HAIRS ON THE UNICORN
Fine structure of monopoles and other solitons

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Abstract

Intrinsically stable or ‘fundamental’ solitons may be decorated with conserved charges which are pieces of those carried by elementary particles in the same medium. These ‘hairs’ are always significant in principle, and in the strong-coupling regime (where solitons and particles exchange roles) may become major factors in dynamics.

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I. Monopoles and unicorns

Long ago my developing fascination with magnetic monopoles led to an awareness of a kinship between the monopole and another mythical beast, the unicorn. As we shall see shortly, the monopole (which was my personal entrée into the subject of solitons) occupies a very special place, as the only ‘fundamental’ soliton able to move freely in our (3+1)-dimensional world.

Before looking at the parallel between monopole and unicorn, let us go into another aspect of the monopole. From Dirac’s quantization argument we know that the magnetic charge $g$ of a pole is quantized, and has a superstrong value. Strong coupling is incompatible with point structure of the coupled particle, and so the pole must have internal geometrical dimension greater than its Compton wavelength by a factor of order $g^2$. Because the natural quantum length scale is small compared to the size of the object, this is a powerful indication that one should be able to describe the pole as a classical field configuration, so that quantum consistency conditions have brought us back to classical physics, and hence to view the monopole as a soliton. However, the quantum consistency conditions for pure QED, or even $SU(2) \times U(1)$ electroweak theory, still require quantized $g$, so clearly the charge cannot be spread out over a length scale larger than the minimum for which the standard model is accurate. That is the basis for the claim below that existing or planned particle accelerators won’t have the energy to produce monopoles.

Now to the title:

The unicorn is a wonderful creature. It has many features in common with the monopole, including

1. Origin in medieval Europe

The concept of a single pole, and the recognition that north and south magnetic poles always come paired together, goes back at least to Peter the Pilgrim in 1269. I don’t know a first date for publication on unicorns, but it cannot have been much later. In both cases, one can find earlier but vaguer antecedents in Asia.
2. **Subject of a vast literature**

   Until the last couple of decades, the literature on unicorns surely was larger than that on monopoles. Now the latest SLAC Spires listing contains 1125 titles with the word ‘monopole’, so the balance may have shifted. In any case, interest in both subjects remains strong.

3. **Never confirmed or captured**

   While there have been claims of finding either entity, they have never been substantiated and accepted. Nevertheless, the methods used both in seeking them and in describing the results often have remarkable appeal and show great insight.

4. **Unique unity, not usual duplexity**

   Like ordinary dipole magnets, fascinating and graceful two-horned animals are commonplace, despite the fact that no known principle excludes the one-horned possibility.

5. **Illuminates much about the world**

   By studying a physical theory to see if and how it accommodates monopoles, one can learn a great deal about its structure, and about subtle consistency conditions which otherwise might be overlooked. In a similar way, contemplating how people react to the concept of the unicorn teaches a great deal about human nature, if not about animal biology.

6. **Beautiful**

   It is hard to imagine someone who could look at the Bayeaux tapestries and not be entranced by the loveliness of the unicorn. The perfect spherical symmetry of the monopole, and the delicacy of its interaction with electrical charges, has a charm which only seems to increase with acquaintance.

7. **Poor cousins exist**

   A monopole can be imitated in several ways. Coulomb used the interaction between nearby ends of two long magnets whose other ends were widely separated to establish the inverse square law for magnetic poles with higher accuracy than the corresponding law
for electric charges. Poincaré [3] realized that the interaction of an electric charge with a magnetic pole gives the same equation of motion as for the purely mechanical system in which a rapidly spinning top is kept aligned with the radial direction from some center to the location of a massive particle. For the unicorn, second-class imitations which come to mind include the rhinoceros and the swordfish.

8. Instantly recognizable

A monopole could be identified unambiguously by its ability to change the quantized flux threading a superconducting loop. For the unicorn, to see it is to know it. Of course, in both cases one must pay attention to effects which might be fake indicators, but that is true in every aspect of life as well as science.

9. Still hope of discovery

For monopoles, we know from the success of the standard model that any conceivable pole must have a mass greater than about 100 TeV, so they won’t be found at particle accelerators in the foreseeable future. As theories exist which are consistent with all observations at current energies but which have monopoles at higher energies, they still could appear – there might be a very low flux of massive primordial monopoles detectable on earth. For unicorns, the possibility of finding one in some remote corner seems exactly that – remote! However, new breeding techniques with sophisticated genetic manipulation conceivably might produce them. Ironically, the best hope for monopoles lies in observation, but for unicorns in experimentation!

