Chen Software Reliability Growth Model

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Abstract
Software reliability analysis is very vital in software development. Software manufacturers assess the quality of their developed software through this analysis. This has triggered the development of reliability models. Software reliability growth models have been used extensively to examine the quality of manufactured software before they are sent to the market. This study presents a new software reliability growth model using Chen distribution. The Chen software reliability growth model is then used to establish sequential probability ratio test limits for determining whether a manufactured software is reliable or unreliable. The applications of the proposed model revealed that it performs better than some of the existing software reliability growth models for the given datasets.

Keywords: Software reliability; growth model; Chen; SPRT; quality control.

Abbreviations
- NHPP Non-Homogeneous Poisson Process
- SRGM Software Reliability Growth Model
- G-O Goel Okumoto

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1 Introduction

Quality control in the production of goods and services come up handy on regular bases due to the fact that the world is evolving into a global village [1]. More producers have products and goods they need to send to the market with each passing day. Thus, when customers realize these products are not of high quality, demand of products decrease [2]. Quality control is one of the most important ways of examining quality for products before they are sent to the market [3]. Products come in the form of tangible and intangible goods [4]. Ranging from tangible products such as clothing, food, building materials and so on, to intangible products such as software [5]. Software are a set of instructions that teach a computer what to do [6]. In today’s economy, software quality is becoming increasingly critical. This is due to the fact that the advantages of its usage outweighs the detrimental aspect of it [7]. Spanning from performing basic mathematics to performing complex computations, software make living easier. According to ANSI Standard (ANSI/ASQC A3/1978): "Quality is the totality of features and characteristics of a product or a service that thrives on its ability to satisfy the given needs". Users of software are aware of software as a tool for assisting them in their endeavors whiles producers see software as a means yielding profits [8]. Software reliability growth models (SRGM) are implementations of statistical methods as a quality control technique, in verifying the quality and durability of a software. These models are able to model various forms of failure data, by collecting information of the behavior of software over a period. It stems from employing techniques such as the Non-homogeneous Poisson Process (NHPP), the Sequential Probability Ratio testing (SPRT) and so on. During the production of software, the monitoring process is the phase where the performance of the software is tracked. The SPRT provides a rule for stopping or continuing the monitoring process of a software during production [9]. It gives a general perspective of whether a software should be deployed to the market or not. This is done by comparing the limits of the SPRT against cumulative number of failures at a given time (this could be number of hours, days, weeks, months or years). This study seeks to propose a Chen SRGM that incorporates environmental factors and the limits of its SPRT. The remainder of the paper is organized as follows: Section two presents the SPRT for the NHPP, Section three presents the proposed Chen SRGM, Section four presents the application of the proposed model and lastly Section five presents the conclusion of the study.

2 Sequential Probability Ratio Test for NHPP

Sequential sampling is a sampling technique where items are drawn one at a time and the consequent result at every stage determines if sampling or testing should stop. Hence, any sampling procedure where the number of observations is a random variable can be regarded as sequential sampling. The term sequential test has its name of origin from the fact that the sample size is not determined in advance but allowed to "float" with a decision as to whether it should be accepted, rejected, or the test must continue after each trial or data point [10].

SPRT is a ratio test method that follows the sequential sampling process. The SPRT is similar to the Maximum Likelihood Estimation used for constructing tests. It is formulated by taking the ratio of the sample densities under $H_1$ over $H_0$. Consider the NHPP $\{N(t), t \in [0, \infty)\}$. Suppose we are interested in testing the hypothesis in terms of the mean value function $m(t)$:
Suppose \( X \) and \( Y \) are two positive constants where \( X > Y \). Then at every stage of the SPRT, the likelihood ratio (LR), \( LR = \frac{P_0}{P_1} \) is estimated. Thus, based on the value of the likelihood ratio, the following decisions are arrived at:

1. Stop sampling and reject \( H_0 \) as soon as \( LR \geq X \).
2. Stop sampling and accept \( H_0 \) as soon as \( LR \leq Y \).
3. Continue sampling as long as \( Y < LR < X \).

