Flexible Multidimensional Scaling for Human Smart Development Analysis in EU Countries

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Abstract:

**Purpose:** The aim of the paper is to propose a flexible procedure to Multidimensional Scaling, allowing to calculate the input distance matrix based on slightly different set of variables for each pair of objects.

**Design/Methodology/Approach:** The procedure starts from the classical standardization of each variable. Before the calculation of flexible distance between two objects, we eliminate the variable with the biggest absolute value in the first object, and the same we do for the second object. So, we have two variables less in the list for these two objects. If by chance the same variable is pointed for elimination by both objects, the next variable with the biggest (out of both objects) absolute standardized value should be eliminated. With this procedure, each element of distance matrix is based on the same number of variables, but some of actual variables can be different.

**Findings:** As an example – Flexible Multidimensional Scaling is performed on the list of 17 variables describing so called smart society, for 28 European Union countries. It shows how the proposed procedure works in practice.

**Practical Implications:** The proposed flexible procedure can be used for the analysis of any problem treated by Multidimensional Scaling.

**Originality/Value:** Flexible Multidimensional Scaling (FMDS) is a new idea and method. It eliminates some elements of subjective choice of initial variables and seems to be more robust against outliers than classical Multidimensional Scaling (MDS).

**Keywords:** Multidimensional scaling, flexibility, smart society, European Union countries.

**JEL codes:** C39, I21, O52, O57.

**Paper type:** Research paper.

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1. Introduction

The flexibility idea has been introduced to the multivariate data analysis by Drewnowski (1966). He suggested flexibility criterion as one of the ways to evaluate methods for linear ordering of multidimensional objects. The classification is flexible if the slight change in the list of variables (by adding or omitting variables) does not strongly affect the final ordering of objects. In some applications – especially in economics – extremely skewed variables can influence results strongly, making research vulnerable to variable choice decision.

Multidimensional Scaling (MDS) is a procedure which allows to present the configuration of objects or variables from multidimensional data in lower-order space – usually two-dimensional, on the plane. The configuration on the plane should produce a distance matrix most like the distance matrix calculated in the original multidimensional space. This similarity is usually assessed by STRESS measure proposed by Kruskal (1964). There are many books describing in details - concepts, methods and algorithms used in MDS, e.g. (Kruskal and Wish, 1978; Schiffman et al., 1981; Young, 1987; Borg and Groenen, 2010; Ding, 2018). Interesting review of basic MDS ideas and literature until 1980 can be found in Davinson (1983).

Complex phenomena are naturally multidimensional, and can be analysed from different points of view, but the general, overall approach is usually more attractive. In social science and economics such categories as development, welfare, poverty, quality of life, human and intellectual capital, globalization, economic crises are examples of such phenomena. They cannot be expressed by just one variable, and are obvious subjects for multivariate analysis such as linear ordering, composite indicators, cluster analysis, classification and dimensionality reduction.

Such multivariate approach has been shown by A. Murawska and others (Murawska et al., 2020) in multidimensional analysis of the relationship between sustainable living conditions and long life in good health, in EU countries. A methodology for the study of multidimensional and longitudinal aspects of poverty and deprivation was proposed by G. Betti and V. Verma (Berri and Verma, 2004). Gumpert (2019) analysed the extension of the core-peripheral Krugman model toward the inclusion of many economic aspects. This paper is the continuation of research on educated and smart society development. Education and human capital were discussed by many notable scientists, such as T.W. Schultz (1961; 1981), G. Becker (1962; 1993), A. Sen (1993; 1994; 1999; 2002), and Borowiecki et al. (2021).

The results and quality of multivariate analysis heavily depend on the choice of variables. This choice is always - to some extend - subjective, and usually limited by the availability of data. So why, studies on flexibility which is understood as the robustness against small changes in the list of diagnostic variables, seems to be important. We have already proposed the flexible approach to cluster analysis (Sokołowski and Markowska, 2021). Many analyses have been done to the European
Union countries with dynamic cluster analysis (Markowska et al., 2021). Multidimensional Scaling provides interesting, and rather simple, graphical illustration for multidimensional problems, so introducing flexibility aspect looks promising here. In this paper we used the proposed methodology to analyse educated and smart society development in the European Union.

