Extend Special Relativity to the Superluminal Case

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First, we extend the special relativity into the superluminal case and put forward a superluminal theory of kinematics, in which we show that the temporal coordinate need exchanging with one of the spatial coordinates in a superluminal inertial frame, and that the coordinate transformations from any superluminal inertial frame to the rest frame (here rest just says in a relative sense) are the same as the Lorentz transformations from some normal inertial frame to the rest frame. Consequently, the causality can not be violated. Secondly, we investigate the superluminal theory of dynamics and find that the total energy of any object moving at a speed of \( v \) (faster than the speed of light in vacuum \( c \)) is equal to the total energy of that object moving at a speed of \( u \) \((u < c)\) provided that the product of two speeds satisfy \( uv = c^2 \). Lastly, we conjecture that this superluminal theory can give a novel interpretation to the essence of matter waves put forward by de Broglie.

Since the special relativity (SR) was put forward by Einstein [1] in 1905, the speed of light in vacuum \( c \) has always been regarded as the limit of particle velocity. As is so often pointed out, the causality will be violated if there are tachyons in our world [2]. It is notable that, using gain-assisted absorption line and linear gain lines, or tunnelling barriers [5, 6, 7, 8], some experiments have exhibited many superluminal phenomena to us, especially in astronomy [3, 4], but they do not imply the genuine existence of superluminal objects. Furthermore, there exist various proposals for observing faster-than-\( c \) propagation of light using anomalous dispersion near an absorption line and linear gain lines, or tunnelling barriers, but these proposals can not be explicitly interpreted as the motion of tachyons. The same case also occurs in the propagation of localized microwaves [10]. All these seem to suggest that the speed of any moving object cannot exceed \( c \) indeed. But virtually, Einstein just tells us that \( c \) is the limit of particle velocity in vacuum. As we know, the limit may indicate the upper bound of the particle velocity which corresponds to the SR, while it may mean the lower bound. In our paper, we aim to develop a superluminal theory which abides by causality and does not contradict most excellent achievements of SR.

Let \( S \) denote the rest frame in which \( x, y \) and \( z \) are three spatial coordinates and \( t \) is the temporal coordinate. \( S' \) is an inertial frame moving at the speed of \( v \) along \( x \)-axis of \( S \). Denote \( x', y', z' \) and \( t' \) as three spatial coordinates and \( t' \) coordinate respectively in \( S' \). At the initial time \( t = t' = 0 \), the origins of two frames are superposed each other. In terms of SR, the Lorentz transformations between \( S \) and \( S' \) can be expressed as follow:

\[
\begin{align*}
\begin{cases}
x' = \frac{1}{\sqrt{1-(v/c)^2}}(x - \frac{v}{c}ct) \\
ct' = \frac{1}{\sqrt{1-(v/c)^2}}(ct - \frac{v}{c}x)
\end{cases} 
\quad (i)
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
y' = y \\
z' = z
\end{cases} 
\quad (ii)
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
x' - axis : ct' = 0 \Rightarrow x = \frac{v}{c}ct \\
ct' - axis : x' = 0 \Rightarrow x = \frac{v}{c}ct
\end{cases} 
\quad (3)
\end{align*}
\]

Eq. (2) is valid if \( v < c \). Eq. (3) tells us that we merely need to consider the relation between the pairs \((x, ct)\) and \((x', ct')\). Their geometrical relation is shown in Fig.1. The positions of \( x' \)-axis and \( ct' \)-axis in the coordinate frame \((x, ct)\) are obtained:

\[
\Delta s^2 = x'^2 + (ict')^2 = x'^2 + (ict')^2, 
\quad (4)
\]

where \( c \) is the light speed in vacuum and \( v < c \). Eq. (i) and Eq. (ii) tell us that we merely need to consider the relation between the pairs \((x, ct)\) and \((x', ct')\). Their geometrical relation is shown in Fig.1. The positions of \( x' \)-axis and \( ct' \)-axis in the coordinate frame \((x, ct)\) are obtained:

\[
\begin{align*}
\begin{cases}
x' = \frac{1}{i\sqrt{(v/c)^2-1}}(x - \frac{v}{c}ct) \\
ct' = \frac{1}{i\sqrt{(v/c)^2-1}}(ct - \frac{v}{c}x)
\end{cases} 
\quad (i)
\end{align*}
\]

\[
\begin{align*}
\begin{cases}
y' = y \\
z' = z
\end{cases} 
\quad (ii)
\end{align*}
\]

Consider an invariant of Minkowski space-time:

\[
\Delta s^2 = x'^2 + (ict)^2 = x'^2 + (ict)^2, 
\quad (4)
\]

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where the terms $y^2, z^2, y'z', y'^2$, and $z'^2$ are omitted for the sake of Eq. (iii) and Eq. (iv). Eq. (iii) tells us that the subtle distinction between space and time of $S$ is that, in the expression of the invariant $\Delta s^2$, the spatial coordinate $x$ is a real number and the temporal term $ict$ is a purely imaginary number. This character can be extracted as a criterion to distinguish between the concepts of space and time.

