Application of Dynamic Sarma Method to Stability Analysis of Rock Slope

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Abstract. In this paper, the shortcoming of Sarma method, a slope safety factor calculation method based on pseudo-static method, are improved by adding real seismic force, calculating acceleration amplification coefficient of each block by using numerical simulation and drawing the time-history of safety factor by using matlab software. The dynamic Sarma method is proposed. Through the calculation of an example, the rationality of the calculation result of slope safety factor by using dynamic Sarma method is verified.

1. Introduction
Sarma method was introduced in 1975[1]. The basic idea is that the rock and soil of the slope can only move as a complete rigid body unless it slips along an ideal circular arc. Otherwise, the rock and soil must be destroyed into several relatively sliding blocks before sliding which means that shear stress occurs inside the stroke. This method is actually an analytical method that satisfies both the balance of force and the balance of torque. After several decades of research and practice by various experts and scholars, the Sarma method has become one of the mainstream methods to calculate the safety factor of slope. Sarma method was used by Zhang Qiangyong[2] to study the stability of high slope with anchorage, Ma Shuzhi[3] uses Sarma method to analyze the excavation process of anchorage slope. The Sarma method was improved by Zhang Li[4], and it can be applied to the stability calculation of tunnel surrounding rock. Based on the original Sarma method, Li Jianming[5] adds the vertical seismic force, and the slope stability under different direction seismic force could be calculated and analyzed. The seismic force of Sarma method is calculated by quasi-static method, so the defects of Sarma method is obvious. Seed said[6]: (1) the inertia force is not permanent or unidirectional, but has rapid fluctuations in magnitude and direction, (2) even if the stability coefficient of the slope is temporarily less than 1, it will not necessarily lead to the overall instability of the slope, but will only lead to a certain permanent displacement of the slope. In addition, the seismic acceleration varies with time, and the distribution of seismic horizontal acceleration on the slope at a certain time is also uneven, which has obvious amplification effect with the increase of slope height. The vertical seismic acceleration also has the amplification effect, but when the height is higher than 2/3 of the slope, it is less than the horizontal seismic acceleration, which is opposite in the middle and lower part of the slip mass[2].

Based on this, the Sarma method should be improved. First, the real seismic force is added. Second, acceleration magnification factor of each block is analyzed by numerical simulation. The peak value of seismic acceleration of different blocks is properly amplified, and it can satisfy the law of the distribution of seismic acceleration on the slope. The time-history diagram of slope safety factor is drawn by using matlab software, which can reflect the spectrum and time-holding characteristics of slope safety factor under the continuous action of earthquake.
2. Computational Model of Dynamics Sarma Mechanics

A rock slope is divided into several sliders, and the indirect contact surface of the block is inclined. The Fig.1 is the calculation diagram of any time of the improved Sarma method. The horizontal seismic force $Q_i$ and the vertical seismic force $H_i$ is added on the basis of the original method. So, there are two kinds of seismic forces acting on the slope. One is the inertial force $K_i W_i$ of horizontal seismic force by pseudo-static method. Another one is the horizontal and vertical seismic forces at any time of earthquake.

Figure 1. Calculation diagram of dynamic Sarma method

3. Derivation of the dynamic Sarma method

According to the balance of the vertical and horizontal forces of the block, the equation can be obtained:

$$T_i \cos \alpha_i - N_i \sin \alpha_i = K_i W_i + X_{i+1} \cos \delta_{i+1} + X_i \sin \delta_i + E_i \cos \delta_i - E_{i+1} \sin \delta_i + Q_i - F_i \sin \theta_i$$

$$N_i \cos \alpha_i + T_i \sin \alpha_i = W_i + X_{i+1} \cos \delta_{i+1} - X_i \cos \delta_i - E_{i+1} \sin \delta_i - E_i \sin \delta_i - H_i + F_i \cos \theta_i$$

