ADIABATIC SURFACES FROM THE LATTICE: EXCITED GLUONIC POTENTIALS

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ABSTRACT

I describe the results from lattice studies of probing the vacuum with static colour sources. Topics include

• Twang the flux tube: string models and hybrid mesons.
• Charge up the flux tube: adjoint and sextet sources.
• Several sources: how does the colour flux get arranged?

1. Introduction

The non-perturbative sector of QCD is a very interesting area of study. Experimental investigation is difficult since it is not possible to make many controlled changes to explore the response. Lattice QCD allows a much more comprehensive set of questions to be asked. In particular, we can vary the quark masses. Moreover the simplifying approximation of removing quark contributions to the vacuum (the so-called quenched approximation) allows phenomenological models to be tested in this context also.

The origin of string models in particle physics is from the hadronic string that grew out of dual models. It is worthwhile to explore the region of applicability of hadronic string-like models. The lattice study of the response of the vacuum to static colour sources is an ideal testing ground. For a pair of sources in the fundamental representation, there has been much progress in extracting the continuum spectrum from the lattice. As well as the ground state which has been well known for a long time, there are now comprehensive results for excited gluonic levels. I compare these lattice results with string models and also discuss the small separation limit.

Hybrid mesons are states beyond the quark model in which the gluonic degrees of freedom contribute in a non-trivial way. One of the clearest ways to study this area theoretically is from the excited gluonic potentials. This gives predictions for hybrid meson spectroscopy with heavy quarks. A brief aside will cover the situation with hybrid mesons containing light quarks: this is of topical relevance since there are new experimental data on these hybrid mesons.

I cover a range of other situations which have been studied using static sources. These include the bound state spectrum of a single adjoint source (the gluelump), the potential between adjoint sources and between sextet sources and string-breaking.
Some preliminary results are described of relevance to baryons: with 3 static quarks. In this case one can compare models with minimal flux length to a simple sum of two-body interactions. Leading to a study of hadron-hadron interactions, I present a summary of results for the energy of four static sources \((QQ\bar{Q}\bar{Q})\) and some new results for the interaction energy between two B mesons.

It is worth recalling the weakness as well as the strength of the lattice approach to QCD. Any quantity which can be expressed as a vacuum expectation value of fields can be extracted straightforwardly in lattice studies. Thus masses and matrix elements of operators can be determined. What is not so easy is to explore hadronic decays. This is difficult because the lattice, using Euclidean time, has no concept of in and out states. About the only feasible strategies are to evaluate the mixing between states of the same energy - so giving some information on on-shell hadronic decay matrices, or to make a model dependent analysis of lattice results.

2. **Gluonic Excitations between Static Quarks in the Fundamental Representation**

In a lattice regularisation, static sources in the fundamental colour representation are readily implemented. In the spirit of the heavy quark effective theory, they correspond to heavy quarks or anti-quarks. Thus in the experimental realm they give insight into \(b\) quark physics. Because of the lattice regularisation, the self energy of these static sources is unphysical. Energy differences, however, are physical and it is possible to obtain the continuum limit (ie extrapolate the lattice spacing to zero).

Before focussing on excitations of the colour flux between static quarks, I recall briefly the great insights that have come from the ground state potential itself. Some of these topics are also covered by other presentations. The ground-state potential energy \(V(R)\) between a static quark and antiquark at separation \(R\) has been measured accurately on the lattice: the continuum limit can be obtained with precision in the quenched approximation when the quark contributions to the vacuum are neglected. These results enable discussion of confinement at large \(R\), of heavy quark bound states and provide one way to determine the running coupling non-perturbatively. The distribution of colour electric and magnetic fields in the flux tube have also been determined from a lattice. This again has impact on models of confinement. Another insight obtained from the ground state potential is its spin dependence - in particular for the long range confining component which proves to be ‘scalar’ exchange.

We now consider gluonic excitations of the potential between static quarks: they will play a rôle in hybrid mesons, for example. An analysis of the colour representation of the quark and antiquark is not useful when they are at different space positions since their combined colour is not gauge invariant. A better criterion is to focus on the spatial symmetry of the gluonic flux. The ground state distribution is symmetric - it is rotationally symmetric about the separation axis, and is symmetric about end to end inversion. As well as this symmetric ground state of the colour flux between two static quarks, there will be excited states with different symmetries.

These can be classified, in the continuum, by the group \(D_{\infty h}\). This classification is well known in molecular applications. The representations are labelled for \(J_z = 1, 2, 3\) as \(\Sigma, \Pi, \Delta\) respectively where \(z\) is the axis of separation of the fundamental sources.
which are $R$ apart. The $J_z \neq 0$ representations are two-dimensional. The other labels of the representations are $g, u$ for $CP = \pm 1$ and, for the one-dimensional $\Sigma$ states only, an additional $\pm$ label indicating whether the state is even/odd under reflection in the plane containing the $z$-axis.

