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Dark matter as a dynamic effect due to a non-minimal gravitational coupling with matter (II): Numerical results

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Abstract. Following the previous contribution discussing the rich phenomenology of models possessing a non-minimal coupling between matter and geometry, with emphasis on its characteristics and analytical results, the obtained “dark matter” mimicking mechanism is numerically studied. This allows for ascertaining the order of magnitude of the relevant parameters, leading to a validation of the analytical results and the discussion of possible cosmological implications and deviation from universality.

1. Introduction

This work aims to confront the available observational evidence, namely the rotation curves of galaxies, with the analytical results of the “dark matter” mimicking mechanism described in a previous contribution to this volume (see also Ref. [1] for the original discussion and extended results). This will allow for the determination of the relevant model parameter $R_0$, via the associated lengthscale $r_0$, and the prediction of possible cosmological implications of the assumed power-law non-minimal coupling.

One chooses to fit numerically several rotation curves of galaxies of E0 type — these are selected since they exhibit an approximate spherical symmetry adequate for comparison with the analytical results obtained in the previous contribution. Seven galaxy rotation curves, are used, with the division into visible and dark matter components as reported in Refs. [2, 3, 4, 5]: NGC 2434, NGC 5846, NGC 6703, NGC 7145, NGC 7192, NGC 7507 and NGC 7626 galaxies.

As stated in the preceding section, particular interest is assigned for $n = -1$ and $n = -1/3$, which correspond to the isothermal sphere or NFW dark matter density profiles. Furthermore, one may recall that the power-law profile $f_2(R) = \left(R/R_0\right)^n$ was discussed in the context of a possible series expansion of a more general non-minimal coupling; hence, there is no reason to select a priori either case $n = -1$ or $n = -1/3$: instead, one can successfully take advantage of a composite coupling given by

$$f_2(R) = \sqrt[3]{\frac{R_3}{R}} + \frac{R_1}{R},$$

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so that both profiles may be added in order to describe the dark matter density profile and the related galaxy rotation curve.

The proposed non-minimal coupling leads to a more complex expression for the trace of the field equations

\[ R = \frac{1}{2\kappa} \left[ 1 + 3 \frac{R_1}{R} + \frac{5}{3} \left( \frac{R_3}{R} \right)^{1/3} \right] \rho + \frac{1}{\kappa} \frac{1}{\rho} \left( \left[ 3 \frac{R_1}{R} + \left( \frac{R_3}{R} \right)^{1/3} \right] \rho \right). \]  

(2)

One can no longer resort to the identification of a “static” solution, as before, since the non-linearity of the above equation indicates that the solution for this composite scenario is not given by the mere addition of individual solutions; by the same token, no clear equation of state or identification of the mimicked dark matter profile with a perfect fluid arises. Nevertheless, one expects that the main feature of the model are kept, namely the overall mimicking of dark matter density profiles according to the scaling relation between visible matter \( \rho \) and “dark matter” \( \rho_{dm} \) densities,

\[ \rho_{dm} = \frac{1 - n}{1 - 4n} \rho_0 \left[ (1 - 2n) \frac{\rho}{\rho_0} \right]^{1/(1-n)}, \]  

(3)

and the dominance of the gradient term on the r.h.s. of the above equation.

2. Numerical study

This stated, one proceeds as follows: each visible matter density profile \( \rho(r) \) is derived from the visible component of the corresponding galaxy’s rotation curve; two added Hernquist profiles are used, one for the extended gas and another for the core matter distribution. This density profile is then used as an input to Eq. (2), and a best fit scenario for the dark matter component of the rotation curve is obtained by varying the parameters \( R_1^{1/2} \) and \( R_3^{1/2} \). Although these should be universal quantities, one resorts to individual fits of each galaxy, allowing for a posterior discussion of order of magnitude, deviation between obtained values, any abnormal case or possible trends, etc.

