Research Article

Theoretical Analysis and Experimental Validation on Galloping of Iced Transmission Lines in a Moderating Airflow

Bing Huo, 1 Xuliang Li, 1 Fujiang Cui, 2 and Shuo Yang 3

1 School of Mechanical Engineering, Tianjin University of Science and Technology, Tianjin 300222, China
2 Department of Mechanics, Tianjin University, Tianjin 300072, China
3 Tianjin Key Lab of Integrated Design and On-line Monitoring for Light Industry & Food Machinery and Equipment, Tianjin 300222, China

Correspondence should be addressed to Bing Huo; huobing@tust.edu.cn

Received 19 March 2021; Accepted 20 May 2021; Published 1 June 2021

Academic Editor: Roberto Nascimbene

Copyright © 2021 Bing Huo et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Galloping of an iced transmission line subjected to a moderating airflow has been analysed in this study, and a new form of galloping is discovered both theoretically and experimentally. The partial differential equations of the iced transmission line are established based on the Hamilton theory. The Galerkin method is then applied on the continuous model, and a discrete model is derived along with its first two in-plane and torsional modes. A trapezoidal wind field model is built through the superposition of simple harmonic waves. The vibrational amplitude is generally observed to be more violent when the wind velocity decreases, except in the 2nd in-plane mode. Furthermore, the influence of the declining wind velocity rates on galloping is analysed using different postdecline wind velocities and the duration of the decline in wind velocities. Subsequently, an experiment has been carried out on a continuous model of an iced conductor in the wind tunnel dedicated for galloping. The first two in-plane modal profiles are observed, along with their response to the moderating airflow. Different declining rates of the wind velocity are also verified in the wind tunnel, which show good agreement with the results simulated by the mathematical model. The sudden increase in the galloping amplitude poses a significant threat to the transmission system, which also improves the damage mechanism associated with the galloping of a slender, a long structure with a noncircular cross-section.

1. Introduction

Long, slender structures are widely employed in engineering applications wherein the external environment plays a major role, including high-voltage transmission lines, suspended cables, inclined cables, and deep-sea mooring. Under adverse weather conditions such as rainfall or snowfall, the cross-section of these structures becomes noncircular, which causes unstable aerodynamic forces in the airflow and leads to galloping. Since these structures exhibit a light mass, a small damping ratio, and nonlinear factors with regard to their geometry and aerodynamic forces, the galloping trend is observed to be diverse. The galloping of iced transmission lines is the main research topic in this paper.

Since wind flow is the main factor that results in galloping, the mean wind velocity has been extensively analysed to acquire aerodynamic coefficients [1–5] along with its influence on the galloping model [6–9], which led to the elucidation of the galloping mechanism. Nonuniform wind velocities were then investigated to promote and enrich the galloping analyses of iced transmission lines including the turbulent [10–12], stochastic [13, 14], fluctuating [15], and sinusoidal wind fields [16]. However, the actual condition of a transient period associated with wind stopping has been rarely analysed and simulated.

Wind tunnel test is a reliable method to analyse the impacts of the wind loads acting on the conductor. On one hand, these tests mainly focused on the aerodynamic coefficients with different cross-sectional shapes [17, 18], ice thicknesses [19], wind attack angles [20, 21], and cross-section areas [22]. On the other hand, the tests were mostly performed on a truncated conductor model [23, 24], which cannot accurately reflect the galloping profiles of different galloping modes; however, conductor galloping involves a
hybrid vibration of different modes, which may even show
different responses to the declining wind velocity.

Therefore, galloping of the iced transmission line sub-
ject to a moderating wind field is analysed, and a wind
tunnel test is performed on a continuous conductor model in
this study. Theoretical analyses have been elucidated in
Section 2; a theoretical galloping model used for determin-
ing the coupling motion of the in-plane and torsional modes is
described in Section 2.1. In Section 2.2, a trapezoidal wind
field is modelled to simulate the decline in the wind velocity.
Numerical simulations are implemented in Section 2.3,
where the declining rate of the wind velocity that effects
galloping is also analysed. A wind tunnel test has been
conducted on a full galloping model, which is described in
Section 3 to validate the theoretically obtained results. The
study has been summarized in Section 4.

