Dark Matter Neutrinos Must Come with Degenerate Masses

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Abstract

It has been known that there are two schemes in the framework of three flavor neutrinos to accommodate the global features of the hot dark matter neutrinos, the solar neutrino deficit and the atmospheric neutrino anomaly in a manner consistent with terrestrial neutrino experiments, i.e., hierarchical mass neutrinos and almost degenerate neutrinos. We demonstrate that the recent result by the CHOOZ experiment excludes the scheme of hierarchical neutrinos. We also point out in the scheme of almost degenerate neutrinos that if neutrinos are Majorana particles then the double $\beta$ decay experiments must see positive signals on their way to reach a limit more stringent than the present one by a factor of 5.

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I. INTRODUCTION

Massive neutrinos are the only known particle candidates for the dark matter in the universe [1]. While it is unlikely that they are responsible for the dark matter as a whole in the cosmos, they serve as the best candidates for the hot component in the mixed (cold + hot) dark matter model [2]. The model is one of the rival models that can account for the observed power spectrum of density fluctuations over wide length scales, from galaxy-galaxy to much longer scales probed by the COBE observation [3].

The remaining unresolved issue in the model is how many flavors of neutrinos are acting as dark matter, e.g., only the tau neutrino or all three of them. The parameter $\Omega_\nu$, the fraction of the density of neutrinos in the universe, is sensitive to the sum of neutrino masses but not to the neutrinos mass spectrum. The problem of which mass pattern of neutrino is favored has been discussed based on the goodness of fit to various cosmological observables [2]. However, the definitive conclusion does not appear to be reached yet.

Here we make a different approach to the same problem based on a particle physics point of view. We demand that our scheme of the hot dark matter neutrinos satisfy the following requirements in the framework of three-flavor neutrinos:

1. Neutrinos exist in the standard three-flavor mixing framework without sterile species.
2. The three-flavor mixing framework must accommodate the deficit of the solar neutrinos [4] and the anomaly in the ratio $R = \frac{(\nu_\mu/\nu_e)_{\text{observed}}}{(\nu_\mu/\nu_e)_{\text{expected}}}$ observed in the atmospheric neutrino experiments [5].
3. The three-flavor mixing framework must be consistent with the constraints imposed by all the reactor and the accelerator experiments [2].

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1 In this paper, we take a conservative attitude to the experimental results obtained by the LSND group [6] who claims to observe the appearance signals consistent with neutrino oscillations. We feel that we should wait for confirmation by other experiments, KARMEN and Fermilab, before concluding it as an evidence for neutrino oscillations. The similar attitude is shared by the author.
Several remarks are in order on the requirements (1)-(3).

(i) The dark matter neutrino hypothesis requires neutrino masses but not flavor mixings. However, it is a legitimate assumption in particle physics that neutrino’s gauge and mass eigenstates differ if they are massive. It is a natural assumption because we know that the flavor mixing occurs in the quark sector [8].

(ii) It appears that the existence of the solar neutrino deficit and the atmospheric neutrino anomaly are robust, as confirmed by the recent high-statistics measurements by Superkamiokande [9]. The most likely explanation of these anomalies is due to the neutrino masses and the flavor mixing.

(iii) Among other terrestrial experiments the recent $\bar{\nu}_e$ disappearance experiment done by the CHOOZ group [10] puts stringent constraints on the three-flavor framework we take in this paper.

In the framework of the three flavor neutrinos two schemes of the hot dark matter neutrinos have been known which satisfy the requirements (1) – (3). One is the scheme of almost degenerate neutrinos (ADN) [11], in which the mass squared differences $\Delta m^2$ of neutrinos are much smaller than several eV$^2$. The other one is that of hierarchical mass neutrinos (HMN) [12–14], in which at least one of $\Delta m^2$ reflects the dark matter mass scale, $\sim$ a few eV. In this paper we show that the recent result by the CHOOZ experiment excludes the HMN scheme. We also show that the CHOOZ result puts a strong constraint on the ADN scheme and that the ADN scenario predicts a positive signal in neutrinoless double $\beta$ decay experiments in the near future.

In section 2 we summarize the constraints from the accelerator and reactor experiments. In section 3 we perform a qualitative analysis in which we only rely on the global features of the anomalies. In section 4 we analyze in detail the goodness of fit of the HMN scheme to the solar and the atmospheric neutrino data, taking into account more sophisticated issues, such of Ref. [7]
as the zenith-angle distribution of the atmospheric neutrino data. In section 5 we discuss the constraints on the hot dark matter scenario with the ADN scheme by the CHOOZ result and the neutrinoless double $\beta$ decay experiments. In section 6 we summarize our conclusions and give remarks on possible signals in the near future experiments.

