Gauge symmetric $\Delta(1232)$ couplings and the radiative muon capture in hydrogen

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Using the difference between the gauge symmetric and standard $\pi N \Delta$ couplings, a contact $\pi \pi NN$ term, quadratic in the $\pi N \Delta$ coupling, is explicitly constructed. Besides, a contribution from the $\Delta$ excitation mechanism to the photon spectrum for the radiative muon capture in hydrogen is derived from the gauge symmetric $\pi N \Delta$ and $\gamma N \Delta$ couplings. It is shown for the photon spectrum, studied recently experimentally, that the new spectrum is for the photon momentums $k \geq 60$ MeV by 4-10 % smaller than the one obtained from standardly used couplings with the on–shell $\Delta$’s.

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I. INTRODUCTION

The photon spectrum in the radiative muon capture in hydrogen

$$\mu^- + p \rightarrow \nu\mu + \gamma + n,$$

(1.1)

has recently been calculated by several authors [1–5] in the search for a process enhancing the high energy part of the photon spectrum, calculated earlier in Ref. [6]. It was concluded in [3], where the study was made both within the small scale expansion scheme [7] and in the heavy baryon chiral perturbation theory [8], that a combination of various small effects could explain the experimental spectrum [9,10]. However, actual size of some of these effects such as of the charge symmetry breaking, are to be considered in a more detail. On the other hand, this spectrum was calculated in [5] using amplitudes derived from Lagrangians possessing the hidden local $SU(2)_L \times SU(2)_R$ symmetry [11,12]. In particular, the vertices containing the $\Delta(1232)$ isobar field were chosen as

$$\mathcal{L}_{\Delta \pi \rho a} = \frac{f_{\pi N \Delta}}{m_\pi} \bar{\Psi}_\mu \bar{T} \mathcal{O}_{\mu \nu}(C(Z)) \Psi \cdot (\partial_\nu \vec{a}) + 2f_\pi g_\rho \bar{a}_\nu - g_\rho \frac{G_1}{M} \bar{\Psi}_\mu \bar{T} \mathcal{O}_{\mu \eta}(C(Y)) \gamma_5 \gamma_\eta \Psi \cdot \bar{\rho}_\nu \ 	ext{h.c.}$$

(1.2)

Here $\bar{T}$ is the operator of the isospin $1/2 \rightarrow 3/2$ transition. The operator $\mathcal{O}_{\mu \nu}(C(B))$ is taken in a form [13–15]

$$\mathcal{O}_{\mu \nu}(C(B)) = \delta_{\mu \nu} + C(B) \gamma_\mu \gamma_\nu,$$

(1.3)

$$C(B) = -\left(\frac{1}{2} + B\right).$$

(1.4)
The parameters $Y$ and $Z$ do not influence the on–shell properties of the $\Delta$ isobar, hence they are called off-shell parameters. The vertices (1.2) has frequently been used [13–17] to study the $\pi N$ reactions and the pion photo- and electroproduction on nucleon and the parameters of the model, including $Y$ and $Z$, were extracted from the data.

On the other hand, one can find also an attempt [18] to show that the off–shell parameters are redundant within the framework of effective fields theories. For this purpose, Tang and Ellis consider the Lagrangian of the $\pi N\Delta$ system with the $\pi N\Delta$ interaction of the type (1.2). After integrating out the $\Delta$ isobar field, they obtain a nonlocal $\pi N$ Lagrangian where the $Z$ dependence is contained in couplings. This leads them to conclude that these couplings can be redefine so that the $Z$ dependence disappears and therefore, this parameter is physically irrelevant. However, after finding that it is difficult to manage the nonlocal part of the resulting Lagrangian, Tang and Ellis return to the starting $\pi N\Delta$ Lagrangian containing the $\Delta$ field explicitly and recommend to use it with some convenient choice of the parameter $Z$, since it is not relevant to the physics. On the other hand, they do not consider any mechanism to compensate the $Z$ dependence of the observables. Indeed, if such an independence on a parameter should take place, one should provide a mechanism to compensate it if it appears due to a particular process, that can generally happen.

