Anneal-path correction in flux qubits

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Quantum annealers require accurate control and optimized operation schemes to reduce noise levels, in order to eventually demonstrate a computational advantage over classical algorithms. We study a high coherence four-junction capacitively shunted flux qubit (CSFQ), using dispersive measurements to extract system parameters and model the device. We confirm the multi-level structure of the circuit model of our CSFQ by annealing it through small spectral gaps and observing quantum signatures of energy level crossings. Josephson junction asymmetry inherent to the device causes a deleterious nonlinear cross-talk when annealing the qubit. We implement a nonlinear annealing path to correct the asymmetry in-situ, resulting in a 50% improvement in the qubit performance. Our results demonstrate a low-level quantum control scheme which enhances the success probability of a quantum annealer.

Introduction.— Quantum annealing (QA) began as a quantum-inspired classical optimization method [1–3] and was eventually proposed as a type of analog, adiabatic quantum computing algorithm [4–6]. Flux qubits [7] are a natural choice for implementing QA. The quantum states are characterized by persistent supercurrents flowing in opposite directions, and these currents can be mapped onto the binary spin variables used in QA [8]. The qubits are initialized in a state with a single persistent potential well and no persistent current. A double well is created toward the end of the anneal, with states in each well corresponding to persistent currents that circulate in opposing directions. The direction of these currents is measured to determine the final qubit state.

The relaxation and coherence times of a flux qubit are strongly dependent on the magnitude of the persistent current (Ip), scaling as T1 \sim 1/I_p^2 and T2 \sim 1/I_p, respectively [9]. D-Wave Systems has performed much of the pioneering work in this field [10–12] using niobium-based qubits with relatively high persistent currents (Ip \sim 3 \mu A), which limits the relaxation and coherence times to \sim 20 ns [13]. Our work is performed using capacitively-shunted flux qubits (CSFQs) [14, 15] fabricated at MIT Lincoln Laboratory by patterning high-quality aluminum on a silicon substrate. They are designed to have small persistent currents (Ip \sim 170 nA) and exhibit \geq 100 times longer T1 and T2 [15–17].

A key challenge for flux qubits is their sensitivity to fabrication variations of the Josephson junction critical currents. In particular, junctions in a SQUID loop often come out different despite identical design. This junction asymmetry causes nonlinear crosstalk between the qubit control fluxes that, if left uncompensated, has significant adverse effects on operational fidelity. One mitigation technique is to use compound junctions [11], replacing each junction with a SQUID loop of two junctions. Flux biasing these loops allows to tune the effective junctions to have identical critical currents. The tradeoff is higher complexity, increased flux noise sensitivity (thus reducing T1 and T2), control overhead for the additional bias lines, and more challenging crosstalk calibration (which scales quadratically with the number of bias lines). In this work we demonstrate an alternative and simpler approach with a CSFQ: using dispersive measurements to quantify the asymmetry in the qubit junctions, we devise corrected annealing paths by dynamically canceling the nonlinear crosstalk effect. This yields a twofold reduction in the “s-curve” transition width between the qubit wells as a function of applied tilt bias, without adding any additional circuit elements. We observe population transfer to higher excited states when the qubit is annealed through small gaps, and develop a model (fit to independently measured spectroscopy data) to accurately predict the population exchanges and qualitatively explain the open system dynamics.

System and model.— We use a four-junction CSFQ [15], controlled with two flux bias lines that thread external fluxes into the loops of the qubit [inset of Fig. 1(a)]. The CSFQ is coupled to a dispersive readout resonator at \omega_r/2\pi = 7.1876 GHz, and is also equipped with a persistent current readout that measures the direction of the circulating current in the large loop (see SM). We also use the dispersive resonator to calibrate the linear crosstalk between the x- and z-flux bias lines [17], and to send microwave pulses to the qubit.

The Hamiltonian of the CSFQ circuit can be written
as (see SM for derivation)

\[ H = \frac{e^2}{2C_{sh} + (4\alpha + 1)C_z}(2\hat{n}_0 + \hat{n}_1)^2 + \frac{e^2}{C_z}\hat{n}_1^2 \]

\[ -2I_z\frac{\Phi_0}{2\pi}\cos(\varphi_1 - \varphi_0/2)\cos(\varphi_0/2 - \varphi_z/2) \]

\[ -2\alphaI_z\frac{\Phi_0}{2\pi}\cos(\varphi_z/2)\sqrt{1 + \tan^2(\varphi_d)}\cos(\varphi_0 - \varphi_d), \]

