Ring-analysis flow measurements of sunspot outflows

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Abstract. We present a local helioseismological analysis of the convective flows around sunspots. The flow deductions were obtained from MDI Dopplergrams using ring-analysis techniques to measure Doppler shifts induced in solar acoustic oscillations. A novel multi-scale 3-D inversion procedure was used to self-consistently combine all ring-analysis data taken from a mosaic of analysis tiles spanning the solar disk. The inversion is multi-scale because in addition to folding together information from tiles located at different positions on the solar surface, it is capable of incorporating tiles of different sizes, thus, enabling fine control of the horizontal resolution and the probing depth. The inversion is based on sensitivity kernels computed with the Born approximation. Our inversion results indicate that outflow observed at the surface surrounding sunspots persists to unexpectedly deep depths (~7 Mm). These outflows appear to have two components, being comprised of a superficial moat flow and a deeper outflow whose strength peaks around 5 Mm.

1. Introduction
Highly vigorous convection, driven by strong radiative cooling, occurs on many horizontal scales in the upper convection zone, ranging from granules to supergranules. Ring analysis, a form of local helioseismology, has been widely used to sample this region with data from the Michelson Doppler instrument (MDI) aboard the SOHO spacecraft, revealing much about the near-surface nature of the plasma flows. Ring-analysis studies have revealed the presence of large-scale flows, now called solar subsurface weather (SSW) that may be the combined signature of deep giant-cell convection and global-scale circulations [7]. As these evolving flows in the upper convection zone exhibit strong linkage with photospheric magnetism, and likely have a significant role in the advection and redistribution of magnetism, much attention has been devoted to the understanding the properties of flow in active regions and around magnetic structures [8, 9].

Ring analysis has also been used to study the subsurface thermodynamic structure of sunspots [2] and the surface outflows around sunspots [9, 11]. The outflows around sunspots with depth,
however, remain unexplored using this technique, but have been studied extensively using time-distance [13, 6, 14]. Curiously, these studies yield inconsistent results concerning the deep flows around sunspots. We outline a method for the 3-dimensional (3-D) inversion of ring-analysis results incorporating measurements made using multiple mode orders and multiple-analysis region sizes that tile the full solar disk. As a demonstration, we apply this technique to the inversion of flows beneath sunspots using MDI data from January of 2002 and compare our results to those found using time-distance.

2. Inversion Technique

Ring-analysis techniques assess subsurface flow velocities from the Doppler shifts that flows induce in the sun’s acoustic oscillations or $p$ modes. These shifts manifest in power spectra made from the surface Doppler signal obtained within a patch on the sun as shifts in frequency of the resonant oscillations. Flow measurements made in this way represent spatial averages over the flow beneath that patch of the sun. Measurements are accordingly carried out over analysis regions of different sizes to obtain reconstructions of the solar flow field with varying levels of horizontal resolution. Smaller measurement regions provide a more horizontally localized, near-surface average of the flow, while measurements from larger regions permit the analysis of longer-wavelength, more deeply penetrating modes, allowing one to assess the deeper subsurface flows with albeit lower horizontal resolution.

One challenge in such schemes has been the reconciliation of measurements resulting from ring analyses of multiple region sizes. Moreover, for a given analysis-region size, complementary measurements are produced using different radial mode orders with different spatial wavenumbers. These measurements represent unique averages over the solar subsurface flow field and flow maps obtained from different subsets of this data will necessarily disagree with one another. To that end, we have developed a 3-D inversion technique that allows us to simultaneously invert data from a number of different modes and region sizes to obtain a self-consistent map of the horizontal flow of the sun over a range of depths.

The aim of our inversion algorithm is to deduce the solar flow field over one large patch of the solar disk, $v(\mathbf{r})$, from the ring measurements, $u_i$, and their errors, $\sigma_i$. We accomplish this through a regularized-least-squares inversion (RLS)[4] that trades off between the solution’s goodness of fit to the data and the smoothness of the solution. We wish to invert for the flow field over the entire solar disk at once, but owing to computational constraints, we can presently only invert regions $45^\circ$ square in a reasonable amount of time. Instead, a full-disk solution must be constructed by tiling the disk with smaller inversion regions that overlap by an arbitrary amount and “stitching” these inversion regions together. We are thus faced with an additional constraint that we want the solution for some region of the solar disk, $\alpha$, to join smoothly to the different overlapping regions $\beta$ for which we also perform inversions. We accomplish this with an iterative inversion procedure, solving for the $v(\mathbf{r})$ that minimizes the functional

