Application and Simulation of Fourier Analysis in Communication Circuit

Zhang Jing¹*, Tao Binbin¹
School of Electrical and Information, Wanjiang University of Technology, Maanshan, Anhui, 243031, China
e-mail: tbv1977@dingtalk.com

Abstract. This article mainly studies the application of Fourier analysis theory in wireless communication circuits. Firstly, the relevant basic theories of Fourier analysis are introduced, including the decomposition of periodic signal, the modulation characteristic of Fourier transform, and the system function; then the relevant applications of Fourier theory in communication circuits are introduced, such as high-frequency resonant power amplifier, amplitude modulation, demodulation and mixing, low-pass filter. Finally, the application of Fourier theory in communication circuits is simulated and analyzed through Multisim circuit simulation software.

1. Introduction
Fourier analysis is an important branch of analytical mathematics. From the history of mathematics development, as early as the beginning of the 18th century, D. Bernoulli, L. Euler and others once discussed trigonometric series in their work. However, the really important step was taken by the French mathematician J. Fourier. In 1822, he systematically used trigonometric series and trigonometric integrals to deal with heat conduction problems in his book "The Analytical Theory of Heat"[1]. After that, scientists such as Dirichlet, Riemann, Lipschitz and Jordan have been engaged in research in this field. Their research results not only made up for the deficiencies in Fourier's work, but also greatly developed the series theory and expanded the Fourier analysis. The application range of Fourier analysis theory has been developed rapidly.

Fourier analysis is not only a mathematical tool, but its vigorous development has made it widely used in many disciplines such as physics, electronic science, statistics, signal processing, cryptography, optics, medical, oceanography, structural mechanics[2]-[8], etc. It plays an extremely important role not only in the fields of communication and control, but also in the fields of earthquake, nuclear science, biomedicine, and electric power engineering. In the information age, communication is inseparable from people's lives. Communication technology has been greatly developed, and the development of communication technology is accompanied by the careful application of Fourier analysis! Such as modulation, power amplification, filters, sampling, frequency division multiplexing and so on.

This article mainly studies the application of Fourier analysis theory in wireless communication circuits, and uses Multisim circuit simulation software to simulate and analyze the specific applications of Fourier analysis in communication circuits, and then understand the application of Fourier analysis theory in communication circuits intuitively and vividly. The article is mainly divided into three parts: one is the introduction of Fourier analysis theory, the other is the application theory of Fourier analysis in communication circuits, and finally, the application and analysis of Fourier
transform in communication circuits are visually demonstrated through Multisim simulation. Multisim circuit simulation can not only carry out circuit design, but also carry out various analyses[13].

2. Basic theory of Fourier analysis

2.1. Decomposition of periodic signals

Suppose a periodic signal \( f(t) \) has a period of \( T \) and an angular frequency of \( \omega = \frac{2\pi}{T} \).

When this signal satisfies Dirichlet condition, it can be decomposed into the following trigonometric series - referred to as Fourier series. series-called the Fourier series[10].

\[
f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\Omega t) + \sum_{n=1}^{\infty} b_n \sin(n\Omega t)
\]

\( a_n, b_n \) are called Fourier coefficients.

\[
a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos(n\Omega t) \, dt \quad b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin(n\Omega t) \, dt
\]

Combining formula (1) with the frequency term can be written as:

\[
f(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos(n\Omega t + \varphi_n)
\]

Where \( A_0 = a_0 \), \( A_n = \sqrt{a_n^2 + b_n^2} \), \( \varphi_n = -\arctan \frac{b_n}{a_n} \).

Equation (3) shows that the periodic signal \( f(t) \) can be decomposed into the form of superposition of DC and many cosine components.