2. Theoretical Model

2.1. Establishment of the Dynamic Model. The transmission
line is modelled as a flexible cable with length \( l \), as shown in
Figure 1(a). Further, \( T_0 \) represents the initial configuration as
per the following expression:
\[
y_0 = -\frac{2T_0}{mg} \sin\left(\frac{mg(l - x)}{2T_0}\right).
\]
(1)

\( \Gamma \) denotes the configuration when galloping occurs. \( v(x, t) \)
and \( \theta(x, t) \) represent the in-plane and torsional displace-
ments, respectively. The out-of-plane model has not been
considered in this study because it exhibits a small amplitude
during galloping, which is also not clearly observed in the
subsequent experiment. The ice accretion is assumed to
uniformly attach along the cable; its cross-section is shown
in Figure 1(b). The airflow \( U \) acts in the out-of-plane di-
rection. \( U_r \) represents the wind velocity relative to the
conductor. The relationship between \( U \) and \( U_r \) can be de-
duced as follows:
\[
U_r = \sqrt{U^2 + \left(\frac{D\dot{\theta}}{2}\right)^2}.
\]
(2)

\( F_L \) and \( F_D \) are the aerodynamic lift and drag forces acting
on the conductor under the relative wind; respectively; the
sum of their projection on the \( y \) axis is \( F_y \), which is rep-
resented along with the torsional aerodynamic force \( M \) as
the following matrix forms:
\[
\begin{pmatrix}
F_y \\
M
\end{pmatrix} = \frac{1}{2} \rho U_r^2 D
\begin{pmatrix}
1 & 0 \\
0 & D
\end{pmatrix}
\begin{pmatrix}
r_{y1} & r_{y2} & r_{y3} \\
r_{M1} & r_{M2} & r_{M3}
\end{pmatrix}
\begin{pmatrix}
\alpha \\
\alpha^2
\end{pmatrix},
\]
(3)

where \( D \) is the diameter of the cross-section; \( r_s \) and \( r_M \) are the
aerodynamic coefficients acquired from the wind tunnel test
[3]; \( \alpha \) is the wind attack angle deduced as follows:
\[
\alpha = \theta_0 + \frac{\dot{\theta}}{U} - \frac{D\dot{\theta}}{2U},
\]
(4)

where \( \theta_0 \) is the initial angle of the ice accretion. The partial
differential equations of the iced transmission line, which
have been established based on the Hamilton theory, are of
the following form [25]:

\[
m\ddot{v} + \left(1 + y_{0x}^2\right)^{-1/2} \left(\frac{3}{4} EA(15y_{0x}^2 - 2)y_{0x}v_{xx} + \frac{3}{16} EAy_{0x}(16 - 40y_{0x}^2 - y_{0x}^4)\right)v_{xx} \\
- \left[\frac{1}{16} EA\left(16y_{0x}^2 + 9y_{0x}^4\right) + \frac{1}{2} T_0(2 - 3y_{0x}^2)\right]v_{xx} - F_y = 0,
\]
(5)

where the parameters above are interpreted with the values
given in Table 1.

The Galerkin method is employed to transform the
partial differential equations into ordinary differential
equations. Consequently, the following assumptions are
made:
\[
\begin{align*}
v(x, t) = & \sum_{i=1}^{L} V_i(t) \sin \left(\frac{\pi}{L} x\right), \\
\theta(x, t) = & \sum_{k=1}^{N} \Theta_k(t) \sin \left(\frac{k\pi}{L} x\right),
\end{align*}
\]
(6)