$$
\sum_i \left( \frac{1}{\sigma_i^2} \left( u_i^\alpha - \int K_i(\mathbf{r}) v_k^\alpha(\mathbf{r}) d\mathbf{r} \right)^2 + \lambda \int \hat{L} v_k^\alpha(\mathbf{r}) d\mathbf{r} + R_k^\alpha = M \right)
$$

at each iteration $k$. Unique measurements are indicated by the index $i$, and $K_i$ is the corresponding flow sensitivity kernel. The smoothness trade-off parameter is denoted by $\lambda$ and the smoothing function by $\hat{L}$, which we take to be

$$
\hat{L} v(\mathbf{r}) = |\nabla v(\mathbf{r})|.
$$

Minimizing the functional $M$ is the standard RLS approach with the exception of the $R$ term. This additional term ensures the smoothness of the solution across regional boundaries, and we
Figure 1. Flow field realized through 3-D inversion of ring-analysis measurements of MDI obtained in January 2002. Horizontal velocity vectors (blue) overlay the magnetogram for that day (positive field in red, negative in green). At a depth of 0.2 Mm (a), the cellular structure of supergranulation is apparent. Strong outflows are visible around the two sunspots at longitude 215° and 225° down to a depth of 7 Mm (b, c). Below 7 Mm (d), flows are decidedly larger scale and exhibit no outflows from the sunspots.

Choose to define it as

\[ R_k^\alpha = \Lambda \sum_{\beta \neq \alpha} \int (v_k^\alpha(r) - v_{k-1}^\beta(r))^2 \, dr, \]  

where \( \Lambda \) is an additional trade-off parameter. This term penalizes the mismatch between the solution at the current iteration and the overlapping solutions from the prior iteration. This procedure requires the inversion of two matrices: one including \( R \) and one without it (for the first iteration). Convergence is typically achieved in 3-5 iterations.

The results presented in these proceedings have been constructed from power spectra of MDI.
Representative sunspot outflows from CR 1985 at a depth of 6 Mm. Such outflows are typical of the flow field at this depth in the vicinity of sunspots for this rotation.

Dopplergrams using three different region sizes of 2°, 4°, and 16° in heliographic angle. These Dopplergrams have been differentially tracked in latitude at the Snodgrass rotation rate [12] and apodized to 1.875°, 3.75°, and 15° respectively before creating power spectra.

We have incorporated measurements from radial mode orders \( n \) from zero to six. Our inversion grid extends in depth from the surface to a depth of 25 Mm. We employ a variable horizontal resolution with depth, adopting 0.9375° at the surface, 1.875° at the mid-depths, and 7.5° near the bottom of the domain. Inversion regions for these analyses are 37.5° on a side, overlapping nearest-neighbor regions by 7.5°. Nine such regions were arranged in a 3x3 matrix to cover the solar disk. The sensitivity kernels employed in this analysis have been computed using solar model S [3] and using the Born approximation, thereby accounting for the effects of wave scattering. Further details concerning the sensitivity kernels may be found in [1].

3. Flows around Sunspots

As a demonstration of the flow fields obtained through our 3-D ring-analysis inversions, we have examined MDI data from Carrington Rotation (CR) 1985. The existence of several magnetic active regions during this rotation allows us to examine the nature of subsurface flows around sunspots. The resulting flow field for a small portion of the solar disk (30° on a side) from 17 January 2002, is shown in figure 1 at four depths. The cellular pattern of supergranulation is clearly visible at the near-surface depth (0.2 Mm), and regions of divergence and convergence formed from the intersection of these cells tend to correspond with the presence of magnetic features (colored underlay). As the depth increases, the size scale of the sampled flows increases and the speed of the flows falls.

One of the more interesting features in this map is the presence of a pair of outflows centered on the sunspots at longitudes 215° and 225°. These outflows, corresponding to the moat flow observed at the surface, persist from the surface down to a depth of about 7 Mm. By 11 Mm, outflows around these sunspots give way to larger-scale meandering motions that freely pass through regions of strong magnetism. The presence of deep subsurface outflows around sunspots has proven to be an ubiquitous feature of the sunspots in our inversions of MDI data from CR 1985. Other representative examples of sunspot outflow from this same rotation are shown in Figure 2 (here at a depth of 6 Mm). In all cases, an unmistakable outflow signature is visible at depth, although the outflows around trailing spots are less organized.

