Task-space coordinated tracking of multiple heterogeneous manipulators via controller-estimator approaches

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Abstract

This paper studies the task-space coordinated tracking of a time-varying leader for multiple heterogeneous manipulators (MHMs), containing redundant manipulators and nonredundant ones. Different from the traditional coordinated control, distributed controller-estimator algorithms (DCEA), which consist of local algorithms and networked algorithms, are developed for MHMs with parametric uncertainties and input disturbances. By invoking differential inclusions, nonsmooth analysis, and input-to-state stability, some conditions (including sufficient conditions, necessary and sufficient conditions) on the asymptotic stability of the task-space tracking errors and the subtask errors are developed. Simulation results are given to show the effectiveness of the presented DCEA.

Keywords: task-space coordinated tracking, multiple heterogeneous manipulators (MHMs), redundant manipulator, distributed controller-estimator algorithm (DCEA).
1. Introduction

Coordinated control has been diffusely invoked in many practical applications of multiple manipulators, including coordination of bilateral human-swarm systems [1], single-master-multiple-slaves teleoperation [2], dual-user shared teleoperation [3]-[6], multi-robot teleoperation [7]-[10], multi-fingered grasping and manipulation [11, 12], due to their prominent superiority comparing with traditional centralized control, such as stronger stability, less energy consumption, greater operational efficiency [13]-[24].

Existing works focused on joint-space synchronization of two manipulators, namely, master-slave bilateral teleoperators, using various control technologies, such as adaptive control, passivity-based control, proportional-derivative control [25]-[28]. However, the single-master-single-slave framework containing two manipulators is already too simple for some complicated practical tasks. By invoking cooperative concurrent control, synchronization of interconnected Lagrangian systems had been investigated [29]. Distributed containment control had been developed for nonlinear multi-agent systems in the presence of dynamical uncertainties and external disturbances and applied to Lagrangian networks [30]. However, the kinematics of robotic manipulators had not been taken into consideration in the above literatures.

In reality, task-space algorithms considering kinematics of manipulators are more practical and applicative comparing with joint-space algorithms. It thus motivates a group of researches on task-space algorithms. In presence of kinematic and dynamic uncertainties, task-space synchronization had been addressed for multiple manipulators under strong connected graphs by invoking passivity control [31] and adaptive control [32]. Note that the above literatures mainly focuses on motion control of kinematically identical and nonredundant robotic manipulators with parametric uncertainties.

Redundant manipulators can achieve more performance benefits in contrast to nonredundant ones [33, 34]. On the other hand, multiple manipulators containing both redundant and nonredundant individuals, namely, multiple heterogeneous manipulators (MHMs), are necessary and inevitable in some natural and man-made systems, e.g., human hands and multi-fingered hands [11, 12]. Inspired by conceivable performance benefits of MHMs, task-space synchronization of MHMs under balanced connected topologies had been achieved via the passivity control technology in [35], which can synchronize the combination signals of task-space position/velocity tracking errors to zero.
In this paper, we present some novel DCEA to achieve the task-space coordinated tracking of a time-varying leader for MHMs with parametric uncertainties and input disturbances, and the interaction topology of the MHMs are assumed to contain a spanning tree. The main contributions are summarised as follows. 1) Different from the coordination algorithms considering dynamics of Euler-Lagrange systems [29, 30], both the dynamics and kinematics are considered, which is more practical and challenging. 2) Different from the coordination algorithms for identical manipulators with a constant agreement value [31, 32], the tracking of a time-varying leader for MHMs is studied. 3) Different from balanced interaction topologies studied in [35], digraphs containing a spanning tree are invoked to describe the interaction topology. 4) The novel controller-estimator structure provides a theoretical guidance for coordinated control of various networked multi-agent systems, whose dynamics are complex and strong nonlinear.

The rest of the paper is structured as follows. In Section 2, the preliminaries are presented. In Section 3, the main results are presented. In Section 4, the simulations are given. In Section 5, the conclusion is proposed.

Notation: $\mathbb{R}^n$ represents the $n$-dimensional Euclidean space, $I_n$ denotes the $n \times n$ identity matrix, $1_n = [1, \cdots, 1]^T$ represents the $n$-dimensional column vector, $\| \cdot \|$ represents the Euclidean norm, $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of the corresponding matrix.

2. Preliminaries

2.1. Dynamics and Kinematics

The dynamics and kinematics of the $i$th individual in the MHMs with input disturbances are given as follows [33]:

$$\begin{cases} H_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) + d_i(t) = u_i, \\ x_i = \psi_i(q_i), \quad \dot{x}_i = J_i(q_i)\dot{q}_i, \end{cases}$$

(1)

where $i \in \mathcal{V} = \{1, \ldots, n\}$, $t \in \mathcal{Q} = [t_0, \infty)$, $t_0 \geq 0$ is the initial time, $q_i \in \mathbb{R}^{p_i}$ denotes the position in the joint space, $p_i \geq 2$ represents the degrees-of-freedom (DOF) of the $i$th manipulator, $H_i(q_i) \in \mathbb{R}^{p_i \times p_i}$ is the inertia matrix, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{p_i \times p_i}$ represents the centripetal-Coriolis matrix, $g_i(q_i) \in \mathbb{R}^{p_i}$ denotes the gravity vector, $d_i(t) \in \mathbb{R}^{p_i}$ represents the input disturbance, $u_i \in \mathbb{R}^{p_i}$ stands for the torque input, $x_i \in \Omega_x^i \subseteq \mathbb{R}^m$ denotes the
task-space position, $\Omega^i_x$ represents the work space, $\psi_i(q_i) \in \mathbb{R}^m$ denotes the forward kinematics, $m \geq 2$ denotes the task-space dimension and is thus the minimum number of DOF required to perform a given end-effector task, $J_i(q_i) = \partial \psi_i(q_i)/\partial q_i \in \mathbb{R}^{m \times p_i}$ stands for the Jacobian matrix.

