The Behaviour of Wave Propagation for Structural System Identification: A Comparative Study

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ABSTRACT

The way a wave behaves, while propagating across a medium, varies with the wave type and the medium. So, knowledge of the behaviour of a wave in a system with a different form of damage, and behaviour of different types of waves in a particular system is an essential prerequisite for almost all activities in structural system identification and mainly for damage detection and localization of damage. This paper presents a comparative study of various wave propagations, that has been done by researchers in various structural systems. Further, a numerical model of an isotropic plate using finite element is created both with and without damage. The behaviour of waves has been studied. Finally, the comparative result is shown. This paper offers a new perspective for ongoing research by providing the most recent developments, difficulties, and prospects of wave propagation behaviours for damage detection and localization in the commonly used structural systems and structural elements. While propagating through different structural systems and components, the most used waves, which are (a) Shear wave, (b) Rayleigh wave, (c) Ultrasonic wave and (d) Lamb wave, have been thoroughly investigated. Along with several difficult problems for future growth, the summarized observations are provided.

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1. INTRODUCTION

A wave is a form of disturbance that is carried forward by the particles of the system through vibration. During wave propagation, only energy is transferred and not particles. As, the behaviour of waves changes with change in medium, this property has been used by researchers for system identification. This method is called, “Wave Propagation based System Identification”. During 19th century, to determine the presence of damage on train wheels, wheel-tappers generated the waves by tapping a railroad wheel with a hammer. They determine the presence of damage based on the sound waves produced by the wheel. The various rail irregularities, in the wheel-rail system, have also been analyzed by Yang et al. [1].

The vibration based structural system identification, for damage identification and localization, has also been used by many researchers. But, not all cases can be adequately assessed using this method [2]. When damage starts occurring and further when it grows, the stiffness is reduced and the stability of the structure is also affected. Also, if the change in stiffness is small, the change in natural frequencies is small. In that case, the vibration based technique is not very efficient [3]. Also, the Timoshenko cantilever beam supported by spring was examined by Aydin et al. [4]. According to the authors, the different beam vibrations will result in varied distributions of elastic supports supported by the beams. Internal forces can occasionally exceed yield limits, which is significant for damage. Also, the sensitivity of the behaviour parameters could be used for the damage assessment. For the same beam, the optimum locations and the amounts of the springs were investigated for the first and second modes [5]. This technique may be used for finding the damage location. When damage is far from the support as in the case of the cantilever beam, the vibration based feature may not detect the damage accurately. Golafshani et al. [6] have also studied the vibration feature in the system.

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In metallic and composite constructions, wave propagation has been very effective for damage detection and localisation. So, it can also be used in various other structural systems such as trusses, multi-storey frames, bridges etc. In the previous studies, it was observed that in many cases, the wave packet overlap, due to which the identification of damage signal becomes more complex. The diffraction wave packets expand in the time domain. Consequently, the crack imaging result based on the signals presents a reasonably low resolution. Also, the wave’s multimodal behaviour, dispersion, scattering and attenuation have an impact on damage signature. As a result, it is more difficult to use the wave technique. But with the right knowledge of the wave’s properties and behaviour, the right choice of transducers, and signal processing, this complexity can be minimized. Hence, a broad classification has been provided for various forms of waves in section 2. For better understanding of how different types of waves behave, while propagating through different structural systems and components, the most commonly used waves, are (a) Shear wave, (b) Rayleigh wave, (c) Ultrasonic wave and (d) Lamb wave, which have been thoroughly investigated in section 3.

As, the objective of this paper is to study the behaviour of wave in a system with different form of damage, so for that total four different damage cases in a particular system were considered for numerical case studies in section 4. As per the literature, the behaviour of waves can be clearly seen in the plate, as compared to any other structural element. Also, in plate various forms of damage occur. So, for the numerical case study, the most common form of damage that can occur in the plate (damage in the form of hole, fatigue crack, near edge crack at 2 different locations) has been considered. The comparative result has been provided.

2. CLASSIFICATION OF WAVE

The broad classification of various types of waves, including waves that can be utilized for structural system identification.

2. 1. Depending upon the Motion of the Wave

Depending on the motion of the wave, it can be classified as mechanical waves, matter waves and electromagnetic waves.