It can be seen [5,15] that the $Z$ dependence of the amplitudes appears in the form of contact terms. As it has recently been discussed in Refs. [19,20], the contact nature of the $\Delta$ excitations appears if the interaction vertices contain the projection operators onto the spin 1/2 space, that leads to the contribution of this space. Indeed, the $\Delta$ propagator has been discussed in Refs. [19,20], the contact nature of the $\Delta$ excitations amplitudes appears if the interaction vertices starting with the $\pi N$ ones. The vertices (1.2) has frequently been used [13–17] to study the $\pi N$ reactions and the pion photo- and electroproduction on nucleon and the parameters of the model, including $Y$ and $Z$, were extracted from the data.

In its turn, the operator (1.3) reads

$$S^\mu\nu = \frac{1}{i \not\! \! \! p + M_\Delta} \left[ \delta_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu + \frac{2}{3 M_\Delta^2} p_\mu p_\nu + \frac{1}{3 M_\Delta} (\gamma_\mu p_\nu - \gamma_\nu p_\mu) \right]$$

$$= -\frac{1}{i \not\! \! \! p + M_\Delta} (P^{3/2})_{\mu\nu} + \frac{1}{\sqrt{3} M_\Delta} [(P^{1/2})_{12,\mu\nu} + (P^{1/2})_{21,\mu\nu}] + \frac{2}{3 M_\Delta} (i \not\! \! \! p - M_\Delta) (P^{1/2})_{22,\mu\nu}. \tag{1.5}$$

Explicit form of the projection operators can be found in Ref. [16]. It is seen that if the $\Delta$ propagator $S^\mu\nu$ is redefined so that the $Z$ dependence disappears and therefore, this parameter is physically irrelevant. However, after finding that it is difficult to manage the nonlocal part of the resulting Lagrangian, Tang and Ellis return to the starting $\pi N\Delta$ Lagrangian containing the $\Delta$ field explicitly and recommend to use it with some convenient choice of the parameter $Z$, since it is not relevant to the physics. On the other hand, they do not consider any mechanism to compensate the $Z$ dependence of the observables. Indeed, if such an independence on a parameter should take place, one should provide a mechanism to compensate it if it appears due to a particular process, that can generally happen.

In Refs. [19,20], new $\pi N\Delta$ and $\gamma N\Delta$ Lagrangians are proposed. They possess the property that with the proper choice of couplings, the new and traditional Lagrangians provide identical $\pi N\Delta$ and $\gamma N\Delta$ vertices for the on–shell particles. Further, using the redefinition (1.7), it is shown in [20] that the new and traditional $\pi N\Delta$ couplings differ by a contact term quadratic in the coupling constant that can be associated with the contribution of the 1/2 spin space involved due to the traditional $\pi N\Delta$ coupling.

In Sect. II, we show how one can construct such a contact term directly. For this purpose, we first use an identity to show that the new and traditional $\pi N\Delta$ couplings differ by a sum of the $\pi N\Delta$ terms that vanish for the $\Delta$ isobar on–shell. Next we construct, in the tree approximation, the contribution of the $\Delta$ excitation to the $\pi N$ scattering amplitude and we show that these $\pi N\Delta$ terms give rise to a contact term quadratic in the $\pi N\Delta$ coupling constant. In Sect. III, we use the new $\pi N\Delta$ and $\gamma N\Delta$ couplings to calculate the $\Delta$ excitation contribution to the photon spectrum.

1Recent review of some aspects of the theory of massive Rarita-Schwinger fields can be found in [21].
for the reaction (1.1). We show for the recently measured spectrum [9,10] that it is suppressed in comparison with the one obtained earlier with the use of the traditional couplings. In Sect. IV, we discuss the obtained results and we present our conclusions.

