Emission by dielectric with oscillating refractive index

V Hizhnyakov, H Kaasik
Institute of Physics, University of Tartu, Riia 142, 51014 Tartu, Estonia
e-mail: hizh@fi.tartu.ee

Abstract. We consider the two-photon emission by a dielectric where refractive index $n$ oscillates in time. The oscillations of $n$ cause analogous oscillations of the optical length. The velocity of the latter oscillations, in principle, may be arbitrarily large. If the maximal velocity of these oscillations $V$ is much less than the light velocity $c$ then the intensity of the emission will increase with $V$. However, if it strongly exceeds $c$ then the opposite dependence takes place as the zero-point fluctuations cannot appreciably react on such a fast oscillations. Therefore, the crossover for the dependence of the emission on $V$ exists at $V_r \sim c$ and a resonant enhancement of it is observed at this value of $V$. Our calculation gives $V_r \approx 3c$.

1. Introduction
The usual nonlinear optical processes account for the stimulated emission [1]. The contribution of spontaneous emission to these processes also exists but usually is very small. Nevertheless one of processes of this type, - the two-photon emission of a dielectric medium with the time-dependent refractive index $n(t)$ attracts last years a noticeable attention (see, e.g. [2–7]). The reason is that this emission, being a response of the zero-point state to its temporal change gives a direct manifestation of this state [8] (usually it is called the dynamic Casimir effect). Another interesting heuristic aspect of this emission is that the change of $n$, linear in time can be considered as an analog of the accelerating reference frame [2]; therefore the two-photon emission in this case has close analogy to the thermal-like radiation seen by an observer when it moves with the constant acceleration [9]. This thermal-like radiation is the close analog of the well-known Hawking radiation of black hole [10].

The two-photon emission may have remarkable intensity if the change of $n$ in time takes place in a large area and is large and fast [3]. Strong time dependence of $n$ can be realized only in a small "spot" for a very short time [2]. Unlikely to that, the periodic oscillations of $n$ in time can be activated in a medium of a large size by applying a monochromatic laser beam. The size factor is of a primary importance here. Indeed, the oscillations of the refractive index in time lead to subsequent oscillations of the optical length of the medium $l$. Taking $n(t) = n_0 + n'_0 \cos \omega_0 t$, we get for the time-dependent part of the optical length $l(t) = l_0 \cos \omega_0 t$, where $l_0 = n'_0 L^{(0)}$. Consequently, if $L^{(0)}$ is large then one can achieve large amplitude of the oscillations of the optical length $l_0$ even if $n'_0$ is not too large. Therefore, the maximal velocity of the oscillations of the optical length $V = \omega_0 l_0$ can also get large values which may be comparable to or even exceed the velocity of light in vacuum $c$. This has an important consequence: strong enhancement of the quantum emission if the maximal velocity $V$ of the oscillations of the optical length approaches $c$.

The reason for this resonant enhancement can be elucidated as follows. If the maximal velocity of the oscillations of the optical length is small ($V \ll c$) then the intensity of the emission increases with
However, if $V \gg c$, then the opposite dependence takes place, as the zero-point fluctuations cannot appreciably react on such fast oscillations. Therefore, the crossover for the dependence of the emission on $V$ exists at $V \approx \frac{c}{2}$ and a resonant enhancement of it is observed at $V_r \approx 3c$.

A possibility of the enhancement of the quantum emission in a dielectric medium with time-dependent parameters has been also pointed out in [4–7] (see also [8]). However, in all previous studies this enhancement has been found to take place only in short resonator, which has discrete modes with well-separated frequencies. The emission in resonator is non-stationary: the number of generated photons increases in time exponentially. On the contrary, in our case the emission is stationary. These differences result from the different mechanisms of the enhancement. In the case of a short resonator this is the parametric resonance for a time-dependent electromagnetic mode. The stimulated emission plays in this mechanism an essential role: all generated photons remain in the resonator and essentially exert the process, resulting in the exponential increase of the number of photons in time. However in the case under consideration there is a continuum of modes which, in general, all are excited due to oscillations of $n$. All generated photons leave the region of generation and do not influence the further process. The strong increase of the emission at $V \approx 3c$ takes place due to the resonance of the zero-point state with the oscillations of the optical length.

