NEW GAUGE INVARIANT VARIABLES  
FOR YANG-MILLS THEORY

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Abstract

A new set of gauge invariant variables is defined to describe the physical Hilbert 
space of $d = 3 + 1$ $SU(2)$ Yang-Mills theory in the fixed-time canonical formalism. A 
natural geometric interpretation arises due to the $GL(3)$ covariance found to hold for 
the basic equations and commutators of the theory in the canonical formalism. We 
emphasize, however, that we are not interested in and do not consider the coupling of 
the theory to gravity. We concentrate here on a technical difficulty of this approach, the 
calculation of the electric field energy. This in turn hinges on the well-definedness of the 
transformation of variables, an issue which is settled through degenerate perturbation 
theory arguments.

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1. Introduction

While there is no doubt nowadays that Quantum Chromodynamics is the governing theory of the strong interactions, and that confinement and chiral symmetry breaking are the essential mechanisms responsible for the low-energy spectrum of mesons and baryons observed experimentally, there is on the other hand no established and systematic procedure to treat the theory analytically in this low-energy regime, where a perturbative expansion in the gauge coupling is not possible. Moreover, since the early days of QCD there has even been a plausible explanation of what could be responsible for the basic mechanisms of low-energy QCD, namely, the dual superconductor picture of the vacuum and magnetic monopole condensation. Still, as far as analytical treatment goes, no well-defined way to proceed further has yet been determined, and one has been mostly left with general pictures rather than specific calculations with definitive results.

At the same time, it is generally accepted that, whatever the physical picture may be in whatever energy regime, a correct treatment must include proper determination of the gauge invariant degrees of freedom and implementation of gauge invariance. This is true of the high energy perturbative regime, and is also what the dual superconductor picture attempts to do.

In this paper we would like to propose a systematic implementation of this gauge invariance principle, with a view to study the low-energy regime of QCD. In fact, our basic setting will be more limited: if one is content with considering color confinement separately from chiral symmetry breaking, and if one considers that it is gluons and not quarks that are essential in generating confinement, then one may as well study the simplified model of pure SU(2) Yang-Mills theory, and this is what we do here. Without quarks, there is no chiral symmetry to speak of, let alone its breaking, and SU(2) is not the gauge group of QCD, and thus whatever we study will admittedly not be the QCD vacuum and probably not even a good approximation to it. However, we are motivated by the fact that this simplified model does present the feature of color confinement, and a sufficiently intricate and interesting vacuum structure, radically different from that of perturbative or of non-gauge theories.

The general way we propose to implement gauge invariance is simple enough to state: in the fixed-time canonical formalism, rather than using the vector potential \( A^a_i \) as basic Hilbert space variables, we would like to use instead variables which are invariant under gauge transformations. Wavefunctionals which are functions exclusively of these variables will then automatically implement Gauss’ law, \( i.e., \) will be gauge invariant. This is unfortunately and by far not the end of the story: if these variables span too small a sample of Hilbert space, if they do not represent the correct number of gauge invariant degrees of freedom, if the transformation of variables is ill-defined for certain field configurations, then this way to proceed might not be feasible.

In Sec. 2, we will briefly mention one such set of variables, which is inappropriate due to the presence of Wu-Yang ambiguities. We then describe a new set of gauge invariant variables which is free of these ambiguities. As opposed to the Wu-Yang ambiguous variables, these new variables depend nonlocally on \( A^a_i \), and form an “overcomplete” set, much like Wilson loops. A natural geometrical description will emerge, without any coupling to gravity, due
to the $GL(3)$ symmetry already present in all the basic formulas and commutators of the canonical formalism except for the Hamiltonian itself. We exploit this geometry in Sec. 3, where we consider the transformation of the theory to the new variables. In Sec. 4, for the sake of illustration, we consider a specific class of configurations, namely, those for which the new gauge invariant variables describe an Einstein geometry. Geometries with a high degree of symmetry such as this one may at first seem to present a problem for our new variables because of some subtleties regarding the 0-modes of a certain operator. In Sec. 5 we turn to these subtleties. They relate in particular to the computation of the electric field energy, a technically more involved point in this approach, and the well-definedness of the transformation of variables. Settling these issues is our main objective here; we will not dwell on any details of calculation since these can be found in previous work along these lines[1].

We also refer the interested reader to [1] for two issues that will not be considered here, namely, the introduction of static point color sources in this formalism, and the extension to gauge group $SU(3)$.

