A New Method for the Aggregate Proportion Calculation and Gradation Optimization of Asphalt-Treated Base (ATB-25)

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Asphalt-treated base (ATB-25) is a widely used flexible base material. The composition and gradation of mineral aggregate are important factors affecting pavement performance of asphalt treated base. In this study, two new methods were proposed to address the problems of existing aggregate proportion calculation for asphalt mixtures: (1) the combination of generalized inverse solution of the normal equation and spreadsheet trial and (2) quadratic programming. Both methods can calculate mass ratios of various aggregates in a quick and accurate manner. The orthogonal test was used to design nine aggregate gradations within the range of asphalt treated base (ATB-25) stated in the industrial standard. The aggregate proportion was calculated by two new methods. The Marshall test, water weight test, rutting test, and water-soaked Marshall test were carried out on the asphalt mixture specimens. The pavement performance test results were fuzzified using the fuzzy mathematics method, and the weights of pavement performance evaluation indexes were determined through the analytic hierarchy process. Taking the fuzzy comprehensive evaluation values as the objective function, test results were analyzed and evaluated. Finally, the optimal aggregate gradation was determined considering factors of compactness, high-temperature rutting resistance, and water stability.

1. Introduction

Asphalt-treated base (ATB-25) is a widely used flexible base material, which has the characteristics of small stiffness, high shear strength, flexural tensile strength, fatigue resistance, and not easy to produce shrinkage cracking and water damage. The design composition of the asphalt mixture determines the optimal mixing ratio of the coarse aggregate, fine aggregate, mineral powder, and asphalt to be used in the mixture to meet the performance requirements of the road. Because the use of waste material can reduce the cost of construction and increase the strength, steel slag and coconut shell were used in the asphalt mixture [1–3]. Owing to different gradations of the aggregates added, the asphalt mixtures have different composition and structures and thus exhibit different physical and mechanical properties and qualities during use. The characteristics of the arrangement and aggregate gradation have a significant effect on the structure and performance of the asphalt mixture. Previous studies [4–6] indicated that aggregate gradation has a significant effect on the rutting resistance of the asphalt mixture at a high temperature. It was also concluded that aggregate gradation has a significant impact on volumetric indicators such as the voids in mineral aggregates (VMA) and voids filled with asphalt (VFA) of the asphalt mixture [7]. Further, it was reported that aggregate gradation significantly affects the resistance of the asphalt mixture to permanent deformation [8]. In the literature [9, 10], it was indicated that the tensile strength, shear strength, and horizontal tensile strain of the asphalt mixture are significantly affected by the aggregate gradation. Husain et al. studied semiflexible pavements and concluded that different aggregate gradations have a significant impact on the pavement performance of semiflexible pavements [11]. Roberts et al. concluded that aggregate gradation is the main factor that affects the stiffness, stability, durability, permeability, fatigue resistance, and water damage resistance of the asphalt mixture [12]. These studies indicate that aggregate gradation can...
2. Raw Material Test and Aggregate Gradation Design

2.1. Raw Material Test. The asphalt was produced by Panjin Northern Asphalt Co., Ltd. The test was conducted according to regulation [20], whereas the specification values were based on the industrial standard [19]. The test results are presented in Table 1.

The aggregate was limestone, whereas the filler was limestone powder. The test was conducted based on regulation [21], whereas the specification values were based on the industrial standard [19]. The test results are presented in Table 2.

2.2. Aggregate Gradation of Orthogonal Design. In this study, within the range for aggregate gradation specified by the industrial standard [19], the impact of aggregate size (if the diameter of the aggregate exceeds 4.75 mm, it is considered coarse; otherwise, it is considered fine) and three positions of the gradation curve within the particular range for the ATB-25 were investigated in terms of compactness, high-temperature rutting resistance, and water stability. The three positions of the gradation curve are such that the upper position is the bisector between the upper limit and median line, the medium position is the median line of the gradation range, and the lower position is the bisector between the median and lower limit lines; these positions are detailed in Table 3.

The cumulative passing rate of each gradation conforms to the $L_0$ ($3^4$) orthogonal array, and the nine gradations obtained in the design are depicted in Figure 1.

