Effects of molecule force on free vibration for a micro electromagnetic harmonic drive system

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Abstract. In this paper, a micro electromagnetic harmonic drive system is proposed. Considering Van der Waals force, dynamics equation of the flexible ring for the micro drive system is deduced and resolved. Using the equations, the effects of the molecule force on the natural frequencies and vibration modes of the drive system are investigated. Results show that considering molecule force, natural frequencies of the flexible ring are reduced and its vibration modes are changed. For lower order modes, smaller clearance between the flexible ring and stator, smaller thickness of the flexible ring and larger radius of the flexible ring, the effects of the molecule force on the natural frequencies and vibration modes are more obvious.

Keywords: Electromagnetic harmonic drive system / micro drive / Van der Waals force / free vibration / natural frequency / vibration modes

1 Introduction

Electromagnetic harmonic drive is a kind of mechatronics drive device combining rotating magnetic field with harmonic drive. It is widely used in the driving mechanism of machine tools and instruments, and is suitable for submarine navigation, aerospace and transportation fields [1–5].

Herdeg proposed the electromagnetic harmonic drive, and successfully developed the experimental prototype of the drive with external magnetic poles [6]. Janes developed an electromagnetic harmonic drive system with built-in electromagnetic winding, which is compact in structure and can transmit more torque per unit space [7–9]. Rens developed a new type of permanent magnet harmonic drive prototype, which is suitable for the requirements of large transmission ratio [10,11]. Tjahjowidodo proposed a harmonic drive model using statistical measures of variation and analyzed its reliability under different conditions [12,13]. Reinhard investigated small harmonic gear drive and developed the harmonic reducer with metal gears which was used in robot driver for semiconductor chip packaging [14,15]. Jose designed harmonic drive with low temperature magnetic superconductor to improve the service life [16]. Chigira proposed a harmonic drive with a stackable structure that is easy to assemble. By adjusting the structure of the magnetic gear, the maximum transfer torque can be increased [17]. Based on studies of the electromagnetic harmonic drive, Xu proposed an electromagnetic harmonic movable tooth drive system and investigated its output torque [18–20]. Afanas'ev calculated the electromagnetic moments of an electromagnetic gear reducer by the energy method [21]. Ando proposed a new harmonic gear with stackable structure easy to assemble and studied the effect of stack on the maximum transmit torque by experiments [22]. Koji proposed a new magnetic harmonic gear which has the stackable structure in which the maximum transmission torque of the gear was improved approximately 5.96 times and the torque density was improved about 3.82 times [23]. Liu et al. investigated eccentric harmonic magnetic gear and presented an analytical method for predicting the distribution of magnetic field in the air gap of harmonic gear [24]. Jing proposed a new type of eccentric harmonic magnetic gear and calculated corresponding magnetic field and static torque with the finite element analysis [25]. With the development of MEMS technology, the size of the driving link is more and more limited, and the micron scale drive technology is urgently needed [26].

Therefore, the Authors propose a micro electromagnetic harmonic drive system. It has advantages such as small volume, light weight, simple and compact structure, large speed ratio and small inertia, etc. Stator components with electrode segments and special piezoelectric ceramic materials are not needed, which is more conducive to the miniaturization.

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In the drive system, the electromagnetic force has a decisive role for its operation behavior. However, with further reducing the size of the system, the effect of molecular force is becoming more and more significant. The Van der Waals force is an important molecular force, will significantly influence the dynamic performance of the micro electromagnetic harmonic drive system.

In this paper, considering Van der Waals force, dynamics equation of the flexible ring for the micro drive system is deduced and resolved. Using the equations, effects of the molecule force on the natural frequencies and vibration modes of the drive system are investigated. Results show that considering molecule force, natural frequencies of the flexible ring are reduced and its vibration modes are changed. The research is useful in design of the dynamics performance for the micro electromagnetic harmonic drive system.

2 Structure and operation principle

Figure 1 shows a micro electromagnetic harmonic drive system. It consists of micro flexible ring and stator. When the external magnetic field is applied sequentially, the rotating magnetic field will cause the flexible wheel to undergo periodic elastic deformation, thus driving the associated supporting shaft to rotate.

Here, \( r \) is radius of the flexible ring, \( l \) is the length of the flexible ring, \( t_0 \) is the clearance between the flexible ring and stator. The stator material is no magnetic and the flexible ring material is magnetic. The flexible ring under electromagnetic force and molecule force is given in Figure 2. In electromagnetic field, distributed electromagnetic force occurs on the flexible ring. \( q_{re} \) is electromagnetic force per unit length. Meanwhile, molecule force occurs at the angle range \( \left[ -\frac{\pi}{4}, \frac{\pi}{4} \right] \). \( q_{r3} \) is Van der Waals force per unit length.

3 Static displacement of the flexible ring

The electromagnetic force per unit length on the flexible wheel is

\[
q_{re} = \frac{n_B^2 \mu_0 R_I^4 I^2}{8(R_I^2 + x_B^2)^3} I^2 \cos \theta
\]

where \( \mu_0 \) – vacuum permeability, \( \mu_0 = 4 \pi \times 10^{-7} \text{N} \cdot \text{A}^{-2} \); \( I \) – current in magnetic coil; \( n_B \) – number of turns per unit length; \( R_I \) – magnetic coil radius;
Distance from the flexible ring to the magnetic coil plane; position angle of the flexible ring.

Equation (1) can be simplified to

\[ q_{re} = q_{r0} \cos \theta \]

(2)

where \( q_{r0} = \frac{n^2 \mu_0 R_1^4 l}{8(R_1^2 + x_B^2)} f_1^2 \).

The van der Waals force per unit length is [27]

\[ q_{r3} = \frac{A I}{6\pi(t_0 - u_0)^3} \]

(3)

Where \( A \) is Hamaker constant, \( A = 6.58 \times 10^{-20} \text{J} \).

The total force per unit length on the flexible ring is

\[ q_{rs} = q_{re} + q_{r3} \]

(4)

The distribution of forces on the flexible wheel is as follows:

\[ q_{rs} = q_{r0} \cos \theta + q_{r3} = \frac{n^2 \mu_0 R_1^4 l}{8(R_1^2 + x_B^2)} f_1^2 \cos \theta + q_{r3} \]

\[ \left\{ \begin{array}{l}
q_{rs} = q_{r0} \cos \theta \\
q_{rs} = -q_{r0} \cos \theta + q_{r3} = -\frac{n^2 \mu_0 R_1^4 l}{8(R_1^2 + x_B^2)} f_1^2 \cos \theta + q_{r3} \\
q_{rs} = -q_{r0} \cos \theta \\
q_{rs} = q_{r0} \cos \theta 
\end{array} \right. \]  

\[ \left( -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4} \right) \]

(5)

\[ \left( \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}, -\frac{\pi}{2} \leq \theta \leq -\frac{\pi}{4} \right) \]

\[ \left( \frac{3\pi}{4} \leq \theta \leq \frac{5\pi}{4} \right) \]

\[ \left( \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{4}, \frac{4\pi}{4} \leq \theta \leq \frac{3\pi}{2} \right) \]

The load is expanded as a Fourier series

\[ q_{rs} = q_0 + \sum_{k=1}^{\infty} q_k \cos k \theta \]

(6)

The coefficients of the series satisfy the following conditions

\[ \text{See equation (7) below.} \]

Thus

\[ q_0 = \frac{2q_{r0} + q_{r3}}{\pi} \]

(8)

To determine the coefficient \( q_k \), multiply the above integral equation by \( \cos k \theta \), the following relation is obtained

\[ \text{See equation (9) below.} \]

\[ \int_{0}^{2\pi} \left[ q_0 + \sum_{k=1}^{\infty} q_k \cos k \theta \right] d\theta = \int_{0}^{\pi} \frac{4}{\pi} (q_{r0} \cos \theta + q_{r3}) d\theta + \int_{0}^{\pi} \frac{2}{\pi} (q_{r0} \cos \theta) d\theta \]

\[ + \int_{\pi}^{2\pi} \frac{4}{\pi} (-q_{r0} \cos \theta + q_{r3}) d\theta + \int_{\pi}^{2\pi} \frac{4}{\pi} (-q_{r0} \cos \theta) d\theta \]

(7)

\[ \int_{0}^{2\pi} \left[ q_0 + \sum_{k=1}^{\infty} q_k \cos k \theta \right] \cos k \theta d\theta = \int_{0}^{\pi} \frac{4}{\pi} (q_{r0} \cos \theta + q_{r3}) \cos k \theta d\theta + \int_{0}^{\pi} \frac{2}{\pi} (q_{r0} \cos \theta) \cos k \theta d\theta \]

\[ + \int_{\pi}^{2\pi} \frac{4}{\pi} (-q_{r0} \cos \theta + q_{r3}) \cos k \theta d\theta + \int_{\pi}^{2\pi} \frac{4}{\pi} (-q_{r0} \cos \theta) \cos k \theta d\theta \]

(9)
In order to make the load symmetric with respect to the vertical axis and the horizontal axis, only take the coefficient of the cosine series to be even, yields

\[ q_k = \frac{2q_{r0}}{\pi} \left[ \frac{\sin \left( \frac{(k+1)\pi}{2} \right)}{k+1} + \frac{\sin \left( \frac{(k-1)\pi}{2} \right)}{k-1} \right] + \frac{4q_{r3}}{\pi k} \sin \frac{k\pi}{4} \quad (k \neq 1) \]

Equation (10) can be changed into following form

\[ q_{rs} = \frac{2q_{r0}}{\pi} + \frac{q_{r3}}{2} + \sum_{k=2,4,6,\ldots} \left\{ \frac{2q_{r0}}{\pi} \left[ \frac{\sin \left( \frac{(k+1)\pi}{2} \right)}{k+1} + \frac{\sin \left( \frac{(k-1)\pi}{2} \right)}{k-1} \right] + \frac{4q_{r3}}{\pi k} \sin \frac{k\pi}{4} \right\} \cos k\theta \]

