Nonlocality degradation of two-mode squeezed vacuum in absorbing optical fibers

Radim Filip, Ladislav Mišta Jr.
Department of Optics, Palacký University, 17. listopadu 50, 772 00 Olomouc, Czech Republic
(Dated: March 31, 2022)

Transfer of nonlocal two-mode squeezed vacuum state through symmetrical and asymmetrical lossy channel is analysed and we demonstrate that the nonlocality is more robust against losses, than it has been previously suggested. It can be important for security of continuous-variable quantum cryptography employing entangled states.

PACS numbers: 03.65.Ud

I. INTRODUCTION

Quantum mechanical entanglement and nonlocality are the inherent features of microworld having many practical applications in quantum information processing and communication [1]. The quantum information protocols developed originally in the context of qubit systems have been successfully extended to continuous-variable (CV) domain [2]. Recently, both CV analogues of BB84 and Ekert’s quantum cryptography [3] protocols have been suggested [4]. In experimental implementations of CV quantum cryptography protocols with entangled states the two-mode squeezed vacuum state

\[ \rho_{\text{NOPA}} = (1 - \lambda_1^2) \sum_{m,n=0}^{\infty} \lambda_1^{m+n} |m,m\rangle\langle n,n|, \]

where \( \lambda_1 = \tanh r \) and \( \sinh^2(r) = \langle n \rangle_r \) is the mean number of signal photons (\( r \) is the squeezing parameter), commonly serves as a quantum channel. This state [5] can be generated either by using nondegenerate parametric amplifier (NOPA) [6, 7] or by mixing two independent single-mode squeezed fields at a beam splitter (or in fiber coupler) [8, 9]. The quantum key distribution goes as follows.

The NOPA source emits two fields: one is distributed to Alice (A) and the other propagates towards Bob (B). Alice and Bob randomly select to measure one of two conjugate field quadrature amplitudes \( Q \), \( P \) and, as the squeezing parameter \( r \) increases, the correlation (anti-correlation) between the results of the same quadrature measurements on Alice’s and Bob’s side grows. Communicating their choices of the measurements through a public classical channel and keeping only the results when both of them measured the same quadrature the key is established between Alice and Bob.

In reality, transfer of the entanglement between two distant partners, usually realized by means of optical fibers, suffers by undesirable losses which strongly reduce the amount of the entanglement. These losses can be induced either by the fiber imperfections or they can be introduced deliberately by an eavesdropper Eve (E). If the losses are modelled by reservoir at zero temperature, one can show that the entanglement of the state [5] does not vanish completely for any extinction coefficient of fibers [10]. On the other hand, the nonlocal correlations between communicating parties Alice and Bob guarantee the security against the individual attacks [11]. We employ a CV analogue of this statement and assume that quantum channel has a sufficient quality if it preserves the nonlocality of distributed NOPA states. Thus, knowing the fiber link transmittivity, one can judge its convenience before experimental implementation.

In practice, it might be based on the Bell inequalities [10, 11] for CV systems [12, 13, 14]. The nonlocality of the two-mode squeezed state under damping is deteriorated as has been recently demonstrated utilizing the Banaszek-Wódkiewicz inequality [15]. Qualitatively, it was shown that for fixed fiber losses the larger the initial squeezing is the more rapidly the nonlocality is degraded irrespective of the entanglement preservation. In this report, we investigate the nonlocality of the two-mode squeezed vacuum state in vacuum environment employing more appropriate CHSH Bell inequalities than those used in [15]. By introducing the “spin-1/2” operators in infinite-dimensional Hilbert space one obtains the two-qubit form of the CV Bell inequalities [14] to which the Horodecki’s nonlocality criterion [12] can be directly applied. This procedure is shown to be more efficient to test the NOPA state nonlocality even when the local losses are involved. In the absence of losses the maximal Bell factor rapidly converges to maximal value \( B_{\text{max}} = 2\sqrt{2} \) with increasing squeezing parameter \( r \). If the losses are involved, then we confirm the qualitative statement that the stronger initial squeezing induces more rapid nonlocality degradation [15]. However, it is proved that the transmitted NOPA state does not admit a local realistic description for much more pronounced losses than it has been pointed out in [15]. This fact is important for secure long-distance quantum communication based on the shared two-mode squeezed vacuum state.

