Study of semileptonic and nonleptonic decays of the $B_c^-$ meson

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Received: date / Revised version: date

Abstract. We evaluate semileptonic and two–meson nonleptonic decays of the $B_c^-$ meson in the framework of a nonrelativistic quark model. The former are done in spectator approximation using one–body current operators at the quark level. Our model reproduces the constraints of heavy quark spin symmetry obtained in the limit of infinite heavy quark mass. For the two–meson nonleptonic decays we work in factorization approximation. We compare our results to the ones obtained in different relativistic approaches.

PACS. 12.39.Hg – 12.39.Jh – 13.20.Fc – 13.20.He

1 Introduction

In this work we have studied, in the framework of a nonrelativistic quark model, exclusive semileptonic and two–meson nonleptonic decays of the $B_c^-$ meson driven by a $b \to c$ or $\bar{c} \to \bar{d}, \bar{s}$ transitions at the quark level. We have not considered semileptonic processes driven by the quark $b \to u$ transition to avoid known problems both at too high $q^2$ transfers, where one might need to include the exchange of $B_c^*$ resonances, and at too low $q^2$ where recoil effects could be important.¹

In order to check the sensitivity of our results to the interquark interaction we have used five different potentials taken from refs. ²⁻⁴. All the potentials used have a confinement term plus coulomb and hyperfine terms coming from one–gluon exchange, and differ from one another in the power of the confining term or in the use of different form factors in the coulomb and hyperfine terms. Their free parameters had been adjusted to reproduce the light and heavy–light meson spectra. Our central results have been obtained with the AL1 potential of ref. ³, while our errors show the spread of the results when using the other four potentials.

A detailed account of the full contents of this work is now available in ref. ².

2 Semileptonic decays

We have made our calculations in the spectator approximation using one–body current operators. In table I we show our branching ratios for semileptonic $B_c^-$ decays and compare them to the results obtained by Ivanov et al. ⁵ and, within a relativistic quark model calculation, and Ebert et al. ⁶⁻⁷, within the quasipotential approach to the relativistic quark model. The three calculations give similar results.

| $B_c^- \to \eta_c l^- \bar{\nu}_l$ | This work | ² | ²
|-------------------------------|-----------|---|---|
| $B_c^- \to \eta_c \mu^- \bar{\nu}_\mu$ | 0.48⁺⁻⁰.₀₂ | 0.81 | 0.42 |
| $B_c^- \to \chi_{c0} l^- \bar{\nu}_l$ | 0.17⁺⁻⁰.₀₁ | 0.22 | |
| $B_c^- \to \chi_{c0} \mu^- \bar{\nu}_\mu$ | 0.11⁺⁻⁰.₀₁ | 0.17 | |
| $B_c^- \to J/\Psi l^- \bar{\nu}_l$ | 1.54⁺⁻⁰.₀₈ | 2.07 | 1.23 |
| $B_c^- \to J/\Psi \mu^- \bar{\nu}_\mu$ | 0.41⁺⁻⁰.₀₂ | 0.49 | |
| $B_c^- \to \chi_{c1} l^- \bar{\nu}_l$ | 0.066⁺⁻⁰.₀₀₃ | 0.092 | |
| $B_c^- \to \chi_{c1} \tau^- \bar{\nu}_\tau$ | 0.007⁺⁻⁰.₀₀₁ | 0.0089 | |
| $B_c^- \to h_c l^- \bar{\nu}_l$ | 0.17⁺⁻⁰.₀₂ | 0.27 | |
| $B_c^- \to h_c \tau^- \bar{\nu}_\tau$ | 0.15⁺⁻⁰.₀₁ | 0.17 | |
| $B_c^- \to \chi_{c2} l^- \bar{\nu}_l$ | 0.13⁺⁻⁰.₀₁ | 0.17 | |
| $B_c^- \to \chi_{c2} \tau^- \bar{\nu}_\tau$ | 0.08⁰⁻⁻⁰.₀₁ | 0.08⁰⁻⁻⁰.₀₁ | |
| $B_c^- \to \Psi (3836) l^- \bar{\nu}_l$ | 0.00⁰⁻⁻⁰.₀₀⁰⁰⁶ | 0.00⁰⁻⁻⁰.₀₀⁰⁰⁶ | |
| $B_c^- \to \Psi (3836) \tau^- \bar{\nu}_\tau$ | 0.00⁰⁻⁻⁰.₀₀⁰⁰⁰⁰ⁱ⁰ | 0.00⁰⁻⁻⁰.₀₀⁰⁰⁰⁰⁰⁰⁹⁹ | |