2. Spontaneous emission by a dielectric with oscillating refractive index

2.1. Field equations

To prove the assertion we consider a dielectric exposed in a standing laser wave. We describe the wave classically and account for the time-dependent part of the nonlinear polarization operator $\hat{P}_{nl}$.

The equation for the field operator $\hat{A}$ then reads [1]:

$$\hat{A} - \left(\frac{c}{n_0}\right)^2 \nabla^2 \hat{A} = -4\pi \hat{P}_{nl}.$$  

Taking $\hat{P}_{nl}(t) = \eta(t) \hat{A}(t) / 4\pi$ with $\eta(t) = \eta_0 \cos \omega t$, one gets

$$\hat{A}' - \left(\frac{c}{n(t)}\right)^2 \nabla^2 \hat{A}' = 0,$$  

where $\hat{A}' = \hat{A}(1 + \eta(t))$, $n(t) \approx n_0 + n'_0 \cos \omega t$, $n'_0 \approx \eta_0 / 2n_0$ (we suppose that $\eta_0 \ll 1$). Equation (1) is the wave equation with a time-dependent refractive index.

We consider the case when the dielectric is placed in a large resonator; the planes of the interface are parallel to the mirrors of the resonator. We restrict ourselves to the case when $n'_0$ very slowly changes in space. Then, according to the theory of the wave-optics [11] the waves $\hat{A}_q(x,t) \propto \exp(iqx)$ are the solutions of this equation outside the medium where $\hat{A}' = \hat{A}$; the allowed values of $q$ satisfy the condition $qL(t) = 2\pi k$, $k = k = 0,1,2,...$ Here $L(t) = L_0 + l(t)$, $L_0$ is the optical length (eikonal) of the resonator + dielectric in the absence of the excitation (i.e. in the case when $n'_0 = 0$). Therefore the field operator outside the medium can be presented in the form $\hat{A} = \sum_k \sin(\pi k x / L(t)) \hat{A}_k$. Inserting this equation for $\hat{A}$ into the wave equation in vacuum we get in the $L_0 \to \infty$ limit

$$\sum_k \left[ \frac{\ddot{A}_k + \omega_k^2 \hat{A}_k}{2} - \left(\frac{\pi k x / L}{L_0}\right)^2 \left(2L_0 \ddot{A}_k + \dot{L} \dot{A}_k\right) \cot\left(\pi k x / L\right) \right] = 0,$$

where $\omega_k = \pi k c / L_0$ (the terms $\propto L^{-m}$ with $m > 2$ are neglected). Using the identity

$$x = -\left(\frac{2L}{\pi}\right) \sum_{j=1}^{\infty} (-1)^j j^{-1} \sin(\pi j x / L), \quad -L \leq x \leq L,$$

one gets for $\ddot{A}_k = (-1)^k \hat{A}_k \sqrt{2 \omega_k / \hbar}$ the equation
\[ \ddot{b}_k + \omega_k^2 \dot{b}_k = \omega_k \dot{B}_k, \quad (2) \]

where \( \dot{B}_k = \frac{2}{\pi c} \sum_{j \neq k} j(2 \hat{L}_j + \hat{E}_j)/(j^2 - k^2) \). The integral form of the equation (2) reads
\[ \dot{b}_k(t) = \dot{b}_k^{(0)}(t) + \int_0^t dt_1 \sin(\omega_k(t-t_1)) \dot{B}_k(t_1), \quad (3) \]
where \( \dot{b}_k^{(0)} \) is the unperturbed operator \( \dot{b}_k \).