where \( V_i(t) \) \((i = 1, \ldots, L)\) and \( \Theta_k(t) \) \((k = 1, \ldots, N)\) are the
displacements of each mode for the in-plane and torsional
motions, respectively. The first two in-plane modes are
reserved since they are easily stimulated, and their ampli-
tudes are relatively large [25]. Although the displacement of
the torsional mode is small, especially those of the higher-
order torsional modes, the torsional modes have significant
impacts on their corresponding in-plane counterparts [26].
Therefore, in this study, \( L = N = 2 \) and equation (6) is substituted into equation (5). The substituted equations are
then multiplied by each vibrational modal function, on
which integrations are finally performed from 0 to \( l \); the
integral equations are written in the following forms:
\[ \begin{aligned}
\dot{V}_1 + \omega_{v1}^2 V_1 + 2\zeta_v \omega_{v1} \dot{V}_1 &= a_1 V_1^2 + a_2 V_2^2 + a_3 V_3^2 + a_4 V_1 V_2^2 + \frac{1}{s_1} \int_0^l \sin \left( \frac{\pi x}{T} \right) \left( 1 + y_{0x}^2 \right)^{1/2} dx, \\
\dot{V}_2 + \omega_{v2}^2 V_2 + 2\zeta_v \omega_{v2} \dot{V}_2 &= b_1 V_1 V_2 + b_2 V_2^3 + b_3 V_1 V_2^2 + \frac{1}{s_2} \int_0^l \sin \left( \frac{2\pi x}{l} \right) \left( 1 + y_{0x}^2 \right)^{1/2} dx, \\
\dot{\Theta}_1 + \omega_{\theta1}^2 \Theta_1 + 2\zeta_{\theta} \omega_{\theta1} \dot{\Theta}_1 &= \frac{1}{s_1} M \cdot \int_0^l \sin \left( \frac{\pi x}{T} \right) \left( 1 + y_{0x}^2 \right)^{1/2} dx, \\
\dot{\Theta}_2 + \omega_{\theta2}^2 \Theta_2 + 2\zeta_{\theta} \omega_{\theta2} \dot{\Theta}_2 &= \frac{1}{s_2} M \cdot \int_0^l \sin \left( \frac{2\pi x}{l} \right) \left( 1 + y_{0x}^2 \right)^{1/2} dx,
\end{aligned} \]

where \( \omega_{v1}, \omega_{v2}, \omega_{\theta1}, \) and \( \omega_{\theta2} \) are the natural frequencies of the first two in-plane and torsional modes, respectively. \( a_1 - a_4, \)
\( b_1 - b_3, \) and \( s_1 \) and \( s_2 \) are the integral coefficients, as seen in Appendix.

2.2. Modelling of Moderating Wind Field. Trapezoidal wind field is employed in this study to analyse the galloping feature, especially when the wind velocity declines. Consequently, the wind field used is expressed as follows:
\[ U(t) = U_0 + \frac{2h}{\pi \omega t_d} \sum_{n=1}^{\infty} \frac{\sin((2n-1)\omega t_d)}{(2n-1)^2} \cdot \sin((2n-1)\omega t), \]  

(8)

where \( U_0 \) is the mean wind velocity shown in Figure 2; \( h = U_1 - U_2 \), and \( U_1 \) represents the previous mean wind velocity. The postdecline wind velocity is denoted by \( U_2 \); \( \omega = 2\pi/T \), and \( t_d \) is the elapsed time of the wind decline.

2.3. Results and Discussion. The Runge–Kutta method has been employed to simulate equations (7a)–(7d). The galloping behaviours in the moderating wind field from 8 m/s to 0 m/s within 2 seconds are shown in Figure 3. The left \( y \) axis represents the displacement associated with galloping, whereas the right \( y \) axis denotes the wind velocity. The initial values of each mode are selected as 0.01 in the numerical simulation. At the beginning, the wind velocity increases sharply, whereas the amplitudes of the first two in-plane and torsional modes increase extremely slowly. Further, the 1st in-plane mode takes about 1500 s to achieve its maximum values. This denotes that galloping requires energy accumulation along with a constant mean wind velocity.

Nevertheless, all the modal amplitudes (except that of the 2nd in-plane) sharply increase within 2 s when the wind velocity declines. A sudden increase in the amplitude poses a greater threat to the transmission system; however, no obvious impacts are observed in the 2nd in-plane mode. As shown in Figure 4, a sudden increase in the amplitude leads to a minor hysteresis that is associated with the decline in wind velocity. Furthermore, it can be determined that the increased amplitude is elicited by the decline in wind velocity. The amplitude of the in-plane mode increases from 0.215 m to 0.271 m, which is 1.3 times the value in the mean wind velocity. The amplitudes of the first two torsional modes are 32.7 and 4.92 times the previous values before the decline in wind velocity, respectively, which are of greater influences. Figure 5 shows the galloping profiles of the first two in-plane modes. The 1st in-plane mode comprises one crest, while the 2nd in-plane mode has two crests and one node, which are also observed in the subsequent experiments. The blue lines in Figure 5 represent the galloping profiles before the decline in wind velocity. The galloping profiles associated with the decrease in wind velocities are denoted by orange lines. The amplitudes of the galloping profiles are larger than the previous amplitudes in the 1st in-plane vibration, while smaller than those in the 2nd in-plane vibration.

