Scalability of Controlling Heterogenous Stress-Engineered MEMS Microrobots (MicroStressBots) through Common Control Signal using Electrostatic Hysteresis

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Abstract—In this paper we present control strategies for implementing reconfigurable planar microassembly using multiple stress-engineered MEMS microrobots (MicroStressBots). A MicroStressBot is an electrostatic microrobot that consists of an untethered scratch drive actuator (USDA) that provides forward motion, and a steering-arm actuator that determines whether the robot moves in straight line or turns. The steering-arm is actuated through electrostatic pull-down to the substrate initiated by the applied global power delivery and control signal. Control of multiple MicroStressBots is achieved by varying the geometry of the steering-arm, and hence affecting its electrostatic pull-down and/or release voltages. Independent control of many MicroStressBots is achieved by fabricating the arms of the individual microrobots in such a way that the robots move differently from one another during portions of the global control signal. In this paper we analyze the scalability of control in an obstacle free configuration space. Based on robust control strategies, we derive the control signals that command some of the robots to make progress toward the goal, while the others stay in small orbits, for several classes of steering-arm geometries. We also present a comprehensive analysis and comparison between the numbers of required independent pull-down and release voltages, demonstrating significant improvement in terms of the efficiency as well as the size of the control signal presented in past work. Our analysis presents an important step for developing multi-microrobot control of MicroStressBots.

Keywords: multi-microrobot systems control, MEMS, underactuated system.

I. INTRODUCTION

Microscale robotic systems have many applications in areas such as biomedicine [1], surveillance [2], or microassembly [3]. In [4, 5, 6] a globally controllable 240 μm × 60 μm × 10 μm mobile stress-engineered microelectromechanical systems (MEMS) microrobot (MicroStressBot) is presented. A MicroStressBot contains an untethered scratch drive actuator (USDA) [7] which provides forward propulsion, and a steering-arm actuator which controls when the robot moves in straight line or turns. All these envisioned microrobotic applications rely on the combined actions of large number microrobots. The high level of underactuation presents in such systems (all robots are controlled by a single global control signal), makes the simultaneous control of several microrobots significantly more challenging than control of single microrobot.

Controlling a distributed system of many devices that differ in behavior falls under the concept of Ensemble Control (EC) [8, 9] and Global Control Selective Response (GCSR) [6]. In Ensemble Control (EC) the robots are modeled as nonholonomic unicycles with inhomogeneity in turning and linear velocity. By using state feedback control policy, globally asymptotically stable ensemble of unicycles controlled by uniform control inputs, is achieved. It has been shown in [9] that the ensemble of nonholonomic unicycles is asymptotically stable by using a suitable Lyapunov function. Although EC provides promising control policy to any number of robots in theory but in practice it is not successful for more than ten of robots due to the control error (system noise), which cancels the inhomogeneity effect. Also EC needs perfect state estimation and the controllers required at worst a matrix inversion and at best a summation over all robot states which is not practical for large number of robots. In [9], the control policy is based on the robots local coordinate and the trajectory of each robot is independent and disregards collision, which is impractical for any microrobotic system due to Stiction effects.

However, in this paper, we present the theory and proof of a novel control voltage methods for multiple heterogeneous stress-engineered Microrobots (MicroStressBots) that provide highly underactuated, reconfigurable, time-efficient and multi-shapes microassembly system. Furthermore, this is the first technique relying on inhomogeneity that the control primitives can always be achieved with a constant number of control primitives and, unlike the previous techniques, do not increase with the size of the system, enabling the implementation of the control strategy presented in [6].

The paper is structured as follows: in Sec. II, we introduce the stress-engineered Microrobot (MicroStressBot). The general approach to controlling multiple MicroStressBots is discussed in Sec. III. Sec. IV describes the theory, proof, and scalability analysis for String-Cluster System and ESATC systems. Concluding discussion regarding the scalability of these different systems is described in Sec. V.

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II. STRESS-ENGINEERED MICrorobot

The stress-engineered MEMS microrobot, MicroStressBot, for short, consists of an untethered scratch drive actuator (USDA) [7] that provides forward motion and a curved steering-arm actuator that determines whether the robot moves straight or turns. Fig. 1 shows the schematic of the MicroStressBot. The USDA is composed of a 120 μm × 60 μm backplate and a 1.5-μm-tall bushing. The steering-arm actuator consists of a 120 to 160 μm long cantilever beam with a circular pad and a 0.75 μm-deep dimple.

The MicroStressBots are fabricated using surface micromachining PolyMUMPS foundry process [10]. The initially planar steering arms are curved out-of-plane (upwards) using a stress engineering process [11]. This process adds a patterned layer of a stressor material (Chromium, Cr) with high compressive stress to provide an upward curvature. The thickness of the deposited material and the area covered by the stressor layer must be precisely defined, such that the steering arm is deflected at the preset actuation voltage.