where the operators \( \hat{\varphi}_k \) and \( \hat{n}_k \) are, respectively, the superconducting phase and number of Cooper pairs at nodes \( k = 0, 1 \), satisfying the commutation relation \([\hat{\varphi}_k, \hat{n}_1] = i\delta_{kl}\). Note that phase and flux are related through \( \varphi_{x,z} = 2\pi\Phi_{x,z}/\Phi_0 \). Here \( \Phi_0 \) is the magnetic flux quantum, \( \varphi_x \) and \( \varphi_z \) are the barrier and tilt, respectively, also referred to as the \( x \) and \( z \) (flux) bias (see Fig. 1). \( C_{sh} \) is the shunt capacitance, \( C_z \) is the capacitance of the \( z \)-loop junction whose critical current is \( I_z \). The \( x \)-loop junctions are on average \( \alpha \) times smaller than the \( z \)-loop junctions, such that \( (I_{z1} + I_{z2})/2 = \alpha I_z \) and \( (C_{z1} + C_{z2})/2 = \alpha C_z \), where \( C_{z_i} \) is the capacitance of the \( i \)th \( x \)-loop junction. A central role is played in our experiments by the asymmetry between the two \( x \)-loop junctions. We define an asymmetry parameter as \( d \equiv (I_{z1} - I_{z2})/(I_{z1} + I_{z2}) \). Its corresponding phase shift is

\[ \varphi_d = \arctan|d\tan(\varphi_z/2)|. \]

Eq. (1) shows that the asymmetry of the \( x \)-loop junctions rescales the total current through them by \( \sqrt{1 + \tan^2(\varphi_d)} \) and also shifts the \( z \)-loop bias as \( \varphi_z \rightarrow \varphi_z - \varphi_d \). This \( \varphi_z \)-dependent shift of the \( z \)-bias is a nonlinear quantum crosstalk that must be taken into account when operating the CSFQ.

We note that the standard QA Hamiltonian of a single qubit is obtained from the circuit Hamiltonian (1) by retaining only the lowest two energy eigenstates (see SM), which yields:

\[ H_q(t) = A(t)\sigma_x + B(t)\sigma_z, \]

where \( \sigma_x \) and \( \sigma_z \) are the Pauli matrices representing the transverse and longitudinal fields, respectively, and \( A(t) \) and \( B(t) \) are the time-dependent annealing schedules, with \( t \in [0, t_f] \). Time-dependent paths in flux space control the transverse and longitudinal fields of the annealing schedule. Such two-level reduction works only if the lowest two eigenstates have support in both wells of the potential, which imposes an upper bound on \( |\varphi_z| \), as illustrated in Fig. 1(b). Under this assumption we provide expressions for the annealing schedules in terms of the circuit Hamiltonian parameters in the SM.

**Asymmetry measurement.**—We measure \( d \) by noting that the qubit’s minimum gap occurs at \( \varphi_z^{\text{min}} = \varphi_d \) when \( \pi \leq \varphi_z \leq 3\pi \) (see SM). As illustrated in Fig. 2, we scan...
the $x$ and $z$-biases around the qubit’s minimum gap and measure the demodulated signal of the dispersive readout resonator, corresponding to an energy eigenbasis measurement of $H$. For a fixed $\varphi_x$, the dispersive readout signal is symmetric as a function of $\varphi_z$ relative to the minimum gap position (symmetry point). We fit a Gaussian to the readout signal along $\varphi_z$ to extract this position, and repeat for all values of $\varphi_x$ (filled green circles in Fig. 2). We then fit this data to $\varphi_z = \min(d, \varphi_x)$ (dashed line in Fig. 2) to extract the asymmetry parameter, albeit with offset parameters on both fluxes that are fitted as well to account for flux drifts and/or offsets. We obtain $d = 0.102 \pm 0.005$, where the value was determined by systematically varying the fitting regions and using resampling to compute the 1σ confidence interval.

**S-curve width reduction via annealing path control.**—
To characterize our device for use in QA experiments, we perform a so-called “s-curve” measurement [9–11] on our CSFQ. This is a single-qubit annealing experiment, where the CSFQ starts in the single-well regime [$A(0) \gg B(0)$] with a variable initial tilt $\varphi_z$, then the barrier $\varphi_x$ is raised (at fixed $\varphi_z$) to put the qubit in a tilted double-well regime, with negligible tunneling between the two wells. This is illustrated in Fig. 1(a) (also see SM). Finally, a persistent current measurement is performed to determine which of the wells is occupied, corresponding to a computational basis measurement of $\sigma_z$. Ideally, the s-curve would be a step function. In actuality, one obtains a curve that resembles an $S$ shape with a characteristic width for transitioning between left and right circulating currents at the degeneracy point. The width $w$ can be found by fitting the right-well population $P$ to a phenomenological model [11]:

$$P = \frac{1}{2} \left[ 1 + \tanh \left( \frac{\varphi_z - \varphi_0}{w} \right) \right].$$

(4)

The width, which should be minimized, depends on the rate at which the barrier is raised, thermalization between the states in left and right wells, and flux noise in the tilt bias near the minimum gap [18].

Nonlinear crosstalk also acts to increase $w$. Namely, when the $z$-bias (barrier) is tuned during an s-curve measurement, the junction asymmetry causes an extra tilt of the potential if the $z$-bias is kept constant. This shifts the center of the s-curve away from the degeneracy point, and broadens its width; see the blue dashed curve in Fig. 3. To cancel this effect, we correct the annealing path with respect to the junction asymmetry by applying an additive $z$-bias correction of $+\varphi_d$ [Eq. (2)], to undo the asymmetry-induced shift of $\varphi_z \rightarrow \varphi_z - \varphi_d$. Note that this amounts to a nonlinear annealing path in the $(\varphi_z, \varphi_x)$ plane.

