Abstract—In this paper, we design an optimal sensor collaboration strategy among neighboring nodes while tracking a time-varying parameter using wireless sensor networks in the presence of imperfect communication channels. The sensor network is assumed to be self-powered, where sensors are equipped with energy harvesters that replenish energy from the environment. In order to minimize the mean square estimation error of parameter tracking, we propose an online sensor collaboration policy subject to real-time energy harvesting constraints. The proposed energy allocation strategy is computationally light and only relies on the second-order statistics of the system parameters. For this, we first consider an offline non-convex optimization problem, which is solved exactly using semidefinite programming. Based on the offline solution, we design an online power allocation policy that requires minimal online computation and satisfies the dynamics of energy flow at each sensor. We prove that the proposed online policy is asymptotically equivalent to the optimal offline solution and show its convergence rate and robustness. We empirically show that the estimation performance of the proposed online scheme is better than that of the online scheme when channel state information about the dynamical system is available in the low SNR regime. Numerical results are conducted to demonstrate the effectiveness of our approach.

Index Terms—Wireless sensor networks, parameter tracking, node collaboration, energy harvesting, semidefinite programming.

I. INTRODUCTION

Recent advances in wireless communications and electronics have enabled the development of low-cost, low-power, multifunctional sensor nodes that are deployed as networks for many applications such as environment monitoring, source localization and target tracking [1]–[3]. These sensor nodes sense and measure some attributes of the targets of interest, and are able to exchange their information through in-network communications [4]. In this paper, we study the problem of parameter tracking in the presence of inter-sensor communications that is referred to as sensor collaboration. Here sensors are equipped with energy harvesters, which can replenish energy from the environment, such as sun light and wind. The act of sensor collaboration allows sensors to update their measurements by taking a linear combination of the measurements of those they interact with, prior to transmission to a fusion center (FC). As suggested in [5], collaboration smooths out the observation noise, thereby improving the quality of the signal transmitted to the FC and eventual estimation performance.

In the absence of sensor collaboration, the proposed estimation architecture reduces to a classical distributed estimation system that uses an amplify-and-forward transmission strategy [6]–[8]. The power allocation problem for distributed estimation under an orthogonal multiple access channel (MAC) model was studied in [6], where the optimal power amplifying factors were found by minimizing the estimation distortion subject to energy constraints. In [7], the same problem was studied under a different communication model, coherent MAC, where sensors coherently form a beam into a common channel received at the FC. In [8], the design of optimal power allocation strategies was addressed over orthogonal fading wireless channels for source estimation in a network of energy-harvesting sensors. In [9], [10], the problem of power allocation for parameter tracking was studied where the (scalar) parameter of interest was modeled as a Gauss-Markov process. In the aforementioned literature [6]–[10], inter-sensor communication was not considered. Moreover, the work in [6]–[8] only focused on one-snapshot estimation, while the work [9], [10] only studied the conventional sensor network with battery-limited sensors. In contrast, here we seek an optimal sensor collaboration scheme for a dynamic parameter tracking using energy-harvesting sensors.

The problem of distributed estimation with sensor collaboration has attracted recent attention [5], [11]–[15]. In [11], the network topology was assumed to be fully connected, where all the sensors are allowed to share their observations. It was shown that the optimal strategy is to transmit the processed signal (after collaboration) over the best available channels with power levels consistent with the channel qualities. In [12], the optimal collaboration strategy was designed under star, branch and linear network topologies. In [5], [13], [14], the problem of collaboration network topology design was studied by incorporating the cost of sensor collaboration. It was shown that a partially connected network can yield estimation performance close to that of a fully connected network. However, the aforementioned literature on collaborative estimation focused only on static networks, and the parameter to be estimated is temporally invariant. In [15], the sensor collaboration strategy was designed for the estimation of temporally correlated parameters. However, in [15] a batch-based linear estimator was
used, where all the historical measurements have to be stored for decision making. Different from [15], a Kalman filter-like estimator is adopted in this paper, and a computationally inexpensive but efficient real-time sensor collaboration scheme is developed in conjunction with parameter tracking.

In the existing work on power allocation for collaborative estimation [5], [11]–[15], sensor networks were composed of (conventional) battery-limited sensors. Motivated by the rapid advances in the emerging energy-harvesting device technology [16], it is attractive to perform collaborative estimation using energy harvesting sensors. In this context, each sensor can replenish energy from the environment without the need of battery replacement.

The effect of energy harvesting on parameter estimation was considered recently in [17]–[20]. In [17], a single sensor equipped with an energy harvester was used for parameter estimation, and the optimal power allocation scheme was designed under causal and non-causal side information of energy harvesting. In [18], a deterministic energy harvesting model was proposed for parameter estimation over an orthogonal MAC. In [19], the optimal communication (sensors to the FC) strategy was obtained for remote estimation with energy harvesting sensors. In [17]–[19], the power allocation policy was designed in the absence of sensor collaboration. In [20], the problem of energy allocation and storage control for collaborative estimation was addressed by solving nonconvex optimization problems through convex relaxations. However, the strategy developed in [20] only provides an offline solution for one snapshot estimation. In contrast, we propose a sensor collaboration policy that obeys the real-time energy harvesting constraints.

Another difference between our work and the previous ones is that the previous studies [5], [11]–[15], [20] hinge on the noise-free inter-sensor communication model, while the noiseless assumption is relaxed in this paper. Moreover, sensors communicate with neighbors using point to point orthogonal channels in [5], [11]–[15], [20], while sensors exchange their information with neighbors through a coherent MAC in this paper. Multiple-access channel communication introduces collaboration noise, which has to be taken into account while designing the optimal sensor collaboration scheme.

In a preliminary version of this paper [21], we studied the sensor collaboration problem for static parameter estimation in the absence of collaboration noise. In this paper, we focus on tracking of a scalar dynamic parameter with noise-corrupted sensor collaboration. The assumption of scalar random process enables us to obtain explicit expressions for the estimation distortion and the transmission cost, and to explore the asymptotic behavior of the estimation error with respect to the energy constraint. The assumption of scalar random process was also used in other power allocation problems [5], [6], [10], [11]. However, different from [5], [6], [10], [11], we consider a different problem in which a scalar random process is tracked in the presence of noise-corrupted inter-sensor collaboration and energy-harvesting sensors. More specifically, we have the following new contributions in this paper.

- We consider tracking of a correlated parameter sequence.
- We incorporate a recursive linear minimum mean squared error estimator (R-LMMSE) to track the parameter in the presence of multiplicative noise.
- We consider noise-corrupted sensor collaboration accomplished through a coherent MAC in a network of energy harvesting sensors for parameter tracking.
- We propose an optimal offline sensor collaboration scheme under the assumption that the energy constraint is on an average basis, and show that although the resulting optimization problem is non-convex, a globally optimal solution can be found via semidefinite programming. Also, we prove the stability of this scheme.
- We propose an online sensor collaboration policy that satisfies the real-time energy harvesting constraints with minimal online computational complexity, and prove that its resulting long-term estimation performance is the same as that of the optimal offline solution. Moreover, we show the convergence rate and robustness of the proposed optimal online scheme.
- We demonstrate the efficiency of our methodology by comparing it with a greedy optimal online policy when the channel state information (CSI) regarding the dynamical system is assumed to be known causally at the FC.

The main generalization compared to [21] is that the parameter to be estimated in this paper is a correlated sequence compared to the independent and identically distributed (i.i.d.) sequence in [21]. Thus in [21], the estimation procedure required estimating the parameter via only the data obtained at that time. In this paper, we incorporate an R-LMMSE to track and estimate the parameter because of the correlation in time. Moreover, it becomes a distributed R-LMMSE with online constraints (due to energy harvesting) and multiplication of the system parameters with random coefficients (due to channel gains). Thus, technically it is much more complicated and different from [21]. The optimization problem solved also is much more complicated than in [21].

The rest of the paper is organized as follows. In Section II, we introduce the collaborative tracking system, and present the general statement of the sensor collaboration problem. In Section III, we formulate the offline sensor collaboration problem for parameter tracking, where only partial knowledge of the second-order statistics of system parameters is required. In Section IV, we solve the non-convex offline sensor collaboration problem and obtain its globally optimal solution via semidefinite programming. We also design an online sensor collaboration policy. In Section V, for comparison, we study a sensor collaboration problem with known channel state information. In Section VI, we demonstrate the effectiveness of our online solution obtained in Section V through numerical examples and compare with the greedy optimal solution obtained in Section IV. Finally, in Section VII, we summarize our work and discuss future research directions.

II. PROBLEM STATEMENT

In this section, we formally state the problem of sensor collaboration for parameter tracking using a network of
energy harvesting sensors. In the wireless sensor network under consideration, each sensor is equipped with an energy harvesting device which replenishes itself from a renewable energy source. In the overall inference system, sensors first acquire raw measurements of the phenomenon of interest via a linear sensing model, and then perform spatial collaboration via a coherent MAC. A coherent MAC channel can be realized through transmit beamforming [22], where sensor nodes simultaneously transmit a common message and the phases of their transmissions are controlled so that the signals constructively combine at the FC. This requires that each of the sensor nodes knows its channel gains. It has been shown in [9] that the coherent MAC can be realized by the distributed synchronization schemes described under moderate amounts of phase error [23], [24]. An advantage of a coherent MAC is that all the sensors use the same channel to transmit their observations. We also remark that the coherent MAC model has been widely used in the study of the power allocation problem for distributed estimation [5], [7], [10], [15], [21].

After collaboration, the signals are transmitted to the FC, which tracks the random process. We show the collaborative estimation system to be studied in Fig.1. A list of the system parameters is provided in Table I.

![Fig. 1: Collaborative tracking system.](image)

### A. Sensor collaboration via coherent MAC

Let \( \{ \theta_k \in \mathbb{R} \} \) be the discrete-time random process to be estimated, which evolves as a first-order Gauss-Markov process

\[
\theta_k = \alpha \theta_{k-1} + \tau_k, \quad k = 1, 2, \ldots, \tag{1}
\]

where \( \theta_k \) denotes the random parameter at time \( k \), \( \alpha \) is the known state transition coefficient, and \( \tau_k \) is i.i.d with zero mean and variance \( \sigma^2 \). We also assume that \( \theta_k \) has zero mean, and \( |\alpha| < 1 \). Then, \( \theta_k \) is an ergodic Markov chain with a unique stationary distribution. In addition, if \( \tau_k \) has a positive density on whole of real line and \( \mathbb{E}[|\tau_k|] < \infty \), then \( \theta_k \) is geometrically ergodic [25]. Many practically relevant distributions satisfy these conditions: Gaussian, mixtures of Gaussians and some heavy tailed stable distributions modeling electromagnetic interference.

