**Abstract**

High strain rate events, such as high velocity impacts and high explosive detonation, require the use of first principles physics based codes to accurately predict the formation of debris during material fracture and fragmentation. However, material fracture behavior is a very complex phenomenon to model due to its stochastic nature and the variability in material properties under dynamic loading conditions (versus static). In addition, physics based codes present challenges in predicting debris pieces that are smaller than the mesh resolution for a large problem domain with a finite number of elements. Yet, these characteristics are important in applications for missile intercepts, satellite collisions and warhead fragmentation, where the debris generated can range in size from large (on the order of meters) to very small (micron-sized). Previous fragmentation models have been implemented in physics based codes, such as the Grady-Kipp energy-based fragmentation model in the CTH Eulerian shock physics package, that produce debris smaller than the numerical grid size. However, recent theories developed by Grady combine the energy-based model with Mott’s statistical based theory to more accurately describe the distribution of debris created under fracture. Using Grady’s theories, we have developed a methodology that predicts sub-grid debris information from the Lagrangian-based hydro-structural code, Velodyne. This methodology utilizes the strain rate data from the Velodyne simulation to determine further breakup of the finite elements. Although there is limited test data available characterizing debris below 1 gram, we show some comparisons of the debris predictions with explosive filled cylinder experiments. © 2012 Published by Elsevier Ltd. Selection and/or peer-review under responsibility of the Hypervelocity Impact Society.

**Keywords**: first principles physics; hydro-structural codes; high strain rate; material fracture; sub-grid numerical techniques.

| Nomenclature |
|--------------|
| c | sound speed (m/sec) |
| m | Johnson Cook strength material constant |
| n | Johnson Cook strength material constant |
| x₀ | average particle size (m) |
| A | Johnson Cook strength material constant (dynes/cm²) |
| B | Johnson Cook strength material constant (dynes/cm²) |
| C | Johnson Cook strength material constant |
| D | accumulative damage |
| D₁ | Johnson Cook damage material constant |
| D₂ | Johnson Cook damage material constant |
| D₃ | Johnson Cook damage material constant |
| D₄ | Johnson Cook damage material constant |
| D₅ | Johnson Cook damage material constant |
| Kᵥ | fracture toughness (Pa·m¹/²) |
| T | temperature (K) |

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1. Introduction

Dynamic fracture and fragmentation are complex phenomena that have been studied at length because of their importance in DoD applications. Extensive fracturing of a material leads to the creation of fragments, or debris, that are typical of high velocity impacts (> 1 km/sec) or other high strain rate events (> 10^3 sec^-1). An example of a high strain rate event is a fragmenting warhead, where the casing material begins to fracture (i.e. crack) due to the expansion of the high explosive, and then fragment (i.e. form debris) as the casing continues to break apart. Examples of high velocity impact scenarios include missile intercepts, satellite collisions and bullet penetration, where both the target and projectile undergo severe material deformation and fracture. Over the years, the ability to model the fracture in these types of events has matured significantly, and as the fracture models become more accurate, the ability to characterize the resulting debris distributions also becomes more accurate.

Analytical models of fracture and fragmentation can be traced back to 1946, where Mott [1] presented a statistics-based theory to explain the fragmentation of an expanding ring for a cylindrically shaped bomb. Later in the 1980’s Grady et al [2] and Kipp and Grady [3] pursued an energy-based theory, determining fragment distributions based on the strain rate of the material. Recently both of these theories have been combined by Grady [4] [5] to capture the statistical characteristics of Mott and the fracture energy requirements of Grady and Kipp.

In this paper, we will explore the use of Grady’s revised theory in the application to the scenario of a naturally fragmenting warhead filled with high explosive. This work builds upon efforts produced by Crowe et al [6] and Mock and Holt [7] to characterize the fragment distributions of an explosive filled Armco iron cylinder. Here, we reproduce the distribution for the larger debris pieces using the first principles physics based code, Velodyne, and expand the debris distribution below the mesh resolution size (referred to as sub-grid) using Grady’s fragmentation theories. In this report we provide an overview of the codes and theories, and we describe the simulation setup and solution. Finally, we compare our predictions with the experimental data and discuss the results.

