Quantum Gravity Corrections for Schwarzschild Black Holes

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We consider the Matrix theory proposal describing eleven-dimensional Schwarzschild black holes. We argue that the Newtonian potential between two black holes receives a genuine long range quantum gravity correction, which is finite and can be computed from the supergravity point of view. The result agrees with Matrix theory up to a numerical factor which we have not computed.

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1. Introduction

Matrix theory [1] is the non-perturbative formulation of the eleven dimensional M-theory. The non-perturbative nature of black hole physics indicates that Matrix theory might be an ideal framework to address questions in black hole physics. A concrete model describing Schwarzschild black holes in diverse dimensions was proposed by Horowitz and Martinec [2] and Banks, Fischler, Klebanov and Susskind [3] which describes the black hole in terms of a Bolzmann gas of distinguishable D0-branes. This model reproduces correctly the Bekenstein-Hawking entropy, the size, the mass, as well as the leading term of the static Newtonian potential between two black holes. Liu and Tseytlin [5] have generalized this picture to include all loop large N supersymmetric Yang-Mills (SYM) corrections, or equivalently corrections from general relativity.

In this paper we shall be interested in the Newton potential between a pair of Schwarzschild black holes described by the model [2], [3] and [5]. Our main interest is the existence of a genuine quantum gravity correction to the gravitational potential, which is finite and can be computed from the supergravity point of view. It is the leading long range quantum gravity correction that can be computed from Matrix theory as well. Happily, we will see that both results indeed agree up to a numerical factor that we have not computed.

The existence of finite quantum gravity corrections to the four-dimensional Newtonian potential has been emphasized by Donoghue [6]. In a series of interesting papers he showed that the leading long distance quantum gravity correction to Newton’s potential can be reliably calculated in a quantum theory of gravity. Let us briefly review the general idea.

The leading term of the four-dimensional Newtonian potential for two particles with mass $M_1$ and $M_2$ separated by a distance $r$ is given by

$$V(r) = -G \frac{M_1 M_2}{r},$$  \hspace{1cm} (1.1)

where $G$ is Newton’s constant. This expression is of course only approximately valid. For large masses or large velocities there are relativistic corrections which can be computed in a

\footnote{A different proposal for a microscopic derivation of the Bekenstein-Hawking entropy for Schwarzschild black holes is presented in [4].}
post-Newtonian expansion and have been verified experimentally. For a small test particle $M_2$ this can be seen from the expansion of the time component of the Schwarzschild metric:

$$g_{00} = \frac{1 - \frac{GM_1}{rc^2}}{1 + \frac{GM_1}{rc^2}} \approx 1 - \frac{2GM_1}{rc^2} \left( 1 - \frac{GM_1}{rc^2} + \ldots \right). \quad (1.2)$$

In a quantum theory of gravity we expect that the potential will be corrected by quantum effects. It is well known that when trying to unify quantum mechanics and general relativity we will face the problem that this is a non-renormalizable theory. Although one can quantize the theory on smooth enough backgrounds the divergences appearing in particular diagrams are such that they cannot be absorbed into the coupling constants of a minimal general relativity. If one introduces new coupling constants to absorb the divergences one is led to an infinite number of free parameters and thus to a lack of predictivity. Despite this situation the leading long distance quantum corrections can be reliably calculated in ‘quantum’ general relativity.

To leading order in the distance two massive objects will interact through a Newtonian potential of the form:

$$V(r) = -\frac{G M_1 M_2}{r} \left( 1 + a \frac{G(M_1 + M_2)}{rc^2} + b \frac{\hbar}{r^2 c^3} + \ldots \right), \quad (1.3)$$

where $a$ and $b$ are some finite numerical coefficients. The first correction is due to general relativity which is the term appearing in (1.2). The correction proportional to $\hbar$ is a true quantum gravity effect. Its overall form can be fixed by dimensional analysis while the numerical coefficient can be calculated if we treat gravity as an effective field theory where we can make an expansion in the energy. This procedure is familiar from chiral perturbation theory which represents the low energy limit of QCD.

