Detection and analysis of coherent structures in the near-field of a turbulent annular swirling jet: an unsteady Reynolds-averaged Navier–Stokes (URANS) simulation

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Abstract. This paper analyses the capabilities of unsteady Reynolds-averaged Navier–Stokes (URANS) simulations to predict the coherent structures found in a swirling jet undergoing vortex breakdown. Recently, tomographic particle image velocimetry experiments of an annular swirling jet at a Reynolds number of 8500 at moderate swirl showed the presence of a double helical structure in the flow field (Vanierschot et al., Physical Review Fluids, 2018). This structure corresponds to the double helix vortex breakdown mode and is rarely observed in turbulent flows. In this study, the same flow topology is simulated using Computational Fluid Dynamics (CFD). Turbulence is modeled using the unsteady RANS methodology with a RNG $k-\varepsilon$ turbulence model. The coherent structures in the flow field were analysed using the spectral proper orthogonal decomposition technique. Despite the fact that it is known that steady-state RANS simulations using two-equation isotropic turbulence models have problems in accurately describing swirling flows which are highly anisotropic, this study shows that the unsteady variant was able to predict the large scale flow structures and their associated dynamics reasonable well. In particular, similar to the experiments, a double helical structure was found in the flow field. The structure has windings in the counter-swirl direction and it is wrapped around the central breakdown bubble. To the authors knowledge, this study is the first one to show the ability of unsteady RANS to predict not only the presence of the double helix vortex breakdown in the flow field, but also the spatial and temporal structure of it.

1. Introduction
Turbulent swirling flows are ubiquitous in nature, such as in tornadoes and typhoons and in engineering technical applications, for instance, the vortices shedding from the leading edge of aircraft and the rotational motion in combustion chambers. In combustion systems, swirl is added to the flow to enhance the fuel-air mixing rate and to reduce the emission of nitrogen oxides through the recirculation zone, which is caused by the centrifugal effects or adverse pressure gradients above a certain level of swirl intensity [1]. Other prevalent features of turbulent swirling jets are the so-called ‘precessing vortex core’ (PVC) and ‘vortex breakdown’ (VB), which is referred to as the abrupt change in the structure of a vortex core. Both coherent structures have been intensively studied during the past several decades and different theories.
have been introduced to explain their occurrence. However, there is currently not a unifying
to which is accepted by the community [2].

Although Reynolds-averaged Navier-Stokes (RANS) modeling is not as accurate as large eddy
simulation or direct numerical simulation, it is widely adopted to model turbulent flows as it
has low computational cost and can be used for highly complex flow geometries. Investigations
have been conducted to assess the capability of different RANS turbulent models to simulate
turbulent swirling flows, both in steady and unsteady cases [3–10]. Their results show that
the $k$–$\varepsilon$ family and Reynolds stress model are capable of capturing the highly unsteady flow
dynamics such as the PVC and its precessing frequency.

Recently, Vanierschot et al. [11] conducted experimental studies on a turbulent annular
swirling jet by tomographic particle image velocimetry, and not only single helix but also double
helix vortex breakdown was identified. In this work, an unsteady RANS simulation is used to
model the turbulent annular swirling jet flow firstly. Then, we apply a modal decomposition to
identify the coherent structures and their dynamics and compare the results with those from
literature.

2. Computational setup

2.1. Governing equations and turbulence model

The governing equations are the three-dimensional incompressible Reynolds-averaged Navier–
Stokes equations for averaged velocity $\bar{u}_i$ and averaged pressure $\bar{p}$:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0,$$

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial u'_i u'_j}{\partial x_j},$$

where $x_i$, $t$, $\rho$ and $\nu$ denote distance, time, density and molecular viscosity, respectively. The
Reynolds stress tensor $u'_i u'_j$ is modeled through the turbulent-viscosity hypothesis introduced
by Boussinesq:

$$u'_i u'_j = \frac{2}{3} k \delta_{ij} - \nu_t \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right),$$

which assumes that the deviatoric Reynolds stress is proportional to the mean rate of strain.
Here, $\nu_t$ is the turbulent or eddy viscosity. In the present work, the RNG $k$–$\varepsilon$ model is chosen
to simulate the flow. Compared to the standard $k$–$\varepsilon$ model, the effect of swirl on turbulence is
included in the RNG $k$–$\varepsilon$ model, which improves accuracy for swirling flows. The eddy viscosity
coefficient is computed from turbulent kinetic energy and dissipation rate $\nu_t = C_\mu k^2/\varepsilon$, where
$C_\mu = 0.09$ is a model constant.

