Hierarchy Problem and a new Bound State*

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Abstract

Instead of solving fine-tuning problems by some automatic method or by cancelling the quadratic divergencies in the hierarchy problem by a symmetry (such as SUSY), we rather propose to look for a unification of the different fine-tuning problems. Our unified fine-tuning postulate is the so-called Multiple Point Principle, according to which there exist many vacuum states with approximately the same energy density (i.e. zero cosmological constant). Our main point here is to suggest a scenario, using only the pure Standard Model, in which an exponentially large ratio of the electroweak scale to the Planck scale results. This huge scale ratio occurs due to the required degeneracy of three suggested vacuum states. The scenario is built on the hypothesis that a bound state formed from 6 top quarks and 6 anti-top quarks, held together mainly by Higgs particle exchange, is so strongly bound that it can become tachyonic and condense in one of the three suggested vacua. If we live in this vacuum, the new bound state would be seen via its mixing with the Higgs particle. It would have essentially the same decay branching ratios as a Higgs particle of the same mass, but the total lifetime and production rate would deviate from those of a genuine Higgs particle. Possible effects on the $\rho$ parameter are discussed.

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1 Introduction

There are several problems in high energy physics and cosmology of a fine-tuning nature, such as the cosmological constant problem or the problem of why the electroweak scale is lower than the Planck energy scale by a huge factor of the order of $10^{17}$. In renormalisable theories, such fine-tuning problems reappear order by order in perturbation theory. Divergent, or rather cut-off dependent, contributions (diagrams) have to be compensated by wildly different bare parameters order by order. The most well-known example is the hierarchy problem in a non-supersymmetric theory like the pure Standard Model, which is the model we consider in this article. New quadratic divergencies occur order by order in the square of the Standard Model Higgs mass, requiring the bare Higgs mass squared to be fine-tuned again and again as the calculation proceeds order by order. If, as we shall assume, the cut-off reflects new physics entering near the Planck scale $\Lambda_{\text{Planck}}$, these quadratic divergencies become about $10^{34}$ times bigger than the final mass squared of the Higgs particle. Clearly an explanation for such a fine-tuning by 34 digits is needed. Supersymmetry can tame these divergencies by having a cancellation between fermion and boson contributions, thereby solving the technical hierarchy problem. However the problem of the origin of the huge scale ratio still remains, in the form of understanding why the $\mu$-term and the soft supersymmetry breaking terms are so small compared to the fundamental mass scale $\Lambda_{\text{Planck}}$.

In addition to the fine-tuning problems, there are circa 20 parameters in the Standard Model characterising the couplings and masses of the fundamental particles, whose values can only be understood in speculative models extending the Standard Model. On the other hand, the only direct evidence for physics beyond the Standard Model comes from neutrino oscillations and various cosmological and astrophysical phenomena. The latter allude to dark matter, the baryon number asymmetry and the need for an inflaton field or some other physics to generate inflation. In first approximation one might ignore such indications of new physics and consider the possibility that the Standard Model represents physics well, order of magnitudewise, up to the Planck scale.

In the short term, rather than a new extended model with new fields, we have the need for an extra principle that can specify the values of the fine-tuned parameters and give predictions for theoretically unknown parameters. Of course we do need new fields or particles, such as dark matter, heavy
see-saw neutrinos and the inflaton, but at present they constitute a rather weak source of inspiration for constructing the model beyond the Standard Model. On the other hand there is a strong call for an understanding of the parameters, e.g. the cosmological constant or the Higgs particle mass, in the already well working Standard Model.

Since these problems are only fine-tuning problems, it would a priori seem that we should look for some fine-tuning principle. In a renormalisable theory, a fine-tuning requirement should concern renormalised parameters rather than bare ones. This is well illustrated by the cosmological constant problem, where simply requiring a small value for the bare cosmological constant will not solve the phenomenological problem. There are several contributions (e.g. from electroweak symmetry breaking) to the observed value, renormalising it so to speak, which are huge compared to the phenomenological value.

2 A Fine-tuning Principle

In the spirit of renormalisable theories, it is natural to formulate a fine-tuning postulate in terms of quantities that are at least in principle experimentally accessible. So one is led to consider n-point functions or scattering amplitudes, which are functions of the 4-momenta of the external particles. However, in order to specify the value of such a quantity for a given configuration of particles, it would be necessary to specify all the external momenta of the proposed configuration. One might consider taking some integral or some average over all the external momenta in a clever way. However, for several external particles, it really looks rather hard to invent a fine-tuning postulate that is simple enough to serve as a fundamental principle to be fulfilled by Nature in choosing the coupling constants and masses. But the situation becomes much simpler if we think of formulating a fine-tuning principle for a zero-point function! The zero-point function is really just the vacuum energy density or the value of the dressed cosmological constant $\Lambda_{\text{cosmo}}$. The cosmological constant is of course a good idea for our purpose, in as far as the cosmological constant problem itself would come into the fine-tuning scheme immediately if we make the postulate that the zero-point function should vanish. Nowadays its fitted value is not precisely zero, in as far as about 73% of the energy density in the Universe is in the form of dark energy or a cosmological constant. However this value of the cosmological constant is anyway very tiny compared to the a priori expected Planck scale or even
compared to the electroweak or QCD scales.

