Scaling and dynamics of special machines

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Abstract. Rotary screw machines (RSM) are widely known as multipurpose vehicles. RSM easily overcome impassable swamps, snow-covered, off-road, ice surface. A significant problem for the creators of vehicles with rotors is the high vibration loads on the operator and machine units, which require multiple experimental studies. A scaled-down model of a vehicle is often used for experiments. The model of the RSM is included in the experiment, replacing not only the object of research, but also the conditions in which it is studied. Strengthening the role of the theoretical side of research, there is a connection between the formulation of the experiment and its results with the object of research. Calculation and calculation of the elements of proportionality is a hard necessity. The article proposes a complex for calculating scale factors when testing vibration loading of machines equipped with screw rotors.

1. Introduction

The research of the similarity of oscillatory processes means that the data on the flow of oscillatory phenomena obtained when measuring specific vibratory displacement, can be extended to all cases of measurements such as this one. Amplitude-frequency and phase-frequency characteristics of the phenomena of displacement of screw rotors in a group of similar phenomena can be calculated by a simple review of the characteristics and scaling.

2. Mathematical model

Dimensional analysis of physical quantities in the studied processes will be based on the theory of similarity. We can get physical quantity of displacements \( \Psi \) only if we compare it with the quantity of the same physical nature \( \psi_1 \). Ration rating in this case - \( \Psi_1 \), and \( \psi_1 \) is the unit of measurement, i.e.:

\[
\frac{\Psi}{\psi_1} = \Psi_1
\]

(1)

If it is necessary to change the unit of measurement a certain number of times - \( N \), the same factor will increase or decrease the number\( \Psi_1 = \Psi_1 \cdot N \).

In classical physics the units of measurement are basic, if they are with arbitrary choice of size and derivatives. The system of units is a set of basic units and derivatives formed by certain rules. Increase in the number of physical dimensional constants increases the number of basic units of measurement.

In this study, it is necessary to make the equation of proportions of vibratory displacement quantities that is the basic unit of measurement of the mechanical system - acceleration, which in turn depends on the mass, distance and time. Dependence of the derivative from the main unit is a formula of dimension of a quantity, which reproduces the product of the raised to the power dimensions of the
basic components of the unit. The dimensions of the basic formulas of vibratory displacements are shown in Equation 2:

- **acceleration** \( (\text{m}/\text{c}^2) \) \( a = \frac{dv}{dt} \), \( |a| = [L]/[T]^2 \)
- **power** \( (\text{Newtons} \cdot \text{kg} \cdot \text{m}^2)/\text{c}^2 \) \( F = \frac{d(mv)}{dt} \), \( |F| = [M]/[L][T]^2 \)
- **frequency** (Hz, \( 1/c \)) \( v = \frac{n}{T} \), \( |v| = [T]^{-1} \)
- **oscillation period** (c) \( T = \frac{1}{v} \), \( |T| = [T]^1 \)
- **amplitude of oscillation** (m) \( A \), \( |A| = [A]^1 \)

In the experiments, it is necessary to make recalculation of parameters and, therefore, to move to a different system of measurement in which the units are:

\[
\begin{align*}
[M_1] &= X[M] \\
[L_1] &= Y[L] \\
[T_1] &= Z[T]
\end{align*}
\] (3)

The equations of the derived quantities based on equations 2 taking into account equations 3 take look like:

- **acceleration** \( (\text{m}/\text{c}^2) \) \( a = \frac{dv}{dt} \), \( |a_1| = YZ^{-2}[a] = N_1[a] \)
- **power** \( (\text{Newtons} \cdot \text{kg} \cdot \text{m}^2)/\text{c}^2 \) \( F = \frac{d(mv)}{dt} \), \( |F_1| = XYZ^{-2}[F] = N_2[F] \)
- **frequency** (Hz, \( 1/c \)) \( v = \frac{n}{T_1} \), \( |v_1| = Z^{-1}[v] = N_3[v] \)
- **oscillation period** (c) \( T = \frac{1}{v} \), \( |T_1| = Z[T] = N_4[T] \)
- **amplitude of oscillation** (m) \( A \), \( |A_1| = N_5[A] \)

Thus, measuring process parameters of vibratory displacement of the body of the studied rotary screw machine are: acceleration, frequency and oscillation period, and then the received and amplitude-frequency and phase-frequency characteristics.

