Running gravitational couplings, decoupling, and curved spacetime renormalization

Antonio Ferreiro and Jose Navarro-Salas

1 Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC. Facultad de Física, Universidad de Valencia, Burjassot-46100, Valencia, Spain.

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We propose to slightly generalize the DeWitt-Schwinger adiabatic renormalization subtractions in curved space to include an arbitrary renormalization mass scale $\mu$. The new predicted running for the gravitational couplings are fully consistent with decoupling of heavy massive fields. This is a somewhat improvement with respect to the more standard treatment of minimal (DeWitt-Schwinger) subtractions via dimensional regularization. Because of the natural decoupling, the proposed runnings could shed new light on the quantum cosmological constant problem. We also show how the vacuum metamorphosis model emerges from the running couplings.

I. INTRODUCTION

One of the cornerstones in quantum field theory has been the design of regularization/renormalization schemes that allows us to overcome ultraviolet divergences when computing physical observables [1–3]. In perturbative quantum electrodynamics we thus obtain reliable, well-proven results such as the Lamb shift and the running of the electromagnetic coupling constant due to vacuum polarization. The renormalization process always involves an arbitrary mass parameter $\mu$ and the possibility of rescaling it. There is also much arbitrariness in the choice of the finite part of the renormalization counterterms. This is also reflected in the predicted running of the coupling constant. However, when the masses can be neglected the leading order beta function is uniquely fixed and one obtains $\beta_e \sim e^3/12\pi^2$ for large $\mu/m$. In general, when masses are not negligible, the beta function inherits a dependence on the chosen subtraction scheme.

Another relevant feature of renormalization is the expected decoupling of higher massive particles, as enforced by the Appelquist-Carazzone theorem [4]. This means that particles with mass higher than the relevant physical energy scale should not contribute to any computed observable. This ensures that for low energy physics we do not need to know about the related very high energy physics, hence supporting the effective field theory framework. The minimal subtraction (MS) scheme in dimensional regularization [5–6] is a very efficient method to evaluate the behavior of the running couplings. However, MS does not fulfill the decoupling theorem and one needs to resort to a mass-dependent scheme to capture the low energy behavior of the beta function.

Renormalization theory has also been extended to quantized fields in curved spacetime from the early seventies, as reported in [7–8]. Here the main focus was the renormalization of the energy-momentum tensor and the evaluation of the effective action in a way consistent with general covariance. One of the major tools is the heat-kernel or proper-time expansion of the Feynman propagator [9–11]. As in the case of perturbative computations in Minkowski space, quantized fields in curved space are also plagued with ultraviolet divergences. The DeWitt-Schwinger expansion serves to identify the emerging ultraviolet divergences, some of which are intrinsically tied to the spacetime curvature and are absent in flat space. In the evaluation of the renormalized effective action the removal of the divergences can also be done using a mass independent scheme, like MS in dimensional regularization [12]. This introduces the usual $\mu$ parameter and the associate running of the gravitational coupling constants (see, for instance, [7–8]). As expected, the obtained runnings do not fulfill the Appelquist-Carazzone theorem and in consequence makes it difficult to arrive at any physical interpretation in the cosmic infrared regime. This is specially important in discussing the cosmological constant problem and the running of Newton’s constant [13–14].

In this work we propose to re-evaluate the effective action, and the associated beta functions, by re-expressing the conventional DeWitt-Schwinger adiabatic expansion with the introduction of a novel $\mu$ scale parameter in the definition of the adiabatic subtraction terms. The $\mu$ parameter is introduced in such a way that a natural decoupling emerges in the running couplings. This allows us to physically interpret the quantum contributions at the low energy limit, somewhat alleviating the cosmological constant problem. We also show how the vacuum metamorphosis model [15–17], one of the most appealing models to account for dark energy [18–19] and to soften the measured $H_0$ tension [20], emerges when the $\mu$ parameter is interpreted in terms of the Ricci scalar.

