Gravitomagnetic Analogs of Electric Transformers

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Abstract

Linearized general relativity admits a formulation in terms of gravitoelectric and gravitomagnetic fields that closely parallels the description of the electromagnetic field by Maxwell’s equations. For steady mass currents, this formalism has been used to understand gravitomagnetic effects like the Lense-Thirring dragging of inertial frames. For time-varying mass-energy currents, the analog of Faraday’s law suggests new effects based on the gravitational equivalent of a transformer where such currents take the place of electrical currents. New experimental possibilities are suggested including a novel coupling mechanism of electromagnetism to gravity, new tests of general relativity in the ultrarelativistic limit using particle beams in the LHC, and searches for a materials exhibiting the gravitational analog of ferromagnetism.

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I. INTRODUCTION

General relativity can be treated in the weak-field (linearized) limit via a set of equations formally analogous to Maxwell’s equations for electromagnetism. These are derived (following the conventions of [1]; see also [2]) by expanding $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ where $\eta_{\mu\nu}$ is the flat Minkowski metric and $h_{\mu\nu}$ represents first order perturbations. Defining $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$ with $h = \text{trace}(h_{\mu\nu})$ and indices raised and lowered with $\eta_{\mu\nu}$, and neglecting terms of order $c^{-4}$, one finds the metric

$$ds^2 = -c^2 \left( 1 - 2 \frac{\Phi}{c^2} \right) dt^2 - \frac{4}{c} \left( \vec{A} \cdot d\vec{x} \right) dt + \left( 1 + 2 \frac{\Phi}{c^2} \right) \delta_{ij} dx^i dx^j$$  

(1)

The quantities $\Phi$ and $\vec{A}$ are the gravitoelectromagnetic analogs of the scalar and vector potential in electrodynamics. One can define an electrogravitic field $\vec{E}$ and gravitomagnetic field $\vec{B}$ by $\vec{E} = -\vec{\nabla}\Phi - \frac{1}{c} \frac{\partial}{\partial t} \left( \frac{1}{2} \vec{A} \right)$ and $\vec{B} = \vec{\nabla} \times \vec{A}$. With the gauge condition $\frac{1}{c} \frac{\partial \Phi}{\partial t} + \vec{\nabla} \cdot \left( \frac{1}{2} \vec{A} \right) = 0$ which is connected to the conservation of $T^\mu_\nu$, $\vec{E}$ and $\vec{B}$ satisfy the Maxwell-like equations:

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial}{\partial t} \left( \frac{1}{2} \vec{B} \right), \quad \vec{\nabla} \cdot \left( \frac{1}{2} \vec{B} \right) = 0$$  

(2)

and

$$\vec{\nabla} \times \left( \frac{1}{2} \vec{B} \right) = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi G}{c} \vec{j}, \quad \vec{\nabla} \cdot \vec{E} = 4\pi G \rho$$  

(3)

where $T^{00} = \rho c^2$, $T^{0i} = cj^i$, and $j^\mu = (c\rho, \vec{j})$ is the mass-energy current of the source, $T^\mu_\nu$ is the stress-energy-momentum tensor, and $G$ is Newton’s constant. Within the approximations used, this allows one to transcribe results from classical electrodynamics to general relativity provided one defines sources of mass $M$ to have gravitoelectric charge $Q_E = GM$ and gravitomagnetic charge $Q_B = 2GM$. Test particles are assigned charges of the opposite sign to ensure that gravity is attractive.

Most calculations of gravitomagnetic near-field (i.e. excluding gravitational radiation) effects in linearized gravity have assumed steady mass currents. For example, the Lense-Thirring [6] dragging of inertial frames can be thought of as the effect of a steady gravitomagnetic field (for a simple derivation, see [7]).
Experimental consequences of Faraday-law (the first expression in equation 2) effects have been considered in perturbations of orbits \[8\] and effects of a massive rotating object rotating moving near a torsional oscillator \[9\].

Remarkably, however, the simple transcription of an electrical transformer \[3\] into a gravitational one does not seem to have been mentioned explicitly in the literature, so I claim:

There is a gravitational analog of an electrical transformer with the time-varying electrical currents through wire windings replaced by mass-energy currents through suitable conduits. Such a transformer can be used to step up or step down the “gravitomotive force” \(G\) (the line integral of \(\vec{E}\) defined above by analogy with electromotive force).

Bini et al. \[1\] consider the gravitational Faraday effect for two connected, concentric tubes of fluid encircling a rotating mass whose angular momentum changes linearly in time, but there is no genuine primary winding. Forward \[4\] considers gravitomagnetic fields produced by a time-varying mass current in order to generate dipole “antigravity” type fields, but there is no secondary winding.

Note that the operation of such a transformer is quite independent of the value of Newton’s constant, just as the operation of a normal transformer is independent of the coupling strength of the electromagnetic field to charge.

Such a device may be difficult to construct in the laboratory, but there is an astrophysical analog if there are flows of discrete masses. Note the difference between a steady current (analogous to a steadily rotating mass) and the flow of discrete masses in orbit around a central mass – the effect described here would be missed by approximating a flow of discrete chunks of mass by the flow of a continuous fluid, much as a transformer will work with a pulsed DC current (which clearly has oscillating Fourier AC components) but not with a continuous DC current. In particular, one can imagine the gravitational analog of betatron-type acceleration resulting from changing gravitomagnetic fluxes due to astrophysical processes involving the motion of large masses. Unlike gravitational radiation, the effect here is a near-field one which avoids suppressions by powers of \(v/c\) (where \(v\) is a typical speed and \(c\) the speed of light) due to retardation effects and does not require rapidly changing mass quadrupole moments \[10\].

