Global Lightning Quanta

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Abstract  The World Meteorological Organization recently declared lightning an essential climate variable which makes the global lightning flash rate density a key quantity, currently assessed by geostationary satellites and ground-based lightning location networks. Yet, no theory has been put forward to explain the physical relationships between the thermodynamic temperature of the Earth’s atmosphere $T$, the global lightning flash occurrence frequency $f_g$, and its radiant energy $E$ of resonant electromagnetic waves within the Earth ionosphere cavity. These three parameters are combined here by adapting the rigorous framework of quantum physics. The minimum amount of radiant energy produced by the lightning flash occurrence frequency is the global lightning quantum $E = hf_g$ $h$ being Planck’s constant. The superposition of numerous global lightning quanta distributes its radiant energy around the world as Earth ionosphere cavity resonances. The novel theory is in agreement with measurements using a radiometer at Arrival Heights, Antarctica, as part of the Stanford ELF/VLF Radio Noise Survey. It is found that the measurements agree with the theory within ~30%. The operation of the theory is illustrated with an interpretation of the measurements for an exemplary thermodynamic energy. In this case, the measurements correspond to a radiant temperature ~30°C akin to the mixed phase region of thunderclouds where lightning discharges are initiated. The theory can help to assess the mutual impact of climate change and global lightning on each other as proposed by the World Meteorological Organization.

Plain Language Summary  The World Meteorological Organization recently declared lightning flashes to be an essential climate variable which helps to assess the mutual impact of global climate change and global lightning activity on each other. Lightning flashes around the world occur at an average rate of 46 1/s and emit radio waves. Some of these radio waves bounce back and forth between the Earth’s surface and the reflective ionized upper atmosphere at ~100 km height and propagate several times around the world in the blink of an eye. This contribution brings together the atmospheric temperature, the global lightning flash rate and the emitted radio waves in just one single theory. The theory adapts the basic principles of quantum physics to explain the distribution of electromagnetic energy from lightning flashes around the world. The theory agrees with extremely sensitive measurements in the Antarctic within ~30%. It is found that he derived brightness temperature is ~30°C where water and ice particles collide to produce the charge separation that is needed to initiate lightning flashes inside thunderclouds. The total energy from all lightning flashes around the world is ~0.5 J which is similar to the energy of a tennis ball at half-a-meter height.

1. Introduction

The World Meteorological Organization (WMO) recently declared lightning flashes to be an Essential Climate Variable (ECV) (World Meteorological Organization, 2019). Essential climate variables are quantities that contribute significantly to the characterization of the Earth’s climate by providing empirical evidence for the benefit of climate services and to inform policy (Bojinski et al., 2014). The WMO’s declaration was based on a recommendation by the Task Team on Lightning Observation for Climate Applications (TT-LOCA) as part of the Atmospheric Observation Panel for Climate (AOPC) of the WMO and the Global Climate Observing System (GCOS) (Aich et al., 2018). The declaration elevates lightning flashes to a key quantity that is currently mainly assessed by geostationary satellites with the Global Lightning Monitor (GLM) (Goodman et al., 2013) which significantly extends space borne lightning observations from low Earth orbit with the Optical Transient Detector (OTD) and the Lightning Imaging Sensor (LIS) (Christian et al., 2003). More recently, a Lightning Imaging Sensor was installed on the International Space Station (ISS) to observe
lightning flashes from near Earth orbit (Blakeslee et al., 2020, and references therein) and in parallel the Atmosphere Space Interaction Monitor (ASIM) payload for sprite research (Neubert et al., 2019). Lightning flash occurrences, their locations, and intensities are also assessed by remote sensing their electromagnetic radiation with global lightning detection networks (e.g., Dowden et al., 2002; Rodger et al., 2006; Rudolfsky, 2015; Said et al., 2013; and references therein). Global lightning activity is also measured with discrete excitations of Earth ionosphere cavity, or Schumann, resonances (e.g., Burke & Jones, 1995; Füllekrug & Constable, 2000; Huang et al., 1999; Sato & Fukunishi, 2003; and references therein) or using the continuum background radiation of Earth ionosphere cavity resonances (e.g., Chrissan & Fraser-Smith, 1996; Fraser-Smith & Bowen, 1992; Fraser-Smith et al., 1991; Sátori et al., 2013; and references therein). Therefore, the Atmospheric Observation Panel for Climate Observations (AOPC) accepted Earth ionosphere cavity resonances as an emerging essential climate variable for lightning activity (World Meteorological Organization, 2019, Section 6.5). Global climate change potentially results in an increase of the number of extreme thunderstorm events (Zipser et al., 2006) associated with an increase of lightning discharge occurrences (e.g., Romps et al., 2014; Williams, 2005, 1992, and references therein), albeit a decrease of tropical lightning discharge occurrences has also been predicted (Finney et al., 2018). The WMO also suggested that lightning could potentially contribute to climate change through atmospheric chemical constituents which form as byproducts of lightning flashes such as lightning nitrogen oxides, or lightning NOx (Aich et al., 2018; World Meteorological Organization, 2019). For this purpose, a large variety of different lightning parametrization schemes have been proposed which use the thermodynamic properties of the atmosphere and thunderclouds to explain lightning flash occurrence frequencies (e.g., Clark et al., 2017; Gordillo-Vázquez et al., 2019; and references therein). The general trend is that the majority of lightning flashes occur in the tropics as a result of deep convection driven by the pole to equator temperature increase and the Clausius Clapeyron equation (e.g., Price, 2000; Williams, 1994, Section 4d) that describes the phase transition between water vapor and liquid water content in the boundary layer. The Clausius Clapeyron equation also describes the coexistence curves of phase transitions near the triple point of water in the mixed phase region of thunderclouds which are of critical importance for the electrical charging and subsequent dielectric breakdown of air in the form of lightning flashes to occur (Emersic & Saunders, 2020; Saunders et al., 1991). However, research in this arena tackles in essence three atmospheric variables, (1) the atmospheric temperature which leads to charge separation in thunderclouds, (2) the resulting lightning flash occurrence frequency, and (3) the electromagnetic waves produced by the lightning discharges. The situation can therefore be compared to the dawn of quantum physics when it was of interest to combine heat, atomic energy transitions, and electromagnetic radiation into a single theory, nowadays known as blackbody, or thermal, radiation (e.g., Longair, 2006; Loudon, 2010; Rybicki and Lightman, 2004, and references therein). In Earth system science, the thermal state of the atmosphere falls into the area of physical meteorology, lightning resides in the field of atmospheric electricity, and electromagnetic radiation falls into the area of radio science. It is therefore interesting to ask the question whether these three different fields of work could potentially be combined by adapting the theoretical framework originally developed for quantum theory. Of course, it is not at all necessary to use quantum theory. For example, the electromagnetic radiation from lightning discharges around the globe can very well be explained using Maxwell’s equations alone with the normal mode expansion (e.g., Sentman, 1990, 1996, and references therein) or the residue series formula (Burke & Llanwyn Jones, 1996, and references therein) for a quasi-static lightning current moment. However, the lack of an immediate necessity does not mean that it is not possible to describe Earth ionosphere cavity resonances as a quantum system. It is therefore interesting to explore the potential benefits arising from a treatment of Earth ionosphere cavity resonances by adapting the fundamental framework of quantum theory to electromagnetic radiation trapped in the spherical shell around the Earth which is bounded by the conductive surface of the Earth and the lower ionosphere. The main aim of this contribution is to develop a new theoretical approach that combines three key properties of the atmosphere: (1) the thermodynamic temperature that leads to the charge separation in thunderclouds, (2) the global lightning flash occurrence frequency, and (3) the radiant energy of Earth ionosphere cavity resonances. The theory is allied with a comparison to real-world measurements of Earth ionosphere cavity resonances to corroborate the theory. Section 2 is dedicated to the development of the novel theory. Section 3 compares the theory with real-world measurements of Earth ionosphere cavity resonances and an exemplary interpretation of the measurements is given in Section 4. Section 5 discusses the results in the context of the well-known blackbody, or thermal, radiation in optics. Note that the novel theory developed in Section 2 depends on several
parameters such that a unique inversion requires the determination of boundary conditions from measured data to explore the analytic relationships between atmospheric temperature, lightning flash occurrences and electromagnetic radiation in future studies.

2. Global Lightning Quantum System

Global lightning activity is characterized by the global lightning flash rate density which is measured by the number of lightning flashes occurring per km² per year around the world, i.e., fl - km⁻² - yr⁻¹ (Christian et al., 2003). This global lightning flash rate density is used here to define a global lightning flash occurrence frequency $f_g$ by integrating the global lightning flash rate density over the surface area of the entire Earth, multiplying with the number of seconds in a year and dividing by one flash. The resulting average global lightning flash occurrence frequency has then the unit s⁻¹, or Hz, and it is found to be $f_g \approx 46$ s⁻¹ (Cecil et al., 2014) with an uncertainty ± 5 s⁻¹ (Christian et al., 2003).

