Abstract—Resistive crossbars have attracted significant interest in the design of Deep Neural Network (DNN) accelerators due to their ability to natively execute massively parallel vector-matrix multiplications within dense memory arrays. However, crossbar-based computations face a major challenge due to a variety of device and circuit-level non-idealities, which manifest as errors in the vector-matrix multiplications and eventually degrade DNN accuracy. To address this challenge, there is a need for tools that can model the functional impact of non-idealities on DNN training and inference. Existing efforts towards this goal are either limited to inference, or are too slow to be used for large-scale DNN training. We propose TxSim, a fast and customizable modeling framework to functionally evaluate DNN training on crossbar-based hardware considering the impact of non-idealities. The key features of TxSim that differentiate it from prior efforts are: (i) It comprehensively models non-idealities during all training operations (forward propagation, backward propagation, and weight update) and (ii) it achieves computational efficiency by mapping crossbar evaluations to well-optimized BLAS routines and incorporates speedup techniques to further reduce simulation time with minimal impact on accuracy. TxSim achieves orders-of-magnitude improvement in simulation speed over prior works, and thereby makes it feasible to evaluate training of large-scale DNNs on crossbars. Our experiments using TxSim reveal that the accuracy degradation in DNN training due to non-idealities can be substantial (3%-10%) for large-scale DNNs, underscoring the need for further research in mitigation techniques. We also analyze the impact of various device and circuit-level parameters and the associated non-idealities to provide key insights that can guide the design of crossbar-based DNN training accelerators.

I. INTRODUCTION

Deep Neural Networks (DNNs) have greatly advanced the state-of-the-art in a wide variety of machine learning tasks [1], [2]. However, these benefits come at the cost of extremely high computation and storage requirements. GPUs and digital CMOS-based accelerators [3], [4] have enabled faster and more energy-efficient realization of DNNs. However, the continuing growth in network complexities and volumes of data processed have led to the quest for further improvements in hardware. For example, training state-of-the-art DNNs requires exa-ops of compute and can take days to weeks on a GPU [5], while Neural Architecture Search (NAS) [6] further increases computation requirements to zetta-ops.

Resistive Crossbars have emerged as promising building blocks for future DNN accelerators. They are designed using emerging non-volatile memory technologies such as PCM [7] and ReRAM [8] that can enable high-density memory arrays, while also realizing massive parallel vector-matrix multiplications (the dominant compute kernel of DNNs) within these arrays. Thus, crossbar-based architectures promise to overcome the data transfer and memory capacity bottlenecks that are present in current DNN hardware platforms. Consequently, many efforts have explored the design of crossbar-based accelerators [9], [10]. We specifically focus on crossbar-based architectures for training DNNs [11]–[13], which have attracted increasing interest in recent years.

Crossbar-based systems face a major challenge due to numerous device and circuit-level non-idealities, viz., driver and sensing resistances, analog-to-digital converter (ADC) and digital-to-analog converter (DAC) non-linearity, interconnect resistances, process variations, noise in synaptic devices, imperfect write and update operations, and sneak paths [12], [14]–[16]. Unless addressed, these non-idealities can significantly degrade DNN accuracy, threatening the viability of crossbar-based hardware [10]. To quantitatively evaluate and address this challenge, there is a need for tools that can model the impact of all non-idealities on each step of DNN training (forward propagation, backward propagation, and weight update). DNN training on native hardware (e.g., GPUs) is already very slow, and software simulation of DNN training on crossbar-based systems (emulated hardware) will further slow down the training process considerably. Therefore, it is extremely important that the modeling tool maintains high simulation speed (same order-of-magnitude as DNN training on native hardware). The tool should also be customizable and support a wide variety of device and circuit parameters and DNN topologies.

In this work, we propose TxSim, a tool to functionally evaluate DNN training on crossbar-based systems, which meets the aforementioned requirements. TxSim utilizes a three-stage vector-matrix multiplication model to capture the impact of non-idealities during forward and backward propagation operations with good fidelity and simulation speed. The first stage consists of a non-linear conversion of digital inputs to voltages considering DAC non-idealities. The second stage models the non-idealities within the core crossbar array (interconnect parasitics, sneak paths and process variations) as a series of linear-algebraic transformations wherein ideal conductance matrices are converted to non-ideal conductance matrices. The final stage consists of the non-linear transformation of the currents back to digital outputs considering ADC non-idealities. Such an approach to modeling allows us to seamlessly utilize highly-optimized BLAS routines present in standard ML frameworks (e.g., PyTorch, Tensorflow and
Caffe). TxSim also models the weight update non-idealities (stochastic noise and update non-linearity) using optimized BLAS routines. TxSim also utilizes speedup techniques that further reduce simulation time without impacting modeling fidelity.