Our first key result is a reduction of the the s-curve width by nearly 50% when comparing the standard (fixed $\varphi_z$) s-curve protocol to our nonlinear protocol, as shown by the orange solid line in Fig. 3. This substantial improvement is made possible by two key capabilities: first, the independent extraction of the asymmetry parameter $d$ via dispersive measurement, and second the independent individual control we have over the flux biases, which enables an accurate traversal of the optimal, nonlinear annealing path shown in Fig. 2.

**Signatures of level crossing.**— Superconducting circuits, including CSFQs, are inherently multilevel quantum systems. To validate this picture, we anneal the CSFQ through small gaps to diabatically transfer the population from the ground state into excited states. We compare the results with our circuit model and find the circuit parameters of the Hamiltonian (1) by fitting the two lowest ground state transition frequencies $\omega_{01}$ and $\omega_{02}$ to our experimental spectroscopy data (see SM for extracted parameter values). We perform spectroscopy by sweeping $\varphi_z$ near the degeneracy point for multiple fixed $\varphi_x$ and measuring the resonance frequency of the microwave drive applied to the qubit (see SM).

To investigate the multilevel circuit model, we perform a modified s-curve measurement where in addition to raising the barrier $\varphi_x$, we also linearly increase the tilt $\varphi_z$ during each anneal, and repeat for different initial values $\varphi_z(0)$ (illustrated by the slanted lines in Fig. 2). The persistent current measurement results obtained at the end of each anneal are shown by the solid black line in Fig. 4(a). The overall behavior resembles the s-curve of Fig. 3 (nonlinear annealing path), but now exhibits a much wider transition domain, accompanied by multiple sharp features [19]. We proceed to establish that these features represent resonances between the quantized higher energy levels of the CSFQ.
FIG. 3. S-curve without (blue) and with (orange) annealing-path correction. In the former each datapoint is obtained by sweeping only $\phi_z$ at fixed $\varphi_x$. In the latter, we added $\varphi_d(\varphi_z)$ to $\varphi_z$. Both anneals occur in 20 ns. The uncorrected anneal (squares) results in a width of $2.59 \pm 0.05$ m$\Phi_0$ (fit, dashed line). Applying the correction (circles) narrows the s-curve by nearly 50% to $1.38 \pm 0.06$ m$\Phi_0$ (fit, solid line). While both sets of data have been shifted and centered for ease of comparison, the corrected anneal should center the curve around the degeneracy point, which can be used to calibrate offsets in the $z$ bias. Error bars are calculated from binomial counting statistics, AWG voltage resolution, and quasistatic noise.

To explain the resonances theoretically we calculate the spectrum of the Hamiltonian (1) along the same annealing paths as implemented experimentally, using the aforementioned independently extracted circuit parameters. The CSFQ is initially in its ground state, but as shown in Fig. 4(c) a cascade of avoided level crossings takes place during the anneal, so that the population is diabatically transferred to higher levels. The initial tilt bias $\varphi_z(0)$ determines the most-populated level at the end of the anneal, as can be seen in Fig. 4(b). The locations of the experimental peaks closely coincide with the avoided level crossings, while those crossings where there are no experimental peaks are unavoidable (i.e., actual level crossings). This is indicated by the green circles in Fig. 4(a), which correspond to the locations of the avoided crossings, calculated using extracted circuit parameters and Eq. (1). The error bars are due to uncertainty in the fitted circuit parameters as given above. We emphasize that the location of features in this experiment is calculated using circuit parameters that are extracted via the independent spectroscopy measurement.

This population transfer mechanism explains the sharp features seen in Fig. 4(a): as we vary $\varphi_z(0)$, a previously unoccupied eigenstate crosses with the occupied eigenstate and suddenly acquires its population (a resonance). This leads to a sudden change in the result of the persistent-current readout, because the right-well population measured at the end of each anneal depends on the population in each eigenstate, the persistent-current value associated with that eigenstate, and the current readout resolution. Only avoided level crossings yield a persistent-current feature that is observable in the experiment, since for actual level crossings the population is completely transferred to other eigenstates and the total persistent current of the CSFQ does not change enough to yield an observable feature.

To account for open system effects, we simulate the dynamics of the circuit described by Eq. (1) using the adiabatic master equation (AME) [20]. We used the same annealing paths and assumed an Ohmic bath at 10 mK that is weakly coupled to the the system ($\gamma g^2 = 3 \times 10^{-6}$; see SM), and a high-frequency cutoff $\omega_c/2\pi = 15$GHz. We also add a 2 ns idle time at the end of each anneal to mimic the effect of delay before the persistent-current readout in the experiment, which allows for relaxation (without this delay the features manifest as plateaus; see SM). The result is the blue dashed line in Fig. 4(a) that accurately predicts the locations of the resonances and qualitatively captures their behavior. However, the linewidth of the experimental features is broadened due to low-frequency noise in the flux biases, which cannot be modeled by the AME with an Ohmic bath. This is demonstrated by convolving the AME results with a Gaussian that has a width of 1 m$\Phi_0$, approximately the amount of quasi-static noise observed in the tilt bias [18]. The orange dot-dashed line in Fig. 4(a) illustrates the effect of broadening due to this slow noise. Closer quantitative agreement requires more realistic noise models, that capture the dominant effect of $1/f$ noise in superconducting qubits [9, 21–23], along with open-system simulation tools that can handle such noise environments [24, 25]. This is a subject for future work.