2.2. Computational domain, boundary conditions and control parameters

The cross section of the computational domain is shown in figure 1. The working fluid flows
through an annular tube firstly and then mixes with the ambient medium in a large cylindrical
domain whose length and diameter are $15D_o$ and $16D_o$, respectively. Here, $D_o = 27mm$ is the
outer diameter of the annular tube, whereas the length $L$ and inner diameter $D_i$ are $54mm$ and
$18mm$, respectively.
The swirling flow is generated by setting an axial-tangential fluid entry, and the constant inlet axial and azimuthal velocities are given by $U_0$ and $U_\theta$, respectively. The no-slip velocity boundary condition is applied to the inner and outer wall of the annular pipe, and the end wall of the central bluff-body. The entrained fluid, a co-flow, is fed in by a mild velocity $U_c = 4\% U_0$. As the boundaries of the jet are sufficiently far from the nozzle outlet, symmetry boundary conditions are employed at the lateral wall of the domain and zero gradient conditions for fluid leaving the computational domain [5, 12]. For the pressure boundary conditions, ambient pressure is imposed at the far field boundary, whereas zero gradient boundary conditions are applied to all other boundaries. The turbulent kinetic energy and dissipation rate at the inlet are calculated from the turbulence intensity (15%) and integral length scale (half the hydraulic diameter, $(D_o-D_i)/2$) and zero gradient conditions across the faces of the computational domain are applied. The behavior close to solid walls is modeled with scalable wall functions.

The flow is characterized by two control parameters. The first one being the Reynolds number $Re = \frac{U_0 D_h}{\nu}$ based on the axial inlet velocity $U_0 = 0.94m/s$, and the hydraulic diameter $D_h = 9mm$ of the annular tube. The working fluid is water with kinematic viscosity $\nu = 1.0048 \times 10^{-6} m^2/s$. The other parameter is the swirl number $S = \frac{M_\theta}{M_x \overline{U}_x}$, which is the ratio between axial flux of tangential momentum $M_\theta = \int_{D_i}^{D_o} 2\pi \rho \overline{U}_x \overline{U}_\theta r^2 dr$ and axial flux of axial momentum $M_x = \int_{D_i}^{D_o} 2\pi \rho (\overline{U}_x)^2 dr$, where $\overline{U}_x$ and $\overline{U}_\theta$ are the mean axial and tangential velocity measured at the end of the annular pipe, respectively. The computed Reynolds number $Re$ is 8500, and the swirl number $S$ is calculated as 0.46.

### 2.3. Grids and discretization methods

The numerical grid is constructed using the software Pointwise V18.0. Figures 2(a) and 2(b) display the computational grid close to the nozzle exit. All grid cells are hexahedral and the grid is finer in the core region. The SIMPLEC (Semi-Implicit Method for Pressure Linked Equations-Consistent) algorithm is employed to solve the governing equations. The transient term is discretised using second order implicit time differencing. The least square method is used for the gradient terms and the QUICK (Quadratic Upstream Interpolation for Convective Kinematics) scheme is employed for spatial discretization. We use scalable wall functions to model the flow field close to solid walls. The computations are done by the commercial software package ANSYS Fluent 18.2.
Figure 2. Computational grid close to the nozzle exit: (a) cross section in the $x-y$ plane, (b) symmetry plane perpendicular to the axis of the jet.

Figure 3. Time-averaged velocity profiles at different axial locations normalized by $U_0$: (a) axial velocity and (b) tangential velocity. The streamlines are depicted by the grey lines in the background.