2. Research Methods

Multidimensional Scaling is finally performed on distance matrix, and the knowledge of original data is even not mandatory. But in the proposed procedure we start from the data on all variables. While calculating distances between pairs of objects, two variables (one for each object) with most outlying values are eliminated. The proposed approach can be defined in the following steps:

- Define the set of objects to be analyzed,
- Set the initial list of $m$ variables,
- Standardize variables,
- Calculate distance matrix $D$ as “per variable” taking $d_{lk}/m$,
- Perform MDS on $m$ variables,
- For each object $j$ find variable $i^*$ with the highest module of standardized value and variable $i^{**}$ with the second highest module,
- Calculate distance matrix $D^*$ with distance between objects $l$ and $k$ is calculated using $m$-2 variables, omitting variables $i^*_j$ and $i^*_k$,
- If both objects $l$ and $k$ point out the same variable $i^*$, select one variable with the higher module, from $i^{**}_j$ and $i^{**}_k$ (it will be the second variable eliminated),
- Divide each element of $D^*$ by $m$-2, making it as “per variable”,
- Perform MDS on $D^*$ distance matrix.

The degree of flexibility (of the MDS based on original list of variables) can be evaluated by measure $M$. It compares distance matrix $D_{MDS}$ calculated for the two-dimensional MDS result based on original full list of variables with $D_{MDS}^*$ matrix based two-dimensional MDS result calculated using flexibility procedure. The degree of flexibility measure ($M$) takes value between 0 and 1 – the bigger the more flexible are results of the original MDS (less vulnerable to omitting some variables):

$$M = 1 - \frac{\sum_{l=1}^{n} \sum_{k=1}^{n} (d_{lk} - d_{lk}^*)^2}{2n^2},$$

where:
$n$ – number of objects,
$l$ – row number in a distance matrix,
$k$ – column number in a distance matrix,
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d_{lk} – elements of distance matrix of two-dimensional configuration of points resulted from MDS based on full list of variables,
d_{lk}^* – elements of distance matrix of two-dimensional configuration of points resulted from MDS based on flexibility procedure.

All calculations presented in this paper were done on STATISTICA ver. 13 software.

3. Research Data

In this paper, flexible multidimensional scaling (FMDS) is illustrated by the analysis of 28 European countries characterized by 17 variables describing human smart development. The data covers year 2017. Here is the list of variables (https://strateg.stat.gov.pl/dashboard/#/56 polityka-spojnosci/1):

X_1 – Expenditures on R&D as % of GDP,
X_2 – % of population using internet at least once a week,
X_3 – Number of inventions reported to EPO (European Patent Office) per 1 million population,
X_4 – PISA (Programme for International Student Assessment) test – % of students on high levels in reading and interpretation,
X_5 – HDI (Human Development Index),
X_6 – PISA test - % of students on high levels in mathematics,
X_7 – PISA test - % of students on high levels in science,
X_8 – Government and higher education sector expenditures on R&D as % of GDP,
X_9 – Corporate expenditures on R&D as % of GDP,
X_{10} – % of small and medium enterprises adopting product or process innovations,
X_{11} – % of population with high internet skills,
X_{12} – % of employees working in R&D,
X_{13} – % of young people not working nor learning,
X_{14} – % of teenagers who quit education,
X_{15} – % of 25-64 population still active in education,
X_{16} – % of 30-34 population with higher education completed,
X_{17} – % of population with at least basic computer skills.

Most of the variables used in the analysis are stimulants (the bigger, the better), and only two (X_{13} and X_{14}) are destimulants (the smaller, the better). Basic descriptive statistics of variables describing human smart development are presented in Table 1 (Part 1 and 2).

