Analysis of Hadronic Properties at SPS energies from a Statistical Model

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Using a statistical model of multiparticle production by Chou, Yang and Yen, which looks similar to the thermal model and is known to account for the single particle distributions in e+ e− collisions, we fit the rapidity and transverse mass spectra of pions and kaons measured in Pb+Pb collisions by NA49 collaboration. This model nicely fits both the rapidity and the transverse mass spectra of each particle species with a few parameters. However, fitting all the particles with a single value of T_p is not possible. This analysis shows that the success of the thermal models does not necessarily mean the thermalization of the system in collision.

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I. INTRODUCTION

It has been of great interest to find out whether a hot and dense nuclear matter has been formed during the high energy nuclear collisions[1], and when the temperature and/or density is high enough one expects a system of free quarks and gluons, namely quark-gluon plasma. Main issue here is the thermalization of the partonic or hadronic system formed just after the collisions and throughout the evolution of the following hadronic system. Thus probing the thermalization of the system in collision is of great importance. The large elliptic flow and the jet quenching effect indicate a collective behavior. However those are not yet the direct experimental evidence of thermalization. Best one can do so far is to assume thermalization and to show the experimental data is not inconsistent with the assumption of thermalization. In this case one has to show that assuming thermalization is the only way to explain the data in order to claim the thermalization. We will show in this paper that a simple statistical model can fit both the rapidity and the transverse momentum spectra of various hadrons.

In the relativistic heavy-ion collisions at AGS, SPS and RHIC, hadrons are measured quite accurately, especially for pions, kaons and protons. Usually the hadronic data are presented in many different ways: (1) total multiplicities of particles or the ratios of the multiplicities of different hadrons, (2) single differentials such as the rapidity distribution dN/dy, and the transverse mass spectra dN/m_⊥ dm_⊥ , (3) the double differentials such as B-E correlation function and (4) the averages or moments of certain physical quantities such as fluctuations and flow coefficients which is the second moment of the azimuthal angle, φ. The total number of particles can be obtained either by integrating the rapidity distribution over the whole range of the rapidity, i.e. N_ℓ = ∫ dy dN_ℓ /dy or by integrating over the whole p_⊥ range, i.e. N_ℓ = ∫ m_⊥ dm_⊥ dN/m_⊥ dm_⊥.

Models assuming thermal and chemical equilibrium fit each pieces of data very nicely[2,3,4,5,6,7,8,9] and explain many features of data. Thermal models assuming chemical equilibrium[2,3,4,5] when applied to the ratios of various hadron multiplicities fit the data impressively well with only a few parameters such as the temperature T_c, baryonic chemical potential, μ_b and/or the strange chemical potential, μ_s.

Fireball models with strong radial flow[6] fits the transverse mass spectra quite nicely with a few parameters, T_β, μ_b and radial expansion velocity at the surface, β_v.

The rapidity distributions are usually Gaussian. Spherically expanding fireball model applied to pion data at AGS energy[10] results in too narrow width compared to the data, and in the cylindrically expanding fireball model[8] fast longitudinal expansion makes the rapidity width wider and seems to fit the data when the system sizes are small. However, expanding fireball models with any geometry seems to have difficulty in fitting the rapidity distributions of pions and kaons, simultaneously, in the Pb+Pb collision at SPS.

The two different sets of freeze-out parameters from the ratio analysis and the transverse momentum spectra analysis leads to the conjecture that the chemical components freezes earlier at high temperature, T_{ch}, and the thermal equilibrium is still maintained until T_{th}, keeping the number of certain particle species constant[8].

However, calculation by Broniowski and Florkowski[7] claim that both the ratios and transverse mass spectra can be fitted with a single set of freeze-out parameters. As the models seem to be the similar, independent calculations are needed to check which one is right.

