Anonymous multi-party quantum computation with a third party

Zhaoxu Ji†, Peiru Fan, Atta Ur Rahman, Huanguo Zhang

Key Laboratory of Aerospace Information Security and Trusted Computing, Ministry of Education, School of Cyber Science and Engineering, Wuhan University, Wuhan 430072 China

jizhaoxu@whu.edu.cn

Abstract

We reconsider and modify the second secure multi-party quantum addition protocol proposed in our original work. We show that the protocol is an anonymous multi-party quantum addition protocol rather than a secure multi-party quantum addition protocol. Through small changes, we develop the protocol to propose, for the first time, anonymous multiparty quantum computation with a third party, who faithfully executes protocol processes, but is interested in the identity of the data owners. Further, we propose a new anonymous multiparty quantum protocol based on our original protocol. We calculate the success probability of the proposed protocols, which is also a modification of the success probability of the original protocols.

PACS: 03.65.Ud, 03.67.-a, 03.67.Dd

Keywords: quantum cryptography, secure multi-party quantum computation, anonymous multi-party quantum computation, entanglement swapping

1 Introduction

It is known that the rapid development of quantum information science and technology has seriously threatened the security of classical cryptography. Unconditional security is a significant advantage of quantum cryptography over its classical counterpart, which is provided by the principles of quantum mechanics [1, 2]. Since the first quantum cryptography protocol (the first quantum key distribution protocol, i.e., the well-known BB84 protocol) was proposed, it has attracted extensive attention [1, 2].

As the counterpart of classical secure multi-party computation [3] in quantum mechanics, secure multi-party quantum computation (SMPQC) is an important part of quantum cryptography, which was first studied by Crepeau et al. by proposing a protocol based on verifiable secret sharing [4]. Secure multi-party quantum modulo addition, addition and multiplication are the fundamental operations for the design and analysis of quantum cryptography protocols, especially in the ones of SMPQC protocols. SMPQC means that \( n (n \geq 2) \) users, \( U_1, U_2, \ldots, U_n \), use quantum mechanics as resources to complete the calculation of the value of a public function \( f(x_1, x_2, \ldots, x_n) \) without disclosing their own secret data \( x_1, x_2, \ldots, x_n \), where \( f \) is the function of \( n \) non-negative integer-valued variables of bounded range [4]. So far, SMPQC includes a wealth of research topics, such as quantum secret sharing and quantum private comparison [5].

Anonymous multi-party quantum computation (AMPQC) is another important part of quantum cryptography. The main difference between it and SMPQC is that the users’ data is public to some users or to the public, while the identity of the data owners is confidential. The practical problems solved by AMPQC seem to be more familiar to the public than those solved by SMPQC, such as anonymous voting, ranking and surveying [5], all of which are closely related to public life. In 2007, Vaccaro et al. first studied the AMPQC protocols for anonymous voting and surveying, where each player makes a vote by applying a phase shifting operation on his site [6]. Afterwards, based on the work of Cabello et al. [7], Wang et al. proposed the first self-tallying quantum anonymous voting protocol and generalized it to AMPQC [8], where all players can complete anonymous computation without the help of a third party (e.g., a tally man).

Recently, we proposed two secure multi-party quantum addition protocols based on the entanglement swapping of a d-level cat state and Bell states [9], where multi-party multi-group data addition and multi-party one-group data addition are realized respectively. We calculated the corresponding success probability, and described the generalizations of secure quantum addition in quantum anonymous voting and quantum anonymous ranking, respectively. In this paper, we remark that the second quantum addition protocol in our original work is not a secure multi-party addition protocol but an anonymous multi-party addition protocol. We further develop the protocol to propose
two AMPQC protocols. One is to directly transform the protocol into a protocol for anonymous multi-party multi-
group-data quantum computation (AMMQC); The other is a protocol for anonymous multi-party single-
group-data quantum computation (AMSQLC), which, likes Wang et al.’s work [8], is also based on the work of Cabello et al. [7].

