Linear state estimation and bad data detection for power systems with RTU and PMU measurements

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Abstract: The reliability of state estimation is of vital importance for secure operation of a power system. The key features of any state estimator are its accuracy, robustness against bad data and the associated computational burden. In order to ensure full system observability and maximise accuracy, a state estimation algorithm needs to take into account all available measurements. A significant number of hybrid estimators that can process both the conventional measurements that have been used in transmission systems for decades, as well as the synchrophasor measurements, can be found in literature. However, these algorithms are predominantly non-linear, thus requiring iterative solving procedures. In this study, a novel linear algorithm is proposed for state estimation including bad data detection of power systems that are monitored both by conventional and synchrophasor measurements. The proposed estimator is based on the linear weighted least square framework. As a result, the computational burden is rendered extremely low since there is no need for an iterative solving procedure, and standardised post-processing tools for bad data detection can be used. To validate the accuracy and robustness of the proposed algorithm, an extensive number of test cases of different sizes are solved and the results are presented and discussed.

1 Introduction

State estimation (SE) is a mathematical algorithm that has a crucial role in power system monitoring. SE processes raw measurement data collected from measurement devices installed throughout the grid and provides an estimate of the state variables of the system states, i.e. voltage magnitudes and angles for all system buses. Based on these estimates, the system operator gains insight into the actual operating state of the grid.

The measurement data have been obtained for decades by remote terminal units (RTUs), which typically measure voltage magnitudes and angles for all system buses. To estimate the state of the system, these types of data are most commonly processed by a weighted least square (WLS) algorithm, as initially proposed in [1]. WLS minimises the weighted mismatch between the measured values and their corresponding measurement functions that relate them to the system states. These functions are based on power flow equations, and are therefore highly non-linear.

The SE area has been a matter of great research interest lately, due to an increased utilisation of phasor measurement units (PMUs) in transmission networks, which diversified the available measurement set. PMUs provide highly accurate measurements of current and voltage phasors. Additionally, the measured values are synchronised in time via global positioning system, which further improves the accuracy. Therefore, achieving full system observability solely by PMUs would be an ideal scenario, since the states could be estimated with a high precision. Furthermore, this would render the SE problem linear [2, 3], which drastically improves the computational efficiency. However, this scenario is unlikely to happen in the foreseeable future. Consequently, novel hybrid SE methods are needed that will be able to include measurements provided both by RTUs and PMUs, thereby leveraging the broad system coverage by the former, and increased accuracy of the latter.

Many hybrid estimators were proposed in literature to address the aforementioned problem. The multi-stage estimators in [4–6] process conventional and synchrophasor measurements separately. In [4, 5] this is done sequentially, while the algorithm in [6] utilises two separate estimators for RTU and PMU measurements that are executed in parallel, which is followed by a fusion stage to produce final estimates. Another group of hybrid SE methods are single-stage estimators that account for all types of measurements simultaneously. Different approaches for PMU current measurement transformation are used in [7, 8], while the algorithm in [9] incorporates all measurements directly, which is enabled by the expansion of the state vector to include current magnitudes and angles. All these approaches are based on the non-linear WLS algorithm, and therefore need to employ iterative solution methods.

Recent advances in the field of power system simulation include the efforts to apply concepts of circuit theory to a range of power system related problems [10, 11]. By leveraging this idea, novel hybrid circuit-based estimators were derived in [12, 13]. These are characterised by simultaneous treatment of conventional and synchrophasor measurements, and by states being estimated in rectangular coordinates. Furthermore, the method in [13] proposes a fully linear estimation algorithm, resulting in a substantially decreased computational time. However, circuit-based estimators are not robust against bad data, and cannot resort to the conventional post-processing tools for bad data detection.

The estimators proposed in [14, 15] are also formulated using a linear framework. Although these papers are focused primarily on RTU measurements, the prospect of including PMU data is emphasised. In [14], a non-iterative algorithm is proposed by utilising quadratic variables that represent various product pairs of real and imaginary bus voltages. Linear measurement functions are derived with respect to these quadratic variables. However, the construction of the bus voltage product pairs, the number of the so-obtained variables that is substantially larger than the number of individual bus voltages, and the fact that a post-processing step is needed to calculate the individual bus voltages yields a considerable computational burden. Consequently, the computational speed is comparable to the conventional non-linear WLS approach, even when parallel computing is employed. The linear algorithm in [15] is formulated in terms of complex variables, and the states of a system, in this case complex bus voltages, are estimated via a two-stage algorithm. The first stage estimates voltage phase angles based on the least squares approach, and these values are leveraged in the WLS-based second stage to estimate complex bus voltages. This sequential algorithm structure is utilised in order to achieve linearity.
Raw measurement data contain random errors due to imperfections of the measuring equipment and disturbances in the communication channels. Filtering these errors is the primary task of any state estimator. However, the capability of the SE algorithm to detect gross measurement errors and suppress their negative effect on the estimation accuracy is also an important feature. All previously listed algorithms, including the circuit-based estimators, are inherently bad data resilient. In general, the lack of inherent robustness against bad data is a well-known drawback of all estimators based on the WLS approach. To overcome this issue, a post-processing step called the largest normalised residual (LNR) test is most commonly utilised to detect and identify bad data [16]. This approach was used in [17] for the non-linear WLS-based estimator which treats both RTU and PMU data. The main drawback of the LNR test is that it detects only one bad measurement per iteration. Therefore, the estimation process has to be repeated until all bad data have been detected, which, in combination with the inherent non-linearity of the traditional WLS-based estimators, results in a significant computational burden. The LNR test is also used in the linear estimator in [15].

To avoid the use of the post-processing step, a separate class of estimators that are robust against outliers are proposed in [18–20]. These are based on the non-linear least absolute value (LAV) approach, which in general has a very high computational complexity [16]. A linear robust estimator is proposed in [21] for systems with conventional measurements, although the possibility of inclusion of PMU measurements is also discussed. The algorithm introduces a set of variables that represent bus voltage product pairs, to allow for the derivation of linear measurement functions. The values of these variables are estimated by solving a linear program (LP), and bus voltages are then recovered from these estimates in the subsequent stage. However, regardless of the established linearity, the structure of the algorithm's optimisation problem still imposes considerable computational burden. A linear LAV-based hybrid estimator, which builds upon the work in [15], was recently proposed in [22] for systems observed by RTU and PMU measurements. The states are estimated in two stages by solving two LP problems, such that the phase angles are estimated by the first LP, while the second provides the estimates of complex bus voltages. While this approach has a clear advantage over the traditional non-linear formulation of the LAV-based SE problem with respect to computational time, it is still computationally significantly more expensive compared to the linear WLS formulation of the SE problem. Alternatively, the robust method proposed in [23] employs separate maximum correntropy criterion-based estimators for RTU and PMU data, followed by a fusion stage to generate final estimates. An iterative solution approach is needed in these algorithms.