2. Computational Approach

2.1. Hydro-structural codes overview

Hydro-structural codes are first principles physics based tools typically used to solve large-scale numerical analyses of high strain rate and high velocity impact events by capturing both the hydrodynamic and structural response [8] [9]. All hydro-structural codes rest on principles from the branch of physics known as continuum mechanics. Their algorithms utilize some form of the partial differential equations that govern conservation of mass, momentum, and energy. These conservation equations are solved in conjunction with material characterizations that include material equations of state (pressure-volume response) and constitutive relationships (strength and fracture models). The solution to these differential equations constitutes the concept of “First Principles Physics”.
In comparison to other Modeling and Simulation (M&S) tools, such as analytical or empirical engineering codes, hydro-structural codes are higher fidelity but also require ample computational resources and extensive runtimes. For the simulation the problem domain is first discretized or broken up into a finite number of elements where the conservation equations are solved. The solution is then calculated on high performance computers consisting of hundreds of CPUs working together to provide output that is predictive in nature. The results include a simulation of the event, the state of the materials during deformation, and a variety of other information that is useful for understanding what happened during the event.

Since 2007, Corvid Technologies has been developing the multi-physics/multi-numerics based hydro-structural code Velodyne to solve a variety of challenging problems using physics-based computational methodologies. The tool emerged to address some of the limitations when applying commercial codes to large geometry domains and high strain rate events, such as missile intercepts and underbody blasts to vehicles. The advances in Velodyne development include optimization and parallelization, allowing solutions for large models comprised of millions of elements in much shorter runtimes. In addition, we have incorporated physics and numerics well suited to high strain events like hypervelocity impacts or high explosive fragmentation. This includes the ability to convert distorted Lagrangian elements to the Smoothed Particle Hydrodynamics (SPH) formulation [10], new material models for advanced fracture mechanics and phase changes, and more accurate and efficient auto-contact methods. A coupled Lagrangian/Eulerian (CLE) methodology is also available to model working fluids and their interaction with surrounding structures [11], which is useful for capturing the blast effects from explosives; however, this paper is focused on the fragmentation and debris aspects of warheads.

The various numerical treatments of hydro-structural codes are described in Fig. 1. In the Eulerian finite-volume formulation, the problem geometry is embedded within a grid, and as the problem evolves, the materials flow from cell to cell through the grid. This makes the Eulerian formulation more robust because it can accommodate large material distortions, but it also causes the numerics to break down late in time (on the order of milliseconds) as material mixture within Eulerian cells can make the calculation intractable. In the Lagrangian finite-element formulation, the mesh remains attached to the material and deforms with it as time proceeds. This method is more economical and efficient, as compared to the Eulerian approach, because the mesh is located solely on the material where it is needed. In addition, the Lagrangian formulation allows for precise material interfaces (or boundaries) that are necessary for correct body-to-body contact and for tracking individual debris pieces. However, large material distortions tend to cause mesh entanglement issues, which can prematurely terminate the calculation unless other numerical techniques are applied.

![Fig. 1. Schematic comparing the time evolution (top) and treatment of material interfaces (bottom) for the various mesh formulations.](image)

The third numerical technique, SPH, represents an alternative Lagrangian formulation. Unlike traditional Lagrangian elements, the SPH formulation requires no grid and uses particles to represent the material, which removes any issues concerning mesh entanglement. Although the SPH formulation is meshless, the particles can interact with one another and transmit loads to Lagrangian elements or other particles. This particle-to-particle interaction can often be computationally intensive and respond poorly to intense tensile stresses combined with distension. However, in general, SPH users find the method efficient (compared with Eulerian codes) and robust (compared with standard Lagrangian).

A key feature for conducting complex numerical simulations of high strain rate events is the capability of modeling severe material deformation through the use of SPH in conjunction with the Lagrangian solid element formulation.
Whereas most Lagrangian solvers would fail due to large element distortions, Velodyne converts highly distorted elements or failed material to SPH particles which can fully interact with each other and the parent mesh via robust contact algorithms. The user-specified conversion criterion is based on one or more of the following: geometric features (minimum volume, maximum volume, relative skew, and relative aspect ratio), numerical derivations (minimum time step which is a function of element dimensions and material properties), or constitutive laws (failure strain, damage, and failure pressure). Through this conversion process, Velodyne is able to conserve mass, momentum, and energy in the computational domain, where traditionally the distorted elements would be deleted. This results in the ability to capture the hydrodynamic regime of the problem and provides a more accurate representation of the high velocity impact event.