Where, \( A_0 / 2 \) is the DC component. \( A_1 \cos(\Omega t + \varphi_1) \) is called the fundamental or first harmonic, The angular frequency \( \Omega \) is the same as the original periodic signal frequency, \( A_2 \cos(2\Omega t + \varphi_2) \) is called the second harmonic component, In turn, \( A_n \cos(n\Omega t + \varphi_n) \) is called the Nth harmonic component.

2.2. Fourier transform frequency shift characteristics-modulation properties

If the Fourier transform of signal \( f(t) \) is \( F(j\omega) \), denoted as \( f(t) \longleftrightarrow F(j\omega) \), then

\[
f(t) e^{j\omega_0 t} \longleftrightarrow F[j(\omega - \omega_0)]
\]

\[
f(t) e^{-j\omega_0 t} \longleftrightarrow F[j(\omega + \omega_0)]
\]

(4) and (5) are called the frequency shift characteristics of Fourier transform. That is: the time domain is multiplied by the imaginary exponential factor \( e^{j\omega_0 t} \) or \( e^{-j\omega_0 t} \), and the frequency domain is shifted by \( \pm \omega_0 \).

Euler's formula:

\[
\cos \omega_0 t = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}
\]

So

\[
f(t) \cos \omega_0 t = f(t) \left[ \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t} \right] \longleftrightarrow \frac{1}{2} F[j(\omega - \omega_0)] + \frac{1}{2} F[j(\omega + \omega_0)]
\]
It can be seen from the formula (7) that if an arbitrary signal \( f(t) \) is multiplied by a cosine signal \( \cos(\omega_0 t) \), the frequency spectrum of the original signal is shifted to the left and right by \( \omega_0 \), and the amplitude is halved.

### 2.3. Frequency response function-system function (transfer function)

The system function \( H(j\omega) \), also called the transfer function \( H(j\omega) \), and the frequency response function \( \tilde{H}(j\omega) \), is the ratio of the Fourier transform of the zero-state response \( y_{zs}(t) \) to the Fourier transform of the input \( f(t) \), that is,

\[
H(j\omega) = \frac{Y(j\omega)}{F(j\omega)},
\]

The amplitude-frequency characteristics and phase-frequency characteristics of an ideal low-pass filter are shown in figure 1.

\[
H(j\omega) = \begin{cases} 
\frac{e^{-j\omega C \tau}}{1-\frac{j\omega}{\omega_c}}, & |\omega| < \omega_c \\
0, & |\omega| > \omega_c 
\end{cases} 
\]

\[
|H(j\omega)| = \begin{cases} 
1, & |\omega| < \omega_c \\
0, & |\omega| > \omega_c 
\end{cases} 
\]

Figure 1. Low-pass filter amplitude-frequency characteristic diagram

### 3. Basic composition of communication system

The wireless communication system mainly includes modulation, demodulation, high frequency resonant power amplifier, high frequency small signal amplifier, mixer, oscillator, frequency multiplier, filter. Among them, amplitude modulation, demodulation, and frequency mixing are all modulation characteristics of Fourier transform. High-frequency resonant power amplifier and frequency multiplier use the series decomposition theory of periodic signals. The filter is designed using the system function and the principle of low-pass filter. Fourier theory is fully applied in various functional circuits of wireless communication.

#### 3.1. High frequency resonant power amplifier

High-frequency power amplifier, also known as Class C power amplifier, is an important part of the transmitting end of the wireless communication system. Its function is to amplify the signal to be transmitted to generate the required power to meet the antenna and load requirements [11] [12]. The high-frequency resonant power amplifier is an energy conversion device that can convert the DC energy supplied by the power supply into a high-frequency AC output.

##### 3.1.1. Circuit composition

The working principle of the high-frequency power amplifier is shown in Figure 2. It mainly includes three parts: input loop, amplifier, and collector output loop. In the base loop, \( V_{BBI} \) is the base bias voltage, which is used to make the amplifier work in a Class C state, and its value is usually zero or
negative. In the collector circuit, the load selects the LC resonant circuit, which is tuned to the center frequency of the input signal to achieve high-frequency amplification. Tuned to an integer multiple of the input signal frequency, it can be used as a frequency multiplier. $R_L$ is the external equivalent load resistance of the power amplifier. $V_{CC}$ is the collector DC power supply.