2.2. Correlation function and emission rate

We consider the \( t \to \infty \) limit and present the hermitian operator \( \dot{b}_k \) in the form \( \dot{b}_k = \dot{\alpha}_k + \dot{\alpha}_k^\dagger \), where \( \dot{\alpha}_k \) and \( \dot{\alpha}_k^\dagger \) are the operators with the positive and the negative time-dependence, respectively. Then the equation (3) for \( \dot{\alpha}_k(t) \) includes under the integral only the terms \( \propto e^{-i\omega_k(t-t_1)} \dot{\alpha}_k \dot{\alpha}_k^\dagger \), where \( \dot{\alpha}_k^\dagger = -(v/\pi) \omega_k^{1/2} \sum_j j \omega_j^{-1/2} \left( 2i \dot{\alpha}_j^\dagger + \omega_j \dot{\alpha}_j^\dagger \right) / \left( j^2 - k^2 \right) \). Therefore
\[ \dot{\alpha}_k(t) = e^{-i\omega_k t} \left[ \dot{\alpha}_k^{(0)} + (i/2) \int_0^t dt_1 e^{i(\omega_k - \omega_j) t_1} \dot{\alpha}_j^\dagger(t_1) \right], \quad (4) \]
where \( \dot{\alpha}_k^{(0)} \) is the initial destruction operator, \( v = V/c \) is the dimensionless velocity. Here it has been taken into account that in the \( t \to \infty \) limit the main contribution to the integral in the equation (3) comes from the large \( t_1 \) when the operator \( \dot{\alpha}_k^\dagger(t_1) \) in \( \dot{\alpha}_k^\dagger(t_1) \) depends on \( t_1 \) as \( e^{i\omega_1 \delta} \). Besides, in this limit one can take \( 2i \dot{\alpha}_j^\dagger + \omega_j \dot{\alpha}_j^\dagger = (\omega_0 - 2\omega_j) \dot{\alpha}_j^\dagger \), and \( \dot{\alpha}_j = \omega_0 - \omega_k \). This means that in the \( t \to \infty \) limit one has \( \omega_0(2\omega_j - \omega_k) = \omega_j^2 - \omega_k^2 \propto j^2 - k^2 \). As a result, the factor \( j^2 - k^2 \) in the equation for \( \dot{\alpha}_k^\dagger \) cancels and one gets \( \dot{\alpha}_k^\dagger = (v/\pi) \omega_k^{1/2} \dot{Q}_k^\dagger \), where
\[ \dot{Q}_k^\dagger = \kappa_0^{-1} \sum_{j=1}^{\kappa_0} \omega_j^{1/2} \dot{\alpha}_j^\dagger, \quad (5) \]
\( \kappa_0 \) is the integer part of \( \omega_0 L_0/\pi c \). The factor \( e^{-i\omega_1 \delta} \) of the equation (4) means that in the large time limit \( \dot{\alpha}_k(t) \) is the destruction operator. Therefore the number of photons at large time equals \( N_k(t) = \langle 0 | \dot{a}_k^\dagger(t) \dot{a}_k(t) | 0 \rangle \). Inserting here equation (4) and taking into account that \( \dot{\alpha}_k^{(0)} | 0 \rangle = 0 \), we get
\[ N_k(t) = (\omega_k v^2 / 4\pi^2) \int_0^t dt_1 \int_0^t dt_2 e^{i(\omega_k - \omega_j)(t_1 - t_2)} D(t_1, t_2), \quad (6) \]
where \( D(t_1, t_2) = \langle 0 | \dot{Q}(t_2) \dot{Q}_k^\dagger(t_1) | 0 \rangle \). The rate of creation of photons is given by the derivative \( \dot{N}_k = dN_k(t)/dt \). Thus, to find the rate of emission by a medium with oscillating in time refractive index, one needs to know the correlation function \( D(t_1, t_2) \) for large \( t_1 \) and \( t_2 \). In this limit this function depends only on the time difference [12]: \( D(t_1, t_2) = D(t_1 - t_2) \). Inserting this relation to the equation (6) we get in the \( t \to \infty \) limit \( N_k(t) = t N_k^\prime \), where
\[ N_k^\prime = (\omega_k v^2 / 4\pi^2) D(\omega_k) \]
\( D(\omega_k) \) is the emission rate which in this limit does not depend on \( t \),
\[ D(\omega_k) = \int_{-\infty}^{\infty} d\tau e^{i(\omega_0 - \omega_k) \tau} D(\tau) \]
is the Fourier transform of the correlation function $D(\tau)$. The constant rate of the emission is expected: the emission in the $L,t \to \infty$ limit is stationary.

