CONDUCTANCE OF LUTTINGER-LIQUID WIRES CONNECTED TO RESERVOIRS

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We show that the dc conductance of a quantum wire containing a Luttinger liquid and attached to noninteracting leads is given by \(e^2/h\) per spin orientation, regardless of the interactions in the wire. It is also shown that weak disorder in the wire results in the temperature- or length-dependent corrections to the conductance. The exponents of these dependences are determined by the interaction strength in the wire and, in the leading order, are not affected by the presence of the non-interacting leads. These results explain recent experiments on quasiballistic GaAs quantum wires.

I. INTRODUCTION

For non-interacting electrons, the conductance of narrow ballistic quantum wires connected to wide reservoirs is quantized in units of \(e^2/h\) \([1,2]\). When the effects of interactions are included this result is expected to be modified. In particular, when the electrons in the wire form a one-dimensional Luttinger liquid (LL) \([3]\), the conductance is believed to be \(Ke^2/h\) per spin orientation \([4–6]\), where \(K\) is the interaction dependent parameter characterizing the LL. For non-interacting electrons \(K = 1\). For repulsive interactions \(K < 1\), and the conductance should be reduced.

A recent experiment on very long GaAs high mobility quantum wires \([7]\) casts doubt on this picture. It is known that the same parameter \(K\) enters the temperature dependence of the impurity correction to the conductance, and using this the authors of \([7]\) where able to estimate \(K\) to be about 0.7 for the electron gas in their wires, implying a conductance reduction of 30% in the ballistic limit. The actual reductions observed are only a few percent of \(e^2/h\) however.

This observation has led us to re-assess the conventional analysis of the charge transport in Luttinger wires. In this paper we will argue that the conductance of a quantum wire attached to one-dimensional non-interacting leads [which are intended to model the higher-dimensional Fermi-liquid (FL) reservoirs] is \(e^2/h\) regardless of the interactions in the wire itself\[^1\]. This is because the finite resistance of a ballistic wire is a contact resistance \([9–13]\) and comes entirely

\[^1\]The result announced above was obtained by us \([8]\) simultaneously and independently with Safi and Schulz \([14]\). By now, there are at least three more papers \([15–17]\) which, each in its own way, confirm this result.
from processes that take place outside the wire, where the electrons are not in the LL state (cf. Fig. 1a, top).

FIG. 1. a) Top: a narrow wire containing a Luttinger liquid and connected to Fermi-liquid leads. Arrows show a typical electron trajectory giving rise to a finite conductance $g = e^2/h$. Bottom: the same for a wire with impurities (depicted by asterisks). Dashed arrows show a typical trajectory giving rise to a correction to $e^2/h$. b) Effective 1D model for the system shown in (a) [top].

While it has been generally asserted in the literature that $g = K e^2/h$ is the correct result for interacting electrons \[4–6\] (at least for wires longer than the Fermi wavelength), we must remark that there have been previous comments supporting the result $g = e^2/h$. In their earlier paper \[18\], Kane and Fisher remark that the ac conductance will cross over to the non-interacting value at frequencies lower than $\omega = v_F/L$, where $v_F$ is the Fermi velocity and $L$ is the wire length. Matveev and Glazman \[19\] make a similar remark when discussing the conductance of multi-mode interacting wires.

Having established the absence of conductance renormalization in pure wires, we will ask what would happen if we connect a weakly disordered wire to the leads. In particular, we are interested in whether the temperature dependence of the conductance reflects the presence of the LL-state in the wire or it is affected by the presence of the leads as well. We will show that the main contribution to the temperature-dependent part of the conductance is determined by the local strength of interactions in the wire and is the same as in the absence of the leads. This result can be anticipated from the following simple picture \[20\]. Consider a weakly disordered wire containing an LL and adiabatically connected to the FL leads (cf. Fig. 1a, bottom). As has been said above, in the absence of disorder, the finite resistance of a perfect wire ($= h/e^2$) is entirely due to contact resistance \[3–13\]: some of the electrons coming from the wide leads are reflected as the channel becomes narrower. This reflection takes place outside the wire, where the electrons are in the FL state, therefore, the contact resistance is not affected by the interactions in the wire. Weak disorder in the wire gives rise to an additional contribution to the resistance. This contribution is determined by the scattering in the wire, where electrons are in the LL-state, and therefore this contribution has features typical of an LL but not of an FL.

