Dynamical Coupling of a Mean-field Dynamo and Its Wind: Feedback Loop over a Stellar Activity Cycle

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Abstract

We focus on the connection between the internal dynamo magnetic field and the stellar wind. If the star has a cyclic dynamo, the modulations of the magnetic field can affect the wind, which, in turn, can back-react on the boundary conditions of the star, creating a feedback loop. We have developed a 2.5D numerical setup to model this essential coupling. We have implemented an alpha–omega mean-field dynamo in the PLUTO code and then coupled it to a spherical polytropic wind model via an interface composed of four grid layers with dedicated boundary conditions.

We present here a dynamo model close to a young Sun with cyclic magnetic activity. First, we show how this model allows one to track the influence of the dynamo activity on the corona by displaying the correlation between the activity cycle, the coronal structure, and the time evolution of integrated quantities. Then we add the feedback of the wind on the dynamo and discuss the changes observed in the dynamo symmetry and wind variations. We explain these changes in terms of dynamo modes; in this parameter regime, the feedback loop leads to a coupling between the dynamo families via a preferred growth of the quadrupolar mode. We also study our interface in terms of magnetic helicity and show that it leads to a small injection in the dynamo. This model confirms the importance of coupling physically internal and external stellar layers, as it has a direct impact on both the dynamo and the wind.

Unified Astronomy Thesaurus concepts: Solar cycle (1487); Solar corona (1483); Solar dynamo (2001); Solar wind (1534); Magnetohydrodynamics (1964)

1. Introduction

Stars are complex objects that require the combination of many fields of physics to be understood as a whole. In particular, the stellar interiors have traditionally been associated with fluid dynamics and, later, magnetohydrodynamics (MHD), while stellar winds and atmospheres are usually described using plasma physics. To go beyond this first approach, these various communities had to gather their knowledge to produce the most consistent models possible. The best example of this is, of course, the closest star to us and hence the easiest one to observe in thorough detail: the Sun.

For more than four centuries, the Sun has been observed displaying cyclic activity through sunspot observations. Later on, Hale (1908) linked the sunspots to magnetic activity, which led to the realization that it has an 11 yr period for amplitude and 22 yr period for polarity. The general framework adopted nowadays to explain such a generation of a large-scale magnetic field is the interface dynamo (Moffatt 1978; Parker 1993; Browning et al. 2006); the differential rotation profile in the convection zone of the star (Schou & Bogart 1998; Thompson et al. 2003) leads to the generation of strong toroidal fields at the tachocline (Spiegel & Zahn 1992), which, in turn, is used to regenerate poloidal fields thanks to the combination of turbulence, buoyancy, and Coriolis force at the surface. This dynamo loop allows for the amplification of the initial magnetic field until saturation, thus sustaining it against ohmic dissipation (Miesch 2005; Brun & Browning 2017). To analyze the dynamo field, it can be decomposed into modes characterized by degrees \( \ell \) and \( m \) by projecting it on the spherical harmonics base. Under the assumption of axisymmetry (e.g., \( m = 0 \)), odd \( \ell \) are called dipolar modes and constitute the primary dynamo family, while even \( \ell \) are called quadrupolar modes and constitute the secondary family (McFadden et al. 1991). A simplified yet efficient approach to modeling solar magnetism is the mean-field dynamo (Roberts 1972; Krause & Raedler 1980), focusing on large-scale fields and assuming axisymmetry. The generation of a toroidal field through differential rotation is then deemed the \( \Omega \) effect, and the regeneration of the poloidal or toroidal field via turbulence is deemed the \( \alpha \) effect. The combination of these two effects can give \( \alpha - \Omega \), \( \alpha^2 \), or \( \alpha^2 - \Omega \) dynamo loops. This description has the advantages of being easily implemented in MHD simulations with low computational costs and yielding realistic results (Tobias 2019; Charbonneau 2020).

On the other hand, it was not until the observations of the Mariner 2 spacecraft that the existence of a dynamic solar atmosphere being accelerated into a transonic and transalfvénic wind was accepted by the community (Neugebauer & Snyder 1962). The first hydrodynamical description was given by Parker (1958), and magnetism was added by Schatzman (1962), Weber & Davis (1967), Mesel (1968), and Sakurai (1985) to yield a better description of the corresponding torque applied to the star. The solar wind can be described using empirical models such as the WSA model (Wang & Sheeley 1990; Pinto & Rouillard 2017), in 2D with axisymmetry (Keppens & Goedbloed 1999; Matt & Pudritz 2008; Réville et al. 2015), or in 3D with a full description of the corona (Tóth et al. 2012; Usmanov et al. 2014; Riley et al. 2015; Réville & Brun 2017). The wind can even be better described using a multifluid approach (Usmanov et al. 2000;
Hollweg & Isenberg 2002) to take into account the influence of the different populations of particles. There are still open problems like the precise mechanism behind the heating of the corona, which can be approximated by a polytropic state with a low adiabatic index but is described better with detailed heating mechanisms such as magnetic reconnection (Parker 1988) or the turbulent dissipation of waves (Cranmer et al. 2007; Réville et al. 2018; Shoda et al. 2020) in a nontrivial way.

Recent observations by the satellite Ulysses for cycle 22 and the beginning of cycle 23 (McComas et al. 2003, 2008) or the satellite OMNI for cycles 23 and 24 (Ovens et al. 2017) have shown that there is a correlation between the 11 yr dynamo cycle and the evolution of the corona. During a minimum of activity, the magnetic field is low in amplitude, and its topology is mostly dipolar. The corona is also very structured, with fast wind at the poles (around 800 km s$^{-1}$) associated with coronal holes and slow wind at the equator (around 400 km s$^{-1}$) associated with streamers (Wang et al. 2007). During a maximum of activity, the magnetic field is high in amplitude, and its topology is a mixture of high modes dominated by the quadrupolar modes with equatorial symmetry (DeRosa et al. 2012), while fast and slow solar streams can be found at all colatitudes. Complementary observations from Earth using interplanetary scintillations (see Hewish et al. 1964 and Asai et al. 1998) can help reconstruct the latitudinal evolution of the solar wind speed over time. They display an 11 yr pattern in the form of a cross due to the correlation between the activity cycle and the latitudinal distribution of slow and fast streams (Tokumaru et al. 2010; Sokół et al. 2015). This suggests that there is a coupling operating between the interior and exterior of the Sun, but the exact physical mechanisms at play, as well as the relevant timescales, are still unclear (Wedemeyer-Böhm et al. 2009).

From a theoretical and numerical point of view, it is, however, very difficult to study all of these layers simultaneously. The magnetic field and the wind evolve over a broad range of scales; the magnetic field evolves over 11 yr with various scales ranging from several megameters to a solar radius (Brun & Browning 2017), while the solar wind adapts in a few hours, accelerating over a couple of tens of solar radii but extending up to several astronomical units (Meyer-Vernet 2007). The physical properties of their respective environments are also very different; the interior of the star is characterized by convection (Rieutord & Rincon 2010) and large-scale flows (Thompson et al. 2003; Basu & Antia 2010), while the exterior witnesses eruptive events in a dynamical atmosphere (Schrijver 2005). The rapidly changing $\beta$ plasma parameter (ratio of the thermal pressure over the magnetic pressure) at the surface of the star or the Mach number going from subsonic to supersonic in the corona are also evidence that the relative importance of physical phenomena can vary between the interior of the star and its extended atmosphere (Gary 2001). All of these disparities make the modeling of this coupling a numerical challenge.

There have been many attempts to resolve this problem with various approaches. A first approach is to use a quasi-static approach, meaning that the coupling is realized through a series of wind-relaxed states corresponding to a sequence of magnetic field configurations evolving in time. These models can be data-driven using a series of magnetic field observations (Luhmann et al. 2002; Merkin et al. 2016; Réville & Brun 2017) or rely on the numerical coupling between two codes dedicated, respectively, to the inner and outer layers of the Sun (Pinto et al. 2011; Perri et al. 2018). Another approach is to focus on the surface with a numerical box of a few tens of megameters to capture the small time and spatial scales. This means that this approach can include all of the relevant physical processes (for example, convection or radiative transfer at the surface) but for a short period of time and only for a specific region of the Sun (Skartlien et al. 2000; Vögler et al. 2005; Stein & Nordlund 2006; Wedemeyer-Böhm et al. 2009), so they lack global coupling. Finally, there have been some attempts to model a dynamical coupling on a global scale, for example, in von Rekowskii & Brandenburg (2006) or Warnecke et al. (2016). The first study focused on T Tauri–like stars with an accretion disk and showed that the presence of a cyclic magnetic field in the star greatly affects its wind and its interaction with the magnetized disk. The second study focused on the influence of a realistic transition region on solar-like convective zones and showed changes in the differential rotation and magnetic field evolution, which means that the modeling of the stellar surface can be crucial for its dynamics.

We wish to follow a similar approach of full dynamical coupling of the inner and outer layers of a star with an emphasis on improving the treatment of the physical interface between these two layers. Our aim is to focus on the large-scale field by using mean-field and axisymmetry assumptions. We design a 2.5D numerical model including the star and its corona and an interface to control the interaction between the two zones. For this first study, we will focus on young solar-like stars to reduce the discrepancies of the timescales between the dynamo and the wind. In Section 2, we describe the choices we made for the modeling of the dynamo and the wind, as well as the interface between the two computational domains. In Section 3, we present an application of our model to a case of a young Sun with a short dynamo period. First, we show only the influence of the dynamo-generated magnetic field on the structure of the corona and wind-integrated quantities such as the Alfvén radius and mass loss. Then, in Section 4, we allow the wind to back-react on the dynamo in a two-way coupling and quantify its impact. We discuss the feedback loop in our simulations by analyzing it in terms of dynamo modes and helicity generation. Finally, in Section 5, we summarize our main results and discuss the next steps for future studies.

2. Numerical Dynamo–Wind Setup

We begin by presenting the equations solved and our numerical setup dedicated to the study of dynamo–wind coupling. We use the PLUTO code (Mignone et al. 2007), which is a compressible multiphysics and multisolver code with an extended MHD module. We first present the wind model in Section 2.1, already described in Perri et al. (2018). Then we present the implementation of the mean-field dynamo and its validation in Section 2.2. Finally, we describe the numerical interface between the two computational zones in Section 2.3. We will limit ourselves here to an axisymmetric (2.5D) description of the dynamo–wind coupling. The main consequence is that any magnetic flux emerging through the stellar boundary is axisymmetric, and we cannot cope with magnetic structures localized in longitude, such as starspots and active regions. This limitation does not prevent us from assessing how the coupling operates on a large-scale basis, and
we leave the extension to fully 3D mean-field models (Karak & Miesch 2017; Kumar et al. 2019) for future work.

