Precision studies of Casimir force and short-range gravity employing prototypes of interferometric gravitational wave detectors

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Abstract

We discuss experimental schemes to measure the Casimir force and short-range forces from hypothetical modified gravity with unprecedented sensitivity using highly sensitive prototype gravitational wave detectors as displacement sensors. The finite temperature effects of the Casimir force would be detectable with a sensitivity better than 1% for separation exceeding 30 μm. Constraints on short-range modifications to gravity can be improved in the distance range of 10–100 μm.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

In the last decade, experimental investigations of the Casimir force [1–10] have been taken up with renewed interest especially because of its significance for cosmology and grand unification theories [11–18]. Considerable attention has also been devoted to theoretical investigations of the Casimir force that take into account realistic situations of experiments [17, 19–22]. More recently, the lateral Casimir force in corrugated structures has also been measured [23, 24]. However, the large finite temperature effect of the Casimir force so far remains elusive in experiments due to the lack of sensitivity in the distance range, where the effect is significant, beyond about 5 μm. In contrast, recently the Casimir–Polder force between atoms and a surface has been measured including the finite temperature effects remarkably well [25]. Accurate theoretical/numerical calculations of the Casimir force are essential to be able to compare with experiments and look for new forces in the sub-micron regime. Diverse experimental apparatus ranging from atomic force microscope (AFM) to micro-electro-mechanical systems (MEMS) and highly sensitive torsional pendulums have been used in these experiments.
Extensions to the standard model of particle physics predict the existence of new particles that mediate new forces. String and M-theories that attempt to unify the fundamental forces close to the Planck scale or the theories with large extra dimensions that attempt the unification of the fundamental forces at the TeV scale of electro-weak symmetry breaking predict variations from the inverse-square law of gravity at sub-mm distances. A review of the experimental and the theoretical status of inverse-square-law tests can be found in [14, 26, 27]. These deviations are usually parametrized by the addition of a Yukawa-type correction term to the Newtonian potential. Thus,

\[ U(z) = -\frac{GM}{z} \left( 1 + \alpha e^{-\frac{z}{\lambda}} \right), \]  

where \( G \) is the Newtonian gravitational constant, \( M \) is the mass, \( z \) is the distance in three-dimensional space, \( \alpha \) represents the coupling strength of the new interaction and \( \lambda \) represents its range. One of the more popular extra-dimensional theories is the Randall–Sundrum (RS) brane-world model with five dimensions [28, 29]. In this model, the corrected potential is given by

\[ U_{RS}(z) \approx \frac{GM}{z} \left( 1 + \frac{l_s^2}{z^2} \right), \]  

where \( l_s \) is the RS parameter. Deviations at distance scales larger than millimeter have been ruled out by astrophysical bounds and laboratory experiments. In the sub-mm range the present constraints are less stringent. A laboratory measurement of gravitational force in this range is the best way to place bounds on the predictions of the new theories. Measurements of the Casimir force provide the best constraints in the sub micron distance regime. Compared to these forces of modified gravity, the Casimir force is much larger, given by

\[ F_C = \frac{\pi^2 \hbar c}{240 \lambda^4} \simeq \frac{10^{-7}}{z(\mu m)^2} N. \]  

Therefore, careful subtraction or shielding of the Casimir force is required to arrive at constraints on modified gravity at a scale below 100 \( \mu m \).

Another field of experimental gravitation that has progressed significantly in the last two decades is that of gravitational wave detectors. Interferometric detectors have reached sensitivity levels for displacements of the order of \( 10^{-19} \) m Hz\(^{-1/2} \), limited only by shot noise [30–32] in the frequency band above 100 Hz or so. The proposed advanced detectors will try to beat this limit using quantum squeezing techniques [33–35]. In a low frequency region (<100 Hz), the sensitivity is limited by radiation pressure noise and seismic noise. The high frequency limitation is photon shot noise that scales as \( 1/\sqrt{N} \). The detectors are Michelson-type interferometers with Fabry–Perot cavities in the two arms to amplify the tiny relative displacement of the suspended mirrors. Including the Fabry–Perot enhancement due to the number of foldings equal to the finesse \( F_{FP} \) and the reduction in shot noise due to power recycling with finesse \( F_{PR} \) we can write the shot noise limited displacement sensitivity for an incident light power \( P \) as

\[ \delta x \simeq \frac{\lambda/2}{\sqrt{P/\hbar \nu}} \frac{1}{F_{FP}} \frac{1}{\sqrt{F_{PR}}}. \]  