Note that this discussion of excited gluonic configurations between heavy quarks is very similar to that of electronic wavefunctions in a di-atomic molecule. Indeed the oxygen molecule has contributions from non-trivial representations of $D_{\infty h}$.

On a lattice with the separation axis along a lattice axis, the relevant symmetry group is $D_{4h}$ and its representations are labelled such that $A, E, \text{ and } B$ are related to the $J_z = 0, 1 \text{ and } 2$ excitations., etc. The lattice data for the lowest gluonic excited states have been determined by the Liverpool group using the Wilson gauge action for SU(2) colour and for SU(3) colour. The lowest non-trivial excitation was found to be in the $E_u$ $(\Pi_u)$ representation (corresponding to flux states from an operator which is the difference of U-shaped paths from quark to antiquark of the form $\Uparrow - \Downarrow$).

Although on a lattice, the self energy of the static sources is unphysical, the energy differences between excited gluonic potentials and the ground state are physical and have a continuum limit. An illustration of this first gluonic excitation is shown in fig.[1].

Recently, a comprehensive study of this area has been undertaken using an asymmetric lattice.[10] The advantage of using a relatively large spatial lattice spacing is that large $R$ can be reached, while a smaller temporal lattice spacing allows a more precise determination of the energy values. The disadvantage of a coarse spatial lattice spacing is that lattice corrections are increased but this can be offset by using improved lattice discretisations with smaller corrections than the Wilson lattice action which has order $a^2$. Recent results extend out to $R$-values of 4fm which is most impressive. By using several different lattice spacings and also by considering source separations in other directions than the lattice axes, it is possible to extract continuum results for the excited gluonic potentials. Because of the lattice dependence of the self-energy, only energy differences can be obtained.

In order to set the scale in lattice calculations, it is now conventional to use $r_0$. This is defined[11] in terms of the force between static sources implicitly as $r_0^2 f(r_0) = 1.65$ where $f(r) = dv(r)/dr$. From relating the interquark potential to that needed phenomenologically for heavy quark bound states, we know that $r_0 \approx 0.5$fm. Since this potential can be very accurately measured on a lattice, this is an appropriate observable to use to set the energy scale for comparisons of lattice results. Thus the lattice excited potential $\tilde{V}(r)$ expressed as $R_0(\tilde{V}(R) - V(R_0))$ where $R_0 = r_0/a$ is dimensionless and, in the limit of small lattice spacing, will be equal to the continuum expression $r_0(\tilde{v}(r) - v(r_0))$ up to corrections of order $a^2$ for the Wilson lattice discretisation. This is thus a suitable way to present and compare results from different lattice studies. We present some of the recent comprehensive results[12] in fig. 2.

It is worth exploring what is to be expected for these gluonic excitations between static sources at separation $R$.

**Stability:** The most obvious criterion is that if the excited gluonic energy lies below the energy of all states with the same symmetry made from a glueball together with a lower-lying potential, then the excited gluonic potential will be stable, at least
Fig. 1. Potentials \( V(R) \) between static quarks at separation \( R \) (in units of \( r_0 \approx 0.5 \text{fm} \)) for the ground state (\( \square \) and *) and for the \( \Pi_u(E_u) \) symmetry which corresponds to the first excited state of the gluonic flux (octagons and diamonds). Results from quenched calculations \(^9\) with SU(3) colour are shown by symbols corresponding to different lattice spacings. The \( R = 0 \) data points \(^19\) for the excited state are discussed in the text. For the ground state potential the continuous curve is an interpolation of the lattice data while the dotted curve with enhanced Coulomb term fits the spectrum and yields the masses shown. For the excited gluonic potential, the continuous curve shows the string excitation expression of eq. 1. The hadronic bound states of \( b \) quarks in these potentials are shown, including the lightest hybrid level in the excited gluonic potential.

in the quenched approximation. Since the lightest glueballs are found, in quenched QCD, to have energies \(^{13,14} \) \( m(0^{++})r_0 = 4.33(5) \) and \( m(2^{++})r_0 = 6.0(6) \), this criterion is satisfied in most cases, especially at larger \( R \).