Countries with the biggest number of “the best” values (first or second maximum of stimulants, first or second minimum of destimulants are: Sweden (eight variables: X_1, X_2, X_3, X_8, X_9, X_{11}, X_{13}, X_{15}), Finland (four variables: X_4, X_7, X_{12}, X_{15}), Denmark (three variables: X_2, X_3 and X_8) and Holland (three variables: X_6, X_{13}, X_{17}). Two variables
with the best values were observed in Estonia (X₆–X₇), Ireland (X₄ and X₅), Lithuania (X₁₁, X₁₆), Luxembourg (X₂, X₁₇) and Austria (X₁ and X₉). Definitely the biggest number of “the worst” values (first or second minimum in stimulants, first or second maximum in destimulants) characterize Romania – 14 variables (X₁, X₂, X₄–X₈, X₁₀–X₁₂, X₁₄–X₁₇). There were seven such variables in Bulgaria (X₂, X₃, X₅, X₈, X₁₂, X₁₅ and X₁₇), and three in Latvia (X₁, X₉, X₁₀). Two less favourable variable characterize Malta (X₉, X₁₁), Italy (X₁₃, X₁₆), Croatia (X₃, X₁₇) Greece (X₇, X₁₃) and Cyprus (X₆, X₇).

Table 1. Descriptive statistics of variables – part 1

| Variable | Max 1 | Country | Max 2 | Country | Min 1 | Country | Min 2 | Country |
|----------|-------|---------|-------|---------|-------|---------|-------|---------|
| X₁       | 3.33  | SE      | 3.16  | AT      | 0.50  | RO      | 0.51  | LV      |
| X₂       | 96.00 | LU      | 95.00 | SE, DK  | 61.00 | BG      | 4.80  | HR      |
| X₃       | 283.46| SE      | 246.61| DK      | 4.13  | RO      | 2.50  | SK      |
| X₄       | 13.90 | FI      | 13.80 | IE      | 1.90  | RO      | 0.81  | BG      |
| X₅       | 0.94  | IE      | 0.94  | DE      | 0.81  | RO      | 3.60  | RO      |
| X₆       | 15.00 | EE      | 14.80 | NL      | 3.45  | RO      | 1.80  | EL, CY  |
| X₇       | 13.70 | EE      | 13.10 | FI      | 0.60  | RO      | 0.21  | BG      |
| X₈       | 0.97  | SE      | 0.95  | DK      | 0.21  | RO      | 0.21  | BG      |
| X₉       | 2.35  | SE      | 2.22  | AT      | 0.00  | MT      | 0.14  | LV      |
| X₁₀      | 57.95 | BE      | 47.87 | PT      | 2.57  | RO      | 11.78 | LV      |
| X₁₁      | 42.00 | SE      | 36.00 | LT      | 6.00  | RO      | 6.40  | MT      |
| X₁₂      | 3.74  | EL      | 3.24  | FI      | 0.53  | RO      | 0.86  | BG      |
| X₁₃      | 25.50 | IT      | 24.20 | EL      | 6.90  | SE      | 7.30  | NL      |
| X₁₄      | 18.30 | ES      | 18.10 | RO      | 3.10  | HR      | 4.30  | SI      |
| X₁₅      | 30.40 | SE      | 27.40 | FI      | 1.10  | RO      | 2.30  | BG      |
| X₁₆      | 58.00 | LT      | 55.90 | CY      | 26.30 | RO      | 26.90 | IT      |
| X₁₇      | 85.00 | LU      | 79.00 | NL      | 29.00 | BG, RO  | 41.00 | RO      |

Source: Own calculations based on data from Statistics Poland, https://strateg.stat.gov.pl/dashboard/#/56 polityka-spojnosci/1.