The displacement sensitivity of a typical gravitational wave interferometer (GWI) is shown in figure 1. While the sensitivity to the strain depends on the arm length of the interferometric detector, the displacement sensitivity is determined by the basic optical and noise-isolation design and even a prototype gravitational wave detector with a short arm length of a few meters is capable of measurements of displacement with a sensitivity below \( 10^{-18} \) m Hz\(^{-1/2} \)}
Figure 1. Typical displacement sensitivity of long baseline gravity wave detectors [32, 36]. They are ideally suited for unprecedented high precision measurements of short-range forces that can be modulated at frequencies above 100 Hz or so for an extended period, as in the case of the Casimir force. With phase sensitive integration of the signal for a few hours, a displacement signal of $10^{-20}$ m can be pulled above the shot noise. This is the basis of our proposed experiment to measure the Casimir force and gravity in the sub-mm range using such detectors as the sensitive force transducer.

2. The experimental proposal

Prototype interferometer detectors have suspended end mirrors that are a few kilograms in weight, determined by considerations of cavity losses, thermal noise and stability of suspension. A static force of $F$ would result in a static angular deflection of $\theta = F/mg$ and a static displacement of $\delta l_s \approx \theta l = Fl/m g$, where $l$ is the length of the mirror suspension. The zero-temperature Casimir force, for example, at the relatively large separation of 100 $\mu$m, where no measurement has been possible, or even foreseen, is about $10^{-15}$ N for 1 cm$^2$ of surface and the static displacement of $\delta l_s \approx 10^{-17}$ m, for a 5 kg mirror on 0.5 m suspension, is well above the sensitivity of the interferometric detector. The finite temperature effect is even larger, about $4 \times 10^{-14}$ N. However, near zero frequency the interferometer detector is noisy and its useful sensitivity starts from about 100 Hz or so. If the force is modulated at high frequency, the response of the mirror decreases as $\delta l_s/\omega^2$, but the sensitivity remains sufficiently high for measurements of the modulated force with very good precision in a range of hitherto unexplored separation, 5–30 $\mu$m. With longer integration, the measurement can be extended to 100 $\mu$m. Such a measurement is significant and important on two counts: (a) it covers a range of distances that has never been explored in any previous measurements and will remain outside the scope of earlier techniques, (b) the finite temperature effect will be detected and studied in great detail for the first time.

The experiment we propose is to measure the force of attraction between one of the suspended mirrors of the GWI and another movable mirror/plate of smaller diameter (which we call the test plate) that is fixed close to it. The separation between the mirror and the
Figure 2. Experimental schematic: one coated plate of large radius is freely suspended while the other coated plate of smaller radius is fixed firmly at the tip of a cantilever spring. The spring is pushed by piezo-electric actuators in order to change the separation between the two plates.

test plate can be adjusted using piezo-electric actuators. Figure 2 shows a schematic of the experimental arrangement. This arrangement would require that the interferometer mirror be coated on both sides; alternatively, a coated lip can be attached to the suspended mirror. In either of the configurations, the equilibrium position of the suspended mirror would be affected by the force of attraction between the suspended mirror and the test plate. When the test plate is modulated about a fixed position, the suspended mirror would be displaced and this would constitute a signal in the GWI. In the following sections we will model the force acting on the suspended mirror and derive the expected displacement.

3. Forces on the suspended mirror

3.1. Casimir force between coated parallel plates

The Casimir force per unit area between parallel metal surfaces at absolute zero temperature is

\[
F_c(z) = -\frac{\pi^2 \hbar c}{240z^4}.
\]

(5)

At any finite temperature, the force per unit area is given by

\[
F_c^T(z) = -\frac{k_B T}{4\pi z^3} \sum_{n=0}^\infty \int_{\nu_n}^\infty dy \frac{y^2}{e^y - 1}, \quad \text{where} \quad x \equiv 4\pi k_B T z / \hbar c
\]

(6)
FT \propto \frac{\xi(3)}{4 \pi} \zeta(3) k_B T^4 z^3 \quad \text{at high } T \quad \text{(i.e. } x \gg 1) \quad \text{with } \xi(3) \approx 1.20206.

Thus, the distance dependence of the force changes from $z^{-4}$ for zero temperature to $z^{-3}$ in the case of finite temperature. The important non-dimensional parameter that distinguishes the domains of high and low temperature is $x = 4 \pi k_B T z / \hbar c$. The finite temperature effect becomes dominant for separations $z$, greater than the thermal wavelength $\lambda_T \approx \hbar c / k_B T$.

Detailed calculations of the finite temperature effects for various configurations have been presented in [19, 20]. These calculations also include corrections due to finite conductivity and surface roughness. All details on the calculation of the Casimir force between real material bodies can be found in the recent monograph [37].

