Current Account Sustainability in Latin America Considering Nonlinearities

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Abstract

We test current account sustainability based on the framework developed by Hakkio and Rush [1991] and Husted [1992] using a two-regime threshold vector error correction model. This methodology allows us to characterize short-run nonlinearities in the current account. We estimate the model for four Latin American economies: Chile, Brazil, Colombia, and Mexico. We find a long-run relationship between the current account components, which implies strong sustainability for Chile and Mexico and weak sustainability for Colombia and Brazil. For the first two countries, the predominant regime is associated with a current account surplus. In contrast, for Colombia and Brazil, the prevailing regime corresponds to a situation in which there is a long-run deficit. In general, the impulse response analysis shows that expenditure and income shocks have positive and significant responses in the predominant regime for both series.

Keywords. Threshold vector error correction; current account sustainability; generalized impulse response function.

JEL Classification. C32, C52, F32.

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1 Introduction

One of the policy makers’ main concerns is the magnitude and persistence of current account deficits. In particular, it is important to know the level of sustainability of the current account and its macroeconomic implications.

This study analyzes the sustainability of the current account in four Latin American countries under a nonlinear model. Latin American economies have been characterized by an export orientation and high vulnerability to external shocks. The dynamics of the current account of these countries show deficits of considerable and persistent magnitudes. Moreover, these economies have experienced current account reversals that caused large contractions in GDP and important macroeconomic adjustments.

Current account sustainability is also fundamental to understanding the effects of recent drops in commodity prices. In most Latin American countries, this shock led to a contagion effect on the economies in countries exporting such commodities. There are two important implications to highlight: first, a contraction in external revenues due to a fall in prices; and second, a drop in the trade-flow between these countries. Both implications led to abrupt changes in relative prices (real depreciation) that were not reflected in significant improvements in the trade balance. On the contrary, current account deficits became persistent or adjusted slowly at the cost of lower economic growth.

In this study, we use a two-regime threshold vector error correction (TVEC) to model the current account sustainability. We base the model on the notion of linear cointegration with short-run nonlinearities. To the best of our knowledge, we are the first to study the nonlinear sustainability of the current account from a multivariate point of view. We estimate the model for four Latin American countries: Colombia, Chile, Mexico, and Brazil.

The results show that the current account for Chile and Mexico is sustainable in the strong sense between 1996 and 2016. For Colombia and Brazil, we find a long-run relationship between income and expenditure of the current account for 1996 to 2015. However, this result suggests a long-run deficit that is consistent with the notion of weak sustainability [Quintos, 1995].

TVEC estimation yields a predominant and non-predominant regime for each
country. Chile and Mexico’s non-predominant regimes are characterized by periods in which expenditure exceeds income. These regimes are infrequent and of short durations. Meanwhile, in Brazil and Colombia, the non-predominant regimes are associated with periods where income is greater or very close to expenditure. Interestingly in the case of Brazil, the non-predominant regimes remain for long periods of time.

The rest of the paper proceeds as follows. Section two presents a review of different methodologies to evaluate current account sustainability along with the theoretical model. Section three presents the empirical methodology. In section four, we present the empirical results, and section five concludes.

2 Theoretical framework

2.1 Methodological Review

From a methodological perspective, researchers use two types of frameworks to evaluate current account sustainability. The first is accounting in the current account. Under this approach, the current account is sustainable if the ratio of net external assets as a percentage of GDP is stable or decreasing over time. Some approaches in this vein consider the valuation effects of changes in the asset prices of international portfolios and their effects on the current account dynamics, see for example Lane and Milesi-Ferretti [2012] and Gourinchas and Rey [2007].

The second set of approximations are empirical and use both linear and non-linear models. The linear category includes univariate models as proposed by Trehan and Walsh [1991] and Husted [1992]. They test whether the current account as a percentage of GNP has a unit root. In this sense, the current account as percentage of GNP is sustainable if the series is integrated of order zero. A disadvantage of this methodology is that it assumes that the current account income and expenditure are cointegrated with a cointegration vector given by (1, −1). We use a methodology that does not impose the latter restriction.

1We define the non-predominant regime in line with unstable processes as those were the modulus of the maximum eigenvalue of the companion matrix in the model (11) is greater than 1. Altissimo and Violante [2001] provide a further discussion of ergodicity and stability in threshold models.
From the multivariate point of view, vector error correction models (VEC) of current account income and expenditure are useful to explain the notion of sustainability. Nevertheless, the dynamics of spreads, risk perceptions, and portfolio decisions could be the most important determinants of nonlinearities in the current account. An important aspect of assuming nonlinearities is that even if the current account sustainability condition is fulfilled in the long term, persistent short-run deficits can still compromise future sustainability [Raybaudi et al., 2004]. Nonlinearities are important from the policy making perspective because they might change the characterization of current account sustainability. For example, Chortareas et al. [2004] find statistical evidence suggesting external debt sustainability in Latin American countries under nonlinear models despite the fact that most traditional linear tests suggest the opposite.2

2.2 Theoretical Model

To evaluate current account sustainability, we test the intertemporal budget constraint following Hakkio and Rush [1991] and Husted [1992]. Recent empirical applications include that by ¨Onel and Utkulu [2006], among others.

The framework is based on the intertemporal model of consumption. In particular, we assume an infinitely lived sovereign planner capable of borrowing and lending in international markets using one-period financial instruments. The objective of the central planner is to maximize the discounted sum of expected social utility subject to the intertemporal constraint

\[
\max_{\{C_t, B_t\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \beta^t [U(C_t)]
\]

\[Y_t + B_t = C_t + (1 + r_t) B_{t-1}, \tag{1}\]

where \(C_t\) represents consumption, \(B_t\) denotes international borrowing, \(Y_t\) is output, \(r_t\) is the one-period world interest rate, and \((1 + r_t) B_{t-1}\) represents the country’s foreign debt.

