NON-LINEAR EQUATION OF STATE, COSMIC ACCELERATION AND DECELERATION DURING PHANTOM-DOMINANCE

S.K. Srivastava\textsuperscript{1} and J. Dutta\textsuperscript{1,2}

\textsuperscript{1} Department of Mathematics, North Eastern Hill University, NEHU Campus, Shillong - 793022 (INDIA )

\textsuperscript{2} Department of Mathematics, St. Edmund’s College, Shillong-793003 (INDIA).

Abstract

Here, RS-II model of brane-gravity is considered for phantom universe using non-linear equation of state. Phantom fluid is known to violate the weak energy condition. In this paper, it is found that this characteristic of phantom energy is affected drastically by the negative brane-tension $\lambda$ of the RS-II model. It is interesting to see that up to a certain value of energy density $\rho$ satisfying $\rho/\lambda < 1$, weak energy condition is violated and universe super-accelerates. But as $\rho$ increases more, only strong energy condition is violated and universe accelerates.

Electronic addresses-

S.K. Srivastava: srivastava@nehu.ac.in; sushilsrivastava52@gmail.com

J. Dutta: jibi_dutta@yahoo.co.in

1
When $1 < \rho/\lambda < 2$, even strong energy condition is not violated and universe decelerates. Expansion of the universe stops, when $\rho = 2\lambda$. This is contrary to earlier results of phantom universe exhibiting acceleration only.

Keywords: RS-II model, non-linear equation of state, phantom cosmology, acceleration and deceleration.

1. Introduction

Astrophysical observations, in the recent past, have conclusive evidence in favor of late cosmic acceleration[1, 2, 3]. This revolutionary observation challenged cosmologists to develop an appropriate cosmological model explaining acceleration in the late universe. Also, observations support homogeneous and flat model of the universe [4]. In such a model, Friedmann equations show that cosmic dynamics can exhibit acceleration only when $\rho + 3p < 0$ with $\rho$ being the energy density and $p$ being the pressure density [5]. It shows violation of the cosmic strong energy condition (SEC) and indicates dominance of exotic fluid in the late universe. The condition $\rho + 3p < 0$ implies $-1/3 > w > -1$ with EOS parameter $w = p/\rho$. In 2002-2003, Caldwell found the case $w < -1$ better fit for the observed astrophysical data and advocated for this case, which violates the weak energy condition (WEC) too [6, 7]. This fluid, is known as phantom. The phantom model explains the present and future acceleration of the universe, but it is plagued with the problem of big-rip singularity (singularity in finite future time when energy density, pressure and the scale factor diverge). Thus, phantom was another exotic matter suggested by Caldwell. Different sources of exotic matter violating SEC
[8, 9, 10, 11, 12, 13, 15, 16, 17] and WEC were proposed in the recent past [6, 7, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. A comprehensive review of these contributions is available in [5]. Later on, it was proposed that curvature could be a possible source of dark energy. These models are known as $f(R)$– dark energy models [33, for review]. Recently, $f(R)$– dark energy models are criticized in [34, 35] on the ground that these models do not produce matter in the late universe needed for formation of large scale structure in the universe. In another review, Nojiri and Odintsov have discussed dark matter in the late universe refuting criticism against their work [36]. Apart from these, curvature-induced dark energy models, different from $f(R)$– dark energy models, were proposed in [37, 38, 39].

In the race to investigate a viable cosmological model, satisfying observational constraints and explaining present cosmic acceleration, brane-gravity was also drawn into service and brane-cosmology was developed. A review on brane-gravity and its various applications with special attention to cosmology is available in [40, 41, 42, 43]. After development of M-theory bringing different string-theories under one umbrella, this theory stemmed from low energy string theory when Hořava and Witten proposed that 11-dimensional supergravity, being a supermembrane theory, could be obtained as low-energy limit of 11-dimensional M-theory. They discussed that it could be done on a particular orbifold $R^{10} \otimes S^1/Z_2$ with $R^{10}$ being the 10-dimensional spacetime and $S^1/Z_2$ being the 1-dimensional space having $x^{11} \equiv -x^{11}$ symmetry [44]. According to this solution, when the six extra-dimensions on $(1+9)$-branes are compactified on very small scale close to the fundamental scale, their effect is realized on $(1+3)$-dimensional brane
located at the ends of $S^1/Z_2$. Thus, Hořava - Witten solution provided an effective 5-dimensional model where extra-dimension can be large relative to the fundamental scale in contrast to Kaluza-Klein theory, where extra-dimension is very small [45, 46]. This solution was used by L.Randall and R.Sundrum in their seminal paper to solve the “hierarchy problem” by a warped or curved dimension showing that fundamental scale could be brought down from the Planck scale to 100 GeV. Thus, Randall-Sundrum approach brought the theory to scales below 100 GeV being the electroweak scale( so far results could be verified experimentally upto this scale only). In this model, extra-dimension is large having (1 + 3)-branes at its ends. These branes are $Z_2$-symmetric (have mirror symmetry) and have tension to counter the negative cosmological constant in the “bulk”, which is AdS$_5$. The model, having two (1 + 3)-branes at the ends of the orbifold $S^1/Z_2$, is known as RS-I model [47].

In another paper, in the same year, these authors proposed another brane-model as an alternative to compactification . In this model, there is only one (1 + 3)-brane at one end of the extra-dimension and the other end tends to infinity. This model is known as RS-II model [48]. Thus, Hořava - Witten solution and RS- theory yielded brane-gravity originating from low-energy string theory, which explained weakness of gravity also in the observable universe. In case, the extra-dimension is time-dependent, brane-gravity induced Friedmann equation (giving dynamics of the universe) contains a correction term $-4\pi G \rho^2 / 3\lambda$ with $\lambda$ being the brane-tension [40, 41, 42, 43]. In RS-I model, $\lambda$ is positive, whereas $\lambda$ is negative in RS-II model.
So, apart from general relativity (GR)-based models and $f(R)$-models, brane-gravity (BG)-based cosmological models were also tried upon to explain acceleration in the late universe. In particular, RS-II model got much attention due to its simple and rich conceptual base [49, 50, 51, 52, 53, 54, 55, 56, 57, 58]. In [59], it is found that RS-II model of brane-gravity yields a phantom model giving transient acceleration (where acceleration stops after sometime in future) and avoiding big-rip singularity. Later on, avoidance of big-rip singularity was shown in GR-based model too, if the dominating fluid behaves as barotropic phantom fluid and generalized Chaplygin gas simultaneously [18].

