A holographic relation between the deconfinement temperature and gluon condensate

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Abstract

We derive a holographic prediction of the deconfinement temperature $T_c$ at vanishing chemical potential within a simplest AdS/QCD model with dynamical dilaton. Our analysis leads to a linear relation between $T_c^4$ and the gluon condensate. After normalizing this relation to the lattice data for $SU(3)$ pure gauge theory, the standard phenomenological value of gluon condensate from QCD sum rules leads to the prediction $T_c = 156$ MeV which is in a perfect agreement with the modern lattice results and freeze-out temperature measured by the ALICE Collaboration.

1 Introduction

Quantum Chromodynamics (QCD) predicts that at high temperatures and/or hadron densities strongly interacting matter exhibits a transition which separates the hadronic, confined phase and the quark-gluon plasma (QGP) phase. The quantitative mapping of QCD phase diagram stays among the major challenges of the physics of strong interaction. The ongoing heavy ion collision experiments at RHIC at Brookhaven National Laboratory, ALICE and SPS at the Large Hadron Collider (LHC) are trying to quantify the properties of the deconfined phase of strongly interacting matter — the phase in which our early universe existed. A special interesting region on the phase portrait is situated near the onset of deconfinement transition at vanishing or small baryon (or quark) chemical potential. The matter is that this region can be studied directly from QCD using lattice simulations and the corresponding critical temperature $T_c$ for deconfinement transition can be calculated. The two leading lattice collaborations in this field reported the values $T_c = 156 \pm 9$ MeV [1] and $T_c = 154 \pm 9$ MeV [2]. At physical quark masses $T_c$ represents in fact a pseudo-critical temperature of crossover region between the hadron phase and QGP. From the experimental side, $T_c$ is believed to be very close to the temperature of chemical freeze-out $T_{cf}$ which is

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extracted from a thermal analysis of the data on relativistic nuclear collisions. The recent precise measurement by ALICE Collaboration at LHC resulted in the value $T_{cf} = 156.5 \pm 1.5$ MeV \[3\]. Thus $T_c$ from lattice simulations and $T_{cf}$ remarkably agree within errors.

From the theoretical viewpoint, however, the lattice calculations represent a kind of ”black box” whose results cannot be checked analytically. On the other hand, the strict relation between $T_c$ and $T_{cf}$ is an open problem. It is therefore highly desirable to have independent estimates for $T_c$ from models which were successful in description of QCD phenomenology. A strong point of these complementary approaches is that they can be able to relate $T_c$ to some important dimensional quantity from low-energy QCD phenomenology. Looking from the opposite side, a correct prediction of the value of deconfinement temperature represents an important test for any model of this kind.

The deconfinement transition in QCD is a non-perturbative phenomenon occurring at strong coupling. A promising modern theoretical tool for dealing with strongly coupled gauge theories represents the idea of gauge/gravity duality \[4,5\]. The application of this idea to low-energy phenomenology of QCD, started in Refs. \[6,7\], turned out to be unexpectedly useful. The constructed bottom-up holographic models link several successful approaches — QCD sum rules, vector meson dominance, and chiral perturbation theory — into one framework through the gauge/gravity duality.

The deconfinement transition is described in holographic models as a first order Hawking-Page type phase transition \[8\] between two different gravitational backgrounds — the thermal Anti-de Sitter (AdS) space and the Schwarzschild-AdS black hole \[9\]. This idea was used by Herzog in Ref. \[10\] to calculate $T_c$ within the framework of two simplest holographic QCD models — the Hard Wall (HW) \[6\] and Soft Wall (SW) \[7\] model. The phenomenological value of $T_c$ was reproduced with about 20% accuracy ($T_c = 122$ MeV for HW and $T_c = 191$ MeV for SW model \[10\]). The given analysis triggered a large activity in the field. Perhaps the main advantage of Herzog’s calculation is its remarkable simplicity. On the other hand, this calculation used rather crude approximations. First, the parameters of models were normalized to the $\rho$-meson mass although the original actions did not contain something related to real hadrons. A more consistent interpretation would be to consider this analysis as a calculation of deconfinement temperature in pure gluodynamics, let us denote it as $T^{gl}_c$, since the thermodynamics was assumed to be governed by the gravitational part of the action. Indeed, one can show that if parameters of HW and generalized SW models are normalized to the mass of lightest glueball from lattice simulations then the lattice value $T^{gl}_c \simeq 260 \pm 10$ MeV \[11,13\] is reproduced in both approaches \[14\].
addition, the deconfinement phase transition is of first order in the $SU(N)$
gluodynamics at $N \geq 3$ and becoming increasingly abrupt with growing
$N$ [15]. This means improving consistency: In the limit $N \to \infty$, the phase
transition becomes of the same type as the Hawking–Page first order transition.