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2As Pippenger and Goering [2000] suggest, unit root and cointegration tests under threshold processes have low power against nonlinear alternatives.
For simplicity, we assume an endowment economy with no investment. The central planner chooses the paths of consumption \( C_t \) and international borrowing \( B_t \) to maximize the present value of the expected utility. The Euler equation entails

\[
E_t \frac{U(C_t)}{U(C_{t+1})} = \beta E_t (1 + r_{t+1})
\]  

(2)

Since \( Y_t - C_t = X_t^s - M_t \), we can express the budget constraint in terms of the trade balance as:

\[
B_t + X_t^s = M_t + (1 + r_t) B_{t-1}
\]  

(3)

Where \( X_t^s \) denotes exports and \( M_t \) imports. Assuming \( r_t \) is stationary around a mean \( r \) with \( r > 0 \), we can add and subtract \( rB_{t-1} \) on the right hand side of equation (3) and introduce the auxiliary variable \( F_t \equiv M_t + (r_t - r) B_{t-1} \), which represents deviations of the interest rate from its unconditional mean. Then, the budget constraint can be rewritten as

\[
B_t + X_t^s = F_t + (1 + r) B_{t-1}
\]  

(4)

Furthermore, assuming that \( X_t^s \) and \( F_t \) follow random walks with drift \( \Delta X_t^s = \eta_1 + \varepsilon_{1,t} \) and \( \Delta F_t = \eta_2 + \varepsilon_{2,t} \), where \( \varepsilon_{1,t}, \varepsilon_{2,t} \) are independent white noise processes. The first difference of equation (4) entails

\[
\Delta B_t + \Delta X_t^s = \Delta F_t + (1 + r) \Delta B_{t-1}
\]  

(5)

Using forward-looking integration to solve for \( \Delta B_t \), we have:

\[
\Delta B_t = \sum_{j=1}^{T} \left[ \left( \frac{1}{1 + r} \right)^j \left( \Delta X_{t+j}^s - \Delta F_{t+j} \right) \right] + \left( \frac{1}{1 + r} \right)^T \Delta B_{t+T}
\]  

(6)

Taking the limit as \( T \to \infty \) and noting that both components \( (X_t^s, F_t) \) are random walks, the changes in the foreign assets can be expressed as follows:

\[
\Delta B_t = (\eta_1 - \eta_2) \sum_{j=1}^{\infty} \left( \frac{1}{1 + r} \right)^j + \left[ \sum_{j=1}^{\infty} \left( \frac{1}{1 + r} \right)^j (\varepsilon_{1,t+j} - \varepsilon_{2,t+j}) \right] + \lim_{T \to \infty} \left( \frac{1}{1 + r} \right)^T \Delta B_{t+T}
\]  

(7)

Using the convergence criteria for a geometric series, the term \( \sum_{j=1}^{\infty} \left( \frac{1}{1 + r} \right)^j \) converges to \( \frac{1}{r} \). Therefore, the current account can be expressed as
△B_t = a + e_t + \lim_{T \to \infty} \left( \frac{1}{1 + r} \right)^T \Delta B_{t+T}, \quad (8)

where \( a = \frac{(\eta_1 - \eta_2)}{r} \) and \( e_t = \left[ \sum_{j=1}^{\infty} \left( \frac{1}{1 + r} \right)^j (\varepsilon_{1,t+j} - \varepsilon_{2,t+j}) \right] \).

Given that \( e_t \) is \( I(0) \) and assuming the limit term in (8) is equal to 0, if the current account, \( \Delta B_t \), is \( I(0) \), the intertemporal budget constraint holds.

**Current Account Sustainability Test**

In practice, the current account can be expressed in terms of net exports and interest rate payments by replacing equation (3) in (8) and assuming that the limit term is zero:

\[ \Delta B_t = MM_t - X^*_t, \quad (9) \]

where \( MM_t \equiv M_t + r_t B_{t-1} \) and we assume that the series of exports \( X^*_t \) is \( I(1) \). A necessary condition to show that \( \Delta B_t \) is \( I(0) \) is that \( X^*_t \) and \( MM_t \) are jointly \( CI(1,1) \) with cointegrating vector \((1, -1)\). We can test the latter condition by evaluating \( \beta = 1 \) in the following regression in a cointegration framework

\[ MM_t - \beta X^*_t = a^* + e^*_t \quad (10) \]

If there is no cointegration between \( X^*_t \) and \( MM_t \), the intertemporal budget constraint is not satisfied. On the other hand, if both series are cointegrated and \( \beta = 1 \), then the current account is sustainable in strong sense. Finally, if the series are cointegrated and \( \beta \neq 1 \), the current account is sustainable in a weak sense.3

As Hakkio and Rush [1991] point out, weak sustainability is consistent with the intertemporal budget constraint (solvency condition). However, in this case the undiscounted value of debt grows to infinity and the incentive for the government to default increases.

### 3 Empirical Methodology

Hansen and Seo [2002] first proposed the estimation for threshold vector error correction (TVEC) models using the maximum likelihood method. However, the properties

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3 Note that we normalize the coefficient of imports plus interest rate payments in equation (10). Hence, our criteria for weak sustainability is the inverse of that proposed by Quintos [1995].
of the estimators of the model were not established. Alternatively, Seo [2011] proposed the smoothed least squares estimation method (SLS) for TVEC models and developed the asymptotic theory for both the least squares (LS) and the SLS method. We adopt the latter estimation method.

Let $x_t$ be a $k$-dimensional $I(1)$ vector of a series that is cointegrated with a normalized cointegrating vector $\alpha = (1, \beta')'$; then, the corresponding error correction term can be written as $z_t(\beta) = x_t' \alpha$.

We can express the two-regime TVEC model as

\[ \Delta x_t = A_1'X_{t-1}(\beta) + A_2'X_{t-1}(\beta)I(z_{t-1}(\beta) > \gamma) + \epsilon_t \quad \forall t \in [p+1, T], \]  

(11)

where $X_{t-1}(\beta) \equiv (1, z_{t-1}(\beta), \Delta_{t-p})'$ with $\Delta_{t-p}$ denoting the vector of the lagged difference terms $(\Delta x_{t-1}', \Delta x_{t-2}', ..., \Delta x_{t-p}')'$, $A_1$ and $A_2$ are $(2 + kp) \times k$ matrices of coefficients, $\gamma$ is the threshold parameter, $I(\cdot)$ represents the usual indicator function, and $\epsilon_t$ is an i.i.d sequence with $E(\epsilon_t) = 0$ and $E(\epsilon_t \epsilon_t') = \Sigma$ a positive definite matrix.

