Nonlinear Parameter Estimation and Experimental Verification of Gravity Sensor Considering Quadratic Error Parameters

Tianyi Xie1, Shuhai Lu2, Juliang Cao1*, and Ruihang Yu1

1College of Intelligence Science and Technology, National University of Defense Technology, Changsha, China
2Baise Bama Airport, Baise, China

*Corresponding author: juliangcao@nudt.edu.cn

Abstract. With the rapid development of inertial navigation technology, the proportion of inertial navigation technology in navigation and positioning is getting higher and higher. The measurement accuracy requirements of inertial components are getting higher and higher. Based on the mature linear calibration method, this article calibrates the quadratic error of the accelerometer component in the inertial component and compensates the error term. In theory, the least squares estimation is used to solve the quadratic parameter. After considering the quadratic term, the measurement accuracy of the gravity sensor has been improved, and the laboratory gravimeter has been used for marine experiments. The experimental results show that the positioning deviation considering the quadratic error term is smaller than the positioning deviation considering only the first error term.

Keywords: Accelerometer, calibration, quadratic error term, parameter estimation.

1. Introduction
With the gradual maturity of today's high-precision inertial sensor technology, high-precision inertial navigation technology has become an important part of modern navigation and positioning technology [1]. The inertial navigation solution obtains angular increment and velocity increment information by integrating the specific force and angular acceleration measured by the inertial components [2]. Therefore, the accuracy of the inertial sensor as a measuring element is an important prerequisite for high-precision navigation. However, only relying on the improvement of device processing technology, the measurement accuracy is usually difficult to meet the theoretically strict requirements [3]. The algorithm compensates for the measurement error to improve performance and measurement accuracy. Due to its low-cost and high-efficiency nature, it can overcome the technical difficulties of slow development of inertial test equipment.

At present, there are a series of mature inertial device parameter calibration methods at home and abroad [4]. An important method that can improve the accuracy of the specific force measurement of the quartz flexible accelerometer is the more mature linear measurement model [5]. However, the linear model alone cannot meet the requirements of measurement accuracy. Therefore, this article adds quadratic term calibration on the basis of linear calibration, and contrasts with traditional linear
calibration. Using inertial navigation to solve and compare the performance of position and velocity errors, we can know that considering the quadratic term can indeed improve the measurement performance of the gravity sensor, which verifies the correctness and practicability of the quadratic term algorithm.

2. Accelerometer linear calibration method using gyro attitude auxiliary measurement

This paper uses the method of laser gyro auxiliary measurement to calibrate the accelerometer model. First, level the turntable before the experiment operation to ensure that the rotation of the outer frame of the turntable is strictly in the direction of the gravity vector. Then, the constraint condition can be established by the projection of the gravity unit vector in the carrier system consistent with the projection of the rotation axis unit vector in the carrier system when calibrating the gyroscope, so as to establish simultaneous equations.

The linear calibration model of the accelerometer is as follows:

\[
\begin{bmatrix}
  P_x^a \\
  P_y^a \\
  P_z^a
\end{bmatrix}
/ T_a =
\begin{bmatrix}
  S_x^a & 0 & 0 \\
  0 & S_y^a & 0 \\
  0 & 0 & S_z^a
\end{bmatrix}
\begin{bmatrix}
  x^a \cdot x^b & x^a \cdot y^b & x^a \cdot z^b \\
  y^a \cdot x^b & y^a \cdot y^b & y^a \cdot z^b \\
  z^a \cdot x^b & z^a \cdot y^b & z^a \cdot z^b
\end{bmatrix}
\begin{bmatrix}
  f_x^b \\
  f_y^b \\
  f_z^b
\end{bmatrix}
+ \begin{bmatrix}
  f_{x0} \\
  f_{y0} \\
  f_{z0}
\end{bmatrix}
\]

(1)

Let the accelerometer numbers be x, y, z. Among them, \( f^b = [f_x^b, f_y^b, f_z^b]^T \) is the component of the specific force vector on the three axes of the body frame system, the measurement time is \( T_a \), \( P^a = [P_x^a, P_y^a, P_z^a]^T \) is the pulse output of the accelerometer at time, \( f_{x0}^a \) and \( S^a_0 \) are the zero bias and scale factor of the three axes of the accelerometer. The measurement model of the gyro is:

\[
\begin{bmatrix}
  \omega_x^b \\
  \omega_y^b \\
  \omega_z^b
\end{bmatrix}
= \begin{bmatrix}
  G_{x1} & 0 & 0 \\
  G_{y1} & G_{y2} & 0 \\
  G_{z1} & G_{z2} & G_{z3}
\end{bmatrix}
\begin{bmatrix}
  N_x^a / T - \omega_{x0}^b \\
  N_y^a / T - \omega_{y0}^b \\
  N_z^a / T - \omega_{z0}^b
\end{bmatrix}
\]

