Multi-objective optimization of laminar flow characteristics in helical coiled tubes based on NSGA-II

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Abstract. To meet the requirement of energy conservation and optimize the design of helical coils in laminar flow, the pitch, the coil radius and the average inlet velocity were selected as design parameters to obtain the tendency of the heat transfer and flow resistance characteristics of the helical coils. Heat transfer coefficient and pressure drop per unit length of helical coils were taken as the optimization objective functions. A multi-objective optimization model was established, three candidate points A, B and C were obtained based on Non-dominated Sorted Genetic Algorithm-II (NSGA-II). The results showed that heat transfer coefficient and pressure drop per unit length increase with the pitch and inlet velocity whilst them decrease with coil radius, among which the influence of inlet velocity is the most obvious and the pitch is the least. It was found that the coil radius of most points in Pareto front was relatively concentrated. The comprehensive performance of candidate points A, B, C had been improved significantly by 16.3%, 28.3% and 19.6% respectively.

1. Introduction
Helically coiled heat exchangers have many advantages, such as compact structure, high pressure resistance, high heat transfer capability, which have been widely employed in petrochemical engineering, nuclear industry, waste heat recovery system, refrigeration, food industry and many other areas. The secondary flow[1] perpendicular to the main flow in helical coiled tube is much stronger than that in the straight tube. Many scholars have made numerous studies[2-4] on the heat transfer coefficient and friction coefficient in the tube because the mixed enhanced heat transfer has a significant meaning in the passive enhancement of heat transfer. Compared to the straight tube, the critical Reynold number from laminar to turbulent flow in the helically coiled tube is higher[5-8].

The special construction enables helical coiled heat exchangers to have more efficient heat transfer performance. As energy conservation and emission reduction has been an important principle in the development of modern industry, thus the optimization of helically coiled tube heat exchanger becomes popular research direction. In response to the requirement of energy conservation and emission reduction, a multi-objective optimization model based on genetic algorithm is established in this study, which takes the heat transfer coefficient and pressure drop in the helically coiled tube as two objective functions. The optimal construction and the average inlet velocity in laminar flow was investigated to provide further assistant for the optimization and design of helically coiled heat exchangers afterwards.

2. Physical model and model validation
The Reynold number is an important parameter to judge the status of the fluid. The critical Reynold number of the smooth tube is about 2300, and the flow is laminar if the Reynold number is less than the
critical Reynold number. Many scholars had investigated the critical Reynold number in the helically coiled tube. Schmidt[6] gave the critical Reynold number calculating formula in the helically coiled tube based on the straight smooth tube:

$$Re_{cr} = 2300 \left[ 1 + 8.6(0.5d / R)^{0.45} \right]$$

(1)

2.1. Physical model and model validation

The helically coiled tubes are made by winding straight tube. Figure 1 shows four key parameters which determine the constructure of the helically coiled tube: helically coiled tube diameter \( d \), pitch \( H \), coil radius \( R \), and height of the tube \( L \).

The helically coiled tube is vertical as show in figure 1. The water flow into the helically coiled tube at the bottom, assume that the properties of the fluid are considered as constant. The boundary conditions are set as the following: (1) The inlet is set as velocity inlet. (2) The outlet is set as pressure outlet and the gauge pressure is set as zero. (3) The wall is set as the boundary conditions of the first type, the temperature of the wall is 373K. (4) The gravity coefficient is -9.8 N/kg. The numerical model is set as laminar. The original parameters and the range of parameters are shown in Table 1.

| Parameters                  | Original value | Range     |
|-----------------------------|----------------|-----------|
| Inner tube diameter: \( d \) (m) | 0.018          | —         |
| Inlet velocity: \( u \) (m s\(^{-1}\)) | 0.3            | 0.055–0.36|
| Coil radius: \( R \) (m)      | 0.25           | 150–300   |
| Coil pitch: \( H \) (m)       | 0.066          | 60–100    |
| Effective length: \( L \) (m) | 0.198          | —         |