The microrobot operates on a grid of insulated interdigitated electrodes. When voltage is applied between sets of electrodes, the electrodes and the conductive chassis of the microrobot form a capacitive circuit, and an electric potential is induced on the microrobot. This potential causes the microrobot body to be bent to the substrate, and the scratch-drive converts this vertical motion into a forward step. This voltage (waveform) changes over time to provide power to the USDA and to control the state of the steering arm. This waveform is called the control waveform. The waveform is divided into two parts:

a) Control cycle: containing control pulses, that sets the state of the steering-arm actuator, and

b) power-delivery cycle: that provides power to the USDA. The power-delivery cycle consisting of stepping pulses, changing between a maximum \( V_{\text{high}} \) and a minimum \( V_{\text{low}} \). In order for the USDA to actuate, \( V_{\text{high}} \) must be greater than the minimum voltage \( V_{\text{f Mex}} \) at which the backplate of the USDA obtains enough curvature to produce

\[ b) \quad \text{a forward step, while } V_{\text{low}} \text{ must be less than the maximum voltage } (V_{\text{rel}}) \text{ at which that curvature is sufficiently relieved to generate forward motion. } V_{\text{f Mex}} \text{ and } V_{\text{rel}} \text{ are described in more detail in [12, 13].} \]

Similar to an electrostatic cantilever beam [14], the steering arm of each microrobot has two distinct voltage levels at which the arm suddenly changes states. This is the snap-down voltage at which the arm is pulled in contact with the substrate as the robot is turning and the release voltage at which the arm is released and the robot is commanded to move straight. We call these voltage levels the transition voltages of the steering arm. The transition voltages are determined by the steering-arm designs. The steering arm can be either raised to cause the robot to move in a straight line, or lowered to cause the robot to turn. We call the position of the steering-arm actuator the hysteresis state of the microrobot (arm raised, hysteresis state = 0; arm lowered, hysteresis state = 1). A system of \( n \) MicroStressBots contains \( 2^n \) possible hysteresis states (all \( 2^n \) combinations of \( n \) steering arms being raised or lowered).

III. GCSR: A STRATEGY FOR MULTI-MICrorobot CONTROL AND ASSEMBLY

As mentioned in [6], GCSR is a strategy to control and maneuver robots to complete the microassembly. GCSR uses design-induced heterogeneity of MicroStressBots and resulting differences between their trajectories to maneuver the robots from an initial to a target goal configuration. As stated in [6], GCSR uses the control matrix to control the single robot sequentially while the other robots confined to the circular trajectories. A mapping between the control primitives (waveforms that program the hysteresis states of the system) and the motion of the individual robots is defined using a control matrix, where each entry contains the hysteresis state of one of the microrobot during the applied control primitive. The resulting sequence of control primitives is called the control sequence, usually denoted by \( S \). Fig. 2 shows the trajectory of a microrobot \( D_j \) from initial (i) to target (ii) configuration (docking with a seed-shape (iii)), during the application of control sequence \( S = \{P_0, P_2, P_6, P_8, P_3, P_5, P, P_1\} \). During this partial assembly robot \( D_j \) orbits without advancing to goal. This type of STRING (STRtically Non-nested hysteresis Gap) GCSR is designed to be sequential (i.e. one robot at a time is maneuvered towards the goal) to increase the number of controllable microrobot and enable robust collision avoidance. The control matrix denoting the correspondences between control primitives in a STRING microassembly is called a STRING control matrix. It has been shown in [6] that microassembly can be implemented on a group of MicroStressBots if a STRING matrix can be generated for such system.