These two results—the absence of conductance renormalization in perfect wires and the temperature dependence of conductance determined by the local strength of interactions in disordered wires—resolve the controversy around the experimental observations by Tarucha et al. \[7\] and suggest that these observations can be considered as an indication of the LL-state in GaAs quantum wires.

This paper is organized as follows. In Sec. II, we prove the absence of conductance renormalization by using the equation of motion for an LL in an external electric field. In Sec. III, we obtain the same result by using the linear response theory. In Sec. IV, we find the temperature-
and length-dependent corrections to the conductance due to weak disorder in the wire. Our conclusions are given in Sec. V.

II. LUTTINGER LIQUID UNDER EXTERNAL ELECTRIC FIELD: EQUATIONS OF MOTION

We begin with replacing the original system containing an one-dimensional (1D) wire connected adiabatically to two-dimensional (2D) leads (cf. Fig. 1a) by the effective 1D system (cf. Fig. 1b). This 1D system is an infinite LL which is described by the inhomogeneous effective interaction parameter \( K(x) \). As is shown in Fig. 1a, the interaction strength starts from the value \( K_L = 1 \), e.g., in the left lead, goes through an arbitrary variation in the middle part of system (“the wire”) and approaches again the asymptotic value of \( K_L = 1 \) in the right lead. The asymptotic value \( K_L = 1 \) describes a non-interacting LL and reflects the fact that interactions in the leads result in the formation of the FL-state, which for the present purposes can be viewed as the Fermi-gas state. Similarly, we allow for an arbitrary variation of the density wave velocity \( v(x) \) in the wire requiring only that it take asymptotic values \( v_L \) in the leads, which coincide with the Fermi-velocity of the non-interacting electrons. The electrostatic potential difference applied to the system produces largest electric field in the narrowest part of the system, i.e., in the wire. Accordingly, we assume that the electric field \( E(x) \) is zero outside the wire and undergoes an arbitrary variation in the wire.

The (real-time) bosonic action for a spinless LL is

\[
S = \frac{\hbar}{2} \int d^2x \frac{1}{K(x)} \left\{ v(x)(\partial_x \phi)^2 - \frac{1}{v(x)}(\partial_t \phi)^2 \right\},
\]

(1)

We have chosen to normalize the \( \phi \) field so that density of the electrons (minus the background density) and the (number) current are given by

\[
\rho = \partial_x \phi/\sqrt{\pi}, \quad j = -\partial_t \phi/\sqrt{\pi}.
\]

(2)

The interaction with an external electromagnetic field \( A_\mu \) is given by (the charge of the electron is taken to be \( -e \))

\[
S_{int} = \frac{e}{\sqrt{\pi}} \int d^2x \left\{ A_0 \partial_x \phi - A_1 \partial_t \phi \right\},
\]

(3)

so the equation of motion for the field (classical or quantum) is

\[
\partial_t \left( \frac{1}{Kv} \partial_t \phi \right) - \partial_x \left( \frac{v}{K} \partial_x \phi \right) = \frac{e}{\sqrt{\pi} \hbar} E(x,t),
\]

(4)

where \( E = -\partial_x A_0 + \partial_t A_1 \) is the electric field. We assume that the electric field is switched on at \( t = 0 \), so that \( E(x,t) = 0 \) for \( t < 0 \) and \( E(x,t) = E(x) \) for \( t \geq 0 \). As we see, the problem reduces now to that of determining the profile of an infinite elastic string under the external force. In this language, \( \phi(x,t) \) gives the displacement of the string at point \( x \) and at time \( t \), while the number current \( j = -\partial_t \phi/\sqrt{\pi} \) is proportional to the vertical velocity of the string.

