Structures and dynamics of fine particles in fine particle plasmas under microgravity and friction between two-dimensional layers of charges

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Abstract. Low temperature structures and dynamics of fine (dust) particles in the ambient (background) plasma with the cylindrical symmetry are analyzed by numerical simulations and theory. Fine particles are modeled as Yukawa particles and the one- and two-component cases are considered. In the one-component case, particles form well-defined concentric cylindrical shells and the structures are expressed by simple interpolation formulas to a good accuracy. When we introduce the difference into charges of particles forming a shell, particles are separated into a smaller shell of larger charges and a larger shell of smaller charges. In the two-component Yukawa system, we usually have structures composed of charge-separated shells where larger and smaller charges appear alternately in the order of radius. When we have charge-dependent forces along the symmetry axis, shells of larger and smaller charges can move relative to each other keeping their own structures and the mobility for the relative motion can be defined. We give examples obtained for some combinations of parameters. It is found that, when appropriate conditions are satisfied, concentric shells form a kind of ‘fine-particle (dust) screw’ and the mobility is increased.

1. Introduction
Fine particles in fine particle (dusty, complex) plasmas, mixtures of fine (dust) particles and the ambient plasma, provide us with a field where various phenomena in strongly coupled charge systems can be directly observed at the kinetic level. One of typical examples is the formation of static structures generally called dust (Coulomb) crystals[1, 2, 3, 4]. Structures are closely related to mutual interactions between particles and expected to give us basic information on the latter. In order to obtain such information and have insight into the nature of these structure formations, observations under the condition of high symmetry will be helpful.

Next to the homogeneous and isotropic systems which are difficult to realize, we have those with spherical or cylindrical symmetry or with restricted degrees of freedom. In experiments on the ground, however, we have to use the electric field in the sheath to levitate fine particles against strong influence of gravitation. Fine particles are usually distributed around the
minimum of the total potential forming horizontal layers whose number is controlled by the
strength of mutual repulsion relative to the strength of vertical confinement[5]. It is therefore
difficult to have not only homogeneous systems but also with the ideal spherical or cylindrical
symmetry. In addition, the supersonic ion flow in the sheath possibly modify the structures.

Long straight discharge chambers under microgravity seem to be one of candidates for such
observations in systems with ideal cylindrical symmetry. Recently, systematic analyses of fine
particle plasmas in such a chamber are in progress[6]. This apparatus is planned to succeed
the highly successful parallel-plate type one now operated in the International Space Station
(ISS)[7] and the structure formation has been observed on the ground[8]. We may thus expect
to observe phenomena in the system with the ideal cylindrical symmetry under microgravity.

Colloid particles in charge stabilized colloidal suspensions also enable us to make kinetic level
observations of strongly coupled charge systems such as colloidal (molecular) crystals[9]. Colloid
particles confined between plates[10] or in two-dimensional channels[9] have been extensively
investigated. They are free from the effect of gravity and their structures may be applied as
microstructures in photonic or phononic crystals. Colloid particles in cylindrically symmetric
systems also seem to be worth serious experimental investigations.

We here analyze the structure formation of Yukawa particles in a system with the cylindrical
symmetry by numerical simulations and theory. Fine particles interacting via the Yukawa
repulsion embedded in the ambient (background) plasma of electrons and ions can be regarded
as a simple model of fine particle plasmas[11, 12, 13]. We assume that the background plasma
is uniformly distributed in a cylinder of radius $R_0$ and the overall charge neutrality is satisfied.
When one neglect the hard core in the Derjaguin-Landau-Verwey-Overbeek potential, Yukawa
system can also work as a model of colloidal suspensions.

In various ion traps, we also have nearly cylindrical systems of ions confined by a combination
of magnetic field and electrodes or by electrodes. We can regard these confinements as equivalent
to the existence of the uniform background charges in a cylinder. We give a general picture for
the structure formation in cylindrical systems covering these Coulombic systems where structure
formations have been investigated[14, 15, 16, 17].

In Section 2, the interaction energy and the potential due to the ambient (background)
plasma are described. The structure formation in the case of one-component system is shown
in Section 3. Structures in the two-component system are given in Section 4 with emphasis on
the separation of charges. Possible relative motion of different species is presented in Section 5.
Concluding remarks are given in Section 6.