II. CONSTRAINTS FROM ACCELERATOR AND THE REACTOR EXPERIMENTS

The mass-squared differences $\Delta m^2$ greater than $1$ eV$^2$ are subject to the constraints by the short-baseline accelerator and the reactor experiments. Therefore, the mixing parameters of the HMN scenario are tightly constrained by them. This point has recently been revealed explicitly by the authors of Refs. [12,13,17] in a generic three-flavor mixing framework, whereas it may have been implicitly noticed before. In particular, the resulting constraint is quantitatively worked out by Fogli, Lisi and Scioscia [17].

To make the discussion clearer we take a particular representation of the neutrino mixing matrix $U$, which connects the gauge and the mass eigenstates as $\nu_\alpha = U_{\alpha i} \nu_i$ ($\alpha = e, \mu, \tau, i = 1, 2, 3$),

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix}. \quad (1)$$

The constraint by the short-baseline experiments is powerful enough to allow only the three regions on plane spanned by mixing angles $\theta_{13}$ and $\theta_{23}$.

(a) small-$s_{13}$ and small-$s_{23}$
(b) large-$s_{13}$ and arbitrary $s_{23}$
(c) small-$s_{13}$ and large-$s_{23}$,

Each region can be characterized by approximate decoupling of a neutrino state, $\nu_\tau, \nu_e, \nu_\mu$ for (a), (b) and (c) respectively. If we (naturally) allow the remaining angle $\theta_{12}$ to be large we have large-angle mixing between $\nu_\mu \leftrightarrow \nu_e, \nu_\mu \leftrightarrow \nu_\tau,$ and $\nu_e \leftrightarrow \nu_\tau$ in regions (a),
(b) and (c), respectively. Since we attempt to implement the atmospheric neutrino anomaly into our framework we cannot live in the region (c). It also implies that in the regions (a) and (b) the atmospheric neutrino anomaly is due to almost pure $\nu_\mu \leftrightarrow \nu_e$ and $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, respectively.

In summary, HMN scenario is strongly constrained by the short-baseline experiments but ADN is free from such constraints.\footnote{A mild restriction on $\theta_{13}$ in ADN scenario arises from the Bugey \cite{Bugey} and the Krasnoyarsk \cite{Krasnoyarsk} experiments if the scale of $\Delta m^2$ relevant to the atmospheric neutrino anomaly falls in region $\Delta m^2 \gtrsim 10^{-2} \text{eV}^2$.}

The recent CHOOZ experiment has drastically altered the above situation. The experiment measured the $\bar{\nu}_e$ beam attenuation from the reactors by the Gd-loaded liquid scintillator detector located at about 1 km from the reactors. Due to the long-baseline of the experiment they are able to probe the wide region of $\Delta m^2 \gtrsim 10^{-3} \text{eV}^2$, which essentially cover most of the region relevant to the atmospheric neutrino anomaly. The negative result in the CHOOZ experiment put stringent constraints on the mixing parameters as we will discuss below.

Let us discuss the constraints on the mixing parameters implied by the CHOOZ result. We start by describing the simplification that occurs to the neutrino oscillation probabilities under the mass hierarchy, which will play an important role in the following discussions. For definiteness of the notation we deal with the mass hierarchy depicted in Fig. 1. Namely, the larger mass squared difference $\Delta m_{31}^2 \simeq \Delta m_{32}^2$, which we denote as $\Delta M^2$ in this paper, is much greater than the smaller one, $\Delta m^2 \equiv \Delta m_{21}^2$. If the neutrino masses obey the hierarchy $\Delta M^2 \gg \Delta m^2$, the disappearance probability can be approximately written as

$$1 - P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha) = 4|U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2) \sin^2 \left( \frac{\Delta M^2 L}{4E} \right) + 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2 \left( \frac{\Delta m^2 L}{4E} \right).$$

(2)

The correction terms to (2) are negligible provided that $\Delta M^2 L/4E$ is so large that oscillations due to $\sin(\Delta M^2 L/4E)$ is averaged to zero, or $\sin(\Delta m^2 L/4E)$ is sufficiently small. A
convenient formula to estimate these quantities is:

\[
\frac{\Delta m^2 L}{4E} = 1.27 \left( \frac{\Delta m^2}{10^{-3} \text{eV}^2} \right) \left( \frac{L}{1 \text{km}} \right) \left( \frac{E}{1 \text{MeV}} \right)^{-1}.
\]