To make the paper self-contained we first introduce the
DeWitt-Schwinger (proper-time) expansion and briefly summarize the derivation of the well-known running for the couplings in dimensional regularization with the minimal prescription. To better explain the main ideas we consider a quantized complex scalar field coupled to external gravitational and electromagnetic fields. The introduction of the external electromagnetic field is somewhat tangential to the main topic of the paper. However, we introduce it in the discussion for pedagogical purposes, since the running of the effective electric charge is a well-established theoretical and experimental result. This permits to compare the one-loop electromagnetic behavior with analogous results in gravity. We use units for which $c = 1 = h$. Our sign conventions for the signature of the metric and the curvature tensor follow Ref. \[7, 8\].

II. EFFECTIVE ACTION, DEWITT-SCHWINGER EXPANSION, AND DIMENSIONAL REGULARIZATION

We start from the classical Einstein-Maxwell theory

$$ S = \int d^4 x \sqrt{-g} \left( -\Lambda + \frac{R}{16\pi G} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) + S_M $$

(1)

coupled to a quantized charged scalar field described by the action

$$ S_M = \int d^4 x \sqrt{-g} \left( (D_\mu \phi) D^\mu \phi + m^2 |\phi|^2 + \xi | R | |\phi|^2 \right) \, , $$

(2)

with $D_\mu = \nabla_\mu + iA_\mu$. The most relevant physical objects are the renormalized energy-momentum tensor $\langle T_{\mu\nu} \rangle$ and the one-loop effective action $S_{\text{eff}}$ for the matter field, related by $\frac{2}{\sqrt{-g}} \partial_s G_\mu = \langle T_{\mu\nu} \rangle$. The effective action can be formally expressed in terms of the Feynman propagator $S_{\text{eff}} = -i \operatorname{Tr} \log (-G_F)$. The propagator satisfies the Klein-Gordon type equation

$$ (\Box + m^2 + \xi | R |) G_F (x, x') = -\delta (x - x') \, . $$

(3)

In general, above formal expression for the effective action is divergent. To explicitly identify the ultraviolet divergences, one can express the Feynman propagator as an integral in the proper time $s$

$$ G_F (x, x') = -i \int_0^\infty ds \, e^{-im^2 s} \langle x, s | x', 0 \rangle \, , $$

(4)

where $m^2$ is understood to have an infinitesimal negative imaginary part ($m^2 \equiv m^2 - i\epsilon$). The kernel $\langle x, s | x', 0 \rangle$ can be expanded in powers of the proper time as follows

$$ \langle x, s | x', 0 \rangle = i \frac{\Delta^{1/2} (x, x')}{(4\pi i)^2 (is)^2} \exp \left( \frac{\sigma (x, x')}{2is} \right) \sum_{j=0}^\infty a_j (x, x')(is)^j \, , $$

(5)

$[\Delta (x, x')$ is the Van Vleck-Morette determinant and $\sigma (x, x')$ is the proper distance along the geodesic from $x'$ to $x$. Therefore, the effective Lagrangian, defined as $S_{\text{eff}} = \int d^4 x \sqrt{-g} L_{\text{eff}}$, has the following expansion

$$ L_{\text{eff}} = \frac{2i}{2(4\pi)^2} \sum_{j=0}^\infty a_j (x) \int_0^\infty e^{-ism^2} (is)^{-3} ds \, . $$

(6)

The first coefficients $a_n (x, x')$ are given, in the coincidence limit $x \to x'$, by \[7, 8\]

$$ a_0 (x) = 1 \, , \quad a_1 (x) = -\xi R \, , $$

$$ a_2 (x) = \frac{1}{180} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - \frac{1}{180} R^{\alpha\beta} R_{\alpha\beta} - \frac{1}{6} \left( \frac{1}{5} - \xi \right) \square R + \frac{1}{2} \xi^2 R^2 - \frac{1}{12} F_{\mu\nu} F^{\mu\nu} \, , $$

(7)

where $\xi = \xi - \frac{1}{n}$. We observe that the ultraviolet divergences of (6) are isolated in the first three terms of the DeWitt-Schwinger expansion. The removal of divergences is usually done via dimensional regularization and minimal subtraction.

