Structural Performance of Slender RC Columns with Cross and Square-Shaped under Compression Load

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Abstract: The idea of using slender Reinforced Concrete (RC) columns with cross-shaped (+-shaped) instead of columns with square-shaped was discussed in this paper. The use of +-shaped columns provides many architectural and structural advantages, such as avoiding prominent columns edges and improved the structural response of member. Therefore, this study explores the structural response of slender +-shaped columns experimentally and numerically by nonlinear finite element analysis using Abaqus simulation tools. The results showed an excellent convergence in strength between numerical and test results with an average standard deviation of 0.05 and 0.07. Besides that, the use of +-shaped columns led to improve the ultimate strength and reduce deflections in all stages of loading, especially with the slenderness ratio more than 40, as compared with square-shaped column. Two design approaches were suggested to evaluate the ultimate strength of +-shaped columns with different slenderness ratios. Briefly, the results showed good structural response of +-shaped columns as compared with square-shaped columns, but further studies are needed to establish the behavior of this type of column, particularly with varying states of loading.

1. Introduction:
Columns are structural members which they are predominantly exposed to the axial compressive force and bending moment. The total load is transferred through columns from the superstructure to substructure, therefore, columns play an essential role in building. Most Reinforced Concrete (RC) columns in the world constructed with rectangular/square or circular shapes. Mostly, the cross-section size of these columns is much larger than the thickness of their walls, as a result, part of these columns is protruded out the wall. Reducing the cross-section size by using high strength concrete will result in long columns, thus the stability problem may be faced [1][2]. The overcoming of this problem is through the usage of columns with special cross-section in which the SR is less than that of rectangular/square columns of the same cross-sectional area. Increasing the moment of area of the column cross-section by using +-shaped section leads to reduction in SR and increasing flexural stiffness of RC column.

Using of +-shaped columns as internal columns in a building (as shown in figure 1.A) instead of rectangular or square-shaped columns (as shown in figure 1.B) results in many advantages such as avoiding prominent columns edges, increasing the usable space or floor area, easy in fashioning, more light efficiency, reducing the dead load of a building, increasing the flexural stiffness of member and cost-saving [3][4][5].
The idea of using RC columns with non-rectangular shaped is an old idea, back in the 1970s, some researchers studied the bearing capacity of T, L, U and + -shaped columns by numerical and experimental research such as [4][6][7][8][9][10][11][12]. All of these researchers focused their studies on the behavior of short columns. In contrast, the behavior of slender columns with non-rectangular shaped was not well understood, except Tsao W. H. in 1993. Tsao W. H. developed a computer program to evaluate the strength and load-deformation behavior of slender columns with L and square shaped [12]. However, there is a lack of known about the structural behavior of + -shaped column. Where there is not obtainable data about it, except Marin’s study in 1978, Marin J. has presented 32 interaction diagrams for short + -shaped columns loaded on one axis of symmetry or the diagonal direction [10].

In the present research, the structural performance of slender + -shaped columns was studied experimentally and numerically via nonlinear Finite Element (FE) analysis using Abaqus simulation tools. The behavior of + -shaped column was compared with the equivalent square-shaped of the same area of concrete and reinforcement and value of SR.

2. Experimental Program
A total of six slender RC column specimens were tested, three with square-shaped (sq.) and there with + -shaped. Their column SR equal to 34, 40 and 50. Shapes and dimensions of column specimens are shown in figure 2. The cross-sectional area of specimens was 180 cm$^2$. The specimens contained 12Φ6mm diameter deformed bars as longitudinal bars (steel ratio of 1.8%) had the yield and ultimate strength of 400 and 600 MPa, respectively. 3mm diameter mild, smooth bars used as lateral reinforcing (stirrups) spaced every 75 mm. Additional steel bars were provided for both ends of specimens to prevent ends failure.
All specimens were cast in the horizontal position. The average compressive strength of concrete was 48 MPa. Figure 3 show a column specimen placed in the loading frame. Both ends of the specimen were supported by two pinned-end conditions. Monotonically load was applied at each end of a column with no eccentricity. The lateral deflection was measured at mid-height of the specimen by Linear Variable Deflection Transformer (LVDT) of 100mm capacity.