Now an interesting question arises concerning the detailed form of this zero cosmological constant postulate, in the case when there are several candidate vacuum states. One would then namely ask: should the zero cosmological constant postulate apply just to one possible vacuum state or should we postulate that all the candidate vacua should have their \textit{a priori} different cosmological constants set equal to zero (approximately)? It is our main point here to answer this question by extending the zero cosmological constant postulate to all the candidate vacua! In fact this form of the zero cosmological constant postulate unifies\footnote{We thank L. Susskind for pointing out to us that the cosmological constant being zero can be naturally incorporated into the Multiple Point Principle} the cosmological constant problem with our so-called Multiple Point Principle \cite{1}, which states that there exist several vacua having approximately the same energy density.

In principle, for each proposed method for explaining why the cosmological constant is approximately zero, we can ask whether it works for just the vacuum that is truly realised or whether it will make several vacuum candidates zero by the same mechanism. For example, one would expect that the proposal of Guendelman \cite{2}, of using an unusual measure on space-time, would indeed easily give several vacua with zero energy density rather than only one. However for a method like that of Tsamis and Woodard \cite{3}, in which it is the actual time development of the Universe that brings about the effectively zero cosmological constant, one would only expect it to work in the actual vacuum. Similarly if one uses the anthropic principle \cite{4}, one would only expect to get zero cosmological constant for that vacuum in which we, the human beings, live.

The main point of the present talk is to emphasize that the Multiple Point Principle, which can be considered as a consequence of solving the cosmological constant problem in many vacua, can be helpful in solving other fine-tuning problems; in particular the problem of the electroweak scale being so tiny compared to the Planck scale.
3 Approaching the Large Scale Ratio Problem

We consider here the problem of the hierarchy between the Planck scale and the electroweak scale. This scale ratio is so huge that it is natural to express it as the exponential of a large number. In fact we might look for inspiration at another scale ratio problem for which we already have a good explanation: the ratio of the fundamental (Planck) scale to the QCD scale. The QCD scale Λ_{QCD} is the energy scale at which the QCD fine structure constant formally diverges. It is believed that the scale ratio Λ_{Planck}/Λ_{QCD} is determined by the renormalisation group running of the QCD fine structure constant α_s(µ), with the scale ratio being essentially equal to the exponent of the inverse of the value of the fine structure constant 1/α_s(Λ_{Planck}) at the Planck scale. So we might anticipate explaining the Planck to electroweak scale ratio in terms of the renormalisation group and the Standard Model running coupling constants at the fundamental scale and the electroweak scale.

At first sight, it looks difficult to get such an explanation by fine-tuning a running coupling – e.g. the top quark Yukawa coupling – at the electroweak scale, using our requirement of having vacua with degenerate energy densities. The difficulty is that, from simple dimensional arguments, the energy density or cosmological constant tends to become dominated by the very highest frequencies and wave numbers relevant in the quantum field theory under consideration – the Planck scale in our case. In fact the energy density has the dimension of energy to the fourth power, so that modes with Planck scale frequencies contribute typically (10^{17})^4 times more than those at the electroweak scale. The only hope of having any sensitivity to electroweak scale physics would, therefore, seem to be the existence of two degenerate phases, which are identical with respect to the state of all the modes corresponding to higher than electroweak scale frequencies. They should, so to speak, only deviate by their physics at the electroweak scale and perhaps at lower scales in energy. In such a case it could be that the energy density difference between the two phases would only depend on the electroweak scale physics and, thus, could more easily depend on the running couplings taken at the electroweak scale. It is, namely, only for the modes of this electroweak scale that the running couplings at this scale are relevant.

So, in order to “solve” the large scale ratio problem using our Multiple Point Principle, we need to have a model with two different phases that only
deviate by the physics at the electroweak scale. So what could that now be? Different phases are most easily obtained by having different expectation values of some scalar field, which really means different amounts of some Bose-Einstein condensate. A nice way to have such a condensate only involve physics at a certain low energy scale, the electroweak scale say, consists in having a condensate of bound states made out of some Standard Model particles – we shall actually propose top quarks and anti-top quarks. Such bound states could now naturally have sizes of the order of the electroweak length scale. Such a picture would really only make intuitive sense, when the density is not large compared to the scale given by the size of the bound states; otherwise they would lie on top of each other and completely disturb the binding. One might say that the physical situation for the binding would become drastically changed, when the density in the bound state condensate gets so high as to have huge multiple overlap. Presumably one could naturally get a condensate with a density which is not so far from the scale given by the size and, thus, the electroweak scale. In the next section we shall spell out this idea of making a bound state condensate in more detail. We shall then return to the large scale ratio problem in section 5 and explain how the Multiple Point Principle is used to determine the top quark Yukawa coupling constant at the Planck scale, in terms of the electroweak gauge coupling constants, by postulating the existence of a third degenerate vacuum.

4 The Bound State

4.1 The Idea of a Bound State Condensate

So we are led to consider some strongly bound states, made out of e.g. top-quarks and using Higgs fields or other particles to bind them, such that the energy scale of a condensate formed from them is - by dimensional arguments - connected to the scale of the Standard Model Higgs field vacuum expectation value or VEV (which is of course what one usually calls the electroweak scale). For dimensional reasons this condensate has now a density of an order of magnitude given by this electroweak scale. Then the frequencies or energies of the involved modes of vibration are also of this order, in the sense that it is the modes with energies of this order that make the difference (between two phases say). It is therefore also the running couplings at this scale that are the directly relevant parameters! If we now impose some condition, like
the degeneracy of two phases resulting from this bound state condensation
dynamics, it should result in some relation or requirement concerning the
running couplings at the electroweak scale.

