Nucleon QCD sum rules in nuclear matter including radiative corrections

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Abstract

We calculate the nucleon parameters in nuclear matter using the QCD sum rules method. The radiative corrections to the leading operator product expansion terms are included, with the corrections of the order $\alpha_s$ beyond the logarithmic approximation taken into account. The density dependence of the influence of radiative corrections on the nucleon parameters is obtained. At saturation density the radiative corrections increase the values of vector and scalar self-energies by about 40 MeV, and 30 MeV correspondingly. The results appear to be stable with respect to possible variations of the value of $\Lambda_{QCD}$.

1 Introduction

In nuclear physics the nucleon parameters (self-energies) in nuclear medium are expressed in terms of meson exchange. In QCD sum rules (SR) approach to the problem the self-energies are expressed in terms of exchanges by systems of weakly correlated quarks. Until now these were just uncorrelated quarks. The correlations can be presented through the QCD radiative corrections. In the present paper we investigate the influence of these corrections on the values of nucleon self-energies in symmetric nuclear matter.

The QCD sum rules (SR) method, suggested by Shifman et al. [1] succeeded in expressing hadron characteristics in terms of expectation values of QCD operators. This approach was initially used for mesons. Later it was expanded by Ioffe et al. [2, 3] to description of baryons (see also [4]). In the SR method one considers the function $\Pi(q)$ which describes the propagation of the system with the quantum numbers of a hadron

$$\Pi(q) = i \int d^4x e^{iq\cdot x} \langle 0 | T j(x) \bar{j}(0) | 0 \rangle ,$$

with the local operator $j(x)$ carrying the quantum numbers of the hadron. We shall consider the SR for proton. In this case

$$\Pi(q) = q_\mu \gamma^\mu \Pi^V(q^2) + i \Pi^I(q^2)$$

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with $\gamma_\mu$ and $I$ standing for the Dirac and unit $4 \times 4$ matrices.

The key point of the SR approach is the analysis of dispersion relations

$$\Pi^i(q^2) = \frac{1}{\pi} \int \frac{\text{Im} \Pi^i(k^2)}{k^2 - q^2} \, dk^2$$  \hspace{1cm} (3)

for the functions $\Pi^i(q^2)$. These equations are considered at large values of $|q^2|$, while $q^2 < 0$. The left-hand sides (LHS) of Eq. (3) are presented as power series of $q^{-2}$, with the QCD condensates being the coefficients of the expansion. This presentation is know as the operator product expansion (OPE) [5]. The spectral densities $\text{Im} \Pi^i(k^2)$ on the right-hand side (RHS) of Eq. (3) are usually approximated by the “pole+continuum” model

$$\text{Im} \Pi^i(q^2) = \lambda_N^2 \delta(k^2 - m^2) + \frac{1}{\pi} \theta(k^2 - W^2) \text{Im} \Pi^i_{\text{OPE}}(k^2).$$  \hspace{1cm} (4)

Thus the position of the lowest pole $m_1$, its residue $\lambda_N^2$ and the effective continuum threshold $W_1^2$ are the unknowns of the SR equations in vacuum. Usually the Borel transform of both sides of Eq. (3) is carried out.

Several lowest order OPE terms have been found in [2, 3]. These were the contribution of free three-quark loop and the terms containing the vacuum condensates $\langle 0|\bar{q}q|0\rangle$, $\langle 0|\bar{q}G_\mu^aG_\nu^{a\mu}|0\rangle$ and $\langle 0|\bar{q}G_\mu^a\gamma_\nu q|0\rangle$, with $q$ and $G_\mu^a$ the quark operators and the gluon field tensor.

Later this approach was expanded for description of nucleons in nuclear matter [6, 7, 8] (see also [9] and references therein). In this case there are three structure of the polarization operator

$$\Pi_m(q, p) = i \int d^4x e^{i(qx)} \langle M|\bar{j}(x)\gamma_\mu j(0)|M\rangle = q_\mu \gamma_\mu \Pi_m^0(q, p) + p_\mu \gamma_\mu \Pi_m^p(q, p) + \Pi_m^I(q, p),$$  \hspace{1cm} (5)

with $|M\rangle$ and $p$ – the vector of state of the matter and its four-momentum; $p = (m, 0)$ in the rest frame of the matter. The dispersion relations for the functions $\Pi^i(q^2) = \Pi^i(q^2, s)$ at fixed value of $s = (p + q)^2$

$$\Pi^i_m(q^2, s) = \frac{1}{\pi} \int \frac{\text{Im} \Pi^i_m(k^2, s)}{k^2 - q^2} \, dk^2$$  \hspace{1cm} (6)

