Nonrelativistic phase in $\gamma$-ray burst afterglows

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Abstract: The discovery of multiband afterglows definitely shows that most $\gamma$-ray bursts are of cosmological origin. $\gamma$-ray bursts are found to be one of the most violent explosive phenomena in the Universe, in which astonishing ultra-relativistic motions are involved. In this article, the multiband observational characteristics of $\gamma$-ray bursts and their afterglows are briefly reviewed. The standard model of $\gamma$-ray bursts, i.e. the fireball model, is described. Emphasis is then put on the importance of the nonrelativistic phase of afterglows. The concept of deep Newtonian phase is elaborated. A generic dynamical model that is applicable in both the relativistic and nonrelativistic phases is introduced. Based on these elaborations, the overall afterglow behaviors, from the very early stages to the very late stages, can be conveniently calculated.

Keywords: gamma-ray bursts, afterglows, jets, shock waves, relativity

1 Introduction

$\gamma$-ray bursts (GRBs) are brief bursts of high energy radiation from the sky. They were serendipitously discovered by Vela satellites in late 1960s, and were first publicly reported by Klebesadel et al. in 1973 [1]. In their classic paper, Klebesadel et al. reported 16 events simultaneously detected by at least two satellites. Their duration varies from less than 0.1 s to about 30 s, and the light curve can be very complicated. The energies of emitted photons are mainly between 0.2 MeV and 1.5 MeV.

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Such strong bursts in γ-rays are completely unexpected to astronomers, thus received wide attention from the beginning. However, the enigma of GRBs once puzzled us for a long time, because the observational data were so coarse at early times that many basic problems that are closely related to observations are uncertain. The most representative problem is how far are GRBs from us. Since the localization ability of γ-ray detectors is very poor, astronomers could not find the counterparts of GRBs in optical, infrared, and radio bands. It is then impossible to measure the distances of GRBs directly.

In 1980s, many researchers tended to believe that GRBs are within our Galaxy. In April 1991, an unprecedentedly sensitive high energy satellite, Compton Gamma-Ray Observatory (CGRO), were launched by NASA, leading to a rapid increase in the number of observed GRBs. The BATSE instrument onboard CGRO unexpectedly found that GRBs are isotropically distributed in the sky and they do not show any corelation with the structure of our Galaxy. This strongly hints that GRBs are of cosmological origin. A direct measure of the distance becomes even pressing. The final breakthrough were made in 1997: thanks to the successful operation of the X-ray telescope onboard the Italian-Dutch BeppoSAX satellite, a few GRBs were rapidly and accurately localilzed, leading to the discovery of their X-ray, optical and radio counterparts (i.e., afterglows), and even their host galaxies. The enigma of GRBs began to be unveiled [2, 3, 4].

Rapid progresses have been made in the field of GRBs since the discovery of afterglows. Important results include: the association of long GRBs with type Ic supernovae, the observations of early afterglows, and the detection of a high redshift GRB. Today, it has been well known that GRBs occur at the far end of our Universe. The energy release is so enormous that GRBs are believed to be the most violent explosions since the Big Bang. The distance problem is resolved, but theorists are now confronted with an even more tough task of explaining the nature of GRBs.

A few important review articles are available for GRBs [5, 6, 7, 8, 9, 10, 11, 12]. In this article, we mainly concentrate on the nonrelativisitic phase of GRB afterglows. For completeness, we first introduce the observational aspects of GRBs and afterglows in Section 2, and their standard theoretical interpretations in Sections 3. Section 4 is devoted to the nonrelativistic phase of afterglows. Finally, Section 5 is our conclusion and discussion.
2 Observational Characteristics

GRBs could only be detected shortly in $\gamma$-rays before 1997, which seriously prohibited researchers from understanding their nature. Beginning from 1997, people find that GRBs are followed by long-lived low energy (i.e., X-ray, optical, infrared and radio) afterglows. Such observations provide key clues for our understanding of these enigmatic objects. Below, we briefly review the observational properties of GRBs and their afterglows.

