An asymptotic restoration procedure is applied for analyzing bound–state overlap functions, separation energies and single–nucleon spectroscopic factors by means of a model one–body density matrix emerging from the Jastrow correlation method in its lowest order approximation for $^{16}O$ and $^{40}Ca$ nuclei. Comparison is made with available experimental data and mean–field and natural orbital representation results.

The first term $K$ is kinematical factor , $\sigma_{ep}$ is the off–shell electron–proton scattering cross–section [24] and the nuclear structure component $| \phi_{\alpha}(k) |^2$ is the squared Fourier transform of the overlap function between the ground state of the target nucleus $\Psi^{(A)}$ and the final state of the residual nucleus $\Psi^{(A-1)}$: $\phi_\alpha(r) = \langle \Psi^{(A-1)} | a(r) | \Psi^{(A)} \rangle$, $\alpha$ being an annihilation operator for a nucleon with spatial coordinate $r$ (spin and isospin coordinates are not put in evidence). The overlap functions $\phi_\alpha(r)$ are not orthonormalized. Their norm defines the spectroscopic factor of the level $\alpha$

\[ S_{\alpha} = \langle \phi_{\alpha} | \phi_{\alpha} \rangle \]  \hspace{1cm} (3)

and the normalized overlap function

\[ \tilde{\phi}_{\alpha}(r) = S_{\alpha}^{-1/2} \phi_{\alpha}(r) . \]  \hspace{1cm} (4)

Usually, $\tilde{\phi}_{\alpha}(r)$ is calculated from an empirical Saxon–Woods potential with a distinct potential radius for each separate transition $\alpha$. Quantitative estimates are then deduced by fitting both the potential radius and the spectroscopic factor $S_{\alpha}$ in order to obtain a good agreement between the experimentally measured cross sections $(d\sigma/d\Omega)_{\exp}$ and those predicted from appropriate calculations for the reaction process.

The full theoretical description of the experiments mentioned above has many components. We should like to mention among them the proper account of the reaction mechanism, of the distortion effects (including the distortion due to the final state interaction), of the meson exchange currents contributions, the study of the $A$–dependence and others (see e.g. [22,12]). Obviously, however, the theoretical estimates of the overlap functions $\phi_\alpha(r)$ and the spectroscopic factors $S_{\alpha}$ are of crucial importance for the adequate evaluation of recent ($e, ep$), $(d,^3He)$ and $(\gamma, p)$ experiments. The problem is that the normalized overlap functions $\phi_\alpha(r)$ cannot be identified with a phenomenological shell–model single–particle wave function especially for energies farther from the Fermi energy, sometimes even within the valence shell [16]. It is known that generally, the independent–particle shell model cannot explain the fragmentation or spreading of the hole strength [21,22]. This is because, due to the residual interaction, the hole state of the target nucleus is not an eigenstate of the $(A – 1)$–nucleon system and its strength is distributed over several states of the final system. Possible modifications going beyond the uncorrelated shell–model approximation quickly become rather involved.

Recently, it has been shown [27] that the knowledge of the ground–state one–body density matrix of the target nucleus is sufficient to determine, at least in principle, the overlap functions, spectroscopic factors and separation energies of the bound $(A – 1)$–particle eigenstates. The aim of the present paper is to apply the procedure suggested in [27] using the model one–body density matrix [23] in which the short–range correlation terms of the Jastrow correlation method are taken into account. The resulting quantitative estimates allow us to make instructive conclusions for the properties of the overlap
Due to the spherical symmetry, the spectroscopic factor \( N_{nlj} \) reads
\[
\phi_{nlj}(r) \to C_{nlj} \exp(-k_{nlj} r)/r ,
\]
where
\[
k_{nlj} = \hbar^{-1} \sqrt{2m(E_{nlj}^{(A-1)} - E_0^{(A)})}
\]
depends on the separation energy \( \epsilon_{nlj} = E_{nlj}^{(A-1)} - E_0^{(A)} \). Obviously, the higher excited states have faster decay. Therefore, for large values \( r = a \to \infty \), eqs. (11) and (12) lead to the asymptotic relation
\[
\rho_j(r,a) \to \phi_{nlj}(r) \times C_{nlj} \exp(-k_{nlj} a)/a . \tag{13}
\]

The procedure of eqs. (12) is based on the generally accepted asymptotic behavior of the neutron overlap functions associated with the bound states of the \((A - 1)\) system.

