Mathematical Simulation of local transfer for non-Newtonian fluid in porous fabrics

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Abstract. Many processes in the polymer composite, textile, food, and pharmaceutical industries are associated with the flow of non-Newtonian fluids in porous media. This paper considers the mathematical model of a multi-scale process for the filtration of non-Newtonian fluid in periodic porous fabrics. The model should be based on a three-dimensional Navier-Stokes equation with non-Newtonian Carreau-Yasuda viscosity using the asymptotic averaging method. A numerical algorithm was developed for solving local problems of non-Newtonian fluids in periodic cells, and the distribution of velocity, pressure and non-Newtonian viscosity in a single pore was obtained. The algorithm for calculating the permeability tensor is developed, and the effects of fluid rheology are also highlighted.

1. Introduction
The study and simulation of the flow of non-Newtonian fluids in porous media is very important in many biological systems or processes of the petroleum, pharmaceutical, food, cosmetic, textile, paper and polymer composite industries [1]. In the above-mentioned fields, the flowing fluids involved generally exhibit complex behaviors such as shear thinning/thickening effects, elasticity, anisotropy, and yield stress [1, 4]. Therefore, it is extremely important to adequately simulate the flow of a liquid binder in a porous composite structure having a complex space geometry. Modeling various process problems in porous systems, especially with reference to the mechanics of composite materials, is quite interesting. Various aspects of modeling the technological processes of composite materials production are considered in [2 − 4, 6].

In most existing literature, the permeability coefficients of a porous medium are determined experimentally or by using various empirical and approximate relationships to determine the local filtration processes. Thus, a rough estimates of the actual processes occurring within the pores with complex geometries are obtained, which results in large deviations in the permeability measurement. Therefore, an important part of the filtering study is to use the solution of the non-Newtonian Navier-Stokes equation, and derive the average filtration equation from the "first principle" to analyze the local process of fluid space flow in a single pore, rather than based on phenomenological theory. This approach to the modeling of filtration in porous structures is called a microscale simulation, which is a joint study of the liquid flow in individual pores (a local problem).

However, the non-Newtonian characteristics of the fluid pose a significant challenge to the numerical simulation of the local transmission process. Although there has been a small amount
of research on local transport of non-Newtonian fluids in regular geometries, this is clearly insufficient. In order to better understand the above complex fluid flows, we performed a multi-scale simulation for the flow of non-Newtonian fluids in a porous fabrics. The effect of non-Newtonian viscosity on the local transport process is highlighted in this study.

2. Mathematical model

2.1. Main assumptions of the model

It is assumed that the composite structure is characterized by periodicity. And individual microscopic periodic cells also have geometric mirror symmetry.

Take the following assumptions regarding the properties of the phases:

(1) fluids are isotropic non-Newtonian viscous incompressible media;
(2) the porous skeleton is assumed to be non-deformable, i.e. his movement is not considered;
(3) fluid movement is isothermal;
(4) mass forces are absent.

2.2. Principal equations

The motion of non-Newtonian fluid in the composite occupying the pore, within the framework of the assumptions made, is described by the system of the incompressible Navier-Stokes equations, which, together with the adhesion conditions on a solid surface, have the following form [6]:

\[ \nabla \cdot \mathbf{v} = 0 \]  
(1)
\[ \rho_0 \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \nabla \cdot 2\mu \mathbf{D}; \]  
(2)
\[ \mathbf{v}|_{\mathcal{G}} = 0; \]  
(3)
\[ p|_{t=t_0} = p_0. \]  
(4)

where \( \rho_0 \) -density; \( \mathbf{v} \) -velocity of fluid; \( p \) - pressure; and \( \mu \) - non-Newtonian viscosity.

The strain rate tensor:

\[ \mathbf{D} = \frac{1}{2}(\nabla \mathbf{v} + \nabla \mathbf{v}^T). \]  
(5)

The \( A_I \) and \( A_V \) models of Voigt isotropic viscous media [6] are considered, and non-Newtonian viscosity satisfies the following defining relation [1, 9]:

\[ \frac{\mu - \mu_\infty}{\mu_0 - \mu_\infty} = (1 + (\lambda I_2)^{\frac{1-n}{2}}. \]  
(6)

which depends on the second invariant of the strain rate tensor [5]:

\[ I_2(\mathbf{D}) = \sqrt{2\mathbf{D} \cdot \mathbf{D}}. \]  
(7)

Then we used a dimensionless form, here:

\[ \tilde{\mathbf{v}} = \frac{\mathbf{v}}{v_0}, \quad \tilde{p} = \frac{p}{p_0}, \quad \tilde{t} = \frac{t}{t_0}, \quad \tilde{x} = \frac{x}{x_0}, \quad t_0 = \frac{v_0 x_0}{v_0}. \]  
(8)