Figure 2. Schematic diagram of high-frequency power amplifier

Figure 3. Transistor transfer characteristic curve

3.1.2. Voltage and current waveforms

Set the voltage $u_i = V_{in} \cos \omega t$ of the input signal, then the effective voltage applied between the base of the transistor and the emitter is $u_{BE} = u_i + V_{BB} = V_{in} \cos \omega t + V_{BB}$, which can be seen from the transfer characteristic curve of the transistor (Figure 3): when $u_{BE} < V_{BZ}$, $i_c = 0$; $u_{BE} > V_{BZ}$, $i_c = g_c (V_{in} \cos \omega t + V_{BB} - V_{BZ})$, $i_c$ is the cosine pulse.

\[
\text{at that time } \omega t = \theta, i_c = 0, \text{ at that time } \omega t = 0, i_c = i_{c_{\text{max}}}
\]

\[
\cdots i_c = g_c \left[ V_{in} \cos \omega t - (V_{BZ} - V_{BB}) \right] = g_c \left[ V_{in} \cos \omega t - V_{BZ} \cos \theta \right] = g_c V_{in} \left[ \cos \omega t - \cos \theta \right]
\]

Mathematical expression of peaked cosine pulse:

Figure 3 shows that the input is a cosine signal, and the output is a periodic cosine pulse signal.

The cosine pulse is a periodic signal, which satisfies the Dirichlet condition, so the Fourier series expansion is

\[
i_c = I_{c0} + I_{c1} \cos \omega t + I_{c2} \cos 2\omega t + \cdots + I_{cn} \cos n\omega t + \cdots
\]

Using the calculation formula of Fourier coefficients, the coefficients can be obtained as:

\[
I_{c0} = \frac{1}{2\pi} \int_{-\theta}^{\theta} i_c \cos \omega t \, d(\omega t) = \frac{i_{c_{\text{max}}}}{\pi} - \frac{\sin \theta - \theta \cos \theta}{1 - \cos \theta} = i_{c_{\text{max}}} \alpha_0(\theta_c)
\]

\[
I_{c1} = \frac{1}{2\pi} \int_{-\theta}^{\theta} i_c \cos \omega t \, d(\omega t) = i_{c_{\text{max}}} \left( \frac{1}{\pi} - \frac{\theta - i_m \cos \theta}{1 - \cos \theta} \right) = i_{c_{\text{max}}} \alpha_1(\theta_c)
\]

\[
I_{c2} = \frac{1}{2\pi} \int_{-\theta}^{\theta} i_c \cos 2\omega t \, d(\omega t) = i_{c_{\text{max}}} \left( \frac{2}{\pi} \frac{\sin \theta \cos \theta - \theta \sin \theta}{n^2 - 1} \right) = i_{c_{\text{max}}} \alpha_2(\theta_c)
\]

In formulas (11)-(13), $\alpha_0(\theta_c)$, $\alpha_1(\theta_c)$, $\alpha_n(\theta_c)$ are called the decomposition coefficient of the peak cosine pulse. Usually you can look up the table based on the value of $\theta_c$ to find the value of each decomposition coefficient.

$I_{c0}$, $I_{c1}$, and $I_{cn}$ are the amplitudes of DC, fundamental and nth harmonics respectively.

Through the collector LC frequency selection network, the fundamental wave signal can be taken out, and high-order harmonics and direct current can be filtered out to obtain a distortion-free cosine signal,
that is, the fundamental frequency signal. Compared with the input signal, the fundamental frequency signal has the same frequency and reverse phase, and the amplitude is amplified.

