A Universal Vertical Stellar Density Distribution Law for the Galaxy

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Abstract We reduced the observational logarithmic space densities in the vertical direction up to 8 kpc from the galactic plane, for stars with absolute magnitudes $(5,6]$, $(6,7]$ and $[5,10]$ in the fields #0952+5245 and SA114, to a single exponential density law. One of the three parameters in the quadratic expression of the density law corresponds to the local space density for stars with absolute magnitudes in question. There is no need of any definition for scaleheights or population types. We confirm with the arguments of nondiscrete thin and thick discs for our Galaxy and propose a single structure up to several kiloparsecs from the galactic plane. The logarithmic space densities evaluated by this law for the ELAIS field fit to the observational ones. Whereas, there are considerable offsets for the logarithmic space densities produced by two sets of classical galactic model parameters from the observational ones, for the same field.

Keywords Galaxy: structure – Galaxy: fundamental parameters – Galaxy: stellar content

1 Introduction

For some years, a disagreement exists among the researchers about the formation history of our Galaxy.

Yet there has been a large improvement about this topic since the pioneering work of (Eggen, Lynden-Bell & Sandage 1962) who argued that the Galaxy collapsed in a free-fall time ($\sim 2 \times 10^8$ yr). We know that the Galaxy collapsed over many Gyr (e.g. Yoshii & Saio 1979; Norris, Bessell, & Pickles 1985; Norris 1986; Sandage & Fouts 1987; Carney, Latham & Laird 1990; Norris & Ryan 1991; Beers & Sommer-Larsen 1995) and at least some of the components are formed from the merger or accretion of numerous fragments, such as dwarf type galaxies (cf. Sandage & Fouts 1987; Freeman & Bland-Hawthorn 1992 and references therein). Also, the number of population components of the Galaxy increased by one, complicating interpretations of any data set. The new component (the thick disc) was introduced by Gilmore & Reid (1983) in order to explain the observations that star counts towards the south galactic pole were not in agreement with a single-disc (thin-disc) component, but rather could be much better represented by two such components. This was the simplest combination of free parameters giving a satisfactory fit, and simplicity can play a major role in astrophysical fits. Different parameterization followed the work of Gilmore & Reid (1983). For example, Kuijken & Gilmore (1989) showed that the vertical structure of our Galaxy could be best explained by a multitude of quasi-isothermal components, i.e. a large number of sech$^2$ isothermal discs, together making up a more sharply peaked sech or exponential distribution. Also, we quote the work of de Grijs, Peletier & van der Kruit (1997) who made clear that in order to build up a sech distribution, one needs multiple components. Finally, we quote our work (Karaali 2006) where the galactic structure were parameterized by two exponentials up to $z \sim 10$ kpc, covering thin disc, thick disc and inner halo.

Although different parameterization were tried by many researchers, only the one of Gilmore & Wyse...
Disc structures are usually parameterized in cylindrical coordinates by radial and vertical exponentials,

\[ D_i(x, z) = n_i \exp(-z/H_i) \exp(-(x - R_o)/h_i), \]

(1)

where \( z \) is the distance from galactic plane, \( x \) is the planar distance from the galactic center, \( R_o \) is the solar distance to the galactic center (8 kpc), \( H_i \) and \( h_i \) are the scaleheight and scalelength respectively, and \( n_i \) is the normalized local space density. The suffix \( i \) takes the values 1 and 2, as long as thin and thick discs are considered. A similar form uses the sech² (or sech) function to parameterize the vertical distribution for thin disc,

\[ D_i(x, z) = n_i \text{sech}^2(z/H_i') \exp(-(x - R_o)/h_i). \]

(2)

Because the sech function is the sum of two exponentials, \( H_i' \) is not really a scaleheight, but it has to be compared with \( H_i \) by multiplying it with 2: \( H_i' = H_i/2 \) (van der Kruit & Searle 1981[a,b, 1982a,b; van der Kruit 1988]). However, in order to build up such a distribution, one needs multiple components (Kuijken & Gilmore 1989, de Grijs, Peletier & van der Kruit 1997).

