Exchange enhancement of quasiparticle and ESR spin-gap in symmetric and asymmetric narrow-gap quantum wells

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Abstract. We report a study of electron-electron (e-e) interaction effect on spin-gap in n-type narrow-gap quantum well (QW) heterostructures. By using the Hartree-Fock approximation (HFA) and generalized single-mode approximation (GSMA) based on the 8-band \( k \cdot p \) Hamiltonian we demonstrate that spin-orbit interaction and the mixing between \(|S>\) and \(|P>\) states in the conduction and valence bands affect significantly many-body corrections to the spin-gap in symmetric and asymmetric QWs in the integer and fractional Quantum Hall regime. The spin-gap values estimated for 2D electron gas (2DEG) in InAs/AlSb QWs are compared with the experimental results obtained by magnetotransport and by electrically detected electron spin resonance (ESR).

1. Introduction

The recent focus on making use of the spin degree of freedom has also revived an interest in basic studies of the enhanced g-factor in QW heterostructures. The g-factor values in 2DEG obtained in magnetotransport and ‘magnetooptic’ experiments greatly differ from each others that result from the different contribution of e-e interaction effects into ‘magnetostatic’ and ‘magnetooptic’ phenomena. The ‘magnetostatic’ effects are related to the exchange renormalization of quasiparticle energy spectrum [1] determined by the 2DEG ground state. The 2DEG investigation techniques involving magnetotransport, capacitance spectroscopy of Landau levels (LLs) [2, 3], thermal capacity measurements [4, 5] and resonant tunneling in a magnetic field [6] allow one to directly determine the spin-gap at the Fermi level that characterizes quasiparticle g-factor.

In ‘magnetooptic’ experiments with 2D systems the resonant absorption of an electromagnetic wave gives rise to the quasiparticle-pair excitations formed by electrons that are excited onto an unfilled or partially filled LL, and quasiholes appearing simultaneously at the electron’s vacated level, which cause the 2D system state to change from the ground state to the excited one. The simplest 2DEG excitation in a magnetic field is a spin exciton (spin wave) arising at the electron transition between the LLs spin-split through Zeeman interaction. The long-wave limit of the spin exciton energy determines the resonant frequency of absorption, measured in ESR. The Larmor theorem states that e-e interaction does not affect ESR energy in a 2D system with rotation invariance in the spin.

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space. So, if the effects related to spin-orbit interaction (SOI) in QW heterostructures are rather weak, the g-factor determined from the ESR measurements is the ‘single-electron’ one, i.e. unaffected by the e-e interaction [7].

Since typical narrow-gap QW heterostructures based on InSb, HgTe and InAs are characterized by pronounced SOI, which leads to breaking of rotational symmetry in spin space, the Larmor theorem is violated in these structures [8]. Our earlier studies [8-11] have shown the SOI and mixing between $|S\rangle$- and $|P\rangle$-states in the $\Gamma_6$, $\Gamma_7$, and $\Gamma_8$ bands to largely influence the e-e interaction effects such as quasiparticle and ESR g-factor enhancement in 2DEG. It was shown that exchange enhancement of the quasiparticle g-factor in narrow-gap QWs arises for any LL filling factors [9-11] in contrast to the QWs with parabolic electronic subbands [1]. In [8] we demonstrated that the ESR g-factor and quasiparticle g-factor, measured in magnetotransport, coincide at even-valued LL filling factors in this heterostructures.

In this paper we extend the limits of previous works to theoretical study of exchange enhancement of spin-gap observed in ESR and magnetotransport in asymmetric narrow-gap QWs within Hartree-Fock approximation (HFA). The spin-gap values calculated for 2DEG in InAs/AlSb QW heterostructures [12-18] are compared with the experimental data obtained by magnetotransport [17, 18] and by ESR in moderate magnetic field. We also predict additional correlation-induced enhancement of ESR spin-gap for polarized Fractional Quantum Hall states (FQHS) as compared with the values obtained within the HFA. A non-linear dependence of ESR energies on LL filling factor in ultra-quantum limit is demonstrated.

