Spin-orbit interaction induced spin selective transmission through a multi-terminal mesoscopic ring

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Spin dependent transport in a multi-terminal mesoscopic ring is investigated in presence of Rashba and Dresselhaus spin-orbit interactions. Within a tight-binding framework we use a general spin density matrix formalism to evaluate all three components ($P_x$, $P_y$, and $P_z$) of the polarization vector associated with the charge current through the outgoing leads. It explores the dynamics of the spin polarization vector of current propagating through the system subjected to the Rashba and/or the Dresselhaus spin-orbit couplings. The sensitivity of the polarization components on the electrode-ring interface geometry is discussed in detail. Our present analysis provides an understanding of the coupled electron transport in mesoscopic bridge systems.

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I. INTRODUCTION

One of the major goals of spintronic applications has always been to manipulate electron’s spin degree of freedom to create a new paradigm in the fields of quantum information processing. Spin-$\frac{1}{2}$ particles are a natural choice for a qubit in quantum computers. So, generation of spin polarized beam is a highly significant issue as far as spintronic applications are concerned. A more or less usual way of realization of spin filtering action is by using ferromagnetic leads or by external magnetic field. But, in the first case, spin injection from ferromagnetic lead is difficult due to large resistivity mismatch and for the second one, the difficulty is to confine a very strong magnetic field into a small region like a quantum ring. Therefore, attention is being paid for modeling of spin filter using the intrinsic properties of mesoscopic systems such as spin-orbit (SO) interaction. Originating from the relativistic correction to the Schrödinger equation, SO interaction provides an all electrical way to generate and manipulate spin current in a far precise way rather than the usual magnetic field based spin control.

The main source of SO coupling in mesoscopic systems comes either from magnetic impurities (extrinsic type), or from bulk asymmetry or structural inversion asymmetry in the confining potential of the system (intrinsic type) yielding Dresselhaus or Rashba type of SO interaction. Studies on Rashba or Dresselhaus kind of interactions has made a significant impact in semiconductor spintronics as far as the control of spin dynamics is concerned.

Since SO interaction couples the spin degree of freedom with the momentum of an electron, so it is possible to achieve spin polarized currents in output terminals of a multi-terminal mesoscopic ring when an unpolarized electron beam is injected into its input terminal, though Kramer’s degeneracy suggests that due to preservation of time reversal symmetry only SO interaction can never induce in a two-terminal system, whereas, in case of multi-terminal system the condition gets relaxed. Additionally, in a multi-terminal system the degree of spin coherence can also be manipulated even when a polarized beam is injected.

FIG. 1: (Color online). Schematic view of a mesoscopic ring, subjected to Rashba and Dresselhaus SO interactions, with one input and two output terminals, where the arrows represent the movement of electrons. Two different configurations of electrode-ring interface geometry are taken to explore quantum interference effect on spin polarization components.

 Till date a lot of theoretical work has been done to model spin selective transmission. In 2003, Kislev and Kim have proposed that a planar T-shaped structure with a ring resonator can be highly efficient in producing spin polarized currents in different output arms in presence of Rashba SO interaction. Following that in 2005, Shelykh et al. investigated analytically by S-matrix theory both the effects of magnetic flux and Rashba coupling on charge and spin transport in a multi-terminal quantum ring. Then, in 2006, Peeters et al. have shown that a mesoscopic semiconductor ring with one input and two outputs can act as an electron spin beam splitter due to Quantum interference effect and Rashba SO interac-
tion\textsuperscript{16}. In another work, Rabani \textit{et al.} has designed a spin filter and spin splitter considering a two-terminal device in presence of external magnetic field\textsuperscript{17}.

Thus, the studies involving spin dependent transport in multi-terminal mesoscopic rings have already generated a wealth of literature, but to the best of our knowledge there is still a need to look deeper into the problem to address several important issues those have not been well analyzed earlier, for example, the understanding of the effect of Rashba and Dresselhaus SO couplings on all three components \(P_x, P_y, \) and \(P_z\) of the polarization vector associated with the charge current through the outgoing leads, and also the sensitivity of the polarization components on different electrode-ring interface configurations.

In the present work we mainly concentrate on these issues. Here we analyze the dependence of the polarization components \(P_x, P_y, \) and \(P_z\) in the two output terminals of a three-terminal mesoscopic ring subjected to Rashba and Dresselhaus SO interactions. Within a tight-binding (TB) framework we evaluate these quantities and Dresselhaus SO interactions. Within a tight-binding of a three-terminal mesoscopic ring subjected to Rashba coefficients of outgoing electrons through the quantum ring, which introduce spin flipping in the system and thus, the studies involving spin dependent transport have been well analyzed earlier, for example, the understanding of the effect of Rashba and Dresselhaus SO couplings was first approached by Nikolic and co-workers\textsuperscript{18} in 2004.