The matter waves’ wavelength is too tiny to have a noticeable practical effect on daily living. An atom’s wavelength is roughly $3 \times 10^{-11}$ metres. It is difficult to encapsulate a wavelength of this size in a structure. Electromagnetic wave does not require any medium for propagation. Both the magnetic and electric fields couple together to form this wave. These waves have been used by researchers for damage detection in metallic and composite materials. However, the generation of electromagnetic waves is not as simple as the generation of mechanical waves in a structural system. Thermal waves have also been used by many researchers for damage assessment. Parvez et al. [7] have shown that, as this wave is generated in the system, the temperature rises and this response is further quantified for building relation for damage assessment. The medium must have a minimal amount of friction between its particles as well as elasticity and inertia to produce mechanical waves. These waves can be refracted, diffracted and reflected. They could be alter in velocity at material discontinuity or when there are foreign particles inside the medium or on its surface. This characteristic will be useful in identifying damage and pinpointing the damage to a specific structural system or structural component.

2. 2. Depending upon the Dynamic Deformation of Particle

Waves can be elastic waves, plastic waves, or shock waves depending on how dynamic deformation in particle occurs.

Recent analyses of plastic wave profiles in metals like Beryllium30, Copper29, and Aluminum28 reveal viscous effects within the shock front and strength qualities at the Hugoniot state that are peculiar in behaviour and difficult to understand. Shock waves also require a medium to propagate, similar to the mechanical wave. Also, the medium required is elastic. But, the speed of propagation is faster than the local speed of sound. Elastic waves are the most commonly used, it can be further classified as:

1. Longitudinal wave/ P Wave
2. Transverse wave/ Distortional wave/ Secondary wave/ Shear wave/ S wave
3. Bending/ Flexural wave
4. Love wave
5. Stonley wave/ Interfacial wave
6. Rayleigh wave/ Surface wave

2. 3. Based on Region of Propagation

Waves can be further classified as Guided Waves and Bulk Waves, based on the region of propagation.

The bulk wave can be generated in an infinite homogenous body. This wave can propagate without getting interrupted by the boundaries or the interface. The guided wave is further classified as a Lamb wave and ultrasonic wave. Lamb waves can be symmetric, asymmetric or have a combined mode.

3. VARIOUS WAVE PROPAGATION BEHAVIOUR IN STRUCTURAL SYSTEMS AND ELEMENTS

The way a wave behaves, while propagating across a medium, varies with the wave type and the medium. Therefore, understanding how various types of waves behave in a given system is a requirement for nearly all actions related to wave propagation based structural...
health monitoring, mainly for system identification for various structural systems and structural elements. So, for the above, various literature review are presented in this section.

3.1 Shear Wave

This wave is generated in a system that has shearing properties, such as solids. In the case of a shear horizontal wave, the particle of the medium executes a simple harmonic motion about its mean position in the direction perpendicular to the direction of propagation of the wave. If the layer interface is too close to the wavelength, it was impossible to discern between the body wave and the surface wave [8]. Mei et al. [9] mentioned in their research that, the shear horizontal waves were very sensitive to damage at the interface. Delamination in composites is considered here. These waves are mild dispersive in composites compared to the lamb wave, so they are simpler for damage diagnosis. This wave has been generated at the layer of a composite beam, with both ends clamped. The symmetric and asymmetric modes of lamb wave along with shear horizontal wave are generated in the system, the damage identification becomes complex in that case. Further, the pure shear horizontal wave has been generated that are non-dispersive and the wave packets are strong. The damage was successfully identified and located with an error of 3.7%. Nazeer et al. [10] have done the damage identification in the plate, having damage in the form of transverse cracks running along the bend and delamination in the bend. The shear mode in the bend is dispersive in comparison to the SH0 wave in a plate, and the curvature of the bend influences its wavefield properties. The scattering investigations revealed that, in contrast to inner surface cracks, the wave is more sensitive to outer surface fractures in the bend as frequency increases, because there is transverse shear tension at the bend. Choi et al. [11] have also used the MIRA, a shear-wave-based ultrasonic system of low frequency (20–100 kHz) multifunctional phased array, which is used to detect damage in concrete related structures. Damage in the form of the changes that occurred in adhesion property at the interfacial adhesion was considered [12]. Further, the damage assessment in an isotropic plate with thickness crack was done by using the shear horizontal guided wave mode [13]. Rajagopal and Lowe [13] have shown that the shear horizontal guided wave gets scattered at the crack location in an isotropic plate. Hashemi and Ghasem Alaskari [14] have also studied the behaviour of shear waves in a laterally complex medium. They have taken the displacement vector of shear waves in two different orientations and showed that this will be best suited for dynamic cases.