II. THE $\pi N\Delta$ COUPLINGS

According to Ref. [19], the gauge symmetric $\pi N\Delta$ coupling is

$$L_{\pi N\Delta}^{g.s.} = f \varepsilon_{\mu \nu \alpha \beta} (\partial_\mu \bar{\Psi}_\nu) \bar{T}^{\gamma_5 \gamma_\alpha} \Psi \cdot (\partial_\beta \bar{\pi}) + h.c.$$  \hspace{1cm} (2.1)

With the choice

$$f = \frac{f_{\pi N\Delta}}{m_\pi M_\Delta},$$  \hspace{1cm} (2.2)

and for the $\Delta$ isobar on–shell ($Z = -1/2$), this coupling is equivalent to the traditional one

$$L_{\pi N\Delta}(Z = -1/2) = \frac{f_{\pi N\Delta}}{m_\pi} \bar{\Psi}_\mu \bar{T} O_{\mu \nu}(C(Z = -1/2)) \Psi \cdot (\partial_\nu \bar{\pi}) + h.c.$$  \hspace{1cm} (2.3)

Using the identity

$$\varepsilon_{\mu \nu \alpha \beta} \gamma_5 \gamma_\alpha = -\delta_{\mu \nu} \gamma_\beta + \delta_{\beta \nu} \gamma_\mu - \delta_{\beta \mu} \gamma_\nu + \gamma_\nu \gamma_\mu \gamma_\beta,$$  \hspace{1cm} (2.4)

one can show that

$$L_{\pi N\Delta}^{a,s.} = L_{\pi N\Delta}(Z) + \delta L_{\pi N\Delta}(Z),$$  \hspace{1cm} (2.5)

where

$$\delta L_{\pi N\Delta}(Z) = f \left\{ - (\partial_\mu \bar{\Psi}_\mu) \bar{T}_\gamma \gamma_\nu \Psi - (\partial_\nu \bar{\Psi}_\alpha \gamma_\alpha) \bar{T} \Psi + (\partial_\mu \bar{\Psi}_\alpha) \gamma_\alpha \bar{T} \gamma_\mu \gamma_\nu \Psi \\
+ \bar{\Psi}_\nu [(-\gamma_\mu \partial_\mu) \bar{T} \Psi - C(Z) M_\Delta \bar{\Psi}_\mu \gamma_\mu \bar{T} \gamma_\nu \Psi] \right\} \cdot \partial_\nu \bar{\pi}.$$  \hspace{1cm} (2.6)

It holds for the $\Delta$ isobar on–shell that

$$\partial_\mu \bar{\Psi}_\mu(x) = \gamma_\mu \bar{\Psi}_\mu(x) = [\partial + M_\Delta] \bar{\Psi}(x) = 0.$$  \hspace{1cm} (2.7)

It follows from these equations that for the $\Delta$ isobar on–shell $\delta L_{\pi N\Delta}(Z) = 0.$

In the tree approximation, the $\pi N$ scattering via the $\Delta$ isobar excitation is described by the Feynman graphs of Figs. 1a and 1b.

Fig. 1. The $\pi N$ scattering amplitudes in the tree approximation; a, b- the $\Delta$ excitation amplitudes, c- the contact term.
One can calculate the S-matrix element corresponding to Fig. 1a either using the left hand side of Eq. (2.5) or, equivalently, its right hand side. If one considers a part of the S-matrix element, \( S_p \), given by the sum of the partial S-matrix elements, calculated with the choice

\[
A = \mathcal{L}_{\pi N \Delta}(Z), \quad B = \delta \mathcal{L}_{\pi N \Delta}^+(Z), \quad (2.8)
\]

\[
A = \delta \mathcal{L}_{\pi N \Delta}(Z), \quad B = \mathcal{L}_{\pi N \Delta}^+(Z), \quad (2.9)
\]

\[
A = \delta \mathcal{L}_{\pi N \Delta}(Z), \quad B = \delta \mathcal{L}_{\pi N \Delta}^+(Z), \quad (2.10)
\]

one obtains the difference between the S-matrix elements, calculated first with the new Lagrangian \( \mathcal{L}_{\pi N \Delta}^{a,n} \) and then only with the traditional Lagrangian \( \mathcal{L}_{\pi N \Delta}(Z) \). Explicit calculations yield \( S_p \) in the form of the \( \pi \pi NN \) contact graph of Fig. 1c. Defining

