X-Ray through Very High Energy Intrabinary Shock Emission from Black Widows and Redbacks

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Abstract

Black widow and redback systems are compact binaries in which a millisecond pulsar heats and may even ablate its low-mass companion by its intense wind of relativistic particles and radiation. In such systems, an intrabinary shock can form as a site of particle acceleration and associated nonthermal emission. We model the X-ray and gamma-ray synchrotron and inverse Compton spectral components for select spider binaries, including diffusion, convection, and radiative energy losses in an axially symmetric, steady-state approach. Our new multizone code simultaneously yields energy-dependent light curves and orbital-phase-resolved spectra. Using parameter studies and matching the observed X-ray spectra and light curves, as well as Fermi Large Area Telescope spectra where available, with a synchrotron component, we can constrain certain model parameters. For PSR J1723–2837 these are notably the magnetic field and bulk flow speed of plasma moving along the shock tangent, the shock acceleration efficiency, and the multiplicity and spectrum of pairs accelerated by the pulsar. This affords a more robust prediction of the expected high-energy and very high energy gamma-ray flux. We find that nearby pulsars with hot or flaring companions may be promising targets for the future Cerenkov Telescope Array. Moreover, many spiders are likely to be of significant interest to future MeV-band missions such as AMEGO and e-ASTROGAM.

Unified Astronomy Thesaurus concepts: Millisecond pulsars (1062); Close binary stars (254); Non-thermal radiation sources (1119); Gamma-ray astronomy (628); Stellar winds (1636)

1. Introduction

Rotation-powered millisecond pulsars (MSPs) that have retained a low-mass stellar binary companion can exhibit a rich set of phenomenology and physics in a relatively well-constrained system. The Fermi Large Area Telescope (LAT) has detected about two dozen of these systems, and the number of MSPs is also expected to grow significantly with continued operation of Fermi and with next-generation radio facilities. Two subsets of binary MSPs in nearly circular, short (typically, \( \lesssim 1 \) day) orbital periods are the “spiders”: black widows (BWs) and redbacks (RBs; Roberts 2011). These spiders are chiefly differentiated by the mass \( M_c \) of their tidally locked companion, with the mass of RB companions \( M_c \gtrsim 0.1 M_\odot \), while for BWs \( M_c \ll 0.1 M_\odot \). In RB systems, the companion is usually bloated to nearly the Roche limit, well beyond the radius of an isolated main-sequence star of similar mass. Since the MSP is often well timed in the radio or by the LAT, the pulsar mass function is precisely known. Likewise, optical studies of the companion can constrain the companion mass function and orbital inclination (e.g., Reynolds et al. 2007; Romani & Shaw 2011; van Kerkwijk et al. 2011; Breton et al. 2013; Romani et al. 2015; Bellm et al. 2016; Draghis & Romani 2018). Therefore, all the orbital elements can be adequately constrained, enabling these systems to be excellent laboratories for the study of pulsar winds and particle acceleration in relativistic magnetized outflows.

When in a rotation-powered state, many spiders exhibit a nonthermal (synchrotron; SR) orbitally modulated emission component in the X-ray band, attributed to particle acceleration in an intrabinary pulsar wind termination shock (IBS) and Doppler boosting of a bulk flow along the shock tangent (e.g., Wadiasingh et al. 2015, 2017; Romani & Sanchez 2016). This type of geometry/radiation picture was identified over two decades ago for the BW system involving the MSP B1957+20 (Arons & Tavani 1993) and later for the gamma-ray-emitting PSR B1259-63/Be star binary system (Tavani & Arons 1997). The existence of a shock is also indicated by orbital-phase- and frequency-dependent radio eclipses of the MSP (since plasma structures at the shock attenuates the radio), where in RBs the eclipse fraction can be \( \gtrsim 50\% \) (e.g., Archibald et al. 2009, 2013; Broderick et al. 2016; Miraval Zanon et al. 2018) of the orbital phase (while the MSP remains largely un eclipsed at inferior conjunction of the pulsar). The hard power laws inferred in X-rays imply emission due to an energetic electron population. The X-ray spectra may furthermore extend to at least \( \gtrsim 50 \) keV with no suggestion of spectral cutoffs (e.g., Tendulkar et al. 2014; Kong et al. 2017b; Al Noori et al. 2018), constraining the shock magnetic field to \( B_{sh} \gtrsim 1 \) G (Wadiasingh et al. 2017). For a given pulsar spin-down power \( E_{\text{SD}} \sim 10^{34}–10^{35} \) erg s\(^{-1}\), the magnetic field \( B_{sh} \) at
the shock can be bounded using the Poynting flux $B_{\text{sh}}^2 c/4\pi$ for a magnetic field $B_{\text{sh}} \propto R_{\text{sh}}^{-1}$ in the striped wind outside the light cylinder. The flux can be integrated over a spherical surface of area $4\pi R_{\text{sh}}^2$ at the shock radius $R_{\text{sh}}$ to yield an isotropic electromagnetic luminosity $B_{\text{sh}}^2 R_{\text{sh}}^2 c \sim E_{\text{SD}}$. One therefore arrives at the constraint $B_{\text{sh}} < B_{\text{sh}} \sim (E_{\text{SD}} c)/R_{\text{sh}}$, which is detailed in Equation (1), being generally less than 100 G. This may be recast as a condition that the ratio of electromagnetic energy density to particle pressure, $\sigma$, is less than unity, i.e., that $E_{\text{SD}}$ is dominated by the plasma wind contribution. As will be apparent in due course (see Section 3), if the particle acceleration is as fast and efficient (attaining the synchrotron burn-off limit) as in pulsar wind nebulae, MeV synchrotron components may be observable by future medium-energy, gamma-ray Compton/pair telescopes, such as e-ASTROGAM (De Angelis et al. 2017) and AMEGO.8

Owing to irradiation and heating powered by the pulsar, many companions are optically bright, implying a rich target feature of many spiders is that the intrabinary shock appears to different work. Owing to irradiation and heating powered by the pulsar, many companions are optically bright, implying a rich target

8 AMEGO: https://asd.gsfc.nasa.gov/amego/index.html (McEnery et al. 2019).

9 This is perhaps a selection effect for the brightest sources.

2. Formalism and Model Assumptions

There is some debate about the source of pressure balance for the formation and orientation of IBSs in spider binaries, which impacts the three-dimensional shock shape. For instance, if wind ram pressures govern the shock, approximate thin-shell bow shock models such as those by Wilkin (1996), Canto et al. (1996), and Rodríguez et al. (1998) for the collision of two isotropic or isotropic-parallel winds might be decent approximations. Conversely, if a companion magnetosphere (e.g., Harding & Gaisser 1990; Wadiasingh et al. 2018) supports the pressure balance in the shock, the morphology may be dramatically different from the wind scenario. The anisotropy of the pulsar wind and its dependence on pulsar magnetic obliquity will also likely impact the final shock structures realized in the population.
of spider binaries. However, for expediency, we assume an infinitely thin, hemispherical shock morphology as a zeroth-order approximation for a shock surrounding either the pulsar or the companion (Figures 1 and 2). Such an “umbrella”-like symmetry and hemispherical structure should be generic for any nonpathological shock structure close to the shock apex (“nose”) and therefore is a reasonable approximation at this juncture. We implement azimuthal symmetry about the line joining the pulsar and companion (∂/∂φ = 0), as well as a steady-state regime (∂/∂t = 0). Relativistic electrons are injected by the pulsar and captured by the shock, where a substantial magnetic field B_{sh} leads to SR. Isotropic blackbody (BB) emission at a temperature T from the facing hemisphere of the companion in the observer frame is presumed as a soft-photon target field for IC. Following our
evaluation of the transport of particles, we calculate SR and IC emissivities but neglect synchrotron self-Compton (SSC) emission, as it is expected to be negligible for spider binaries (Appendix C).

2.1. Treatment of Particle Acceleration and Injection in UMBRELA

The upstream time-averaged Poynting flux in the pulsar wind is a moderate function of the magnetic inclination angle \( \alpha \) (e.g., Tchekhovskoy et al. 2016), the variation being roughly a factor of two from an aligned to orthogonal rotational, noting that the plasma component loads the magnetosphere and contributes significantly to the spin-down torque on the pulsar (e.g., Spitkovsky 2006). The pulsar wind is “striped,” as field lines of opposite polarity make up the undulating current sheet. At large distances the stripes may disrupt as magnetic field energy is converted to particle energy (thus lowering \( \sigma \)). Magnetohydrodynamic (MHD) and kinetic plasma instabilities generate hydromagnetic and plasma turbulence and precipitate localized and dynamic magnetic reconnection and associated heating of charges. Thus, the entropy increases, and these decoherence effects tend to isotropize the MHD flow in the wind frame. To simplify the treatment here, in particular for considerations of the energy budget in spatial zones along the IBS (Figure 1), we assume that the pulsar wind is isotropic and omit detailed consideration of modest anisotropies identified in Tchekhovskoy et al. (2016). To reduce the number of free model parameters, we assume that a fraction \( \eta_p < 1 \) of the pulsar’s spin-down luminosity \( \dot{E}_{\text{SD}} \) is converted into particle/pair luminosity \( \dot{L}_{\text{pair}} \). We do not model the microphysics of particle acceleration at the bow shock. The maximum Lorentz factor

\[
\gamma_{e,\text{max}} = \frac{eR_{\text{sh}}B_{\text{sh}}}{m_e c^2} \sim 2 \times 10^8 \left( \frac{R_{\text{sh}}}{10^{10} \text{ cm}} \right) \left( \frac{B_{\text{sh}}}{10 \text{ G}} \right)
\]

(5)

(1) The Hillas criterion for when the shock radius \( R_{\text{sh}} \) and the lepton’s gyroradius \( r_g = \gamma p/eB_{\text{sh}} \) are equal, yielding (Hillas 1984)

(2) Balancing the diffusive acceleration rate at the shock with the SR loss rate timescale (e.g., De Jager et al. 1996),

\[
\gamma_{e,\text{acc}} \sim \frac{3}{2} \sqrt{\frac{\sigma_{\text{acc}} B_{\text{sh}}}{\alpha_I}} \sim 4 \times 10^7 \left( \frac{B_{\text{sh}}}{10 \text{ G}} \right)^{-1/2}.
\]

(6)
Table 1
Model Parameters for Illustrative Cases

| Parameters                      | Symbols | Values |
|---------------------------------|---------|--------|
| Orbital separation*            | a (×10^{11} cm) | 1.95  1.95  2.90  2.90  1.25  1.25  0.597 |
| Orbital period                 | P_orb (hr) | 9.17  9.17  14.8  14.8  4.6  4.6  1.56 |
| Mass ratio                     | q        | 70    70    3.5    3.5    18.2  18.2  180 |
| Pulsar mass                    | M_{psr} (M_{\odot}) | 1.7    1.7    1.7    1.7    1.7    1.7    1.7 |
| Pulsar radius                  | R_{psr} (10^{6} cm) | 1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
| Pulsar period                  | P (ms)  | 1.60  1.60  1.86  1.86  2.89  2.89  2.56 |
| Pulsar period derivative       | P (×10^{-20} s^{-1}) | 1.7    1.7    0.75   0.75   1.4    1.4    2.1 |
| Moment of inertia of pulsar    | I (×10^{45}) | 1.0  1.0  1.0   1.0   1.0    1.0    1.0 |
| Inclination angle              | i (deg) | 65    85    40    40    54    60    60 |
| Distance                       | d (kpc) | 1.40  1.40  0.93  0.93  1.10  1.10  1.4 |
| Companion temperature          | T_{comp} (K) | 8500  8500  6000  6000  6000  12,000  12,000 |

