The abstraction process of junior high school students

R Amelia, S Chotimah, and G Kadarisma
Institut Keguruan dan Ilmu Pendidikan Siliwangi, Jl. Terusan Jenderal Sudirman, Cimahi 40526, Indonesia
Email: chotimahsiti@ikipsiliwangi.ac.id

Abstract. The background of this study is in fact that students’ abstraction process still low in geometry. Straight-line equations are a prerequisite in understanding geometry material further. Therefore, the objective of this study is to measure the process of student abstraction in straight line equations. The population in this study are Eight Graders from one of the junior high schools in Cimahi City, with the selection of research subjects, namely schools in the medium cluster category in Cimahi City, and the samples in this study are 3 students that categorized on high, medium and low abilities. This research is a descriptive qualitative study. The instrument of the research is a test that is analysed to see and measure the student's abstraction process which appears in straight line equations. The results of the analysis and observations show that the abstraction process from 3 students who categorized based on high, medium and low ability, shows different ability on abstraction process on the criteria of verbal reasoning, numerical ability, and abstract reasoning. Students with high ability category have high variable reasoning ability, high numerical ability and high abstract reasoning as well. While students with medium abilities have high variable reasoning abilities, moderate numerical abilities and medium abstract reasoning. For students with low ability categories have low variable reasoning ability, moderate numerical ability and low abstract reasoning. So this process of abstraction can be seen from students with high ability categories. As for students of medium and low ability, need more guidance so that their abstraction process can improve.

1. Introduction
Mathematics is an important part that cannot be separated from human life. In the world of education, mathematics is often attached to other subjects. Therefore, mathematics is called as applied science. One of the goals of mathematics learning is to educate the life of the nation [1]. According to the Ministry of Education [2], the purpose of learning mathematics is to train thinking systematically, logically, critically, creatively and consistently.

Mathematics is a basic subject taught both in elementary and secondary schools. Mathematics tends to be abstract, the words abstract related to things that are not concrete and not visible. Therefore, there are still students who think that mathematics is difficult, especially in learning mathematics, it requires reasoning in the thinking process [3]. The process of mathematical thinking is the basic needs of students in learning mathematics, the process of abstraction is one of the thinking processes that students need to poses. The fact found in the real world that many students still focus in achieving the final results, regardless of the basic processes and concepts that must be mastered at each problems. Many students memorize the concept, without knowing the process of discovering the concept. Abstraction thinking ability is inseparable from knowledge of concepts, because abstraction requires the ability to imagine and describe objects and events that are not always physically present. The nature of abstraction is the ability to operate symbols, figures, and formulas, especially in the level of analysis and interpretation in giving opinions. Gray and Tall [4] argues that abstraction is the process of describing a particular
situation into a concept that can be thought through a construction. On the other hands Nurhasanah also said that the essence of mathematics is abstract concepts and abstractions [5].

Abstraction is a basic process in mathematics. some mathematicians interpret abstraction as a process of generalization and contextualization (a process that brings knowledge out of context) [6]. The main aspect in the abstraction ability is the use of affective of concepts and symbols in dealing with various situations, especially in solving a problem. This is confirmed by Ruseffendi, generalizing means making prediction based on the knowledge obtained in the form of examples [7]. Abstraction is an ability to find ways to solve problems without the existence of real problems, abstractions are divided into two types; empirical and theoretical abstractions [8]. Empirical abstraction is closely related to empirical experience. Understanding an object that is abstract based on the children’s social and physical experience. While the theoretical abstraction is the formation of concepts based on a theory.

The process of abstraction is an activity when a person becomes sensitive to the same characteristics of the experience, then the similarity of characteristics is used as the basis for conducting a classification so that one can recognize a new experience by comparing it to the class that has formed in his mind earlier [9]. To distinguish abstraction as an activity and abstraction as a result, the results of abstraction from the abstraction process are then referred to as concepts. The indicators of empirical abstraction are making generalizations, forming mathematical concepts related to other concepts, forming further mathematical objects, and formalizing mathematical objects. While the theoretical abstraction indicator is the process of manipulating symbols [10].

