Research article

Finite element analysis of mixed convection flow in a trapezoidal cavity with non-uniform temperature

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A R T I C L E   I N F O

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A B S T R A C T

A two dimensional flow analysis in a cavity shaped isosceles trapezium is carried out. Non-parallel sides of a trapezium are adiabatic. A varying sinusoidal temperature is applied to the lower wall while the upper wall is at constant temperature. Upper wall of the cavity moves with a velocity \( \eta_2 \) in the positive x-direction. Also, \( B_i \) is constant magnetic field of strength aligned in the same x-direction and Newtonian fluid is considered. The values of magnetic field parameter used are \( H_a = 0.50 \), the Richardson number is \( Re = 0.1, 1, 10 \), \( Re = 100 \) is Reynolds number used for the analysis, the amplitude of sinusoidal temperature is \( m = 0.25, 0.5, 1 \). The impacts of different leading parameters are analyzed by plotting streamlines for flow fields and isotherm contours for temperature of the flow dynamics. The graphs that signify the variation of average Nusselt number and local Nusselt number are sketched for both lower and upper walls of the cavity. Result indicated that with constant temperature the top wall of the boundary layer thickness decreases as Richardson number \( R_i \) increases and for bottom wall with variable temperature. The Nusselt number gets higher with an increment in the amplitude of the oscillation of temperature function. Furthermore, the study revealed that the average Nusselt number gets reduced as the intensity of magnetic field is enhanced. The variation in transit of heat at the bottom wall is similar but the maximum value of heat transfer at the bottom wall shows a variation from 3.8 to 20 when \( H_a = 0 \) and from 3 to 18 when \( H_a = 50 \). The accuracy of the present numerical algorithms is also established.

1. Introduction

The study of flow in lid-driven cavity of different geometrical shaped body is a hot topic of inquiry in the area of fluid dynamics. Because of its numerous inferences in metallurgy, in food processing, in petroleum production, solar pond, lake and reservoirs and others, the examination on cavity flow is paramount. Many researchers are actively engaged in conducting research on the lid-driven cavity under different forces and flow configurations. The first work on flow in a cavity was presented by Torrance et al. [1] followed by Shanker and Deshpande [2]. Later on, a series of many papers appeared on reputable journals in the area [3, 4, 5, 6, 7] taking into consideration the dynamics in the cavity using the geometries such as square, triangle and rhombus. Study conducted by Antar et al. [8] examined the impact of thermal radiation on lid-driven cavity of rectangular region when its bottom walls were moving with varying velocity. Experimental as well as numerical examination of lid-driven cavity of triangular region when its three sides have different temperature has been discussed by [9]. In addition Bakar et al. [10] made analysis of the power of magnetic field in making a difference on the streamlines and contours of the vortices formed in the cavity.

Recently, study about flow in cavity with different geometrical obstacles were simulated by plotting the streamlines, contour lines and other flow indicating diagrams as indicated in the references [11, 12, 13]. Zhu et al. [14] have presented the salient features of vortex dynamic in rectangular cavity with oscillatory temperature function and focusing on depth to width ratio of the cavity. Their result indicates that the form of flow is a unique function of Stokes number and Reynolds number of the flow. Further, Hamid et al. [15] studied a cavity filled with a carbon nanotube with water as base fluid with an obstacle shaped in cylinder inserted the flow region. They plotted the streamline and isotherms for such a complex flow region to display the outstanding flow characteristics. They also computed the average and local Nusselt number for the flow. They concluded that the corner is the place where the maximum local Nusselt number is situated. Hamid et al. [16] have also presented analysis of transport of heat through trapezoidal cavity filled with Casson fluid. Their result indicates that at the middle of the
Nomenclature

\begin{align*}
B_0 & \quad \text{Magnetic field strength} \\
g & \quad \text{Gravitational acceleration} \\
u, v & \quad \text{Velocity components} \\
T & \quad \text{Temperature} \\
c_p & \quad \text{Specific heat capacity} \\
L & \quad \text{Cavity height} \\
m & \quad \text{sinusoidal function amplitude} \\
Ha & \quad \text{Hartmann number} \\
Re & \quad \text{Reynolds number} \\
Gr & \quad \text{Grashof number} \\
Nu & \quad \text{Local Nusselt number} \\
Nu_{avg} & \quad \text{Average Nusselt number} \\
P_r & \quad \text{Prandtl number}
\end{align*}