2.1 GRBs

The main burst phase of GRBs is usually very short, during which the emitted photons are mainly $\gamma$-rays. The basic features are summarized as follows:\footnote{5}:

- Event rate. During its operation period (1991 — 2000), BATSE could detect one or two GRBs almost every day, indicating that they are not rare events. Till now, more than 3000 GRBs have been observed, most of which were contributed from BATSE observations. Since we have known that GRBs occur at cosmological distances, it can be easily derived that a GRB will happen in one typical galaxy every $10^4$—$10^5$ years (assuming that the radiation is isotropic).
2704 BATSE Gamma-Ray Bursts

Figure 2: The distribution of 2704 GRBs observed by BATSE on the sky (http://www.batse.msfc.nasa.gov/batse/grb/skymap/).

- Durations. The distribution of GRB durations is bimodal, i.e., GRBs can be divided into two sub-groups: long bursts with durations clustered at $\sim 20$ s, and short bursts with durations clustered at $\sim 0.2$ s$^{13}$. Generally speaking, the spectra of short bursts are slightly harder than those of long bursts. So these two sub-groups are usually called short/hard bursts, and long/soft bursts respectively. Long bursts are about three times as many as short ones.

- Light curves. The temporal structure of GRBs is very complex. The light curves of some GRBs are smooth, but in most cases the light curves are highly variable. Figure 1 gives some examples. It has been specially noted that in a few events, the $\gamma$-ray flux increases rapidly from the background to the peak in less than $\delta t \sim 0.2$ ms. This provides a strong constraint that the size of the radiation zone should be less than $c\delta t \sim 60$ km if the emitter’s bulk velocity is much less than the speed of light.

- Spectral features. The photon energy of a GRB is typically between 200 keV and several MeV. In some rare cases, it can even extend to $\sim 20$ GeV. In the typical energy range, the spectrum is obviously non-thermal, and can be well
approximated by a power-law or a broken power-law function \(^{(14)}\).

- Energetics. The typical fluence of GRBs is between \(10^{-9} \text{ to } 10^{-6} \text{ J/m}^2\). The total radiated energy will be \(\sim 10^{45} \text{ J}\) if the emission is isotropic. In some extreme cases, the energy can even be as large as \(10^{47} \text{ J}\), or \(\sim 2M_\odot c^2\).

- Sky distribution. BATSE found that the distribution of GRBs is isotropic in the sky \(^{(15)}\), as illustrated in Figure 2. No correlation can be found between GRBs and the structure of our Galaxy. It provides the first clue that GRBs should be of extra-galactic origin. Additionally BATSE also found that there are too few weak GRBs as compared with original expectation. It can be interpreted as the effect of the expansion of our Universe. Due to these BATSE discoveries, the opinion that GRBs should be of cosmological origin became more and more popular in the 1990s.

Here we must note that there are actually two kinds of GRBs: soft \(\gamma\)-ray repeaters (SGRs) and classical GRBs. SGRs can repeatedly but randomly burst out. On the contrary, classical GRBs do not show any repetition activities. Only 5 SGR sources have been found till now. More than 100 bursts have been observed from the most active SGR, while less than 10 events were observed from the most inactive one. Another feature of SGRs is that the average photon energy is low, i.e., \(\sim 30\ \text{keV}\). It has been generally accepted that SGRs are neutron stars with super-strong magnetic field (i.e., magnetars) in local galaxies \(^{(16)}\). In this article, all “GRBs” are referred to classical GRBs unless declared explicitly otherwise.

2.2 Afterglows

It has long been expected that GRBs should be associated with afterglows. However, due to the awkward localization ability, afterglow was detected for the first time only till 1997. In the operation period of BeppoSAX (1997 — 2000), only one or two GRBs could be localized each month. On Dec 20, 2004, the Swift satellite was launched \(^{(17)}\). It can localize nearly 200 GRBs every year, leading to a rapid increase in the afterglow sample. Till the end of January 2007, \(\sim 480\) GRBs have been rapidly localized, of which \(\sim 80\%\) are detected in X-rays, \(\sim 40\%\) (i.e. 176 events) are detected in optical or infrared bands, and \(\sim 10\%\) are detected in radio. Redshifts have been measured for \(\sim 20\%\) (111) GRBs, with the highest record being \(z = 6.29\) for GRB 050904 \(^{(18, 19, 20)}\).

