Searching for light WIMPS in view of neutron decay to dark matter

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In the present work we examine the implications on dark matter searches of the possibility of a partial decay of a neutron into a dark matter particle, slightly lighter than itself. Such a scenario recently proposed is required to bridge the discrepancy between the results of two different experiments measuring the life time of the neutron. It was subsequently suggested that this light dark matter candidate could make up the whole of dark matter in the universe. We thus first compute the nucleon cross section based on such models. Then we proceed explore the various signatures appearing in dark matter searches involving nuclear targets, in the case of such a light dark matter candidate.

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I. INTRODUCTION

The combined MAXIMA-1 [1], [2], [3], BOOMERANG [4], [5] DASI [6] and COBE/DMR Cosmic Microwave Background (CMB) observations [7] imply that the Universe is flat [8] and that most of the matter in the universe is Dark [9], i.e. exotic. These results have been confirmed and improved by the recent WMAP [10] and the original as well as the recent Planck data [11]. Combining the data of these quite precise measurements [11] one finds:

$$\Omega_b = 0.04867 \pm 0.00062, \quad \Omega_{\text{CDM}} = 0.2688 \pm 0.01152, \quad \Omega_\Lambda = 0.685_{-0.016}^{+0.018}$$

In addition there exists firm indirect evidence for a halo of dark matter in galaxies and dwarf galaxies from the observed rotational curves, see e.g. the review [12]. Therefore there is room for cold dark matter candidates or WIMPs (Weakly Interacting Massive Particles). Anyway even though there exists indirect evidence for the existence of dark matter at all scales, it is essential to directly detect such matter in order to unravel the nature of the constituents of dark matter. The possibility of such detection, however, depends on the nature of the dark matter constituents and their interactions.

The WIMP’s are expected to have a velocity distribution with an average velocity which is close to the velocity of the sun around the enter of the galaxy, \(v_0 \approx 10^{-3}c\), i.e. they are completely non relativistic. In fact a Maxwell-Boltzmann leads to a maximum energy transfer which is close to the average WIMP kinetic energy \(\langle T \rangle \approx 0.4 \times 10^{-6}mc^2\). Thus for GeV WIMPs the average energy is in the KeV regime, not high enough to excite the nucleus, except in very few cases, but sufficient to make possible the measurement of the nuclear recoil energy. For light dark matter particles in the MeV region, which are also called WIMPs, the average energy that can be transferred is in the eV region.

The event rate for a WIMP-nucleus scattering process can be computed from the following usual ingredients [13]:

i) The elementary WIMP-nucleon cross section.

ii) The knowledge of the WIMP particle density in our vicinity. This is extracted from WIMP energy density in the neighborhood of the solar system, obtained from the rotation curves, dividing it by the assumed WIMP mass.

iii) The WIMP velocity distribution. In the present work we will consider a Maxwell-Boltzmann (MB) distribution in the galactic frame, with a characteristic velocity \(v_0 = 220 \text{ km/s}\) and an upper velocity cut off (escape velocity) of \(2.84v_0\). Then the WIMP velocity is appropriately transformed into the local frame.

In all recoil experiments, like the nuclear measurements first proposed more than 30 years ago [14], one has to face the problem that the process of interest does not have a characteristic feature to distinguish it from the background. So since low counting rates are expected the background is a formidable problem. Some special features of the WIMP-interaction can be exploited to reduce the background problems, such as:
a) the modulation effect: This yields a periodic signal due to the motion of the earth around the sun. Unfortunately this effect, also proposed a long time ago [15] and subsequently studied by many authors [13, 16–24], is small in the case of nuclear recoils.

b) Directional experiments. These are experiments such that not only the energy but also the direction of the recoiling nucleus is measured. The signature now is that there exists a correlation between the data in a given direction of observation and the sun’s direction of motion. As a result, due to the rotation of Earth around its own axes, one may observe a diurnal variation of the data, see e.g. [27]. Such directional data, given enough counts, can be used to discriminate against background, since no known background can show such a correlation.

In spite of the above problems many experimental teams undertook the important task of detecting nuclear recoils in WIMP-nucleus scattering, see e.g. [26–36]. None has been detected, but very stringent limits on the nucleon cross section have been set which can be found in a recent review [37].