Shock radius                     | R_{sh}/a | 0.4    0.2   0.4   0.4   0.3   0.3   0.4 |
| Magnetic field at shock         | B_{sh} (G) | 1.2    7.0    0.8   0.8   0.8   1.0  1.0 |
| Pair multiplicity               | M_{pair} | 9000   9000  4000  2000  500   600  600 |
| Maximum particle conversion efficiency | \eta_p | 0.6   0.8   0.9   0.8   0.8   0.8   0.8 |
| Acceleration efficiency         | \epsilon_{acc} = r_{g}/\lambda | 0.001  0.001  0.08  0.001  0.001  0.001  0.001 |
| Index of injected spectrum      | p        | 2.5   2.4   2.5   1.4   1.4   1.4   1.4 |
| Bulk flow momentum              | (\beta \Gamma)_{max} | 4.0    5.0   6.0   7.0   7.0   7.0   5.0 |
| Maximum shock angle             | \theta_{max} | 61    68    60   50    55    65   60 |

Note.
* This is a derived value using \( a = (GM_{\text{NS}}M_{\text{comp}})^{1/2} / (1 + q)^{1/3} \).

(3) The pulsar polar cap (PC) voltage drop \( \Phi_{\text{open}} \), limiting the maximum primary electron energy to

\[
\gamma_{e,max}^{P} = \frac{e\Phi_{\text{open}}}{m_{e}c^{2}} \sim 5 \times 10^{9} \left( \frac{P}{5 \times 10^{-3} \text{ s}} \right)^{-2} \times \left( \frac{R_{\text{PSR}}}{10^{6} \text{ cm}} \right)^{3} \left( \frac{B_{\text{PSR}}}{10^{9} \text{ G}} \right).
\]

Here \( e \) is the elementary charge, \( m_{e} \) is the electron mass, \( c \) is the speed of light in vacuum, \( \alpha_{e} = e^{2} / \hbar c \) is the fine-structure constant, and \( B_{e} = m_{e}^{2}c^{3}/\hbar e \approx 4.41 \times 10^{13} \text{ G} \) is the quantum critical magnetic field, at which the cyclotron energy \( \hbar \omega_{B} \) of the electron equals its rest-mass energy \( m_{e}c^{2} \). Here \( \omega_{B} = eB/m_{e}c \) is the electron cyclotron frequency, which defines the scale for the rate of gyroresonant diffusive acceleration. The dimensionless parameter \( \epsilon_{acc} \equiv r_{g}/\lambda(\gamma_{e}) \leq 1 \) expresses the diffusive mean free path \( \lambda(\gamma_{e}) \) in units of the electron’s gyroradius \( r_{g} \). It describes the complexities of diffusion near the shock and how they impact the acceleration rate; \( \epsilon_{acc} \) is around unity in the so-called Bohm limit when lepton diffusion is quasi-isotropic and on the gyroradius scale, and it can be much less than unity if field turbulence that seeds diffusion is at a low level (see, e.g., Baring et al., 2017, for blazar contexts).

The third estimate applies for a pulsar of radius \( R_{\text{PSR}} \), surface polar field strength \( B_{\text{PSR}} \), and a rotational period \( P \), corresponding to an angular frequency \( \Omega = 2\pi/P \), Goldreich & Julian (1969). Evaluating the resulting potential drop for the open-field-line region leads to \( \Phi_{\text{open}} = \Omega^{2}R_{\text{PSR}}B_{\text{PSR}}/2c^{2} \), yielding a maximum \( \gamma_{e} \sim 10^{9} \); see Equation (7), which defines the most optimistic acceleration case. We enforce \( \gamma_{e,max} \equiv \min(\gamma_{e,max}^{H}, \gamma_{e,max}^{acc}, \gamma_{e,max}^{P}) \). The third estimate of the Lorentz factor is typically an absolute upper limit; however, in our case we choose \( B_{sh} \) independently, so we have to take the minimum of these factors as the maximum particle energy.

We normalize the particle injection spectrum by requiring (Sefako & De Jager 2003) the pulsar wind to be the sole supplier of pair flux incident on the IBS:

\[
\int_{\gamma_{e,min}}^{\infty} Q_{PSR} d\gamma_{e} = (M_{\text{pair}} + 1) N_{GJ}, \quad N_{GJ} \sim \frac{4\pi^{2}B_{PSR}R_{PSR}^{3}}{Pc^{2}c^{4}}.
\]

This pair flux is thereby benchmarked using the Goldreich–Julian primary particle injection rate \( N_{GJ} = I_{GJ}/e \) for pulsar magnetospheres, which is expressed in terms of the current \( I_{GJ} \sim \sqrt{E_{SD}} e \) of the Goldreich–Julian current \( I_{GJ} \) is determined as follows. The charge density is \( \rho \approx -\Omega \cdot B / (2\pi c) \), so that \( |\rho| \approx B_{PSR} / P_{c} \) at the surface. For small PC sizes, the area of both PCs that are proximate to this charge is \( A_{\text{cap}} \approx 2\pi R_{PSR}^{2} = (4\pi^{2}B_{PSR}^{3} / P_{c}) \). The flux of charge through this area is then \( I_{GJ} = |\rho|A_{\text{cap}} c \) and reduces to \( eN_{GJ} \) with \( N_{GJ} \) as given in Equation (8). The primary flux is enhanced by the pair multiplicity \( M_{\text{pair}} \) in a magnetospheric pair cascade, i.e., the number of pairs spawned per primary accelerated in the pulsar’s PC \( E_{ij} \) field. Typically \( M_{\text{pair}} \sim 10^{2} \) results from pulsar models; see just below.

A parallel constraint is that the power in pairs impinging on the shock not exceed the pulsar spin-down power \( E_{SD} \). Thus, we introduce a conversion efficiency \( \eta_{p} < 1 \) that is defined by
the pair luminosity relation

\[ L_{\text{pair}} \equiv m_e c^2 \int_{\gamma_{e,\text{min}}}^{\infty} \gamma e \mathcal{Q}_{\text{PSR}} \, d\gamma_e = \eta_p E_{\text{SD}}, \quad E_{\text{SD}} = \frac{4\pi^2 I^2}{p^3}, \]

with \( I \) the moment of inertia and \( \dot{P} \) the time derivative of the pulsar period. The value of \( \eta_p < 1 \) includes the density compression across the relativistic intrabinary shock. It is evident, given the form for \( Q_{\text{PSR}} \) in Equation (4), that these pair flux and power equations are coupled. Thus, by fixing \( \gamma, M_{\text{pair}}, \) and \( \eta_p \) and integrating up to \( \gamma_{e,\text{max}} \), one may self-consistently solve for \( Q_0 \), as well as the minimum particle energy \( \gamma_{e,\text{min}} \) (Venter et al. 2015). Attributing the spin-down to losses in rotational kinetic energy in the usual manner, one can estimate the pulsar surface polar magnetic field using the standard vacuum orthogonal rotator formula: \( B_{\text{PSR}} \approx 6.4 \times 10^{19} \sqrt{\dot{P} P} \) G. Since plasma contributions to the electromagnetic torque are generally comparable to the vacuum ones (see, e.g., Spitkovsky 2006, for force-free MHD considerations), these formulae as estimates are generally secure. For instance, the magnetic obliquity of the MSP (or additional multipolar components) generally has an order-of-unity influence on inferred parameters (e.g., Tchekhovskoy et al. 2016). Thus, given this closure, we may specify \( Q_{\text{PSR}} \).

2.2. Pair Cascades and Pair Multiplicity

PC pair cascades are thought to occur in all pulsars, both for generating coherent radio emission (Philippov et al. 2020) and for supplying plasma for the current closure in the magnetosphere (Brambilla et al. 2018). Strong electric fields that develop above the PCs accelerate particles to Lorentz factors exceeding \( 10^7 \), and their curvature radiation photons can create electron–positron pairs via magnetic photon pair production \( (\gamma \rightarrow e^+e^-) \) in the strong magnetic field (Daugherty & Harding 1983). The pairs screen the electric field and short out the gap potential, which again builds up in cycles as the pairs clear the gap (Timokhin & Arons 2013). The bursts of pairs from the gap then continue to produce further generations of pairs through SR and IC radiation above the gap to build up the full multiplicity \( M_{\text{pair}} \) (Daugherty & Harding 1982; Timokhin & Harding 2015). The maximum multiplicity can reach \( \sim 10^5 \) (Timokhin & Harding 2019) and most likely increases with pulsar spin-down power, because both higher magnetic fields and smaller field line radi of curvature enhance the magnetic pair creation rate. However, the detailed dependence is still an open question.

Monte Carlo simulations have modeled the pair cascade spectra, which are hard power laws with low- and high-energy cutoffs that depend on the pulsar period and period derivative (Harding & Muslimov 2011). The pair spectral index is roughly \( \gamma_{\text{pair}} \sim 1.5 \) near the low-energy cutoff, and the spectrum is curved so that the index increases with energy. Interestingly, such a hard index for the underlying electron population is suggested by the spectrum of the orbitally modulated X-ray emission in RBs. This index may or may not be modified by energization at the IBS, depending on the intrinsic acceleration characteristics of that shock. If the shock is weak enough to naturally generate power laws with indices \( p > 1.6–1.7 \), then the injection distribution from the pulsar will maintain its index \( \gamma_{\text{pair}} \) but be boosted up to higher Lorentz factors (e.g., Jones & Ellison 1991). If, instead, the shock naturally generates very flat nonthermal distributions with \( 1 < p < 1.5 \), for example, through efficient shock drift energization in weak shock layer turbulence (Summerlin & Baring 2012), then this is the index that is appropriate for the injection function \( Q_{\text{PSR}} \). Note that in this study the index \( p \) is a free parameter, so that we do not consider specific details of the shock acceleration characteristics. For young pulsars, low-energy cutoffs are around \( \gamma_{\text{pair,min}} \sim 100 \) and high-energy cutoffs vary between \( \gamma_{\text{pair,max}} \sim 10^5 \) and \( \gamma_{\text{pair,max}} \sim 10^6 \). For MSPs, low-energy cutoffs are around \( \gamma_{\text{pair,min}} \sim 10^6 \) and high-energy cutoffs vary between \( \gamma_{\text{pair,max}} \sim 10^6 \) and \( \gamma_{\text{pair,max}} \sim 10^7 \). The different behaviors result from the much lower surface magnetic fields of MSPs, where the photon energies must be higher in order to produce pairs that have energies roughly equal to half of the parent photon energy.