Considering all available observations, we can rewrite the full model in (11)

\[ y = [(X(\beta), X_\gamma(\beta)) \otimes I_k] \lambda + \epsilon, \]  

(12)

where $\epsilon$ stacks $\epsilon_t$, $X(\beta)$ is the matrix that stacks $X_{t-1}(\beta)$, and $X_\gamma(\beta)$ stacks $X_{t-1}(\beta) I(z_{t-1}(\beta) > \gamma)$, $y$ stacks $\Delta x_t$, $\lambda$ is equal to $vec((A_1', A_2')')$, and $vec$ is the operator that stacks the columns of a matrix.

We can obtain the SLS estimator by replacing $I(z_t(\beta) > \gamma)$ with a function $\kappa(s_t(\beta, \gamma))$, where $s_t(\beta, \gamma)$ is given by $s_t(\beta, \gamma) = \left( \frac{z_t(\beta) - \gamma}{h} \right)$ and $h \to 0$ as $T \to \infty$. Furthermore, the function $\kappa(\cdot)$ must satisfy

\[ \lim_{n \to -\infty} \kappa(n) = 0, \quad \lim_{n \to \infty} \kappa(n) = 1 \]

In addition to the last two conditions, the function $\kappa(\cdot)$ must also fulfill certain technical requirements, such as those imposed in Seo and Linton [2007].

We obtain the smoothed model by replacing $X_\gamma(\beta)$ with $X_\gamma(\beta)^*$ in (12), where the latter is the matrix that stacks $X_{t-1}(\beta) \kappa(s_{t-1}(\beta, \gamma))$. Then, the SLS estimator is defined as

\[ \text{Argmin}_{\theta \in \Theta} S_T(\theta) \]  

where $S_T(\theta) := (y - [(X(\beta), X_\gamma(\beta)^*) \otimes I_k] \lambda)'(y - [(X(\beta), X_\gamma(\beta)^*) \otimes I_k] \lambda),$
where $\theta = (\beta', \gamma, \lambda')'$ and $\Theta$ is a compact parameter space.

Hansen and Seo [2002] solve the previous optimization problem in two stages. In the first stage, they consider a grid search for the parameters $(\beta', \gamma)$. In the second stage, given this parameters, we can obtain the estimator of $\lambda$ using LS.

Under some assumptions, Seo [2011] shows that the asymptotic distribution of the SLS estimator is

$$
\sqrt{T}(\hat{\lambda} - \lambda) \Rightarrow N\left(0, \left[\mathbb{E}\left(\begin{pmatrix} 1 & I_{t-1} \\ I_{t-1} & I_{t-1} \end{pmatrix} \otimes X_{t-1}X_{t-1}' \right) \right]^{-1} \otimes \Sigma \right),
$$

where $T$ is the sample size, $\Rightarrow$ denotes convergence in distribution, $W$ denotes a standard Brownian motion, and the last $k - 1$ components of $\frac{x_{Ts}}{\sqrt{T}}$ converge to a Brownian motion vector $B$ with covariance matrix $\Omega$. On the other hand, $\hat{\sigma}_v^2$ and $\hat{\sigma}_q^2$ are given by

$$
\hat{\sigma}_v^2 = \frac{1}{T} \sum t \left( \frac{1}{2\sqrt{h}} X_{t-1}(\hat{\beta})' A_2 \frac{dK(s_{t-1}(\hat{\beta}, \hat{\gamma}))}{ds_{t-1}} \hat{\epsilon}_t \right)^2,
$$

$$
\hat{\sigma}_q^2 = \frac{h}{2T} Q_{Tkk}(\hat{\theta}),
$$

where $Q_{Tkk}$ is the diagonal element associated with $\gamma$ of the Hessian matrix $Q_T(\theta) = \frac{\partial^2 S_T(\theta)}{\partial \theta \partial \theta'}$.

Note that the asymptotic distribution of $(\beta', \gamma)$ depends on the smoothing parameter $h$, which has to fulfill certain assumptions. We use $h = \hat{\sigma}_E \frac{T}{2} \log T$ as in Wang et al. [2016] and Seo [2011]. Furthermore, we must also consider that the vectors $(\beta', \gamma)'$ and $\lambda$ are asymptotically independent, so constructing the confidence intervals for the parameters in $\lambda$ is straightforward.

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4The grid for $\beta'$ can be centered around a preliminary estimate, such as that obtained from a linear VEC model.

5See Seo [2011].
4 Empirical Results

4.1 Data

Our data covers the period between the first quarter of 1996 through the second quarter of 2016 for Chile and Mexico, and until the last quarter and the third quarter of 2015, for Colombia and Brazil, respectively.

Following Herzberg [2005], we consider the current account components directly to test for sustainability. This disaggregation has been fundamental in explaining the current account dynamics in Latin American countries because factor income and remittances serve as buffers for export fluctuations.

We construct the current account income and expenditure for each country by considering the sum of goods and services, primary and secondary income records of the current account, including debit and credit records, respectively. Income and expenditure are measured as a percentage of GDP.

4.2 Results

Our modeling strategy consists of five steps. In the first step, we test for the presence of unit roots in the series of current account income and expenditure. The second step is to test the existence of a cointegration relationship between the aforementioned series. In the third, step we perform a test to choose between a TVEC model and the usual VEC model. Then, we estimate the TVEC model and we show an analysis of the model residuals in the fourth step. Finally, we compute the general-

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\footnote{We consider GDP instead of GNP since the total production that occurs outside the countries’ borders is not significant. On average, the ratio between GNP/GDP is around 94% for Colombia, 93% for Brazil, 95% for Mexico, and 93% for Chile during 1996 – 2015.}

\footnote{The series that we use in the empirical exercise are not exactly the same implied by the theoretical framework. However, the exports and imports of goods and services are accounted in primary income (credit and debit). Moreover, interest payments are part of the current account records. In addition, exports and imports plus interest payments represent a high percentage of current account income and expenditure, respectively. More precisely, exports represent between 77% and 97% of the current account income, while imports plus interest payments represent between 63% and 99% of the expenditure through all quarters. On average, they represent 84% and 89% for Colombia, 90% and 83% for Chile, 90% and 85% for Mexico, and 93% and 79% for Brazil. Considering these characteristics, the current account series that we use in the empirical exercise are consistent with the theoretical framework developed in section 2.}
ized impulse response function (GIRF) of the nonlinear model as the last step.