\textbf{String models:} We consider now simple phenomenological models for the excited gluonic potentials: one such model is the hadronic string. A bosonic string fixed at two points \( R \) apart and with no intrinsic width will give a parameter-free prediction although the tachyonic problem implies that this prediction will be unphysical for the lowest energy excitation at small string length \( R \). The lattice excited potentials can be compared with this bosonic string model provided an appropriate expression \(^7\) is used.
Fig. 2. Potentials $E(r)$ between static quarks at separation $r$ (in units of $2r_0 \approx 1.0\text{fm}$) for different symmetry representations from Juge et al. The two lines shown in each case represent an estimate of the error on the determination of the continuum value.

for excited level $n$:

$$V_n(R) = \left(\sigma^2 R^2 - \frac{\pi \sigma}{6} + 2\pi n\sigma\right)^\frac{1}{2}. \tag{1}$$

If one uses this expression for the energy differences of excited levels from the ground state, there are no free parameters since the string tension $\sigma$ is determined by the ground state potential with $n = 0$. At large $R$, this expression has the simple consequence that gluonic excitations are at multiples of $\pi/R$ in energy higher than the
ground state. The relationship between the excitation symmetry and \( n \) is that the \( n = 0, 1 \) levels are given by \( \Sigma^+_g, \Pi_u \), while \( n = 2 \) excites \( \Sigma^+_{g'}, \Pi_g \) and \( \Delta_g \), and \( n = 3 \) excites \( \Sigma^\pm_u, \Pi'_{u} \), and \( \Delta_{u} \).

In order to make the string expression well behaved at small \( R \), the string has to be regulated in some phenomenological way related to its intrinsic width - for instance as in the Isgur-Paton flux tube model or in the Hungarian bag model. Nevertheless, the expression of eq. 1 was found to agree quite well with lattice spectra obtained for \( R \) above 0.5fm for both SU(2) and SU(3). Indeed the curve drawn in fig. 1 for the \( \Pi_u \) excited potential is just that given by the expression of eq. 1 for the difference from the ground state (\( \Sigma^+_g \)). Comparison with the larger \( R \) lattice data now available again gives good qualitative agreement at moderate \( R \) values of 1 to 2fm. The ordering at the largest \( R \) available (2 to 4fm) agrees very well with that from the string excitation approach but the energy differences agree less well than they do below 2fm.

This is surprising since the string model should be most reliable at the largest \( R \). However, it should be kept in mind that the systematic errors in the lattice determinations are largest at large \( R \) - coming both from the difficulty of separating the many excited states of a given symmetry which lie close together and from possible finite lattice spacing effects since a coarse spatial lattice spacing is used to reach large \( R \). A further possible source of error is from the transverse size of the lattice which is assumed to be much larger than the string length \( R \) in the string model whereas it is comparable to \( R \) in some of the lattice analyses. So, it is possible that the systematic errors in the lattice results are somewhat underestimated. This comparison between the hadronic string model and the large \( R \) excited potentials is discussed in detail by Morningstar.

The limit \( R \to 0 \): At \( R = 0 \), the symmetry considerations are different and this imposes constraints on the excited gluonic spectra that help to explain some of the more significant departures from the string model. In the limit as \( R \to 0 \), the static source and anti-source will be at the same site and hence their colour can be combined in a gauge invariant way - creating an adjoint colour source and a singlet (glueball correlator). Pictorially this is shown in figure 3. This former situation has been studied previously: the gluelump is an adjoint source in the presence of a gluonic field - it is rotationally invariant and is described by \( J^{PC} \). Thus the Wilson loop correlation satisfies

\[
\lim_{R \to 0} W(R, t) = ce^{-M_{\text{gluelump}}t} + \frac{1}{3} c' e^{-M_{\text{glueball}}t}
\]

In the large \( t \) limit, the lighter of the two states will dominate the correlation function. In most cases of present interest, the gluelump state is lighter than the glueball. Thus, we can obtain a relationship between the gluonically-excited states of the generalised Wilson loop, in the limit \( R \to 0 \), and those measured in the gluelump spectrum. This relationship in the continuum is obtained by subducing the rotation group representations appropriate to the gluelump to the \( D_{\infty} \) representations appropriate to the generalised Wilson loop when \( R \neq 0 \). By subducing the irreducible representation of the gluelump with \( J^{PC} \) we will find \( D_{\infty} \) representations with \( J_z = -J, \ldots , J \); labels
Fig. 3. Relation of $R \to 0$ Wilson loop with adjoint source (gluelump) and glueball correlations

$g$, $u$ given by $CP$ and, for any $J_z = 0$ states an additional label given by $P(-1)^J$. These relationships are given in Table I.