Now substituting Eq. (3) into Eq. (4), we obtain the expression of $\Delta s^2$ in $S'$:

$$\Delta s^2 = \left[-i\frac{1}{\sqrt{(v/c)^2 - 1}}(x - \frac{v}{c}ct)^2 + \frac{1}{\sqrt{(v/c)^2 - 1}}(ct - \frac{v}{c}x)^2\right].$$

Considering our criterion, we redefine the concepts of space and time in $S'$:

$$\begin{align*}
  \tilde{ct}' &= -\frac{1}{\sqrt{(v/c)^2 - 1}}(x - \frac{v}{c}ct) \quad \text{(i)} \\
  \tilde{x}' &= \frac{1}{\sqrt{(v/c)^2 - 1}}(ct - \frac{v}{c}x) \quad \text{(ii)}.
\end{align*}$$

Now suppose there is another inertial frame $S''$ moving at the speed of $u$ ($u = \frac{c}{v} < c$) along $x$-axis of $S$. The Lorentz transformations between $S$ and $S''$ is:

$$\begin{align*}
  x'' &= \frac{1}{\sqrt{1 - (u/c)^2}}(x - \frac{u}{c}ct) \\
  ct'' &= \frac{1}{\sqrt{1 - (u/c)^2}}(ct - \frac{u}{c}x).
\end{align*}$$

Substituting $u = \frac{c^2}{v} < c$ into Eq. (9), we obtain:

$$\begin{align*}
  x'' &= -\frac{1}{\sqrt{(v/c)^2 - 1}}(ct - \frac{v}{c}x) \\
  ct'' &= -\frac{1}{\sqrt{(v/c)^2 - 1}}(x - \frac{v}{c}ct).
\end{align*}$$

Obviously, Eq. (10) coincides with Eq. (9), which means that the coordinate transformations between $S$ and $S'(v > c)$ are the same as the Lorentz transformations between $S$ and $S''(u < c)$. As is known, the causality is abided by in the inertial frame $S''$ since $u < c$. Therefore, it can not be violated in the inertial frame $S'$ when $v > c$ as long as we redefine the concepts of space and time in $S'$ according to Eq. (9).

Now let us turn to the superluminal dynamics. Based on SR, we can construct another invariant $\Psi$:

$$\Psi = xp_x + (ict)(iE/c),$$

where $p_x$ is the $x$-direction momentum of an object observed in some inertial frame and $E$ is the total energy. Here we still omit the magnitudes involving $y$ and $z$. It is evident that the position of $x$ is equivalent to that of $p_x$ in the expression $\Psi$. The case is available to $ct$ and $E/c$. Therefore, we need to redefine the concepts of momentums and energy in $S'$ if $v > c$. Imitating Eq. (9), we define

$$\begin{align*}
  \tilde{E}'/c &= -\frac{1}{\sqrt{(v/c)^2 - 1}}(p_x - \frac{u}{c}E) \quad \text{(i)} \\
  \tilde{p}_x' &= -\frac{1}{\sqrt{(v/c)^2 - 1}}(E/c - \frac{u}{c}p_x) \quad \text{(ii)}.
\end{align*}$$

Assume there is an object moving at the speed of $v$ ($v > c$) along $x$-axis of $S$. Fixing $S'$ on this object, we know that $p_x' = 0$, and denote $E_0 = \tilde{E}'$ called the rest energy. Eq. (12) suggests

$$E_v = E_0/\sqrt{1 - (v/c)^2},$$

where the symbol $E$ in Eq. (12) has been replaced by $E_v$.

In fact, from SR we have known that the total energy and the rest energy has the following relation

$$E_u = E_0/\sqrt{1 - (u/c)^2},$$

when an object moves at the speed of $u$ ($u < c$) along $x$-axis of $S$. In the same way, here we use $E_u$ to replace the symbol $E$. Obviously, if $u = \frac{c^2}{v}$, then $E_v = E_u$, which
means the total energy of an object moving at a speed of \( v \) (\( v > c \)) is equal to the total energy of that object moving at a speed of \( u \) (\( u < c \)), and the product of two speeds is \( uv = c^2 \). Considering SR and Eq. (13), we can plot the curve shown in Fig. 2 to describe the total energy \( E \) of an object varying with the velocity \( v \) (here \( 0 < v < +\infty \) and \( v \neq c \)) if the rest energy \( E_0 > 0 \). As is shown in Fig. 2, the total energy \( E \) increases with the increasing of \( v \) if \( v < c \); and decreases with the increasing of \( v \) if \( v > c \). If \( v \rightarrow c \), then \( E \rightarrow +\infty \). As to the mechanism that the object switches from the lower-than-\( c \) state to faster-than-\( c \) state, it is a mystery.

The expression \( uv = c^2 \) is very familiar to us. Actually, we have learned it in the de Broglie hypothesis of matter waves. This hypothesis implies that the group velocity \( u_g \) and the phase velocity \( v_p \) of matter waves satisfy the same expression \( u_g v_p = c^2 \) [11]. If this is not just a coincidence, the further development our theory will give another novel interpretation to the essence of matter waves.

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FIG. 1: The geometrical relation between the pairs \((x, ct)\) and \((x', ct')\).

FIG. 2: The total energy \(E\) of an object varying with the velocity \(v\).