Table 1. Descriptive statistics of variables – part 2

| Variable | Median | Average | Standard deviation | Coefficient of variation | Max/min | Max-min |
|----------|--------|---------|--------------------|--------------------------|---------|---------|
| X₁       | 1.31   | 1.57    | 0.88               | 56.02                    | 6.66    | 2.83    |
| X₂       | 79.50  | 80.14   | 10.11              | 12.61                    | 1.57    | 35.00   |
| X₃       | 34.67  | 82.88   | 91.76              | 110.72                   | 68.63   | 279.33  |
| X₄       | 6.15   | 7.21    | 3.53               | 49.03                    | 7.32    | 12.00   |
| X₅       | 0.88   | 0.88    | 0.04               | 4.25                     | 1.16    | 0.13    |
| X₆       | 10.25  | 9.83    | 3.69               | 37.56                    | 4.35    | 11.55   |
| X₇       | 6.65   | 6.72    | 3.53               | 52.52                    | 22.83   | 13.10   |
| X₈       | 0.56   | 0.57    | 0.24               | 41.82                    | 4.62    | 0.76    |
| X₉       | 0.71   | 0.97    | 0.67               | 68.86                    | -       | 2.35    |
| X₁₀      | 32.32  | 29.57   | 12.91              | 43.65                    | 22.55   | 55.38   |
| X₁₁      | 15.50  | 16.75   | 8.75               | 52.24                    | 7.00    | 36.00   |
| X₁₂      | 2.07   | 2.03    | 0.83               | 40.96                    | 7.06    | 3.21    |
| X₁₃      | 13.15  | 13.95   | 4.76               | 34.10                    | 3.70    | 18.60   |
Values of percentage coefficient of variance greater than 50, allow to identify variables mostly differing EU countries, as: number of inventions reported to EPO per 1 million population, corporate expenditures on R&D as % of GDP, % of 25-64 population still active in education, expenditures on R&D as % of GDP, PISA test – % of students on high levels in science and % of population with high internet skills. Clearly, the smallest variability has been found in HDI. It is also backed by the relations of maximum to minimum for this variable. In Table 2, variables which were identified to be eliminated while calculating matrix \( D^* \), are listed for each EU country. Plus indicates that the standardized value under the module was positive, and minus that it was negative.

**Table 2. Extreme value variables for EU countries**

| Country       | Standardized variable with the highest absolute value | Standardized variable with the second highest absolute value |
|---------------|-------------------------------------------------------|-------------------------------------------------------------|
| Austria       | \( X_9 \) – Corporate R&D expenditures (+)           | \( X_1 \) – Expenditures on R&D (+)                         |
| Belgium       | \( X_{10} \) – SME with innovations (+)             | \( X_9 \) – Corporate R&D expenditures (+)                 |
| Bulgaria      | \( X_{17} \) – Computer skills (-)                  | \( X_5 \) – HDI (-)                                        |
| Cyprus        | \( X_6 \) – PISA mathematics (-)                    | \( X_{16} \) – 30-34 with higher education (+)             |
| Czechia       | \( X_6 \) – PISA mathematics (+)                    | \( X_{11} \) – High internet skills (-)                   |
| Germany       | \( X_9 \) – Corporate R&D expenditures (+)          | \( X_1 \) – Expenditures on R&D (+)                        |
| Denmark       | \( X_{15} \) – Adults active in education (+)      | \( X_3 \) – Inventions in EPO (+)                          |
| Estonia       | \( X_3 \) – PISA science (+)                        | \( X_6 \) – PISA mathematics (+)                           |
| Spain         | \( X_{14} \) – Teenagers quitting education (+)    | \( X_4 \) – PISA reading and interpret. (-)               |
| Finland       | \( X_{15} \) – Adults active in education (+)      | \( X_4 \) – PISA reading and interpret. (+)               |
| France        | \( X_4 \) – PISA reading and interpret. (+)        | \( X_{11} \) – High internet skills (-)                   |
| Great Britain | \( X_2 \) – Internet activity (+)                   | \( X_7 \) – PISA science (+)                               |
| Greece        | \( X_{15} \) – Teenagers quitting education (-)    | \( X_2 \) – Internet activity (-)                          |
| Croatia       | \( X_{14} \) – Teenagers quitting education (-)    | \( X_2 \) – Internet activity (-)                          |
| Hungary       | \( X_5 \) – HDI (-)                                 | \( X_{16} \) – 30-34 with higher education (+)             |
| Ireland       | \( X_4 \) – PISA reading and interpret. (+)        | \( X_5 \) – HDI (+)                                        |
| Italy         | \( X_{13} \) – Young not working/ learning (+)     | \( X_{16} \) – 30-34 with higher education (-)             |
| Lithuania     | \( X_{11} \) – High internet skills (+)            | \( X_{16} \) – 30-34 with higher education (+)             |
| Luxembourg    | \( X_{17} \) – Computer skills (+)                 | \( X_2 \) – Internet activity (+)                          |
| Latvia        | \( X_{10} \) – SME with innovations (-)            | \( X_9 \) – Corporate R&D expenditures (-)                 |
| Malta         | \( X_{14} \) – Teenagers quitting education (+)    | \( X_9 \) – Corporate R&D expenditures (-)                 |
| Netherlands   | \( X_{17} \) – Computer skills (+)                 | \( X_{13} \) – Young not working/ learning (-)             |
| Poland        | \( X_{10} \) – SME with innovations (-)            | \( X_{14} \) – Teenagers quitting education (-)            |
| Portugal      | \( X_9 \) – SME with innovations (-)               | \( X_5 \) – HDI (-)                                        |
| Romania       | \( X_{14} \) – Teenagers quitting education (+)    | \( X_{10} \) – SME with innovations (-)                   |
| Sweden        | \( X_{11} \) – High internet skills (+)            | \( X_{15} \) – Adults active in education (+)              |
| Slovenia      | \( X_{14} \) – Teenagers quitting education (-)    | \( X_6 \) – PISA mathematics (+)                           |
| Slovakia      | \( X_4 \) – PISA reading and interpret. (-)        | \( X_{10} \) – SME with innovations (-)                   |
While interpreting the list given in Table 2, one must keep in mind that positive values of some variables are in fact negative from the point of view of human social development. One example of such variable is $X_{14}$ – Teenagers who quit education. This variable appeared six times among two most outlying variables for countries.

