Analysis of expression for determination of specific capacitive conductivity of power lines and refinement of calculations of earth fault currents in networks with isolated neutral

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Abstract. Topic relevance is due to necessity of the correct mathematical expressions during the calculation of constants of equivalent circuit of power supply system, including the overhead transmission lines. Research purpose is the allowances thanks to which formula for capacity admittance calculation and of ground fault current in electrical grids with insulated neutral determination were deduced and made common use. As exemplified by numerical computation accepted allowances can lead to significant error (more than 15%), for example, during the determination of single phase-to-ground fault current in electrical grids with insulated neutral.

1. Introduction

To calculate the charging capacity of transmission lines (TML) and earth fault current, the following formula for determining the capacitive conductivity of lines is widely used [1]:

\[ b = \frac{7.58 \cdot 10^{-6}}{\lg \left( \frac{D}{R} \right)} \]  

where R – wire radius, D – average compound distance between the two phase wires of TML

This equation is presented in not only specified but in educational literature also. Here with conditions of allowable use of this expression are not being mentioned or considered.

Development of this expression is shown to the fullest extend in [2], however in this case allowances that were made during the development of this expression are not being mentioned too. There is no answer to the following issues:

- use of this expression in case of real transmission in one way or the other is different from that one for which the expression was made;
- errors that must be expected;
- is it correct to use this expression not only for overhead power lines, but also for cable.

2. Formulation of the problem

Purpose of the study is analysis of allowances of the formula (1) for capacity admittance calculation and calculation refinement of ground fault current in electrical grids with insulated neutral determination were deduced and made common used.

For implementation of stated goal, it is necessary to complete the tasks:
1. Determine allowances (requirements for TML) using which the expression for determination of specific capacity admittance of TML was made, and to do that the development of this expression should be examined carefully.

2. If real TML doesn’t meet these requirements. Then it is necessary to estimate the calculating error using this expression for every specific case.

3. Theory

The system of equations can be written for electrostatic field’s charged cylindrical or spherical bodies which are situated nearby the conducting plane. The system connects the charges of the body (τ) and potential (φ):

\[
\begin{align*}
\phi_1 &= \tau_1 \cdot \alpha_{11} + \tau_2 \cdot \alpha_{12} + \tau_3 \cdot \alpha_{13} + \cdots + \tau_n \cdot \alpha_{1n} \\
\phi_2 &= \tau_1 \cdot \alpha_{21} + \tau_2 \cdot \alpha_{22} + \tau_3 \cdot \alpha_{23} + \cdots + \tau_n \cdot \alpha_{2n} \\
\phi_3 &= \tau_1 \cdot \alpha_{31} + \tau_2 \cdot \alpha_{32} + \tau_3 \cdot \alpha_{33} + \cdots + \tau_n \cdot \alpha_{3n} \\
&\vdots \\
\phi_n &= \tau_1 \cdot \alpha_{nn} + \tau_2 \cdot \alpha_{n2} + \tau_3 \cdot \alpha_{n3} + \cdots + \tau_n \cdot \alpha_{nn}
\end{align*}
\]  

(2)

The values of the coefficients \( \alpha \) are determined by the formulas:

\[
\alpha_{nn} = \frac{1}{2 \pi \varepsilon_n} \ln \frac{2 h_n}{r_n} ; \quad \alpha_{nk} = \frac{1}{2 \pi \varepsilon_n} \ln \frac{b_{nk}}{a_{nk}}
\]  

(3)

where: \( \alpha_{nn} \) – eigene potential coefficient, \( \alpha_{nk} \) – bilateral potential coefficient; \( h_n \) – distance between the center of the body N and its mirrored image relatively to the conducting plane; \( r_n \) – radius of the body N; \( b_{nk} \) – distance between the center of the body N and mirrored image of the center of the body K; \( a_{nk} \) – distance between the centers of bodies N and K; \( \varepsilon_n \) – absolute dielectric capacitance of working substance, equal 8.85 \times 10^{-12} \text{ F/m}.