IV. BEYOND STRING CONTROL: ENABLELING GCSR THROUGH CONTROL VOLTAGE DIFFERENTIATION

Successful implementation of GCSR control relies on a set of control primitives that couple the motion of the microrobots in
Several steering-arm design configurations (NHG, STRING, and SESat) have been proposed in [6] to achieve GCSR. The objective of all these designs is to reduce the control bandwidth size with respect to the number of robots, i.e., maximize the number of independently controllable microrobots for a given microrobotic system. Per [15], Nested Hysteresis Gaps (NHG) is the system of n steering arms, sorted according to ascending $V_{di}$, when $(V_{di} + 2\delta_{v} > V_{dj})$ and $(V_{ui} - 2\delta_{v} > V_{uj})$, for all $i < j$. NHG systems can access all $2^n$ hysteresis states. However each device requires two unique control voltage levels, and so the control voltage bandwidth requirement of this system is $\xi_n = 2n$. However, NHG is sufficient but not necessary to achieve GCSR. Strictly non-nested hysteresis gaps (STRING) [6] is an n - microrobotic system, primarily sorted according to ascending values of $V_{di}$, and secondarily sorted according to ascending values of $V_{ui}$, has non-nested hysteresis gaps if $(V_{di} \leq V_{dj})$ and $(V_{ui} \leq V_{uj})$, for all $i < j$. It has been shown in [6] that STRING system can access $n + 1$ hysteresis states and the control bandwidth requirement for a STRING system is $\xi_n = n + 1$. Although STRING system cannot not access all the $2^n$ states, as sequential microassembly algorithm makes the system controllable when implementing microassembly. STRING reduces the control bandwidth requirements from $\xi_n = 2n$ (NHG) to $\xi_n = n + 1$. Further, sublinear reduction $\xi_n$ has been done under the SESat control strategy [6]. It has been shown in [6] that SESat needs $\xi_n = [2\sqrt{n}]$ while it can access at least $n + 1$ hysteresis states which are essential for sequential microassembly algorithm. Although SESat could reduce the control bandwidth requirements to $O(2\sqrt{n})$, only 3 out of the 4 hysteresis states for the first two robots forming the seed-shape are accessible, making their independent control challenging. In this paper we present new set of control strategies (String-cluster System and Electromechanically SATurated cluster system (ESATC)) that not only has the control voltage bandwidth $\xi_n = O(n)$ and $\xi_n = O(2\sqrt{n})$ but also are capable of controlling two microrobots simultaneously.

### A. String-Cluster System

In this section we introduce a control strategy that is capable to control two microrobots simultaneously at each iteration of assembly process. We start with the following definitions:

**Definition. I:** Nested-Group-Microrobots (NGM) set:

Set of all groups of two Microrobots that forms a Nested-Hysteresis-Gaps (NHG) structure is Nested-Group-Microrobots (NGM). $M$ is the set of all microrobots in the system. $M = \{D_1, D_2, ..., D_n\}$; where $D_i$ is ith Microrobot in the system.

$$\text{NGM} = \{(D_i, D_j) \mid (D_i, D_j) \in M, V_{dj} > V_{di}, V_{ui} > V_{uj}, i, j \in N\}$$  

(1)

where $V_d$ and $V_u$ are snap-down and release voltages.

**Definition. II:** Nested-Group-Microrobots Cluster (Cluster):

Each member of NGM is called Cluster.

$$\text{Cluster} = (D_i, D_j) \in \text{NGM}; i, j \in N$$  

(2)

In each Cluster, we define $V_{ui} = \min(V_{di}, V_{dj})$, $V_{uh} = \max(V_{di}, V_{dj}), V_{ui} = \min(V_{ui}, V_{uj})$ and $V_{uh} = \max(V_{ui}, V_{uj})$, respectively. Fig. 3. Shows the relation between the transition voltages in a Cluster, the snap-down and release voltages are shown as circles and rectangles, respectively.
Definition. III: STRictly Non-nested hysteresis-Gaps (String)-Cluster (String-Cluster) system:
This system consists of Clusters where each two microrobots in two different Clusters form a String system. This system is formed by Equation. 3.

It is convenient for us to define a lexicographic sorting of the robots, using two keys. In general, an M-Clusters microrobotic system, primarily sorted according to ascending values of $V_{d1}$, $V_{dh}$ and secondarily sorted according to ascending values of $V_{u1}$, $V_{uh}$ has non-nested hysteresis gaps between each two microrobots belong to different Clusters. However, in the case when $V_{d1j} - V_{dh,i} < 2\delta v$, $V_{dh,j} - V_{d1,i} < 2\delta v$ and $V_{u1j} - V_{uh,i} < 2\delta v$, $V_{uh,j} - V_{u1,i} < 2\delta v$, the behavior of robots of Cluster, and Cluster, is indistinguishable, and four such microrobots in Cluster, and Cluster, cannot be controlled independently. We call such two Clusters a degenerate Cluster pair. Fig. 4 shows String-Cluster system.

Lemma. 1: An M-String-Cluster system has exactly “n+3=2M+3” accessible hysteresis states, where $M=\text{no. Clusters}$, $n=\text{no. Microrobots and M=n/2}$.

Proof: By Induction:
Base condition: An String-Cluster system with M=2 has seven accessible states.

In each Cluster we have two microrobots sorted in an NHG system format. Each of the microrobots got 2 states: (0=arm up) and (1= arm down). Hence we have $2^2=4$ states (00= arm up, arm up), (01= arm up, arm down), (10= arm down, arm up) and (11= arm down, arm down). Let M=2 Clusters microrobotic system, $C_1$ and $C_2$, where each Cluster $C_i$ consists of two microrobots {$D_i$, $D_2$}. Without loss of generality, $V_{dh,1} \leq V_{u1,2}$ and $V_{uh,1} \leq V_{u2,2}$. Fig. 5 shows the ranges for transition voltages of Cluster ($C_2$), such that the M=2 Clusters microrobotic system becomes String-Cluster.

