Technicolor models with coupled systems of Schwinger-Dyson equations

A. Doff\textsuperscript{1} \textsuperscript{*} and A. A. Natale\textsuperscript{2} \textsuperscript{†}

\textsuperscript{1} Universidade Tecnológica Federal do Paraná - UTFPR - DAFIS
Av Monteiro Lobato Km 04, 84016-210, Ponta Grossa, PR, Brazil
\textsuperscript{2} Instituto de Física Teórica - UNESP, Rua Dr. Bento T. Ferraz, 271,
Bloco II, 01140-070, São Paulo, SP, Brazil

When Technicolor (TC), QCD, Extended Technicolor (ETC) and other interactions become coupled through their different Schwinger-Dyson equations, the solution of these equations are modified in comparison with the ones of the isolated equations. The change in the self-energies is similar to the one obtained in the presence of four-fermion interactions, but without their \textit{ad hoc} inclusion in the theory. In this case TC and QCD self-energies decrease logarithmically with the momenta, what allow us to build models where ETC boson masses can be pushed to very high energies, and their effects will barely appear at present energies. Here we present a detailed discussion of this class of TC models. We first review the Schwinger-Dyson TC and QCD coupled equations, explaining the origin of the asymptotic self-energies. We develop the basic ideas of how viable TC models may be built along this line, where ordinary lepton masses appear naturally lighter than quark masses. One specific unified TC model associated with a necessary horizontal (or family) symmetry is described. The values of scalar and pseudo-Goldstone boson masses in this class of models are also discussed, as well as the value of the trilinear scalar coupling, and the consistency of the models with the experimental constraints.

I. INTRODUCTION

Throughout the past years there were many attempts to solve some of the Standard Model (SM) drawbacks related to the presence of a fundamental scalar boson (like the hierarchy problem, triviality, etc.). Some of the proposals along this direction are interesting due to the fact that fundamental scalar bosons fit naturally in these models, as in supersymmetric models \cite{ref1,ref2} and asymptotically safe SM extensions \cite{ref3}. However no signal of these theories have appeared up to now. The Higgs particle found at the LHC \cite{ref4,ref5} can be the first signal of a fundamental scalar boson, although the possibility of this boson to be a composite one has not yet been discarded, and in this case some of the SM problems commented above may be alleviated.

Scalar bosons are essential to the mechanisms of chiral and gauge symmetry breaking in the SM, but it should be remembered that most that we have learned about the mechanisms of spontaneous symmetry breaking are based on the presence of composite or pair correlated scalar states, as happens in the Nambu-Jona-Lasinio model, in the QCD chiral symmetry breaking, and in the microscopic BCS theory of superconductivity. For instance, chiral symmetry breaking is promoted in QCD by a non-trivial vacuum expectation value of a fermion bi-linear operator and the Higgs boson role is played by the composite \textit{σ} meson. Gauge theory models along this idea, dubbed Technicolor (TC), were proposed forty years ago \cite{ref6,ref7} and reviewed in Refs. \cite{ref8,ref9}. The many variations of these models continue to be studied \cite{ref10,ref11,ref12,ref13,ref14,ref15,ref16,ref17}, but no phenomenologically viable model has been found up to now.

It is clear that to build SM extensions in order to solve unknown questions, like the origin of the fermionic mass spectra, is easier in the case where we deal with fundamental scalar bosons, than in the case when the spontaneous symmetry breaking is promoted by composite scalars, even if we are far from solving the problems related to the existence of fundamental scalar bosons. The difficulty in models with composite scalar boson resides in knowing the dynamics of the non-Abelian gauge theory responsible for their formation.

We may say that the root of most of the TC problems lies in the way the ordinary fermions acquire their masses, which appears through the diagram shown in Fig.(1), where an ordinary fermion (f) couples to a technifermion (F) mediated by an Extended Technicolor (ETC) boson.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Ordinary fermion mass (f) in ETC models}
\end{figure}

Assuming a standard non-Abelian TC self-energy ($\Sigma_T$) given by \cite{ref18}

\begin{equation}
\Sigma_T(p^2) \propto \frac{\mu_{TC}^3}{p^2} \left( \frac{p}{\mu_{TC}} \right)^{\gamma_m},
\end{equation}

where $\mu_{TC}$ is the characteristic TC dynamical mass at zero momentum (of order of the Fermi mass) and $\gamma_m$ the anomalous mass dimension, which depends on the TC coupling constant, and for an asymptotically free theory

\textsuperscript{*} agomes@utfpr.edu.br
\textsuperscript{†} adriano.natale@unesp.br
has a small value; the ordinary fermion mass turn out to be
\[ m_f \propto \frac{\mu^2_{TC}}{M^2_e}, \]  
(2)
where \( M_e \) is the ETC gauge boson mass. In order to explain the top quark gauge we need a small \( M_e \) value, and since ETC is one interaction that changes flavor, the simplest model that we can imagine will inevitably lead, among other problems, to flavor changing neutral currents incompatible with the experimental data.

Solutions to the above dilemma seem to require a large \( \gamma_m \) value \[19\] leading to a TC self-energy with a harder momentum behavior, and many models along this idea can be found in the literature \[20, 32\]. In particular we may quote the work of Takeuchi \[33\] where the TC Schwinger-Dyson equation (SDE) was solved with the introduction of an \textit{ad hoc} four-fermion interaction, which can lead to the following expression for the TC self-energy
\[ \Sigma_T(p^2 \to \infty) \propto \ln^{-\delta}(p^2/\mu^2_{TC}), \]  
(3)
where \( \delta \) is a function of the many parameters of the model. The Takeuchi solution, when dominated by the four-fermion interaction, is not different from the behavior of the self-energy when a bare mass is introduced into the theory, or from the SDE solution called as irregular \[18\].

In Ref. \[35\] we verified analytically that the radiative corrections shown in Fig. (2) act as an effective bare mass. In the case of ordinary quarks the second diagram \((b_2)\) on the right hand of Fig. (2) originates an effective mass due to TC condensation, on the other hand the techniquarks obtain a tiny effective mass due to QCD condensation (see diagram \((a_2)\) in Fig. (2)), and even a larger mass due to the other diagrams \(((a_3) \text{ and } (a_4))\). Therefore, the TC self-energy can be described by
\[ \Sigma_T(p^2) \approx \mu_{TC} \left[ 1 + \delta_1 \ln \left( \frac{(p^2 + \mu^2_{TC})}{\mu^2_{TC}} \right) \right]^{-\delta_2}, \]  
(4)
where \( \delta_1 \text{ and } \delta_2 \) are parameters that will depend on the many possible SDE radiative corrections depicted in Fig. (2), in particular the dominant correction to the techniquark masses will be generated by diagrams \((a_3) \text{ and } (a_4)\) of Fig. (2), and by diagram \((b_2)\) in the case of ordinary fermion masses. A similar expression also results for ordinary quarks, and it should be noticed that the \textit{isolated} infrared TC and QCD self-energy behavior is the traditional one (the one associated to the regular solution, or Eq. \((1)\)) with dynamical masses of order of \( \mu_{TC} \approx O(1) TeV \) and \( \mu_{QCD} \approx 250 MeV \) respectively, i.e. the coupled SDE system is a combination of the regular and irregular self-energy solutions \[18\]. It is interesting to recall that such behavior is indeed the one that minimize the vacuum energy in gauge theories \[32\], and not different from the Takeuchi’s result, but originated from known interactions (QCD, for example).

