Re} (\alpha_{ct} \gamma_{u2}^*) - \text{Im} (\beta_{tu} \gamma_{u2}^*) \\
+ 56 |\gamma_{u1}|^2 + 26 |\gamma_{u2}|^2 + 35 |\gamma_{u3}|^2 \frac{1}{\Lambda^4} \text{ pb}
\end{array} \right.
\]

(29)

and also

\[
\sigma_{g \bar{c} \to \bar{t}} = \left\{ \begin{array}{l}
50 |\alpha_{ct} + \alpha_{tc}^*|^2 + 791 \left( |\beta_{tc}|^2 + |\beta_{ct}|^2 \right) + 398 \text{Im} \{ (\alpha_{ct} + \alpha_{tc}^*) \beta_{tc} \} \frac{1}{\Lambda^4} \text{ pb} \\
14 |\alpha_{ct} + \alpha_{tc}^*|^2 + 221 \left( |\beta_{tc}|^2 + |\beta_{ct}|^2 \right) + 111 \text{Im} \{ (\alpha_{ct} + \alpha_{tc}^*) \beta_{tc} \} \frac{1}{\Lambda^4} \text{ pb} \\
25 |\alpha_{ct} + \alpha_{tc}^*|^2 + 395 \left( |\beta_{tc}|^2 + |\beta_{ct}|^2 \right) + 199 \text{Im} \{ (\alpha_{ct} + \alpha_{tc}^*) \beta_{tc} \} \frac{1}{\Lambda^4} \text{ pb} \\
20 |\alpha_{ct}|^2 + 20 |\alpha_{tc}^*|^2 - 12 \text{Re} (\alpha_{ct} \alpha_{tc}) + 53 \text{Im} (\alpha_{ct} \beta_{tc}) - 55 \text{Im} (\alpha_{tc} \beta_{tc}) \\
+ 107 \left( |\beta_{tc}|^2 + |\beta_{ct}|^2 \right) - 32 \text{Re} (\alpha_{ct} \gamma_{c1}) + 36 \text{Re} (\alpha_{tc} \gamma_{c1}^*) + 8 \text{Im} (\beta_{tc} \gamma_{c1}^*) \\
+ 7 \text{Re} (\alpha_{ct} \gamma_{c2}) - 7 \text{Re} (\alpha_{tc} \gamma_{c2}^*) + 0.3 \text{Im} (\beta_{tc} \gamma_{c2}^*) \\
+ 82 |\gamma_{c1}|^2 + 29 |\gamma_{c2}|^2 + 29 |\gamma_{c3}|^2 \frac{1}{\Lambda^4} \text{ pb} 
\end{array} \right.
\]

(30)

For completeness, the cross sections for production of an anti-top alongside with a bottom quark are (leaving out terms which are too small compared with the others)

\[
\sigma_{t+b}^{(u)} = \left[ \begin{array}{l}
1 |\alpha_{ut}|^2 + 1 |\alpha_{tu}|^2 + 0.4 \text{Re} (\alpha_{ut} \alpha_{tu}) + 5 \text{Im} (\alpha_{ut} \beta_{tu}) - 4 \text{Im} (\alpha_{ut} \beta_{tu}^*) \\
+ 10 \left( |\beta_{tu}|^2 + |\beta_{ut}|^2 \right) - 2 \text{Re} (\alpha_{ut} \gamma_{u1}) + 1 \text{Re} (\alpha_{tu} \gamma_{u1}^*) - 0.5 \text{Im} (\beta_{tu} \gamma_{u1}) \\
+ 0.2 \text{Re} (\alpha_{ut} \gamma_{u2}) - 0.2 \text{Re} (\alpha_{tu} \gamma_{u2}^*) + 2 |\gamma_{u1}|^2 \\
+ 0.5 |\gamma_{u2}|^2 + 2 |\gamma_{u3}|^2 \frac{1}{\Lambda^4} \text{ pb}
\end{array} \right.
\]

\[
\sigma_{t+b}^{(c)} = \left[ \begin{array}{l}
0.4 |\alpha_{at}|^2 + 0.6 |\alpha_{ct}|^2 + 0.2 \text{Re} (\alpha_{at} \alpha_{tc}) + 3 \text{Im} (\alpha_{at} \beta_{tc}) - 2 \text{Im} (\alpha_{at} \beta_{tc}^*) \\
+ 5 \left( |\beta_{tc}|^2 + |\beta_{ct}|^2 \right) - 0.7 \text{Re} (\alpha_{at} \gamma_{c1}) + 0.5 \text{Re} (\alpha_{tc} \gamma_{c1}^*) - 0.5 \text{Im} (\beta_{tc} \gamma_{c1}^*) \\
+ 0.8 |\gamma_{c1}|^2 + 0.2 |\gamma_{c2}|^2 + 0.6 |\gamma_{c3}|^2 \frac{1}{\Lambda^4} \text{ pb} 
\end{array} \right.
\]