2. Interaction energy of Yukawa particles
2.1. Total interaction energy
We take $z$ along the symmetry axis and express coordinates as $r = (R, z)$. The interaction
between Yukawa particles with the charge $-Qe$ is given by

$$\frac{(-Qe)^2}{r} \exp(-r/\lambda),$$ (1)

where $\lambda$ is the screening length. In the case of fine particle plasmas, the screening length is given
by

$$\frac{1}{\lambda^2} = \frac{4\pi n_e e^2}{k_B T_e} + \frac{4\pi n_i e^2}{k_B T_i},$$ (2)

where the number density, the charge, and the temperature of electrons and ions in the ambient
plasma are denoted by $(n_e, -e, T_e)$ and $(n_i, +e, T_i)$, respectively. The total interaction
energy including particle-particle, particle-background, and background-background interactions
is written in the form [13]

\[
\int \int d\mathbf{r} d\mathbf{r}' \exp\left(-\frac{|\mathbf{r} - \mathbf{r}'|/\lambda}{2} \right) \rho(\mathbf{r}) \rho(\mathbf{r}') - \frac{(Qe)^2}{2} \sum_i \int \int d\mathbf{r} d\mathbf{r}' \exp\left(-\frac{|\mathbf{r} - \mathbf{r}'|/\lambda}{|\mathbf{r} - \mathbf{r}'|} \right) \delta(\mathbf{r} - \mathbf{r}_i) \delta(\mathbf{r}' - \mathbf{r}_i). 
\]

(3)

Here \(\rho(\mathbf{r})\) is the total charge density of Yukawa particles and the background. Since the latter charge density is \(\rho_0 = (-e)n_e + en_0\) for \(R < R_0\) and 0 otherwise, we have (\(\theta(\cdot)\) being the step function)

\[
\rho(\mathbf{r}) = \sum_i (-Qe)b(\mathbf{r} - \mathbf{r}_i) + \rho_0 \theta(R_0 - R).
\]

(4)

In (3), self-interactions formally included in the first term are subtracted.

2.2. Potential due to background with cylindrical symmetry

When the charge density of the ambient plasma is \(\rho_0(\mathbf{r})\), its potential \(\Phi(\mathbf{r})\) is given by

\[
\Phi(\mathbf{r}) = \int d\mathbf{r}_1 \frac{\rho_0(\mathbf{r}_1)}{|\mathbf{r} - \mathbf{r}_1|} \exp\left(-\frac{|\mathbf{r} - \mathbf{r}_1|/\lambda}{\lambda} \right).
\]

(5)

In our system with cylindrical symmetry, \(\rho_0(\mathbf{r}) = \rho_0(R)\) and \(\Phi(\mathbf{r}) = \Phi(R)\). For uniform distribution of the ambient plasma inside of \(R_0\) with the density \(\rho_0\), we have \(\rho_0(R) = \rho_0 \theta(R_0 - R)\) and

\[
\Phi(R) - \Phi(R = 0) = 4\pi \rho_0 R_0^2 \phi(R/R_0; R_0'),
\]

(6)

where

\[
\phi(x; R_0') = \frac{K_1(R_0')}{R_0^2} \left[1 - I_0(xR_0')\right] \quad \text{for } x = \frac{R}{R_0} < 1,
\]

(7)

\[
\phi(x; R_0') = I_1(R_0') - \frac{1}{(R_0')^2} \quad \text{for } x = \frac{R}{R_0} > 1,
\]

(8)

\(R'_0 = R_0/\lambda\), and \(K_\nu\) and \(I_\nu\) are modified Bessel functions.

For Yukawa particles with the negative charge \(-Qe\), the background works as a source of the confining potential \((-Qe)\Phi(R)\). The values of the function \(-\Phi/4\pi \rho_0 R_0^2\) is shown in Fig.1. With the increase of the screening, the potential becomes shallower and flatter in accordance with the decrease of mutual repulsion between particles as in the case of uniformly filled spheres [13]. In the limit \(\lambda \to \infty\), \(\Phi\) reduces to the Coulombic case as

\[
\Phi(R) - \Phi(R = 0) = -\pi \rho_0 R^2 \quad \text{for } R < R_0,
\]

(9)

\[
\Phi(R) - \Phi(R = 0) = -\pi \rho_0 R_0^2 \left[1 + 2 \ln(R/R_0)\right] \quad \text{for } R > R_0.
\]

(10)

Since particles are distributed within the radius \(R_0\) at low temperatures, the potential outside of \(R_0\) has no effect on their structures.