are considered now instead of Eq. (3). The same approximations as in vacuum, i.e. the OPE and ”pole+continuum” model were used for the LHS and RHS of the SR. The LHS contains now contributions of two types. There are expectation values of the same operators as in the case of vacuum, averaged over the ground state of nuclear matter ($\langle M|\bar{q}q|M\rangle$, etc.). There are also the QCD condensates which vanish in vacuum. In the leading OPE terms these are the vector condensate $\langle M|\bar{q}(0)\gamma_\mu q(0)|M\rangle$ and the expectation value $\langle M|\bar{q}(0)\gamma_\mu D_\nu q(0)|M\rangle$ caused by the nonlocal vector operator $\langle M|\bar{q}(0)\gamma_\mu q(x)|M\rangle$.

The in-medium characteristics of the nucleons, i.e. the vector self-energy $\Sigma_v$ and the Dirac effective mass $m^*$ are unknowns of the SR equations in nuclear matter. Two other unknowns are the residue at the nucleon pole and the continuum threshold, which obtain new values $\lambda^2_m$ and $W^2_m$. The SR approach [6, 7, 8, 9] provided reasonable values of $\Sigma_v \approx 200$ MeV, $m^* - m \approx -300$ MeV, which are consistent with the results obtained by nuclear physics methods. Similar results have been obtained in other modifications of the SR approach [10, 11].
Interactions between the quarks in polarization operators (1) and (5) manifest themselves in radiative corrections, which contain the powers of the QCD coupling constant $\alpha_s$. In the asymptotics $q^2 \to -\infty$ the terms in which $\alpha_s$ is enhanced by the “large logarithm” $\ln(q^2)$ are the most important ones. The corrections $(\alpha_s \ln q^2)^n$ have been obtained in all orders for the leading OPE terms \cite{12} and included into calculations carried out in \cite{2, 3, 6, 7, 9}. The SR are considered at finite values of the Borel mass $M^2$, i.e.

$$0.8 \text{ GeV}^2 < M^2 < 1.4 \text{ GeV}^2,$$

where $\alpha_s$ is small enough for perturbative treatment of the radiative corrections. Perturbative calculation of such corrections in vacuum beyond the logarithmic approximation \cite{13, 14} provided a numerically large coefficient of the lowest radiative correction to the leading OPE terms. Analysis of the role of radiative corrections in vacuum Borel transformed SR method which includes also corrections to the four-quark condensate \cite{14} was carried out in \cite{15}. It was shown that at least for the standard current \cite{16}

$$j(x) = \varepsilon_{abc}[u^T_a(x)C\gamma_\mu u_b(x)]\gamma_5\gamma^\mu d_c(x)$$

these corrections modify mainly the value of the nucleon residue $\lambda_N^2$, while that of the nucleon mass suffers minor changes. In the present paper we are also using this current.

Recent calculations \cite{17} demonstrated that the lowest order radiative corrections to the leading OPE terms of the in-medium polarization operator (5) contain the coupling constant $\alpha_s$ multiplied by large coefficients, which are 7/2 in $\Pi_m$ structure and 15/4 in $\Pi_p$ structure. Since $\alpha_s/\pi \approx 0.15$ at the values of the Borel mass $M^2 \approx 1 \text{ GeV}^2$, new analysis of the sum rules with inclusion of radiative corrections is needed.

We present the polarization operator $\Pi_m$ determined by Eq. (5) as

$$\Pi_m(q^2, s) = \Pi(q^2) + \Pi_\rho(q^2, s),$$

with the vacuum term $\Pi$ determined by Eq. (1) while $\Pi_\rho(q^2, s)$ describes interaction with the nuclear matter. We include radiative corrections to the vacuum term up to the contributions $\sim q^{-2}$ of OPE. We include also radiative corrections to the leading OPE terms provided by interaction with nuclear matter. We compare the results provided by inclusion of radiative corrections in several ways.

It is well known that the sum of the terms $(\alpha_s \ln q^2)^n$ can be presented in terms of the function

$$L(q^2) = \frac{\ln q^2/\Lambda^2}{\ln \mu^2/\Lambda^2}$$

with $\Lambda = \Lambda_{QCD}$ while $\mu = 500 \text{ MeV}$ is the normalization point of the characteristic involved. The sum of corrections $(\alpha_s \ln q^2)^n$ to a certain term multiplies it by a factor $L^{-\gamma}$, with $\gamma$ reflecting the “anomalous dimension” of corresponding operator – see, i.e. \cite{18}.