The above results combined with theoretical motivations stimulated interest in lower mass WIMPs involving exiting experiments with targets which contain light element, see e.g. CRESST-II [38], CRESST 2017 surface run [39] and CRESST-III [40]. The advent of recent technologies in spherical TPC detectors [41, 42] gave rise to new possibilities for dark matter low threshold detectors, see e.g NEWS-G [43] and TREX-DM [44, 45] with their status summarized in [46].

There has been recently a new interesting scenario suggesting that the whole dark matter can be explained in terms of WIMPs, which are slightly lighter than the neutron. This is due to a new proposal that seems to settle the long inconsistency between two qualitatively different types of direct neutron lifetime measurements known as "bottle" and "beam" experiments. In the first method by averaging [47] various experiments the value:

\[ t_n(\text{bottle}) = (879.6 \pm 0.6) \text{s}. \]  

is reported.

In the second, beam method, the same authors provide an average [48, 49] given by

\[ t_n(\text{beam}) = (888 \pm 2.0) \text{s}. \]  

We will view the discrepancy between the two types of experiments as being at the 4.0\(\sigma\) level as reported. Another source of discrepancy may arise from systematic errors, as emphasized in ref. [49], but we will not elaborate here.

This issue is settled in a scenario in which a free neutron can decay via a baryon violating process to a dark matter fermion with an appropriate branching ratio [47]. This model has been later extended [50] to make this scenario consistent with neutron stars of size larger than two solar masses. In doing so, however, it was possible to show that all dark matter in the universe can, in principle, be made of the type of dark matter needed for neutron decay.

This novel possibility of having WIMPs with precisely known mass very close to the neutron mass, opens the possibility of making the analysis of the detection rates employing in standard detectors much simpler and more precise. It disfavors, of course, heavy nuclear targets due to the fact that the corresponding maximum recoil energy expected in this case is lower than their current energy threshold. Light elements are used, however, in many targets. Even very light WIMPs in the keV region can be detected employing Superfluid Helium [51].

We should mention that, since in this scheme the communication of dark matter with ordinary matter occurs via Higgs exchange, the relevant analysis can benefit from the rich experience gained in the past, when the favored dark matter candidate had been the LSP (Lightest Supersymmetric Particle).

Even, though, the Higgs mechanism can, in principle, lead to both nuclear and electron recoils, in this paper will limit our study to the former.

II. THE PARTICLE MODEL.

Even though the new channel entering neutron decay is accommodated by a heavy scalar \( \Phi \) [47], we will consider a more general model that involves new interactions [50] given by a Lagrangian of the form:

\[ L = \lambda_{q} e^{ijk} u_{Li} u_{Rj} \Phi_k + \lambda_{\chi} \Phi^* \chi \Phi u_{Li} + \lambda_{\phi} \chi \chi + \mu H H \phi + g_{\lambda} \chi \phi + \text{hc}, \]  

(3)
where $\epsilon^{ijk}$ is the completely anti-symmetric symbol, $d_{Rj}$ and $u^c_{Li}$ are the standard model singlet quarks of charge $-1/3$ and $2/3$ respectively and color indices $j$ and $i$. The neutron decay to dark matter takes place via the first 3 interactions, described by the dimensionless couplings $\lambda_q$, $\lambda_\chi$ and $\lambda_\phi$ as originally proposed [47], associated with the scalar fields $\Phi_k$ and its conjugate $\Phi^*$. The fields $\chi$ and $\tilde{\chi}$ are fermion fields. The additional two terms were introduced [50] to make the model viable by curing conflicts with the observation of neutron stars of size larger than two solar masses. The field $H$ is for the Higgs particle and $\phi$ is an additional scalar. The coupling $\mu$ has dimensions of energy, while $g_\chi$ is dimensionless. For a more complete specification of the quantum numbers of the fields the reader is referred to the original work [50].

This model involves a lot of particles beyond the standard model (SM), but its proposers made a proper analysis and it does not lead to any conflicts with existing limits in physics and cosmology.