In spite of the success of thermal models for hadrons produced in the relativistic heavy-ion collisions, it is known that the statistical model of multiparticle production leads to the Boltzmann factor[11,12,13] and can explain the hadronic data produced in e+ e− and pp collisions. In the series of papers by T. T. Chou, C. N. Yang and E. Yen[11] a statistical model for multiparticle production is derived and applied successfully to the rapidity distribution and transverse momentum distribution of measured hadrons in e+ e− collisions by TASSO collaboration. This model has been further modified and studied by Hoang[12] to include the Lorentz-boosted Boltzmann factor in order to mimic the shift of the peak of the rapidity distribution in the asymmetric collisions. It should be emphasized that this statistical model does neither re-
quire the thermalization of the system nor the concept of freeze-out. Recently the same argument has been re-
vived \[13, 14, 12\], but application of the statistical model to the relativistic heavy-ion collision data has not been attempted yet.

Thus it is very interesting to study whether the statistical model which successfully fit e\(^+\)e\(^-\) collision data can also fit the rapidity distributions and transverse momentum spectra of pions and kaons produced in the relativistic heavy-ion collisions, especially whether it can fit all of them with a single set of parameters. A priori it is not clear whether pions and kaons have the same parameter "\(T\)" in the statistical model. However, there is a claim that e\(^+\)e\(^-\) collision data could be fit with only one "temperature" for various hadrons \[11\] \[12\].

In this paper we will show that a simple statistical model can describe both the rapidity and transverse mass spectra of various hadrons measured in the heavy-ion collisions. In our previous work \[18\], the rapidity distribution of hadrons are analyzed using the same model.

In the next section the statistical model by chou, Yang and Yen is reviewed and in sec. III the result of our analysis of the rapidity distributions and transverse mass spectra is given, after integration over the rapidity \(y\). A by the NA49 collaboration \[13, 14, 15\] is presented and finally we summarize in sec. IV.

**II. STATISTICAL MODEL**

In this section we follow the paper by Chou, Yang and Yen \[11\] for the statistical model invented for e\(^+\)e\(^-\) collisions which can be used for the heavy-ion collisions with minor modifications.

We will assume the multiparticle production in the relativistic heavy-ion collisions is stochastic but subject to a number of conditions: (a) energy conservation, (b) leading baryon effect which means that leading baryons in both the projectile and target region do not participate in the collision, (c) \(d^3p/E\) probability for each particle, and (d) transverse momentum cutoff factor \(g(p_\perp)\). Then the probability for non-leading particles in the collision zone can be taken as

\[
\delta \left( \sum_i E_i - E_0 h \right) \prod_i \left( d^3p_i/E_i \right) g(p_\perp) \tag{1}
\]

where \(E_0 = \sqrt{s}\) and \(h\) is the fraction of the total energy \(E_0\) used for the particle production in the central region.

It was pointed out that the probability distribution is exactly the same as in the microcanonical ensemble in the statistical mechanics. By the well-known Darwin-Fowler method the single particle distribution of such an ensemble can be converted into a distribution in the canonical ensemble:

\[
\text{Probability} = (d^3p/E) g(p_\perp) \exp(-E/T_p) \tag{2}
\]

where \(T_p\) is called the partition temperature. Noting that

\[
d^3p = dp_\perp dp_\perp d\phi = Edy dp_\perp d\phi, \tag{3}
\]

one gets the equation for the rapidity distribution by integrating over the transverse momentum.

\[
dN_i/dy = 2\pi K \int_0^{p_{\max}} p_\perp dp_\perp g_i(p_\perp) \exp(-E_i/T_p) \tag{4}
\]

where \(K\) is a normalization constant, and \(p_{\max} = E_0 h\) which can be taken as infinity in the relativistic heavy-ion collisions. \(E_i = m_T \cosh y\), and \(m_T = \sqrt{m_i^2 + p_i^2}\). We will use for the high momentum cutoff factor as in Ref. \[11\],

\[
g_i(p_\perp) = \exp(-\alpha_i p_\perp) \tag{5}
\]

and in fitting e\(^+\)e\(^-\) data, the value of \(\alpha\) was taken from the measured data.