\[
S_p = -i(2\pi)^4 \delta^{(4)}(p_f - P_i)(\chi^b)^+ T_{p}^{ba}(s) \chi^a, \quad (2.11)
\]

we obtain for the amplitude \( T_{p}^{ba}(s) \) the following equation

\[
T_{p}^{ba}(s) = f^2 M_{\Delta} \bar{u}(p_1) \langle p_2 \gamma_\nu \gamma_\mu \rangle \left[ \delta_{\nu \mu} + \frac{1}{3} \gamma_\nu \gamma_\mu + \frac{i}{3M_{\Delta}} (3 \bar{P} \delta_{\nu \mu} + \gamma_\nu P \gamma_\mu - P \gamma_\nu - P \gamma_\mu - P \gamma_{\nu\gamma_\mu}) \right] u(p_2)
\]

\[
+ \frac{2}{3} C(Z) \left\{ \beta_2 \beta_2 + \frac{i}{M_{\Delta}} [ \bar{P} \gamma_\nu \gamma_\mu \beta_2 + \beta_2 \gamma_\mu \beta_2] \right\}
\]

\[
+ \frac{2}{3} C(Z) \beta_2 \left( 2 + \frac{P}{M_{\Delta}} \right) \beta_2 \right) (T^+) \cdot T^a \cdot u(p_1). \quad (2.12)
\]

Here \( P = p_1 + p_2 = p_1^+ + p_2^+, (T^+) \cdot T^a = \frac{2}{3} \delta_{ba} - \frac{1}{3} \tau^b \tau^c \) and \( \tau^c \) are the isospin Pauli matrices. In deriving Eq. (2.12) we supposed \( C(Z) \) to be a real function of the variable \( Z \).

The amplitude \( T_{p}^{ba}(s) \) corresponds to an effective contact Lagrangian

\[
\mathcal{L}_{\pi \pi NN}(Z) = -f^2 M_{\Delta} (\partial_\nu \pi^b) \bar{\Psi} \left\{ \delta_{\nu \mu} - \frac{1}{3} \left[ 1 + 2 C(Z) + 4 C^2(Z) \right] \gamma_\nu \gamma_\mu \right. \left. + \frac{1}{M_{\Delta}} \delta_{\nu \mu} \beta \right.
\]

\[
- \left. \frac{1}{3M_{\Delta}} \left[ 1 + 2 C^2(Z) \right] \gamma_\nu \gamma_\mu \beta \right. \left. + \frac{1}{3M_{\Delta}} \left[ 1 - 2 C(Z) \right] (\gamma_\nu \partial_\mu + \gamma_\mu \partial_\nu) \right\} (T^+) \cdot T^a \cdot \bar{\Psi} (\partial_\mu \pi^a). \quad (2.13)
\]

Let us write this equation in the form

\[
\mathcal{L}_{\pi \pi NN}(Z) = -f^2 M_{\Delta} (\partial_\nu \pi^b) \bar{\Psi} \mathcal{T}_{\nu \mu} (C(Z)) (T^+) \cdot T^a \cdot \bar{\Psi} (\partial_\mu \pi^a). \quad (2.14)
\]

The straightforward calculations yield the amplitude \( \mathcal{T}_{\nu \mu} (C(Z)) \) in the factorized form

\[
\mathcal{T}_{\nu \mu} (C(Z)) = \mathcal{O}_{\nu \alpha}(C(Z)) \mathcal{T}_{\alpha \beta}(C(Z) = 0) \mathcal{O}_{\beta \mu}(C(Z)). \quad (2.15)
\]

It follows from the form of the operator vertex (1.3) that

\[
\mathcal{O}_{\nu \alpha}(a) \mathcal{O}_{\alpha \mu}(b) = \mathcal{O}_{\nu \alpha}(b) \mathcal{O}_{\alpha \mu}(a) = \mathcal{O}_{\nu \mu}(a + b + 4ab) = \delta_{\nu \mu}, \quad (2.16)
\]

if the parameters \( a \) and \( b \) satisfy the equation

\[
a + b + 4ab = 0. \quad (2.17)
\]

