Kaon Meson Condensation and Δ resonance of Hyperonized Star with relativistic mean-field model

Fu Ma¹, Chen Wu² and Wenjun Guo¹

1. University of Shanghai for Science and Technology, Shanghai 200093, China
2. Shanghai Advanced Research Institute, Chinese Academy of Sciences, Shanghai 201210, China

We study the equation of state of dense baryon matter within the relativistic mean-field model, and we include Δ(1232) isobars into IUFSU model with hyperons and consider the possibility of kaon meson condensation. We find that it is necessary to consider the Δ resonance state inside the massive neutron star. The critical density of Kaon mesons and hyperons is shifted to a higher density region, in this respect an early appearance of Δ resonances is crucial to guarantee the stability of the branch of hyperonized star with the difference of the coupling parameter $x_{s,\Delta}$ constrained based on the QCD rules in nuclear matter. The Δ resonance produces a softer equation of state in the low density region, which makes the tidal deformability and radius consistent with the observation of GW170817. As the addition of new degrees of freedom will lead to a softening of the equation of state, the σ-cut scheme, which states the decrease of neutron star mass can be lowered if one assumes a limited decrease of the σ-meson strength at $\rho_B(\rho_B > \rho_0)$, finally we get a maximum mass neutron star with Δ resonance heavier than $2M_\odot$.

I. INTRODUCTION

Astronomical observations and gravitational wave data over the past decade have placed a series of constraints on a range of properties of neutron stars (mass, radius, deformabilities, e.g.). The massive neutron stars (NSs) observed, e.g., PSR J1614-2230 with $M = 1.908 \pm 0.016M_\odot$ [1–4], have established strong constraints on the equation of state (EOS) of nuclear matter. PSR J0348+0432 with $M = 2.01 \pm 0.04M_\odot$ [5], MSP J0740+6620, with $M = 2.08^{+0.07}_{-0.07}M_\odot$ [6, 7], and radius $12.39^{+1.30}_{-0.98}$ km obtained from NICER data [8]. The recent observation of gravitational waves from the binary neutron star merger event GW170817 suggests that the dimensionless combined tidal deformability $\Lambda$ is considered to be less than 720 at 90% confidence level based on low spin priors [9], while a lower limit with $\Lambda \geq 197$ is obtained based on electromagnetic observations of the transient counterpart AT2017gfo [10]. These astronomical observations constrain the tidal deformability of a 1.4 $M_\odot$ mass neutron star and thus strong interactions in dense nuclear matter. These upper limits indicate that the EOS of stellar material is softened at this (intermediate) density. One way to solve this problem is to introduce new degrees of freedom (hyperons [11, 12], Δ resonance [13–17], kaon meson condensation [18–22]), as the density of nucleons increases, the appearance of hyperons, Δ resonance, and kaon condensation inevitably softens the equation of state, resulting in a neutron star with a mass radius consistent with astronomical observations.

As the density of nucleons increases, hadron degrees of freedom inside the neutron star are excited into strangeness-bearing hyperons, they affect the stellar structure and evolution in various ways [23, 24]. Although the existence of hyperons inside neutron stars is inevitable, its appearance will significantly lead to a softening of the equation of state, resulting in a decrease in the maximum mass of the neutron star, which does not correspond to the observation of massive neutron stars ($2M_\odot$), which is known as hyperons puzzle [25–27]. In order to guarantee a stiffen EOS and massive neutron stars, density covariant functional theory has been chosen to study neutron stars containing hyperons [17, 28–33]. However, with the constraints on tidal deformation and radii imposed by astronomical observations, the application of this theoretical model is subject to some limitations [34, 35].

Although there are many speculations about the existence of hyperons inside neutron stars, there is little discussion about the Δ resonance. One reason is that early work suggested that the critical densities of Δ resonances in the RMF model with the same strength of the meson field as the nucleon case have exceeded the densities of the core of typical neutron stars [36, 37], which is considered out of the realm of astrophysics, and another reason is that the occurrence of Δ resonance leads to a softening of the equation of state, which has become a Δ puzzle [17] in some literature, the same as the hyperon puzzle. However, recent work has shown that considering the Δ resonance inside a neutron star reduces the radius with a standard mass of 1.4$M_\odot$ NS, and that the equation of state does not change significantly [33, 38, 39].

Another new degree of freedom for non-nucleons in dense stars includes various mesons (kaon, pion) condensates. Kaplan and Nelson have suggested that the ground state of hadronic matter might form a negatively charged Kaon Bose-Einstein condensation above a certain critical density [40, 41]. In the interior of a neutron star, as the density of neutrons increases, the electronic chemical potential will increase to keep the matter in β-equilibrium. When the electronic chemical potential exceeds the mass of muons, muons appear. And when the vacuum mass of the meson (pion, Kaon) is exceeded, as the density increases, negatively charged mesons begin to appear, which helps to maintain electrical neutrality. However, the $s$-wave $\pi N$ scattering potential repels the ground state mass of the $\pi$ meson and prevents the generation of
the π meson [36]. With the increase of density, the energy ω_K of a test Kaon in the pure normal phase can be computed as a function of the nucleon density. The Kaon energy will decrease while the chemical potential of Kaon increases with the density. When the condition ω_K = μK is achieved, the Kaon will occupy a small fraction of the total volume, then K− will be more advantageous than electrons as a neutralizer for positive charges, and this will open the possibility of the appearance of Kaon condensates.

Although many scholars have previously proposed various density covariant functional theories or realistic nuclear potential in order to obtain more massive neutron stars containing hyperons. However these theories are usually used to consider neutrons, protons and leptons (n, p, e, μ−) because of the hyperon puzzle, and considering hyperons in the RMF framework leads to a reduction of the maximum mass of neutron stars and thus does not satisfy astronomical observations. However, the σ-cut scheme [42], which point out that in the small scale range where the density ρ_B > ρ_0, a sharp decrease in the strength of the σ meson reduces the decrease in the effective mass of the nucleon, which eventually stiffens the EOS and still yields neutron stars of more than 2M⊙ after considering the hyperon degrees of freedom [43, 44].

