Spatial distribution of spin-wave modes in cylindrical nanowires of finite aspect ratio

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Abstract

The spin-wave modes of cylindrical nanowires of moderate diameter-to-length ratios are investigated in this paper. Based on three-dimensional simulations and analytical calculations we determine the spatial structures of the modes. We show that standing spin waves and localized edge modes form the discrete spectrum of the nanowires. Using a simple analytical model we infer an extended dispersion relation for spin waves in cylinders. Considering the variation of the demagnetizing (internal) field we show that the localized dipole-exchange modes at the edges are always present.

1. Introduction

Magnetic nanostructures are intensively studied nowadays for fundamental and practical purposes [1]. Understanding the magnetism in low dimensions presents a fundamental interest, while achieving smaller data storage media and magnetic memory is of utmost technological importance. Large periodic arrays of magnetic nanowires have been easily produced with inexpensive techniques like electrodeposition [2]. For such structures, the manipulation of magnetization in very short times is very important for high-speed applications and requires a complete understanding of the nature of magnetic excitations, magnons or spin waves, and their dependence on the geometry.

The spin-wave spectrum for ellipsoidal samples has been known for some time [3, 4]. In uniform magnetized ellipsoidal samples the demagnetizing field is uniform. As many of the magnetic nanostructures studied nowadays have in general a non-ellipsoidal form, the demagnetizing field inside them is nonuniform even if they are uniformly magnetized [5]. Thus the shape can drastically affect the dynamic properties and the spectrum of the spin waves will be modified (see [6] and references therein). As the dimensions of the wires decrease, the exchange interaction becomes important. We consider here that both dipolar and exchange interactions contribute to the spin-wave spectrum. Obtaining a general theory is challenging, but approximate analytical solutions and numerical results can be derived for certain geometries.

Reducing the dimensions of magnetic nanostructures is the natural trend to obtain smaller devices. Until now, only very long nanowires with diameter-to-length ratios \(d/L\) \(\ll 1\) have been studied. In this paper, we investigate nanowires of moderate aspect ratios magnetized along the axial direction. We develop the analytical theory and present numerical results for the spatial distribution of spin waves modes in this type of nanostructure. We will outline the important features of taking into account the inhomogeneity of the demagnetizing field for the confined geometry under study. Previous studies of cylindrical geometry neglected the nonuniformity of the demagnetizing fields considering the cylinders as semi-infinite [7, 8].

2. Analytical model

We start with a ferromagnetic cylindrical sample, magnetized to saturation along the axial direction (\(z\) direction), which ensures that the longitudinal component of \(M\) is much larger than the transverse ones, \(M = (m_x, m_y, M_s)\). A static magnetic field \(H\) is applied along \(z\) and an rf field \(h(r, t) = h(r)e^{-i\omega t}\) is applied in the perpendicular direction, \(H_{\text{ext}} = (h_x, h_y, H)\). We assume that the rf field and the saturation magnetization \(M_s\) are uniform in the sample, even if the
state of uniform magnetization is not actually realized in this geometry. Two main approaches are used for solving the spectrum of spin waves in confined structures. In both, one solves simultaneously the linearized Landau–Lifschitz equation of motion for the magnetization together with the Maxwell equations satisfying the electromagnetic boundary conditions and the exchange boundary conditions [9]. The two methods are equivalent. We choose here the method of magnetic potential where the relation \( \mathbf{m}(\mathbf{h}) \) is found first from the equation of motion and then we search for the solutions of the Maxwell equations which satisfy the boundary conditions.