Thus, to find the rate of emission by a medium with oscillating in time refractive index, one needs to find the function $D(\omega)$. The equation for this function has been derived in [13, 14]. The result is as follows:

$$D(\omega) = \frac{D^{(0)}(\omega)}{|1 - v^2 \omega^2 G(\omega) G^*(\omega_0 - \omega)|^2}. \quad (7)$$

Here $D^{(0)}(\omega) = h \omega / 2 \pi^2 c^2 \omega_0^2$ is the Fourier transform of the correlation function $D(t)$ in the unperturbed case, $G(\omega)$ is the Fourier transform of the Green function

$$G(t) = (\Theta(t) / \pi \kappa_0^2) \sum_{k=1}^{\kappa_0} k \sin \omega_k t \quad (8)$$

describing the time evolution of the operator $\hat{Q}$ in the unperturbed case. In the $L_0 \to \infty$ limit

$$G(\omega) = \pi^{-1} \left[ 1 + (\omega/2) \ln \left( (1 - \omega)/(1 + \omega) \right) \right] + (i\omega/2) \Theta(- |\omega|) \quad (9)$$

(we take $\omega_0 = 1$ for the frequency unit). One sees that the effect of oscillations of the refractive index on the correlation function $D(\omega)$ is described in equation (7) by the resolvent, which gives a remarkable contribution only if the dimensionless velocity $v$ is not small.

2.3. Results

According to the equations (6) and (7) the rate of emission of photons per unit frequency equals:

$$\dot{N}(\omega) = \frac{v^2 \omega (1 - \omega)}{|1 - v^2 G^*(\omega) G(1 - \omega)|^2}; \quad 0 \leq \omega \leq 1. \quad (10)$$

This expression describes the emission under consideration for any value of $v$. If $v \ll 1$, then the intensity of the emission increases quadratically with $v$. However if $v \gg 1$ then the dependence on $v$ is the opposite: $\dot{N} \propto v^{-2}$. If $v$ approaches the value $v_\text{r} \approx 4\pi \sqrt{\pi^2 + (4 - \ln 3)^2} \approx 3$ then the resolvent at $\omega = 1/2$ diverges and the emission is resonantly enhanced (see figures 1 and 2).

**Figure 1.** The logarithm of the rate of the emission of photons $\dot{N}$ by a medium with periodically time-dependent refractive index $n(t) = n_0 + n' \cos(\omega_0 t)$ as a function of the frequency of emission $\omega$ and maximal velocity $v$ of the optical length (the units $\omega_0 = 1$ and $c = 1$ are used).
Figure 2. The rate of the emission of photons $\Rate = \int \Rate(\omega) d\omega$ by a medium with periodically time-dependent refractive index as a function of the maximal dimensionless velocity $v$ of the optical length (the same units as in the figure 1 are used).

3. Possibilities of experimental verification

To estimate the intensity of a laser beam which can give $v = n_0' \omega_0 I(0)/c$ of the order of 1 and which can cause the resonant enhancement of the two-photon emission, we take $n_0' - n(2) I$, where $I$ is the intensity of the laser light, $n(2) = 10^{-15}$ cm$^2$/W, a typical non-resonant value of $n(2)$ in crystals [1]. We also take $\omega_0 I(0)/c \sim 10^5$ and $n(2) = 10^{-15}$ cm$^2$/W. We get $I \sim 10^{10}$ W/cm$^2$. Such intensity of the laser light is experimentally achievable.

In our consideration we describe the emission in the direction $x$, which was chosen arbitrarily. This means that photons are emitted in all directions. The positive and the negative direction along the $x$ axis are not distinguished in our case. Therefore the pairs of photons with the wave vectors $q \pm G$ along the $x$ axis are equally generated by the medium (here $\omega = \omega_0/(|\omega - 1|)$).