3. Calculation Method for Aggregate Proportion

3.1. Establishment of Mathematical Model. Assuming that the number of hole types for sieving the aggregates of ATB-25 is $m$, the upper, lower, and median values of the designed gradation curve are expressed, respectively, as the following matrices:

$$ G = [g_1, g_2, \ldots, g_m]^T, $$

$$ L = [l_1, l_2, \ldots, l_m]^T, $$

$$ B = [b_1, b_2, \ldots, b_m]^T, $$

where $g_i$, $l_i$, and $b_i$ ($i = 1, 2, \ldots, m$) are the respective cumulative passing rates of the upper, lower, and median values of the design gradation curve in the $i^{th}$ sieve hole type.

Assuming that the aggregate mixture has $n$ types of raw materials, the results for each raw material after sieving are expressed in matrix form as follows:

$$ P = \begin{bmatrix}
  p_{11} & p_{12} & \cdots & p_{1n} \\
  p_{21} & p_{22} & \cdots & p_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  p_{m1} & p_{m2} & \cdots & p_{mn}
\end{bmatrix}, $$

where $p_{ij}$ represents the percentage of the $i^{th}$ raw material size in the $j^{th}$ aggregate mixture.
where $p_{ij} (i=1, 2, \ldots, m; j=1, 2, \ldots, n)$ are the cumulative passing rates of the $j^{th}$ raw material in the $i^{th}$ sieve hole type.

### Table 1: Main technical indicators of asphalt.

| Technical indicators          | Units      | Measured values | Specification values |
|------------------------------|------------|-----------------|----------------------|
| Penetration (25°C)           | 0.1 mm     | 67.1            | 60 ~ 80              |
| Penetration index            | —          | -0.65           | -1.5 ~ +1.0          |
| Softening point              | °C         | 51.7            | ≥45                  |
| Ductility (15°C)             | cm         | 67.5            | ≥40                  |
| Density (15°C)               | g/cm$^3$   | 1.0066          | Measured value       |
| Mass loss after aging        | %          | 0.05            | -0.8 ~ +0.8          |
| Penetration ratio after aging| %          | 62.5            | 61                   |
| Residual ductility (10°C)    | cm         | 23              | 8                    |

### Table 2: Main technical indicators of aggregate.

| Technical indicators                                | Units | Measured values | Specification values |
|-----------------------------------------------------|-------|-----------------|----------------------|
| Crushing value of coarse aggregate                 | %     | 20.96           | ≤28                  |
| Los Angeles abrasion value of coarse aggregate      | %     | 24.32           | ≤30                  |
| Adhesion between asphalt and aggregate              | Level | 4               | —                    |
| Relative density of 19–26.5 mm limestone            | —     | 2.661           | —                    |
| Relative density of 9.5–19 mm limestone             | —     | 2.641           | —                    |
| Relative density of 4.75–9.5 mm limestone           | —     | 2.636           | —                    |
| Relative density of 2.36–4.75 mm limestone          | —     | 2.571           | —                    |
| Relative density of 0–2.36 mm limestone             | —     | 2.685           | —                    |
| Relative density of limestone mineral powder        | —     | 2.70            | —                    |

### Table 3: Design factors and levels of aggregate gradation.

| Factors                               | Cumulative passing rate of coarse aggregate (%) | Cumulative passing rate of fine aggregate (%) |
|---------------------------------------|------------------------------------------------|---------------------------------------------|
| sieve size (mm)                       |                                                |                                             |
| Level 1                               | 31.5 26.5 19 16 13.2 9.5 23.6 18.1 13 9 6.5 4 |                                             |
| Level 2                               | 100 95 70 58 52 42 23.5 17.5 13 9.5 6.5 4    |                                             |
| Level 3                               | 100 92.5 65 53 47 37 19.25 13.75 10.5 7.25 4.75 3 |                                             |

The $n$ known types of raw materials are used to prepare a mixture that meets the design gradation. The mass ratio of the various raw materials in the mixture is calculated and expressed in a matrix as follows:

$$X = [x_1, x_2, \ldots, x_n]^T,$$

where $x_j (j=1, 2, \ldots, n)$ is the mass ratio of the $j^{th}$ raw material in the aggregate asphalt mixture.