2.1. Wind Model

Our wind model is adapted from Réville et al. (2015). We solve the set of conservative ideal MHD equations composed of the continuity equation for the density $\rho$, the momentum equation for the velocity field $\mathbf{u}$ with its momentum written $m = \rho \mathbf{u}$, the energy equation with the energy noted $E$, and the induction equation for the magnetic field $\mathbf{B}$,

$$\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (1)$$

$$\frac{\partial}{\partial t} m + \nabla \cdot \left( mu - \frac{BB}{4\pi} + Ip \right) = \rho \mathbf{u}, \quad (2)$$

$$\frac{\partial}{\partial t} E + \nabla \cdot \left( (E + p)\mathbf{u} - \frac{B}{4\pi}(\mathbf{u} \cdot \mathbf{B}) \right) = m \cdot \mathbf{a} - \frac{1}{4\pi} \nabla \cdot [(\eta_\omega(\mathbf{\nabla} \times \mathbf{B}) \times \mathbf{B})], \quad (3)$$

$$\frac{\partial}{\partial t} \mathbf{B} + \nabla \cdot (\mathbf{uB} - \mathbf{Bu}) = -\nabla \times [(\eta_\omega \nabla \times \mathbf{B})], \quad (4)$$

where $p$ is the total pressure (thermal and magnetic), $I$ is the identity matrix, $\mathbf{a}$ is a source term (gravitational acceleration, in our case), and $\eta_\omega$ is the resistivity in the wind domain. We use the ideal equation of state,

$$\rho \varepsilon = \rho_{th}/(\gamma - 1), \quad (5)$$

where $\rho_{th}$ is the thermal pressure, $\varepsilon$ is the internal energy per mass, and $\gamma$ is the adiabatic exponent. This gives $E = \rho \varepsilon + m^2/(2\rho) + B^2/(8\pi)$ for the energy.

PLUTO solves normalized equations using three quantities to set all of the others: length $R_0$, density $\rho_0$, and speed $U_0$. If we note the parameters related to the star with an asterisk and the parameters related to the normalization with zero, we have $R_\ast/R_0 = 1$, $\rho_\ast/\rho_0 = 1$, and $u_{\text{cep}}/U_0 = \sqrt{GM/R_0}/U_0 = 1$, where $u_{\text{cep}}$ is the Keplerian speed at the stellar surface, $G$ is the gravitational constant, and $M_\ast$ is the stellar mass. By choosing the physical values of $R_0$, $\rho_0$, and $U_0$, one can deduce all of the other values given by the code in physical units. In our setup, we choose $R_0 = R_\odot = 6.96 \times 10^{10}$ cm, $\rho_0 = \rho_\odot = 1.68 \times 10^{-6}$ g cm$^{-3}$ (which corresponds to the density in the solar corona above 2.5 Mm; see Vernazza et al. 1981), and $U_0 = u_{\text{cep}}, U_\odot = 4.37 \times 10^3$ km s$^{-1}$. Our wind simulations are then controlled by three parameters: the adiabatic exponent $\gamma$ for the polytropic wind, the rotation of the star normalized by the escape velocity $u_{\text{esc}}/u_{\text{esc}}$, and the sound speed also normalized by the escape velocity $c_s/u_{\text{esc}}$. Note that the escape velocity is defined as $u_{\text{esc}} = \sqrt{2} u_{\text{cep}} = \sqrt{2GM/R_\odot}$. For the rotation speed, we take the solar value, which gives $u_{\varpi}/u_{\text{esc}} = 2.93 \times 10^{-3}$. We choose to set $c_s/u_{\text{esc}} = 0.243$, which corresponds to a $1.3 \times 10^4$ K hot corona for solar parameters and $\gamma = 1.05$. This choice of $\gamma$ is dictated by the need to maintain an almost constant temperature as the wind expands, which is what is observed in the solar wind. Hence, choosing $\gamma = 5/3$ is a simplified way of taking into account heating, which is not modeled here. An evolution of this model handling the complex problem of coronal heating with Alfvén waves can be found in Réville et al. (2020) and Hazra et al. (2021). We have a final control parameter, $u_A/u_{\text{esc}}$, which corresponds to the Alfvén speed $u_A$ normalized by the escape velocity to control the amplitude of the initial magnetic field. The value of this parameter depends on the dynamo parameters, as explained in Appendix C, and thus will be presented in Section 3.

We assume axisymmetry but solve for the three components of flow and magnetic field. We use the spherical coordinates ($r$, $\theta$). We choose a finite-volume method using an approximate Riemann solver (here the HLL solver; see Einfeldt 1988). PLUTO uses a reconstruct–solve–average approach using a set of primitive variables ($\rho$, $\mathbf{u}$, $p$, $\mathbf{B}$) to solve the Riemann problem corresponding to the previous set of equations. The time evolution is then implemented via a second-order Runge–Kutta method. To enforce the divergence-free property of the field, we use a hyperbolic divergence cleaning, which means that the induction equation is coupled to a generalized Lagrange multiplier in order to compensate for the deviations from a divergence-free field (Dedner et al. 2002).

The numerical domain dedicated to the wind computation is an annular meridional cut with a colatitude $\theta \in [0, \pi]$ and radius $r \in [1.01, 20] R_\odot$, as shown in the left panel of Figure 1. We use a uniform grid in latitude with 256 points and a stretched grid in radius with 400 points, with the grid spacing geometrically increasing from $\Delta r/R_\odot = 0.0016$ at the surface of the star to $\Delta r/R_\odot = 0.02$ at the outer boundary. At the latitudinal boundaries ($\theta = 0$ and $\pi$), we set axisymmetric boundary conditions. At the top radial boundary ($r = 20 R_\odot$), we set an outflow boundary condition that corresponds to $\partial/\partial r = 0$ for all variables, except for the radial magnetic field, where we enforce $\partial(rB_r)/\partial r = 0$. Because the wind has opened the field lines and under the assumption of axisymmetry, this ensures the divergence-free property of the magnetic field. The bottom boundary conditions that we would use with the wind computational domain only are shown in the right panel of Figure 1. In the ghost cells (shown in green), the density $\rho$ and pressure $p$ are set to a polytropic profile, the rotation is uniform, and the poloidal speed $U_{pol}$ is aligned with the poloidal magnetic field $B_{pol}$. The latter is held fixed, while the toroidal magnetic field $B_t$ is linearly extrapolated from the numerical domain. In the first point of the computational domain (shown in blue), all physical quantities are free to evolve, except for the poloidal speed $u_{pol}$, which is forced to be aligned with the poloidal magnetic field $B_{pol}$ to minimize the generation of currents at the surface of the star and keep it as close as possible to a perfect conductor. We initialize the velocity field with a spherically symmetric polytropic wind solution. In the gray area, we do not yet solve the dynamo, which we will now describe.

2.2. Dynamo Model

As we have seen in Equation (4), PLUTO solves the full nonlinear induction equation. For the dynamo inside the star, we will consider that the magnetic field $\mathbf{B}$ is the large-scale mean field and the velocity $\mathbf{u}$ is now only composed of a toroidal component $\mathbf{u}_t$ (we neglect the meridional flow for now); thus, we implement an alternative form of the induction equation,

$$\frac{\partial}{\partial t} \mathbf{B} + \nabla \cdot (u_t \mathbf{B} - \mathbf{Bu}_t) = \nabla \times (\alpha \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}), \quad (6)$$

where $\eta$ is the effective magnetic diffusivity and $\alpha$ is a parameterized coefficient (the $\alpha$ effect, which can be obtained through the first-order smoothing approximation of the electromotive force; Pouquet et al. 1976). Hence, the $\Omega$ effect
is taken into account with the second term and the $\alpha$ effect with the third one. We can choose to have an $\alpha$ or $\alpha^2$ effect by disabling or enabling the third term for the equation on $B_\phi$. This form of the induction equation is only active inside the star ($r < R_\ast$). Equations (1)–(3) are not solved inside the star.

This new induction equation follows the same normalization as described before. However, the mean-field dynamo community usually refers to the control parameters $C_\alpha$ and $C_{\Omega 1}$ obtained with a different normalization. To make it more convenient, we will use in this paper the traditional control parameters of the dynamo models when discussing our solution, but we describe in Appendix A the difference between the two and hence which conversion factor we apply in our code to respect the PLUTO normalization.

The flow in the dynamo domain will consist here only of a differential rotation such that $u_\phi = \Omega_0 r \sin \theta \Omega(r, \theta)$. The rotation in this zone is solar-like (Thompson et al. 2003) with a solid-body rotation below 0.66 $R_\ast$ and differential rotation above, with the equator rotating faster than the poles (25 versus 35 days). We use the following normalized profile:

$$\Omega(r, \theta) = \Omega_c + \frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{r - r_c}{d}\right)\right) \left(1 - \Omega_c - c_2 \cos^2 \theta\right),$$

(7)

where $\Omega_c = 0.92$, $r_c = 0.7 R_\ast$, $d = 0.02 R_\ast$, and $c_2 = 0.2$. The corresponding control number $C_{\Omega 1} = \Omega_0 R_\ast^2 / \eta$ is always fixed at $1.4 \times 10^{11}$, which corresponds to an equatorial rotation rate of $\Omega_0 / 2\pi = 456$ Hz for $\eta_\ast = 10^{11}$ cm$^2$ s$^{-1}$, in good agreement with the observation of the actual solar rotation rate.