Similar identities as $R \to 0$ also apply to the lattice discretisation. Then the $O_h$ representations appropriate for the gluelump can be subduced into the $D_{4h}$ representations appropriate for the generalised Wilson loop with $R \neq 0$. Thus, as has been emphasised previously, the ground state gluelump with $1^+-$ ($T_1^{+ -}$) implies that as $R \to 0$ there must be a degeneracy of the two-dimensional $\Pi_u$ state ($E_u$) and a $\Sigma_u^-$ state ($A_{1u}$). This explains why the $\Sigma_u^-$ state which is high lying at large $R$ (since it is a $n = 3$ string excitation) is seen in fig. 2 to decrease in energy rapidly at small $R$ since it has to be degenerate at $R = 0$ with the $\Pi_u$ state which is a $n = 1$ string excitation and so is lower lying in energy. The lowest lying gluelump states have been determined to be $1^+$, $1^-$ and $2^-$ with energy differences in units of $r_0$ of 0.93(2) and 1.44(3) from the $1^+-$ respectively. This implies that the $R = 0$ limit of the excited gluonic potentials should have degenerate states as given in Table I with energy differences as quoted above. These considerations explain why the $\Sigma_u^-$, which is a $n = 4$ string level, also departs significantly (see fig. 2) from the string-like expectation since it has to be degenerate at $R = 0$ with the $n = 2$ string level $\Delta_g$.

One gluon exchange: Although the above group-theoretical identities are a good guide to the behaviour of the excited gluonic potentials at small $R$, the limit as $R \to 0$ of the excited gluonic potential is not trivial to extract from lattice data with $R = a, R = 2a, \ldots$. A guide is to consider the gluon exchange contributions perturbatively. One way to investigate this is to consider the self energies of the contributions: $2E_F$ at $R \neq 0$ and $E_A$ at $R = 0$, where $F$ and $A$ label fundamental and adjoint colours. Since, to lowest order, $E_A = 9E_F/4$ for SU(3) of colour, there will be a mismatch and one might expect the energy to increase as $R \to 0$ since the adjoint self-energy is larger. Another way to investigate this, is to imagine that as $R \approx 0$, there is a gluonic field in the adjoint representation, so that the heavy quark and anti-quark are also in an adjoint and hence will have a Coulombic interaction energy given by $-1/8$ of the Coulombic energy between a quark and antiquark in the fundamental representation (which is approximately given by $-0.25/R$ in lattice quenched studies). This again suggests that the excited gluonic potentials should rise as $R \to 0$, here as $0.03/R$.

Lattice data for the $E_u$ representation for small $R$ from SU(2) colour studies at $\beta = 2.4$ with values of $aV_{Eu}(R) = 1.31, 1.32$ and 1.38 for $R = 3a$, $2a$ and $a$ respectively.
do qualitatively support these estimates and are consistent with a limit as $R \to 0$ which agrees with the lattice glue lump energy\(^\text{20}\) of $aE_{\text{gluelump}} = 1.50$. Also in fig. [1], we show the $R = 0$ point from the SU(3) glue lump analysis\(^\text{3}\) which fits in well with the above considerations.

Table 1. Connection between glue lump and two-body potential as $R \to 0$.

| Gluelump $J^{PC}$ | Two-body potential states |
|-------------------|---------------------------|
| $1^{+-}$          | $\Sigma^-_u, \Pi_u$      |
| $1^{--}$          | $\Sigma^+_g, \Pi_g$      |
| $2^{--}$          | $\Sigma^-_g, \Pi_g, \Delta_g$ |

It is possible to measure the distribution of the colour flux around an excited gluonic state. Results have been presented\(^\text{21}\) for the $\Pi_u$ and for the first excited $\Sigma^+_g$ states. The excited states show a wider transverse distribution than for the ground state, as would be expected. There is some evidence for a node in the transverse distribution of the first excited $\Sigma^+_g$ state.

2.1. Hybrid Mesons with Heavy Quarks

The potential $V(R)$ between static sources obtained from the lattice (it is approximately of the form $V(R) = -e/R + \sigma R$) can be used to determine the spectrum of $b\bar{b}$ mesons by solving Schrödinger’s equation since the motion is reasonably approximated as non-relativistic. The result from quenched lattices is similar to the experimental $\Upsilon$ spectrum: see fig. [1]. The main difference is that the Coulombic part ($e$) is effectively too small (0.28 rather than 0.4). This produces\(^\text{9}\) a ratio of mass differences $(1P - 1S)/(2S - 1S)$ of 0.71 to be compared with the experimental ratio of 0.78. This difference is understandable as a consequence of the Coulombic force at short distances which would be increased by $33/(33 - 2N_f)$ in perturbation theory in full QCD compared to quenched QCD. Indeed lattice studies\(^\text{22}\) including sea-quark effects in the vacuum do see evidence for an increase of the Coulombic component of the potential, although they have not yet reached sea-quark masses as small as those in nature.