Conclusions.—We have demonstrated a hardware-level quantum control approach to overcome the nonlinear crosstalk between control fluxes arising from fabrication variation of Josephson junctions in flux qubits. We have shown that a linear correction to the tilt bias is insufficient for mitigating the asymmetry-induced s-curve broadening. Our approach implements the necessary nonlinear anneal-path correction, while avoiding introducing additional control lines or circuit elements. These results pave the way towards achieving high-fidelity annealing operation of high-coherence flux qubits, a critical enabling step in constructing quantum annealers exhibiting a quantum advantage.

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FIG. 4. Resonant features of quantum multi-level structure of the CSFQ. (a) Persistent current readout giving the right-well probability as a function of the initial tilt bias, for a linear anneal in both $\varphi_x$ and $\varphi_z$, as illustrated by the slanted arrows in Fig. 2. Tilt bias anneal amplitude is $\text{amp} = 0.326\pi$, i.e., $\varphi_x(t) = \varphi_x(0) + \text{amp} \times (t/t_f)$, where $t_f = 60$ ns is the anneal time. The black solid line shows the experimental result, the dashed blue line is the the AME result for the CSFQ circuit with parameters extracted from spectroscopy, and the orange dot-dashed line shows Gaussian broadening of the AME results (see text). Experimental error bars are calculated as in Fig. 3. (b) Population of CSFQ circuit eigenstates (indicated by the numbers in the legend, with 0 being the ground state) at the end of each anneal, along with the experimental result (exp) also shown in (a). Only avoided level crossings lead to observable features in the persistent-current readout, indicated by the green circles in (a). (c) An example of the CSFQ spectrum vs normalized anneal time $s = t/t_f$, for an initial tilt bias corresponding to the grey vertical dashed line in panels (a) and (b). The blue arrow marks the avoided level crossing between levels 5 and 6 at the end of the anneal, which corresponds to the population exchange between these two levels in panel (b), and the corresponding experimental resonant feature. Cascaded level crossings that transfer the population are visible throughout the anneal.

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Supplementary material for
“Anneal-path correction in flux qubits”

![Diagram](image)

FIG. S1. Schematic of the experimental set up. Capacitively shunted flux qubit in the center, with curved arrows showing $x$ and $z$ fluxes threading their corresponding loops. Nodes 0 and 1 used in the Hamiltonian (S6) are marked with filled circles. The qubit is coupled to a dispersive readout resonator, and also has a dedicated persistent-current readout that measures the direction of the circulating current in the $z$ loop.

DERIVATION OF THE CSFQ HAMILTONIAN

In this section we give a detailed derivation of the CSFQ Hamiltonian [Eq. (1) in the main text]. For clarity and completeness, we repeat some of the details given there.

The capacitively shunted flux qubit (CSFQ) has two superconducting loops, each terminated with two junctions, shunted with a large capacitance (Fig. S1). The $x$-loop is threaded with an external flux $\Phi_x = \Phi_0 \hat{n}_x/2\pi$, which controls the height of the barrier in the double well potential. The larger $z$-loop is threaded with $\Phi_z = \Phi_0 \hat{n}_z/2\pi$, which tilts the double-well potential. In our experiment, the qubit is coupled to a dispersive readout resonator, and also has persistent-current readout that can measure the direction of the circulating current in the $z$-loop. The nodes 0 and 1 used for derivation of the Hamiltonian of this circuit are marked with filled circles in Fig. S1.

The capacitance matrix of the above circuit can be written as

$$C = \begin{pmatrix} C_{\text{sh}} + C_{x1} + C_{x2} + C_z & -C_z \\ -C_z & C_z + C_z \end{pmatrix}$$

$$= \begin{pmatrix} C_{\text{sh}} + (2\alpha + 1)C_z & -C_z \\ -C_z & 2C_z \end{pmatrix},$$

where $C_{\text{sh}}$ is the shunt capacitance, and $C_z$ is the capacitance of the $z$-loop junction that has a critical current of $I_z$. The $x$-loop junctions are on average $\alpha$ times smaller than the $z$-loop junctions, such that $(I_{x1} + I_{x2})/2 = \alpha I_z$ and $(C_{x1} + C_{x2})/2 = \alpha C_z$, where $C_{xi}$ and $I_{xi}$ are the capacitance and critical current of the $i$th $x$-loop junction respectively. The kinetic energy of the circuit is then

$$K_{2D} = \frac{1}{2} (2e)^2 \hat{n}^T \cdot C^{-1} \cdot \hat{n} = e^2 C_z \hat{n}_1^2 + \frac{e^2}{2C_{\text{sh}} + (4\alpha + 1)C_z} (2\hat{n}_0 + \hat{n}_1)^2,$$

where $\hat{n} = (n_0, n_1)$ is a column vector of the number of Cooper pairs at each node.

