Measurement of the spectral function for the $\tau^- \rightarrow K^-K_S\nu_\tau$ decay in BABAR experiment

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Abstract

The decay $\tau^- \rightarrow K^-K_S\nu_\tau$ has been studied using $430 \times 10^6$ $e^+e^- \rightarrow \tau^+\tau^-$ events produced at a center-of-mass energy around 10.6 GeV at the PEP-II collider and studied with the BABAR detector. The mass spectrum of the $K^-K_S$ system has been measured and the spectral function has been obtained. The measured branching fraction $B(\tau^- \rightarrow K^-K_S\nu_\tau) = (0.739 \pm 0.011\text{(stat.)} \pm 0.020\text{(syst.)}) \times 10^{-3}$ is found to be in agreement with earlier measurements.

1 Introduction

The $\tau$ lepton provides a remarkable laboratory for studying many open questions in particle physics. With a large statistics of about $10^9 \tau_s$ produced in $e^+e^-$ annihilation at the BABAR experiment, various aspects can be studied, for example, improving the precision of spectral functions describing the mass distribution of the hadronic decays of the $\tau$. In this work, we analyze the $\tau^- \rightarrow K^-K_S\nu_\tau$ decay and measure the spectral function of this channel defined as

$$V(q) = \frac{m_\tau^8}{12\pi C(q)} \frac{B(\tau^- \rightarrow K^-K_S\nu_\tau)}{B(\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau)} \frac{dN}{dq},$$

(1)

where $m_\tau$ is the $\tau$ mass, $q \equiv m_{K^-K_S}$ is the invariant mass of the $K^-K_S$ system, $V_{ud}$ is an element of the CKM (Cabibbo-Kobayashi-Maskava) matrix, $(dN/dq)/N$ is the normalized $K^-K_S$ mass spectrum, and $C(q)$ is the phase space correction factor given by the following formula:

$$C(q) = q(m_\tau^2 - q^2)^2(m_\tau^2 + 2q^2).$$

(2)

The branching fraction for the $\tau^- \rightarrow K^-K_S\nu_\tau$ decay has been measured with relatively high (3%) precision by the Belle experiment. The $K^-K_S$ mass spectrum was measured

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1 Throughout this paper, inclusion of charge-conjugated channels is implied.
by the CLEO experiment [4]. In the CLEO analysis, a data set of $2.7 \times 10^6$ produced $\tau$ pairs was used, and about 100 events in the decay channel $\tau^- \rightarrow K^- K_S \nu_{\tau}$ were selected. In this work [5], using about $\sim 10^9 \tau$ leptons, we significantly improve upon the measurement of the spectral function for the $\tau^- \rightarrow K^- K_S \nu_{\tau}$ decay.

2 Data used in the analysis

We analyze a data sample corresponding to an integrated luminosity of 468 fb$^{-1}$ recorded with the BABAR detector [6, 7] at the SLAC PEP-II asymmetric-energy $e^+e^-$ collider. In the laboratory frame, the energy of electron and positron beams is 9 and 3.1 GeV, respectively.

For simulation of $e^+e^- \rightarrow \tau^+\tau^-$ events the KK2f Monte Carlo generator [8] is used, which includes higher-order radiative corrections to the Born-level process. Decays of $\tau$ leptons are simulated using the Tauola package [9]. Two separate samples of simulated $e^+e^- \rightarrow \tau^+\tau^-$ events are used: a generic sample with $\tau$ decaying to all significant final states, and the signal channel where $\tau^+ \rightarrow l^+\nu_l\bar{\nu}_\tau$, $l = e$ or $\mu$ and $\tau^- \rightarrow K^- K_S \nu_{\tau}$. To estimate backgrounds, we use a sample of simulated generic $e^+e^- \rightarrow \tau^+\tau^-$ events after excluding the signal decay channel ($\tau^+\tau^-$ background) and a sample containing all events arising from $e^+e^- \rightarrow q\bar{q}$, $q = u,d,s,c$ and $e^+e^- \rightarrow BB$ processes ($q\bar{q}$ background). The $q\bar{q}$ background events with $q = u,d,s,c$ are generated using the JETSET generator [10], while $BB$ events are simulated with EVTGEN [11]. The detector response is simulated with GEANT4 [12]. The equivalent luminosity of the simulated sample is 2-3 times higher than the integrated luminosity in data.
3 Event selection

