Design and Performance Analysis of Low Reynolds Number Propeller using Analytical Methods by Varying Blades Alpha Design

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Abstract. In the past few years, the usage of Unmanned Aerial Vehicles (UAVs) aircraft has increased significantly, but the need for UAVs with higher performance is increasing also. Therefore, all systems of the UAVs must be developed to have high efficiency, including the propulsion system. It is necessary to perform some design and analysis tasks on low Reynolds number propeller, which is a commonly used propulsion system on UAVs, to have higher efficiency and improve the overall UAVs performance. In this paper, low Reynolds number propellers are designed using the Larrabee Design analytical method by referring to the "Master AirScrew 10x7E" propeller. Meanwhile, performance of the designed propellers is analysed by using analytical method of Inversing Larrabee Design method. At the design stage of low Reynolds number propeller, 42 variations of alpha design (α_{des}) was inputted along the blades with uniform, linear, and quadratic distributions. Furthermore, the best design from these 42 variations was selected by performing two-stage scoring. At first stage scoring, 42 designs are selected to obtain 5 designs with the highest on-design efficiency. Then, performance of these 5 selected designs are analysed analytically along a certain range of advance ratio. Finally, second stage scoring is performed to obtain 1 design with the best overall performance.

Keywords: low Reynolds number propeller, efficiency, propeller design, alpha design, Larrabee method.

1. Introduction
The usage of Unmanned Aerial Vehicles (UAVs) aircraft have boomed in the past few years, both for military or commercial application. The need for UAVs with higher performance is also increasing. Therefore, performance of all systems of the UAVs must be increased to achieve UAV with higher efficiency, including the propulsion system. Propulsion system that is commonly used on UAVs aircraft is non-conventional propeller which has small diameter, operating at lower Reynolds number (based on chord at 0.75 radius location) , low axial speed, and high revolution per minute (RPM). The researches on this type of propeller are relatively rare. Therefore, it is very important to design and analyse the performance of the low Reynolds number propeller to have higher efficiency, so that the UAV aircraft has reliable performance.
There are several common methods that can be used to perform design and analysis tasks on propeller performance. These methods are analytical methods, numerical simulations using computational tools, and experimental methods. In analytical methods, design and performance analysis of propeller is performed based on the development of various propeller theories, such as vortex theory, blade element theory, momentum theory, and a combination thereof. In numerical simulation methods, propeller performance analysis is carried out by utilizing the Reynold Average Navier Stoke (RANS) equation to solve physical problem of propeller phenomena using Computational Fluid Dynamics (CFD). Whereas in the experimental methods, the propeller performance analysis is carried out by testing the propeller using wind tunnel, both in static conditions and along a certain range of advance ratio.

This research focuses on design and performance analysis of low Reynolds number propeller using analytical method by varying alpha design ($\alpha_{des}$). Several studies have been carried out by other researchers previously related to this research. Brand et al. [1], have carried out experimental studies using wind tunnel to determine the performance of 79 low Reynolds number propellers ranging in diameter 9 to 11 inch. The experiment was conducted to see the impact of Reynolds numbers and advance ratio on propeller performance. The study shows that propeller performance is increased with increasing of Reynolds number. Similar research was conducted by Merchant et al. [2], who performed experimental studies using wind tunnel to determine performance of low Reynolds number propellers at ranges of Reynolds number from 30,000 to 300,000. Kutty, et al. [3] performed a numerical CFD simulation to predict the performance of small-sized propeller 'APC Slow Flyer' using ANSYS Fluent solver. The results of numerical simulations were then compared with the 'APC Slow Flyer' propeller experimental data. The results show that the propeller performance data that were predicted using numerical simulations are close enough to the propeller performance data that were obtained experimentally. Kumar et al. [4] also conducted an analysis of 9 inches diameter propeller using the numerical CFD simulation method. Before analyzing with CFD, CAD data of propeller was obtained using the reverse engineering method. The thrust value from numerical simulation method was then compared to the thrust value from the analytical calculation. The difference in the value of thrust resulting from these two methods is 27.12%. Research that is related to the design and analysis of the performance of low Reynolds number propellers still requires further development.