To write the potential energy, we choose a gauge that splits (symmetrizes) the control fluxes over both of its junctions, to get:

$$U_{2D} = -\frac{\Phi_0}{2\pi} [I_{x1} \cos(\varphi_0 - \varphi_x/2) + I_{x2} \cos(\varphi_0 + \varphi_x/2) + I_z \cos(\varphi_0 - \varphi_1 - \varphi_x/2) + I_z \cos(\varphi_1 - \varphi_x/2)],$$

where $\varphi_0$ and $\varphi_1$ are the superconducting phases at nodes 0 and 1, satisfying commutation relation $[\varphi_k, \hat{n}_l] = i\delta_{kl}$. Note that phase and flux are related through $\varphi = 2\pi \Phi_0 / \Phi_0$, where $\Phi_0$ is the magnetic flux quantum. By defining the qubit asymmetry parameter as $d = (I_{x1} - I_{x2})/(I_{x1} + I_{x2})$ and its corresponding phase shift as

$$\tan(\varphi_d) \equiv d \tan(\varphi_x/2),$$

after some algebra we can simplify the potential energy as:

$$U_{2D} = -2I_z \frac{\Phi_0}{2\pi} \cos(\varphi_1 - \varphi_0/2) \cos(\varphi_0/2 - \varphi_x/2) - 2\alpha I_z \frac{\Phi_0}{2\pi} \cos(\varphi_x/2) \sqrt{1 + \tan^2(\varphi_d)} \cos(\varphi_0 - \varphi_d).$$

The Hamiltonian of the CSFQ circuit can then be written as

$$H_{2D} = K_{2D} + U_{2D}$$

$$= e^2 C_z \hat{n}_1^2 + \frac{e^2}{2C_{\text{sh}} + (4\alpha + 1)C_z} (2\hat{n}_0 + \hat{n}_1)^2 - 2I_z \frac{\Phi_0}{2\pi} \cos(\varphi_1 - \varphi_0/2) \cos(\varphi_0/2 - \varphi_x/2) - 2\alpha I_z \frac{\Phi_0}{2\pi} \cos(\varphi_x/2) \sqrt{1 + \tan^2(\varphi_d)} \cos(\varphi_0 - \varphi_d).$$
We can transform the coordinates in (S6) to diagonalize the kinetic part of the Hamiltonian. This will allow us to identify and separate fast and slow degrees of freedom in our Hamiltonian and further simplify our circuit model. The coordinate transformation that satisfies the commutation relations is

\[
\begin{pmatrix}
    n_0' \\
    n_1'
\end{pmatrix} = \begin{pmatrix}
    0 & 1 \\
    2 & 1
\end{pmatrix}
\begin{pmatrix}
    n_0 \\
    n_1
\end{pmatrix} = \begin{pmatrix}
    n_1 \\
    2n_0 + n_1
\end{pmatrix}, \tag{S7a}
\]

and the Hamiltonian in the transformed coordinates can be written as

\[
H'_{2D} = \frac{e^2}{C_z} \hat{n}_0^2 + \frac{e^2}{2C_{sh} + (4\alpha + 1)C_z} \hat{n}_1^2
- 2I_z \Phi_0 \frac{\pi}{2} \cos(\varphi_0') \cos(\varphi'_1 - \varphi_z/2)
- 2\alpha I_z \Phi_0 \frac{\pi}{2} \cos(\varphi_z/2) \sqrt{1 + \tan^2(\varphi_d)} \cos(2\varphi_d' - \varphi_d). \tag{S8}
\]

We note that in CSFQs the junction capacitance is much smaller than the shunt capacitance \(C_z \ll C_{sh}\), and therefore the mode corresponding to \(\{\varphi_0', n_0'\}\) has a plasma frequency that is much larger than the other mode. Therefore, we can neglect this fast oscillating degree of freedom to reduce the number of modes in our model, i.e., we can perform a Born-Oppenheimer approximation [S1] which assumes the fast degree of freedom is always in its ground state. To do this we take \(C_z \to 0\) and fix \(\varphi_0' = 0\), which is the phase value that minimizes the potential energy of (S8) with respect to \(\varphi'_0\). The resulting simplified Hamiltonian then becomes

\[
H_{1D} = K_{1D} + U_{1D} \tag{S9a}
\]

\[
K_{1D} = \frac{e^2}{2C_{sh}} \hat{n}_0^2 \tag{S9b}
\]

\[
U_{1D} = -2I_z \Phi_0 \frac{\pi}{2} \cos(\varphi - \varphi_z/2) \tag{S9c}
\]

\[-2\alpha I_z \Phi_0 \frac{\pi}{2} \cos(\varphi_z/2) \sqrt{1 + \tan^2(\varphi_d)} \cos(2\varphi_d' - \varphi_d),
\]

where we have dropped the subscript and prime for brevity.