Here \( \tilde{\mathbf{v}}, \tilde{p} \) are the dimensionless velocity and pressure of fluid, respectively; \( v_0, p_0 \) are their typical magnitudes. \( t_0 \) is the dimensionless time. The equations for the flow of an incompressible fluid can be written in a dimensionless form as follows (the symbol is omitted below):

\[ \nabla \cdot \mathbf{v} = 0 \]  
(9)
\[ \frac{\partial \mathbf{v}}{\partial \tilde{t}} + \mathbf{v} \cdot \nabla \mathbf{v} = -\mathcal{E}u \nabla p + \frac{1}{\mathcal{R}e} \nabla \cdot 2\mu \mathbf{D}; \]  
(10)
\( \mu = s + (1 - s)(1 + (CuI_2)^2)^{\frac{1 + \kappa}{2}} \), \( s = \frac{\mu_{\infty}}{\mu_0} \) \quad (11)

\( \mathbf{D} = \frac{1}{2}(\nabla \mathbf{v} + \nabla \mathbf{v}^T), \quad I_2(\mathbf{D}) = \sqrt{2\mathbf{D} \cdot \mathbf{D}} \) \quad (12)

\( \mathbf{v}|_\Sigma = 0; \) \quad (13)

\( p|_{t=t_0} = p_0. \) \quad (14)

where \( Eu = \frac{p_0}{\rho_0 u_0^2} \) - Euler number, \( Re = \frac{\rho_0 u_0^2}{\mu_0} \) - Reynolds number, \( Cu = \frac{\lambda_0}{x_0} \) - Carreau number.

3. Statement of the local problem

In accordance with the method of asymptotical averaging [3, 4, 7, 8], composite structure can be presented by a periodical structure consisting of repeating elements, i.e., periodicity cells \( V_\xi \).

Local coordinates \( \xi \) introduced as

\[ \xi = \frac{x}{\kappa}. \] \quad (15)

can change within limits of periodicity cells \( V_\xi \), where \( \mathbf{\bar{x}} = \frac{x}{x_0} \) are the nondimensional global coordinates changing inside the whole region \( V \) occupied by the material; \( x_0 \) is the characteristic global size of the whole body \( V \); \( l_0 \) is the characteristic size of the periodicity cell \( V_\xi \); \( \kappa \) is the small parameter:

\[ \kappa = \frac{l_0}{x_0} << 1. \] \quad (16)

Then all main functions \( f \) describing a fluid flow in pores can be considered to be quasi-periodic (i.e. depending on \( \xi \) and \( \mathbf{\bar{x}} \)) and periodic in \( \xi \). Differentiation of the functions is realized by the following rule:

\[ \nabla f = \nabla_x f + \frac{1}{\kappa} \nabla_\xi f. \] \quad (17)

where \( \nabla_x, \nabla_\xi \) are nabla-operators over coordinates \( x, \xi \).

We introduce the operator of the mean value \( \langle - \rangle \) over the periodicity cells \( V_\xi \) for quasi-periodic in local \( \xi \in V_\xi \) coordinates of the functions \( f \):

\[ \langle f \rangle = \frac{1}{\varphi_p} \int_{V_\xi} f dV; \quad \varphi_p = \int_{V_\xi} dV. \] \quad (18)

However, due to periodicity of the structure, the solution may be represented in the form of asymptotic expansion in terms of parameter \( \kappa \) [7, 8]:

\[ \mathbf{v} = \mathbf{v}^{(0)}(\mathbf{x}, \xi) + \kappa \mathbf{v}^{(1)}(\mathbf{x}, \xi) + \kappa^2 \mathbf{v}^{(2)}(\mathbf{x}, \xi) + \cdots; \] \quad (19)

\[ p = p^{(0)}(\mathbf{x}, \xi) + \kappa p^{(1)}(\mathbf{x}, \xi) + \kappa^2 p^{(2)}(\mathbf{x}, \xi) + \cdots; \] \quad (20)

\[ \mu = \mu^{(0)}(\mathbf{x}, \xi) + \kappa \mu^{(1)}(\mathbf{x}, \xi) + \kappa^2 \mu^{(2)}(\mathbf{x}, \xi) + \cdots. \] \quad (21)

Having substituted expansions (19) – (21) into Equations (9) – (14), then having collected in them terms at the same powers of \( \kappa \) and putting terms at the lowest powers of \( \kappa \) equal to zero, we obtain the local problems of the zero level 'over the periodicity cell':

\[ \nabla_\xi \cdot \mathbf{v}^{(0)} = 0 \] \quad (22)

\[ -\nabla_\xi p^{(1)} + \eta_0 \nabla_\xi \cdot 2\mu^{(0)} \mathbf{D}^{(0)} = -\nabla_x p^{(0)}; \] \quad (23)

\[ \mathbf{D}^{(0)} = \frac{1}{2}(\nabla_\xi \mathbf{v}^{(0)} + \nabla_\xi \mathbf{v}^{(0)T}); \] \quad (24)

\[ \mu^{(0)} = s + (1 - s)(1 + (Cu^0)^2)Y^{(0)} \frac{2s - 1}{s}, \quad Y^{(0)} = 2\mathbf{D}^{(0)} \cdot \mathbf{D}^{(0)}; \] \quad (25)

\[ \mathbf{v}^{(0)}|_{\Sigma_\xi} = 0, \quad \langle p^{(1)} \rangle = 0. \quad [[\mathbf{v}^{(0)}]] = 0, \quad [[p^{(1)}]] = 0. \] \quad (26)
where unknown $v^{(0)}$ and $p^{(1)}$, the symbol $[[\cdot]]$ denotes the periodicity conditions. $\nabla_x p^{(0)}$ is considered as "input data". And $Re\kappa^0 = Re^0 = O(1)$, $Eu\kappa^2 = Eu^0 = O(1)$ and $Cu = \kappa^0 Cu^0$. Then $\eta_0 = \frac{1} {Re^0 Eu^0}$.

