4π Decay Modes of the $f_0(1500)$ Resonance

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Abstract

We investigate the two-body decay modes $\rho\rho$, $\pi\pi^*$ (1300) and $\sigma\sigma$ of the $f_0(1500)$, all leading to the $4\pi$ decay channel, in a three-state mixing scheme, where the $f_0(1500)$ is a mixture of the lowest-lying scalar glueball with the nearby isoscalar states of the $0^{++}Q\bar{Q}$ nonet. In the leading order of this scheme, the decay mechanism of the $f_0(1500)$ proceeds dominantly via its quarkonia components, which can be described in the framework of the $3P_0\,Q\bar{Q}$ pair creation model. We predict the hierarchy of decay branching ratios $B$ with $B(\rho\rho) > B(\pi\pi^*) > B(\sigma\sigma) > B(\pi\pi)$, providing a key signature for the proposed mixing scheme in this leading order approach.

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Recently, there has been growing attention, both experimental and theoretical, on the possible existence of glueball states. A scalar-isoscalar resonance $f_0(1500)$ is clearly established by Crystal Barrel at LEAR \[1, 2\] in proton-antiproton annihilation, which is regarded as a rich gluon production mechanism with favored coupling to glueballs \[3\]. The main interest in the $f_0(1500)$ as a candidate reflecting the intrusion of a glueball state in the scalar-isoscalar meson spectrum rests on several phenomenological and theoretical observations. Lattice QCD, in the quenched approximation \[4\], predicts the lightest glueball to be a scalar, lying in the mass range of 1500-1700 MeV. Although the decay pattern of the $f_0(1500)$ into two pseudoscalar mesons is compatible with a quarkonium state $n\bar{n}$ identified with the scalar $Q\bar{Q}$ nonet, the observed total width of $\Gamma \approx 120$ MeV is in conflict with naive quark model expectations of $\Gamma \geq 500$ MeV \[5\]. Moreover, it is now believed \[1, 6\], that the $f_0(1370)$ state occupies the corresponding position in the scalar nonet, with decay branching ratios and total width consistent with a dominant $n\bar{n}$ structure. As the $f_0(1500)$ cannot be accommodated as a radial excitation of the scalar $3P_0$ nonet \[7\], it appears as an excess state in the $Q\bar{Q}$ systematics.

In the analysis of the partial decay widths into two pseudoscalar mesons, several schemes have been proposed to attribute at least a partial glueball nature to the $f_0(1500)$ state \[8, 9\], although quantitative predictions for its glueball content differ sizably. In the scenario proposed by Ref. \[6\] the pure glueball is mixed with the $n\bar{n}$ and $s\bar{s}$ states of the scalar $3P_0$ nonet. The mixture leads to three scalar states with nearly equal glueball strength, identified with the $f_0(1370), f_0(1500)$ and an additional state around 1700 MeV which can possibly be the $f_{J=0,2}(1710)$. It is argued that in the leading order of strong coupling QCD, the hadronic two meson decay proceeds dominantly via the $Q\bar{Q}$- components of the $f_0(1500)$, if the Fock states of final state isoscalar mesons contain no gluonic component. Neglecting an intrinsic violation of flavor symmetry, the model can explain the decay pattern of the $f_0(1500)$ into two pseudoscalar mesons. In particular, the experimental weak $K\bar{K}$ decay mode is obtained from the destructive interference between the admixed $n\bar{n}$ and $s\bar{s}$ modes.

There are hints that the scalar $s\bar{s}$ state may have a lower mass than the scalar glueball motivating another three-state mixing model with this level ordering \[10\]. In this scenario the glueball component dominantly resides in the $f_{J=2}(1710)$ with the $f_0(1500)$ remaining as a nearly pure quarkonium state, but the destructive interference between the $n\bar{n}$ and $s\bar{s}$ flavors for the $f_0(1500)$ still remains. For this reason, both analyses of Refs. \[3\] and \[10\] are very similar for the decay of $f_0(1500)$ into two pseudoscalar mesons \[10\].

