QUADRATIC SOLITONS IN CUBIC CRYSTALS

Boris V. Gisin
Department of Electrical Engineering - Physical Electronics, Faculty of Engineering,
Tel-Aviv University, Tel-Aviv 69978, Israel

Boris A. Malomed
Department of Interdisciplinary Studies, Faculty of Engineering, Tel Aviv University, Tel
Aviv 69978, Israel

Abstract

Starting from the Maxwell’s equations and without resort to the paraxial approxima-
tion, we derive equations describing stationary (1+1)-dimensional beams propagating at an
arbitrary direction in an optical crystal with cubic symmetry and purely quadratic ($\chi^{(2)}$)
nonlinearity. The equations are derived separately for beams with the TE and TM polar-
izations. In both cases, they contain $\chi^{(2)}$ and cubic ($\chi^{(3)}$) nonlinear terms, the latter ones
generated via the cascading mechanism. The final TE equations and soliton solutions to
them are quite similar to those in previously known models with mixed $\chi^{(2)}$-$\chi^{(3)}$ nonlin-
earities. On the contrary to this, the TM model is very different from previously known
ones. It consists of four first-order equations for transverse and longitudinal components of
the electric field at the fundamental and second harmonics. Fundamental-soliton solutions
of the TM model are also drastically different from the usual $\chi^{(2)}$ solitons, in terms of the
parity of their components. In particular, the transverse and longitudinal components of the
electric field at the fundamental harmonic in the fundamental TM solitons are described,
respectively, by odd and single-humped even functions of the transverse coordinate. Amplitu-
des of the longitudinal and transverse fields become comparable for very narrow solitons,
whose width is commensurate to the carrier wavelength.
1 Introduction

Optical crystals with the cubic symmetry are the basis of contemporary photonics. In particular, spatial optical solitons generated by the $\chi^{(2)}$ (quadratic) nonlinearity, see reviews [1, 2], are possible in these media. In the simplest case, these are two-component pulses composed of fundamental-frequency (FF) and second-harmonic (SH) waves.

Usually, solitons in noncentrosymmetric crystals are considered when directions of the light propagation and polarization coincide with the crystal axes. However, in cubic crystals it is possible to investigate solitons carried by a beam which is propagating in arbitrary direction, the situation being similar to that known in electrooptics [3]. In the present work, our objective is to study spatial solitons of both TE and TM (transverse-electric and transverse-magnetic) types with the arbitrary propagation direction of the carrier wave. First, we derive a corresponding general model (actually, we obtain a system of ODEs to describe a stationary shape of the (1+1)-dimensional solitons). The model is derived directly from the Maxwell’s equations, without using the paraxial approximation, so that the final equations apply as well to very narrow (subwavelength) spatial solitons.

In the case of the TE polarization, which is considered in section 2, the obtained model differs from the well-known simplest model giving rise to the $\chi^{(2)}$ solitons by a single additional cubic ($\chi^{(3)}$) term in the equation for the transverse FF component, which is generated by the underlying quadratic nonlinearities via the cascading mechanism (see a review [4]). As solitons in this TE model are very close to those in the known $\chi^{(2)}-\chi^{(3)}$ models, we consider them, also in section 2, only briefly.

Unlike the TE case, the final model for the TM solitons, which is derived in section 3, turns out to be essentially novel, consisting of four first-order equations. Two equations for transverse FF and SH electric fields contain only quadratic nonlinear terms, while two other equations, for longitudinal electric fields, contain cascading-generated cubic terms too. Then, in section 4 we find, in a numerical form, stationary solutions for both fundamental and higher-order solitons in the TM model. The fundamental TM solitons turn out to be drastically different, as concerns the parity of their components, from solitons in the traditional $\chi^{(2)}$ models: their longitudinal FF electric field, which has no counterpart among components of the usual $\chi^{(2)}$ solitons, is described by an even single-humped function, while the transverse TE electric field, which is an analog of the FF field in the usual solitons, is an odd function of the transverse coordinate $x$. Moreover, the transverse SH electric field, although being an even function, is not monotonically decaying in the interval $0 < x < \infty$, but rather changes its sign at a finite $x$.

