Breaking the Single Clock Symmetry: measuring single-field inflation non-Gaussian features.

Daniele Bertacca,1,2,3 Raul Jimenez,4,5 Sabino Matarrese,1,2,3,6 and Licia Verde4,5

1Dipartimento di Fisica e Astronomia Galileo Galilei, Università di Padova, 35131 Padova, Italy
2INFN, Sezione di Padova, via F Marzolo 8, I-35131 Padova, Italy
3INAF - Osservatorio Astronomico di Padova, vicolo dell'Osservatorio 5, I-35122 Padova, Italy.
4ICC, University of Barcelona, Marti i Franques 1, 08028 Barcelona, Spain.
5ICREA, Pg. Lluís Companys 23, Barcelona, E-08010, Spain.
6Gran Sasso Science Institute, viale F. Crispi 7, I-67100 L'Aquila, Italy.

(Dated: Wednesday 20th October, 2021)

The Universe is not just cold dark matter and dark energy, it also contains baryons, radiation and neutrinos. The presence of these components, beyond the pressure-less cold dark matter and the quasi-uniform dark energy ones, imply that the single clock assumption from inflation is no longer preserved. Here we quantify this effect and show that the single-clock symmetry is ensured only on scales where baryonic effects, neutrinos effects, or sound speed are zero. These scales depend on the cosmic epoch and the Universe composition. Hence for all use and purposes of interpreting state-of-the-art and possibly forthcoming surveys, in the accessible scales, single clock symmetry cannot be said to be satisfied. Breaking the single-clock symmetry has key consequences for the study of non-Gaussian features generated by pure single-field inflation which arise from non-linearities in the metric yielding non-Gaussianities of the local type: the $n_s - 1$ and the relativistic $-5/3$ term.

INTRODUCTION

The most economical model to describe the early Universe is a quasi-de Sitter state, i.e., inflation driven by a single field that is undergoing slow-rolling. Strong empirical support for inflation comes from two observational facts: the discovery of super-horizon fluctuations [1] and the near-scale-invariant spectrum of scalar fluctuations with a red tilt [2, 3]. In the simplest inflationary scenario, one single field dominates the quasi-exponential expansion of the space-time. This can happen because, at the energy scale of inflation, no other fields are light enough as to be able to provide enough e-folds on the expansion and are effectively frozen. This lack of other fields that are relevant during the inflation period indicates that one could describe inflation as a single clock with translational invariance of the field\(^1\). Under this symmetry, it is natural that one expects no coupling between the different fields and thus no deviation from Gaussian fluctuations as produced by the modes leaving the horizon without interaction. Self-coupling of the inflaton will produce a negligible non-Gaussian signal, of order $O(\epsilon^2)$ where $\epsilon$ denotes the slow roll parameter. However, non-linearities of the single field paint a different story.

Non-linearities in the metric induced by the non-linearities of the single-field while slow rolling, give rise to two very well known effects [5–9]. The first is a non-Gaussianity contribution with a local shape component with amplitude parameter $f_{\text{NL}}^{\text{local}} \propto n_s - 1$ and an equilateral shape component of amplitude parameter $f_{\text{NL}}^{\text{equil}} \propto \epsilon^2$. The second effect is due to General Relativity and translates effectively into a non-Gaussian contribution of the local type with $f_{\text{NL}}^{\text{local}} \approx -5/3$.

The inclusion of gravity in the analysis is crucial as this non-Gaussian effect is generated by the non-linearities in the metric. In Ref. [10] a method was recently proposed to look at the effect of graviton interactions with scalar fluctuations of the inflaton field. This opens the possibility to study observational signatures of single field inflation via the generation of gravitons, which is again a pure gravity effect that can only happen during a phase of accelerated expansion.

Each of the above effects, primordial non-Gaussianity and relativistic effects, if measured, would open a new window into the physics of the early Universe. Despite some early claims that GR-effects driven non-Gaussian signatures were simply gauge artifacts [11–14], extensive work e.g., [15–18], has shown that indeed these non-Gaussianities are measurable and physical\(^2\). In addition, it has been shown, from a quantum information theory point of view [19].

---

\(^1\) For a description of the role of time during inflation see Ref. [4]

\(^2\) One interesting and intuitive argument that shows that these higher order correlations cannot be gauged away is that if this was the case, one could also gauge away the 2-point correlation function (the power spectrum). This is very simply to see by using the Mukhanov-Sasaki variable in the in-in formalism with the proposed gauge transformations in Ref. [11–14].
that single field non-Gaussianities cannot be gauge artifacts. Only in the unphysical case of exact \( k = 0 \) can these non-Gaussianities be gauged away.

In this work we take the calculation a step further and show that, in addition to the above arguments, the presence in the Universe of pressure(full) terms (e.g. baryons, radiation, neutrinos etc...) do break the single clock symmetry and automatically imply that the above non-Gaussianities cannot be gauged away during the entire history of the evolution of the patch i.e., on scales that are not significantly larger than the sound horizon at radiation drag. This impossibility to synchronize clocks across cosmic epochs and separate patches, which size approaches the size of the current horizon, has important consequences for the detectability of primordial non-Gaussianity signatures.