In the present work, we go beyond the two pseudoscalar meson decay modes and study
the predictions of the three-state mixing schemes of Refs. [6, 9] in the $4\pi$ decay modes. We derive the decay pattern of the $f_0(1500)$ into the two-body decay channels $\rho\rho$, $\pi\pi^*(1300)$ and $\sigma\sigma$ ($\sigma$ is the broad $\pi\pi$ S-wave resonance). They all contribute to the $4\pi$ decay rate, and we calculate their relative ratios to that of the $2\pi$ decay channel. The two-body decay modes leading to the $4\pi$ final state have the advantage that, in the decay dynamics, no $SU(3)$ flavor symmetry breaking and mixing angles for the meson nonets in the decay channel are needed. This reduces the uncertainties occurring in the analysis of the two pseudoscalar meson decays.

The model - As mentioned earlier, the hadronic decay of the admixed $f_0(1500)$ is dominated by the $Q\bar{Q}$ components. The coupling of the scalar $Q\bar{Q}$ components to the final two-meson states can be calculated in the non-relativistic $Q\bar{Q}$ pair creation $^3P_0$ model [10], in which the $Q\bar{Q}$ pairs are created with vacuum quantum numbers. The relevant OZI-allowed quark line diagram, describing the flavor flux from the initial $Q\bar{Q}$ component of the $f_0(1500)$ to the two meson final states BC (=$\pi\pi$, $\rho\rho$, $\sigma\sigma$ and $\pi\pi^*(1300)$) is shown in the figure. The details of all the following formulae can be found in Refs. [5, 7, 10, 11]. The transition dynamics is governed by the nonperturbative $Q\bar{Q}$ $^3P_0$ vertex:

$$V_{3P_0}^{(34)} = \lambda \delta(p_4 + p_3) \left[ \mathcal{Y}_{1\mu}^*(p_4 - p_3) \otimes \sigma_{-\mu}^{(43)} \right]_{00} \mathbf{1}_F^{(43)} \mathbf{1}_C^{(43)},$$

$p_{3(4)}$ are the momenta of the created $Q\bar{Q}$ pair and $\mathcal{Y}_{1\mu}^*(p) = |p|Y_{1\mu}^*(\hat{p})$. The identity operators $\mathbf{1}_F$ and $\mathbf{1}_C$ project onto singlet states in flavor and color space, respectively. The dimensionless parameter $\lambda$ corresponds to the strength of the transition, which in turn is related to the probability for $Q\bar{Q}$ pair creation out of the hadronic vacuum. Since for the considered final states BC no $s\bar{s}$ pair creation is involved $\lambda$ is flavor independent. With harmonic oscillator wave functions for the quark clusters, the scalar $Q\bar{Q}$-component of the $f_0(1500)$ is given as:

$$\Psi_{f_0}(p_1, p_2) = \psi_{J_{f_0}=1, J_{f_0}=0} (p_1, p_2) \chi_{S=1}^{f_0} \chi_C^{f_0}$$

$$= (-i) \left[ \frac{2R_{f_0}^2}{3\sqrt{\pi}} \right]^{\frac{1}{2}} \delta(p_1 + p_2) \exp \left\{ -\frac{1}{8} R_{f_0}^2 (p_1 - p_2)^2 \right\}$$

$$\cdot \left[ \chi_{S=1}(12) \otimes \mathcal{Y}_{l=1}^i (p_1 - p_2) \right]_{J=0} \chi_{C=0}^{f_0} \chi_C^{f_0}.$$
\[ \Psi_{\sigma}(p_i, p_j) = \psi_{S_{\pi}=1, J_{\pi}=1, J_{\sigma}=0}(p_i, p_j) \chi_{\sigma}^F \chi_{C}^\rho \]

\[ = (-i) \left[ \frac{2R_{\pi}^5}{3\sqrt{\pi}} \right]^{1/2} \delta(p_i + p_j - K) \exp \left\{ -\frac{1}{8} R_{\pi}^2 (p_i - p_j)^2 \right\} \]

\[ \cdot \left[ \chi_{S_{\sigma}=1(ij) \otimes Y_{J_{\sigma}=1}(p_i - p_j)} \right]_{J_{\sigma}=0} \chi_{F}^\sigma \chi_{C}^\rho , \] (3)

\[ \Psi_{\pi^*}(p_i, p_j) = \psi_{S_{\pi^*}=1, J_{\pi^*}=0, J_{\rho^*}=0}(p_i, p_j) \chi_{\pi^*}^\rho \chi_{C}^\pi^* \]

