Compact and Efficient Encodings for Planning in Factored State and Action Spaces with Learned Binarized Neural Network Transition Models

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Abstract

In this paper, we leverage the efficiency of Binarized Neural Networks (BNNs) to learn complex state transition models of planning domains with discretized factored state and action spaces. In order to directly exploit this transition structure for planning, we present two novel compilations of the learned factored planning problem with BNNs based on reductions to Weighted Partial Maximum Boolean Satisfiability (FD-SAT-Plan+) as well as Binary Linear Programming (FD-BLP-Plan+). Theoretically, we show that our SAT-based Bi-Directional Neuron Activation Encoding is asymptotically the most compact encoding relative to the current literature and supports Unit Propagation (UP) – an important property that facilitates efficiency in SAT solvers. Experimentally, we validate the computational efficiency of our Bi-Directional Neuron Activation Encoding in comparison to an existing neuron activation encoding and demonstrate the ability to learn complex transition models with BNNs. We test the runtime efficiency of both FD-SAT-Plan+ and FD-BLP-Plan+ on the learned factored planning problem showing that FD-SAT-Plan+ scales better with increasing BNN size and complexity. Finally, we present a finite-time incremental constraint generation algorithm based on generalized landmark constraints to improve the planning accuracy of our encodings through simulated or real-world
1. Introduction

Deep neural networks (DNNs) have significantly improved the ability of autonomous systems to perform complex tasks, such as image recognition [2], speech recognition [3] and natural language processing [4], and can outperform humans and human-designed super-human systems (i.e., systems that can achieve better performance than humans) in complex planning tasks such as Go [5] and Chess [6].

In the area of learning and planning, recent work on HD-MILP-Plan [7] has explored a two-stage framework that (i) learns transition models from data with ReLU-based DNNs and (ii) plans optimally with respect to the learned transition models using Mixed-Integer Linear Programming, but did not provide encodings that are able to learn and plan with discrete state variables. As an alternative to ReLU-based DNNs, Binarized Neural Networks (BNNs) [8] have been introduced with the specific ability to learn compact models over discrete variables, providing a new formalism for transition learning and planning in factored discretized state and action spaces that we explore in this paper. However, planning with these BNN transition models poses two non-trivial questions: (i) What is the most efficient compilation of BNNs for planning in domains with factored state and (concurrent) action spaces? (ii) Given that BNNs may learn incorrect domain models and a planner can sometimes have limited access to real-world (or simulated) feedback, how can the planner repair BNN compilations to improve their planning accuracy?

To answer question (i), we present two novel compilations of the learned factored planning problem with BNNs based on reductions to Weighted Partial Maximum Boolean Satisfiability (FD-SAT-Plan+) and Binary Linear Programming (FD-BLP-Plan+). Theoretically, we show that the SAT-based Bi-
Directional Neuron Activation Encoding is asymptotically the most compact encoding relative to the current literature and is efficient with Unit Propagation (UP). Experimentally, we demonstrate the computational efficiency of our Bi-Directional Neuron Activation Encoding compared to the existing neuron activation encoding. Then, we test the effectiveness of learning complex state transition models with BNNs, and test the runtime efficiency of both FD-SAT-Plan+ and FD-BLP-Plan+ on the learned factored planning problems over four factored planning domains with multiple size and horizon settings.

While there are methods for learning PDDL models from data and excellent PDDL planners, we remark that BNNs are strictly more expressive than PDDL-based learning paradigms for learning concurrent effects in factored action spaces that may depend on the joint execution of one or more actions. Furthermore, while Monte Carlo Tree Search (MCTS) methods including AlphaGo and AlphaGoZero could technically plan with a BNN-learned black box model of transition dynamics, unlike this work, they do not exploit the BNN transition structure (i.e., they simply sample it as a black box) and they do not provide optimality guarantees with respect to the learned model (unless they exhaustively sample all trajectories). Other methods that combine deep learning and planning such as ASNets focus on exploiting (P)PDDL domain structure for learning policies, but do not learn BNN transition models from raw data and then plan with these BNN models as we focus on here.

To answer question (ii), that is, for the additional scenario when the planner has further access to real-world (or simulated) feedback, we introduce a finite-time constraint generation algorithm based on generalized landmark constraints from the decomposition-based cost-optimal classical planner, where we detect and constrain invalid sets of action selections from the decision space of the planners and efficiently improve their planning accuracy through simulated or real-world interaction.

In sum, this work provides the first two planners capable of learning complex transition models in domains with mixed (continuous and discrete) factored state and action spaces as BNNs, and capable of exploiting their structure in
Weighted Partial Maximum Boolean Satisfiability and Binary Linear Programming encodings for planning purposes. Theoretically, we show the compactness and efficiency of our SAT-based encoding, and the finite-time convergence of our incremental algorithm. Empirical results show the computational efficiency of our new Bi-Directional Neuron Activation Encoding demonstrates strong performance for FD-SAT-Plan+ and FD-BLP-Plan+ in both the learned and original domains, and provide a new transition learning and planning formalism to the data-driven model-based planning community.

2. Preliminaries

Before we present the Weighted Partial Maximum Boolean Satisfiability (WP-MaxSAT) and Binary Linear Programming (BLP) compilations of the learned planning problem, we review the preliminaries motivating this work. We begin this section by describing the formal notation and the problem definition that is used in this work.

2.1. Problem Definition

A deterministic factored [9] planning problem is a tuple $\Pi = \langle S, A, C, T, I, G, R \rangle$ where $S = \{ s_1, \ldots, s_{n_1} \}$ and $A = \{ a_1, \ldots, a_{n_2} \}$ are sets of state and action variables for positive integers $n_1, n_2$ with domains $D_{s_1}, \ldots, D_{s_{|S|}}$ and $D_{a_1}, \ldots, D_{a_{|A|}}$ respectively, $C : D_{s_1} \times \cdots \times D_{s_{|S|}} \times D_{a_1} \times \cdots \times D_{a_{|A|}} \rightarrow \{\text{true, false}\}$ is the global function representing the properties of state and action variables that must be true for all time steps, $T : D_{s_1} \times \cdots \times D_{s_{|S|}} \times D_{a_1} \times \cdots \times D_{a_{|A|}} \rightarrow D_{s_1} \times \cdots \times D_{s_{|S|}}$ denotes the stationary transition function between two time steps, and $R : D_{s_1} \times \cdots \times D_{s_{|S|}} \times D_{a_1} \times \cdots \times D_{a_{|A|}} \rightarrow \mathbb{R}$ is the reward function. Finally, $I : D_{s_1} \times \cdots \times D_{s_{|S|}} \rightarrow \{\text{true, false}\}$ is the initial state function that defines the initial values of state variables, and $G : D_{s_1} \times \cdots \times D_{s_{|S|}} \rightarrow \{\text{true, false}\}$ is the goal state function that defines the properties of state variables that must be true at the last time step. In this work, we assume the initial value $V_i \in \mathbb{R}$ of each state variable $s_i \in S$ is known.
Given a planning horizon $H$, a solution $\pi = \langle \bar{A}^1, \ldots, \bar{A}^H \rangle$ (i.e. plan) to problem $\Pi$ is a tuple of values $\bar{A}^t = \langle \bar{a}^t_1, \ldots, \bar{a}^t_{|A|} \rangle \in D_{a_1} \times \cdots \times D_{a_{|A|}}$ for all action variables $A$ and time steps $t \in \{1, \ldots, H\}$ (and a tuple of values $\bar{S}^t = \langle \bar{s}^t_1, \ldots, \bar{s}^t_{|S|} \rangle \in D_{s_1} \times \cdots \times D_{s_{|S|}}$ for all state variables $S$ and time steps $t \in \{1, \ldots, H + 1\}$) such that $T((\bar{s}^t_1, \ldots, \bar{s}^t_{|S|}, \bar{a}^t_1, \ldots, \bar{a}^t_{|A|})) = \bar{S}^{t+1}$ and $C((\bar{s}^t_1, \ldots, \bar{s}^t_{|S|}, \bar{a}^t_1, \ldots, \bar{a}^t_{|A|})) = true$ for all time steps $t \in \{1, \ldots, H\}$, and $I(\bar{S}^1) = true$ for time step $t = 1$ and $G(\bar{S}^{H+1}) = true$ for time step $t = H + 1$, where $\bar{x}^t$ denotes the value of variable $x \in A \cup S$ at time step $t$. Similarly, an optimal solution to $\Pi$ is a plan such that the total reward $\sum_{t=1}^{H} R((\bar{s}^{t+1}_1, \ldots, \bar{s}^{t+1}_{|S|}, \bar{a}^t_1, \ldots, \bar{a}^t_{|A|}))$ is maximized. For notational simplicity, we denote the tuple of variables $\langle x_{e_1}, \ldots, x_{e_{|E|}} \rangle$ as $\langle x_e | e \in E \rangle$ given set $E$, and use the symbol $\sim$ for the concatenation of two tuples. Next, we introduce an example domain with a complex transition structure.

### 2.2. Example Domain: Cellda

Influenced by the famous video game *The Legend of Zelda* [19], the Cellda domain models an agent who must escape from a two dimensional ($N$-by-$N$) dungeon cell through an initially locked door by obtaining a key without getting hit by the enemy. Next, we describe the Cellda domain as a factored planning problem $\Pi$ and illustrate two planning scenarios in Figure 1.

- Location of the agent, location of the enemy, whether the key is obtained or not and whether the agent is alive or not are represented by six state variables $S = \{s_1, \ldots, s_6\}$ where state variables $s_1$ and $s_2$ represent the horizontal and vertical locations of the agent with integer domains, state variables $s_3$ and $s_4$ represent the horizontal and vertical locations of the enemy with integer domains, state variable $s_5$ represents whether the key is obtained or not with a Boolean domain, and state variable $s_6$ represents whether the agent is alive or not with a Boolean domain.

- Intended movement of the agent is represented by four action variables $A = \{a_1, a_2, a_3, a_4\}$ with Boolean domains where action variables $a_1, a_2,$
Figure 1: Visualization of two different attempts of solving the automated planning problem, namely Cellda. In all figures, the character (i.e., Cellda) that is controlled by the planner is represented by a humanoid character wearing a golden crown, the other character (i.e., the enemy) is represented by the green monster. The planner must control Cellda so that she obtains the key and escapes through the door without getting hit by the enemy. The top three figures (a-c) represent an unsuccessful attempt of solving the problem whereas the bottom three figures (d-f) represent a successful attempt of solving the problem. In the unsuccessful attempt, Cellda moves to obtain the key by moving up three times followed by a move to the right. Once the key is obtained, Cellda keeps moving to the right but gets intercepted by the enemy. In the successful attempt, Cellda waits for two time steps and allows the enemy to approach her. Once the enemy is located underneath the block (i.e., represented by the red squares), Cellda starts moving up three times and to the right in order to first obtain the key, and then moves right towards the door to escape. In order to solve this complex automated planning problem, the planner must make long-term decisions while accounting for the unknown adversarial behaviour of the enemy.
$a_3$ and $a_4$ represent whether the agent intends to move up, down, right or left, respectively.