To develop some intuition into the solution of Eq. (4), we first solve it in the homogeneous case, when \( K = \text{const} \), \( v = \text{const} \), and \( E(x) = \text{const} \) for \( |x| \leq 0 \) and is equal to zero otherwise. In this case, the solution of Eq. (4) for late times, i.e., when \( t > L/v \), has the following form
\[
\phi(x, t) = \frac{KeV}{2\sqrt{\pi}\hbar} \begin{cases} 
\frac{t - x^2 + L^2/4}{L^2} & \text{for } |x| \leq L/2; \\
\frac{t - |x|/v}{L^2} & \text{for } L/2 \leq |x| \leq vt - L/2 \\
\frac{\frac{v}{2\pi}(t - \frac{|x| - L/2}{v})^2}{L^2} & \text{for } vt - L/2 \leq |x| \leq vt + L/2 \\
0 & \text{for } |x| \geq vt + L/2,
\end{cases}
\]

where \( V = E \cdot L \) is the total voltage drop. This solution is depicted in Fig. 2a.

![Figure 2](image)

**FIG. 2.** a) Solution of the wave equation in the homogeneous case for \( t = 5L/v \). b) Schematic solution in the inhomogeneous case for \( t \gg L/v \).

The profile of the string consists of two segments (I and II in Fig. 2a) whose widths—equal to \( (vt - L) \)—grow with time, and of three segments (III, IV and V in Fig. 2a) whose widths are constant in time and equal to \( L \). In segments I and II, the profile of the string \( \phi(x, t) \) is linear in \( x \), and therefore, being the solution of the wave equation, also in \( t \); in segments III-V, the profile is parabolic. Outside segments IV and V, the string is not perturbed yet and \( \phi(x, t) = 0 \). As time goes on, the larger and larger part of the profile becomes linear. Thus, for late times, the motion of the string can be described as follows: the pulse produced by the force spreads outwards with the velocity \( v \), involving the yet unperturbed parts of the string in motion; simultaneously, in all but narrow segments in the middle and at the heads of the pulse, the string moves upwards with the \( t \)- and \( x \)-independent “velocity" \( \partial_t \phi = KeV/2\sqrt{\pi}\hbar \). In terms of the original transport problem, it means that the charge current \( I = -ej \) is constant outside the wire (but not too close to the edges of the regions of where the electron density is not yet perturbed by the electric field) and given by \( I = Ke^2V/h \). Therefore, the conductance (per spin orientation) is \( g = Ke^2/h \).

We now turn to the inhomogeneous case. As in the homogeneous case, the profile consists of several characteristic segments (cf. Fig. 2b). In segments III-V, the profile is affected by the inhomogeneities in \( K(x) \), \( v(x) \), and \( E(x) \) and depends on the particular choice of the \( x \)-dependences in all these quantities. In segments I and II however, the profile, being the solution of the free wave equation, is again linear in \( x \) (and in \( t \)). Requiring the slopes of the string be equal and opposite in segments I and II (which is consistent with the condition of the current conservation), the solution in these regions can be written as \( \phi(x, t) = A(t - |x|/v_L) \). The constant \( A \) can be found by integrating Eq. (4) between two symmetric points \( \pm a \) chosen outside the wire

\[
-\int_{-a}^{+a} dx \partial_x \left( \frac{v}{K} \partial_x \phi \right) = \frac{e}{\sqrt{\pi}\hbar} \int_{-a}^{+a} dx E(x) = \frac{eV}{\sqrt{\pi}\hbar}.
\]

Outside the wire, \( K(x) = K_L \) and \( v(x) = v_L \), thus \( A = K_L eV/2\sqrt{\pi}\hbar \). Calculating the current, we get \( g = K_L e^2/h \) and, recalling that \( K_L = 1 \), we finally arrive at \( g = e^2/h \). Thus, the
conductance is not renormalized by the interactions in the wire. The reason for this is that at late times ($t \gg L/v$) the motion of the major part of the string is not affected by the local inhomogeneities in the parameters $K$, $v$, and $L$. It is this late-behavior of the solution, which one needs to determine the $dc$ conductance.

III. KUBO FORMULA: PERFECT WIRE

We are now going to derive the result that $g = e^2/h$ regardless of the interactions in the wire in a different way, by using the Kubo formula for conductivity. To do so, we adopt a simple model, in which the interaction parameter $K$ changes stepwise from the value $K_L = 1$ in the leads (for $|x| > L/2$) to the value $K_W$ in the wire (for $|x| \leq L/2$). The electric field is assumed to be zero outside the wire but can take an arbitrary variation in the wire.

