Inflationary and Deflationary Branches in Extended Pre–Big Bang Cosmology

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Abstract

The pre–big bang cosmological scenario is studied within the context of the Brans–Dicke theory of gravity. An epoch of superinflationary expansion may occur in the pre–big bang phase of the Universe’s history in a certain region of parameter space. Two models are considered that contain a cosmological constant in the gravitational and matter sectors of the theory, respectively. Classical pre– and post–big bang solutions are found for both models. The existence of a curvature singularity forbids a classical transition between the two branches. On the other hand, a quantum cosmological approach based on the tunneling boundary condition results in a non–zero transition probability. The transition may be interpreted as a spatial reflection of the wavefunction in minisuperspace.

PACS Number(s): 98.80.Bp, 98.80.Cq, 98.80.Hw

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1 Introduction

Inflationary cosmology provides a plausible and, in principle, experimentally testable picture for the very early Universe [1]. As well as resolving the well known problems of the hot big bang model – such as the horizon and flatness problems – it results in the quantum mechanical generation of primordial scalar (density) and tensor (gravitational wave) fluctuations [2, 3, 4]. It is widely thought that these perturbations may be responsible for the observed anisotropy in the temperature distribution of the cosmic microwave background radiation [5]. Moreover, the growth of the scalar fluctuations via gravitational instability may have led to the formation of large-scale structure in the Universe. (For recent reviews see, e.g., Refs. [6]).

The defining feature of inflation is the acceleration of the cosmological scale factor, $\ddot{a} > 0$. This is in contrast to the decelerating expansion that is characteristic of the big bang model. In the standard, chaotic inflationary Universe, the acceleration is driven by the potential energy of a self–interacting scalar field [7]. The potential plays the role of an effective cosmological constant, thereby leading to a quasi–exponential expansion of the Universe.

An interesting alternative to the standard inflationary Universe has recently been developed within the context of string theory [8]. It is generically referred to as pre–big bang cosmology. It is known that the tree–level, $(d+1)$–dimensional (super)string effective action exhibits an $O(d,d)$ symmetry [9]. Embedded in this group is a symmetry known as scale factor duality [10]. This symmetry applies when the Universe is spatially flat and homogeneous and leaves the action invariant under the simultaneous interchange $\tilde{\alpha} = 1/a$ and $\tilde{\Phi} = \Phi - 2d \ln a$, where $\Phi$ represents the dilaton field.

Scale factor duality therefore relates expanding cosmologies to contracting ones. Since the theory is also invariant under time reversal, $\tilde{t} = -t$, the contracting branch may itself be mapped onto a new, expanding branch. These two expanding solutions are referred to as the pre– and post–big bang branches, respectively. They are conventionally defined over the negative and positive halves of the time axis and are separated by a curvature singularity at $t = 0$.

Applying the duality transformation simultaneously with time reversal implies that the Hubble expansion parameter $H \equiv d(\ln a)/dt$ remains invariant, $\tilde{H}(-t) = H(t)$, whilst its first derivative changes sign, $\tilde{\dot{H}}(-t) = -\dot{H}(t)$. A decelerating, post–big bang solution – characterized by $\dot{a} > 0$, $\ddot{a} < 0$ and $\dot{H} < 0$ – is therefore mapped onto a pre–big bang phase of inflationary expansion, since $\ddot{a}/a = \dot{H} + H^2 > 0$. The Hubble radius $H^{-1}$ decreases with increasing time and the expansion is therefore superinflationary. It is driven by the kinetic energy of the dilaton field.

Since the two branches are separated by a curvature singularity, however, it is not clear how the transition between the pre– and post–big bang phases might proceed. This is the graceful exit problem of the pre–big bang scenario. Such a problem is characteristic of more general inflationary scenarios that are driven by the dilaton’s kinetic energy [11]. One possible solution is to introduce a suitable dilaton potential.
that modifies the classical solution around $t \approx 0$ \cite{12}. It has recently been shown, however, that a branch change can not occur for a realistic dilaton potential if one is limited to the lowest–order expansion of the string action \cite{13}. An alternative approach would be to employ the techniques of conformal field theory, where all higher–order terms in the action are considered \cite{14}. Unfortunately, however, the appropriate conformal background is presently unknown.

In view of this, it has recently been proposed that the graceful exit problem might be addressed by employing a quantum cosmological approach \cite{15,16}. It is reasonable to suppose that quantum gravitational effects should become important in the high curvature regime and, indeed, the relationship between quantum cosmology and string theory has been considered previously by a number of authors Ref. \cite{17}. The significance of scale factor duality in this approach has also been discussed \cite{18,19} and an $O(d,d)$–invariant Wheeler–DeWitt equation was recently derived by Gasperini, Maharana and Veneziano \cite{13} and by Kehagias and Lukas \cite{20}.

Gasperini et al. considered the minisuperspace model for the spatially, homogeneous Bianchi I Universe \cite{15}. They found that the wavefunction of the Universe could be expanded in terms of plane waves in minisuperspace, where the configurations associated with the pre– and post–big bang branches corresponded to waves moving in opposite directions along the effective spatial coordinates. This is important, because it suggests that a transition between the pre– and post–big bang phases might be possible if the wavefunction undergoes a spatial reflection in minisuperspace.

Motivated by these considerations, we investigate whether the concept of pre–big bang cosmology can be extended beyond the truncated string effective action to include more general dilaton–graviton systems. Theories of this type are interesting in their own right and they also place the results and predictions of string cosmology in a wider setting. We shall consider the vacuum Brans–Dicke theory of gravity \cite{21}, whose action is given by

$$S = \int d^4 x \sqrt{-g} e^{-\Phi} \left[ R - \omega (\nabla \Phi)^2 - 2\Lambda(\Phi) \right]$$ \hspace{1cm} (1.1)

where the metric $g_{\mu\nu}$ has signature $(-,+,+,+)$, $R$ is the Ricci curvature scalar and $g \equiv \det g_{\mu\nu}$. The parameter $\omega$ determines the strength of the coupling between the dilaton and graviton degrees of freedom and is assumed to be a space–time constant. The function $\Lambda(\Phi)$ determines the self–interactions of the dilaton. The truncated string effective action is given by Eq. (1.1) with $\omega = -1$ and constant $\Lambda$.