Pair production by MSPs has been puzzling, since most have surface magnetic fields that are too low to initiate pair cascades in purely dipolar fields. As a result, studies of pair cascades have long suggested the presence of nondipolar magnetic fields near the stellar surface to produce the pairs needed for radio emission (Ruderman & Sutherland 1975; Arons & Scharlemann 1979). Harding & Muslimov (2011) found that deviations from a dipole field could enable pair cascades in older pulsars, including MSPs, and significantly increase the pair multiplicities. The recent result from the Neutron Star Interior Composition Explorer (NICER) inferring a heated spot geometry on the surface of MSP J0030+0451 (Miller et al. 2019; Riley et al. 2019) that is far from antipodal strongly suggests the presence of nondipolar fields, as well as a severely offset dipole (Bilous et al. 2019). This gives direct support to the idea that MSP surface fields are highly nondipolar, giving them the ability to produce high pair multiplicity, though probably lower than for young rotation-powered pulsars. As we demonstrate in Section 3, the data seem to be consistent with theoretical constraints on particle energies and high pair multiplicities for pair injection from MSPs. This may have implications for the locally measured energetic positron excess (Venter et al. 2015).

2.3. Shock Orientation and Division into Spatial Zones

Shock orientation is a key parameter choice in UMBRELA. In BWs such as B1957+20, observations suggest that the shock wraps around the companion. In contrast, in RBs this orientation appears to be reversed. This influences how much of the pulsar wind is captured by and interacts with the shock, and hence affects the radiative power of the system in the X-ray band and beyond. Expectations for the RB case are then that the IC flux will be lower than in the BW case (given a lower photon energy density, but also depending on the companion temperature) and that the SR flux will be higher (for a higher magnetic field), and these are borne out by model output (Section 3). Furthermore, the velocity tangent to the spherical shock is changed from \( u_i \) to \( -u_i \) in the RB case (Section 2.6). The effect of this is a 0.5 phase change in the position of the light-curve peak compared to the BW case. We assume that the particle flow is initially radial upstream of the termination shock, but then along the shock tangent downstream, which is taken to be azimuthally symmetric (Figure 2). We denote the
azimuthal angle about the $z$-axis that is oriented along the line joining the two stellar centers by $\phi_i$.

We divide the shock into multiple zones of equal width in $\mu \equiv \cos \theta$, with $\theta$ the angle measured from the companion center with respect to the $z$-axis. When the shock is wrapped around the pulsar, let us label the latitude as measured from the pulsar by $\lambda$, the shock radius from the pulsar center by $R_{sh}$, the orbital separation by $a$, and the distance from the companion to the shock as $\rho$ (see Figure 1). Using elementary trigonometry, we find

$$\rho(\theta) = \sqrt{R_{sh}^2 + a^2 - 2R_{sh}a \cos \theta}, \quad (10)$$

$$\lambda(\theta) = \sin^{-1}\left(\frac{\rho(\theta) \sin \theta}{R_{sh}}\right). \quad (11)$$

In the BW case (shock around companion) the origin is centered on the companion, with $\theta$ running clockwise and being used to define the spatial zones along the shock. The distance from the companion center to a particular zone is always $R_{sh}$. However, when we consider the RB case (shock around pulsar; see Figure 2), the origin is centered on the pulsar using $\lambda(\theta)$ (Equation (11)) to indicate the spatial zones along the shock. To calculate the distance from the companion center to a particular zone, we then use a cosine rule as seen in Equation (10). Most of the power of the pulsar wind (if spin and orbital axes are aligned) is confined to the equator at $\theta_{\text{PSR}} = \pi/2$ or $\lambda = 0$, corresponding to the nose of the shock. For small values of $\lambda$, we approximate the injection spectrum assuming isotropy of the pulsar wind near the nose. Furthermore, the $i$th zone in the shock intercepts a fraction of the total injected wind $Q_{\text{PSR}}$ such that $Q_i = \frac{d\Omega(i)}{4\pi} Q_{\text{PSR}}$, as illustrated by large arrows in Figure 3, with $d\Omega(i)$ the solid angle subtended by the $i$th zone.

For zones $i > 1$, there is also a contribution of particles diffusing isotropically from the previous zone into the current one, as indicated by the small arrows in Figure 3. It thus follows that the injection spectrum for the first zone is

$$Q_i = \left(\frac{1}{4\pi} \int_0^{2\pi} d\phi \int_{\lambda_i}^{\lambda_{i+1}} \sin \lambda \ d\lambda\right) Q_{\text{PSR}}$$

$$= \frac{1}{2} (\cos \lambda_i - \cos \lambda_{i+1}) Q_{\text{PSR}}, \quad (12)$$

while the injection spectra for other zones ($i = 2, \ldots N$) are

$$Q_i = \frac{N_{i,i-1}}{t_{\text{diff},i-1}} + \frac{1}{2} (\cos \lambda_i - \cos \lambda_{i+1}) Q_{\text{PSR}}, \quad (13)$$

with $t_{\text{diff}}$ the diffusion timescale defined in Section 2.4.

### 2.4. The Convection–Diffusion and Particle Transport Equation

We implement a bulk flow of relativistic particles along the shock tangent and parameterize the bulk speed such that (Wadiasingh et al. 2017)

$$(\beta\Gamma)_i = (\beta\Gamma)_{\text{max}} \left(\frac{\theta_i}{\theta_{\text{max}}}\right) \quad (14)$$

with $\beta = (\beta\Gamma)/\sqrt{1 + (\beta\Gamma)^2}$. The maximum bulk momentum required to describe the X-ray light curves is generally low, $(\beta\Gamma)_{\text{max}} \lesssim 10$. Given that the particles in the comoving frame are themselves ultrarelativistic, $\gamma_e \gtrsim 10^4$, such a bulk flow may be regarded as a small anisotropy in momentum space of otherwise nearly isotropic relativistic plasma. We note that in thin-shell models (e.g., Canto et al. 1996; Wilkin 1996), a linear dependence on momentum is recovered by performing a Taylor series expansion near the shock nose, and therefore such a linear term is a zeroth-order approximation to a generalized curved shock tangent velocity. Implementing a bulk flow along the shock tangent implies that the effects of convection and adiabatic losses in our transport equation are important. Below, we approximately solve a Boltzmann-type convection–diffusion equation for particle transport, including the effect of radiative losses, isotropic in momentum space in each zone in the comoving frame, applicable to relativistic particle flow and a spatially independent diffusion coefficient $\kappa(\gamma_e)$ (e.g., van Rensburg et al. 2018):

$$\frac{\partial N_e}{\partial t} = -V \cdot (\nabla N_e) + \kappa(\gamma_e) \nabla^2 N_e$$

$$\quad + \frac{\partial}{\partial \gamma_e} (\gamma_{e,\text{tot}} N_e) - (V \cdot V) N_e + Q. \quad (15)$$

with $N_e$ the differential particle distribution function per energy interval in units of $\text{erg}^{-1} \text{cm}^{-3}$, $V = \beta c$ the bulk velocity (assumed to be directed along the shock tangent), $\gamma_e$ the particle energy, $\gamma_{e,\text{tot}} = \gamma_{e,\text{rad}} + \gamma_{e,\text{ad}}^+$ the adiabatic losses, and $\gamma_{e,\text{rad}}$ the total radiation (SR and IC) losses. In a steady-state approach, $\partial N_e/\partial t \equiv 0$. Under various assumptions (see
Appendix A), one may simplify the above transport equation to find

$$0 = -\frac{N_{e,i}}{\tau_{ad,i}} - \frac{N_{e,i}}{\tau_{diff,i}} - \frac{N_{e,i}}{\tau_{fl,i}} - \frac{N_{e,i}}{\tau_{rad,i}} + Q_i$$

$$\Rightarrow N_{e,i} = Q_i \tau_{eff,i},$$

(16)

where

$$\tau_{diff,i}^{-1} = \tau_{ad,i}^{-1} + \tau_{fl,i}^{-1} + \tau_{rad,i}^{-1} + \tau_{i,i},$$

(17)

and the respective timescales are defined in Appendix A. We use a parametric form for the spatial diffusion coefficient

$$\kappa = \kappa_0(\gamma_0)^{\alpha_d},$$

in this paper, we investigate Bohm diffusion ($\alpha_D = 1$ and $\kappa_0 = c(3eB)^{-1}$), corresponding to very turbulent plasmas.

2.5. Radiation Losses

The radiation loss term is composed of SR and IC losses:

$$\dot{\gamma}_\text{rad} = \dot{\gamma}_\text{SR} + \dot{\gamma}_\text{IC}.$$ The SR loss rate for an isotropic distribution of pitch angles is

$$\dot{\gamma}_\text{SR} = \frac{4\sigma_T c U_B \gamma_e^2}{3 m_e c^2},$$

(18)

with $\sigma_T$ the Thomson cross section, $\gamma_e$ the electron Lorentz factor, and $U_B = B_{\perp}^2 / 8\pi$, with $B_{\perp}^2$ assumed to be constant at the shock radius $R_{sh}$. For this initial study, we assume that $B_{\perp}^2$ does not transform between comoving and lab frames (i.e., it is roughly parallel to the downstream comoving flow). For the IC loss rate, we take the Klein–Nishina effects approximately into account using (Schlickeiser & Ruppel 2010)

$$\dot{\gamma}_\text{IC} = \frac{4\sigma_T c U_B \gamma_e^2}{3 m_e c^2} \frac{\gamma_{KN}^2}{\gamma_{KN}^2 + \gamma_e^2},$$

(19)

with

$$\gamma_{KN} = \frac{3\sqrt{3} m_e c^2}{8\pi k_B T},$$

(20)

where $k_B$ is the Boltzmann constant and $T$ the soft-photon temperature. The total BB energy density in the observer frame is

$$U = \frac{2\sigma_{SB} T^4}{c} \left(\frac{R_s}{\rho}\right)^2 \approx 5 \times 10^{16} \left(\frac{T}{5 \times 10^3 \text{K}}\right)^4 \text{eV cm}^{-3}$$

(21)

for the dayside hemisphere and for $R_s / \rho \approx 0.2$, with $\sigma_{SB} = 5.6704 \times 10^{-5} \text{erg cm}^{-2} \text{s}^{-1} \text{K}^{-4}$ the Stefan–Boltzmann constant. For the BW case, $\rho = R_{sh}$ since the shock is wrapped around the companion at a fixed radius. In the RB case, $\rho$ is determined using the cosine rule to calculate the distance the particular zone is from the source of soft photons. We include the cosmic microwave background (CMB) as an additional source of soft photons for completeness (using an energy density $U_{\text{CMB}} = 0.265 \text{eV cm}^{-3}$), but this has little effect on the predicted IC spectrum. We use Equation (21) to normalize the photon number density in the comoving frame:

$$n_{\gamma}(\epsilon_\gamma) = \frac{30\sigma_{SB} h}{(\pi k_B)^4} \left(\frac{R_s}{R_{sh}}\right)^2 \frac{\gamma_e^2}{\epsilon_\gamma^4} \exp \left(\frac{\epsilon_\gamma}{\delta k_B T} - 1\right),$$

(22)

where $\epsilon_\gamma$ is the photon energy. This is very similar to the expression by Dubus et al. (2015), and we see that $n_{\gamma} \sim U / \delta^2$ (see Section 2.6). See Appendix B for the derivation of this expression.