**METHODOLOGY**

Let us start by considering a Universe with a Friedmann-Lemaître-Robertson-Walker (FLRW) space-time at the background level, setting to zero the constant curvature \( K \) (for simplicity), and assume General Relativity (GR) as the theory of gravity. The unperturbed energy-momentum tensor \( T_{\mu \nu} \) of matter contains ordinary (baryonic) matter, radiation, neutrinos, Cold Dark Matter (CDM) and Dark Energy (DE) or, the cosmological constant \( \Lambda \). At the background level, \( T_{\mu \nu} \) describes a perfect fluid where the total energy density \( \rho^{(0)} \), and the total pressure \( p^{(0)} \) depend only on time. The Einstein and continuity background equations are

\[
\frac{\dot{a}}{a} = \frac{8\pi G}{3} \bar{\rho}^{(0)} , \quad \frac{\dot{\rho}}{\rho} = 4\pi G a^2 (\rho^{(0)} + p^{(0)}) \quad \text{and} \quad \rho^{(0)} = -3\frac{\dot{H}}{a} (\rho^{(0)} + \rho^{(0)}) , \tag{1}
\]

where \( \dot{\mathcal{H}} = a' / a = a\dot{H} \) is the comoving Hubble scale and prime denotes the derivative with respect to \( \eta \). Here \( \eta \) is the conformal time, i.e., \( \eta = \int \frac{dt}{a(t)} \), where \( a \) is the scale factor. It is useful to define (see Ref. [20, 21]) a quantity obtained by the ratio between \( 0 - 0 \) and \( i - j \) components of Einstein’s equations

\[
\theta_M = \left. \frac{1}{a} \frac{1}{(1 + \rho^{(0)} / \rho^{(0)}')^{1/2}} \right|_{a} = \frac{\rho^{(0)} / a}{\rho^{(0)} / a} \exp \left[ - \frac{3}{2} \int \frac{\mathcal{H}}{(1 + c_M^2 \mathcal{H})} d\eta \right] , \tag{2}
\]

that depends (at the functional level, i.e. \( \theta_M = \theta_M[c_M] \)) on the speed of sound \( c_M^2 = \rho^{(0)} / \rho^{(0)}' \). Here we define two models: the spatially flat \( \Lambda \)CDM model, which components are, as it is customary, ordinary (baryonic) matter, radiation, neutrinos, Cold Dark Matter (CDM) and the cosmological constant; we also define a pressureless counterpart of this model, the 0p\( \Lambda \)CDM model, which does not contain (or contains only a fully negligible contribution of) baryons, radiation and neutrinos. Hence the speed of sound just introduced is related to any possible effect that describes the physics beyond the pressure-less 0p\( \Lambda \)CDM model\(^4\). Using Eq. (2) we note, immediately, that \( \mathcal{H}^2 - \mathcal{H}' = (3/2) \mathcal{H}^2 / a^2 \theta_M^2 \).

We expand the background around the pressureless case; using the bar to imply quantities in the the corresponding 0p\( \Lambda \)CDM model, we have \( \mathcal{H}^2 = 8\pi G a^2 \bar{\rho}^{(0)} / 3 \), and \( \mathcal{H}^2 - \mathcal{H}' = 4\pi G a^2 (\rho^{(0)} + p^{(0)}) = 4\pi G a^2 \bar{\rho}^{(0)} + \bar{p}^{(0)} + 3\mathcal{H} \rho^{(0)} \rho^{(0)} \), where \( \rho^{(0)} = \rho^{(0)} + \rho^{(0)} \), \( \rho^{(0)} \rho^{(0)} = \rho^{(0)} \rho^{(0)} \), \( \bar{p}^{(0)} = \bar{p}^{(0)} \), and \( \bar{p}^{(0)} = \bar{p}^{(0)} \). (Note that in general if today \( a(\eta = \eta_0) = a_0 = 1 \), then \( a_0 \neq 1 \). Of course, in a \( \Lambda \)CDM model with massless neutrinos, one could adjust the values of the model’s parameters to ensure that \( \Omega_{\text{CDM}} \) in the 0p\( \Lambda \)CDM coincides with \( \Omega_{\text{m}} \) in the \( \Lambda \)CDM one, where baryons and cold dark matter are all included in the matter density parameter. In this case the scale factor evolution after recombination and for negligible amount of radiation, would effectively be the same and \( \bar{a} \simeq a \). Even in this hyper-simplified \( \Lambda \)CDM, \( c_M \) is still non zero, but the effects can only be seen in the perturbations (see below) and not in the background. However for non-massless neutrinos, different neutrino masses transition from relativistic to non-relativistic at different times so if at a starting point, even if well after matter radiation equality, we set \( a = \bar{a} \), in general at later times this will not hold in details. Moreover, if the dark energy were not to be a cosmological constant, the dark energy component could also contribute to \( c_M \) in principle. For our analytical calculation, we have to consider the initial conditions, which we denote with subscript (in). We set initial conditions in the matter dominated era, and we consider two cases, one well before recombination and one well after recombination.

---

\(^3\) The superscript (0) denotes quantities at background.

\(^4\) Here we have defined \( c_M^2 = \mathcal{H}(\eta) \) and \( \mathcal{H} = \mathcal{H}(\eta) \). Following Ref. [21], the constant of integration in Eq. (2) corresponds to an unphysical solution and, from now, it will be discarded.
In addition, at the background level (in FLRW space-time), we assume that the parameter expansion \( a \) at \( \eta_{in} \) coincides in the 0pCDM and \( \Lambda \)CDM model\(^5\).

For a generic quantity, let us call it \( C^{(0)} \) which in this section we take to be a background quantity, we define, at the linear level, the deviations from 0pCDM due to the presence of a small pressure component as

\[
\delta C^{(0)} = C^{(0)} - \bar{C}^{(0)}, \quad \text{where} \quad \left| \frac{\delta C^{(0)}}{\bar{C}^{(0)}} \right| \ll 1. \tag{3}
\]

Note that for the pressure component to be small, the size of the patch of the Universe under consideration needs to be much larger than the largest scale reached by the sound horizon (the Jean’s length) from the initial conditions on. Hence, for initial conditions set before recombination, that corresponds to scales much larger than the sound horizon at radiation drag. If we instead set initial conditions after recombination, this scales needs to be larger than massive neutrinos free streaming length, or for massless neutrinos, larger than the scale where baryonic effects become important. Recall that the sound horizon at radiation drag is \( \sim 150 \) Mpc, the neutrinos free streaming length corresponds to \( k \sim 0.01h \) Mpc\(^{-1}\), and baryonic effects are important on scales of few Mpc.