We will assume that $g_{\mu\nu}$ represents the space–time of the physical Universe and will therefore work in the Jordan frame rather than the Einstein frame. We will consider the spatially flat, isotropic and homogeneous cosmology with a line element $ds^2 = -N^2(t) dt^2 + e^{2\alpha(t)} dx^2$, where $e^{\alpha(t)}$ represents the scale factor of the Universe and is a function of cosmic time $t$ only and $N(t)$ is the lapse function. We further assume that the dilaton field is constant on the surfaces of homogeneity and consider the region of parameter space where $\omega > -3/2$ and $\Lambda(\Phi) \geq 0$. Thus, the weak energy condition is always satisfied.
We begin in Section 2 by considering the special case where the dilaton potential vanishes, $\Lambda = 0$. Such a model provides a useful framework within which the key ideas of duality and branch changing can be discussed. We identify the region of parameter space that leads to a superinflationary, pre–big bang phase. We then employ a generalized scale factor duality to illustrate how the pre– and post–big bang branches are related at both the classical and quantum levels. In particular, it is shown how the two branches correspond to left– and right–moving waves in minisuperspace. A reflection of the wavefunction is only possible, however, if a dilaton potential is included. We therefore introduce a cosmological constant into the model in Section 3 by specifying $\Lambda = \text{constant}$. This model is quantized in Section 4 and the conditional probability for a reflection of the wavefunction is calculated. We then proceed in Section 5 to consider a second model where $\omega = -1/2$ and $\Lambda \propto e^{\Phi}$. This potential plays the role of a cosmological constant in the matter sector of the theory. It is found that the probability for a transition between the pre– and post–big bang branches is also non–zero. We conclude in Section 6.

Units are chosen such that $\hbar = c = 1$.

## 2 Scale Factor Duality and Pre–Big Bang Cosmology in Brans–Dicke Theory

Eq. (1.1) simplifies to

$$ S = \int dt e^{2\alpha - \Phi} \left[ \frac{1}{N} \left( -6\dot{\alpha}^2 + 6\dot{\alpha}\dot{\Phi} + \omega \dot{\Phi}^2 \right) - 2N\Lambda(\Phi) \right] $$

(2.1)

after integration over the spatial variables, where it has been assumed that the spatial sections have finite volume, a boundary term has been neglected and the dilaton has been rescaled $\Phi \rightarrow \Phi - \ln \int d^3x$. It has recently been shown that kinetic sector of this action is invariant under a generalized scale factor duality

$$\alpha = \left( \frac{2 + 3\omega}{4 + 3\omega} \right) \tilde{\alpha} - \left( \frac{2(1 + \omega)}{4 + 3\omega} \right) \tilde{\Phi}$$

$$\Phi = - \left( \frac{6}{4 + 3\omega} \right) \tilde{\alpha} - \left( \frac{2 + 3\omega}{4 + 3\omega} \right) \tilde{\Phi}$$

(2.2)

when $\omega \neq -4/3$ [19]. This discrete symmetry is embedded within a more general, continuous Noether symmetry that exists in the cosmological field equations of Brans–Dicke theory [22]. It reduces to the scale factor duality associated with the truncated string effective action when $\omega = -1$.

The free field model ($\Lambda = 0$) represents the limiting case of a more general class of models in which the dilaton potential becomes negligible near the curvature singularity. It is also relevant to the recently studied kinetic inflationary scenario, where the acceleration of the scale factor is driven by the kinetic energy of the dilaton field.
rather than its potential energy \cite{1,23,24}. The classical field equations derived from action (2.1) for \( \Lambda = 0 \) are given by

\begin{align}
\ddot{\Phi} - \dot{\alpha} \dot{\Phi} + 3 \dot{\alpha} \Phi &= 0 \\
\dot{\alpha}^2 - \dot{\alpha} \dot{\Phi} - \frac{\omega}{6} \dot{\Phi}^2 &= 0
\end{align}

for \( N = 1 \). They admit the power–law solution

\[ e^\alpha = e^{\alpha_0} |t|^{p_\pm}, \quad e^\Phi = e^{\Phi_0} |t|^{3p_\pm - 1} \]

where

\[ p_\pm \equiv \frac{1}{4 + 3\omega} \left[ 1 + \omega \pm \left( 1 + \frac{2\omega}{3} \right)^{1/2} \right] \]  

The integration constants have been chosen so that the curvature singularity is at \( t = 0 \). There are two distinct solutions depending on whether \( p_\pm \) is chosen. It may be verified by direct substitution that the two branches are related by the duality transformation (2.2) and the duality is effectively generated by the simultaneous interchange \( p_+ \leftrightarrow p_- \).

The square root of the Friedmann equation (2.4) implies that the Hubble expansion parameter is given by

\[ H = \frac{\dot{\Phi}(1 \pm f)}{2}, \quad f \equiv [1 + 2\omega/3]^{1/2}. \]

It can then be shown that the scale factor accelerates if and only if \( f \pm 1 < 0 \) \cite{24}. Thus, kinetic inflation can only occur if the negative root is chosen and the coupling constant is negative, \( \omega < 0 \). One may further show that

\[ \dot{H} = -\frac{1}{4} \dot{\Phi}^2 (1 \pm 3f)(1 \pm f) \]

thereby implying that superinflation with \( \dot{H} > 0 \) is possible when \(-4/3 < \omega < 0\). It is this region of parameter space that is of relevance to the pre–big bang scenario. The time–reversed negative root \( (t < 0, p = p_-) \) corresponds to a superinflationary, pre–big bang branch. It is equivalent to the pole–law inflation considered by Pollock and Sahdev \cite{25}. The positive root \( (p = p_+) \) then represents the deflationary, post–big bang phase. The two branches are shown in Figure 1. We shall refer to them as the (+)– and (−)–branches, respectively.

