Weisskopf units for neutron-proton pair transfers

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Abstract

We introduce the concept of neutron-proton two-particle units (\textit{np}-Weisskopf units) to be used in the analysis of the \((^3\text{He},p)\) and \((p,^3\text{He})\) reactions on nuclei along the \(N=Z\) line. These are presented for the conditions relevant to the \((n,j,f)\) orbits expected from \(^{16}\text{O}\) to \(^{100}\text{Sn}\). As is the case of the Weisskopf units for electromagnetic transitions, the \textit{np}-WU’s will provide a simple, yet robust, measure of isoscalar and isovector \textit{np} pairing collective effects.

1. Introduction

In 1958, Bohr, Mottelson and Pines \cite{1} suggested a pairing mechanism in the atomic nucleus analogous to that observed in superconductors \cite{2}. Since the publication of that seminal paper, a wealth of experimental data have been accumulated, supporting the important role played by neutron-neutron and proton-proton “Cooper pairs” in modifying many nuclear properties such as deformation, moments of inertia, alignments, etc. \cite{3,4,5}. Driven by advances in experimental techniques, the development of sensitive and highly efficient instruments and the availability of radioactive beams, the study of pairing correlations in exotic nuclei is a subject of active research in nuclear physics. Of particular interest is the competition between isovector and isoscalar “Cooper pairs” expected to occur in \(N \approx Z\) nuclei \cite{6}.

The dominant pairing in almost all known nuclei with \(N > Z\) is that in which “superconducting” pairs of neutrons (\textit{nn}) and protons (\textit{pp}) couple to a state with angular momentum \(J = 0\) and isospin \(\tau = 1\), known as isovector or spin-singlet pairing. However, for nuclei with \(N \approx Z\), neutrons and protons occupy the same single-particle orbits at their respective Fermi surfaces and pairs, consisting of a neutron and a proton (\textit{np}), may form. These types of pairs couple in either isovector or isoscalar (spin-triplet with angular momentum \(J = 1\)) and isospin \(\tau = 0\) modes, the latter being allowed by the Pauli principle. Since the nuclear force is charge independent, we expect to observe the effects of the standard \(\tau = 1\) pairing on an equal footing between the \(\tau_z = 0\) (\textit{np}) and \(\tau_z = 1\) mode, \textit{aa} (\textit{nn} and \textit{pp}) components. Furthermore, given that the nuclear force is stronger in the\( T = 0\) channel, a priori arguments suggest the existence of correlated isoscalar \textit{np} pairs. However, the effectiveness of the in-medium \(T = 0\) correlations in giving rise to a “deuteron-like condensate” remains a controversial and fascinating topic in nuclear structure physics \cite{6,7}.

2. The Experimental probe

Two-neutron transfer reactions such as \((p,t)\) and \((t,p)\) have provided a unique tool to understand neutron pairing correlations in nuclei \cite{8,9,10}. Based on the formal analogy between pairing distortions and quadrupole shape fluctuation \cite{10,11,12}, where an important measure of collective effects is provided by the reduced transition probabilities \(B(E2)'s\), one can associate a similar role to the transition operators \(\langle f|a^\dagger a^\dagger|i\rangle\) and \(\langle f|a a|i\rangle\) in the two-particle transfer mechanism between the initial \(|i\rangle\) and final \(|f\rangle\) states.

Thus, it seems natural to consider the transfer of an \textit{np} pair from even-even to odd-odd self-conjugate nuclei as a sensitive probe to study \textit{np} correlations \cite{13,14,15}. Of the possible direct reactions we could envision, the \((^3\text{He},p)\) and \((p,^3\text{He})\) are perhaps the best choice since both isoscalar and isovector transfers are allowed. As schematically showed in Fig.\,1 exclusive forward center of mass angles \((L = 0)\) cross sections, \(d\sigma_T/d\Omega(0)\)
the ratio of spin-triplet and spin-singlet superfluidity. Specifically, servable to quantify the nature of and interplay between = final odd-odd N region of superfluid nuclei between 56
56 Ni and 100 Sn. Ini-
100 Sn. Initial measurements of these reactions in N
ments which will be featured prominently at new beams in reverse kinematics done at ATLAS
sions and pair transfer, the cross-section factorizes in a structure part, , and a DWBA reaction part usually calculated with codes such as DWUCK [21] or FRESCO [22],
population of the ground states, the cross-section for two-neutron transfers from the nucleus to is approxi-
Let us start by recalling that in superfluid nuclei, where the BCS theory provides a good representa-
together with the measured cross-sections factorizes in a structure part, , and a DWBA reaction part usually calculated with codes such as DWUCK [21] or FRESCO [22],
C_{^3He,p} \approx \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle \langle 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from the Eqs. above that the limit

tion equal to \((S)\)

units, we further assume transfers of 2 nucleons in a Gordan coe-
ture information from the heavy ion. As we assume a

be obtained from the following approximation. Since the energy difference of the \(T = 0\) and \(T = 1\) low-lying states in the odd-odd final nucleus is small compared to the reaction and binding energy scales, it is expected that the reaction kinematics part will cancel out, leaving only the different probabilities of finding in the \(( nf)\) configuration, an \(np\) pair in relative \(3S_1\) or \(1S_0\) states entering in the pair form factor.

