Piston Error Evaluation and Correction for Multi-aperture Imaging System

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Abstract. In this paper, the imaging quality of a three-aperture Golay3 imaging system is studied. The formula of point spread function (PSF) is derived when piston errors exist among each subaperture. The 1951 resolution target is used to analyze the imaging quality under the influence of different piston errors. To evaluate the imaging quality quantificationally, the Tenengrad function based on image gradient definition is employed. The fast steering mirror (FSM) controlled by PZT is used to eliminate the phase error in the optical structure of the Golay3 imaging system. The principle and formula of piston error correction are introduced. Using the Tenengrad function, the relationship between the movement of PZT and the image sharpness is obtained. Through the simulation of piston error correction, the piston error $p = \lambda / 2$ in the system can be completely corrected when the PZT movement $p' = 0.35\lambda$.

Keywords: Imaging quality; Co-phase error; Multi-aperture imaging system; Error correction

1. Introduction

To increase the angular resolution of a telescope system, a primary mirror with large aperture is requested. However, the development of large aperture telescope system is restricted by many factors, such as optical fabrication, uniformity of material, mechanical supporting and control. Multi-aperture imaging system is one of an efficient technique to overcome the problems. But, a good multi-aperture imaging system requires strict co-phase control on the subapertures, which presents unique multidisciplinary challenges in the areas of structural dynamics, controls and multi-aperture phasing active optics. The co-phase error among subapertures is inevitable in the process of manufacturing, installation and transportation, and it has significant influence on the imaging quality. If the co-phase condition among subapertures cannot be ensured, the multi-aperture system cannot improve but even reduce the resolution. Therefore, co-phase control on the subapertures in the multi-aperture imaging system is one of the most important tasks for increasing the imaging quality[1].

In 2007, Nicholas J Miller et al. analyzed the point spread function (PSF) and modulation transfer function (MTF) when the subapertures of a Golay-9 system had piston and tilt errors [2]. In 2016, Xiaojun He et al. carried out simulated imaging of the subapertures in the two-aperture sparse aperture system with piston and tilt errors [3]. At present, the three-aperture system’s pupil function with piston error is rarely studied and few reports have elaborated and simulated the relationship between the...
compensation amount of the system and the imaging quality when piston error existed in the three-aperture system. In this paper, the formula of the total pupil function of a Golay3 system with piston errors is deduced. An optical delay line is designed to correct the piston error of the sub-aperture. Consider the pupil function with piston error, the mathematical relationship between displacement of PZT and pupil function, compensation of PZT and imaging quality are analyzed. Furthermore, a control method based on piezoceramics is proposed.

2. Theoretical Analysis of Multi-aperture Imaging System in Ideal Conditions

The total pupil of a multi-aperture system is composed of multiple sub-pupils. The pupil function is expressed as \( P(x, y) \), then the total pupil can be expressed as:

\[
P_a(x, y) = \sum_{i=1}^{n} P_i(x-x_i, y-y_i) = P(x, y) \ast \sum_{i=1}^{n} \delta(x-x_i, y-y_i)
\]

(1)

If the entrance pupil is a circle:

\[
P(x, y) = circ(2d(x^2 + y^2))d^{-1}
\]

(2)

where \( d \) is the pupil diameter, \( circ(\bullet) \) is the circular domain function, \( (x, y) \) is the central coordinate of the sub-aperture, \( n \) is the total number of sub-apertures, and \( \ast \) represents the convolution operation. According to the Fourier transform, the complex amplitude of the multi-aperture system is:

\[
A(\xi, \eta) = F \{ P_a(x, y) \} = F \{ P(x, y) \ast \sum_{i=1}^{n} \delta(x-x_i, y-y_i) \}
\]

(3)

The point spread function can be obtained by taking the square of the complex amplitude:

\[
PSF(x, y) = |A(\xi, \eta)|^2 = |F \{ P(x, y) \ast \sum_{i=1}^{n} \delta(x-x_i, y-y_i) \}|^2
\]

(4)

Similarly, MTF of multi-aperture system can be deduced according to the point spread function:

\[
MTF(\xi, \eta) = |F \{ PSF(x, y) \}| = MTF_{sa} + n^2MTF_{sub} \ast \sum_{i=1}^{n} \sum_{j=1}^{n} [\xi - (x_i - x_j)(\lambda f)^{-1}, \eta - (y_i - y_j)(\lambda f)^{-1}]
\]

(5)