In $n$ spacetime dimensions the corresponding expression \[6\] can be expanded as

$$ L_{\text{eff}} \approx \frac{2i}{2(4\pi)^n/2} \left( \frac{m}{\mu} \right)^{n-4} \sum_{j=0}^\infty a_j (m) m^{4-3j} \Gamma (j - \frac{n}{2}) \, , $$

(8)

where one has introduced an arbitrary mass scale $\mu$ to maintain the initial units of $L_{\text{eff}}$ as (length)$^3$. As $n \to 4$, the first three terms diverge with simple poles in $1/(n-4)$. Subtracting the terms with poles one obtains the renormalized effective Lagrangian. This also requires that the original classical Lagrangian be modified, up to total derivatives, by the addition of higher derivative terms of the form $a_1 (C^2 + a_2 R^2)$, where $a_1$ and $a_2$ are dimensionless coupling constants. Here $C^2 \equiv R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 2R_{\mu\nu} R^{\mu\nu} + \frac{1}{4} R^2$ is the square of the Weyl tensor. Demanding that the total effective Lagrangian, including the classical part, be $\mu$-independent leads to the following beta-functions (see for instance \[13\])

$$ \beta_{\Lambda}^{DR} = \frac{m^4}{16\pi^2} \, , \quad \beta_{\kappa}^{DR} = -\frac{m^2 \xi}{4\pi^2} \, , \quad \beta_{\phi}^{DR} = \frac{q^3}{48\pi^2} \, , $$

$$ \beta_{a_1}^{DR} = -\frac{1}{1920\pi^2} \, , \quad \beta_{a_2}^{DR} = -\frac{1}{16\pi^2} \xi^2 \, , $$

(9)

where $\kappa^{-1} = 8\pi G$.

III. ADIABATIC DEWITT-SCHWINGER SUBTRACTIONS

The DeWitt-Schwinger expansion can also be regarded as an adiabatic expansion in number of derivatives of the metric and the external fields. This is even more explicit in its counterpart expansion in local-momentum space \[21\]. Therefore, the renormalization of the effective action can also be performed simply by subtracting off all
(DeWitt-Schwinger) terms up and including the fourth adiabatic order [7]

\[ L_{\text{div}} = \frac{2i}{2(4\pi)^2} \sum_{j=0}^{\infty} a_j(x) \int_0^\infty e^{-ism^2(is)^j} \text{d}s . \]  

(10)

Our purpose now is to evaluate the running of the coupling constants within the above subtraction prescription. To this end we have to introduce a mass scale parameter \( \mu \) in the DeWitt-Schwinger framework. This can be easily done from the following observation. We can replace the mass parameter \( m^2 \) in (10) by an arbitrary \( \mu^2 \) parameter and redefine the DeWitt coefficients \( a_i \rightarrow \tilde{a}_i \) to keep consistency within each adiabatic order. The new proposed \( L_{\text{div}}(\mu) \) reads

\[ L_{\text{div}}(\mu) = \frac{2i}{2(4\pi)^2} \sum_{j=0}^{\infty} \tilde{a}_j(x) \int_0^\infty e^{-is\mu^2(is)^j} \text{d}s , \]  

(11)

where now the first coefficients of the expansion are

\[ \tilde{a}_0(x) = 1 , \quad \tilde{a}_1(x) = a_1(x) + \mu^2 - m^2 \]

\[ \tilde{a}_2(x) = a_2(x) + \xi R (\mu^2 - m^2) + \frac{1}{2} (\mu^2 - m^2)^2 . \]  

(12)

Now we can separate from expression (11) a \( \mu \)-independent divergent term and a finite \( \mu \)-dependent term by computing the finite expression

\[ L_{\text{div}}(\mu) - L_{\text{div}}(m) = \delta_\lambda + \delta_G R + \delta_\sigma a_2 , \]  

(13)

where

\[ \delta_\lambda = \frac{1}{(8\pi)^2} \left\{ 4m^2(\mu^2 - m^2) - (\mu^4 - m^4) - 2m^4 \log \left( \frac{\mu^2}{m^2} \right) \right\} \]

\[ \delta_G = \frac{1}{16\pi^2} \xi \left( \mu^2 - m^2 - m^2 \log \left( \frac{\mu^2}{m^2} \right) \right) \]

\[ \delta_\sigma = -\frac{1}{16\pi^2} \log \left( \frac{\mu^2}{m^2} \right) . \]  

