A Game-Theoretic Approach to Energy-Efficient Modulation in CDMA Networks with Delay QoS Constraints

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Abstract—A game-theoretic framework is used to study the effect of constellation size on the energy efficiency of wireless networks for M-QAM modulation. A non-cooperative game is proposed in which each user seeks to choose its transmit power (and possibly transmit symbol rate) as well as the constellation size in order to maximize its own utility while satisfying its delay quality-of-service (QoS) constraint. The utility function used here measures the number of reliable bits transmitted per joule of energy consumed, and is particularly suitable for energy-constrained networks. The best-response strategies and Nash equilibrium solution for the proposed game are derived. It is shown that in order to maximize its utility (in bits per joule), a user must choose the lowest constellation size that can accommodate the user’s delay constraint. This strategy is different from one that would maximize spectral efficiency. Using this framework, the tradeoffs among energy efficiency, delay, throughput and constellation size are also studied and quantified. In addition, the effect of trellis-coded modulation on energy efficiency is discussed.

Index Terms—Energy efficiency, M-QAM modulation, trellis-coded modulation, game theory, utility function, Nash equilibrium, delay, quality-of-service (QoS), cross-layer design.

I. INTRODUCTION

Wireless networks are expected to support a variety of applications with diverse quality-of-service (QoS) requirements. Because of the scarcity of network resources (i.e., energy and bandwidth), radio resource management is crucial to the performance of wireless networks. The goal is to use the network resources as efficiently as possible while providing the required QoS to the users. Adaptive modulation has been shown to be an effective method for improving the spectral efficiency in wireless networks (see for example [1]–[4]). However, the focus of many of the studies to date has been on maximizing the throughput of the network, and the impact of the modulation order on energy efficiency has not been studied to the same extent. Recently, the authors of [5] have studied modulation optimization for an energy-constrained time-division-multiple-access (TDMA) network. For such a network, they have used a convex-optimization approach to obtain the best modulation strategy that minimizes the total energy consumption under throughput and delay constraints.

Game-theoretic approaches to power control have recently attracted considereable attention (see, for example, [6]–[16]). In [6], the authors provide motivations for using game theory to study communication systems, and in particular power control. In [7], power control is modeled as a non-cooperative game in which users choose their transmit powers in order to maximize their utilities, where utility is defined as the ratio of throughput to transmit power. A game-theoretic approach to joint power control and receiver design is presented in [13], and power control for multicarrier systems is studied in [14]. The authors in [8] use pricing to obtain a more efficient solution for the power control game. Similar approaches are taken in [9]–[12] for different utility functions. Game-theoretic approaches to power control in delay-constrained networks are proposed in [15], [16].

In this work, we use a game-theoretic approach to study the effects of modulation on energy efficiency of code-division-multiple-access (CDMA) networks in a competitive multiuser setting. Focusing on M-QAM modulation, we propose a non-cooperative game in which each user chooses its strategy, which includes the choice of the transmit power, symbol rate and constellation size, in order to maximize its own utility while satisfying its QoS constraints. The utility function used here measures the number of reliable bits transmitted per joule of energy consumed, and is particularly suitable for energy-constrained networks. We derive the best-response strategies and Nash equilibrium solution for the proposed game. In addition, using our non-cooperative game-theoretic framework, we quantify the tradeoffs among energy efficiency, delay, throughput and modulation order. The effect of coding on energy efficiency is also studied and quantified using the proposed game-theoretic approach. In addition, our framework allows us to illustrate the tradeoff between energy efficiency and spectral efficiency.

The remainder of this paper is organized as follows. The system model and definition of the utility function are given in Section III. We then present a power control game with no delay constraints in Section III and derive the corresponding...
Nash equilibrium solution. A delay-constrained power control game is proposed in Section IV and the corresponding best-response strategies and Nash equilibrium solution are derived. The analysis is extended to coded systems in Section V. Numerical results and conclusions are given in Sections VI and VII respectively.

II. SYSTEM MODEL

We consider a direct-sequence CDMA (DS-CDMA) wireless network in which the users′ terminals are transmitting to a common concentration point (e.g., a cellular base station or an access point). The system bandwidth is assumed to be \( B \) Hz. Let \( R_{s,k} \) and \( p_k \) be the symbol rate and the transmit power for user \( k \), respectively. In this work, we focus on M-QAM modulation. Hence, each symbol is assumed to be complex to represent the in-phase and quadrature components. For the M-QAM modulation, the number of bits transmitted per symbol is given by

\[
b = \log_2 M.
\]

Since there is a one-to-one mapping between \( M \) and \( b \), we sometimes refer to \( b \) as the constellation size. We focus on square M-QAM modulation, i.e., \( M \in \{4, 16, 64, \ldots\} \) or equivalently \( b \in \{2, 4, 6, \ldots\} \), since there are exact expressions for the symbol error probability of square M-QAM modulation (see [17]). We can easily generalize our analysis to include odd values of \( b \) by using an approximate expression for the symbol error probability.