| $B_c^- \to B_s^- e^- \bar{\nu}_e$ | This work | ² | ²
|-------------------------------|-----------|---|---|
| $B_c^- \to B_s^- \mu^- \bar{\nu}_\mu$ | 0.046⁺⁻⁰.₀₀⁴ | 0.071 | 0.042 |
| $B_c^- \to B_s^- \tau^- \bar{\nu}_\tau$ | 0.04⁰⁻⁻⁰.₀₀⁰⁵ | 1.10 | 0.84 |
| $B_c^- \to B_s^- \mu^- \bar{\nu}_\mu$ | 0.10⁺⁻⁰.₀⁰⁴ | 0.063 | 0.12 |
| $B_c^- \to B_s^- \tau^- \bar{\nu}_\tau$ | 0.11⁺⁻⁰.₀₀¹ | 0.23⁰⁻⁻⁰.₀¹⁰ | 0.23⁰⁻⁻⁰.₀¹⁰ |
| $B_c^- \to B_s^- \mu^- \bar{\nu}_\mu$ | 0.11⁺⁻⁰.₀₀¹ | 1.10 | 0.84 |
| $B_c^- \to B_s^- \tau^- \bar{\nu}_\tau$ | 0.35⁺⁻⁰.₀¹⁴ | 2.3⁰⁻⁻⁰.₀₁⁰ | 1.75 |
| $B_c^- \to B_s^- \mu^- \bar{\nu}_\mu$ | 0.22⁺⁻⁰.₀₁² | 2.3⁰⁻⁻⁰.₀₁⁰ | |

In table II we show the forward-backward asymmetry, with the forward direction being defined by the three–momentum of the final meson, of the charged lepton measured in the leptons center of mass frame. Our results are

(continued on next page)
Forward-backward asymmetry $A_{FB}$ of the final charged lepton ($e$, $\mu$ or $\tau$) measured in the leptons center of mass frame.

| Decay    | $A_{FB}(e)$       | $A_{FB}(\mu)$       | $A_{FB}(\tau)$ |
|----------|-------------------|---------------------|----------------|
| $B_c \to \eta_c$ | $0.60^{+0.01}_{-0.01} \cdot 10^{-6}$ | $0.13^{+0.01}_{-0.01} \cdot 10^{-1}$ | $0.35$ |
| This work | $0.95^{+0.03}_{-0.03} \cdot 10^{-6}$ | $0.36$ | |
| $B_c \to \chi_{c0}$ | $0.72^{+0.02}_{-0.02} \cdot 10^{-6}$ | $0.15 \cdot 10^{-1}$ | $0.40$ |
| This work | $1.31 \cdot 10^{-6}$ | $0.39$ | |
| $B_c \to J/\Psi$ | $-0.19$ | $-0.18_{-0.01}$ | $-0.35^{+0.02}_{-0.02} \cdot 10^{-1}$ |
| This work | $-0.21$ | $-0.48 \cdot 10^{-1}$ | |
| $B_c \to \chi_{c1}$ | $-0.60_{-0.01}$ | $-0.60_{-0.01}$ | $-0.46$ |
| This work | $0.19$ | $0.34$ | |
| $B_c \to h_c$ | $-0.83_{-0.05} \cdot 10^{-2}$ | $0.97_{-0.05}^{+0.01} \cdot 10^{-2}$ | $0.35$ |
| This work | $-3.6 \cdot 10^{-2}$ | $0.31$ | |
| $B_c \to \chi_{c2}$ | $-0.14$ | $-0.13$ | $0.55^{+0.02}_{-0.02} \cdot 10^{-1}$ |
| This work | $-0.16$ | $0.44 \cdot 10^{-1}$ | |
| $B_c \to \Psi(3836)$ | $-0.59$ | $-0.59$ | $-0.42$ |
| This work | $0.21$ | $0.41$ | |

