Transducer finite aperture effects in sound transmission near leaky Lamb modes in elastic plates at normal incidence

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Abstract

The interaction of ultrasonic waves with fluid-embedded viscoelastic plates, pipes, and shells, have been subject to extensive theoretical and experimental studies over several decades. In normal-incidence through-transmission measurements of a water-embedded solid plate using ultrasonic piezoelectric transducer sound fields, significant deviations from plane wave theory have recently been observed. To quantitatively describe such measured phenomena, finite element modeling (FEM), also combined with an angular spectrum method (ASM), have been used for three-dimensional (3D) simulation of the voltage-to-sound-pressure signal propagation through the electro-acoustic measurement system consisting of the piezoelectric transducer, the water-embedded steel plate, and the fluid regions at both sides of the plate. The observed phenomena of frequency downshift of the plate resonance, increased sound pressure level through the plate, and beam narrowing / widening, are ascribed to the finite angular spectrum of the beam, that excites a region of negative and zero group velocity for the leaky Lamb mode in question.

SUMMARY: The interaction of ultrasonic waves with fluid-embedded viscoelastic plates, pipes, and shells, have been subject to extensive theoretical and experimental studies over several decades. Ultrasonic guided waves (GUW) represent important scientific and industrial fields [1-26]. Examples include non-destructive testing and evaluation, measurement of elastic properties of solid materials, wall thickness measurement and corrosion measurement in pipes, deposit detection, integrity evaluation, and fluid flow measurement. In such applications the plane-wave theory of fluid-embedded viscoelastic plates is often used, in design of measurement methods as well as for signal interpretation [1-4].

In normal-incidence through-transmission measurements of a water-embedded solid plate using ultrasonic piezoelectric transducer sound fields, significant deviations from plane wave theory have been observed. Downward frequency shift for some plate resonances and enhanced signal transmission as compared to plane-wave theory have been demonstrated in measurements and simulations [15-19]. These phenomena may be accompanied by beam narrowing (and in other cases beam widening) in the transmitted field [18,19]. Such observations are made e.g. in the frequency region of the important
fundamental thickness-extensional (TE) mode of a steel plate, corresponding to the cutoff frequency of the $S_1$ Lamb mode.\(^1\)

To quantitatively describe such measured phenomena, finite element modeling (FEM), also combined with an angular spectrum method (ASM), have been used for three-dimensional (3D) simulation of the voltage-to-sound-pressure signal propagation through the electro-acoustic measurement system consisting of the piezoelectric transducer, the water-embedded steel plate, and the fluid regions at both sides of the plate [15-19]. Theoretical and measured results show that the transmission through the plate can be studied quantitatively for excitation of leaky Lamb modes in the plate using real acoustic beams. For more generic studies, to avoid relying on the particular transducer being used in these measurements, a simplified model has also been used, in which the 3D transducer sound field incident to the plate is approximated by the far field radiated by a baffled and uniformly vibrating, circular and planar piston source. In earlier works fair agreement has been obtained on the acoustic axis in comparison of this model with measurements and the more complete simulation models [15-19].

In the present work, the latter model is used to investigate the observed deviations between measurements and plane-wave theory, by systematically varying the Poisson’s ratio in the plate, in a frequency band covering the lower leaky Lamb modes of the plate. A 21.1 mm diameter piston transducer is radiating at normal beam incidence to a $d = 6.05$ mm thick water-embedded solid plate, at a distance $z_0 = 270$ mm from the plate. At 478 kHz the $ka$ number is about 21, and the Rayleigh distance is about 112 mm, well below $z_0$. The Poisson’s ratio of the plate, $\sigma$, is varied over the range 0 - 0.5 by setting the plate’s compressional-wave velocity to a fixed value, $c_p = 5780$ m/s (corresponding to steel), and varying the shear-wave velocity, $c_t$ [19]. For this incident beam field, the sound pressure field transmitted through the plate is studied as a function of the Poisson’s ratio, $\sigma$, and the signal frequency, $f$. This includes the pressure-to-pressure transfer function of the plate, $H_{pp}^{plate}$, defined as the ratio of the transmitted axial sound pressure at the lower surface, to the incident free-field axial sound pressure at the upper surface of the plate, at the frequency $f$.