They presented an elegant way of expressing all three components \(P_x, P_y, \) and \(P_z\) of outgoing spin polarization vector in terms of spin resolved transmission matrices within the framework of Landauer-Büttiker formalism. This formalism encompasses both pure and mixed states as incoming beam and provides knowledge to investigate how polarization evolves\textsuperscript{19,20} in outgoing current due to Rashba or Dresselhaus interaction, scattering at lead-conductor interface and spin dependent scattering off impurities in both the weak and strong disordered regimes. The sensitivity of the polarization components on the electrode-ring interface geometry is also described in detail to make the present communication a self contained study.

The rest of the paper is organized as follows. In Section II, we present the model and theoretical formulation to obtain spin polarization components of the output currents in terms of transmission probabilities. The numerical results are illustrated in Section III. Finally, in Section IV, the results are summarized.

\section{II. THEORETICAL FRAMEWORK}

In this section we describe the model quantum system within a TB framework and express the spin polarization components \(P_x, P_y, \) and \(P_z\) in terms of transmission coefficients of outgoing electrons through the quantum ring following spin density matrix formalism.

\subsection{A. Model and Hamiltonian}

Let us start with Fig. 1 where a mesoscopic ring subjected to Rashba and Dresselhaus SO interactions is attached with one input and two output terminals. A simple lattice model within the framework of TB approximation assuming only nearest-neighbor coupling is used to describe the ring and side-attached leads. The TB Hamiltonian describing the entire system gets the form:

\begin{equation}
H = H_{\text{ring}} + H_{\text{leads}} + H_{\text{tun}}.
\end{equation}

The first term \(H_{\text{ring}}\) describes the Hamiltonian of the ring and for a \(N\)-site ring it reads,

\begin{equation}
H_{\text{ring}} = \sum_{n=1}^{N} (c_n^\dagger \epsilon c_n) - \sum_{n=1}^{N} t_{n,n+1} c_n^\dagger c_{n+1} + h.c. \quad (2)
\end{equation}

where,

\begin{equation}
c_n = \left( \begin{array}{c} c_{n,\uparrow}^\dagger \\ c_{n,\downarrow} \end{array} \right) ; \quad \epsilon_n = \left( \begin{array}{c} \epsilon \\ 0 \\ 0 \end{array} \right) \quad (3)
\end{equation}

Here, \(c_{n,\sigma}^\dagger\) and \(c_{n,\sigma}\) are the creation and annihilation operators, respectively, at the \(n\)-th site for an electron with spin \(\sigma (\uparrow, \downarrow). \) \(\epsilon\) being the on-site energy.

The factor \(t_{n,n+1}\) is the sum of three terms as follows,

\begin{equation}
t_{n,n+1} = t_{\text{R}}^n + t_{\text{T}}^n + t_{\text{D}}^n.
\end{equation}

Here\textsuperscript{21,22},

\begin{align}
t_{\text{R}}^n &= t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\
t_{\text{T}}^n &= -it_{\text{T}} \left( \begin{array}{cc} \cos \frac{\phi_n + \phi_{n+1}}{2} & \sin \frac{\phi_n + \phi_{n+1}}{2} \\ \sin \frac{\phi_n + \phi_{n+1}}{2} & \cos \frac{\phi_n + \phi_{n+1}}{2} \end{array} \right), \\
t_{\text{D}}^n &= -it_{\text{D}} \left( \begin{array}{cc} \sin \frac{\phi_n + \phi_{n+1}}{2} & \cos \frac{\phi_n + \phi_{n+1}}{2} \\ \cos \frac{\phi_n + \phi_{n+1}}{2} & \sin \frac{\phi_n + \phi_{n+1}}{2} \end{array} \right).
\end{align}

In these above expressions \(t\) is the isotropic nearest-neighbor coupling strength, whereas, \(t_{\text{R}}^n\) and \(t_{\text{T}}^n\) correspond to the spin dependent terms with \(t_{\text{R}}\) and \(t_{\text{T}}\) being the nearest-neighbor hopping integrals due to Rashba and Dresselhaus SO interactions, respectively, which introduce spin flipping in the system and \(\phi_n\) is the azimuthal angle for the \(n\)-th site. Mathematically, it can be expressed as \(\phi_n = 2\pi(n-1)/N.\)

In our formulation we assume that the one-dimensional (1D) semi-infinite leads are free from any kind of disorder and SO interactions. They can be expressed as,

\begin{equation}
H_{\text{leads}} = \sum_{\alpha} H_{\alpha}
\end{equation}

where,

\begin{equation}
H_{\alpha} = \sum_{n} \epsilon_n c_n^\dagger c_n + \sum_{\langle mn\rangle} t_{\alpha} c_m^\dagger c_n.
\end{equation}

Similarly, the ring-to-lead coupling is described by the following Hamiltonian.

\begin{equation}
H_{\text{tun}} = \sum_{\alpha} H_{\text{tun},\alpha}.
\end{equation}

Here,

\begin{equation}
H_{\text{tun},\alpha} = t_c [c_i^\dagger c_m + c_m^\dagger c_i] (7)
\end{equation}
In these above equations (Eqs. 4-7), the index \( \alpha \) signifies the number of leads attached to the ring. It can be two or three or even more depending on the number of outgoing leads in addition to the incoming one. \( c_i \) and \( t_i \) stand for the site energy and nearest-neighbor hopping between the sites of the leads. The coupling between the leads and the ring is denoted by the hopping integral \( t_e \). In Eq. 7, \( i \) and \( m \) belong to the boundary sites of the ring and the leads, respectively.