Students' mathematical abstraction become crucial due to the process of initiating the initial concepts of mathematics, this become the concern of researchers to make the improvements of student abstraction. abstraction in mathematics is a process to obtain the essence of mathematical concepts and eliminate dependence on real-world objects which may initially be interrelated [11]. The Research conducted by Noorafshar found that 39% of 12th-year students of Toowamba High School Australia did not like mathematics. In this study it was revealed that the proportion of students' concern for mathematics is low [12]. If students unable to solve a particular mathematical topic at school, and continues at home then it is sure that they cannot solve the problem. The purpose of examining process of abstraction is to see how students discover problem solving using the abstraction ability by applying the concepts and principles that students have learned. the test of abstraction ability objected to show the ability in problem solving without the presence of the object, it means that students do symbolic and imaginative thinking activities on the object both with diagrams, patterns, and images. In addition, abstraction test also aims to measure how easy students are in solving the problems.

![Abstraction Thinking Process Framework](image)

**Figure 1.** Abstraction Thinking Process Framework

Based on Figure 1, researchers will see and measure the abstraction process of students in mathematical thinking under the subject straight line equations based on criteria of verbal reasoning, numerical ability and abstract reasoning to measure how far the students' ability solving the problems in straight line equation based on criteria and how the results of learning and the ability of students to think abstraction on straight line equations. This research also expected to be a benchmark or preliminary study for further research in developing students' mathematical abstraction process so that students' mathematical abstraction ability can improve.
2. Method
The method in this study is qualitative-descriptive. This qualitative method is used to explore and obtain a more detailed picture of the situation and behaviour of students during the test seen from the answers that students give. Data collected both from test and interviews were analysed descriptively. The instruments in this study were test which were analysed to measure students’ mathematical abstraction process based on verbal reasoning, numerical abilities and abstract reasoning. The population in this study were class VIII students of one of the junior high schools in Cimahi, and the samples in this study were 3 students based on the ability of students in the high, medium and low categories. By taking a sample, it can be seen from the values of the assignments and students’ mathematics tests based on the KKM at the school.

3. Result and Discussion
Data analysis that conducted namely an analysis on the questions of straight-line equation along with observations or direct observations and interviews with students. The test was tested on third grader of junior high school students of class VIII A in one of the junior high schools in Cimahi who categorized on high, medium and low ability according to students’ initial abilities that taken from previous rankings and scores. Analysis of the process on students’ abstraction seen from three criteria, namely verbal reasoning, numerical ability, and abstract reasoning. The questions tested are as follows (Figure 2):

![Figure 2](image)

1. What is the position of two lines equation of $2x + 4y = 0$ and $6x-3y-1 = 0$?
2. The line g has the equation $y = 4x +6$. Determine the graph of the equation of the line!
3. Determine the intersection of the line equation through $2x + 4y = 0$ and $3x-2y-1 = 0$!

From the result,

| Abstraction Thinking Ability Criteria | Indicator                      | Classification                              | Problem 1 | Problem 2 | Problem 3 |
|--------------------------------------|--------------------------------|--------------------------------------------|-----------|-----------|-----------|
| Variable reasoning                   | Explain mathematics in the form of words | Able to explain correctly                  | S1-S2     | S1        |           |
|                                      |                                | Able to explain but incorrect              | S3        | S2        | S1        |
|                                      |                                | Unable to explain                          |           | S3        | S2-S3     |
| Numerical ability                    | Complete using the formula     | Able to finish using the formula correctly | S1        |           | S1-S2     |
|                                      |                                | Able to finish using formulas but wrong    | S2-S3     | S1-S2     | S3        |

Table 1. Recapitulation of Percentage of Answers Based on Indicators and Classifications Thinking of Student Abstractions
Based on the data presented in Table 1. It shows the abstraction ability of students’ on each criterion according to the level of students' abilities. Students who have high ability of variable reasoning with indicators explain mathematics in the form of words in questions number 1 and number 2, students with high ability are able to explain answers in words that are systematic and as expected. Whereas in question number 3 high-ability students have been able to explain systematically but still incorrect and not as expected. Meanwhile, students who are capable in number 1 students have been able to explain correctly, while in question number 2 and number 3 students have been able to explain but are incorrect or not as expected. As for students with low ability on question number 1 students can explain but still not in line with expectations and in questions number 2 and number 3 students are unable to explain in words.

