The confinement of electrons to nanoscale regions in semiconductor devices leads to the formation of low-dimensional quantum systems which are highly susceptible to quantum fluctuations. Electron-electron interactions can then have dramatic effects. Indeed, experimental studies of nanoscale semiconductor devices have uncovered evidence of many very interesting strongly-correlation phenomena including the spin-charge separation\[2\] and the “0.7 effect”\[3\].

The 0.7 effect refers to a series of anomalous features that are observed in the conductance of quantum point contact (QPC) devices\[3, 4\]. The QPC is a simple device, in which a split gate is used to confine the electrons in a two-dimensional electron gas into a quasi one-dimensional (1D) channel. The anomalous conductance features observed are believed to arise from electron-electron interactions in this quasi-1D geometry, but the question of how interactions lead to these conductance anomalies is hotly debated. Many theoretical models have been proposed to account for the observations\[5, 6, 7, 8, 9\]: including spontaneous spin polarisation\[6, 10\], the Kondo effect\[4, 7\], and the spin-incoherent Luttinger liquid\[8\]. These theories make similar predictions for the conductance, the property that is usually measured, precluding a conclusive experimental distinction between them.

In this paper we show that the nature of the electronic state responsible for the 0.7 effect can be uncovered through a variant of nuclear magnetic resonance (NMR). NMR is a very powerful tool that is widely used to study strongly-correlated electronic phases in bulk materials. The very small active volume in a QPC would make it extremely difficult to perform a conventional NMR measurement, owing to the small number of nuclei coupled to the electrons. Here we describe how NMR can be performed on a QPC by generating and detecting a local non-equilibrium nuclear spin polarisation. We then turn to discuss the nuclear spin relaxation rate in the vicinity of the 0.7 effect. We show that different interaction mechanisms that can lead to similar features in conductance have very different effects on the nuclear spin relaxation rate. We identify clear experimental signatures which distinguish between different proposed scenarios for the 0.7 effect. Our work shows how electrical manipulation of local nuclear spin polarisation opens the possibility of performing NMR in nanoscale electronic systems.

We describe how a local non-equilibrium nuclear polarisation can be generated and detected by electrical means in a semiconductor quantum point contact device. We show that measurements of the nuclear spin relaxation rate will provide clear signatures of the interaction mechanism underlying the “0.7” conductance anomaly. Our analysis illustrates how nuclear magnetic resonance methods, which are used extensively to study strongly-correlated electron phases in bulk materials, can be made to play a similarly important role in nanoscale devices.
nuclei in the QPC may be measured. Nuclear spin relaxation is dominated by coupling to the electrons, and is determined by their low-frequency spin dynamics, via

$$T_1^{-1}(R) = \frac{A^2}{2\hbar^2} \int_{-\infty}^{\infty} dt \langle S^+(R, t) S^- (R, 0) \rangle$$

where the angled brackets denote thermal and quantum averages. Since there are many nuclei per electron (of order $10^6$) the gradual nuclear depolarisation leads to a smooth evolution of $G$ over the timescale $T_1$; this time is much longer than electronic timescales (see below) so many electrons pass through the QPC and contribute to the measurement of $G$.

We shall calculate the nuclear spin relaxation rate for an electron gas on the first conductance riser, $0 \leq G \leq 2e^2/h$ [21], where experiments show the appearance of the anomalous conductance features of the “0.7 effect” [3]. We first calculate the nuclear spin relaxation rate for a non-interacting electron gas, before turning to consider the effects of electron-electron interactions within several theoretical models of the 0.7 effect.

(ii) Non-interacting 1D electron gas

We consider a non-interacting electron gas described by (1). Restricting attention to the lowest subband ($s = 0$), and focusing on nuclei at the centre of the quantum wire, we find from (4)

$$T_1^{-1} = \frac{2\pi A^2 m}{\hbar^3 w_y w_z}$$

is a characteristic rate and $w_y w_z$ is the root mean square transverse area of the lowest subband. For a typical GaAs QPC, $\Gamma_0 \approx 0.5$Hz. The value of $\Gamma_0$ is very sensitive to the value of $w_y w_z$. Our main results, below, concern the dependence of $T_1^{-1}$ on gate voltage and temperature and are independent of this overall scale.

In Fig. 2(a) we show the conductance $G$ and relaxation rate $T_1^{-1}$ as a function of electron density (controlled by gate voltage) for a small Zeeman energy ($Z_e \ll k_B T$). There is a maximum in $T_1^{-1}$ close to the midpoint of the conductance riser. Increasing the electronic Zeeman energy to $Z_e \sim k_B T$, Fig. 2(b), leads to the emergence of a plateau at $G = e^2/h$; this is accompanied by a suppression of the peak in $T_1^{-1}$.