(31)
We have thus far presented the complete expressions for the cross sections but, as was discussed earlier and is made manifest by equation (6), some of the operators we considered are not independent. In fact, eq. (6) implies that we can choose two of the couplings \{\alpha_{ut}, \alpha_{tu}, \beta_{ut}, \beta_{tu}, \gamma_{u_1}\} to be equal to zero. Notice that \gamma_{u_2} and \gamma_{u_3} are not included in this choice, as the respective operators do not enter into equations (6). A similar conclusion may be drawn, of course, about the couplings \{\alpha_{ct}, \alpha_{tc}, \beta_{ct}, \beta_{tc}, \gamma_{c_1}\}. We choose to set \beta_{tu} and \gamma_{u_1} to zero, as this choice eliminates many of the interference terms of the cross sections. Summing all of the different contributions, we obtain, for the single top production cross section, the following results:

\[
\sigma^{(u)}_{\text{single } t} = \left[ 756 |\alpha_{ut}|^2 + 764 |\alpha_{tu}|^2 + 994 \text{Re}(\alpha_{ut} \alpha_{tu}) + 9942 |\beta_{ut}|^2 \\
-17 \text{Re}(\alpha_{ut} \gamma_{u_2}) + 17 \text{Re}(\alpha_{tu} \gamma_{u_2}^*) + 94 |\gamma_{u_2}|^2 + 88 |\gamma_{u_3}|^2 \right] \frac{1}{\Lambda^4} \text{ pb },
\]

\[
\sigma^{(c)}_{\text{single } t} = \left[ 109 |\alpha_{ct}|^2 + 109 |\alpha_{tc}|^2 + 166 \text{Re}(\alpha_{ct} \alpha_{tc}) + 1514 |\beta_{ct}|^2 \\
-3 \text{Re}(\alpha_{ct} \gamma_{c_2}) + 3 \text{Re}(\alpha_{tc} \gamma_{c_2}^*) + 24 |\gamma_{c_2}|^2 + 27 |\gamma_{c_3}|^2 \right] \frac{1}{\Lambda^4} \text{ pb }. \quad (32)
\]

For anti-top production,

\[
\sigma^{(u)}_{\text{single } \bar{t}} = \left[ 174 |\alpha_{ut}|^2 + 174 |\alpha_{tu}|^2 + 265 \text{Re}(\alpha_{ut} \alpha_{tu}) + 2422 |\beta_{ut}|^2 \\
+3 \text{Re}(\alpha_{ut} \gamma_{u_2}) - \text{Re}(\alpha_{tu} \gamma_{u_2}^*) + 26 |\gamma_{u_2}|^2 + 35 |\gamma_{u_3}|^2 \right] \frac{1}{\Lambda^4} \text{ pb }
\]

\[
\sigma^{(c)}_{\text{single } \bar{t}} = \left[ 109 |\alpha_{ct}|^2 + 109 |\alpha_{tc}|^2 + 166 \text{Re}(\alpha_{ct} \alpha_{tc}) + 1514 |\beta_{ct}|^2 \\
+7 \text{Re}(\alpha_{ct} \gamma_{c_2}) - 7 \text{Re}(\alpha_{tc} \gamma_{c_2}^*) + 29 |\gamma_{c_2}|^2 + 29 |\gamma_{c_3}|^2 \right] \frac{1}{\Lambda^4} \text{ pb }. \quad (33)
\]

As mentioned earlier, there are also interference terms between our gluonic operators and the electroweak processes of single top production in the SM. These depend on \Lambda^{-2} but are very small, namely

\[
\sigma^{\text{int}}_{\text{single } t} = 0.81 \frac{|\text{Re}(\beta_{ut})|}{\Lambda^2} + 0.27 \frac{|\text{Re}(\beta_{ct})|}{\Lambda^2} \text{ pb}. \quad (34)
\]

Given the different \Lambda dependence on these interference terms and our results for the cross sections (32), it is worth asking what is the domain of values of \Lambda for which the \Lambda^{-4} terms are superior to those of eq. (34). To obtain a reasonable estimate, we consider only the terms proportional to |\beta|^2 in the cross sections (32) and (33). We further simplify the estimation by taking \beta_{ut} = \beta_{ct} |V_{ub}/V_{cb}|. In figure (7) we plot the value of the cross sections (32) and (34) versus \Lambda/|\beta_{ct}|. We see that they only cross for very large values of this variable, of about 30 TeV. For scales of new physics inferior to, say, \sim 30 TeV, the cross sections presented in this paper are almost exactly the full contributions from the set of effective operators we chose to single top production.