\section*{B. A brief introduction to spin density matrix formalism}

Most of the quantum interference phenomena observed in different experiments, e.g., Aharonov-Bohm effect, weak localization effect, etc., within the mesoscopic regime deal with the aspect of orbital quantum coherence of electronic states. At much low temperatures \( (T < 1 \text{ K}) \) and for the systems having \( L < L_e \) \((L_e \approx 1 \mu \text{m})\), inelastic scattering processes get suppressed so that an electron can be described by a single orbital wave function within the system. Now if the spin degree of freedom of the electron is taken into account then two separate vector spaces are multiplied tensorially to get the full Hilbert space of the quantum states i.e.,

\[ \mathcal{H} = \mathcal{H}_0 \otimes \mathcal{H}_s. \] (8)

Here, \( \mathcal{H}_0 \) spans over the orbital degrees of freedom while \( \mathcal{H}_s \) operates in spin space only. Therefore, any arbitrary state \( \ket{\psi} \in \mathcal{H} \) can be written as a linear combination of \( \ket{\phi_\alpha} \otimes \ket{\sigma} \), where \( \ket{\phi_\alpha} \)'s are the basis vectors of \( \mathcal{H}_0 \) and \( \ket{\sigma} \)'s are the eigenstates of \( \sigma_z \). \( \sigma_z \) being the Pauli spin matrix and \( \hat{u} \) is the unit vector along the direction of spin quantization axis. Thus, we can write the most general form of \( \ket{\psi} \) as,

\[ \ket{\psi} = \sum_{\alpha,\sigma} C_{\alpha,\sigma} \ket{\phi_\alpha} \otimes \ket{\sigma}. \] (9)

Here, we choose the quantization direction along the +ve \( Z \) direction, and accordingly, \( \ket{\sigma} \)'s are the eigenstates of \( \sigma_z \) operator i.e., \( \ket{\uparrow} = \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \) and \( \ket{\downarrow} = \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \).

The corresponding density matrix operator \( \rho \) for the state \( \ket{\psi} \) becomes,

\[ \rho = \ket{\psi}\bra{\psi}. \] (10)

This state can describe a pure one or a mixture of different quantum states. Below we consider both these two cases to get a complete picture.

**Coherent beam of electrons:** First, we consider the case where the state \( \ket{\psi} \) is a pure one indicating a coherent beam of electrons.

If the electron is free from any kind of spin-dependent interactions, then spin and charge coherences are independent of each other, resulting a separable state \( \ket{\psi} \). In that case \( \ket{\psi} \) can be written as,

\[ \ket{\psi} = \ket{\Phi} \otimes \ket{\Sigma} \] (11)

where, \( \ket{\Phi} \) and \( \ket{\Sigma} \) are the orbital and spin parts, respectively, of the total wave function \( \ket{\psi} \) and they are expressed as follows.

\[ \ket{\Phi} = \sum_{\alpha} a_\alpha \ket{\phi_\alpha}, \]
\[ \ket{\Sigma} = \left( a_\uparrow \ket{\uparrow} + a_\downarrow \ket{\downarrow} \right). \] (12)

The corresponding density matrix operator is thus given by,

\[ \rho = \ket{\psi}\bra{\psi} = \ket{\Phi}\bra{\Phi} \otimes \ket{\Sigma}\bra{\Sigma} = \rho_0 \otimes \rho_s. \] (13)

Here, \( \rho_0 \) and \( \rho_s \) are the reduced density matrices in the orbital and spin spaces, respectively.

Now, in presence of SO interaction, the state \( \ket{\psi} \) is no longer separable, and hence, individual orbital and spin parts lose their coherence and become entangled and the state \( \ket{\psi} \) can be written in a form as expressed in Eq. 11.

In this situation,

\[ \rho = \ket{\psi}\bra{\psi} \neq \rho_0 \otimes \rho_s. \] (14)

**Incoherent beam of electrons:** Next, we consider the case where the electronic beam is an incoherent mixture i.e., statistical superposition of different quantum states. Here, the state cannot be expressed like Eqs. 9 and 11.

In this situation the state is best represented by the density matrix operator as,

\[ \rho = \sum_i w_i \ket{\psi_i}\bra{\psi_i} \] (15)

where, \( w_i \) gives the probability for the ensemble to be found in the quantum state \( \ket{\psi_i} \).

