Are Bartik Regressions Ever Robust to Heterogeneous Treatment Effects?∗

Clément de Chaisemartin† Ziteng Lei‡

December 12, 2022

Abstract

Bartik regressions use locations’ differential exposure to aggregate shocks as an instrument to estimate the effect of a treatment on an outcome. We derive several decomposition results, all showing that Bartik regressions identify non-convex combinations of location-specific treatment effects. Some of our decompositions rely on arguably strong assumptions, including a linear first-stage model with homogeneous effects, or a randomly-assigned-shocks assumption. Therefore, even under strong assumptions, Bartik regressions are not robust to heterogeneous effects. We propose alternative correlated-random-coefficient estimators that are more, though not fully, robust to heterogeneous effects. We use our results to revisit Autor et al. (2013).

∗We are very grateful to Xavier D’Haultfœuille, Paul Goldsmith-Pinkham, Peter Hull, Michal Kolesár, Félix Pasquier, Aureo de Paula, Isaac Sorkin, and seminar participants at CREST, Tilburg University, and the Sao Paulo School of Economics for their helpful comments.
†Economics Department, Sciences Po, clement.dechaisemartin@sciencespo.fr.
‡School of Labor and Human Resources, Renmin University of China, leiziteng@ruc.edu.cn.
1 Introduction

The “Bartik instrument”, also known as the “shift-share instrument”, is a popular method to estimate the effect of a treatment evolution $\Delta D_g$ on an outcome evolution $\Delta Y_g$. It has first been proposed by Bartik (1991). Since then it has been applied in many fields, including labor (see Altonji & Card 1991, Card 2001, 2009), international trade (see Autor et al. 2013, 2020), and finance (see Greenstone et al. 2020).

For instance, Autor et al. (2013) study how the evolution of imports from China in US commuting-zone (CZ) $g$ affects the evolution of manufacturing employment in that same CZ. They could estimate an OLS regression of CZs’ evolution of manufacturing employment on their evolution of Chinese imports, but they worry that the latter may be endogenous. For example, the evolution of Chinese imports may be correlated with demand shocks. Instead, they estimate a 2SLS regression, instrumenting imports’ evolution by the so-called Bartik instrument. Assume manufacturing is divided into $S$ sectors indexed by $s$. Let $\Delta Z_s$ denote the evolution of Chinese imports in sector $s$, in a group of high-income countries similar to the US. Then their instrument is

$$\Delta Z_g = \sum_{s=1}^{S} Q_{s,g} \Delta Z_s,$$

where $Q_{s,g}$ is the share sector $s$ accounts for in CZ $g$’s manufacturing employment. $\Delta Z_g$ represents the Chinese-imports evolution that CZ $g$ would have experienced if all its sectors had experienced the same imports evolution as in non-US high-income countries.

Throughout the paper, we say that the Bartik 2SLS regression is robust to heterogeneous effects if it identifies a weighted average of CZ-specific treatment effects $\alpha_g$, with positive weights. This ensures that sign reversal phenomena, where, say, $\alpha_g \geq 0$ for all $g$ but the Bartik coefficient is negative, cannot happen. This is arguably a rather weak robustness criterion, so all our negative results still hold under stronger criteria.

This paper derives three decomposition results, all showing that Bartik regressions are not robust to heterogeneous effects. Our first decomposition shows that if the Bartik instrument is uncorrelated with CZs’ outcome evolutions without treatment $\Delta Y_g(0)$, the Bartik 2SLS regression identifies a weighted sum of CZ-specific treatment effects $\alpha_g$, where the weights can be estimated and may be negative. Our second decomposition shows that if we further assume a linear first-stage model, Bartik regressions are still not robust to heterogeneous effects, even if we assume a fully homogeneous first-stage effect of the instrument $\Delta Z_g$ on the treatment evolution $\Delta D_g$. Our third decomposition shows that if we further assume that the shocks $\Delta Z_s$ are as-good-as randomly assigned, as in Borusyak et al. (2022) and Adão et al. (2019), Bartik regressions are still not robust to heterogeneous effects. Under this randomly-assigned shocks assumption, Borusyak et al. (2022) and Adão et al. (2019) have shown that a Bartik 2SLS regression with no intercept and demeaned shocks identifies a weighted average of CZ-specific treatment effects, with only positive weights. We show that this result does not extend to the more-commonly
used 2SLS regression with an intercept. Accordingly, researchers invoking the randomly-assigned shocks assumption to motivate their Bartik regression should estimate it with no intercept and demeaned shocks.

Then, we propose correlated-random-coefficient (CRC) estimators inspired from Chamberlain (1992), that can be used if there are at least three time periods in the data. They are more robust to heterogeneous effects than Bartik regressions, though they still rely on strong assumptions on effects’ heterogeneity. Our first estimator allows for CZ-specific first- and second-stage effects, but assumes homogeneous second-stage effects over time and homogeneous first-stage effects across sectors and over time. Our second estimator allows for CZ- and time-specific first- and second-stage effects, provided the first- and second-stage effects follow the same evolution over time in every CZ. Due to an incidental-parameters problem, it may be consistent only if the number of periods goes to infinity. Unfortunately, the number of periods is often low in Bartik applications. Our third estimator allows for period- and CZ-specific second-stage effects, and for period-specific first-stage effects, but assumes constant first-stage effects across CZs and sectors. It therefore relies on stronger assumptions than our second estimator, but still allows for period-specific effects and can be used with a low number of periods.

We use our results to revisit Autor et al. (2013). In this application, we reject the randomly-assigned shocks assumption. Under this assumption, sectoral shocks should be uncorrelated with sectors’ characteristics. In practice, those shocks are strongly correlated with some sectors’ characteristics, and in particular with \( \frac{1}{G} \sum_{g=1}^{G} Q_{s,g} \), sectors’ average share across CZs. On the other hand, we do not reject a placebo test of the assumptions underlying our first two decompositions of Bartik regressions and our alternative estimators. This placebo test assesses if \( \Delta Z_g \) is correlated with CZs’ evolution of manufacturing employment before the China shock, and is similar to a placebo test previously proposed by Autor et al. (2013). Our first decomposition indicates that in this application, the Bartik regression estimates a weighted sum of effects where more than 50% of effects are weighted negatively, and where negative weights sum to \(-0.16\). Weights are negatively correlated with CZs’ share of college graduates, which may themselves be correlated with their treatment effects, thus inducing a bias of the Bartik regression. Our second decomposition shows that the Bartik regression is still not robust to heterogeneous effects, even under a linear first-stage model with a fully homogeneous effect. We also find that the 2SLS Bartik regressions with and without an intercept yield very different results. Finally, our alternative CRC estimators are more negative than the Bartik estimator.

Based on our results, here is the course of action we recommend to applied researchers using Bartik regressions in applications where treatment effects may be heterogeneous. First, they should assess the plausibility of the randomly-assigned-shocks assumption, regressing shocks on several industry characteristics, and in particular on sectors’ average share. If those tests are conclusive, they should estimate a Bartik regression without an intercept and with demeaned shocks: this is the regression that is robust to heterogeneous
effects under the randomly-assigned-shocks assumption. If those tests are inconclusive, they can use our first and/or our second decomposition to assess if their Bartik regression is robust to heterogeneous effects, under weaker assumptions than the randomly-assigned-shocks assumption. If those decompositions indicate that their Bartik regression is not robust, they may use instead one of our alternative estimators. When their data contains a time period with no shock, as in Autor et al. (2013), they should also implement a placebo test of the assumptions underlying our decompositions of Bartik regressions and our alternative estimators.

The paper is organized as follows. Section 2 presents our decompositions of Bartik regressions. Section 3 presents our alternative estimators. Section 4 presents our re-analysis of Autor et al. (2013). To preserve space, a second application, where we re-analyse the canonical Bartik setting is in Section A of the Web Appendix.

Related literature

Our paper is connected to three recent papers that have considerably improved our understanding of Bartik regressions, namely Goldsmith-Pinkham et al. (2020), Borusyak et al. (2022), and Adão et al. (2019). Our paper builds upon their pioneering work. We are solely concerned with the robustness of Bartik regressions to heterogeneous effects. Accordingly, our paper has no bearing on all the results derived under homogeneous effects in those three papers. Heterogeneous effects is less central in those three papers, though Goldsmith-Pinkham et al. (2020) still discuss it in an extension, Borusyak et al. (2022) in their online appendix, and Adão et al. (2019) in the main sections of their paper. We now briefly discuss the connections and the differences between our and their results with heterogeneous effects. We give further details on those comparisons in Section 2.

Our first decomposition of Bartik regressions (see Theorem 1) is obtained under strictly weaker assumptions than those in Goldsmith-Pinkham et al. (2020). In their Equation (10), Goldsmith-Pinkham et al. (2020) also show that Bartik regressions may identify a weighted sum of effects, potentially with some negative weights. Their decomposition is related to, but different from, our second decomposition (see Theorem 2). In particular, the weights in their decomposition depend on the so-called Rotemberg weights (see Rotemberg 1983), while the weights in our decomposition do not depend on the Rotemberg weights. The difference between our decompositions stems from the fact our first-stage assumptions are different and almost incompatible. Goldsmith-Pinkham et al. (2020) assume $S$ linear first-stage models, where the first-stage effect of only one sectoral shock appears explicitly, thus implying that other shocks are in the residual. We instead assume a single linear first-stage model, where the treatment evolution depends on all sectoral shocks (see Assumption 3). Under our first-stage model, we show that it is hard to rationalize the first-stage exogeneity conditions in Goldsmith-Pinkham et al. (2020). Accordingly, whenever our linear first-stage model seems plausible, the first-stage assumptions in Goldsmith-Pinkham et al. (2020) are unlikely to hold, and accordingly
their decomposition of Bartik regressions is also unlikely to hold.

Our two first decompositions of Bartik regressions (Theorems 1 and 2 below) are obtained under strictly weaker identifying assumptions than those in Borusyak et al. (2022), and Adão et al. (2019). In particular, those two decompositions do not rely on their randomly-assigned shocks assumption. In their Proposition 1, Adão et al. (2019) show, under assumptions similar to those underlying our third decomposition (see Theorem 3), that a first-stage Bartik regression without an intercept identifies a weighted average of first-stage effects. Equation (2.12) in our Theorem 3 is a straightforward generalization of their result to a 2SLS Bartik regression without an intercept. When they allow for heterogeneous treatment effects in their Appendix A.1, Borusyak et al. (2022) also consider a Bartik regression without an intercept (see their Equation (A2)). Adão et al. (2019) argue that omitting the intercept from the regression is without loss of generality, because the treatment evolution in their regression has been previously demeaned. However, the linear first-stage equation underlying their result is not invariant to demeaning. Therefore, their Proposition 1 applies to a Bartik regression with an intercept only if one imposes linear models directly on the demeaned treatment and outcome evolutions, which may be both unnatural and implausible. If one instead imposes linear models on the non-demeaned treatment and outcome evolutions, as seems more natural to us, then Equation (2.11) in our Theorem 3 shows that a 2SLS Bartik regression with an intercept is not robust to heterogeneous effects, even under the randomly-assigned shocks assumption.

Our decompositions of Bartik regression coefficients are also related to an older literature on IV-CRC models (see Wooldridge 1997, Heckman & Vytlacil 1998). Our alternative estimators are almost direct applications of the CRC estimator in Chamberlain (1992).

2 Bartik regressions with heterogeneous effects

2.1 Setup and notation

Setup. We consider a data set with $G$ locations, indexed by $g \in \{1, ..., G\}$. To make exposition as simple as possible, for now we assume the data has two time periods indexed by $t \in \{1, 2\}$. Locations are typically geographical regions, for instance counties, states or commuting zones (CZs). Let $R_{g,t}$ denote the value of a generic variable $R$ in location $g$ and period $t$. Then, let $\Delta R_g = R_{g,2} - R_{g,1}$ denote the change of that variable from period 1 to 2 in location $g$. We are interested in how the evolution of a treatment variable $\Delta D_g$ affects the evolution of an outcome variable $\Delta Y_g$. For instance, Autor et al. (2013) have studied how the evolution of imports from China in CZ $g$ affects the evolution of manufacturing employment there. We could estimate an OLS regression of $\Delta Y_g$ on $\Delta D_g$, but we worry that the treatment evolution may be endogenous. For example, the evolution of Chinese imports may be correlated with demand shocks.
Bartik instrument. Instead, it has been proposed to estimate a 2SLS regression of $\Delta Y_g$ on $\Delta D_g$, instrumenting $\Delta D_g$ by the so-called Bartik instrument. Assume there are $S$ sectors indexed by $s \in \{1, \ldots, S\}$. Sectors could for instance be industries. Let $R_{s,t}$ denote the value of a generic variable $R$ in sector $s$ and period $t$. Then, let $\Delta R_s = R_{s,2} - R_{s,1}$ denote the change of that variable from period 1 to 2 in sector $s$. For every $s \in \{1, \ldots, S\}$, let $\Delta Z_s$ denote a shock affecting sector $s$ between periods 1 and 2. For example, in Autor et al. (2013), $\Delta Z_s$ denotes the growth of Chinese imports in sector $s$, in a group of high-income countries similar to the US.

Definition 1 Bartik Instrument: The Bartik instrument $\Delta Z_g$ is:

$$\Delta Z_g = \sum_{s=1}^{S} Q_{s,g} \Delta Z_s.$$ 

For every $g$, $Q_{s,g}$ are positive weights summing to 1 or less, reflecting the importance of sector $s$ in location $g$ at period 1. For instance, $Q_{s,g}$ could be the share that sector $s$ accounts for in location $g$’s employment at period 1. In Autor et al. (2013), $\Delta Z_g$ represents the Chinese import evolution that location $g$ would have experienced if all its sectors had experienced the same imports evolution as in non-US high income countries.

Potential outcomes. Let $\Delta Y_g(d_g)$ denote the potential outcome evolution that location $g$ will experience if $\Delta D_g = d_g$. $\Delta Y_g(0)$ is location $g$’s potential outcome evolution without any treatment change. This notation implicitly assumes that the shocks have no direct effect on the outcome evolution, they can only affect the outcome evolution through their effect on the treatment evolution, see Angrist & Imbens (1995). As noted by Goldsmith-Pinkham et al. (2020), this notation also assumes that the outcome evolution can only depend on the treatment evolution, not on the level of the treatment at period 1.

Assumption 1 Linear Treatment Effect: for all $g \in \{1, \ldots, G\}$, there exists $\alpha_g$ such that for any $d_g \in \mathbb{R}$:

$$\Delta Y_g(d_g) = \Delta Y_g(0) + \alpha_g d_g.$$ 

Assumption 1 is a linear treatment effect assumption. It is analogous to Equation (7) in Goldsmith-Pinkham et al. (2020) without control variables, and to Equation (30) in Adão et al. (2019) allowing for location-specific treatment effects. Under Assumption 1, $\Delta Y_g = \Delta Y_g(0) + \alpha_g \Delta D_g.$ (2.1)

Sampling and sources of uncertainty. Our assumptions below are tailored to be compatible with two sampling schemes. In the first one, hereafter referred to as the fixed-shocks approach, the shocks $\Delta Z_s$ are fixed (or conditioned upon), and locations are an
independent and identically distributed (iid) sample drawn from a super population of locations. Then, the vectors \((\Delta Z_g, \Delta D_g, \Delta Y_g)\) are iid. This approach is similar to that in Goldsmith-Pinkham et al. (2020). In the second one, hereafter referred to as the fixed-locations approach, the locations are fixed (or conditioned upon), and the shocks \(\Delta Z_s\) are drawn independently across sectors. This approach is similar to that in Borusyak et al. (2022) and Adão et al. (2019). In Sections 2.2 and 2.3 below, we do not take a strong stand on which of these two approaches may be more applicable: the decompositions therein are derived under assumptions compatible with both approaches.

**Estimator and estimand.** Let \(\Delta Z = \frac{1}{G} \sum_{g=1}^{G} \Delta Z_g\).

**Definition 2** 2SLS Bartik regression.\(^1\) let

\[
\hat{\theta}^b = \frac{\sum_{g=1}^{G} \Delta Y_g (\Delta Z_g - \Delta Z)}{\sum_{g=1}^{G} \Delta D_g (\Delta Z_g - \Delta Z)} \quad (2.2)
\]

\[
\theta^b = \frac{\sum_{g=1}^{G} E(\Delta Y_g (\Delta Z_g - \Delta Z))}{\sum_{g=1}^{G} E(\Delta D_g (\Delta Z_g - \Delta Z))} \quad (2.3)
\]

\(\hat{\theta}^b\) is just the sample coefficient from a 2SLS regression of \(\Delta Y_g\) on an intercept and \(\Delta D_g\), using \(\Delta Z_g\) as the instrument. One can show that \(\theta^b\) is the probability limit of \(\hat{\theta}^b\), both in the fixed shocks approach of Goldsmith-Pinkham et al. (2020) when \(G \to +\infty\), and in the fixed locations approach of Borusyak et al. (2022) and Adão et al. (2019) when \(S \to +\infty\) (on that point, see e.g. Proposition 1 in Adão et al. 2019, who consider an estimand similar to \(\theta^b\)).