2.2. Criteria of heat transfer and flow resistance in helically coiled tubes

The whole heat transfer coefficient and pressure drop per unit length are calculated to evaluate the performance of helically coiled tube. The heat transfer coefficient can be computed by:

$$h = \frac{c_p \dot{m}(T_{out} - T_{in})}{A \Delta T_{ln}}$$

(2)

Where the \( c_p \) is the specific heat, \( \dot{m} \) is the mass flow rate, \( T_{out} \) is the temperature of outlet, \( T_{in} \) is the temperature of inlet, \( A \) is the inner surface area of helically coiled tube, \( \Delta T_{ln} \) is the logarithmic mean temperature difference. Transform the heat transfer coefficient to the dimensionless number \( Nu \):

$$Nu = \frac{hd}{\lambda}$$

(3)

Where the \( \lambda \) is liquid thermal conductivity coefficient. The pressure drop per unit length can be calculated by:

$$\Delta p = \frac{p_{in} - p_{out}}{l}$$

(4)

Where the \( l \) is the length that the helically coiled tube transforming to a straight tube, which can be computed by:

$$l = \frac{L}{H} \sqrt{(2\pi R)^2 + H^2}$$

(5)

Turn the pressure drop into the Darcy friction factor in the tube:

$$f = \frac{d}{2\rho u^2} \Delta p$$

(6)

Where the \( \rho \) is the liquid density.

2.3. Grids independence validation

The grids independence is investigated with original geometry as an example. There are seven groups of grids (the grids number of them are 247000, 371000, 646000, 994000, 1659000, 2221000, 3278000).
As shown in figure 2, the simulation result is tending to be stable when the sixth group of grids is applied. Thus, the follow-up computation employs the sixth group of grids.

2.4. Model validation

Manlapaz\cite{9} studied the fully developed laminar in the helically coiled tube and gave the empirical correlation:

\[
f = \frac{16}{Re} \left[1 - 0.18 \left(1 + \left(\frac{35}{He}\right)^{1/2}\right)^{1/2} + \left(1 + \frac{d}{3R}\right)^{2} \frac{He}{88.33}\right]^{1/2}
\]

(7)

\[
Nu = \left[\frac{48}{11} + \frac{51}{11} \left(1 + \left(\frac{1342}{PrHe^{2}}\right)^{2}\right)^{3/2} + 1.816 \left(\frac{He}{1+1.15/Pr}\right)^{3/2}\right]^{3/2}
\]

(8)

\[
Re = \frac{\rho ud}{\mu}, \quad Dn = Re(\frac{d}{2R})^{1/2}, \quad He = Dn / \left[1 + \left(\frac{H}{2\pi R}\right)^{2}\right]^{1/2}
\]

(9)

Where the \( Pr \) is Prandtl number, it is a const when the properties are fixed. \( Re \) is Reynold number. \( Dn \) is Dean number. \( He \) is spiral number. \( \mu \) is the fluid dynamic viscosity coefficient. When \( Dn<20, 20<Dn<40 \) and \( 40<Dn \), the value of \( n \) is 2, 1 and 0. In the present study, the range of \( Dn \) is much larger than 40.

All the computation results are taken into the Manlapaz empirical correlation, and the comparison is shown in figure 3 and figure 4. The error of \( Nu \) and \( f \) is within the acceptable range.

3. Multi-objective optimization genetic algorithm

The genetic algorithm\cite{10} was an evolution algorithm based on biogenetic evolution, proposed by John
holland from the America. The present study uses the Ansys Workbench optimization model to design: Choose the MOGA algorithm, and randomly generate a certain number of initial sample points within the constraints. New sample points are generated by crossover, mutation, and selection. Maximum allowable pareto percentage is set as 70 to search as many solutions meet requirements as possible, which is a judge convergence basis. Take the heat transfer coefficient and pressure drop per unit length as two optimization objectives and propose a multi-objective optimization model as:

\[
\begin{align*}
\text{Maximum } h \\
\text{Minimum } \Delta p
\end{align*}
\]

(10)

4. Results and discussions

4.1. Effects of flow resistance and heat transfer in the helically coiled tube

As shown in figure 5. The study investigates effects of the \( H \), \( R \) and \( u \) on the pressure drop and heat transfer in the helically coiled tube. Furthermore, in order to reduce the \( \Delta p \) while strengthening the \( h \) as much as possible, the structure parameters of helically coiled tube have to be optimized.