The $\chi^{(2)}$ contribution to the polarization induced by the electric field with real vectorial components $E_n$ in noncentrosymmetric dielectric crystals can be represented in the form $P^{(2)} = d_{lmn}E_mE_n$ (summation over repeated subscripts is assumed), where $d_{lmn}$ is a nonlinear susceptibility tensor [5]. Obviously, it is symmetric with respect to the permutation of the subscripts $m$ and $n$. For cubic noncentrosymmetric crystals in the crystallographic coordinates $\tilde{x}, \tilde{y}, \tilde{z}$, nonvanishing components of the tensor are $d_{123} = d_{213} = d_{312} \equiv d [6]$.

We will consider light propagation in an arbitrary direction, relative to the axes $\tilde{x}, \tilde{y}, \tilde{z}$, in a new coordinate frame $(x, y, z)$, the $z$-axis being aligned with the propagation direction. The new frame may be generated by consecutive rotations of the initial one by angles $\varphi$, $\vartheta$ and $\gamma$ around the $\tilde{z}$, $\tilde{x}$, and $z$ axes, respectively. The matrix $(a_{mn})$ of the orthogonal transformation
to the new frame, \( x_k = a_k t \), where \( \bar{x}_k \) and \( x_k \) stand for the sets \((\bar{x}, \bar{y}, \bar{z})\) and \((x, y, z)\), is

\[
\begin{pmatrix}
\cos \varphi \cos \gamma - \cos \vartheta \sin \varphi \sin \gamma & \sin \varphi \cos \gamma + \cos \vartheta \cos \varphi \sin \gamma & -\sin \vartheta \sin \varphi \\
-\cos \varphi \sin \gamma - \cos \vartheta \sin \varphi \cos \gamma & -\sin \varphi \sin \gamma + \cos \vartheta \cos \varphi \cos \gamma & -\sin \vartheta \cos \gamma \\
-\sin \vartheta \sin \varphi & \sin \vartheta \cos \varphi & \cos \vartheta
\end{pmatrix}
\]

(1)

Components of the dielectric displacement vector in the new frame are

\[
D_k = \varepsilon E_k + 2d_{kmn}E_mE_n,
\]

(2)

where \( \varepsilon \) is the dielectric constant, and

\[
d_{kmn} = a_{kl}a_{mp}a_{nq}d_{lpq} \equiv 2d(a_{k1}a_{m2}a_{n3} + a_{k2}a_{m1}a_{n3} + a_{k3}a_{m1}a_{n2})
\]

is the \( \chi^{(2)} \) susceptibility tensor in the new frame. In the general case, all the 27 tensor components are different from zero, but they depend only on one constant \( d \) and three angles \( \varphi, \vartheta \) and \( \gamma \).

Below, the full electric and magnetic fields, including both FF and SH components, will be sought for in the form

\[
E_k = a_k \cos (2\Phi) + b_k \sin (2\Phi) + A_k \cos \Phi + B_k \sin \Phi,
\]

(3)

\[
H_k = p_k \cos (2\Phi) + q_k \sin (2\Phi) + P_k \cos \Phi + Q_k \sin \Phi.
\]

(4)

Here, the FF phase is \( \Phi = \beta z - \omega t \), \( \omega \) is the frequency, \( t \) is time, and \( \beta \) is the propagation constant, the small and capital letters standing for the SH and FF amplitudes, respectively. In the general case, with the carrier wave propagating in an arbitrary direction, solutions are of the mixed TE-TM type, i.e., they have all the electric and magnetic field components. However, pure TE and TM modes will be found below for certain angles \( \varphi, \vartheta \) and \( \gamma \), including nontrivial cases when the propagation and polarization directions are not aligned with the crystallographic axes \((\bar{x}, \bar{y}, \bar{z})\).