The system of equations (2) is strictly valid for electrostatic field, but also can be used for alternating electric field, because alternating electric field can be considered electrostatic in every moment of time with specific condition. For this to happen it is necessary and enough for oscillation phase of field’s density to stay constant in between the limits of linear dimensions of bodies. In other words, for this to happen it is necessary and enough for linear dimensions of bodies to be inappreciable in relation to the length of the electromagnetic wave.

The length of the electromagnetic wave is 6000 km for commercial frequency which is far more the length of distribution circuit’s transmission lines with straining of 6 – 35 kV and also is far more than most of transfers with further higher straining.

Looking into equations (2) for the case of alternating electric field, charge and potential should be understood to mean the instantaneous values. The potential \( \phi \) should be understood to mean the potential between the phase conductor and the conducting plane (earth), i.e. line-to-neutral voltage. Since values change with the harmonic law, the equations (2) could be written in a symbolic form for complex virtual charges \( \hat{\tau} \) and line-to-neutral voltages \( \hat{\phi} \). For the single-circuit power line the system of equations would have the following form:

\[
\begin{align*}
\hat{\phi}_1 &= \hat{\tau}_1 \cdot \alpha_{11} + \hat{\tau}_2 \cdot \alpha_{12} + \hat{\tau}_3 \cdot \alpha_{13} \\
\hat{\phi}_2 &= \hat{\tau}_1 \cdot \alpha_{21} + \hat{\tau}_2 \cdot \alpha_{22} + \hat{\tau}_3 \cdot \alpha_{23} \\
\hat{\phi}_3 &= \hat{\tau}_1 \cdot \alpha_{31} + \hat{\tau}_2 \cdot \alpha_{32} + \hat{\tau}_3 \cdot \alpha_{33}
\end{align*}
\]  

(4)

The phase voltages will be considered to form a symmetric system, i.e. the conductor transposition will be taken into account. With this condition, the current values of potential coefficients \( \alpha_{nn} \) and \( \alpha_{nk} \) could be replaced with its average by the line’s length values:

\[
\alpha_{nn} = \frac{\alpha_{11} + \alpha_{22} + \alpha_{33}}{3} ; \quad \alpha_{nk} = \frac{\alpha_{12} + \alpha_{23} + \alpha_{13}}{3}
\]

In this case, the symmetric electrotransmission will be obtained with the symmetric voltages system. With this electrotransmission \( \hat{\tau}_1, \hat{\tau}_2, \hat{\tau}_3 \) form a symmetric system as well, i.e.:

\[\hat{\tau}_2 = a^2 \cdot \hat{\tau}_1 \; ; \; \hat{\tau}_3 = \dot{a} \cdot \hat{\tau}_1,\]

where \( a \) – complex factor.
With the understanding that \( a = -\frac{1}{2} + \frac{j\sqrt{3}}{2} \); \( a^2 = -\frac{1}{2} - \frac{j\sqrt{3}}{2} \), equations of symmetric electrotransmission will transform into:

\[
\hat{\phi}_1 = \hat{t}_1 \cdot \alpha_{nus} + \hat{t}_2 \cdot \alpha_{skc} + \hat{t}_3 \cdot \alpha_{skc} = \\
= \hat{t}_1 \left( \alpha_{nus} + \left(a + a^2\right) \alpha_{skc} \right) = \hat{t}_1 \left( \alpha_{nus} - \alpha_{skc} \right) \\
\hat{\phi}_2 = \hat{t}_2 \left( \alpha_{nus} - \alpha_{skc} \right) \\
\hat{\phi}_3 = \hat{t}_3 \left( \alpha_{nus} - \alpha_{skc} \right)
\]

The expression \( \frac{1}{\alpha_{nus} - \alpha_{skc}} \) is an operating capacitance \( C_r \) or the capacitance of the line’s phase.