In this article, we use the IUFSU model [45] to study NS matter including hyperons, Δ resonance and Kaon condensates with σ-cut scheme.

This paper is organized as follows. First, the theoretical framework is presented. Then we will study the effects of Kaon meson condensation and Δ resonance contains hyperons with the σ-cut scheme. Finally, some conclusions are provided.

II. THEORETICAL FRAMEWORK

In this section, we introduce the IUFSU model to study the properties of the Δ resonance and phase transition from hadronic to Kaon condensed matter. For the baryons matter we have considered nucleons (n and p), and hyperons (Λ, Σ, and Ξ), Δ resonance (Δ^+, Δ^0, Δ^-), kaon (K^-). The exchanged mesons include the isoscalar scalar meson (σ), the isoscalar vector meson (ω), the isovector vector meson (ρ), and strange vector meson (φ), the starting point of the extended IUFSU model is the Lagrangian density:

\[
\mathcal{L} = \sum_B \bar{\psi}_B [i \gamma^\mu \partial_\mu - m_B + g_\sigma B \sigma - g_\omega B \gamma^\mu \omega_\mu - g_\phi B \gamma^\mu \phi_\mu - g_\rho B \gamma^\mu \rho_\mu - \frac{g_\rho B}{2} \gamma^\mu \gamma^\nu \rho_\mu \rho_\nu] \psi_B + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma \\
- \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{\kappa}{3!} (g_\sigma N \sigma)^3 - \frac{\lambda}{4!} (g_\sigma N \sigma)^4 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \Phi_{\mu \nu} \Phi^{\mu \nu} + \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu \\
+ \frac{\xi}{4!} (g_\omega N \omega^\mu \omega^\mu)^2 + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^\mu - \frac{1}{4} G_{\mu \nu} G^{\mu \nu} + \Lambda_\mu (g_\rho N \bar{\rho}_\mu \cdot \rho^\nu)(g_\rho N \omega_\nu \omega^\nu) + \sum_i \bar{\psi}_i [i \gamma^\mu \partial_\mu - m_i] \psi_i,
\]

with the field tensors

\[
F_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \\
\Phi_{\mu \nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu \\
G_{\mu \nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu.
\]

The model contains following quantities, the baryon octet and two leptons(n, p, e, μ, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-), Δ resonances (Δ^+, Δ^0, Δ^-), isoscalar-scalar σ, isoscalar-vector ω, φ and isoscalar-vector ρ with the masses and coupling constants. Where Λ_μ is introduced to modify the density dependence of symmetry energy. The isoscalar meson self-interactions (via κ, λ and ξ terms) are necessary for the appropriate EOS of symmetric nuclear matter, respectively. In RMF models, the operators of meson fields are replaced by their expectation values by the mean field approximation. We take the Lagrangian of Kaon condensation as the same that is Ref. [46] and [47], which reads

\[
\mathcal{L}_K = D_\mu K^* D^\mu K - m_K^2 K^* K,
\]

where D_μ = ∂_μ + ig_ω K ω_μ + ig_φ K φ + i g_ρ K ρ_μ is the covariant derivative and the Kaon effective mass is defined as m_K^2 = m_K - g_σ K σ.

Finally, with the Euler-Lagrange equation, the equations of motion for baryons and mesons are obtained:
measured in IUFSU models are tabulated in Table. 

\[ g = 2 \text{ for baryons and } g = 4 \text{ for } \Delta \text{ resonance. where } \]

\[ E_{fB} = \sqrt{k_{fB}^2 + M^2}, \text{ the kaon density } \]

\[ \rho_K = 2(\omega_K + g_{\omega K} \sigma + g_{\phi K} \phi + \frac{g_{\rho K}}{2} \rho)K^*K, \]  

(6)

Now, we are in a position to discuss the coupling parameters between baryons (nucleons, hyperons and \( \Delta \)) or \( K^- \) and meson fields. The masses of nucleons and mesons, and the coupling constants between nucleon and mesons in IUFSU models are tabulated in Table.I.

For the meson-hyperon couplings, we take those in the SU(6) symmetry for the vector couplings constants:

\[ g_{\rho \Lambda} = 0, g_{\rho \Sigma} = 2g_{\rho \Xi} = 2g_{\rho N} \]

\[ g_{\omega \Lambda} = g_{\rho \Sigma} = 2g_{\omega \Xi} = \frac{2}{3}g_{\omega N} \]

\[ 2g_{\phi \Lambda} = 2g_{\phi \Sigma} = g_{\phi \Xi} = -\frac{2\sqrt{2}}{3}g_{\omega N} \]  

(7)

While the nucleons do not couple to the strange mesons, \( g_{\rho N} = 0 \), and mass of meson \( \phi \) takes as \( M_{\phi}=1020\text{MeV} \). The scalar couplings are usually fixed by fitting hyperon potentials with \( U^{(1)}_{\omega(N)} = g_{\omega N} \omega_0 - g_{\omega Y} \sigma_0 \), where \( \sigma_0 \) and \( \omega_0 \) are the values of the scalar and vector meson strengths at saturation density [48]. We choose the hyperon-nucleon potentials of \( \Lambda, \Sigma \) and \( \Xi \) as \( U^{\Lambda}_N = -30\text{MeV}, U^{\Sigma}_N = 30\text{MeV} \) and \( U^{\Xi}_N = -18\text{MeV} \) [49-51]. Table. II provides the numerical values of the meson hyperon couplings at nuclear saturation density, where \( x_{\omega Y} = g_{\omega Y}/g_{\omega N} \).