Substituting \( q_{rs} \) into the dynamics equation of the flexible wheel:

\[ \frac{\partial^2 u}{\partial \theta^2} + 2 \frac{\partial^3 u}{\partial \theta^3} + \frac{\partial u}{\partial \theta} = \frac{r^4}{EI} \frac{\partial^4 u}{\partial \theta^4} - \frac{r^4}{EI} \frac{\rho A}{E} \frac{\partial \dot{u}}{\partial \theta} \quad (12) \]

This is a static solution, so the derivative of the displacement with respect to time is zero, that is, \( \partial \dot{u} / \partial \theta = 0 \), yields

\[ \frac{\partial^2 u}{\partial \theta^2} + 2 \frac{\partial^3 u}{\partial \theta^3} + \frac{\partial u}{\partial \theta} = -\frac{r^4}{EI} \frac{1}{\pi} \sum_{k=2,4,6,\ldots} \left\{ k \frac{\sin \left( \frac{(k+1)\pi}{2} \right)}{k+1} + k \frac{\sin \left( \frac{(k-1)\pi}{2} \right)}{k-1} + 4q_{r3} \sin \frac{k\pi}{4} \right\} \sin k\theta \quad (13) \]

Write the solution of equation (13) as a series as follows

\[ u = \sum_{k=2,4,6,\ldots} C_k \cos k\theta \quad (14) \]

Substituting equation (14) into equation (13) to obtain

\[ \sum_{k=2,4,6,\ldots} C_k (-k^5 + 2k^3 - k) \sin k\theta = -\frac{r^4}{EI} \frac{1}{\pi} \sum_{k=2,4,6,\ldots} \left\{ \frac{2q_{r0}}{\pi} \left[ \frac{\sin \left( \frac{(k+1)\pi}{2} \right)}{k+1} + \frac{\sin \left( \frac{(k-1)\pi}{2} \right)}{k-1} \right] + \frac{4q_{r3}}{\pi} \sin \frac{k\pi}{4} \right\} \sin k\theta \quad (15) \]

On both sides of equation (15), if the corresponding terms of the same \( k \) value are equal, then

\[ C_k = \frac{2q_{r0}}{\pi} \frac{\sin \left( \frac{(k+1)\pi}{2} \right)}{k+1} + \frac{\sin \left( \frac{(k-1)\pi}{2} \right)}{k-1} + \frac{4q_{r3}}{\pi} \sin \frac{k\pi}{4} \]

\[ \frac{r^4}{EI} \frac{1}{\pi} \sum_{k=2,4,6,\ldots} k(k^2 - 1)^2 \]

So

\[ \text{See equation (17) below.} \]

Letting \( u_0 \) be equal to the average value of the displacement \( u \) at the angle range \(-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}\), using equation (17), \( u_0 \) can be obtained

\[ \text{See equation (18) below.} \]

\[ u = \frac{r^4}{\pi EI} \sum_{k=2,4,6,\ldots} \left[ \frac{(k-1)\sin \left( \frac{(k+1)\pi}{2} \right) + (k+1)\sin \left( \frac{(k-1)\pi}{2} \right)}{k^2 - 1} + 4q_{r3} \sin \frac{k\pi}{4} \right] \cos k\theta \quad (17) \]

\[ u_0 = \frac{2}{\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} u d\theta \]

\[ = \frac{8r^4}{\pi^2 EI} \sum_{k=2,4,6,\ldots} \left[ q_{r0} \frac{(k-1)\sin \left( \frac{(k+1)\pi}{2} \right) + (k+1)\sin \left( \frac{(k-1)\pi}{2} \right)}{k(k^2 - 1)} + 2q_{r3} \sin \frac{k\pi}{4} \right] \sin \frac{k\pi}{4} \quad (18) \]
4 Solution of the dynamics equation for flexible ring

4.1 Mode function

Let the radial displacement $u$ of the flexible ring be composed of static displacement $u_0$ and dynamic displacement $\Delta u$

$$u = u_0 + \Delta u$$  \hspace{1cm} (19)

The radial load $q_r$ is composed of static load $q_{rs}$ and dynamic load $\Delta q_r$

$$q_r = q_{rs} + \Delta q_r$$  \hspace{1cm} (20)

Substituting equation (20) into equation (12), yields

$$\frac{\partial^6 \Delta u}{\partial \theta^6} + 2 \frac{\partial^4 \Delta u}{\partial \theta^4} + \frac{\partial^2 \Delta u}{\partial \theta^2} + \Delta u = \frac{r^4}{E I_x} \rho A_e \partial u$$  \hspace{1cm} (21)

where $\partial u$ is the second derivative of the dynamic displacement $\Delta u$ with respect to time, $E$ is the elastic modulus of the flexible ring material, $\rho$ is density of the flexible ring, $A_e$ is the section area of the flexible ring, $I_x$ is the cross section modulus of the flexible ring.

Equation (21) can be changed into following form

$$\frac{\partial^6 \Delta u}{\partial \theta^6} + 2 \frac{\partial^4 \Delta u}{\partial \theta^4} + \frac{\partial^2 \Delta u}{\partial \theta^2} + \Delta u = \frac{r^4}{E I_x} \rho A_e \partial u$$  \hspace{1cm} (22)

where $\Delta u$ is the dynamic load, $\Delta q_r = \frac{dq_{rs}}{du} \Delta u$, from equations (1) and (3), the dynamic load can be determined by:

- Only considering electromagnetic force:

$$\Delta q_{rs} = \frac{dq_{rs}}{du} \Delta u = 0$$  \hspace{1cm} (23)

- Considering electromagnetic force and van der Waals force:

$$\left\{ \begin{array}{l}
\Delta q_{rs} = \frac{dq_{rs}}{du} \Delta u = \frac{Al}{2\pi(t_0 - u_0)} \Delta u \\
(0 \leq \theta \leq \frac{\pi}{4}) \\
\Delta q_3 = 0 \\
(\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2})
\end{array} \right.$$  \hspace{1cm} (24)

Substituting equations (23) and (24) into (22), the dynamics equation of the micro electromagnetic harmonic drive system is as follows:

- Only considering electromagnetic force:

$$\frac{\partial^6 \Delta u_e}{\partial \theta^6} + 2 \frac{\partial^4 \Delta u_e}{\partial \theta^4} + \frac{\partial^2 \Delta u_e}{\partial \theta^2} + \Delta u_e = \frac{r^4}{E I_x} \rho A_e \partial u$$  \hspace{1cm} (25)

- Considering electromagnetic force and van der Waals force:

$$\text{See equation (26) below.}$$

Letting

$$\Delta u = \phi(\theta) q(t)$$  \hspace{1cm} (27)

Then

- Only considering electromagnetic force:

$$\frac{\partial(\phi(\theta))}{\partial \theta} + 2 \frac{\partial^2(\phi(\theta))}{\partial \theta^2} + \phi(\theta) = \frac{r^4}{E I_x} \rho A_e \partial u$$  \hspace{1cm} (28)

From equation (28), it is given

$$\phi(\theta) + \phi''(\theta) = 0$$  \hspace{1cm} (29)

$$\phi''(\theta) = \frac{2 \phi''(\theta)}{\phi(\theta)}$$

where $Q = 1 - \frac{r^4}{E I_x} \epsilon^2$

Letting $\phi(\theta) = e^\epsilon$ and substituting it into (30), yields

$$\lambda^4 + 2 \lambda^2 + Q = 0$$  \hspace{1cm} (31)

The four eigen values can be obtained as follows

$$\pm \sqrt{-1 + \sqrt{1 - Q}} \text{ and } \pm i \sqrt{1 - Q}$$

and then the mode function can be given by

$$\phi_e(\theta) = B_1 \cos k_1 \theta + B_2 \sin k_1 \theta + B_3 \sin k_2 \theta + B_4 \sin k_2 \theta$$  \hspace{1cm} (32)

where $k_1 = \sqrt{1 + \sqrt{1 - Q}}$ and $k_2 = \sqrt{1 - \sqrt{1 - Q}}$

The integral constants $B_j (j = 1, 2, 3, 4)$ and frequency equation can be determined by symmetry and continuity conditions of the flexible ring.

- Considering electromagnetic force and van der Waals force:

$$\text{at } 0 \leq \theta \leq \frac{\pi}{4}$$

In a same manner, following equation can be given

$$\frac{\partial(\phi(\theta))}{\partial \theta} + 2 \frac{\partial^2(\phi(\theta))}{\partial \theta^2} + \phi(\theta) = \frac{r^4}{E I_x} \rho A_e \partial u$$  \hspace{1cm} (33)

where $P = 1 - \frac{r^4}{2E I_x \pi(t_0 - u_0)}$

$$\left\{ \begin{array}{l}
\frac{\partial^6 \Delta u_{13}}{\partial \theta^6} + 2 \frac{\partial^4 \Delta u_{13}}{\partial \theta^4} + \frac{\partial^2 \Delta u_{13}}{\partial \theta^2} + \Delta u_{13} = \frac{Al}{2\pi(t_0 - u_0)} \Delta u_{13} - \frac{r^4}{E I_x} \rho A_e \partial u_{13} \\
(0 \leq \theta \leq \frac{\pi}{4}) \\
\frac{\partial^6 \Delta u_{23}}{\partial \theta^6} + 2 \frac{\partial^4 \Delta u_{23}}{\partial \theta^4} + \frac{\partial^2 \Delta u_{23}}{\partial \theta^2} + \Delta u_{23} = -\frac{r^4}{E I_x} \rho A_e \partial u_{23} \\
(\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2})
\end{array} \right.$$  \hspace{1cm} (26)
From equation (33), it is obtained

$$\phi_{31}^{(i)}(\theta) + 2\phi_{32}^{(i)}(\theta) + R\phi_{31}(\theta) = 0$$  \hspace{1cm} (34)

where \( R = P - \frac{\rho A_4 r^4}{EI} \omega^2 \).