Prior efforts on functional modeling of crossbar-based DNN hardware can be broadly classified into efforts that model inference [14]–[16] and efforts that model training [12], [14]. Inference models are not sufficient for evaluating DNN training, since training includes additional backward propagation and weight update operations. As elaborated in Section II, TxSim’s modeling approach and speedup techniques make it 108x-2000x faster than prior efforts to model DNN training on crossbars [12], [14]. It achieves this while also being more comprehensive in the non-idealities modeled (e.g., sneak paths and wiring parasitics), and being customizable to different DNN topologies and circuit and device parameters.

II. RELATED WORK

In this section, we discuss prior efforts to modeling inference and training on crossbar-based systems, as well as training algorithms/methodologies for such systems.

Inference modeling. PytorX [15] and RxNN [16] are modeling tools that consider the impact of non-idealities in crossbar-based inference. Other works [17], [18] propose methods to compensate for accuracy degradation. However, these tools are not directly applicable to crossbar-based training, which involves modeling the non-idealities in the backward propagation and weight update phases as well.

Training accelerators. Various architectures [11]–[13] have been proposed that perform DNN training on crossbar-based hardware. MNSIM [19] is a tool for early design space exploration of such architectures. The major focus of these works have been on area, speed and energy while ignoring, or assuming very primitive error models for, accuracy evaluation.

Modeling crossbar-based training. Two noteworthy efforts that model crossbar-based training are CrossSim [12] and NeuroSim [13]. Table I compares our work with these efforts along two important dimensions – the fidelity in modeling non-idealities and the simulation time. CrossSim considers only the errors due to device updates and peripheral circuits, and reports results only on a simple 3-layer network and small data set (MNIST). NeuroSim supports training only with MLPs (fully connected networks). In contrast, TxSim considers all circuit and device level non-idealities and is capable of evaluating more complex networks. When considering simulation time, CrossSim requires about a week to train a simple 3-layer network on MNIST. TxSim is 108x faster than CrossSim for the same task. NeuroSim takes around 5 minutes to perform inference for a single image. Projected to the full CIFAR 10 validation dataset, this would translate to 34 days for inference alone (and training would take much longer). In contrast, our framework is able to train Alexnet on CIFAR100 with only 0.0136 seconds per image, which is around 2000x faster.

Training algorithms. Given the errors intrinsic in crossbar-based computing, it is important to come up with the right kind of algorithms for training to converge to a good accuracy. To this end, previous efforts [20], [21] propose enhanced algorithms that help overcome non-idealities such as device non-linearity, asymmetry and stochastic noise. Our work complements these efforts by providing a generic modeling tool that provides an accurate estimate of the degradation due to non-idealities. We expect such tools to further enable future development of crossbar-based training architectures and algorithms.

III. PRELIMINARIES

In this section, we present a brief overview of the non-idealities in the crossbar during all phases of training (forward propagation, backward propagation and weight update).

Peripheral circuitry. The analog computation in the crossbar array requires digital to analog (DAC) and analog to digital (ADC) converters. The DACs and the ADCs are non-linear and limited in precision to keep their area and power overheads low.

Circuit non-idealities. The wire resistances, source resistances, sink resistances and sneak paths in the crossbar array impact the column currents, causing errors in the vector-matrix multiplication.

Device non-idealities. The synaptic elements within the crossbar array are inherently stochastic. Existing device technologies can only support precisions up to 6 bits. These devices also exhibit a non-linear and asymmetric behaviour, suffer from process variations, drift and limited endurance, which can affect the overall classification accuracy.

IV. TXSIM MODELING FRAMEWORK

TxSim is a highly customizable and scalable modeling tool that evaluates the application-level accuracy of DNNs trained on crossbar-based hardware. Figure I outlines the TxSim modeling process. TxSim takes three main inputs: (i) the network architecture that defines the number of layers, and the numbers and sizes of input/output channels and kernels, (ii) the hardware architecture parameters such as weight and activation precisions, mapping strategy, level of non-ideality modeling, etc., and (iii) the crossbar parameters, including DAC/ADC models, crossbar dimensions, synaptic device characteristics.
etc. The rest of the section describes TxSim by going over each component in detail.