We define the utility function of a user as the ratio of its throughput to its transmit power, i.e.,

\[
u_k = \frac{T_k}{p_k}.
\]

This utility function is similar to the one used in [7] and [8]. Throughput in (1) is defined as the net number of information bits that are transmitted without error per unit time (it is sometimes referred to as goodput), and is expressed as

\[
T_k = R_k f(\gamma_k)
\]

where \( R_k = b_k R_{s,k} \) is the transmission rate, \( \gamma_k \) is the output signal-to-interference-plus-noise ratio (SIR) for user \( k \), and \( f(\gamma_k) \) is the “efficiency function” which represents the packet success rate (PSR) for the \( k \)th user. We require that \( f(0) = 0 \) to ensure that \( u_k = 0 \) when \( p_k = 0 \). In general, the efficiency function depends on the modulation, coding and packet size.

We assume an automatic-repeat-request (ARQ) mechanism in which the user keeps retransmitting a packet until the packet is received at the access point without any errors. Based on (1) and (2), the utility function for user \( k \) can be written as

\[
u_k = \frac{R_k f(\gamma_k)}{p_k}.
\]

This utility function, which has units of bits/joule, measures the number of reliable bits that are transmitted per joule of energy consumed, and is particularly suitable for energy-constrained networks.

Let us for now focus on a specific user and drop the subscript \( k \). Assuming a packet size of \( L \) bits, the packet success rate for square M-QAM modulation is given by

\[
P_{\text{success}}(b, \gamma) = \left(1 - \alpha_b Q(\sqrt{\beta_b \gamma})\right)^{\frac{2}{bk}}
\]

where

\[
\alpha_b = 2 \left(1 - 2^{-\frac{b}{2}}\right),
\]

and

\[
\beta_b = \frac{3}{2^b - 1}.
\]

Here, \( \gamma \) represents the symbol SIR and \( Q(\cdot) \) is the complementary cumulative distribution function of the standard Gaussian random variable. Note that at \( \gamma = 0 \), we have \( P_{\text{success}} = 2^{-L} \neq 0 \). Since we require the efficiency function to be zero at zero transmit power, we define

\[
f_b(\gamma) = \left(1 - \alpha_b Q(\sqrt{\beta_b \gamma})\right)^{\frac{2}{bk}} - 2^{-L}.
\]

Note that \( 2^{-L} \approx 0 \) when \( L \) is large (e.g., \( L = 100 \)).

A non-cooperative power control game, in general, can be expressed as \( G = [K, \{A_k\}, \{u_k\}] \) where \( K = \{1, \ldots, K\} \) is the set of users/players, \( A_k \) is the strategy set for the \( k \)th user, and \( u_k \) is the utility function given by (3). Each user decides what strategy to choose from its strategy set in order to maximize its own utility. Hence, the best-response (i.e., utility-maximizing) strategy of user \( k \) is given by the solution of

\[
\max_{a_k \in A_k} u_k = \max_{a_k \in A_k} \frac{R_k f(\gamma_k)}{p_k}.
\]

For this game, a Nash equilibrium is a set of strategies \( (a_1^*, \ldots, a_K^*) \) such that no user can unilaterally improve its own utility [18], that is,

\[
u_k(a_k^*, a_{-k}^*) \geq u_k(a_k, a_{-k}^*), \quad \text{for all } a_k \in A_k \text{ and } k = 1, \ldots, K.
\]

In this work, we propose non-cooperative games in which the actions open to each user are the choice of transmit power, and possibly transmit symbol rate, as well as the choice of constellation size.

III. POWER CONTROL GAME WITH M-QAM MODULATION

Consider a DS-CDMA network with \( K \) users and express the transmission rate of user \( k \) as

\[
R_k = b_k R_{s,k}
\]

where \( b_k \) is the number of information bits per symbol and \( R_{s,k} \) is the symbol rate. Let us for now assume that users have no delay constraints. We propose a power control game in which each user seeks to choose its constellation size and transmit power in order to maximize its own utility, i.e.,

\[
\max_{p_k \geq 0} \frac{R_k f_b(\gamma_k)}{p_k} \quad \text{for } k = 1, \ldots, K,
\]

where \( b_k \in \{2, 4, 6, \ldots\} \) and \( p_k \in [0, P_{\text{max}}] \) with \( P_{\text{max}} \) being the maximum allowed transmit power. Throughout this work, we assume \( P_{\text{max}} \) is large.
For all linear receivers, the output SIR for user $k$ can be written as
\[ \gamma_k = \left( \frac{B}{R_{s,k}} \right) p_k \hat{h}_k \] (12)
where $B$ is the system bandwidth and $\hat{h}_k$ is the effective channel gain which depends on the channel gain of user $k$ and on the channel gains and transmit powers of other users in the network but is independent of the transmit power and rate of user $k$. For example, for a matched-filter receiver, $\hat{h}_k$ is given by
\[ \hat{h}_k = \frac{h_k}{\sigma^2 + \sum_{j \neq k} p_j h_j}, \]
where $h_k$ is the channel gain for user $k$, and $\sigma^2$ is the noise power. Therefore, the utility function in (11) can be written as
\[ u_k = B \hat{h}_k b_k f_b(\gamma_k) \frac{f_b'(\gamma_k)}{\gamma_k}. \] (13)
Based on (13), and by dropping the subscript $k$ for convenience, the maximization in (11) can be written as
\[ \max_{b,\gamma} B \hat{h} b f_b(\gamma) \frac{f_b'(\gamma)}{\gamma}. \] (14)
Since for a given user, $B$ and $\hat{h}$ are fixed, maximizing the user’s utility is equivalent to maximizing $bf_b(\gamma)/\gamma$ for that user. It is important to observe that, for a given $b$, specifying the operating SIR completely specifies the utility function. Let us now fix the symbol rate $R_s$ and the constellation size.