$A_{FB}(\mu)$ for $B_c \to \eta_c$, $J/\Psi$ and $\Psi(3836)$:

| Decay    | $A_{FB}(\mu)$ |
|----------|----------------|
| $B_c \to \eta_c$ | $0.67^{+0.02}_{-0.02} \cdot 10^{-5}$ |
| This work | $0.82^{+0.01}_{-0.01} \cdot 10^{-1}$ |
| $B_c \to J/\Psi$ | $0.89^{+0.01}_{-0.01} \cdot 10^{-5}$ |
| This work | $0.96^{+0.01}_{-0.01} \cdot 10^{-1}$ |
| $B_c \to \eta_c$ | $0.17_{-0.01}$ |
| This work | $0.19_{-0.01}$ |
| $B_c \to \eta_c$ | $0.14_{-0.01}$ |
| This work | $0.16^{+0.01}_{-0.01}$ |

$\alpha^*$ angular asymmetry for the $\mu^+ \mu^-$ pair in the decay $B_c^- \to \mu^+ \mu^- (J/\Psi) \bar{\nu}_\tau$.

| Decay    | $\alpha^*(e)$ | $\alpha^*(\mu)$ | $\alpha^*(\tau)$ |
|----------|----------------|-----------------|------------------|
| $B_c \to J/\Psi$ | $-0.29_{-0.01}$ | $-0.29$ | $-0.19$ |
| This work | $-0.34$ | $-0.24$ | |

Table 3. $\alpha^*$ angular asymmetry for the $\mu^+ \mu^-$ pair in the decay $B_c^- \to \mu^+ \mu^- (J/\Psi) \bar{\nu}_\tau$. in reasonable agreement with the ones of ref. [5] with two exceptions corresponding to final $\chi_{c1}$ and $\Psi(3836)$ where not even the sign of the asymmetry is the same.

For the decay $B_c^- \to J/\Psi \bar{\nu}_\tau$ with the $J/\Psi$ decaying into a $\mu^+ \mu^-$ pair one can evaluate another angular asymmetry. Calling $x_\mu$ the cosine of the polar angle of the $\mu^+ \mu^-$ pair, measured in the $\mu^+ \mu^-$ rest frame relative to the momentum of the decaying $J/\Psi$, we have that the differential decay width $d\Gamma_{B_c^- \to \mu^+ \mu^- (J/\Psi) \bar{\nu}_\tau}/dx_\mu = 1 + \alpha^* x_\mu$. Our results for the asymmetry parameter $\alpha^*$ are given in table 4. They are in reasonable agreement with the ones obtained by Ivanov et al. [5].

2.1 Heavy quark spin symmetry

While one can not apply ordinary heavy quark symmetry to hadrons with two heavy quarks, there is a symmetry that survives for those systems which is heavy quark spin symmetry (HQSS). This symmetry reflects the fact that for infinite heavy quark masses the spins of the two heavy quarks decouple.

We have checked that our model reproduces the constraints imposed by HQSS in the infinite heavy quark mass limit. Those constraints relate the form factors for $B_c$ semileptonic decays into final $0^-$ and $1^-$ mesons near the zero recoil point [5].