2. Theoretical Formulation

2.1. Larrabee Design Method

Larrabee design is an analytical method that is used to design a propeller. This method is based on momentum theory, blade element theory, and vortex theory. Larrabee design method calculates the behavior of a propeller by dividing the propeller blades into small parts. Each part of this blade acts as an airfoil that produces two-dimensional aerodynamic forces. This method requires power or thrust coefficient desired ($C_{p_{des}}$ or $C_{T_{des}}$), propeller diameter ($D$), hub to tip ratio (HTR), number of propeller blades ($B$), operating condition of propeller ($V_0$, RPM, $h$, $\rho$, $\mu$), alpha design ($\alpha_{des}$) and airfoil data ($c_1$, $c_d$) as inputs. Equation of thrust and power on each blade element in this method are presented in Eq. (1) and (2),

$$
\frac{dT_e}{d\xi} = 4\xi F \frac{x^2(1 + \xi)}{(1 + \frac{1}{2}\xi [(1 + \xi)^2 + x^2])}
\left[1 - \frac{D}{L} \frac{1}{2}(1 + \frac{1}{2}\xi [(1 + \xi)^2 + x^2]) - \frac{1}{2}\xi x^2\right]
$$

(1)

$$
\frac{dP_e}{d\xi} = 4\xi F \frac{x^2(1 + \xi)}{(1 + \frac{1}{2}\xi [(1 + \xi)^2 + x^2])}
\left[1 + \frac{D}{L} \frac{1}{2}(1 + \frac{1}{2}\xi [(1 + \xi)^2 + x^2]) - \frac{1}{2}\xi x^2\right]
$$

(2)
Based on this method, propeller performance at on-design condition can be obtained.

2.2. Inversing Larrabee Design Method

Inversing Larrabee design method is an analytical method to analyze performance of propeller. This method requires propeller diameter \((D)\), hub to tip ratio \((HTR)\), number of propeller blades \((B)\), chord distribution \((c/R)\), twist distribution \((\theta)\), operating condition of propeller \((V_0, h, \rho, \mu)\), and airfoil data \((c_l, c_d)\) as inputs. Propeller performance analysis is performed by comparing the thrust value from general momentum theory and blade element theory iteratively. Iteration is executed continuously until thrust that is calculated from these 2 theories generates the same value. Thrust on each propeller blade element based on the general momentum theory is obtained by using the following equation:

\[
\frac{dT_c}{d\xi} = \frac{B}{\pi} \left( \frac{W}{V} \right)^2 \frac{c}{R} [C_l \cos \theta - C_d \sin \theta]
\]  

(3)

Propeller performance is commonly expressed as non-dimensional quantities. The general parameters used to analyze propeller performance are thrust coefficient, power coefficient, and propeller efficiency, which are obtained using the following equations,

\[
C_T = \frac{T}{\rho n^2 D^4}
\]  

(4)

\[
C_p = \frac{P}{\rho n^3 D^5}
\]  

(5)

\[
\eta = \frac{C_T}{C_p}
\]  

(6)

while \(n\) states number of propeller revolutions per second and \(f\) is an advance ratio which states the relationship between axial speed, propeller rotational speed, and propeller diameter with the following equation,

\[
n = \frac{RPM}{60}
\]  

(7)

\[
f = \frac{V_0}{nD}
\]  

(8)

2.3. Scoring

Scoring is a method used to select the best design from the overall design by giving a score to each design. The design with the highest score is considered as the best design. In this study, the best design selection was obtained by conducting two scoring stages which will be explained as follows,

First stage scoring. On this stage, scoring is carried out on the design advance ratio \((f=0.643)\) by comparing the performance of the designed propeller with the performance of the 'Master AirScrew 10x7E' reference propeller. The formula used on first stage scoring is as follows,

\[
S_1 = \frac{\eta_{design}}{10} \left( f_p \vert_{design} + 4 f_T \vert_{design} + 5 f_n \vert_{design} \right)
\]  

(9)

with,

\[
f_p \vert_{design} = \begin{cases} 0 & C_{p,design} \vert_{design} - C_{p,Master AirScrew} \vert_{design} \geq 0 \\
1 & C_{p,design} \vert_{design} - C_{p,Master AirScrew} \vert_{design} < 0 
\end{cases}
\]
From this stage, 5 designs with the highest score is chosen to be selected in scoring stage 2.

