Abstract

We consider different types of Q-balls as self-interacting dark matter. For the Q-balls to act as the dark matter of the universe they should not evaporate, which requires them to carry very large charges; depending on the type, the minimum charge could be as high as $Q \sim 10^{33}$ or the Q-ball coupling to ordinary matter as small as $\sim 10^{-35}$. The cross-section-to-mass ratio needed for self-interacting dark matter implies a mass scale of $m \sim \mathcal{O}(1) \text{ MeV}$ for the quanta that the Q-balls consist of, which is very difficult to achieve in the MSSM.
1 Introduction

Dark matter is widely expected to form a significant portion of the total energy density of the universe, regardless of the lack of direct experimental evidence. Indirect experimental evidence, on the other hand, for the existence of dark matter is well established: galactic rotation curves, dynamics of galaxy clusters, and large scale flows all indicate that large amounts of dark matter must be present on galactic halo scales. In addition, the present cosmic microwave background (CMB) observations strongly support a cosmological model with a significant dark matter fraction [1].

The cosmological model that is in good agreement with the CMB observations and large scale structure, is based on collisionless cold dark matter. A number discrepancies, however, between observations and numerical simulations have been noted on galactic and sub-galactic scales. The halo density profiles, and the number density of satellite galaxies are examples of where the collisionless cold dark matter models are in disagreement with observations [2, 3].

The discrepancies between expected and observed distribution of dark matter can be alleviated, and possibly resolved, by allowing interactions between the dark matter constituents [3]. The scattering of dark matter particles in high density regions leads to smoothing out of the density distribution, randomises the velocity distribution of the dark matter particles, and can lead to enhanced destruction of halo substructure. All of these processes help to resolve the problems associated with collisionless cold dark matter models.

To have the appropriate properties, the self-interaction cross-section and mass of the dark matter particles undergoing elastic scattering need to satisfy [3, 4],

\[ s = \frac{\sigma_{DD}}{m_{DM}} \approx 2 \times 10^{3} - 3 \times 10^{4} \text{ GeV}^{-3} = 0.5 - 6 \text{ cm}^{2} \text{ g}^{-1}, \]

where \( \sigma_{DD} \) is the self-interaction cross-section, and \( m_{DM} \) the mass of the dark matter particle. Hence the required interaction is relatively strong and one talks about strongly interacting dark matter (SIDM).

Note that here only elastic scattering are considered and if other types of processes are studied, these values may be somewhat different. As an example, in [5], an effective annihilation cross section per unit mass is found to be \( 0.03 \text{ cm}^{2}/\text{g} \) in a model where dark matter undergoes both elastic scattering and annihilation.

Q-balls [6] have been recently proposed as a candidate for the self-interacting dark matter [4]. Q-balls are non-topological solitons [7] that can exist in theories with scalar fields carrying a conserved \( U(1) \)-charge [6]. The Q-ball is the ground state of the theory in the sector of fixed charge, i.e. the energy of the Q-ball configuration is less than that of a collection free scalars carrying an equal amount charge as the Q-ball. The phase of the Q-ball field, \( \phi \), rotates uniformly with frequency \( \omega \), and we can write

\[ \phi = \varphi e^{i\omega t}, \]

where \( \varphi \) is a spherically symmetric, monotonically decreasing, positive function [6]. The energy and charge of a Q-ball are given by

\[ E = \int dx^{3} [\omega^{2} \varphi^{2} + |\nabla \varphi|^{2} + U(\varphi^{2})], \]

\[ Q = 2\omega \int dx^{3} \varphi^{2}, \]
where $U(|\phi|^2)$ is the potential that has a global minimum at the origin and is invariant under $U(1)$-transformations of the $\phi$-field. For the Q-ball to be energetically stable, condition

$$E < mQ,$$

where $m$ is the mass of the free $\phi$ scalar, must hold. This condition is met whenever the potential $U(\varphi^2)$ grows slower than $\varphi^2$.

