The Effect of Martian Ionospheric Dispersion on SAR Imaging

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When passing through the ionosphere, the high-frequency (HF) pulse signal of the Mars Exploration Radar is affected by the dispersion effect error, which results in signal attenuation and time delay and brings about a phase advance in such a way that the echo cannot be matched and filtered. In this paper, a high-order phase model is built to overcome the above problems and enable echo matching and filtering. Most studies on the dispersion effect approximate the additional phase after the effect, assuming that the ionosphere is a thin-layer structure. In this paper, an effective model for the HF waveband is constructed to analyze the change of signal propagation paths in the ionosphere. The additional phase is expanded in a Taylor series and retained these expansions as high-order terms to calculate the cumulative additional phase along the path. We show the range-offset variables of signal frequency, bandwidth, and electron density, simulate the effects of the ionosphere under different conditions, and conclude that the model can effectively estimate Mars without considering the effects of magnetic fields and anomalous solar activity and the effect of the ionosphere on synthetic aperture radar (SAR) echoes. The results obtained using ray tracing calculations are different from those obtained by simplifying assumptions, and we can simulate the Martian ionospheric effects by the former.

1. Introduction

The subsurface of Mars records important historical information on the formation and evolution of Mars. As an ionized medium, the Martian ionosphere plays a special role in radio wave propagation and is directly related to the local communication on Mars and the communication between Mars and Earth. Therefore, the information on the subsurface and the Martian ionosphere provides a scientific basis for understanding and exploring Mars, as well as for studying the history of geological evolution.

The multiband low-frequency down-looking synthetic aperture radar (SAR) mounted on the Mars Orbiter can emit low-frequency radio waves that can penetrate the surface of Mars and propagate downwards. It inverts the physical properties such as the dielectric constant of subsurface material and thereby reveals the water content and distribution in the subsurface. SAR has already been used in Mars missions. For example, the United States launched the Mars Reconnaissance Orbiter (MRO) with the Shallow Subsurface Radar (SHARAD). For SHARAD, its vertical resolution in a vacuum and in the subsurface can reach 15 m and 10–20 m, respectively, and it can penetrate about 1 km of the subsurface structure and detect the internal structure of the ice cap in the polar regions of Mars to look for possible liquid water [1, 2]. The European Space Agency launched the Mars Express (MEX) in 2003 [3], which has been communicating with the lander and several rovers on the surface. During the communication, a double traverse of radio signals through the Mars ionosphere caused effects [4, 5]. On the side facing the sun (the electron density is $2 \times 10^5 \text{ cm}^{-3}$ by calculation), the plasma cutoff frequency of the Martian ionosphere is 4.02 MHz, namely that signals below this frequency cannot propagate in the Martian ionosphere; on the dark side (the electron density is $5 \times 10^3 \text{ cm}^{-3}$ by calculation), the cutoff frequency of the Martian ionosphere is 0.65 MHz. Therefore, this study is mainly aimed at the carrier frequency in HF bands to ensure that SAR can work on Martian days and nights. When a signal passes through the Martian ionosphere, it is affected by the ionosphere, which results in...
dispersion and scintillation effects. Ionospheric dispersion has the effects such as signal distortion, turbulence amplitude, and phase fluctuations, and these effects reduce the range resolution of the radar and seriously affect its detection capability [6]. Although there are often various scales of electron density irregularities in the ionosphere, which can also cause the dispersion of a received signal, the large-scale background ionospheric dispersion term is the main research object of this paper.