**Instrument relevance.** Throughout the paper, we assume that the instrument is relevant: \(\sum_{g=1}^{G} E(\Delta D_g (\Delta Z_g - \Delta Z)) \neq 0\). Without loss of generality we can further assume that \(\sum_{g=1}^{G} E(\Delta D_g (\Delta Z_g - \Delta Z)) > 0\): the population first-stage is strictly positive.

**Definition of robustness to heterogeneous effects** In this paper, we say that \(\theta^b\) is robust to heterogeneous effects if and only if \(\theta^b = E \left( \sum_{g=1}^{G} w_g \alpha_g \right)\), with \(E \left( \sum_{g=1}^{G} w_g \right) = 1\) and \(w_g \geq 0\) almost surely. One may find this definition too weak, and one may argue that \(\theta^b\) is only robust to heterogeneous effects if \(\theta^b = E \left( \frac{1}{G} \sum_{g=1}^{G} \alpha_g \right)\), meaning that \(\theta^b\) identifies the average treatment effect (ATE). Under this stricter robustness criterion, all our negative results below still hold. Requiring that the weights are almost surely positive is important. Having that \(E(w_g) \geq 0\) is not enough to prevent a so-called sign reversal, where, say, \(\alpha_g \geq 0\) almost surely for all \(g\), but \(\theta^b < 0\). Under our definition of robustness, such sign reversals cannot happen.

\(^1\)Throughout the paper, we consider Bartik regressions that are not weighted, say, by locations’ population. It is straightforward to extend all our results to weighted Bartik regressions.
2.2 Bartik is not robust to heterogeneous effects under a linear treatment effect model

Identifying assumption. In this section, we make the following assumption.

Assumption 2 1. For all \( g \in \{1, ..., G\} \), \( \text{cov}(\Delta Z_g, \Delta Y_g(0)) = 0 \).

2. For all \( g \neq g' \in \{1, ..., G\}^2 \), \( \text{cov}(\Delta Z_{g'}, \Delta Y_g(0)) = 0 \).

3. \( \frac{1}{G} \sum_{g=1}^{G} E(\Delta Y_g(0)) \left( E(\Delta Z_g) - E(\Delta Z) \right) = 0 \).

The first point of Assumption 2 requires that location \( g \)’s potential outcome evolution without any treatment be uncorrelated with its Bartik instrument. Because \( \Delta Y_g(0) \) is a potential outcome evolution, this condition may be interpreted as a parallel trends assumption. The second point of Assumption 2 requires that \( \Delta Y_g(0) \) be uncorrelated with the Bartik instruments of other locations. The third point of Assumption 2 requires that across locations, there is no correlation between \( g \)’s expected outcome evolution without any treatment and the expectation of \( g \)’s Bartik instrument.

Assumption 2 is weaker than the assumptions in Goldsmith-Pinkham et al. (2020). Goldsmith-Pinkham et al. (2020) consider the shocks \( (\Delta Z_s)_{1 \leq s \leq S} \) as non-stochastic, and their Assumption 2 requires that for all \((s, g)\), \( \text{cov}(Q_{s,g}, \Delta Y_g(0)) = 0 \). This implies that \( \text{cov}(\Delta Z_g, \Delta Y_g(0)) = 0 \). Points 2 and 3 of Assumption 2 trivially follow from the fact they assume that all location-level variables are iid, see their Section I.A.

Assumption 2 is also weaker than the assumptions in Borusyak et al. (2022) and Adão et al. (2019). Without control variables, Assumption 4.ii) in Adão et al. (2019) requires that \( E(\Delta Z_s|\Delta Y_g(0), (Q_{s,g})_{s \in \{1, ..., S\}}) = 0 \) for all \((s, g)\). This implies that \( E(\Delta Z_g) = 0 \), so

\[
\text{cov}(\Delta Z_g, \Delta Y_g(0)) = E \left( \Delta Y_g(0) \sum_{s=1}^{S} Q_{s,g} E \left( \Delta Z_s|\Delta Y_g(0), (Q_{s,g})_{s \in \{1, ..., S\}} \right) \right) = 0.
\]

Similarly, \( \text{cov}(\Delta Z_{g'}, \Delta Y_g(0)) = 0 \). Finally, Point 3 of Assumption 2 trivially follows from the fact \( E(\Delta Z_g) = 0 \). Therefore, Assumption 2 is implied by the assumptions in Adão et al. (2019). In their Appendix A.1, where they allow for heterogeneous effects, Borusyak et al. (2022) also assume that \( E(\Delta Z_s|\Delta Y_g(0), (Q_{s,g})_{s \in \{1, ..., S\}}) = E(\Delta Z_s) = 0 \) for all \((s, g)\), so Assumption 2 is implied by their assumptions as well.

Decomposition of \( \theta^b \) under Assumptions 1-2.

Theorem 1 Suppose Assumptions 1-2 hold. Then,

\[
\theta^b = E \left( \sum_{g=1}^{G} \frac{\Delta D_g(\Delta Z_g - \Delta Z)}{E \left( \sum_{g'=1}^{G} \Delta D_{g'}(\Delta Z_{g'} - \Delta Z) \right)} \alpha_g \right).
\]
Proof of Theorem 1

\[ E \left( \sum_{g=1}^{G} \Delta Y_g (\Delta Z_g - \Delta Z) \right) \]

\[ = E \left( \sum_{g=1}^{G} (\Delta Y_g(0) + \alpha_g \Delta D_g) (\Delta Z_g - \Delta Z) \right) \]

\[ = E \left( \sum_{g=1}^{G} (\Delta Y_g(0) (\Delta Z_g - \Delta Z)) + \sum_{g=1}^{G} \Delta D_g (\Delta Z_g - \Delta Z) \alpha_g \right) \]

\[ = \sum_{g=1}^{G} E (\Delta Y_g(0)) (E (\Delta Z_g) - E (\Delta Z)) + E \left( \sum_{g=1}^{G} \Delta D_g (\Delta Z_g - \Delta Z) \alpha_g \right) \]

\[ = E \left( \sum_{g=1}^{G} \Delta D_g (\Delta Z_g - \Delta Z) \alpha_g \right) \quad (2.4) \]

The first equality follows from Assumption 1. The third equality follows from Points 1 and 2 of Assumption 2. The fourth equality follows from Point 3 of Assumption 2. Then, plugging (2.4) into (2.3) yields the result. QED.

Consequences of Theorem 1. Theorem 1 shows that under Assumption 2, \( \theta^b \) is equal to the expectation of a weighted sum of the treatment effects \( \alpha_g \), with weights

\[ \frac{\Delta D_g (\Delta Z_g - \Delta Z)}{E \left( \sum_{g'=1}^{G} \Delta D_{g'} (\Delta Z_{g'} - \Delta Z) \right)} \quad (2.5) \]

that may not all be positive. Thus, \( \theta^b \) is not robust to heterogeneous effects according to our definition. Locations whose \( \alpha_g \)s get weighted negatively are those for which \( \Delta Z_g - \Delta Z \) is of a different sign than \( \Delta D_g \). Therefore, \( \theta^b \) is robust to heterogeneous effects if and only if \( \Delta Z_g - \Delta Z \) and \( \Delta D_g \) are almost surely of the same sign for all \( g \), a condition that is unlikely to often hold. Whenever there is at least one \( g \) in the data such that \( \Delta Z_g - \Delta Z \) and \( \Delta D_g \) are of a different sign, this condition must be violated.

Practical use of Theorem 1. The weights in Theorem 1 can be estimated. Estimating the weights, and assessing if many are negative, can be used to assess the robustness of the Bartik regression to heterogeneous effects. Moreover, when one observes some locations’ characteristics that are likely to be correlated with their \( \alpha_g \), one can test if those characteristics are correlated with the weights. If they are not, it may be plausible to assume that \( \alpha_g \) and \( \Delta D_g (\Delta Z_g - \Delta Z) \) are independent, in which case one can show that \( \theta^b \) identifies the ATE. On the other hand, if the weights are correlated with those characteristics, it may not be plausible to assume that \( \alpha_g \) and \( \Delta D_g (\Delta Z_g - \Delta Z) \) are independent, and \( \theta^b \) may not identify a meaningful parameter.
Connection with previous literature. In the fixed-shocks approach with iid locations, Theorem 1 reduces to
\[ \theta^b = E \left( \frac{\Delta D(\Delta Z - E(\Delta Z))}{E(\Delta D(\Delta Z - E(\Delta Z)))} \right)^{\alpha}, \]
where we can drop the subscript because locations are iid. This is a well-known result (see e.g. Equation (3) in Benson et al. 2022), that applies to any 2SLS regression under a linear treatment effect model. Theorem 1 is just restating this result under an exogeneity assumption compatible both with the fixed-shocks and fixed-locations approaches to Bartik. Theorem 1 says that with heterogeneous effects, instrument-exogeneity is not enough for 2SLS to identify something meaningful. A natural and well-known next step, first proposed by Imbens & Angrist (1994), is to see if 2SLS identifies something more meaningful if one further assumes a first-stage model relating the treatment to the instrument.

2.3 Bartik is still not robust if one further assumes a linear first-stage model

Linear first-stage model. For any \((\delta_1, \ldots, \delta_s) \in \mathbb{R}^S\), let \(\Delta D_g(\delta_1, \ldots, \delta_s)\) denote the potential treatment evolution that location \(g\) will experience if \((\Delta Z_1, \ldots, \Delta Z_s) = (\delta_1, \ldots, \delta_s)\). And let \(\Delta D_g(0) = \Delta D_g(0, \ldots, 0)\) denote the potential treatment evolution that location \(g\) will experience in the absence of any shocks. The actual treatment evolution is \(\Delta D_g = \Delta D_g(\Delta Z_1, \ldots, \Delta Z_s)\). We make the following assumption:

**Assumption 3** Linear First-Stage Model: for all \(g \in \{1, \ldots, G\}\), there exists \((\beta_{s,g})_{s \in \{1, \ldots, S\}}\) such that for any \((\delta_1, \ldots, \delta_s) \in \mathbb{R}^S\):
\[ \Delta D_g(\delta_1, \ldots, \delta_s) = \Delta D_g(0) + \sum_{s=1}^{S} Q_{s,g} \delta_s \beta_{s,g}. \]
Assumption 3 requires that the effect of the shocks on the treatment evolution be linear: increasing \(\Delta Z_s\) by 1 unit, holding all other shocks constant, leads the treatment of location \(g\) to increase by \(Q_{s,g} \beta_{s,g}\) units. Similar assumptions are also made by Adão et al. (2019) (see their Equation (11)) and Goldsmith-Pinkham et al. (2020) (see their Equation (8), which we discuss in more details later). Under Assumption 3,
\[ \Delta D_g = \Delta D_g(0) + \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g}. \]

Identifying assumptions with a first-stage model. With our first-stage model in hand, we can make a slightly different identifying assumption than Assumption 2, replacing the potential outcome evolution without treatment \(\Delta Y_g(0)\), the residual in the outcome equation, by the potential treatment evolution without any shocks \(\Delta Y_g(\Delta D_g(0))\), the residual in the reduced-form equation.
Assumption 4 1. For all \( g \in \{1, ..., G\} \), \( \text{cov}(\Delta Z_g, \Delta Y_g(\Delta D_g(0))) = 0 \).

2. For all \( g \neq g' \in \{1, ..., G\} \), \( \text{cov}(\Delta Z_{g'}, \Delta Y_g(\Delta D_g(0))) = 0 \).

3. \( \frac{1}{G} \sum_{g=1}^{G} E(\Delta Y_g(\Delta D_g(0))) (E(\Delta Z_g) - E(\Delta Z)) = 0 \).

We also need to assume that the instrument is exogenous with respect to the first-stage residual \( \Delta D_g(0) \).

Assumption 5 1. For all \( g \in \{1, ..., G\} \), \( \text{cov}(\Delta Z_g, \Delta D_g(0)) = 0 \).

2. For all \( g \neq g' \in \{1, ..., G\} \), \( \text{cov}(\Delta Z_{g'}, \Delta D_g(0)) = 0 \).

3. \( \frac{1}{G} \sum_{g=1}^{G} E(\Delta D_g(0)) (E(\Delta Z_g) - E(\Delta Z)) = 0 \).

Like Assumption 2, Assumptions 4 and 5 are implied by the identifying assumptions in Borusyak et al. (2022) and Adão et al. (2019).

Comparing Assumptions 2 and Assumptions 4-5. If \( E(\Delta D_g(0)) = 0 \), Assumptions 2, 4, and 5 can jointly hold under no restrictions on the joint distribution of \( \alpha_g \) and \( \Delta Z_g \). For instance, if Point 1 of Assumption 2 holds and \( E(\Delta D_g(0)|\alpha_g, \Delta Z_g) = 0 \), then Point 1 of Assumption 4 holds. On the other hand, if \( E(\Delta D_g(0)) \neq 0 \), imposing jointly Assumptions 2 and 4 is essentially equivalent to assuming that \( \text{cov}(\Delta Z_g, \alpha_g) = 0 \), a strong requirement, unless one is ready to assume that the Bartik instrument is as-good-as randomly assigned to locations. Then, one should decide whether to work under Assumption 2 or under Assumptions 4-5. In our view, an important advantage of Assumptions 4-5 is that they are “placebo testable”, if the data contains a prior time period without any shocks, as is sometimes the case in practice (see e.g. Autor et al. 2013). Then, one can assess if locations’ outcome and treatment evolutions prior to the shocks are uncorrelated with their Bartik instruments. If so, that lends credibility to Point 1 of Assumptions 4 and 5. Assumption 2, on the other hand, may rarely be placebo testable: even when the data contains a prior time period without any shocks, locations may still experience changes in their treatment at that time period.

Decompositions of \( \theta^b \) under Assumptions 1 and 3-5.

Theorem 2 Suppose Assumptions 1 and 3-5 hold.

1. Then,

\[
\theta^b = E \left( \sum_{g=1}^{G} \frac{\sum_{s=1}^{S} Q_{s,g}\Delta Z_s(\Delta Z_g - \Delta Z_s)\beta_{s,g}}{E \left( \sum_{g'=1}^{G} \sum_{s'=1}^{S} Q_{s',g'}\Delta Z_{g'}(\Delta Z_{g'} - \Delta Z_s)\beta_{s',g'} \right)} \alpha_g \right).
\]

2. If \( \Delta Z_s \geq 0 \) for all \( s \), and one further assumes that \( \beta_{s,g} \geq 0 \) for all \( (s, g) \), \( \alpha_g \) is weighted negatively in the decomposition in Point 1 if and only if \( \Delta Z_g - \Delta Z \leq 0 \).
3. If one further assumes $\beta_{s,g} = \beta$, 

$$
\theta^b = E \left( \sum_{g=1}^{G} \frac{\Delta Z_g(\Delta Z_g - \Delta Z)}{E \left( \sum_{g'=1}^{G} \Delta Z_{g'}(\Delta Z_{g'} - \Delta Z) \right)} \alpha_g \right). 
$$

Consequences of Point 1 of Theorem 2  
Point 1 of Theorem 2 shows that under Assumptions 1 and 3-5, $\theta^b$ identifies a weighted sum of the second-stage effects $\alpha_g$, with weights 

$$
\frac{\sum_{s=1}^{S} Q_{s,g} \Delta Z_s(\Delta Z_g - \Delta Z) \beta_{s,g}}{E \left( \sum_{g'=1}^{G} \sum_{s'=1}^{S} Q_{s',g'} \Delta Z_{s'}(\Delta Z_{g'} - \Delta Z) \beta_{s',g'} \right)} 
$$

that may not all be positive. Contrary to the weights in Equation (2.5), those in Equation (2.7) cannot be estimated, as they depend on the first stage effects $\beta_{s,g}$. Locations whose $\alpha_g$ get weighted negatively are those for which $\Delta Z_g - \Delta Z$ is of a different sign than the effect of the shocks on their treatment evolution $\sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g}$.

Consequences of Point 2 of Theorem 2  
Point 2 of Theorem 2 shows that in applications where all the shocks $\Delta Z_s$ are positive, if one further assumes that the first-stage effects $\beta_{s,g}$ are all positive, an assumption similar to the monotonicity condition in Imbens & Angrist (1994), then locations whose $\alpha_g$ get weighted negatively are those for which $\Delta Z_g$ is strictly lower than $\Delta Z$. This has two important implications. First, in designs where $\Delta Z_s \geq 0$, even under a monotonicity condition à la Imbens & Angrist (1994), there must be locations whose second-stage effects are weighted negatively, so $\theta^b$ is never robust to heterogeneous effects. Second, one can estimate the set of locations whose second-stage effects are weighted negatively. Comparing the characteristics of those locations to the characteristics of locations whose second-stage effects are weighted positively may be a way to assess if those two groups are likely to have different second-stage effects, and the likely direction of the bias induced by the negative weights.

Consequences of Point 3 of Theorem 2  
Point 3 of Theorem 2 shows that even if one assumes that $\beta_{s,g} = \beta$, namely homogeneous first-stage effects across sectors and locations, $\theta^b$ may still not be robust to heterogeneous effects. Note that the weights in that last decomposition can be estimated. With respect to those in Equation (2.5), they are proportional to $\Delta Z_g(\Delta Z_g - \Delta Z)$ instead of $\Delta D_g(\Delta Z_g - \Delta Z)$.