As a first simple approximation, the $\alpha$ effect is constant in the convection zone and zero in the radiative zone below with a smooth transition between the two zones. We use the following profile studied by the benchmark of Jouve et al. (2008):

$$\alpha(r, \theta) = \alpha_0 \frac{3\sqrt{3}}{4} \sin^2 \theta \cos \theta \left(1 + \operatorname{erf} \left(\frac{r - r_c}{d}\right)\right) \frac{1}{t_{\text{quench}}},$$

(8)

The physical amplitude of the $\alpha$ effect is given by the $C_\alpha = C_\alpha R_\ast / \eta_\ast$ parameter, the value of which will vary. The quenching term $t_{\text{quench}}$ allows for saturation at the reference magnetic field value $B_{\text{quench}}$:

$$t_{\text{quench}} = 1 + \left(\frac{B_\phi(r, \theta, t)}{B_{\text{quench}}}\right)^2.$$  

(9)

We have a swift transition in diffusivity between the radiative and convection zones of 2 orders of magnitude following this profile:

$$\eta(r) = \eta_c + \frac{1}{2} \left(\eta_h - \eta_c\right) \left(1 + \operatorname{erf} \left(\frac{r - r_c}{d}\right)\right),$$

(10)

with $\eta_c = 10^8$ and $\eta_h = 10^{11}$ cm$^2$ s$^{-1}$.

The magnetic field is initialized with a dipole confined in the convection zone,

$$A_\phi = A_0 R_\ast^2 \sin \theta \left(1 - \frac{1}{2} \left(1.0 + \tanh \left(\frac{r - r_c}{d}\right)\right)\right),$$

(11)

where $A_0 = \eta_\ast \sqrt{4\pi \rho_\ast R_\ast}$. Hence, $B_\phi$ is initially equal to zero.

The numerical domain dedicated to the dynamo computation is an annular meridional cut with a colatitude $\theta \in [0, \pi]$ and radius $r \in [0.6, 1.01] R_\ast$, as shown in the left panel of Figure 2. We use a uniform grid in latitude with 256 points and a uniform grid in radius with 200 points, which yields a grid spacing of $\Delta r / R_\ast = 0.002$. At the latitudinal boundaries ($\theta = 0$ and $\pi$), we set axisymmetric boundary conditions. For the bottom boundary condition ($r = 0.65 R_\ast$), we use a perfect conductor condition:

$$A_\phi = 0, \quad \frac{\partial (r B_\phi)}{\partial r} = 0.$$  

(12)

The top boundary conditions ($r = R_\ast$) that we use when only the dynamo model is running are shown in the right panel of Figure 2. The poloidal magnetic field $B_{\text{pol}}$ is matched to a potential field following the method described in Appendix B, while the toroidal magnetic field $B_\phi$ is set to zero.
Since this kind of kinematic dynamo had never been implemented before in the PLUTO code, we validated our model by comparing it with the benchmark described in Jouve et al. (2008). Table 1 compares the critical threshold for the dynamo number \( C_{\text{crit}} \) to have a dynamo solution growing exponentially and the period \( \omega \) of the corresponding cycle obtained in our dynamo setup to the published benchmark cases. The letters A and B refer to cases without and with a jump in diffusivity. Cases with a prime refer to radial boundary conditions, while cases without refer to potential boundary conditions. Our cases show good agreement within the error bars found in the benchmark, and we are thus confident that our dynamo setup is numerically robust.

### 2.3. Wind–Dynamo Interface

The interface buffer between the dynamo and wind computational domains is shown in Figure 3. This interface is crucial to control and understand the interactions between the two domains. We will describe the chosen interface in this section, combining the traditional dynamo and wind boundary conditions shown in Figures 1 and 2. Now both gray zones are becoming active.

The interface is divided into four layers of one grid point each because of the two-point stencil used for our linear reconstruction. That way, the first two layers constitute the boundary condition for the dynamo, and the last two layers constitute the boundary condition for the wind. We also describe the first two points of the wind computational domain (blue in Figure 3), where we add extra conditions for more stability.

The first layer is very similar to the dynamo boundary conditions shown in the right panel of Figure 2, except that now we prescribe the density and pressure to follow a polytropic law (but continuous with the constant value inside the star). The magnetic field is extrapolated as a potential field. On the right is a scheme of the top boundary conditions used for the dynamo.

The second layer is special because we can enforce two types of boundary conditions. It can be similar to the first layer, so closer to the dynamo boundary condition; if we do so, the wind computational region has absolutely no impact on the dynamo boundary conditions. This leads to what we call **one-way coupling**, where only the dynamo can influence the wind, without feedback from the latter. On the contrary, we can choose to enforce a condition similar to the last two layers, so closer to the wind boundary condition; if we do so, the wind can back-react on the dynamo boundary conditions via the \( B_{\phi} \) condition, which is derived from the wind computational domain. This is what we call **two-way coupling**, where both the dynamo and the wind influence each other.

In the first two layers of the wind computational domain, we still impose the poloidal speed to limit the generation of currents. We also impose some diffusivity a bit further than the star surface using the following expression:

\[
\eta_w = \frac{1}{2} \eta \left( 1 + \tanh \left( \frac{r_{\eta} - R_*}{d_{\eta}} \right) \right),
\]

with \( r_{\eta} = 1.015 \) \( R_* \) and \( d_{\eta} = 0.003 \) \( R_* \). This allows the diffusivity to drop only after around 10 grid points above the interface. As we explain in Appendix C, this additional
diffusivity helps with the stability of our coupled dynamo–wind model and allows the information from the dynamo domain to be smoothly transmitted to the wind domain. The final simulation box obtained is thus a combination of a dynamo computational domain, an interface buffer region, and a wind computational domain, as can be seen in Figure 4. Thus, the poloidal magnetic field generated by the dynamo effect inside the star can directly impact the corotating idealized corona, with propagating timescales characterized in Appendix C.

3. Consistent Evolution of Dynamo and Wind Quantities along an Activity Cycle

To validate the coupling from a theoretical point of view, we focus on a model that develops a fast magnetic cycle and is thus probably more related to a young Sun with the same rotation rate but enhanced activity. This allows us to reduce the discrepancy between the dynamo and wind timescales and thus compute the evolution along several activity cycles using available computation resources. The main characteristics of this model, which we will now refer to as DW (for dynamo–wind), are given in Table 2. We use the magnetic diffusivity to set the cycle period by adjusting the diffusive time $\Delta t = \frac{R_*^2}{\eta}$. Here it corresponds to a 5 day period for the magnetic cycle. Then, we set $C_\alpha$ to have the solar rotation rate, and finally, we set $C_\alpha$ to have the proper dynamo number $D = C_\Omega \times C_\alpha$ to trigger the dynamo. This yields a case where $C_\Omega$ is set to a smaller value than $C_\alpha$ (which is less frequent than the reverse), which corresponds to a strong generation of the poloidal field due to turbulence. Because of this parameter range, we initialize the magnetic field with a small value to prevent growth up to unrealistic values, with a value of $u_A/u_{\text{esc}}$ of 0.05.

Another important parameter to set is the quenching value of the toroidal field $B_{\text{quench}}$. It is indeed crucial to ensure that the large-scale magnetic field does not reach unrealistic values of $u_A/u_{\text{esc}}$ (see Réville et al. 2015). We have thus derived an empirical criterion to stabilize the poloidal maximum surface
energy close to the input value of the simulation. To do so, we have run four simulations with the same dynamo number $D$ but different ratios of $C_\alpha$ and $C_\Omega$, and found the following relationship:

$$B_{\text{quench}} = 0.17 B_{\text{pol}} \sqrt{\frac{C_\alpha}{C_\Omega}}.$$

(14)

We can then use our input parameters to compute an estimate of $B_{\text{quench}}$ given a desired surface poloidal field.

We will first present a one-way coupling version of the DW model, which we will call DW1. This allows us to show the impact of the dynamo-generated magnetic field on the corona. For the first time, we obtain a self-consistent solution using one simulation domain. This model is indeed already a great improvement in regard to the quasi-static methods (Perri et al. 2018) because it allows for a dynamical influence of the dynamo magnetic field on the wind without any risk of breaking causality (given that we respect the criteria derived in Appendix C).

We start by presenting the dynamo solution. The butterfly diagram obtained through this one-way coupling is presented in the left panel of Figure 4. We have chosen to display the evolution between approximately $1.07 t_\eta$ and $1.23 t_\eta$ to avoid initial transients and focus on typical structures. The top panel shows the toroidal magnetic field $B_t$ at the base of the tachocline (0.7 $R_*$), the middle panel shows the radial magnetic field $B_r$ at the surface of the star, and the bottom panel shows the latitudinal magnetic field $B_\phi$, also at the surface of the star. They are all in Gauss units. At the base of the tachocline (top panel), we can clearly see the reversal of polarity of the toroidal magnetic field; the surface poloidal field also follows a cyclic evolution with polarity reversals (bottom panels). The period of the dynamo cycle is approximately $0.04 t_\eta$. The cycle displays both equatorial and polar branches at the surface due to our choice of $\alpha$ effect. It corresponds to a so-called Parker–Yoshimura dynamo wave (Yoshimura 1975). Over the temporal range considered, the cyclic magnetic field is always antisymmetric with respect to the equator. The right panel of Figure 4 shows meridional cuts taken at moments of interest during the evolution of the solution. We can see the norm of the poloidal magnetic field over the first 1.5 solar radii in code units ($B_0 = 2$ G). The surface of the star is marked by a black dashed line, while the magnetic field lines are displayed in white (solid line when clockwise, dashed when counterclockwise). Snapshot (a) allows us to see the regeneration of the poloidal magnetic field due to the dynamo effect inside the star, more precisely at the tachocline (0.7 $R_*$), due to our distributed $\alpha$ effect. The magnetic structure appears mostly dipolar at large scales but more octopolar closer to the surface (both of antisymmetric parity). We can already see the field lines crossing the interface continuously into the corona. Snapshot (b) shows the advection of the magnetic structures and their transmission to the corona thanks to our potential field boundary condition. Once in the corona, the structures interact with the stellar wind, which tends to open the field lines and quickly decrease in amplitude. We begin to see the regeneration of the next cycle structures inside the tachocline, with the opposite polarity. The corresponding times are shown as black dashed lines on the butterfly diagram in the left panel.