To conclude the introduction, it is relevant to mention several other models of the \( \chi^{(2)} \) type which take into regard the polarization of the fields. They describe bimodal fields in an isotropic medium \([\text{I}]\), or the so-called type-II interactions between the FF and SH waves, with a single (see, e.g., Refs. \([\text{II}]\)) or double \([\text{III}]\) parametric resonance between them. However, none of these previously studied models included longitudinal components of the fields, nor magnetic field was explicitly introduced in them.

## 2 The TE case

First, we consider a simpler case with the FF wave of the TE type, i.e., when the electric FF field has only one component \( A_2 \), which is a function of \( x \), but directed along the \( y \) axis. Then, the only nonzero FF components are \( A_2 \equiv A \), \( P_1 \equiv P \), and \( Q_3 \equiv Q \), the latter one being longitudinal magnetic field. Substituting \( E_k \) and \( H_k \) from Eqs. (3) and (4) into the Maxwell’s equations, it is possible to eliminate the magnetic-field components and obtain a
set of differential equations for the electric components only:

\[ A'' = \left( \beta^2 - \beta_1^2 \right) A - 2\beta_0^2 A(d_{221} a_1 + d_{222} a_2 + d_{223} a_3), \]
\[ a''_1 = 4 \left( \beta^2 - \beta_2^2 \right) a_1 - 4\beta_0^2 d_{122} A^2 - \varepsilon_2^{-1} d_{122}(A^2)'', \]
\[ a''_2 = 4 \left( \beta^2 - \beta_2^2 \right) a_2 - 4\beta_0^2 d_{222} A^2, \]
\[ a''_3 = 4 \left( \beta^2 - \beta_2^2 \right) \left( a_3 + \varepsilon_2^{-1} d_{322} A^2 \right), \]
\[ b''_1 = 4 \left( \beta^2 - \beta_2^2 \right) b_1 - 2\beta\varepsilon_2^{-1} d_{322}(A^2)', \]
\[ b''_3 = 4 \left( \beta^2 - \beta_2^2 \right) b_3 + 2\beta\varepsilon_2^{-1} d_{122}(A^2)', \]
\[ b''_2 = 4 \left( \beta^2 - \beta_2^2 \right) b_2, \]

where the prime stands for \(d/dx\), \(\beta_0 \equiv \omega/c\) is the FF propagation constant in vacuum, \(c\) being the light velocity in vacuum, \(\beta_{1,2}^2 \equiv \varepsilon_{1,2}\beta_0^2\), and \(\varepsilon_{1,2}\) are FF and SH dielectric permeabilities. From the condition that the fields must vanish at \(x = \pm \infty\), it immediately follows from the last equation in the system (3) that \(b_2 \equiv 0\), and it can be shown too that \(a_1 = p_3 \equiv 0\).

Note that, while \(\beta\) is the actual propagation constant for the FF component of the soliton-carrying beam, \(\beta_1\) and \(2\beta_2\) are the wave numbers of linear plane waves propagating in the medium at the frequencies \(\omega\) and \(2\omega\). In fact, \(\beta\), which is different from \(\beta_1\) because of the nonlinearity and finite transverse size of the soliton beam, is a parameter of the soliton family at fixed values of the carrier-wave propagation constants \(\beta_{1,2}\).

In addition to the differential equations (5), it also follows from the Maxwell’s equations that the SH electric filed must satisfy a set of linear algebraic relations, \(d_{12m} a_m = d_{32m} a_m = d_{12m} b_m = d_{22m} b_m = d_{32m} b_m = 0\). We do not display details of analysis which shows that Eqs. (3) are compatible with the extra linear relations at some special values of the angles \((\varphi, \theta, \gamma)\). After lengthy transformations, in all such cases Eqs. (3) can be reduced to a simple system of two equations,

\[ \frac{d^2 R}{d\eta^2} = R - uR + \delta \cdot R^3, \]  
\[ \frac{d^2 u}{d\eta^2} = 4\left( \frac{\beta^2 - \beta_2^2}{\beta^2 - \beta_1^2} \right) u \pm R^2, \]

where \(R\) and \(u\) are certain combinations of the FF and SH electric fields, \(\eta \equiv \sqrt{\beta^2 - \beta_1^2} x\), and \(\delta\) is some other constant.