\[ = (-) \left[ \frac{8R_{\pi^*}^3}{3\sqrt{\pi}} \right]^{1/2} \delta(p_i + p_j - K) \exp \left\{ -\frac{1}{8} R_{\pi^*}^2 (p_i - p_j)^2 \right\} \]

\[ \cdot \left[ L_{n_{\pi^*}}^{1/2} \left( \frac{R_{\pi^*}^2}{4} (p_i - p_j)^2 \right) \chi_{S_{\pi^*}=0(ij) \otimes Y_{J_{\pi^*}=0}(p_i - p_j)} \right]_{J_{\pi^*}=0} \chi_{F}^\pi^* \chi_{C}^\pi^* , \] (4)

with the Laguerre polynomial \( L_{n_{\pi^*}}^{1/2}(p) = 3/2 - p \). Here, \( \sigma \) is effectively described by an isoscalar \( 3P_0 \) \( Q\bar{Q} \) state. The S-wave mesons \( \pi \) and \( \rho \) are described by

\[ \Psi_{\pi(\rho)}(p_i, p_j) = \psi_{S_{\pi(\rho)}=0, J_{\pi(\rho)}=0}(p_i, p_j) \chi_{\pi(\rho)}^\rho \chi_{C}^\pi(\rho) \]

\[ = \left[ \frac{4R_{\pi(\rho)}^3}{\sqrt{\pi}} \right]^{1/2} \delta(p_i + p_j - K) \exp \left\{ -\frac{1}{8} R_{\pi(\rho)}^2 (p_i - p_j)^2 \right\} \]

\[ \cdot \left[ \chi_{S_{\pi(\rho)}=0(ij) \otimes Y_{J_{\pi(\rho)}=0}(p_i - p_j)} \right]_{J_{\pi(\rho)}} \chi_{F}^\pi(\rho) \chi_{C}^\pi(\rho) , \] (5)

with \( S_{\pi} = J_{\pi} = 0, S_{\rho} = J_{\rho} = 1 \). The size parameters for \( \pi \) and \( \pi^* \) have to fulfill the relation \( R_{\pi} = R_{\pi^*} \) due to orthogonality of the wave functions.

The transition amplitude for the process \( f_0(1500) \rightarrow BC \) of the figure is given by

\[ T = \langle \Psi_{\pi^*}(p_1, p_2) | V_{f_0}^{(3)}(p_0) | \Psi_{f_0}(p_1, p_2) \rangle \]

\[ = T_{SS}^{f_0 \rightarrow BC} T_{F}^{f_0 \rightarrow BC} T_{C}^{f_0 \rightarrow BC} \] (6)

The color part \( T_C \) is \( \frac{1}{M} \), and the spin-spatial part \( T_{SS}^{f_0 \rightarrow BC} \) is given with the appropriate wave functions of Eqs. (2)-(5) as

\[ T_{SS}^{f_0 \rightarrow BC} = \int \prod_{i=1, \ldots, 4} dp_i \psi_{S_{f_0}^\dagger J_{f_0}^I}(p_1, p_3) \psi_{S_{C}^\dagger J_{C}}(p_4, p_2) V_{f_0}^{(3)}(p_0) \psi_{S_{f_0}^0=1, J_{f_0}=0}(p_1, p_2) , \] (7)

where the internal quark momenta are integrated over and evaluation of the integral (7) is done analytically [10]. The various flavor factors \( T_{F}^{f_0 \rightarrow BC} \) are indicated in the table. Using relativistic phase space, the partial decay width \( \Gamma_{f_0 \rightarrow BC} \) is written as

\[ \Gamma_{f_0 \rightarrow BC} = 2\pi \frac{E_B E_C}{M_{f_0}} K \int d\Omega_K |T(K)|^2 \] (8)
with decay momentum \(|K| = K\) and where \(E_i = \sqrt{M_i^2 + \mathbf{K}^2}\) and \(M_i\) (i=B,C) are energy and mass of the outgoing meson \(i\). Analytical evaluation of Eq. (8) leads to

\[
\Gamma_{f_0 \rightarrow \sigma \sigma} = \lambda^2 \frac{E_\sigma^2}{M_{f_0}} \frac{16}{81 \pi} \frac{R_{f_0}^5 R_{\sigma}^6}{(R_{f_0}^2 + 2R_{\sigma}^2)^7} \exp \left\{ -\frac{1}{2} \left( \frac{R_{f_0}^2 R_{\sigma}^2}{R_{f_0}^2 + 2R_{\sigma}^2} \right) K^2 \right\} 
\]