In this paper, a novel linear state estimation algorithm, including bad data detection is derived for systems that comprise both conventional and synchrophasor measurements. Both types of measured data are processed simultaneously. The entire system is modelled in terms of voltages and currents in rectangular coordinates. The states, i.e. bus voltages, are estimated in rectangular coordinates within a single estimation step from a set of linear equations. Therefore, an iterative solving procedure and initial guess are not needed. The proposed estimator utilises the equality-constrained linear WLS framework, which yields a very high computational speed and scalability of the algorithm. This is enabled by representing RTU data by equivalent measurements that are linearly related to the system states. Null power injections are taken into account via equality constraints. Finally, LNR testing is performed in the post-processing stage to suppress the negative effect of bad data that might occur in the measurement set.

The remainder of the paper is structured as follows. Section 2 gives an overview of the conventional WLS method, along with the description of the LNR test for bad data detection. Furthermore, the approach for linear modelling of RTU data used in circuit-based estimators is explained. The proposed linear SE algorithm is presented in Section 3. Section 4 showcases simulation results, and the main conclusions are summarised in Section 5.

2 Technical background

2.1 Conventional WLS-based state estimation

Power system state estimation is traditionally formulated as a WLS problem. The vectors of states that are estimated comprises angles ($\theta$) and magnitudes ($V$) of the bus voltages

\[ x = \begin{bmatrix} \theta \\ V \end{bmatrix} \]  

(1)

The voltage angle of an arbitrarily chosen bus is usually set to zero to serve as a reference, and is therefore removed from the state vector.

A measurement set $z$ is represented as

\[ z = h(x) + e \]  

(2)

where $h(x)$ are the measurement functions relating measurement values to the state variables, and $e$ is the vector of measurement errors, which are assumed to be normally distributed and uncorrelated.

By minimising the weighted sum of squares of the measurement residuals, i.e. by solving the following optimisation problem:

\[ \min_{x} J(x) = (z - h(x))^T R^{-1} [z - h(x)] \]  

(3)

the most probable state of the system can be found. Therein, $R$ is a diagonal measurement error covariance matrix and for each measurement $i$, $R_i$ is equal to the square of the standard deviation $\sigma_i$ of the corresponding measurement. Hence, $R^{-1}$ comprises the weight coefficients for all measurements.

In order to find the optimal solution, i.e. the optimal state vector, the first-order optimality conditions are derived for (3). Measurement functions $h(x)$ for the measurements of injected powers and line flows are derived from the power flow equations and are therefore non-linear, thus rendering the entire optimisation problem non-linear. As a result, an iterative procedure has to be utilised to solve it. The following equation holds at each iteration $k$:

\[ G(x^k) \Delta x^k = H^T (x^k) R^{-1} [z - h(x^k)] \]  

(4)

where $H(x^k) = \partial h(x^k) / \partial x^k$, $\Delta x^k = x^{k+1} - x^k$ and $G(x^k) = H^T(x^k) R^{-1} H(x^k)$. Equation (4) is solved for $\Delta x^k$ at each iteration, until its value is smaller than a predefined convergence threshold.

Alternatively, if the WLS problem is linear, e.g. if the measurement set comprises only PMU measurements of voltage and current phasors and the state vector is formulated in rectangular coordinates [2], there is no need to execute an iterative procedure, as the measurement functions $h(x)$ are linear, i.e. $h(x) = Hx$, and the system states can be estimated directly by solving the following equation for $x$:

\[ x = (H^T R^{-1} H)^{-1} H^T R^{-1} z \]  

(5)

where $H$ is a constant Jacobian matrix.

Treatment of buses with null power injections is an important segment in any state estimator. A technique to represent the null injections might be to treat them as measurements with a very high weight coefficient. However, this can lead to an ill-conditioned problem. Instead, the null injections can be treated within an iterative procedure, as the measurement functions $h(x)$ are uncorrelated. However, this can lead to an ill-conditioned problem. Instead, the null injections can be treated within an iterative procedure, as the measurement functions $h(x)$ are uncorrelated.
The normalised residual vector estimation process is completed. However, if estimation problem in [12] for the first time. The resulting non-

largest normalised residual, executed again until no bad data is detected.

The SE procedure is repeated after the correction is made. For

more details, the reader is referred to [16].

Alternatively, if the measurement b is identified as bad data, instead of removing it from the measurement set, it can be corrected by subtracting the estimated value of its error from the actual measured value $\hat{z}_b$. The corrected value of the measurement

$b$ is, therefore, calculated as

$$
\hat{z}_b^\text{corr} = \hat{z}_b - \frac{\hat{r}_b^N}{\Omega}\hat{r}_b
$$

The SE procedure is repeated after the correction is made. For more details, the reader is referred to [16].

2.2 Bad data detection and identification

As mentioned, certain measurements may contain large errors, which can occur due to various reasons. Therefore, one of the main tasks of any state estimator is to detect the existence of outliers in the measurement set and suppress their negative effect on the accuracy of the state estimates.

A common method to detect bad data in WLS-based SE algorithms is using normalised residuals [16]. The first step is to solve the SE problem and calculate the measurement residuals $r_i$ for each measurement $i$:

$$
r_i = z_i - h_i(\hat{x})
$$

where $\hat{x}$ is the vector of estimated states. Then, normalised residuals are calculated as

$$
r_i^N = \frac{|r_i|}{\sqrt{\Omega_{ii}}}
$$

where $\Omega$ is the residual covariance matrix, computed in equality-

constrained WLS estimation as $\Omega = R - HEH^T$ [24]. If $N$ is the number of buses, the size of matrix $E$ is $(2N - 1) \times (2N - 1)$, and it can be obtained from the upper left corner of the inverse of the coefficient matrix from (7)

$${H^T R^{-1}}H - D^{-1}_N = E_{(N-1)\times(N-1)} \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdOTS
$$

The normalised residual vector $r^N$ has a standard normal distribution, since it is assumed that measurement errors have Gaussian distribution and are uncorrelated. Thus, the existence of bad data in the measurement set can be detected by finding the largest normalised residual, $r^N_{\text{max}}$, and comparing its value against a predefined threshold $q$. If $r^N_{\text{max}} < q$, no bad data is detected and the estimation process is completed. However, if $r^N_{\text{max}} > q$, the corresponding measurement is identified as bad data and is removed from the measurement set. The SE algorithm is then executed again until no bad data is detected.