Now we will present the response of the corona to this cyclic dynamo for the one-way coupling. The left panel of Figure 5 shows meridional cuts taken at moments of interest during the evolution of the DW1 solution. We can see the Mach number projected on the normalized magnetic field up to $20 R_\ast$. This allows us to see at the same time the various streams of the stellar wind and the corresponding polarity of the magnetic field. The black line shows the Alfvén surface that corresponds to the distance at which the wind speed becomes equal to the

![Figure 5](image.png)

**Figure 5.** Left panel: meridional cuts of the dynamo–wind solution at two different times for case DW1. We show the quantity $u \cdot B / (c_\alpha |B|)$, which corresponds to the Mach number projected on the normalized magnetic field. The black line is the Alfvén surface, which is the distance at which the wind speed becomes equal to the Alfvén speed. The surface of the star is indicated by a black dashed line. We clearly notice various polarity sectors. Right panel: time–latitude diagram of the wind solution for case DW1. We show the wind speed $u_\text{esc}$ at the far end of our computational box ($r = 20 R_\ast$) in kilometers per second. The corresponding times for the meridional cuts in the left panel are shown as black dashed lines on the wind diagram.

| Name | $\eta_\text{f}$ (cm$^2$ s$^{-1}$) | $C_\alpha$ | $C_\Omega$ | $B_0$ (G) | $u_\text{esc}/u_\text{esc}$ | $\gamma$ | $c_\alpha/u_\text{esc}$ | $\rho_\text{es}$ |
|------|------------------|-----------|-----------|----------|-----------------|--------|----------------|----------|
| DW   | $3.7 \times 10^{14}$ | $1.44 \times 10^7$ | $3.4 \times 10^7$ | $3.0 \times 10^{-3}$ | 0.05 | 1.05 | 0.243 | 0.55 |

This table shows the main parameters for the test case DW.
Alfvén speed. The surface of the star is marked by a black dashed line. We can see that the corona is highly dynamical with transients generated at the surface of the star and carried by the wind to the external boundary of the computational domain. Due to the cyclic nature of the dynamo, the hemispheres reverse in polarity. In snapshot (c), the northern hemisphere is divided between two zones, one positive near the pole and one negative near the equator. In snapshot (d), however, we have a similar disposition but with reverse polarity; the coronal hole near the pole is now negative, and the equator is positive. This behavior is the same with opposite polarity in the southern hemisphere. Finally, we can notice that the Alfvén surface is displaying various shapes along the cycle, which is a marker of the topological changes of the dynamo field passing into the corona. Occasionally, this surface can come very close to the star surface. This is due to both the low value of the magnetic field that we have to maintain with the quenching to ensure the correct coupling of the stellar interior and exterior and the dynamical evolution of the dynamo that can generate a localized region of a very low amplitude surface field. We now turn to studying the long-term evolution of the corona by considering the time–latitude diagram of the wind speed in the right panel of Figure 5. The radial velocity $u_r$ is taken at the outer boundary of the domain (20 $R_\odot$) and shown in kilometers per second. The corresponding times for the meridional cuts in the left panel are shown as black dashed lines on the wind diagram. We observe latitudinal variations as a response to the coupling with the dynamo-generated magnetic field. They also present themselves as cyclic with a period that is half of the dynamo cycle (around 0.02$T_\text{r}$ versus 0.04$T_\text{r}$) because it is dictated by the topological changes rather than the polarity changes. We observe an alternation of slow (blue) and fast (red) streams at the poles and equator. Because of the rapidly changing magnetic field, we also observe regimes where fast streams are at all latitudes while the corona is adapting to the stellar interior. This is reminiscent of the solar wind observed during the minimum and maximum of the solar cycle (McComas et al. 2003); we do not have the same structures because we do not model the Sun, but we still achieve a coupling between the dynamo field and the wind structures. The speeds reached are between 200 and 270 km s$^{-1}$ at 20 $R_\odot$, which corresponds to between 400 and 520 km s$^{-1}$ if we extrapolate them at 1 au. Thus, we mostly recover the slow component of the wind, which is typical of polytropic winds and expected given our choice of $c_s/H_{\text{esc}}$.

We now want to understand how the one-way coupling operates. Figure 6 shows time–radius diagrams of $B_r$ and $B_\theta$ at two latitudes; the top panel is at colatitude $\theta = 30^\circ$ (closer to the northern pole), and the bottom panel is at colatitude $\theta = 60^\circ$ (mid-latitude in the northern hemisphere). This allows us to focus on the transmission of the magnetic field from the stellar interior to the corona, with the stellar surface marked by a black dashed line at 1.0 $R_\odot$. The times corresponding to the meridional cuts shown in Figures 4 and 5 are shown as black dashed lines on the time–radius diagrams with the associated letter. We can clearly see the magnetic structures being generated at the base of the convection zone at 0.7 $R_\odot$, then advected to the stellar surface, and finally transmitted to the corona with the potential field boundary condition for $B_{\phi}$ in the left panel. The structures are then influenced by the solar wind to the end of the computational domain but also quickly decrease in intensity. We can also see the reversals in polarity due to the cyclic dynamo that translates into reversals of polarity in the corona as well, but with a delay due to the transmission time. The various latitudes show that the continuity of the stellar corona is different depending on the strength and geometry of the field. For example, $\theta = 60^\circ$ corresponds to less intense dynamo structures that decrease rapidly at the surface of the star, so the transmission leads to less intense structures in the wind. For $B_{\phi}$, we can clearly see the discontinuity created by our interface; the dynamo converges to a zero-intensity toroidal field at the surface, while the wind needs to have a stellar surface with small electric currents. This leads, however, to the periodic generation of toroidal magnetic structures, which happens at the same time as the reversal in polarity for $B_r$. It is thus very likely a result of the adjustment of the corona because of our condition on $B_{\phi}$ and the necessity of keeping a divergence-free field in the domain. In this case, this leads to the fact that the polarity of the structures of $B_{\phi}$ in the corona is synchronized with the one from the interior. Here in the one-way coupling, there is no...
feedback from the wind, so the dynamo itself is not different from a model without atmosphere.

To complement our analysis of the DW1 case, we can further compute some key quantities of our stellar dynamo model.

We start with the magnetic characteristic length scale \( l_b \), defined as

\[
l_b = \frac{\sum_f \beta_{f,0}^2}{\sum_f \beta_{f,0}^2},
\]

where \( B_f(r = R_\star, \theta) = \sum_{m=0}^{\infty} \beta_{f,m} Y_m^0(\theta) \), so that the coefficients \( \beta_{f,0} \) correspond to the decomposition of the surface radial magnetic field on the spherical harmonics. We only use the \( m = 0 \) modes because in our 2.5D simulations, we consider only axisymmetric fields. This length scale allows us to characterize the dominant scale in the magnetic energy power spectrum.

We can also check the energy associated with a certain mode of the surface radial field, defined as follows:

\[
f_f = \frac{\beta_{f,0}^2}{\sum_{f=1}^{\infty} \beta_{f,0}^2}.
\]

We will use these two quantities to characterize the minima and maxima of the activity in our dynamo solution.

The time evolutions of the magnetic characteristic length scale \( l_b \) (black) and the fraction of energy in the dipolar component \( f_1 \) (red) for case DW1. The minima of activity are marked using red downward triangles derived from \( f_1 \), and the maxima are marked with black upward triangles derived from \( l_b \).

Figure 7. Time evolution of the magnetic characteristic length scale \( l_b \) (black) and dipole coefficient \( f_1 \) (red) for case DW1. The minima of activity are marked using red downward triangles derived from \( f_1 \), and the maxima are marked with black upward triangles derived from \( l_b \).

The time evolutions of the magnetic characteristic length scale \( l_b \) (black) and dipole coefficient \( f_1 \) (red) for case DW1. The minima of activity are marked using red downward triangles derived from \( f_1 \), and the maxima are marked with black upward triangles derived from \( l_b \).

The mass loss is defined as

\[
M = 2\pi R^2 \int_0^\pi \rho u_\theta \sin \theta \, d\theta,
\]

where \( R \) is the radius at which the mass loss is computed by integration through a spherical surface. In theory, when a stationary state is reached, the mass loss should be independent of \( R \), as long as it is big enough to include all of the closed magnetic structures. In our simulations, we perform an average of the mass losses computed with \( R \in [9; 20] \) \( R_\star \) to have a more precise value.

The temporal evolution of these quantities is shown in Figure 8. We have added the minima and maxima of activity as defined previously by \( l_b \) and \( f_1 \) with the same symbols as before: minima are marked using red downward triangles, and maxima are marked with black upward triangles. The average Alfvén radius varies between 1.5 and 3.0 \( R_\star \). This is smaller than what was found in all of the previous studies cited above, but this is mainly because we did not try to use solar parameters and because of the criterion (Equation (C3); see Appendix C for more details) that limits us to weaker magnetic fields. The mass loss varies between 4.5 and 6.4 \( \times 10^{-15} \) \( M_\odot \) \( \text{yr}^{-1} \), which corresponds to a 42% variation. For comparison, the Sun has a variation of about 35% (McComas et al. 2008; Réville & Brun 2017; Finley et al. 2019).

In this study, we are more interested in the relative variations of these quantities than their values. We see modulations in time with the evolution of the activity cycles, obtained for the first time in a completely self-consistent way. Based on similar studies, at a minimum of activity, we expect a bigger Alfvén radius but a smaller mass loss. At a maximum of activity, it is the opposite trend. The large-scale modulations are mostly in agreement with the cycles. At a minimum of activity, when the field is mostly dipolar, we indeed see a higher Alfvén radius and lower mass loss. The maximum of activity is slightly less related to the integrated quantities, but most of the time it still does correspond to a smaller Alfvén radius and bigger mass loss. We also see many small-scale variations corresponding to fluctuations and transients, which explains why the correlation is not always perfect. This may be due to the fact that we have a rapidly changing magnetic field at the surface of the star, which means the wind is constantly adjusting to the magnetic configuration.
4. Feedback Loop between Dynamo and Wind

We will now present what happens when we enable the feedback of the wind, which means that the wind is now able to back-react on the dynamo via the second layer of the interface and its condition on $B_f$ (see Figure 3). We will run the same model as presented in Table 2 but with full two-way coupling, so now we call this two-way coupling model DW2. We will begin by showing the evolution of the global quantities, as was done for DW1 in the last section. Then we will analyze the differences with the one-way coupling and quantify them in terms of dynamo modes and helicity.