Instead of simply a bound state condensing, one could \textit{a priori} also hope
for some other nonlinear effect taking place in a way involving essentially
only the modes/physics at the electroweak scale. The crux of the matter
is that, at short distances compared to the electroweak length scale, the
non-perturbative effect in question would hardly be felt. Consequently, the
huge contributions to the energy density from the short distance modes can
be cancelled out between the two phases, in imposing our Multiple Point
Principle (MPP). But the bound state idea is in a way the most natural and
simple, since bound states are already well-known to occur in many places
in quantum physics.

4.2 A Bound State of 6 top and 6 anti-top quarks?

Of course, when we look for bound states in the Standard Model, we know
immediately that there is a huge number of hadronic bound states consisting
of mesons, baryons and glueballs, i.e. from QCD. These bound states typically
have the size given by the QCD scale parameter $\Lambda_{QCD}$, which means
lengths of the order of an inverse GeV. That is to say the strong scale rather
than the electroweak one. Nevertheless you could \textit{a priori} hope that some
phase transition, involving quarks and caused by QCD, could determine a
certain quark mass by the MPP requirement of being at the border between
two phases of the vacuum. That could then in turn lead to a fixing of the
Standard Model Higgs VEV, which is known to be responsible for all the
quark masses. But, from dimensional considerations, you would expect to
get that the quark mass needed by such an MPP mechanism would be of the
order of the strong scale. So it may work this way if the strange quark were
the one to be used, since it namely has a mass of the order of the strong scale.
But these speculated QCD-caused phase transitions are not quite what we
require. We rather seek a condensation getting its scale from the Standard
Model Higgs VEV and want to avoid making severe use of QCD. However, at
first sight, the other gauge couplings and even the top quark Yukawa coupling
(let alone the other smaller Yukawa couplings) seem rather small for making
strongly bound states; with a binding so strong, in fact, as to make the
bound state tachyonic and to condense in the vacuum. Indeed, if you think
of bound states consisting of a couple of particles, it is really pretty hopeless
to find any case of such a strong binding except in QCD. But now scalar particle exchange has an important special feature. Unlike the exchange of gauge particles, which lead to alternating signs of the interaction when many constituents are put together, scalar particle exchange leads to attraction in all cases: particles attract both other particles and antiparticles and the attraction of quarks, say by Higgs exchange, is independent of colour.

The only hope of getting very strong binding without using QCD, so as to obtain tachyonic bound states in the Standard Model, is to have many particles bound together and acting cooperatively – and then practically the use of a scalar exchange is unavoidable. So we are driven towards looking for bound states caused dominantly by the exchange of Higgs particles, since the Higgs particle is the only scalar in the Standard Model and we take the attitude of minimising the amount of new physics. Since the Yukawa couplings of the other quarks are so small, our suggestion is to imagine some top quarks and/or anti-top quarks binding together into an exotic meson. It better be a boson and thus a “meson”, since we want it to condense.

There are, of course, bound states of say a top quark and an anti-top quark which are mainly bound by gluon exchange, although comparably by Higgs exchange. However these are rather loosely bound resonances compared to the top quark mass. But, if we now add more top or anti-top quarks to such a state, the Higgs exchange continues to attract while the gluon exchange saturates and gets less significant. This means that the Higgs exchange binding potential for the whole system gets proportional to the number of pairs of constituents, rather than to the number of constituents itself. So, at least a priori by having sufficiently many constituents, one might foresee the binding energy exceeding the constituent mass of the system.

In order to get the maximal binding, one needs to put each of the added quarks or anti-quarks into an S-wave state. Basically we can use the same technique as in the calculation of the binding energy of the electron to the atomic nucleus in the hydrogen atom. Once the S-wave states are filled, we must go over to the P-wave and so on. But the P-wave binding in a Coulomb shaped potential only provides one quarter of the binding energy of the S-wave. In the case of a scalar exchange, an added particle gets a binding energy to each of the other particles already there. However, once the S-wave states are filled, these binding energies go down by a factor of about four in strength and it becomes less profitable energetically to add another particle. Depending on the strength of the coupling, it can therefore very easily turn out to be most profitable to fill the S-wave states and then
Now, when we use top quarks and anti-top quarks, one can easily count the number of constituents, by thinking of the S-wave as meaning that essentially all the particles are in the same state in geometrical space. Then there are 12 different “internal” states into which these S-wave quarks/antiquarks can go: each quark or anti-quark can be in two spin states and three colour states, making up all together $2 \times 2 \times 3 = 12$ states. Thus we can have 6 top quarks and 6 anti-top quarks in the bound state, before it gets necessary to use the P-wave. We shall here make the hypothesis, which to some extent we check below, that indeed the strongly bound state which we seek is precisely this one consisting of just 12 particles.

So we now turn to the question of whether or not this exotic 6 top quark and 6 anti-top quark state is bound sufficiently strongly to become tachyonic, i.e. to get a negative mass squared. Actually, in order to confirm our proposed MPP fine-tuning mechanism, we need that the experimentally measured top quark Yukawa coupling should coincide with the borderline value between a condensate of this almost tachyonic exotic meson being formed or not being formed. On the basis of the following crude estimate, we want to claim that such a coincidence is indeed not excluded.

