Entanglement and Berry Phase in Two Interacting Qubits

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Abstract

Entanglement and Berry phase are investigated in two interacting qubit systems. The XXZ spin interaction model with a slowly rotating magnetic field is employed for the interaction between the two qubits. We show how the anisotropy of interaction reveals unique relations between the Berry phases and the entanglements for the eigenstates of the system.

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**Introduction.** Entanglement of a quantum system have been intensively studied in a wide range of research areas. Various systems have been considered to investigate a key role of entanglement in quantum phenomenon such as quantum phase transitions [1, 2, 3], many-body effects [4, 5], quantum information processing [6, 7, 8], and quantum transport [9, 10, 11]. During the periodic time evolution of the system, i.e., the system Hamiltonian is varied slowly and eventually brought back to its initial form, the entanglement has been shown to affect on the geometric phase of the system. For entangled bipartite systems, especially, geometric phases have been studied [12, 13, 14]. It has been shown in bipartite systems that a prior entanglement influences on the geometric phase although there are no interactions during the cyclic evolution of the system.

Recently, advanced quantum technologies have made it possible to manipulate a quantum system in a controllable manner. The interactions between the subsystems of a composite system can be controlled by varying the system parameters [15]. For example, superconducting qubits have been realized in experiments and various types of interactions between them have been demonstrated such as Ising-[16, 17, 18], XY-[19, 20, 21], and XXZ-type [22] interactions for two qubit systems. In addition, a possible way to generate a Berry phase has been shown theoretically in flux qubits [23]. Thus, in such quantum systems, the interaction strengths between the subsystems of a composite system may be adjusted by controlling system parameters. Varying the interaction strengths may make the entanglement of the subsystems changing [24, 25]. Also, phase dynamics of the subsystems can be changed by varying interaction strengths between the subsystems [26].

Of particular interest are the Berry phases [27] of a composite system because its applications are to be the implementation of quantum information processing [28, 29, 30, 31]. This raises questions of how the interactions among the subsystems changes the Berry phase and entanglement of the composite system and what the relation is between the Berry phase of the composite system and the entanglement of the two subsystems. In this Brief Report, we consider a composite system consisting of two interacting qubits (spin-\(\frac{1}{2}\) s). We focus on the effects of interactions on relations between the Berry phases and entanglements for the eigenstates of the system. To do this, the XXZ-type of spin exchange interaction is employed to describe the interaction between two qubits. We investigate the behavior of the Berry phase and entanglement of two interacting qubits due to a rotating magnetic field. The spin exchange interaction effects on the Berry phase and entanglement are discussed. A relation between the Berry phase and entanglement for the eigenstates of the systems is found to be unique as the interaction strengths vary.

**Model.** We start with two interacting qubits corresponding to two spin-\(\frac{1}{2}\) s, \(S_1\) and \(S_2\), in an
external magnetic field. The system is described by the XXZ spin Hamiltonian,

\[ H = H_x + H_z + H_B, \]  

where \( H_x = J_x(S^x_1 S^x_2 + S^x_2 S^x_1), H_z = J_z S^z_1 S^z_2, \) and \( H_B = \mu (S^1 + S^2) \cdot \mathbf{B}(t). \) Here, \( \mu \) is the gyromagnetic ratio and \( J_x \) and \( J_z \) are the spin exchange interaction strengths between the spins. The slowly rotating magnetic field \( \mathbf{B}(t) = B \hat{n}(t) \) is chosen with the unit vector \( \hat{n}(t) = (\sin \theta \cos \phi(t), \sin \theta \sin \phi(t), \cos \theta) \). It is assumed that the frequency of rotating magnetic field is a constant \( \omega \) and then \( \phi(t) = \omega t \). For a period of time \( T \), the \( \phi \) is slowly changing from \( \phi(0) = 0 \) to \( \phi(T) = 2\pi \).

Eigenstates of the system. There are four eigenvalues \( E_n (n \in \{0, 1, 2, 3\}) \) of the Hamiltonian. One of them is unique, i.e., \( E_0 = -(J_z + 2J_x) \). This eigenstate is nothing but the spin singlet state, \( |\Psi_0\rangle = \frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle) \). Note that the singlet state is not a function of the interaction parameters. Since the Bell states are maximally entangled states for two spin (qubit) systems, this singlet state is maximally entangled for all interaction parameter regimes. The singlet state does not capture any extra phase, i.e., its Berry phase is zero, during the adiabatic and cyclic evolution of the system.