2. Canonical Formalism and Gauge Invariant Variables

In the fixed-time canonical formalism in $A_0^a = 0$ gauge, the Hamiltonian is

$$ H = \frac{1}{2} \int d^3x \left( \frac{B^{ia}B_{ia}}{g_s^2} + g_s^2E^{ia}E_{ia} \right), \quad (1) $$

where the color magnetic field $B^{ia}$ is defined as a function of the vector potential $A_i^a$:

$$ B^{ka}[A] = \varepsilon^{kij}F_{ij}^a[A] = \varepsilon^{kij}(\partial_iA_j^a + \frac{1}{2} \varepsilon^{abc}A_i^bA_j^c), \quad (2) $$

and the color electric field $E^{ia}$ is the canonical momentum conjugate to $A_i^a$, such that:

$$ [ A_i^a(x), E^{jb}(y) ] = i\delta^{ab}\delta^j_\ell\delta(x-y). \quad (3) $$

In the particular realization we will use of the above commutators the electric field is simply a functional derivative with respect to $A_i^a$. The Gauss law generator is,

$$ \mathcal{G}^a(x) = D_iE^{ia}(x) \equiv \partial_iE^{ia}(x) + \varepsilon^{abc}A_i^b(x)E^{ic}(x), \quad (4) $$

and it generates the following infinitesimal gauge transformations:

$$ [ A_i^a(x), \mathcal{G}_b^b(y) ] = -i\delta^{ab}\partial_\ell\delta(x-y) + i\varepsilon^{abc}A_\ell^c(x)\delta(x-y) \quad (5) $$

$$ [ E^{ia}(x), \mathcal{G}_b^b(y) ] = i\varepsilon^{abc}E^{ic}(x)\delta(x-y) \quad (6) $$

$$ [ H, \mathcal{G}^a(x) ] = 0, \quad (7) $$

with $B^{ia}$ transforming identically to $E^{ia}$. These generators furthermore form a local algebra

$$ [ \mathcal{G}^a(x), \mathcal{G}_b^b(y) ] = i\varepsilon^{abc}\mathcal{G}^c(x)\delta(x-y). \quad (8) $$

2The reason for the peculiar placement of the spatial indices will become clear in what follows.
What we are looking for, then, are variables which represent the correct number of gauge invariant degrees of freedom at each point, and which are annihilated by $G^a$. The simplest possibility is of course to consider the magnetic field. As the above shows, unlike $A^a_i$ it simply rotates under $G^a$, and so one can easily build something gauge invariant from that: $\phi^{ij} \equiv B^{ia} B^{ja}$. Furthermore, $\phi^{ij}$ defined thus is a $3 \times 3$ symmetric matrix, which has 6 independent entries, i.e., just the correct number of gauge invariant degrees of freedom for $SU(2)$. Unfortunately, this approach does not work, due to the fact that gauge field configurations in general have Wu-Yang ambiguities\cite{2}. These are gauge-unrelated configurations of $A^a_i$ which lead to the same magnetic field. If $\phi^{ij}$ are used as basic variables, they are blind to Wu-Yang ambiguous configurations and thus such configurations will not be integrated over when they actually should. There does not seem to be any simple way out of this problem, and thus we discard this first possibility, and turn to one which does not suffer from these ambiguities.

To motivate our definition of gauge invariant variables, let us note first of all that Eqs. (2)-(6),(8) are covariant under $GL(3)$ transformations if we require $A^a_i$ to transform as a covariant vector and $E^{ia}$ as a contravariant vector density under $GL(3)$:

$$E'^{ia}(x') = \left| \frac{\partial x^j}{\partial x'^i} \right| \partial x'^i E^{ja}(x) \hspace{1cm} (9)$$

$$A'^a_i(x') = \frac{\partial x'^j}{\partial x^i} A^a_j(x) \hspace{1cm} (10)$$

where $x^i \to x'^i(x)$ is some coordinate reparametrization, and $\partial x'^i/\partial x^j$ is a $GL(3)$ matrix. In fact, we can see that the only failure of the canonical formalism to fulfill this symmetry comes in the Hamiltonian, where the space indices are summed with a Kronecker $\delta_{ij}$. This symmetry will serve as a useful tool in what follows. We now wish to define a set of new variables $u^a_i$ that will respect not only the $SU(2)$ gauge symmetry but also this $GL(3)$ symmetry. We define them as follows, through a system of linear first order p.d.e.’s:

$$\varepsilon^{ijk} D_j u^a_k = \varepsilon^{ijk} (\partial_j u^a_k + \varepsilon^{abc} A^b_j u^c_k) = 0 \hspace{1cm} (11)$$