In particular, a nontrivial compatibility case can be obtained by choosing \(\cos \vartheta = \cos \gamma = \cos(2\varphi) = 0\). One then has \(d_{122} = \pm d, a_2 = a_3 = b_1 = 0\), and

\[ R = \sqrt{2D} (\beta/\beta_2) A, \quad u = D(a_1 + A^2/\varepsilon_2), \quad b_3 = -\left( \sqrt{\beta^2 - \beta_1^2}/2\beta D \right) \frac{du}{d\eta}, \]

with \(\delta \equiv (\beta^2 - \beta_1^2)/4\beta^2, D \equiv 2d_{122}\omega^2/c\).

A special case when all the equations are compatible and \(\delta = 0\) (no effective cubic nonlinearity) corresponds (for instance) to \(\cos(2\vartheta) = \sin \varphi = \sin \gamma = 0\), which again yields \(d_{122} = \pm d\). In this case, Eqs. (4) and (8) are equivalent to the simplest version of the
stationary $\chi^{(2)}$ model, hence they have well-known soliton solutions $[2]$. If $\delta \neq 0$, the system differs from the simplest one by the extra $\chi^{(3)}$ term in Eq. (4).

Soliton-generating models combining $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities are well known too, see, e.g., Refs. $[9]$. Equations (3) and (8) are somewhat different from them, as they contain the cubic term solely in the FF equation. In models of this type, soliton solutions can be easily found $[9]$. A typical example of a TE fundamental (single-humped) soliton obtained from Eqs. (3), (4) and (9) is displayed in Fig. 1.

Cases when all the equations are compatible and, eventually, amount to the simplest system of Eqs. (7) and (8) are possible not only for discrete sets of values of the three angles, as in the two examples shown above, but also for certain one-parameter families, when two equations are imposed on the three angles. Examples are

$$\sin^2 \varphi = 2\sin^2 \gamma, \quad \sin^2 \vartheta \cos^2 \gamma = 1; \tag{10}$$

$$\cos^2 \varphi = 2\sin^2 \gamma, \quad \cos \vartheta = \tan \varphi \cdot \tan \gamma. \tag{11}$$

These examples do not exhaust all the compatibility cases. A complete analysis of the compatibility conditions for the TE type of the fields in the cubic crystal proves to be very tedious.

### 3 The TM case

We now proceed to a more interesting TM model, when nonzero FF amplitudes are $A_1 \equiv A$, $A_3$, and $P_2 \equiv P$, see Eqs. (4) and (5). For this model, stationary equations for the magnetic- and electric-field amplitudes can be derived in a general form, which, however, turns out to be very cumbersome. Therefore, we here concentrate on a particular (but quite nontrivial) case,

$$\sin 2\varphi = \sin \gamma = \cos 2\vartheta = 0. \tag{12}$$

In the present case, equations for nonvanishing fields can be cast into the following form:

$$\beta P = -\beta_0 (\varepsilon_1 A + 2dA_3b_3),$$

$$P' = \beta_0 [\varepsilon_1 A_3 + 2d(AB_3 - A_3a_1)],$$

$$\beta_0 P' = -\beta A - A'_3,$$

$$\beta p_2 = -\beta_0 (\varepsilon_2 a_1 - dA_3^2),$$

$$p'_2 = 2\beta_0 (\varepsilon_2 b_3 + 2dAA_3),$$

$$2\beta_0 p_2 = -2\beta a_1 - b'_3. \tag{13}$$

It is convenient to define renormalized transverse ($E_z, e_z$) and longitudinal ($E, e$) FF and SH electric fields,

$$\frac{2d\beta^2}{\sqrt{\beta^2 - \beta_1^2}} A_3 \equiv -E_z, \quad \frac{2d\beta^2}{\beta^2} A \equiv E,$$  

$$\frac{2d\beta^2}{\sqrt{\beta^2 - \beta_1^2}} b_3 \equiv -e_z, \quad \frac{2d\beta^2}{\beta^2} a_1 \equiv e. \tag{14}$$
Finally, the magnetic fields $P$ and $p_2$ can be eliminated from Eqs. (13), which yields a final fourth-order system for the electric fields (where, again, $\eta \equiv \sqrt{\beta^2 - \beta_1^2} x$),