For other eigenvalues, there is a cubic function having a form,

\[ F(\varepsilon, J) = \varepsilon^3 - 2J \varepsilon^2 - 4B_0^2 \varepsilon + 8JB_0^2 \cos^2 \theta, \]

where \( B_0 = \mu B/2 \). When \( Q = 4(J^2 + 3B_0^2)/9 \) and \( R = 4J(2J^2 + 9B_0^2(1 - 3 \cos^2 \theta))/27 \) are defined, for \( R^2 \leq Q^3 \), the three real roots of \( F(\varepsilon, J) = 0 \) are given by

\[ \varepsilon_n = 2 \sqrt{Q} \cos \left( \frac{P + 2n\pi}{3} \right) + \frac{2}{3}J, \]

where \( P = \cos^{-1} \left( R/\sqrt{Q^3} \right) \). Then, the eigenvalues \( E_n = \varepsilon_n + J_z \) of the Hamiltonian in Eq. 1 satisfy

\[ F(\varepsilon_n = E_n - J_z, J = J_x - J_z) = 0. \]

For the eigenvalues \( E_n (n \in \{1, 2, 3\}) \), generally, the other instantaneous eigenstates are in a superposition of the three triplet states,

\[ |\Psi_n\rangle = a_n e^{-i\phi} |\uparrow \uparrow\rangle + \frac{b_n}{\sqrt{2}} \left( |\uparrow \downarrow\rangle + |\downarrow \uparrow\rangle \right) + c_n e^{i\phi} |\downarrow \downarrow\rangle, \]  

\[ (5a) \]
where the coefficients in terms of the energy $\varepsilon_n$ are given by

$$a_n = -\frac{2}{\sqrt{d_n}} B_0 \sin \theta (\varepsilon_n + 2B_0 \cos \theta),$$  \hspace{1cm} (5b)

$$b_n = -\frac{2}{\sqrt{d_n}} (\varepsilon_n + 2B_0 \cos \theta) (\varepsilon_n - 2B_0 \cos \theta),$$  \hspace{1cm} (5c)

$$c_n = -\frac{2}{\sqrt{d_n}} B_0 \sin \theta (\varepsilon_n - 2B_0 \cos \theta)$$  \hspace{1cm} (5d)

with $d_n = 2(\varepsilon_n^4 - 2(1 + 3 \cos 2\theta)B_0^2 \varepsilon_n^2 + 16B_0^4 \cos^2 \theta)$. It should be noticed that the coefficients are a function of the shifted energies as $\varepsilon_n = E_n - J_z$. $\varepsilon_n$ is a function of the exchange energy difference between $J_x$ and $J_z$, i.e., $\varepsilon_n(J)$, where $J = J_x - J_z$. Actually, the $\varepsilon_n$ determines the behaviors of the coefficients as the interactions vary. Then, the anisotropy of the exchange interactions plays a significant role for entanglement and Berry phase.

Note that the coefficients have a property of $a_n(\theta) = c_n(\pi - \theta)$ and $b_n(\theta) = b_n(\pi - \theta)$. In addition, when the energy $\varepsilon_n$ changes to $-\varepsilon_n$, the coefficients hold the relations of $a_n(\varepsilon_n) = -c_n(-\varepsilon_n)$ and $b_n(\varepsilon_n) = b_n(-\varepsilon_n)$. As a consequence, the coefficients satisfy $a_n(\varepsilon_n, \theta) = -a_n(-\varepsilon_n, \pi - \theta)$ and $b_n(\varepsilon_n, \theta) = b_n(-\varepsilon_n, \pi - \theta)$.

To help understanding of the model Hamiltonian in the interaction parameter space, a schematic diagram is drawn in Fig. 1. In the absence of magnetic field $B(t) = 0$, i.e., the $\alpha$-plane in Fig. 1, the system Hamiltonian reduces to the XXZ model without magnetic fields, $H_{XXZ} = J_x(S_1^x S_2^x + S_1^y S_2^y) + J_z S_1^z S_2^z$. Then, the eigenstates are to be a spin singlet state and three triplet states, i.e., $|\Psi_n\rangle \in \{ |\uparrow\uparrow\rangle, \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle), |\downarrow\downarrow\rangle \}$. The eigenstates of the XXZ model without magnetic fields do not depend on the interaction parameters $J_x$ and $J_z$. It is shown that, by varying the interactions between two qubits, the entanglements of the eigenstates for the XXZ spin exchange interaction without magnetic fields can not be manipulated.