Greek symbols

\begin{align*}
\beta & \quad \text{Fluid particle interaction parameter} \\
\sigma & \quad \text{Electrical conductivity} \\
\nu & \quad \text{Kinematic viscosity} \\
\rho & \quad \text{Density} \\
\theta & \quad \text{Dimensionless temperature} \\
\eta, \zeta & \quad \text{Dimensionless velocities} \\
\psi & \quad \text{Dimensionless stream function} \\
\eta_0 & \quad \text{Lid velocity} \\
a & \quad \text{Thermal diffusivity} \\
\kappa & \quad \text{Thermal Conductivity}
\end{align*}

Subscripts

\begin{itemize}
\item \(c\) \quad \text{Cold wall}
\end{itemize}

cavity the local Nusselt number is high when the value of Casson parameter is lowered.

Pioneer work on analysis of the effects of change in temperature on the flow in a cavity was examined by Al-Amiri et al. [17]. Their study implied that the local Nusselt number was in the form of wave with varying amplitude. The study of flow analysis of the cavity extended to nanofluid as described by Sheikholeslami and Rokni [18]. They examined the effect of magnetic field and convective heat transfer on porous cavity using the model of non-equilibrium. Further, Nazari et al. [19] presented numerical simulation of flow analysis of non-Newtonian fluid with nanoparticle in lid-driven square cavity embedded in porous media. They indicated that the streamlines curve is highly sensitive to Darcy number when Richardson numbers is small. Synakos et al. [20] and Toufik et al. [21] studied the flow of Bingham fluid inside the square cavity using finite volume method by plotting streamlines and contour lines for various parameters such as Richardson number and Prandtl number. Most of the studies about the cavity flow inside complex geometry with the help of finite volume method are available in [22].

Furthermore, Alsabery et al. [23] studied the free convection flow in a trapezoidal cavity one part filled with Newtonian nanofluid and the other part filled with non-Newtonian nanofluid by means of heat line simulation technique. Moreover, Faridzadeh et al. [24] presented analysis of forced and free convection flow of nanofluid inside inclined square cavity using the method of artificial neural network. In addition, the researchers in references [25, 26, 27, 28] computed the analysis of nanofluid inside different cavity enclosures such as square under different flow condition. Furthermore, Goodarzi et al. [29] and Izadi et al. [30] produced a numerical simulation inside cavity with a shape in square with different aspect ratio using hybrid nanofluids. Geridonmez and Oztop [31] presented the analysis on cavity flow with consideration of magnetic field and mixed convection.

Here, the trapezoidal cavity flow analysis when the bottom wall is exposed to sinusoidal temperature using finite element analysis with Comsol multiphysics software is scrutinized. The flow in the cavity is induced by lid-driven and buoyancy forces emanating from the movement of the top wall.

2. Physical model and governing equations

The system of interest in this article is an isosceles trapezoidal cavity (Fig. 1). The cavity is filled with incompressible Newtonian fluid and laminar flow type. The height of the trapezium hole under consideration is \(L\) and the corresponding length of the bottom wall is \(2L\) and top wall is \(L/2\). A varying temperature in sinusoidal form is considered on the bottom wall while the top wall is fixed with constant temperature \(T_e\). Adiabatic temperature scenario on the right and left sides of the hole is assumed. The top lid is moved with a velocity \(\eta_0\) in the positive \(x\)-axis direction and magnetic field of strength \(B_0\) is applied in the same positive \(x\)-axis.

The Navier-Stokes equation of flow problem with dimension is given by [4, 5]:

\begin{align}
\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} &= 0 \quad (1) \\
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2) \\
\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta(T - T_e) - \frac{\sigma B^2}{\rho} \frac{\partial u}{\partial y} \quad (3) \\
\frac{\partial T}{\partial y} + \frac{\partial T}{\partial x} &= a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)
\end{align}

The quantities \(T, p, v, u\) are temperature, pressure, \(y\)- and \(x\)-directions of components of the velocity, respectively. \(\rho, v, \sigma, g, \beta\) are density, kinematic viscosity, the electrical conductivity, acceleration due to gravity, fluid thermal expansion coefficient, respectively. \(c\) is the specific heat capacity, \(\kappa\) is the thermal conductivity, \(a = \frac{c}{\rho \kappa}\) is the thermal diffusivity.