The present density of dark energy is found to be $0.73\rho^0_{cr}$ with $\rho^0_{cr} = 2.5 \times 10^{-47}$ GeV$^4$ (present critical energy density) [1, 2]. If late universe is dominated by phantom, present phantom energy density is $0.73\rho^0_{cr}$. As discussed above, Friedmann equation, obtained from RS-II model of brane-gravity, contains the energy term as $(8\pi G\rho)/3[1 - (\rho/2\lambda)]$, where $\lambda = 48\pi G/k_5^2 = 48\pi /M_P^2 k_5^2$ with $k_5^2 = 8\pi G_5 = 8\pi Gl = 8\pi l/M_P^2$, Newtonian gravitational constant $G = M_P^{-2}$ in natural units given below ($M_P$ being the Planck mass) and $l$ being length of the extradimension of the 5-dimensional bulk [41, 42, 43]. As an example, if we set $k_5^2 = 1$ GeV$^{-3}$ as taken in ref. [42], $\lambda = 48\pi /M_P^2 = 6.03 \times 10^{10}\rho^0_{cr}$ and $\rho/2\lambda \sim 10^{-10}$ in the present universe. This example shows that the brane-correction term, in the Friedmann equation, may not be effective in the present universe unless length of the extra-dimension is sufficiently small. But, energy density of the phantom fluid will increase with expansion of the universe due to EOS parameter $w < -1$ in this case. So, even if the correction term is not effective in the present universe, it will be effective in future phantom universe.
In what follows, three situations are obtained. In the present universe, brane-corrections are not effective (as obtained above). This situation will continue until $\rho$ will grow sufficiently. During this period WEC is violated and phantom universe will super-accelerate. As far as $\rho << 2\lambda$, universe will super-accelerate in future and $\rho$ will grow with $a(t)$. It is reasonable to believe $\rho/\lambda \gtrsim 1$ in future due to rapid increase in $a(t)$ caused by super-acceleration. Increase in $\rho$ will still continue with growing $a(t)$. On further increase in $\rho$, brane-corrections will be effective and only SEC will be violated upto a certain value of phantom energy density. As a consequence, acceleration of the phantom universe will become comparatively slow. It means that, in this situation, phantom universe will accelerate, but it will not super-accelerate. It is because, $\ddot{a}/a > 8\pi G \rho/3$ when WEC is violated and $0 < \ddot{a}/a < 8\pi G \rho/3$, when only SEC is violated. This is the intermediate state. When $\rho$ will increase more, none of SEC and WEC will be violated due to strong effect of brane-corrections and phantom universe will decelerate. Acceleration and super-acceleration manifest anti-gravity effect of dark energy. So, it is found that that brane-corrections, in RS-II model, counter anti-gravity effect of phantom dark energy. In the case of quintessence, $\rho$ decreases with expansion of the universe due to $w > -1$, so brane-corrections can not be effective in RS-II model-based present and future quintessence universe.

As discussed above, it is found that $w = -1$ divides the cases violating SEC and WEC. It is known as phantom divide. It means that ideal EOS $p = -\rho$ needs a correction term being linear or non-linear function of $\rho$ causing deviation from the ideal situation as $p = -\rho \pm f(\rho)$. Here, only phantom fluid is considered, so we take the negative sign yielding
$w < -1$. Moreover, $f(\rho)$ in the proposed non-linear EOS implies dependence of $w$ on $\rho$. In some earlier investigations [60, 61, 62], these types of EOS were used considering time-dependent viscosity with correction terms dependent on $\rho$ and $H = \dot{a}/a (\dot{a} = da/dt)$.

In a recent paper [63], EOS $p = -|w|\rho$ for phantom fluid with constant $w < -1$ has been considered in RS-II model based Friedmann equation and it is found that brane-gravity corrections suppress the phantom characteristic to violate WEC and SEC, when $\rho$ increases sufficiently with expansion of the universe. As a consequence, this model expands with acceleration up to some finite time explaining present cosmic acceleration, but it decelerates later on. In paper [63], $f(\rho)$ is a linear function of $\rho$. So, it is natural to study effect of brane-corrections taking non-linear $f(\rho)$ too. Aim of the present paper is to extend the work of [63] taking EOS $p = -\rho - f(\rho)$ with $f(\rho)$ being non-linear functional of $\rho$.

The paper is organized as follows. In section 2, effective EOS is obtained with brane-gravity corrections. Section 3 contains discussion on acceleration and deceleration of the model. Section 4 summarizes the work.

Here, natural units $\hbar = c = 1$ are used, where $\hbar$ and $c$ have their standard meaning.

2. Effective equation of state

Observations support homogeneous and isotropic model of the late universe, given by the line-element [4]

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]$$  \hspace{1cm} (2.1)

where $a(t)$ is the scale factor.
In this space-time, RS-II model based Friedmann equation is obtained as [41, 57, 58]

\[ H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho \left[ 1 - \frac{\rho}{2\lambda} \right] \]  

(2.2)

with \( G, \rho \) and \( \lambda \) defined above.

In the brane-cosmology also, conservation equation is given as [41]

\[ \dot{\rho} + 3H(\rho + p) = 0. \]  

(2.3)

Connecting (2.2) and (2.3), it is obtained that [57, 58]

\[ \frac{\ddot{a}}{a} = -4\pi G (\rho + p) \left[ 1 - \frac{\rho}{\lambda} \right] + \frac{8\pi G}{3} \rho \left[ 1 - \frac{\rho}{2\lambda} \right] \]  

(2.4)

Here, the non-linear equation of state for phantom fluid is taken as

\[ p = -\rho - f(\rho) \]  

(2.5)

As \( \rho + p < 0 \), (2.3) yields \( \dot{\rho} > 0 \). It shows that phantom energy density will increase in future with growing \( a(t) \). Moreover, (2.2) shows that \( a(t) \) will be maximum when \( \rho = 2\lambda \).

In GR-based theory, Friedmann equation is obtained as

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} [\rho + 3P]. \]  

(2.6)

Comparing (2.4) and (2.6), effective EOS with brane gravity corrections is obtained as

\[ P = -\rho - f \left[ 1 - \frac{\rho}{\lambda} \right] + \frac{\rho^2}{3\lambda} \]  

(2.7)

using eq.(2.5).