(2)

\[
\begin{bmatrix}
  G_{x1} & 0 & 0 \\
  G_{y1} & G_{y2} & 0 \\
  G_{z1} & G_{z2} & G_{z3}
\end{bmatrix}
\]
replaces the inverse of the product of the gyro component scale factor matrix and the installation error matrix. \( [\omega_{x0}^b, \omega_{y0}^b, \omega_{z0}^b]^T \) is gyro zero bias.

We use the gyro attitude auxiliary measurement to establish the equation, and eliminate the gyro zero bias by forward and reverse. Another position is selected and the equation is established according to the above formula. The subtraction of these two equations can eliminate the accelerometer bias. Both sides of the equation are unitized, and there is an equation:

\[
\begin{bmatrix}
  A_{x1} & A_{x2} & A_{x3} \\
  A_{y1} & A_{y2} & A_{y3} \\
  A_{z1} & A_{z2} & A_{z3}
\end{bmatrix}
\begin{bmatrix}
  (P_x^a)^T / T_a - f_{x0}^a \\
  (P_y^a)^T / T_a - f_{y0}^a \\
  (P_z^a)^T / T_a - f_{z0}^a
\end{bmatrix}
\frac{1}{g}
\]

\[
\begin{bmatrix}
  G_{x1} & 0 & 0 \\
  G_{y1} & G_{y2} & 0 \\
  G_{z1} & G_{z2} & G_{z3}
\end{bmatrix}
\begin{bmatrix}
  (N_x^a)_{x^a} - (N_x^a)_{y^a} \\
  ((N_y^a)_{x^a} - (N_y^a)_{y^a}) \\
  ((N_z^a)_{x^a} - (N_z^a)_{y^a})
\end{bmatrix}
\frac{1}{4n\pi}
\]

(3)
\[
\begin{bmatrix}
A_{x1} & A_{x2} & A_{x3} \\
A_{y1} & A_{y2} & A_{y3} \\
A_{z1} & A_{z2} & A_{z3}
\end{bmatrix}
\]
is the undetermined parameter and is the inverse of the product of the scale factor matrix and the installation error matrix of the accelerometer assembly. Continue to rotate the IMU so that the IMU is in the other two sets of positions, we can get the data of three sets of positions (6 positions). So we can get a total of 15 (C^2_6 = 15) equations. Then establish the least square relationship, you can solve the accelerometer scale factor calibration matrix.

### 3. Nonlinear calibration method of accelerometer for gyro attitude auxiliary measurement considering quadratic term error

Same as the linear calibration, the equations are also established with the aid of the gyro attitude, and the corresponding specific force input is obtained through the projection of the plumb shaft on the carrier coordinate system.

The projection of the gravity unit vector in the inertial component carrier coordinate system can be obtained by normalizing the accelerometer relative to the gravity value \(G\):

\[
\vec{e}^b_i = \begin{bmatrix}
\frac{e_{x^i}}{g} \\
\frac{e_{y^i}}{g} \\
\frac{e_{z^i}}{g}
\end{bmatrix} = \begin{bmatrix}
f_{x^i} \\
f_{y^i} \\
f_{z^i}
\end{bmatrix} = \begin{bmatrix}
\frac{k_{axx}}{g} \\
\frac{k_{axy}}{g} \\
\frac{k_{axz}}{g}
\end{bmatrix}
\begin{bmatrix}
P^i_x \\
P^i_y \\
P^i_z
\end{bmatrix} - \begin{bmatrix}
P_{x0} \\
P_{y0} \\
P_{z0}
\end{bmatrix} - \frac{1}{T^2}
\begin{bmatrix}
k_{2xx} \left( P^i_x \right)^2 \\
k_{2xy} \left( P^i_y \right)^2 \\
k_{2xz} \left( P^i_z \right)^2
\end{bmatrix}
\]  

(4)