2 The proposed AMPQC protocols

Before describing the two AMPQC protocols, let us introduce the quantum resources adopted. The quantum
Fourier transform on a $d$-level single-particle state is defined as

$$F |k\rangle = \frac{1}{\sqrt{d}} \sum_{r=0}^{d-1} \zeta^{kr} |r\rangle, \quad k = 0, 1, \ldots, d - 1,$$

where $\zeta = e^{2\pi i/d}$. One can construct two sets of non-orthogonal bases

$$\begin{align*}
\mathcal{B}_1 &= \{ |0\rangle, |1\rangle, \ldots, |d - 1\rangle \}
\mathcal{B}_2 &= \{ F |0\rangle, F |1\rangle, \ldots, F |d - 1\rangle \}.
\end{align*}$$

The $d$-level Bell states, which forms an orthonormal basis in qudit systems, can be expressed as

$$|\phi(u, v)\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \zeta^{ju} |j\rangle |j \oplus v\rangle,$$

where the variables $u$ and $v$ run from 0 to $d - 1$, and the symbol $\oplus$ denotes addition modulo $d$ throughout this paper. Easily, one can get

$$|\phi(0, 0)\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} |j\rangle |j\rangle.$$

The Bell state $|\phi(u, v)\rangle$ can be generated by acting on $|\phi(0, 0)\rangle$ with $|U_{(u,v)}\rangle$,

$$|U_{(u,v)}\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \zeta^{ju} |j \oplus v\rangle |j\rangle.$$

Specifically,

$$(I \otimes |U_{(u,v)}\rangle) |\phi(0, 0)\rangle = |\phi(u, v)\rangle,$$

where $I$ is the identity matrix and the symbol $\otimes$ denotes the tensor product throughout this paper.

The $d$-level $m$-particle cat states have the form

$$|\phi(u_1, u_2, \ldots, u_m)\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \zeta^{j u_1} |j \oplus u_2, j \oplus u_3, \ldots, j \oplus u_m\rangle,$$

where each of the labels $u_1, u_2, \ldots, u_m$ runs from 0 to $d - 1$. These cat states are orthonormal and complete [10, 11].

$$\begin{align*}
\{ |\phi(u_0, u_1, \ldots, u_m)\rangle &|\phi(u'_0, u'_1, \ldots, u'_m)\rangle \} = \delta_{u_0u'_0} \delta_{u_1u'_1} \ldots \delta_{u_mu'_m}, \\
|u_0, u_1, \ldots, u_m\rangle &= \frac{1}{\sqrt{d}} \sum_{l=0}^{d-1} \zeta^{-lu_0} |\phi(l, u_1 \oplus u_0, u_2 \oplus u_0, \ldots, u_m \oplus u_0)\rangle.
\end{align*}$$

Swapping the particle with the mark $u_1^l$ in the cat state and the particle with the mark $u_2^l$ in the Bell state, one can get (see Ref. [11] for more details)

$$|\phi(u_0, u_1^l, \ldots, u_m^l)\rangle \otimes |\phi(u_0^l, u_2^l)\rangle$$

$$= \frac{1}{d} \sum_{(u_0^l, u_1^l, \ldots, u_m^l)} \zeta^{u_0^lu_0^l} |\phi(u_0 \oplus u_0^l \otimes v_0, u_1^l, u_2^l \oplus u_0^l \otimes v_1, \ldots, u_m^l \oplus v_m)\rangle \otimes |\phi(v_0, v_1)\rangle,$$

where the symbol $\oplus$ denotes subtraction modulo $d$ throughout this paper.

The expression of $\kappa$-particle $\kappa$-level singlet states [7] is give by

$$|\Psi\rangle = \frac{1}{\sqrt{k!}} \sum_{l_1, l_2, \ldots, l_k \in \mathbb{S}} (-1)^{\omega_0} |l_1 l_2 \cdots l_k\rangle,$$

where $\mathbb{S}$ denotes the permutation of $\{0, 1, \ldots, \kappa - 1\}$, and $\omega_0$ the inverse number of $\mathbb{S}$ (i.e., the number of transpositions of pairs of elements in $\mathbb{S}$ which are used to place the elements in ascending order.)
3 Protocol I: AMMQC

Let us introduce the preliminaries of the protocol. There are $n$ mutually distrustful participants labeled $P_1, P_2, \ldots, P_n$, and that each party $P_i$ has a string of secret data $S_i = \{x_i^1, x_i^2, \ldots, x_i^l\}$, where $l_i$ denotes the number of secret data in $S_i$ and 

$$x_i^j \in \{0, 1, 2, \ldots, \xi\}, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, l_i. \quad (11)$$