Stable Q-balls exist in many theories, and in particular in supersymmetric extensions of the Standard Model, where Q-balls are made of squarks and sleptons.

The purpose of the present paper is to study the general circumstances under which Q-balls can act as strongly interacting dark matter. The key point here is that although the Q-balls typically consist of weakly interacting quanta, they are large objects and can have large interaction rates. This was first pointed out in a recent paper by Kusenko and Steinhardt [4], where the idea of self-interacting Q-ball dark matter was proposed. In this paper we study the proposal in more detail, taking into account the commonly considered Q-ball types and the implications of a primordial Q-ball charge distribution. We also discuss the importance of evaporation and thermal processes, which need to be accounted for in any realistic detailed model. In Section 2 we recall the salient features of the three main types of Q-balls: thin-wall, thick-wall in flat potentials, and thick-wall in logarithmic potentials. In Section 3 we study experimental constraints on a dark matter Q-balls by studying their interaction cross sections and number distribution. In Section 4 we discuss how Q-ball evolution, in particular evaporation of charge from Q-balls and the surrounding thermal bath, constrain the properties of Q-ball dark matter. Section 5 contains our conclusions.

2 Different Q-ball Types

2.1 Type I: Thin-wall Q-balls

The Q-balls considered in the literature have very different properties, which depend on the details of the potential. These differences are also reflected in their scattering cross sections and hence in dark matter properties. The Q-balls may either have a narrow, well-defined edge, in which case they are called thin-wall Q-balls, or their boundaries are not localised in a narrow region, in which case they are called thick-wall Q-balls. Both types may exist within a same theory.

Let us first consider thin-wall Q-balls, which arise for any suitable potential that allows Q-balls to exist and grows faster than $\varphi^2$ in the large $\varphi$ limit. These solutions are approximated by the profile $\varphi(r) \approx \varphi_0 \theta(r-R)$. The energy to charge ratio of such a configuration is (neglecting all surface terms)

$$\frac{E}{Q} = \min \sqrt{\frac{U(\varphi^2)}{\varphi^2}} \equiv \omega_c,$$

i.e. energy grows linearly with charge. Note that the radius of such a Q-ball can be very large and is related to the charge simply by

$$Q = 2\omega_c \varphi_0^2 V = \frac{8}{3} \pi R^3 \omega_c \varphi_0^2.$$


2.2 Type II: Thick-wall Q-balls in flat potentials

In addition to the thin-wall Q-balls, two other types of Q-balls have been commonly considered. These arise e.g. in supersymmetric theories with gauge and gravity mediated supersymmetry breaking scenarios.

In flat potentials the mass of a Q-ball can grow more slowly that in the thin-wall case. If the potential has an absolutely flat plateau at large $\varphi$, $U(\varphi) \sim m^4$, as has been studied in association with the gauge mediated supersymmetry breaking, the energy of such a Q-ball grows as $E \approx \frac{4\sqrt{2}}{3} m\pi Q^{3/4}$. (8)

The radius and value of the field inside the Q-ball are given by

\begin{align*}
R &\approx \frac{1}{\sqrt{2m}} Q^{1/4}, \\
\varphi_0 &\approx \frac{m}{\sqrt{2\pi}} Q^{1/4}.
\end{align*}

(9) (10)

2.3 Type III: Thick-wall Q-balls in logarithmic potentials

The potential may also grow only slightly slower that bare $\varphi^2$ term. E.g. in the gravity mediated supersymmetry breaking scenario the scalar potential grows like

\[ U(\varphi) = m^2 (1 + K \log(\frac{\varphi^2}{M^2})) \varphi^2, \]

(11)

where $K < 0$ and $M$ is a large mass scale. In potentials of this form the Q-ball profile can be solved:

\[ \varphi(r) = M e^{-\left(1-\frac{\varphi^2}{2m^2} - 2K\right)/(2K)} e^{K m^2 r^2/2} \equiv \tilde{M} e^{K m^2 r^2/2}. \]

(12)

The energy charge relation is approximately

\[ E \approx mQ. \]

(13)

The radius of the Q-ball remains constant with increasing charge in potentials of this form, $R \equiv |K|^{-1/2} m^{-1}$.