Taking the derivative of (14) with respect to $\gamma$, and equating it to zero, we conclude that the utility of a user is maximized when its output SIR, $\gamma$, is equal to $\gamma^*$, which is the (positive) solution of
\[ f_b(\gamma) = \gamma f_b'(\gamma). \] (15)
It is shown in [19] that for an S-shaped (sigmoidal) efficiency function, $f(\gamma) = \gamma f'(\gamma)$ has a unique solution. It can easily be verified that $f_b(\gamma)$ given by (12) is S-shaped. Note that $\gamma^*$ is (uniquely) determined by physical-layer parameters such as packet size, modulation, and coding.

Assuming that $\gamma^*$ is feasible, the maximum utility is hence given by
\[ u_k^* = B \hat{h} b f_b(\gamma^*) \frac{f_b'(\gamma^*)}{\gamma^*}. \] (16)
Based on (17), it can be shown that $\gamma^*$ is (approximately) given by the solution of
\[ \alpha_b L \frac{\beta_b \gamma}{\sqrt{2\pi}} e^{-\frac{\beta_b \gamma}{2}} + \alpha_b Q(\sqrt{\beta_b \gamma}) = 1. \]

We can compute $\gamma^*$ numerically for different values of $b$. Table I summarizes the results for a system with $L = 100$ bits (i.e., 100 bits per packet). It is observed from Table I that the user’s utility is maximized when $b = 2$ (i.e., QPSK modulation). This is because, as $b$ increases, the linear increase in the throughput is dominated by the exponential increase in the required transmit power (which results from the exponential increase in $\gamma^*$). Therefore, it is best for a user to use QPSK modulation.

\begin{table}[h]
\centering
\caption{Summary of the parameters corresponding to the utility-maximizing solutions for different modulation orders for an uncoded system with 100 bits per packet}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$b$ & $\alpha_b$ & $\beta_b$ & $\gamma^*$ (dB) & $\gamma^* f_b(\gamma^*)$ dB & $b/\gamma^*$ (dB) \\
\hline
2 & 1 & 1 & 9.1 & 0.801 & -6.1 & 0.1978 \\
4 & 1.5 & 0.2 & 15.7 & 0.775 & -9.7 & 0.0846 \\
6 & 1.75 & 0.0476 & 21.6 & 0.775 & -13.8 & 0.0322 \\
8 & 1.875 & 0.0118 & 27.3 & 0.757 & -18.3 & 0.0112 \\
10 & 1.9375 & 0.0029 & 33.0 & 0.743 & -23.0 & 0.0037 \\
\hline
\end{tabular}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{Efficiency function (packet success rate) as a function of SIR for different constellation sizes.}
\end{figure}

So far, we have shown that at Nash equilibrium (if it exists), QPSK modulation must be used by each user (otherwise a user can always improve its utility by switching to QPSK and reducing its transmit power), and each user’s transmit power is chosen to achieve the $\gamma^*$ corresponding to the QPSK modulation.
modulation at the output of the receiver (i.e., 9.1 dB according to Table [I]). The existence of the Nash equilibrium for the proposed game can be shown via the quasiconcavity of each user’s utility function in its own power [18]. Furthermore, because of the uniqueness of $\gamma^+$ and the one-to-one correspondence between the transmit power and the output SIR (see (17)), this equilibrium is unique.

In addition, we observe that the energy-efficient strategy is not spectrally-efficient. Incorporating the choice of the modulation order into our utility maximization allows us to trade off energy efficiency with spectral efficiency. For the same bandwidth and symbol rate, as a user switches to a higher-order modulation, the spectral efficiency for the user improves but its energy efficiency degrades.

IV. DELAY-CONSTRAINED POWER AND RATE CONTROL GAME WITH M-QAM MODULATION

In Section [III] we showed that for our utility function, it is best for a user to use the lowest-order modulation. We now extend the analysis to the case in which the users have delay QoS requirements. Our goal in this part is to study the effects of constellation size on energy efficiency and delay. We consider a game in which each user seeks to choose its transmit power, symbol rate and constellation size to maximize its own utility while satisfying its delay QoS constraint. The delay QoS constraint considered here is in terms of the average packet delay, i.e., we require

$$W \leq D.$$  \hspace{1cm} (23)

We specify the delay QoS constraint of a user by an upper bound on the average packet delay, i.e., we require

$$W \leq D.$$  \hspace{1cm} (23)

A. Delay Model

Let us assume that the incoming packets for user $k$ have a Poisson distribution with parameter $\lambda_k$ which represents the average packet arrival rate with each packet consisting of $L$ bits. The source rate (in bits per second) is hence given by $L\lambda_k$. The user transmits the arriving packets at a rate $R_k = b_k R_{s,k}$ (bps) and with a transmit power equal to $p_k$ Watts. We assume an ARQ mechanism for packet transmission. Also, the incoming packets are assumed to be stored in a queue and transmitted in a first-in-first-out (FIFO) fashion. The packet success probability (per transmission) as before is represented by the efficiency function $f_b(\gamma)$.