4.3 The Binding Energy Estimate

We now make a crude estimate of the binding energy of the proposed 12 quark/anti-quark bound state. As a first step we consider the binding energy $E_1$ of one of them to the remaining 11 constituents treated as just one collective particle, analogous to the nucleus in the hydrogen atom. Provided that the radius of the system turns out to be sufficiently small compared to the Compton wavelength of the Standard Model Higgs particle, we can take this to be given by the well-known Bohr formula for the ground state binding energy of a one electron atom. It is simply necessary to replace the electric charge of the electron $e$ by the top quark Yukawa coupling $g_t/\sqrt{2}$, in the normalisation where the running mass of the top quark is given by the formula $m_t = g_t \times 174$ GeV, and to take the atomic number to be $Z = 11$:

$$E_1 = - \left( \frac{11g_t^2/2}{4\pi} \right)^2 \frac{11m_t}{24}$$

Here we have used $m_t^{\text{reduced}} = 11m_t/12$ as the reduced mass of the top quark.
In order to obtain the full binding energy for the 12 particle system, we should multiply the above expression by 12 and divide by 2 to avoid double-counting the pairwise binding contributions. However this analogy with the atomic system only takes into account the $t$-channel exchange of a Higgs particle between the constituents. A simple estimate of the $u$-channel Higgs exchange contribution \(^{[5]}\) increases the binding energy by a further factor of \((16/11)^2\). So the expression for the total non-relativistic binding energy due to Higgs particle exchange interactions becomes:

$$E_{\text{binding}} = \left( \frac{11g_t^4}{\pi^2} \right) m_t$$  \hspace{1cm} (2)

We have here neglected the attraction due to gluon exchange and the even smaller electroweak gauge field forces. However the gluon attraction is rather a small effect compared to the Higgs particle exchange, in spite of the fact that the QCD coupling $\alpha_s(M_Z) = g_s^2(M_Z)/4\pi = 0.118$. This value of the QCD fine structure constant corresponds to an effective gluon top anti-top coupling constant squared of:

$$e_{tt}^2 = \frac{4}{3}g_s^2 \simeq \frac{4}{3}1.5 \simeq 2.0$$  \hspace{1cm} (3)

We have to compare this gluon coupling strength $e_{tt}^2 \simeq 2$ with $Zg_t^2/2 \simeq 11/2 \times 1.0$ from the Higgs particle. This leads to an increase of the binding energy by a factor of $(15/11)^2$ due to gluon exchange, giving our final result for the non-relativistic binding energy:

$$E_{\text{binding}} = \left( \frac{225g_t^4}{11\pi^2} \right) m_t$$  \hspace{1cm} (4)

The correction from $W$-exchange will be smaller than that from gluon exchange by a multiplicative factor of about \((\frac{\alpha_s(M_Z)^2}{\alpha_s(M_Z)^3})^2 \simeq \frac{1}{25}\), and the weak hypercharge exchange is further reduced by a factor of $\sin^4 \theta_W$. Also the $s$-channel Higgs exchange diagrams will give a contribution in the same direction. There are however several effects going in the opposite direction, such as the Higgs particle not being truly massless and that we have over-estimated the concentration of the 11 constituents forming the “nucleus”. Furthermore we should consider relativistic corrections, but we postpone a discussion of their effects to ref. \([6]\).
4.4 Estimation of Phase Transition Coupling

From consideration of a series of Feynman diagrams or the Bethe-Salpeter equation for the 12 particle bound state, we would expect that the mass squared of the bound state, $m_{bound}^2$, should be a more analytic function of $g_t^2$ than $m_{bound}$ itself. So we now write a Taylor expansion in $g_t^2$ for the mass squared of the bound state, crudely estimated from our non-relativistic binding energy formula:

$$m_{bound}^2 = (12m_t)^2 - 2 (12m_t) \times E_{binding} + ...$$  \hspace{1cm} (5)

$$= (12m_t)^2 \left(1 - \frac{225}{66\pi^2} g_t^4 + ...ight)$$  \hspace{1cm} (6)

We now assume that, to first approximation, the above formal Taylor expansion (6) can be trusted even for large $g_t$ and with the neglect of higher order terms in the mass squared of the bound state. Then the condition that the bound state should become tachyonic, $m_{bound}^2 < 0$, is that the top quark Yukawa coupling should be greater than the value given by the vanishing of equation (6):

$$0 = 1 - \frac{225}{66\pi^2} g_t^4 + ...$$  \hspace{1cm} (7)

We expect that once the bound state becomes a tachyon, we should be in a vacuum state in which the effective field, $\phi_{bound}$, describing the bound state has a non-zero expectation value. Thus we expect a phase transition just when the bound state mass squared passes zero, which roughly occurs when the running top quark Yukawa coupling at the electroweak scale, $g_t(\mu_{weak})$, satisfies the condition (7) or:

$$g_t|_{phase\ transition} = \left(\frac{66\pi^2}{225}\right)^{1/4} \simeq 1.3$$  \hspace{1cm} (8)

We can make an estimate of one source of uncertainty, by considering the effect of using a leading order Taylor expansion in $g_t^2$ for $m_{bound}$ instead of for $m_{bound}^2$. This would have led to difference of a factor of 2 in the binding strength and hence a correction by a factor of the fourth root of 2 in the top quark Yukawa coupling at the phase boundary; this means a 20% uncertainty in $g_t|_{phase\ transition}$. Within an uncertainty of this order of 20%, we have a $\frac{225}{66\pi^2}$ In fact the phase transition (degenerate vacuum condition) could easily occur for a small positive value of $m_{bound}$ and hence a somewhat smaller value of $g_t^2$.  