(14)

The beta functions are obtained by requiring \( \mu \)-independence of the effective Lagrangian

\[ L_{\text{eff}} = -\Lambda(\mu) + \frac{1}{2} \kappa(\mu) R - \frac{1}{4q^2(\mu)} F_{\mu\nu} F^{\mu\nu} \]

\[ + \alpha_i(\mu) R_i + \frac{1}{\mu^4} E \]

\[ - \delta_\lambda(\mu) - \delta_G(\mu) R + \delta_\sigma(\mu) a_2 + \ldots . \]  

(15)

\( E = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4R^2 \) is the integrand of the Gauss-Bonnet topological invariant. Note that the omitted terms in the third line of (15) are independent of \( \mu \). The results for the beta functions are

\[ \beta^{DS}_\lambda = \frac{(\mu^2 - m^2)^2}{16\pi^2} \quad \beta^{DS}_\kappa = -\frac{\xi(\mu^2 - m^2)}{4\pi^2} \]

\[ \beta^{DS}_q = -\frac{93}{48\pi^2} \quad \beta^{DS}_1 = -\frac{1}{960\pi^2} \quad \beta^{DS}_2 = -\frac{\xi^2}{16\pi^2} \]

\[ \beta^{DS}_3 = \frac{1}{2880\pi^2} \quad \beta^{DS}_4 = -\frac{1}{48\pi^2} . \]  

(16)

We have included for completeness all coupling constants. This agrees with the results obtained in [22] for Friedmann-Lemaître-Robertson-Walker spacetimes using a similar generalization of the usual adiabatic regularization method [23] via the introduction of an analogous off-shell scale \( \mu \). (For a recent use of this generalization see [24].) We also have exact agreement for the dimensionless coupling constants obtained in dimensional regularization, as displayed in [22]. Hadamard renormalization also leads to a similar result for the running of the electric coupling constant [25, 26].

IV. DECOUPLING AND RUNNING GRAVITATIONAL CONSTANTS

The unsatisfactory point of the above results (11) and also (9) is the absence of decoupling for heavy massive fields. For several charged (scalar) matter fields we have \( \beta_q = \sum_i \frac{q_i^3}{48\pi^2} \mu^2 \), irrespective of the masses. On the other hand, for \( \mu \ll m_i \) the mass dependent \( \mu \)-scheme [2, 3] in scalar electrodynamics leads to \( \beta_q \sim \sum_i \frac{q_i^3}{48\pi^2} \mu^2 \). Hence the contribution to the running of particles with masses much bigger than the physical scale \( \mu \) is negligible. The same happens for the dimensionless couplings \( \alpha_k \).

To incorporate a decoupling mechanism we introduce a simple and natural modification of (11). We propose to choose, instead of (11),

\[ L_{\text{div}}(\mu) = \frac{2i}{2(4\pi)^2} \sum_{j=0}^{\infty} \tilde{a}_j(x) \int_0^\infty e^{-is(m^2 + \mu^2)(is)^j} \text{d}s , \]  

(17)

and hence \( \tilde{a}_0(x) = 1, \tilde{a}_1(x) = a_1(x) + \mu^2, \tilde{a}_2(x) = a_2(x) + \xi R \mu^2 + \frac{1}{4} \mu^4 \). For several charged scalar fields we have

\[ L_{\text{div}}(\mu) = \frac{2i}{2(4\pi)^2} \sum_{i} \sum_{j=0}^{\infty} \tilde{a}_j(x) \int_0^\infty e^{-is(m^2 + \mu^2)(is)^j} \text{d}s , \]  

(18)

where the index \( i \) is for any particle species that appears in the Lagrangian and \( \mu \) is the single renormalization scale of the overall theory.