In the present work, we want to determine the components \( P_x, P_y \) and \( P_z \) of output currents propagating through the leads attached to the ring subjected to SO interactions. Using Pauli spin operators they are expressed as follows.

\[ P_x = \langle \sigma_x \rangle ; P_y = \langle \sigma_y \rangle ; P_z = \langle \sigma_z \rangle. \]

Now, following quantum statistics, measurement of any spin observable \( O_s \) is accomplished by the following way,

\[ \langle O_s \rangle = Tr[\rho \cdot O_s] \] (16)

where \( O_s \) is the matrix form of the operator \( O_s \) and \( \rho \) is the spin density matrix which is obtained by taking
partial trace over the orbital degrees of freedom of the full density matrix \( \rho \),

\[
\rho_s = Tr_\sigma(\rho) = \sum_\alpha \langle \phi_\alpha | \rho | \phi_\alpha \rangle.
\]

(17)

Thus, in our case spin density matrix plays the central role in understanding the quantum dynamics of a spin sub-system, subjected to SO interactions and attached to the environment through ideal (free from any kind of charge or spin dependent interaction) leads. For electrons (i.e., spin-1/2 particles) \( \rho_s \) has a simple \( 2 \times 2 \) representation in a chosen basis \( | \uparrow \rangle, | \downarrow \rangle \) in \( \mathcal{H}_s \) which reads,

\[
\rho_s = \begin{pmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{pmatrix}.
\]

(18)

Here, the diagonal elements represent the probabilities of finding an electron with spin \( | \uparrow \rangle \) or \( | \downarrow \rangle \), whereas the off-diagonal elements describe the probabilities of coherent superposition of \( | \uparrow \rangle \) and \( | \downarrow \rangle \) states due to quantum interference effect.

It can also be represented as,

\[
\rho_s = I_s + \vec{P} \cdot \vec{\sigma}/2.
\]

(19)

Here \( \vec{P} \) is the polarization vector whose components are evaluated from the following relations.

\[
P_x = Tr[\rho_s \sigma_x],
\]

\[
P_y = Tr[\rho_s \sigma_y],
\]

\[
P_z = Tr[\rho_s \sigma_z].
\]

(20)

For a completely unpolarized electron beam i.e., an incoherent mixture of up and down spin electrons, \( | \vec{P} \rangle = 0 \).

In this case, the spin density operator \( \rho_s \) becomes,

\[
\rho_s = \rho_{\uparrow\uparrow} | \uparrow \rangle \langle \uparrow | + \rho_{\downarrow\downarrow} | \downarrow \rangle \langle \downarrow |.
\]

(21)

It is seen that for an incoherent mixture of up and down spin electrons the off-diagonal elements of \( \rho_s \) are zero.

Therefore, to determine the polarization components of outgoing currents we need to construct the spin density matrix for the outgoing charge current.

\section{C. General expressions of \( P_x, P_y \) and \( P_z \) in terms of transmission matrices for a mesoscopic ring coupled to source and drain leads having \( M \) channels in each lead}

In this sub-section we present a general scheme for evaluating spin polarization components of the charge current through the outgoing leads, considering a mesoscopic ring subjected to SO interactions, where each lead contains \( M \) number of channels.

In our problem, we assume that a beam of incoherent i.e., unpolarized electrons is injected from the source to the ring. Therefore, the incident beam is best represented by the spin density operator as,

\[
\rho_s^i = n_\uparrow | \uparrow \rangle \langle \uparrow | + n_\downarrow | \downarrow \rangle \langle \downarrow |.
\]

(22)

For a fully unpolarized beam, \( n_\uparrow = n_\downarrow = \frac{1}{2} \) and hence the polarization components are evaluated as- (using En. [20]):

\[
P_x = 0,
\]

\[
P_y = 0,
\]

\[
P_z = (n_\uparrow - n_\downarrow) = 0.
\]

Now, in presence of SO interaction, individual spin and orbital parts become entangled, and thus, the corresponding outgoing beam through the lead \( j \), can be described in terms of reduced spin density matrices \( \rho_{s,j}^{\text{out}} \) by taking partial trace over all the orbital degrees of freedom.

\[
\rho_{s,j}^{\text{out}} = \frac{e^2}{\hbar} \sum_{\alpha,\beta} \left( \begin{array}{cc} \alpha & \beta \\ \gamma & \delta \end{array} \right)
\]

(23)

where,

\[
\zeta = n_\uparrow (G_{ji}^{\uparrow\uparrow} + G_{ji}^{\uparrow\downarrow}) + n_\downarrow (G_{ji}^{\downarrow\uparrow} + G_{ji}^{\downarrow\downarrow}),
\]

\[
\alpha = n_\uparrow |t_{ji}^\uparrow|^2 + n_\downarrow |t_{ji}^\downarrow|^2,
\]

\[
\beta = n_\uparrow |t_{ji}^{\uparrow \downarrow}|^2 + n_\downarrow |t_{ji}^{\downarrow \uparrow}|^2,
\]

\[
\gamma = n_\uparrow |t_{ji}^{\uparrow \downarrow}|^2 + n_\downarrow |t_{ji}^{\downarrow \uparrow}|^2,
\]

\[
\delta = n_\uparrow |t_{ji}^{\uparrow \downarrow}|^2 + n_\downarrow |t_{ji}^{\downarrow \uparrow}|^2.
\]