A few comments are in order with respect to this definition. Firstly, as opposed to the previous gauge covariant variables $B^{ai}[A]$, $A^a_i$ is built locally as a function of $u^a_i$ (in fact, the explicit relation $A^a_i[u]$ is simple to find from the above), whereas it is $u^a_i$ that is built nonlocally as a function of $A^a_i$. This manifestly eliminates Wu-Yang ambiguities, because a single $u^a_i$ cannot give rise to two different $A^a_i$ configurations. Secondly, it is also true on the other hand that there will always be many configurations $u^a_i$ for a single $A^a_i$: if $u^a_i$ is a solution, then so is $\lambda u^a_i$ for $\lambda$ any real number, and there might even be many other solutions in special cases, as we shall see in what follows. Finally, we see that this definition is both $SU(2)$ and $GL(3)$ covariant if $u^a_i$ transforms as an adjoint $SU(2)$ vector and $GL(3)$ covariant vector.

From the $u^a_i$ we are now ready to build our gauge invariant variables. They are:

$$g_{ij} = u^a_i u^a_j \hspace{1cm} (12)$$
3. Geometry and Yang-Mills Fields

From Eq. (11), it is straightforward to show that
\[ \partial_i u^a_j + \varepsilon^{abc} A^b_i u^c_j - \Gamma^k_{ij} u^a_k = 0 , \]  
where \( \Gamma^i_{jk} \) is the Christoffel connection built from \( g_{ij} \). A geometry has thus entered the formalism without the introduction of any extraneous coordinate reparametrization invariance. From the above, we see that \( u^a_i \) is playing the role of a 3-bein, \( g_{ij} \) that of a metric, and \( \omega^a_{ic} = \varepsilon^{abc} A^b_i \) that of a spin connection.

We now list the expressions for the relevant Yang-Mills quantities in terms of \( u^a_i \), where the geometrical nature of the formalism becomes manifest. We do not present any deductions here, as they can all be found in [1]. The magnetic field is found from the gauge Ricci identity, expressing it in terms of the commutator of two covariant derivatives. One finds
\[ B^{ai} = \sqrt{\text{det} u^a_i} G^{ij} , \]
where by \( \sqrt{\text{det} u^a_i} \) we mean \( \sqrt{\text{det} u^a_i} = \sqrt{\text{det} g_{ij}} \), and \( G_{ij} \) is the Einstein tensor built from the metric \( g_{ij} \). The gauge Bianchi identity reads:
\[ 0 = D_i B^{ai} = \sqrt{\text{det} u^a_i} \left( \nabla_i G^{ij} \right) u^a_j , \]
so that it stands in mutual implication with the geometric Bianchi identity on the right hand side above.

In order to find the electric field we define the tensor \( e^{ij} \):
\[ \frac{\delta}{\delta A^a_i} \equiv \sqrt{\text{det} u^a_i} e^{ij} . \]
In the geometrical variables, Gauss’ law becomes
\[ i\mathcal{G}_a = D_i \left( \frac{\delta}{\delta A^a_i} \right) = \sqrt{\text{det} u^a_i} (\nabla_i e^{ij}) . \]
Thus, if \( \Psi \) is a gauge invariant wavefunctional, then
\[ \nabla_i e^{ij} \Psi = 0 , \]
and vice-versa. It is also possible to show [1] that if \( \Psi \) is only a function of \( g_{ij} \) then it is gauge invariant, and that if it is gauge invariant, then it is only a function of \( g_{ij} \).

In order to calculate the electric field, we consider the response of a wavefunctional to a small variation \( \delta A^a_i \) and attempt to express that in terms of variations in the “metric” \( \delta g_{ij} \). We find, for wavefunctionals depending solely on \( g_{ij} \):
\[ \frac{\delta \Psi}{\delta g_{ij}} = \frac{\varepsilon^{mn}}{2} \nabla_m e^{ij} \Psi , \]
where
\[ e^i_j \equiv e^i_j - \frac{1}{2} \delta^i_j e^s_s . \] (20)

It would now remain to invert the operator \( T = \varepsilon \nabla \) on the right hand side above to find the electric field in terms of metric variations. Formally, one may consider the eigenvalue problem for this operator:
\[ T^i_n \phi^{n_j}_\alpha \equiv \frac{1}{\sqrt{g}} \varepsilon^{im} n_m \phi^{n_j}_\alpha = \lambda_\alpha \phi^{n_j}_\alpha . \] (21)