\textbf{Figure 1}

It proves convenient to define the new coordinate pair

\[ \beta \equiv \sqrt[6]{\frac{6}{4 + 3\omega}} [\alpha + (1 + \omega)\Phi] \]

\[ \sigma \equiv \kappa^{-1}(\Phi - 3\alpha), \quad \kappa \equiv \sqrt[6]{\frac{4 + 3\omega}{6 + 4\omega}} \]
It follows that the scale factor duality transformation (2.2) is generated by the simultaneous interchange \( \tilde{\sigma} = \sigma \) and \( \tilde{\beta} = -\beta \). The pre– and post–big bang classical trajectories (2.5) are then given by \( \beta = \beta_0 \pm \kappa^{-1} \ln(\pm t) \) and \( \sigma = \sigma_0 - \kappa^{-1} \ln(\pm t) \), respectively, where \( \{ \beta_0, \sigma_0 \} \) are related to \( \{ \alpha_0, \Phi_0 \} \). Action (2.1) simplifies to \( S = \frac{1}{2} \int dt e^{-\kappa \sigma} [\dot{\beta}^2 - \dot{\sigma}^2] \) and the momenta conjugate to \( \beta \) and \( \sigma \) are given by \( p_\beta = \dot{\beta} e^{-\kappa \sigma} \) and \( p_\sigma = -\dot{\sigma} e^{-\kappa \sigma} \), respectively. We may conclude, therefore, that since \( \sigma \) is invariant under scale factor duality, the pre– and post–big bang branches have equal and opposite momentum along the \( \sigma \)–axis, i.e., \( p_\beta^+(t) = -p_\sigma^-(t) = -1/\kappa \). (We specify \( \sigma_0 = 0 \) for simplicity).

It is this feature that allows us to interpret the two branches at the quantum level in terms of left– and right–moving waves. The Wheeler-DeWitt equation is the operator version of the Hamiltonian constraint \( NH_0 = p_\beta \dot{\beta} + p_\sigma \dot{\sigma} - L = 0 \), where \( L \) is the Lagrangian density [26]. It is derived in the usual fashion by identifying the conjugate momenta \( p^\mu \) with the operators \( p_\mu = \pm i \partial / \partial q_\mu \). The wavefunction of the Universe is an eigenvector of the Hamiltonian operator and physical states have zero eigenvalue, i.e., \( H_0 \Psi = 0 \). If one ignores ambiguities in the operator ordering, this constraint takes the form of the one–dimensional wave equation

\[
\left[ \frac{\partial^2}{\partial \sigma^2} - \frac{\partial^2}{\partial \beta^2} \right] \Psi = 0 \tag{2.9}
\]

We may therefore view the wavefunction as a free, bosonic particle propagating over \((1 + 1)\)–dimensional Minkowski space–time, where \( \beta \) and \( \sigma \) represent the time–like and space–like variables, respectively.

The general solution to this equation has two components \( \Psi = \Psi^+(u_+) + \Psi^-(u_-) \), where \( \Psi^{(\pm)} \) are arbitrary functions of \( u_\pm \equiv \beta \mp \sigma \). We may consider wavefunctions of the form \( \Psi^{(\pm)} = \exp[iS_{\pm}] \), where \( S_{\pm} = S_{\pm}(u_\pm) \) are solutions to the Hamilton–Jacobi equation \( (\partial_\sigma S_{\pm})^2 = (\partial_\beta S_{\pm})^2 \). These wavefunctions are peaked around the classical trajectories given by \( p_\beta^{(\pm)} = \partial S_{\pm} / \partial \beta \) and \( p_\sigma^{(\pm)} = \partial S_{\pm} / \partial \sigma \). Specifying \( S_{\pm} = \kappa^{-1} u_{\pm} \) therefore implies that \( \Psi^+ \) and \( \Psi^- \) are the wavefunctions for the pre– and post–big bang branches, respectively. Moreover, they may be viewed as plane waves moving in opposite spatial directions through minisuperspace. This suggests that a spatial reflection of the wavefunction in minisuperspace could result in a transition between the pre– and post–big bang branches. Such a reflection can not proceed for this model, however, because the dilaton field is free. The simplest extension is to allow this field to self–interact. This interaction will manifest itself as an effective potential in the Wheeler–DeWitt equation and it is this potential that may lead to a reflection of the wavefunction.
3 A Cosmological Constant in the Gravitational Sector

In this Section we introduce a cosmological constant into the gravitational sector of the theory by assuming that \( \Lambda(\Phi) \equiv \Lambda \) is a constant in Eq. (2.1). The generalized scale factor duality (2.2) is respected by this model since the potential energy in the Lagrangian has the form \( e^{-\kappa \sigma} \).

The field equations derived from action (2.1) are

\[ \ddot{\alpha} + 3 \dot{\alpha}^2 - \dot{\alpha} \dot{\Phi} = \frac{2(1 + \omega)\Lambda}{2\omega + 3} \]  
\[ \ddot{\Phi} - \dot{\Phi}^2 + 3 \dot{\alpha} \dot{\Phi} = -\frac{2\Lambda}{2\omega + 3} \]  
\[ \dot{\alpha}^2 - \dot{\alpha} \dot{\Phi} - \frac{\omega}{6} \dot{\Phi}^2 = \frac{\Lambda}{3} \]  

Now, multiplying Eq. (3.1) by 3 and subtracting Eq. (3.2) implies that

\[ \frac{d^2}{dt^2} (e^{-\phi}) = \eta^2 e^{-\phi} \]  

where

\[ \eta \equiv \left[ 2\Lambda \left( \frac{4 + 3\omega}{2\omega + 3} \right) \right]^{1/2} \]  

and \( \phi \equiv \kappa \sigma \) represents a shifted dilaton field \cite{27, 28}. The general solution to Eq. (3.4) is given by

\[ e^{-\phi} = Ae^{\eta t} + Be^{-\eta t} \]  

where \( A \) and \( B \) are arbitrary constants.