The action describing gravity is then organized in terms of powers of the curvature

$$S_{\text{grav}} = \int d^4x \sqrt{-g} \left( \frac{2}{\kappa^2} R + \alpha R^2 + \ldots \right), \quad (1.4)$$

where $\kappa^2 \sim G$ and $\alpha$ is a constant. Derivatives are related to the momentum $\partial_\mu \sim p_\mu$, so that the curvature is of order $p^2$, two powers of curvature are of order $p^4$, and so on. At low energies or equivalently long distances terms of order $p^4$ can be ignored compared terms of order $p^2$. 
The quantum gravity correction (1.3) is the leading long distance quantum gravity effect which is due to the long range propagation of massless particles such as the graviton. Only the lowest order coupling contained in the Einstein action is needed to describe this effect since higher order interactions like $R^2 + \ldots$ contain too many powers of the momentum and are negligible at large distances. Since this quantum correction to (1.3) is finite it might be the first and simplest quantum gravity corrections that could actually be tested by Matrix theory. This is the purpose of our paper.

There are certainly many contributions to the effective action in a quantum theory of gravity that are divergent like for example the $R^4$ contribution considered in [7] [8] [9]. These are not the effects that we would like to consider here. Matrix theory regularizes the divergent contributions in M-theory by fixing the value of the cutoff and there is no finite result on the gravity side with which one could compare.

Let us now come to the Matrix theory version of the story. A number of important papers suggested that Schwarzschild black holes could be described in string theory and Matrix theory [10]. Horowitz and Martinec [2] and Banks, Fischler, Klebanov and Susskind [3] have proposed a beautiful model whose aim is to describe Schwarzschild black holes in diverse dimensions. Many interesting features of black hole physics are correctly reproduced by this model. The basic idea is that Schwarzschild black holes in diverse dimensions can be described in terms of a system of D0-branes in toroidally compactified space. We shall be interested in the eleven-dimensional case. One might have thought that this case is especially difficult, because very little is known about the SYM theory for general $N$. However, as we shall see, enough is known to determine the quantum gravity correction we are interested in.

In this paper we would like to see if the Matrix theory description is able to reproduce the leading long distance quantum gravity correction to Newton’s potential of an eleven-dimensional Schwarzschild black hole. We will use crude methods in the sense that we will not compute any numerical coefficients. All the dependences turn out to work out correctly so that precise agreement between Matrix theory and supergravity can be found.

This paper is organized as follows. In the section 2 we would like to consider the Matrix model point of view. We will recapitulate the result for the leading term of Newton’s
potential and compute relativistic corrections. We will see that the leading long distance quantum gravity effect follows from a one loop term in the Matrix model. In section 3 we consider the supergravity side of the story, which we then compare with Matrix theory. Finally, in section 4 we will present our conclusions.

2. The Newtonian Potential from Matrix Theory

Let us start by reviewing some relevant features of the Matrix theory model for Schwarzschild black holes. From now on we will only take into account general dependences but not any numerical coefficients. The basic idea of [2] and [3] is that a $D$-dimensional Schwarzschild black hole can be described in terms of a Boltzmann gas of distinguishable D0-branes with two body interactions given by the leading term in the one-loop SYM effective action compactified on $T^d$ ($D = 11 - d$). We will be interested in the case $D = 11$ where the Lagrangian is

$$L_{\text{eff}} = \frac{Nv^2}{R} + \frac{G_{11}N^2}{R^3} \frac{v^4}{r^7}, \quad (2.1)$$

where $G_{11}$ is the eleven-dimensional Newton’s constant, $R$ is the radius of the eleven dimension, $N$ is the number of D0-branes and $v$ and $r$ are the relative velocity and distance respectively. This Lagrangian has a holographic scaling property [1]. We would like to rescale the relative transverse coordinate $r$ and the time coordinate $t$ as

$$r \to N^{\frac{1}{3}} r, \quad t \to \frac{1}{R} N^{\frac{2}{3}} t. \quad (2.2)$$