We first discuss the constraints imposed on the ADN scenario. In this scenario role of the dark matter is played by all the three neutrinos with almost equal masses of the order of a few eV, with the mass difference roughly of the order of \(\Delta M^2 = \Delta m^2_{\text{atm}} \simeq 10^{-3} - 10^{-2} \text{eV}^2\). The smaller \(\Delta m^2\) depends upon whether we take the MSW solution of the solar neutrino problem \([20]\) for which \(\Delta m^2 = \Delta m^2_{\text{solar}} \simeq 10^{-6} - 10^{-5} \text{eV}^2\), or the just-so vacuum oscillation solution \([21]\), \(\Delta m^2 \simeq 10^{-10} \text{eV}^2\). For notational convenience, we use in the rest of the paper the symbols \(\Delta m^2_{\text{atm}}\) and \(\Delta m^2_{\text{solar}}\), which stands for the above mass scales relevant to the atmospheric neutrino anomaly and the solar neutrino deficit, respectively.

Since the neutrino oscillation phenomenon is sensitive only to the squared-mass difference, not to the absolute mass values, the following discussion of the constraints on the ADN case also applies to the case of no dark matter mass neutrinos, e.g., \(m_1 \ll m_2 \sim 10^{-3} - 10^{-2} \text{eV}\), and \(m_3 \sim 0.03 - 0.1 \text{eV}\), to which we will refer as ELN (extremely light neutrinos).

In ADN type \(\Delta m^2\)-hierarchy the disappearing rate in \(\bar{\nu}_e \rightarrow \bar{\nu}_e\) experiment can be written as

\[
1 - P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 4 c_{13}^2 s_{13}^2 \sin^2 \left( \frac{\Delta M^2 L}{4E} \right),
\]

because the second term in (3) is negligible for \(\Delta m^2 = \Delta m^2_{\text{solar}}\). It is easy to observe that the bound obtained by the CHOOZ group can be easily translated into the one for \(s_{13}\). We have calculated the allowed region for the \(\bar{\nu}_e \leftrightarrow \bar{\nu}_e\) disappearance experiments at 90 % confidence level by combining the CHOOZ, the older Bugey and the Krasnoyarsk results. The result is presented in Fig.2.
III. QUALITATIVE ANALYSIS OF THE HMN SCHEME

We perform a qualitative analysis of the HMN scenario in which we rely on global features of the anomalies of the solar and the atmospheric neutrino data. Namely, we require for solar neutrinos the suppression of the flux by at least factor of 2. It is a milder requirement than requiring the consistency with the varying suppression rate observed in the different experiments which seem to indicate that the depletion rate of the solar neutrino flux is strongly energy-dependent. Similarly, we only require the consistency with the gross deviation of the ratio $R$ in the atmospheric neutrino data from unity.

In the HMN scenario $\Delta M^2 = \Delta m^2_{\text{DM}} \sim$ several eV$^2$, and $\Delta m^2 = \Delta m^2_{\text{atm}}$ or $\Delta m^2 = \Delta m^2_{\text{solar}}$ as required by the condition (2). If we exploit the second option for $\Delta m^2$ in order to incorporate naturally the solar neutrino deficit, there is no way of accommodating the atmospheric neutrino anomaly.\footnote{There is a marginal solution that can take into account of the atmospheric neutrino anomaly with the choice of $\Delta m^2 = \Delta m^2_{\text{solar}}$, as proposed by Cardall and Fuller \cite{22} and recently confirmed by more elaborate analysis by Fogli et al. \cite{23}. However, it is questionable \cite{24} if the solution survives after the zenith angle distribution is taken into account. Furthermore, the solution survives only with a small value of $\Delta M^2 \simeq 0.45$ eV$^2$, a too small value for the model of hierarchical mass neutrino as hot dark matter, while it is consistent with the LSND data from which their original motivation comes.} This point is in fact well known and is recently summarized in Ref. \cite{25}. It will be discussed more quantitatively below. For this reason we employ the first option, $\Delta m^2 = \Delta m^2_{\text{atm}} \simeq 10^{-3} - 10^{-2}$ eV$^2$. This mass hierarchy has recently been discussed in Refs. \cite{12,14}.