3.2 Rayleigh Wave

Masserey and Fromme [15] have shown that it can be interpreted as the superposition of the first anti-symmetric and symmetric Lamb mode. These are also sensitive to extremely small cracks. The wave alters its behaviour even at a 120 μm size crack. Wang et al. [16] have considered damage in the form of crack, that is present near the surface, in any homogeneous, isotropic and elastic system. The wave showed a change in behaviour at the crack length, that are smaller than the wavelength of the Rayleigh Wave. Also, damage identification is more accurate in such cases. Cook and Berthelot [17] have considered the damage in the form of fatigue crack, on the surface of the flat bar specimen. A small amount of wave gets reflected at the notch, and also at the growing fatigue crack. Both the wave signals interfered. The amplitude of the reflected wave signal of the notch is much larger than that of the fatigue crack. That property is used to identify the damage. If the wavelength is smaller than the radius of curvature then it can not detect the little distortion. Zeng et al. [18], considered the inclined surface crack in an aluminium plate. When the wave reaches the inclined defect, it splits into two parts. Thus, these waves can only be employed for the identification of near-surface fractures and flaws, according to the aforementioned literature.

3.3 The Ultrasonic Guided Wave

This wave can be generated in the cantilever beam. Damage may be in the form of crack, discontinuity or the presence of a foreign particle. All of these cases create change in the medium. So, in this region, the velocity of the wave is affected, if scatters, refracts or diffracts. Hence, the wave signal when compared with the baseline signal indicate the existence and location of the damage. Arundas and Dewangan [19] have used the ultrasonic pulse velocity test to find the crack location in a cantilever beam. Mu et al. [20], have done the real-time performance of health monitoring in ocean platform. Carbon Steel, Q235, of 500 mm (length) × 500 mm (width) × 5 mm (thickness) was taken for damage assessment. Damage is in the form of a hole of 2 mm in depth with a 3 mm diameter. Initially, when there was no damage, there is no reflected wave, so all the wave packets at the sensors are the same. When a wave comes across the damage, the signal scatters, resulting in the formation of minor wave packets, as shown in Figures 4(a) and 4(b) [20]. Mu et al. [21] have given a damage localization approach based on diffracted wave property of the ultrasonic wave. In this paper, Carbon Steel, Q235, of 1250 mm (length) × 1250 mm (width) × 2 mm (thickness) is taken for analysis. Damage is in the form of a fatigue crack. When the wave reaches the damage region, it gets reflected, scattered and diffracted. When the obstruction’s size is far less than the sound wave’s wavelength, it still propagates as it would in the absence of the obstacle, hence damage could not be captured in such case. The sound wave diffracts around the obstruction and propagates to the other side of the obstacle if the obstacle length is similar to the
wavelength and is primarily reflected and the region behind the obstruction is silent when the obstacle's size is significantly larger, hence in these two conditions the damage affect wave signal. Also, the ultrasonic guided wave propagation method was utilized by Malik, and Chinchilla [22] to detect and locate damage in a composite beam. The damage is defined as delamination and transverse crack. Delamination was located between the 2nd and 3rd layers of the beam. The exact damage location was 0.45 meters, but the Bayesian inverse problem methodology has given 0.4494 meters. Thus, the deviation was only 0.13%. Further, on a 6.4 m tube, by El Mountassir et al. [23], they have carried out the damage identification and localization using a sparse estimating technique. The actual position of damage was at 3 meters but this technique estimated at 3 meters ± 0.41 mm. Here, ± 0.41 mm is the error of localization. Structural system when discretized to the higher order finite element, the ultrasonic wave was able to capture the damage, which was reported by Schmicker et al. [24]. This wave can also capture the damage caused by improper manufacturing, in carbon fibre reinforced plastic rods, that are used in gliders [25]. Also, the ultrasonic wave propagation technique for damage detection can be used in the polycrystalline medium stated by Adithya et al. [26].