Owing to the weak solar radiation flux, the distance between Mars and the sun is long, and the plasma density in the ionosphere of Mars is low. The ionosphere of Mars begins above the surface and extends upward for hundreds of kilometers, where data are directly measured by the Viking Lander 2 [7]. The primary peak of electron density occurs at an altitude of about 130 kilometers, known as the M2 layer, and a secondary peak occurs around 100 km and is called the M1 layer. According to the expected behavior of the Chapman layer, the peak electron density varies with the solar zenith angle (SZA), except for high SZA values (SZA = 90°). Other than the M1 and M2 layers, the transition layer ranges from 145 to 195 km, and the top layer is at an altitude of 200–240 km. On the basis of the theoretical analysis of previous research [8–10], we combined the predicted values of the Martian ionosphere model with the data obtained from Mars ignoring the influence of the solar wind and electromagnetic field and delineated the basic parameters of the model.

The ionosphere of Mars does not have strong global magnetic confinement. Its plasma density is an order of magnitude lower than that of Earth, with the peak density during the solar maximum similar to that of the night surface ionosphere on Earth during the solar minimum. By integrating the solar ionospheric profile, we can obtain a total electron content (TEC) of 4.0 × 10^{11} \text{ cm}^2 [11], which is 50 times the value of the ionospheric TEC of Earth, but even in this case, the interference of radar pulse signals of the Martian ionosphere cannot be ignored.

The ionosphere of a planet imposes a delay upon the radio transmission from an orbiting artificial satellite to a ground receiving station, which leads to ranging errors in systems designed for precise positioning. Allnut [11] discovered that radio waves at frequencies below 10 GHz are affected by the ionosphere, especially those at frequencies below 1 GHz. Ishimaru et al. [12] analyzed the effect of the ionosphere on the spaceborne SAR at carrier frequencies of 100 MHz–2 GHz and calculated the effects on SAR images. Liu et al. [13, 14] proved that the ionosphere interferes seriously with the range and azimuth resolution of low-frequency electromagnetic waves, and the influence of the ionosphere on electromagnetic signals increases as the frequency of radio waves decreases. They have developed a numerical model to simulation ionospheric effects on SAR signals. In the case of Mars, recent studies have dealt with some impacts of the Martian ionosphere upon HF radio signal propagation. Wang et al. [15] have explored how the Martian ionosphere would affect on SAR imaging, based on the Taylor series approximation. Safaeinili et al. [16, 17] examined how the ionosphere would impact an orbital radar at low frequencies; they have elaborated on the influence of the Martian ionosphere on radio waves in their article and proposed methods for signal correction. Meyer and Watkins [18] and Meyer [19] outlined the problem of ionospheric effects in SAR data and showed that SAR data are fairly sensitive to ionospheric TEC changes.

We research the characteristics of ionospheric distortion and develop statistical models to analyze the impacts on radar data. Specifically, this paper simulates signal transmission paths to estimate integrated phases by the Martian ionospheric model. The phase estimation of signals is carried out by using the high-order number expansion of the refractive index. By the ambiguity model, propagation functions and ionospheric effects on radar are derived.

In Section 2, the ionospheric dispersion effect and signal path change in the ionosphere are introduced. In Section 3, mathematical and statistical methods are applied to describe ionospheric impacts on echoes. An ionospheric phase simulator and signal parameter changes under different conditions are presented in Section 4. Section 5 summarizes the research findings.

### 2. Ionospheric Dispersion Effect

The ionosphere is a special dispersive medium with anisotropic characteristics. For a radio signal with a wide frequency spectrum, different frequency components of the signal propagate at different phase velocities in the ionosphere, and thus, different frequency components have different phase relationships. The signal will be distorted, and the pulse is broadened in time and space. This is the dispersion phenomenon of the ionosphere. The dispersion effect will cause signal amplitude attenuation and phase advance, which will directly affect the effect of echo processing such as distance pulse compression and correction. At the same time, the chromatic dispersion effect will also broaden signal pulse and cause difficulties in filtering and windowing.