The main consequence of the results of Refs. \[34\] and \[32\] (i.e. Eq. \((4)\)) is that the dynamically generated masses will be barely dependent on the ETC scale \( M_e \). In Ref. \[34\] we verified numerically that the ordinary quark masses behave as
\[ m_q \propto \lambda_E \mu_{TC} [1 + \kappa_1 \ln \left( \frac{M^2_e}{\mu^2_{TC}} \right) ]^{-\kappa_2}, \]  
(5)
where \( \lambda_E \) involves ETC couplings, a Casimir operator eigenvalue and other constants, \( \kappa_i \) are related to the self-energies that enter in the calculation of the generated masses, what is compatible with a quark mass computed with the help of Eq. \((4)\). Looking at Eq. \((4)\) it is clear that we can push the ETC scale up to the grand unification scale (or even the Planck scale) without large variation of the \( m_q \) values with \( M_e \). It is also clear that the ordinary fermionic mass hierarchy will not come out from different \( M_e \) scales! The purpose of the present work it to discuss how viable TC models can be built in this context, as well as to verify the phenomenological consequences of these models, and to show how they can be consistent with the existent high energy data.

It is opportune to recall that the SDE are nowadays studied at a high sophisticated level, taking into account gluon mass generation and possibly confinement \[37, 41\] as well as a complex vertex structure \[42, 43\]. However the solutions discussed in Refs. \[34, 35\] and in this work are related to the asymptotic behavior produced by the effective mass of the coupled SDE, and not affected by the infrared intricacies of the strongly interacting theories.

Recently we solved numerically the coupled TC (based on a \( SU(2) \) group) and QCD gap equations \[34\], which are depicted in the Fig. (2). It turned out that both self-energies have the same asymptotic behavior of Eq. \((5)\). It is not difficult to understand the origin of such behavior.

FIG. 2. The coupled system of SDEs for TC (\(T\equiv\)technifermion) and QCD (\(Q\equiv\)quark) including ETC and electroweak or other corrections. \( G \) (\( g \)) indicate technigluons (gluons).
The distribution of our work is the following: In Section II we present one specific TC model, which is just a example of many models that can be built along the line sketched in that section. It is discussed that a horizontal symmetry is necessary in this scheme. In Section III we discuss how a composite scalar boson can be lighter than the typical composition scale of the theory responsible for this particular state. In Section IV we determine the order of magnitude of pseudo-Goldstone masses. In Section V we compare the value of the TC condensate in our model with the one expected in walking TC theories. Section VI contains a brief discussion of possible experimental consequences of the models discussed in Section II, and in Section VII we comment what can be expected about the trilinear scalar coupling. Section VIII contains our conclusions.

II. BUILDING TC MODELS

In Ref. [34] we briefly proposed one specific TC model, which will be detailed here. As will be discussed at the end of this section, there is a large class of models that can be built along the same line of the model delineated in the sequence. The model discussed in Ref. [34] is based on the following group structure

$$SU(9)_U \otimes SU(3)_H,$$

where the $SU(9)_U$ is a non-Abelian Grand Unified Theory (GUT) containing the SM and a $SU(4)_{TC}$ group. The $SU(3)_H$ is a horizontal or family symmetry that is important to generate the hierarchy of fermion masses.

There are several reasons for this particular choice. First, the $SU(9)_U$ GUT will make the ETC role, because the generated fermion masses will be weakly dependent on the GUT boson masses (here acting as “ETC” boson masses) as shown in Eq. (5). This group also contains the standard $SU(5)_{GC}$ Georgi-Glashow GUT [44]. Secondly, the $SU(4)_{TC}$ group contained in the GUT will condensate before QCD, generating an appropriate Fermi scale necessary to break the electroweak group. Note that this choice is based on the most attractive channel hypothesis (MAC) [15, 17], but it can be relaxed if the GUT breaking can be promoted at very high energies, where even fundamental scalar bosons may be natural due to the presence of supersymmetry [1, 2]. In this case we could not neglect the possibility of a small TC group (perhaps $SU(2)$) that condensates at one mass scale larger than the QCD one. Third, the horizontal or family symmetry is necessary to prevent the first and second generation ordinary fermions coupling to TC. The third fermionic generation will obtain masses due to diagrams like the one of Fig.(1), and will be of order of $\lambda_{E/\mu_{TC}}$ as will be detailed in the sequence.

The $SU(9)_U$ has the following anomaly free fermionic representations [47]

$$5 \otimes [9, 8]_i \oplus 1 \otimes [9, 2]_i,$$

where the $[8]$ and $[2]$ are antisymmetric under $SU(9)_U$, and $i = 1, 2, 3$ is the horizontal index necessary for the replication of the $SU(3)_H$ families. The decomposition of these representations under $SU(4)_{TC} \otimes SU(5)_{GC}$ are

$$[9, 2]_i,$$

$$\begin{pmatrix} 0 & u_iB & -\bar{u}_iY & -u_iR & -d_iR \\ -\bar{u}_iB & 0 & u_iR & -u_iY & -d_iY \\ \bar{u}_iY & -\bar{u}_iR & 0 & -u_iB & -d_iB \\ u_iR & u_iY & u_iB & 0 & e_i \\ d_iR & d_iY & d_iB & 0 & -e_i \\ \end{pmatrix}$$

$$(1, 10) = \begin{pmatrix} T_{i1} \ \\
T_{iY} \\
T_{iB} \\
\bar{L}_i \\
N_i \end{pmatrix}_{TC}, \ (\bar{6}, 1) = N_i$$

$$(4, 5) = \begin{pmatrix} \bar{d}_{iR} \\
\bar{d}_{iY} \\
\bar{d}_{iB} \\
e_i \\
\nu_e \end{pmatrix}_{TC,} \ (1, \bar{5}) = \begin{pmatrix} \bar{X}_{Bk} \\
\bar{X}_{E_k} \\
\bar{X}_{B_k} \\
E_k \\
N_{E_k} \end{pmatrix}_i$$

$$(\bar{4}, 1) = \bar{T}_{i\varepsilon}, L_i, N_{iL}.$$
whose interaction is mediated by one $SU(9)$ gauge boson. Apart from the logarithmic term appearing in Eq. (8) this mass is

$$m_3 \approx 2\lambda_9 \mu_{TC},$$

where we can assume roughly $\lambda_9 \approx 0.1$, which is the product of the $SU(9)$ coupling constant times some Casimir operator eigenvalue, the factor 2 is due to account both diagrams of Fig.(3), and $\mu_{TC}$ can be assumed to be of $O(1)$TeV. The $SU(9)$ interaction is playing the role of the ETC interaction. These naive assumptions will lead to a top quark of approximately $200\text{GeV}$. The logarithmic term appearing in Eq.(8), and neglected in Eq.(8), provides a small change, decreasing the value of our rough estimate.

Note that the first and second charge $2/3$ quarks do not couple directly to the techniquarks due to the different $SU(3)_C$ quantum numbers, and at this level they remain massless.

We can now see how the first generation obtain their mass. In Fig.(4) we show the diagrams that are responsible for the $u$ quark mass. This quark does not couple to techniquarks at leading order, but does couple with other ordinary quark and with itself due to the bosons of the unified theory and the horizontal one. Its mass can be approximated from Eq.(5) as we did to obtain Eq.(8) and will be given by

$$m_1 \approx \lambda_5 \mu_{QCD},$$

where we can assume naively the $SU(5)_{GG}$ factor $\lambda_5 \approx 0.1$ and $\mu_{QCD} \approx 200\text{MeV}$, obtaining roughly a mass of order of $20\text{MeV}$. Here we are not introducing a factor 2 in Eq.(9) due to the presence of the two diagrams in Fig.(4), because the $c$ quark condensate (in the second diagram of Fig.(4)) may be smaller than the $u$ and $d$ condensate.

In Eq.(8) and Eq.(9) we probably super-estimated the results when we neglected the logarithmic dependence on the unified or “ETC” boson masses. These are very simple calculations. To have better estimates we must solve the coupled SDE obtaining good fits to the self-energies, what means to obtain reasonable values of the parameters $\delta_1$ and $\delta_2$ in the approximate expression of Eq.(4). It is clear that this is far away from the scope of this work.