(2.7) yields

\[ \rho + P = -f \left[ 1 - \frac{\rho}{\lambda} \right] + \frac{\rho^2}{3\lambda} \]  

(2.8)

and

\[ \rho + 3P = -2\rho - 3f \left[ 1 - \frac{\rho}{\lambda} \right] + \frac{\rho^2}{\lambda}. \]  

(2.9)
It is interesting to see from (2.8) and (2.9) that
\[ \rho + P = -f\left[1 + \frac{\rho}{\lambda}\right] - \frac{\rho^2}{3\lambda} < 0 \]
showing violation of WEC and
\[ \rho + 3P = -2\rho - 3f\left[1 + \frac{\rho}{\lambda}\right] - \frac{\rho^2}{\lambda} < 0 \]
showing violation of SEC, if \( \lambda > 0 \), which is the case of RS-I model. But, in the RS-II model being addressed here, we find certain situations when these cosmic conditions are not violated due to effect of brane-gravity corrections.

Here, the case of RS-II model is analyzed taking following three cases yielding different non-linear EOS (2.5) for the phantom fluid

(I) \( f(\rho) = A\rho^\alpha \)

(II) \( f(\rho) = \frac{A\rho^\alpha}{\sqrt{1 - \frac{\rho}{\rho_0}}} \)

(III) \( f(\rho) = \frac{A\rho^{\frac{\alpha}{2}}\ln(\rho/\rho_0)}{\sqrt{1 - \frac{\rho}{\rho_0}}} \)

with \( \alpha \) being a real number and \( A \) being a coupling constant having dimension \((\text{mass})^{4-4\alpha}\) in cases (I) and (II). Moreover, \( A \) has dimension \((\text{mass})^2\) in case (III).

| Case | \( f(\rho) = A\rho^\alpha \) |
|------|-------------------------------|
| I    | \( f(\rho) = A\rho^\alpha \) |
| II   | \( f(\rho) = \frac{A\rho^\alpha}{\sqrt{1 - \frac{\rho}{\rho_0}}} \) |
| III  | \( f(\rho) = \frac{A\rho^{\frac{\alpha}{2}}\ln(\rho/\rho_0)}{\sqrt{1 - \frac{\rho}{\rho_0}}} \) |

In this case, (2.8) implies that
\[ P = -\rho - A\rho^\alpha\left[1 - \frac{\rho}{\lambda}\right] + \frac{\rho^2}{3\lambda}. \]  \( (2.10) \)

(2.10) yields effective pressure \( P < 0 \) for
\[ \rho < 3\lambda\left[1 + A\rho^{(\alpha-1)}\left\{1 - \frac{\rho}{\lambda}\right\}\right]. \]  \( (2.11) \)

Further, it is found that
\[ \rho + P = -A\rho^\alpha\left[1 - \frac{\rho}{\lambda}\right] + \frac{\rho^2}{3\lambda} < 0 \]
till

$$\rho_0 < \rho < 3\lambda A\rho^{(\alpha-1)}\left[1 - \frac{\rho}{\lambda}\right]$$

(2.12)

with $\rho_0$ being the present energy density. This result shows that WEC will be violated till $\rho$ will satisfy the inequality (2.12). It will not be violated when

$$\rho > 3\lambda A\rho^{(\alpha-1)}\left[1 - \frac{\rho}{\lambda}\right].$$

(2.13)

It means that phantom fluid will behave effectively as phantom dark energy till $\rho$ will obey the inequality (2.12). It will not behave effectively as phantom when $\rho$ will increase more and will obey the inequality (2.13).

Moreover, (2.9) shows that SEC will be violated till

$$\rho < \lambda\left[2 + 3A\rho^{(\alpha-1)}\left\{1 - \frac{\rho}{\lambda}\right\}\right].$$

(2.14)

It shows that when $\rho$ will increase such that

$$3\lambda A\rho^{(\alpha-1)}\left[1 - \frac{\rho}{\lambda}\right] < \rho < \lambda\left[2 + 3A\rho^{(\alpha-1)}\left\{1 - \frac{\rho}{\lambda}\right\}\right],$$

(2.15)

only SEC will be violated. It shows that when $\rho$ will satisfy the inequality (2.15), phantom characteristic to violate WEC will be suppressed by brane-gravity effects for negative brane tension and phantom fluid will behave effectively as quintessence. These results yield effective phantom divide at

$$\rho = \rho_{\text{phd}} = 3\lambda A\rho_{\text{phd}}^{(\alpha-1)}\left[1 - \frac{\rho_{\text{phd}}}{\lambda}\right].$$

(2.16)

It is interesting to see that even SEC will not be violated when

$$\lambda\left[2 + 3A\rho^{(\alpha-1)}\left\{1 - \frac{\rho}{\lambda}\right\}\right] < \rho < 2\lambda.$$ 

(2.17)

implying that, during the range (2.17), dark energy characteristic to violate SEC and WEC will be suppressed completely by brane-corrections in RS-II model.
Thus, the above analysis shows that universe will accelerate till $\rho$ will satisfy the inequality (2.14) and it will decelerate during the range of $\rho$ given by (2.17).

**Case II:** $f(\rho) = A\rho^\alpha/\sqrt{1 - \rho^2/2\lambda}$

In this case, (2.8) implies that

$$P = -\rho - \frac{A\rho^\alpha}{\sqrt{1 - \rho^2/2\lambda}} \left[ 1 - \frac{\rho}{\lambda} \right] + \frac{\rho^2}{3\lambda}.$$  

(2.18)

This equation yields effective pressure $P < 0$ for

$$\rho < 3\lambda \left[ 1 + \frac{A\rho^{(\alpha-1)}}{\sqrt{1 - \rho^2/2\lambda}} \left( 1 - \frac{\rho}{\lambda} \right) \right].$$  

(2.19)

Further, it is found that

$$\rho + P = -\frac{A\rho^\alpha}{\sqrt{1 - \rho^2/2\lambda}} \left[ 1 - \frac{\rho}{\lambda} \right] + \frac{\rho^2}{3\lambda} < 0$$

till

$$\rho_0 < \rho < 3\lambda \frac{A\rho^\alpha}{\sqrt{1 - \rho^2/2\lambda}} \left[ 1 - \frac{\rho}{\lambda} \right].$$  

(2.20)

This result shows that WEC is violated till $\rho$ will satisfy the inequality (2.20). It will not be violated when

$$\rho > 3\lambda A\rho^{(\alpha-1)} \left[ 1 - \frac{\rho}{\lambda} \right].$$  

(2.21)

So like the case I, in this case too, we find that phantom fluid will not behave effectively as phantom when $\rho$ will satisfy the inequality (2.21) due to brane-corrections.