The corresponding beta function for the electric charge obtained from (17) is

\[ \beta_q = \frac{q^3}{48\pi^2} \frac{\mu^2}{m^2 + \mu^2} , \]  

(19)

while the result for the dimensionless gravitational constants are similarly

\[ \beta_1 = -\frac{1}{960\pi^2} \frac{\mu^2}{m^2 + \mu^2} \quad \beta_2 = -\frac{\xi^2}{16\pi^2} \frac{\mu^2}{m^2 + \mu^2} \]

\[ \beta_3 = \frac{1}{2880\pi^2} \frac{\mu^2}{m^2 + \mu^2} \quad \beta_4 = -\frac{1}{48\pi^2} \frac{\mu^2}{m^2 + \mu^2} . \]  

(20)
The difference between (19)–(20) and (16) is that the former approaches the latter in the limit \( \mu \gg m \) while it approaches to zero quadratically in the limit \( \mu \ll m \). This is equivalent to the decoupling of very massive charged particles in scalar electrodynamics.

Concerning the dimensionful gravitational constants, the decoupling is also absent in dimensional regularization. This makes it not trivial to assign some physical meaning to the \( \mu \) parameter. However, within the proposed DeWitt-Schwinger framework and from (17) we get the following beta functions

\[
\beta_D^{\lambda} = \frac{1}{16\pi^2} \frac{\mu^6}{\mu^2 + \mu^2} \quad \beta_D^{\kappa} = \frac{1}{4\pi^2} \frac{\mu^4}{\mu^2 + \mu^2}.
\]  

(21)

For large values of the scale \( \mu \gg m \) the mass can be ignored, while heavy particles \( m \gg \mu \) decouple and the beta functions tens to zero. Note that for dimensionful coupling constants the decoupling is a stronger restriction since the running from (14) possesses quadratic and quartic mass terms.

The running of the cosmological and Newton’s gravitational constants are given by \( \Lambda = \Lambda_c/8\pi G \), where \( \Lambda_c \) is the traditional cosmological constant. For completeness we also give the results for fields of various spins. Details will be given elsewhere.

\[
\begin{align*}
\Lambda(\mu) &= \Lambda_0 + \frac{\gamma}{128\pi^2}((\mu^4 - \mu_0^4) - 2m^2(\mu^2 - \mu_0^2)) \\
& \quad + 2m^4 \log \left( \frac{m^2 + \mu^2}{\mu_0^2 + m^2} \right) \\
G(\mu) &= \frac{G_0}{1 + \kappa_0 \pi G_0 (\mu^2 - \mu_0^2 - m^2 \log \left( \frac{m^2 + \mu^2}{\mu_0^2 + m^2} \right))},
\end{align*}
\]

(22)

(23)

while the running for the dimensionless gravitational constants are

\[
\alpha_i(\mu) = \alpha_{i0} + \frac{\sigma_i}{\pi^2} \left( \frac{m^2 + \mu^2}{m_0^2 + \mu_0^2} \right).
\]

(24)

In Table I we give the values of the numerical constants \( \gamma, \rho \) and \( \sigma_i \) for fields of spin \( s = 0, \frac{1}{2}, 1 \). In the calculation we have used the DeWitt coefficients for the various fields given [8].

It is interesting to briefly consider the massless limit for the predicted running for the Newton constant, as given by (23):

\[
G(\mu) = G_0 (1 + \rho \frac{G_0}{G_0} (\mu^2 - \mu_0^2)^{-1}.
\]

This expression has the same form as the one obtained within a very different approach. The asymptotic safety framework of quantum gravity predicts a similar behavior for the running of Newton’s constant [27] (see also [28]).

Even though the above renormalization prescription does not give us a uniquely physical interpretation for \( \mu \), it supports the idea that indeed it can be linked to some physical scale, such as the conventional momentum rescaling in flat space particle scattering. One possible way of choosing a natural mass/length scale in a cosmological setting is to make \( \mu \) proportional to the Hubble parameter \( H \), or \( \mu^2 \) to be proportional to the Ricci scalar \( R \). Then the running obtained in (23) is somewhat similar to the generic form of the running proposed in the running vacuum models (RVM’s) [29] (see also [13, 24] and [30] for a connection with cosmological observations and smoothing of data tensions).