In the $\beta$-plane ($J_x = J_z$) of Fig. 1, when magnetic fields $B(t) \neq 0$ is applied and the spin exchange interaction is isotropic, the model Hamiltonian reduces to the Heisenberg model

$$H_H = J_H (S_1^x S_2^x + S_1^y S_2^y + S_1^z S_2^z)$$

with an external magnetic field $B(t)$. In this case, the energy $\varepsilon_n(J)$ is independent of the spin exchange interactions because $J = J_x - J_z = 0$, i.e., $\varepsilon_n(0)$. Then the coefficients of the eigenstates are not a function of the spin exchange interactions. As a consequence, in terms of the bases $|\chi_+\rangle = \begin{pmatrix} e^{-i\phi} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} \end{pmatrix}$ and $|\chi_-\rangle = \begin{pmatrix} -\sin \frac{\theta}{2} \\ e^{i\phi} \cos \frac{\theta}{2} \end{pmatrix}$ for a single spin, the eigenstates are given in the states $|\Psi_n\rangle \in \{ |\chi_+ \chi_+\rangle, \frac{1}{\sqrt{2}} (|\chi_+ \chi_-\rangle \pm |\chi_- \chi_+\rangle), |\chi_- \chi_-\rangle \}$. They are nothing but a rotated one of the eigenstates of the XXZ Hamiltonian for $B = 0$. 
FIG. 1: (Color online) Interaction parameter space for XXZ spin model with an external magnetic field. $J_x$ and $J_z$ are the strengths of spin exchange interactions. $B_0$ is the interaction energy between the spins and the applied magnetic field. There are four characteristic parameter planes for the two interacting spins. Note that, (i) for $B_0 = 0$, i.e., the $\alpha$-plane, the XXZ Hamiltonian without the magnetic field has the eigenstates which do not depend on the spin exchange interactions. (ii) Also, for the isotropic spin interaction ($J_x = J_z$) with the magnetic field $B_0 \neq 0$, i.e., the $\beta$-plane, the eigenstates of the Heisenberg Hamiltonian do not depend on the strengths of the spin exchange interaction and the magnetic field. (iii) For $J_x = 0$, i.e., the $\lambda$-plane, the system is described by the Ising Model with the magnetic field. (iv) On the other hand, for $J_z = 0$, i.e., the $\omega$-plane, the XX model with the magnetic field describes the two interacting qubit systems. Controlling the interaction parameters will turn out a unique relation between entanglement and Berry phase for the eigenstates of the system based on the characteristic properties of the eigenstates in each interaction parameter plane.

**Berry phase and concurrence.**—After the system undergoes an adiabatic and cyclic evolution with an initial state $|\Psi_n(0)\rangle$, the eigenstates have an additional phase, which comes from a geometrical feature, i.e., the Berry phase obtained by

$$\gamma_n = \int_0^{2\pi} d\phi \langle \Psi_n | i \partial_\phi | \Psi_n \rangle = 2\pi \left( |a_n|^2 - |c_n|^2 \right).$$

(6)

It is shown that the Berry phase is determined by the coefficients $a_n$ and $c_n$ of the eigenstates. From the relations of the coefficients, in general, the Berry phases satisfy $\gamma_n(\theta) = -\gamma_n(\pi - \theta)$ and $\gamma_n(\epsilon_n) = -\gamma_n(-\epsilon_n)$. Also, the Berry phase holds a symmetry giving the relation $\gamma_n(\epsilon_n, \theta) = \gamma_n(-\epsilon_n, \pi - \theta)$.

The entanglement of a pure general bipartite state can be quantified by introducing the concurrence [32]. For the eigenstates, the concurrences are given by

$$C_n = \left| \langle \Psi_n | \sigma_y \otimes \sigma_y | \Psi_n \rangle \right| = \left| 2a_n c_n - b_n^2 \right|,$$

(7)

where $\sigma_y$ is the pauli matrix. The concurrences $C_n$ range from 0 (an unentangled product state) to 1 (a maximally entangled state). Note that the entanglement of the eigenstates is not changed during
very much anisotropic, i.e., (ii) the eigenenergies of the XX model can be approximated to the XX and Ising models, respectively. 

Table I shows the relation between the eigenstates for the XX model with an external magnetic field. The XX spin Hamiltonian with the magnetic field for the spins \( S_1 \) and \( S_2 \) is \( H = H_x + H_B \), where \( H_x = J_x(S_1^x S_2^x + S_1^y S_2^y) \) and \( H_B = \mu(S_1 + S_2) \cdot B(t) \). In the intermediate regime \( J_x \sim B_0 \), ‘\( \cdots \)’ indicates a superposition of the three triplet states.