2.6. Beaming and Emission

In what follows, we denote observer-frame quantities without a prime, but quantities in the comoving frame will be primed. We calculate the steady-state particle distributions in the observer frame by solving Equation (16). The particle transport calculation is performed for the full shock, i.e., for $\theta \in (0, \theta_{\text{max}})$ and for all azimuthal angles $\phi_x$. The resulting particle spectrum is thus observer independent. We neglect any change to the steady-state particle spectrum between the comoving and lab frames since the particle energies $\gamma_e$ are much larger than the imposed bulk motion:

$$\frac{dN_e}{d\gamma_e} \approx \frac{dN_e'}{d\gamma_e'}. $$

(23)

We also assume that the emitted flux is isotropic in the comoving frame, such that $\Omega_{\text{beam}} = 4\pi$.

Importantly, the eventual observed radiative flux will be observer dependent, since the observer makes a particular slice through the shock and the emission is Doppler boosted in the observer direction (Figure 4). Let us define the unit velocity vector as the polar shock tangent

$$u_x = \sin \theta,$n

(24)

$$u_y = \cos \theta \cos \phi_x,$n

(25)

$$u_z = \cos \theta \sin \phi_x.$$n

(26)

We rotate the velocity vector by the binary phase $\Omega_{sh}$ and inclination angle $i$:

$$u = \Lambda_i \Lambda_{\Omega_{sh}} u' = \begin{pmatrix} \sin i & 0 & \cos i \\ 0 & 1 & 0 \\ -\cos i & 0 & \sin i \end{pmatrix} \begin{pmatrix} \cos(\Omega_{sh} t) & -\sin(\Omega_{sh} t) & 0 \\ \sin(\Omega_{sh} t) & \cos(\Omega_{sh} t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_x' \\ u_y' \\ u_z' \end{pmatrix}.$$n

(27)

Without loss of generality, we choose the vector pointing in the observer direction as $u = (1, 0, 0)$. Using the above, we calculate the Doppler beaming factor

$$\delta = \frac{1}{\Gamma(1 - \beta u \cdot \hat{u})}.$$n

(28)

We compute the emitted photon spectrum in each spatial zone in the comoving frame using the following expressions from
Kopp et al. (2013). For IC scattering

\[
\frac{dN_{\text{IC}}}{dE_{\gamma}}(E_\gamma, R_{sh}) = \frac{8g_{\text{IC}}}{AE_0^2} \sum_{j=0}^{k-1} \int n_{\text{IC}}(R_{sh}, \epsilon_j, T_j) \\
\times \frac{N_0(\gamma_0, R_{sh})}{\epsilon_0} \frac{\zeta(\gamma_0, E_\gamma, \epsilon_0)}{\epsilon_0^2} d\epsilon d\gamma_0, \tag{29}
\]

where \( E_0 = m_\gamma c^2 \) and \( T_j \) is the BB temperature of the \( j \)th component (we assume both the CMB and the heated companion surface as sources of target photons), \( g_{\text{IC}} = 2\pi e^2 \) (with \( g_{\text{IC}}/E_0^2 \propto \sigma_T \)), \( n_{\text{IC}}(\epsilon_j) \) is defined in Equation (22), and \( A = 4\pi d^2 \), with \( d \) being the distance from the source. The function \( \zeta(\gamma_0, E_\gamma, \epsilon_0) \) is proportional to the collision rate and is split into four cases, depending on \( E_\gamma \) (Jones 1968):

\[
\zeta(\gamma_0, E_\gamma, \epsilon_0) = \begin{cases} 
0 & \text{if } E_\gamma < \frac{\epsilon_0}{4\epsilon_0}, \\
\frac{E_\gamma}{\epsilon_0} - \frac{1}{4\epsilon_0} & \text{if } \frac{\epsilon_0}{4\epsilon_0} \leq E_\gamma \leq \epsilon_0, \\
f(q, g_0) & \text{if } \epsilon_0 \leq E_\gamma \leq \frac{4\epsilon_0^2}{1+4\epsilon_0/E_0}, \\
0 & \text{if } E_\gamma > \frac{4\epsilon_0^2}{1+4\epsilon_0/E_0}.
\end{cases} \tag{30}
\]

The function \( f(q, g_0) \) is given by

\[
f(q, g_0) = 2q \ln q + (1 - q)(1 + q(2 + g_0)), \tag{31}
\]

with \( q = E_\gamma^2/(4\epsilon_0^2(E_\gamma - E_0)) \) and \( g_0 = 2\epsilon_0 E_\gamma/E_0^2 \). The emitted SR photon spectrum in each spatial zone is calculated using

\[
\frac{dN_{\text{SR}}}{dE_\gamma}(E_\gamma, R_{sh}) = \frac{1}{Ah^2E_\gamma E_0} \int_0^{\sqrt{3}E_\gamma B(R_{sh})} f^{3/2} \sin^2 \Theta d\Theta d\gamma_0, \tag{32}
\]

where \( \nu_\epsilon \) stands for the critical frequency (with pitch angle \( \Theta \))

\[
\nu_\epsilon(\gamma_0, \Theta) = \frac{3e^2}{4\pi E_0} \gamma_0^2 B \sin \Theta. \tag{33}
\]

The function \( \kappa \) (where \( K_{5/3}(y) \) is modified Bessel function of order 5/3)

\[
\kappa(x) = x \int_x^{\infty} K_{5/3}(y) dy \tag{34}
\]

and is computed using the algorithm given by MacLeod (2000). We note that as a matter of simplification, our IC calculation assumes isotropic radiation, rather than anisotropic, an improvement that we defer to future work. We lastly transform the photon energy flux (luminosity) from the comoving to the lab frame via the standard form (e.g., Böttcher et al. 2012)

\[
\nu F_\nu = \delta^3 \nu F_\nu'. \tag{35}
\]

Similarly, the photon energy becomes \( E_\gamma = \delta E_\gamma' \).

We ensure that the resulting observer-dependent flux (the flux in the observer frame that depends on the observer’s line of sight) is grid independent by scaling the (azimuthally independent) flux by \( d\phi/2\pi \), with \( d\phi \) the bin size of the azimuthal coordinate measured about the line joining the two stars. The photon flux is already weighted by \( \theta \) for each zone, since the injection spectrum reflects the zone size (see Equation (4)). It is not necessary to weight this flux by the phase bin size \( d\Omega_h \), since we are calculating the \( \nu F_\nu \) flux at a specific orbital phase. Our calculation utilizes \( N_{\text{zones}} = 50 \) zones along the shock surface to compute the steady-state spectrum \( dN_{\nu}/d\nu \), 300 bins for the SR and IC energies, 300 azimuthal bins, and 300 orbital-phase bins. Our flux predictions do not change by more than 2% when choosing a finer grid. Furthermore, the number of photon energy bins we use ensures a smooth IC high-energy tail that exhibits some numerical instabilities when this number is too low (see Figure 5). We have also tested energy conservation in this code by considering the energy input/output per zone due to particle injection, escape, and radiation losses. We confirmed energy
conservation to a good level, the discrepancy being lower than 20% for all zones, which gives more confidence in the results the model produces. This discrepancy stems from the numerical approximations we make to solve the transport equation, as well as the fact that we imposed a parametric macroscopic bulk flow profile but did not take this into account in our energy conservation calculation, since we do not know all the details of the microphysics of the acceleration and bulk motion processes. We thus obtain the radiated energy flux as a function of orbital phase and photon energy. Making cuts along constant orbital phase yields spectra, while making cuts at certain constant photon energies yields light curves.

2.7. Shadowing

Obscuration by the companion of the shock emission zones (“shadowing”) can be an important effect for cases where the shock surrounds the companion and the inclination is close to edge-on, especially around pulsar superior conjunction. The implementation of such shadowing by the companion follows a protocol similar to that promulgated in Appendix A of Wadiasingh et al. (2017). In this work, we adopt an expedient approximation that the companion is spherical, with a sharp terminator via a Heaviside step function. The shadowing function, which multiplies the observer-sampled emissivity from each zone of the shock, is

\[
\Theta = \begin{cases} 
0 & r_s - r_{k,j} < 0 \land (r_j - r_{k,j})^2 + (r_z - r_{k,j})^2 < R_S^2 \\
1 & \text{otherwise},
\end{cases}
\]

where \( r_j \) and \( r_{k,j} \) are the transformed (via the matrices above) position vectors of each zone of the shock and the companion, respectively. This expression terminates emission when (i) the companion is in front of the shock and (ii) emission is directed toward the observer but is being blocked by the projection of the companion surface (at a particular orbital phase).

Although we have implemented this condition in what follows, we find this to be a negligible effect for most parameter choices. In particular, the shadowing effect is small for small \( i \), when \( \theta_{\text{max}} \) is large, or when the ratio \( R_{\text{comp}}/R_{\text{sh}} \) is small. This effect is also small in the RB case, when the shock is wrapped around a pulsar.

These conditions are typical for the fits below, so we include this small effect only for completeness.

3. Results

In this section, we present several predictions and survey illustrative output from the model. First, we study the different timescales as a function of particle energy to aid us in interpreting the underlying physics. Second, we probe the parameter space using PSR J1723–2837 as a case study, to uncover trends and diagnose the code’s sensitivity to parameter choices. Finally, we present and discuss illustrative fits to the broadband spectra and light curves of four binaries.

3.1. Timescales versus Particle Energy

In Figure 6 we plot relevant timescales for the first and final spatial zones of the IBS for RB PSR J1723–2837, where the shock wraps around the pulsar. We note that the IC timescale (blue line) is much shorter for the first zone than for the last, given the reduction of soft-photon energy density with distance from the companion star. The SR timescale (red dashed line) is the same for both zones, given the spatially constant shock magnetic field we assumed. The green line is the effective radiation loss timescale. The yellow line indicates the diffusion timescale \( t_{\text{diff},i} = R_S^2/2\kappa \). The adiabatic timescale (purple line) is similar as for similar zone sizes (our zones were chosen for constant \( d\mu \) intervals, so zones at higher \( \theta \) are slightly larger), but it also depends on the local bulk flow velocity, which grows linearly with \( \theta \) (see Equations (14) and (A5)). Thus, we see that the spatial timescales (i.e., the adiabatic one, which is very similar in magnitude to the convective timescales, as well as diffusion) dominate the radiation (IC and SR) ones in this case. This leads to relatively short residence times of particles in the spatial zones, and thus reduced emission (especially for long IC timescales); conversely, a relatively large bulk flow speed increases the Doppler factor in each zone, compensating for this effect, and leading to flux levels that should be accessible for some instruments. The total transport timescale (black line) is the shortest timescale between all those associated with the different processes (Equation (17)). We have conducted a more exhaustive study of the behavior of the timescales for the different sources we have modeled below, detailed in Appendix E, where the shock orientation is also seen to have
a significant effect on the relative magnitudes of these timescales.