From (2.2) it follows that the velocity gets rescaled as

$$v \to R(NG_{11})^{-\frac{1}{3}} v. \quad (2.3)$$

The dependence on Newton’s constant in formula (2.3) can be determined by dimensional analysis. The Lagrangian (2.1) becomes (up to an overall factor) independent of $N$ and $R$. Applying a qualitative analysis based on the virial theorem, i.e. equating the kinetic and potential term of (2.1) one can derive a relation between $N$ and the size of the bound state
\[ G_{11}^{-1} R_s^9 = N. \]  

Since one is treating the D0-branes as a Boltzmann gas its entropy is of order \( N \) so that the above relation is precisely the Bekenstein-Hawking area law for a Schwarzschild black hole \[2], \[3\] and \[11\], where \( R_s \) corresponds to the Schwarzschild radius. Using the standard relation between the light cone energy and mass one can determine the scaling of the mass \( M \) as a function of \( N \)

\[ M \sim G_{11}^{-1/9} N^{8/9}. \]  

This means the Matrix model correctly reproduces the scaling of the mass in terms of the Schwarzschild radius.

Finally, one can show that the static Newtonian potential between two equal mass black holes can be be completely understood in terms of the velocity dependent potential between the D0-branes. For this purpose we use the connection between the light cone energy and the rest mass plus the potential \((2.1)\) and scaling relations that we just mentioned. The result for the (leading term of the) eleven-dimensional Newton potential is \[3\]

\[ V(r) \sim G_{11} \frac{M^2}{r^7} (G_{11} N)^{-1/9} = G_{10} \frac{M^2}{r^7}. \]  

The last factor on the left hand side of \((2.6)\) comes from the averaging over the longitudinal direction. This is because this factor is \(1/R_s\) and if one is working for \( S \sim N \) then one can set \( R_s \sim R \). The reason for this is that the longitudinal box expands in such a way that the black hole fits into it (see the discussion in \[3\] and \[4\]). We have further used \(G_{10} \sim G_{11}/R\).

Let us now discuss general relativity corrections to the previous gravitational potential. Liu and Tseytlin \[3\] have proposed a generalization of the action \((2.1)\) which includes all loop large \( N \) SYM corrections. The basic idea is to consider the classical Born-Infeld action for a D0-brane probe moving in a supergravity background produced by a D0-brane source. The all loop large \( N \) SYM corrections to \((2.1)\) can be obtained if one formally extrapolates this action to the short distance or near horizon region. The Lagrangian obtained in this
way is

\[ L_{\text{eff}} = \frac{r^7 R}{G_{11}} \left[ \sqrt{1 - G_{11} \frac{N v^2}{R^2 r^7}} - 1 \right]. \]  

(2.7)

Expanding (2.7) one gets (2.1) plus corrections

\[ L_{\text{eff}} = \frac{N v^2}{R} + \frac{G_{11} N^2 v^4}{R^3 r^7} + \frac{G_{11}^2 N^3 v^6}{R^5 r^{14}} + \ldots. \]  

(2.8)

Let us consider the third term appearing in (2.8). If we follow the same steps as for the leading contribution we conclude that this expression corrects the gravitational potential in the following way

\[ \Delta V(r) \sim G_{10}^2 \frac{M^3}{r^{14}}. \]  

(2.9)

This is precisely the form of a relativistic correction to Newton’s potential in \( D = 11 \) as can be seen for example by analogy to the four dimensional formula (1.3) that we mentioned in the introduction. By the same argument it is easy to see that higher order corrections in (2.8) reproduce the correct form of higher order relativistic corrections to Newton’s potential.

Let us now come to our main point: the leading long distance quantum gravity correction. By now it is well known that the effective action of Matrix theory at a given number of loops is a double expansion in the relative velocity and distance between D0-branes. The leading one loop term, i.e. the \( v^4 \)-term appearing in (2.1), is not renormalized at higher loops \([12][13]\). The \( v^6 \)-term has a very special property: it vanishes at one loop and it is not renormalized beyond two loops \([14]\). From these considerations we come to the conclusion that the leading long distance quantum gravity correction to the one-loop \( v^4 \)-term corresponds to the \( v^8 \)-term at one loop. Let us have a closer look at this contribution.