It is instructive to compare these results with the conventional Korringa expression for the nuclear spin relaxation rate of a metal, $T_1^{-1} \propto \rho_1 \rho_1 T$, where $\rho_1$ are the densities of states for the two spin species at the Fermi level. The Korringa expression applies when $k_B T \ll \mu$, which for the quantum wire implies that $G \gg 2e^2/h$. In this regime, we do find that $T_1^{-1} \propto (1/\mu^2) T$, consistent with the Korringa expression; increasing the Zeeman energy leads to a small increase in $T_1^{-1}$, consistent with
FIG. 2: Conductance (top panels) and nuclear spin relaxation rate (bottom panels) for a quasi-1D electron gas on the first conductance riser. (a) Non-interacting electron gas, with small Zeeman energy, $Z_0 = 0.001 \hbar \omega_y$. (b) Non-interacting electron gas, with larger Zeeman energy, $Z_0 = 0.05 \hbar \omega_y$. (c) Electron gas with exchange-enhanced spin-splitting, Eqn. (7) with $Z_0 = 0.001 \hbar \omega_y$ and $\gamma = 0.1 \hbar \omega_y / n_0$. The electron density $n$ is in units of $n_0 \equiv \sqrt{m \omega_y / \hbar^2}$. A typical quantum wire has subband spacing $\hbar \omega_y = 20 K$, for which the illustrated temperatures are $T = 40, 100, 200$ and 400mK.

an expected increase in $\rho_1 \rho_1$ at fixed $n$. However, the Korringa expression does not account for the most dramatic signatures in $T_1^{-1}$. These occur on the conductance riser, $0 \lesssim G \lesssim 2e^2/h$ where $k_B T \sim \mu$. In this regime, we find that $T_1^{-1}$ increases more slowly than linear in $T$. An increase in the Zeeman energy leads to a dramatic decrease in the height of the peak in $T_1^{-1}$; at the same time, the position of the maximum shifts to lie in the regime $e^2/h \lesssim G \lesssim 2e^2/h$ where both spin species are occupied [compare Figs. 2(a),(b)].

(ii) Exchange-enhanced spin-splitting
Ref. [6, 10] have provided a phenomenological theory that can successfully reproduce many features of the 0.7 effect. The electron gas is assumed to experience a density-dependent exchange splitting, leading to an effective Zeeman energy

$$Z_{\text{eff}} = Z_e + \gamma n$$

where $\gamma$ is a phenomenological parameter. (We note that a linear dependence of exchange energy on $n$ is expected for Coulomb interactions, with $\gamma \propto e^2/\epsilon$.) Treating the system as a non-interacting gas with this exchange-enhanced Zeeman splitting leads to the results shown in Fig. 2(c), where $\gamma$ has been chosen to give conductance features similar to those of the 0.7 effect [6]. Comparing the results for $T_1^{-1}$ with those for non-interacting electrons at the same bare Zeeman energy, Fig. 2(a), one sees that the main effect of exchange is a strong suppression of the peak in $T_1^{-1}$. This is consistent with the result discussed above, that increasing the Zeeman energy leads to a suppression of $T_1^{-1}$. However, the exchange enhancement of the Zeeman energy leads to a qualitatively new feature: there are now two peaks in $T_1^{-1}$ as a function of density. At very small densities exchange interactions are negligible and $T_1^{-1}$ rises as for the non-interacting gas with small Zeeman energy, Fig. 2(a); at higher densities the increase of exchange splitting at first causes a reduction in $T_1$, leading to a second peak similar to that for a large Zeeman energy, Fig. 2(b). The observation of a double-peak structure in $T_1^{-1}$ as a function of density (gate voltage) is a clear signature of a density dependent exchange-enhanced spin-splitting.

(iii) “Kondo” model
Within the “Kondo” model for the 0.7 effect [4, 7], one of the electrons is assumed to become trapped in a quasi-bound state, and to behave as a spin-1/2 “impurity” exchange-coupled to the rest of the electron gas [4, 8]. This exchange coupling, $J_K$, leads to a low energy scale, the Kondo temperature, $k_B T_K \sim \epsilon_F e^{-1/J_K \rho(\epsilon)}$ [$\rho(\epsilon)$ is the density of states at the Fermi level]. For $Z_e \ll k_B T_K$, which is the regime that we shall consider here, the conductance for the QPC shows an interesting temperature-dependence, with a crossover from $G \sim 2e^2/h$ for $T > T_K$ to $G \approx 2e^2/h$ for $T \ll T_K$ [4, 8].