The refractive index of electromagnetic wave propagation in the Martian ionosphere can be expressed as the Appleton-Hartree formula. In other words, the signal transmission relationship in the ionosphere is written as follows [11]:

\[
\begin{align*}
\frac{n^2 - 1}{n^2 + 1} &= \frac{X}{1 - jZ - A \pm B}, \\
A &= \frac{Y^2 \sin \theta}{2(1 - X - jY)}, \\
B &= \sqrt{A^2 + Y^2 \cos^2 \theta}.
\end{align*}
\]

Here, \(X, Y,\) and \(Z\) are the Appleton parameters, which can be expressed as

\[
X = \frac{\omega_p^2}{\omega^2}, \quad Y = \pm \frac{\omega_p B}{\omega}, \quad Z = \frac{v}{\omega},
\]

where \(\theta\) is the angle between the normal direction of the electric wave and the geomagnetic field, \(v\) is the collision
frequency of free electrons and heavy particles, $\omega_p$ is the plasma pulsation, and $\omega_H$ is the magnetic rotation frequency, whose expressions are as follows:

$$
\omega_H = \left| \frac{q_e B_0}{m} \right|, \quad \omega_p^2 = \frac{q_e^2 N_e}{m_0},
$$

(5)

where $B_0$ is the strength of the Earth’s magnetic field, $q_e$ is the electron charge (Coulomb), $q_e = 1.6 \times 10^{-19}$, $m_e$ is the electron mass (kg), $m_e = 9.11 \times 10^{-31}$, $N_e$ is the electron density ($m^{-3}$), and $e_0$ is the free-space permittivity ($F/m$), $e_0 = 8.854 \times 10^{-12}$. When the signal frequency is less than 30 MHz, $f_0$ is the center frequency of the radar signal band. We perform the Taylor series expansion on the refractive index $n$ and then integrate each item of the expansion, and thus we have

$$
n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} = 1 - \frac{1}{2} \left( \frac{\omega_p^2}{\omega^2} \right) - \frac{1}{8} \left( \frac{\omega_p^2}{\omega^2} \right)^2 - \frac{1}{16} \left( \frac{\omega_p^2}{\omega^2} \right)^3,
$$

(6)

where $\omega_p = 2\pi f_p$ and $f_p$ (Hz) is expressed as follows:

$$
f_p = \frac{q_e}{2\pi \sqrt{\varepsilon_0 m_e}} \frac{\omega}{\sqrt{n_e}} \approx 8.98 \sqrt{n_e}.
$$

(7)

It can be seen from Equation (7) that the refractive index is a function of the frequency and the electron density. Considering the working frequency band (MHz) of the Mars Exploration Radar, the high-order terms of the refractive index cannot be ignored.

3. The Influence of the Mars Ionospheric Dispersion Effect

3.1. Effect on Echo Pulse Compression. In a radar system, the range resolution is an important performance index that determines the minimum distance between two units that the radar can distinguish. Pulse compression technology can ensure sufficient detection range and improve range resolution. However, owing to the time delay, phase error, and amplitude attenuation caused by the ionospheric dispersion, echoes cannot match the matched filter function, which directly leads to the degradation of the image quality after pulse compression.

When electromagnetic waves propagate in a homogeneous medium, the magnitude of the electric field strength is expressed as

$$
E = E_0 \times \exp \left( j\omega t - jk_0 n r \right),
$$

(8)

where $E_0$ is the amplitude of the electromagnetic wave, $\omega$ the angular frequency, $r$ the path length of signal propagation, $k_0$ is the propagation constant of the vacuum, $k_0 = \omega/c$, and $n$ is the refractive index in the propagation medium. However, the ionosphere is a medium with uneven ion concentration, and $n$ changes with the transmission path. In this situation, the phase depends on the integral of the refractive index on the path. The transmission function of the electromagnetic wave can be expressed as follows:

$$
E = E_0 \times \exp \left[ \left( j\omega t - k_0 \int n(l) dl \right) \right].
$$

(9)