\[
|T_{f_0 \rightarrow \sigma \sigma}|^2 K \left[ 60 + \frac{11R_{f_0}^4 + 4R_{\sigma}^2(R_{f_0}^2 + R_{\sigma}^2)}{(R_{f_0}^2 + 2R_{\sigma}^2)} K^2 - \frac{R_{f_0}^4 R_{\sigma}^2(R_{f_0}^2 + R_{\sigma}^2)}{(R_{f_0}^2 + 2R_{\sigma}^2)^2} K^4 \right] 
\]  

(9)

\[
\Gamma_{f_0 \rightarrow \pi \pi} = \lambda^2 \frac{E_{\pi \pi}^2}{M_{f_0}} \frac{8}{27 \sqrt{\pi}} \frac{R_{f_0}^5 R_{\pi}^6}{(R_{f_0}^2 + 2R_{\pi}^2)^7} \exp \left\{ -\frac{1}{2} \left( \frac{R_{f_0}^2 R_{\pi}^2}{R_{f_0}^2 + 2R_{\pi}^2} \right) K^2 \right\} 
\]

\[
|T_{f_0 \rightarrow \pi \pi}|^2 K \left[ (18R_{f_0}^2 - 24R_{\pi}^2) - \frac{R_{f_0}^2 R_{\pi}^2(13R_{f_0}^2 + 6R_{\pi}^2)}{(R_{f_0}^2 + 2R_{\pi}^2)} K^2 + \frac{R_{f_0}^4 R_{\pi}^2(R_{f_0}^2 + R_{\pi}^2)}{(R_{f_0}^2 + 2R_{\pi}^2)^2} K^4 \right]^2 
\]

(10)

and

\[
\Gamma_{f_0 \rightarrow \pi \pi(\rho)} = \lambda^2 \frac{E_{\pi(\rho)}^2}{M_{f_0}} \frac{64}{A \sqrt{\pi}} \frac{R_{f_0}^5 R_{\pi(\rho)}^6}{(R_{f_0}^2 + 2R_{\pi(\rho)}^2)^5} \exp \left\{ -\frac{1}{2} \left( \frac{R_{f_0}^2 R_{\pi(\rho)}^2}{R_{f_0}^2 + 2R_{\pi(\rho)}^2} \right) K_{\pi(\rho)}^2 \right\} 
\]

\[
|T_{f_0 \rightarrow \pi \pi(\rho)}|^2 K_{\pi(\rho)} \left[ 1 - \frac{2}{3} \left( \frac{R_{\pi(\rho)}^2 R_{f_0}^2 + R_{\pi(\rho)}^2}{R_{f_0}^2 + 2R_{\pi(\rho)}^2} \right) K_{\pi(\rho)}^2 \right. 
\]

\[+ \left. \frac{1}{B} \left( \frac{R_{\pi(\rho)}^2 R_{f_0}^2 + R_{\pi(\rho)}^2}{R_{f_0}^2 + 2R_{\pi(\rho)}^2} \right) K_{\pi(\rho)}^4 \right] 
\]

(11)

with \(A = 1, B = 9\) for \(f_0 \rightarrow \pi \pi\) and \(A = 3, B = 1\) for \(f_0 \rightarrow \rho \rho\). For the broad mesons \(f_0(1500), \pi^*(1300), \rho(770)\) and \(\sigma(760)\) we average over the mass spectrum \(f(m)\):

\[
\Gamma_{f_0 \rightarrow BC} = \int dm_{f_0} dm_B dm_C \Gamma_{f_0 \rightarrow BC}(m_{f_0}, m_B, m_C) f(m_{f_0}) f(m_B) f(m_C) 
\]