The Landau–Lifschitz (LL) equation of motion for the magnetization neglecting damping is

\[
\frac{d\mathbf{M}}{dt}(r, t) = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} \tag{1}
\]

where \( \gamma \) is the gyromagnetic ratio. The effective field inside the sample represents the sum of the external applied field \( \mathbf{H}_{\text{ext}} \), the exchange field \( \mathbf{H}_{\text{exch}} = D\nabla^2 \mathbf{M} \) and the demagnetizing field \( \mathbf{H}_{\text{demag}} = -\hat{N}\mathbf{M} \) (excluding crystal anisotropy):

\[
\mathbf{H}_{\text{eff}} = \mathbf{H}_{\text{ext}} + \mathbf{H}_{\text{exch}} + \mathbf{H}_{\text{demag}}
\]

where \( D \) is the exchange stiffness and \( \hat{N} \) is the demagnetizing tensor field. For non-ellipsoidal bodies the demagnetizing tensor field (in first order) is a function of position and is defined in the Fourier space as in [10]:

\[
N_{ij}(\mathbf{k}) = \frac{D(\mathbf{k})}{k^2} k_i k_j
\]

with \( D(\mathbf{k}) \) the shape function. The LL equation (equation (1)) describes a uniform precession of the magnetization around the effective field \( \mathbf{H}_{\text{eff}} \). To find the normal modes of the magnetization we search for nonzero solutions of the LL equation. Considering the dynamical magnetization uniform in the sample we find

\[
\begin{align*}
\text{i} \omega m_x &= \gamma m_x [H_i - D \nabla^2] - \gamma M_s h_x \\
\text{i} \omega m_y &= \gamma m_y [H_i - D \nabla^2] - \gamma M_s h_y
\end{align*}
\]

(4)

with \( N_{zz} \) and \( N_{rr} \) the longitudinal (dc) and transverse (ac) demagnetizing factors and \( H_i = H - 4\pi (N_{zz} - N_{rr})M_s \). Here, we will consider as first order approximation that the demagnetizing tensor field can be diagonalized as \( N_{zz} + 2N_{rr} = 4\pi \).

The demagnetizing field is determined by the Maxwell equations in the magnetostatic limit:

\[
\nabla \times \mathbf{H}_{\text{int}} = 0
\]

\[
\nabla B = \nabla (\mathbf{H}_{\text{int}} + 4\pi \mathbf{M}) = 0
\]

where the internal field is \( \mathbf{H}_{\text{int}} = \mathbf{h} - N_{rr} \mathbf{m} + H - N_{zz} M_s \). From equation (5), the magnetic field may be written as \( \mathbf{h} = -\nabla \Psi_m \), where \( \Psi_m \) is the scalar magnetic potential. Replacing the scalar potential in equation (6) we obtain

\[
-\nabla^2 \Psi_m + (4\pi - N_{rr}) \left( \frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} \right) - N_{zz} \frac{\partial M_s}{\partial z} = 0.
\]

(7)

Equation (7) has the form of an anisotropic Laplace equation or Walker type equation for cylinders with the last term coming from the variation of the demagnetization (internal) field near the edges. In the center of the cylinder, where the demagnetization field is almost constant (here the longitudinal demagnetization field is zero) the Walker equation reduces to the classical equation for cylinders [8]. We considered here only the variation of the longitudinal demagnetizing factor \( N_{zz} \) along \( z \) (see the appendix for more details). The nonuniformity of \( N_{zz} \) strongly affects the mode frequencies as each mode feels a different demagnetizing factor. The full analysis should also take into account the variation of \( N_{rr} \) in the radial direction \( r \), which we neglect here as \( N_{rr} \) is almost constant with \( r \) due to the fact that our cylinders have a reduced aspect ratio (diameter-to-length less than 0.1) and such an analysis will introduce a high degree of analytical complexity.

We state that we made the following simplifying assumptions: the static magnetization is considered as constant and independent of the external field and the dc and ac demagnetizing fields are the same and can be calculated with the expressions given in equation (3). Even if the non-diagonal elements of the demagnetizing tensor field are not zero, they can be overlooked as they are negligible (the \( N_{zz} \) component is less than \( 10^{-4} \)). The variation of the demagnetizing factors [10] along the axial direction is shown in the figure 1.