We begin our analysis by testing for the presence of unit roots in the series of current account income and expenditure for each country. For this purpose, in Table 1, we report the results for the variance ratio statistic proposed in Breitung [2002]. The null hypothesis of the test is that the series has a unit root, while the alternative implies that the series is stationary.

Table 1: Breitung [2002] Unit Root Tests

|          | Chile | Mexico | Colombia | Brazil |
|----------|-------|--------|----------|--------|
| Expenditure | 0.047 | 0.075  | 0.061    | 0.030  |
| Income   | 0.043 | 0.083  | 0.037    | 0.028  |

Authors’ calculations. *, **, and *** indicate significance at the 10%, 5 %, and 1% levels, respectively. $H_0 : y_t$ is $I(1)$
$H_a : y_t$ is $I(0)$.

Second, we conduct a test to confirm the existence of a cointegration relationship between current account income and expenditure. In Table 2, we present the results of two rank tests used to examine cointegration as proposed by Breitung [2001]. We correct both a Kolmogorov-Smirnov type ($\kappa^*$ statistic) and a Cramer-Von Mises type ($\epsilon^*$ statistic) to account for the correlation between the two series, as in Breitung [2001]. These tests are considered over the traditional cointegration tests following the results of Pippenger and Goering [2000], who showed that the latter have low power under threshold processes. The null hypothesis of the test is that the series under consideration are not cointegrated. Based on our results, we find statistical evidence to reject the null hypothesis of no cointegration in all cases at traditional levels of significance.

It is worth noting that each step assumes that the previous step has a certain result. For example, the cointegration step requires that the series are non-stationary.
Table 2: Cointegration Tests

| Country   | $\kappa^*$-Statistic | $\epsilon^*$-Statistic |
|-----------|-----------------------|------------------------|
| Chile     | 0.201***              | 0.006***               |
| Mexico    | 0.266***              | 0.005***               |
| Colombia  | 0.222***              | 0.008***               |
| Brazil    | 0.223***              | 0.009***               |

Authors’ calculations. *, **, and *** indicate significance at the 10%, 5 %, and 1% levels, respectively. $H_0 : x_1, x_2$ are not cointegrated $H_a : x_1, x_2$ are cointegrated.

Third, we compute the Hansen and Seo [2002] two-regime threshold cointegration test in VEC models. Under the null hypothesis, we can approximate the data generating process by a linear VEC model, while we consider a two-regime TVEC model under the alternative. Given the results in Table 3, we reject the null hypothesis of a linear VEC model at the 5% level of significance in all cases.

Table 3: Hansen Test Results for TVEC Models

|       | Chile   | Mexico  | Colombia | Brazil  |
|-------|---------|---------|----------|---------|
| Statistic | 32.39   | 34.29   | 33.02    | 35.28   |
| P-value  | 0.00    | 0.03    | 0.01     | 0.03    |

Authors’ calculations. P-values calculated using a residual based bootstrap method. $H_0 : \text{VEC}$ $H_a : \text{two-regime TVEC}$.

The results of the Hansen and Seo [2002] test confirm the presence of nonlinearities in the VEC representation of the cointegration relationships in each of the countries. For this reason, we conduct the estimation for the TVEC model described in section 3.
Table 4 presents the estimation results for the cointegrating coefficients of the TVEC models. This implies a long-run relationship between current account income and expenditure in all countries. In these cases, the current account is sustainable. In particular, there is strong sustainability for Chile and Mexico since the null hypothesis that $\beta = 1$ cannot be rejected at the 1% significance level. On the other hand, there is weak sustainability in Colombia and Brazil since this hypothesis is rejected.

To evaluate the proposed models, we consider several diagnostic tools, including the multivariate portmanteau and normality tests suggested in Lütkepohl [2005], as well as the multivariate autoregressive conditional heteroskedasticity (ARCH) tests in Tsay [2014]. On the other hand, to evaluate the stability of the models, we report the maximum eigenvalues of the companion matrix for both regimes of the TVEC models. Additionally, we evaluate the remaining nonlinearity in the residuals using the Broock et al. [1996] (BDS) test. We report the results of these tests in Appendix B, which shows no evidence of misspecification in the estimated models.

In Figure 1, we present the current account income and expenditure dynamics for Chile, Mexico, Colombia, and Brazil. Both series are measured as a percentage of GDP. Additionally, we show the estimated error correction term (EC) and threshold ($\hat{\gamma}$). We define the EC as $(Expenditure_t - \hat{\beta}Income_t)$. The threshold determines the transition between regimes (predominant and non-predominant).

For Chile and Mexico, the predominant regimes are periods where the EC is below the threshold. On the other hand, for Colombia and Brazil, the predominant regimes occur when the EC exceeds the threshold. The periods of the non-predominant regime are plotted in grey areas.

Table 4: Estimation Results for the Cointegrating Coefficients

|       | Chile | Colombia | Mexico | Brazil |
|-------|-------|----------|--------|--------|
| $\beta$ | 1.026 | 1.178    | 1.069  | 1.232  |
| St Dev | 0.011 | 0.033    | 0.040  | 0.023  |

Authors’ calculations. St Dev denotes the standard deviation.

