Positive Operation Valued Measurement Based Multi-User Detection in DS-CDMA Systems

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Abstract

3G and 4G mobile are based on CDMA technology. In order to increase the effectiveness of CDMA receivers large amount of effort is invested to develop suitable multi-user detector techniques. However, at this moment there are only suboptimal solutions available because of the rather high complexity of optimal detectors. One of the possible receiver technologies can be the quantum assisted computing devices which allows high level parallelism in computation. The first commercial devices are estimated by 2004, which meets the advent of 3G and 4G systems. In this paper we introduce a novel quantum computation based Quantum Multi-user detection (QMUD) algorithm, employing simple Positive Operation Valued Measurement (POVM), which provides optimal solution. The proposed algorithm is robust to any kind of noise.

Keywords: Multi-user detection, Positive Operation Valued Measurement, Quantum computing, Quantum Signal Processing

1 Introduction

The subscribers of next generation wireless systems will communicate simultaneously, sharing the same frequency band. All around the world in 3G mobile systems apply Direct Sequence - Code Division Multiple Access (DS-CDMA) promising due to its high capacity and inherent resistance to interference, hence it comes into the limelight in many communication systems. Another physical layer scheme, Orthogonal Frequency Division Access (OFDM), is also often used e.g. for Wireless LANs (WLAN) or HiperLAN, where the subscriber’s signal is transmitted via a group of orthogonal frequencies, providing Inter Channel Interference (ICI) exemption. Nevertheless due to the frequency selective property of the channel, in case of CDMA communication the orthogonality between user codes at the receiver is lost, which leads to performance degradation. Single-User detectors were overtaxed and showed rather poor performance even in multi-path environment [1]. To overcome this problem, in recent years Multi-User Detection (MUD) has received considerable attention and become one of the most important signal processing task in wireless communication.

Verdu [2] has proven that the optimal solution is consistent with the optimization of a quadratic function, which yields in MLSE (Maximum-Likelihood Sequence Estimation) receiver. However, to
find the optimum is a \(NP\)-hard problem as the number of users grows. Many authors proposed sub-optimal linear and nonlinear solutions such as Decorrelating Detector, MMSE (Minimum Mean Square Error) detector, Multistage Detector, Hopfield neural network or Stochastic Hopfield neural network \([1, 2, 3, 4]\), and the references therein. One can find a comparison of the performance of the above mentioned algorithms in \([3]\).

Nonlinear sub-optimal solutions provide quite good performance, however, only asymptotically. Quantum computation based algorithms seem to be able to fill this long-felt gap. Beside the classical description, which we recently use, researchers in the early 20\(^{th}\) century raised the idea of quantum theory, which nowadays becomes remarkable in coding theory, information theory and for signal processing \([3]\).

Nowadays, every scientist applies classical computation, using sequential computers. Taking into account that Moore’s law can not be held for the next ten years because silicon chip transistors reach atomic scale, therefore new technology is required. Intel, IBM and other companies invest large amount of research to develop devices based on quantum principle. Successful experiments show up that within 3-4 years quantum computation (QC) assisted devices will be available on the market as enabling technology for 3G and 4G systems.

This paper is organized as follows: in Section 2 we shortly review the necessity of multi-user detection, whereas in Section 3 the applied quantum computation method is shown. In Section 4 we discuss the used system model. In Section 5 the novel MUD algorithm is introduced and finally we conclude our paper in Section 6.

## 2 Multi-User Detection

One of the major attributes of CDMA systems is the multiple usage of a common frequency and time slot. Despite the interference caused by the multiple access property, the users can be distinguished by their codes. Let us investigate a DS-DCDMA system, where the \(z\)th symbol of the \(k\)th \((k = 1, 2, \ldots, K)\) user is denoted by \(b_k(z)\). Applying BPSK modulation, the output signal of the \(k\)th user, denoted by \(q_k(t)\), is given as

\[
q_k(t) = \sqrt{E_k} \sum_{z=-L}^{L} b_k(i)s_k(t - zT),
\]

where \(s_k(t)\) and \(E_k\) is the continuous signature signal and energy associated to the \(k\)th user, \(T\) is the time period of one symbol, and \((2L + 1)\) is the size of a block, respectively. For the sake of simplicity we assume one path propagation channel, so the channel distortion for the \(k\)th user is modeled by a simple attenuation factor \(h_k(t) = a_k\). This model, however, can be easily applied to more sophisticated channel models, as well.