2. Theory

To take into consideration SOI and the mixing of the conduction band ($\Gamma_6$) with the light- and heavy-hole bands ($\Gamma_8$) and with the split-off band ($\Gamma_7$) exactly, the 8-band $k\cdot p$ Hamiltonian as a single-particle kinetic energy operator is used [9]. It directly takes into account the influence of nonparabolicity, lattice-mismatch deformation and SOI on electron energy spectrum. As all calculations in this paper are performed for InAs/AlSb QW heterostructures, we also take into consideration the ‘built-in’ electric field of ionized donors in GaSb cap layer [14-16] in single-particle Hamiltonian. The Coulomb Green function was obtained by solving the electrostatic problem in three-layered system with different permittivity values in QW and in barriers [9].

2.1. Quasiparticle spin-gap versus ESR energy

Many-body corrections to quasiparticle spin-gap were obtained in the ‘screened’ Hartree-Fock approximation (HFA) [9]. To take into account the LL broadening under the influence of a random potential caused by defects in actual structures, we used a Gaussian profile for the DOS of each LL:

$$D(E) = \frac{1}{2\pi a_B^2} \frac{2}{\sqrt{2\pi \Gamma}} \exp \left( -2 \frac{E^2}{\Gamma^2} \right), \quad (1)$$

where $a_B$ is the magnetic length ($a_B^2 = \hbar c/eB$), $\Gamma$ is the width of the DOS. By analogy with the Born approximation for the 8-correlated random potential [19] we assume that

$$\Gamma = \Gamma_0 \sqrt{\frac{B}{B_0}}, \quad \quad \quad (2)$$

where $B_0 = 0.5 \, T$ is the magnetic field in which the Shubnikov-de Haas oscillations show up [17, 18] and $\Gamma_0$ is free parameter.

Figure 1 illustrates the quasiparticle and ESR spin-gap calculations versus magnetic field in InAs/AlSb heterostructures with a 15 nm wide QW. The step-like features observed at even-valued LL filling factors appear due to the electronic subband nonparabolicity in InAs/AlSb QW [8-11]. As the magnetic field increases, the Fermi level ‘jumps’ from one pair of spin-split LLs to the lower-lying pair with a greater Zeeman splitting. This causes a sharp rise of the Zeeman energy at the Fermi level. It is seen that the single-particle spin-gap in a symmetric QW is determined by the Zeeman splitting of
LLs demonstrating a linear dependence on magnetic field in the region of weak magnetic fields. The Rashba spin splitting in an asymmetric QW leads to distortion of the monotonic dependence of single-particle spin-gap on magnetic field and appearance of a pronounced minimum in the low magnetic fields region. In this case the spin splitting at the Fermi level is determined by two contributors: the Rashba- and the Zeeman splitting [20]. As the magnetic field increases from zero, the single-particle spin-gap quickly drops, going smoothly over to the Zeeman splitting. Note that for high LL indices \( n \) (in weak magnetic fields) the single-particle spin-gap can be obtained analytically [21]:

\[
\Delta_0 \approx \left[ \hbar \omega_c - g(\text{e}) \mu_B B \right]^2 + \Delta_R^2 \right]^{1/2} - \hbar \omega_c ,
\]

where \( g(\text{e}) \) is a single-particle g-factor [5, 6], \( \omega_c = eB/m^*c \), \( \Delta_R \) and \( m^* \) are the Rashba splitting and the electron effective mass at the Fermi wavevector in a zero magnetic field, respectively. Thus, the single-particle spin-gap decreases linearly with \( B \) in weak magnetic fields.

![Figure 1](image.png)

**Figure 1**: Spin-gap in a 15 nm wide InAs/AlSb QW with symmetric (left panel) and asymmetric (right panel) profile versus magnetic field at 2DEG concentration of 7.65\( \times \)10\(^{11} \) cm\(^{-2} \). Black curve is the single-particle spin-gap calculated within the Hartree approximation. Red and green curves correspond to the quasiparticle spin-gap for LL broadening values \( \Gamma_0 = 0.3 \) meV and 0.5 meV respectively. Blue curve is ESR spin-gap. The arrows indicate the magnetic field values corresponding to the integer-valued LL filling factors.