The mass of the second quark generation will involve necessarily the horizontal symmetry, where the coupling to techniquarks will appear only at two-loop order. The $c$ quark mass will be generated by diagrams like the ones shown in Fig.(5), and it is expected to be one order of magnitude below the typical mass of the third quark generation, due to an extra factor $\lambda_{3H} \approx 0.1$ that contains the $SU(3)_H$ coupling constant. In this way we verify that the horizontal or family symmetry is fundamental to generate a quark mass matrix with the Fritzsch texture.[56, 57]

$$m_q = \begin{pmatrix} 0 & m_1 & 0 \\ m_1^* & 0 & m_2 \\ 0 & m_2^* & m_3 \end{pmatrix},$$

which has several good qualities of the experimentally known quark mass matrix.

Lepton masses will appear in the same way as quark masses. The $\tau$ lepton is the only one that will couple with techniquarks at leading order, due to the appropriate choice of quantum numbers of the horizontal symmetry. As a consequence, the mass matrix for the leptonic sector is similar to the one described above, although lepton masses should be naturally smaller than quark masses, because quarks end up coupling to two different condensates and with a larger number of diagrams contributing to their mass. It is not difficult to verify the

---

1 Note that the self-energy and the condensate values are inti-

---

FIG. 3. Diagrams contributing to the top quark mass.

FIG. 4. Diagrams contributing to the light quarks masses

FIG. 5. Diagrams contributing to the $c$ quark mass.
We have not discussed the $SU(9)_U$ and horizontal symmetry breaking, which we just assume happens at the unification scale $\Lambda_{SU(9)}$, which can possibly be naturally promoted by fundamental scalars bosons. The breaking of the GUT symmetry can also be used to produce a larger splitting in the third fermionic generation. For instance, if in the $SU(9)_U$ breaking besides the standard model interactions and the TC one we leave an extra $U(1)$ interaction, we could have quantum numbers such that only the top quark would be allowed to couple to the TC condensate at leading order. In fact, the splitting $(S_{(t-b)})$ between the $t$ and $b$ quarks

$$S_{(t-b)} = \frac{m_b}{m_t} \approx \frac{1}{40},$$

is quite large and it would be interesting that the $b$ quark and the $\tau$ lepton could couple at a larger order in the coupling constant (possibly $(\alpha_s^2)$), what could be accomplished by this remaining $U(1)$ interaction that we referred above. More sophisticated models where large fermionic mass splittings and even neutrino masses could be generated were presented in Refs. [58–62].

At this point we hope that we have made clear the necessity of introducing a horizontal or family symmetry. It is necessary to prevent the first and second generation of ordinary fermions to obtain large masses coupling with TC at leading order. This symmetry can be a local one, but a global symmetry is not necessarily discarded. If the family symmetry is local, its breaking can also happens at very high energies and (again) even promoted by fundamental scalars at the GUT or Planck scale, producing feeble effects at lower energies.

When building a TC model the existence of grand unification is also welcome. For example, in the model described here a $SU(5)_{\text{GUT}}$ gauge boson interaction is fundamental to give the electron a mass, which appears due to the electron coupling to the first generation quark, with exactly the same interaction that may mediate proton decay in the $SU(5)_{\text{GUT}}$, theory. There are more diagrams contributing to the first generation quark masses than there are for the electron mass, what may explain why leptons are less massive than quarks.

Concerning the possible class of models following the idea presented here, it is also clear that the full and precise mass spectra determination is quite complex. Once a GUT involving the SM and TC is proposed we have also to choose the horizontal symmetry. The coupled SDE of such model has to be solved determining all self-energies with their specific infrared and ultraviolet expressions. Of course, simple estimates can be performed approximating the calculation of each specific fermion mass diagram, by the product of the dynamical mass involved in the diagram (TC or QCD) with the respective coupling constants and Casimir operator eigenvalues, as performed in Eqs. [8] and [9] where a logarithmic term was neglected.

### III. SCALAR MASS

The common lore about theories with a composite scalar boson is that its mass should be of the order of the dynamical mass scale that forms such particle. This concept may be reminiscent from the work of Nambu-Jona-Lasinio [63], as also discussed for the $\sigma$ meson in QCD [64], where the scalar composite mass appearing in one strongly interacting theory is given by

$$m_\sigma = 2 \mu_{\text{QCD}}.$$

Eq. (12) comes from the fact that at leading order the SDE for the quark propagator is similar to the homogeneous Bethe-Salpeter equation (BSE) for a massless pseudoscalar bound state $(\Phi^p_{\text{BS}}(p,q)|_{q\to0})$ (the pion), and a scalar p-wave bound state $(\Phi^S_{\text{BS}}(p,q)|_{q^2=4\mu^2})$ (the sigma meson or the $f_0(500)$ [62]), i.e.

$$\Sigma(p^2) \simeq \Phi^p_{\text{BS}}(p,q)|_{q\to0} \simeq \Phi^S_{\text{BS}}(p,q)|_{q^2=4\mu^2}_{\text{QCD}}.$$

Eq. (13) tell us that in QCD the $\sigma$ meson must have a mass $2\mu_{\text{QCD}} \approx 500 \text{MeV}$. In TC we should expect a scalar boson with a mass of 2 TeV, which is clearly not the case of the observed Higgs boson [3–4].

There are two subtle points concerning the result of Eq. (12) and the determination of the scalar composite mass. The first one is that Eq. (12) was determined using the homogeneous BSE. There is nothing wrong with this. However this gives the right result if the fermionic self-energy that enters into the BSE is a soft one. When the self-energy decreases slowly (as the one of Eq. (11)) the scalar mass is modified by the normalization condition of the inhomogeneous BSE. This modification lowers the composite scalar mass as a consequence of Eq. (11). The second point about Eq. (12) that we would like to comment is not exactly about the equation itself, but about the value of the dynamical QCD and TC mass scales that enters in such scale. The QCD dynamical mass scale is usually extracted from the hadronic spectra, for instance, expected to be 1/3 of the nucleon mass or 1/2 of the sigma meson mass. However it is not clear up to now how much this spectra is affected by gluons (or techniquarks in the TC case) and mixing among different particles. These points will be discussed in the following subsections.

#### A. Normalization condition and the scalar mass

The BSE normalization condition in the case of a non-Abelian gauge theory is given by [18]

$$2\nu_{q_\mu} = i^2 \int d^4p \text{Tr} \left\{ \mathcal{P}(p,p+q) \left[ \frac{\partial}{\partial q^\mu} F(p,q) \right] \mathcal{P}(p,p+q) \right\}$$

$$- i^2 \int d^4p d^4k \text{Tr} \left\{ \mathcal{P}(k,k+q) \left[ \frac{\partial}{\partial q^\mu} K(p,k,q) \right] \mathcal{P}(p,p+q) \right\}$$
where

\[ K'(p, k, q) = \frac{1}{(2\pi)^4} K(p, k, q) , \]

\[ F(p, q) = \frac{1}{(2\pi)^4} S^{-1}(p + q)S^{-1}(p) , \]

where \( \mathcal{P}(p, p + q) \) is a solution of the homogeneous BSE and \( K(p, k, q) \) is the fermion-antifermion scattering kernel in the ladder approximation. When the internal momentum \( q_\mu \to 0 \) the wave function \( \mathcal{P}(p, p + q) \) can be determined only through the knowledge of the fermionic propagator:

\[ \mathcal{P}(p) = S(p)\gamma_5 \frac{\Sigma(p)}{F_{\Pi}} S(p) , \]

(14)

where \( \Sigma(p) \) will describe the technifermion self-energy and it should be noticed that \( F_{\Pi} \) describes the technion decay constant associated to normalization condition in the rainbow approximation as \( K_{\Pi} \) and \( \Sigma_{\Pi} \).