2.3. Characteristic parameters

We denote the linear density of Yukawa particles by \(n\). Due to the charge neutrality, we have

\[
(-Qe)n + \pi R_0^2 \rho_0 = 0
\]

(11)

and (6) is rewritten as

\[
\Phi(R) - \Phi(R = 0) = 4Qen \phi(R/R_0; R'_0).
\]

(12)
Figure 1. Potential for fine particles in the field of a uniform cylinder of neutralizing ambient (background) plasma with radius $R_0$.

Noting that $\rho_0/(Qe)$ is the average three-dimensional density of particles, we define the mean distance $a$ by

$$a = \left(\frac{4\pi \rho_0}{3Qe}\right)^{-1/3}.$$

The radius of the background $R_0$ is expressed by $a$ as $R_0/a = (4/3)^{1/2}(na)^{1/2}$. With the mean distance $a$, we express the strength of screening by the parameter $\xi = a/\lambda$.

Rewriting the parameters in the function as $\phi[(R_i/a); (na)^{1/2}, \xi]$, we have the potential energy of particles in the form

$$\frac{(Qe)^2}{a} \left[\frac{1}{2} \sum_{i,j} \exp(-\xi|\mathbf{r}_i - \mathbf{r}_j|/a) - 4na \sum_i \phi[(R_i/a); (na)^{1/2}, \xi]\right].$$

Our system at low temperatures is characterized by two independent dimensionless parameters and we take the set

$$(na, \xi),$$

the degrees of packing and screening. It will be shown that the structure is directly related to the former but is almost insensitive to the latter when scaled by the mean distance $a$.

Hitherto we have implicitly assumed that the system contains fine particles of a single species. Characteristic parameters in multi-component cases will be considered in sections 4.

3. Structures of one-component Yukawa and Coulomb systems in cylindrically symmetric background

3.1. Structure formation at low temperatures

In order to obtain low temperature structures of the system, we perform Monte Carlo simulations based on the usual Metropolis importance sampling. The periodic boundary condition is imposed along the $z$-axis and independent particles between $N = 200$ and $N = 1000$ are placed in the basic cell. We confirm these numbers are sufficient. In this section, we briefly summarize the results[18].

Some examples of the structures are shown in Fig.2 where particles are projected onto the $xy$-plane. Particles are organized into well-defined thin concentric shells. We number the shells
by \( m = 1, 2, \ldots \) from outside and denote their radii by \( R_m \). On each shell, particles form a triangular lattice with defects as shown in Fig.3.

Because of the periodic boundary condition, we need to use larger number of independent particles in order to simulate the case of multiple layers accurately. We adopt the number of independent particles so that the length of the basic cell along the axis is long enough compared with the lattice constant of quasi-two-dimensional lattices on shells.

With the increase of \( na \), the number of concentric shells increases. As is shown in Fig.4 in Ref.18, values of \( R_m/a \) \((m = 1, 2, \ldots)\) linearly increase with \((na)^{1/2}\) and the spacing between them is almost constant. In addition, the value of \( \xi \) varies from 0.115 to 0.531 in these simulations but the structures are almost independent of the value of \( \xi \). We note that the results of the Coulombic system[15] which correspond to the case of \( \xi = 0 \) are also covered by these \( \xi \)-insensitive values.

The surface density of particles on shells is plotted in Fig.5 in Ref.18 including the case of \( \xi = 0\)[15]. We observe that the surface density is almost common for various cases except for the innermost shell of radius smaller than \( a \). The average (common) surface density for shells with radii larger than \( a \) is given by

\[
\sigma a^2 \approx 0.352. \tag{16}
\]

3.2. Summary of structure formation

With the decrease of the temperature, particles are organized into well-defined thin concentric cylindrical shells and, on each shell, particles form a triangular lattice with defects. Structures measured in the units of \( a \) are almost insensitive to the strength of screening. Shell radius has a linear \((na)^{1/2}\)-dependence and the spacing is almost common when measured in \( a \). The number of shells increases discretely with the increase of \((na)^{1/2}\) and the critical values for the appearance of new shells \((na)^{1/2}_m\) are linearly dependent on the number of shells. The surface density of particles on shells measured in \( a \) is almost common except for shells with small radius compared with \( a \).

