Improved radiative corrections and proton charge form factor from the Rosenbluth separation technique

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We investigate whether the apparent discrepancy between proton electric form factor from measurements using the Rosenbluth separation technique and polarization transfer method is due to the standard approximations employed in radiative correction procedures. Inaccuracies due to both the peaking approximation and the soft-photon approximation have been removed in our simulation approach. In contrast to results from (e, e′p) experiments, we find them in this case to be too small to explain the discrepancy.

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I. INTRODUCTION

Knowledge of the electromagnetic form factors of the proton and the neutron, \(G_{ep}, G_{mp}, G_{en}, \) and \(G_{mn}\), is important for an understanding of the inner structure of the nucleon. Until recently the proton form factors \(G_p\) and the neutron, \(G_n\), have been determined from \(e-p\) cross section measurements using the Rosenbluth technique, i.e. by measuring cross sections at constant momentum transfers \(Q^2 = -q^2\) at forward and backward scattering angles. More recently, the polarization technique has become available for proton form factor measurements; the recoil proton polarization in \(e-g, \vec{e}-p\) scattering yields the ratio \(G_{ep}/G_{mp}\).

The Rosenbluth separation technique is based on the assumption that the interaction between electron and proton occurs via a single-photon exchange (Born approximation). This assumption leads to an \(e-p\) cross section from which \(G_{ep}\) and \(G_{mp}\) can be deduced as follows [1]. Defining the variable \(\varepsilon\) as

\[
\varepsilon^{-1} \equiv 1 + 2(1 + \tau) \tan^2 \frac{\theta}{2},
\]

where \(\tau \equiv -q^2/(4M^2)\), with \(M\) being the proton mass and \(q\) the (four) momentum transfer, the \(e-p\) cross section in terms of the Mott cross section yields

\[
\frac{d\sigma}{d\Omega_e} = \frac{d\sigma}{d\Omega_e}^{\text{Mott}} \frac{\tau G_{en}^2 + \varepsilon G_{ep}^2}{\varepsilon(1 + \tau)}. \tag{2}
\]

We then define the so-called reduced cross section by

\[
\sigma_{\text{red}} \equiv \frac{d\sigma}{d\Omega_e} \frac{\varepsilon(1 + \tau)}{d\sigma/d\Omega_e}^{\text{Mott}}. \tag{3}
\]

Inserting cross section (2) into (3), the reduced cross section becomes a linear function in \(\varepsilon\) [1],

\[
\sigma_{\text{red}} = \tau G_{mp}^2 + \varepsilon G_{ep}^2. \tag{4}
\]

The slope of this linear function equals \(G_{ep}^2\) and its intercept is \(\tau G_{mp}^2\). Comparing forward and backward scattering, each set of measurements at constant \(Q^2\) yields one data point of the form factors \(G_{ep}^2(Q^2)\) and \(G_{mp}^2(Q^2)\) at the chosen momentum transfer \(Q^2\).

Cross sections depend on the proton electric and magnetic form factors simultaneously which poses a problem at higher values of \(Q^2\) where the respective contributions of \(G_{ep}\) and \(G_{mp}\) to the reduced cross section [4] are distributed unevenly among the two form factors. At \(e.g. Q^2 = 5\ \text{GeV}^2\), the electric form factor contribution to the reduced cross section is down to 8% and it further decreases with increasing \(Q^2\). Hence the slope of the measured reduced cross section [4] becomes extremely small and thus very sensitive to systematic errors.

Form factor measurements are often parameterised using the so-called dipole form factor

\[
G_d(q^2) = \left( \frac{1}{1 - \frac{q^2}{\Lambda^2}} \right)^2, \tag{5}
\]

where the term 'dipole' refers to the two poles of the denominator; \(\Lambda\) is a constant of the order of 1 GeV. To date all Rosenbluth measurements are approximately compatible with scaling, i.e. all Rosenbluth experiments indicate that

\[
G_{ep} \sim G_d \quad \text{or} \quad G_{ep} \sim G_{mp}/\mu_p, \tag{6}
\]

respectively [2, 3, 4], where \(\mu_p\) is the proton magnetic moment.