The $E, X$ are normal force and tangential force acting on the block, The $W_i$ is the gravity of block, $H_i$ and $Q_i$ are the vertical and horizontal seismic forces at any time, $F_i$ is the external force applying on the block such as anchoring force, The $\alpha_i$ is the angle between the bottom surface and the horizontal plane of block, The $\delta_i$ is the dip angle of the left interface of the block. And taking the lead straight line as the starting line, clockwise as the positive, counterclockwise as the negative, $\theta$ is as the angle between the external force and the vertical plane. And taking the lead straight line as the starting point, clockwise as the negative, counterclockwise as the positive. According to the Mohr-Coulomb failure criterion, the equation can be obtained:

$$T_i = (N_i - U_i) \tan \phi_i + c_i b_i \sec \alpha_i$$

$$X_i = (E_i - PW_i) \tan \phi_i / c_i / d_i$$

$$X_{i+1} = (E_{i+1} - PW_{i+1}) \tan \phi_i / c_i / d_{i+1}$$

The $\Phi_i$ is the average internal friction angle at the left interface of the block. The $\Phi_i$ is the average internal friction angle at the bottom of the block. The $c_i$ is the average cohesive force at the left interface of the block, The $c_i$ is the average cohesive force on the bottom surface of block. The $b_i$ is the bottom width of block. The $d_i$ is the length of the left interface of a block. The $U_i$ is the water pressure acting on the bottom of a block, The $PW_i$ is the water pressure acting on the block interface. Adding
equation (3)–(5) into equation (1) and equation (2) to eliminate $T_i, X_i, X_{i+1}, N_i$, the equation could be obtained:

$$ E_{i+1} = a_i + E_i e_i - p_i K_c $$

The equation (6) is a cyclic formula, which can be obtained:

$$ E_{n+1} = a_n - p_n K_c + e_n E_n 
= a_n - p_n K_c + e_n (a_{n-1} - p_{n-1} K_c + e_{n-1} E_{n-1}) 
= (a_n + a_{n-1} e_n) - (p_n + p_{n-1} e_n) K_c + e_ne_{n-1} E_{n-1} $$

Further calculation is obtained:

$$ E_{n+1} = (a_n + a_{n-1} e_n + a_{n-2} e_n e_{n-1} + \cdots + a_1 e_n e_{n-1} \cdots e_2) 
- K_c (p_n + p_{n-1} e_n + \cdots + p_1 e_n e_{n-1} \cdots e_2) + E ne_{n-1} \cdots e_1 $$

Assuming no external load, the $K_c$ could be calculated:

$$ K_c = \frac{a_n + a_{n-1} e_n + a_{n-2} e_n e_{n-1} + \cdots + a_1 e_n e_{n-1} \cdots e_2}{p_n + p_{n-1} e_n + p_{n-2} e_n e_{n-1} + \cdots + p_1 e_n e_{n-1} \cdots e_2} $$

The $K_c$ of the original Sarma method is the acceleration factor. Because of the real seismic force applied in the seismic dynamic calculation, the seismic force on the slope block is left with the horizontal and vertical seismic force acting on the block at any time when $K_c=0$. The equation could be obtained:

$$ a_n + a_{n-1} e_n + a_{n-2} e_n e_{n-1} + \cdots + a_1 e_n e_{n-1} \cdots e_2 = 0 $$

Among them:

$$ a_i = \frac{(W_i + F_i \cos \theta_i - H_i \sin \phi_i - \delta_i) \sin(\phi_i - \alpha_i) + S_{i+1} \sin(\phi_i - \alpha_i - \delta_{i+1})}{\cos(\phi_i - \alpha_i + \phi_{i+1}^/- \delta_{i+1}) \sec \phi_{i+1}^/-} 
- (Q_i - F_i \sin \theta_i) \cos(\phi_i - \alpha_i) - R_i \cos \phi_i + S_i \sin(\phi_i - \alpha_i - \delta_i) \cos(\phi_i - \alpha_i + \phi_{i+1}^/- \delta_{i+1}) \sec \phi_{i+1}^/- $$