II. NONLOCALITY DEGRADATION

We consider a lossy two-line optical-fiber transmission channel injected by the NOPA state [5]. Simulating the losses by Markovian reservoir at zero temperature and eliminating free oscillations the evolution of the NOPA state can be effectively described by the following standard master equation for the density matrix

\[ \frac{d\rho}{dt} = -i[H,\rho] + \Gamma_A (2a_A \rho a_A^\dagger - a_A^\dagger a_A \rho - \rho a_A^\dagger a_A) + \]

with \( H = \frac{\lambda_1^2}{\Gamma_A} (2a^\dagger a) \) and 

\[ \Gamma_A = \frac{\lambda_1^2 + \lambda_1^2}{\Gamma} \]

the damping rate of the NOPA state.
where \( \Gamma_i, i = A, B \) are damping constants and \( a_i, a_i^\dagger \) are annihilation and creation operators of modes \( A \) and \( B \). Equivalently, the following nonunitary evolution can induce an eavesdropper Eve by mixing the modes \( A \) and \( B \) with vacua at two beam splitters with reflectivities \( R_i = \sqrt{1 - \exp(-\Gamma_i t)}, i = 1, 2 \). Solving the equation \( \rho = \rho \) for initial NOPA state \( \| \) one finds the following solution

\[
\rho = (1 - \lambda^2) \sum_{m,n=0}^{\infty} \left( \lambda \sqrt{1 - R_A^2} \sqrt{1 - R_B^2} \right)^{m+n} \times \\
\times \sum_{k=0}^{\infty} \sqrt{\left( \frac{m}{k} \right) \left( \frac{n}{l} \right)} \left( \frac{R_A}{\sqrt{1 - R_A^2}} \right)^{2k} \left( \frac{R_B}{\sqrt{1 - R_B^2}} \right)^{2l} \times \\
\times |m - k, m - l\rangle \langle n - k, n - l|. \tag{3}
\]

Note that if the losses are introduced only on the Bob’s side \( (R_A = 0) \), the \((A, E)\) state can be obtained from the state \( \| \) by exchanges \( R_B \to \sqrt{1 - R_B^2} \) and \( \sqrt{1 - R_B^2} \to -R_B \).

To turn the infinite dimensional nonlocality problem into the two-qubit problem, one can simply introduce the following single-mode realization of the Pauli matrices \( \| \)

\[
S_1 = \sum_{m=0}^{\infty} (|2m\rangle \langle 2m + 1| + |2m + 1\rangle \langle 2m|),
\]

\[
S_2 = i \sum_{m=0}^{\infty} (|2m + 1\rangle \langle 2m| - |2m\rangle \langle 2m + 1|),
\]

\[
S_3 = \sum_{m=0}^{\infty} (|2m\rangle \langle 2m| - |2m + 1\rangle \langle 2m + 1|), \tag{4}
\]
satisfying the Pauli matrix algebra

\[
[S_i, S_j] = 2i\epsilon_{ijk}S_k, \quad (S_i)^2 = 1, \tag{5}
\]

where \( i, j, k = 1, 2, 3; \) \( \epsilon_{ijk} \) is the totally antisymmetric tensor with \( \epsilon_{123} = +1 \). Due to the commutation rules \( \| \), the nonlocality of our state \( \| \) can be investigated by means of the standard two-qubit CHSH Bell inequalities in which the Pauli matrices are replaced by the operators \( \| \)