Approaches with the Lagrangian formulation in the hydro-structural code Velodyne provide a suitable methodology for predicting fragmentation and late time debris formation in high velocity impact and high strain rate events. The finite element numerical technique allows debris surfaces to be accurately measured and tracked through the deforming mesh. In addition, mass erosion is avoided by converting to the SPH formulation. Furthermore, the SPH formulation provides a framework for predicting debris sizes below the mesh resolution, a technique that is described in this paper for the case of a naturally fragmenting warhead.

2.2. Modeling natural fragmentation

Modeling natural fragmentation of a material requires advanced numerical techniques and appropriate material characterization in order to accurately capture the behavior. The first requirement is a material model that can capture the dynamic behavior of a material as a function of the strain rate. In most cases, material properties (e.g. stress-strain curve, fracture strength) behave differently at lower strain rates in quasi-static events (10^0 sec^-1) versus high strain rate events (10^3 sec^-1). A variety of material models are available to capture these effects, including Johnson Cook [12], Steinberg Guinan [13], and Zerilli-Armstrong [14]. In this paper we focus on the use of the Johnson Cook material model, which includes both a strength and fracture model, because of its wide availability of material properties.

The Johnson Cook strength model characterizes the stress-strain relationship of a material at low and high strain rates, with the inclusion of work hardening at large strains and material softening at high temperatures. In the model, the von Mises flow stress, \( \sigma \), is expressed as a function of the equivalent plastic strain, \( \varepsilon \), the dimensionless plastic strain rate, \( \dot{\varepsilon}^* \), and the homologous temperature, \( T^* \):

\[
\sigma = [A + B\varepsilon^n][1 + C \ln \dot{\varepsilon}^*][1 - T'^m]
\]  

where \( A, B, n, C, \) and \( m \) are material constants derived from empirical data. Each of the individual brackets in Equation (1) illustrates how the flow stress is dependent on the strain, strain rate, and temperature. The dimensionless plastic strain rate and the homologous temperature are defined, respectively, as:

\[
\dot{\varepsilon}^* = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}
\]

\[
T^* = \frac{T - T_{room}}{T_{melt} - T_{room}}
\]

where \( \dot{\varepsilon}_0 = 1.0 \) sec^{-1}.

In addition to the strength model, Johnson and Cook have developed a damage model to capture the fracture of a material based on the strain, strain rate, temperature and pressure in a simplified form. Within this model, the strain at fracture, \( \varepsilon_f \), is expressed as:

\[
\varepsilon_f = [D_1 + D_2 \exp(D_3 \sigma^*)][1 + D_4 \ln \dot{\varepsilon}^*][1 - D_5 T^*]
\]

where \( D_1 \) through \( D_5 \) are material constants derived from empirical data, and \( \sigma^* \) is the dimensionless pressure-stress ratio, or hydrostatic tension, equal to the pressure divided by the von Mises equivalent stress. Actual fracture occurs when the accumulative damage, \( D \), equals 1. The damage is defined as:

\[
D = \sum \Delta \varepsilon / \varepsilon_f
\]
where $\Delta \varepsilon$ equals the change in equivalent plastic strain during one integration cycle. Although a relatively simplistic fracture model, the Johnson Cook damage model in Equation (4) expresses the fracture strain as a function of the hydrostatic tension, strain rate, and temperature, similar to the strength model in Equation (1). Of these variables, hydrostatic tension is the most significant contributor to fracture; as the hydrostatic tension increases, the strain to fracture decreases.

In addition to parameterizing the fracture, another important component of material modeling is capturing the inhomogeneity of material properties, especially strain at fracture. Experimental evidence has shown that material failure occurs for a range of strains that tend to follow a weibull probability distribution function (pdf) for an explosive fracture event [15]. The three term weibull pdf is described as:

$$f(\varepsilon) = \left( \frac{\beta}{\eta} \right) \left( \frac{\varepsilon - \varepsilon_0}{\eta} \right)^{\beta-1} \exp \left[ - \left( \frac{\varepsilon - \varepsilon_0}{\eta} \right)^{\beta} \right], \quad \varepsilon > \varepsilon_0$$

where $\beta$ and $\eta$ are the shape and scale parameters, and $\varepsilon_0$ is the cutoff failure strain, which gives a lower bound of the failure strain. This method is adapted to the Johnson Cook damage model in Velodyne to allow a weibull pdf for the strain at fracture in Equation (4). The effects of not using a weibull distribution on the fracture properties for an explosively driven cylinder are shown in Fig. 2. Without some variability in the material properties, the fragmentation becomes mesh dependent, and in this case, follows the mapped structure of the mesh.