The second weak point is that both HW and SW holographic models are
not solutions of Einstein equations and this drawback is inherited in many
other holographic calculations of $T_c$ followed by Ref. [10]. The reason lies in
a need to introduce a finite scale associated with the fifth direction in order
to describe the Hawking-Page transition for infinite boundary volume. The
corresponding scale (associated with the confinement scale) is inserted into
the HW model via a hard cut-off of holographic coordinate and into SW
model through a certain static dilaton background.

In this Letter, we propose a somewhat new scheme for holographic calcu-
lation of $T_c$. Our analysis retains a conceptual simplicity of Herzog’s calcu-
lation for the HW holographic model but is free from aforementioned crude
approximations and closer to phenomenological gluodynamics. The main
proposal consists in replacing the empty thermal AdS$_5$ space with a hard
cut-off imposed on the holographic coordinate by the simplest gravity-dilaton
system in AdS$_5$ whose analytical solution is known and where the cut-off
emerges dynamically. The dilaton in holographic description of QCD is usu-
ally associated with a field dual to the gluon condensate $\langle G^2 \rangle = \langle \text{Tr} G^2_{\mu \nu} \rangle$ — an important phenomenological quantity parametrizing the mass gap in
gluodynamics that appears due to a dynamical violation of scale invariance
in massless QCD and measures the QCD vacuum energy density. We will get
a simple relation between $T_c$ and $\langle G^2 \rangle$ and demonstrate its phenomenological
viability.

Physically the gluon condensate in our analysis will play the role of order
parameter for deconfinement. This is consistent with the lattice results [16]:
The renorminvariant vacuum average $-(\beta G^2)$, where $\beta = \beta(\alpha_s)$ denotes the
QCD $\beta$-function, is almost temperature-independent in the confined phase
and drops sharply near the critical temperature (strictly speaking, to negative
values signaling instability of original theory). In our 5D dual description,
this effect is simulated as the Hawking–Page first order phase transition from
a gravitational background where the thermal AdS$_5$ space is distorted by the
dilaton to the AdS$_5$ space distorted by a black hole.

In order to make our analysis self-contained, in Section 2 we remind the
reader the main steps of Herzog’s holographic calculation of deconfinement
temperature in the HW model. Our derivation of $T_c$ is given in Section 3.
Section 4 contains some discussions and phenomenological fits. We conclude
in Section 5.
2 Deconfinement temperature in the HW model

By assumption, the pure gravitational part of 5D action of holographic dual theory for a 4D $SU(N)$ gauge theory has the form (the Euclidean signature is used)

$$S = -\frac{1}{2\kappa^2} \int d^5 x \sqrt{g} \left( R + \frac{12}{L^2} \right).$$

(1)

Here $g = \det g_{MN}$, $\kappa$ is the coefficient proportional to the 5D Newton constant, $R$ is the Ricci scalar and $L$ represents the radius of AdS$_5$ space defined below. The gravitational coupling scales as $\kappa \sim 1/N$.

The deconfinement in holographic models of QCD occurs as the Hawking–Page phase transition between the following two gravitational backgrounds. The first is the thermal AdS$_5$ space with a line element

$$ds^2 = \frac{L^2}{z^2} \left( d\tau^2 + d\vec{x}^2 + dz^2 \right),$$

(2)

and the Euclidean time $\tau$ restrained to a finite interval $0 \leq \tau \leq \beta$. Here $z$ represents the holographic coordinate with physical meaning of inverse energy scale. The second background is the AdS$_5$ black hole$^2$ that describes the deconfined phase,

$$ds^2 = \frac{L^2}{z^2} \left( f(z) d\tau^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right),$$

(3)

where $f(z) = 1 - (z/z_h)^4$ and $z_h$ denotes the horizon of the black hole. The corresponding Hawking temperature is related to the horizon as $T = 1/(\pi z_h)$.