Following \[30\], the cross sections for stripping \((^3\text{He}, p)\) or pick-up \((p, ^3\text{He})\) can be reduced to the same formula:

\[
d\sigma/d\Omega_{\text{stripping}} = \frac{k_f}{2 \ell + 1} \left| \frac{d\sigma}{d\Omega_0} \right| \quad \text{(6)}
\]

\[
d\sigma/d\Omega_{\text{pick-up}} = \frac{k_f}{2 \ell + 1} \left| \frac{d\sigma}{d\Omega_0} \right| \quad \text{(7)}
\]

where:

\[
\frac{d\sigma}{d\Omega_0} = \frac{\hbar^2}{(2\pi\hbar^2)^2} \sum_{L,S_J} |C_{A,B}^{\ell,nf,j0}|^2 \sum_{M} G_{J\ell}^{\ell,nf,j0} B_{\ell}^{\ell,nf,j0}.
\]

Here, \(b_{J\ell}^{\ell,nf,j0}\) collects all spectroscopic factors and isospin coefficients of Table \[1\] of the light ions. In this case it is always 1/2 as it absorbs an extra \((2S + 1)^{-1}\) factor \[30\]. Notice also that \(L = 0\) reduces all summation to one unique term except that of \(N\). \(B_{\ell}^{\ell,nf,j0}\) is an integral of the corresponding distorted waves, the wavefunction of the particle transferred and the interaction responsible of the transfer. This is so, as this calculation is a no-remnant calculation and can be understood as an approximation of the full second-order calculation shown here. Furthermore, this integral includes all the reaction kinematics part that we will assume to be identical in both \(T = 0\) and \(T = 1\) cases.

The remaining ingredient: \(G_{J\ell}^{\ell,nf,j0}\) englobe all the structure information from the heavy ion. As we assume a single \(\ell^2\) configuration, this factor can be retained proportional to:

\[
G_{J\ell}^{\ell,nf,j0} \propto (2J + 1) \sqrt{2S + 1} \left\{ \begin{array}{c} \ell \\ \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2} \end{array} \right\} (10\ell 0; 0|n\ell t; 0)
\]

where \(|\rangle\) is a 9-\(j\) coefficient and \(|\rangle\) is a Moshinsky-Talmi bracket, where \(N = 2n + \ell\) due to selection rules so that it will also be unique. Notice that this Moshinsky-Talmi bracket is the same for both \(T = 0\) and \(T = 1\) cases.

Finally, taking into account all these factors, one can

| Reaction | \((I,T)\) | \(C_{A,B}^{\ell,nf,j0}\)^2 | \(S_{\text{DW}}^{\ell,nf,j0}\) | \(C_{A,B}^{\ell,nf,j0}\)^2 |
|----------|------------|----------------|--------------------|----------------|
| \(^3\text{He}, p\) | (0,1) | \(1/2\) | \(1/2\) | 1 |
| (1,0) | \(1/2\) | \(1/2\) | 1 |
| \((p, ^3\text{He})\) | (0,1) | \(1/2\) | \(1/2\) | 1 |
| (1,0) | \(1/2\) | \(1/2\) | 1 |
arrive to the result that:

$$\mathcal{R}_{01,2sp}^0 \approx \mathcal{R}_{01,2sp}^- \approx \frac{1}{9} \begin{vmatrix} \ell & 1/2 & j \\ \ell & 1/2 & j \\ 0 & 0 & 0 \end{vmatrix}^2$$

(8)

the estimates from this approximation are also shown in Fig. 2 to compare with those from Eq. (5). Approximated ratios find an overall agreement with calculated ones. It could be noted that $\mathcal{R}_{01,2sp}^0$ and $\mathcal{R}_{01,2sp}^-$, which are approximately equal, are different due to kinematical aspects not included in the approximation. Furthermore, an inspection of Fig. 3 where we present cross-sections and np WU’s as a function of the bumparding energy plus the Q-value to the 1$^+$ state, for representative cases of the $s_{1/2}$ and $d_{3/2}$ orbits, confirms that the ratios are stable even when the cross-sections change by factors of 10-100, and thus reflect a measure of the structural properties.

4. Summary

Inspired by the works of Refs. [19, 20] we have introduced the concept of np two-particle units (or np-Weisskopf units) to empirically assess, from the measured cross-sections and/or ratios, enhancement effects due to np-pairing correlations and the competition between the isoscalar and isovector channels.

Unit cross-sections and ratios were presented for the conditions relevant to the expected filling of the different $(n, j, \ell)$ orbits along the $N=Z$ line, from $^{16}$O to $^{100}$Sn. We believe that these units, used in the analysis of the ($^3$He,$p$) and ($p$,$^3$He) reactions, will provide a simple and robust measure of np pairing collectivity, much in the same way as the Weisskopf units for electromagnetic transitions.

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