![Fig. 2. Comparison of simulation without (left) and with (right) weibull pdf.](image)

As described here, adequate material characterization plays a key role in providing an accurate analysis of high strain rate events, such as a naturally fragmenting warhead filled with high explosive. This includes the ability to model the change in material properties at various strain rates, including strength and fracture, as well as the inhomogeneity of materials due to impurities and defects in the crystalline structure. These factors were considered for the present fragmenting warhead analysis in order to accurately determine the debris distributions during the hydro-structural code simulation.

2.3. Sub-grid techniques

An artifact of the SPH methodology employed within Velodyne is an excess of nearly identical particles generated from the relatively uniform Lagrangian mesh. In calculations where a significant fraction of elements are converted to SPH particles, one tends to observe a “pile up” of particles of the same mass and size due to the one-to-one correspondence between Lagrangian elements and SPH particles for a given material. For applications where the mesh resolution is much smaller than the impact debris sizes of interest, the particle excess does not pose a problem. Applications that require debris model accuracy at or near the mesh resolution limit (typically a few mm) have motivated the development of a post-processing fragmentation methodology to further sub-divide particles beyond their nominal mass and size as defined by the original finite element model mesh resolution.

The SPH particle sub-division methodology developed is based primarily on the theory of dynamic fragmentation summarized by Grady [4]. This theory builds upon a statistical-based fragmentation model [1] and an energy-based model [2] [3]. The unified theory developed by Grady [4] led to an estimate of particle size that is based upon the principle in-plane stretching rates at failure. Moreover, the particle aspect ratio and area can be estimated from the orthogonal, in-plane stretching rates.
Within the Velodyne calculation, the full strain rate tensor of each particle is saved at the time of conversion from element to particle. The strain rate is used to define the average size, $x_0$, for that particle based on the characteristic fracture spacing from Grady as shown in Equation (7) where $K_f$ is the fracture toughness, $\rho$ is the density, and $c$ is the sound speed.

$$x_0 = \left( \frac{\sqrt{12K_f}}{\rho c} \right)^{\frac{2}{3}} \quad (7)$$

Rather than sub-dividing particles into smaller, uniform debris pieces corresponding to this average size, the sizes of the new particles are distributed according to a power-law consistent with the slope defined by the larger fragments made of multiple Lagrangian solid elements. An example of this slope fit to the mass distribution of the fragments is shown in Fig. 3. The figure also shows the “pile-up” of the original SPH particles at the mesh resolution limit.

![Fig. 3. An example of the mass distribution between fragments and SPH particles that are output from Velodyne and the new sub-grid particles that are formed by sub-dividing the original SPH particles.](image)

In order to perform the new particle distribution, the average size, $x_0$, of the original particle is first converted to an average mass using the material density. The bounds of the power-law mass distribution are defined such that the average mass (in log space) of the power-law distribution is equal to the calculated Grady average mass and the maximum mass is equal to or smaller than the original particle mass. The minimum mass is found using the fragment slope described above assuming that the average mass is also the median mass (again in log space). The number of new particles and the corresponding mass distribution are calculated based on mass conservation. A draw from the power-law distribution is performed and velocities are assigned to the new particles such that momentum and energy are conserved individually based on the original parent particle. The distribution of new particles is shown with the fragments and original SPH particles in the figure using this method.

The method behind the development of the new particles is based on a combination of the Grady fragmentation theory and trends seen in the distribution of the larger fragment pieces. There is still some uncertainty in the results expected for the low mass end of the distribution of pieces due to the lack of field data in this regime. Most field tests return the majority of high mass pieces but only a fraction of the low mass pieces due to the logistics of collecting the debris. The methodology for this sub-grid technique continues to evolve as additional experimental data becomes available.