We select $e^+e^-\rightarrow\tau^+\tau^-$ events with the $\tau^+$ decaying leptonically ($\tau^+\rightarrow l^+\nu l\bar{\nu}_{\tau}, l = e$ or $\mu$) and the $\tau^-$ decaying to $K^-\bar{K}_S\nu_{\tau}$. Such events referred to as signal events below. The $K_S$ candidate is detected in the $K_S\rightarrow\pi^+\pi^-$ decay mode. The topology of events to be selected is shown in Fig. 1. Unless otherwise stated, all quantities are measured in the laboratory frame. The selected events must satisfy the following requirements:

- The total number of charged tracks, $N_{trk}$, must be four and the total charge of the event must be zero.
- Among the four charged tracks there must be an identified lepton (electron or muon) and an identified kaon of opposite charge.
- To reject non-$\tau^+\tau^-$ signal backgrounds, the lepton candidate must have a momentum above 1.2 GeV/c, the momentum in the center-of-mass frame (c.m. momentum) must be smaller than 4.5 GeV/c, and the cosine of the lepton polar angle $|\cos\theta_l|$ must be below 0.9.
- To suppress background from charged pions, the charged kaon candidate must have a momentum, $p_K$, above 0.4 GeV/c and below 5 GeV/c, and the cosine of its polar angle must lie between -0.7374 and 0.9005.
- The two remaining tracks, assumed to be pions, form the $K_S$ candidate. The $\pi^+\pi^-$ invariant mass must lie within 25 MeV/$c^2$ of the nominal $K_S$ mass, 497.6 MeV/$c^2$. The $K_S$ flight length $r_{K_S}$, measured as the distance between the $\pi^+\pi^-$ vertex and the collision point, must be larger than 1 cm.
- The total energy in neutral clusters, $\sum E_{\gamma}$, must be less than 2 GeV. Here, a neutral cluster is defined as a local energy deposit in the calorimeter with energy above 20 MeV and no associated charged track.
- The magnitude of the thrust [13][14] for the event, calculated using charged tracks only, must be greater than 0.875.
- The angle between the momentum of the lepton and the direction of the hadronic final state in the c.m. frame should be between 110 and 180 degrees.

The chosen selection requirements are close to those used in previous $\tau$ studies in BABAR [15]. As a result of applying these cuts the $\tau$ background is suppressed by 3.5 orders of magnitude, and the $q\bar{q}$ background by 5.5 orders.

4 Detection efficiency

The detection efficiency obtained after applying the selection criteria is calculated using signal Monte Carlo simulation as a function of the true $m_{K^-K_S}$ mass. The efficiency is weakly dependent on $m_{K^-K_S}$. The average efficiency over the mass spectrum is about 13%. It should be noted that the $K^-K_S$ mass resolution is about 2-3 MeV/$c^2$, significantly smaller
than the size of the mass bin (40 MeV/c^2) used in our analysis. Therefore, in the following we neglect the effects of the finite \(K^- K_S\) mass resolution.

To correct for the imperfect simulation of the kaon identification requirement, the particle identification PID efficiencies have been compared for data and simulation on high purity control samples of kaons from \(D^*^+ \rightarrow \pi^+ D^0\), \(D^0 \rightarrow K^- \pi^+\) decays \cite{16}. We correct the simulated efficiency using the measured ratios of the efficiencies measured in data and Monte Carlo, in bins of the kaon candidate momentum and polar angle. The resulting correction factor is small \(\sim 1\%\) and weakly depends on \(m_{K^- K_S}\).

5 Subtraction of non-\(K_S\) background

The \(\pi^+ \pi^-\) mass spectra for \(K_S\) candidates in data and simulated signal events are shown in Fig. 2. The data spectrum consists of a peak at the \(K_S\) mass and a flat background. To subtract the non-\(K_S\) background, the following procedure is used. The signal region is set to \(\pi^+ \pi^-\) masses within 0.0125 GeV/c^2 of the \(K_S\) mass (indicated by arrows in Fig. 2), and the sidebands are set to between 0.0125 and 0.0250 GeV/c^2 away from the nominal \(K_S\) mass. Let \(\beta\) be the fraction of events with a true \(K_S\) that fall in the sidebands, and let \(\alpha\) be the fraction of non-\(K_S\) events that fall in the sidebands. The total number of events in the signal region plus the sidebands, \(N\), and the number of events in the sidebands, \(N_{sb}\), depend on the number of true \(K_S\), \(N_{K_S}\), and the number of non-\(K_S\) background events, \(N_b\) according to the following relation:

\[
N = N_{K_S} + N_b, \tag{3a}
\]

\[
N_{sb} = \alpha \cdot N_b + \beta \cdot N_{K_S}. \tag{3b}
\]