$$ p_i = \frac{W_i \cos(\phi_i - \alpha_i)}{\cos(\phi_i - \alpha_i + \phi_{i+1}^/- \delta_{i+1}) \sec \phi_{i+1}^/-} $$

$$ e_i = \frac{\cos(\phi_i - \alpha_i + \phi_{i+1}^/- \delta_i) \sec \phi_{i+1}^/-}{\cos(\phi_i - \alpha_i + \phi_{i+1}^/- \delta_{i+1}) \sec \phi_{i+1}^/-} $$

$$ R_i = c b_i \sec \alpha_i - U_i \tan \phi_i $$

$$ S_i = c i d_i - PW_i \tan \phi_i $$

$$ \phi_i = \phi_i^/- \delta_i = \phi_i^/- \delta_{i+1} $$

The real seismic force is difficult to calculate. So, in order to facilitate the calculation, the quasi-static force is used to represent the horizontal and vertical seismic force.

$$ Q_i = W_i K_j $$

$$ H_i = W_i K_j $$

The $K_i$ is the horizontal seismic acceleration coefficient of block. The $K_j$ is the vertical seismic acceleration coefficient of block.

4. Calculation of safety and stability factor

The $k$ which is safety and stability factor is defined such a value if the $c$ and $\Phi$ values of the discretionary shear strength index are reduced to $c_e$ and $\tan (\Phi)$. The slope is at its limit of slide[7].

$$ c_e = c / k $$

$$ \tan(\phi_e) = \tan(\phi) / k $$

The safety factor appears as a implicit equation in the equations, and it needs to be calculated
iteratively. If the direction of seismic acceleration which points to the inside of slope is defined to be negative, the function relationship between the safety factor and the seismic acceleration is monotone decreasing[8]. Therefore the method of dichotomy could be used in dynamic Sarma method to iterative calculation. Calculation program is compiled by Matlab software to calculate the slope safety factor at each time during the earthquake.

Seismic forces have not only adverse effects but also favorable effects on the slope. In order to obtain a more conservative safety factor, analyzing the adverse effects of earthquakes on the slope is the most important thing. Therefore, the safety factor value is removed which is greater than the safety factor value without earthquake. The time history diagram of the revised safety factor is drawn, and the comprehensive safety factor is calculated by equation (21).

\[ k = \frac{1}{T} \int_0^T k(t) dt \]  

The \( T \) is the dynamic duration and the \( k(t) \) is the function of dynamic safety factor time history.

5. Example operations

5.1. Example

In this paper, a rock slope engineering example is used as the model of calculation and analysis. The rock mass parameters and profile structure of the slope are derived from geological survey data. The finally obtained numerical calculation model is shown in figure 4.

![Figure 2. FLAC3D calculation model](image)

Mohr-Coulomb plastic constitutive model is used to calculate the numerical simulation. According to the engineering data and referring to the relevant specifications, the average value of relevant parameters is used as the standard of rock. The detailed parameters of rock mass are detailed in table 1.

| Number | \( \rho \) kg/m\(^3\) | \( E \) MPa | \( \mu \) | \( c \) MPa | \( \phi \) (°) |
|--------|----------------|----------|--------|--------|--------|
| 1      | 2710           | 19       | 0.25   | 0.505  | 43.55  |
| 2      | 1970           | 3        | 0.35   | 0.036  | 10     |
| 3      | 2390           | 10       | 0.28   | 0.265  | 29.43  |
| 4      | 2100           | 4        | 0.32   | 0.0774 | 25.8   |
| 5      | 2180           | 2.5      | 0.38   | 0.042  | 25.65  |
| 6      | 1920           | 0.75     | 0.35   | 0.037  | 23.48  |

Table 1. Physical parameters of rock mass
5.2. Calculation process

The sliding surface is determined by the displacement variable cloud map of the slope obtained by numerical simulation calculation. The landslide is simplified and divided into blocks, and the calculation model is established as shown in figure 3.