We call the Hamiltonian of Eq. (S6) the 2D model and the Hamiltonian of Eq. (S9) the 1D model. We fit both of these models to our qubit spectroscopy data, and the fitted parameters are presented in Table S1.

| qubit model | \(I_z\) (nA) | \(C_{sh}\) (fF) | \(C_z\) (fF) | \(\alpha\) |
|-------------|-------------|-------------|-------------|---------|
| 2D CSFQ    | 242 ± 3     | 62 ± 1      | 4.85 ± 0.07 | 0.423 ± 0.001 |
| 1D CSFQ    | 228 ± 3     | 70 ± 1      | N/A         | 0.452 ± 0.001 |

TABLE S1. Fit parameters for qubit models. Asymmetry is fixed at \(d = 0.102\) for both models.

We numerically calculate the 0 ↔ 1 transition frequency of the circuit as a function of control biases. The result is shown in Fig. S3. The qubit gap is a periodic function of the
master equation (AME) [S2]. The system is coupled to the bath via the persistent-current operator, defined as $\hat{I}_p = -\partial U/\partial \varphi_z$, where $U$ is the CSFQ potential, given by Eqs. (S5) and (S9) for the 2D and 1D models, respectively. The persistent-current operator for each model is as follows:

$$\hat{I}_p^{2D} = \hat{I}_z \frac{\Phi_0}{2\pi} \cos(\hat{\varphi}_1 - \hat{\varphi}_0/2) \sin(\hat{\varphi}_0/2 - \varphi_z/2),$$

$$\hat{I}_p^{1D} = \hat{I}_z \frac{\Phi_0}{2\pi} \sin(\hat{\varphi} - \varphi_z/2).$$

The density operator of the circuit evolves according to the adiabatic master equation as

$$\dot{\rho} = -i[H + H_{LS}, \rho] + \sum_\omega \gamma(\omega) \left[ L_\omega \rho L^\dagger_\omega - \frac{1}{2} \{ L^\dagger_\omega L_\omega, \rho \} \right],$$

where

$$\gamma(\omega) = \eta g^2 \frac{1}{\omega e^{-|\omega|/\omega_c} - 1},$$

is the Ohmic bath spectral function, with a high frequency cut-off at $\omega_c/2\pi = 15$ GHz, and is in thermal equilibrium at $T = 1/k_BT = 10$ mK. Conforming to the notations in Ref. [S2], $\eta g^2 = 3 \times 10^{-6}$ is the system-bath coupling strength, where $\eta g^2/h$ has units of 1/energy$^2$. The Lindblad operators are calculated as

$$L_\omega = \sum_{\varepsilon_k - \varepsilon_a = \omega} \langle \varepsilon_a | \hat{I}_p | \varepsilon_b \rangle \langle \varepsilon_a | \varepsilon_b | \rangle = L^\dagger_{-\omega},$$

where $\varepsilon_k$ and $| \varepsilon_k \rangle$ are eigenvalues and eigenvectors of the Hamiltonian respectively. $H_{LS}$ denotes the Lamb shift, which is calculated as

$$H_{LS} = \sum_\omega L^\dagger_\omega L_\omega S(\omega),$$

with

$$S(\omega) = \int_0^\infty \frac{d\omega'}{2\pi} \gamma(\omega') P \left( \frac{1}{\omega - \omega'} \right),$$

where $P$ denotes the Cauchy principal value. We note that in order to keep the computations for the multilevel circuit manageable, at each time step of the ODE solver we rotate the density matrix into the instantaneous eigenbasis of the Hamiltonian that is truncated (e.g., truncated at 10 eigenlevels), calculate all the above terms for AME, and then rotate it back into its initial basis.

**PERSISTENT-CURRENT READOUT**

The persistent-current readout uses a quantum flux parametron (QFP), which is positioned between the control fluxes, and annealing paths could be chosen from any of the periodic “cells” [Fig. S3(a)]. The asymmetry extraction procedure discussed in the main text uses one of these cells, which is shown in Fig. S3(b) for the same flux ranges as in Fig. 2 of the main text. It can be seen that the minimum gap occurs at $\varphi_z = \varphi_d$ for $\pi \leq \varphi_z \leq 3\pi$, indicated by the white dashed line in Fig. S3(b).

**MASTER EQUATION SIMULATIONS**

To simulate the open system behavior of the qubit for linearly corrected anneal paths we use the adiabatic
qubit and an rf-SQUID resonator and inductively coupled to both (Fig. S1). The QFP, which is a larger flux- 
qubit-like device operated in a classical regime, ampli- 
fies the persistent-current signal and isolates the CSFQ 
from the resonator. This reduces the Purcell effect and 
increases $T_1$ [S3, S4]. At the end of each anneal, the cir-
culating current in the qubit creates an effective tilt bias 
on the QFP that changes the direction of its circulating 
current, which in turn shifts the rf-SQUID resonator fre-
quency that can be measured to infer the direction of the 
circulating currents.