The coupling constants between the vector meson and the Kaon \( g_{\rho K}, g_{\rho \phi K} \) are determined by the meson SU(3) symmetry as \( g_{\rho K} = g_{\rho\Sigma}/3, g_{\rho \phi K} = g_{\rhoN} \) [22], and \( g_{\phi K} = 4.27 \) for the \( \phi \) meson [52]. The scalar coupling constant \( g_{\sigma K} \) is fixed to the optical potential of the \( K^- \) at saturated nuclear matter,

\[ U_K(\rho_0) = -g_{\sigma K} \sigma(\rho_0) - g_{\omega K} \omega(\rho_0), \]  

(8)

and in this paper, we carry out our calculations with a series of optical potentials ranging from -160 MeV to -120 MeV. The \( g_{\sigma K} \) can be related to the potential of Kaon at the saturated density through Eq.(8). \( g_{\sigma K} \) values corresponding to several values of \( U_K \) are listed in Table III.

Because experimental data on the \( \Delta \) resonance is scarce, the coupling parameters between the \( \Delta \) resonances and meson fields are uncertain, so we limit ourselves to considering only the couplings with \( \sigma \) meson fields, which are explored in the literature [53, 54]. We assume the scalar coupling ratio \( x_{\sigma \Delta} = g_{\sigma \Delta}/g_{\sigma N} > 1 \) with a value close to the mass ratio of the \( \Delta \) and the nucleon [55], and adopt three different choices (\( x_{\sigma \Delta} = 1.05, x_{\sigma \Delta} = 1.1 \) and \( x_{\sigma \Delta} = 1.15 \)) [56], for \( x_{\omega \Delta} \) and \( x_{\rho \Delta} \) we take as \( x_{\omega \Delta} = g_{\omega \Delta}/g_{\omega N} = 1.1 \) and \( x_{\rho \Delta} = g_{\rho \Delta}/g_{\rho N} = 1 \) [57]. Similar to the nucleons, \( \Delta \) resonances do not couple to meson \( \phi \), so \( g_{\phi \Delta} = 0 \).

By solving the Euler-Lagrangian equation of Kaon we obtain equation of motion: \[ [D_{\mu}D^\mu + m^2_{\phi}] K = 0. \]  

We can then derive the dispersion relation for the Bose-condensation of \( K^- \), which reads

\[ \omega_K = m_K - g_{\sigma K} \sigma - g_{\omega K} \omega - g_{\phi K} \phi - \frac{g_{\rho K}}{2} \rho, \]  

(9)

For the neutron matter with baryons and charged leptons, the \( \beta \)-equilibrium conditions are guaranteed with the following relations of chemical potentials for different particles:

\[ \mu_\rho = \mu_{\Sigma^+} = \mu_{\Delta^+} \]

\[ \mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Sigma^0} = \mu_{\Delta^0} = \mu_n \]

\[ \mu_{\Sigma^-} = \mu_{\Sigma^-} = \mu_{\Delta^-} = 2\mu_n - \mu_\rho \]

\[ \mu_{\Delta^{++}} = 2\mu_\rho - \mu_n \]

\[ \mu_\mu = \mu_\mu \]

and the charge neutrality condition is fulfilled by:

\[ \sum_B q_B \rho_B + \sum_D q_D \rho_D - \rho_K - \rho_e - \rho_\mu = 0. \]  

(11)

The chemical potential of baryons, \( \Delta \) and leptons read:

\[ \mu_i = \sqrt{k^2_F + m_i^2} + g_{\omega i} \omega + g_{\phi i} \phi + g_{\rho i} \tau_3 \rho, \quad i = B, D. \]  

(12)

\[ \mu_i = \sqrt{k^2_F + m_i^2}, \]  

(13)

where \( k^2_F \) is the Fermi momentum and the \( m_i^* \) is the effective mass of baryon and \( \Delta \) resonances, which can be
\[ \frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r) dr \]  

(18)

The tidal deformability of a neutron star is reduced as a dimensionless form \cite{59, 60}.

\[ \Lambda = \frac{2}{3}k_2C^{-5} \]  

(19)

where \( C = GM/R \), the second love number \( k_2 \) can be fixed simultaneously with the structures of compact stars \cite{61}.

The \( \sigma \)-cut scheme \cite{42}, which is able to stiffen the EOS above saturation density, adds in the original Lagrangian density, the function \cite{42, 62, 63}

\[ \Delta U(\sigma) = \alpha \ln(1 + e^{\beta(f - f_{s,\text{core}})}) \]  

(20)

where \( f = g_\sigma \Sigma/M \) and \( f_{s,\text{core}} = f_0 + c_\sigma(1 - f_0) \). \( M \) is the nucleon mass. \( f_0 \) is the value of \( f \) at saturation density, equal to 0.31 for the IUFSU model. \( c_\sigma \) is a positive parameter that we can adjust. The smaller \( c_\sigma \) is, the stronger the effect of the \( \sigma \)-cut scheme becomes. However, we must be careful that this scheme would not affect the saturation properties of nuclear matter, in our previous work we discussed in detail the choice of parameter \( c_\sigma \) \cite{43}. In this paper, we take \( c_\sigma = 0.15 \) to satisfy the maximum mass constraint. \( \alpha \) and \( \beta \) are constants, taken to be \( 4.822 \times 10^{-4} M_5^{1/3} \) and 120 as in Ref.\cite{42}. This scheme stiffens the EOS by quenching the decreasing of the effective mass of the nucleon \( M_\Lambda = M_N(1 - f) \) at high density.

