A THEORETICAL CASE FOR NEGATIVE MASS-SQUARE FOR SUB-eV PARTICLES

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Electroweak gauge bosons have masses of the order of $10^2$ GeV/$c^2$, while masses of additional bosons involved in gravito-electroweak unification are expected to be still higher. These are at least eleven orders of magnitude higher than sub-eV range indications for neutrino masses. Under these circumstances we suspect that the sub-eV particles are created in a spacetime where gravitational effects of massive gauge bosons may become important. The question that we thus ask is: What is the spacetime group around a gravito-electroweak vertex? Modeling it as de-Sitter we find that sub-eV particles may carry a negative mass square of the order of $-\left(\frac{3}{8\pi^3}\right)\left(\frac{M_{\text{unit.}}}{M_{\text{Planck}}}\right)^4M_{\text{Planck}}^2$. Neutrino oscillation data then hints at $30 - 75$ TeV scale for $M_{\text{unit.}}$, where $M_{\text{unit.}}$ characterizes gravito-electroweak unification scale.

1. Introduction

One of the lessons from E. P. Wigner’s early work[1] is that the notion of mass is not an arbitrary physical construct but takes its origin from constancy of speed of light for all inertial observers. The latter implies description of physical states in terms of the Casimir invariants associated with the Poincaré group:

$$C_1 = P_\mu P^\mu, \quad C_2 = W_\mu W^\mu,$$

(1)

with Pauli-Lubanski pseudovector, $W_\mu$, defined as

$$W_\mu = -\frac{1}{2}f_{\mu\rho\sigma}J^{\rho\sigma}P^\sigma.$$  

(2)

Here we use the notation of Ref. [2]. Each representation space is then characterized by eigenvalues of these Casimir operators. Representation spaces of the type $(j, 0) \oplus (0, j)$ are characterized by

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(a) Positive definite mass, and
(b) Single spin-$j$

while spaces of the type $[(j,0) \oplus (0,j)](1/2,1/2)$ carry interpretation of

(i) Positive definite mass, but
(ii) Indefinite/multiple spin.

Attempts to force a single-spin interpretation – as for Rarita-Schwinger framework – result in well-known problems. Efforts to implement a single-spin interpretation on $[(j,0) \oplus (0,j)](1/2,1/2)$ spaces are akin to insisting on a “particle” interpretation for the Dirac’s $(1/2,0) \oplus (0,1/2)$ representation space by imposing a covariant constraint which throws away the “negative energy” sector, i.e., the antiparticles. Or, at least this is a view we have put forward in Refs. [3, 5].

Now, with the discovery of massive gauge bosons of electroweak interactions a new situation has arisen. This, as we shall now argue, may question positive/real definiteness of mass for sub-eV particles.

The massive gauge bosons have masses of the order of $10^2$ GeV/$c^2$. These are at least eleven orders of magnitude higher than sub-eV range indication for neutrino masses. Our thesis arises from the possibility that the sub-eV particles are created in a spacetime where gravitational effects of massive gauge bosons may become important.

The question that we thus ask is: What is the spacetime group around a gravito-electroweak vertex? In the context of Refs. [18, 19], if we impose the requirements of (a) spherical symmetry, (b) dominant energy condition for a source term, (c) regularity of density, and (d) finiteness of mass; then, the answer is de Sitter-Schwarzschild geometry. Thus, in the interaction region the spacetime symmetry group is de Sitter. This may be taken as our fundamental working assumption.