Connection between Point 3 of Theorem 2 and results in Wooldridge (1997) and Heckman & Vytlacil (1998). Wooldridge (1998) and Heckman & Vytlacil (1998) consider a linear correlated-random-coefficient model similar to that in Assumption 1, and show that with an instrument independent of $(\Delta D_g(0), \Delta Y_g(0), \alpha_g)$ and with an homogeneous first-stage effect, 2SLS identifies the average treatment effect $E \left( \frac{1}{G} \sum_{g=1}^{G} \alpha_g \right)$. Point 3 of Theorem 2 may seem to contradict that result, as there as well we assume
an homogeneous first-stage effect. The difference between our and their results stems from the fact we do not assume that the Bartik instrument is independent of the effects \( \alpha_g \), an assumption that may be strong, unless one is ready to assume that the Bartik instrument is as-good-as randomly assigned to locations. Instead, Point 3 of Theorem 2 only relies on Assumptions 4 and 5, namely instrument-exogeneity assumptions akin to parallel trends assumptions in the context of Bartik regressions, and that can sometimes be placebo-tested.

Connection between Point 1 of Theorem 2 and Equation (10) in Goldsmith-Pinkham et al. (2020). In their Equation (10), Goldsmith-Pinkham et al. (2020) also show that \( \theta^b \) identifies a weighted sum of treatment effects, potentially with some negative weights. The weights in their and our decomposition differ. Expressed in our notation, the weight assigned to \( \alpha_g \) in their decomposition is

\[
\sum_{s=1}^S \left( \Delta Z_s \left( \sum_{g'=1}^G Q_{s,g'} (\Delta D_{g'} - \Delta D_s) \right) \right)^2 \Delta Z_s \beta_{s,g},
\]

where

\[
\frac{\Delta Z_s \left( \sum_{g'=1}^G Q_{s,g'} (\Delta D_{g'} - \Delta D_s) \right)}{\sum_{s=1}^S \Delta Z_s \left( \sum_{g'=1}^G Q_{s,g'} (\Delta D_{g'} - \Delta D_s) \right)}
\]

is the so-called Rotemberg weight (see Rotemberg 1983). The weights in our decomposition do not depend on the Rotemberg weights. The difference between our decompositions stems from the fact our first-stage assumptions are different and almost incompatible. In what follows, we assume that shocks are non-stochastic, as in Goldsmith-Pinkham et al. (2020). Then, using our notation, and assuming the regression has no control variables, the first-stage assumptions in Goldsmith-Pinkham et al. (2020) (see Equation (8) and Assumption 3 therein) require that for all \( (s, g) \),

\[
\Delta D_g = \mu^D + Q_{s,g} \Delta Z_s \beta_{s,g} + u_{s,g},
\]

(2.8)

with \( E(Q_{s,g} u_{s,g} \alpha_g) = 0 \). (2.9)

Equation (2.8) is a linear first-stage equation similar to our Assumption 3, but where the first-stage effect of only one sector explicitly appears. Under Assumption 3, it is difficult to rationalize Equations (2.8) and (2.9). Indeed, under Assumption 3, Equations (2.6) and (2.8) imply that

\[
u_{s,g} = \Delta D_g(0) - \mu^D + \sum_{s' \neq s} Q_{s',g} \Delta Z_{s'} \beta_{s',g},\]

so

\[
E(Q_{s,g} u_{s,g} \alpha_g) = E(Q_{s,g} (\Delta D_g(0) - \mu^D) \alpha_g) + \sum_{s' \neq s} E(Q_{s,g} Q_{s',g} \alpha_g \beta_{s',g}) \Delta Z_{s'}.\]

13
Then, Equation (2.9) is hard to rationalize. For instance, if for all \((s, g)\) \(\Delta Z_s > 0\), \(\beta_{s,g} > 0\), \(\alpha_g > 0\), and \(Q_{s,g} > 0\), then \(\sum_{s' \neq s} E(Q_{s,g}Q_{s',g}\alpha_g\beta_{s',g})\Delta Z_{s'} > 0\), so Equation (2.9) can only hold if the first and second terms in the right-hand-side of the previous display compensate each other exactly. Overall, whenever the linear first-stage model in Assumption 3 seems plausible, the first-stage assumptions in Goldsmith-Pinkham et al. (2020) are unlikely to hold, and accordingly their decomposition of \(\theta^b\) under heterogeneous effects is also unlikely to hold. Note that heterogeneous effects is not a central issue in Goldsmith-Pinkham et al. (2020). Except for their Proposition 4 and Equation (10), all their other results assume homogeneous effects and do not rest on their Equation (8) and Assumption 3.

2.4 Bartik may still not be robust if one further assumes that shocks are as-good-as randomly assigned

The randomly-assigned-shocks assumption.

Assumption 6 Let \(\mathcal{F} = (\Delta Y_g(0), \Delta D_g(0), \alpha_g, (\beta_{s,g}, Q_{s,g})_{s \in \{1, \ldots, S\}})_{g \in \{1, \ldots, G\}}\).

1. \(E(\Delta Z_s|\mathcal{F}) = E(\Delta Z_s)\).
2. \(E(\Delta Z_s) = 0\) for all \(s\).
3. The shocks \(\Delta Z_s\) are mutually independent across \(s\), conditional on \(\mathcal{F}\).

Point 1 of Assumption 6 requires that shocks be mean independent of locations’ potential outcome (resp. treatment) evolutions without treatment (resp. shocks), and of locations’ shares and first- and second-stage effects. Point 2 requires that all sector-level shocks have the same expectation. Further assuming that this common expectation is equal to zero is without loss of generality: throughout this section we follow Borusyak et al. (2022) and Adão et al. (2019) and assume that shocks have been demeaned. Point 3 requires that shocks are independent across sectors.

Assumption 6 is equivalent to assumptions in Borusyak et al. (2022) and Adão et al. (2019). Points 1 and 2 of Assumption 6 are equivalent to Assumption 4.ii) in Adão et al. (2019) without control variables. Point 3 is identical to their Assumption 2.i). In their Appendix A.1, where they allow for heterogeneous effects, Borusyak et al. (2022) also make an assumption equivalent to Assumption 6.

Testability of the randomly-assigned-shocks assumption. First, Point 1 of Assumption 6 implies that \(E\left(\Delta Z_s|\frac{1}{G}\sum_{s=1}^{S} Q_{s,g}\right) = E(\Delta Z_s)\): shocks should be mean independent of the average share of sector \(s\) across locations. This can be tested, for instance by regressing shocks on the average share of the corresponding sector. Second, Point 2 of
Assumption 6 implies that shocks’ expectation should not vary with industry-level characteristics, which can for instance be tested by regressing the shocks on such characteristics, a similar test as that already proposed by Borusyak et al. (2022). Finally, Assumption 6 implies that \( E(\Delta Z_g | \Delta Y_g(0), \Delta D_g(0), \alpha_g, (\beta_{s,g})_{s \in \{1, \ldots, s\}}) = 0 \). \( E(\Delta Z_g | \Delta Y_g(0), \Delta D_g(0)) = 0 \) is similar in spirit to our Assumptions 2 and 5, but Assumption 6 also implies that the Bartik instrument is mean independent of locations’ first- and second-stage effects. This latter condition can be placebo-tested, by regressing \( \Delta Z_g \) on location-level characteristics likely to be correlated with their first- and second-stage effects.

**Bartik regression without an intercept.** Under the randomly-assigned-shocks assumption, we also consider a second estimand, 

\[
\theta_{NI}^b = \frac{\sum_{g=1}^{G} E(\Delta Y_g \Delta Z_g)}{\sum_{g=1}^{G} E(\Delta D_g \Delta Z_g)},
\]

Equation (2.10) shows that adding the randomly-assigned-shocks assumption is still not enough to ensure that \( \theta^b \) is robust to heterogeneous effects, even if \( \beta_{s,g} \geq 0 \) for all \((s, g)\). If shares sum to 1 for all \( g \), each location must be such that \( Q_{s,g} - Q_{s,} > 0 \) for some \( s \), and \( Q_{s,g} - Q_{s,} < 0 \) for some other \( s \).\(^2\) Sectors such that \( Q_{s,g} - Q_{s,} > 0 \) (resp. \( Q_{s,g} - Q_{s,} < 0 \)) are over-(resp. under-)represented in \( g \). Then, \( \alpha_g \) may be weighted negatively, if the sectors under-represented in \( g \) have a larger value of \( Q_{s,g} \beta_{s,g} \Delta Z_s \times \Delta Z_s \), the first-stage effect of shock \( \Delta Z_s \) in location \( g \) multiplied by \( \Delta Z_s \), than the sectors over-represented in \( g \). On the other hand, Equation (2.12) shows that with randomly-assigned shocks, \( \theta_{NI}^b \) is robust to heterogeneous effects. Thus, under Assumption 6 it may be preferable to estimate \( \theta_{NI}^b \). However, in practice, applied researchers rarely estimate Bartik regressions without an intercept, even when they invoke

\(^2\)Unless \( Q_{s,g} = Q_{s,} \) for all \( s \), in which case \( \alpha_g \) receives a weight of zero in the decomposition.
shocks’ randomness to justify their design. Note that this positive result on $\theta_{NI}'$ relies on the randomly-assigned-shocks assumption: if one instead makes Assumptions 4-5, $\theta_{NI}'$ is no longer robust to heterogeneous effects.

**Connection with the results in Adão et al. (2019) and Borusyak et al. (2022).** In their Proposition 1, Adão et al. (2019) show, under assumptions similar to Assumptions 3 and 6, that the first-stage Bartik regression without an intercept identifies a weighted average of first-stage effects. Equation (2.12) is a straightforward generalization of that result to the 2SLS Bartik regression without an intercept. Adão et al. (2019) argue that omitting the intercept from the regression is without loss of generality, because the treatment evolution in their regression $\Delta \tilde{D}_g$ has been previously demeaned: $\Delta \tilde{D}_g = \Delta D_g - 1/G \sum_{g'=1}^G \Delta D_{g'}$ using our notation. However, the linear first-stage equation underlying their result is not invariant to demeaning. For their Proposition 1 to apply to $\theta^b$, the demeaned treatment should be generated by a first-stage model linear in $\Delta Z_s, Q_{s,g}$, as in their Equation (11), the equivalent of our Assumption 3. However, under Assumption 3,

$$\Delta \tilde{D}_g = \Delta D_g - \frac{1}{G} \sum_{g'=1}^G \Delta D_{g'}$$

$$= \Delta D_g(0) + \sum_{s=1}^S Q_{s,g} \Delta Z_s \beta_{s,g} - \frac{1}{G} \sum_{g'=1}^G \Delta D_{g'}(0) - \frac{1}{G} \sum_{g'=1}^G \sum_{s=1}^S Q_{s,g'} \Delta Z_s \beta_{s,g'}$$

$$= \Delta \tilde{D}_g(0) + \sum_{s=1}^S \Delta Z_s \left( Q_{s,g} \beta_{s,g} - \frac{1}{G} \sum_{g'=1}^G Q_{s,g'} \beta_{s,g'} \right),$$

so the demeaned treatment does not satisfy a first-stage model linear in $\Delta Z_s, Q_{s,g}$, or in $\Delta Z_s \left( Q_{s,g} - \frac{1}{G} \sum_{g'=1}^G Q_{s,g'} \right)$. Therefore, their Proposition 1 applies to the first-stage Bartik regression with an intercept only if one imposes a linear first-stage effect model directly on the demeaned treatment evolution, which may be both unnatural and implausible.\(^3\) The previous discussion also applies to the results in Adão et al. (2019) on

\(^3\)Adão et al. (2019) also motivate their focus on a regression without an intercept by assuming that the shocks have been demeaned. Demeaning the shocks does not ensure that the regression without an intercept and demeaned shocks is equivalent to the regression with an intercept and the original shocks. Indeed, the average Bartik instrument with demeaned shocks is not equal to zero: $\frac{1}{S} \sum_{g=1}^G \sum_{s=1}^S Q_{s,g} \left( \Delta Z_s - 1/S \sum_{s'=1}^S \Delta Z_{s'} \right) \neq 0$. If the shocks are demeaned with respect to the weighted average of shocks $\Delta Z = \sum_{s=1}^S \Delta Z_s \sum_{g=1}^G Q_{s,g}$, then the regression without an intercept and the demeaned shocks $\Delta \tilde{Z}_s = \Delta Z_s - \Delta Z$, is equivalent to the regression with an intercept and the original shocks. However, for Proposition 1 in Adão et al. (2019) to apply to this regression, their assumptions need to hold for the demeaned shocks $\Delta \tilde{Z}_s$. Then, their Assumption 1.ii) requires that $E \left( \Delta \tilde{Z}_s \right) = 0$, or equivalently that $E \left( \Delta \tilde{Z}_s \Delta D_g(0) + \Delta Z \sum_{s=1}^S Q_{s,g} \beta_{s,g} \right) = 0$. This condition is unlikely to hold. For instance, assume that $\Delta D_g(0)$, and $(\beta_{s,g}, Q_{s,g})_{s \in \{1, \ldots, S\}}$ are non-stochastic, and that the shocks $\Delta Z_s$ are iid. Then, the previous condition reduces to $E \left( \Delta \tilde{Z}_s \Delta Z \right) = 0$. This is unlikely to hold, because $\Delta Z$ is a weighted average of the shocks $\Delta Z_s$. For sectors $s$
reduced-form and 2SLS Bartik regressions with heterogeneous effects: those results as well do not apply to regressions with an intercept. When they allow for heterogeneous treatment effects in their Appendix A.1, Borusyak et al. (2022) also consider a Bartik regression without an intercept (see their Equation (A2)). Overall, Theorem 3 shows that with randomly-assigned shocks, Bartik regressions without an intercept are robust to heterogeneous effects, while Bartik regressions with an intercept are not. Then, it may be preferable to estimate the former rather than the latter.

3 Alternative estimators

In this section, we propose alternative estimators to Bartik regressions. They are almost direct applications of the correlated-random-coefficients (CRC) estimator proposed by Chamberlain (1992). They can be used when the data has at least three periods⁴ so we begin by adapting our notation and assumptions to the case where there are multiple time periods indexed by \( t \in \{1, ..., T\} \). Our decompositions of Bartik regressions in Theorems 1-2 readily extend to applications with multiple time periods: to preserve space those extensions are presented in Appendix B.

3.1 Setup and identifying assumptions

Setup. For all \( t \geq 2 \) and any location-level variable \( R_{g,t} \), let \( \Delta R_{g,t} = R_{g,t} - R_{g,t-1} \), and let \( \Delta R_g = (\Delta R_{g,2}, ..., \Delta R_{g,T}) \) be a vector stacking the \( T - 1 \) first differences of \( R \). Let \( \Delta Z_{s,t} \) denote the shock affecting sector \( s \) between periods \( t - 1 \) and \( t \).

Definition 3 Bartik Instrument: The Bartik instrument \( \Delta Z_{g,t} \) is:

\[
\Delta Z_{g,t} = \sum_{s=1}^{S} Q_{s,g} \Delta Z_{s,t}.
\]

With more than two periods, our definition of the Bartik instrument assumes that the shares \( Q_{s,g} \) do not change over time. In practice, applied researchers sometimes use time-varying shares to define the Bartik instrument. Our results readily generalize to time-varying shares.

Potential treatments and outcomes. For any \((\delta_1, ..., \delta_S) \in \mathbb{R}^S\), let \( \Delta D_{g,t}(\delta_1, ..., \delta_S) \) denote the potential treatment evolution that location \( g \) will experience from period \( t-1 \) to \( t \) if \((\Delta Z_{1,t}, ..., \Delta Z_{S,t}) = (\delta_1, ..., \delta_S) \). And let \( \Delta D_{g,t}(0) = \Delta D_{g,t}(0, ..., 0) \) denote the potential treatment evolution that location \( g \) will experience in the absence of any shocks. Finally, that receive a large weight in \( \Delta Z \), when \( \Delta Z < E(\Delta Z) \) it is more likely that \( \Delta Z_s < \Delta Z \). For instance, if \( S = 2 \) and \( Z_1 \) and \( Z_2 \) are distributed as fair dices, \( E(Z_1|Z_1+1/3Z_2=5/3) = 3/2 < 5/3 \).

⁴With two periods, the estimation method we propose cannot be used, but one may then be able to follow a similar estimation strategy as that proposed in Graham & Powell (2012).
let $\Delta Y_{g,t}(d_{g,t})$ denote the potential outcome evolution that location $g$ will experience from period $t - 1$ to $t$ if $\Delta D_{g,t} = d_{g,t}$. The assumptions below generalize Assumptions 3 and 1 to instances with multiple periods.

**Assumption 7** Linear First-Stage Equation: for all $g \in \{1, ..., G\}$, $t \in \{2, ..., T\}$, there exists $(\beta_{s,g,t})_{s \in \{1, ..., S\}}$ such that for any $(\delta_1, ..., \delta_S) \in \mathbb{R}^S$:

$$\Delta D_{g,t}(\delta_1, ..., \delta_S) = \Delta D_{g,t}(0) + \sum_{s=1}^{S} Q_{s,g} \delta_s \beta_{s,g,t}.$$ 

**Assumption 8** Linear Second-Stage Equation: for all $g \in \{1, ..., G\}$, $t \in \{2, ..., T\}$, there exists $\alpha_{g,t}$ such that for any $d_{g,t} \in \mathbb{R}$:

$$\Delta Y_{g,t}(d_{g,t}) = \Delta Y_{g,t}(0) + \alpha_{g,t} d_{g,t}.$$ 

Identifying assumptions underlying our alternative estimators.