The variables in this study are non-dimensionalized as:

\begin{align}
X &= \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \eta = \frac{u}{\eta_0}, \quad \zeta = \frac{v}{\eta_0}, \quad P = \frac{p}{\rho \eta_0^2}, \quad \theta = \frac{T - T_e}{\Delta T} \quad (5)
\end{align}

By plugging the nondimensional variables in Eq. (5) into the Eqs. (1)-(4), the equations without dimension are:
\[
\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}
\left(\frac{\partial \eta}{\partial x} + \frac{\partial \zeta}{\partial y}\right) = 0,
\]
\[
\zeta \frac{\partial \eta}{\partial t} + \eta \frac{\partial \eta}{\partial x} - \frac{\partial p}{\partial x} + \frac{1}{Re} \left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right),
\]
\[
\zeta \frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial y}
\left(\frac{\partial \eta}{\partial x} + \frac{\partial \zeta}{\partial y}\right) = \frac{\partial p}{\partial y} + \frac{1}{Re} \left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right) + \frac{Gr}{Re^2} \frac{\partial^2 \zeta}{\partial y^2} + \frac{\partial^2 \eta^2}{\partial y^2},
\]
\[
\frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial x} + \frac{\partial \eta}{\partial x} \frac{\partial \theta}{\partial x} + \frac{\partial \zeta}{\partial y} \frac{\partial \theta}{\partial y} = \frac{Gr}{Re^2} \frac{\partial^2 \zeta}{\partial y^2} + \frac{\partial^2 \eta^2}{\partial y^2}.
\]
where \( Pr, Gr, Re \) and \( Ha \) stand for Prandtl, Reynolds, Hartmann and Grashof number respectively, and formulated as:

\[
Pr = \frac{v}{a}, \quad Re = \frac{\eta_0 L}{v}, \quad Ha = B_0 \sqrt{\frac{\sigma L}{\mu}}, \quad Gr = \frac{g \beta \Delta L^3}{v^2}.
\]
The boundary conditions for Eqs. (6 - 9) are given by:

On the inclined walls: \( \eta = \zeta = \frac{w}{m} = 0 \), where \( m \) shows direction the normal to a plane

On the top wall: \( \eta = 1, \zeta = \theta = 0 \)

On the bottom wall: \( \eta = \zeta = 0, \theta = m \sin(2\pi x) \), where \( m \) is the amplitude of sinusoidal temperature.

The local rate of transfer of heat from the surface given as,

\[
Nu_L = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}.
\]
The average rate of heat transfer from the surface can be evaluated by the average Nusselt numbers, which is defined as,

\[
Nu_{avg} = \frac{1}{2} \int_0^2 \frac{\partial \theta}{\partial y} \, dY.
\]

3. Numerical method

Finite element method is employed to simulate the flow dynamics in the cavity induced by the movement of the top wall of the cavity with the help of Comsol-Multiphysics commercial software. This software is the state of the art in modern computational fluid dynamics (CFD) simulation. It uses the finite element method algorithm for simulation of flow dynamics.

In this study, finite element algorithm was used to solve the Eqs (6 - 9). Eq (6) was employed as a control due to conservation of mass and this control was used to get the distribution of pressure. To obtain numerical solution of Eqs (7 - 9), the method of finite element with penalty was employed and pressure \( P \) was reduced by using penalty parameter \( \gamma \) and the conditions for incompressibility was given by Eq (6) which resulted in

\[
P = -\gamma \left(\frac{\partial \eta}{\partial x} + \frac{\partial \zeta}{\partial y}\right).
\]
Eq (6) is exactly fulfilled when the amount of \( \gamma \) is very big. Distinctive amount of \( \gamma \) that yield reliable solution is \( 10^7 \) as used by Basak et al. [4].