II. ‘Fundamental’ and ‘complementary’ solitons

Solitons have become an important part of physics, and yet their role remains a bit confusing. At one level, things are quite simple: a soliton is simply a massive object which can be described accurately as a classical nonlinear field configuration, perhaps absolutely stable or perhaps metastable. This is an extremely attractive notion, because it may be the only alternative to introducing a particle as an elementary object, a quantum of some field. With the soliton, the relation between particle and field suddenly develops
a new possibility, where the particle is constructed from the field rather than vice versa. However, not all solitons are equal:

Historically one of the first soliton-particles was the skyrmion – a description of the nucleon as a map with winding number unity from ordinary space onto the three-dimensional surface of a sphere in four abstract Euclidean dimensions[^4]. Skyrme realized that this system could be quantized either with integer spin and isospin or half-integer spin and isospin. The latter of course is what makes it a candidate to describe the nucleon. It also is an example of fractional soliton charge, as the soliton possesses isospin eigenvalues which are half-integer, unlike the integer values for the elementary pion field from which it is formally constructed. Further, it is a fermion where the pions are bosons. Can you imagine a more stunning way to get something for nothing? One might think scepticism about this ‘free lunch’ explains why Skyrme got so little response when he first proposed the skyrmion, but the real reason seems to have been a near-total incomprehension of the concept that a particle could be described as a soliton at all.

One can only wonder what would have happened if there had been a more positive immediate reception to Skyrme's idea. As we now know, it has an astonishing degree of success in describing the properties and interactions of nucleons with pions and even with each other. Conceivably such successes could have slowed down substantially the developments which led to QCD, now recognized as the underlying theory to which Skyrme dynamics can be a superb approximation in the domain of long-distance and low-energy interactions. In this setting, it clearly is worthwhile to exploit the skyrmion for all it is worth, while still recognizing that ultimately it is a derivative from the fundamental theory. In particular, the fractional charge and novel spin-statistics are not created by the Skyrme dynamics, only compatible with it, instead being based on the foundation of elementary quarks, with the peculiar charges and statistics resulting because in QCD the number of quark colors happens to be odd[^5]. Michael Mattis has given here a beautiful exposition of the subtle but tight connection between the Skyrme and QCD actions[^6].
This example of a very important ‘derivative’ soliton naturally leads one to ask “Could there be such a thing as a ‘fundamental’ soliton, and what would that mean?” At first this seems like a contradiction in terms, because the soliton is described with a more or less elaborate structure expressed through field configurations, so that it would appear in principle to be a composite. However, if the soliton has a unique type of charge, and if no conceivable variation of the high-energy, short-distance dynamics of the fields used in its description could destroy it, then one would be justified in calling it ‘fundamental’.

The interesting point here is that only a few solitons can pass that test [7]. In one space dimension a ‘kink’ has this property, meaning that some field can minimize vacuum energy density for more than one value of the field, and the kink interpolates between different allowed vacuum values at $x = \pm \infty$. In three space dimensions a magnetic monopole again passes the test, because the Gauss law for magnetism assures that an isolated pole cannot be created or destroyed. Such a long-range field is the only known way to assure conservation of a charge in more than one space dimension, and as electromagnetism is the only field of that sort we have, the monopole really is the only option. [An electrically-charged soliton would be stable if its electric charge were not an integer multiple of the smallest elementary charge unit, but in any theory which at least potentially could contain magnetic monopoles, this would be impossible to contrive.] The focus in the remainder of this discussion will be on fundamental solitons, and since we experience three space dimensions in our world, therefore especially on monopoles.