In the present paper we consider the Borel transformed sum rules for the nucleon in nuclear matter, considering the dispersion relations for the operator $\Pi_\rho$ defined by Eq. (8), employing the current $j(x)$ suggested in \cite{16}. We follow our paper \cite{9} adding the radiative corrections to
the analysis carried out there. We include the logarithmic corrections and the correction of the
order $\alpha_s$ beyond the leading logarithmic approximation (LLA) to the condensates of dimension
d $= 3$, which are the vector and scalar condensates. We include also the LLA corrections to
the main condensates of dimension $d = 4$, which are due to nonlocal structure of the vector condensate.

We demonstrate that while the LLA corrections provide substantial changes of the values
of vector and scalar nucleon self-energies $\Sigma_v$ and $m^* - m$, these changes are to large extent
compensated by the terms $\sim \alpha_s$, which are beyond the LLA. For example, at saturation value
of nucleon density this composition of the radiative corrections adds 40 MeV and 30 MeV to $\Sigma_v$ and
$m^* - m$ correspondingly. These corrections effect mostly the values of the vector self-energy
and of the nucleon residue. The values of $m^* - m$ and of the continuum threshold suffer minor
changes.

Our main numerical results are obtained for $\Lambda_{\text{QCD}} = 150$ MeV corresponding to
$\alpha_s(1 \text{ GeV}^2) = 0.37$ in one-loop approximation. We used these values in our previous papers on the subject.
Somewhat larger values of $\Lambda_{\text{QCD}}$ and $\alpha_s(1 \text{ GeV}^2)$ are often used nowadays – see, e.g. [4]. The
possible modifications affect mostly the value of the nucleon residue, while the results for the
self energies appear to be stable within several percent. We recall the results of inclusion of
the radiative corrections to vacuum sum rules in Sect. II adding also some new data. The sum
rules in nuclear matter with radiative corrections are considered in Sect. III. The results are
discussed in Sect. IV.

\section{Radiative correction in vacuum}

Following [15] we present the lowest OPE terms of the operators $\Pi^q(q^2)$ and $\Pi^I(q^2)$ as

\begin{equation}
\Pi^q = A_0 + A_4 + A_6 + A_8; \quad \Pi^I = B_3 + B_7 + B_9, \tag{10}
\end{equation}

with lower indices showing the dimensions of the condensates, $A_0$ is the contribution of the free
3-quark loop. Perturbative inclusion of the lowest order $\alpha_s$ corrections provides

\begin{align}
A_0 &= -\frac{1}{64\pi^4} Q^4 \ln \frac{Q^2}{\mu^2} \left( 1 + \frac{71}{12} \frac{\alpha_s}{\pi} - \frac{1}{2} \frac{\alpha_s}{\pi} \ln \frac{Q^2}{\mu^2} \right); \\
A_6 &= \frac{2}{3} \frac{\langle 0|\bar{q}q\bar{q}q|0 \rangle}{Q^2} \left( 1 - \frac{5}{6} \frac{\alpha_s}{\pi} - \frac{1}{3} \frac{\alpha_s}{\pi} \ln \frac{Q^2}{\mu^2} \right); \\
B_3 &= -\frac{\langle 0|\bar{q}q|0 \rangle}{4\pi^2} Q^2 \ln \frac{Q^2}{\mu^2} \left( 1 + \frac{3\alpha_s}{2\pi} \right), \tag{11}
\end{align}

with $Q^2 = -q^2 > 0$. Radiative corrections to the terms $A_4$ and $B_7$ which are numerically small
[2, 3], and to the terms $A_8$ and $B_9$, which are known with poor accuracy are not included.

The Borel transformed nucleon sum rules [2, 3] can be written as

\begin{equation}
\mathcal{L}^q(M^2, W^2) = \mathcal{R}^q(M^2), \quad \mathcal{L}^I(M^2, W^2) = \mathcal{R}^I(M^2), \tag{12}
\end{equation}

\[\text{4}\]
with
\[ R^q(M^2) = \lambda^2 e^{-m^2/M^2}, \quad R^I(M^2) = \lambda^2 m e^{-m^2/M^2} \] (13)
describing the contributions of nucleon pole \((\lambda^2 = 32\pi^4\lambda_\pi^2)\), while
\[ L^q = \tilde{A}_0 + \tilde{A}_4 + \tilde{A}_6 + \tilde{A}_8, \quad L^I = \tilde{B}_3 + \tilde{B}_7 + \tilde{B}_9. \] (14)
Here \(\tilde{A}_i(\tilde{B}_i)\) denote the Borel transforms of the terms \(A_i(B_i)\) on the RHS of Eq. (10), multiplied also by \(32\pi^4\).