Once one has the spectrum, inversion can be obtained through the standard spectral representation. The presence of 0-modes of \( T \), however, apparently leads to a problem: the inversion can only be made in the subspace orthogonal to the 0-modes of \( T \), and it seems there is an indetermination in the components of the electric field in the direction of 0-modes of \( T \), which would lead to spurious constraints. We investigate this more carefully in Sec. 5. For now, let us outline how this problem is circumvented: Eq. (19) is gotten essentially through the chain rule
\[ \frac{\delta \Psi}{\delta g_{ij}} = \frac{\delta u^a_k}{\delta g_{ij}} \cdot \frac{\delta \Psi}{\delta A^b_\ell} . \] (22)

It is the 0-modes of \( \delta A^b_\ell / \delta u^a_k \) which cause the problem. The chain rule in the opposite direction, to isolate \( \delta \Psi / \delta A^b_\ell \) would require a well-defined expression for \( \delta u^a_k / \delta A^b_\ell \). Strictly speaking, however, while \( A^a_i \) is a function of \( u^a_i \), \( u^a_i \) is not a function of \( A^a_i \), since there are many \( u^a_i \) which lead to the same \( A^a_i \). This leads to the indetermination in the electric field mentioned above. To cure it, we will need a systematic prescription that assigns a definite variation \( \delta u^a_k \) to a variation \( \delta A^b_\ell \). This as we will see is gotten through standard quantum mechanical degenerate perturbation theory.

In the following section we consider the spectrum of \( T \) for a special class of configurations, those for which \( g_{ij} \) describes an Einstein, or constant curvature, space. This will in particular illustrate the 0-mode problems appearing in the inversion of \( T \).

4. Einstein Space Configurations

To have an idea of what the spectrum of \( T \) looks like, we consider in this section those configurations whose associated metric describes an Einstein space (i.e., \( R_{ij} \propto g_{ij} \), which in \( d = 3 \) implies a constant curvature space). It is possible to show that if \( g_{ij} \) is Einstein with positive curvature (i.e., the sphere \( S_3 \)), then in some gauge \( u^a_i = a^{-1} A^a_i \), where \( a^{-1} \) is the radius of the sphere. This in turn implies that for these configurations, \( A^a_i = \frac{1}{2} \bar{A}^a_i \), where \( \bar{A} \) is pure gauge. Throughout this section we consider only the sphere \( S_3 \); the hyperbolic case works in a similar way.

To find the spectrum of \( T \), we decompose \( \phi^{i\bar{j}} \) as follows:
\[ \phi^{i\bar{j}} = S^{i\bar{j}} + \frac{1}{2} \frac{\varepsilon^{i\bar{j}k}}{\sqrt{g}} V_k + \frac{1}{3} g^{i\bar{j}} \bar{\phi} , \] (23)

where \( S^{i\bar{j}} \) is symmetric and traceless. By trying different \( \text{ansätze} \) for \( S^{i\bar{j}}, V_k \) and \( \bar{\phi} \) one then finds the following eigenvectors and eigenvalues of \( T \):
1. \( \phi_{ij} = \frac{1}{2a_N^2} \left[ \nabla_i \nabla_j + s_N^2 g_{ij} - \sqrt{s_N^2 - a^2} \frac{\varepsilon_{ijk}}{\sqrt{g}} \nabla^k \right] Y_{Nlm} \), with eigenvalues \( \lambda = \pm \sqrt{s_N^2 - a^2} \), where \( -s_N^2 = -a^2 N(N + 2), N = 1, 2, ... \) is the spectrum of the Laplacian acting on scalars on \( S_3 \) (with radius \( a^{-1} \)), and \( Y_{Nlm} \) are the associated hyperspherical harmonics.

2. \( \phi_{ij} = [\nabla_i \nabla_j + a^2 g_{ij}] Y_{Nlm} \), all with eigenvalue \( \lambda = 0 \).

3. \( \phi_{ij} \) is symmetric, traceless and covariantly divergence-free, and is an eigenvector of the Laplacian acting on rank 2 tensors. The associated eigenvalue is \( \lambda = \pm \sqrt{t_N^2 + 3a^2} \), where \( t_N^2 = -s_N^2 + 2a^2 \) is the spectrum of the Laplacian acting on rank 2 tensors on \( S_3 \).

