Self-Organizing Neutrino Mixing Matrix

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Abstract

A new and novel idea for a predictive neutrino mass matrix is presented, using the non-Abelian discrete symmetry $A_4$ and the seesaw mechanism with only two heavy neutral fermion singlets. Given the components of the one necessarily massless neutrino eigenstate, the other two massive states are automatically generated. A realistic example is discussed with predictions of a normal hierarchy of neutrino masses and maximal $CP$ violation.
To understand the observed neutrino mixing pattern in terms of a symmetry, the charged-lepton mass matrix and the neutrino mass matrix must be considered at the same time. Given that $m_{e,\mu,\tau}$ are all different, it is by no means trivial to find a symmetry which predicts a leptonic mixing matrix as the mismatch between the unitary matrices which diagonalize the respective mass matrices in the two different sectors. This was successfully done using the non-Abelian discrete symmetry $A_4$ [1] [2] [3] and applied [4] to the case of tribimaximal mixing. Whereas the specific prediction of $\theta_{13} = 0$ is now refuted by data [5, 6], it does not mean that $A_4$ itself is not valid, only those additional assumptions beyond $A_4$ which are used to enforce the tribimaximal hypothesis. Two variations [7, 8] of the original $A_4$ model [4] are in fact completely consistent with $\sin^2 2\theta_{13} = 0.1$.

In this paper, an entirely different application of $A_4$ is presented for a predictive neutrino mass matrix. It is based on an earlier proposal [9] which works very well if $\sin^2 2\theta_{13}$ is small [10] but not with present data [5, 6]. The new and novel idea is to combine the $A_4$ texture with the seesaw mechanism using only two heavy neutral fermion singlets. As a result, a massless neutrino eigenstate must appear. If it is identified with $\nu_1$, then $\nu_2$ and $\nu_3$ are generated with $m_2 = \sqrt{\Delta m^2_{21}}$ and $m_3 = \sqrt{\Delta m^2_{31}}$. The tribimaximal case may in fact be derived this way in a certain symmetry limit. Here it will be shown how a realistic pattern of masses and angles emerges, with predictions of the Dirac phase $\delta_{CP}$ for leptonic $CP$ violation and the effective mass $m_{ee}$ in neutrinoless double beta decay.

Before showing how $A_4$ allows this to happen, consider the end result, i.e.

$$\mathcal{M}_\nu = \begin{pmatrix}
(2A + B)u^2_1 & (-A - B + iC)u_1u_2 & (-A - B - iC)u_1u_3 \\
(-A - B + iC)u_1u_2 & (2A - B - iC)u^2_2 & (-A + 2B)u_2u_3 \\
(-A - B - iC)u_1u_3 & (-A + 2B)u_2u_3 & (2A - B + iC)u^2_3
\end{pmatrix}. \quad (1)$$

Note that in this basis, the charged-lepton mass matrix is diagonal, which is not an assumption but a consequence of the $A_4$ symmetry. It is clear from the above that there is one massless eigenstate, i.e.

$$\nu_1 = (u^{-1}_1, u^{-1}_2, u^{-1}_3)/\sqrt{u^{-2}_1 + u^{-2}_2 + u^{-2}_3} \quad (2)$$
for any $A, B, C$. Let $\nu_{1,2,3}$ be defined by the tribimaximal basis, i.e.

$$
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix} = \begin{pmatrix}
\sqrt{2/3} & -\sqrt{1/6} & -\sqrt{1/6} \\
1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\
0 & -1/\sqrt{2} & 1/\sqrt{2}
\end{pmatrix}
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix},
$$

(3)

then for $u_1 = 1/2, u_2 = u_3 = -1$,

$$
\mathcal{M}^{(1,2,3)}_\nu = \begin{pmatrix}
0 & 0 & 0 \\
0 & 3(B + A)/2 & i\sqrt{3/2}C \\
0 & i\sqrt{3/2}C & -3(B - A)
\end{pmatrix}.
$$

(4)

This shows that for $C = 0$, tribimaximal mixing is obtained with

$$
m_1 = 0, \quad m_2 = 3(B + A)/2, \quad m_3 = -3(B - A).
$$

(5)

However, since $C$ is in general not zero, deviation from tribimaximal mixing will occur, as shown below.