- Mutual exclusion on the intended movement of the agent, the boundaries of the maze and requirement that the agent must be alive are represented by global constraint function $C$ where $C((s_1, \ldots, s_6, a_1, \ldots, a_4)) = true$ if and only if $a_1 + a_2 + a_3 + a_4 \leq 1$, $0 \leq s_1, s_2 < N$ and $s_6 = true$, $C((s_1, \ldots, s_6, a_1, \ldots, a_4)) = false$ otherwise.

- The starting location of the agent, location of the enemy, whether the key is currently obtained or not, and whether the agent is currently alive or not are represented by the initial state function $I$ where $I((s_1, \ldots, s_6)) = true$ if and only if $s_1 = V_{s_1}$, $s_2 = V_{s_2}$, $s_3 = V_{s_3}$, $s_4 = V_{s_4}$, $s_5 = false$ and $s_6 = true$; $I((s_1, \ldots, s_{N^2})) = false$ otherwise where $V_{s_1}$ and $V_{s_2}$ denote the location of the agent, and $V_{s_3}$ and $V_{s_4}$ denote the location of the enemy.

- Final location of the agent (i.e., the location of the door), requirement that the agent must be alive and requirement that the key must be obtained are represented by the goal state function $G$ where $G((s_1, \ldots, s_6)) = true$ if and only if $s_1 = V'_{s_1}$, $s_2 = V'_{s_2}$, $s_3 = V'_{s_3}$, $s_4 = V'_{s_4}$, $s_5 = true$ and $s_6 = true$ $G((s_1, \ldots, s_6)) = false$ otherwise where $V'_{s_1}$ and $V'_{s_2}$ denote the goal location of the agent.

- The cost objective is to minimize total number of intended movements by the agent and is represented by the reward function $R$ where $R((s_1, \ldots, s_6, a_1, \ldots, a_4)) = a_1 + a_2 + a_3 + a_4$.

- Next location of the agent, next location of the enemy, whether the key will be obtained or not, and whether the agent will be alive or not are represented by the state transition function $T$ that is a nonlinear function of state and action variables $s_1, \ldots, s_6, a_1, \ldots, a_4$. The next location of the agent is a function of whether it is alive or not, its previous location and its intended movement. The next location of the enemy is a function of its previous location and the previous location of the agent and the intended movement of the agent. Whether the agent will be alive or not
is a function of whether it was previously alive, and the locations of the agent and the enemy. Finally, whether Cellda will have the key or not is a function of whether it previously had the key and its location.

Realistically, the enemy has an adversarial deterministic policy that is unknown to Cellda which will try to minimize the total Manhattan distance between itself and Cellda by breaking the symmetry first in vertical axis. The complete description of this domain can be found in Appendix C. Given that the state transition function $T$ that describes the location of the enemy must be learned, a planner that fails to learn the adversarial policy of the enemy (e.g., $\pi_1$ as visualized in Figure 1a-1c) can get hit by the enemy. In contrast, a planner that learns the adversarial policy of the enemy (e.g., $\pi_2$ as visualized in Figure 1d-1f) avoids getting hit by the enemy in this scenario by waiting for two time steps to trap her enemy, who will try to move up for the remaining time steps and fail to intercept Cellda. To solve this problem, next we describe a learning and planning framework that (i) learns an unknown transition function $T$ from data, and (ii) plans optimally with respect to the learned deterministic factored planning problem.

2.3. Factored Planning with Learned Transition Models using Deep Neural Networks and Mixed-Integer Linear Programming

Factored planning with DNN learned transition models is a two-stage framework for learning and solving nonlinear factored planning problems as first introduced in HD-MILP-Plan [7] that we briefly review now. Given samples of state transition data, the first stage of HD-MILP-Plan learns the transition function $\tilde{T}$ using a DNN with Rectified Linear Units (ReLUs) [20] and linear activation units. In the second stage, the learned transition function $\tilde{T}$ is used to construct the learned factored planning problem $\tilde{\Pi} = (S, A, C, \tilde{T}, I, G, R)$. That is, the trained DNN with fixed weights is used to predict the values $\tilde{S}^{t+1}$ of state variables at time step $t+1$ for values $\tilde{S}^t$ of state variables and values $\tilde{A}^t$ of action variables at time step $t$ such that $\tilde{T}(\tilde{S}^t, \tilde{A}^t) = \tilde{S}^{t+1}$. As visualized
in Figure 2, the learned transition function $\bar{T}$ is sequentially chained over horizon $t \in \{1, \ldots, H\}$, and compiled into a Mixed-Integer Linear Program yielding the planner HD-MILP-Plan [7]. Since HD-MILP-Plan utilizes only ReLUs and linear activation units in its learned transition models, the state variables $s \in S$ are restricted to have only continuous domains $D_s \subseteq \mathbb{R}$.

Next, we describe an efficient DNN structure for learning discrete models, namely Binarized Neural Networks (BNNs) [8].

**2.4. Binarized Neural Networks**

Binarized Neural Networks (BNNs) are neural networks with binary weights and activation functions [8]. As a result, BNNs naturally learn discrete models by replacing most arithmetic operations with bit-wise operations. Before we describe how BNN-learned transitions relate to HD-MILP-Plan in Figure 2, we first provide a technical description of the BNN architecture, where BNN layers are stacked in the following order:

*Real or Binary Input Layer.* Binary units in all layers, with the exception of the first layer, receive binary input. When the inputs of the first layer are non-binary domains, signed binary representations up to $m$ bits of precision are used [8]. For example, the integer value $\tilde{x} = -93$ can be represented using $m = 8$ bits of
precision as $\langle x^8 = 1, x^7 = 0, x^6 = 1, x^5 = 0, x^4 = 0, x^3 = 0, x^2 = 1, x^1 = 1 \rangle$ using the formula $\hat{x} = -2^{m-1}x^m + \sum_{i=1}^{m-1} 2^{i-1}x^i$. Therefore in the remaining of the paper, we assume the inputs of the BNNs have Boolean domains representing the binarized domains of the original inputs with $m$ bits of precision.

**Binarization Layer.** Given input $x_{j,l}$ of binary unit $j \in J(l)$ at layer $l \in \{1, \ldots, L\}$ the deterministic activation function used to compute output $y_{j,l}$ is: $y_{j,l} = 1$ if $x_{j,l} \geq 0$, $-1$ otherwise, where $L$ denotes the number of layers and $J(l)$ denotes the set of binary units in layer $l \in \{1, \ldots, L\}$.

**Batch Normalization Layer.** For all layers $l \in \{1, \ldots, L\}$, Batch Normalization [21] is a method for transforming the weighted sum of outputs at layer $l-1$ in $\Delta_{j,l} = \sum_{i \in J(l-1)} w_{i,j,l-1}y_{i,l-1}$ to input $x_{j,l}$ of binary unit $j \in J(l)$ at layer $l$ using the formula: $x_{j,l} = \frac{\Delta_{j,l} - \mu_{j,l}}{\sqrt{\sigma^2_{j,l} + \epsilon_{j,l}}} \gamma_{j,l} + \beta_{j,l}$ where parameters $w_{i,j,l-1}$, $\mu_{j,l}$, $\sigma^2_{j,l}$, $\epsilon_{j,l}$, $\gamma_{j,l}$ and $\beta_{j,l}$ denote the weight, input mean, input variance, numerical stability constant (i.e., epsilon), input scaling and input bias respectively, where all parameters are computed at training time.

In order to place BNN-learned transition models in the same planning and learning framework of HD-MILP-Plan [7], we simply note that once the above BNN layers are learned, the Batch Normalization layers reduce to simple linear transforms (i.e., as we will show in Section 4.3.3 once parameters $w_{i,j,l-1}$, $\mu_{j,l}$, $\sigma^2_{j,l}$, $\epsilon_{j,l}$, $\gamma_{j,l}$ and $\beta_{j,l}$ are fixed, $x_{j,l}$ is a linear function of $y_{i,l-1}$). This results in a BNN with layers as visualized in Figure 2 where (i) all weights $w$ are restricted to either +1 or -1 and (ii) all nonlinear transfer functions at BNN units are restricted to thresholded counts of inputs. The benefit of the BNN over the ReLU-based DNNs for HD-MILP-Plan [7] is that it can directly model discrete variable transitions and BNNs can be translated to both Binary Linear Programming (BLP) and Weighted Partial Maximum Boolean Satisfiability (WP-MaxSAT) problems discussed next.
2.5. Weighted Partial Maximum Boolean Satisfiability Problem

In this work, one of the planning encodings that we focus on is Weighted Partial Maximum Boolean Satisfiability (WP-MaxSAT). WP-MaxSAT is the problem of finding a value assignment to the variables of a Boolean formula that consists of hard clauses and weighted soft clauses such that:

1. all hard clauses evaluate to true (i.e., standard SAT) \[22\], and
2. the total weight of the unsatisfied soft clauses is minimized.

While WP-MaxSAT is known to be \textit{NP-hard}, state-of-the-art WP-MaxSAT solvers are experimentally shown to scale well for large instances \[23\].

2.6. Cardinality Networks

When compiling BNNs to satisfiability encodings, it is critical to encode the counting (cardinality) threshold of the binarization layer as compactly as possible since smaller encoding sizes positively impact both compilation and optimization times. Cardinality Networks $CN_{k}^{=}(\langle x_1, \ldots, x_n \rangle \rightarrow \langle c_1, \ldots, c_k \rangle)$ provide an efficient encoding in conjunctive normal form (CNF) for counting the number of true assignments to Boolean variables $x_1, \ldots, x_n$ using auxiliary counting variables $c_i$ such that $\min(\sum_{j=1}^{n} x_j, i) = \sum_{j=1}^{i} c_j$ holds for all $i \in \{1, \ldots, k\}$ where $k$ is selected to be the smallest power of 2 such that $k > p$ \[24\]. As visualized in Figure 3, $CN_{k}^{=}$ is made up of three smaller building blocks, namely Half Merging (HM) Networks, Half Sorting (HS) Networks and Simplified Merging (SM) Networks, that recursively sort and merge the input Boolean variables $x_1, \ldots, x_n$ with respect to their values. The detailed CNF encoding of $CN_{k}^{=}$ is outlined in Appendix A.
Figure 3: Visualization of Cardinality Networks (on the left) \cite{24} that consist of (i) Simplified Merge Networks (SM) and (ii) Half Sorting Networks (HS), and each HS further consists of Half Merging Networks (HM). An example Cardinality Network $CN = 4$ takes in the tuple of variables $\langle x_1, \ldots, x_8 \rangle$ and counts up to $k = 4$ variables that are assigned to true using additional auxiliary counting variables and respective hard clauses.

2.7. Boolean Cardinality Constraints

A Boolean cardinality constraint $\text{Exactly}_p(\langle x_1, \ldots, x_n \rangle)$ describes the number of Boolean variables that are allowed to be true, and is in the form of $\sum_{i=1}^{n} x_i = p$. Given $CN_k^\leq$, $\text{Exactly}_p(\langle x_1, \ldots, x_n \rangle)$ is defined as:

$$\text{Exactly}_p(\langle x_1, \ldots, x_n \rangle) = \bigwedge_{i=n+1}^{r} (\neg x_i) \land (\neg c_{p+1}) \land (c_p)$$

$$\land CN_k^\leq (\langle x_1, \ldots, x_{n+r} \rangle \rightarrow \langle c_1, \ldots, c_k \rangle) \quad (1)$$

where $r$ is the smallest size of additional input variables needed to ensure the number of input variables is a multiple of $k$. Boolean cardinality constraint $\text{Exactly}_p(\langle x_1, \ldots, x_n \rangle)$ is encoded using $O(n\log_2^2 k)$ number of variables and hard clauses \cite{24}.