### III. RESULTS

First, we studied the effect of the \( \sigma \)-cut scheme on IUFSU model. In Fig.1, we plot the ratio of the effective mass of nucleons to the rest mass and the \( \sigma \) meson strength as a function of the baryon density, where \( \rho_0 \) is the saturation density, and we choose \( x_{\sigma,\Delta} = 1.05 \) and \( U_K = -160 \text{ MeV} \) to consider \( \Delta \) resonance and Kaon condensation. From the left panel, we can see that when \( \rho \leq \rho_0 \), the effect mass is almost same as nucleons-only matter and unchanged by the \( \sigma \)-cut scheme, when \( \rho > \rho_0 \), the effect mass dropped to around \( 0.55M_N \). And it is obviously observed that under the \( \sigma \)-cut scheme considering \( \Delta \) and \( K^- \) in the EOS or not has very tiny effect on the effective mass of nucleons. From right panel, when \( \rho > \rho_0 \), the \( \sigma \) meson field strength is quenched at high baryon density, this is what we want by using the \( \sigma \)-cut scheme.

In Fig.2, we plot the chemical potential of \( K^- \) and \( e^- \) as a function of baryon density. With the increase of

\begin{table}

| Model   | \( g_\sigma \) | \( g_\omega \) | \( g_\rho \) | \( \kappa \) | \( \lambda \) | \( \xi \) | \( \Lambda \) |
|---------|-------------|-------------|-------------|-----------|-----------|--------|--------|
| IUFSU   | 9.9713      | 13.0321     | 13.5899     | 3.37685   | 0.000268  | 0.03   | 0.046  |

\end{table}

\begin{table}

| \( U_K \) (MeV) | \( g_{\sigma K} \)        | \( \Lambda \) | \( \Sigma \) | \( \Xi \) |
|-----------------|------------------------|--------------|------------|--------|
| -120            | 0.600417               | 0.615796     | 0.45219    | 0.305171 |
| -140            | 1.144204               |              |            |         |
| -160            | 1.68799                |              |            |         |

\end{table}

related to the scalar meson field as \( m_1^2 = m_i - g_{\sigma i} \sigma \), and \( k_F^i \) is the Fermi momentum of the lepton \( l(\mu, e) \).

The total energy density of the system with Kaon condensation reads then \( \varepsilon = \varepsilon_{B,D} + \varepsilon_K \), where \( \varepsilon_{B,D} \) is the energy density of baryons and \( \Delta \) resonances, can be given as

\[ \varepsilon_{B,D} = \sum_{l=B,D} \gamma_l (2\pi)^3 \int_0^{k_F} \sqrt{m_i^2 + k^2} dk + \frac{1}{2} m_\omega^2 \omega^2 
+ \xi \sigma_0^A + \frac{4}{3} m_\sigma^2 \sigma^2 + \frac{5}{6} g_{\sigma N}^4 \sigma^3 + \frac{\lambda}{24} g_{\sigma N}^4 \sigma^4 
+ \frac{1}{2} m_\rho^2 \rho^2 + 3 \Lambda \sigma_0^A g_{\rho N}^2 g_{\omega N}^2 \omega^2 \rho^2 + \frac{1}{2} m_\tau^2 \tau^2 
+ \frac{1}{\pi^2} \int_0^{k_F} \sqrt{k^2 + m_i^2} dk, \]  

(14)

And the energy contributed by the Kaon condensation \( \varepsilon_K \) is

\[ \varepsilon_K = 2m_K^2 K* K = m_K^2 \rho_K, \]  

(15)

The Kaon does not contribute directly to the pressure as it is a (s-wave) Bose condensate so that the expression of pressure reads

\[ P = \sum_{l=B,D} \mu_l \rho_l + \sum_{l=\mu, e} \mu_l \rho_l - \varepsilon, \]  

(16)

With the obtained \( \varepsilon \) and \( P \), the mass-radius relation and other relevant quantities of neutron star can be obtained by solving the Oppenheimer and Volkoff equation \cite{58}.
density, the energy $\omega_K$ of a test Kaon in the pure normal phase can be computed as a function of the nucleon density. The Kaon energy($\omega_K$) will decrease while the electron chemical potential ($\mu_e$) increases with the density. When the condition $\omega_K = \mu_e$ is achieved, the Kaon will occupy a small fraction of the total volume. We can see that both $x_{\sigma \Delta}$ and $U_K$ affect the Kaon meson condensation. For $x_{\sigma \Delta} = 1.05$ and $1.1$, there is no intersection between $\omega_K$ and $\mu_e$ when $U_k = -120$ MeV and -140 MeV. The intersection of $\omega_K$ and $\mu_e$ is only possible when $U_K=-160$MeV, which means that the smaller the optical potential of the $K^-$ at saturated nuclear matter is, the greater the possibility of the Koan condensation is. When we choose $\sigma$-cut scheme(Fig.3), there is no intersection between $\omega_K$ and $\mu_e$. The decrease of $\sigma$ meson field strength slows down the decline of $\omega_K$, and makes the appearance of $K^-$ difficult. We list the threshold densities $n_{cr}$ for Kaon condensation for different values of $K^-$ optical potential depths $U_K$ in Table IV.

Fig.4 shows the relative population of particles versus baryon density with $x_{\sigma \Delta}=1.05$, $x_{\sigma \Delta}=1.1$ and $x_{\sigma \Delta}=1.15$, $U_K=-160$MeV. We find that the presence of $\Delta$ resonances shift threshold density of $\Lambda^0$ to higher densities when $x_{\sigma \Delta}=1.05$ and $x_{\sigma \Delta}=1.05$, we conclude that the $\Delta$ resonance squeezes the critical density of hyperons to higher densities, this is well illustrated by the case $\Lambda^0$ in Fig.4, especially, as increase of $x_{\sigma \Delta}$, it also delays the appearance of $K^-$. When $x_{\sigma \Delta}=1.05$, the mixed phase $K^-$ occurs at $\sim 5.24\rho_0$, for $x_{\sigma \Delta}=1.1$, $K^-$ occurs at $\sim 6.79\rho_0$. But when $x_{\sigma \Delta}=1.15$, all the $\Delta$ resonance disappear and $\Lambda^0$, $K^-$ move to lower densities.