This crossover should be accompanied by dramatic changes in the nuclear spin relaxation rate. The nuclear spin relaxation in the QPC is dominated by the fluctuations of the impurity spin [22]. The fastest rate is for those nuclei located close to the impurity, which are coupled to the impurity spin with an energy scale $A_0 \sim A_0/(w_x w_y w_z)$ where $w_x w_y w_z$ is the mean volume of the impurity. Relating [11] to the impurity dynamical susceptibility, we can make use of known results in limiting cases. For $T \gg T_K$, the coupling of the impurity spin...
to the electron gas is relatively weak. Using the results of Ref. \[23\] we find

$$T^{-1}_1 = \frac{2A_s^2 S(S+1)}{3\pi \hbar (k_{\text{B}}T)[J_{K}\rho(\epsilon_F)]^2},$$

(8)

where $S = 1/2$ for the spin-1/2 impurity $^{24}$. For $T \ll T_K$, the Kondo singlet is well-formed and the system behaves as a local Fermi liquid. From Ref. $^{25}$, one then recovers a Korringa law for the nuclear spin relaxation rate, with

$$T^{-1}_1 = \frac{2\pi(k_{\text{B}}T)A_s^2}{h(g_s\mu_B)^2} \chi_{\text{imp}},$$

(9)

where $\chi_{\text{imp}}$ is the static Kondo impurity susceptibility, which is a universal function of $T/T_K$ and tends to a constant as $T \to 0$. The nuclear spin relaxation rate is a non-monotonic function of $T$, passing through a maximum at $T \sim T_K$ with a maximum rate of order

$$\Gamma_{\text{Kondo}} \simeq \frac{A_s^2}{\hbar k_{\text{B}}T_K} = \frac{A_s^2}{\hbar k_{\text{B}}T_K(w_xw_yw_z)^2}$$

(10)

This non-monotonic temperature dependence of $T^{-1}_1$ is characteristic of the Kondo physics. It is qualitatively distinct from the case of non-interacting electrons, or electrons with exchange-enhanced Zeeman energy, for which $T^{-1}_1$ increases monotonically with $T$.

(iv) Spin-incoherent Luttinger Liquid

Finally, we consider the possibility that the electron system in the QPC behaves as a strongly interacting 1D wire. Strong repulsive interactions lead to pronounced local charge density wave order, and a suppression of the exchange interaction energy scale $J_{\text{LL}}$, with $J_{\text{LL}} \ll \epsilon_F$.\[8\]

At low temperatures, $T \ll J_{\text{LL}} \ll \epsilon_F$ the system should behave as a Luttinger liquid. Nevertheless, since it is coupled to Fermi liquid leads the conductance is $G = 2e^2/h$, and is insensitive to the electron-electron interactions.\[27\] Applying the general approach of bosonisation to the spin susceptibility of the repulsive 1D electron gas leads to the prediction\[28\] that as $T \to 0$, $T^{-1}_1 \sim T^{K_L}$, with $K_o < 1$ for repulsive interactions. Thus, the nuclear spin relaxation rate is sensitive to the formation of a Luttinger liquid.

As temperature is increased, the 1D electron gas enters the regime of the “spin-incoherent” Luttinger liquid\[8, 29\]. $J_{\text{LL}} \ll k_{\text{B}}T \ll \epsilon_F$. The conductance is then expected\[8\] to be $G \simeq e^2/h$. The spin-incoherent Luttinger liquid is characterised by an enhanced nuclear spin relaxation rate. This arises from the existence of low energy spin-flip excitations, of bandwidth $J_{\text{LL}}$, which couple from the electronic motion. Treating the spin-flip excitations as a spin-chain with lattice constant $1/n$ and exchange energy $J_{\text{LL}}$, one finds for $k_{\text{B}}T \gg J_{\text{LL}}$

$$T^{-1}_1 \sim \Gamma_{\text{SLL}} = \frac{IA^2_{\text{nucl}}^2}{\hbar g_s^2 \omega_{\text{nucl}}^2 J_{\text{LL}}},$$

(11)

Since $J_{\text{LL}} \ll \epsilon_F$, the relaxation rate\[11\] is parametrically enhanced as compared to that for the non-interacting electron gas.\[6\] In the spin-incoherent Luttinger liquid regime $T^{-1}_1$ is expected to be large and weakly temperature-dependent.

In summary, we have described methods by which a non-equilibrium nuclear polarisation can be generated and detected in a QPC device. Measurements of the nuclear spin relaxation rate $T^{-1}_1$ are sensitive to the electronic system in the point contact region, and will show distinctive signatures of electron-electron interactions that can be used to distinguish between different proposed scenarios for the 0.7 effect. Our study shows how NMR methods can be used to explore novel electronic phenomena in nanoscale semiconductor devices.

