Improvement of the vessel traffic control system for accident-free electronic navigation in the port area

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Abstract. The increase in the intensity of navigation leads to unsafe navigation, which necessitates the improvement of existing measures to ensure safe navigation using specific mathematical models and methods. The configuration of the mathematical model of the traffic flow of ships obtained in this study is realizable on modern computer technology and can be applied by embedding advanced ship traffic control systems, which is an object of the infrastructure of a modern seaport.

1. Introduction
In the water area of the port's operational zone, there is an active environment, unsafe navigation due to the extensive crowding of ships at the entrance and exit, and coastal navigation on the approaches to the terminals. There is a multidimensional vector of many configurations of safe control systems for the movement of ships in the water area of ports. As a variation, the mathematical problem of describing and evaluating the traffic flow in the transport network of the dynamic system of vessel movement on the course along the ship's route is investigated.

2. Problem statement
The mathematical model assumes control actions on the ship in TP and TS, as a multidimensional vector in the sections of the transport network [1]. Signal plans are changed as a result of parametric analysis of traffic flows, as averaging over a given time $T_i$ and obtaining an estimate in the form:

$$X_i(T_y) = \frac{1}{n} \sum_{i=1}^{n} X_i \cdot n^{-1}$$

where: $n$ – the number of measurements during $T_i$; $X_i$ – multidimensional vector, as a result of changes in the transport stream parameters, in the given sections of the transport network.

After obtaining the estimate $\hat{X}_i$, we will select the nearest multidimensional control vector $\hat{X}_k(i)$, from a given set $(i = 1, 2, ..., M)$, at the minimum of the vector $\hat{R}_{min}$:

$$\hat{X}_i(T_y) R < \hat{X}_k(i) = \hat{R}_{min}$$

(2)
We consider the presence of a transition interval $T_{\text{conv.}}$, with the effect of coordinated control tending to zero [2]. Therefore, a change from a coordination plan $Y_i$ to a plan $Y_j$ is accepted on condition:

$$F_{\alpha j} (Y_j)T_{\alpha j} - F_{\alpha i} (Y_i)(T_{\alpha i} - T_{\text{conv.}})$$

where: $F_{\alpha j} (Y_j)$ – efficiency of management, per unit of time, with the current plan; $F_{\alpha i} (Y_i)$ – the effectiveness of the new plan.

The duration $T_{\text{conv.}}$ in operating systems is two to three control cycles.

Functional estimates $F_{\alpha j} (Y_j)$ and $F_{\alpha i} (Y_i)$ are obtained by modeling. Duration $T_{\alpha i}$ accepted according to the criterion of the minimum averaging error of the measured parameters. Due to the finiteness of the number and types of sensors and motion detectors in the control unit, we select a representative subset of measurement points and find, in the form:

$$\frac{dX^*}{dt} = dX (t)$$

in equality, the left side is the full derivative of the change in the norm of the vector of parameters at the measurement points, and the right is the total derivative of the norm of the vector of parameters on all permitted directions of the ship's movement in the navigation safety zone, then:

$$\frac{dX^*}{dt} - \frac{dX (t)}{dt} \leq C$$

where $C$ – permissible deviation.

To conduct the assessment, we use the target functions of the controlled object as the quality of the state:

$$G = G(m,u)$$

Let us investigate the optimization problem, in the meaning $mM$ that will provide the maximum or minimum of the function:

$$G = G(m,u)$$

Measuring the parametrization of traffic flows $(X)$ and correcting on the analysis of the effectiveness of the current plans – $F_{\alpha}$, and comparing with the reference values $F_{\alpha k}(X)$. When analyzing and testing, we get:

$$F_{\alpha k} (X) - F_{\alpha} (X) \geq A$$

We recalculate the knowledge base of coordination management $Y$ and control values of their action time $t_i$. The plan $Y_i$ is put into operation when the real-time coincides $t_i$ with the control one $t_k$ [3]. Adaptive systems for controlling the movement of ships along the ship’s lanes, we optimize in real-time functionality, in the form of:

$$Q (\dot{X}, \dot{U}) = \min$$

where $X$ – vector of the state of controlled objects; $\dot{U}$ – vector of state of control actions.

We aggregate macroscopic traffic flow models by the interaction of transport units using macroscopic variables:

$v(x, t)$ – average speed, in m / h; $p(x, t)$ – density of TP, per court/mile; $q(x, t)$ – the intensity of the traffic, to the court / h, where $x$, $t$ – the variables of space and time.
Then the microscopic variables will be derivatives in the cross-section of the connection at the moment and will be the average distance and the number of ships [4].

Where from:

$$\rho = \frac{N_{vech}}{L} = \frac{N_{vech}}{L} \cdot \sum_{i=1}^{N_{vech}} d_i = 1/\bar{d}$$  \hspace{1cm} (10)

where $L$ – the length of the considered segment of the transport network.

