Scaling laws and phase space analysis of a geomagnetic domino model

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Abstract. The geomagnetic field is among the most striking features of the Earth. By far the most important ingredient of it is generate in the fluid conductive outer core and it is known as the main field. It is characterized by a strong dipolar component as measured on the Earth's surface. It is well established the fact that the dipolar component has reversed polarity many times, a phenomenon dubbed as dipolar field reversal (DFR). There have been proposed numerous models focused on describing the statistical features of the occurrence of such phenomena. One of them is the domino model, a simple toy model that despite its simplicity displays a very rich dynamic. This model incorporates several aspects of the outer core dynamics like the effect of rotation of Earth, the appearance of convective columns which create their own magnetic field, etc. In this paper we analyse the phase space of parameters of the model and identify several regimes. The two main regimes are the polarity changing one and the regime where the polarity remains the same. Also, we draw some scaling laws that characterize the relationship between the parameters and the mean time between reversals (\(mtr\)), the main output of the model.

Key words: Domino model, Heisenberg spin model, dipolar geomagnetic field, dipolar field reversals, convective columns, fluid outer core

1. Introduction
The Earth’s magnetic field (henceforth geomagnetic field) is a complex system and a primary example of a planetary magnetic field [1-6]. Actually, magnetic fields are ubiquitous in the Universe and seem to be associated with all astrophysical bodies ranging from planets to stars and even galaxies [1,3,6]. By far, the geomagnetic field is the most extensively studied. It is well known that this field is a superposition of magnetic fields originating from two groups of sources: the external field created by the current systems around Earth [2,3], and the internal field whose absolute majority of it originates in the outer core of the planet [1,4,5]. This planetary layer is made mostly of molten iron and additional lighter elements [1]. However, the presence of an electrically conducting fluid under rotation and in a temperature gradient from the Inner Core Boundary (ICB) to the uppermost Core–Mantle Boundary (CMB) are crucial to generate and maintain the geomagnetic field [5-7]. The mechanism, dubbed the dynamo mechanism, is very complicated and is an active field of research. There is a general consensus that the convective motion in the outer core compensates for the Ohmic losses of the medium, thus fulfilling the necessary energy requirements for the whole dynamo...
mechanism to work [1,6]. The geomagnetic ingredient generated by this mechanism is known as the main field or the geomagnetic dipolar field [3,4]; this particular field is our focus in the present study.

The theory behind the dynamo mechanism is widely explored and various types of dynamo models have been constructed focusing either on the thermal convection being dominant [7,8], or the chemical convection being the main driver of convection in the outer core [7]. All dynamo models consist of several equations that are appropriately brought into their non-dimensional forms. One typical dynamo model is provided below [9]:

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \tag{1}
\]

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla \left( p - \frac{1}{2} \mathbf{\Omega} \times \mathbf{r}^2 \right) + \rho \nabla^2 \mathbf{v} + \rho \mathbf{g} - 2\rho \mathbf{\Omega} \times \mathbf{v} + \mathbf{j} \times \mathbf{B} \tag{2}
\]

\[
\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \kappa \nabla^2 T + \varepsilon_T \tag{3}
\]

The equation (1-3) are in their physical form and constitute the magnetohydrodynamics (MHD) of the incompressible fluid of the outer core. In these equations, \( \mathbf{B} \) is the magnetic field, \( \mathbf{v} \) is the velocity field, \( T \) is the absolute temperature, \( \rho \) is the density considered constant under the Boussinesq approximation [9,10], \( p \) is the thermodynamic pressure, \( \mathbf{\Omega} \) is the angular velocity, \( \mathbf{j} \) is the current density and \( \mathbf{g} \) is the gravitational field. Meanwhile there are also the coefficients of magnetic diffusivity \( \eta \), the kinematic viscosity \( \nu \), the thermal conductivity \( \kappa \), and the heat source in the core \( \varepsilon_T \).

Equation (1) is named the induction equation and shows that the changing magnetic field is the outcome of the balancing between the source and dissipative terms (first and second terms on the RHS). Equation (2) is the well-known Navier-Stokes equation where the backreaction of the magnetic field is present. Also, there are included the Coriolis effect (second last term on RHS) and the centrifugal effect (which results in the decrease of the pressure \( p \)). Lastly, equation (3) is the energy equation which models the heat distribution in the outer core. This equation implies that thermal convection is considered well [11,12,13,14].