Therefore:

\[
N_{K_S} = (\alpha N - N_{sb})/(\alpha - \beta). \tag{4}
\]

The value of \(\beta\) is determined using \(\tau\) signal simulation. It is found to be nearly independent of the \(m_{K^- K_S}\) mass and is equal to 0.0315 \(\pm\) 0.0015. The value of \(\alpha\) is expected to be 0.5 for a uniformly distributed background. This is consistent with the value 0.499 \(\pm\) 0.005 obtained on simulated \(\pi^+ \pi^-\) background events. The non-\(K_S\) background is subtracted in each \(m_{K^- K_S}\) bin. Its fraction is found to be about 10% of the selected events with \(m_{K^- K_S}\) near and below 1.3 GeV/c^2 and increases up to 50% above 1.6 GeV/c^2.

6 Subtraction of \(\tau\)-background with a \(\pi^0\)

Although the studied process \(\tau^- \rightarrow K^- K_S \nu_{\tau}\) is not supposed to contain a \(\pi^0\) in the final state, some events from background processes with a \(\pi^0\) pass the selection criteria. In the following, we describe how the \(\pi^0\) background contribution is subtracted.

According to the simulation, the number of signal and \(\tau\)-background events are of the same order of magnitude. The \(\pi^+ \tau^-\) background consists of events with the decay \(\tau^- \rightarrow K^- K_S \pi^0 \nu_{\tau}\) (79%), events with a misidentified kaon from decays \(\tau^- \rightarrow \pi^- K_S \nu_{\tau}\) (10%) and \(\tau^- \rightarrow \pi^- K_S \pi^0 \nu_{\tau}\) (3%), and events with a misidentified lepton mainly from the decays \(\tau^+ \rightarrow\)
Figure 3: The probabilities $\epsilon_s$ and $\epsilon_b$ used in Eqs. (5a, 5b) as functions of the $K^- K_S$ mass, measured on simulated events.

Figure 4: Measured $m_{K^- K_S}$ spectra for signal events in comparison with the Monte Carlo simulation.

$\pi^+ \bar{\nu}_\tau$ and $\tau^+ \rightarrow \pi^+ \pi^0 \bar{\nu}_\tau$ (7%). Thus, more than 80% of the background events contain a $\pi^0$ in the final state. The hadronic mass spectra for $\tau$ decays with a $\pi^0$ are not well known, so we use the experimental data to subtract this background.

The $\tau$ background without a $\pi^0$ ($\tau^- \rightarrow \pi^- K_S \nu_\tau$, $\tau^+ \rightarrow \pi^+ \bar{\nu}_\tau$) and $q\bar{q}$ background are simulating well. Therefore, this background is subtracted using Monte Carlo simulation.

To subtract the $\pi^0$ background, the selected events are divided into two classes, without and with a $\pi^0$ candidate, which is defined as a pair of photons with an invariant mass in the range $100 - 160$ MeV/$c^2$.

On the resulting sample, the numbers of signal ($N_s$) and background $\tau^+ \tau^-$ events containing a $\pi^0$ candidate ($N_b$) are obtained in each $m_{K^- K_S}$ bin:

$$\begin{align*}
N_{0\pi^0} &= (1 - \epsilon_s)N_s + (1 - \epsilon_b)N_b, \\
N_{1\pi^0} &= \epsilon_sN_s + \epsilon_bN_b,
\end{align*}$$

(5a)

(5b)

where $N_{0\pi^0}$ and $N_{1\pi^0}$ are the numbers of selected data events with zero and at least one $\pi^0$ candidate, and $\epsilon_s$ ($\epsilon_b$) is the probability for signal (background) $\tau^+ \tau^-$ events to be found in events with at least one $\pi^0$ candidate calculated using Monte Carlo simulation. The values $\epsilon_s$ and $\epsilon_b$ for each bin in $m_{K^- K_S}$ are measured in Monte Carlo by counting how many signal and background event candidates contain a $\pi^0$ candidate. Figure 3 shows the $\epsilon_s$ and $\epsilon_b$ measured in Monte Carlo as a function of $m_{K^- K_S}$. These efficiencies are corrected to take into account the difference between data and Monte Carlo.