The choice of selection of the estimates of \( X \) and \( Y \) are chosen such that the SPRT has the needed strength \((\alpha, \beta)\). Thus, with the above test, suggested by [11] \( X \) and \( Y \) can be expressed as follows: \( Y = \frac{1-\beta}{\alpha} \) and \( X = \frac{1-\beta}{\alpha} \). The basis for \( \alpha \) and \( \beta \) respectively are, therefore,

\[
P[LR > X | H_0] = \alpha \tag{1.3}
\]

and

\[
P[LR < X | H_0] = \beta, \tag{1.4}
\]

where \( \alpha \) and \( \beta \) are the type I and type II errors respectively. To make computation more simplified the logarithm form of the likelihood ratio is used often, that is \( \log Y < \log \frac{P_0}{P_1} < \log X \). Hence, using the SPRT in reliability analysis of software that follows the NHPP, the following decisions may be arrived at: the given software is reliable, the software is unreliable or the testing process should be continued by adding more observations in the failure data.

The likelihood ratio \( \frac{P_0}{P_1} \) at a given time \( t \) is considered as an estimate for a decision to take with regards to \( m_0(t) \) or \( m_1(t) \), for a sequence of time \( t_1 < t_2 \cdots < t_n \) with their corresponding realizations \( N(t_1), N(t_2), \ldots, N(t_n) \). By using the likelihood ratio, a software failure data which follows a NHPP is reliable if,

\[
\frac{P_0}{P_1} = \frac{e^{-m_0(t)}[m_1(t)]^{N(t)}}{e^{-m_0(t)}[m_0(t)]^{N(t)}} \leq Y. \tag{1.5}
\]

Implying that,

\[
N(t) \leq \frac{\log(\frac{e^{-m_0(t)}}{e^{-m_0(t)}} + m_1(t) - m_0(t))}{\log\left(\frac{m_1(t)}{m_0(t)}\right)}. \tag{1.6}
\]

A software failure data which follows a NHPP is unreliable if,

\[
\frac{P_0}{P_1} = \frac{e^{-m_1(t)}[m_1(t)]^{N(t)}}{e^{-m_0(t)}[m_0(t)]^{N(t)}} \geq X. \tag{1.7}
\]

Implying that,

\[
N(t) \geq \frac{\log(\frac{e^{-m_0(t)}}{e^{-m_0(t)}} + m_1(t) - m_0(t))}{\log\left(\frac{m_1(t)}{m_0(t)}\right)}. \tag{1.8}
\]

The testing process is continued as long as,

\[
\frac{\log(\frac{e^{-m_0(t)}}{e^{-m_0(t)}} + m_1(t) - m_0(t))}{\log\left(\frac{m_1(t)}{m_0(t)}\right)} < N < \frac{\log(\frac{e^{-m_0(t)}}{e^{-m_0(t)}} + m_1(t) - m_0(t))}{\log\left(\frac{m_1(t)}{m_0(t)}\right)}. \tag{1.9}
\]
3 The Proposed Chen SRGM

Software developers often times ignore the effects environmental conditions could have on their software. When the location of the software is changed from its original environment, this could result in a type I or II error [12].

[13] proposed a generalized NHPP incorporating the uncertainty of environmental conditions with all existing assumptions of the NHPP in place and the following assumptions;

i. The occurrence of software failures follows an NHPP.

ii. Software can fail during execution, caused by faults in the software.

iii. The software failure detection rate at any time is proportional to the number of remaining faults in the software at that time.

iv. When a software failure occurs, a debugging effort removes the faults immediately.

v. For each debugging effort, regardless of whether the faults are successfully removed, some new faults may be introduced into the software system.

vi. The environment affects the unit failure detection rate, \( c(t) \), by multiplicative factor \( \omega \).

Thus a generalized NHPP SRGM can be obtained as,

\[
\frac{dm(t)}{dt} = \omega[c(t)(N - m(t))] \quad (1.10)
\]

where \( \omega \) represents the uncertainty of the system fault detection rate in the operating environments with a probability density function \( f \), \( N \) is the expected number of faults that exists in the software before testing, \( c(t) \) is the fault detection rate function, which also represents the average failure rate of a fault, and \( m(t) \) is the mean value function (the expected number of errors detected by time \( t \)).