We consider a linear measurement model

\[
x_k = h_k \theta_k + \epsilon_k, \quad k = 1, 2, \ldots, \tag{2}
\]

where \( x_k \in \mathbb{R}^N \) is the vector of measurements acquired by \( N \) sensors at time \( k \), \( h_k \in \mathbb{R}^N \) is the vector of observation gains with known second-order statistics, \( \mathbb{E}[h_k] = h \) and \( \text{cov}(h_k) = \Sigma_h \); \( \epsilon_k \in \mathbb{R}^N \) is the vector of measurement noises with zero mean and covariance \( \mathbb{E}[\epsilon_k, \epsilon_j^T] = \Sigma _h I (k_1 - k_2) \). We also assume that \( \{ h_k \}, \{ \theta_k \} \) and \( \{ \epsilon_k \} \) are independent of each other.

After linear sensing, each sensor may pass its observation to other sensors for collaboration, prior to transmission to the FC. The sharing of measurements is accomplished via a coherent MAC. Collaboration among sensors is represented by a fixed topology matrix \( A \) with binary entries, namely, \( A_{mn} \in \{0, 1\} \) for \( m \in [M] \) and \( n \in [N] \), where for ease of notation, \( [M] \) denotes \( \{1, 2, \ldots, M\} \). Here \( A_{mn} = 1 \) signifies that there exists a communication link from the \( n \)th sensor to the \( m \)th sensor; otherwise, \( A_{mn} = 0 \). The neighborhood structure of the sensor network can be determined by the communication range of each sensor in the network. Note that \( A \) is essentially a truncated adjacency matrix with a self loop because each sensor can collaborate with itself, namely, \( A_{mm} = 1 \). We note that if \( A_{mn} = 0 \) for all \( m \neq n \), the resulting sensor collaboration strategy reduces to the amplify-and-forward transmission strategy considered in [7], [10].

With a relabeling of sensors, we assume that the first \( M \) sensors (out of a total of \( N \) sensor nodes) communicate with the FC. The condition \( M \leq N \) is motivated by the need to reduce possible high transmission energy expense. The prior knowledge about the network topology \( A \) is commonly determined based on the quality of observation and channel gains, and the pairwise sensing distance [5].

Given the network topology, the collaborative signal received at the \( i \)th sensor at time \( k \) is modeled as

\[
z_{k,i} = W_k(i,i)x_{k,i} + \left( \sum_{j \in \mathcal{N}_i} W_k(i,j)x_{k,j} + \kappa_{k,i} \right), \tag{3}
\]

where \( x_{k,i} \) is the \( i \)th entry of the observation vector \( x_k \), \( \mathcal{N}_i \) is the set of immediate neighbors of the \( i \)th sensor, \( W_k(i,i) \) is a power amplifying factor used for transmitting the measurement of the \( i \)th sensor to the FC, \( W_k(i,j) \) is the power amplifying factor used for transmitting the measurement of the \( j \)th sensor to the \( i \)th sensor, and \( \{ \kappa_{k,i} \mid k \geq 0 \} \) denotes the communication noise associated with a coherent MAC for inter-sensor collaboration, which is an independent, identically distributed (i.i.d) sequence, independent of other system parameters, with zero mean and variance \( \sigma^2 \).

The sensor collaboration model (3) can also be written in a compact form

\[
z_k = W_k x_k + \kappa_k, \quad W_k \circ (1_M 1_N^T - A) = 0, \tag{4}
\]

where \( z_k = [z_{k,1}, \ldots, z_{k,M}]^T \in \mathbb{R}^M \) denotes the collaborative signals at time \( k \), \( W_k \in \mathbb{R}^{M \times N} \) is the collaboration matrix that contains collaboration weights (based on the allocated energy) used to combine sensor measurements, \( W_k(i,j) \) is the \((i,j)\)th entry of \( W_k \), \( \kappa_k = [\kappa_{k,1}, \ldots, \kappa_{k,M}]^T \) is the vector of receiver communication noises for sensor collaboration, \( \circ \) denotes the Hadamard product, \( 1_N \) is the \( N \times 1 \) vector of all ones, and \( 0 \) is the \( M \times N \) matrix of all zeros. In what
follows, while referring to vectors of all ones and all zeros, their dimensions will be omitted for simplicity but can be inferred from the context. In (4), the second identity implies that \( W_k \) preserves the sparsity structure of the topology matrix \( A \).

After sensor collaboration, the collaborative signal \( z_k \) is transmitted through a coherent MAC so that the received signal \( y_k \) at the FC is a coherent sum

\[
y_k = g_k^T z_k + \zeta_k, \quad k = 1, 2, \ldots
\]

where \( g_k \in \mathbb{R}^M \) is the vector of channel gains with known second-order statistics \( \mathbb{E}[g_k^T g_k] = g \) and \( \text{cov}(g_k) = \Sigma_g \), and \( \{\zeta_k\} \) is an i.i.d noise sequence independent of \( \{g_k\} \) and \( \{z_k\} \) with zero mean and variance \( \sigma^2 \). We assume that the FC knows the second-order statistics of the observation gain, channel gain, and additive noises. Our goal is to design the optimal energy allocation scheme \( W_k \) such that the estimation distortion of \( \theta_k \) is optimized under real-time energy harvesting constraints. Prior to formally stating our problem, we next introduce the transmission costs, energy-harvesting sensors, and the estimation approach.

### B. Transmission cost

In the entire estimation system, sensor energy is consumed for both inter-sensor communication and sensor-to-FC communication. Based on (3), the cost of energy consumed at sensor \( j \in [N] \) and time \( k \) for spatial collaboration is

\[
T^{(k)}_{k,j}(W_k) := \sum_{i \in \mathcal{N}_j} (W_k(i,j)^T x_{k,j})^2,
\]

and its expectation over stochastic parameters is given by

\[
\mathbb{E}[T^{(k)}_{k,j}(W_k)] = (e_j^T \mathbb{E}[x_k x_k^T] e_j) \sum_{i=1,i \neq j}^M W_k^2(i,j) - e_j^T (W_k \circ \mathbf{1})^T(W_k \circ \mathbf{1}) e_j,
\]

where \( \mathbb{E}[x_k x_k^T] = \Sigma_x + \sigma^2 (\mathbf{1} \mathbf{1}^T + \Sigma_h) \), the equality (a) holds since \( W_k(i,j) = 0 \) if \( i \notin \mathcal{N}_j \) and \( i \neq j \), \( \circ \) denotes the Hadamard product, \( \mathbf{1} := \mathbf{1}M - [\mathbf{1}_M, \mathbf{0}_{M \times (N-M)}] \), and \( e_j \) is a basis vector with 1 at the \( j \)th coordinate and 0s elsewhere.

Moreover from (4), the cost of energy consumed at sensor \( i \in [M] \) and time \( k \) for sensor-to-FC communication becomes

\[
T^{(k)}_{k,i}(W_k) = z_{k,i}^2
\]

and its expectation is given by

\[
\mathbb{E}[T^{(k)}_{k,i}(W_k)] = e_i^T W_k \mathbb{E}[x_k x_k^T] W_k^T e_i + \sigma^2.
\]

The total transmission cost can now be represented as

\[
T_{k,i}(W_k) = \begin{cases} 
T^{(k)}_{k,i}(W_k) + T^{(k)}_{k,i}(W_k), & i \leq M, \\
T^{(k)}_{k,i}(W_k), & M < i \leq N,
\end{cases}
\]

where we recall that only the first \( M \) sensors are used to communicate with the FC, and thus \( T^{(k)}_{k,i}(W_k) = 0 \) if \( i > M \).

### C. Energy flow of energy harvesting sensors

Let \( H_{k,i} \) denote the harvested energy of the \( i \)th sensor at time \( k \). Let \( H_k = \{H_{k,i}, i = 1, 2, \ldots, N\} \). Following [26], [27], we assume that \( \{H_{k,i}, k \geq 0\} \) satisfy the strong law of large numbers (SLLN) with the limiting mean \( \mathbb{E}[H_{k,i}] = \mu_i \): \( \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^n H_{k,i} = \mu_i \) almost surely. An asymptotically stationary and ergodic process \( \{H_k\} \) satisfies this assumption. This energy generation process is general enough to cover the assumptions usually made in the literature [23], [32], and is quite realistic. In particular, this allows correlation in time of the energy harvested at a sensor as well as spatial correlation in the energy harvested at neighboring sensors. Moreover, we assume that the energy harvested at the end of each time step, namely \( H_{k,i} \) is not available until the beginning of time \( k+1 \). The energy storage buffer of each sensor is considered to be large enough (here assumed with infinite capacity). And each sensor is supposed to know the energy it has in its buffer at a given time. We denote by \( S_{k,i} \) the energy available in the \( i \)th sensor at time \( k \), which evolves as

\[
S_{k+1,i} = (S_{k,i} - T_{k,i})^+ + H_{k,i}, \quad i \in [N],
\]

where for notational simplicity, we use \( T_{k,i} \) to denote the transmission cost given by (10) and \((x)^+ = \max\{0, x\}\).

### D. Problem Formulation

In this setup, the goal is to solve the following problem: At each time \( k \), find an energy allocation scheme \( W_k \) such that \( \sum_{k=1}^\infty \mathbb{E}[\{\theta_k - \theta_k\}^2] \) is minimized with the real-time energy constraints \( T_{k,i} \leq S_{k,i}, \forall i = 1, 2, \ldots, N \) and \( \forall k \geq 1 \).