![Figure 3. Loading frame, two hinged support, LVDT positions](image.png)

3. Finite Element Simulation of Specimens
There are two types of analysis available in Abaqus program, linear and nonlinear analysis. Linear analysis is based on two main assumptions, material and geometric linearity. Nonlinear analysis is occurring when the stress-strain behavior of materials exceeds the elastic limit and the material properties is changed. In the present study, only non-linear analysis of specimens could not be enough to express the real behavior of slender columns under concentric load, for both static general and dynamic analysis available in Abaqus because the buckling behavior cannot be obtained [13][14]. This phenomenon is occurred due to two main reasons. The first reason is that all equations of FE method are based on the equilibrium of the stresses and compatibility of strains, in other word, these equations cannot be solved due to discontinuous response at the point of buckling. Secondly, the FE model is represented as a perfect column up to complete failure.

Abaqus solved a discontinuous problem by turn it into a problem with a continuous response instead of bifurcation, which can be accomplished by presenting a geometric imperfection pattern in the perfect geometry of the column model. Abaqus offers three methods to define an imperfection. One of these methods is to apply imperfection in the input file by using the * IMPERFECTION keyword directly. This requires data, such as eigenvalue and buckling mode, these data were provided by linear elastic buckling analysis [14].

Table 1 shows characteristics of FE model for linear and nonlinear analysis. The plain concrete was molded using the concrete damage plasticity model taking into account the damage parameters. The uniaxial behavior of concrete was modeled based on the relationships proposed by Euro-code in section 3.1.5 for compression and the relation proposed by Belrbi and Hsu for tension [15] [16]. In addition, the bilinear model was used to describe the stress-strain curve of reinforcing bars using the plasticity model provided in Abaqus.
Table 1. Characteristics FE simulation models

| Characteristic          | Elastic buckling analysis                  | Post-buckling analysis                     |
|-------------------------|--------------------------------------------|--------------------------------------------|
| Materials               |                                            |                                            |
| Concrete                | Linear elastic model                       | Damage-plasticity model                    |
| Steel bars              |                                            | Plasticity model                           |
| Section                 |                                            |                                            |
| Concrete                | Solid homogenous                           | Solid Homogenous                            |
| Steel bars              | Beam as truss                              | Beam as truss                              |
| Analysis procedure      |                                            |                                            |
| Steel bars              | Embedded region in concrete                | Embedded region in concrete                |
| Load                    | Pressure load at top surface               | Displacement-control at centroid           |
| Boundary conditions     | Pinned-pinned ends as center line in strong direction | Pinned-pinned ends in centroid            |
| Mesh                    |                                            |                                            |
| Concrete                | C3D4 with size 30 mm                       | C3D4 with size 30 mm                       |
| Main bars               | D3T2 with size 30 mm                       | D3T2 with size 30 mm                       |
| Tie bars                | D3T2 with size 15 mm                       | D3T2 with size 15 mm                       |

4. Verification of Numerical Analysis Results

The failure modes of the FE analysis were compared with the observed collapse in the test as shown in figure 4. The failure in FE analysis appeared as compression damage in concrete elements, where the red color refers to maximum compression damage in concrete elements.

Besides that, the strength and lateral deflection (U) of test and FE analysis of column specimens were displayed in table 2. The ultimate strength of FE analysis (PFE) was compared with the test results (PExp). The average values of convergence were 97% and 96% and the values of standard deviation were 0.05 and 0.07 for sq- and +-shaped columns, as well as, the FE load-deflection curves were compared with the experimental load-deflection diagrams as shown in figure 5. These results show an acceptable convergence between experimental and numerical results.