In the normal operating conditions, the capacitive charge current is calculated using the operating capacitance, thus the operating capacitance is also called the charging capacitance. The operating capacitance is an equivalent of the capacitance between the conductor and earth \( C_0 \) and the capacitance between conductors \( C_{mf} \) (Figure 1.) [3]:

\[
C_r = 3C_{mf} + C_0
\]

where: \( C_{mf} \) – specific capacitance between the phases, \( C_0 \) – specific capacitance between phases and earth.

![Figure 1. To the definition of the line’s operating capacitance](image)

Values \( \alpha_{nus} \) and \( \alpha_{skc} \) for line’s unit of length will be written replacing \( r_n \) with conductor’s radius \( R \):

\[
\alpha_{nus} = \frac{1}{2\pi\varepsilon_0} \cdot \frac{1}{3} \left( \ln \frac{2h_1}{R} + \ln \frac{2h_2}{R} + \ln \frac{2h_3}{R} \right) \\
\alpha_{skc} = \frac{1}{2\pi\varepsilon_0} \cdot \frac{1}{3} \left( \ln \frac{b_{12}}{a_{12}} + \ln \frac{b_{23}}{a_{23}} + \ln \frac{b_{13}}{a_{13}} \right)
\]

Then the \( C_r \) value for the unit of length will be determined according to the equation:

\[
C_r = \frac{1}{\alpha_{nus} - \alpha_{skc}} = \frac{2\pi\varepsilon_0}{\ln \left( \frac{2}{R} \left( \frac{h_1 \cdot h_2 \cdot h_3 \cdot (a_{12} \cdot a_{23} \cdot a_{13})}{b_{12} \cdot b_{23} \cdot b_{13}} \right)^{\frac{1}{3}} \right)}
\]
If the line’s phase conductors are settled on tops of the equilateral triangle, according to the Figure 2:
\[ h_1 = h_5 = h; \quad h_2 = h + \frac{a}{2} \sqrt{3}; \quad b_{13} = \sqrt{\left(2h + \frac{a}{2} \sqrt{3}\right)^2 + \left(\frac{a}{2}\right)^2}; \quad h_3 = \sqrt{4h^2 + a^2}; \]
\[ a_{12} = a_{23} = a_{13} = a, \]

Then
\[
C_r = \frac{2\pi\varepsilon_a}{\ln \frac{\frac{2}{R}}{\left(\frac{h^2\left(h + \frac{a}{2} \sqrt{3}\right)^3}{2} + \left(\frac{a}{2}\right)^2 \sqrt{4h^2 + a^2}\right)^\frac{1}{2}}}.
\]

If one assumes that \(a << h\), i.e. the effect of earth is omitted, then after the conversion (6) the following equation will be obtained:
\[
C_r = \frac{2\pi\varepsilon_a}{\ln \frac{a}{R}}.
\]

If the conductors are configured horizontally, according to the Figure 3:
\[ a_{12} = a_{23} = a; \quad a_{13} = 2 \cdot a; \quad h_1 = h_2 = h_3 = h; \quad b_{12} = b_{23} = \sqrt{4h^2 + a^2}; \quad b_{13} = \sqrt{4h^2 + 4a^2}. \]

Then
\[
C_r = \frac{2\pi\varepsilon_a}{\ln \frac{2ha^{\frac{1}{3}}}{R \left(4h^2 + a^2\right)^\frac{1}{3} \sqrt{4h^2 + a^2}}}.
\]

**Figure 2.** To the definition of the line’s operating capacitance The geometry of the power line, if the conductors are settled on tops of the equilateral triangle
Introducing the previous condition \( a \ll h \) we will obtain the equation

\[
C_s = \frac{2\pi\varepsilon_s}{\ln \left( \frac{2^{\frac{1}{3}} \cdot a}{R} \right)}
\]  

(9)

Using equations (7) and (8) the equation for capacitive conductance of the electrotransmission’s unit of length:

\[
b_0 = \omega \cdot C_s = 2\pi f C_s.
\]

After the substitution of numerical values of constants and going from hyperbolic logarithm to the common logarithm \( \lg(N) = \ln(e) \cdot \ln(N) \), the equation (1) will be obtained.