The density law form for the spheroid (halo) component is parameterized in different forms. The most common is the de Vaucouleurs (1948) spheroid used to describe the surface brightness profile of elliptical galaxies. This law has been deprojected into three dimensions by Young (1976) as

\[ D_s(R) = n_s \exp[-7.669(R/R_e)^{1/4}]/(R/R_e)^{7/8}, \]

(3)

where \( R \) is the (uncorrected) galactocentric distance in spherical coordinates, \( R_e \) is the effective radius and \( n_s \) is the normalized local space density. \( R \) has to be corrected for axial ratio \( \kappa = c/a \),

\[ R = [x^2 + (z/\kappa)^2]^{1/2}, \]

(4)
where,

\[ x = \left[ R_o^2 + \left( \frac{z}{\tan b} \right)^2 - 2R_o \frac{z}{\tan b} \cos l \right]^{1/2}, \]  

(5)

where \( a_0 \) is the core radius.

If one restricts the work to the vertical direction, then the third factor in Eqs. (1) and (2) can be neglected.

3 A unified density law for the thin disc, thick disc and halo

As already mentioned in Section 1, the introduction of the thick disc component into the studies was in order to better representation of the observed star counts in the south galactic pole. However, the metallicities and kinematical data of thin and thick discs overlap. This is contradictory to the arguments related to two discrete components. What we mean, when we restrict our work to vertical direction by canonical approach, is the space density which decreases by amount equivalent to \( \exp(-1) \) at each distance adopted for the thin disc up to a \( z \)-distance from the galactic plane where the thin disc is dominant. Whereas where thick disc dominates, the space density decreases with smaller gradient at larger \( z \)-distances.

Thus, two components such as thin and thick discs were arbitrarily adopted just to fit the observational data, up to a few kpc. Then, we can approach in a similar way, i.e. we can introduce a density law which fits to the observational data up to a distance from the galactic plane, without defining any population type, however. Also, there are some indications that, a significant fraction of material with \( [Fe/H] < -1 \) dex has disc like structure \[ Norris 1984 \]. Hence, we expect a larger \( z \)-distance interval for the new density law we propose in the following.

Our new density law covers the canonical density law, for densities in the vertical direction and it contains no constant scaleheight. In the vertical direction, the density law form for thin and thick discs takes the form as,

\[ D(z) = n \exp\left(\frac{-z}{H}\right). \]  

(7)

which can also be written as,

\[ D(z) = \exp\left[-(a_1 z + a_0)\right]. \]  

(8)

where \( a_1 = 1/H \) and \( n = \exp(-a_0) \). The parameter \( a_0 \) is constant due to its definition. However, we let \( a_1 \) which is defined as the reciprocal of the scaleheight \( H \) in situ, to change with \( z \)-distance. Hence, we assume that the scaleheight changes continuously rather than as a step function. This is the main difference between the arguments of our work and the ones in situ.

When we examine Eq. (8), we see that the space density in vertical direction changes exponentially as a linear function of \( z \). We consider a quadratic function of \( z \) could be better matched to observational data. Hence, we obtained the final density law by adding a quadratic term to Eq. (8):

\[ D(z) = \exp\left[-(a_2 z^2 + a_1 z + a_0)\right]. \]  

(9)

For the estimation of the coefficients \( a_i \) (i=0,1,2), the following procedure was adopted. Eq. (9) is written in logarithmic form:

\[ a_2 z^2 + a_1 z + a_0 = - \ln D(z). \]  

(10)

Here, we used Hipparcos’ local space density \[ Jahreiss & Wielen 1997 \] for a specific absolute magnitude interval \( M_1 - M_2 \) and a sequence of observed \( D(z) \) space densities and estimated the corresponding coefficients.