Under the mass spectrum we just specified the arguments of the first and the second sine functions in (2) are of order unity for the short- and the long-baseline experiments, respectively, if the neutrino beam energy is of order $\sim 1$ GeV. Then, the short- and the long-baseline neutrino experiments can give rise to independent different constraints on mixing
parameters. Let us discuss it more explicitly by restricting to $\bar{\nu}_e$ disappearance experiment. With use of the definition of the mixing angles in (1) the $\bar{\nu}_e$ disappearance probability at long-baseline can be written as

$$1 - P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 2s_{13}^2c_{13}^2 + 4c_{12}s_{12}c_{13}s_{13}^2 \sin^2 \left( \frac{\Delta m^2 L}{4E} \right).$$  

(4)

To indicate the point let us take $\Delta M^2 = 5$ eV$^2$. Then, the parameter $s_{13}^2c_{13}^2$ is constrained to be $s_{13}^2c_{13}^2 \leq 2 \times 10^{-2}$ and thus the first term in (2) is smaller than $4 \times 10^{-2}$. Therefore, the long-baseline $\bar{\nu}_e$ disappearance experiment primarily constrains the angle $\theta_{12}$.

Let us discuss the parameter region (a) and (b) separately.

The region (a): small $s_{13}$ and small $s_{23}$

In this region $c_{13}$ is approximately unity. Then, the neutrino disappearance probability is effectively of the type of two-flavor mixing. One can translate the bound on $\sin^2 2\theta$ obtained by the CHOOZ group to that on $\sin^2 2\theta_{12}$. The most conservative bound is obtained for $s_{12}^2$ by ignoring the first term of (2) and it coincides with the one obtained for $s_{13}^2$ in the preceding analysis of the ADN scenario.

The region (b): large $s_{13}$ and arbitrary $s_{23}$

In this region $c_{13}$ is small, and the Bugey experiment [18] implies $c_{13}^2 \leq 2 \times 10^{-2}$. If we take the smaller mass squared difference $\Delta m^2 = 2 \times 10^{-3}$ eV$^2$ then the CHOOZ experiment yields a little stronger constraint on $c_{13}$ in this region (See section 4).

We now address the messages that are most important in this paper; In the case of HMN the gross deficit of the solar neutrino flux cannot be achieved in any of the parameter regions (a) and (b). In the region (a) because of the bound on $s_{12}^2$ the solar neutrino deficit is at most $\sim 20\%$ if $\Delta m^2_{atm} \geq 2 \times 10^{-3}$ eV$^2$. We will address the case of smaller $\Delta m^2_{atm}$ in our quantitative analysis later.

In the region (b) the solar neutrino deficit does not occur as was pointed out in Ref. [15,16]. One can easily understand this by noticing the approximate expression of the three-flavor oscillation probability,

$$P(\nu_e \rightarrow \nu_e) = 1 - 2(c_{13}^4s_{12}^2c_{12}^2 + s_{13}^2c_{13}^2)$$

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\[ \simeq 1, \]

which is obtained from (4) after taking average over rapid oscillations, since the smallest mass squared difference \( \Delta m^2 \) is much larger than \( \Delta m^2_{\text{solar}} \). It implies that the large \( s_{13} \) region (b) cannot accommodate the gross deficit of solar neutrino flux.

Even worse one cannot have enough atmospheric neutrino anomaly in the region (a). To achieve a qualitative understanding of this fact prior to the quantitative analysis pursued later we ignore the matter effect and write down the oscillation probability at long-baseline as

\[
P(\nu_\mu \to \nu_e) = 2c_{13}^2 s_{13}^2 s_{23}^2 + \left[ 4c_{12}^2 s_{12}^2 c_{13}^2 (c_{23}^2 - s_{23}^2 s_{13}^2) + 4(c_{12}^2 - s_{12}^2) J \cot \delta \right] \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) + 2J \sin \left( \frac{\Delta m^2 L}{2E} \right),
\]

where \( J = c_{12} s_{12} c_{13} s_{13} c_{23} s_{23} \sin \delta \) denotes the leptonic Jarlskog factor. At \( \Delta M^2 = 5 \text{eV}^2 \) the first term is small, \( \leq 1.6 \times 10^{-4} \). The third term is also small, \( \leq 5 \times 10^{-3} \). By imposing the bound obtained by CHOOZ group [10] on the coefficient of the second term one can show that it is smaller than 0.21. Because of the dominance of \( \nu_\mu \to \nu_e \) oscillation we obtain, as a rough estimation of the ratio \( R \), \( R = (1 - 0.1)/(1 + 0.1) \simeq 0.8 \), which may not be sufficient to explain the data of atmospheric neutrino observation.

This concludes our qualitative discussions to demonstrate that the HMN dark matter cannot be reconciled with the requirements (2) and (3). We therefore conclude that the dark matter neutrinos must come with almost degenerate mass spectrum.