Note that the cardinality constraint $\sum_{i=1}^{n} x_i = p$ is equivalent to $\sum_{i=1}^{n} (1 - x_i) = n - p$. Since Cardinality Networks require the value of $p$ to be less than or equal to $\frac{n}{2}$, Boolean cardinality constraints of the form $\sum_{i=1}^{n} x_i = p$ with $p > \frac{n}{2}$ must be converted into $\text{Exactly}_{n-p}(\langle \neg x_1, \ldots, \neg x_n \rangle)$.

Finally, a Boolean cardinality constraint encoded in CNF is said to be efficient if it allows efficient algorithms, such as Unit Propagation (UP) \cite{25}, to
deduce the values of its variables whenever possible. A Boolean cardinality constraint encoded in CNF, such as $\text{Exactly}_p(x_1, \ldots, x_n)$, is said to be efficient with UP if and only if the CNF encoding allows UP to deduce:

1. when exactly $p$ variables from the set $\{x_1, \ldots, x_n\}$ are assigned to true, assignment of the remaining (unassigned) $n - p$ variables to false,
2. when exactly $n - p$ variables from the set $\{x_1, \ldots, x_n\}$ are assigned to false, assignment of the remaining (unassigned) $p$ variables to true,
3. when at least $p + 1$ variables from the set $\{x_1, \ldots, x_n\}$ are assigned to true, the Boolean cardinality constraint is not satisfiable, and
4. when at least $n - p + 1$ variables from the set $\{x_1, \ldots, x_n\}$ are assigned to false, the Boolean cardinality constraint is not satisfiable.

In practice, the ability to be efficient with algorithms such as UP (as opposed to search) is one of the most important properties for the efficiency of a Boolean cardinality constraint encoded in CNF [26, 27, 24, 28]. It has been shown that $\text{Exactly}_p(x_1, \ldots, x_n)$ encoding is efficient with UP [24].

2.8. Binary Linear Programming Problem

As an alternative to WP-MaxSAT encodings of BNN transition models, we can also leverage Binary Linear Programs (BLPs). The BLP problem requires finding the optimal value assignment to the variables of a mathematical model with linear constraints, linear objective function, and binary decision variables. Similar to WP-MaxSAT, BLP is $NP$-hard. The state-of-the-art BLP solvers [29] utilize branch-and-bound algorithms and can handle cardinality constraints efficiently in the size of its encoding.

2.9. Generalized Landmark Constraints

In this section, we review generalized landmark constraints that are necessary for improving the planning accuracy of the learned models through simulated or real-world interaction when the plans for the learned planning problem $\tilde{\Pi}$ are infeasible for the planning problem $\Pi$. A generalized landmark constraint is in the form of $\bigvee_{a \in A}(z_a \geq k_a)$ where the decision variable $z_a \in \mathbb{N}$
counts the total number of times action $a \in A$ is executed in the plan, and $k_a$ denotes the minimum number of times action $a$ must occur in a plan [IS]. The decomposition-based planner, $OpSeq$ [IS], incrementally updates generalized landmark constraints to find cost-optimal plans to classical planning problems.

3. Model Assumptions

Before we describe our compilation-based planners for solving the learned planning problem $\tilde{\Pi}$, we present the set of assumptions used to model $\Pi$.

- The deterministic factored planning problem $\Pi$ is static (i.e., $\Pi$ does not change between the time data was collected, planning, and simulation or execution).

- Boolean-valued functions $I$, $C$ and $G$ only take in arguments with Boolean domains representing the domains of state and action variables with $m$ bits of precision where the value of $m$ is selected prior to the training time of $\tilde{T}$, and is assumed to be known. Since functions $I$, $C$ and $G$ must always be satisfied by $\pi$, we further assume $I$, $C$ and $G$ can be equivalently represented by a finite set of constraints with $m$ bits of precision. Specifically, $I$ can be equivalently represented by a finite set of equality constraints (i.e., $I$ sets the value of every state variable $s_i \in S$ to their respective initial value $V_i$), and $C$ and $G$ can be represented by a finite set of linear constraints which are in the form of $\sum_{i=1}^{n} a_ix_i \leq p$ for state and action variables $x_i \in S \cup A$ where $a_i \in \mathbb{N}$ and $p \in \mathbb{Z}_{\geq 0}$.

- The reward function $R$ only takes in arguments with Boolean domains and is in the form of $\sum_{i=1}^{n} b_ix_i$ for state and action variables $x_i \in S \cup A$ with $m$ bits of precision where $a_i \in \mathbb{N}$ and $b_i \in \mathbb{R}_{\geq 0}$.
4. Weighted Partial Maximum Boolean Satisfiability Compilation of the Learned Factored Planning Problem

In this section, we show how to reduce the learned factored planning problem \( \tilde{\Pi} \) with BNNs into WP-MaxSAT, which we denote as Factored Deep SAT Planner (FD-SAT-Plan+). FD-SAT-Plan+ uses the same learning and planning framework as HD-MILP-Plan \[7\] that is visualized in Figure 2 where the ReLU-based DNN is replaced by a BNN \[8\] and the compilation of \( \tilde{\Pi} \) is a WP-MaxSAT instead of a Mixed-Integer Linear Program (MILP).

4.1. Propositional Variables

First, we describe the set of propositional variables used in FD-SAT-Plan+. We use three sets of propositional variables: action decision variables, state decision variables and BNN binary unit decision variables, where we use signed binary representation upto \( m \) bits of precision for action and state variables with non-binary domains.

- \( X_{a,t}^{i} \) denotes if \( i \)-th bit of action variable \( a \in A \) is executed at time step \( t \in \{1, \ldots, H\} \) (i.e., each bit of an action variable corresponds to a red circle in Figure 2).
- \( Y_{s,t}^{i} \) denotes if \( i \)-th bit of state variable \( s \in S \) is true at time step \( t \in \{1, \ldots, H+1\} \) (i.e., each bit of a state variable corresponds to a blue circle in Figure 2).
- \( Z_{j,l,t} \) denotes if BNN binary unit \( j \in J(l) \) at layer \( l \in \{1, \ldots, L\} \) is activated at time step \( t \in \{1, \ldots, H\} \) (i.e., each BNN binary unit corresponds to a gray circle in Figure 2).

4.2. Constants and Indexing Functions

Next we define the additional constants and indexing functions used in FD-SAT-Plan+.

- \( V_{s}^{i} \) is the initial (i.e., at \( t = 1 \)) value of the \( i \)-th bit of state variable \( s \in S \).
• $In(x, i)$ is the function that maps the $i$-th bit of a state or an action variable $x \in S \cup A$ to the corresponding binary unit in the input layer of the BNN such that $In(x, i) = j$ where $j \in J(1)$.

• $Out(s, i)$ is the function that maps the $i$-th bit of a state variable $s \in S$ to the corresponding binary unit in the output layer of the BNN such that $Out(s, i) = j$ where $j \in J(L)$.

4.3. The WP-MaxSAT Compilation

Below, we define the WP-MaxSAT encoding of the learned factored planning problem $\tilde{\Pi}$ with BNNs. First, we present the hard clauses (i.e., clauses that must be satisfied) used in FD-SAT-Plan+.

4.3.1. Initial State Clauses

The following conjunction of hard clauses encodes the initial state function $I$.

\[
\bigwedge_{s \in S} \bigwedge_{1 \leq i \leq m} (Y^i_{s,1} \leftrightarrow V^i_{s})
\]  

(2)

where hard clause (2) sets the initial values of the state variables at time step $t = 1$.

4.3.2. Bi-Directional Neuron Activation Encoding

In this section, we present a CNF encoding to model the activation behaviour of a BNN binary unit $j \in J(l), l \in \{1, \ldots, L\}$ that requires only $O(n\log_2^2 k)$ variables and hard clauses, and is efficient with Unit Propagation (UP) where $k$ is selected to be the smallest power of 2 such that $k > p$.

Given the input variables $x_1, \ldots, x_n$ and the binary activation function with the activation threshold $p$, the output of a binary unit can be efficiently encoded in CNF as follows. First, the Boolean variable $v$ is defined to represent the activation of a binary unit such that $v = true$ if and only if the binary unit is activated, and $v = false$ otherwise. Then, Cardinality Networks $CN_k^=\ Thor$ are used to count the input variables $x_1, \ldots, x_n$, and an additional bi-directional relation
(i.e., \(v \leftrightarrow c_p\)) is used to relate the counting variable \(c_p\) to the output variable \(v\) as follows.

\[
\begin{align*}
\text{Act}_p(v, \langle x_1, \ldots, x_n \rangle) &= \bigwedge_{i=n+1}^{r} (\neg x_i) \land (v \lor \neg c_p) \land (\neg v \lor c_p) \\
&\land CN^{-}_k \langle \langle x_1, \ldots, x_{n+r} \rangle \rightarrow \langle c_1, \ldots, c_k \rangle \rangle
\end{align*}
\]

In hard clause (3), we remind that the purpose of the additional input variables and the respective unit clauses is to ensure the number of input variables is a multiple of \(k\) such that \(r\) denotes the smallest size of the additional input variables needed. Note that \(\text{Act}_p(v, \langle x_1, \ldots, x_n \rangle)\) simply replaces hard clauses \((c_p)\) and \((\neg c_{p+1})\) with hard clauses \((v \lor \neg c_p)\) and \((\neg v \lor c_p)\) from \(\text{Exactly}_p(\langle x_1, \ldots, x_n \rangle)\) to model the bi-directional relation \((v \leftrightarrow c_p)\) (i.e., a neuron is activated \(v = \text{true}\) if and only if the value of at least \(p\) input variables is true \(c_p = \text{true}\)).

As a result, unlike the previous work that uses \(O(np)\) number of variables and hard clauses for encoding neuron activations in CNF, the encoding we present here uses only \(O(n \log^2 k)\) number of variables and hard clauses. For notational clarity, we will refer to the neuron activation encoding that is presented in this section as the Bi-Directional Neuron Activation Encoding, and the previous neuron activation encoding \([1]\) as the Uni-Directional Neuron Activation Encoding\(^\dagger\).

Finally, the Uni-Directional Neuron Activation Encoding has been shown not to be efficient with UP \([10]\) in contrast to the Bi-Directional Neuron Activation Encoding which we show is efficient with UP in Theorem \([\dagger]\). Given the Bi-Directional Neuron Activation Encoding, next we present the CNF clauses that model learned BNN transition models.