The charge current $I$ is related to the field by

$$I(x, t) = \int_{-L/2}^{L/2} dx' \int \frac{d\omega}{2\pi} e^{-i\omega t} \sigma_\omega(x, x') \bar{E}_\omega(x'),$$

where $\bar{E}_\omega(x)$ is the temporal Fourier component of the electric field and $\sigma_\omega(x, x')$ is the non-local ac conductivity. In the Matsubara representation, $\sigma_\omega(x, x')$ is expressed via the (imaginary time) current-current correlation function by the usual Kubo formula

$$\sigma_\omega(x, x') = \frac{i e^2}{\hbar \omega} \int_0^\beta d\tau (T^\ast_\tau j(x, \tau) j(x', 0)) e^{-i\omega \tau} \bigg|_{\omega = i\omega - \epsilon}. \tag{8}$$

In the bosonized form, the particle-number current is given by $j = -i\partial_\tau \phi/\sqrt{\pi}$ and Eq. (8) reduces to

$$\sigma_\omega(x, x') = \frac{e^2 i \omega^2}{\hbar \pi \omega} G^0_\omega(x, x') \big|_{\omega \rightarrow i\omega - \epsilon}, \tag{9a}$$

$$G^0_\omega(x, x') = \int_0^\beta d\tau (T^\ast_\tau \phi(x, \tau) \phi(x', 0)) e^{-i\omega \tau}, \tag{9b}$$

where $G^0_\omega$ is the propagator of the boson field $\phi$. The propagator $G^0_\omega(x, x')$ satisfies the equation

$$\left\{ - \partial_x \left( \frac{v(x)}{K(x)} \partial_x \right) + \omega^2 \right\} G^0_\omega(x, x') = \delta(x - x'), \tag{10}$$

complemented by the following boundary conditions: i) $G^0_\omega(x, x')$ is continuous at $x = \pm L/2$ and $x = x'$, ii) $[v(x)/K(x)]\partial_x G^0_\omega(x, x')$ is continuous at $x = \pm L/2$ but iii) undergoes a jump of unit height at $x = x'$, i.e.,

$$- \frac{v(x)}{K(x)} \partial_x G^0_\omega(x, x') \bigg|_{x = x'}^{x = x' + 0} = 1. \tag{11}$$

In addition, we assume that the infinitesimal dissipation is present in the leads, so that $G^0_\omega(\pm \infty, x') = 0$. (In a real-time formulation this corresponds to outgoing wave boundary conditions.) We are interested in the dc limit of $\sigma_\omega(x, x')$ for which we need to determine the

2Safi and Schulz \[14\] have shown that the ac conductance is renormalized by the interactions in the wire in a non-trivial way.
behavior of $G^0_\omega(x,x')$ for $\omega \to 0$. Note that according to Eq. (7), one needs to know $G^0_\omega(x,x')$ only for $|x'| \leq L/2$. In this region, the low-frequency asymptotic form of the propagator can readily be shown to be independent of $x,x'$ and given by $G^0_\omega(x,x') \to iK_L/2\omega$. Consequently, Eq. (9a) gives

$$\lim_{\omega \to 0} \sigma_\omega(x,x') = \frac{K_Le^2}{h}.$$  

(12)

For a static electric field, $\bar{E}_\omega(x) = 2\pi \delta(\omega) E(x)$, and Eq. (7) gives the $x$- and $t$-independent current

$$I = \frac{K_Le^2}{h} \int_0^L dx' E(x') = \frac{K_Le^2}{h} V,$$

(13)

from which we see that the conductance is

$$g = K_L \frac{e^2}{h}.$$  

(14)

Thus the conductance is determined by the value of $K_L$ in the leads and does not depend on the value of $K_W$ in the wire. Recalling that $K_L = 1$, we get $g = e^2/h$. We have thus confirmed the result obtained in Sec. II.

Notice that in this calculation we have implicitly taken the limit $\omega \to 0$ while keeping $q$ finite [21–23]. The traditional order of limits is $q \to 0$ before $\omega \to 0$. The latter yields the Drude formula, which has a divergent dc limit for perfect systems. The former produces a finite (two terminal Landauer-Büttiker) dc conductance even for a perfect system. It furthermore corresponds to the experimental situation where a static field is applied over a finite region.