Choosing the scalar potential in the form \( \Psi_m(r, \varphi, z) = J_n(qr) \exp(i\varphi + ikz) \), where \( J_n(qr) \) is the Bessel function of order \( n \), we obtain the transcendental equation (see the appendix)

\[
\begin{align*}
\omega^2 \left( k_z^2 + q^2 - M_s \frac{\partial N_{zz}}{\partial z} \right) &= \gamma^2 D^2 \left( k_z^2 + q^2 \right)^3 \\
+ \gamma^2 D \left( k_z^2 + q^2 \right)^2 [H_i + (4\pi - N_{rr})M_s] \\
+ \gamma^2 D \left( k_z^2 + q^2 \right) H_i [H_i + (4\pi - N_{rr})M_s] \\
- k_z^2 \gamma^2 M_s (4\pi - N_{rr}) H_i - \gamma^2 M_s D (4\pi - N_{rr}) \\
\times k_z^2 (k_z^2 + q^2) - \frac{\partial N_{zz}}{\partial z} \gamma^2 M_s H_i^2 + \frac{\partial^3 N_{zz}}{\partial z^3} 2\gamma^2 M_s D H_i \\
- \gamma^2 D^2 M_s \frac{\partial^5 N_{zz}}{\partial z^5}.
\end{align*}
\]

(8)

In the limit of very long wavelength, when the wavevector has all components equal to zero, we derive the following expression:

\[
-\frac{\partial N_{zz}}{\partial z} \omega_0^2 = -\frac{\partial N_{zz}}{\partial z} \gamma^2 H_i^2 + \frac{\partial^3 N_{zz}}{\partial z^3} 2\gamma^2 M_s D H_i - \gamma^2 D^2 \frac{\partial^5 N_{zz}}{\partial z^5}.
\]

(9)

which takes the form of the Kittel uniform mode [11] if we neglect the higher exchange terms (derivatives of order three and five) and keep only the first derivative term: \( \omega_0 = \gamma^2 H_i \). However, no real uniform mode exists in cylinders of finite aspect ratio as the internal field varies near the edges. In the case of a nonuniform magnetic field we usually assume that a spin wave can propagate with continuously changing wavevector \( k_z(z) \) [12]. At certain points (surface), the wavevector becomes zero and the condition of quasi-static variation of the internal field is no longer satisfied, thus the
The derivatives of the volume of the cylinder, between the turning surfaces at magnetization has a plane-wave character. In almost all the changes in the internal field profile. The dynamic of the magnet and the spin wavevector varies to accommodate the micromagnetic simulations. We can use the mean value of the wavevector of each mode which can be evaluated as \( k_z^l = \frac{l\pi}{L} \), where \( L_z \) is the effective length where the mode is localized in the sample. The spectrum of the spin wave consists of a series of dispersion curves characterized by two quantization numbers \( n \) and \( l \). The quantization parameter \( l \) should be taken with care, it provides here a qualitative description of longitudinal modes which is to be compared with the internal field. This effect, called spin-wave well, was demonstrated in stripes recently [13]. For samples of finite size, the dispersion relation is shape dependent. The complete spin-wave spectrum and the mode shapes depend strongly on the boundary conditions. We expect some degree of pinning at the edges, and this degree of pinning is determined by a competition between the dipolar magnetostatic energy and exchange energy. Apart from the Maxwell boundary conditions, continuity of the magnetization and the magnetic potential, usually a Rado–Weertman type of boundary condition [14, 15] is considered at the cylinder surface:

\[
\frac{\partial m_z}{\partial z} = \pm p \times m = 0
\]

at \( z = \pm L/2 \) with \( L \) the length of the cylinder and \( p \) the so-called pinning parameter which is determined by the effective surface anisotropy \( K_s \) and the exchange stiffness constant \( D \): \( p \propto K_s / D \). This condition implies stationary waves in the \( z \) direction with the particular solution of the type \( m_z = A \sin k_z z + B \cos k_z z \).