Similarly the transverse momentum or transverse mass spectra is given, after integration over the rapidity, as

\[
dN_i/m_\perp dm_\perp = 2\pi K g_i(p_\perp) \int dy \exp(-E_i/T_p) \tag{6}
\]

Eq. (3) can be integrated for the massless particles and the role of parameters can be easily understood.

\[
dN_i/dy = \frac{2\pi K}{(\alpha_i + \cosh y/T_p)^2} \tag{7}
\]

whose maximum at \(y = 0\) is

\[
(dN_i/dy)_{\max} = \frac{2\pi K}{(\alpha_i + 1/T_p)^2} = \frac{2\pi KT_p^2}{(\alpha_i T_p + 1)^2} \tag{8}
\]

and the width \(\delta y_i\) at the half maximum is given from the relation

\[
\cosh \delta y_i = (\sqrt{2} - 1)\alpha_i T_p + \sqrt{2} \tag{9}
\]

For the massless particle the width is governed by the factor \(\alpha_i T_p\). Thus when the value \(\alpha\) is taken from experiment, the parameter \(T_p\) is a measure of the width of \(dN/dy\) in this model. This is quite different from the thermal models, where \(T_p\) determines the slope of the transverse momentum spectra and in order to fit the width of the rapidity distribution one further need the concept of longitudinal expansion of the system.

This can be understood again from Eq. (6), which describes an exponential shape. Contrary to the naive expectation, the slope of the exponential is determined not by \(T_p\), but by \(1/\alpha\) through Eq. (8). This is because the fitted value of \(T_p\) is very large (\(\sim 1\) GeV) compared to the average transverse momentum, which usually translates into the temperature. Thus in this model the main
slope of the exponential shape is put in by hand as a high momentum cut-off factor, Eq. (4) and the integration in Eq. (6) gives a modification to the simple exponential shape.

In Eq. (6) and Eq. (6), \( K, T_p, \) and \( \alpha_i \)'s are parameters. In the next section results of fitting of the rapidity distributions and transverse mass spectra of \( \pi^+, \pi^-, K^+ \) and \( K^- \) in Pb+Pb collisions at 158 GeV-A by the NA49 collaborations.

### III. ANALYSIS

Main results of the fitting of the rapidity distributions and transverse mass spectra of \( \pi^+, \pi^-, K^+ \) and \( K^- \) in Pb+Pb collisions at 158 GeV-A measured by NA49 collaboration [12,13] are shown in Fig. 1-4 and the resulting parameters are tabulated in Table 1. In Table 1, we have fitted each species separately and thus the resulting parameters are all different. \( < p_\perp > \) is calculated from \( \alpha \) using Eq. (5). One may use experimental value of \( \alpha \) and then \( K \) and \( T_p \) are the only parameters in this model. It is amazing that a very simple model with only two parameters can fit both of the rapidity and transverse momentum spectra of each particle species.

In this model there is no a priori reason for the common value of \( T_p \) for different particle species. Simultaneous fitting of the rapidity and transverse mass spectra of pions and kaons with a single value \( T_p \) is not possible, which is contrary to the case of \( e^+ e^- \) collisions [12]. However, in Ref. [12] Lorentz-boosted exponential was used and direct comparison is not possible and further studies with the Lorentz-boosted exponential is needed to check whether in the relativistic heavy-ion collisions a single value of \( T_p \) can describe the particle spectra of all the different species.

In Ref. [12] we have successfully fitted only the rapidity distribution of pions and kaons with a single \( T_p \) but different \( \alpha_i \)'s. We get \( K = 1212, \ T_p = 0.98 \) GeV, \( \alpha_{\pi^+} = 5.98 \ c/\text{GeV} \), \( \alpha_{\pi^-} = 5.67 \ c/\text{GeV} \), \( \alpha_{K^+} = 11.4 \ c/\text{GeV} \), and \( \alpha_{K^-} = 17.1 \ c/\text{GeV} \). The large values of \( \alpha_i \)'s spoils the interpretation of \( \alpha \) in terms of the average transverse momentum through Eq. (6). This means that even though all the rapidity distributions are fitted simultaneously, the transverse mass spectra are far off. However, one can also fit the rapidity distributions of each particle species separately by fixing the \( \alpha_i \) values as the experimental values and the resulting values from the least square fit are approximately the same as those in the Table 1, meaning that by fitting only the rapidity distribution one can fit the transverse mass spectrum also. Concentrating only for the rapidity distributions, one can either fit them with a single value of \( T_p \) with the transverse mass spectra far off or fit both of the rapidity and transverse mass spectrum with different \( T_p \) values for different particle species.