Here, the value of $\xi$ is selected according to an actual situation. For example, if the full score of all subjects in an exam is 100, then set $\xi = 100$. $P_i$ arranges the data values in descending order as follows:

$$y_i^1, y_i^2, \ldots, y_i^k_i, \quad k_i = \{z \in \mathbb{Z}^+, \ 1 \leq z \leq l_i\},$$

where the superscript $k_i$ indicates the number of different numbers in $S_i$. $P_i$ counts the number of data equal to $y_i^1, y_i^2, \ldots, y_i^k_i$, respectively, and marks the numbers as

$$\eta_i^1, \eta_i^2, \ldots, \eta_i^k_i.$$

A semi-honest third party (named TP throughout this paper), who executes the protocol honestly but is curious about the identity of the data owners, help the participants calculate the public function

$$f \{x_1^1, x_1^2, \ldots, x_1^l_1, x_2^1, x_2^2, \ldots, x_2^l_2, \ldots, x_n^1, x_n^2, \ldots, x_n^l_n\},$$

in which the value of $f$ is invariant under the permutation of the data. We now describe the protocol steps.

**Step 1.** $P_i$ prepares $\xi + 1$ copies of $d$-level Bell state $|\phi(0, 0)\rangle$, where 

$$d \in \{z \in \mathbb{Z}^+, \ 1 > \max \{l_1, l_2, \ldots, l_n\}\},$$

and marks them by 

$$|\phi(0, 0)\rangle_\xi,$$

where $r = 0, 1, \ldots, \xi$ throughout this protocol.

TP prepares $\xi + 1$ copies of $d$-level $(n + 1)$-particle cat states $|\phi(0_1, 0_2, \ldots, 0_n)\rangle$, and marks them by 

$$|\phi(0_1, 0_2, \ldots, 0_n)\rangle_\xi.$$

Then, TP takes the particles with the marks $u_1^0, u_1^1, \ldots, u_n^0$ out from $|\phi(0_1, 0_2, \ldots, 0_n)\rangle$, and arrange them as follows:

$$\left(\begin{array}{c}
(0_1, 0_2, \ldots, 0_1) \\
(0_2, 0_2, \ldots, 0_2) \\
\vdots \\
(0_n, 0_n, \ldots, 0_n)
\end{array}\right), \left(\begin{array}{c}
(0_1, 1_2, \ldots, 0_1) \\
(0_2, 1_2, \ldots, 0_2) \\
\vdots \\
(0_n, 1_n, \ldots, 0_n)
\end{array}\right), \ldots, \left(\begin{array}{c}
(0_1, \xi_2, \ldots, 0_1) \\
(0_2, \xi_2, \ldots, 0_2) \\
\vdots \\
(0_n, \xi_n, \ldots, 0_n)
\end{array}\right),$$

in turn, and denotes them by $S_1, S_2, \ldots, S_n$, respectively.

**Step 2.** TP prepares $n$ sets of decoy photons where each decoy photon randomly in one of the states from the set $\mathbb{B}_1$ or $\mathbb{B}_2$. Then he inserts the $n$ sets of decoy photons into $S_1, S_2, \ldots, S_n$, respectively, at random positions and records their insertion positions. Then TP sends the new sequences to $P_1, P_2, \ldots, P_n$ through quantum channels, respectively.

**Step 3.** TP and $P_i$ use the decoy photons to check the channel security in the process of transmitting $S_i$. If he finds eavesdroppers, he terminates the protocol and restarts it. Otherwise, he proceeds to the next step.

**Step 4.** After confirming that there are no eavesdroppers, $P_i$ removes the decoy photons and encodes his secret data which equal to the value $y_i^j$, where $i = 1, 2, \ldots, k_i$. Specifically, he chooses the $d$-level Bell state with the subscript which equals to $y_i^j$, and establishes a variable $w_i^{j}$ and sets $w_i^0 = \eta_i^j$. For the rest of $d$-level Bell states with the subscripts $h_i \in \{0, 1, \ldots, \xi\} \setminus \{y_i^1, y_i^2, \ldots, y_i^k_i\}$, $P_i$ sets $w_i^h = 0$. Subsequently, he generates the $d$-level Bell state

$$|\phi(y_i^j, w_i^j)\rangle = \frac{1}{\sqrt{d}} \sum_{l=0}^{d-1} e^{2\pi i l y_i^j} |l\rangle \oplus |w_i^j\rangle,$$

using the unitary operation shown in Eq. [6].
Step 5. \( P_i \) performs the \( d \)-level Bell state measurement on the particle with the marks \( u'_i \) in the cat state and the particles with the marks \( v'_i \) in his Bell states, and denotes the measurement results as
\[
|\phi(u'_i, \gamma'_i)\rangle. \tag{20}
\]
\( P_i \) sends particles that have not been measured in the Bell states to TP by using decoy photons.