There is a common terminology which needs to be related to the terms introduced above. Solitons may be described as ‘topological’ or ‘nontopological’. The fundamental solitons all are topological. This is obvious for those in one space dimension. For a monopole, it follows because a Dirac string [1] emanates from the pole and can only terminate on an antipole, or (in different language) on every closed surface surrounding the pole the gauge field corresponds to a nontrivial fiber bundle [8]. On the other hand, as discussed above, the skyrmion certainly is a topological soliton, even though it is
not fundamental. Another important term is ‘duality’: Electric-magnetic duality allows description in the weak-coupling domain where purely electric particles are treated as elementary, and monopoles are treated as solitons, while in the strong-coupling domain the roles of the particles and solitons are interchanged. From this point of view, ‘complementarity’ is the best term to describe the relationship of the Skyrme and QCD pictures. The one gives a good first-order description of low-energy, long-distance phenomena, while the other does the same for high energies and short distances. However, unlike charges and poles, the two types do not coexist on either scale – it is a matter of one or the other but not both. A striking illustration of complementarity is the hybrid or ‘Cheshire cat’ model, where the inside of the nucleon is a chiral quark bag, and the outside is a Skyrme field configuration [9]. There is no comparable division for a fundamental soliton, emphasizing that the two types are qualitatively different.

III. Fractional and peculiar soliton charges

Jackiw and Rebbi [10] observed that a soliton could carry charges which are pieces of the charges carried by elementary particles. We are talking here about conserved, isolable charges, which have sharp eigenvalues [11, 12, 13, 14]. Evidently, only fundamental solitons could carry such peculiar charges; otherwise, disappearance of a nonconserved soliton would leave the charges with nowhere to go! For the same reason, fundamental solitons and their antisolitons must carry opposite values of the peculiar charges, modulo the units found on an elementary particle. Even for fundamental solitons, one would like to understand the mechanism by which the peculiar charges are acquired, as it cannot be by simple binding of elementary particles to the soliton. What alternative is there? Goldstone and Wilczek [15] introduced a systematic way to study this question, by considering a class of theories with an adjustable parameter, such that for some values of this parameter there would be negligible coupling between the particle and the soliton, and therefore no peculiar charge. As the parameter changes slowly, the only way that the conserved charge can arrive is by adiabatic flow in from infinity, and they found quantitatively what the
flow would be for the kinks and monopoles studied by Jackiw and Rebbi, verifying in particular the latter’s claim of half-integer fermion charge. However, in the GW method there is no necessity to reproduce another feature of the original JR discussion, namely a charge conjugation symmetry implying the existence of a zero-energy fermion bound state in the presence of the soliton.

Let us focus on the JR monopole example, which I tried repeatedly but unsuccessfully to disprove, in the process nevertheless learning much which is valuable and correct. Here let me emphasize what still makes this example mysterious, and how one can at least partly penetrate the mystery.

First, let us review what they did. JR [10] considered two examples of a quantum Fermi field coupled to a specific classical, static Bose field configuration (a soliton). In both cases the Dirac equation for the Fermi field includes a single mode with zero frequency, and the rest of the spectrum is completely symmetrical between positive and negative frequencies. Consequently, the state in which the zero-frequency mode is occupied and that in which it is not ought to be charge conjugate to each other. Since they differ by one unit in fermion number \( F \), they should be characterized by \( F = \pm \frac{1}{2} \). The term ‘fermion number’ is potentially confusing, as at least in 3+1 dimensions the number of spin-\( \frac{1}{2} \) particles may possess only integer eigenvalues. However, one could imagine a new kind of ‘photon’ coupled to all fermions with a weight 1 and to all antifermions with a weight -1. Then the value of the ‘fermion charge’ measured by this photon could in principle be fractional for some special object which polarizes the fermion vacuum in a particular way. Thus, fermion charge coincides with fermion number for collections of elementary fermions, but may have no simple relation to fermion number for one of the special objects, the solitons.

For the first JR example, where the Fermi field is Yukawa-coupled to a sine-Gordon soliton in 1 + 1 dimensions, the treatment of Fermi and Bose fields can be made symmetrical by bosonization of the Fermi field. This makes it possible to account systematically for possible back-reaction of the fermion on the boson degrees of freedom, and confirms
the suggestion that the object carries half-integer $F$ \cite{16}.

For the monopole example, JR also noted a case where an extra label distinguishes two zero modes, one corresponding to spin $+\frac{1}{2}$ in some direction and one to spin $-\frac{1}{2}$. This is an example of charge \textit{dissociation} - such a soliton may exist in any of four nearly degenerate states, characterized by $F = \pm 1$, spin $S = 0$ (spin singlet), or $F = 0, S = \frac{1}{2}$ (spin doublet). Two charges (fermion ‘number’ and spin) which characterize a free fermion have been ‘torn apart’ so that soliton states carry one charge or the other but not both. Note that in this example the states with $F = \pm 1$ should behave like bosons, in which case those with $F = 0$ would behave like fermions! Su, Schrieffer, and Heeger \cite{17} independently discovered charge dissociation in their model of polyacetylene, with zero modes for two species of fermion, electrons with spin up and electrons with spin down.