3.2. Modulation, demodulation, and mixing theory

Modulation is an indispensable and very important module in wireless communication. It linearly moves low-frequency signals to high-frequency places, which not only facilitates antenna transmission, but also facilitates multiplexing.

Let the modulation signal (low frequency signal) be a cosine signal with a single frequency, the modulation voltage is $u_m(t) = U_m \cos \Omega t$, the carrier voltage is $u_c(t) = U_c \cos \omega_c t$, usually the carrier frequency meets the modulation signal $\Omega \gg \omega_c$.

According to the definition of amplitude modulation, during amplitude modulation, the frequency and phase of the carrier remain unchanged, and there is a change $\Delta u$ in amplitude, and this change changes linearly with the modulation signal $u_m(t)$, that is, $\Delta u$ is proportional to the modulation signal $u_m(t)$. The amplitude of the modulated signal:

$$u_{AM}(t) = U_c(1 + m_u \cos \Omega t) \cos \omega_c t$$

among them, $m_u = \frac{k_u U_m}{U_c}$, is the modulation index, $U_c(1 + m_u \cos \Omega t)$ is called the envelope of the amplitude modulation signal, which is consistent with the baseband signal (modulation signal). In the formula $u_{AM}(t) = U_c \cos \omega_c t + m_u U_c \cos \Omega t \cos \omega_c t$, the low-frequency signal $\cos \omega_c t$ is multiplied by the high-frequency signal $\cos \Omega t$, and the baseband signal is multiplied by $\cos \omega_c t$ in the time domain, and the frequency domain is equivalent to shifting left and right by $\omega_c$, thereby realizing modulation and spectrum shifting.

The formula (16) are spread:

$$u_{AM}(t) = U_c \cos \omega_c t + \frac{1}{2} m_u U_c \cos(\omega_c + \Omega) t + \frac{1}{2} m_u U_c \cos(\omega_c - \Omega) t$$

Explain that the modulation signal contains three parts: carrier frequency $\omega_c$, upper frequency $\omega_c + \Omega$, and lower frequency $\omega_c - \Omega$. That is, $\Omega$ moved to $\omega_c$.

Demodulation is to move the frequency spectrum of the modulated signal from the high frequency to the low frequency, which is a linear movement of the frequency spectrum. So just multiply the AM signal by $\cos \omega_c t$ to achieve demodulation. Mathematical expression:

$$u_{AM}(t) \cdot \cos \omega_c = U_c(1 + m_u \cos \Omega t) \cos \omega_c t \cdot \cos \omega_c t$$

$$= U_c(1 + m_u \cos \Omega t) \cdot \frac{1}{2}(1 + \cos 2\omega_c t)$$

The modulated signal can be recovered by passing the signal of formula (17) through the low-pass filter.

Mixing only changes the center frequency of the modulated wave signal and does not change the spectrum structure, but only moves the modulated wave signal from the high frequency to the intermediate frequency. The mathematical expression is
By passing equation (18) through one side band filter, the mixed intermediate frequency signal can be obtained.

Modulation, demodulation, and mixing all use the frequency shift characteristics (modulation characteristics) of the Fourier transform to realize the shift of the frequency domain spectrum by multiplying the cosine signal in the time domain.

3.3. Low-pass filter

Low-pass filters are commonly used filters to remove noise. Butterworth low-pass filters have maximum flat amplitude-frequency response curves in the passband, and there is no ripple in the passband, so they are widely used in signal denoising.

The amplitude-frequency characteristics of Butterworth low-pass filter are[9]:

$$\left| \frac{A_u(j\omega)}{A_{uo}} \right| = \frac{1}{\sqrt{1 + \left( \frac{\omega}{\omega_c} \right)^{2n}}}$$

Among them, $\omega_c$ is the cut-off angle frequency, $A_{uo}$ is the voltage amplification in the passband, and $n$ is the order of the filter. It can be found from equation (19) that when $\omega = 0$, it has the maximum value. When $\omega = \omega_c$, equation (19) is equal to 0.707, that is, $A_u(j\omega)$ is attenuated by 3dB. The larger the $n$, the faster the attenuation, and the more ideal the amplitude-frequency characteristics.