Step 6. All participants cooperate together to calculate the
\[
\mathcal{A'} = \sum_{i=1}^{n} \gamma'_i. \tag{21}
\]
and then announce \( \mathcal{A'} \) to TP publicly. Here, if all participants gather together, they can directly complete the calculation, otherwise they use secure quantum channels to complete the joint calculation.

Step 7. After receiving all cat states, TP measures each cat state and obtains the marks
\[
\left( u'_0 + \sum_{i=1}^{n} k'_i - \sum_{i=1}^{n} l'_i \right) \mod d, u'_1 \oplus w'_1 \oplus \gamma'_1, u'_2 \oplus w'_2 \oplus \gamma'_2, \ldots, u'_n \oplus w'_n \oplus \gamma'_n. \tag{22}
\]
Then he calculates
\[
\sum_{i=1}^{n} (u'_i \oplus w'_i \oplus \gamma'_i) - \sum_{i=1}^{n} u'_i + \mathcal{A'}, \tag{23}
\]
and marks the calculation results as \( \mathcal{R'} \), which means the number of all participants’ data that is equal to \( r \) is \( \mathcal{R'} \).

By adding subscripts, the data of all participants can be marked as
\[
\begin{cases}
0_1, 0_2, \ldots, 0_{\gamma_1}, \\
1_1, 1_2, \ldots, 1_{\gamma_1}, \\
\vdots \\
\xi_1, \xi_2, \ldots, \xi_{\gamma_1}
\end{cases} \tag{24}
\]
such that TP can calculate the function
\[
f(0_1, 0_2, \ldots, 0_{\gamma_1}; 1_1, 1_2, \ldots, 1_{\gamma_1}; \ldots; \xi_1, \xi_2, \ldots, \xi_{\gamma_1}). \tag{25}
\]

4 Protocol II: AMSQC

As before, let us first introduce the preliminaries of the protocol. Suppose that there are \( n \) mutually distrustful participants, \( P_1, P_2, \ldots, P_n \), who have secret data \( x_1, x_2, \ldots, x_n \), respectively, where
\[
x_i \in \{0, 1, 2, \ldots, \xi\}, \ i = 1, 2, \ldots, n. \tag{26}
\]
TP helps the participants complete the calculation of the public function
\[
f(x_1, x_2, \ldots, x_n), \tag{27}
\]
in which the value of \( f \) is invariant under the permutation of the data. The steps of the protocol are given as follows.

Step 1. TP prepares \( n \) groups of decoy photons, and \( n\tau + 1 \) copies of the \( n \)-particle \( n \)-level singlet state \( |P\rangle \) labeled
\[
|P^1\rangle, |P^2\rangle, \ldots, |P^n\rangle. \tag{28}
\]
He takes out the \( i \)-th particle in each state in turn, and then sends them to \( P_i \) with one group of decoy photons.

Step 2. TP and \( P_i \) use the decoy photons to check channel security, and randomly choose \( \tau \) singlet states to verify their authenticity by the entanglement correlation of the singlet state. After confirming that there are no eavesdroppers and that the singlet states are real, all participants perform single-particle measurements on the remaining singlet state. The measurement result is labeled \( \mathcal{M}_i \).
Step 3. TP prepares \( n \) copies of \( d \)-level \((n + 1)\)-particle cat states \( \ket{\phi(u_0', u_1', \ldots, u_n')} \), where \( r = 1, 2, \ldots, n, \)
\[
d \in \{ z \in \mathbb{Z}^+, z > \max |x_1, x_2, \ldots, x_n| \},
\]
and takes \( u_1', u_2', \ldots, u_n' \) out from \( \ket{\phi(u_0', u_1', \ldots, u_n')} \) to constructs the groups
\[
(u_1', u_1'^2, \ldots, u_n'), (u_2', u_2'^2, \ldots, u_n'), \ldots, (u_n', u_n'^2, \ldots, u_n'),
\]
labeled \( S_1, S_2, \ldots, S_n \). Then he sends \( S_i \) to \( P_i \) after inserting decoy photons.