For dark matter searches the important terms are: i) the last term which connects the dark matter to the scalar, ii) the term before it, which leads to a cubic coupling to the Higgs after the Higgs acquires a vacuum expectation value $v$ with the coupling $\mu v$. In addition we have the coupling of Higgs to the matter fermions with a coupling $m_f/v$. This type of communication via Higgs has previously been studied [52].

The amplitude connected with the diagram of Fig. 1 in the case of quarks takes the form

$$A = g_\chi \frac{\mu}{q^2 + m_\phi^2} \frac{m_q}{m_H^2},$$

with an analogous expression for electrons.

![Diagram](image)

**FIG. 1:** (a) The Feynman diagram associated with the quark interaction of dark matter via a cumulative effect of the scalar $\phi$ coupled to the standard Higgs scalar. Note that the amplitude is independent of the vacuum expectation value $v$ of the Higgs. (b) The corresponding diagram for electron-WIMP scattering.

Using this diagram in the case of the nucleon the cross section is:

$$d\sigma_N = \frac{1}{v} g_\chi^2 \left( \frac{\mu m_N}{v} \right)^2 \frac{1}{m_H^4} \left( \sum_q \frac{m_q}{m_N} |N_q| \right)^2 \frac{1}{(2\pi)^2} q^2 dq d\Omega \delta \left( qv \xi - \frac{q^2}{2\mu_r(p)} \right).$$

where now $v$ is the WIMP velocity and $\xi$ is the cosine of the angle between the velocity of the incoming WIMP and the momentum of the outgoing nucleon, while $\mu_r(p)$ is the reduced mass of the WIMP-nucleon system approximately the nucleon mass $(1/2)m_N$.

Before proceeding further we recall that frequently some authors in dark matter searches, especially in the case of electron detectors [53–56], prefer to use as input some average cross section. This averaging is, of course, important when the amplitude at the quark level is momentum dependent like it is in our case.
Performing the integration over the angles using $\xi$ via the $\delta$ function we obtain:

$$d\sigma_N = \frac{1}{\upsilon^2 g^2} \left( \frac{\mu m_N}{q^2 + m_\phi^2} \right)^2 \frac{1}{m_H} \frac{1}{2\pi} \frac{1}{q dq}. \quad (6)$$

Performing the integration up to an average momentum $\bar{q}$ we find

$$\sigma_N = \frac{\sigma_0^0 g_{\chi H}^2 (\bar{q})}{g_{\chi H}^2 (\bar{q})} \approx \frac{1}{\upsilon^2 g^2} \frac{1}{8} \frac{\mu^2}{m_\phi^2} \frac{\bar{q}^2}{q^2 + m_\phi^2},$$

$$\sigma_N^0 = \frac{m_N^2}{m_H} \left[ \langle N| \sum_q m_q | N \rangle \right]^2 \frac{1}{\pi}. \quad (7)$$

We have replaced $\upsilon$ by $\bar{\upsilon}$, its average over the velocity distribution. Thus for a Maxwell-Boltzmann distribution is given by:

$$\frac{1}{\bar{\upsilon}^2} \approx 0.8 \times 10^6.$$

There may uncertainties arising from the velocity distribution as discussed in section IV. Since $m_\phi$ has been found to be 0.1 eV, while $\bar{q}$ is much larger, we get:

$$g_{\chi H}^2 (\bar{q}) \approx 10^5 g_{\chi m}^2 \frac{\mu^2}{m_\phi^2}. \quad (8)$$

Noting now that in the model employed here $\mu$ takes the value of 0.4 eV and $g_\chi \approx 4 \times 10^{-4}$ we obtain

$$g_{\chi H}^2 (\bar{q}) \approx 25. \quad (9)$$

Thus we can now rewrite the nucleon cross section as

$$\sigma_N = 0.25 \frac{1}{\pi} \frac{m_N^2}{m_H} \left[ \langle N| \sum_q m_q | N \rangle \right]^2. \quad (10)$$