The concurrences are a symmetry function with the axis \( \theta = \pi/2 \), i.e., \( C_n(\theta) = C_n(\pi - \theta) \). Changing the energy \( \epsilon_n \) to \( -\epsilon_n \) gives the relation \( C_n(\epsilon_n) = C_n(-\epsilon_n) \). In general, then, the entanglement of the eigenstates hold \( C_n(\epsilon_n, \theta) = C_n(-\epsilon_n, \pi - \theta) \).

Relation between Berry phase and entanglement. When the spin exchange interaction becomes very much anisotropic, i.e., (ii) \( J_z \ll J_x, B_0 \) for the \( \omega \)-plane and (iii) \( J_x \ll J_z, B_0 \) for the \( \lambda \)-plane in Fig. 1, the XXZ model can be approximated to the XX and Ising models, respectively.

Let us discuss the eigenstates for the XX model \( H = H_x + H_B \), i.e., \( J_z = 0 \) \( (J > 0) \). The eigenenergies \( E_n^\pm \) satisfy \( F(\epsilon_n^+ = E_n^+, J_x) = 0 \). For \( J_x \ll B_0 \), the eigenstates become \( |\Psi_1\rangle \approx |\Psi_{-\epsilon_-, \theta}\rangle \), \( |\Psi_2\rangle \approx \frac{1}{\sqrt{2}}(|\Psi_{+\epsilon_+, \theta}\rangle + |\Psi_{-\epsilon_+\theta}\rangle) \), and \( |\Psi_3\rangle \approx -|\Psi_{+\epsilon_+\theta}\rangle \). 

In the limit of \( J_x \gg B_0 \), the eigenstates are to be \( |\Psi_1\rangle \approx |\downarrow\downarrow\rangle \), \( |\Psi_2\rangle \approx -|\uparrow\uparrow\rangle \), and \( |\Psi_3\rangle \approx -\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \). Table II shows the relation between the Berry phase and concurrence for the limiting values. The coefficients of the eigen wavefunctions are a monotonic function of \( \epsilon_n \) and the interactions. In Fig. 2 (a), the overall behaviors of the relations are schematically drawn as the ratio of \( J_x \) to \( B_0 \) varies for the eigenstates once the polar angle \( \theta \) is fixed. As \( J_x \) varies from \( B_0 \gg J_x \), \(|\Psi_2\rangle \) \( (|\Psi_3\rangle) \) reaches gradually to a product state.
It was confirmed numerically. It is shown that the Berry phase is zero when an eigenstate becomes a maximally entangled state.

\[ \gamma \] between the Berry phases for the XX and Ising models, Heisenberg model (XXZ model) reduces to the Ising model, as mentioned, the eigenstates are not dependent of the coefficients of the eigenstates of the XX model when \( J_z \) is replaced with \(-J_z\) and \( e_{1(3)}^x \) and \( e_2^x \) are replaced with \(-e_{3(1)}^x\) and \(-e_2^x\), respectively. One finds the relations between the Berry phases for the XX and Ising models,

\[ \gamma_{1(3)}^x(e_{1(3)}^x) = -\gamma_{3(1)}^x(e_{3(1)}^x), \quad \text{and} \quad \gamma_2^x(e_2^x) = -\gamma_2^x(e_2^x). \]

Also the relations between the concurrences for the XX and Ising models are given by

\[ C_{1(3)}^x(e_{1(3)}^x) = C_{3(1)}^x(e_{3(1)}^x), \quad \text{and} \quad C_2^x(e_2^x) = C_2^x(e_2^x). \]

Such a symmetrical property of the Berry phases and concurrences is shown by directly comparing Fig. 2(a) for the XX model and (b) for the Ising model.

Now, let us discuss more general cases. When the spin exchange interaction is isotropic, i.e., Heisenberg model (\( J_x = J_z \)), or the spin exchange interaction is very much anisotropic, i.e., XX (\( J_x \gg J_z \)) and Ising (\( J_x \ll J_z \)) models, as mentioned, the eigenstates are not dependent of the

(\[ \gamma \] maximally entangled state) from a maximally entangled state (product state), while \(|\Psi_1\rangle\) changes from a product state to another product state. During the adiabatic and cyclic evolutions, \(|\Psi_2\rangle\) (\(|\Psi_3\rangle\)) captures a Berry phase up to zero (2\(\pi\)) from \(\cos \theta\) (zero) as \(J_x\) varies from \(B_0 \gg J_x\). While \(|\Psi_1\rangle\) takes a Berry phase from \(-\cos \theta\) to \(-2\pi\). \(|\Psi_1\rangle\) does not reach any maximally entangled state. It is shown that the Berry phase is zero when an eigenstate becomes a maximally entangled state. It was confirmed numerically.