There is an extensive literature on the subject of single top production [17]. For the LHC, the SM prediction is usually considered to be 319.7 \pm 19.3 \text{ pb} [15]. Considering the large numbers we are obtaining in the expressions above - specially the coefficients of the \beta couplings, though the others are not in any way negligible - we can see that even a small deviation from the SM framework will produce a potentially large effect in this cross section. It is indeed a good observable to test new physics, as it seems so sensible to its presence. Alternatively, if the cross section for single top production at the LHC is measured in the years to come and is found to be in complete agreement with the SM predicted value, then we will be able to set extremely stringent bounds on the couplings \{\alpha, \beta, \gamma\} - on new physics in general - precisely for the same reasons.
Figure 7: Interference cross section (solid line) and total cross section, from eq. (32) (dashed line), versus \( \Lambda / \sqrt{|\beta_{ct}|} \). Notice that the two curves only cross for very large values of the new physics energy scale.

In conclusion, we have calculated the contributions from a large set of dimension six operators to cross sections of several processes of single top production at the LHC. All cross sections involving gluons in the initial or final states are proportional to branching ratios of rare top quark decays. This makes these processes extremely sensitive to new physics, since those branching ratios may vary by as much as eight orders of magnitude in the SM and extended models. The four-fermion operators we chose break this proportionality so that, even if the branching ratios of the top quark conform to those of the SM, we may still have an excess of single top production at the LHC, stemming from those same operators. One of the advantages of working in a fully gauge-invariant manner is the possibility of using the equations of motion to introduce relations between the operators and thus reduce the number of independent parameters. One possible further simplification, if one so wishes, would be to consider each generation’s couplings related by the SM CKM matrix elements, so that, for instance, \( \alpha_{tu} = \alpha_{tc} |V_{ub}/V_{cb}| \). This should constitute a reasonable estimate of the difference in magnitude between each generations’ couplings. Finally, in this paper we presented both the total anomalous cross sections for single top production and those of the individual processes that contribute to it. If there is any experimental method - through kinematical cuts or jet analysis - to distinguish between each of the possible partonic channels (direct top production; gluon-quark fusion; gluon-gluon fusion; quark-quark scattering), the several expressions we presented here will allow a direct comparison between theory and experiment. At this point a thorough detector simulation of these processes is needed to establish under which conditions, if any, they might be observed at the LHC, and what precision one might expect to obtain on bounds on the couplings \( \{\alpha, \beta, \gamma\} \).

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References.

[1] M. Beneke et al., hep-ph/0003033
   D. Chakraborty, J. Konigsberg and D. Rainwater, Ann. Rev. Nucl. Part. Sci. 53 (2003) 301;
   W. Wagner, Rept. Prog. Phys. 68 (2005) 2409.

[2] P.M. Ferreira, O. Oliveira and R. Santos, Phys. Rev. D73 (2006) 034011.

[3] W. Buchmüller and D. Wyler, Nucl. Phys. B268 (1986) 621.

[4] E. Malkawi and T. Tait, Phys. Rev. D54 (1996) 5758;
   T. Han, K. Whisnant, B.L. Young and X. Zhang, Phys. Lett. B385 (1996) 311;
   T. Han, M. Hosch, K. Whisnant, B.L. Young and X. Zhang, Phys. Rev. D55 (1997) 7241;
   K. Whisnant, J.M. Yang, B.L. Young and X. Zhang, Phys. Rev. D56 (1997) 467;
   M. Hosch, K. Whisnant and B.L. Young, Phys. Rev. D56 (1997) 5725;
   T. Han, M. Hosch, K. Whisnant, B.L. Young and X. Zhang, Phys. Rev. D58 (1998) 073008;
   K. Hikasa, K. Whisnant, J.M. Yang and B.L. Young, Phys. Rev. D58 (1998) 114003.

[5] F. del Águila and J.A. Aguilar-Saavedra, Phys. Rev. D67 (2003) 014009.

[6] T. Tait and C. P. Yuan, Phys. Rev. D63, (2001) 014018;
   D. O. Carlson, E. Malkawi, and C. P. Yuan, Phys. Lett. B337, (1994) 145;
   G. L. Kane, G. A. Ladinsky, and C. P. Yuan, Phys. Rev. D45, (1992) 124;
   T. G. Rizzo, Phys. Rev. D53, (1996) 6218;
   T. Tait and C. P. Yuan, Phys. Rev. D55, (1997) 7300;
   A. Datta and X. Zhang, Phys. Rev. D55, (1997) 2530;
   E. Boos, L. Dudko, and T. Ohl, Eur. Phys. J. C11, (1999) 473;
   D. Espriu and J. Manzano, Phys. Rev. D65, (2002) 073005.