For the actual heavy quark masses we find small deviations from the infinite heavy quark mass limit relations for the $B_c \to \eta_c$, $J/\Psi$ case, while corrections are large for some of the form factors in the $B_c \to \eta_c$, $\eta_c$, $\eta_c$ cases. See ref. [2] for details.

3 Two-meson nonleptonic decays

Neglecting penguin contributions, the effective Hamiltonians used for these calculations are given by local four-quark operators of the current–current type [6,10]. We work, as it is usually done, in factorization approximation in which the amplitude only contains contributions as the ones depicted in fig. 1. To evaluate the amplitude we need different meson decay constants that we take from experiment or lattice data. For the $\eta_c$ we use our own theoretical result obtained with the model described in ref. [11].

In table 4 we show results for decays that include a $c\bar{c}$ meson in the final state. For decays with a $\pi^-$, $\rho^-$, $K^-$, $K^+$ meson in the final state we find good agreement with the data by Ebert et al. [8] while our results are roughly a factor of two smaller than the ones obtained by Ivanov et al. [12].

| Decay    | $\alpha^*$ |
|----------|------------|
| $B_c \to \eta_c$ | $0.59$ |
| $B_c \to J/\Psi$ | $0.34$ |

Table 4. $\alpha^*$ angular asymmetry for the $\mu^+ \mu^-$ pair in the decay $B_c^- \to \mu^+ \mu^- (J/\Psi) \bar{\nu}_\tau$. in reasonable agreement with the ones of ref. [5] with two exceptions corresponding to final $\chi_{c1}$ and $\Psi(3836)$ where not even the sign of the asymmetry is the same.

For the decay $B_c^- \to J/\Psi \bar{\nu}_\tau$ with the $J/\Psi$ decaying into a $\mu^+ \mu^-$ pair one can evaluate another angular asymmetry. Calling $x_\mu$ the cosine of the polar angle of the $\mu^+ \mu^-$ pair, measured in the $\mu^+ \mu^-$ rest frame relative to the momentum of the decaying $J/\Psi$, we have that the differential decay width $d\Gamma_{B_c^- \to \mu^+ \mu^- (J/\Psi) \bar{\nu}_\tau}/dx_\mu = 1 + \alpha^* x_\mu$. Our results for the asymmetry parameter $\alpha^*$ are given in table 4. They are in reasonable agreement with the ones obtained by Ivanov et al. [5].

Fig. 1. Diagrammatic representation of $B_c^-$ two-meson nonleptonic decay in the factorization approximation.

In table 5 we show results for decays that include a $b\bar{b}$ meson in the final state. For decays with a $D^-$, $D^*$, $D_s^-$, $D_s^*$ meson in the final state we are in good agreement with the results by El-Hady et al. [12], obtained using the Bethe–Salpeter equation, and the ones by Kiselev [13], obtained within the three point sum rules of QCD and nonrelativistic QCD.

In table 6 we show results for decays that include a meson with a $b$ quark. Our branching ratios for decays with a $B^0$, $B_\tau^0$ meson in the final state are in very good agreement, being $B_c^- \to B^0 \pi^-$ the exception, with the
Table 4. Branching ratios in % for exclusive two-meson nonleptonic decays of the $B_s^-$ meson that include a $c\bar{c}$ meson in the final state.

| Decay                | This work | LHCb | SE          |
|----------------------|-----------|------|-------------|
| $B_s^- \to \eta_c D^-$| 0.014$^{+0.001}_{-0.002}$ | 0.19 | 0.014       |
| $B_s^- \to \eta_c D^{*-}$| 0.015$^{+0.001}_{-0.002}$ | 0.19 | 0.013       |
| $B_s^- \to J/\psi D^-$| 0.0083$^{+0.0005}_{-0.0005}$ | 0.15 | 0.009       |
| $B_s^- \to J/\psi D^{*-}$| 0.031$^{+0.001}_{-0.002}$ | 0.15 | 0.028       |
| $B_s^- \to \eta_c D_s^0$| 0.44$^{+0.02}_{-0.02}$ | 0.44 | 0.26        |
| $B_s^- \to \eta_c D_s^{*-}$| 0.24$^{+0.02}_{-0.02}$ | 0.37 | 0.24        |
| $B_s^- \to J/\psi D_s^0$| 0.24$^{+0.02}_{-0.02}$ | 0.34 | 0.15        |
| $B_s^- \to J/\psi D_s^{*-}$| 0.68$^{+0.03}$ | 0.97 | 0.55        |