Solving the differential equation from equation (1.10) and setting the initial condition of \( m(0) = 0 \) results in,

\[
m(t) = \int_{\omega} N(1 - e^{-\gamma \int_0^t b(s)ds}) dg(\omega). \quad (1.11)
\]

The mean value function can be derived from equation (1.11) given that the random variable \( \omega \) has a generalized probability density function \( g \) with two parameters \( \rho \) and \( \tau \), and is given by,

\[
m(t) = N \left( 1 - \frac{\tau}{\tau + \int_0^t b(s)ds} \right)^\rho, \quad (1.12)
\]

where \( b(s) \) is the fault detection rate [14].

Now, assuming the underlying distribution of the failure time data is the Chen distribution and its fault rate is given as,

\[
b(t) = \gamma \nu \nu^{-1} e^{-e^{\nu t}}, t > 0, \theta > 0, \gamma > 0, \nu > 0.
\]

Let the expected number of failures in the software be \( N \), then the mean value function when the environmental uncertainties are factored in can be given as,

\[
N \left( 1 - \frac{\tau}{\tau + \gamma(e^{\nu t} - 1)} \right)^\rho, \quad (1.13)
\]

where \( \gamma, \tau, \rho \) and \( \nu \) are positive parameters of the mean value function.
3.1 SPRT Limits for Chen SRGM

This section presents the limits of the SPRT of the Chen SRGM. Assuming our interest is in testing the null hypothesis \( H_0 : m(t) = m_0(t) \) against the alternative hypothesis \( H_1 : m(t) = m_1(t) (> m_0(t)) \). Suppose the focus parameters here is \( \tau \), then for a Chen SRGM which follows the NHPP having a mean value function given in equation (1.13), the probability distribution when \( H_0 \) is true and when \( H_1 \) is true are respectively given as,

\[
P_0 = \frac{\exp \left( -N \left( 1 - \frac{\tau}{\tau_0 + \gamma (t^{e_1} - 1)} \right)^{\alpha} \right) \left( 1 - \frac{\tau}{\tau_0 + \gamma (t^{e_1} - 1)} \right)^{\beta}}{N!},
\]

(1.14)

and

\[
P_1 = \frac{\exp \left( -N \left( 1 - \frac{\tau}{\tau_1 + \gamma (t^{e_1} - 1)} \right)^{\alpha} \right) \left( 1 - \frac{\tau}{\tau_1 + \gamma (t^{e_1} - 1)} \right)^{\beta}}{N!},
\]

(1.15)

where the parameter specifications are; for \( \gamma \) and \( \tau_0 \). Therefore, the acceptance region for a reliable software with the Chen mean value function is established as,

\[
N(t) \leq \log\left( \frac{\beta}{1 - \alpha} \right) + N \left( \left( 1 - \frac{\tau_1}{\tau_1 + \gamma (t^{e_1} - 1)} \right)^{\alpha} - \left( 1 - \frac{\tau_0}{\tau_0 + \gamma (t^{e_1} - 1)} \right)^{\alpha} \right).
\]

Similarly, for unreliable software, the rejection region is given by,

\[
N(t) \geq \log\left( \frac{\beta}{1 - \alpha} \right) + N \left( \left( 1 - \frac{\tau_1}{\tau_1 + \gamma (t^{e_1} - 1)} \right)^{\alpha} - \left( 1 - \frac{\tau_0}{\tau_0 + \gamma (t^{e_1} - 1)} \right)^{\alpha} \right).
\]

(1.16)

The continuation region is given by,

\[
\log\left( \frac{\beta}{1 - \alpha} \right) + N \left( \left( 1 - \frac{\tau_1}{\tau_1 + \gamma (t^{e_1} - 1)} \right)^{\alpha} - \left( 1 - \frac{\tau_0}{\tau_0 + \gamma (t^{e_1} - 1)} \right)^{\alpha} \right) < N(t) < \log\left( \frac{1 - \beta}{1 - \alpha} \right) + N \left( \left( 1 - \frac{\tau_1}{\tau_1 + \gamma (t^{e_1} - 1)} \right)^{\alpha} - \left( 1 - \frac{\tau_0}{\tau_0 + \gamma (t^{e_1} - 1)} \right)^{\alpha} \right).
\]

4 Application of the Chen SRGM

This section presents the application of the Chen SRGM in real life using two data sets. The first data set was reported by [15] based on system test data for a telecommunication system data (TSD). System test data consisting of two releases, that is Phase 1 and Phase 2. In both tests, automated test and human involved tests are executed on multiple test beds. Phase two was considered in this study.