The mass ratio of the various raw materials should satisfy the following equation:

$$P \cdot X \leq G,$$

$$P \cdot X \geq L,$$

$$\sum_{j=1}^{n} x_j = 1, \quad j = 1, 2, \ldots, n,$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n.$$

Formulas (4) and (5) represent the upper and lower bounds of the gradation of the mixture. Formula (6) indicates that the sum of the mass ratios of all the raw materials is equal to 1. Formula (7) restricts the result of the calculation to a nonnegative number.
3.2. Problem-Solving Methods

3.2.1. Normal Equation Method. The fundamentals of normal equation method are the principle of least squares. The synthetic aggregate gradations do not meet the conditions of equations (4) and (5). However, the quadratic sum of the deviation between the sieve weight value of each sieve hole and the target design grading value should be minimized. The mass ratios of raw materials in synthetic aggregates should satisfy the following equation:

\[ P \cdot X = B. \]  (8)

The passing percentage of aggregates at any sieve hole is the theoretical design grading value \( b_j \), which is equal to the sum of the passing percentage of various aggregates at each sieve hole multiplied by the amount of various aggregates in the mixture; that is,

\[ \sum_{j=1}^{n} p_{ij}x_j = 1. \]  (9)

The least square principle was used to minimize the quadratic sum of the deviation between the sieve weight value of each sieve hole and the theoretical design grading value; that is,

\[ \min F(x) = \sum_{i=1}^{m} \left( \sum_{j=1}^{n} p_{ij}x_j - b_i \right)^2. \]  (10)

Under the condition that the sieving results of various aggregates were linearly independent, the extreme value condition was used:

\[ \frac{\partial F(x)}{\partial x_j} = 0, \quad j = 1, 2, \ldots n. \]  (11)

The normal equations of \( x_j \) were obtained as follows:

\[ \sum_{j=1}^{n} \left( \sum_{k=1}^{m} p_{ki}p_{kj} \right) x_j = \sum_{k=1}^{m} p_{ki}b_k, \quad i = 1, 2, \ldots n. \]  (12)

Equation (12) is expressed in matrix as

\[ P^T P X = P^T B, \]  (13)

where

\[ P^T = \begin{bmatrix} p_{k1} & p_{k1} & p_{k1} & \cdots & p_{k1} \\ p_{k2} & p_{k2} & p_{k2} & \cdots & p_{k2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{kn} & p_{kn} & p_{kn} & \cdots & p_{kn} \end{bmatrix}, \]

\[ P^T P = \begin{bmatrix} \sum_{k=1}^{m} p_{k1} & \sum_{k=1}^{m} p_{k1} & \sum_{k=1}^{m} p_{k1} & \cdots & \sum_{k=1}^{m} p_{k1} \\ \sum_{k=1}^{m} p_{k2} & \sum_{k=1}^{m} p_{k2} & \sum_{k=1}^{m} p_{k2} & \cdots & \sum_{k=1}^{m} p_{k2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{k=1}^{m} p_{kn} & \sum_{k=1}^{m} p_{kn} & p_{kn} & \cdots & \sum_{k=1}^{m} p_{kn} \end{bmatrix}, \]

\[ P^T B = \begin{bmatrix} \sum_{k=1}^{m} p_{k1}b_1 \\ \sum_{k=1}^{m} p_{k2}b_2 \\ \vdots \\ \sum_{k=1}^{m} p_{kn}b_n \end{bmatrix}. \]

When \( P^T P \) is invertible, equation (13) has a unique solution.

\[ X^\theta = (P^T P)^{-1} P^T B. \]  (15)

When \( P^T P \) is not invertible, the generalized inverse function is used for the solution.

\[ X^0 = \text{pinv}(P) \cdot B. \]  (16)

3.2.2. Spreadsheet Trial Method. The retained percentage \( a_i \) is the percentage of residual mass on the \( i^{th} \) sieve in the total mass of the sample, which can be calculated by the following equation:

\[ a_i = \frac{m_i}{m_0}, \]  (17)

where \( m_i \) is the mass retained on the \( i^{th} \) sieve (g); \( m_0 \) is the total mass of the specimen (g).