In the above problem, \( \hat{\theta}_k \) is the estimate of \( \theta_k \). However, this problem may not remain finite for all choices of \( W_k \). Therefore, we consider the minimization of

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^N \mathbb{E}[\{\theta_k - \hat{\theta}_k\}^2].
\]

This is close to the standard average cost Markov decision problem (MDP) [33] except that we have real-time constraints, which are not considered by the usual constrained MDP [34]. Thus, we may consider a greedy approach: At each time \( k \), find \( W_k \) that minimizes \( \mathbb{E}[\{\theta_k - \hat{\theta}_k\}^2] \) such that \( T_{k,i} \leq S_{k,i} \) for each \( i \). This will imply that we should take \( T_{k,i} = S_{k,i} - \eta \), \( \eta \) is a small positive constant. Also, for computational reasons, because we need to perform the computations at each time \( k \), we consider a linear estimator. Furthermore, ours is a distributed setup: the energy constraint \( S_{k,i} \) will only be

### Table I: List of System Parameters

| Parameter                          | Symbol | Mean | Covariance |
|------------------------------------|--------|------|------------|
| Parameter of interest              | \( \theta_k \) | 0    | \( \sigma \) |
| State transition coefficient       | \( \alpha \)  | -    | -          |
| Parameter model noise              | \( \tau_k \)  | 0    | \( \sigma^2 \) |
| Observation gain vector            | \( v_k \)   | \( h \) | \( \Sigma_h \) |
| Measurement noise vector           | \( \epsilon_k \) | 0  | \( \Sigma_x \) |
| Collaboration noise at sensor \( i \) | \( \kappa_{k,i} \) | 0    | \( \sigma^2 \) |
| Channel gain vector                | \( g_k \)   | \( g \) | \( \Sigma_g \) |
| Transmission noise                 | \( \zeta_k \) | 0    | \( \sigma^2 \) |
| Rayleigh fading channel parameter | \( \xi \)    | \( \xi \sqrt{\frac{2}{\pi}} \) | \( \frac{1-e^{-2\xi^2}}{2} \) |
known to sensor $i$ at time $k$ and is randomly changing with time. Also, we would like the FC to compute $W_k$ because it has the maximum computational power and is assumed to be connected to a regular power grid. However, sending the needed information $\mathbf{h}_k$, $g_k$ and $c_k$ at each time $k$ will incur too much communication cost. Also, after computation, the FC will need to transmit $W_k$ to all the sensors for them to decide on the collaboration strategy.

Taking the above considerations into account, in the following, we first formulate an offline optimization problem that can be solved using only the second-order statistics of $\mathbf{h}_k$, $g_k$, $\mathbf{H}_k$, $\alpha$ and the variances of the receiver noises at different sensor nodes. This solution $W^*$ will not necessarily satisfy the real-time energy constraint at the sensor nodes. The solution $W^*$ is provided to all the sensor nodes and the FC before hand. Based on this solution, each sensor can obtain a real-time solution $W_k$ using only its own energy information $S_{k,i}$ at time $k$. We will show that asymptotically the mean squared error (MSE) of the real-time solution matches that obtained using $W^*$. Furthermore, at the end, via simulations, we will also show that the performance of this scheme is close to that of an optimal linear estimator computed at the FC at a far more computational cost and using all the current information of $\mathbf{h}_k$ and $g_k$.

Remark 1: The second-order statistics can be estimated using robust mean and variance estimators \cite{35}. Most commonly used robust estimators, e.g., $M$-estimators and $S$-estimators, to estimate the covariance matrix have been shown to have very low estimation error with a reasonable number of samples \cite{35}.

III. OFFLINE OPTIMIZATION PROBLEM

Based on (1)–(5), we consider the state space model at the FC,

$$
\begin{align*}
\theta_k &= \alpha \theta_{k-1} + \tau_k, \\
y_k &= g_k^T W_k \mathbf{h}_k + v_k,
\end{align*}
$$

(13)

where $v_k = g_k^T W_k c_k + g_k^T \xi_k + \eta_k$. Here $v_k$ can be regarded as the measurement noise with statistics $\mathbb{E}[v_k] = 0$ and $\text{cov}(v_k) = \text{tr}(W_k \Sigma_v g_k^T g_k) + \sigma_v^2 \text{tr}(\mathbb{E}[g_k g_k^T]) + \sigma_k^2$.

$$
\begin{align*}
\text{cov}(v_k) &= \text{tr}(W_k \Sigma_v g_k^T g_k) + \sigma_k^2 \text{tr}(\Lambda_g) + \sigma_v^2,
\end{align*}
$$

(14)

where $\Lambda_g = \mathbb{E}[g_k g_k^T] = gg^T + \Sigma_g$ from (5).

We will consider $W_k = \mathbf{W}$ in this section and find $\mathbf{W}$ that minimizes (12). Since we are considering a linear estimator, we can formulate it as a Kalman filter. However, even with $W_k = \mathbf{W}$, the filter for the state space model given in (13) is not a standard Kalman filter because of the multiplicative noise $g_k^T W_k c_k$ in the measurement model. A recursive linear minimum mean squared error estimator (R-LMMSE) in the presence of multiplicative noise has been considered in \cite{36, 37}. Motivated by the results in \cite{36, 37} and since we have already assumed that $|\alpha| < 1$, the following Lemma shows that (13) satisfies the assumption in \cite{37} and hence that the R-LMMSE to be derived is stable. In the following, we denote $u_k := g_k^T W_k c_k$.

Lemma 1: Given the state space model in (13), we have the following results: $u_k$ is uncorrelated with $v_k$. Moreover, $\{v_k\}$ is a zero mean, uncorrelated sequence.

Proof: See Appendix [A] \[\blacksquare\]

Based on Lemma 1, R-LMMSE \cite{37} is employed to track the random parameter $\theta_k$, which is shown in Algorithm [1].

Algorithm 1 R-LMMSE with known second-order statistics of the system.

Inputs: $\theta_0 = 0$, $P_0$ assumed known

1) Prediction step:

$$
\hat{\theta}_k|_{k-1} = \alpha \hat{\theta}_{k-1},
$$

(15)

with the predicted estimation error

$$
P_{k|k-1} = \alpha^2 P_{k-1} + \sigma_v^2.
$$

(16)

2) Updating step:

$$
\hat{\theta}_k = \hat{\theta}_{k|k-1} + d_k (y_k - \mathbb{E}[u_k \hat{\theta}_{k|k-1}]),
$$

(17)

with the updated estimation error

$$
P_k = (1 - d_k \mathbb{E}[u_k]) P_{k|k-1},
$$

(18)

where $d_k$ is the estimator gain

$$
d_k = \frac{P_{k|k-1} \mathbb{E}[u_k]}{(\mathbb{E}[u_k])^2 P_{k|k-1} + \text{cov}(v_k) + \text{cov}(u_k)} - 1
$$

(19)

and

$$
s_k := \text{cov}(\theta_k), s_k = \sigma_k^2
$$

(20)

with known initial condition $s_0$, $\mathbb{E}[u_k] = g^T W_k c_k$ and

$$
\mathbb{E}[u_k u_j] = \begin{cases}
\text{tr}(W_k \Lambda_g W_k^T \Lambda_g) & k = j, \\
0 & k \neq j.
\end{cases}
$$

(21)

Using the fact that $\alpha^2 < 1$, we note that $s_\infty = \frac{\sigma^2}{1 - \alpha^2}$. Moreover, $P_\infty$ satisfies the Riccati equation

$$
P_\infty = \alpha^2 \frac{P_\infty d}{\sigma^2} + \frac{\sigma^2}{\alpha d} + \sigma^2,
$$

(23)

where $\bar{c} = g^T W_k c_k$, and $\bar{d} = \text{tr}(W_k \Sigma_v W_k^T \Lambda_g) + \sigma_v^2 \text{tr}(\Lambda_g) + \sigma_k^2 \text{tr}(W_k \Lambda_g W_k^T \Lambda_g)s_\infty - (g^T W_k c_k)\sigma^2$. Also, \cite{32} equals $P(\infty)$. Thus, we would like to obtain the scheme $W$ that minimizes $P(\infty)$ which at the same time also satisfies the energy constraint in the mean, i.e., $\mathbb{E}[T_i(W)] \leq \mu_i - \eta_i$ for all $i$ and a small positive $\eta$, where $T_i(\cdot)$ denotes the total transmission cost as $k \to \infty$.

It is known from \cite{9} Lemma 1 that $P_\infty$ is a decreasing function of $\sigma^2/d$. Thus, the optimal $W$ can be obtained by solving the following optimization problem:

$$
\begin{align*}
\text{maximize} \quad & f(W) \\
\text{subject to} \quad & W \odot (11^T - \Lambda) = 0, \\
& \mathbb{E}[T_i(W)] \leq \mu_i - \eta_i, \quad i \in [N],
\end{align*}
$$

(24)
where $T_1(W)$ is the transmission cost given by (10), the expectation is with respect to the stationary distribution of $\theta_k$, $\mu_i$ is the mean of the harvested energy at sensor $i$, $\eta > 0$ is a given small constant and

$$f(W) = \frac{(g^T W h)^2}{\text{tr}(W|A_h s_\infty + \sum_i|W^T \Lambda_g| + \sigma^2_w \text{tr}(\Lambda_g) + \sigma^2_z t)},$$

where $s_\infty = \frac{\sigma^2_w}{1 - \alpha^2}$. In the offline optimization problem (24), the last inequality constraint implies that the energy cost cannot exceed the amount of harvested energy on an average, where $\eta$ can be regarded as the remaining average energy that is deliberately kept in the energy storage buffer for further usage [21]. We observe that $\{\theta_k\}$ is a geometrically ergodic Markov chain and the regeneration epochs of $\{\theta_k\}$ and $\{\{T_k(W), \theta_k\}\}$ are the same. Therefore, $\{\{T_k(W), \theta_k\}\}$ is also a geometrically ergodic Markov chain. In particular, $T_k(W)$ satisfies SLLN.

**Remark 3:** Note that the stated optimization problem in (24) is an offline sensor collaboration scheme. Therefore, it may not satisfy real-time energy constraints.

This optimal offline sensor collaboration scheme needs to be determined only once, and thus has significant computational merit. In the next section, we use the offline solution to derive an online solution that asymptotically achieves the optimal $P_\infty$ subject to the real-time energy constraints. Some key equations are summarized in Table II.

### IV. Optimal Sensor Collaboration: From Offline to Online Solution

In this section to solve the offline problem (24), we begin by concatenating the nonzero entries of a collaboration matrix into a collaboration vector. There are two benefits of using matrix vectorization: a) the topology constraint can be eliminated without loss of performance, which results in a less complex problem; b) the structure of nonconvexity is easily characterized via such a reformulation. We then derive the optimal solution of offline problem (24) with the aid of semidefinite programming. Given the optimal offline sensor collaboration scheme, we provide an online scheme which is asymptotically equivalent to the optimal offline solution.

In the offline problem (24), the only optimization variables are the nonzero entries of the collaboration matrix. To solve this problem, we begin by concatenating the nonzero entries of $W$ (columnwise) into the collaboration vector $w = [w_1, w_2, \ldots, w_L]^T \in \mathbb{R}^L$, where $L$ is the total number of nonzero entries in the topology matrix $A$. We also note that there exists a one-to-one correspondence between the elements of $w$ and $W$, that is, there exists a row index $m_l$ and a column index $n_l$ such that $w_l = [W]_{m_l n_l}$, where $[X]_{ij}$ (or $X_{ij}$) denotes the $(i, j)$th entry of a matrix $X$.

Another important observation from the offline problem (24) is that both the objective function and the energy constraint involve quadratic matrix functions. Inspired by this fact, we explore the relationship between $W$ and $w$ in Proposition 1.