Second stage scoring. Second stage scoring is carried out by comparing the performance of designed propeller and performance of the 'Master AirScrew 10 x7E' reference propeller at each values of advance ratio ranging from 0.147 to 0.712. That advance ratio range is chosen based on the availability of experimental performance data of the 'Master AirScrew 10x7E' propeller. Before second stage scoring is executed, propeller performance analysis was carried out on 5 selected designs at ranges of advance ratio from 0.147 to 0.712 using inversing Larrabee method. The formula used for Second stage scoring is as follows,

\[ S_2 = \sum_{i=1}^{9} k_i \left[ \eta_{\text{design}} |_{J_i} \left( f_p |_{J_i} + 4 f_T |_{J_i} + 5 f_\eta |_{J_i} \right) \right] \]

with,

- \( f_p |_{J_i} = \begin{cases} 0, & C_{\text{p,design}} |_{J_i} - C_{\text{p,Master AirScrew}} |_{J_i} \geq 0 \\ 1, & C_{\text{p,design}} |_{J_i} - C_{\text{p,Master AirScrew}} |_{J_i} < 0 \end{cases} \)

- \( f_T |_{J_i} = \begin{cases} 0, & C_{\text{T,design}} |_{J_i} - C_{\text{T,Master AirScrew}} |_{J_i} \leq 0 \\ 1, & C_{\text{T,design}} |_{J_i} - C_{\text{T,Master AirScrew}} |_{J_i} > 0 \end{cases} \)

- \( f_\eta |_{J_i} = \begin{cases} 0, & \eta_{\text{design}} |_{J_i} - \eta_{\text{Master AirScrew}} |_{J_i} \leq 0 \\ 1, & \eta_{\text{design}} |_{J_i} - \eta_{\text{Master AirScrew}} |_{J_i} > 0 \end{cases} \)

and,

Table 1. The Value of Multiplier Factor at Each Advance Ratio

| \( l \) | \( J \) | \( k \) |
|---|---|---|
| 1 | 0.147 | 12.5 |
| 2 | 0.218 | 25 |
| 3 | 0.286 | 37.5 |
| 4 | 0.360 | 50 |
| 5 | 0.423 | 62.5 |
| 6 | 0.504 | 75 |
| 7 | 0.575 | 87.5 |
| 8 | 0.643 | 100 |
| 9 | 0.712 | 87.5 |

From Table 1, it can be seen that multiplier factor \( k \) is set so that it has gradual values along the ranges of advance ratio which are analyzed in this scoring stage. So, propeller performance on the advance ratio value
which is closer to on-design operating conditions \((J = 0.643)\) has a greater effect on scoring compared to the advance ratio value which is further from on-design operating conditions. For example, it can be seen that multiplier factor at \(J = 0.147\) is smaller than multiplier factor at \(J = 0.575\). Meanwhile, the multiplier factor on the on-design condition \((J = 0.643)\) has the biggest value.

3. Low Reynolds Number Propeller Design

In this research, Design Requirement and Objectives (DRO) of propeller refers to the geometrical characteristics of low Reynolds number propeller 'Master Air Screw 10x7 E' and its sea-level performance at 4008 RPM that was obtained through experimental results conducted by Brandt [5]. Fig. 1 show experimental results of sea level performance of 'Master Air Screw 10x7 E' propeller at 4008 RPM,

![](image)

**Figure 1.** Sea-Level Performance of 'Master Air Screw 10x7E' Propeller at 4008 RPM, (a) \(C_T \) vs \(J\), (b) \(C_p \) vs \(J\), and (c) \(\eta \) vs \(J\)

From Fig. 1(c), it can be seen that the maximum efficiency of the "Master AirScrew 10x7E" propeller based on experimental results at 4008 RPM is obtained at the advance ratio of 0.643 with an efficiency value of 0.629. On that advance ratio, it can be obtained propeller thrust and power coefficient value based on Fig. 1(a) and (b) respectively is 0.052 and 0.053. Then using Eq. (8), the axial velocity of the propeller on that advance ratio is 10.91 m/s. Then, it is obtained DRO specifications that are used in this study are as follows,

| Specification of Low Reynolds Number Propeller |
|-----------------------------------------------|
| **Axial velocity (\(V_0\))**                  |
| 10.91 m/s                                     |
| **Rotational speed (RPM)**                    |
| 4008                                          |
| **Diameter (\(D\))**                          |
| 10 in                                         |
| **Number of blades (\(B\))**                  |
| 2                                             |
| Hub to tip ratio (HTR) | 0.15 |
|-----------------------|------|
| Altitude (h)          | 0 m (sea-level) |
| Advance Ratio (β)     | 0.643 |

**Design Objectives**

| Power coefficient desired ($C_{p_{des}}$) | 0.053 |

The type airfoil that is used by the propeller design in this study is NACA 4412.