3. 4. Lamb Wave Guided waves have been widely employed in the structural health monitoring and system identification of composite constructions, because of their long-distance propagation in complicated structures with low energy loss. Staszewski et al. [27] presented a damage detection methodology for composites based on active and passive approach. Lamb waves were used for online monitoring. However, there is propagation complexity of these waves which results in difficult analysis and interpretation. Requirements for baseline measurements are mainly the problem. Ng and Veidt [28] have conducted the numerical case study on the laminate composite square plate of 0.250 mm and thickness of 0.0016 mm. Damage is modelled as through-hole. In the output, the out-of-plane displacement of waves is shown on the plate. The demonstration of lamb wave propagating in all directions was seen at 15.8 µs. Further, at 26.8 µs and 36.8 µs, the scattering of the wave at the through-hole has been observed. Now, the signal with damage and undamaged case, are combine plotted to identify the presence of damage. This plot has represented the presence of the damage. Mishra et al. [29] have shown the displacement along the y-direction of the beam in crack identification.

4. NUMERICAL CASE STUDY

4. 1. Finite Element Model In this section, the finite element model of a plate having a density of 2500 Kg/m³, Young’s modulus of elasticity 62 GPa and Poisson’s ratio 0.33 is modelled in ABAQUS CAE. The length of the plate is 100 mm, and the width of 100 mm has a thickness of 1 mm. To study the behaviour of a wave in a system with different forms of damage four most common forms of damage are considered in a particular system.

Case 1: Damage in the form of a hole
Case 2: Damage in the form of fatigue crack
Case 3: Damage in the form of crack near bottom edge
Case 4: Damage in the form of crack near the right edge

The simulation model of the initial state and all four damage cases are shown in Figures 1(a), 1(b), 1(c), 1(d) and 1(e), respectively.

4. 2. Simulation of Finite Element Model An impulse signal, which is applied as concentrated force(CF3) in normal direction on excitor node to generate lamb wave is shown in Figure 2. The wave field model of the initial state is shown in Figure 3. and for all four damage cases are shown in Figure 4 to 7.

4. 3. Analysis of Simulation Result In all the damaged model and the initial model, the exciter S1 is actuated, and the sensor S2 collect the wave motion. The signals collected by the sensor are shown in Figure 3 to

![Figure 1](image-url) Simulation model of (a) Initial state (b) Damage state in Case 1 (c) Damage state in Case 2 (d) Damage state in Case 3 (e) Damage state in Case 4
Figure 2. Impulse signal applied on the exciter.

Figure 3. Wave field of baseline model at (a) 9μs, (b) 48μs, (c) 66μs, (d) 96μs

Figure 4. Wave field of damage model at (a) 9μs, (b) 48μs, (c) 66μs, (d) 96μs

Figure 5. Wave field of damage model, at (a) 9μs, (b) 48μs, (c) 66μs, (d) 96μs

Figure 6. Wave field of damage model, at (a) 9μs, (b) 48μs, (c) 66μs, (d) 96μs

Figure 7. Initially, the wave has not reached the damage location, when the signal was collected by S2. So, it is visualized that the wave packets in all five models shown in Figures 3(a), 4(a), 5(a), 6(a), and 7(a) are identical in shape. The wave pattern at 48 μs, in Figures 6(b) and 7(b) is still the same as that of initial stage Figure 3(b), this is because the wave has not reached the damage portion.
Whereas, the wave when interacting with the damage case 1 and case 2, as shown in Figures 4(b) and 5(b); it is observed that, it gets reflected in this region. Due to this, the phase change occurs and the intensity of the wave is also affected. The wave field at this time interval when compared with the initial model, will provide the existence of damage. Further at time 66 μs (approximately) the minor wave packet formation is observed, which appears only in the damage stage and not in the initial stage. As shown in Figures 4(c) and 6(c) is due to phase change. The red mark is shown in figures is clearly showing the phase change. Further, when the wave reaches the boundary of the plate, then again due to phase change the wave packets are formed at each boundary. After some time all the wave packets are merged. If the sensor capture signal at this interval the complex wave signal may not give an accurate result. If there is a crack at the edge, then the sensor must be present near it. At the crack tip, the wave pattern is affected, and also it forms another wave packet.