It has been found that the aspect ratio of the Lagrangian solid element fragments correlates nicely with the mass of the fragment. Using the mass-aspect ratio correlation established by the Lagrangian fragments, the aspect ratios of the new particles are assigned for their given mass to ensure consistency with the other debris fragments. A further development of this post-processing methodology will apply the Grady methodology to define the aspect ratios of the new particles using the material properties and full strain tensor.
3. Example

3.1. Experiment Overview

The exploding cylinder experiments used in this analysis were performed at NSWD/DL in the early 1980’s. Information regarding the experimental setup was acquired from Crowe et al [6] and Mock and Holt [7]. The setup, illustrated in Fig. 4, consisted of an Armco iron cylinder filled with cast Composition B explosive. The explosive was extended 8.89 cm forward and 6.35 cm aft of the casing to reduce end effects. A detonator was positioned at the center of the forward end of the explosive.

![Image](https://example.com/image.png)

Fig. 4. Illustration of experimental setup.

A total of three experiments were performed for the Armco iron cylinder and are summarized in Table 1. Tests 1 and 2 were repeated saw dust pit experiments at the NSWC Sawdust Pit Facility to collect the fragments and characterize their distribution. In these experiments, the fragments from the cylinder were captured in saw dust and recovered using various separation techniques. All fragments with a mass greater than 1 grain (0.0648 gram) were weighed and cataloged individually. Fragments less than 1 grain were weighed as an aggregate because of their small size and high count. Over 99 percent of the total cylinder mass was collected in each test.

| Test Number | Test Type       | Inner Diameter (cm) | Outer Diameter (cm) | Cylinder Length (cm) | Original Cylinder Mass (kg) | Total Mass of Fragments (kg) | HE Mass (kg) |
|-------------|-----------------|---------------------|---------------------|----------------------|-----------------------------|----------------------------|-------------|
| 1           | Pit             | 7.62                | 11.43               | 20.3                 | 9.13                        | 9.080                      | 2.77        |
| 2           | Pit             | 7.62                | 11.43               | 20.3                 | 9.16                        | 9.119                      | 2.76        |
| 3           | Framing Camera  | 7.62                | 11.43               | 38.1                 | N/A                         | N/A                        | 4.10        |

Test 5 was a framing camera experiment performed to measure the cylinder expansion under the high explosive loading. The cylinder length was extended to 38.1 cm for this case, as compared to 20.3 cm in Tests 1 and 2. This allowed extended observation time of the cylinder expansion and fracture process. The outer radius was determined from the framing camera photographs taken at 4.17 μsec intervals. The point of measurement was 178 mm from the front end of the cylinder, which corresponds to the left side of Fig. 4. The expansion measurements from these experiments are compared to the simulation results to understand the accuracy of the hydro-structural code and give some insight into the code’s ability to predict debris velocities.

3.2. Simulation Overview

The information from the experimental setup in Crowe et al [6] and Mock and Holt [7] was used to build a finite element model of the Armco iron cylinder and Comp B explosive for the Velodyne simulation. The same model was used for Tests 1 and 2 with approximately 400,000 elements at an average resolution of 2 mm. Ten elements were used through the thickness of the casing, and a structured mapped mesh was utilized for uniformity. The mesh is shown in Fig. 5a. The mass of the explosive in the analysis was 2.77 kg (as compared to 2.77 kg for Test 1 and 2.76 kg for Test 2), and the mass of the cylinder was 9.13 kg (as compared to 9.13 kg for Test 1 and 9.16 kg for Test 2). The small discrepancies in the explosive masses could be the result of variations in the Comp B cast fill for RDX/TNT and paraffin wax, while the differences in the cylinder casing could be the result of manufacturing tolerances.
An additional finite element model was built for the Velodyne simulation of Test 5 because of the differences in the cylinder length. The mapped mesh is provided in Fig. 5b. An average resolution of 2 mm was again used to generate the mesh, resulting in a total of 650,000 elements. The mass of the charge was 4.16 kg, compared to 4.10 kg in the test, while the Armco Iron casing weighed approximately 17.13 kg.

To model the detonation of the high explosive, a programmed burn model was used, which propagates an ideal detonation wave through the explosive. The initiation point is specified at the center of the front end of the explosive in the simulation to represent the detonator in the test. For Composition B, the detonation velocity and pressure were set to 7.98 km/sec and 29.5 GPa. A Jones-Wilkins-Lee (JWL) equation of state was used for the reactive products, and the parameters were taken from Urtiew et al. [16].