The MHMs contain redundant manipulators and nonredundant ones, which means $\mathcal{V}$ consists of two subsets $\mathcal{E} = \{ i \in \mathcal{V} \mid p_i = m \}$ and $\mathcal{F} = \{ i \in \mathcal{V} \mid p_i > m \}$, i.e., the manipulators in $\mathcal{E}$ are nonredundant and the ones in $\mathcal{F}$ are redundant. It is worthy to point out that a nonredundant manipulator has just a necessary number of DOF to perform a given end-effector task (i.e., main task) and a redundant manipulator has more DOF than the necessary number, which results in an infinite number of joint configurations to the inverse-kinematics problem. The redundancy can improve the functionality and flexibility of robotic manipulators and many subtasks (including manipulability enhancement, mechanical limit avoidance, and obstacle avoidance) can be obtained by choosing appropriate joint configurations. Then the following algebraic operation, that will be invoked hereinafter, is given by

$$J^*_i = \begin{cases} J^{-1}_i, & i \in \mathcal{E}, \\ J^\dagger_i, & i \in \mathcal{F}, \end{cases}$$

where for $i \in \mathcal{E}$, $J^*_i = J^{-1}_i \in \mathbb{R}^{m \times m}$ denotes the normal inverse of $J_i(q_i)$; for $i \in \mathcal{F}$, $J^*_i = J^\dagger_i = J^T_i(J_iJ^T_i)^{-1} \in \mathbb{R}^{p_i \times m}$ represents the pseudoinverse of $J_i(q_i)$ and satisfies the well-known Moore-Penrose conditions [36]. The following lemma that will be used hereinafter is given.

**Lemma 1.** For $i \in \mathcal{F}$, the algebraic operation $J^*_i$ satisfies

$$J_i(I_{p_i} - J^*_i J_i) = 0, \quad (I_{p_i} - J^*_i J_i)J^*_i = 0,$$

$$I_{p_i} - J^*_i J_i = I_{p_i} - J^*_i J_i.$$ 

For any $i \in \mathcal{V}$, the properties of system (11) are given as follows [35, 37].

(P1) $H_i(q_i)$ is positive definite. $H_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric;
(P2) The dynamic terms $H_i(q_i), g_i(q_i), d_i(t)$ are bounded for all possible $q_i$, and $\|C_i(q_i, \dot{q}_i)\| \leq \bar{c}_i\|\dot{q}_i\|$, where $\bar{c}_i > 0$ is a positive constant;
(P3) The dynamic terms can be parameterized, i.e., $H_i(q_i) x + C_i(q_i, \dot{q}_i) y + g_i(q_i) = Y_i(q_i, \dot{q}_i, y, x) \vartheta_i$, where $x, y$ are any proper vectors, $Y_i(q_i, \dot{q}_i, y, x)$ is the regressor and $\vartheta_i$ is a set of constant dynamic parameters.
Remark 1. By the actual characteristics of MHMs, we assume that the kinematic terms $J_i(q_i)$ and $J_i^\#(q_i)$ are bounded; the kinematic singularities are avoided, i.e., $\text{rank}(J_i^\#(q_i)) = m$, for $i \in V$. The above assumptions are the general properties of multiple manipulators, see [33, 37] for details.

2.2. Graph Theory

The interaction of the MHMs is denoted by a digraph $G = \{V, \xi, A\}$, where $V$ is the node set, $\xi \subseteq V \times V$ is the edge set, $A = [\varepsilon_{ij}]_{n \times n}$ is the weighted adjacency matrix with nonnegative adjacency elements. An edge in $G$ is denoted by an ordered pair $(v_i, v_j)$. $(v_i, v_j) \in \xi$ if and only if node $j$ (i.e., the $j$th manipulator) can directly access the information of node $i$. $N_j = \{i \in V | (v_i, v_j) \in \xi\}$ denotes the neighbor set of node $j$. $A$ is defined as $i \in N_j \iff \varepsilon_{ji} > 0$, otherwise $\varepsilon_{ji} = 0$, $\forall i, j \in V$. $D = \text{diag}\{d_1, \ldots, d_n\}$ is the degree matrix, where $d_i = \sum_{j \in N_i} \varepsilon_{ij}$. The Laplacian matrix is defined as $L = D - A$. A directed path is an ordered sequence $v_1, v_2, \ldots, v_{\omega}$ satisfying that any ordered pair of vertices appearing consecutively in the sequence is an edge of the digraph $G$. A digraph contains a spanning tree if there exists a root node that has a directed path to all the other nodes. $B = [b_1, \ldots, b_n]^T$ is the weight vector between the $n$ nodes and the leader, where $b_i > 0$ if the states of the leader is available to node $i$, namely, node $i$ is pinned; $b_i = 0$ otherwise. The states of the leader (node 0) $x_0, v_0$ and $a_0 \in \mathbb{R}^m$ satisfy $\dot{x}_0 = v_0, \dot{v}_0 = a_0$. The following assumptions that will be used hereinafter are given.

(A1) The leader has a directed path to all the nodes in $G$ under $B$;
(A2) $\|v_0\|_\infty < \beta_1$, $\|a_0\|_\infty < \beta_2$ and $\|\dot{a}_0\|_\infty < \beta_3$, where $\beta_1, \beta_2$ and $\beta_3$ are positive constants.

Remark 2. By Assumption A2, the derivatives of the states of the leader $x_0$, $v_0$ and $a_0$ are bounded, which happens to be the actual characteristics of the trajectories that can be reachable by the real-world manipulators described by Euler-Lagrange equations [38].

2.3. Problem Statement

The control tasks of this paper are presented in this section. It can be seen that for given $\dot{x}_i, \dot{x}_i = J_i(q_i)\dot{q}_i$ admits an infinite number of $\dot{q}_i$ when $i \in \mathcal{F}$ and only a single solution when $i \in \mathcal{E}$. Thus, to accomplish a given end-effector task, there exist only a single joint configuration for nonredundant manipulators and an infinite number of joint configurations for redundant
manipulators. Let the given end-effector task be called maintask and the tracking of a class of special joint configuration be called subtasks. Then the control tasks in this paper can be divided into the maintask and the subtask. Moreover, note that the nonredundant manipulators which have only one joint configuration for a given end-effector task (i.e., maintask) cannot accomplish the subtasks.