The analytical solution of the system (1-3) is momentarily out of question and therefore only the numerical methods allow their integration. However, this task is extremely expensive computationally—wise and other venues of numerical integration are explored. In the literature there are examples of stochastic models constructed to study the dipolar field and its main features [15,16]. In the present paper we study another type of models, namely the domino model [9,17-19]. This is a toy model and basically is a simplified version of the system (1-3). In other words, this model is an effort to quantify some of the crucial processes occurring in the outer core and at the same time being simple enough to allow fast computation. However, there is not up to date a direct mathematical way that relates the domino model with the system (1-3) [9]. Furthermore, there should be not any illusion that this model may somehow substitute the respective MHD equations.

The domino model is relatively simple and has an amenable theory. It is basically a modification of the well-known Heisenberg spin model which is particularly useful in the low temperature physics [9 and reference therein]. The Lagrange function of the model offers flexibility in the sense that we can use different terms in more imaginative ways. Thus, the model can be easily adapted into more complicated geometries than the one we discuss in this paper.

The domino model has been discussed in various details by several authors [17-19]. They have focused specifically on the ability to describe and reproduce the features of the dipolar geomagnetic field. More attention is devoted to the geomagnetic reversals [10-16], i.e. the complete reversal of the polarity of the dipolar moment that creates the dipolar field. In spite of the simplicity of the domino model, it has successfully reproduced some of the features of the dipolar field like the energy distribution against frequency (power spectral density) [18,19], the chron distribution (see below for the meaning of chron) [4,20], the dipolar moment magnitude distribution [17-19], etc.
In the present paper we focus more on the model itself. We believe it is very important to determine which region of the parameters’ space we should explore in order to obtain real Earth–like simulations. The determination of such domain is one of the aims of this study. The dynamics of the domino model is sharply affected by the changes in each of the parameters. In this paper we draw some scaling laws which will be useful in scaling the outputs of the model in different situations.

The paper is organized as follows: section 2 is devoted to the domino model and its theory; section 3 provides some results; in section 4 we make some discussion regarding the model and in section 5 we provide some conclusions.

2. The domino model and the physics behind it

The theory of the domino model is discussed extensively in several papers. The basic idea of the model is to generate a time series of the dipolar field which arises due to the collective interaction of the dynamo elements, i.e. macro spins [17-19]. Each of these elements creates its own magnetic field. Before discussing the model, we have to explain the physical assumption that lead to its construction in the first place. Thus, it will be more straightforward to interpret all the terms that appear in the Lagrange function of the domino model.

2.1. The physics behind the model

The electrically conduction fluid in the outer core is constantly under rotation. The fluid is considered to be highly conductive despite the debate about the precise magnitude of its conductivity [1,6,7]. However, for practical application the fluid can be considered to be ideally conductive, and with a very low viscosity. Under these conditions the Proudman–Taylor theorem is approximately valid [9,17]. As a consequence, the fluid flow tends toward being two dimensional and is organized into convective columnar or sheet-like structures also identified in MHD simulations [11,21,22]. These columns create their own magnetic field, whose polarity depends on the sense of rotation of the fluid in the column. Therefore, a term that describes this tendency of the flow should be present in the model’s Lagrange function.

There is a magnetic dipolar moment associated to each column and they do interact. One can model these interactions between dipolar moments in the same way as for spins (macro spins in this case) just like in a Heisenberg–like or Ising–like spin model [9]. In the Lagrange function there should be a term the models the aforementioned interaction. A crucial factor to be considered is how macro spins interact and two main version arise: only neighboring macro spins interact or all of them interact simultaneously. In the first case we obtain the Short–range Coupled Spins (SCS) domino model. This type is described thoroughly in many papers [17-19]. The other version, known as Long–range Coupled Spins (LCS) domino model is analyzed here. In has been examined elsewhere and actually it has been shown that LCS does better reproduce specific features of the dipolar field than the SCS model [18,19]. The present paper marks the first extensive study of the parameter space of the LCS model.

The Lagrange function of the LCS model contains also the kinetic energy term which physically represents the energy of motion of the fluid in the outer core. Recall that the columns are not rigid and during the convective motion of the fluid they may deform; this fact is modeled as the rotation of the macro spin around the axis of the system (see section 2.2).