Moreover, (2.9) shows that SEC will be violated till

$$\rho < \lambda \left[ 2 + \frac{3A\rho^{(\alpha-1)}}{\sqrt{1 - \rho^2/2\lambda}} \left( 1 - \frac{\rho}{\lambda} \right) \right].$$  

(2.22)

but as $\rho$ will increase with time such that

$$3\lambda \frac{A\rho^\alpha}{\sqrt{1 - \rho^2/2\lambda}} \left[ 1 - \frac{\rho}{\lambda} \right] < \rho < \lambda \left[ 2 + \frac{3A\rho^{(\alpha-1)}}{\sqrt{1 - \rho^2/2\lambda}} \left( 1 - \frac{\rho}{\lambda} \right) \right],$$  

(2.23)
only SEC will be violated. It shows that when $\rho$ will satisfy the inequality (2.23), phantom characteristic to violate WEC will be suppressed by brane-gravity effects for negative brane tension and phantom fluid will behave effectively like quintessence. These results suggest effective phantom divide at

$$\rho = \rho_{phd} = 3\lambda A \rho_{phd}^{(\alpha - 1)} \left[ 1 - \frac{\rho_{phd}}{\lambda} \right]. \quad (2.24)$$

It is interesting to see that even SEC will not be violated for

$$\lambda \left[ 2 + \frac{3A\rho^{(\alpha - 1)}}{\sqrt{1 - \rho/2\lambda}} \left\{ 1 - \frac{\rho}{\lambda} \right\} \right] < \rho < 2\lambda. \quad (2.25)$$

implying that, during the range (2.25), dark energy characteristic to violate SEC and WEC will be suppressed completely by brane-corrections in RS-II model.

Thus the above analysis shows that universe will accelerate till $\rho$ will satisfy the inequality (2.22) and it will decelerate during the range of $\rho$ given by (2.25).

**Case III:**

$$f(\rho) = A\rho^{1/2}ln(\rho/\rho_0)/\sqrt{1 - \rho/2\lambda}$$

In this case, (2.8) implies that

$$P = -\rho - \frac{A\rho^{1/2}ln(\rho/\rho_0)}{\sqrt{1 - \rho/2\lambda}} \left[ 1 - \frac{\rho}{\lambda} \right] + \frac{\rho^2}{3\lambda}. \quad (2.26)$$

This equation yields effective pressure $P < 0$ for

$$\rho < 3\lambda \left[ 1 + \frac{A\rho^{1/2}ln(\rho/\rho_0)}{\sqrt{1 - \rho/2\lambda}} \left\{ 1 - \frac{\rho}{\lambda} \right\} \right]. \quad (2.27)$$

Further, it is found that

$$\rho + P = -\frac{A\rho^{1/2}ln(\rho/\rho_0)}{\sqrt{1 - \rho/2\lambda}} \left[ 1 - \frac{\rho}{\lambda} \right] + \frac{\rho^2}{3\lambda} < 0$$

till

$$\rho_0 < \rho < 3\lambda \rho^{-1/2} \frac{A\rho^{1/2}ln(\rho/\rho_0)}{\sqrt{1 - \rho/2\lambda}} \left[ 1 - \frac{\rho}{\lambda} \right]. \quad (2.28)$$
This result shows that WEC will be violated till \( \rho \) will satisfy the inequality (2.28). It will not be violated when

\[
\rho > 3\lambda \frac{A\rho^{-1/2}ln(\rho/\rho_0)}{\sqrt{1 - \frac{\rho}{2\lambda}}} [1 - \frac{\rho}{\lambda}].
\]  

(2.29)

So like the case I and II, in this case also, we find that phantom fluid will not behave effectively as phantom when \( \rho \) will satisfy the inequality (2.29) due to brane-corrections.

Moreover, (2.9) shows that, in this case, SEC will be violated till

\[
\rho < \lambda \left[ 2 + \frac{3A\rho^{-1/2}ln(\rho/\rho_0)}{\sqrt{1 - \rho/2\lambda}} \left\{ 1 - \frac{\rho}{\lambda} \right\} \right].
\]  

(2.30)

It shows that as \( \rho \) will increase with time such that

\[
3\lambda \frac{A\rho^{-1/2}ln(\rho/\rho_0)}{\sqrt{1 - \frac{\rho}{2\lambda}}} \left[ 1 - \frac{\rho}{\lambda} \right] < \rho < \lambda \left[ 2 + \frac{3A\rho^{-1/2}ln(\rho/\rho_0)}{\sqrt{1 - \rho/2\lambda}} \left\{ 1 - \frac{\rho}{\lambda} \right\} \right],
\]  

(2.31)

only SEC will be violated. It shows that when \( \rho \) will satisfy the inequality (2.31), phantom characteristic to violate WEC will be suppressed by brane-gravity effects for negative brane tension and phantom fluid will behave effectively as quintessence. These results suggest effective phantom divide at

\[
\rho = \rho_{phd}^{3/2} = 3\lambda \frac{Aln(\rho_{phd}/\rho_0)}{\sqrt{1 - \frac{\rho_{phd}}{2\lambda}}} \left[ 1 - \frac{\rho_{phd}}{\lambda} \right].
\]  

(2.32)

It is interesting to see that even SEC will not be violated for

\[
\lambda \left[ 2 + \frac{3A\rho^{-1/2}ln(\rho/\rho_0)}{\sqrt{1 - \rho/2\lambda}} \left\{ 1 - \frac{\rho}{\lambda} \right\} \right] < \rho < 2\lambda.
\]  

(2.33)

implying that, during the range (2.33), dark energy characteristic to violate SEC and WEC will be suppressed completely by brane-corrections in RS-II model.
Thus, the above analysis shows that universe will accelerate till $\rho$ satisfies the inequality (2.38) and it will decelerate during the range of $\rho$ given by (2.33).

3. Cosmic expansion with acceleration and deceleration in RS-II model

In the preceding section, we obtained different conditions for changes in the behaviour of phantom fluid dominating the RS-II model-based universe due to brane-gravity corrections. In what follows, we derive scale factor $a(t)$ solving Friedmann equation (2.2) and conservation equation (2.3). It helps to find time period, during which, WEC and SEC will be violated and time period, during which, these will not be violated.