In the proposed experiment, the mirror and the test plate would be at temperatures close to about 300 K; finite temperature corrections become appreciable beyond about 3 \( \mu \)m. Hence, in the distance range of 10–100 \( \mu \)m where we propose to perform the experiment, the Casimir force on the suspended mirror due to the test plate would be given by equation (6). Correction due to the finite conductivity and surface roughness is negligibly small over the range of the proposed experiment. As shown in [20] for gold coatings, the correction due to finite conductivity is about 0.66% at 10 \( \mu \)m separation and scales down to 0.066% at 100 \( \mu \)m separation. Also in this distance range, the Casimir force will be the strongest force acting on the suspended mirror.

3.2. Gravitational force between coated parallel plates

Next to the Casimir force, gravitational interaction between the plates will be the most important. If the suspended mirror is of radius $R$ and thickness $T$ and the test plate is of radius $r$ and thickness $t$, such that $T \gg t$, then in the simplest case, the gravitational force between the plates will be independent of the distance between them and would depend only on their respective densities $\rho_1$, $\rho_2$ and thickness. To the leading order in $T$ and $t$ it is given by

$$F_{\text{grav}}(z) \approx 2 \pi r^2 G \rho_1 \rho_2 T t.$$  

(8)

The mirrors are typically made of a glass substrate coated with metal or dielectric layers whose densities are very different from the substrate. Consider such a mirror of radius $R$ with substrate density $\rho_{\text{sub}}$ and thickness $T$ and coating density $\rho_{\text{coat}}$ and thickness $\Delta$; the gravitational potential due to the plate at a distance, $z$ (figure 3), from the surface of the plate is obtained by integrating the contribution due to various mass elements on the mirror:

$$U_{\text{grav}}(z) = -G \int \frac{\rho(z)}{\sqrt{r^2 + z^2}} 2 \pi r \, dr \, dz$$

(9)

$$= -2 \pi G \int_0^R \frac{dr}{\int_z^{z+T+\Delta} dz} \rho(z) \sqrt{r^2 + z^2}$$

(10)

$$= -2 \pi G \int_z^{z+T+\Delta} dz \rho(z) \left[ \sqrt{R^2 + z^2} - z \right].$$

(11)

For $R \gg z$, the potential is given by

$$U_{\text{grav}}(z) \simeq -2 \pi G R (\rho_{\text{sub}} T + \rho_{\text{coat}} \Delta) - \frac{\pi G \rho_{\text{sub}}}{R} [z + \Delta]^3 - (z + \Delta + T)^3]$$

$$\quad - \frac{\pi G \rho_{\text{coat}}}{R} \left[ z^3 - (z + \Delta)^3 \right] - 2 \pi G [\rho_{\text{sub}} (T^2 + 2(z + \Delta)T) + \rho_{\text{coat}} (\Delta^2 + 2z \Delta)].$$

(12)
The force due to this potential at point \( z \) is

\[
f_{\text{grav}}(z) = -\frac{\partial U}{\partial z}
\]

\[
\simeq -2\pi G \left( \rho_{\text{sub}} z T + \rho_{\text{coat}} z \Delta \right) + \pi G \rho_{\text{sub}} \left( 2T - \frac{2\Delta T + T^2}{R} \right) + \pi G \rho_{\text{coat}} \left( 2\Delta - \frac{\Delta^2}{R} \right).
\]

The gravitational force between this and the test plate of radius \( r \) with substrate density \( \rho_{\text{sub}} \) and thickness \( t \) and coating density \( \rho_{\text{coat}} \) and thickness \( \delta \), placed at a distance \( z \) from the mirror surface, would be obtained by integrating this force over the volume of the test plate. Thus, the gravitational force between the plates will be given by

\[
F_{\text{grav}}(z) = \int f_{\text{grav}}(z') \rho(z') \, dV
\]

\[
= \int_z^{z+t+\delta} dz' \int_0^r \frac{2\pi r \, dr}{2\pi r \, dr}
\]

\[
\simeq \pi Gr^2 \left[ \left( \rho_{\text{sub}} t + \rho_{\text{coat}} \delta \right) \cdot \left\{ \rho_{\text{sub}} \left( 2T - \frac{2\Delta T + T^2}{R} \right) + \rho_{\text{coat}} \left( 2\Delta - \frac{\Delta^2}{R} \right) \right\} \right]
\]

As expected for an inverse-square law, the gravitational force is largely independent of the distance between the plates for \( R \gg z, r \). The dependence on \( z \) is largely due to edge effects and the leading contribution goes as \( z/R \).