Explicit expression for \(\tilde{A}_i\) and \(\tilde{B}_i\) are given in [15]. Here we focus on the terms which contain the radiative corrections. Including the contributions \((\alpha_s \ln q^2)^n\) exactly and treating the corrections \(\sim \alpha_s\) beyond the logarithms perturbatively, we can write
\[ \tilde{A}_0 = \frac{M^6 E_2(W^2/M^2)r_0}{L^{4/9}}; \quad \tilde{B}_3 = \frac{2a(M^2)M^4 E_4(W^2/M^2)r_3}{L^{4/9}}; \]
\[ \tilde{A}_6 = \frac{4 a_4(M^2)}{L^{4/9}} r_6. \] (15)
Here \(E_1(x) = 1 - (1 + x)e^{-x}; \quad E_2(x) = 1 - (1 + x + x^2/2)e^{-x}\). The function
\[ \tilde{L}(M^2) = \frac{\ln M^2/\Lambda^2}{\ln \mu^2/\Lambda^2} \] (16)
comes from logarithmic corrections to the current \(j(x)\) and to condensates. Also in Eq. (15)
\[ a = -(2\pi)^2 \langle 0|\bar{q}q|0 \rangle; \quad a_4 = (2\pi)^4 \langle 0|\bar{q}qq\bar{q}|0 \rangle. \] (17)
For \(i = 0, 3\) the factors
\[ r_i = 1 + \frac{\alpha_s}{\pi} c_i; \quad c_0 = \frac{53}{12}, \quad c_3 = \frac{3}{2} \] (18)
include the radiative corrections of the order \(\alpha_s\) beyond the LLA [14].

The situation with the contribution \(\tilde{A}_6\) is a bit more complicated. The four-quark condensate is presented usually under factorization assumption [1]
\[ \langle 0|\bar{q}qq\bar{q}|0 \rangle = (\langle 0|\bar{q}q|0 \rangle)^2, \quad a_4 = a^2. \] (19)
However, one should clarify the point, where Eq. (19) is postulated. Assuming, following [1] that the factorization \(a_4(k^2) = a^2(k^2)\) to be true at certain \(k^2 = M_0^2\), with \(M_0^2\) belonging to the interval determined by Eq. (7) and employing \(a(M_0^2) = a(\mu^2)\tilde{L}^{4/9}\) one finds in the LLA
\[ a_4(M_0^2) = a^2(\mu^2)\tilde{L}(M_0^2)^{8/9}. \] (20)
On the other hand \(M^2\) dependence of the condensate \(a_4\) in framework of the factorization hypothesis was found in [14]
\[ a_4(M^2) = a_4(M_0^2) \left(1 - \frac{\alpha_s}{3\pi} \ln \frac{M^2}{M_0^2}\right). \] (21)
Of course, $\ln \frac{M^2}{M_0^2}$ is not a "large logarithm" and hence is included perturbatively. Thus we can write

$$\tilde{B}_3 = 2a(\mu^2)M^4E_1(W^2/M^2)r_3, \quad \tilde{A}_6 = \frac{4}{3}a^2(\mu^2)\tilde{L}(M_0^2)^{4/9}r_6(M^2, W^2). \quad (22)$$

Here

$$r_6(M^2, W^2) = 1 - \frac{\alpha_s}{3\pi}\left(\frac{5}{2} + \ln \frac{W^2}{M_0^2} + \mathcal{E}\left(-\frac{W^2}{M^2}\right)\right), \quad (23)$$

with

$$\mathcal{E}(x) = \sum_{n=1} x^n n \cdot n!$$

includes the lowest correction beyond the LLA.

In Eqs. (15) and (22) we assumed that the corrections of the order $(\alpha_s \ln M^2)^n$ and of the order $\alpha_s$ compose independent factors. This can be justified by the fact that the former terms come from integration over momenta $\mu^2 \ll k^2 \ll M^2$, while the latter come from $k^2 \sim M^2$.

In Table I we present the values of characteristics $m$, $\lambda^2$ and $W^2$ obtained with the radiative corrections being included in various ways. We show the results with the radiative corrections totally neglected (i.e. all $\tilde{L} = 1, r_i = 1$), the results in LLA based on Eq.(22) with $\tilde{L}$ determined by Eq.(16), while $r_i = 1$. We include the corrections beyond LLA, with $r_i$ determined by Eqs.(18) and (23). We show also the results for all perturbative inclusion of radiative corrections (PRIC)- see Eq.(11). We assume the factorization point for the four-quark condensate (Eq.(19)) to be $M_0^2 = 1$ GeV$^2$. We present also the results corresponding to factorization point $M_0 = \mu = 0.5$ GeV obtained earlier in [15].