Recognizing that peculiar charge must be a vacuum polarization effect allows us to make a link with a long-familiar phenomenon: In the presence of a medium, the charge of an elementary excitation may be renormalized. For example, an electron in an insulator with dielectric constant $\epsilon$ carries a charge $-e/\epsilon$, a fraction of the charge residing on that same electron in vacuum. One would not say that only a fraction of an electron is present in the medium, just that the manifestation of the electron’s presence is different from what it would be in vacuum.

Recently it has been argued that the quasiparticles of the fractional quantum Hall effect (FQHE) should be treated as electrons dressed by the FQHE medium, so that their fractional charge is fractional in exactly the same sense as for an electron inside an insulator \cite{18}. There still is something special about these quasiparticles, which can be described in two alternate ways: 1) The Aharonov-Bohm or Lorentz-force charge is renormalized by the same factor as the local or Gauss-law charge \cite{13}. In ordinary insulators, only the latter charge is renormalized. 2) The electromagnetic field strength is renormalized by the same factor as the Gauss-law charge \cite{18}. This renormalization would be undefinable in ordinary three-dimensional systems, but here refers to the ratio of the
effective field acting on excitations moving in the FQHE layer to the field as measured in standard ways just outside the layer. Whichever description one might prefer, the quasiparticles are the unique light electrically charged excitations in the FQHE medium, so that their charge can be considered fractional only with respect to elementary excitations in other media, such as the QED vacuum.

IV. Conditions for integer $F$

Accepting that fractional soliton charge is restricted to monopoles and kinks, are there any further limitations on fractional values in these cases? To address this issue, at least with respect to fermion charge $F$ in situations with charge conjugation symmetry like that of Jackiw and Rebbi, let us begin by recalling the

Theorem of Jackiw and Schrieffer: A soliton whose spectrum is invariant under a charge conjugation symmetry $C$ which reverses the sign of $F$ may have integer $F$, or half-integer $F$, but no other fractional value is allowed.

Proof: For an isolated soliton, the only way $F$ (assumed conserved) can change is by scattering processes in which the number of fermions incident differs from the number emerging. This means that two allowed values of $F$ must differ by an integer. $C$ symmetry implies that for every allowed $F$, $-F$ also is allowed. Therefore the difference $2F$ must be an integer, and $F$ must be either an integer or a half-integer. If one allowed $F$ is an integer (half-integer), so must be all the others, since they differ by integers.

This theorem shows that to answer our question we need find only what conditions must supplement charge conjugation symmetry to exclude half-integer eigenvalues. It also suggests a strategy for determining those conditions. If absorbing a fermion changes other conserved charges of the soliton as well as $F$, then extra consistency requirements may follow. Therefore let us proceed by attending to other charges carried by fermions. The problem naturally divides fundamental solitons into two classes, (A) magnetic monopoles (interacting with fermions whose electric charge has the minimum magnitude $e$ allowed by the Dirac quantization condition [1]), and (B) all others.
The reason for this division is that according to the spin-statistics connection a fermion in $3 + 1$ dimensions must carry half-integer spin. However, for a minimal electric charge in the field of a magnetic monopole there is an extra electromagnetic angular momentum which also is half-integer. Consequently, it is possible for the monopole to absorb a fermion with the appropriate magnitude of electric charge without absorbing angular momentum. For any other kind of soliton, or for a monopole interacting with fermions whose electric charges are even multiples of the minimum Dirac unit, absorbing a fermion requires the absorption of a half-integer unit of angular momentum. Therefore spin is a suitable candidate for the extra quantum number associated with depositing a fermion on a soliton for all cases in category B. For category A, the only apparent (and inevitably open) option for the extra quantum number is electric charge. Let us quote the results, leaving the rather long proof for case A to be presented elsewhere [20].

**Integer $F$ Theorem A:** If the electromagnetic vacuum angle $\theta$ vanishes, then a minimal-strength Dirac magnetic monopole symmetric under fermion conjugation $C$ must carry integer $F$.

