A comparative analysis of control allocation methods applied to autonomous underwater vehicles

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Abstract. The report presents a comparative analysis of various control allocation methods by taking into account an over-actuated propulsion system and actuator constraints for autonomous underwater vehicles. Comparing methods by the size of a feasible control region, compute time, consumed power, and control error is presented. It is shown that the choice of an allocation method significantly affects the quality of control of an underwater vehicle and consumed power during maneuvers.

1. Introduction
The vital task of the motion control of an autonomous underwater vehicle (AUV) is the control allocation between propulsion system elements. This task becomes much more complicated, taking into account the physical limitations of each of the thrusters. Also, AUV’s over-actuated propulsion system leads to a formally infinite variant of combinations of control actions distribution that satisfy the formed AUV motion control law. During working with autonomous underwater vehicles, the last point is significant. In this case, the distribution efficiency directly affects the survey range and time. A large number of articles are devoted to the study [1, 2, 3, 4].

The research aims to analyze and quantitatively compare various approaches to the control allocation of an over-actuated propulsion system according to several parameters: the feasible control region, the compute time of the algorithm, power consumption, and the control error.

2. Problem statement
The design of a control algorithm for an underwater vehicle is often divided into several levels [5]. First, a high-level motion control algorithm is designed to compute a vector of virtual unbounded inputs to the vehicle \( F^c \in \mathbb{R}^m \) from the target and current vehicle states and the control type where \( m \) is the number of independent control axes. Second, the control allocation algorithm is designed to map the vector of virtual input force and torque \( F^c \) to individual thruster forces \( t = [t_1, \ldots, t_p]^T \) (\( p \) is number of actuators) such that the total force and torque generated by all thrusters \( F = Bt \) amount to the commanded virtual input \( F^c \).

The control allocation problem in the general case is reduced to solving an equation of the following form.

\[
F^c = Bt
\]
The searched variable is \( t \in \mathbb{T}_p \), where \( \mathbb{T}_p \) defines a feasible region of the thrust allocation. The feasible region is a hypercube with vertices in all such vectors \( t \) where \( t_i \) is \( t_i^{\text{min}} \) or \( t_i^{\text{max}} \). \( t_i^{\text{min}} \) and \( t_i^{\text{max}} \) represent, respectively, the minimum and maximum thrust of the thruster \( i \).

\( B \) is the thruster configuration matrix. It contains the location and orientation of all thrusters in the vehicle-body fixed frame.

Let the vector of virtual inputs computed by the high-level motion control or vehicle operator be denoted by the generalized force vector \( \mathbf{F}_c = (f_x, f_y, f_z, m_x, m_y, m_z)^T \), here \( f_i \) is the projection of the force onto the \( i \)-axis, and \( m_i \) is the projection of the torque onto the \( i \)-axis in the body-fixed coordinate frame.

The \( x \)-axis is directed along the longitudinal vehicle axis from the vehicle’s stern to forward. The \( y \)-axis leads along the latitudinal vehicle axis from the vehicle’s left side to the right. The \( z \)-axis completes the frame to a right-handed coordinate system.

This leads to the following scalar relationship between \( t \) and the control inputs \( \mathbf{F}_c \) (Equation 2):

\[
\begin{pmatrix}
  f_x \\
  f_y \\
  f_z \\
  m_x \\
  m_y \\
  m_z
\end{pmatrix} =
\begin{pmatrix}
  C^1_x & C^2_x & \ldots & C^p_x \\
  C^1_y & C^2_y & \ldots & C^p_y \\
  C^1_z & C^2_z & \ldots & C^p_z \\
  {[C^1 \times P^1]_x} & {[C^2 \times P^2]_x} & \ldots & {[C^p \times P^p]_x} \\
  {[C^1 \times P^1]_y} & {[C^2 \times P^2]_y} & \ldots & {[C^p \times P^p]_y} \\
  {[C^1 \times P^1]_z} & {[C^2 \times P^2]_z} & \ldots & {[C^p \times P^p]_z}
\end{pmatrix}
\begin{pmatrix}
  t_1 \\
  t_2 \\
  \ldots \\
  t_p
\end{pmatrix}
\]

where:

- \( C^i_j \) is direction cosine of the thruster \( i \) onto \( j \)-axis;
- \( P_i \) is position of thruster \( i \) in the body-fixed coordinate frame.