Keep \( R=0.25 \) m and \( u=0.36 \) m/s to study the effects of different \( H \) on the performance of helically coiled tube. \( \Delta p \) and \( h \) increase with the increase of \( H \) in the research range. As the \( H \) increases from 0.066m to 0.096m, \( \Delta p \) and \( h \) increase in the percentage of 9.71% and 5.69%.

Set \( H=0.066 \) m and \( u=0.36 \) m/s to investigate the effects of different \( R \) on the performance of helically coiled tube. \( \Delta p \) and \( h \) decrease with the increase of \( R \). When the \( R \) increases from 0.15m to 0.3m, \( \Delta p \) and \( h \) decrease in the percentage of 21.44% and 27.90%.

Choose \( H=0.066 \) m and \( R=0.25 \) m to research the effects of different \( u \) on the performance of helically coiled tube. \( \Delta p \) and \( h \) increase with the increase of \( u \) in the study range. When the \( u \) increases from 0.06m/s to 0.36m/s, \( \Delta p \) and \( h \) increase in the percentage of 152.91% and 1083.65%.

![Figure 5. Effects of flow resistance and heat transfer in the helically coiled tube](image)

4.2. Multi-objective optimization results

4.2.1. Sample space and distribution of optimal points. Figure 6 illustrates the distribution of all design points in the sample space. The sample points are uniformly distributed in the sample space based on the entropy generation maximization. The pareto points are generated by genetic algorithm. As displayed in the figure 7, most pareto points are distributed at the boundary of sample space and gathered at \( R = 0.15 \)–\( 0.17 \) m.

| Table 2. The candidate optimization points. |
|-----------------|---------------|---------------|----------------|----------------|----------------|
| \( R / \) m     | \( H / \) m   | \( u / \) m/s | \( \Delta p / \) Pa \( \) m\(^{-1}\) | \( h / \) Wm\(^{-2}\)K\(^{-1}\) | \( h/\Delta p^{1/3}\) |
| Original value  | 0.25          | 0.066         | 0.3            | 108.13         | 1224.8         | 257.2          |
| Candidate point A | 0.15237       | 0.09645       | 0.16024        | 54.635         | 1134.6         | 299.0          |
| Candidate point B | 0.16725       | 0.09270       | 0.27115        | 112.26         | 1592.1         | 330.0          |
| Candidate point C | 0.15253       | 0.09051       | 0.21091        | 79.237         | 1321.6         | 307.7          |
4.2.2. **Optimization recommendation results.** The pareto points are numerous and the best point is determined by the designer’s engineering experience. Workbench proposes three typical optimal points given in the Table 2. The performance effect criteria\(^{[11]}\) are defined as \(h/\Delta p^{1/3}\) (PEC). The higher PEC means better comprehensive performance. Compared to the initial point, \(\Delta p\) of A point decreases sharply while \(h\) decreases slightly and PEC increases by 16.3\%. \(\Delta p\) of B point increases a little while \(h\) increases a lot and PEC increases by 28.3\%. C point is balanced respectively: compared to the original value, \(\Delta p\) decreases while \(h\) increases and PEC increases by 19.6\%.

[Figure 6. Distribution of the sample space.]

[Figure 7. Mapping of sample space to objective function distribution.]

5. **Conclusion**

(1) Through numerical simulation, the heat transfer coefficient and pressure drop per unit length of the laminar flow in the helically coiled tube increases with the increase of velocity of inlet and pitch, decreases with the increase of coil radius.

(2) Through analyzing the effects of design parameters to the objective functions, the most influential parameter is velocity, next is coil radius and the least is pitch.

(3) The distribution of optimal points is plotted. It illustrates that most pareto points are gathering at the range of lower coil radius.

(4) The comprehensive performance of helically coil tube has improved obviously after the optimization of genetic algorithm. The PEC of three candidate optimization points A, B, and C increases by 16.3\%, 28.3\% and 19.6\%.

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