Where \(G\) is the gravity scalar value, \(a\) is the component representation of the specific force vector in the three axes of the body frame system, the measurement time is, \(e_i\) is the pulse output of the accelerometer in time, and \(e\) is the accelerometer bias. In the same way as for the model with only linear calibration, the rotation axis of the outer frame should be kept consistent with the gravity direction as far as possible. The rotation angles of the outer axis are \(\varphi = 0\), \(\varphi = \frac{\pi}{2}\), \(\varphi = \pi\) and \(\varphi = \frac{3\pi}{2}\). The average value of these four groups of data is taken as \(\vec{e}^b_i\):

\[
\vec{e}^b_i = \frac{1}{4} \left( e^{b,i}_{x^i} g_{x^i} + e^{b,i}_{y^i} g_{y^i} + e^{b,i}_{z^i} g_{z^i} \right)
\]

(5)

To specify some variables:

\[
\sum P^i_x = P^{i,0}_x + P^{i,y}_x + P^{i,\pi}_x + P^{i,3\pi}_x \quad \sum \left( P^i_x \right)^2 = \left( P^{i,0}_x \right)^2 + \left( P^{i,y}_x \right)^2 + \left( P^{i,\pi}_x \right)^2 + \left( P^{i,3\pi}_x \right)^2
\]

\[
\sum P^i_y = P^{i,0}_y + P^{i,y}_y + P^{i,\pi}_y + P^{i,3\pi}_y \quad \sum \left( P^i_y \right)^2 = \left( P^{i,0}_y \right)^2 + \left( P^{i,y}_y \right)^2 + \left( P^{i,\pi}_y \right)^2 + \left( P^{i,3\pi}_y \right)^2
\]

\[
\sum P^i_z = P^{i,0}_z + P^{i,y}_z + P^{i,\pi}_z + P^{i,3\pi}_z \quad \sum \left( P^i_z \right)^2 = \left( P^{i,0}_z \right)^2 + \left( P^{i,y}_z \right)^2 + \left( P^{i,\pi}_z \right)^2 + \left( P^{i,3\pi}_z \right)^2
\]

(6)

We can get the observation equation of accelerometer:
In the same way as the linear calibration method, we eliminate the gyro bias by the forward and reverse rotation of the gyroscope:

\[
e^j_g = \begin{bmatrix} e^j_{g_x} \\ e^j_{g_y} \\ e^j_{g_z} \end{bmatrix} = \frac{1}{g} \begin{bmatrix} \sum_{i} e_i x_i^{j} \\ \sum_{i} e_i y_i^{j} \\ \sum_{i} e_i z_i^{j} \end{bmatrix} = \frac{1}{g} \begin{bmatrix} k_{gx} & k_{gy} & k_{gz} \\ k_{gx} & k_{gy} & k_{gz} \\ k_{gx} & k_{gy} & k_{gz} \end{bmatrix} \begin{bmatrix} P_{0x} \\ P_{0y} \\ P_{0z} \end{bmatrix} - \frac{1}{4T} \begin{bmatrix} k_{2xx} \sum_{i} P_{ixi}^2 \\ k_{2yy} \sum_{i} P_{iyi}^2 \\ k_{2zz} \sum_{i} P_{izi}^2 \end{bmatrix} \]

(7)

The relationship (7) and (8) are combined to realize the nonlinear calibration of accelerometer by gyro attitude aided measurement:

\[
e^j_g = e^j
\]

(9)

The least square method is used to solve the parameters of three axes:

\[
F_x \theta_x = P_x ; F_y \theta_y = P_y ; F_z \theta_z = P_z
\]

(10)

In the above formula, the elements are as follows:

\[
\theta_x = \begin{bmatrix} k_{xx}, k_{xy}, k_{xz}, P_{0x}, k_{2xx} \end{bmatrix}^T \\
\theta_y = \begin{bmatrix} k_{yx}, k_{yy}, k_{yz}, P_{0y}, k_{2yy} \end{bmatrix}^T \\
\theta_z = \begin{bmatrix} k_{zx}, k_{zy}, k_{zz}, P_{0z}, k_{2zz} \end{bmatrix}^T
\]

(11)

\[
P_{0x} = \sum_{i} (P_{ixi})^2 / 4T^2 ; P_{0y} = \sum_{i} (P_{iyi})^2 / 4T^2 ; P_{0z} = \sum_{i} (P_{izi})^2 / 4T^2
\]

(12)

\[
F_x = \begin{bmatrix} g e^{bix} & g e^{bix} & g e^{bix} \\ 1 & \sum_{i} (P_{ixi})^2 / 4T^2 \end{bmatrix} \\
F_y = \begin{bmatrix} g e^{biy} & g e^{biy} & g e^{biy} \\ 1 & \sum_{i} (P_{iyi})^2 / 4T^2 \end{bmatrix} \\
F_z = \begin{bmatrix} g e^{biz} & g e^{biz} & g e^{biz} \\ 1 & \sum_{i} (P_{izi})^2 / 4T^2 \end{bmatrix}
\]