When done in this way, in case of conductors being settled on tops of the equilateral triangle \( D_{aa} = (a \cdot a \cdot a)^{\frac{1}{3}} = a \), and with horizontal configuration of the conductors \( D_{aa} = (a \cdot a \cdot 2a)^{\frac{1}{3}} = a2^{\frac{1}{3}} \).

Thus, the expression (1) can be correctly used for single-chain overhead power lines at a height of suspension of wires above the ground much more than the distance between the wires.

However, if the researcher faces a different task, for example, determining the single-phase current \([4\text{-}7]\), the use of the expression (1) can lead to sufficiently large errors. To illustrate this, let us conduct a numerical experiment.

Capacitances of overhead line wires can be determined by Maxwell’s formulas. For a line without cables, the capacity of the wire is determined:

\[
C_1 = C_{11} + C_{12} + C_{13}
\]  

(10)

where \( C_{11} \) is the intrinsic partial capacitance of the wire relative to the ground; \( C_{12} \) and \( C_{13} \) are mutual partial capacitances relative to the second and third wire.

Partial capacities are determined by the formula:

\[
C_{11} = \frac{\Lambda_{11}}{\Delta}; C_{12} = \left| \frac{\Lambda_{12}}{\Delta} \right|; C_{13} = \frac{\Lambda_{13}}{\Delta}.
\]  

(11)

Here in \( \Delta \) denotes a determinant, composed of self and mutual potential coefficients:

\[
\Delta = \begin{vmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{13} \\
\alpha_{21} & \alpha_{22} & \alpha_{23} \\
\alpha_{31} & \alpha_{32} & \alpha_{33}
\end{vmatrix}
\]

(12)

where \( \Lambda_{11}, \Lambda_{12}, \text{etc.} \) – cofactor of the elements of the determinant \( \Delta \).

Eigenvalue and mutual potential coefficients are calculated using expressions (3).

Based on the expressions (10) and (11), the following analytical expressions can be obtained to determine the capacitance of single-chain three-phase power lines with arbitrary wire suspension:
$$C_1 = \frac{\alpha_{13}^2 - \alpha_{13} \cdot \alpha_{23} + \alpha_{13} \cdot \alpha_{12} + \alpha_{13} \cdot \alpha_{13} - \alpha_{12} \cdot \alpha_{23} - \alpha_{12} \cdot \alpha_{13}}{\alpha_{13} \cdot \alpha_{12} - 2 \cdot \alpha_{12} \cdot \alpha_{13} + \alpha_{12} \cdot \alpha_{23} + \alpha_{12} \cdot \alpha_{13} - \alpha_{13} \cdot \alpha_{23} - \alpha_{13} \cdot \alpha_{12}};$$  \hfill (13)  

$$C_2 = \frac{\alpha_{12}^2 + \alpha_{13} \cdot \alpha_{23} - \alpha_{12} \cdot \alpha_{13} - \alpha_{12} \cdot \alpha_{13} + \alpha_{12} \cdot \alpha_{23} - \alpha_{12} \cdot \alpha_{13}}{\alpha_{13} \cdot \alpha_{12} - 2 \cdot \alpha_{12} \cdot \alpha_{13} + \alpha_{12} \cdot \alpha_{23} + \alpha_{12} \cdot \alpha_{13} - \alpha_{13} \cdot \alpha_{23} - \alpha_{13} \cdot \alpha_{12}};$$  \hfill (14)  

$$C_3 = \frac{\alpha_{12}^2 - \alpha_{13} \cdot \alpha_{23} - \alpha_{12} \cdot \alpha_{13} + \alpha_{12} \cdot \alpha_{13} + \alpha_{12} \cdot \alpha_{23} + \alpha_{12} \cdot \alpha_{13} - \alpha_{13} \cdot \alpha_{23} - \alpha_{13} \cdot \alpha_{12}}{\alpha_{13} \cdot \alpha_{12} - 2 \cdot \alpha_{12} \cdot \alpha_{13} + \alpha_{12} \cdot \alpha_{23} + \alpha_{12} \cdot \alpha_{13} - \alpha_{13} \cdot \alpha_{23} - \alpha_{13} \cdot \alpha_{12}}.$$  \hfill (15)  