The factor $\langle N| \sum_q \frac{m_q}{m_N} | N \rangle$, the matrix element of the quark mass operator with the nucleon wave function, enters in going from the quark to the nucleon level. It is quite familiar from the days when the LSP was the favored WIMP candidate, see, e.g., ref. 57, the review 58 and references therein. It is a rather complicated procedure, since the heavy quarks contribute significantly, in spite of the fact that they appear with small probability in the nucleon. As a result many calculations have been made to accurately determine it, including lattice gauge techniques, some of them claiming that the uncertainties have been reduced 59. We will select the reasonable value $\langle N| \sum_q \frac{m_q}{m_N} | N \rangle = 0.370$ recently suggested 60 and used in our basic reference 50. This is not far from the value 0.308 obtained by phenomenological and lattice calculations 59. Then using $m_H = 126$ GeV we get

$$\sigma_N = 1.5 \times 10^{-38} \text{cm}^2 = 1.5 \times 10^{-2} \text{pb}. \quad (11)$$

This value appears to be very large compared to that extracted from the exclusion plots of various experiments, see e.g. ref. 37 for a recent review. These exclusions, however, involved heavier WIMPs. On the other hand it is consistent with the value $\sigma_N < 0.8 \times 10^{-37} \text{cm}^2$ presented in a recent analysis of Emken and Kouvaris 61, shown in their FIG. 4, dotted line, appropriate for WIMPs with a mass very close to the nucleon mass. They indicate that they have obtained this value by taking into account the limits extracted in the experiments CRESST-II 38, CRESST 2017 surface run 39, DAMIC 62, XENON1T 34 and the recent CRESST-III results 40, in conjunction with constraints from XQC 63 and CMB 64.

We will thus adopt our calculated value in evaluating the WIMP- nucleus cross sections.
III. ELASTIC WIMP-NUCLEUS SCATTERING

The elastic WIMP-nucleus scattering has been extensively studied. For the benefit of the reader and in order to establish notation we will exhibit the main features of the relevant formalism here.

The differential cross section can be cast in the form:

\[ \frac{d\sigma}{d\Omega} = \frac{1}{\mu_r^2} \frac{\pi \sigma_N}{q^2} \left( \frac{p}{2\mu_r(A)} \right)^2 \left( qv\xi - \frac{q^2}{2\mu_r(A)} \right) \left( \frac{p}{2\mu_r(A)} \right)^2 \left( \frac{2\pi}{2\mu_r(A)} \right)^2 q^2 dq d\xi \]  
(12)

The cross section \( \sigma_N \) is assumed to be known and \( \mu_r(p) \) is the reduced mass of the WIMP-nucleon system and \( \mu_r(A) \) the reduced mass of the WIMP-nucleus, which from now on will be indicated simply as \( \mu_r \). In our case for the coherent scattering, we have

\[ |M(q^2)|^2 = A^2 \left( F \left( \frac{1}{2} b^2 q^2 \right) \right)^2, \]  
(13)

where \( A \) is the number of nucleons in the nucleus, \( b \) is the nuclear size parameter and \( F \) is the nuclear form factor.

Then integrating over the angles, using the \( \delta \) function in the case of \( \xi \), we obtain

\[ d\sigma = \frac{\pi \sigma_N}{\mu_r^2(p)} \frac{1}{2\pi v^2} A^2 F^2(u) m_A dE_R, \quad u = \left( \frac{E_R}{\hbar \omega} \right), \]  
(14)

where \( E_R \) is the recoiling energy of the nucleus and \( \hbar \omega \) the nuclear harmonic oscillator energy. Analogous is the expression for the form factor, if it is determined phenomenologically, e.g. a Helm form factor.

In the present study the nuclear form factor is not important even in the case of a heavy target nucleus. This is due to the fact that the recoil energy is quite low, since the reduced mass is quite small, \( \mu_r \approx m_N \). So it will be neglected.

IV. EXPRESSIONS FOR THE EVENT RATES

Next one has to fold the cross section with the WIMP velocity distribution. We do not, of course, know what exactly the velocity distribution is. In the present work we used a Maxwell- Boltzmann (M-B) distribution with respect to the galactic center, with an upper cut off (escape velocity) as described in the introduction. We know from the past that other velocity distributions obtained in halo models yield distributions that resemble M-B. This has been confirmed by considering velocity distributions obtained in the Eddington approach for reasonable halow models, see e.g. [25] and references therein. We have not used other velocity distributions in this current work, but we are confident that the M-B distribution is accurate to better than 25%.