2.1. Global Lightning Quanta

The global lightning flash occurrence frequency $f_g = 46$ s⁻¹ and the global lightning flash rate density inferred from the Optical Transient Detector and Lightning Imaging Sensor on board of satellites (Cecil, 2015) can be understood as a probability density function for lightning flashes to occur at a given location around the Earth within one second. It is therefore possible to parametrize global lightning flash occurrences with a superposition of 46 Dirac delta functions within each second which are found at various locations around the globe in line with the probability density function. The delta functions thereby describe the stochastic generation of a collective electromagnetic energy radiated by the 46 lightning flashes in every second. The collective minimum energy of the global lightning flash occurrence frequency is defined here as $E = h f_g$, named global lightning quanta, where $h$ is the electromagnetic action related to the radiant energy, named the Planck constant. This definition reflects the fact that the contributions of individual lightning flashes to the energy observed at a receiver are practically impossible to distinguish. The general concept of global lightning quanta is explained in more detail below in section 2.2 with an accompanying artistic illustration. The definition of global lightning quanta $E = h f_g$ arising from a purely stochastic process is fundamentally different to the definition of light quanta, known as photons $E = h\nu$, which draw their energy from the frequency $\nu$ of an electromagnetic wave emitted by an atomic or molecular energy transition. A more detailed comparison of global lightning quanta and photons is discussed in Section 5.

2.2. The General Concept of Global Lightning Quanta

The electromagnetic energy produced by the global lightning flash occurrence frequency is a stochastic process that is initially caused by electrons that are accelerated by the electrostatic field change of the lightning flash. The accelerated electrons produce electromagnetic waves which propagate away from the lightning flash. Subsequently, the waves are reflected between the conductive Earth and the lower ionosphere, known as the Earth ionosphere cavity, such that the electromagnetic energy flux, or Poynting vector, propagates along the great circle path around the Earth to the receiver (Füllek rug & Constable, 2000, Figure 2). For an exemplary single modal frequency of degree $n = 1$, the electromagnetic energy has a distinct distribution around the Earth that can be described by spherical harmonic functions (Sentman, 1996, Equations 31 and 34), similar to atomic orbitals in quantum physics (Figure 1a). However, the observed spatial energy variations in real-world measurements are much less pronounced as evidenced by the striking similarity of Earth ionosphere cavity spectra measured at numerous locations around the Earth. This surprising effect is caused by the superposition of many lightning flash occurrences around the entire globe with many modal frequencies contained in their average spectra (see Sentman, 1996, p. 9484, the paragraph before Equation 38). It is therefore conceivable that a large number of independent measurements at disparate locations around the Earth can be used to calculate one single spectrum which is representative for the entire globe. At extremely low frequencies from ~4 to 100 Hz the electromagnetic waves propagate several times around the Earth for a few hundreds of milliseconds such that it is practically impossible to distinguish the contributions from individual lightning flashes with an average interarrival time of $\delta t = 1/f_g \approx 21.7$ ms. The electromagnetic waves from different lightning flashes exhibit slightly varying amplitudes, frequencies and
phases such that the wave field around the Earth produced by all lightning flashes constitutes an incoherent superposition of electromagnetic waves, yet with a distinct observable collective electromagnetic energy (Figure 1b). The electromagnetic energy distribution around the Earth can be described with spherical harmonic functions for an exemplary single modal frequency (4-leaf clover like structure around the Earth). Shortly afterward, two almost simultaneously occurring lightning flashes in India and Malaysia launch electromagnetic waves which will arrive at later times at the receiver in the Antarctic (dashed and dotted white lines). A global lightning flash occurrence frequency of \( f_g = 46 \pm 5 \, \text{s}^{-1} \) means that on average every ∼ 21.7 ms new electromagnetic waves from different lightning flashes arrive at the receiver. The waves from individual lightning flashes exhibit slightly varying amplitudes, frequencies and phases (colored lines) such that their superposition results in a seemingly erratic temporal evolution of wave amplitudes observed with a receiver (black solid line). As a result, it is practically impossible to distinguish the energies from individual lightning flashes from observations such that their collective energy is described in this contribution by global lightning quanta.

**2.3. Properties of Global Lightning Quanta**

The likelihood for a specific global lightning flash occurrence frequency to occur can be described in first order approximation by a Poisson distribution (Chrissan and Fraser-Smith, 2000). The probability mass function of the Poisson distribution is given by

\[
P_k(\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad [\%]
\]

where \( \lambda = 46 \) is the average number of lightning flashes in one second and \( k \) is the actually observed number of lightning flashes in a specific second. For example, the likelihood of the global lightning flash occurrence frequency to be exactly 46 s\(^{-1}\) is only ∼ 6%, whereas the likelihood of the flash occurrence frequency to vary between 41 and 51 s\(^{-1}\) is ∼ 58% and ∼ 88% for a variation of the flash occurrence frequency between 36 and 56 s\(^{-1}\). The likelihood for the global lightning flash occurrence frequency to reach its absolute minimum for \( f_g = 1 \, \text{s}^{-1} \) or \( f_g = 0 \, \text{s}^{-1} \) is practically nil. The superposition of numerous global lightning quanta \( E \) can be understood as a radiant energy. In this picture, global lightning quanta correspond to a collection of delta
functions at the lightning flash locations in space and time and their collective impulse response describes the electromagnetic wave field around the Earth. It is possible to describe global lightning quanta by adapting the methodology developed for quantum systems, because a system composed of numerous quantum systems is still a quantum system. In this contribution, we make the Ansatz that this radiant energy from global lightning quanta is ultimately caused by the thermodynamic energy $E_T$ of the atmosphere which is thought to be in thermodynamic equilibrium within the Earth ionosphere cavity. This quantum system is then characterized by the balance between the radiant energy of the global lightning flash occurrence frequency and the thermodynamic energy such that $\frac{E}{E_T} = \frac{h}{h_T}$. This parametrized balance means that if the ratio is constant, an increase of the temperature results in an increase of the global lightning flash occurrence frequency and a decrease of the temperature results in a decrease of the global lightning flash occurrence frequency. The radiant surface energy density $u(f_g)$ of numerous global lightning quanta in thermodynamic equilibrium describes the amount of energy that is observable at a ground-based detector with an approximate footprint of say $\sim 1 \text{ m}^2$. This radiant energy density is proportional to (1) the average radiant energy of global lightning quanta $E_g$ which is discussed in Section 2.4, (2) the number of possible quantum states in the system $Z$ which is discussed in Section 2.5, and (3) the inverse of the Earth’s surface area $\Omega$ which is discussed in Section 2.6. The radiant energy density detectable on the surface of the Earth is then the product of these three factors $u(f_g) = E_gE_T/\Omega$.

### 2.4. Average Energy of Global Lightning Quanta

A quantum system in thermodynamic equilibrium is characterized by its dimensionless partition function

$$P = \lim_{n \to \infty} \frac{\prod_{n=0}^L e^{-nE/\Omega}}{\prod_{l=0}^M e^{-lE/\Omega}} \approx \sum_{l=0}^\infty e^{-lE/\Omega}, \quad [1]$$

(Loudon, 2010, p. 10), where $L_0$ is the very large maximum number of global lightning quanta which places an upper bound on the total energy within the Earth ionosphere cavity. The quantum energies $E_l = lE = \hbar f_g$ in the system are countable multiples of the basic global lightning quantum $E = \hbar f_g$ with count number $l$ which varies up to its maximum $L_0$. This maximum number is considered to be very large because the energy conveyed by electromagnetic waves requires a very large number of global lightning quanta. The average energy $E_0$ of the global lightning quanta $E$ is given by the first moment of the partition function

$$E_0 = \frac{\sum_{l=0}^\infty lE e^{-lE/\Omega}}{\sum_{l=0}^\infty e^{-lE/\Omega}} = \frac{E \sum_{l=0}^\infty l e^{-lE/\Omega}}{\sum_{l=0}^\infty e^{-lE/\Omega}} \approx \frac{E \sum_{l=0}^\infty l^2}{\sum_{l=0}^\infty l^2} = \frac{E \sum_{l=0}^\infty l^2/z}{\sum_{l=0}^\infty l^2/z}, \quad [2]$$

using the substitution $z = e^{-\hbar f_g}$. The average energy can then be calculated explicitly by an interpretation of the sums as geometric series such that

$$E_0 = \frac{E}{z - 1} \frac{1}{(1 - z)^2} = E e^{-E/\Omega} \frac{1}{z^{-1} - 1} = E e^{-E/\Omega} - 1. \quad [3]$$

### 2.5. Number of Quantum States in the System

The radiant energy caused by global lightning quanta spreads around the Earth with an electromagnetic wave field that remains mainly confined within the Earth ionosphere cavity. It is common practice to expand the magnetic field, or the vector potential, of a single lightning flash in a series of spherical harmonic functions (Sentman, 1987, 1989) and to superpose the corresponding spectra of many lightning flashes occurring around the Earth (Nickolaenko et al., 2006). In quantum physics, the electromagnetic fields are rather superposed in the time domain prior to a calculation of the radiant energy (e.g., Longair, 2006, pp. 369–370). In this case, the radiant energy depends on the geographic coordinates, and it can be expanded in a series of spherical harmonic functions $Y_n^m = P_n^m e^{im\phi}$ with the associated Legendre polynomials $P_n^m$ of degree $n$ and order $m$. 

