Does There Really Exist the Problem 3mm of the Dark Matter in Spiral Galaxies?

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Abstract

A simple model for the dust media describing evolution of the system like spiral galaxy is considered. In contrast to previous considerations we show that the initial density fold should be quasi-one-dimensional (bar-like) instead of disc-like. The disc component of the galaxies appears only during the evolution. The model naturally reproduces some essential features of the galaxies. In particular, it reproduces all the observed typical forms of the rotation curves for the spiral galaxies with a characteristic minimum and plateau. It appears that the plateau corresponds to escaping the matter (external spiral arms, due to initial conditions, have too large velocities to be confined by the gravitational field of the galaxy). Such a scenario of the galaxy evolution leads to the conclusion that the hypothesis of the dark matter is not necessary (at least, for the spiral galaxies).

Key words: virial paradox, kinetics, galaxy, relaxation, rotation curve, dark matter, self-organization.

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1. Introduction

A so-called "virial paradox" arises naturally from observations of stellar systems, galaxies etc. If we suppose that a given system moves stationary then the virial theorem should be valid

\[ 2T = -U, \]

where \( T \) is an averaged kinetic energy of the system (excluding center of mass motion) and \( U \) is an averaged value of its potential (gravitational) energy. Generally speaking, these averages are assumed over a time. But, in the astronomy we can fix the ensemble averaged values only. So, usually peculiar "astronomic" ergodic hypothesis is accepted. In this case averaged kinetic energy may be written in the form

\[ 2T = M \langle v^2 \rangle, \]

where \( M \) is a total mass of the system and \( \langle v^2 \rangle \) is averaged squared velocity. The values \( U = -GM^2/R \) and \( \langle v^2 \rangle \) may be estimated from the observations (here \( R \) is mean radius of the ensemble). It should be noted that in reality we find a component of the velocity directed to the observer, because the only Doppler effect may be used to calculate this value independently. Using (1.1) and (1.2) we find an estimation for total mass of the system:

\[ M_V = R \langle v^2 \rangle / 2G \]  

(1.3)

The value \( M_V \) obtained from (1.3) is called "virial mass" of the system. Of course, there is uncertainty in determining \( M_V \) connected with estimation of the mean radius of the system \( R \). Nevertheless numerous observation yield the value of \( M_V \) which appears to be much more greater than the mass \( M_o \) estimated from stars, dust and other types of visible matter. Ordinary, \( M_V / M_o \sim 10 \) but there are systems where this ratio may be of order of \( 10^2 \). This fact is the "virial paradox" mentioned above. It is interesting to note a specific "scale invariance" property. The paradox takes place starting from the star system forming a nearest vicinity of the Sun up to all highest scales (namely for the star clusters in our Galaxy, the galaxies by themselves and for the galaxy clusters). The same conclusion may be found studying a kinematics of the spiral galaxies. Indeed, let us suppose that the main masses forming the galaxy rotate stationary around the galaxy center. So, their orbits are close to the circles. If an edge of the galaxy is visible we can plot a so-called rotation curve. It is a dependence of the star velocity component directed to the observer on the distance between this star and galaxy center. Best known rotation curve for the M31 galaxy (Andromeda nebula) is strongly non-monotonous and has a "strange" plateau on galaxy periphery (see de fragment on the figure 22 in book [1], for example). Central parts of the curve (like \( ab \) and \( bc \) on the same figure) may be invisible for some other galaxies due to large distance or dust presence near their center. But, the plateaus are characteristic features for the other spiral galaxies too. This fact forces astronomers to assert on existence of some additional invisible masses ("dark matter") in the galaxies. Indeed, if we suppose that the main mass of galaxy is localized near its center an obvious law

\[ v^2 \propto 1/r \]

(1.4)