4 Estimation of the parameters in the new density law

The Eq. (10) applied to the data taken from \[ Karaali et al. 2004 \]. The first set were evaluated for stars with absolute magnitude intervals (5,6] and (6,7], for the field #0952+5245 (Table 1), whereas the second set covers the space densities for a unique absolute magnitude interval \( M_1 - M_2 \) and a sequence of observed \( D(z) \) space densities and estimated the corresponding coefficients.
Table 3  Distance-to the galactic plane- dependant $a_i$ ($i=0, 1, 2$) parameters for different samples of absolute magnitude intervals (5,6] and (6,7] for the field #0952+5245.

| $M_g$ $\rightarrow$ | (5,6] | $M_g$ $\rightarrow$ | (6,7] |
|---------------------|-------|---------------------|-------|
| Sample No | $< z^* >$ | $z^*$ | $D^*$ | Sample No | $< z^* >$ | $z^*$ | $D^*$ |
| 1 | 1.53 | 0.00 | 7.47 | $a_2 = -0.3705(0.0034)$ | 1 | 1.19 | 0.00 | 7.47 | $a_2 = -0.6180(0.0026)$ |
| | | 0.86 | 6.47 | $a_1 = 2.9923(0.0077)$ | | | 0.61 | 6.62 | $a_1 = 3.5886(0.0050)$ |
| | | 1.48 | 5.90 | $a_0 = 5.8262(0.0034)$ | | | 1.10 | 6.08 | $a_0 = 5.8251(0.0020)$ |
| | | 2.25 | 5.36 | | | | 1.86 | 5.50 |
| 2 | 2.24 | 0.00 | 7.47 | $a_2 = -0.3095(0.0305)$ | 2 | 1.85 | 0.00 | 7.47 | $a_2 = -0.4710(0.0761)$ |
| | | 1.48 | 5.90 | $a_1 = 2.8734(0.0922)$ | | | 1.10 | 6.08 | $a_1 = 3.3561(0.2026)$ |
| | | 2.25 | 5.36 | $a_0 = 5.8315(0.0601)$ | | | 1.86 | 5.50 | $a_0 = 5.8338(0.1136)$ |
| | | 2.99 | 4.93 | | | | 2.59 | 5.05 |
| 3 | 2.99 | 0.00 | 7.47 | $a_2 = -0.2546(0.0105)$ | 3 | 2.58 | 0.00 | 7.47 | $a_2 = -0.3040(0.0453)$ |
| | | 2.25 | 5.36 | $a_1 = 2.7246(0.0384)$ | | | 1.86 | 5.50 | $a_1 = 2.2685(0.1502)$ |
| | | 2.99 | 4.93 | $a_0 = 5.8271(0.0296)$ | | | 2.59 | 5.05 | $a_0 = 5.8329(0.1076)$ |
| | | 3.73 | 4.59 | | | | 3.34 | 4.62 |
| 4 | 3.79 | 0.00 | 7.47 | $a_2 = -0.2183(0.0099)$ | 4 | 3.34 | 0.00 | 7.47 | $a_2 = -0.2062(0.0222)$ |
| | | 2.99 | 4.93 | $a_1 = 2.6006(0.0447)$ | | | 2.59 | 5.05 | $a_1 = 2.6699(0.0886)$ |
| | | 3.73 | 4.59 | $a_0 = 5.8272(0.0419)$ | | | 3.34 | 4.62 | $a_0 = 5.8286(0.0726)$ |
| | | 4.66 | 4.26 | | | | 4.10 | 4.21 |
| 5 | 4.85 | 0.00 | 7.47 | $a_2 = -0.1461(0.0213)$ | | | | |
| | | 3.73 | 4.59 | $a_1 = 2.2933(0.1288)$ | | | | |
| | | 4.66 | 4.26 | $a_0 = 5.8336(0.1704)$ | | | | |
| | | 6.17 | 3.72 | | | | | |

Table 1  Distance to the galactic plane ($z^*$ in kpc) and the logarithmic space density ($D^*$) data for two absolute magnitude intervals for the field #0952+5245 ($l = 83.38\,^\circ, b = 48.35\,^\circ$).