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\[ E(\theta, \phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} c_n^m Y_n^m(\theta, \phi) \approx \sum_{n=0}^{n_m} \sum_{m=-n}^{n} c_n^m Y_n^m(\theta, \phi), \quad [J] \] (5)

where \( c_n^m \) are the expansion coefficients

\[ c_n^m = \frac{1}{\Omega} \int_{\Omega} E(\theta, \phi) Y_n^m(\theta, \phi) d\Omega = \frac{2\pi}{0} \int_{0}^{2\pi} \int_{0}^{\pi} \sin \theta E(\theta, \phi) Y_n^m(\theta, \phi) d\theta d\phi, \quad [J] \] (6)

\( \Omega \) is the surface area over a unit sphere representing the Earth, \( \theta \) is the co-latitude, \( \phi \) is the longitude, and \( a \) is the equivolumetric radius of the Earth. The expansion of the radiant energy in spherical harmonics can be truncated when a maximum degree \( n_m \) is reached which does not change the approximation of the radiant energy significantly. In this case, the order \( n \) and the degree \( m \) are both limited, such that the number of quantum states \( Z \) can be calculated from the maximum order \( n_m \) of the spherical harmonics used

\[ Z = \sum_{n=0}^{n_m} n_m \sum_{m=-n}^{n} m = \sum_{n=0}^{n_m} 2n + 1 \approx \sum_{n=0}^{n_m} 2n = 2 \sum_{n=0}^{n_m} n = 2 \frac{n_m(n_m + 1)}{2} = n_m(n_m + 1) \quad [1] \] (7)

for large \( n_m \). The relationship between the number of quantum states \( Z \) and the radiant energy of a single global lightning quantum \( E \) can be determined from the maximum momentum \( p \) and the wave propagation velocity \( v \) of a single electromagnetic wave in the Earth ionosphere cavity. A single electromagnetic wave is produced, for example, by a single global lightning quantum when all the delta functions attributed to lightning flashes of that global lightning quantum are thought to coincide in space and time. In this case,

\[ E = pv = hvk = h \frac{\sqrt{n_m(n_m + 1)}}{2\pi a} \sqrt{h_1 \over h_2} \quad [J] \] (8)

(e.g., Füllekrug et al., 2002, Equations 2 and 3), where \( h \) is the reduced Planck constant, \( k \) is the wave number of a global lightning quantum in the state \( n_m \) with an approximate wavelength \( \lambda \approx 2\pi a / \sqrt{n_m(n_m + 1)} \), \( a \) is the equivolumetric radius of the Earth, \( v = c/n \), is the wave propagation velocity of the electromagnetic wave, where \( c \) is the speed of light, and \( n \) is the refractive index of the atmosphere within the Earth ionosphere cavity such that \( v = c \sqrt{h_1 / h_2} \), where \( h_1 \approx 50 \text{ km} \) is the conduction boundary and \( h_2 \approx 100 \text{ km} \) is the reflection boundary for electromagnetic waves in the Earth ionosphere cavity. Rearranging Equation 8 to solve for \( \sqrt{n_m(n_m + 1)} \) finally enables a calculation of the number of possible quantum states

\[ \sqrt{n_m(n_m + 1)} = E \frac{2\pi a}{hc} \sqrt{h_1 \over h_2} \rightarrow Z = n_m(n_m + 1) = E \frac{2\pi^2 a^2}{h^2 c^2} \quad [1] \] (9)

as a function of the radiant energy of a global lightning quantum.

2.6. Radiant Energy Density at the Earth’s Surface

The surface area of the Earth \( \Omega \) can be calculated from

\[ \Omega = \int_{0}^{2\pi} \int_{0}^{\pi} a^2 \sin \theta d\theta d\phi = 4\pi a^2 \quad [m^2] \] (10)

where \( a \) is the equivolumetric radius of the Earth. The radiant energy density at the surface of the Earth is then composed of the three elements given in Equations 4, 9 and 10

\[ u(f_g) = \frac{E}{E_{\text{eq}}} \frac{1}{\Omega} = \frac{E}{E_{\text{eq}}} \left( \frac{E}{E_{\text{eq}}} \right) \frac{1}{\Omega} = 2\pi a \frac{E^3}{\hbar^2 c^2} e^{E_{\text{eq}}/E} - 1. \quad [\text{Jm}^{-2}] \] (11)
The resulting radiant energy density is thought to be a distribution function resulting from global lightning quanta such that it is sensible to apply a variable substitution to replace \( f_g \) by \( E \). The resulting distribution function \( F \) can be calculated using the Jacobian for the variable substitution

\[
F(E)dE = u(f_g)df_g \quad [\text{Jm}^{-2}\text{s}^{-1}]
\]

\[
\rightarrow \quad F(E) = u(f_g) \frac{df_g}{dE} = u(f_g) \frac{d}{dE} \frac{E}{h} = u(f_g) \frac{1}{h}, \quad [\text{m}^{-2}\text{s}^{-1}]
\]

such that the final result is

\[
F(E) = \frac{2\pi}{h^2c^2} \frac{E^3}{e^{E/E_T} - 1} \quad [\text{Jm}^{-2}\text{s}^{-1}]
\]

where \( F(E) \) has the units of a flux, i.e., \( \text{m}^{-2}\text{s}^{-1} \). This flux is not an observable quantity. As a result, Equation 14 needs to be modified for a comparison with the probability density function of energies that are actually observed. This aim is achieved in three steps. (1) The most likely energy to occur is determined in Section 2.7, (2) the flux is reformulated relative to this most likely energy in Section 2.8 and (3) the resulting distribution function is normalized in Section 2.9. These three steps then enable a variety of opportunities to conduct temperature measurements which are discussed in Section 2.10 and the relation to electromagnetic field measurements is established in Section 2.11 using the quantization volume discussed in Section 2.12.

### 2.7. Maximum of the Radiant Energy Density at the Earth’s Surface

To identify the maximum of the flux \( F(E) \), the limiting values for large and small energies are calculated. For large energies \( E \rightarrow \infty \)

\[
\lim_{E \rightarrow \infty} F(E) = \frac{2\pi}{h^2c^2} \lim_{E \rightarrow \infty} \frac{E^3}{e^{E/E_T} - 1} = 0 \quad [\text{m}^{-2}\text{s}^{-1}]
\]

because \( e^{E/E_T} \) grows faster than \( E^3 \). For small energies \( E \rightarrow 0 \) the second ratio in Equation 14 is not defined such that de l’Hôpital’s rule needs to be applied which results in

\[
\lim_{E \rightarrow 0} F(E) = \frac{2\pi}{h^2c^2} \lim_{E \rightarrow 0} \frac{3E^2}{e^{E/E_T} - 1} = 0 \quad [\text{m}^{-2}\text{s}^{-1}]
\]

because \( e^{E/E_T} \) decreases less fast than \( E^2 \). As a result, the flux \( F(E) \) has at least one local maximum because it has only positive values. The radiant energy of global lightning quanta which defines this maximum is important because it constitutes an observable value that can be calculated from real-world measurements. The maximum can be found by rewriting Equation 14 as a function of the new variable \( x = E/E_T \)

\[
F(E) = \frac{2\pi}{h^2c^2} \frac{E^3}{e^{E/E_T} - 1} \quad [\text{m}^{-2}\text{s}^{-1}]
\]

The corresponding variable substitution \( x = E/E_T \) results in

\[
F(x)dx = F(E)dE \quad [\text{Jm}^{-2}\text{s}^{-1}]
\]

\[
\rightarrow \quad F(x) = F(E) \frac{dE}{dx} = F(E) \frac{d}{dx} xE_T = F(E)E_T, \quad [\text{Jm}^{-2}\text{s}^{-1}]
\]

such that
\[ F(x) = \frac{2\pi}{h^2 c^2} E_I^3 \frac{x^3}{e^x - 1} E_T = \frac{2\pi}{h^2 c^2} E_T^4 \frac{x^3}{e^x - 1}. \quad [\text{J}m^{-2}s^{-1}] \quad (20) \]

The maximum can now be found by setting the first derivative of the energy flux \( F(x) \) to zero

\[ \frac{d}{dx} F(x) = 0 \quad [\text{J}m^{-2}s^{-1}] \quad (21) \]

\[ \rightarrow \frac{d}{dx} \frac{x^3}{e^x - 1} = \frac{3x^2}{e^x - 1} - \frac{x^3 e^x}{(e^x - 1)^2} = 0. \quad [1] \quad (22) \]

Rearranging Equation 22 and using the maximum value \( x_m \) results in

\[ 3(e^{x_m} - 1) = x_m e^{x_m} \quad \rightarrow \quad -3 = (x_m - 3)e^{x_m} \quad [1] \quad (23) \]

and using the variable substitution

\[ y = x_m - 3 \quad \rightarrow \quad -3 = ye^y \quad \rightarrow \quad -3e^{-y} = ye^y. \quad [1] \quad (24) \]