The received signal is the sum of arriving signals plus a Gaussian noise component and thus can be written as follows:

\[
r(t) = \sum_{k=1}^{K} h_k(t) * q_k(t) + n(t) = \\
= \sum_{k=1}^{K} \sum_{l=-L}^{L} \sqrt{E_k a_k} b_k(l)s_k(t - lT) + n(t), \tag{1}
\]

where \(K\) is the number of users using the same band, \(n(t)\) is a white Gaussian noise with a constant \(N_0\) spectral density. In case of signatures limited to one symbol length and if the system is properly synchronized, then there is no intersymbol interference. Consequently, index \(z\) can then be omitted from \([3]\), yielding:

\[
r(t) = \sum_{k=1}^{K} \sqrt{E_k a_k} b_k s_k(t) + n(t), t \in [0, T].
\]
A conventional detector contains \( k = 1, \ldots, K \) filters which are matched to the corresponding signature waveforms and channels and calculates the following decision variable:

\[
\tilde{b}_k = \sqrt{E_k}a_k \int_0^T r(t)s_k(t)dt.
\]  

The traditional "single-user" detector (SUD) simply calculate the sign of expression (2) yielding \( \tilde{b}_k^{SUD} = \text{sign}\{b_k\} \). This method results in poor performance, as \( \tilde{b}_k \) contains not only the signal transmitted by the \( k^{th} \) user but an interference term generated by the other users:

\[
\tilde{b}_k = b_k \theta_{kk} + \sum_{l=1, l \neq k}^{K} b_l \theta_{kl} + n_k,
\]

where \( \theta_{kl} \) is defined as follows:

\[
\theta_{kl} = \sqrt{E_k} \sqrt{E_l} \alpha_k \alpha_l \int_0^T s_k(t)s_l(t)dt.
\]

and \( n_k = \int_0^T s_k(t)n(t)dt \) is a zero mean white Gaussian noise due to linear transformation. The output of the matched filter in vector form is:

\[
\tilde{b} = Rb + n,
\]

where \( \tilde{b} = [\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_K]^T \), \( b = [b_1, b_2, \ldots, b_K]^T \) and \( n = [n_1, n_2, \ldots, n_K]^T \), whereas \( R = [\theta_{kl}, k = 1,2,\ldots,K \atop l = 1,2,\ldots,K] \) is a symmetric matrix with property of diagonal dominance (\( \theta_{zz} > \theta_{zj} \forall j = 1,2,\ldots,K \atop j \neq i \)).

Based on the model introduced above, we derive now the optimal MUD. The MUD algorithm processes vector \( \tilde{b} \), which is the output of the matched filter. To obtain optimal solution based on Bayesian decision rule one wants to chose the maximal probability binary sequence conditioned by the received data series. The optimal Bayesian detection reduces to the following minimization problem [1]:

\[
\tilde{b}^{opt} = \min_{y \in \{-1, +1\}^K} (\tilde{b} - Ry)^T R^{-1}(\tilde{b} - Ry).
\]

Unfortunately, the search for the global optimum of (3) usually proves to be rather tiresome, which prevents real time detection (its complexity by exhaustive search is \( \mathcal{O}(2^K) \)). Therefore, our objective is to develop new, powerful detection technique, which paves the way toward real time MUD even in highly loaded system.

3 Quantum Computation Theory

Quantum theory is a mathematical model of a physical system. To describe such a model we need to specify the representation of a system. Every physical system can be characterized by means of its states in the Hilbert vector space over the complex numbers \( \mathbb{C} \). The vectors will be denoted as \( |\varphi\rangle \). The inner product \( \langle \psi | \varphi \rangle \) maps the ordered pair of vectors to \( \mathbb{C} \) with the properties [1]

- Positivity: \( \langle \psi | \psi \rangle > 0 \) for \( |\psi\rangle = 0 \),
- Linearity: \( \langle \varphi | (a|\psi_1 \rangle + b|\psi_2 \rangle) = a\langle \varphi | \psi_1 \rangle + b\langle \varphi | \psi_2 \rangle, \)
- Skew symmetry: \( \langle \varphi | \psi \rangle = \langle \varphi | \psi \rangle^* \).