The form of the neutrino mass matrix of Eq. (1) is diagonalized by the unitary matrix $U$, i.e.

$$
U \mathcal{M}_\nu U^T = \begin{pmatrix}
0 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_3
\end{pmatrix},
$$

(6)

where

$$
\nu_1 = \begin{pmatrix}
\sqrt{2/3}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}
\end{pmatrix},
$$

(7)

$$
\nu_2 = \frac{1}{\sqrt{1 + 3\zeta^2}} \begin{pmatrix}
\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} + i\sqrt{3/2}\zeta, \frac{1}{\sqrt{3}} - i\sqrt{3/2}\zeta
\end{pmatrix},
$$

(8)

$$
\nu_3 = \frac{1}{\sqrt{1 + 3\zeta^2}} \begin{pmatrix}
-i\zeta, -\frac{1}{\sqrt{2}} - i\zeta, \frac{1}{\sqrt{2}} - i\zeta
\end{pmatrix}.
$$

(9)

This solution is obtained with

$$
B + A = \frac{2}{1 + 3\zeta^2} \left( \frac{m_2}{3} - \zeta^2 m_3 \right),
$$

(10)

$$
B - A = -\frac{1}{1 + 3\zeta^2} \left( \frac{m_3}{3} - \zeta^2 m_2 \right),
$$

(11)

$$
C = -\frac{\sqrt{2}\zeta}{1 + 3\zeta^2} (m_3 + m_2).
$$

(12)
Using Eqs. (7),(8),(9), the mixing angles in the conventional definition are given by

\[
\sin \theta_{13} = \frac{\zeta}{\sqrt{1 + 3\zeta^2}}, \quad \sin \theta_{12} = \frac{1}{\sqrt{3\sqrt{1 + 2\zeta^2}}}, \quad \sin \theta_{23} = -\frac{1}{\sqrt{2}}.
\] (13)

As for CP violation, using the Jarlskog invariant, it is easily shown that

\[
\sin \delta_{CP} = 1,
\] (14)
i.e. maximal CP violation. Note that Eqs. (10),(11),(12) allow complex values of \(m_2\) and \(m_3\), but Eqs. (7),(8),(9) remain the same, and so thus Eqs. (13) and (14). In other words, this model’s three mixing angles and one Dirac phase are independent of the Majorana phases of \(\nu_{2,3}\).

Using the experimental constraints [11]

\[
|m_2|^2 = 7.50 \pm 0.20 \times 10^{-5} \text{ eV}^2, \quad (15)\\
|m_3|^2 = 2.32 + 0.12(-0.08) \times 10^{-3} \text{ eV}^2, \quad (16)
\]

and assuming \(m_{2,3}\) to be real, the two cases of \(m_2 = \pm 0.00866 \text{ eV}\) (with \(m_3 = 0.04817 \text{ eV}\)) are considered, as well as \(\zeta = 0.165\) from \(\sin^2 2\theta_{13} = 0.098\). The parameter values of this model are then determined to be

\[
A = 0.08769 \text{ eV}, \quad B = -0.00586 \text{ eV}, \quad C = -0.01226 \text{ eV}, \quad (17)\\
A = 0.00365 \text{ eV}, \quad B = -0.01141 \text{ eV}, \quad C = -0.00852 \text{ eV}, \quad (18)
\]

respectively. The effective neutrino mass in neutrinoless double beta decay is \(m_{ee} = |A + B|/2 = 0.04\) or \(0.004 \text{ eV}\). They represent the maximum and minimum values of \(m_{ee}\) in the presence of arbitrary Majorana phases. Note also that \(\theta_{13}\) is related to \(\theta_{12}\) by

\[
\tan^2 \theta_{12} = \frac{1 - 3\sin^2 \theta_{13}}{2} < 1/2. \quad (19)
\]

This is a generic consequence of any model which has \(\nu_1 \sim (2, -1, -1)\) and is favored by data. In another class of models where \(\nu_2 \sim (1, 1, 1)\), the relationship becomes

\[
\tan^2 \theta_{12} = \frac{1}{2 - 3\sin^2 \theta_{13}} > 1/2, \quad (20)
\]
which is disfavored by data.