\(^\dagger\)The names of the encodings are selected to reflect the fact that the Uni-Directional encoding separately encodes uni-directional the constraints \(v \rightarrow (\sum_{i=1}^{n} x_i \geq p)\) and \(v \leftarrow (\sum_{i=1}^{n} x_i \geq p)\) while the Bi-Directional Neuron Activation Encoding compactly encodes the bi-directional constraint \(v \leftrightarrow (\sum_{i=1}^{n} x_i \geq p)\).
4.3.3. BNN Clauses

Given the efficient CNF encoding \( \text{Act}_p(u, (x_1, \ldots, x_n)) \), we present the conjunction of hard clauses to model the complete BNN model.

\[
\bigwedge_{1 \leq t \leq H} \bigwedge_{s \in S} \bigwedge_{1 \leq i \leq m} (Y_{s,t}^i \leftrightarrow Z_{In(s,i),1,t}) \quad (4)
\]

\[
\bigwedge_{1 \leq t \leq H} \bigwedge_{a \in A} \bigwedge_{1 \leq i \leq m} (X_{a,t}^i \leftrightarrow Z_{In(a,i),1,t}) \quad (5)
\]

\[
\bigwedge_{1 \leq t \leq H} \bigwedge_{s \in S} \bigwedge_{1 \leq i \leq m} (Y_{s,t+1}^i \leftrightarrow Z_{Out(s,i),L,t}) \quad (6)
\]

\[
\bigwedge_{1 \leq t \leq H} \bigwedge_{2 \leq l \leq L} \bigwedge_{j \in J(l), \ p_j^* \leq \left\lfloor \frac{|J(l-1)|}{2} \right\rfloor} \text{Act}_p(j, Z_{j,l,t}, (Z_{i,l-1,t}^i | i \in J(l-1), w_{i,j,l-1} = 1)}
\]

\[
\wedge (\neg Z_{i,l-1,t}^i | i \in J(l-1), w_{i,j,l-1} = 1)) \quad (7)
\]

\[
\bigwedge_{1 \leq t \leq H} \bigwedge_{2 \leq l \leq L} \bigwedge_{j \in J(l), \ p_j^* > \left\lfloor \frac{|J(l-1)|}{2} \right\rfloor} \text{Act}_p(j, Z_{i,l-1,t}^i | i \in J(l-1), w_{i,j,l-1} = 1 = 1)
\]

\[
\wedge (\neg Z_{i,l-1,t}^i | i \in J(l-1), w_{i,j,l-1} = 1)) \quad (8)
\]

where activation constant \( p_j \) in hard clauses (7-8) are computed using the batch normalization parameters for binary unit \( j \in J(l) \) in layer \( l \in \{2, \ldots, L\} \) at training time such that:

\[
p_j^* = \left\lfloor \frac{|J(l-1)|}{2} \pm \frac{\mu_{j,l} - \mu_{j,l}}{\gamma_{j,l}} \right\rfloor
\]

if \( p_j^* \leq \left\lfloor \frac{|J(l-1)|}{2} \right\rfloor \) then \( p_j = p_j^* \)

else \( p_j = |J(l-1)| - p_j^* + 1 \)

where \( |x| \) denotes the size of set \( x \). The computation of the activation constant \( p_j, j \in J(l) \) ensures that \( p_j \) is less than or equal to the half size of the previous layer \( |J(l-1)| \), as Bi-Directional Neuron Activation Encoding only counts up to \( \left\lfloor \frac{|J(l-1)|}{2} \right\rfloor \).

Hard clauses (4-5) map the binary units at the input layer of the BNN (i.e., \( l = 1 \)) to a unique state or action variable, respectively. Similarly, hard
clause (6) maps the binary units at the output layer of the BNN (i.e., \( l = L \)) to a unique state variable. Note that because each state and action variable uniquely maps onto an input and/or an output BNN binary unit, constraint functions \( I, C \) and \( G \) limit the feasible set of values input and output units of the BNN can take. Hard clauses (7-8) encode the binary activation of every unit \( j \in J(l) \) in the BNN given its activation constant \( p_j \) and weights \( w_{i,j,l-1} \) such that
\[
Z_{j,l,t} \leftrightarrow \left( \sum_{i \in J(l-1), \frac{w_{i,j,l-1}}{w_{i,j,l-1}} = 1} Z_{i,l-1,t} + \sum_{i \in J(l-1), \frac{w_{i,j,l-1}}{w_{i,j,l-1}} = -1} \left( 1 - Z_{i,l-1,t} \right) \right) \geq p_j.
\]

### 4.3.4. Global Constraint Clauses

The following conjunction of hard clauses encodes the global function \( C \).
\[
\bigwedge_{1 \leq t \leq H} C(\langle Y_{i,s,t}^i | s \in S, 1 \leq i \leq m \rangle) \land (X_{a,t}^i | a \in A, 1 \leq i \leq m))
\]
where hard clause (9) represents domain-dependent global function on state and action variables. Some common examples of function \( C \), such as exactly one Boolean action or state variable must be true, are respectively encoded by hard clauses (10-11) as follows.
\[
\bigwedge_{1 \leq t \leq H} \text{Exactly} \, (\langle X_{a,t}^i | a \in A \rangle)
\]
\[
\bigwedge_{1 \leq t \leq H} \text{Exactly} \, (\langle Y_{s,t}^i | s \in S \rangle)
\]

In general, linear constraints in the form of \( \sum_{i=1}^n a_i x_i \leq p \), such as bounds on state and action variables, can be encoded in CNF where \( a_i \) are positive integer coefficients and \( x_i \) are decision variables with non-negative integer domains.

### 4.3.5. Goal State Clauses

The following conjunction of hard clauses encodes the goal state function \( G \).
\[
G(\langle Y_{s,H+1}^i | s \in S, 1 \leq i \leq m \rangle)
\]
where hard clause (12) sets the goal constraints on the state variables \( S \) at time step \( t = H + 1 \).
4.3.6. Reward Clauses

Given the reward function \( R \) for each time step \( t \) is in the form of \( \sum_{s \in S} \sum_{i=1}^{m} f_i Y_{s,t+1}^i + \sum_{a \in A} \sum_{i=1}^{m} g_a^i X_{a,t}^i \), the following weighted soft clauses (i.e., optional weighted clauses that may or may not be satisfied where each weight corresponds to the penalty of not satisfying a clause):

\[
\bigwedge_{1 \leq t \leq H} \bigwedge_{1 \leq i \leq m} \left( \bigwedge_{s \in S} (f_i^s, Y_{s,t+1}^i) \land \bigwedge_{a \in A} (g_a^i, X_{a,t}^i) \right)
\]  

(13)

can be written to represent \( R \) where \( f_i^s, g_a^i \in \mathbb{R}_{\geq 0} \) are the weights of the soft clauses for each bit of state and action variables, respectively.

5. Binary Linear Programming Compilation of the Learned Factored Planning Problem

Given FD-SAT-Plan+, we present the Binary Linear Programming (BLP) compilation of the learned factored planning problem \( \tilde{\Pi} \) with BNNs, which we denote as Factored Deep BLP Planner (FD-BLP-Plan+).

5.1. Binary Variables and Parameters

FD-BLP-Plan+ uses the same set of decision variables and parameters as FD-SAT-Plan+.

5.2. The BLP Compilation

FD-BLP-Plan+ replaces hard clauses (2) and (4-6) with equivalent linear constraints as follows.

\[
Y_{s,1}^i = V_s^i \quad \forall s \in S, 1 \leq i \leq m
\]

(14)

\[
Y_{s,t}^i = Z_{In(s,i),1,t} \quad \forall 1 \leq t \leq H, s \in S, 1 \leq i \leq m
\]

(15)

\[
X_{a,t}^i = Z_{In(a,i),1,t} \quad \forall 1 \leq t \leq H, a \in A, 1 \leq i \leq m
\]

(16)

\[
Y_{s,t+1}^i = Z_{Out(s,i),L,t} \quad \forall 1 \leq t \leq H, s \in S, 1 \leq i \leq m
\]

(17)
Given the activation constant $p^*_j$ of binary unit $j \in J(l)$ in layer $l \in \{2, \ldots, L\}$, FD-BLP-Plan+ replaces hard clauses (7-8) representing the activation of binary unit $j$ with the following linear constraints:

\begin{align}
    p^*_j Z_{j,l,t} &\leq \sum_{i \in J(l-1), \atop w_{i,j,l-1} = 1} Z_{i,l-1,t} + \sum_{i \in J(l-1), \atop w_{i,j,l-1} = -1} (1 - Z_{i,l-1,t}) \\
    \forall 1 \leq t \leq H, 2 \leq l \leq L, j \in J(l) \quad (18) \\
    p^*_j (1 - Z_{j,l,t}) &\leq \sum_{i \in J(l-1), \atop w_{i,j,l-1} = -1} Z_{i,l-1,t} + \sum_{i \in J(l-1), \atop w_{i,j,l-1} = 1} (1 - Z_{i,l-1,t}) \\
    \forall 1 \leq t \leq H, 2 \leq l \leq L, j \in J(l) \quad (19)
\end{align}

where $p'_j = |J(l-1)| - p^*_j + 1$.

Global constraint hard clauses (9) and goal state hard clauses (12) are compiled into linear constraints given they are in the form of $\sum_{i=1}^n a_i x_i \leq p$. Finally, the reward function $R$ with linear expressions is maximized over time steps $1 \leq t \leq H$ such that:

$$\max \sum_{t=1}^H \sum_{i=1}^m (\sum_{s \in S} f^i_s Y_{s,t+1} + \sum_{a \in A} g^i_a X_{a,t})$$

(20)

6. Incremental Factored Planning Algorithm for FD-SAT-Plan+ and FD-BLP-Plan+

In this section, we extend the capabilities of our data-driven planners and place them in a planning scenario where the planners have access to limited (and potentially expensive) simulated or real-world interaction. Given that the plans found for the learned factored planning problem $\tilde{\Pi}$ by FD-SAT-Plan+ and FD-BLP-Plan+ can be infeasible to the factored planning problem $\Pi$, we introduce an incremental algorithm for finding plans for $\Pi$ by iteratively excluding invalid plans from the search space of our planners. That is, we add hard clauses or constraints to the encodings of our planners to exclude infeasible plans from their respective search space. Similar to $\text{Ops}\text{eq}$ [10], FD-SAT-Plan+ and FD-BLP-Plan+ are updated with the following generalized landmark hard clauses
or constraints:

\[
\bigvee_{1 \leq t \leq H} \bigg( \bigvee_{a \in A} \big( \bigvee_{1 \leq i \leq m} (\neg X^i_{a,t}) \lor \bigvee_{1 \leq i \leq m} (X^i_{a,t}) \big) \bigg) \quad (21)
\]

\[
\sum_{t=1}^{H} \sum_{a \in A} \left( \sum_{i=1, (t,a,i) \in \mathcal{L}_n}^{m} (1 - X^i_{a,t}) + \sum_{i=1, (t,a,i) \notin \mathcal{L}_n}^{m} X^i_{a,t} \right) \geq 1 \quad (22)
\]

respectively, where \( \mathcal{L}_n \) is the set of \( 1 \leq i \leq m \) bits of action variables \( a \in A \) executed at time steps \( 1 \leq t \leq H \) (i.e., \( \bar{X}^i_{a,t} = 1 \)) at the \( n \)-th iteration of Algorithm 1 that is outlined below.