As an additional preparatory step, we introduce two equations based on Kirchhoff’s current law (KCL), one for real and one for imaginary currents, for each RTU bus $k$:

$$
I_{R,k} = \frac{I_k}{V_k} \cos(\phi_k) V_{R,k} + \frac{I_k}{V_k} \sin(\phi_k) V_{I,k}
$$

$$
I_{I,k} = \frac{I_k}{V_k} \cos(\phi_k) V_{I,k} - \frac{I_k}{V_k} \sin(\phi_k) V_{R,k}
$$

2.3 Linear modelling of RTU measurements

An equivalent circuit formulation (ECF) for the power flow problem has been recently proposed in [10, 11]. The main idea of this method is that an entire power system can be represented by an equivalent circuit in terms of voltage and current state variables in rectangular coordinates. As a result, the equivalent circuit consists of two coupled sub-circuits, where the first sub-circuit represents the real voltages and currents, while the second one comprises their imaginary parts. The ECF approach was applied to the state estimation problem in [12] for the first time. The resulting non-

linear estimator treated RTU and PMU measurements simultaneously and provided state estimates in rectangular coordinates. The only non-linearity in this formulation was associated with the circuit representation of RTU measurements. This work was leveraged in [13] to derive a fully linear estimator by reformulating the circuit models for RTU measurements. A brief overview of the linear modelling approach for RTU measurements, used in the circuit-based estimator, is given below.

Directly deriving measurement functions for the RTU measurements yield non-linear relations if the modelling framework is based on voltages and currents in the rectangular coordinates. Hence, if an RTU bus $k$ is observed and the measurement set comprises bus voltage magnitude ($V_k$), and active ($P_k$) and reactive ($Q_k$) power injections, the following relations between injected current and voltage at this bus are derived [13]:

$$
I_{R,k} = \frac{P_k}{V_k} V_{R,k} + \frac{Q_k}{V_k} V_{I,k}
$$

$$
I_{I,k} = \frac{P_k}{V_k} V_{I,k} - \frac{Q_k}{V_k} V_{R,k}
$$

where $V_{R,k}$ and $V_{I,k}$ are the real and imaginary voltages at bus $k$, and $I_{R,k}$ and $I_{I,k}$ are the real and imaginary injected currents, respectively [The expressions (12) and (13) were derived for the case where the reference directions of measured voltage and current correspond to load conditions]. Since $V_k$, $P_k$ and $Q_k$ are the measurement values, the given relations between injected currents and bus voltages are linear. Alternatively, if an RTU located at bus $k$ is providing measurements of voltage magnitude ($V_k$), current injection magnitude ($I_k$), and the phase angle between voltage and current ($\phi_k$), the linear relations between bus voltages and injected currents in rectangular coordinates can be expressed as follows [13]:

$$
I_{R,k} = \frac{I_k}{V_k} \cos(\phi_k) V_{R,k} + \frac{I_k}{V_k} \sin(\phi_k) V_{I,k}
$$

$$
I_{I,k} = \frac{I_k}{V_k} \cos(\phi_k) V_{I,k} - \frac{I_k}{V_k} \sin(\phi_k) V_{R,k}
$$

3 Proposed linear state estimator

To achieve a linear SE formulation, the state vector in the proposed algorithm consists of bus voltages in rectangular coordinates.
where \( V_R \) and \( V_I \) are vectors of real and imaginary bus voltages. For a system with \( N \) buses, the size of vector \( V_R \) is \( N \), while there are \( N - 1 \) elements in \( V_I \) since one bus serves as the reference for all angles, and its imaginary voltage is therefore fixed. The proposed SE method utilises a linear WLS framework with equality constraints, and therefore, obtains the estimate of the system states by solving (7) for \( x \). The structure of individual terms in (7) used in the proposed method will be discussed below.

3.1 Measurement vector \( z \)

Conventional and synchrophasor measurements are treated simultaneously, and the vector of measurements which is processed by the proposed method is

\[
z_{\text{org}} = \begin{bmatrix} z_{\text{PMU}} \\ z_{\text{RTU}} \end{bmatrix} \]

(19)

where \( z_{\text{PMU}} \) is the vector of PMU measurements, consisting of voltages \( (V_{R,\text{PMU}}, V_{I,\text{PMU}}) \) and currents \( (I_{R,\text{PMU}}, I_{I,\text{PMU}}) \) in rectangular coordinates, while \( z_{\text{RTU}} \) is the vector of RTU measurements comprising voltage magnitudes \( (V) \), as well as active and reactive injections \( (P_{\text{inj}}, Q_{\text{inj}}) \) and line flows \( (P_{\text{flow}}, Q_{\text{flow}}) \). It is important to emphasise that the vector of originally available measurements \( z_{\text{org}} \) is not the same as \( z \) as we use in (7). While both comprise raw PMU measured values, conventional measurements are accounted for in \( z \) by introducing equivalent measurements to represent the original RTU data from \( z_{\text{org}} \):

\[
z = \begin{bmatrix} z_{\text{PMU}} \\ z_{\text{RTU,eq}} \end{bmatrix}
\]

(20)

The main reason for this approach is the fact that relations between individual RTU measurements, either voltage magnitudes, active and reactive injections or line flows, and bus voltages in rectangular coordinates are non-linear. To avoid non-linearity in measurement functions, equivalent measurements that are functions of the actual measurements are used to represent a set of original RTU data. Namely, for each RTU bus, each available group of measurements comprising: (i) bus voltage magnitude, active and reactive power injections; or (ii) bus voltage magnitude, and active and reactive line flows in any line incident to the respective bus, is represented in \( z_{\text{RTU,eq}} \) with two equivalent measurements that are equal to zero. It is assumed that a voltage magnitude measurement is available for each RTU bus. To clarify the idea, we consider the example of an RTU bus with three lines incident to it, shown in Fig. 1, and the available measurement set consists of voltage magnitude, as well as active and reactive power injections and line flows in all lines. Therefore, in this case \( z_{\text{RTU}} = [V, P_{\text{inj}}, Q_{\text{inj}}, P_{\text{flow}}, Q_{\text{flow}}] \). In the proposed algorithm, the subset of original RTU measurements comprising voltage magnitude \( V \) and power injections \( P_{\text{inj}} \) and \( Q_{\text{inj}} \) is represented with two equivalent measurements equal to zero. For line 1, the subset of original RTU measurements that consists of voltage magnitude \( V \) and line flows \( P_{\text{flow}} \) and \( Q_{\text{flow}} \) is also represented with two equivalent measurements equal to zero. The same applies to lines 2 and 3. Hence, for the measurement set shown in Fig. 1 the resulting vector of equivalent measurements consists of eight zero entries, i.e. \( z_{\text{RTU,eq}} = [0, 0, 0, 0, 0, 0, 0, 0] \). The fact that the voltage magnitude measurement appears in all subsets is the reason for the assumption that it is available at all RTU buses. The RTU measurements of voltage magnitude are only considered in combination with the corresponding measurements of either power injections or line flows, as explained above, and cannot be taken into account individually.