9 Appendix A presents the threshold parameters and the short-run coefficient matrices.
10 We report $\hat{\beta}$ and $\hat{\gamma}$ in Table 4 and Table 5 of Appendix A, respectively.
Figure 1: Error Correction Dynamics

The model equation is described in (11), ECT denotes the error correction term $x_{t}'\hat{\alpha}$, threshold denotes $\gamma$, the non-predominant regimes are those where the modulus of the maximum eigenvalue of the companion matrix is greater than 1.
4.2.1 Chile

Chile’s current account has remained in balance for most of the years in the sample, as in the top-left panel of Figure [1]. This is partly due to the fact that the levels of public sector external indebtedness have been relatively low through the last 20 years.

The Chilean economy is characterized by high imports of hydrocarbons and copper exports. The behavior of the current account from 2004 onwards can be explained by positive shocks to the terms of trade due to an increase in international copper prices. Figure [1] suggests that this shock temporarily reversed during the financial crisis of 2007-2008, but was offset by the subsequent fall in oil prices.

The terms of trade remained high until 2011, when they began a gradual decline that persists until today. The latter effect is associated with the reduction of global demand (mainly in China), along with the reduction in the quality of copper exports.

The TVEC model identifies the non-predominant regime when the error correction term is above the threshold. For the Chilean economy, the non-predominant regime is associated with periods in which current account expenditure exceeds current account income. These periods were frequent and of short duration in the late 1990s, during which Chile had large capital inflows (1996-1997) that generated current account deficits. They were followed by contractionary monetary policies, low domestic liquidity, and a reduction in the terms of trade that put more pressure on the current account. On the other hand, the gradual decline in the international price of copper along with the increase in mining investments and strong growth in private domestic demand explain the other non-predominant periods during 2012-2013.

4.2.2 Mexico

The evolution of current account expenditure and income for the Mexican economy appears in the top-right panel of Figure [1]. We can analyze the dynamics of the external expenditures by accounting for the changes in consumption and investment explained by shocks to the terms of trade. These shocks are highly associated with oil production and exports along with the trade balance with the U.S.

Similar to Chile, the non-predominant regime in Mexico is related to periods where current account expenditure exceeds income. These periods occurred in the late 1990s and coincided with a contraction in the U.S. economy, stagnant non-oil
exports, and weaker domestic demand. The current account deficit in the late 1990s was financed mostly by foreign direct investment (FDI), primarily in financial sector investments. More recently, a fall in the international price of oil partly explains the brief non-prevalent period during 2015-2016.

4.2.3 Colombia

The lower-left panel in Figure 1 shows the evolution in current account expenditure and income for Colombia. Unlike Chile and Mexico, the Colombian current account deficit has been historically persistent. This is reflected in the estimation results for the cointegration vector in Table 1, which suggest a long-run deficit. In this case, the non-predominant regime corresponds to periods where the current account income and expenditure were very close.

Colombia accumulated imbalances in the external sector in the second half of the 1990s. This led to an economic adjustment between the end of the 1990s and the beginning of the 2000s. Following the adjustment, Colombia experienced an economic downfall and a temporary current account surplus. The other periods of the non-predominant regime observed during 2004-2005 are associated with improvements in the terms of trade, mainly from higher external demand in Venezuela.

Finally, the price of oil declined significantly in 2014. This led to a persistent negative shock to the terms of trade, which explains the increase in the current account deficit from then onward.

4.2.4 Brazil

The lower-right panel of Figure 1 shows the dynamics of Brazil’s current account. We can explain the dynamics by considering three historical phases.

The first phase begins in 1995 with the implementation of the ”Real Plan” and ends in 2002. During this period, the current account deficits increased and reached a maximum of 4.32% of GDP in 2002. The second phase occurred from 2004 to 2010 and was characterized by economic policies aimed to expand the domestic market and improve international trade conditions. This is reflected in current account surpluses since there were increases in commodity prices and in the export volume to
South American trading partners.

The third phase covers the period between 2010 and 2015 and is characterized by persistent current account deficits. This occurred for two reasons. First, a decline in the international trade of manufactured goods in response to the 2008 international financial crisis; second, a sharp increase in domestic demand generated by favorable terms of trade conditions in the previous years.

The estimated model suggests a non-predominant regime in the second phase. This regime was characterized by a surplus or balance in the current account. Similar to the case of Colombia, we found statistical evidence of a long-run deficit.

4.3 Generalized impulse response function

In this section, we evaluate the response of current account income and expenditure to a shock in both series by conducting a generalized impulse response function (GIRF) analysis as proposed by Koop et al. [1996]. The advantage of the GIRF over the traditional impulse response function is that it accounts for nonlinear dynamics and allows for asymmetries related to the shock size and the shock sign, as well as different behaviors depending on when the shock occurs.

For a given horizon \( (h) \), a given shock size \( (\delta) \), and a given history \( (\omega) \), the GIRF is defined as

\[
GIRF(h, \delta, \omega) := E[y_{t+h}|\delta, \omega] - E[y_{t+h}|\omega]
\]  

We can interpret the expectations of the equation above as the optimal forecasts of \( y_{t+h} \) at time \( t \) with and without a shock of size \( \delta \). For nonlinear models, we can calculate these expectations as in Granger and Teräsvirta [1993].

In our empirical exercise, the estimated GIRF includes all observations in the sample as histories. We also considered the following set of shocks: \( \left\{ \frac{\delta_i}{\sigma_i} \right\} \), where \( \sigma_i \) is the standard deviation of the \( i-th \) residual series taken from an evenly spaced grid between \([-3, 3]\) with grid distance of 0.1, \( i = expenditure, income \). We calculate the expectations in equation (15) using Monte Carlo methods, assuming normal distribution for all countries except Brazil. In this case, we employed bootstrap methods considering the non-normality of the residuals.

\[\text{Table 10 reports the results of the normality tests.}\]
The GIRF distribution can show multimodal behaviors in nonlinear models. We therefore considered high density regions (HDR), as proposed by Hyndman [1995].

We evaluate the response of current account income and expenditure to positive shocks between 1 and 3 standard deviations. We compute the GIRF for each country for predominant and non-predominant regimes and report the results in Appendix C. In each figure, the horizontal and vertical axes represent the time horizon and the magnitude of the response, respectively. The responses are given as a percentage of GDP. We considered 95% (light grey) and 90% (dark grey) regions for the HDR boxplots of the GIRF. The black points outside the boxes identify outliers.