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1.5 standard deviation difference between the phase transition (and thus the MPP predicted) coupling, $g_t \simeq 1.3$, and the measured one, $g_t \simeq 1.0$, corresponding to a physical top quark mass of about 173 GeV. We thus see that it is quite conceivable within our very crude calculations that, with the experimental value of the top quark Yukawa coupling constant, the pure Standard Model could lie on the boundary to a new phase; this phase is characterised by a Bose-Einstein condensate of bound states of the described type, consisting of 6 top quarks and 6 anti-top quarks!

4.5 Mixing between the Bound State and the Higgs Particle

Strictly speaking, if the above scenario is correct, it is not at all obvious in which of the two vacua we live. If we live in the phase in which the bound state condensate is present, the interaction of the bound state particle with the Standard Model Higgs particle can cause a bound state particle to be pulled out of the vacuum condensate and then to function as a normal particle. This effect will mean that the normal Higgs particle will mix with the bound state, in a similar way as one has mixing between the photon and the $Z^0$ gauge boson, or between $\eta$ and $\eta'$. This means that the two observed particles would actually be superpositions, each with some amplitude for being the bound state and with some amplitude for being the original Higgs particle. Both can have expectation values, or rather the expectation value is described by some abstract vector denoting the two different components. Also both superpositions would be exchanged and contribute to the binding of the bound state. Taking this two component nature of the effective Higgs particles into account makes the discussion more complicated than with a single Higgs particle.

Really there are three types of experimentally accessible parameters for which we at first want to predict a relation from our bound state model:

1. The top quark mass is given in the simplest case by the top quark Yukawa coupling times the Higgs VEV. However, in the two effective Higgs picture (one of them being the bound state essentially just mixed somewhat with the Higgs particle), the top quark mass becomes of the form $h_1 v_1 + h_2 v_2$. Here $h_1$ and $h_2$ are the Yukawa couplings of the top quark for the two superpositions, whose VEVs are denoted by $v_1$ and $v_2$. 

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2. The gauge boson masses: for example the $W$ boson mass in the two effective Higgs picture becomes $M_{WW}^2 = g_2^2(v_1^2 + v_2^2)$. Here, for simplicity, we have taken both the fields to be doublets with weak hypercharge $y/2 = -1/2$ like the original Higgs field. We reconsider the irreducible representation content of the bound state field in section 6.2 where we discuss the $\rho$ parameter problem.

3. The binding strength parameter for the bound state which determines the vacuum phase in which the energy density is the lowest. Even if this parameter is hard to determine experimentally, we may at least relate it to our MPP from which it can essentially be predicted. In the simplest case with a single Higgs particle, the binding strength parameter is just the top quark Yukawa coupling squared $g_t^2$ as discussed in section 4.4. However with two effective Higgs particles, the parameter $g_t^2$ would to first approximation be replaced by $h_1^2 + h_2^2$.

We now remark that the three quantities listed above are related by a Schwarz inequality, namely:

$$|h_1v_1 + h_2v_2|^2 \leq (v_1^2 + v_2^2)(h_1^2 + h_2^2)$$  \hspace{1cm} (9)

(written as if we had only real numbers, but we could use complex ones also). With this correction due to the mixing, we lose our strict prediction of the top quark mass corresponding to two degenerate phases, with and without a bound state condensate respectively, unless we can estimate the mixing. In fact such an estimate is not entirely out of question, because we know the coupling of the Higgs to the bound state and can potentially also estimate the density of the condensate. Qualitatively we just predict that the resulting top quark mass will be somewhat smaller than the estimate made in section 4.4 but we expect it to remain of a similar order of magnitude.

Such a mixing correction would seem to be welcome, in order to improve the agreement of the experimental top quark Yukawa coupling with the estimated phase transition value. We namely tend to predict a top quark mass, which is too large by a factor of about 1.3, without including any mixing correction. However this disagreement should not be taken too seriously, as it is within the accuracy of our calculation. Nonetheless, if we do live in the phase containing the bound state condensate, the mixing correction would be good for repairing this weak disagreement with experiment.
5 Return to the Large Scale Ratio Problem

5.1 Three degenerate vacua in the pure Standard Model

As discussed at Portoroz by Colin Froggatt [7], it is possible to determine the value of the top quark running Yukawa coupling $g_t(\mu)$ at the fundamental scale $\mu_{\text{fundamental}} = \Lambda_{\text{Planck}}$ by using the Multiple Point Principle to postulate the existence of a third degenerate vacuum, in which the Standard Model Higgs field has a VEV of order the Planck scale [8]. This requires that the renormalisation group improved effective potential for the Standard Model Higgs field should have a second minimum near the Planck scale, where the potential should essentially vanish. This in turn means that the Higgs self-coupling constant $\lambda(\mu)$ and its beta function $\beta_\lambda(\mu)$ should both vanish near the fundamental scale, giving the following relationship between the top quark Yukawa coupling $g_t(\mu_{\text{fundamental}})$ and the electroweak $SU(2) \times U(1)$ gauge coupling constants $g_2(\mu_{\text{fundamental}})$ and $g_1(\mu_{\text{fundamental}})$:

$$g_t^4 = \frac{1}{48} \left( 9g_2^4 + 6g_2^2g_1^2 + 3g_1^4 \right)$$  \hfill (10)

If we now input the experimental values of the gauge coupling constants, extrapolated to the Planck scale using the Standard Model renormalisation group equations, we obtain $g_t(\mu_{\text{fundamental}}) \simeq 0.4$. However we note that the numerical value of $g_t(\mu)$, determined from the expression on the right hand side of eq. (10), is rather insensitive to the scale, varying by approximately 10% between $\mu = 246$ GeV and $\mu = 10^{19}$ GeV.