As was discussed in the previous section \( T_p \) is above 1 GeV and thus in Eq. (6), \( 1/\alpha \) plays the role of the slope parameter and the integral in Eq. (6) acts as the high energy cut-off.

For pions we have used \( K, T_p, \alpha_{\pi^+} \) and \( \alpha_{\pi^-} \) as parameters. One may use the same \( T_p \) and \( \alpha \) but different \( K \) values for \( \pi^+ \), \( \pi^- \). Or one may further introduce the charge chemical potential \( \mu_c \) to write \( K_{\pi^+} = K \exp(\mu_c/T_p) \) and \( K_{\pi^-} = K \exp(-\mu_c/T_p) \). In this way one gets \( K = 625.0, \ T_p = 1.21 \) GeV, \( \alpha = 4.17 \ c/\text{GeV} \) and \( \mu_c = 0.043 \) GeV. The quality of the fitting is more or less the same. Similarly for kaons one can introduce the strange chemical potential \( \mu_s \) to write \( K_{K^+} = K \exp(-\mu_s/T_p) \) and \( K_{K^-} = K \exp(\mu_s/T_p) \). Then the rapidity and transverse spectra of both the \( K^+ \) and \( K^- \) can be fitted simultaneously with \( K = 50.7, \ T_p = 1.43 \) GeV, \( \alpha_{K^+} = 2.6c/\text{GeV}, \ \alpha_{K^-} = 2.9c/\text{GeV} \) and \( \mu_s = -0.415 \) GeV.

### IV. CONCLUSION

We have fitted the rapidity and transverse mass spectra of \( \pi^+, \pi^-, K^+ \) and \( K^- \) in Pb+Pb collisions at 158 GeV-A measured by NA49 collaboration with a simple statistical model. For each particle species both the rapidity and the transverse mass spectra are nicely fitted with only three parameters. However, simultaneous fitting for all the pions and kaons with a single value of \( T_p \) is not possible.

Thus the success of thermal models for the hadron multiplicities and/or transverse momentum spectra, by itself alone, does not necessarily mean the thermalization of the system.

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| \( K \) | \( T_p \) \( \text{GeV} \) | \( \alpha \) \( \text{c/GeV} \) | \( < p_\perp > \) \( \text{GeV/c} \) | \( \chi^2/n \) |
|---|---|---|---|---|
| \( \pi^+ \) | 599.9 | 1.27 | 4.23 | 0.47 | 5.4 |
| \( \pi^- \) | 649.8 | 1.19 | 4.15 | 0.48 | 15.0 |
| \( K^+ \) | 86.86 | 1.57 | 2.69 | 0.74 | 2.5 |
| \( K^- \) | 37.82 | 1.35 | 2.79 | 0.72 | 2.7 |

[1] Quark Matter 2001, Nucl. Phys. A (2002) and references therein.

[2] P. Braun-Munzinger, I. Heppe and J. Stachel, Phys. Lett.
FIG. 1: Rapidity distribution and transverse mass spectra of \(\pi^+\) in Pb+Pb collisions at 158 GeV A by NA49 collaborations. Lines are the fittings with Eq. (3) and Eq. (6) with parameters in Tab. (1). In the right figure from the top y bin is 0.0 ± 0.1 and the second one is for y = 0.2 ± 0.1, and so forth.

[3] F. Becattini, Z. Phys. C 69, 485 (1996).
[4] J. Rafelski and J. Letessier, nucl-th/0209084.
FIG. 2: Same as in Fig1. for $\pi^-$. 

FIG. 3: Same as in Fig1. for $K^+$. 

FIG. 4: Same as in Fig1. for $K^-$. 

$Y - Y_{\text{mid}}$