The frequency of an eigenmode is constant throughout the magnet and the spin wavevector varies to accommodate the changes in the internal field profile. The dynamic magnetization has a plane-wave character. In almost all the volume of the cylinder, between the turning surfaces at both ends (excluding the edge domains), the variation of the magnetization field is small and usually the averaged value of the components of the magnetization field is used. The derivatives of the \( N_z \) in equation (8) are negligible. For moderate aspect ratio cylinders we can use a simple and intuitive model where the dynamical magnetization takes the form

\[
m_{ln}(r, z) \sim J_0(qr) \cos(k_z^l z)
\]

where \( k_z^l \) is the longitudinal wavenumber for the \( l \)th longitudinal mode (backward geometry) and \( n \) indexes the radial modes. This relation is valid between the turning surfaces, where the internal field is almost constant. In this region, stationary waves in the axial and radial directions are formed and we can have mixed modes. The distance between the turning surfaces, or effective length \( \Delta z \), is considered to be approximately 0.8L (with \( L \) the length of the cylinder). The dispersion curves are characterized by two quantization numbers \( n \) and \( l \). The quantization parameter \( l \) should be taken with care, it provides here a qualitative description of longitudinal modes which is to be compared with the internal field. This effect, called spin-wave well, was demonstrated in stripes recently [13]. For samples of finite size, the dispersion relation is shape dependent. The complete spin-wave spectrum and the mode shapes depend strongly on the boundary conditions. We expect some degree of pinning at the edges, and this degree of pinning is determined by a competition between the dipolar magnetostatic energy and exchange energy. Apart from the Maxwell boundary conditions, continuity of the magnetization and the magnetic potential, usually a Rado–Weertman type of boundary condition [14, 15] is considered at the cylinder surface:

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\[
m_{ln}(r, z) \sim J_0(qr) \cos(k_z^l z)
\]
obtained in the first step (at a given dc magnetic field) and we calculate the spatial distribution of the allowed values for the wavenumber in the edge region. From the plot we observe that the edge mode is not restricted to the actual surface but goes underneath the surface in a parabolic fashion.

3. Micromagnetic simulation

In order to understand the mode structure and to compare the analytical results of section 2.1, a micromagnetic simulation was conducted with the nmag package [16] for a cylinder with the aspect ratio of \( d/L = 0.1 \) \((L = 300 \text{ nm})\). The cylinder was discretized with a cell size of 5 nm. First, the magnetization was relaxed to equilibrium using a large damping parameter \( \alpha = 0.5 \). The equilibrium configuration was then excited adiabatically with a small 20 GHz rf magnetic field in the perpendicular direction, while a constant magnetic field was applied along the cylinder axis. The values used for the parameters are the same as used in the analytical simulation and are those of Ni: \( D = 2 \times 10^{-9} \text{ G cm}^{-2} \), \( \gamma = 188.5 \text{ GHz T}^{-1} \) (g factor of 2.15) and \( M_s = 480 \text{ emu cm}^{-3} \) [8, 17] and a damping parameter of 0.015. We subsequently computed the spatial distribution of the oscillation of the dynamic magnetization by carrying out Fourier transforms on a number of cells along the \( z \) and \( x \) axes of the cylinder. For comparison with previous results, we kept the frequency of the rf magnetic field fixed and we varied the amplitude of the dc magnetic field.

To determine the spatial distribution of the longitudinal modes in a cylindrical nanowire we calculated the amplitude of oscillation of the magnetization at different magnetic fields. The values of the static magnetic field were chosen to correspond to longitudinal modes as calculated with equation (8). In figure 3(a), we show the average of the magnetization profiles along the \( z \)-axis for one dc magnetic field which should correspond to the fifth longitudinal spin-wave mode determined from the analytical model. These profiles are calculated by carrying out Fourier transforms on the magnetization for each nm in the \( z \) direction and the \( x \) direction and extracting the values at 20 GHz. Basically, we compute the amplitude of the dynamical magnetization variation in an \( \text{xz} \)-plane passing through the center of the cylinder (figure 3(b)) and then we average between the curves in the \( x \) direction. This corresponds to the average of a contour plot in an \( \text{xz} \)-plane. As observed, the calculated profile corresponds well with the one expected at this applied external field from the analytical model. The spins which form the longitudinal mode precess mainly in the effective length \( \Delta z \). Another mode appears at the edges of the cylinder where the spins precess at a lower frequency.

