Formation of optimal-order necklace modes in one-dimensional random photonic superlattices

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We study the appearance of resonantly coupled optical modes, optical necklaces, in Anderson localized one-dimensional random superlattices through numerical calculations of the accumulated phase. The evolution of the optimal necklace order \( m^\ast \) shows a gradual shift towards higher orders with increasing the sample size. We derive an empirical formula that predicts \( m^\ast \) and discuss the situation when in a sample length \( L \) the number of degenerate in energy resonances exceeds the optimal one. We show how the extra resonances are pushed out to the miniband edges of the necklace, thus reducing the order of the latter by multiples of two.

Wave interference phenomena play a crucial role in transport properties of various physical systems from periodic \(^1\) to disordered \(^2,3\). Among these, originally studied for electronic systems, Anderson localization seems the more intriguing one \(^4\). It predicts a phase transition form metallic-like conductivity to an insulating regime, when the transport can come to halt with increasing the randomness. On the other hand, in a one-dimensional (1D) disordered system, such a transition happens when the sample size \( L \) extends beyond the so-called \textit{localization length} \( \xi \). In such conditions, the wavefunctions become localized within an extension \( \xi \) and decay exponentially with distance \( l \). Being essentially an interference phenomenon, Anderson localization has been studied as for electromagnetic and acoustic waves \(^5\), as well as for degenerate atomic gases \(^6\). Very recently, Anderson localization of optical waves in the microwave regime has been demonstrated in experiments on 1D random multilayer dielectric stacks \(^7,8\).

Initially, it has been widely accepted that the conductivity (transmittivity) of a disordered chain is mainly supported by states which are closer situated to the sample center \(^9\). Later, this was questioned, since for long enough specimen, the states in the center possess significantly reduced probability to support a two-step hopping transport through these states. In late 80’s, Pendry \(^10\) and Tartakovskii et al. \(^11\), independently, argued that the conductivity in such systems should be dominated by so-called necklaces – few homogeneously distributed through the sample states, degenerate in energy, and coupled resonantly to delocalize and extend through the chain. The number \( m \) of resonant states forming a necklace, was calculated to scale as \( L^{1/2} \), while the probability of their occurrence was shown to drop as \( \exp(-L^{1/2}) \), thus predicting them to be increasingly improbable events in long samples \(^10\). Therefore, for a certain sample length, a trade-off between the expected number \( m \) and their occurrence probability would determine the optimal order of the necklace.

In this Letter we study the appearance of optical necklaces in finite 1D random superlattices through numerical calculations of the accumulated phase and follow the evolution of the optimal necklace order when increasing the sample size. We suggest an empirical formula which predicts the optimal necklace order \( m^\ast \) and discuss the situation when in a sample length \( L \) the number of degenerate in energy resonances exceeds \( m^\ast \). We show that the extra resonances, which are spaced by a distance less than the optimal one for a certain sample length, are coupled strongly enough to split and be pushed out to the miniband edges of the necklace, thus reducing the order of the latter by multiples of two.

We studied random binary multilayer stacks composed by \( N \) layers of \( A \) and \( B \)-type dielectric materials (refractive indices chosen to be \( n_A = 1.3 \) and \( n_B = 2.1 \)). Positionally random sequences were generated by giving equal weights to each type of layer. Two types of random multilayer stacks were investigated. In one type, the physical thicknesses \( d_A \) and \( d_B \) of the constituent layers were generated randomly through the sequence, thus a complete randomness was achieved. For the other type, \( d_A \) and \( d_B \) were chosen to be resonant at some wavelength \( \lambda_0 \) (quarter-wave layers), therefore, the condition \( n_A d_A = n_B d_B = \lambda_0/4 \) was considered. The quarter-wave condition makes the system not perfectly random and, in particular, maximizes the constructive interference effects at the resonant wavelength.