The cumulative retained percentage \( A_i \) is the sum of the retained percentages of the \( i^{th} \) sieve hole and sieve holes with a larger size than that of the \( i^{th} \) sieve hole. It can be calculated as follows:

\[ A_i = a_1 + a_2 + \cdots + a_i, \]  (18)

where \( a_1, a_2, \ldots a_i \) are the retained percentage (%) for each sieve.

The passing rate \( p_i \) represents the percentage of the mass passing the \( i^{th} \) sieve hole in the total mass of the specimen, which is the difference between 100 and the cumulative percentage of sieve residue on the \( i^{th} \) sieve hole. It can be obtained using the following equation:

\[ p_i = 100 - A_i. \]  (19)

The results obtained from equation (15) or equation (16) were substituted into the spreadsheet prepared using equations (17)–(19) for trial. Only minor adjustments were required to obtain optimal mixing mass ratios for various raw materials.

3.2.3. Quadratic Programming Method. Let the passing rates of the synthetic aggregate gradations at the \( i^{th} \) sieve hole be \( f_i(X) \), and let \( P_i = \{ p_{i1}, p_{i2}, \ldots, p_{im} \} \). Replace \( b_i \) with \( f_i(X) \). Therefore, equation (8) was transformed into the following form:

\[ f_i(X) = P_iX. \]  (20)

A fuzzy mathematical membership function \( A_i(f_i(X)) \) was introduced to fuzzify the passing rates of synthetic aggregate gradations at various sieve holes with the following equation:

\[ A_i(f_i(X)) = \frac{(f_i(X) - g_i)(f_i(X) - l_i)}{(b_i - g_i)(b_i - l_i)}, \quad l_i \leq f_i(X) \leq g_i, \]

\[ A_i(f_i(X)) = 0, \quad \text{else.} \]  (21)
Let $U_i(X) = A_i \left( f_i(X) \right)$ be substituted into equation (15), and then substitute equation (13) into equation (16) to obtain

$$
U_i(X) = \begin{cases} 
\frac{(P_i X - g_i)(P_i X - l_i)}{(b_i - g_i)(b_i - l_i)} & i \leq P_i X \leq g_i, \\
0 & \text{else.}
\end{cases}
$$

(22)

Let $U(X) = \sum_{i=1}^{m} (P_i X - g_i)(P_i X - l_i)/(b_i - g_i)(b_i - l_i)$, which can be further modified into a standard quadratic form for the following programming problem:

$$
-U(X) = \frac{1}{2} X^T H X + F X + C,
$$

(23)

where $H = \sum_{i=1}^{m} 2 P_i P_i/(b_i - g_i)(b_i - l_i)$, $F = \sum_{i=1}^{m} (l_i + g_i)P_i/(b_i - g_i)(b_i - l_i)$, and $C = \sum_{i=1}^{m} -l_i g_i/(b_i - g_i)(b_i - l_i)$.

### 4. Fuzzy Comprehensive Evaluation Method

Since Zadeh published the paper on fuzzy mathematics [22], it has been widely used in many fields to solve engineering problems. In this study, different aggregate gradations were obtained through the orthogonal design, and then the corresponding pavement performance test results were fuzzified. The experimental data were processed using the fuzzy mathematics method. Taking the fuzzy comprehensive evaluation value as the objective function, the test results were analyzed and evaluated to obtain the optimal aggregate gradation. The void ratio was used as the index to evaluate the compactness of the asphalt mixtures. The dynamic stability was used as the index to assess the high-temperature rutting resistance of the asphalt mixtures. The residual stability of the water-soaked Marshall test was used to evaluate the water stability of the asphalt mixtures. The evaluation index set was established based on the above three indexes. The pavement performance test results of the nine aggregate gradations obtained through the orthogonal test design were used to determine the evaluation object set.