If we identify \( \Sigma(p^2) \equiv \mu_{TC} f(p^2) \) we can write the normalization condition in the rainbow approximation as

\[ 2i \left( \frac{F_{\Pi}}{\mu_{TC}} \right)^2 q_\mu = \frac{i^2}{(2\pi)^4} \times \]

\[ \left[ \int d^4p Tr \left\{ S(p)f(p)\gamma_5 S(p) \left( \frac{\partial}{\partial q^\mu} S^{-1}(p + q)S^{-1}(p) \right) \right\} \right] \]

\[ S(p)f(p)\gamma_5 S(p) \]

\[ f(k)\gamma_5 S(k) \left( \frac{\partial}{\partial q^\mu} K(p, k, q) \right) S(p)f(p)\gamma_5 S(p) \}

(15)

Eq.\ref{eq:15} is quite complicated but it can be separated into two parts:

\[ 2i \left( \frac{F_{\Pi}}{\mu_{TC}} \right)^2 q_\mu = I_\mu^0 + I_\mu^K , \]

(16)

corresponding, respectively, to the two integrals in the right hand side of Eq.\ref{eq:15}. The fermion propagator given by \( S(p) = 1/\mu - \Sigma(p) \) can be written as

\[ \frac{\partial}{\partial q^\mu} S^{-1}(p + q) = \gamma_\mu - \frac{\partial}{\partial q^\mu} \Sigma(p + q) , \]

(17)

and the term \( \frac{\partial}{\partial q^\mu} \Sigma(p + q) \) in the above expression may be written as

\[ \frac{\partial \Sigma(p + q)}{\partial q^\mu} = (p + q)_\mu \frac{d\Sigma(Q^2)}{dQ^2} \]

(18)

where \( Q^2 = (p + q)_\mu (p + q)^\mu \). Considering the angle approximation we transform the term \( \frac{d\Sigma(Q^2)}{dQ^2} \) as

\[ \frac{d\Sigma(Q^2)}{dQ^2} = \frac{d\Sigma(p^2)}{dp^2} \Theta(p^2 - q^2) + \frac{d\Sigma(q^2)}{dq^2} \Theta(q^2 - p^2) \]

(19)

where \( \Theta \) is the Heaviside step function. We can finally contract Eq.\ref{eq:10} with \( q^\mu \) and compute it at \( q^2 = M_H^2 \) in order to obtain

\[ M_H^2 = 4\mu_{TC}^2 \left\{ \int d^4p \frac{f^2(p)\Sigma(p)}{8\pi^4} \times \right\} \]

\[ \left[ \left( \frac{\mu_{TC}}{F_{\Pi}} \right)^2 + 4\pi \right] S(dq) \right\} \}

(20)

where \( n_f \) is the number of technifermions, \( N_{TC} \) is the number of technicolors and \( g_{TC} \) is the technicolor coupling constant.

An expression similar to Eq.\ref{eq:20} was already obtained by us in Ref.\ref{ref:10}. In that work we just assumed, in a totally ad hoc fashion, a hard momentum behavior for the TC self-energy. The calculation here will differ not only in the origin of the self-energy but also in the approach we follow to determine the \( M_H \) value. Considering the work of Ref.\ref{ref:35} it becomes evident that the behavior of \( M_H \) is a result that will fundamentally depend on the boundary conditions satisfied by the coupled system described in Fig.(2). In Eq.\ref{eq:20} the UV behavior of the term

\[ (UV) \lim_{p^2 \to \infty} - p^2 \frac{d\Sigma(p)}{dp^2} \]

will be affected by the effective mass generated by the diagrams (\( a_2, a_3, a_4 \)) in Eq.(2). In Ref.\ref{ref:10} we verified that the UV behavior of the term in Eq.(21) is modified as \( a_E \) is different or equal to zero, and we shall return to comment this term later.

\( M_H \) will be computed solving numerically the differential coupled equations shown in Eqs.(11) and (12) of Ref.\ref{ref:35}, fitting the resulting solutions (all fits with \( R^2 = 0.98 \)) and inserting the fits into Eq.(20). We considered the following TC gauge groups \( SU(2)_{TC}, SU(3)_{TC} \) and \( SU(4)_{TC} \), with \( n_f = 5 \) fermions in the fundamental representation, \( \mu_{TC} = 1TeV \), the MAC hypothesis to constrain the TC gauge coupling and Casimir eigenvalue. Hereafter we follow Refs.\ref{ref:34} \ref{ref:35} using a Casimir eigenvalue \( C_E = 1 \) and gauge coupling constant \( a_E = 0.032 \) which are quantities related to the ETC gauge theory.

Our results for \( M_H \) are shown in Table 1, where we can see that the normalization condition lowers the scalar mass by a factor of \( O(1/10) \). The results are consistent with the ones of Ref.\ref{ref:60} obtained with the naive assumption of the irregular solution for the TC self-energy. Therefore, the effect of radiative corrections in coupled SDE involving a TC theory, act in order to produce a scalar composite boson with a mass compatible with the one of the observed Higgs boson.

B. Dynamical mass scales and mixing

The most precise quantity to constrain the dynamical mass scale in the QCD case is the pion decay constant, which is a function of the quark self-energy. In
TABLE I. The last column contain the composite scalar mass determined through Eq.(20), where we plugged the TC self energy obtained solving the coupled SDE system. The different factors and couplings of the gap equations are described in the text.

| SU(N) | n_f | M_H (GeV) |
|-------|-----|-----------|
| 2     | 5   | 105.3     |
| 3     | 5   | 141.5     |
| 4     | 5   | 148.8     |

The problem is that the result of Eq. (approximately the value of the QCD mass scale (Λ_{QCD})), in the TC case the technipion decay constant is related to the W and Z gauge boson masses. However, in both cases that quantity depend on the dynamical mass scale as well as the functional expression of the self-energy. Therefore we have some freedom to pin-point the dynamical mass scale. Even the numerical determination of the self-energy through SDE solutions include the introduction of a cutoff and specific approximations. We conclude that the calculation of the scalar boson mass depend on the functional form of the self-energy and on the dynamical mass scale. It is curious that in the past the scalar boson mass was considered in order to constrain the dynamical mass scale, i.e. in QCD the scalar meson mass has led to the usual value μ_{QCD} ≈ 250 MeV, which is also approximately the value of the QCD mass scale (Λ_{QCD}). The problem is that the result of Eq (12) is not modified only by the inhomogeneous BSE condition but by many other effects as we discuss in the following.

The dynamical QCD mass scale is also thought to be related to the nucleon mass, but even this is not certain since we do not know how much gluons contribute to the nucleon mass. It is also not clear yet how much of the sigma meson mass comes from mixing with heavier quark-anti-quark scalars and with glueballs, and the same is true if we just exchange QCD by TC, meaning that the scales μ_{QCD} and μ_{TC} may be smaller than usually thought, leading to a smaller scalar composite mass (i.e. the σ and the “Higgs” mass). The scalar mass can also be modified by the effect of radiative loop corrections due to the presence of heavy fermions as described in Ref. [55].

These are not the only effects that modify the scalar mass, leading to a new relation of the scalar mass with the dynamical mass scale. There is still another effect that is intimately related to the type of dynamical symmetry breaking model that we discussed in the previous section.

In Section II we discussed a model with two composite scalar states responsible for the chiral (and gauge) symmetry breaking: the scalars belonging to the 6 and 3 representations of the horizontal group, formed respectively by technifermions and quarks. The different scalars may mix among themselves due to electroweak or other interactions, as already pointed out in Ref. [34].