3.2 Measurement functions \( h(x) \) and Jacobian \( H \)

The complex admittance of the series branch is

\[
Y_{\text{branch}} = \frac{1}{R + jX} = \frac{r}{r^2 + x^2} - j \frac{x}{r^2 + x^2},
\]

while the admittance of the shunt branch is \( Y_{\text{sh}} = jB \). After incorporating these real and imaginary admittance terms into (21), the following expressions for real and imaginary series and shunt branch currents are obtained:

\[
I_{R,\text{ser,km}} = \frac{R}{R^2 + X^2} V_{R,\text{ser}} \quad \text{and} \quad I_{I,\text{ser,km}} = \frac{X}{R^2 + X^2} V_{I,\text{ser}} \quad (22)
\]

\[
I_{R,\text{sh,k}} = -B h V_{I,k} \quad \text{and} \quad I_{I,\text{sh,k}} = B h V_{R,k} \quad (24)
\]

\[
I_{R,\text{sh,m}} = -B h V_{I,m} \quad \text{and} \quad I_{I,\text{sh,m}} = B h V_{R,m} \quad (26)
\]

Fig. 1 RTU bus with nine measurements

Fig. 2 Pi-model of a transmission line

This approach lays the foundation for the derivation of linear measurement functions that relate the RTU equivalent measurements to states, based on the idea discussed in Section 2.3. The derivation of these functions will be discussed in the following section.
3.2.1 PMU voltage: It is assumed in this paper that PMU current and voltage measurements are available in rectangular coordinates. Also, the state vector comprises bus voltages in the same form. Hence, the measurement functions for real and imaginary PMU voltages at bus \(k\) are trivial

\[
\begin{align*}
    h_{\text{PMU}}^{\text{R},k} &= V_{R,k} \\
    h_{\text{PMU}}^{\text{I},k} &= V_{I,k}
\end{align*}
\]  

(28) (29)

3.2.2 PMU injected current: The measurement functions for the PMU real and imaginary injection currents at bus \(k\) are defined according to the KCL

\[
\begin{align*}
    h_{\text{PMU}}^{\text{R},k,i} &= \sum_{i \in \mathcal{N}_{\text{in},k}} I_{R,i} \\
    h_{\text{PMU}}^{\text{I},k,i} &= \sum_{i \in \mathcal{N}_{\text{in},k}} I_{I,i}
\end{align*}
\]  

(30) (31)

where \(\mathcal{N}_{\text{in},k}\) is the set of all network components incident to bus \(k\) and \(I_{C,i}, C \in [R, I]\), are the currents flowing through these components. As explained above, these currents are linear functions of the system states. Each current \(i\) has a positive sign if it flows out of the bus, while it is negative if it is directed into the bus. The opposite applies to the measurement of the value in vector \(z\). Thus, if the measured injection current flows into the bus, it has a positive sign.

3.2.3 PMU line flow: In case a PMU located at bus \(k\) measures real and imaginary currents injected into the line connecting this bus to bus \(m\), the corresponding measurement functions are defined as

\[
\begin{align*}
    h_{\text{PMU}}^{\text{R},k,m} &= I_{R,\text{ser},km} + I_{R,\text{sh},k} \\
    h_{\text{PMU}}^{\text{I},k,m} &= I_{I,\text{ser},km} + I_{I,\text{sh},k}
\end{align*}
\]  

(32) (33)

where \(I_{C,\text{ser},km}, C \in [R, I]\), is the current flowing through the series branch from bus \(k\) to \(m\), and \(I_{C,\text{sh},k}\) is the current flowing through the shunt branch incident to bus \(k\). The same above-mentioned sign convention still holds. The same approach is used for the derivation of measurement functions if there is a transformer between buses \(k\) and \(m\) instead of a transmission line.

3.2.4 RTU power injections: As it was explained in Section 3.1, if the measurements of voltage magnitude (\(V_i\)) and active (\(P_i\)) and reactive (\(Q_i\)) power injections are available at bus \(k\), these data are represented in the measurement vector \(z\) by two equivalent measurements equal to zero. These are related to the states based on the approach presented in Section 2.3, thus the measurement functions are derived based on the KCL equations for real and imaginary currents at bus \(k\). Please note that we introduce these equivalent measurements to enable a linear formulation of the measurement functions, which is not possible with the original RTU measurements. We now use the analogy between a modified form of the general measurement model in (2) and KCL equations (16) and (17), which are given again here for the ease of understanding

\[
\begin{align*}
    z_{\text{RTU},\text{eq}} &= h(x,z_{\text{RTU}}) + e \\
    0 &= I_{R,k} + \sum_{i \in \mathcal{N}_{\text{in},k}} I_{R,i} + I_{R,\text{err},k} \\
    0 &= I_{I,k} + \sum_{i \in \mathcal{N}_{\text{in},k}} I_{I,i} + I_{I,\text{err},k}
\end{align*}
\]  

(34a) (34b) (34c)

The measurement model in (34a) is essentially equivalent to the general model given in (2). Zero values in the KCL equations (34b)–(34c) correspond to two zero-valued RTU equivalent measurements in \(z_{\text{RTU},\text{eq}}\), hence one equivalent measurement is related to the KCL equation for the real currents, and the other corresponds to the KCL equation for imaginary currents. Varnces of the equivalent measurements are calculated based on the original measurements \(V_i, P_i\) and \(Q_i\) and their variances, as described in Section 3.4. Then, the measurement functions for RTU equivalent measurements, \(h(x,z_{\text{RTU}})\), are expressed with respect to the state variables, i.e. bus voltages in rectangular coordinates, by utilizing original measurement values as known parameters. The error term \(e\) exists since all measurement values contain a certain degree of error. Mismatch currents \(I_{C,\text{err},k}, C \in [R, I]\), correlate to the error terms \(e\), while \(I_{C,k} + \sum_{i \in \mathcal{N}_{\text{in},k}} I_{C,i}\) terms essentially represent the measurement functions \(h(x,z_{\text{RTU}})\). Finally, if \(I_{R,k}\) and \(I_{I,k}\) are replaced with the expressions in (12) and (13), the following relations are obtained for the measurement functions relating real and imaginary RTU injection equivalent measurements at bus \(k\) to the system states:

\[
\begin{align*}
    h_{\text{RTU}}^{\text{R},k,i} &= \frac{P_{\text{km}}}{V_k} V_{R,k} + \frac{Q_{\text{km}}}{V_k} V_{I,k} + \sum_{i \in \mathcal{N}_{\text{in},k}} I_{R,i} \\
    h_{\text{RTU}}^{\text{I},k,i} &= \frac{P_{\text{km}}}{V_k} V_{R,k} - \frac{Q_{\text{km}}}{V_k} V_{I,k} + \sum_{i \in \mathcal{N}_{\text{in},k}} I_{I,i}
\end{align*}
\]  

(35) (36)

All currents in expressions (35) and (36) have a positive sign if they are directed away from the bus. If the value of \(P_{\text{km}}\) denotes the generation conditions, the term \(\frac{P_{\text{km}}}{V_k} V_{I,k}\) will be negative. The same applies to the reactive power and the sign of \(\frac{Q_{\text{km}}}{V_k} V_{I,k}\). The currents \(I_{R,i}\) and \(I_{I,i}\) are real and imaginary currents flowing through the network components that are incident to bus \(k\), i.e. branches of transmission lines, transformers and shunts connected to bus \(k\). These currents are linear functions of states, as it was explained in (22)–(27) for the case of a transmission line. Therefore, the measurement functions in (35) and (36) are linear.