Next we examine the effect of the $\sigma$-cut scheme on the particle population. This is plotted in the Fig.5. From Fig.3, we determined that no $K^-$ is generated when using $\sigma$-cut scheme, as there is no intersection between $\omega_K$ and $\mu_e$, but the $\Delta$ resonance has some interesting variations. With the disappearance of $K^-$, the $\Delta^{++}$ appears, which leads the percentage of leptons to drop rapidly due to the charge neutrality condition, especially $\mu^-$, the presence of $\Xi^-$ slows down the descent of electrons. As $x_{\sigma \Delta}$ increases from 1.05 to 1.1, the critical value of $\Delta$ resonance

---

**TABLE IV.** Threshold densities $n_{cr}$ (in units of $\rho/\rho_0$) for Kaon condensation in dense nuclear matter for different values of $K^-$ optical potential depths $U_K$ (in units of MeV) without $\sigma$-cut scheme.

| $U_K$ (MeV) | $n_{cr}(K^-)$ |
|-------------|----------------|
| -120        | none           |
| -140        | none           |
| -160        | 5.24           |

---

**FIG. 1.** Effective mass and $\sigma$ meson strength of nucleons versus baryon density in NS matter using and not using $\sigma$-cut scheme.

**FIG. 2.** Kaon meson energy($\omega_K$) and electronic chemical potential($\mu_e$) as a function of $\rho_B$ with different $x_{\sigma \Delta}$ and $U_K$, and without $\sigma$-cut scheme.

**FIG. 3.** Kaon energy($\omega_K$) and electron chemical potential($\mu_e$) as a function of baryon density with different $x_{\sigma \Delta}$ and $U_K$, $c_\sigma=0.15$.
FIG. 4. Relative population of particles versus baryon density without $\sigma$-cut scheme with $x_{\sigma} \Delta = 1.05, x_{\sigma} \Delta = 1.1, x_{\sigma} \Delta = 1.15$ and $K^-$ potential depth of $U_K = -160$ MeV, dashed lines denote $K^-$ and $\Delta$ resonance.

FIG. 5. Relative population of particles versus baryon density with $\sigma$-cut scheme ($c_{\sigma} = 0.15$), $x_{\sigma} \Delta = 1.05, x_{\sigma} \Delta = 1.1, x_{\sigma} \Delta = 1.15$ and $K^-$ potential depth of $U_K = -160$ MeV, dashed lines denote $K^-$ and $\Delta$ resonance.

FIG. 6. Pressure versus energy density without the $\sigma$-cut scheme. The solid line is for n, p, leptons and hyperons whereas others are with additional $\Delta$ resonance, dashed lines contain $K^-$ mesons and exhibit $U_K = -160$ MeV.

shifts to lower density. From these figures, it can be concluded that the appearance of Kaon meson condensation is more likely to suppress the hyperon production than the $\Delta$ resonance. When $x_{\sigma} \Delta = 1.15$, which is analogous to Fig. 4, $\Delta$ resonance disappears and the critical density of hyperons returns from the high-density region to the normal density range. $\Lambda^0$ at $\sim 2.26 \rho_0$, $\Sigma^-$ at $\sim 6.14 \rho_0$ and $\Xi^-$ at $\sim 2.52 \rho_0$.

Then we can discuss some properties of the neutron star. Fig. 6 shows pressure as a function of energy density in NS matter containing $\Delta$ resonance and $K^-$ without $\sigma$-cut scheme. We can see from the enlarged area in the figure that the addition of $K^-$ softens the equation of state to some extent, although this is not particularly significant with the onset of the $\Delta$ resonance. And $\Delta$ resonance softens the EOS when the energy density is between 300 MeV/fm$^3$ and 600 MeV/fm$^3$, and intensifies with increasing $x_{\sigma} \Delta (1.05 \to 1.1)$, which suggests the existence of a softer EOS in the low-density region, and the recent constraints on tidal deformation and radius point to this. As the energy density increases, the EOS of NS matter that contains Kaon mesons and $\Delta$ resonance will...
we determined that there is no $K^-$ when using $\sigma$-cut scheme, so the composition contains only hyperons and $\Delta$, we can see that $\sigma$-cut scheme significantly stiffens the EOS, and it is the truncated intensity of $\sigma$ meson field strength in Fig. 1 that leads to this result, and still retains the EOS softening feature in the low density region. The EOS obtained by this way can generate heavier $2M_\odot$ NS by solving the TOV equation, in order to eliminate “hyperon puzzle”.

The results of mass-radius relation for NS discussed here and shown in Fig. 8. The constraints from the observables of massive neutron stars, PSR J1614-2230 [1–4] and PSR J0348+0432 [5] are also shown as the shaded bands. The Neutron star Interior Composition Explorer (NICER) collaboration reported an accurate measurement of mass and radius of PSR J0030+0451 [64] in 2019, and MSP J0740 + 6620 in 2021 [6]. For the solid lines without $\sigma$-cut, different coupling parameters $x_{\sigma\Delta}$ have a significant effect on the maximum mass and radius of NS, it shows that the $\Delta$ resonance increases the maximum mass of NS and decreases the radius. As the increase of $x_{\sigma\Delta}(1.05 \rightarrow 1.1)$, the maximum mass decreases, but is still greater than in the case of pure hyperons. The dashed lines denote $c_\sigma=0.15$, this scheme can significantly increase the maximum mass of the neutron star and make it heavier than $2M_\odot$, also accords with the constraints from gravitational wave and NICER(2021). Note that there is no appearance of $K^-$ when $c_\sigma=0.15$ from Fig. 3. We list the simultaneous measurement of radius for MSP J0740 + 6620 and PSR J0030 - 0451 by the NICER data and maximum mass of the neutron star for various values of $x_{\sigma\Delta}$ in Table V.

