Grand Unified Theories and Higgs Physics

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The grand unified theories are theoretically well motivated, but they typically have less direct indications on the low energy physics and it is not easy to test them. Here, we discuss a scenario of them which naturally solves the so-called doublet-triplet splitting problem and, at the same time, generally predicts characteristic collider phenomenology. Then, we may get a hint on the breaking of the grand unified symmetry at the on-going and next-generation collider experiments.

I. INTRODUCTION

Since a resonance consistent with the standard model (SM) Higgs field was reported [1], most people consider that the SM is now being confirmed, at least as an effective theory valid below the TeV scale. Then, the next question that we ask is what will come as the physics beyond the SM. The reason we ask it is that the SM still has some problems and puzzles, such as the hierarchy problem and the charge quantization problem. Here, we emphasize that the latter requires a tuning at least as fine as $10^{-10}$ to explain why the hydrogen atom is (almost) neutral. Thus, if we mind the former problem (as often happens), the latter also should be taken care. A simple solution to the latter is to extend the gauge group to a semi-simple one. Looking back the history of the physics, which is that of the unification, it is reasonable to take the idea of the grand unification [2] seriously.

Supposing the unification of the three forces of the SM, the matter fields are also to be unified. This unification works perfectly for the SM fermions: the five multiplets in each generation are unified into two in $SU(5)$ grand unified theories (GUTs). This is not trivial at all actually, as it becomes clear when people try to unify the electroweak $SU(2) \times U(1)$ symmetry into $SU(3)$. Thus, this success strongly supports the idea of the grand unification. It is also to be commented that the idea can easily accommodate other ideas to solve other SM problems: the supersymmetry (SUSY) for the hierarchy problem, the conventional seesaw mechanism [3] for the tiny neutrino masses and the Leptogenesis [4] for the baryon asymmetry of the universe. And if the $R$-parity is assumed, as usual in SUSY models, the candidate dark matter is supplied. In addition, in the minimal model, the three running gauge couplings become almost the same value at a superheavy scale, called the GUT scale: $2 \times 10^{16}$ GeV. This success of the gauge coupling unification (GCU) is so impressive that many people tend to believe the SUSY-GUTs. In this way, GUTs, especially SUSY-GUTs, potentially solve many of the problems/puzzles in the SM and give an amazing by-product.

On the other hand, the grand unification fails to unify the Yukawa interactions and the Higgs field. The former is insisted as a consequence of the fermion unification and is not necessarily bad for the third generations but not good for the lighter fermions. This issue is often called the wrong GUT relation and is to be taken care in model building, while it is relatively easy to solve (see for example Ref. [5]). The latter requires a $SU(5)$ partner of the SM doublet Higgs fields. The minimal choice is to introduce a color-triplet partner to embed them into the fundamental representation of the $SU(5)$ group. In SUSY models [6], the triplet partner generates effective dimension five operators that contribute to the nucleon decay [7]. In order to make the proton lifetime long enough without tuning, the triplet partner should be much heavier than the GUT scale. It is not an easy task to realize naturally the mass splitting between such a superheavy triplet and the weak scale doublet which originate from a common ($SU(5)$) multiplet. This rather severe issue is called the doublet-triplet (DT) splitting problem and is one of the biggest problem in the SUSY-GUTs. When we consider the grand unification seriously, these issues have to be dealt with.

Since the idea of the grand unification is so attractive, this problem has been attacked by many researchers for long time, and several solutions have been proposed [7–11]. They, however, all require some extension of the matter content, the grand unified gauge group and/or spacetime geometry. These extensions bring rather large ambiguity on the gauge couplings around the GUT scale due to the threshold corrections and so on. Then, what usually done are just to forget the ambiguity, to assume the corrections are aligned not to affect the GCU or, at most, to make models so that the GCU is kept. At this stage, the GCU is no longer a success but just a constraint in model building. Here, however, we would like to stress that the success of the GCU is just a by-product, and even without it the idea of the grand unification is attractive enough, as mentioned above.

Next, let us discuss indications of the grand unification on the low energy physics that we can detect. The most famous one is the nucleon decay. It is actually impressive prediction, but the information that we would get will be rather little and thus it would be nice if there are some characteristic predictions on the collider...
physics in addition. Unfortunately, since the GUT scale is so high, the decoupling theorem \[12\] makes it hopeless to detect the effects in most of the SUSY-GUTs.

Here, we would like to introduce a scenario of the SUSY-GUTs where the DT splitting problem is naturally solved and an extraordinary collider phenomenology is generally predicted \[13\]. Interestingly, this scenario, in a sense, can be regarded as an effective field theoretical description of the SUSY-GUTs embedded into the heterotic string theory \[14\] which could treat the quantum gravity and explain the numbers of our spacetime \[13\] and of the generations \[10\]. It is quite exiting if we can get some informations on the GUT breaking which might indicate the string theory at the on-going and next-generation collider experiments.

