Calculation of disc friction power in pumps at operation on viscoelastic fluids

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Abstract. The flow of viscoelastic fluid in the gap between the impeller disc and the pump wall for laminar flow mode is considered. Waste water sludge (WWS) was used as a working fluid. Its flow is described by viscoplastic fluid model. The results of theoretical and experimental studies of disk friction at WWS are presented and formulas are proposed for calculation of disk friction power between wheel disks and pump body.

1. Introduction

There are inevitable and high energy consumption for friction of impellers disks against liquid at operation of centrifugal pumps on any liquid [1 - 3]. This is especially for viscous and viscous plastic liquids. Disk losses can be up to 30% of energy consumption in a total pump energy balance [4, 5].

There are dependencies to determine the disc friction coefficient, confirmed by numerous experimental studies for Newtonian viscous liquids. There are formulas for determining the coefficient of disk friction not taking into account the second rheological factor - limit shear stress. These formulas are used for viscoelastic fluid [5, 6].

2. Calculation of disk friction power between the wheel disk and the pump body

Let us consider the flow of viscoplastic fluid in the gap between the impeller disc and the pump wall for laminar flow mode.

As it is known, disc losses depend on the flow mode, i.e. on the Reynolds number in the gap, gap width, the amount of leakage through the gap, the surface area contacting the liquid and the amount of roughness [7].

In our case, the flow state will be determined by rheological properties, i.e. depends on two similarity criteria: Reynolds number and Saint-Venant number.

The effect of disc roughness in laminar mode does not depend on roughness. The effect of overflows has not been studied in our case because there was no fluid overflows as a result of the use in the installation of Patent Decision No. 139427 "Shaft Seal" [7].

The effect on the disc friction coefficient of rheological properties of viscoplastic fluid and gap width between the wheel disc and the pump body was thus studied.
Figure 1 shows the flow diagram in the gap between the pump body and the impeller disc. Figure 1 a shows velocity distribution in the narrow gap between the disc and the body. In this case we have Cuett's laminar flow with linear velocity profile. Distribution of circumferential velocities is calculated by the following formula:

\[ w_r = \omega r = \frac{S}{\mu} (\tau - \tau_m) \]  

(1)

in case of \( S \ll \delta \), where \( \mu \) is viscosity and \( \omega \) is the angular velocity.

In this case, tangential stress at the distance from the axis is calculated by the following relation:

\[ \tau_\phi = \tau_m + \frac{u \mu}{\delta} \]  

(2)

where \( u \) is the circumferential velocity.

As the power of disc friction is calculated by following formula:

\[ N = \frac{1}{4} \rho \omega^3 r_q^5 c \]  

(3)

where \( \rho \) is the density, \( r_q \) is the disk's radius and \( c \) is disk friction coefficient.

So \( c \) disk friction coefficient is:

\[ c = \frac{4N}{\rho \omega^3 r_q^5} \]  

(4)

Capacity of the disc friction of one side of pump disc is calculated by following formula:

\[ N = 2\pi \int_0^r \tau_\phi r_q w_r \, dr = \frac{\pi}{2} \omega^2 r_q^4 (1 + \frac{4\tau_m \delta}{3u\mu}) \frac{\mu}{\delta} \]  

(5)

So the disc friction coefficient is calculated as:
\[ C = \frac{2 \cdot \pi \cdot \omega^2 \cdot r_q^4 \cdot \left( 1 + \frac{4}{3} \cdot \frac{\tau_m}{u} \right) \cdot \mu}{\delta} = \frac{2 \cdot \pi \cdot \left( 1 + \frac{4}{3} \cdot \frac{\tau_m}{u} \right) \cdot \mu}{\rho \cdot \omega \cdot r_q \cdot \delta} = \frac{2 \cdot \pi \cdot r_q}{Re \cdot \delta} \left( 1 + \frac{4}{3} \cdot S_e \right) \]

Where \( Re = \rho \cdot u \cdot r / \mu \) is the Reynolds number and \( S_e = \tau_m \cdot \delta / u \cdot \mu \) is the Saint-Venant number.

There are separate boundary layers on the disc and the pump body with sufficiently large gaps between the pump body and the disc wheel [6, 7].

There will also be the movement of a fluid in separate boundary layers as in the first case. The movement is on the disc from the center to disc's periphery and on the wall of the body is to the center.

We will consider laminar flow of viscoplastic fluid at sufficient gaps exceeding thickness of boundary layer as circulating with interlayer between boundary layers. This interlayer rotates as a rigid body at an angular velocity less than disc’s angular velocity. This is for viscous fluids.