IV. KUBO FORMULA: WEAKLY DISORDERED WIRE

We now consider the case when a weak disorder potential $V(x)$ is present in the wire. The action now acquires the term describing the backscattering by disorder

$$S_i = \frac{2}{a} \int dx \int_0^\beta d\tau V(x) \cos(2k_Fx + 2\sqrt{\pi}\phi),$$  

(15)

in which $a$ is the microscopic length cut-off. In what follows, we will determine the leading disorder-induced corrections to $\sigma_\omega$ directly from the Kubo formula (8) via perturbation theory in $S_i$. The first non-vanishing correction to the ensemble-averaged propagator is given by

$$\delta G(X,X') = \frac{1}{a^2} \int dX_1 dX_2 V(x_1)V(x_2) \cos(2k_F(x_1 - x_2))$$

$$\left\{ \langle \phi(X)\phi(X')Q(X_1,X_2) \rangle_0 - \langle \phi(X)\phi(X') \rangle_0 \langle Q(X_1,X_2) \rangle_0 \right\}$$  

(16)

where $X \equiv \{x,\tau\}$, $\ldots$ stands for ensemble averaging over disorder realizations, $\langle \ldots \rangle_0$ for thermodynamic averaging and

$$Q(X_1,X_2) = e^{i\sqrt{\pi}[\phi(X_1)-\phi(X_2)]}$$

(17)

$^3$The assumption of weak disorder is adequate for the experimental situation of Ref. [7], where $L$ was at least $\simeq 6$ times smaller than the elastic mean free path of the unbounded 2DEG and the total observed change in the conductance was $1 - 5\%$ of $e^2/h$ depending on the wire length.
At this stage, for the sake of simplicity, we choose \( V(x) \) in the form of white-noise: \( \overline{V(x)} = 0 \) and \( \overline{V(x_1)V(x_2)} = n_i u^2 \delta(x_1 - x_2) \), where \( n_i \) is the concentration of “impurities”, and \( u \) is the “impurity” strength. The effective elastic mean free path \( \ell \) (in the absence of the interactions) can then be defined as \( 1/\ell = n_i u^2/\omega^2 \), where \( \omega_F \) is the (non-universal) energy ultraviolet cut-off (of the order of the Fermi energy)\(^4\). We then obtain the correction to the non-local conductivity

\[
- \delta \sigma(x, x') = \frac{2ie^2 \bar{\omega}^2 \omega^2_F}{\pi \ell \omega} \int_{-L/2}^{L/2} d\bar{x} G^0_\omega(x, \bar{x}) G^0_\omega(\bar{x}, x') \left[F_0(\bar{x}) - F_\omega(\bar{x})\right] |\bar{\omega} \rightarrow i\omega - \epsilon,
\]

where \( F_\omega(x) \) is the \( \tau \)-Fourier transform of the (inhomogeneous) \( 2k_F \) density-density correlation function

\[
F(x, \tau) = \langle Q(x\tau, x0) \rangle_0 = \exp \left[ - \frac{4\pi \kappa^2}{\bar{\omega}} \sum_{\omega_0} (1 - e^{-i\omega_0 \tau}) G^0_\omega(x, x) \right].
\]

To evaluate the function \( F \) in Eq. (19), one needs to know \( G^0_\omega(x, x') \) only for \(-L/2 \leq x = x' \leq L/2\). Straightforward, albeit lengthy, algebra leads to

\[
G^0_\omega(x, x) = \frac{K_W}{2|\omega|} + \frac{K_W \kappa^2 e^{-L/L_\omega}}{|\omega|} e^{L/L_\omega} \kappa^2_{\pm} - e^{-L/L_\omega} \kappa^2_{\pm},
\]

where \( L_\omega = v_W/|\omega| \), \( v_W \) is the density-wave velocity in the wire and \( \kappa^2_{\pm} = 1/K_W \pm 1/K_L \). We now consider separately the cases of “high” \((v_W/L \ll T \ll \omega_F)\) and “low” \((T \ll v_W/L)\) temperatures. (The quotations marks here are implied to mean that, say, “low” \(-T\) case can be alternatively viewed as the “long” \(-L\) case and vice versa.)