**Proposition 1:** Given a matrix $W \in \mathbb{R}^{M \times N}$ and its columnwise vector $w \in \mathbb{R}^L$ that only contains the nonzero elements of $W$, the expressions of $b^T W$ and $\text{tr}(CWDW^T)$ can be equivalently expressed as functions of $w$.

$$b^T W = w^T B,$$

$$\text{tr}(CWDW^T) = w^T E w,$$

where $B \in \mathbb{R}^{L \times N}$ and $E \in \mathbb{R}^{L \times L}$ are given by

$$B_{m_l n_l} = \begin{cases} b_{m_l n_l}, & n = n_l \\ 0, & \text{otherwise} \end{cases},$$

$$E_{kl} = [C]_{m_k n_l} [D]_{n_n n_l},$$

for $n \in [N], k \in [L]$ and $l \in [L]$, where the indices $m_l$ and $n_l$ satisfies $w_l = W_{m_l n_l}$ for $l \in [L]$.

**Proof:** See Appendix II.

Based on Proposition 1, we can simplify the offline problem (24) by: a) eliminating the network topology constraint, and b) expressing the objective function $f(W)$ and the transmission cost $\mathbb{E}[T_1(W)]$ in the forms that only involve quadratic vector functions. Consequently, the offline problem (24) becomes

$$\max_w w^T \Omega w$$

subject to $w^T \Omega_{D,i} w + \sigma^2_i \text{tr}(\Lambda_g) + \sigma^2_z / \alpha^2 \leq \mu_i - \eta, \quad i \leq M,$

$$w^T \Omega_{C,i} w \leq \mu_i - \eta, \quad M < i \leq N,$$

where $\Omega_N, \Omega_D, \Omega_{T,i}$ and $\Omega_{C,i}$ are all positive semidefinite matrices. We elaborate on the expressions of the aforementioned coefficient matrices in Appendix II.

We remark that problem (31) is a nonconvex optimization problem as it requires the maximization of a ratio of convex quadratic functions. However, we will show that semidefinite programming (SDP) can be used to find the globally optimal solution of problem (31).

By introducing a new variable $t$, together with a new constraint $1/t^2 = w^T \Omega_{D} w + \sigma^2_i g^T g + \sigma^2_z / \alpha^2$, problem (31) can be rewritten as

$$\max_{w,t} t^2 w^T \Omega_N w,$$

subject to $t^2 w^T \Omega_{T,i} w + t^2 \sigma^2_i \leq t^2 (\mu_i - \eta), \quad i \leq M,$

$$t^2 w^T \Omega_{C,i} w \leq t^2 (\mu_i - \eta), \quad M < i \leq N,$$

$$t^2 w^T \Omega_{D} w + t^2 \sigma^2_i g^T g + t^2 \sigma^2_z / \alpha^2 = 1,$$

where $t$ and $w$ are optimization variables.

Upon defining $\bar{w} = [tw^T, t]^T \in \mathbb{R}^{L+1}$, problem (32) can be rewritten as

$$\max_{\bar{w}} \bar{w}^T Q_{0} \bar{w},$$

subject to $\bar{w}^T Q_{1,i} \bar{w} \leq 0, \quad i \leq M,$

$$\bar{w}^T Q_{2,i} \bar{w} \leq 0, \quad M < i \leq N,$$

$\bar{w}$ is a concatenation of $w$ and $t$, where $[Q_{0}]_{ij}$, $[Q_{1,i}]_{ij}$, $[Q_{2,i}]_{ij}$ denote the $(i, j)$th element of the coefficient matrices $Q_0, Q_1, Q_2$, respectively.

1 Quadratic matrix function is a function $f : \mathbb{R}^{n \times r} \rightarrow \mathbb{R}$ of the form $f(X) = \text{tr}(X^T A X) + 2 \text{tr}(B^T X) + c$ for certain coefficient matrices $A$, $B$ and $c$. 

---

**TABLE II:** Key Equations

| Equation Number | Problem Statement |
|-----------------|------------------|
| (24)            | Offline optimization problem |
| (36)            | Constructed online energy allocation policy |
| (38)            | Statistics based optimization problem |
| (42)            | Full knowledge based online policy |
where \( \bar{w}^T Q_{N+1} \bar{w} \leq 1 \),
where \( \bar{w} \) is the optimization variable, and
\[
Q_0 = \begin{bmatrix} \Omega_N & 0 \\ 0^T & 0 \end{bmatrix}, \quad Q_{i,1} = \begin{bmatrix} 0 & 0 \\ 0^T \end{bmatrix}, \quad Q_{i,2} = \begin{bmatrix} \bar{\Omega}_i & 0 \\ 0^T \end{bmatrix}, \quad Q_{N+1} = \begin{bmatrix} \Omega_0 + g^T g + \sigma_2^2 \end{bmatrix}.
\]

In problem (33), we have replaced \( \bar{w}^T Q_{N+1} \bar{w} = 1 \) with its inequality counterpart without loss of performance since the inequality constraint is satisfied with equality at the solution while maximizing a convex quadratic function. We also note that the solution of (31) can be obtained from the solution of problem (33) via \( w = [\bar{w}]_L/|\bar{w}|_L + 1 \), where \([\bar{w}]_L\) denotes the vector that consists of the first \( L \) entries of \( \bar{w} \), and \([\bar{w}]_L + 1 \) is the \((L+1)\)th entry of \( \bar{w} \).

Problem (33) now contains a special nonconvex structure: maximization of a convex homogeneous quadratic function (i.e., no linear term with respect to \( w \) is involved) subject to homogeneous quadratic constraints. Although problem (33) is not convex, its globally optimal solution can be obtained via SDP. We provide the solution of problem (33) in Proposition 2. In the following Proposition, \([W^*]_L = s^*(s^*)^T\) denotes the submatrix of \( W^* \) formed by deleting its \((L+1)\)st row and column. It is rank 1.

**Proposition 2:** Let \( W^* \) be the solution of the following SDP problem,
\[
\begin{align*}
\text{maximize} & \quad \text{tr}(Q_0 W), \\
\text{subject to} & \quad \text{tr}(Q_{i,1} W) \leq 0, \quad i \leq M, \\
& \quad \text{tr}(Q_{i,2} W) \leq 0, \quad M < i \leq N, \\
& \quad \text{tr}(Q_{N+1} W) \leq 1, \quad W \succeq 0,
\end{align*}
\]
where \( W \in \mathbb{R}^{(L+1) \times (L+1)} \) is the optimization variable. The solution of problem (31) is then given by
\[
W^* = s^*/\sqrt{[W^*]_L + 1},
\]
where \([W^*]_L + 1 \) is the \((L+1)\)th diagonal element of \( W^* \).

**Proof:** See Appendix C.

The optimal offline sensor collaboration matrix, namely, the solution of the offline problem (24), can now be found by using Proposition 2 and the sparsity pattern of the network toplogy matrix \( A \). In what follows, we will utilize the optimal offline sensor collaboration scheme to design an online sensor collaboration scheme \( W_k \) that satisfies the real-time energy constraint \( T_k,i(W_k) \leq S_k,i \) for every time \( k \) and every sensor \( i \). Here we recall that at time \( k \), each sensor has access only to its stored energy \( S_{k,n} \) given by (11).

We construct the following online allocation policy: \( W_k = W^* \) if \( T_k,i(W^*) \leq S_k,i \), \( i \in [N] \), \( W_k = \beta_k \circ W^* \), otherwise, where \( W^* \) denotes the optimal offline sensor collaboration matrix, \( \circ \) denotes the Hadamard product, \( \beta_k = \min_i \{ \beta_{k,i} \} \), and \( \beta_{k,i} = \min \{ 1, \sigma_i(S_k,i) / T_k,i(W^*) \} \). Once \( W_k \) is obtained from (36), the transmission cost at time \( k \) is then given by \( T_k,i = T_k,i(W_k) \). Subsequently, the status of the energy storage buffer at the next time step is updated as \( S_{k+1,i} \) according to (11).

**Remark 4:** Note that the computation of \( \beta_k \) requires the global knowledge \( \beta_{k,i}, i \in [N] \). However, we can use the max-consensus protocol such that the computation \( \beta_k = \max_i \{-\beta_{k,i} \} \) can be performed in a decentralized setting via inter-sensor message passing.

In Theorem 1 we show that the proposed online sensor collaboration policy is asymptotically consistent with the optimal offline solution in the mean-squared-error sense under the framework of R-LMMSE. To show this asymptotic consistency, we first present Lemma 2.

**Lemma 2:** As \( k \to \infty \), the following results hold:
1) \( \theta_k \overset{k}{\to} \theta_\infty, E[\theta_k] \to E[\theta_\infty] \) for \( \gamma = 1, 2 \), where \( \overset{k}{\to} \) denotes convergence in distribution.
2) \( E[T_k,i(W^*)] \to E[T_i(W^*)] \) for each \( i \in [N] \).

**Proof:** See Appendix D.

**Theorem 1:** Under the R-LMMSE framework (15)–(18), let \( \theta_{k|k} \) and \( \theta_{k|k}^* \) denote the estimates when using the online sensor collaboration scheme and the offline policy \( W^* \). Then
\[
\lim_{k \to \infty} E[(\theta_{k|k} - \theta_{k|k}^*)^2] = 0.
\]

**Proof:** See Appendix E.

In practice, the system parameters \( \alpha, \sigma_0^2, \sigma_2^2, \Sigma_c, \Lambda_c, \Lambda_h \) needed for obtaining the online scheme \( W_k \) in (36), will need to be estimated. This will incur estimation error. The following proposition shows that the MSE \( P_\infty \) of the offline scheme \( W^* \) is robust with respect to the estimation error. The same can be shown for \( P_k \).

**Proposition 3:** The asymptotic MSE \( P_\infty \) is a continuous function of the system statistics parameters \( \alpha, \sigma_0^2, \sigma_2^2, \Sigma_c, \Lambda_c, \Lambda_h \).

**Proof:** See Appendix F.

It is also of interest to get the rate of convergence of \( E[(\theta_{k|k} - \theta_k)^2] \) to \( P_\infty \) depends on the rate \( P((1/k) \sum_{j=1}^k (H_j,i - T_j,i(W^*)) \leq \omega) \) and \( P(T_{k,i}(W^*)) > \omega \) to go to zero as \( k \) tends to infinity, where \( \omega \) is a small positive constant less than \( \eta \).