Using Eq. (10), Eqs. (7) and (8) were reduced to:

\[
\eta \frac{\partial \eta}{\partial x} + \zeta \frac{\partial \zeta}{\partial y} = \frac{\partial}{\partial x}
\left(\frac{\partial \eta}{\partial x} + \frac{\partial \zeta}{\partial y}\right) + \frac{1}{Re} \left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right),
\]
\[
\eta \frac{\partial \zeta}{\partial x} + \zeta \frac{\partial \zeta}{\partial y} = \frac{\partial}{\partial y}
\left(\frac{\partial \eta}{\partial x} + \frac{\partial \zeta}{\partial y}\right) + \frac{1}{Re} \left(\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2}\right)
\]
\[
+ \frac{Gr}{Re^2} \frac{\partial^2 \zeta}{\partial y^2} + \frac{\partial^2 \eta^2}{\partial y^2}.
\]
Expanding the components of velocity (\( \eta, \zeta \)) and temperature (\( \theta \)) using basis set \( \{\Phi_i\}_{i=1}^N \) as,
of three leading parameters on streamlines, isotherms and heat transfer rates for the analysis. These parameters are the Richardson number $Ri = \frac{\partial \theta}{\partial x}$, Hartmann number ($Ha$), and amplitude of sinusoidal function $m$. Throughout this paper, the Prandtl number, $Pr = 6.2$, and Reynolds number, $Re = 100$ are fixed. The type of convection is determined by the Richardson number. Specifically, when $Ri < 0.1$ the free convection is insignificant, while $Ri > 10$, forced convection is insignificant, and lastly when $0.1 < Ri < 10$ both convection are significant amount.

In this study numerical simulations were performed when the Richardson numbers are 0.1, 1 and 10, the Hartmann numbers are 0 and 50, and the amplitude $m$ for sinusoidal function is 0.25, 0.5 and 1 respectively.

### 4.1. Effects of Richardson number

The flow characteristics for this problem were depicted in the form of streamlines and temperature distribution which was given in isotherms for various values of detecting parameters such as Richardson number, Hartmann number and amplitude of the sinusoidal function (Fig. 3–Fig. 8).

Fig. 3(a–f) was plotted to describe the streamlines to the left and isotherms to the right for different values of Richardson number $Ri$. Fig. 3 (a) was sketched for small value of Richardson number $Ri = 0.1$ in the absence of the effect of magnetic field for unit value of the amplitude of the oscillating temperature function. With these parameter values a big clockwise circulation eddy filled the cavity is formed close to upper wall of the cavity and two small eddies at both left and right, the corner of the cavity oriented in anticlockwise direction as the upper wall moves in positive x-direction due to the lid velocity toward the right direction which created shear force on a flow field. Due to the motion of upper wall, a very little fluids pulled up towards the left side of the cavity. In this case, three circulations were formed, a dominating clockwise oriented circulation with high strength near the top of the cavity and two small circulations with anticlockwise direction with small strength at the bottom edge corners of the cavity. By increasing the values of $Ri$ to 1 and 10 consecutively, two more circulations were formed to show the distribution of the vortices. As the values of $Ri$ increases from 0.1 to 10, the number of vortices increases from three to four. Physically, an increment in Richardson number $Ri$ associated with an increment in the buoyancy force as compared to the force caused by lid-driven one, consequently the number of circulating eddies in the cavity increases. Therefore, the effect of buoyancy force is to increase the number of circulations in the cavity. Physically, in this case the direction of two forces, applied magnetic force and lid-driven force oriented toward the positive x-direction are the same and enhanced the strength of eddies so that they fill the hollow. As the values of $Ri$ increase from 0.1 to 10, the buoyancy force effect is more than the lid-driven force, as a result the position of the major vortex slightly moved down to the center of the cavity with the formation of new small vortex at the middle and two vortices at the corners of the trapezium as shown in Fig. 3 (c) and Fig. 3 (c) respectively. In all of the three cases, the big vortex is oriented in the clockwise direction. This is due to the fact that direction of lid-driven force and the force due to magnetic field are in the same direction.