In contrast to Rosenbluth measurements, polarization transfer experiments use polarized electron beams or polarized proton targets. In nuclear physics nomenclature they are denoted \(e, e'p\) reactions. Polarization transfer experiments with maximum values of \(Q^2\) large enough to exhibit a discrepancy with results from
According to the polarization transfer measurementsthe ratio $\mu_p G_{ep}/G_{mp}$ decreases approximately linearly with $Q^2$, reaching a value of 0.2 at $Q^2 = 6\, \text{GeV}^2$, in contrast to the scaling behaviour seen in Rosenbluth measurements. A linear fit to the polarization transfer data yields

$$\mu_p G_{ep}/G_{mp} = 1 - 0.13(Q^2 - 0.04),$$  \hspace{1cm} (7)

shown in Fig. 2 together with the Rosenbluth data. As a consequence of this discrepancy an improved Rosenbluth measurement (called 'SuperRosenbluth') was carried out at TjNAF $^{[8, 9]}$. In order to reduce the effects of radiative corrections and other systematic uncertainties due to beam fluctuations, the SuperRosenbluth experiment measured the $H(e, p)$ cross section. In addition, the world Rosenbluth data was re-analyzed $^{[2]}$ as well as the world polarization transfer data, but neither data set revealed internal inconsistencies.

The apparent discrepancy between the proton form factor measurements from Rosenbluth separation technique and from polarization transfer experiments has led to different possible explanations. Most approaches aim to explain the discrepancy in terms of the contribution from the two-photon exchange TPE diagrams (sometimes also called 'box diagrams'), see Fig. 1 $^{[10]}$ which can only be included into the cross section approximately due to inelasticities on the proton side of the two TPE diagrams $^{[2, 10, 11, 12]}$. Blunden et al. can reduce the apparent discrepancy in $G_{ep}/G_{mp}$ by roughly a factor of 2 when including the proton ground-state only, with the Rosenbluth data moving down towards the polarization data $^{[10]}$. Further hadronic calculations of the TPE contribution, involving more intermediate states (e.g. the $\Delta$ resonance), are model dependent and only valid for small and intermediate values of $Q^2$, since for larger values of the momentum transfer more and more intermediate states have to be taken into account $^{[10]}$. Calculations involving all intermediate states can be carried out using generalized parton distributions, relating them to virtual Compton processes on the nucleon. They are valid for values of $Q^2 \approx 1\, \text{GeV}^2$ and larger when a virtual photon starts to resolve point-like partons $^{[17]}$.

The Rosenbluth technique is very sensitive to corrections depending on $\varepsilon$ and the TPE is such a correction. While the effects from TPE are merely at the level of around 5%, the contribution of $G_{ep}$ is also just a few percent at high $Q^2$, rendering the impact on $G_{ep}$ much larger. So the effects from TPE correction are magnified considerably by the fact that the slope of the reduced cross section $\sigma_{\text{red}}$ as a function of $\varepsilon$ is so small. While the TPE diagrams can usually be neglected in the cross section, they render the Born approximation invalid in the case of the Rosenbluth technique $^{[10]}$, aiming at small values of $G_{ep}$.

Rosenbluth measurements were carried out at TjNAF in 1998 $^{[8]}$. They covered momentum transfers from $Q^2 = 0.5\, \text{GeV}^2$ to $3.5\, \text{GeV}^2$. By swapping spectrometers and using a calorimeter for electron detection, higher momentum transfers of up to $5.6\, \text{GeV}^2$ became accessible later $^{[8]}$.

FIG. 1: Feynman diagrams beyond the leading order. $\mathcal{M}_{\text{Born}}^{(1)}$ and $\mathcal{M}_{\text{Born}}^{(2)}$ constitute the Born approximation. The latter amplitude includes the internal radiative corrections from vacuum polarization, vertex corrections, and self-energy diagrams; and the external radiative corrections referred to as bremsstrahlung from $\mathcal{M}_{\text{brems}}$. Two-photon exchange (TPE) contributions are not included in the Born approximation.