In the equation, the phase after the signal passes through the ionosphere is $k_0 \int n(l) dl$; $l$ represents the signal propagation path, and $dl = dh/\cos \theta$, where $\theta$ is the zenith angle of the point of the radar point. Applying Snell’s law, i.e., $nr \cos \theta = n_l r_0 \cos \theta$, we can obtain the phase shift change caused by the dispersion effect:

$$
\Delta R_p = \int_L (n - 1) dl.
$$

(10)

It can be seen from Equation (8) that the dispersion effect shortens the phase path length, and a two-way phase advance is added to the signal relative to free space as follows:

$$
\Delta \phi(f) = 2\pi \times \left[ \frac{\Delta I_p}{A} \right].
$$

(11)

Substituting Equations (4) and (5) into Equation (9), we have

$$
\Delta \phi(f) = \frac{4\pi \sec \theta}{c} \left[ \frac{40.32}{f} \int n_l dh + \frac{812.851}{f^2} \right] n_l^3 dh
$$

+ \frac{32774.2}{f^3} \int n_l^3 dh.
$$

(12)

Different incident angles are used to simulate the path tracking situation. $x$ is the horizontal distance from the entry point of the incident signal to the position of the target point projected on the ground, $R_s$ is the radius of Mars, and $H_s$ is the height of the entry point from the ground.

After the primary term is transformed into the time-domain, pulse compression will cause a distance shift, and even-order phase errors will lead to a broader main lobe of the signal, a lower peak point energy, and greater side lobes. The odd-order phase errors lead to asymmetrical distortion of the sidelobes after pulse compression of the point target.

From Equation (12), the additional phase of the signal after the signal passes through the ionosphere base on the integral amount of ion concentrations over paths. When the radar signal passes through the ionosphere, the incident direction of the signal cannot be guaranteed to be vertical. If the incident direction is inclined to a certain extent, the integral over the height should be changed to the integral over the path presented by $l$. Figure 1 shows that after applying Snell’s law of refraction, we have $dl = dh/\cos \theta$, where $\theta$ is the inclination of the radar signal in the direction of propagation at a certain point. In view of the above analysis, the following research can analyze the simulation paths according to different incident angles. $R_s$ is the radius of Mars, and $H_s$ is the height of that certain point above the ground.
Therefore, the signal expression under the influence of the ionosphere is as follows:

$$\Psi_1(f, x) = \Psi(f, x) \times \exp(j \Delta \Phi(f)) \left| f - f_c \right| \leq B/2. \quad (13)$$

It can be seen from the above expression that the original matched filter function remains unchanged, but the echo under the influence of the ionosphere changes, which results in a mismatch between the received signal and the original matched filter function. Moreover, the image quality is degraded.

### 3.2. Effect on the Range Resolution

As the refractive index changes with frequency and position, the SAR signal deviates from the normal signal in a vacuum, which affects the result of SAR imaging. The Martian ionosphere is constantly changing and has a certain degree of randomness, which causes the echo phase to be random and indeterminate. Therefore, it is necessary to use statistical models to study the influence of the Martian ionosphere on SAR imaging. The SAR signal of the ground position can be expressed as follows [15].

$$u(r') = \int S(r) \chi(r, r') ds, \quad (14)$$

where $S(r)$ is the reflection coefficient at the ground position and $\chi(r, r')$, the ambiguity function at the focus $r'$, is given by

$$\chi(r, r') = \sum_n \frac{1}{2\pi} \int \overline{g_n}(\omega, r_n) f_n(\omega, r_n') d\omega, \quad (15)$$

$$\overline{g_n}(\omega, r_n) = \overline{\pi_i(\omega) G_0(\omega, r_n)}, \quad (16)$$

$$f_n(\omega, r_n') = \overline{\pi_i(\omega)} \exp \left( j \frac{\omega}{c} 2r_n' \right), \quad (17)$$

