Abstract

Quasi-set theory is a first order theory without identity, which allows us to cope with non-individuals in a sense. A weaker equivalence relation called “indistinguishability” is an extension of identity in the sense that if $x$ is identical to $y$ then $x$ and $y$ are indistinguishable, although the reciprocal is not always valid. The interesting point is that quasi-set theory provides us a useful mathematical background for dealing with collections of indistinguishable elementary quantum particles. In the present paper, however, we show that even in quasi-set theory it is possible to label objects that are considered as non-individuals. We intend to prove that individuality has nothing to do with any labelling process at all, as suggested by some authors. We discuss the physical interpretation of our results.

1 Introduction

Concerning non-individuality in quantum mechanics, the problems raised by this have provided many papers in the literature. See, for example, the references in French (2004).

Elementary particles that share the same set of state-independent (intrinsic) properties are sometimes said to be indistinguishable. Although classical particles can share all their intrinsic properties, we might say that they ‘have’ some kind of quid which makes them individuals. Hence, we are able to follow the trajectories of classical particles, at least in principle. That allows us to identify them. In quantum physics this is not possible, i.e., it is not possible, a priori, to keep track of individual particles in order to distinguish among them when they share the same set of intrinsic properties. In other words, it is not possible to label quantum particles. And this non-individuality plays a very important role in quantum mechanics (Sakurai, 1994).

On the possibility that collections of such indistinguishable entities should not be considered as sets in the usual sense, Manin (1976) proposed the search for axioms which should allow to deal with indiscernible objects. As he said,

I would like to point out that it [standard set theory] is rather an extrapolation of common-place physics, where we can distinguish things, count them,
put them in some order, etc. New quantum physics has shown us models of entities with quite different behavior. Even \textit{sets} of photons in a looking-glass box, or of electrons in a nickel piece are much less Cantorian than the \textit{sets} of grains of sand.

We are using the philosophical jargon in saying that ‘indistinguishable’ objects are objects that share their properties, while ‘identical’ objects means ‘the very same object’. Nevertheless, in considering the behavior of the ensembles of such particles, there is a fundamental difference between classical and quantum statistics. In classical statistical mechanics, particles are treated like individuals. In quantum statistics, on the other hand, the Indistinguishability Postulate asserts that if a permutation is applied to any state for an assembly of particles, then there is no way of distinguishing the resulting permuted state-function from the original one by means of any observation at any time. The Indistinguishability Postulate (IP) seems to be one of the most basic principles of quantum theory and implies that permutations of quantum particles are not regarded as an observable.

Usually, IP has been interpreted in two basic ways: the first assumes that IP implies that quantum particles cannot be regarded as ‘individuals’, since an ‘individual’ should be something having properties similar to those of usual (macroscopic) bodies. This interpretation is closely related to what is assumed in the context of quantum field theory, since, roughly speaking, quantum field theories do not deal with ‘individuals’. The second way regards particles as individuals in a sense, and the non-classical counting of quantum statistics are then viewed as resulting from the restrictions imposed to the set of the possible states accessible to the particles. In short, only symmetrical and anti-symmetrical states are available, and the initially attached individuality of particles is then ‘veiled’ by such a criterion. Both alternatives, albeit used in current literature, present problems from the ‘foundational’ point of view. There is some obscurity lurking in the concept of individuality in quantum physics. The idea of considering ‘non-individuals’ is weird, and in general other metaphysical packages are used instead. For instance, that one which assumes that quantum objects are individuals of a sort, despite quite distinct from the usual objects described by classical mechanics (Sant’Anna and Krause, 1997).

Let us recall that some authors like Hermann Weyl expressed the calculation with ‘aggregates’ so that some of the basic assumptions of quantum theory can be reached in an adequate way. Weyl’s efforts were done in the sense of finding an alternative manner to express the procedure physicists implicitly use in treating indistinguishable particles, namely, the assumption that there is a \textit{set} $S$ of (hence, distinguishable) objects (say, $n$ objects) endowed with an equivalence relation $\sim$. Then the ‘desired result’, according to Weyl, is to obtain the \textit{ordered decomposition} $n = n_1 + \cdots + n_k$, where $n_i$ are the cardinalities of the equivalence classes $C_i$, $i = 1, \ldots, k$ of the quotient set $S/\sim$. But, as it is easy to note, this procedure ‘veils’ the very nature of the elements of the set $S$, that is, veils the fact that they are individual objects since they are members of a \textit{set}. We would like to emphasize that there is no scape. Classical logic and mathematics are committed with a conception of identity which does not make any distinction between identity and indistinguishability: indistinguishable things are the very same thing and conversely.