(24)

Following the prescription of En. [20], the spin polarization components of the outgoing through lead \( j \), current can be expressed as,

\[
P_{x,j}^{\text{out}} = \frac{2e^2}{\hbar} Re \left[ Tr \left( t_{ji}^{\uparrow \uparrow} \dagger \left( \left( t_{ji}^{\uparrow \downarrow} \dagger \right)^\dagger \left( t_{ji}^{\downarrow \uparrow} \right)^\dagger \right) \left( t_{ji}^{\downarrow \downarrow} \dagger \right) \right) \right],
\]

\[
P_{y,j}^{\text{out}} = \frac{2e^2}{\hbar} Im \left[ Tr \left( t_{ji}^{\uparrow \uparrow} \dagger \left( \left( t_{ji}^{\uparrow \downarrow} \dagger \right)^\dagger \left( t_{ji}^{\downarrow \uparrow} \right)^\dagger \right) \left( t_{ji}^{\downarrow \downarrow} \dagger \right) \right) \right],
\]

\[
P_{z,j}^{\text{out}} = \frac{G_{ji}^{\uparrow\uparrow}}{G_{ji}^{\uparrow\uparrow} + G_{ji}^{\uparrow\downarrow}} - \frac{G_{ji}^{\downarrow\uparrow}}{G_{ji}^{\downarrow\uparrow} + G_{ji}^{\downarrow\downarrow}}.
\]

(25)

where, \( G_{ji} = (G_{ji}^{\uparrow\uparrow} + G_{ji}^{\downright\downright}) + G_{ji}^{\uparrow\downright} + G_{ji}^{\downright\uparrow} \).

Here, \( |t_{ji}^{\sigma\sigma'}\rangle \) is the transmission matrix, having dimension \( M \times M \), for an electron injected from the lead \( i \), with spin \( \sigma \) and transmitted through the lead \( j \) with \( \sigma' \). Thus, for a single channel lead \( |t_{ji}^{\sigma\sigma'}\rangle \) becomes a simple element rather than being a matrix. \( G_{ji}^{\sigma\sigma'} \) is the conductance of the ring, defined by the Landauer formula, and at low bias voltage it gets the form,

\[
G_{ji}^{\sigma\sigma'} = \frac{e^2}{\hbar} Tr \left[ |t_{ji}^{\sigma\sigma'}\rangle \langle t_{ji}^{\sigma\sigma'}| \right].
\]

(26)

Below we describe the way of determining the transmission matrix and its relation to the conductance.
D. Evaluation of transmission matrix

The transmission matrix is obtained from the well known relation\(^{27,30}\),

\[
t_{ji}^{\sigma \sigma'} = \sqrt{\Gamma_{ji,\text{red}}} G_{ij}^{\sigma \sigma'} \sqrt{\Gamma_{ji,\text{red}}}. \quad (26)
\]

Here, \(G_{ij}^{\sigma \sigma'}\) is the retarded Green’s function (in matrix form) in the reduced dimension \(M \times M\), connecting \(i\)-th and \(j\)-th leads (each lead contains \(M\) channels) i.e., \(G_{ij}^{\sigma \sigma'} = \langle i, \sigma | j, \sigma' \rangle\). \(\Gamma_{ji,\text{red}}\) are the coupling matrices in the same reduced dimension.

The single particle Green’s function describing the complete system i.e., the ring with side-attached leads for an electron with energy \(E\) is defined as,

\[
G = (E - H + i\eta)^{-1} \quad (27)
\]

where, \(\eta \rightarrow 0^+\).

The problem of finding \(G\) in the full Hilbert space of \(H\) taking the matrix forms of \(H\) and \(G\) can be mapped exactly to a Green’s function \(G_{\text{ring}}\) which represents an effective Hamiltonian in the reduced Hilbert space of the mesoscopic ring. It is expressed like,

\[
G = G_{\text{ring}} = \left(E - H_{\text{ring}} - \sum_{\alpha, \sigma} \Sigma_{\alpha}^{\sigma}\right)^{-1} \quad (28)
\]

where,

\[
\Sigma_{\alpha}^{\sigma} = H_{\text{ring}, \alpha}^{\dagger} G_{\alpha} H_{\text{ring}, \alpha}. \quad (29)
\]