We may rewrite Eq. (3.2) in terms of the shifted dialton:

\[ \frac{d}{dt} \left( \frac{d\Phi}{dt} e^{-\phi} \right) = -\frac{2\Lambda}{2\omega + 3} e^{-\phi} \]  

This equation admits the first integral

\[ \dot{\Phi} = \frac{1}{4 + 3\omega} \dot{\varphi} + ce^{\varphi} \]  

where \( c \) is a constant and substituting this result into the Friedmann equation (3.3) implies that

\[ c^2 = -\frac{24AB}{4 + 3\omega} \Lambda \]  

We will consider the specific solution \( A = -B \), since this implies that the singularity may be located at \( t = 0 \) without loss of generality. Eq. (3.6) then implies that

\[ \varphi = \varphi_0 - \ln \sinh |\eta t| \]
where $e^{-\psi_0} \equiv 2A$. The functional forms of $\alpha(t)$ and $\Phi(t)$ may now be determined by substituting this result into Eq. (3.8) and integrating. The solution that corresponds to an expanding Universe with a vanishing scale factor in the limit $t \to 0^+$ is given by

$$
e^{\alpha} = e^{\alpha_0} \left[ \sinh(\eta t/2) \right]^{p_+} \left[ \cosh(\eta t/2) \right]^{p_-}$$
$$e^{\Phi} = e^{\Phi_0} \left[ \sinh(\eta t/2) \right]^{3p_+ - 1} \left[ \cosh(\eta t/2) \right]^{3p_- - 1}$$

where $\{\alpha_0, \Phi_0\}$ are integration constants. This solution is defined over the positive half of the time axis and there is a singularity in the curvature at $t = 0$.

We may now apply the scale factor duality to solution (3.11) to generate the second branch. It follows that

$$e^{\tilde{\alpha}} = e^{\tilde{\alpha}_0} \left[ \sinh(-\eta t/2) \right]^{p_-} \left[ \cosh(-\eta t/2) \right]^{p_+}$$
$$e^{\tilde{\Phi}} = e^{\tilde{\Phi}_0} \left[ \sinh(-\eta t/2) \right]^{3p_+ - 1} \left[ \cosh(-\eta t/2) \right]^{3p_- - 1}$$

where we have also performed the time reversal. This solution is related to Eq. (3.11) by the simultaneous interchange $p_+ \leftrightarrow p_-$. This corresponds, in effect, to choosing the negative square root of Eq. (3.9). The cosmology given by (3.12) approaches a singularity as $t \to 0^-$. There is also a singularity in the effective gravitational coupling $G_{\text{eff}} \equiv e^{\tilde{\Phi}}$ in this limit. The solutions (3.11) and (3.12) may therefore be viewed as two distinct branches corresponding to the post– and pre–big bang phases of the Universe’s history, respectively.

The qualitative behaviour of solution (3.11) and its dual (3.12) is shown in Figures 2a–2c for $-4/3 < \omega < 0$. The behaviour in the limit $|t| \to \infty$ is given by

$$\alpha_\infty \propto \left( \frac{2\Lambda}{(4 + 3\omega)(2\omega + 3)} \right)^{1/2} (1 + \omega)|t|$$
$$\Phi_\infty \propto - \left( \frac{2\Lambda}{(4 + 3\omega)(3 + 2\omega)} \right)^{1/2} |t|$$

This represents the analogue of the de Sitter solution. Near the singularity, on the other hand, the solutions simplify to the free–field solution (2.37), since the dilaton’s kinetic energy dominates the energy density of the Universe in this regime. For $-1 < \omega < 0$, the post–big bang branch is initially deflationary, but inflates at later times when the cosmological constant begins to dominate the dynamics. The corresponding pre–big bang branch is a bouncing solution. It collapses from infinity, reaches a minimum size and then re–expands to infinity as $t \to 0^-$. When $\omega = -1$, both branches expand monotonically, as shown in Figure 2b. For $-4/3 < \omega < -1$, the pre–big bang branch expands monotonically to infinity, while its dual expands initially, but recollapses at later times.

The behaviour of the effective gravitational coupling $e^{\Phi}$ is shown in Figure 3 for both branches. The coupling vanishes initially in the pre–big bang branch and
becomes infinitely strong as $t \to 0^-$. In the post–big bang solution, the coupling also vanishes initially, but falls back to zero after a certain time has elapsed.

**Figures 2 & 3**

In the following Section, we will argue that a transition between the pre– and post–big bang branches may proceed at the quantum level even though such a process is classically forbidden.

## 4 Quantum Transitions

The cosmology may be quantized by rewriting the system in terms of the variables (2.8). Action (2.1) then takes the canonical form

$$S = \int dt e^{-\kappa \sigma} \left( \frac{1}{2} \dot{\beta}^2 - \frac{1}{2} \dot{\sigma}^2 - 2\Lambda \right)$$

(4.1)

and the momenta conjugate to $\beta$ and $\sigma$ are given by

$$p_\beta = \dot{\beta} e^{-\kappa \sigma} \equiv k$$

(4.2)

$$p_\sigma = -\dot{\sigma} e^{-\kappa \sigma}$$

(4.3)

respectively. Substitution of Eqs. (3.11) and (3.12) into Eq. (4.2) implies that

$$k = 2\sqrt{\Lambda} e^{-\varphi_0} = \left( \frac{\eta}{\kappa} \right) e^{-\varphi_0}.$$ 

Since the scale factor duality (2.2) is respected by action (2.1), $p_\beta$ is invariant under duality and time reversal, whilst $p_\sigma$ changes sign. Indeed, we find that the momenta for the pre– and post big bang branches are given by $p_\beta^{(+)} = p_\beta^{(-)} = k$ and $p_\sigma^{(\pm)} = \mp k \cosh (\mp \eta t)$

(4.4)

It follows that $p_\sigma^{(\pm)}$ asymptotically approaches a constant value near to the singularity ($|t| \to 0$):

$$\lim_{\sigma \to +\infty} p_\sigma^{(\pm)} = \mp k$$

(4.5)

whereas we find that

$$\lim_{\sigma \to -\infty} p_\sigma^{(\pm)} = \mp k e^{\varphi_0 - \kappa \sigma}$$

(4.6)

in the low energy regime ($|t| \to \infty$, $\sigma \to -\infty$).