takes place. This law contradicts to the \( de \) part of the rotation curve. We may treat \( de \) line as a horizontal one approximately. In this case it is necessary to have an additional mass spherically
distributed with a density $\rho \propto r$. However, observation do not give other evidences of its existence in spiral galaxies. Hypothesis of the dark matter is accepted by majority of the astrophysicists (see, however, [3], where it was shown that at least for some spiral galaxies the hypothesis of massive galo of dark matter contradicts to observed motion of the galaxy satellites). Theoretically, however, two other variants of a solution of this problem have been proposed. They are: 1) possible violation of the gravitation law at large (galactical) scales; relativistic variant of this hypothesis was analyzed recently in the work [4], where authors conclude that this supposition does not effective; 2) V.A.Ambartsumyan hypothesis [5] that the star clusters, galaxies and their clusters are strongly unstable (flying away) objects which arose as the results of an explosion of some protostar substance. This hypothesis takes away the virial paradox, obviously. But, it meets with many essential difficulties connected with the physical interpretation of the protostar objects, the reasons of their explosion with extremely high energy, etc. (see critical comments on this conception in Ref. [6]). From our point of view, a general idea of the instability of star (and more generally - self-gravitating) systems is a most fruitful one in V.A.Ambartsumyan hypothesis. In turn, the instability should not be related to any exotic hypothesis of the ”explosion”, etc. Conclusion about run away of the peripheral parts of the galaxies does not require any supposition outside standard physical theory. Moreover, we intend to show that such a behaviour is a necessary scenario at general form of initial conditions which are characteristic ones at galaxy formation from the dust medium.

Recently, new kinetic approach to study of the dust medium evolution has been proposed by one of authors [7], [8]. Below it will be shown that using this approach virial paradox may be solved quite naturally. In short our solution will be based on the fact (stably observable in numerical simulations using the model [7], [8]) that a part of the substance in the galaxy really runs away. It means that the virial theorem can not be utilized here in the form of (1.1). Moreover, our numerical experiments provide extremely strong agreement between calculated and observed ”rotation curves” for spiral galaxies. Really, as it will be clear below,these curves do not describe rotation but instead correspond to some complicated kinetic processes in the system. They are running away with ”twisting out” for peripheral branches and compressing with ”twisting in” for internal regions of system. As a result, peripheral plateau and other features of the rotation curves arise naturally and does not require latent mass hypothesis to be explained.

2. Model and results

A simple kinetic model of dust media evolution has been proposed in [7], [8]. Let us repeat briefly the main ideas used and some general features of the model which may be interesting in context of present article.

Numerical study of three (and more) body problem shows that the dynamic chaos is a typical behaviour of the system [8], [10]. This behaviour even leads to instability of the system with small number of bodies. Chaotic behaviour allows use a statistical approach to many body problem. Dynamic equations with random noise source may be used instead of an exact ones. For sufficiently large system simplest form of $\delta$-correlated noise may be used. In its turn, some of the interactions with regularly moving objects should be accounted in the equation directly [7], [8].

Formal use of the noise leads to energy production in the system. The equation of motion
should have a dissipative term to restore the energy conservation. Dust evolution leads to some energy dissipation. Dissipative effect in the equation should be taken larger than effect of the noise to simulate this real dissipation. Below, we return to this question.

Above notes lead to the equation

\[
d^2 \mathbf{r}_k / dt^2 = -\gamma d\mathbf{r}_k / dt - f(t, \mathbf{r}_k) - G \sum_j m_j \frac{\mathbf{r}_k - \mathbf{R}_j}{|\mathbf{r}_k - \mathbf{R}_j|^3} \tag{2.5}
\]

where vectors \( \mathbf{r}_k = (x_k, y_k, z_k) \) are the positions of the particles, \( \mathbf{R}_j \) are the positions of the regularly moving objects with the masses \( m_j \) respectively and (or) the positions of the mass centers of the dust clouds. White \( \delta \)-correlated noise

\[
\langle f(t, \mathbf{r}_k) \rangle = 0; \langle f(t', \mathbf{r}_k') f(t, \mathbf{r}_k) \rangle = D \delta(t - t') \delta(\mathbf{r} - \mathbf{r}') \tag{2.6}
\]

is taken for a simplicity. In this note we concentrate on typical scenarios of the galaxy evolution. Simplicity of model allows simulate both: formation of the galaxies from trial density folds and their collisions. In this case the \( \mathbf{R}_j \) values are the mass centers of the galaxies. Limiting ourselves by one galaxy arising and calculating at each step its mass center:

\[
\mathbf{R} = \frac{\sum_k m_k \mathbf{r}_k}{\sum_k m_k} \left( = \frac{\sum \mathbf{r}_k}{N} \text{ at } m_k = 1 \right) \tag{2.7}
\]

one has a very compact equation

\[
d^2 \mathbf{r}_k / dt^2 = -\gamma d\mathbf{r}_k / dt - f(t, \mathbf{r}_k) - G \frac{\mathbf{r}_k - \mathbf{R}}{|\mathbf{r}_k - \mathbf{R}|^3}. \tag{2.8}
\]