Note that in the thick-wall cases where the potential grows more slowly than a mass term, $m^2 \varphi^2$, the non-renormalizable terms will begin to dominate at some large value of $\varphi$ after which the Q-ball solution approaches the thin-wall type. Since the potentials associated with the Type II and Type III Q-balls represent the extremes (strong binding and weak binding, respectively), any thick-wall Q-ball should fall into a category that is somewhere in between. Hence it is sufficient to consider only the above three types separately.

3 Flux limits on Q-ball dark matter

3.1 Scattering Cross-sections

The scattering cross-section of Q-balls has been studied for different types of Q-balls in [9]-[13]. Collisions of thick-wall Q-balls associated with potentials in the gauge
and gravity mediated SUSY breaking scenarios have been studied in detail in [12, 13]. There it was found that for like Q-balls, the average fusion and charge transfer cross-sections are somewhat larger than the geometric cross-section in the studied charge range. It was also found that the type of the collision process: merger, elastic scattering, or charge transfer between the Q-balls, is dependent on the relative phase of the colliding Q-balls. If the Q-balls are in phase when they collide, they will typically fuse into one large Q-ball whereas if their phase difference is \( \pi \), they will repel each other. All of these properties were noted at velocities of the order of typical galactic velocities. At higher velocities, the scattering cross-section typically decreases due to a reduced interaction time as was noted in [13].

In the study of collision processes, it was also noted that when two equally sized Q-balls collide, most of the charge can be transferred to one of the Q-balls, so that one of the Q-balls in the final state is small compared to its initial charge. In such a process the small Q-ball can acquire a large velocity due to momentum and energy conservation. In the simulations a ten-fold increase in velocity was not uncommon and hence such processes can reduce the number of dark matter Q-balls in the galaxy as the small Q-ball can escape from the galactic halo.

On the basis of the numerical simulations [12, 13] we may write \( \sigma_{DD} = \xi \pi R^2 \), where \( \xi \) represents the deviation of the scattering cross-section from the geometric one. Typically one finds that \( \xi \simeq 2 \) for the gauge mediated and \( \xi \simeq 4 \) for the gravity mediated case. We take \( \xi = 1 \) for the thin-wall case. The scattering cross-section to mass ratio, \( s = \sigma_{DD}/E_Q \), of the different types of Q-balls are given by:

\[
\begin{align*}
    s_I &= \frac{\xi \pi R^2}{\omega_c Q} = \left( \frac{9\pi}{64\omega_c^5 \varphi_0^4} \right)^{1/3} Q^{-1/3} 	ext{ (thin – wall)} \\
    s_{II} &\simeq \frac{3\xi}{4\sqrt{2}m^3} Q^{-1/4} \text{ (thick – wall, flat } U) \\
    s_{III} &\simeq \frac{4\pi \xi}{|K|m^3} Q^{-1} \text{ (thick – wall, log } U). \tag{14}
\end{align*}
\]

Inserting the allowed values for \( s \) from (1), we obtain the range of acceptable charges:

\[
\begin{align*}
    \text{I : } & \quad 3 \times 10^{13} < \frac{\omega_c^5 \varphi_0^4}{(\text{MeV})^9} Q < 5 \times 10^{16} \\
    \text{II : } & \quad 3 \times 10^{18} < \left( \frac{m}{\text{MeV}} \right)^{12} Q < 7 \times 10^{22} \\
    \text{III : } & \quad 2 \times 10^6 < |K|(\frac{m}{\text{MeV}})^3 Q < 2 \times 10^7. \tag{15}
\end{align*}
\]