FIG. 2: Proton electromagnetic form factor data $^{[7]}$ for $\mu_p G_{ep}/G_{mp}$ as obtained via the Rosenbluth separation (red circles) and via the polarisation transfer technique (blue dots). The Rosenbluth data indicates scaling (see Eq. (6)) whereas the polarisation transfer data can be fitted linearly according to Eq. (7).
The non-negligible TPE contribution to the reduced cross section \( \sigma \) destroys its linearity towards small values of \( \varepsilon \). But no indications for such non-linearities have been found so far in Rosenbluth measurements. Refs. 21, 25 set limits on the non-linearities. Ref. 25 does not rule out the non-linearities predicted by some calculations. However, these tests do not constrain the linear part of the correction which can modify \( G_{ep}/G_{mp} \). A very clean experimental access to the TPE contribution would be provided by positron scattering. But suitable positron beams with the necessary luminosities are not yet available.

The TPE effect on the Rosenbluth data only providing a partial resolution of the discrepancy raises the question: which other corrections to the reduced cross section exhibit an \( \varepsilon \)-dependence leading to a sizable effect?

In this letter we study radiative corrections to \( e-p \) scattering as a possible source for the discrepancy described above. While these radiative corrections are usually approximated we here apply an improved radiative correction procedure to Rosenbluth data in order to evaluate the effect of the approximations on the discrepancy. In Sec. II we introduce radiative corrections to \( e-p \) scattering, highlighting the most common approximations used in radiative correction procedures. In Sec. III we sketch an improved correction procedure which partially removes these approximations. In Sec. IV we apply the improved radiative corrections to Rosenbluth data, showing that the approximations usually made in the treatment of radiative corrections to \( e-p \) scattering data have little effect on the proton electric form factor as measured in Rosenbluth type experiments.

Full calculations of radiative corrections to order \( \alpha^1 \) have already been calculated for radiation originating from the incident and the outgoing electron [22, 23, 24]. But the improved radiative corrections shown here go beyond order \( \alpha^1 \).

In accordance with Refs. 22, 23, 24 we find that the improved corrections are small and do not contribute significantly. Even though this is a negative outcome it provides an important clarification, as it has been discussed as a possible explanation for the discrepancy between the two approaches. In addition it was found that the improvement is important in other observables.

II. RADIATIVE CORRECTIONS TO \( e-p \) SCATTERING

For practical purposes radiative corrections (see Fig. 1) to \( e-p \) scattering data are usually carried out using approximations [13, 26, 27, 28]. While hadronic contributions to the radiative corrections can still be included to a good accuracy, most procedures employ two approximations, the soft-photon approximation (SPA) and the peaking approximation (PA) in order to simplify the calculations 28, 29, 30.

The SPA assumes that the emitted bremsstrahlung photon has no effect on the hard scattering; this is justified (only) in the limit where the bremsstrahlung photon has vanishing energy \( \omega^0 \). Consequently, in SPA, the cross section for emitting one soft bremsstrahlung photon factorizes into the elastic first-order Born cross section \( M^{(1)}_{\text{Born}} \), times the probability for emitting a soft bremsstrahlung photon 31, 32, 33. This factorization also applies in the case of multi-photon bremsstrahlung 34, where it translates into an exponentiation of the soft-photon contribution to the cross section 28, 31, 32, 35. The exponentiation of the soft-photon contribution renders radiative correction procedures much more straightforward, considerably simplifying data analyses 28, 30, 37. However, in practice the SPA is applied to scattering events accompanied by the multiple emission of photons with energies which cannot be considered as ‘soft’ photons any more 37.

The PA is based on the SPA. It further simplifies radiative correction procedures by assuming that the momenta of all bremsstrahlung photons are aligned with the emitting particles; it was first introduced by Schiff in 1952 29 for \( (e,e') \) experiments. Later it was extended to inclusive \( (e,e',p) \) scattering by Ent et al. 28. The PA is inspired by the observation that \( H(e,e',p) \) data indeed show that the bremsstrahlung photons are emitted mostly along the directions of the incident electron \( (e) \), the scattered electron \( (e') \), and the recoiling proton \( (p) \). But the PA overestimates the amplitudes of the photon peaks and cannot appropriately treat the kinematics and the evaluation of the form factors for those bremsstrahlung photons which deviate from the \( e-, e' \), and \( p \)-directions 36, 37.

While SPA and PA both exhibit shortcomings, together these two approximations considerably simplify the numerical treatment of the radiative corrections to \( e-p \) scattering data. For many purposes the two approximations are of good quality and may be used without harm; but there are also experimental settings for which the approximative application of radiative corrections do lead to inaccuracies 28, 30, 37.