**Proof:** See Appendix G.

Now we provide some conditions on the distributions of the system parameters to obtain some rates of convergence of \( E[(\theta_{k|k} - \theta_k)^2] \) to \( P_\infty \). We can show that \( \theta_\infty \) has an exponential moment if \( \tau_k \) has a positive density everywhere and has finite moment generating function in a neighborhood of 0. Then if \( h_{k,i}, \theta_\infty \), and \( \epsilon_k \) have light tails, \( T_{k,i} \) has light tail. If \( h_{k,i} \) is finite valued, then \( h_{k,i} \theta_\infty \) is light tailed. Otherwise, this condition can get violated even if \( h_{k,i} \) is light tailed. If \( H_k \) is also strongly aperiodic, geometrically ergodic Markov chain, we can, using concentration inequalities for Markov processes and the above Lemma, show that we obtain exponential rate of convergence of \( E[(\theta_{k|k} - \theta_k)^2] \to P(\infty) \). Otherwise, one may only get a polynomial rate of convergence.
We emphasize that the proposed online policy has a significant computational merit in that it is only required to solve the sensor collaboration offline problem (34) once and then minimal extra computation (36) is required at each time. Theorem 1 relates the performance of the online strategy with that of the optimal offline strategy.

Note that we can also directly obtain a real-time optimal online sensor collaboration scheme by minimizing $P_k$, the estimation error when using R-LMMSE, subject to the expected transmission cost and scaling its solution using (36). The corresponding optimization problem is given as

$$\text{minimize } P_k \text{ subject to } \mathbf{W}_k \circ (11^T - \mathbf{A}) = 0,$$

where

$$P_k = \frac{(g^T \mathbf{W}_k \mathbf{h})^2}{\text{tr}(\mathbf{W}_k \Sigma_k \mathbf{h}_k) + \sum_i \text{tr}(\mathbf{A}_i) + \sigma^2_z}.$$

By converting the collaboration matrix to the collaboration vector, statistics based problem (38) can be solved using the optimization approach proposed in Section IV. However, the computational complexity for this filter is very high since we need to solve the optimization problem at each time instant. We do not know the stability of this filter either. Thus, unlike the optimal online filter obtained above, this filter is not asymptotically optimal, but rather a greedy optimal scheme.

We will see in Section VII that this filter does not perform better than the above proposed online filter.

To demonstrate the efficacy of the proposed online policy, in Section VII we will also present a comparison between our proposed sensor collaboration scheme with the online solution based on the full knowledge about the dynamical system, i.e., the realizations of observation and channels gains at each time.

V. COMPARISON WITH CSI BASED SENSOR COLLABORATION

In this section, for the purpose of comparison, we assume that the channel state information, i.e., $\mathbf{h}_k$ and $\mathbf{g}_k$, of the state-space model (13) is available at the FC at time $k$. In this ideal case, R-LMMSE simplifies to the standard Kalman filter shown in Algorithm 2.

Algorithm 2 R-LMMSE with CSI.

Inputs: $\hat{\theta}_0 = 0, P_0$ assumed known.

1) Prediction: (15) and (16)

2) Update:

$$\hat{\theta}_k = \hat{\theta}_{k|k-1} + q_k(y_k - g_k^T \mathbf{W}_k \mathbf{h}_k \hat{\theta}_{k|k-1}),$$

with the updated estimation error

$$P_k = (1 - q_k g_k^T \mathbf{W}_k \mathbf{h}_k) P_{k|k-1},$$

where $q_k$ is the Kalman filter gain

$$q_k = \frac{P_{k|k-1} g_k^T \mathbf{W}_k \mathbf{h}_k}{P_{k|k-1} g_k^T \mathbf{W}_k g_k + \sigma^2_k \Sigma_k},$$

where $E[\sigma^2_k] = g_k^T \mathbf{W}_k \Sigma_k \mathbf{W}_k^T \mathbf{g}_k + \sigma^2_k$. The sensor collaboration problem given the CSI about the estimation system is cast as

$$\text{maximize } \mathbf{w}_k \frac{(g_k^T \mathbf{W}_k \mathbf{h}_k)^2}{g_k^T \mathbf{W}_k \Sigma_k \mathbf{W}_k^T \mathbf{g}_k + \sigma^2_k \Sigma_k}$$

subject to

$$E[T_{k,i}(\mathbf{W}_k)] \leq \mu_k - \eta, \quad i \in [N],$$

$$\mathbf{W}_k \circ (1 + 1^T - \mathbf{A}) = 0.$$

Similarily, by converting the collaboration matrix to the collaboration vector, the CSI based problem (42) can be solved using the optimization approach proposed in Section IV. However, the stability of the Kalman filter for the CSI based case has not been established in the literature. Let $\mathbf{W}_k$ be the solution of the CSI based problem (42) at time $k$. We then use (36) to construct the real-time sensor collaboration scheme $\mathbf{W}_k$.

This Kalman filter is a greedy scheme and not necessarily even asymptotically optimal. This is because in this setup we cannot use our approach used for the offline filter to optimize $P_{\infty}$ since stability of this Kalman filter is not known. Another possible approach to obtain an optimal filter could be via Markov decision theory. But this would be computationally infeasible even for a small number of sensors. Also, such CSI at the FC is not available in practice. Instead, we usually have access only to the second-order statistics of the system parameters for parameter inference as in Section IV. In the next section, we will empirically show that in terms of estimation accuracy, the sensor collaboration policy proposed in Section IV performs better than the CSI based approach at low signal-to-noise ratios (SNRs), where the SNR refers to SNRs in terms of measurement noise and collaboration noise. Moreover, the former is computationally light, while the latter in addition to requiring much more information, also requires the solution of the CSI based problem (42) at every time step, which thus introduces high computational complexity.

VI. NUMERICAL RESULTS

In this section, we demonstrate the efficacy of our proposed sensor collaboration schemes through numerical examples. The collaborative tracking system is shown in Fig. 1 where the vector of observation gains $\mathbf{h}_k$ and channel gains $\mathbf{g}_k$ are chosen randomly and independently from Rayleigh($\xi$) distribution with $\xi = 1$. Therefore, we obtain $\mathbf{h} = \mathbf{g} = \sqrt{\pi/21}$ and $\Sigma_h = \Sigma_g = (4 - \pi)/21$. The observation noise $\epsilon_k$ and the channel noise $\zeta_k$ are drawn from zero-mean Gaussian distributions with variances $\Sigma_{\epsilon_k} = 0.5(1 + 1^T)$ and $\sigma^2_\zeta = 1$. Similarly, the vector of collaboration noises $\zeta_k$ at time $k$ is assumed to be Gaussian with covariance $\Sigma_{\zeta_k} = \sigma^2_\zeta \mathbf{I}$, where unless specified otherwise, we set $\sigma^2_\zeta = 0.5$. The parameter $\theta_k$ evolves as a Gauss-Markov process with Gaussian noise $\epsilon_k$ of variance $\sigma^2_\epsilon = 0.5$. The state transition parameter is set as $\alpha = 0.8$. The initialized variance of $\theta_k$, denoted by $\theta_0$, is 100. We assume that the spatial placement and neighborhood structure of the sensor network is modeled as a random geometric graph, RGG($N, r$) [5, 41], where sensors are uniformly distributed over a unit square meter area and bidirectional communication links are possible only for pairwise distances at most $r$ meters, where $r$ refers to collaboration radius. Namely, $A_{mn} = 1$ if the distance between the $n$th sensor and the $m$th sensor is less
than \( r \), otherwise it is 0. Without loss of generality, we assume that \( N = 10 \) sensors are uniformly distributed over the region of interest and unless otherwise specified, we will assume \( M = 10 \) and \( r = 0.6m \). At each sensor, the harvested energy \( H_{k,i} \) is modeled as an Exponential random variable with mean \( \mu_i = 10 \) for \( i \in [N] \). In the considered sensor collaboration schemes, the empirical mean squared error (MSE) is computed by simulating over 500 samples and taking average.

The proposed offline sensor collaboration scheme \( W^* \) is the solution of problem (24).

The proposed online sensor collaboration scheme \( W_k \) is defined in (36).

The online sensor collaboration scheme based on temporally-dependent statistics of system parameters \( W_k^1 \) is the solution of problem (38) that satisfies (36).

The CSI based online greedy sensor collaboration scheme \( W_k^2 \) is the solution of problem (42) that satisfies (36).

We remark that compared to our proposed sensor collaboration schemes \( W^* \) and \( W_k \), the schemes \( W_k^1 \) and \( W_k^2 \) require the solution of the optimization problems at every time step. This naturally leads to high computational cost. The tracking performance for the above four schemes is measured through the empirical MSE, which is computed by simulating over 500 samples and taking average.

In Fig. 2, we present the empirical MSE when using the aforementioned sensor collaboration schemes as a function of time \( k \) with \( \eta = 0.01 \). As we can see, the empirical MSE decreases as \( k \) increases, and it converges to a steady-state value by \( k = 10 \). Note that the average variance of the parameter is decreasing as \( k \) increases and eventually converges to a constant which implies that the average SNR is decreasing as \( k \) increases; see (22). As we can see, our proposed sensor collaboration schemes \( W^* \) and \( W_k \) yield MSE that is better than the greedy CSI based scheme \( W_k^2 \) at low SNR. This indicates that our proposed scheme \( W_k \) is efficient not only in computation but also in estimation performance at low SNR. The empirical MSE against SNRs with respect to measurement noise and collaboration noise will be explored in the following. We also remark that the offline scheme \( W^* \) performs as well as the online schemes. But the
former might not obey the real-time energy constraint.

In Fig. 3 we present the 95% confidence intervals of $\hat{\theta}_k$ for the proposed optimal online scheme under $\eta = 0.01$ using two tailed t-test with a 1000 number of samples. As we can see, our proposed online scheme performs very well as $k$ increases.

In Fig. 4 we show the impact of the choice of $\eta$ on the tracking performance for the proposed online scheme. We recall that the parameter $\eta$ is the remaining average energy that
and the proposed offline scheme.

W energy at the first sensor for the proposed online schemes and our theoretical result in Theorem 1.

\[ \eta = 0 \]

is deliberately kept in the energy storage buffer. As we can see, the smaller the \( \eta \) is, the better the tracking performance we obtain since more energy is available to be used in sensor collaboration. As a result, a larger \( \eta \) leads to a less effective energy allocation scheme.

In Fig. 5, we show the asymptotic performance of the proposed online scheme and the optimal offline scheme under \( \eta = 0.01 \), namely, when \( k \) is large enough, the estimation performance of the proposed online scheme is the same as the offline scheme. As expected, the empirical MSE between \( W_k \) and \( W^* \) converges to 0 as \( k \) increases which is consistent with our theoretical result in Theorem 1.