2. **Experimental research**

In order to study the vibratory displacement of the rotary screw machine on a viscoelastic suspension (previously there were constructions on a rigid or semi-rigid suspension only) we built a scaled model of transport and technological means. The designed mechanical system (Figures 1 and 2) was a complete simulation of physical processes occurring in the natural machine. The similarity of vibratory displacement of the model and the full-scale sample was achieved by proportional correspondence of parameters of the two systems of rotary screw machines.

![Figure 1. Structural diagram of the rotary-screw machine](image-url)
\[ Z_i/S_i = m_i \] (5)

where \( Z_i \) and \( S_i \) – are uniform parameters of the rotary screw machine individual points vibration process \( Z \) and \( S \); \( m_i \) – scale or similarity coefficient of the oscillating system.

\[ \eta_1 + \eta_2 + \cdots + \eta_n = \sum_{i=1}^{n} \eta_i = 0 \]
\[ \gamma_1 + \gamma_2 + \cdots + \gamma_n = \sum_{i=1}^{n} \gamma_i = 0 \] (6)

\[ \gamma_n \eta_n \] are not equal to zero and equation 6 is rewritten as:
\[ 1 + \frac{\eta_1}{\eta_n} + \frac{\eta_2}{\eta_n} + \cdots + \frac{\eta_{n-1}}{\eta_n} = 0 \]
\[ 1 + \frac{\gamma_1}{\gamma_n} + \frac{\gamma_2}{\gamma_n} + \cdots + \frac{\gamma_{n-1}}{\gamma_n} = 0 \] (7)

We know, that function \( \eta_i = \eta_i(Z_1, Z_2, \ldots, Z_m) \), \( \gamma_i = \gamma_i(S_1, S_2, \ldots, S_m) \) have nonzero dimension, where \( Z_i S_i \) (\( i = 1, 2, \ldots, m \)) – are identical parameters of such processes of similarity with criterion of identity \( m_i \) by formula 5. So, the equations are true: \( Z_i = m_i \cdot S_i \), \( \omega t_i = \gamma_i \cdot N_i \), where \( N_1 = N_2 = \cdots = N_n = N \). Taking into account these equations in systems 5.6 and 5.7, we get identical expressions:
\[ \frac{\eta_2}{\eta_n} = \frac{\eta_1}{\eta_n}, \quad \frac{\eta_2}{\eta_n} = \frac{\eta_1}{\eta_n}, \quad \frac{\eta_{n-1}}{\eta_n} = \frac{\eta_{n-1}}{\eta_n} \] and in generalized form: \( \eta_i/\eta_n = idem \) (8)

In a study of oscillations of the machine (in this case, describing the support surface we take \( \eta_i = \sin \omega t \), then \( \omega t = idem \). If we consider several types of similarity, we introduce similarity criterion - \( \pi_k \), where \( k \) – is a number of the criterion.
Any full equation of the physical process must be submitted according to the criteria of similarity of the members of the equation parameters.

![Image](image_url)

**Figure 3.** Suspension elements of the rotary screw machine - hydraulic shock absorbers and a spring performed by physical scaling.

When calculating the stiffness of the spring scale model the following equation is applied:

\[ C = \frac{G d_F^2}{8 d_D^4} n \]  

where \( d_D \) – diameter of the wire, m; \( d_F \) – winding diameter, as measured from the axis of the wire, m; \( n \) – number of turns; \( G \) – shear modulus, Pa \( \left( \frac{kg}{m \cdot s^2} \right) \).

Equation 8 is a function of the parameters:

\[ f(d_D, d_F, n, G) \] or in the general form:

\[ f(Z_1, Z_2, ..., Z_m) = 0 \]  

\[ Z_{01}, Z_{02}, ..., Z_{0m} \] - characteristic quantities that are related to the parameters of the general equation \( Z_1, Z_2, ..., Z_m \). Equality 9 takes the form:

\[ f \left( \frac{Z_1}{Z_{01}}, \frac{Z_2}{Z_{02}}, ..., \frac{Z_m}{Z_{0m}} \right) = 0 \]  

In equation 8, the number of basic units is 4, and they are studied, and the rest will be their functions:

\[ Z_{0s} = Z_{01}^{u_1} \cdot Z_{02}^{u_2} \cdot Z_{03}^{u_3} \cdot Z_{04}^{u_4} \]  

(5 ≤ \( s \) ≤ \( m \))  

(11)

The ratio of the determinants of the system coefficients allows to obtain the exponents \( u_i \).