At “high” temperatures, the second term in Eq. (20) is exponentially small \([\propto \exp(-(L - 2x)/L_\omega)]\) unless \( \bar{\omega} = 0 \). The term \( \bar{\omega} = 0 \) gives zero contribution to the sum in Eq. (19), as can be seen by performing the infrared regularization of the propagator \(|\omega| \rightarrow \sqrt{|\omega|^2 + m^2}|\) and then letting \( m \rightarrow 0 \). Thus, in this case only the first term in Eq. (20) has to be taken into account. This term is precisely the same, however, as in the case of a homogeneous Luttinger liquid with the parameter \( K_W \). Already at this stage the result that the \( T\)-scaling of the conductance is determined by \( K_W \) can be anticipated. The function \( F \) is now \( x\)-independent and is given by

\[
F = \left[ \frac{2\pi/\omega_F \beta}{\sin \pi \tau/\beta} \right]^{2K_W}
\]

After analytic continuation \( \bar{\omega} \rightarrow i\omega - \epsilon \), the limit \( \omega \rightarrow 0 \) can be taken. As we saw in Sec. III, the remaining two propagators in Eq. (18) are \( x\)- and \( x' \)-independent in this limit: \( \lim_{\omega \rightarrow 0} G^0_\omega(x, x') = iK_L/2\omega \). The rest of the calculations proceeds exactly as in the homogeneous case. Recalling the result for the conductance of a perfect wire Eq. (14), the final result for the conductance can be written as

\[
\overline{g} = K_L e^2 \hbar \{ 1 - C K_L \frac{L}{\ell} \frac{\hbar \omega_F}{2\pi T} \}^{2(1 - K_W)}.
\]

\(^4\)Although in the real \( GaAs \) system the impurity potential is long-ranged, the expectation is that this simplification is not going to affect the final results significantly, as soon as \( \ell \) is replaced by the correct mean free path for a more realistic potential.
where
\[ C = 8\sqrt{\frac{\pi}{2}}\sin(\pi K_W)\frac{\Gamma(1 - K_W)}{\Gamma(\frac{1}{2} + K_W)}\left[\Gamma(K_W)\right]^2. \] (23)

Note that \( K_L \) enters only the prefactors, while the exponent of \( T \)-scaling is determined by \( K_W \).

Tracing back the calculations, we can now see the reason for this. The \( K_L \)-dependence comes from the propagators in the prefactor of Eq. (18) which depend on the frequency of the external field \( \omega \). In the limit \( \omega \to 0 \), these propagators become long-ranged and “know” only about \( K_L \) but not \( K_W \). The \( T \)-scaling comes the propagator entering the function \( F \). This propagator does not depend on \( \omega \). In the “high” temperature limit, it becomes short-ranged and “knows” only about \( K_W \) but not \( K_L \).

In the case of “low” temperatures, the analysis is much more cumbersome. Referring the reader for details to Ref. [20], we give only the final expression the real part of the conductance
\[ \overline{\text{Reg}} = \frac{e^2}{h} - \frac{e^2}{h}2\pi^2\Gamma(K_W)\frac{L}{L} \left[ \frac{2L\hbar\omega_F}{v_W} \right]^{2(1-K_W)}. \] (24)

Comparing this result with the analogous result for the homogeneous case [4,6], we see that the exponent of the length-dependence is the same as if the leads would be absent [9]. The \( L \)-scaling is still symmetric to \( T \)-scaling upon replacement \( T \leftrightarrow v/L \) (apart from the additional factor of \( L \) entering in the combination with the mean free path). Contrary to the homogeneous case however, this symmetry exists only if the leads are not interacting, i.e., \( K_L = 1 \).

V. DISCUSSION AND CONCLUSIONS

In Secs. II and III, we have shown that the conductance of a perfect wire containing a Luttinger liquid and connected to noninteracting leads is not renormalized by the interactions in the wire. It remains at the noninteracting value \( g = \frac{e^2}{h} \) per spin orientation. In Sec. IV, we have shown that weak disorder in the wire leads to the temperature- or length-dependent corrections to the conductance, the exponent of which being determined (in the leading order) by the interaction strength in the wire and independent of the presence of the leads.

These results are consistent with the recent experimental observations by Tarucha et al. [7], who observed the anomalous temperature dependence of the conductance at lower temperatures with exponent corresponding to \( K_\rho \approx 0.7 \) but no renormalization of conductance at higher temperatures. According to the previous theory, this value of \( K_\rho \) extracted from the temperature dependence should have implied a 30% renormalization of \( g \) from the value of \( e^2/h \) at higher temperatures, which clearly contradicts to the data.