It can be seen from Eqs. (2.16) and (2.17) that the amplitude \( \mathcal{T}_{\nu \mu} (C(Z)) \) can be factorized in an infinite number of forms. Choosing \( a = -1 \) and \( b = -1/3 \), we can write Eq. (2.15) as
In Ref. [20], applying the field redefinition (1.7) with a particular choice of the field

\[ \partial_{\mu} \xi = -\frac{1}{M_{\Delta}} \mathcal{O}_{\rho \mu}(C(Z)) = -1/3 \mathcal{O}_{\rho \mu}(C(Z)) T^{a} \bar{\Psi} \phi^{a}, \]  

(2.19)

in the \( Z \) dependent Lagrangian (2.3), besides the gauge symmetric Lagrangian \( \mathcal{L}_{\pi N\Delta}(Z) \) of the form (2.14) was obtained, where the amplitude \( T_{\nu \mu}(C(Z)) \) is given by Eq. (2.18).

In the next section, we apply the gauge symmetric Lagrangians to the calculation of the contribution of the \( \Delta \) excitation process to the photon spectrum in the radiative muon capture in the hydrogen.

### III. THE PHOTON SPECTRUM IN THE RADIATIVE MUON CAPTURE IN HYDROGEN

The new Lagrangians, needed for the calculations of the photon spectrum, that are derived from the gauge symmetric ones [19] read

\[ \mathcal{L}_{\pi N\Delta a 1}^{g,s} = f \frac{\varepsilon_{\mu \alpha \beta}}{2} \left[ (\partial_{\mu} \bar{\Psi} \right) \bar{T} \gamma_{5} \gamma_{\alpha} \Psi \right] \cdot \left( \partial_{\beta} \bar{\pi} + 2 f_{\pi} g_{\rho} \bar{a}_{\beta} \right) + h.c., \]  

(3.1)

\[ \mathcal{L}_{\rho N\Delta}^{g,s} = f_{\rho} g_{\rho} \frac{\varepsilon_{\mu \alpha \beta}}{2} \left[ (\partial_{\mu} \bar{\Psi} \right) \bar{T} \gamma_{5} \gamma_{\alpha} \Psi \right] \cdot \bar{\rho}_{\lambda \beta} + f_{\rho} g_{\rho} \left[ (\partial_{\mu} \bar{\Psi} \right) \bar{T} \gamma_{5} \gamma_{\alpha} \Psi \right] \cdot \bar{\rho}_{\lambda \nu} + h.c. \]  

(3.2)

Here \( f_{\rho} = \frac{G_{F}}{M_{\Delta}} \) is obtained from the condition that the new (3.2) and the standard \( \rho N\Delta \) couplings (1.2) are equivalent for the \( \Delta \) isobar on-shell. The notations of this section coincide with the notations of Refs. [5,11], where one can also find more detailed discussion of the reaction (1.1).

The contribution from the \( \Delta \) excitation processes to the photon spectrum for the reaction (1.1) is presented in Fig. 2.

From various form factors, calculated in Sect. II.B of Ref. [5], we need consider

\[ \Delta g_{2} = -\frac{8}{9 M_{\Delta}} f_{\pi N\Delta} G_{1} \frac{f_{\pi}}{m_{\pi}} \eta k \left[ -(1 + 2R) + 2(1 - 2R) C(Y) + 2(1 - R) C(Z) + 4(2 - R) C(Y) C(Z) \right], \]  

(3.3)

and

\[ \Delta g_{3} = -\frac{16}{9} \lambda f_{\pi N\Delta} G_{1} \frac{f_{\pi}}{m_{\pi}} \eta k \frac{1}{M_{\Delta} - M} \left\{ 1 + (1 - R) \left[ C(Y) + C(Z) + 2(2 + R) C(Y) C(Z) \right] \right\}, \]  

(3.4)

Here \( \lambda \) is the photon polarization, \( k \) is the photon momentum, \( \eta = \frac{m_{\pi}}{M_{\Delta}} \) and \( R = M/M_{\Delta} \). As it can be seen from Eq. (3.3), this form factor is of the contact origin, whereas inspecting of the Eq. (3.4) shows that the dependence of the form factor on the off-shell parameters \( Y \) and \( Z \) is entirely located in the contact part of the form factor.