In the formula, \( (x_i - x_j, y_i - y_j) \) represents the relative position of each subaperture, \( MTF_{sa} \) represents the modulation transfer function of the subaperture, which can be expressed as:

\[
MTF_{sa}(\rho) = \begin{cases} \pi & \text{if } \rho < \frac{d}{\lambda f} \\ \frac{\pi}{\rho^2} \left[ \arccos \left( \frac{\rho \lambda f}{d} \right) - \left( \frac{\lambda f}{d} \right) \sqrt{1 - \left( \frac{\rho \lambda f}{d} \right)^2} \right] & \text{if } \rho > \frac{d}{\lambda f} \\ 0 & \text{otherwise} \end{cases}
\]

(6)

where, \( \rho = \sqrt{\xi^2 + \eta^2} \) represents the radius of any vector in the frequency domain.

It can be seen from the above formulas that the combination of multiple \( MTF_{sa} \) in the frequency domain constitutes the MTF of the multi-aperture imaging system. At the central zero frequency, \( MTF_{sa} \) of every subaperture superimposes on each other. The components of \( MTF_{sa} \) outside zero frequency distribute in different regions of frequency domain with different positions. Therefore, changing the relative position of any one subaperture in the multi-aperture system, \( MTF(\xi, \eta) \) will be changed correspondingly. Obviously, the distribution of \( MTF(\xi, \eta) \) is also depended on the array forms of subapertures. The Golay3 structure studied in this paper is shown in Figure 1.

The pupil function of Golay3 structure is:

\[
P(x, y) = circ(d/2)^{-1} \left[ \pm \sqrt{\frac{d}{2}}(D-d)^{-1} \left[ (x - \sqrt{\frac{d}{2}}(D-d)) + (y - \sqrt{\frac{d}{2}}(D-d)) \right] \right]^{1/2} + circ(d/2)^{-1} \left[ \pm \sqrt{\frac{d}{2}}(D-d)^{-1} \left[ (x + \sqrt{\frac{d}{2}}(D-d)) + (y + \sqrt{\frac{d}{2}}(D-d)) \right] \right]^{1/2} + circ(d/2)^{-1} \left[ \pm \sqrt{\frac{d}{2}}(D-d)^{-1} \left[ (x - \sqrt{\frac{d}{2}}(D-d)) + (y + \sqrt{\frac{d}{2}}(D-d)) \right] \right]^{1/2}
\]

(7)

The modulation transfer function is:

\[
MTF_{Golay} = |P(x, y)| \otimes |P(x, y)| = MTF_{sa} + \frac{1}{3}MTF_{sub} \left[ \pm \sqrt{\frac{d}{2}}(D-d)^{-1} \left[ (x - \sqrt{\frac{d}{2}}(D-d)) + (y + \sqrt{\frac{d}{2}}(D-d)) \right] \right]^{1/2} + \frac{1}{3}MTF_{sub} \left[ \pm \sqrt{\frac{d}{2}}(D-d)^{-1} \left[ (x - \sqrt{\frac{d}{2}}(D-d)) + (y - \sqrt{\frac{d}{2}}(D-d)) \right] \right]^{1/2}
\]

(8)

According to the analysis, the distribution of MTF and PSF of the Golay3 system is shown in Figure 2.
3. Influence of Piston Error on Imaging Quality

The piston error represents the phase delay of the sub-aperture relative to standard reference plane. Because of the influence factors such as machining, assembling, vibration and environmental temperature change, piston error, tilt error and pupil surface matching error exist among the sub-aperture beams [6]. Fig. 3 left shows the position error $p$ between the sub-apertures, and Fig. 3 right shows the influence of piston error $p$ on the entrance pupil surface of the beam combiner.

Ideally, Eq. (7) gives the PSF mathematical expression of the multi-aperture system. When piston error exists in the system, the pupil function of a single sub-aperture becomes:

$$P(x, y) = \text{circ}(\delta y) \exp \left( i \frac{2\pi}{\lambda} p \right)$$

Therefore, the pupil function of the three-aperture imaging system becomes:

$$P(x, y) = \text{circ}(d/2) \left[ x + \frac{\sqrt{3}}{2} (D - d) \right] \times \exp(2\pi i p_1)$$

Because piston error is a relative value, $P_i$ and $P_j$ represent the piston error value of subaperture 2 and subaperture 3 relative to subaperture 1. The point spread function of the three-aperture system can be obtained by Fourier transform of the optical pupil function of the system. To simplify the calculation and process, it is assumed that the subapertures 2 and 3 have the same piston error, i.e., $P = P_2 = P_3$. Then the point spread function in this case can be deduced from the pupil function:
where, $FT(\cdot)$ represents the Fourier transform character, and $PSF(x,y)$ is the point spread function of a single subaperture system. Analysing the above formula, the point spread function of the system is periodic. When the piston error $p = (N + 0.5)\lambda$ (where N is an integer), the value of the system point spread function is the lowest.