These equations should be completed by initial conditions. Fold catastrophe formation in a dust medium density is known as a probable initial stage of galaxy formation \cite{11, 12, 13} and namely such dust configurations were used as initial conditions in \cite{7, 8}. They give time transformations of the initial fold-like configuration which are very close to the typically observed. Few intermediate states obtained at numerical simulations are shown on the Fig.1. Dust beams in initial fold do not parallel in general case. It leads to a rotation of the system. A dispersion of the velocities leads to a formation of the typical structure depicted. As a rule it tends to increase galaxy volume. It is seen directly from the Fig.1 that final galaxy-like structure is bigger then starting fold. In turn, dissipation effect tends to decrease phase volume of the system. Dissipation has maximum at maximal velocities near mass center. Last fact even makes a model more self-consistent, because system has maximal density in this region which should favor to the dissipation too. As result the particles can not return from vicinity of mass center to more large distances \cite{14} and total system forms galaxy-like picture. According to general thermodynamic principles (see \cite{15} and references there) temp of relaxation tends to its minimum. System goes to a stationary dissipative attractor. In works \cite{7, 8} this fact was treated as the main reason of stable formation of galaxy-like structures from quite arbitrary configuration of dust in the space. Lowest relaxation corresponds to a motion along special saddle-like trajectory (so-called ”large river”). This trajectory separates two different kinds of flow lines. First of them are lines having more high velocities then \( v_c \), going outside the separatrix and second ones with \( v < v_c \) shooting to the center (here \( v_c \) denotes the velocity on
circular orbit where the tempo of relaxation is minimal). Let us plot a specific phase pattern of the system on the coordinates $K = v^2/2; |U| = 1/r$. Fig.2 presents a developed intermediate stage of the evolution shown on above coordinates. For convenience two characteristic straight lines corresponding to the energy balance $K = |U|$ and to the virial relation $2K = |U|$ (lines A and B respectively) are shown also. Three different families of the "star population" may be separated here.

1) Above the line B the points running away are located. Their configuration preserves partially a structure of initial configuration. On the figure presented it has been taken in the form of long density fold with tangential velocities slowly decreasing with distance from the center of structure. Evolution shifts this part of density distribution to more small $|U|$ and $K$ values. It corresponds obviously to a run away of the external branches. They are quite visible on the peripheral regions of the insert to the Fig.2. reproducing the galaxy structure in real space corresponding to given phase portrait.

2) In right hand side of the Fig.2 the points moving relatively close to the center are shown. They correspond to the developed spiral-like structure shown on the insert. It should be noted specially that this structure is not a real spiral trajectory of the points. It is a current distribution of the particles only. In reality the points forming this spiral move around the center along (quasi-) elliptic mechanical trajectories. Numerical simulation gives "paradoxical" evolution of this central spiral. It seems like generating and untwisting from the center, but averaged radius of this central part of massive decreases with time.

Internal branches are more strongly twisted than external ones. Being the parts of the distorted mechanical trajectories they sometimes intersect forming pictures which seems like spiral galaxies with bars (or bridges).

The phase portrait (on $K, |U|$ coordinates) of the central spiral structure tends at $t \to \infty$ to a straight line parallel to the line A. It corresponds to a simple conservation law $K + U = const$ due to balance between noise and relaxation.

3) One of the most interesting region on the Fig.2 is a fragment of the distribution located near the line $B$. It corresponds to the vicinity of the separatix where points move with velocities close to the $v_c$. At exactly circular velocity virial theorem is automatically satisfied for each point and $2K_j = |U_j|$. Projection to the plane $(K, |U|)$ gives for them a line close and parallel to the line $B$. Initial stages of the evolution transform trial fold configuration to the vicinity of this line as to the dissipative attractor. At this initial stage system relaxes along the line $B$. This relaxation produces first, second and other internal branches of spiral starting from the line B and forms a developed picture shown on Fig.2.

At relatively late stages of the process all points are strongly separated into 1)-st and 2)-nd classes. Vicinity of the separatix (class 3)) becomes very impoverished. Galaxy takes the form shown on the Fig.3 (in two projections). Visually it seems like elliptic or spiral (with strongly twisted branches) galaxy having couple of satellites jointed with the central "galaxy" by two very thin channels. These configurations were found by the astronomers quite often producing some artificial explanations of the reasons for the satellites formation. In its turn, numerically calculated final picture like Fig.3 is produced by the system as a very typical too. It seems like very natural theoretical explanation of such galactic configurations in the frame of present kinetic model.