### 3.3. Force due to Yukawa-type deviations to gravity

The Yukawa-type correction to Newtonian gravity will lead to a force that will fall exponentially as the range of the interaction \( \lambda \); the coupling strength would be modified
by a factor $\alpha$, the coupling constant of the ‘new force’. Thus, in the simplest case, the force between two plates would be

$$F_{\text{Yuk}}(z) = 2\pi^2 r^2 G\alpha \lambda^2 e^{-\pi/\lambda} \left[ \rho_1 (1 + e^{\pi/\lambda}) \right]\left[ \rho_1 (1 + e^{\pi/\lambda}) \right].$$  \hspace{1cm} (18)

In the presence of coating, this force can be derived from the potential following the same procedure as that of the gravitational force. Thus,

$$F_{\text{Yuk}}(z) = 2\pi^2 r^2 G\alpha \lambda^2 e^{-\pi/\lambda} \left[ \rho_{\text{sub}} e^{-\Delta z/\lambda} (1 - e^{-T/\lambda}) + \rho_{\text{coat}} (1 - e^{-\Delta z/\lambda}) \right]$$

$$\times \left[ \rho_{\text{coat}} (1 - e^{-T/\lambda}) + \rho_{\text{sub}} e^{-T/\lambda} (1 - e^{-T/\lambda}) \right].$$  \hspace{1cm} (19)

### 3.4. Force due to RS-type correction to gravity

The RS-type modification to gravity will give rise to a force that is diminished by $l_0^2$ as compared to gravity and has a slow logarithmic dependence on the separation. The RS-correction term for uncoated plates will be

$$F_{\text{RS}}(z) = 2\pi^2 r^2 G l_0^2 \cdot \rho_1 \ln \left( \frac{z + T + t}{z + T} \right) \cdot \rho_2 \ln \left( \frac{z + t}{z} \right).$$  \hspace{1cm} (20)

For experimental mirrors with coated surfaces, the force can be shown to be

$$F_{\text{RS}}(z) = -2\pi^2 r^2 G l_0^2 \left[ \rho_{\text{coat}}^2 \left\{ \ln \left( \frac{z + T + t}{z + T} \right) - \ln \left( \frac{z + \Delta}{z} \right) \right\} \right]$$

$$+ \rho_{\text{sub}} \rho_{\text{coat}} \left\{ \ln \left( \frac{z + T + \Delta}{z + T + \Delta} \right) - \ln \left( \frac{z + \Delta}{z + \Delta} \right) \right\}$$

$$+ \ln \left( \frac{z + t + \Delta}{z + \Delta} \right) - \ln \left( \frac{z + \Delta}{z + \Delta} \right)$$

$$+ \rho_{\text{sub}}^2 \left\{ \log \left( \frac{z + T + t}{z + \Delta} \right) - \log \left( \frac{z + T + \Delta}{z + \Delta} \right) \right\}. \hspace{1cm} (21)$$

### 4. Expected signals

The forces described above are plotted in figure 4 when the suspended mirror has a radius $R = 10$ cm and is made of glass of thickness $T = 5$ cm, coated with a gold layer of thickness $\Delta = 30$ $\mu$m and the test mirror has a radius $r = 3$ cm and is made of glass of thickness $t = 0.1$ mm, coated with a gold layer of thickness $\delta = 30$ $\mu$m. The displacement of the mirror due to this force would be $\text{Force} \times \frac{L}{m}$ g. GW interferometers are designed to look at changes in the mirror position; however, their noise floor at DC and low frequencies is too large for the signal to be measurable. Thus, in order to create a measurable signal, the separation between the plates will be modulated about a fixed separation. The displacement measured will be proportional to the spatial derivative of the force and the amplitude of the modulation. When we modulate at a frequency $f$, far from the resonance of the detector, the amplitude of the signal is suppressed by $1/f^2$. Figure 5 shows the expected displacement of a mirror of mass $m = 10$ kg, with a suspension length $L = 0.5$ m due to the various forces for a modulation amplitude of $2$ $\mu$m at 200 Hz. The dominant signal is due to the Casimir force with expected displacements at $10$ $\mu$m separation between the plates that are ten times the sensitivity of the GWI. Even though the gravitational force is stronger than the Casimir force for separations larger than $\sim 20$ $\mu$m, it does not contribute to the signal even at separations of $\sim 90$ $\mu$m as it depends very weakly on $z$. The signal to noise ratio will increase
Figure 4. Forces acting on the interferometer mirror.