Let $V_{\alpha}$, ..., $V_{\phi}$ be significantly independent transition voltage levels, ordered such that $V_{\delta} < V_{\gamma} < V_{\beta} < V_{\alpha} < V_{\zeta} < V_{\eta} < V_{\lambda} < V_{\eta} < V_{\phi} < V_{\alpha} < V_{\psi}$ with $|V_{\delta} - V_{\phi}| = 26\delta v$ and $|V_{\zeta} - V_{\eta}| = 2\delta v$. Let $V_{u1,1} = V_{\alpha}$, $V_{dh,1} = V_{\phi}$ and $V_{u2,1} = V_{\beta}$, $V_{uh,1} = V_{\gamma}$. It follows that the snap-down voltage $V_{u1,2}$ can have value $V_1 \in (V_{\phi}, V_{\eta}]$, or voltage $V_2 = V_{\eta}$ and the snap-down voltage $V_{dh,2}$ can have value $V_3 \in (V_{\alpha}, V_{\psi}]$, or voltage $V_4 = V_{\alpha}$ and $|V_{u1,2} - V_{u1,2}| \geq 26\delta v$. Similarly, the release voltage, $V_{uh,2}$ can have the value $V_5 \in (V_{\delta}, V_{\ref} - 2\delta v)) or voltage $V_6 = V_{\delta}$, and the release voltage $V_{u1,2}$ can have the value $V_7 \in (V_{\alpha}, V_{\beta}) or voltage $V_8 = V_{\alpha}$ and $|V_{u1,2} - V_{u1,2}| \geq 2\delta v$. Consequently, for the M=1 Clusters microrobotic system to remain String-Cluster, one of the following combinations of the snap-down and release voltages for $C_2$ must hold: $(V_1, V_5, V_3, V_7)$, $(V_1, V_5, V_3, V_8)$, $(V_1, V_6, V_3, V_9)$, $(V_2, V_5, V_3, V_7)$, $(V_2, V_5, V_3, V_8)$, $(V_2, V_6, V_3, V_9)$, $(V_2, V_5, V_4, V_7)$, $(V_2, V_5, V_4, V_8)$ and $(V_2, V_6, V_4, V_9)$.

We examine each case separately:

$(V_1, V_5, V_3, V_7)$: because of $V_{u1,2}, V_{dh,2}$ are greater than the snap-down voltages of $C_1$, ($V_{u1,2} > V_{u1,1}, V_{dh,2} > V_{dh,1}$), and ($V_{uh,2} > V_{u1,1}, V_{uh,2} > V_{u1,1}$), we can only snap down the arms of $D_1$ and $D_2$ in $C_2$ after we snap down the arms of $C_1$.

Since $V_{u1,2}, V_{uh,2}$ is greater than the release voltages of $C_1$, ($V_{u1,2} > V_{u1,1}, V_{uh,2} > V_{uh,1}$), and ($V_{uh,2} > V_{u1,1}$ and $V_{uh,2} > V_{uh,1}$), we can only release the arms of $C_1$ after we have released the arms of $D_1$ and $D_2$ in $C_2$. Because the $(V_{dh,2} > V_{u1,2} > V_{uh,2})$, we can snap down $C_1$ and $D_1$ of $C_2$ while
$D_2$ of $C_2$ is released. Since the $V_{ul,2} > V_{ul,1} > V_{ul,1,2}$, we can release $D_1$ of $C_2$ while $D_2$ and all other clusters are snapped down. Consequently, we can change the states of $C_2$ to “01”, “10” or “11” when $D_1$ are in state “11”. During all other states of the system the state of $C_2$ must remain “00”. Consequently, the number of accessible hysteresis states increase by exactly 3.

$V_{1,5,3,8}$: This case is similar to $V_{1,5,3,7}$, except that the arm of $D_1$ of Cluster $C_1$ is released at the same time as the arm of $D_2$ of Cluster $C_2$. As long as $V_{ul,2} > V_{ul,1}$, we can snap down the arm of $D_2$ of $C_2$ only after all other clusters are in state “11”. As a consequence the number of accessible hysteresis states increase by exactly 3.