Since galloping exhibits a synchronous behaviour for each mode (except the 2nd in-plane mode) during the decline in wind velocity, the 1st in-plane mode is used in the subsequent simulation due to its significant threat to the transmission system. The influence of the declining wind velocity rate on the galloping behaviour is discussed in this section. Herein, the previous wind velocity \( U_1 \) and the elapsed time \( t_d \) are kept constant, and the postdecline wind velocity \( U_2 \) is selected as 0 m/s, 0.5 m/s, and 1 m/s (Figure 6). To reduce the time taken for performing the calculations, the initial value in the simulation is selected as 0.2. The galloping trends in the three conditions are qualitatively consistent. In addition, a smaller \( U_2 \) value leads to larger wind velocity differences, thereby eliciting larger amplitude increase. Subsequently, \( t_d \) is selected as 0.5 s, 2 s, and 5 s for simulating the decrease in wind velocity from 8 m/s to 0 m/s. Figure 7 shows that the decline in wind velocity for a shorter duration causes a more violent galloping, i.e., it exhibits an amplitude that is up to 1.4 times the previous amplitude. According to the above analysis, significant differences in the wind velocity and the shorter duration of the decline in wind velocity lead to a significant decrease in the declining rate of the wind velocity, which can release more galloping energy. Therefore, it can be deduced that an increase in amplitude is directly proportional to the energy release.

3. Experimental Validation

3.1. Experimental Platform. An iced conductor experiment is performed on the galloping platform (Figure 8). Since a scaled model is difficult to achieve under laboratory conditions, critical factors of galloping are considered and simulated to qualitatively verify the theoretical results obtained above. The essential elements of galloping are slender, long structure, noncircular cross-section, and constant airflow, based on which the experimental system is composed of three parts: a continuous cable model with a noncircular cross-section; a wind tunnel which can provide a stable airflow; and data acquisition system which can measure the displacements on different positions of the cable model.

3.1.1. Continuous Model of the Iced Transmission Line. A steel wire rope is used to simulate the continuous transmission line with the length of 4.8 m. As shown in Figure 9, since the cable with an excessive mass per length is hard to gallop, the diameter of the steel wire rope is selected as 1 mm. However, stimulating a cable with a small diameter is also difficult. Therefore, to demonstrate an evident galloping phenomenon, the diameter is enlarged to 8 mm by using a polystyrene rod, which is light and wrapped on the steel wire rope. Polystyrene is chosen for fabricating the ice accretion, which adheres to the surface of the cable model with a thickness of 2 mm.
Figure 3: Time histories (0–3000 s). (a) 1st in-plane mode. (b) 2nd in-plane mode. (c) 1st torsional mode. (d) 2nd torsional mode.

Figure 4: Continued.
3.1.2. Wind Tunnel. The wind tunnel is streamlined, as shown in Figure 10. The airflow is generated by using 12 three-phase asynchronous motor fans in two rows that are installed at the back of the wind tunnel. Two honeycombs are installed in front of the motor fans to ensure the uniformity and stability of the airflow. The rotational speed of the motor fans is controlled via an electric control gear. Further, the wind velocity can be adjusted from 0 m/s to 5 m/s, which is measured by an anemometer. The cable model is placed in the test section. One end of the cable is fixed to the

![Figure 4: Time histories (1500–1650 s). (a) 1st in-plane mode. (b) 2nd in-plane mode. (c) 1st torsional mode. (d) 2nd torsional mode.](image)

![Figure 5: Galloping profiles. (a) 1st in-plane mode. (b) 2nd in-plane mode.](image)

![Figure 6: Time history of the first in-plane modal galloping with wind speed reduction at $U_1 = 8$ m/s and $t_d = 2$ s. (a) $U_2 = 1$ m/s. (b) $U_2 = 0.5$ m/s. (c) $U_2 = 0$ m/s.](image)
Figure 7: Time history of the first in-plane modal galloping with wind speed reduction at $U_1 = 8$ m/s and $U_2 = 0$ m/s. (a) $t_d = 0.5$ s. (b) $t_d = 2$ s. (c) $t_d = 5$ s.

Figure 8: Galloping platform.

Figure 9: Cross-section of the model.