The tidal deformability $\Lambda$, as a function of neutron stars is shown in Fig. 9. From the gravitation wave of BNS merger in GW170817, it was extracted as $\Lambda_{1.4} = 190^{+390}_{-120}$ at $1.4M_\odot$ [65]. From the figure we can see that the $\sigma$-cut scheme with the stiffer EOS has the larger $\Lambda_{1.4}$ and heavier masses, whose $\Lambda_{1.4}$ are out the constraint of GW170817, while the softer EOS satisfies the constraints of GW170817 and has smaller radii without $\sigma$-cut scheme. With the strong constraint on the compositions of compact stars by the observational tidal deformability, the $\Delta$ resonance inside neutron stars cannot be ignored,
TABLE V. The maximum mass (in unit of solar mass $M_{\odot}$) and radius (km) in NS matter including hyperons, $\Delta$ resonance and $K^-$ using and not using $\sigma$-cut scheme with potential $U_K=-160\text{MeV}$.

| $(n,p,H)$ | $x_{\sigma\Delta}=1.05(n,p,H,D,(K^-))$ | $x_{\sigma\Delta}=1.1(n,p,H,D,(K^-))$ | $x_{\sigma\Delta}=1.15(n,p,H,D,(K^-))$ | MSP J0740+6620 [6] | PSR J0030-0451 [8] |
|-----------|---------------------------------|---------------------------------|---------------------------------|----------------|----------------|
| M         | R                               | M                              | R                               | M              | R              |
| without $\sigma$-cut |                                 |                                 |                                 | 1.51           | 11.47          |
| $\sigma_\Delta=0.15$ |                                 |                                 |                                 | 2.2            | 13.65          |
| 1.58      | 10.37                           | 2.12                           | 12.93                           | $2.08 \pm 0.07$ | $12.39_{-0.98}^{+1.3}$ |
| 1.54      | 10.27                           | 2.09                           | 12.84                           | $1.34_{-0.16}^{+0.15}$ | $12.71_{-1.19}^{+1.14}$ |
| 1.67      | 10.59                           | 2.27                           | 13.39                           |                |                |

and the tidal deformability of the neutron star at $2.0M_{\odot}$, which is expected to be measured in the future gravitational wave events from the binary neutron-star merger.

IV. SUMMARY

In this paper, we have discussed the $\Delta$ resonance and Kaon meson condensation inside the neutron star under the IUFSU model, due to the recent rapid results of astronomical observations on the radii and tidal deformations of compact stars. However, the maximum masses of neutron stars generated by the softer EOS (hyperon puzzle) cannot approach $2.0M_{\odot}$, which did not satisfy the constraints from the massive neutron stars observed, so we take $\sigma$-cut scheme and got the maximum mass heavier than $2M_{\odot}$.

We find that the Kaon condensation cannot appear in the hyperons and $\Delta$ resonance with our parameter $U_K=-120\text{MeV}$ and $-140\text{MeV}$, it occurs only at $U_K=-160\text{MeV}$, and the $\Delta$ resonance also shifts the Kaon meson toward the high-density region. In NS matter containing hyperons and $\Delta$ resonances, the effect of Kaon meson on EOS is very insignificant.

On the other hand, we investigated the effect of $x_{\sigma\Delta}$ on the $\Delta$ resonance, for the $\Delta$ coupling constants, we take $x_{\sigma\Delta}=1.05,1.1$ and 1.15. We find that the inclusion of $\Delta$ resonance shifts the critical density of hyperons towards the high density region, and the value of $x_{\sigma\Delta}$ has great influence on the relative population of particles as a function of the baryon density. And the $\Delta$ resonance suggests the existence of a softer EOS in the low-density region, while the softer EOS satisfies the constraints of GW170817 and has smaller radii. This shows that it is necessary to consider $\Delta$ resonance inside NSs.

[1] Paul Demorest, Tim Pennucci, Scott Ransom, Mallory Roberts, and Jason Hessels. Shapiro Delay Measurement of A Two Solar Mass Neutron Star. Nature, 467:1081–1083, 2010.
[2] Zaven Arzoumanian et al. The NANOGrav 11-year Data Set: High-precision timing of 45 Millisecond Pulses. Astrophys. J. Suppl., 235(2):37, 2018.
[3] Emmanuel Fonseca et al. The NANOGrav Nine-year Data Set: Mass and Geometric Measurements of Binary Millisecond Pulses. Astrophys. J., 832(2):167, 2016.
[4] Feryal Ozel, Dimitrios Psaltis, Scott Ransom, Paul Demorest, and Mark Alford. The Massive Pulsar PSR J1614-2230: Linking Quantum Chromodynamics, Gamma-ray Bursts, and Gravitational Wave Astronomy. Astrophys. J. Lett., 724:L199–L202, 2010.
[5] John Antoniadis, Paulo CC Freire, Norbert Wex, Thomas M Tauris, Ryan S Lynch, Marten H Van Kerckwijk, Michael Kramer, Cees Bassa, Vik S Dhillon, Thomas Driebe, et al. A massive pulsar in a compact relativistic binary. Science, 340(6131):1233232, 2013.
[6] Emmanuel Fonseca, H Thankful Cromartie, Timothy T Pennucci, Paul S Raim, A Yu Kirichenko, Scott M Ransom, Paul B Demorest, Ingrid H Stairs, Zaven Arzoumanian, Lucas Guillemot, et al. Refined mass and geometric measurements of the high-mass psr j0740+ 6620. The Astrophysical Journal Letters, 915(1):L12, 2021.
[7] H. T. Cromartie et al. Relativistic Shapiro delay measurements of an extremely massive millisecond pulsar. Nature Astron., 4(1):72–76, 2019.
[8] Thomas E Riley, Anna L Watts, Paul S Ray, Slavko Bogdanov, Sebastien Guillot, Sharon M Morsink, Anna V Bilous, Zaven Arzoumanian, Devarshi Choudhury, Julia S Deneva, et al. A nicer view of the massive pulsar psr j0740+ 6620 informed by radio timing and xmm-newton spectroscopy. The Astrophysical Journal Letters, 918(2):L27, 2021.
[9] B. P. Abbott et al. Properties of the binary neutron star merger GW170817. Phys. Rev. X, 9(1):011001, 2019.
[10] Michael W. Coughlin et al. Constraints on the neutron star equation of state from AT2017gfo using radiative transfer simulations. Mon. Not. Roy. Astron. Soc., 480(3):3871–3878, 2018.
[11] Jurgen Schaffner and Igor N. Mishustin. Hyperon rich matter in neutron stars. Phys. Rev. C, 53:1416–1429, 1996.
[12] Chen Wu and Zhongzhou Ren. Strange hadronic stars in relativistic mean-field theory with the FSUGold parameter set. Phys. Rev. C, 83:025805, 2011.
[13] Ting-Ting Sun, Shi-Sheng Zhang, Qiu-Lan Zhang, and Cheng-Jun Xia. Strangeness and $\Delta$ resonance in compact stars with relativistic-mean-field models. Phys. Rev. D, 99(2):023004, 2019.
[14] K. A. Maslov, E. E. Kolomeitsev, and D. N. Voskresensky. $\Delta$ resonances and charged $\rho$ mesons in neutron stars. J. Phys. Conf. Ser., 932(1):012040, 2017.
[15] E. E. Kolomeitsev, K. A. Maslov, and D. N. Voskresensky. Strangeness and $\Delta$ resonance in compact stars with relativistic-mean-field models. Phys. Rev. D, 99(2):023004, 2019.
sky. Delta isobars in relativistic mean-field models with σ -scaled hadron masses and couplings. *Nucl. Phys. A*, 961:106–141, 2017.