This also can be seen from the results of interviews between researchers and students and student answers as follows:

Q: For number 1, what do you have in mind about the position of the line with these equations?
S1: first find the gradient of the two equations, from the equation 2x + 4y = 0 the gradient = -1/2, and from the second equation 6x-3y-1 = 0 the gradient = 2 is obtained, the conditions for two lines intersect perpendicularly if first line gradient and second line equal to -1 and from the result m1.m2 produces -1, then both lines are perpendicular to each other.
S2: For the position or position of 2 lines, the gradient must be searched for and determined from the two equation lines, seen from the equation the first gradient line is -1/2 and the second gradient line is 2. because m1.m2 = -1 then position 2 the line is perpendicular to each other.
S3: I think the position of the two lines is parallel, because seeing the equation the coefficients of x and y are different.

P: Then what is your response and explanation about question number 2?
S1: To make a line from an existing equation, we must first find the value from the point x and point y. By making x = 0, what is the y value? And by making y = 0, what is the y value? From the equation y = 4x + 6, then if y = 0 the value x = -6/4, and if x = 0 then y = 6. Then connect the two points, form the desired line drawing.
S2: I usually first draw the chart diagram and then specify the point x and point y, if seen from the equation x it is 4 and y is 1. So there is a point x = 4 and y = 1.
S3: I can't get the picture, does the line have a graph?

From the results of the interview, students with all three categories have been able to explain with words (verbal reasoning) it can be seen in question number 1. All students have been able to explain in words even though students whose low ability category is incorrect in explaining and not yet directed towards the correct answer.

For high-ability students on numerical ability criteria in question number 1 and number 3 students are able to solve questions using the formula correctly, while for question number 2 students are able to
answer the question using the formula but the answer is not as expected. This can be seen from students' answers as shown in Figure 3.

![Figure 3. High Category Student Answers in Question Number 2](image)

Whereas for students who are moderately capable of numerical ability criteria in question number 3 students have been able to solve questions using the formula correctly but in question number 1 and number 2 students are able to use the formula but the answer is still wrong, this can be seen in figure 4.

![Figure 4. Student Answers Medium Category in Question number 3](image)

In Figure 5, students who have low ability on the criteria for abstract reasoning both on questions number 1, 2 and 3 have not been able to find mathematical principles and have not been able to change questions into mathematical models. This can be seen from the results of student answers as follows.

![Figure 5. Low Category Student Answers in Question Number 3](image)

From the results of the observations, it can be concluded that the ability of the abstraction process of students varies based on the categories of high, medium, and low abilities seen from their abilities on verbal reasoning, numerical abilities, and abstract reasoning criteria. Students with high ability category have high variable reasoning ability, high numerical ability and high abstract reasoning as well. While students with medium abilities have high variable reasoning abilities, moderate numerical abilities and medium abstract reasoning. For students with low ability categories have low variable reasoning ability,
moderate numerical ability and low abstract reasoning. The process of abstraction of students is an important process for creating meaningful mathematics learning. But this process is not easy, requiring the role of the teacher. The teacher must find the best learning flow, which is rich-context so that there is no cognitive leap by students. The teacher himself must go through a concrete process that is really not easy, the abstract concept patterns that the teacher has had to find a contextual path that will help the students' abstraction process. Improving the abstraction process by students is not easy, it requires scaffolding assistance provided by the teacher in the learning process. The abstract position of most Indonesian teachers regarding mathematics material requires the ability of concretization to find a learning flow so that the abstraction process occurs by students. This is in line with opinions[13] Mathematical learning is meaningful by involving students' abstraction processes, directing students to actively discover mathematical concepts. Learning by increasing the ability of abstraction turns out to be very effective when applied especially to students who face difficulties learning mathematics.

Straight-line equation is a material that tends to be difficult for students to learn, even though this material is very important because many mathematical topics must be solved by straight-line equations [14]. Therefore, in studying the material of straight line equations, a good process of abstraction is needed, if students have a good abstraction process, a difficult mathematical material will be mastered. The process of abstraction is a fundamental process in internalizing the initial concepts of mathematics. This process creates mathematical abstraction capabilities, namely the ability to solve mathematical problems without having to present the object in real terms [15].