$V_{2,5,3,7}$: The snap-down voltage $D_1$ of $C_2$ is equal to the snap-down voltage of $D_2$ of $C_1, (V_{ul,2} = V_{ul,1})$. In this case, the arm of $D_1$ of $C_2$ is snapped down at the same time as the arm of $D_2$ of $C_2$. Because the release voltage of $D_1$ of $C_2$ is greater than the release voltages of $C_1, (V_{ul,2} > V_{ul,1}, V_{ul,2})$, we can only release the arm $D_2$ of $C_2$ after we release the arm $D_1$ of $C_2$. As in the $V_{1,5,3,7}$ case, the state of $C_2$ must be “00” except when $C_1$ is snapped down, then $D_2$ of $C_2$ can be either “00”, “01”, “10” or “11” by varying the release voltages. Consequently, the number of accessible hysteresis states increases by exactly 3.

$V_{1,5,3,8}$: This case is similar to $V_{1,5,3,7}$.

$V_{2,5,3,8}$: This case is similar to $V_{2,5,3,7}$.

$V_{2,5,4,8}$: This case is similar to $V_{2,5,3,8}$.

$V_{2,6,4,8}$: This case is similar to $V_{2,6,3,8}$.

Inductive step: adding a Cluster (changing the size of the system from $M$ to $M+1$). This extends the number of accessible control states by exactly 3, provided that both $M$ and $M+1$ Clusters microrobotic system remain String-Cluster. Let $M$ Clusters microrobotic system, $C_1, ..., C_M$, where each $C_i$ consist of two microrobots $D_i$, $D_2$, be a String-Cluster system sorted according to $V_{ul,1}, V_{ul,1}, V_{ul,1}$... Without loss of generality, $V_{sh,M} < V_{ul,M} + 1$ and $V_{sh,M} < V_{ul,M,1}$. Fig. 6 shows the ranges for transition voltages of $Clustering M+1$ ($C_{M+1}$), such that the new $M+1$ Clusters microrobotic system remain String-Cluster.

Let $V_5, ..., V_8$ be significantly independent transition voltage levels, ordered such that $V_6 < V_7 < V_9 < V_8 < V_9$ with $|V_9 - V_9| = 2\delta v$ and $|V_9 - V_9| = 2\delta v$. Let $V_{al,M} = V_7$, $V_{dm,M} = V_9$ and $V_{ul,M} = V_9$, $V_{sh,M} = V_9$. It follows that the snap-down voltage $V_{al,M+1}$ can have value $V_9$, $V_{al,M+1}$ or voltage $V_9$ and the snap-down voltage $V_{sh,M+1}$ can have value $V_9$, $V_9$. Similarly, the release voltage, $V_{sh,M+1}$ can have the value $V_9$, $V_{sh,M+1}$ or voltage $V_9$, $V_9$ and the release voltage $V_{ul,M+1}$ can have the value $V_9$, $V_{ul,M+1}$ or voltage $V_9$, $V_9$. Consequently, for the $M+1$ Clusters microrobotic system to remain String-Cluster, one of the following combinations of the snap-down and release voltages for $C_{M+1}$ must hold: $V_9, V_9, V_9$, $V_9, V_9, V_9$, $V_9, V_9, V_9$, $V_9, V_9, V_9$, $V_9, V_9, V_9$, $V_9, V_9, V_9$.

We examine each case separately:

$V_1, V_5, V_3, V_7$: because of $V_{ul,M+1}, V_{sh,M+1}$ are greater than the snap-down voltages of $C_{sh}, ..., C_{sh}, (V_{ul,M+1} > V_{ul,1}, V_{sh,1})$ and $V_{sh,M+1} > V_{al,1}, V_{sh,1}$, $i \in ZM$ where $ZM = \{1, ..., M\}$ we can only snap down the arm of $D_1$ and $D_2$ in $C_{M+1}$ after we snap down the arms of all other clusters.

Since $V_{al,M+1}, V_{sh,M+1}$ is greater than the release voltages of $C_{sh}, ..., C_{sh}, (V_{al,M+1} > V_{ul,1}, V_{sh,1})$... and $V_{sh,M+1} > V_{ul,1}, V_{sh,1}$... $i \in ZM$ where $ZM = \{1, ..., M\}$, we can release the arms of all $C_{sh}, ..., C_{sh}$ after we have released the arm of $D_1$ and $D_2$ in $C_{M+1}$. Because the $V_{sh,M+1} > V_{al,M+1} > V_{al,1}, V_{sh,1}$, $i \in ZM$ where $ZM = \{1, ..., M\}$, we can release $D_1$ of $C_{M+1}$ while $D_2$ and all other clusters are snapped down. Consequently, we can change the states of $C_{M+1}$ to “01”, “10” or “11” when $C_{sh}, ..., C_{sh}$ are in state “11”. During all other states of the system the state of $C_{M+1}$ must remain “00”. Consequently, the number of accessible hysteresis states increase by exactly 3.