Immediately we note that \( \delta C^{(0)} \) depends on \( c_s^2 \) and it is possible to identify explicitly \( \delta p^{(0)} \) from the definition of the speed of sound. Indeed we obtain

\[
\delta p^{(0)}[c_s^2] = \delta p^{(0)} + \int_{\eta_{in}}^\eta c_s^2 \rho^{(0)'} \, d\eta. \tag{4}
\]

From \( a = \bar{a} + \delta a \)

\[
\mathcal{H}^2 = \bar{\mathcal{H}}^2 \left( 1 + 2 \frac{\delta \mathcal{H}}{\bar{\mathcal{H}}} \right) = \frac{8\pi G}{3} \bar{a}^2 \rho^{(0)} \left( 1 + \frac{\delta a}{\bar{a}} \right) \left( 1 + \frac{\delta \rho^{(0)}}{\rho^{(0)}} \right), \tag{5}
\]

and

\[
\mathcal{H} = (\bar{\mathcal{H}} + \delta \mathcal{H}) = \left( 1 - \frac{\delta a}{\bar{a}} \right) \left( \bar{\mathcal{H}} + \frac{\delta a'}{\bar{a}} \right), \tag{6}
\]

we have the following relations for \( \delta \mathcal{H} \)

\[
\delta \mathcal{H} = \frac{\bar{\mathcal{H}}}{2} \left( \frac{\delta \rho^{(0)}}{\rho^{(0)}} + 2 \frac{\delta a}{\bar{a}} \right), \quad \delta \mathcal{H} = \frac{\delta a'}{\bar{a}} - \bar{\mathcal{H}} \frac{\delta a}{\bar{a}} \tag{7}
\]

or, equivalently, combining the above relations

\[
2\delta \mathcal{H} = \frac{\delta a'}{\bar{a}} + \frac{\bar{\mathcal{H}}}{2} \frac{\delta \rho^{(0)}}{\rho^{(0)}}, \quad \frac{d}{da} \left( \frac{\delta a}{\bar{a}}^2 \right) = \frac{1}{2a^2} \frac{\delta \rho^{(0)}}{\rho^{(0)}}, \tag{8}
\]

From the continuity equation we find

\[
\delta \rho^{(0)'} + 3 \bar{\mathcal{H}} \left( \delta \rho^{(0)} + \rho^{(0)} \right) = -3(\dot{\rho}^{(0)} + \ddot{\rho}^{(0)}) \delta \mathcal{H}. \tag{9}
\]

Here we note that as long as we are only concerned with times well after neutrino decoupling, it is possible to go from the 0pCDM model to the \( \Lambda \)CDM model in a continuous and differentiable way, i.e. the background quantities \( C \) are continuous and differentiable.

This equation, using Eq. (8), can be rewritten as

\[
\frac{d^2}{da^2} \frac{\delta \rho^{(0)}}{\rho^{(0)}} + \frac{3}{2a} (\ddot{\bar{w}} + \bar{w}) \frac{d}{da} \frac{\delta \rho^{(0)}}{\rho^{(0)}} + \frac{3}{2a^2} (4 + 3(\bar{w} + 1)^2) \delta \rho^{(0)} + \frac{6}{a^2} \bar{w} \delta \rho^{(0)} + \frac{3}{a} \frac{d}{da} \delta \rho^{(0)} = 0, \tag{10}
\]

where we defined \( \bar{w} = \dot{\rho}^{(0)}/\ddot{\rho}^{(0)} \). This is the complete and correct equation to solve in the generic case. However it has no analytic solution and is therefore not too transparent. We proceed by considering a simplified case, which, however,

\(^{5}\) In principle we could use an alternative condition, i.e., the parameter expansion could coincide both for 0pCDM and \( \Lambda \)CDM model today at \( z = 0 \). In the main text we have used the condition during matter epoch in order to match our considerations with approach used in separate universe technique. See also the discussion at the end of the next section.
and, taking into account Eq. (9), we have

\[ \rho^{(0)}(z) = \frac{3H_0^2}{8\pi G} \left( \Omega_{\text{CDM}0} + \Omega_{b0} \right) (1 + z)^3 + \Omega_{\gamma0}(1 + z)^4 \left[ 1 + 0.2271N_{\text{eff}} f \left( \frac{m_\nu}{1 + (1 + z)T_{\nu0}} \right) \right] + \Omega_{\Lambda0}, \]

where \( f \) is a suitable function in which \( (\Omega_{\gamma}/a^4)0.2271N_{\text{eff}}f(m_\nu/a^3) \rightarrow \Omega_{\gamma}/a^3 \) for \( a \rightarrow \infty \) (e.g. see [22]). In the massless neutrinos case \( f = 1 \). Here the values of different cosmological parameters for the \( \Lambda \)CDM model are constrained in Ref. [3] and for the 0p\( \Lambda \)CDM the reader can think of absorbing the baryonic component into the cold component to keep \( \Omega_m \) the same.