#### 4.1. Establishment of Membership Functions and Fuzzy Matrix

From the industrial standard [19], it is known that the recommended void ratio range is 3%–6%. The void ratio is an intermediate indicator, and the degree of membership can be calculated using the following equation:

$$
r_{ij} = \frac{\min(u_{ij}, 4.5\%) - \min(u_{ij}, 4.5\%)}{\max(u_{ij}, 4.5\%) - \min(u_{ij}, 4.5\%)}
$$

(24)

where $i = 1, 2, ..., 9$ and $j = 1$.

It is known from the industrial standard [19] that the dynamic stability is not less than 800 times/mm and that the residual stability of the water-soaked Marshall test is not less than 80%. Both of them are partial large indexes. The degree of membership can be calculated based on the following equation:

$$
r_{ij} = \frac{\max(u_{ij})}{\max(u_{ij}, 4.5\%)}
$$

(25)

where $i = 1, 2, ..., 9$ and $j = 2, 3$.

The degree of membership values calculated from equation (24) and equation (25) form the fuzzy relationship matrix.

$$
\begin{bmatrix}
    r_{1,1} & r_{1,2} & r_{1,3} \\
    r_{2,1} & r_{2,2} & r_{2,3} \\
    \vdots & \vdots & \vdots \\
    r_{9,1} & r_{9,2} & r_{9,3}
\end{bmatrix}
$$

(26)

#### 4.2. The Determination of Evaluation Index Weights

The comparison matrix was constructed according to the scales of pairwise comparisons (see Table 4) and judgment principles proposed by the literature [23]. The scale values of 2, 4, 6, and 8 denoted the median of two adjacent scale value comparisons, respectively. $D_{ij}$ $(i = 1, 2, 3; j = 1, 2, 3)$ denotes the importance comparison of $D_i$ and $D_j$, and $1/D_{ij}$ denotes the importance comparison of $D_j$ and $D_i$.

Let the comparison matrix constructed by the pairwise comparison of three pavement performance evaluation indexes be $D$. By definition,

$$
D = \begin{bmatrix}
    D_{11} & D_{12} & D_{13} \\
    D_{21} & D_{22} & D_{23} \\
    D_{31} & D_{32} & D_{33}
\end{bmatrix} = \begin{bmatrix}
    D_1 & D_1 & D_1 \\
    D_2 & D_2 & D_2 \\
    D_3 & D_3 & D_3
\end{bmatrix}
$$

(27)

The square root method was used to solve for the maximum eigenvalue $\lambda_{\text{max}}$ and the eigenvector $W$ of the comparison matrix $D$. The elements of the comparison matrix $D$ were multiplied by rows to obtain the product $M_i$ $(i = 1, 2, 3)$ of each element.

$$
M_i = \prod_{j=1}^{3} W_{ij}
$$

(28)

Calculate the square root of $M_i$.

$$
W_i = \sqrt{M_i}
$$

(29)

Normalize the vector $W$.

$$
W_i = \frac{W_i}{\sum_{j=1}^{3} W_{ij}}
$$

(30)

Calculate the maximum eigenvalue of the comparison matrix.

$$
\lambda_{\text{max}} = \sum_{i=1}^{3} \frac{(DW)_i}{3 \cdot W_i}
$$

(31)
The consistency test of the comparison matrix is as follows:

\[
CR = \frac{C_I}{RI}
\]

(32)

where \(C_I = \lambda_{max} - 1/3 - 1\) is the consistency test indicator and \(RI\) is the average random consistency indicator proposed by the literature [23]. According to the comparison matrix \(D\) with an order of 3 \((n = 3)\), \(RI = 0.58\).

When \(CR < 0.1\), the consistency of the comparison matrix \(D\) is generally acceptable. Otherwise, the comparison matrix \(D\) requires modification until the consistency test is passed.