An even more interesting possibility is that:

One can replace the primary of such a device with a mass flow produced by non-gravitational means, such as an electrically driven current of charged particles. In this case
the output gravitomotive force $G$ per turn of secondary winding would be (following \[2\])

$$G = -\frac{2}{c} \frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$

(4)

where the integral on the right hand side is the gravomagnetic flux produced by the source from the first equation in \[3\] that links the secondary. For example, a time-varying flow of charged massive particles could be driven through the primary by electromagnetic forces. This would give rise not only to the usual Maxwell magnetic fields, but also to gravitomagnetic fields. Effectively one has a transformer which converts a time-varying electromotive force into a time-varying gravitomotive force which can, in principle, be made arbitrarily large by increasing the number of turns on the secondary.

Just as is the case for electrical transformers\[3\], one can imagine tuning the primary and/or secondary to (electrical or mechanical) resonance with the driving force. This technique is used commonly for efficient high-frequency air-core transformers including Tesla coils\[11\] and would be natural for situations where there is no gravitational analog of a ferromagnetic or even ferrite core. Any electromagnetic Faraday-type couplings between primary and secondary can be prevented by magnetically shielding them from each other allowing the gravitomagnetic inductive effects to be clearly separated. In addition, the flows in the secondary need not be of electrically charged particles, again eliminating any effects due to conventional electromagnetic induction. Clearly the situation with primary and secondary exchanging roles can also be envisaged, although it is hard to envision significant laboratory sources of gravitationally driven electrically neutral matter.

Extremely relativistic cases such as a pulsed beam of (laser) light or the pulsed beam of particles in an accelerator as the primary are particularly interesting. Approximations for the Maxwell-like description neglecting powers of $v/c$ may not be valid \[12\] and new and distinct effects might be found experimentally before being predicted theoretically. Braginsky et al.\[9\] consider experiments to measure the gravitational kick delivered by a relativistic particle bunch as it passes a suitable detector, but this is distinct from the induction effect suggested here.

Laboratory experiments to test relativistic gravitational effects\[9\] are typically hampered by the difficulty of making large masses move at large speeds. For beams in an accelerator such as the LHC\[13\] one has particles moving very close to the speed of light so all effects usually suppressed by powers of $v/c$ (including the leading suppression of magnetic over
electric effects by one power of \(v/c\) are now unsuppressed. In addition, very high accelerations (very high angular velocities in the case of circular motion) and thus high rates of change of gravitomagnetic flux are possible which could not be achieved with normal matter: the up to 7 TeV protons in the LHC are \(10^{11}\) protons per bunch times 2808 bunches with a relativistic \(\gamma = \frac{1}{\sqrt{1-v^2/c^2}} \approx 7000\) revolving in a 27 km circle at a rate of 40 MHz, held together in a way that no ordinary matter could via external electromagnetic fields.

The LHC is cooled by superfluid liquid helium at 1.9 K, so much of the infrastructure for superconducting magnetic shielding or cryogenic detectors - in particular superconducting[14] or superfluid[15] Josephson interferometers - is already present. Detailed proposals for various experimental scenarios are in preparation[16].

The production of high frequency gravitational waves by accelerators such as the LHC[13] or the Tevatron[18] has been considered previously[17]. Taking the source of magnetogravitic flux to be a gravitational wave produced by an accelerator, the secondary of the proposed transformer would essentially be a loop antenna, but it should be noted that what is proposed here a new effect which is strictly near-field and distinct from gravitational radiation. Gravitational synchrotron radiation produced would necessarily be accompanied by much larger electromagnetic synchrotron radiation losses in any readily conceivable device so that fact that the effect considered here is near-field (not radiation) is very important.

Eddy current losses due to induced EMF’s can, of course, be made very small independent of any near-field gravitational effects via standard techniques[3].

As noted long ago by Forward[4, 5], electrical transformers are remarkably efficient at transferring power. A large part of this efficiency is due to the existence of ferromagnetic materials with very high magnetic permeability which can be used to make essentially all the magnetic flux produced by the primary link the secondary. He suggested that a search be made for the gravitational analog of such materials, pointing out that ferromagnetic levels of permeability are ultimately due to cooperative effects of spins. Whether or not spin coupling to torsion[19] might play a role could be open here to experimental tests. Torsion is zero in standard general relativity. Certainly gravitational paramagnetic and diamagnetic effects have been considered in the literature[20, 21]. Thus:

One can use a transformer of the kind described above (for example with pulsed electric current or particle beam in the primary and a superfluid or superconducting secondary) to measure, or at least set bounds on, the gravitational equivalent of magnetic permeability, and
to search for the gravitational equivalent of ferromagnetic materials by using them as cores.

So far attempts have only been made using wedges of material placed between laboratory sources of gravitational waves and Weber-type detectors.

One should keep in mind that ferromagnetism is a very unusual state of condensed matter and its early discovery was very fortuitous, relying on the fact that some materials exhibiting it were easy to find in nature. Without this good luck, magnetism itself might have been very hard to discover.

Other surprising and unforeseen phenomena in bulk condensed matter systems such as superconductivity and superfluidity were found much later, with colossal magnetoresistance only discovered in the 1950’s, and Bose-Einstein condensation in 1995.

Aside from analogs of ferromagnetism, other effects such as effective $\vec{E} \cdot \vec{B}$ couplings in condensed matter might also be looked for. However unlikely such effects might seem at the moment, there are simply no constraints on their existence at all at present. Given that there are no predictions even of the magnetic permeability of ferromagnetic materials from first principles, one might be optimistic that an as-yet-undiscovered gravitational analog might be found - only experiment can tell!

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