\[
2 \geq |\langle a \cdot S^A |b \cdot S^B \rangle| + |\langle a' \cdot S^A |b \cdot S^B \rangle| + |\langle a \cdot S^A |b' \cdot S^B \rangle| - |\langle a' \cdot S^A |b' \cdot S^B \rangle|, \tag{6}
\]

where \( a, a', b, b' \) are unit vectors in the real three-dimensional space and the angle brackets denote the average over the matrix \( \rho \). Now, according to the Horodecki nonlocality criterion \( \| \), if

\[
B_{\text{max}} = 2\sqrt{u + u'} > 2, \tag{7}
\]

where \( u \) and \( u' \) are two greater eigenvalues of the matrix \( U = V^T V \) \( (V_{ij} = \text{Tr}(\rho S^A_i S^B_j) \) and \( T \) stands for the transposition), then the density matrix \( \| \) violates the inequalities \( \| \) for some vectors \( a, a', b \) and \( b' \).

Employing \( \| \) and \( \| \) one finds after some algebra, that the matrix \( U \) is diagonal with two-fold eigenvalue \( \alpha^2 \) and single eigenvalue \( \beta^2 \)

\[
\alpha = 2(1 - \lambda^2) \sum_{m=0}^{\infty} (m + 1) \times \\
\times \left( \lambda \sqrt{1 - R_A^2} \sqrt{1 - R_B^2} \right)^{2m+1} \Lambda_A(m)\Lambda_B(m), \tag{8}
\]

\[
\Lambda_i(m) = \sum_{k=0}^{[m/2]} \left( \frac{m}{2k} \right) \frac{1}{\sqrt{2k+1}} \left( \frac{R_i}{\sqrt{1 - R_i^2}} \right)^{2m-4k}, \tag{9}
\]

where \( i = A, B \) and

\[
\beta = \frac{1 - \lambda^2}{1 - \lambda^2(1 - 2R_A^2)(1 - 2R_B^2)}. \tag{10}
\]

The maximal Bell factor then reads

\[
B_{\text{max}} = 2\sqrt{\alpha^2 + \max(\alpha^2, \beta^2)}. \tag{11}
\]

In the following we investigate two special cases: (i) \( R_A = R_B = R \) and (ii) \( R_A = R, R_B = 0 \). The first case corresponds to the symmetric location of Alice and Bob with respect to the source of the NOPA state, whereas the second case is connected with asymmetric arrangement, in which Bob resides in the close vicinity of the source.

The nonlocality analysis of the transmitted state \( \| \) is based on the numerical calculation of the value of the maximal Bell parameter \( B_{\text{max}} \| \). We confirm the qualitative result of the previous paper \( \| \) that the rapidity of nonlocality degradation of the NOPA state increases with increasing initial squeezing, as can be seen in Fig. 1. However, there is pronounced difference in the rapidity of the nonlocality destruction in comparison with the result found in \( \| \). Our analysis reveals that the nonlocality is preserved even if stronger losses are taken into account. In the following we perform a quantitative comparison of our results with results obtained in \( \| \). If \( r = 1 \), then the Fig. 1 demonstrates that the nonlocality threshold for the symmetric case (i) occurs for \( R \approx 0.42 \) in contrast to the threshold value \( R \approx 0.13 \) calculated in \( \| \). In our analysis the threshold value \( R \approx 0.13 \) is achieved for larger squeezing \( r > 2 \). Better results are obtained in the asymmetric case (ii). If \( r = 2 \), then the threshold is shifted to value \( R \approx 0.24 \) as can be seen in Fig. 2.