3.3. Interpolation formulas for structures

Based on the above observations, we express \( R_m/a \) in the form

\[
R_m/a = c_0[(na)^{1/2} - (na)^{1/2}_m] \quad (na)^{1/2} > (na)^{1/2}_m. \tag{17}
\]

Figure 2. Examples of the distribution in the \( xy\)-plane. Dotted lines are \( R = R_0 \). (a) \( na = 10 \), \( N = 800 \), (b) \( na = 20 \), \( N = 800 \), and (c) \( na = 49.3 \), \( N = 1000 \).
with $c_0 = 1.138$. The relative errors are within 2%. Since the critical values for the appearance of the shell $m$ are expressed as

$$\left(\frac{na}{m}\right)^{1/2} = c_1 (m - c_2)$$

with constants $c_1 = 1.411$ and $c_2 = 0.501$, we have

$$R_m/a = 1.138 \left(\frac{na}{m}\right)^{1/2} - 1.411 (m - 0.501)$$

within relative error of 4%. By the common surface density of particle on shells, the number of particles on a shell of length $L$ is given in the form

$$N_m = 2\pi \sigma R_m L = 2\pi (\sigma a^2) \frac{R_m L}{a} = 2\pi \cdot 0.352 \left[1.138 \left(\frac{na}{m}\right)^{1/2} - 1.411 (m - 0.501)\right] \frac{L}{a}.$$

When we assume that the spacing between shells is a constant $d$, we have a heuristic formula for the positions of shells in the form

$$R_m/a = (4/3)^{1/2} \left(\frac{na}{m}\right)^{1/2} - \frac{d}{a} (m - 0.5).$$

(21)

Compared with this expression, the results of simulations summarized in the form (19) give 0.501 instead of 0.5, 1.138 instead of $(4/3)^{1/2} = 1.1547$, and

$$\frac{d}{a} = 1.411.$$

(22)

The formula (19) is thus consistent with the result of this heuristic reasoning.

As for the value of $d/a$, we have $(d/a)^{\text{FP}} = 1.477$ in the three-dimensional closest packing (cp) structure. With this value, (21) takes the form

$$R_m/a = (4/3)^{1/2} \left(\frac{na}{m}\right)^{1/2} - 1.477 (m - 0.5).$$

(23)

In the results of simulations (19), we have $1.411$ which is close to 1.477. We observe that (23) reproduces results of simulations when $m$ is small or in the case of outer shells as expected. The average surface density $\sigma a^2$ from our simulation 0.352 is also close to the value for the lattice plane in the closest packing structure $$(\sigma a^2)^{\text{FP}} = 3^{1/6} / (2\pi)^{2/3} = 0.3527.$$ We may thus regard the local structure of one-component dust particles in cylindrical symmetry as being similar to the closest packing with equally spaced triangular lattices.

The structures are almost independent of the strength of screening also in the case of uniform spherical background where we have[13]

$$\frac{R_m}{a} = N^{1/3} - 1.474 (m - 0.488),$$

(24)

**Figure 3.** Examples of particle distribution in shells in the case of the structure Fig.2(c). (a) The first ($m = 1$) and (b) the second ($m = 2$) outermost shells. The abscissa is the azimuthal position, $\theta$ being the angle.
\[
\frac{d}{a} = 1.474, \quad (25)
\]
\[
\sigma a^2 = 0.356, \quad (26)
\]
and
\[
N_m = 4\pi \sigma R_m^2 = 4\pi \cdot 0.356 \left[ N^{1/3} - 1.474 (m - 0.488) \right]^2 \quad (27)
\]
with the mean distance \(a\) assuming a uniform distribution in the sphere. The local structure is also close to the three-dimensional closest packing structure.

4. **Structures of two-component Yukawa system in cylindrically symmetric background**

The charge on a fine particle is approximately proportional to its radius and the electron temperature. When we have fine particles of different radii and therefore different charges, the parameter space for static structures has additional dimensions, the ratios of charges and their concentrations. (As far as the gravity has no significant effect, the static structures are generally independent of the mass of particles. In our case, the mass ratio is not independent of the charge ratio and does not compose a new dimension even with the effect of gravity.)