Case I: $f(\rho) = A\rho^\alpha$

In this case, connecting (2.3) and (2.5), we obtain

$$\dot{\rho} - 3A \frac{\dot{a}}{a} \rho^\alpha = 0. \quad (3.1)$$

It integrates to

$$\rho = \left[ \rho_0^{1-\alpha} + 3A(1-\alpha)\ln(a/a_0) \right]^{\frac{1}{1-\alpha}}, \quad (3.2)$$

where $\rho_0 \leq \rho \leq 2\lambda$.

(2.2), (2.3) and (2.5) yield

$$\dot{\rho} - 3A \sqrt{\frac{8\pi G}{3}} \rho^{\alpha+\frac{1}{2}} \sqrt{1 - \frac{\rho}{2\lambda}} = 0, \quad (3.3)$$

where $\rho_0 \leq \rho \leq 2\lambda$.

Exact solution of this equation is obtained as

$$t = \frac{1}{A\sqrt{24\pi G}} \left[ t_0 + 2(2\lambda)^{(1/2) - \alpha} \left\{ \sqrt{1 - \frac{\rho_0}{2\lambda}} \frac{1}{2\lambda} \right\} F_1 \left( \frac{1}{2}, \frac{1}{2} + \alpha, \frac{3}{2}, 1 - \frac{\rho}{2\lambda} \right) - \sqrt{1 - \frac{\rho}{2\lambda}} \frac{1}{2\lambda} \right], \quad (3.4)$$
where \( 2F_1(a, b, c, x) \) is the hypergeometric function. Further, using (3.2) in (3.4), we get a relation between time \( t \) and the scale factor \( a(t) \).

As maximum value of \( \rho \) is \( 2\lambda \), so phantom universe will expand upto time \( t_m \) given as

\[
t_m = \frac{1}{A\sqrt{24\pi G}} \left[ t_0 + 2(2\lambda)^{(1/2) - \alpha} \sqrt{1 - \frac{\rho_0}{2\lambda}} \, 2F_1 \left( \frac{1}{2}, \frac{1}{2} + \alpha, \frac{3}{2}, 1 - \frac{\rho_0}{2\lambda} \right) \right]
\]

(3.5)

with \( t_0 \) being the present time. Moreover, from (3.2), it is obtained that

\[
3(1 - \alpha)A\ln(a_m/a_0) = (2\lambda)^{1-\alpha} - \rho_0^{1-\alpha},
\]

(3.6)

where \( a_m = a(t_m) \). This equation shows that if \( \alpha \geq 1, 2\lambda > \rho_0 \) as \( a_m > a_0 \).

From (2.16) and (3.4), we obtain effective phantom divide at time

\[
t = t_{phd} = \frac{1}{A\sqrt{24\pi G}} \left[ t_0 + 2(2\lambda)^{(1/2) - \alpha} \left\{ \sqrt{1 - \frac{\rho_0}{2\lambda}} \, 2F_1 \left( \frac{1}{2}, \frac{1}{2} + \alpha, \frac{3}{2}, 1 - \frac{\rho_0}{2\lambda} \right) 
- \frac{\rho_{phd}}{2\lambda} \, 2F_1 \left( \frac{1}{2}, \frac{1}{2} + \alpha, \frac{3}{2}, 1 - \frac{\rho_{phd}}{2\lambda} \right) \right\} \right]
\]

(3.7)

Inequalities (2.15) and (2.17) show that for \( \rho \) satisfying

\[
\rho < \lambda \left[ 2 + 3A\rho^{(\alpha-1)\left\{ 1 - \frac{\rho}{\lambda} \right\}} \right],
\]

(3.8)

SEC will be violated. It means that the universe will accelerate till \( \rho \) will obey (3.8). But as \( \rho \) will grow more and it will satisfy

\[
\rho > \lambda \left[ 2 + 3A\rho^{(\alpha-1)\left\{ 1 - \frac{\rho}{\lambda} \right\}} \right],
\]

(3.9)

SEC will not be violated. It means that the universe will decelerate when \( \rho \) will obey the inequality (3.9). It shows a transition from acceleration to deceleration at \( \rho = \rho_{tr} \) given by the equation

\[
\rho_{tr} = \lambda \left[ 2 + 3A\rho_{tr}^{(\alpha-1)\left\{ 1 - \frac{\rho_{tr}}{\lambda} \right\}} \right].
\]

(3.10)
Connecting (3.4) and (3.10), we obtain that this transition will take place at time

\[ t = t_{tr} = \frac{1}{A\sqrt{24\pi G}} \left[ t_0 + 2(2\lambda)^{(1/2)} - \alpha \left( \sqrt{1 - \frac{\rho_0}{2\lambda}} {}_2F_1 \left( \frac{1}{2}, \frac{1}{2} + \alpha, \frac{3}{2}, 1 - \frac{\rho_0}{2\lambda} \right) \right. \right. \]

\[ \left. - \sqrt{1 - \frac{\rho_0}{2\lambda}} 2F_1 \left( \frac{1}{2} + \alpha, \frac{3}{2}, 1 - \frac{\rho_0}{2\lambda} \right) \right] \right]. \tag{3.11} \]

If \( \alpha = 1 \), (3.3) integrates to

\[ \rho = \left[ \frac{1}{2\lambda} + \left\{ \sqrt{\frac{1}{\rho_0} - \frac{1}{2\lambda} - A\sqrt{6\pi G(t - t_0)}} \right\}^2 \right]^{-1}, \tag{3.12} \]

where \( \rho_0 < 2\lambda \) is the current energy density of the DE. It shows that phantom energy density will increase with time, which is consistent with results of GR- based theory.

From (2.2) and (3.12) it is obtained that

\[ H^2 = \frac{8\pi G}{3} \left[ \frac{1}{2\lambda} + \left\{ \sqrt{\frac{1}{\rho_0} - \frac{1}{2\lambda} - A\sqrt{6\pi G(t - t_0)}} \right\}^2 \right], \tag{3.13} \]

The solution of (3.13) is

\[ a(t) = a_0 \rho_0^{-\frac{1}{3}} \left[ \frac{1}{2\lambda} + \left\{ \sqrt{\frac{1}{\rho_0} - \frac{1}{2\lambda} - A\sqrt{6\pi G(t - t_0)}} \right\}^2 \right]^{-\frac{1}{3}} \tag{3.14} \]

Using \( \rho = 2\lambda \), in (3.12), it is obtained that phantom era will end at time

\[ t_e = t_0 + \frac{1}{A\sqrt{6\pi G}} \sqrt{\frac{1}{\rho_0} - \frac{1}{2\lambda}} \tag{3.15} \]