An order of magnitude estimate of these mixing diagrams is quite lengthy, but the most important fact is that the scalars coupling to the electroweak bosons is going to be enhanced, when compared to this coupling calculated when the TC self-energy is soft. Note that this effective coupling happens when scalars and W bosons couple through a ordinary fermion or technifermion loop. The W coupling to fermions is the SM one, while the scalar composite coupling to ordinary fermions was shown by Carpenter et al. [70] to be proportional to 2g_{s0}W \Sigma, where \Sigma is the fermionic self-energy, which now is a slowly decreasing function with the momentum and enhance the effective coupling. If we denote a composite scalar by φ, it is possible to show that the ϕWW effective coupling will be proportional to [70]

\[ \Gamma_{\phi\phi WW} \propto \frac{g_{\phi W}^4}{M_W^2} \frac{g_{\mu\nu}}{32\pi^2} \int dq^2 \Sigma_{\phi}^2, \]

where \Sigma_{\phi} has to be substituted by the TC or QCD self-energy depending of which fermion the composite scalar is formed. Of course, the complete calculation of the mixing diagrams is quite model dependent, but, as commented in Ref. [34], the origin of this mixing is another way to see how a full Fritsch matrix pattern of fermion masses can be generated in the type of model that we are proposing here. It is due to this type of coupling that the second generation fermion masses are generated in models with fundamental scalar bosons [48–51]. Finally, in a context where all SM symmetry breaking is promoted by composite scalars we cannot even say how much of the σ (or f_0(500)) meson mass is due to a possible mixing with a composite Higgs boson.

IV. PSEUDO-GOLDSTONE BOSONS

In the condensation of the SU(4)_TC group a large number of Goldstone bosons are formed. Even if we consider other TC groups only three of the Goldstone bosons are absorbed in the SM gauge breaking, and no matter which is the theory we may end up with several light composite states resulting from the chiral symmetry breaking of the strong sector.

These pseudo-Goldstone bosons (or technipions) in the model of Section II may differ by their quantum numbers. They may be colored bosons (Q\gamma_5\lambda^aQ), where \lambda^a is a color group generator, charged bosons (L\gamma_5Q) and neutral pseudo-Goldstone bosons (N\gamma_5N). These bosons receive masses through radiative corrections, and we will verify that, as a consequence of the logarithmic TC self-energy, they will be heavier than usually thought, what is desired in view of the stringent limits on light technipions.

In Ref. [34] we briefly commented that the technipions masses (m_{tl}) are enhanced in comparison with models where the TC self-energy is not of Eq (34) form. One of the arguments is quite simple: The technifermions obtain an effective mass (m_F) of several GeV through the diagram (a_3) and (a_4) of Fig.(2). Note that in our case the condensation effect is not soft, and the calculation of
these diagrams will result in a mass not different from the ones of the third ordinary fermionic family. Particularly in our model there will be several contributions to these type of diagrams. Even the neutral technifermion $N$ will receive contributions from TC condensation mediated by the electroweak $Z$ boson, and from QCD condensation due to $SU(9)$ GUT bosons. These masses, apart from small logarithmic terms, will be roughly of order

$$m_F \approx \sum_i \lambda_i \mu_{TC},$$  

(23)

where $\lambda_i$ represents the product of some coupling constant times Casimir operator eigenvalue contained in any diagram of the type $(a_3)$ or $(a_4)$ contributing to the technifermion mass. For the colored and charged technifermions we cannot even discard a mass as heavy or higher than the top quark mass. These masses will generate quite heavy technipions as can be verified using the Gell-Mann-Oakes-Renner relation

$$m_{\Pi}^2 \approx m_F \frac{\langle \bar{\psi}_T \psi_T \rangle}{2F_{\Pi}},$$  

(24)

where $\langle \bar{\psi}_T \psi_T \rangle$ is the TC condensate and $F_{\Pi}$ the technipion decay constant. With $m_F$ of order of several GeV and standard values for the condensate and technipion decay constant the technipion masses turn out to be of order of 100GeV or higher, as discussed in Ref. [34].

Another way to see that technipion masses are enhanced can be observed in the calculation of a diagram that was already shown in Ref. [34] (see Fig.(4) of that reference). Any radiative boson exchange within a technipion modifying its mass will necessarily involve the technipion vertex connecting with technifermions ($\Gamma_{\Pi F}$). However this vertex is proportional to the technipion wave function ($\Phi_{BS}^\Pi(p, q)$), which at leading order is also related to the TC self-energy as

$$\Phi_{BS}^\Pi(p, q)|_{q=0} \approx \Sigma_T(p^2),$$  

(25)

what is responsible for an enhancement of this radiative correction. An order of magnitude calculation of such diagram was presented in Ref. [34], and we will comment later on the phenomenology of technipions with masses not so much different from the one known for the Higgs boson.

V. TC CONDENSATE

In the previous section and throughout this work we have commented about the different condensates (TC and QCD), and it is interesting to make a parallel between the several studies about the TC condensate value based on walking TC [53] and the one we are discussing here. The TC condensate at one high energy scale $\Lambda$ is related to its value at one scale $\mu$ by

$$\langle \bar{\psi}_T \psi_T \rangle_\Lambda = Z_m^{-1} \langle \bar{\psi}_T \psi_T \rangle_\mu,$$  

(26)

where $Z_m^{-1}$ is a renormalization constant which is given by

$$Z_m^{-1} \approx \left( \frac{\Lambda}{\mu} \right)^{\gamma_m},$$

where $\gamma_m$ is the condensate operator anomalous dimension.

It is possible to compare the condensate values for a theory where the anomalous dimension is perturbative and small at high energy, i.e. $\gamma_m \to 0$ and the one with a non-trivial large anomalous dimension, for instance, in the extreme walking case where $\gamma_m \to 2$. We can define the following ratio that measures the difference between condensates in the walking and non-walking regime

$$R_w = \frac{\langle \bar{\psi}_T \psi_T \rangle_\Lambda_{\gamma_m - 2}}{\langle \bar{\psi}_T \psi_T \rangle_\Lambda_{\gamma_m - 0}},$$  

(27)

Considering these extreme cases this ratio is proportional to

$$R_{w|\gamma_m - 2} \approx \left( \frac{\Lambda}{\mu} \right)^2,$$  

(28)

and this expression serves as an indicator of how much the theory is modified by the non-trivial anomalous dimension. This kind of relation can also be used to verify how radiative corrections appearing in Fig.(2) change the TC behavior.

The UV boundary conditions of the differential TC gap equations modified by the radiative corrections (as can be seen in Ref. [35]) is given by

$$p^2 \frac{d\Sigma(p)}{dp^2} \Big|_{\lambda \to \infty} = -a \int_0^{\Lambda^2} dk^2 \frac{\Sigma(k)}{k^2 + \Sigma^2(k)},$$  

(29)

where $a$ is a factor involving the gauge coupling constant and Casimir operator eigenvalue related to the interaction that induces the radiative correction (e.g., constants related to one of the diagrams $a_2$, $a_3$ or $a_4$ in Fig.(2)). On the other hand we recall that in a $SU(N)$ gauge theory the condensate can be represented by

$$\langle \bar{\psi}_T \psi_T \rangle_\Lambda = -\frac{N}{4\pi^2} \int_0^{\Lambda^2} dk^2 \frac{\Sigma(k)}{k^2 + \Sigma^2(k)}.$$  

(30)

These relations allow us to redefine the ratio shown in Eq.(27) where the condensates values are determined with and without radiative corrections, i.e. when they are calculated with the coupled SDE system ($\alpha_E \neq 0$) and with the values of the isolated condensates ($\alpha_E = 0$)

$$R_{w|rad.cor.} = \frac{\langle \bar{\psi}_T \psi_T \rangle_{\Lambda, \alpha_E \neq 0}}{\langle \bar{\psi}_T \psi_T \rangle_{\Lambda, \alpha_E = 0}} \approx \frac{\langle \bar{\psi}_T \psi_T \rangle_{\Lambda, \alpha_E \neq 0}}{\langle \bar{\psi}_T \psi_T \rangle_{\Lambda, \alpha_E = 0}}.$$  

(31)