Letting \( \phi_{31}(\theta) = e^{i\theta} \) and substituting it into (34), yields

$$\lambda^4 + 2\lambda^2 + R = 0$$  \hspace{1cm} (35)

Thus, the mode function can be given as

$$\phi_{31}(\theta) = A_1 \cos m_1 \theta + A_2 \sin m_1 \theta + A_3 \sin m_2 \theta$$

$$+ A_4 \sin m_3 \theta$$  \hspace{1cm} (36)

where \( m_1 = \sqrt{1 + \sqrt{1 - R}} \), \( m_2 = \sqrt{-1 + \sqrt{1 - R}} \).

In a same manner, following equation can be given

$$\ddot{q}(t) = -\frac{\rho A_4 r^4}{EI}\phi_{32}(\theta)$$  \hspace{1cm} (37)

From equation (37), it is obtained

$$\phi_{32}^{(i)}(\theta) + 2\phi_{32}^{(i)}(\theta) + S\phi_{32}(\theta) = 0$$  \hspace{1cm} (38)

where \( S = 1 - \frac{\rho A_4 r^4}{EI} \).

Letting \( \phi_{32}(\theta) = e^{i\theta} \) and substituting it into (38), yields

$$\lambda^4 + 2\lambda^2 + S = 0$$  \hspace{1cm} (39)

Thus, the mode function can be given as

$$\phi_{32}(\theta) = A_5 \cos n_1 \theta + A_6 \sin n_1 \theta + A_7 \sin n_2 \theta$$

$$+ A_8 \sin n_3 \theta$$  \hspace{1cm} (40)

where \( n_1 = \sqrt{1 + \sqrt{1 - S}} \) and \( n_2 = \sqrt{-1 + \sqrt{1 - S}} \).

The integral constants \( A_j \); \( j = 1, 2, 3, 4, 5, 6, 7, 8 \) and frequency equation can be determined by symmetry and continuity conditions of the flexible ring.

4.2 Natural frequencies

From symmetry and continuity conditions of the flexible ring, both angle \( \varphi \) and tangent displacement \( v \) are zero at \( \theta = 0 \) and \( \theta = \frac{\pi}{2} \), so

$$\phi_{31}(\theta) = 0$$

$$\phi_{32}(\theta) = 0$$

- Only considering electromagnetic force:

Substituting equation (41) into (32), yields

$$\text{See equation (42) below.}$$

From equation (42), it can be known that \( B_2 = 0 \) and \( B_4 = 0 \), so equation (42) can be changed into following form

$$C_e X_e = D_e$$  \hspace{1cm} (43)

where

$$C_e = \begin{bmatrix}
-k_1 \sin\left(\frac{\pi}{2} k_1\right) & k_2 \sinh\left(\frac{\pi}{2} k_2\right) \\
-k_1^3 \sin\left(\frac{\pi}{2} k_1\right) & k_2^3 \sinh\left(\frac{\pi}{2} k_2\right)
\end{bmatrix}$$

$$X_e = [B_1 B_3]^T D_e = [0 \ 0]^T$$

If there are non-zero solutions for equation (43), the determinant of is zero, that is

$$\begin{vmatrix}
-k_1 \sin\left(\frac{\pi}{2} k_1\right) & k_2 \sinh\left(\frac{\pi}{2} k_2\right) \\
-k_1^3 \sin\left(\frac{\pi}{2} k_1\right) & k_2^3 \sinh\left(\frac{\pi}{2} k_2\right)
\end{vmatrix} = 0$$  \hspace{1cm} (44)

- Considering electromagnetic force and van der Waals force:

From continuity conditions of the flexible ring, following relationship can be given at \( \theta = \frac{\pi}{4} \)

$$\begin{cases}
\phi_{31}(\theta) = \phi_{32}(\theta) \\
\phi_{31}(\theta) = \phi_{32}(\theta) \\
\phi_{31}(\theta) = \phi_{32}(\theta) \\
\phi_{31}(\theta) = \phi_{32}(\theta)
\end{cases}$$  \hspace{1cm} (45)

$$B_2 k_1 + B_4 k_2 = 0$$

$$-B_1 k_1 \sin\left(\frac{\pi}{2} k_1\right) + B_2 k_1 \cos\left(\frac{\pi}{2} k_1\right) + B_3 k_2 \sinh\left(\frac{\pi}{2} k_2\right) + B_4 k_2 \sin\left(\frac{\pi}{2} k_2\right) = 0$$

$$-B_2 k_1^3 + B_4 k_2^3 = 0$$

$$B_1 k_1^3 \sin\left(\frac{\pi}{2} k_1\right) - B_2 k_1^3 \cos\left(\frac{\pi}{2} k_1\right) + B_3 k_2^3 \sinh\left(\frac{\pi}{2} k_2\right) + B_4 k_2^3 \sin\left(\frac{\pi}{2} k_2\right) = 0$$
Substituting equations (36) and (40) into (45), yields the following form

\( C_1 X_1 = D_1 \) \( (47) \)

where

From equation (46), it can be known that \( A_2 = 0 \) and \( A_4 = 0 \), so equation (46) can be changed into

\[
\begin{align*}
A_2 m_1 + A_4 m_2 &= 0 \\
-A_5 n_1 \sin \left( \frac{\pi}{2} n_1 \right) + A_6 n_1 \cos \left( \frac{\pi}{2} n_1 \right) + A_7 n_2 \sinh \left( \frac{\pi}{2} n_2 \right) + A_8 n_2 \cosh \left( \frac{\pi}{2} n_2 \right) &= 0 \quad (46) \\
-A_2 m_1^3 + A_4 m_2^3 &= 0 \\
A_5 n_1^3 \sin \left( \frac{\pi}{2} n_1 \right) - A_6 n_1^3 \cos \left( \frac{\pi}{2} n_1 \right) + A_7 n_2^3 \sinh \left( \frac{\pi}{2} n_2 \right) + A_8 n_2^3 \cosh \left( \frac{\pi}{2} n_2 \right) &= 0 \\
A_1 \cos \left( \frac{\pi}{4} m_1 \right) + A_2 \sin \left( \frac{\pi}{4} m_1 \right) + A_3 \cosh \left( \frac{\pi}{4} m_2 \right) + A_4 \sinh \left( \frac{\pi}{4} m_2 \right) - A_5 \cos \left( \frac{\pi}{4} n_1 \right) - A_6 \sin \left( \frac{\pi}{4} n_1 \right) - A_7 \cosh \left( \frac{\pi}{4} n_2 \right) - A_8 \sinh \left( \frac{\pi}{4} n_2 \right) &= 0 \\
-A_1 m_1 \sin \left( \frac{\pi}{4} m_1 \right) + A_2 m_1 \cos \left( \frac{\pi}{4} m_1 \right) + A_3 m_2 \sinh \left( \frac{\pi}{4} m_2 \right) + A_4 m_2 \cosh \left( \frac{\pi}{4} m_2 \right) \\
+A_5 n_1 \sin \left( \frac{\pi}{4} n_1 \right) - A_6 n_1 \cos \left( \frac{\pi}{4} n_1 \right) - A_7 n_2 \sinh \left( \frac{\pi}{4} n_2 \right) - A_8 n_2 \cosh \left( \frac{\pi}{4} n_2 \right) &= 0 \\
-A_1 m_1^2 \cos \left( \frac{\pi}{4} m_1 \right) - A_2 m_1^2 \sin \left( \frac{\pi}{4} m_1 \right) + A_3 m_2^2 \cosh \left( \frac{\pi}{4} m_2 \right) + A_4 m_2^2 \sinh \left( \frac{\pi}{4} m_2 \right) \\
+A_5 n_1^2 \cos \left( \frac{\pi}{4} n_1 \right) + A_6 n_1^2 \sin \left( \frac{\pi}{4} n_1 \right) - A_7 n_2^2 \cosh \left( \frac{\pi}{4} n_2 \right) - A_8 n_2^2 \sinh \left( \frac{\pi}{4} n_2 \right) &= 0 \\
A_1 m_1^2 \sin \left( \frac{\pi}{4} m_1 \right) - A_2 m_1^2 \cos \left( \frac{\pi}{4} m_1 \right) + A_3 m_2^2 \sinh \left( \frac{\pi}{4} m_2 \right) + A_4 m_2^2 \cosh \left( \frac{\pi}{4} m_2 \right) \\
-A_5 n_1^2 \sin \left( \frac{\pi}{4} n_1 \right) + A_6 n_1^2 \cos \left( \frac{\pi}{4} n_1 \right) - A_7 n_2^2 \sinh \left( \frac{\pi}{4} n_2 \right) - A_8 n_2^2 \cosh \left( \frac{\pi}{4} n_2 \right) &= 0
\end{align*}
\]