![Figure 3. calculation model of dynamic Samra method](image)

The EI seismic wave is used in dynamic calculation. The acceleration magnification of each block is obtained by simulation calculation. And the values of acceleration magnification from bottom to top are 1.09, 1.31, 1.16, 1.23, 1.6.

Cutting out the first 30s of seismic wave, the peak value of vertical seismic acceleration is third less than the peak value of a horizontal seismic acceleration. The time history diagram of slope safety factor is calculated by matlab, shown as Fig.4. The maximum safety factor appears at 2.46s, and the value is 1.90. The minimum safety factor appears at 2.14s, and the value is 0.77. The average value is 1.186.

![Figure 4. time-history of safety factor](image)

Without earthquake, the safety factor value of the slope calculated by Sarma method is 1.178. The values larger than the static safety factor in figure 4 are replaced with the value of static safety factor, and the modified time history diagram of safety factor is obtained, such as fig.5.
According to equation (21), the value of modified comprehensive safety factor is 1.08. Compared to the revised comprehensive safety factor calculated by the dynamic Sarma with the safety factor obtained by other methods, the table is shown as Table 2.

Table 2. Comparison of safety factors

| Calculation method                  | Safety factor |
|------------------------------------|---------------|
| Static method                      | 1.1783        |
| Quasi-static method                | 1.02          |
| Minimum safety factor              | 0.77          |
| Average safety factor              | 1.186         |
| Dynamic Sarma method               | 1.08          |

6. Conclusion

First, the time-history diagram of safety factor takes the static safety factor as the midpoint to round-trip vibration under earthquake action, and the amplitude and frequency of vibration are affected by the amplitude and frequency of earthquake.

Second, the value of safety factor calculated by dynamic Sarma method is between static method and quasi-static method, which would not overestimate slope stability or underestimate slope stability. It is more suitable to evaluate the stability of slope under earthquake action.

References

[1] Sarma S K. Stability analysis of embankments and slopes[J]. Geotechnique, 1973, 23(3): 423–433.
[2] ZHANG Qiang-yong, LIU Da-wen and CAI De-wen. Application of Sarma Method to Stability Evaluation Analysis of A Large Anchored Rockmass Slope[J]. Chinese Journal of Rock Mechanics and Engineering, 2005(18):3368-3372.
[3] MA Shu-zhi, LIU Xiao-lang, XI Ren-shuang and Wang Zhe. Analysis of the Influence of Excavation on Stability of High Lithological Bedding Slope and the Reinforcement Effect of Anchor Cable[J]. Journal of Earth Sciences and Environment, 2018, 40(5):637-644.
[4] ZHANG Li. Calculation Method and Application for Wedge Rock-Mass in Surrounding Rock of Tunnel Based on the Theory of Sarma[J]. Technology of Highway and Transport, 2018, 34(03):75-79.
[5] LI Jian-ming, ZHANG Yan-jun, HUANG Xian-lon, Zhang Qing and Qu Cheng-song. Revised of Sarma Method for Different Directions of Seismic Force[J]. Journal of Engineering
Geology, 2011, 19(05): 725-731.

[6] SEED H B. Stability of earth and rockfill dams during earthquakes[C]//Embankment-dam Engineering (Casagrande blume) New York John Wiley and Sons Inc., 1973: 239-269.

[7] CHEN W F. Limit Analysis and Soil Plasticity[M]. New York: Elsevier Scientific Publishing Co., 1975.

[8] SU Yong-hua, ZHAO Ming-hua, ZOU Zhi-peng and Ouyang Guang-qian. Sarma Model for Slope Stability Analysis and its Reliability Degree Calculation Method[J]. Journal of Hydraulic Engineering, 2006(04): 457-463.