When

$$\omega > \omega_c,$$ (20)

Formula (20) takes the logarithm to obtain

$$20 \left| \frac{A_u(j\omega)}{A_{uo}} \right| \approx -20 n \log_{10} \frac{\omega}{\omega_c}$$

The attenuation frequency of the stop band is 20ndB every 10 times the frequency.

Figure 4 is a second-order low-pass active filter. High-order filters can be cascaded through low-order filters.
As shown in Figure 4:
Passband magnification:
\[
A_{up} = \frac{U_0}{U_p} = 1 + \frac{R_2}{R_1} \quad (f = 0)
\]
Transfer function:
\[
A_e = \frac{A_{up}}{1 - \left(\frac{f}{f_0}\right)^2 + j(3-A_{up})\frac{f}{f_0}} = \frac{A_{up}}{1 - \left(\frac{f}{f_0}\right)^2 + \frac{1}{Q}\frac{f}{f_0}}
\]

among them, Quality factor \( Q = \frac{1}{3-A_{up}} \), Cut-off frequency \( f_0 = \frac{1}{2\pi RC} \) \( (C_1 = C_2 = C) \);

4. Multisim simulation

4.1. High frequency resonant power amplifier
Figure 5 is the circuit diagram of the designed high-frequency resonant power amplifier. The circuit includes three parts: the basic input circuit, the amplifier, and the collector circuit. The collector load adopts two kinds of loads, the switch of pure resistance load and LC resonant circuit is controlled by single-pole double-throw switch.

Turn the switch to pure resistance \( R_2 \) and observe the collector output waveform as shown in Figure 6. It can be seen from figure 6 that the input is a cosine signal, and the collector output is a periodic cosine pulse with the same period as the input signal. This periodic pulse satisfies the Dirichlet condition and can be decomposed into a Fourier series, decomposed into a form of superposition of many cosine signals. Through Multisim-Analysis-Fourier analysis, you can observe the decomposed frequency spectrum as shown in figure 8. From figure 7 and figure 8, it can be found that the input signal is a single frequency, and the output signal contains many frequency components and is an integer multiple of the input signal.

Set the switch to the LC resonant circuit, click Simulate, and observe the output waveform as shown in figure 9. From figure 9, it can be found that the output signal and the input signal have the same waveform, the same cosine signal, and the frequency is the same, both are 1MHz, and the output signal has a larger amplitude than the input signal, achieving high-frequency amplification. The output spectrum is shown in figure 10, which is a single-frequency signal. Because the LC resonant circuit has a frequency selection function, the fundamental frequency signal is amplified without distortion, so the output and input signal frequency is the same, and the output signal amplitude becomes larger.
Figure 5. Circuit diagram of high frequency resonant power amplifier

Figure 6. Collector output signal waveform (load is pure resistance)

Figure 7. The frequency spectrum of the input signal
Figure 8. Collector output signal spectrum

Figure 9. Collector output signal waveform
4.2. Modulation

Figure 11 shows the diode balance modulation circuit, the modulation signal (low frequency signal) is 1kHz, and the high frequency carrier signal is 100kHz. Click Simulate to get the AM amplitude modulation signal waveform as shown in figure 12. It can be seen from the waveforms in figure 12 that the envelope of the AM amplitude modulation signal is consistent with the low frequency signal, achieving amplitude modulation. Use Multisim-analyses-Fourieranalysis to observe the frequency spectrum of the input low-frequency signal and output AM signal, as shown in figure 13 and figure 14. It can be seen from figure 13 and figure 14 that the input is a single spectrum, and the output AM signal spectrum is two ways that the input signal spectrum is linearly shifted to the high-frequency signal.