Fig. 3(b), Fig. 3(d) and Fig. 3(f) represent isotherms graphs under the influence of Richardson number for the cavity flow. Isotherms are the lines that connect two points in the cavity which have the same temperature. As the values of $Ri$ varies in a multiple of ten i.e. 0.1, 1, 10, temperature distribution near the four walls of the cavity gets intensified and the temperature distribution at the center of the cavity reduced. The wall of the cavity exposed to sinusoidal temperature has got more heat as compared to the wall with constant temperature. It can be seen that on the right wall, the shear forces and buoyancy forces are in concurrent direction whereas on the left wall of the cavity the two forces are in opposite direction to each other. Therefore, small energy is perceived to be carried away from the sliding top wall into the cavity and as a result the conduction heat transfer becomes a significant mode of energy transport in the cavity. Therefore, as the values of $Ri$ increase the isotherm at the walls are more stronger than at the center of the cavity. The part of a cavity exposed to sinusoidal temperature shows oscillating thermal boundary layer. The isotherm has a maximum value of 0.94.
Fig. 3. Isotherms and streamlines for \( m = 1 \), \( Ha = 0 \).

Fig. 4. Isotherms and streamlines for \( m = 0.5 \), \( Ha = 0 \).

Fig. 4(a-f) was plotted to display the pattern of streamlines and isotherms when the amplitude \( m = 0.5 \) for different values of Richardson number \( Ri = 0.1, 1, 10 \). Fig. 4(a),(c) and (e) showed the variation and formation of streamlines due to a reduction in the amplitude of sinusoidal temperature. The form and numbers of the streamlines are almost the same as with the case when \( m = 1 \). Fig. 4(b),(d) and (f) showed the distribution of isotherms when the amplitude \( m \) reduced by half. The maximum value of the isotherms in this case is 0.47 which is exactly half of the isotherms in Fig. 3(b),(d) and (f). This is related with the reduction in amplitude of the sinusoidal temperature. Here, the temperature is in the form of sin function and varies periodically with amplitude, as a result the isotherm variation is high at the bottom.
From the simulation it can be observed that as Ri increases, the temperature on the top wall of the cavity is increased but at the center of the cavity still it is negligible. This is due to the motion of upper wall of the cavity, the shear and buoyancy forces that generate more heat at the upper wall. Almost all graphs of streamlines showed the same pattern and the same number of vortices Fig. 3. But the isotherms have shown variations with maximum value of 0.47 which is when \( m = 1 \) with similar pattern of distribution of heat. From the graphs it can be observed that the mode of heat transfer is predominantly by conduction, since the distribution of the isotherms are concentrated to the wall of the cavity rather than at the center of the cavity.
Fig. 5(a-f) was drawn to display the pattern of streamline and isotherms when the amplitude is $m = 0.25$ for different values of Richardson number $R_i = 0.1, 1, 10$ and with the absence of magnetic field, i.e. $H_a = 0$. When $H_a = 0$, the forces that contribute for the flow in the cavity are shear and buoyancy force, as Richardson number $R_i$ increases, the isotherms is enhanced in the cavity because the heat transfer becomes almost a conduction one. Almost all the graphs of streamlines showed the same pattern and the same number of vortices Fig. 4. But the isotherms have shown moderate variations with similar pattern of flow. Further, the values of the isotherm were much reduced as the amplitude of the temperature variation reduced.
Fig. 9. Effect of Richardson number on velocities $\eta$ and $\zeta$ respectively for $m = 1, H_a = 0$.

Fig. 10. Effect of Richardson number on velocities $\eta$ and $\zeta$ respectively for $m = 1, H_a = 50$. 
Fig. 6–Fig. 8 were sketched to capture the effect of amplitude, magnetic field parameter and Richardson numbers on the streamlines and isotherms of the cavity flow. Fig. 6 (a),(c) and (e), Fig. 7 (a),(c) and (e) and Fig. 8(a),(c) and (e) depict the streamlines when the magnetic field strength is enhanced. As the magnetic field strength is getting stronger, the number, the position and orientation of the vortices are changed. This is due to the fact that the Lorentz force has more effect than lid-driven force which adversely affect the eddy formation inside the cavity. This variation conforms with the change in Richardson number. On the other hand, Fig. 6(b),(d) and (f), Fig. 7 (b),(d) and (f) and Fig. 8(b),(d) and (f) show isotherms of the temperature distribution in the cavity. As the values of magnetic field is fixed to 50, both the amplitude of the temperature variation and the values of Richardson number increase, thereby the isotherms fill the region of cavity.