With the comparison matrix satisfying the consistency condition, the set of weights assigned to the evaluation indicators is obtained as follows:

\[
W = \begin{bmatrix} w_1, & w_2, & w_3 \end{bmatrix}
\]

(33)

4.3. Calculation of Fuzzy Evaluation Values. A fuzzy subset \(E\), called the evaluation set, was introduced based on the evaluation index set. Its fuzzy evaluation values were obtained based on the fuzzy matrix \(R\) and the weight allocation set \(W\) according to the following equation:

\[
E = R \times W^T = \begin{bmatrix} e_1, & e_2, & \cdots, & e_i \end{bmatrix}^T.
\]

(34)

According to the principle of maximum degree of membership, the combination of coarse and fine aggregates is more reasonable, and the pavement performance of corresponding aggregate gradation improves as \(e_i (i = 1, 2, \ldots, 9)\) becomes larger.

5. Engineering Examples

Asphalt treated base (ATB-25) was used for the lower surface course of the Ji-Cao Expressway. The total number of aggregate and mineral powder types used was 6 \((n = 6)\), and the sieving results of the raw materials are presented in Table 5.

The aggregate proportion calculation was carried out using 1# gradation as the target gradation (Table 6).

5.1. Results of the Aggregate Proportion Calculation. The normal equation, table trial, and quadratic programming methods were used to calculate the mass ratios of various raw materials in the mineral aggregates. The results are presented in Table 6.

Mineral aggregates were prepared according to the mixing mass ratios of each raw material as presented in Table 6 and the synthetic aggregate gradations were calculated. The results are presented in Table 7.

Methods 1, 2, and 3 in Tables 6 and 7 were the normal equation, table trial, and quadratic programming methods, respectively. As can be seen from Table 6, the calculation results of the normal equation method cannot satisfy the condition that the sum of raw material mixing mass ratios should be equal to 1 (equation (7)). The calculation results satisfying equation (7) and other constraints can be obtained quickly after minor adjustments using the table trial method. As can be seen from Table 7, the quadratic sum of deviation between synthetic aggregate gradation and design aggregate gradation obtained from the formal equation method was the smallest. However, the calculated cumulative passing percentage for the sieve size of 31.5 mm was 100.75%, which substantially exceeded the upper limit of the design gradation curve.

From Table 6, it can be seen that the calculated results of the quadratic programming method satisfied the constraints. According to Table 7, the calculated cumulative passing percentage for the sieve size of 31.5 mm was 100.01%, which exceeded the upper limit of the design gradation curve of 100%. However, the deviation was quite small and thus acceptable for practical application. The calculation results of quadratic programming method and table trial method showed that the aggregate gradations obtained by both methods are in good agreement with the target gradations and, thus, meet the design requirements.

5.2. Fuzzy Comprehensive Evaluation of Pavement Performance. Marshall tests were carried out according to the specification [20]. The optimum asphalt aggregate ratio for each gradation was determined by plotting the Marshall stability, flow value, relative density, VFA, and voids in mineral aggregate of the specimens against the asphalt aggregate ratio. Results are presented in Table 8. With the optimum asphalt aggregate ratio, the Marshall test, water weight test, rutting test, and water-soaked Marshall test were carried out, and the results are presented in Table 8. The test results in Table 8 were substituted into equation (24) and equation (25) to calculate the degree of membership of each index \(r_{ij}\). The results are presented in Table 8.

According to the fundamentals of analytic hierarchy process, literature review, and communications with experts in the road industry, the comparison matrix \(D\) was constructed, and corresponding weights were calculated according to equation (27) to equation (30). The results are presented in Table 9.
Table 5: The sieving results of the raw materials.

| Sieve size (mm) | Aggregate 1 | Aggregate 2 | Aggregate 3 | Aggregate 4 | Aggregate 5 | Mineral power |
|-----------------|-------------|-------------|-------------|-------------|-------------|---------------|
| 31.5            | 100         | 100         | 100         | 100         | 100         | 100           |
| 26.5            | 84.33       | 100         | 100         | 100         | 100         | 100           |
| 19.0            | 19.75       | 100         | 100         | 100         | 100         | 100           |
| 16.0            | 5.05        | 88.57       | 100         | 100         | 100         | 100           |
| 13.2            | 0.88        | 6.77        | 100         | 100         | 100         | 100           |
| 9.5             | 0           | 0.1         | 93.11       | 100         | 100         | 100           |
| 4.75            | 0           | 0           | 11.69       | 99.62       | 100         | 100           |
| 2.36            | 0           | 0           | 0.09        | 8.2         | 92.74       | 100           |
| 1.18            | 0           | 0           | 0           | 0.96        | 72.35       | 100           |
| 0.6             | 0           | 0           | 0           | 0           | 51.22       | 100           |
| 0.3             | 0           | 0           | 0           | 0           | 35.94       | 100           |
| 0.15            | 0           | 0           | 0           | 0           | 20.13       | 100           |
| 0.075           | 0           | 0           | 0           | 0           | 0           | 79.12         |