II. SUSY GRAND GAUGE-HIGGS UNIFICATION

In this section we review the scenario proposed in Ref. \[13\]. In the scenario, the Hosotani mechanism \[17\], which works in higher dimensional gauge theories, is applied to break the GUT symmetry \[18\]. In the Hosotani mechanism, the symmetry breaking occurs by the extradimensional component of the gauge field which is a higher dimensional vector field. Thus, in this model, the Higgs field is unified with the gauge field, and it is often called the gauge-Higgs unification especially when it is applied to the electroweak symmetry breaking. In the present scenario, it is applied to the grand unified symmetry breaking \[27\] and named as grand gauge-Higgs unification \[13\].

An important point is that, in this case, the order parameter is not the extradimensional component itself which is valued on the algebra, but the Wilson loop which is valued on the group and thus free from the traceless condition. Because of it, interestingly, a kind of the so-called missing VEV \[8\] can be realized and the DT splitting problem is naturally solved even in \(SU(5)\) models \[13\]. In this way, the application of the Hosotani mechanism to the GUT breaking in SUSY-GUTs looks attractive.

Naively thinking, the application to the GUT breaking seems reasonable since the Higgs field that is unified with the gauge field behaves as an adjoint field. Actually, at the first stage of the study of this mechanism, it was applied to the GUT breaking (or simpler toy model) \[17\]. Unfortunately, however, chiral fermions can not be accommodated in these models and thus these are phenomenologically less interesting. After the orbifold symmetry breaking \[11\] becomes famous among researchers who works on phenomenological model building, this mechanism have been applied mainly to the electroweak symmetry breaking \[20\]. This is because the orbifold symmetry breaking can extract fundamental-representational components (with respect to the remaining subgroup of the original gauge group) from the adjoint representation (with respect to the original group), besides chiral fermions from the higher-dimensional fermions. Furthermore, in such models, the Higgs field are free from the quadratically divergent radiative corrections to the mass term, thanks to the higher dimensional gauge symmetry \[21\]. In any case, now we know the orbifold symmetry breaking to realize chiral fermions, and thus it is interesting to examine the application to the GUT breaking. It might seem straightforward, but we immediately meet a difficulty. Namely, the adjoint scalar fields (with respect to the remaining gauge symmetry) originated from the extra-dimensional components tend to be projected out by the orbifold action when chiral fermions are realized.

This difficulty is shared with the heterotic string theory \[14\] and, fortunately, a method, called diagonal embedding method \[22\], to evade the difficulty is known. In Ref. \[18\], it is pointed out that the same method can be applied in a field theoretical setup and thus we have an advantage that it is much easier to calculate the quantum corrections that tell us the positions of vacua. By this, the symmetry breaking pattern is controlled by the dynamics described by the field theory irrelevantly to the ultraviolet theory, in contrast to the orbifold breaking where it is chosen by hand. In addition, as mentioned above, the DT splitting problem, when the SUSY version is considered, is naturally solved. Thus, this scenario is theoretically well motivated.

Interestingly, this scenario generically gives particular predictions also on the collider phenomenology. It is existence of light adjoint chiral multiplets with masses of the SUSY-breaking scale. The reason is as follows. The adjoint Higgs field is a part of the gauge field and thus massless at the tree level. Since the symmetry that ensures the masslessness is broken by the compactification, the adjoint field gets mass corrections via the quantum corrections. As the radiative corrections would be vanishing when the SUSY was not broken, the mass corrections are proportional to the SUSY breaking scale. In SUSY models, there are the SUSY partners of the adjoint scalar which would be degenerate with the scalar if the SUSY is exact and thus again has masses of at most the SUSY-breaking scale. Then, the whole the adjoint chiral multiplet is predicted to be light (if the SUSY-breaking scale is around the electroweak scale, as often expected) while the components of \(SU(5)/SU(3) \times SU(2) \times U(1)\) are eaten when the unified \(SU(5)\) gauge group is broken. Thus, color octet, weak triplet and singlet chiral multiplets \[28\] will appear in the effective theory below the compactification scale (which is assumed to be around the GUT scale), and they may be observed in the on-going and next-generation experiments.
collider experiments.

An immediate consequence of the adjoint chiral multiplets is that the GCU is disturbed. As mentioned above, however, the GCU should be treated just as a constraint instead of a success. It is easy to recover the GCU by adding further chiral multiplets. An example which is easily realized in this scenario and we consider here is two vectorlike pairs of $(1, 2)_{-1/2}$, one of $(3, 1)_{-2/3}$ and one of $(1, 1)_{1}$, with which the GCU is realized at the GUT scale and the unified gauge coupling is remains in the perturbative region: $\alpha_G \sim 0.3$.