In general, if we choose \( \delta a_{(in)} = 0 \), using Eq. (7), we obtain

\[ \delta H_{(in)} = \frac{\bar{H}_{(in)} \delta \rho^{(0)}_{(in)}}{2 \bar{\rho}^{(0)}_{(in)}}. \]

Using Eq. (8), we find

\[ \left( \frac{\delta a}{a^2} \right) = \frac{1}{2} \int a^2 \frac{\delta \rho^{(0)}_{(in)}}{\bar{\rho}^{(0)}_{(in)}} \, d\bar{a}, \]

and, taking into account Eq. (9), we have

\[ \delta \rho_{(in)}^{(0)} = -\frac{3 + \bar{\omega}_{(in)}}{2} \delta \rho_{(in)}^{(0)} - \frac{\delta \rho_{(in)}^{(0)}}{3\bar{H}_{(in)}}. \]

Finally, from Eq. (2), the correction for \( \theta_M \) becomes

\[ \delta \theta_M = \bar{\theta}_M \left[ \frac{1}{2} \left( \frac{\delta \rho^{(0)}}{\bar{\rho}^{(0)}} - \frac{\delta \rho^{(0)} + \delta \rho^{(0)}}{\bar{\rho}^{(0)} + \bar{\rho}^{(0)}} \right) - \frac{\delta a}{a} \right], \]

where

\[ \bar{\theta}_M = \frac{1}{\bar{\rho}^{(0)}} \left( 1 + \bar{\rho}^{(0)} / \bar{\rho}^{(0)} \right)^{1/2} = \frac{1}{\bar{\rho}^{(0)}} \left( \bar{\rho}^{(0)}_{\text{CDM}} \right)^{1/2}. \]

Here \( \bar{\rho}^{(0)}_{\text{CDM}} \) denotes the density of cold dark matter and we have assumed that for dark energy \( p = -\rho \).

While at the background level baryons, radiation and neutrinos (massless, fully relativistic or massive and non-relativistic) have a negligible effect, it is important to bear in mind that at the perturbation level massive neutrinos and baryons have important effects even at linear scales. Baryons are known to have effects on the mildly non-linear regime (the highly feared baryonic effects) which, however at linear scales are at the few percent level. Massive neutrinos on the other hand, have non zero velocity dispersion which acts as an effective sound speed; because of this, they have effects on the power spectrum at scales \( k > 0.01 h \, \text{Mpc}^{-1} \) comparable to the free streaming length; the effect is scale- and redshift-dependent. Even in the linear regime, at \( k \sim 0.1 h \, \text{Mpc}^{-1} \) neutrinos produce a suppression on the power spectrum of \( \sim 6f_\nu \) where \( f_\nu \) denotes the neutrino mass fraction. At the minimum neutrino mass imposed by oscillation experiments, the linear power spectrum suppression is \( \sim 6\% \) at \( z = 0 \). It is well known that massive neutrinos also introduce a scale-dependent bias [23] and affect peculiar velocities. Hence their effects likely cannot be neglected in the perturbations. In particular while in the absence of massive neutrinos the matter density perturbations in the linear regime evolve \( \propto a \), in the presence of massive neutrinos evolve as \( a^{1−3/5f_\nu} \).

\footnote{For the minimum neutrino mass allowed by oscillation experiments, and for the concordance cosmology, \( f_\nu \) is \( \sim 0.01 \).}
Consider the correction in the scalar metric perturbations in the longitudinal gauge [24–27] (or also called Conformal Newtonian Gauge [20, 21, 28]). In this gauge\(^7\)

\[
\frac{\text{d}s^2}{\Lambda} = a(\eta)^2 \left[ -(1 + 2\Phi) \, \text{d}\eta^2 + c_i^2 (1 - 2\Psi) \, \text{d}x^i \, \text{d}x^j \right].
\]  

(17)

Let us introduce the following variables (already used in [20, 21]):\(^8\)

\[
u_M = \exp \left[ \frac{3}{2} \int_0^\eta \text{d}\eta' \left( 1 + \bar{c}_i^2 \right) \frac{H'}{H} \right] \Psi = \frac{a \theta_M}{(\rho(0))^{1/2}} \, \Psi,
\]

(18)

\[
\zeta_M = \Psi - \frac{H^2}{\dot{H} - H^2} \left( \Psi + \Psi' \right) = \frac{2}{3} \left[ \frac{3}{8\pi G} \right]^{1/2} \theta_M^{2} \left( \frac{u_M}{\theta_M} \right)'
\]

(19)

and the comoving curvature perturbation that can be written in terms of \(\zeta_M\) and \(D_M\)

\[
\mathcal{R} = \Psi - \frac{H^2}{\dot{H} - H^2} \left( \Phi + \frac{\Psi'}{H} \right) = \zeta_M + \frac{2}{3} a^2 \theta_M^2 D_M,
\]

(20)

where \(D_M = \Phi - \Psi\). (Of course, in \(0\)p\(A\)CDM model, we have \(D_M = 0\) and \(\mathcal{R}\) coincides with \(\zeta_M\)). Using the linear perturbation of the spatial part of the stress energy tensor

\[
T_j^{(1)} = p^{(1)} \delta_j + D^j \Pi,
\]

where \(D_{ij} = \partial_i \partial_j - \delta_{ij} \nabla^2 / 3\), \(p^{(1)}\) is the perturbation of isotropic pressure\(^9\) and \(\Pi\) is the trace-free scalar part of total anisotropic stress tensor [27], thorough the Einstein equation we can write

\[
D_M = -8\pi G a^2 \Pi.
\]

(21)

Finally, using (19), Eq. (20) becomes

\[
\mathcal{R} = \zeta_M - \frac{16\pi G}{3} a^2 \frac{\rho(0)}{(\rho(0) + \rho(0))} \Pi.
\]

(22)

Deviations from the isotropic pressure can be split in the adiabatic perturbations (which is proportional to \(c_s^2\) and the intrinsic non-adiabatic pressure perturbation\(^10\) [24, 25] where \(\Gamma\) is the non-adiabatic component of the equation of state and the \(\delta\) is the adiabatic component.

\[
p^{(1)} = p^{(0)} \left( \Gamma + \frac{c_s^2}{w} \delta \right),
\]

(23)

and we can easily generalise the prescription made in Ref. [20, 21]. Indeed, with some algebra, we can obtain

\[
\left[ \theta_M^{2} \left( \frac{u_M}{\theta_M} \right)' + \left( \frac{8\pi G}{3} \right)^{1/2} a^2 \theta_M^2 D_M \right]' = c_s^2 \theta_M^2 \nabla^2 \left( \frac{u_M}{\theta_M} \right) - \frac{a^2 \theta_M^2}{3 \rho(0)^{1/2}} \nabla^2 D_M + 4\pi G \frac{a^2 \theta_M^2 (\rho^{(0)})^{1/2}}{3 \rho(0)^{1/2}} \Gamma.
\]

(24)

Without \(D_M\) and \(\Gamma\) this is identical to the results of [21], hence this is the generalization of that calculation in presence of pressure terms.