Maintask: The maintask is to design distributed input $u_i$ with the states of node $i$ (the $i$th manipulator) and its neighbour set such that the task-space coordinated tracking can be accomplished, i.e., $x_i \rightarrow x_0$ and $\dot{x}_i \rightarrow v_0$ as $t \rightarrow \infty$, $\forall i \in V$.

Subtask: The subtask, e.g., manipulability enhancement, mechanical limit avoidance, and obstacle avoidance, can only be accomplished by redundant manipulators. For different subtasks with respect to different redundant manipulators, we can construct a corresponding auxiliary vector $\varphi_i(t)$ to denote the gradients of some performance indices with respect to the corresponding subtask. Then the subtask for the $i$th manipulator is said to be achieved if $e_{si} \rightarrow 0$ as $t \rightarrow \infty$, where $i \in F$, $e_{si} = (I_{pi} - J_i^T J_i)(\dot{q}_i - \varphi_i)$ denotes the subtask tracking error [39].

Remark 3. The subtasks give more functional constraints on joint configurations and thus cannot be accomplished by nonredundant manipulators because they have only one joint configuration for their inverse-kinematics. Redundant manipulators have an additional DOF to accomplish the subtasks, which gives higher robustness and wider operational space comparing with non-redundant ones. However, the control design for redundant manipulators is more complex and challenging. Human arms, elephant trunks, and snakes are some examples of this kind of redundant system. The redundant is applicable in many practical applications and tough challenging in theoretical analysis [11, 12, 31].

3. Task-space Coordinated Tracking of Multiple Heterogeneous Manipulators

This section studies the task-space coordinated tracking for MHMs with a time-varying leader, in which the position vector of the leader in the generalized coordinate is assumed to be bounded up to its third derivative, which is also invoked in [38].
3.1. Distributed Controller-Estimator Algorithms

In this section, DCEA for task-space coordinated tracking of MHMs are developed. Let $\dot{x}_i$, $\ddot{x}_i$ and $\hat{a}_i \in \mathbb{R}^m$ be, respectively, the estimated value of $x_0$, $v_0$ and $a_0$ for the $i$th manipulator. Considering the heterogeneity of the MHMs, a joint-space auxiliary velocity $\dot{\hat{x}} \in \mathbb{R}^n$ is given by

$$\dot{\hat{x}}_i = J_i^T (\hat{v}_i - \alpha_i (x_i - \hat{x}_i)) + (I_{p_i} - J_i^2 J_i) \varphi_i,$$

where $\alpha_i$ is a positive constant, $\varphi_i \in \mathbb{R}^m$ is the gradient of some performance indices with respect to the redundant manipulators. Then the joint-space auxiliary acceleration $\ddot{\hat{x}}_i$ is defined as

$$\ddot{\hat{x}}_i = J_i^T (\hat{v}_i - \alpha_i (x_i - \hat{x}_i)) + J_i^T (\hat{a}_i - \alpha_i (\hat{x}_i - \hat{v}_i)) + \frac{d}{dt} [(I_{p_i} - J_i^2 J_i) \varphi_i].$$

Let $\zeta_i = \text{col}(\hat{x}_i, \hat{v}_i, \hat{a}_i) \in \mathbb{R}^3m$. Then for $i, j \in \mathcal{V}$, we define

$$\sigma_{ij} = \zeta_i - \zeta_j,$$

especially, $\sigma_i = \zeta_i - \text{col}(x_0, v_0, a_0)$.

Let $\bar{s}_i = \dot{\hat{x}}_i - \hat{\varphi}$. In the presence of uncertain dynamics and input disturbance, the DCEA for MHMs consist of the control law

$$u_i = Y_i(q_i, \dot{q}_i, \ddot{q}_i, \hat{v}_i, \hat{\varphi}) \hat{v}_i - J_i^T \varphi_i \bar{s}_i - \varphi_i \bar{s}_i - \varphi_i \text{sgn}(\bar{s}_i),$$

and the distributed adaptive estimators

$$\begin{cases}
\dot{\hat{v}}_i = -T_i Y_i^T(q_i, \dot{q}_i, \dot{\hat{v}}_i, \ddot{\hat{v}}_i, \hat{\varphi}), \\
\dot{\zeta}_i = - \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \otimes I_m \text{sgn} \left( \sum_{j \in \mathcal{N}_i} \varepsilon_{ij} \sigma_{ij} + b_i \sigma_i \right),
\end{cases}$$

where $\beta_1$, $\beta_2$ and $\beta_3$ are given in Assumption A2, $\otimes$ denotes the Kronecker product, $\sigma_{ij}$ is given in (4), $\dot{\varphi}_i$ is the estimated value of $\dot{\varphi}_i$, $Y_i(q_i, \dot{q}_i, \dot{\hat{v}}_i, \ddot{\hat{v}}_i, \hat{\varphi}) = \dot{\hat{\varphi}}_i = \hat{C}_i(q_i, \dot{q}_i) \dot{\hat{q}}_i + \hat{\varphi}_i$, and $\varphi_i$ are the estimates of $\varphi_i$, $C_i(q_i, \dot{q}_i)$, and $g_i(q_i)$ respectively, $\varphi_i \in \mathbb{R}^m$, $\varphi_i \in \mathbb{R}^{n \times m}$ and $\varphi_i \in \mathbb{R}^{m \times m}$ are positive definite matrices, $T_i$ is a designed diagonal positive definite matrix with an appropriate dimension.
Remark 4. It can be seen from the definition of $J_1^i$ that $I_{p_i} - J_1^i J_i = 0$ for $i \in \mathcal{E}$. Therefore, (2) implies that for redundant manipulators and nonredundant ones, $\dot{q}_{ri}$ is distinguishing, namely,

$$\dot{q}_{ri} = \begin{cases} J_i^{-1}(\dot{v}_i - \alpha_i(x_i - \hat{x}_i)), & i \in \mathcal{E}, \\ J_i^i(\dot{v}_i - \alpha_i(x_i - \hat{x}_i)) + (I_{p_i} - J_i^i J_i)\varphi_i, & i \in \mathcal{F}. \end{cases}$$

Besides, $\ddot{q}_{ri}$ is distinguishing with respect to redundant manipulators and nonredundant ones. It follows that the control law (5) is distinguishingly designed with respect to different sets of manipulators.