In general, in a 1D dielectric multilayer stack, the phase suffers a \( \pi \) shift each time it meets a resonance. Therefore, the phase measurement is a valid tool to isolate the spectrum singularities in either periodic \(^12\) or random systems \(^8\). This way, in Ref. \(^8\), localized states and, more importantly, optical necklaces have been identified through interferometric measurements on Anderson localized 1D random superlattices. When \( m > 2 \) degenerate in energy states couple weakly to form a necklace \(^13\), the accumulated phase increases smoothly through the transmission band built up by these latter, summing up to \( m\pi \). On the contrary, when strong coupling occurs, the phase suffers single \( \pi \) jumps through well defined spectrally separate peaks.

Hence, through this study, the order of the necklaces has been assigned to the multiples of \( \pi \) in the smoothly changing phase shift through the transmittance
panels in Fig. 1 report the counts of different order necklaces. We observe that at smaller thicknesses (panel A, \(L \approx 103 \mu m\)) the major number of necklaces is of second order and fewer third order ones occur. With increasing sample size, the number of second order necklaces diminishes and the third order ones firstly start to dominate at \(L \approx 184 \mu m\), obtaining a maximum at \(L \approx 220 \mu m\). With a further increase in sample size (panel E, \(L \approx 253 \mu m\)) higher orders shoulder starts to rise in the necklace counts histogram, indicating that the samples become enough thick to fit more than three degenerate resonances.

The order of the necklace, which fits optimally inside the sample length \(L\) to have the highest and most compact transmission band, can be empirically calculated as

\[
m^* = \left\lfloor \frac{L - \beta \xi}{\alpha \xi} \right\rfloor + 1, \tag{1}\]

where \(\lfloor \cdot \rfloor\) stand for the floor-function and denote the greatest integer part of the expression inside. Here, the parameter \(\alpha\) counts the distance (in the units of \(\xi\)) between two neighboring resonances efficiently coupling to form a necklace, while \(\beta\) considers the coupling of first and last resonances to the environment (see Fig. 2). In reality, the step function of Eq. 1 considers the order of the most frequent necklace from the histogram of all observed orders. In our case, a reasonable fit to the numerical results was found with \(\alpha = 7\) and \(\beta = 2.2\).

The power law scaling of the order of necklaces in disordered systems and the very low probability of the higher orders occurrence predicts a trade-off scenario, which should reduce the number of actually observed ones \cite{10}. In the following, we address this issue to understand how this reduction happens and reveal interesting physical picture of the phenomenon.

Let us consider the case of an optimal \(m\)th order necklace stretching through sample length \(L\). The most compact and, in the meanwhile, high-transmission band formed by this should occur when \(m\) resonances, degenerate in energy, are distributed homogeneously through the sample to favor similar coupling between neighboring states. This situation is an ideal one and is very improbable for higher order necklaces. Since the resonances occur randomly in the depth of the sample, it becomes more probable that some of them will appear at a distance closer than the optimal separation \(l^* = \alpha \xi\) between two neighboring cavities. When this happens, the spatially close resonances couple strongly and a larger mode

\[
\frac{1}{2} \beta \xi \quad l^* = \alpha \xi
\]

FIG. 2: A simplified model of formation of an optimal-order necklace through the sample length \(L\).

\(\text{FIG. 1: (Color online) The logarithm of integrated transmission versus sample length for two cases of non-resonant (○) and resonant (■) layers. The solid and dashed lines are linear fits to the data for non-resonant and resonant layers, respectively. The fits give localization lengths of } \sim 8.5 \mu m \text{ and } \sim 10.7 \mu m \text{ respectively. The histograms on the right show the counts of the order of observed necklaces for five different thicknesses of samples with resonant } \lambda/4 \text{ layers.}\)
FIG. 3: (Color online) The light intensity distribution inside a periodic superlattice of four coupled cavities (left panels) and the corresponding transmission and phase spectra (right panels). All the spectra are plotted in a narrow frequency range around $\nu_0 = c/\lambda_0 = 200$ THz. (a) homogeneously distributed cavities (an artificial $m=4$ necklace) form the highest transmission band through which the phase accumulates smoothly a $4\pi$ increase, (b)-(d) moving the two central cavities closer to each other, the light transmission around $\lambda_0$ attenuates by orders of magnitude and the smooth phase jump counts only $2\pi$, reducing the necklace order.

repulsion pushes them farther from the miniband center.