Before 2005, afterglows were usually detected several hours after the main bursts due to the limited localization ability. At this stage, the optical/infrared afterglow
Figure 3: R band afterglow light curves of 24 GRBs\cite{23}.

generally decays as a power-law function of time, $S_\nu \propto t^{-1.0} - t^{-1.4}$. Several days later, the decay becomes steeper as $S_\nu \propto t^{-2.0} - t^{-2.5}$, so that a break can be observed in the afterglow light curve\cite{21,22}. This break can be well explained by the jet effect, which will be further illustrated later. The optical/infra red afterglow light curves are generally smooth (see Fig. 3 for some examples\cite{23}), although rebrightening can also be occasionally seen in a few events. In the radio bands, afterglow behaviors are some what different. When the observer’s time ($t$) is less than 20 — 40 days, the radio emission is in a brightening phase. Fast variability can also be observed, which should be scintillation due to the scatter by the interstellar medium. When $t > 20 — 40$ days, the radio afterglow decays as a power-law of time, and the scintillation also disappears\cite{4}.

Thanks to the quick response of Swift satellite, very early afterglows are observed from many GRBs since 2005. In a few cases, afterglows are observed even for $t < 20$ s. At the very early stage ($t \leq 100 — 300$ s), the X-ray afterglow decays steeply, then it enters a shallow decay phase that may last for a few hours, and finally connects with the later afterglow as described in the above paragraph\cite{24}. An unexpected result of Swift is that X-ray flares were detected during the period of 100 s $\leq t \leq 10000$ s in many GRBs\cite{25,26}. These enigmatic flares can hopefully provide useful clues for our understanding of the central engine of GRBs.
Figure 4: A Cartoon illustration of the X-ray afterglow of GRBs [27].

The X-ray afterglows of GRBs show some general characteristics [27, 28], as illustrated in Fig. 4. In this cartoon figure, the segment marked with “0” indicates emission of the main burst phase; Segment “I” indicates the early fast decay phase, usually with $S_\nu \propto t^{-3} - t^{-5}$; Segment “II” indicates the subsequent shallow decay phase, with $S_\nu \propto t^{-0.2} - t^{-0.8}$; Segment “III” is the late normal decay phase, $S_\nu \propto t^{-1.0} - t^{-1.3}$; Segment “IV” is the late fast decay phase, $S_\nu \propto t^{-2.0} - t^{-2.5}$; The dashed segment “V” refers to possible flares at early stage. Note that multiple flares may be observed in a single event. It should be pointed out that not every GRB has all the five segments. Actually most GRBs lack one or two of these components, and very few events can have all the five components.

3 Fireball Model

The discovery of afterglows leads to a final solution for the famous distance problem that once troubled astronomers for about 30 years. The enigma of GRBs is now being unveiled. We now know that in a typical GRB event, a huge amount of energy ($10^{43} - 10^{47}$ J) will be released from a small volume (on the scale of tens to hundreds of kilometers) in a very short period (tens of seconds). This will inevitably give birth to
a fireball whose optical depth will be much larger than 1. Emission from the surface of the fireball cannot account for the observed GRB, since the luminosity will be too low and also its thermal spectrum is not consistent with the observed non-thermal spectrum. However, we can imagine that the radiation pressure inside the fireball should be enormous, which will drive the fireball to expand outward. If the fireball is mainly composed of electron-positron pairs and photons so that baryon pollution is small (with a static mass less than $10^{-7} \sim 10^{-3} M_\odot$), then the fireball material will be accelerated to an ultra-relativistic speed at the radius of $R \sim 10^5 \sim 10^8$ km, forming a thin shell with the bulk Lorentz factor $\gamma > 100 \sim 1000$. At $R \sim 10^{11}$ km, the shell will be decelerated by interstellar medium, producing strong blastwaves. These blastwaves are called external shocks since they originate from the interaction of the fireball material with the external medium. Electrons are accelerated by shocks, whose emission may give birth to the observed GRB.