Only a spontaneous emission was considered above. If $Nq$ photons with the wave-vector $q$ and the frequency $\omega = cq < \omega_0$ fall into a medium with $n$ oscillating in time, then the stimulated processes give a contribution to the emission. E.g. the photons with the wave vector $q$ and wave number $q = \omega_0/2c$ stimulate the emission of photons not only with the wave vectors $q$ but also with the wave vector $-q$. This is a well-known fact [15]: a medium with a refractive index oscillating in time performs the phase-conjugated reflection of photons with half a frequency. In the resonance condition it takes place with a high efficiency.

Finally, we note that the emission under consideration can be generated by any strong coherent excitation, which periodically modulates $n$. Presented here theory may be of interest to the physics of condensed matter, e.g. to the generation of phonon pairs in a semiconductor by a strong microwave or to a two-phonon decay of a strong coherent optical phonon wave with small wave-vector generated in CARS experiments. A similar emission of other bosons as well as the resonant enhancement of this emission is also possible in a periodically time-dependent medium, analogously to the above-described resonant enhancement of the two-photon emission. It appears that the rate of emission of particles - antiparticle pairs of the Klein-Gordon field is also described by the equation (9) if $G(\omega)$ is replaced by the difference $G(\omega) - G_i(\omega)$, where $G_i(\omega)$ can be obtained from $G(\omega)$ by replacing $\omega$ by $\omega - 2m + 1$ (in the $\omega = c = h = 1$ units); this emission exists only if the rest mass of the particle-antiparticle pair $2m$ does not exceed $\hbar \omega_0/2c^2$. The resonant enhancement of the particle-antiparticle emission may offer interest for the astrophysics as a possible mechanism of a powerful directed emission of particles.
4. Summary

A solution of the problem of the two-photon emission of a dielectric with periodically time-dependent refractive index has been given. It has been found that if the maximal velocity of the oscillations of the optical length approaches the critical value $V_\tau \approx 3c$, then a strong enhancement of the two-photon emission takes place. The proposed theory may be also applied for description of the two-phonon decay of strong coherent optical phonon waves with small wave-vector and to other two-quantum emissions in solids in the case of strong long-wave coherent excitation by light or by the microwaves.

Acknowledgement

The research was supported by the ETF Grant No 6534 and by the US NRC Twinning Program.

References

[1] Shen Y R 1984 The principles of nonlinear optics, University of California, Berkeley, John Wiley and Sons, Inc.
[2] Yablonovitch E 1989 Phys. Rev. Lett. 62 1742
[3] Hizhnyakov V 1992 Quantum Opt. 4 277
[4] Lobashov A A and Mostepanenko V V 1991 Teor. Mat. Fiz. 86 438; 1991 88 913 (1991 Theor. and Math. Phys. 86 303; 1991 88 340).
[5] Dodonov V V, Klimov A B and Nikonov D E 1993 Phys. Rev. A 47 4422
[6] Johnston H and Sarkar S 1995 Phys. Rev. A 51 4109
[7] Saito H and Hyuga H 1996 J. Phys. Soc. Japan, 65, 1139; 3513
[8] Bordag M (editor) 1999 The Casimir Effect 50 Years Later, Proc. of the 4th Workshop on Quantum Field Theory under the Influence of External Conditions (World Scientific)
[9] Unruh W G 1967 Phys. Rev. D 14 870
[10] Birrell N D and Davies P C W 1982 Quantum Fields in Curved Space, Cambridge University Press, Cambridge
[11] Solimeno S, Crosignani B and DiPorto P 1998 Guiding, Diffraction, and Confinement of Optical Radiation, Academic Press, Inc.
[12] Hizhnyakov V 1996 Phys.Rev. B 53 13981
[13] Hizhnyakov V 1999 Europhys. Lett. 45 508
[14] Hizhnyakov V, Kaasik H and Tehver I 2002 Eur. Phys. J. B 28 271
[15] Pepper D M 1982 Nonlinear optical phase conjugation Opt. Engineering 21 156