In this case, (2.4) reduces to

\[ \frac{\ddot{a}}{a} = \frac{4\pi G\rho}{3} \left[ 3A \left( 1 - \frac{\rho}{\lambda} \right) + 2 \left( 1 - \frac{\rho}{2\lambda} \right) \right], \tag{3.16} \]

which yields

\[ \ddot{a} \gtrless 0 \quad \text{when} \quad \frac{\rho}{\lambda} \gtrless \frac{3A + 2}{1 + 3A} \tag{3.17} \]

(3.12) and (3.16) show that \( \ddot{a} = 0 \) when

\[ \frac{3A + 1}{(3A + 2)\lambda} = \frac{1}{2\lambda} + \left\{ \sqrt{\frac{1}{\rho_0} - \frac{1}{2\lambda} - A\sqrt{6\pi G(t_{vm} - t_0)}} \right\}^2. \tag{3.18} \]
It yields time for transition from acceleration to deceleration as

\[ t_{vm} = t_0 + \frac{1}{A\sqrt{6\pi G}} \left[ \sqrt{\frac{1}{\rho_0} - \frac{1}{2\lambda}} - \sqrt{\frac{3A}{2(3A+2)\lambda}} \right] \]  \hspace{1cm} (3.19)

From eq.(3.12), it is also obtained that \( \ddot{a} < 0 \) when \( 2\lambda \geq \rho > \frac{(3A+2)\lambda}{3A+1} \). It means that during the time interval

\[ t_0 + \frac{1}{A\sqrt{6\pi G}} \sqrt{\frac{1}{\rho_0} - \frac{1}{2\lambda}} \geq t > t_0 + \frac{1}{A\sqrt{6\pi G}} \left[ \sqrt{\frac{1}{\rho_0} - \frac{1}{2\lambda}} - \sqrt{\frac{3A}{2(3A+2)\lambda}} \right] \]  \hspace{1cm} (3.20a)

phantom universe will decelerate in case \( \alpha = 1 \). It gives deceleration period

\[ \sqrt{\frac{M_p^2}{4(3A+2)\pi G\lambda}}, \]  \hspace{1cm} (3.20b)

which depends on magnitude of negative brane tension. There is no deceleration if \( \lambda \gg M_p^2 \). So role of brane tension is very crucial here[39]

\[ CaseII : f(\rho) = \frac{A\rho^2}{\sqrt{1 - \rho}}. \]

In this case, connecting (2.3) and (2.5), we obtain

\[ \dot{\rho} - 3A\frac{\dot{a}}{a} \rho^{\alpha-1} = 0, \]  \hspace{1cm} (3.21)

which integrates to

\[ 3Aln\left(\frac{a}{a_0}\right) = \frac{1}{3} 2^{-\alpha} \lambda^{1-\alpha} \left[ \left(1 - \frac{\rho_0}{2\lambda}\right)^{3/2} 2F_1\left(\frac{3}{2}, \alpha, \frac{5}{2}, 1 - \frac{\rho}{2\lambda}\right) \right. \]

\[ \left. - \left(1 - \frac{\rho}{2\lambda}\right)^{3/2} 2F_1\left(\frac{3}{2}, \alpha, \frac{5}{2}, 1 - \frac{\rho}{2\lambda}\right) \right], \]  \hspace{1cm} (3.22)

where \( \rho_0 \leq \rho \leq 2\lambda \).

(2.2), (2.3) and (2.5) yield

\[ \dot{\rho} - 3A\sqrt{\frac{8\pi G}{3}} \rho^{\alpha+\frac{1}{2}} = 0 \]  \hspace{1cm} (3.23)

Exact solution of this equation is obtained as

\[ \rho = \left[ \rho_0^{(1-2\alpha)/2} + \frac{1}{2}(1 - 2\alpha)A\sqrt{6\pi G}(t - t_0) \right]^{2/(1-2\alpha)}. \]  \hspace{1cm} (3.24)
Thus, $(3.22)$ and $(3.24)$ yield the scale factor $a(t)$ as function of time $t$.

As in RS-II model expansion stops at $\rho = 2\lambda$, here phantom driven universe will expand upto time $t_m$ given as

$$
t_m = t_0 + \frac{2}{A(1-2\alpha)\sqrt{6\pi G}} \left[(2\lambda)^{(1-2\alpha)/2} - \rho_0^{(1-2\alpha)/2}\right] \tag{3.25}
$$

In this case, WEC will be violated till

$$
t < t_0 + \frac{2}{A(1-2\alpha)\sqrt{6\pi G}} \left[\rho^{(1-2\alpha)/2} - \rho_0^{(1-2\alpha)/2}\right], \quad (3.26a)
$$

and $\rho$ will satisfy the inequality $(2.20)$. It is found that WEC will not be violated when

$$
t > t_0 + \frac{2}{A(1-2\alpha)\sqrt{6\pi G}} \left[\rho^{(1-2\alpha)/2} - \rho_0^{(1-2\alpha)/2}\right], \quad (3.26b)
$$

and $\rho$ will satisfy the inequality $(2.21)$. So, effective phantom divide is obtained at time

$$
t_{\text{phd}} = t_0 + \frac{2}{A(1-2\alpha)\sqrt{6\pi G}} \left[\rho_{\text{phd}}^{(1-2\alpha)/2} - \rho_0^{(1-2\alpha)/2}\right], \quad (3.26c)
$$

where $\rho_{\text{phd}}$ given by $(2.24)$.

The inequality $(2.23)$ and $(3.24)$ yield the time period during which SEC will be violated. It shows that phantom fluid will behave effectively as quintessence fluid due to brane-corrections in this case. So, universe will accelerate till

$$
t < t_{\text{ae}} = t_0 + \frac{2}{A(1-2\alpha)\sqrt{6\pi G}} \left[\rho_{\text{ae}}^{(1-2\alpha)/2} - \rho_0^{(1-2\alpha)/2}\right], \quad (3.27a)
$$

where

$$
\rho = \rho_{\text{ae}} = \lambda \left[2 + \frac{3A\rho^{(\alpha-1)/2}}{\sqrt{1 - \rho_{\text{ae}}/2\lambda}} \left\{1 - \frac{\rho_{\text{ae}}}{\lambda}\right\}\right]. \quad (3.27b)
$$

Similarly, inequalities $(2.25)$ and $(3.27b)$ as well as $(3.24)$ and $(3.27a)$ yield the time period $t_{\text{ae}} < t < t_m$, during which brane-corrections of RS-II model will be so effective that neither SEC nor WEC will be
violated for phantom fluid. As a consequence, universe will decelerate during this period.