Using Eqs. (18), (19), (22), (21) and the gauge-invariant definition of the Newtonian Poisson equation

\[
\nabla^2 \Psi = 4\pi G a^2 \rho^{(0)} \Delta_{\text{com}} = 4\pi G a^2 p^{(1)}_{\text{com}},
\]

(25)

\(^7\) Here \(\Phi = \Phi_A Q^{(0)}\) and \(\Psi = -\Phi_H Q^{(0)}\), where \(\Phi_A\) and \(\Phi_H\) are the well known Bardeen potentials [24].

\(^8\) The constants of integration arising in these formulae correspond to unphysical solution that can be removed.

\(^9\) Here \(p^{(1)}\) is not the same \(dp\) defined in [20].

\(^10\) Note that \(p^{(1)}\) is the pressure perturbation in the longitudinal gauge.
where $\Delta_{\text{com}}$ is the density contrast and $\rho^{(1)}_{\text{com}}$ is density perturbation in the comoving orthogonal gauge, we obtain the usual relation

$$\left(\frac{p^{(0)} + \rho^{(0)}}{\mathcal{H}}\right) R' = c_s^2 \rho^{(1)}_{\text{com}} + \frac{2}{3} \nabla^2 \Pi + p^{(0)} \Gamma . \tag{26}$$

The time evolution of $R$ in a spatially flat background is dictated by three components: the sound speed modulated by the comoving density perturbation, the spatial variation (second derivative) of the shear and non-adiabatic pressure perturbations: $R' = 0$ on scales and epochs where $c_s$, $\Gamma$ and $\nabla^2 \Pi$ are negligible. Before recombination this happens on scales $\gg 150$ Mpc, after recombination depending on the neutrino mass the scales are well above tens or few megaparsecs (respectively).

Some other important comments are in order here:

- Due to the breaking of the Single Clock Symmetry, for each point in space we have a non zero acceleration due to $c_s$, $\Pi$ and/or $\Gamma$ and we cannot build an exact observers proper reference frame described by the local (normal) coordinate within the patch along all the observer world line.

- If we wanted to build a local volume expansion it is necessary to describe the physics inside this patch with the coordinates of an observer that are comoving with matter. For this purpose, the comoving gauge is the most appropriate.

In this gauge the curvature potential is $R$ and the lapse perturbation are described by the 00 metric perturbation $\xi = g_{00}^{(1)}|_{\text{com}}$ which can be written in terms of pressure perturbation and anisotropic stress

$$\left(\frac{p^{(0)} + \rho^{(0)}}{\mathcal{H}}\right) \xi = c_s^2 \rho^{(1)}_{\text{com}} + \frac{2}{3} \nabla^2 \Pi + p^{(0)} \Gamma . \tag{27}$$

Then, using Eq. (26), we have $\xi = R'/\mathcal{H}$. We note immediately that we can follow the separate Universe prescription (e.g., see also [29]) and using the above equations we are able to compute the curvature evolution equation within the patch. We find [29]

$$\frac{K'_{\text{patch}}}{\mathcal{H}} = -\frac{2}{3} \nabla^2 \xi = -\frac{2}{3} \left(\frac{\rho^{(0)} + \rho^{(0)}}{\rho^{(0)} + \rho^{(0)}}\right) \nabla^2 \left[ c_s^2 \rho^{(1)}_{\text{com}} + \frac{2}{3} \nabla^2 \Pi + p^{(0)} \Gamma \right] . \tag{28}$$

Note that $K_{\text{patch}}$ is not conserved if there is flow of material in or out of the patch. For example, for a patch of 5 Mpc corresponding roughly to the Lagrangian radius of a massive elliptical, non-linear evolution would yield a lot of material to enter this radius through the surrounding filaments, even for a 0pΛCDM Universe. Therefore the curvature $K_{\text{patch}}$ is conserved if the size of the patch is large enough and/or for $\rho^{(1)}_{\text{com}}$ and $\Pi$ equal to zero, i.e. in the 0pΛCDM model. Consequently, if $K_{\text{patch}}$ is not conserved the lapse perturbation is not zero. This is another way to see that the single clock symmetry is broken.

- As an example, it is useful quantify the contribution of $\Pi$. For example the well-know effect due to neutrinos is

$$D_M = -\frac{2}{5} R_v \Phi = -\frac{2R_v/5}{1 + 2R_v/5} \Psi , \tag{29}$$

where [22]

$$R_v = \frac{0.2271 N_{\text{eff}} f (m_\nu a/T_{v,0})}{1 + 0.2271 N_{\text{eff}} f (m_\nu a/T_{v,0})} . \tag{30}$$