The allowed range of charge for different types of Q-balls have been plotted in Fig. 4 for different values of the mass parameter \( m \). From the allowed range of charges, we see that the commonly considered supersymmetric Q-balls that carry baryon number [14, 15] are unacceptable as candidates of self-interacting dark matter: In the gauge mediated case (Type II), \( m \sim 10^2 \sim 10^4 \text{ GeV} \), which clearly leads to unacceptable values of charge. This is also true for the gravity mediated case (Type III) with \( m \sim 10^2 \text{ GeV} \), \( \omega_c \sim m \) and \( |K| \sim 0.01 \sim 0.1 \). In the thin-walled case (Type I), \( \omega_c \) is typically of the order of the supersymmetry breaking scale \( \sim 10^2 \text{ GeV} \) and \( \varphi_0 \) is at
least of the same order (and can be much larger), so that also in this case the baryon number carrying supersymmetric Q-balls are not acceptable SIDM candidates.

Q-balls which satisfy the boundaries (13) can, however, be included into models where the U(1) -symmetry is not associated to baryon or lepton number. If the supersymmetry breaking is not setting the scale of parameters ($\omega_c, \varphi_0, m$), they may be low enough to allow appropriate values of charge $Q$. These depend, however, crucially on the details of the particular model, e.g. on the couplings of the Q-balls to the ordinary matter.

![Figure 1: Acceptable values (between lines) of $Q$ for different Q-ball types for parameter values $\varphi_0 = m$, $\omega = m$, $K = -0.1$](image)

### 3.2 Thermally distributed dark matter

The cross section considered in the previous section do not as such represent a realistic situation, where one expects a distribution of Q-balls with different charges. This was considered both analytically and in a numerical simulation in [21] in the context of Type III Q-balls, where it was argued that a Q-ball ensemble that originates from a fragmentation of a scalar condensate soon achieves thermal equilibrium. Let us assume that this also holds true in a general case. The one-particle partition function reads

$$Z_1 = \int_{V_D} \frac{d^3 x d^3 p}{(2\pi)^3} \int_{-\infty}^{\infty} dQ \ e^{-\beta E + \mu Q}, \quad (16)$$

where $\mu$ is the chemical potential and $\beta^{-1} = T$ is the temperature of the Q-ball ensemble, which is related to the average energy (or average charge) of the system. Assuming that the distribution is such that the charge both in Q-balls and in anti-Q-balls is much larger than the net charge of the distribution i.e. we set $Q_+, |Q_-| \gg |Q_+ + Q_-|$, we may approximate $\mu \approx 0$, as was the case in [20].
Let us write the relation of the mass of the Q-ball to charge as \( M_Q = A Q^B \), where \( A \) and \( B \) are constants. Then the one particle partition function reads

\[
Z_1 = \frac{gV_D}{\pi^2 B A^{1/B}} \beta^{-3-1/B} G_1(B, 0),
\]

where the function \( G_i \) is defined as

\[
G_1(B, x) = \int_x^\infty dt \, t^{1+\frac{\beta}{B}} K_2(t).
\]

\( K_n(x) \) is the modified Bessel function. For the Type II \( B = 3/4 \) (see Eq. (8)) and for the Type III \( B = 1 \) (see Eq. (13)), \( G_1(B, 0) \) can be evaluated: \( G_1(1, 0) = 3\pi/2 \) and \( G_1(3/4, 0) = (2^{4/3} 10/9) \Gamma(2/3)^2 \).

We know that the energy density of galactic DM is

\[
\rho_{DM} \approx 0.3 \text{ GeV cm}^{-3} = 2.3 \times 10^{-42} \text{ GeV}^4.
\]

The flux of Q-balls is constrained by a number of experiments probing different values of Q-ball masses,

\[
\frac{1}{4\pi} n_{DM} v < F_{ex},
\]

where \( n_{DM} \) is the number density of the dark matter constituents, \( v \sim 10^{-3} c \) is their velocity, and \( F_{ex} \) is an experimental constraint. Note that here it is assumed that the velocity distribution of the dark matter Q-balls in the galactic halo is uniform, only the mass distribution is assumed to be thermal.