$$G_0(\omega, r_n) = \exp \left( -j2 \int \beta(\omega) ds \right), \quad (18)$$

where $u_i(\omega)$ is the transmitted signal, $f_n$ is the Fourier transform of the matched filter, $G_0$ is the Green formula, and $\beta(\omega)$ is the propagation constant on the propagation path. The phase fluctuations caused by the background ionosphere are included in the path integral, and the path integral $\beta(\omega)$ is expanded in the Taylor series near the carrier frequency $\omega_0$:

$$\beta(\omega) = \beta(\omega_0) + (\omega - \omega_0) \beta'(\omega_0) + \frac{(\omega - \omega_0)^2}{2} \beta''(\omega_0). \quad (19)$$

Taking the Fourier transform of the chirp signal and substituting it into the ambiguity function, we can obtain

$$\chi(r, r') = \sum_n \frac{1}{2\pi} \int \overline{\pi_i(\omega)} \exp \left( -j2 \int \beta(\omega) ds + j(\omega/c) 2r_n' \right) d\omega. \quad (20)$$
Table 1: The ion concentration of Martian and Earth’s ionospheres.

| Ion concentration | Mars $n_0\, (cm^{-3})$ | $f_0\, (MHz)$ | Earth $n_0\, (cm^{-3})$ | $f_0\, (MHz)$ |
|-------------------|------------------------|--------------|-------------------------|--------------|
| Dayside Solar max. | $2.5 \times 10^5$ | 4.5          | $2.0 \times 10^6$      | 12.7         |
|       Solar min.   | $1.0 \times 10^5$ | 2.9          | $5.0 \times 10^5$      | 6.3          |
| Nightside Solar max. | $5.0 \times 10^7$ | 0.6          | $2.0 \times 10^5$      | 4.0          |
| Dayside TEC       | $4.0 \times 10^{11} \, cm^{-2}$ |              | $2.0 \times 10^{13} \, cm^{-2}$ |              |

No nightside ionospheric data are available for Mars during the solar maximum.

Table 2: Usable critical frequency for various launch angles.

| Launch angle $\theta$ | Maximum usable frequency (MHz) |
|-----------------------|--------------------------------|
|                       | 0°  | 15° | 30° | 45° | 60° | 75° |
| Maximum usable frequency (MHz) | 4.0 | 4.14| 4.62| 5.66| 8.0 | 15.5|

Figure 4: The relationship among the phase error, incident angle, and height. (a–c) The variation curves in the high, moderate, and low solar activity periods, respectively.
The input signal \( u_i(\omega) \) is generally a linear frequency modulation signal, which is written after the Fourier transform as follows:

\[
S_{LFM}(\omega) \approx \text{rect}\left(\frac{\omega}{2\pi KT}\right) \sqrt{\frac{2\pi K}{K}} \exp \left[ -\frac{(\omega - \omega_c)^2}{2K} \right].
\] (21)

Finally, the normalized ambiguity function is expressed as

\[
\chi_n(r, r') = \frac{1}{(4\pi r')^2} \exp (i\Phi_0(\omega_0)) \times \int \exp (i(\omega - \omega_0)\Phi_1) \, d\omega + (\omega - \omega_0)^2 \Phi_2 \right) \, d\omega,
\] (22)

\[
\Phi_0(\omega_0) = \frac{2}{c} \left( r'_n - r_n \right) + 2 \frac{40.3(2\pi)^2}{c\omega_0^6} \int n_c \, dl + 2 \frac{812.85(2\pi)^2}{c\omega_0^6} \int n_c^3 \, dl,
\] (23)

\[
\Phi_1(\omega_0) = \frac{2}{c} \left( r'_n - r_n \right) - 2 \frac{40.3(2\pi)^2}{c\omega_0^6} \int n_c \, dl + 2 \frac{812.85(2\pi)^2}{c\omega_0^6} \int n_c^3 \, dl,
\] (24)

\[
\Phi_2(\omega_0) = \frac{2}{c} \frac{40.3(2\pi)^2}{c\omega_0^6} \int n_c \, dl,
\] (25)

where \( \Phi_0 \) is the change in azimuth, \( \Phi_1 \) is the group delay, and \( \Phi_2 \) is the degree of beam broadening. The signal position of distance output is given by \( \Phi_1 \), where \( 2(r_0 - r_n)/c \) is the signal position under ideal conditions.