3 Radiative corrections in nuclear matter

1. Sum rules without radiative corrections

Following [6, 7, 9] we consider the SR for

$$\Pi_\rho^q = A_{3\rho} + A_{4\rho} + A_{6\rho}; \quad \Pi_\rho^I = B_{3\rho} + B_{6\rho}; \quad \Pi_\rho^p = P_{3\rho} + P_{4\rho} + P_{6\rho}. \quad (24)$$

The Borel transformed SR are

$$\mathcal{L}_\rho^q(M^2, W_m^2) = \tilde{A}_{3\rho} + \tilde{A}_{4\rho} + \tilde{A}_{6\rho} = \Lambda_m(M^2) - \Lambda(M^2); \quad (25)$$

$$\mathcal{L}_\rho^I(M^2, W_m^2) = \tilde{B}_{3\rho} + \tilde{B}_{6\rho} = m^*\Lambda_m(M^2) - m\Lambda(M^2); \quad (26)$$

$$\mathcal{L}_\rho^p(M^2, W_m^2) = \tilde{P}_{3\rho} + \tilde{P}_{4\rho} + \tilde{P}_{6\rho} = -\Sigma_v\Lambda_m(M^2), \quad (27)$$

with $\Lambda_m(M^2) = \lambda^2_m e^{-m^2_m/M^2}$, $\Lambda(M^2) = \lambda^2 e^{-m^2/M^2}$; $\lambda^2_m$ and $W_m^2$ are the effective value of the residue and of the continuum threshold in nuclear matter; $m^2_m$ can be presented in terms of $\Sigma_v$ and $m^*$. [9].
Here \( q = q - p \alpha, Q^2 = -q^2, Q'^2 = -q'^2 \), the functions \( \eta_{a,b}(\alpha) \) are defined following \[8\ 9\]

\[
\langle M|\bar{q}(0)\gamma_{\mu}q(x)|M\rangle = \left( \frac{p_\mu}{m} \int_0^1 d\alpha e^{-i\alpha(px)} \eta_a(\alpha) + ix_\mu m \int_0^1 d\alpha e^{-i\alpha(px)} \eta_b(\alpha) \right) \rho,
\]

for \( (i = u, d) \) and

\[
A_{3\rho} = \frac{\langle M| \sum_i \bar{q}_i \gamma_0 q_i |M\rangle}{6\pi^2 m} (pq) \ln Q^2;
\]

\[
P_{3\rho} = -\frac{\langle M| \sum_i \bar{q}_i \gamma_0 q_i |M\rangle Q^2 \ln Q^2}{3\pi^2 m},
\]

\[
B_{3\rho} = \frac{-\langle N| \sum_i \bar{q}_i q_i |N\rangle \rho}{8\pi^2} Q^2 \ln Q^2, \quad \langle N| \sum_i \bar{q}_i q_i |N\rangle \approx 8.
\]

Since the SR are considered at \( s = (p + q)^2 = \text{const} = 4m^2 \) \[6\ 7\ 8\ 9\], we must put \( pq = (s - m^2 - q^2)/2 \) in Eq. (28). From Eqs. (28), (29) we obtain \[9\]

\[
\tilde{A}_{3\rho} = -8\pi^2 \langle M| \sum_i \bar{q}_i \gamma_0 q_i |M\rangle \frac{(s - m^2)E_0(M^2) - M^2 E_1(M^2)}{3m};
\]

\[
\tilde{B}_{3\rho} = -4\pi^2 \langle N| \sum_i \bar{q}_i q_i |N\rangle M^4 E_1(M^2) \rho;
\]

\[
\tilde{P}_{3\rho} = -\frac{32\pi^2}{3} \langle M| \sum_i \bar{q}_i \gamma_0 q_i |M\rangle M^4 E_1(M^2).
\]

The contributions of the fourth dimension \( A_{4\rho} \) and \( P_{4\rho} \) come mainly from the nonlocality of the vector condensate \( \langle 0| \sum_i \bar{q}_i(0)\gamma_0 D_{\mu} q_i(x = 0)|0\rangle \) (there is also a small contribution \( A^q_{4\rho} \) caused by the in-medium gluon condensate – see \[9\]). Such contributions can be expressed in terms of the nucleon structure functions \[6\ 7\ 9\]

\[
A_{4\rho} = A^q_{4\rho} + A^v_{4\rho};
\]

\[
A^v_{4\rho} = \frac{m}{6\pi^2} \ln \frac{Q^2}{\Lambda^2} \int_0^1 d\alpha \left( -\alpha \eta_a(\alpha) + 9\eta_b(\alpha) \right) \rho
\]