Figure 10: Wind tunnel.
vertical steel frame; the other end is fixed to the horizontal steel frame around the pulley, which connects a turnbuckle with an S-shaped tension sensor. The turnbuckle is designed to adjust the initial tension, which can be measured using the tension sensor.

3.1.3. Data Acquisition System. Contact sensors have significant impacts on the measurement due to the light mass of the conductor model. Therefore, two noncontact laser sensors (HL-G103-S-J) are used under the cable model. To verify the theoretical results mentioned above, the first two modal profiles should be measured. Based on the existing theory, one sensor should be placed at a point that represents 1/2 of the cable length, where the largest amplitude of the 1\textsuperscript{st} mode occurs. The other sensor is installed at a point that represents 1/4 of the cable length, where the maximum displacement of the 2\textsuperscript{nd} mode appears. Due to the limitations associated with the experimental conditions, the measurement of torsional vibration is yet to be attained.

3.2. Experimental Results. When the mean wind velocity is 0.85 m/s and 1.207 m/s, the iced conductor model is observed to undergo the 1\textsuperscript{st} and 2\textsuperscript{nd} in-plane modal galloping, respectively, whose profiles are shown in Figure 11. The 1\textsuperscript{st}
in-plane modal galloping exhibits a crest and valley at a cable length of 1/2, which becomes a node in the 2\textsuperscript{nd} in-plane modal galloping. Further, the crest and valley of the 2\textsuperscript{nd} in-plane mode show their appearances at cable lengths of 1/4 and 3/4. The profiles are consistence with those depicted by the theoretical equation in Figure 5.

The galloping data of the first two in-plane modes are then collected by the sensor at cable lengths of 1/2 and 1/4, which are shown in Figures 12 and 13, respectively. The galloping frequency of the 2\textsuperscript{nd} in-plane mode is twice that of the 1\textsuperscript{st} in-plane mode (Figures 12(a) and 13(a)); this result is also in accordance with the calculation in the previous work [25]. Subsequently, a sudden increase in the 1\textsuperscript{st} in-plane amplitude is observed when the wind velocity decreases to 0 m/s, which is 1.49 times the previous value (Figure 12(b)). In comparison, the 2\textsuperscript{nd} in-plane mode decays with the decline in wind velocity (Figure 13(b)). Although the torsional modes are yet to be measured, the galloping is found to be unstable beyond the in-plane vibration during the decline in wind velocity. The phenomenon observed in the wind tunnel test is consistent with that obtained via theoretical analysis.

Since the control gear of the motor fans can only adjust the wind velocity, the durations for which the wind velocity changes are yet to be controlled and measured. Therefore, different values of the postdecline wind velocity $U_2$ are used to verify the theoretical results. During the test, the wind velocity is allowed to rapidly decline in order to avoid the influence of the elapsed time $t_d$. As shown in Figure 14, a small $U_2$ value causes a high declining rate of the wind velocity, which leads to a more violent galloping process. This characteristic of galloping is consistent with that obtained via numerical simulation.

4. Conclusions

In this study, a mathematical galloping model of an iced transmission line has been established under a moderating airflow, and its first two in-plane and torsional modes are elucidated. Numerical simulations are used to determine the response of galloping to the declining wind velocity. A wind tunnel test, which has been designed for a continuous model of the iced conductor, is conducted to validate the simulated results mentioned above. The theoretical analysis and experimental data are in accordance with each other qualitatively. The main results of this study are summarized as follows:

1. Galloping can only occur when energy accumulation takes place under a constant mean wind velocity. Meanwhile, a sharp decline in the wind velocity...
causes violent galloping, especially for the torsional modes; however, this decline in the wind velocity rarely influences the 2nd in-plane mode.

(2) Different postdecline wind velocities and the elapsed times corresponding to the decline in wind velocities are used to interpret that a higher declining rate of the wind velocity leads to a more violent galloping.

(3) The simulated results qualitatively agree with the experimental observations for the first two in-plane modes, with regard to the galloping profile, vibrational frequency relationship, galloping feature of each mode under a moderating airflow, and the influence of the declining rate of the wind velocity on the galloping. Therefore, the mathematical model is verified to be feasible and can be used for simulating and predicting the galloping of long, slender structures with noncircular cross-sections.