Focusing on a specific user and dropping the subscript $k$, we can represent the combination of the user’s queue and wireless link as an M/G/1 queue (as shown in Fig. 3) where the service time, $S$, has the following probability mass function (PMF):

$$\Pr\{S = m\tau\} = f_b(\gamma)(1 - f_b(\gamma))^{m-1} \quad \text{for } m = 1, 2, \ldots, \hspace{1cm} (19)$$

with $\tau$ being the packet transmission time which is given by

$$\tau = \frac{L}{bR_s} + \epsilon \approx \frac{L}{bR_s}. \hspace{1cm} (20)$$

Here, $\epsilon$ represents the time taken for the user to receive an ACK/NACK from the access point. We assume $\epsilon$ is negligible compared to $\frac{L}{bR_s}$. Based on (19), the service rate, $\mu$, is given by

$$\mu = \frac{1}{\mathbb{E}\{S\}} = \frac{f_b(\gamma)}{\tau} = R_s \frac{b f_b(\gamma)}{L}, \hspace{1cm} (21)$$

and the load factor $\rho = \frac{\lambda}{\mu} = \frac{\lambda \tau}{f_b(\gamma)\tau}$. Therefore, the average service rate is affected by the constellation size through a linear factor $b$ as well as the efficiency function $f_b(\gamma)$. To keep the queue stable, we must have $\rho < 1$ or $f_b(\gamma) > \lambda \tau$.

Now, let $W$ be a random variable representing the total packet delay for the user. The delay includes both transmission and queuing delays. Using the Pollaczek-Khintchine formula for the M/G/1 queue considered here, the average packet delay is given by (see [21])

$$W = \tau \left( \frac{1 - \frac{\lambda \tau}{f_b(\gamma)\tau}}{f_b(\gamma) - \lambda \tau} \right) \quad \text{with } f_b(\gamma) > \lambda \tau. \hspace{1cm} (22)$$

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This delay constraint can equivalently be expressed as

$$f_b(\gamma) \geq \eta_b$$  \hspace{1cm} (24)

where

$$\eta_b = \frac{L \lambda}{b R_s} + \frac{L}{b R_s D} - \frac{L^2 \lambda}{2 b^2 R_s^2 D^2}.$$  \hspace{1cm} (25)

Note that (24) is equivalent to the condition

$$\gamma \geq \tilde{\gamma}_b$$

where

$$\tilde{\gamma}_b = f_b^{-1}(\eta_b).$$  \hspace{1cm} (26)

Based on (7), $\tilde{\gamma}_b$ is given by

$$\tilde{\gamma}_b \simeq \frac{1}{\beta_b} \left[ Q^{-1} \left( \frac{1 - \eta_b}{\alpha_b} \right) \right]^2.$$  \hspace{1cm} (27)

This means that the delay constraint in (23) translates into a lower bound on the output SIR. It should be noted that since the upper bound on the average delay must be at least as large as the transmission time, i.e., $D \geq \tau$, we must have that $b R_s \geq L/D$. This automatically implies that $\eta_b > 0$. Also, since $0 \leq f_b(\gamma) \leq 1$, (24) is possible only if $\eta_b < 1$.

**B. The Proposed Game**

We propose a game in which each user chooses its transmit power and symbol rate as well as its constellation size in order to maximize its own utility while satisfying its delay requirement. Fixing the other users’ transmit powers and rates, the best-response strategy for the user of interest is given by the solution of the following constrained maximization:

$$\max_{\eta, R_s, b} u \quad \text{s.t.} \quad \tilde{W} \leq D,$$  \hspace{1cm} (28)

or equivalently

$$\max_{\gamma, R_s, b} b f_b(\gamma) \quad \text{s.t.} \quad \gamma \geq \tilde{\gamma}_b \quad \text{and} \quad 0 \leq \eta_b < 1.$$  \hspace{1cm} (29)

**Proposition 1:** For a fixed $b$, the source rate $\lambda$ and the delay constraint $D$ are feasible if and only if

$$\frac{L \lambda}{b B} + \frac{L}{b BD} - \frac{L^2 \lambda}{2 b^2 B^2 D} < 1,$$  \hspace{1cm} (30)

where $B$ is the system bandwidth.

**Proof:** For $\lambda$ and $D$ to be feasible, we must have $\eta_b < 1$ where $\eta_b$ is given by (25). Also, since $\eta_b$ is a decreasing function of $R_s$, the lowest possible value of $\eta_b$ is achieved when $R_s = B$. Hence, it is straightforward to see that the source rate $\lambda$ and the delay constraint $D$ are feasible if and only if (30) is satisfied.