The second data set employed yields from a modest on-line data entry software program available in Japan since 1980 [16]. According to the International Business Machines (IBM) journal, the
software was about 40,000 lines of code long. The duration of the testing was calculated using the amount of shifts spent executing test cases and evaluating results of the matched couples.

It is very important to explore the behavior of the failure rate of the data sets employed in this study. Table 1 shows descriptive statistics of the failure rate of the TSD-Phase two and IBM data sets. They both have a minimum of 1 and a maximum of 21 weeks. They both have negative excess kurtosis that indicates the distribution of the failure rate is flat with thin tails. They also both have a skewness of 0 that indicates the failure rate is perfectly symmetrical.

| Statistic            | TSD-Phase two data | IBM data |
|----------------------|--------------------|----------|
| Minimum              | 1.00               | 1.00     |
| Maximum              | 21.00              | 21.00    |
| Mean                 | 11.00              | 11.00    |
| Median               | 11.00              | 11.00    |
| Standard deviation   | 6.205              | 6.205    |
| Skewness             | 0                  | 0        |
| Kurtosis             | -1.37              | -1.37    |

The total test on time (TTT) plot is employed to investigate the empirical behavior of the data sets. Fig. 1 presents the failure rate of both data sets. It shows that they both have an increasing failure since the curve is concave above the 45° line.

Fig. 1. Plot of TTT of the TSD-Phase Two and IBM failure rate

Tables 2 and 3 summarize the least square estimates of parameters for the TSD-Phase two and the IBM data sets respectively for the Chen SRGM and the other existing competing SRGMs. The estimated parameters of the Chen SRGM are all significant at 5%. This can be verified by using the *p-values* of the parameters.

Three model selection criteria are used to compare the Chen SRGM and other existing SRGM for the data sets. This is to help identify which model fits the data sets better. For the three criteria, the smaller the value, the closer the model fits relative to other models [17].
The Akaike information criterion (AIC) is a model selection criterion test used to evaluate how well a model fits the data it is meant to describe. The test statistic is given by,

$$AIC = -2 \log L(\hat{\theta}) + 2k,$$

where $k$ is the number of estimated parameters for the model.

2. Bayesian Information Criterion (BIC)

The BIC is a model selection criterion that employs the Bayes factor assumption. It is defined as,

$$BIC = -2 \log L(\hat{\theta}) + k \log(n),$$

where $n$ is the sample size.

3. Predictive Ratio Risk (PRR)

The PRR indicates the distance of model estimates from actual data with respect to the model estimates. Its test statistic is given as,

$$PRR = \sum_{i=1}^{n} \left( \frac{\hat{m}(t_i) - y_i}{\hat{m}(t_i)} \right)^2.$$ 

Tables 4 and 5 shows the AIC, BIC, and PRR of the fitted models for the TSD-Phase two and the IBM data set respectively. Since the Chen SRGM has the smallest AIC, BIC and PRR values for both data sets, it means that the Chen SRGM performs better than the other existing models.
### Table 3. Estimates of parameters of fitted models for IBM data