The folding procedure is well know, see e.g. [52]. Thus the differential event rate is given by

\[ dR = \rho_x \frac{m_N}{m_0} v_0 N_A \sigma_N \left( \frac{m_N}{\mu_r(p)} \right)^2 A^2 \psi(T)dT, \psi(T) = \frac{\rho}{x} (F(u(T)))^2 \Psi_0(y_{min}(T, A, x)), \]  
(15)

with \( T \) the recoil energy in keV and

\[ u(T) = 2.5 \times 10^{-5} A^{4/3}, \quad \rho = \frac{1}{m_N v_0^2} \approx 1, \quad x = \frac{m_x}{A m_N}. \]  
(16)

Furthermore \( y_{min} = \frac{v_{min}}{v_0} \), while \( v_{min} \) is the minimum WIMP velocity required to have an energy \( T \) given \( x \). \( \Psi_0(y_{min}(T, A, X)) \) is obtained from the function

\[ \Psi_0(z) = \frac{1}{2} \left( \text{erf}(1 - z) + \text{erf}(z + 1) + \text{erfc}(1 - y_{esc}) + \text{erfc}(y_{esc} + 1) - 2 \right) \]  
(17)

by setting \( z \rightarrow y_{min}(T, A, x) \) in it, while \( y_{esc} = 2.84 \) is the escape velocity in units of \( v_0 \) and

\[ y_{min}(T, A, x) = \sqrt{\frac{\rho T}{A}} (1 + \frac{1}{x}). \]  
(18)
The function $\Psi_0(z)$ was obtained by folding the velocity distribution in the local frame ignoring the motion of the Earth. erf stands for the error function and erfc stands for the compliment of the error function.

We must mention that one cannot ignore the motion of the Earth. In fact expects a periodic signal in the rate due to the motion of the earth around the sun (the effect of the Earth’s rotation is negligible here). This effect, proposed a long time ago [15], has subsequently been studied by many authors [13, 16–24], see e.g. [65] for details. The modulation of the rate is a bit more complicated, but one finds

$$dR_1 = \frac{\rho_x}{m_N} v_0 N_A \sigma_N \left( \frac{m_N}{\mu_r(p)} \right)^2 A^2 \cos \alpha \psi_m(T, \delta) dT, \quad \psi_m(T, \delta) = \frac{\rho_x}{x} (F(u(T)))^2 \Psi_1(y_{min}(T, A, x, \delta)),$$

where $\alpha$ is the phase of the earth ($\alpha = 0$ around June 3rd) and $\Psi_1(y_{min}(T, A, X, \delta))$ is obtained by setting $z \rightarrow y_{min}(T, A, x)$ in the following function

$$\Psi_1(z, \delta) = \frac{1}{2} \left[ \frac{\text{erf}(1 - z) - \text{erf}(z + 1) - \text{erfc}(y_{min} - 1) - \text{erfc}(y_{max} + 1)}{2} \right. + \frac{e^{-(z-1)^2}}{\sqrt{\pi}} + \frac{e^{-(z+1)^2}}{\sqrt{\pi}} - \frac{e^{-(y_{min} - 1)^2}}{\sqrt{\pi}} - \frac{e^{-(y_{max} + 1)^2}}{\sqrt{\pi}} + 1 \right]$$

$\delta$ is the ratio of the Earth’s velocity around the sun by the velocity of the sun around the center of the galaxy, $\delta \approx 0.135$. Thus

$$\frac{dR}{dT} = \frac{dR_0}{dT} + \frac{dR_m}{dT} \cos \alpha,$$

$$\frac{dR_0}{dT} = \frac{\rho_x}{m_N} v_0 N_A \sigma_N \left( \frac{m_N}{\mu_r(p)} \right)^2 A^2 \psi(T),$$

$$\frac{dR_m}{dT} = \frac{\rho_x}{m_N} v_0 N_A \sigma_N \left( \frac{m_N}{\mu_r(p)} \right)^2 A^2 \psi_m(\delta, T).$$