The results derived from the numerical solution of Eq. (2) are depicted in Table 1, and shown in Fig.1: as can be seen, the composite non-minimal coupling Eq. (1) provides close fits for all rotation curves in the outer region — of much higher quality than those obtainable if one assumed individual couplings \( f_2(R) = R/R_1 \) or \( f_2(R) = \sqrt{R/R_3} \) (see additional graphs in Ref. [1]). For some galaxies, the overall quality of fit is disturbed by a discrepancy in the inner galactic region. This could result from an uncertainty in the original derivation of the rotation curves, or perhaps due to the deviation from purely spherical symmetry.

From a fundamental standpoint, this deviation could also indicate that the non-minimal coupling should include an additional term that acquires relevance in a higher density environment: recalling an earlier work in the high density context of the Sun, one could admit a linear addition \( \delta f_2(R) \propto R \), for instance. Of course, the considered model could be degenerate to the approximation of the pure curvature term given by the GR prescription \( f_1(R) = 2\kappa R \), and a more complex picture involving both non-trivial \( f_1(R) \) and \( f_2(R) \) couplings is in order.

Although details are here suppressed, for brevity, after obtaining the solution to Eq. (2) it is straightforward to compute the gradient term on its r.h.s.: as discussed before, this is indeed dominant, so that the mimicked dark matter density profile is almost completely overlapped with it.
3. Characteristic density, background matching and mimicked “dark matter” inter-dominance

The selected galaxies have their rotation curves fitted by characteristic lengthscales of the order $r_1 \sim 10$ Gpc and $r_3 \sim (10^5 - 10^6)$ Gpc. Although the analytic results derived in the preceding sections do not hold strictly in the assumed composite coupling scenario of Eq. (1), the expression

$$ r_\infty = \left\{ \frac{2}{3} \times 3^n(1 - 2n) \right\}^{1/4} \left( \frac{c}{r_0 H} \right)^{(1-n)/2} \left( r_s r_0 d \right)^{1/4}, \quad (4) $$

should nevertheless provide a quantitative figure for the background matching distances,

$$ r_\infty = \left\{ \frac{2}{3} \times 3^n(1 - 2n) \right\}^{1/4} \left( \frac{c}{r_0 H} \right)^{(1-n)/2} \left( r_s r_0 d \right)^{1/4} \approx \left\{ \begin{array}{ll} \left( \frac{3.8 \text{ Gpc}}{r_3} \right)^{2/3} \left( r_s r_0 d \right)^{1/4}, & n = -1/3 \\ \left( \frac{3.8 \text{ Gpc}}{r_1} \right) \left( r_s r_0 d \right)^{1/4}, & n = -1 \end{array} \right. \quad (5) $$

after inserting the Hubble constant $H_0 = 70.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Notice that this expression was obtained with the assumption of a single Hernquist profile for the visible matter density, and not for the two component model used in the current fitting session; using the dominant contribution stemming from the extended gas distribution, one obtains the values displayed in the last columns of Table 1.

Since $r_\infty$ may be regarded as signaling the radius of the mimicked dark matter haloes, Table 1 shows that the $n = -1/3$ NFW haloes are always smaller than those with the $n = -1$ isothermal sphere profile — with the exception of NGC 2434 that, however, stands out from the set since it is best fitted by a single NFW “dark matter” component. Also, $r_\infty$ is always larger than the range of the rotation curves, so that the isothermal sphere “dark matter” extends further than the observationally inferred; on the contrary, the value of $r_\infty$ goes from about $0.57L$ (NGC 7145) to $1.73L$ (NGC 7507), $L$ being the full distance considered for the rotation curves.