Rotation curve calculated should reflect the separation of the "star population" into three families also. Doppler effect allows to plot such a curve for real galaxies using one direction...
(relatively accidental) of the velocity only. Model gives formally more complete information to plot a map of the velocities and compare it with the maps partially available from astrophysical literature \[1\]. For example, we may compare analogous curve calculated with a plot of the \(z\)-component of the velocity (orthogonal to the galaxy plane). It is shown on Fig.4. More regular (and representative) characteristics is a mean square of the velocity which shown on Fig.5.

It is seen directly that \(z\)-component of the velocity is generated by its small perturbations near the galactic center. It produces \(z\)-component of the coordinate with maximum slightly shifted from zero \(r\). At given time moment its value depends on a structure of initial dust configuration and on intensity of noise (mutual interactions). Combining them one may produce very natural galaxy-like pictures. For example, already mentioned Fig.1 displaces a picture where two different components of the population are accounted. Bold points here simulate strongly interacting "stars" from relatively compact central part of starting fold and small points correspond to a dust component of the system respectively.

The Figs.4 and 5 are plotted for different initial velocity distributions (see the sequences of the inserts there showing different time steps of the evolution). It allows to see both: universal features of the process in central part of the system and some memory about initial conditions conserved on the external branches. All these different structures of rotation curves are observed for different real galaxies. For example, insert c) on Fig.4 demonstrates unexpectedly good coincidence with already mentioned data for Andromeda galaxy (see \[1\]). The insert a) on Fig.5 corresponds to the typical rotation curve for spiral galaxies, whereas the inserts b),c) and d) on Fig.5 demonstrate very close correspondence with observed rotation curves for the galaxies: NGC 7541, NGC 2998 and NGC 801 respectively \[16\], \[17\]. Thin line on the main plot of Fig.5 shows an integral mass distribution as function of \(r\), which is also close to the estimated from the observations.

Analyzing numerical data one may extract some averaged information about the system. Most interesting for us here is a relation between \(2K\) and \(|U|\) used in virial theorem. Fig.6 shows a typical time dependence for this relation in the frame of model (curve \(A\)). It may be proved directly that for most of the scenarios leading to the typical galaxy-like space distributions initial ratio \(2K/|U|\) should be much more larger than 1 required by the virial theorem for stationary moving objects. Our numerical experiments give the typical relations: \(2K/|U| = 2 \div 10\). Value of \(2K/|U|\) decreases with the time and at \(t \to \infty\) tends to limit value which is defined by a balance between noise and relaxation in the model. For stationary object this relation is expected to be \(2K/|U| = 1\) exactly. It gives good self-consistent criterion used by us to fix a relation between \(g\) and \(D\) constants in the Eqs. (2.1) and (2.2). Moreover, it puts a natural limit of the model validity, because a physical dissipation can not be accounted after reaching of this limit.

To complete the discussion of the Fig.6 let us note that the evolution of the perturbations of the \(z\)-component of the velocity already discussed above leads to growth of averaged \(z\)-coordinate of the particles in central part of the system. Curve \(B\) on the Fig.6 shows time dependence of the ratio between averaged \((x, y)\) and \(z\) coordinates in the central subsystem. It is interesting to note that the curve has a typical maximum at relatively small \(t\). It is supported by direct observations of the system evolution. Indeed, at small \(t\) large angular momentum of initial dust configuration leads to increasing the "planarity" of the system. This process affects internal parts of the system too. But then, after some evolution, growth of \(z\)-coordinate leads to an isotropization of the central part of distribution producing an elliptic (or toroidal) galaxy as result.
3. Summary

It is found that different observed features of the galaxies may be explained without artificial suppositions like: formation of trial dust disk in the space; its rotation like solid system and other additional hypothesis. The only density folds stably generated by the dust randomly moving in the space are sufficient sources for galaxy-like structure arise.

We used a simple model of dust media evolution which is a combination of mechanical and statphysical approaches. It accounts a nonstationarity of the systems and describes an evolution the galaxy-like systems in the terms of their relaxation to dissipative attractors. This model reproduces some features of the galaxies quite naturally. These features may be summarized as follows.