![Figure 4. Experimental and FE final failure modes of specimens](image-url)
Table 2. Results of experimental test and finite element analysis

| Sp. ID | PExp. | UExp. | Failure mode | PFE | UFE. | Failure mode | P □□□ |
|--------|-------|-------|--------------|-----|------|--------------|-------|
| Sq.34  | 832   | 1.97  | C. Failure   | 769 | 1.84 | C. Failure   | 0.92  |
| Sq.40  | 716   | 3.15  | C. Failure   | 736 | 3.02 | CB. Failure  | 1.03  |
| Sq.50  | 696   | 5.91  | B. Failure   | 667 | 4.89 | CB. Failure  | 0.96  |
| Average|       |       |              |     |      |              | 0.97  |
| Standard of Deviation |       |       |              |     |      |              | 0.05  |
| Cr.34  | 829   | 2.81  | C. Failure   | 741 | 2.12 | C. Failure   | 0.89  |
| Cr.40  | 787   | 4.01  | CB. Failure  | 741 | 2.54 | C. Failure   | 0.94  |
| Cr.50  | 725   | 2.74  | C. Failure   | 747 | 5    | CB. Failure  | 1.03  |
| Average|       |       |              |     |      |              | 0.96  |
| Standard of Deviation |       |       |              |     |      |              | 0.07  |

Figure 5. Numerical and experimental load-deflection curves of specimens
5. Parametric Study

Other three limits of column SR equal to 25, 65 and 80 were studied to provide a better understanding of the behavior of slender +shaped columns. All results of these variables were shown in Table 3.

6. Investigation Study of Results

Before detailing any results, the structural behavior of columns under pure compression must be displayed. Figure 6 shows the main concept of column stability, where the critical stresses (σ) were plotted as a function of SR. The figure shown three types of columns: short, intermediate and long columns [17]. The stresses (σ) of the FE results were calculated by dividing the ultimate strength (PEF) on cross-sectional area of specimens, as shown in table 3. To provide a better understanding of behavior of these columns, figure 6 re-plotted based on FE results, as shown in figure 7. This figure shown two main points: First, the +shaped column resists more stresses than the column with sq-shaped, especially when the SR is more than 40. Secondly, when the SR is less than or equal to 40, the pure compression failure (C) happened with small values of deflection (reach to 2 mm) except +40 combined failure (Compression-Bending (CB)) occurred due to some eccentricity during the test. While sq-50 and +50 were failed with buckle (B) and compression, respectively. The compression failure has occurred in +50 because the +shaped has more stiffness than sq-shaped column. On the other hand, the buckling failure happened with large deflection (reach to 15mm) with SR is more than 65. All the FE failure modes are shown in figure 8.

![Figure 6. Critical stresses-slenderness ratio curve](image)

![Figure 7. Comparison of stress between +shaped and sq-shaped columns](image)

| Table 3. FE results of specimens |
|---------------------------------|
| Characteristic | PFE | σ | U | M | Failure |
| Sym. | L (cm) | I. factor | kN | MPa | mm | kN.m | Mode |
| Sq.25 | 100 | 10 | 774 | 43 | 1.11 | 0.86 | C |
| Sq.34 | 130 | 10 | 769 | 43 | 1.84 | 1.41 | C |
| Sq.40 | 150 | 10 | 736 | 41 | 3.02 | 2.22 | CB |
| Sq.50 | 195 | 10 | 667 | 37 | 4.89 | 3.26 | CB |
| Sq.65 | 255 | 10 | 595 | 33 | 8.52 | 5.07 | B |
| Sq.80 | 310 | 10 | 504 | 28 | 14.83 | 7.47 | B |
| +.25 | 105 | 10 | 809 | 45 | 1.26 | 1.02 | C |
| +.34 | 140 | 10 | 741 | 41 | 2.12 | 1.57 | C |
| +.40 | 165 | 9 | 742 | 41 | 2.54 | 1.88 | CB |
| +.50 | 210 | 6 | 747 | 42 | 5 | 3.74 | CB |
| +.65 | 270 | 8 | 666 | 37 | 9.81 | 6.53 | B |
| +.80 | 333 | 8 | 535 | 30 | 15.1 | 8.08 | B |