3.2. Boundedness Analysis

In this section, by analyzing the boundedness of the system states, the simplification of the close-loop dynamics is given.

The normal variables $\dot{q}_{ri}$, $\ddot{q}_{ri}$ and $s_i$ with respect to the auxiliary variables $\dot{q}_{ri}$, $\ddot{q}_{ri}$ and $\dot{s}_i$, containing estimated states, are defined as

$$\dot{q}_{ri} = J_i^i(v_0 - \alpha_i(x_i - x_0)) + (I_{p_i} - J_i^i J_i)\varphi_i,$$

$$\ddot{q}_{ri} = J_i^i(v_0 - \alpha_i(x_i - x_0)) + J_i^i(a_0 - \alpha_i(\dot{x}_i - v_0)) + \frac{d}{dt}[(I_{p_i} - J_i^i J_i)\varphi_i], \quad (7)$$

$$s_i = \dot{q}_i - \dot{q}_{ri},$$

where the normal variables are formed based on the estimated variables by replacing $\hat{x}_i$, $\dot{v}_i$ and $\dot{a}_i$ with $x_0$, $v_0$ and $a_0$ respectively. Then we define

$$\dot{q}_{ri} = \dot{q}_{ri} - \dot{q}_{ri} = J_i^i(\dot{v}_i - v_0 + \alpha_i(\hat{x}_i - x_0)), \quad (8)$$

and

$$\ddot{q}_{ri} = \ddot{q}_{ri} - \ddot{q}_{ri} = J_i^i(\dot{v}_i - v_0 + \alpha_i(\hat{x}_i - x_0)) + J_i^i(\dot{a}_i - a_0 + \alpha_i(\dot{v}_i - v_0)). \quad (9)$$

Let $\ddot{s}_i = \ddot{s}_i - \ddot{s}_i = -\dot{q}_{ri}$. Substituting (5) into (1) yields the following
close-loop dynamics
\[
H_i(q_i)\dot{s}_i + C_i(q_i, \dot{q}_i)s_i + d_i(t) + J_i^T K_{xi} J_i s_i + \mathcal{K}_{si} s_i + \mathcal{K}_{ri} \text{sgn}(s_i)
\]
\[= Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{q}_{ri}) \tilde{\phi}_i + \tilde{f}_i(t),
\]
where \(\tilde{\phi}_i = \hat{\phi}_i - \phi_i\) and \(\tilde{f}_i(t) = \dot{H}_i(q_i) \tilde{q}_{ri} + \dot{C}_i(q_i, \dot{q}_i) \tilde{q}_{ri} - J_i^T K_{xi} J_i \tilde{s}_i - \mathcal{K}_{si} \tilde{s}_i - \mathcal{K}_{ri} \text{sgn}(\tilde{s}_i) - \text{sgn}(s_i)).\) The boundedness of the states \(q_i(t), \dot{q}_i(t), s_i(t),\) and \(\dot{s}_i(t),\) which will be used hereinafter, is analyzed in the following theorem.

**Theorem 1.** Suppose that Assumptions A1 and A2 hold. The distributed sliding-mode estimator (6b) guarantees that there exists a settle time \(t_f \in (t_0, \infty)\) such that \(\tilde{f}_i(t) = 0\) when \(t \geq t_f, \forall i \in \mathcal{V}.\) Moreover, using the DCEA (2) and (6) for (1), the states \(q_i(t), \dot{q}_i(t), s_i(t),\) and \(\dot{s}_i(t),\) will remain bounded for bounded initial values when \(t \in Q_f = [t_0, t_f).\)

**Proof.** For the first presentation, we prove that there exists a finite time \(t_f \in (t_0, \infty)\) such that \(\tilde{f}_i(t) = 0\) when \(t \geq t_f.\) Because the right-hand side of (6b) is discontinuous, the differential inclusions and nonsmooth analysis are invoked for convergence analysis of (6b) [40, 41]. The error dynamics of (6b) can be rewritten as

\[
\dot{\sigma}_i \in a.e. \mathbb{K} \left\{ - \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \end{bmatrix} \otimes I_m \right\} \text{sgn} \left( \sum_{j \in N_i} \varepsilon_{ij} \sigma_{ij} + b_i \sigma_i \right) - \begin{bmatrix} \dot{x}_i \varepsilon \sigma_i \dot{x}_i \varepsilon \sigma_i \dot{x}_i \varepsilon \sigma_i \end{bmatrix},
\]

where \(a.e.\) stands for “almost everywhere” refer to almost all \(t \in Q,\) and \(\mathbb{K}\{\cdot\}\) denotes the differential inclusion [41]. By Theorem 3.1 in [14], under Assumptions A1 and A2, it can be obtained that \(\sigma_i = 0\) when \(t \geq t_f\) and \(t_f\) is bounded by

\[
t_f \leq t_{f_{\max}} := \max(t_{f1}, t_{f2}, t_{f3}) < \infty,
\]

\[
t_{f1} = t_0 + \max_{x \in \mathcal{V}} \| \dot{x}_i(t_0) - x_0(t_0) \|_{\infty},
\]

\[
t_{f2} = t_0 + \max_{x \in \mathcal{V}} \| \dot{x}_i(t_0) - x_0(t_0) \|_{\infty},
\]

\[
t_{f3} = t_0 + \max_{x \in \mathcal{V}} \| \dot{x}_i(t_0) - x_0(t_0) \|_{\infty},
\]

where \(\max(\cdot)\) and \(\sup(\cdot)\) denote the maximal value and the supremum respectively. It follows from (8) and (9) that \(\dot{q}_{ri} = 0\) and \(\dot{q}_{ri} = 0\) when \(t \geq t_f.\)
Therefore, $\tilde{f}_i(t) = 0$ when $t \geq t_f$, $\forall i \in \mathcal{V}$.