These \(\Sigma_{\alpha}^{\sigma}\)’s are the self-energies introduced to incorporate the effect of coupling of the ring to the leads. It is evident from Eq. \(^{28}\) that the form of the self-energies are independent of the ring itself through which spin transmission is studied. \(\Sigma_{\alpha}^{\sigma}\)’s describe the coupling between the ring and the leads and they are mathematically defined as,

\[
\Gamma_{\alpha}^{\sigma} = i \left[ \Sigma_{\alpha}^{\sigma} - \Sigma_{\alpha}^{\sigma \dagger} \right], \quad (30)
\]

where, \(\Sigma_{\alpha}^{\sigma}\) and \(\Sigma_{\alpha}^{\sigma \dagger}\) are the retarded and advanced self-energies associated with the \(\alpha\)-th lead, respectively. This self-energy term is again expressed\(^{31,32}\) as a sum of real and imaginary parts,

\[
\Sigma_{\alpha}^{\sigma} = \Lambda_{\alpha}^{\sigma} - i \Delta_{\alpha}^{\sigma}, \quad (31)
\]

where, they describe the energy shift and broadening of the energy levels of the ring, respectively. The finite imaginary part is obtained due to inclusion of semi-infinite leads having continuous energy spectrum. Therefore, the coupling matrices can easily be determined from the self-energy expression and is expressed in the form,

\[
\Gamma_{\alpha}^{\sigma} = -2 \text{Im}(\Sigma_{\alpha}^{\sigma}). \quad (32)
\]

From this relation (Eq. \(^{32}\)) the reduced coupling matrices \(\Gamma_{\alpha,\text{red}}\)’s are constructed.

Following the reference\(^{33}\), the self-energy matrices (\(\Sigma_{\alpha}^{\sigma}\)’s) are evaluated in the reduced Hilbert space of the ring. The details are also available in other articles\(^{34-37}\).

Although the theoretical formulation presented above is based on a general approach considering finite width leads having \(M\) number of channels, for completeness of the theoretical description, but in this article we present all the numerical results (Sec. III) considering single-channel leads.

III. NUMERICAL RESULTS AND DISCUSSION

In what follows we restrict ourselves to absolute zero temperature and use the units where \(e = h = c = 1\). Throughout the numerical calculations we choose \(\epsilon = \epsilon_l = 0\) and \(t = t_l = t_c = -1\). The energy scale is measured in unit of \(t\) and the SO coupling strengths \((t_R\) and \(t_D\)) are also scaled in unit of \(t\).

A. Two-terminal transport

Before addressing the central problem i.e., the possibilities of getting spin polarized currents in two output terminals of a three-terminal mesoscopic ring from a completely unpolarized incident electron beam, first we analyze the results for a simple system where a mesoscopic ring, subjected to Rashba and Dresselhaus SO interactions, is coupled to two leads. In Fig. 2 we show the variation of spin polarization components \(P_x\), \(P_y\) and \(P_z\) as a function of energy \(E\) for a symmetrically connected mesoscopic ring with \(N = 40\), where we set \(t_R = 0.2\) and \(t_D = 0\).

![FIG. 2: (Color online). \(P_x\), \(P_y\) and \(P_z\) as function of energy \(E\) for a symmetrically connected mesoscopic ring with \(N = 40\), where we set \(t_R = 0.2\) and \(t_D = 0\).](attachment:fig2.png)
lead-ring interface geometries i.e., for asymmetrical connections and find the identical behavior of these three components. Thus we can emphasize that in absence of any magnetic impurity or external magnetic field, shown in their recent work that only SO interaction cannot remove the degeneracies of the transmission eigenvalues. Thus, our numerical results exactly corroborate with these arguments. A similar nature of the polarization components in this two-terminal geometry is also be obtained when they are plotted as a function of Dresselhaus SO coupling setting $t_R = 0$.

B. Three-terminal transport

In this sub-section we discuss the central results of our present investigation i.e., the interplay of Rashba and Dresselhaus SO couplings and lead-ring interface geometry on the polarization components of outgoing currents in a three-terminal mesoscopic ring. We analyze the results for two distinct configurations of lead-ring geometry as schematically illustrated in Fig. 1. In one configuration the outgoing leads are coupled symmetrically with

![FIG. 3: (Color online). $P_x$, $P_y$ and $P_z$ as a function of Rashba SO coupling strength $t_R$ at a particular energy $E = -0.2$ for the identical lead-ring configuration taken in Fig. 2. The ring size $N$ and the Dresselhaus SO coupling strength are also same as Fig. 2.](image)

![FIG. 4: (Color online). Polarization components of outgoing currents as a function of energy in a three-terminal mesoscopic ring ($N = 40$) when the output leads are connected symmetrically with respect to the source lead. The red and blue lines correspond to the results for the leads 2 and 3, respectively. Here we set $t_R = 0.2$ and $t_D = 0$.](image)

![FIG. 5: (Color online). Same as Fig. 4 with $t_R = 0$ and $t_D = 0$.](image)

![FIG. 6: (Color online). Momentum vectors and their components at the two equivalent sites for the two arms of the ring.](image)
respect to the source lead, while in the other case they are connected asymmetrically.