4.3.1 Chile

In general, the response to both shocks is positive and statistically significant as Figure 2 and Figure 3 of Appendix C show. Moreover, the magnitude of the response is very similar in both the predominant and non-predominant regimes. In particular, for current account expenditure shocks, the magnitude of the responses in both series stabilizes around 2% in 5 years. However, the responses in income and expenditure to an income shock are slightly higher (3%) and have greater uncertainty.

Overall, the results suggest a short-term reversal from deficit (non-predominant regime) to surplus periods (predominant regime). A possible explanation is the countercyclical economic policy in the Chilean economy. This allows for macroeconomic buffers that soften the business cycle.

4.3.2 Mexico

We find that the responses to current account expenditure shocks are asymmetrical and statistically significant when these shocks occur in the predominant regime (Figure 4 of Appendix C). On the other hand, with respect to income shocks, the response in both series is positive and significant. The responses to both shocks are similar (less than 1%) in magnitude.

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12 HDR are such that for a given confidence level $\alpha$, the probability that the random variable falls within the region is greater than $1 - \alpha$. Furthermore, HDR satisfy two properties: first, given a confidence level, the region occupies the smallest possible volume in the sample space. Second, every point inside the region has a density at least as large as every point outside the region.
Conditioning over the non-predominant regime (Figure 5 of Appendix C), we find that the responses of expenditure and income to expenditure shocks are not significant and show multimodal behavior after 3 years. The responses of both series to income shocks are again not significant.

4.3.3 Colombia

As Figure 7 of Appendix C shows, the response to an expenditure shock in periods of current account deficits (predominant regime) is positive and statistically significant for both income and expenditure. The magnitude varies between 3% and 1%.

Furthermore, the responses to income shocks in the predominant regimes are significant at 10%. The expenditure responses are slightly smaller than income responses. The latter is uplifting since this might suggest that income shocks can generate pressure on the balance in current account deficit scenarios.

Moreover, the results in Figure 6 of Appendix C indicate that the responses to both shocks are only statistically significant in the first period after the shock during a current account surplus (or balance).

4.3.4 Brazil

For Brazil, we find that an expenditure shock has a positive effect on both series in current account deficit periods (predominant regime), as Figure 9 of Appendix C shows. In this case, the magnitude of the response for both current account components ranges from 1 to 3% over a 5-year horizon.

However, unlike the other countries, the responses of income and expenditure to an income shock are not statistically significant. Furthermore, as Figure 8 of Appendix C shows, the responses of both series to expenditure shocks are only significant up to the third year in periods of surplus (or balance). In the case of an income shock, the responses in both series are positive but only significant one or two periods after the shock.
4.4 Brief summary of GIRF results

In the case of Chile and Mexico, the predominant regime is characterized by a current account surplus, while for Colombia and Brazil, this regime is characterized by a current account deficit. Moreover, when the current account expenditure and income shocks occur in the predominant regime, the response of the current account components is positive, significant, and permanent for Chile, Colombia, and Mexico. In Brazil, we find that the response to income shocks is not statistically significant. The magnitude of these responses varies between 5% and 1% of GDP, except for Mexico, whose response is less than 1% of GDP.

On the other hand, the response to income and expenditure shocks in the non-predominant regime is not statistically significant in Mexico and Colombia. In Chile, the response to these shocks is almost equal to the response when the shock occurs in the predominant regime. For Brazil, the response is temporal and only significant for expenditure shocks.

5 Concluding remarks

In this study, we examine current account sustainability in a nonlinear framework by estimating a two-regime TVEC model for Chile, Colombia, Mexico, and Brazil. We find a long-run relationship between the current account components, which suggests strong sustainability for Chile and Mexico and weak sustainability for Colombia and Brazil.

In this context, strong and weak sustainability imply that the current account components have a common trend. In the strong case, a unit shock in expenditures is followed by an equivalent response in income in the long run. However, in the weak case, the response has a different magnitude. For Colombia and Brazil, we find that this response is less than one. Therefore, if expenditure increases by 1%, income will increase by less than 1% in the long term. Then, despite the long-run relationship between the current account components, the economy accumulates external imbalances over time.

The results of the model also indicate that the first regime is associated with current account surplus periods while the second is associated with current account deficits. For Chile and Mexico, the first regime associated with current account
surplus is predominant, while current account deficit periods are few and of short duration.

On the other hand, the regime related to current account deficit prevails for Colombia and Brazil. Despite this imbalance, there is a long-run relationship since both series are cointegrated. This result suggests a long-run deficit in the current account.

In the short run, the responses of both current account components to expenditure shocks are positive and significant for all countries in the predominant regime. However, in this regime, the response to income shocks is generally not significant for Brazil. In the non-predominant regime, we find that only in Chile are the responses to both shocks significant beyond two quarters.

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Appendix A  Estimation Results

The two-regime TVEC model can be written as

\[ \Delta x_t = A_1' X_{t-1}(\beta) + A_2' X_{t-1}(\beta) I(z_{t-1}(\beta) > \gamma) + \epsilon_t \]

where \( X_{t-1}(\beta) \equiv (1, z_{t-1}(\beta), \Delta_{t-p}')' \) with \( \Delta_{t-p} \) denoting the vector of the lagged difference terms \( (\Delta x_{t-1}', \Delta x_{t-2}', ..., \Delta x_{t-p}')', z_t(\beta) = x_t', \alpha, \) and \( \alpha = (1, \beta')' \).

Table 5: Estimation Results for Threshold Parameter

|         | Chile   | Colombia | Mexico  | Brazil |
|---------|---------|----------|---------|--------|
| \( \gamma \) | 0.028   | -0.033   | 0.002   | -0.006 |
| \( St \; Dev \) | 0.066   | 0.236    | 0.245   | 0.007  |

Authors’ calculations. \( St \; Dev \) denotes the standard deviation.