**Assumption 9** For all $t \in \{2, ..., T\}$, there are real numbers $\mu_t^D$ such that $\forall g \in \{1, ..., G\}$, $E(\Delta D_{g,t}(0)|\Delta Z_g) = \mu_t^D$.

**Assumption 10** For all $t \in \{2, ..., T\}$, there are real numbers $\mu_t^Y$ such that $\forall g \in \{1, ..., G\}$, $E(\Delta Y_{g,t}(\Delta D_{g,t}(0))|\Delta Z_g) = \mu_t^Y$.

Interpreting our alternative estimators’ identifying assumptions. Assumptions 9 and 10 require that locations’ potential treatment and outcome evolution without any shock be mean-independent of the full sequence of their Bartik instruments. Assumptions 9 (resp. 10) strengthens the first point of Assumption 5 (resp. 4), by requiring mean independence rather than zero-correlation, and by requiring that $\Delta D_{g,t}(0)$ (resp. $\Delta Y_{g,t}(\Delta D_{g,t}(0))$) be also unrelated with the Bartik instrument at other dates than $t$.

Testability of the assumptions underlying our alternative estimators. Assume that the data contains a period $t_0 \in \{2, ..., T\}$ where $\Delta Z_{s,t_0} = 0$ for all $s$, meaning that no shocks arise. Then, $\Delta D_{g,t_0}(0)$ and $\Delta Y_{g,t_0}(\Delta D_{g,t_0}(0))$ are observed, and Assumptions 9 and 10 respectively have the following testable implications:

$$E(\Delta D_{g,t_0}|\Delta Z_g) = E(\Delta D_{g,t_0}) \quad (3.1)$$

$$E(\Delta Y_{g,t_0}|\Delta Z_g) = E(\Delta Y_{g,t_0}). \quad (3.2)$$

To test Equation (3.1) (resp. (3.2)), one can for instance regress $\Delta D_{g,t_0}$ (resp. $\Delta Y_{g,t_0}$) on $\Delta Z_{g,t'}$ for any $t' \neq t_0$, which is the placebo test of Assumption 5 (resp. 4) we suggested in Section 2.3. One could also regress $\Delta D_{g,t_0}$ (resp. $\Delta Y_{g,t_0}$) on a polynomial in $(\Delta Z_{g,2}, ..., \Delta Z_{g,t_0-1}, \Delta Z_{g,t_0+1}, ..., \Delta Z_{g,T})$, to take advantage of the fact mean-independence has stronger implications than just linear-independence.
3.2 Estimator robust to heterogeneous effects across locations

Restrictions on first- and second-stage effects underlying our first estimator.

Assumption 11 For all \( g \in \{1, \ldots, G\} \), there exists \( \beta_g \) and \( \alpha_g \) such that \( \beta_{s,g,t} = \beta_g \) for every \( s \in \{1, \ldots, S\} \) and \( t \in \{2, \ldots, T\} \) and \( \alpha_{g,t} = \alpha_g \) for every \( t \in \{2, \ldots, T\} \).

Assumption 11 requires that first-stage effects be constant across sectors and over time, and that treatment effects be constant over time. Assumption 11 undoubtedly imposes strong restrictions on effects heterogeneity. Under Assumption 11, for every \( g \in \{1, \ldots, G\} \), let \( \gamma_g = \alpha_g \beta_g \) denote the reduced-form effect of the instrument on the outcome.

Identification result. Let

\[
\begin{align*}
\mu^D &= (\mu_2^D, \ldots, \mu_T^D)' \\
\mu^Y &= (\mu_2^Y, \ldots, \mu_T^Y)'
\end{align*}
\]

be \((T-1) \times 1\) vectors stacking together the common trends affecting the treatment and the outcome, defined in Assumptions 9 and 10. For every \( g \in \{1, \ldots, G\} \), let

\[
M_g = I - \frac{1}{\Delta Z'_g \Delta Z_g} \Delta Z_g \Delta Z'_g,
\]

where \( I \) denotes the \((T-1) \times (T-1)\) identity matrix.

**Theorem 4** Suppose that Assumptions 7, 8, 9, 10, and 11 hold, \( E\left(\frac{1}{G} \sum_{g=1}^{G} M'_g M_g\right) \) is invertible, and with probability 1 \( \Delta Z_g \neq 0 \) for every \( g \in \{1, \ldots, G\} \). Then:

\[
\begin{align*}
\mu^D &= E\left(\frac{1}{G} \sum_{g=1}^{G} M'_g M_g\right)^{-1} E\left(\frac{1}{G} \sum_{g=1}^{G} M'_g M_g \Delta D_g\right) \\
\mu^Y &= E\left(\frac{1}{G} \sum_{g=1}^{G} M'_g M_g\right)^{-1} E\left(\frac{1}{G} \sum_{g=1}^{G} M'_g M_g \Delta Y_g\right)
\end{align*}
\]

\[
E\left(\frac{1}{G} \sum_{g=1}^{G} \beta_g\right) = E\left(\frac{1}{G} \sum_{g=1}^{G} \frac{\Delta Z'_g (\Delta D_g - \mu^D)}{\Delta Z'_g \Delta Z_g}\right)
\]

\[
E\left(\frac{1}{G} \sum_{g=1}^{G} \gamma_g\right) = E\left(\frac{1}{G} \sum_{g=1}^{G} \frac{\Delta Z'_g (\Delta Y_g - \mu^Y)}{\Delta Z'_g \Delta Z_g}\right)
\]

\[
E\left(\frac{1}{G} \sum_{g=1}^{G} \frac{\beta_g}{\sum_{g=1}^{G} \beta_g} \alpha_g\right) = \frac{E\left(\frac{1}{G} \sum_{g=1}^{G} \frac{\Delta Z'_g (\Delta Y_g - \mu^Y)}{\Delta Z'_g \Delta Z_g}\right)}{E\left(\frac{1}{G} \sum_{g=1}^{G} \frac{\Delta Z'_g (\Delta D_g - \mu^D)}{\Delta Z'_g \Delta Z_g}\right)}.\]

**Outline of the proof of Theorem 4.** Under Assumptions 7, 9, and 11,

\[
E(\Delta D_g | \Delta Z_g) = \mu^D + E(\beta_g | \Delta Z_g) \Delta Z_g,
\]

(3.8)
an equation that is additively separable in the location-specific coefficient \( E(\beta_g | \Delta Z_g) \) and the common trends \( \mu^D \), and thus falls into the class of semi-parametric models studied in Chamberlain (1992). Then, identification of \( E \left( \frac{1}{G} \sum_{g=1}^{G} \beta_g \right) \) follows from the same steps as in Chamberlain (1992). Identification of \( E \left( \frac{1}{G} \sum_{g=1}^{G} \gamma_g \right) \) follows similarly. Finally, Equation (3.7) directly follows from Equations (3.5) and (3.6) and the definition of \( \gamma_g \).

**Intuition of the correlated-random-coefficients estimator.** Notice that

\[
\frac{\Delta Z^*_g (\Delta D_g - \mu^D)}{\Delta Z^*_g \Delta Z_g}
\]

is the coefficient of \( \Delta Z_{g,t} \) in the regression, within group \( g \), of \( \Delta D_{g,t} - \mu^D_t \) on \( \Delta Z_{g,t} \) without an intercept. Accordingly, once the common trends \( \mu^D_t \) have been identified, Chamberlain’s estimator of the average first-stage effect \( E \left( \frac{1}{G} \sum_{g=1}^{G} \beta_g \right) \) amounts to regressing \( \Delta D_{g,t} - \mu^D_t \) on \( \Delta Z_{g,t} \) without an intercept in every group, and then averaging those coefficients across groups.

**Interpretation of the causal effect identified, and comparison with Bartik regressions.** Under the assumption that all first-stage effects are positive,

\[
\alpha_w = E \left( \sum_{g=1}^{G} \frac{\beta_g}{\sum_{g=1}^{G} \beta_g} \alpha_g \right)
\]

is a weighted average of location-specific second stage effects, that gives a higher weight to locations with a higher first-stage effect. Therefore, Equation (3.7) implies that under Assumption 11 and if \( \beta_g \geq 0 \), our correlated-random-coefficient estimator is robust to heterogeneous effects. Point 4 of Theorem B.1 in the Web Appendix shows that under the same assumptions, the Bartik coefficient \( \theta^b \) may not be robust.

**Computation of the estimators in Theorem 4.** Operationally, estimators of \( \mu^D \), \( \mu^Y \), \( E \left( \frac{1}{G} \sum_{g=1}^{G} \beta_g \right) \), and \( E \left( \frac{1}{G} \sum_{g=1}^{G} \gamma_g \right) \) can be computed using the Generalized Method of Moments (GMM). Indeed, one has

\[
E \left( \frac{1}{G} \sum_{g=1}^{G} M_g (\Delta D_g - \mu^D) \right) = 0
\]

\[
E \left( \frac{1}{G} \sum_{g=1}^{G} M_g (\Delta Y_g - \mu^Y) \right) = 0
\]

\[
E \left( \frac{1}{G} \sum_{g=1}^{G} \frac{\Delta Z^*_g (\Delta D_g - \mu^D)}{\Delta Z^*_g \Delta Z_g} \right) - E \left( \frac{1}{G} \sum_{g=1}^{G} \beta_g \right) = 0
\]

\[
E \left( \frac{1}{G} \sum_{g=1}^{G} \frac{\Delta Z^*_g (\Delta Y_g - \mu^Y)}{\Delta Z^*_g \Delta Z_g} \right) - E \left( \frac{1}{G} \sum_{g=1}^{G} \gamma_g \right) = 0,
\] (3.9)
a just-identified system with $2T$ moment conditions and $2T$ parameters.\(^5\)

**Inference.** We suggest a method to draw inference on $\alpha_w$ under Assumption 12.

**Assumption 12** Conditional on $(\Delta Z_{s,t})_{(s,t)\in\{1,...,S\}\times\{2,...,T\}}$, the vectors $(\Delta Z_g, \Delta D_g, \Delta Y_g)$ are iid across $g$.

Assumption 12 requires that conditional on shocks, locations’ variables be iid. Therefore, our inference method is compatible with the fixed-shocks approach, not with the fixed-locations approach. If the treatment and outcome are also influenced by unobserved industry-level shocks, as hypothesized in Borusyak et al. (2022) and Adão et al. (2019), such unobserved shocks also need to be conditioned upon for Assumption 12 to be plausible. Under Assumption 12, to perform inference on $\alpha_w$ conditional on the shocks, one may use the heteroskedasticity-robust standard errors attached to the GMM system in (3.9). The entire time series of each location enters in the system’s $2T$ moment conditions, so those heteroskedasticity-robust standard errors do not assume that the vectors $(\Delta Y_{g,t}, \Delta D_{g,t}, \Delta Z_{g,t})$ are independent across $t$: they only require that conditional on the shocks, the vectors $(\Delta Z_g, \Delta D_g, \Delta Y_g)$ are iid across $g$, as stated in Assumption 12. Those standard errors do not account for the variance arising from the shocks. Accounting for it would require extending the approach in Adão et al. (2019) to the estimators in Theorem 4, without making the randomly-assigned-shocks assumption. This important extension is left for future work. Similarly, in our applications, to draw inference on Bartik regression coefficients we use standard errors clustered at the location level: those do not account for the variance arising from the shocks, but they give valid estimators of the coefficients’ standard errors conditional on the shocks under Assumption 12.

### 3.3 Estimator robust to heterogeneous effects across locations and over time?

**Restrictions on first- and second-stage effects underlying our second estimator.**

**Assumption 13** For all $g \in \{1,...,G\}$ and $t \in \{2,...,T\}$, there exists $\beta_g$, $\alpha_g$, $\lambda^D_t$ and $\lambda^Y_t$ such that $\beta_{s,g,t} = \beta_g + \lambda^D_t$ for every $s \in \{1,...,S\}$, and $\alpha_{g,t} = \alpha_g + \lambda^Y_t$.

$\lambda^D_t$ and $\lambda^Y_t$ respectively represent the change in the first- and second-stage effects from $t-1$ to $t$, which is assumed to be constant across locations. Accordingly, Assumption

\(^5\)All the moment conditions in (3.9) actually hold conditional on $(\Delta Z_g)_{g\in\{1,...,G\}}$, thus implying that the parameters could be estimated using conditional rather than unconditional GMM. Applying results in Chamberlain (1992), one can derive the optimal estimator of $\mu^D$ and $\mu^Y$ attached to the first two equations in (3.9). An issue, however, is that Chamberlain’s optimality results do not apply to the estimators of $E\left(\frac{1}{G} \sum_{g=1}^{G} \beta_g\right)$ and $E\left(\frac{1}{G} \sum_{g=1}^{G} \gamma_g\right)$, the building blocks of our target parameter. Moreover, the computation of the optimal estimator requires a non-parametric first-stage estimation. To our knowledge, no data-driven method has been proposed to choose the tuning parameters involved in this first stage. Accordingly, we prefer to stick with the unconditional GMM estimator above.
maybe be interpreted as a parallel trends assumption on the first- and second-stage effects. Without loss of generality, we can assume that \( \lambda_d^g = \lambda_y^g = 0 \).

### Identifying assumption underlying our second alternative estimator.

Our second alternative estimator relies on a slightly stronger assumption than Assumption 10, which requires that \( \Delta Y_{g,t}(\Delta D_{g,t}(0)) \) be mean independent of both \( \Delta Z_g \) and \( \beta_g \):

**Assumption 14** For all \( t \in \{2, ..., T\} \), there are real numbers \( \mu_i^g \) such that \( \forall g \in \{1, ..., G\} \), \( E(\Delta Y_{g,t}(\Delta D_{g,t}(0))|\Delta Z_g, \beta_g) = \mu_i^g \).

### Identification result.

Let \( \theta^D = (\mu_2^g, \mu_3^g, \lambda_3^g, ..., \mu_T^g, \lambda_T^g) \) and \( \theta^Y = (\mu_2^g, \mu_3^g, \lambda_3^g, ..., \mu_T^g, \lambda_T^g) \), let \( 0_k \) denote a vector of \( k \) zeros, let \( \Delta \tilde{Z}_{g,t} = \Delta Z_{g,t}(\beta_g + \lambda_T^g) \), and let

\[
P_g = \begin{pmatrix}
1, 0_{2T-4} \\
0, 1, \Delta Z_{g,3}, 0_{2T-6} \\
0, 1, \Delta Z_{g,4}, 0_{2T-8} \\
\vdots \\
0, 1, \Delta Z_{g,T}
\end{pmatrix}
\]

let

\[
\tilde{M}_g = I - \frac{1}{\Delta \tilde{Z}_g \Delta \tilde{Z}_g'} \Delta \tilde{Z}_g \Delta \tilde{Z}_g'
\]

and let

\[
\tilde{P}_g = \begin{pmatrix}
1, 0_{2T-4} \\
0, 1, \Delta \tilde{Z}_{g,3}, 0_{2T-6} \\
0, 1, \Delta \tilde{Z}_{g,4}, 0_{2T-8} \\
\vdots \\
0, 1, \Delta \tilde{Z}_{g,T}
\end{pmatrix}
\]

**Theorem 5** Suppose that Assumptions 7, 8, 9, 13, and 14 hold, \( E\left(\frac{1}{G} \sum_{g=1}^{G} P_g' M_g P_g\right) \) and \( E\left(\frac{1}{G} \sum_{g=1}^{G} \tilde{P}_g' \tilde{M}_g \tilde{P}_g\right) \) are invertible, and with probability 1 \( \Delta Z_g \neq 0 \) and \( \Delta \tilde{Z}_g \neq 0 \) for every \( g \in \{1, ..., G\} \). Then:

\[
\theta^D = E\left(\frac{1}{G} \sum_{g=1}^{G} P_g' M_g P_g\right)^{-1} E\left(\frac{1}{G} \sum_{g=1}^{G} P_g' M_g \Delta D_g\right) \quad (3.10)
\]

\[
E(\beta_g) = E\left(\frac{\Delta Z_g' (\Delta D_g - P_g \theta^D)}{\Delta Z_g' \Delta Z_g}\right) \quad (3.11)
\]

\[
\theta^Y = E\left(\frac{1}{G} \sum_{g=1}^{G} P_g' M_g \Delta Y_g\right)^{-1} E\left(\frac{1}{G} \sum_{g=1}^{G} P_g' M_g \Delta Y_g\right) \quad (3.12)
\]

\[
E\left(\frac{1}{G} \sum_{g=1}^{G} \alpha_g\right) = E\left(\frac{1}{G} \sum_{g=1}^{G} \frac{\Delta \tilde{Z}_g' (\Delta Y_g - \tilde{P}_g \theta^Y)}{\Delta \tilde{Z}_g' \Delta \tilde{Z}_g}\right) \quad (3.13)
\]
Outline of the proof of Theorem 5. Identification of $\theta^D$ and $\beta_g$ follows from similar steps as the identification of $\mu^D$ and $\beta_g$ in Theorem 4. Once $\theta^D$ and $\beta_g$ are identified, $\Delta \tilde{Z}_{g,t}$ is identified. Then, under Assumptions 7, 8, 9, 13, and 14,

$$E(\Delta Y_{g,t}|\Delta \tilde{Z}_g) = \mu^Y_t + \lambda^Y_t \Delta \tilde{Z}_{g,t} + E(\alpha_g|\Delta \tilde{Z}_g)\Delta \tilde{Z}_{g,t}.$$  

Therefore, 

$$E(\Delta Y_g|\Delta Z_g) = \tilde{P}_g \theta^Y + E(\alpha_g|\Delta \tilde{Z}_g)\Delta \tilde{Z}_g,$$  

(3.14)

an equation again additively separable in the location-specific coefficient $E(\alpha_g|\Delta \tilde{Z}_g)$ and the common trends $\theta^Y$, so identification follows from Chamberlain (1992).