**Algorithm 1** Incremental Factored Planning Algorithm

1. \( n = 1 \), planner = FD-SAT-Plan+ or FD-BLP-Plan+
2. \( \mathcal{L}_n \leftarrow \text{Solve } \tilde{\Pi} \text{ using the planner.} \)
3. if \( \mathcal{L}_n \) is empty (i.e., infeasibility) or \( \mathcal{L}_n \) is a plan for \( \Pi \) then
   4. return \( \mathcal{L}_n \)
   5. else
5. if planner = FD-SAT-Plan+ then
   6. planner \leftarrow \text{hard clause (21)}
   7. else
     8. planner \leftarrow \text{constraint (22)}
  9. end
10. \( n \leftarrow n + 1 \), go to line 2.

For a given horizon \( H \), Algorithm 1 iteratively computes a set of executed action variables \( \mathcal{L}_n \), or returns infeasibility for the learned factored planning problem \( \tilde{\Pi} \). If the set of executed action variables \( \mathcal{L}_n \) is non-empty, we evaluate whether \( \mathcal{L}_n \) is a valid plan for the original factored planning problem \( \Pi \) (i.e., line 3) either in the actual domain or using a high fidelity domain simulator – in our case RDDLsim [31]. If the set of executed action variables \( \mathcal{L}_n \) constitutes a plan for \( \Pi \), Algorithm 1 returns \( \mathcal{L}_n \) as a plan. Otherwise, the planner is updated with the new set of generalized landmarks to exclude \( \mathcal{L}_n \) and the loop repeats. Since the original action space is discretized and represented up to \( m \) bits of precision, Algorithm 1 can be shown to terminate in no more than \( n = 2^{|A| \cdot m} \cdot H \) iterations by constructing an inductive proof similar to the termination criteria of OpSeq. The outline of the proof can be found in Appendix B. Next, we present the
theoretical analysis of Bi-Directional Neuron Activation Encoding.

7. Theoretical Results

We now present our theoretical results on Bi-Directional Neuron Activation Encoding, and prove that Bi-Directional Neuron Activation Encoding is efficient with Unit Propagation (UP), which is considered to be one of the most important theoretical properties that facilitate the efficiency of a Boolean cardinality constraint encoded in CNF \[26, 27, 24, 28\].

**Definition 1** (Unit Propagation Efficiency of Neuron Activation Encoding). A neuron activation encoding \(v \leftrightarrow (\sum_{i=1}^{n} x_i \geq p)\) is efficient with Unit Propagation (UP) if and only if UP is sufficient to deduce the following:

1. For any set \(X' \subset \{x_1, \ldots, x_n\}\) with size \(|X'| = n - p\), value assignment to variables \(v = true\), and \(x_i = false\) for all \(x_i \in X'\), the remaining \(p\) variables from the set \(\{x_1, \ldots, x_n\} \setminus X'\) are assigned to true,
2. For any set \(X' \subset \{x_1, \ldots, x_n\}\) with size \(|X'| = p - 1\), value assignment to variables \(v = false\), and \(x_i = true\) for all \(x_i \in X'\), the remaining \(n - p + 1\) variables from the set \(\{x_1, \ldots, x_n\} \setminus X'\) are assigned to false,
3. Partial value assignment of \(p\) variables from \(\{x_1, \ldots, x_n\}\) to true assigns variable \(v = true\), and
4. Partial value assignment of \(n - p + 1\) variables from \(\{x_1, \ldots, x_n\}\) to false assigns variable \(v = false\)

where \(|x|\) denotes the size of set \(x\).

**Theorem 1** (Unit Propagation Efficiency of \(\text{Act}_p(v, \langle x_1, \ldots, x_n \rangle)\)). Bi-Directional Neuron Activation Encoding \(\text{Act}_p(v, \langle x_1, \ldots, x_n \rangle)\) is efficient with Unit Propagation.

*Proof.* To show \(\text{Act}_p(v, \langle x_1, \ldots, x_n \rangle)\) is efficient with Unit Propagation (UP), we need to show it exhaustively for all four cases of Definition 1.

Case 1 (\(\forall X' \subset \{x_1, \ldots, x_n\}\) where \(|X'| = n - p\), \(v = true\) and \(x_i = false\) \(\forall x_i \in X' \to x_i = true\) \(\forall x_i \in \{x_1, \ldots, x_n\} \setminus X'\) by UP): When \(v = true\), UP
assigns \( c_p = true \) using the hard clause \((\neg v \lor c_p)\). Given value assignment \( x_i = false \) to variables \( x_i \in X' \) for any set \( X' \subset \{x_1, \ldots, x_n\} \) with size \( |X'| = n - p \), it has been shown that UP will set the remaining \( p \) variables from the set \( \{x_1, \ldots, x_n\} \setminus X' \) to true using the conjunction of hard clauses that encode \( CN_k^= \) [24].

**Case 2** (\( \forall X' \subset \{x_1, \ldots, x_n\} \) where \( |X'| = p - 1, v = false \) and \( x_i = true \) \( \forall x_i \in X' \rightarrow x_i = false \forall x_i \in \{x_1, \ldots, x_n\} \setminus X' \) by UP): When \( v = false \), UP assigns \( c_p = false \) using the hard clause \((v \lor \neg c_p)\). Given value assignment \( x_i = true \) to variables \( x_i \in X' \) for any set \( X' \subset \{x_1, \ldots, x_n\} \) with size \( |X'| = p - 1 \), it has been shown that UP will set the remaining \( n - p + 1 \) variables from the set \( \{x_1, \ldots, x_n\} \setminus X' \) to false using the conjunction of hard clauses that encode \( CN_k^= \) [24].

**Cases 3** (\( \forall X' \subset \{x_1, \ldots, x_n\} \) where \( |X'| = p, x_i = true \forall x_i \in X' \rightarrow v = true \) by UP) When \( p \) variables from the set \( \{x_1, \ldots, x_n\} \) are set to true, it has been shown that UP assigns the counting variable \( c_p = true \) using the conjunction of hard clauses that encode \( CN_k^= \) [24]. Given the assignment \( c_p = true \), UP assigns \( v = true \) using the hard clause \((v \lor \neg c_p)\).

**Cases 4** (\( \forall X' \subset \{x_1, \ldots, x_n\} \) where \( |X'| = n - p + 1, x_i = false \forall x_i \in X' \rightarrow v = false \) by UP) When \( n - p + 1 \) variables from the set \( \{x_1, \ldots, x_n\} \) are set to false, it has been shown that UP assigns the counting variable \( c_p = false \) using the conjunction of hard clauses that encode \( CN_k^= \) [24]. Given the assignment \( c_p = false \), UP assigns \( v = false \) using the hard clause \((\neg v \lor c_p)\).

We now discuss the importance of our theoretical result in the context of both related work and the contributions of our paper. Amongst the state-of-the-art CNF encodings [10] that are efficient with UP for constraint \( v \rightarrow (\sum_{i=1}^n x_i \geq p) \), Bi-Directional Neuron Activation Encoding uses the smallest number of variables and hard clauses. The previous state-of-the-art CNF encoding for constraint \( v \rightarrow (\sum_{i=1}^n x_i \geq p) \) is an extension of the Sorting Networks [32] and uses \( O(n \log_2 n) \) number of variables and hard clauses [10]. In contrast, Bi-Directional Neuron Activation Encoding is an extension of the Cardinality
Networks [24], and only uses $O(n\log^2 k)$ number of variables and hard clauses, and is efficient with UP as per Theorem [1].

8. Experimental Results

In this section, we evaluate the effectiveness of factored planning with BNNs. First, we present the benchmark domains used to test the efficiency of our learning and factored planning framework with BNNs. Second, we present the accuracy of BNNs to learn complex state transition models for factored planning problems. Third, we compare the runtime efficiency of Bi-Directional Neuron Activation Encoding against the existing Uni-Directional Neuron Activation Encoding [1]. Fourth, we test the efficiency and scalability of planning with FD-SAT-Plan+ and FD-BLP-Plan+ on the learned factored planning problems $\tilde{\Pi}$ across multiple problem sizes and horizon settings. Finally, we demonstrate the effectiveness of Algorithm 1 to find a plan for the factored planning problem $\Pi$.

8.1. Domain Descriptions

The description of four automated planning problems that are designed to test the planning performance of FD-BLP-Plan+ and FD-SAT-Plan+ with non-linear state transition functions, and state and action variables with discrete domains, namely Navigation [33], Inventory Control [34], System Administrator [35][33], and Cellda [19], follows.

**Navigation**: The Navigation [33] task for an agent in a two-dimensional ($N$-by-$N$ where $N \in \mathbb{Z}^+$) maze is cast as an automated planning problem as follows.

- Location of the agent is represented by $N^2$ state variables $S = \{s_1, \ldots, s_{N^2}\}$ with Boolean domains where state variable $s_i$ represents whether the agent is located at position $i$ or not.

- Intended movement of the agent is represented by four action variables $A = \{a_1, a_2, a_3, a_4\}$ with Boolean domains where action variables $a_1$, $a_2$, $a_3$, and $a_4$ represent

Location of the agent is represented by $N^2$ state variables $S = \{s_1, \ldots, s_{N^2}\}$ with Boolean domains where state variable $s_i$ represents whether the agent is located at position $i$ or not.

- Intended movement of the agent is represented by four action variables $A = \{a_1, a_2, a_3, a_4\}$ with Boolean domains where action variables $a_1$, $a_2$, $a_3$, and $a_4$ represent
\(a_3\) and \(a_4\) represent whether the agent intends to move up, down, right or left, respectively.

- Mutual exclusion on the intended movement of the agent is represented by global function \(C\) where 
  \[C(\langle s_1, \ldots, s_{N^2}, a_1, \ldots, a_4 \rangle) = \text{true} \text{ if and only if } a_1 + a_2 + a_3 + a_4 \leq 1,\]
  \[C(\langle s_1, \ldots, s_{N^2}, a_1, \ldots, a_4 \rangle) = \text{false} \text{ otherwise.}\]

- Current location of the agent is represented by the initial state function \(I\) where 
  \[I(\langle s_1, \ldots, s_{N^2} \rangle) = \text{true} \text{ if and only if } s_i = V_i \text{ for all positions } i \in \{1, \ldots, N^2\}, I(\langle s_1, \ldots, s_{N^2} \rangle) = \text{false} \text{ otherwise.}\]

- Final location of the agent is represented by the goal state function \(G\) where 
  \[G(\langle s_1, \ldots, s_{N^2} \rangle) = \text{true} \text{ if and only if } s_i = V'_i \text{ for all positions } i \in \{1, \ldots, N^2\}, G(\langle s_1, \ldots, s_{N^2} \rangle) = \text{false} \text{ otherwise where } V'_i \text{ denotes the goal position of the agent (i.e., } V'_i = \text{true} \text{ if and only if position } i \in \{1, \ldots, N^2\} \text{ is the final location, } V'_i = \text{false} \text{ otherwise).}\]

- Objective is to minimize total number of intended movements by the agent and is represented by the reward function \(R\) where 
  \[R(\langle s_1, \ldots, s_{N^2}, a_1, \ldots, a_4 \rangle) = a_1 + a_2 + a_3 + a_4.\]

- Next location of the agent is represented by the state transition function \(T\) that is a nonlinear function of state and action variables \(s_1, \ldots, s_{N^2}, a_1, \ldots, a_4\). For each position \(i \in \{1, \ldots, N^2\}\), next location of the agent is defined by the function 
  \[T_i(\langle s_1, \ldots, s_{N^2}, a_1, \ldots, a_4 \rangle) = \text{if } r(i, j, k) \land s_j \land a_k \text{ then } \text{true}, \text{ otherwise } \text{false} \text{ where } r(i, j, k) \text{ denotes whether position } i \text{ can be reached from position } j \in \{1, \ldots, N^2\} \text{ by intended movement } k \in \{1, 2, 3, 4\}.\]

We report the results on maze sizes \(N = 3, 4, 5\) over planning horizons \(H = 4, \ldots, 10\). Note that this automated planning problem is a deterministic version of its original from IPPC2011 [33].