Independent variables are equal: \( Z_{01} = Z_{01}; \ Z_{02} = Z_{02}; \ Z_{03} = Z_{03}; \ Z_{04} = Z_{04} \). Equality 10 is transformed as follows:

\[ f \left( 1, 1, 1, 1, \frac{Z_s}{Z_{01}^2 \cdot Z_{02}^2 \cdot Z_{03}^2 \cdot Z_{04}^2}, ..., \frac{Z_m}{Z_{01}^2 \cdot Z_{02}^2 \cdot Z_{03}^2 \cdot Z_{04}^2} \right) = 0 \]  

(12)

Dimension of the fractions elements are equal, then the equality is:

\[ f \left( 1, 1, 1, 1, \pi_1, ..., \pi_{m-4} \right) = 0 \]  

(13)

We have the following description of similarity: if there is \( m - 5 \) an independent criterion, under which the dependent criterion is satisfied, and the formula is:

\[ \pi_1 = \Phi(\pi_2, \pi_3, ..., \pi_{m-4}) = 0 \]  

(14)

Proportionality of the congruent parameters in terms of uniqueness, and equality of similarity criteria of the oscillatory process is a necessary and sufficient condition for similarity.

Getting similarity criteria in the study of oscillatory system of the rotary screw machine.

Oscillatory system, which is rotary screw machine in all its details is unrealistic cumbersome and complicated for a possible solution of the equations, and the analysis of the results obtained is difficult. The equivalent system is simplified as far as statement of a problem permits. Its view in particular cases leads to the scheme for calculating the dynamics of rotary screw machine in the longitudinal and diametric planes.
Dynamic system equivalent to rotary screw machine consists of several masses: rotors, body, engine, cab et al., interconnected through elastic connections and dampers. A generalized equation of oscillatory process of the rotary screw machine on the viscoelastic suspension is as follows:

$$M_0 \ddot{Z} + k_0 \dot{Z} + cZ = H_x(t) \sin(\omega t)$$ (15)

That is function

$$f(M_0, H_x, \omega, Z, k, c, t)$$ (16)

where $M_0$ – Sprung mass, kg; $H_x$ – the power of external influences kg m/s$^2$; $\omega$ – cyclic frequency, $1/c^2$; $Z$ – displacements, m; $k$ – coefficient of resistance of shock absorbers, N/m; $c$ – spring rate, N/m; $t$ – time, c.

Matrix components of the machine oscillation parameters in the system of unit, which takes the form:

$$
\begin{bmatrix}
M_0 & 1 & 0 & 0 \\
H_x & 1 & 1 & -2 \\
\omega & 0 & 0 & -1 \\
Z & 0 & 1 & 0 \\
k & 1 & 0 & -1 \\
c & 1 & 0 & -2 \\
t & 0 & 0 & 1
\end{bmatrix}
$$

For this matrix it is easy to calculate the determinants of the third order. The quantities, of which determinants are different from zero, are independent combinations. Determinant of quantities $M_0, H_x, Z$ equals 2, so they are an independent combination. Combinations: {\omega, k, c}; {M_0, c, t}; {M_0, k, t} – calculation of determinants of which is zero are dependent of each other. Quantities $Z, c, t$ are expressed through independent values of combinations $M_0, H_x, \omega$. It is accepted: $Z = M_0^{1/3} H_x^{1/2} \omega^{3/4}$. We calculate the determinants derived by replacing of the lines. It is determined that:

$$Z = M_0^{-1} H_x^{1/2} \omega^{-2}; c = M_0^{1/2} H_x^{1/2} \omega^2; k = M_0^{1} H_x^{0} \omega^1; t = M_0^{0} H_x^{0} \omega^{-1}$$ (17)

The result of the study is similarity criteria:

$$\pi_1 = \omega t; \pi_2 = \frac{Z}{M_0^{1/4} H_x^{1/2} \omega^{-2}}; \pi_3 = \frac{c}{M_0^{1/2} \omega^{3/2}}; \pi_4 = \frac{c}{M_0^{0} \omega^{1}}$$ (18)

4. Conclusion

Thus, the effects are similar, if the determining criteria are invariant (the same). We set similarity of rotary screw machine the oscillatory system, undetectable criteria are considered to be same - due to their similarity. The similarity theory allows, without integrating the differential equations derived from them, to get similarity criteria and to set the criteria dependence displayed by equations 18 from of the experimental data.

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