(12)

\[
f(m) \propto \frac{(\Gamma_i/2)^2}{(m - M_i)^2 + (\Gamma_i/2)^2} 
\]

with a proper threshold cutoff introduced as in Ref. [2]. Masses \(M_i\) and widths \(\Gamma_i\) are taken from the Particle Data Group. They are well defined except \(\Gamma_{\pi^*}\) which is \((400 \pm 200)\) MeV. We parameterize the mass distribution of the scalar-isoscalar \(\sigma\) meson as in Ref. [3] \((M_\sigma = 760\) MeV, \(\Gamma_\sigma = 640\) MeV). The partial decay widths of all \(4\pi\) channels and also the \(2\pi\) mode scale with \((\xi \cdot \lambda)^2\). The predictions for relative branching ratios are therefore independent of
any particular three-state mixing scheme \cite{6,9}, specified by the explicit value of $\xi$. The only parameters left in the model are the size parameters $R_\pi, R_\rho, R_\sigma$ and $R_{f_0}$ of the meson wave functions. Assuming a common oscillator parameter for all mesons, meson decay analyses yield $R = 2.5\, GeV^{-1}$ \cite{5,11}. These values will be refered to as set 1. In Ref. \cite{6}, for the two pseudoscalar meson decay modes, differences in size parameters for the $Q\bar{Q}$ states are taken into account, with $R(3P_0) = 2.0\, GeV^{-1}$ and $R(1S_0) = 2.5\, GeV^{-1}$ (set 2). A recent full scale analysis of tensor meson decays \cite{14} yields $R_\pi = R_\rho = 3.69\, GeV^{-1}$, suggesting $R(3P_0) \approx 3.7\, GeV^{-1}$ (set 3). There is no reliable constraint for $R_\sigma$; since $\sigma$ represents a broad $\pi\pi$ S-wave resonance, $R_\sigma \geq R_\pi$ is reasonable. We resort to these different sets of values to study the sensitivity of the different observables to this input leading to an effective error band of the decay model.

Results - If we adopt $R_\pi = R_\sigma$ and $\Gamma_\pi^* = 400\, MeV$ we obtain for the $f_0 \to BC$ decays the following ratios:

$$B(\pi\pi) : B(\rho\rho) : B(\sigma\sigma) : B(\pi^*\pi) = \begin{cases} 1 : 1.62 & : 0.93 & : 0.33 & \text{with set 1} \\ 1 : 1.04 & : 0.76 & : 0.56 & \text{with set 2} \\ 1 : 0.92 & : 0.28 & : 0.19 & \text{with set 3} \end{cases}$$

We can see that all decay rates reveal a rather strong dependence on the size parameters. However, even when we vary the size parameters in a rather wide range, the $4\pi$ decay modes of the $f_0(1500)$ always follow the decay pattern

$$B(\pi^*\pi) < B(\sigma\sigma) < B(\pi\pi) < B(\rho\rho)$$

This hierarchy of the decay modes remains also unaltered when taking into account the experimental uncertainty of $\Gamma_\pi^*$ by $\pm 200\, MeV$. Similarly, a variation of the $\sigma$ size parameter in the range $R_\pi \leq R_\sigma \leq 6.0\, GeV^{-1}$ does not change the order \cite{14}. This constitutes a stable prediction of the model. Experimental data concerning the $4\pi$ decay mode of the $f_0(1500)$ and its separation into the contributions of the individual two-body channels are still incomplete. Recently, Crystal Barrel observed the $f_0(1500)$ in its $4\pi^0$ decay mode in the annihilation reaction $p\bar{p} \to 5\pi^0$ at rest \cite{15}. With their results for $f_0(1500) \to \pi\pi$, given by a single channel ($p\bar{p} \to 3\pi^0$) \cite{11} and, more consistently, by a coupled channel analysis ($p\bar{p} \to \pi^0\eta\pi^0\eta, \pi^0\pi^0\eta$ and $3\pi^0$) \cite{16}, the ratio $r$ is found to be

$$r = \frac{B(f_0(1500) \to 4\pi)}{B(f_0(1500) \to 2\pi)} = \begin{cases} 3.4 \pm 0.8 & \text{[15]} \\ 2.1 \pm 0.6 & \text{[17]} \end{cases}$$