Then using the Newtonian Poisson equation Eq. (25) we find

$$\frac{2}{3} \nabla^2 \Pi = \frac{3}{3} \left(\frac{2R_v/5}{1 + 2R_v/5}\right) \rho^{(1)}_{\text{com}} \tag{31}$$

and we immediately note that the the single clock is explicitly broken due to the neutrinos

$$\left(\frac{p^{(0)} + \rho^{(0)}}{\mathcal{H}}\right) \xi = \left[ c_s^2 + \frac{1}{3} \left(\frac{2R_v/5}{1 + 2R_v/5}\right)\right] \rho^{(1)}_{\text{com}} + p^{(0)} \Gamma . \tag{32}$$
These points have a fundamental consequence. In order to quantify the size of the patch which preserves the curvature $K_{\text{patch}}$ at a given point of the space we have to consider the entire evolution of the separate Universe from $a_{\text{in}}$, until today. (Note that our initial condition can be chosen so that $a_{\text{patch, in}} = a_{\text{in}}$, where $a_{\text{patch}}$ is the local scale factor of the patch local Universe, see also [29]). The patch radius $R_{\text{patch}}$ enclosing constant mass has to be above the Jeans scale along all the observer’s world line. For example, assuming massive neutrino, we should take scales with $\tilde{k} < k_{\text{nr}}$. Here, for individual neutrino masses $m_\nu$ from 0.046 to 0.46 eV, the scale $k_{\text{nr}}$ ranges from $2.1 \times 10^{-3}h\text{Mpc}^{-1}$ to $6.7 \times 10^{-3}h\text{Mpc}^{-1}$ [30] (see also [31, 32]). Of course at recombination the Jeans length for baryons is the sound horizon which is comparable to the horizon. This point confirms what was previously discussed earlier, i.e. these patches are the only places where clocks can be synchronized, but they are the ones with the most cosmic variance.

In the next subsection we analytically calculate the corrections that break the validity of the separate Universe prescription (see also Ref. [29]). As we will see, this implies that the single clock from inflation is no longer preserved.

**Correction to linear order perturbations**

Denoting again the concordance model quantities with an over-bar, we split the linear order perturbations, e.g. let us call $C^{(1)}$, in the following way

$$C^{(1)} = \bar{C}^{(1)} + \delta C^{(1)},$$

(33)

where

$$\delta C^{(1)} = \int \delta \rho^{(0)} \left( \frac{\delta C^{(1)}}{\delta \rho^{(0)}} \right) \, d\tilde{\eta} + \int \varepsilon_{a}^{2} \left( \frac{\delta C^{(1)}}{\delta \varepsilon_{a}^{2}} \right) \, d\tilde{\eta}$$

$$+ \int \Gamma \left( \frac{\delta C^{(1)}}{\delta \Gamma} \right) \, d\tilde{\eta} \, d^{3}\tilde{x} + \int \Pi \left( \frac{\delta C^{(1)}}{\delta \Pi} \right) \, d\tilde{\eta} \, d^{3}\tilde{x} + \ldots .$$

(34)

Here, for simplicity, we are assuming that the correction, i.e., the contribution due to $\{\delta \rho^{(0)} = \rho^{(0)} - \bar{\rho}^{(0)}, \varepsilon_{a}^{2}, \Gamma, \Pi\}$, is only linear. In principle, it is easy generalise this point. Immediately we note that from Eq. (26) we have $\mathcal{R}^\prime = 0$,

$$\delta \mathcal{R} = \delta \mathcal{R}^{(in)} + \frac{3}{2} \left( \frac{3}{8\pi G} \right)^{1/2} \int_{\text{in}} \varepsilon_{a}^{2} \tilde{\theta}_{M}(\tilde{\eta}) \nabla^{2} \tilde{u}_{M}(\tilde{\eta}, \tilde{x}) \, d\tilde{\eta}$$

$$+ \int_{\text{in}} \frac{\tilde{\mathcal{H}}(\tilde{\eta})}{\bar{\rho}^{(0)}(\tilde{\eta}) + \bar{\rho}^{(0)}(\tilde{\eta})} \left[ \frac{2}{3} \nabla^{2} \Pi(\tilde{\eta}, \tilde{x}) + \bar{\rho}^{(0)} \Gamma(\tilde{\eta}, \tilde{x}) \right] \, d\tilde{\eta}$$

(35)

and, using Eqs. (20) and (22), we obtain

$$\tilde{\zeta}_{M} = \mathcal{R} \quad \text{and} \quad \delta \zeta_{M} = \delta \mathcal{R} - \frac{2}{3} \left( \frac{\bar{\rho}^{(0)}}{\bar{\rho}^{(0)} + \bar{\rho}^{(0)}} \right) D_{M} = \delta \mathcal{R} + \frac{16\pi G}{3} a^{2} \left( \frac{\bar{\rho}^{(0)}}{\bar{\rho}^{(0)} + \bar{\rho}^{(0)}} \right) \Pi .$$

(36)

The leading contribution to $\Pi$ depends again on the scale, the cosmic epoch and the Universe composition. Before recombination is dominant inside the horizon. After recombination, for massive neutrinos, free streaming is not negligible even on linear scales; for baryons, it is important on cluster scales and below.

We note that Eq. (36) is our main result, and at the end of this section we compute the initial conditions. It is transparent that it is not possible to completely eliminate the $\varepsilon_{a}$ dependence from the perturbed variable, except in the (academic) case of infinite wavelength $k = 0$.

Now, splitting each term in Eq. (19), we find immediately the solution for $\delta \bar{u}_{M}$:

$$\bar{u}_{M} = \frac{3}{2} \left( \frac{3}{8\pi G} \right)^{1/2} \tilde{\theta}_{M} \int_{\text{in}} \tilde{\zeta}_{M}(\tilde{\eta}, \tilde{x}) \, d\tilde{\eta} + \frac{\tilde{\theta}}{\tilde{\theta}_{M}^{(in)}} \tilde{u}_{M}^{(in)}, \quad \text{for } \Lambda \text{CDM model, and}$$