The solution of this equation for \( y \) is the Lambert-W function such that

\[ y = W(-3e^{-3}) \quad \rightarrow \quad x_m = y + 3 = W(-3e^{-3}) + 3 \approx 2.82. \quad [1] \quad (25) \]

It is somewhat surprising that this maximum has a fixed numerical value which places a strong boundary condition on the relationship between the maximum radiant energy of global lightning quanta \( E_m \) and the thermodynamic energy of the atmosphere \( E_T \) within the Earth ionosphere cavity

\[ x = \frac{E}{E_T} \quad \rightarrow \quad x_m = \frac{E_m}{E_T} \quad [1] \quad (26) \]

\[ \rightarrow \quad E_T = E_m / x_m. \quad [1] \quad (27) \]

### 2.8. Radiant Energy Density Distribution Function

The expression for the thermodynamic energy in Equation 26 can be used to reformulate Equation 17 in terms of the maximum radiant energy \( E_m \) such that

\[ F(E) = \frac{2\pi}{h^2 c^2} \left( \frac{E_m}{x_m} \right)^3 \frac{(x_m E / E_m)^3}{e^{x_m E/E_m} - 1}. \quad [\text{m}^{-2}\text{s}^{-1}] \quad (28) \]

The main advantage of this result is that the maximum energy \( E_m \) is an observable value that can be calculated from real-world measurements. On the contrary, the flux \( F(E) \) is not an observable quantity. Yet, Equation 28 can be used to calculate the total energy flux \( S_0 \) by integrating \( F(E) \) over all energies

\[ S_0 = \int_0^\infty F(E)dE = 2\pi \left( \frac{E_m}{x_m} \right)^3 \frac{1}{e^{x_m E/E_m} - 1} \int_0^\infty x_m E/E_m dE. \quad [\text{Wm}^{-2}] \quad (29) \]

Subsequently, the total energy flux can be used to normalize the flux such that the resulting distribution function \( p(E) \) becomes an observable normalized energy distribution function

\[ p(E) = \frac{F(E)}{S_0} \quad [\text{J}^{-1}] \quad (30) \]

\[ \rightarrow \int_0^\infty p(E)dE = 1. \quad [1] \quad (31) \]
2.9. Normalized Radiant Energy Distribution Function

The integral for the total flux in Equation 29 can be calculated using the variable substitution \( x = x_m E / E_m \) to calculate \( p(x) \)

\[
p(x)dx = F(E)dE \quad [\text{Jm}^{-2}\text{s}^{-1}]
\]  

\[
\rightarrow p(x) = \frac{dE}{dx} = \frac{dE}{dx} \frac{x_m}{E_m} x = \frac{F(E) E_m}{x_m} \quad [\text{Jm}^{-2}\text{s}^{-1}]
\]  

such that

\[
p(x) = \frac{2\pi}{h^2 c^2} \left( \frac{E_m}{x_m} \right)^3 x^3 \quad [\text{Jm}^{-2}\text{s}^{-1}]
\]  

and

\[
S_0 = \int_0^\infty p(x)dx = \frac{2\pi}{h^2 c^2} \left( \frac{E_m}{x_m} \right)^4 \int_0^{\infty} x^3 e^{-x} \, dx \quad [\text{Jm}^{-2}\text{s}^{-1}]
\]

\[
\rightarrow S_0 = \frac{2\pi}{h^2 c^2} \left( \frac{E_m}{x_m} \right)^4 \pi^4 \frac{\pi^3}{15} = \frac{2\pi^5}{15h^2 c^2} \left( \frac{E_m}{x_m} \right)^4 \quad [\text{Wm}^{-2}]
\]

This total flux can now be used to normalize the flux in Equation 28 such that

\[
p(E) = \frac{F(E)}{S_0} = \frac{2\pi}{h^2 c^2} \left( \frac{E_m}{x_m} \right)^3 \left( \frac{x_m E / E_m}{e^{x_m E / E_m} - 1} \right)^3 \frac{15h^3 c^2}{2\pi^5} \left( \frac{x_m}{E_m} \right)^4 \quad [\text{J}^{-1}]
\]

which finally results in

\[
p(E) = \frac{15}{\pi^4} \left( \frac{x_m E / E_m}{e^{x_m E / E_m} - 1} \right)^3 \left( \frac{E_m}{x_m} \right)^4 \frac{E_m^3}{e^{x_m E / E_m} - 1} \quad [\text{J}^{-1}]
\]

The radiant energy distribution function \( p(E) \) reaches its maximum \( p_m \) at the energy \( E_m \). This maximum can be calculated from Equation 38

\[
p_m = p(E_m) = \frac{15}{\pi^4} \left( \frac{x_m}{E_m} \right)^4 \frac{E_m^3}{e^{x_m E / E_m} - 1} = \frac{15}{\pi^4} \frac{x_m^3}{E_m^4} \quad [\text{J}^{-1}]
\]

The main advantage of these mathematical descriptions is that the key properties of the radiant energy distribution function \( E_m \) in Equation 26, \( p(E) \) in Equation 38 and \( p_m \) in Equation 39 are observable quantities. This is important because actual measurements enable a comparison with the theoretical description. One key application is for example that the peak value of the energy distribution function can be used to calculate the total energy flux in Equation 35 in a more convenient way without any integration. To achieve this aim, Equation 39 is reformulated

\[
\frac{E_m}{x_m} = \frac{15}{\pi^4} \frac{x_m^3}{e^{x_m E / E_m} - 1} \quad [\text{J}]
\]

where \( c_p \approx 0.22 \) is a dimensionless constant. This equation can then be used to calculate the integrated energy flux in Equation 36 in simplified form

\[
S_0 = \frac{2\pi^5}{15h^2 c^2} \left( \frac{E_m}{x_m} \right)^4 = \frac{2\pi^5 c_p^4}{15h^2 c^2 p_m^4} \quad [\text{Wm}^{-2}]
\]
where the maximum of the radiant energy distribution \( p_m \) is the only variable quantity. This interesting result means that the integrated energy flux \( S_0 \) is entirely determined by the peak likelihood \( p_m \) of the radiant energy \( E_m \) to occur.

### 2.10. Temperature Measurements

The main results of the preceding sections can now be reformulated using the thermodynamic energy of the Earth ionosphere cavity \( E_T = E_m/x_m \) used in Equation 26. In this case, the radiant energy distribution function in Equation 38 becomes

\[
p(E) = \frac{15}{\pi^2} \frac{1}{E_T^4} \frac{E^3}{e^{E/E_T} - 1}, \quad [J^{-1}] \tag{42}
\]

the maximum of the distribution function function in Equations 39 and 40 is

\[
p_m = \frac{15}{\pi^4} \frac{x_m^3}{e^{x_m} - 1} E_T = c_p \frac{1}{E_T} \quad [J^{-1}] \tag{43}
\]

\[
\rightarrow E_T = c_p \frac{1}{p_m}, \quad [J] \tag{44}
\]

and the integrated flux in Equation 41 becomes

\[
S_0 = \frac{2\pi^5}{15h^2 c^2} E_T^4. \quad [Wm^{-2}] \tag{45}
\]

The left-hand sides of Equations 42 and 43 are both observable quantities whereas the only parameter on the right hand side is the thermodynamic energy of the Earth ionosphere cavity. As a result, it is possible to infer the thermodynamic energy from the observations. For example, an increase of the thermodynamic energy \( E_T \) in Equation 43 results in a decrease of the peak value \( p_m \) and a decrease of the thermodynamic energy results in an increase of the peak value. This process can be described by

\[
E_T = \frac{c_p}{p_m} \rightarrow E_T + \delta E_T = \frac{c_p}{p_m - \delta p_m} \quad [J] \tag{46}
\]

such that

\[
E_T \left( 1 + \frac{\delta E_T}{E_T} \right) = \frac{c_p}{p_m} \frac{1}{1 - \delta p_m/p_m} \approx \frac{c_p}{p_m} \left( 1 + 1 + \frac{\delta p_m}{p_m} \right) \quad [J] \tag{47}
\]

for small changes \( \delta p_m \) and it follows that

\[
1 + \frac{\delta E_T}{E_T} = 1 + \frac{\delta p_m}{p_m} \quad [1] \tag{48}
\]

\[
\rightarrow \delta E_T = \frac{\delta p_m}{p_m} E_T \quad [J] \tag{49}
\]

which enables a calculation of the relative changes of the thermodynamic energy from the observed maxima of the radiant energy distribution function. The thermodynamic energy and its variability depends in one way or the other on the temperature \( T \) in the Earth’s atmosphere with an analytic function \( E_T(T) \) which remains to be determined. For example, it would possible to assume that the thermodynamic energy is in first order approximation an average multiple \( N_0 \) of the basic thermodynamic energy for one degree
of freedom $k_B T$, where $k_B$ is the Boltzmann constant, similar to the average radiant energy trapped in the Earth ionosphere cavity, which is assumed to be an average integer multiple $L_0$ of the basic global lightning quantum $hf_q$. In this case