However, the large radii of the external shocks indicate that they cannot produce a highly variable light curve as observed in the main burst phase. It has been suggested that the central engine may be active for tens of seconds, producing many thin shells with different velocities, like a geyser. These shells will collide with each other at $R \sim 10^8$ km and produce a sequence of shocks, giving birth to a highly variable GRB. These shocks are called internal shocks since they come from the interaction of the material within the fireball. Now the most popular view is that the main bursts are produced by internal shocks, and afterglows are produced by long-lasting external shocks. This is the so called standard fireball model\[29, 30, 31, 32, 33, 34, 35\].

The Fireball model can explain the observed afterglows well. For example, in Fig. 4, the fast decay of Segment I comes from the delayed high latitude emission when the internal shocks quenched; The shallow decay of Segment II may be due to continuous energy injection from the central engine; The normal decay of Segment III can be naturally explained by considering the deceleration of the external shock; The fast decay of Segment IV indicates that the GRB ejecta is not isotropic, but should be a jet with a typical half-opening angle of $\sim 0.1$; Finally, the flares of Segment V can be interpreted as late explosions of the central engine. Additionally, the giant amplitude scintillation of radio afterglows at early stage can be naturally regarded as a proof for ultra-relativistic motion. The radius of the fireball is relatively small at first, so that the scattering of interstellar medium is serious. But tens of days later, the fireball has increased markedly due to ultra-relativistic expansion, then scintillation becomes insignificantly.
The above explanation of afterglows is generally satisfactory and the physics of afterglows are relatively clear. However, we still know very few about the central engine of GRBs. After the initial acceleration phase, the fireball loses most of its memory to the progenitor and we can hardly find any direct clues from afterglows. From the fireball model, we only know that the central engine must satisfy the following requirements: (i) Can release a huge amount of radiation energy (typically $\sim 10^{45}$ J, but can also as large as $10^{47}$ J in some extreme cases); (ii) The energy is released from a small volume (less than 100 km); (iii) The energy release process should last for several seconds to tens of seconds, and should be intermittent; (iv) The fireball should be free of baryon pollution, i.e., the static mass of baryons associated with every $\sim 10^{45}$ J of energy should be less than $\sim 10^{-5} M_\odot$; (v) The event rate should be one GRB per typical galaxy every $\sim 10^6 — 10^7$ year. The above discussion is based on the assumption that the radiation of GRB is isotropic. If GRB emission is highly collimated, then some requirements will be changed. For example, the intrinsic energy may be lower by two magnitudes, but the event rate per galaxy should be correspondingly higher.

In any case, the above requirements are rigorous and can be satisfied only by very few objects. Currently there are mainly two kinds of models: (i) Merge of compact stars, including the merge of two neutron stars, or a neutron star with a black hole; (ii) Collapse to a black hole of a dying massive star ($M \geq 40 M_\odot$). A similar structure will be formed in these two processes, a quickly rotating black hole (or neutron star) surrounded by an accretion disk, plus two highly collimated jets moving outward perpendicular to the disk. In Model (i), the lifetimes of the accretion disk and jets are very short, much less than 1 second. It is suitable for producing short GRBs. On the contrary, the lifetimes of disk and jets in Model (ii) can be tens of seconds and it is proper for long GRBs. Now it has been widely believed that long GRBs should be due to massive star collapses and short GRBs be due to mergers of compact stars. This viewpoint is further supported by the observational facts that long GRBs usually occur in star forming regions and are frequently associated with type Ic supernovae, while short GRBs generally deviate from star forming regions.