As an example, Fig. 1a shows $|H_{pp}^{plate}|$ calculated over the frequency band 382 to 535 kHz, for Poisson’s ratio in the range 0 - 0.5. The frequency axis is normalized to the fundamental TE mode of the corresponding vacuum-embedded plate, $c_l/2d \approx 478$ kHz. Note that only a limited frequency range is shown in this example, covering the range of the fundamental TE mode. The plane-wave cutoff frequencies for Lamb modes of the corresponding vacuum-embedded plate, $f_c$, are shown using white curves. The white vertical line designates the fundamental TE mode, and non-vertical white curves designate thickness-shear (TS) modes. Solid and dotted curves represent symmetric (S) and anti-symmetric (A) modes, respectively. The black curve with white circled markers gives $f_0$, the zero-group velocity (ZGV) frequency of the $S_1$ Lamb mode in the vacuum-embedded plate. From the figure, it is observed that for all $\sigma$, a frequency of maximum $|H_{pp}^{plate}|$ (i.e., maximum beam transmission), denoted $f_{br}$, exists in-between the ZGV frequency, $f_0$, and the nearest cutoff frequency, $f_c$. Over the $\sigma$ region 0 - 0.5, resonance frequency downshifts in the range of approximately 0.92 to 0.98 may be experienced, relative to the nearest $f_c$. The frequency downshift appears to be largest for $\sigma$ close to 1/3.

Further details are given in Fig. 1b, for a steel plate with $\sigma = 0.2925$ (AISI 316L stainless steel). $|H_{pp}^{plate}|$ is shown as a function of frequency (red curve), in comparison with the magnitude of the corresponding plane-wave pressure transmission coefficient for a

\(^1\) Here, Lamb modes are numbered according to increasing cutoff frequency, and “S” denotes a symmetric mode.
plane-wave at $1^\circ$ incidence (blue curve). As in Fig. 1a, it is noted that $f_b$ is located in the region between $f_0$ and $f_c$, at about $0.95f_c$. In addition, an enhanced axial signal transmission through the plate of about $+4.1$ dB is observed at $f_b$. This means that for the finite beam in question, at the frequency $f_b$, the axial sound pressure level (SPL) is about 4.1 dB higher at the plate’s lower surface than at the upper surface.

Fig. 1. (a) The magnitude of the pressure-pressure transfer function for beam transmission through a water-embedded plate, $|H_{pp}^{\text{plate}}|$, shown as a function of the normalized frequency and the Poisson’s ratio in the plate, over a limited frequency span. The cutoff frequencies of Lamb modes in the associated vacuum-embedded plate, $f_c$, are shown using white curves. The ZGV frequency of the $S_1$ Lamb mode, $f_0$, is shown using a black curve with white circled markers. 
(b) $|H_{pp}^{\text{plate}}|$ for beam transmission through a steel plate with Poisson’s ratio equal to 0.2925 (red curve), compared with the plane-wave pressure transmission coefficient of the plate, $|T|$, for a plane-wave at $1^\circ$ incidence (blue curve). The frequencies $f_0$, $f_b$ and $f_c$ of the $S_1$ Lamb mode are indicated.
The observed phenomena of frequency downshift of the plate resonance, increased sound pressure level through the plate, and beam narrowing / widening (not illustrated in the example shown here), are ascribed to the finite angular spectrum of the beam, that excites a region of negative and zero group velocity for the leaky Lamb mode in question [19]. The finite beam, combined with the negative group velocity (NGV) region, appears to result in a concentration of the beam energy in the plate, close to the acoustic axis. The plate is put into resonance at a lower frequency than the Lamb mode’s cutoff frequency, with re-radiation of sound into the fluid at a higher axial SPL than that of the incident field. The phenomena addressed here may have important influences e.g. for ultrasonic methods for material characterization and thickness measurement.

In the literature, regions of negative group velocity for certain Lamb modes have been associated with backward wave propagation, phase and group velocities of opposite sign, and energy transport in the negative direction [20-26]. The apparent correlation between such Lamb wave phenomena discussed in the literature, and the finite beam diffraction Lamb wave phenomena described here, will be addressed in further work.

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