$$\delta \bar{u}_{M} = \frac{\delta \theta_{M}}{\delta M} \bar{u}_{M} + \frac{3}{2} \left( \frac{3}{8\pi G} \right)^{1/2} \tilde{\theta}_{M} \int_{\text{in}} \left[ -2 \frac{\delta \theta_{M}(\tilde{\eta})}{\tilde{\theta}_{M}^{(in)}} \tilde{\zeta}_{M}(\tilde{\eta}, \tilde{x}) + \frac{\delta \zeta_{M}(\tilde{\eta}, \tilde{x})}{\tilde{\zeta}_{M}^{(in)}} \right] \, d\tilde{\eta} + \tilde{\theta}_{M} \left( \frac{\delta \bar{u}_{M}^{(in)}}{\tilde{\theta}_{M}^{(in)}} - \frac{\delta \theta_{M}^{(in)}}{\tilde{\theta}_{M}^{(in)}} \frac{\delta \bar{u}_{M}^{(in)}}{\tilde{\theta}_{M}^{(in)}} \right) .$$
From Eq. (18), we get
\[ \Psi = \left( \frac{\bar{\rho}(0)}{a \bar{\theta}_M} \right)^{1/2} \bar{u}_M = \left( \frac{\bar{\rho}(0) + \bar{\rho}(0)}{a \bar{\theta}_M} \right)^{1/2} \bar{u}_M \] and
\[ \delta \Psi = \left( \frac{\bar{\rho}(0) + \bar{\rho}(0)}{2 (\bar{\rho}(0) + \bar{\rho}(0))} \right)^{1/2} \bar{u}_M + \frac{1}{2} \left( \frac{\bar{\rho}(0) + \bar{\rho}(0)}{\bar{\rho}(0) + \bar{\rho}(0)} \right)^{1/2} \bar{u}_M , \] (38)
and from Eq. (21), we obtain
\[ \delta \Phi = \delta \Psi + D_M = \delta \Psi - 8 \pi G a^2 \Pi . \] (39)

Finally from the Newtonian Poisson Eq. (25) we deduce
\[ \Delta_{\text{com}} = \frac{1}{4 \pi G a^2 \bar{\rho}(0)} \nabla^2 \bar{\Psi} \quad \text{and} \quad \delta \Delta_{\text{com}} = \frac{1}{4 \pi G a^2 \bar{\rho}(0)} \nabla^2 \delta \Psi = \left( \frac{2 \delta a}{a} + \frac{\delta \bar{\rho}}{\bar{\rho}(0)} \right) \Delta_{\text{com}} . \] (40)

We suggest to set the initial conditions by the following approach. From \( \delta a_{\text{in}} = 0 \), we can choose \( \delta \Psi_{\text{in}} \equiv \Psi_{\text{in}} - \bar{\Psi}_{\text{in}} \)\) where \( \Psi_{\text{in}} = 4 \pi G a^2 (\rho_{\text{in}}(0)) \nabla^{-2} \Delta_{\text{com}} \) and \( \bar{\Psi}_{\text{in}} = 4 \pi G a^2 (\rho_{\text{in}}(0)) \nabla^{-2} \Delta_{\text{com}} \). Using these relations and Eqs. (36)-(39), we get immediately \( \delta a_{\text{M}}^{\text{in}}, \delta \bar{\Psi}_{\text{M}}^{\text{in}} \) and \( \delta \bar{R}(\text{in}) \).

CONCLUSIONS

We have explicitly computed, for the first time, at the perturbation level, the effect of pressure from baryons, radiation, neutrinos and any general equation of state with sound speed \( c_s \) different from zero on the perturbations that arise from single-field inflation which include (small) non-linearities in the metric. We have shown that \( c_s, \Pi, \Gamma \) appear in the resulting perturbations thus, on scales/epochs when they are non zero these terms induce breaking the single clock symmetry from single field inflation.

The claims of Ref. \[11-14\] translated in asserting that one could synchronize (observers) clocks to the inflaton one in patches of arbitrary size in the Universe. This in itself is very counter intuitive as General Relativity forbids this in the presence of curvature (gravity) as explicitly shown in Ref. \[18\]. We have shown here is that, in addition, the presence of pressure also breaks the synchronization of clocks in patches smaller than the Jeans length. These patches must be much larger than tens of Mpc and larger than neutrinos free streaming length in the presence of massive neutrinos. This impossibility to synchronize clocks across separate patches opens up the possibility to measure the primordial non-Gaussianity from single-field inflation of the local shape which include a term \( \sim n_s - 1 \) and the General Relativity \( \sim -5/3 \) term arising from modifications of the Poisson equation at the horizon scale. Current and ongoing surveys like DESI, Spherex and LSST are forecasted to have enough statistical power to detect a non-gaussian contribution of the local type of an amplitude \( f_{\text{loc}}^{\text{NL}} \sim 1 \) \[33, 34\] making them well suited to access the inflationary signature.

DB and SM acknowledge partial financial support by ASI Grant No. 2016-24-H.0. Funding for this work was partially provided by project PGC2018-098866- B-I00 MCIN/AEI/10.13039/501100011033 y FEDER “Una manera de hacer Europa”, and the “Center of Excellence Maria de Maeztu 2020-2023” award to the ICCUB (CEX2019- 000918-M funded by MCIN/AEI/10.13039/501100011033). LV acknowledges support from the European Union Horizon 2020 research and innovation program ERC (BePreSySe, grant agreement 725327).

* Electronic address: daniele.bertacca@unipd.it
[1] H. V. Peiris et al. [WMAP], Astrophys. J. Suppl. 148 (2003), 213-231 doi:10.1086/377228 [arXiv:astro-ph/0302225 [astro-ph]].
[2] D. N. Spergel et al. [WMAP], Astrophys. J. Suppl. 148 (2003), 175-194 doi:10.1086/377226 [arXiv:astro-ph/0302209 [astro-ph]].
[3] N. Aghanim et al. [Planck], Astron. Astrophys. 641, A6 (2020) [erratum: Astron. Astrophys. 652, C4 (2021)] doi:10.1051/0004-6361/201833910 [arXiv:1807.06209 [astro-ph.CO]].
[4] C. Gomez and R. Jimenez, “Model Independent Prediction of the Spectral Index of Primordial Quantum Fluctuations,” JCAP, 10 (2021), 052 [arXiv:2103.10144 [hep-th]].