**IV. QUANTITATIVE ANALYSIS OF THE HMN SCHEME**

We quantify our discussion by contrasting the various hypothesis against the experimental data. For the regions (a) – (c), we optimize the combined \( \chi^2 \):

\[
\chi^2_{\text{tot}} = \chi^2_{\text{solar}} + \chi^2_{\text{atm}},
\]
where $\chi^2_{\text{solar}}$ and $\chi^2_{\text{atm}}$ are the $\chi^2$ for the solar and the atmospheric neutrino data, respectively. To make our discussions concrete, let us first take the smaller mass squared difference $\Delta m^2 = 2 \times 10^{-3}\text{eV}^2$. We will discuss the case for $\Delta m^2 \neq 2 \times 10^{-3}\text{eV}^2$ later.

To compute $\chi^2_{\text{solar}}$ we adopt the theoretical predictions by [26] and the solar neutrino data quoted in the table in [27] with the 6 day-night bins of the Kamiokande data [28] (# of the data = 6 (Kamiokande) + 1 (Superkamiokande) + 1 (Cl) + 1 (Ga, combined) = 9) and we minimize the $\chi^2_{\text{solar}}$ by varying the suppression probability $P(\nu_e \rightarrow \nu_e)$ and the normalization factor $f_B$ of the $^8\text{B}$ neutrino flux as in Ref. [13]. The theoretical predictions to the atmospheric neutrino data are computed by using a code developed by one of the authors [32]. We perform separate analyses of the Kamiokande [33] and the Superkamiokande data [9] of atmospheric neutrino observation. We use the Kamiokande atmospheric neutrino data (# of the data = 10 (multi-GeV, zenith angle) + 5 (sub-GeV, zenith angle) + 20 (sub-GeV, energy spectrum) = 35) and the Superkamiokande data (# of the data = 10 (multi-GeV, zenith angle) + 10 (sub-GeV, zenith angle) + 25 (sub-GeV, energy spectrum) = 45) in the following.

In the region (a) we have a poor fit to both the solar and atmospheric neutrino data. The most stringent constraint on $s_{13}^2$ comes from the CHOOZ experiment [10]. Namely, using the disappearance probability (4) with $\Delta m^2 = 2 \times 10^{-3}\text{eV}^2$, the allowed region at 90 % confidence level can be obtained as $s_{13}^2 \leq 3 \times 10^{-2}$ and $\sin^2 2\theta_{12} \leq 0.4$. The most stringent bound on $s_{23}^2$ comes from the E531 experiment ($\nu_{\mu} \rightarrow \nu_{\tau}$ appearance) [34]: $s_{23}^2 \leq 1 \times 10^{-3}$. Since we would like to demonstrate that our argument also holds for all the regions of $\Delta M^2 > 5 \text{eV}^2$, we use less severe constraint from the CDHSW experiment ($\nu_{\mu} \leftrightarrow \nu_{\mu}$ disappearance) [11].

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4The most recent solar neutrino data of Superkamiokande and GALLEX are slightly different from those quoted in [27], but it turns out that the conclusions below do not change. Also our conclusions below do not change even if we adopt other theoretical predictions [24–31] for solar neutrinos.
here: $s_{23}^2 \leq 2 \times 10^{-2}$. For $\Delta M^2 > 5 \text{eV}^2$ the bound on $s_{23}^2$ from the E531 experiment becomes stronger. Under these constraints we obtain

$$
\chi^2_{\text{solar}} \geq 81.3 \quad (9 \text{ d.f.})
$$

$$
\chi^2_{\text{atm}} \geq \begin{cases} 105.8 \quad \text{(Kamiokande; 35 d.f.)} \\ 175.8 \quad \text{(Superkamiokande; 45 d.f.)} \end{cases}
$$

so that we have

$$
\chi^2_{\text{tot}} \geq \begin{cases} 187.1 \quad \text{(with Kamiokande atm.}\ \nu; \ 38 \text{ d.f.)} \\ 257.1 \quad \text{(with Superkamiokande atm.}\ \nu; \ 48 \text{ d.f.)} \end{cases} \quad \text{for region (a),} \quad (5)
$$

where we have subtracted the number (=6) of degrees of freedom of the parameters ($\Delta m_{21}^2$, $\Delta m_{32}^2$, $\theta_{12}$, $\theta_{13}$, $\theta_{23}$, $f_B$) in (5). We note in passing that the reason that a fit to the Superkamiokande atmospheric neutrino data is much worse is because our scheme (a) gives dominant $\nu_\mu \leftrightarrow \nu_e$ oscillations which are disfavored by the Superkamiokande data. Thus we conclude that the region (a) is excluded at $(1 - 1 \times 10^{-21}) \text{CL}$ (or 9.5 standard deviation) in case of $\chi^2_{\text{solar}} + \chi^2_{\text{atm}}$ (Kamiokande) and at $(1 - 2 \times 10^{-30}) \text{CL}$ (or 11.5 standard deviation) in case of $\chi^2_{\text{solar}} + \chi^2_{\text{atm}}$ (Superkamiokande), respectively.