3. Results and discussion

3.1. Mean flow fields
Figures 3(a) and 3(b) provide the mean axial and azimuthal velocity profiles at different axial locations. It is seen that the axial velocity profiles are wake-like in the near-field of the annular jet, which is similar to the observation in experimental measurements [13]. The first reason is due to flow separation behind the central bluff-body, which causes a central toroidal recirculation zone. The other one comes from the bubble caused by vortex breakdown further downstream at $x/D_o > 0.4$. 
3.2. Spectral proper orthogonal decomposition analysis

One of the key issues in this research field is to identify the coherent structures from flow fields, which involves the construction of empirical modes that dominate the flow dynamics. The most popular methods, such as energy-ranked proper orthogonal decomposition (POD) and frequency-ranked fast Fourier decomposition or dynamic mode decomposition (DMD), are not applicable when coherent structures occur with low energies or fluctuating frequencies. However, the recently introduced spectral proper orthogonal decomposition (SPOD) approach allows to detect the coherent structures both in spatial and in frequency domains especially when the flow dynamics is rather complicated [14].

Figure 4 displays the extracted SPOD mode pairs and their energy content. The first four ranked pairs with the highest harmonic correlation are labeled with numbers. Each dot corresponds to one mode pair and the size of it represents the harmonic correlation between the modes. Here, the Strouhal number $St$ is defined as $St = fD_h/U_0$, where $f$ is the numerically obtained precessing frequency. The SPOD results show that the first two mode pairs account for 23.6% and 6.7% of the total turbulent kinetic energy, respectively. Hence, they represent the dominant coherent structures in the flow field. The evolution of their temporal coefficients is given in figures 5(a) and 5(b), which have the same content except for a phase difference of $\pi/2$.

The power spectral density (PSD) of the harmonic mode coefficient shown in figures 5(c) and 5(d) indicates that their spectra have a peak at Strouhal numbers 0.23 and 0.53, corresponding to frequencies of 24.4Hz and 54.8Hz, respectively. Both POD analysis of the pressure fields [15] and phase-averaged velocity fields [11] in turbulent annular swirling jet have shown similar results.

![Figure 4](image)

**Figure 4.** The contribution to the total turbulent kinetic energy (TKE) by the identified SPOD mode pairs.

3.3. Single and double helical structures

The reconstructed velocity fields are presented in figures 6(a) and 6(b). Here, the $Q$ criterion [16] is applied to identify vortical structures in the flow field. The results show that not only a single helical but also a double helical vortical structure is found, and they both correspond to the spiral type of vortex breakdown. These spiral-shaped structures have high vorticity wrapped around the vortex breakdown bubble, and they are not always present in the flow field according to the mode coefficients shown in figures 5(a) 5(b). It should be noted that the double helix structure is rarely observed in swirling jets, and it is in line with the one detected in a similar turbulent annular swirling jet by phase averaging PIV measurements [11].
Figure 5. The time evolution (a-b) and power spectral density (c-d) of the first two ranked SPOD mode pairs. The time $t$ is given in seconds.

Figure 6. Reconstructed velocity field with the fist (a) and second (b) ranked mode pair: slices with counters of axial velocity in $x - y$ plane, and $Q$ criterion vortices visualization with isosurfaces of positive $Q = 4 \times 10^4 \ 1/s^2$. 
4. Summary and conclusions
In this paper, a turbulent annular swirling jet at Reynolds number $Re = 8500$ and swirl number $S = 0.46$ is simulated using the unsteady Reynolds-averaged Navier-Stokes approach, where the RNG $k$–$\varepsilon$ turbulence model is used. The spectral proper orthogonal decomposition methodology is applied to investigate the coherent structures in the flow field. It is found that, apart from the single helical vortical structure usually reported in turbulent swirling flows, a double helical vortical structure is also identified. These single and double spiral type coherent structures with excessive vorticity are wrapped around the vortex breakdown bubble in the counter-swirl direction and rotate in the swirl direction with Strouhal numbers equal to 0.23 and 0.53, respectively. This result is in agreement with the recent experimental observations by Vanierschot et al. [11]. Although it is widely known that the $k$–$\varepsilon$ model has disadvantages such as over-prediction of eddy viscosity and is not capable to capture the anisotropic turbulence of swirling flows, the findings in this work indicate that the unsteady RNG $k$–$\varepsilon$ two-equation model can predict the periodic large scale coherent structures in turbulent annular swirling jet flow both temporally and spatially with good accuracy.

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