The fluid flow in the outer core is considered to happen with very low viscosity. However, this is an approximation and due to the real limitations, the flow is not strictly two dimensional. There exist parasite fluid currents that do enable additional interaction between macro spins and obstruct the motion of the fluid in the columns. We model this effect as the usual drag force experienced while moving immersed in a fluid [18].

The thermal convection is considered to be the main factor in the core, although there are hints about chemical convection as well [8,9]. Their effect is harder to be modelled in the framework of the domino model especially regarding the spatial structure. Here we focus on the temporal behavior and considering that all the heat flux at the ICB and CMB varies in a complicated fashion [10,12,13,14],

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its effect on the flow is modelled as a random term. As a standard choice, the random term satisfies the normal distribution, while other distributions have been considered without much change [17,18]. Actually, there are other processes taking place in the outer core that should be considered in a more detailed way. In the discussion section are offered more insights.

2.2. The domino model
The physical background of the previous section is actually a guideline that one should follow to construct the domino model because up to date, there is not formulated a clear mathematical path that quantifies these physical processes into Lagrange function’s terms. As stated above, we focus on the LCS model which is more realistic regarding the real dipolar geomagnetic field.

The macro spins \( S_i \), are considered to have unit length and all of them are immersed in a medium with unit angular velocity \( \Omega \). Considering all the physical processes discussed in section 2.1, we formulate the Lagrange function of the domino model as below [9,18]:

\[
L = T - U = \frac{1}{2} \sum_{i=1}^{N} \dot{\theta}_i^2 - \gamma \sum_{i=1}^{N} (\Omega \cdot S_i) - \frac{\lambda}{2N} \sum_{i=1}^{N} \sum_{j \neq i} (S_i \cdot S_j),
\]  

(4)

where \( \gamma \) measures the tendency of the columns to be oriented to the axis of rotation, \( \lambda \) is the coupling coefficient that describes the interaction among macro spins, \( N \) is the number of macro spins and \( 2N \) on the RHS is a normalizing factor. Also, in (4) is excluded the meaningless self-interaction of the macro spins. In the case of the SCS model, the coupling term is simpler and is of the form

\[
\lambda \sum_{i=1}^{N} (S_i \cdot S_{i+1}),
\]

where the interaction is confined only to the closest neighboring macro spins. Due to the geometry of the outer core, it is practically forced to arrange the columns in a ring on the equator (figure 1). This implies automatically that the boundary condition will be \( \theta_{N+1} = \theta_1 \).

\[\text{Figure 1. A sketch of the domino model. The macro spins are positioned in the equatorial ring and the equilibrium alignment would be parallel to the axis of rotation. They can rotate around their central point and with } \theta_i \text{ we measure the angle each of them makes with the rotation axis at any instant of time. The figure is reproduced from } [17,18,19].\]

To include friction and random forcing (see section 2.1) we set up a Langevin-type equation as follows:

\[
\frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{\theta}_i} \right) = \frac{\partial L}{\partial \theta_i} - \kappa \dot{\theta}_i + \frac{\epsilon \chi_i}{\sqrt{\tau}},
\]

(5)
where $\kappa$ models the drag or "friction" experienced by the moving fluid and $\varepsilon$ is a measure of the random forcing. In (5) $\chi_i$ is a random number that satisfies normal distribution and $\tau$ is a regularizing parameter which is taken to be equal to the time step. Thus, we have considered gaussian forcing [17]. By substituting (4) into (5) and performing long but straightforward calculations, we obtain the differential equations of motion

$$\dot{\theta}_i - 2\gamma \cos \theta_i \sin \theta_i + \frac{\lambda}{2N} \sum_{j \neq i} (\cos \theta_j \sin \theta_j, -\cos \theta_i \sin \theta_i) + \kappa \dot{\theta}_i \frac{S\chi_i}{\sqrt{\tau}} = 0. \quad (6)$$

In equation (6) the index $i$ runs from 1 to $N$. In order to solve numerically the system of $N$ equations (6) we employ ode45 in MATLAB and convert them into a system of $2N$ first order ODEs.