For the de Sitter-Schwarzschild case, the stress-energy tensor evolves smoothly from de Sitter vacuum $T_{\mu\nu} = \rho_0 c^2 g_{\mu\nu}$ at $r = 0$ to Minkowski vacuum $T_{\mu\nu} = 0$ at infinity. Here $\rho_0$ is the mass density at the origin, and shall be identified with the gravito-electroweak scale. The induced metric is given by

$$ds^2 = \left(1 - \frac{2GM(r)}{c^2r}\right)c^2dt^2 - \left(1 - \frac{2GM(r)}{c^2r}\right)^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2,$$

with

$$M(r) = 4\pi \int_0^r \rho(x)x^2 dx.$$  \hspace{1cm} (3)

This approximation neglects spin of the massive gauge bosons which may, when properly accounted for, may be responsible for explaining neutrino mixing matrix.
For any density profile satisfying the requirements (b)-(d) enumerated above, asymptotic behavior in the $r \to 0$ region is dictated by (b) and is de Sitter vacuum:

$$ds^2 = \left(1 - \frac{r^2}{r_0^2}\right)c^2dt^2 - \left(1 - \frac{r^2}{r_0^2}\right)^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2,$$

with

$$r_0^2 = \frac{3c^2}{8\pi G\rho_0}.$$  \hspace{1cm} (6)

For $r \to \infty$, the asymptotic is Schwarzschild

$$ds^2 = \left(1 - \frac{2Gm}{c^2r}\right)c^2dt^2 - \left(1 - \frac{2Gm}{c^2r}\right)^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2,$$

where $m = M(r \to \infty)$ is the ADM mass.

2. Negative mass-square for sub-eV particles

We now envisage that a sub-eV particle, which we may think of as neutrino to be concrete, is created in the de Sitter region and later propagates to $r \to \infty$ (i.e., to $r \gg r_0$ region) where spacetime is Minkowskian to a good approximation. In the creation region, such a particle is characterized as an eigenstate of the de Sitter Casimir invariants, $|I_1', I_2'\rangle$. The $I_1', I_2'$ are eigenvalues, respectively, of the de Sitter Casimir operators $^{21}$:

$$I_1 = \Pi_\mu \Pi^\mu - \frac{1}{2r_0^2}J_{\mu\nu}J^{\mu\nu},$$

and $I_2$ (see, Ref. $^{21}$, for its definition). In going from Poincaré to de Sitter symmetries, the notion of mass must undergo an unavoidable change. To investigate this modification, we shall concentrate on $I_1$ only. The $\Pi_\mu$ is defined as,

$$\Pi_\mu = \left(1 + \frac{r^2 - c^2t^2}{4r_0^2}\right)P_\mu + \frac{1}{2r_0^2}x^\nu J_{\mu\nu}. $$

In the interaction region $r^2 - c^2t^2 \ll r_0^2$, $^c$ So, $I_1$ approximates to:

$$I_1 \approx P_\mu P^\mu - \frac{1}{r_0^2}(J^2 - K^2).$$

Or, equivalently

$$I_1 \approx C_1 - \frac{1}{r_0^2}(J^2 - K^2),$$

where, we remind, $C_1$ is the first Casimir operator of the Poincaré group. Eigenvalue of $C_1$, up to a multiplicative factor of $c^2$, is identified with square of the mass of the particle, $\mu^2$.

$^b$Note that $\eta_{\mu\nu}$ of Ref. $^{21}$ and that used here differ by a minus sign.

$^c$It shall be confirmed explicitly below towards the end of next section.
In order to study implications for sub-eV neutrinos, we now evaluate the dragged $I_1$ for $(1/2,0)$ and $(0,1/2)$ representations spaces. The right handed and left handed fields inhabit these spaces, respectively. For the $(1/2,0)$ representation space, we have

$$J = \frac{\hbar}{2} \sigma, \quad K = -i\frac{\hbar}{2} \sigma,$$

while for the $(0,1/2)$ representation space,

$$J = \frac{\hbar}{2} \sigma, \quad K = +i\frac{\hbar}{2} \sigma.$$  \hfill (13)

This, immediately yields:

$$I_1 \approx P^\mu P_{\mu} - \frac{\hbar^2}{2r_0^2} \sigma^2.$$  \hfill (14)

Its eigenvalues are:

$$I'_1 = \mu^2 c^2 - \frac{3\hbar^2}{2r_0^2}.$$  \hfill (15)

It is now explicit that the notion of mass is modified in going form one spacetime symmetry group to another. As we shall shortly see, this modification allows for negative mas-square for sub-eV particles if gravito-electroweak unification occurs at TeV scales.