4. Numerical Simulation
Next, the finite element method will be used to solve the local problems (22) – (26). We only consider finite elements that meet LBB conditions [10]. Here, the chemical industry raw material benzene is taken as an example for numerical study, so $\eta_0 = 0.0652$. Applying the physical symmetry of the periodic cells in the hypothesis, the distribution of the results of $1/8$ periodic cells can be solved.

4.1. Distribution of results on periodic cells
Here we consider reinforced composites and the composites are isotropic. Here we consider the power-law index $n = 0.25$.

**Figure 1.** Distribution of the velocity component $v^{(0)}_1$ in $1/8$ of the tissue structure periodicity cell.

**Figure 2.** Distribution of the velocity component $v^{(0)}_2$ in $1/8$ of the tissue structure periodicity cell.

**Figure 3.** Distribution of the velocity component $v^{(0)}_3$ in $1/8$ of the tissue structure periodicity cell.

**Figure 4.** Distribution of pressure $p^{(1)}$ in $1/8$ of the tissue structure periodicity cell.
4.2. Compare the permeability of Newtonian fluid

The averaging of the local equations of Newtonian fluids allows one to obtain the classical Darcy’s law of filtration:

\[ <v> = -K \nabla p. \]  

(27)

where \( <v> \) - the filtration velocity, \( K \) - the permeability tensor of the porous medium.

For Newtonian fluids, the components of the seepage rate tensor is constant. For non-Newtonian fluids, however, the components of the permeability tensor are a function related to the constitutive equation of viscosity. To describe this difference, we compared the seepage rate and viscosity changes of non-Newtonian fluids and Newtonian fluids in porous media.

5. Conclusions

The proposed method allows calculation of the distribution of pressure and non-Newtonian viscosity micro-regions, as well as the filtration velocity component within a single pore, and can also numerically calculate the permeability coefficient of the porous medium without any additional experimental research. For this purpose, modeling of non-Newtonian viscosity resin flow in the periodicity cells of composite structure using an asymptotic homogenization method. And the finite element method is used to calculate the local problem. The results of numerical modeling of non-Newtonian fluid local spatial flows on the periodicity cell of composite structures are obtained, showing the effectiveness of the proposed algorithm for solving local problems. Comparing the difference between the filtration speed of Newtonian fluid and non-Newtonian fluid, the particularity of the local transmission process of non-Newtonian fluid is fully verified.
6. References

[1] Bird R.B, Armstrong R.C. and Hassager O. 1987 Dynamics of polymeric liquids. Vol. 1: Fluid mechanics (New York: John-Wiley) P 649.
[2] Golovatov D., Mikhaylov M., Bosov A. 2016 Optimization of Technological Parameters of Impregnation of Load-Bearing Rod Elements of Reflector made of Polymer Composite Materials by Transfer Molding Method Indian J. of Science and Technology. 9(46).
[3] Idris Z., Orgeas L., Geindreau C., Bloch J.F. and Auriault J.L. 2004 Microstructural effects on the flow law of power-law fluids through fibrous media Model. Simul. Mater. Sc. 12(5) 995.
[4] Orgeas L., Geindreau C., Auriault J.L. and Bloch J.F. 2007 Upscaling the flow of generalised Newtonian fluids through anisotropic porous media J. Non-Newtonian Fluid Mech. 145 15-29.
[5] Dimitrienko Yu.I. 2002 Tensor analysis and nonlinear tensor functions (Berlin: Springer) P 662.
[6] Dimitrienko Yu.I. 2010 Nonlinear continuum mechanics and large inelastic deformations (Berlin: Springer) P 747.
[7] Dimitrienko Yu.I. 2015 Thermomechanics of composites under high temperatures (Berlin: Springer) P 434.
[8] Dimitrienko Yu.I. and Dimitrienko I.D. 2013 Simulation of local transfer in periodic porous media Eur. J. Mech. B-Fluid. 37 174-179.
[9] Dimitrienko Yu.I. and Li S.G. 2018 Mathematical simulation of non-isothermal steady flow of non-Newtonian fluid by finite element method Mathematical Modeling and Computational Methods. 2 70-95.
[10] Zienkiewicz O.C., Taylor R.L. and Zhu J.Z. 2005 The Finite Element Method: Its Basis and Fundamentals: Its Basis and Fundamentals (Amsterdam: Elsevier) P 733.