4.1. Impact of Two-way Coupling on Global Quantities

The corresponding butterfly diagram for the two-way coupling DW2 case is displayed in the left panel of Figure 9 with the same disposition as in Figure 4. This new butterfly diagram is slightly different from the DW1 case. The toroidal magnetic field at the tachocline now tends to be more symmetrical with respect to the equator, starting at $1.12 \eta$, indicating that the quadrupolar modes (symmetric with respect to the equator) are dominating the dynamics of the dynamo. At the surface, the radial magnetic field is showing reversals of polarity but in an irregular way. It now presents north–south asymmetry most of the time and is alternating between symmetrical and antisymmetrical configurations, which suggests an effective coupling of the dynamo families (DeRosa et al. 2012). The same effect is present as well for $B_\theta$. This asymmetry is highlighted by the snapshots in the right panel of Figure 9. With the same presentation as in Figure 4, we can see in snapshot (e) a more symmetrical configuration when the poloidal field is generated at the tachocline. But when the field rises in snapshot (f), we see that the magnetic field lines crossing the surface and influencing the corona have an asymmetric structure. The field exhibits more deformed closed loops in the northern hemisphere because of the influence of the wind than in the southern hemisphere. The magnetic structures inside the star are also not symmetrical, with the
northern hemisphere rising faster than the southern one. The corresponding times are shown as black dashed lines on the butterfly diagram in the left panel.

In Figure 10, we present the corresponding time–latitude diagram of the radial wind velocity \( u_r \) for the two-way coupling model. We do not display the corresponding meridional snapshots of the corona like in Figure 5 because the evolution of the corona is very similar, apart from the north–south asymmetry we have already reported. The evolution of the corona is still highly dynamical with transients. When the butterfly diagram is more symmetrical at the surface (for example, between 1.15\( t_\eta \) and 1.25\( t_\eta \)), we can see that the coronal structure becomes more irregular; there is a big patch of slow wind at the northern pole at 1.17\( t_\eta \) that is not reproduced elsewhere. Fast streams seem to also be more dominant in the southern hemisphere over this period of time considered. This already shows that the feedback is introducing more variability in both the dynamo and the associated wind.

Figure 11 shows time–radius diagrams of \( B_r \) and \( B_\theta \) (in code units, \( B_0 = 2 \) G) for case DW2 at different latitudes; the top panel is at colatitude \( \theta = 30^\circ \) (closer to the northern pole), and the bottom panel is at colatitude \( \theta = 60^\circ \) (mid-latitude in the northern hemisphere). The stellar surface is marked by a dashed black line at 1 \( R_* \). We only show the radial evolution from 0.6 and 1.5 \( R_* \) to focus on the most intense structures. The times corresponding to the meridional cuts shown in Figure 9 are shown as black dashed lines on the time–radius diagrams with the associated letter.

Figure 10. Time–latitude diagram of the wind solution for case DW2. We show the wind speed \( u_r \) at the far end of our computational box \((r = 20 \ R_*)\) in kilometers per second. We clearly see the alternation of slower and faster wind streams and a huge degree of asymmetry compared to Figure 5.

Figure 11. Time–radius diagrams of \( B_r \) and \( B_\theta \) for case DW2 with two-way coupling. The times corresponding to the meridional cuts shown in Figure 9 are shown as black dashed lines on the time–radius diagrams with the associated letter. Once again, we clearly see the alternation of polarity and the efficient transmission of \( B_r \) at \( \theta = 30^\circ \). We can also see that the feedback of the wind has modified the dynamo boundary conditions, so that at \( \theta = 60^\circ \), the magnetic structures do not decrease as much in intensity. For \( B_\theta \), we are now left with only one point where \( B_\theta = 0 \); the other points depend on the dynamics of the wind. This results in more continuous and extended radial magnetic field structures at the star surface, especially at \( \theta = 60^\circ \). These structures are once again correlated with the changes of polarity of \( B_r \) in the corona. In case DW2,
this leads to an opposition of polarity between the structures in the star and those in the corona. This is due to the specific period we have selected, which focuses on a time with visible north–south asymmetry. We have verified that at different times when the quadrupole modes are weaker, the polarity of $B_p$ over the interface matches as in Figure 6.

Figure 12 shows the time evolution of $l_b$ and $f_1$ for case DW2 with the two-way coupling. In this case, $l_b$ reaches slightly higher values, going up to 5.5. This is an indication that the cycle is slightly more multipolar than in case DW1. We see that the two curves are still in anticorrelation over the considered time period; however, the relative positions of the minima and maxima are affected by the two-way coupling. We can still see a phase quadrature between them at the beginning when the cycle is more antisymmetrical, like in case DW1. However, when we start to be more symmetrical (at 1.15$t_f$ when $f_1$ reaches only 40% at maximum instead of 50%), the minima and maxima of activity derived begin to be in anticorrelation because $l_b$ is also affected by the two-way coupling and the feedback of the wind.

We can then see in Figure 13 the evolution of the integrated quantities defined above, namely the average Alfvén radius and the mass loss as the various cycles proceed. The Alfvén radius evolves between 1.5 and 2.75 $R_*$, which is a bit smaller as an interval with a smaller maximum value compared to case DW1. The mass loss evolves between 4.7 and $6.4 \times 10^{-15} M_\odot$ yr$^{-1}$, which is again a bit smaller with a higher minimum value compared to case DW1. The correlation with the minima and maxima of activity is still working well when the cycle is symmetrical, but when it tends to be more symmetrical (between 1.15$t_f$ and 1.25$t_f$), the trends are not so clear anymore. This could show that the trends derived in previous studies (described in Section 3) are not generically true when the symmetry of the dynamo cycle changes, as $f_1$ stops being the most relevant marker of the dynamics of the cycle.

4.2. Impact of the Interface Region on Dynamo Modes

As explained in the previous section, we see a clear difference between the two butterfly diagrams presented in Figures 4 and 9. Without the wind influence, the tachocline toroidal and surface radial magnetic fields are equatorially antisymmetric, and the cycle is regular with a period of 0.04$t_f$. With the influence of the wind, the cycle has the same period but evolves to a more equatorially symmetric shape after 1.15$t_f$ before returning to a more antisymmetric pattern after 1.25$t_f$. However, it never reaches full antisymmetry, which suggests a coupling between the dynamo families (DeRosa et al. 2012).

To understand this difference, we show in Figure 14 the evolution of the dipolar and quadrupolar modes ($\ell = 1$ and 2) for these two cases. The magnetic field is taken at the surface of the dynamo computational domain. We also show it for the entirety of the simulation, between 0.0$t_f$ and 1.75$t_f$, while previous figures only focused on the time period between 1.0$t_f$ and 1.3$t_f$. Without the influence of the wind (left panel), the dipolar mode has a rather constant maximal amplitude around 0.2 code units with cyclic variations with a period of approximately 0.04$t_f$ (same as the dynamo cycle), while the quadrupolar mode has an amplitude 4 orders of magnitude smaller. Hence, although it is continuously growing, the symmetric family remains negligible. But with the influence of the wind in case DW2, the quadrupolar mode grows to an amplitude equivalent of the dipolar mode (0.1 versus 0.2). This is, however, not constant because we can clearly see a long-term modulation on the quadrupolar mode with periods of high intensity (between 1.15$t_f$ and 1.25$t_f$ as noted before, for example) and periods of very low values compared to the dipolar mode (between 1.0$t_f$ and 1.15$t_f$, for example). The feedback of the wind in this case has a visible influence by favoring the growth of the symmetric family by influencing the boundary conditions of the dynamo. This is a very interesting result, demonstrating the need to take into account the dynamical coupling between the dynamo and the wind with its feedback loop.

We nevertheless recall that the considered case has unusual values for a dynamo with $C_{\alpha} \gg C_{\Omega}$. Tavakol et al. (1995) showed that in such parameter regimes, the dynamo is highly nonlinear and can switch easily from symmetric to antisymmetric regimes because of quenching or an asymmetry that results in the coupling of the dynamo families. We have also performed a threshold study similar to Jouve & Brun (2007) and determined that for this set of parameters, the quadrupolar mode has a growth threshold lower than the dipolar mode ($C_{\alpha} = 80$ versus $C_{\Omega} = 100$). This case is therefore mostly a proof of concept to show that the feedback loop between the dynamo and the wind plays an important role in our simulations. More investigation is needed to assess whether this feedback would be as efficient in other parameter regimes or for other types of dynamos. Another possible explanation is the treatment of magnetic helicity in our interface, which we will investigate in the next section.

4.3. Analysis of Helicity in Coupled Dynamo–Wind Simulations

To quantify the exchange of information between the stellar interior and atmosphere, we now turn to magnetic helicity. Magnetic helicity is the measure of the warping and coiling of the magnetic field and is defined as

$$H = \int A \cdot B dV,$$

where $B$ is the magnetic field and $A$ is the vector potential associated with $B$, such as $B = \nabla \times A$. There is no unicity of the vector potential $A$, which means that magnetic helicity has to be computed with respect to a certain gauge. To avoid this problem, we can define the relative magnetic helicity, which is
gauge-invariant (Berger 1999):

$$H_{\text{rel}} = \int (A + A_p) \cdot (B - B_p) dV,$$

(20)

where $B_p$ is a reference component, chosen to be potential \((\nabla \times B_p = 0)\) such that its components match the components of $B$ at the boundaries of the domain \((B_p \cdot n|_S = B \cdot n|_S)\). Here $A_p$ is the vector potential associated with $B_p$, such as $B_p = \nabla \times A_p$.

To compute $B_p$ and $A_p$, we use the fact that $B_p$ is chosen to be a potential field, which implies that it is derived from a scalar magnetic potential $\phi$ \((B_p = \Delta \phi)\). Here $\phi$ can be projected onto spherical harmonics, and the boundary conditions then give us enough constraints to determine it. To avoid discontinuity problems at the interface, we compute a reference field for the stellar interior and another for the corona, and we check after computation that the two match. To compute $A$, we combine direct integration of the magnetic field and a projection onto spherical harmonics to solve equations on the current $j$. For more details, please refer to Appendix D.