[7] Gad Eilam, Mariana Frank and Ismail Turan, hep-ph/0601253
   J. Guasch, W. Hollik, S. Peñaranda and J. Solà, hep-ph/0601218

[8] J. J. Liu, C. S. Li, L. L. Yang and L. G. Jin, Phys. Rev. D72 (2005) 074018;
   L.L. Yang, C. S. Li, Y. Gao and J. J. Liu, hep-ph/0601180

[9] C.T. Hill and S.J. Parke, Phys. Rev. D49 (1994) 4454;
   G. J. Gounaris, D. T. Papadamou, F. M. Renard, Z. Phys. C76 (1997) 3 33.

[10] B. Grzadkowski, J.F. Gunion and P. Krawczyk, Phys. Lett. B268 (1991) 106;
    G. Eilam, J.L. Hewett and A. Soni, Phys. Rev. D44 (1991) 1473;
    T.P. Cheng and M. Sher, Phys. Rev. D35 (1987) 3484;
    L.J. Hall and S. Weinberg, Phys. Rev. D48 (1993) R979.
[11] B. Grzadkowski, Z. Hioki, K. Ohkuma and J. Wudka, *Nucl. Phys.* B689 (2004) 108.
[12] V.M. Budnev, I.F. Ginzbirg, G.V. Meldon and V.G. Serfo, *Phys. Rep.* 15C (1975) 183; R. Cutler and D. Sivers, *Phys. Rev.* D17 (1978) 17.
[13] A. Denner and T. Sack, *Nucl. Phys.* B358 (1991) 46; G. Eilam, R.R. Mendel, R. Migneron and A. Soni, *Phys. Rev. Lett.* 66 (1991) 3105; A. Czarnecki and K. Melnikov, *Nucl. Phys.* B554 (1999) 520; K.G. Chetyrkin, R. Harlander, T. Seidensticker and M. Steinhauser, *Phys. Rev.* D60 (1999) 114015; S.M. Oliveira, L. Brücher, R. Santos and A. Barroso, *Phys. Rev.* D64 (2001) 017301.
[14] J. Pumplin *et al.*, *JHEP* 0207 (2002) 012.
[15] A. Belyaev and E. Boos, *Phys. Rev.* D63 (2001) 034012; Z. Sullivan, *Phys. Rev.* D70 (2004) 114012.
[16] M.E. Luke and M.J. Savage, *Phys. Lett.* B307 (1993) 387; D. Atwood, L. Reina and A. Soni, *Phys. Rev.* D55 (1997) 3156; J.M. Yang, B.L. Young and X. Zhang, *Phys. Rev.* D58 (1998) 055001; J. Guasch and J. Solà, *Nucl. Phys.* B562 (1999) 3; D. Delepine and S. Khalil, *Phys. Lett.* B599 (2004) 62; J.J. Liu, C.S. Li, L.L. Yang and L.G. Jin, *Phys. Lett.* B599 (2004) 92; J.A. Aguilar-Saavedra, *Acta Phys. Pol.* B35 (2004) 2695.
[17] S. S. Willenbrock and D. A. Dicus, *Phys. Rev.* D34 (1986) 155; C. P. Yuan, *Phys. Rev.* D41, (1990) 42; R. K. Ellis and S. Parke, *Phys. Rev.* D46, (1992) 3785; D. O. Carlson and C. P. Yuan, *Phys. Lett.* B206, (1993) 386; A. P. Heinson, A. S. Belyaev and E. E. Boos, *Phys. Rev.* D 56, (1997) 3114; G. Bordes and B. van Eijk, *Nucl. Phys.* B435, (1995) 23; T. Stelzer, Z. Sullivan and S. Willenbrock, *Phys. Rev.* D56 (1997) 5919; B. W. Harris, E. Laenen, L. Phaf, Z. Sullivan, and S. Weinzierl, *Phys. Rev.* D66, (2002) 054024; S. Cortese and R. Petronzio, *Phys. Lett.* B253, (1991) 494; T. Stelzer and S. Willenbrock, *Phys. Lett.* B357, (1995) 125; Martin C. Smith and S. Willenbrock, *Phys. Rev.* D54 (1996) 6696; S. Mrenna and C. P. Yuan, *Phys. Lett.* B416, (1998) 200; B. W. Harris, E. Laenen, L. Phaf, Z. Sullivan, and S. Weinzierl, *Int. J. Mod. Phys.* A16S1A, (2001) 379; T. M. P. Tait, *Phys. Rev.* D61, (2000) 034001; S. Moretti, *Phys. Rev.* D56, (1997) 7427; S. Zhu, *Phys. Lett.* B524 (2002) 283; *Erratum-ibid.* B537 (2002) 351; T. Stelzer, Z. Sullivan, and S. Willenbrock, *Phys. Rev.* D58 , (1998) 094021.