In figure 4, the 2D spatial distribution of the magnetization is shown corresponding to the edge modes obtained at two different dc magnetic fields: (a) 1.5 kOe and (b) 2.9 kOe.
In figure 5, a snapshot of the 3D profile of the y component of magnetization is shown after 30 ps. We clearly observe the variation of the dynamical magnetization along the z direction (long axis). The mode is the 3D longitudinal mode, as expected, even though some hybridization effects may exist, the standing wave picture being simplistic [18]. The edge mode is clearly seen, it corresponds to the spins rotating to the transverse right and left handed magnetization density operators) we can rewrite equation (7) as

\[
\nabla^2 \Psi_m - \frac{4\pi - N_r}{2} \left( \frac{\partial}{\partial x} - i \frac{\partial}{\partial y} \right) m_+ + \left( \frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right) m_- + M_s \frac{\partial N_{zz}}{\partial z} = 0. 
\]

(A.1)

Applying the operators \( \left( \frac{\partial}{\partial x} - i \frac{\partial}{\partial y}\right) m_+ \) and \( \left( \frac{\partial}{\partial x} + i \frac{\partial}{\partial y}\right) m_- \) to equation (5), it can be easily shown that

\[
[\omega^2 - \gamma^2 (H_z - D\nabla^2)^2 - (4\pi - N_r)\gamma^2 M_s(H_z - D\nabla^2)] \times \nabla^2 \Psi_m + [\omega^2 - \gamma^2 (H_z - D\nabla^2)^2] M_s \frac{\partial N_{zz}}{\partial z} + (4\pi - N_r)\gamma^2 M_s(H_z - D\nabla^2) \nabla^2 \Psi_m = 0 \quad \text{ (A.2)}
\]

with \( \nabla^2 = \nabla^2 - \frac{\partial^2}{\partial x^2} \).

Replacing the scalar potential in equation (A.2) we obtain equation (8).

**4. Discussion and conclusion**

The spatial distribution of longitudinal and edge modes in cylinders of moderate aspect ratio was calculated in this paper. It provides new insight into the spatial distribution of spin waves in cylindrical nanowires. We used a 2D analytical model and obtained the spin-wave mode frequencies and profiles for the edge modes. The validity of the 2D model was tested through comparison with 3D micromagnetic results. The values for the resonances agree quantitatively although the profiles of the edge modes from the 3D micromagnetics are not exactly parabolic and seem to penetrate for a longer distance beneath the surface (30–50 nm). The differences between the two results are probably due to the assumptions and approximations that were made: in the analytical model the magnetization is considered uniform in the whole cylinder and a correction in the z direction is obtained but the variation of the radial demagnetizing field is not considered (which can be a useful extension of the theory), while in the 3D micromagnetics we break the cylinder into small tetrahedral cells (discretization length of 4.8 nm) and consider that the magnetization is continuous and varies linearly and then the demagnetizing field is computed.

The nanowires used in the simulation have a finite diameter-to-length ratio \( d/L = 0.1 \) and thus the longitudinal modes were well separated and their spatial distribution was resolved. We tried to resolve the spatial distribution of longitudinal modes for a cylinder with \( d/L = 0.02 \) without success, as the modes were hybridized (closer in frequency) and formed a quasi-continuum band. Furthermore, as the demagnetizing field is inhomogeneous it confines the spin waves at the boundaries of the cylinder: the edge modes are always present in finite aspect ratio cylinders. These modes can be observed in experiments. Until now, only modes of ensembles of nanowires have been investigated [17]. To measure the spectrum of an individual nanowire a particularly well suited technique is (ferro)magnetic resonance force microscopy [19, 20]. This local probe technique could validate our results.

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