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\[1\] D. Goldhaber-Gordon \emph{et al.}, Nature \textbf{391}, 156 (1998).
\[2\] O. M. Auslaender \emph{et al.}, Science \textbf{308}, 88 (2005).
\[3\] K. J. Thomas \emph{et al.}, Phys. Rev. Lett. \textbf{77}, 135 (1996).
\[4\] S. M. Cronenwett \emph{et al.}, Phys. Rev. Lett. \textbf{88}, 226805 (2002).
\[5\] C.-K. Wang and K.-F. Berggren, Phys. Rev. B \textbf{54}, R14257 (1996); B. Spivak and F. Zhou, Phys. Rev. B \textbf{61}, 16730 (2000); H. Bruus, V. V. Cheianov, and K. Flensberg, Physica E \textbf{10}, 97 (2001).
\[6\] D. J. Reilly, Phys. Rev. B \textbf{72}, 033309 (2005).
\[7\] Y. Meir, K. Hirose, and N. S. Wingreen, Phys. Rev. Lett. \textbf{89}, 196802 (2002).
\[8\] K. A. Matveev, Phys. Rev. Lett. \textbf{92}, 106801 (2004).
\[9\] T. Rejec and Y. Meir, Nature \textbf{442}, 900 (2006).
\[10\] D. J. Reilly \emph{et al.}, Phys. Rev. Lett. \textbf{89}, 246801 (2002).
\[11\] S. Kronmüller \emph{et al.}, Phys. Rev. Lett. \textbf{81}, 2526 (1998).
\[12\] G. Yusa \emph{et al.}, Nature \textbf{434}, 1001 (2005).
\[13\] K. R. Wald \emph{et al.}, Phys. Rev. Lett. \textbf{73}, 1011 (1994).
\[14\] D. C. Dixon, K. R. Wald, P. L. McEuen, and M. R. Melloch, Phys. Rev. B \textbf{56}, 4743 (1997).
\[15\] T. Machida \emph{et al.}, Phys. Rev. B \textbf{65}, 233304 (2002).
\[16\] The decay of nuclear polarisation may be suppressed by depleting the electron gas in the QPC, limiting the loss of polarisation to nuclear spin diffusion. For GaAs the timescale for diffusion over typical device dimensions can be several hours [D. Paget, Phys. Rev. B \textbf{25}, 4444 (1982)].
\[17\] V. Tripathi, A. Cheung, and N. R. Cooper, \texttt{arxiv/0709.0333} (unpublished).
\[18\] We take $I = 3/2$, $n_{\text{nucl}} = 4.5 \times 10^{22}$ cm$^{-3}$, and the (average) hyperfine coupling $A_s = 3.3 \times 10^{-12}$ J m$^{-3}$ [34]. A typical GaAs QPC \cite{21} has $\omega_{\text{nucl}} \sim 2$ meV, $\omega_z \sim 15$ meV, such that $\sqrt{\omega_{\text{nucl}}\omega_z} \sim 15$ nm.
\[19\] A. C. Graham \emph{et al.}, Phys. Rev. Lett. \textbf{91}, 136404 (2003).
\[20\] A. Narath, Physica Scripta \textbf{11}, 237 (1975).
\[21\] In general, $T_1$ will depend on the nuclear polarisation itself, through the Overhauser shift \cite{22}. For simplicity,
we consider the (change in) nuclear polarisation to be small and \( Z \) to be constant.

[22] Nuclei within \( w_x \) of the impurity spin are directly coupled to it. Those at a distance \( R \gtrsim w_x \) remain coupled by RKKY interactions. We find that RKKY coupling to the fluctuating impurity dominates the nuclear relaxation rate for \( R \lesssim \epsilon_F/(k_B T K k_F) \) when \( T \lesssim T_K \), and for \( R \lesssim \epsilon_F/[k_B T k_F] \) when \( T \gtrsim T_K \).

[23] W. Götze and P. Wölfle, J. Low Temp. Phys. 5, 575 (1971).

[24] This result is valid for \( Z_e \ll k_B T [J_K \rho(\epsilon_F)]^2 \). In the opposite limit, we find \( T_1^{-1} = \pi (k_B T) A_1^2 S^2 [J_K \rho(\epsilon_F)]^2 / (\hbar Z_e^2) \). At temperatures high compared to the impurity binding energy, \( T_1^{-1} \) will increase with \( T \) and approach that of a non-interacting gas.

[25] H. Shiba, Prog. Theor. Phys. 54, 967 (1975).

[26] A. C. Hewson, The Kondo Problem to Heavy Fermions (Cambridge University Press, 1997), Ch. 4.

[27] D. L. Maslov and M. Stone, Phys. Rev. B 52, R5539 (1995).

[28] R. Chitra and T. Giamarchi, Phys. Rev. B 55, 5816 (1997); A. Y. Zavidonov and D. Brinkmann, Phys. Rev. B 61, 3282 (2000).

[29] G. A. Fiete, Rev. Mod. Phys. 79, 801 (2007).

[30] D. Paget, G. Lampel, B. Sapoval, and V. I. Safarov, Phys. Rev. B 15, 5780 (1977).