The both solutions lead to the curvature $R = -20/L^2$. Dividing out by the volume of $\vec{x}$ space one gets the free energy densities,

$$V_1(\epsilon) = \frac{4L^3}{\kappa^2} \int_0^\beta d\tau \int_\epsilon^{z_m} \frac{dz}{z^5},$$

(4)

$$V_2(\epsilon) = \frac{4L^3}{\kappa^2} \int_0^{z_h} d\tau \int_\epsilon^{\min(z_m,z_h)} \frac{dz}{z^5},$$

(5)

where $z_m$ represents the infrared cut-off of the HW model and an ultraviolet cut-off $z = \epsilon$ is introduced to regulate the arising infinity. The two geometries are compared at $z = \epsilon$ where the periodicity in the time direction is locally

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$^2$There are two different black holes in the AdS space — a ”small” one and a ”big” one [17]. A contribution of small black hole to thermodynamics can be neglected [8]. One can show that the small one disappears at all in the Poincaré patch (2) of global AdS$_5$ space and one automatically deals with the big black hole in (3).
the same, i.e. $\beta = \pi z_h \sqrt{f(\epsilon)}$. The order parameter for the phase transition is
\[
\Delta V = \lim_{\epsilon \to 0} \Delta V(\epsilon) = \lim_{\epsilon \to 0} (V_2(\epsilon) - V_1(\epsilon)). \tag{6}
\]
The thermal AdS space is stable when $\Delta V > 0$, otherwise the black hole is stable. The condition $\Delta V = 0$ defines the critical temperature $T_c$ at which the transition between the two phases happens. It is easy to see that $\Delta V > 0$ if $z_m < z_h$. But if $z_m > z_h$ one has
\[
\frac{\kappa^2}{L^3 \pi z_h} \Delta V(\epsilon) = \left( \frac{1}{\epsilon^4} - \frac{1}{z_h^2} \right) - \left( \frac{1}{\epsilon^4} - \frac{1}{z_m^2} - \frac{1}{2 z_h^2} \right). \tag{7}
\]
Thus the phase transition takes place at $z_m^4 = 2 z_h^4$ corresponding to a temperature \[10\]
\[
T_c = \frac{2^{1/4}}{\pi z_m}. \tag{8}
\]
The numerical values depend on a choice of infrared cut-off $z_m$ which is usually taken from the hadron phenomenology and is of the order of $1/\Lambda_{\text{QCD}}$.

3 Deconfinement temperature in a solvable gravity-dilaton system

The simplest extension of action (1) to a gravity-dilaton system consists in adding a dilaton kinetic term,
\[
S_d = -\frac{1}{2 \kappa^2} \int d^5 x \sqrt{g} \left( R + \frac{12}{L^2} - \frac{1}{2} \partial_M \phi \partial^M \phi \right). \tag{9}
\]
We assume that this action is dual to pure gluodynamics. The backreaction of dilaton to the AdS$_5$ metric will describe holographically the dynamical violation of scale invariance in gluodynamics leading to emergence of positive gluon condensate.

Let us analyze the Hawking–Page phase transition between the gravitational background following from (9) and AdS$_5$ black hole. First of all we need the solution of Einstein equations for metric and dilaton profile. The corresponding solution was found in Refs. [18,19], after continuation to Euclidean signature it takes the form
\[
ds^2 = \frac{L^2}{z^2} \left( \sqrt{1 - c^2 z^4} \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right), \tag{10}
\]
\[
\phi = \sqrt{\frac{3}{2}} \log \frac{1 + cz^4}{1 - cz^4} + \phi_0, \tag{11}
\]
where \( c \) and \( \phi_0 \) are some constants. The behavior near the boundary \( z \to 0, \phi = \sqrt{6}cz^4 + \phi_0 \), shows that according to AdS/CFT prescriptions \([20]\) the constant \( c \) is proportional to the gluon condensate \( \langle G^2 \rangle \). The proportionality factor can be found from matching the leading term in the OPE of the gluon operator \( G^2_{\mu\nu} \) to the corresponding holographic calculation, the result in the large-\( N \) limit is \([19]\)

\[
\langle G^2 \rangle = \frac{8\sqrt{3}N}{\pi} c. \tag{12}
\]