\[
+ \frac{1}{6\pi^2 m} \int_0^1 d\alpha \ln \frac{Q^2}{\bar{Q}^2} \left( (pq') \eta_a(\alpha) + 9m^2 \eta_b(\alpha) \right) \rho
\]

\[
(31)
\]

and

\[
P^v_{4\rho} = -\frac{1}{6\pi^2 m} \ln \frac{Q^2}{\Lambda^2} \int_0^1 d\alpha \left[ 5\alpha(pq') + 2\alpha^2 m^2 \eta_a(\alpha) + 9m^2 \alpha \eta_b(\alpha) \right] \rho +
\]

\[
+ \frac{1}{6\pi^2 m} \int_0^1 d\alpha \ln \frac{Q^2}{\bar{Q}^2} \left[ -\alpha(pq') - 2Q^2 \eta_a(\alpha) - 9m^2 \alpha \eta_b(\alpha) \right] \rho.
\]

\[
(32)
\]
with $\eta_a(\alpha)$ the standard deep inelastic nucleon structure function, the moments of the function $\eta_b(\alpha)$ can be expressed in terms of those of the function $\eta_a(\alpha)$ \cite{8}. The cut off $\Lambda_c$ will be eliminated by the Borel transform.

For the Borel transforms of the left hand sides of Eqs. (31) and (32) (multiplied by $32\pi^4$) we find

$$\tilde{A}_{\alpha\beta}(M^2) = u^\alpha(M^2) + h^\alpha(M^2); \quad \tilde{P}_{\alpha\beta}(M^2) = u^\beta(M^2) + h^\beta(M^2), \tag{33}$$

with the contributions $u^\alpha$ and $u^\beta$ coming from the first terms on the right hand sides of Eqs. (31) and (32), while $h^\alpha(M^2)$ and $h^\beta(M^2)$ come from the second terms. Expressions for $u^\alpha$ and $u^\beta$ are presented in \cite{9}, while $h^\alpha$ and $h^\beta$ require a special treatment. Due to the terms $\ln Q^2/Q^2$ they have finite cuts, corresponding to the singularities in the $u$ channel of the interaction between the baryon current and the quarks belonging to the nucleon of matter, i.e. by the exchange terms. In paper \cite{9} these terms have been neglected. This was justified by claiming for the description of the nucleon in the Hartree approximation.

Here we include the terms $h^\alpha$ and $h^\beta$. Performing the the Borel transform of

$$\ln \frac{Q^2}{Q^2} = \ln \left(1 + \alpha\frac{(Q^2 + X^2(\alpha))}{Q^2}\right),$$

with $X^2(\alpha) = \frac{\alpha}{1+\alpha}(s-m^2-m^2\alpha)$ and using the numerical values of the moments of the structure functions $\eta_a$ \cite{19}, we find the coefficients of $M^{-2}$ expansion of the terms containing $\eta_a$ to be of the same order of magnitude. Thus the OPE in powers of $M^{-2}$ series for these terms exhibits a poor convergence. Hence, following \cite{8}, we employ the explicit expressions

$$h^\alpha(M^2) = \frac{8\pi^2}{3m} \int_0^1 d\alpha \left[ \left( (s-m^2-2m^2\alpha)G_0(M^2,\alpha) - G_1(M^2,\alpha) \right) \eta_a(\alpha) - 18m^2\left( (s-m^2\alpha) \right) \eta_b(\alpha) \right], \tag{34}$$

$$h^\beta(M^2) = \frac{8\pi^2}{3m} \int_0^1 d\alpha \left[ \left( -(5s-m^2)\alpha + 6m^2\alpha^2 \right)G_0(M^2,\alpha) + \right.$$

$$\left. + (4+5\alpha)G_1(M^2,\alpha) \right) \eta_a(\alpha) - 18m^2\alpha\eta_b(\alpha) \right]. \tag{35}$$

Here $G_0(M^2,\alpha) = M^2(1-e^{-X^2(\alpha)/M^2})$; $G_1(M^2,\alpha) = M^4 \left( (1 + X^2(\alpha)/M^2) e^{-X^2(\alpha)/M^2} \right)$. We included only the lowest moments of the function $\eta_b(\alpha)$, which are related to the moments of the function $\eta_a(\alpha)$ by the equations, presented in \cite{8}. The values of these moments enable to expect the convergence of the latter expansion.