For \(J_x = 0\) (\(J < 0\)), the XXZ model reduces to the Ising model, \(H = H_x + H_B\). The eigenenergies \(E_n^z\) satisfy \(F(e_n^z = E_n^z - J_z, -J_z) = 0\). For \(J_z \ll B_0\), the eigenstates become \(|\Psi_1\rangle \approx |\chi_-,\chi_\rangle\), \(|\Psi_2\rangle \approx \frac{1}{\sqrt{2}}(|\chi_+\chi_\rangle + |\chi_-\chi_\rangle)\), and \(|\Psi_3\rangle \approx -|\chi_+\chi_\rangle\). In the limit of \(J_z \gg B_0\), the eigenstates are to be \(|\Psi_1\rangle \approx -\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)\), \(|\Psi_2\rangle \approx |\downarrow\downarrow\rangle\), and \(|\Psi_3\rangle \approx -|\uparrow\uparrow\rangle\). Table II shows the relation between the Berry phase and concurrence for the limiting values. Since the cubic function in Eq. 2 has a property \(F(e_n, J) = -F(-e_n, -J)\), the coefficients of the eigenstates of the Ising model can be written in terms of the coefficients of the eigenstates of the XX model when \(J_x\) is replaced with \(-J_z\) and \(e_{1(3)}^x\) and \(e_2^x\) are replaced with \(-e_{3(1)}^x\) and \(-e_2^x\), respectively. One finds the relations between the Berry phases for the XX and Ising models,

\[ \gamma_{1(3)}^x(e_{1(3)}^x) = -\gamma_{3(1)}^x(e_{3(1)}^x), \quad \text{and} \quad \gamma_2^x(e_2^x) = -\gamma_2^x(e_2^x). \]

Also the relations between the concurrences for the XX and Ising models are given by

\[ C_{1(3)}^x(e_{1(3)}^x) = C_{3(1)}^x(e_{3(1)}^x), \quad \text{and} \quad C_2^x(e_2^x) = C_2^x(e_2^x). \]

Such a symmetrical property of the Berry phases and concurrences is shown by directly comparing Fig. 2(a) for the XX model and (b) for the Ising model.
TABLE II: Comparison for eigenstates $|\Psi_n\rangle$ ($n \in \{1, 2, 3\}$), their concurrences $C_n$ and Berry phases $\gamma_n$ in varying interaction parameters $J_x$ and $B_0$ of the Ising model with an external magnetic field. The Ising spin Hamiltonian with the magnetic field for the spins $S_1$ and $S_2$ is $H = H_z + H_B$, where $H_z = J_z S_1^z S_2^z$ and $H_B = \mu (S_1 + S_2) \cdot B(t)$. In the intermediate regime $J_z \sim B_0$, ‘···’ indicates a superposition of the three triplet states.