A. Non-ideal conductance generator

The non-ideal conductance generator analyzes the non-idealities associated with the core crossbar array, viz., the wire resistances, sink and source resistances, sneak paths, and process variations. It takes an ideal conductance matrix as an input and converts it into a non-ideal conductance matrix that incorporates these core array non-idealities. First, the ideal conductance matrix \((G_{\text{ideal}} - \text{updated})\) is mapped to one or more synaptic devices based on the on-off ratio and the precision of each device. Next, the ideal conductance matrix is partitioned into crossbar instances based on the specified crossbar dimensions. Within each crossbar instance, the positive and negative conductances may be further mapped onto separate crossbars, obtaining two different currents that are subtracted. Lastly, process variations are applied to each synaptic element based on the specified variation profile. The ideal conductance matrices are converted to non-ideal conductance matrices by applying FCM, which was originally proposed for modeling inference [16]. Although this process is very accurate, it causes slowdown during the training simulation because it needs to be used after every minibatch iteration due to weight updates. Therefore, we also propose speedup techniques that approximate FCM while maintaining good modeling fidelity (discussed in section IV-D). Finally, the crossbar instances are stitched back together to obtain the non-ideal conductance matrix. Note that a copy of the ideal conductance matrix is always preserved and used to obtain the \((G_{\text{ideal}} - \text{updated})\) for the next minibatch iterations.

B. Three-stage vector-matrix multiplication model

Once we obtain the non-ideal conductance matrix, we utilize a three-stage model (shown in Figure 1) to perform the forward and backward passes. The incoming digital inputs of each crossbar are converted to voltages depending on the user’s choice of DAC. The voltages and the non-ideal conductance matrix are fed to the underlying BLAS functions to obtain column currents. Subsequently, the column currents are fed to the ADC model and propagated to the next layer. The maximum current through ADCs is data dependent and obtained by collecting output distribution statistics over multiple training epochs. Peripheral operations such as ReLU, sigmoid, batchnorm, and pooling are computed in the digital domain and are hence unimpaired by crossbar non-idealities.

C. Update model

For efficient weight updates, various parallel update schemes have been proposed [12], [22], wherein inputs and errors are converted to voltages and fed to the rows and columns of the crossbar simultaneously. The change in synaptic conductance is proportional to the product of the voltages, which translates to the weight update operation. The voltages are either converted to time and magnitude based pulses [12] or modeled as stochastic bit streams [22] whose coincidence
yields a multiplicative effect. To convert digital gradients to $\Delta G_{\text{ideal}}$, for every layer, a pre-determined scaling factor ($\text{Scale}_{\text{Layer}-k}$) is used (shown in Figure 2). $\text{Scale}_{\text{Layer}-k}$ is determined using weight ($W_{\text{max}}$) and gradient ($\Delta W_{\text{max}}$) statistics from native DNN training. The major non-idealities during update operations are stochastic noise of synaptic devices and the asymmetric write non-linearity [12], which are both modeled in TxSim.

The update model, shown in equation (1), depends on the sign of the update. It depends on the current conductance state ($G$), the minimum conductance ($G_{\text{min}}$), the maximum conductance ($G_{\text{max}}$), the ideal conductance conductance ($\Delta G_{\text{ideal}}$) and the update non-linearity factor ($\gamma$). Due to the non-linear nature of the device, the updated conductances deviate from the original values based on $\gamma$. Another source of non-ideality is the write noise which arises due to the stochastic nature of the device [12]. $G_{\text{non-ideal}}$ is sampled from a Gaussian distribution whose standard deviation is $\gamma \sqrt{(G_{\text{max}} - G_{\text{min}}) \cdot \gamma_{\text{ideal}}}$, where $\gamma$ is the write noise factor. The write noise is directly proportional to the size of the gradient and higher write noise factor translates to more write noise being applied to $G_{\text{ideal}}$. After obtaining $\Delta G_{\text{non-ideal}}$ (see Figure 2), it is passed the optimizer (such as SGD or Adam). $G_{\text{ideal-previous}}$, stored for every layer is now updated to obtain a new conductance matrix $G_{\text{ideal-updated}}$, and subsequently passed to the non-ideal conductance generator to obtain the next set of non-ideal conductances. This process is repeated for each DNN layer over multiple epochs until training converges.

\[
\begin{align*}
\text{Case 1: } & \Delta G_{\text{ideal}} > 0 \\
& \Delta G_{\text{non-ideal}} = \frac{G_{\text{max}} - G_{\text{min}}}{1 - e^{-\gamma}} + G_{\text{min}} - G) (1 - e^{-\gamma})
\end{align*}
\]

\[
\begin{align*}
\text{Case 2: } & \Delta G_{\text{ideal}} < 0 \\
& \Delta G_{\text{non-ideal}} = \frac{G_{\text{max}} - G_{\text{min}}}{1 - e^{-\gamma}} - G_{\text{max}} - G) (1 - e^{-\gamma})
\end{align*}
\]

\[D. \text{ Speedup Techniques}\]

As mentioned earlier, the generation of the non-ideal conductance matrices is very slow and, while acceptable for inference (where it is one-time), does scale to DNN training (where it needs to be invoked after each minibatch, when weights change). Therefore, we present two complementary speedup techniques that significantly accelerate training simulation while preserving good modeling fidelity.