A control allocation method aims to make such control input \( t \) for a propulsion system elements that will ensure \( \mathbf{F} = \mathbf{F}_c \). If a feasible \( t \) can not be found, the control allocation algorithm is usually required to degrade its performance objectives and search for a control input \( t \) that minimizes the allocation error \( |\mathbf{F} - \mathbf{F}_c| \) (3).

\[
\begin{align*}
\text{min} \quad & ||Q_s|| \\
\text{subject to} \quad & \mathbf{F}_c - B \cdot t = s, t \in \mathbb{T}_p
\end{align*}
\]

where \( s \) is slack variable and \( Q_s \) is a some weight matrix that prioritizes the requirements that should be honored in case the commanded virtual input \( \mathbf{F}_c \) can not be achieved.

For autonomous vehicles, the matrix is often singular due to propulsion system over-actuation. Moreover, the feasible set of thrust \( \mathbb{T}_p \) imposes linear restrictions onto control allocation problem. Given the presented constraints, there are various approaches to solving the allocation problem. Some of them are presented in the next section.

3. Description of Used Methods

In this paper, four approaches to solving a control allocation problem for an underwater vehicle are considered.
3.1. Thrust Reservation

The thrust reservation method’s main feature is the separation of power resource of each thruster in equal parts for each control action where thruster is somehow involved [6, 7]. The control allocation along this control axis is realized within this reservation. This method allows for bypassing thruster limitations by significantly reducing the feasible control region. The method can be written as follows:

\[ t^j_i = \begin{cases} 
B^+_i F^c_i, & B^+_i F^c_i \in \mathbb{T}_i/q \\
\mathbb{T}_i/q, & B^+_i F^c_i \notin \mathbb{T}_i/q
\end{cases} \]

where:
- \( t^j_i \) is contribution to the thruster \( i \) of the control axis \( j \);
- \( q \) is the number of control axes where thruster is involved;
- \( B^+ \) is pseudo-inverse matrix to \( B \) obtained by Moore–Penrose inverse method;
- \( T_i/q \) is the part of thruster \( i \) at \( q \) control axes.

After this operation, a thrust of each thruster is formed by summing over the \( j \) index by the following formula: \( t_i = \sum_{j=1}^{p} t^j_i \).

3.2. Linear Constraints

The method of linear constraints implies the search for such a set of linear compression coefficients \( \mathbf{a} = [a_1, \ldots, a_m] \) for control vector \( F^c \) that ensures localization of \( t \) within feasible control region[8].

\[ F^{lim}_i = \begin{cases} 
F^c_i, & t \in \mathbb{T}^p, t = B^+ F^c \\
\mathbb{R} \setminus \mathbb{T}^p, & t = B^+ F^c
\end{cases} \]

where \( i \) is a separate control axis and \( \mathbf{a} \) is set of compression coefficients within range of \([0, 1]\).

3.3. Linear Optimization

This approach involves optimal solving of the allocation problem by minimizing a linear function under given conditions and constraints [9].

The use of slack variable \( s \) in the equation below ensures that a feasible solution always exists. The slack variable \( s \) has the same dimension as \( F^c \).

\[ \min_{t, s} \left( \sum_{i=1}^{m} q_i s_i + \sum_{j=1}^{p} w_j t_j \right) \times \times \]

subject to

\[ Bt = F^c \]

\[ t_{min} < t < t_{max} \]

where:
- \( q_1, \ldots, q_m \) are positive definite weight coefficients of control axes;
- \( w_1, \ldots, w_p \) are positive definite weight coefficients of propulsion system elements.
3.4. Quadratic Optimization

This approach involves optimal solving of the allocation problem by minimizing a quadratic function under given conditions and constraints. The standard quadratic functional is given below [10].

\[ \min \left( \frac{1}{2} (t^t, s^t) H (t, s) \right) \]

subject to

\[ B t = F_c \]

\[ t_{min} < t < t_{max} \]

where \( H = 2 \cdot \text{diag}(w_1, \cdots, w_p, q_1, \cdots, q_m) \) is diagonal matrix of positive definite weight coefficients of propulsion system elements and control axes.