3.2. Case Study: Parameter Variation for PSR J1723−2837

In what follows, we indicate the effect on model predictions of varying a single parameter at a time (as indicated in the figures below) while keeping others fixed (for a list of parameters, see Table 1), and we discuss the physical reasons for this behavior. In the plots in this section, we include spectral measurements from soft X-ray observatories (Chandra and XMM-Newton; Bogdanov et al. 2014), NuSTAR (Kong et al. 2017a) and Fermi LAT (Hui et al. 2014), represented using black spectral butterflies. We also include continuum sensitivities for AMEGO (3 yr, 20% duty cycle) in the MeV band and H.E.S.S. and the Cerenkov Telescope Array (CTA)\(^\text{10}\) in the > 100 GeV band. As an ensemble, these sensitivities facilitate comparison with the models so as to identify the main impacts of varying different parameters on source visibility. We note that, for efficiency, we used a lower resolution for the photon energy grid for our parameter study than for the source-specific cases. This led to some numerical instability in the high-energy tail of the IC spectrum. In the plots shown in this section, the SEDs show the flux for PSR J1723−2837 with the IBS around the pulsar at orbital phase of \(\Omega_{d} = 180^\circ\).

The magnetic field at the shock, \(B_{sh}\), is a crucial parameter for fitting the X-ray spectral data, with which we calibrate our model. As can be seen in Figure 7, slight changes to \(B_{sh}\) result in large changes to the SR flux since \(\gamma_{SR} \propto B_{sh}^{2}\). The IC flux is not significantly altered, since the SR loss rate typically dominates that of IC. The acceleration efficiency \(\epsilon_{\text{acc}}\) is used to calculate the maximum energy of particles, \(\gamma_{e,\text{max}} \propto \epsilon_{\text{acc}}^{1/2}\). It can be seen in Figure 7 that this parameter determines the maximum energy cutoff for the SR component and can be constrained by GeV-band LAT data. It also sets an upper limit on the IC high-energy cutoff.

The particle injection spectral index \(p\) is another crucial parameter and is constrained by the X-ray spectral data. As Figure 8 indicates, decreasing \(p\) (hardening the particle spectrum) also leads to altering the particle spectrum’s low-energy cutoff \(\gamma_{e,\text{min}}\) by Equation (8). This parameter also plays an important role in determining the peak IC flux and very high energy (VHE) spectral shape. The pair multiplicity \(M_{\text{pair}}\)

\(^{10}\) https://www.cta-observatory.org/science/
determines how many particles need to share the total available power (see Equation (4)). Thus, increasing $M_{\text{pair}}$ results in broader SR and IC spectra since more particles share the available power, and thus $g_{\text{min}}$ is lower in this case (Venter et al. 2015). While we vary individual parameters in this survey stage of our presentation, it should be noted that there exist parameter correlations and degeneracies. Moreover, the parameter variations may also be coupled. For example, the conditions promoting an increase in the number of cascade generations and therefore higher $M_{\text{pair}}$ should yield lower $g_{\text{min}}$ and $g_{\text{max}}$ and significant changes to the value of $p_{\text{pair}}$ (Daugherty & Harding 1982; Harding & Muslimov 2011; Timokhin & Harding 2015); this is discussed further in association with Figure 9.

The distance of the IBS from either the pulsar (RB case) or companion (BW case) is set by $R_{\text{sh}}$. Varying this parameter only has a significant effect on the SR flux for this RB case. This is because an increase in $R_{\text{sh}}$, while keeping the number of spatial zones fixed, leads to physically larger zones. Thus, while the same number of particles are injected into a particular zone (see Equation (12)), since the solid angle remains constant, the residence time of particles in that zone is longer, since this scales linearly with $R_{\text{sh}}$. An increased number of particles therefore leads to a higher SR flux. On the other hand, the reason that we do not see a significant change in IC flux is more complicated. One would naively expect the IC flux to increase for a larger value of $R_{\text{sh}}$, because the shock and associated energetic pairs are then closer to the source of soft photons, if one assumes that $u_{\text{sp}} \propto R_{\text{sh}}^2$. Also, the size of the spatial zones is larger, thus increasing the residence time of the particles in each zone, as indicated above. However, the scaling of $u_{\text{sp}}$ is associated with a cosine rule used to calculate the distance from the companion to a specific zone along the shock, leading to a more complicated behavior for $u_{\text{sp}}$. The zone-to-companion distance is given by $r_{\text{m}} = R_{\text{sh}} \cos \mu_{\text{mid}}$, where $\mu_{\text{mid}}$ is the angle associated with the middle of a specific zone (see Equation (10)). This causes $u_{\text{sp}}$ to be larger at zones close to the shock nose (as would be expected for a simple $1/R_{\text{sh}}^2$ scaling), but then it drops off much quicker than $1/R_{\text{sh}}^2$ for zones farther from the shock nose. This is indicated in Figure 10(b). The net result is an approximate cancellation of these effects of first increasing and then decreasing the IC flux, with the IC flux then dropping only slightly for an increase in $R_{\text{sh}}$. 

Figure 8. Model SED plots for PSR J1723–2837 depicting the effect of varying (a) injection spectral index $p$ and (b) pulsar pair multiplicity $M_{\text{pair}}$.

Figure 9. Model SED plots (a) for PSR J1723–2837 depicting the effect of varying $|\beta|$ and (b) for PSR J2339–0533 indicating two different fitting scenarios: (1) large $g_{\text{min}}$ so that we fit X-ray data using the intrinsic single-particle SR spectral slope of $4/3$ (black line), and (2) using a lower $g_{\text{min}}$ so that the X-ray spectrum is matched by varying the particle spectral index $p$ (gray line).
The companion’s dayside temperature, $T_{\text{comp}}$, only has a major influence on the IC flux (and dominates the contribution from the CMB). This dependence is also seen in Equations (19) and (20), where the IC flux scales\(^{11}\) as \(\exp(-\epsilon_b/k_B T_{\text{comp}})\). This relation has two limits—for low energies the flux is linearly proportional to the temperature. At high energies, the flux becomes proportional to \(\exp(-\epsilon_b/k_B T_{\text{comp}})\). These limits explain the monotonic but nonlinear dependence of the IC flux on the temperature.

The mass ratio $q$ is used to calculate the interbinary separation between the MSP and its companion, with $a \sim [(1 + q)/q]^{1/3}$. This parameter is also used to calculate the radius of the companion using $R_{\text{ch}}/a \approx 0.46(1 + q)^{-1/3}$ (Paczyński 1971). This characteristic size is a good approximation for many BWs and RB companions, which are generally bloated and far brighter than main-sequence stars of similar mass. For example, J1723–2847 and others have Roche “filling factor” fractions near unity (see Table 6 in Strader et al. 2019). Thus, smaller values of $q$ imply a slightly larger orbital separation, but also a larger $R_{\text{ch}}$. This implies that the soft-photon energy density should stay roughly the same, but a slightly longer residence time is expected for particles in the larger spatial zones (since these scale linearly with $a$ and thus with $R_{\text{ch}}$), which leads to a slight increase in the IC flux as $q$ is lowered, as seen in Figure 5(b).

A change in the relativistic bulk flow momentum of particles along the shock tangent, $\beta\Gamma$, impacts the velocity at which the particles are flowing along the IBS and thus will have an effect on the factor $\delta^3$ with which emission is boosted into the observer frame. Thus, a higher bulk momentum leads to higher fluxes, which would demand an accompanying adjustment, for example, lowering the magnetic field, in order to describe the data well.

Next, we note that there are two distinct physical scenarios associated with different values of the minimum particle energy not yet distinguishable by the data. Thus, ambiguity exists in the injection parameters used to describe the X-ray data we use to anchor our model. Figure 9 shows the two different scenarios we considered, namely, (1) low pair multiplicity ($M_{\text{pair}} = 500$) and soft particle injection spectrum ($\gamma_{\text{min}} = 2.7$), and (2) high pair multiplicity ($M_{\text{pair}} = 3000$) and harder particle injection spectrum ($\gamma_{\text{min}} = 2.0$), represented by the black and gray spectra, respectively.

These parameter combinations lead to a relatively high and low value for $\gamma_{\text{min}}$ respectively. For the black spectra we may match the X-ray data with the intrinsic single-particle SR spectral slope of 4/3, while for the gray spectra the X-ray data demand a hard $\gamma_{\text{min}}$. The latter scenario yields an upper limit on $\gamma_{\text{min}}$.

### 3.3. Application of the Model to Individual Sources

In this work, we constrain our model parameters by using a manual fitting procedure to simultaneously reproduce both SED and light-curve data. An automated minimization or multidimensional parameter sampler is deferred to a future work. For the SED, we first anchor the models to the empirical X-ray spectral fits of the nonthermal power-law component (and Fermi LAT data if available), and then we predict the IC flux of the system. As a first approximation for the light curve, we match the available light-curve data making sure the peak multiplicity (i.e., the number of peaks in the light curve) and position in phase roughly coincide with that found in the literature for these sources. We note that we do not include a (free) constant background term in our models, leading to artificially high modulation fractions in this case. Despite these simplifications, matching the light curves yields additional constraints on $\theta_{\text{max}}$ and $\beta\Gamma$. The value of $\theta_{\text{max}}$ is allowed to vary (see Table 1 for source model values), truncating the hemispherical shock into a cap. This latitude in the choice of $\theta_{\text{max}}$ is desirable since it affords an improved ability to fit the light curves: $\theta_{\text{max}}$ affects the peak multiplicity (typically a smaller value effectively leads to a shallower shock geometry, leading to double peaks) and enables us to produce either a single peak or double peaks. The value of $\beta\Gamma$ (in combination with $\theta_{\text{max}}$) has an effect on the width of the light curves via the Doppler boosting, with higher $\beta\Gamma$ values decreasing the width and thus leading to sharper peaks. In the parameter source-specific cases illustrated below, we indicate whether the double peaks of each system are located at the pulsar inferior conjunction, i.e., pulsar between companion and observer.