$V_1, V_5, V_3, V_7$: This case is similar to $V_1, V_5, V_3, V_7$, except that the arm of $D_1$ of Cluster $C_M$ is released at the same time as the arm of $D_2$ of Cluster $C_{M+1}$. As long as $V_{sh,M+1} > V_{al,1}$, we can snap down the arm of $D_2$ of $C_{M+1}$ only after all

Fig. 6. Illustration of the proof of Lemma 1
other clusters are in state “11”. As a consequence the number of accessible hysteresis states increase by exactly 3.

\[(V_2, V_5, V_3, V_2): \text{ The snap-down voltage } D_1 \text{ of } C_{M+1} \text{ is equal to the snap-down voltage of } D_2 \text{ of } C_M, V_{alt,M+1} = V_{ah,M}. \text{ In this case, the arm of } D_1 \text{ of } C_{M+1} \text{ is snapped down at the same time as the arm of } D_2 \text{ of } C_M. Because the release voltage of } D_1 \text{ of } C_{M+1} \text{ is greater than the release voltages of } C_i, ..., C_M, V_{uh,M+1} > V_{ul,i}, \text{ then } i \in Z_M, \text{ we can only release the arm } D_1 \text{ of } C_M \text{ after we release the arm } D_1 \text{ of } C_{M+1}. \text{ As in the } (V_1, V_5, V_3, V_2) \text{ case, the state of } C_{M+1} \text{ must be “00” except when } C_i, ..., C_M \text{ are all snapped down, then } D_1 \text{ of } C_{M+1} \text{ can be either “00”, “01”, “10” or “11” by varying the release voltages. Consequently, the number of accessible hysteresis states increase by exactly 3.} \]

\[(V_2, V_8, V_3, V_8): \text{ This case is similar to } (V_1, V_5, V_3, V_7), \text{ except that The snap-down voltage } D_1 \text{ of } C_{M+1} \text{ is equal to the snap-down voltage of } D_2 \text{ of } C_M, V_{alt,M+1} = V_{ah,M}. \text{ In this case, the arm of } D_1 \text{ of } C_{M+1} \text{ is snapped down at the same time as the arm of } D_2 \text{ of } C_M. Because the release voltage of } D_1 \text{ of } C_{M+1} \text{ is greater than the release voltages of } C_i, ..., C_M, V_{uh,M+1} > V_{ul,i}, \text{ we can only release the arm } D_2 \text{ of } C_M \text{ after we release the arm } D_1 \text{ of } C_{M+1}. \text{ As in the } (V_1, V_5, V_3, V_7) \text{ case, the state of } C_{M+1} \text{ must be “00” except when } C_i, ..., C_M \text{ are all snapped down, then } D_1 \text{ of } C_{M+1} \text{ can be either “00”, “01”, “10” or “11” by varying the release voltages. Consequently, the number of accessible hysteresis states increase by exactly 3.} \]

\[(V_2, V_6, V_3, V_8): \text{ This case is similar to } (V_2, V_5, V_3, V_8), \text{ except that The snap-down voltage } D_1 \text{ of } C_{M+1} \text{ is equal to the snap-down voltage of } D_2 \text{ of } C_M, V_{alt,M+1} = V_{ah,M}. \text{ In this case, the arm of } D_1 \text{ of } C_{M+1} \text{ is snapped down at the same time as the arm of } D_2 \text{ of } C_M. Because the release voltage of } D_1 \text{ of } C_{M+1} \text{ is greater than the release voltages of } C_i, ..., C_M, V_{uh,M+1} > V_{ul,i}, \text{ we can only release the arm } D_2 \text{ of } C_M \text{ after we release the arm } D_1 \text{ of } C_{M+1}. \text{ As in the } (V_2, V_5, V_3, V_8) \text{ case, the state of } C_{M+1} \text{ must be “00” except when } C_i, ..., C_M \text{ are all snapped down, then } D_1 \text{ of } C_{M+1} \text{ can be either “00”, “01”, “10” or “11” by varying the release voltages. Consequently, the number of accessible hysteresis states increase by exactly 3.} \]

We have shown that adding a Cluster to a String-Cluster, such that the resulting system remains a String-Cluster, increases the number of accessible hysteresis states by exactly 3. Combined with the base case (\(n = 1\), four hysteresis states), it follows by induction that every \(M\)-String-Cluster system has exactly “\(n+3=2M+3\)” accessible hysteresis states, where \(M= \text{number of Clusters}, n \text{ number of Micro-robots and } M=n/2\).