Interpretation of the causal effect identified. Let 

$$\alpha_{ate} \equiv \frac{1}{T-1} \sum_{t=2}^{T} \frac{1}{G} \sum_{g=1}^{G} (\alpha_g + \lambda^Y_t)$$  

(3.15)

denote the average second-stage effect across all periods and locations. Once $\frac{1}{G} \sum_{g=1}^{G} \alpha_g$ and $\theta^Y$ are identified, $\alpha_{ate}$ is also identified.

Estimation challenges. To estimate $\alpha_{ate}$, one can proceed as follows:

1. Compute $\hat{\theta}^D = \left( \frac{1}{G} \sum_{g=1}^{G} P'_g M_g P_g \right)^{-1} \left( \frac{1}{G} \sum_{g=1}^{G} P'_g M_g \Delta D_g \right)$.

2. Compute $\hat{\beta}_g = \frac{\Delta \hat{Z}'_g(\Delta D_g - \hat{P}_g \hat{\theta}^D)}{\Delta \hat{Z}_g \Delta \hat{Z}_g}$.

3. Compute $\Delta \hat{Z}_{g,t} = \Delta Z_{g,t}(\hat{\beta}_g + \hat{\lambda}_t^D)$ and define $\hat{P}_g$ and $\hat{M}_g$ accordingly.

4. Compute $\hat{\theta}^Y = \left( \frac{1}{G} \sum_{g=1}^{G} \hat{P}'_g \hat{M}_g \hat{P}_g \right)^{-1} \left( \frac{1}{G} \sum_{g=1}^{G} \hat{P}'_g \hat{M}_g \Delta Y_g \right)$.

5. Finally, compute $\hat{\alpha}_g = \frac{\Delta \hat{Z}'_g(\Delta Y_g - \hat{P}_g \hat{\theta}^Y)}{\Delta \hat{Z}_g \Delta \hat{Z}_g}$.

In this estimation procedure, $\beta_g$ are incidental parameters that need to be estimated in a first step so as to be able to estimate $\alpha_{ate}$. If $T$ is fixed and the number of locations $G$ goes to infinity, the estimators $\hat{\beta}_g$ are inconsistent. Accordingly, $\hat{\alpha}_{ate}$ is also inconsistent. It is only if both $T$ and $G$ go to infinity that $\hat{\alpha}_{ate}$ may be consistent. However, Bartik applications typically have a low number of time periods. Moreover, we found an example where the sign of the bias of $\hat{\alpha}_{ate}$ changes with $T$, thus implying that automated bias-reduction methods to deal with incidental-parameter problems in short panels, like the split-panel jacknife method proposed by Dhaene & Jochmans (2015), cannot be used here. No previous paper seems to have characterized analytically the asymptotic bias of an
estimator nesting ours, which would then allow us to correct that bias. An analytic characterization of the asymptotic bias of $\hat{\alpha}_{LTE}$ when $T$ is fixed and the number of locations $G$ goes to infinity goes beyond the scope of this paper and is left for future work.

A third alternative estimator allowing for time-varying first- and second-stage effects, but assuming homogeneous first-stage effects across CZs. If one wants to allow for time-varying effects while avoiding the incidental-parameters problem affecting $\hat{\alpha}_{LTE}$, one may assume that $\beta_{s,g,t} = \beta + \lambda^D_t$, and $\alpha_{g,t} = \alpha_g + \lambda^Y_t$, meaning that the first-stage effect does not vary across CZs. This gives rise to a CRC estimator very similar to that in Theorem 5, replacing Equation (3.11) by

$$\beta = E \left( \frac{1}{G} \sum_{g=1}^{G} \frac{\Delta Z_g' \left( \Delta D_g - P_g \theta^D \right)}{\Delta Z' \Delta Z_g} \right),$$

and redefining $\Delta \tilde{Z}_{g,t}$ as $\Delta Z_{g,t}(\beta + \lambda^D_t)$. The resulting estimator is a standard GMM estimator, that does not suffer from an incidental-parameters problem and is consistent when $T$ is fixed and $G$ goes to infinity. Its asymptotic variance can be estimated using Stata or R’s GMM command.

4 Empirical application: China shock

In this section, we revisit Autor et al. (2013), who use the Bartik instrument to estimate the effects of exposure to Chinese imports on manufacturing employment in the US. We also revisit the canonical application in Bartik (1991) in Section A of our Web Appendix.

4.1 Data

We use the replication dataset of Autor et al. (2013). They use a commuting-zone (CZ) panel data set, with 722 CZs and 3 periods (1990, 2000, and 2007). The outcome variable $\Delta Y_{g,t}$ is the change in the manufacturing employment share of the working age population in CZ $g$ between two consecutive periods. The treatment variable $\Delta D_{g,t}$ is the change in per-worker Chinese imports in CZ $g$ between two consecutive periods. The sectoral shocks $\Delta Z_{s,t}$ are the change in per-worker imports from China to other high-income countries in sector $s$, for 397 manufacturing sectors. The shares $Q_{s,g}$ are the employment shares of sector $s$ in location $g$, and the shares are lagged by 1 period.\footnote{We use the same data as Autor et al. (2013).}

\footnote{We use the same data as Autor et al. (2013).}

The two closest papers are Fernández-Val & Vella (2011) and Fernández-Val & Weidner (2016). Fernández-Val & Vella (2011) consider linear estimators with incidental first-step estimates, but they do not allow for time fixed effects. Fernández-Val & Weidner (2016) allow for both unit and time fixed effects, but they do not allow for a first-step estimation.

\footnote{The shares do not sum up to 1 in each location, because there is employment in non-manufacturing sectors as well. Under their random-shocks assumption, Borusyak et al. (2022) argue that it is important to control for the sum of shares when it is not always equal to 1 (see their section 4.2). Whether shares do not sum up to 1 affects the estimates, but not the interpretation of the results.}

\footnote{The shares do not sum up to 1 in each location, because there is employment in non-manufacturing sectors as well. Under their random-shocks assumption, Borusyak et al. (2022) argue that it is important to control for the sum of shares when it is not always equal to 1 (see their section 4.2). Whether shares do not sum up to 1 affects the estimates, but not the interpretation of the results.}
variable definitions as Borusyak et al. (2022). The replication dataset of Autor et al. (2013) does not contain the shock and share variables. We obtained those variables from the replication dataset of Borusyak et al. (2022).

4.2 Tests of the identifying assumptions

4.2.1 The randomly-assigned shocks assumption is rejected

Shocks are correlated to sectors’ average share. Point 1 of Assumption 6 implies that $E \left( \Delta Z_s \mid \sum_{s=1}^{S} Q_{s,g} \right) = E (\Delta Z_s)$: shock should be mean independent of the average share of sector $s$ across locations. We test this by regressing shocks on $\frac{1}{G} \sum_{s=1}^{S} Q_{s,g}$ in Panel A of Table 1. In Column (1), we regress the shocks from 1990 to 2000 on the sector’s average share; in Column (2) we regress the shocks from 2000 to 2007 on the sector’s average share. We follow Table 3 Panel A in Borusyak et al. (2022), and cluster standard errors at the level of three-digit SIC codes, but results are very similar when one uses robust standard errors. In Columns (1) and (2), we strongly reject the null, with t-stats respectively equal to -1.96 and -4.43: large shocks are significantly more likely to arise in sectors with a lower average share.

Shocks are correlated to sectors’ characteristics. Point 2 of Assumption 6 implies that shocks’ expectation should not vary with sector-level characteristics. We test this by regressing the shocks on such characteristics. We use the five sector characteristics in Acemoglu et al. (2016) that are in the replication dataset of Borusyak et al. (2022). Panel B of Table 1 show regressions of sectoral shocks from 1990 to 2000 and from 2000 to 2007 on these characteristics. We follow Table 3 Panel A in Borusyak et al. (2022) and weight the regressions by the average sector shares, but the results are very similar when the regressions are not weighted. We find that large import shocks tend to appear in sectors with low wages and more high-tech equipment investment, and we can reject the hypothesis that the import shocks are not correlated with any sector characteristic (p-value < 0.001 in Column (1), p-value = 0.017 in Column (2)).

Comparison with the test of Assumption 6 in Borusyak et al. (2022). Our second test of Assumption 6 is inspired from and closely related to that in Table 3 Panel A in Borusyak et al. (2022). The only difference is that they separately regress each sector characteristic on the shocks, while we regress the shocks on all the sector characteristics. Borusyak et al. (2022) find no significant correlation between sectors’ characteristics and shocks. Reverting the dependent and the independent variables in their Table 3 Panel A sum to one or not does not seem to affect the robustness of Bartik regressions to heterogeneous effects: our Theorem 3, derived under their random-shocks assumption, holds irrespective of whether shares sum to 1.

The shares are time-varying in Autor et al. (2013), and we use the sector’s average share in the corresponding time period as the independent variable in the analysis.
would leave their t-stats unchanged, so the difference between our and their results is that
they regress the shocks on each characteristic individually, while we regress the shocks
jointly on the five characteristics. It follows from standard formulas relating coefficients
in short- and long- OLS regressions that the null in our test is stronger than the null in
their test: if the coefficients of all characteristics are equal to zero in our long regression,
then the coefficients of all characteristics are equal to zero in their short regressions. The
fact that we test a stronger implication of Assumption 6 may explain why our test is
rejected while theirs is not.

| Variables                                                                 | (1) | (2)       |
|--------------------------------------------------------------------------|-----|-----------|
| ∆Z_{s,t}: 1990-2000                                                      | -567.488 | -3,798.060 |
| ∆Z_{s,t}: 2000-2007                                                      | (289.280) | (857.583) |
| Sector’s average share across commuting zones in the Bartik instrument in Autor et al. (2013). |
| Observations                                                             | 397 | 397 |

Panel A: Shocks uncorrelated to sectors’ average share?

Panel B: Shocks uncorrelated to sectors’ characteristics?

| Variables                                                                 | (1) | (2)       |
|--------------------------------------------------------------------------|-----|-----------|
| Production workers’ share of employment_{1991}                           | 0.178 | 1.528     |
| Ratio of capital to value-added_{1991}                                   | 0.209 | 8.389     |
| Log real wage (2007 USD)_{1991}                                          | -8.946 | -11.514   |
| Computer investment as share of total investment_{1990}                  | 0.103 | 1.149     |
| High-tech equipment as share of total investment_{1990}                  | 0.299 | 1.095     |
| F-test P-value                                                           | 0.0000 | 0.0172    |
| Observations                                                             | 397 | 397 |

Notes: The table shows regressions of the sector-level change in per-worker imports from China to
other high-income countries on a set of sectors’ characteristics. The dependent variable in Column (1)
(resp. (2)) is the change in per-worker imports from China to other high-income countries from 1990 to
2000 (resp. from 2000 to 2007). In Panel A, the independent variable is sectors’ average share across
commuting zones in the Bartik instrument in Autor et al. (2013). In Panel B, the independent variables
are five sector characteristics obtained from Acemoglu et al. (2016): sectors’ share of production workers
in employment in 1991, sectors’ ratios of capital to value-added in 1991, sectors’ log real wages in 1991,
sectors’ share of investment devoted to computers in 1990, and sectors’ share of high-tech equipment in
total investment in 1990. Standard errors clustered at the level of three-digit SIC codes are shown in
parentheses. The regressions in Panel B are weighted by the average sector exposure shares. The F-test
p-value in Panel B is the p-value of the joint test that all the coefficients of the sector characteristics are
equal to 0.
4.2.2 Assumptions 4 and 10 are not rejected

Testing procedure. As discussed in Autor et al. (2013), the growth in Chinese imports to the US was very small prior to 1990. Accordingly, 1990 and prior periods satisfy, or nearly satisfy, \( \Delta Z_{s,t} = 0 \) for every sector. In their Table 2, Autor et al. (2013) leverage this to implement a placebo test of their Bartik regression. They average the 1990-to-2000 and 2000-to-2007 Bartik instrument of each CZ, and then estimate a 2SLS regression of 1970-to-1980 and 1980-to-1990 manufacturing employment growths on 1990-to-2000 and 2000-to-2007 Chinese import exposure growth, using the average Bartik as the instrument. Instead, in Table 2 we test Equation (3.2) directly, thus giving us a test of Assumption 10, and a placebo test of Assumption 4. In Column (1) (resp. (2)), we regress CZs 1970-to-1980 and 1980-to-1990 manufacturing employment growths on their 1990-to-2000 (resp. 2000-to-2007) Bartik instrument. Unlike Autor et al. (2013), we do not weight the regression by CZ’s population, but results remain similar with weighting. We also cluster our standard errors at the CZ rather than at the state level, but results are similar if we cluster at the state level. Finally, we do not average the 1990-to-2000 and 2000-to-2007 instruments, but again results are similar if we instead use the average of the 1990-to-2000 and 2000-to-2007 instruments.

Test’s results. Table 2 shows that the placebo reduced-form coefficients of the 1990-to-2000 and 2000-2007 Bartik instruments are small, and insignificant at the 5% level. Confidence intervals are relatively small, so we can reject pre-trends much smaller than the actual reduced-form effect shown in Column (2) of Table 3 below, so the potential issue with pre-trends tests highlighted in Roth (2022) seems relatively mild here. Overall, Assumptions 4 and 10 seem plausible in this application. Unfortunately, we are unable to implement a similar test of Assumptions 5 and 9, because trade data with China is unavailable before 1990, as explained by Autor et al. (2013).

Robustness checks. Equation (3.2) requires mean-independence between \( \Delta Y_{g,t}, t \in \{1980, 1990\} \) and \( (\Delta Z_{g,2000}, \Delta Z_{g,2007}) \), which is stronger than just linear-independence between \( \Delta Y_{g,t}, t \in \{1980, 1990\} \) and each element of the vector. To investigate if stronger implications of Equation (3.2) may be rejected, we regress \( \Delta Y_{g,t}, t \in \{1980, 1990\} \) on polynomials of orders 1 and 2 in \( (\Delta Z_{g,2000}, \Delta Z_{g,2007}) \), and run F-tests of the joint nullity of the coefficients of \( (\Delta Z_{g,2000}, \Delta Z_{g,2007}) \). These F-tests’ p-values are respectively equal to 0.183 and 0.264: stronger testable implications of Equation (3.2) are not rejected.
Table 2: Testing Assumptions 4 and 10

| Dependent Variable | Column (1) | Column (2) |
|--------------------|------------|------------|
| $\Delta Z_{g,2000}$ | 0.038      | 0.053      |
| (0.077)            | (0.031)    |
| $\Delta Z_{g,2007}$ |            |            |
| Observations       | 1,444      | 1,444      |

Notes: The table reports estimates of regressions using a US commuting-zone (CZ) level panel data set with five periods, 1970, 1980, 1990, 2000, and 2007. The dependent variable is the change of the manufacturing employment share in CZ $g$, from 1970 to 1980 and from 1980 to 1990. In Column (1) (resp. (2)), the independent variables are the 1990-to-2000 (resp. 2000-to-2007) Bartik instrument of each CZ, and an indicator equal to 1 if the independent variable is measured from 1980 to 1990. The construction of the Bartik instrument is detailed in the text. Standard errors clustered at the CZ level are shown in parentheses. All regressions are unweighted.