Let us analyze with more details the consequences of the assumption \( \mu^2 \propto R \). For computational purposes it is convenient to choose \( \mu^2 = \xi R \). We also select the referent point \( \mu_0 = 0 \) and assume that

\[
\Lambda_0 = 0, \quad \alpha_{i0} = 0,
\]

and keep \( \kappa_0 = (1/8\pi G_0) \), where \( G_0 \) is the measured Newton’s constant. The effective Lagrangian is well-approximated by (here we are considering a single real scalar field)

\[
L_{eff} = -\Lambda(\mu) + \frac{1}{2} \kappa(\mu) R + \alpha_1(\mu) \xi R^2 + \alpha_2(\mu) R^2 + \alpha_3(\mu) E + \alpha_4(\mu) \Box R.
\]

(25)

Taking into account the running derived in (24), for all gravitational coupling constants and the conditions [23], the above effective action can be rewritten in the form

\[
L_{eff} = \frac{1}{2} \kappa_0 R + \frac{1}{64\pi^2} \left\{ m^2 \xi R + \frac{3}{2} \xi^2 R^2 \ight. \\
- (m^2 + 2m^2 \xi R + 2a_2) \log \left( \frac{m^2 + \xi R}{m^2} \right). \}
\]

(27)

Remarkably, this coincides with the action proposed by Parker and Raval in [16, 17] on the basis of the \( R \)-summed form of the Feynman propagator [31, 32]. Here only the measured Newton’s constant \( G_0 \) appears in the action. \( a_2 \) is the DeWitt coefficient given in [7].

V. CONCLUSIONS AND FINAL COMMENTS

We have generalized the DeWitt-Schwinger renormalization subtractions to include an arbitrary renormalization mass scale \( \mu \), and in such a way to ensure the
decoupling of heavy masses. This is a somewhat improvement with respect to the more common treatment of the DeWitt-Schwinger expansion via dimensional regularization and minimal subtraction. We have also analyzed the new predicted running for the gravitational couplings.

As a byproduct of our proposal, and because of the natural decoupling, the obtained runnings soften the standard quantum cosmological constant problem. To see this in the conventional way let us assume that Λ = 0. Following the standard approach, i.e., dimensional regularization, any massive particle will contribute as ΛDR(μ) ∼ m^4 log (μ^2/m^2) (see, for instance, [13, 14]). Taking for m a characteristic mass of the Standard Model, such as Higgs mass (m_H ∼ 125 GeV), and for μ the Hubble rate μ_H ∼ H_0 ∼ 10^{-33}eV, the above expression gives the well-known extremely high contribution ΛDR(H_0) ∼ 10^{46}eV^4. This is in conflict with the observed current energy density Λ_{obs} ∼ 10^{-11}eV^4 (see [13] for a detailed discussion). However, if we now use (22) we obtain an extremely low value. More generally, in the limit of large masses m ≫ μ_c (all the standard model particles) the term m^4 decouple and we get Λ^{DS}(μ) ∼ μ^6/m^2 + O(1/m^3). Note that other scale fixing is possible, for instance one can take μ = √EνH/2 ∼ 2 × 10^{-16}eV where Eν is the energy of the supernova photon in the context of the local measures of H_0, as advocated in [14]. However, since the decoupling is valid as long as m ≫ μ the contribution for this last energy scale will still be negligible. This heuristic discussion suggests that the origin of the accelerated expansion could be more naturally found in ultra-low masses. This requires the identification of μ^2 as a time-dependent scale proportional to the Ricci scalar, as also reinforced in the more quantitative arguments displayed in this work.