### 4. Study of Alpha Design ($\alpha_{des}$) Variations

In this research, propeller design are varied by giving 42 different $\alpha_{des}$ inputs. From these 42 $\alpha_{des}$ inputs, there are 6 inputs with $\alpha_{des}$ characteristics that are uniform along the blade span, 8 inputs with the characteristics of $\alpha_{des}$ are linear distribution, and 28 inputs with $\alpha_{des}$ characteristics are quadratic distribution. All variations of $\alpha_{des}$ that were inputted in this research are shown in Fig. 2 below.

![Figure 2](image-url)  
**Figure 2.** Alpha Design Along the Blade Span, (a) Uniform Distribution, (b) Linear Distribution, (c) Quadratic Distribution
5. Results
The on-design performances of propellers are obtaining which include thrust coefficient, power coefficient, and efficiency of each design as follows.

Table 3. On-Design Performance of All Designed Propellers

| Design | $C_T$ | $C_P$ | $\eta$ | Design | $C_T$ | $C_P$ | $\eta$ | Design | $C_T$ | $C_P$ | $\eta$ | Design | $C_T$ | $C_P$ | $\eta$ |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1      | 0.060  | 0.053  | 0.721  | 15     | 0.065  | 0.053  | 0.783  | 29      | 0.054  | 0.053  | 0.651  | 1       | 0.060  | 0.053  | 0.721  |
| 2      | 0.063  | 0.053  | 0.766  | 16     | -      | -      | -      | 30      | 0.061  | 0.053  | 0.733  | 15      | 0.065  | 0.053  | 0.783  |
| 3      | 0.065  | 0.053  | 0.782  | 17     | 0.064  | 0.053  | 0.768  | 31      | 0.063  | 0.053  | 0.763  | 29      | 0.065  | 0.053  | 0.783  |
| 4      | 0.065  | 0.053  | 0.783  | 18     | 0.065  | 0.053  | 0.782  | 32      | 0.059  | 0.053  | 0.708  | 29      | 0.065  | 0.053  | 0.783  |
| 5      | 0.065  | 0.053  | 0.780  | 19     | 0.064  | 0.053  | 0.769  | 33      | 0.064  | 0.053  | 0.769  | 31      | 0.064  | 0.053  | 0.763  |
| 6      | 0.064  | 0.053  | 0.769  | 20     | 0.049  | 0.053  | 0.592  | 34      | 0.065  | 0.053  | 0.782  | 31      | 0.064  | 0.053  | 0.763  |
| 7      | -      | -      | -      | 21     | 0.057  | 0.053  | 0.684  | 35      | 0.058  | 0.053  | 0.707  | 36      | 0.061  | 0.053  | 0.760  |
| 8      | 0.065  | 0.053  | 0.782  | 22     | 0.064  | 0.053  | 0.768  | 36      | 0.063  | 0.053  | 0.760  | 29      | 0.065  | 0.053  | 0.783  |
| 9      | 0.061  | 0.053  | 0.740  | 23     | 0.055  | 0.053  | 0.665  | 37      | 0.064  | 0.053  | 0.769  | 30      | 0.061  | 0.053  | 0.763  |
| 10     | 0.062  | 0.053  | 0.755  | 24     | 0.061  | 0.053  | 0.733  | 38      | 0.062  | 0.053  | 0.748  | 31      | 0.062  | 0.053  | 0.763  |
| 11     | 0.061  | 0.053  | 0.734  | 25     | -      | -      | -      | 39      | 0.065  | 0.053  | 0.782  | 32      | 0.064  | 0.053  | 0.772  |
| 12     | 0.064  | 0.053  | 0.776  | 26     | 0.064  | 0.053  | 0.778  | 40      | 0.061  | 0.053  | 0.738  | 33      | 0.064  | 0.053  | 0.769  |
| 13     | 0.057  | 0.053  | 0.692  | 27     | 0.065  | 0.053  | 0.783  | 41      | 0.064  | 0.053  | 0.772  | 34      | 0.064  | 0.053  | 0.760  |
| 14     | 0.064  | 0.053  | 0.772  | 28     | 0.064  | 0.053  | 0.773  | 42      | 0.063  | 0.053  | 0.756  | 35      | 0.064  | 0.053  | 0.782  |

Design or 7, 16 and 25 were not analysed because they gave the results that were considered irrational.