\[
C_1 = \begin{bmatrix}
0 & 0 & -n_1 \sin \left( \frac{\pi}{2} n_1 \right) & n_1 \cos \left( \frac{\pi}{2} n_1 \right) & n_2 \sinh \left( \frac{\pi}{2} n_2 \right) & n_2 \cosh \left( \frac{\pi}{2} n_2 \right) \\
0 & 0 & n_1^3 \sin \left( \frac{\pi}{2} n_1 \right) & -n_1^3 \cos \left( \frac{\pi}{2} n_1 \right) & n_2^3 \sinh \left( \frac{\pi}{2} n_2 \right) & n_2^3 \cosh \left( \frac{\pi}{2} n_2 \right) \\
\cos \left( \frac{\pi}{4} m_1 \right) & \cosh \left( \frac{\pi}{4} m_2 \right) & -\cos \left( \frac{\pi}{4} n_1 \right) & -\sin \left( \frac{\pi}{4} n_1 \right) & -\cosh \left( \frac{\pi}{4} n_2 \right) & -\sinh \left( \frac{\pi}{4} n_2 \right) \\
-m_1 \sin \left( \frac{\pi}{4} m_1 \right) & m_2 \sinh \left( \frac{\pi}{4} m_2 \right) & n_1 \sin \left( \frac{\pi}{4} n_1 \right) & -n_1 \cos \left( \frac{\pi}{4} n_1 \right) & -n_2 \sinh \left( \frac{\pi}{4} n_2 \right) & -n_2 \cosh \left( \frac{\pi}{4} n_2 \right) \\
-m_1^2 \cos \left( \frac{\pi}{4} m_1 \right) & m_2^2 \cosh \left( \frac{\pi}{4} m_2 \right) & n_1^2 \cos \left( \frac{\pi}{4} n_1 \right) & n_1^2 \sin \left( \frac{\pi}{4} n_1 \right) & -n_2^2 \cosh \left( \frac{\pi}{4} n_2 \right) & -n_2^2 \sinh \left( \frac{\pi}{4} n_2 \right) \\
m_1^3 \sin \left( \frac{\pi}{4} m_1 \right) & m_2^3 \sinh \left( \frac{\pi}{4} m_2 \right) & -n_1^3 \sin \left( \frac{\pi}{4} n_1 \right) & n_1^3 \cos \left( \frac{\pi}{4} n_1 \right) & -n_2^3 \sinh \left( \frac{\pi}{4} n_2 \right) & -n_2^3 \cosh \left( \frac{\pi}{4} n_2 \right)
\end{bmatrix}
\]

\[
X_1 = [A_1 \ A_3 \ A_5 \ A_6 \ A_7 \ A_8]^T, \quad D_1 = [0 \ 0 \ 0 \ 0 \ 0 \ 0]^T
\]
If there are non-zero solutions for equation (47), the corresponding determinant is zero, that is

\[
\text{See equation (48) below.}
\]

Using equations (42) and (46), the constants \(B_j\) and \(A_j\), of mode functions can be determined. Using equation (44) and (48), natural frequencies of the flexible ring can be determined. Then, the primary mass \(M_{pi}\) of the first order can be calculated by

\[
M_{pi} = \int_0^\beta \rho[\phi_1'(\theta)]^2 \text{Ard}\theta + \int_0^{\pi/2} \rho[\phi_2'(\theta)]^2 \text{Ard}\theta
\]

(49)

Multiplying \(\phi(\theta)\) by \(M_{pi}^{-1/2}\), the normal mode function of the flexible ring is obtained.

5 Effects of friction and air damping

Under the electromagnetic force, the flexible wheel after deformation contacts with the outer ring stator at the angle range \([-\pi/4, \pi/4]\). Here, the friction force occurs. At other angle range, the air damping force is applied to the flexible wheel. Letting \(q_f\) denote friction force per unit length on the flexible wheel, \(q_p\) denote damping force per unit length on the flexible wheel.

From friction torque equation of the flexible wheel, the friction force \(q_f\), considering electromagnetic force and Van der Walls force can be given by

\[
q_f = \frac{\mu (q_{o1} Y_1 + 2 q_{o2} Y_2)}{2 Y_2} - \frac{\mu t_0 \pi EI_x}{4 r_4 Y_2} + \mu q_{o3}
\]

(50)

This friction force is expanded as a Fourier series and high order terms are neglected, the dynamic friction force can be obtained

\[
\Delta q_f = \frac{\mu A l}{2 \pi (t_0 - u_0)^4} \Delta u_p
\]

(51)

The air damping force \(q_p\) per unit length on the flexible wheel is [28]

\[
q_p = \frac{\eta \rho^3}{(t_0 - u_0)^3} \frac{\partial u}{\partial t}
\]

(52)

where \(\eta\) is the air dynamic viscosity, \(\eta = 1.86 \times 10^{-5}\) N · m⁻².

This air damping force is expanded as a Fourier series and high order terms are neglected, the dynamic air damping force can be obtained

\[
q_p = \frac{\eta \rho^3}{(t_0 - u_0)^3} \frac{\partial \Delta u}{\partial t}
\]

(53)

Considering tangent friction force, the dynamics equation (12) of the flexible wheel is changed into following form

\[
\frac{d^2 \Delta u}{d \theta^2} + \frac{2 \Delta u}{d \theta^2} + \frac{\partial \Delta u}{\partial \theta} = \frac{r^4}{E I_x} \left( \frac{\partial \Delta q_f}{\partial \theta} + \Delta q_l \right) - \frac{r^4 \rho A_s}{E I_x} \frac{\partial \Delta \dot{u}}{\partial \theta}
\]

(54)

The forces on the flexible wheel include radial one and tangent one, and the force distribution can be given by

\[
\text{See equation (55) below.}
\]

\[
\begin{vmatrix}
0 & 0 & -n_1 \sin \left( \frac{\pi}{2} n_1 \right) & n_1 \cos \left( \frac{\pi}{2} n_1 \right) & n_2 \sin \left( \frac{\pi}{2} n_2 \right) & n_2 \cos \left( \frac{\pi}{2} n_2 \right) \\
0 & 0 & n_1^3 \sin \left( \frac{\pi}{2} n_1 \right) & -n_1^3 \cos \left( \frac{\pi}{2} n_1 \right) & n_2^3 \sin \left( \frac{\pi}{2} n_2 \right) & n_2^3 \cos \left( \frac{\pi}{2} n_2 \right) \\
\cos \left( \frac{\pi}{4} m_1 \right) & \sin \left( \frac{\pi}{4} m_2 \right) & \cos \left( \frac{\pi}{4} n_1 \right) & -\sin \left( \frac{\pi}{4} n_1 \right) & -\sin \left( \frac{\pi}{4} n_2 \right) & \sin \left( \frac{\pi}{4} n_2 \right) \\
-\sin \left( \frac{\pi}{4} m_1 \right) & -\cos \left( \frac{\pi}{4} m_2 \right) & \sin \left( \frac{\pi}{4} n_1 \right) & -\cos \left( \frac{\pi}{4} n_1 \right) & -\cos \left( \frac{\pi}{4} n_2 \right) & -\cos \left( \frac{\pi}{4} n_2 \right) \\
-m_1 \sin \left( \frac{\pi}{4} m_1 \right) & m_2 \cos \left( \frac{\pi}{4} m_2 \right) & \sin \left( \frac{\pi}{4} n_1 \right) & -\cos \left( \frac{\pi}{4} n_1 \right) & -\sin \left( \frac{\pi}{4} n_2 \right) & -\cos \left( \frac{\pi}{4} n_2 \right) \\
m_1^3 \sin \left( \frac{\pi}{4} m_1 \right) & m_2^3 \cos \left( \frac{\pi}{4} m_2 \right) & -n_1^3 \sin \left( \frac{\pi}{4} n_1 \right) & n_1^3 \cos \left( \frac{\pi}{4} n_1 \right) & -n_2^3 \sin \left( \frac{\pi}{4} n_2 \right) & -n_2^3 \cos \left( \frac{\pi}{4} n_2 \right)
\end{vmatrix} = 0
\]

(48)

\[
\begin{align*}
\Delta q_r &= \frac{A l}{2 \pi (t_0 - u_0)^4} \Delta u_p \\
\Delta q_i &= \frac{\mu A l}{2 \pi (t_0 - u_0)^4} \Delta u_p \\
\Delta q_r &= -\frac{\eta \rho^3}{(t_0 - u_0)^3} \frac{\partial \Delta u_p}{\partial t} \\
\Delta q_i &= 0
\end{align*}
\]

(55)
Combining equation (54) with (55), yields

\[ \Delta u = \phi(\theta)q(t) \quad (57) \]

Substituting equation (57) into (56), yields:

\[ \text{See equation (56) below.} \]

Letting

\[ \lambda = \phi(\theta)q(t) \]

The four eigenvalues can be obtained

\[ (\pm \sqrt{-1+1-V} \text{ and } \pm i \sqrt{1+1-V}), \]

and then the mode function can be given by

\[ \phi_{p2}(\theta) = C_0 \cos \alpha_1 \theta + C_1 \sin \alpha_1 \theta + C_2 \sin \alpha_2 \theta \]

where \( \alpha_1 = \sqrt{1+1-V} \) and \( \alpha_2 = \sqrt{-1+1-V} \)

From symmetry and continuity conditions of the flexible ring, both angle \( \phi \), tangent displacement \( u \), and shear force are zero at \( \theta = 0 \) and \( \theta = \frac{\pi}{2} \), so

\[ \begin{align*}
\phi_{p1}(\theta)_{\theta=0} &= 0 \\
\phi_{p2}(\theta)_{\theta=\pi/2} &= 0 \\
\phi_{p1}(\theta)_{\theta=\pi/2} &= 0 \\
\phi_{p2}(\theta)_{\theta=\pi/2} &= 0 
\end{align*} \quad (65) \]

From continuity conditions of the flexible ring, following relationship can be given at \( \theta = \frac{\pi}{4} \)

\[ \begin{align*}
\phi_{p1}(\theta) &= \phi_{p2}(\theta) \\
\phi_{p1}'(\theta) &= \phi_{p2}'(\theta) \\
\phi_{p1}'(\theta) &= \phi_{p2}'(\theta) \\
\phi_{p1}(\theta) &= \phi_{p2}(\theta) \\
\phi_{p1}(\theta) &= \phi_{p2}(\theta) 
\end{align*} \quad (66) \]

where \( V = 1 - \frac{\rho A_1 r_1}{E I_x} \omega^2 \)