$$E_f = N_0 j k_B T = \frac{E_m}{x_m} = \frac{L_0 hf_q}{x_m} \text{ [J]} \quad (50)$$

and

$$\rightarrow T = \frac{h}{x_m k_B N_0} f_q = \frac{L_0}{N_0} f_q^* \text{ [K]} \quad (51)$$

where $q_T = h/(x_m k_B) \approx 1.70 \cdot 10^{-11}$ Ks is the quantum of thermodynamic action related to the radiant energy of global lightning quanta and $L_0/N_0 \approx 3.67 \cdot 10^{11}$ for a global lightning flash occurrence frequency $f_q \approx 46 \text{ s}^{-1}$ with an average equilibrium temperature $T \approx 287 \text{ K}$ at the Earth’s surface. This means that an increase of the temperature results in an increase of the global lightning flash occurrence frequency and a decrease of the temperature results in a decrease of the global lightning flash occurrence frequency when $q_T L_0/N_0$ is constant. However, it remains to be seen whether this first order approximation with a linear model for the thermodynamic energy $E_T = N_0 j k_B T$ can describe the atmosphere in the Earth ionosphere cavity with a reasonable degree of accuracy in future studies. It is likely that the thermodynamic energy leading to lightning flashes is a nonlinear function of the temperature described by the Clausius Clapeyron equation as a result of the phase transition between water vapor and liquid water content in the boundary layer as explained in the introduction (Section 1). If the thermodynamic energy has been measured, for example using Equation 44, $E_T$ can be expanded into a Taylor polynomial

$$E_T(T) = \sum_{k=0}^{K} \frac{E_T^{(k)}(T_0)}{k!}(T - T_0)^k + \mathcal{O}((T - T_0)^{k+1}) \text{ [J]} \quad (52)$$

to account for nonlinear relationships, where $K$ is the largest derivative of interest, $T_0$ is the thermodynamic equilibrium temperature for electromagnetic radiation to occur, and $E_T^{(k)}(T_0)/k!$ are scaled derivatives which constitute the expansion coefficients for the thermodynamic energy. In this case, the linear model described in Equation 50 would correspond to the first order approximation in Equation 52.

### 2.11. Electromagnetic Field Measurements

The radiant energy caused by global lightning quanta is distributed via an electromagnetic wave field around the Earth ionosphere cavity, formed by the Earth’s surface and the ionosphere, without major losses $<0.1\%$ (Simoes et al., 2011), such that more or less the same amount of radiant energy can be observed around the Earth. This energy can be described by quantum states with normalized probability density functions. When a quantum field operator is applied to the quantum state an observable electromagnetic field materializes as described in quantum field theory. It is therefore interesting to explore whether meaningful information can be gained from quantum field theory, even though an analytic description of the quantum states of global lightning quanta is currently not known. The quantum field operator for the magnetic field is given by

$$\hat{B}(r,t) = \sum_{k,l} k \times e_{k,l} \sqrt{\frac{h}{2e_0 f_q V_q}} \hat{a}_{k,l} \exp[i\chi_k(r,t)] \text{ [Vs m}^{-2}] \quad (53)$$

(Loudon, 2010, Equation 4.4.18, p. 142), where $\hat{B}$ is the magnetic field operator, $r$ is the spatial position vector, $t$ is time, $k,l$ are the modes of the radiation field in the quantization volume $V_q$, $k$ is the dimensionless wave number vector in the direction of the Poynting vector of the electromagnetic wave, $e_{k,l}$ is the polarization vector for left- and right-hand polarization, $e_0$ is the vacuum permittivity, $\hat{a}_{k,l}$ is the quantum destruction operator, $i$ is the imaginary unit, and $\chi_k$ is the phase angle function for the electromagnetic wave. Similarly, the quantum field operator for the electric field is

$$\hat{E}(r,t) = \sum_{k,l} k \times e_{k,l} \sqrt{\frac{hf_q}{2e_0 V_q}} \hat{a}_{k,l} \exp[i\chi_k(r,t)] \text{ [V m}^{-1}] \quad (54)$$
(Loudon, 2010, Equation 4.4.15, p. 141), such that the electromagnetic radiation Hamiltonian operator is independent of the global lightning flash occurrence frequency

\[
H = \frac{1}{2} \int_0^\infty \int_0^\infty \int_0^\infty e^{i b a^2} \left[ \varepsilon_0 \hat{E}(\mathbf{r},t) \cdot \hat{E}(\mathbf{r},t) + \frac{1}{\mu_0} \hat{B}(\mathbf{r},t) \cdot \hat{B}(\mathbf{r},t) \right] d\mathbf{r} d\mathbf{a} \phi \left[ \text{Jm}^{-3} \right] \quad (55)
\]

(Loudon, 2010, Equation 4.4.21, p. 142) when the electric and magnetic quantum field operators are linearly related to each other akin to the classical electromagnetic field (Loudon, 2010, Equation 4.1.28, p. 129). The quantum field operators in Equations 53–55 are given in the Heisenberg picture, where the operators carry a time dependence when they are applied to quantum states. These quantum states must therefore bear information on the time varying global lightning flash occurrence frequency. Similarly, the observed classical electromagnetic fields within the Earth ionosphere cavity depend on the specific characteristics of the source, the medium in which the fields propagate, the dimensions of the cavity, and the geometric distance between the source and the receiver. All of the above factors would need to be considered when a quantum field operator is applied to a quantum state. Yet, an analytical description of the quantum states of global lightning quanta is currently not known and should be the subject of future studies. Nevertheless, it is possible to calculate the total electromagnetic energy in the Earth ionosphere cavity as it will be shown in the results Section 3.2.

2.12. Quantization Volume

The calculation of the quantum field operators for the electric and magnetic fields in Equations 53 and 54 requires a quantitative assessment of the quantization volume where the measurement instrument influences the quantities to be measured. This quantization volume \( V_q \) is the small space around a receiver which influences the electromagnetic fields of real-world measurements and where the radiant energy of global lightning quanta is transduced into electrical signals in the electronic circuits of the receiver. Practitioners of the craft of measuring electromagnetic fields in the frequency range \( \sim 4–100 \) Hz typically use receivers with an approximate footprint of \( \sim 1 \) m\(^2\). Once the receiver is physically installed, the practitioner tends to swing a key ring or another small metal object at nearby distances \( \sim 5–7 \) m from the receiver to ascertain whether this quasi-static electromotive force produces a sufficient distortion of the electric and/or magnetic field lines to ensure that the receiver is operational. The quantization volume can also be characterized by the coherency of the observed electromagnetic fields. Nowadays receivers are synchronized by Global Positioning Satellite clocks with an accuracy \( \sim 20 \) ns which corresponds to a location uncertainty \( \sim 6 \) m (Füllekrug, 2010). In other words, a second receiver which is placed more than \( d \approx 6 \) m away from the first receiver would record noticeable phase differences which correspond to a small, yet meaningful incoherence. It is therefore thought that a receiver integrates the radiant energy density over a volume surrounding the instrument that can be approximated by a cube with an edge length \( d/2 \approx 3 \) m. Half of the location uncertainty is used here because it would be practically impossible to distinguish the measurements taken with two independently operating receivers within the corresponding quantization volume \( V_q \approx 3^3 \text{ m}^3 = 27 \text{ m}^3 \). This means that the measurements would be fully coherent within the quantization volume. This estimate for the quantization volume is thought to be a reasonable first order approximation for the foreseeable future, given that there are currently no physical mechanisms known which would enable a reduction of the geometric form factors of existing receivers or offer significant improvements on the timing uncertainty.

The quantization volume can now be used to compare the theory with actual measurements using relevant quantities of electromagnetic fields published in the scientific literature. For example, the magnetic field of real-world measurements is on the order of \( B \approx 1 \) pT such that the locally measured radiant energy density from numerous global lightning quanta is \( U_m = BH = B^2/\mu_0 \approx 7.96 \cdot 10^{-19} \text{ J/m}^3 \), where \( H \) is the magnetic field strength and \( \mu_0 \) is the magnetic field constant. This observed energy density needs to be adjusted by the scaling factor \( V_0/V_q = 1/27 \) resulting from the spatial integration by the receiver explained above, where \( V_q \) is the quantization volume and \( V_0 \approx 1 \) m\(^3\) is the reference volume above the 1 m\(^2\) footprint of the receiver such that the observed radiant energy is now \( u_m = U_m V_0^2 / V_q \approx 2.95 \cdot 10^{-20} \) J. These scaled measurements can now be equated to the theoretical radiant energy.
\[ u_m = E_m = L_0 h f_g \quad [\text{J}] \quad (56) \]

\[ \Rightarrow L_0 = \frac{u_m}{h f_g} \quad [1] \quad (57) \]

which determines the average number of global lightning quanta \( L_0 \) within the Earth ionosphere cavity when \( f_g \) is known at the observation time. In this case, the multiple \( N_0 \) of the basic thermodynamic energy for one degree of freedom \( k_B T \) can be calculated when Equation 51 is used

\[ T = q_T \frac{L_0}{N_0 f_g} \quad [\text{K}] \quad (58) \]

\[ \Rightarrow N_0 = q_T L_0 \frac{f_g}{T} \quad [1] \quad (59) \]

such that

\[ u_m = E_m = x_m E_T = x_m N_0 k_B T. \quad [\text{J}] \quad (60) \]

The theory developed in Section 2 is now compared to experimental observations in Section 3.