However, there are still many unsolved problems. For example, why the central engine behaves like a geyser, as required by the internal shocks? How can the central engine avoid the baryon pollution problem, so that the fireball material can be accelerated to ultra-relativistic speeds? How are highly collimated jets produced? How can the central engine be active for hundreds or even thousands of seconds, so as to produce the observed flares (Segment V in Fig. 4) and continuous energy ejection (Segment II
in Fig. 4)? Why X-ray and optical afterglows behave quite differently in some GRBs? Why there are many dark GRBs, i.e., GRBs with afterglows detected in X-rays but not in optical bands? What is the true relation between GRBs and supernovae? Additionally, the connections between GRBs and cosmology, cosmic rays, gravitational waves are all attracting problems and need many hard work.

4 Afterglows in the Nonrelativistic Phase

The fireball model is most successful in explaining Segments III and IV in Fig. 4, which are all due to external shocks. According to the standard fireball model, the external shock should be ultra-relativistic (Lorentz factor $\gamma \gg 1$) initially. In a homogeneous external medium, it decelerates as $\gamma \propto t^{-3/8}$, $R \propto t^{1/4}$, $\gamma \propto R^{-3/2}$ in the adiabatic case. Then it is easy to derive that synchrotron radiation from the external shock should decay as a power-law of time. For a highly collimated jet with a half opening angle $\theta$, lateral expansion will be significant when $\gamma < 1/\theta$, leading to a more quick deceleration. At the same time, the edge of the jet becomes visible. These two factors act together to make the afterglow decay more steeply. This is the reason for the observed light curve break (the so called “jet break”) between Segments III and IV \[36\].

As mentioned above, a basic assumption of the standard fireball model is that the external shocks are ultra-relativistic ($\gamma \gg 1$). However, we will point out below that this assumption is correct only in a very limited period.

4.1 Importance of the Nonrelativistic Phase

GRBs are most impressive for two characteristics: the huge energy release and the ultra-relativistic motion. Misled by the energetics, people once believe that the fireball should be ultra-relativistic for a long time in the afterglow phase. As such, the explanation of the fireball model to afterglows is based on the assumption of $\gamma \gg 1$. However, a careful study of the deceleration of the external shock reveals that it is not true. The fireball generally decelerates as $\gamma \propto t^{-3/8}$, with the detailed expression given by

$$\gamma \approx (200 - 400)E_{44}^{1/8}n_0^{-1/8}t_s^{-3/8},$$

where $E_{44}$ is isotropic kinetic energy in units of $10^{44}$ J, $n_0$ is the number density of the interstellar medium in units of $1 \text{ cm}^{-3}$, and $t_s$ is observer’s time in units of second. In typical cases, we would find that $\gamma \approx 2.8 - 5.6$ for $t = 1 \text{ day}$, $\gamma \approx 1.2 - 2.4$ for $t = 10 \text{ day}$. For $t = 30 \text{ day}$, the Lorentz factor will even be less than 1, which is of course
un-physical. In fact, Eq. (1) is an approximate expression that is applicable only when \( \gamma \gg 1 \). In realistic case, the condition of \( \gamma \gg 1 \) may no longer be satisfied several days or teens of days later, then Eq. (1) cannot be used to calculate the afterglows.

Observationally, X-ray afterglows usually last for one or two weeks, optical afterglows can last for tens of days to several months, and radio afterglows can even last for more than 1000 days\(^3\!\!7, 38, 39\). To account for these observations, we will inevitably need to consider the dynamics and radiation in the nonrelativistic phase\(^4\!\!0\). In fact, the importance of nonrelativistic phase has been pointed out by Huang et al. as early as in 1998, i.e., soon after the discovery of GRB afterglows\(^4\!\!1, 42\).