Letting \( \phi_{p2}(\theta) = \cos \beta \) and substituting it into (62), yields

\[ \lambda^4 + 2\lambda^2 + V = 0 \quad (63) \]

\[ \begin{align*}
\frac{\partial^5 \Delta u_{1p}}{\partial \theta^5} + 2 \frac{\partial^3 \Delta u_{1p}}{\partial \theta^3} + \frac{\partial \Delta u_{1p}}{\partial \theta} &= \frac{r^4}{E I_x} \left( \frac{A_l}{2\pi(t_0 - w_0)^4} \frac{\partial \Delta u_{1p}}{\partial \theta} + \frac{\mu A_l}{2\pi(t_0 - w_0)^4} \Delta u_{1p} \right) - \frac{r^4 \rho A_s \partial \Delta u_{1p}}{E I_x} \frac{\partial \theta}{\theta} \quad (0 \leq \theta \leq \frac{\pi}{4}) \quad (56) \\
\frac{\partial^4 \Delta u_{2p}}{\partial \theta^4} + 2 \frac{\partial^2 \Delta u_{2p}}{\partial \theta^2} + \Delta u_{2p} &= \frac{r^4}{E I_x} \left( \frac{\eta^3}{(t_0 - w_0)^3} \frac{\partial \Delta u_{2p}}{\partial t} - \frac{r^4 \rho A_s \Delta u_{2p}}{E I_x} \frac{\partial \theta}{\theta} \quad \left( \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2} \right) 
\end{align*} \]

\[ \begin{align*}
\dot{q}(t) &= \frac{\phi_{p1}(\theta) + 2\phi_{p1}'(\theta) + (1 - T)\phi_{p1}(\theta) - \mu T \phi_{p1}(\theta)}{\rho A_s r^4 \frac{\partial}{\partial \theta} \phi_{p1}(\theta)} \quad (0 \leq \theta \leq \frac{\pi}{4}) \quad (58 - a) \\
\ddot{q}(t) + \frac{\eta^3}{(t_0 - w_0)^3} \dot{q}(t) &= \frac{\phi_{p1}(\theta) + 2\phi_{p1}'(\theta) + \phi_{p1}(\theta)}{\rho A_s r^4 \frac{\partial}{\partial \theta} \phi_{p1}(\theta)} \quad \left( \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2} \right) \quad (58 - b) 
\end{align*} \]
Substituting equations (61) and (64) into (65) and (66), we have:

\[ C_1 r_1 + C_2 r_2 + C_3 r_3 + C_4 r_4 + C_5 r_5 = 0 \]

\[ -C_6 \alpha_1 \sin \left( \frac{\pi}{2} \alpha_1 \right) + C_7 \alpha_1 \cos \left( \frac{\pi}{2} \alpha_1 \right) + C_8 \alpha_2 \sin \left( \frac{\pi}{2} \alpha_2 \right) + C_9 \alpha_2 \cos \left( \frac{\pi}{2} \alpha_2 \right) = 0 \]

Equation (67) can be changed into a following form:

\[ C X_1 = D_1 \]
where

\[
C = \begin{bmatrix}
  r_1 & r_2 & r_3 & r_4 & r_5 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & -\alpha_1 \sin\left(\frac{\pi}{2} \alpha_1\right) & \alpha_1 \cos\left(\frac{\pi}{2} \alpha_1\right) & \alpha_2 \sinh\left(\frac{\pi}{2} \alpha_2\right) & \alpha_2 \cosh\left(\frac{\pi}{2} \alpha_2\right) \\
  r_1^2 & H_{11} & H_{12} & H_{13} & H_{14} & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & -\alpha_1^2 \sin\left(\frac{\pi}{2} \alpha_1\right) & -\alpha_1^2 \cos\left(\frac{\pi}{2} \alpha_1\right) & \alpha_2^2 \sinh\left(\frac{\pi}{2} \alpha_2\right) & \alpha_2^2 \cosh\left(\frac{\pi}{2} \alpha_2\right) \\
  \frac{\pi}{e^4} r_1 & H_{21} & H_{22} & H_{23} & H_{24} & -\cos\left(\frac{\pi}{4} \alpha_1\right) & -\sin\left(\frac{\pi}{4} \alpha_1\right) & -\alpha_1^2 \cosh\left(\frac{\pi}{4} \alpha_2\right) & -\alpha_2^2 \cosh\left(\frac{\pi}{4} \alpha_2\right) \\
  r_1^2 e^4 & H_{31} & H_{32} & H_{33} & H_{34} & \alpha_1 \sin\left(\frac{\pi}{4} \alpha_1\right) & -\alpha_1 \cos\left(\frac{\pi}{4} \alpha_1\right) & -\alpha_2 \sinh\left(\frac{\pi}{4} \alpha_2\right) & -\alpha_2 \cosh\left(\frac{\pi}{4} \alpha_2\right) \\
  \frac{\pi}{e^4} r_1 & H_{41} & H_{42} & H_{43} & H_{44} & \alpha_1^2 \cos\left(\frac{\pi}{4} \alpha_1\right) & \alpha_1^2 \sin\left(\frac{\pi}{4} \alpha_1\right) & -\alpha_2^2 \cosh\left(\frac{\pi}{4} \alpha_2\right) & -\alpha_2^2 \cosh\left(\frac{\pi}{4} \alpha_2\right) \\
  r_1^2 e^4 & H_{51} & H_{52} & H_{53} & H_{54} & -\alpha_1^3 \sin\left(\frac{\pi}{4} \alpha_1\right) & \alpha_1^3 \cos\left(\frac{\pi}{4} \alpha_1\right) & -\alpha_2^3 \sinh\left(\frac{\pi}{4} \alpha_2\right) & -\alpha_2^3 \cosh\left(\frac{\pi}{4} \alpha_2\right) \\
  r_1^3 e^4 & H_{61} & H_{62} & H_{63} & H_{64} & -\alpha_1^4 \cos\left(\frac{\pi}{4} \alpha_1\right) & \alpha_1^4 \sin\left(\frac{\pi}{4} \alpha_1\right) & -\alpha_2^4 \sinh\left(\frac{\pi}{4} \alpha_2\right) & -\alpha_2^4 \cosh\left(\frac{\pi}{4} \alpha_2\right)
\end{bmatrix}
\]

Here

\[
H_{61} = e^{x_2} \left( (4r_2 r_3^2 - 4r_2 r_3^2) \sin\left(\frac{\pi}{4} r_3\right) + (r_2^2 - 6r_2 r_3^2 + r_3^4) \cos\left(\frac{\pi}{4} r_3\right) \right),
\]

\[
H_{62} = e^{x_2} \left( (r_2^3 - 6r_2^2 r_3^2 + r_3^4) \sin\left(\frac{\pi}{4} r_3\right) + (4r_2^2 r_3^2 - 4r_2 r_3^2) \cos\left(\frac{\pi}{4} r_3\right) \right),
\]

\[
H_{63} = e^{x_4} \left( (4r_4 r_3^2 - 4r_4 r_3^2) \sin\left(\frac{\pi}{4} r_3\right) + (r_4^2 - 6r_4^2 r_3^2 + r_3^4) \cos\left(\frac{\pi}{4} r_3\right) \right),
\]

\[
H_{64} = e^{x_4} \left( (r_4^2 - 6r_4^2 r_3^2 + r_3^4) \sin\left(\frac{\pi}{4} r_3\right) + (4r_4^2 r_3^2 - 4r_4 r_3^2) \cos\left(\frac{\pi}{4} r_3\right) \right),
\]

\[
H_{51} = e^{x_2} \left( (r_3^2 - 3r_2^2 r_3) \sin\left(\frac{\pi}{4} r_3\right) + (r_2^3 - 3r_2 r_3^2) \cos\left(\frac{\pi}{4} r_3\right) \right),
\]

\[
H_{52} = e^{x_2} \left( (r_2^3 - 3r_2 r_3^2) \sin\left(\frac{\pi}{4} r_3\right) + (3r_2^2 r_3 - r_3^3) \cos\left(\frac{\pi}{4} r_3\right) \right),
\]

\[
H_{53} = e^{x_4} \left( (r_3^2 - 3r_2^2 r_3) \sin\left(\frac{\pi}{4} r_3\right) + (r_3^3 - 3r_2 r_3^2) \cos\left(\frac{\pi}{4} r_3\right) \right),
\]

\[
H_{54} = e^{x_4} \left( (r_3^2 - 3r_2^2 r_3) \sin\left(\frac{\pi}{4} r_3\right) + (3r_2^2 r_3 - r_3^3) \cos\left(\frac{\pi}{4} r_3\right) \right),
\]

\[
H_{41} = e^{x_2} \left( (r_2^2 - r_3^2) \cos\left(\frac{\pi}{4} r_3\right) - 2r_2 r_3 \sin\left(\frac{\pi}{4} r_3\right) \right),
\]

\[
H_{42} = e^{x_2} \left( (r_2^2 - r_3^2) \sin\left(\frac{\pi}{4} r_3\right) + 2r_2 r_3 \cos\left(\frac{\pi}{4} r_3\right) \right).
\]
If there are non-zero solutions for equation (68), the determinant is zero, that is

\( \text{See equation (69) below.} \)

Using equation (68), the constants \( C_j \) of mode functions can be determined. Using equation (69), natural frequencies of the flexible ring can be determined. Substituting the natural frequencies into following equation

\[ \omega_p = \omega \sqrt{1 - \xi^2} \quad (70) \]

where \( \xi = \frac{n^2}{2\alpha(b_0 - w_0)\rho A_s} \).
From equation (70), the effects of the air damping on the natural frequencies can be given.

