Pair productions from gauge bosons collisions

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Abstract. Pair production is a process of creating a particle and its antiparticle from two gauge bosons collision. In this work, we suppose the pair production can occur from the collision between a spin-two graviton and a spin-one gauge boson. We focus on the electron-positron pair production between gravitons which is determined from the vielbein field and coupling to a photon in minimal coupling scheme. The collision cross section is evaluated and compared to those of the same creation types at the energy 0.71 MeV. We found that the total cross section of the graviton-photon interaction is less than that of photon-photon about $10^{41}$ times but that is greater than that of graviton-graviton about $10^{42}$ times.

1. Introduction
Pair production is a process of creating a particle and its antiparticle. We can observe this from cosmic rays or accelerator physics experiments. The simplest case is the electron-positron pair creation from two photons collision because their rest energy is low (0.51 MeV) [1]. Other examples are $W^+$, $W^-$ and $Z$ bosons pair creations in the weak interaction regime [2]. Graviton is a spin-two gauge boson which we are not familiar with in the study of pair production process. We are interested in the question about the probability of pair production from graviton-photon collision when compared to pair production from photon-photon collision.

We present our work in this article as follows. In Section 2, we introduce the weak-field limit of graviton-electron interactions, using vielbein field formulation. The Feynman rules for field propagators and the interaction vertex are read out. In Section 3, the amplitudes for electron-positron pair production for graviton-photon and graviton-graviton collision are evaluated. The corresponding cross sections are calculated in terms of scattering energy. In Section 4, the comparisons of the two processes at the collision energy of 0.71 MeV are shown. Conclusion of our work appears in Section 5.

2. Graviton and Fermionic fields interaction
2.1. Fermionic fields in curved spacetime
A fermionic field or spin-$\frac{1}{2}$ field can be determined on a flat spacetime manifold from Dirac Lagrangian [3],

$$\mathcal{L}_D = \bar{\psi}(i\gamma^a \partial_a - m)\psi,$$

where $a = 0, 1, 2, 3$ are Lorentz index, $\psi$ is Dirac spinor and $\gamma^a$ are Dirac gamma matrices.
On a curved spacetime manifold, Dirac spinor $\psi$ is intrinsically coupling to gravity according to Einstein’s theory of general relativity. The spinor Lagrangian density appears in the form [4, 5]

$$\mathcal{L} = \frac{1}{2\kappa^2} eR + e\bar{\psi}(ie_a^\mu\gamma^a\mathcal{D}_\mu - m)\psi,$$

where $e_a^\mu$ is a vielbein field with $\mu$ being a tensor index on curved manifold relating to $g_{\mu\nu}$ by $g_{\mu\nu} = e_a^\mu e_b^\nu \eta_{ab}$, $e = \sqrt{-g}$, $g$ is the determinant of the metric tensor $g_{\mu\nu}$, $R$ is the Ricci scalar, $\eta_{ab}$ is the Lorentzian metric, $\mathcal{D}_\mu$ is the covariant derivative which is equal to $\partial_\mu + \omega_\mu$, and $\omega_\mu$ is the spin connection.

2.2. Graviton with fermionic fields

From the linearlized gravity $e_a^\mu = \bar{e}_a^\mu + \kappa c_a^\mu$, $\kappa = \sqrt{8\pi G}$ and $G$ are the gravitational constant, the Lagrangian in equation (2), up to the first order of $\kappa$, becomes

$$\mathcal{L} = \frac{1}{2\kappa^2} \bar{e}R + \bar{e}\psi(ie_a^\mu\gamma^a\partial_\mu - m)\psi + \kappa \left( \frac{1}{2} \bar{\psi} i\epsilon^a\partial_\mu \psi + \frac{1}{2} \bar{\psi}(ic\gamma^a\partial_\mu - m)\psi \right).$$

The graviton-fermion coupling appears with the first order of $\kappa$, with the coupling vertex in momentum space

$$V^{ab}(p, p', m) = \frac{\kappa}{2} \left( \gamma^a(p + p')^b + \gamma^b(p + p')^a + \frac{\eta^{ab}}{2}(\not{p} + \not{p'}) - m \right),$$

where $\not{p} = \gamma^a p_a$.