We denote the charge and linear number density of species \(\alpha\) by \((-Q_\alpha e, n_\alpha)\). The condition of the charge neutrality is written as
\[
\sum_\alpha (-Q_\alpha e)n_\alpha + \pi R_0^2 \rho_0 = \sum_\alpha (-Q_\alpha e)n_\alpha + (-e)n_e + e n_i = 0. \quad (28)
\]

We restrict ourselves within the minimum multi-component system or two-component mixtures. Even in this case, we have three characteristic parameters, putting aside the strength of screening which is expected to be not effective, and the overall survey is difficult. We give some examples which may help us predict characteristic properties of multi-component systems.

4.1. **Parameters for two-component system**

We here analyze the systems of two species of particles, 1 and 2, with charges and linear densities \((-Q_1 e, n_1)\) and \((-Q_2 e, n_2)\), respectively. By the total linear density \(n\),
\[
n = n_1 + n_2, \quad (29)
\]

we define the average charge on a fine particle \(-Q e\) and the mean distance \(a\) by
\[
\bar{Q} e = \frac{Q_1 e n_1 + Q_2 e n_2}{n_1 + n_2}, \quad (30)
\]
and
\[
a = \left( \frac{4\pi \rho_0}{3 \bar{Q} e} \right)^{-1/3}, \quad (31)
\]
respectively. Without loss of generality, we assume \(Q_2 > Q_1\). Now the system has two more parameters, the charge ratio \(Q_2/Q_1\) and the concentration ratio \(n_2/n_1\), in addition to \(na\) and \(\xi\). We thus have three effective characteristic parameters, \(na\), \(Q_2/Q_1\), and \(n_2/n_1\).
Figure 4. Examples of separation of shells by the magnitudes of charges. Larger charges form the shell of smaller radius. The ratio of densities is \( n_2 = n_1 \).

4.2. Separation of shells: Inner shell of larger charges and outer shells of smaller charges

We first analyze how the structure changes when the one-component system is continuously modified into the two-component system. When \( na = 2.66 \) (\( nR_0 = 5 \)), the one-component system forms only one shell. Let us now increase the charges on half of the particles and decrease those of the rest keeping the average charge unchanged so that the background needs not to be changed. By large or small of charges we refers to their magnitudes irrespective of their signs.

Examples in Fig.4 indicate that larger charges move inward and smaller ones move outward forming two separate shells. The difference of the two radii depends on the ratio of charges as shown in Fig.5. The difference increases with the increase of \( Q_2/Q_1 - 1 \) starting from the linear dependence and, when \( Q_2/Q_1 > 10 \), the changes of both radii are almost saturated.

In Fig.6, we show the cases where the ratio of the number densities is changed in comparison with the one-component case composed of the average charge. The ratio \( n_2/n_1 \) is changed form 0.125 to 0.875. Larger charges are located inside of the corresponding one-component case and the radius becomes close to the latter when the ratio \( n_2/n_1 > 1 \). The radius of the shell of smaller charges, on the other hand, is larger than the radius in the one-component case and is almost independent of \( n_2/n_1 - 1 \).

Figure 7 shows examples which start from the multiple-shell structures of the one-component case where we have one, two, and three shells for \( na = 2.66 \) (\( nR_0 = 5 \)), 12.3 (\( nR_0 = 50 \)), and 19.6 (\( nR_0 = 100 \)), respectively. Other parameters are \( Q_2/Q_1 = 100 \) and \( n_2/n_1 = 1 \). We observe that each shell in the one-component case is separated into two shells and we have alternate shells of large and small charges in the order of increasing radius. From the charge-ratio dependence shown in Fig.5, we expect to have similar structure also in the cases of smaller charge ratios.

Though our simulations are not exhaustive, we find no example where shells of the same charge are adjacent. In other words, we have always alternate shells of different charges within our exploration of the parameter space.

The results our simulations of two-component systems on static structures are summarized as follows. [A] Each shell in the one-component case is separated into the one of smaller radius with larger charges and the one of larger radius with smaller charges. [B] We have shells of one-by-one alternating charges in the order of increasing radius.

