Valence shell excitations in even-even spherical nuclei within a microscopic model

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Abstract. The structure of some low-lying states in the N=80 and N=84 isotones is investigated within the quasiparticle-phonon model. The calculations reproduce the experimental levels, the electromagnetic transition strengths, and the selection rules. They account also for the M1 splittings observed in \textsuperscript{138}Ce, \textsuperscript{144}Nd and \textsuperscript{142}Ce and relate them to correlations specific of each nucleus.

1. Introduction
Low-energy spectra in nuclei are of crucial importance for understanding the correlations among valence nucleons. They include elementary modes, like the collective lowest $2^+$, but also excitations of complex nature.

These spectra fit well in the proton-neutron interacting boson model (IBM-2), which classifies the states according to the number of $s$ ($L = 0$) and $d$ ($L = 2$) bosons and the symmetry with respect to the exchange between proton and neutron bosons [1]. Strong $E2$ transitions connect states of the same symmetry (the same $F$ spin) differing by one boson, while strong $M1$ transitions connect symmetric $F = F_{\text{max}}$ to mixed symmetry ($F = F_{\text{max}} - 1$) states with the same number of bosons.

More detailed investigations have been carried out within the shell model (SM) [2, 3] and, more thoroughly, the quasiparticle-phonon model (QPM) [4, 5, 6, 7]. The focus of our QPM studies here will be the nuclei around $N = 82$, object of recent detailed experimental investigations [8, 9, 10, 11].

2. A sketch of the QPM

The QPM [12] adopts a Hamiltonian of the form

$$H = H_{sp} + V_{\text{pair}} + V_{\text{ph}}^{ph} + V_{\text{SM}}^{ph} + V_{M}^{pp}.$$  \hspace{1cm} (1)

$H_{sp}$ is a Woods-Saxon one-body Hamiltonian, $V_{\text{pair}}$ the monopole pairing, $V_{\text{ph}}^{ph}$ and $V_{\text{SM}}^{ph}$ are respectively sums of separable multipole and spin-multipole particle-hole potentials, and $V_{M}^{pp}$ is the sum of multipole pairing potentials.
In the QPM, one first expresses the Hamiltonian in terms of the quasiparticle creation and annihilation operators $a^\dagger_{jm}$ ($a_{jm}$) and, then, generates the QRPA energies $\omega_{i\lambda}$ and the corresponding phonons

$$Q_{i\lambda\mu}^\dagger = \frac{1}{2} \sum_{jj'} \left\{ \langle j|a_{j'}^\dagger a_{j}\rangle_{\lambda\mu} - (-1)^{\lambda-\mu} \langle j'|a_{j}\rangle_{\lambda'\mu'}. \right\}.$$  \hspace{1cm} (2)

These allow to reduce the quasiparticle separable Hamiltonian to the phonon form

$$H_{QPM} = \sum_{i\mu} \omega_{i\lambda} Q_{i\lambda\mu}^\dagger Q_{i\lambda\mu} + H_{eq},$$  \hspace{1cm} (3)

where the first term is the unperturbed phonon Hamiltonian and $H_{eq}$ is a phonon-coupling piece whose exact expression can be found in Ref. [12].

The phonon Hamiltonian is accordingly diagonalized in a space spanned by states composed of one, two, and three QRPA phonons. The eigenfunctions have the structure

$$\Psi_{\nu}(JM) = \sum_i R_i(\nu J) Q_{iJM}^0|0 \rangle + \sum_{i\lambda_1} P_{i\lambda_2\lambda_3}^{i\lambda_1}(\nu J) \left[ Q_{i\lambda_1\lambda_2}^\dagger \otimes Q_{i\lambda_2\lambda_3}\right]_J |0 \rangle + \sum_{i\lambda_1\lambda_2\lambda_3} T_{i\lambda_1\lambda_2\lambda_3}(\nu J) \times \left[ Q_{i\lambda_1\lambda_2}^\dagger \otimes Q_{i\lambda_2\lambda_3}\right]_J |0 \rangle,$$  \hspace{1cm} (4)

where $\nu$ labels the specific QPM excited state of total spin $JM$. The above wave functions are properly antisymmetrized according to the procedure outlined in [5].