### 4. Result of an experiment

As is well known, 35 kV electrotransmissions are executed without transpositions of phase conductors, thus it is asymmetric and does not fulfill the conditions mentioned above. Let us calculate the capacitive earth fault currents and the unbalance rate of the capacitance between a conductor and earth by the example of type P 35-1 transmission tower (Figure 4.)

Initial data for the calculation according to the dimensions of the support:

- \( h_1 = 18 \) m; \( h_2 = h_1 = 15 \) m; \( d_{13} = 3 \) m; \( d_{23} = 4 \) m; \( L_1 = 2 \) m; \( L_2 = 2 \) m; \( L_3 = 3 \) m;
- \( \varepsilon_o = 8.86 \cdot 10^{-9} \) F/m; \( \varepsilon_r = 1.00059 \) F/m.

Wire type AS-120 with conductor’s diameter \( d=15.2 \) mm;

The designation of the calculated dimensions are shown in the Figure 5.

Let us determine medium’s absolute capacitivity and conductor’s radius:

\[
\varepsilon = \frac{1}{2\pi\varepsilon_r} = \frac{1}{2 \cdot 3.14 \cdot 8.86 \cdot 10^{-9} \cdot 1} = 1.795 \cdot 10^7 \text{ F/km}
\]

\[
r = 7.6 \cdot 10^{-3} \text{ m};
\]

\[
L_t = h_1 - h_2 = 18 - 15 = 3 \text{ m}
\]

**Figure 4.** Type P 35-1 transmission tower

Distances between phases according to picture 5:

\[
d_{13} = \sqrt{L_1^2 + (L_3 - L_2)^2} = \sqrt{3^2 - 1.3^2} = 3.27 \text{ m}
\]

\[
d_{23} = L_3 + L_2 = 5.3 \text{ m}
\]

\[
d_{12} = \sqrt{L_1^2 + (2L_2)^2} = \sqrt{3^2 + 4^2} = 5 \text{ m}
\]

Let us determine the potential coefficient of conductor 1:

\[
\alpha_{11} = \varepsilon \cdot \ln \left( 2 \cdot \frac{h_1}{r} \right) = 1.795 \cdot 10^7 \cdot \ln \left( 2 \cdot \frac{18}{7.6 \cdot 10^{-3}} \right) = 1.519 \cdot 10^8 \text{ F/km}
\]
Figure 5. Geometrical arrangement of conductors’ phases

Distance from center of conductor 1 to center of mirror image of conductor 2:

\[ b_{12} = \sqrt{(h_1 + h_2)^2 + (L_4 + L_2)^2} = \sqrt{33^2 + 4^2} = 33.242 \, \text{m} \]

Distance between the centers of conductors 1 and 2:

\[ a_{12} = d_{12} \]

Mutual potential coefficient between the first and second conductor

\[ \alpha_{12} = \varepsilon \cdot \ln \left( \frac{b_{12}}{d_{kn12}} \right) = 1.795 \cdot 10^7 \cdot \ln \left( \frac{33.242}{5} \right) = 3.4 \cdot 10^7 \, \text{F/km} \]

Similarly, the geometric parameters and potential coefficients for the second and third conductor are determined.