Consider now the hadrons corresponding to bound states of the excited gluonic potentials. The lightest such gluonic excitation ($\Pi_u$) corresponds to a component of angular momentum of one unit along the quark-antiquark axis. Since the energy scale associated with the gluonic excitation is much larger than the energy scale associated with orbital or radial excitations, it is a reasonable approximation to use the adiabatic approximation. Thus one solves for the spectrum of hybrid mesons in the excited potential using the Schrödinger equation. For those excitations with a non-zero angular momentum $J_z$ about the separation axis, an extra centrifugal term has to be added. The spatial wave function necessarily has non-zero angular
momentum and corresponds to $L^{PC} = 1^{+-}$ and $1^{-+}$. Combining with the quark and antiquark spins then yields a set of 8 degenerate hybrid meson states with $J^{PC} = 1^{--}$, $0^{-+}$, $1^{++}$, $0^{++}$, $1^{+-}$, $0^{+-}$, $2^{--}$ respectively. These contain the spin-exotic states with $J^{PC} = 1^{++}$, $0^{+-}$ and $2^{+-}$ which will be of special interest since they do not occur as $\bar{q}q$ states and so any experimental evidence for a resonance with these quantum numbers is a strong suggestion for the existence of a hybrid meson.

Since the lattice calculation of the ground state and hybrid masses allows a direct prediction for their difference, the result for this 8-fold degenerate hybrid level is illustrated in fig. and corresponds to masses of 10.81(25) GeV for $b\bar{b}$ and 4.19(15) GeV for $c\bar{c}$. Here the errors take into account the uncertainty in setting the ground state mass (i.e., $\Upsilon$ or $\psi$) using the quenched potential as described above. Using the recent comprehensive results confirms the results above and the preliminary values quoted for the lightest hybrid meson is 10.8 GeV for $b\bar{b}$ with no error estimate given.

The quenched lattice results show that the lightest hybrid mesons lie above the open $BB$ threshold and hence are likely to be relatively wide resonances. This could also be checked by comparing with quenched masses for the $B$ meson itself but at present there are quite large uncertainties on that mass determination. The very flat potential implies a very extended wavefunction: this has the implication that the wavefunction at the origin will be small, so hybrid vector states will be weakly produced from $e^+e^-$. An explicit evaluation of the hybrid wavefunction shows that it is very extended: being significant out to a radius of 1fm.

It would be useful to explore the splitting among the 8 degenerate $J^{PC}$ values obtained. This could come from different excitation energies in the $L^{PC} = 1^{+-}$ (magnetic) and $1^{-+}$ (pseudo-electric) gluonic excitations, from spin-orbit terms, as well as from mixing between hybrid states and $Q\bar{Q}$ mesons with non-exotic spin. One way to study this on a lattice is to use the NRQCD formulation which describes non-static heavy quarks which propagate non-relativistically. Preliminary results for hybrid excitations from several groups give at present similar results to those with the static approximation as described above, but future results may be more precise and able to measure splittings among different states.

### 2.2. Hybrid Mesons with Light Quarks

Unlike very heavy quarks, light quark propagation in the gluonic vacuum sample is very computationally intensive — involving inversion of huge ($10^7 \times 10^7$) sparse matrices. Current computer power is sufficient to study light quark physics thoroughly in the quenched approximation. The state of the art is the Japanese CPPACS Collaboration who are able to study a range of large lattices (up to about $64^4$) with a range of light quark masses. Qualitatively the meson and baryon spectrum of states made of light and strange quarks is reproduced with discrepancies of order 10% in the quenched approximation.

Here I will focus on hybrid mesons made from light quarks. In the quenched approximation, there will be no mixing involving spin-exotic hybrid mesons and so these are of special interest. The first study of this area was by the UKQCD Collaboration who used operators motivated by the static ($Q\bar{Q}$) studies referred to above. Using non-local operators, they studied all 8 $J^{PC}$ values coming from $L^{PC} = 1^{+-}$ and $1^{-+}$
The masses in GeV of states of $J^{PC}$ built from hybrid operators with strange quarks, spin-exotic ($\ast$) and non-exotic ($\asterm$). The dot-dashed lines are the mass values found for $s\bar{s}$ operators. Excitations. The resulting mass spectrum is shown in fig. 4 where the $J^{PC} = 1^{-+}$ state is seen to be the lightest spin-exotic state with a statistical significance of 1 standard deviation. The statistical error on the mass of this lightest spin-exotic meson is 7% but to take account of systematic errors from the lattice determination, a mass of 2000(200) MeV is quoted for the $s\bar{s}$ meson. Although not directly measured, the corresponding light quark meson would be expected to be around 120 MeV lighter. In view of the small statistical error, it seems unlikely that the $1^{-+}$ meson in the quenched approximation could lie as light as 1.4 GeV where there are experimental indications for such a state. There are also recent experimental claims for a second $1^{--}$ spin exotic meson at 1.6 GeV. Beyond the quenched approximation, there will be mixing between such a hybrid meson and $q\bar{q}q\bar{q}$ states such as $\eta\pi$ and this may be significant in explaining the apparent discrepancy between experiment and lattice.