The persistent-current readout has an effective pos-
tive operator valued measure (POVM) for calculating 
the probability of measuring the right circulating cur-
rent, which can be written as

$$\hat{M}_r = \sum_{\lambda} f \left( \frac{I_{\lambda}}{\Delta I} \right) |\lambda\rangle \langle \lambda|, \quad (S17)$$

where $I_{\lambda}$ and $|\lambda\rangle$ are the eigenvalues and eigenvectors 
of the persistent-current operator respectively, $f(x) = \lfloor \tanh(x) + 1 \rfloor / 2$ is a filter function, and $\Delta I$ is the sen-
titivity of the persistent-current readout device, which 
in our QFP-based system is $\Delta I = 10 \text{nA}$. The proba-
bility of measuring the right circulating current is then 
$P_r = \text{Tr}(\rho \hat{M}_r)$, where $\rho$ is the qubit density matrix.

**MAPPING CIRCUIT TO ISING SPIN**

In order to map the multi-level circuit Hamiltonian 
into an Ising spin model with only two levels, we keep the 
two lowest eigenenergies of the CSFQ, as these are the 
two levels we use for representing a qubit. Furthermore, 
because we perform a persistent-current measurement at 
the end of each anneal, we would like the computational 
basis to be the eigenstates of the persistent-current op-
erator. Therefore, we first write the persistent-current 
operator in the low-energy subspace as

$$I_p^{\text{low}} = \begin{pmatrix} \langle g | \hat{I}_p | g \rangle & \langle g | \hat{I}_p | e \rangle \\ \langle e | \hat{I}_p | g \rangle & \langle e | \hat{I}_p | e \rangle \end{pmatrix}, \quad (S18)$$

where $\{|g\rangle, |e\rangle\}$ are the ground and exited eigenstates of the 
circuit Hamiltonian with eigenenergies $\{E_g, E_e\}$ re-
spectively, and $\hat{I}_p$ is the persistent-current operator.

Let $U$ be the unitary basis transformation that diag-
onalizes $I_p^{\text{low}}$, or in other words, transforms the energy 
basis into the computational (persistent-current) basis. 
$U$ is formed from the eigenstates of $I_p^{\text{low}}$ as its columns. 
The computational basis $\{|0\rangle, |1\rangle\}$ is then

$$\begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} = U^\dagger \begin{pmatrix} |g\rangle \\ |e\rangle \end{pmatrix}, \quad (S19)$$

and the effective Hamiltonian in the computational basis is

$$H_{\text{eff}} = U^\dagger \begin{pmatrix} E_g & 0 \\ 0 & E_e \end{pmatrix} U. \quad (S20)$$

We extract the Ising coefficients by rewriting the effective 
Hamiltonian as

$$H_{\text{eff}} = \alpha_x \sigma_x + \alpha_y \sigma_y + \alpha_z \sigma_z + \alpha_I I. \quad (S21)$$

For simplicity, the following constraints are usually im-
posed on the effective Hamiltonian by applying an addi-
tional unitary transformation to the computational basis:

1. $\alpha_y$ is set to zero.
2. $\alpha_x$ is always positive.
3. $\alpha_z$ is positive for $\varphi_z > \varphi_d$ and is negative for $\varphi_z < \varphi_d$,

where $\varphi_d$ is given in Eq. (S4). After imposing the above 
constraints, we can write the effective Hamiltonian as a 
standard transverse field Ising Hamiltonian of the form

$$H_{\text{eff}} = A \sigma_x + B \sigma_z. \quad (S22)$$

This procedure leads to Fig. 2 in the main text.

As an example, and to make the connection between 
the s-curve measurements and the qubit picture, we cal-
culate the $A$ and $B$ coefficients for two of the asymmetry-
corrected anneal paths that were used for the measure-
ments in Fig. 3 of the main text. The result is shown in 
Fig. S4, where the solid lines correspond to the an-
neal path with $\varphi_z(0)/\pi = 0.005$, and the dashed lines 
correspond to the path with $\varphi_z(0)/\pi = 0.01$. 

![FIG. S4. Transverse field Ising Hamiltonian coefficients for two of the asymmetry-corrected anneal paths, vs normalized 
anneal time $t/t_a$. Solid lines correspond to the anneal path 
with $\varphi_z(0)/\pi = 0.005$, dashed lines correspond to the path 
with $\varphi_z(0)/\pi = 0.01$. For our system $E_J/2\pi \approx 100 \text{GHz}$.](image)
THE EFFECT OF IDLING POST ANNEAL

As illustrated in Fig. 4 of the main text, peaks in the s-curve appear when the anneal path traverses level crossings, leading to diabatic population transfer. The AME simulations reproduce these peaks only when an idle time is added between the end of the qubit anneal and readout. Without any delay, the theory predicts that instead the s-curve will exhibit plateaus. This delay allows for relaxation to occur, redistributing population between levels in either well. However, note that the AME with an Ohmic bath produces large transition rates at small gaps [S5], which means the peaks will rise faster than they do in the experiment. Nevertheless, the AME can qualitatively predict the effect of an idle time after the anneals, as seen in Fig. S5 and Fig. 4(a) of the main text.