\(^1\)Say ket \( \varphi \).
In the classical information theory the smallest conveying information unit is the bit. The
counterpart unit in quantum information is called the "quantum bit" the qubit. Its state can
be described by means of the state \(|\varphi\rangle, \varphi = \alpha|0\rangle + \beta|1\rangle\), where \(\alpha, \beta \in \mathbb{C}\) refers to the complex
probability amplitudes and \(|\alpha|^2 + |\beta|^2 = 1\). The expression \(|\alpha|^2\) denotes the probability that
after measuring the qubit it can be found in \(|0\rangle\) computational base, and \(|\beta|^2\) shows the probability
to be in computational base \(|1\rangle\). In more general description an \(N\)-bit qregister \(|\varphi\rangle\) is set up from
qubits spanned by \(|i\rangle\) \(i = 0 \ldots (M - 1)\) computational basis, where \(M = 2^N\) states can be stored in
the same time.

\[|\varphi\rangle = \sum_{i=0}^{M-1} \varphi_i|i\rangle \quad \varphi_i \in \mathbb{C}, \quad (6)\]

where \(M\) denotes the number of states and \(\forall i \neq j \langle i|j\rangle = 0, \langle i|i\rangle = 1, \sum |\varphi_i|^2 = 1\), respectively. It is worth mention, that a \(U\) transformation on a qregister is executed parallel on all \(M\) stored
states, which is called quantum parallelization. To the irreversibility of transformation, \(U\) must be unity \(U^{-1} = U^T\). The quantum registers can be set in a general state using quantum gates which
can be represented by means of a unitary operation, described by a quadratic matrix. Applying
four basic gates any states can be prepared.

### 4 System Model

In DS-CDMA systems an information bearing bit is encoded by means of a user specific code with
length of the processing gain \((PG)\). In case of uplink communication we assume perfect power
control. In the receiver side it is not required synchronization between input signals and user
specific codes, however, chip synchronization is necessary, that allows using multipath propagation
channels.

Since classical multi-user detection schemes only try to minimize the probability of error in
noisy and high interference environment, they, even also optimal solutions, can commit an error.
Actually, these classical approaches make compromise between computational complexity, probability
of error and time barrier required for efficient working. On the other hand, QMUD does not
make error in detection, at most QMUD can not make a decision in certain cases, furthermore, it
indicate us this symbol in order to correct in a higher layer.

#### 4.1 Representation of Possible Received Sequences in Qregisters

We quantize every chip of the \(k^{th}\) user’s codeword in a qregister of length \(N_{ch}\), where the number
representation is not significant at the evaluation of the received symbol, because with increasing
the number of the alphabet, the precision of number notation grows exponentially. In our model
we prepare for user \(k\) two quantum register \(|\varphi_1^k\rangle\) and \(|\varphi_0^k\rangle\) each corresponding to transmitted bit
"1" and "0" with an overall length \(N_Q = N_{ch} \cdot PG\). It is important to notice that the effects of
a multi-path channel and the additive noise are contained in the registers, moreover, the density
function of the noise does not need to be known \(a-priori\), however, the knowledge of noise can reduce the number of qubits in a qregister. This uncertainty may not influence the exact decision.

Let \(V\) denotes a vector space spanned by \(|v_i\rangle, i = 1 \ldots 2^N_Q\) orthonormal computational base states,
where \(\langle v_i|v_j\rangle = 0\) for \(\forall i \neq j\) and \(\langle v_i|v_i\rangle = 1\) for \(\forall i = j\) is hold. The number of stored states in quantum
registers \(|\varphi_1^k\rangle\) or \(|\varphi_0^k\rangle\) is denoted with \(N_{s1}\) or \(N_{s0}\), respectively. If the register \(|\varphi_1^k\rangle\) contains the
desired state \(|v_i\rangle\), then

\[\varphi_1^k(i) \equiv \langle \varphi_1^k|v_i\rangle = \begin{cases} \frac{1}{\sqrt{N_{s1}}} & \text{if } |v_i\rangle \in |\varphi_1^k\rangle \equiv a_i \neq 0 \\ 0 & \text{otherwise,} \end{cases} \quad (7)\]

that fulfills the stipulation \(\sum_{i=1}^{2^N_Q} |\varphi_1^k(i)|^2 = 1\).
4.2 Preparation of quantum register states

Due to the effect of multi-path propagation it is required to form any delayed version of chip sequences of user \( k \). This operation can be made via the so called swap gate, which changes the position of two qubits in a register. In general, it can be seen as a quantum shift register. One can think, all the possible states should be computed before doing quantum multi-user detection. It is true, however, using classical sequential computers, this operation could take rather long time, whereas quantum computation exploits the quantum parallelism. Applying this feature a transformation on \( N \) states stored in a register can be done in one single step, that provides fast, efficient preparation of \( |\varphi_k^1\rangle \) and \( |\varphi_k^0\rangle \).