We computed Eq.(31) considering the solutions of the coupled and isolated SDE system in the case of the $SU(3)$
TC group, with $\mu = 1 TeV$, $\alpha_E = 0.032$, $\alpha_{TC} = 0.87$ and $C_{TC} = 4/3$. The self-energies were obtained in terms of the variable $x = p^2 / \mu^2$ for each ETC scale $M_E$, and the condensates integrated from $x = 10^2$ up to the UV cut-off $x_A = \Lambda^2 / \mu^2 \sim 10^7$. The ratio $R_w^{rad.cor.}$ was fitted with $R^2 = 0.999$ in the form $a_1 \ln(M_E^2 / \mu^2) + a_2$ and the result is

$$R_w^{rad.cor.} \propto 7.87 \times 10^6 \ln(M_E^2 / \mu^2)^{-4.3}.$$  \hspace{1cm} (32)

If we consider the value of our cut-off ($\Lambda^2 / \mu^2 = 10^7$) we verify that the effect of the radiative correction is not exactly the one of the extreme walking case shown in Eq.(28), but still quite large. We again see that the effect of radiative corrections is not too much different from the effect of the ad hoc four-fermion interactions determined by Takeuchi [33]. Moreover, if we compute the generated quark mass ($m_Q$) as a function of the TC condensate we obtain

$$m_Q \approx \frac{\langle \bar{\psi} T \psi \rangle_\Lambda^{\alpha_E \neq 0}}{\Lambda^2} \approx C \ln(M_E^2 / \mu^2)^{-\kappa_2},$$ \hspace{1cm} (33)

where the constant $C \sim O(\mu)$. This behavior is consistent with the one of Eq.(19).

VI. EXPERIMENTAL CONSTRAINTS

A. $S$ parameter

The $S$ parameter provides an important test for new beyond standard models [79]. This parameter can be described by the absorptive part of the vector-vector minus axial-vector-axial-vector vacuum polarization in the following form in the case of a TC model with new composite vector and axial-vector mesons with masses $M_V$ and $M_A$ and respective decay constants ($F_V$ and $F_A$) [79]:

$$S = 4 \int_0^\infty \frac{ds}{s} Im \Pi(s) = 4\pi \left[ \frac{F^2}{M^2} - 1 - \frac{F^2}{M_A^2} \right].$$  \hspace{1cm} (34)

An interesting analysis of the $S$ parameter in TC theories was performed in Ref. [80] with the use of the Weinberg sum rules, where the case of a conformal theory was considered. In our case we have a TC model which is just a scaled QCD theory, with effective masses due to the different SDE contributions shown if Fig.(2), besides its dynamical mass of $O(1) TeV$. There are no reasons to expect modifications of Eq. (34) for this type of theory, as well as the simple extension to TC of the first and second Weinberg sum rules, which are respectively

$$F_V^2 - F_A^2 = F_{11}^2,$$ \hspace{1cm} (35)

and

$$F_{11}^2 M_V^2 - F_A^2 M_A^2 = 0,$$ \hspace{1cm} (36)

which lead to

$$S = 4\pi F_{11}^2 \left[ \frac{1}{M_V^2} + \frac{1}{M_A^2} \right].$$ \hspace{1cm} (37)

We can also apply to Eq. (37) the result of vector meson dominance [81], implying that $M_V^2 \approx 2 M_A^2$. This relation is not exact even in QCD, but considering it we will be at most overestimating the $S$ parameter, which will be now given by

$$S \approx \frac{6\pi F_{11}^2}{M_V^2}.$$

The TC tecnipion decay constant is usually assumed to be $F_{11} \approx 246 GeV$.

To determine the $S$ value shown in Eq.(38) we must have one estimate of the vector meson mass. It should be remembered that the vector boson mass is quite massive due to the spin-spin part of the hyperfine interactions. We can determine the vector boson mass using the hyperfine splitting calculation performed in the heavy quarkonium context in Ref. [82]

$$M(3 S_1) - M(1 S_0) \approx \frac{8}{9} g^2(0)|\psi(0)|^2 / \mu^2,$$ \hspace{1cm} (39)

where $M(3 S_1)$ and $M(1 S_0)$ describe respectively the masses of vector and scalar lighter bosons. In Eq. (39) $|\psi(0)|^2$ is the meson wave function at the origin, describing a vector boson formed by techniquarks with dynamical mass $\mu_{TC}$. Eq. (39) seems to be reasonable even when the vector boson constituents are light [83].

Making the following assumptions: 1) The TC theory has an infrared frozen coupling constant $g^2(0)/4\pi \approx 0.5$, whose value can be similar to several determinations of this quantity in the QCD case (see, for instance, Ref. [84]); 2) the lightest TC scalar boson has the mass of the Higgs boson found at the LHC, i.e $M(1 S_0) = 125 GeV$, 3) the wave function is approximated by $|\psi(0)|^2 \approx \mu_{TC}^2 \approx 1 TeV^3$, consistent with the other BSE wave functions proportional to the dynamical fermion mass (see Eq.(13)). As a consequence we obtain a vector boson mass $M_V \approx 5.71 TeV$, leading to

$$S \approx 0.035,$$ \hspace{1cm} (40)

whose value has probably been overestimated but still consistent with the experimental data ($S = 0.02 \pm 0.07$) [62].

B. Horizontal symmetry

A necessary condition for the type of model that we are proposing here is the presence of the horizontal (or family) symmetry. This symmetry can be local, and is necessary to enforce the connection of the TC sector only with the third ordinary fermionic generation, i.e the $t$ and
b quarks, the τ and its neutrino. This symmetry in general lead to flavor violations at undesirable level, however in the scheme proposed here the masses of the horizontal gauge bosons can be quite heavy, affecting only logarithmic corrections to the fermion masses, and not producing significant tree level reactions that may be severely constrained by the experimental data. On the other hand there are hints of B decay anomalies \[87\, 88\], which, if confirmed, could also set a mass scale for our horizontal symmetry.

One of the anomalies in B decays appears in the measurement of the ratio between the branching fractions of the \( B^0 \to K^{*0} \mu^+ \mu^- \) and \( B^0 \to K^{*0} e^+ e^- \), which in the small dilepton invariant mass region is given by

\[
R(K^*) = \frac{B^0 \to K^{*0} \mu^+ \mu^-}{B^0 \to K^{*0} e^+ e^-} = 0.66^{+0.14}_{-0.09} \pm 0.03, \tag{41}
\]

which is around 2.2 standard deviations with the SM expectation.

If such deviation is confirmed in the future, it could be explained by a current-current interaction described by the following effective Lagrangian

\[
L_h \propto \alpha_h \frac{\lambda_{\mu} C^{\mu\nu}}{M_h^2} \langle \bar{\gamma}_\nu P_L b \rangle \langle \gamma^\nu \mu \rangle, \tag{42}
\]

where \( \alpha_h \) is the horizontal gauge coupling, \( \lambda_{\mu} \) are mixing angles, \( M_h \) the horizontal gauge boson mass and \( C^{\mu\nu} \) is a Wilson coefficient. Assuming roughly the results of a \( SU(3)_h \) horizontal model of Ref. \[90\] for these several constants we can roughly estimate that \( M_h \) should be greater than 10 TeV. However, this is only a guess because, as repeatedly said in the previous sections, the horizontal gauge boson masses can be quite heavy, and this scale can be set to these masses only if the anomalies remain discrepant with the SM expectation. Otherwise the dependence on the factor \( 1/M_h^2 \) in all observables of this kind will lessen experimental constraints originated from horizontal symmetries.