4.3. Origin of charge separation

The most important phenomenon characterizing structures of two-component systems is the separation of shells according to charges: Starting from the same shell, particles with increased (decreased) charges form a shell with smaller (larger) radius. In order to understand this separation, let us consider what happens when we introduce a small difference of charges into
Figure 5. The separation of shells vs. charge ratio. The ratio of densities is $n_2 = n_1$.

Figure 6. Separation of shells vs. number density ratio for $na = 2.66$ ($nR_0 = 5$) and $Q_2 = 100Q_1$. Thin dotted circle is the position of the shell in the one-component case.

Figure 7. Separation of shells. The parameter $na$ is changed from 2.66 to 19.6 ($n_2 = n_1$, $Q_2 = 100Q_1$).

particles which are in the equilibrium distribution of the one-component system. We slightly modify the charge of the $i$-th particle as

$$Q_i = -Q(1 + \Delta_i),$$ (32)

where

$$\Delta_i = \Delta \text{ or } -\Delta \quad (1 \gg \Delta > 0).$$ (33)

We assume

$$\sum_i \Delta_i = 0,$$ (34)

so that we do not need to change the density of the background.
Due to these changes, the balance of forces acting on each particle will be lost. The force on the $i$-th particle is given by

$$ f_i = Q_i e (E_{\Phi,i} + \sum_{j \neq i} Q_{j} e f_{ij}). \quad (35) $$

Here $Q_i e E_{\Phi,i}$ is due to the background and $Q_i e^2 f_{ij}$ is due to the other particle $j$;

$$ E_{\Phi,i} = E_{\Phi}(R_i) = -\frac{\partial}{\partial R_i} \Phi, \quad (36) $$

$$ f_{ij} = -\frac{\partial}{\partial R_i} \frac{1}{r_{ij}} \exp(-r_{ij}/\lambda). \quad (37) $$

We rewrite (35) into the form

$$ f_i = Q_i e (E_{\Phi,i} - Q e \sum_{j \neq i} f_{ij}) + Q^2 e^2 (1 + \Delta_i) \sum_{j \neq i} \Delta_j f_{ij}. \quad (38) $$

Since we start from the equilibrium state of the one-component system, we have

$$ E_{\Phi,i} - Q e \sum_{j \neq i} f_{ij} = 0 \quad (39) $$

and the first term of (38) vanishes. If the distribution of $\{\Delta_i\}$ is not mutually correlated, the second term also vanishes on the average and the replacement of charges has no effect on the radius of the shell.

Since we are considering the low temperature structures, the distribution of $\{\Delta_i\}$ and the positions of particles are determined so as to minimize the total potential energy

$$ \sum_i Q_i e \Phi(R_i) + \frac{1}{2} \sum_{i \neq j} Q_i Q_j e^2 \frac{\Delta_i \Delta_j}{r_{ij}} \exp(-r_{ij}/\lambda). \quad (40) $$

We consider the effect of $\{\Delta_i\}$ in two steps: We first distribute $\{\Delta_i\}$ keeping the positions of particles unchanged and then consider the effect of rearrangement of positions.

In the first step, the first term of (40) is not changed. We rewrite the second term into

$$ \frac{1}{2} \sum_{i \neq j} Q_i^2 e^2 \frac{\Delta_i}{r_{ij}} \exp(-r_{ij}/\lambda) + \sum_i \Delta_i \sum_{j \neq i} Q_i^2 e^2 \frac{\Delta_j}{r_{ij}} \exp(-r_{ij}/\lambda) + \frac{Q^2 e^2}{2} \sum_{i \neq j} \frac{\Delta_i \Delta_j}{r_{ij}} \exp(-r_{ij}/\lambda). \quad (41) $$