Similarly the total rates result by integrating the differential rates from $T_{th}$ to $T_{max}$ where $T_{th}$ is the energy threshold in units of keV, which depends on the detector. $T_{max}$ the maximum energy that can be extracted from the WIMP, which depends on the WIMP mass and the velocity distribution mainly via the value of the escape velocity $v_{esc}$. We thus write:

$$R = R_0 + R_1(\alpha), \quad R_1(\alpha) = R_m \cos \alpha,$$

$$R_0 = \frac{\rho_x}{m_N} v_0 N_A \sigma_N \left( \frac{m_N}{\mu_r(p)} \right)^2 A^2 \int_{T_{th}}^{T_{max}} dT \psi(T),$$

$$R_m = \frac{\rho_x}{m_N} v_0 N_A \sigma_N \left( \frac{m_N}{\mu_r(p)} \right)^2 A^2 \int_{T_{th}}^{T_{max}} dT \psi_m(\delta, T).$$

In view of the oscillation of the rates by the $\cos \alpha$, the quantities $\frac{dR}{dT}$ and $R_m$ will be called amplitudes for the differential and total rates respectively.

V. SOME RESULTS

Using the model discussed in section III we can proceed to obtain some numerical results. To proceed further to numerical calculations the following information is needed

$$n = \frac{0.3\text{GeV/cm}^{-3}}{m_N} = 0.32\text{cm}^{-3}, \quad \Phi = n v_0 = 0.32\text{cm}^{-3} \times 220\text{km/s} = 7.0 \times 10^6\text{s}^{-1}\text{cm}^{-2} = 2.2 \times 10^{14} \text{y}^{-1}\text{cm}^{-2}.$$

$$\sigma_N = 1.5 \times 10^{-38}\text{cm}^2, \quad N_A = \frac{\text{Kg}}{A \times 1.67 \times 10^{-27}\text{Kg}} = 6.0 \times 10^{26} \frac{\text{Kg}}{A}.$$
FIG. 2: The time averaged differential event rates $dR_0/dT$ as a function of the recoil energy in keV. (a) for a light system, e.g. $A=3$, (b) for $A=23$, which is a component of the NaI detector, (c) and (d) for the $A=73$ and $A=131$ targets respectively.

| Target (A) | Maximum Recoil Energy (keV) |
|------------|-----------------------------|
| $A=3$      | 1.63                        |
| $A=4$      | 1.39                        |
| $A=16$     | 0.480                       |
| $A=19$     | 0.412                       |
| $A=20$     | 0.393                       |
| $A=23$     | 0.346                       |
| $A=40$     | 0.206                       |
| $A=73$     | 0.116                       |
| $A=131$    | 0.065                       |

Thus for a kg of target and one year of running we find:

$$\frac{\rho_x}{m_N} v_0 N_A \sigma_N = \frac{r_0}{A}, \quad r_0 = 2.0 \times 10^3 \text{y}^{-1}.$$ 

It is well known that, unfortunately for experiments, the time average differential rates do not show any particular structure to differentiate them from the background, see e.g. [52] for heavy WIMPs. We have, however, decided to present the obtained time average differential event rates in Fig. 2 not just for the reader’s convenience, but in order to guide the experiments in the region of low recoil energies, which is crucial for WIMPs with a mass close to the nucleon mass. For purposes of comparison of the expected rates, we present our results for heavy targets, even though they are not expected to be useful for light WIMPs, due to the fact that thresholds achieved are very high, in the few keV regime.

The corresponding modulated event rates do show some structure, which becomes more important at low recoil energies. The obtained results are shown in Fig. 3.

We should note that the amplitude $\frac{dR_1}{dT}$ gets negative at low recoil energies. Then at such recoil energies the maximum occurs in December, rather than in June as expected.

Next we proceed to exhibit the total event rate, which, as we have mentioned, is given by Eq. [22]. The time independent part $R_0$ is shown as a function of the energy threshold of the detector in Fig. 4. One sees that the energy threshold is very crucial for light WIMPs. This is not surprising, since the maximum allowed recoil energies for the 1 GeV WIMP are shown in table I for some nuclei of interest.