From the overall designed propeller, then first stage scoring is executed. The results of first stage scoring are summarized in Table 4 below.

Table 4. The Results of First Stage Scoring

| Design | $S_1$ | Design | $S_1$ | Design | $S_1$ | Design | $S_1$ | Design | $S_1$ | Design | $S_1$ | Design | $S_1$ | Design | $S_1$ |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1      | 0.721  | 8      | 0.782  | 15     | 0.783  | 22     | 0.692  | 29      | 0.586  | 36      | 0.760  | 36      | 0.760  | 16      | 0.766  |
| 2      | 0.766  | 9      | 0.740  | 16     | -      | 17     | 0.768  | 24      | 0.733  | 31      | 0.763  | 31      | 0.763  | 17      | 0.768  |
| 3      | 0.782  | 10     | 0.755  | 17     | 0.768  | 24     | 0.733  | 31      | 0.763  | 38      | 0.748  | 38      | 0.748  | 18      | 0.782  |
| 4      | 0.705  | 11     | 0.734  | 18     | 0.782  | 25     | -      | 32      | 0.637  | 39      | 0.782  | 39      | 0.782  | 25      | 0.782  |
| 5      | 0.780  | 12     | 0.776  | 19     | 0.769  | 26     | 0.778  | 33      | 0.692  | 40      | 0.665  | 40      | 0.665  | 26      | 0.778  |
| 6      | 0.769  | 13     | 0.692  | 20     | 0.355  | 27     | 0.705  | 34      | 0.782  | 41      | 0.772  | 41      | 0.772  | 27      | 0.705  |
| 7      | -      | 14     | 0.772  | 21     | 0.684  | 28     | 0.696  | 35      | 0.636  | 42      | 0.756  | 42      | 0.756  | 28      | 0.696  |

From Table 4, it can be seen that 5 propeller designs that have the highest on-design performances are design 3, 8, 15, 34, and 39. Furthermore, the propeller performances of the 5 selected designs are then analyzed along a range of advance ratio from 0.147 to 0.712 using inversing Larrabee design method. The plot of the propeller performances in the form of thrust coefficient, power coefficient, and efficiency of the selected propeller designs are shown in Fig. 3 below.
Finally, second stage scoring was carried out on 5 selected designs. The results of second stage scoring are summarized in Table 5 below.

Table 5. The Results of Second Stage Scoring

| Design | $S_2$  |
|--------|--------|
| 3      | 0.636  |
| 8      | 0.647  |
| 15     | 0.622  |
| 34     | 0.664  |
| 39     | 0.619  |

Table 5 shows that design 34 has the highest overall performances.
6. Analysis

![Figure 4](image)

**Figure 4.** The Scheme How $\alpha_{des}$ Affects the Propellers Performance

Based on the study of $\alpha_{des}$ variations that have been carried out, it can be concluded that $\alpha_{des}$ affects the performance of the propellers with the scheme as shown in Fig. 4. It can be seen that $\alpha_{des}$ along the blade span will affect the output of the chord distribution that is generated at the propeller design stage. Then, the chord distribution will affect the Reynolds number that works along the propeller blades. The Reynolds number that works on the propeller blade affects the aerodynamic characteristics of the airfoil blades. In addition, the aerodynamic characteristics of the airfoil blades are also directly affected by the $\alpha_{des}$ distribution along the blade span. Finally, airfoil aerodynamic characteristics affect the performance of the propeller design.

Propeller that provides the best overall performance, which in this research is design 34, is a propeller that has the best aerodynamic efficiency along the advance ratio, which is affected either directly or indirectly by $\alpha_{des}$. It can be seen from Fig. 2(c) that design 34 has quadratic distribution of $\alpha_{des}$ along the blade span. But in general, $\alpha_{des}$ with quadratic distribution is not always provides the best overall performance, because the results that are obtained in this research are also influenced by other inputs, such as type of airfoil. It's difficult to get an alpha design distribution which produces the best overall propeller performance in all design cases. So, if a similar research is carried out but using different input, for example, type of airfoil, then the results obtained may be different from the results of this research.

7. Conclusions

This study conclude that the alpha design ($\alpha_{des}$) affects the performance of the propeller, where design 34 provides the best overall performance of the propeller. The distribution of $\alpha_{des}$ which produces the best overall propeller performances is still uncertainty as the performance are also influenced by other inputs. Further study on airfoil variations used for propeller blades is needed.

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