L: length of specimen, I. Factor: Imperfection factor used in FE analysis.
The secondary moment (M) at mid-height of the specimen due to the second-order effect (P-Delta) was calculated by multiplying PFE with U corresponding to the maximum load. The values of M increased with increase of SR values, as shown in table 3. The results of strength and lateral deflection at ultimate load exhibited by column specimen and listed in table 3 can be summarized as follows:

1. Increasing the SR from 25 to 80 resulted in:
   - Reduction in ultimate strength. Where the ultimate strength reached by sq-80 and +- 80 were less than those reached by sq-25 and +- 25 by approximately 34%.
   - Increasing in deflection, especially with specimens that fail by buckling as shown in figure 9.
   
   The figure shown that all stages of loading, the column with a certain value of SR has more deflection than a column with less SR.

Reduction in strength and increasing in deformations were occurring because of the P-Delta effect. Where the P-Delta effect increases with the increasing of SR which produces the additional moments as shown in table 3.

2. The use of +-shaped columns led to improve the strength and reduce deformations at all stages of loading as compared with equivalent sq-shaped columns. It was observed that the strength of column was affected slightly with SR less than 40, while this effect has begun large with SR more than 40.

Therefore, the concept of equivalent-column can be used to evaluate the strength of +-shaped column with SR does not exceed 40. The strength of +-65 was increased by about 12% as compared with Sq.65. On the other hand, the deflection of +- shaped column was less than the deflection of sq-shaped column, as shown in figure 10.
7. An Improved Design Formulas for +−-Shaped Column

According to the present study, two design approaches were suggested to evaluate the strength of +−-shaped column (\(N_{cr}\)) under pure compression with different SR based on the equation of ACI 318-19 [18]. As follow:

\[
N_{cr} = Y(0.85f - A + Af) \quad (1)
\]

\[
N_{cr} = \Psi(0.85f^\beta - A + Af) \quad (2)
\]
Where: the reduction factor was calculated as the ratio between $P_{FE}$ to the strength which calculated by ACI 318-19 (Normalized with ACI). and Coefficients of slenderness ratio and section-shape. These coefficients are shown in table 4. It is hoped that these approaches provided design aids for engineers. Eq. (2) is more conservative as compared with Eq. (3), where the ultimate strength found by Eq. (3) increasing by about 9% and 13% as a maximum for Sq and +-shaped columns.

**Table 4. Coefficients of two proposed equations**

| Slenderness Ratio | Sq-Shaped $Y = P_{FE}/P_{ACI}$ | $\Psi$ Coefficient | $\lambda$ Coefficient |
|-------------------|---------------------------------|---------------------|----------------------|
| 25                | 0.9                             | 1                   | 1                    |
| 34                | 0.9                             | 0.99                | 0.92                 |
| 40                | 0.86                            | 0.9                 | 0.92                 |
| 50                | 0.78                            | 0.84                | 0.89                 |
| 65                | 0.69                            | 0.77                | 0.82                 |
| 80                | 0.59                            | 0.65                | 0.66                 |

$A_n$, Gross section area of concrete, $A_{cn}$, $f_y$ Area and yield strength of main steel

**8. Conclusions**

The main objective of the present research is to study the structural response of slender RC columns with +-shaped and compared their response with the response of equivalent square-shaped columns. According to the test and FE results the following conclusions were listed:

i. Using +-shaped columns show good results especially with high SR as compared with equivalent sq-shaped columns. Where the strength was increased, and deformations were reduced at all stages of loading.

ii. When the SR less than 40, +-shaped has less influence on the strength, while this effect was gradually increased with SR more than 40.

iii. The increasing of SR (with the same cross-section shaped) led to reduction in ultimate strength and increasing in deflections. The maximum loss in strength was 34% in sq.80 and +.80 as compared with sq.25, +.25, respectively.

Finally, +-shaped columns shown a good structural response as compared with equivalent sq-shaped columns, but further studies are required to evaluate the structural performance of this type of columns, particularly with varying states of loading.

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