| $M_g$ $\rightarrow$ | (5,6] | (6,7] |
|---------------------|-------|-------|
| ID | $< z^* >$ | $z^*$ | $D^*$ | $< z^* >$ | $z^*$ | $D^*$ |
| 1 | 0.00 | 7.47 | 0.00 | 7.47 |
| 2 | 0.86 | 6.47 | 0.61 | 6.62 |
| 3 | 1.48 | 5.90 | 1.10 | 6.08 |
| 4 | 2.25 | 5.36 | 1.86 | 5.50 |
| 5 | 2.99 | 4.93 | 2.59 | 5.05 |
| 6 | 3.73 | 4.59 | 3.34 | 4.62 |
| 7 | 4.66 | 4.26 | 4.10 | 4.21 |
| 8 | 6.17 | 3.72 | | |

Table 2  Distance to the galactic plane ($z^*$ in kpc) and logarithmic space density, $D^* = \log D+10$, per unit absolute magnitude interval for stars with $5 < M_g \leq 10$ for the field SA 114 ($l = 68.38\,^\circ, b = -48.38\,^\circ$).

| ID | $< z^* >$ | $z^*$ | $D^*$ |
|-----|-------|-------|-------|
| 1 | 0.00 | 7.52 |
| 2 | 0.41 | 6.90 |
| 3 | 0.63 | 6.62 |
| 4 | 0.93 | 6.24 |
| 5 | 1.30 | 5.91 |
| 6 | 1.68 | 5.62 |
| 7 | 2.06 | 5.42 |
| 8 | 2.60 | 5.10 |
| 9 | 3.36 | 4.71 |
| 10 | 4.66 | 4.29 |
| 11 | 6.46 | 3.81 |
| 12 | 8.28 | 3.37 |
| 13 | 10.21 | 3.11 |
### Table 4: Distance-to the galactic plane-dependent $a_i$ ($i=0, 1, 2$) parameters for different samples of absolute magnitude interval (5,10] for the field SA 114.

| Sample No | $<z^*>$ | $z^*$ | $D^*$   | Sample No | $<z^*>$ | $z^*$ | $D^*$   |
|-----------|---------|-------|---------|-----------|---------|-------|---------|
| 1         | 0.99    | 0.00  | 7.52    | 2         | 0.6939(0.0522) | 2.87 | 0.00  | 7.52    |
|           | 0.41    | 6.90  | 3.7701(0.0928) | 1         | 2.9160(0.1492) | 1.68 | 5.62  |
|           | 0.63    | 6.62  | 5.7082(0.0344) | 2         | 5.7740(0.1725) | 2.06 | 5.42  |
|           | 0.93    | 6.24  | 5.7064(0.0354) |           | 2.60    | 5.10  |
|           | 1.30    | 5.91  | 3.36    |           | 3.36    | 4.71  |
|           | 1.68    | 5.62  | 4.66    |           | 4.66    | 4.29  |
| 2         | 1.32    | 0.00  | 7.52    | 4         | 0.6832(0.0342) | 3.83 | 0.00  | 7.52    |
|           | 0.63    | 6.62  | 3.7569(0.0742) | 3         | 2.5807(0.1739) | 2.06 | 5.42  |
|           | 0.93    | 6.24  | 5.7072(0.0354) |           | 2.60    | 5.10  |
|           | 1.30    | 5.91  | 3.36    |           | 3.36    | 4.71  |
|           | 1.68    | 5.62  | 4.66    |           | 4.66    | 4.29  |
|           | 2.06    | 5.42  | 6.46    |           | 6.46    | 3.81  |
| 3         | 1.71    | 0.00  | 7.52    | 5         | 0.5462(0.0633) | 5.07 | 0.00  | 7.52    |
|           | 0.93    | 6.24  | 3.5190(0.1720) | 4         | 2.2799(0.2196) | 2.60 | 5.10  |
|           | 1.30    | 5.91  | 5.7495(0.1085) |           | 3.36    | 4.71  |
|           | 1.68    | 5.62  | 4.66    |           | 4.66    | 4.29  |
|           | 2.06    | 5.42  | 6.46    |           | 6.46    | 3.81  |
|           | 2.60    | 5.10  | 8.28    |           | 8.28    | 3.37  |
| 4         | 2.20    | 0.00  | 7.52    | 6         | 0.3891(0.0536) | 6.59 | 0.00  | 7.52    |
|           | 1.30    | 5.91  | 3.1897(0.1889) | 5         | 2.0541(0.1901) | 3.36 | 4.71  |
|           | 1.68    | 5.62  | 5.7616(0.1586) |           | 4.66    | 4.29  |
|           | 2.06    | 5.42  | 6.46    |           | 6.46    | 3.81  |
|           | 2.60    | 5.10  | 8.28    |           | 8.28    | 3.37  |
|           | 3.36    | 4.71  | 3.36    |           | 3.36    | 4.71  |
|           |         |       |         | 10.21    | 3.11    |       |         |
4.1 Estimation of the parameters for the absolute magnitude interval (5,6)