Table 6: Calculation results of the mass ratios.

| Calculation methods | Aggregate 1 | Aggregate 2 | Aggregate 3 | Aggregate 4 | Aggregate 5 | Mineral power |
|---------------------|-------------|-------------|-------------|-------------|-------------|---------------|
| Method 1            | 0.4055      | 0.1864      | 0.1298      | 0.0512      | 0.2046      | 0.0300        |
| Method 2            | 0.4055      | 0.1864      | 0.1290      | 0.0412      | 0.2046      | 0.0333        |
| Method 3            | 0.3925      | 0.1946      | 0.1236      | 0.0751      | 0.1708      | 0.0435        |

Table 7: Calculation results of the synthetic aggregate gradation.

| Sieve size (mm) | 1# gradation (%) | 2# gradation (%) | 3# gradation (%) | 4# gradation (%) | 5# gradation (%) | 6# gradation (%) | 7# gradation (%) | 8# gradation (%) | 9# gradation (%) |
|-----------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 31.5            | 100              | 100              | 100              | 100              | 100              | 100              | 100              | 100              | 100              |
| 26.5            | 70               | 68.21            | 67.46            | 68.12            | 67.46            | 68.12            | 67.46            | 68.12            | 67.46            |
| 19.0            | 58               | 60.12            | 59.37            | 60.12            | 59.37            | 60.12            | 59.37            | 60.12            | 59.37            |
| 16.0            | 52               | 51.53            | 50.78            | 51.53            | 50.78            | 51.53            | 50.78            | 51.53            | 50.78            |
| 13.2            | 42               | 41.97            | 41.23            | 41.97            | 41.23            | 41.97            | 41.23            | 41.97            | 41.23            |
| 9.5             | 30               | 30.10            | 29.42            | 30.10            | 29.42            | 30.10            | 29.42            | 30.10            | 29.42            |
| 4.75            | 23.5             | 22.41            | 21.76            | 22.41            | 21.76            | 22.41            | 21.76            | 22.41            | 21.76            |
| 2.36            | 17.5             | 17.85            | 16.47            | 17.85            | 16.47            | 17.85            | 16.47            | 17.85            | 16.47            |
| 1.18            | 13               | 13.48            | 12.74            | 13.48            | 12.74            | 13.48            | 12.74            | 13.48            | 12.74            |
| 0.6             | 9.5              | 10.35            | 9.71             | 10.35            | 9.71             | 10.35            | 9.71             | 10.35            | 9.71             |
| 0.3             | 6.5              | 7.12             | 6.61             | 7.12             | 6.61             | 7.12             | 6.61             | 7.12             | 6.61             |
| 0.15            | 4                | 2.38             | 2.38             | 2.38             | 2.38             | 2.38             | 2.38             | 2.38             | 2.38             |
| 0.075           |                  |                  |                  |                  |                  |                  |                  |                  |                  |
| The quadratic sum of deviation | 14.15 | 18.57 | 20.79 |

Table 8: Results of pavement performance test and fuzzy comprehensive evaluation value.