The dependence of the threshold damping parameter \( R_{\text{max}} \) on the squeezing parameter \( r \) for symmetric and asymmetric setups is depicted in Fig. 3 and Fig. 4, respectively. The inequality \( R < R_{\text{max}} \) represents sufficient condition for nonlocality of the NOPA state to be
preserved during the transfer through lossy optical fibers. If the squeezing $r$ is sufficiently large ($r \geq 1.5$), then one can find the following fits for the symmetric case (Fig. 3)

$$R_{\text{max}} \approx 1.64e^{-r}$$

and for the asymmetric case (Fig. 4)

$$R_{\text{max}} \approx 1.2e^{-r}.$$  \hspace{1cm} (13)

These approximative rules enable us to check simply the ability of the lossy fiber communication channel to transfer the nonlocality of the NOPA state.

The losses can be described by the dimensionless absorption coefficient

$$\gamma = \frac{\Gamma L}{v_f} = \ln \left( \frac{P(0)}{P(L)} \right),$$

where $P(0)$ is input optical power, $P(L)$ is output optical power, $L$ is length of the fiber and $v_f$ is the field velocity in the fiber. According to the results obtained in [12] the initial squeezing $r = 1$ ($B_{\text{max}} = 2.19$) leads to the following threshold absorption coefficient $\gamma_{\text{max}} \approx 8.5 \times 10^{-3}$. On the other hand, our analysis gives the following threshold absorption coefficient $\gamma_{\text{max}} \approx 9.7 \times 10^{-2}$. Thus, it is important fact for the experimentalist that the nonlocality of the NOPA state with $r = 1$ is preserved at ten times longer transmission length than it was suggested earlier [12]. Thus the quantum key can be securely distributed even under the influence of stronger losses.

III. CONCLUSION

We analyse the nonlocality evolution of the two-mode squeezed vacuum state transferred through symmetric or asymmetric lossy channels. The nonlocality preservation is important to ensure quality of quantum channel for CV cryptography with entangled states. We confirm qualitative statement about pronounced nonlocality degradation with increasing initial squeezing for fixed losses. However, utilizing the stronger nonlocality criterion in the infinite-dimensional Hilbert space, we demonstrate the nonlocality preservation in the presence of substantially larger losses, than it was suggested previously. Maximal allowable losses preserving the nonlocality can be approximately expressed by the simple formulas (12) and (13), which can be useful for the experimentalist to quickly determine the possibility of nonlocality transfer. The theoretical nonlocality proof presented here is more simple in comparison with the proof based on the generalized Banaszek-Wódkiewicz (B-W) version of Bell inequalities which requires the numerical maximization in the eight-dimensional space. On the other hand, it is not clear as to measure the spin-like operators [9] experimentally. Thus, our approach is appropriate for theoretical calculations, whereas the generalized B-W inequalities are more suitable for the experimental test of nonlocality.

IV. ACKNOWLEDGMENTS

The authors would like to thank J. Fiurášek for useful discussions. This work was supported by project LN00A015 and project CEZ:J14/98 of the Czech Ministry of Education and by the EU grant under QIPC, project IST-1999-13071 (QUICOV).