The system is quantized by identifying the conjugate momenta with the operators $\hat{p}_\beta \equiv i\partial/\partial \beta$ and $\hat{p}_\sigma \equiv i\partial/\partial \sigma$, respectively. The question of operator ordering now

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1. The unconventional sign choice of Gasperini et al. is chosen in the differential operators [15]. This ensures that the positive momentum waves move from negative to positive values of the effective spatial variable in minisuperspace [16].
arises. Since $\beta$ is related to the logarithm of the scale factor, we may consider the semi–general operator ordering proposed by Hartle and Hawking [31]:

$$\hat{p}_\beta^2 = -e^{p\beta} \frac{\partial}{\partial \beta} e^{-p\beta} \frac{\partial}{\partial \beta}$$  \hspace{1cm} (4.7)

where $p$ is an arbitrary constant. Our choice of $p$ is motivated by the scale factor duality (2.2) associated with the kinetic sector of the classical action (2.1). It is reasonable to suppose that the Wheeler-DeWitt equation should respect this symmetry and should therefore be invariant under the interchange $\beta = -\beta$. In view of this, we should specify $p = 0$ and this implies that the Wheeler-DeWitt equation is then given by

$$\left[ \frac{\partial^2}{\partial \sigma^2} - \frac{\partial^2}{\partial \beta^2} + 4\Lambda e^{-2\sigma} \right] \Psi = 0$$  \hspace{1cm} (4.8)

Furthermore, Eq. (4.7) implies that $\beta$ is a monotonically increasing function. It may therefore be viewed as the effective time coordinate of the minisuperspace. The space–like variable may then be identified with $\sigma$.

The wavefunction is an eigenstate of the momentum operator $\hat{p}_\beta$ and we specify the eigenvalue to be $k$, in agreement with Eq. (4.2). The general, separable solution to Eq. (4.8) is then given by

$$\Psi = Z_{\pm ik/\kappa}(z)e^{-ik\beta}$$  \hspace{1cm} (4.9)

where $z \equiv (2\sqrt{\Lambda}/\kappa)e^{-\kappa\sigma}$ and $Z_{\pm ik/\kappa}$ is a linear combination of Bessel functions of order $\pm ik/\kappa$.

The specific solution appropriate to the pre–big bang scenario is determined from the tunneling boundary condition [15, 32, 33]. Vilenkin has formulated this condition as a boundary condition on the superspace of all three–metrics and matter configurations [33]. In this formulation, the wavefunction of the Universe should be everywhere bounded and consist only of outgoing modes at the singular boundaries of superspace that correspond to singularities in the four–geometry. The minisuperspace considered in this model is $(1 + 1)$–dimensional Minkowski space–time and the outward (inward) trajectories at the singular boundary of this minisuperspace correspond to the pre– (post–) big bang branches, respectively. The tunneling boundary condition is therefore consistent with the pre–big bang initial conditions.

In the vicinity of the singularity ($\sigma \to +\infty$), the effective potential in the Wheeler–DeWitt equation (4.8) becomes negligible and this equation therefore reduces to the free wave equation (2.9). Consequently, the general solution of Eq. (4.8) in this limit may be expanded in terms of plane waves, i.e., $\Psi^{(\pm)}_{+\infty} \propto \exp[-ik(\beta \mp \sigma)]$. It follows that the wavefunction is also an eigenvector of the momentum operator $\hat{p}_\sigma$, i.e., $\lim_{\sigma \to +\infty} \hat{p}_\sigma \Psi^{(\pm)}_{+\infty} = \mp k \Psi^{(\pm)}_{+\infty}$. We may conclude from Eq. (4.5), therefore, that $\Psi^{(+)}_{+\infty}$ and $\Psi^{(-)}_{+\infty}$ represent wavefunctions for the pre– and post–big bang branches in the high energy limit.
The wavefunction must therefore reduce to \( \Psi = \Psi_+^{(+) \infty} \) at the singular boundary of minisuperspace in order to satisfy the tunneling boundary condition. It is this condition that then determines the order of the Bessel function in Eq. (1.9). In the limit that \( z \to 0 \), \( J_p(z) \propto z^p \), and this implies that we should choose \( p = -ik/\kappa \). The solution to Eq. (1.8) that represents the quantum version of the pre–big bang branch on the approach to the curvature singularity is therefore given by

\[
\Psi = J_{-ik/\kappa} \left( 2\sqrt{\Lambda/\kappa} e^{-\kappa \sigma} \right) e^{-ik\beta}
\]  

(4.10)

modulo a constant of proportionality.

We may now discuss the low energy limit \((z \to +\infty)\). The form of the Bessel function in this limit implies that the wavefunction may be expanded as a linear superposition of left– and right–moving modes of the form \( \Psi \mid \psi \rangle \),

\[
\lim_{\sigma \to -\infty} \Psi = \Psi_+^{(+ \infty)} + \Psi_-^{(- \infty)}
\]  

(4.11)

where

\[
\Psi_{\pm}^{(\pm)} = (2\pi z)^{-1/2} \exp \left[ -ik\beta \mp iz \pm \frac{\pi}{4} \pm \frac{\pi k}{2\kappa} \right]
\]  

(4.12)

The wavefunctions (4.12) are eigenstates of the momentum operator \( \hat{p}_\sigma \), since the prefactor is a slowly varying function. We find that

\[
\lim_{\sigma \to -\infty} \hat{p}_\sigma \Psi_{\pm}^{(\pm)} = \mp ke^{\pm\kappa \sigma} \Psi_{\pm}^{(\pm)}
\]  

(4.13)

and comparison with Eq. (4.6) then implies that \( \Psi_{\pm}^{(\pm)} \) represent the wavefunctions for the pre– and post–big bang phases in the low energy, weak coupling limit.