Step 4. Confirming that there are no eavesdroppers, \( P_i \) removes the decoy photons. \( P_i \) prepares \( n \) copies of \( d \)-level Bell state \( \ket{\phi(0, 0)} \), labeled \( \ket{\phi(0, 0)}_i \). He chooses the \( M_i \)-th \( d \)-level Bell state, and establishes a variable \( w_i^h = x_i \). For the rest of \( d \)-level Bell states with the subscripts \( h_i \in \{1, 2, \ldots, n\} \setminus \{ M_i \} \), \( P_i \) sets \( w_i = 0 \). Subsequently, he uses the unitary operation in Eq. (26) to generate the \( d \)-level Bell state
\[
\ket{\phi(v_i', w_i')} = \frac{1}{\sqrt{d}} \sum_{l=0}^{d-1} e^{i l v_i'} \ket{l} \otimes \ket{w_i'}.
\]

Step 5. Like protocol I, \( P_i \) performs the \( d \)-level Bell state measurement on the particle with the marks \( u_i' \) in the cat state and the particles with the marks \( v_i' \) in his Bell states, and denotes the measurement results as \( \ket{\phi(u_i', v_i')} \). \( P_i \) sends particles that have not been measured in the Bell states to TP by using decoy photons.

Step 6. All participants use classical channels to calculate
\[
\mathcal{R}' = \sum_{i=1}^{n} \gamma'_i,
\]
and then announce \( \mathcal{R}' \) to TP.

Step 7. TP makes the same measurements and calculations as protocol I, and denote the calculation results as \( \mathcal{R}'' \). Then, TP calculates the function
\[
f(\mathcal{R}'_1, \mathcal{R}'_2, \ldots, \mathcal{R}'_n),
\]
which is equal to \( f(x_1, x_2, \ldots, x_n) \).

5 Discussion

In what follows we will calculate the success probability of the proposed protocols, and show the applications in several directions of AMPQC.

5.1 Success probability

Let us compute the success probability for protocol I and II, labeled \( \varphi_I \) and \( \varphi_{II} \). We can get
\[
\varphi_I = \prod_{r=0}^{\xi} \prod_{i=1}^{n} \frac{(d - u_i') (u_i' + w_i')}{d^2},
\]
and
\[
\varphi_{II} = \prod_{r=1}^{n} \prod_{i=1}^{n} \frac{(d - u_i') (u_i' + w_i')}{d^2}.
\]
\( \varphi_I \) decreases with the increase of \( d \), thus we let \( d = \max |l_1, l_2, \ldots, l_n| + 1 \) in protocol I according to Eq. 15 and
\( d = \max \{x_1, x_2, \ldots, x_n\} + 1 \) in protocol II according to Eq. 22.
5.2 Several examples

1. Quantum anonymous ranking.
   Quantum anonymous ranking aim to rank the data of at least two participants anonymously [12]. The ranking results are made public at the end of a protocol. The proposed protocol I can be used to rank multi-group data, while protocol II can be used to rank one-group data.

2. Quantum anonymous voting.
   Quantum anonymous voting [6] can be described as follows: n voters labeled \( V_1, V_2, \ldots, V_n \) vote for m candidates labeled \( C_1, C_2, \ldots, C_m \). Protocol I can be applied when each voter has multiple votes, while protocol II only allows each voter to cast one vote for one candidate.

3. Quantum anonymous surveying.
   Quantum anonymous survey [6] is to complete the anonymous addition of data, which can be used to complete social statistics. Different from the binary addition realized by quantum anonymous voting, the data in anonymous survey can be any positive integer.

6 Remarks

We remark that in some cases, the difference between AMPQC protocols and SMPQC protocols is ambiguous. When both have a third party, if the third party in a AMPQC protocol only publishes the calculation results of the public function and does not disclose the data, then the protocol is also a SMPQC protocol. For example, in quantum anonymous voting and quantum anonymous survey protocols, the third party publishes the results after completing the data addition calculation without publishing the value of each data, such that the protocols also belong to secure multi-party quantum addition protocols. We point out that secure multi-party addition is first implemented in the quantum anonymous voting and surveying protocol proposed by Alice et al. Recently, we formally proposed secure multi-party addition as a new primitive in quantum cryptography [9]. Further, we point out that the first secure multi-party quantum addition protocol proposed in our original work is the first secure multi-party quantum addition protocol, while the second addition protocol is both a secure multi-party quantum addition protocol and an anonymous multi-party quantum addition protocol. We call such a protocol which can not only meet the security requirements of SMPQC, but also meet the security requirements of AMPQC as private multi-party quantum computation.