The output of the model is the axial projection of the net dipolar moment, which by itself is calculated as the vector sum of the macro spins. The projection, labeled as magnetization, is calculated by the formula

$$M = \frac{1}{N} \sum_{i=1}^{N} (\mathbf{\Omega} \cdot \mathbf{S}_i) = \frac{1}{N} \sum_{i=1}^{N} \cos \theta_i. \quad (7)$$

One can analyze also the whole magnetic dipolar moment if needed as follows [18]:

$$M(t) = \sqrt{\left(\frac{1}{N} \sum_{i=1}^{N} \cos \theta_i\right)^2 + \left(\frac{1}{N} \sum_{i=1}^{N} \sin \theta_i\right)^2}. \quad (8)$$

However, we will consider in our analysis only (7), because (8) is more appropriate to be analyzed when considering full 3D problems of geomagnetism.

### 3. Results

Let us analyze some results of the several simulations. Each time series contained 75,000 entries and only one in 10 values was accepted during the simulations. The aim is to eliminate any long–range correlation that may arise during the numerical calculations [17]. All the time series contain reversals ranging from 18 to several hundreds and even over one thousand when $\gamma$ and $\lambda$ have positive values. In geomagnetism, a period of same polarity is known as chron [8,18]. In each simulation we measure the length for all chron and estimate the output quantity mean time between reversals ($\text{mtr}$). Clearly this quantity is also the mean length of chron and represents a simple mathematical average.

In figure 2 is shown a typical run of the LCS model. In the series are clear the intermittent reversals that occur in a seemingly random fashion [17,18]. In the lower panel is provided a smaller portion of the time series where are visible the typical variations of the dipolar moment observed also in several palaeomagnetic models [18 and reference therein]. The parameters we have chosen for this specific run are $\gamma = -1.0$, $\lambda = -1.5$, $\kappa = 0.10$, $\varepsilon = 0.40$ and $N = 8$. As we have mentioned in section 2, there is not yet a direct way to quantify the physical processes in the core and these parameters are arbitrarily chosen. This is actually a delicate aspect of the model that needs to be discussed (see discussion section).

We tested the LCS model for certain ranges of the parameters in order to analyze the parameter space and determine possible domains where certain regimes are established. In subsection 3.1 we discuss the results regarding this topic. In subsection 3.2 we analyze some scaling laws regarding the $\text{mtr}$ quantity.

#### 3.1. Parameter space analysis

The parameters space of the LCS model is quite complex because there are five independent parameters: $\gamma$, $\lambda$, $\kappa$, $\varepsilon$ and $N$. Thus, a visualization of the 5D parameters space is impossible here. Therefore, we will mention the ranges of the intervals for which we have seen reversals. We have to stress the fact that the analysis of the parameters space is far from being exhaustive and the conclusions drawn here are limited for the ranges we have analyzed in this paper.
Some of the parameters should have negative values, like $\gamma$ and $\lambda$. The positive values are physically erroneous because in that case the respective terms in (4) would respectively describe the tendency of the flow to be purely 3D and the repulsive interaction between convective columns. Regarding the other parameters, $\kappa$, $\varepsilon$ and $N$, they take only positive values because the friction always damps the mechanical energy of the flow and a random forcing has no physical meaning. When numerically integrating the differential equations of the model we kept all the parameters but one of them unchanged. We found for each parameter of the LCS model the respective range for which reversals occur and the results are listed in table 1. A similar process can be done also for the SCS model [17,18]. However, one should be very careful when considering these ranges because they were found by varying only one parameter. As an example, [18,19] have found the set of parameters: $\gamma = -2.1$, $\lambda = -2.0$, $\kappa = 0.015$, $\varepsilon = 0.20$ and $N = 10$, actually produces a reversal despite the fact that according to table 1 something like this should have not happened. This indicates that the analysis made thus far is incomplete and other interesting domains with their respective regimes need to be investigated.

| Parameter | Range            |
|-----------|------------------|
| $\gamma$  | $[-1.25, 0]$     |
| $\lambda$ | $[-1.50, 0]$     |
| $\kappa$  | $[0.0200, 0.100]$|
| $\varepsilon$ | $[0.400, 0.900]$|
| $N$       | $[1, 100]$       |

Table 1. Range of parameters for which reversals are observed. The analysis is inconclusive.

![Figure 2. A typical run of the LCS model. In the upper panel is shown the full run that spans 75,000 units in the time scale of the model. In the lower panel is shown the portion of the same series from](image-url)
38,000–39,500 in which a reversal occurs. The typical variation of the dipolar field follows after the reversal ends.