1. Symmetric configuration

We start by discussing the variation of spin polarization components $P_x$, $P_y$ and $P_z$ of outgoing currents in a three-terminal mesoscopic ring with Rashba SO coupling only, that is, setting the Dresselhaus SO coupling to zero. The results for a 40-site ring with $t_R = 0.2$ and $t_D = 0$ are shown in Fig. 8 where the red lines describe the results for the output lead 2, while for the other lead (lead 3) they are presented by the blue lines. From the spectra it is observed that the $X$ and $Z$ components of the spin polarization vectors in two symmetrically coupled output leads are exactly identical in magnitude but they carry opposite signs for each value of the injecting electron energy $E$. On the other hand, the component $P_y$ exhibits identical sign in both these two leads providing vanishingly small amplitudes. These features can be explained from the following arguments.

The $Z$ component of spin polarization essentially describes the normalized difference between the charge currents of up and down spin electrons flowing through the output leads (see Eq. 24), since in our present scheme we choose the quantization direction along the +ve $Z$ axis. Now, it is well known that the up and down spin electrons scatter in opposite directions when they traverse through a conductor subjected to a SO interaction which is the aspect of visualizing mesoscopic spin Hall effect and accumulation of opposite spins on the opposite edges. Therefore, spin polarization with opposite signs for the component $P_z$ is expected in two output leads those are attached symmetrically to the mesoscopic ring with respect to the input lead.

The above argument cannot be given to explain the characteristic features of the other two components $P_x$ and $P_y$ since we select +ve $Z$ axis as the quantization direction. In our theoretical framework we have already stated that $P_x$ and $P_y$ are evaluated from the expectation values of $\sigma_x$ and $\sigma_y$. Now, we know from the Rashba Hamiltonian that $\sigma_z$ is coupled with the $Y$-component of the momentum of an electron which gets opposite signs at two equivalent atomic sites in the two arms of the ring resulting opposite spin polarization $P_x$ in the output leads. But, for $P_y$ the situation is somewhat different as $\sigma_y$ is coupled with the $X$-component of the momentum which shows identical sign at the equivalent points (see Fig. 9). Hence, a destructive interference takes place and it leads to a vanishingly small spin polarization at the output leads. Based on the symmetry arguments...
of the S-matrix elements Kim et al.  have shown analytically that in a Y-shaped conductor subjected to SO interaction the transmission amplitudes for the $X$ and $Z$ components get equal magnitude and opposite phases for symmetrically connected leads which provide opposite spin polarization for these two components. While, for the $Y$ component almost zero polarization is achieved with Rashba SO interaction only. Below we justify these results theoretically with respect to the source lead. The red and green polarizations of outgoing currents in two leads when the ring is described with only Dresselhaus SO coupling. These expressions Eqs. (34) and (35) yield the reason for interchanging the features of $P_x$ and $P_y$ in the mesoscopic ring with only Dresselhaus SO coupling.

Finally, in Figs. 7-9 we present the variations of $P_x$, $P_y$ and $P_z$ as a function of Rashba SO coupling for a typical energy $E = 0.5$ considering the identical ring size used in Figs. 4 and 5. The results are computed for two different values of $t_D$ to explore the combined effect of Rashba and Dresselhaus SO interactions on the spin polarization components. When $t_D = 0$, the components $P_x$ and $P_z$ provide outgoing currents with equal magnitude and opposite phases, while the other component $P_y$ almost drops to zero. The situation is somewhat interesting when Dresselhaus SO coupling is included in addition to the Rashba term. For the non-zero value of $t_D$, all these three components get finite values in output leads and the magnitudes of individual components also differ in these two leads. These phenomena can be well explained from the analysis described above. Very interestingly we see that, at the particular case when the strengths of these two SO couplings are identical i.e., $t_R = t_D$, the three polarization components drop to zero. (shown by the green dashed lines in Figs. 7-9). It is already said that $P_z$ in presence of Rashba coupling is just equal in magnitude but opposite in sign with $P_y$ in presence of Dresselhaus coupling, so when both the interactions are present in equal strength net polarization must vanish. For the other components we argue that when $t_R = t_D$, the total Hamiltonian commutes with $(\sigma_x + \sigma_y)$, so $U = (\sigma_x + \sigma_y)/\sqrt{2}$. Therefore, if $|\psi\rangle$ is the eigenstate of $H_R$ corresponding to a particular energy eigenvalue and $|\psi'\rangle$ is the eigenstate of the transformed Hamiltonian $H_D$, then we can write $|\psi'\rangle = U|\psi\rangle$. Following this transformation the Z component of spin polarization (Eq. 20) in presence of only Dresselhaus SO interaction gets the form:

$$P_z|_D = \langle \sigma_z \rangle = \langle \psi' | \sigma_z | \psi \rangle = \langle \psi | U^\dagger \sigma_z U | \psi \rangle = \langle \psi | (-\sigma_z) | \psi \rangle = -\langle \psi | \sigma_z | \psi \rangle = -\langle \sigma_z \rangle = -P_z|_R.$$

This equation (Eq. 33) clearly illustrates the reason behind the sign reversal of $P_z$ of outgoing currents in two leads when the ring is described with only Dresselhaus SO term compared to the other case i.e., the ring with only Rashba SO interaction.