One feature clearly seen in fig. 4 is that non spin-exotic mesons created by hybrid meson operators have masses which are very similar to those found when the states are created by $q\bar{q}$ operators. This suggests that there is a significant coupling between hybrid and $q\bar{q}$ mesons even in the quenched approximation. This lattice result is difficult to quantify in terms of decay widths but does imply that the $\pi(1800)$ is unlikely to be...
a pure hybrid, for example.

A second lattice group has also evaluated hybrid meson spectra from light quarks. They obtain masses with statistical and various systematic errors for the $1^{-+}$ state of $1970(90)(300)$ MeV, $2170(80)(100)(100)$ MeV and $4390(80)(200)$ MeV for $n\bar{n}$, $s\bar{s}$ and $c\bar{c}$ quarks respectively. For the $0^{+-}$ spin-exotic they have a noisier signal but evidence that it is heavier. They also explore mixing matrix elements between spin-exotic hybrid states and 4 quark operators.

3. The String Self-Energy

The expression for the energy of the ground state string mode ($n = 0$) will have a contribution from string fluctuation as well as the linear string tension component, namely \[ V(R) \approx \sigma R - \pi/(12R) \] as $R \to \infty$. In practice this $\pi/(12R)$ string fluctuation term for a bosonic string is very hard to disentangle from the Coulomb term $e/R$ in the potential. One way to get round this in lattice studies is to consider a hadronic string that encircles the periodic spatial boundaries of length $L$. Then there are no sources and hence no Coulomb component. The appropriate string fluctuation term in the energy of this state, called the torelon, is then given by $E(L) = \sigma L - \pi/(3L)$ as $L \to \infty$. By plotting $E(L)/L$ against $L$, see fig. 5, lattice studies have confirmed the presence of this string fluctuation term with its coefficient determined to agree with the expected value and to be determined quite accurately (as $(\pi/3) \pm 0.03$). This is impressive evidence that the hadronic string is a good model of the energy of the colour flux tube at large distances.

4. Excited Colour Sources

4.1. Adjoint Sources

Table 2. Energy differences from the ground ($1^{+-}$) state of gluelump states with $J^{PC}$.

| Excited State | $\Delta(M_{0}r_{0})$ | Energy (MeV) |
|---------------|----------------------|-------------|
| $1^{-+}$      | 0.933(18)            | 368(7)      |
| $2^{--}$      | 1.438(25)            | 584(10)     |
| $0^{++}$      | 2.771(72)            | 1092(28)    |

One adjoint source: The ‘hydrogen atom’ of pure gauge QCD has a static adjoint source with a gluon field making it an overall colour singlet. This is the gluelump whose spectrum has already been mentioned above. It has been explored in SU(2).
Fig. 5. Relationship between energy $E$ and length $L$ around the periodic spatial boundary for a torelon in lattice units. The continuous line shows the bosonic string result.

and SU(3) quenched lattice studies. The ‘magnetic gluon’ with $J^{PC} = 1^{-+}$ is found to be the ground state with the $1^{--}$ state as the first excited state. See table 2.

This spectrum would be accessible experimentally should a gluino exist which is sufficiently heavy for the static approximation to be valid and which lives long enough to allow spectroscopic energy levels to be determined.

As well as exploring the spectrum, the distribution of colour fields has been measured. They are found to extend out to a radius of about 0.5fm. This is consistent with the result (see below) that the adjoint string breaks into two gluelumps at a separation of around 1fm - that is when the two gluelumps are just touching. More detailed results give some evidence of the nature of the colour fields in the low-lying states: disk-like for the $1^{+-}$ and toroidal for the $1^{--}$.