For the second presentation, using the DCEA \((\ref{eq:5})\) and \((\ref{eq:6})\) for \((\ref{eq:1})\), it is shown that the states $q_i(t)$, $\dot{q}_i(t)$, and $s_i(t)$ will remain bounded for bounded initial values when $t \in Q_f$. Note that

$$
\left\| \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \otimes I_m \right\|_\infty \leq \max(\beta_1, \beta_2, \beta_3),
$$

and

$$
\left\| \text{sgn} \left( \sum_{j \in \mathcal{N}_i} \varepsilon_{ij} \sigma_{ij} + b_i \sigma_i \right) \right\|_\infty \leq 1.
$$

It thus follows from \((\ref{eq:6b})\) that

$$
\left\| \dot{\zeta}_i \right\|_\infty \leq \max(\beta_1, \beta_2, \beta_3), \quad \forall i \in \mathcal{V},
$$

which means

$$
\sup_{t \in Q_f} \left\| \zeta_i(t) \right\|_\infty \leq \max(\beta_1, \beta_2, \beta_3)(t_f - t_0) < \infty, \quad \forall i \in \mathcal{V}.
$$

It follows that $\hat{x}_i(t)$, $\hat{v}_i(t)$, and $\hat{a}_i(t)$ remain bounded for bounded initial values $\hat{x}_i(t_0)$, $\hat{v}_i(t_0)$, and $\hat{a}_i(t_0)$ when $t \in Q_f$. For bounded $q_i$ and $\dot{q}_i$, the kinematics in \((\ref{eq:1})\) implies that $x_i$ and $\dot{x}_i$ remain bounded. Then \((\ref{eq:2})\) and \((\ref{eq:3})\) implies that $\hat{q}_ri$, $\ddot{q}_ri$, and $\dot{s}_i$ remain bounded for bounded $q_i$ and $\dot{q}_i$. By \((\ref{eq:7})\)-\((\ref{eq:9})\), $\dot{q}_ri$, $\ddot{q}_ri$, $s_i$, $\hat{\dot{q}}_ri$, $\ddot{\dot{q}}_ri$, and $\dot{s}_i$ remain bounded for bounded $q_i$ and $\dot{q}_i$. By Property P2 and Remark 1, $Y_i(q_i, \dot{q}_i, \ddot{q}_ri, \dot{s}_i)$ are bounded for bounded $q_i$ and $\dot{q}_i$. Then \((\ref{eq:6a})\) implies that $\hat{q}_i(t)$ remain bounded when $t \in Q_f$ for bounded initial value $\hat{q}_i(t_0)$. It follows that $\tilde{f}_i(t)$ remain bounded for bounded $q_i$ and $\dot{q}_i$. By \((\ref{eq:10})\), $\dot{s}_i$ remains bounded for bounded $q_i$ and $\dot{q}_i$. Therefore, we can get that for bounded initial values $q_i(t_0)$ and $\dot{q}_i(t_0)$, the states $q_i(t)$, $\dot{q}_i(t)$, and $s_i(t)$ remain bounded when $t \in Q_f$. This completes the proof.

### 3.3. Convergence Analysis

In this section, based on Theorem \((\ref{thm:1})\) the convergence of the close-loop dynamics is analyzed. Differential inclusion and Filippov solution \([41]\) are invoked because the presented DCEA are nonsmooth.
By Theorem 1, \( \dot{s}_i = s_i \) and \( \tilde{f}_i(t) = 0 \) when \( t \in \tilde{Q}_f = [t_f, \infty) \). Then when \( t \in \tilde{Q}_f \), the combination of (9) and (10) yields the following cascade system

\[
\begin{aligned}
\dot{s}_i &= H_i^{-1}(q_i)[-C_i(q_i, \dot{q}_i)s_i - J_i^T K_{xi} J_i s_i - K_{si} s_i \\
&\quad + Y_i(q_i, \dot{q}_i, \dot{q}_ri, \dot{q}_ri) \tilde{v}_i + \tilde{h}_i(t)], \\
x_i &= \psi_i(q_i), \quad \hat{x}_i = J_i \hat{q}_i, \\
\dot{\tilde{v}}_i &= -T_i Y_i^T(q_i, \dot{q}_i, \dot{q}_ri, \dot{q}_ri) s_i,
\end{aligned}
\]  

(11)

where \( q_i(t_f), \dot{q}_i(t_f), x_i(t_f), \dot{x}_i(t_f), \) and \( s_i(t_f) \) are bounded, \( \tilde{h}_i(t) = -d_i(t) - \lambda(T) s_i - K_{si} \) denotes the combination of the nonsmooth and uncertain terms in the system dynamics. Then the following theorem is given.

**Theorem 2.** Suppose that Assumptions A1 and A2 hold. Using the DCEA (2) and (3) for (4), if \( \lambda_{min}(K_{xi}) > 0, \lambda_{min}(K_{si}) > 0, \lambda_{min}(K_{ri}) \geq \sup_{t \in Q} \|d_i(t)\|, \) and \( \lambda_{min}(T_i) > 0 \), then the control tasks in this paper can be achieved, namely, \( x_i \to x_0 \) and \( \dot{x}_i \to v_0 \) as \( t \to \infty \), \( \forall i \in \mathcal{V} \); \( e_{si} \to 0 \) as \( t \to \infty \), \( \forall i \in \mathcal{F} \).

**Proof.** The proof proceeds in three steps. First, the passivity of the close-loop dynamics (11) is analyzed. Second, the convergence of \( s_i \) and \( J_i s_i \) is obtained using differential inclusions and nonsmooth analysis. Finally, the stability of \( x_i \) and \( e_{si} \) is shown invoking kinematic analysis of nonredundant and redundant manipulators.