Strong SOI and mixing between \(|S>\) and \(|P>\)-states in the conduction and valence bands in the InAs/AlSb QWs also leads to considerable enhancement of quasiparticle and ESR spin-gaps. It is clear that the Rashba spin splitting in the asymmetric QWs resulted in a noticeable effect on the enhanced spin gap (quasiparticle and ESR) in weak magnetic fields only. It should be noted that, in asymmetric QWs in low magnetic fields, both spin-gaps enhanced by exchange interaction tend to Rashba splitting; in symmetric QWs, the exchange enhanced spin-gaps tend to zero. It can be shown that HFA corrections to the energy of spin-gaps at even-valued LL filling factors in symmetric [8-11] and asymmetric [18] narrow-gap QWs have a square-root dependence on the magnetic field resulting in the g-factor divergence in low magnetic field region [8, 10].

### 2.2. ESR spin-gap for FQHS

HFA completely ignores the correlation interaction between 2D electrons, and it is a fairly good approximation for spin-gap calculation at integer LL filling factors or in the case of large number occupied LLs only. To take into account the correlated nature of 2DEG ground state the generalized single-mode approximation (GSMA) instead of HFA should be used [22, 23]. In this section by using GSMA, we perform the calculations ESR spin-gap at filling factors \( \nu = 1/M \) (where \( M \) is an odd integer), corresponding to the polarized Laughlin states and at \( \nu = 1/2 \) for polarized Composite Fermi Sea (CFS) and Pfaffian states [24]. By using the 8-band \( k \cdot p \) Hamiltonian for single-particle states for
narrow-gap QW heterostructures, the many-body corrections to the ESR spin-gap for the FQHS have the following form [25]:

\[
\Delta_{\text{ESR}}^{(\text{GSM})} = \int \frac{d^2 q}{(2\pi)^2} \hat{h}_0^{(a)}(q) \left[ \hat{V}^{(b,a,b,a)}_{-1,0,-1,0}(\vec{q}) - \hat{V}^{(a,a,a,a)}_{0,0,0,0}(\vec{q}) + \hat{V}^{(a,h,b,a)}_{0,-1,-1,0}(\vec{q}) \right],
\]

where \( \hat{V}^{(b,a,b,a)}_{-1,0,-1,0}(\vec{q}) \), \( \hat{V}^{(a,a,a,a)}_{0,0,0,0}(\vec{q}) \), and \( \hat{V}^{(a,h,b,a)}_{0,-1,-1,0}(\vec{q}) \) are the matrix elements of the e-e interaction explicitly given in [8] and \( \hat{h}_0^{(a)}(q) \) is defined by the Fourier component of the pair distribution function \( g^{(c)}_n(r) \) for the electrons in LL \((n, i)\):

\[
\hat{h}_0^{(c)}(q) = \nu^{(c)}_n \int d^2 \vec{r} \exp(-i\vec{q}\vec{r}) \left[ g^{(c)}_n(r) - 1 \right],
\]

where \( \nu^{(c)}_n \) is filling factor for LL \((n, i)\) [8-10]. To calculate the ESR spin-gap values for the FQHS, we use the parameterization of \( g^{(c)}_n(r) \) proposed by Girvin for the Laughlin states [22], and the parameterization for \( \nu^{(c)}_n = 1/2 \) obtained in [26] by fitting the pair distribution functions for polarized CFS and Pfaffian states [24]. By employing the particle-hole symmetry that takes place in homogeneous 2D systems, we find \( \Delta_{\text{ESR}}^{(\text{GSM})} \) for the conjugate states \( \nu^{(c)}_n = 1 - 1/M \) as well.

![Figure 2: ESR spin-gap in a 15 nm wide InAs/AlSb QW with symmetric profile, calculated at a fixed magnetic field of 10 T. Brown dotted curve corresponds to Hartree approximation. Green curve is the result of calculations taking into account the e-e interaction within HFA. Black symbols correspond to the ESR spin-gap obtained in the GSMA for Laughlin states \(1/M\) and \(1-1/M\) \((M=1,3,5,7,9)\). The corresponding energies for polarized CFS and Pfaffian states are marked in blue and red, respectively.](image)

One can also see from figure 2 that, by taking into account the correlation interaction in the GSMA calculations, we obtain not only a greater, as compared to the ‘mean-field’ HFA values, enhancement of the ESR spin-gap for FQHS, but also a nonlinear dependence of the ESR energy on an LL filling factor. Note that the correlation corrections are the dominating contributor in the ESR spin-gap for FQHE at LL filling factor close to zero. For the filling factors approaching unity both approximations, HFA and GSMA, yield the same results.