We separated the eight \((z^*, D^*)\) couples for the absolute magnitude interval \((5,6)\) in Table 1 into five samples, i.e. \((1,2,3,4)\); \((1,3,4,5)\); \((1,4,5,6)\); \((1,5,6,7)\) and \((1,6,7,8)\), each of which involves the couple \((0,7,47)\) and estimated the \(a_i\) \((i=0,1,2)\) parameters in Eq. (10) by the least square method (Table 3). Since \(a_0\) corresponds to the local space density of the stars with absolute magnitude \((5,6)\), its value must be constant. Hence, we re-produced it for each sample and we found that the numerical value of the parameter \(a_0\) is the same for each sample and equals to the Hipparcos’ local space density, for the same absolute magnitude interval \((5,6)\). This result confirms suitability of our procedure we followed for the estimation of the \(a_1\) parameters.

However, the trends of \(a_1\) and \(a_2\) are different than \(a_0\) and from each other. For instance, \(a_1\) decreases with increasing \(z^*\) to the galactic plane, whereas \(a_2\) increases in the same direction. This result also confirms the argument that the scaleheight of a population can not be adopted as a constant, but it should be (distance to galactic plane) dependent (Karaali 2006). If \(a_2\) is omitted in Eq. (10), the reciprocal of \(a_1\) corresponds to scaleheight in Eq. (8) and increases with distance to galactic plane.

4.2 Estimation of the parameters for the absolute magnitude interval (6,7)

The seven \((z^*, D^*)\) couples for the absolute magnitude interval \((6,7)\) in Table 1 were separated into four samples and the corresponding \(a_i\) \((i=0,1,2)\) parameters were estimated by the same procedure as was done and explained for the absolute magnitude interval \((5,6)\). Also, these results are given in Table 3. There, \(a_0\) is found to be constant and equal the Hipparcos local space density, for the same absolute magnitude interval \((6,7)\), \(a_0 = 7.47\). On the other hand, the trends of \(a_1\) and \(a_2\) are the same as the corresponding ones for the absolute magnitude interval \((5,6)\).

4.3 Estimation of the parameters for stars with absolute magnitudes \((5,10)\)

The number of \((z^*, D^*)\) couples in Table 2 are 13 in total and more than in Table 1. We used this advantage to increase the sample numbers and the couple numbers in each sample. Thus, we separated the \((z^*, D^*)\) couples into eight samples, i.e. \((1, 2, 3, 4, 5, 6)\); \((1, 3, 4, 5, 6, 7)\); \((1, 4, 5, 6, 7, 8)\); \((1, 5, 6, 7, 8, 9)\); \((1, 6, 7, 8, 9, 10)\); \((1, 7, 8, 9, 10, 11)\); \((1, 8, 9, 10, 11, 12)\) and \((1, 9, 10, 11, 12, 13)\), each of which involves the couple \((0,7,52)\) and estimated the \(a_i\) \((i=0,1,2)\) parameters in Eq. (10) by the procedure explained in section 4.1. The results are given in Table 4.

Although the trends of \(a_2\) and \(a_1\) are the same as the trends of \(a_2\) and \(a_1\) for the data given in the previous sections, the value of \(a_0\) is not constant, but it corresponds to a local space density within a range \(7.43 \leq D^*(0) \leq 7.52\). The lower values correspond to the local space densities of brighter stars in the Hipparcos’ catalogue. That is, the estimated local space density deviates from the mean local space density \(< D^* >= 7.52\) of stars with \((5,10)\) as one goes to large vertical distances. This discrepancy is due to a bias effect, i.e. the galactic model parameters varies with distance (Bilir et al. 2006; Ak et al. 2007; Cabrera-Lavers et al. 2007; Karaali et al. 2007; Bilir et al. 2008).