5 Quantum Multi-User Detector

The decision rule of classical multi-user detector becomes a measurement in quantum world. In our case we have to find out that the received and quantized signal vector of user \( k \) \( |r_k\rangle = |v_i\rangle \) is either in the register \( |\varphi_k^1\rangle \) or \( |\varphi_k^0\rangle \) or both. In more mathematical description

\[
\langle \varphi_k^1 | r_k \rangle \neq 0 \quad \text{i.e.} \quad \varphi_k^1(i) \neq 0.
\]

(8)

Because of the multi-path propagation and the noise the same state \( |v_i\rangle \) could be found in both registers that makes the detection impossible. It shall be emphasized, however, that QMUD is able to recognize this event allowing higher layer protocols to perform error correction, hence it will never made false decision, as classical MUD algorithms (independently whether it is suboptimal or optimal) may do. On the other hand this can not be seen as feebleness of QMUD since the classical MUD is also unable to make proper decision in such a situation. The decision rules of QMUD are showed in Table 1. From now onward we only focus on \( |\varphi_k^1\rangle \), the operations on \( |\varphi_k^0\rangle \) are analogous.

| \( \langle \varphi_k^1 | r_k \rangle \) | \( \langle \varphi_k^0 | r_k \rangle \) | decision |
|----------------|----------------|---------|
| 0 | 0 | no message was sent |
| 0 | \( \neq 0 \) | the bit ”0” was sent |
| \( \neq 0 \) | 0 | the bit ”1” was sent |
| \( \neq 0 \) | \( \neq 0 \) | no decision is possible |

5.1 Evaluation of \( \langle \varphi_k^1 | r_k \rangle \) - The measurement

The evaluation of \( \langle \varphi_k^1 | r_k \rangle \) is not a trivial task as this is not an unitary operation, as discussed in Section 3. In the register \( |\varphi_k^1\rangle \) there is only one state \( |r_k\rangle = |v_i\rangle \) we are interested in. However, from measurement point of view the overall state of the quantum register being is state \( |\varphi_k^1\rangle \) can be regarded as a qubit. This qubit can be written as \( \alpha |0\rangle + \langle \varphi_k^1 | r_k \rangle |1\rangle \), where \( \alpha = \sqrt{\sum_{j=1,j \neq i}^{N-1} |\varphi_k^1(j)|^2} = \langle \varphi_k^1 | v_j \rangle \). This qubit contains two states \( |\eta_1\rangle = |0\rangle \) and \( |\eta_2\rangle = \sqrt{N_{\text{all}}}/N_{\text{all}} |0\rangle + \sqrt{N_{\text{all}}}/N_{\text{all}} |1\rangle \) corresponding to the probability amplitude of \( |v_i\rangle \) is in the register or not. It can be simply proved that \( |\eta_1\rangle \) and \( |\eta_2\rangle \) are not unambiguously distinguishable, because \( \langle \eta_1 | \eta_2 \rangle \neq 0 \).
However, we can extend the computational bases and apply the so called Positive Operation Valued Measurement (POVM-see Appendix). We introduce three positive operators

\[
E_1 = \alpha |1\rangle \langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & \alpha \end{pmatrix},
\]

(9)

\[
E_2 = \beta \left[ \frac{1}{N_{s1}} |0\rangle - \sqrt{1 - \frac{1}{N_{s1}}} |1\rangle \right] \left[ \sqrt{1 - \frac{1}{N_{s1}}} |0\rangle - \sqrt{1 - \frac{1}{N_{s1}}} |1\rangle \right] = \begin{pmatrix} \frac{1}{N_{s1}} & \beta \frac{1}{N_{s1}} \\ -\beta \sqrt{\frac{N_{s1} - 1}{N_{s1}}} & \beta \left( 1 - \frac{1}{N_{s1}} \right) \end{pmatrix} \quad \text{and}
\]

(10)

\[
E_3 = I - E_1 - E_2 = \begin{pmatrix} 1 - \frac{1}{N_{s1}} & \beta \sqrt{\frac{N_{s1} - 1}{N_{s1}}} \\ \beta \sqrt{\frac{N_{s1} - 1}{N_{s1}}} & 1 - \alpha - \beta \left( 1 - \frac{1}{N_{s1}} \right) \end{pmatrix},
\]