This is the central result of this essay and may offer a natural explanation for certain anomalous results which have come to be known as “negative mass squared problem” for $\nu_e$ \cite{23, 27} and $\nu_\mu$ \cite{28, 30} even though, as outlined in the Addenda, efforts in data analysis tend toward imposing by hand the requirement of physical $m^2_\nu > 0$.

Thus the negative mass-squared values for sub-eV particles, and neutrinos in particular, may be expected to be governed by parameter:

$$m^2_{\text{neg.}} = -\frac{3\hbar^2}{2r_0^2}.$$  \hfill (16)

3. Hint for a TeV scale gravito-electroweak unification

If $\rho_0$ is identified with an (yet unknown) electroweak-gravitation mass scale $M_{\text{unif.}}$, then, on recalling the definition of $r_0$ from Eq. \cite{9}, we have

$$m^2_{\text{neg.}} = -\frac{4\pi G \hbar^2}{c^4} \rho_0,$$  \hfill (17)

\hfill \footnote{See, Ref. \cite{13}, for the definition of a dragged Casimir. Also, recall that, $J_{ij} = -J_{ji} = \epsilon_{ijk} J_k$ and $J_{0i} = -J_{0i} = -K_i$, with each of the $i, j, k$ taking the values 1, 2, 3. The $J$ are then generators of Lorentz rotations and $K$ are generators of Lorentz boosts.}

\hfill \footnote{Note, it corrects Ref. \cite{23}.}
\[-\frac{4\pi G h^2}{c^4} \left( M_{\text{unif.}} \left/ \left[ \left( \frac{4\pi}{3} \right) \left( \frac{2\pi \hbar}{M_{\text{unif.}} c} \right)^3 \right] \right. \right), \quad (18)\]

\[-\frac{3}{8\pi^3} \frac{G}{\hbar c} M_{\text{unif.}}^4. \quad (19)\]

Identifying, $\sqrt{\frac{\hbar c}{G}}$ with $M_{\text{Planck}}$, the above expression becomes,

\[m_{\text{neg.}}^2 = -\frac{3}{8\pi^3} \left( \frac{M_{\text{unif.}}}{M_{\text{Planck}}} \right)^4 M_{\text{Planck}}^2 \quad (20)\]

If $M_{\text{unif.}}$ is set to be 100 GeV, i.e. of the order of masses for electroweak gauge bosons $W^\pm$ and $Z$, one immediately sees that $m_{\text{neg.}}^2 \approx -8.4 \times 10^{-15} \text{eV}^2$. Existing data on neutrino masses rules out this identification because it is natural to expect that $m_{\text{neg.}}^2 \sim -\Delta m^2$. Where, $\Delta m^2$ as derived from atmospheric and solar neutrino data is $\Delta m^2_{\text{ATM}} = 2.5 \times 10^{-3} \text{eV}^2/[c^4], \Delta m^2_{\text{SOL}} = 6.9 \times 10^{-5} \text{eV}^2/[c^4]$.

However, $(M_{\text{unif.}}/M_{\text{Planck}})^4$ sensitivity of $m_{\text{neg.}}^2$ suggests a TeV scale for $M_{\text{unif.}}$. This can be seen explicitly by setting $m_{\text{neg.}}^2 \sim -\Delta m^2$. Then $M_{\text{unif.}}$ reads:

\[M_{\text{unif.}} \sim \left( \frac{8\pi^3}{3} \frac{\Delta m^2}{M_{\text{Planck}}^2} \right)^{1/4} M_{\text{Planck}} \quad (21)\]

The atmospheric neutrino data implies a $M_{\text{unif.}} \approx 74 \text{ TeV}$ while the cited solar neutrino mass-squared difference yields, $M_{\text{unif.}} \approx 30 \text{ TeV}$. These $M_{\text{unif.}}$, correspond, respectively, to $r_0 \approx 5 \times 10^{-4} \text{ cm}$, and $r_0 \approx 3 \times 10^{-3} \text{ cm}$.