\[ b_{13} = \sqrt{(h_1 + h_2)^2 + (L_3 - L_1)^2} = \sqrt{33^2 + 1^2} = 33.026 \, \text{m} \]

\[ \alpha_{13} = \varepsilon \cdot \ln \left( \frac{b_{13}}{d_{kn13}} \right) = 1.795 \cdot 10^7 \cdot \ln \left( \frac{33.026}{3.27} \right) = 4.152 \cdot 10^7 \, \text{F/km} \]

\[ \alpha_{31} = \alpha_{43} \]

\[ \alpha_{22} = \varepsilon \cdot \ln \left( 2 \cdot \frac{h_2}{r} \right) = 1.795 \cdot 10^7 \cdot \ln \left( 2 \cdot \frac{15}{7.6 \cdot 10^{-3}} \right) = 1.487 \cdot 10^8 \, \text{F/km} \]

\[ b_{23} = \sqrt{(2h_2)^2 + (L_3 + L_2)^2} = \sqrt{30^2 + 5^2} = 33.465 \, \text{m} \]

\[ \alpha_{23} = \varepsilon \cdot \ln \left( \frac{b_{23}}{d_{kn23}} \right) = 1.795 \cdot 10^7 \cdot \ln \left( \frac{33.465}{5.3} \right) = 3.14 \cdot 10^7 \, \text{F/km} \]

\[ \alpha_{32} = \alpha_{23} \]

\[ \alpha_{33} = \varepsilon \cdot \ln \left( 2 \cdot \frac{h_3}{r} \right) = 1.795 \cdot 10^7 \cdot \ln \left( 2 \cdot \frac{15}{7.6 \cdot 10^{-3}} \right) = 1.487 \cdot 10^8 \, \text{F/km} \]

\[ \alpha_{32} = \alpha_{23} \]

For a line without cables, the capacity of the conductors is determined according to (13), (14), (15):

\[ C_1 = 4.273 \cdot 10^{-9} \, \text{F/km} \]

\[ C_2 = 4.794 \cdot 10^{-9} \, \text{F/km} \]

\[ C_3 = 4.521 \cdot 10^{-9} \, \text{F/km} \]

In practice, approximate formulas derived from expression (1) are widely used to determine earth fault currents. For overhead power line it has the following form:

\[ I_s = \frac{U_{\text{nom}} \cdot L}{350} \quad (16) \]
Let's compare the results of the calculation of the short-circuit current for the presented type of supports used for the voltage of 35 kV obtained by (16) and by formulas (13-15), taking into account the real geometry of the wires. The calculation results are given for one kilometer of the line.

According to the formula (16):
\[ I_s = \frac{35 \cdot 1}{350} = 0.1 \text{ A} \]

Phase-by-phase calculation of short-circuit currents using the obtained values of wire capacitances gives the following results:
\[
\begin{align*}
I_A &= 35 \cdot 314 \cdot C_1 \cdot 10^3 / \sqrt{3} = 0.027 \text{ A} \\
I_B &= 35 \cdot 314 \cdot C_2 \cdot 10^3 / \sqrt{3} = 0.03 \text{ A} \\
I_C &= 35 \cdot 314 \cdot C_3 \cdot 10^3 / \sqrt{3} = 0.02868 \text{ A}
\end{align*}
\]

Therefore, the value of the fault current is:
\[
I_{\text{sum}} = I_A + I_B + I_C = 0.027 + 0.03 + 0.02868 = 0.086 \text{ A}
\]

Thus the calculation error between the approximate calculation and the refined calculation:
\[
\Delta I = \frac{I_{\text{sum}} - I_s}{I_{\text{sum}}} \cdot 100 = \frac{0.086 - 0.1}{0.086} \cdot 100 = -16 \%
\]

5. Conclusion

Application of an equation (1) for specific capacitance conductance determination is correct under following conditions:
1. The power transmission line is single-circuited and has no overhead earth wire;
2. Length of the power transmission line must be much smaller than the wavelength \(l < \lambda\);
3. The power transmission line must be symmetric (transposition is executed on the line) with the symmetric system of line-to-neutral voltages;
4. The height of the power transmission line’s wiring above the earth must be much bigger than the distance between the conductors \(h \gg a\).

If a real electrotransmission does not correspond with these conditions, then the application of equation (1) will lead to a calculation error. Magnitude of this error depends on a problem that needs to be solved and could be significant. For example, a ground fault current in 35 kV power transmission line without a transposition of conductors would be determined with an error of about 16%.

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