6 Results and discussion

Using equations given in this paper, the free vibration of the micro electromagnetic harmonic drive system is investigated. Parameters of the system are shown in Table 1.

Table 1. Parameters of the system.

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| $N$        | 200    | $R_1$ (mm) | 10     |
| $x_0$ (mm) | 10     | $t_0$ (μm) | 0.5    |
| $r$ (mm)   | 1.4    | $l$ (mm)   | 0.3    |
| $d$ (μm)   | 50     | $E$ (MPa)  | 95     |

Table 2. Changes of natural frequency with $t_0$ (rad/s).

| $t_0$/μm | 0.4 | 0.5 | 0.7 | 0.8 | 1  | 2  |
|----------|-----|-----|-----|-----|----|----|
| Order one |     |     |     |     |    |    |
| $ω_1$    | 6986.3 | 6986.3 | 6986.3 | 6986.3 | 6986.3 | 6986.3 |
| $ω_3$    | 2328.7 | 4206.9 | 5688.3 | 6081.9 | 6554 | 6956.6 |
| $ω_3 - ω_1$ | 4657.6 | 2279.4 | 1298 | 904.4 | 432.3 | 29.7 |
| $ω_c$    | 34931.3 | 34931.3 | 34931.3 | 34931.3 | 34931.3 | 34931.3 |
| Order two |     |     |     |     |    |    |
| $ω_1$    | 31041.3 | 33379.2 | 34532.5 | 34698 | 34835.9 | 34925.3 |
| $ω_3 - ω_1$ | 3890 | 1552.1 | 398.8 | 233.3 | 95.4 | 6 |
| $ω_c$    | 81506.3 | 81506.3 | 81506.3 | 81506.3 | 81506.3 | 81506.3 |
| Order three |    |     |     |     |    |    |
| $ω_1$    | 79937.4 | 80857.2 | 81336.5 | 81406.7 | 81465.5 | 81503.8 |
| $ω_3 - ω_1$ | 1568.9 | 649.1 | 169.8 | 99.6 | 40.8 | 2.5 |
| $ω_c$    | 146711.4 | 146711.4 | 146711.4 | 146711.4 | 146711.4 | 146711.4 |
| Order four |     |     |     |     |    |    |
| $ω_1$    | 145823.2 | 146348 | 146616.9 | 146656 | 146688.7 | 146710 |
| $ω_3 - ω_1$ | 888.2 | 363.4 | 94.5 | 55.4 | 22.7 | 1.4 |

Table 3. Changes of natural frequency with $r$ (rad/s).

| $r$/mm | 0.8 | 1 | 1.2 | 1.4 | 1.7 |
|--------|-----|---|-----|-----|-----|
| Order one |     |   |     |     |    |
| $ω_1$    | 21395.4 | 13693.1 | 9509.1 | 6986.3 | 4738.1 |
| $ω_3$    | 19300.8 | 11149 | 6854.8 | 4206.9 | 949.9 |
| $ω_3 - ω_1$ | 2094.6 | 2544.1 | 2654.3 | 2279.4 | 3788.2 |
| $ω_c$    | 106977.1 | 68465.3 | 47545.4 | 34931.3 | 23690.4 |
| Order two |     |   |     |     |    |
| $ω_1$    | 106478.2 | 67683.8 | 46414.4 | 33379.2 | 21352.4 |
| $ω_3 - ω_1$ | 498.9 | 781.5 | 1131 | 1552.1 | 2338 |
| $ω_c$    | 249613.1 | 159752.4 | 110939.2 | 81506.3 | 55277.6 |
| Order three |    |     |     |     |    |
| $ω_1$    | 249399.9 | 159419.5 | 110460.8 | 80857.2 | 54328.3 |
| $ω_3 - ω_1$ | 213.2 | 332.9 | 478.4 | 649.1 | 949.3 |
| $ω_c$    | 449303.7 | 287554.3 | 199690.5 | 146711.4 | 99499.8 |
| Order four |     |     |     |     |    |
| $ω_1$    | 449185.1 | 287369.1 | 199423.6 | 146348 | 98963.4 |
| $ω_3 - ω_1$ | 118.6 | 185.2 | 266.9 | 363.4 | 536.4 |

From equation (70), the effects of the air damping on the natural frequencies can be given.

6 Results and discussion

Using equations given in this paper, the free vibration of the micro electromagnetic harmonic drive system is investigated. Parameters of the system are shown in Table 1. Table 1 shows that the clearance between the flexible ring and stator is 500 nm and the effects of the Van der Walls force on the natural frequencies are quite significant and should be considered. Tables 2–4 give the first four orders of the natural frequencies and their changes along with system parameters. Results show:
- Considering molecule force, the natural frequencies of the flexible ring are decreased. With decreasing clearance
between the flexible ring and stator, the natural frequencies of the flexible ring are decreased more rapidly. This is due to the increased effects of electromagnetic force and molecular force on the flexible ring, resulting in the stiffness reduction of the coupling system.

As clearance between the flexible ring and stator is relatively large ($t_0 > 1 \mu m$), the decrease of the natural frequencies of the flexible ring is not obvious. At $t_0 = 1 \mu m$, the relative error between the first order of the natural frequencies with and without considering molecule force is equal to $(\omega_{0} - \omega_{0})/\omega_{0} = 6.2\%$. At $t_0 = 2 \mu m$, the relative error is $(\omega_{0} - \omega_{0})/\omega_{0} = 0.4\%$. At $t_0 = 0.8 \mu m$, the relative error between natural frequencies with and without considering molecule force is equal to: $(\omega_{0} - \omega_{0})/\omega_{0} = 12.9\%$. At $t_0 = 0.5 \mu m$ and $t_0 = 0.4 \mu m$, the relative error between natural frequencies is equal to: $(\omega_{0} - \omega_{0})/\omega_{0} = 32.6\%$. $(t_0 = 0.5 \mu m)$ and $t_0 = 0.4 \mu m$, and $(\omega_{0} - \omega_{0})/\omega_{0} = 66.7\%$. $(t_0 = 0.4 \mu m)$.

As the order number of the vibration modes increases, effects of the molecule force on the natural frequencies of the flexible ring becomes weak as well. For example, at $t_0 = 0.4 \mu m$, the relative error between the second order of the natural frequencies with and without considering molecule force is reduced to be 11.1%; the relative error between the third order of the natural frequencies is reduced to be 1.9%; and the relative error between the fourth order of the natural frequencies is reduced to be 0.6%.

With increasing radius of the flexible ring, its natural frequencies are decreased. For a relatively large radius of the flexible ring, the relative error between the natural frequencies with and without considering molecule force becomes large. It shows that effects of the molecule force on the natural frequencies of the flexible ring increases with increasing radius of the flexible ring. As the order number of the vibration modes increases, effects of the molecule force on relationship between the natural frequencies and radius of the flexible ring becomes weak.

At radius $r = 1.4 \text{ mm}$ decrease of the natural frequencies caused by molecule force is 6.2% for mode one, 0.3% for mode two, 0.05% for mode three, and 0.02% for mode 4.

With increasing thickness of the flexible ring, the natural frequencies of the flexible ring are increased. For a relatively large thickness of the flexible ring, the relative error between the natural frequencies with and without considering molecule force becomes small. It shows that effects of the molecule force on the natural frequencies of the flexible ring decreases with increasing thickness of the flexible ring. As the order number of the vibration modes increases, effects of the molecule force on relationship between the natural frequencies and thickness of the flexible ring becomes weak. At thickness $d = 40 \text{ mm}$, decrease of the natural frequencies caused by molecule force is 66.7% for mode one, 8.8% for mode two, 1.55% for mode three, and 0.48% for mode 4.

In a word, in dynamics performance design of the drive system, to determine its natural frequency accurately, the effects of the molecule force on the natural frequencies should be considered for smaller clearance between the flexible ring and stator, smaller thickness of the flexible ring and larger radius of the flexible ring.

Substituting above natural frequencies into equations (32), (36) and (40), the first four orders of the vibration modes can be obtained (see Fig. 3). It shows:

- As the molecule force is considered, for mode 1, at $\theta = 0$ and $\theta = \pi$, the amplitudes of the flexible ring vibrations decrease; at $\theta = \pi/2$ and $\theta = 3\pi/2$, the amplitudes of the flexible ring vibrations increase obviously. It is because there is larger molecule force near $\theta = 0$ and $\theta = \pi$ than other places.

- As the order number of the vibration modes increases, effects of the molecule force on the vibration modes of the flexible ring becomes weak as well. For mode 1, difference between modes with and without molecule force is 0.8376; for mode 2, 3 and 4, the difference between modes are 0.0464, 0.0168, and 0.0025, respectively.

### Table 4. Changes of natural frequency with $d$ (rad/s).

| $d/\mu m$ | $\omega_1$ | $\omega_2$ | $\omega_3$ | $\omega_4$ |
|-----------|-----------|-----------|-----------|-----------|
| 30        | 4191.8    | 5589      | 6986.3    | 11178     | 13972.5   | 20958.8   |
| 40        | 1954.1    | 1863      | 4206.9    | 9182.3    | 12418.9   | 20157.8   |
| 50        | 3237.7    | 3726      | 2279.4    | 1995.7    | 1553.6    | 801       |
| 80        | 20958.8   | 27945     | 34931.3   | 55890.1   | 69862.6   | 104793.9  |
| 100       |           |           |           |           |           |           |
| 150       |           |           |           |           |           |           |

- As clearance between the flexible ring and stator is relatively large ($t_0 > 1 \mu m$), the decrease of the natural frequencies of the flexible ring is not obvious. At $t_0 = 1 \mu m$, the relative error between the first order of the natural frequencies with and without considering molecule force is equal to $(\omega_{0} - \omega_{0})/\omega_{0} = 6.2\%$. At $t_0 = 2 \mu m$, the relative error is $(\omega_{0} - \omega_{0})/\omega_{0} = 0.4\%$. At $t_0 = 0.8 \mu m$, the relative error between natural frequencies with and without considering molecule force is equal to: $(\omega_{0} - \omega_{0})/\omega_{0} = 12.9\%$. At $t_0 = 0.5 \mu m$ and $t_0 = 0.4 \mu m$, the relative error between natural frequencies is equal to: $(\omega_{0} - \omega_{0})/\omega_{0} = 32.6\%$. $(t_0 = 0.5 \mu m)$ and $t_0 = 0.4 \mu m$, and $(\omega_{0} - \omega_{0})/\omega_{0} = 66.7\%$. $(t_0 = 0.4 \mu m)$.