4. \( \phi_{ij} = -\frac{1}{2}(\lambda - a^2/\lambda)^{-1}(\nabla_i V_j + \nabla_j V_i) + \frac{1}{2} \frac{\varepsilon_{ijk}}{\sqrt{g}} V^k \), where \( V_i \) is a covariantly divergence-free eigenvector of the Laplacian acting on vectors. The associated eigenvalues are \( \lambda = \pm \sqrt{v_N^2 + 2a^2 \pm \sqrt{v_N^2 - 2a^2}} \), where \( -v_N^2 = -s_N^2 + a^2 \) is the spectrum of the Laplacian acting on vectors on \( S_3 \).

We believe this exhausts the spectrum of \( T \) on \( S_3 \), but we have no complete proof of this. Two features of this spectrum are worth mentioning: first, there is an infinite set of exact 0-modes. To this infinite set, there corresponds an infinite set of solutions of Eq. (11) (cf. [1]). Second, some of the eigenvalues in 4. above approach 0 as \( 1/N \) for \( N \to \infty \). Thus, there is an accumulation of modes near \( \lambda = 0 \). Under an infinitesimal variation of \( A_i^a \), these exact- and near-0 modes mix to give the variation in the solution to Eq. (11).

5. Electric Energy and Degenerate Perturbation Theory

If we look at the variation of \( A_i^a \) under a variation in \( u_i^a \),

\[
\delta A_i^a(x) = \frac{\delta A_i^a(x)}{\delta u_j^b(y)} \cdot \delta u_j^b(y)
\]

it immediately seems that if \( \delta A_i^a(x)/\delta u_j^b(y) \) has left zero modes (i.e., zero modes upon action of the operator to the left), there will be constraints on \( \delta A_i^a(x) \): apparently not all \( \delta A_i^a(x) \) can be generated from all \( \delta u_j^b(x) \). Since the electric field is the response of the wavefunctional to a variation \( \delta A_i^a(x) \), this would mean in particular that, in the \( u \)-variables, the electric field could not be measured along all directions. These statements being true would not be a good starting point for these variables. We shall now see, however, that because of the enormous degeneracy of zero and near-zero modes, through degenerate perturbation theory considerations one can actually prove the transformation of variables is well-defined, and the electric field can be determined in all directions in the \( u \)-variables.

Let us reconsider Eq. (11) in the light of the fact that it has many solutions for any \( A_i^a \), and let us label these solutions as \( u_\alpha \), where we omit all space and color indices; \( \alpha \) spans the different solutions. Picking one particular \( \alpha = \bar{\alpha} \), one then builds a metric \( g_{ij}^\alpha \), from which one can study the fully \( GL(3) \) covariant eigenvalue problem for the nonzero modes:

\[
\frac{1}{\sqrt{g}} \varepsilon^{ijk} D_j u_k^a = \lambda w^a.
\]
Again omitting all color and space indices, we can label these eigenvectors by some index $a$, $w_a$, and the corresponding eigenvalue by $\lambda_a$. These states can be orthonormalized:

\[
(u_\alpha, u_{\alpha'}) \equiv \int d^3x \sqrt{g} g^{ij} (u_\alpha_i)^a (u_{\alpha'})_j^a = \delta_{\alpha\alpha'} \tag{26}
\]

\[
(w_a, w_{a'}) = \delta_{a a'} \tag{27}
\]

\[
(u_\alpha, w_a) = 0 . \tag{28}
\]

The $\delta$’s on the right hand side may be Dirac or Kronecker deltas, depending on whether their arguments are continuous or discrete.

We now consider “perturbations” $\delta A^a_i(x)$. If $\tilde{u}_\alpha$ denotes the perturbed $u_\alpha$, and $\delta \lambda_\alpha$ the perturbed eigenvalue, then:

\[
\varepsilon D \tilde{u}_\alpha + \Delta A \tilde{u}_\alpha = \delta \lambda_\alpha \sqrt{g} \tilde{u}_\alpha , \tag{29}
\]

where $\Delta A \equiv \varepsilon \varepsilon \delta A$ and all space and color indices are omitted.