(11)

where \(I\) is the identity matrix. The operator \(\mathbf{1}\) provides

\[
\sum_{j=1}^{3} p(E_j) |\eta_1\rangle = \sum_{j=1}^{3} p(E_j) |\eta_2\rangle = 1,
\]

(12)

besides the first two POVM measurement operators in \(\mathbf{1}, \mathbf{1}\) are orthogonal to \(|\eta_1\rangle\) and \(|\eta_2\rangle\), respectively, making the probabilities of measuring \(E_1\) and \(E_2\)

\[
P(E_1)|\eta_1\rangle = \langle \eta_1 |E_1 |\eta_1\rangle = 0,
\]

\[
P(E_2)|\eta_2\rangle = \langle \eta_2 |E_2 |\eta_2\rangle = 0,
\]

(13)

where \(P(E_i)|\eta_j\rangle\) refers to the probability of the event the \(E_i\) was measured if \(|\eta_j\rangle\) had been received. In other words, if our instrument indicates \(E_1\), only the information corresponding to the state \(\eta_2\) could be sent, otherwise if \(E_2\) is indicated the received state must be \(\eta_1\). It is appreciable that the scale of uncertainty arising from POVM measurement is a function of \(E_3\). It is important to emphasize that detecting \(E_3\) we do not any false detection. To reduce this effect the free variables \(\alpha\) and \(\beta\) in \(E_3\) should be set to zero which makes it to identity matrix. Unfortunately, in that case, the resulting matrix becomes to a non-positive definite one.

### 5.2 Setting the variables \(\alpha\) and \(\beta\)

The operator \(E\) is positive if \(\langle \varphi |E|\varphi\rangle \geq 0\) for any \(|\varphi\rangle\). A positive definite matrix has the form

\[
E_3 = \langle A|B\rangle \langle A, B\rangle,
\]

where in our case \(A = \sqrt{1 - \beta \frac{1}{N_{s1}}}\) and \(B = \sqrt{1 - \alpha - \beta \left( 1 - \frac{1}{N_{s1}} \right)}\), moreover, according to \(\mathbf{1}\) the product should satisfy

\[
AB = \sqrt{1 - \beta \frac{1}{N_{s1}}} \cdot \sqrt{1 - \alpha - \beta \left( 1 - \frac{1}{N_{s1}} \right)} = \beta \sqrt{\frac{N_{s1} - 1}{N_{s1}}},
\]

(14)

which leads to

\[
\alpha = \frac{1 - \beta}{1 - \frac{\beta}{N_{s1}}},
\]

(15)

that makes \(E_3\) positive.

We assume at the moment the symbols "1" and "0" are transmitted with equal probabilities, therefore it is worth choosing the measurement probabilities \(P(E_1)|\eta_2\rangle\) and \(P(E_2)|\eta_1\rangle\) to be equal.
Lemma 1. If the probabilities measurement \( P(E_1||\eta_2) = P(E_2||\eta_1) \) then \( \alpha = \beta \) furthermore \( \alpha = \frac{1}{2} \).

Proof.

\[
\begin{align*}
P(E_1||\eta_2) &= \langle \eta_2 | E_1 | \eta_2 \rangle = \frac{\beta}{N_{s1}}, \\
and \\
\quad P(E_2||\eta_1) &= \langle \eta_1 | E_2 | \eta_1 \rangle = \frac{\alpha}{N_{s1}}
\end{align*}
\]

Substituting \( \alpha = \beta \) in (15) one gets a quadratic function with roots of \( \alpha_1 = N_{s1} - \sqrt{N_{s1} (N_{s1} - 1)} \) and \( \alpha_2 = 2N_{s1} \), where the latter one is impossible since probability can not become greater than 1. Moreover, \( \alpha_1 \) converges very fast to \( \frac{1}{2} \) as \( N_{s1} \) goes to infinity.

However, as the length \( N_{s1} \) of a register grows the resulting probability of detection \( \frac{\beta}{N_{s1}} \) becomes always smaller, due to the small angle between \( |\eta_1\rangle \) and \( |\eta_2\rangle \). POVM [10] is typically used in such situations, where the measurement can not be repeated. In our case, however, in our system the content of the registers \( |\varphi_{k1}\rangle \) and \( |\varphi_{k0}\rangle \) are constant during detection, allowing multiple measurements or even parallelization of them, which makes \( P(E_3) \) smaller at every step.