- As the order number of the vibration modes increases, effects of the molecule force on the natural frequencies of the flexible ring becomes weak as well. For example, at $t_0 = 0.4 \mu m$, the relative error between the second order of the natural frequencies with and without considering molecule force is reduced to be 11.1%; the relative error between the third order of the natural frequencies is reduced to be 1.9%; and the relative error between the fourth order of the natural frequencies is reduced to be 0.6%.

- With increasing radius of the flexible ring, its natural frequencies are decreased. For a relatively large radius of the flexible ring, the relative error between the natural frequencies with and without considering molecule force becomes large. It shows that effects of the molecule force on the natural frequencies of the flexible ring increases with increasing radius of the flexible ring. As the order number of the vibration modes increases, effects of the molecule force on relationship between the natural frequencies and radius of the flexible ring becomes weak.
From mode analysis, it can be known that effects of the molecule force on the vibration modes are obvious only for modes 1 and 2. Here, for modes 1 and 2, effects of the molecule force on relationship between the vibration modes and other parameters are investigated (see Figs. 4–6). Results show:

– As the clearance between the flexible ring and stator is relatively small, effects of the molecule force on vibration amplitudes of the flexible ring become more obvious. At the clearance $t_0 = 0.4 \mu m$, the difference between the maximum vibration amplitudes for mode one with and without considering molecule force is 2.287. At the clearance $t_0 = 0.5 \mu m$, $t_0 = 1 \mu m$ and $t_0 = 2 \mu m$, the difference between the maximum vibration amplitudes is 1.700, 0.164 and 0.008, respectively. For mode two, the difference between the maximum vibration amplitudes is 0.136 for $t_0 = 0.4$, 0.047 for, 0.0002 for $t_0 = 2 \mu m$.

– As the radius $r$ of the flexible ring increases (from 0.8 to 1.7 mm), vibration amplitudes of the flexible ring increase as well. Near $\theta = \pi/2$ and $\theta = 3\pi/2$, vibration amplitudes of the flexible ring increase are increased more obviously. Near $\theta = \pi/2$ and $\theta = 3\pi/2$, the difference between the maximum vibration amplitudes with and without considering molecule force is large as well. For mode 1, the difference between the maximum vibration amplitudes with and without considering molecule force is 0.2861 at $r = 0.8$ mm, 0.69 at $r = 1$ mm, and 2.045 at $r = 1.7$ mm. For mode 2, the difference between the maximum vibration amplitudes with and without considering molecule force is 0.005 at $r = 0.8$ mm, 0.012 at $r = 1$ mm, and 0.112 at $r = 1.7$ mm.

The results show that effects of the molecule force on vibration amplitudes increases with increasing the radius $r$ of the flexible ring.

– As the thickness $d$ of the flexible ring increases (from $30 \mu m$ to $150 \mu m$), vibration amplitudes of the flexible ring decrease. As the thickness $d$ of the flexible ring increases, effects of the molecule force on vibration amplitudes of the flexible ring become weak. For mode 1, the difference between the maximum vibration amplitudes with and without considering molecule force is 2.509 at $d = 30 \mu m$, 1.7 at $d = 50 \mu m$, and 0.09547 at $d = 150 \mu m$. For mode 2, the difference between the maximum vibration amplitudes with and without considering molecule force is 0.3066 at $d = 30 \mu m$, 0.04698 at $d = 150 \mu m$, and 0.0056 at $d = 150 \mu m$. 

Fig. 3. Differences between modes with and without molecule force. (a) Mode 1 (b) mode 2 (c) mode 3 (d) mode 4.
It can be seen that effects of the molecule force on the maximum vibration amplitudes increases with decreasing the thickness $d$ of the flexible ring and order number of the modes.

Using equations (69) and (70), the effects of the friction force and air damping on the natural frequencies are investigated. The parameters of the drive system are shown in Table 1. The effects of the friction force are given in Tables 5–8. The effects of the damping are given in Tables 9–12. Results show:

- When considering friction force, the natural frequencies of the flexible ring are decreased. With increasing order number of the modes, the effects of the friction force on the natural frequencies reduce rapidly and the effects can be neglected when the order number of the modes is above 2.
- When the clearance between the flexible ring and stator is decreased, the effects of the friction force on the natural frequencies become more significant. When the radius of the flexible ring is increased, the effects of the friction force on the natural frequencies decrease.
- When the thickness of the flexible ring increases, the frequency difference $(\omega_3-\omega_1)$ increases. This shows that the effects of the friction force on the natural frequencies become more significant for large thickness of the flexible ring. When the current in the coils increases, the frequency difference $(\omega_3-\omega_1)$ decreases. This shows that the effects of the friction force on the natural frequencies reduce for large coil current.
- When considering air damping force, the natural frequencies of the flexible ring are also decreased. With increasing order number of the modes, the effects of the air damping force on the natural frequencies reduce as well.
- When the clearance between the flexible ring and stator is decreased, the frequency difference $(\omega_3-\omega_1)$ increases which shows that the effects of the air damping force on the natural frequencies increase with decreasing the clearance.
- When the radius of the flexible ring is increased, the frequency difference $(\omega_3-\omega_1)$ increases which shows that the effects of the air damping force on the natural frequencies increase with decreasing the radius.
- When the thickness of the flexible ring increases, the frequency difference $(\omega_3-\omega_1)$ decreases which shows that...
Table 5. Changes of natural frequency with $t_0$ with and without friction force (rad/s).

| $t_0/\mu m$ | 0.3 | 0.4 | 0.5 | 0.6 | 0.8 | 1  |
|------------|-----|-----|-----|-----|-----|----|
| $\omega_3$ | 5711.9 | 6634.5 | 6848.1 | 6921 | 6966 | 6978.1 |
| $\omega_t$ | 2328.7 | 2454.9 | 2372.4 | 2348.6 | 2334.8 | 2331.2 |
| $\omega_3 - \omega_t$ | 3383.2 | 4179.6 | 4475.7 | 4572.4 | 4631.2 | 4646.9 |
| $\omega_3$ | 34544.2 | 34855.3 | 34902.9 | 34918.1 | 34927.2 | 34929.6 |
| \begin{tabular}{l}
Order 2
\end{tabular} | \begin{tabular}{l}
$\omega_t$
\end{tabular} | 34544.2 | 34855.3 | 34902.9 | 34918.1 | 34927.2 |
| $\omega_3 - \omega_t$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_3$ | 81341.5 | 81473.8 | 81494.2 | 81500.7 | 81504.6 | 81505.6 |
| \begin{tabular}{l}
Order 3
\end{tabular} | \begin{tabular}{l}
$\omega_t$
\end{tabular} | 81341.5 | 81473.8 | 81494.2 | 81500.7 | 81504.6 |
| $\omega_3 - \omega_t$ | 0 | 0 | 0 | 0 | 0 | 0 |

Table 6. Changes of natural frequency with $r$ with and without friction force (rad/s).

| $r/mm$ | 0.8 | 1 | 1.2 | 1.3 | 1.4 | 1.45 |
|--------|-----|---|-----|-----|-----|------|
| $\omega_3$ | 21069.1 | 13163.8 | 8863.7 | 7094.5 | 5711.9 | 4988.4 |
| $\omega_t$ | 7233.2 | 4744.6 | 3522.8 | 3288.4 | 3288.4 | 3170.9 |
| $\omega_3 - \omega_t$ | 13835.9 | 8419.2 | 5170.9 | 3806.1 | 3383.2 | 2817.5 |
| $\omega_3$ | 106910.4 | 68353.2 | 47356.4 | 40255.5 | 34544.2 | 32007.7 |
| \begin{tabular}{l}
Order 2
\end{tabular} | \begin{tabular}{l}
$\omega_t$
\end{tabular} | 106910.4 | 68353.2 | 47356.4 | 40255.5 | 34544.2 |
| $\omega_3 - \omega_t$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_3$ | 249584.6 | 159704.4 | 110858.4 | 94418.5 | 81341.5 | 75745.5 |
| \begin{tabular}{l}
Order 3
\end{tabular} | \begin{tabular}{l}
$\omega_t$
\end{tabular} | 249584.6 | 159704.4 | 110858.4 | 94418.5 | 81341.5 |
| $\omega_3 - \omega_t$ | 0 | 0 | 0 | 0 | 0 | 0 |

Table 7. Changes of natural frequency with $d$ with and without friction force (rad/s).