3. Numerical details

All the strength parameters of the different pieces of the Hamiltonian have been fixed by an independent fit procedure [7]. In this respect, the calculation is to be considered parameter free.

The occurrence of at least one low-energy isovector collective QRPA $2^+$, in addition to the lowest isoscalar $2^+$, is a preliminary condition for getting QPM states that can be considered the counterpart of the IBM symmetric and mixed symmetry (MS) states. To test such a symmetry we have computed the ratio

$$\beta(2^+) = \frac{|(2^+) \sum_k r_k^2 Y_{2\mu}(\Omega k) - \sum_k r_k^2 Y_{2\mu}(\Omega k)|_{g.s.}|^2}{|2^+| \sum_k r_k^2 Y_{2\mu}(\Omega k) + \sum_k r_k^2 Y_{2\mu}(\Omega k)|_{g.s.}|^2}. \hspace{1cm} (5)$$

Clearly, a state is isoscalar ($[2^+_i]_{RPA}$) or isovector ($[2^+_i]_{RPA}$) according that $\beta(2^+) < 1$ or $\beta(2^+) > 1$. The value of $\beta(2^+)$ depends critically on the ratio $G^{(2)}/\kappa_0^{(2)}$ between the coupling constants of the quadrupole pairing and particle-hole interactions. For realistic $G^{(2)}$ and $\kappa_0^{(2)}$, at least one isovector $[2^+_i]_{RPA}$ occurs at sufficiently low energy.

3.1. $N = 80$ isotones

The second lowest $2^+$ in $^{134}$Xe and $^{136}$Ba is fairly collective and has a MS character, with the main proton and neutron amplitudes in opposition of phase. In $^{138}$Ce, instead, there are two $2^+$ states which get an appreciable $E2$ strength and have a dominant MS character. Both $2^+_2$ and $2^+_3$ states play a role and are to be treated on equal footing.

The different behavior of $^{138}$Ce with respect to the other two isotones is understood if we analyze the energies of the three lowest two-quasiparticle proton states. These are all closely spaced in $^{138}$Ce. In $^{134}$Xe and $^{136}$Ba, instead, one level is well above the other two. This is a
Figure 1. QPM versus Experimental M1 strength distribution in $^{134}$Xe, $^{136}$Ba, and $^{138}$Ce.

Figure 2. QPM $E2$ strength distribution in $^{134}$Xe and $^{138}$Ce. $^{136}$Ba has a similar behavior.

clear shell effect. In $^{138}$Ce, indeed, the $1g_{7/2}$ proton subshell is filled. Because of the diffuse Fermi surface, due to pairing, several proton configurations can enter the $2\frac{1}{2}^+$. This does not
occur in $^{134}$Xe and $^{136}$Ba, where the chemical potential is within the $1g7/2$ proton subshell, which is only partially filled.

In all three nuclei, the first isoscalar $2^+_1$ has a prominent QRPA one-phonon component, with an appreciable two-phonon piece, especially in $^{136}$Ba. The second QPM state has a dominant two-phonon component in all three nuclei.

A marked difference between the $^{138}$Ce and the other two isotones, $^{134}$Xe and $^{136}$Ba, is registered for the other states. In $^{134}$Xe and $^{136}$Ba, the third $2^+_3$ is dominated by the MS $[2^n_2]_{RPA}$ . In $^{138}$Ce, both $2^+_1$ and the $2^+_1$ contain the MS QRPA $[2^n_2]_{RPA}$ with an appreciable amplitude. We have pointed out already that in $^{138}$Ce the residual quadrupole collectivity was shared by the second and third QRPA $2^+$ states.

Apparently, the interaction between these two fairly collective states lead to the generation of two QPM states with MS character. This is reflected in the $M1$ transitions. While in $^{134}$Xe and $^{136}$Ba we have one strong $M1$ peak (first and second panels of Fig. 1), in $^{138}$Ce the $M1$ strength splits into two peaks (third panel of Fig. 1). This small splitting in $^{138}$Ce is related to the filling of the proton $1g7/2$ subshell closure in correspondence of $Z = 58$. Because of the gap with the other subshells, the low-lying proton excitations are made possible only because of the diffuse Fermi surface induced by pairing.