Remember that $D$ cannot be smaller than $\tau$. Hence, it can be shown that the condition $0 \leq \eta_b < 1$ is equivalent to $R_s \geq \Omega^\infty / b$ where

$$\Omega^\infty = \left( \frac{L}{D} \right) 1 + D \lambda + \sqrt{1 + D^2 \lambda^2}.$$  \hspace{1cm} (31)

Also, let us define $\Omega^*_{b}$ as the rate for which $\hat{\gamma}_b = \gamma^*_b$, i.e.,

$$\Omega^*_{b} = \left( \frac{L}{D} \right) \frac{1 + D \lambda + \sqrt{1 + D^2 \lambda^2}}{2 f_b^*}.$$  \hspace{1cm} (32)

where $f_b^* = f_b(\gamma^*_b)$.

**Proposition 2:** For given values of $\lambda$ and $D$, the best-response strategy for a user (i.e., the solution of (28)) is any combination of $p$ and $R_s$ such that

$$\min \left\{ \Omega^*_b / \hat{b}, \hat{B} \right\} \leq R_s \leq B$$  \hspace{1cm} (33)

and

$$\gamma = \begin{cases} \gamma^*_b & \text{if } \Omega^*_b / \hat{b} \leq \hat{B}; \\ \hat{\gamma}_b & \text{if } \Omega^*_b / \hat{b} > \hat{B}, \end{cases}$$

where $\hat{b}$ is the lowest constellation size for which $\lambda$ and $D$ are feasible, $\gamma^*_b$ is the solution of (15), and $\hat{\gamma}_b$ is given by (26). Proof of Proposition 2 requires the following lemma.

**Lemma 1:** If $\hat{\eta}_b > 2^{L/2}$, then $\hat{\eta}_b$ is an increasing function of $b$ and the increase is exponential.

**Proof:** Let us define $x_b = \frac{1 - \eta_b^{1/2L}}{\alpha_b}$. Given (3), we can express $x_b$ as

$$x_b = \frac{1}{2} \left[ \frac{1 - (\hat{\gamma}_b)^{1/2L}}{1 - (\hat{\eta}_b)^{1/2L}} \right].$$

Define $\tilde{\eta}_b = \eta_b^{1/2L}$. If $\eta_b > 2^{-L}$, then $\tilde{\eta}_b > 1/\sqrt{2}$. Now, if $b' > b''$, then, it is easy to show that

$$\frac{1 - (\tilde{\eta}_b)^{b'}}{1 - (1/\sqrt{2})^{b'}} > \frac{1 - (\tilde{\eta}_b)^{b''}}{1 - (1/\sqrt{2})^{b''}}.$$

Also, since $\eta_b < \eta_b''$, we have

$$\frac{1 - (\tilde{\eta}_b)^{b'}}{1 - (1/\sqrt{2})^{b'}} > \frac{1 - (\tilde{\eta}_b)^{b''}}{1 - (1/\sqrt{2})^{b''}}.$$ As a result,

$$\frac{1 - (\tilde{\eta}_b)^{b'}}{1 - (1/\sqrt{2})^{b'}} > \frac{1 - (\tilde{\eta}_b)^{b''}}{1 - (1/\sqrt{2})^{b''}},$$

which implies that $x_b$ is an increasing function of $b$. According to (3), we have

$$\hat{\gamma}_b = \frac{1}{\beta_b} \left[ Q^{-1}(x_b) \right]^2.$$

Since $x_b$ is an increasing function of $b$, and $\beta_b$ is a decreasing function of $b$, $\hat{\gamma}_b$ is an increasing function of $b$. Furthermore, since $\left[ Q^{-1}(x_b) \right]^2$ is increasing in $b$ and $1/\beta_b = (2^b - 1)/3$ is exponentially increasing in $b$, $\gamma^*_b$ is also exponentially increasing in $b$.

The $\eta_b > 2^{-L}$ assumption is consistent with good design practice. In particular, since $\eta_b$ represents the packet success probability, $\eta_b \leq 2^{-L}$ would correspond to a very poorly-designed system. In fact, in such a case, there would not be any need to transmit the packets since random guessing at the receiver would give a PSR of $2^{-L}$. We now give the proof for Proposition 2.

**Proof:** [Proof of Proposition 2] We showed in Section III that for the unconstrained optimization problem and for a fixed

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4Note that $f(\gamma) = 1$ requires an infinite SIR which is not practical.
Therefore, the user must always choose the lowest constellation size for which the user’s QoS constraint can be satisfied.

b, the utility is maximized when the user’s SIR is equal to \( \gamma \), which is the solution of \( f_b(\gamma) = \gamma f_b'(\gamma) \). It is straightforward to show that \( \gamma_b \) is a decreasing function of \( R_s \) for all \( R_s \geq \Omega^\infty / b \). Therefore, for all \( \Omega^\infty / b \leq R_s < \Omega^\infty / b, \gamma_b > \hat{\gamma}_b \). This means that based on \( \Omega^\infty / b \), a user has no incentive to transmit at a symbol rate smaller than \( \Omega^\infty / b \). Hence, for a fixed constellation size, any combination of \( p \) and \( R_s \geq \Omega^\infty / b \) that results in an output SIR equal to \( \gamma \) maximizes the utility and satisfies the delay constraint. If \( \Omega^\infty / b > B \), then \( \gamma_b \) cannot be made equal to \( \gamma_b \). In this case, the user must transmit at the maximum symbol rate (i.e., \( R_s = B \)) and choose its transmit power such that \( \gamma = \gamma_b < \gamma_b \) in order to meet its delay constraint.