**Approximate analytical model (AAM).** The current at the output column in a non-ideal crossbar can be viewed as a sum of many terms, each corresponding to a path through the crossbar. In the AAM model, we consider a subset of these paths (typically the shorter paths from each row to each column), while ignoring the longer paths (as shown in Figure 3(a)). The current for each path is computed considering the source ($r_{\text{source}}$), sink ($r_{\text{sense}}$), and wire resistances ($r_{\text{row}}$ and $r_{\text{col}}$). The AAM model allows us to seamlessly trade-off efficiency for accuracy by simply considering more or fewer paths.

We plot the modeling error of AAM with respect to FCM for 64x64 crossbars with different $R_{\text{min}}-R_{\text{max}}$ ranges in

**Interpolated-FCM.** In this speedup technique, we perform FCM selectively – only once every L minibatch iterations (as opposed to each iteration). Every time FCM is performed, the net synaptic conductance distortion due to non-idealities ($\langle G_{\text{ideal}} - G_{\text{non-ideal}} \rangle$) is profiled and stored. For the subsequent L-1 iterations, the $G_{\text{non-ideal}}$ is computed using the stored distortion profile. Figure 3(b) shows the application-level accuracy for various models –FCM, AAM, and Interpolated-FCM for the LetNet-5 DNN on MNIST dataset. As shown, the speedup techniques can effectively model DNN training without much loss in modeling fidelity (note the highly magnified y-axis range).

**V. EXPERIMENTAL METHODOLOGY**

In this section, we briefly describe the methodology used to evaluate TxSim. The synaptic device used was a Ag/Si ReRAM technology [23] with $R_{\text{min}}=100k\Omega$, $R_{\text{max}}=1M\Omega$, and read voltage of 0.5V. The DAC and ADC models are calibrated with SPICE based on designs obtained from [24] and [25]. The row and column resistances are derived from circuit layout and found to be 1$\Omega$ and 4.6$\Omega$, respectively. We
conservatively assume 32-bit precision for all data structures based on the scheme proposed in [26], since it provides classification accuracy close to floating-point training. We use 64x64 crossbar arrays with 2-bit synaptic devices. We assume DAC and ADC precision to be 16 bits for our simulations.

VI. RESULTS
In this section, we present results from applying TxSim to evaluate the impact of non-idealities on the accuracy of DNNs trained using crossbar-based systems. We also analyze DNN training sensitivity to various device and circuit-level parameters to provide insights for future research.

A. Simulation speed
To quantify the advantage of TxSim in simulation speed, we first compare it to prior frameworks that model training, viz., NeuroSim [14] and CrossSim [12]. We achieve 108x and 2000x simulation speedup compared to CrossSim [12] and NeuroSim [14], respectively. Next, we compare the simulation speed ofTxSim to native fixed-point training on a NVIDIA GeForce GTX 1080 Ti GPU. For these experiments, we use a batch size of 128 and a crossbar size of 64x64. From Figure 5 we can observe that software simulation of DNN training on crossbar-based system is only 14x slower compared to Fxp training on GPU. This is reasonable considering the fact that TxSim emulates DNN training on crossbar-based system with high modeling fidelity by considering all crossbar non-idealities during the forward, backward, and update operations.

B. Application-level accuracy
To evaluate the impact of crossbar non-idealities on DNN training, we trained image classification networks on three different datasets – LeNet-5 on MNIST, AlexNet on CIFAR-10, and AlexNet on CIFAR-100, on an ideal crossbar system (Cross-Ideal) without any non-idealities and a non-ideal crossbar system (Cross-NI) with all crossbar non-idealities. The test accuracy vs. training epochs are reported in Figure 6. We used a 64x64 crossbar with \( R_{\text{min}}=100\,\Omega \), \( R_{\text{max}}=1\,\mathrm{M\Omega} \), non-linearity factor \( (v) = 0.01 \), and stochastic noise factor \( (\gamma) = 5 \). The accuracy degradation due to crossbar non-idealities (Cross-NI) is observed to be 4.5%-8.2% across the benchmarks. Moreover, the impact of non-idealities is found to be more prominent on the more complex AlexNet DNN than the simple LeNet-5 DNN. Clearly, there is a need to bridge the accuracy gap due to crossbar non-idealities to enable adoption of crossbar-based system for training DNNs. To guide potential solutions to this challenge, we next perform sensitivity analysis to provide insights into the impact of device and circuit-level parameters on accuracy degradation.