We can expand $\tilde{u}_\alpha$ in the $\{u_\alpha, w_a\}$ basis:

\[
\tilde{u}_\alpha = P_{\alpha\alpha'} u_{\alpha'} + Q_{aa} w_a . \tag{30}
\]

Here and below, repeated indices are summed over unless explicitly indicated otherwise. The coefficients $P$ are of order 1, while $Q$ are of order $\delta A$. We now need solutions for $P$ and $Q$. Inserting this in Eq. (29), and using orthonormality, we obtain the system

\[
P_{\alpha\beta'} (\Delta A)_{\beta\beta'} + Q_{aa} (\Delta A)_{\beta a} = \delta \lambda_\alpha P_{\alpha\beta} \quad (\alpha, \beta \text{ fixed}) \tag{31}
\]

\[
P_{\alpha\beta'} (\Delta A)_{b\beta'} + \lambda_b Q_{ab} + Q_{aa} (\Delta A)_{ba} = \delta \lambda_\alpha Q_{ab} \quad (\alpha, b \text{ fixed}) , \tag{32}
\]

where $(\Delta A)_{\beta\beta'} = (u_\beta, (\Delta A) u_{\beta'})$. The last two terms of the second equation above are of higher order and we drop them, to find:

\[
Q_{ab} = -\frac{1}{\lambda_b} P_{\alpha\beta'} (\Delta A)_{b\beta'} \quad (\alpha, b \text{ fixed}). \tag{33}
\]

This is inserted in the first equation (it is not difficult to see that the difference of the two “$P$-terms” is of the same order as the “$Q$-term”, and so this latter one should not be dropped):

\[
\left[(\Delta A)_{\beta\beta'} - (\Delta A)_{\beta a} \frac{1}{\lambda_a} (\Delta A)_{a\beta'}\right] P_{\alpha\beta'} = \delta \lambda_\alpha P_{\alpha\beta} \quad (\alpha, \beta \text{ fixed}). \tag{34}
\]

If we now diagonalize $(\Delta A)_{\alpha\beta}$:

\[
S^T_{\mu\alpha} (\Delta A)_{\alpha\beta} S_{\beta\nu} = (\Delta A)_{\mu\nu} \delta_{\mu\nu} \quad (\mu, \nu \text{ fixed}), \tag{35}
\]

we have in the transformed system:

\[
\tilde{P}_{\alpha\nu}(\Delta A)_\nu - \tilde{P}_{\alpha\mu}(\Delta A)_\mu = \delta \lambda_\alpha \tilde{P}_{\alpha\nu} \quad (\alpha, \nu \text{ fixed}), \tag{36}
\]

where $\tilde{P}_{\alpha\nu} = P_{\alpha\beta} S_{\beta\nu}$.
Since we are assuming there is a solution for every $A$-configuration, there must be an $\alpha$, call it $\alpha = 0$, such that $\delta \lambda_\alpha = 0$. That is the one we are interested in. Also, there is a $\mu$, call it $\mu = 0$, such that $(\Delta A)_\mu$ is minimum (no $(\Delta A)_\mu$ can be zero, since otherwise there would be a linear combination of $u_\alpha$ that would be a solution to both $A$ and $A + \delta A$ simultaneously, which we have shown previously to be impossible). Now we can determine $P$ uniquely: the ansatz $\tilde{P}_{0\mu} = \delta_{0\mu} + \mathcal{O}(\delta A)$ solves the above to lowest order for the component $\alpha = 0$:

$$\tilde{P}_{0\nu} = \frac{1}{(\Delta A)_\nu} (\Delta A)_{\nu a} \frac{1}{\lambda_a} (\Delta A)_{a0} \quad (\nu \text{ fixed}) \quad (37)$$

$$Q_{0b} = -\frac{1}{\lambda_b} (\Delta A)_{b0} \quad (b \text{ fixed}) \quad (38)$$

This is the result we need: by substituting this above in Eq. (30), we have a prescription for the unique variation in $u_i^a$, $\delta u_i^a$, given a generic variation $\delta A_i^a$. This amounts to having $\delta u_i^a/\delta A_j^b$, from which one can then unequivocally calculate the electric field.

6. Conclusions

It is virtually impossible to present in a short space all the details of a long project such as this. Many of our results, which may appear too telegraphic here (or even do not appear here at all), can be found in [1], while others will be left for future publications. Here our main concern was with establishing the fact that the transformation of variables and the electric field in the variables $u_i^a$ are well-defined even for those geometries which, because of a high degree of symmetry, lead to 0-modes of the operator $T$. Beyond this fact, it remains to be seen whether the technical difficulties associated to the calculation of the electric field can be overcome at a practical level, in order to allow for a numerical treatment of some sort. Work along these lines is in progress.

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