Since the one-component system at low temperature can be regarded as a lattice,

$$ \sum_{j \neq i} Q_i^2 e^2 \frac{\Delta_j}{r_{ij}} \exp(-r_{ij}/\lambda) \quad (42) $$

is independent of $i$ and the second term vanishes due to (34). In order to minimize the last term, the distribution of $\{\Delta_i\}$ has to be correlated so that $\Delta_i \Delta_j < 0$ for near neighbors. As a result, particles with positive $\Delta_i$ (‘plus’ particles) are surrounded by particles with negative $\Delta_i$ (‘minus’ particles) and vise versa. Since the shell has a small but finite curvature,

$$ \sum_{j \neq i} \Delta_j f_{ij} \quad (43) $$

tends to have the inward (outward) direction when $i$ is a ‘plus’ particle (‘minus’ particle). The shell thus tends to be separated into the shell of larger charges with smaller radius and the one of smaller charges with larger radius in the first order in $\Delta$. In the second step where positions of particles are relaxed, particles are reorganized. The changes, however, are of the order of $\Delta$ when normalized by the original positions and their effect to the results obtained in the first step is of the second order. In this way, we may understand the directions of the separation of radius of shells at least when $\Delta$ is small.
5. Two-component system: Dynamics
Fine particles in fine particle plasmas also serve as the important source of the information on dynamics of strongly coupled systems. Among many interesting dynamical aspects of strongly coupled charge systems, such as collective modes and transport coefficients, we here focus on the relative motion of different species of charges. In strongly coupled systems there exists a possibility that different components of charges keep their own structures and, at the same time, move relative to other components. This kind of motion may be characterized by the mobility, the friction coefficient, and the stopping power. The system with the cylindrical symmetry seems to fit particularly the observation of this phenomenon.

In two-dimensional systems, we can have the cases where different components are spatially separated. For example, we have systems of electrons (holes) in adjacent layers in semiconductor microstructures and the mutual friction between electrons (holes) in adjacent layers may be applied to control electronic devices. Our cylindrically symmetric system of two-component fine particles with separated shells might be regarded as a kind of two-dimensional system with spatially separated components.

The motion of fine particle against the rest of the system can be excited by irradiating the laser beam. The excitation, however, will be limited to a small group of particles and it seems to be difficult to have collective motions of some specific component in the system. In fine particle plasmas, we have systems with different species of particles. Different species of particles have different radius and therefore different charges and there occur a variety of structures even in the case of two components. We may therefore expect the mutual friction between different species may be directly observed in fine particle plasmas with multiple species.

Since fine particles are embedded in the ambient plasma, one may wonder how to excite the relative collective motion. Even if the direct (full) application of the electric field is difficult due to the screening effect by the ambient plasma, some small part of the applied electric field may penetrate and give different accelerations to different species of particles. One may also induce some flow of the neutral gas or the ambient plasma in order to give species-dependent forces. We here analyze the behavior of the two-component systems under species-dependent accelerations.

5.1. Equations of motion
The equation of motion for the particle \( i \) is written as

\[
m_i \frac{d^2 r_i}{dt^2} = Q_i e \mathbf{E}(\mathbf{R}_i) + Q_i e \sum_{j \neq i} Q_j e \mathbf{f}_{ij} + \mathbf{F}_{i}^{ext} - \nu_i m_i \frac{dr_i}{dt}.
\]

Here \( m_i \) is the mass, \( \mathbf{F}_{i}^{ext} \) is the external force, and \( \nu_i \) is the friction coefficient which is usually due mainly to collisions with neutral atoms. We assume the gas species is Ar and estimate the coefficient \( \nu_i \) by\[19, 20, 21]\n
\[
\nu_i \sim c n_n (d_i/2)^{2} \frac{m_n}{m_i} \nu_{th,n},
\]

where \( n_n \) is the number density of neutral atoms, \( d_i/2 \) the radius of the particle, \( \nu_{th,n} \) the thermal velocity of neutral atoms, and \( c \) a factor of the order unity. Measuring the pressure in Pa and the temperature in eV and expressing the particle radius \( d_i/2 \) in \( \mu m \), we have for Ar

\[
\nu_i \sim 5.93 p_n \text{[Pa]} (d_i/2[\mu m])^{-1} \left( \frac{T_n \text{[eV]}}{0.03} \right)^{-1/2} \text{s}^{-1}.
\]

Here \( c = 1.4 \times (8/3)(2\pi)^{1/2} \) is adopted\[21\].
5.2. **Effect of axial electric field**

Let us assume that the neutral gas is Ar with $T_n = 0.03\text{eV}$ and typically $p_n \sim 10\text{Pa}$ and consider a mixture with $d_1/2 = d_0/2 = 1\mu\text{m}$ for the species 1 and $d_2/2 = 10(d_0/2)$ for the species 2. The mobility is inversely proportional to the neutral gas pressure and we can scale the results accordingly. At low temperatures and under appropriate conditions, particles form two separate shells, outer shell of charge 1 and inner shell of charge 2.

