Recent results on the gapless 2SC phase are reviewed. These include: the thermal stability under the constraint of the local charge neutrality condition, the properties at zero and nonzero temperatures, and the color screening properties.

1. Introduction

Because the interaction between two quarks in the color anti-triplet channel is attractive, sufficiently cold dense quark matter is a color superconductor. It is very likely that a color superconducting phase may exist in cores of compact stars, where bulk matter should satisfy the charge neutrality condition as well as $\beta$-equilibrium. For a three-flavor quark system, when the strange quark mass is small, the color-flavor-locked (CFL) phase is favorable. For the two-flavor quark system, the charge neutrality condition plays a nontrivial role. In the ideal two-flavor color superconducting (2SC) phase, the paired $u$ and $d$ quarks have the same Fermi momenta. Because $u$ quark carries electrical charge $2/3$, and $d$ quark carries electrical charge $-1/3$, it is easy to check that quark matter in the ideal 2SC phase is positively charged. To satisfy the electrical charge neutrality condition, roughly speaking, twice as many $d$ quarks as $u$ quarks are needed. This induces a large difference between the Fermi surfaces of the two pairing quarks, i.e., $\mu_d - \mu_u = \mu_e \approx \mu/4$, where $\mu, \mu_e$ are chemical potentials for quark and electron, respectively. Naively, one would expect that the requirement of the charge neutrality condition will destroy the $ud$ Cooper pairing in the 2SC phase.

However, it was found in Ref. 4 that a charge neutral two-flavor color superconducting (N2SC) phase does exist. Comparing with the ideal 2SC
phase, the N2SC phase found in Ref. 4 has a largely reduced diquark gap parameter, and the pairing quarks have different number densities. The latter contradicts the paring ansatz. It is natural to think that the N2SC phase found in Ref. 4 is an unstable Sarma state. In Ref. 7, it was shown that the N2SC phase is a thermally stable state when the local charge neutrality condition is enforced. As a by-product, which comes out as a very important feature, it was found that the quasi-particle spectrum has zero-energy excitations. Thus, this phase was named the gapless 2SC (g2SC) phase. In the following, we first show the thermal stability of the g2SC phase, then we discuss its properties at zero as well as at nonzero temperatures, at last, we present our most recent results about the chromomagnetic instability in the g2SC phase.

2. The gapless 2SC phase and its thermal stability

Bulk quark matter inside the neutron star should be neutral with respect to the color charge as well as the electrical charge. The color superconducting phase in full QCD is automatically color neutral. In the Nambu–Jona-Lasinio (NJL) type model, the color neutrality can be satisfied by tuning the chemical potential $\mu_8$ for the color charge. The value of $\mu_8$ for a charge neutral 2SC phase is very small. Correspondingly, the electrical charge neutrality can be satisfied by tuning the chemical potential $\mu_e$ for the electrical charge. The value of $\mu_e$ is determined by the electrical charge neutrality condition.

The ground state is determined by solving the gap equation together with the charge neutrality condition. It is found that the ground state of charge neutral two-flavor quark matter is very sensitive to the diquark coupling constant $G_D$:

$$
G_D/G_S \gtrsim 0.8, \quad \Delta > \delta\mu, \quad 2SC,
$$

$$
0.7 \lesssim G_D/G_S \lesssim 0.8, \quad \Delta < \delta\mu, \quad g2SC,
$$

$$
G_D/G_S \lesssim 0.7, \quad \Delta = 0, \quad \text{NQM}. \quad (1)
$$

Where $\delta\mu \equiv \mu_e/2$, $G_S$ is the quark-antiquark coupling constant, and “NQM” indicates the normal phase of quark matter. The most interesting case is the g2SC phase, which exists in the diquark coupling regime $0.7 \lesssim G_D/G_S \lesssim 0.8$. Even though this regime is narrow, it is worth to mention that, either from the Fierz transformation ($G_D/G_S = 0.75$) or from fitting the vacuum baryon mass ($G_D/G_S \simeq 2.26/3$), the value of the ratio $G_D/G_S$ is inside this regime.
The g2SC phase, indicated by the order parameter $\Delta < \delta \mu$, resembles the unstable Sarma state. For the flavor asymmetric $ud$ quark system, i.e., when $\mu_e$ is a free parameter and there is no constraint from the charge neutrality condition, the solution $\Delta < \delta \mu$ of the gap equation indeed corresponds to a maximum of the thermodynamical potential $\Omega_{u,d,e}$.

However, bulk quark matter inside neutron stars should be charge neutral. A nonzero net electrical charge density $n_Q$ will cause an extra energy $\Omega_{\text{Coulomb}} \sim n_Q^2 V^{2/3}$ ($V$ is the volume of the system) by the repulsive Coulomb interaction. The total thermodynamical potential of the whole system is given by $\Omega = \Omega_{\text{Coulomb}} + \Omega_{u,d,e}$. The energy density grows with increasing the volume of the system, as a result, it is impossible for matter inside stars to remain charged over macroscopic distances. So, the proper way to find the ground state of the homogeneous neutral $u, d$ quark matter is to minimize the thermodynamical potential along the neutrality line $\Omega|_{n_Q=0} = \Omega_{u,d,e}|_{n_Q=0}$ with $\Omega_{\text{Coulomb}}|_{n_Q=0} = 0$. The g2SC phase corresponds to the global minimum of the thermodynamical potential along the charge neutrality line, thus it is a stable state under the restriction of the charge neutrality condition.