[1] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres and W.K. Wootters, Phys. Rev. Lett. 70 1895 (1993); M. Zukowski, A. Zeilinger, M.A. Horne and A. Ekert, Phys. Rev. Lett. 71, 4287 (1993); J.-W. Pan, D. Bouwmeester, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett. 80 3891 (1998); C.H. Bennett and S.J. Wiesner, Phys. Rev. Lett. 69 2881 (1992); A. Ekert, Phys. Rev. Lett. 67 661 (1991); A. Barenco, D. Deutsch, A. Ekert and R. Jozsa, Phys. Rev. Lett. 74 4083 (1995).
[2] S.L. Braunstein and H.J. Kimble, Phys. Rev. Lett. 80 869 (1998); S.L. Braunstein and H.J. Kimble, Phys. Rev. A 61 042302 (2000); R.E.S. Polkinghorne and T.C. Ralph, Phys. Rev. Lett. 83 2095 (1999).
[3] C. Bennett, G. Brassard, in: Proceedings of the Int. Conf. on Computer, System and Signal Processing, Bangalore, India (IEEE, New York, 1984), p. 175; A. Ekert, Phys. Rev. Lett. 67 91 (1991); C. Bennett, G. Brassard, and N.D. Mermin, Phys. Rev. Lett. 68 557 (1992); Ch.A. Fuchs, N. Gisin, R.B. Griffiths, Ch.S. Niu, and A. Peres, Phys. Rev. A 56 1163 (1997).
[4] T.C. Ralph, Phys. Rev. A 61 010303(R) (2000); M. Hillery, Phys. Rev. A 61 022309 (2000); M.D. Reid, Phys. Rev. A 62 062308 (2000); N.J. Cerf, A. Ipe, and X. Rotenberg, Phys. Rev. Lett. 85 1754 (2000); N.J. Cerf, M. Lévy, and G. Van Assche, Phys. Rev. A 63 052311 (2001); [5] Z.Y. Ou, S.F. Pereira, H.J. Kimble and C.K. Peng, Phys. Rev. Lett. 68, 3663 (1992).
[6] A. Furusawa, J.L. Sørensen, S.L. Braunstein, C.A. Fuchs, H.J. Kimble and E.S. Polzik, Science 282 706 (1998).
[7] Ch. Silberhorn, P.K. Lam, O. Weiss, F. König, N. Korolkova and G. Leuchs, Phys. Rev. Lett. 86 4267 (2001).
[8] S. Scheel, T. Opatrný, D.-G. Welsch, Opt. Spectrosc. 91 411 (2001).
[9] N. Gisin, and B. Huttner, Phys. Lett. A 228, 13 (1997); [10] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. A 47 777 (1935); D. Bohm, Quantum Theory (Prentice Hall,
Englewood Cliffs, NJ, 1951). J.S. Bell, Physics (Long Island City, N.Y.) 1 195 (1965); J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt, Phys. Rev. Lett. 23 880 (1969).

[11] R. Horodecki, P. Horodecki, and M. Horodecki, Phys. Lett. A 200, 340 (1995).

[12] M.D. Reid, Phys. Rev. A 40 913 (1989); M.D. Reid, and P. Drummond, Phys. Rev. Lett. 60 2731 (1988); W.J. Munro, and G.J. Milburn, Phys. Rev. Lett. 81 4285 (1998); A. Gilchrist, P. Deuar, and M.D. Reid, Phys. Rev. Lett. 80 3169 (1998); Phys. Rev. A 60 4259 (1999); K. Banaszek and K. Wódkiewicz, Phys. Rev. A 58 4345 (1998); W.J. Munro, Phys. Rev. A 59 4197 (1999); M.S. Kim and J. Lee, Phys. Rev. A 61 042102 (2000).

[13] B.S. Cirel’son, Lett. Math. Phys. 4 93 (1980); D. Wilson, H. Jeong, and M.S. Kim, arXiv: quant-ph/0109121 (2001).

[14] Z.-B. Chen, J.-W. Pan, G. Hou, and Y.-D. Zhang, Phys. Rev. Lett. 88 040406 (2002).

[15] H. Jeong, J. Lee and M.S. Kim, Phys. Rev. A 61 (2000) 052101.

[16] In our knowledge, this was firstly discussed in paper: H. Halvorson, Lett. Math. Phys. 53, 321 (2000).

FIG. 1: Violation of the Bell inequalities for symmetrically distributed NOPA state in dependence on the damping parameter $R$.  

FIG. 2: Violation of the Bell inequalities for asymmetrically distributed NOPA state in dependence on the damping parameter $R$.  

FIG. 3: Threshold losses $R_{\text{max}}$ for symmetrically distributed NOPA state in dependence on the squeezing parameter $r$: the solid line represents exact numerical calculations and the dotted line represents the approximative result.
FIG. 4: Threshold losses $R_{\text{max}}$ for asymmetrically distributed NOPA state in dependence on the squeezing parameter $r$: the solid line represents exact numerical calculations and the dotted line represents the approximative result.