The mechanism governing the increase in amplitude during the decline in wind velocity is still unclear. However, energy release has been concluded as the reason on account of the galloping trend responding to the wind velocity difference and elapsed time; this aspect will be further researched in subsequent studies.

**Appendix**

\[
\begin{align*}
    s_k &= \int_0^l \sin^2\left(\frac{\pi x}{l}\right) \sqrt{1 + y_0^2} \, dx, \\
    \omega_{v1} &= \frac{1}{ms_1} \int_0^l \sin\left(\frac{\pi x}{l}\right) g_c \, dx, \\
    \omega_{v2} &= \frac{1}{ms_2} \int_0^l \sin\left(\frac{2\pi x}{l}\right) g_c \, dx, \\
    \omega_{d1} &= \frac{1}{l^2 s_1} \int_0^l \sin^2\left(\frac{\pi x}{l}\right) \left(-101\pi^2 \sqrt{1 + y_0^2}\right) \, dx, \\
    \omega_{d2} &= \frac{1}{l^2 s_2} \int_0^l \sin^2\left(\frac{2\pi x}{l}\right) \left(-404\pi^2 \sqrt{1 + y_0^2}\right) \, dx, \\
    a_p &= \frac{1}{ms_1} \int_0^l \sin\left(\frac{\pi x}{l}\right) g_{op} \, dx, \quad p = 1 \sim 3, \\
    b_q &= \frac{1}{ms_2} \int_0^l \sin\left(\frac{2\pi x}{l}\right) g_{eq} \, dx, \quad q = 1 \sim 3, \\
    g_o &= \frac{1}{l^2} EA \pi^2 y_{0x}^2 \sin^2\left(\frac{\pi x}{l}\right) \left(\frac{3}{2} y_0^2 - 1 - \frac{1}{2} y_0^4\right) + \frac{1}{l^2} \pi^2 T_0 \sin^2\left(\frac{\pi x}{l}\right) \left(\frac{3}{2} y_0^2 - 1\right), \\
    g_{o1} &= \frac{9}{2l^4} EA \pi \pi^2 y_{0x}^2 \sin^2\left(\frac{\pi x}{l}\right) \left(2 - 3 y_0^2 + y_0^4\right) \left[1 - \cos^2\left(\frac{\pi x}{l}\right)\right] + \frac{1}{l^2} T_0 \pi^2 \sin^2\left(\frac{\pi x}{l}\right) \left[-36 \cos^2\left(\frac{\pi x}{l}\right)\right] + 9 - \frac{27}{2} y_0^2 + 54 y_0^4 + 10 y_0^4, \\
    g_{o2} &= \frac{3}{4l^4} \pi^2 y_{0x} \sin^2\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi x}{l}\right) \left[EA \left(-4 + 10 y_0^2 - 5 y_0^4\right) + 4 T_0\right], \\
    g_{o3} &= \frac{1}{l^2} \pi^2 y_{0x} \sin^2\left(\frac{\pi x}{l}\right) \cos^2\left(\frac{\pi x}{l}\right) \left[EA \left(-120 y_0^2 + 48 - 96 \cos^2\left(\frac{\pi x}{l}\right) + 240 y_0^2 \cos^2\left(\frac{\pi x}{l}\right) - 120 y_0^4 \cos^2\left(\frac{\pi x}{l}\right) + 60 y_0^4\right]\right] + T_0 \left[96 \cos^2\left(\frac{\pi x}{l}\right) - 48\right].
\end{align*}
\]
$g_{o4} = \frac{1}{l} \pi^3 y_{ox} \sin \left( \frac{\pi x}{l} \right) \cos \left( \frac{\pi x}{l} \right) \left[ EA \left[ -1620 y_{ox}^4 \cos^4 \left( \frac{\pi x}{l} \right) + 1296 \cos^2 \left( \frac{\pi x}{l} \right) - 1296 \cos^4 \left( \frac{\pi x}{l} \right) + 1620 y_{ox}^4 \cos^2 \left( \frac{\pi x}{l} \right) \right] \ight.$
\[+ 3240 y_{ox}^4 \cos^4 \left( \frac{\pi x}{l} \right) - \frac{1215}{4} y_{ox}^4 + \frac{1215}{2} y_{ox}^2 \]
\[- 243 - 3240 y_{ox}^4 \cos^2 \left( \frac{\pi x}{l} \right) + T_0 \left[ 1296 \cos^2 \left( \frac{\pi x}{l} \right) - 1296 \cos^4 \left( \frac{\pi x}{l} \right) + 243 \right] \right],
\]
$g_{e1} = \frac{4}{l} \pi^2 \sin \left( \frac{\pi x}{l} \right) \cos \left( \frac{\pi x}{l} \right) \left[ EA y_{ox}^2 \left( -2 - y_{ox}^4 + 3 y_{ox}^2 + T_0 \left( 3 y_{ox}^2 - 2 \right) \right) \right],
\]
$g_{e2} = \frac{1}{l^3} \pi^3 y_{ox} \sin \left( \frac{\pi x}{l} \right) \cos \left( \frac{\pi x}{l} \right) \left[ EA \left[ -45 y_{ox}^4 \cos^2 \left( \frac{\pi x}{l} \right) + 6 + 90 y_{ox}^2 \cos^2 \left( \frac{\pi x}{l} \right) + \frac{15}{2} y_{ox}^4 - 36 \cos^2 \left( \frac{\pi x}{l} \right) - 15 y_{ox}^2 \right] \right.
\[+ T_0 \left[ 36 \cos^2 \left( \frac{\pi x}{l} \right) - 6 \right] \],
\]
$g_{e3} = \frac{1}{l^3} \pi^3 y_{ox} \sin \left( \frac{\pi x}{l} \right) \cos \left( \frac{\pi x}{l} \right) \left[ EA \left[ -54 \cos \left( \frac{\pi x}{l} \right) + 540 \cos^3 \left( \frac{\pi x}{l} \right) - 720 \cos^5 \left( \frac{\pi x}{l} \right) + 135 y_{ox}^2 - 1350 y_{ox}^4 \cos^2 \left( \frac{\pi x}{l} \right) \right] \right.
\[+ 1800 y_{ox}^4 \cos^4 \left( \frac{\pi x}{l} \right) + 675 y_{ox}^6 \cos^2 \left( \frac{\pi x}{l} \right) - \frac{135}{2} y_{ox}^4 - 900 y_{ox}^6 \cos^2 \left( \frac{\pi x}{l} \right) \left] \right] \left] \right],
\]
$g_{e4} = \frac{1}{l} EA \pi^3 y_{ox} \sin \left( \frac{\pi x}{l} \right) \cos \left( \frac{\pi x}{l} \right) \left[ -3780 y_{ox}^4 \cos^2 \left( \frac{\pi x}{l} \right) + 108 + 135 y_{ox}^4 - 3024 \cos^2 \left( \frac{\pi x}{l} \right) - 25200 y_{ox}^4 \cos^4 \left( \frac{\pi x}{l} \right) \right.
\[+ 12600 y_{ox}^4 \cos^2 \left( \frac{\pi x}{l} \right) + 10800 \cos^2 \left( \frac{\pi x}{l} \right) - 270 y_{ox}^2 - 7560 y_{ox}^4 \cos^2 \left( \frac{\pi x}{l} \right) \right].
\]
\[\text{(A.1)}\]