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interaction parameters $J_x$ and $J_z$. Then, as $J_z$ varies, for $B_0 \ll J_x$, the behavior of the relation between the Berry phase and concurrence is summarized in the three characteristic limits in Table III. In Fig. 3, the schematic diagrams are then drawn the relations of the Berry phases and concurrences of the eigenstates as the exchange interaction $J_z$ varies from zero. The arrows on the curves of the relations indicate the direction of increasing $J_z$ from zero. At $J_z = 0$, the values of the Berry phase and concurrence are on the relation for the case of XX model in Fig. 2(a). For instance, the A ($J_x < B_0$) and B ($J_x > B_0$) are shown for $|\Psi_1\rangle$ in Fig. 2(a). The values at these points A and B are the same in Fig. 3(b). The reason is why the energy $\varepsilon_n(J)$ determining the Berry phases and concurrences is a function of the exchange energy difference $J = J_x - J_z$. In other words, once the polar angle $\theta$ is fixed, the relation between Berry phase and concurrence is uniquely determined by only one curve whatever we choose the values of the exchange energies and the magnetic energy $B_0$. Then if $B_0 \gg J_x$, the system corresponds to the Ising model in Fig. 2(b). As a result, we find that (i) for $J_x > J_z$, the behavior of the relation between the Berry phases and concurrences is similar to the XX model, (ii) while the behavior is similar to the Ising model.
| Interactions        | $J_z \sim B_0 \ll J_x$ | $B_0 \ll J_z < J_x$ | $B_0 \ll J_z = J_x$ | $B_0 \ll J_x < J_z$ | $B_0 \ll J_x \ll J_z$ |
|---------------------|------------------------|---------------------|---------------------|---------------------|------------------------|
| Hamiltonian         | $H \approx H_x$        | $H \approx H_H$     | $H \approx H_H$     | $H \approx H_z$     | $H \approx H_z$        |
| $|\Psi_1\rangle$    | $|\downarrow\downarrow\rangle$ | $|\chi_-\chi_-\rangle$ | $-\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ |  |
| $C_1$               | 0                      | 0                   | 0                   | 1                   | 1                      |
| $\gamma_1[2\pi]$   | -1                     | $-\cos \theta$     | 0                   | 0                   | 0                      |
| $|\Psi_2\rangle$    | $-|\uparrow\uparrow\rangle$ | $\frac{1}{\sqrt{2}}(|\chi_+\chi_-\rangle + |\chi_-\chi_+\rangle)$ | $|\downarrow\downarrow\rangle$ |  |
| $C_2$               | 0                      | 1                   | 0                   | 0                   | 0                      |
| $\gamma_2[2\pi]$   | 1                      | 0                   | $-1$                | 0                   | 0                      |
| $|\Psi_3\rangle$    | $-\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ | $-|\chi_+\chi_+\rangle$ | $-|\uparrow\uparrow\rangle$ |  |
| $C_3$               | 0                      | 0                   | 0                   | 0                   | 0                      |
| $\gamma_3[2\pi]$   | 0                      | $\cos \theta$      | $-1$                | 0                   | 0                      |

TABLE III: Comparison for eigenstates $|\Psi_n\rangle$ ($n \in \{1, 2, 3\}$), their concurrences $C_n$ and Berry phases $\gamma_n$ in varying interaction parameters $J_z$ for $B_0 \ll J_x$. The XXZ Hamiltonian with the magnetic field for the spins $S_1$ and $S_2$ is $H = J_x(S_1^x S_2^x + S_1^y S_2^y) + J_z S_1^z S_2^z + \mu (S_1 + S_2) \cdot B(t)$. (i) For $J_z \ll J_x$, the two spin Hamiltonian can be approximated to the XX model $H \approx J_x(S_1^x S_2^x + S_1^y S_2^y)$. (ii) For $J_z = J_x$, the Heisenberg model $H \approx J(S_1^x S_2^x + S_1^y S_2^y + S_1^z S_2^z)$ can describe the system. (iii) For $J_z \gg J_x$, the two spin Hamiltonian becomes the Ising model $H \approx J_z S_1^z S_2^z$. In the intermediate regimes, ‘⋅⋅⋅’ indicates a superposition of the three triplet states.

for $J_x < J_z$. Therefore, the anisotropy of the exchange interaction determines the behavior of the relation between the Berry phase and entanglement in two interacting qubits (spins).

Summary. The interaction effects on the entanglement and Berry phase are investigated in two qubits (spins). It is found that the anisotropy of the interaction plays an important role in determining the unique relation between the Berry phase and concurrence for the eigenstates of two interacting qubits. Also, it is shown that when the eigenstates become a maximally entangled state their Berry phases are zero. During the period time evolution of the system, unentangled eigenstates, as product states, can capture a finite value of Berry phase.

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FIG. 3: (Color online) Relations between Berry phases $\gamma_n$ and concurrences $C_n$ of the eigenstates (a) $|\Psi_1\rangle$, (b) $|\Psi_2\rangle$, and (c) $|\Psi_3\rangle$ in varying $J_z$ from zero for the XXZ model. The arrows on the curves indicate the increasing direction of $J_z$ from zero. When $J_z = 0$, the starting point of the relation between the Berry phase and concurrence depends on the ratio $J_x$ to $B_0$. In Fig. 1 the $l$ ($J_x < B_0$) and $m$ ($J_x > B_0$) show the direction of increasing $J_z$ from zero. For instance, $A$ ($J_x < B_0$) and $B$ ($J_x > B_0$) shows that the starting points at $J_z = 0$ is determined by the ratio $J_x$ to $B_0$. As $J_z$ varies from zero to $J_z = J_x$, the relation curve is the same with the case of XX model in Fig. 2 (a). For $J_z > J_x$, the relation curve is the same with the case of Ising model in Fig. 2 (b).

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