Coalescence of Multielectron Bubbles in Liquid Helium

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Abstract. In 1977, Volodin et al. observed that the electrohydrodynamic instability of charged helium surfaces can lead to the loss of the electrons from the surface in the form of bubbles. These are Multielectron Bubbles (MEBs) which contain electrons pinned on their inner surfaces. The MEBs form a model system for studying electrons on curved surfaces, which are predicted to have many interesting properties. Recent experiments showed that above the Lambda point, MEBs could be trapped using a Paul-trap and their sizes are mainly determined by the amount of vapour present inside. Here, we report the experimental observation of the coalescence of two MEBs which were moving upward in bulk liquid helium-4. The charge and radii of the MEBs were determined before and after the coalescence. The merging of two similar charged bubbles was possible because of the presence of vapour inside the merging MEBs.

1. Introduction

An electron present above a liquid helium surface are attracted to the liquid, which originates due to the image charges induced due to the finite polarizability of helium. However, to enter inside the bulk liquid, the electron has to overcome a potential barrier of 1 eV which arises due to Pauli exclusion principle. Hence all the low energy electrons are confined on the helium surface, and form a model two dimensional electron system. Compared to two dimensional electron systems on other semiconductor systems, 2DES on the helium surface are unique as they are governed by classical statistics. If the electron density on the helium surface is taken beyond a critical value of $2 \times 10^9$ electrons/cm$^2$, an electrohydrodynamic (EHD) instability develops\[1\], which results in charge loss from the surface in the form of bubbles. These bubbles contain a layer of electrons pressed on their inner surfaces and are known as multielectron bubbles (MEBs), which were first observed by Volodin [2] in 1977. Unlike the flat helium surface, the electron density of MEBs can be varied over a wide range, which could necessitate their behavior to be modeled by quantum statistics at high densities. This and many interesting electronic phases possible with MEBs have been predicted\[3\]. Of particular interest is the question whether these objects are stable against shape fluctuations, for which there are conflicting theoretical predictions\[4, 5, 6, 7\].

After the first observation of MEBs, there were very few experiments on MEBs[8, 9], where the properties such as charge, velocity, radius and drag of the MEBs were measured in a destructive measurement. There have also been reports of their generation[10, 11, 12, 13] using alternate...
methods. A scheme to observe and study MEBs non-destructively has been developed by our group\cite{14, 15} using a Paul trap above the lambda point. Using the Paul trap, we were able to trap the MEBs for a longer period of time (a few hundred milliseconds), and the charge and radius of the MEBs were measured by analyzing the trajectories. The results showed that the MEBs were filled with vapour, and the size of the MEB was mainly determined by the amount of vapour inside. As we observe in our experiments, the MEBs collapse as the vapour condenses with time. This phenomenon has been modeled, and the time scale was found to match\cite{16} with experimentally observed values of the collapse time.

In the Paul trap experiments, the charge of the MEBs were typically low, around $2 \times 10^3 - 4 \times 10^4$ electrons, implying as the vapour condenses they become too small to be resolved using our imaging setup. In our recent experiments\cite{17}, it was shown that the splitting of MEBs using high electric fields could result in the generation of MEBs with higher charge density. Here we describe coalescence, which is the opposite of splitting phenomena, to obtain MEBs with lower charge density.

2. Experimental Setup

Experiments were carried out at 2.5 K inside an optical cryostat. The experimental chamber was made of stainless steel with optical access from four sides. The schematic of the setup inside the chamber is shown in figure 1, which was similar to our previous experiments\cite{17}. Electrons were generated on the helium surface by field emission from the tungsten tip. The emitted electrons were confined using a plastic tube of inner diameter 3 mm around the tungsten tip to achieve the electron densities required for the MEB generation. Two metal plates, top and bottom, were kept parallel to each other for generating the electric field required for controlling the motion of MEBs.

To create field emission, high voltage pulses in the order of -2 to -3.5 kV were applied to tip for duration varying between 30 to 100 milliseconds. When the charge density on the helium surface exceeded beyond critical value, an EHD instability developed on the surface. At the instability point, charged helium surface broke, and gave off MEBs. Negative voltage present on the tip pushed MEBs into the bulk of the liquid. Charge of the MEB was measured by applying
appropriate voltage pulses to top and bottom metal plates, using a method described later. The motion of MEBs between the plates occurred within few milliseconds, hence we used a high speed camera with suitable light source for capturing the entire sequence of events. For the experiments mentioned here, the high speed camera was operated at a frame rate of 3600 fps. To obtain the trajectories of the MEBs, videos were processed using Matlab image processing tools. The number of electrons inside the MEB was estimated by analyzing the trajectories.