### 4.2 Generic Dynamical Model

Now we describe the overall evolution of the external shocks. At first, the shock is ultra-relativistic, with \( \gamma > 100 — 1000 \). The shock is also highly radiative since the synchrotron-induced energy loss rate (\( P_{\text{syn}} \)) is much higher than the rate due to adiabatic expansion (\( P_{\text{ex}} \)), leading to a high radiation efficiency of \( \epsilon \sim P_{\text{syn}}/(P_{\text{syn}} + P_{\text{ex}}) \approx 1 \). The deceleration of such a highly radiative ultra-relativistic shock in a homogeneous external medium is \( \gamma \propto t^{-3/7} \), \( R \propto t^{1/7} \), \( \gamma \propto R^{-3} \). With the increase of radius, the ratio of \( P_{\text{syn}} \) to \( P_{\text{ex}} \) decreases. Several hours later, \( \epsilon \) will be very close to 0 so that the shock becomes adiabatic, but it is still ultra-relativistic (\( \gamma \gg 1 \)). The evolution laws then become \( \gamma \propto t^{-3/8} \), \( R \propto t^{1/4} \), \( \gamma \propto R^{-3/2} \). Several days later, \( \gamma \) will be decreased to be between 2 and 5. At this stage, the approximate equations under \( \gamma \gg 1 \) assumption may still be marginally applicable. But teens or tens of days later, \( \gamma \) will be so close to 1 that the shock will no longer be highly relativistic. It enters the nonrelativistic phase. According to the well-known Sedov solution\(^4\!\!3\), the evolution of such a nonrelativistic adiabatic shock should be \( v \propto t^{-3/5} \), \( R \propto t^{2/5} \), \( v \propto R^{-3/2} \), where \( v \) is the velocity of the shock.

From the above description, we see that there are two transitions in the process. One is the transition from highly radiative regime to adiabatic regime at about a few hours, the other is the transition from relativistic phase to nonrelativistic phase at about teens of days. The overall evolution can then be divided into three stages: (i) ultra-relativistic and highly radiative stage (\( t \leq \) a few hours), \( \gamma \gg 1, \epsilon \approx 1 \); (ii) ultra-relativistic and adiabatic stage (a few hours < \( t \) < a few days), \( \gamma \gg 1, \epsilon \approx 0 \); (iii) nonrelativistic and adiabatic stage (\( t \geq \) teens of days), \( v \ll c, \gamma \approx 1, \epsilon \approx 0 \). To calculate the afterglows, a conventional method is to use approximate expressions within each stage separately. The overall light curve can then be obtained by connecting the three segments. However, a natural problem in the process is that the light curve may be
discontinuous at the joint points. Additionally, if the GRB ejecta is a jet, we must go further to consider the effect of the lateral expansion. This will lead to a fourth segment. If the cooling-induced broken power-law distribution of electrons is taken into account, then many additional segments will be added. What makes things even more complicated is that the order of all these segments depends on parameters. Finally, the analytical expressions for the overall light curve become very complicated.[11]

Figure 5: Lorentz factor versus radius for an isotropic fireball. The dashed line corresponds to the Sedov limit, the dash-dotted line is plot according to an earlier dynamical model suggested by others, which deviates from the Sedov limit markedly. The solid line is plotted according to Eq. (2), which is correct in both the relativistic and nonrelativistic phases.[47]

To relax the problem, Huang et al. proposed a generic dynamical model to depict the overall evolution of the external shock,[44],

\[
\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_{ej} + \epsilon m + 2(1 - \epsilon)\gamma m}, \tag{2}
\]

where \(m\) is the mass of the swept-up medium and \(M_{ej}\) is the initial mass of the GRB ejecta. Taking \(\epsilon = 1\), Eq. (2) describes a highly radiative shock; while taking \(\epsilon = 0\), Eq. (2) describes an adiabatic shock. This equation is applicable in both the ultra-relativistic and nonrelativistic phases.
Eq. (2), together with the following equations about the evolution of swept-up mass, shock radius, and jet opening angle, can give a thorough description of the dynamics of external shocks\[^{44, 45, 46}\],

\[
\frac{dm}{dR} = 2\pi R^2 (1 - \cos \theta) n m_p, \tag{3}
\]

\[
\frac{dR}{dt} = \beta c \gamma \left( \gamma + \sqrt{\gamma^2 - 1} \right), \tag{4}
\]

\[
\frac{d\theta}{dt} = \frac{c_s (\gamma + \sqrt{\gamma^2 - 1})}{R}, \tag{5}
\]

where $\beta = v/c$, $m_p$ is the proton mass, and $c_s$ is the sound speed in co-moving frame.