The one–body density matrix has a quite similar form in its natural orbital representation
\[
\rho(r,r') = \sum_{\alpha} N_{\alpha} \psi_{\alpha}^*(r) \psi_{\alpha}(r') , \tag{7}
\]
where the natural orbitals \( \psi_{\alpha} \) are defined as a complete orthonormal set of functions which diagonalize the one-body density matrix \( \rho \) with eigenvalues \( N_{\alpha} \) called natural occupation numbers.

In the case of a spherical ground state \( \Psi^{(A)} \) with \( 0^+ \) angular momentum and parity, each eigenstate \( \Psi^{(A-1)}_{\alpha} \) is characterized by the ‘single–particle’ quantum numbers \( ljm \), i.e., \( \alpha \equiv nljm \) with \( n \) being the number of the \( ljm \)–state. The overlap functions and the natural orbitals then factorize into radial and spin–angular parts
\[
\phi_{\alpha}(r) = \phi_{nlj}(r) \ Y_{ljm}(\Omega,\sigma) , \quad \psi_{\alpha}(r) = \psi_{nlj}(r) \ Y_{ljm}(\Omega,\sigma) ,
\]
where \( Y_{ljm}(\Omega,\sigma) = [Y_{lj}(\Omega) \times \chi^{1/2}(\sigma)]_{lj} \) and \( \sigma \) is the spin variable. Using \( \rho(r,r') \), the one–body density matrix reads
\[
\rho(r,r') = \sum_{lj} \rho_{lj}(r,r') \sum_{m\sigma} Y_{ljm}^*(\Omega,\sigma) Y_{ljm}(\Omega',\sigma) .
\]

Due to the spherical symmetry, the spectroscopic factor \( S_{nlj} \) and the radial contributions \( \rho_{lj}(r,r') \) entering the one–body density matrix \( \rho \) do not depend on the magnetic quantum number \( m \). From eqs. (3) and (4) then follows that in each \( lj \)–subspace the spectroscopic factor \( S_{nlj} \) is smaller than the largest natural occupation number \( N_{nlj}^{\text{max}} \) with the same \( lj \), i.e.,
\[
S_{nlj} \leq N_{nlj}^{\text{max}} . \tag{10}
\]

The present calculations of the bound–state overlap functions, separation energies and spectroscopic factors have been performed applying the recipe (13) - (18) to a
model one–body matrix [28–30] obtained within the Jas-trow correlation method in its low–order approximation for 

\[ ^{16}O \] and \[ ^{40}Ca \] nuclei. The model is based on harmonic oscillator single Slater determinant and Gaussian–like state–independent correlation factor. Although the resulting density matrix has a simple analytical form, it is physically significant that the short–range correlations are incorporated in it to a large extent. In addition, its natural orbital representation is well investigated [28–30].

Here we should like to mention that the procedure suggested in [27] requires accurate values of the one–body density matrix at large \( r \). In principle this limits the practical application of the method. In our opinion, however, the analytical expressions of the one–body density matrix obtained within some correlation methods give a basis for an easier numerical application of the procedure. This is the case in the present work. Using the model one–body density matrix from [28–30] we have to avoid, however, another difficulty arising from its Gaussian asymptotic behavior. This is achieved by applying the recipe (13) – (15) not for a single asymptotic point \( a \) but within an asymptotic region \((a_L, a_U)\) bracketing the point \( a \) and sustained after the point where the diagonal element \( \rho_{lj}(r, r) \) is less than 10 percent from its maximal value. We are looking for such a radial contribution \( \rho_{nlj}(r, r') = \phi_{nlj}(r') \phi_{nlj}(r) \) whose diagonal part is less than or equal to \( \rho_{lj}(r, r) \) at each point \( r \) and which minimizes the trace \( \text{Tr} \left[ (\rho_{lj} - \rho_{nlj})^2 \right] \). Then \( a_L, a \) and \( a_U \) as well as the unknown \( C_{nlj} \) and \( n_{nlj} \) are uniquely determined by the requirement that the overlap function (16) satisfies eqs. (13) and (14) simultaneously with minimal least–squared deviation within the region \((a_L, a_U)\). We should mention that the procedure suggested is not the unique one, but this problem does not exist when a realistic one-body density matrix with a correct exponential asymptotic behavior is considered.