With these corrected values for $\epsilon_s$ and $\epsilon_b$ we solve Eqs. (5a, 5b) for each $K^- K_S$ mass bin and obtain mass spectra for signal ($N_s$) and background ($N_b$). The efficiency corrected signal mass spectrum is shown in Fig. 4 in comparison with the simulation. We find a substantial difference between data and simulation for the signal spectrum. The result is not affected by inaccuracies of the simulation since it doesn’t depend on the normalization of the simulated $m_{K^- K_S}$ spectrum.
Figure 5: Normalized $K^-K_S$ invariant mass spectrum for the $\tau^- \rightarrow K^-K_S\nu_{\tau}$ decay measured in this work (filled circles) compared to the CLEO measurement [4] (empty squares). Only statistical uncertainties are shown.

Figure 6: Measured spectral function for the $\tau^- \rightarrow K^-K_S\nu_{\tau}$ decay. Only statistical uncertainties are shown.

7 Systematic uncertainties

The uncertainty from non-$K_S$ background subtraction (0.4%) is estimated by varying the coefficients of $\alpha$ and $\beta$ within their uncertainties. This uncertainty is independent on the $K^-K_S$ mass. The PID correction uncertainty due to data-Monte Carlo simulation difference in particle identification is taken to be 0.5%, independent of the $K^-K_S$ mass. The uncertainty on how well the Monte Carlo simulates the tracking efficiency is estimated to be 1%. We take the observed difference between data and Monte Carlo near the end point $M_{K^-K_S} = m_\tau$ as an uncertainty on the $q\bar{q}$ background. This leads to an uncertainty on $B(\tau^- \rightarrow K^-K_S\nu_{\tau})$ of 0.5%. The uncertainty associated with the subtraction of the $\tau^+\tau^-$ background with $\pi^0_S$ is estimated to be 2.3%.

The systematic uncertainties from different sources are combined in quadrature. The total systematic uncertainty for the branching fraction $B(\tau^- \rightarrow K^-K_S\nu_{\tau})$ is 2.7%. The systematic uncertainties for the mass spectrum are listed in Table 1. They gradually decrease from $\approx$9% at $m_{K^-K_S} = 1$ GeV/$c^2$ to 1.5% at $m_{K^-K_S} = m_\tau$. Near the maximum of the mass spectrum (1.3 GeV/$c^2$) the uncertainty is about 2.5%.
Table 1: Measured spectral function \( V \) of the \( \tau^- \rightarrow K^- K_S \nu_\tau \) decay, in bins of \( m_{K^- K_S} \). The columns report: the range of the bins, the normalized number of events, the value of the spectral function. The first error is statistical, the second systematic.

| \( m_{K^- K_S} \) (GeV/c\(^2\)) | \( N_s/N_{tot} \times 10^3 \) | \( V \times 10^3 \) |
|-----------------------------|-------------------------------|----------------|
| 0.98 – 1.02                | 5.6 ± 1.4                    | 0.071 ± 0.018 ± 0.006 |
| 1.02 – 1.06                | 26.0 ± 2.7                   | 0.331 ± 0.034 ± 0.026 |
| 1.06 – 1.10                | 46.0 ± 3.2                   | 0.593 ± 0.042 ± 0.042 |
| 1.10 – 1.14                | 70.8 ± 3.5                   | 0.934 ± 0.046 ± 0.056 |
| 1.14 – 1.18                | 84.4 ± 3.4                   | 1.148 ± 0.047 ± 0.057 |
| 1.18 – 1.22                | 92.3 ± 3.3                   | 1.309 ± 0.046 ± 0.052 |
| 1.22 – 1.26                | 98.2 ± 3.2                   | 1.468 ± 0.048 ± 0.044 |
| 1.26 – 1.30                | 98.4 ± 3.2                   | 1.569 ± 0.050 ± 0.042 |
| 1.30 – 1.34                | 96.3 ± 3.0                   | 1.663 ± 0.052 ± 0.042 |
| 1.34 – 1.38                | 90.2 ± 2.9                   | 1.715 ± 0.052 ± 0.039 |
| 1.38 – 1.42                | 87.8 ± 3.1                   | 1.873 ± 0.066 ± 0.039 |
| 1.42 – 1.46                | 65.1 ± 2.6                   | 1.597 ± 0.064 ± 0.032 |
| 1.46 – 1.50                | 57.3 ± 2.5                   | 1.666 ± 0.073 ± 0.032 |
| 1.50 – 1.54                | 38.1 ± 2.5                   | 1.361 ± 0.090 ± 0.023 |
| 1.54 – 1.66                | 36.9 ± 2.4                   | 0.785 ± 0.049 ± 0.013 |
| 1.66 – 1.78                | 6.6 ± 10.2                   | 0.986 ± 1.520 ± 0.014 |

