Primordial Nucleosynthesis: an updated comparison of observational light nuclei abundances with theoretical predictions

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An up to date review of Standard Big Bang Nucleosynthesis predictions vs the astrophysical estimates of light nuclei abundances is here presented. In particular the analysis reports the expected ranges for baryon fraction and effective number of neutrinos as obtained by BBN only.

1. Introduction

The nucleosynthesis taking place in the primordial plasma plays a twofold role: it is certainly one of the observational pillars of the hot Big Bang model, being indeed known simply as “Big Bang Nucleosynthesis” (BBN); at the same time, it provides one of the earliest direct cosmological probe nowadays available, constraining the properties of the universe when it was a few seconds old, or equivalently at the MeV temperature scale.

The basic framework of the BBN emerged in the decade between the seminal Alpher-Bethe-Gamow (known as $\alpha\beta\gamma$) paper in 1948 \cite{1} and the essential settlement of the paradigm of the stellar nucleosynthesis of elements heavier than $^7\text{Li}$ with the $\text{B}^2\text{FH}$ paper \cite{2}. This pioneering period—an account of which can be found in \cite{3}—established the basic picture that sees the four light-elements $^2\text{H}$, $^3\text{He}$, $^4\text{He}$ and $^7\text{Li}$ as products of the early fireball, and virtually all the rest produced in stars or as a consequence of stellar explosions.

In the following decades, the emphasis on the role played by BBN has evolved significantly. In the simplest scenario, the only free parameters in primordial nucleosynthesis are the baryon to photon ratio $\eta$ (equivalently, the baryon density of the universe) and the neutrino chemical potentials, $\mu_{\nu_\alpha}$. However, only neutrino chemical potentials larger than $\eta$ by many orders of magnitude have appreciable effects. This is why the simple case where all $\mu_{\nu_\alpha}$’s are assumed to be negligibly small (e.g., of the same order of $\eta$) is typically denoted as Standard BBN (SBBN). However, it is usual to add to $\eta$, as an additional parameter, the so-called effective number of neutrinos, $N_{\text{eff}}$, which measure the amount of relativistic d.o.f. at the time of BBN.

2. Standard BBN theoretical predictions versus data

The goal of a theoretical analysis of BBN is to obtain a reliable estimate of the model parameters, once the experimental data on primordial abundances are known. In this Section we will consider only the case of standard BBN, where the only two free parameters are the value of the baryon energy density parameter $\Omega_B h^2$ (or equivalently the baryon to photon number density, $\eta$) and possibly, a non-standard value for the relativistic energy content during BBN. The latter, after $e^\pm$ annihilation can be parameterized in terms of the effective number of neutrinos

$$\rho_R = \left(1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right) \rho_\gamma .$$

As it is well known, $N_{\text{eff}}$ can differ from zero also via the non thermal equilibrium terms which characterize the relic neutrino distributions. We
Figure 1. Frozen distortions of the flavor neutrino spectra as a function of the comoving momentum, for the best fit solar and atmospheric mixing parameters. R is the scale factor. In the case where we allow for θ_{13} ≠ 0 consistently with present bounds (blue dotted lines), one can distinguish the distortions for ν_µ and ν_τ (middle and lower, respectively). From [4].

In particular, the $N_{\text{eff}}$ reported in Table 1 is the contribution of neutrinos to the whole radiation energy budget, but only at the very end of neutrino decoupling. Hence, not all the $\Delta N_{\text{eff}}$ there reported will be really contributing to BBN processes. In order to clarify this subtle point, we report in Table 2 the effect on all light nuclides, of the non instantaneous neutrino decoupling in the simple scenario of no neutrino oscillation, and compare this column with the simple prescription of adding a fix $\Delta N_{\text{eff}} = 0.013$ contribution to radiation. Even though $Y_p$ is reproduced (by construction), this is not the case for the other nuclear yields. Similar analysis have been recently presented by various groups, which might be slightly different depending on the adopted values of $Y_p$ and/or $^3\text{H}/\text{H}$ experimental determination, see e.g. [6–16].

In the minimal BBN scenario the parameters reduces to the baryon density only, since

| Case                        | $N_{\text{eff}}$ | $\Delta Y_p$       |
|-----------------------------|------------------|---------------------|
| No mixing (no QED)          | 3.035            | $1.47 \times 10^{-4}$ |
| No mixing                   | 3.046            | $1.71 \times 10^{-4}$ |
| Mixing, $\theta_{13} = 0$  | 3.046            | 2.07$ \times 10^{-4}$ |
| Mixing, sin^2($\theta_{13}$) = 0.047 | 3.046 | 2.12$ \times 10^{-4}$ |
| Mixing, Bimaximal, $\theta_{13} = 0$ | 3.045 | 2.13$ \times 10^{-4}$ |

Table 1
$N_{\text{eff}}$ and $\Delta Y_p$ obtained for different cases, with and without neutrino oscillations, as reported in [4].