4.3 Results

4.3.1 Bartik regressions

Columns (1) to (3) of Table 3 below show the results of the Bartik first-stage, reduced-form, and 2SLS regressions. In Column (1), the first-stage coefficient is 0.867. In Column (2), the reduced form coefficient is -0.539. Finally, in Column (3), the 2SLS coefficient is -0.622. Robust standard errors clustered at the CZ level are shown between parentheses. All coefficients are statistically significant. The 2SLS coefficient slightly differs from that in Table 2 Column (3) in Autor et al. (2013), because our 2SLS regression is not weighted by CZ’s population. This is just to be consistent with Sections 2.1 to 3, where we consider unweighted Bartik regressions. Column (4) shows $\hat{\theta}_{NI}^b$, the Bartik 2SLS coefficient without an intercept and with demeaned shocks, which is robust to heterogeneous effects under the randomly-assigned-shocks assumption (see Theorem 3). $\hat{\theta}_{NI}^b$ is much more negative than $\hat{\theta}^b$, insignificantly different from zero, and very noisy, perhaps reflecting the fact that the randomly-assigned-shocks assumption does not hold in this application.
Table 3: Effects of the China shock on US employment using the data in Autor et al. (2013)

| Dependent Variable | FS (1) | RF (2) | 2SLS (3) | 2SLS, no intercept (4) | Chamberlain (5) |
|--------------------|--------|--------|----------|-----------------------|-----------------|
| Bartik Instrument  | ∆Dg,t  | ∆Yg,t  | ∆Yg,t    | ∆Yg,t                 |                 |
| ∆Dg,t              |        |        |          |                       |                 |
| Observations       | 1,444  | 1,444  | 1,444    | 1,444                 | 722             |

Notes: Columns (1) to (3) respectively report estimates of the first-stage, reduced-form, and 2SLS Bartik regressions with period fixed effects, using a US commuting-zone (CZ) level panel data set with T = 3 periods, 1990, 2000, and 2007. ∆Yg,t is the change of the manufacturing employment share in CZ g, from 1990 to 2000 for t = 2000, and from 2000 to 2007 for t = 2007. ∆Dg,t is the change in exposure to Chinese imports in CZ g from 1990 to 2000 for t = 2000, and from 2000 to 2007 for t = 2007. ∆Zg,t is the Bartik instrument, whose construction is detailed in the text. Column (4) shows the 2SLS coefficient in a regression with no period fixed effects and no intercept. Standard errors clustered at the CZ level are shown in parentheses. All regressions are unweighted. Column (5) reports an alternative estimate of the second-stage effect, using the GMM Stata command, and with the system of moment conditions in Equation (3.9). The 2T = 6 moment conditions have one observation per CZ, hence the number of observations. Heteroskedasticity-robust standard errors are shown in parentheses.

4.3.2 Decompositions of the 2SLS Bartik regression

Decomposition under Assumptions 8 and B.1. We follow Theorem B.1 in the Web Appendix, a straightforward generalization of Theorem 1 to more than two periods, to estimate the weights attached to the Bartik 2SLS regression under Assumptions 8 and B.1 (the last assumption is a generalization of Assumption 2 to more than two periods). Column (1) of Table 4 shows that under those assumptions, θb estimates a weighted sum of 1,444 effects a_g,t, where 571 weights are positive, 873 weights are strictly negative, and where negative weights sum to −0.161. Therefore, θb is not robust to heterogeneous effects. The weights are negatively correlated with CZs’ shares of college-educated workers in 1990 (correlation= −0.068, p-value< 0.01), so θb may differ from E(1/T\sum a_g,t), if the effects a_g,t are correlated with CZs’ educational levels.

Decompositions under Assumptions 7, 8, B.3, and B.2. We follow Theorem B.2 in the Web Appendix, a straightforward generalization of Theorem 2 to applications with more than two periods, to estimate the weights attached to the Bartik 2SLS regression under Assumptions 7, 8, B.3, and B.2 (the last two assumptions are generalizations of Assumptions 4 and 5 to more than two periods). Column (2) shows that under the assumption that first-stage effects do not vary across sectors and are all positive (βs,g,t = βg,t ≥ 0), θb estimates a weighted sum of 1,444 second-stage effects, where 854 effects receive a strictly negative weight. The weights depend on the unobserved first-stage
effects $\beta_{g,t}$, so we cannot estimate the sum of the negative weights. Similarly, Column (3) shows that under Assumption 11 ($\beta_{s,g,t} = \beta_g \geq 0$, and $\alpha_{g,t} = \alpha_g$), which underlies our first alternative estimator, $\theta^b$ estimates a weighted sum of 722 second-stage effects, where 390 effects receive a strictly negative weight. Finally, Column (4) shows that if the first-stage effect is fully homogeneous ($\beta_{s,g,t} = \beta$), $\theta^b$ estimates a weighted sum of 1,444 second-stage effects, where 854 effects receive a strictly negative weight, and where negative weights sum to -0.084. Thus, $\theta^b$ is not robust to heterogeneous effects, even under a first-stage model with fully homogeneous effects.

### Table 4: Summary Statistics on the Weights Attached to Bartik Regressions in Table 3

| Assumption on first-stage effects | None | $\beta_{s,g,t} = \beta_{g,t} \geq 0$ | $\beta_{s,g,t} = \beta_g \geq 0$ | $\beta_{s,g,t} = \beta$ |
|----------------------------------|------|-------------------------------------|---------------------------------|------------------|
| Assumption on second-stage effects | None | None | $\alpha_{g,t} = \alpha_g$ | None |
| Number of strictly negative weights | 873 | 854 | 390 | 854 |
| Number of positive weights | 571 | 590 | 332 | 590 |
| Sum of negative weights | -0.161 | ? | ? | -0.084 |

Notes: The table reports summary statistics on the weights attached to the 2SLS regression in Column (3) of Table 3. The weights in Column (1) are estimated following Theorem B.1, those in Columns (2) to (4) are estimated following Theorem B.2. In Column (1), no assumption is made on the first- and second-stage effects. Column (2) assumes that the first-stage effects do not vary across sectors and are positive ($\beta_{s,g,t} = \beta_{g,t} \geq 0$). Column (3) assumes that the first-stage effects do not vary across sectors and over time and are positive ($\beta_{s,g,t} = \beta_g \geq 0$), and that the second-stage effects do not vary over time ($\alpha_{g,t} = \alpha_g$). Finally, Column (4) assumes that the first-stage effects are fully homogeneous ($\beta_{s,g,t} = \beta$). Question marks indicate that the quantity under consideration cannot be estimated.

### 4.3.3 Alternative estimators

**Estimator robust to heterogeneous first- and second-stage effects across CZs**

In Column (5) of Table 3, we follow Equation (3.7) and present our first alternative estimator of the second stage effect. The system of moment conditions has one observation per CZ, hence the number of observations. Because the estimation only uses one observation per CZ, the heteroskedasticity-robust standard error shown between parenthesis below the estimate relies on the assumption that observations are independent across CZ, and is therefore comparable to the CZ-clustered standard errors in Columns (1) to (4). Our first alternative estimator is of the same sign as the 2SLS coefficient in Table 3, and is almost twice as large in absolute value.

**Estimator robust to heterogeneous first- and second-stage effects across CZs and over time**

While it is more robust to heterogeneous second-stage effects than the Bartik regression, our alternative estimator in Column (5) of Table 3 still assumes that
the first- and second-stage effects are time invariant, and that the first-stage effects do not vary across sectors. Both assumptions are strong, in particular the first one: economic conditions in the US changed substantially over the period, and the effect of Chinese competition on US employment may have evolved over time. To alleviate this concern, we tentatively compute our second estimator $\hat{\alpha}_{ate}$ introduced in Section 3.3, which allows for heterogeneous first- and second-stage effects over time, provided those follow the same evolution in all CZs. It is important to keep in mind that due to an incidental parameter problem, $\hat{\alpha}_{ate}$ might be biased in this short panel ($T = 3$). We find that $\hat{\alpha}_{ate} = -0.867$, in-between the 2SLS Bartik regression coefficient and our first alternative estimator.

**Estimator allowing for time-varying first- and second-stage effects, but assuming homogeneous first-stage effects across CZs.** Finally, we compute our third alternative estimator, and find that it is equal to $-0.903$ (s.e.=0.910). In this application, this third estimator is very noisy, but this is not always the case: for instance, in Appendix A where we revisit the canonical Bartik design, this estimator’s standard error is comparable to that of our first CRC estimator, and to that of the Bartik regression. Overall, the admittedly imperfect evidence we can provide suggests that if we allow for time-varying first- and second-stage effects, we still get a more negative estimate of the effect of Chinese imports on US employment than in Autor et al. (2013).

5 Conclusion

Bartik regressions use locations’ differential exposure to nationwide sector-level shocks as an instrument to estimate the effect of a treatment on an outcome. We derive several decomposition results, all showing that Bartik regressions identify weighted sums of location-specific treatment effects, potentially with some negative weights. Due to the negative weights, the Bartik regression coefficient could be, say, negative, even if all location-specific effects are positive. Some of our decompositions rely on arguably strong assumptions, including a linear first-stage model with homogeneous effects, or a randomly-assigned-shocks assumption. Therefore, our results imply that Bartik regressions are in general not robust to heterogeneous effects. We propose alternative correlated-random-coefficient estimators that are more, though not fully, robust to heterogeneous effects. We use our results to revisit Autor et al. (2013). We find that the Bartik regression in this application estimates a weighted sum of location-specific effects where more than half of the effects are weighted negatively, and where the negative weights are rather large, as they sum to $-0.16$. Our alternative correlated-random-coefficient estimators are more negative than the Bartik regression coefficient.
References

Acemoglu, D., Autor, D., Dorn, D., Hanson, G. H. & Price, B. (2016), ‘Import competition and the great US employment sag of the 2000s’, *Journal of Labor Economics* **34**(S1), S141–S198.

Adão, R., Kolesár, M. & Morales, E. (2019), ‘Shift-share designs: Theory and inference’, *The Quarterly Journal of Economics* **134**(4), 1949–2010.

Altonji, J. G. & Card, D. (1991), The effects of immigration on the labor market outcomes of less-skilled natives, Technical report, National Bureau of Economic Research.

Angrist, J. D. & Imbens, G. W. (1995), ‘Two-stage least squares estimation of average causal effects in models with variable treatment intensity’, *Journal of the American Statistical Association* **90**(430), 431–442.

Autor, D., Dorn, D., Hanson, G. & Majlesi, K. (2020), ‘Importing political polarization? the electoral consequences of rising trade exposure’, *American Economic Review* **110**(10), 3139–3183.

Autor, D. H. & Dorn, D. (2013), ‘The growth of low-skill service jobs and the polarization of the US labor market’, *American Economic Review* **103**(5), 1553–97.

Autor, D. H., Dorn, D. & Hanson, G. H. (2013), ‘The China syndrome: Local labor market effects of import competition in the United States’, *American Economic Review* **103**(6), 2121–68.

Bartik, T. J. (1991), ‘Who benefits from state and local economic development policies?’. 

Benson, D., Masten, M. A. & Torgovitsky, A. (2022), ‘iverc: An instrumental-variables estimator for the correlated random-coefficients model’, *The Stata Journal* **22**(3), 469–495.

Borusyak, K., Hull, P. & Jaravel, X. (2022), ‘Quasi-experimental shift-share research designs’, *The Review of Economic Studies* **89**(1), 181–213.

Card, D. (2001), ‘Immigrant inflows, native outflows, and the local labor market impacts of higher immigration’, *Journal of Labor Economics* **19**(1), 22–64.

Card, D. (2009), ‘Immigration and inequality’, *American Economic Review* **99**(2), 1–21.

Chamberlain, G. (1992), ‘Efficiency bounds for semiparametric regression’, *Econometrica: Journal of the Econometric Society* pp. 567–596.

Dhaene, G. & Jochmans, K. (2015), ‘Split-panel jackknife estimation of fixed-effect models’, *The Review of Economic Studies* **82**(3), 991–1030.
Fernández-Val, I. & Vella, F. (2011), ‘Bias corrections for two-step fixed effects panel data estimators’, *Journal of Econometrics* **163**(2), 144–162.

Fernández-Val, I. & Weidner, M. (2016), ‘Individual and time effects in nonlinear panel models with large n, t’, *Journal of Econometrics* **192**(1), 291–312.

Goldsmith-Pinkham, P., Sorkin, I. & Swift, H. (2020), ‘Bartik instruments: What, when, why, and how’, *American Economic Review* **110**(8), 2586–2624.

Graham, B. S. & Powell, J. L. (2012), ‘Identification and estimation of average partial effects in “irregular” correlated random coefficient panel data models’, *Econometrica* **80**(5), 2105–2152.

Greenstone, M., Mas, A. & Nguyen, H.-L. (2020), ‘Do credit market shocks affect the real economy? quasi-experimental evidence from the great recession and” normal” economic times’, *American Economic Journal: Economic Policy* **12**(1), 200–225.

Heckman, J. & Vytlacil, E. (1998), ‘Instrumental variables methods for the correlated random coefficient model: Estimating the average rate of return to schooling when the return is correlated with schooling’, *Journal of Human Resources* pp. 974–987.

Imbens, G. W. & Angrist, J. D. (1994), ‘Identification and estimation of local average treatment effects’, *Econometrica: Journal of the Econometric Society* pp. 467–475.

Rotemberg, J. (1983), ‘Instrument variable estimation of misspecified models’.

Roth, J. (2022), ‘Pretest with caution: Event-study estimates after testing for parallel trends’, *American Economic Review: Insights* **4**(3), 305–22.

Ruggles, S., Flood, S., Goeken, R., Grover, J., Meyer, E., Pacas, J. & Sobek, M. (2019), ‘Ipums usa: Version 9.0 [dataset]’. Minneapolis, MN: IPUMS, 2019. https://doi.org/10.18128/D010.V9.0.

Wooldridge, J. M. (1997), ‘On two stage least squares estimation of the average treatment effect in a random coefficient model’, *Economics letters* **56**(2), 129–133.
6 Proofs

6.1 Theorem 2

Note that

$$\Delta Y_g = \Delta Y_g(0) + \alpha_g \Delta D_g$$

$$= \Delta Y_g(0) + \alpha_g \Delta D_g(0) + \alpha_g \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g}$$

$$= \Delta Y_g(0) + \alpha_g \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g}.$$  \hspace{1cm} (6.1)

The first equality follows from Assumption 1. The second equality follows from Assumption 3. The third equality follows from Assumption 1. Then

$$E \left( \sum_{g=1}^{G} \Delta Y_g (\Delta Z_g - \Delta Z) \right)$$

$$= E \left( \sum_{g=1}^{G} \left( \Delta Y_g(\Delta D_g(0)) + \alpha_g \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} \right) (\Delta Z_g - \Delta Z) \right)$$

$$= \sum_{g=1}^{G} E(\Delta Y_g(\Delta D_g(0)))(\Delta Z_g - \Delta Z) + E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} (\Delta Z_g - \Delta Z) \alpha_g \right)$$

$$= \sum_{g=1}^{G} E(\Delta Y_g(\Delta D_g(0)))(E(\Delta Z_g) - E(\Delta Z)) + E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} (\Delta Z_g - \Delta Z) \alpha_g \right)$$

$$= E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} (\Delta Z_g - \Delta Z) \alpha_g \right).$$ \hspace{1cm} (6.2)

The third equality follows from Points 1 and 2 of Assumption 4. The fourth equality follows from Point 3 of Assumption 4. Also note that

$$E \left( \sum_{g=1}^{G} \Delta D_g (\Delta Z_g - \Delta Z) \right)$$

$$= E \left( \sum_{g=1}^{G} \left( \Delta D_g(0) + \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} \right) (\Delta Z_g - \Delta Z) \right)$$

$$= \sum_{g=1}^{G} E(\Delta D_g(0))(\Delta Z_g - \Delta Z) + E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} (\Delta Z_g - \Delta Z) \right)$$

$$= \sum_{g=1}^{G} E(\Delta D_g(0))(E(\Delta Z_g) - E(\Delta Z)) + E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} (\Delta Z_g - \Delta Z) \right)$$

$$= E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} (\Delta Z_g - \Delta Z) \right).$$ \hspace{1cm} (6.3)
The first equality follows from Assumption 3. The third equality follows from Points 1 and 2 of Assumption 5. The fourth equality follows from Point 3 of Assumption 5. Then, plugging (6.2) and (6.3) into (2.3) yields the result in Point 1. Points 2 and 3 directly follows from Point 1.

6.2 Proof of Theorem 3

Proof of Equation (2.11)

Note that for all \( g \),

\[
E(\Delta Z_g|\mathcal{F}) = E\left(\sum_{s=1}^{S} Q_{s,g} \Delta Z_s|\mathcal{F}\right) = \sum_{s=1}^{S} Q_{s,g} E(\Delta Z_s|\mathcal{F}) = 0. \tag{6.4}
\]

The last equality follows from Points 1 and 2 of Assumption 6. Therefore, \( E(\Delta Z|\mathcal{F}) = E\left(\frac{1}{G} \sum_{g=1}^{G} \Delta Z_g|\mathcal{F}\right) = 0 \). Then,

\[
\begin{align*}
E \left( \sum_{g=1}^{G} \Delta Y_g (\Delta D_g(0))(\Delta Z_g - \Delta Z.) \right) \\
= E \left( E \left( \sum_{g=1}^{G} \Delta Y_g (\Delta D_g(0))(\Delta Z_g - \Delta Z.)|\mathcal{F} \right) \right) \\
= E \left( \sum_{g=1}^{G} (\Delta Y_g(0) + \alpha_g \Delta D_g(0)) E(\Delta Z_g - \Delta Z.|\mathcal{F}) \right) \\
= 0. \tag{6.5}
\end{align*}
\]
The first equality follows from law of iterated expectation. The second equality follows from Assumption 1. The last equality follows from (6.4). Therefore,

\[
E \left( \sum_{g=1}^{G} \Delta Y_g (\Delta Z_g - \Delta Z) \right)
\]

\[= E \left( \sum_{g=1}^{G} \left( \Delta Y_g (\Delta D_g(0)) + \alpha_g \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} \right) (\Delta Z_g - \Delta Z) \right)
\]

\[= E \left( \sum_{g=1}^{G} \Delta Y_g (\Delta D_g(0)) (\Delta Z_g - \Delta Z) \right) + E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} (\Delta Z_g - \Delta Z) \alpha_g \right)
\]

\[= E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} (\Delta Z_g - \Delta Z) \alpha_g \right)
\]

\[= E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} \left( \sum_{s'=1}^{S} Q_{s',g} \Delta Z_{s'} - \frac{1}{G} \sum_{g'=1}^{G} \sum_{s'=1}^{S} Q_{s',g'} \Delta Z_{s'} \right) \alpha_g \right)
\]

\[= E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} \left( Q_{s,g} \Delta Z_s - \frac{1}{G} \sum_{g'=1}^{G} Q_{s,g'} \Delta Z_{s'} \right) \alpha_g \right)
\]

\[= E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \left( Q_{s,g} - Q_s \right) \Delta Z_s^2 \beta_{s,g} \alpha_g \right). \tag{6.6}
\]

The first equality follows from (6.1). The third equality follows from (6.5). The fifth equality follows from the law of iterated expectations, and the fact that \( \forall s' \neq s \in \{1, ..., S\}, E (\Delta Z_s \Delta Z_{s'} | \mathcal{F}) = 0 \) by Points 1-3 of Assumption 6.