To explore the depth dependence of these outflows in more detail, we plot the outflow as a function of depth for the sunspot in figure 1 located at longitude 225°. In figure 3a we show the radial outflow from the sunspot’s center averaged in azimuth about the sunspot for
Figure 3. (a) Average outflow with depth around the sunspot from the sunspot in figure 1 located at 225° in longitude. Shown is the outflow (averaged in azimuth and over one day’s worth of data) at a distance of 17 Mm (black), 23 Mm (red), and 28 Mm (blue) from the sunspot’s center. Error bounds are denoted by dotted lines. Outflows are visible at all depths, but decay sharply below 7 Mm. Representative averaging kernels versus depth are shown in b-d with the horizontal line indicating the kernel’s target depth. Averaging kernels are much narrower near the surface than at depth.

three different radial distances. Outflow is strongest near the surface and closest to the umbral-penumbra boundary. We note that the outflow is positive down to about 10 Mm, becoming weakly negative below this depth, and approaching zero beyond 15 Mm (where our averaging kernels become poor). The outflow structure transitions relatively sharply from strong outflow around 7 Mm to insignificant outflow at 10 Mm. Outflows exhibit two maxima; one near the surface and one near 5 Mm at the radii shown in Figure 3a. We may thus be seeing the combined signature of the surface moat flow and a distinct deeper outflow. All the sunspots we have examined in this Carrington rotation display this two component structure.

In Figure 3(b-d) we also show depth cuts through the accompanying averaging kernels resulting from this inversion. For target depths near the surface, these kernels are well localized, but broaden considerably as the target depth deepens. In some cases, narrow negative sidelobes do appear. These horizontal widths of our averaging kernels widen with depth as well, transitioning from a width of about 1° at the surface where the measurements from our 2° regions largely determine the solution, to about 8° at depth where the 16° regions govern the solution. The vertical width of the kernels is such that the true transition between strong and weak outflow (between 7 and 10 Mm) may in fact be much narrower than indicated by figure 3a.

4. Discussion
We have outlined a novel technique for the inversion of ring-analysis results that allows for the simultaneous inversion of measurements spanning the solar disk by tiling it with several smaller regions. This method allows for the self-consistent inversion of measurements made from a variety of analysis-region sizes, allowing us to incorporate both high- and low-resolution data into our inversion.

One of the more notable features we find is the ubiquitous outflow from sunspots. Zhao et al. [13, 14] have investigated the nature of outflow and inflows around sunspots using p-mode
time-distance procedures. These studies suggest that the surface outflow observed in earlier f-mode time-distance studies [6] around sunspots is a surface effect, persisting to a depth of \( \sim 1.5 \) Mm. Below this depth, and down to about 5 Mm, this trend reverses and an inflow is seen. This picture is, however, inconsistent with the findings of [6], who found outflows down to a depth of \( \sim 5 \) Mm using time-distance measurements incorporating both f-mode and p-mode data to study a sunspot in NOAA Region 9787.

Our ring analysis has found outflows that span a broad range of depths for essentially all observed sunspots, and seems consistent with the results of [6]. The lack of any outflow feature observed below a depth of about 9 Mm in the measurements of Zhao et al. [13] also agrees with our findings. The regularization used in our inversion might allow the imprinting of surface flows into the solution at depth. To minimize this effect, we repeated our inversion around the sunspots in figure 1 using only p-mode measurements. We found persistent outflows with depth even in the absence of the f-mode surface flow data.

The presence of two flow components, a superficial moat flow and a more deeply rooted outflow, is unexpected, particularly given that the time-distance studies of [6] have not reported such a subsurface structure (though they did indicate that outflows increased steadily from the surface to a depth of about 5 Mm). As the moat flow decays more sharply with radius than does the deeper outflow, these two flow structures may be driven by different physics that affect time-distance and ring-analysis techniques differently. We plan to pursue a more extensive study of the subsurface structure of sunspot flows in [5].

5. Acknowledgements
The authors would like to thank Juri Toomre, Deborah Haber, and Aaron Birch for many useful discussions and insights. Support for this work was provided through NASA grants NNG06GD97G, NNX07AH82G, NNX08AJ08G, NNX08AQ28G, and NNX09AB04G.

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