\[
\begin{align*}
\frac{dE}{d\eta} &= E_z + E e_z - \left(3\frac{\beta^2}{\beta_1^2} - 2\frac{\beta_2^2}{\beta_1^2}\right) E_z e - \frac{\beta^2 - \beta_1^2}{\beta_1^2} E_z (E_z^2 - e_z^2), \\
\frac{dE_z}{d\eta} &= E - E_z e_z, \\
\frac{de}{d\eta} &= 2E_z + 3\frac{\beta^2 - \beta_1^2}{\beta_2^2} E E_z - \frac{\beta^2 - \beta_1^2}{\beta_2^2} E_z^2 e_z, \\
\frac{de_z}{d\eta} &= 2\frac{\beta^2 - \beta_1^2}{\beta^2 - \beta_1^2} e + E_z^2.
\end{align*}
\]

This model with mixed $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities appears to be novel, as compared to various known models of the $\chi^{(2)}$-$\chi^{(3)}$ type [9].

It is relevant to stress that, since both the TE and TM models, composed of Eqs. (7), (8) and (16), (17), respectively, have been derived directly from the Maxwell’s equations, without resorting to the paraxial approximation [2], the applicability of these models is not limited to the case of broad solitons, whose width is much larger than the carrier wavelength. In other words, the final equations derived above apply as well to very narrow subwavelength solitons, which have recently attracted considerable attention in spatial and temporal models with the Kerr nonlinearity [10], but were not thus far considered in $\chi^{(2)}$ systems.

4 TM solitons

As the system of equations (16) and (17) is essentially different from the standard models, it is necessary to investigate its soliton solutions, which we have done by means of the usual shooting technique. Fundamental and higher-order solitons were found in a broad parametric region.

The first noteworthy result is that fundamental solitons are characterized by a single-humped profile of the longitudinal electric FF field, corresponding to an even single-humped function $E_z(x)$, while the transverse FF field is described by an odd function $E(z)$. In other words, only the distribution of the squared longitudinal electric FF field, $E_z^2(x)$, is single-humped in the fundamental TM soliton, while the distribution of the squared transverse FF field, $E^2(x)$, is double-humped, with $E^2(0) = 0$. As for the SH components, it is evident from Eqs. (17) that the corresponding transverse and longitudinal fields $e$ and $e_z$ must be, respectively, even and odd functions of $x$, whatever parity of the FF fields $E$ and $E_z$. This feature is confirmed by Fig. 2; however, a nontrivial property of the fundamental soliton is that the even function $e(x)$ is not monotonic in the interval $0 < x < \infty$, giving rise to three humps and two zeros in the distribution of the corresponding squared field $e^2(x)$.

Note that the opposite parities of the FF transverse and longitudinal fields $E(x)$ and $E_z(x)$ is an obvious consequence of Eqs. (16). Evidently, besides the structure seen in Fig. 2, with the odd transverse and even longitudinal FF fields, the latter condition is also compatible with the opposite case, when $E(x)$ and $E_z(x)$ would be, respectively, even and odd. However, numerical solutions have never turned up TM solitons of that type.
Lastly, Fig. 2 clearly suggests a conclusion which is also supported by numerical solutions obtained at many other values of the parameters: in terms of the renormalized variables defined by Eqs. (14) and (15), the maximum values of the longitudinal and transverse fields are nearly equal. To compare the maximum values of the physical fields, one should undo the rescalings (14) and (15). An obvious result is that the amplitudes of the longitudinal FF and SH electric fields remain much smaller than the amplitudes of the transverse fields if $\sqrt{\beta^2 - \beta_1^2} \ll \beta$, which is the case for the usual solitons whose width is much larger than the carrier wavelength. For narrow solitons, whose width is comparable to or smaller than the wavelength, the physical amplitudes of the longitudinal and transverse fields become comparable too.