In Fig. 6, we present the real-time stored, and consumed energy at the first sensor for the proposed online schemes \( W_k \) and the proposed offline scheme \( W^* \) with \( \eta = 0.01 \). Here the stored energy and the consumed energy at sensor 1 and time \( k \) are denoted by \( S_{k,1} \) and \( T_{k,1} \), respectively. As we can see, the consumed energy \( T_{k,1} \) is always less than or equal to the stored energy \( S_{k,1} \) for the proposed efficient online scheme. Such dynamics of the energy flow is consistent with (11). We have verified that the online schemes \( W^*_k \) and \( W^2_k \) also follow such energy dynamics. Moreover, we can see that the offline scheme violates real-time energy constraints.

In Fig. 7, we show the empirical MSE as a function of the average SNR in terms of measurement noise with \( \eta = 0.01 \) and \( k = 20 \). This \( k \) is large enough to provide the converged estimation error. As we can see, the empirical MSE decreases when the average signal to measurement noise ratio increases. This is expected. By fixing the average signal to measurement noise ratio, we can see that the proposed online scheme \( W_k \) never performs worse than \( W^1_k \) which requires much more computational effort. Also, it performs better than the full information filter \( W^2_k \) for signal to measurement noise ratio less than 15 dB.

In Fig. 8, we present the normalized empirical MSE as a function of the collaboration radius \( r \) at time step \( k = 20 \) with \( \eta = 0.01 \). As we can see, the normalized empirical MSE decreases significantly as \( r \) increases, since more collaboration links are established. Moreover, the normalized empirical MSE decreases to significantly decrease when \( r > 0.7 \). This indicates that a large part of the performance improvement is achieved only through partial collaboration.

In Fig. 9, we show the empirical MSE as a function of \( \alpha \) with \( \eta = 0.01 \) and \( k = 20 \) for signal to measurement noise ratio less than 3 dB. The empirical MSE increases when \( \alpha \) increases. This is expected because the estimation error is a monotonic function of \( \alpha \) that is the state transition coefficient of the parameter model; see (18) - (20). By fixing \( \alpha \), we can see that the proposed online scheme \( W_k \) performs close to or even better than the statistics based online scheme \( W^1_k \). Moreover, the proposed online scheme \( W_k \) performs better than the full knowledge based online scheme \( W^2_k \).

In Fig. 10, we present the empirical MSE as a function of the number of sensors \( N \) under \( \eta = 0.01 \) and \( k = 20 \). As we can see, the empirical MSE decreases when \( N \) increases since more information is transmitted to the FC. Moreover, we can see the proposed online scheme \( W_k \) performs better than the full knowledge based online scheme \( W^2_k \).

In Fig. 11, we show the empirical MSE as a function of \( M \), the number of sensors that communicate with the FC, for the proposed online scheme at \( \eta = 0.01 \) and \( k = 20 \). As we can see, the empirical MSE decreases when \( M \) increases since more sensors are used for communication to the FC and, therefore the FC can acquire more information.

VII. CONCLUSION

In this paper, we studied the problem of sensor collaboration for dynamic parameter tracking in energy harvesting sensor networks. We incorporated the effect of sensor collaboration noise while designing the online collaboration scheme. Based on the second-order statistics of the system parameters, we developed an online sensor collaboration policy that is built on the solution of the offline sensor collaboration problem. Although the latter is a complex non-convex optimization problem, we showed that it can be solved exactly via semidefinite programming. We also proved that the proposed online policy is asymptotically consistent with the optimal offline solution over an infinite time horizon. We have also shown via simulations that at low SNRs, in the region of interest in sensor networks, our online scheme actually outperforms a greedy optimal online policy which has full information at the fusion center about the channel gains and energy at the sensors at any given time. Numerical results have illustrated the efficiency of our approach and the impact of energy harvesting and collaboration noise on the performance of collaborative estimation.
In future work, it would be desirable to consider a more practical sensor collaboration model by taking into account the impact of pairwise distances between sensors on their channel gain. Also, it is interesting to take into account the energy consumption and delay due to the estimation of second-order statistics. Moreover, the scalar random parameter can be generalized to a vector of random parameters. Also, one can study quantization-based inter-sensor collaboration. Lastly, the design of network topologies could be taken into account under the current framework.

**APPENDIX A**

**PROOF OF LEMMA**

Since \( u_k = g_k^T W h_k \) and \( v_k = g_k^T W e_k + g_k^T \kappa_k + \varsigma_k \), we have
\[
E[u_k v_k] = E[g_k^T W h_k (g_k^T W e_k + g_k^T \kappa_k + \varsigma_k)] = E[g_k^T W h_k g_k^T W e_k + E[g_k^T W h_k g_k^T \kappa_k] + E[g_k^T W h_k \varsigma_k].
\]

The first term on the right hand side (RHS) of (43) yields
\[
E[g_k^T W h_k g_k^T W e_k] = E[g_k^T W h_k g_k^T W] \times E[e_k] = 0.
\]

Applying similar analysis for other terms on the RHS of (43), we can conclude that \( u_k \) is uncorrelated with \( v_k \). Likewise, we can show that \( \{v_k\} \) is an uncorrelated sequence.

**APPENDIX B**

**QUADRATIC VECTOR FUNCTIONS**

Based on Proposition 1, the offline problem (24) can be simplified such that it only contains quadratic vector functions with respect to \( w \). Using the identities (27) and (28), we can rewrite the quadratic matrix functions in \( f(W) \) as
\[
(g^T W h)^2 = w^T \Omega_n w,
\]
where \( \Omega_n = G h h^T G^T \) is defined as in (29) that satisfies
\[
\Omega_n^T w = w^T \Omega_D w,
\]
and \( \Omega_D \) is defined as in (30), namely, \( \Omega_{D,i} = [\Lambda_g m_{k,m} | A h s_{\infty} + \Sigma_e]_n_{k,n} \).

Recall the expected transmission cost for sensor-to-FC in (9).
\[
E[T_q^{(2)}(W)] = e_i^T W [A h s_{\infty} + \Sigma_e] W^T e_i + \sigma_{e,i}^2
\]
where \( T_q^{(2)} \) is also obtained according to (30), \( \Omega_q^{(2)}_{k,l} = [e_i e_i^T]_{m_{k,m},m_{l,m}} [A h s_{\infty} + \Sigma_e]_{n_{k,n},n_{l,n}} \).

According to (7), the expected transmission cost for inter-sensor communication is a function of \( \mathbf{W} \circ \mathbf{I} \), where \( \mathbf{I} = 11^T - [1M, 0_{M \times (N-M)}] \). We further define \( \mathbf{W} = \mathbf{W} \circ (11^T - [1M, 0_{M \times (N-M)}]) \), and one can view the matrix \( \mathbf{W} \) as the collaboration matrix \( \mathbf{W} \) with zero diagonals. Therefore,
\[
E[T_q^{(1)}(W)] = \text{tr}(e_i e_i^T[A h s_{\infty} + \Sigma_e]) \text{tr}(\mathbf{W} e_i e_i^T W^T).
\]

Similar to \( w \), let \( \mathbf{w} \) be the vector that consists of the nonzero entries (columnwise) of \( \mathbf{W} \), namely, \( \mathbf{w}_l = W_{m_{l,n}} \) for certain indices \( m_l \in [M], n_l \in [N] \) and \( m_l \neq n_l \). Clearly, \( \mathbf{w} \) is a vector of \((L-M)\) dimension, and it can be readily expressed as a linear function of \( w \), namely, \( \mathbf{w} = F w \) with \( \mathbf{F} \in \mathbb{R}^{(L-M) \times L} \), given by
\[
F_{ij} = \begin{cases} 1, & K(i) = j, \ j \in [L], i \in [L-M], \\ 0, & \text{otherwise} \end{cases}
\]
where \( K = \{ l|W_{m_{l,n}}, m_l \neq n_l \} \).

We elaborate this linear transformation through the following example.
\[
A_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \ \mathbf{w} = \begin{bmatrix} W_{11} \\ W_{31} \\ W_{22} \\ W_{23} \\ W_{23} \\ W_{33} \end{bmatrix}, \ \mathbf{w} = \begin{bmatrix} W_{31} \\ W_{13} \\ W_{23} \end{bmatrix}
\]

Given \( A_1 \), we can obtain that \( K = \{2, 4, 5\} \). Therefore, \( F_{12} = 1, F_{24} = 1 \) and \( F_{35} = 1, \text{and otherwise} F_{ij} = 0 \). We have
\[
F_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

By concatenating \( \mathbf{W} \) to \( \mathbf{w} \) and using the identity (28) as well as the linear transformation \( \mathbf{w} = F w \), we can rewrite the quadratic matrix function in (47) as
\[
\text{tr}(e_i e_i^T[A h s_{\infty} + \Sigma_e]) \text{tr}(\mathbf{W} e_i e_i^T W^T) = \mathbf{w}^T \Omega_i^{(0)} \mathbf{w},
\]
where \( \tilde{k}, \tilde{l} \)th entry of \( \Omega_i^{(0)} \) is also obtained according to (30), namely, \( \Omega_i^{(0)}_{k,l} = [e_i e_i^T]_{m_{k,m},m_{l,m}} [A h s_{\infty} + \Sigma_e]_{n_{k,n_{},n_{l,n}}} \), here \( k, l \in [L-M] \) and \( \Omega_i^{(1)} = F^T \Omega_i^{(0)} F \).

Upon defining \( \Omega_C,i = \Omega_i^{(1)} \) as the characterization of expected transmission cost consumed for inter-sensor communication, and \( \Omega_T,i = \Omega_i^{(1)} + \Omega_i^{(2)} \) as the characterization of both the expected transmission cost consumed for inter-sensor communication and sensor-to-FC, we eventually obtain the problem (31). In what follows, we summarize the coefficients precisely correspond to the problem (31).

\[
\begin{align*}
\Omega_N &= G h h^T G^T, \\
\Omega_D &= 0, \\
\Omega_C,i &= \Omega_i^{(1)}, \\
\Omega_T,i &= \Omega_i^{(1)} + \Omega_i^{(2)},
\end{align*}
\]

where \( \Omega_D[k,l] = [A_g m_{k,m} | A h s_{\infty} + \Sigma_e]_{n_{k,n_{},n_{l,n}}} \).

\( \Omega_i^{(1)} = F^T \Omega_i^{(0)} F, \Omega_i^{(2)}_{k,l} = [e_i e_i^T]_{m_{k,m},m_{l,m}} [A h s_{\infty} + \Sigma_e]_{n_{k,n_{},n_{l,n}}} \), and \( \Omega_i^{(2)}_{k,l} = [e_i e_i^T]_{m_{k,m},m_{l,m}} [A h s_{\infty} + \Sigma_e]_{n_{k,n_{},n_{l,n}}} \), for \( k, l \in [L-M] \) and \( \Omega_T,i = \Omega_i^{(1)} + \Omega_i^{(2)} \).