4. Conclusion
Students' abstraction process based on high, medium and low abilities have different capabilities to create an abstraction seen from their abilities on verbal reasoning, numerical ability, and abstract reasoning. Students with high ability category have high variable reasoning ability, high numerical ability and high abstract reasoning as well. While students with medium abilities have high variable reasoning abilities, moderate numerical abilities and medium abstract reasoning. For students with low ability categories have low variable reasoning ability, moderate numerical ability and low abstract reasoning. So this process of abstraction can be seen from students with high ability categories. As for students of medium and low ability, need more guidance so that their abstraction process can improve.

To improve the process of student abstraction the teacher needs to create an effective learning process by applying an appropriate approach.

5. Acknowledgments
The author would like to deliver a gratitude to all parties that involved in writing this article, enthusiasm, motivation and support from all parties that the author cannot mention one by one is very helpful in completing this paper. Finally, hopefully this paper is useful for all readers and writers in particular.

6. References
[1] Chotimah S, Bernard M and Wulandari, Sukma M 2018 Contextual approach using VBA learning media to improve students’ mathematical displacement and disposition ability J. Phys. Conf. Ser. 948 012025
[2] Kadarisma G 2016 Improving students’ logical thinking mathematic skill through learning cycle 5e and discovery learning Proceeding Of 3rd International Conference on Research Implementation Education Of Mathematics and Science (Yogyakarta: UNY) pp.351-356
[3] Kadarisma G, Nurjaman A, Sari I P and Amelia R 2019 Gender and mathematical reasoning ability J. Phys. Conf. Ser. 1157 042109
[4] Marsi N N, Candiasa I M, Kom M I, Kirna I M and Si M 2014 Pengaruh model pembelajaran kooperatif tipe STAD dan kemampuan abstraksi terhadap prestasi belajar matematika siswa. Jurnal Teknologi Pembelajaran Indonesia. 4 1-14
[5] Komala E 2018 Analysis of students’mathematical abstraction ability by using discursive approach integrated peer instruction of structure algebra II Infinity. J. 7 25–34
[6] Ferrari P L 2013 Abstraction in mathematics Philosophical Transactions of the Royal Society
London Series B: Biological Sciences 358 1225-1230

[7] Ruseffendi E 2006 Hakekat pendidikan matematika dan perkembangannya (Bandung: Tarsito)

[8] Mitchelmore M and White P 2017 Abstraction in mathematics learning Math. Educ. Res. J. 19 1–9

[9] Yulianti A 2013 Penerapan pendekatan concrete-representational-abstract (cra) untuk meningkatkan kemampuan abstraksi matematis siswa smp dalam belajar geometri (Bandung: Universitas Pendidikan Indonesia)

[10] Tata 2015 Peningkatan kemampuan pemodelan dan abstraksi matematis serta motivasi belajar siswa sekolah menengah pertama melalui pembelajaran kontekstual kolaboratif (Bandung: Universitas Pendidikan Indonesia)

[11] Harari O 2006 Methexis and geometrical reasoning in proclus’ commentary on euclid’s elements Oxford Studies in Ancient Philosophy 30 361-368

[12] Nooriafshar M 2002 The use innovate teaching method for “maximizing” the enjoyment form learning Int. J. Math. Teach. Learn 10 1-11

[13] Merliza P (2016) Peranan kemampuan abstraksi peserta didik dalam pembelajaran matematika melalui soal rich context persamaan linear dua variabel PRISMA, Prosiding Seminar Nasional Matematika pp 104–110.

[14] Kadarisma G and Amelia R 2018 Epistemological obstacles in solving equation of straight line problems International Conference on Mathematics and Science Education of Universitas Pendidikan Indonesia pp 905–10

[15] Merliza P 2016 Peranan Kemampuan Abstraksi Peserta Didik dalam Pembelajaran Matematika Melalui Soal Rich Context Persamaan Linear Dua Variabe PRISMA, Prosiding Seminar Nasional Matematika pp 104–10