In the region (b) we have a poor fit to the solar neutrino data. The CHOOZ experiment again gives the strongest bound on $\theta_{13}^2$ and $c_{13}^2 \leq 1 \times 10^{-2}$. With this condition we find

$$
\chi^2_{\text{solar}} \geq 127.8 \quad (9 \text{ d.f.}).
$$

The fit to the atmospheric neutrino data is moderate:

$$
\chi^2_{\text{atm}} \geq \begin{cases} 47.4 \quad \text{(Kamiokande; 35 d.f.)} \\ 78.2 \quad \text{(Superkamiokande; 45 d.f.)} \end{cases}
$$

Therefore we have

$$
\chi^2_{\text{tot}} \geq \begin{cases} 175.2 \quad \text{(with Kamiokande atm.}\ \nu; \ 38 \text{ d.f.)} \\ 206.0 \quad \text{(with Superkamiokande atm.}\ \nu; \ 48 \text{ d.f.)} \end{cases} \quad \text{for region (b),}
$$
and we conclude that the region (b) is excluded at $(1 - 2 \times 10^{-19})$ CL (or 9.0 standard deviation) in case of $\chi^2_{\text{solar}} + \chi^2_{\text{atm}}$ (Kamiokande) and at $(1 - 2 \times 10^{-21})$ CL (or 9.5 standard deviation) in case of $\chi^2_{\text{solar}} + \chi^2_{\text{atm}}$ (Superkamiokande), respectively.

In the region (c) a fit to the solar neutrino data is moderate and a fit to the atmospheric neutrino data is poor. The severest constraints on $s_{13}^2$ come from the E776 experiment ($\nu_\mu \rightarrow \nu_e$ appearance) \cite{6} and we have $s_{13}^2 \leq 5 \times 10^{-4}$. Although the E531 experiment ($\nu_\mu \rightarrow \nu_\tau$ appearance) \cite{44} gives the strongest bound on $s_{23}^2$, we use less tight bound from the CDHSW experiment ($\nu_\mu \leftrightarrow \nu_\mu$ disappearance) \cite{35} which gives $c_{23}^2 \leq 2 \times 10^{-2}$. Under these conditions we get

$$\chi^2_{\text{solar}} \geq 29.8 \quad (9 \text{ d.f.}),$$
$$\chi^2_{\text{atm}} \geq \begin{cases} 74.2 & (\text{Kamiokande; 35 d.f.}) \\ 144.4 & (\text{Superkamiokande; 45 d.f.}) \end{cases}$$

So we have

$$\chi^2_{\text{tot}} \geq \begin{cases} 104.0 & (\text{with Kamiokande atm. } \nu; \ 38 \text{ d.f.}) \\ 174.2 & (\text{with Superkamiokande atm. } \nu; \ 48 \text{ d.f.}) \end{cases}$$

for region (c), and we conclude that the region (c) is excluded at $(1 - 5 \times 10^{-8})$ CL (or 5.5 standard deviation) in case of $\chi^2_{\text{solar}} + \chi^2_{\text{atm}}$ (Kamiokande) and at $(1 - 3 \times 10^{-16})$ CL (or 8.2 standard deviation) in case of $\chi^2_{\text{solar}} + \chi^2_{\text{atm}}$ (Superkamiokande), respectively. As we have seen above, irrespective of which atmospheric neutrino data we use, Kamiokande or Superkamiokande, the regions (a), (b) and (c) are completely excluded for $\Delta m^2 = 2 \times 10^{-3}\text{eV}^2$.

We have also investigated the case for $\Delta m^2 < 2 \times 10^{-3}\text{eV}^2$ and for $\Delta m^2 > 2 \times 10^{-3}\text{eV}^2$. For $\Delta m^2 > 2 \times 10^{-3}\text{eV}^2$ the CHOOZ result gives even severer constraint on $\sin^2 2\theta_{12}$, so a fit to the solar and atmospheric neutrino data becomes even worse for all the regions (a) – (c), and hence (a) – (c) are excluded for $\Delta m^2 > 2 \times 10^{-3}\text{eV}^2$. For $\Delta m^2 < 2 \times 10^{-3}\text{eV}^2$ we have basically no constraint from the CHOOZ result on $s_{12}$, so a fit to the solar neutrino data becomes good in the region (a). On the other hand, the zenith angle dependence cannot be reproduced for the atmospheric neutrino data as $\Delta m^2$ becomes smaller, so a fit to the atmospheric neutrino data becomes worse. We found that
\[ \chi^2_{\text{tot}} > \begin{cases} 120 & \text{(with Kamiokande atm. } \nu; \text{ 38 d.f.)} \\ 220 & \text{(with Superkamiokande atm. } \nu; \text{ 48 d.f.)} \end{cases} \text{ for region (a)}, \]