Depending on the details of the Q-balls, some of the Q-balls can be unstable and hence only a fraction of the total distribution contributes to the galactic dark matter content. The charge of the smallest stable Q-ball is denoted here by \( Q_{\text{stab}} \). The average mass and number density of stable Q-balls are easily calculated from Eq. (16). By approximating \( \rho_{DM} \approx n_{\text{stab}} \langle M_{\text{stab}} \rangle \) and \( \rho_{DM} = \rho_{\text{stab}} \), we can write the constraint Eq. (20) as

\[
\frac{\beta G_1(B, \beta A Q_{\text{stab}}^B)}{G_2(B, \beta A Q_{\text{stab}}^B)} < \frac{4\pi F_{ex}}{\rho_{DM} v}.
\]

The expectation value of \( s \) can be evaluated in the limit of small stable Q-balls, \( Q_{\text{stab}} = 0 \) we find

\[
\langle s_I \rangle \approx 0.82 \left( \frac{\beta}{\omega_c^4 \varphi_0^2} \right)^{1/3} \quad \text{(thin - wall)}
\]

\[
\langle s_{II} \rangle \approx 1.76 \left( \frac{\beta m}{m^*} \right)^{1/3} \quad \text{(thick - wall, flatU)}.
\]

For Q-balls in logarithmic potentials, \( \langle s \rangle \) is not calculable in the stable Q-ball limit. This is due to the fact that the radius of the Q-ball is assumed to constant, regardless of the charge of the Q-ball, i.e. even a zero-charged Q-ball has a constant radius. This leads to a divergent \( s \), which also makes \( \langle s \rangle \) diverge in the small \( Q_{\text{stab}} \) limit. Actually \( s \) also diverges in the two other cases as \( Q \) tends to zero, but the divergence is milder as can be seen from Eq. (14).
To estimate the value of $\langle s \rangle$ in the $Q_{stab} = 0$ limit in the logarithmic case, we approximate

$$\langle s \rangle \approx \frac{\sigma_{DD}}{\langle M_{stab} \rangle}.$$  \hfill (23)

To constrain the flux of Q-balls experimentally we study the flux of strongly-interacting dark matter particles, which is constrained by several experiments (see e.g. [13]). To study what parameter regions are allowed for thermally distributed Q-ball dark matter, we adopt a scheme similar to the one in [4], i.e. we assume that the interaction of a Q-ball with a nucleon is mediated by a heavy boson $Z'$ so that the interaction cross-section is given by

$$\sigma_{Qp} \sim \frac{g}{M_{Z'}}^2 Q^2.$$  \hfill (24)

The form factor $F$ is of order one if the radius of the Q-ball, $R_Q$, is less than that of the nucleus, $R_n$, and of order $(R_n/R_Q)^3$ if $R_Q \gg R_n$.

For each Q-ball type we can then calculate the expectation value of the mass of the Q-ball, $M_Q$, and of the interaction cross-section, $\sigma_{Qp}$. These have been plotted in Fig. 2 for $g = 0.1$, $M_{Z'} = 1$ TeV. The shaded areas are excluded regions in all of the graphs and the allowed parameter regions are between the solid lines (except in the thin-wall case, where the allowed region for a fixed $\varphi_0$ is just the line). In all of the cases in Fig. 2, we have assumed that $Q_{stab} = 0$. A non-zero $Q_{stab}$ shifts curves towards larger $\langle M_Q \rangle$s as one would expect.

From Fig. 2 it is apparent that the experimentally allowed window for thermally distributed is narrow in all of the cases. The most promising case appears to be the thin-walled type Q-ball with a $\varphi_0 \gtrsim O$(MeV). The thick-walled Q-balls have less free parameters than in the thin-walled case and the experiments exclude most of the parameter space. Again, an appropriate mass parameter is of the order of MeV.

4 Constraints from Q-ball evolution

4.1 Evaporation

When one considers Q-balls created in the early universe, e.g. by the fragmentation of the AD-condensate [14, 15], as candidates of SIDM, the question of Q-ball evaporation as well as thermal effects need to be addressed.