Now let us include the effect of constellation size. Let \( b' > b'' \) and consider the following cases.

- If \( \Omega^\infty_{b''} / b'' \leq B \), then we will have \( \Omega^\infty_{b'} / b' \leq B \). This means both \( \gamma_{b''} \) and \( \gamma_{b''} \) are feasible. However, the user’s utility will drop if the user moves to a higher-order modulation. This is because the linear gain in utility due to an increase in \( b \) is dominated by the exponential increase in the optimum operating SIR as shown in Section III. Therefore, in this case, the user would choose the smallest \( b \).

- If \( \Omega^\infty_{b''} / b'' > B \) but \( \lambda \) and \( D \) are feasible with \( b'' \) (see Proposition 1), then the user’s utility is maximized when the symbol rate is equal to \( B \) and the SIR is equal to \( \gamma_{b''} \). On the other hand, the user can switch to \( b' > b'' \). In that case, \( \gamma_{b''} \) is smallest when the symbol rate is equal to \( B \). However, based on Lemma 1 with \( R_s = B \) and \( b' > b'' \), we have \( \gamma_{b''} > \gamma_{b'} \). Furthermore, the increase in \( \gamma_{b''} \) is exponential. Since the exponential increase in the SIR would dominate the linear increase in the rate caused by an increase in \( b \), it is best for the user to use \( b'' \) (i.e., the smaller constellation size).

- If \( \Omega^\infty_{b''} / b'' > B \) and \( \lambda \) and \( D \) are not feasible, the user must switch to a higher constellation size and a similar argument as above would follow.

Therefore, the user must always choose the lowest constellation size for which the user’s QoS constraint can be satisfied.

Proposition 2 implies that, in terms of energy efficiency, choosing the lowest-order modulation (i.e., QPSK) is the best strategy unless the user’s delay constraint is too tight. In other words, the user would jump to a higher-order modulation only when it is transmitting at the highest symbol rate (i.e., \( R_s = B \)) and still cannot meet the delay requirement. Also, the proposition suggests that if \( \Omega^\infty / b < B \), the user has infinitely many best-response strategies. In particular, the user chooses the lowest constellation size that can accommodate the delay constraint. Then, for that constellation, any combination of \( p_k \) and \( R_{s,k} \) for which \( \gamma_k = \gamma_b \) and \( R_{s,k} \geq \Omega^\infty / b \) is a best-response strategy.

C. Nash Equilibrium

At Nash equilibrium, the transmit powers, symbol rates and constellation sizes of all the users have to satisfy Proposition 2 simultaneously. There are, therefore, cases where we have infinitely many Nash equilibria. For a matched filter, for example, the best-response transmit power of user \( k \) is given by

\[
p_k = \frac{\sigma^2}{h_k} \left( \frac{\Phi_k}{1 - \sum_{j=1}^K \Phi_j} \right),
\]

where

\[
\Phi_k = \left( 1 + \frac{B}{R_{s,k} \gamma_k} \right)^{-1},
\]

and \( \gamma_k \) and \( R_{s,k} \) are determined according to Proposition 2 for \( k = 1, \ldots, K \). We refer to \( \Phi_k \) as “size” of user \( k \). \( \Phi_k \) is a measure of the amount of network resources that is consumed by the \( k \)th user. Note that \( R_{s,k} \)’s and \( \gamma_k \)’s are feasible if and only if

\[
\sum_{j=1}^K \Phi_j < 1.
\]

Combining (35) with (36), the utility of user \( k \) at Nash equilibrium is given by

\[
u_k = \frac{B f(\gamma_k) h_k}{\sigma^2 \gamma_k} \left( 1 - \sum_{j \neq k} \Phi_j \right).
\]

Therefore, the Nash equilibrium with the smallest \( R_{s,k} \) achieves the largest utility. A higher symbol rate (i.e., smaller processing gain) for a user requires a larger transmit power by that user to achieve the required SIR. This causes more interference for other users in the network and forces them to raise their transmit powers as well. As a result, the level of interference in the system increases and the users’ utilities decrease. This means that the Nash equilibrium with \( R_{s,k} = \min\{ \Omega_{b_k} / \hat{b}_k, B \} \) and \( p_k \) given by (35) for \( k = 1, \ldots, K \) is the Pareto-dominant Nash equilibrium.