1. Arbitrary density fold evolves to the structures close to the observed for real galaxies [1], [8], [9].
2. Phase portrait of the system is transformed to the attractor form providing with dissipation minimum.
3. At least three characteristic families of the moving points in the galaxy structure may be separated:
   (i). Central part of the system having densely parked and strongly twisted branches. This subsystem tends to the elliptic form at large stages of the evolution;
   (ii). Peripheral part of the system with (typically) two branches running away. These branches are twisted much more weakly than central ones;
   (iii). Intermediate region corresponding to a separatrix regime. Particles move here with a lowest velocity providing with a minimum of dissipation. With the time all points tend to be strongly separated into internal and external classes. Vicinity of the separatrix becomes very impoverished and system takes (typical) form of the central galaxy with two satellites.
4. Model reproduces typical forms of rotation curves known from the observations. In particular, standard separation of the system into three regions reproduces a rotation curve with a characteristic minimum in the separatrix region. It should be stressed that the term "rotation curve" does not correspond to the real motion of galaxy: only particles near the separatrix can be considered as rotating around the center of mass. This separatrix corresponds to the minimum on the observed "rotation curve".
5. For the scenarios leading to the galaxy-like structures the initial relation between doubled kinetic and potential energy $2K/|U|$ should be larger than the condition $2K/|U| = 1$ required by the virial theorem for stationary moving objects. Our numerical experiments give the typical relations: $2K/|U| = 2 \div 10$. This result quite naturally takes away a virial paradox without any supposition about dark matter or other additional hypothesis.
6. The spiral arms structure arises quite naturally in our model as dynamical attractors of the particles moving in Newtonian gravitational potential. The "external" and "internal" arms have different origin and different evolution: the first one are escaping from galaxy, whereas the second
one become more twisted and dense during the time. Note that we need not any (rather artificial) hypothesis, like density waves, for explaining the origin of spiral arms.

7. It is clear that generic initial density fold should be quasi-one-dimensional (bar-like), in contrast to the commonly accepted opinion that initial fold is disc-like. This leads to the conclusion that disc component is not necessary for spiral galaxies. Indeed there are peculiar non-complanar galaxies with arms but without disc (see bright examples of such “exotic” non-complanar spiral galaxies in [1]). In our approach the non-complanarity is explained merely by appropriate choosing of the initial velocities in the fold (clearly, these velocities need not be complanar, in general). In turn, absence of initial disc allows to resolves the problem of the origin of galaxy rotation (in standard scenarios with initial disc this problem is crucial).

8. A wide spectrum of observed forms of spiral galaxies is explained by:

(i) difference of initial conditions for the density fold;

(ii) time evolution: the form changes during the evolution. Moreover the spiral galaxies are essentially non-stationar objects contrary to commonly accepted point of view.

9. We see from the scenario that usual alternative for resolving the viral paradox - either completely bound, stationar system, or completely unbound, exploding one - is not correct. The situation may be much more complicated: coexistence of two components - bound and unbound. What is more important is essential non-stationarity of the matter distribution in the galaxy leading to the conclusion that in general, the virial theorem should not be valid for gravitational systems. It is interesting to note that recently the same conclusion was made studying the model of three interacted galaxies [20].

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Captions to figures

Fig.1 (a-d) Typical stages of the galaxy evolution at $\gamma = 10^{-2}, G = 0.15, D = 10^{-4}$. Two different types of the points are described in the main text. On the insert a projection to the galaxy plane orthogonal is presented for the last of the shown stages.

Fig.2 Phase pattern of the galaxy-like distribution shown on the $K, |U|$ space. Lines A and B correspond to the equalities $K = |U|$ and $2K = |U|$ respectively. The same galaxy-like structure in physical $x, y, (z = 0)$ space is shown on the insert to the picture.

Fig.3 Two projections (orthogonal to the $z = 0$ plane and close this plane respectively) of the configuration having naturally generated satellites.

Fig.4 Transformations of rotation curve (upper points) with a time shown as a projection of the numerical data on the $v_x$ and $r$ coordinates. Down points correspond to the same evolution of the $v_z(r)$ dependence. Series of the inserts show different time stages of the evolution of one initial configuration. Bold line on the main picture presents an average $v$ at each given value of radius $r$.

Fig.5 The same as it is on the Fig.4, but for the absolute velocity $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ instead of the $v_x$.

Fig.6 Time dependencies of the relation $2K = |U|$ (line A) and the ratio between averaged $(x, y)$ and $z$ coordinates (ellipticity) in the central region of the system (line B).

Fig.7 Different "peculiar" forms of the galaxies obtained in the frame of model. Near each calculated picture the real galaxies having close qualitative forms are given.
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