We confirm this effect experimentally by varying the idle time after the anneals, as shown in Fig. S5. The anneal is depicted in Fig. 2 of the main text, where the anneal traverses a “tilted” path in flux space due to a large amplitude applied to the tilt bias. As the idle time increases from 2 ns to 600 ns, plateau-like features in the s-curve become peaks, as expected from the theory.

We performed similar delay studies with asymmetry-corrected anneal paths, and neither the plateaus nor the peaks appeared (not shown). This suggests that fewer excitations into higher-energy states occurred.

COMPARING S-CURVE WIDTHS

The width of the s-curve depends on the rate at which the barrier is raised, thermalization between the states in left and right wells, and flux noise in the tilt bias, specifically around the minimum gap [S8]. Therefore the width is a characteristic of the noise environment of the qubit, assuming the anneals are slow enough to avoid broadening due to nonadiabatic effects. In order to compare the results between multiple platforms and qubit designs, we associate an effective temperature to the qubit’s s-curve width by multiplying it by the persistent current of the qubit at the end of the anneal to get

\[ T_{\text{eff}} = \frac{wI_p}{k_B}. \] (S23)

Note that near the degeneracy point, \( wI_p \) is the effective longitudinal field (\( \sigma_z \) coefficient) in the Ising spin model of qubits.

Since both \( w \) and \( I_p \) should be minimized (recall that \( T_1 \sim 1/I_p^2 \) and \( T_2 \sim 1/I_p \)), a smaller \( T_{\text{eff}} \) is preferable. The relevant dimensionless quantity is \( T_{\text{eff}} \) scaled by the dilution fridge temperature, \( T_{\text{fridge}} \). Table S2 shows a summary of \( T_{\text{eff}}/T_{\text{fridge}} \) values across different flux qubit designs.

EXPERIMENTAL PULSES

In this section we describe in more detail the actual pulses used to perform the correction, as well as how different pulse parameters affect the s-curve width.

Note that for experimental parameters, we use real flux values in units of \( \Phi_0 \), and recall the relation to phase:

\[ \varphi_{x,z} = 2\pi \Phi_{x,z} / \Phi_0. \]

As implied by Eq. (2) in the main text, we parametrize the \( z \)-flux in terms of the \( z \)-flux. We first decide on a functional form and duration for \( \Phi_z(t) \), and then Eq. (2) is used to determine \( \Phi_z(t) \) from those values of \( \Phi_x \). We found that a gaussian pulse shape for the \( x \)-flux produced better results than a linear ramp, likely due to a reduction of pulse distortion in the lines. Future studies could explore using more sophisticated techniques like DRAG, black-box optimization, or other methods to find more performant pulses.

Figure S6 illustrates the pulses used to correct for an asymmetry of \( d = 0.102 \). The top plot shows each independent control line as a function of time. The shaded gray area indicates the \( 5 - 95\% \) rise time of 20 ns, which is the parameter used when defining the “anneal time” for a particular experiment. The bottom plot then combines these two pulse to illustrate the actual annealing...
| Group               | $w$ ($\mu\Phi_0$) | $I_p$ ($\mu$A) | $T_{fridge}$ (mK) | $T_{eff}$ (mK) | $T_{eff}/T_{fridge}$ |
|---------------------|-------------------|----------------|-------------------|----------------|---------------------|
| D-Wave              | 45 [S6]           | 2.6 [S7]       | 8 [S6]            | 17.5           | 2.2                 |
| Google              | 100 [S8]          | 0.87 [S8]      | 10 [S9]           | 13             | 1.3                 |
| QEO (this work)     | 1400              | 0.17           | 20                | 35.7           | 1.8                 |
| QEO (improved)      | 760               | 0.17           | 15                | 19.4           | 1.3                 |

TABLE S2. Comparison of s-curve widths across experimental groups. The raw width is multiplied by the qubit persistent current, $I_p$, to convert it to an effective temperature, $T_{eff}$. This is then divided by the dilution refrigerator operating temperature, $T_{fridge}$, to create a dimensionless quantity for cross-platform comparison. The improved QEO width was measured on a colder fridge with better line filtering, and will be discussed in a future publication.

FIG. S6. Experimental pulses used to correct for junction asymmetry of magnitude $d = 0.102$. Top: $x$ [$\Phi_x(t)$, green circles] and $z$ [$\Phi_z(t)$, purple circles] flux pulses as a function of time. The hitch after the first point is due to truncation of the gaussian pulse, and can be smoothed in software. The gray shaded area indicates the 20 ns rise time. Bottom: The corrected pulse visualized in flux space, where each orange circle corresponds to one time step from the top plot.

We also studied the effect of changing the value of $d$ when applying the correction. Fig. S7 highlights the reduction in s-curve width when the value of $d$ is near the value of $\approx 0.102$ measured during calibration (and verified in simulation).

The difference in trends between the two anneal times suggests that there is potentially more room for optimization by exploring the entire space of annealing time in addition to the path trajectory.

We also note that the data in Fig. S7 were taken when the fridge base temperature was 20% higher (24 mK) than that in the main text (20 mK). This accounts for the overall increase in measured s-curve widths.

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