Table 6: Estimation Results for the TVEC(3) Model for Chile

| \( A_1 \) | Expenditure | Income |
|-----------|-------------|--------|
| \( Est \) | \( St \; Dev \) | \( Est \) | \( St \; Dev \) |
| \( EC_{t-1} \) | -0.20** | 0.08 | -0.03 | 0.08 | 0.03 | 0.21 | 0.06 | 0.20 |
| \( Expenditure_{t-1} \) | 0.12 | 0.13 | 0.06 | 0.12 | -1.07*** | 0.40 | -0.44 | 0.39 |
| \( Income_{t-1} \) | -0.15 | 0.16 | -0.23 | 0.16 | 0.91 | 0.92 | -0.04 | 0.90 |

| \( A_2 \) | Expenditure | Income |
|-----------|-------------|--------|
| \( Est \) | \( St \; Dev \) | \( Est \) | \( St \; Dev \) |
| \( Expenditure_{t-2} \) | 0.00 | 0.13 | -0.20 | 0.12 | -0.43 | 0.41 | -0.16 | 0.40 |
| \( Income_{t-2} \) | -0.12 | 0.16 | 0.12 | 0.16 | -1.41** | 0.63 | -1.16* | 0.61 |
| \( Expenditure_{t-3} \) | -0.19 | 0.12 | -0.31*** | 0.12 | -0.62 | 0.54 | 0.23 | 0.52 |
| \( Income_{t-3} \) | -0.08 | 0.16 | 0.28* | 0.16 | -0.65 | 0.46 | -0.65 | 0.45 |

Authors’ calculations. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively. \( Est \) denotes the estimated coefficient, \( St \; Dev \) denotes the standard deviation, and \( EC \) denotes the error correction term \( (z) \).
### Table 7: Estimation Results for the TVEC(5) Model for Mexico

|                      | $A_1$                      |                      | $A_2$                      |                      |
|----------------------|----------------------------|----------------------|----------------------------|----------------------|
|                      | **Expenditure**            | **Income**           | **Expenditure**            | **Income**           |
|                      | $\text{Est}$              | $\text{St Dev}$     | $\text{Est}$              | $\text{St Dev}$     |
| $EC_{t-1}$           | -0.40**                    | 0.18                 | -0.19                     | 0.17                 |
| Expenditure$_{(t-1)}$| 0.06                       | 0.26                 | 0.24                      | 0.24                 |
| Income$_{(t-1)}$     | -0.27                      | 0.30                 | -0.28                     | 0.28                 |
| Expenditure$_{(t-2)}$| -0.19                      | 0.26                 | -0.02                     | 0.24                 |
| Income$_{(t-2)}$     | 0.18                       | 0.29                 | 0.06                      | 0.27                 |
| Expenditure$_{(t-3)}$| -0.03                      | 0.26                 | -0.07                     | 0.24                 |
| Income$_{(t-3)}$     | -0.02                      | 0.28                 | 0.00                      | 0.26                 |
| Expenditure$_{(t-4)}$| -0.14                      | 0.25                 | 0.10                      | 0.24                 |
| Income$_{(t-4)}$     | -0.12                      | 0.27                 | -0.32                     | 0.26                 |
| Expenditure$_{(t-5)}$| -0.08                      | 0.25                 | -0.08                     | 0.23                 |
| Income$_{(t-5)}$     | 0.00                       | 0.27                 | 0.04                      | 0.26                 |

Authors’ calculations. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively. Est denotes the estimated coefficient, St Dev denotes the standard deviation, and EC denotes the error correction term ($z$).

### Table 8: Estimation Results for the TVEC(6) Model for Colombia

|                      | $A_1$                      |                      | $A_2$                      |                      |
|----------------------|----------------------------|----------------------|----------------------------|----------------------|
|                      | **Expenditure**            | **Income**           | **Expenditure**            | **Income**           |
|                      | $\text{Est}$              | $\text{St Dev}$     | $\text{Est}$              | $\text{St Dev}$     |
| $EC_{t-1}$           | -0.63**                    | 0.25                 | -0.25                     | 0.20                 |
| Expenditure$_{(t-1)}$| 0.26                       | 0.31                 | 0.02                      | 0.25                 |
| Income$_{(t-1)}$     | -0.76**                    | 0.36                 | -0.71**                   | 0.28                 |
| Expenditure$_{(t-2)}$| -0.75***                   | 0.28                 | -0.87***                  | 0.22                 |
| Income$_{(t-2)}$     | 0.51                       | 0.40                 | 0.16                      | 0.31                 |
| Expenditure$_{(t-3)}$| 0.22                       | 0.48                 | -0.18                     | 0.38                 |
| Income$_{(t-3)}$     | -1.06*                     | 0.57                 | -0.72                     | 0.45                 |
| Expenditure$_{(t-4)}$| -0.88***                   | 0.33                 | -0.88***                  | 0.26                 |
| Income$_{(t-4)}$     | 0.67                       | 0.51                 | 0.49                      | 0.40                 |
| Expenditure$_{(t-5)}$| 1.13                       | 0.73                 | 0.46                      | 0.57                 |
| Income$_{(t-5)}$     | 1.95**                     | 0.81                 | -1.19*                    | 0.64                 |
| Expenditure$_{(t-6)}$| -0.02                      | 0.37                 | -0.40                     | 0.29                 |
| Income$_{(t-6)}$     | -0.79                      | 0.62                 | -0.29                     | 0.49                 |