for \( \Delta m^2 < 2 \times 10^{-3}\text{eV}^2 \), and hence the region (a) is excluded at \((1 - 2 \times 10^{-10}) \text{ CL} \) (or 6.4 standard deviation) in case of \( \chi^2_{\text{solar}} + \chi^2_{\text{atm}} \) (Kamiokande) and at \((1 - 7 \times 10^{-24}) \text{ CL} \) (or 10.1 standard deviation) in case of \( \chi^2_{\text{solar}} + \chi^2_{\text{atm}} \) (Superkamiokande), respectively. In the regions (b) and (c) we do not have any improvement on a fit to the solar neutrino data for \( \Delta m^2 < 2 \times 10^{-3}\text{eV}^2 \), whereas a fit to the atmospheric neutrino data becomes poorer. Therefore we can exclude the regions (b) and (c) also for \( \Delta m^2 < 2 \times 10^{-3}\text{eV}^2 \).

In the discussions above we have taken the larger mass squared difference \( \Delta M^2 = 5\text{eV}^2 \) as a reference value. For \( \Delta M^2 > 5 \text{ eV}^2 \) the constraint on \( s_{23}^2 \) from the E531 experiment becomes even tighter while the bound on \( s_{13}^2 \) remain the same as \( \Delta M^2 = 5 \text{ eV}^2 \). A fit to the solar neutrino and atmospheric neutrino data is as poor as the case for \( \Delta M^2 = 5 \text{ eV}^2 \), because all the factors \( \sin^2(\Delta m^2_{ij} L/4E) \) are averaged to give \( 1/2 \). So our conclusions still hold for \( \Delta M^2 > 5 \text{ eV}^2 \). For \( 1 \text{ eV}^2 \lesssim \Delta M^2 \lesssim 5 \text{ eV}^2 \), the bound on \( s_{23}^2 \) becomes weaker, but it turns out that a fit to the solar and atmospheric neutrino data does not improve, since we are considering the energy independent solution to the solar neutrino problem and the zenith angle dependence of the atmospheric neutrino data is not sensitive to the value of the larger mass squared difference \( \Delta M^2 \). Thus our conclusions do not change as long as the larger mass squared difference satisfies \( \Delta M^2 \gtrsim 1 \text{ eV}^2 \), which covers the region suggested by the mixed dark matter scenario [2].

To summarize, we have shown quantitatively that our hierarchical schemes in the regions (a) – (c) are all excluded.

V. ANALYSIS OF THE ADN SCENARIO

We further discuss the meaning of the constraint imposed on the ADN scenario by the CHOOZ experiment. We point out that if neutrinos are Majorana particles the neutrinoless double \( \beta \) decay experiments further tighten the parameters of the ADN scenario. It has
been pointed out [37,38] that the CP violating phases have to be large for the ADN scheme to be consistent with the neutrinoless double $\beta$ decay experiments. In particular we have derived in Ref. [38] the inequality that is valid in the ADN scenario:

$$r \equiv \frac{\langle m_{\nu e} \rangle}{m} \geq \left| c_{13}^2 \sqrt{1 - \sin^2 \beta \sin^2 2\theta_{12} - s_{13}^2} \right|$$

where $m \equiv m_1 \simeq m_2 \simeq m_3$ and $\beta$ is an extra Majorana CP phase. The most conservative bound is obtained for $\sin \beta = 1$.

If $\Delta m^2_{\text{atm}} \geq 2 \times 10^{-3}\text{eV}^2$ the constraint by the CHOOZ experiment implies that $s_{13}^2 \leq 0.05$. We further assume the MSW mechanism [20] as a solution to the solar neutrino problem. By looking into Fig.1 of Ref. [38] one can understand if $r < 0.16$ there is no consistent solution with the 90% confidence level allowed region of the solar neutrino deficit derived with the MSW mechanism [39]. This implies the bound for dark matter neutrino mass

$$m \leq \frac{\langle m_{\nu e} \rangle}{0.16}.$$ 

The experimental bound on $\langle m_{\nu e} \rangle$ obtained by the Heidelberg-Moscow group [40], $\langle m_{\nu e} \rangle \leq 0.45\text{eV}$ [41] can be translated into the constraint on the degenerate neutrino mass as $m \leq 2.8\text{eV}$. If we include a factor of 2 uncertainty of the nuclear matrix elements the bound may be relaxed to $m \leq 5.6\text{eV}$. These bounds can be expressed into the ones for $\Omega_\nu$,

$$\Omega_\nu = \frac{\sum_i m_i}{91.5\text{eV}} h^{-2} \leq 0.1(0.2) h^{-2}$$

without (with) uncertainty of nuclear matrix element, where $h$ indicates the Hubble parameter measured in units of 100 km/s·Mpc.