We could do the same for the modulated amplitude, but we prefer to exhibit this effect in the ratio of the time dependent $R_1(\alpha)$ divided by the time average rate.
FIG. 3: The same as in Fig. 2 for the modulated differential amplitude \( \frac{dR_m}{dT} \). One sees that the modulated amplitude changes sign at some energy transfer. This has an effect on the time of location of the maximum of the modulation signal.

FIG. 4: The total time averaged event rate per kg-y as a function of the detector energy threshold in keV. The fine-solid, the thick-solid, the short-dashed and the long-dashed curves correspond to \( A=3, A=23, A=73 \) and \( A=131 \).

In other words we write:

\[
R = R_0 \left( 1 + \frac{R_1(\alpha)}{R_0} \right) = R_0(1 + h \cos \alpha), \quad h = \frac{R_m}{R_0}.
\]  

(23)
In our case it turns out that $h$ is pretty independent of the target, since the form factor is negligible and all the other factors are common to both $R_0$ and $R_m$. It does, however, depend on the threshold energy. The total modulation $h \cos \alpha$ is exhibited in Fig. 5. As expected the modulation is the usual one, maximum at the beginning of June, minimum in December. This happens because in integrating the differential amplitude over the energy, the positive part wins out. This is not always true, e.g. when the reduced WIMP-nucleus mass is large the total modulation amplitude is negative, i.e. the maximum of the rate occurs in December.

The directional event rates, i.e. those registered in a given direction of the nuclear recoil, may also be used to discriminate against background. They are expected to follow for light WIMPs the usual trend, i.e. a maximum in a direction opposite to the sun’s velocity and a minimum in the direction of the sun’s velocity, and we will not elaborate further here. The observed data will exhibit diurnal variation, due to the rotation of the earth around its axis, which depends on the inclination of the direction of observation. The interested reader is referred to the literature, e.g. [65].

VI. DISCUSSION

We investigated the WIMP-nuclear scattering event rates using an elementary nucleon cross section derived in a model proposed recently [50], which suggests that the neutron decay can be understood assuming a small partial neutron decay width to a dark matter particle with a mass slightly less than that of the neutron. This model is consistent with the assumption that all dark matter present in the universe could be composed of this candidate alone. Using the parameters of this novel proposal we find a nucleon cross section of a reasonable size in agreement with a recent analysis [61], appropriate for WIMPs with a mass very close to the nucleon mass. These authors have obtained this value by considering the limits extracted from experiments, like XENON1T [34], CRESST-II [38], CRESST 2017 surface run [39], DAMIC [62] and CRESST-III [40].

Some interesting features appear in this scenario, which may affect the experiments. The nuclear recoil energies tend to be smaller in this case, since the reduced mass is quite small, close to the nucleon mass. In fact for the nuclei already used as targets the current energy threshold values encountered are above the maximum allowed recoil energy. As a result, in spite of the very large nucleon cross sections, such WIMPs could not have been seen. As one can see in Fig. 4 in the case of light targets detection of the WIMP considered becomes feasible. This, e.g., is the case of the CRESST-III experiment, if the claimed energy threshold of 0.3 keV is reached. This is below the maximum recoil energy of 0.5 keV coming from the large oxygen component in the target. In the case of the experiments NEWS-G [43] and TREX-DM [46], employing low threshold spherical TPC detectors, this can also be achieved using relatively light elements. In still lighter targets, such as $^3$He, many events are expected.

The model considered here also permits WIMP-electron interactions with a cross section $\approx 4 \times 10^{-9}$ pb. With such a cross section, the detection of electrons is not hard [66], especially since the WIMP is much heavier than the electron.

![FIG. 5: The quantity $\frac{R_1(\alpha)}{R_0} = h \cos \alpha$ entering the modulated total event rate (a) near zero threshold energy and (b) at $E_{th} = 0.3$ keV. We note that the relative modulation $h$ slightly increases with threshold (in fact both the modulated and the non-modulated decrease with threshold, but the modulated decreases less).](image-url)
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