Figure 5. Expected displacement of the mirror for 2 \( \mu \)m modulation at 200 Hz.
by a factor of 100 if we integrate the signal for $10^4$ s. Hence, even at separations as large as $30 \, \mu m$ the Casimir force can be measured to an accuracy of better than 1%. This would be the most accurate measurement of the Casimir force so far at separations greater than 1 $\mu m$. It might be possible to improve this even further if the useful frequency band of the detector can be brought down to 100 Hz, which is feasible with low frequency isolation techniques [38, 39].

Earlier measurements of the Casimir force at separations larger than 1 $\mu m$ were plagued by electrostatic forces arising due to patch fields, contact potentials and static charges. As documented by every experimenter trying to measure the Casimir force at micron scale separations, the electrostatic forces have to be carefully measured and subtracted to reveal the presence of the Casimir force. Some of the serious concerns regarding this issue have been expressed by Speake *et al* [40], who derive an expression for the dependence of the force due to random variation of the electrostatic potential, on the separation between the plates. Decca *et al* [41] have also shown that the correction due to these effects can be as small as 0.037% even at separations as small as 160 nm and already reduces to 0.027% at 170 nm separation. Thus, for separations larger than 10 $\mu m$ and for the configuration of the plates in the proposed experiment, the gradient of the stray electrostatic forces will be small and hence its contribution to the signal will be almost at the level of the gravitational force. We expect that patch field effects can be measured and corrected for at the required level, at the relatively large separations we plan to make the measurements. At separations beyond 10 $\mu m$, the mirror can be parallel transported once it is aligned parallel by fixing it on a dual flexure stage. The parallelism can also be monitored interferometrically.

The parameters of predicted inverse-square law violating interactions can be constrained by comparing the measured force with the theoretically expected force. A 1% measurement of Casimir forces in the 10–60 $\mu m$ range would place constraints of order $1 \times 10^{-13}$ N, corresponding to an $\alpha$ of 1 for $\lambda = 30 \, \mu m$ on the corrections to the inverse-square law. The existing limits of the violation parameters derived from experiments are shown in figure 6. The parameter space above the curves is excluded by experiments. The existing constraints on Yukawa-type interaction in the range of our experiment are indicated by $\alpha \leq 200$ at $\lambda = 20 \, \mu m$. Our proposed experiment will place limits in the level of $\alpha = 1$ at $\lambda = 30 \, \mu m$, and study the inverse-square law in the range 10–30 $\mu m$. This range of the parameters ($\alpha, \lambda$) has so far not been probed with sensitivity sufficient to significantly constrain particle physics models with implications for the gravitational interaction. By mounting a thin conducting membrane between the two mirrors at a fixed distance, say 5 $\mu m$ from the suspended mirror, the Casimir force between the mirrors can be kept constant while varying the distance to the movable mirror to measure the inverse-square dependence of gravity. By modulating the plate at 200 Hz and integrating for $10^4$ s, Yukawa-type interaction with $\alpha = 200, \lambda = 30 \, \mu m$ would give rise to signals that can be detected to about 5% indicating significant improvement over previous measurements (figure 5).

The best constraints on $\alpha$ in the range of $\lambda$ below 10 $\mu m$ are from measurements of the Casimir force [1, 9]. To improve these constraints it is necessary to measure the Casimir force in this distance range with accuracies better than 0.1%. There is considerable practical difficulty in measurements in this distance range with flat plates since maintaining parallelism of relatively large plates to much less than 1% of their separation, namely 1–10 $\mu m$, is a difficult task. This difficulty can be avoided by replacing the test plate by a convex surface of large radius of curvature, at the cost of the displacement signal. The forces between plates in this scheme can be derived accurately enough for comparison with precision experiments by applying the proximity force approximation [48]. Expressions for the Casimir force, gravity and Yukawa-type correction to gravity for this geometry are derived in [9, 49, 50].
5. Conclusion

We have presented an experimental scheme employing high sensitivity prototype GW interferometer detectors to measure the Casimir force at separations of 10–100 \( \mu \text{m} \) with unprecedented accuracy. Finite temperature effects can be detected and explored in detail by studying the force law as a function of separation and by making measurement at various temperatures between room temperature of about 25 \( ^\circ\text{C} \) and 100 \( ^\circ\text{C} \). Other aspects of the Casimir force, including peculiarities arising with corrugated surfaces that break translational symmetry [23, 51], are expected to be accessible with better sensitivity in our scheme. An experiment to search for hypothetical modifications to the inverse-square law of gravity in the 10–100 \( \mu \text{m} \) range can be performed, and improved limits can be placed on the parameters of inverse-square law violating interactions. These experiments will be taken up in our proposed 3 m interferometer in the Indian gravitational wave research initiative [52].

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