When the external electric field is applied along the axis, two species of particles feel the forces of different magnitudes. They also have different coefficients for the friction in the motion among the neutral gas. Since particles are strongly coupled, each species are expected to move as a solid at least when the external field is sufficiently small. When the external field is sufficiently large, on the other hand, the difference in the forces acting on different species may lead either to a relative motion between different species with each structure kept (as in the case of weak field), or a destruction of the structure of either (or both) species. We analyze the response of two-component systems by molecular dynamics simulations.

5.3. **Mobility for relative motion and formation of ‘dust screw’**

The results of simulations are summarized in Figs.8, 9, and 10. With the increase of the axial electric field, the relative velocity between the charges 2 and 1 increases. The dependency is locally linear and we may define a kind of mobility by the ratio of the $z$-component of the relative velocity $v_{rel}$ to the $z$-component of the electric field $E_z$ as

$$\mu_{rel} = \frac{v_{rel}}{E_z}. \quad (47)$$

For small electric field, the mobility seems to have small dependence on the structure. The value is given by

$$\mu_{rel} \approx \frac{9.2}{p_n[\text{Pa}]} \text{cm}^2/\text{Vs}. \quad (48)$$

When the electric field (or the relative velocity) exceeds some critical value, the mobility increases stepwise for a structure while unchanged for other structures. the increased value is given by

$$\mu_{rel} \approx \frac{18.3}{p_n[\text{Pa}]} \text{cm}^2/\text{Vs}. \quad (49)$$

We note that these values are much smaller than those found in two-dimensional systems of electrons or holes in semiconductor microstructures such as the inversion layers used in electronic devices.

In the case of stepwise increase, we observe that, corresponding to this increase of the mobility, the structure changes into a kind of ‘screw’ as shown in Fig.9. Figure 10 shows the motion of particles projected onto the $xy$-plane. The applied electric field thus induces the rotations of charges 1 and 2 in opposite directions. It is clear that these rotations help to have larger relative velocity under a given electric field. One might be tempted to understand these rotations in opposite directions in terms of the conservation of the total angular momentum. Fine particles, however, collide with neutral atoms and the total angular momentum is not conserved in our system.

6. **Concluding remarks**

We have analyzed structures and dynamics of fine particles in fine particle plasmas with the cylindrical symmetry. Such a system of high symmetry is expected to be realized in a long discharge chamber under the condition of microgravity.
Figure 8. Relative velocity vs. electric field. The change in the slope corresponds to the structural change shown in Fig. 9. Values multiplied by $p_n [\text{Pa}]$ are shown.

Figure 9. The structural change shown in the $(\theta, z)$-plane. In the case of $na = 2.29$, the ‘screw’ structure is formed when the electric field is increased, changing from the one in the left panel to the one in the center panel. In the case of $na = 2.66$ (and $na = 3.0$), we have no such change as shown in the right panel.

Figure 10. Velocity of particles projected onto the $xy$-plane. Particles in the inner shell and those in the outer shell rotate in opposite directions. The magnitudes of velocities of particle in the inner shell are magnified by the factor 4 for clarity.
In the case of one species, well-defined concentric shells are formed at low temperatures and the structures are expressed by simple but accurate interpolation formulas covering the case of Coulomb particles. In the case of two components, shells of the one-component system are separated by charges. Larger and smaller charges form shells of smaller and larger radii, respectively, and an explanation for this separation is given.

As for dynamics, the relative motion of separated shells in the two-component system is analyzed. It is shown that, by applying an appropriate force along the axis of the cylinder, the relative motion of separated shells is induced and we can define the mobility for the relative motion. An example of the formation of ‘screw’ by fine particles and corresponding reduction of mutual friction is shown.

In these analyses, some simplifications are made. For example, the density of the ambient (background) plasma is assumed to be uniform in a cylinder and the mutual interaction between fine particles are expressed by the Yukawa potential. We expect there exist the domain of parameters where these simplifications are allowed and the results obtained here are expected to serve as a basis of detailed investigations.

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