Two adjoint sources: Another route to explore confinement is to measure on a lattice the potential energy between two static sources in the adjoint representation of the colour group. Here three regions are expected as $R$ varies. At small $R$ one gluon exchange should give a Coulombic region with strength proportional to $C_A/R$ where $C_A$ is the Casimir appropriate to the colour representation; at intermediate $R$ an effective string tension may be discernible, while at large $R$ the adjoint potential $V_A(R)$ must become independent of $R$ because each adjoint colour source can be screened by a
Fig. 6. The potentials in units of $r_0$ between static sources for quenched SU(3) with fundamental, adjoint and sextet sources. From quenched $\beta = 5.712^{3}24$ lattices, setting the scale using $r_0 \approx 0.5$fm. Also shown are $2M_{\text{gluelump}}$ and $2M(Q\bar{q})$ and the Casimir ratios $\frac{9}{4}V_{F}(R)$ and $\frac{5}{2}V_{F}(R)$.

Glumonic field. Indeed at large $R$, $V_{A} \rightarrow 2m_{\text{gluelump}}$ where the gluelump is the ground state hadron with a gluon field around a static adjoint source discussed above. Of interest to model builders is the adjoint potential at moderate $R$ values: the effective string tension region. Some theories of confinement have suggested that the effective adjoint string tension is given by the Casimir ratio (i.e. $\sigma_{A}/\sigma_{F} = C_{A}/C_{F}$): for instance from assuming that confinement in four-dimensional QCD acts like two-dimensional QCD - which has Casimir scaling exactly.

Precise data exist for $SU(2)$ of colour and they do show a region of linear rise, although with a slope less steep than that given by the Casimir ratio (namely $\frac{2}{3}V(R)$ where $V(R)$ is the fundamental colour source potential discussed previously). For $SU(3)$ of colour, I have determined the adjoint potential to large $R$ and the result is shown in fig. Again the Casimir ratio, here $\frac{4}{3}V(R)$, is an underestimate except at very small $R$ where the Coulomb contribution is expected to have this ratio. A rule that $V_{A}(R) \approx 2V(R)$ seems to be a better guide to this intermediate $R$ region.

The $R$ value where the two gluelump state becomes degenerate in energy with the measured adjoint potential is found for both 2 and 3 colours to be near 1.2fm. For
SU(2) colour, a variational basis\textsuperscript{20} with both colour-flux and two-gluelump basis states was used which allowed the mixing in this string breaking region to be studied more readily.

Lattice studies have also been made\textsuperscript{24} of the colour field distributions for the adjoint potential. This allows a comparison of the distribution of the adjoint colour field with the fundamental case. The longitudinal distribution at intermediate $r$-values shows a ratio of field strengths falling below the Casimir ratio so in agreement with the discussion above. The transverse distribution is surprisingly similar in the two cases - which may be interpreted as giving evidence that the two colour flux tubes involved in the adjoint case are attracted to each other - as in a type I superconductor.

4.2. Sextet Sources

For SU(3) of colour, I have determined the sextet potential to large $R$ and the result is also shown in fig. 6. Again the Casimir ratio (here $\frac{5}{2}V(R)$) is an underestimate except at very small $R$ where the Coulomb contribution is expected to have this ratio. At large $R$, the sextet source can be screened by gluons to form an effective fundamental source. Thus one expects the sextet potential to have a slope of the fundamental string tension $\sigma$ at large $R$. There is no sign of this, but the data are also consistent with a change of slope beyond 1.2fm.

4.3. Fundamental string breaking

In passing, we point out that the fundamental potential itself, in full QCD, will show string breaking at $R$-values where $V(R) > 2M(Q\bar{q})$ where by $Q\bar{q}$ we mean the ground state meson with a light quark $q$ bound to a static heavy quark $Q$. This comparison of the energies of the static potential and the two meson system can be explored in the quenched approximation - although there will be no actual mixing between these systems in that case. The situation is illustrated in fig. 6 and again breaking would be expected to occur at an $R$-value near 1.2fm where there is a degeneracy. Attempts to look for this mixing explicitly using $V(R)$ determined from full QCD simulations\textsuperscript{22,35} have not yet achieved sufficient precision to address this topic.

5. Multiple Sources

5.1. Baryonic Applications

Early attempts to study the potential with three static quarks were rather exploratory\textsuperscript{36} Indeed at moderate $R$ values, the baryonic potential was found to be given quite well by an average of the three two-body potentials. In contrast, in a flux tube approach, one would expect an important contribution from a three-body term with a flux tube of minimal length (ie with a star-like configuration): see fig. 7. This would give a different behaviour of the energy levels as a function of the position of the three sources than the case of a two-body sum. The above studies had rather small interquark separations of 0.5fm or less which may be an explanation for the apparent dominance of the two-body terms. Some recent preliminary result\textsuperscript{37} give some support to the three-body flux-tube picture for the special case of 3 sources at the corners of an equilateral triangle.
5.2. $Q\bar{Q}Q\bar{Q}$

To study the residual strong force between hadrons on a lattice is difficult since it is so much weaker than the colour force between quarks. For this reason, preliminary studies have either used static quarks and/or SU(2) of colour to simplify the computations. There has been extensive work\textsuperscript{38} using static quarks in SU(2) colour lattice studies - this has a bearing on the meson-meson force. Of special interest is whether the measured 4-body potential can be expressed as a sum of 2-body components. It turns out that a description in terms of the mixing of different 2-body assignments as shown in fig. \textsuperscript{8} is possible but that the mixing coefficient is geometry-dependent. In other words there is a four-body force.