| Galaxy  | Composite | Single |
|---------|-----------|--------|
| (NGC)   | $r_1$    | $r_3$  | $r_\infty 1$ | $r_\infty 3$ | $r_1$  | $r_3$  |
| 2434    | $\infty$ | 0.9    | 0           | 33.1         | 4.1     | 0.9  |
| 5846    | 37       | $\infty$ | 138         | 0            | 37      | 34.9 |
| 6703    | 22       | $\infty$ | 61.2        | 0            | 22      | 26.2 |
| 7145    | 22.3     | 47.3   | 60.9        | 14.2         | 19.9    | 23.8 |
| 7192    | 14.8     | 24     | 86.0        | 18.3         | 14.5    | 7.3  |
| 7507    | 4.9      | 2.9    | 178         | 31.1         | 4.3     | 1.1  |
| 7626    | 28       | 9.6    | 124         | 42.5         | 16.0    | 7.1  |

Table 1. Best fit values for the characteristic lengthscales $r_1$ and $r_3$ for the composite and separate fits of the galaxy rotation curves with the $n = -1$ isothermal sphere and $n = -1/3$ NFW mimicked “dark matter” scenarios, together with background matching distances $r_\infty 1$ ($n = -1$) and $r_\infty 3$ ($n = -1/3$) for the composite non-minimal coupling. The limit $r_1 = \infty$ for the composite scenario indicates that the corresponding scale $R_i = 0$. The units used are $r_1$ (Gpc), $r_3$ ($10^5$ Gpc), $r_\infty 1$ and $r_\infty 3$ (kpc).
Figure 1. Observed rotation curve (dashed full), decomposed into visible (dotted) and dark matter (dashed grey) contributions [2], superimposed with the mimicked dark matter profile (full grey) arising from the composite non-minimal coupling and resulting full rotation curve (full).

This indicates that the isothermal sphere halo is more significant that its NFW equivalent:
even the exceptional case of NGC 2434, where no isothermal sphere component appears in the best fit scenario, yields a matching distance \( r_{\infty} \approx 33.1 \text{ kpc} \) — only about twice the endpoint of the corresponding rotation curve, \( L \sim 17 \text{ kpc} \).

This is confirmed by a simple algebraic computation: since the NFW profile falls as \( r^{-3} \), while the isothermal sphere one displays a shallower \( r^{-2} \) long-range dependence, and the respective characteristic densities obey \( \rho_3 \ll \rho_1 \), it is easy to conclude that the isothermal sphere eventually becomes the dominant dark matter contribution. Indeed, equating both contributions yields

\[
\frac{\rho_{dm\ 3}}{\rho_{dm\ 1}} = \left( \frac{\sigma^2}{a r_3} \right)^{1/2} \left( \frac{r_{H}}{a} \right)^{1/4} \left( \frac{a}{r} \right) \sim \frac{1}{10} \frac{a}{r},
\]

so that this dominance occurs at distances larger than \( r = a/10 \sim 1 \text{ kpc} \). Thus, the considered galactic rotation curves are found to be “mostly flat”.

4. Cosmological relevance

As discussed already, the results obtained above are naturally tuned towards the corresponding astrophysical setting, if one assumes that the chosen form for the (composite) non-minimal coupling represents the leading terms of a series expansion of a more evolved expression for \( f_2(R) \), suitable in the mentioned range for the scalar curvature \( R \). Hence, there is nothing mandatory concerning a possible cosmological relevance of the composite coupling Eq. (1), with the obtained values for \( R_1 \) and \( R_3 \): it might be the case that a suitable mechanism applicable in cosmology corresponds to another term of the hypothetical series expansion of \( f_2(R) \).

Nevertheless, it is natural to assess if the considered cases \( n = -1 \) and \( n = -1/3 \) give rise to implications for cosmology, since this would present a natural candidate for a unification model of dark matter and dark energy. From the discussion of paragraph 3, one may write the non-minimal coupling Eq. (1), in a cosmological context, as

\[
f_2 = \sqrt{\frac{R_3}{R}} + \frac{R_1}{R} \sim \left( \frac{r_H}{r_3} \right)^{2/3} + \left( \frac{r_H}{r_1} \right)^2 \sim 10^{-4} + 10^{-1},
\]

having used \( R = 3/r_H^2 \), with \( r_H = c/H = 4.2 \text{ Gpc} \) the Hubble radius, and inserting the best fit orders of magnitude \( r_1 \sim 10 \text{ Gpc} \) and \( r_3 \sim (10^3 - 10^6) \text{ Gpc} \).

