Influence of External Magnetic Field on Potential Differences of Long Josephson Junction

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Abstract. The soliton phenomenon in a long Josephson junction was studied by using the particular solution of Sine-Gordon equation. Using analytical method, sine-Gordon equation was normalized and two-soliton solutions correlated to phase, and potential differences of long Josephson junction were obtained. Based on the two-soliton solutions, the influence of external magnetics fields on the dynamics of phase and potential differences of long Josephson junction were numerically studied.

Key word: Josephson Effect, Long Josephson Junction, Sine-Gordon Equation, Soliton.

1. Introduction

Long Josephson junction is a device that consists of superconductor-insulator/metal-superconductor (SIS) junction, which in this junction there is a Josephson effect. Because of that effect, phase and potential differences of long Josephson junction are occurred [1]. Based on the theory of electrodynamics and quantum mechanics, it is revealed that the physical equation of the long Josephson junction corresponds to sine-Gordon equation. Kink-antikink soliton solutions of sine-Gordon equation in a long Josephson junction is obtained from [2-3]. Some researchers had been done to study Josephson junction in technology, for example, thermal transport, SQUID, quantum phase and solitons [4-8].

Dynamics phase of long Josephson junction is expressed in equation (1)

\[
\frac{1}{v_{ph}^2} \frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{\lambda_J^2} \sin(\phi)
\]  

(1)

If it is defined new variables \( t' = \frac{v_{ph}}{\lambda_J} t \) and \( z' = \frac{1}{\lambda_J} z \), thus the above equation become sine-Gordon equation,

\[
\frac{\partial^2 \phi}{\partial t'^2} - \frac{\partial^2 \phi}{\partial z'^2} = \sin(\phi)
\]  

(2)

Using kink-antikink solutions; equation (3),
\phi(z', t') = 4 \tan^{-1} \left( \frac{u(z')}{v(t')} \right) \tag{3}

as a soliton solution of equation (2). Thus, we obtained

\begin{align}
\frac{(u(z'))^2}{v(t')} \frac{\partial^2 v(t')}{\partial t'^2} + \frac{(v(t'))^2}{u(z')} \frac{\partial^2 u(z')}{\partial z'^2} &= \\
\left[ (v(t'))^2 - v(t') \frac{\partial^2 v(t')}{\partial t'^2} + 2 \left( \frac{\partial v(t')}{\partial t'} \right)^2 \right] - \left[ (u(z'))^2 + u(z') \frac{\partial^2 u(z')}{\partial z'^2} - 2 \left( \frac{\partial u(z')}{\partial z'} \right)^2 \right] \tag{4}
\end{align}

equation (4) applies for all space and time. Therefore, left and right-hand side of equation (4) is permitted to derive partially with respect to \( z' \) and \( t' \). So it gives,

\begin{align}
\frac{1}{v(t')} \frac{\partial}{\partial t'} \left( \frac{\partial^2 v(t')}{\partial t'^2} \right) &= - \frac{1}{u(z')} \frac{\partial}{\partial z'} \left( \frac{\partial^2 u(z')}{\partial z'^2} \right) \tag{5}
\end{align}

Because left and right-hand side of equation (5) are differential equations having different variables, so it should be a constant.

\begin{align}
\frac{1}{v(t')} \frac{\partial}{\partial t'} \left( \frac{\partial^2 v(t')}{\partial t'^2} \right) &= - \frac{1}{u(z')} \frac{\partial}{\partial z'} \left( \frac{\partial^2 u(z')}{\partial z'^2} \right) = -6k^2 \tag{6}
\end{align}

Using ordinary differential method, from equation (6), we obtain two following equations:

\begin{align}
\frac{\partial^2 v(t')}{\partial t'^2} &= -3k^2 v^3(t') + Av(t') \tag{7}
\end{align}

and

\begin{align}
\frac{\partial^2 u(z')}{\partial z'^2} &= 3k^2 u^3(z') + Bu(z') \tag{8}
\end{align}

where \( A \) and \( B \) are constant.

The two last equations can be changed in two new equations:

\begin{align}
\left( \frac{\partial v(t')}{\partial t'} \right)^2 &= 2C - \frac{3}{2} k^2 v^4(t') + Av^2(t') \tag{9}
\end{align}

and
The goals of this work are to investigate numerically the dynamic of phase and potential differences in long Josephson junction under the influence of external magnetic field parameter.