For all other solitons, including all condensed-matter defects, we have

**Integer $F$ Theorem B:** An object, other than a minimal strength Dirac magnetic monopole, symmetric under a unitary charge conjugation symmetry $C$, with fermion number and spin both sharp, must carry integer $F$.

**Proof:** We are omitting the case of Theorem A, so that adding a fermion to the object does change its spin by a half-integer. $C$ is assumed to commute with rotations, so that the unitarity of $C$ implies its commutation with angular momentum and spin. Therefore charge conjugate states must have the same spin. This means that $2F$ must be an even integer, as otherwise the states would differ in spin by a half-integer. Hence $F$ is an integer.

The essence of the proof for Theorem A [20] is that, if $C$ reverses electric charge $Q$ as well as $F$, then $F = \frac{1}{2}$ implies $Q = \frac{1}{2}$. However, fractional $Q$ is equivalent to fractional
vacuum angle \[21\]. If \(C\) does not reverse \(Q\), then the same reasoning as in Theorem B precludes fractional \(F\).

Except for even-charge monopoles, all the candidate solitons for charge \(\frac{1}{2}\) under Theorem B must be kinks in one space dimension. Examples of these are discussed in \[22, 20\].

V. Questions about and applications of the Jackiw-Rebbi monopole

In the semiclassical approximation studied by JR \[10\], there are several symmetries, in particular a charge conjugation symmetry \(CP\) which reverses both \(F\) and \(Q\), and a symmetry \(G\) which only reverses \(F\). Neither of these can be a simple symmetry if there is a single zero mode, because the two states connected by the zero mode must differ in sign of the manifestly conserved operator \(U = e^{2\pi i T_a}\), where \(a\) labels an arbitrary axis in isospin space \[23\]. This does not invalidate the JR value for \(F\), but draws attention to a dilemma for the fully quantized description. If the mass of the fermion is very small compared to that of the vector boson which determines the characteristic radius of the monopole, then the interaction of the fermion with the pole can be described in terms of chiral ‘boundary conditions’ on the Dirac wave functions at the location of the pole, at least if electric Coulomb interactions are negligible. If there is to be exactly one zero-energy bound state, then is \(u\) or \(d\) bound?

Seiberg and Witten \[23\], provide an important clue to solving this mystery, once again by looking at a constraint associated with a very long distance scale, as in the work of Goldstone and Wilczek \[15\]: Suppose that the fermion isodoublet has both an isoscalar and an isovector contribution to its mass, such that (say) the \(u\) fermion has negligible mass compared to \(d\). Then the boundary conditions at the monopole combine with requirements of energy conservation to assure that, for small negative \(u\) mass, \(u\) but not \(d\) has a zero-energy bound state. Further, as the \(u\) mass goes from slightly positive to slightly negative, there is an inevitable jump \(\Delta \theta = \pm \frac{\pi}{2} mod \pi\) in the vacuum angle. Thus, if that angle were zero before, now all the conditions are obeyed for two states with \(F = Q = \pm \frac{1}{2}\). Evidently, a similar jump in \(\theta\) should occur if instead the \(d\) mass went
through zero, and this time \( d \) would have a zero mode. Precisely at the point where the two masses are equal in magnitude but opposite in sign, we have the ambiguous condition of the original JR system. It seems likely that if these masses are small compared to the vector boson mass this point is a cusp, where the structure jumps, in such a way that electric charge is exchanged between the bosonic dyon rotor degree of freedom and the fermion vacuum polarization. However, if the fermion and boson masses are comparable, then the transition probably is smooth. The difficulty of this case comes because there is no large length scale which can be used to simplify the problem. These issues are addressed more fully in [20].

VI. Conclusions

The fact that fundamental solitons indeed can carry pieces of the charges of elementary particles is interesting, but does it have wider implications? At least one important positive indication is the mutual reinforcement of JR fermion zero modes with the concepts of electric-magnetic duality and supersymmetry [24]. Further, the monopole’s fermion charge is significant in the strong-coupling domain where a monopole condensate may emerge to enforce both color confinement and chiral symmetry breaking at the same time [23]. These developments are on top of results for condensed matter physics, illustrated in [17, 22]. Even though fundamental solitons are few, their role (and that of their peculiar charges) in the conceptual understanding of physics is large.

VII. Acknowledgements

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