| SRGM     | Parameters | Estimates     | Standard error | Z value | p-value |
|----------|------------|---------------|----------------|---------|---------|
|          | a          | 23.1984       | 1.5430 x 10^-1 | 1.5026  | 2.2 x 10^-15 |
|          | b          | 1.0480 x 10^6 | 4.955 x 10^-13 | 7.1781  | 2.2 x 10^-15 |
|          | c          | 1.050 x 10^5  | 4.513 x 10^-13 | 4.1868  | 2.2 x 10^-15 |
|          | d          | 10.00         | 5.62 x 10^-10  | 1.79 x 10^-13 | 2.2 x 10^-15 |
|          | 0.001 x 10^-5 | 5.62 x 10^-10 | 7.78 x 10^-13 | 2.2 x 10^-15 |
|          | YID        | 0.21973       | 0.0293         | 7.3825  | 1.5 x 10^-14 |
|          | a          | 0.34286       | 0.0703         | 4.8471  | 1.25 x 10^-14 |
|          | b          | 12.14272      | 1.5548         | 7.8104  | 5.69 x 10^-15 |
|          | DUM        | 49.60         | 1.061 x 10^-1  | 4.8551  | 2.2 x 10^-15 |
|          | 1.5310 x 10^-1 | 3.995 x 10^-13 | 7.08 x 10^-15 | 2.2 x 10^-15 |
|          | Obba       | 23.190        | 0.1848         | 130.42  | 2.2 x 10^-15 |
|          | TS         | 1.0446 x 10^5 | 5.806 x 10^-14 | 5.28 x 10^-15 | 2.2 x 10^-15 |
|          | 0.001 x 10^-5 | 5.27 x 10^-10 | 4.20 x 10^-13 | 2.2 x 10^-15 |
|          | 5.92 x 10^-11 | 8.41 x 10^-13 | 2.2 x 10^-15 |
|          | 99.886     | 2.364 x 10^-4 | 4.23 x 10^-15 |
|          | TS         | 0.446 x 10^-4 | 3.329 x 10^-10 | 3.713 x 10^-15 | 2.2 x 10^-15 |
|          | 1.01 x 10^-2 | 2.37 x 10^-10 | 4.20 x 10^-13 | 2.2 x 10^-15 |
|          | 9.3 x 10^-13 | 8.41 x 10^-13 | 2.2 x 10^-15 |
|          | 0.001 x 10^-5 | 5.27 x 10^-10 | 4.20 x 10^-13 | 2.2 x 10^-15 |
|          | 10.00      | 5.92 x 10^-11 | 8.41 x 10^-13 | 2.2 x 10^-15 |
|          | Obba       | 23.190        | 0.1848         | 130.42  | 2.2 x 10^-15 |
|          | 1.0446 x 10^5 | 5.806 x 10^-14 | 5.28 x 10^-15 | 2.2 x 10^-15 |
|          | 0.001 x 10^-5 | 5.27 x 10^-10 | 4.20 x 10^-13 | 2.2 x 10^-15 |
|          | 5.92 x 10^-11 | 8.41 x 10^-13 | 2.2 x 10^-15 |
|          | 99.886     | 2.364 x 10^-4 | 4.23 x 10^-15 |
|          | TS         | 0.446 x 10^-4 | 3.329 x 10^-10 | 3.713 x 10^-15 | 2.2 x 10^-15 |
|          | 1.01 x 10^-2 | 2.37 x 10^-10 | 4.20 x 10^-13 | 2.2 x 10^-15 |
|          | 9.3 x 10^-13 | 8.41 x 10^-13 | 2.2 x 10^-15 |
|          | 0.001 x 10^-5 | 5.27 x 10^-10 | 4.20 x 10^-13 | 2.2 x 10^-15 |
|          | 5.92 x 10^-11 | 8.41 x 10^-13 | 2.2 x 10^-15 |
|          | 99.886     | 2.364 x 10^-4 | 4.23 x 10^-15 |

* denotes significant at the 0.01 significance level

### 4.1 Sensitivity Analysis

In understanding the effect of the model parameters on the model, sensitivity analyses were carried out on the two data sets. Tables 6 and 7 shows that the parameters of the Chen SRGM which vary the most are $\tau$ and $\gamma$ for the TSD-Phase two and IBM data sets respectively. This means that $\tau$ and $\gamma$ have a higher effect on the model than the other parameters.

### Table 4. The AIC, BIC, and PRR of existing models with the TSD-Phase two data set in comparison with the proposed model

| Model                          | AIC    | BIC    | PRR    |
|--------------------------------|--------|--------|--------|
| Goel Okumoto                   | 599.229| 601.318| 762.2693|
| Delayed Shaped                 | 314.666| 316.755| 14106.45|
| Yamada imperfect debugging     | 395.0934| 398.227| 203.7502|
| Testing coverage               | 439.8727| 445.09534| 113.5076|
| Dependent parameter model      | 247.4809| 249.57| 232.6096|
| Obba                           | 213.5285| 216.6594| 192.7347|
| Vtub-shaped fault detection rate| 11640| 11644.18| 353.0258|
| Chen SRGM                      | 197.1243| 202.3469| 77.22079|
4.2 Application of the SPRT