8 The results

The branching ratio of the \( \tau^- \rightarrow K^- K_S \nu_\tau \) decay is obtained using the following expression:

\[
B(\tau^- \rightarrow K^- K_S \nu_\tau) = \frac{N_{exp}}{2L_{lep}\sigma_{\tau\tau}} = (0.739 \pm 0.011 \pm 0.020) \times 10^{-3},
\]

where \( N_{exp} = 223741 \pm 3461 \) (error is statistical) is the total number of signal events in the spectrum in Fig. 5. \( L = 468.0 \pm 2.5 \) fb\(^{-1}\) is the BABAR integrated luminosity \[20\], \( \sigma_{\tau\tau} = 0.919 \pm 0.003 \) nb is the \( e^+ e^- \rightarrow \tau^+ \tau^- \) cross section at 10.58 GeV \[8\] and \( B_{lep} = 0.3521 \pm 0.0006 \) is the world average sum of electronic and muonic branching fractions of the \( \tau \) lepton \[2\]. The first uncertainty in \[6\] is the statistical, the second is systematic. Our result agrees well with the Particle Data Group (PDG) value \((0.740 \pm 0.025) \times 10^{-3} \) \[2\], which is determined mainly by the recent Belle measurement \((0.740 \pm 0.007 \pm 0.027) \times 10^{-3} \) \[3\].

The measured mass spectrum \( m_{K^- K_S} \) for the \( \tau^- \rightarrow K^- K_S \nu_\tau \) decay is shown in Fig. 5 and listed in Table 1. Our \( m_{K^- K_S} \) spectrum is compared with the CLEO measurement \[4\]. The BABAR and CLEO spectra are in good agreement. The spectral function \( V(q) \) calculated using Eq. \[1\] is shown in Fig. 6 and listed in Table 4. Due to the large error in the mass interval 1.66-1.78 GeV/c\(^2\), which exceeds the scale of Fig. 6, the value of \( V(q) \) in this interval is not shown in Fig. 6.
9 Conclusions

The $K^-K_S$ mass spectrum and vector spectral function in the $\tau^- \to K^-K_S\nu_\tau$ decay have been measured by the BABAR experiment. The measured $K^-K_S$ mass spectrum is far more precise than CLEO measurement \cite{4} and the branching fraction $(0.739 \pm 0.011 \pm 0.020) \times 10^{-3}$ is comparable to Belle’s measurement \cite{3}.

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References

\cite{1} Y. S. Tsai, \textit{Decay Correlations of Heavy Leptons in $e^+e^- \to l^+l^-$}, Phys. Rev. D 4, 2821 (1971), [Erratum-ibid. D 13, 771 (1976), \url{https://doi.org/10.1103/PhysRevD.4.2821}]
\cite{2} C. Patrignani \textit{et al.} (Particle Data Group), \textit{Review of Particle Physics}, Chin. Phys. C 40, 100001 (2016), \url{https://doi.org/10.1088/1674-1137/40/10/100001}
\cite{3} S. Ryu \textit{et al.} (Belle Collaboration), \textit{Measurements of Branching Fractions of $\tau$ Lepton Decays with one or more $K^0_S$}, Phys. Rev. D 89, 072009 (2014), \url{https://doi.org/10.1103/PhysRevD.89.072009}.
\cite{4} T. E. Coan \textit{et al.} (CLEO Collaboration), \textit{Decays of tau leptons to final states containing $K^0_S$ mesons}, Phys. Rev. D 53, 6037 (1996), \url{https://doi.org/10.1103/PhysRevD.53.6037}.
\cite{5} J. P. Lees \textit{et al.} (BABAR Collaboration), \textit{Measurement of the spectral function for the $\tau^- \to K^-K_S\nu_\tau$ decay}, Phys. Rev. D 98, 032010 (2018), \url{https://doi.org/10.1103/PhysRevD.98.032010}.
\cite{6} B. Aubert \textit{et al.} (BABAR Collaboration), \textit{The BaBar detector}, Nucl. Instr. and Meth. A 479, 1 (2003), \url{https://doi.org/10.1016/S0168-9002(01)02012-5}.
\cite{7} B. Aubert \textit{et al.} (BABAR Collaboration), \textit{The BABAR Detector: Upgrades, Operation and Performance}, Nucl. Instr. and Meth. A 729, 615 (2013), \url{https://doi.org/10.1016/j.nima.2013.05.107}.
\cite{8} S. Jadach, B. F. Ward, and Z. Was, \textit{The Precision Monte Carlo event generator KK for two fermion final states in $e^+e^-$ collision}, Comput. Phys. Commun. 130, 260 (2000), \url{https://doi.org/10.1016/S0010-4655(00)00048-5}.
\cite{9} S. Jadach \textit{et al.}, \textit{The tau decay library TAUOLA: Version 2.4}, Comput. Phys. Commun. 76, 361 (1993), \url{https://doi.org/10.1016/0010-4655(93)90061-G}.
[10] T. Sjöstrand, S. Mrenna, and P. Skands, *PYTHIA 6.4 Physics and Manual*, JHEP **05**, 026 (2006), [https://doi.org/10.1088/1126-6708/2006/05/026](https://doi.org/10.1088/1126-6708/2006/05/026).