[5] Alejandro Gangui, Francesco Lucchin, Sabino Matarrese, and Silvia Mollerach. The Three point correlation function of the cosmic microwave background in inflationary models. Astrophys. J., 430:447–457, 1994.

[6] Alejandro Gangui and Jerome Martin. Cosmic microwave background bispectrum and slow roll inflation. Mon. Not. Roy. Astron. Soc., 313:233, 2000.

[7] Li-Min Wang and Marc Kamionkowski. The Cosmic microwave background bispectrum and inflation. Phys. Rev. D, 61:063504, 2000.

[8] Viviana Acquaviva, Nicola Bartolo, Sabino Matarrese, and Antonio Riotto. Second order cosmological perturbations from inflation. Nucl. Phys. B, 667:119–148, 2003.

[9] Juan Martin Maldacena. Non-Gaussian features of primordial fluctuations in single field inflationary models. JHEP, 05:013, 2003.

[10] N. Bellomo, N. Bartolo, R. Jimenez, S. Matarrese and L. Verde, JCAP 11 (2018), 043 doi:10.1088/1475-7516/2018/11/043 [arXiv:1809.07113 [astro-ph.CO]].

[11] Takahiro Tanaka and Yuko Urakawa. Dominance of gauge artifact in the consistency relation for the primordial bispectrum. Journal of Cosmology and Astroparticle Physics, 2011(05):014, May 2011.

[12] Roland de Putter, Olivier Doré, and Daniel Green. Is There Scale-Dependent Bias in Single-Field Inflation? JCAP, 1510(10):024, 2015.

[13] Enrico Pajer, Fabian Schmidt, and Matias Zaldarriaga. The Observed Squeezed Limit of Cosmological Three-Point Functions. Phys. Rev., D88(8):083502, 2013.

[14] Giovanni Cabass, Enrico Pajer, and Fabian Schmidt. How Gaussian can our Universe be? JCAP, 1701(01):003, 2017.

[15] D. Bertacca, N. Bartolo, M. Bruni, K. Koyama, R. Maartens, S. Matarrese, M. Sasaki and D. Wands, Class. Quant. Grav. 32, no.17, 175019 (2015) doi:10.1088/0264-9381/32/17/175019 [arXiv:1501.03163 [astro-ph.CO]].

[16] N. Bartolo, D. Bertacca, M. Bruni, K. Koyama, R. Maartens, S. Matarrese, M. Sasaki, L. Verde and D. Wands, Class. Quant. Grav. 32, no.17, 175019 (2015) doi:10.1088/0264-9381/32/17/175019 [arXiv:1501.03163 [astro-ph.CO]].

[17] A. A. Abolhasani, H. Firouzjahi, A. Naruko and M. Sasaki, doi:10.1142/10953

[18] S. Matarrese, L. Pilo and R. Rollo, JCAP 01 (2021), 062 doi:10.1088/1475-7516/2021/01/062 [arXiv:2007.08877 [astro-ph.CO]].

[19] C. Gomez and R. Jimenez, JCAP 07 (2020), 047 doi:10.1088/1475-7516/2020/07/047 [arXiv:2005.09506 [astro-ph.CO]].

[20] V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rept. 215 (1992), 203-333 doi:10.1016/0370-1573(92)90044-Z

[21] V. Mukhanov, “Physical Foundations of Cosmology,” Cambridge, 2005

[22] E. Komatsu et al. [WMAP], Astrophys. J. Suppl. 192, 18 (2011) doi:10.1088/0067-0049/192/2/18 [arXiv:1001.4538 [astro-ph.CO]].

[23] Shiveshwarkar, C., Jamieson, D., Loverde, M. 2021. Scale-dependent halo bias and the squeezed limit bispectrum in the presence of radiation. Physical Review D 103. doi:10.1103/PhysRevD.103.103503

[24] J. M. Bardeen, Phys. Rev. D 22, 1882-1905 (1980) doi:10.1103/PhysRevD.22.1882

[25] H. Kodama and M. Sasaki, Prog. Theor. Phys. Suppl. 78 (1984) 1.

[26] E. Bertschinger, astro-ph/9503125.

[27] K. A. Malik and D. Wands, Phys. Rept. 475 (2009) 1 [arXiv:0809.4944 [astro-ph]].

[28] C. P. Ma and E. Bertschinger, Astrophys. J. 455 (1995) 7 [astro-ph/9506072].

[29] W. Hu and A. Joyce, Phys. Rev. D 95 (2017) no.4, 043529 doi:10.1103/PhysRevD.95.043529 [arXiv:1612.02454 [astro-ph.CO]].

[30] J. Lesgourgues and S. Pastor, Adv. High Energy Phys. 2012, 608515 (2012) doi:10.1155/2012/608515 [arXiv:1212.6154 [hep-ph]].

[31] R. Jimenez, C. P. Garay and L. Verde, Phys. Dark Univ. 15, 31-34 (2017) doi:10.1016/j.dark.2016.11.004 [arXiv:1602.08430 [astro-ph.CO]].

[32] K. N. Abazajian et al. [Topical Conveners: K.N. Abazajian, J.E. Carlstrom, A.T. Lee], Astropart. Phys. 63, 66-80 (2015) doi:10.1016/j.astropartphys.2014.05.014 [arXiv:1309.5383 [astro-ph.CO]].

[33] C. Carbone, L. Verde and S. Matarrese, Astrophys. J. Lett. 684 (2008), L1-L4 doi:10.1086/592020 [arXiv:0806.1950 [astro-ph]].

[34] D. Karagiannis, A. Lazanu, M. Liguori, A. Raccanelli, N. Bartolo and L. Verde, Mon. Not. Roy. Astron. Soc. 478 (2018) no.1, 1341-1376 doi:10.1093/mnras/sty1029 [arXiv:1801.09280 [astro-ph.CO]].