We now construct the control primitives and corresponding control matrix that can access the \(n + 3\) hysteresis states of an \(M\)-String-Cluster system. The ordering of the clusters is determined by the transition voltages of the steering arms. We construct the control primitive \(P_j(S)\) such that it assigns the state “11” to all clusters \(C_i\) for \(i < j\), and “00” to all cluster \(C_i\) for \(i > j\), and base on the value of \(S\), it can assign the states “01”, “10” or “11” to \(C_j\). \(P_j\) is defined by a control cycle containing two control pulses, \(P_j(S) = V_{a,1}, V_{a,2}\) with a decision variable \(S\). Consider the String-Cluster system shown in Fig. 7, where \(V_6, ..., V_0\) represent significantly independent control voltage levels. \((S)\) selects the Hysteresis state of \(C_j\):
We refer to $A$ as the String-Cluster control matrix, the $n + 3$ control primitives contained in $M$ as the String-Cluster control primitives, and the $n + 3$ hysteresis states accessible using these control primitives as the String-Cluster hysteresis states. Note that because adding three new control states to a String-Cluster system requires the addition of two independent voltage levels (per Lemma 1), the control bandwidth requirement for a String-Cluster system is $\xi_n = n + 2$.

As it can be seen, String-Cluster system needs a control voltage bandwidth of order $n$ but as compared to [6] String (which could control one robot at a time), it can control and maneuver two robots simultaneously which results in time efficient and also reconfigurable system because the first two robots that create the assembly point need not to be in a specific configuration.

For such an order, there exists a formula $P_J(S)$, shown in equation (8), which generates all $(n + 3)$ String-Cluster control primitives. $P_J(S)$ is defined by a control cycle containing six control pulses, $P_J(S) = (V_{a,1}, V_{a,2}, V_{a,3}, V_{a,4}, V_{a,5}, V_{a,6})$ with a decision variable $S$. Unlike the previous techniques, the new control primitives do not increase with population size, enabling the implementation of the control presented in [6]. The control cycle for each control primitive defined by equation (8) contains a sequence exactly 6 control pulses. Again $S$ selects the Hysteresis state of $C_j$ in Equation. 7. We construct the control primitive $P_J(S)$ in Equation. 8. Where $V_{\text{max}} = \text{MAX}\{V_{ah}\}, I \leq j \leq n$; $V_{u,l,j} = V_{ul,j} + \delta v, V_{uh,j} = V_{uh,j} + \delta v, V_{u,l,j} = V_{ul,j} - \delta v$ and $V_{d,l,j} = V_{dl,j} - \delta v$.

$P_J(S)$ generates $n + 3$ control primitives that form a String-Cluster matrix, by causing all clusters $C_i (i < j)$ to be in the state “11”, and all cluster $C_j (i > j)$ to be in the state “00”, while based on the value of $S$ it assigns the states “01”, “10” or “11” to $C_j$. Consider the base case, where all $C_j, (j \in Z_M)$ are in state “00”.

**Proof:**

We define Group $(G_i)$, $(i \in \mu = \{1, ..., M/|K| - 1\})$; Where $M$ is the number of clusters and $K$ is the number of independent snap-down voltages) to be the set of all clusters $C_j, (j \in Z_M)$ with equal $V_{ul,j}$ and $V_{uh,j}$.

$$G_i = \left\{ \text{Cluster}, \forall \text{Cluster}_{k}, \text{Cluster}_m \in G_i, \begin{cases} V_{ul,j} = V_{ul,k}, & V_{uh,j} = V_{uh,k} \end{cases}, m, k \in Z_M \right\}$$

We make the inductive argument:

**Base condition:** Base case keeps all cluster $C_j, (j \in Z_M)$ in state “00”.

**Inductive step:** after applying of the first two control pulses ($V_{\text{max}}, V_{ul,f}$), all groups $(G_i, ..., G_{i})$ are in state “11” and all cluster $G_i, (i > j)$ are in state “00”. We will show that by applying the sequence of four primitive control voltages shown in Eq. (9), the system will be in one of the three states of String-Cluster system, where Cluster ($C_j$) will be in state “01”, “10” or “11” based on the value of $S$ while all clusters ($C_i, ..., C_{j}$) are in state “11” and all cluster $C_i, (i$
Case (a), (b): If \( V_{dl,j} \) is greater than \( V_{ul,j} \), the \( C_j \) cluster is kept in state "00". If \( V_{dl,j} \) is greater than \( V_{ul,j} \), the \( C_j \) cluster is set to state "01" and will release \( D_i \) of all clusters \( C_i (i < j) \) with \( V_{uh,i} = V_{uh,j} \). Finally, \( V_{dl,j} \) and \( V_{ul,j} \) will set \( C_j \) to state "11" again while keeping \( C_k \) in state if \( S = 0 \) then release \( D_i \) of all clusters \( C_i (i < j) \) with \( V_{uh,i} = V_{uh,j} \). By applying the remaining control primitives \( V_{dl,j} \), \( V_{uh,j} \), \( V_{ul,j} \), \( V_{ul,j} \), and \( V_{dl,j} \), all \( D_i \) of all clusters \( C_i (i < j) \) with \( V_{uh,i} = V_{uh,j} \) will be released while keeping the state of \( C_j \) in "00". If \( S = 1 \) then