3. Experiment

The InAs/AlSb QW heterostructures to be studied were grown by molecular beam epitaxy (MBE) on a semi-insulating GaAs (001) substrate with a metamorphic GaSb buffer layer. The order and conditions of growth of the layers are described in detail elsewhere [13, 17]. Magnetotransport measurements were performed at the temperature 250 mK, with sweeping the magnetic field. In electron transport measurements, a low-frequency (9 Hz) alternating current of about 1 µA was passed through the B824 sample. The parameters of the B824 structure at liquid-helium temperature are given in [17]. The sample in the form of a Hall bar was placed at the center of a superconducting solenoid in a dilution cryostat. To measure the longitudinal resistance \( (R_{xx}) \), we used the standard locked-in detection scheme. Figure 3 is the experimental and calculation results of quasiparticle spin-gap studies.
performed in B824 sample. It can be seen that the experimental data for the B824 sample are in good agreement with the results of theoretical calculations performed for the case of the most asymmetrical QW profile. Agreement between the theory and experiment supports the conclusion [14, 15] that the main ‘suppliers’ of 2D electrons to the InAs QW are surface donors in the GaSb coating layer and suggests that, in InAs/AlSb heterostructures, the ‘built-in’ electric field is highly asymmetric.

Figure 3: Quasiparticle spin-gap in the B824 sample \( (d_{QW} = 15 \text{ nm}, \quad n_S = 7.67 \cdot 10^{11} \text{ cm}^{-2}) \). Black curve is the single-particle spin-gap calculated within the Hartree approximation. Red and green curves correspond to the quasiparticle spin-gap for LL broadening values \( \Gamma_0 = 0.3 \text{ meV} \) and 0.5 meV respectively. The arrows indicate the magnetic field values corresponding to the integer-valued LL filling factors. The blue symbols are experimental values obtained from magnetotransport data [18].

ESR measurements were carried out at 4.2 K using electrical detection technique [7]. Magnetic field was oriented perpendicular to the sample plane. As a radiation source the backward-wave tube which covers the range of 250-500 GHz was used. The measurements were performed on two samples T340 and T338 with a QW width of 15 nm and 18 nm, respectively. The 2DEG concentration and mobility in the samples at \( T = 4.2 \text{ K} \) were \( 6.70 \cdot 10^{11} \text{ cm}^{-2} \) and \( 4.5 \cdot 10^5 \text{ cm}^2/\text{V}\cdot\text{s} \) for T340 sample and \( 7.65 \cdot 10^{11} \text{ cm}^{-2} \) and \( 4.6 \cdot 10^5 \text{ cm}^2/\text{V}\cdot\text{s} \) for T338 sample. The experimental values are presented in figure 4 by symbols. One can see a good agreement between theoretical predictions and experimental data if the asymmetric ‘built-in’ electric field of the surface donors in GaSb cap layer [14, 15] resulting in the Rashba splitting [16] is taken into account. A small discrepancy between calculated curve and experimental data can be attributed, in our opinion, to the influence of dynamic polarization (Overhauser shift [27]) on the ESR line shape and position.

Figure 4: ESR spin-gap in T340 (left panel) and T338 (right panel) samples. The symbols are experimental data in the samples. Magnetic fields corresponding to even-valued LL filling factors are indicated by arrows. Black and brown curves correspond to the Hartree approximation in a asymmetric and symmetric QWs. Red and blue curves are the calculation within HFA in InAs/AlSb QW with a symmetric and asymmetric profile, respectively.
This work demonstrates that subband structure related to the single-particle wave functions and SOI affects significantly the exchange enhancement of quasiparticle and ESR spin-gap in symmetric and asymmetric narrow-gap QWs both in the case of integer and fractional Quantum Hall regime. We show that ESR can serve as a tool to probe the e-e interaction effects in narrow-gap QWs. We hope that our results will stimulate further experimental ‘magnetostatic’ and ‘magnetooptic’ studies of narrow-gap QW heterostructures and will provide an in-depth understanding of the physical mechanisms responsible for the exchange enhancement of spin-gap in 2DEG.

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