From our assumption of the existence of three degenerate vacua in the Standard Model, our Multiple Point Principle has provided predictions for the values of the top quark Yukawa coupling constant at the electroweak scale, $g_t(\mu_{\text{weak}}) \simeq 1.3$, and at the fundamental scale, $g_t(\mu_{\text{fundamental}}) \simeq 0.4$. So we can now calculate a Multiple Point Principle prediction for the ratio of these scales $\mu_{\text{fundamental}}/\mu_{\text{weak}}$, using the Standard Model renormalisation group equations.

5.2 Estimation of the logarithm of the scale ratio

We now estimate the fundamental to electroweak scale ratio by using the leading order beta function for the Standard Model top Yukawa coupling.
constant $g_t(\mu)$:

$$\beta_{g_t} = \frac{d g_t}{d \ln \mu} = \frac{g_t}{16\pi^2} \left( \frac{9}{2} g_t^2 - 8 g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 \right)$$

(11)

where the $SU(3) \times SU(2) \times U(1)$ gauge coupling constants are considered as given at the fundamental scale, $\mu_{\text{fundamental}} = \Lambda_{\text{Planck}}$. It should be noticed that, due to the relative smallness of the fine structure constants $\alpha_i = g_i^2 / 4\pi$ and particularly of $\alpha_3(\mu_{\text{fundamental}})$, the beta function $\beta_{g_t}$ is numerically rather small at the Planck scale. So the logarithm of the scale ratio $\ln \mu_{\text{fundamental}} / \mu_{\text{weak}}$ needed to generate the required amount of renormalisation group running, between the values $g_t(\mu_{\text{fundamental}}) \simeq 0.4$ and $g_t(\mu_{\text{weak}}) \simeq 1.3$, must be a large number. Hence the scale ratio itself must be huge and in this way we explain why the electroweak scale $\mu_{\text{weak}}$ is so low compared to the fundamental scale $\mu_{\text{fundamental}}$. In practice the Multiple Point Principle only gives the order of magnitude of the logarithm of the scale ratio, predicting $\mu_{\text{fundamental}} / \mu_{\text{weak}} \sim 10^{16} - 10^{20}$.

We note that as the strong scale is approached, $\mu \to \Lambda_{\text{QCD}}$, $g_3(\mu)$ and the rate of logarithmic running of $g_t(\mu)$ becomes large. So the strong scale $\Lambda_{\text{QCD}}$ provides an upper limit to the scale ratio predicted by the Multiple Point Principle. Indeed the predicted ratio naturally tends to give an electroweak scale within a few orders of magnitude from the strong scale.

6 How to see the bound state?

6.1 Mixing with the Higgs Particle

Such a strongly bound state as we propose, consisting of 12 constituents, will practically act as a conserved type of particle, because energy conservation forbids its destruction by having a few of its constituents decay. The point is that the mass of the remaining bound state or resonance, made up of the leftover constituents, would be larger than that of the original strongly bound state. Considering its interaction with the relatively light particles of the Standard Model, the bound state would therefore still be present after the interaction. This means that the most important effective couplings, involving an effective scalar bound state field and Standard Model fields, would have two or four external bound state attachments. If we further restrict ourselves to a renormalisable effective theory, we would be left with
the bound state scalar field only having interactions involving scalar and
gauge fields. An interaction between two fermions and two scalar fields would
already make up a dimension five operator, which is non-renormalisable.

If we live in the phase without the condensate of new bound state particles,
these considerations imply that the bound state must be long lived; it could
only decay into a channel in which all 12 constituents disappeared together.
The production cross section for such a particle would also be expected to be
very low, if it were just crudely related to the cross section for producing 6
top quarks and 6 anti-top quarks. However, if we live in the phase with the
condensate, there exists the possibility that the bound state particle could
disappear into the condensate, which has of course an uncertain number of
bound state particles in it. Since, as is readily seen, there is a significant
coupling of the Standard Model Higgs particle to two bound state attach-
ments - a three scalar coupling vertex - we can achieve such a disappearance
very easily by means of this vertex. This disappearance results in the bound
state obtaining an effective transition mass term into the Standard Model
Higgs particle. Such a transition means that the Higgs particle and the new
bound state will - provided we are in the condensate phase - mix with each
other! This has very important consequences for the observability of the
bound state. We shall seemingly get two Higgs particles sharing the strength
of the fundamental Higgs particle, by each being a superposition of the latter
and of the bound state.

So, provided that we presently live in the phase with the bound state
condensate, we predict that at the LHC we shall apparently see two Higgs
particles! They will each behave just like the normal Higgs particle, except
that all of its couplings will be reduced by a mixing angle factor, common of
course for all the different decay modes of the usual Standard Model Higgs
particle, but different for the two observed Higgs particles.