### 3. Comparison of Theory With Observations

The theory in Section 2 makes numerous assumptions, approximations, and simplifications all of which might result in a theoretical description that bears little, if any, resemblance with real-world measurements. Instead of discussing each of these potential shortcomings in turn, the approach taken here is to compare the theory with real-world measurements with the aim to demonstrate that there is a reasonable agreement between the measurements and theory. The measurements are described in Section 3.1, the experimental results are discussed in Section 3.2 and an exemplary interpretation of the results is given in Section 3.3.

#### 3.1. Observations

Magnetic field variations in the frequency range from 4 to 49 Hz are continuously recorded with a radiometer in the geographic north-south direction at Arrival Heights (78° S, 167° E), Antarctica, from 1985 to 2000 as part of the Stanford ELF/VLF Radio Noise Survey (Fraser-Smith & Bowen, 1992). The half hourly synoptic recordings at 4 and 34 min past the hour can be used to extract 52 s long time series which results in 48 time series per day and 1440 time series in 30 days. For example, the time series recorded from 07:04:00 to 07:04:52 UTC on July 7, 1995, is shown in Figure 2a. The discrete excitations of Earth ionosphere cavity resonances from particularly intense lightning discharges around the world are superimposed on a background continuum radiation produced by the more numerous smaller lightning discharges. The observed magnetic field variations of Earth ionosphere cavity resonances are scaled for a calculation of the radiant energy as described in Section 2.12. Subsequently, the radiant energy distribution is calculated from an application of the jacknife method to the continuum radiation of Earth ionosphere cavity resonances, similar to an application of the bootstrap method used in previous work (Füllekrug et al., 2018, Section 4). The jacknife is a common resampling method in statistics that uses randomly chosen subsets from the data to calculate robust estimates of statistical moments of distribution functions (e.g., Tukey, 1958, and references therein). In this contribution, the jacknife is used to calculate the measured radiant energy distribution for comparison with a least squares fit of the theoretical radiant energy distribution because the fitting is computationally faster and less expensive when compared to the bootstrap method. This pragmatic approach is intended to enable the practical calculation of numerous radiant energy distributions for the entire recordings at Arrival Heights without the need of high performance computing in future studies.
3.2. Results

The experimentally observed and theoretically calculated radiant energy distribution functions are in excellent agreement (Figure 2b). The average difference between observations and theory is as small as $\sim 30\%$ when the deviation between the observations and theory is characterized by the merit function

$$
\chi = \frac{1}{N} \left( \sum_{i=1}^{N} \left| \frac{p_{obs,i} - p_{th,i}}{p_{th,i}} \right| \right) \times 100, \ [\%]
$$

where $N$ is the number of sample points $i$ of the observed and theoretical probability density functions $p_{obs,i}$ and $p_{th,i}$, respectively. This agreement between observations and theory is further corroborated by randomly selecting one time series for each year from 1988 to 2000 and by applying the jackknife to all the data from these 13 years (Figure 3a). In this case, the agreement is better than for a single time series because 13 times more data has been used for the calculation of the radiant energy distribution. A similar reduction of the uncertainty would be expected when more than one of the six electromagnetic field components are used. Another method of corroborating the agreement between observations and theory is using a comparison of the individual normalized radiant energy distributions from the 13 years of observation (Figure 3b). The normalization scales the energy $E$ by the most likely energy $E_m$, i.e., $E/E_m$, and by multiplying the likelihood $p(E)$ with the energy resolution $dE$ of the radiant energy distributions, i.e., $p(E)dE$. This normalization effectively removes the natural variability of $E_m$, $p_m$, and $p(E)$ and makes the observed radiant energy distributions comparable. In this case, the agreement is also $\sim 30\%$, i.e., the same as for the single time series in Figure 2b. It is therefore safe to state that the theoretical description is appropriate to explain the observations. The best fitting theoretical radiant energy distribution in Figure 2b is characterized by its peak value $p_m$ (star) at the energy $E_m$ (dot).

$$
E_{total} = \frac{2 \pi \tau a + b}{a} \int_{0}^{2\pi} \frac{E_m r^2 \sin \theta dr d\theta}{V_0} = \frac{4}{3} \pi [(a + h)^3 - a^3] \frac{E_m}{V_0}, \ [J]
$$

Figure 2. Magnetic field measurements and their radiant energy distribution. (a) Magnetic field measurements of Earth ionosphere cavity resonances in the frequency range from 4 to 49 Hz at Arrival Heights, Antarctica, from 07:04:00 to 07:04:52 UTC on July 7, 1995, as part of the Stanford ELF/VLF Radio Noise Survey. Discrete excitations of the Earth ionosphere cavity resonances are superimposed on their background continuum radiation. (b) The probability density function $p(E)$ of the radiant energy $E$ is calculated from the magnetic field measurements shown on the left. The theoretical radiant energy distribution function (solid line) is fitted to the measurements (circles) for comparison and agrees with the measurements on average within $\sim 30\%$. The radiant energy distribution is characterized by its peak value $p_m$ (star) at the energy $E_m$ (dot).
where \( a \approx 6,371 \text{ km} \) is the equivolumetric radius of the Earth, \( h_2 \approx 100 \text{ km} \) is the ionospheric reflection height for electromagnetic waves trapped in the Earth ionosphere cavity, and the observed energy density \( E_m/V_0 \) is considered to be in first order approximation a constant observable around the entire Earth. In this case, the bracket in Equation 62 can be expanded into a Taylor polynomial

\[
(a + h_2)^3 - a^3 = a^3(1 + 3h_2/a + O((h_2/a)^2)) - a^3 \approx 3h_2a^2 \quad [\text{m}^3]
\]

with an accuracy \( \sim 1.5\% \) such that

\[
E_{\text{total}} = \frac{4}{3} \pi 3h_2a^3 \frac{E_m}{V_0} = 4\pi a^2h_2 \frac{E_m}{V_0} \approx 0.5 \text{ J}
\]

which roughly corresponds to the potential energy of a \( \sim 100 \text{ g} \) heavy tennis ball elevated at \( \sim 0.5 \text{ m} \) height.

4. Exemplary Interpretation

The radiant energy \( E_m \) recorded with a receiver at a single location reflects the thermodynamic temperature within the Earth ionosphere cavity which causes the production of global lightning quanta. The calculation of this thermodynamic temperature requires either an analytic description of the thermodynamic energy \( E_T \) or corresponding observations, neither of which are sufficiently well known at present as explained in Section 2.10. However, it is interesting to illustrate an exemplary interpretation of the results, even though there is currently no experimentally confirmed analytic description of the thermodynamic energy within the Earth ionosphere cavity. A first order approximation is used here for illustration, where it is assumed that the thermodynamic energy is determined by one degree of freedom such that \( E_T = k_B T \). This Ansatz can be regarded as a standard reference that uses a first order linear approximation to a nonlinear function of the temperature as explained at the end of Section 2.10. In this case, the thermodynamic temperature can be estimated from Equation 44 such that

\[
T = \frac{c_p}{k_B} \frac{1}{P_m} \approx 242 \text{ K}.
\]
or \( T \approx -31^\circ C \), which roughly corresponds to the temperature in the mixed phase region of a thundercloud around \( \sim 4-8 \) km height where the charge separation occurs that is required for the initiation of lightning discharges. From a technical point of view, such an interpretation is known as an inversion, where a model parameter is inferred from the observations. This inversion is independent of the agreement between theory and observations described in the results (Section 3.2), because the merit function for the agreement compares the theoretical and measured probability densities with each other which are independent from the model parameters. An inversion requires knowledge of at least two of the three input parameters, i.e., the chosen model for the thermodynamic temperature which leads to charge separation in thunderclouds, the global lightning flash occurrence frequency, and the electromagnetic energy of Earth ionosphere cavity resonances as described by Equation 42 in Section 2.10. The exemplary interpretation here relates the observed temperature to the source region of lightning discharges which are not evenly distributed across the globe (Christian et al., 2003). Most of the lightning flashes occur in the tropical belt around the world and practically no lightning flashes occur in the polar regions. As a result, it will be interesting to map the global distribution and temporal evolution of inferred temperatures to investigate its link to global climate change in future studies. Note that, this thermodynamic temperature is inferred from electromagnetic observations, and it is therefore akin to a radiation, or brightness, temperature which is a common concept used in radio science at higher frequencies. The use of other models for the thermodynamic energy might lead to different radiation temperatures. For example, the assumption of a thermodynamic energy that depends on three degrees of freedom, i.e., one for each spatial dimension such that \( E_T = 3k_B T \), would result in a radiation temperature that is one third of the radiation temperature for one degree of freedom. As the dependence of the thermodynamic energy on the radiation temperature remains uncertain at the present time, this contribution uses the first order approximation \( E_T = k_B T \) which might be regarded as a standard reference for future studies. The challenge of finding a suitable thermodynamic energy is therefore twofold, as the thermodynamic energy needs to incorporate suitable theoretical descriptions for both, the thermodynamic temperature and the radiation temperature. As a result, more extended comparisons between radiation temperatures inferred from Earth ionosphere cavity resonances and temperatures measured by other means are clearly warranted.