If the Heidelberg-Moscow group continues to see no neutrinoless double $\beta$ decay event in the present setting the constraint on $\langle m_{\nu e} \rangle$ will be tighten to $\langle m_{\nu e} \rangle \leq 0.1$ [41]. If this happened, it implies the bound on neutrino mass $m \leq 0.6 \text{eV} (1.2 \text{eV})$ without (with) the extra uncertainty of the nuclear matrix elements. These lead to $\Omega_\nu \leq 0.02(0.04) h^{-2}$ which may be too small to meet the demand by the mixed dark matter cosmology. This leads to
the important conclusion that if the hot dark matter neutrinos are Majorana particles and they exist in nature in a manner satisfying the requirements (1)-(3) mentioned in section 1 then the double $\beta$ decay experiments must see positive signals on the way to reach the sensitivity down to $\langle m_{\nu e} \rangle = 0.1$ eV.

VI. CONCLUSIONS

We have shown in the framework of three flavor neutrino oscillations that all schemes with hierarchical mass scales in which the larger and the smaller mass squared differences are of order several eV$^2$ and $10^{-3} - 10^{-2}$ eV$^2$, respectively are excluded on the firm statistical ground of more than 5 $\sigma$. We have also shown that the almost degenerate neutrino scenario can be tested in neutrinoless double $\beta$ decay experiments in the near future if neutrinos are of the Majorana type.

Finally several remarks are in order:

(1) In a previous communication [42] we have argued that if the ratio $R_{lb} = (\text{No. of observed electron events}) / (\text{No. of expected muon events} - \text{No. of observed muon events})$ that can be measured by the long-baseline neutrino experiments falls into the region $0.02 \leq R_{lb} \leq 0.87$ (for $\Delta M^2 \geq 5$ eV$^2$) then the hierarchical mass dark matter neutrino hypothesis can be rejected. We have shown in this paper that the new constraint imposed by the CHOOZ experiment and the requirement of accommodating the solar neutrino deficit implies that it must be the case.

(2) On the other hand, if the CHORUS [43] or the NOMAD [44] experiments observe the appearance signals it turns out that our conclusion is wrong. It then indicates that at least one of our assumptions (1) - (3) is incorrect. It would imply a strong indication for unexpected new features in physics of neutrinos.

(3) In general the new constraint from the CHOOZ experiment makes it more difficult to observe CP violation in neutrino oscillation experiments. But a closer examination reveals that the situation is not so simple and in fact it depends upon the scenarios of neutrino mass
hierarchies. In the ADN or the ELN type mass hierarchies, the estimation of the magnitude of CP violation in long-baseline neutrino oscillation experiments done by Arafune, Koike and Sato [45] remains unchanged because the (pin-pointed) mixing parameters they used is consistent with the bound from the CHOOZ data. In the HMN case, however, we cannot allow all the $s_{12}^2$ regions analyzed in Refs. [46], but should restrict to the region $s_{12}^2 \leq 0.05$, assuming that $\Delta m^2_{\text{atm}} \geq 2 \times 10^{-3} \text{ eV}^2$. The magnitude of the CP violating effect thereby decreases depending upon the value of other mixing angles.

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FIGURE CAPTIONS

Fig.1 A schematic illustration of the mass pattern with hierarchy in differences of squared masses discussed in this paper. We deal with the two mass patterns (A) and (B) simultaneously while they can be distinguished by the matter effect in the earth.

Fig.2 The allowed region ($\Delta m^2$, $\sin^2 2\theta$) in a framework of two flavor neutrino oscillations obtained by combining the CHOOZ [10], the Bugey [18], and the Krasnoyarsk [19] experiments. The abscissa can be regarded as $\sin^2 2\theta_{12}$ in the HMN scenario in its parameter region (a), as discussed in the text.
\[ \Delta m_{21}^2 = \Delta m^2 \quad \Delta m_{32}^2 \approx \Delta M^2 \]

(A)

\[ \Delta m_{13}^2 \approx \Delta M^2 \quad \Delta m_{21}^2 = \Delta m^2 \]

(B)
CHOOZ + Bugey + Krasnoyarsk 90% CL

Fig. 2