The behavior of velocities along vertical axis η and horizontal axis ζ under the influence of Richardson and Hartmann number are demonstrated in Fig. 9 and Fig. 10. When the value of Ri = 0.1, the intensity of vertical velocity is concentrated at the top of the cavity, but as the value of Richardson number increased ten times i.e. Ri = 1, the strength of the velocity further progressed to the center whereas when the value of Ri = 10 the distribution of velocity along the vertical axis fills the whole cavity. The number of vortices created inside the cavity enhanced along with the values of the Richardson number. It is also noticed that as the value of Hartman number increases the intensity of the vertical velocity decreases due to the fact that the Lorentz force is working against the vertical velocity. On the other hand when the velocity along the horizontal axis is i.e. ζ two vortices were formed along the symmetric line with respect to the middle line of the cavity. Similarly along with vertical axis, the velocity distribution inside the cavity is increased with the parameter Ri. When Ri = 0, the horizontal eddies are concentrated at the top wall of the cavity with less number of eddies at the center and at the bottom. As the values of Ri is increased to 10, the horizontal velocity intensity enhances and fills almost whole part of the cavity. Furthermore, the impact of Hartman number on horizontal axis is very significant as indicated in the Fig. 9.

Fig. 11–Fig. 13 illustrate the variation of local heat transfer rate along the bottom and top wall for various values of leading parameters like Richardson number, Hartmann number and amplitude of sinusoidal function on the bottom wall of the trapezoidal cavity.

Fig. 11 (a),(c) and (e) were plotted for Nusselt number with out the effect of magnetic field for bottom wall left part and Fig. 11 (b),(d) and (f) for top wall in the right part. In both cases set of graphs were sketched for Nusselt number along horizontal axis. The graphs for bottom wall are sinusoidal with different amplitude, but set of graphs of Nusselt number are curvilinear for upper wall and their curvature increase with an increment in the Richardson number. The graphs form a boundary layer structure and the boundary layer thickness increase as the values of Ri increase.

Fig. 11(a-f) and Fig. 12 (a-f) display the variation of local Nusselt number when there is no magnetic field and with the existence of applied magnetic field for the top and bottom walls of the cavity. In both case, with existence and non-existence of magnetic field, the variation of heat transfer at the bottom wall were similar but the maximum value of heat transfer at the bottom wall shows a variation from 3.8 to 20 when HA = 0 and from 3 to 18 when HA = 50. Similarly, the variation of top wall of the cavity has a boundary layer structure with decreasing boundary layer thickness.
Fig. 12. Variation of Nusselt number with length of bottom wall and top wall respectively for $H_a = 50$.

Fig. 13(a) and (b) present the variation of average Nusselt number for both top and bottom surface of the cavity for different values of Hartmann number against Richardson number $R_i$. As the values of Richardson number increase, the average Nusselt number increases for both surfaces. But the average Nusselt number reduced monotonically as Hartmann number increases from 0 to 50. The graphs of average Nusselt number for both constant and variable temperature surface show similar characteristics. The reason for an increment in the average Nusselt number is due to the fact that an increment in convection heat transfer inside the cavity.

To show the accuracy of the method used the average Nusselt number is calculated when the temperature of the bottom wall is constant and compared with the result computed by [32] as shown in Table 2. The present computation is in good harmony with the material available in literature. In this comparison we didn't get the same geometry but we tried to compare the square cavity which is available in the lit-
erature. There is some tolerable error due to a difference in geometry. Table 2 verifies the accuracy of the current code.

5. Conclusions
Flow simulation has been carried out with the help of Consol-multisurface physics for the analysis of transfer of heat and flow of fluid in a trapezoidal hollow region with sin function temperature variation at the bottom wall. Different graphs illustrate the variation of the streamlines, isothenr and Nusselt number. Inquiry Outcomes are presented in concise form as follows:

- Hartmann number and the number of vortices formed have positive correlation.
- The upper side wall of a cavity with constant temperature shows the boundary layer structure which reduces as Richardson number increases.
- Hartmann number has negative impact on the average Nusselt number.
- Richardson number has a positive effect on average Nusselt number for both constant and variables surface temperature.
- The maximum value of local Nusselt number was obtained as the amplitude of the sinusoidal temperature increases.
- As the intensity of Richardson number Ri is upgraded, the vortices formed in the cavity increase.
- The Richardson number and the velocity along vertical axis are positively correlated.
- The Hartman number significantly influenced the vertical and horizontal velocities.