6
where the contribution of the $\rho\rho$ intermediate state is not included in $B(f_0(1500) \to 4\pi)$. However, an alternative multi-channel analysis [18], resulting in a partial decay width of $\Gamma_{\pi\pi} = 60 \pm 12$ MeV and a total width of $\Gamma = 132 \pm 15$ MeV for the $f_0(1500)$ seems to indicate a ratio of $r$ closer to 1. For the three parameter sets and using $\Gamma_{\pi^*} = 400$ MeV, $B(\pi^* \to \sigma\pi) = 0.6$, and $R_\sigma = R_\pi$, our calculation gives

$$r = \frac{B(f_0 \to 4\pi)}{B(f_0 \to 2\pi)} = \begin{cases} 1.13 & \text{for set 1} \\ 1.09 & \text{for set 2} \\ 0.40 & \text{for set 3} \end{cases}$$

(16)

to be compared to the experimental number of Eq. (13). Again, taking into account the experimental uncertainties in $\Gamma_{\pi^*}$ and in $B(\pi^* \to \sigma\pi)$ by $\pm 0.2$ the results (16) are changed by less than 10%. A variation of $R_\sigma$ from the value of $R_\pi$ to 6 GeV$^{-1}$ leads to ranges of predictions for $r$ of $0.41 - 1.21$, $0.32 - 1.20$ and $0.26 - 0.45$, for the three parameter sets respectively. Even when one allows strong variations in the size parameters the theoretical predictions for $r$ are lower than the experimental result by at least a factor of two. Although in Ref. [15], the final states $\sigma\sigma$ and $\pi\pi^*$ contribute with similar intensities to the $4\pi^0$ mode, precise values for the respective decay branching ratios cannot be given due to interference effects. Furthermore, preliminary results [19] of an analysis of $p\bar{p} \to 5\pi$ including charged state combinations of $\pi$, seem to indicate that the $\sigma\sigma$ decay is dominating over the $\rho\rho$ decay.

In summary, we have investigated the glueball-quarkonia mixing scheme of Refs. [6, 9] in the $f_0(1500)$ decay into $\sigma\sigma, \rho\rho, \pi\pi^*$ and $\pi\pi$ final states. The leading order hadronic decay mechanism, as described in Ref. [6], proceeds both by the $Q\bar{Q}$ and the gluonic component of the $f_0(1500)$; latter process, where the glueball decays into a pair of glueballs, can feed final states with isoscalar mesons, possessing a sizable gluonic component. Given the modelling of $\sigma$ as a $^3P_0$ $Q\bar{Q}$ state, where gluonic components are neglected, for the leading order decay mechanism we predict a decay pattern of $4\pi$ modes, where the $\rho\rho$ channel dominates. This is in conflict with preliminary experimental results [19], where the $\rho\rho$ decay mode is suppressed. For the $4\pi/2\pi$ decay ratio the result we obtain is insufficient to fully explain the experimental result of Eq. (13). Another analysis of the $f_0(1500)$ decays [18] seems to indicate that the ratio of Eq. (13) is considerably lower, reducing the disagreement with the theoretical results. We stress that the decay pattern developed here is common to a simple $n\bar{n}$ state and to the mixed $f_0(1500)$, where its $s\bar{s}$ configuration does not couple to the $4\pi$ modes. If the experimental qualitative ordering of the $f_0(1500)$ decay rates $B(4\pi) > B(2\pi)$ and $B(\sigma\sigma) > B(\rho\rho)$ is confirmed, the decay scheme developed here should be modified by taking
into account an additional direct coupling of the gluonic component of the $f_0(1500)$ state to the $4\pi$ decay channels; for example, leading order contributions can enhance the $\sigma\sigma$ decay mode, while higher order decay mechanisms can contribute to all $4\pi$ decay channels. Thus an accurate experimental determination of the $4\pi$ decay modes of the $f_0(1500)$ provides an additional sensitive signal for the need to go beyond a pure $Q\bar{Q}$ consideration of the $f_0(1500)$, possibly revealing the significant glueball configuration claimed to reside in this state.

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Table: Flavor matrix elements for the decay modes $f_0 \to BC$.

| decay           | $|T_{f_0 \to BC}^{(f_0 \to BC)}|^2 / \xi^2$ |
|-----------------|------------------------------------------|
| $f_0 \to \pi\pi(\rho\rho)$ | 3                                        |
| $f_0 \to \pi\pi^*$         | 6                                        |
| $f_0 \to \sigma\sigma$    | 1                                        |
Figure: Decay of the $Q\bar{Q}$ component of the $f_0$ into two mesons BC. The dot indicates the $Q\bar{Q} \ ^3P_0$ vertex.