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\(^{11}\) The energy-integrated total photon number density scales as $n_b \propto T_{\text{comp}}^{-1}$. However, the spectrum $n_b(\epsilon_b)$ is normalized using an input value for the total energy density $\rho \propto T_{\text{comp}}^4$. Therefore, $n_b(\epsilon_b)$ scales as $\exp(-\epsilon_b/k_B T_{\text{comp}}) - 1)^{-1}$, see Appendix B.
3.3.1. PSR J1723−2837 (RB)—ICDP

PSR J1723−2837 is an ICDP RB system. We adopt a pulsar mass $M_{\text{ps}} = 1.7 M_\odot$, period $P = 1.86$ ms, and $\dot{P} = 7.61 \times 10^{-21}$ s. This system is located at a distance of $d = 0.93$ kpc and has an orbital period of $P_b = 14.8$ hr. In Crawford et al. (2013) the inclination angle was derived from radial velocity data and assuming a pulsar mass range of $1.4–2.0 M_\odot$, which gives an inferred companion mass range of $0.4–0.7 M_\odot$. These mass ranges then yield an orbital inclination angle of $30^\circ–41^\circ$. For this work we choose $i = 40^\circ$ and $q = 3.5$, which implies that the companion has a mass of $M_{\text{comp}} \approx 0.57 M_\odot$. Optical spectroscopy indicates the companion to have a temperature of $T_{\text{comp}} \approx 6000$ K (Crawford et al. 2013). We have used XMM-Newton, Chandra, NuSTAR, and Fermi LAT spectral data (Hui et al. 2014; Kong et al. 2017a) to anchor our models for this system. We are able to match the X-ray data for PSR J1723−2837 in two distinct ways. Both fits require a minimum energy break $\gamma_{e,\text{min}}$ and cannot be described with the intrinsic single-particle SR slope of 4/3. The black line is a model that has a very soft spectral index ($p = 2.5$) and low pair multiplicity ($M_{\text{pair}} = 400$). These parameter choices enable us to match the Fermi LAT data that may be associated with this source. Additionally, a second model is shown without matching the Fermi LAT data (gray line) as no orbital modulation has been demonstrated in this component. This model has a very hard spectral index ($p = 1.4$) and a much larger pair multiplicity ($M_{\text{pair}} = 2000$). Both illustrative models indicate that this source may be detectable by AMEGO, H.E.S.S., and CTA. In particular, CTA measurements of the IC component above 100 GeV would enable constraints mainly on the bulk Lorentz factor at the intrabinary shock and the temperature of the companion. Further in the future, a mission like AMEGO that is sensitive in the MeV band would probe the maximum Lorentz factor and magnetic field strength in the shock environs.

3.3.2. J1311−3430 (BW)—ICDP

PSR J1311−3430 is a BW system, but we interpret the orbital modulation in the X-ray band as shown in An et al. (2017), as implying that the shock is wrapping around the pulsar. This implies high magnetic fields $\mathcal{O}(10^4)$ G on the companion as a source of the pressure balance and shock orientation. For our modeling, we adopt a pulsar mass $M_{\text{ps}} = 1.7 M_\odot$, period $P = 2.56$ ms, and $\dot{P} = 2.1 \times 10^{-20}$. This system is located at a distance of $d = 1.40$ kpc, exhibiting an orbital period of $P_b = 1.56$ hr and an inclination of $i \approx 60^\circ$, as inferred from light-curve fitting described in Romani et al. (2012). The companion has inferred surface temperatures of $T_{\text{comp}} = 12,000$ and $40,000$ K, respectively, in the quiescent and flaring states as reported by Romani et al. (2015). We adopt these two characteristic temperatures as case studies on quiescent or flaring states. Since UMBRELLA is a time-independent approach, the illustrative models on the two states should not be overinterpreted. A companion mass of $M_{\text{comp}} \sim 0.01 M_\odot$ (An et al. 2017) yields a particularly high mass ratio of $q \sim 180$. To model the quiescent state (black line), we employ XMM-Newton, Chandra, Suzaku, Swift-XRT, and Fermi LAT spectral data (An et al. 2017) to constrain as many of our model parameters as possible. For the flaring state we use the associated XMM-Newton and Suzaku spectral data obtained by An et al. (2017). We consider two possible scenarios for the flaring state as shown in Figure 11b. The first is for a higher companion temperature, pair multiplicity, and bulk flow along the shock tangent (solid gray line). For the second, we keep all parameters identical to the quiescent state and only increase the magnetic field at the IBS to $B_{\text{sh}} = 3$ G and the companion temperature again to $T_{\text{comp}} = 45,000$ K (dashed gray line). It can be seen that this significant rise in temperature has a large effect on the predicted IC flux from this system because of its dependence of temperature as described in Section 3.2 in the
context of Figure 5. Once again, the energy-dependent SR and IC light-curve shapes are indicated, for both the quiescent and flaring state. One can see that the predicted peaks become more widely separated as energy is increased. The SR and IC pulse shapes are similar, but there are minor differences. Our model is able to describe the GeV modulations reported by An et al. (2017) for the 1–100 GeV bin and for the off-pulse pulsar phases reasonably well, although our SR-band peaks are not completely aligned with the data. Future MeV and TeV data may constrain different scenarios for the character, and perhaps physical origin (i.e., magnetic reconnection in the companion magnetosphere), of the optical/X-ray flares.

3.3.3. J2339–0533 (RB)—ICDP

PSR J2339–0533 is an ICDP RB system. The companion is nondegenerate with a temperature of $T_{\text{comp}} \approx 6000$ K. The mass ratio is $q \sim 18.2$, which implies a companion mass of $M_{\text{comp}} \sim 0.1 M_\odot$ (Romani & Shaw 2011). We use a pulsar mass of $M_{\text{NS}} = 1.7 M_\odot$, period $P = 2.88$ ms, $\dot{P} = 1.41 \times 10^{-20}$, distance $d = 1.10$ kpc, orbital period $P_b = 4.6$ hr, and an inclination of $i = 54^\circ$ as reported by Kandel et al. (2019). We make use of publicly reported Chandra spectral fits from Romani & Shaw (2011). Note that we are able to match the X-ray data for PSR J2339–0533 in two model scenarios, as was shown in Figure 9(b). The model in Figure 12(a) is similar to the gray models in Figure 9(b) but with slightly different parameters (see Table 1), leading to a slightly higher VHE flux and better match to the X-ray spectrum (we chose to incorporate a high minimum energy break in the particle spectrum). This model has a particle spectral index of $p = 1.8$ and low pair multiplicity ($M_{\text{pair}} = 500$). The model indicates that this source may be detectable by AMEGO.

We noted in Section 1 that An et al. (2020) recently reported an orbitally modulated gamma-ray light curve that is offset by 180° in binary phase from the X-ray double-peaked light curve. This implies that this GeV component measured by Fermi LAT must arise from a different energetic leptonic population than the IBS emission that we consider in this work. In this work, our gamma-ray and X-ray light curves are predicted to be phase aligned, as they originate from the same underlying particle population. Moreover, our predicted SR spectrum cuts off before the Fermi LAT energy range for the parameters we used, and thus our predicted SR light curves will only be detectable by AMEGO in this case. We leave the modeling of the extra spectral component that has now been measured by Fermi LAT to a future work.

3.3.4. J1959+2048 (B1957+20) (BW)—SCDP

PSR B1957+20 is the original and famous BW system (Fruchter et al. 1988). We adopt $M_{\text{NS}} = 1.7 M_\odot$, period $P = 1.60$ ms, and $P = 1.69 \times 10^{-20}$. The companion has a temperature of $T_{\text{comp}} \sim 8500$ K (Reynolds et al. 2007; van Kerckwijk et al. 2011). This system is located at a distance of $d \sim 1.40$ kpc, has an orbital period of $P_b = 9.17$ hr, and has an inclination of either $i = 65^\circ$ or $i = 85^\circ$ (Reynolds et al. 2007; van Kerckwijk et al. 2011; Johnson et al. 2014; Wadiasingh et al. 2017). Considering the lower limit on the mass of the companion, $M_{\text{comp}} \gtrsim 0.03 M_\odot$ (Reynolds et al. 2007), we use $M_{\text{comp}} \sim 0.03 M_\odot$, yielding a mass ratio $q \sim 70$. We model Chandra data from Huang et al. (2012). We are able to match the X-ray spectral data for PSR B1957+20 for both inclination angles. Both models require a low-energy break in the particle spectrum, and the X-ray data cannot be described with the intrinsic single-particle SR slope of 4/3. The gray line ($i = 65^\circ$) is a model that has a very soft particle spectral index ($p = 2.5$) and very high pair multiplicity ($M_{\text{pair}} = 9000$). The black model has similar $p$ and $M_{\text{pair}}$ values, but for a much higher magnetic field at the shock $B_{\text{sh}} = 7$ G and smaller shock radius $R_{\text{sh}} = 0.2a$ compared to $B_{\text{sh}} = 1.5$ G and $R_{\text{sh}} = 0.4a$ for the $i = 65^\circ$ case. The IC component satisfies the VHE upper limits obtained by MAGIC (Ahnen et al. 2017). Our predicted light curves are double peaked and provide a reasonable match to the X-ray data from Kandel et al. (2019), which are not fine-tuned to our fixed energy of 10 keV. One way to improve the fit would be to lower the bulk flow magnitude; alternatively, one could diminish the latitudinal variation of the bulk Lorentz factor constituted by Equation (14). This would modify the Doppler beaming in an array of directions, reducing the “pulse fraction,” i.e., the amplitudes between maximum and minimum light. The considerable scatter in the light-curve data in Figure 13 (and also in Figure 14 for PSR J1723–2837) limits the insights to be gleaned from taking such a step currently. Wu et al. (2012) reported a low-significance modulation above 2.7 GeV at the orbital period using Fermi LAT data. Although we can fit these GeV light curve data invoking the shock IC spectral component, our model predicts double peaks and not single peaks. This perhaps suggests a nonshock emission component, from a different electron population than that of the
IBS, for example the IC scattering of optical photons by leptons in the upstream pulsar wind.

4. Summary and Outlook

The single-peaked or double-peaked orbitally modulated X-ray emission, along with orbital-phase- and frequency-dependent radio eclipses of MSPs observed in spider binary systems, imply the existence of an IBS. This shock is believed to be a site of particle acceleration, with the predicted X-ray SR being Doppler boosted owing to a bulk flow of plasma along the shock tangent. These bulk motions include a dependence on the distance from the pulsar-companion axis, with an increase in speed away from the shock nose to its periphery. In this study, we used our newly developed UMBRELA code to predict energy-dependent light curves and phase-resolved spectra from these binary systems. This multizone code solves a simplified transport equation that includes diffusion, convection, and radiative energy losses in an axially symmetric, steady-state approach. The emissivities for SR and IC, including Doppler beaming, are then calculated for each spatial zone as the particles move along the shock surface. Modeling the expected SR and IC emission from the intrabinary shock and constraining our model parameters using observational data on these sources enabled us to investigate the underlying physics of these systems.

In this work, we present models of the spectra and light curves in the X-ray through the VHE band from two BW (PSR J1311−3430, PSR B1957+20) and two RB (PSR J1723−2837, PSR J2339−0533) sources. We employed reported X-ray spectral fits for all four of these systems to approximately anchor our SR spectra, enabling us to make a more robust prediction for the expected high-energy to VHE flux. We ascertained that the general character of our model light curves is fairly similar to the observed data for PSR B1957+20 and PSR J1723−2837, the systems most archetypal with among the best determinations of their orbital flux modulations.

We find that the predicted light curves and spectral models are currently quite unconstrained solely by X-ray and optical information. More broadband detections are needed, especially by VHE instruments, in order to better constrain the model parameter space. This prompted us to explore the parameter space, finding that the most important parameters for reproducing the SR spectra are the injection spectral index ($p$), pair multiplicity ($M_{\text{pair}}$), magnetic field at the shock ($B_{\text{sh}}$), and acceleration efficiency ($\epsilon_{\text{acc}}$). Additionally, detectability of these sources by instruments such as H.E.S.S. and the future
CTA telescope is highly dependent on the companion temperature ($T_{\text{comp}}$), the bulk flow momentum of particles along the shock tangent ($\beta'$), and also the spectral index $p$. We thus find that BWs and RBs may be a promising class of VHE targets for detecting modulated IC flux, especially for nearby, flaring sources. Constraining the IC emission by future data from ground-based Cerenkov telescopes may probe the particle acceleration at the shock, as well as the pulsar wind content. Likewise, measurement of the SR component and its curvature in the MeV band will constrain the efficiency of particle acceleration in the termination shock.