3.2.5 RTU line flows: If an RTU located at bus \(k\) measures voltage magnitude (\(V_i\)) as well as active (\(P_{\text{km}}\)) and reactive (\(Q_{\text{km}}\)) line flows in the transmission line connecting this bus to bus \(m\), this measurement set will be represented in the measurement vector \(z\) by two equivalent measurements equal to zero. These are related to the system states by applying the same approach as in the case of RTU power injections. Hence, the corresponding measurement functions are

\[
\begin{align*}
    h_{\text{RTU}}^{\text{R},k,m} &= \frac{P_{\text{km}}}{V_k} V_{R,k} + \frac{Q_{\text{km}}}{V_k} V_{I,k} + I_{R,\text{ser},km} + I_{R,\text{sh},k} \\
    h_{\text{RTU}}^{\text{I},k,m} &= \frac{P_{\text{km}}}{V_k} V_{R,k} - \frac{Q_{\text{km}}}{V_k} V_{I,k} + I_{I,\text{ser},km} + I_{I,\text{sh},k}
\end{align*}
\]  

(37) (38)

The sign convention is the same as in the case of power injections. The expressions for currents \(I_{R,\text{ser},km}, I_{I,\text{ser},km}\) \(I_{R,\text{sh},k}\) and \(I_{I,\text{sh},k}\) are given in (22)–(25) and are linear functions of states. The same approach is used for the derivation of measurement functions if there is a transformer between buses \(k\) and \(m\) instead of a transmission line.

3.2.6 Measurement Jacobian: A very important feature of the proposed estimator is that all measurement functions are linear, which is achieved by estimating voltages in rectangular coordinates and transforming RTU measurements to yield equivalent measurements that are linearly related to the states. Finally, the measurement Jacobian is calculated as \(H = \partial h(x)/\partial x\), where \(h(x)\)
is the vector of measurement functions, while \( x \) is the vector of states. Since all measurement functions are linear, \( H \) is a constant matrix. The size of \( H \) is \( M \times (2N - 1) \), where \( M \) is the number of measurements in \( z \), and \( N \) is the number of buses.

3.3 Equality constraints matrix \( D \)

In the proposed algorithm, null power injections are modelled as equality constraints that need to be satisfied by the estimator’s optimisation problem. The matrix \( D \) is constructed for this purpose.

For each bus that is known not to have any generation or load attached to it, two rows are added to matrix \( D \). One row represents the real KCL equation for the respective bus, while the second corresponds to the imaginary KCL equation. Therefore, the size of \( D \) is \( 2F \times (2N-1) \), where \( F \) is the number of buses with null injections, and \( N \) is the number of all buses in the system. To clarify this further, if bus \( k \) has neither generation nor load, and rows \( k \) \( \) and \( k \) \( \) in matrix \( D \) correspond to bus \( k \), the elements in rows \( k \) \( \) and \( k \) \( \) are obtained so that the following equations are satisfied:

\[
D_{k_1}x = \sum_{i \in [N_\text{inc,}k]} I_{i,k_1}
\]

(39)

\[
D_{k_2}x = \sum_{i \in [N_\text{inc,}k]} l_{i,k_2}
\]

(40)

where \( N_\text{inc,}k \) is the set of all network components incident to bus \( k \) and \( I_{i,\text{c},k}, C \in \{R, I\} \), are the currents flowing through these components. The currents \( I_{i,\text{c}} \) are linear functions of states, as it was shown in (22)–(27) for the case of a transmission line.

3.4 Measurement error covariance matrix \( R \)

Same as in conventional WLS, the measurement errors are assumed to be normally distributed and uncorrelated. Thus, \( R \) is a diagonal matrix in the proposed algorithm, where for each measurement \( i \) the corresponding diagonal entry \( R_{ii} \) denotes the variance of the measurement. The size of \( R \) is \( M \times M \), where \( M \) is the number of measurements in \( z \).

For each PMU measurement with standard deviation \( \sigma_i \), the corresponding variance is simply calculated as \( \sigma_i^2 \). On the contrary, variances of the RTU equivalent measurements are calculated based on their measurement functions and the error propagation theory [25]. If bus \( k \) is observed and the measurement set consists of voltage magnitude \( V_i(k) \) and active \( P_i(k) \) and reactive \( Q_i(k) \) power injections, variances of the terms \( P_i/k_i V_i(k) \) and \( Q_i/k_i V_i(k) \) can be calculated based on the following general rule:

\[
f = ABx \Rightarrow f_j^2 = \left[ \frac{\sigma_k^2}{A} \right] + \left[ \frac{\sigma_f^2}{B} \right]
\]

where the measurements \( A \) and \( B \) are transformed to the equivalent measurement \( f_j \), and \( \sigma_k \) and \( \sigma_f \) are standard deviations of \( A \), \( B \) and \( f_j \), respectively. Let \( \sigma_k \) be the obtained variance of the term \( P_i/k_i V_i(k) \) and let \( \sigma_f \) denote the calculated variance of \( Q_i/k_i V_i(k) \). Then, the variances of the real and imaginary RTU equivalent measurements that represent the observed measurement set can be calculated based on their measurement functions (35) and (36).

Two different approaches are proposed. According to the error propagation theory, the properly calculated variance of the real RTU equivalent measurement would be equal to \( V_{\text{r}}(k) \sigma_k(P_i/k_i V_i(k)) + V_{\text{i}}(k) \sigma_k(Q_i/k_i V_i(k)) \), while the variance of the imaginary equivalent measurement would be equal to \( V_{\text{r}}(k) \sigma_k(Q_i/k_i V_i(k)) + V_{\text{i}}(k) \sigma_k(P_i/k_i V_i(k)) \). However, values of rectangular bus voltages are states and are not known prior to the execution of the algorithm. Nevertheless, one can use the values of \( V_{\text{r}}(k) \) and \( V_{\text{i}}(k) \) obtained at the previous instance of the state estimation algorithm to calculate variances of the RTU equivalent measurements at the current instance. For this purpose, the estimated values of states from the previous instance of state estimation need to be available. We refer to this approach as the enhanced approach. A simplified approach implies that the variances of the real and imaginary equivalent measurements are set to \( \sigma_k^2 \) and \( \sigma_f^2 \), respectively. These values are chosen since, in general, real voltages are substantially higher than their imaginary counterparts, and therefore the corresponding terms in the measurement functions predominantly define the appropriate weight coefficient, i.e. the variance. However, it should be noted that this is not always the case, e.g. in large systems imaginary voltages at certain buses might be higher than their real counterparts. Therefore, the enhanced approach is expected to yield a higher estimation accuracy, provided that the estimated values of states from the previous instance of state estimation are available.