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

**Acknowledgments**

The authors gratefully acknowledge the support of the National Natural Science Foundation of China (no. 51808389) and the Natural Science Foundation of Tianjin City (nos. 18JCQNJC08000 and 18JCQNJC75300).

**References**

[1] J. P. D. Hartog, "Transmission line vibration due to sleet," *Electrical Engineering*, vol. 51, no. 4, pp. 1074–1076, 2013.
[2] O. Nigol and P. Buchan, "Conductor galloping-Part II torsional mechanism," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-100, no. 2, pp. 708–720, 1981.
[3] Q. Zhang, N. Popplewell, and A. H. Shah, "Galloping of bundle conductor," *Journal of Sound and Vibration*, vol. 234, no. 1, pp. 115–134, 2000.
[4] M. He and J. H. G. Macdonald, "An analytical solution for the galloping stability of a 3 degree-of-freedom system based on quasi-steady theory," *Journal of Fluids and Structures*, vol. 60, pp. 23–36, 2016.
[5] G. Chen, Y. Yang, Y. Yang et al., "Study on galloping oscillation of iced catenary system under cross winds," *Shock and Vibration*, vol. 2017, Article ID 1634292, 16 pages, 2017.
[6] P. Y. M. Desai, A. H. Shah, and N. Popplewell, “Three-degree-of-freedom model for galloping, part i: formulation,” *Journal of Engineering Mechanics*, vol. 119, no. 12, pp. 2404–2425, 1993.