| Nuclide | Exact (No $\nu$-oscillations) | Fixed $\Delta N_{\text{eff}} = 0.013$ |
|---------|--------------------------------|--------------------------------------|
| $\Delta Y_p$ | $1.71 \times 10^{-4}$ | $1.76 \times 10^{-4}$ |
| $\Delta(^3\text{H}/\text{H})$ | $-0.0068 \times 10^{-5}$ | $+0.0044 \times 10^{-5}$ |
| $\Delta(^4\text{He}/\text{H})$ | $-0.0011 \times 10^{-5}$ | $+0.0007 \times 10^{-5}$ |
| $\Delta(^7\text{Li}/\text{H})$ | $+0.0214 \times 10^{-10}$ | $-0.0058 \times 10^{-10}$ |

Table 2
Comparison of the exact BBN results with a fixed-$\Delta N_{\text{eff}}$ approximation. From [4].
ΔN_{eff} is just produced by the above non thermal distortions. Fig. 2 shows the dependence on $\eta_{10}$ of $\Delta N_{\text{eff}}$. The hatched blue bands represent the experimental determination with $1 - \sigma$ statistical errors on $Y_p$, $^2H$, and $^7Li$, while the red band is the upper bound obtained in Ref. [17]. Note that for high value of $\eta_{10}$ all $^7Li$ comes from $^7Be$ radioactive decay via electron capture. The vertical green band represents WMAP 5-year result $\Omega_B h^2 = 0.02273 \pm 0.00062$ [18].

$\Delta N_{\text{eff}}$ is just produced by the above non thermal distortions. Fig. 2 shows the dependence on $\eta_{10} \equiv \eta \cdot 10^{10}$ of the final value of the primordial yields, calculated using the nucleosynthesis code PArthEnOPE [19], along with the experimental values of the abundances and their corresponding uncertainties, as discussed in details in Ref. [20]. Just to summarize, according to the analysis of [20] the inferred primordial abundances result to be

\begin{align*}
^2H/H &= (2.87^{+0.22}_{-0.21}) \times 10^{-5} , \\
^3He/H &< (1.1 \pm 0.2) \times 10^{-5} , \\
Y_p &= 0.247 \pm 0.002_{\text{stat}} \pm 0.004_{\text{syst}} ,
\end{align*}

To get confidence intervals for $\eta$, one construct a likelihood function

$$
\mathcal{L}(\eta) \propto \exp \left(-\chi^2(\eta)/2\right) ,
$$

with

$$
\chi^2(\eta) = \sum_{ij} [X_i(\eta) - X_i^{\text{obs}}] W_{ij}(\eta) [X_j(\eta) - X_j^{\text{obs}}] .
$$

The proportionality constant can be obtained by requiring normalization to unity, and $W_{ij}(\eta)$ denotes the inverse covariance matrix,

$$
W_{ij}(\eta) = [\sigma_{ij}^2 + \sigma_{i,\text{exp}}^2 \delta_{ij} + \sigma_{ij,\text{other}}^2]^{-1} ,
$$

where $\sigma_{ij}$ and $\sigma_{i,\text{exp}}$ represent the nuclear rate uncertainties and experimental uncertainties of nuclide abundance $X_i$, respectively (we use the nuclear rate uncertainties as in Ref. [21]), while by $\sigma_{ij,\text{other}}$ we denote the propagated squared error matrix due to all other input parameter uncertainties ($\tau_n$, $G_N$, etc.).

We first consider $^2H$ abundance alone, to illustrate the role of deuterium as an excellent baryometer. In this case the best fits found are $\Omega_B h^2 = 0.021 \pm 0.001 (\eta_{10} = 5.7 \pm 0.3)$ at 68% C.L., and $\Omega_B h^2 = 0.021 \pm 0.002$ at 95% C.L.. A similar analysis can be performed using $^4He$. In case only statistical error is considered we get $\Omega_B h^2 = 0.021^{+0.005}_{-0.004} (\eta_{10} = 5.7^{+1.4}_{-1.1})$ at 68% C.L., and $\Omega_B h^2 = 0.021^{+0.010}_{-0.006}$ at 95% C.L.. Fig. 3 shows the two likelihood profiles, which nicely agree. When accounting for

$$
\frac{^7Li}{H} = (1.86^{+1.30}_{-1.10}) \times 10^{-10} .
$$

Table 3

| $\Omega_B h^2$ | 0.017 | 0.019 | 0.021 | 0.023 |
|----------------|------|------|------|------|
| $^2H/H (10^{-5})$ | 4.00 | 3.36 | 2.87 | 2.48 |
| $^4He/H (10^{-5})$ | 1.22 | 1.14 | 1.07 | 1.01 |
| $^6Li/H (10^{-12})$ | 1.72 | 1.45 | 1.25 | 1.08 |
| $^7Li/H (10^{-10})$ | 2.53 | 3.22 | 3.99 | 4.83 |
| $^7Be/H (10^{-10})$ | 2.15 | 2.89 | 3.69 | 4.56 |