5 Testing the unified density law

We tested the unified density law by the \((z^*, D^*)\) data for the absolute magnitude intervals \((5,6)\), \((6,7)\) and \((5,10)\) for the ELAIS field making use of the following procedure. A mean \(< z^* >\) distance were attributed to each sample in Table 3 and Table 4. Then, \(a_i\) \((i=0,1,2)\) parameters were evaluated for the given \(z^*\) distance by interpolation/extrapolation of the corresponding \(a_i\) \((i=0,1,2)\) in Table 3 and Table 4. Finally, the space densities were evaluated according to Eq. (10). The results are presented in Table 5, Table 6 and Table7, for the absolute magnitude intervals \((5,6)\), \((6,7)\) and \((5,10)\), respectively. For clarification, we state that the numbers in the sixth column are the sample numbers in Table 3 or Table 4, the \(z^*\) distances in seventh column are the corresponding mean \(z^*\) distances for these samples, \(a_i\) \((i=0,1,2)\) are the interpolated/extrapolated parameters for the \(z^*\) on the same line, \(D^*\) is the original logarithmic space density taken from [Bilir, Karaali & Gilmore (2006, hereafter, BKG), and \(D^*_w\), is the logarithmic space density evaluated by the interpolated/ extrapolated \(a_i\) \((i=0,1,2)\) parameters. The \(\Delta D^*\) offsets of the evaluated logarithmic space densities from the original ones in the last columns in three tables mentioned above are rather small, confirming that the procedure could be applied efficiently.

We reproduced the \(D^*\) logarithmic space densities for the \(z^*\) distances given in Tables 5-7, for stars with absolute magnitudes \((5,6)\), \((6,7)\) and \((5,10)\), making use of the calibrations of Phleps et al. (2000) and Jurić et al. (2008), and we compared the \(\Delta D^*\) offsets of the evaluated logarithmic space densities from the original ones with the corresponding offsets obtained.
by means of the unified density law. It is interesting to compare the results of [Phleps et al. (2000)] with the results we obtained. Since they estimated galactic model parameters in the vertical direction of the Galaxy, as was done in the present study. The galactic model parameters of [Phleps et al. (2000)] are as follows: Normalized local space densities for thin and thick discs \( n_1 = 2.725 \times 10^{-3}, n_2 = 2.229 \times 10^{-4}, \) scaleheights for thin and thick discs \( H_1 = 280 \) pc, \( H_2 = 1267 \) pc, respectively. [Jurić et al. (2008)] estimated the following galactic model parameters both in vertical and longitudinal directions and they found for normalized local space densities \( n_1 = 2.951 \times 10^{-3}, n_2 = 3.160 \times 10^{-4}, \) \( H_1 = 300 \) pc, \( H_2 = 900 \) pc, \( (c/a)=0.64. \) The evaluated logarithmic space densities, \( D_{ev}, \) and their offsets from the original ones taken from BKG are given in Tables 8, 9 and 10. The offsets were plotted in Fig. 1 and compared with the ones resulted by using the unified density law. One can see that there is a systematic deviation in the dispersions of the offsets evaluated via the calibrations of [Phleps et al. (2000)] and [Jurić et al. (2008)], favouring the unified density law.

### 6 Discussion

We used a unified density law for the thin disc, thick disc and halo to match the observational logarithmic space densities evaluated for the fields #0952+5245 and SA 114 in the vertical direction up to 8 kpc from the galactic plane. The exponent of the density law is adopted as a quadratic function of the distance from the galactic plane. The absolute magnitude dependent parameters estimated for a set of distance from the galactic plane, were interpolated/extrapolated for three given sets of \( z \)-distances and applied to the ELAIS field. The offsets of the logarithmic space densities evaluated