4. Propulsion System Model

An over-actuated model of the AUV “Galtel” (Figure 1) propulsion system was used for algorithms simulation. The propulsion system consists of five thrusters: four thrusters located at the stern of the vehicle at an angle of \( \alpha = 22.5^\circ \) to the longitudinal axis, and a vertical tunnel thruster located at the forward part of the vehicle.

The propulsion system is conditioned to the need not only for the cruise motion at a velocity of 1.0-1.5 m/s during the side-sonar survey but for position mode during the photo survey at a velocity of 0.0-0.7 m/s as well. Since only cruising modes of operation were investigated within the research, the tunnel thruster was not taken into account in the calculations. Only one longitudinal and two rotational types of movement are possible with the propulsion system and the operating mode. In that case the matrix \( B \) (Equation 2) takes the form described below.

\[
\begin{pmatrix}
  t_x \\
  m_y \\
  m_z
\end{pmatrix} = \cos \alpha \begin{pmatrix}
  1 & 0 & 0 \\
  0 & 1 & 0 \\
  P_x^l - P_y^l & P_x^r - P_y^r & P_x^z - P_y^z
\end{pmatrix} \begin{pmatrix}
  t_l \\
  t_r \\
  t_d \\
  t_u
\end{pmatrix}
\]

where:

- \( t_x, m_y, m_z \) are the force along the longitudinal axis, the moment around the transverse, and normal axis of the vehicle accordingly;
- \( P_i^j \) is projection of the position of the thruster \( j \) onto the axis \( i \) of the coordinate system;
- \( t_l, t_r, t_d, t_u \) are thrusts generated by the left, right, down and up thruster accordingly.
Figure 2. Feasible regions of control for various control allocation methods in the $t_x - m_z$ projection.

5. Results of Computer Simulation

For convenience, a pair of control actions $t_x$ and $m_z$ was considered during the numerical simulation. Figure 2 shows feasible regions of control for various control allocation methods in the $t_x - m_z$ projection which can be obtained by various control allocation methods.

Table 1 illustrates valuations of the operation of the methods in MATLAB by four main parameters: the size of a feasible region, compute time, consumed power, and target force error.

All valuations are given relative to the minimum value of each parameter. The valuations were calculated by the following equation:

$$P_r = \left( \frac{P_a}{P_{a_{\text{min}}}} - 1 \right) \times 100\%$$

where:
- $P_r$ is a value of a parameter in relative form;
- $P_a$ is a value of a parameter in absolute form;
- $P_{a_{\text{min}}}$ is the minimum value of the parameter among different control allocation methods.

The following formula was used to calculate the power consumption[2]

$$P = \sum_{i=1}^{p} k_i t_i^{3/2}$$

where $k_i$ are some coefficients.
The Euclidean norm of the difference between the target and resultant control vectors $||F_c - Bt||$ was used as the norm of the control error.

| Comparison parameter          | Thrust reservation | Linear constraints | Linear optimization | Quadratic optimization |
|------------------------------|--------------------|--------------------|---------------------|------------------------|
| Compute time, %              | 0%                 | +4.6%              | +282.4%             | +51.7%                 |
| Feasible region volume, %    | 0%                 | +68.8%             | +162.1%             | +158.4%                |
| Average power consumption, % | 0%                 | +1.3%              | +18.4%              | +0.6%                  |
| Average Control error, %     | +0.1%              | +0.1%              | +4.6%               | 0%                     |

Table 1. Results of computer simulation.

6. Conclusion
Comparing methods by the size of a feasible set, compute time, consumed power, and control error is presented in the research. It is shown that the choice of an allocation method significantly affects the quality of control of an underwater vehicle and consumed power during maneuvers. It is shown that the control allocation algorithm based on quadratic optimization significantly increases the feasible region of control relative to the thrust reservation and the linear restriction method. Also, it is more energy efficient in comparison with the control allocation method based on linear optimization.

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