Figure 5: The relationship among the phase error, frequency, and height. (a–c) The variation curves in the high, moderate, and low solar activity periods, respectively.
Equation (26) represents the signal translation introduced by the ionosphere, and it is proportional to the path integral of each order of ion concentration and inversely proportional to the carrier frequency. It can be seen that a greater carrier center frequency of the signal is accompanied by a smaller signal translation, and a greater ion concentration leads to more difficulty in calibrating the signal and a more serious offset.

4. Simulation of Ionospheric Dispersion Effect

4.1. Model Design. SAR signals are usually modulated chirp pulses. The frequency and period of the pulses are proportional to the duration of the normal pulse period. As the Martian ionosphere is a dispersive medium, the working frequency band of the Martian surface is different from that of Earth. First, a chirp signal in the band of 10–13 MHz is selected. Assume that the single pulse signal emitted by SAR under ideal conditions is written as

\[
f_t(t) = \text{rect}\left(\frac{t}{T}\right) \exp\left(j\omega_0 t + \frac{1}{2}K\pi t^2\right). \tag{27}\]

The echo after the ionospheric dispersion effect is expressed as

\[
f(t) = \frac{1}{2\pi} \int F(\omega) \exp\left(-jk_0(\omega) \int n(\omega, l) dl\right) d\omega. \tag{28}\]

The key step is to establish the spatial distribution of the refractive index and determine the true influence of signal propagation on the SAR echo. According to Equation (6), the spatial distribution of the refractive index can be determined by the spatial distribution of electron density and signal frequency. The signal propagation path can be obtained by path tracking technology.

On the basis of the above analysis, the actual simulation steps are as follows:

1. Build an ion concentration distribution model based on ionospheric data of Mars.
2. According to the system simulation parameters and \( \text{Ne} \) (Martian ionospheric model of different solar activity periods and different zenith angles), use the path tracking method to calculate the two-way phase advance caused by the dispersion effect.
3. Multiply the ideal signal and the additional phase advance in the range frequency-domain. The frequency-domain signal is affected by the ionosphere.
4. Perform inverse Fourier transform on the frequency-domain signal to obtain the affected signal in the time-domain, and then compare it with the ideal signal.

The simulation process is shown in Figure 2.

4.2. Simulation Parameter Selection. First, according to the ion concentration distribution data of Mars \([20, 21]\), the Chapman model is used to build the relationship model.
between the ion concentration and the height of the Martian ionosphere (this model is simulated by the same project which is shown in Figure 3 and is built by Mingyan Huang when the Martian ionosphere is in the high solar activity period). Statistical studies based on available data [7] indicate that the dayside Martian ionosphere can be generally described using a simple Chapman layer model. In addition, the recent development proves that the Chapman model is suitable for describing the Martian ionosphere [8, 22–24].

According to the above process, we first simulated the impact of the point target and analyzed the changes in the phase error caused by the ionosphere under different conditions. Then, we analyzed the impact on echo processing and discussed the signal attenuation offset, signal broadening, and system parameters. Finally, we analyzed the SAR imaging simulations through the calculation of imaging quality indicators to simulate the position shift of the detection target caused by the dispersion effect.

4.3. Simulation Analysis of the Phase Error. The phase error is related not only to the ion concentration and frequency of the ionosphere but also to the selection of the incident angle. In the simulation, it is necessary to ensure that the signal can penetrate the ionosphere and that there will be no signal loss due to ionospheric reflection. The critical frequency of the Martian ionosphere is $f_0(MHz) = 9.0 \times 10^{-6} \sqrt{N_0(m^{-3})}$. It is 4.0 MHz at normal incidence, which is three times lower than the critical frequency of Earth’s ionosphere. For oblique incident waves, the critical frequency available is $f_0(MHz) = 4.0 MHz/\cos \theta$, where $\theta$ is the initial wave launch angle.