Using new Lagrangians (3.1) and (3.2) and performing calculations, identical to those presented in Sect. II.B of Ref. [5], one obtains

\[ \Delta g_{3}^{g,s} = -\frac{16}{9} \lambda f_{\pi N\Delta} G_{1} \frac{f_{\pi}}{m_{\pi}} \left( \frac{M}{M_{\Delta}} \right)^{2} \eta k \frac{1}{M_{\Delta} - M}. \]  

(3.5)
Fig. 2. The Δ excitation amplitudes contributing to the radiative muon capture in hydrogen. The weak hadron current \( j_{W, \mu} \), interacting with the nucleon, is exciting it and the Δ isobar in the intermediate state appears that decays into the final state nucleon and photon.

In contrast to the calculations with the standard couplings, now the form factor \( \Delta g_{3}^{g.s.} \) contains only the Δ pole contribution. This follows from the fact that if the propagator (1.5), given in terms of the projection operators, is sandwiched between gauge symmetric Lagrangians, only the part proportional to the projection operator \( (P_{3/2})_{\mu \nu} \), accompanied by the Δ isobar pole, contributes to the matrix element [20]. This simplifies the calculations of matrix elements considerably.

Let us write down the contribution \( \Delta g_{3}^{0} \), given in Eq. (3.4), for the Δ isobar on–shell \( (Y = Z = -1/2) \)

\[
\Delta g_{3}^{0} = -\frac{16}{9} \lambda f_{\pi N} G_{1} \frac{f_{\pi}}{m_{\pi}} \eta k \frac{1}{M_{\Delta} - M},
\]

(3.6)

and calculate the difference with the form factor \( \Delta g_{3}^{g.s.} \) from Eq. (3.5)

\[
\Delta g_{3}^{g.s.} - \Delta g_{3}^{0} = -\frac{16}{9} \lambda f_{\pi N} G_{1} \frac{f_{\pi}}{m_{\pi}} \eta k \left( 1 + \frac{M}{M_{\Delta}} \right).
\]

(3.7)

As it is seen, the difference is a contact term, in agreement with the more general discussion that took place above.

In Fig. 3, we present the change in the photon spectrum calculated as

\[
\text{[spectrum calculated using } \Delta g_{3}^{0} \text{ from Eq. (3.6)] - [spectrum calculated using } \Delta g_{3}^{g.s.} \text{ from Eq. (3.5)]} / \text{[spectrum calculated using } \Delta g_{3}^{0} \text{ from Eq. (3.6)]}.
\]

Other contributions are the same as in Ref. [5]. The spectrum measured in the TRIUMF experiment [9,10] is given as

\[
S_{T} = 0.061S_{s} + 0.854S_{o} + 0.085S_{p}.
\]

(3.8)