The singularity at

\[
z_0 = \frac{1}{c^{1/4}}, \tag{13}
\]

provides a natural infrared cut-off for the holographic coordinate which emerges dynamically.

Now we apply Herzog’s analysis outlined in the previous Section to the Hawking-Page phase transition between the AdS black hole geometry \([3]\) and the thermal AdS dilaton-gravity geometry \([10]\) with the dilaton profile \([11]\). The two geometries are compared at \( z = \epsilon \) where the periodicity in the time direction is locally the same, i.e. \( \beta = \pi z_h \sqrt{f(\epsilon)}/(1 - c^2\epsilon^8)^{1/4} \). In the limit \( \epsilon \to 0 \), the correction from \( \epsilon^8 \) will not contribute, hence, the comparison will be at \( \beta = \pi z_h \sqrt{f(\epsilon)} \) as before.

The calculation of Ricci scalar for the geometry \([10]\) yields

\[
R = -\frac{20}{L^2} + \frac{48c^2\epsilon^8}{L^2(1 - c^2\epsilon^8)^2}. \tag{14}
\]

It is convenient to divide the contributions to free energy \( V_1 \) into three parts,

\[
V_1 = V_1^{(c)} + \Delta V_R + \Delta V_\phi, \tag{15}
\]

where \( V_1^{(c)} \) stems from the first term in \([13]\), \( \Delta V_R \) goes from the second one in \([14]\), and \( \Delta V_\phi \) arises from the dilaton \([11]\). Using \( \sqrt{g} = L^5(1 - c^2\epsilon^8)/\epsilon^5 \) from the geometry \([10]\), we get

\[
\Delta V_R(\epsilon) = -\frac{24L^3c^2}{\kappa^2} \int_0^\beta d\tau \int_\epsilon^{z_0} \frac{z^3dz}{1 - c^2z^8}, \tag{16}
\]

On the other hand, the solution \([10]\), \([11]\) gives \( \partial_M\phi\partial^M\phi = g^{zz}(\partial_z\phi)^2 = 96c^2\epsilon^4/(L^2(1 - c^2\epsilon^8)^2) \), that leads to

\[
\Delta V_\phi(\epsilon) = \frac{24L^3c^2}{\kappa^2} \int_0^\beta d\tau \int_\epsilon^{z_0} \frac{z^3dz}{1 - c^2z^8}. \tag{17}
\]

3We recall that the 5D mass of a scalar field \( \phi \) dual to some 4D gauge theory operator \( \mathcal{O} \) having the canonical dimension \( \Delta \) is \( m_5^2L^2 = \Delta(\Delta - 4) \). The asymptotics of \( \phi \) at \( z \to 0 \) becomes \( \phi = c_1z^{4-\Delta} + c_2z^\Delta \), where up to renormalization constants \( c_1 \) corresponds to the source of \( \mathcal{O} \) and \( c_2 \) becomes proportional to v.e.v. \( \langle \mathcal{O} \rangle \) \([20]\).
Comparing (16) and (17) we see that the last two contributions in (15) cancel each other. This important cancellation removes otherwise emerging logarithmic divergence at $\epsilon = 0$.