III. IMPROVED RADIATIVE CORRECTIONS

As mentioned above the SPA considerably simplifies the treatment of multi-photon bremsstrahlung. In fact an exact treatment of multi-photon bremsstrahlung without SPA is not feasible since higher-order bremsstrahlung diagrams cannot be included into radiative
correction calculations to arbitrary order (in $\alpha$). It has, however, been shown that the SPA can partially be removed from multi-photon radiative correction procedures by treating one ‘hard’ bremsstrahlung photon exactly while calculating the remaining photons in SPA \[37\]. Because this novel approach combines bremsstrahlung photons treated exactly with a kind of bremsstrahlung ‘background’ which is treated in SPA, it is here referred to as the combined calculation. It has been shown that this combined calculation is invariant under different methods of selecting the ‘hard’ photon, which is treated exactly, from a given multi-photon event \[37\].

It has further been shown that the PA can fully be removed from $e$-$p$ data analyses without large computational expense by introducing a full angular treatment of the bremsstrahlung photons \[36\]. The full angular treatment renders the assumption, that all bremsstrahlung photons are either emitted in $e$-, $e'$-, or $p$-direction, unnecessary. Removing the PA leads to improved kinematical treatment and to a more systematic evaluation of the form factor \[36\].

Both improvements – the partial removal of the SPA and the complete removal of the PA – can be done simultaneously and have simultaneously been applied to $(e, e'p)$ data \[37\]. Together they are here referred to as the improved radiative correction treatment. The improved radiative correction treatment reproduces experimental $(e, e'p)$ data more accurately than correction procedures fully relying on SPA and PA \[37\] and the question arises whether this effect can also be seen in Rosenbluth experiments.

IV. RESULTS AND DISCUSSION

In order to recompute Rosenbluth plots using the improved radiative corrections we used an empirical fit to the world Rosenbluth data \[38\] and generated the reduced cross section \[3\] with $Q^2 = 2.0\text{ GeV}^2$, with $Q^2 = 4.0\text{ GeV}^2$, and with $Q^2 = 6.0\text{ GeV}^2$. The comparison between existing $(e, e'p)$ Rosenbluth results with a calculation based on the improved approach described here was done by multiplying the reduced cross section \[3\] with a correction factor accounting for the differences between the two radiative correction treatments.

Our calculations of the Rosenbluth results using standard radiative corrections (with full SPA and PA) and of the improved approach was done separately for each momentum transfer squared. A Monte Carlo generator was used to sample multiple bremsstrahlung photons per scattering event (see also \[37\]). Using the techniques and assumptions described above the four-momenta of the scattered electron and the recoiling proton were calculated subsequent to the emission of

\[FIG. 3: \text{Rosenbluth plots: reduced cross section as a function of } \varepsilon \text{ at three different values of the momentum transfer (}Q^2 = 2.0\text{ GeV}^2,\ Q^2 = 4.0\text{ GeV}^2,\ \text{and }Q^2 = 6.0\text{ GeV}^2\). The red lines (squares) show the reduced cross sections calculated using the improved radiative corrections. The black lines (circles) are hardly distinguishable from the red lines. They depict the reduced cross sections calculated using SPA and PA. One can clearly see that the improved radiative corrections have no sizable impact on the Rosenbluth plots and hence on } G_{ep}. \]
bremsstrahlung photons. Geometrically the detectors were treated as rectangular windows. And the detectors were given a momentum acceptance. Once an electron (proton) passed through the "window" of the electron (proton) detector its three-momentum was computed in order to check whether it was within the detector’s acceptance range. Events with electrons (protons) outside the acceptance were given zero weights. Weights of events with electrons (protons) not passing through the "windows" were also set to zero.

In order to compare the two approaches the missing energy was considered, binning the two event weights (one coming from the approximate procedure, the other calculated via the improved procedure) in the vicinity of the total missing energy for each event. As \((e, e')\) Rosenbluth experiments only consider the elastic peak of the missing energy \(E_m\) up to energies of the order of 20 to 50 MeV, we integrated the two missing-energy distributions, obtaining the two total cross sections

\[
\sigma_{\text{tot}}^\text{impr.}(E_m \leq 50 \text{ MeV}),
\]

and

\[
\sigma_{\text{tot}}^\text{approx.}(E_m \leq 50 \text{ MeV}).
\]