[7] P. Y. M. Desai, N. Popplewell, and A. H. Shah, “Three-degree-of-freedom model for galloping, Part II: solutions,” *Journal of Engineering Mechanics*, vol. 119, no. 12, pp. 2426–2448, 1993.

[8] Z. Yan, Z. Yan, Z. Li, and T. Tan, “Nonlinear galloping of internally resonant iced transmission lines considering eccentricity,” *Journal of Sound and Vibration*, vol. 331, no. 15, pp. 3599–3616, 2012.

[9] B. Liu, K. J. Zhu, X. Q. Sun et al., “A contrast on conductor galloping amplitude calculated by three mathematical models with different DOFs,” *Shock and Vibration*, vol. 2014, Article ID 781304, 10 pages, 2014.

[10] Q. S. Li, J. Q. Fang, and A. P. Jeary, “Evaluation of 2D coupled galloping oscillations of slender structures,” *Computers & Structures*, vol. 66, no. 5, pp. 513–523, 1998.

[11] M. Cai, Q. Xu, L. Zhou, X. Liu, and H. Huang, “Aerodynamic characteristics of iced 8-bundle conductors under different turbulence intensities,” *KSCE Journal of Civil Engineering*, vol. 23, no. 11, pp. 4812–4823, 2019.

[12] Z. H. Liu, C. H. Ding, and J. Qin, “The nonlinear galloping of iced transmission conductor under uniform and turbulence wind,” *Structural Engineering and Mechanics*, vol. 75, no. 4, pp. 465–475, 2020.

[13] M. Cai, X. Yang, H. Huang, and L. Zhou, “Investigation on galloping of d-shape iced 6-bundle conductors in transmission tower line,” *KSCE Journal of Civil Engineering*, vol. 24, no. 6, pp. 1799–1809, 2020.

[14] B. Yan, X. Liu, X. Lv, and L. Zhou, “Investigation into galloping characteristics of iced quad bundle conductors,” *Journal of Vibration and Control*, vol. 22, no. 4, pp. 965–987, 2016.

[15] J.-W. Kim and J.-H. Sohn, “Galloping simulation of the power transmission line under the fluctuating wind,” *International Journal of Precision Engineering and Manufacturing*, vol. 19, no. 9, pp. 1393–1398, 2018.

[16] X. P. Yang, J. Ren, Z. Zhang et al., “Dynamic aerodynamic characteristics of iced conductor with crescent shape under sine wind field,” *Engineer Journal of Wuhan University*, vol. 50, no. 2, pp. 285–289, 2017.

[17] J. Lu, Q. Wang, L. Wang et al., “Study on wind tunnel test and galloping of iced quad bundle conductor,” *Cold Regions Science and Technology*, vol. 160, pp. 273–287, 2019.

[18] G. Alonso, J. Meseguer, A. Sanz-Andrés et al., “On the galloping instability of two-dimensional bodies having elliptical cross-sections,” *Journal of Wind Engineering & Industrial Aerodynamics*, vol. 98, no. 8-9, pp. 438–448, 2010.

[19] X. Liu, M. Zou, C. Wu et al., “Galloping stability and aerodynamic characteristic of iced transmission line based on 3-DOF,” *Shock and Vibration*, vol. 2020, Article ID 8828319, 15 pages, 2020.

[20] X. M. Li, X. C. Nie, Y. K. Zhu et al., “Wind tunnel tests on aerodynamic characteristics of ice-coated 4-bundled conductors,” *Mathematical Problems in Engineering*, vol. 2017, Article ID 1628173, 11 pages, 2017.

[21] W. J. Lou, D. G. Wu, H. W. Xu et al., “Galloping stability criterion for 3-dof coupled motion of an ice-accreted conductor,” *Journal of Structural Engineering*, vol. 146, no. 5, 11 pages, Article ID 04020071, 2020.

[22] J.-X. Li, J. Sun, Y. Ma, S.-H. Wang, and X. Fu, “Study on the aerodynamic characteristics and galloping stability of conductors covered with sector-shaped ice by a wind tunnel test,” *International Journal of Structural Stability and Dynamics*, vol. 20, no. 6, Article ID 2040016, 2020.