Figure 10. Collector output signal spectrum

Figure 11. Diode balance modulation circuit

Figure 12. AM amplitude modulation signal waveform
4.3. Butterworth low-pass filter

Figure 15 shows the second-order Butterworth low-pass filter with a cut-off frequency of 1kHz. Observe the transfer function and amplitude-frequency characteristic curve with a Baud meter. The cut-off frequency is calculated by the formula, 

\[ f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 3.4 \times 10^9 \times 47 \times 10^{-9}} = 996\, \text{Hz} \]

Close to 1kHz. Gain:

\[ G = 1 + \frac{R_4}{R_3} = 1.5882(4.018\, \text{dB}) \]

Figure 16 shows the amplitude-frequency characteristic curve of the Butterworth filter. From the amplitude-frequency diagram, it can be seen that the passband is flat without ripples, and the stopband drops slowly. The amplitude at 996Hz drops 1.016V=15.7272mV at 10 times the stopband, which is 37dB lower than that at 1kHz. It can be seen from the scanning curve that it is 1.1204V at 1kHz and 1.1228V at 10kHz. The signal amplitude in the passband is 1.5882V, and the cut-off frequency should be 1.5882*0.707=1.1228V. It can be seen from the scanning curve that it is 1.1204V at 1kHz and 15.7272mV at 10 times the stopband, which is 37dB lower than that at 1kHz. 40dB lower than the passband.
5. conclusion

Fourier analysis theory has extremely important applications in various fields. This paper studies the related theory of Fourier analysis and its application in wireless communication circuits, and simulates and analyzes the application of Fourier analysis theory in communication circuits. Through Multisim circuit simulation software, and the application of Fourier theory in communication circuits is displayed intuitively and vividly.

References

[1] Pan Wenjie. (2000) Fourier analysis and its application, Peking University Press, Beijing.
[2] Liu Jun. (2017) Fourier analysis and wavelet de-noising test of danger signals of landslides and debris flows, Science and Technology Bulletin, 3, 33, 141-144
[3] Dong Liang, Yao Zhilin, Ge Caigang. (2021) Fourier analysis of the characteristics of pipe-to-ground potential fluctuation under the interference of subway stray current. Demonstration technique, 2, 50: 294-303
[4] Zhang Jie, Wang Zhuo, (2010) Correlation analysis of stock market UHF data based on Fourier analysis, Mathematical Practice and Understanding, 7, 40: 63-67.
[5] Liu Zhengxian, Wang Shengling. (2019) The application of spatial Fourier analysis in the recognition of centrifugal impeller stall signal, Journal of Tianjin University (Natural Science and Engineering Technology Edition), 4, 52: 353-360
[6] Wu Yunlong, Nie Jinsong, Shao Li, Sun Xiaoquan. (2017) Using Fourier analysis to study the far-field propagation characteristics of Airy beams, Journal of Photonics, 3, 46: 1-12.
[7] Zhang Leiyan. (2020) Application of Short-Time Fourier Transform in Electrical Engineering, Information Technology and Informatization, 10: 113-114

[8] Huang Guo. (2020) Application of Discrete Fourier Transform in Medical Image, Electronic World, 11: 163-164

[9] Li Yanzhe, Guan Enming, Zhang Xianmin. (2010) Butterworth active low-pass filter design, Heilongjiang Science and Technology Information, 15,27: 15

[10] Wu Dazheng. (2005) Signal and Linear System Analysis, Higher Education Press, Beijing

[11] Zhang Suwen. (2009) High-frequency electronic circuit. 5th edition, Higher Education Press, Beijing.

[12] Zeng Xingwen. (2017) High-frequency electronic circuits. 3rd edition, Higher Education Press, Beijing.

[13] Liang Qing, Hou Chuanjiao, etc. (2012) Multisim11 circuit simulation and practice. Tsinghua University Press, Beijing.