| Gradation | Optimum asphalt aggregate ratio (%) | Void ratio (%) | Dynamic stability (times/mm) | Marshall remnant stability ratio (%) | $r_1$ | $r_2$ | $r_3$ | $c_1$ |
|-----------|------------------------------------|----------------|------------------------------|-------------------------------------|------|------|------|------|
| 1#        | 3.8                                | 4.0            | 1909                         | 81.3                                | 0.889| 0.814| 0.767| 0.840|
| 2#        | 3.9                                | 4.2            | 1350                         | 83.4                                | 0.933| 0.575| 0.787| 0.807|
| 3#        | 4.0                                | 4.4            | 1536                         | 86.5                                | 0.978| 0.655| 0.816| 0.857|
| 4#        | 3.9                                | 4.5            | 2298                         | 106.0                               | 1.000| 0.980| 1.000| 0.995|
| 5#        | 3.9                                | 4.7            | 1853                         | 82.9                                | 0.957| 0.790| 0.782| 0.872|
| 6#        | 4.0                                | 5.8            | 2346                         | 83.9                                | 0.776| 1.000| 0.792| 0.836|
| 7#        | 3.9                                | 3.7            | 1813                         | 80.1                                | 0.822| 0.733| 0.756| 0.793|
| 8#        | 3.9                                | 4.4            | 1468                         | 97.3                                | 0.978| 0.626| 0.918| 0.875|
| 9#        | 4.0                                | 5.2            | 1654                         | 81.6                                | 0.865| 0.705| 0.770| 0.801|
Table 9: The comparison matrix D and weights.

|     | $D_1$ | $D_2$ | $D_3$ | Weights |
|-----|-------|-------|-------|---------|
| $D_1$ | 1     | 2     | 2     | 0.50    |
| $D_2$ | 0.5   | 1     | 1     | 0.25    |
| $D_3$ | 0.5   | 1     | 1     | 0.25    |

According to the weights of each evaluation index, the maximum eigenvalue was calculated ($\lambda_{\text{max}} = 3.00$) from equation (31), which was then substituted into equation (32). It was found that $Cp = 0$. Because $Cp < 0.1$, the comparison matrix passed the consistency test. In other words, the weight set $W$ of the evaluation indexes was acceptable. According to equation (34), the fuzzy evaluation values $e_i$ were calculated for the pavement performance of nine aggregate gradations, and the results are presented in Table 8. As presented in Table 8, the comprehensive pavement performance of asphalt mixtures prepared with $1\#-9\#$ aggregate gradations was ranked as follows: $4\# > 8\# > 5\# > 3\# > 1\# > 6\# > 2\# > 9\# > 7\#$ according to the principle of maximum degree of membership. The ATB-25 formed by $4\#$ aggregate gradation had the optimal overall performance in terms of compactness, high-temperature rutting resistance, and water stability.

6. Conclusions

New methods for the aggregate proportion calculation and gradation optimization were proposed in this study and then verified by example calculations using ATB-25. The main conclusions are as follows:

1. Using the generalized inverse solution of the normal equation, raw material mass ratios of the synthetic mineral aggregates were calculated, which were substituted into the spreadsheet as the initial values. Minor adjustments were required to obtain the calculation results satisfying the constraint conditions. The equation of this method was simple and did not require programming. Therefore, it has great potential for engineering practice.

2. The degree of membership function in fuzzy mathematics was introduced into the aggregate proportion calculation of asphalt mixtures. The range of design gradations was taken as the domain, and the values of synthetic gradations were fuzzified on the domain. The linear programming was transformed into nonlinear quadratic programming. Through the programming module, satisfactory raw material mass ratios and synthetic aggregate gradations were calculated. From the example calculations, it was found that the quadratic programming method has the advantages of high calculation efficiency and accurate calculation results. Also, it can be used to analyze the problem from different angles. Therefore, it has the potential to be widely used.

3. Fuzzy mathematics and analytic hierarchy process were applied to evaluate the pavement performance of ATB-25. Through the establishment of fuzzy matrix and the determination of corresponding weights, the fuzzy comprehensive evaluation values were calculated. Finally, the $4\#$ aggregate gradation was determined as the optimal choice considering factors of compactness, high-temperature rutting resistance, and water stability was selected.

4. In the design of asphalt mixtures, the influence of various factors was fully considered using the orthogonal experimental design and fuzzy mathematics. The analytic hierarchy process was employed to determine the weight distribution set. In this way, the aggregate gradation was optimized.

Data Availability

All the data included in this study are available upon request by contact with the corresponding author.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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