3. Coalescence of MEBs

![Figure 2](image-url)

**Figure 2.** Series of images showing the coalescence of MEBs. Initially the two MEBs were separated by a distance of 0.53 mm. As they move up during the negative pulse, they came nearby and fused with each other. Each frame corresponds to a time scale of 0.277 milliseconds, total frame width is 1.2 mm and height is 2.4 mm.

After the generation, we applied a series of high voltage pulses to the bottom plate as shown in figure 1b for measuring the charge and radius of MEBs. During the experimental trials, we observed the coalescence two MEBs even though they were separated by a few millimetres initially. These MEBs were moving up during negative pulse, and when they were sufficiently close to each other, they merged. It appears their proximity during their upward motion is a random event made possible by a somewhat chaotic flow field inside the bulk. However, we can not rule out the fact that the MEBs may interact with each other through hydrodynamic interactions and this may bring them together. The sequence of images of the two MEBs fusing with each other is shown in figure 2. Just before coalescence, the MEBs were having radii 90 $\mu$m and 95 $\mu$m moving with velocities 16 cm/s and 18 cm/s respectively. The resultant MEB had a radius of 120 $\mu$m. After the coalescence, the resultant MEB showed a particular pattern of oscillation. This is shown in figure 3. Amplitude of the oscillations decayed in a few milliseconds.

For a detailed analysis of coalescence, charges of the MEBs before and after coalescence event were estimated. We used a methodology, where all the forces on the MEB are equated at the point where the MEB makes a turn (zero velocity). Total force acting on the MEB at any instant can be related to its acceleration, and is given by
\[ F_b + F_E - F_{\text{drag}} = m_{\text{MEB}a} \] (1)

Here, \( F_b \) is the buoyant force acting on the bubble and given by \( \frac{4}{3} \pi \rho R^3 g \), \( m_{\text{MEB}} \) is the effective hydrodynamic mass of the MEB moving through the liquid with acceleration \( a \), and given by \( \frac{2}{3} \pi \rho R^3 \), where \( R \) is the effective radius of the MEB, \( \rho \) is the density of liquid helium. We compared the forces at the point where the MEB turned, implying it was a point of zero velocity and therefore \( F_{\text{drag}} = 0 \). The only unknown in this equation is the charge of the MEB, which can be determined by knowing the electric field at zero velocity point. We used a Comsol model for obtaining the electric field value at zero velocity point for the applied electrical conditions.

We followed this procedure and found that MEBs were having charges \( 1.38 \times 10^5 \) electrons and \( 3 \times 10^5 \) electrons before coalescence. After the coalescence, the resultant MEB contained approximately \( 5 \times 10^5 \) electrons, which proved the consistency of the method. The equilibrium radius of an MEB containing \( Z \) electrons with no vapour is \( R_{eq} = \left( \frac{Z^2 e^2}{4\pi \kappa \sigma} \right)^{1/3} \), where \( \sigma \) is the surface tension of helium at 2.5 K. For a given \( Z \) of \( 5 \times 10^5 \), \( R_{eq} \) is 16.1 \( \mu \)m. This value is much smaller than the experimentally observed value of 120 \( \mu \)m, which indicates that the MEB was filled with vapour.

Since MEBs were having similar charges, their union was somewhat surprising. This can be explained by looking at different energy scales involved in the process. The MEB with a radius of 90 \( \mu \)m was moving with a velocity 16 cm/s, and contained \( 1.38 \times 10^5 \) number of electrons. For this MEB, the electrostatic and kinetic energies were 0.02 pJ and 2.8 pJ respectively. Similarly for the other MEB which moved at 18 cm/s and contained \( 3 \times 10^5 \) number of electrons, these values were 0.1 pJ and 4.2 pJ respectively. For coalescence to occur, they needed to overcome the electrostatic repulsion, which resulted in an energy barrier of around 0.05 pJ. This was estimated by assuming the electrons in the individual MEBs to be effectively point charges and separated by a distance which was the sum of their individual radii. This was a justifiable assumption since the size of the MEBs were effectively governed by the vapor inside, which allowed reorganization of the electrons inside the bubbles with minimal energy cost. As the kinetic energy of the individual MEBs were sufficiently larger than the repulsive barrier of 0.05 pJ, coalescence was possible. The oscillation modes induced in the coalesced MEB, as shown in figure 3, was a consequence of the excess energy getting dissipated into the liquid.

4. Conclusion
We present experimental evidence of coalescence of two MEBs. The radii and charges of the MEBs were determined before and after the coalescence, which suggested significant amount of helium vapor inside the bubbles. The estimate of different energies of the MEBs suggest that coalescence of these like-charged systems was indeed energetically allowed.
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