After conducting the sensitivity analysis, it showed that $\tau$ and $\nu$ were the most sensitive parameters for the TSD-Phase two and IBM data sets respectively. Thus, the focus of the SPRT are on these two parameters. Table 8 shows the specifications of the cases for the application of the SPRT. Hence, the parameters of interest are set as, $\tau_0 = \tau - \delta$, $\tau_1 = \tau + \delta$ and $\nu_1 = \nu + \delta$, $\nu_0 = \nu - \delta$. The values of $\alpha$ and $\beta$ are kept low. This is due to the fact that these values increase the effectiveness of the SPRT.

Table 10 presents the SPRT for the parameter $\tau$ for the TSD-Phase two data set. From 1 cumulative failure for serial number 15 indicates we should terminate the experiment and conclude that the associated software the data was extracted from is not reliable.

Likewise from 9 which presents the SPRT for parameter $\tau$ for the IBM data set. The cumulative failure for serial number 16 indicates the experiment should be terminated and the conclusion drawn that the software is not reliable.
Table 8. Case for applying SPRT

| Case          | δ     | α (Type I Error) | β (Type II Error) |
|---------------|-------|------------------|-------------------|
| Case 1 (for parameter τ) | 0.33  | 0.1              | 0.1               |
| Case 2 (for parameter ν)  | 0.9   | 0.1              | 0.1               |

Table 9. SPRT results for the IBM data (Case 1, parameter τ)

| T  | N(t) | Acceptance region | Rejection region | Decision |
|----|------|-------------------|------------------|----------|
| 1  | 2    | -36.7727          | 36.7907          | continue |
| 2  | 3    | -36.8335          | 36.7511          | continue |
| 3  | 4    | -36.8206          | 36.7414          | continue |
| 4  | 5    | -36.8148          | 36.7302          | continue |
| 5  | 6    | -36.8115          | 36.7174          | continue |
| 6  | 7    | -36.8099          | 36.7196          | continue |
| 7  | 11   | -36.8092          | 36.7286          | continue |
| 8  | 12   | -36.6949          | 36.7620          | continue |
| 9  | 19   | -36.6944          | 36.7635          | continue |
| 10 | 22   | -36.6943          | 36.8024          | continue |
| 11 | 23   | -36.6942          | 36.8023          | continue |
| 12 | 25   | -36.6942          | 36.8024          | continue |
| 13 | 27   | -36.6942          | 36.8024          | continue |
| 14 | 31   | -36.6942          | 36.8155          | continue |
| 15 | 32   | -36.6942          | 36.8204          | continue |
| 16 | 38   | -36.6942          | 36.8319          | continue |
| 17 | 39   | -36.6942          | 36.8363          | continue |
| 18 | 42   | -36.6942          | 36.8429          | continue |
| 19 | 44   | -36.6942          | 36.8435          | continue |
| 20 | 46   | -36.6942          | 36.8501          | continue |
| 21 | 47   | -36.6942          | 36.8508          | continue |

Table 10. SPRT results for the TSD-Phase two (Case 1, parameter τ)

| T  | N(t) | Acceptance region | Rejection region | Decision |
|----|------|-------------------|------------------|----------|
| 1  | 2    | -36.7724          | 36.8013          | continue |
| 2  | 4    | -36.6195          | 36.7855          | continue |
| 3  | 4    | -36.6216          | 36.7744          | continue |
| 4  | 7    | -36.6149          | 36.7797          | continue |
| 5  | 9    | -36.8108          | 36.7918          | continue |
| 6  | 9    | -36.8099          | 36.7936          | continue |
| 7  | 10   | -36.8096          | 36.8016          | continue |
| 8  | 13   | -36.6949          | 36.7796          | continue |
| 9  | 17   | -36.6944          | 36.8064          | continue |
| 10 | 23   | -36.6943          | 36.8061          | continue |
| 11 | 25   | -36.6942          | 36.8031          | continue |
| 12 | 30   | -36.6942          | 36.8222          | continue |
| 13 | 32   | -36.6942          | 36.8249          | continue |
| 14 | 36   | -36.6942          | 36.8358          | continue |
| 15 | 37   | -36.6942          | 36.8356          | continue |
| 16 | 39   | -36.6942          | 36.8382          | continue |
| 17 | 39   | -36.6942          | 36.8384          | continue |
| 18 | 39   | -36.6942          | 36.8399          | continue |
| 19 | 39   | -36.6942          | 36.8398          | continue |
| 20 | 42   | -36.6942          | 36.8384          | continue |
| 21 | 43   | -36.6942          | 36.8393          | continue |