We have to stress the fact that the ranges shown in table 1 are not conclusive. This is especially true for the range of the number of dipoles because the provided range is about the simulations we have performed thus far. We have studied more in detail up to $N = 9$ and in the present paper we report the results for the series up to this value. A more extensive analysis is reported elsewhere for the SCS model [17] which is cheaper to compute because the model’s equations contain considerably less terms.

Interesting details emerge about the statistical features of the simulated series by analyzing the respective power spectra (figure 3). We show here the results for $N$, but a similar picture is observed for the other parameters as well. By increasing the number of macro spins, the frequency of reversals drops because when you have more macro spins it becomes harder to flip the majority of them. This explains the reduction of the power for increasing $N$. the most affected region of the spectrum pertains to the low-to-middle frequencies. This suggests that the dipolar moment reversals are specifically associated with this band of frequencies. Furthermore, as $N$ increases there are developed two different slopes in the low-middle frequencies region. The part of the belt that pertains to the lowest frequencies attains the new slope while the slope of the remaining portion remains unchanged. We interpret this occurrence by associating the reversals to the lower end of the band and other time changes with the remaining part of the frequency section. It seems that by adding more macro spins, there are added more equations which contain more terms that mitigate the effect of the random forces, like the dissipative term, the spin-spin interaction terms and the column orientation terms. This explains the drop in power and also the development of two frequencies indicates that the reversals and the other time changes are affected differently. Regarding the other sections of the spectrum, namely the low and high frequencies belts, there are not observed significant changes.

![Figure 3. Power Spectral Density for eight runs of the LCS model. All the parameters are fixed except for the number of dipoles, N. The markers are described in the legend.](image-url)
Similar developments are observed for the other parameters as well. Again, the affected section of the spectrum pertains to the Low-to-middle frequencies where by decreasing all of them (γ and λ have negative values), the power drops because the reversals become less frequent. Interestingly though the slope of the spectrum does not change in a noticeable way. When the values of the parameters change there are not added new terms or removed already existing ones. Thus, there is no reason for the slope to change. Naturally the matter is different when the number of macro spins changes.

3.2. Scaling laws
In this section we focus exclusively on analyzing the scaling of mtr with respect to each of the parameters of the LCS model. The “standard” simulation, described at the beginning of section 3, has a length of 75,000 units in model’s timescale. We equate this timespan with the robust reversals data of the last 150 Myr (read million years) [4,23]. There is some consensus that the minimal chron length is about 20,000 years [24,25], that is equivalent to 10 model’s time units. This enables us to consider this numerical value as the threshold to distinguish real chron. This analysis is carried out for all the simulations and mtr is calculated as the chron’s length mathematical average. All the time series contain from 21 up to several hundreds or even some thousands of reversals. Thus, the estimation of mtr is acceptably robust. For series that contain less reversals, one can either make the simulation longer or generate several shorter series and statistically analyze all of them simultaneously.

![Scaling laws graphs](image)

**Figure 4.** The scaling laws for the parameters N, γ and λ. The coefficients for each fit are calculated within 95% accuracy interval.

We perform several simulations by changing only one parameter, for all five parameters of the model. They take values in the ranges shown in table 1. Each set of time series contains from six to
eight simulations. We are aware of the fact that the limited number of data points make the fitting procedure somehow delicate and we intend to make the calculation robust by increasing the data points in the future. However, the results we have found so far are shown in figures 4 and 5. All the coefficients of the fitting lines have been calculated up to 95% accuracy.

The clue that the mtr is scaled exponentially with the parameters is obtained when plotting the data in semi log frame with mtr in the logarithmic axis (not shown here). In all cases the data manifest a linear dependence against the respective parameter. We are aware of the fact that the data quantity is small and the results shown here may be not robust enough. Therefore, a future objective is to increase the resolution by choosing more values for each parameter and possibly extend the ranges provided in table 1.