We have performed the above numerical procedure separately for each set of quantum numbers \( nl \) (the model does not split the states with respect to \( j = l \pm 1/2 \)). It leads to predictions for the neutron separation energies \( \epsilon_{\alpha} \), spectroscopic factors \( S_{\alpha} \) and the overlap functions \( \phi_{\alpha} \) which are given in Table I and Fig. 1, respectively.

It is seen from Table I that the calculated separation energies \( \epsilon_{\alpha} \) are in acceptable agreement with the self–consistent Hartree–Fock (HF) results [28] and the available experimental data. The calculated spectroscopic factors \( S_{\alpha} \), however, differ significantly from their mean–field values. Due to the short–range correlations a depletion of the states below and a filling of the states above the Fermi level results. At the same time the calculated values of \( S_{\alpha} \) are consistent with experimentally deduced spectroscopic factors [3]. In general, the values of \( S_{\alpha} \) emerging from the present restoration procedure are larger than the experimental ones. This fact is most probably related to the crude approximation for the density matrix used. The reason is the same for the larger value of \( S_{2s1/2} \) in comparison with the spectroscopic fac-
tor of the lower \( 1s \)-state in \( ^{40}Ca \).

In Table I we compare \( S_{\alpha} \) also with the natural occupation numbers \( N_{\alpha} \) derived after diagonalizing the same model one–body density matrix [28–30]. The comparison shows that our numerical procedure satisfies the general requirement (11). The trend of the calculated spectroscopic factors \( S_{\alpha} \) follows that of the natural occupation numbers. This result becomes more transparent realizing that the overlap functions \( \phi_{\alpha} \) are rather close to the natural orbitals \( \psi_{\alpha} \) as it is seen from Fig. 1.

From Fig. 1 it can be also seen that all three functions, the overlap, mean–field and natural orbital wave functions, are quite similar for the hole states in nuclei. This justifies the use of shell–model orbitals instead of overlap functions within PWIA calculations (1) for such kind of nuclear states. This approximation, however, is no longer valid for the particle nuclear states where the mean–field wave–functions significantly differ from the overlap functions (see the \( 1f \)-state in Fig. 1). The latter take some intermediate position between the natural orbitals and the HF wave–functions. It should be stressed that our model one–body density matrix is completely different from the Hartree–Fock (HF) one. It has been demonstrated in [28,29,35] that due to the short–range correlations the correlated particle–state orbitals are much more localized than the particle–state mean–field single–particle wave functions. This is the reason why the HF orbitals associated with the particle–states go further out than the overlap functions (or the natural orbitals) associated with the correlated one–body density matrix. The place of the correlated particle-state asymptotic region is affected by the SRC while the HF particle-state orbitals have a larger spread although the orbit is more strongly bound.

The instructive conclusion is that neither natural orbitals nor shell–model wave–function can be used as particle–state overlap functions in the theoretical analysis of the experimental data. The present restoration procedure gives a possible solution of the problem if it is applied to some realistic ground–state one–body density matrices.

Concluding, we have demonstrated in this paper the possibility for restoring the separation energies, spectroscopic factors and overlap functions for bound \((A–1)\)-particle eigenstates on the basis of the ground–state one–body density matrix of the target \( A \)-particle system. Although we have used quite crude approximation for the one–body density matrix [28–30], the asymptotic restoring procedure [28] leads to acceptable quantitative results. Thus one obtains a method for estimating such important quantities as spectroscopic factors and overlap functions which is supplemental to the more involved approaches [10]. For this purpose, one has simply to apply the present restoring procedure to more sophisticated one–body density matrices as for example the ones emerging from Brueckner–Hartree–Fock [10,11], variational Monte–Carlo [12,13] or simplectic model calculations [13]. The resulting bound–state spectroscopic fac-
tors and overlap functions will have more realistic properties and can be used for reliable description of the characteristics of the \((e,e'p), (d,^3He), (\gamma, p)\) and other one-nucleon removal nuclear processes.