6.2 The Rho Parameter; a Problem?

If we do not live in the phase with the condensate we can naturally not ex-
pect to observe any effects of this condensate, but if we live in the phase
with the condensate then one might look for the effects of this condensate.
An effect that at first seems to be there, and is perhaps likely to prevent
the model from being phenomenologically viable, is that the condensate of
bound states is not invariant under the $SU(2) \times U(1)$ gauge group for the
electro-weak interactions. In fact this condensate will a priori begin to “help” the
Standard Model Higgs field giving masses to the \( W \) and \( Z^0 \) particles. Now, however, since we imagine the bound state to exist in the background of the usual Higgs condensate and as only being bound due to the effects of this surrounding medium, the bound state is strongly influenced by the \( SU(2) \times U(1) \) breaking effects of these surroundings. Thus we cannot at first consider the bound state as belonging to any definite irreducible representation of this electroweak group. Rather we must either describe it by a series of fields belonging to different irreducible representations of this group or simply describe it by a single effective field that does not have any definite electroweak quantum numbers. But this fact means that the condensate of bound states has to be expressed by several such fields having non-zero expectation values. These different fields of different irreducible representations will not give the same mass ratio for the \( W \) and the \( Z^0 \) bosons. Thus, provided the bound state condensate is of such an order of magnitude that its effect on the gauge boson masses is not negligible, it will in general generate a \( \rho \) parameter in disagreement with experiment.

So far our calculations have not supported the hope that, by some mathematical accident, the \( \rho \) parameter comes out to be essentially equal to unity. Rather it seems that, in order for our model to be consistent with the remarkably good agreement of the Standard Model predictions with experiment, we require one of the following situations to occur:

1) We do not live in the phase with the condensate but rather in the one without the condensate.

2) The contribution of the condensate expectation value to the gauge boson masses is simply very much smaller than that of the genuine Standard Model Higgs field.

3) For irreducible representations other than the singlet and the doublet with weak hypercharge \( y/2 = 1/2 \), some self-interaction or renormalisation group effect has made the irreducible representation content of the bound state very small or vanishing.

As we shall see below, there is some weak evidence that our model favours the idea that we actually live in the phase \textit{with} the bound state condensate and even with an appreciable expectation value compared to that of the genuine Higgs condensate. So it would seem to fit our model best, if we could get the third of the above possibilities to work and thereby avoid causing problems for the value of the \( \rho \) parameter.
7  In which phase do we live?

In a model like ours, where there are many vacuum states, one must identify which of those states is the vacuum around us. We definitely live in a phase with a remarkably small Higgs field VEV compared to the “fundamental” scale or natural unit for Higgs field VEVs, which we take to be the Planck scale in our model. Among the three Standard Model vacua discussed above, there are thus two possibilities corresponding to the phases with the low value of the Standard Model Higgs VEV. So what remains to be decided is whether or not there is a condensate of the bound states in the vacuum in which we live.

In section 6 we discussed possible observational effects related to the bound state, which could discriminate between the two phases. Here we shall investigate which vacuum phase is likely to emerge from the Big Bang and then assume that it survives to the present epoch.

There is, however, no a priori reason to believe in the absence of vacuum phase transitions since the first minutes after the Big Bang. They might even have occurred in the era when stars and galaxies were already present, but then one could imagine that there should be astrophysical signatures revealing such transitions. Indeed one might even wonder if the claims for a time variation of the fine structure constant, indicated by some spectral investigations, could be a consequence of such phase transitions. But it must be admitted that the domain walls between phases would have such a huge energy per unit surface area that they might be expected to disturb all of cosmology as we understand it. So it seems likely that there were no later phase transitions and that we do live in the phase that emerged after the first minutes of the Big Bang. If the other vacuum phases are to occur anywhere or anytime at all, it must then be in the future.

We now turn to the question: what phase is likely to have come out of the Big Bang? Of course the phase that emerges depends very sensitively on the vacuum energy density. The higher energy density vacua are expected to decay into the one with the lowest energy density, provided though that sufficient thermal energy is present to surmount any energy barriers between the vacua. Hence the question of which vacuum emerges will be settled at the epoch when the temperature is still just high enough that the phase border can be passed, i.e. when it is still possible to produce the walls between the phases by thermal fluctuations.

At that epoch it is the Helmholtz free energy density $f$ rather than the
true energy density \( u \) that matters. The difference is the term \(-sT\), where \( T \) is the temperature and \( s \) is the entropy density. Assuming the true energy density is exactly the same in the two phases, the emergent phase should be the one having the highest entropy density \( s \) at the temperature in question. That in turn should be the phase with highest number of light species. Now, in the Standard Model, the known fermions and gauge bosons, \( W \) and \( Z^0 \), get their masses from the Higgs VEV in the vacuum in question. So the emergent phase should be the one with the lowest Higgs VEV, when these particles have the smallest masses, giving in turn the larger entropy, and then the lower free energy density. Now the presence of the many bound states in the condensate tends to reduce the Higgs field VEV. So it is indeed the phase with the condensate, which is expected to come out from the early Universe. Our tentative conclusion is thus that we should live in the phase with the bound state condensate, provided of course that we are correct in assuming that a new phase did not take over at a later epoch.

Phenomenologically this phase with the bound state condensate present today is the more interesting possibility, in as far as it leads to the mixing of the Higgs particle and the bound state. It thereby gives us the possibility of seeing this bound state much more easily, namely as another “Higgs” particle. However, in this case we must face up to the challenge of calculating the \( \rho \) parameter.