4. Results

Multidimensional Scaling was performed first on the full set of variables and then on its Flexible MDS proposed in the paper. Configuration of countries are presented on Figure 1. Blue circles mark the position of a country in all variables MDS, and red square show the position at Flexible MDS. Both points of the same country are joined with the line if they do not just overlap.

**Figure 1. Classical (blue circles) and Flexible (red squares) MDS results**

[Diagram showing positions of countries in MDS space]

*Source: Own composition.*

The degree of flexibility measured by $M$ equals to 0.7573. $M$ compares distances in original MDS results (on the plane) with Flexible MDS. The M the more flexible is MDS, less vulnerable to changes in list of variables, and omitting “most extreme” one for each pair of objects. It is too early – with just one application – to propose qualitative interpretation to $M$.

5. Discussion

In Multidimensional Scaling, it is usually not easy to identify meanings/interpretation of both axes on the plane. In our example the meaning of horizontal axis seems rather obvious. Sokołowski and Markowska (2019) calculated Composite Smart Society Index, based on the same data that is used here. Table 3 presents the ranking of
countries together with the value of Composite Indicator, which takes value from [0;100] interval.

**Table 3. Composite Smart Society Index**

| Position | Country    | Composite Indicator | Position | Country    | Composite Indicator |
|----------|------------|---------------------|----------|------------|---------------------|
| 1        | Sweden     | 85                  | 15       | Lithuania  | 44                  |
| 2        | Finland    | 78                  | 16       | Portugal   | 40                  |
| 3        | Netherlands| 74                  | 17       | Spain      | 34                  |
| 4        | Denmark    | 73                  | 17       | Greece     | 34                  |
| 5        | Austria    | 68                  | 19       | Malta      | 33                  |
| 6        | Germany    | 67                  | 19       | Poland     | 33                  |
| 7        | Belgium    | 65                  | 21       | Italy      | 30                  |
| 8        | France     | 60                  | 21       | Latvia     | 30                  |
| 9        | Luxemburg  | 59                  | 23       | Cyprus     | 29                  |
| 10       | Great Britain| 58              | 23       | Slovakia   | 29                  |
| 11       | Slovenia   | 57                  | 25       | Hungary    | 28                  |
| 12       | Estonia    | 54                  | 26       | Croatia    | 24                  |
| 12       | Ireland    | 54                  | 27       | Bulgaria   | 14                  |
| 14       | Czech      | 50                  | 28       | Romania    | 3                   |

*Source: Sokolowski and Markowska, 2019.*

It can be easily noticed that the distribution of countries along horizontal level on MDS graph closely follow the ranking presented in Table 3. Principal Component Analysis (PCA) with Varimax rotation has been performed to identify the vertical scale. With Kaiser’s criterion on eigenvalues, four factors have been found. Loading is given in Table 4.