Like the case I, here too, we take $\alpha = 1$ as an example. In what follows, above results are analyzed if $\alpha = 1$. Using (3.26c) time for effective phantom divide will be obtained at

$$ t_{\text{phd}} = t_0 + \frac{2}{A\sqrt{6\pi G}} \left[ \rho_0^{-1/2} - \rho_{\text{phd}}^{-1-2/2} \right], \quad (3.28a) $$

where

$$ \rho_{\text{phd}} = \frac{3\lambda A}{1 + 3A}. \quad (3.28b) $$

According to (3.25), universe will expand till

$$ t_m = t_0 + \frac{2}{A\sqrt{6\pi G}} \left[ \rho_0^{-1/2} - (2\lambda)^{-1/2} \right]. \quad (3.29) $$

(3.22) and (3.24) show that at time

$$ t_{\text{br}} = t_0 + 2/A\sqrt{6\pi G\rho_0} \quad (3.30) $$

$\rho$ is divergent and $a(t)$ is complex. It is an unphysical situation.

Connecting (3.29) and (3.30), it is obtained that

$$ t_m = t_{\text{br}} - \frac{1}{A\sqrt{3\pi G\lambda}}. \quad (3.31) $$

It shows that expansion of the universe will stop before encountering the unphysical situation occurring at time $t_{\text{br}}$ given by (3.30).

**Case III:** $f(\rho) = A\rho^{1/2}\ln(\rho/\rho_0)/\sqrt{1 - \rho/2\lambda}$

In this case, using (2.2), (2.3) and (2.5), we get

$$ \dot{\rho} - 3A\sqrt{8\pi G/3} \rho \ln \rho = 0, \quad (3.32) $$

which integrates to

$$ \ln(\rho/\rho_0) = \sqrt{24\pi G}(t - t_0). \quad (3.33) $$
The effective phantom divide is obtained at time

\[ t = t_{\text{phd}} = t_0 + \rho_{\text{phd}}/A\rho_0\sqrt{24\pi G}. \] (3.34)

It shows that, at \( t < t_{\text{phd}} \), the phantom fluid will violate WEC, but, at \( t > t_{\text{phd}} \) WEC will not be violated.

(2.30) shows that even SEC will be violated till \( \rho < \rho_{ae} \) and it will not be violated when \( \rho > \rho_{ae} \), where

\[ \rho_{ae} = \lambda \left[ 2 + \frac{3A\rho_{ae}^{-1/2}ln(\rho_{ae}/\rho_0)}{\sqrt{1-\rho_{ae}/2\lambda}} \left\{ 1 - \frac{\rho_{ae}}{\lambda} \right\} \right]. \] (3.35)

Phantom energy will acquire the value \( \rho_{ae} \) at time

\[ t_{ae} = t_0 + \rho_{\text{phd}}/A\rho_0\sqrt{24\pi G} \] (3.36)

being obtained from (3.33) and (3.35).

Like above cases, in this case, \( t_m \) is obtained as

\[ t_m = t_0 + 2\lambda/\sqrt{24\pi G} \] (3.37)

Results (3.36) and (3.37) show that universe will accelerate till \( t_0 \leq t < t_{ae} \) and will decelerate for \( t_{ae} < t < t_m \).

4. Summary

In this paper, we analyze the behaviour of phantom fluid in RS-II model of brane-gravity having negative brane-tension \( \lambda \). Here, three cases of non-linear equations of state for the phantom fluid are taken. It is found that, contrary to RS-I model, in RS-II model, brane-corrections make drastic changes in the behaviour of phantom fluid, which is characterized by violation of WEC and accelerating universe ending up in big-rip singularity in most of the models. Interestingly, RS-II model based phantom cosmology is found different from the usual picture of phantom universe. Energy conservation for phantom fluid yields that phantom energy density increases as universe expands. Above results
suggest that behaviour of phantom fluid will change in the future universe as energy density will grow with expansion. Here, the model of the future universe begins at time $t_0$ being the present age of the universe and it stops expanding when phantom energy density $\rho$ grows to $2\lambda$ by the time $t_m$. The above analysis shows that, during the period $t_0 \leq t < t_m$, two transitions will take place. The first one will take place at $t_{phd}$ being the time of transition from violation of WEC to non-violation of WEC and violation of SEC. The second one will take place at $t_{ae}$ being the time of transition from violation of SEC to non-violation of SEC. These transitions are caused by brane-corrections due to negative brane-tension in RS-II model-based universe. As a consequence, it is found that the present model of the universe will accelerate during the period $t_0 \leq t < t_{ae}$ and decelerate during the period $t_{ae} < t < t_m$. Moreover, the model is free from big-rip problem. Thus, it is found that the role of brane-tension is very crucial. When it is negative, it causes drastic changes in the behaviour of phantom dark energy, but phantom fluid has usual behaviour when brane-tension is positive.