In future, we will improve our fitting procedure by incorporating an automated minimization or multidimensional parameter sampler. The use of Markov Chain Monte Carlo techniques is an obvious future path. Likewise, measurements of MeV–GeV orbitally modulated emission components (see Table 2 in Torres & Li 2020) can provide important constraints for our models. Following the findings of An et al. (2020) for PSR J2339–0533, we will also incorporate nonshock emission components in our modeling, e.g., anisotropic IC emission from a cold, upstream wind. We will furthermore implement an improved shock geometry, shock sweepback, and spatially dependent acceleration and injection; refine our transport calculations; and use our code to make predictions for the expected high-energy to VHE flux for several more spider binaries. One could also calculate the escaping positron flux from these systems that may contribute to the local positron excess seen above 10 GeV (Adriani et al. 2015; Venter et al. 2015). MHD calculations may further elucidate the wind—wind or wind—magnetosphere interactions, which will give more insight into the underlying physics found in these unique systems, and the plasma’s spatial variations along the intrabinary shock surface. Continued observation of these binaries in both the time and energy domains will provide improved model constraints and aid in the scrutiny of pulsar wind physics in BW and RB contexts.

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### Appendix A

#### Particle Transport

In this appendix, we simplify Equation (15), which is repeated here for convenience:

$$\frac{\partial N_e}{\partial t} = -\mathbf{V} \cdot (\nabla N_e) + \kappa(\gamma_e) \nabla^2 N_e + \frac{\partial}{\partial \gamma_e} (\gamma_{\text{tot}, N_e}) - (\nabla \cdot \mathbf{V}) N_e + \mathcal{Q}. $$

We make a number of assumptions. First, we assume that $N_e$ has only colatitudinal and energy dependence, with $N_e \propto \mu = \cos \theta$ (i.e., to model the fact that we expect to have a high density of particles at the shock nose, but that this density decreases as particles move along the shock tangent and eventually escapes from the last zone). The conventional definition of colatitude is such that it increases counterclockwise (see Figure 2). Our definition is such that $\theta = -\theta'$, with $\theta'$ the standard definition of colatitude. This implies (neglecting $\partial^2 N_e / \partial \mu^2$, since $N_e$ is assumed to be linear in $\mu$)

$$\frac{\partial N_e}{\partial \theta'} = -\sin \theta \frac{\partial N_e}{\partial \mu} \frac{\partial \theta}{\partial \theta'} = \sin \theta \frac{N_e}{\mu} = \tan \theta N_e,$$

$$\frac{\partial^2 N_e}{\partial (\theta')^2} = \frac{\partial}{\partial \mu} \left( \frac{\partial N_e}{\partial \mu} \frac{\partial \theta}{\partial \theta'} \right)^2 = \frac{\gamma e_{\text{diff}}}{\gamma_{\text{tot}}} = -N_e.$$  \hspace{1cm} (A1)

We assume that the bulk flow only has a colatitudinal dependence $\mathbf{V} = c \beta(\theta) \hat{\mathbf{r}}$. The first term on the right-hand side of Equation (15) then becomes (with $\theta$ increasing clockwise)

$$-\mathbf{V} \cdot (\nabla N_e) = -V(\theta) \hat{\mathbf{r}} \cdot \left( \frac{1}{R_0} \frac{\partial N_e(\theta)}{\partial \theta} \right) \hat{\mathbf{r}} = -\frac{c \beta(\theta)}{R_0} \tan \theta N_e \equiv -\frac{N_e}{\tau_1},$$

with $\gamma_1 = R_0 / (c \beta(\theta) \tan \theta)$.

The second term on the right-hand side of Equation (15) becomes (again neglecting $\partial^2 N_e / \partial \mu^2$)

$$\kappa(\gamma_e) \nabla^2 N_e = \frac{\kappa}{R_0^2 \sin \theta} \left( \cos \theta \frac{\partial N_e}{\partial \theta'} + \sin \theta \frac{\partial^2 N_e}{\partial (\theta')^2} \right).$$  \hspace{1cm} (A2)

$$= -\frac{\kappa}{R_0^2 \sin \theta} (\cos \theta (\tan \theta N_e) - \sin \theta (N_e)),$$

$$= -\frac{2 \kappa N_e}{R_0^2 \sin \theta} \equiv -\frac{N_e}{\tau_{\text{diff}}}.$$  \hspace{1cm} (A3)
with $\tau_{\text{diff}} = R_2^2/2\kappa$.

Let us assume that the particles lose energy owing to adiabatic and radiation losses: $\dot{\gamma}_{\text{e, tot}} = \dot{\gamma}_{\text{e, ad}} + \dot{\gamma}_{\text{e, rad}}$. The adiabatic loss (cooling) rate is given by $\dot{\gamma}_{\text{e, ad}} = -\gamma_e (\mathbf{\nabla} \cdot \mathbf{V})/3$. Splitting the third term on the right-hand side of Equation (15), we find

$$\frac{\partial}{\partial \gamma_e} (\gamma_{e, \text{ad}} N_e) = -\frac{1}{3} (\mathbf{\nabla} \cdot \mathbf{V}) N_e \equiv -\frac{N_e}{\tau_{\text{ad}}}.$$  (A4)

Using

$$\mathbf{\nabla} \cdot \mathbf{V} = \frac{1}{R_0 \sin \theta'} \frac{\partial}{\partial \theta'} (c/\beta(\theta) \sin \theta') = -\frac{1}{R_0 \sin \theta} \frac{\partial}{\partial \theta} (c/\beta(\theta) \sin \theta) = -\frac{c}{R_0 \sin \theta} \left[ \frac{\partial \beta}{\partial \theta} \sin \theta + \beta \cos \theta \right],$$

we obtain

$$\tau_{\text{ad}} = \frac{3 R_0}{c} \left[ \frac{\partial \beta}{\partial \theta} + \beta \cot \theta \right]^{-1}. $$  (A5)

The penultimate term on the right-hand side of Equation (15) is just three times larger in absolute magnitude than the adiabatic loss term. We can thus replace this with $-N_e/\tau_2$ and define $\tau_2 = \tau_{\text{ad}}/3$. To be explicit, we kept $\tau_{\text{ad}}$ and $\tau_2$ separate, but these could be combined into one term. Furthermore, the adiabatic losses and convection terms all depend on there being a nonzero bulk flow of particles and thus share the same basic physical origin.

Let us next assume that $N_e \dot{\gamma}_{\text{e, rad}} \propto \gamma_e^{-p}$, such that

$$\frac{\partial}{\partial \gamma_e} (\gamma_{e, \text{rad}} N_e) = -p N_e \gamma_{e, \text{rad}} \frac{\gamma_{e, \text{rad}}}{\gamma_e} \equiv -\frac{N_e}{\tau_{\text{rad}}}.$$  (A6)

If $p \approx 1$, we have $\tau_{\text{rad}} = \gamma_e/\gamma_{e, \text{rad}}$.

### Appendix B

**Derivation of the Expression for the Photon Number Density in the Comoving Frame**

The total soft-photon energy density in the comoving frame is (Böttcher et al. 2012)

$$u_{e, \text{em}} = \int_0^\infty d\nu \int_4^\infty d\Omega_{e, \text{em}} \frac{u_{e, \text{em}}}{\Gamma^4(1 + \beta \mu_{e, \text{em}})}.$$  (B1)

where $\nu$ is the photon frequency, $d\Omega$ the solid angle, $\Gamma$ the bulk flow Lorentz factor, $\beta_\Gamma = \nu/c$ the normalized bulk flow speed, $\mu = n \cdot u$ the cosine of the angle between the flow and observer directions (see Equation (28)), $u_{e, \text{em}}(\Omega)$ the energy density per solid angle and frequency interval, $u_\nu$ the energy density per frequency interval, and $u = \int_0^\infty d\nu u_\nu$ the total energy density; the superscripts “em” and “rec” respectively refer to the comoving (emission) and observer (lab) frame, previously indicated by primed and unprimed quantities in Section 2.6, to conform to the usage of Böttcher et al. (2012).

We assume that the radiation field in the observer (rec) frame is isotropic,

$$u_{\nu, \text{rec}}(\Omega_{\text{rec}}) = \frac{u_{\nu, \text{rec}}}{4\pi}.$$  (B2)

Noting that $\delta = \Gamma(1 - \beta \mu_{\text{rec}})^{-1} = \Gamma(1 + \beta \mu_{\text{em}})$, where $\mu_{\text{rec}} = \cos(\alpha - \pi/2)$ is the angle between the velocity of a particle (tangent to the shock) and the direction of a soft photon originating from the companion (taken to be along the direction pointing from the companion center to a particular spatial zone or particle position), with $\alpha = \pi - \lambda - \theta$ (see Figure 1), we have

$$u_{e, \text{em}} = \frac{u_{\nu, \text{rec}}}{4\pi} \int d\Omega_{\nu, \text{rec}} \left( \frac{d\Omega_{e, \text{em}}}{d\Omega_{\text{rec}}} \right)^{-1} \delta^{-4}.$$  (B3)

We assume that the isotropic BB emission is beamed into a cone of width $\mu_e = \cos \theta_e$ in the observer frame (see Figure 15):

$$u_{e, \text{em}} = \frac{1}{2} u_{\nu, \text{rec}} \int_{\mu_e}^1 d\mu_{\text{rec}} (\delta^2 \delta^{-4}$$  (B4)

$$= \frac{1}{2} \Gamma^2 u_{\nu, \text{rec}} \int_{\mu_e}^1 d\mu_{\text{rec}} (1 - \beta \mu_{\text{rec}})^2.$$  (B5)
Assuming $\mu \approx \mu_c$ due to the beamed nature of the BB emission (as “seen” by the particle) and normalizing this expression using the BB energy density in the observer frame, we can evaluate the above integral to finally obtain an expression for the total soft-photon density in the comoving frame (divided by a factor of 2 since we are only assuming radiation from the facing side of the companion):

$$u_{sp} \approx \frac{2\sigma_{SB} T^4}{c} \frac{1 - \mu_c}{[1 - (1 - \beta \mu_{sec})^2]}.$$  \hspace{1cm} (B6)

The expression for the spectral photon number density in the comoving frame can then be written as

$$n_s(\epsilon_s) = \frac{30\sigma_{SB} \hbar}{(\pi k_B)^4} (1 - \mu_c) \epsilon_s^2 \delta^{-2} \left[ \exp \left( \frac{\epsilon_s}{k_B T_{\delta}} \right) - 1 \right]^{-1}.$$  \hspace{1cm} (B7)

A Taylor expansion of the $1 - \mu_c$ term yields

$$n_s(\epsilon_s) = \frac{30\sigma_{SB} \hbar}{(\pi k_B)^4} \left( \frac{R_*}{R_{dh}} \right)^2 \epsilon_s^2 \delta^{-2} \left[ \exp \left( \frac{\epsilon_s}{k_B T_{\delta}} \right) - 1 \right]^{-1},$$  \hspace{1cm} (B8)

where $\hbar$ is Planck’s constant and $\epsilon_s$ is the photon energy. This final expression is similar to the expression for the photon number density used in Dubus et al. (2015).