### Table 5 Space densities evaluated by the unified density law for stars with absolute magnitudes (5,6) in the ELAIS field. The columns give: \( (z^* \) distance from the galactic plane in kpc, \( (2) \) the logarithmic space density \( D^* \) taken from BKG, \( (3), (4) \) and \( (5) \): \( a_2, a_1 \) and \( a_0 \) parameters interpolated for the \( z^* \) distance on the same line, \( (6) \) no of samples used for interpolation of \( a_i \) \( (i=1, 2, 3), \) \( (7) \) mean \( z^* \) distances for the samples used for the interpolation, \( (8) \) evaluated logarithmic space density \( D^*_{ev}, \) and \( (9) \) the difference between the evaluated logarithmic space density and the adopted one from BKG.

| \( z^* \) | \( D^* \) | \( D^*_{ev} \) | \( \Delta D^* \) | \( \Delta D^* \) | \( \Delta D^* \) |
|----------|----------|--------|--------|--------|--------|
| 1.24     | 5.94     | 6.08   | -0.14  | 6.09   | -0.15  |
| 1.58     | 5.63     | 5.88   | -0.25  | 5.84   | -0.21  |
| 1.94     | 5.31     | 5.73   | -0.42  | 5.62   | -0.31  |
| 2.45     | 5.11     | 5.54   | -0.43  | 5.33   | -0.22  |
| 3.18     | 4.73     | 5.30   | -0.57  | 4.93   | -0.20  |

### Table 6 Space densities evaluated by the unified density law for stars with absolute magnitudes (6,7) in the ELAIS field. The symbols are as in Table 5.

| \( z^* \) | \( D^* \) | \( D^*_{ev} \) | \( \Delta D^* \) | \( \Delta D^* \) | \( \Delta D^* \) |
|----------|----------|--------|--------|--------|--------|
| 1.24     | 5.94     | 6.08   | -0.14  | 6.09   | -0.15  |
| 1.58     | 5.63     | 5.88   | -0.25  | 5.84   | -0.21  |
| 1.94     | 5.31     | 5.73   | -0.42  | 5.62   | -0.31  |
| 2.45     | 5.11     | 5.54   | -0.43  | 5.33   | -0.22  |
| 3.18     | 4.73     | 5.30   | -0.57  | 4.93   | -0.20  |

### Table 7 Space densities evaluated by the unified density law for thin disc, thick disc and halo components of the Galaxy for stars with absolute magnitudes (5,10) in the ELAIS field. The symbols are as in Table 5.

| \( z^* \) | \( D^* \) | \( D^*_{ev} \) | \( \Delta D^* \) | \( \Delta D^* \) | \( \Delta D^* \) |
|----------|----------|--------|--------|--------|--------|
| 1.24     | 5.94     | 6.08   | -0.14  | 6.09   | -0.15  |
| 1.58     | 5.63     | 5.88   | -0.25  | 5.84   | -0.21  |
| 1.94     | 5.31     | 5.73   | -0.42  | 5.62   | -0.31  |
| 2.45     | 5.11     | 5.54   | -0.43  | 5.33   | -0.22  |
| 3.18     | 4.73     | 5.30   | -0.57  | 4.93   | -0.20  |

### Table 8 Comparison of the original logarithmic space densities taken from BKG \( D^* \) and the evaluated ones \( D^*_{ev} \) by means of the Galactic model parameters of [Phleps et al. (2000)] (columns 3 and 4) and [Jurić et al. (2008)] (columns 5 and 6) for the absolute magnitude interval (5,6).

| \( z^* \) | \( D^* \) | \( D^*_{ev} \) | \( \Delta D^* \) | \( \Delta D^* \) | \( \Delta D^* \) |
|----------|----------|--------|--------|--------|--------|
| 1.60     | 5.66     | 5.87   | -0.21  | 5.83   | -0.17  |
| 1.93     | 5.51     | 5.73   | -0.22  | 5.62   | -0.11  |
| 2.46     | 5.17     | 5.54   | -0.37  | 5.33   | -0.16  |
| 3.15     | 4.77     | 5.31   | -0.54  | 4.98   | -0.21  |
| 4.35     | 4.17     | 4.92   | -0.75  | 4.40   | -0.23  |
| 6.15     | 3.80     | 4.34   | -0.54  | 3.60   | +0.20  |