The small value of the \( n = -1/3 \) non-minimal coupling indicates that this should not be significant at a cosmological level; the higher value of the \( n = -1 \) power-law coupling (when compared to unity) hints that a simple \( R_1/R \) coupling could prove interesting for cosmology.

5. Universality

The obtained values for \( r_1 \) and \( r_3 \), shown in Table 1, display an undesired variation between galaxies, since these parameters should be universal. Indeed, \( r_1 \) averages \( \bar{r}_1 = 21.5 \text{ Gpc} \) with a standard deviation \( \sigma_1 = 10.0 \text{ Gpc} \), while \( r_3 \) presents \( \bar{r}_3 = 1.69 \times 10^5 \text{ Gpc} \) and \( \sigma_3 = 1.72 \times 10^6 \text{ Gpc} \). This deviation from universality is not unexpected, and several causes could be hinted as culprits: although small, the existing asymmetry in the selected type E0 galaxies due to the \( 1 - b/a \lesssim 0.1 \) semi-axis deviation from pure sphericity could translate into the difference in obtained values for \( r_1 \) and \( r_3 \). This shift from the assumed symmetry could be enhanced by localized features and inhomogeneities, which could also act as “seeds” for the dynamical generation of the mimicked “dark matter” component. Furthermore, the reported dominance of the gradient term in the r.h.s. of Eq. (2) could boost this effect, if these perturbations vary significantly on a short lengthscale.

By the same token, the relevance of the gradient term could lead to the deviation from universality if the chosen profiles (Hernquist for visible matter, NFW and isothermal sphere for mimicked dark matter) prove not to be so suitable as hoped: this mismatch between the
real density profiles and the fitted ones does not need to be large, if a large deviation of its first and second derivatives lies in very localized regions: this could act as a sort of “fitting inhomogeneities” with the same effect as of those discussed above.

Aside from the symmetry and fitting functions selected, more fundamental issues could be behind the variation of the obtained values for \( r_1 \) and \( r_3 \): in fact, the non-minimal coupling Eq. (1) could be too simplistic, and a more complex model could provide a more universal fit of the galaxy rotation curves.

Also (as discussed with respect to the relatively less successful fits of the inner galactic regions), the simplifying assumption of a linear curvature term \( f_1(R) = 2\kappa R \) might also contribute to this deviation from universality, since one knows that power-law terms \( f_1(R) \propto R^n \) can also induce a dynamical “dark matter” [6].

Finally, a more interesting idea could lie behind the obtained discrepancy, namely that the visible matter Lagrangian density itself may be different from the assumed perfect fluid form, \( \mathcal{L}_m = -\rho [7, 8] \): this functional dependence is inadequate in the context of the model here addressed, and a more adequate form \( \mathcal{L}_m = \mathcal{L}_m(\rho) \) should be considered — perhaps even including other variables related to the thermodynamical description of the matter distribution (see Ref. [9] for an in-depth discussion based upon a similar consideration).

6. Conclusions

In the two contributions to this volume, the possibility of obtaining a solution to the dark matter puzzle, embodied by the flattening of galaxy rotation curves, was approached by resorting to the main phenomenological implications of models possessing a non-minimal coupling of matter to curvature, following results reported in Ref. [1].

As a first attempt, one first examined the non-conservation of the energy-momentum tensor of matter and the implied deviation from geodesic motion, and ascertained what coupling \( f_2(R) \) should be so that the derived extra force would lead to the reported flattening of the rotation curves. This requires a logarithmic coupling of the form \( \lambda f_2(R) = -\nu^2/(m \log(R/R_\ast)) \) (where \( m \) is the outer slope of the visible matter density \( \rho \)), which can be approximated by a simpler power-law \( \lambda f_2(R) \approx (R_\ast/R)^\alpha \), with the asymptotic velocity given by \( v_\infty^2 = \alpha m \). However, this solution yields an almost universal \( v_\infty \), prompting for another possible mechanism for the dynamical mimicking of “dark matter”.