## Appendix C

### Estimate of Synchrotron Self-Compton Flux Level

In this appendix, we show that the SSC flux is expected to be orders of magnitude lower than the IC flux, and this emission process can therefore safely be neglected.

Let us write the normalized photon energy as

$$\chi \equiv \frac{h \nu}{m_e c^2}.$$  \hspace{1cm} (C1)

To make a rough estimate of the expected SSC flux, we focus on SR photons with energies of $E_{SR} = 100 \text{ MeV} = 1.6 \times 10^{-4} \text{ erg}$, which means that $\chi \sim 200 \gg 1$ and implies that we are in the Klein–Nishina regime. The expected SSC flux is approximated by

$$(\nu F_\nu)_{SCC} \approx \epsilon n_{ph} \frac{\sigma_{KN}}{4\pi d^2} \frac{dN}{dE_c}. \hspace{1cm} (C2)$$

where (Blumenthal & Gould 1970)

$$\sigma_{KN} \approx \frac{3}{8} \sigma_T \left[ \ln(2\chi) + \frac{1}{2} \left( \frac{1}{\chi} \right) \right] \sim 8 \times 10^{-27} \text{ cm}^2.$$  \hspace{1cm} (C3)
If we then consider a pulsar with a spin-down luminosity of $\dot{E}_{\text{rot}} = 10^{34} \text{ erg s}^{-1}$ and very conservatively assume that 50% of $\dot{E}_{\text{rot}}$ is converted into SR (in reality this number is much lower in our model fits, typically a few fractions of a percent, but this depends on the assumed parameters that determine the particle transport), we can approximate the SR luminosity as being $L_{\text{SR}} \lesssim 5 \times 10^{33} \text{ erg s}^{-1}$. Assuming the size of the emission region (thus assuming the emission to come primarily from the first spatial zone) to be $A \sim 2\pi R_0^2 (1 - \mu_0) \sim 10^{18} \text{ cm}^2$, one can then estimate the photon number density as

$$n_{\text{ph}} \lesssim \frac{L_{\text{SR}}}{cA\varepsilon_{\text{SR}}} \sim 10^9 \text{ ph cm}^{-3}. \quad (C4)$$

Consider a source at a distance of $d = 2 \text{ kpc}$. Estimating the steady-state particle spectrum to be roughly $dN/dE_e \sim 10^{33} \text{ erg}^{-1}$ and assuming the typical electron energy to be $\gamma_e \approx 2 \times 10^6$, we estimate the SSC flux as (using a cross section $\sigma_{\text{KN}} \approx 8 \times 10^{-27} \text{ cm}^2$ as in Equation (C3))

$$(\nu F_\nu)_{\text{SSC}} \lesssim 10^{-18} \text{ erg s}^{-1} \text{ cm}^{-2}. \quad (C5)$$

Next, we estimate the IC flux by considering only the scattering of soft photons produced by the companion, with an energy $E_{\text{soft}} \sim 2.7kT \approx 1 \text{ eV} \sim 2 \times 10^{-12} \text{ erg}$, which means that $\chi \ll 1$, implying that we are now in the Thomson regime. The expected IC flux is approximated by

$$(\nu F_\nu)_{\text{IC}} \approx c n_{\text{ph}} \frac{\sigma_{\gamma \gamma}}{4\pi d^2} \frac{dN}{dE_e} E_{IC}, \quad (C6)$$

where $n_{\text{ph}}$ is now given by

$$n_{\text{ph}} \sim \frac{n_{\text{BB}}}{E_{\text{soft}}} \sim \frac{1}{E_{\text{soft}}} 4\pi \sigma_{\text{SR}} T^4 \left( \frac{R_e}{r} \right)^2 \sim 3 \times 10^{11} \text{ ph cm}^{-3}, \quad (C7)$$

using $T \sim 6 \times 10^3 \text{ K}$, $R_e \sim 8 \times 10^{10} \text{ cm}$, and $r = a - R_0 \sim 2 \times 10^{11} \text{ cm}$. Now using $\gamma_e = 2 \times 10^6$ and $E_{\text{IC}} \sim \gamma_e^2 E_{\text{soft}} = 10 \text{ erg}$, we estimate the IC flux to be

$$(\nu F_\nu)_{\text{IC}} \sim 10^{-13} \text{ erg s}^{-1} \text{ cm}^{-2}. \quad (C8)$$

Upon comparing Equations (C5) and (C8), it is clear that the SSC flux is negligible compared to the IC flux (as can also be seen from more detailed modeling in Section 3).

### Appendix D

#### $\gamma \gamma$ Absorption

TeV photons produced by IC scattering of the companion’s soft-photon field may be may be attenuated by the same (optical) photon field. The angle-dependent two-photon pair production (Breit-Wheeler) cross section peaks at $\varepsilon_1 \varepsilon_2 (1 - \mu_{12})/2 \approx 2$, where $\varepsilon_{1,2}$ are the photon energies and $\mu_{12}$ is the cosine of the interaction angle in the lab frame. Correspondingly, optical photons of a few eV preferentially interact with TeV photons for nearly all interaction angles relevant in this work. The optical depth for absorption of TeV photons is maximized for propagation directions directed toward the observer of photons traveling near pulsar superior conjunction. Such opacity is not all that relevant for Doppler-boosted photons that arise from wing locales of intrabinary shocks that surround the pulsar, as is the case for most of the sources in this work. Nevertheless, it is instructive to estimate the influence of $\gamma \gamma$ absorption here.

The characteristic optical depth at the peak of the cross section is $\tau_{\gamma \gamma} \sim n_\gamma \sigma_{\gamma \gamma} n_\gamma$, where $\sigma_{\gamma \gamma} \sim \sigma_{\gamma \gamma}$ and $n_\gamma$ is the characteristic photon number density. Here we may estimate $n_\gamma \sim \zeta(3)(\Theta' / \lambda)^3 (R_e / a)^2 / \pi^2$, where $\Theta' = k_B T / (m_e c^2)$ and $\lambda$ is the reduced Compton wavelength. Then (for the most optimal case),

$$\tau_{\gamma \gamma} \sim \zeta(3) \left( \frac{8\zeta(3)}{3\pi} \right) \left( \frac{R_e}{a} \right)^2 \left( \frac{\Theta'}{\lambda} \right)^3$$

$$\sim \begin{cases} 
0.03 \left( \frac{T}{6 \times 10^3 \text{ K}} \right)^3 \left( \frac{a}{2 \times 10^{11} \text{ cm}} \right)^2 \left( \frac{R_e}{a} \right)^2 & \text{J1723–2837 – like RB systems} \\
0.05 \left( \frac{T}{3 \times 10^4 \text{ K}} \right)^3 \left( \frac{a}{5 \times 10^{10} \text{ cm}} \right)^2 \left( \frac{R_e}{a} \right)^2 & \text{J1311–3430 – like BW systems} \end{cases} \quad (D1)$$

The above estimate is for head-on photon angles and thus mostly operative for orbital phases near pulsar superior conjunction. For other phases, $\tau_{\gamma \gamma}$ is much smaller. Thus, we see that $\gamma \gamma$ absorption ($\propto \exp(-\tau_{\gamma \gamma})$) may have at most a few percent influence on the TeV flux in both RBs and hot systems such as PSR J1311–3430, except in exceptionally flaring states, owing to the $T^3$ scaling of the photon number density in those cases.
Appendix E

Transport Timescales

In Figure 16 we plot relevant timescales for the first, middle, and final spatial zones of the shock for PSR J1723–2837. These timescales were described in some detail in Section 3, and the plots and discussion are repeated here for convenience and as a basis for comparison with results of other sources. The IC timescale (blue line) is much shorter for the first zone than for the last, given the reduction of soft-photon energy density with distance (increasing zone). The SR timescale (red line) is the same, given the constant shock magnetic field. The green line represents the effective radiation loss timescale \( \tau_{\text{rad}} = \tau_{\text{SR}} + \tau_{\text{RC}}^{-1} \), while the yellow line indicates the diffusion timescale. This is similar for zones of similar sizes (and a constant magnetic field), as is the adiabatic timescale (purple line). Thus, the spatial (adiabatic and diffusion) timescales dominate the radiation ones in this case. As mentioned before, on the one hand, convection leads to relatively short residence times of particles in particular zones, and thus reduced emission, but on the other hand, a large bulk flow speed increases the Doppler factor in each zone, compensating for this effect. The effective timescale (black line) finds the shortest timescale between all the different processes.

In Figure 17 we plot relevant timescales for the first, middle, and final spatial zones for PSR J2339–0533. We note that the IC timescale (blue line) is only slightly shorter for the first zone than for the last. This is because we model this source with a smaller shock radius than for PSR J1723–2837, so the particles are removed farther from the companion source of soft photons. For larger-distance zones, the particles are even farther away from the source of soft photons, giving a reduction of soft-photon energy density and thus an increase in the IC loss timescale. We see that the spatial (adiabatic and diffusion) timescales dominate the radiation ones for this source as well. The other timescales behave in a similar way to those for PSR J1723–2837.

In Figure 18 we plot relevant timescales for the first, middle, and final spatial zones for PSR J1311–3430. We note that the IC timescale (blue line) is much shorter for the first zone than for the last. Note that we have modeled this source with the shock wrapping around the pulsar. The spatial timescales again dominate the radiation ones, and this seems to be a characteristic for the RB systems we have modeled. The other timescales behave in a similar way to those described for PSR J1723–2837.

In Figure 19 we plot relevant timescales for the first, middle, and final spatial zones for PSR B1957+20. The IC timescale (blue line) is much longer for the first zone than for the last. This is because the shock is wrapping around the companion for this BW system and thus the shock is close to the soft-photon source for all zones. The reason for the shorter timescales for later zones is that the bulk flow speed increases for each consecutive zone, increasing the Doppler factor and thus the comoving soft-photon number density in these zones. The timescale for IC is even shorter for the higher inclination angle because in this case \( R_0 \) is relatively smaller, so the shock is even closer to the companion and thus closer to the soft-photon field. The radiation timescales now dominate the spatial ones. This behavior may thus be a distinguishing feature between BWs and RBs.

![Figure 16. Timescale plots for PSR J1723–2837 depicting the (a) first, (b) middle, and (c) last spatial zones.](image1)

![Figure 17. Timescale plots for PSR J2339–0533 depicting the (a) first, (b) middle, and (c) last spatial zones.](image2)
Figure 18. Timescale plots for the quiescent state of PSR J1311–3430 depicting the (a) first, (b) middle, and (c) last spatial zones.

Figure 19. Timescale plots for PSR B1957+20 depicting the (a) first, (b) middle, and (c) last spatial zones for inclination angle $i = 64^\circ$. Similarly, panels (d)–(f) are for the alternative fit with inclination angle of $i = 84^\circ$.

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22
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