\textbf{Inventory Control:} The Inventory Control\textsuperscript{34} is the task of managing inventory of a product with demand cycle length \(N \in \mathbb{Z}^+\), and is cast as an automated planning problem as follows.

\textit{Inventory Control}: The Inventory Control\textsuperscript{34} is the task of managing inventory of a product with demand cycle length \(N \in \mathbb{Z}^+\), and is cast as an automated planning problem as follows.
Inventory level of the product, phase of the demand cycle and whether inventory demand level is met or not are represented by three state variables \( S = \{ s_1, s_2, s_3 \} \) where state variables \( s_1, s_2 \) have non-negative integer domains and state variable \( s_3 \) has a Boolean domain, respectively.

Ordering fixed amount of inventory is represented by an action variable \( A = \{ a_1 \} \) with a Boolean domain.

Meeting the inventory demand level is represented by global function \( C \) where \( C((s_1, s_2, s_3, a_1)) = true \) if and only if \( s_3 = true \), \( C((s_1, s_2, s_3, a_1)) = false \) otherwise.

Current inventory level, current step of the demand cycle and meeting the current inventory demand level are represented by the initial state function \( I \) where \( I((s_1, s_2, s_3)) = true \) if and only if \( s_1 = V, s_2 = 0 \) and \( s_3 = true \), \( I((s_1, s_2, s_3)) = false \) otherwise where \( V \) denotes the current inventory level.

Meeting the final inventory demand level is represented by goal state function \( G \) where \( G((s_1, s_2, s_3)) = true \) if and only if \( s_3 = true \), \( G((s_1, s_2, s_3)) = false \) otherwise.

Objective is to minimize total inventory storage cost and is represented by the reward function \( R \) where \( R((s_1, s_2, s_3, a_1)) = cs_1 \) and \( c \) denotes the unit storage cost of inventory.

Next inventory level, next step of the demand cycle and whether the next inventory demand level is met or not are represented by the state transition function \( T \) that is a nonlinear function of state and action variables \( s_1, s_2, s_3, a_1 \). The next inventory level is defined by the function \( T_1((s_1, s_2, s_3, a_1)) = \max(ra_1 + s_1 - d(s_2), 0) \) where \( r \) and \( d(i) \) are the fixed order amount and the demand at the \( i \)-th step of the demand cycle, respectively. The next step of the demand cycle is defined by the function \( T_2((s_1, s_2, s_3, a_1)) = \) if \( s_2 < N \) then \( s_2 + 1 \), otherwise \( 0 \). Finally, whether
the next inventory demand level is met or not is defined by the function
\[ T_3(s_1, s_2, s_3, a_1) = \begin{cases} 
false & \text{if } r a_1 + s_1 - d(s_2) \leq d^{\min} \\
true & \text{otherwise}
\end{cases} \]
where \( d^{\min} \) is the minimum allowable unmet demand.

We report the results on Inventory Control tasks with two demand cycle
lengths \( N \in \{2, 4\} \) over planning horizons \( H = 5, \ldots, 8 \).

System Administrator: The System Administrator \cite{33, 32} is the maintenance
task of a computer network of size \( N \) and is cast as an automated planning
problem as follows.

- The age of computer \( i \in \{1, \ldots, N\} \), and whether computer \( i \in \{1, \ldots, N\} \)
is running or not, are represented by \( 2N \) state variables \( S = \{s_1, \ldots, s_{2N}\} \) with non-negative integer domains and Boolean domains, respectively.

- Rebooting computers \( i \in \{1, \ldots, N\} \) are represented by \( N \) action variables \( A = \{a_1, \ldots, a_N\} \) with Boolean domains.

- Bound on the number of computers that can be rebooted and the require-
ment that all computers must be running are represented by global function \( C \) where \( C(s_1, \ldots, s_{2N}, a_1, \ldots, a_N) = true \) if and only if \( \sum_{i=1}^{N} a_i \leq a^{max} \) and \( s_{N+1}, \ldots, s_{2N} = true, C(s_1, \ldots, s_{2N}, a_1, \ldots, a_N) = false \) otherwise where \( a^{max} \) is the bound on the number of computers that can be
rebooted at a time.

- Current age of computer \( i \in \{1, \ldots, N\} \), and whether computer \( i \in \{1, \ldots, N\} \) is currently running or not are represented by the initial state
function \( I \) where \( I(s_1, \ldots, s_{2N}) = true \) if and only if \( s_1, \ldots, s_N = 0 \) and \( s_{N+1}, \ldots, s_{2N} = true, I(s_1, \ldots, s_{2N}) = false \) otherwise.

- Final requirement that all computers must be running is represented by
the goal state function \( G \) where \( G(s_1, \ldots, s_{2N}) = true \) if and only if \( s_{N+1}, \ldots, s_{2N} = true, G(s_1, \ldots, s_{2N}) = false \) otherwise.
Objective is to minimize total number of computer reboots and is represented by the reward function $R$ where

$$R(\langle s_1, \ldots, s_{2N}, a_1, \ldots, a_N \rangle) = \sum_{i=1}^{N} a_i.$$ 

Next age of computer $i \in \{1, \ldots, N\}$ and whether computer $i \in \{1, \ldots, N\}$ will be running or not, are represented by the state transition function $T$ that is a nonlinear function of state and action variables $s_1, s_{2N}, a_1, \ldots, a_N$.

For each computer $i \in \{1, \ldots, N\}$, next age of computer is defined by the function

$$T_i(\langle s_1, \ldots, s_{2N}, a_1, \ldots, a_N \rangle) = \text{if } \neg s_i + N \lor a_i \text{ then } 0, \text{ otherwise } s_i + 1.$$ 

For each computer $i \in \{1, \ldots, N\}$, whether the computer will be running or not is defined by the function

$$T_i + N(\langle s_1, \ldots, s_{2N}, a_1, \ldots, a_N \rangle) = \text{if } a_i \lor (s_i + N \land s_i \leq s^{\text{max}}) \lor (s_i + N \land s_i \leq s^{\text{max}}) \cdot \left(1 - \frac{\sum_{j=1}^{N} c(i,j) s_j + N}{1 + \sum_{j=1}^{N} c(i,j)}\right) \leq d^{\text{max}} \text{ then true, otherwise false}$$

where $c(i,j)$ denotes whether computers $i$ and $j \in \{1, \ldots, N\}$ are connected or not, $d^{\text{max}}$ denotes the network density threshold, and $s^{\text{max}}$ is the maximum computer age.

We report the results on System Administrator tasks with $N \in \{4, 5\}$ computers over planning horizons $H = 2, 3, 4$.

**Cellda:** As previously described in Section 2.2, the agent Cellda must escape a dungeon through an initially locked door by obtaining its key without getting hit by her enemy. The gridworld-like dungeon is made up of two types of cells: (i) regular cells on which Cellda and her enemy can move from/to deterministically up, down, right, left, or wait on, and (ii) blocks that neither Cellda nor her enemy can stand on.

We report the results on maze size $N = 4$ over planning horizons $H = 8, \ldots, 12$ with two different enemy policies.

### 8.2. Transition Learning Performance

In Table 4, we present test errors for different configurations of the BNNs on each domain instance where the sample data was generated from the RDDL-based domain simulator RDDLsim [31] using the code available for stochastic
exploration policy with concurrent actions. For each instance of a domain, total of 200,000 state transition samples were collected and the data was treated as independent and identically distributed. After random permutation, the data was split into training and test sets with 9:1 ratio. The BNNs with the feed-forward structure described in Section 2.4 were trained on MacBookPro with 2.8 GHz Intel Core i7 16 GB memory using the code available [8]. For each instance of a domain, the smallest BNN size (i.e., the BNN with the least number of neurons) that achieved less than a preselected test error threshold (i.e., 3% test error) was chosen using a grid-search over preselected network structure hyperparameters, namely width (i.e., 36, 96, 128) and depth (i.e., 1,2,3). The selected BNN structure for each instance is detailed in Table 1. Overall, Navigation instances required the smallest BNN structures for learning due to their purely Boolean state and action spaces, while both Inventory, SysAdmin and Cellda instances required larger BNN structures for accurate learning, owing to their non-Boolean state spaces.

Table 1: Transition Learning Performance Table measured by error on test data (in %) for all domains and instances.

| Domain      | Network Structure | Test Error (%) |
|-------------|-------------------|---------------|
| Navigation(3) | 13:36:36:9        | 0.0           |
| Navigation(4) | 20:96:96:16       | 0.0           |
| Navigation(5) | 29:128:128:25     | 0.0           |
| Inventory(2)  | 7:96:96:5         | 0.018         |
| Inventory(4)  | 8:128:128:5       | 0.34          |
| SysAdmin(4)   | 16:128:128:12     | 2.965         |
| SysAdmin(5)   | 20:128:128:128:15 | 0.984         |
| Cellda(x)     | 12:128:128:4      | 0.645         |
| Cellda(y)     | 12:128:128:4      | 0.65          |
8.3. Planning Performance on the Learned Factored Planning Problems

In this section, we present the results of two computational comparisons. First, we test the efficiency of Bi-Directional Neuron Activation Encoding to the existing Uni-Directional Neuron Activation Encoding [1] to select the best WP-MaxSAT-based encoding for FD-SAT-Plan+. Second, we compare the effectiveness of using the selected WP-MaxSAT-based encoding against a BLP-based encoding, namely FD-SAT-Plan+ and FD-BLP-Plan+, to find plans for the learned factored planning problem $\tilde{\Pi}$. We ran the experiments on a MacBookPro with 2.8 GHz Intel Core i7 16GB memory. For FD-SAT-Plan+ and FD-BLP-Plan+, we used MaxHS [23] with underlying LP-solver CPLEX 12.7.1 [29], and CPLEX 12.7.1 solvers respectively, with 1 hour total time limit per domain instance.
8.3.1. Comparison of neuron activation encodings

Figure 4: Timing comparison between for FD-SAT-Plan+ with Sequential Counters [26] and Uni-Directional Encoding [1] (x-axis) and Cardinality Networks [24] and Bi-Directional Encoding (y-axis). Over all problem settings, FD-SAT-Plan+ with Cardinality Networks and Bi-Directional Encoding significantly outperformed FD-SAT-Plan+ with Sequential Counters and Uni-Directional Encoding on all problem instances due to its (i) smaller encoding size, and (ii) UP efficiency property.