Let us now provide some interpretation about the scaling laws shown in figures 4 and 5. When \( \gamma \) increases in magnitude (become more negative) the mtr should decrease because the flow has more tendency to be 2D; the convective columns have a stronger tendency to align with the rotational axis. The decrease of the mtr as \( \lambda \) becomes more negative suggests that a stronger interaction between the convective columns actually has a stabilizing effect; a stronger interaction makes it harder to flip the columns. The increase of the mtr when \( \kappa \) increases can be interpreted as the stabilizing effect of the

![Figure 5. The scaling laws for the parameters \( \kappa \) and \( \epsilon \). The coefficients for each fit are calculated within 95% accuracy interval.](image-url)
friction force which impedes the flipping of the columns. Meanwhile the increase of the vigour of the random forcing (increasing \( e \)) makes the flipping easier and the reversals more frequent, thus reducing the \( m_{tr} \). Finally, by increasing the number of dipoles there are added more stabilizing terms (pertaining to rotation axis alignment, interaction and friction) rather than destabilizing ones (random forcing) which explains the drop in \( m_{tr} \). However, there is some indication for the SCS model that after \( N = 10 \) the \( m_{tr} \) decreases [18]. We have yet to analyse what happens when \( N \) further increases for the LCS model.

4. Discussion and future perspective

The domino model in all its versions is considered a toy model, i.e. a compressed model that contains simplified mathematical equation about the physical processes that occur in the outer core and are fundamental for the dynamo mechanism. This is an important observation because it means that some of the typical strongly nonlinear effects in the core may be unwittingly omitted from the model. While the LCS model produces consistent results regarding the physics involved, it is nonetheless limited in its actual form. In the following we list some of the concerns that need to be addressed in order to make this model much more suitable and valuable.

One of the processes deemed as important element of the dynamo mechanism is the differential rotation in the outer core, i.e. the angular velocity is different for different points [1,6,26]. This process, also known as the \( \alpha \)-effect, is completed neglected here. Meanwhile, in the outer core there is a backreaction of the induced magnetic field via Faraday’s law on the fluid flow, thus creating currents which generate magnetic field on their own. This whole process is known as the \( \alpha \)-effect [6,26]. In the LCS model it is incorporated into the random forcing and friction terms. Thus, we can assert that the actual model studied here is a simplified version of an \( \alpha^2 \) dynamo [6].

The friction term is rather mechanistical in nature and surely it does not reflect the reality of the fluid flow in the core where there is evidence of turbulent motion [20]. In order to make the model more accurate and realistic it is mandatory to cast the friction term in a much more suitable form that takes into consideration turbulence and all the rest. The same can be said for the random forcing term which is actually very simplistic. It is needed more specification that considers not only the temporal variations but more importantly the spatial variations which may be responsible for multiple observed features of the geomagnetic reversals [10,12,13].

Finally, the macro spins interaction terms are directly borrowed from statistical physics. If this type of interaction works fine there, application to geomagnetism brings some concerns. The convective columns are not rigid structures. Instead they are fluid and embedded in turbulence and the real “flip” actually consists in the destruction and re-establishment of a given column. Furthermore, these structures are not always observed but specifically under the conditions of weakly-driven dynamos, where the thermal convection is not that vigorous [21,22]. In order to make the LCS model with \( e \) broader target, this term needs appropriate modifications.

5. Conclusions

In the present paper we analysed the parameters space of the domino model in the LCS version. We determined empirically, though not in a conclusive way, the ranges of the parameters for which dipolar field reversals are observed. The quest is open not only to resolve the details of such ranges but also to extend them whenever it is possible.

We observed that while changing a given parameter the statistical properties of the simulated series change. When we change each of the parameters except \( N \), so that the reversals become less frequent, the spectral power is reduced especially in the low-middle band. This specific part of the spectrum apparently is associated to reversals (the low frequency part of the band) and some other variations of comparable timescale (middle frequency part of the band). The slope of the spectrum in this band remains the same suggesting that the power drops uniformly for all the concerned variations.

The situation is somehow different with \( N \). While \( N \) increases and the reversals rate decreases, there occurs a transition in the low-middle frequency band from one slope to two slopes. This means
that the reversals and the other time variations are affected differently. We are convinced that this different dynamic is due to the change in the amount of terms in the equations that are solved numerically.

We have drawn some scaling laws of the mean time between reversals with respect to each of the parameters of the model. All of them are exponential laws indicating a sharp change of \( mtr \) at one end of the range or at the other. We need to have better gauged ranges will help to obtain more accurate laws which on their turn will allow a better choice of parameters while simulating the geomagnetic field.

The domino model allows flexibility due to its simplicity. We intend to modify it in future works not only by providing better suited mathematical structures for the existing terms, but also by introducing new ones. Possibly this can lead to a better modelling of the dipolar geomagnetic field, its variation and geomagnetic reversals.

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