results by Ebert et al. [8], while for the case with a $B_s^0$ meson in the final state we are in very good agreement with the results by Ivanov et al. [10]. Finally for the case with a $B^-$, $B^{*-}$ meson in the final state we find, with just one exception ($B_s^- \to B^{*-} \pi^0$), very good agreement with the results by Ebert et al. [8].

This research was supported by DGI and FEDER funds, under contracts FIS2005-00810, BMF2003-00856 and FPA2004-05616, by Junta de Andalucía and Junta de Castilla y León under contracts FQM0225 and SA104/04, and it is part of the EU integrated infrastructure initiative Hadron Physics Project under contract number RI3-CT-2004-506078. J. M. V.-V. acknowledges a contract E.P.I.F. with the University of Salamanca.

Table 5. Branching ratios in % for exclusive two-meson nonleptonic decays of the $B_c^-$ meson that include a $\bar{B}$, $B$ meson in the final state.

| Decay                | This work | LHCb | SE          |
|----------------------|-----------|------|-------------|
| $B_c^- \to \bar{B}^+ \pi^-$| 0.11$^{+0.01}_{-0.01}$ | 0.20 | 0.10        |
| $B_c^- \to \bar{B}^0 \rho^-$| 0.14$^{+0.02}_{-0.02}$ | 0.20 | 0.13        |
| $B_c^- \to \bar{B}^0 K^-$| 0.010$^{+0.001}_{-0.001}$ | 0.015 | 0.009 |
| $B_c^- \to \bar{B}^0 K^*$| 0.0039$^{+0.0005}_{-0.0005}$ | 0.0048 | 0.0044     |
| $B_c^- \to \bar{B}^0 \pi^-$| 0.072$^{+0.012}_{-0.009}$ | 0.057 | 0.026      |
| $B_c^- \to \bar{B}^0 \rho^-$| 0.58$^{+0.08}_{-0.08}$ | 0.30 | 0.67        |
| $B_c^- \to \bar{B}^0 K^*$| 0.0049$^{+0.0008}_{-0.0008}$ | 0.0036 | 0.0044     |
| $B_c^- \to \bar{B}^0 K^*$| 0.30$^{+0.002}_{-0.002}$ | 0.13 | 0.032      |
| $B_c^- \to \bar{B}^0 \pi^-$| 3.51$^{+0.06}_{-0.06}$ | 3.9 | 2.46       |
| $B_c^- \to \bar{B}^0 \rho^-$| 2.34$^{+0.06}_{-0.06}$ | 2.3 | 1.38       |
| $B_c^- \to \bar{B}^0 K^-$| 0.29$^{+0.01}_{-0.01}$ | 0.29 | 0.21        |
| $B_c^- \to \bar{B}^0 K^*$| 0.10$^{+0.02}_{-0.02}$ | 0.11 | 0.0030       |
| $B_c^- \to \bar{B}^0 \pi^-$| 2.34$^{+0.14}_{-0.14}$ | 2.1 | 1.58       |
| $B_c^- \to \bar{B}^0 \rho^-$| 13.4$^{+0.6}_{-0.6}$ | 11 | 10.8       |
| $B_c^- \to \bar{B}^0 K^*$| 0.13$^{+0.02}_{-0.02}$ | 0.13 | 0.11       |

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