[16] Armen Sedrakian and Arus Harutyunyan. Delta-resonances and hyperons in proto-neutron stars and merger remnants. *Eur. Phys. J. A*, 58(7):137, 2022.

[17] Alessandro Drago, Andrea Lavagno, Giuseppe Pagliara, and Daniele Pigato. Early appearance of Δ isobars in neutron stars. *Phys. Rev. C*, 90(6):065809, 2014.

[18] Vivek Baruah Thapa and Monika Sinha. Dense matter equation of state of a massive neutron star with antikaon condensation. *Phys. Rev. D*, 102(12):123007, 2020.

[19] Toshiki Maruyama, Toshitaka Tatsumi, Dmitri N. Voskresensky, Tomonori Tanigawa, Tomoki Endo, and Satoshi Chiba. Finite size effects on kaonic pasta structures. *Phys. Rev. C*, 73:035802, 2006.

[20] G. E. Brown, Chang-Hwan Lee, Hong-Jo Park, and Manque Rho. Study of strangeness condensation by expanding about the fixed point of the Harada-Yamawaki vector manifestation. *Phys. Rev. Lett.*, 96:062303, 2006.

[21] Guo-yun Shao and Yu-xin Liu. Influence of the isovector-scalar channel interaction on neutron star matter with hyperons and antikaon condensation. *Phys. Rev. C*, 82:055801, 2010.

[22] Prasanta Char and Sarmistha Banik. Massive Neutron Stars with Antikaon Condensates in a Density Dependent Hadron Field Theory. *Phys. Rev. C*, 90(1):015801, 2014.

[23] G. F. Burgio, H. J. Schulze, I. Vidana, and J. B. Wei. Neutron stars and the nuclear equation of state. *Prog. Part. Nucl. Phys.*, 120:103879, 2021.

[24] Domenico Logoteta. Hyperons in Neutron Stars. *Universe*, 7(11):408, 2021.

[25] GF Burgio, H-J Schulze, A Li, et al. Hyperon stars at finite temperature in the brueckner theory. *Physical Review C*, 83(2):025804, 2011.

[26] Diego Lamardoni, Alessandro Lovato, Stefano Gandolfi, and Francesco Pederiva. Hyperon Puzzle: Hints from Quantum Monte Carlo Calculations. *Phys. Rev. Lett.*, 114(9):092301, 2015.

[27] Ignazio Bombaci. The Hyperon Puzzle in Neutron Stars. *JPS Conf. Proc.*, 17:101002, 2017.

[28] Luca Bonanno and Armen Sedrakian. Composition and stability of hybrid stars with hyperons and quark color-superconductivity. *Astronomy & Astrophysics*, 539:A16, 2012.

[29] Giuseppe Colucci and Armen Sedrakian. Equation of state of hypernuclear matter: impact of hyperon–scalar–color meson couplings. *Phys. Rev. C*, 87:055806, 2013.

[30] M. Fortin, C. Providencia, A. R. Raduta, F. Guinlinelli, J. L Zdunik, P. Haensel, and M. Bejger. Neutron star radii and crusts: uncertainties and unified equations of state. *Phys. Rev. C*, 94(3):035804, 2016.

[31] Yanjun Chen, Hua Guo, and Yuxin Liu. Neutrino scattering rates in neutron star matter with Delta isobars. *Phys. Rev. C*, 75:035806, 2007.

[32] Jia Jie Li, Armen Sedrakian, and Mark Alford. Relativistic hybrid stars with sequential first-order phase transitions and heavy-baryon envelopes. *Phys. Rev. D*, 101(6):063022, 2020.

[33] Jia Jie Li and Armen Sedrakian. Implications from gw170817 for δ-isobar admixed hypernuclear compact stars. *The Astrophysical Journal Letters*, 874(2):L22, 2019.

[34] Zhen-Yu Zhu, En-Ping Zhou, and Ang Li. Neutron Star Equation of State from the Quark Level in Light of GW170817. *Astrophys. J.*, 862(2):98, 2018.