### 3.2. $N = 84$ isotones

A more pronounced $M1$ splitting is observed in $^{142}$Ce, having the same proton number as $^{138}$Ce, and also in $^{144}$Nd, with two more protons. Since $^{142}$Ce and $^{144}$Nd have the same neutron number, it is tempting to relate the alike $M1$ responses in these two nuclei to the two valence neutrons in excess with respect to the $N = 82$ shell closure.

A QPM study [14], which refines and expands a previous investigation [15], has been performed on these nuclei. In both $^{142}$Ce and $^{144}$Nd, the QRPA yields two low-lying collective $2^+$ states. The lowest one is $n\rho$ symmetric, while the second has mixed symmetry. The proton and neutron components are in phase in the first $2^+$ state and out of phase in the second.

Few QPM observables are compared with the experimental data in Figs. 3 and 4. As shown pictorially in Fig. 3, the strength distribution of the $E2$ transitions to the ground and $2^+_1$ states is quite similar in $^{142}$Ce and $^{144}$Nd. Concerning the $E2$ transitions to the ground state, most of the strength is concentrated into the lowest $2^+_1$ state of each nucleus at 0.641 MeV and 0.696 MeV, respectively. The remaining $E2$ strength, about 5.4 W.u. and 4.2 W.u., respectively, is distributed mainly among two or three $2^+$ states in the energy range $2 - 2.6$ MeV.

The QPM underestimates the second large peak in $^{142}$Ce and overestimates the lowest large peak in $^{144}$Nd. The overall strength is comparable with experiments in both nuclei. It is remarkable, in any case, that the calculation accounts for the main properties of the spectra.

The immediate reason of the $M1$ splitting is to be found in the mentioned composite phonon structure of the QPM states. The first prominent peak is due to the collective MS $[2^n_2]_{RPA}$ component present with large amplitude in $2^+_3$ state. The second arises from the combined presence in the fourth $2^+_1$ of the same one-phonon MS $[2^n_2]_{RPA}$ plus the two-phonon MS component $[2^+_1 \otimes 2^+_1]_{RPA}$. The two components are respectively coupled by the $M1$ operator to the symmetric one-phonon $[2^+_1]_{RPA}$ and $[2^+_1 \otimes 2^+_1]_{RPA}$, the two main components of the isoscalar QPM $2^+_1$ state.

The one- to two-phonon matrix elements of the coupling Hamiltonian are comparable in the $N=80$ isotope $^{138}$Ce and the $N=84$ isotones $^{142}$Ce and $^{144}$Nd. In $^{142}$Ce the energy of the second RPA $[2^n_2]_{RPA}$ is almost twice the energy of the lowest RPA quadrupole phonon. This small energy difference between the MS one-phonon and the symmetric two-phonon components, in both nuclei $^{142}$Ce and $^{144}$Nd, enhances considerably the coupling between these configurations and, therefore, their admixture in the QPM states leading finally to the fragmentation of the $M1$ strength. In $^{138}$Ce, instead, the energy difference is slightly larger. Moreover, the collectivity is
Figure 3. Experimental [9, 10] and QPM strength distributions of the $2^+_1 \rightarrow 0^+_g$ and $2^+_1 \rightarrow 2^+_1$ $E2$ transitions in $^{142}$Ce and $^{144}$Nd.

shared among the second and third RPA phonons $[2^+_2]_{RPA}$ and $[2^+_3]_{RPA}$, very close in energy and, therefore, strongly interacting. This mitigates their coupling with the two-phonon configuration $[2^+_1 \times 2^+_1]_{RPA}$.

In summary, the splitting mechanism in the $N=84$ isotones $^{142}$Ce and $^{144}$Nd is different from the one in $^{138}$Ce. In the latter case is associated to the filling of the proton $g_{7/2}$ subshell and the diffuse Fermi surface induced by pairing, while in $^{142}$Ce and $^{144}$Nd is a result of the phonon coupling.

The different strengths fragmentation in $N = 80$ and $N = 84$ isotones, respectively, are ultimately to be ascribed to the asymmetry between particle and hole spectra with respect to the closed shell $N = 82$. 
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