The evaporation of Q-balls can lead to the washing out of primordial Q-balls, depending on the details of the theory. The evaporation rate of a thin-walled Q-ball is bounded from above by [17]

$$\frac{dQ}{dAdt} \leq \frac{\omega^3}{192\pi^2}.$$  \hfill (25)

Evaporation rates for realistic profiles have been considered in [15]. The evaporation rate of thin-walled Q-balls is dependent on the combination $g\varphi_0/\omega$, where $g$ represents the coupling between the Q-ball field and the (massless) fermions that it decays into. A thin-walled Q-ball with $\omega \sim m_\varphi$ then decays at a rate

$$\frac{dQ}{dt} \approx \frac{m_\varphi^3}{48\pi} R^2 = \frac{1}{48} \left( \frac{m_\varphi^7}{\pi^8\varphi^4} \right)^{1/3} Q^{2/3}$$  \hfill (26)
so that its decay time can be roughly estimated by

$$\Delta t \sim \frac{Q}{dQ/dt} = 48\left(\frac{\pi^5\varphi_0^4}{m_\varphi^7}\right)^{1/3}Q^{1/3}. \quad (27)$$

If one takes \( m_\varphi \sim \varphi_0 \), then \( \Delta t \sim 10^2Q^{1/3}/m_\varphi \). Assuming, say, that Q-balls must still be around at about million years from the Big Bang, \( \Delta t_{gw} \sim 10^{37} \text{ GeV}^{-1} \). It is then clear that Q-balls of this type need to be very large not to have evaporated too early in order to have an effect on the galaxy formation.

In the above calculation it has been assumed, however, that \( g\varphi_0/\omega \gtrsim 1 \). If, on the other hand, the Q-ball field is very weakly coupled to the fields that it can decay into, the lifetime of a Q-ball can be long enough for it to play role in galaxy formation. The small \( g \) limit has been studied in \( [17] \), where it was found that the evaporation rate tends to

$$\frac{dQ}{dtdA} = \frac{g\varphi_0\omega^2}{64\pi} \quad (28)$$

as \( g \to 0 \). The lifetime of a Q-ball in this limit is of the order

$$\Delta t \sim \frac{70}{g}\left(\frac{\varphi_0}{m_\varphi}\right)^{1/3}Q^{1/3}. \quad (29)$$

Hence the coupling constant \( g \) needs to small enough to allow long lived Q-balls. This can be accomplished by fine-tuning or by having the interaction to be mediated by heavy bosons.

Equations (23) and (28) may be used to get order-of-magnitude approximations for thick-walled Q-balls, too. Indeed, according to the calculations made in \( [18] \) we know that it gives reasonable order-of-magnitude approximation at least for the logarithmic potential.

For the thick-wall, flat \( U \) Q-ball, applying the bound for the evaporation rate Eq. (23) gives

$$\frac{dQ}{dt} \leq \frac{\sqrt{2}\pi^2}{48}mQ^{-1/4}, \quad (30)$$

i.e. large Q-balls evaporate more slowly than small ones. Assuming that the bound is satisfied for all \( Q \) and integrating Eq. (30) we get a Q-ball lifetime of

$$\Delta t = \frac{192}{5\sqrt{2}\pi^2m}Q^{5/4}. \quad (31)$$

Again assuming that \( \Delta t \sim 10^{37} \text{ GeV}^{-1} \), we get a lower bound on Q-ball charge of \( Q \sim 10^{29}(m/\text{ GeV}) \). However, if realistic Q-balls are much smaller, say \( Q \sim 10^{20} \), as simulations suggest \( [4, 19, 20] \), the coupling \( g \) should be \( \sim 10^{-22}\text{GeV}/m \).