Figure 15 shows the time evolution of the relative magnetic helicity for cases DW1 (left) and DW2 (right). We have selected a different time interval from what was previously shown, this time from $1.3t_f$ to $1.58t_f$. This is to avoid the region where the dynamo modes are too coupled to really study only the influence of the interface on magnetic helicity in the most basic situation. We have divided it into a stellar interior (blue/green) and exterior (red/orange). We also decompose the helicity between its northern and southern components, respectively, in blue and green for the interior and orange and red for the exterior. This allows us to check the most basic property of magnetic helicity, which is its conservation in the numerical domain when magnetic diffusion is neglected. In the figure, we can see that indeed, the northern and southern components of helicity apparently compensate each other relatively well at each time for the star interior, and this is for both coupling cases. This shows that the diffusivity inside the star is not dominant enough to suppress this property on the timescales considered here. We can also see that the helicity is slightly more dominant in the exterior of the star, with 1 order of magnitude of difference with the interior. However, in the stellar corona, the conservation of relative helicity is visibly not as good as inside the star. We can see in the bottom panel, which shows the amplitude of the difference between northern and southern hemisphere relative helicity for the stellar interior (blue) and exterior (red). That the conservation of helicity in the exterior is not achieved in DW1 or DW2. Ohmic diffusion does affect the relative helicity inside the star but is unlikely to affect it in the corona, where it acts weakly only in the very first points of the wind domain. This points to a possible loss or injection of helicity at the interface and/or outer boundary. We will then analyze the fluxes of helicity to check this assumption.

Our setup was designed to let only the poloidal field go through the dynamo–wind interface. Now we want to see how our boundary condition affects magnetic helicity transfer into the idealized corona, as has been advocated by, for instance, Low (2013). To do so, we compute the evolution equation for $H_{\text{rel}}$ (Barnes 1988),

$$\frac{\partial H_{\text{rel}}}{\partial t} = \frac{8\pi \eta}{c} \int j \cdot B dV + 2F_H,$$

(21)

where $j$ is the current associated with the magnetic field $B$.

Based on this equation, we can define a helicity flux through a surface associated with a temporal variation of helicity. Under the assumptions of axisymmetry, 2.5D, and spherical coordinates of this study, this yields

$$F_H = 2\pi \int_0^{\pi} ((u_\phi B_\phi - u_\theta B_\theta) - \eta(\nabla \times B)_\theta)A_{p,\phi}R^2 \sin \theta d\theta,$$

(22)

where $R$ is the radius of the spherical surface through which the flux is computed. For more details about the helicity budget, please refer to Appendix D.

Figure 16 shows the time evolution of $F_H$ at three different radii for cases DW1 (left) and DW2 (right). In blue, we have the top boundary condition for the wind at $r = 20R_*$; in green, we have the bottom of the wind domain (just above the interface at $r = R_*$); and, finally, in red, we have the top of the dynamo domain (just below the interface at $r = R_*$). The top panels show the fluxes for the helicity budget in the wind domain; a positive (negative) value means that the wind helicity decreases. The bottom panels show the flux at the top of the dynamo domain; a positive (negative) value means that the dynamo helicity decreases. In all cases, we show the
decomposition of the helicity fluxes in the northern (dashed lines) and southern (dotted lines) hemispheres. As a sanity check, we have evaluated the flux loss at the bottom boundary condition of the dynamo and found it to be completely negligible. At the top wind boundary condition, helicity is lost due to the modified outflow boundary condition for the wind. We find a loss of the order of $5 \times 10^{-5}$ (in code units) at maximum in both cases. With these numbers in mind, we can then see that for case DW1, the injection at the interface is negligible, being practically zero at the top of the dynamo and reaching only $10^{-5}$ at maximum at the bottom of the wind domain. However, for case DW2, helicity transfer is enhanced at the interface with typical amplitudes of $10^{-4}$ at the bottom of the wind domain. We also notice that the northern and southern fluxes are different from one another in case DW2 due to the asymmetry discussed before (see Section 4.2), while they compensate perfectly in case DW1. Note that in both cases, we have verified that ohmic dissipation plays a negligible role, as shown in Appendix D.

We therefore see that our interface plays a nontrivial role in helicity conservation and injection. The net helicity flux at the top of the dynamo domain is small in case DW1, but a substantial helicity is injected at the bottom of the wind domain due to our boundary condition on $B_\phi$. In case DW2, this effect is even more pronounced, as our boundary dictates the helicity budget in the wind domain. It furthermore leads to helicity injection in the dynamo domain, which adapts strongly in case DW2 with the two-way coupling. This is probably another factor that explains why the coupling of the modes is so strong in this case. This means that we need to find a more appropriate way to limit the generation of currents at the bottom of our wind domain without jeopardizing the conservation of helicity. We plan to thus further investigate this condition in the interface for a more solar parameter space where the physical quantities may be less sensitive to this effect.

5. Conclusion and Discussion

We have presented here one of the first examples of self-consistent numerical coupling between an $\alpha$–$\Omega$ stellar dynamo and a thermally driven stellar wind. We used the PLUTO code (Mignone et al. 2007) to couple the wind setup of Réville et al. (2015) adapted for spherical coordinates in Perri et al. (2018) and the dynamo model described in the benchmark of Jouve et al. (2008) and implemented for the first time in the PLUTO code. The key point of this model is the interface between the
two physical domains; instead of being a software coupling between two independent codes, we have used a single numerical domain, where a four-mesh thick layer structure at the surface of the star has been implemented. This allows us to solve self-consistently for the dynamo–wind coupling inside a computational domain encompassing the star’s interior and atmosphere from 0.6 to 20 $R_\odot$. We can thus dynamically study the physical exchanges between the dynamo and the wind along a dynamo cycle. The magnetic field generated inside the star is extrapolated using a potential field extrapolation across the interface layers to be transmitted to the corona and affect the structure of the wind by its changes of amplitude and topology. We present here one of the first models able to quantify the feedback loop between the wind and the internal magnetic field. 

Here we lay out only the first developments of this model, which must be seen as a proof of concept to demonstrate the efficiency and interest of this method. To do so, we presented a model with an activity cycle having a much shorter period than the Sun. This model demonstrates cyclic variations of the corona along the dynamo cycle. These variations are associated with the opening and closing of coronal holes with the corresponding generation of streamers and variations of the wind-integrated quantities, such as the average Alfvén radius or the mass loss. These correlate with the minima and maxima of activity of the cycles, reproducing results obtained so far only by data-driven models (Merkin et al. 2016; Réville & Brun 2017) or quasi-static approaches (Pinto et al. 2011; Perri et al. 2018). This approach, however, may generate transients from which it is difficult to evaluate whether they carry physical information. By adjusting the interface layer, we are able to enable a one- or two-way coupling between the dynamo and the wind; thus, we are able to illustrate a possible feedback loop between the dynamo and the wind. By enabling the feedback of the wind, we observe a different dynamo behavior where the wind feedback loop seems to enhance the secondary family modes (Ossendrijver 2003; Svalgaard & Kamide 2013). We also demonstrate that our interface region may generate additional helicity due to the various conditions on $B_\theta$ and are exploring new methods to modify this property. By focusing next on slower dynamo cycles, we may have less transients and reconnection, which could also lead to more stable models.

There are many different directions to broaden the accuracy and applications of this model. To have a better description of the physics, we have begun to implement a Babcock–Leighton mechanism with flux transport for the dynamo; this means that instead of the deep-seated $\alpha$ effect, we take into account the generation of strong toroidal magnetic tubes at the base of the convective zone and their rise in this model by buoyancy, then twist by the Coriolis force, and then allow the regeneration of a poloidal field at the surface. The meridional circulation redistributes the poloidal field at the base of the convective zone to be sheared by the differential rotation (Babcock 1961; Leighton 1969; Dikpati & Charbonneau 1999; Jouve & Brun 2007). This model needs the implementation of a nonlocal term, which is more complex given the architecture of the PLUTO code; we will report this new dynamo setup in future work. The description of the wind can also be improved by using a more realistic heating; instead of a polytropic equation, there are models based on turbulence and Alfven-wave heating that give results in good agreement with observations (Riley et al. 2015; Réville et al. 2020; Hazra et al. 2021). For our next study, we will add these new elements and tackle the description of a coupled model between a solar-like dynamo with an 11 yr cycle and a solar-like turbulent corona with an interface better designed to avoid injection of helicity. This is, of course, a numerical challenge given the discrepancy of the timescales with a time step of the order of an hour to simulate at least 11 yr of solar activity. On the other hand, it will allow easier comparison with observational data to better calibrate the model (McComas et al. 2008). The parameter space description given in Appendix C will help us ensure that the coupling is operating in an efficient and physical way. One of the more ambitious objectives would be to more precisely describe the transition region between the photosphere and the corona. In our model, the resolution is too coarse to allow a separate description of the photosphere, chromosphere, and corona and the transitions in density and
temperature between these regions (Gary 2001; Meyer-Vernet 2007). Finally, we remind the reader that this model could actually be very easily used for other stars of F, G, or K spectral type with an internal structure similar to that of the Sun (with a convective zone at the surface and a radiative zone deep inside) and a thermal-driven wind. With elements such as the rotation profile, internal poloidal flows, or dynamo mechanism prescribed with input parameters, we can easily change them to reproduce other stars. For example, the two cases described in this paper were chosen purely theoretically but can be assimilated to young stars of T Tauri type, with results similar to the work of von Rekowski & Brandenburg (2006).

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Appendix A
Normalization of the Equations

Dynamo and wind numerical codes usually have different normalizations for the MHD equations, leading to different control parameters for simulations. For the coupling, as the dynamo and wind happen in the same numerical domain, we had to choose one normalization and convert the corresponding parameters. We chose the wind normalization to focus on only the induction equation in the PLUTO code. In this section, the two different normalizations are presented along with the conversion factors between the two for the induction equation.