From the systematic expansion derived in \([15]\) the concrete form of this term is

\[ \Delta L_{\text{eff}} = \frac{N^2}{R^7 M_{pl}^{21}} v^8 = G_{10}^2 \frac{N^2}{M_{pl}^3 R^5} v^8, \]  

(2.10)

where \( M_{pl} \) is the eleven-dimensional Planck mass. We would like to add this correction to the Lagrangian (2.1) and compute the result for the gravitational interaction. Using the

\[ \text{Of course } c = 1 \text{ in string theory conventions.} \]
same formulas that we used for the leading term and the relativistic corrections we obtain

\[ V(r) \sim G_{10}^2 \frac{1}{R^3 M_{pl}^3} \frac{M^2}{r^{15}}. \]  \hfill (2.11)

We can rewrite this expression in terms of the string coupling constant \( g_s \) and the ten-dimensional gravitational constant. Recall that the eleven-dimensional Planck length is related to the string coupling constant and \( \alpha' \) as

\[ l_{pl} = (2\pi g_s)^{1/3} \sqrt{\alpha'}, \]  \hfill (2.12)

while the compactification radius is

\[ R = g_s \sqrt{\alpha'}. \]  \hfill (2.13)

This means that the string coupling constant can be expressed in terms of \( R \) and \( M_{pl} \) as

\[ g_s^2 = \frac{2\pi R^3}{l_{pl}^3} = 2\pi R^3 M_{pl}^3. \]  \hfill (2.14)

From this it follows

\[ G_{10} \sim \frac{G_{11}}{R} \sim g_s^2 \alpha'^4. \]  \hfill (2.15)

The dimensionless coupling constant appearing in the correction to Newton’s potential (2.11) is nothing but the string coupling constant

\[ V(r) \sim g_s^2 \alpha'^8 \frac{M^2}{r^{15}} \sim \frac{G_{10}^2 M^2}{g_s^2 r^{15}}. \]  \hfill (2.16)

This result looks similar to the finite quantum gravity correction appearing in the four-dimensional formula (1.3). In string theory conventions we can set \( \hbar = 1 \) or equivalently \( \hbar \) can be shifted away with a scale transformation of the metric. This is the origin of the dependence on the string coupling constant in the above formula. We will see this in more detail in the next section. The fact that integer powers of Newton’s constant and \( g_s \) appear is indicating that this is a finite correction from the supergravity point of view. Otherwise fractional powers of \( G_{10} \) would be present due to the existence of a dimensionful parameter, the cutoff. The above quantum gravity correction is the leading quantum effect as other quantum effects are higher order in \( 1/r \).
To summarize, the leading terms of the eleven-dimensional Newton’s potential from
the Matrix theory point of view take the form
\[ V(r) \sim \frac{G_{10}M^2}{r^7} \left( 1 + \frac{G_{10}M}{g_s^2 r^8} + \ldots \right). \]  
(2.17)

This formula has an interesting analogy with (1.3). The different powers in \( r \) are due to the
fact that the dimensions of Newton’s constant are different in ten and in four dimensions.
Let us now have a closer look at the supergravity side of the story.

3. The Newtonian Potential from Supergravity

On the supergravity side we can follow closely the four-dimensional calculation [6] though we will not determine any numerical coefficients. Our aim is to illustrate which
Feynman diagrams contribute to the different terms in the gravitational potential.

