I present a very simplistic toy model for the inflationary paradigm where the size of the universe undergoes a period of exponential growth. The basic assumption I make use of is that black holes might have a quantized area (mass) spectrum with a stable ground state and that the universe has started with a tightly packed collection of these objects alone.

I. INTRODUCTION

Inflationary paradigm has been successful in explaining away the basic structural problems of the standard model of cosmology, namely the horizon, flatness, homogeneity and monopole problems. It is then a necessary exercise to look for ways it can occur. In what follows I present a very naive toy model in which the scale factor may grow exponentially. I will borrow from loop quantum gravity where the area operator has been shown to have a discrete spectrum with sizes starting from Planck area. The basic physical assumption I will make is that whatever interaction these objects might have, the outcome of it will have to be a state present in the spectrum.

II. THE MODEL AND ITS APPLICATION

It is interesting to think of the possibility of sequences in the spectrum of black holes such that the mass eigenvalues satisfy,

\[ 2m_j = m_k . \] (1)

Such a relation is possible as presented in loop quantum gravity. It then follows that two black holes can fuse into one conserving mass. This interaction will increase the entropy if we assume Hawking’s result will hold for black holes of any size. The crucial consequence of this type of interaction is that the initial states have a radii \( r_j \) and that the final state has radius \( 2r_j \). Now let us assume that we start with \( N(0) \) ground state black holes and also assume that they are tightly packed (say optimum filling for hard spheres). Then, it is possible that two neighboring black holes will fuse into one and that this happens almost simultaneously for the whole collection. This means that we end up with half the number we started but the radius of each element grew twice. Let us also assume that this can be reiterated. Then we find the following,

\[ N(k) = N(0) 2^{-k} , \] (2)

\[ V_H(k) = \frac{4\pi}{3} N(0) r_0^3 2^{2k} . \] (3)

Here \( V_H \) denotes the total volume behind the horizons, it can be used to estimate the size of the volume in which the collection is packed. With the introduction of a filling ratio \( \gamma \) the radius of the collection is given by,

\[ R(k) = [(1 + \gamma)N(0)]^{1/3} r_0 2^{2k/3} . \] (4)

This represents an exponential growth although the parameter \( k \) does not necessarily have a one-to-one correspondence to the actual time variable \( t \). I will try to estimate the time it takes for a single step of the mechanism above in the next section.

Now, the initial mass of the collection is about \( N(0) \) times the Planck mass. Assuming that the mass of the universe today is about \( 10^{22} \) solar masses we get the following

\[ N(0) \approx 10^{60} . \] (5)

Which will result in,

\[ R(0) \approx 1 \text{ Fermi}. \] (6)

This seems large, but let us remember that most of the volume is behind horizons and that the empty spaces between black holes is still of the size of Planck length.

It is clear that this process of halving the number of black holes can not continue indefinitely. At the extreme case it should stop when there is only one black hole left. To estimate the final number of black holes let us assume that the mechanism stops at a value \( k_f \) where \( R(k_f) \) becomes about 1 cm. This will give

\[ k_f \approx 65 . \] (7)

At this value of \( k \) the size of the black holes and hence the inter spacing between them is of the order
of 1 Fermi. It then starts to become possible to create proton-antiproton pairs in the inter black hole space without both of them disappearing behind horizons. It is at this stage that I assume the mechanism above stops and all the remaining \( N(k_f) \approx 10^{40} \) black holes explode to release particles of all sorts (presumably starting with larger mass particles and, when the inter black hole spacing grows, proceeding to include lighter particles) such that finally they will settle down to their ground states \( 8 \). If we borrow that the observed size of the universe today is about 10 billion light years we get the following relic density of Planck size black holes

\[
\rho_{\text{relic}} \approx 10^{-39} \text{ m}^{-3} , \quad (8)
\]

that is one relic per a volume of radius about the distance from the sun to Neptune ; small enough for not having been observed (if they are evenly distributed in cosmos).

To estimate the temperature we can first eliminate the \( t \) dependence of the extensive variables and get the equation of state

\[
S^2 = \kappa EV \ . \quad (9)
\]

Here \( \kappa \) is a number of order unity times Planck mass squared, (an exact calculation of this number requires the knowledge of the filling ratio). Then the temperature is calculated by \( T^{-1} = (\partial S/\partial E)^{1/2} \) after which we reemphasize the \( k \) dependence to get

\[
T \approx 10^{19} 2^{-t} \text{ Gev} \ . \quad (10)
\]

Thus the universe starts growing at around the Planck temperature and the above scenario ends at a temperature of about 1 GeV. The temperature follows the size of the inter black hole spacing.