Thus, the properties of the TM soliton are drastically different from, and essentially more nontrivial than those of the TE soliton. Indeed, the latter one has the transverse FF electric field described by an even function $A_2(x)$, which is monotonically decaying in the interval $0 < x < \infty$. Another strong difference is that, although the SH transverse electric field in both the TM and TE solitons is described by an even function, only in the latter case this function is monotonic at $0 < x < \infty$.

It is also natural to compare the TE and TM solitons found above with solitons in the usual $\chi^{(2)}$ models, which, as a matter of fact, are also of the TE type. As well as the TE soliton shown in Fig. 1, fundamental solitons in the usual models are characterized by even distributions of both FF and SH electric fields, with the single hump at the center of the soliton. Thus, the fundamental TM solitons found in this work are drastically different from the traditional $\chi^{(2)}$ solitons.

The numerical integration of Eqs. (16) and (17) readily generates, alongside the fundamental TM solitons, higher-order ones. A typical example of a second-order TM soliton, in which each component has an additional extremum inside the interval $0 < x < \infty$, is displayed in Fig. 3. It is quite easy to find solitons of still higher orders too.

It is relevant to mention that higher-order solitons are well known in usual $\chi^{(2)}$ models, both in the simplest two-component systems (see, e.g., Refs. [11]) and in more sophisticated three- [12] and four-component [13] models. In most cases, higher-order solitons are subject to strong dynamical instability. Nevertheless, they were found to be (numerically) stable in three- and four-component models combining the $\chi^{(2)}$ nonlinearity and resonant reflections on a Bragg grating [13]. Direct analysis of the soliton stability in the present system is quite difficult and is beyond the scope of this work, as it is necessary to simulate the full system of the Maxwell’s equations. Nevertheless, it seems very plausible that both the TE and TM fundamental solitons may be stable, while the higher-order ones are unstable.

5 Conclusion

We have derived, starting from the Maxwell’s equations and without resort to the paraxial approximation, equations describing stationary (1+1)-dimensional beams propagating at an arbitrary direction in an optical crystal with cubic symmetry and purely quadratic $\chi^{(2)}$ nonlinearity. The equations were derived separately for beams of the TE and TM types. In both cases, they contain $\chi^{(2)}$ and $\chi^{(3)}$ nonlinear terms, the latter ones being generated via the cascading mechanism. The final TE equations and soliton solutions to them are
quite similar to those in previously known models with mixed $\chi^{(2)}-\chi^{(3)}$ nonlinearities. On the contrary to this, the TM model turns out to be very different from the previously known ones. It consists of four first-order equations for transverse and longitudinal components of the electric field at the fundamental and second frequencies. Fundamental-soliton solutions to the TM model are also drastically different from the usual $\chi^{(2)}$ solitons, in terms of the parity of their components. Most noteworthy, the transverse and longitudinal components of the electric field at the fundamental frequency in these solitons have, respectively, odd and single-humped even shapes. Besides that, it was also concluded that maximum values of the longitudinal and transverse fields become comparable for very narrow solitons, whose width is commensurate to or smaller than the carrier wavelength.
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Figure Captions

Fig. 1. A typical example of a bright spatial soliton with the TE polarization at values of the parameters \((\beta^2 - \beta_2^2) / (\beta^2 - \beta_1^2) = 0.8\) and \(\delta = 0.1\). Shown in this figure are the fields \(R\) and \(u\), obtained from Eqs. (7) and (8), vs. the normalized transverse coordinate \(\eta\).

Fig. 2. A typical example of a fundamental bright spatial soliton with the TM polarization. Here and in the subsequent figure, the continuous and dashed lines show, respectively, the transverse and longitudinal normalized fields: (a) the FF fields \(E\) and \(E_3\), and (b) the SH fields \(e\) and \(e_3\). The parameters are \((\beta_1/\beta)^2 = 0.60\), \((\beta_2/\beta)^2 = 0.80\).

Fig. 3. A typical example of a second-order bright spatial soliton with the TM polarization. The parameters are \((\beta_1/\beta)^2 = (\beta_2/\beta)^2 = 0.35\).