5. Discussion

The main aim of this contribution was to develop a novel theory with the ability to bring the thermodynamic temperature of the atmosphere, the global lightning flash occurrence frequency and the electromagnetic energy of Earth ionosphere cavity resonances together in a single theory allied by a comparison with real-world measurements. The potential implications of the promising initial results obtained to date are now discussed in more detail in the following Sections 5.1–5.6.

5.1. Overview

The theory developed in Section 2 considers the thermodynamic temperature of the Earth’s atmosphere which causes the electrostatic charging of thunderclouds. When dielectric breakdown of air occurs inside thunderclouds around the Earth, a global lightning flash occurrence frequency is created with a minimum energy of a global lightning quantum \( E = hf_g \). In this picture, the superposition of numerous global lightning quanta correspond to a collection of delta functions in space and time and their collective impulse response describes an electromagnetic wave field around the Earth with a corresponding radiant energy. The radiant energy is distributed within the Earth ionosphere cavity without major losses to its boundaries, i.e., the lower ionosphere and the conductive surface of the Earth, such that the same radiant energy of the global lightning quanta can be observed by receivers located anywhere within the Earth ionosphere cavity. An individual receiver integrates the locally available radiant energy of global lightning quanta within the quantization volume and the sensor transduces the radiant energy into detectable currents within the electronic circuits of the receiver. The recorded signals are subsequently digitized and summarized in observable radiant energy distribution functions. The observed radiant energy distribution fits the theoretically predicted radiant energy distribution with an uncertainty \( \sim 30\% \) and thereby agrees with the developed quantum theory of the Earth ionosphere cavity.
5.2. Temperature

An exemplary interpretation of the observed parameters of the radiant energy distributions assumes in first order approximation that the thermodynamic energy in the Earth ionosphere cavity can be described by one thermodynamic degree of freedom. This assumption results in a corresponding thermodynamic temperature of \( \sim -30°C \). However, this first order approximation is currently not corroborated by measured temperature data or compared to potential analytic models for the thermodynamic temperature and thereby represents the largest uncertainty in the interpretation of the experimental results.

However, it is possible to extract first order approximation estimates for the variabilities of the temperature, global lightning flash occurrence frequency and Earth ionosphere cavity resonances from the published scientific literature to test whether the developed theory can combine these three parameters in a meaningful way. For climate change studies, it is common practice to quantify the relative change of the global lightning flash occurrence frequency for one degree temperature change, i.e., \( f' \cdot df / dT \), which is on the order of a few % per K (e.g., Price, 2000; Price & Rind, 1994; Reeve & Toumi, 1999; Williams, 2012; Williams, 1994, and references therein). This quantity can be calculated for one degree of thermodynamic freedom \( N_0 = 1 \) by rearranging Equation 51

\[
T' = \frac{T}{q_T L_0} \quad [s^{-1}] \tag{66}
\]

such that

\[
\frac{1}{f'_e} dT = \left| \frac{q_T L_0}{T} \cdot \frac{1}{q_T L_0} - \frac{T}{q_T L_0} \cdot \frac{1}{L_0} \right| \approx \frac{1}{T} \frac{\Delta L_0}{\Delta T} \approx \frac{1}{\Delta T} \frac{\Delta L_0}{L_0} \quad [% \cdot K^{-1}] \tag{67}
\]

where the inverse average thermodynamic equilibrium temperature \( 1/T \approx 0.3 % \cdot K^{-1} \), for \( T \approx 287 \) K is neglected because it is much smaller than the leading term \( \Delta L_0/(\Delta TL_0) \) such that only the relative temperature change \( \Delta T \) and the relative change of global lightning quanta \( \Delta L_0/L_0 \) needs to be estimated. For example, for the annual temperature variation in the tropics, where \( \sim 65% \) of all global lightning flashes occur (Williams, 1992), \( \Delta L_0/L_0 \) can be approximated by the relative annual variability of spectral densities from calibrated Earth ionosphere cavity resonance measurements that are corrected for the source receiver proximity effect (Füllekrug & Fraser-Smith, 1997, Figure 3, right). In this case, \( T \approx 297.90 \) K (24.75°C), \( \Delta T \approx \pm 1.25 \) K, and \( \Delta L_0/L_0 \approx \pm 30 \) fT/Hz\(^{-1/2} \). A similar estimate can be obtained for North America, where \( T \approx 285.65 \) K (12.50°C), \( \Delta T \approx \pm 12.5 \) K, and \( \Delta L_0/L_0 \approx \pm 30 \) fT/Hz\(^{-1/2} \). Future studies might expand on these results by developing a more sophisticated analytic model of the thermodynamic energy that reflects measured temperature data.

5.3. Long Wave Photons

One obvious question to ask is why more conventional resonant long wave photons (Goldhaber & Nieto, 2010, Section 4, and references therein) cannot easily be used to explain Earth ionosphere cavity resonances. There are two main reasons for this. First, the brightness temperature of blackbody radiation for an electromagnetic wave at an example frequency of \( \nu_m = 10 \) Hz is

\[
T = \frac{h \nu_m}{k_B \nu_m} \approx 1.7 \cdot 10^{-10} \text{ K} \tag{68}
\]

(Rybicki and Lightman, 2004, Equation 1.56a, p. 24) which seems unrelated to the physical properties of the Earth ionosphere cavity. Second, the maximum spectral irradiance \( F_m = F(\nu_m) \) of black body radiation at \( \nu_m = 10 \) Hz is
(Rybicki and Lightman, 2004, Equation 1.51, p. 22). On the contrary, the observed spectral irradiance \( s_m = s(v_m) \) of Earth ionosphere cavity resonances can be estimated from

\[
s(v_m) = c u_0 \left( \frac{v_0}{v_m} \right)^2 \approx 3 \cdot 10^{-10} \text{ Wm}^{-2}\text{Hz}^{-1},
\]

where \( u_0 \approx 10^{-16} \text{ Jm}^{-3} \text{ Hz}^{-1} \) and \( v_0 = 1 \text{ Hz} \) is a scaling constant (Füllekrug & Fraser-Smith, 2011, Section 3). The discrepancy between the two estimates is more than 38 orders of magnitude. It is therefore safe to conclude that resonant long wave photons cannot explain observations of Earth ionosphere cavity resonances as conventional black body radiation. This conclusion does not preclude the possibility to describe the electromagnetic waves emitted by lightning flashes with nonresonant long wave photons \( E = h \nu \). However, to the best of our knowledge, no such theory does exist at present. Yet, the production of a single electromagnetic wave can be thought to result from spatially and temporally coinciding delta functions attributed to lightning flashes of a single global lightning quantum, as described shortly before Equation 8. In this case, the energies of a nonresonant long wave photon and a global lightning quantum are equivalent for a suitable frequency and a suitable number of lightning flashes.

### 5.4. Radiant Energy Flux

The thermodynamic temperature within a cavity causes electromagnetic waves at optical wavelengths, widely known as black body, or thermal, radiation (e.g., Longair, 2006, case study V). It is therefore not particularly surprising that the shape of the radiant energy distribution in the Earth ionosphere cavity studied in this contribution is similar to the black body radiation in optics, simply because it is based on the adapted fundamental framework of quantum physics. The corresponding total radiant energy can be calculated from Equation 41 using the observed peak value \( p_m \approx 6.59 \cdot 10^{19} \text{ J}^{-1} \) as described in the results Section 3.2. In this case, the total radiant flux is surprisingly large, i.e., \( S_0 \approx 190 \text{ Wm}^{-2} \). This total radiant energy flux of global lightning quanta is much larger than the energy, or Poynting flux, \( S_P = |\mathbf{E} \times \mathbf{H}| \) of the instantaneous electric and magnetic fields \( \mathbf{E} \) and \( \mathbf{H} \) of Earth ionosphere cavity resonances. Taking typical values for \( H = B/\mu_0 \approx 1 \text{ pT}/\mu_0 \approx 8 \cdot 10^{-7} \text{ A/m} \) and \( E = cB \approx 3 \cdot 10^{-4} \text{ V/m} \) results in \( S_P \approx 2.4 \cdot 10^{-10} \text{ W/m}^2 \). The reason for this difference is attributed to the fact that the total radiant energy flux of global lightning quanta \( S_0 \) is calculated from the integral over all possible quantum states in Equation 29 and \( S_P \) is not integrated over all frequencies. It is also surprising that the total radiant energy flux of global lightning quanta calculated from Equation 41 is similar to the outgoing long wave radiation of the Earth (R. A. Hanel and Conrath, 1970, 1969; Kiehl & Trenberth, 1996). For an average equilibrium temperature of \( T \approx 287 \text{ K} \) at the surface of the Earth, the peak frequency in the black body radiation spectrum can be calculated from rearranging Equation 68 such that

\[
v_m = x_m \frac{k_B T}{h} \approx 1.7 \cdot 10^{13} \text{ Hz}.
\]