8 Conclusion

In this talk, we have put forward a scenario for how the huge hierarchy in energy scale comes about between a supposed fundamental scale, taken as the Planck scale, and the electroweak scale, meaning the scale of the \( W \) and \( Z^0 \) particles and the Higgs particle etc. This consists of introducing a fine-tuning postulate – the Multiple Point Principle – according to which there are many different vacua, in each of which the cosmological constant or energy density is very small. In this way our main fundamental assumption is that the cosmological constant problem is solved, in some way or other, several times. The remarkable result of the present article is that, as well as fine-tuning the cosmological constants, this principle can lead to a solution of a separate mystery, namely of why the electroweak scale of energy is so low compared to the Planck scale. This problem, which is separate from but closely related to the technical hierarchy problem, gets solved in our scenario.
to the degree that we even obtain a crude value for the logarithm of the large scale ratio. We even get a suggestive explanation for why, compared to its logarithmic distance from the Planck scale, the electroweak scale is relatively close to the strong scale, $\Lambda_{QCD}$. We, of course, have to input the large ratio of the Planck to QCD scales, in the form of the value of the QCD coupling constant at the fundamental scale.

In our scenario the pure Standard Model is assumed to be valid up close to the Planck scale, apart from a possible minor modification at the neutrino see-saw scale. We then postulate that there are just three vacuum states all having, to first approximation, zero energy density. In addition to specifying information about the bare cosmological constant, this postulate leads to two more restrictions between the parameters of the Standard Model. They are, in principle, complicated relations between all the coupling constants and masses and it is non-trivial to evaluate their consequences. However we took the values of the gauge coupling constants, which are anyway less crucial, from experiment and these two relations then gave values for the top quark Yukawa coupling constant at the electroweak scale and the “fundamental” scale respectively: $g_t(\mu_{\text{weak}}) \approx 1.3$ and $g_t(\mu_{\text{fundamental}}) \approx 0.4$.

The main point then is that we need an appreciable running of the top quark Yukawa coupling, in order to make the two different values compatible. That is to say we need a huge scale ratio, since the running is rather slow due to the smallness of the Standard Model coupling constants in general from the renormalisation group point of view. This is our suggested explanation for the mysterious huge hierarchy found empirically between the Planck and electroweak scales. Indeed it even leads to an approximately correct value for the logarithm of the huge scale ratio!

It is crucial for our scenario that there should exist the possibility of a phase with a certain bound state condensing in the vacuum. The existence of such a bound state is a priori a purely calculational problem, in which no fundamentally new physics comes in. We suggest that this bound state should be composed of 6 top quarks and 6 anti-top quarks held together by Higgs exchange and, maybe to some extent, also by the exchange of the bound state itself – in a bootstrap-like way. If we live in the vacuum without a bound state condensate, it would be difficult to obtain direct experimental evidence for the bound state. However if we live in the vacuum with a bound state condensate, which actually seems to be the most likely situation in our scenario, it should be possible to see the effects of this condensate. There should then be a significant mixing between the bound state and the Higgs
particle. This implies the existence of two physical particles, sharing the coupling strength and having the same decay branching ratios as the conventional Standard Model Higgs particle. The resulting effective 2 Higgs doublet model deviates from supersymmetry inspired models, by both “Higgs” particles having the same ratio of the couplings to the $-\frac{1}{3}$ charged quarks and the $\frac{2}{3}$ charged quarks. This distinguishing feature puts a high premium on being able to detect the charm anti-charm quark decay modes as well as the bottom anti-bottom quark decays of Higgs particles at the LHC. It should also be possible to calculate the contribution of the bound state to the $\rho$ parameter, but this seems to be rather difficult in practice.

At present the strongest evidence in favour of our scenario is that the experimental top quark Yukawa coupling constant is, within the crude accuracy of our calculations, in agreement with the value at which the phase transition between the two vacua should take place. If this agreement should persist with a more accurate calculation of the phase transition coupling, it would provide strong evidence in support of our scenario.

There is, of course, a need for some physical mechanism underlying our model, which could be responsible for the needed fine-tuning. It seems likely that some kind of non-locality, through space-time foam or otherwise, is needed.

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References

[1] D.L. Bennett, C.D. Froggatt and H.B. Nielsen, Proceedings of the 27th International Conference on High Energy Physics, p. 557, ed. P. Bussey and I. Knowles (IOP Publishing Ltd, 1995); Perspectives in Particle Physics ’94, p. 255, ed. D. Klabin, I. Picek and D. Tadić (World Scientific, 1995) [arXiv:hep-ph/9504294].

[2] E.I. Guendelman, Mod. Phys. Lett. A 14, 1043 (1999) [arXiv:gr-qc/9901017].

[3] N.C. Tsamis and R.P. Woodard, Ann. Phys. 238, 1 (1995).
[4] S. Weinberg, Sources and detection of dark matter and dark energy in the universe: Proceedings, ed. D. Cline Springer Verlag, 2001 [arXiv:astro-ph/0005265].

[5] C.D. Froggatt and H.B. Nielsen, Surv. High Energy Phys. 18, 55 (2003) [arXiv:hep-ph/0308144].

[6] C.D. Froggatt, L.V. Laperashvili and H.B. Nielsen, Hierarchy Problem and Multiple Point Principle (in preparation).

[7] C.D. Froggatt, These Proceedings [arXiv:hep-ph/0312220].

[8] C.D. Froggatt and H.B. Nielsen, Phys. Lett. B 368, 96 (1996) [arXiv:hep-ph/9511371];
C.D. Froggatt, H.B. Nielsen and Y. Takanishi, Phys. Rev. D 64, 113014 (2001) [arXiv:hep-ph/0104161].