REFERENCES

[1] Perlmutter, S. J. et al.,(1999), Astrophys. J. 517,565; (1998) astro-ph/9812133.
[2] Spergel, D. N. et al.,(2003) , Astrophys J. Suppl. 148 175[ astro-ph/0302209] and references therein.
[3] Riess, A. G. et al.,(2004), Astrophys. J. 607, 665 [ astro-ph/0402512].
[4] Miller, A.D. et al.,(1999), Astrophys. J. Lett. 524 L1.
[5] E.J.Copeland, M.Sami and S. Tsujikawa, Int. J. Mod. Phys. D, 15,(2006)1753 [hep-th/0603057] and references therein.
[6] Caldwell, R.R.(2002), Phys.Lett.B,545,23.
[7] Caldwell, R.R., Kamionkowski, M. and Weinberg, N.N.(2003), Phys. Rev. Lett., 91, 071301.
[8] Overduin, J.M., Cooperstock, F.I., (1998), Phys. Rev. D 58, 043506.
[9] Armendariz-Picon, C., Damour,T. and Mukhanov,V. ,(1999), Phys. Lett. B 458, 209.
[10] Chiba,T., Okabe, T. and Yamaguchi, M. ,(2000), Phys. Rev. D 62, 023511.
[11] Sen, A. , (2002), J. High Energy Phys. 04, 048; (2002) 07, 065; (2002), Mod. Phys. Lett. A 17, (2002)1799.
[12] Garousi, M. R., (2000),Nucl. Phys. B 584, 284;(2003) J. High Energy Phys. 04, 027.
[13] Singh, Parampreet ,Sami, M. , Dadhich, Naresh (2003) ,Phys.Rev. D 68, 023522 and references therein.
[14] Panda , S., Perez-Lorenzana, A.(2001) , Nucl. Phys. B 584, (2001) 284.
[15] Srivastava, S.K.,(2004), gr-qc/040974.
[16] Sami, M. , Toporensky, A. , Tretjakov, P.V. , Tsujikawa, S,(2005) , Phys. Lett. B 619 193 [hep-th/0504155].
[17] Calcagni, G. , Tsujikawa, S., Sami, M.,(2005), Class. Quan. Grav. 22 3977 [hep-th/0505193].
[18] Srivastava, S.K.; (2005), Phys. Lett. B 619 1 [astro-ph/0407048].
[19] Jackiw, R. (2000) , physics/0010042.
[20] Bertolami, O. et al, (2004),Mon. Not.R.Astron.Soc. 353 329 [astro-ph/0402387].
[21] Bento, M. C. , Bertolami,O. ,Sen, A.A., (2002),Phys. Rev. D 66 043507 [gr-qc/0202064].
[22] Capozziello, S.,(2002), Int.J.Mod.Phys.D 11483.
[23] Capozziello, S., Carloni,S., Troisi, A., (2003), astro-ph/0303041.
[24] Caroll, S.M., Duvvuri,V., Trodden, M., Turner, M.S.,(2004), Phys.Rev. D 70043528, astro-ph/0306438.
[25] Dolgov,A.D., Kawasaki,M.,(2003), Phys.Lett.B 573 1 ; astro-ph/0307285.
[26] Soussa, M.E., Woodard, R.P.,(2004), Gen. Relativ. Gravit. 36 855, astro-ph/0308114.
[27] Nojiri, S., Odintsov, S.D.,(2004), Phys.Lett.A 19(2004)627; hep-th/0310045.
[28] Nojiri, S., Odintsov, S.D., (2003), Phys.Rev. D 68123512, hep-th/0307228.
[29] Abdalaa, M.C.B., Nojiri, S., Odintsov, S.D.,(2005), Class. Quan. Grav. 22 L35, hep-th/0409117.
[30] Mena,O., Santiago, J., Weller, J.,(2006), Phys.Rev.Lett. 96(2006)041103.
[31] Atazadeh, K. ,Sepangi, H.R.(2006)., gr-qc/0602028.
[32] Bouhmadi-Lopez, M. et al,(2005), astro-ph/0512124; (2006) gr-qc/0612135; (2007) arxiv:0707.2390; (2007) arXiv:0706.3896[astro-ph].
[33] Nojiri, S., Odintsov, S.D.,(2007),Int.J. Geom. Meth. Mod. Phys. 4,115 [ hep-th/0601213].
[34] Amendola, L , Polarski, D., Tsujikawa, S.(2007) , Phys.Rev. Lett. 98 131302 [astro-ph/0603703].
[35] Amendola, L , Polarski, D., Gannouji,R., Tsujikawa, S,(2007), Phys.Rev.D, 75 083504 [gr-qc/0612180].
[36] S. Nojiri and S.D.Odintsov, arXiv:0801.4843[astro-ph].
[37] Srivastava, S.K., (2005) astro-ph/0511167; (2006),astro-ph/0602116; (2007) Int.J.Mod.Phys.A 22 (6), 1123-1134 [hep-th/0605019].
[38] Srivastava, S.K., (2006), Phys.Lett. B 643 1-4 [astro-ph/0608241].
[39] Srivastava, S.K., (2007) Phys.Lett. B 648 119-126 [astro-ph/0603601].
[40] Rubakov, V. A., (2001), Phys. Usp. 44, 871 [hep-ph/0104152].
[41] Maartens, R., (2004) Living Rev. Relativity, 7, 7, [(2003) gr-qc/0312059].
[42] Brax, P. et al., (2004), Rep. Prog. Phys. 67, 2183 [hep-th/0404011].
[43] Csaki, C., (2004) [hep-ph/0404096]
[44] Hořava, P., Witten, E., (1996), Nucl. Phys.B 460, 506.
[45] Appelquist, T., Cheng, A., Freund, P.G.O., (1987) “Modern Kaluza-Klein Theories”, Addison-Wesley, Reading, MA.
[46] Srivastava, S.K., Sinha, K.P., (1998), “Aspects of Gravitational Interactions”, Nova Science Publishers, Inc., Commack, NY (1998).
[47] Randall, L., Sundrum, R., (1999), Phys. Rev. Lett., 83, 3370.
[48] Randall, L., Sundrum, R., (1999), Phys. Rev. Lett., 83, 4690.
[49] Bine' truy, P., Deffayet, C., Langlois, D., (2000), Nucl. Phys. B 565, 269 [hep-th/9905012]
[50] Bine' truy, P., Deffayet, Ellwanger, U., C., Langlois, D., (2000) Phys. Lett. B 477, 289 [hep-th/9910219].
[51] Csa' ki, C., Graesser, M., Kolda, C., Terning, (1999) J. Phys. Lett.B 462, 34 [hep-ph/9906513].
[52] Cline, J.M., Grojean, C., Servant, G., (1999), Phys. Rev. Lett. 83, 4245 [hep-ph/9906523].
[53] Maartens, R., Wands, D., Bassett, B.A., Heard, I.P.C., (2000), Phys. Rev. D 62, 023509 [hep-ph/9912464].
[54] Copeland, E.J., Liddle, A.R., Lidsey, J.E., (2001), Phys. Rev. D 64, 041301 [astro-ph/0006421]
[55] Calcagni, G., (2004), Phys. Rev. D 69, 103508.
[56] Apostolopoulos, P.S., Tetradis, N., (2006) Phys. Lett. B 633, 409.
[57] Nojiri, S., Odintsov, S.D., (2004), Phys. Lett. B 595, 1 [(2004) hep-th/0408170].
[58] Srivastava, S.K., (2007), Gen. Rel. Grav., 39, 241.
[59] Sahni, V., Shtanov, Y., (2003), JCAP, 0311, 014 [astro-ph/0202346].
[60] Nojiri, S., Odintsov, S.D., Tsujikawa, S., (2005), Phys. Rev. D, 71, 063004.
[61] Štefančič, H., (2005) Phys. Rev. D, 71, 084024.
[62] Nojiri, S., Odintsov, S.D., (2005), Phys. Rev. D, 72, 023002.
[63] Srivastava, S.K., (2007); arXiv:0707.1376 [gr-qc].