The theoretical values of the nuclear abundances for some value of $\Omega_B h^2$. 

\begin{align*}
^7Li/H &= 10^{-5} \times 10^{\eta_{10}} , \\
Y_p &= 0.247 \pm 0.002_{\text{stat}} \pm 0.004_{\text{syst}} ,
\end{align*}
Figure 3. Likelihood functions for $^2$H/H (narrow) and $Y_p$ (broad), using only the statistical error for the $^4$He measurement.

the largest possible systematic error on $Y_p$, the determination of $\Omega_B h^2$ becomes even more dominated by deuterium. In any case, the result is compatible at 2-$\sigma$ with WMAP 5-year result $\Omega_B h^2 = 0.02273 \pm 0.00062$ \cite{18}. The slight disagreement might have some impact on the determination from CMB anisotropies of the primordial scalar perturbation spectral index $n_s$, as noticed in \cite{22}, where the BBN determination of $\Omega_B h^2$ from deuterium is used as a prior in the analysis of the five year data of WMAP.

In Table\ref{table3} we report the values of some relevant abundances for three different baryon densities, evaluated using PArthENoPE \cite{19}. Notice the very low prediction for $^6$Li and that, for these values of baryon density, almost all $^7$Li is produced by $^7$Be via its eventual electron capture process.

If one relaxes the hypothesis of a standard number of relativistic degrees of freedom, it is possible to obtain bounds on the largest (or smallest) amount of radiation present at the BBN epoch, in the form of decoupled relativistic particles, or non standard features of active neutrinos. Fig. \ref{fig4} displays the contour plot of the total likelihood function, in the plane $(\Omega_B h^2, N_{\text{eff}})$, beautifully centered extremely close to the standard value $N_{\text{eff}} = 3.0$. After marginalization one gets $\Omega_B h^2 = 0.021 \pm 0.001$ and $N_{\text{eff}} = 2.97 \pm 0.14$ at 68% C.L., and $\Omega_B h^2 = 0.021 \pm 0.002$ and $N_{\text{eff}} = 2.97^{-0.29}_{+0.27}$ at 95% C.L.. Note that in this case we are using for $Y_p$ the statistical uncertainty only. More conservatively, if one also considers the systematic uncertainty on $Y_p$, the allowed range for $N_{\text{eff}}$ becomes broader, $N_{\text{eff}} = 3.0 \pm 0.3_{\text{stat}}(2\sigma) \pm 0.3_{\text{syst}}$.

Figure 4. Contours at 68 and 95 % C.L. of the total likelihood function for deuterium and $^4$He in the plane $(\Omega_B h^2, N_{\text{eff}})$. The bands show the 95% C.L. regions from deuterium (almost vertical) and Helium-4 (horizontal), neglecting possible systematic uncertainty on $Y_p$. We also show the 95 % C.L. allowed region from $Y_p$ with systematic error included (dashed lines).
3. Conclusions

The “classical parameter” constrained by BBN is the baryon to photon ratio, $\eta$, or equivalently the baryon abundance, $\Omega_B h^2$. At present, the constraint is dominated by the deuterium determination, and we find $\Omega_B h^2 = 0.021 \pm 0.001(1\sigma)$. This determination is consistent with the upper limit on primordial $^3\text{He}/\text{H}$ (which provides a lower limit to $\eta$), as well as with the range selected by $^4\text{He}$ determinations, which however ever neglecting systematic errors provides a constraint 5 times weaker. The agreement within 2σ with the WMAP determination, $\Omega_B h^2 = 0.02273 \pm 0.00062$, represents a remarkable success of the Standard Cosmological Model. On the other hand, using this value as an input, a factor $\sim 3$ discrepancy remains with $^7\text{Li}$ determinations, which can hardly be reconciled even accounting for a conservative error budget in both observations and nuclear inputs. Even more puzzling are some detections of traces of $^6\text{Li}$ at a level far above the one expected from Standard BBN. Both nuclides indicate that either their present observations do not reflect their primordial values, and should thus be discarded for cosmological purposes, or that the early cosmology is more complicated and exciting than the Standard BBN lore. Neither a non-standard number of massless degrees of freedom in the plasma (parameterized via $N_{\text{eff}}$) or a lepton asymmetry $\xi_e$ (all asymmetries assumed equal) can reconcile the discrepancy. Current bounds on both quantities come basically from the $^4\text{He}$ measurement, $N_{\text{eff}} = 3.0 \pm 0.3_{\text{stat}}(2\sigma) \pm 0.3_{\text{syst}}$ and $\xi_e = 0.004 \pm 0.017_{\text{stat}}(2\sigma) \pm 0.017_{\text{syst}}$.

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