4. Conclusion

In view of these considerations, and additional and earlier work of Simicevic,\textsuperscript{37} it appears that to discard experimentally indicated $m_{\nu_e}^2 < 0$ for electron and muon neutrinos may be unwise. The best route may be to look at the data and experiments afresh and allow that it may indeed be that

\[m_{\nu_e}^2 < 0, \quad m_{\nu_\mu}^2 < 0. \quad (22)\]

At the same time a global analysis of data on neutrinos – specifically data on neutrino oscillations, data on neutrino-less double beta decay, data on the end point of tritium beta decay, and $\pi^+ \rightarrow \mu^+ + \nu_\mu$ – must be done to allow for $m_{\nu_e}^2 < 0$. If negative mass-square is finally established for a sub-eV particle it would be necessary to device experiments which may distinguish between various proposals\textsuperscript{24–35} which suggest, or attempt to accommodate, negative mass squares.

Addenda

We think that the following additional information may be helpful to the reader of our essay:

(Anti)Electron neutrino mass

The latest publication on $m_{\nu_e}^2$ at the time of sending this essay to IJMPD seems
to be from Lobashev. It gives \( m_{\mu e}^2 = -2.5 \pm 2.5 \pm 2.0 \text{ eV}^2 \). Particle data group gives: \( m_{\mu e}^2 = -2.5 \pm 3.3 \text{ eV}^2 \) and only includes results of Lobashev and Weinheim with the following two observations:

(1) The data were corrected for electron trapping effects in the source, eliminating the dependence of the fitted neutrino mass on the fit interval. The analysis assuming a pure beta spectrum yields significantly negative fitted \( m_{\nu}^2 \approx -(20-10) \text{ eV}^2 \). This problem is attributed to a discrete spectral anomaly of about \( 6 \times 10^{-11} \text{ intensity with a time-dependent energy of } 5-15 \text{ eV below the endpoint} \). The data analysis accounts for this anomaly by introducing two extra phenomenological fit parameters resulting in a best fit of \( m_{\nu}^2 = -1.9 \pm 3.4 \pm 2.2 \text{ eV}^2 \) which is used to derive a neutrino mass limit. However, the introduction of phenomenological fit parameters which are correlated with the derived \( m_{\nu}^2 \) limit makes unambiguous interpretation of this result difficult.

(2) We do not use the following data for averages, fits, limits, etc. . . .

\[ m_{\nu}^2 = -130 \pm 20 \pm 15 \text{ eV}^2 \ldots \text{Robertson} \]
\[ m_{\nu}^2 = -147 \pm 68 \pm 41 \text{ eV}^2. \]

In order to put all these matters in perspective one of us has taken liberty of asking one of the early experimentalists about his views on his own experiment. His candid reply reads,

We still have no clue why there was an excess of counts close to the beta-endpoint in our spectra. The experiment is “mothballed” since 1993 and has not been operated since. We were not able to come up with a satisfactory explanation for the bump at the end of the spectrum. In the meantime, The neutrino group in . . . improved their spectrometers. They had initially similar puzzling result, but in the meantime, the neutrino mass extracted from their spectra is consistent with zero or a very small value.

and further strengthens the need for careful experiments without any prejudice in data analysis for \( m_{\nu}^2 > 0 \).

**Muon neutrino mass**

Additional hint for negative mass-square for neutrino masses resides in two possible values — determined by which value of pion mass one uses — for \( m_{\mu e}^2 \).

\[
\text{Solution B: } -0.016 \pm 0.023 \text{ MeV}^2.
\]
\[
\text{Solution A: } -0.143 \pm 0.024 \text{ MeV}^2.
\]

\(^{1}\)Taking \( m_{\mu e}^2 \sim -10^2 \text{ eV}^2 \) yields \( M_{\text{unif}} \sim 10^3 \text{ TeV} \).

\(^{8}\)Taking \( m_{\mu e}^2 \sim -0.143 \text{ MeV}^2 \) yields \( M_{\text{unif}} \simeq 2.0 \times 10^5 \text{ TeV} \).
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