(13)

All parameters except cross coupling are obtained:

\[
\theta_x = \left( F_x^T F_x \right)^{-1} F_x^T P_x ; \theta_y = \left( F_y^T F_y \right)^{-1} F_y^T P_y ; \theta_z = \left( F_z^T F_z \right)^{-1} F_z^T P_z
\]

(14)

Thus, the calibration of the model with quadratic coefficients is finished.
4. Specific scheme of calibration parameters
The position arrangement of the gyroscope and accelerometer is as follows:

| position | Inner frame (degrees) | Middle frame (degrees) | Outer frame rotation | Outer frame positioning (degrees) | Accelerometer static data collection time |
|----------|-----------------------|------------------------|----------------------|-----------------------------------|-------------------------------------------|
| 1        | 0                     | 0                      |                      |                                   | 4*60S                                     |
| 2        | 180                   | 0                      | First turn forward 5 turns, then reverse 5 turns. | 0, 90, 180, 270                   | 4*60S                                     |
| 3        | 270                   | 0                      |                      |                                   | 4*60S                                     |
| 4        | 90                    | 0                      |                      |                                   | 4*60S                                     |
| 5        | 0                     | 90                     |                      |                                   | 4*60S                                     |
| 6        | 0                     | 270                    |                      |                                   | 4*60S                                     |

A 24-position arrangement scheme is used to provide observation information for the least squares algorithm to solve the parameters.

5. Experimental verification analysis
This experiment mainly wants to verify the effect of adding calibration of quadratic parameters. The sea test was conducted this time, and the experimental instrument used strap-down gravimeter SGA-WZ03. The sailing direction is from southeast to northwest, collecting GNSS data and IMU data during this process. Position reference and speed information are provided by GNSS as the basis for inertial navigation settlement comparison. The calibration method with only linear calibration is compared with that considering quadratic term, the conclusion is as follows:

(a) Traditional linear calibration scheme
(b) Accelerometer calibration scheme considering quadratic term

Fig. 1. Northbound position deviation

It can be seen that after considering the quadratic coefficients, the north position deviation is smaller than the linear calibration scheme. At 100 minutes, the position value error is about 150 meters smaller, and the accuracy is significantly improved. The speed calculation results are as follows:

(a) Traditional linear calibration scheme   (b) Accelerometer calibration scheme considering quadratic term

Fig 2. Speed information solution result

Compared with the traditional linear calibration method, the pure inertial navigation solution can be found that the calibration method considering the quadratic term is closer to the benchmark. The two trajectories are compared as follows:
Fig 3. The trajectory diagram of the inertial navigation solution

It can be seen that after considering the quadratic term, the inertial navigation solution accuracy is obviously closer to the reference trajectory than only the linear calibration method.

6. Conclusion

From the theoretical derivation and experimental results, the measurement and calculation accuracy of inertial navigation components has been significantly improved after the quadratic term is considered. In comparison with the benchmark information provided by GNSS, we can draw conclusions intuitively. Moreover, this algorithm uses the least square algorithm, adopts a 24-position arrangement method, and combines multiple position information to calculate parameters. The algorithm structure is simple and easy to understand and use, which can be popularized and used for calibration work.

Acknowledgments
This work was financially supported by the National Key Research and Development Program of China under Grant NO.2016YFC0303002. The corresponding author of this article is Juliang Cao.

References
[1] Chatfield A B. Fundamentals of High Accuracy Inertial Navigation[M]. American Institute of Aeronautics and Astronautics, Inc, 1997.
[2] Wu Y. Research on Dual-Quaternion Navigation Algorithm and Nonlinear Gaussian Filtering [D]. Changsha: National University of Defence Technology, 2005.
[3] Titterton D H, Weston J L. Strapdown Inertial Navigation Technology(2nd Ed)[M]. London, United Kingdom: Peter Peregrinus Ltd. on behalf of the Institute of Electrical Engineers, 2004.
[4] Niu X. Micromachined attitude measurement unit with application in satellite TV antenna stabilization[D]. Beijing: Tsinghua University, 2002.
[5] IEEE Std1293-1998, IEEE Standard Specification Format Guide and Test Procedure for Linear, Single-Axis, Nongyroscopic Accelerometers[S]. IEEE Standards Board, Approved September 25, 1998.