**APPENDIX C**

**PROOF OF PROPOSITION**

By introducing a new variable \( \mathbf{W} \in \mathbb{R}^{(L+1) \times (L+1)} \) together with the constraint \( \mathbf{W} = \mathbf{w} \mathbf{w}^T \), problem (33) can be reformulated as another quadratic problem, which is then solved using the same approach as before.
mulated as
\[
\text{maximize} \quad \text{tr}(Q_0 W),
\]
subject to
\[
\text{tr}(Q_1 W) \leq 0, \quad i \leq M,
\]
\[
\text{tr}(Q_i^2 W) \leq 0, \quad M < i \leq N,
\]
\[
\text{tr}(Q_{N+1} W) \leq 1, \quad \text{rank}(W) = 1, \quad W \succeq 0,
\]
where we have used the fact that the constraint \(\text{rank}(W) = 1\) together with \(W \succeq 0\) is equivalent to the constraint \(W = \bar{w}w^T\), and \(W \succeq 0\) indicates that \(W\) is positive semidefinite. After dropping the (nonconvex) rank-one constraint, problem (52) is relaxed to the semidefinite program (SDP) (34).

It is clear from (33)-(34) that the solution of (31) is achievable based on the solution of problem (34) if the latter renders a rank-one solution. Similar to (10) Theorem 1 and (38) Theorem 1.4, the rank-one property of \(W^*\) can be explored by studying the KKT conditions of problem (34). Details of the proof are omitted here for the sake of brevity. Since \(W^*\) is of rank one, we can immediately construct (35) as the solution of problem (31).

**APPENDIX D**

**PROOF OF Lemma 2**

1) Since \(|\alpha| < 1\), \(\theta(k) = (1 + \alpha)^{k}\) where \(\theta_{\infty}\) is an a.s. finite random variable [42]. Also, then \(\theta(k) \rightarrow \theta_{\infty} + \tau(k)\), where \(\theta_{\infty}\) and \(\tau(k)\) are independent, and \(\tau(k)\) has the distribution of \(\tau_1\) and \(\tilde{\tau}\) denotes equality in distribution. Therefore, \(E[\theta_1] = E[\theta_{\infty}] + E[\tau]\) and hence \(E[\theta_{\infty}] = E[\tau]/(1 - \alpha)\). Also, \(E[\theta_2] = E[\tau_1^2]/(1 - \alpha)\). Therefore, \(E[\theta_2] = E[\tau_2^2]/(1 - \alpha^2)\).

From the dynamical equation (1),
\[
\bar{y}_k = \bar{y}_0 + \bar{w}k + \alpha \tau_k + \alpha^2 \tau_{k-1} + \ldots + \alpha^{k-1} \tau_1.
\]
Therefore,
\[
E[\theta_k] = \alpha^k E[\theta_0] + (1 + \alpha + \ldots + \alpha^{k-1}) E[\tau_1] \rightarrow E[\tau_1]/(1 - \alpha), \quad \text{as } k \rightarrow \infty \quad \text{and}
\]
\[
E[\theta_2] = E[\tau_1^2]/(1 - \alpha^2).
\]

2) From (7) and (9), we have
\[
E[T_{k,i}^1(W^*)] = e_i^T (E[\theta_2^2]) (hh^T + \Sigma_\epsilon) e_i,
\]
\[
\times [\bar{\Sigma} (W^* - \Sigma_\epsilon) (W^* - \Sigma_\epsilon)] e_i,
\]
\[
E[T_{k,i}^2(W^*)] = e_i^T W^* (E[\theta_2]) (hh^T + \Sigma_\epsilon) e_i,
\]
\[
\times [\bar{\Sigma} (W^* - \Sigma_\epsilon)] e_i + \sigma^2 e_i.
\]
From the total transmission cost in (10), we obtain
\[
E[T_{k,i}^1(W^*)] \rightarrow E[T_i^1(W^*)], \quad \text{since from 1) } E[\theta_2^2] \rightarrow E[\theta_2^2_{\infty}].
\]

**APPENDIX E**

**PROOF OF Theorem 1**

In the following, we consider a space \(H\) of all real valued random variables on the common underlying probability space, with \(E[|x|^2] < \infty\). This is a Hilbert space with inner product \(E[xy] = E[|x|^2]\). Let \(\{y_k\}\) and \(\{y'_k\}\) denote the observations received at the FC by using the online sensor collaboration scheme (36) and the offline policy, respectively.

The estimates of \(\theta_k\) based on \(\{y_k\}\) and \(\{y'_k\}\) are given by the projection operator \(P_o\) in the Hilbert space \(H\):
\[
\hat{\theta}_k = P_o(\bar{y}_k | y_1, \ldots, y_k, \hat{\theta}_0),
\]
\[
\tilde{\theta}_k = P_o(\bar{y}_k | y'_1, \ldots, y'_k, \hat{\theta}_0).
\]

Consider the equation \(S_{k+1,i}^*(\bar{W}^*) = (S_{k+1,i}^* - T_{k,i}(W^*)) + H_{k,i}\) for \(i \in [N]\), with \(S_{0,i}^* = S_{0,i}\). We have \(\{H_{k,i}, k \geq 0\}\) and \(\{S_{k,i}(W^*), k \geq 0\}\), independent, each satisfying SLLN. Also, from Lemma 2, \(E[T_{k,i}(W^*)] \rightarrow E[T_i(W^*)]\). From offline problem (24), we have \(E[T_i(W^*)] \leq \mu_i - \eta\). Therefore, since \(S_{k+1,i}^* \geq \sum_{i=1} E(T_{k,i}(W^*))\), \(S_{k+1,i}^* \rightarrow \infty\) almost surely (a.s.) for all \(i\) as \(k \rightarrow \infty\). Since the actual energy consumption for sensor \(i\) at time \(k\) is \(T_{k,i}(W^*) \leq kT_{k,i}(W^*)\), we also have \(S_{k+1,i} \rightarrow \infty\) a.s. for all \(i\). Therefore, from (36), we obtain \(T_{k,i}(W^*) \rightarrow T_i(W^*)\) a.s. for all \(i\). Accordingly, we obtain that \(W_k \rightarrow W^*\) a.s. as \(k \rightarrow \infty\).

We will use \(||x||\) to denote \(\sqrt{E[x^T x]}\) for a random vector \(x\).

Since \(W_k \leq W^*\), we also obtain \(||W_k - W^*|| \rightarrow 0\) as \(k \rightarrow \infty\). From (13), we have
\[
||y_k - y'_k|| \leq \sqrt{E[\|y_k - y'_k\|^2]}.
\]

Since \(E[T_i(W^*)] \rightarrow E[T_i(W^*)],\) \(\|y_k - y'_k\|\) and \(\|\bar{y}_k\|\) (namely, \(s_\alpha\)) are bounded and \(||W_k - W^*|| \rightarrow 0\) and \(\|\bar{y}_k - \bar{y}_0\| \rightarrow 0\), as \(k \rightarrow \infty\), we obtain
\[
||y_k - y'_k|| \rightarrow 0, \quad \text{for } k \rightarrow \infty.
\]

In what follows, we will show that
\[
||P_o(\bar{y}_k | y_1, \ldots, y_k, \hat{\theta}_0) - P_o(\bar{y}_k | y'_1, \ldots, y'_k, \hat{\theta}_0)|| \rightarrow 0, \quad (57)
\]
for \(k \rightarrow \infty\).

From (1), we have \(\theta_k = \alpha^N \theta_{k-N} + \tau_k + \alpha \tau_{k-1} + \ldots + \alpha^{N-1} \tau_{k-N+1}\). Therefore, we get
\[
P_o(\bar{y}_k | y_1, \ldots, y_k, \hat{\theta}_0) = P_o(\alpha^N \theta_{k-N} + \tau_k + \alpha \tau_{k-1} + \ldots + \alpha^{N-1} \tau_{k-N+1} | y_1, \ldots, y_k, \hat{\theta}_0).
\]

Since \(\tau_1, \ldots, \tau_{N+1}\) are zero mean and independent of \(y_{k-N+1}, \ldots, y_k, \hat{\theta}_0\), from (58), we obtain that
\[
P_o(\tau_k + \alpha \tau_{k-1} + \ldots + \alpha^{N-1} \tau_{k-N+1} | y_1, y_2, \ldots, y_k, \hat{\theta}_0) + (59)
\]
\[
P_o(\tau_k + \alpha \tau_{k-1} + \ldots + \alpha^{N-1} \tau_{k-N+1} | y_1, y_2, \ldots, y_k, \hat{\theta}_0).
\]

For the first term in the RHS of (59), we note that
\[
\alpha^N P_o(\tau_k | y_1, \ldots, y_k, \hat{\theta}_0) \leq \alpha^N E[\theta_{k-N}^2] < \infty,
\]
where we have used the fact that \(E[\|x\|^2] \leq \|x\|\). Moreover, from Lemma 2 we obtain that \(E[\theta_{k-N}^2] < \infty\). Therefore, RHS of (60) can be taken arbitrarily small by taking \(N\) large enough. In what follows, we keep \(N\) fixed.

Now, let us focus on the second term in (59). For ease of notation, we define
\[
\bar{\tau}_k := \tau_k + \alpha \tau_{k-1} + \ldots + \alpha^{N-1} \tau_{k-N+1},
\]
\[
\bar{\bar{y}}_k := [y_1, \ldots, y_{k-N+1}],
\]
\[
\bar{\bar{y}}'_k := [y'_1, \ldots, y'_{k-N+1}].
\]
From (56), we obtain that
\[
\|\bar{\bar{y}}_k - \bar{\bar{y}}'_k\| \rightarrow 0, \quad \text{for } k \rightarrow \infty.
\]
Then, we have
\[ \|P_{\bar{r}_k}(y_k) - P_{\bar{r}_k}(\bar{y}_k)\| = \frac{1}{\|y_k\|^2} \|E[\bar{r}_k^T y_k] - E[\bar{r}_k^T \bar{y}_k]\| \leq \frac{1}{\|y_k\|^2} \|E[\bar{r}_k^T y_k] - E[\bar{r}_k^T \bar{y}_k]\| \leq \frac{1}{\|y_k\|^2} \|E[\bar{r}_k^T y_k] - E[\bar{r}_k^T \bar{y}_k]\|. \] (63)