The peak ion concentration and critical frequency values of Martian and Earth’s ionospheres during different solar activity periods are shown in Table 1.

Table 2 shows the relationship between the critical frequency and the incident angle. It can be seen that as the incident direction of the signal deviates from the normal direction, the minimum frequency required for the signal penetration
to pass through the ionosphere increases. In this experimental simulation, the detection signal selects the frequency band of 10–30 MHz. The path tracking method is used to simulate the path of the detection signal refracted in the ionosphere, and the relationship between the incident angle and the critical frequency is fully simulated.

Table 4: SAR simulation parameters.

| Parameter               | Value     |
|-------------------------|-----------|
| Track height            | 400 km    |
| Signal carrier frequency| 10–30 MHz |
| Bandwidth               | 10 MHz    |
| Angle of incidence      | 45°       |
| Pulse repetition frequency| 600 Hz   |
| Signal form             | Chirp signal |

Table 5: Simulation parameters of the ionospheric dispersion effect.

| TEC ($10^{14}$ m$^{-2}$) | Carrier frequency (MHz) | Bandwidth (MHz) |
|---------------------------|-------------------------|-----------------|
| 1                         | 1, 1.2, 1.5, and 2      | 5               |
| 2                         | 1.5                     | 10, 20, 25, and 30 |
| 3                         | 1.5                     | 5               | 1, 2, and 3 |
It can be seen from the data above that the incident angle is directly proportional to the critical frequency. Selecting an appropriate incident angle for the signal can increase the critical frequency, and thus, the signal can carry more information and ensure that the radar echo within a certain range is received by the receiver.

A phase error is an important cause of serious distortion of signal waveforms, which can deteriorate image quality, and its value varies with frequency and bandwidth. For the first-order phase, its value only affects the position of the target distance and does not affect imaging quality. When calculating the phase error, we only consider the nonlinear phase error, and in the simulation, the phase error changes under different carrier frequencies and different incident angles. Excessive phase errors can result in a decrease in the resolution and an increase in the peak sidelobe ratio and integral sidelobe ratio. The simulation diagrams of phase errors, incident angles, and detection signal frequency in different solar activity periods are as follows:

As far as the Martian ionosphere is concerned, the experimental results show that the effect of incident angles on phase errors is an essential factor. A larger angle of incidence of the signal leads to a greater phase error and higher critical frequency for ionosphere penetration. As the propagation direction of the incident wave deviates from the normal direction of the ionosphere, where the point target is located, the phase error and the critical frequency for ionosphere penetration increase. It can be seen from Figure 4 that for the HF band of the Mars detection SAR, the impact of ionospheric dispersion on SAR imaging must be considered.

Figure 5 shows that the phase error and frequency are not strictly proportional. The phase error declines rapidly as the carrier frequency grows. The phase error is relatively large, and with the carrier frequency of 10 MHz and the incident angle of 75°, the phase error reaches 5000°, which seriously affects imaging quality.

The phase error causes the match failure of the echo and the matched filter; thereby, the filtered and demodulated signal is aliased and out of focus, and the imaging quality is significantly degraded. The simulations are shown in Figure 6.

It can be seen that the phase error caused by the ionospheric dispersion effect brings about different degrees of time-domain frequency shift, which presents difficulties in pulse compression and echo correction. This paper uses the simulation of the pulse compression processing mode of the point target echo signal to simulate the SAR echo.
processing. The simulation parameters of the four-point target SAR echo model are shown in Table 3.

Figure 7 shows the ionospheric dispersion effect on the pulse echo in the broadband HF band of SAR under different TEC values.