**Table 4. PCA loadings**

| Variable                         | R&D  | Internet | Skills | Education |
|----------------------------------|------|----------|--------|-----------|
| X1 – expenditures on R&D         | 0.898| 0.003    | 0.371  | 0.024     |
| X2 – Internet activity           | 0.400| 0.234    | 0.827  | 0.116     |
| X3 – Inventions in EPO           | 0.831| 0.129    | 0.437  | 0.013     |
| X4 – PISA reading/interpr.       | 0.350| -0.277   | 0.581  | 0.452     |
| X5 – HDI                         | 0.600| -0.038   | 0.606  | 0.328     |
| X6 – PISA maths                  | 0.303| -0.508   | 0.641  | 0.135     |
| X7 – PISA science                | 0.360| -0.145   | 0.797  | 0.124     |
| X8 – R&D public spendings.       | 0.828| 0.139    | 0.412  | 0.081     |
| X9 – R&D corporate expen.        | 0.882| -0.038   | 0.325  | 0.025     |
| X10 – SME with innovations       | 0.657| -0.003   | 0.166  | 0.424     |
| X11 – high internet skills      | 0.176| 0.856    | 0.115  | 0.119     |
| X12 – R&D employment            | 0.747| 0.083    | 0.139  | 0.448     |
| X13 – Young not work/learn       | -0.202| -0.140  | -0.805 | -0.115    |
| X14 – Teens quit. education     | -0.131| -0.026  | -0.047 | -0.851    |
| X15 – Adults active educat.      | 0.558| 0.329    | 0.650  | 0.025     |
| X16 – 30-34 higher educat.       | -0.017| 0.463   | 0.433  | 0.668     |
| X17 – Computer skills            | 0.462| 0.272    | 0.741  | 0.142     |
Loadings with module higher than 0.6 are reported in bold italics. Names for the four factors identified were imposed in Tab 3 to summarize the general meaning of factors. Coordinates of countries on MDS map were correlated with factor values. Correlation coefficients with respective p-values are given in Table 5. Variable \( X_{14} – \text{Teenagers qui. educations} \) are the most responsible for vertical coordinate on Figure 1 and on country position changes between classical and flexible MDS. Movements of countries are usually along vertical axis when changing from classical to flexible MDS.

### Table 5. Correlation coefficients and p-values (in brackets) between MDS coordinates and Factors

| Coordinate axis       | R&D          | Internet     | Skills       | Education    |
|-----------------------|--------------|--------------|--------------|--------------|
| Classical horizontal  | **0.695 (0.000)** | 0.121 (0.541) | **0.664 (0.000)** | 0.238 (0.222) |
| Classical vertical    | 0.133 (0.499) | **-0.491 (0.008)** | 0.228 (0.243) | **-0.771 (0.000)** |
| Flexible horizontal   | **0.731 (0.000)** | 0.126 (0.524) | **0.627 (0.000)** | 0.224 (0.252) |
| Flexible vertical     | 0.194 (0.324) | **-0.382 (0.045)** | 0.127 (0.520) | **-0.726 (0.000)** |

Source: Own calculations.

Variability along horizontal axis is much higher (range roughly from -2 to +2) than on vertical axis (from -1 to 0.8). It can relate to the portion of overall variance explained by the first and the third factor and their correlations.

### 6. Conclusions

The proposed procedure of Flexible Multidimensional Scaling (FMDS) is a new idea with the aim to introduce element of objectivity to the selection of variables for MDS. With this approach you can observe how vulnerable are your MDS results to slight changes in the list of variables. In FMDS distances between pair of objects are calculated based on the list of variables which can be not the same for each pair objects. The elimination of two variables is based on standardized modules, and distances are finally calculated as “per variable”. The degree of flexibility can be assessed by the proposed measure.

Human Smart Social Development of EU countries serves not only as an example of FMDS application. With composite indicator and PCA the broader look into the problem has been offered. There is no definite and generally acceptable list of variables which characterize the analyzed phenomenon. In this sense, the flexibility approach helps to optimize the choice of variables. Out of the variables used in our study, the importance of percentage of teenagers quitting education is one of the most interesting findings in this research. For FMDS idea more applications are needed (or maybe some simulation studies) specially to scale flexibility measure M.
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