These cross sections were used to correct the standard reduced cross section \([8]\) by multiplication with the correction factor

\[
\sigma_{\text{red}}^\text{ex}(\varepsilon) = \frac{\sigma_{\text{tot}}^\text{impr.}(E_m \leq 50 \text{ MeV})}{\sigma_{\text{tot}}^\text{approx.}(E_m \leq 50 \text{ MeV})} \sigma_{\text{red}}^\text{approx.}(\varepsilon).
\]

The results are presented in Fig. 3 and in Tab. 1. As one can see the correction factor (ratio on the r.h.s. of Eq. (10)) is very close to unity for the three kinematic settings considered here. Given the large errors usually appearing in Rosenbluth measurements we can conclude here, that the improved radiative corrections have no visible impact on \(e-p\) Rosenbluth data.

Improved radiative corrections to \((e, p)\) measurements such as the SuperRosenbluth experiment are more difficult than the corrections to \((e, e')\) and \((e, e'p)\) experiments since the scattered electron’s momentum (which is not measured) would have to be generated very efficiently in an additional Monte Carlo generator. However, since the \((e, p)\) SuperRosenbluth data do not exhibit any deviations from other Rosenbluth measurements \([2, 3]\), the improved radiative corrections should lead to similarly small (or even smaller) corrections as the ones shown here in Tab. 1.

In conclusion, we observe that the approximate radiative correction treatment (using SPA and PA) does not have a sizable impact on the reduced cross section as obtained via the Rosenbluth technique. This may be due to the fact that the Rosenbluth technique considers a narrow range around the elastic scattering peaks where bremsstrahlung does not play the role it has in radiative tails.

### Table I: Impact of the improved radiative corrections on the proton electric form factor

| \(Q^2\) (GeV\(^2\)) | \((\sigma_{\text{impr.}}^\text{approx.})^2\) | \((\sigma_{\text{impr.}}^\text{impr.})^2\) | deviation |
|------------------------|------------------|------------------|-----------|
| 2.0 GeV\(^2\)         | 4.772 × 10\(^{-3}\) | 4.816 × 10\(^{-3}\) | +0.92%    |
| 4.0 GeV\(^2\)         | 6.055 × 10\(^{-4}\) | 6.081 × 10\(^{-4}\) | +0.43%    |
| 6.0 GeV\(^2\)         | 1.660 × 10\(^{-4}\) | 1.662 × 10\(^{-4}\) | +0.12%    |

\(G_\text{ep}^2\) are usually large such that the deviations shown in this table are entirely negligible.

V. OUTLOOK AND FURTHER INVESTIGATIONS

Removing or partially removing approximations such as PA and SPA from radiative corrections in the way sketched here and elaborated further in Refs. \([36, 37]\) could be carried over to radiative corrections to initial-state radiation (ISR) experiments. These collider mode experiments could yield independent measurements of \(G_\text{ep}^2\) and \(G_\text{mp}^2\) by studying the

\[
e^+e^- \rightarrow \gamma pp
\]

reaction, the photon being emitted by one of the leptons. The ISR matrix element is smaller than the \(e^+e^- \rightarrow pp\) matrix element by a factor of \(\mathcal{O}(\alpha)\). But 'meson factories’ like DA\(\ddot{o}\)NE, CESR, and KEK-B are pushing the luminosity frontier far enough to compensate for this suppression \([39, 40]\). ISR experiments usually detect the hardest initial-state photon, its angular distribution being found to be mostly aligned with the electron or the positron three-momentum \([41]\), as the bremsstrahlung photons in \(e-p\) scattering. Softer initial-state photons constitute a background, as well as final-state radiation (FSR). The latter can be suppressed by suitably chosen cuts. FSR seems to be negligible for \(pp\) production, except for the region close to threshold, where the Coulomb interaction becomes dominant \([39]\). Radiative corrections to \(e^+e^-\) experiments, relying on PA and SPA, have been carried to next-to-leading order using Monte Carlo simulations \([39, 42, 43]\). As in the case of \(e-p\) scattering, PA and SPA used in ISR experiments may be (partially) removable: the PA could be replaced by a full angular treatment \([36]\) and the SPA could partially be removed by calculating one hard photon from a given multi-photon event exactly; and the remaining photons could be accounted for resorting to the SPA \([37]\).
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