The wavefunction in this regime is therefore a linear superposition of the pre– and post–big bang components. Since these components are moving in opposite spatial directions through minisuperspace, we may consider the probability for the wavefunction to undergo a spatial reflection \([15]\). This is given by the reflection coefficient and is defined as the ratio of the current density of the reflected modes to the current density of the incident modes:

\[
R \equiv \frac{\left| \Psi_{-\infty}^{(-)} \right|^2}{\left| \Psi_{-\infty}^{(+) \infty} \right|^2}
\]  

(4.14)

This ratio represents the probability for a branch change to occur between the pre– and post–big bang phases and for the model considered in this Section, we find that

\[
R = e^{-2\pi k/\kappa}
\]  

(4.15)

The approximate dependence of the transition probability on the cosmological constant may be estimated from the definition of \( k \) given by Eq. (4.2). We evaluate
this expression at a time $t_s < 0$ defined by the condition $H(-t_s) = \mathcal{O}(1)$ \cite{13}. We will assume that $|\eta t_s| \ll 1$ and this is valid if $\Lambda$ is not too large. It follows from Eqs. (4.2) and (2.8) that

$$k = \frac{\dot{\beta}_s \Omega(\alpha_s)}{g_s^2} \quad (4.16)$$

where $g_s^2 \equiv e^{\Phi_s}$ is the effective coupling at the scale $t = t_s$ and $\Omega(\alpha_s)$ is the proper spatial volume at that time. The approximate form for $\beta$ is evaluated from Eqs. (2.8) and (3.12):

$$\beta_s \approx -\kappa^{-1} \ln (-\eta t_s/2) \quad (4.17)$$

and the direct dependence on $t_s$ is eliminated by evaluating $H(-t_s)$. The proper volume $\Omega_s$ is calculated by normalizing to the initial proper volume $\Omega_i$ at $t = -\infty$ and employing Eq. (3.12). Substituting the result into Eq. (4.16) then implies that

$$R \approx \exp \left[ 2\pi \frac{\Omega_i}{p_- g_s^2} \left( \frac{4\omega + 6}{4 + 3\omega} \right) \left( -\eta p_- \right)^{3p_-} \right] \quad (4.18)$$

We recall that $p_- < 0$ when $-4/3 < \omega < 0$. The dependence of the probability on $\Lambda$ is therefore

$$R \approx \exp \left[ -\frac{1}{\Lambda^{3p_-}} \right] \quad (4.19)$$

and this suggests that larger values of the cosmological constant are favoured. We should emphasize, however, that this expression is only valid for small $\Lambda$ and a more accurate calculation would be required to determine the general dependence. When $\omega = -1$, Eq. (4.19) exhibits the same $\Lambda$-dependence (in the small $\Lambda$ limit) as that found in the string model \cite{10}.

This concludes our discussion on the model containing a cosmological constant in the gravitational sector of the theory. In the following Section, we will consider a second model that contains a pre–big bang phase.

5 A Cosmological Constant in the Matter Sector

In this Section, we consider a model with an effective cosmological constant in the matter sector of the theory. Such a term is present when a second, self–interacting scalar field becomes trapped in a metastable, false vacuum state. This energy is formally equivalent to a dilaton potential of the form $\Lambda = \lambda e^{\Phi}$ in Eq. (2.1), where $\lambda$ is an arbitrary, positive constant.

The field equations derived from Eq. (2.1) for this form of $\Lambda(\Phi)$ are given by

$$6\dot{\alpha}^2 - 4\dot{\alpha} \dot{\Phi} + (2 + \omega)\Phi^2 + 4\dot{\alpha} - 2\dot{\Phi} = 2\lambda e^\Phi \quad (5.1)$$

$$12\dot{\alpha}^2 + 6\omega \dot{\alpha} \dot{\Phi} - \omega \dot{\Phi}^2 + 6\dot{\alpha} + 2\omega \dot{\Phi} = 0 \quad (5.2)$$

$$6\alpha^2 - 6\dot{\alpha} \dot{\Phi} - \omega \Phi^2 = 2\lambda e^\Phi \quad (5.3)$$
Subtracting Eq. (5.3) from (5.1) eliminates any direct dependence on the cosmological constant:

\[2\ddot{\alpha} - \ddot{\Phi} + \dot{\alpha}\dot{\Phi} + (1 + \omega)\dot{\Phi}^2 = 0\] (5.4)

Subtracting Eq. (5.2) from (5.4) then implies that

\[4\ddot{\alpha} + (1 + 2\omega)\ddot{\Phi} + 12\dot{\alpha}^2 + (6\omega - 1)\dot{\alpha}\dot{\Phi} - (1 + 2\omega)\dot{\Phi}^2 = 0\] (5.5)

Eq. (5.5) admits the first integral

\[e^{3\alpha - \Phi} \left[4\dot{\alpha} + (1 + 2\omega)\dot{\Phi}\right] = \gamma\] (5.6)

where \(\gamma\) is an arbitrary constant. This constraint is very useful, because it implies that a new variable \(\chi\), proportional to the term in the square brackets, may be introduced. The constant \(\gamma\) may then be interpreted as the momentum conjugate to this variable. Consequently, the Lagrangian will be independent of \(\chi\), since \(\gamma\) is time–independent. Thus, the effective potential in the corresponding Wheeler–DeWitt equation will also be independent of \(\chi\) and this implies that the wavefunction of the Universe will be an eigenstate of the momentum operator associated with \(\chi\).