Following similar steps,

\[
E \left( \sum_{g=1}^{G} \Delta D_g (\Delta Z_g - \Delta Z) \right)
\]

\[= E \left( \sum_{g=1}^{G} \left( \Delta D_g(0) + \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \beta_{s,g} \right) (\Delta Z_g - \Delta Z) \right)
\]

\[= E \left( \sum_{g=1}^{G} \Delta D_g(0) (\Delta Z_g - \Delta Z) \right) + E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s (\Delta Z_g - \Delta Z) \beta_{s,g} \right)
\]

\[= E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s (\Delta Z_g - \Delta Z) \beta_{s,g} \right)
\]

\[= E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \left( \sum_{s'=1}^{S} Q_{s',g} \Delta Z_{s'} - \frac{1}{G} \sum_{g'=1}^{G} \sum_{s'=1}^{S} Q_{s',g'} \Delta Z_{s'} \right) \beta_{s,g} \right)
\]

\[= E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \Delta Z_s \left( Q_{s,g} \Delta Z_s - \frac{1}{G} \sum_{g'=1}^{G} Q_{s,g'} \Delta Z_{s'} \right) \beta_{s,g} \right)
\]

\[= E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g} \left( Q_{s,g} - Q_s \right) \Delta Z_s^2 \beta_{s,g} \right). \tag{6.7}
\]
Plugging (6.6) and (6.7) into (2.3) yields the result.

**Proof of Equation (2.12)**

Similarly, one can show that

\[
E \left( \sum_{g=1}^{G} \Delta Y_g \Delta Z_g \right) = E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g}^2 \Delta Z_s^2 \beta_{s,g} \alpha_g \right)
\]

(6.8)

and

\[
E \left( \sum_{g=1}^{G} \Delta D_g \Delta Z_g \right) = E \left( \sum_{g=1}^{G} \sum_{s=1}^{S} Q_{s,g}^2 \Delta Z_s^2 \beta_{s,g} \right).
\]

(6.9)

Plugging (6.8) and (6.9) into (2.10) yields the result.

**6.3 Theorem 4**

Under Assumptions 7, 9, and 11,

\[
E(\Delta D_g|\Delta Z_g) = \mu^D + E(\beta_g|\Delta Z_g)\Delta Z_g.
\]

(6.10)

Because \(M_g\Delta Z_g = 0\) and \(M_g\) is a function of \(\Delta Z_g\), we left-multiply Equation (6.10) by \(M_g\), and it follows that

\[
E(M_g \Delta D_g|\Delta Z_g) = M_g \mu^D.
\]

Also note that

\[
M_g' M_g = \left( I - \frac{1}{\Delta Z_g' \Delta Z_g} \Delta Z_g \Delta Z_g' \right) \left( I - \frac{1}{\Delta Z_g' \Delta Z_g} \Delta Z_g \Delta Z_g' \right) = M_g,
\]

therefore, it follows that

\[
E(M_g' M_g \Delta D_g|\Delta Z_g) = M_g' M_g \mu^D.
\]

Therefore, by the law of iterated expectation:

\[
E \left( \frac{1}{G} \sum_{g=1}^{G} M_g' M_g \Delta D_g \right) = E \left( \frac{1}{G} \sum_{g=1}^{G} M_g' M_g \right) \mu^D.
\]

So Equation (3.3) holds.

Similarly, we left-multiply Equation (6.10) by \(\Delta Z_g'\), and it follows that

\[
E(\Delta Z_g' \Delta D_g|\Delta Z_g) = \Delta Z_g' \mu^D + E(\beta_g|\Delta Z_g)\Delta Z_g' \Delta Z_g.
\]

Therefore,

\[
E \left( \frac{1}{\Delta Z_g' \Delta Z_g} \Delta Z_g' (\Delta D_g - \mu^D) \mid \Delta Z_g \right) = E(\beta_g|\Delta Z_g).
\]

37
Then by the law of iterated expectation:

\[
E \left( \frac{1}{G} \sum_{g=1}^{G} \beta_g \right) = E \left( \frac{1}{G} \sum_{g=1}^{G} \frac{1}{\Delta Z_g' \Delta Z_g} \Delta Z_g' (\Delta D_g - \mu^D) \right).
\]

So Equation (3.5) holds. The proofs of Equations (3.4) and (3.6) are similar: one simply needs to replace \( \Delta D_g, \mu^D \), and \( \beta_g \) by \( \Delta Y_g, \mu^Y \), and \( \gamma_g \).

### 6.4 Theorem 5

Under Assumptions 7, 9, and 13, and because \( P_g \) is a function of \( \Delta Z_g \),

\[
E(\Delta D_g|\Delta Z_g) = P_g \theta^D + \beta_g \Delta Z_g,
\]

(6.11)

Because \( M_g \Delta Z_g = 0 \) and \( M_g \) is a function of \( \Delta Z_g \), we left-multiply Equation (6.10) by \( M_g \), and it follows that

\[
E(M_g \Delta D_g|\Delta Z_g) = M_g P_g \theta^D,
\]

which in turn implies

\[
E(P_g' M_g \Delta D_g|\Delta Z_g) = P_g' M_g P_g \theta^D.
\]

Therefore, by the law of iterated expectation:

\[
E \left( \frac{1}{G} \sum_{g=1}^{G} P_g' M_g \Delta D_g \right) = E \left( \frac{1}{G} \sum_{g=1}^{G} P_g' M_g P_g \right) \theta^D.
\]

So Equation (3.10) holds.

Similarly, we left-multiply Equation (6.11) by \( \Delta Z_g' \), and it follows that

\[
E(\Delta Z_g' \Delta D_g|\Delta Z_g) = \Delta Z_g' P_g \theta^D + \beta_g \Delta Z_g' \Delta Z_g.
\]

Equation (3.11) follows from rearranging and the law of iterated expectations.

Similarly, under Assumptions 7, 8, 9, 13, and 14, and because \( \tilde{P}_g \) is a function of \( \Delta \tilde{Z}_g \),

\[
E(\Delta Y_g|\Delta \tilde{Z}_g) = \tilde{P}_g \theta^Y + \alpha_g \Delta \tilde{Z}_g.
\]

(6.12)

Then, the proofs of Equations (3.12) and (3.13) are similar to those of Equations (3.10) and (3.11).
Web Appendix: not for publication

A Empirical application: canonical Bartik design

In this section, we revisit the canonical application in Bartik (1991), where the Bartik instrument is used to estimate the inverse elasticity of labor supply.

A.1 Data

Our data construction closely follows Goldsmith-Pinkham et al. (2020). We construct a decennial continental US commuting-zone (CZ) level panel data set, from 1990 to 2010, with CZ wages and employment levels. For 1990 and 2000, we use the 5% IPUMS sample of the U.S. Census. For 2010, we pool the 2009-2011 ACSs (Ruggles et al. 2019). Sectors are IND1990 sectors. We follow Autor & Dorn (2013) to reallocate Public Use Micro Areas level observations of Census data to the CZ level. We also follow Autor et al. (2013) to aggregate the Census sector code ind1990 to a balanced panel of sectors for the 1990 and 2000 Censuses and the 2009-2011 ACS, with new sector code ind1990dd. In our final dataset, we have 3 periods, 722 CZs and 212 sectors.

The outcome variable $\Delta Y_{g,t}$ is the change in log wages in CZ $g$ from $t-10$ to $t$, for $t \in \{2000, 2010\}$. The treatment variable $\Delta D_{g,t}$ is the change in log employment in CZ $g$ from $t-10$ to $t$. We use people aged 18 and older who are employed and report usually working at least 30 hours per week in the previous year to generate employment and average wages. We define $Q_{s,g}$ as the employment share of sector $s$ in CZ $g$ in 1990, and then construct the Bartik instrument using 1990-2000 and 2000-2010 leave-one-out sectoral employment growth rates. Specifically, to construct the nationwide employment growth rate of sector $s$ for CZ $g$, we use the change in log employment in sector $s$ over all CZs excluding CZ $g$, following Adão et al. (2019).

A.2 Tests of the identifying assumptions

A.2.1 The randomly-assigned shocks assumption is rejected

**Shocks are correlated to sectors’ average share.** Point 1 of Assumption 6 implies that $E(\Delta Z_s|\bar{G} \sum_{s=1}^{S} Q_{s,g}) = E(\Delta Z_s)$: shock should be mean independent of the average employment share. 

\[\text{Crosswalk files are available online at https://www.ddorn.net/data.htm. The original crosswalk file for sector code only creates a balanced panel of sectors up to the 2006-2008 ACSs. We extend the crosswalk approach to one additional sector (shoe repair shops, crosswalked into miscellaneous personal services) to create a balanced panel of sectors up to the 2009-2011 ACSs.}\]

\[\text{As discussed in Adão et al. (2019) and Goldsmith-Pinkham et al. (2020), we use the leave-one-out definition to construct the national growth rates, in order to avoid the finite sample bias that comes from using own-observation information. In practice, because we have 722 locations, whether one uses leave-one-out or not to estimate the national growth rates barely changes the results.}\]
share of sector $s$ across locations. We test this by regressing shocks, namely the employment growth of each sector, on $\frac{1}{G} \sum_{s=1}^{S} Q_{s,g}$ in Panel A of Table A.1. In Column (1), we regress the shocks from 1990 to 2000 on the sector’s average share; in Column (2) we regress the shocks from 2000 to 2010 on the sector’s average share. The t-stats in columns (1) and (2) are equal to 1.15 and 2.56 respectively, and the null is strongly rejected in column (2), even multiplying the 0.01 p-value by 2 to account for multiple testing.

**Shocks are correlated to sectors’ characteristics.** Point 2 of Assumption 6 implies that shocks’ expectation should not vary with sector-level characteristics. We test this by regressing the shocks on a set of pre-determined sector characteristics measured in 1990. These sector characteristics are the log of average wages, the proportion of male workers, the proportion of white workers, the average age of workers, and the proportion of workers with some college education. Panel B of Table A.1 shows that large employment shocks tend to appear in sectors with low average wages and more educated workers, and in Columns (1) and (2) we can reject the hypothesis that the employment shocks are not correlated with any sector characteristic (p-value < 0.001). Therefore, there are sectors with certain characteristics that make them more likely to receive a large employment shock, and the random-shock assumption is rejected.

---

A.3 When constructing the sector characteristics, we only use the workers from continental U.S. CZs, in order to be consistent with our main sample.
Table A.1: Testing the Random Shock Assumption

| Variables                        | (1) Δ log \( E \): 1990-2000 | (2) Δ log \( E \): 2000-2010 |
|----------------------------------|-------------------------------|-------------------------------|
| **Panel A: Shocks uncorrelated to sectors’ average share?** |                               |                               |
| Sector’s average share           | 1.936                         | 4.706                         |
|                                  | (1.685)                       | (1.842)                       |
| Observations                     | 212                           | 212                           |
| **Panel B: Shocks uncorrelated to sectors’ characteristics?** |                               |                               |
| \( \log w_{1990} \)            | -0.372                        | -0.375                        |
|                                  | (0.093)                       | (0.081)                       |
| Male\(_{1990}\)                 | 0.212                         | 0.337                         |
|                                  | (0.200)                       | (0.146)                       |
| White\(_{1990}\)                | -0.738                        | -0.182                        |
|                                  | (0.547)                       | (0.472)                       |
| Age\(_{1990}\)                  | -0.027                        | 0.009                         |
|                                  | (0.015)                       | (0.011)                       |
| Some College\(_{1990}\)        | 1.095                         | 1.122                         |
|                                  | (0.229)                       | (0.199)                       |
| F-test P-value                   | 0.0000                        | 0.0000                        |
| Observations                     | 212                           | 212                           |

Notes: The table reports estimates of regressions of sector-level employment growth on a set of pre-determined sector characteristics. The dependent variable in Column (1) (resp. (2)) is the change in log nationwide employment in the sector from 1990 to 2000 (resp. from 2000 to 2010). In Panel A, the independent variable is sectors’ average share across commuting zones in the Bartik instrument. In Panel B, the independent variables are a set of pre-determined sector characteristics measured in 1990: log average wages in the sector \( \log w_{1990} \), the proportion of male workers in the sector Male\(_{1990}\), the proportion of white workers in the sector White\(_{1990}\), the average age of workers in the sector Age\(_{1990}\), the proportion of workers with at least some college education in the sector Some College\(_{1990}\). Robust standard errors are shown in parentheses. The F-test p-value in Panel B is the p-value of the joint test that all the coefficients of the sector characteristics are equal to 0.

**A.2.2 Assumptions 4 and 10 are not testable**

Unfortunately, in this application we cannot implement the tests of Assumptions 4 and 10, as there are no consecutive time periods where the nationwide employment remains stable in every sector.
A.3 Results

A.3.1 Bartik regressions

Columns (1) to (3) of Table A.2 below show the results of the first-stage, reduced-form, and 2SLS Bartik regressions. In Column (1), the first-stage coefficient is 0.818. In Column (2), the reduced form coefficient is 0.390. Finally, in Column (3), the 2SLS coefficient is 0.477. If interpreted causally, this 2SLS coefficient means that a 1% increase in employment leads to a 0.477% increase in wages. Robust standard errors clustered at the CZ level are shown between parentheses. All coefficients are statistically significant. Column (4) shows \( \hat{\theta}_{NI} \), the Bartik 2SLS coefficient without an intercept and with demeaned shocks, which is robust to heterogeneous effects under the randomly-assigned-shocks assumption (see Theorem 3).\(^{A.4}\) \( \hat{\theta}_{NI} \) is implausibly large, perhaps reflecting the fact that the randomly-assigned-shocks assumption does not hold in this application.

| Dependent Variable | FS (1) | RF (2) | 2SLS (3) | 2SLS, no intercept (4) | Chamberlain (5) |
|-------------------|--------|--------|---------|------------------------|-----------------|
| Bartik Instrument | \( \Delta D_{g,t} \) | \( \Delta Y_{g,t} \) | \( \Delta Y_{g,t} \) | \( \Delta Y_{g,t} \) |
| \( \Delta D_{g,t} \) | 0.818  | 0.390  | 0.477   | 3.292                  | 0.483           |
|                   | (0.055)| (0.031)| (0.039) | (0.131)                | (0.061)         |
| Observations      | 1,444  | 1,444  | 1,444   | 1,444                  | 722             |

Notes: Columns (1) to (3) respectively report estimates of Bartik regressions with period fixed effects, using a decennial US commuting-zone (CZ) level panel data set from 1990 to 2010. \( \Delta Y_{g,t} \) is the change in log wages in CZ \( g \) from \( t - 10 \) to \( t \), for \( t \in \{2000, 2010\} \). \( \Delta D_{g,t} \) is the change in log employment in CZ \( g \) from \( t - 10 \) to \( t \). \( \Delta Z_{g,t} \) is the Bartik instrument, whose construction is detailed in the text. Columns (1), (2), and (3) respectively report estimates of the first-stage, reduced-form, and 2SLS Bartik regression coefficients. Column (4) shows the 2SLS coefficient in a regression with no period fixed effects and no intercept. Standard errors clustered at the CZ level are shown in parentheses. Column (5) reports an alternative estimate of the second-stage effect, using the GMM Stata command, and with the system of moment conditions in Equation (3.9). The \( 2T = 6 \) moment conditions have one observation per CZ, hence the number of observations. Heteroskedasticity-robust standard errors are shown in parentheses.

A.3.2 Decompositions of the 2SLS Bartik regression

Decomposition under Assumptions 8 and B.1. We follow Theorem B.1 in the Web Appendix, a straightforward generalization of Theorem 1 to more than two periods, to estimate the weights attached to the Bartik 2SLS regression under Assumptions 8 and

\(^{A.4}\)Note that leave-one-out sectoral employment growth rates are used when constructing the Bartik instrument, so the shocks are demeaned with respect to all \((s, g)\) instead of with respect to all \( s \), since locations do not have the same value of shock in a given sector.
B.1 (the last assumption is a generalization of Assumption 2 to more than two periods). Column (1) of Table A.3 shows that under those assumptions, $\theta^b$ estimates a weighted sum of 1,444 effects $\alpha_{g,t}$, where 897 weights are positive, 547 weights are strictly negative, and where negative weights sum to $-0.494$. Therefore, $\theta^b$ is not robust to heterogeneous effects. The weights are positively correlated with CZs’ shares of college-educated workers in 1990 (correlation= 0.208, p-value< 0.01), so $\theta^b$ may differ from $E\left(\frac{1}{GT} \sum_{g,t} \alpha_{g,t}\right)$, if the effects $\alpha_{g,t}$ are correlated with CZs’ educational levels.