In the weak coupling limit we get

$$\frac{dQ}{dt} = \left(\frac{\pi}{8}\right)^{3/2}gmQ^{1/4}, \quad (32)$$

which translates into a decay time of

$$\Delta t = \frac{4}{3}\left(\frac{8}{\pi}\right)^{3/2}\frac{1}{gm}Q^{3/4}. \quad (33)$$
The radius of the thick-wall, log $U$ Q-ball is independent of charge. Taking $\omega \sim m$, the decay rate is bound by
\[
\frac{dQ}{dt} \lesssim \frac{1}{48\pi} \frac{m}{|K|},
\]
and the weak coupling limit gives
\[
\frac{dQ}{dt} \sim \frac{1}{16} \frac{g\varphi_0}{|K|}.
\]
Taking $|K| \sim 0.1$, and assuming that the bound Eq. (34) is satisfied, a Q-ball that survives until $\Delta t_{gf}$ must have a minimum charge of $Q \sim 10^{36} m/\text{GeV}$. Again, if $Q \sim 10^{20}$, the coupling $g$ should be smaller than $\sim 10^{-26} \text{GeV}/M$.

From these considerations we can conclude that evaporation can typically destroy Q-balls before they can affect galaxy formation, unless the decay channels of the quanta that the Q-balls consist of are greatly suppressed. As the best candidates then are possibly the Q-balls interacting only gravitationally, because suppressed interactions arise there naturally.

### 4.2 Thermal Effects

The thermal bath of the early universe can have an effect on the distribution of primordial Q-balls by thermally erasing them. Thermal effects on Q-balls have been considered for different types of Q-balls by utilising various methods [14, 15, 21, 22, 23]. In Refs. [21, 22] Purely thermodynamical considerations have been utilised to estimate thermal evaporation rate and diffusion rate.

A different approach has been adopted in [14, 15, 23], where collisions of thermal background particles with a Q-ball have been considered. In these consideration, two processes are important: dissociation and dissolution. In dissociation a thermal particle hits a Q-ball and transfers energy to it. If the rate of energy transfer into the Q-ball is larger than the emission rate of extra energy, excess energy builds up and can overcome the binding energy of the Q-ball. In dissolution, a Q-ball loses its charge from the edge of the Q-ball to the surrounding plasma. A thermal equilibrium exists at the surface of a thick-walled Q-ball and charge can leave by diffusion.

All of the described thermal processes can in principle alter the initial distribution. An important factor in the evolution of the Q-ball distribution is then the reheat temperature, which if large can effectively destroy the Q-ball distribution. For comparison, for baryonic Q-balls in the gravity-mediated scenario, it has been estimated that the reheat temperature should be at most $T_{RH} \lesssim Q^{1/4}$ GeV. Note, however, that the coupling strentght of the thermal particles to the Q-ball is crucial: if the coupling is very small, even small Q-balls can survive the high temperature bath of the early universe. If Q-balls are to survive evaporation, as discussed above, the coupling should be weak. Indeed, as was pointed out in Sect. 3.1., SIDM Q-balls cannot carry have Standard Model interactions (see also [4]). The possible candidates for such Q-ball fields should be searched for either in the hidden sector, coupled to the Standard Model only via gravity, or in the SM singlet sector of the extensions of the MSSM.
5 Conclusions and discussion

The question of the initial Q-ball distribution is obviously important when considering the possibility of Q-ball SIDM. Numerical simulations have shown that if Q-balls form from the fragmentation of an Affleck-Dine condensate, the initial conditions are important in deciding the characteristics of the Q-ball distribution [19, 2, 20]. If the charge and energy of the condensate are roughly equal, the following distribution consists only of Q-balls. However, if the condensate carries excess energy compared to the charge, a large number anti-Q-balls also appear. This has been observed in simulations by two separate groups [4, 24]. Appearing anti-Q-balls obviously lead one to consider the possibility of dark matter annihilations and its effect on galaxy formation.

We should also mention that there exist other suggestions to alleviate the problems of standard CDM models. Among these are the ideas of decaying dark matter [24] and annihilating dark matter [5]. In the decaying dark matter models some of the dark matter particles decay into relativistic particles by \( z = 0 \) so that small dwarf galaxies fail to form. In [5] the evolution of an isolated dark matter halo which undergoes both scattering and annihilation was considered.