| $d/\mu m$ | 48 | 50 | 60 | 80 | 100 | 150 |
|----------|----|----|----|----|-----|-----|
| $\omega_3$ | 5200.3 | 5711.9 | 7605.4 | 10774.3 | 13721.5 | 20849.6 |
| $\omega_t$ | 2235.6 | 2328.7 | 3147.1 | 3862.0 | 4736.2 | 7019.0 |
| $\omega_3 - \omega_t$ | 2964.7 | 3383.2 | 4458.3 | 6912.3 | 8985.3 | 13830.6 |
| $\omega_3$ | 33002.9 | 34544.2 | 41734.4 | 55804.9 | 69811.1 | 104771.9 |
| \begin{tabular}{l}
Order 2
\end{tabular} | \begin{tabular}{l}
$\omega_t$
\end{tabular} | 33002.9 | 34544.2 | 41734.4 | 55804.9 | 69811.1 |
| $\omega_3 - \omega_t$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_3$ | 78020.4 | 81341.5 | 97729.3 | 130373.7 | 162990.6 | 244509.6 |
| \begin{tabular}{l}
Order 3
\end{tabular} | \begin{tabular}{l}
$\omega_t$
\end{tabular} | 78020.4 | 81341.5 | 97729.3 | 130373.7 | 162990.6 |
| $\omega_3 - \omega_t$ | 0 | 0 | 0 | 0 | 0 | 0 |

Table 8. Changes of natural frequency with $I$ with and without friction force (rad/s).

| $I/A$ | 0.05 | 0.1 | 0.13 | 0.15 | 0.16 |
|-------|------|-----|------|------|------|
| $\omega_3$ | 5850.7 | 5711.9 | 5546.1 | 5347 | 5125.5 |
| $\omega_t$ | 2328.7 | 2328.7 | 2328.7 | 2328.7 | 2328.7 |
| $\omega_3 - \omega_t$ | 3522.0 | 3383.2 | 3217.4 | 3018.3 | 2796.8 |
| $\omega_3$ | 34608.2 | 34544.2 | 34456.1 | 34332.1 | 34171.1 |
| \begin{tabular}{l}
Order 2
\end{tabular} | \begin{tabular}{l}
$\omega_t$
\end{tabular} | 34608.2 | 34544.2 | 34456.1 | 34332.1 | 34171.1 |
| $\omega_3 - \omega_t$ | 0 | 0 | 0 | 0 | 0 |
| $\omega_3$ | 81368.6 | 81341.5 | 81304.2 | 81251.9 | 81184.4 |
| \begin{tabular}{l}
Order 3
\end{tabular} | \begin{tabular}{l}
$\omega_t$
\end{tabular} | 81368.6 | 81341.5 | 81304.2 | 81251.9 | 81184.4 |
| $\omega_3 - \omega_t$ | 0 | 0 | 0 | 0 | 0 |
Table 9. Changes of natural frequency with $t_0$ with and without damping (rad/s).

| $t_0/\mu$m | 0.3   | 0.4   | 0.5   | 0.6   | 0.8   | 1    |
|------------|-------|-------|-------|-------|-------|------|
| $\omega_t$ | 2328.7| 2454.9| 2372.4| 2348.6| 2334.8| 2331.2|
| $\omega_p$ | 2312.4| 2451.9| 2371.2| 2348.0| 2334.6| 2331.1|
| $\omega_t - \omega_p$ | 16.3  | 3.0   | 1.2   | 0.6   | 0.2   | 0.1  |
| $\omega_t$ | 34544.2| 34855.3| 34902.9| 34918.1| 34927.2| 34929.6|
| $\omega_p$ | 34543.1| 34855.1| 34902.8| 34918.1| 34927.2| 34929.6|
| $\omega_t - \omega_p$ | 1.1   | 0.2   | 0.1   | 0     | 0     | 0    |
| $\omega_t$ | 81341.5| 81473.8| 81494.2| 81500.7| 81504.6| 81505.6|
| $\omega_p$ | 81341.0| 81473.7| 81494.2| 81500.7| 81504.6| 81505.6|
| $\omega_t - \omega_p$ | 0.5   | 0.1   | 0     | 0     | 0     | 0    |

Table 10. Changes of natural frequency with $r$ with and without damping (rad/s).

| $r/$mm | 0.8 | 1   | 1.2  | 1.3  | 1.4  | 1.45 |
|--------|-----|-----|------|------|------|------|
| $\omega_t$ | 7233.2| 4744.6| 3522.8| 3288.4| 2328.7| 2170.9|
| $\omega_p$ | 7230.4| 4740.0| 3515.6| 3279.5| 2312.4| 2147.4|
| $\omega_t - \omega_p$ | 2.8   | 4.6   | 7.2   | 8.9   | 16.3  | 23.5 |
| $\omega_t$ | 106910.4| 68353.2| 47356.4| 40255.5| 34544.2| 32007.7|
| $\omega_p$ | 106910.2| 68352.9| 47355.9| 40254.8| 34543.1| 32006.1|
| $\omega_t - \omega_p$ | 0.2   | 0.3   | 0.5   | 0.7   | 1.1   | 1.6  |
| $\omega_t$ | 249584.6| 159704.4| 110858.4| 94418.5| 81341.5| 75745.5|
| $\omega_p$ | 249584.5| 159704.3| 110858.2| 94418.2| 81341.0| 75744.8|
| $\omega_t - \omega_p$ | 0.1   | 0.1   | 0.2   | 0.3   | 0.5   | 0.7  |

Table 11. Changes of natural frequency with $d$ with and without damping (rad/s).

| $d/$mm | 48   | 50   | 60   | 80   | 100  | 150  |
|--------|------|------|------|------|------|------|
| $\omega_t$ | 2235.6| 2283.7| 3147.1| 3862.0| 4736.2| 7019.0|
| $\omega_p$ | 2212.2| 2312.4| 3141.4| 3859.8| 4735.1| 7018.7|
| $\omega_t - \omega_p$ | 23.4  | 16.3  | 5.7   | 2.2   | 1.1   | 0.3  |
| $\omega_t$ | 33002.9| 34544.2| 41734.4| 55804.9| 69811.1| 104771.9|
| $\omega_p$ | 33001.3| 34543.1| 41734.0| 55804.7| 69811.0| 104771.9|
| $\omega_t - \omega_p$ | 1.6   | 1.1   | 0.4   | 0.2   | 0.1   | 0    |
| $\omega_t$ | 78020.4| 81341.5| 97729.3| 130373.7| 162990.6| 244509.6|
| $\omega_p$ | 78019.7| 81341.0| 97729.1| 130373.6| 162990.6| 244509.6|
| $\omega_t - \omega_p$ | 0.7   | 0.5   | 0.2   | 0.1   | 0     | 0    |

Table 12. Changes of natural frequency with $I$ with and without damping (rad/s).

| $I/A$ | 0.05 | 0.1  | 0.13 | 0.15 | 0.16 |
|-------|------|------|------|------|------|
| $\omega_t$ | 2328.6| 2328.7| 2328.7| 2328.7| 2328.7|
| $\omega_p$ | 2315.0| 2312.4| 2308.6| 2303.4| 2296.6|
| $\omega_t - \omega_p$ | 13.6  | 16.3  | 20.1  | 25.3  | 32.1  |
| $\omega_t$ | 34608.2| 34544.2| 34456.1| 34332.1| 34171.1|
| $\omega_p$ | 34607.3| 34543.1| 34454.8| 34330.4| 34168.9|
| $\omega_t - \omega_p$ | 0.9   | 1.1   | 1.3   | 1.7   | 2.2   |
| $\omega_t$ | 81368.6| 81341.5| 81304.2| 81251.9| 81184.4|
| $\omega_p$ | 81368.2| 81341.0| 81303.6| 81251.2| 81183.5|
| $\omega_t - \omega_p$ | 0.4   | 0.5   | 0.6   | 0.7   | 0.9   |
the effects of the air damping force on the natural frequencies become weak for large thickness of the flexible ring.

- When the current in the coils increases, the frequency difference \( \omega_3 - \omega_t \) increases which shows that the effects of the air damping force on the natural frequencies increases for large coil current.

To illustrate the theoretical analysis, FEM software, ANSYS, is used to simulate the dynamics performance of the flexible ring. The simulating process is as follows:

- FEM model of the flexible ring is produced. The density, Young’s modulus and Poisson’s ratio of the flexible ring material are set.
- At \( \theta = 0 \) and \( \theta = \pi/2 \), the movement and rotation in other directions of the flexible ring are restricted, only the movement in radial direction is allowed.
- Calculated electromagnetic force and molecular force are converted into the force per unit area applied to the flexible ring. The electromagnetic force and molecular force are 0.0785 and 0.2156 N/m², respectively. The direction of the forces is radial to the outside of the flexible wheel.

Using the FEM model, the natural frequencies of the flexible wheel are obtained and compared with the calculated ones (see Tab. 13). Table 13 show:

| Order number | Calculated (Hz) | Simulated (Hz) | Error (%) |
|--------------|----------------|----------------|-----------|
| 1            | 909.077        | 1095.1         | 16.99     |
| 2            | 5497.88        | 5921.1         | 7.15      |
| 3            | 12945.9        | 13977          | 7.38      |
| 4            | 23335.23       | 25132          | 7.15      |

7 Conclusions

In this paper, considering Van der Waals force, dynamics equation of the flexible ring for the micro electromagnetic
A harmonic drive system is proposed. Using the equations, effects of the molecule force on the natural frequencies and vibration modes of the drive system are investigated. Results show:

- Considering molecule force, natural frequencies of the flexible ring are reduced. For lower order modes, the effects of the molecule force on the natural frequencies are more obvious. For smaller clearance between the flexible ring and stator, smaller thickness of the flexible ring and larger radius of the flexible ring, the effects of the molecule force on the natural frequencies are more obvious.

- Considering molecule force, vibration modes of the flexible ring are changed. At some positions, the vibration amplitudes are decreased; at other positions, the vibration amplitudes are increased. For lower order modes, the effects of the molecule force on the vibration modes are more obvious. For smaller clearance between the flexible ring and stator, smaller thickness of the flexible ring and larger radius of the flexible ring, the effects of the molecule force on the vibration modes are more obvious.

- When considering friction force and air damping force, the natural frequencies of the flexible ring are decreased.

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