The first presentation presents the passivity of the close-loop dynamics. For system (11), consider the following storage function

\[
V_{i1} = \frac{1}{2}(s_i^T H_i(q_i) s_i + \tilde{v}_i^T T_i^{-1} \tilde{v}_i).
\]

Differentiating the storage function along (11) gives

\[
\dot{V}_{i1} = \frac{1}{2} s_i^T \dot{H}_i(q_i) s_i + s_i^T H_i(q_i) \dot{s}_i + \tilde{v}_i^T T_i^{-1} \tilde{v}_i
\]

\[
= -s_i^T K_{si} s_i - s_i^T J_i^T(q_i) K_{xi} J_i(q_i) s_i + s_i^T \tilde{h}_i
\]

\[
\leq s_i^T \tilde{h}_i,
\]

where Property P1 is invoked to obtain the second equation. Integrating
both sides of the above inequality with respect to time \( t \in \bar{Q}_f \) provides

\[
V_{i1}(t) - V_{i1}(t_f) \leq \int_{t_f}^{t} s_i^T(\omega)\hat{h}_i(\omega)d\omega.
\] (12)

It thus follows that system (11) is passive with the mapping from the input \( \hat{h}_i(t) \) to the state \( s_i, \forall i \in \mathcal{V}, t \in \bar{Q}_f \) [12].

The second presentation analyze the convergence of \( s_i \) and \( J_is_i \) using nonsmooth analysis and the passivity property given in the first presentation.

Let \( \eta_i \in (0, \lambda_{\min}(K_{ri})) \) be a positive scalar. Then when \( t \in \bar{Q}_f \), consider the Lyapunov function candidate for system (11) as

\[
V_i = V_{i1} + \eta_i \int_{t_f}^{t} s_i^T(\omega)J_i^T(q_i(\omega))J_i(q_i(\omega))s_i(\omega)d\omega.
\]

Considering that \( \hat{h}_i(t) \) is nonsmooth and uncertain, the generalized time derivative is invoked to carry out more formal mathematical analysis; additionally, considering that the signum function is measurable and locally essentially bounded, the Filippov solution exists for (11) [40, 41]. Taking the generalized time derivative of \( V_i \) along (11) gives that

\[
\dot{V}_i = \mathbb{K}\{\frac{1}{2}s_i^T\dot{\mathcal{H}}_i(q_i)s_i + s_i^T[-C_i(q_i, \dot{q}_i)s_i - J_i^T(K_{xi})J_is_i - K_{si}s_i
\]

\[
+ Y_i(q_i, \dot{q}_i, \theta_r, \dot{\theta}_r)\tilde{\theta}_i + \tilde{h}_i(t)] - \tilde{Y}_iJ_i^T(q_i, \dot{q}_i, \theta_r, \dot{\theta}_r)s_i
\]

\[
+ \eta_i s_i^T J_i^T J_i s_i\}
\]

\[
\leq \mathbb{K}\{-s_i^T K_{si}s_i - s_i^T J_i^T(K_{xi} - \eta_i I_m)J_is_i - s_i^T K_{ri}\text{sgn}(s_i) - s_i^T d_i\}
\]

\[
\leq -s_i^T K_{si}s_i - s_i^T J_i^T(K_{xi} - \eta_i I_m)J_is_i - \|s_i\|\|\lambda_{\min}(K_{ri}) - \|d_i\|\|
\]

\[
\leq -s_i^T K_{si}s_i - s_i^T J_i^T(K_{xi} - \eta_i I_m)J_is_i - \|s_i\|\|\lambda_{\min}(K_{ri}) - \sup_{t \in Q} \|d_i(t)\|\|
\]

\[
\leq -s_i^T K_{si}s_i - s_i^T J_i^T(K_{xi} - \eta_i I_m)J_is_i,
\]

where (12), \( \lambda_{\min}(K_{ri}) \geq \sup_{t \in Q} \|d_i(t)\| \), Property P1-P3, and the fact that \( \mathbb{K}\{f\} \equiv \{f\} \) if \( f \) is continuous [41] are invoked to obtain the above inequalities. Provided \( \lambda_{\min}(K_{xi}) > 0 \) and \( \lambda_{\min}(K_{si}) > 0 \), \( \dot{V}_i \) is negative definite, \( \forall i \in \mathcal{V} \). It follows that \( V_i \in \mathcal{L}_\infty \) then \( s_i, J_is_i, \tilde{\theta}_i \in \mathcal{L}_\infty, \forall i \in \mathcal{V} \). \( \tilde{\theta}_i \in \mathcal{L}_\infty \) implies that \( \tilde{\theta}_i \in \mathcal{L}_\infty \) because \( \tilde{\theta}_i \) is a vector of constant dynamic parameters,
∀ \in V, s_i, J_i s_i \in L_\infty gives that Y_i(q_i, \dot{q}_i, \ddot{q}_ri) \in L_\infty, \forall i \in V. Then \ref{eq:5} and \ref{eq:6} imply that the control input \( u_i \) is bounded. Thus, the closed-loop dynamics \ref{eq:11} implies that \( \dot{s_i} \in L_\infty; \) then \( s_i \) is uniformly continuous, \( \forall i \in V. \) Then invoking Corollary 2 in \cite{41}, \( s_i \to 0 \) and \( J_i s_i \to 0 \) as \( t \to \infty, \forall i \in V. \)

The third presentation shows that \( e_{si} \to 0 \) (\( \forall i \in F \)) as \( t \to \infty \) under the DCEA. Let \( e_i = x_i - x_0. \) \ref{eq:7} gives that

\[
\dot{e}_i = -\alpha_i e_i + J_i s_i. \tag{13}
\]

It thus follows from \cite{42} that \ref{eq:13} is input-to-state stable with respect to the input \( J_i(q_i)s_i \) and the state \( e_i; \) hence, \( J_i s_i \to 0 \) as \( t \to \infty \) implies that \( e_i \to 0 \) as \( t \to \infty, \) which means \( x_i \to x_0 \) and \( \dot{x}_i \to v_0 \) as \( t \to \infty, \forall i \in V, i.e., \) the main task is addressed. Additionally, by \ref{eq:7} and Lemma 1, for the redundant manipulators, namely, for \( i \in F, \)

\[
e_{si} = (I_{pi} - J_i^T J_i) (\dot{q}_i - \phi_i)
\]

\[
= (I_{pi} - J_i^T J_i) [(\dot{q}_i - J_i^T (v_0 - \alpha_i (x_i - x_0)) - (I_{pi} - J_i^T J_i) \phi_i]
\]

\[
= (I_{pi} - J_i^T J_i) (\dot{q}_i - \dot{q}_ri)
\]

\[
= (I_{pi} - J_i^T J_i) s_i.
\]

Therefore, \( s_i \to 0 \) as \( t \to \infty \) implies that \( e_{si} \to 0 \) as \( t \to \infty, \forall i \in F, i.e., \) the subtask is addressed. This completes the proof. \( \square \)

Note that the necessary and sufficient condition can be obtained by some simple transformation for Theorem 2. The proof of Corollary \ref{cor:1} can be easily obtained by contradiction and is omitted here.