[35] Tuhin Malik, N. Alam, M. Fortin, C. Providencia, B. K. Agrawal, T. K. Jha, Bharat Kumar, and S. K. Patra. GW170817: constraining the nuclear matter equation of state from the neutron star tidal deformability. *Phys. Rev. C*, 98(3):035804, 2018.

[36] N. K. Glendenning. Neutron Stars Are Giant Hypernuclei? *Astrophys. J.*, 293:470–493, 1985.

[37] N. K. Glendenning and S. A. Moszkowksi. Reconciliation of neutron star masses and binding of the lambda in hypernuclei. *Phys. Rev. Lett.*, 67:2414–2417, 1991.

[38] Jia Jie Li, Armen Sedrakian, and Fridolin Weber. Competition between delta isobars and hyperons and properties of compact stars. *Phys. Lett. B*, 783:234–240, 2018.

[39] Patricia Ribes, Angels Ramos, Laura Tolos, Claudia Gonzalez-Boquera, and Mario Centelles. Interplay between Δ Particles and Hyperons in Neutron Stars. *Astrophys. J.*, 883:168, 2019.

[40] D. B. Kaplan and A. E. Nelson. Strange Goings on in Dense Nucleonic Matter. *Phys. Lett. B*, 175:57–63, 1986.

[41] Ann E. Nelson and David B. Kaplan. Strange Condensate Realignment in Relativistic Heavy Ion Collisions. *Phys. Lett. B*, 192:193, 1987.

[42] K. A. Maslov, E. E. Kolomeitsev, and D. N. Voskresensky. Making a soft relativistic mean-field equation of state stiffer at high density. *Phys. Rev. C*, 92(5):052801, 2015.

[43] Fu Ma, Wenjun Guo, and Chen Wu. Kaon meson condensate in neutron star matter including hyperons. *Phys. Rev. C*, 105(1):015807, 2022.

[44] Fei Wu and Chen Wu. Hyperonized neutron stars within the framework of the σ-cut scheme. *Eur. Phys. J. A*, 56(1):20, 2020.

[45] F. J. Fattoyev, C. J. Horowitz, J. Piekarewicz, and G. Shen. Relativistic effective interaction for nuclei, giant resonances, and neutron stars. *Phys. Rev. C*, 82:055803, 2010.

[46] Norman K. Glendenning and Jurgen Schaffner-Bielich. First order kaon condensate. *Phys. Rev. C*, 60:025803, 1999.

[47] Norman K. Glendenning and Jurgen Schaffner-Bielich. Kaon condensation and dynamical nucleons in neutron stars. *Phys. Rev. Lett.*, 81:4564–4567, 1998.

[48] Jurgen Schaffner, Carl B. Dover, Avraham Gal, Carsten Greiner, D. John Millener, and Horst Stoecker. Multiply charged Ξ hypernuclei. *Phys. Rev. Lett.*, 78:234–240, 1997.

[49] Jinniu Hu, Ying Zhang, and Hong Shen. The Ξ hypernuclear matter within a relativistic mean field framework. *Eur. Phys. J. A*, 539:A16, 2019.

[50] Jinniu Hu, Ying Zhang, and Hong Shen. The Ξ hypernuclear matter within a relativistic mean field framework. *Eur. Phys. J. A*, 539:A16, 2019.
matter. *Phys. Rev. C*, 51:2260–2263, 1995.

[55] D. S. Kosov, C. Fuchs, B. V. Martemyanov, and A. Faessler. Constraints on the coupling constants of Sigma and Omega mesons to Delta isobars. *Phys. Lett. B*, 421:37–40, 1998.

[56] Alessandro Drago, Andrea Lavagno, and Giuseppe Pagliara. Can very compact and very massive neutron stars both exist? *Phys. Rev. D*, 89(4):043014, 2014.

[57] Vivek Baruah Thapa, Monika Sinha, Jia Jie Li, and Armen Sedrakian. Massive $\Delta$-resonance admixed hypernuclear stars with antikaon condensations. *Phys. Rev. D*, 103(6):063004, 2021.

[58] M. Baldo, I. Bombaci, and G. F. Burgio. Microscopic nuclear equation of state with three-body forces and neutron star structure. *Astron. Astrophys.*, 328:274–282, 1997.

[59] Tanja Hinderer. Tidal love numbers of neutron stars. *The Astrophysical Journal*, 677(2):1216, 2008.

[60] Sergey Postnikov, Madappa Prakash, and James M. Lattimer. Tidal Love Numbers of Neutron and Self-Bound Quark Stars. *Phys. Rev. D*, 82:024016, 2010.

[61] Tanja Hinderer, Benjamin D. Lackey, Ryan N. Lang, and Jocelyn S. Read. Tidal deformability of neutron stars with realistic equations of state and their gravitational wave signatures in binary inspiral. *Phys. Rev. D*, 81:123016, 2010.

[62] Young-Ho Song, Rimantas Lazauskas, and Vladimir Gudkov. Time Reversal Invariance Violating and Parity Conserving effects in Neutron Deuteron Scattering. *Phys. Rev. C*, 84:025501, 2011. [Erratum: Phys.Rev.C 93, 049901 (2016)].

[63] E. E. Kolomeitsev, K. A. Maslov, and D. N. Voskresensky. Hyperon puzzle and the RMF model with scaled hadron masses and coupling constants. *J. Phys. Conf. Ser.*, 668(1):012064, 2016.

[64] Thomas E Riley, Anna L Watts, Slavko Bogdanov, Paul S Ray, Renee M Ludlam, Sebastien Guillot, Zaven Arzoumanian, Charles L Baker, Anna V Bilous, Deepto Chakrabarty, et al. A nicer view of psr j0030+ 0451: Millisecound pulsar parameter estimation. *The Astrophysical Journal Letters*, 887(1):L21, 2019.

[65] B. P. Abbott et al. GW170817: Measurements of neutron star radii and equation of state. *Phys. Rev. Lett.*, 121(16):161101, 2018.