**Decompositions under Assumptions 7, 8, B.3, and B.2.** We follow Theorem B.2 in the Web Appendix, a straightforward generalization of Theorem 2 to applications with more than two periods, to estimate the weights attached to the Bartik 2SLS regression under Assumptions 7, 8, B.3, and B.2 (the last two assumptions are generalizations of Assumptions 4 and 5 to more than two periods). Column (2) shows that under the assumption that first-stage effects do not vary across sectors and are all positive ($\beta_{s,g,t} = \beta_{g,t} \geq 0$), $\theta^b$ estimates a weighted sum of 1,444 second-stage effects, where 446 effects receive a strictly negative weight. The weights depend on the unobserved first-stage effects $\beta_{g,t}$, so we cannot estimate the sum of the negative weights. Similarly, Column (3) shows that under Assumption 11 ($\beta_{s,g,t} = \beta_{g} \geq 0$, and $\alpha_{g,t} = \alpha_{g}$), which underlies our first alternative estimator, $\theta^b$ estimates a weighted sum of 722 second-stage effects, where 75 effects receive a strictly negative weight. Finally, Column (4) shows that if the first-stage effect is fully homogeneous ($\beta_{s,g,t} = \beta$), $\theta^b$ estimates a weighted sum of 1,444 second-stage effects, where 446 effects receive a strictly negative weight, and where negative weights sum to $-0.040$. Thus, $\theta^b$ is not robust to heterogeneous effects, even under a first-stage model with fully homogeneous effects.
Table A.3: Summary Statistics on the Weights Attached to Bartik Regressions in Table A.2

| Assumption on first-stage effects | β_{s,g,t} = β_{g,t} ≥ 0 | β_{s,g,t} = β_{g} ≥ 0 | β_{s,g,t} = β |
| Assumption on second-stage effects | None | None | α_{g,t} = α_{g} | None |
| Number of strictly negative weights | 547 | 446 | 75 | 446 |
| Number of positive weights | 897 | 998 | 647 | 998 |
| Sum of negative weights | -0.494 | ? | ? | -0.040 |

Notes: The table reports summary statistics on the weights attached to the 2SLS regression in Table A.2. The weights in Column (1) are estimated following Theorem B.1, those in Columns (2) to (4) are estimated following Theorem B.2. In Column (1), no assumption is made on first- and second-stage effects. In Column (2), it is assumed that first-stage effects do not vary across sectors and are positive (β_{s,g,t} = β_{g,t} ≥ 0). In Column (3), it is assumed that first-stage effects do not vary across sectors and over time and are positive (β_{s,g,t} = β_{g} ≥ 0), and that second-stage effects do not vary over time (α_{g,t} = α_{g}). Finally, in Column (4) it is assumed that the first-stage effects are fully homogeneous (β_{s,g,t} = β). Question marks indicate that the quantity under consideration cannot be estimated.

A.3.3 Alternative estimators

**Estimator robust to heterogeneous first- and second-stage effects across CZs**

In Column (5) of Table A.2, we follow Equation (3.7) and present our first alternative estimator of the second stage effect. The system of moment conditions has one observation per CZ, hence the number of observations. Because the estimation only uses one observation per CZ, the heteroskedasticity-robust standard error shown between parenthesis below the estimate relies on the assumption that observations are independent across CZ, and is therefore comparable to the CZ-clustered standard errors in Columns (1) to (4). Our first alternative estimator is equal to 0.483, very close to the 2SLS coefficient in Column (3) of Table A.2.

**Estimator robust to heterogeneous first- and second-stage effects across CZs and over time**

While it is more robust to heterogeneous second-stage effects than the Bartik regression, our alternative estimator in Column (5) of Table A.2 still assumes that the first- and second-stage effects are time invariant, and that the first-stage effects do not vary across sectors. Both assumptions are strong, in particular the first one: economic conditions in the US changed substantially over the period, and the effect of employment growth on wage growth may have evolved over time. To alleviate this concern, we tentatively compute our second estimator \( \hat{\alpha}_{ate} \) introduced in Section 3.3, which allows for heterogeneous first- and second-stage effects over time, provided those follow the same evolution in all CZs. It is important to keep in mind that due to an incidental parameter
problem, \( \hat{\alpha}_{ate} \) might be biased in this short panel \((T = 3)\). We find that \( \hat{\alpha}_{ate} = 0.464 \), still very close to the 2SLS coefficient in Table A.2 and our first alternative estimator.

**Estimator allowing for time-varying first- and second-stage effects, but assuming homogeneous first-stage effects across CZs.** Finally, we compute our third alternative estimator, and find that it is equal to 0.457 \((s.e.=0.064)\). Overall, allowing for time-varying first- and second-stage effects, we still get a similar point estimate.
B Decompositions of Bartik regressions with multiple periods

In this section, we use the same notation and definitions as in Section 3 of the paper and we extend our decompositions of Bartik regressions in Theorems 1-2 to multiple periods. With several periods, the analog of the 2SLS regression with a constant is a 2SLS regression with period fixed effects.

Estimator and estimand. Let \( \Delta Z_{g,t} = \frac{1}{G} \sum_{g=1}^{G} \Delta Z_{g,t} \).

**Definition B.1** 2SLS Bartik regression with multiple periods: let

\[
\hat{\theta}^b = \frac{\sum_{g=1}^{G} \sum_{t=2}^{T} \Delta Y_{g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right)}{\sum_{g=1}^{G} \sum_{t=2}^{T} \Delta D_{g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right)} \tag{B.1}
\]

\[
\theta^b = \frac{\sum_{g=1}^{G} \sum_{t=2}^{T} E \left( \Delta Y_{g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right)}{\sum_{g=1}^{G} \sum_{t=2}^{T} E \left( \Delta D_{g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right)}. \tag{B.2}
\]

**Identifying assumptions.** The following assumption generalizes Assumption 2 to the case with multiple periods.

**Assumption B.1** 1. For all \( g \in \{1, \ldots, G\} \), \( t \in \{2, \ldots, T\} \), \( \text{cov}(\Delta Z_{g,t}, \Delta Y_{g,t}(0)) = 0 \)

2. For all \( g \neq g' \in \{1, \ldots, G\}^2 \), \( t \in \{2, \ldots, T\} \), \( \text{cov}(\Delta Z_{g',t}, \Delta Y_{g,t}(0)) = 0 \).

3. For all \( t \in \{2, \ldots, T\} \), \( \frac{1}{G} \sum_{g=1}^{G} E \left( \Delta Y_{g,t}(0) \right) \left( E \left( \Delta Z_{g,t} \right) - E \left( \Delta Z_{.,t} \right) \right) = 0 \).

The following assumptions generalizes Assumptions 4 and 5 to the case with multiple periods.

**Assumption B.2** 1. For all \( g \in \{1, \ldots, G\} \), \( t \in \{2, \ldots, T\} \), \( \text{cov}(\Delta Z_{g,t}, \Delta Y_{g,t}(\Delta D_{g,t}(0))) = 0 \).

2. For all \( g \neq g' \in \{1, \ldots, G\}^2 \), \( t \in \{2, \ldots, T\} \), \( \text{cov}(\Delta Z_{g',t}, \Delta Y_{g,t}(\Delta D_{g,t}(0))) = 0 \).

3. For all \( t \in \{2, \ldots, T\} \), \( \frac{1}{G} \sum_{g=1}^{G} E \left( \Delta Y_{g,t}(\Delta D_{g,t}(0)) \right) \left( E \left( \Delta Z_{g,t} \right) - E \left( \Delta Z_{.,t} \right) \right) = 0 \).

**Assumption B.3** 1. For all \( g \in \{1, \ldots, G\} \), \( t \in \{2, \ldots, T\} \), \( \text{cov}(\Delta Z_{g,t}, \Delta D_{g,t}(0)) = 0 \).

2. For all \( g \neq g' \in \{1, \ldots, G\}^2 \), \( t \in \{2, \ldots, T\} \), \( \text{cov}(\Delta Z_{g',t}, \Delta D_{g,t}(0)) = 0 \).

3. For all \( t \in \{2, \ldots, T\} \), \( \frac{1}{G} \sum_{g=1}^{G} E \left( \Delta D_{g,t}(0) \right) \left( E \left( \Delta Z_{g,t} \right) - E \left( \Delta Z_{.,t} \right) \right) = 0 \).

**Decomposition of \( \theta^b \) under Assumptions 8 and B.1.**

**Theorem B.1** Suppose Assumptions 8 and B.1 hold. Then,

\[
\theta^b = E \left( \sum_{g=1}^{G} \sum_{t=2}^{T} \frac{\Delta D_{g,t}(\Delta Z_{g,t} - \Delta Z_{.,t})}{\sum_{g'=1}^{G} \sum_{t'=2}^{T} \Delta D_{g',t'}(\Delta Z_{g',t'} - \Delta Z_{.,t'})} a_{g,t} \right). \]
Decompositions of $\theta^b$ under Assumptions 7, 8, B.2 and B.3.

**Theorem B.2** Suppose Assumptions 7, 8, B.2 and B.3 hold.

1. Then,

$$
\theta^b = E \left( \sum_{g=1}^{G} \sum_{t=2}^{T} \frac{\sum_{s=1}^{S} Q_{s,g} \Delta Z_{s,t} (\Delta Z_{g,t} - \Delta Z_{.,t}) \beta_{s,g,t}}{E \left( \sum_{g'=1}^{G} \sum_{t'=2}^{T} \sum_{s'=1}^{S} Q_{s',g'} \Delta Z_{s',t'} (\Delta Z_{g',t'} - \Delta Z_{.,t'}) \beta_{s',g',t'} \right)} \alpha_{g,t} \right).
$$

2. If one further assumes that $\beta_{s,g,t} = \beta_{g,t}$,

$$
\theta^b = E \left( \sum_{g=1}^{G} \sum_{t=2}^{T} \frac{\Delta Z_{g,t} (\Delta Z_{g,t} - \Delta Z_{.,t}) \beta_{g,t}}{E \left( \sum_{g'=1}^{G} \sum_{t'=2}^{T} \Delta Z_{g',t'} (\Delta Z_{g',t'} - \Delta Z_{.,t'}) \beta_{g',t'} \right)} \alpha_{g,t} \right).
$$

3. If one further assumes that $\beta_{s,g,t} = \beta$,

$$
\theta^b = E \left( \sum_{g=1}^{G} \sum_{t=2}^{T} \frac{\Delta Z_{g,t} (\Delta Z_{g,t} - \Delta Z_{.,t})}{E \left( \sum_{g'=1}^{G} \sum_{t'=2}^{T} \Delta Z_{g',t'} (\Delta Z_{g',t'} - \Delta Z_{.,t'}) \beta_{g',t'} \right)} \alpha_{g,t} \right).
$$

4. If one further assumes that $\beta_{s,g,t} = \beta_g$ and $\alpha_{g,t} = \alpha_g$,

$$
\theta^b = E \left( \sum_{g=1}^{G} \frac{\left( \sum_{t=2}^{T} \Delta Z_{g,t} (\Delta Z_{g,t} - \Delta Z_{.,t}) \right) \beta_g}{E \left( \sum_{g'=1}^{G} \left( \sum_{t'=2}^{T} \Delta Z_{g',t'} (\Delta Z_{g',t'} - \Delta Z_{.,t'}) \beta_{g'} \right) \alpha_g \right)} \right).
$$

Points 1 and 3 of Theorem B.2 are generalizations of Points 1 and 3 of Theorem 2. The important new implication that was not present in Theorem 2 is that Bartik regressions are also not robust to heterogeneous effects over time. Point 4 is a new result that was not present in Theorem 2. There, it is assumed that first-stage effects do not vary across sectors and over time, and that second-stage effects do not vary over time ($\beta_{s,g,t} = \beta_g$ and $\alpha_{g,t} = \alpha_g$). It is useful to consider what Bartik regressions estimate under that assumption, because it underlies the first alternative estimator we propose in Section 3. Under that assumption, the second-stage Bartik regression estimates again a weighted sum of the location-specific second stage effects $\alpha_g$, with weights that may be negative. This contrasts with our first alternative estimator: if $\beta_{s,g,t} = \beta_g$ and $\alpha_{g,t} = \alpha_g$, it estimates a weighted average of the second-stage effects $\alpha_g$.  

47
C  Proofs of results in Web Appendix

C.1 Theorem B.1

\[ E \left( \sum_{g=1}^{G} \sum_{t=2}^{T} \Delta Y_{g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right) \]

\[ = \sum_{g=1}^{G} \sum_{t=2}^{T} \left( \sum_{g=1}^{G} \left( \Delta Y_{g,t}(0) + \alpha_{g,t} \Delta D_{g,t} \right) \right) \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \]

\[ = \sum_{g=1}^{G} \sum_{t=2}^{T} E \left( \Delta Y_{g,t}(0) \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right) + \sum_{g=1}^{G} \sum_{t=2}^{T} \Delta D_{g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \alpha_{g,t} \]

\[ = E \left( \sum_{g=1}^{G} \sum_{t=2}^{T} \Delta D_{g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \alpha_{g,t} \right). \quad (C.1) \]

The first equality follows from Assumption 8. The third equality follows from Points 1 and 2 of Assumption B.1. The fourth equality follows from Point 3 of Assumption B.1. Then, plugging (C.1) into (B.2) yields the result.

C.2 Theorem B.2

Note that

\[ \Delta Y_{g,t} = \Delta Y_{g,t}(0) + \alpha_{g,t} \Delta D_{g,t} \]

\[ = \Delta Y_{g,t}(0) + \alpha_{g,t} \Delta D_{g,t}(0) + \alpha_{g,t} \sum_{s=1}^{S} Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t} \]

\[ = \Delta Y_{g,t}(\Delta D_{g,t}(0)) + \alpha_{g,t} \sum_{s=1}^{S} Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t}. \quad (C.2) \]
The first equality follows from Assumption 8. The second equality follows from Assumption 7. The third equality follows from Assumption 8. Then

\[ E \left( \sum_{g=1}^G \sum_{t=2}^T \Delta Y_{g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right) \]

\[ = E \left( \sum_{g=1}^G \sum_{t=2}^T \left( \Delta Y_{g,t}(\Delta D_{g,t}(0)) + \alpha_{g,t} \sum_{s=1}^S Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t} \right) \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right) \]

\[ = \sum_{g=1}^G \sum_{t=2}^T E \left( \Delta Y_{g,t}(\Delta D_{g,t}(0)) \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right) + E \left( \sum_{g=1}^G \sum_{t=2}^T \sum_{s=1}^S Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \alpha_{g,t} \right) \]

\[ + E \left( \sum_{g=1}^G \sum_{t=2}^T \sum_{s=1}^S Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \alpha_{g,t} \right) \]

\[ = \sum_{g=1}^G \sum_{t=2}^T E \left( \Delta Y_{g,t}(\Delta D_{g,t}(0)) \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right) + E \left( \sum_{g=1}^G \sum_{t=2}^T \sum_{s=1}^S Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \alpha_{g,t} \right) \]

\[ = E \left( \sum_{g=1}^G \sum_{t=2}^T \Delta Z_{s,t} \beta_{s,g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \alpha_{g,t} \right) \]  \hspace{1cm} (C.3)  

The third equality follows from Points 1 and 2 of Assumption B.2. The fourth equality follows from Point 3 of Assumption B.2. Also note that

\[ E \left( \sum_{g=1}^G \sum_{t=2}^T \Delta D_{g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right) \]

\[ = E \left( \sum_{g=1}^G \sum_{t=2}^T \left( \Delta D_{g,t}(0) + \sum_{s=1}^S Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t} \right) \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right) \]

\[ = \sum_{g=1}^G \sum_{t=2}^T E \left( \Delta D_{g,t}(0) \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right) + E \left( \sum_{g=1}^G \sum_{t=2}^T \sum_{s=1}^S Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right) \]

\[ = \sum_{t=2}^T \sum_{g=1}^G E \left( \Delta D_{g,t}(0) \left( E \left( \Delta Z_{g,t} \right) - E \left( \Delta Z_{.,t} \right) \right) \right) + E \left( \sum_{g=1}^G \sum_{t=2}^T \sum_{s=1}^S Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right) \]

\[ = E \left( \sum_{g=1}^G \sum_{t=2}^T \sum_{s=1}^S Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t} \left( \Delta Z_{g,t} - \Delta Z_{.,t} \right) \right) \]  \hspace{1cm} (C.4)  

The first equality follows from Assumption 7. The third equality follows from Points 1 and 2 of Assumption B.3. The fourth equality follows from Point 3 of Assumption B.3. Then, plugging (C.3) and (C.4) into (B.2) yields the result in Point 1. Points 2, 3 and 4 directly follow from Point 1.