In case the measurement set comprises measurements of voltage magnitude and active and reactive line flows, the variances of the corresponding real and imaginary equivalent measurements are determined by applying the same approach.

3.5 Bad data processing

Gross errors in either RTU or PMU measurements can severely bias the estimated states. Therefore, it is very important to detect them and suppress their negative effect on the estimation outcome. To this aim, the proposed algorithm utilises the largest normalised residual test that was explained in Section 2.2. Normalised measurement residuals are calculated according to (9) and the value of the largest normalised residual \( r_{\text{max}}^2 \) is compared to the threshold \( q \). Since the normalised measurement residuals have standard normal distribution, the value of the threshold \( q \) is set to 3 [16]. If \( r_{\text{max}}^2 > q \), the corresponding measurement is identified as bad data. Instead of eliminating it from the measurement set, its value is corrected based on (11) and the estimation process is executed again. This process has to be repeated as long as bad data are detected in the measurement set.

The benefit of correcting bad data instead of removing it is that the data structure remains the same. Also, it prevents any RTU equivalent measurement, which essentially represents a set of three RTU measurements, from being eliminated from the estimation process if any of the original measured values that it is related to have a large error. Therefore, the proposed algorithm is robust against bad data in both RTU and PMU measurement sets.

In conventional non-linear WLS estimation, the residual covariance matrix \( \Omega \) needs to be calculated in each iteration of the bad data identification procedure, which is computationally costly [16]. However, as long as the set of available measurements remains unchanged, the matrix \( \Omega \) is constant in the proposed estimator due to the linearity of the measurement functions. Hence, it needs to be calculated only once, which substantially reduces the overall computational burden of the algorithm, even when multiple gross errors are present in the measurement set.

3.6 Comparison to the linear circuit-based state estimation

The algorithm proposed in this paper, as well as the equivalent circuit-based estimator proposed in [13], provide fully linear methods for state estimation of transmission networks observed by conventional and synchrophasor measurements. The main similarity between these algorithms is the fact that linearity is achieved by modelling the entire system, along with the measurements, with respect to voltages and currents in rectangular coordinates. The system states, i.e. bus voltages, are estimated in rectangular coordinates, and certain transformations of RTU measurements are performed to ensure linearity.

While utilising a similar modelling approach, the linear circuit-based estimator differs considerably from the method proposed in this paper. The main idea of the algorithm proposed in [13] is that the entire system, including the available measurements, is modelled by an equivalent circuit. Therefore, all power network components, as well as different sets of PMU and RTU measurements that can potentially be available at a bus, are modelled by the appropriate circuit models [26]. Then, the states are estimated by solving an optimisation problem with linear equality constraints, which essentially represent governing
equations of the system’s equivalent circuit. The optimisation problem minimises the weighted sum of squared differences between measurements and their corresponding variables within the equivalent circuit, as well as the square of particular currents in the PMU circuit models in order to ensure accurate representation of the available PMU measurement sets. For more details, the reader is referred to [13]. Apart from the bus voltages, the vector of variables in the circuit-based SE method comprises a large set of additional variables due to the circuit representation of the entire system, which increases the size of the problem. Also, the utilisation of the slack conductances in the circuit models for the PMU measurements and circuit models for some RTU measurement sets, can affect the estimation accuracy if the values of these conductances are not properly set. Finally, the main drawback of the linear circuit-based estimator from [13] is the fact that it is neither inherently robust against bad data in the measurement set, nor is there an effective method to detect bad data in the post-processing stage.

The algorithm proposed in this paper is based on the conventional linear WLS framework. The states are estimated by solving an optimisation problem that minimises the weighted sum of the squared measurement residuals, i.e. differences between measurements and their corresponding measurement functions. The optimisation problem is subject to a set of equality constraints that are modelling null power injections. Hence, the state estimates are obtained based on (7). Contrary to the circuit-based estimator, the PMU measurements are taken into account individually, hence without any slack components. Also, even though the RTU measurements are initially modelled using a similar approach as in the circuit-based estimator, their treatment within the WLS framework is substantially simpler compared to the case when circuit models are used, especially when the complex circuit models from [26] are taken into account. The only variables in the proposed algorithm are the system states, i.e. bus voltages in rectangular coordinates. Consequently, the proposed method substantially decreases the number of equations and variables compared to the circuit-based approach, thus reducing the computational burden. Finally, the main advantage of the proposed method over the circuit-based estimator is its ability to provide unbiased state estimates when bad data are present in the measurement set. This is done by executing a well-known bad data identification and correction technique, i.e. the largest normalised residual test, which is applicable since the proposed estimator is based on the WLS approach. Furthermore, the proposed linear WLS-based estimator lends itself to deriving even more comprehensive algorithms that can be used for parameter estimation and topology error detection, by combining its inherent linearity and the well-known methods already used for these problems, thus enabling algorithms with high computational efficiency.

4 Numerical results

The performance of the proposed algorithm is evaluated on the IEEE 14, 57 and 118 bus test systems, and the 2869 and 13659 test systems provided by the PEGASE project [27]. Accuracy and computational time are examined for the cases, where (i) the measurement set consists only of measurements with random Gaussian errors and there are no bad data; (ii) bad data are present in the measurement set. The full algorithm is executed in both cases, i.e. both state estimation and bad data detection and correction stages are employed. Furthermore, the effect of different approaches for the calculation of variances of RTU equivalent measurements on the estimation accuracy is studied, and the performance comparison is made between the proposed algorithm and one of the well-known constrained hybrid estimators based on the non-linear WLS framework.

4.1 Test cases without bad data

The measurement set comprises both conventional and synchrophasor measurements for all test systems. The set of RTU measurements consists of bus voltage magnitudes, as well as active and reactive power injections and line flows. It is assumed that each PMU device has a sufficient number of channels to monitor all lines incident to the bus where it is located. Therefore, the set of PMU measurements comprises phasors of bus voltages and line currents in rectangular coordinates. It should be emphasised that PMU measurements in polar form can also be incorporated, by transforming them into rectangular coordinates and calculating the corresponding variances based on the error propagation theory [25]. In each test case, one PMU bus is selected to serve as the slack bus, thus providing the reference for the voltage angles. The structure of the measurement sets for all test systems is presented in Table 1. Naturally, the majority of measurements are provided by RTUs, which corresponds to the current situation in real transmission networks, which are still predominantly monitored by legacy measurements. Locations and number of measurements were selected so that the full system observability is ensured for all test systems.