The comparison of two geometries is thus reduced to comparison of free energies $V_2$ and $V_1 = V_1^{(c)}$ which is identical to the case of previous Section with the obvious replacement $z_m \to z_0 = 1/c^{1/4}$ in the relation (8),

$$T_c = \frac{(2c)^{1/4}}{\pi}.$$  \hspace{1cm} (18)

Inserting the renormalization factor (12) we obtain our final result,

$$T_c^4 = \frac{\sqrt{3} \langle \alpha_s \pi G^2 \rangle}{12 \pi^2 N \alpha_s},$$  \hspace{1cm} (19)

where instead of scale-dependent gluon condensate $\langle G^2 \rangle$ we expressed $T_c$ via an approximately renorminvariant quantity $\langle \alpha_s \pi G^2 \rangle$ (proportional to the one-loop approximation to $-\langle \beta G^2_{\mu\nu} \rangle$) which is usually extracted in QCD sum rules and lattice simulations. This entails the appearance of additional parameter $\alpha_s$ which is not known \textit{a priori} since the energy scale is not fixed.

4 Discussions and phenomenological fits

An important technical point in our derivation was the cancellation of the last two contributions in (15). One can show that if in our analysis the gravity-dilaton system is replaced by AdS$_5$ space with static dilaton background of standard SW holographic model [7], \textit{i.e.} by action proportional to $\int d^5 x \sqrt{g} e^{-cz} (R + 12/L^2)$, then the given cancellation does not happen. In the original Herzog’s calculation of $T_c$ in the SW model [10], this problem was avoided by ad hoc considering black hole in the same background $e^{-cz}$ although such a gravitational solution is not known. This is another one troublesome point of analysis [10].

A qualitative correctness of relation (19) can be motivated by a dimensional analysis: If the vacuum average $\langle \beta G^2 \rangle$ provides the only dimensional and renorminvariant scale in a theory then any dimensional and renorminvariant quantity in this theory can be expressed as an appropriate power of $\langle \beta G^2 \rangle$. Thus we should have $T_c^4 \sim \langle \beta G^2 \rangle$ by dimensionality. We believe that such a natural relation should appear in any ”natural” model describing the deconfinement transition in a non-abelian gauge theory. In this sense, the standard derivation of $T_c$ within the HW and SW holographic models (and in their numerous successors) does not look ”natural”. In the HW model,
the infrared cut-off is not directly related to the gluon condensate since non-perturbative corrections to the leading logarithm in two-point correlators decrease exponentially. The required power-like corrections appear in the SW model and this allows to relate the dimensional parameter of SW-like models to the gluon condensate. But because of aforementioned artificial trick needed to cancel the ultraviolet divergence in the difference of free energies, the resulting relation between $T_c$ and gluon condensate takes a form of transcendental integral equation [10] which can be solved only numerically.

In order to make a phenomenological prediction from the relation (19) we will use the following procedure. The critical temperature $T_c$ in the l.h.s. refers implicitly to pure gluodynamics, i.e. it is $T_c^{gl}$ in the notation of Section 1. Let us take the values of $T_c^{gl}$ and gluon condensate $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ in pure $SU(3)$ gluodynamics from lattice simulations and fix thereby $\alpha_s$ for $N = 3$. To get an estimate for $T_c$ in the real word with physical quarks we will substitute the value of $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ from QCD sum rules. This can be partly justified in the large-$N$ limit we are dealing with in holography: The quark effects are of the order of $O(N)$ while the gluon ones are of the order of $O(N^2)$, hence, the inclusion of quarks in the fundamental representation should give $O(1/N)$ corrections to (19) which are beyond the validity of holographic approach.

We note in passing that on general grounds one expects a decreasing of gluon condensate in presence of quarks in comparison with pure Yang-Mills theory [21]. A similar decreasing was observed in lattice calculation for $T_c$. As far as we know, earlier these two effects have not been related in a manifest way. The obtained relation (19) expresses and explains the given proportionality.

The most quoted relevant lattice results for $SU(3)$ Yang-Mills theory we found are: $T_c^{gl} = 264$ MeV [11] and $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.1$ GeV$^4$ [22]. With these inputs we obtain from (19) the value $\alpha_s = 0.1$. This value of QCD coupling roughly corresponds to a scale of Z-boson mass [23], where the perturbation theory in QCD becomes robust. This might be an interesting prediction on its own.