In Fig. 5, we plot the Lorentz-radius relation calculated according to Eq. (2—5). It can be seen that Eq. (2) is really correct in both relativistic and nonrelativistic phases.

### 4.3 Deep Newtonian Phase

Afterglow mainly comes from the synchrotron radiation from shock-accelerated electrons, although inverse Compton scattering may also play a role in some particular cases. Now we concentrate on the calculation of radiation. The acceleration of electrons is a complicated process. To simplify the problem, we usually assume that the accelerated electrons obey a power-law distribution according to their energies\[^{48, 49}\],

\[
dN_e/d\gamma_e \propto \gamma_e^{-p}, (\gamma_{e, \text{min}} \leq \gamma_e \leq \gamma_{e, \text{max}}).\]

When the cooling effect is taken into account, the power-law function will be broken into several segments\[^{50}\]. Here, $p$ is the power-law index, which is typically between 2 and 3; $\gamma_{e, \text{max}}$ is the maximum Lorentz factor of electrons, depending on the equilibrium between the acceleration and radiation power; $\gamma_{e, \text{min}}$ is the minimum Lorentz factor, which actually is also a measure of the typical Lorentz factor since very few electrons are accelerated to high energy state and most electrons have relatively lower energies. The exact expression for $\gamma_{e, \text{min}}$ is difficult to derive from the acceleration process. Usually we assume an energy equipartition between electrons and protons, then we have,

\[
\gamma_{e, \text{min}} = \xi_e (\gamma - 1) m_p (p - 2) / [m_e (p - 1)] + 1,
\]

where $\xi_e$ is the equipartition factor. Note that the distribution functions discussed here are applicable only to ultra-relativistic electrons.

The generic dynamical equation (2) is applicable even in the very late phase. But to calculate the afterglow radiation, we must pay special attention to a problem that is related to $\gamma_{e, \text{min}}$. According to the energy equipartition between electrons and protons, $\gamma_{e, \text{min}}$ could be much larger than 1 even at the nonrelativistic phase when $\gamma = 1.01$. However, with the further deceleration of the external shock, $\gamma_{e, \text{min}}$ will inevitably approach 1, so that the electron distribution function introduced in the above paragraph
will no longer be applicable. This stage is called the deep Newtonian phase by Huang & Cheng\[51\]. Afterglows may enter the deep Newtonian phase when \( t > \) a few months (see Fig. 6). This factor should be taken into account when explaining the observed afterglows that can last for several months or even several years\[37, 38, 39, 40\].

Huang & Cheng suggested that in the deep Newtonian phase, the basic form of the electron distribution function should be revised as\[51\]

\[
\frac{dN_e'}{d\gamma_e} \propto (\gamma_e - 1)^{-p}, \ (\gamma_{e,\min} \leq \gamma_e \leq \gamma_{e,\max}).
\] (6)

In this equation, electrons are still distributed according to their energies, but the originally approximate expression for energy (i.e. \( \gamma_e \)) is now replaced by the exact expression of \( \gamma_e - 1 \). In the deep Newtonian phase, most electrons are nonrelativistic, but a minor portion of electrons are still relativistic according to Eq. (6). Synchrotron radiation from these relativistic electrons produces the afterglow in the deep Newtonian phase.

Fig. 6 illustrates the theoretical afterglow light curves based on the above considerations\[51\]. We see that for an isotropic fireball, the afterglow decays slightly more rapidly after entering the deep Newtonian phase. In the jet cases, an obvious light curve break can be seen when the afterglow transits from ultra-relativistic phase to midly-relativistic phase. It is actually due to the jet effect, but note that the break usually occurs in the trans-relativistic phase (when \( \gamma \sim 2 - 5 \))\[45, 46, 47\]. In the deep Newtonian phase, the light curve becomes slightly shallower\[51\]. These results are consistent with others’ analytical solutions\[52, 53\].