In ten dimensions two massive objects with mass \( M \) separated by a distance \( r \) interact
to lowest order by the Newtonian potential
\[ V(r) \sim \frac{G_{10}M^2}{r^7}. \]  
(3.1)

If the mass or velocities of these objects get too big the potential receives relativistic
corrections. These would be of the form
\[ V(r) \sim \frac{G_{10}M^2}{r^7} \left( 1 + a \frac{G_{10}M}{g_s^2 r^8} + \ldots \right). \]  
(3.2)

The numerical coefficient \( a \) is calculable in a post-Newtonian expansion. At some point
this expression will be corrected by quantum gravity effects. It is possible to figure out the
general form of these corrections using dimensional analysis. Since they arise from loop
diagrams they will involve an additional power of Newton’s constant \( G_{10} \) and if they are
quantum corrections they will be at least linear in \( \hbar \). As we have already mentioned, we
will be interested in a very particular quantum gravity correction: the leading long distance
quantum gravity effect. It will be a non-analytic effect in the momentum transfer, since
analytic effects correspond to contact terms in coordinate space. Since this effect is due
to long range propagation of massless particles, the other dimensionful parameter is the
distance $r$. The ten-dimensional Newton’s constant has dimension $M^{-1}L^7$ and $\hbar$ has dimensions $ML$. The combination

$$\frac{G_{10}\hbar}{r^8},$$

is dimensionless and provides an expansion parameter for the long distance quantum effects. Altogether, to leading order we expect a modification of the classical potential of the form

$$V(r) \sim \frac{G_{10}M^2}{r^7} \left(1 + a \frac{G_{10}M}{r^7} + b \frac{G_{10}\hbar}{r^8} + \ldots\right).$$

The free parameters $a$ and $b$ are finite constants that can be calculated in a quantum theory of gravity by computing Feynman diagrams. Let us illustrate the general idea.

Our starting point is the ten-dimensional Einstein action

$$S_{\text{grav}} \sim \frac{1}{G_{10}} \int d^{10}x \sqrt{-g}R,$$  \hspace{1cm} (3.5)

coupled to a massive scalar field

$$S_{\text{matter}} \sim \int d^{10}x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} M^2 \varphi^2 \right).$$  \hspace{1cm} (3.6)

The quantum fluctuations of the gravitational field $h_{\mu\nu}$ can be expanded around a flat background

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa_{10} h_{\mu\nu},$$  \hspace{1cm} (3.7)

where $\kappa_{10}^2 = G_{10}$ up to a numerical coefficient. To quantize the fluctuations one needs to fix a gauge and a gauge fixing term has to be added to the Lagrangian. One can then follow the standard rules to quantize the theory by computing Feynman diagrams. We are interested in considering quantum corrections at one loop. For that purpose the Lagrangians have to be expanded to quartic order in the fields.

The leading term of Newton’s potential follows from the single graviton exchange diagram where two scalar particles interact through the exchange of a single graviton

$$M_{12} \sim \kappa_{10}^2 T^{(1)}_{\mu\nu}(q) D^{\mu\nu\alpha\beta} T^{(2)}_{\alpha\beta}(-q).$$  \hspace{1cm} (3.8)

\footnote{We will not be taking fermions nor any other fields of type IIA supergravity besides the graviton into account. These fields will have to be taken into account to determine the precise numerical coefficients of (3.4).}
Here $T_{\mu\nu}$ is the on-shell matrix element of the matter energy-momentum tensor and $D_{\mu\nu\alpha\beta}$ is the graviton propagator [17]. The Newton potential can then be found by Fourier transforming

$$\frac{1}{M^2} M_{12} \sim \frac{G_{10} M^2}{q^2}, \quad (3.9)$$

where the factor $1/M^2$ on the left hand side accounts for the proper normalization. In coordinate space we obtain

$$V(r) \sim G_{10} \int \frac{d^9 q}{(2\pi)^9} e^{-iqr} \frac{M^2}{q^2} \sim \frac{G_{10} M^2}{r^7}. \quad (3.10)$$