Here \( S_{s} \), \( S_{o} \) and \( S_{p} \) correspond to the muon-hydrogen singlet system, and to the ortho- and paramolecular \( p\mu p \) states, respectively. As it is seen from Fig. 3, the spectrum \( S_{T} \), calculated with \( \Delta g_{3}^{g.s.} \) from Eq. (3.5) is in the region \( k > 60 \text{ MeV} \) suppressed, in comparison with the spectrum obtained using the traditional couplings. It means that the new couplings cannot resolve the "\( g_{P} \) puzzle" either. Minor difference between the curves of the same sort comes from omitting the form factor \( \Delta g_{2} \) of Eq. (3.3) in the calculations.
It is seen from Eqs. (3.5) and (3.6) that the form factors ∆g_{3s} and ∆g_{0} differ by the factor \((M/M_\Delta)^2 \approx 0.58\). Such a factor will appear also in the meson exchange current operators with the ∆ excitation. On the other hand, a suppression factor 0.8 has been found to be needed [22] to reduce the effect of the weak axial meson exchange currents with the ∆ excitation in order to explain the experimental value of the Gamow–Teller matrix element for the triton β decay, if the value of the constant \(f_{\pi N\Delta}\) is taken from the constituent quark model. In other words it means that effectively the value of the constant \(f_{\pi N\Delta}\) turns out to be unrealistically small or one should speculate about other processes effectively suppressing the meson exchange current effect [23]. If these weak axial exchange currents are constructed from the new Lagrangians, the factor \((M/M_\Delta)^2\) appears naturally and the value of the constant \(f_{\pi N\Delta}\) can be taken larger and therefore, more realistic. Simultaneously, such a factor will appear also in the vector meson exchange currents with the ∆ excitation. However, the precise data on the radiative capture of neutrons by protons do not demand any damping of the vector meson exchange currents effect [24] and the capture rate for the reaction \(\mu^- + ^3\text{He} \rightarrow \nu_\mu + ^3\text{H}\), that has been measured in the precise experiment [25,26], is by the suppressed weak axial exchange currents underestimated [23]. Precise data, expected from the experiments on the ordinary muon capture in hydrogen and deuterium [27], will be for the axial sector of the weak nuclear interaction very helpful.

### IV. RESULTS AND CONCLUSION

Here we study some aspects of new \(\pi N\Delta\) and \(\gamma N\Delta\) couplings that have recently been proposed [19] by Pascalutsa and Timmermans. In comparison with the traditional couplings, the new ones possess an additional gauge symmetry (1.7) that is present in the kinetic energy term of the ∆ Lagrangian. This symmetry guarantees that the couplings have the same ∆ degrees of freedom as the kinetic energy term. As a consequence, the amplitudes of the processes with the ∆ excitation in the intermediate state do not contain the contact terms coming from the 1/2 spin space.

In Sect. II, we study the difference between the traditional and gauge symmetric \(\pi N\Delta\) couplings. Using an algebraic identity between the gamma matrices, we first express the new coupling as the sum of the traditional coupling and of terms that are zero for the ∆ isobar on–shell. The \(\pi N\) scattering amplitude, constructed from these terms, is a contact term, quadratic in the coupling constant. In Ref. [20], such a term was obtained by imposing the symmetry condition (1.7) on the traditional coupling. We also show that this contact term can be factorized in an infinite number of forms.

In Sect. III, we employ the gauge symmetric \(\pi N\Delta\) and \(\gamma N\Delta\) couplings [19] to calculate the photon spectra for the radiative muon capture in hydrogen. As a result, the new form factor \(\Delta g_3\) contains only the ∆ isobar pole contribution. This form factor differs from the old one, calculated for the ∆ isobar on–shell, by the damping factor \((M/M_\Delta)^2 \approx 0.58\). Consequently, the new photon spectrum, corresponding to the spectrum measured in the TRIUMF experiment, is suppressed in the region \(k > 60\ \text{MeV}\), in comparison with the photon spectrum, calculated from the traditional couplings. Therefore, the problem of extraction of the induced pseudoscalar form factor \(g_P\) from the photon spectrum in the radiative muon capture in hydrogen cannot be solved by employing the gauge symmetric \(\pi N\Delta\) and \(\gamma N\Delta\) couplings.

Let us note that the damping factor \((M/M_\Delta)^2\) will be also present in the meson exchange current operators with the ∆ isobar excitation, if for the construction the gauge symmetric couplings are used. However, the comparison of the existing calculations with the present data on the weak and electromagnetic reactions in few–nucleon systems does not allow to decide uniquely, if this factor is needed or not.

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Fig. 3. The change in the photon spectrum. Solid curve- the spectrum measured in the TRIUMF experiment; dashed curve- the spectrum for the muon–hydrogen triplet state; dotted curve- the spectrum for the muon–hydrogen singlet state.
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