Q-balls can exhibit all of the three phenomena: evaporation, annihilation and scattering, and hence one can speculate that dark matter consisting of interacting Q-balls can exhibit different types of behaviour. How to actually combine all of the elements of Q-ball dynamics so that appropriate galactic halos are produced is a question that clearly requires further study.

Thus, in an attempt to develop a scenario where primordial Q-balls act as self-interacting dark matter, one must then pay attention to several issues. First of all, an efficient mechanism of producing appropriate Q-balls needs to exist. The fragmentation of the AD-condensate is a promising candidate for such a mechanism. Secondly, the produced Q-balls must survive thermal effects if they are to influence galaxy formation. On the other hand, thermal effects can also be responsible for erasing unwanted small Q-balls. One must also keep in mind evaporation processes which can lead to an early decay of Q-balls. This might also offer an exciting scenario where Q-balls that have played a significant role in galaxy formation, have since decayed leaving particle dark matter in their place. The obvious question that requires an answer is the composition and interactions of the dark matter Q-balls. The commonly studied baryonic Q-balls are not acceptable so one must develop a model where the requirements for cross-section and mass are acceptable, while keeping in mind the experimental constraints.

On the basis of the results of this paper, the natural scale of the Q-ball scalar particle appears to be MeV, regardless of the type of the Q-ball considered. The charge of an appropriate dark matter Q-ball depends on the considered Q-ball type and very strongly on the parameter values, as Fig. 1 shows. In each case, the mass parameter must be close to MeV, otherwise charge can become unacceptably small or large. On the other hand, the charge of a dark matter Q-ball can then vary greatly, which obviously has an effect on the possibility of its detection. In [4] it was suggested, by applying naturalness arguments, that the charge of a dark matter Q-ball is of the order of \( 10^{-10^3} \) in the thin-wall case and \( 10^4 - 10^5 \) in the flat potential case. It should be noted that the charge can be much greater and considerations should not be
limited to small values of charge. Furthermore, naturalness arguments which indicate \( \omega_c \sim \varphi_0 \), need to be critically considered in the thin-wall case, where \( \omega \) can differ greatly from \( \varphi_0 \).

In [1], a uniform Q-ball distribution was discussed. If Q-balls form from the fragmentation of the the Affleck-Dine condensate, the initial conditions of the condensate are decisive in determining whether a uniform charge distribution is a good approximation or not. In this paper we have also considered the possibility of a thermal Q-ball charge distribution, which, on the basis of the simulations, is a realistic possibility. The appropriate Q-ball mass parameter obviously depends on the experimental flux limit, but is again naturally of the order of MeV or slightly less. Such a small mass scale might be difficult to achieve naturally in the extensions of MSSM but is not necessarily a problem for hidden sector Q-balls.

The experimental flux limit is dependent on the composition of the Q-balls and their interactions with matter. If, for example, Q-balls reside in the hidden sector and interact only gravitationally, their flux can be very high. If, however, they do have also other interactions with ordinary matter, experimental flux limits can be calculated. In the case of thermally distributed Q-ball dark matter, the parameter space appears to be constrained, especially for thick-walled Q-balls.

As it was discussed, evaporation can be a decisive process in determining whether Q-balls can act as dark matter or not. The exact charge limits coming from evaporation processes obviously again depend on the details, but it seems that if a decay channel to a light particle exists, Q-balls must be large or their couplings extremely suppressed in order to act as dark matter during galaxy formation. Even then, it seems probable that they would have decayed by now, leaving particle dark matter in their place.

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Figure 2: Acceptable values for the thin-wall case with $\omega_c = 1$ keV...1 GeV and $\varphi_0 = 1$ keV...1 GeV (the first two figures), flat thick-wall case (the middle figures) and logarithmic thick-wall case (the last two figures) with $m = 1$ keV...1 GeV, $\varphi_0 = 1$ keV...1 GeV and $|K| = 0.1, 10^{-5}, 10^{-10}$. Shaded areas are excluded.