![Fig. 4 The system point spread function under different piston errors](image)

It can be seen from the figure that the three-aperture imaging system can get clear images of the 1951 resolution target when the piston error of the three-aperture imaging system is zero. The vertical stripe of the obtained image becomes fuzzy and the horizontal resolution decreases when the piston error is $\lambda / 2$, which is coincide with the analysis results under the condition of point target.

4. Imaging Quality Evaluation

Fig. 5 shows the imaging results of resolution targets under different piston errors intuitively. In order to provide quantitative results of the imaging quality, a performance evaluation function should be defined to conduct quantitative analysis on the clarity of the target[9]. In this paper, Tenengrad function based on image gradient is used to evaluate image quality.

The Tenengrad function use Sobel operator to extract the gradient values in the horizontal and vertical directions of pixel points. The Tenengrad function is defined as the sum of squares of the gradient of pixel points and sets a threshold $T$ to regulate the sensitivity of the gradient. The expression is:

$$
PSF(x,y) = FT[P(x,y)] = PSF_0(x,y) \left[ 2\cos\left(\frac{\pi dx}{\lambda f} + \frac{\pi dy}{\lambda p}\right) + 1 \right]^2
$$

(11)
\[ F = \sum_{i,j} (G(x,y))^2 \] (13)

where \( G(x,y) \) is the gradient at pixel \((x, y)\).

\[ G(x,y) = \sqrt{G_x^2(x,y) + G_y^2(x,y)} \] (14)

where \( G_x(x,y) \) and \( G_y(x,y) \) are the gradient values in the horizontal and vertical directions of pixel points:

\[ G_x(x,y) = f(x,y) \odot g_x \] (15)

\[ G_y(x,y) = f(x,y) \odot g_y \] (16)

where \( \odot \) is the convolution symbol. \( g_x, g_y \) is the horizontal template and vertical template of Sobel operator:

\[ g_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \] (17)

\[ g_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \] (18)

Suppose subaperture 1 as the standard reference plane, apertures 2 and 3 have the same piston error, \( p_2 = p_3 = p \), the 1951 resolution target imaging results is shown in Fig. 6 by using Tenengrad function to evaluate sharpness.

As shown in Fig. 6, when the piston error is 0, the evaluation value is the highest, it means the imaging quality is the best. With the increasing of piston error, the evaluation value decreases gradually. When the piston error increases from 0 to 0.3\( \lambda \), the evaluation value is relatively gentle. When the piston error increases from 0.3\( \lambda \) to 0.5\( \lambda \), the evaluation value drops sharply. When the piston error is 0.5\( \lambda \), the evaluation value is the least, which means the worst imaging quality. After the piston error is greater than 0.5\( \lambda \), the evaluation value increases gradually, until the piston error is \( \lambda \), the evaluation value returns to the maximum, and the imaging becomes the most clear again. The above changes are periodic with \( \lambda \). Through the above analysis, it shows that the Tenengrad function can accurately evaluate the imaging quality of surface targets with different piston errors. In practical application for the correction of piston error, the imaging quality can be restored to its clearest by controlling the integer multiple of the optical path of \( \lambda \) between each sub-path.

5. **Correction of Piston Error**

Whether the multi-aperture imaging system can realize high resolution observation ultimately depends on the error correction ability of the optical system. As shown in Fig. 7. The optical path is the solid line abc when there is no piston error in the sub-aperture. When there is a piston error \( p \) in the
sub-aperture, the fast steering mirror FSM driven by PZT is adjusted accordingly to compensate for the piston error \( p \) to ensure that the optical path remains constant. Assuming that the compensation is \( p' \), it can be seen from the figure that no matter what the value of \( p' \) is, there is \( bcd = b'c'd' \). When the motion of \( p' \) can make \( ab = a'b' \), the piston error \( p \) of subaperture can be compensated. In order to make \( ab = a'b' \), it should be: \( p' = p \times \cos 45^{\circ} \), that is, when the compensation value \( p' = p \times \cos 45^{\circ} \) \( abcd = a'b'c'd' \). That is, when the z-axis movement of piezoelectric ceramics \( p' = p \times \cos 45^{\circ} \), the piston error \( p \) of subaperture can be compensated.