In view of this, we define the coordinate pair

\[\chi \equiv 4\alpha + (1 + 2\omega)\Phi\]
\[\psi \equiv \Phi - 6\alpha\] (5.7)

It also proves convenient to define a new time variable

\[\tau \equiv \int dt e^{3\alpha(t)}\] (5.8)

since the Wheeler–DeWitt equation does not depend on the specific choice of lapse function. It follows that action (2.1) transforms to

\[S = \int d\tau \left[\frac{e^{-\psi}}{4(5 + 6\omega)} \left(6\chi'^2 - 2(2\omega + 3)\psi'^2\right) - 2\lambda\right]\] (5.9)

for all \(\omega \neq -5/6\), where a prime denotes differentiation with respect to \(\tau\). The transformation is unphysical if \(\omega = -5/6\) and the significance of this value becomes apparent in the Einstein frame. This model is conformally equivalent to Einstein gravity minimally coupled to a scalar field with an exponential, self–interaction potential. One can show that the functional form of the attractor solution in the Einstein frame changes when \(\omega = -5/6\) \[35\]. For \(\omega < -5/6\), the potential is so steep that it rapidly redshifts to zero and the field effectively becomes massless. For \(\omega > -5/6\), on the other hand, the attractor is given by the well known power law solution, where the kinetic and potential energies of the field redshift at the same rate.
The field equations (5.1)–(5.3) have been solved by Barrow and Maeda in terms of the parametric time $\tau$ for $\omega > -5/6$ \cite{23}. There exist two distinct solutions given by
\begin{equation}
\begin{aligned}
e^\alpha &= \tau^{\delta_\pm} (\tau + \tau_1)^{\delta_\mp} \\
e^{-\Phi} &= \lambda \left(\frac{5 + 6\omega}{2\omega + 3}\right)^{\tau^{\sigma_\pm} (\tau + \tau_1)^{\sigma_\mp}} 
\end{aligned}
\tag{5.10}
\end{equation}
where
\begin{equation}
\begin{aligned}
\delta_\pm &\equiv \frac{\omega}{3 \left(1 + 2\omega \pm \sqrt{1 + 2\omega / 3}\right)} \\
\sigma_\pm &\equiv \frac{1 \pm \sqrt{1 + 2\omega / 3}}{1 + 2\omega \pm \sqrt{1 + 2\omega / 3}} 
\end{aligned}
\tag{5.11}
\end{equation}
and
\begin{equation}
\tau_1 \equiv \mp \frac{\gamma}{\lambda} \left(1 + \frac{2\omega}{3}\right)^{-1/2} \left(\frac{2\omega + 3}{5 + 6\omega}\right) 
\tag{5.12}
\end{equation}
is assumed to be an arbitrary, semi–positive definite constant. It follows that each branch is characterized by equal and opposite values of $\gamma$. As $\tau \to +\infty$, both solutions asymptotically approach the attractor solution $e^\alpha \propto t^{\omega+1/2}$. The attractor is recovered by specifying $\gamma = 0$ in the first integral (5.6). There is a curvature singularity at $\tau = 0$ and Eq. (5.10) approaches the free field solution (2.5) as $\tau \to 0$, i.e., $e^\alpha \propto \tau^{\delta_\pm} \propto t^{\sigma_\mp}$. Both branches therefore show qualitatively different behaviour in the vicinity of the curvature singularity. The constant $\tau_1$ determines the time interval during which the dynamics is dominated by the dilaton’s kinetic energy.

We shall consider the example where $\omega = -1/2$. This is interesting from a physical point of view, because it corresponds to a dimensionally reduced version of six–dimensional Einstein gravity with a two–dimensional, isotropic, Ricci–flat internal space. In this model, the dilaton field is related to the radius of the internal space $b$ by $\Phi = -2 \ln b$ \cite{36}. Eq. (5.6) implies that the scale factor exhibits no turning points when $\omega = -1/2$. Consequently, the two branches in Eq. (5.10) represent cosmologies that either expand or contract indefinitely and the qualitative behaviour of the solutions for all time can therefore be determined from the evolution of the scale factor in the high curvature regime. This is given by $\alpha \propto p_\pm \ln t$, where $p_\pm = -1/(3 \mp \sqrt{24})$, and the $(p_+)$– and $(p_-)$–branches therefore represent expanding and contracting solutions. However, the time reversal of the contracting solution generates a new, expanding solution that is defined over $t < 0$.

This implies that the $(p_-)$– and $(p_+)$–branches may be viewed as pre– and post–big bang solutions, respectively. The qualitative behaviour of the scale factor in each branch is similar to that of the string model considered in Section 3, where $\omega = -1$. The scale factor tends to a finite constant in the low energy limit ($|t| \to \infty$) and
diverges as $t \to 0^-$, as shown in Figure 2b. The two branches are given in terms of $\chi$ and $\psi$ by

$$e^{\chi(\pm)} = (\mp \tau) \pm \sqrt{2/3} (\mp \tau + \tau_1) \pm \sqrt{2/3}$$

and the momenta conjugate to these variables are

$$p^{(\pm)}_{\chi} = (3/2)^{1/2} \lambda \tau_1, \quad p^{(\pm)}_{\psi} = \lambda (2 \mp \tau)$$

It follows, therefore, that the two branches have equal and opposite momentum with respect to the shifted dilaton $\psi$. This momentum tends to a constant value, $p^{(\pm)}_{\psi} \approx \mp \lambda \tau_1$, in the region near to the curvature singularity ($|\tau| \to 0$) and is given by

$$\lim_{\psi \to -\infty} p^{(\pm)}_{\psi} = \mp 2\sqrt{\lambda e^{-\psi(\pm)/2}}$$

in the low energy limit ($|\tau| \to \infty$).

The quantum analysis for this model is similar to that considered in the previous Section. The cosmology is quantized by identifying the conjugate momenta with the operators $\hat{p}_\chi = i\partial/\partial \chi$ and $\hat{p}_\psi = i\partial/\partial \psi$ in the Hamiltonian constraint $H_0 = p_\chi \dot{\chi} + p_\psi \dot{\psi} - L = 0$. The Wheeler–DeWitt equation is then given by

$$\left[ \frac{\partial^2}{\partial \chi^2} - \frac{3}{2} \frac{\partial^2}{\partial \psi^2} - 6\lambda e^{-\psi} \right] \Psi = 0$$