The contributions $A_{\alpha\beta}, B_{\alpha\beta}$ and $P_{\alpha\beta}$ are determined by the 4-quark condensate. Here the calculations require certain model assumptions. We shall use the expressions presented in \cite{9}, based on the calculations of the four-quark condensates \cite{20} in framework of the Perturbative Chiral Quark Model suggested in \cite{21}. Further development of this model is described in \cite{22}.
2. Inclusion of radiative corrections

Now we include the radiative corrections. If the LLA corrections \((\alpha_s \ln q^2)^n\) are included exactly, while the terms beyond the LLA are taken into account in the lowest order of perturbation theory, each term of Eq.(30), obtains a factor

\[
\xi^x = \frac{r^x}{L^{\gamma^x}}
\]

with \(x = q, I, x\), and \(r^x = 1 + \frac{\alpha_s}{\pi} c^x\).

Following [12, 3, 7] we put \(\gamma^q = \gamma^p = \gamma^q = 4/9\); \(\gamma^I = 0\). In the LLA \(c^x_{\pi x} = 0\). Beyond the LLA we employ the recently obtained in-medium parameters \(c^x = 7/2\) and \(c^p = 15/4\) [17] for corrections to the local vector condensate, while \(c^I = 3/2\) [14].

Considering the terms of the higher dimension, we include the factor \(L^{-4/9}\) corresponding to the anomalous dimension of the proton current. Note also that the nucleon structure functions [19] employed in our paper reproduce their moments with the proper anomalous dimensions. Corrections to contributions of the nonlocal vector condensate beyond the logarithmic approximation require calculation of additional set of the Feynman diagrams. As it stands now such corrections are not known.

We do not include the radiative corrections of the contributions \(A_{6\rho}, B_{6\rho}\) and \(P_{6\rho}\) to the four-quark condensates, provided by the nucleons of the matter. This is because these contributions are obtained in framework of a certain model [21], whose accuracy cannot be estimated.

Thus we include the radiative corrections BLLA to the leading OPE contributions containing condensates of dimension \(d = 3\). We include the LLA corrections to the terms containing condensates of dimension \(d = 4\). We do not include corrections to the terms with the condensates of higher dimension. Such approach is possible since the leading OPE terms indeed provide numerically largest contributions. For example, considering Eq.(27) for \(\Sigma_v\) and employing the vacuum values of parameters \(\lambda^2_m\) and \(\omega_2^2\), presented in Table 1 we can see that at the saturation value of nucleon density \(\rho_0\) the local vector condensate contributes approximately 330 MeV to \(\Sigma_v\), while nonlocal corrections and the four-quark condensate subtract about 50 MeV and 70 MeV from this value.

We compare several sets of results, corresponding to the cases, considered for vacuum in Sec.II.

3. Numerical results

Now we present the results of solutions of the SR equations. Following our previous works we put \(\alpha_s(1 \text{ GeV}^2) = 0.37\) which is consistent with the present data [23]. At the saturation density \(\rho_0\) we find the vector self-energy \(\Sigma_v = 230\) MeV, while \(m^* - m = -330\) MeV, if the radiative corrections are totally neglected.

Inclusion of the leading logarithmic corrections diminishes \(\Sigma_v\) and \(m^* - m\) by 70 MeV and 50 MeV correspondingly. Inclusion of radiative corrections beyond the logarithms makes
the values of $\Sigma_v$ and $m^* - m$ closer to those, obtained with total neglection of the radiative corrections. The data at saturation density are presented in Table II. Comparing the results of BLLA calculation and those obtained with perturbative inclusion of all $\alpha_s$ corrections, we see that they differ by about 20%. Thus it is important to include corrections $(\alpha_s \ln q^2)^n$ in all orders. Density dependence of nucleon parameters is shown in Figs. 1, 2.

Somewhat larger values of $\alpha_s(1 \text{GeV}^2)$ are often used in nowadays calculations. The authors of [17] employed $\alpha_s = 0.47$ corresponding to $\Lambda_{QCD} = 230 \text{ MeV}$ in one-loop approximation, while the value $\alpha_s(1 \text{GeV}^2) = 0.55$ corresponding to $\Lambda_{QCD} = 280 \text{ MeV}$ was used in [4]. We carried out calculations with these sets of parameters as well. Assuming $\alpha_s(1 \text{GeV}^2) = 0.55$ we found very small changes of values of the vector self-energy $\Sigma_v \approx 260 \text{ MeV}$ and of the scalar one $m^* - m \approx -290 \text{ MeV}$. The residue $\lambda^2$ suffered largest change, reaching the value $1.8 \text{ GeV}^6$. Recall that in the case of vacuum sum rules variation of the value of $\alpha_s$ also affected mostly the value of $\lambda^2$ [15]. Dependence of the results on the value of $\Lambda_{QCD}$ are shown in Table III.