One manner to cope with the problem of non-individuality in quantum physics is by means of quasi-set theory (Krause, 1992; Krause, Sant’Anna, and Volkov, 1999;
Sant’Anna and Santos, 2000), which is an extension of Zermelo-Fraenkel set theory, that allows to talk about certain indistinguishable objects that are not identical. Such indistinguishable objects are termed as non-individuals. In quasi-set theory identity does not apply to all objects. In other words, there are some kinds of terms in quasi-set theory where the sequence of symbols \( x = y \) is not a well-formed-formula, i.e., it is meaningless. A weaker equivalence relation called “indistinguishability” is an extension of identity in the sense that it allows the existence of two objects that are indistinguishable. In standard mathematics, there is no sense in saying that two objects are identical. If \( x = y \), then we are talking about one single object with two labels, namely, \( x \) and \( y \).

We want to continue our investigations on the use of quasi-set theory in the foundations of quantum mechanics, based on some questions that we think about and that may be interesting. Some of these questions have to do with the notion of levels of individuality, which is introduced in further details in the next sections. Actually, our main mathematical framework is some sort of quasi-set-theoretical predicate for quantum systems, which is a natural extension of Patrick Suppes (2002) ideas about axiomatization. We prove, e.g., that even in quasi-set theory it is possible to prove that objects without individuality (in the sense of the theory) may be labelled if certain conditions are satisfied. We want to investigate the meaning of this labelling process from the point of view of formal logic and we want to study the possibility of a new kind of quasi-set theory where this kind of labelling process cannot be performed.

2 Quasi-sets

This section is strongly based on other works about quasi-set theory (Krause, 1992; Krause, Sant’Anna, and Volkov, 1999; Sant’Anna and Santos, 2000). I use standard logical notation for first-order theories (Mendelson, 1997).

Quasi-set theory \( \mathcal{Q} \) is based on Zermelo-Fraenkel-like axioms and allows the presence of two sorts of atoms (\( \text{Urelemente} \)), termed \( m \)-atoms (micro-atoms) and \( M \)-atoms (macro-atoms). Concerning the \( m \)-atoms, a weaker ‘relation of indistinguishability’ (denoted by the symbol \( \equiv \)), is used instead of identity, and it is postulated that \( \equiv \) has the properties of an equivalence relation. The predicate of equality cannot be applied to the \( m \)-atoms, since no expression of the form \( x = y \) is a formula if \( x \) or \( y \) denote \( m \)-atoms. Hence, there is a precise sense in saying that \( m \)-atoms can be indistinguishable without being identical. This justifies what we said above about the ‘lack of identity’ to some objects.

The universe of \( \mathcal{Q} \) is composed by \( m \)-atoms, \( M \)-atoms and quasi-sets (qsets, for short). The axiomatization is adapted from that of ZFU (Zermelo-Fraenkel with \( \text{Urelemente} \)), and when we restrict the theory to the case which does not consider \( m \)-atoms, quasi-set theory is essentially equivalent to ZFU, and the corresponding quasi-sets can then be termed ‘sets’ (similarly, if also the \( M \)-atoms are ruled out, the theory collapses into ZFC). The \( M \)-atoms play the same role of the \( \text{Urelemente} \) in the sense of ZFU.

In all that follows, \( \exists_Q \) and \( \forall_Q \) are the quantifiers relativized to quasi-sets. That is, \( Q(x) \) reads as ‘\( x \) is a quasi-set’.

In order to preserve the concept of identity for the ‘well-behaved’ objects, an Extentional Equality is defined for those entities which are not \( m \)-atoms on the following
grounds: for all \(x\) and \(y\), if they are not \(m\)-atoms, then

\[ x =_E y := \forall z (z \in x \leftrightarrow z \in y) \lor (M(x) \land M(y) \land x \equiv y). \]

It is possible to prove that \(=_E\) has all the properties of classical identity in a first order theory and so these properties hold regarding \(M\)-atoms and 'sets'. In this text, all references to ‘\(=\)’ (in quasi-set theory) stand for ‘\(\equiv\)’, and similarly ‘\(\leq\)’ and ‘\(\geq\)’ stand, respectively, for ‘\(\leq_E\)’ and ‘\(\geq_E\)’. Among the specific axioms of \(Q\), few of them deserve explanation. The other axioms are adapted from ZFU.

For instance, to form certain elementary quasi-sets, such as those containing ‘two’ objects, we cannot use something like the usual 'pair axiom', since its standard formulation assumes identity; we use the weak relation of indistinguishability instead:

The ‘Weak-Pair’ Axiom - For all \(x\) and \(y\), there exists a quasi-set whose elements are the indistinguishable objects from either \(x\) or \(y\). In symbols,

\[ \forall x \forall y \exists Q \exists t (t \in z \leftrightarrow t \equiv x \lor t \equiv y). \]

We use the standard notation with \('[x, y]'\) and, when \(x \equiv y\), we have \('[x]', by definition. We remark that this quasi-set cannot be regarded as the ‘singleton’ of \(x\), since its elements are all the objects indistinguishable from \(x\), so its ‘cardinality’ (see below) may be greater than 1. A concept of strong singleton, which plays a crucial role in the applications of quasi-set theory, may be defined.

In \(Q\) we also assume a Separation Schema, which intuitively says that from a quasi-set \(x\) and a formula \(\alpha(t)\), we obtain a sub-quasi-set of \(x\) denoted by

\[ [t \in x : \alpha(t)]. \]

We use the standard notation with ‘\{' and ‘\}' instead of ‘[’ and ‘]’ only in the case where the quasi-set is a set.

It is intuitive that the concept of function cannot also be defined in the standard way, so we introduce a weaker concept of quasi-function, which maps collections of indistinguishable objects into collections of indistinguishable objects; when there are no \(m\)-atoms involved, the concept is reduced to that of function as usually understood. Relations (or quasi-relations), however, can be defined in the usual way, although no order relation can be defined on a quasi-set of indistinguishable \(m\)-atoms, since partial and total orders require antisymmetry, which cannot be stated without identity. Asymmetry also cannot be supposed, for if \(x \equiv y\), then for every relation \(R\) such that \(\langle x, y \rangle \in R\), it follows that \(\langle x, y \rangle \equiv_E [x] \equiv_E \langle y, x \rangle \in R\), by force of the axioms of \(Q\).

We remark that \([x]\) is the same (\(=_E\)) as \(\langle x, x \rangle\) by the Kuratowski’s definition.

It is possible to define a translation from the language of ZFU into the language of \(Q\) in such a way that we can obtain a ‘copy’ of ZFU in \(Q\). In this copy, all the usual mathematical concepts (like those of cardinal, ordinal, etc.) can be defined; the ‘sets’ (in reality, the ‘\(Q\)-sets’ which are ‘copies’ of the ZFU-sets) turn out to be those quasi-sets whose transitive closure (this concept is like the usual one) does not contain \(m\)-atoms.

Although some authors like Weyl (1949) sustain that (in what regard cardinals and ordinals) “the concept of ordinal is the primary one”, quantum mechanics seems to present strong arguments for questioning this thesis, and the idea of presenting collections which have a cardinal but not an ordinal is one of the most basic and important assumptions of quasi-set theory.
The concept of quasi-cardinal is taken as primitive in \( Q \), subject to certain axioms that permit us to operate with quasi-cardinals in a similar way to that of cardinals in standard set theories. Among the axioms for quasi-cardinality, we mention those below, but first we recall that in \( Q \), \( qc(x) \) stands for the ‘quasi-cardinal’ of the quasi-set \( x \), while \( Z(x) \) says that \( x \) is a set (in \( Q \)). Furthermore, \( Cd(x) \) and \( card(x) \) mean ‘\( x \) is a cardinal’ and ‘the cardinal of \( x \)’, respectively, defined as usual in the ‘copy’ of ZFU.

Quasi-cardinality - Every qset has an unique quasi-cardinal which is a cardinal (as defined in the ‘ZFU-part’ of the theory) and, if the quasi-set is in particular a set, then this quasi-cardinal is its cardinal stricto sensu:

\[
\forall_{\mathcal{Q}} x \exists_{\mathcal{Q}}! y (Cd(y) \land y =_{E} qc(x) \land (Z(x) \Rightarrow y =_{E} card(x))).
\]

Then, every quasi-cardinal is a cardinal and the above expression ‘there is a unique’ makes sense. Furthermore, from the fact that \( \emptyset \) is a set, it follows that its quasi-cardinal is 0 (zero).

\( Q \) still encompasses an axiom which says that if the quasi-cardinal of a quasi-set \( x \) is \( \alpha \), then for every quasi-cardinal \( \beta \leq \alpha \), there is a sub-quasi-set of \( x \) whose quasi-cardinal is \( \beta \), where the concept of sub-quasi-set is like the usual one. In symbols,

\[
\forall_{\mathcal{Q}} x (qc(x) =_{E} \alpha \Rightarrow \forall \beta (\beta \leq_{E} \alpha \Rightarrow \exists y (y \subseteq x \land qc(y) =_{E} \beta))).
\]

Another axiom states that

The quasi-cardinal of the power quasi-set -

\[
\forall_{\mathcal{Q}} x (qc(P(x)) =_{E} 2^{qc(x)}).
\]

where \( 2^{qc(x)} \) has its usual meaning.

As remarked above, in \( Q \) there may exist qsets whose elements are \( m \)-atoms only, called ‘pure’ qsets. Furthermore, it may be the case that the \( m \)-atoms of a pure qset \( x \) are indistinguishable from one another, in the sense of sharing the indistinguishability relation \( \equiv \). In this case, the axiomatization provides the grounds for saying that nothing in the theory can distinguish among the elements of \( x \). But, in this case, one could ask what it is that sustains the idea that there is more than one entity in \( x \). The answer is obtained through the above mentioned axioms (among others, of course). Since the quasi-cardinal of the power qset of \( x \) has quasi-cardinal \( 2^{qc(x)} \), then if \( qc(x) = \alpha \), for every quasi-cardinal \( \beta \leq \alpha \) there exists a sub-quasi-set \( y \subseteq x \) such that \( qc(y) = \beta \), according to the axiom about the quasi-cardinality of the sub-quasi-sets. Thus, if \( qc(x) = \alpha \neq 0 \), the axiomatization does not forbid the existence of \( \alpha \) sub-quasi-sets of \( x \) which can be regarded as ‘singletons’.

Of course the theory cannot prove that these ‘unitary’ sub-quasi-sets (supposing now that \( qc(x) \geq 2 \) are distinct, since we have no way of ‘identifying’ their elements, but qset theory is compatible with this idea. In other words, it is consistent with \( Q \) to maintain that \( x \) has \( \alpha \) elements, which may be regarded as absolutely indistinguishable objects. Since the elements of \( x \) may share the relation \( \equiv \), they may be further understood as belonging to a same ‘equivalence class’ (for instance, being indistinguishable electrons) but in such a way that we cannot assert either that they are identical or that they are distinct from one another (i.e., they act as ‘identical electrons’ in the physicist’s jargon).
We define $x$ and $y$ as *similar* qsets (in symbols, $Sim(x, y)$) if the elements of one of them are indistinguishable from the elements of the other one, that is, $Sim(x, y)$ if and only if $\forall z \forall t(z \in x \land t \in y \Rightarrow z \equiv t)$. Furthermore, $x$ and $y$ are *Q-Similar* ($QSim(x, y)$) if and only if they are similar and have the same quasi-cardinality. Then, since the quotient qset $x/\equiv$ may be regarded as a collection of equivalence classes of indistinguishable objects, then the ‘weak’ axiom of extensionality is:

**Weak Extensionality** -

\[
\forall x \forall y (\forall z(z \in x/\equiv \Rightarrow \exists t(t \in y/\equiv \land \forall t(t \in y/\equiv \Rightarrow \exists z(z \in x/\equiv \land QSim(t, z)))) \Rightarrow x \equiv y)
\]

In other words, this axiom says that those qsets that have ‘the same quantity of elements of the same sort (in the sense that they belong to the same equivalence class of indistinguishable objects) are indistinguishable.

### 3 Some Applications

Quasi-set theory has found its way in the sense of some applications in quantum physics. Here we list some of them:

1. It has been used (Krause, Sant’Anna, and Volkov, 1999) for an authentic proof of the quantum distributions. By “authentic proof” we mean a proof where elementary quantum particles are really considered as non-individuals right at the start. If the physicist says that some particles are indistinguishable (in a sense) and he/she still uses standard mathematics in order to cope with these particles, then something seems not to be sound. For standard mathematics is based on the concept of individuality, in the sense that it is grounded on the very notion of identity.

2. It has been proved (Sant’Anna and Santos, 2000) that even non-individuals may present a classical distribution like Maxwell-Boltzmann’s. That is another way to say that a Maxwell-Boltzmann distribution in an ensemble of particles does not entail any ontological character concerning such particles, as it was previously advocated by Nick Huggett (1999).

3. Krause, Sant’Anna, and Volkov (1999) also introduced the quasi-set-theoretical version of the wave-function of the atom of Helium, which is a well known example where indistinguishability plays an important role. Other discussions may be found in the cited reference.

### 4 Individualizing Indiscernible Objects

This section presents the main contribution of the present paper. We introduce an algorithm which allows us to label indiscernible objects in the context of quasi-set theory. The algorithm is given below, followed by its interpretation and discussion.

1. INPUT $[x]$
2. DO \( m =_E 0 \)
3. DO \( w =_E \emptyset \)
4. DO \( m := m + 1 \)
5. DO \([x] := [x] - x'\)
6. DO \( w := w \cup [\langle x', m \rangle] \)
7. OUTPUT \( w \)
8. IF \([x] =_E \emptyset \) THEN GO TO 10
9. GO TO 4
10. END

In the first step, we introduce a finite weak singleton \([x]\), i.e., a pure quasi-set with a
finite quasi-cardinality (a finite number of elements) where all its elements are indistin-
guishable objects. Next, we introduce a variable \(m\) with an initial value equal to zero
and another variable which is an empty quasi-set \(w\). In the fourth step we transform \(m\)
into \(m + 1\) (we use as attribution symbol the sign “:=”). In the fifth step we subtract one
of the elements of qset \([x]\) by means of a difference between the weak singleton \([x]\) and
the strong singleton \(x'\) (a weak singleton whose elements are indistinguishable from \(x\)
but such that \(x'\) has actually just one element). Next we create an ordered pair defined
by \(x'\) and by the label \(m\). In step seven, this ordered pair is an output. Actually the
ordered pairs are stored in \(w\). In a sense, this works as a data warehousing process.
Such process repeats from step 4 until step 7 up to the moment when the qset \([x]\) is
empty.

So, in a sense, it is possible to ‘individualize’ (by means of integer labels \(m\)) objects
that have no individuality in principle. We are seriously tempted to refer to micro-atoms
as non-individuals, since the standard identity does not apply to them. Nevertheless,
there is nothing within the scope of quasi-set theory that forbids us to attribute labels
to micro-atoms, even in a finite collection of indistinguishable micro-atoms.

From the physical point of view this is like to say that two electrons, separated by
a large distance between them, may be identified by their coordinates in spacetime.
Another interpretation is the next one. Consider that we have a \(Na\) atom, with its
11 electrons. Consider also that we excite this atom by adding a new electron to it.
When the atom emits an electron, there is no way to know if the emitted electron is
the ‘same’ one used to excite the \(Na\) atom. In this sense, the electrons of the \(Na\)
atom are indistinguishable, they are non-individuals. But when an electron is emitted,
this particle may be labelled as that one which was emitted. For some very restrictive
purposes, it is possible to label even non-individuals. But the most important point is
that this labelling process is possible even in a formal mathematical framework which
was designed specially for applications in quantum theories, namely, quasi-set theory.

So, we advocate that we need to be very careful with respect to the notions of identity
and indistinguishability.
5 A Conclusion

What is individuality? If we answer this, it seems reasonable to consider, at least in principle, that the notion of non-individuality is also settled. That seems to happen because we intend to consider that a given object is a non-individual if it is not an individual at all. In order to talk about the individuality of an object, do we need to talk about its properties, like suggested by some authors (French, 2004)? Or should the individuality of an object be expressed in terms of its haecceity or primitive thisness like suggested by Adams (1979). Actually, I am not concerned with these points in this paper. Otherwise, a much longer discussion should be made. The point that I want to emphasize has to do with the possible relationship between individuality and the labelling process. Dalla Chiara and Toraldo di Francia (1993), for example, refer to quantum physics as the land of anonymity, in the sense that, particles cannot be uniquely labelled. Many other authors have a similar opinion. But I want to advocate a different position on this.

Quasi-set theory is a set theory without identity that allows the existence of collections of objects that are indistinguishable in the sense of the properties of the equivalence relation \( \equiv \). In some cases, this indistinguishability collapses to the usual identity. That means that, in some cases, when we say that \( x \equiv y \), then we are talking about just one object, named \( x \) and \( y \). It is usual to consider that such objects are individuals due to their uniqueness. Nevertheless, quasi-set theory allows also the existence of some objects (some micro-atoms) such that \( x \equiv y \) does not ensure that we are really talking about one single (or unique) object. Any object \( x \) that is indistinguishable from \( y \) in this sense cannot be an individual, since it lacks the notion of uniqueness. Nevertheless, physicists usually consider that elementary particles are not individuals in the sense that they cannot be labelled and we cannot keep track of them in spacetime.

But with our algorithm we have proved that even objects that are considered as non-individuals right at the start can be labelled, at least in a weak sense. By weak sense mean that two ordered pairs of \( w \) can always be distinguished by means of the second element of the pair; but, on the other hand, the first coordinates of these two pairs are still indistinguishable in the quasi-set-theoretical sense. So, we think that we have proved (at least in the context of quasi-set theory) that individuality have nothing to do with any labelling process. The fact that we can label some object does not guarantee that this object is an individual in a precise sense. Obviously, there is a limitation in our algorithm. Since this is a step-by-step algorithm, it works only for finite or denumerable weak singletons \([x]\). By denumerable weak singleton we mean a qset \([x]\) whose quasi-cardinality is the cardinality of the set of natural numbers. But for any physical interpretation of \([x]\), only finite weak singletons make sense. And my concern here is with the notion of individuality (or non-individuality) in physics, with special emphasis on quantum physics.

The ability of labelling non-individuals makes perfect sense from the physical point of view. See, e.g., the Einstein-Podolsky-Rosen Gedanken experiment (Sakurai, 1994), where two distant electrons seem to be strongly connected by some sort of non-local ‘interaction’. In this experiment the two electrons are formally considered as indistinguishable, due to their entangled quantum state. Nevertheless, someone could say that these electrons can be labelled by their coordinates in the three dimensional space. But the fact that these electrons can be labelled by their space coordinates does not affect
the fact that they are truly indistinguishable, i.e., that they are non-individuals. A similar situation happens with electrons in an atom. If an atom has 11 electrons, all of them are considered as non-individuals, although they can be labelled by their respective quantum states, obeying Pauli’s Principle of Exclusion.

6 Open Problems

Here we present two lists of open problems that we consider worth of investigation. Our main goal, in the present Section, is to propose some ideas for future papers related to quasi-set theory and the problems of non-individuality in quantum mechanics.

The first list has to do with our labelling algorithm.

1. Is it possible to create some kind of quasi-quasi-set theory where the algorithm introduced in section 4 does not work? The existence of strong singletons is a crucial aspect in the algorithm. Another point is that the membership relation \( \in \) in quasi-set theory is like the usual one. What should we change in quasi-set theory in order to avoid the labelling process of our algorithm? And if this new theory is possible, then it makes sense to talk about “levels of individuality”. One first level would be in correspondence with the standard view of identity in first order and higher order theories; a second level would be in correspondence to the notion of indistinguishability in quasi-set theory as presented here; and a third level of individuality would be a very weak form where there would exist some kind of indistinguishability such that our algorithm does not apply.

2. If it is possible to create some sort of quasi-quasi-set theory where our algorithm does not work, another question is: can we use this quasi-quasi-set theory in order to ground the foundations of quantum mechanics? Can we derive, e.g., the quantum statistics in this new framework? That would be a way for a better understanding of the meaning of non-individuality among quantum particles.

Our second list concerns related problems about quasi-set theory.

1. Maxwell-Boltzmann statistics can be derived even in a collection of indiscernible particles (Sant’Anna and Santos, 2000). This means that Maxwell-Boltzmann statistics is not necessarily committed to individuality. What are the physical (or metaphysical) implications of this kind of result?

2. How to derive the spin-statistics theorem (Ryder, 1996) within the scope of quasi-set theory?

3. Some authors have considered that there is a close relationship between indistinguishability and non-locality in quantum theory. One of them is Mandel (1991), who achieved some interesting results in the context of the two-slit experiment. On the other hand Einstein-Podolsky-Rosen (EPR) Gedanken experiment is also one of those outstanding results that have allowed many interesting discussions about the concept of non-locality. We speculate if it is possible to extend the notion of metric space in order to replace the notion of identity by the concept of indistinguishability in the axioms of distance, since nonlocal phenomena occur only
between indistinguishable particles (or indistinguishable paths). In metric spaces it is considered that if \( p = q \) then the distance between \( p \) and \( q \) is zero. What if we consider that the distance is zero if and only if \( p \equiv q \)? In this case, nonlocal phenomena would be more reasonable, since the distance (in this new metric space) between indistinguishable particles would be always null. The philosophical issue here is to consider that space and spacetime are concepts that are closely related to physical systems in the sense that space and spacetime are derived concepts and not primitive ones (da Costa and Sant’Anna, 2001; da Costa and Sant’Anna, 2002).

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