2. Numerical Method
The physical system considered in this research is long Josephson junction under external magnetic field. The external magnetic field is uniformly applied along of long Josephson junction. Heun method [Mathews, 2004] is used to determine inverse tan function. Long Josephson Junction phase difference equation that is similar to the sine-Gordon equation is converted into dimensionless equations by means of normalizing each variable.

3. Results and Discussion
Base on Equation (10) and (11), dynamics of phase and potential differences in long Josephson junction under the influence of external magnetic field were investigated. Three conditions were considered. The graphs under the three conditions are obtained by taking penetration depth of magnetic field (λd) is 1760 angstrom, insulator thickness (d) is 2 nm, supercurrent density (Jc) is 5.240.000 A/m² and integral coefficients α and γt are equal to 0,75 and γt=0,133994, respectively.

3.1 Influence of external magnetic field on the phase difference of long Josephson junction for k = 0
while A = B =α and C = D = β.
For this condition, long Josephson junction phase difference function is written as

\[ \phi(z',t') = 4 \tan^{-1} \left( \frac{e^{\sqrt{\alpha}(z'-\gamma_t)} - e^{-\sqrt{\alpha}(z'-\gamma_t)}}{e^{\sqrt{\alpha}(t'-\gamma_t)} - e^{-\sqrt{\alpha}(t'-\gamma_t)}} \right) \]  (11)

and potential differences of the long Josephson junction is written as

\[ V(z',t') = \frac{4\phi_0}{2\pi} \sqrt{\alpha} \left( -e^{\sqrt{\alpha}(z'-\gamma_t)} + e^{-\sqrt{\alpha}(z'-\gamma_t)} \right) \left( e^{\sqrt{\alpha}(t'-\gamma_t)} + e^{-\sqrt{\alpha}(t'-\gamma_t)} \right) \]  (12)

Figure 1 shows graphs of potential difference and external magnetic fields of the long Josephson junction. Based on Figure 1, we conclude that solutions are not solitons, because from t = 0 s to t = 12 \times 10^{-12} s, graphs of the magnetic field and potential differences of long Josephson junction always change. Physically, it means that external magnet fields applied to long Josephson junction are not large enough to generate soliton solutions [10].

\[ \left( \frac{\partial u(z')}{\partial z'} \right)^2 = 2D + \frac{3}{2} k^2 u^4(z') + Bu^2(z') \]  (10)
Figure 1. Graphs of potential difference and external magnetic fields of long Josephson junction for as a function of time; \( t = 0; 5 \times 10^{-12}; 8 \times 10^{-12}; 12 \times 10^{-12} \) seconds, for \( k = 0 \) while \( A = B = \alpha \) and \( C = D = \beta \).

Figure 2. Graphs of potential difference as a function of external magnetic fields of long Josephson junction for a given times: \( t = 0; 5 \times 10^{-12}; 8 \times 10^{-12}; 12 \times 10^{-12} \) seconds, for \( k = 0 \) while \( A = B = \alpha \) and \( C = D = \beta \).

Because of the evolution of potential differences as a function of a magnetic field change in for different given time, the potential differences are non-solitary.

3.2 Influence of external magnetic field on phase difference of long Josephson junction for \( C = D = k = 0 \) while \( A = B \neq 0 \)

For this condition, long Josephson junction phase difference function is written as

\[
\phi(z', t') = 4 \tan^{-1} \left( \exp \left( \frac{z' - \beta t'}{\sqrt{1 - \beta^2}} \right) \right)
\]  

and potential differences of the long Josephson junction is written as
\begin{equation}
V(z',t') = -\frac{\beta}{\sqrt{1-\beta^2}} \frac{1}{2\pi} \frac{4\phi_0}{1 + \left( \exp \left( \frac{-\beta z'}{\sqrt{1-\beta^2}} \right) \right)} \exp \left( \frac{-\beta t'}{\sqrt{1-\beta^2}} \right)
\end{equation}

Figure 3 shows graphs of phase and potential differences of the long Josephson junction. From the graphs shown in Figure 2, we conclude that solutions are solitons, because from \( t = 0 \) s to \( t = 12 \times 10^{-12} \) seconds. Graphs of phase and a potential difference of long Josephson junction do not change in. Physically, it means that external magnet fields applied to long Josephson junction are enough to form soliton solutions.