### Table 9 Comparison of the original logarithmic space densities taken from BKG \( (D^* \) and the evaluated ones \( (D^*_{ev}) \) by means of the Galactic model parameters of [Phleps et al. (2000)] (columns 3 and 4) and [Jurić et al. (2008)] (columns 5 and 6) for the absolute magnitude interval (6,7).

| \( z^* \) | \( D^* \) | \( D^*_{ev} \) | \( \Delta D^* \) | \( \Delta D^* \) | \( \Delta D^* \) |
|----------|----------|--------|--------|--------|--------|
| 1.24     | 5.94     | 6.08   | -0.14  | 6.09   | -0.15  |
| 1.58     | 5.63     | 5.88   | -0.25  | 5.84   | -0.21  |
| 1.94     | 5.31     | 5.73   | -0.42  | 5.62   | -0.31  |
| 2.45     | 5.11     | 5.54   | -0.43  | 5.33   | -0.22  |
| 3.18     | 4.73     | 5.30   | -0.57  | 4.93   | -0.20  |
Table 10 Comparison of the original logarithmic space densities taken from BKG ($D^*$) and the evaluated ones ($D^*_\text{ev}$) by means of the galactic model parameters of Phleps et al. (2000) (columns 3 and 4) and Jurić et al. (2008) (columns 5 and 6) for the absolute magnitude interval (5,10].

| $z^*$ | $D^*$ | $D^*_\text{ev}$ | $\Delta D^*$ | $D^*_\text{ev}$ | $\Delta D^*$ |
|-------|-------|-----------------|--------------|----------------|--------------|
| 0.92  | 6.20  | 6.32            | -0.12        | 6.41           | -0.21        |
| 1.26  | 5.86  | 6.05            | -0.19        | 6.09           | -0.23        |
| 1.61  | 5.55  | 5.85            | -0.30        | 5.83           | -0.28        |
| 1.96  | 5.35  | 5.70            | -0.35        | 5.61           | -0.26        |
| 2.52  | 4.95  | 5.49            | -0.54        | 5.30           | -0.35        |
| 3.22  | 4.59  | 5.25            | -0.66        | 4.94           | -0.35        |
| 4.58  | 4.03  | 4.78            | -0.75        | 4.28           | -0.25        |
| 6.30  | 3.56  | 4.19            | -0.63        | 3.55           | +0.01        |
| 8.04  | 3.15  | 3.59            | -0.44        | 3.02           | +0.13        |

The so-called “unified density law” does not define any population type, up to distances of 8 kpc from the galactic plane. Hence, the space density of the Galaxy could be matched to a unified density law with three parameters. This approach would imply and support the argument of existence of a single disc. Hence, the thin and thick discs discussed for a long time in the literature are not discrete. However, we reiterate that this is not our idea. But we confirmed their arguments (cf. Norris 1987; Ivezić et al. 2008). A significant fraction of material with $[Fe/H] < -1$ dex has disclike structure (Norris 1987). This confirms our argument related to the unified density law.

In the present work, we proposed galactic model parameters as a function of distance from the galactic plane, as was done in the study of Karaali (2006). However, there is a difference between, what was proposed over there and here. Namely, in the former study, scaleheight and scalelength were taken into consideration, whereas in the present study, none of these parameters are taken into consideration. If we omit the coefficient of $z^2$, for instance $a_2$, reciprocal of the coefficient of $z$, $1/a_1$ corresponds to scaleheight for the total density, but not for a specific population.

7 Conclusion

Norris (1987) proposed a Galaxy model where he did not assume thin and thick discs are discrete components. Rather, he assumed that the thin and thick discs form a kinematical and chemical continuum. Ivezić et al. (2008) confirmed this hypothesis by demonstrating the absence of a correlation between the observed velocity and metallicity distributions for disc stars. The present work, reinforces the same argument, making use of different procedure. The density law proposed here can match to the observed vertical space densities up to several kpc without the need to separate the Galactic stars into separate population types.

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