Although, lemma 1. shows that \( \alpha = \beta \) is an expedient choice, with additional thoughts \( P(E_3) \) can be further reduce. Therefore, we assert the next theorem.

Theorem 1. Using appropriate different values for \( \alpha \) and \( \beta \) one can double the probability of detection.

Proof. We apply two measurements parallel, where

\[
\max_{\beta} P(E_1)||\eta_2\rangle \implies \max_{\beta} \alpha
\]

and

\[
\max_{\beta} P(E_2)||\eta_1\rangle.
\]

Focusing again on the former case, of course, \( P(E_2)|\eta_1\rangle \) and also \( P(E_3) \) become very small. Since the bounds of a probability variable \( x \) must satisfy \( 0 < P(x) < 1 \), so the bounds of \( \alpha \), \( 0 < \alpha < N_{s1} \) are known, as well. Issued from (15) \( \alpha \) takes negative values if \( \beta \) becomes greater than 1, and the same is held for the opposite case, respectively. The possible values of \( \alpha \) and \( \beta \) are depicted in Figure 1, where two linear function of \( \beta \) can be seen according to the numerator and denominator of (15).

The maximum value for \( \max \alpha = 1 \) is also conceivable from Figure 1 that makes \( P'(E_2)|\eta_1\rangle = \frac{1}{N_{s1}} \) which is \( 2 \cdot P(E_1)|\eta_2\rangle \). The same techniques can be applied for \( \max P(E_2)|\eta_1\rangle \), and it is enough for decision if one of the two measurements or both results \( E_1 \) or \( E_2 \).
One can make a secure decision whether $E_1$ or $E_2$ or both is indicated as well as the effect of $E_3$ is reducible with repeated measurements.

For a right quantum decision a Measurement Block ($MB^k$) can be built up employing two POVM operations according to Theorem 1, as depicted in Figure 2, where $k$, $i$ refers to the user and computation base ($b$), respectively. In general a detection can be made by a Decision logic operating on the following rule:

1. input$_1 \in \{E_1, E_2, E_3\}$;
2. if $\exists$ input$_1 = E_1$, $i = 1, \ldots, k \Rightarrow$ out $= E_1$
3. else if $\exists$ input$_1 = E_2$, $i = 1, \ldots, k \Rightarrow$ out $= E_2$
4. else no decision.

In our case the Decision logics and the Selector units in Figure 2 and 3 use the decision rule table to be found in Table 2 according to decision rule described above.

It is remarkable that in case of $Out = E_3$ the measurement block can be fed back i.e. the operation can be repeated, which is normally equivalent to switch such a block serial.

Table 2: Measurement block decision rule table

| Input 1 | Input 2 | Decision | Observation                          |
|---------|---------|----------|--------------------------------------|
| $E_1$   | $E_1$   | $E_1$    | the received symbol is surely in $|\varphi^k_1\rangle$ |
| $E_1$   | $E_3$   | $E_1$    | the received symbol is surely in $|\varphi^k_3\rangle$ |
| $E_2$   | $E_2$   | $E_2$    | the received symbol is surely not in $|\varphi^k_2\rangle$ |
| $E_2$   | $E_3$   | $E_2$    | the received symbol is surely not in $|\varphi^k_3\rangle$ |
| $E_3$   | $E_3$   | $E_3$    | no decision                           |
6 Conclusions

In this paper we presented a quantum computation based multi-user detector algorithm, which involves the Positive Operation Valued Measurement. The new method utilizes one of the possible future receiver technologies of 3G and 4G mobile systems, the so called quantum assisted computing. QMUD provides optimal detection in finite time and complexity when classical methods can achieve only suboptimal solutions. Our task is in the future to examine and underline the in this paper given theoretical results with some simulations.

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Appendix 1: POVM-measurement

POVM is a common used type of measurement, which provides a secure decision, however, does not care about the state after the measurement. The probability, notable as \( p(m) = \langle \phi | M_m^\dagger M_m | \phi \rangle \),

where \( E_m \) is positive definite i.e. \( \langle \phi | E_m | \phi \rangle \geq 0 \) and \( \sum_m E_m = 1 \) must be satisfied. One can construct a POVM with three elements/bases \( \{ E_1, E_2, E_3 \} \) in such a way, that in case of \( E_1 \) or \( E_2 \) unambigious decision is possible between two occurrence. If \( E_3 \) is indicated we can not decide, however, in worst case the error correction is handled in a higher layer protocol.