There are other possible flavor changing neutral currents induced by the horizontal symmetry. For instance, the effective Lagrangian

\[
L_h \propto \alpha_h \frac{\lambda_{d}}{M_h^2} \langle \bar{\gamma}_\nu d_L \rangle \langle \gamma^\nu d_R \rangle, \tag{43}
\]

is induced by one gauge boson exchange and contributes to the \( K^0 - \bar{K}^0 \) transition, which for \( \lambda \approx 1/20 \) requires \( M_h \geq 200 \text{ TeV} \) \[91\]. This contribution can be easily evaded in our type of model just increasing the horizontal gauge boson mass scale, what will not affect the mechanism of ordinary fermions mass generation. Therefore, only if the \( B \) decays anomaly is confirmed it will be necessary a careful scrutiny of the gauge symmetry breaking of the horizontal group.

C. Technipion masses

The LHC has already enough data to constrain the existence of light technipions \[77\]. Due to the fact that the technifermions acquire masses of \( O(100) \text{ GeV} \) the resulting pseudo-Goldstone bosons, i.e. the ones generated in the chiral breaking of the \( SU(4)_T \) TC gauge group discussed in Section II, may be heavier than the SM Higgs boson. Moreover, due to the choice of the horizontal symmetry quantum numbers the technipions will mainly couple to the third ordinary fermionic family, i.e. \( t \) and \( b \) quarks and the \( \tau \) lepton, in such a way that may easily evade the limits found in Ref. \[77\] obtained from the SM Higgs boson data decaying into \( \gamma \gamma \) and \( \tau^+ \tau^- \).

The colored and charged technipions will be quite heavy and produced associated with \( t \) and \( b \) quarks. In the case of the decay into \( b \) quarks the branching ratio may be reduced by a possible small coupling between this quark and the technipion, which will happens through the exchange of a quite heavy gauge boson, and their signal may easily be buried in the background. This leave us with the lightest technipions, which should be the neutral ones \((N \gamma_{5} N)\). In this case a neutral technipion may be produced through vector boson fusion and decay through the weak \( ZZ \) channel.

The TC condensate discussion of Section V can be used to estimate the neutral technipion mass \((m_\Pi)\) in a different way than the one of Ref. \[33\]. As considered in Eq.\((33)\) the neutral technifermion mass \((m_N)\) in terms of the TC condensate generated by the diagram \((a_4)\) of Fig.(2) is given by

\[
m_N \sim \frac{\langle \bar{\psi}_T \psi_T \rangle^{\alpha E \neq 0} \Lambda}{\Lambda^2}. \tag{44}
\]

The above equation together with Eq.\((24)\) lead to the following estimate of the neutral technipion mass

\[
m_\Pi^2 \approx \frac{\langle \bar{\psi}_T \psi_T \rangle_{\Lambda}^{\alpha E \neq 0}}{2 E_{\Pi}^2}. \tag{45}
\]

Assuming \( SU(3)_{TC} \) as the TC gauge group, \( \langle \bar{\psi}_T \psi_T \rangle^{\alpha E = 0} \sim \mu^3 \) with \( \mu = 17 \text{ TeV} \), \( R_{w}^{rad.cor.} \approx 7.87 \times 10^9 (E_{\Lambda}^2/\mu^2)^{4.3} \) defined and appearing in Eqs.\((31)\) and \(42\) we obtain

\[
m_\Pi \sim 160 \text{ GeV}, \tag{46}
\]

which is a rough estimate for the smallest pseudo-Goldstone mass of our type of model, and not yet eliminated by the LHC data \[77\].

The fact that in our type of model the technifermions couple preferentially to the third fermionic family, obtain a large effective mass due ETC interactions, and their other couplings to ordinary fermions are always diminished by the exchange of a very heavy horizontal or GUT gauge boson, turn the search for pseudo-Goldstone signals to be quite difficult. The main hope to detect technipions may be the resonant production of the lightest neutral technipion and its decay into neutral weak bosons.
VII. SCALAR BOSON TRILINEAR COUPLING

As already pointed out many years ago [92], the measurement of the Higgs boson trilinear coupling is fundamental to determine the nature of this particle. If the Higgs boson is a composite particle its trilinear coupling may deviate from the SM value of a fundamental scalar boson, and its measurement can even provide a signal of the underlying theory forming the composite state [93].

In TC or any composite scalar model the scalar trilinear coupling is determined through its coupling to fermions. Using Ward identities we can show the couplings of the scalar boson to fermions to be [72]

\[ G^a(p + q, p) = \frac{i g_W}{2 M_W} \left[ \tau^a \Sigma(p) P_R - \Sigma(p + q) \tau^a P_L \right] \]

where \( P_{R,L} = \frac{1}{2} (1 \pm \gamma_5) \), \( \tau^a \) is a \( SU(2) \) matrix, and \( \Sigma \) is the fermionic self-energy in weak-isodoublet space. As in Ref. [73] we assume that the scalar composite Higgs boson coupling to the fermionic self-energy is saturated by the top quark. We also do not differentiate between the two fermion momenta \( p \) and \( p + q \), since, in all situations with which we will be concerned, \( \Sigma(p + q) \approx \Sigma(p) \).

Therefore the coupling between a composite Higgs boson with fermions at large momenta is given by

\[ \lambda_{Hf} (p) \equiv G(p, p) \sim -\frac{g_W}{2 M_W} \Sigma(p^2). \]  

The trilinear coupling of the composite scalar boson is determined by the diagram shown in Fig. (6). Assuming the coupling of the scalar boson to the fermions to be given by Eq. (48) we find that

\[ \lambda_{MN} = \frac{3 g_W^3}{64 \pi^2} \left( \frac{3 n_F}{M_W^3} \right) \int_0^{M^2_W} \frac{\Sigma^4(p^2)p^4 dp^2}{(p^2 + \Sigma^2(p^2))^3}. \]  

where \( n_F \) is the number of technifermions included in the model.

\[ \lambda_{SM} = \frac{M^2_H}{2 v^2}. \]  

Combined with the above normalization the trilinear coupling of Eq. (49) leads to the following scalar trilinear coupling (\( \lambda \))

\[ \lambda = \frac{1}{6 v} \lambda_{SM}. \]  

Considering Eqs. (49) and (51), \( v = F_n \) and the relation

\[ M^2_W = \frac{g_W^2 F_n^2}{4} \]

we obtain for the trilinear coupling

\[ \lambda = \frac{1}{16 \pi^2} \left( \frac{3 n_F}{F_n^3} \right) \int_0^{M^2_W} \frac{\Sigma^4(p^2)p^4 dp^2}{(p^2 + \Sigma^2(p^2))^3}, \]

which is the trilinear scalar composite coupling that can be compared to the SM coupling of Eq. (50).

The SM trilinear scalar coupling value, according to the normalization of Ref. [64], is

\[ \lambda_{SM} = \frac{M^2_H}{2 v^2}. \]  

The comparison of the trilinear composite coupling with the SM one is shown in Fig. (7). The composite trilinear coupling does differ from the SM one, however by small amounts. It is also shown in the figure the existent LHC limits on this coupling obtained in Ref. [64] from the (\( b \bar{b} \gamma \gamma \)), whose...
values are $\lambda < -1.3\lambda_{SM} = -0.169$ (red region) and $\lambda > 8.7\lambda_{SM} = 1.13$ (green region). Fig.(7) is appropriate to recall that the actual result of the scalar trilinear coupling does vary with $M_E$, and this variation should appear when the coupled gap equations are solved taking into account the running of the ETC gauge coupling constant. Of course, this will introduce only a small variation in the curves of that figure. The white region is the one not excluded yet, and this large region show how difficult is going to differentiate one composite scalar boson from a fundamental one just observing this specific coupling.

\section{Conclusions}

In Refs. \[34, 35\] we called attention to the fact that the self-energies of strongly interacting theories are modified when we consider coupled SDE including radiative corrections. The effect of the radiative corrections is not so much different from the \textit{ad hoc} introduction of effective four-fermion interactions as verified many years ago by Takeuchi \[33\], and lead to self-energies decreasing logarithmically with the momentum. This effect is reviewed in the introduction of this work, where it is made clear that the usual TC model building has to be modified, where the ordinary fermion mass hierarchy is not going to be related to different ETC gauge boson masses.