It is certainly tempting to ask whether the observed temperature dependence [7] really comes from the Luttinger–liquid-like behavior of the electrons in the wire or there exists some

5Safi and Schulz [25] have found corrections to the leading \( T \)-dependent term in Eq. (22) which arise from additional scattering at the boundaries between disordered and perfect regions. These corrections become dominant only if the interaction is strongly attractive, which is not the case for the semiconductor heterostructures.

6 For electrons with spin, the exponent \( 2 - 2K_W \) is replaced by \( 2 - K_W^c - K_W^s \) or, if the \( SU(2) \) symmetry of the underlying Hubbard model is preserved, i.e., \( K_W^c = 1 \), by \( 1 - K_W^c \), where \( K_W^c, K_W^s \) are the parameters of the charge (spin) parts of the Luttinger-liquid in the wire.
other plausible explanation for this. One of the possible reasons for the $T$-dependence might be
the phase-breaking scattering of weakly-localized electrons at phonons. The counter-argument
to this assertion is that it would be difficult to explain the anomalous $T$-dependence of the
conductance observed in Ref. [7] ($-\delta g \propto T^{-0.3}$) by the scattering of non-interacting electrons
at three-dimensional phonons.

Even within the Luttinger-liquid model, the impurity scattering is not the only possible
mechanism of the $T$-dependence. The other one is the scattering of electrons from the non-
adiabatic openings of the wire into the leads[4]. One can imitate this mechanism in the 1D
model by putting the point scatterers at the ends of the wire, the $S$-matrices of these scatterers
being equal to that of the non-adiabatic leads. The conductance of such a system is known
to be $T$-dependent [3,20]. However, the $T$-dependent correction to the conductance is length-
independent in this mechanism, whereas the observed deviation of the conductance from $e^2/h$
[7] is definitely more pronounced in longer wires. This gives more support to the disorder
mechanism considered in Sec IV, although the coexistence of two mechanisms is quite plausible.
We hope to see more experiments in the nearest future providing more checks for the current
and new theories.

Finally, we would like to mention that the result of the absence of conductance renormaliza-
tion by the interactions in a Luttinger-liquid wire discussed in this paper should not be applied
to chiral Luttinger liquids [27,28] formed at the edges of fractional quantum Hall effect (FQHE)
systems. The two-terminal conductance of a FQHE system is $g_{FQHE} = \nu e^2/h$, where $\nu$ is the
filling factor. Despite the formal similarity of this expression with the conductance of a homo-
geneous non-chiral Luttinger liquid ($= Ke^2/h$), the physical meanings of these two quantities
are quite different. In FQHE system, $g_{FQHE}$ is the Hall (non-dissipative) conductance [29,28],
whereas the conductance of non-chiral Luttinger liquids is the longitudinal (dissipative) one.
Moreover, the finiteness of $g_{FQHE}$ is not due to the presence of reservoirs (as in the non-chiral
case). The “renormalization” of $g_{FQHE}$ from the value of $e^2/h$ is due to the interactions in the
bulk, and the bosonic description of a chiral Luttinger liquid [27] is constructed in such a way
that the correct result for $g_{FQHE}$ is obtained.

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7We thank A. Gogolin for attracting our attention to this issue.
CONDUCTANCE DANS UN LIQUIDE DE LUTTINGER CONNECTÉ À DES RÉSERVOIRS

Dmitrii L. Maslov et Michael Stone

Nous montrons que la conductance continue dans un fil quantique qui contient un liquide de Luttinger et qui est connecté à des réservoirs non-interactifs est donnée par $e^2/h$ pour chaque orientation de spin. Ce résultat est indépendant de l’interaction dans le fil. Il est aussi montré que la présence d’un faible désordre dans le fil résulte en des corrections de la conductance dépendant de la température et de la longueur du fil. Les exposants de ces corrections sont déterminés par la force de l’interaction dans le fil, et, au premier ordre, ne dépendent pas de la présence des réservoirs non-interactifs. Ces résultats expliquent des expériences récentes sur des fils quantique de GaAs dans le régime quasi-ballistique.