### 3. The g2SC phase at zero and nonzero temperatures

As we already mentioned, at zero temperature, in the g2SC phase, the pairing quarks have different number densities. This is different from the 2SC phase when $\delta \mu < \Delta$.

It is the quasi-particle spectrum that makes the g2SC phase different from the 2SC phase. The excitation spectrum for the ideal 2SC phase ($\delta \mu = 0$) include: two free blue quarks, which do not participate in the Cooper pairing, and four quasi-particle excitations (linear superpositions of $u_{r,g}$ and $d_{r,g}$) with an energy gap $\Delta$. If there is a small mismatch ($\delta \mu < \Delta$) between the Fermi surfaces of the pairing $u$ and $d$ quarks, $\delta \mu$ induces two different branches of quasi-particle excitations. One branch moves up with a larger energy gap $\Delta + \delta \mu$, another branch moves down with a smaller energy gap $\Delta - \delta \mu$. If the mismatch $\delta \mu$ is larger than the gap parameter $\Delta$, the lower dispersion relation for the quasi-particle crosses the zero-energy axis. Thus we call the phase with $\Delta < \delta \mu$ the gapless 2SC (g2SC) phase. In the g2SC phase, there are only two gapped fermionic quasiparticles, and the other four quasiparticles are gapless.

In a superconducting system, when one increases the temperature at a given chemical potential, thermal motion will eventually break up the
quark Cooper pairs. In the weakly interacting Bardeen-Copper-Schrieffer (BCS) theory, the transition between the superconducting and normal phases is usually of second order. The ratio of the critical temperature $T_{c}^{BCS}$ to the zero temperature value of the gap $\Delta_{0}^{BCS}$ is a universal value\textsuperscript{10}

\[ r_{BCS} = \frac{T_{c}^{BCS}}{\Delta_{0}^{BCS}} \approx 0.567. \]

In the ideal 2SC phase, the ratio of the critical temperature to the zero temperature value of the gap is also the same as in the BCS theory\textsuperscript{11}. However, the g2SC phase has very different properties at nonzero temperatures\textsuperscript{12}. The ratio $T_{c}/\Delta_{0}$ is not a universal value. It is infinity when $G_{D}/G_{S} \ll 0.68$ and approaches $r_{BCS}$ when $G_{D}/G_{S}$ increases. The temperature dependence of the gap reveals a nonmonotonic behavior. In some cases, the diquark gap could have sizable values at finite temperature even if it is exactly zero at zero temperature.

4. Chromomagnetic instability in the g2SC phase

Because the g2SC phase has four gapless modes and two gapped modes, one may think that the low energy (large distance scale) properties of the g2SC phase should interpolate between those of the normal phase and those of the 2SC phase. However, its color screening properties do not fit this picture.

One of the most important properties of an ordinary superconductor is the Meissner effect, i.e., the superconductor expels the magnetic field. Using the linear response theory, the induced current $j_{i}^{\text{ind}}$ is related to the external magnetic field $A_{j}$ as $j_{i}^{\text{ind}} = \Pi_{ij}A^{j}$, where the response function $\Pi_{ij}$ is the magnetic part of the photon polarization tensor. The response function has two components, diamagnetic and paramagnetic part. In the static and long-wavelength limit, for the normal metal, the paramagnetic component cancels exactly the diamagnetic component. In the superconducting phase, the paramagnetic component is quenched by the energy gap and produces a net diamagnetic response. Thus the ordinary superconductor is a prefect diamagnet. In cold dense quark matter, the gauge bosons connected with the broken generators obtain masses in the ideal 2SC phase\textsuperscript{13} as well as in the CFL phase\textsuperscript{14}, which indicate the Meissner screening effect in these phases.

However, in the g2SC phase, it is found that the Meissner screening masses of the five gluons, corresponding to the five broken generators of the $SU(3)_{c}$ group, are imaginary\textsuperscript{15}. This is because, in the static and long-wavelength limit, the paramagnetic contribution to the magnetic part of these five gluon polarization tensors becomes dominant. In condensed
matter, this phenomenon is called the paramagnetic Meissner effect (PME)\textsuperscript{16}. The imaginary Meissner screening mass indicates a chromomagnetic instability of the g2SC phase. There are several possibilities to resolve the instability. One is through a gluon condensate, which may not change the structure the g2SC phase. It is also possible that the instability drives the homogeneous system to an inhomogeneous phase, like the crystalline phase or the vortex lattice phase. This problem remains to be clarified in the future. It is also very interesting to know whether the chromomagnetic instability develops in the gapless CFL phase\textsuperscript{17}.

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