**Figure 3.** Graphs of potential difference and external magnetic fields of long Josephson junction for as a function of time; \( t = 0; 5 \times 10^{-12}; 8 \times 10^{-12}; 12 \times 10^{-12} \) seconds, for \( C = D = k = 0 \) while \( A = B \neq 0 \).

**Figure 4.** Graphs of potential difference as a function of external magnetic fields of long Josephson junction for a given times: \( t = 0; 5 \times 10^{-12}; 8 \times 10^{-12}; 12 \times 10^{-12} \) seconds, for \( C = D = k = 0 \) while \( A = B \neq 0 \).
The potential difference of Long Josephson Junction is solitary; consequently, the external magnetic fields allow a solitary shaped pattern. So with a solitary condition, the curve of potential difference Long Josephson Junction as a function of the external magnetic fields forms linear patterns for different times. This condition is called anti-kink soliton.

3.3 Influence of external magnetic field on phase difference of long Josephson junction for \( C = D = 0 \) while \( A = B = \frac{3}{2}k^2 \)

For this condition, long Josephson junction phase difference function is written as

\[
\phi(z', t') = 4 \tan^{-1} \left( \frac{A_2}{A_1} e^{-\frac{\sqrt{2} (z' - r)}{1 + A_1^2 e^{-\frac{t'}{\sqrt{2} k^2}}}} \right) \frac{1 + A_1^2 e^{-\frac{t'}{\sqrt{2} k^2}}}{A_1^2 e^{-\frac{t'}{\sqrt{2} k^2}} - 1} \tag{15}
\]

and potential differences of the long Josephson junction is written as

\[
V(z', t') = \frac{1}{2\pi} \frac{\sqrt{2} A_2}{A_1} \frac{4\phi_0}{1 + \left( \frac{A_2}{A_1} e^{-\frac{\sqrt{2} (z' - r)}{1 + A_1^2 e^{-\frac{t'}{\sqrt{2} k^2}}}} \right)^2 \left( \frac{1 + A_1^2 e^{-\frac{t'}{\sqrt{2} k^2}}}{A_1^2 e^{-\frac{t'}{\sqrt{2} k^2}} - 1} \right)^2} \tag{16}
\]

Figure 5. Curves of potential difference and external magnetic fields of long Josephson junction for as a function of time: \( t = 0; 5 \times 10^{-12}; 8 \times 10^{-12}; 12 \times 10^{-12} \) seconds, for \( C = D = 0 \) while \( A = B = \frac{3}{2}k^2 \)

Figure 5 shows graphs of potential differences and external magnetic fields of the long Josephson junction. From the graphs shown in Figure 5, we conclude that the solutions form the soliton solutions, because from \( t = 0 \) s to \( t = 12 \times 10^{-12} \) seconds, the graphs of phase and a potential difference of long Josephson junction do not change in. Physically, it means that external magnet fields applied to long Josephson junction are enough to form soliton solutions. From the graphs above, the external magnetic field and the potential difference of Long Josephson Junction evolve along of the junction.
and attain the saturation peak. In this condition, the graphs produce the solitary solution in the form of semi fluxon [11].

\[ V(10^{(-15)}) \text{ Volt} \]

\[ B(y)(10^{(-4)}) \text{ Tesla} \]

Figure 6. Curves of potential difference as a function of external magnetic fields of long Josephson junction for a given times: \( t = 0; 5 \times 10^{-12}; 8 \times 10^{-12}; 12 \times 10^{-12} \) seconds, for \( C = D = 0 \) while \( A = B = 3/2k^2 \)

The influence of the external magnetic fields on the generation of a potential differences of Long Josephson junction are linear from time to time, but they attain saturation at a certain time.

4. Conclusion

The information on the variation of phase difference and the potential difference with the external magnetic field in a long Josephson junction is very important for the correct interpretation of the experimental results. In this paper, we investigated the dependence of phase difference and potential difference on the external magnetic field. We found that under certain external magnetic field values applied in a long Josephson junction, the dynamic of potential differences is in the form of solitons. This result gives information the change in the potential difference of Josephson Junction under the influence of external magnetic field change which is useful in SQUID application.

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