We start with the induction equation from the mean-field theory (Moffatt 1978):

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U}_{\text{pol}} \times \mathbf{B} + \mathbf{U}_0 \times \mathbf{B} - \eta \nabla \times \mathbf{B} + \alpha \mathbf{B}). \quad (A1)$$

Each quantity can then be written as a product between a constant physical amplitude with units and a normalized profile describing the dependency on \(r, \theta, \phi, t\):

$$t = t_0 \tilde{t}, \quad r = L_0 \tilde{r}, \quad B = B_0 \tilde{B}, \quad \mathbf{U}_{\text{pol}} = \mathbf{U}_0 \tilde{U},$$

$$\mathbf{U}_0 = \Omega_0 L_0 \tilde{U}_0, \quad \eta = \eta_0 \tilde{\eta}, \quad \alpha = \alpha_0 \tilde{\alpha}. \quad (A2)$$

By injecting the decomposition described in Equation (A2) into Equation (A1), we obtain the following general normalized induction equation:

$$\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}} = \nabla \times \left( \frac{t_0 U_0}{L_0} \tilde{U}_{\text{pol}} \times \tilde{B} + \frac{L_0 \Omega_0}{t_0} \tilde{U}_0 \right) \times \tilde{B} - \frac{t_0 \eta_0}{L_0} \tilde{\eta} \nabla \times \tilde{B} + \frac{\alpha_0 t_0}{L_0} \tilde{\alpha} \tilde{B} \right). \quad (A3)$$

To obtain the usual dynamo normalization, we set the time and length constants to, respectively, the diffusive time and solar radius,

$$t_0 = \frac{R_s^2}{\eta_t}, \quad L_0 = R_s, \quad \eta_t = 10^{11} \text{ cm s}^{-1}, \quad \eta_t = 10^{11} \text{ cm}^2 \text{s}^{-1}.$$

By injecting Equation (A4) into Equation (A3), this leads to the following standard form of the induction equation:

$$\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}} = \nabla \times \left( \frac{U_0 R_s}{\eta_t} \tilde{U}_{\text{pol}} \times \tilde{B} + \frac{\Omega_0 R_s^2}{\eta_t} \tilde{U}_0 \right) \times \tilde{B} - \frac{\eta_0}{\eta_t} \nabla \times \tilde{B} + \frac{\alpha_0 R_s}{\eta_t} \tilde{\alpha} \tilde{B} \right). \quad (A5)$$

with the standard control parameters being the magnetic Reynolds number \(R_s\), the normalized rotation rate \(C_\Omega\), and the normalized \(\alpha\) effect amplitude \(C_\alpha\).

For the wind normalization used in the PLUTO code, we usually set the length, speed, and time constants to, respectively, the solar radius, the Kepler speed at the surface of the star, and the time ratio obtained by these two values,

$$L_0 = R_s, \quad U_0 = U_K = \sqrt{\frac{GM_s}{R_s}}, \quad t_0 = \frac{L_0}{U_0}, \quad (A6)$$

with \(G = 6.67 \times 10^{-8} \text{ dyne cm}^2 \text{ g}^{-2}\) and \(M_s = M_\odot = 1.99 \times 10^{33} \text{ g}\). By injecting Equation (A6) into Equation (A3), we then obtain a new normalization for the induction equation:

$$\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}} = \nabla \times \left( \frac{U_0}{U_K} \tilde{U}_{\text{pol}} \times \tilde{B} + \frac{\Omega_0 R_s}{U_K} \tilde{U}_0 \right) \times \tilde{B} - \frac{\eta_0}{U_K} \tilde{\eta} \nabla \times \tilde{B} + \frac{\alpha_0}{U_K} \tilde{\alpha} \tilde{B} \right). \quad (A7)$$

To switch from dynamo to wind normalization (i.e., from Equation (A5) to Equation (A7)), the following conversion factor can be applied:

$$\frac{\partial \tilde{\mathbf{B}}}{\partial \tilde{t}} = \frac{\eta_0}{R_s U_K} \nabla \times \left( \frac{R_s \tilde{U}_{\text{pol}} \times \tilde{B} + C_\Omega \tilde{U}_0}{R_s} \right) \times \tilde{B} - \frac{\eta_0}{\eta_t} \tilde{\eta} \nabla \times \tilde{B} + C_\alpha \tilde{\alpha} \tilde{B} \right). \quad (A8)$$

Appendix B
Extrapolation of the Poloidal Magnetic Field

In our interface region between the dynamo and the wind computational domains, we need to extrapolate the magnetic field generated by the dynamo inside the star to quantify the impact of its amplitude and topology on the wind. To do so, we have implemented a potential field extrapolation using the following mathematical method.

We choose to perform a potential field extrapolation, meaning that the field at the surface of the star is derived from a scalar potential \(\Phi\) such as \(B = -\nabla \Phi\), which corresponds to a magnetic field in vacuum. Such a magnetic field has the property that both its divergence and curl are equal to zero;
thus, $\Phi$ is a solution to the Laplace equation, which means it can be decomposed as a sum of the spherical harmonics $Y^m_\ell$, 

$$
\Phi(r, \theta, \phi) = \sum_{\ell=0}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} (\Phi^\ell_{m, r} r^\ell + \Phi^\ell_{m, \theta} r^{-(\ell+1)}) \times Y^m_\ell(\theta, \phi),
$$

where $Y^m_\ell(\theta, \phi) = c^m_\ell \mathbf{P}^m_\ell(\theta)e^{im\phi}$, $c^m_\ell = \frac{2\ell+1}{4\pi(\ell-m)!} \sqrt{\frac{\ell-m}{\ell+m}}$, and $\mathbf{P}^m_\ell(\theta)$ are the associated Legendre polynomials. As the first branch in $r^\ell$ diverges, we keep only the second one, choosing this form for the scalar potential:

$$
\Phi(r, \theta, \phi) = \sum_{\ell=0}^{\ell_{\text{max}}} \sum_{m=0}^{\ell} \Phi^\ell_{m, \theta} r^{-(\ell+1)} Y^m_\ell(\theta, \phi). \quad \text{(B1)}
$$

Applying the gradient operator to obtain the magnetic field and projecting onto the vectorial spherical harmonics $R^m_\ell = Y^m_\ell(\theta, \phi)\mathbf{e}_\theta$ and $S^m_\ell = \nabla Y^m_\ell(\theta, \phi) = \frac{\partial Y^m_\ell}{\partial \theta}(\theta, \phi)\mathbf{e}_\theta + \frac{1}{\sin \theta} \frac{\partial Y^m_\ell}{\partial \phi}(\theta, \phi)\mathbf{e}_\phi$ yields

$$
B(r, \theta, \phi) = \sum_{\ell=0}^{\ell_{\text{max}}} \sum_{m=0}^{\ell} \Phi^\ell_{m, \theta} (\ell + 1) r^{-(\ell+2)} \times R^m_\ell(\theta, \phi) - \Phi^\ell_{m, \theta} r^{-(\ell+2)} S^m_\ell(\theta, \phi),
$$

$$
= \sum_{\ell=0}^{\ell_{\text{max}}} \sum_{m=0}^{\ell} \alpha_{\ell, m}(r) R^m_\ell(\theta, \phi) + \frac{\beta_{\ell, m}(r)}{\ell + 1} S^m_\ell(\theta, \phi). \quad \text{(B3)}
$$

We then assume that we know the value of the magnetic field at a certain radius named $R_b$:

$$
B(r = R_b) = \sum_{\ell=0}^{\ell_{\text{max}}} \sum_{m=0}^{\ell} \alpha_{\ell, m}(R_b) R^m_\ell + \frac{\beta_{\ell, m}(R_b)}{\ell + 1} S^m_\ell. \quad \text{(B4)}
$$

This yields a system of two equations but with only one variable that is $\Phi^\ell_{m, \theta}$. We have to choose to base ourselves on either $B_r$ or $B_\theta$ to compute the solution. Based on previous benchmarks like Jouve et al. (2008), we realize that most dynamo models use $B_r$ as a reference, so that is what we choose as well. This yields the solution

$$
\Phi^\ell_{m, \theta} = \frac{\alpha_{\ell, m}(R_b)}{R_b^{\ell+2}}. \quad \text{(B5)}
$$

By injecting Equation (B5) into Equation (B3), this yields, in the end,

$$
\alpha_{\ell, m}(r > R_b) = \frac{\alpha_{\ell, m}(R_b)}{R_b^{\ell+2}},
$$

$$
\beta_{\ell, m}(r > R_b) = -\frac{\alpha_{\ell, m}(R_b)}{R_b^{\ell+2}}. \quad \text{(B7)}
$$

**Appendix C**

Preliminary Study with an Oscillatory Dipole

Before diving into the full coupling with a dynamo cycle, we started with a preliminary study, where we did not yet use the coupling model described above. The goal of this section is to have a more precise idea of the parameter space allowed for our model for the coupling to be operational. This will give a first indirect characterization of our coupling model by discussing the issue of timescales.

In this simplified model, we have only the wind computational domain with the boundary conditions described in Figure 1. The magnetic field is imposed at the surface of the star, but it varies in time; at each time step, it oscillates at a certain frequency $\omega$, which is a control parameter. The amplitude of the field is then given in code units by the following relation:

$$
B_{\text{osc}} = \frac{u_A}{u_{\text{esc}}} \sqrt{8\pi \rho_s (1 + 0.2 \cos t\omega)}, \quad \text{(C1)}
$$

where $u_A/u_{\text{esc}}$ is the Alfvén speed at the equator and the surface of the star divided by the escape velocity, and $\rho_s$ is the density at the surface of the star. We recall that the Alfvén speed $u_A$ is given by $|B|/\sqrt{4\pi \rho}$. That way, the magnetic field never reaches a null amplitude; it oscillates between 80% and 120% of its initial value. These oscillations are shown in the left panel of Figure 17. The wind then adapts to the magnetic field configuration; the right panel of Figure 17 shows the radial speed $u_r$ of the wind at $r = 14.57 R_*$ (so 75% of our numerical box) and three different latitudes ($\theta = 7^\circ$, which is near the pole; $\theta = 45^\circ$, which is at mid-latitude; and $\theta = 84^\circ$, which is near the equator). We can see oscillations with a frequency corresponding to the $\omega$ frequency of the magnetic field given in the left panel. Since we studied only a dipolar topology for the magnetic field, we have a latitudinal dependency, which means that the oscillations are more easily transmitted near the poles, where the magnetic field influence is weaker so that the wind can adapt quicker, while the oscillations are less visible at the equator, where the dipole is strong and thus creates a dead zone where the wind speed is reduced (between 0.45 and 0.48 versus 0.50 and 0.55 in PLUTO units).