We examine the dynamics of such a situation on an example of a periodic photonic structure, where for simplicity four resonant cavities are coupled through Bragg sequences in such a way that a high transmission band of an artificially built necklace is formed. The choice of a periodic structure allows us to manipulate the spatial positions of various cavities, which is an impossible task for random systems. Figure 4 shows the effect of stronger coupling between these two resonances: two well defined resonances (symmetric and anti-symmetric modes) rise out of the miniband core due to the mode repulsion and the channel intensity decreases. The transmission of the miniband peak drops by four orders of magnitude, and the smooth change in the phase now sums up to $2\pi$. If we move these cavities closer (Fig. 3C,D), the peak transmission drops down to $\sim 10^{-8}$. Note that when excluding the two central cavities (while maintaining the same sample length) the transmission drops further down by almost three orders of magnitude to have a peak transmission of $\sim 10^{-11}$.

The intensity map in Fig. 4 contains an interesting feature, to which we would like to draw a special attention. We observe three maxima instead of two through the original transmission miniband, while the smooth change of the phase sums up to $4\pi$. The intensity distribution map, and, therefore, the electric field profile at the miniband center frequency shows many bright and other less intense clumps, but the real number of resonances, directly involved in the light transport through the miniband is less and can be exactly counted through the phase variation.

In order to catch such a situation in a random system, we have performed a number of realizations of a 950 layer sample ($218 \mu m$-thick). In Fig. 4 we show such an event. In the right panel six well-resolved intensity maxima are observed, while the smooth change of the phase sums up to $4\pi$. The intensity distribution map, and, therefore, the electric field profile at the miniband center frequency shows many bright and other less intense clumps, but the real number of resonances, directly involved in the light transport through the miniband is less and can be exactly counted through the phase variation.

Thus, when the number of randomly built degenerate cavities exceeds the optimal order of the necklace for a certain sample length, the extra cavities couple strongly to be pushed out to the edges of the miniband, reducing
the order of the necklace state and its peak transmission. We stress that this reduction is always a multiple of two, independently on the fact whether even or odd number of extra cavities are coupled strongly. This is because a strong coupling of even or odd number of cavities results in a dip or a peak in the spectral line at the resonant wavelength. Therefore, at $\lambda_0$, only multiples of $2\pi$ are filtered out from the phase jump. Nevertheless, the extra cavities play an important role in the formation of necklaces, since their non-zero overlap reduces the reflectivity of inter-cavity regions and contributes positively to link the resonant tunneling transport through the necklace miniband.

Along this, in a fully random system, another interesting situation can happen. Suppose, that next to a necklace band centered at $\lambda_0$ (call $N^0$), for simplicity, two other degenerate states, resonant at some wavelength $\lambda_0 + \Delta$ (or $\lambda_0 - \Delta$), are formed. If these are coupled strongly enough, it can happen that a fortuitous mode splitting will push either the symmetric or the asymmetric mode to fit into the band width of $N^0$. In the case, when the original necklace at $\lambda_0$ is weak, the new state can positively contribute to enhance the necklace transmission, thus increasing the order of $N^0$ by one. This is to say that the phase shift will count only one more resonance, but still the intensity map will show extra nodes, since the new resonance will appear with a double clump. We note, that in a random sample with resonant layers, such a situation cannot occur: the transmission spectrum in this case is always symmetric against $c/\lambda_0$, therefore, the described contribution to the necklace $N^0$ will occur from both sides ($\lambda_0 \pm \Delta$) and consequently will result in a back repulsion of degenerate modes out of the necklace band.

To conclude, we address through this study some peculiarities of the formation of optical necklaces, resonantly coupled degenerate modes, in one-dimensional random multilayer systems. We stress the importance of using the phase-jump method to reveal the exact order of necklaces, in contrast to counting intensity or electric field clumps through the necklace. We show how the extra resonances are filtered out from the necklace when in a sample length $L$ the number of degenerate in energy resonances exceeds the optimal one.

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