In the following section, the signal of initial and damage state is plotted combinely, for the following case:
Case 1: Combine signal of initial state and damage in the form of a hole.
Case 2: Combine signal of initial state and damage in the form of fatigue crack
Case 3: Combine signal of initial state and damage in the form of crack near the right edge
Case 4: Combine signal of initial state and damage in the form of crack near the bottom edge

In all the graphs, it can be visualized that the signal of the initial stage and damage signal is initially overlapping. Only there is a uniform variation in magnitude in fatigue and hole in Figures 8(a) and 8(b). Although this variation is different in both damage cases, the pattern is similar. At the time interval, approximately between 78 μs to 90 μs the magnitude of both signals of Figures 8(a) and 8(b) are showing variation which can be due to some change in the model, that may be due to the presence of damage. The signal in this region is showing variation because it is affected by the difference between the initial and the damage model. The location of both fatigue crack and hole as damage is the same. This is the reason for the variation in the signal at the same time interval. Thus it can be remarked that if the location of the damage is the same then the signal may show variation at a particular time interval irrespective of the type of damage.

If the crack is present near the bottom edge then at the time interval between 55 μs to 78.7 μs (approximately) as shown in Figure 8(d), the signal of damage is not overlapping with the initial stage signal. Thus it can be remarked that it is due to the existence of some damage. If the crack is present near edge of the system, it is observed that the magnitude of the signal due to the crack is showing uniform decrease in comparison to the initial stage signal. As the crack is present far from the sensor near the edge, so the wave reflected at the boundary and the crack are combinedly showing in the signal. Due to this complexity sensor is not able to detect the variation accurately, hence for such a case the infinite boundary condition could provide a better solution.

5. FINAL REMARKS

Damage if present in the purview of the sensor will only change the behaviour of the wave. Also, the finite system
produces wave packets due to reflection at the boundaries, which when combined with the signal of the damage portion, will create a complex signal. Hence infinite boundaries could provide better results. Also, if the location of the damage is the same, then the signal may show variation at a particular time interval irrespective of the type of damage. Only the magnitude will vary. Each damage is creating its own wave packet, this could vary the signal.

Shear wave is commonly studied for monitoring composite structures, periodically layered composite structures with defects and adhesively bonded joints and plate like structures such as laminated composite plates. These waves are very mild dispersive in composites compared to the Lamb wave, so they are simpler for damage diagnostic. The wave is more sensitive to outer surface fractures in the bend as frequency increases. The wave gets scattered at the crack location. In comparison, Rayleigh wave has been used for damage detection in metallic, civil structures, geo-materials and geophysical problems in literature. It involves mostly the propagation of single wave mode and a nearly non-dispersive nature. This wave alters its behaviour even at 120 µm size crack, hence they are also sensitive to extremely small cracks. Also, the wave travels to a very large extent as the speed is independent of the frequency. It easily propagates in the system containing a curved surface portion, with very little distortion if the wavelength is smaller than the radius of curvature. This wave can be used for the detection of near-surface cracks and defects. The ultrasonic wave can propagate through bounded structural media, such as thin plates and shells bounded by stress-free surfaces, composite beams, tubes, and rods. But, the bounded configuration of the waveguide often results in multi-modal, dispersive and attenuation, due to which the signal becomes more complex. In addition, the waves are reflected by the discontinuities and boundaries, leading to the cluttering of the response. Thus, analysis and implementation of this wave propagation of guided wave and their interaction with damage are more complex. The above phenomena often camouflage the damage signature. Which can be sorted out through a proper selection of transducers, a good understanding of the wave characteristics, and signal processing schemes. Lamb wave propagates through thin plates and shells bounded by stress-free surfaces. Also, it can travel as an integral part of an aircraft. But, a major hindrance to the use of Lamb wave for SHM of real-life structures is the complexities associated with the multi-modal nature of Lamb wave propagation. In order to lower the complexities, the frequency of excitation is kept below the cut-off frequencies of the higher Lamb wave modes. The consequence is a decrease in sensitivity to detect minute damages of size smaller than the wavelength of excitation. At higher frequency or in thicker plates, Lamb wave gets converted to Rayleigh wave propagation.

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