Tables 11 and 12 presents the SPRT for the case where the focused parameter is ν for the IBM and TSD-phase two data set respectively. Based on the reliability, using the Chen SRGM, it can be concluded that the software is reliable as the SPRT gave an early decision.
Table 11. SPRT results for the IBM data (Case 2, parameter \( \nu \))

| \( T \) | \( N(t) \) | Acceptance region | Rejection region | Decision |
|-------|-------------|-------------------|------------------|---------|
| 1     | 2           | 2764.07           | –                | Accept  |

Table 12. SPRT results for the TSD-Phase Two data (Case 2, parameter \( \nu \))

| \( T \) | \( N(t) \) | Acceptance region | Rejection region | Decision |
|-------|-------------|-------------------|------------------|---------|
| 1     | 3           | 74.48128          | –                | Accept  |

5 Conclusion

Quality control techniques come in different forms. Software reliability growth model as a quality control technique is one of the most crucial elements in software production. A newly developed Chen SRGM is proposed incorporating environmental uncertainties. We developed the limits of its SPRT and showed the application of these limits. This was done by examining which parameters were the most sensitive ones. The results revealed that the variation of the parameters \( \tau \) and \( \nu \) were the largest. Thus the SPRT was performed based on these parameters. The first case of the SPRT showed the software was not reliable and hence the experiment was terminated, while the second case showed that software was reliable and could be deployed to the market. The results from the model selection criteria showed that the Chen SRGM is suitable for reliability analysis than the other competing models. The Chen SRGM can therefore assist software engineers model reliability data which follow the Chen distribution. The limits of the SPRT developed can be used as a quality control tool in software production during the monitoring phase.

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Competing Interests

Authors have declared that no competing interests exist.

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Appendix

Table 13. IBM data

| Failure rate (weeks) | Failures | Cumulative failures |
|----------------------|----------|---------------------|
| 1                    | 2        | 2                   |
| 2                    | 1        | 3                   |
| 3                    | 1        | 4                   |
| 4                    | 1        | 5                   |
| 5                    | 2        | 7                   |
| 6                    | 2        | 9                   |
| 7                    | 2        | 11                  |
| 8                    | 1        | 12                  |
| 9                    | 7        | 19                  |
| 10                   | 3        | 22                  |
| 11                   | 1        | 23                  |
| 12                   | 2        | 25                  |
| 13                   | 2        | 27                  |
| 14                   | 1        | 31                  |
| 15                   | 1        | 32                  |
| 16                   | 6        | 38                  |
| 17                   | 1        | 39                  |
| 18                   | 3        | 42                  |
| 19                   | 1        | 43                  |
| 20                   | 3        | 46                  |
| 21                   | 1        | 47                  |

Table 14. TSD-Phase two data

| Failure rate (weeks) | Failures | Cumulative failures |
|----------------------|----------|---------------------|
| 1                    | 3        | 3                   |
| 2                    | 1        | 4                   |
| 3                    | 0        | 4                   |
| 4                    | 2        | 6                   |
| 5                    | 2        | 9                   |
| 6                    | 0        | 9                   |
| 7                    | 1        | 10                  |
| 8                    | 3        | 13                  |
| 9                    | 4        | 17                  |
| 10                   | 2        | 19                  |
| 11                   | 4        | 23                  |
| 12                   | 2        | 25                  |
| 13                   | 2        | 27                  |
| 14                   | 2        | 29                  |
| 15                   | 4        | 33                  |
| 16                   | 1        | 34                  |
| 17                   | 2        | 36                  |
| 18                   | 0        | 36                  |
| 19                   | 0        | 36                  |
| 20                   | 3        | 39                  |
| 21                   | 1        | 40                  |

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