The presence of a horizontal symmetry is mandatory in the type of models envisaged in Section II. This symmetry is necessary to give masses only to the third generation of ordinary fermions at leading order. The model discussed in Section II is based on the non-Abelian gauge group structure $SU(9)_U \otimes SU(3)_H$, where the $SU(9)_U$ group contains the SM and a $SU(5)_{CG}$ Georgy-Glashow GUT \[44\] and a $SU(4)_{TC}$ group. The $SU(3)_H$ horizontal symmetry is introduced in such a way that their fermionic quantum numbers allow only the third fermionic generation to be coupled to the technifermions. The other fermions remain massless at leading order. However the first generation fermions obtain their masses due to the coupling with QCD, which also has a slowly decreasing self-energy. This is the most interesting fact of our model:

The different fermionic mass scales are dictated by the different strong interactions present in the model! We have shown some of the diagrams that generate the different masses, and made rough determination of their masses. We believe that a large number of theories can be built along the ideas of the model of Section II. Precise determination of fermion masses in this type of model will demand a lengthy determination of SDE coupled equations, where different self-energies can be fitted by equations like the one of Eq.(5).

The fact that the ETC interactions can be pushed to very high energies apparently seems to open a path for a plethora of TC models capable of describing the ordinary fermionic mass spectra. The determination of fermion masses will involve a delicate balance of different gauge group theories for TC, ETC (or GUT) and horizontal symmetry. The ordinary fermion mass matrix calculation will involve the knowledge of specific Casimir eigenvalues, which will depend on the different fermionic representations of the different gauge groups. Will also involve the different coupling constants values of these theories at different scales, and the far more demanding solutions of the coupled system of Schwinger-Dyson equations even with a minimum of approximations. Therefore, at the same time that a new frontier may be open, not any generic combination of gauge theories and respective fermionic representations will be able to explain the known fermionic spectra, meaning that an enormous engineering work will be necessary for a \textit{precise} calculation of ordinary fermion masses.

In Section III we discussed how the composite scalar boson may have a mass lighter than the characteristic mass scale of the theory that forms the composite particle. This could explain how the observed Higgs boson mass, if composite, is smaller than the Fermi mass scale. Perhaps the most important factor about the mass value of the scalar composite resides in the normalization condition of the inhomogeneous BSE, which has to be taken into account when the self-energy is hard and not decaying as $1/p^2$. The normalization condition, as shown by the results presented in Table 1, is enough to lower the scalar mass by a factor $1/10$. However we list many other effects that may also lower the scalar composite mass.

Section IV contains a brief discussion about pseudo-Goldstone boson masses. It is just a complementary discussion to the one already presented in Refs. \[34, 35\], indicating that their masses should be of the order or higher than the one of the observed Higgs boson. Moreover, the pseudo-Goldstone bosons couple at leading order only to the third generation fermions, what is another fact that will complicate their experimental observation.

In Section V we computed the TC condensate in the coupled SDE scenario. This calculation serves to make a comparison with the enhancement that appears in the TC condensate in walking TC theories. Although the mechanism is totally different, i.e. here the gauge theory is just a running theory, there is also one enhancement in the condensates as a result of a logarithmic decreasing self-energy with the momentum. Again, it is possible to verify that the effect is not qualitatively different from the \textit{ad hoc} inclusion of a four-fermion interaction, which is replaced by genuine radiative corrections of known interactions.

In Section VI we comment on possible experimental constraints on this type of model. The main point is that the ETC gauge boson masses may be pushed to very high energies and unnatural flavor changing events will be absent. The S parameter will be of the expected order, and should not differ from the case of TC as a scaled QCD theory. Complementing the discussion of Section IV with what was presented in Section V, we estimated pseudo-Goldstone masses, verifying that they could not yet be seem at the LHC data according the analysis of Ref. \[77\].
In Section VII we computed the trilinear scalar coupling, verifying that a signal of compositeness is far from being observed with the present data \[94\], and this coupling does not differ by a large amount from the SM value in the case of a fundamental scalar boson. Finally, we may say that in the scenario presented in this work, there is a possibility that the SM gauge symmetry breaking promoted dynamically by composite scalar bosons is still alive.

### ACKNOWLEDGMENTS

This research was partially supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) under the grants 302663/2016-9 (A.D.) and 302884/2014 (A.A.N.).
[63] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).
[64] R. Delbourgo and M. D. Scadron, Phys. Rev. Lett. 48, 379 (1982).
[65] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018).
[66] A. Doff, A. A. Natale and P. S. Rodrigues da Silva, Phys. Rev. D 80, 055005 (2009).
[67] C. Lorcé, arXiv:1811.02803.
[68] J. Nebreda, J. T. Londergan, J. R. Pelaez and A. P. Szczepaniak, arXiv:1403.2790 [hep-ph].
[69] L. Montanet, Nucl. Phys. Proc. Suppl. 86, 381 (2000).
[70] S. Narison, arxiv:0208081 [hep-ph].
[71] H. G. Dosch and S. Narison, Nucl. Phys. Proc. Suppl. 121, 114 (2003).
[72] J. R. Pelaez, Phys. Rev. Lett. 92, 102001 (2004).
[73] G. Mennessier, S. Narison and X.-G. Wang, Phys. Lett. B 696, 40 (2011).
[74] J. D. Carpenter, R. E. Norton and A. Soni, Phys. Lett. B 212, 63 (1988).
[75] J. Carpenter, R. Norton, S. Siegemund-Broka and A. Soni, Phys. Rev. Lett. 65, 153 (1990).
[76] A. Doff and A. A. Natale, Eur. Phys. J. C 32, 417 (2003).
[77] R. S. Chivukula, P. Ittisamai, E. H. Simmons and J. Ren, Phys. Rev. D 84, 115025 (2011); (E) Phys. Rev. D 85, 119903 (2012).
[78] Koichi Yamawaki, Proc. 14th Symposium on Theoretical Physics “Dynamical Symmetry Breaking and Effective Field Theory”, (Cheju, Korea, July 21-26, 1995), arXiv:9603293v1[hep-ph].
[79] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65, 964 (1990); Phys. Rev. D 46, 381 (1992).
[80] T. Appequist and F. Sannino, Phys. Rev. D 59, 067702 (1999).
[81] S. Weinberg, Phys. Rev. Lett. 18, 507 (1967).
[82] E. Eichten et al, Phys. Rev. D 17, 3090 (1978); Phys. Rev. D 21, 203 (1980).
[83] H. Schnitzer, invited talk at Conf. on Hadron Spectroscopy (Univ. Maryland, March 1985), preprint BRX-TH-184.
[84] A. C. Aguilar, A. Mihara and A. A. Natale, Phys. Rev. D 65, 054011 (2002).
[85] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 113, 151601 (2014).
[86] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 115, 111803 (2015); (E), Phys. Rev. Lett. 115, 159901 (2015).
[87] R. Aaij et al. [LHCb Collaboration], JHEP, 1509, 179 (2015).
[88] R. Aaij et al. [LHCb Collaboration], JHEP, 1602, 104 (2016).
[89] R. Aaij et al. [LHCb Collaboration], JHEP, 1708, 055 (2017).
[90] R. Alonso, P. Cox, C. Han and T. T. Yanagida, Phys. Rev. D 96, 071701 (2017).
[91] S. Dimopoulos and J. Ellis, Nucl. Phys. B 182, 505 (1981).
[92] O. J. P. Eboli, G. C. Marques, S. F. Novaes and A. A. Natale, Phys. Lett. B 197, 269 (1987).
[93] A. Doff and A. A. Natale, Phys. Lett. B 641, 198 (2006).
[94] F. Maltoni, D. Pagani, A. Shivaji and X. Zhao, Eur. Phys. J. C 77, 887 (2017).