To quantify the propagation of information, we introduce three characteristic times:

$$
t_A = \frac{\Delta r}{u_A}, \quad t_{\text{cyc}} = \frac{2\pi}{\omega}, \quad t_\eta = \frac{(\Delta r)^2}{\eta}, \quad \text{(C2)}
$$

where $t_A$ is the local Alfvén time, $t_{\text{cyc}}$ is the time associated with the oscillation of the surface magnetic field, and $t_\eta$ is the magnetic diffusive time associated with $\eta$ in the wind computational domain. Without diffusivity, our various simulations yield the following empirical criterion:

$$
t_A < 0.01t_{\text{cyc}}. \quad \text{(C3)}
$$

This is a first criterion concerning the resolution used for the numerical grid (see Equation (C2)). For the tests performed, we fixed the resolution with the following grid: we used a uniform grid of 256 points in $\theta$ between 0 and $\pi$ and a stretch grid of 256 points in $r$ between 1 and 20 $R_*$ with $\Delta r = 0.01$ at the surface of the star. We then used various Alfvén speeds to vary the corresponding Alfvén time. Figure 18 shows how the wind is supposed to react in a case where the criterion (Equation (C3)) is respected, and thus the coupling is operating properly. In the two meridional cuts, we can clearly see how the wind has adapted itself to the dipolar magnetic field; we
have an equatorial streamer up to 4 $R_\ast$, and elsewhere the magnetic field lines (shown in white) are open. The black line is the Alfvén surface, which is the surface at which $u_r = u_A$. This is the major visible change between the two configurations; on the left, the amplitude of the magnetic field is minimal (80% of its initial value), and thus the Alfvén surface is further away from the star (on latitudinal average, the corresponding Alfvén radius $r_A$ is equal to 5.0 $R_\ast$), while on the right, the amplitude of the magnetic field is maximal (120% of its initial value), making the Alfvén surface closer to the star ($r_A = 3.5R_\ast$).

If we choose to add magnetic diffusivity to the wind model, it can help propagate the information from inside the star to the

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4 Please note that this is a misuse of language; the Maxwell laws prohibit the existence of a magnetic monopole, meaning that the magnetic field lines are not really "open," they simply reconnect far away from the star and way outside of our numerical box, thus only appearing open in our simulations.
outside. If the diffusivity is limited to a few grid points above the star surface (like with the profile described in Equation (13)), then it has little to no influence on the corresponding relaxed state of the wind. We still have to respect another criterion to make sure that the wind has time to adapt to the magnetic cycle:

$$t_{\eta} < 0.01 t_{\text{cyc}}.$$  

(C4)

In the cases that did not respect criteria (C3) and (C4), the coupling was not working properly. What we observed was that the magnetic field was evolving too quickly at the star surface, so the wind could not adapt fast enough. This led to an uncoupling reaction of the Lorentz force, which is expressed as

$$F_L = \frac{1}{4\pi} \nabla \times B \times B \Rightarrow 4\pi F_{L,\theta}$$

$$= \left[ \frac{1}{r \partial} (r B_{\theta}) - \frac{1}{r} \frac{\partial}{\partial \theta} (r B_{r}) \right] B_{\theta} - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta B_{\theta}) B_{r}.$$  

(C5)

This force triggers latitudinal flows, and in the end, the wind speed is compressed to negative flows because of it, which leads to accretion at the poles.

Appendix D
Computations for Helicity

D.1. Computation of $A_P$ with Various Boundary Conditions

D.1.1. General Principle

The relative magnetic helicity, which is gauge-invariant, is defined as

$$H_{\text{rel}} = \int (A + A_p) \cdot (B - B_p) dV.$$  

(D1)

where $B_p = \nabla \times A_p$ but also $B_p$ is potential ($\nabla \times B_p = 0$) and a reference component from $B$ ($B_p \cdot n_s = B \cdot n_s$).

Thus, we want to find a potential field $B_p$ derived from a scalar magnetic potential $\phi$ ($B_p = -\nabla \phi$) that we can project on spherical harmonics:

$$\phi(r, \theta, \varphi) = \sum_{\ell,m} (A_{\ell,m}^m r^\ell + B_{\ell,m}^m r^{-(\ell+1)}) Y_{\ell m}^m(\theta, \varphi).$$  

(D2)

Because it is potential, $B_p$ is also poloidal; thus, we can introduce a quantity $C_p$, such as

$$B_p = \nabla \times B \times (C_p e_r) = A_p = \nabla \times (C_p e_r).$$  

(D3)

Thus, to obtain $B_p$ and $A_p$, we only need to compute $C_p$, which can be linked to $\phi$ using the following relation:

$$C_p = -\sum_{\ell,m} \left( A_{\ell,m}^m r^\ell + B_{\ell,m}^m r^{-(\ell+1)} \right) Y_{\ell m}^m.$$  

(D4)

Under the assumption of axisymmetry, we have, in the end,

$$A_p = -\frac{1}{r \partial} (r \partial \phi) e_r, B_p = -\frac{\partial}{\partial r} e_r - \frac{1}{r \partial} (r \partial \phi) e_\theta,$$

$$H_{\text{rel}} = \int (A \cdot B - A \cdot B_p + A_p B_\theta) dV.$$  

(D6)

The computation of $A_{\ell,m}^m$ and $B_{\ell,m}^m$ depends on the boundary conditions of the domain.

D.1.2. Potential Components with Perfect Conductor and Potential Field

For the dynamo, we have a perfect conductor boundary condition at the bottom, which leads to

$$\partial_r \phi(r = r_b) = 0 \Leftrightarrow \ell r_b^{\ell-1} A_{\ell}^m - (\ell + 1) r_b^{-(\ell+2)} B_{\ell}^m = 0.$$  

(D7)

At the top, we have a potential field boundary condition, which implies

$$\partial_r \phi(r = r_t) = -B_r(r = r_t) \Leftrightarrow \ell r_t^{\ell-1} A_{\ell}^m - (\ell + 1) r_t^{-(\ell+2)} B_{\ell}^m = -G_{\ell}^m,$$  

(D8)

with $B_r(r_t) = \sum_{\ell,m} g_{\ell,m}^m Y_{\ell m}$.

We combine these two equations to obtain, in the end,

$$A_{\ell}^m = \frac{G_{\ell}^m}{\ell r_t^{\ell-1} \left[ \left( \frac{\alpha}{\gamma} \right)^{2\ell+1} - 1 \right]} B_{\ell}^m$$

$$= \frac{r_t^{2\ell+1} G_{\ell}^m}{(\ell + 1) r_t^{-(\ell+2)} \left[ \left( \frac{\alpha}{\gamma} \right)^{2\ell+1} - 1 \right]}.$$  

(D9)

D.1.3. Potential Components with Potential Field and Outflow Boundaries

For the wind, we have a potential field boundary at the bottom, which leads to

$$\partial_r \phi(r = r_b) = -B_r(r = r_b) \Leftrightarrow \ell r_b^{\ell-1} A_{\ell}^m - (\ell + 1) r_b^{-(\ell+2)} B_{\ell}^m = -G_{\ell}^m,$$  

(D10)

with $B_r(r_b) = \sum_{\ell,m} g_{\ell,m}^m Y_{\ell m}$.

At the top, we have an outflow boundary condition, which is expressed using a modified outflow boundary condition with $\partial_r (B_r r^2) = 0$ instead of $\partial_r B_r = 0$, which leads to

$$2r_t B_r(r = r_t) + r_t^2 \partial_r B_r(r = r_t) = 0$$

$$\Leftrightarrow 2r_t \partial_r \phi(r = r_t) + r_t^2 \partial^2_r \phi(r = r_t) = 0.$$  

(D11)

With Equation (D2), we obtain

$$\ell r_t^{\ell} A_{\ell}^m + r_t^{-(\ell+1)} B_{\ell}^m = 0.$$  

(D12)

We combine this condition with Equation (D10) to obtain, in the end,

$$B_{\ell}^m = \frac{G_{\ell}^m r_t^{\ell+2}}{(\ell + 1) \left[ \left( \frac{\alpha}{\gamma} \right)^{2\ell+1} - 1 \right]} A_{\ell}^m = -r_t^{-(2\ell+1)} B_{\ell}^m.$$  

(D13)

D.2. Computation of $A$

We have a magnetic field $B$ that derives from a vector potential $A$ such that $B = \nabla \times A$. Knowing $B$, we want to recover $A$ in 2.5D.

To do so, we decompose the magnetic field $B$ in a poloidal and a toroidal term in spherical coordinates:

$$B = \nabla \times (Ce_z) + \nabla \times (Ae_\varphi).$$  

(D14)
To obtain $C$, we can use direct integration, given that

$$B_r = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \, C), \quad B_\theta = -\frac{1}{r} \frac{\partial}{\partial r} (r \, C).$$  \hfill (D15)

To obtain $A$, we use two different methods: direct integration for outside the star and projection on spherical harmonics for inside the star.

With direct integration, we have

$$B_\varphi = -\frac{1}{r} \partial_\varphi A.$$  \hfill (D16)

We can project $A$ onto spherical harmonics, which leads to

$$A = \sum_{\ell,m} A_\ell^m Y_\ell^m.$$  \hfill (D17)

Now, if we take the $r$ component of the projection of the current on spherical harmonics, we obtain

$$\sum_{\ell,m} \frac{\ell (\ell + 1)}{r^2} A_\ell^m Y_\ell^m = \nabla \times B_r.$$  \hfill (D18)

We can then link $A$ and $C$ to $A$ using the following relation:

$$A_r = A, \quad A_\theta = 0, \quad A_\varphi = C.$$  \hfill (D19)

### D.3. Balance for Helicity Fluxes

To make sure that all of the helicity computations were carried out correctly, we have checked the balance described by Equation (21), which we rewrite here in the form

$$\frac{\partial H_{rel}}{\partial t} = \Gamma_d + F_H,$$  \hfill (D20)

where $\Gamma_d$ is the diffusion term due to the electric currents

$$\Gamma_d = -\frac{8\pi\eta}{c} \int j \cdot B dV,$$  \hfill (D21)

and $F_H$ is the helicity flux

$$F_H = 2 \int_{\partial V} \left[ A_r \times (-u \times B + \eta \nabla \times B) \right] \cdot n dS.$$  \hfill (D22)

The results are presented in Figure 19. Case DW1 is on the left, and case DW2 is on the right. The top panel is the budget for the dynamo domain, while the bottom panel is the budget for the wind domain. For each domain, we plot in red $\partial H_{rel}$, in green $\Gamma_d$, in dashed blue the fluxes $F_H$ at the boundaries, and in blue the sum $\Gamma_d + F_H$. The balance is qualitatively realized with the blue and red curves following each other in most instances.