This need led to the second, more evolved approach, which is rooted empirically in the phenomenological Tully-Fisher relation: one may instead assume that geodesical motion is preserved, \( \nabla^\mu T_{\mu\nu} = 0 \) (a condition proved to be self-consistent), but that the metric itself is perturbed: the mimicked dark matter density is then given by the difference \( R/2\kappa - \rho \). By resorting to a power-law coupling with matter \( f_2(R) = (R/R_0)^n \) (with a negative exponent \( n \) yielding the desired effect at low curvatures and long range).

The proposed power-law directly leads to a “dark matter” density that depends on a power of the visible matter density, thus accounting for the Tully-Fisher law in a natural way. The obtained “dark matter” component has a negative pressure, as commonly found in cosmology in dark energy models: this hints the possibility that the non-minimal coupling model might unify dark matter and dark energy.

Given their relevance in the literature, two different scenarios were considered: the NFW and isothermal sphere dark matter profiles \( n = -1/3 \) and \( n = -1 \), respectively. Although separate fits of the selected galaxy rotation curves to each coupling do not yield satisfactory results, a composite coupling of both power-laws produced a much improved adjustment.

The characteristic lengthscales \( r_1 \) and \( r_2 \) were taken as fitting parameters for each individual galaxy, yielding the order of magnitudes \( r_1 \sim 10 \text{ Gpc} \) and \( r_3 \sim 10^5 \text{ Gpc} \). This enabled the computation of the cosmological background matching distances for each galaxy (which depend on their characteristic lengthscale \( a \)) and the obtained astrophysical range showed that the
\( n = -1 \) isothermal sphere “dark matter” halo dominates the \( n = -1/3 \) NFW component. Furthermore, the \( n = -1 \) scenario was shown to have possible relevance in a cosmological context, as \( r_1 \sim r_H \), the latter being the Hubble radius — again hinting at a possible unification of the dark components of the Universe.

The lack of the desirable universality in the model parameters \( R_1 \) and \( R_3 \) was duly noticed, although these quantities are characterized by the same order of magnitude for the best fits obtained; several possible causes for this variation were put forward, from deviation from spherical symmetry to a more complex form for the non-minimal coupling between curvature and matter \( f_2(R) \) or the curvature term \( f_1(R) \), or the need for a more evolved Lagrangian description of the latter.

As a final remark, one concludes that the rich phenomenology that springs from the model with a non-minimal coupling between matter and curvature enables a direct and elegant alternative to standard dark matter scenarios and many modifications to GR, which usually resort to extensive use of additional fields and other \textit{ad-hoc} features. As an example of the latter, one points out the MOfified Newtonian Dynamics (MOND) hypothesis, which is by itself purely phenomenological, and whose underlying Tensor-Vector-Scalar theory is based upon an extensive paraphernalia of vector and scalar fields \[10, 11, 12, 13\] (see Ref. \[14\] for a critical assessment). As a final remark, we point out that the considered model may also be translated into a multi-scalar theory \[15\], with two scalar fields given by

\[ \varphi^1 = \frac{\sqrt{3}}{2} \log \left[ 1 + n \left( \frac{R}{R_0} \right)^n \right] \quad \varphi^2 = R, \]  

(8)

with dynamics driven by a potential

\[ U(\varphi^1, \varphi^2) = \frac{1}{4} \exp \left( -\frac{2\sqrt{3}}{3} \varphi^1 \right) \left[ \varphi^2 - \frac{f_1(\varphi^2)}{2\kappa} \exp \left( -\frac{2\sqrt{3}}{3} \varphi^1 \right) \right]. \]  

(9)

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