Authors’ calculations. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively. Est denotes the estimated coefficient, St Dev denotes the standard deviation, and EC denotes the error correction term ($z$).
|                | Expenditure  | Income  | Expenditure  | Income  | Expenditure  | Income  |
|----------------|--------------|---------|--------------|---------|--------------|---------|
|                | Est  | St Dev | Est  | St Dev | Est  | St Dev | Est  | St Dev |
| **Const**      | 0.01* | 0.01   | 0.01 | 0.01   | -0.01 | 0.01   | -0.01 | 0.01   |
| **EC_{t-1}**  | 0.33** | 0.14   | 0.26* | 0.16   | 0.08  | 0.28   | 0.32  | 0.31   |
| **Expenditure_{t-1}** | -0.19 | 0.31   | 0.14 | 0.35   | -0.51 | 0.48   | -0.84 | 0.53   |
| **Income_{t-1}** | -0.15 | 0.26   | -0.18 | 0.29   | 0.39  | 0.48   | 0.49  | 0.54   |
| **Expenditure_{t-2}** | 0.16  | 0.35   | 0.16 | 0.39   | -0.44 | 0.52   | -0.78 | 0.58   |
| **Income_{t-2}** | -0.52* | 0.28   | -0.42 | 0.31   | 0.64  | 0.49   | 0.80  | 0.55   |
| **Expenditure_{t-3}** | 0.74** | 0.36   | 0.62 | 0.40   | -1.21** | 0.51 | -1.34** | 0.57 |
| **Income_{t-3}** | -1.05*** | 0.29   | -0.78** | 0.33 | 1.03*** | 0.52 | 1.16** | 0.57 |
| **Expenditure_{t-4}** | -0.90** | 0.39   | -1.13*** | 0.43 | 0.28  | 0.50   | 0.54  | 0.55   |
| **Income_{t-4}** | 0.29  | 0.30   | 0.49 | 0.33   | -0.20 | 0.46   | -0.43 | 0.51   |
| **Expenditure_{t-5}** | -0.77* | 0.44   | -0.19 | 0.48   | 0.70  | 0.53   | -0.08 | 0.59   |
| **Income_{t-5}** | 0.40  | 0.31   | 0.11 | 0.35   | -0.58 | 0.46   | -0.27 | 0.51   |
| **Expenditure_{t-6}** | -1.22*** | 0.42   | -1.23*** | 0.46 | 1.11** | 0.49 | 1.35** | 0.54 |
| **Income_{t-6}** | 0.68** | 0.31   | 0.81** | 0.34 | -0.54 | 0.45   | -1.00** | 0.50 |

Authors’ calculations. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively. Est denotes the estimated coefficient, St Dev denotes the standard deviation, and EC denotes the error correction term (z).
## Appendix B  Diagnostic Results

Table 10: ARCH, Normality, and Portmanteau Tests for the Residuals of the TVEC Models

|                | Chile Statistic | Pval | Mexico Statistic | Pval | Colombia Statistic | Pval | Brazil Statistic | Pval |
|----------------|-----------------|------|------------------|------|-------------------|------|------------------|------|
| **ARCH**       |                 |      |                  |      |                    |      |                  |      |
| LM             | 8.18            | 0.61 | 9.58             | 0.48 | 6.12              | 0.81 | 6.39             | 0.78 |
| Rank-Based     | 5.97            | 0.82 | 14.99            | 0.13 | 11.16             | 0.35 | 11.91            | 0.29 |
| Corrected Q    | 54.88           | 0.06 | 55.20            | 0.06 | 28.83             | 0.91 | 38.05            | 0.56 |
| **Normality**  |                 |      |                  |      |                    |      |                  |      |
| JB             | 5.14            | 0.27 | 2.70             | 0.61 | 3.95              | 0.41 | 15.69            | 0.00 |
| **Portmanteau**|                 |      |                  |      |                    |      |                  |      |
| Lag 20         | 59.89           | 0.62 | 53.73            | 0.56 | 49.20             | 0.58 | 46.11            | 0.70 |

Authors’ calculations.

Table 11: Maximum Eigenvalues of the Companion Matrix of the TVEC Models

|          | A<sub>1</sub> |          | A<sub>1</sub> + A<sub>2</sub> |          |
|----------|---------------|----------|-------------------------------|----------|
| Chile    | Mexico        | Colombia | Brazil | Chile | Mexico | Colombia | Brazil |
| 1.00     | 1.00          | 1.26     | 1.04   | 1.32  | 1.27   | 1.00     | 1.00   |
| 0.83     | 0.92          | 1.26     | 1.04   | 1.32  | 1.13   | 0.94     | 0.92   |
| 0.83     | 0.76          | 1.03     | 1.00   | 1.15  | 1.13   | 0.84     | 0.92   |
| 0.68     | 0.76          | 1.03     | 0.97   | 1.00  | 1.00   | 0.84     | 0.91   |
| 0.68     | 0.73          | 1.01     | 0.97   | 0.91  | 0.96   | 0.84     | 0.91   |

Authors’ calculations. Reported modulus for the five maximum eigenvalues.
Table 12: BDS Test for Residuals of the TVEC Models

| Country   | Expenditure | Income |
|-----------|-------------|--------|
| Chile     | 0.21        | 0.63   |
| Mexico    | 0.06        | 0.95   |
| Colombia  | 0.89        | 0.22   |
| Brazil    | 0.78        | 0.04   |

Authors’ calculations. Reported p-values at embedding dimension 2, epsilon value equal to 1.5 standard deviations.
Appendix C  Generalized impulse-response functions

Figure 2: GIRF for the Predominant Regime in Chile

Shock in Expenditure – Response in Expenditure

Shock in Income – Response in Expenditure

Shock in Expenditure – Response in Income

Shock in Income – Response in Income

\begin{align*}
\text{Value} & \quad \text{Value} \\
1 & \quad 0.00 \\
3 & \quad 0.05 \\
5 & \quad 0.10 \\
7 & \quad 0.10 \\
9 & \quad 0.10 \\
12 & \quad 0.10 \\
15 & \quad 0.10 \\
18 & \quad 0.10 \\
\end{align*}

\begin{align*}
\text{Value} & \quad \text{Value} \\
1 & \quad 0.00 \\
3 & \quad 0.05 \\
5 & \quad 0.10 \\
7 & \quad 0.10 \\
9 & \quad 0.10 \\
12 & \quad 0.10 \\
15 & \quad 0.10 \\
18 & \quad 0.10 \\
\end{align*}
Figure 3: GIRF for the Non-predominant Regime in Chile
Figure 4: GIRF for the Predominant Regime in Mexico
Figure 5: GIRF for the Non-predominant Regime in Mexico
Figure 6: GIRF for the Non-predominant Regime in Colombia
Figure 7: GIRF for the Predominant Regime in Colombia
Figure 8: GIRF for the Non-predominant Regime in Brazil
Figure 9: GIRF for the Predominant Regime in Brazil