**Corollary 1.** Suppose that \( \lambda_{min}(\mathcal{K}_{xi}) > 0, \lambda_{min}(\mathcal{K}_{si}) > 0, \lambda_{min}(\mathcal{K}_{ri}) \geq \sup_{t \in \Omega} \|d_i(t)\|, \lambda_{min}(\mathcal{T}_i) > 0, \) and Assumption A2 holds. Using \ref{eq:5} and \ref{eq:6} for \ref{eq:7}, the control tasks are achieved, namely, \( x_i \to x_0 \) and \( \dot{x}_i \to v_0 \) as \( t \to \infty, \forall i \in V; e_{si} \to 0 \) as \( t \to \infty, \forall i \in F, \) if and only if Assumption A1 holds.

It is worthy to point out that the DCEA \ref{eq:5} and \ref{eq:6} can also deal with the consensus problem, namely, leaderless coordination, presented in \cite{20} by some simple changes.
Corollary 2. Suppose that $G$ contains a spanning tree and Assumption A2 holds. Let estimators (6b) be replaced by

$$
\dot{\zeta}_i = - \left( \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \otimes I_m \right) \text{sgn} \left( \sum_{j \in N_i} \varepsilon_{ij} \sigma_{ij} \right).
$$

Using (5), (6a) and the above estimators for the MHMs (1), if $\lambda_{\min}(K_{xi}) > 0$, $\lambda_{\min}(K_{si}) > 0$, $\lambda_{\min}(K_{ri}) \geq \sup_{t \in \mathcal{Q}} \|d_i(t)\|$, and $\lambda_{\min}(T_i) > 0$, then consensus in [20] is achieved, meanwhile, the subtasks are obtained for $i \in \mathcal{F}$.

Let a switching topology $G(t) = \{V, \xi(t), A(t)\}$ be the system interaction. $A(t) = [\varepsilon_{ij}(t)]_{n \times n}$ is the weighted adjacency matrix. $B(t) = [b_1(t), \ldots, b_n(t)]^T$ is the time-varying nonnegative weight vector between the $n$ nodes and the leader. We can obtain the following corollary.

Corollary 3. Suppose that Assumption A2 holds and the leader is reachable to the MHMs under $G(t)$ and $B(t)$. Let (6b) be replaced by

$$
\dot{\zeta}_i = - \left( \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \otimes I_m \right) \text{sgn} \left( \sum_{j \in N_i} \varepsilon_{ij}(t) \sigma_{ij} + b_i(t) \sigma_i \right).
$$

Using (5), (6a) and the above estimators for the MHMs (1), if $\lambda_{\min}(K_{xi}) > 0$, $\lambda_{\min}(K_{si}) > 0$, $\lambda_{\min}(K_{ri}) \geq \sup_{t \in \mathcal{Q}} \|d_i(t)\|$, and $\lambda_{\min}(T_i) > 0$, then $x_i \to x_0$ and $\dot{x}_i \to v_0$ as $t \to \infty$, $\forall i \in V$; $e_{si} \to 0$ as $t \to \infty$, $\forall i \in F$.

Proof. The proof can be easily developed by the combination of Lemma 6 in [43] and Theorem 2 and is omitted here.

Assuming that $G$ is balanced connected or detail balanced, other conditions are identical, the above results still hold, meanwhile, the subtasks are obtained for all $i \in F$.

Remark 5. The DCEA (5) and (6) provide a distributed control framework for various multi-agent networks. Note that (5) and (6a) deal with the tracking of the estimated value of the $i$th manipulator using its states (local states), and are called local algorithms, while (6b) deals with the estimation of the states of the leader using both the states of the $i$th manipulator and its neighbour set, and is called networked algorithms. Thus, the DCEA consists of local algorithms and networked algorithms, which can be easily applied
to various nonlinear multi-agent networks by designing local algorithms and networked algorithms separately.

Remark 6. Note that Assumption A1 is less conservative than many other assumptions in recent works. For details, let $\bar{G}$ be the augmentation digraph by adding the leader and the weight vector $B$ to $G$, Assumption A1 means that $\bar{G}$ contains a spanning tree, which is less conservative than the following assumptions for graphs, including strong connected $[7, 31, 32]$, balanced connected $[33]$, detail balanced $[44]$, which means, less consumption is required to establish and maintain Assumption A1.

Remark 7. Assumption A1 implies that the presented DCEA have the following advantages. (1) If $G$ contains one or more edges, then only partial nodes are required to be pinned, which reflects the superiority of networked control. (2) By weighing the cost of maintaining interaction and pinning nodes, the interaction topology can be optimized.

Remark 8. It is worthy to point out that the leader considered in Theorem 2 is a bounded time-varying signal, which is necessary and inevitable in some natural and man-made systems, e.g., humanoid hands and multi-fingered hands $[11, 12]$.

4. Simulations

In this section, simulations are performed to show the effectiveness of the presented DCEA. The MHMs contain five two-DOF and two three-DOF planar manipulators, as shown in Fig.1 and the physical parameters are given in Tab.1. The dynamics and kinematics of the MHMs are adopted from $[31]$. The input disturbance $d_i(t)$ is a stochastic signal, whose $\infty$-norm is bounded by 40. The maintask is to obtain the tracking of the leader

$$x_0(t) = \begin{bmatrix} 1.2 + 0.5 \sin(\pi t) \\ 1.3 + 0.3 \cos(\pi t) \end{bmatrix}$$

for the end-effectors in the $XY$ plane. The MHMs are interacted invoking the digraph $G$ shown in Fig.2 where node 0 is the leader, nodes 1-5 are two-DOF nonredundant manipulators, nodes 6 and 7 are three-DOF redundant manipulators, and nodes 1, 3, 5, and 7 can access the information of the leader directly. For simplification, if node $i$ can access the information of node $j$
directly, \( \varepsilon_{ij} = 1 \); otherwise \( \varepsilon_{ij} = 0 \), \( \forall i,j = 1, \ldots, 7 \). \( \mathcal{B} = \text{col}(1, 0, 1, 0, 1, 0, 1) \).