The solution that satisfies the tunneling boundary condition at the singular boundary of minisuperspace is given by

$$\Psi = J_p(z) e^{-ik\chi}$$

where $p \equiv -2i\lambda \tau_1$, $k \equiv \sqrt{3/2}\lambda \tau_1$ and $z \equiv 4\sqrt{\lambda e^{-\psi/2}}$. It may be verified that the wavefunction (5.17) is an eigenfunction of both momentum operators $\hat{p}_\chi$ and $\hat{p}_\psi$ in the high energy limit ($\psi \to +\infty$). Indeed, the ratio of the eigenvalues is given by $p_\chi/p_\psi = -\sqrt{3/2}$, in agreement with the classical momenta (5.14) for the pre–big bang phase. Moreover, in the low energy limit ($\psi \to -\infty$), the wavefunction (5.17) may be expressed as a superposition of two components, $\Psi = \Psi^{(\pm)}_{-\infty} + \Psi^{(-\infty)}$, where

$$\Psi^{(\pm)}_{-\infty} = (2\pi z)^{-1/2} \exp \left[ -ik\chi \mp iz \pm \frac{i\pi}{4} \pm \left( \frac{2}{3} \right)^{1/2} \pi k \right]$$

These components are eigenfunctions of $\hat{p}_\psi$:

$$\lim_{\psi \to -\infty} \hat{p}_\psi \Psi^{(\pm)}_{-\infty} = \mp 2\sqrt{\lambda e^{-\psi/2}} \Psi^{(\pm)}_{-\infty}$$

and comparison with Eq. (5.15) implies that they represent the pre– and post–big bang branches, respectively. We may conclude, therefore, that there is a non–zero probability for the quantum transition between the two branches. It is given by

$$R = e^{-\sqrt{32/3\pi k}}$$
6 Conclusion

In this paper we have considered the pre–big bang cosmological scenario within the context of the Brans–Dicke theory of gravity. In the free field model, an epoch of superinflationary expansion may occur during a pre–big bang phase when $-4/3 < \omega < 0$. This is an example of pole–like, kinetic inflation, where the expansion is driven by the kinetic energy of the dilaton.

At the quantum cosmological level, the two branches are represented by wavefunctions that move in opposite spatial directions through the configuration space. A transition between the two branches may be viewed as a spatial reflection of the wavefunction. A non–trivial dilaton potential is required, however, if such a reflection is to proceed.

The pre– and post–big branches of the free field model are related by a generalization of the scale factor duality associated with the string effective action. This generalization allows us to gain further insight into why string theory is related to $\omega = -1$ Brans–Dicke theory. Scale factor duality in string theory extends the $R$–duality of toroidal string compactification \[1\]. For example, one can consider a closed string propagating on a five–dimensional space–time $M_4 \times R^1$, where $M_4$ represents Minkowski space and $R^1$ is a flat circle. $R$–duality arises because there is no limit to the number of times a closed string may wrap itself around the compact fifth dimension. This results in a duality in the metric such that $\tilde{g}_{55} = g_{55}^{-1}$, where $g_{55}$ is the metric component associated with the extra dimension. Eq. (2.2) then implies that invariance under a direct inversion of the metric components is only possible if $\omega = -1$.

The introduction of a cosmological constant into the gravitational sector of the theory preserves the scale factor duality symmetry. We found classical pre– and post–big bang solutions that are related by this duality but are separated by a curvature singularity. We also considered a second model that contained an effective cosmological constant in the matter sector of the theory. In both models, the pre– and post–big bang branches may be viewed classically as particles moving in opposite spatial directions through minisuperspace. At the quantum level, the wavefunction corresponding to the pre–big bang branch was selected by invoking the tunneling boundary condition, as suggested by Gasperini et al. \[15\]. This leads to a non–zero probability for a spatial reflection of the wavefunction to occur. An approximate calculation implied that the probability becomes higher for a larger cosmological constant. In a sense, the reflection coefficient represents the probability for the birth of our Universe (corresponding to the post–big bang branch) out of the pre–big bang phase.

We have not addressed the question of how the semi–classical limit is recovered in this scenario. Although the wavefunction reduces to a superposition of two semi–classical wavefunctions in the low–energy limit, it represents a superposition of two macroscopically different states and this does not correspond to classical behaviour.
It is possible that this question could be resolved by studying the decoherence of the wavefunction \[37\]. Indeed, Lukas and Poppe have recently analyzed decoherence in the string model \((\omega = -1, \Lambda = 0)\) by including the effects of inhomogeneous dilaton fluctuations \[38\]. They have shown that decoherence is possible if certain conditions are satisfied and have suggested that decoherence itself may result in a branch change.

It should be noted that the range of \(\omega\) that allows a superinflationary, pre–big bang phase is inconsistent with primordial nucleosynthesis constraints \[39\] and solar system tests \[10\]. These lead to the lower limit of \(\omega > 500\) at the present epoch. Some modification to the class of models that we have considered in this work is therefore required. One possible extension would be to consider more general scalar–tensor theories of gravity, where \(\omega = \omega(\Phi)\) is assumed to be some function of the dilaton. Alternatively, more complicated dilaton potentials that contain at least one global minimum could be invoked.

Finally, the generation of primordial scalar and tensor perturbations should be considered. It has been suggested that the string scenario may produce a significant quantity of gravitational waves with frequencies accessible to the next generation of gravitational wave detectors \[11\]. This is in contrast to the standard, slow–roll inflationary scenario \[12\]. It would be of interest to investigate whether similar conclusions apply for the extended scenario considered in this paper. The inhomogeneous modes of the dilaton would need to be incorporated into the analysis, however, before definitive conclusions could be drawn.

Acknowledgments

The author is supported by the Particle Physics and Astronomy Research Council (PPARC), UK. It is a pleasure to thank Maurizio Gasperini for helpful communications.

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FIGURE CAPTIONS

Figure 1: The pre– and post–big bang solutions of the free field model for $-4/3 < \omega < 0$.

Figure 2: The pre– and post–big bang solutions when a cosmological constant is introduced into the gravitational sector of the theory. (a) $-1 < \omega < 0$. (b) $\omega = -1$. (c) $-4/3 < \omega < -1$.

Figure 3: The evolution of the effective gravitational coupling $G_{\text{eff}} \propto e^\Phi$ when $\Lambda(\Phi)$ is a constant and $-4/3 < \omega < 0$. 