\[
P_j(S) =
\begin{cases}
(V_{max}, V_{ul}, j^-; V_{dl}, j^+; V_{ul}, j^+; V_{dl}, j^-; V_{uh}, j^-) & \text{if } j \in Z_M, \ S = 0 \\
(V_{max}, V_{ul}, j^-; V_{dh}, j^+; V_{ul}, j^+; V_{dl}, j^-; V_{uh}, j^-) & \text{if } j \in Z_M, \ S = 1 \\
(V_{max}, V_{ul}, j^-; V_{dl}, j^+; V_{uh}, j^+; V_{uh}, j^-) & \text{if } j \in Z_M, \ S = 2 \\
(V_{max}, V_{ul}, j^-; V_{dh}, j^+; V_{ul}, j^+; V_{dl}, j^-; V_{uh}, j^-) & \text{if } j \in Z_M, \ S = 3
\end{cases}
\]
It is clear that in case (a) and (b): \( Vdh, j \) sets cluster \( C_j \) to state “11”, while \( C_{sd}, k > j \) is in state “00”. Consequently, \( Vul, j^+ \) will set \( C_j \) to state “01” and will release \( D_i \) of all clusters \( C_i (i \leq j) \) with \( Vul, i = Vuh, j \). By applying the remaining control primitives \( Vdl, j^- \), \( Vuh, j^+ \), all \( D_i \) of all clusters \( C_i (i \leq j) \) with \( Vul, i = Vuh, j \) will snapped down while keeping the state of \( C_j \) in “01”. Case (c), (d): \( Vdh, j \) sets cluster \( C_j \) to state “11” and \( C_k, k > j \) to “10”. Consequently, \( Vul, j^+ \) will be in state “11” and all clusters \( C_i = (i \leq j) \) will be snapped down.

**Theorem:** An algorithm that can plan the motion (i.e., finds the control sequence) for a String-Cluster system can be applied to plan the motion for any \( ESATC \) system of stress-engineered microrobots.

**Proof:** A consequence of Lemma 2; a string-cluster control matrix can be constructed for any \( M-(ESATC) \) microrobotic system.

Table 1: Comparison of the control voltage bandwidth requirements, \( \xi_n \), the number of control pulses of \( n \)-robot NHG, Reconfigurability and Multi-shapes-Assembly of STRING, SESat systems, String-Cluster and SESATC.

|                  | NHG  | STRING | SESat | String-Cluster | SESATC |
|------------------|------|--------|-------|----------------|--------|
| \( \xi_n \)      | 2n   | \( n+1 \) | \( 2\sqrt{n} \) | \( n+2 \) | \( \left[ 1 + 2\sqrt{n} \right] \) |
| No. control pulses | 1    | 2      | \( O(n) \)  | 2              | 6      |
| Number of robots with \( \xi_n=20 \) | 10   | 19     | 100   | 18             | 90     |
| Multi-shapes-Assembly | YES | NO    | NO    | YES            | YES    |

Case (a), (b): \( Vdl, j \) sets cluster \( C_j \) to state “10”, while \( C_{sd}, k > j \) is in state “00”. Consequently, \( Vul, j^+ \) will keep \( C_j \) in state “00”. Case (c), (d): \( Vdl, j \) sets \( C_j \) to state “10”, while \( C_{sd}, k > j \) is in state “00”. Consequently, \( Vul, j^+ \) will keep \( C_j \) in state “00”. Case (a), (b): \( Vdh, j \) sets cluster \( C_j \) to state “11”, while \( C_{sd}, k > j \) is in state “00”. Consequently, \( Vul, j^+ \) will set \( C_j \) to state “01” and will release \( D_i \) of all clusters \( C_i = (i \leq j) \) with \( Vul, i = Vuh, j \). Finally, by applying \( Vdl, j^+, Vuh, j^+ \),
V. CONCLUSION

In this paper, we presented a comprehensive analysis of the scalability of different methods for differentiating the behavior of MicroStressBots using electrostatic hysteresis. We have shown that by the two new control strategies not only we can have a highly underactuated system but also we will have robust controllable system which could complete any microassembly process started from any configuration. These control methods are sufficient to implement a reconfigurable system. These results lay the theoretical foundation for developing new methods to control of large number of MEMS microrobots.

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