The effects of different ionospheric TEC values on the SAR signal after pulse compression and matched filtering are shown in Figure 8.

It can be seen that pulse compression can effectively separate strong point targets at a relatively close distance, but the phase error makes it impossible to clearly distinguish point targets after echo processing. A larger TEC brings more glitches and overlap on SAR images, and hence, the phase error caused by the dispersion effect must be corrected.

4.4. Effects on the Position of Points Targets. The additional phase caused by the ionospheric dispersion effect of Mars is not only related to the incident angle of the signal but also related to the signal’s parameters such as carrier frequency and bandwidth, and also related to the ionospheric ion concentration. Therefore, the simulated ionospheric dispersion effect on the focus of point targets under different carrier frequencies, bandwidths, and ionospheric TEC values is analyzed. The simulation parameters are shown in Table 4.

The ionospheric group refractive index decreases with the increase in signal frequency, and therefore, signals at different frequencies have different speed in the ionospheric medium. For the identical propagation range, the path delay is different.

First, we simulate the ionospheric dispersion effect on SAR. As the effect of ionospheric dispersion on range signals is related to TEC, carrier frequency, and bandwidth, the simulation of the dispersion effect includes the following three parts:

1. Under fixed carrier frequency and bandwidth, the range signal output changes with TEC
(2) Under fixed TEC, bandwidth, the range signal output changes with carrier frequency

(3) Under TEC, carrier frequency, the range signal output changes with bandwidth

The simulation parameters are shown in Table 5. The simulation results are shown in Figure 9. Through the above simulations, the following conclusions can be drawn:

The influence of the chromatic dispersion effect on the signal is mainly the introduction of phase errors, signal shift and time delay which are shown in Figure 10. As a result, the echo and the matched filter do not match, which leads to problems such as signal broadening and peak attenuation of the main lobe.

A low-frequency signal shift is greatly affected by TEC and carrier frequency. A larger carrier frequency gives rise to a smaller signal shift, and a greater TEC is accompanied by a greater ion concentration and a greater signal shift. It can be seen that the amount of translation is unrelated to the bandwidth, which is consistent with the analysis. For $T = 2 \times 10^{14} \text{ m}^{-2}$ and $f_c = 5 \text{ MHz}$, the image positioning offset is about 100 m.

The broadening of the main lobe of the pulse after the signal is affected is related to the bandwidth, carrier frequency, and TEC. A larger bandwidth causes more severe bandwidth expansion of the main lobe and a sharper peak power drop; a higher carrier frequency leads to smaller bandwidth expansion of the main lobe and a gentler peak power drop; a larger TEC results in a sharper peak power drop. For $TEC = 1 \times 10^{14} \text{ m}^{-2}$, $f_c = 5 \text{ MHz}$, and $B = 3 \text{ MHz}$, the peak height drops by half.

5. Conclusion

In this paper, we analyze the influence of the ionospheric dispersion effect on the single SAR signal and imaging under different bandwidths, carrier frequencies, and path incidence angles in the Martian ionosphere. The work is based on the Mars ionospheric model constructed by the Mars’ real ionospheric data, and the path tracking method is used to obtain the influence of the dispersion effect on the radar signal. The additional phase error of the signal is obtained by simulation of the high-order Taylor series approximation. The pulse broadening and echo caused by the ionosphere are mismatched with the matched filter. Further analysis has been done on the variation of range displacement under different carrier frequencies, bandwidths, and incident angles. The frequency modulation coefficient of the echo is also changed, with the pulse broadened and the filter mismatched.

Data Availability

The data used to support the findings of this study are available from the author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

Bo Wang is responsible for conceptualization and methodology. Xijin Luo is responsible for writing, visualization, investigation, and data curation. Qinghong Sheng is responsible for supervision and project administration. Zhijun Yan is responsible for corresponding and writing—reviewing.

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