All measurement errors are assumed to have a Gaussian distribution with zero mean and are not correlated. For each measurement \( i \), the measured value is randomly selected from the range \([z_i - 3\sigma_i, z_i + 3\sigma_i]\), where \( z_i \) is the true value of the measurement \( i \) obtained from the power flow simulation in MATPOWER, while \( \sigma_i \) is its corresponding standard deviation. To ensure that the measurement errors are normally distributed, 68% of the generated measurement values are within one standard deviation from the true value, while 95% of the measurement values are within two standard deviations. The standard deviations for different types of measurements are given in Table 2, and are based on the values that are typically used in literature [7, 9, 16]. The difference in the order of magnitude of the selected standard deviations for PMU and RTU data is due to the fact that PMU measurements are characterised in practice by significantly higher accuracy compared to RTU data.

Two different performance indices are used in order to evaluate the accuracy of the proposed method. The first one is the sum of the squared differences between the estimated values of states and their true values, divided by the number of estimated states:

\[
\sigma_i = \frac{1}{2N - 1} \sum_{m=1}^{2N-1} (\hat{x}_i - x_i)^2
\]  

(42)

where \( \hat{x} \) and \( x' \) are the estimated and true values, respectively, and \( N \) is the number of buses. The second index essentially quantifies the accuracy of the estimated measurement values with respect to the accuracy of their original values

\[
\xi = \frac{\sum_{i=1}^{M} (\hat{z}_i - z_i)^2}{\sum_{i=1}^{M} (z_i - \hat{z}_i)^2}
\]  

(43)

where \( \hat{z} \), \( z' \) and \( z'' \) are the estimated, true and original measurement values, respectively. \( M \) denotes the number of measurements. Hence, \( \xi < 1 \) indicates that the estimated values are closer to the true values, compared to the raw measured data. All state and
Table 3  Average performance indices with simplified RTU variances

| Test case | $\sigma^2_i$ | $\xi$ | Time [s] |
|-----------|-------------|------|--------|
| 14 buses  | $1.9627 \times 10^{-4}$ | 0.1527 | 0.0003 |
| 57 buses  | $2.7897 \times 10^{-4}$ | 0.3372 | 0.0015 |
| 118 buses | $3.4308 \times 10^{-4}$ | 0.3502 | 0.0007 |
| 2869 buses| $8.9662 \times 10^{-4}$ | 0.4933 | 0.1102 |
| 13659 buses| $4.6172 \times 10^{-7}$ | 0.5981 | 0.6004 |

Fig. 3  State estimation errors for the 14 bus test case

Table 4  Average performance indices with enhanced RTU variances

| Test case | $\sigma^2_i$ | $\xi$ | Time [s] |
|-----------|-------------|------|--------|
| 14 buses  | $1.8516 \times 10^{-4}$ | 0.1569 | 0.0003 |
| 57 buses  | $2.5957 \times 10^{-4}$ | 0.3072 | 0.0015 |
| 118 buses | $3.4249 \times 10^{-4}$ | 0.3367 | 0.0097 |
| 2869 buses| $7.5028 \times 10^{-4}$ | 0.4783 | 0.1107 |
| 13659 buses| $3.2836 \times 10^{-7}$ | 0.5836 | 0.6009 |

Table 5  Average performance indices for non-linear WLS-based hybrid SE

| Test case | $\sigma^2_i$ | $\xi$ | Time [s] |
|-----------|-------------|------|--------|
| 14 buses  | $1.8253 \times 10^{-4}$ | 0.0170 |        |
| 57 buses  | $2.4889 \times 10^{-4}$ | 0.0716 |        |
| 118 buses | $3.3716 \times 10^{-4}$ | 0.2582 |        |
| 2869 buses| $8.2615 \times 10^{-4}$ | 1.1178 |        |
| 13659 buses| $4.1723 \times 10^{-7}$ | 6.0276 |        |

measurement values are expressed in per unit system. The proposed algorithm is implemented in Matlab, and simulations are executed on a PC with an Intel i7-6600 CPU and 16 GB of RAM. One thousand simulations are performed for each test case to obtain averaged results for different measurement values. The measurement allocation is kept the same for all simulations, while the measurement values are randomly selected as it was explained above.

4.1.1 Simplified approach for RTU variances: The first group of simulations is executed so that variances of the RTU equivalent measurements are determined based on the simplified approach, which is explained in Section 3.4. There are no bad data in the measurement set. The average values of all performance indices for all test cases are presented in Table 3. The obtained results demonstrate high accuracy of the proposed method, since the values of all indices are very low for test cases of all sizes. The values of index $\sigma^2_i$ verify that the level of estimation accuracy is similar for all test cases, regardless of their size. A slightly higher values of $\sigma^2_i$ for larger test cases can be attributed to two factors. First, the lower percentage of PMU measurements in these systems, compared to the smaller test cases, might lead to slightly lower accuracy, while the second reason is that the simplified approach for the calculation of RTU variances assumes that the real bus voltages are substantially higher than their imaginary counterparts, which is not always true in large systems. The values of index $\xi$ show that the estimated measurement values are much more accurate than the original ones. The absolute errors of the estimated bus voltages, both real and imaginary, are presented in Fig. 3 for the 14 bus test case. One can observe very high accuracy of each individual estimated voltage value. Furthermore, the average computational time is given for each test case in Table 3. The obtained values are very low, which is the consequence of utilisation of the linear WLS framework. Also, it is shown that the algorithm is fast even for very large systems, which demonstrates the scalability of the proposed method.

4.1.2 Enhanced approach for RTU variances: The second group of simulations aims to examine the performance of the proposed algorithm when the enhanced approach for the calculation of variances of the RTU equivalent measurements is used. The enhanced approach leverages the values of estimated bus voltages from the previous instance of state estimation, as explained in Section 3.4. For this reason, one instance of the algorithm using the simplified approach for RTU variances, which does not require the values of estimated states from the previous instance, is executed first in order to obtain the estimated values of bus voltages, so that they can be used in the next instance for the calculation of RTU variances. Then, one thousand simulations are run using the enhanced approach, and the obtained averaged values of performance indices are shown in Table 4. Comparison of the results presented in Tables 3 and 4 shows that the accuracy of the proposed method for small test cases is almost the same, regardless of the utilised approach for the calculation of variances of the RTU equivalent measurements. On the other hand, the use of the enhanced approach improves the algorithm's accuracy for large systems. This is expected since the simplified approach is based on the assumption that the real part of bus voltages is substantially higher than the imaginary part, which is not always true, especially in large systems. The calculation times remain unchanged.