Since the strong interaction is no longer asymptotically free (irrelevantly to the choice of the additional fields to recover the GCU), the QCD corrections are enhanced and thus the colored particles tend to be rather heavy in this scenario. Although it is also interesting study to examine the extraordinary pattern of the mass spectrum of the colored particles for the hadron colliders, here we concentrate on the colorless fields: the singlet and the triplet. These additional fields couples to the two Higgs doublets of the minimal SUSY SM (MSSM). These couplings push the SM-like Higgs mass by the tree level $F$-term contribution and thus the rather heavy Higgs mass around 125GeV can be easily realized. In addition, they cause mixing between the MSSM doublet Higgs fields and the adjoint fields which result in modification of the coupling of the SM-like Higgs fields \cite{24}. Such corrections may be measured at the linear collider. In the next section, we will discuss these issues in more detail.

## III. PHENOMENOLOGY

In order to examine the colorless sector, it is convenient to consider an effective theory where the Higgs sector of the MSSM is extended with the singlet $S$ and the triplet $\Delta$. The superpotential, in this case, is given as

$$W = \mu H_u H_d + \mu_{\Delta} \text{tr}(\Delta^2) + \frac{\mu_S}{2} S^2 + \lambda_{\Delta} H_u \Delta H_d + \lambda_S S H_u H_d,$$

where $H_u$ and $H_d$ are the MSSM doublet Higgs supermultiplets. Note that there are no self-couplings among $S$ and $\Delta$ although such couplings are allowed by the symmetry at the level of the effective theory. This is because $S$ and $\Delta$ originate from the gauge field. Furthermore, this fact also insists the two additional couplings $\lambda_{\Delta}$ and $\lambda_S$ to be related to the gauge couplings so that they are unified at the GUT scale (with appropriate group theoretical factors). Thus, this model is quite predictive (up to the dimensionful parameters).

For instance, taking the above example of the additional chiral multiplets to recover the GCU, the running of the gauge couplings are determined. The unified gauge coupling is used to fix the boundary values of $\lambda_{\Delta}$ and $\lambda_S$ at the GUT scale, and we get

$$\lambda_{\Delta} = 1.1, \quad \lambda_S = 0.26,$$

at the weak scale (within the 1-loop approximation). Using these predicted parameters, we can calculate the

![FIG. 1: An example of running couplings.](image)
IV. SUMMARY

In this article, we introduce the supersymmetric version \cite{13} of the grand gauge-Higgs unification scenario \cite{18}, where the grand unified gauge symmetry is broken by the Hosotani mechanism \cite{17}. Interestingly, in this scenario, the doublet-triplet splitting problem can be solved naturally even in $SU(5)$ models \cite{13}, thanks to the phase nature of the Hosotani mechanism. In addition, it generally predicts the existence of light adjoint chiral multiplets: the color octet, the weak triplet and the neutral singlet. Their mass is around the supersymmetry-breaking scale, which is often assumed to be the TeV scale, and thus there is a chance to detect them at the on-going and next-generation collider experiments.

Due to the color octet chiral multiplet, the QCD is no longer asymptotic free, and the QCD corrections are typically enhanced. This suggests that the additional colored particles become rather heavy. Thus, we concentrate on the colorless fields \cite{24}, though it is also an interesting work to examine the mass spectrum of the colored particles. Then, the effective theory of this scenario becomes the one with an extended Higgs sector: the neutral triplet and singlet are added. Since these are unified to the gauge field, they do not have self couplings and their couplings are related to the unified gauge coupling. This fact makes the model very predictive. For instance, we can calculate the SM-like Higgs mass, the charged Higgs mass in terms of the CP-odd Higgs mass, deviations of the SM-like Higgs couplings from the corresponding SM values, and so on, up to the ambiguity due to the dimensionful parameters. Although the details are referred to Refs. \cite{24, 25}, we emphasize that the linear collider is expected since the deviations are typically of order a few percents which are in its reach.

Acknowledgments

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[26] For the people who do not mind the fine tuning, the following is not a problem neither.
[27] Note that the Higgs field that is unified with the gauge field is an adjoint Higgs field, in this case, and the SM doublet Higgs field is introduced as a matter field.
[28] In this scenario, models generally have a $Z_2$ symmetry under which these adjoint multiplets change the sign \cite{13}. Thus, this scenario can give a theoretical background of (SU(3) inert models \cite{23}, though we consider a model where this $Z_2$ symmetry is broken.