The runtime efficiency of both neuron activation encodings are tested for the learned factored planning problems over 27 problem instances where we test our Bi-Directional Neuron Activation Encoding that utilizes Cardinality Networks [24] against the previous Uni-Directional Neuron Activation Encod-
ing \[1\] that uses Sequential Counters \[26\].

Figure \[4\] visualizes the runtime comparison of both neuron activation encodings. The inspection of Figure \[4\] clearly demonstrate that FD-SAT-Plan+ with Bi-Directional Neuron Activation Encoding signicantly outperforms FD-SAT-Plan+ with Uni-Directional Neuron Activation Encoding in all problem instances due to its (i) smaller encoding size (i.e., \(O(n \log_2^2 k)\) versus \(O(np)\)) with respect to both the number of variables and the number of hard clauses, and (ii) UP efficiency property as per Theorem 1. Therefore, we use FD-SAT-Plan+ with Bi-Directional Neuron Activation Encoding in the remaining experiments and omit the results for FD-SAT-Plan+ with Uni-Directional Neuron Activation Encoding.

8.3.2. Comparison of FD-SAT-Plan+ and FD-BLP-Plan+

Next, we test the runtime efficiency of FD-SAT-Plan+ and FD-BLP-Plan+ for solving the learned factored planning problem.

Table 2: Summary of the computational results presented in Appendix D including the average runtimes in seconds for both FD-SAT-Plan+ and FD-BLP-Plan+ over all four domains for the learned factored planning problem within 1 hour time limit.

| Domains    | FD-SAT-Plan+ | FD-BLP-Plan+ |
|------------|--------------|--------------|
| Navigation | 529.11       | 1282.82      |
| Inventory  | 54.88        | 0.54         |
| SysAdmin   | 1627.35      | 3006.27      |
| Cellida    | 344.03       | 285.45       |
| Coverage   | 27/27        | 20/27        |
| Optimality Proved | 25/27 | 19/27 |

In Table 2, we present the summary of the computational results including the average runtimes in seconds, the total number of instances for which a feasible solution is returned (i.e., coverage), and the total number of instances for which an optimal solution is returned (i.e., optimality proved), for both FD-
SAT-Plan+ and FD-BLP-Plan+ over all four domains for the learned factored planning problem within 1 hour time limit. The analysis of Table 2 shows that FD-SAT-Plan+ covers all problem instances by returning an incumbent solution to the learned factored planning problem compared to FD-BLP-Plan+ which runs out of 1 hour time limit in 7 out of 27 instances before finding an incumbent solution. Similarly, FD-SAT-Plan+ proves the optimality of the solutions found in 25 out of 27 problem instances compared to FD-BLP-Plan+ which only proves the optimality of 19 out of 27 solutions within 1 hour time limit.
Over all problem settings, FD-BLP-Plan+ performed better on instances that require less than approximately 100 seconds to solve (i.e., computationally easy instances) whereas FD-SAT-Plan+ outperformed FD-BLP-Plan+ on instances that require more than approximately 100 seconds to solve (i.e., computationally hard instances).

In Figure 5, we compare the runtime performances of FD-SAT-Plan+ (x-axis) and FD-BLP-Plan+ (y-axis) per instance labeled by their domain. The analysis of Figure 5 across all 27 instances shows that FD-BLP-Plan+ proved the optimality of problem instances from domains which require less computational demand (e.g., Inventory) more efficiently compared to FD-SAT-Plan+. In contrast, FD-SAT-Plan+ proved the optimality of problem instances from
domains which require more computational demand (e.g., SysAdmin) more efficiently compared to FD-BLP-Plan+. As the instances got harder to solve, FD-BLP-Plan+ timed-out more compared to FD-SAT-Plan+, mainly due to its inability to find incumbent solutions as evident from Table 2.

The detailed inspection of Figure 5 and Table 2 together with Table 1 shows that the computational efforts required to solve the benchmark instances increase significantly more for FD-BLP-Plan+ compared to FD-SAT-Plan+ as the learned BNN structure gets more complex (i.e., from smallest BNN structure of Inventory, to moderate size BNN structures of Navigation and Cellda, to the largest BNN structure of SysAdmin). Detailed presentation of the run time results for each instance are provided in Appendix D.

8.4. Planning Performance on the Factored Planning Problems

Finally, we test the planning efficiency of the incremental factored planning algorithm, namely Algorithm 1, for solving the factored planning problem Π.

Table 3: Summary of the computational results presented in Appendix D including the average runtimes in seconds for both FD-SAT-Plan+ and FD-BLP-Plan+ over all four domains for the factored planning problem within 1 hour time limit.

| Domains    | FD-SAT-Plan+ | FD-BLP-Plan+ |
|------------|--------------|--------------|
| Navigation | 529.11       | 1282.82      |
| Inventory  | 68.88        | 0.66         |
| SysAdmin   | 2463.79      | 3006.27      |
| Cellda     | 512.51       | 524.53       |
| Coverage   | 23/27        | 19/27        |
| Optimality Proved | 23/27 | 19/27 |

In Table 3 we present the summary of the computational results including the average runtimes in seconds, the total number of instances for which a feasible solution is returned (i.e., coverage), and the total number of instances for which an optimal solution is returned (i.e., optimality proved), for both
FD-SAT-Plan+ and FD-BLP-Plan+ using Algorithm 1 over all four domains for the factored planning problem within 1 hour time limit. The analysis of Table 3 shows that FD-SAT-Plan+ with Algorithm 1 covers 23 out of 27 problem instances by returning an incumbent solution to the factored planning problem compared to FD-BLP-Plan+ with Algorithm 1 which runs out of 1 hour time limit in 8 out of 27 instances before finding an incumbent solution. Similarly, FD-SAT-Plan+ with Algorithm 1 proves the optimality of the solutions found in 23 out of 27 problem instances compared to FD-BLP-Plan+ with Algorithm 1 which only proves the optimality of 19 out of 27 solutions within 1 hour time limit.

Overall, the constraint generation algorithm successfully verified the plans found for the factored planning problem Π in three out of four domains with low computational cost. In contrast, the incremental factored planning algorithm spent significantly more time in SysAdmin domain as evident in Table 3. Over all instances, we observed that at most 5 instances required constraint generation to find a plan where the maximum number of constraints required was at least 6, namely for (Sys,4,3) instance. Detailed presentation of the run time results and the number of generalized landmark constraints generated for each instance are provided in Appendix D.

9. Discussion and Directions for Future Work

In this section, we discuss the strengths and limitations of our learning and planning framework, focusing on the topics of (i) domain discretization, (ii) grounded versus lifted model learning, (iii) assumptions on the reward function, (iv) linearity assumptions on functions $C, I, G, R$, (v) the availability of real-world (or simulated) interaction, and (vi) exploration policy choices for data collection. We also discuss opportunities for future work that can relax many of these assumptions.

(i) Discretization of non-binary domains: One of the main assumptions we make in this work is the knowledge of $m$, which denotes the total bits of
precision used to represent the domains of non-binary state and action variables, to learn the known deterministic factored planning problem \( \Pi \). In order to avoid any assumptions on the value of \( m \) in our experiments, we have limited our domain instances to only include state and action variables with binary and/or bounded integer domains. Our framework can be extended to handle variables with bounded continuous domains up to a fixed number of decimals where the value of the number of decimals is chosen at training time to learn an accurate \( \tilde{T} \). An important area of future work here is to improve the accuracy of BNNs through the explicit use of the information about the original known domains of state and action variables. That is, the current training of BNNs treat each input and output unit independently. Therefore, future research that focuses on learning of BNNs with non-Boolean domains can further improve the effectiveness of our framework.

(ii) Grounded versus lifted model learning: In this paper, we learn grounded representations of \( \Pi \), that is, we learn a transition function \( \tilde{T} \) for each instance of the domain. Under this assumption, we only plan in the realized (i.e., grounded) instances of the world over which the data is collected. For future work, we will investigate methods for learning and planning with lifted representations of problem \( \Pi \).

(iii) Assumptions on the reward function \( R \): We have assumed the complete knowledge of \( R \) since in many planning domains (e.g., Navigation, Inventory etc.), the planning objective is user-specified (e.g., in the Navigation domain the agent knows where it wants to go, in the Inventory domain the user knows he/she wants to minimize total cost etc.). Given this assumption, we have learned a transition function \( T \) that is independent of \( R \). As a result, our learning and planning framework is general with respect to \( R \) (i.e., planning with respect to a new \( R \) would simply mean the modification of the set of soft clauses \([13]\) or the objective function \([20]\)). If we assumed \( R \) can also be measured and collected as data, our framework can easily be extended to handle planning for an unknown reward function \( R \).
by also learning \( R \) from training data.

(iv) Linearity assumptions: In order to compile the learned planning problem \( \tilde{\Pi} \) into WP-MaxSAT and BLP, we have assumed that functions \( C, I, G, R \) are linear. As a result, our compilation-based planners can be solved by state-of-the-art WP-MaxSAT \[23\] and BLP solvers \[29\]. This experimental limitation can be lifted by changing our branch-and-solver (i.e., for FD-BLP-Plan+) to a more general solver that can handle nonlinearities with optimality guarantees (e.g., a spatial branch-and-bound solver such as SCIP \[36\]). For example, the model-based metric hybrid planner SCIPPlan \[37, 38\] leverages the spatial branch-and-bound solver of SCIP to plan in domains with nonlinear functions \( C, I, G \), and action and state variables with mixed (continuous and discrete) domains.

(v) Availability of real-world (or simulated) interaction: In this work, we assumed the presence of historical data that enables model learning but do not always assume the availability of online access to real-world (or simulated) feedback. Therefore, we have investigated two distinct scenarios for our learning and planning framework, namely planning, (i) without and (ii) with, the availability of real-world (or simulated) interaction. Scenario (i) assumes the planner must produce a plan without having any feedback from the real-world (or simulator). Under scenario (i), all information available about transition function \( T \) is the learned transition function \( \tilde{T} \). As a result, we assumed \( \tilde{\Pi} \) is an accurate approximation of \( \Pi \) (as demonstrated experimentally in Table 1) and planned optimally with respect to \( \tilde{\Pi} \). Scenario (ii) assumes the limited availability of real-world (or potentially expensive simulated) interaction with \( \Pi \). Under scenario (ii), the planner can correct its plans with respect to the new observations. In this work, we do not employ a re-training technique for \( T \) since (i) \( \Pi \) is static (i.e., \( \Pi \) does not change between the time training data was collected and planning), and (ii) we assume the amount of training data is significantly larger than the newly collected data. As a result, we do not re-train the learned transition function \( T \) but instead correct the planner using the
constraint generation framework introduced in Section 5.