### 4 Summary

We calculated nucleon self-energies $\Sigma_v$ and $m^* - m$ in nuclear matter in framework of the QCD sum rules approach, including the radiative corrections to the leading OPE terms. We demonstrated that there is a large compensation between the logarithmic corrections $(\alpha_s \ln q^2)^n$ and the corrections of the order $\alpha_s$ beyond the logarithmic approximation. We showed that at saturation value of nuclear density simultaneous inclusion of these corrections diminishes the values of the vector self-energy $\Sigma_v$ by about 40 MeV and of the scalar self-energy by about 39 MeV. The threshold value suffers minor changes. The radiative corrections affect mostly the value of the nucleon residue $\lambda^2_m$.

Result for the saturation density are presented in Table II. Density dependence of the parameters is shown in Figs. 1, 2.

We analyzed also stability of the results with respect to variation of $\Lambda_{QCD}$ and $\alpha_s(1 \text{GeV}^2)$ since other sets of values are often used in literature. We found the changes to affect mostly the value of the nucleon residue. The vector self energy $\Sigma_v$ and the scalar self energy $m^* - m$ may vary only by percents at the saturation density. The results are given in Table III.
Table 1: Nucleon parameters in vacuum. Line 1-all radiative corrections are neglected. Line 2-radiative corrections are included in the leading logarithmic approximation (LLA). Line 3-corrections $\sim \alpha_s$ are included beyond the LLA (BLLA). Lines 4-results for perturbative inclusion of radiative corrections (PIRC 1) $\alpha_s$ and $\sim \alpha_s \ln q^2$ with factorization (19) assumed at $M_0^2 = 1 \text{ GeV}^2$. Line 5-(PIRC-2) with factorization assumed at $M_0 = 0.5 \text{ GeV}$

| Rad. corrections            | $m$, GeV | $\lambda^2$, GeV$^6$ | $W^2$, GeV$^2$ |
|-----------------------------|----------|-----------------------|----------------|
| are neglected               | 0.94     | 1.84                  | 2.03           |
| LLA                         | 0.96     | 2.13                  | 2.37           |
| BLLA                        | 0.91     | 2.03                  | 1.96           |
| PIRC 1                      | 0.92     | 1.94                  | 1.85           |
| PIRC 2                      | 0.95     | 2.04                  | 1.91           |

Table 2: Nucleon parameters at the phenomenological saturation value of nucleon density. Notations are the same as in Table. I

| Rad. corrections            | $\Sigma_v$, MeV | $m^* - m$, MeV | $\lambda^2_{m*}$, GeV$^6$ | $W^2_{m*}$, GeV$^2$ |
|-----------------------------|------------------|----------------|--------------------------|-------------------|
| are neglected               | 229              | -329           | 1.10                     | 1.72              |
| LLA                         | 160              | -380           | 1.10                     | 1.77              |
| BLLA                        | 271              | -300           | 1.41                     | 1.75              |
| PIRC 1                      | 334              | -251           | 1.47                     | 1.80              |
| PIRC 2                      | 347              | -249           | 1.48                     | 1.81              |

Table 3: Dependence of the nucleon parameters on the values of $\Lambda_{QCD}$ and $\alpha_s$ at the phenomenological saturation value of nucleon density with radiative corrections included beyond the Leading logarithmic Approximation (BLLA). The results are presented for $\Lambda_{QCD} = 0.23 \text{ GeV}$, $\alpha_s(1\text{GeV}^2) = 0.47$ and $\Lambda_{QCD} = 0.23 \text{ GeV}$, $\alpha_s(1\text{GeV}^2) = 0.47$. The corresponding values of vacuum parameters are given in brackets.

| $\alpha_s(1\text{GeV}^2)$ | $\Sigma_v$, MeV | $m^* - m$, MeV | $\lambda^2_{m}$, GeV$^6$ | $W^2_{m}$, GeV$^2$ |
|---------------------------|------------------|----------------|--------------------------|-------------------|
| 0.47                      | 269              | -291 ($m = 0.93 \text{ GeV}$) | 1.65 (2.35) | 1.89 (2.13) |
| 0.55                      | 264              | -289 ($m = 0.94 \text{ GeV}$) | 1.81 (2.61) | 1.99 (2.26) |
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5 Figure captions

Fig.1 Density dependence of the vector self energy $\Sigma_m$ and of the scalar self energy $m^* - m$. The nuclear matter density $\rho$ is related to its saturation value $\rho_0$. Dotted lines: all radiation corrections are neglected. Dashed lines: radiative corrections are included in the Leading Logarithm Approximation (LLA). Solid lines: Corrections of the order $\alpha_s$ are included perturbatively beyond the LLA.

Fig.2 Density dependence of the nucleon residue $\lambda_2^m$ and of the continuum threshold $W_2^m$. Notations are the same as in Fig.2.
Figure 1:
Figure 2: