Collective states of even Xe isotopes in IBM+MQRPA

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Abstract. A modification of the Quasiparticle Random Phase Approximation (QRPA) with small ground state correlations is suggested. The lowest energy phonon is used as the image of $d$-boson of the Interacting Boson Model 1 (IBM1) and applied to microscopical calculations of the IBM1 parameters. Results are compared with experimental data for Xe isotopes.

The Quasiparticle Random Phase Approximation (QRPA) is one of the widely used methods for description of semimagic and deformed nuclei [1]. Application of this approximation is quite correct in those cases when the Ground State Correlations (GSC) i.e. the distinction between the quasiparticle vacuum (QV) and the RPA-phonon vacuum (PV), exist but are rather small. Quantitatively it means, e.g., that $\Sigma \varphi^2 \approx \varphi^2$. $\psi$, $\varphi$ being amplitudes of the RPA $D_{\mu}$-phonons ($\mu=0, \pm 1, \pm 2$)

$$D_{\mu}^* = \frac{1}{\sqrt{2}} \sum_{\tau=(n,p)} \sum_{i,k} \{\psi_{ik}(\mu) a^*_k a^*_i \pm \varphi_{ik}(\mu) a_k a_i\}. \quad (1)$$

Futher, we’ll work with the QRPA-phonons composed of the Bogoliubov quasiparticles. In Eq. (1), $a^*_k, a_k$ are creation and annihilation quasiparticle operators, $i$ is a single-particle state in a spherical mean field, $\bar{i}$ is a time-conjugated state.

Direct application of QRPA is not justified in the region where the transition takes place from spherical nuclei to deformed ones that is revealed in lowering the energies of the first $2^+$-states (e.g., from 1.2 MeV in Sn isotopes up to 0.32 MeV in $^{120}$Xe) and at the same time increasing $B(E2, 2^+ \rightarrow 0^+)$. Applying the standard QRPA yields the values $\Sigma \varphi^2$ smaller then but comparable with $\Sigma \psi^2$ that necessitates to modify QRPA conformably.

Several modification of RPA, included the particle- hole RPA and QRPA, have been suggested which take into account GSC more consistent [2–8]. The main idea of our modification of QRPA (MQRPA) was formulated in [9, 10] and was developed further in [11, 12]. We construct special equations like RPA for amplitudes $\psi$ and $\varphi$ of the collective quadrupole phonon $D_{\mu}$, Eq. (1), which give very small values of $\Sigma \varphi^2$, i.e. with small GSC in the fermion space. However, after mapping the $D$-phonons and PV onto the boson space the boson Hamiltonian proves to be such that its diagonalization leads to a rather complicated ground state function comprising apart from the boson vacuum (BV) two, three and more quadrupole bosons. Thus, if one returned from the boson space to the fermion one the ground state in this space would turn out to be strongly correlated.

Transition to the boson description is implemented following the Interacting Boson Model (IBM) in its first variant (IBM1 or SU(6)-model) which was first introduced by Janssen et al. [13] and then in another formulation by Arima and Iachello [14]. The IBM1 Hamiltonian, $H_{IBM}$, contains 6 parameters ($e_d, k_1, k_2, C_L$):

$$H_{IBM} = e_d\hat{n}_d + k_1 \hat{p}_1 + k_2 \hat{p}_2 + \frac{1}{2} \sum_{L=0,2,4} C_L \hat{C}_L. \quad (2)$$

where

$$\hat{n}_d = \sum_{\mu} d^\dagger_{\mu} d^\mu, \quad \hat{p}_1 = (d^\dagger d^\mu s^+) s + \text{H.c.;} \quad (3)$$

$$\hat{p}_2 = (d^\dagger d^\mu)^{(2)} s + \text{H.c.;} \quad \hat{C}_L = (d^\dagger d^\mu)^{(L)} (dd)^{(L)}.$$ 

In Eqs. (2)-(3), $d^\dagger$, $d$, $s^+$, $s$ are the quadrupole and scalar boson operators. The seventh parameter is $\Omega$ – the maximum $d$-boson number in boson wave functions.

A microscopical way to calculate the IBM1 parameters was suggested in the first work on IBM [13] where quadrupole $d$-bosons were introduced as an image of the most collective two-quasiparticle phonons composed by both neutron and proton pairs (in the IBM2 there are separately neutron and proton bosons [15]). As the image of $d$-bosons we employ the quadrupole phonons of MQRPA with the lowest energy. As shown by IBM2 [16] and our [12] calculations, in order to obtain the realistic values of IBM parameters one has to renormalize them, taking into account the connection of the collective $D$-phonon or $d$-boson space with phonons of higher energies and angular momenta $L \geq 2$. So all calculations here for Xe isotopes and in Ref. [12] are performed with the renormalized IBM1 parameters. We consider all states inside of the yrast
hand and several ones outside with energies less than the
doubled pairing gap, i.e. \( \lesssim 3.0 \text{ MeV} \), to be collective states
which can be described in the \( D \)-phonon or \( d \)-boson space.

The energy of such state \( E \) is formed by the following three
constituents \[12\]:

\[ E = E_{\text{QV}} + E_{\text{PV}} + E_{\text{IBM}}, \]

(4)

\( E_{\text{QV}} \approx \langle \text{QV}|H|\text{QV} \rangle, E_{\text{PV}} = \langle \text{PV}| : H_F : |\text{PV} \rangle, \) and \( E_{\text{IBM}} = \langle \nu_i|H_{\text{IBM}}|\nu_i \rangle \). The first of them, \( E_{\text{QV}} \), is the energy of \( QV \) and
\( H_F \) is the fermion Hamiltonian. The second term, \( E_{\text{PV}} \), is the energy of \( PV \) which is determined by the GSC,
\( : H_F : \) is the same Hamiltonian but the quasiparticle operators
in it stand in the normal order. \( E_{\text{IBM}} \) is the expectation
value of \( H_{\text{IBM}} \), Eq. (2), \( |\nu_i \rangle \) being a boson wave func-
tion of a state with spin \( I \). Index \( v \) shows which time this
function appears, the larger \( v \) the higher energy of the state
\( |\nu_i \rangle = \sum_n \alpha_{vn}|n \rangle \), where \( |n \rangle \) is a function of the SU(5)-basis.

In Refs. \[11, 12\], we suggested a functional \( \Phi \). Min-
imization of \( \Phi \) yields all the quantities determining \( E \),
Eq. (4), i.e. the Bogoliubov parameters \( (u, v) \), \( D \)-phonon
amplitudes \( (\psi, \varphi) \) and the set of amplitudes \( \{\alpha_{vn}\} \) of boson
wave functions. This functional \( \Phi \) involves \( E \) and several
additional conditions.

At first, such quantities as \( (u, v) \), \( (\psi, \varphi) \) and \( \{\alpha_{vn}\} \)
are to be normalized. Secondly, it is supposed in IBM1
that the corresponding chemical potentials are introduced in
the IBM1 parameters, \( E_{\text{IBM}} \), finds \( \{\alpha_{vn}\} \) from
\( \{\nu_{v_1} \rangle \rangle \) which is highly sensitive to a mutual arrangement of single-particle
levels. However, a change of the spin-orbital potential depth,
which we admit in transitional nuclei such as Xe isotopes,
produces much stronger effect on level positions. Therefore,
we calculate \( k_2 \) but do not consider its variations, i.e.
its influence on the level positions.

At the end, the variations of \( \Phi \) with respect to \( (u, v) \),
(\( \psi, \varphi \)) give a set of coupled equations. They include men-
tioned above EXPV which depends on boson amplitudes
\( \{\alpha_{vn}\} \). Thus, the self-consistence procedure is demanded:
using \( (u, v) \), \( (\psi, \varphi) \) one calculates the IBM1 parameters,
diagonalizes \( H_{\text{IBM}} \), finds \( \{\alpha_{vn}\} \), calculates EXPV and re-
turns to equations for \( (u, v) \), \( (\psi, \varphi) \) and so on until the
self-consistent values of all the unknown quantities are ob-
tained.

The explicit form of components entering \( \Phi \) depends on
the method applied to treat the many-body problem.
In particular, one can use the Hartree-Fock-Bogoliubov
(HFB) method with a realistic effective interaction. In \[11, 12\],
we exploited a rather simplified approach with the
spherical Saxon-Woods mean field and residual effective
forces. In our calculations, matrices of the monopole par-

ticle-particle forces are approximated by the well-known constants \( G_\tau \) \( (\tau = n, p) \) that allow the standard Bogoliubov
transformation \((u, v)\) are numbers for each single-particle state to be employed. The particle-hole and particle-

table forces are factorized in a traditional manner as prod-

ucts of matrix elements of the quadrupole single-particle
operators. We do not include in calculations the spin-

exchange and two-particle spin-orbit interactions. They
can reduce the quadrupole collectivity and to compensate
omitting those forces, our isoscalar strength of the particle-
hole force is by 10% lower than the estimation by A. Bohr
and B. Mottelson \[17\]. Since our approach is rather far
from the universal self-consistency we admit some vari-
ations of the Saxon-Woods potential parameters to repro-
duce experimental data better.
Responding sums at $\omega$ for the phonon energy further from zero is the first pole in the secular equation the character of the phonon collectivity. The more is $\xi$ particle-hole forces. Thereby neutrons a ff However, inclusion the tations with Tamm-Danko tation energy. Thus, we confirm conclusions of calcula-
tions are attenuated with the growth of spin and exci-
tions in isotonic ones as well. Hole forces is worth notice at analysis of the pairing ener-
gies in isotopic chains and, at replacement of protons by
pairing and vice versa. The e tect on the pairing, i.e. the pairing cor-
face number in the ground and excited states
r, Eq. (6). Since $R$ joins to two-quasiparticle energies in $\varepsilon_d$ and $k_j$, it is able to change the character of the phonon collectivity. The more is $\xi$ the further from zero is the first pole in the secular equation for the phonon energy $\omega$ and the smaller are values of corresponding sums at $\omega = 0$. This leads to increasing $\omega$ and, as the result, to a weakening of GSC. Note, that in our approach $\omega$ is always higher than the $d$-boson energy $\varepsilon_d$. Figure 1 shows $r$ as a function of $\xi$. The quasiparticle number in PV, $N_{qp}$, is practically proportional to $r$: $N_{qp}\approx12.1r + 0.1$, where $0.02 < r < 0.08$ $(N_{qp}\rightarrow r \rightarrow 0)$. We did not found unique criterion that could give an un-
doubted value of $N_{qp}$ or $r$. However, there are several arguments narrowing the region of their search. Firstly, it is the fulfilment of the exclusion principle. Addition of $\xi$ to each two-quasiparticle energy draws together and equalizes $D$-phonon amplitudes. Thereby the quasiparticle number $n_j$ in each single-particle $j$-state (given by one phonon) decreases. Therefore, even at large enough boson number in a state $|I,\nu⟩$ the $n_j$ value can remain $< j + 1/2$. However, it is possible only at the value of $\xi \geq 5$ MeV and $r < 0.075$ (Fig. 1). These numbers are found in calcula-
tions for Xe isotopes. Secondly, to achieve the self-
consistency between $(\alpha, \nu)$, $(\psi, \phi)$ and $[\nu_m]$, mentioned above, the values of $\xi$ and $r$ have to be larger than 5 MeV and less than 0.075, respectively. Thirdly, as shown in [5], a more correct boson mapping of the $D$-phonon operators, to get SU(6)-algebra, can be attained at small values of $\sum |\varphi_{nm}^2| = 0.05 \approx r$. If the effective interaction strengths were not subject to small alterations to fit them in with experimental spectra, then we could choose the value of $\xi$ to reproduce better the experimental data as at fixed strengths the yrast spectrum is compressed with increasing $r$ that is shown in Fig. 2.

All our calculations for Xe isotopes are carried out with $r \sim 0.02 \div 0.03$ ($\xi \approx 5$ MeV) (see Fig. 1). These values of $r$ and $\xi$ ensure the fast convergence of the self-
consistency process and fulfilment of the exclusion prin-
ciple. This value of $\xi$ gives rise to the extension of the $D$-phonon space, i.e. to increasing the maximum $d$-boson number $\Omega$ because of increasing a relative role of the high energy two-quasiparticle states in the $D$-phonon structure.

Apart from the interaction strengths in the fermion Hamiltonian we vary the spin-orbital potential depth $V_{ls}$.
which strongly affects the value of parameter \( k_2 \) in \( H_{\text{IBM}} \), Eq. (2). We keep invariable all parameters of the spin-orbital potential taken from Ref. [19] (Table 4, set 1) with the only exception of the depth. We denote the depth as \( \alpha V_0 \) (\( V_0 \) is given in [19]) and slightly change parameter \( \alpha \) (0.8<\( \alpha <1 \)). At a fixed IBM parameter \( e_\alpha \), the growth of \( k_2 \) (it almost linearly follows \( \alpha \)) is accompanied by convergence of the yrast band states and rising the energies of 2\(^+\)-, 3\(^+\)-collective states simultaneously with probabilities of inter-band \( E2\)-transitions (e.g., 2\(^+\) \( \rightarrow \) 0\(^+\)). To compensate the impact of \( \alpha \) (through \( k_2 \)) on the yrast state energies we alter linearly with \( \alpha \) the quadrupole strengths but not more than by 3\%. The dependencies on \( \alpha \) of state energies, \( E2\)-probabilities and the quadrupole moment of the 2\(^+\)-state are given in Figs. 3 and 4. Thus, the value of \( \alpha \) can be fixed for each of Xe isotopes and more or less reasonable agreement with experimental data can be obtained. We did not investigate in detail the influence of the effective forces entering into parameters of \( H_{\text{IBM}} \) on single-particle energies and so we cannot assert that the weakening of the spin-orbital potential (\( \alpha_{\text{average}} \approx 0.9 \)) cannot be imitated by this influence. However, it is undoubted that the presence of the collective quadrupole phonons (bosons) could change the spatial density on the nuclear edge and, as \( V_0 \sim \partial \rho / \partial r \), affects the spin-orbital potential.

Table 1. \( B(E2) \) (in \( e^2\cdot fm^4 \) units) inside the yrast bands in Xe isotopes.

| \( I^+ \rightarrow I^+ \) | \( ^{116}\text{Xe} \) | \( ^{120}\text{Xe} \) |
|------------------------|----------------|----------------|
| \( 2^+_1 \rightarrow 0^+_1 \) | 2470±92 | 3560±160 | 3600 |
| \( 4^+_1 \rightarrow 2^+_1 \) | 4320±190 | 4120±300 | 5290 |
| \( 6^+_1 \rightarrow 4^+_1 \) | 3890±360 | 5100±1240 | 6100 |
| \( 8^+_1 \rightarrow 6^+_1 \) | 3370±450 | 3650±450 | 6590 |
| \( 10^+_1 \rightarrow 8^+_1 \) | 3830±870 | 3230±450 | 6900 |

Table 2. \( B(E2) \) (in \( e^2\cdot fm^4 \) units) and \( Q(2^+_1) \) (in \( e\cdot fm^2 \) units) in \( ^{124}\text{Xe} \).

| \( I^+ \rightarrow I^+ \) | \( ^{124}\text{Xe} \) | \( Q(2^+_1) \) |
|------------------------|----------------|-------------|
| \( 2^+_1 \rightarrow 0^+_1 \) | 2890±120 | 1560±85 | 1580 |
| \( 4^+_1 \rightarrow 2^+_1 \) | 4150±190 | 2270±200 | 2390 |
| \( 6^+_1 \rightarrow 4^+_1 \) | 3820±2120 | 2990±300 | 2860 |
| \( 8^+_1 \rightarrow 6^+_1 \) | 2980±3000 | 3680±450 | 3190 |
| \( 10^+_1 \rightarrow 8^+_1 \) | 4450±3110 | – | – |

Figures 5 and 6 compare the theoretical energies of the yrast band states and of states 2\(^+\), 0\(^+\), 3\(^+\), 4\(^+\) with the experimental data in Xe isotopes. Calculations were performed with \( \Omega \) which are about two times higher than the half of the number of valence particles or holes, that is caused by the choice of \( \xi \). The levels 2\(^+\), 4\(^+\) in all considered isotopes are in the best agreement with experiment. For \( ^{124,126}\text{Xe} \) the very good agreement is observed even up to \( I = 12 \). However, as a rule, the theoretical yrast spectra are stretched. Perhaps that explicit consideration of non-collective two-quasiparticle high-spin states (\( I \geq 6 \)), as it was done in [10], improve the situation. Reasonable description is obtained for 2\(^+\), 4\(^+\)-states. Apparently for adequate description of the low-lying 0\(^+\)-states in \( ^{118,120}\text{Xe} \) one has to consider explicitly monopole pair-vibrational states. The better description of 3\(^+\)-states can be obtained by means of the terms of \( H_{\text{IBM}} \) containing 6 \( d\)-boson operators (i.e. with taking into account “nonaxiality”). Tables 1 and 2 show that \( B(E2) \) values inside the yrast band and those for inter-band transitions in \( ^{124}\text{Xe} \) are in reasonable agreement with experimental data.

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