A widely accepted phenomenological estimation of the gluon condensate in QCD sum rules yields a smaller value $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012$ GeV$^4$ [21]. Substituting this estimation to the relation (19), we obtain $T_c = 156$ MeV in remarkable agreement with the lattice and experimental values mentioned in Section 1.
5 Conclusions

We proposed a new holographic calculation of deconfinement temperature at vanishing chemical potential. Our calculation is free from some internal inconsistencies inherent in many calculations of this sort starting from Herzog’s analysis [10]. In our scheme, the phase transition from the hadron phase to quark-gluon plasma is associated with a sharp disappearance of positive definite gluon condensate in accord with lattice simulations. Motivated by the AdS/CFT correspondence, as a holographic image of pure gluodynamics we considered an exactly solvable gravity-dilaton system in which gravity in AdS$_5$ space (by assumption, dual to a strongly coupled 4D conformal gauge theory) is backreacted by a free massless dilaton field which is dual via AdS/CFT prescriptions to source of gluon condensate. The solution of corresponding Einstein equations is known to lead to a singularity at some value of holographic coordinate that is associated with dynamical cut-off describing holographically the violation of scale invariance in QCD. From the analysis of Hawking-Page phase transition between this gravity-dilaton system and black hole in AdS$_5$ (which is dual to deconfined phase in gauge theory) we got a relation between the deconfinement temperature $T_c$ and gluon condensate. After fixing a free parameter (gauge coupling $\alpha_s$) from lattice data on $T_c$ and gluon condensate in $SU(3)$ pure Yang-Mills theory, the derived relation [19] yields $T_c = 156$ MeV for the accepted value of gluon condensate in QCD sum rules. The obtained estimation of deconfinement temperature agrees perfectly with the modern lattice data $156 \pm 9$ MeV [1] and freeze-out temperature $T_{cf} = 156.5 \pm 1.5$ MeV measured by ALICE Collaboration at LHC [3].

The proposed calculation of deconfinement temperature can be applied in more contrived dynamical gravity-dilaton holographic models (which include a dilaton potential and/or other scalar fields) but most likely at the cost of a loss of exact analytical relations. Also the effects of chemical potentials (normal, chiral and isospin) can be considered.

References

[1] S. Borsanyi et al. [Wuppertal-Budapest], JHEP 09, 073 (2010); S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, Phys. Lett. B 730, 99-104 (2014).

[2] A. Bazavov et al. [HotQCD], Phys. Rev. D 90, 094503 (2014).
A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature 561, no.7723, 321-330 (2018).

J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998); Int. J. Theor. Phys. 38, 1113 (1999).

E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998); S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B 428, 105 (1998).

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 95, 261602 (2005); L. Da Rold and A. Pomarol, Nucl. Phys. B 721, 79 (2005).

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D 74, 015005 (2006).

S. W. Hawking and D. N. Page, Commun. Math. Phys. 87, 577 (1983).

E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998).

C. P. Herzog, Phys. Rev. Lett. 98, 091601 (2007).

G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lutgemeier and B. Petersson, Nucl. Phys. B 469, 419 (1996).

Y. Iwasaki, K. Kanaya, T. Kaneko and T. Yoshie, Nucl. Phys. Proc. Suppl. 53, 429 (1997).

B. Lucini, A. Rago and E. Rinaldi, Phys. Lett. B 712, 279 (2012).

S. S. Afonin and A. D. Katanaeva, Eur. Phys. J. C 74, 3124 (2014); Phys. Rev. D 98, 114027 (2018); Theor. Math. Phys. 200, 1383 (2019).

B. Lucini and M. Panero, Prog. Part. Nucl. Phys. 75, 1 (2014).

G. Boyd and D. E. Miller, [arXiv:hep-ph/9608482 [hep-ph]].

B. Zwiebach, A First Course in String Theory, Cambridge University Press; 2nd edition.

S. S. Gubser, [arXiv:hep-th/9902155], A. Kehagias, K. Sfetsos, Phys. Lett. B 454, 270 (1999).

C. Csaki, M. Reece, JHEP 0705, 062 (2007).

I. R. Klebanov and E. Witten, Nucl. Phys. B 556, 89 (1999).
[21] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B 147, 385 (1979).

[22] M. Campostrini, A. Di Giacomo and Y. Gunduc, Phys. Lett. B 225, 393 (1989).

[23] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018).