### III. TIME

It is evident that a Hamiltonian can not induce transitions between its own stationary states. So the fusion of the black holes has to be due to an interaction between them. This interaction will also change the energy-entropy relation in \( 8 \). If we knew the dynamics we could calculate the time \( \Delta t(k) \) needed for a single step of fusion to occur and from it we could define a time variable

\[
t(k) = \sum_{n=0}^{n=k} \Delta t(k) \ . \quad (11)
\]

We do not yet know what type of interactions the Plank size black holes might have. However interestingly enough if we assume a stringy interaction with a linear potential we get interesting results. Let us assume that the stringy interaction will not extend beyond nearest neighbours since in order to do so the string will have to traverse a black hole. Thus the contribution of the stringy interaction to the total energy will go like

\[
E_{\text{stringy}} = \text{constant} \times N(k) m(k)^2 r(k) \ . \quad (12)
\]

Here the constant of proportionality is related to the packing scheme that is the mean number of nearest neighbours. Remembering the \( 2^k \) dependence of all the quantities it is straightforward to show that the energy density of the stringy interaction is independent of \( k \) and hence of \( t \). So like in the inflationary scenario we get a time independent and homogeneous \( 8 \) energy density. Having an interaction potential we can estimate \( \Delta t(k) \) for the stringy interaction. Newtonian dynamics \((F = ma)\) yields

\[
\Delta t(k) = \text{constant} \times t_{pl} \ , \quad (13)
\]

which means that \( t(k) = \text{constant} \times k \ t_{pl} \) giving an exponential growth in “time” for the scale factor \( R \) of the universe.

Another interesting implication is the following. It is not possible to assume that the stringy interaction dominates when the sizes of the black holes and hence the mean separation between them grows. Assuming at some point Newtonian force law sets in the above scheme of estimation gives

\[
\Delta t(k) = \text{constant} \times t_{pl} 2^k \ , \quad (14)
\]

which results in \( R(t) = \text{constant} \times t^{2/3} \) matter dominated expansion. It is interesting that this emerges as a byproduct \( 10 \).

### IV. FURTHER SPECULATIONS

The numbers presented depend considerably on the total energy (which fixes \( N(0) \)) and on the size of the universe as it exits the inflationary (with the mechanism presented here) period which I took to be around 1 cm. As we have seen, it makes sense to identify the exit point as the point at which the production of particles of mass corresponding to the mean inter black hole spacing starts. The heaviest known particles today are the electroweak gauge bosons with a mass of around 90 GeV. This would correspond to \( k_f \approx 56 \) meaning that \( R(k_f) \approx 10^{-2} \text{ cm} \) and \( N(k_f) \approx 10^{43} \) resulting in a relic density of about a thousand times larger than the number I estimated before; one relic in a volume of radius of roughly twice the distance from the sun to Mars. Much more massive particles (say a GUT monopole) renders the numbers obtained meaningless. It might however be the case that the fusion process of black holes occurs very rapidly at
the beginning such that the heavy particles beyond the scope of standard model of particle physics may not have enough time to materialize in the inter black hole regions.

I assumed throughout that the black holes formed a space and time independent close packing scheme. While this is a useful assumption in estimating numbers it is true that this can not be exactly satisfied. The overall filling ratio may be a constant but the local filling ratio will fluctuate around a mean, which would result in density fluctuations; a necessary ingredient for any cosmological model. On the other hand it is also possible that remaining relic black holes formed local groups and eventually duplicated the scenario presented, resulting in fewer relics, but this somewhat unlikely. Furthermore it could also be the case that the black holes completely evaporates after the inflationary period resulting in no relics at all. Finally the model presented here does not actually need to be the only mechanism for inflation it might possibly be combined with the usual inflationary paradigm.

V. CONCLUSION

In this short letter I outlined a very naive model which incorporates inflationary paradigm. It is my hope that the ideas presented here have a significance beyond the toy model presented and may attract the attention of researchers. Further analysis requires much more elaborate mathematical tools. An effort to study the significance of the local filling ratio fluctuations and to incorporate rigorous interpretation of black hole evaporation is in progress.

I have possibly (not willingly) overlooked relevant literature. I recently became aware that in loop quantum gravity there is a possibility to have a quantized scale factor for the radius of the universe yielding an inflationary period \[\text{[5, 6]}\], the origin of which however, I believe, is not related to the model presented here.

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[1] A.H. Guth, Phys. Rev. D23, 347, (1981).
[2] A. Ashtekar, “Quantum Mechanics of Geometry” \[\text{gr-qc/9901023}\].
[3] S. W. Hawking, Commun. Math. Phys. 43, 199, (1975).
[4] A.H. Guth, “The Inflationary Universe”, Addison-Wesley, (1997), pp. 254.
[5] T. Thiemann, “Lectures on Loop Quantum Gravity”, \[\text{gr-qc/0210094}\].
[6] M. Bojowald, “Inflation from Quantum Gravity”, \[\text{gr-qc/0206054}\].
[7] I do not intend to mean that the relation in Eq. (1) defines the whole spectrum, it is enough for the purposes presented here that such a subspace can be found and it is shown in \[\text{[3]}\] that this is possible.
[8] The actual area spectrum is much richer than the portion of it I used to climb up the ladder. The black holes need not use the same steps to get down to their ground states. This gives more freedom to the evaporation process.
[9] boundary effects may change this energy density slightly.
[10] The contribution of the Newtonian interaction to the total energy can be shown to be a constant (apart from boundary effects) and hence its energy density is decaying like \(2^{-2k}\) or like \(1/t^2\).