3. Pair production calculation

We focus on electron-positron pair creation process from graviton-graviton and graviton-photon collisions. General Feynman diagrams of pair production processes are illustrated in figures 1 and 2.

![Figure 1](image-url)

**Figure 1.** The Feynman diagrams of pair creations by two-graviton $(g_1, g_2)$ collisions.

The amplitude of graviton-graviton pair production is

$$M_{gg} = \epsilon_{ab}\epsilon_{cd}\bar{u}(p_e, m_e)V^{cd}(k, p_e, m_e)\left( \frac{i}{k - m_e} \right) V^{ab}(k, p_e, m_e)v(p_e, m_e)$$

$$- \epsilon_{ab}\epsilon_{cd}\bar{u}(p_e, m_e)V^{cd}(k', p_e, m_e)\left( \frac{i}{k' - m_e} \right) V^{ab}(k', p_e, m_e)v(p_e, m_e).$$

(5)
The amplitude of graviton-photon pair production is

$$\mathcal{M}_{g\gamma} = \epsilon_{ab} \epsilon_{g} \bar{u}(p_{e^{-}}, m_{e}) V^{ab}(k, p_{e^{-}}, m_{e}) \left( \frac{i}{k - m_{e}} \right) (-i e \gamma_{5}) v(p_{e^{+}}, m_{e})$$

$$- \epsilon_{ab} \epsilon_{g} \bar{u}(p_{e^{-}}, m_{e}) (-i e \gamma_{5}) \left( \frac{i}{k' - m_{e}} \right) V^{ab}(k', p_{e^{+}}, m_{e}) v(p_{e^{+}}, m_{e}).$$  \hspace{1cm} (6)

The differential cross sections \( \frac{d\sigma_{cm}}{d\Omega} \) are calculated from the mean amplitude squared \( |\mathcal{M}|^2 = \frac{1}{4} \sum_{polar,s} |\mathcal{M}|^2 \). We assume that the energy of each boson is the same. Therefore, we observe the differential cross sections in the center of momentum frame derived in [3] as

$$\frac{d\sigma_{cm}}{d\Omega} = \frac{1}{64\pi^2 s} \left[ \frac{s - 4m_e^2}{s} \right]^{\frac{3}{2}} |\mathcal{M}|^2$$  \hspace{1cm} (7)

where \( s \) is the Mandelstam variable representing gauge bosons collision energy.

The total cross sections \( \sigma \) is evaluated from the formula

$$\sigma = 2\pi \int_{0}^{\pi} \frac{d\sigma_{cm}}{d\Omega} \sin \theta d\theta.$$  \hspace{1cm} (8)

Total cross sections of graviton-graviton and graviton-photon pair productions are shown in figures 3 and 4, respectively.
4. Discussion
In Section 3, we have derived the graviton-graviton and graviton-photon total cross sections. We observe that the graviton-graviton cross section increases with energy. The graviton-photon cross section is also increasing at high energy but with a slower rate. These behaviors are opposite to that of photon-photon cross section which decreases with energy at high energy as appears in figure 5. This may come from the weak field approximation of gravitational field. Higher-order corrections are needed to clarify this point.

![Graph](image_url)

**Figure 5.** Total cross section of photon-photon collision vs the energy of each photon.

However, we also would like to compare the results at low energy because experiments can be set up more easily. We chose to consider at 0.71 MeV as it yields the highest total cross section in two-photon pair production. we found that the total cross section of graviton-photon collision is less than that of the photon-photon collision by about $10^{41}$ times but it is greater than the graviton-graviton collision by about $10^{42}$ times.

5. Conclusion
We derived the total cross sections of the electron-positron pair creation from the graviton-graviton and graviton-photon collisions and compared the results to the photon-photon collision. They show that the probability of pair production from gravitons is much less than that of the photon-photon collision by about $10^{41}$ times and by $10^{42}$ times less for graviton-photon collision at 0.71 MeV.

Our method is a basic idea to study graviton collision directly. It helps the observers to estimate the probabilities of the graviton observations. Moreover, this method can be applied to study other collision models such as the graviton-Z boson collision in electroweak or can increase the process to add loop corrections for higher-order predictions.

References
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