![Fig.7 Piston error correction principle](image)

The micro displacement of PZT will change the pupil function of sub-aperture. The relationship between the displacement of PZT and the pupil function can be deduced from Formula 9 and expressed as

\[
P(x, y) = \text{circ}(\bullet) \exp \left\{ 2\pi i \lambda^{-1} \left[ p - p'(\cos 45^{\circ})^{-1} \right] \right\} \tag{19}
\]

where, \( p(x, y) \) is pupil function, \( \text{circ}(\bullet) \) is circular domain function, \( p \) is piston error of a single aperture, \( p' \) is displacement of piezoelectric ceramics. According to the formula, when the z-axis displacement of piezoelectric ceramics is \( p' = 0 \) which means there have no piston error compensation, the optical pupil function expressed by Eq. (19) is consistent with that of Eq.(9). When the z-axis displacement of piezoelectric ceramics is \( p' = p \times \cos 45^{\circ} \) which means the piston error of sub-aperture can be fully compensated, the optical pupil function expressed by Eq. (19) is consistent with which of Eq. (2). Taking the piston error \( p = \lambda / 2 \) image as the research target, with Z displacement of the piezoelectric ceramics moves from \( p' = 0 \) to \( p' = \lambda \), the corrected imaging results are shown in Figure 8.

![Fig. 8 Imaging results after piston error correction](image)

Using the image sharpness evaluation function of Tenengrad to evaluate the corrected images, the relationship between the adjustment value \( p' \) of PZT and the evaluation value is obtained.As shown in Fig. 9, the image sharpness is the lowest when the piezoelectric ceramics are not moving \( p' = 0 \). With the motion of piezoelectric ceramics, the image sharpness gradually becomes clearer. When the motion of PZT \( p' = 0.35\lambda \), the image sharpness is the best and the piston error can be completely corrected.
6. Summary
In this paper, the influence of piston error on imaging quality is analyzed. The existence of piston error will lead to the decrease of imaging clarity. When piston error $p = \frac{\lambda}{2}$, the imaging quality is the worst. Through piston error correction design and using the evaluation value of Tenengrad image sharpness evaluation function as the definition evaluation standard, the relationship between PZT correction and image sharpness is obtained. Taking 1951 resolution target imaging results as the research target, when the motion of piezoelectric ceramics $0.35\lambda$ = $0.35\lambda$, the piston error in the system can be completely corrected. The research contents and results of this paper provide some theoretical reference for piston error correction of multi-aperture imaging system and some data support for motion control of piezoelectric ceramics.

References
[1] Zongliang Xie, Haotong Ma, Bo Qi, Ge Ren, Xiaojun He, Li Dong, Yufeng Tan, Shan Qiao. Optical sparse aperture imaging with faint objects using improved spatial modulation diversity[J]. Scientific Reports, 7(2017) 17844-7.
[2] Nicholas J Miller, Matthew P Dierking, Bradley D Duncan. Optical sparse aperture imaging[J]. Appl Opt, 46(2007) 5933-5943.
[3] Xiaojun He, Haotong Ma, Chuanxin Luo, Simulation of co-phase error correction of optical multi-aperture imaging system based on stochastic parallel gradient decent algorithm, C. International Symposium on Advanced Optical Manufacturing and Testing Technologies. 96820 (2016) 1-8.
[4] Soon-Jo, C. and David, W. M., ARGOS testbed: study of multidisciplinary challenges of future space borne interferometric arrays, J. Opt. Eng., 43(2004), 2156-2167.
[5] Junlun Jiang, Weirui Zhao, Baoyun Sun. Piston phasing in segmented telescopes, C. Optics Express, 24 (2016). 19123-19137
[6] E. A. Shields, Phase diversity with undersampled systems via superresolution preprocessing, Opt. Lett., 37 (2012). 2463–2465
[7] Naiguang Lv, Fourier optics. second ed., China Machine Press, BeiJing, 2010.
[8] Zongliang Xie, Haotong Ma, Bo Qi, Ge Ren, Jianliang Shi, Xiaojun He, Yufeng Tan, Li Dong, Zhipeng Wang. Experimental demonstration of enhanced resolution of a Golay3 sparse-aperture telescope, J. Chinese Optics Letters, 15(2017), 0411011-4.
[9] Wang Da-yong, Han Ji, Liu Han-cheng, Fu Xi-yang, Guo Hong-feng, Tao Shi-quan, Imaging of Optical Sparse Aperture Systems and Evaluate Method J. Acta Photonica Sinica, 37(2004), 1208-1212.