Following the above prescription we can also get the relations

$$P_x|_D = P_y|_R \quad \text{and} \quad P_y|_D = P_x|_R$$

since,

$$U^\dagger \sigma_x U = \sigma_y \quad \text{and} \quad U^\dagger \sigma_y U = \sigma_x.$$  

These expressions Eqs. (34) and (35) yield the reason for interchanging the features of $P_x$ and $P_y$ in the mesoscopic ring with only Dresselhaus SO coupling.
\[ \langle \sigma_y \rangle \] i.e., \((P_x + P_y)\) is a conserved quantity, i.e., \((P_x + P_y)\) should have the same sign and same magnitude in both leads, which is possible only when both \(P_x\) and \(P_y\) are individually zero at both the outputs. This phenomenon emphasizes that any one of the SO fields can be predicted precisely if the other one is known. Needless to say, the precise determination of the SO coupling strengths is extremely crucial in the field of spintronics. One can control Rashba SO interaction by a suitable gate voltage, and hence, it can be measured. On the other hand, the control Rashba SO interaction by a suitable gate voltage, an accurate measurement of Dresselhaus SO coupling, an accurate measurement of Dresselhaus term is possible. For a single sample if a finite polarization is achieved for a particular value of \(t_D\), then one can tune Rashba SO coupling by means of gate voltage to get vanishing spin polarization at the output currents. Thus, knowing the Rashba SO coupling, an accurate measurement of Dresselhaus term is possible.

2. Asymmetric configuration

The spin polarization turns out to be sensitive to the lead-ring interface geometry. To this end, we analyze the behavior of spin polarization components in a three-terminal mesoscopic ring where the output leads are attached asymmetrically with respect to the source lead. The results are shown for two different values of \(t_D\), where the red and blue curves represent the results for the leads 2 and 3, respectively.

FIG. 11: (Color online). Same as Fig. 10 with \(t_R = 0\) and \(t_D = 0.2\).

FIG. 12: (Color online). \(P_x\) as a function of Rashba SO coupling in a three-terminal mesoscopic ring \((N = 40)\) for a typical energy \(E = 0.5\) when the output leads are attached asymmetrically with respect to the source lead. The results are shown for two different values of \(t_D\), where the red and blue curves represent the results for the leads 2 and 3, respectively.

FIG. 13: (Color online). \(P_y\) as a function of Rashba SO coupling. The lead-ring interface geometry and all the other parameters are same as Fig. 12.
the spectra we see that, unlike the symmetric configuration, the magnitudes of $P_x$ and $P_y$ in two output leads

are no longer identical even when one SO coupling is set equal to zero. This is solely due to the effect of quantum interference among the electronic waves passing through different arms of the mesoscopic ring. All the other properties, for example, the phase reversals of the polarization components $P_x$ and $P_y$ in two output leads and the vanishingly small amplitude of $P_y$ remain exactly same as discussed earlier in the case of symmetric configuration. The same quantities are also analyzed for this asymmetric configuration considering the ring with only Dresselhaus SO interaction, i.e., setting $t_R = 0$. The results are given in Fig. 11. Except getting different amplitudes of polarizing currents in two output leads, all the other physical phenomena remain unchanged as discussed in Fig. 4.

Before we end this section, in Figs. 12-14 we describe the variations of $P_x$, $P_y$ and $P_z$ as a function of Rashba SO coupling considering two different values of $t_D$ for this asymmetric lead-ring interface geometry to make the present communication a self contained study. The amplitudes of the individual components of spin polarization in two output terminals get changed, as expected, and their magnitudes can also be tuned by including other SO coupling. The physical picture about the possibilities of determining SO fields by observing the vanishing spin polarization of outgoing currents in the limit $t_R = t_D$ remains also invariant for this lead-ring interface geometry, like the symmetric configuration.

IV. CONCLUSION

To conclude, in the present work we have described spin dependent transport through a multi-terminal mesoscopic ring in presence of Rashba and Dresselhaus SO interactions. Within a TB framework we have determined the polarization components $P_x$, $P_y$ and $P_z$ of outgoing currents using a general spin density matrix formalism. The sensitivity of these components on the lead-ring interface geometry has also been analyzed in detail to make the communication is self contained study. From our extensive numerical work we have established that a two-terminal mesoscopic ring with only SO coupling, in absence of any magnetic impurity or external magnetic field, cannot induce spin polarization in the output lead. On the other hand, a multi-terminal geometry, subjected to SO interaction, containing at least two out put leads can generate polarized spin currents from a completely unpolarized electron beam even in absence of any magnetic field or magnetic like impurities. Finally, we have also provided a possible realization of determining Dresselhaus SO coupling knowing the Rashba term in a single sample by observing the vanishing spin polarization of the outgoing currents, and hence facilitates a possible experimental measurement in this line.

Finally we point out that, the presented results in this communication are also valid even for non-zero temperatures ($\sim 300$ K) as the broadening of the energy levels caused by side-attached leads is much higher than the thermal broadening.$^{31,32,38-42}$

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