As the delay constraint of a user becomes tighter, according to Proposition 2, the user will increase its symbol rate. This results in an increase in the user’s “size”. When the symbol rate increases, the users’ utilities decrease and the users’ QoS constraints are satisfied. This results in an exponential increase in the required SIR, which dominates the linear decrease in the symbol rate. Therefore, \( \Phi \) increases again. This shows that the user’s “size” increases as the delay requirement becomes more stringent. The feasibility condition given by (37) determines the maximum number of users that can be accommodated by the network. A tighter delay constraint results in a larger “size” for the user. This, in turn, results in a smaller network capacity.

V. Power Control Games with Trellis-Coded M-QAM Modulation

So far, we have focused on an uncoded system. In this section, we extend our analysis to trellis-coded modulation (TCM). We consider a trellis-coded M-QAM system in which \( b \) information bits are divided into two groups of size \( n \) and \( b - n \) bits, respectively. The first group (with size \( n \))

\[
\Phi_k \text{ here is a generalized version of the definition that has been given in [16].}
\]
is convolutionally encoded into $\ell$ bits which are used by the coset selector to choose one of the $2^\ell$ constellation subsets. The remaining $b - n$ bits are used to choose one of the $2^{b-n}$ signal points in the selected subset (see [22] for more details). The code rate is hence given by $\theta_c = n/\ell$ and the constellation size is increased from $2^b$ to $2^{b+n}$. It is common to use a code rate of $\theta_c = n/(n+1)$ for subset selection. For $b > 2$, $n = 2$ is usually a good choice.\footnote{For $b = 2$, $n$ is equal to one.}

For trellis-coded modulation (TCM), the efficiency function (which represents the packet success probability) is given by

$$f_b^{(c)}(\gamma) \simeq \left(1 - \alpha_b Q(\sqrt{\beta_b \gamma G_b(\gamma)})\right) - 2^{-L},$$

where $b$ is the number of information bits per symbol and $G_b(\cdot)$ is the effective coding gain which in general is a function of SIR and also depends on the modulation level. Recall that in our proposed game, each user chooses its transmit power, symbol rate and modulation level to maximize its own utility function while satisfying its delay constraint. One could potentially follow the same analysis for the coded system as the one presented for the uncoded system by replacing $f_b^{(c)}$ with $f_b^{(c)}(\gamma)$ given in (39). For the coded case, the delay-constrained utility maximization can be written as

$$\max_{\gamma, R, b} f_b^{(c)}(\gamma) \quad \text{s.t.} \quad \gamma \geq \tilde{\gamma}_b \quad \text{and} \quad 0 \leq \eta_b < 1,$$

where $\tilde{\gamma}_b^{(c)} = f_b^{(c)-1}(\eta_b)$. Therefore, the solution of this maximization is heavily dependent on the efficiency function given in (39). While the coding gain can be assumed to be constant in the limit of very large SIRs, the dependence of $G$ on $\gamma$ and $b$ is important for our optimization problem. Since there are no closed-form expressions for $G_b(\gamma)$, we can use the BER curves available in the literature (for example in [3]) to estimate the coding gain as a function of $\gamma$ for different modulation levels. Then, using these discrete values, we can approximate the shape of $G_b(\gamma)$ for different values of $b$. We have found that the following function gives us a reasonable estimate for $G$:

$$\hat{G}_b(\gamma) = A_b + C_b \tan^{-1}\left(\frac{\gamma - \tilde{\gamma}_b}{D_b}\right),$$

where $A_b$, $C_b$, $D_b$, and $\tilde{\gamma}_b$ are constants that only depend on the modulation level and can be determined by trial and error. The function in (41) is plotted in Fig. 4 for different values of $b$ for an 8-state convolutional encoder with rate 2/3. The solid lines are the estimates based on $G_b$, and the dashed lines are piece-wise linear estimates based on the data obtained from the BER curves.

Fig. 4. Coding gain as a function of SIR for an 8-state convolutional encoder with rate 2/3. The solid lines are the estimates based on $G_b$, and the dashed lines are piece-wise linear estimates based on the data obtained from the BER curves.

In this section, we quantify the effect of constellation size on energy efficiency of a user with a delay QoS constraint. The packet size is assumed to be 100 bits, and the source rate (in bps) for the user is assumed to be equal to $0.01B$ where $B$ is the system bandwidth. We further assume that a user chooses its constellation size, symbol rate, and transmit power according to its best-response strategy corresponding to the Pareto-dominant Nash equilibrium (see Section IV-C). For the coded system, we assume an 8-state convolutional encoder with rate 2/3. The code rate for QPSK is chosen to be 1/2. Fig. 5 shows the optimum constellation size, transmit power, throughput, and user’s utility as a function of the delay constraint for both uncoded and coded systems. The results for the coded case are obtained by using (41) as an approximation for the coding gain. For all four plots, the packet delay is normalized by the inverse of the system bandwidth. To keep the spectral efficiency of the two systems the same, we assume that the number of information bits transmitted per symbol is the same for both uncoded and coded systems. The throughput corresponds to the transmission rate for the user which is obtained by multiplying the symbol rate by the number of (information) bits per symbol (i.e., $b$), and is normalized by