It is known that the modulo addition result can be obtained by performing modulo operation after data addition calculation, but not vice versa. That is, for a set of non-negative integers \( x_1, x_2, \ldots, x_n \), we have

\[
\bigoplus_{i=1}^{n} x_i = \left( \sum_{i=1}^{n} x_i \right) \mod d.
\] (36)

Therefore, in this paper, we call our original proposed protocols addition protocols rather than summation protocols, which aims to distinguish our proposed protocols from existing ones. It must be pointed out that all the existing secure multi-party quantum summation protocols only complete modulo addition. Therefore, we call these protocols secure multi-party quantum modulo addition (SMPQMA) protocols here. The first SMPQMA protocol was proposed by Du et al. in 2007 [13]. Later, Yang et al. proposed the SMPQMA based on quantum Fourier transform in their quantum secret sharing scheme [14] (note that the authors of Ref. [15] did the same and repetitive work). Shi et al. studied SMPQMA based on quantum Fourier transform and inverse quantum Fourier transform [16]. At present, SMPQMA has been widely studied, and more work can be found in Ref. [5].

7 Security Analysis

We here provide simple security analysis for the proposed protocols. For detailed one see [9]. Compared with the attack on secure multi-party computing protocol, the attack on anonymous multi-party computing does not seem to attract too much attention. After all, the data in many anonymous multi-party computing situations is public, and the attacker’s attack is just to know who the data owner is. The decoy photon technology used in the proposed protocol is effective for detecting eavesdroppers [9].
8 Conclusion

We have presented two protocols for anonymous multi-party quantum computation, both of which allow multiple mutually distrustful participants to compute a public function for any non-negative integers. We have calculated success probability and analyzed security. We are currently studying more branches of anonymous multi-party quantum computation. More detailed description of the success probability and security analysis of the proposed protocols can be considered.

References

[1] Galindo, A., & Martin-Delgado, M. A. (2002). Information and computation: Classical and quantum aspects. Reviews of Modern Physics, 74(2), 347.
[2] Gisin, N., Ribordy, G., Tittel, W., & Zbinden, H. (2002). Quantum cryptography. Reviews of modern physics, 74(1), 145.
[3] Yao, A. C. (1982, November). Protocols for secure computations. In 23rd annual symposium on foundations of computer science (sfcs 1982) (pp. 160-164). IEEE.
[4] Crepeau, C., Gottesman, D., & Smith, A. (2002, May). Secure multi-party quantum computation. In Proceedings of the thirty-fourth annual ACM symposium on Theory of computing (pp. 643-652).
[5] Zhang, H. G., Ji, Z. X., Wang, H. Z., & Wu, W. Q. (2019). Survey on quantum information security. China Communications, 16(10), 1-36.
[6] Vaccaro, J. A., Spring, J., & Chefles, A. (2007). Quantum protocols for anonymous voting and surveying. Physical Review A, 75(1), 012333.
[7] Cabello, A. (2002). N-particle N-level singlet states: some properties and applications. Physical review letters, 89(10), 100402.
[8] Wang, Q. L., Yu, C., Gao, F., Qi, H., & Wen, Q. (2016). Self-tallying quantum anonymous voting. Physical Review A, 94(2), 022333.
[9] Ji, Z. X., Zhang H. G., Wang, H. Z., Wu, F. S., Jia, J. W., & Wu, W. Q. (2019). Quantum protocols for secure multi-party summation. Quantum Information Processing, 18(6), 168.
[10] Karimipour, V., Bahraminasab, A., & Bagherinezhad, S. (2002). Entanglement swapping of generalized cat states and secret sharing. Physical Review A, 65(4).
[11] Ji, Z. X., Fan, P. R., & Zhang, H. G. (2022). Entanglement swapping for Bell states and Greenberger-Horne-Zeilinger states in qubit systems. Physica A: Statistical Mechanics and its Applications, 585(1), 126400.
[12] Huang, W., Wen, Q. Y., Liu, B., Su, Q., Qin, S. J., & Gao, F. (2014). Quantum anonymous ranking. Physical Review A, 89(3), 032325.
[13] Du, J. Z., Chen, X. B., Wen, Q. Y., & Zhu, F. C. (2007). Secure multiparty quantum summation. Acta Physica Sinica, 56(11), 6214-6219.
[14] Yang, W., Huang, L., Shi, R., & He, L. (2013). Secret sharing based on quantum Fourier transform. Quantum information processing, 12(7), 2465-2474.
[15] Yang, H. Y., & Ye, T. Y. (2018). Secure multi-party quantum summation based on quantum Fourier transform. Quantum Information Processing, 17(6), 1-17.
[16] Shi, R. H., Mu, Y., Zhong, H., Cui, J., & Zhang, S. (2016). Secure multiparty quantum computation for summation and multiplication. Scientific reports, 6(1), 1-9.