Traffic intensity, relative to microscopic variables, over a relative time:

$$q = \frac{N_{vech}}{\Delta t} = \frac{N_{vech}}{L} \cdot \sum_{i=1}^{N_{vech}} h_i = 1/\bar{h}$$  \hspace{1cm} (11)

Models are developed for the mathematical interpretation of the relationship between the speed, density, and intensity of traffic, in the form of:

$$q(x,t) = v(x,t) \cdot \rho(x,t)$$  \hspace{1cm} (12)

then

$$v(x,t) = f(q(x,t), \rho(x,t))$$  \hspace{1cm} (13)

where $f \upharpoonright$ – a function.

3. Materials and methods

The dependence of the traffic intensity on the traffic density is described by the fundamental traffic flow diagram, where the left part of the curve reflects the steady-state of the flow, in which, as the traffic density increases, the phases of free, then discrete saturation pass through. Let us explore the traffic flow as a model LWR. We express the traffic flow intensity as a function of density and a fundamental relationship between macroscopic variables. Then: the law of conservation of mass (number of ships) is fulfilled:

$$v(x,t) = v\left(\rho(x,t)\right)$$  \hspace{1cm} (14)

at the intensity of the traffic flow, as:

$$q(\rho) = \rho V(\rho)$$  \hspace{1cm} (15)

then the law of conservation of masses will be:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial \rho} \cdot \frac{\partial \rho}{\partial x} = 0$$  \hspace{1cm} (16)

Let us assume that the numerical approach is to reduce the partial differential equation to a total differential:

$$\rho(x,t) = f(x - ct, 0)$$  \hspace{1cm} (17)

where $f$ – derivative function, as a combination of initial and boundary conditions.

In a linear wave equation, each initial value, in the form:

$$\rho_0 = \rho(x_0, t_0)$$  \hspace{1cm} (18)

is transferred with an undamped wave at speed:

$$c = \frac{\partial x}{\partial t}$$  \hspace{1cm} (19)

along characteristic straight lines in time and space.

The model is not linear since $\frac{dq}{d\rho}$ depends on the density and speed of the wave, consisting of:
The intersection of the two characteristics is continuous in the distribution density, like those moving at speed:

$$c = q(x_2) - q(x_1) - \rho(x_1)$$  \hspace{1cm} (21)

We find the dependence "speed-density," as an expression, in the form:

$$\rho(v) = \frac{1}{L} + c_1v + c_2v^2$$  \hspace{1cm} (22)

where $L$ is the average length of a transport vessel; $c_1$ – the reaction time of the boat master; $c_2$ – coefficient of proportionality to the stopping distance.

The first-order discrete model is applicable to reduce the computational complexity of continuous algorithms. In this model, the relationship is length, the number of ships in each cell is updated for each time step $t$, the shipping path is divided into many piecewise linear sections $i$. Vessels on a section of the ship's route move to travel to the next section $i+1$. We find the average speed from the values of intensity and density:

$$v(i,k) = \frac{1}{\rho(i,k)} \cdot \left( \max\{\rho(i,k), q_{\text{max}}\} - \rho(i,k) \right)$$  \hspace{1cm} (23)

where:

$$\rho(i,k) = \min\{v(i,k) \times \rho(i,k), q_{\text{max}}\} \times \left( \rho_j - \rho(i,k) \right)$$  \hspace{1cm} (24)

Modifying, we present the equivalent equation:

$$\rho(i,k+1) = \rho(i,k) + \frac{T}{L} \cdot \left[ Y(i,k) - Y(i+1,k) \right]$$  \hspace{1cm} (25)

where: $i$ – index of the section of a piecewise linear ship route; $k$ – time index; $\rho(i,x)$ – density on the site $i$ in the period of the ship's route $k$; $v(i,x)$ – speed on a piecewise linear section of the track; $w(i,x)$ – the speed of the shock wave at the site $i$ in the period $k$; $q_{\text{max}}$ – maximum flow on the site; $\rho_j$ – density when vessels are jammed.

4. Discussion

The study considered the difference in the aggregation time interval for the flow and measurements. The network traffic model has a fixed degree of dispersion of a group of ships on the course and assumes a long time due to the technical difference in stopping characteristics. In network traffic, the structure of the profile depends on the mutual correspondence OUT and GO. In the profile GO, the time of effective movement according to the "traffic-control" resolution $OUT = GO$, in the form of:

$$\begin{align*}
\text{OUT}_i = \begin{cases} 
0, u(t) = 0 \\
\text{GO}_i, u(t) = 1, m_i > 0 \\
\text{IN}_i, u(t) = 1, s_i = 0
\end{cases}
\end{align*}$$  \hspace{1cm} (26)

where: $u(t)$ – binary control function; $m_i$ - the number of vessels on the course section of the ship's route in the time interval $t$.

We find the number of ships in the queue using the iterative procedure:

$$m_i = \max\{m_{i-1} + q_i - s_i\}$$  \hspace{1cm} (27)

where: $q_i$ – the number of ships arriving during the time interval $t$ according to the profile $IN$; $s_i$ – the number of vessels departing during the time $t$ along with the profile $GO$. 
5. Conclusion

Thus, a mathematical model of the traffic flow of ships in the transport network of the port on piecewise linear sections on the course in a complex zone of navigation safety [5] with crowding in the sea operating line of ports is obtained.

References

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