To compute one loop corrections to Newton’s potential we have to consider both vertex corrections and vacuum polarization effects at one loop [6]. Double graviton exchange diagrams contribute to this order as well [18]. The one loop diagrams will have an additional power of $\kappa_{10}^2$ compared to the tree diagram and $\kappa_{10}^2$ has dimension $M^{-8}$. The combination $\kappa_{10}^2 q^8$ is dimensionless. However loop diagrams will also produce non-analytic terms of the form $\log(-q^2)$ and $\sqrt{-q^2}$ which are dimensionless. Such non-analytic effects lead to a power law behavior in coordinate space

$$\int \frac{d^9 q}{(2\pi)^9} e^{-iqr} \sqrt{-q^2} \sim \frac{1}{r^{14}}, \quad (3.11)$$

On the other hand analytic terms correspond to delta functions in coordinate space

$$\int \frac{d^9 q}{(2\pi)^9} e^{-iqr} = \delta^9(r). \quad (3.12)$$

We are interested in precisely the terms (3.11) appearing as one-loop effects. The first term will correspond to the relativistic correction while the second term corresponds to the leading quantum correction. In momentum space the contributions to the potential take the form

$$V(q) \sim G_{10} M^2 \left[ \frac{1}{q^2} + G_{10} \left( (\sqrt{-q^2})^6 \log(-q^2) + \sqrt{-q^2}^5 M \right) \right], \quad (3.13)$$

4 The constant which makes the argument of the logarithm dimensionless gives a contact term in coordinate space which we ignore in our discussion.
with some finite numerical coefficients that can be calculated. After Fourier transforming we obtain the Newtonian potential

\[ V(r) \sim \frac{G_{10} M^2}{r^7} \left( 1 + a \frac{G_{10} M}{r^7} + b \frac{G_{10} \bar{h}}{r^8} + \ldots \right). \] (3.14)

Here we have restored the \( \bar{h} \) dependence by dimensional analysis. This result takes the same form as (2.17). Of course in section 2 we have been using conventions where \( \bar{h} = 1 \) as one usually does in string theory. We can absorb the value of \( \bar{h} \) in (3.14) with a rescaling of the metric [19]. This will introduce the dilaton dependence. Consider the action (3.5). Under a scale transformation where lengths are rescaled by a factor \( t^{-1} \) the metric transforms as

\[ g_{\mu\nu} \rightarrow t^{-2} g_{\mu\nu}. \] (3.15)

Under this transformation the curvature \( R_{\mu\nu} \) will remain invariant so that the ten-dimensional action scales as

\[ S \rightarrow t^{-8} S. \] (3.16)

At the classical level the normalization of the action is not relevant so that general relativity is scale invariant at this level. When we pass to the quantum theory we compute a path integral

\[ Z = \int e^{i \bar{\pi} S}. \] (3.17)

The parameter \( \bar{h} \) appearing in this expression can be absorbed into a transformation of the type (3.15). In other words, under a scale transformation \( \bar{h} \) will transform as \( \bar{h} \rightarrow t^{-8} \bar{h} \). Furthermore, type IIA supergravity has the same classical scale invariance if the dilaton behaves in the following way under scale transformations

\[ \phi \rightarrow t^2 \phi. \] (3.18)

From these considerations it follows that the effect of order \( \bar{h} \) that we have been considering corresponds to a \( \phi^{-4} \)-effect in type IIA supergravity, or equivalently a \( 1/g_s^2 \)-effect. This is what we wanted to show.
4. Conclusions

In this paper we have shown that the Matrix model for Schwarzschild black holes of \cite{2} and \cite{3} describes correctly the leading long range quantum gravity correction to Newton’s potential in eleven dimensions. This effect is finite and can be computed from the supergravity point of view from one-loop Feynman diagrams. We have found agreement up to numerical factors that we have not calculated. We have further shown that relativistic corrections to Newton’s potential are correctly reproduced. Obviously it would be of interest to know if it is possible to match the precise numerical coefficients between the Matrix model and supergravity. Since the model of \cite{2} and \cite{3} purports to describe Schwarzschild black holes in diverse dimensions as toroidal compactifications of the eleven-dimensional model it would be interesting to know if the lower dimensional models also correctly reproduce the finite quantum gravity correction to Newton’s potential.

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