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[1] S. Weinberg, *The Quantum Theory of Fields*, Vol. 1,2, Cambridge University Press, Cambridge, (1995).
[2] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley, Reading MA, (1995).
[3] L. Alvarez-Gaume and M.A. Vazquez-Mozo, *An Invitation to Quantum Field Theory*, Springer-Verlag, Berlin, (2012).
[4] T. Appelquist and J. Carazzone, *Phys. Rev. D* **11**, 2856 (1975).
[5] G. ’t Hooft, *Nucl. Phys. B* **61**, 455 (1973).
[6] G. ’t Hooft and M. J. G. Veltman, *Nucl. Phys. B* **44** (1972) 189.
[7] L. Parker and D. J. Toms, *Quantum Field Theory in Curved Spacetime: Quantized Fields and Gravity*, Cambridge University Press, Cambridge, England (2009).
[8] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge, England (1982).
[9] J. Schwinger, *Phys. Rev.* **82**, 664 (1951).
[10] B. S. DeWitt, *Dynamical theory of groups and fields*, Gordon and Breach, New York (1965).
[11] B. S. DeWitt, *Phys. Rep.* **19**, 295 (1975).
[12] T. S. Bunch, *J. Phys. A* **12**, 4 (1979).
[13] J. Solà, *J. Phys. Conf. Ser.* **453**, 012015 (2013).
[14] J. Martin, *Comptes Rendus Physique* **13**, 566-665 (2012).
[15] S. M. Carroll, *Living Rev. Relativ.* **4**, 1 (2001).
[16] L. Parker and A. Raval, *Phys. Rev. D* **60**, 063512 (1999).
[17] L. Parker and A. Raval, *Phys. Rev. D* **62**, 083503 (2000); *Phys. Rev. Lett.* **86**, 749 (2001).
[18] L. Parker and D. A. T. Vanzella, *Phys. Rev. D* **69**, 104009 (2004).
[19] R.R. Caldwell, W. Komp, L. Parker and D.A.T. Wanzella, *Phys. Rev. D* **73**, 023513 (2006).
[20] E. Di Valentino, E. V. Linder, A. Melchiorri, *Phys.Rev. D* **97**, 4 (2018).
[21] T. S. Bunch and L. Parker, *Phys. Rev. D* **20**, 2499 (1979).
[22] A. Ferreiro and J. Navarro-Salas, *Phys. Lett. B* **792**, 81 (2019).
[23] L. Parker and S. A. Fulling, *Phys. Rev. D* **9**, 341 (1974).
[24] P. R. Anderson and L. Parker, *Phys. Rev. D* **36**, 2963 (1987). I. Aguilo, J. Navarro-Salas, G. J. Olmo, and L. Parker, *Phys. Rev. D* **84**, 107303 (2011).
[25] C. Moreno-Pulido and J. Sola, *Running vacuum in quantum field theory in curved spacetime: renormalizing ρ_{vac}(H) without ~ m^4 terms*, arXiv: 2005.03164.
[26] V. Balakumar, E. Winstanley, *Class. Quant. Grav.* **37**, 056004 (2020).
[27] P. Beltran-Palau, J. Navarro-Salas and S. Pla, *Adiabatic regularization for Dirac fields in time-varying electric backgrounds*, arXiv:2001.08710 (Phys. Rev. D, in press).
[28] M. Niedermaier and M. Reuter, *Living Rev. Rel.* **9**, 5 (2006). M. Reuter, *Newton's constant isn't constant; arXiv: hep-th/0012069*.
[29] A. Polyakov, in *Gravitation and Quantization*, J. Zinn-Justin, B. Julia (Eds.), North-Holland, (1995).
[30] I. L. Shapiro and J. Solà, *JHEP* **02** (2002) 006; *Phys. Lett. B* **475**, 236 (2000); *Phys. Lett. B* **682**, 105 (2009).
[31] J. Solà, A. Gómez-Valent and J. de Cruz Pérez, *Astrophys J.*, 836, 43 (2017). J. Solà, *Int. J. Mod. Phys. A*, 33, 1844009 (2018).
[32] L. Parker and D. J. Toms, *Phys. Rev. D* **31**, 953 (1985).
[33] I. Jack and L. Parker, *Phys. Rev. D* **31**, 2439 (1985).
[34] A. Ferreiro, J. Navarro-Salas and S. Pla, *R-summed form of adiabatic expansions in curved spacetime*, arXiv:2003.09610 (Phys. Rev. D, in press).