In (63), we note that \( \|y_k\| \geq \text{cov}(v_k) > 0 \) due to (13). This also holds for \( \|\bar{y}_k\| \). Moreover, we have \( \sup_k \|y_k\| < \infty \) and \( \sup_k \|\bar{y}_k\| < \infty \) as \( k \to \infty \) due to \( \sup_k \|\theta_k\| < \infty \). Also, we have \( \|y_k\|^2/\|\bar{y}_k\|^2 \to 1 \) as \( k \to \infty \) since \( \|y_k\| - \|\bar{y}_k\| \leq \|y_k - \bar{y}_k\| \to 0 \) as \( k \to \infty \). Therefore, we obtain
\[ \|E[\bar{r}_k^T y_k] - E[\bar{r}_k^T \bar{y}_k]\| \leq \|E[\bar{r}_k^T y_k] - E[\bar{r}_k^T \bar{y}_k]\| \leq \frac{1}{\|y_k\|^2} \|E[\bar{r}_k^T y_k] - E[\bar{r}_k^T \bar{y}_k]\| \to 0, \text{ as } k \to \infty. \] (64)

As a result, RHS of (63) converges to 0 as \( k \to \infty \). Together with (60), we have completed the proof of (57). Therefore, we obtain the result in (57). ■

**APPENDIX F**

**PROOF OF PROPOSITION 1**

Let \( w \in \mathbb{R}^L \) be the vector of column-wise nonzero entries of \( W \in \mathbb{R}^{M \times N} \). The one-to-one mapping between \( w \) and \( W \) ensures that for any \( w_l \), we have a certain pair of indices \( (m_l, n_l) \) such that \( w_l = W_{m_l, n_l} \), where \( m_l \in [M], n_l \in [N], l \in [L] \). Given \( b \in \mathbb{R}^M \), we obtain
\[ b^T W = [b^T W_{1,1} \cdots b^T W_{N,N}], \] (65)
where \( W_j \) is the \( j \)th column of \( W \) and \( b^T W_j = \sum_{i=1}^M b_i W_{ij}, \) for \( j \in [N] \). Similarly, given \( B \in \mathbb{R}^{L \times N} \), we have
\[ w^T B = [w^T B_{1,1} \cdots w^T B_{N,N}], \] (66)
where \( B_{j, j} \) is the \( j \)th column of \( B \) and \( w^T B_{j, j} = \sum_{l=1}^L W_{m_l, n_l} B_{lij}, \) for \( j \in [N] \).

Consider the \( j \)th entry of \( w^T B \), we obtain
\[ [(w^T B)_{j}] = \sum_{l=1}^L W_{m_l, n_l} B_{lij} = \sum_{l=1}^L b_m W_{m_l} \] (67)
\[ = \sum_{m=1}^M b_m W_{mj} = [b^T W]_j, \]
where we have used the facts that \( B_{lij} = b_{nj} \), if \( j = n_l \), and 0 otherwise. Based on (67), we can conclude that \( w^T B = b^T W \).

We next show identity (28). Given \( C \in \mathbb{R}^{M \times M} \) and \( D \in \mathbb{R}^{N \times N} \), we have
\[ \text{tr}(C W D^T) = \sum_{i=1}^n e_i^T C W D^T e_i. \] (68)

Let \( c_i := e_i^T C \). By applying the identity (27), we have \( e_i^T C W = w_i^T C_i \) and \( W D e_i = D_i^T w_i \), where
\[ C_i = \begin{cases} c_i_{ij}, & n = n_i, \\ 0, & \text{otherwise}, \end{cases} \]
and
\[ D_i = \begin{cases} d_i_{nk}, & n = n_i, \\ 0, & \text{otherwise}, \end{cases} \]
for \( l \in [L] \) and \( n \in [N] \).

Therefore,
\[ \text{tr}(C W D^T) = \sum_{i=1}^n e_i^T C_i D_i^T w = w_i^T (\sum_{i=1}^n C_i D_i^T) w_i. \] (69)

Upon defining \( d_n := e_n^T D \) and
\[ C_i D_i = \begin{cases} d_n_{nk}, & m = i, \\ 0, & \text{otherwise}, \end{cases} \]
for \( n \in [N], l \in [L] \), we obtain
\[ \sum_{i=1}^n C_i D_i^T = \sum_{i=1}^n C_i D_i = E, \] (70)
where we have used the fact that \( E_{ik} = [C]_{mj} D_{nk} \) for \( n \in [N], k \in [L] \) and \( l \in [L] \). Based on (70) and (69), we can conclude that \( \text{tr}(C W D^T) = w_i^T E w \). ■

**APPENDIX G**

**PROOF OF PROPOSITION 3**

From Proposition 2, we have the optimal solution \( \hat{W} \) that is of rank one. According to [43] Theorem 2.4, we can obtain that \( \hat{W} \) is unique. Moreover, in Proposition 2, we have shown that the optimal solution of problem (31) is obtained by decomposing the submatrix of \( \hat{W} \) formed by deleting its \((L+1)\)st row and column, namely \( [\hat{W}]_{L+1} = s^*(s^*)^T \). Then, if we choose \( s^* \), we obtain a unique globally optimal solution \( \hat{W} \) for the offline problem.

Since the optimizing function of problem (34) is continuous and the feasibility set is compact for a given parameter set, by [44] Theorem 9.14, the unique optimal solution \( \hat{W} \) of problem (34) is a continuous function of the parameters. Hence, the optimal solution \( \hat{W} \) of problem (31) is also a continuous function.

Next consider the asymptotic mean squared error (MSE) \( P_{\infty} \) in (23) of the offline sensor collaboration scheme (and hence also of the online sensor collaboration scheme). Equation (23) is quadratic in \( P_{\infty} \) and hence \( P_{\infty} \) is a continuous function of its coefficients which are continuous functions of \( \hat{W} \) and the other parameters. Therefore, \( P_{\infty} \) is a continuous function of the system parameters. ■

**APPENDIX H**

**PROOF OF LEMMA 3**

In the following, we show the convergence rate of \( \|\hat{\theta}_{k+1} - \theta_k\| \to P_{\infty} \), where \( \hat{\theta}_{k+1} \) denotes the estimate using the proposed optimal online sensor collaboration scheme \( W_k \) and \( \theta_k \) is the true value.

According to triangle inequality, we have
\[ \|\hat{\theta}_{k+1} - \theta_k\| < \|\hat{\theta}_{k+1} - \theta_{k+1}'\| + \|\theta_{k+1}' - \theta_k\|, \] (71)
where \( \hat{\theta}_{k+1}' \) denote the estimate when using the offline policy \( \hat{W} \).
From [37] Theorem 2], \( \| \theta'_k - \theta_k \| \) converges to \( P_k \) exponentially. Now, we consider \( \| \hat{\theta}_k - \hat{\theta}_k' \| \).

From (65) in Proof of Theorem 1, for any \( N > 0 \), we have
\[
\| \hat{\theta}_k - \hat{\theta}_k' \| \leq \alpha_N E \| \theta^2 \|_{N} + P_\alpha(\hat{\theta}_k - \hat{\theta}_k') \| .
\]
where \( \hat{\theta}_k := \theta_k + \alpha \theta_{k-1} + \ldots + \alpha^{N-1} \theta_{k-N+1} \), \( \theta_k := [\theta_{k-1}, \ldots, \theta_{k-N+1}]^T \), \( \hat{\theta}_k := [\theta_{k-1}, \ldots, \theta_{k-N+1}]^T \) and \( P_\alpha \) is the projection operator which determines the estimate of \( \theta_k \).

By taking \( N \) large enough, the first term on RHS can be taken arbitrarily small. This also happens at an exponential rate.

From (69) and the inequality below it, we have
\[
\| P_\alpha(\hat{\theta}_k / \hat{\theta}_k') - P_\alpha(\hat{\theta}_k / \hat{\theta}_k') \| \leq c \| \hat{y}_k - \hat{y}_k' \|
\]
for \( c \) is an appropriately large constant. Using triangle inequality and Cauchy-Schwarz inequality, we obtain
\[
\| \hat{y}_k - \hat{y}_k' \| \leq \sum_{j=k-N+1}^{k} \| y_j - y_j' \| ,
\]
and
\[
\| y_j - y_j' \|^2 = E[(g_j^T W h_j \theta_j + v_j) - (g_j^T \beta_j \circ W \theta_j + v_j)]^2 \leq \| g_j \|^2 \| h_j \|^2 \| \theta_j \|^2 \| W^* - \beta_j \circ W \|^2 .
\]
Also, \( \| W^* - \beta_k \circ W \|^2 \leq E[1 - \beta_k^2] \| W^* \|^2 .\) Therefore, from (72), we have
\[
\| \hat{y}_k - \hat{y}_k' \| \leq \| g \| \| h \| \| W^* \| \sum_{j=k-N+1}^{k} \| \theta_j \| E[1 - \beta_k^2] .
\]
Since \( \sup_j \| \theta_j \| < \infty \) and \( N \) is taken a finite fixed number, we then need to show rate of convergence of \( E[1 - \beta_k^2] \).

For \( \delta_k > 0 \),
\[
E[1 - \beta_k^2] = E[1 - \beta_k^2] \| \beta_k \leq \delta_k \| P(\beta_k \leq \delta_k) + E[1 - \beta_k^2 \| \beta_k > \delta_k \| P(\beta_k > \delta_k) \leq P(\beta_k \leq \delta_k) + (1 - \delta_k^2) .
\]
Take \( \delta_k = 1 - \frac{\delta}{N} \) for a small \( \delta > 0 \). Then second term \( (1 - \delta_k^2) = \frac{\delta}{N} \) and decays exponentially. Now we consider\( P(\beta_k \leq \delta_k) = P(\beta_k \leq 1 - \frac{1}{\delta_k})
\]
\[
= N \sum_{i=1}^{N} P(\beta_{k,i} \leq 1 - \frac{1}{\delta_k} )^2 T_{k,i} W^* , i = 1, \ldots, N
\]
\[
\leq N \sum_{i=1}^{N} \left[ \sum_{j=1}^{k} T_{k,i} W^* \right] \leq (1 - \frac{1}{\delta_k})^2 T_{k,i} W^* .
\]
\[
\leq N \sum_{i=1}^{N} \left[ \sum_{j=1}^{k} H_{j,i}^* T_{k,i} W^* \right] \leq T_{k,i} W^* \leq \| W^* \|
\]
\[
= N \sum_{i=1}^{N} \left[ \frac{1}{k} \sum_{j=1}^{k} H_{j,i}^* T_{k,i} W^* \right] \leq \frac{T_{k,i} W^*}{k} \leq \| W^* \|
\]
\[
\leq \sum_{i=1}^{N} \left[ \frac{1}{k} \sum_{j=1}^{k} H_{j,i}^* T_{k,i} W^* \right] \leq \frac{T_{k,i} W^*}{k} \leq \| W^* \|
\]
where we choose \( \omega < \eta \). Since \( E[T_i W^*] \leq E[H_i] - \eta \), then
\[
E[H_i] - E[T_i W^*] \geq \eta .
\]

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