5 Conclusions and Discussion

The discovery of afterglows is a major breakthrough in the GRB field. It definitely tells us that at least most GRBs are of cosmological origin. A huge amount of energy is released in each event, and ultra-relativistic motions are involved in the process. The fireball model becomes the standard model, which can explain the basic features of GRBs and afterglows well. Here we have given a brief description of the observations and theories of GRBs and their afterglows. We pay special attention on the importance of the nonrelativistic phase in afterglows. A generic dynamical model for the afterglows is described, and the concept of the deep Newtonian phase is highlighted.

Typically the fireball enters the nonrelativistic phase in several days or teens of days, and may enter the deep Newtonian phase in tens of days or several months. When
Figure 6: The overall afterglow light curve that includes the deep Newtonian phase\textsuperscript{51}. The left panel is plot for isotropic fireballs, and the right panel is for jets. Different line styles correspond to various energy (in units of ergs) and circum-burst medium density (in units of $cm^{-3}$). The decrease of the bulk Lorentz factor $\gamma$ is marked by the open circles, and the full circles indicate the time when the afterglow enters the deep Newtonian phase.

the GRB ejecta is highly collimated, the nonrelativistic phase and deep Newtonian phase will come even earlier\textsuperscript{45, 46}. In some special cases such as when the medium is very dense\textsuperscript{54, 55}, the fireball may even enter the nonrelativistic phase in one day. In some GRB explosions, the fireball may fail to be accelerated to $\gamma \sim 100 - 1000$ since the energy release is not enough or the baryon contamination is too serious. Such a fireball with $1 \ll \gamma \ll 100$ cannot produce a GRB successfully, but can only give birth to a soft $\gamma$-ray burst or an X-ray transient. These events are called failed GRBs by Huang et al.\textsuperscript{56}. In these cases, the fireball will also enter the nonrelativistic and deep Newtonian phases earlier, since the initial Lorentz factor itself is small and the medium density may be high as well.

It is interesting that external shocks similar to those in GRB afterglows may also exist in other explosive phenomena in the cosmos. For example, transient X-ray bursts can be accidently observed from the center of normal galaxies\textsuperscript{57}, which may be due to the tidal disruption of a star by the massive black hole that resides at the galaxy center.
Radiation is generally believed to come from a temporary accretion disk formed in the process. However, Wong et al. suggested that external shocks should also be excited when the jet associated with the accretion disk interacts with external medium. They found that emission from these external shocks can explain the observed X-ray light curves and spectra well\cite{58}. In these processes, the external shock will also enter the nonrelativistic and deep Newtonian phases quickly since the initial Lorentz factor of the jet is rather low (typically $\gamma < 10$).

The study of GRBs is a rapidly developing field. In the past BeppoSAX era (1997—2002), afterglows were discovered from long GRBs for the first time. In the current Swift era (beginning from 2004), many encouraging results are further brought about, such as the discovery of afterglows from short GRBs, the discovery of flares in early afterglows, and the detection of the most distant GRB up to a redshift of $z \sim 6.3$ \cite{12}. However, the enigma of GRBs is far from clear yet. Many key problems are still remained uncertain, as pointed out in Section 3. For example, a suspicious problem is related to the beaming of GRBs. In the BeppoSAX era, jet breaks have been clearly observed in the afterglow light curves of many GRBs. But in the Swift era, when our sample is greatly increased, very few GRBs show such a jet break in their afterglow light curves. Are GRBs really highly collimated? An once resolved problem now seems to be raised again. It is interesting to note that the study of nonrelativistic afterglows may help to solve this puzzle, since the emission in this late phase should be largely isotropic, thus it is easier to derive the intrinsic kinetic energy of the GRB remnant. A comparison of this intrinsic kinetic energy with the prompt gamma-ray energy release then may hopefully give a direct measure of the beaming angle. To do this, we need more events with well sampled afterglow data that extend to months after the GRB trigger. It is encouraging that a powerfull satellite, the Gamma ray Large Area Space Telescope (GLAST), will be launched in early 2008. It will open a new era for the GRB study in the sense that a high energy window (20 — 300 GeV) of GRBs will be open\cite{59}, and more afterglow samples will be available as well. We expect that the enigma of GRBs will be further revealed in the coming GLAST era.

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