\footnote{Optimum here refers to the best-response strategy (i.e., the most energy-efficient solution).}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{SIR (dB)} & \textbf{Coding Gain (dB)} & \textbf{Probability} & \textbf{Utility} \\
\hline
0 & 0.0 & 0.0 & 0.0 \\
1 & 0.1 & 0.1 & 0.1 \\
2 & 0.2 & 0.2 & 0.2 \\
3 & 0.3 & 0.3 & 0.3 \\
4 & 0.4 & 0.4 & 0.4 \\
5 & 0.5 & 0.5 & 0.5 \\
6 & 0.6 & 0.6 & 0.6 \\
7 & 0.7 & 0.7 & 0.7 \\
\hline
\end{tabular}
\caption{Probabilities and utilities of the uncoded and coded systems for different values of $b$ and with no delay constraints. It is seen from the table that the target SIR (i.e., $\gamma^*$) is lower in a coded system. Also, comparing the fourth and seventh columns of Table I, we see that coding improves user’s utility (i.e., energy efficiency).}
\end{table}
the system bandwidth. The transmit power and user’s utility are also normalized by $\hat{h}$ and $B\hat{h}$, respectively.

Let us for now focus on the uncoded system. When the delay constraint is large, QPSK (which is the most energy efficient M-QAM modulation) can accommodate the delay requirement and hence is chosen by the user. As the delay constraint becomes tighter, the user increases its symbol rate and also raises the transmit power to keep the output SIR at 9.1 dB (recall that $\gamma_b^* = 9.1$ dB when $b = 2$). In this case, the user’s utility stays constant. Once the symbol rate becomes equal to the system bandwidth, the user cannot increase it anymore. Hence, as the delay constraint becomes more stringent, the user is forced to aim for a higher target SIR to meet its delay requirement. In this case, the transmission rate stays constant and the transmit power increases. Hence, the user’s utility decreases. As the delay requirement becomes tighter, a point is reached where the spectral efficiency of QPSK is not enough to accommodate the delay constraint. In this case, the user jumps to a higher-order modulation (i.e., 16-QAM) and the process repeats itself. The trends are similar for the coded system except that, due to coding gain, the required transmit power is smaller for the coded system. This results in an increase in the user’s utility. This means that, for the same number of information bits transmitted per symbol, the energy efficiency is higher when TCM is used. In addition, in Fig. 6 we have plotted the utility gain achieved due to TCM as a function of normalized packet delay. It is seen that the gain in energy efficiency due to TCM depends on the delay constraint and fluctuates between 1.5 dB and 3 dB. In general, for the same delay constraint, the uncoded system has to transmit at a slightly higher symbol rate as compared to the coded system. This is because $f_b^c > f_b^u$ (see Table II).

As a result, $\Omega_b^u < \Omega_b^c$. The spikes in Fig. 6 correspond to the cases in which the uncoded system is transmitting at the maximum possible symbol rate and has to increase its target SIR to meet the delay constraint. This results in a drop in the utility of the uncoded system. The coded system may still be able to meet the delay constraint without increasing the target SIR.

Fig. 7 shows the user’s “size”, $\Phi$, corresponding to the Pareto-dominant Nash equilibrium as a function of the packet delay for both uncoded and coded systems. As explained in Section IV-C, $\Phi$ increases as the delay constraint becomes tighter. This makes sense because a user would need to consume more network resources to satisfy a more stringent delay. It is also seen that coding reduces the user’s “size” and, hence, increases the network capacity. This is because
for the same constellation size, the symbol rate and the SIR are smaller for the coded system.

We have seen throughout this paper that the strategy that maximizes the user’s energy efficiency is not spectrally efficient. To illustrate the energy efficiency-spectral efficiency tradeoff, let us fix the symbol rate to be 0.01B. For a fixed constellation size, the user’s utility (energy efficiency) is proportional to \( bR_s/B \). By varying the constellation size, we can quantify the tradeoff between energy efficiency and spectral efficiency. This tradeoff is shown in Fig. 8 for both uncoded and coded systems. In this figure, we have plotted \( bR_s/B \) vs. \( bR_s/B \). Different points on the plot correspond to different values of \( b \). The energy efficiency-spectral efficiency tradeoff is definitely an interesting and important topic that requires more in-depth analysis.

VII. Conclusions

We have studied the effects of modulation order on energy efficiency of wireless networks using a game-theoretic framework. Focusing on M-QAM modulation, we have proposed a non-cooperative game in which each user chooses its strategy in order to maximize its utility while satisfying its delay QoS constraint. The actions open to the users are the choice of the transmit power, transmit symbol rate and constellation size. The utility function measures the number of reliable bits transmitted per joule of energy consumed and is particularly suitable for energy-constrained networks. The best-response strategies and the Nash equilibrium solution for the proposed game have been derived. We have shown that to maximize its utility (i.e., energy efficiency), the user must choose the lowest modulation level that can accommodate the user’s delay constraint. Using our non-cooperative game-theoretic framework, the tradeoffs among energy efficiency, delay, throughput and constellation size have also been studied and quantified.

In addition, we have included the effects of TCM and have shown that, as expected, coding increases energy efficiency. The tradeoff between energy efficiency and spectral efficiency has also been illustrated.

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