ACKNOWLEDGMENTS

This work is supported in part by the Contracts \(\Phi - 406\) and \(\Phi - 527\) with the Bulgarian National Science Foundation.

[1] G. J. Kramer, H. P. Blok, J. F. A. van Hienen, S. Brandenburg, M. N. Harakeh, S. Y. van der Werf, P. W. M. Glaudemans and A. A. Wolters, Nucl. Phys. A477, 55 (1988)
[2] G. van der Steenhoven, H. P. Blok, E. Jans, M. de Jong, L. Lapikás, E. N. M. Quint and P. K. A. de Witt Huberts, Nucl. Phys. A480, 547 (1988); G. van der Steenhoven, H. P. Blok, E. Jans, L. Lapikás, E. N. M. Quint and P. K. A. de Witt Huberts, Nucl. Phys. A484, 445 (1988)
[3] J. W. A. den Herder, H. P. Blok, E. Jans, P. H. M. Keizer, L. Lapikás, E. N. M. Quint, G. van der Steenhoven and P. K. A. de Witt Huberts, Nucl. Phys. A490, 507 (1988)
[4] A. E. L. Dieperink and P. K. A. de Witt Huberts, Annu. Rev. Nucl. Part. Sci. 40, 239 (1990)
[5] P. K. A. de Witt Huberts, J. Phys. G: Nucl. Part. Phys. 16, 507 (1990)
[6] G. J. Kramer, Ph. D. thesis, Universiteit van Amsterdam, 1990
[7] I. Sick and P. K. A. de Witt Huberts, Comm. Nucl. Part. Phys. 20, 177 (1991)
[8] G. van der Steenhoven, Nucl. Phys. A527, 17c (1991)
[9] P. Grabmayer, Prog. Part. Nucl. Phys. 29, 251 (1992)
[10] L. J. de Bever, Ph. D. thesis, Universiteit van Utrecht, 1993
[11] L. Lapikás, Nucl. Phys A553, 297c (1993)
[12] D. G. Ireland and G. van der Steenhoven, Phys. Rev. C49, 2182 (1994)
[13] J. Vernotte, G. Berrier–Rousin, J. Kalifa, R. Tamisier and B. H. Wildenthal, Nucl. Phys. A571, 1 (1994)
[14] I. Bobeldijk, M. Bouwhuis, D. G. Ireland et al., Phys. Rev. Lett. 73, 2684 (1994); I. Bobeldijk, Ph. D. thesis, Universiteit van Utrecht, 1995
[15] K. I. Blomqvist, W. U. Boeglin, R. Böhm et al., Phys. Lett. B344, 85 (1995)
[16] C. Mahaux and R. Sartor, Adv. Nucl. Phys. 20, 1 (1991)
[17] Z. Y. Ma and J. Wambach, Phys. Lett. B256, 1 (1991)
[18] W. H. Dickhoff and H. Müther, Rep. Prog. Phys. 55, 1947 (1992); H. Müther and W. H. Dickhoff, Phys. Rev. C49, R17 (1994); H. Müther, A. Polls and W. H. Dickhoff, Phys. Rev. C51, 3040 (1995);
[19] A. N. Antonov, P. E. Hodgson and I. Zh. Petkov, Nucl. Phys. 20, 1 (1991)
[20] A. N. Antonov, P. E. Hodgson and I. Zh. Petkov, Nucl. Correlations in Nuclei (Spriger–Verlag, Berlin–Heidelberg–New York, 1993)
[21] J. Ryckebusch, M. Vanderhaeghen, K. Heyde and M. Waroquier, Phys. Lett. B350, 1 (1995)
[22] S. Frullany and J. Mougey, Adv. Nucl. Phys. 14, 1 (1984)
[23] T. de Forest Jr., Nucl. Phys. A392, 232 (1983)
[24] T. Berggren, Nucl. Phys. 72, 337 (1965)
[25] J. M. Bang, F. A. Gareev, W. T. Pinkston and J. S. Vaagen, Phys. Rep. 125, 253 (1985)
[26] A. E. L. Dieperink and T. de Forest Jr., Ann. Rev. Nucl. Sci. 25, 1 (1975)
[27] D. Van Neck, M. Waroquier and K. Heyde, Phys. Lett B314, 255 (1993)
[28] M. V. Stoitsov, A. N. Antonov and S. S. Dimitrova, Phys. Rev. C47, R455 (1993)
[29] M. V. Stoitsov, A. N. Antonov and S. S. Dimitrova, Phys. Rev. C48, 74 (1993)
[30] M. V. Stoitsov, A. N. Antonov and S. S. Dimitrova, Z. Phys. A: Atoms and Nuclei, A345, 359 (1993)
[31] P.-O. Löwdin, Phys. Rev. 97, 1474 (1955)
[32] A. N. Antonov, Chr. V. Christov, E. N. Nikolov, I. Zh. Petkov and P. E. Hodgson, Nuovo Cim. A102, 1701 (1989)
[33] A. N. Antonov, I. S. Bonev, Chr. V. Christov, E. N. Nikolov and I. Zh. Petkov, Nuovo Cim. A103, 1287 (1990)
[34] A. N. Antonov, D. N. Kadreov and P. E. Hodgson, Phys. Rev. C50, 164 (1994).
[35] D. S. Lewart, V. R. Pandharipande and S. C. Pieper, Phys. Rev. B37, 4950 (1988)
[36] D. Vaughter and D. M. Brink, Phys. Rev. C5, 626 (1972)
[37] S. Boffi and F. D. Pacati, Nucl. Phys. A204, 485 (1973)
[38] A. Bohr and B. Mottelson, Nuclear Structure (Benjamin, New York, 1969) Vol. 1
[39] C. Mahaux and R. Sartor, Nucl. Phys. A528, 253 (1991)
[40] H. Kümmel, K. H. Lührmann and J. G. Zabolitzky, Phys. Rep. C36, 1 (1978)
[41] R. B. Wiringa, Phys. Rev. C43, 1585 (1991)
[42] S. C. Pieper, R. B. Wiringa and V.R. Pandharipande, Phys. Rev. C46, 1741 (1992)
[43] J. P. Draayer, in Algebraic Approaches to Nuclear Structure (ed. R. F. Casten, Harwood, New York, 1993, p. 423)
TABLE I. Separation energies $\epsilon_\alpha$ and spectroscopic factors $S_\alpha$ calculated on the base of the one–body density matrix [25–30] for $^{16}O$ and $^{40}Ca$. Comparison is made with the Hartree–Fock (HF) single–particle energies (set SkI from [36]), natural occupation numbers $N_\alpha$ [29] and experimental data (EXP).