Declarations

Author contribution statement
Wubeshet Ibrahim & Mohammed Hirpho: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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The authors declare no conflict of interest.

Additional information
No additional information is available for this paper.

References
[1] K. Torrance, R. Davis, K. Eike, P. Gill, D. Gutman, A. Hsu, et al., Cavity flows driven by buoyancy and shear, J. Fluid Mech. 51 (2) (1972) 221–231.
[2] P. Shankar, M. Deshpande, Fluid mechanics in the driven cavity, Annu. Rev. Fluid Mech. 32 (1) (2000) 93–136.
[3] K.M. Khanafar, A.J. Chamkha, Mixed convection flow in a lid-driven enclosure filled with a fluid-saturated porous medium, Int. J. Heat Mass Transf. 42 (13) (1999) 2465–2481.
[4] T. Basak, S. Roy, P.K. Sharma, I. Pop, Analysis of mixed convection flows within a square cavity with linearly heated side wall (s), Int. J. Heat Mass Transf. 52 (9–10) (2009) 2224–2242.
[5] T. Basak, S. Roy, P.K. Sharma, I. Pop, Analysis of mixed convection flows within a square cavity with uniform and non-uniform heating of bottom wall, Int. J. Therm. Sci. 48 (5) (2009) 891–912.
[6] S. Sivasankaran, S. Ananthan, A.A. Hakeem, Mixed convection in a lid-driven cavity with sinusoidal boundary temperature at the bottom wall in the presence of magnetic field, Sci. Trans. B. Mech. Eng. Sci. 23 (3) (2011) 1027.
[7] S. Sivasankaran, H. Cheong, M. Bhuvaneshwari, P. Ganesh, Effect of moving wall direction on mixed convection in an inclined lid-driven square cavity with sinusoidal heating, Numer. Heat Transf., Part A, Appl. 69 (6) (2016) 630–642.
[8] M. Antar, R. Ben-Mansour, S.A. Al-Díini, The effect of thermal radiation on the heat transfer characteristics of lid-driven cavity with a moving surface, Int. J. Numer. Methods Heat Fluid Flow 24 (3) (2014) 679–696.
[9] G. Yesilöz, O. Aydın, Laminar natural convection in right-angled triangular enclosures heated and cooled on adjacent walls, Int. J. Heat Mass Transf. 60 (2013) 365–374.
[10] N. Bakar, A. Karimipour, R. Roslan, Effect of magnetic field on mixed convection heat transfer in a lid-driven square cavity, J. Thermodyn. 2016 (2016) 3487182.
[11] M.I. Hossain, M. Maleque, M.M. Ali, Numerical study of magnetohydrodynamic mixed convection in a partially heated rectangular enclosure with elliptic block, in: AIP Conference Proceedings, vol. 2121, AIP Publishing LLC, 2019, 030010.
[12] M.S. Honze, analysis of natural convection in a trapezoidal cavity with magnetic field and cooled triangular obstacle of different orientations, in: AIP Conference Proceedings, vol. 2121, AIP Publishing LLC, 2019, 030003.
[13] A. Sarkar, M. Alam, M.J.H. Munshi, M. Ali, Numerical study on MHD mixed convection in a lid driven cavity with a wavy top wall and rectangular heaters at the bottom, in: AIP Conference Proceedings, vol. 2121, AIP Publishing LLC, 2019, 030004.
[14] J. Zhu, L.E. Holmedal, H. Wang, D. Myrhaug, Vortex dynamics and flow patterns in a two-dimensional oscillatory lid-driven rectangular cavity, Eur. J. Mech. B, Fluids 79 (2020) 255–269.
[15] M. Hamid, Z. Khan, W. Khan, R. Haq, Natural convection of water-based carbon nano-otubes in a partially heated rectangular fin-shaped cavity with an inner cylindrical obstacle, Phys. Fluids 31 (10) (2019) 103607.
[16] M. Hamid, M. Usman, Z. Khan, R. Haq, W. Wang, Heat transfer and flow analysis of Casson fluid enclosed in a partially heated trapezoidal cavity, Int. Commun. Heat Mass Transf. 108 (2019) 104284.
[17] A. Al-Amiri, K. Khanafar, J. Bull, I. Pop, Effect of sinusoidal wavy bottom surface on mixed convection heat transfer in a lid-driven cavity, Int. J. Heat Mass Transf. 50 (9–10) (2007) 1771–1780.
[18] M. Sheikholeslami, H.B. Rokni, Magnetic nanofluid flow and convective heat transfer in a porous cavity considering Brownian motion effects, Phys. Fluids 30 (1) (2018) 012003.
[19] S. Nazari, R. Ellis, M. Sarafraz, M.R. Safaei, A. Asgari, O.A. Akbari, Numerical study on mixed convection of a non-Newtonian nanofluid with porous media in a two lid-driven square cavity, J. Therm. Anal. Calorim. (2019) 1–25.
[20] A. Syrakos, G.C. Georgiou, A.N. Alexandrou, Solution of the square lid-driven cavity flow of a Bingham plastic using the finite volume method, J. Non-Newton. Fluid Mech. 195 (2013) 19–31.
[21] B. Toulki, M. Morales, Y. Bilal, S. Ferha, A. Said, Mixed convection of Bingham fluid in a two sided lid-driven cavity heated from below, Fluid Dyn. Mater. Proc. 15 (2) (2019) 107–123.
[22] C. Cancêt, P. Omnes, in: Finite Volumes for Complex Applications VIII-Methods and Theoretical Aspects: FVCA 8, Lille, France, June 2017, vol. 199, Springer, 2017.
[23] A. Alkhabary, A. Chamkha, S. Hussain, H. Saleh, I. Hashim, Heatline visualization of natural convection in a trapezoidal cavity partly filled with nanofluid porous layer and partly with non-Newtonian fluid layer, Adv. Powder Technol. 26 (4) (2015) 1230–1244.
[24] M. Faridzadeh, D.S. Toghirose, A. Niroomand, Analysis of laminar mixed convection in an inclined square lid-driven cavity with a nanofluid by using an artificial neural network, Heat Transf. Res. 45 (4) (2014).
[25] M.H. Esf, A.H. Refahi, H. Teimouri, M. Javad Noroozi, M. Afrand, A. Karimipour, Mixed convection fluid flow and heat transfer of the Al2O3-water nanofluid with variable properties in a cavity with an inside quadrilateral obstacle, Heat Transf. Res. 46 (5) (2015).
[26] S. Jani, M. Mahmoody, M. Amini, M. Akbari, Free convection in rectangular enclosures containing nanofluid with nanoparticles of various diameters, Heat Transf. Res. 45 (2) (2014).
[27] S. Natesan, S.K. Arumugam, S. Murugesan, A.J. Chamkha, Heat transfer enhancement of uniformly-linearly heated side wall in a square enclosure utilizing alumina-water nanofluid, Comput. Therm. Sci. 9 (3) (2017).