In order to accurately capture the fragment formation, a Johnson Cook material model was used for the Armco Iron with a Mie Gruneisen equation of state. The parameters for the strength model are given in Johnson and Cook [12] and reproduced in Table 2, and the Mie Gruneisen parameters are taken from Mock and Holt [7]. The Johnson Cook parameters correspond to Equation (1). This model is used to capture the rate effects and thermal softening of the cylinder deformation.

| Material       | Density (g/cc) | A (MPa) | B (MPa) | n  | C   | m  |
|----------------|---------------|---------|---------|----|-----|----|
| Armco Iron     | 7.89          | 175     | 380     | 0.32 | 0.06 | 0.55 |

The material constants for the Johnson Cook damage model are also given in Reference 12, but when used in the simulation, the fracturing of the cylinder casing did not agree with the observed experimental data. This could indicate that the damage parameters were sufficient only for the testing conditions from which they were derived. The experiments used to empirically derive the damage parameters from Reference 12 appear to be mostly quasi-static tensile tests, while the fragmenting warhead case shown here shows much higher strain rates between $10^4$ and $10^5$ sec$^{-1}$. Some discussion on the sensitivity of the damage parameters is given by Kay [17] for titanium and aluminum alloys but the effects are not fully investigated or resolved.

For the fragmenting warhead simulations provided in this report, the damage parameters were adjusted to match the fracture characteristics seen in the Armco Iron cylinder experiments. These modified values are compared to the original values in Table 3. The modifications use the theory that the hydrostatic tension is the most significant contributor to fracture, so D4 and D5 are set to zero. The D3 parameter is assumed to be -1.5, as specified by Hancock and McKenzie [18]. The D1 and D2 parameters were adjusted until cracks began to appear at similar times that were recorded in the framing camera experiment (Test 5). These values were then applied to the fragmentation predictions for Tests 1 and 2.

| Material             | D1   | D2   | D3   | D4  | D5  |
|----------------------|------|------|------|-----|-----|
| Armco Iron           | 2.20 | 5.43 | 0.47 | 0.016 | 0.63 |
| Armco Iron Modified  | 0.15 | 1.08 | -1.5 | 0.0  | 0.0  |
A weibull distribution was also placed on the strain to fracture calculated in Equation (4) to mimic the random variation in fracture caused by material inhomogeneity. A similar distribution was used as specified by Goto et al. for AerMet 100 and 1018 steel cylinders undergoing explosive fragmentation [15]. The pdf is applied as a scale factor for the strain at fracture of each element. The distribution is shown in Fig. 6.

3.3. Results and Comparison

The data collected from the framing camera experiments were used to validate the cylinder expansion and onset of fracture in the Velodyne simulation. Fig. 7a illustrates the change in the cylinder outer radius over time at a position of 178 mm from the front end of the cylinder. The time history begins after the explosive detonation front enters the front end of the cylinder. This occurs, analytically, at 11.27 μsec after the detonation (assuming a detonation velocity of 7.89 km/sec) and is confirmed by the Velodyne simulation with the shock front entering at approximately 11 μsec. The figure compares the cylinder expansion in the simulation with the measurements from the photographs in the experiment.

Overall, the simulation agrees very well with the experimental data; the simulation is within 3% of all the data points. For both the simulation and test data, the plot shows the initial expansion of the cylinder outer surface beginning at 25 μsec. The surface accelerates between 25 and 70 μsec during the initial expansion and then maintains a constant velocity of approximately 1 km/sec after 70 μsec, due to the relief from fracture. A closer look at the simulation velocity data in Fig. 7b shows multiple spikes in the velocity between 25 and 45 μsec, due to the shock reflections inside the cylinder wall.

In addition to the velocity data, we performed a qualitative comparison of the progression of the surface cracks recorded in the framing camera images. Fig 8 compares both the simulation with the recorded data at various times after the detonation front enters the cylinder. The observation window is between 127 and 254 mm from the front end of the
cylinder. The axes indicate the distance along the cylinders length and the cylinder outer radius with the centerline denoted. The cracks (i.e. where damaged occurs) in the simulation are colored in red.

![Diagram](image)

Fig. 8. Time evolution of the cylinder expansion and cracking between 127 and 254 mm along the cylinder length for test data (a) and simulation (b).

At 31 μsec after the detonation front enters the cylinder, both the simulation and experimental data indicate expansion of the cylinder and yielding of the Armco Iron material, but no fracture in the window of interest. At 39 μsec, a few cracks begin to appear as the material reaches its failure strength. In the simulation, this occurs at a strain of approximately 0.35, similar to that measured by Mock et al [7]. As the cylinder expands the cracks increase both in size and number. The non-uniform crack formation in the simulation is produced by the weibull pdf applied to the strain at fracture.