It has been shown that the neutrino mass matrix of Eq. (1) allows it to generate $\nu_2, \nu_3$ once the massless state $\nu_1 \sim (u_1^{-1}, u_2^{-1}, u_3^{-1})$ is decided. It has the simple and verifiable predictions of Eqs. (13) and (14), if $\nu_1 \sim (2, -1, -1)$. To derive Eq. (1), the symmetry $A_4$ is used following Ref. [9]. The lepton and Higgs representations are listed in Table 1. The important departure from Ref. [9] is that $N^c_1 \sim 1$ is now missing. The two $Z_2$ symmetries are used to distinguish the two different sets of Higgs doublets. Because of the $A_4$ multiplication rules [1], the charged-lepton mass matrix is diagonal with

$$
\begin{pmatrix}
m_e \\
m_\mu \\
m_\tau
\end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega
\end{pmatrix} \begin{pmatrix} h_1 v_1 \\ h_2 v_2 \\ h_3 v_3
\end{pmatrix},
$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$ and $v_{1,2,3}$ are the vacuum expectation values of $\phi^0_{1,2,3}$. The Dirac mass matrix linking $\nu_{e,\mu,\tau}$ to $N^c_{2,3}$ is now

$$
\mathcal{M}_D = \begin{pmatrix} f_2u_1 & f_3u_1 \\ f_2\omega u_2 & f_3\omega^2 u_2 \\ f_2\omega^2 u_3 & f_3\omega u_3
\end{pmatrix},
$$

where $u_{1,2,3}$ are the vacuum expectation values of $\eta^0_{1,2,3}$. The most general Majorana mass matrix for $N^c_{2,3}$ is given by

$$
\mathcal{M}_N = \begin{pmatrix} M_2 & M_1 \\ M_1 & M_3
\end{pmatrix}.
$$

Note that $M_1$ is an invariant mass under $A_4$, but $M_{2,3}$ are soft terms which break $A_4$. In this model, $A_4$ is broken spontaneously by $v_{1,2,3}$ and $u_{1,2,3}$ as well as softly in the complete...
Lagrangian. After inverting $\mathcal{M}_N$ and using the seesaw formula $\mathcal{M}_\nu = -\mathcal{M}_D (\mathcal{M}_N)^{-1} \mathcal{M}_D^T$, Eq. (1) is obtained with

$$A = -\frac{f_2 f_3 M_1}{M_1^2 - M_2 M_3}, \quad B = \frac{f_2^2 M_3 + f_3^2 M_2}{2(M_1^2 - M_2 M_3)}, \quad C = \frac{f_2^2 M_3 - f_3^2 M_2}{2(M_1^2 - M_2 M_3)}. \quad (24)$$

The next step is to choose $\nu_1 \sim (u_1^{-1}, u_2^{-1}, u_3^{-1})$. Since $\nu_1$ is guaranteed to be massless, Eq. (1) is reduced to a $2 \times 2$ mass matrix in the basis $\nu'_2 \sim [-u_1 (u_2^{-2} + u_3^{-2}), u_2^{-1}, u_3^{-1}]$ and $\nu'_3 \sim (0, u_2, -u_3)$. Diagonalizing this then yields the two mass eigenstates $\nu_{2,3}$ with $m_{2,3}$ and the corresponding mixing angle and Dirac phase. The special case of $\nu_1 \sim (2, -1, -1)$ has been studied in this paper, but the method may be adapted to any $\nu_1$.

In conclusion, a remarkable form of the neutrino mass matrix has been derived using $A_4$ and a reduced seesaw mechanism. It has a simple solution as shown by Eqs. (13) and (14). It is numerically consistent with all present data and predicts the exciting possibility of maximal $CP$ violation in the neutrino sector.

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