[11] D. J. Lange, *The EvtGen particle decay simulation package*, Nucl. Instr. and Meth. A **462**, 152 (2001), [https://doi.org/10.1016/S0168-9002(01)00089-4](https://doi.org/10.1016/S0168-9002(01)00089-4).

[12] S. Agostinelli *et al.* *GEANT4: A Simulation toolkit*, Nucl. Instr. and Meth. A **506**, 250 (2003), [https://doi.org/10.1016/S0168-9002(03)01368-8](https://doi.org/10.1016/S0168-9002(03)01368-8).

[13] E. Farhi, *A QCD Test for Jets*, Phys. Rev. Lett. **39**, 1587 (1977), [https://doi.org/10.1103/PhysRevLett.39.1587](https://doi.org/10.1103/PhysRevLett.39.1587).

[14] S. Brandt, C. Peyrou, R. Sosnowski, and A. Wroblewski, *The Principal axis of jets. An Attempt to analyze high-energy collisions as two-body processes*, Phys. Lett. **12**, 57 (1964), [https://doi.org/10.1016/0031-9163(64)91176-X](https://doi.org/10.1016/0031-9163(64)91176-X).

[15] J. P. Lees *et al.* (BABAR Collaboration), *The branching fraction of τ− → π−K^0_SK^0_S(π^0)ν_τ decays*, Phys. Rev. D **86**, 092013 (2012), [https://doi.org/10.1103/PhysRevD.86.092013](https://doi.org/10.1103/PhysRevD.86.092013).

[16] D. Boutigny *et al.* The BABAR physics book: Physics at an asymmetric B-factory, (BABAR Collaboration), SLAC-R-0504 (2010).

[17] D. Epifanov *et al.* (Belle Collaboration), *Study of τ− → K^0_Sπ^-ν_τ decay at Belle*, Phys. Lett. B **654**, 65 (2007), [https://doi.org/10.1016/j.physletb.2007.08.045](https://doi.org/10.1016/j.physletb.2007.08.045).

[18] M. Fujikawa *et al.* (Belle Collaboration), *High-Statistics Study of the τ− → π^-π^0ν_τ Decay*, Phys. Rev. D **78**, 072006 (2008), [https://doi.org/10.1103/PhysRevD.78.072006](https://doi.org/10.1103/PhysRevD.78.072006).

[19] B. Aubert *et al.* (BABAR Collaboration), *Study of e^+e^- → π^+π^-π^0 process using initial state radiation with BaBar*, Phys. Rev. D **70**, 072004 (2004), [https://doi.org/10.1103/PhysRevD.70.072004](https://doi.org/10.1103/PhysRevD.70.072004).

[20] J. P. Lees *et al.* (BABAR Collaboration), *Time-Integrated Luminosity Recorded by the BABAR Detector at the PEP-II e^+e^- Collider*, Nucl. Instr. and Meth. A **726**, 203 (2013), [https://doi.org/10.1016/j.nima.2013.04.029](https://doi.org/10.1016/j.nima.2013.04.029).