Figure 1b shows flow diagram at sufficient clearance and velocity distribution diagram. We integrate the known equations of the boundary layer of Newtonian liquid with substitution of Shvedova-Bingham formula into them. It is done to derive the formula of disc friction coefficient:

\[ \left\{ \begin{aligned} \rho \int_0^\delta W_r \cdot \frac{dW_r}{dr} \cdot dz + \frac{\rho}{r} \int_0^\delta W_\varphi^2 \cdot dz = \int_0^\delta \frac{\partial P}{\partial z} \cdot dz - \tau_m \\ \rho \int_0^\delta W_r \cdot \frac{\partial W_\varphi}{\partial r} \cdot dz + \rho \int_0^\delta W_z \cdot W_\varphi \cdot dz = \tau_\varphi \end{aligned} \]  

As a result of integration, we determine tangential stresses on the disc. Pressure gradient is calculated from circumferential velocity of interlayer rotation.

Speed profiles on disc and fixed wall of the body are calculated taking into account the fact that thickness of boundary layer does not depend on disc’s radius and maximum radial speed depends linearly on radius.

Frictional power of the disc against viscoplastic fluid will be:

\[ N = 4 \cdot \pi \cdot \int_0^r \tau_\varphi \cdot r_q \cdot \omega_r \cdot dr^2 = \frac{2 \cdot \pi \cdot \mu \cdot \beta \cdot r_q \cdot (\varepsilon - 1)}{\delta} + \frac{4}{3} \cdot \pi \cdot \tau_m \cdot r_q \cdot dr_q \]

So disk friction coefficient:

\[ c = \frac{4 \cdot \pi \cdot (\varepsilon - 1) \cdot (3 \cdot \xi + 7)^3}{\sqrt{15} \cdot 20 \cdot 1/4 \cdot (\varepsilon - 1)^{3/4} \cdot \xi^{7/2} \cdot \sqrt{Re}} \cdot \sqrt{1 + \frac{2}{3} \cdot S_e \cdot \frac{\xi}{\xi - 1}} \]

where \( \xi \) is angular rate of interlayer; \( \xi = \omega / 2 \) is used for practical use in calculations.

In conclusion

\[ c = \frac{2.82}{\sqrt{Re}} \cdot \sqrt{1 + \frac{4}{3} \cdot S_e} = \frac{2.82}{\sqrt{Re}} \]

where \( \sqrt{Re^*} = \frac{Re}{1 + \frac{4}{3} \cdot S_e} \)

In this case analysis of obtained formulas (6) and (10) shows that disc friction coefficient depends on both the actual Reynolds number and the Saint-Venant number in the presence of both closed and separate boundary layers. Reynolds number increasing reduces disc friction coefficient and Saint-Venant parameter increasing increases it.
As it can be seen from formulae (6) and (10) both equivalent Reynolds numbers coincide. Thus it is possible to use formulae to determine the power of disc friction for Newtonian fluids substituting generalized Reynolds number and to determine disc friction coefficient. 

It should be noted that equations (6) and (10) are also true for Newtonian fluid when Saint-Venant parameter is zero. Thus the relation of disc friction coefficient is universal for both classes of fluids.

3. Results of pilot studies
The installation is a pump body in which a disc mounted on the shaft can rotate [7 - 9]. The aim of experimental studies is to determine disk friction coefficient to confirm calculated relation.

Figure 2 shows the results of experimental studies of disc friction, where generalized Reynolds numbers are on axis X and values of disc friction coefficient are on axis Y. Studies are carried out at the installation where waste water sludge (WWS) is used as viscoplastic fluid.

![Figure 2](image_url)

**Figure 2.** Comparison of experimental data and design values of disk friction coefficient for WWS.

\[ 1 - \frac{C}{2} = \frac{\pi \tau_q}{Re^* \delta} \quad \text{where} \quad 1 - \text{closed interface,} \quad 2 - \text{not closed interface according to experimental results.} \]

Test data match up the calculation data according to equation (9) for Reynolds numbers less than \( 2 \cdot 10^4 \). This fact confirms the theory of individual boundary layers closing. In this case the distribution of circumferential velocities is the same as to the distribution of velocities at Couette flow, as there is Couette laminar flow with a linear velocity profile at \( Re \leq 10^5 \).

There are two separate boundary layers - on the disc and the pump body at \( Re \geq 2 \cdot 10^4 \) around disc wheel at sufficiently large axial clearance.
Figure 3. Dependence of disc friction coefficient on generalized Reynolds number in the gap between disc wheel and body wall.

There is a misrepresentation of experimental data when \( R_e \gg 10^5 \). An increase in a gap leads to an increase in disc friction coefficient which depends on gap width between the disc and the body. Figure 3 shows a graph of \( c \) dependence on generalized \( R_e^* \) for closed boundary layers depending on gap width between the disc and the pump body and \( R_e \). This graph is done to ease the calculation of disc friction coefficient.

4. Conclusions

The relations of disc friction coefficient of viscoplastic fluid for centrifugal pumps are theoretically obtained for the case of separate boundary layers and for the case of closed boundary layers for laminar flow mode. Experimental studies of disc friction in the gap between the impeller and the pump body confirm obtained theoretical dependences for the disk friction coefficient. Generalized similarity criteria are proposed for viscoplastic fluid flow in the gap between the pump impeller and its body. These criteria allow using known dependencies of disc friction coefficient for viscous fluid.

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