The Laplacian matrix with respect to the digraph \( \mathcal{G} \) is given by

\[
L = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & -1 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
\]

The elements of \( q_i(0), \dot{q}_i(0), \text{ and } \zeta_i(0) \) are randomly chosen from \([-5, 5]\). The elements of \( \hat{\vartheta}_i(0) \) are randomly selected from \([0, 5]\). The control parameters for the DCEA (5) and (6) are given by

\[
\beta_1 = 4, \beta_2 = 7, \beta_3 = 21; \\
\alpha_i = 3, \gamma_i = 0.1I, K_{xi} = \text{diag}\{50, 50\}, \forall i = 1, \ldots, 7; \\
K_{si} = \text{diag}\{100, 100\}, K_{ri} = \text{diag}\{60, 60\}, \forall i = 1, \ldots, 5; \\
K_{si} = \text{diag}\{150, 150, 150\}, K_{ri} = \text{diag}\{60, 60, 60\}, \forall i = 6, 7,
\]

where \( I \) denotes the identity matrices of appropriate orders. The sampling period is selected as 10 ms.

| \( i \)-th node | \( m_i \) (kg) | \( l_i \) (m) | \( r_i \) (m) | \( I_i \) (kg m²) |
|----------------|----------------|--------------|--------------|----------------|
| \( i = 1 \)    | 0.8, 0.6       | 1.4, 0.9     | 0.8, 0.45    | 6, 3           |
| \( i = 2 \)    | 1, 0.8         | 1.2, 1.1     | 0.7, 0.5     | 2, 3           |
| \( i = 3 \)    | 0.5, 0.8       | 1.1, 1.3     | 0.4, 0.6     | 5, 3           |
| \( i = 4 \)    | 1.5, 0.8       | 1.1, 1.2     | 0.6, 0.6     | 5, 4           |
| \( i = 5 \)    | 2.3, 0.8       | 1.0, 1.2     | 0.4, 0.7     | 5, 3           |
| \( i = 6 \)    | 0.8, 1.2, 1.4  | 0.8, 1.1, 1.4| 0.4, 0.5, 0.7| 4, 6, 5        |
| \( i = 7 \)    | 1.8, 1.2, 1.4  | 1.1, 1.1, 1.2| 0.6, 0.6, 0.6| 5, 6, 5        |

Table 1: The physical parameters of the MHMs.
Simulation results of the presented DCEA are shown in Fig. 3-7. \( \|v_0\|_\infty \leq 1.571, \|a_0\|_\infty \leq 4.935, \) and \( \|\dot{a}_0\|_\infty \leq 15.504. \) It thus follows from Fig. 2 that Assumptions A1 and A2 holds in this simulation. Then the estimated states \( \hat{x}_i, \hat{v}_i \) and \( \hat{a}_i \) follows the states of the leader in finite time, as shown in Fig. 3. It gives that \( \tilde{f}(t) \) converges to the origin in finite time and Theorem 1 holds, which means the distributed sliding-mode estimator (6b) can drive the system dynamics into the cascade form (11). Moreover, the end-effectors of the manipulators follows the leader asymptotically because \( \lambda_{\min}(\mathcal{K}_{xi}) > 0, \lambda_{\min}(\mathcal{K}_{si}) > 0 \) and \( \lambda_{\min}(\mathcal{K}_{ri}) \geq \sup_{t \in \mathcal{Q}} \|d_i(t)\|, \) as shown in Fig. 4 (the trajectories of the states in time-domain) and Fig. 5 (the trajectories of the states in \( XY \) plane). It means that the maintask can be accomplished under the DCEA in the simulations. The subtask function for the sixth manipulator (node 6) is selected as \( \varphi_6 = \text{col}[0, 9(1 - q_2), 0] \) with respect to the performance indices \( 4.5(q_2 - 1)^2, \) where \( q_2 \) denotes the second joint angle of the sixth manipulator. This subtask function forces the
second joint of the sixth manipulator toward 1 rad. For Fig. 6 in the left picture, the second joint of the sixth manipulator is forced toward 1 rad with subtask control; in the right picture, the subtask cannot be accomplished without subtask control. The subtask function for the seventh manipulator is $\varphi_7 = \frac{\partial}{\partial q} \left( \det(J_7 J_7^T) \right)$, where $q$ denotes the joint position of the seventh manipulator. This subtask function increases the manipulability of the seventh manipulator, as shown in [39]. It can be observed in Fig. 7 that the manipulability of the seventh manipulator is enhanced with subtask control. It means that the subtask can also be accomplished under the DCEA in the simulations.

Figure 3: The estimated value $\hat{x}_i$, $\hat{v}_i$ and $\hat{a}_i$ for the $i$th manipulator.
Figure 4: The task-space states $x_i$ and $\dot{x}_i$ for the $i$th manipulator.

Figure 5: Position (the left picture) and velocity (the right picture) of the end-effectors of MHMs in the $XY$ plane.
Figure 6: Joint-space position of the sixth manipulator with and without subtask control.

Figure 7: Manipulability of the seventh manipulator with and without subtask control.

5. Conclusion

This paper focuses on the task-space coordinated tracking problem of MHMs with parametric uncertainties and input disturbances under digraphs. Especially, maintask and subtask for MHMs are designed and addressed simultaneously. Several conditions (including necessary and sufficient conditions,
sufficient conditions) for driving both the task-space tracking error and the subtask tracking error of MHMs to zero have been derived. The simulations for the DCEA have shown satisfactory performance in the MHMs containing two-DOF and three-DOF manipulators.

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