4.1.3 Comparison with non-linear WLS approach: Another set of simulations is performed in order to evaluate how the performance of the proposed estimator compares to one of the well-known hybrid estimators based on the equality-constrained non-linear WLS framework, proposed in [9]. While this method is also treating PMU and RTU measurements simultaneously, it does not require any transformation of the original RTU data. Instead, the RTU measurements are accounted for directly, and the measurement functions corresponding to the RTU measurements are derived based on the non-linear power flow equations. This algorithm is also applied to the test systems given in Table 1, and the obtained results are presented in Table 5. One can observe comparable levels of accuracy of the non-linear algorithm and the proposed method for all test systems, which proves that treatment of the original RTU data via equivalent measurements in the proposed method does not deteriorate its accuracy. Another important conclusion is that the proposed linear algorithm significantly outperforms the non-linear approach in terms of computational time, which is expected, since the proposed method does not require an iterative solving procedure.

4.2 Test cases with bad data

Capability of the proposed algorithm to accurately estimate the state of the system when bad data are present in the measurement set is evaluated based on the 14 and 13659 bus test cases. For both test systems, the performance of the algorithm is examined when only one measurement has a gross error, as well as when there are multiple bad data in the measurement set. For these experiments, the same measurement sets are used as for the 14 and 13659 bus test systems in the previous section. One thousand simulations are performed to obtain results for different values of the measurements that have small random errors. The values of bad measurements are obtained by modifying the true value by 30%,
and this value is kept the same for all simulations. The variances of the RTU equivalent measurements are obtained based on the corrected value of the corresponding equivalent measurements.

For the 14 bus test case with single bad data, the value of one PMU measurement, namely the real voltage at bus 1, is altered by 30%. The following PMU and RTU measurements are manually changed for the 14 bus test case with multiple bad data: real voltage at bus 1, real line current in the line connecting buses 6 and 5, voltage magnitude at bus 12, active power injection at bus 5, and active and reactive line flows in the line connecting buses 7 and 9. All values are altered by 30%. The obtained numerical results are shown in Table 6. Capability of the algorithm to accurately estimate the states in the presence of bad data is clearly demonstrated, since the values of $\sigma_i^2$ are very similar as in the case when there are no bad data. Furthermore, the absolute errors of real and imaginary estimated voltages for the case of multiple bad data, presented in Fig. 4, show very high accuracy of each individual estimated state. The accuracy of the corrected values of bad measurements for the case with multiple bad data can be assessed based on the results provided in Table 7. It can be verified that the corrected values of bad measurements are very close to their true values. This is mostly confirmed by the obtained variances, since the mean value might be misleading due to averaging effect. Finally, the average computational time, presented in Table 6, is still very low for both test cases, even though multiple iterations of the state estimation and bad data correction stages have to be executed until all bad data are identified and corrected. This is enabled by the linearity of the proposed estimator.

The 13659 bus test case with bad data is simulated in order to check the scalability of the algorithm when bad data are present in the measurement set. For the case with single bad data, the value of real voltage measurement at bus 1, provided by a PMU, is altered by 30%. For the case with multiple bad data, the following PMU and RTU data are manipulated to contain gross error: real voltage at bus 1, imaginary line current in the line connecting buses 11715 and 924, voltage magnitude at bus 5477, active power injection at bus 8705, and reactive line flow in the line connecting buses 4850 and 8031. Therefore, the set of bad measurements comprises all possible types of PMU and RTU measurements. Table 7 summarises the obtained averaged values of the performance indices. The values of index $\sigma_i^2$ verify that the proposed algorithm suppresses the negative effect of bad data and provides reliable state estimates, both for single and multiple bad data cases. Although multiple iterations of the algorithm need to be executed to identify and correct all measurements with gross errors, the resulting computational time still remains low. Furthermore, the corrected values of bad measurements for the case of multiple bad data, shown in Table 9, demonstrate that the algorithm is able to modify bad data so that the corrected values are very close to the true values.

### 5 Conclusion

A novel linear state estimation algorithm is proposed for the synchrophasor systems, which comprise conventional and synchrophasor measurements. The estimator is based on the linear equality-constrained WLS approach, which is enabled by the adequate treatment of the RTU data and network modelling in terms of voltages and currents in a rectangular form. All measurements are accounted for simultaneously and the states are estimated in rectangular coordinates. Equality constraints are used to model null power injections. The obtained results demonstrate very high accuracy of the proposed method, as well as its low computational burden. Furthermore, utilisation of the WLS framework enables the use of the LNR test that renders the algorithm bad data resilient, which comes at a low computational cost due to the linearity of the estimation stage.

### Table 6 Average performance indices for the 14 bus test case with bad data

| Test case       | $\sigma_i^2$ | Time [s] |
|-----------------|--------------|----------|
| single bad data | $1.9178 \times 10^{-8}$ | 0.0004 |
| multiple bad data | $2.1525 \times 10^{-8}$ | 0.0008 |

### Table 7 Accuracy of the corrected values of bad measurements for the 14 bus test case

| Measurement                  | True [pu] | Mean Est. [pu] | $\sigma_i^2_{corr}$ |
|------------------------------|-----------|----------------|---------------------|
| $V_{R,1}$                    | 1.0600    | 1.0601         | $3.1725 \times 10^{-8}$ |
| $I_{R,6-5}$                  | $-0.3809$ | $-0.3809$      | $8.2732 \times 10^{-9}$ |
| $V_{12}$                     | 1.0552    | 1.0551         | $8.3764 \times 10^{-7}$ |
| $P_{m,3}$                    | 0.0760    | 0.0762         | $2.1938 \times 10^{-7}$ |
| $P_{line,7-9}$              | $-0.2807$ | $-0.2810$      | $8.1966 \times 10^{-7}$ |
| $Q_{line,7-9}$              | $-0.0578$ | $-0.0577$      | $4.0694 \times 10^{-7}$ |

### Table 8 Average performance indices for the 13659 bus test case with bad data

| Test case       | $\sigma_i^2$ | Time [s] |
|-----------------|--------------|----------|
| single bad data | $3.3847 \times 10^{-7}$ | 0.8538 |
| multiple bad data | $3.4277 \times 10^{-7}$ | 1.6263 |

### Table 9 Accuracy of the corrected values of bad measurements for the 13659 bus test case

| Measurement                  | True [pu] | Mean Est. [pu] | $\sigma_i^2_{corr}$ |
|------------------------------|-----------|----------------|---------------------|
| $V_{R,1}$                    | 1.0317    | 1.0316         | $3.8261 \times 10^{-6}$ |
| $I_{1,11715 - 924}$         | $-2.0071$ | $-2.0070$      | $2.1465 \times 10^{-8}$ |
| $V_{5477}$                   | 1.0003    | 1.0000         | $2.4773 \times 10^{-6}$ |
| $P_{m,5477}$                | 0.9050    | 0.9053         | $7.7034 \times 10^{-6}$ |
| $Q_{line,4850 - 8031}$      | $-0.3459$ | $-0.3465$      | $9.8219 \times 10^{-6}$ |
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