C. Sensitivity Analysis
Sensitivity to update non-idealities. Figure 7 shows the effect of update non-idealities, viz., write non-linearity and stochastic noise on the application-level accuracy. The crossbar dimensions, on-off ratio and other hardware parameters are kept constant. As non-linearity factor \( (v) \) increases from 0.01 to 0.1, there is almost no drop in accuracy. However, when \( v \) is increased to 0.5 and subsequently to 1, the effect of write non-linearity is very prominent, resulting in large accuracy degradation. Next, the stochastic noise factor \( (\gamma) \), which determines the standard deviation of the Gaussian distribution from which the write noise is sampled, is varied between 1 to 10. For \( \gamma=1 \) and \( \gamma=5 \), the drop in accuracy is almost negligible. However, \( \gamma=10 \) leads to significant (10%) drop in accuracy. From these experiments, we conclude that the non-linearity factor should be maintained between 0.1 to

![Fig. 5: Slowdown w.r.t software training per epoch](image1)

![Fig. 6: Impact of crossbar non-idealities on application-level accuracy](image2)

![Fig. 7: Sensitivity to update non-idealities](image3)
Sensitivity to crossbar dimensions. Figure 8 shows the application-level accuracy with increase in the crossbar dimensions. The accuracy drops slightly for larger crossbars due to increasing impact of all non-idealities including DACs, ADCs, wire resistances and sneak paths. In our experiments, the on-off ratio ($R_{\text{max}}/R_{\text{min}}$) of the synaptic device is orders-of-magnitude higher than the wire resistances, highlighting an important observation that the non-idealities due to wire parasitics is less prominent for devices such as ReRAM and PCM. However, the effect will be very prominent when the resistance range of the synaptic device is closer to the wire resistances, e.g. Spintronic devices.

Sensitivity to on-off ratio. To determine the effect of on-off ratio ($R_{\text{max}}/R_{\text{min}}$) on the application-level accuracy, we fix the inputs to the crossbar and change $R_{\text{min}}$ and $R_{\text{max}}$. We performed two sets of experiments – (i) decrease $R_{\text{min}}$ and fix $R_{\text{max}}$ and (ii) increase $R_{\text{max}}$ and fix $R_{\text{min}}$. From both the experiments, as indicated in Figure 9, we observe that decreasing the on-off ratio can degrade accuracy significantly. However, increasing the $R_{\text{max}}$ values to attain high on-off ratio can make the sensing current low and hence can lead to a decrease in the accuracy due to sensing errors. On the other hand, decreasing $R_{\text{min}}$ can have an even greater effect because of circuit non-idealities. Circuit non-idealities have a greater impact when the synaptic device resistance range is close to wire parasitics. For this particular configuration of the device, when $R_{\text{max}}$ is $1\Omega$, $R_{\text{min}}$ can be decreased to maintain a ratio of 8 for the best classification accuracy. Similarly, for the second experiment, the best accuracy is obtained when the on-off ratio is 5.

![Fig. 8: Sensitivity to crossbar dimensions](image)

Fig. 8: Sensitivity to crossbar dimensions

VII. CONCLUSION

Crossbar-based systems are extremely promising for efficiently executing DNN training. In this work, we propose TxSim, i.e., a scalable and customizable modeling tool that evaluates DNN training on resistive crossbars considering the impact of all computational non-idealities. TxSim models a more comprehensive set of non-idealities than prior works. It seamlessly utilizes well optimized CuBLAS routines to model non-idealities during all DNN training operations, and achieves 108x-2000x speedup over prior frameworks. To further improve the simulation runtime for complex datasets and network architectures, we also propose speedup techniques, viz., approximate analytical model (AAM) and interpolated FCM, that show a good balance between modeling fidelity and simulation runtime. Using TxSim, we evaluate several DNN benchmarks and observe that the accuracy degradation can be considerable (3%-10%). We also perform sensitivity analysis to gain further insights into the impact of various circuit and device level parameters on DNN training.

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