As well as the energy levels for a wide range of geometries, the spatial distribution of the colour flux has been determined for the case of 4 sources at the corners of a square.\textsuperscript{39}

5.3. The $BB$ interaction

The $Q\bar{q}Q\bar{q}$ system with the heavy quarks $Q$ treated as static at separation $R$ is a step towards studying the strong force between realistic hadrons. This case can be
considered as the interaction between two B mesons in the heavy quark limit - where the $b$ quarks are taken as static. With the B mesons separated by distance $R$, the interaction comes from the mutual influence of the light quarks. The relevant diagrams to study are shown in fig. 9. A determination of this potential energy will allow a non-perturbative determination of the spectrum of BB bound states - which are exotic states of considerable interest.

Exploratory studies had been made for the cross diagram only for SU(3) colour and for both diagrams using SU(2) colour. Recently a preliminary study has been made using SU(3) colour for quenched lattices with light quarks of mass near the strange quark mass. Only if the two B mesons each have the same flavour of light quark will the additional cross diagram contribute. Preliminary results for same light flavour and different light flavour are shown in fig. 10. Binding is seen in each case, but it is stronger in the unlike flavour case. This suggests that a $B - B_s$ exotic dimeson might exist. It will also be possible to study the spin-dependence of these potential energy differences on a lattice.

Note that again, at $R = 0$, the system is related to one previously studied. Thus the colour of the two static quarks can be combined to a triplet or sextet and the lightest state will have them in a colour triplet - creating a system like the $\Lambda_b$ baryon but with a static $b$ quark. These baryonic states with one static quark have been studied on the lattice previously and their mass gives the BB energy on the lattice at $R = 0$. Lattice estimates give around 0.3 GeV for this binding at $R = 0$. For light quarks of the same flavour, there is no $\Lambda_b$ state - the lightest baryon will be the $\Sigma_b$ which is somewhat heavier so the binding in this case will be around 0.2 GeV.

In the continuum, in the heavy quark limit, one would expect that the binding of the BB system at $R = 0$ for unlike light quark flavours is given by $2(M_B - m_b) - (M_{\Lambda_b} - m_b)$ where $m_b$ is the $b$-quark mass.

The interaction energy between a $B$ and $\bar{B}$ can be studied similarly. However, this system at separation $\bar{R}$ has a coupling to a state consisting of the flux tube joining the static sources (ie the state which occurs in the static potential $V(R)$) plus a light quark-antiquark meson (or vacuum when the light quark and antiquark have identical flavour). This reflects the physical process that $B\bar{B} \to \Upsilon + q\bar{q}$ and since the $\Upsilon$ is known...
to be tightly bound, this latter channel may have a lower energy at small to moderate $R$ values. Conversely, at large $R$, this transition is essentially that of fundamental string breaking: where it is energetically favourable for the string of length $R$ to break leaving a $B$ and $\bar{B}$. A thorough study of this area thus needs to use a basis with both string (flux) states and meson-antimeson states.

6. Conclusions

The lattice has a lot to offer to phenomenological model builders and those wishing to unravel the mysteries of QCD.

String-like features are seen in the excited gluonic potentials for $R > 1 \text{fm}$. The departure from this behaviour at moderate and small $R$, especially for the $\Sigma^-$ states, can be understood as a consequence of the symmetry requirements at $R = 0$. There is even lattice evidence from closed string studies for the self energy appropriate to a bosonic string.

As well as predictions for hybrid mesons with heavy quarks, recent lattice results
give predictions for the light quark hybrid sector. There seems to be a discrepancy between the lattice expectation of around 1.8 GeV and the experimental candidate at 1.4 GeV.

The response of the gluonic vacuum to static sources in adjoint and sextet representations was discussed. An effective adjoint string tension approximately double that of the fundamental was found. The string breaking for both adjoint and fundamental sources appears to occur for separation of around 1.2 fm.

It is possible to study the residual strong force between colour singlet hadrons on a lattice. Results were presented for this interaction - both among 4 static sources (ie like the Υ - Υ system) and among two heavy-light mesons (ie like the BB system).

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