Table 2. Comparison of present result with [32].

| Ri   | This study |
|------|-----------|
| 1    | 1.34      |
| 0.06 | 3.62      |
| 0.01 | 6.29      |
[28] S. Bezi, A. Campo, N. Ben-Cheikh, B. Ben-Beya, Numerical study of natural convection heat transfer of nano fluids in partially heated semi-annuli, Comput. Therm. Sci. 6 (3) (2014).

[29] H. Goodarzi, O.A. Akbari, M.M. Sarafraz, M.M. Karchegani, M.R. Safaei, G.A. Sheikh Shabani, Numerical simulation of natural convection heat transfer of nano fluid with Cu, MWCNT, and Al₂O₃ nanoparticles in a cavity with different aspect ratios, J. Therm. Sci. Eng. Appl. 11 (6) (2019).

[30] M. Izadi, R. Mohabbi, D. Karimi, M.A. Sheremet, Numerical simulation of natural convection heat transfer inside a shaped cavity filled by a MWCNT-Fe₃O₄/water hybrid nano fluids using LBM, Chem. Eng. Process. - Process Intensif. 125 (2018) 56–66.

[31] B.P. Geridonmez, H.F. Oztop, Mixed convection heat transfer in a lid-driven cavity under the effect of a partial magnetic field, Heat Transf. Eng. (2020) 1–13.

[32] H.F. Oztop, I. Dagtekin, Mixed convection in two-sided lid-driven differentially heated square cavity, Int. J. Heat Mass Transf. 47 (8–9) (2004) 1761–1769.