The corresponding spectral irradiance calculated from Equation 69 is then \( F(v_m) \approx 4.6 \cdot 10^{-12} \text{ Wm}^{-2} \text{Hz}^{-1} \) which is more similar to the spectral irradiance of Earth ionosphere cavity resonances calculated in Equation 70 when compared to the spectral irradiance of resonant long wave photons quoted in Equation 69. Whether this similarity is a mere coincidence or whether there is a physical relationship between global lightning quanta and the photons of outgoing longwave radiation is currently speculation and remains to be determined. From an experimental point of view, it might be useful to match \( S_0 \) with \( S_P \) by scaling the original flux \( F(E) \) distribution with the corresponding Jacobian as described in Equations 12 and 13. Such scaling has no effect on the normalized distribution functions shown in Figures 2b, 3a and 3b.
5.5. Black Body Radiation in Optics

It is interesting to compare in more detail the radiant energy distribution observed here and the black body radiation in optics. For example, if the thermodynamic energy is assumed to be one degree of freedom, i.e., \( E_{\nu} = k_{B}T \), then Equation 45 becomes the Stefan-Boltzmann law

\[
S_{0} = \frac{2\pi^{5}}{15h^{3}c^{2}} E_{\nu}^{4} = \frac{2\pi^{5}k_{B}^{4}}{15h^{3}c^{2}} T^{4} = \sigma T^{4}, \quad [\text{Wm}^{-2}] \tag{72}
\]

where \( \sigma = 5.67 \cdot 10^{-8} \text{Wm}^{-2}\text{K}^{-4} \) is the Stefan-Boltzmann constant. A similar analogy can be drawn between the observed radiant energy in this contribution and the black body radiation of optical spectra (Rybicki and Lightman, 2004, chapter 1.5). In optics, it is currently not possible to measure the temporal variations of the electric and magnetic field vector directly (Loudon, 2010, p. 84). As a result, the optical frequency of light \( \nu_{o} \) can be defined using the radiant energy of light \( E_{o} \) and division by Planck’s constant such that \( \nu_{o} = E_{o}/h \) (Lewis, 1926). This frequency is typically inferred from optical spectra which reflect the possible transitions between quantum energy states of atomic or molecular constituents. On the contrary, the frequency of low frequency electromagnetic waves \( f_{m} \) is defined by integer multiples \( m \) of the base frequency \( \Delta f = 1/T_{1} \), such that \( f_{m} = m\Delta f = m/T_{1} \), where \( T_{1} \) is the duration of the recordings used to calculate the spectrum. As a result, the frequency of low frequency electromagnetic waves has no obvious direct relation to the energy of the electromagnetic waves. Yet, the maximum radiant energy of global lightning quanta \( E_{m} \) can be used to define an equivalent frequency \( \nu_{m} = E_{m}/h \) that takes into account the quantum system approach outlined in Section 2. In this case, \( \nu_{m} = 1.42 \cdot 10^{13} \text{Hz} \) which falls into the mid to far infrared range where the outgoing long wave radiation of the Earth’s atmosphere is situated in the spectrum. This analogy can be extended by recasting the flux \( F(E) \) in Equation 14 as a function of frequency \( F(\nu) \) using the variable substitution

\[
F(\nu)d\nu = F(E)dE \quad [\text{Jm}^{-2}\text{s}^{-1}] \tag{73}
\]

\[
\rightarrow \quad F(\nu) = F(E) \frac{dE}{d\nu} = F(E) \frac{dE}{h\nu} = F(E)h, \quad [\text{Wm}^{-2}\text{Hz}^{-1}] \tag{74}
\]

where \( d\nu = dE/h \) such that the resulting surface energy density at the footprint of a receiver depends on the equivalent frequency

\[
F(\nu) = F(E)h = \frac{2\pi^{5}}{h^{3}c^{2}} E_{\nu}^{3} \quad h = \frac{2\pi^{5}}{h^{3}c^{2}} \left( \frac{h\nu^{3}}{e^{h\nu/k_{B}T} - 1} \right) = \frac{2\pi^{5}h}{h^{3}c^{2}} \left( \frac{\nu^{3}}{e^{h\nu/k_{B}T} - 1} \right), \quad [\text{Wm}^{-2}\text{Hz}^{-1}] \tag{75}
\]
If $B$ represents the amplitude of a sinusoidal oscillation where the energy is usually defined as $E_{\text{in}} = B^2/2$, then $B^2$ in Equation 76 needs to be replaced by $2E_{\text{in}}$ which offers a direct link between the equivalent frequency $\nu$ and the energy of the sinusoid $E_{\text{in}}$. A similar argument applies when $B$ represents a power spectral density $P_{\text{in}} = B^2/\Delta f$. In this case, $B^2$ in Equation 76 needs to be replaced by $P_{\text{in}}\Delta f = P_{\text{in}}/T1$.

### 5.6. Looking ahead

The main aim of this contribution was to combine the temperature $T$, the lightning flash rate $f_L$ and the radiant energy $E_{\text{in}}$ into a single theory to bring together the fields of physical meteorology, atmospheric electricity and radio science. All of these three parameters can be measured, simulated or described with analytic models to study their mutual relationships. For example, the simulation of global lightning quanta starting from a randomized distribution of global lightning flash locations could be attempted. Such a simulation could lead to a better understanding of the two parameters $f_L$ and $E_{\text{in}}$ and subsequently enable an inversion of the remaining parameter $T$. Yet, the mutual relationships between the three parameters might also depend on location and time such as the longitudinal dependence of lightning activity during the day and the latitudinal dependence of lightning activity during the seasons. If such a spatial and temporal breakdown of the three parameters is of interest, it might be possible to describe such variabilities with the auxiliary parameters $L_0$ and $N_0$, i.e., the number of global lightning quanta and multiples of the basic thermodynamic energy for one degree of freedom. In this case, the auxiliary parameters would correspond to some sort of spatial and temporal calibration factors for the theory. It is therefore evident that the theory depends on several parameters such that a unique inversion requires the determination of suitable boundary conditions from measured data. It also seems possible to expand the theory for use in other frequency ranges with non-resonant electromagnetic waves to characterize thermal continuum radiation with corresponding radiation temperatures, commonly known as brightness temperature at higher frequencies. The theory could then be used to distinguish between thermal and nonthermal continuum radiation for practical applications such as the detection of interference. For example, nonthermal continuum radiation could be detected when the theory cannot be used to fit the observations with a reasonable degree of accuracy. However, the prime motivation to develop the theory was to summarize the mutual relationships between essential climate variables and emerging essential climate variables which may initiate and assist future studies of empirical models that use the three measured parameters that are quickly accruing in climate data archives of the World Meteorological Organization.

### 6. Summary and Conclusions

This contribution develops a novel theory to explain the radiant energy of global lightning activity within the Earth ionosphere cavity. The theory adapts the rigorous framework of quantum physics to combine the thermodynamic temperature, the global lightning flash occurrence frequency and the radiant energy of resonant electromagnetic waves within the Earth ionosphere cavity in a single theory. The theory makes the Ansatz that the minimum amount of electromagnetic energy $E$ produced by the variable global lightning flash occurrence frequency $f_L$ is the global lightning quantum $E = hf$, where $h$ is Planck’s constant. Numerous global lightning quanta distribute their radiant energy around the world via resonant electromagnetic waves within the Earth ionosphere cavity. The theory is in agreement with exemplary measurements of radiant energy distributions with a radiometer at Arrival Heights, Antarctica, as part of the Stanford ELF/VLF Radio Noise Survey. The measurements agree with the theory within an average uncertainty $\sim 30\%$. It is found that the radiant energy flux of global lightning quanta is $\sim 190 \text{ W m}^{-2}$ with a total energy of $\sim 0.5 \text{ J}$ stored within the Earth ionosphere cavity. The measured radiant energy distribution exhibits a remarkable similarity with the black body radiation of the Earth when an equivalent frequency $\nu = E_{\text{in}}/h$ is introduced, where $E_{\text{in}}$ is the most likely radiant energy to occur. One exemplary interpretation of the results is that it enables a calculation of the thermodynamic temperature $\sim -30^\circ \text{C}$ for one thermo-
dynamic degree of freedom. This temperature is similar to the temperature in the mixed phase region of thunderclouds where the charge separation occurs that is required for the dielectric breakdown of air initiating lightning discharges. This novel theory can thereby help to assess the mutual impact of climate change on the Earth’s atmosphere and global lightning on each other as recently proposed by the World Meteorological Organization.

Data Availability Statement
The data used for this publication will be available from http://dx.doi.org/10.15125/BATH-00918.

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