| nl  | $\epsilon^{HF}_\alpha$ | $\epsilon_\alpha$ | EXP | $S^{HF}_\alpha$ | $S_\alpha$ | $N_\alpha$ | EXP |
|-----|-------------------------|-------------------|-----|-----------------|------------|------------|-----|
| $^{16}O$ | 1s  | 32.96 | 35.82 | 47.$^a$ | 1  | 0.940 | 0.95 |
|      | 1p  | 20.81 | 17.48 | 21.8$^b$ | 1  | 0.953 | 0.965 |
|      | 1d  | 5.31  | 12.76 | 4.14$^d$ | 0  | 0.004 | 0.006 |
| $^{40}Ca$ | 1s  | 41.04 | 32.85 | 56.$^a$ | 1  | 0.763 | 0.89 | 0.75$^d$ |
|      | 1p  | 32.17 | 29.54 | 40.$^a$ | 1  | 0.89 | 0.938 | 0.72$^d$ |
|      | 1d  | 22.16 | 24.75 | 22.38$^c$ | 1  | 0.907 | 0.946 | 0.74$^d$ |
|      | 2s  | 15.67 | 13.07 | 18.2$^c$ | 1  | 0.953 | 0.958 | 0.64$^d$ |
|      | 1f  | 11.25 | 8.69  | 8.36$^c$ | 0  | 0.01 | 0.013 | 0.11$^d$ |

The energies are in MeV and only the states with $j + \frac{1}{2}$ are displayed. Experimental data are taken: $^a$) from [37], $^b$) from [38], $^c$) from [39], $^d$) from [6].

FIG. 1. Overlap functions (solid line), self-consistent Hartree-Fock single–particle wave functions (dot-dashed line) and natural orbitals (dashed line) for the nucleus $^{40}Ca$. 