Validation of the simulations continues by comparing the fragment mass distributions recorded in Tests 1 and 2. Two Velodyne simulations were performed using different initial seeds for the weibull distribution to mimic the repeatability of the tests. Fig. 9 shows the comparison for the top 50 fragments by mass from Test 2 with the top 50 fragments from each of the simulations. Individual mass measurements of the top 50 pieces were not documented in the report for Test 1. The two simulations are plotted separately to understand the effects of the weibull distribution on the material strength.

As expected, there is some variability in the predicted debris distributions, caused by the application of the weibull pdf on the strain at fracture. The debris curves from both simulations follow the same trend, and the slope of the curve is similar to the test data. However, although these similarities occur, there is a discrepancy in the debris masses when comparing the simulations to the test data. The debris in the simulations weigh an average 50 percent less than the test. This is observed to be the result of using element conversion to create the cracks, which tends to reduce the size of the larger debris surrounding the crack and increases the number of smaller debris pieces generated. Another factor is the mesh resolution, which affects the size of the cracks (i.e. a crack is always at least one element thick). Lastly, the material model may be insufficient to capture the damage initiation and crack nucleation at a microscopic level which leads to material failure and fracture. The hydro-structural codes capture deformation at the meso-scale using continuum mechanics, which may not provide enough fidelity for modeling grain boundaries.

In addition to comparing the top 50 pieces, we analyzed the full distribution of the debris down to 1 grain. Fig. 10 provides a log-log plot of the number of fragments greater than a particular mass. Although the simulations appear to follow the trend in the fragment distribution down to 10 grains, as we approach the mesh resolution limit of approximately 1 grain, there is a pile up of the number of debris pieces. This occurs because the particles generated from the elements comprising the cylinder have a uniform mass and size. This is the effect that is to be countered using the sub-grid methodology.
Figure 11 shows the updated results when the sub-grid methodology is applied. With this methodology, the pile up of SPH particles at the mesh resolution is reduced. This is because each particle is broken into multiple particles of varying sizes based on the strain rate information and Grady’s theory. A small pile up still occurs because some of the fragments that consist of a single Lagrangian solid element may need to be further resolved into multiple smaller fragments. These could be further reduced by continually increasing the mesh resolution of the simulation, but this methodology becomes impracticable for large, complex geometries. In this case, the sub-grid methodology provides an alternate and efficient solution.

This methodology shows that post-processing techniques are available to predict debris distributions below the mesh resolution of a given hydro-structural code simulation without significant increases in runtimes. Although further development could be employed to refine our sub-grid techniques, the methodology has shown feasibility and merit for this application.

4. Conclusions

The analysis provided in this report demonstrates the ability of modeling debris generation using the hydro-structural code Velodyne for a fragmenting warhead scenario. The results illustrate that the code accurately captures the hydrodynamics of the expanding cylinder; the simulations correctly predict the expansion of the cylinder from the high explosive detonation as measured in the test. In addition, we are able to predict the structural response regarding the onset of fracture and the variability using modified Johnson Cook damage parameters and a weibull distribution of the strain at fracture. However, modeling fracture and fragmentation still present some challenge. The predicted debris mass distributions show similar trends as the test data, but there is some discrepancy due to the approach used for modeling material fracture.
Additional investigation into the effects of the mesh resolution applied and the material model utilized should be considered. Other material models, such as Mossfrac [19], have the ability to capture anisotropic cracking in a material, allowing fracture to occur in one direction while being able to handle tension in the other directions. Models that capture crack nucleation at grain boundaries should also be examined. Lastly, further increasing the mesh resolution or using other meshing schemes (such as paved) could yield insight into the mesh dependency of the fracture process.

Regardless of the fracture modeling process, we have proposed a sub-grid methodology for predicting debris below the mesh resolution in a hydro-structural code simulation. When applied to the fragmenting warhead scenario, the sub-grid method further sub-divides the particle debris and follows the trend in the distribution. Modifications can be made to continually refine the methodology (such as further resolution for the single Lagrangian solid elements), but the current approach shows merit and efficiency for predicting below the mesh resolution of a given problem.

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