(vi) Data collection: Data acquisition is an important part of machine learning which can directly effect the quality of the learned model. In this work, we have assumed the availability of data and assumed data as an input parameter to our learning and planning framework. In our experiments, we have considered the scenario where the learned BNN model can be incorrect (i.e., scenario (ii) as described previously). Under scenario (ii), we have investigated how to repair our planner based on its interaction with the real-world (or simulator). Clearly, if the model that is learned does not include any feasible plans \( \pi \) to \( \Pi \) (i.e., there does not exist a plan that is both a solution to \( \Pi \) and \( \tilde{\Pi} \)), then Algorithm 1 terminates, as shown in Appendix B. If this is the case, then it is an interesting direction for future work to consider how to collect data in order to accurately learn \( T \). However, considerations of such methods (e.g., active learning, investigation of different exploration policies etc.) are orthogonal to the scope of the technical contributions of this paper.

10. Conclusion

In this work, we utilized the efficiency and ability of BNNs to learn complex state transition models of factored planning domains with discretized state and action spaces. We introduced two novel compilations, a WP-MaxSAT (FD-SAT-Plan+) and a BLP (FD-BLP-Plan+) encodings, that directly exploit the structure of BNNs to plan for the learned factored planning problem, which provide optimality guarantees with respect to the learned model when they successfully terminate. Theoretically, we have shown that our SAT-based Bi-Directional Neuron Activation Encoding is asymptotically the most compact encoding in the literature, and is efficient with Unit Propagation (UP), which is one of the most important efficiency indicators of a SAT-based encoding.

We further introduced a finite-time incremental factored planning algorithm based on generalized landmark constraints that improve planning accuracy of
both FD-SAT-Plan+ and FD-BLP-Plan+ through simulated or real-world interaction. Experimentally, we demonstrate the computational efficiency of our Bi-Directional Neuron Activation Encoding in comparison to the Uni-Directional Neuron Activation Encoding [1]. Overall, our empirical results showed we can accurately learn complex state transition models using BNNs and demonstrated strong performance in both the learned and original domains. In sum, this work provides two novel and efficient factored state and action transition learning and planning encodings for BNN-learned transition models, thus providing new and effective tools for the data-driven model-based planning community.

11. Acknowledgements

This work has been funded by the Natural Sciences and Engineering Research Council (NSERC) of Canada.

Appendices

Appendix A. CNF Encoding of the Cardinality Networks

The CNF encoding of Cardinality Networks ($CN_k^\neq$) is as follows [24].

Half Merging Networks. Given two inputs of Boolean variables $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$, Half Merging (HM) Networks merge inputs into a single output of size $2n$ using the CNF encoding as follows.

For input size $n = 1$:

$$HM(\langle x_1 \rangle, \langle y_1 \rangle \rightarrow \langle c_1, c_2 \rangle) = (\neg x_1 \lor \neg y_1 \lor c_2) \land (\neg x_1 \lor c_1) \land (\neg y_1 \lor c_1)$$

$$\land (x_1 \lor y_1 \lor \neg c_1) \land (x_1 \lor \neg c_2) \land (y_1 \lor \neg c_2) \quad \text{(A.1)}$$
For input size $n > 1$:

$$HM(\langle x_1, \ldots, x_n \rangle, \langle y_1, \ldots, y_n \rangle \rightarrow \langle d_1, c_2, \ldots, c_{2n-1}, e_n \rangle) = H_o \land H_e \land H'$$

$$H_o = HM(\langle x_1, x_3, \ldots, x_{n-1} \rangle, \langle y_1, y_3, \ldots, y_{n-1} \rangle \rightarrow \langle d_1, \ldots, d_n \rangle)$$

$$H_e = HM(\langle x_2, x_4, \ldots, x_n \rangle, \langle y_2, y_4, \ldots, y_n \rangle \rightarrow \langle e_1, \ldots, e_n \rangle)$$

$$H' = \bigwedge_{i=1}^{n-1} (\neg d_{i+1} \lor e_i \lor c_{2i+1}) \land (\neg d_{i+1} \lor c_{2i}) \land (\neg e_i \lor c_{2i}) \land$$

$$\bigwedge_{i=1}^{n-1} (d_{i+1} \lor e_i \lor \neg c_{2i}) \land (d_{i+1} \lor \neg c_{2i+1}) \land (e_i \lor \neg c_{2i+1})$$

*Half Sorting Networks.* Given an input of Boolean variables $x_1, \ldots, x_{2n}$, Half Sorting (HS) Networks sort the variables with respect to their value assignment as follows.

For input size $2n = 2$:

$$HS(\langle x_1, x_2 \rangle \rightarrow \langle c_1, c_2 \rangle) = HM(\langle x_1 \rangle, \langle x_2 \rangle \rightarrow \langle c_1, c_2 \rangle)$$

For input size $2n > 2$:

$$HS(\langle x_1, \ldots, x_{2n} \rangle \rightarrow \langle c_1, \ldots, c_{2n} \rangle) = H_1 \land H_2 \land H_M$$

$$H_1 = HS(\langle x_1, \ldots, x_n \rangle \rightarrow \langle d_1, \ldots, d_n \rangle)$$

$$H_2 = HS(\langle x_{n+1}, \ldots, x_{2n} \rangle \rightarrow \langle d'_1, \ldots, d'_n \rangle)$$

$$H_M = HM(\langle d_1, \ldots, d_n \rangle, \langle d'_1, \ldots, d'_n \rangle \rightarrow \langle c_1, \ldots, c_{2n} \rangle)$$

*Simplified Merging Networks.* Given two inputs of Boolean variables $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$, Simplified Merging (SM) Networks merge inputs into a single output of size $2n$ using the CNF encoding as follows.

For input size $n = 1$:

$$SM(\langle x_1 \rangle, \langle y_1 \rangle \rightarrow \langle c_1, c_2 \rangle) = HM(\langle x_1 \rangle, \langle y_1 \rangle \rightarrow \langle c_1, c_2 \rangle)$$

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For input size $n > 1$:

$$SM(\langle x_1, \ldots, x_n \rangle, \langle y_1, \ldots, y_n \rangle \rightarrow \langle d_1, c_2, \ldots, c_{n+1} \rangle) = S_o \land S_e \land S'$$ (A.12)

$$S_o = SM(\langle x_1, x_3, \ldots, x_{n-1} \rangle, \langle y_1, y_3, \ldots, y_{n-1} \rangle \rightarrow \langle d_1, \ldots, d_{\frac{n}{2}+1} \rangle)$$ (A.13)

$$S_e = SM(\langle x_2, x_4, \ldots, x_n \rangle, \langle y_2, y_4, \ldots, y_n \rangle \rightarrow \langle e_1, \ldots, e_{\frac{n}{2}+1} \rangle)$$ (A.14)

$$S' = \bigwedge_{i=1}^{n/2} (\neg d_{i+1} \lor e_i \lor c_{2i+1}) \land (\neg d_{i+1} \lor c_{2i}) \land (\neg e_i \lor c_{2i}) \land$$

$$\bigwedge_{i=1}^{n/2} (d_{i+1} \lor e_i \lor \neg c_{2i}) \land (d_{i+1} \lor \neg c_{2i+1}) \land (e_i \lor \neg c_{2i+1})$$ (A.15)

Note that unlike HM, SM counts the number of variables assigned to true from input variables $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ up to $n + 1$ bits instead of 2$n$.

**Cardinality Networks.** Given an input of Boolean variables $x_1, \ldots, x_n$ with $n = kw$ where $p, u \in \mathbb{N}$ and $k$ is the smallest power of 2 such that $k > p$, the CNF encoding of Cardinality Networks ($CN_k^\pm$) is as follows.

For input size $n = k$:

$$CN_k^\pm(\langle x_1, \ldots, x_n \rangle \rightarrow \langle c_1, \ldots, c_n \rangle) = HS(\langle x_1, \ldots, x_n \rangle \rightarrow \langle c_1, \ldots, c_n \rangle)$$ (A.16)

For input size $n > k$:

$$CN_k^\pm(\langle x_1, \ldots, x_n \rangle \rightarrow \langle c_1, \ldots, c_k \rangle) = C_1 \land C_2 \land C_M$$ (A.17)

$$C_1 = CN_k^\pm(\langle x_1, \ldots, x_k \rangle \rightarrow \langle d_1, \ldots, d_k \rangle)$$ (A.18)

$$C_2 = CN_k^\pm(\langle x_{k+1}, \ldots, x_n \rangle \rightarrow \langle d'_{1}, \ldots, d'_{k} \rangle)$$ (A.19)

$$C_M = SM(\langle d_1, \ldots, d_k \rangle, \langle d'_{1}, \ldots, d'_{k} \rangle \rightarrow \langle c_1, \ldots, c_{k+1} \rangle)$$ (A.20)

**Appendix B. Proof for Algorithm 1**

Given hard clauses (4-8) and Theorem 1, Corollary 1 follows.

**Corollary 1 (Forward Pass).** Given the values of state $\bar{Y}_{s,t}$ and action $\bar{X}_{a,t}$ decision variables for all bits $1 \leq i \leq m$ and time step $t \in \{1, \ldots, H \}$, and the learned transition function $\bar{T}$, hard clauses (4-8) deterministically assign values
to all state decision variables \( Y_{s,t}^i \) through Unit Propagation (from Theorem 1) such that \( \tilde{T}(Y_{s,t}^i | s \in S, 1 \leq i \leq m) \rightarrow \langle \bar{X}_{a,t}^i | a \in A, 1 \leq i \leq m \rangle = \langle Y_{s,t+1}^i | s \in S, 1 \leq i \leq m \rangle \).

**Theorem 2** (Finiteness of Algorithm 1). Let \( \tilde{\Pi} = \langle S, A, C, \tilde{T}, I, G, R \rangle \) be the learned deterministic factored planning problem. For a given horizon \( H \) and \( m \)-bit precision, Algorithm 1 terminates in finite number of iterations \( n \leq 2^{|A| \cdot m} \cdot H \).

**Proof by Induction.** Let \( V \) be the set of all value tuples for all action variables \( A \) with \( m \) bits of precision and time steps \( t \in \{1, \ldots, H\} \) such that \( \langle \bar{A}^1, \ldots, \bar{A}^H \rangle \in V \). From the definition of the decision variable \( X_{a,t}^i \) in Section 3.1 and the values \( \bar{A}^t = \langle \bar{a}_t^1, \ldots, \bar{a}_t^{|A|} \rangle \) of action variables \( A \) for all time steps \( t \in \{1, \ldots, H\} \), the value of every action decision variable \( X_{a,t}^i \) is set using the binarization formula such that \( \bar{a}_t^i = -2^{m-1} \bar{X}_{a,t}^m + \sum_{i=1}^{m-1} 2^{i-1} \bar{X}_{a,t}^i \). Given the initial values \( \bar{S}_{t}^i \), hard clause (9) sets the values of state decision variables \( Y_{s,1}^i \) at time step \( t = 1 \) for all bits \( 1 \leq i \leq m \). Given the values of state \( \bar{Y}_{s,1}^i \) and action \( \bar{X}_{a,1}^i \) decision variables at time step \( t = 1 \), the values of state decision variables \( \bar{Y}_{s,2}^i \) are set (from Corollary 1). Similarly, using the values of action decision variables \( \bar{X}_{a,t}^i \) for time steps \( t \in \{2, \ldots, H\} \), the values of state decision variables are set \( \bar{Y}_{s,t}^i \) sequentially for the remaining time steps \( t \in \{3, \ldots, H+1\} \). Given we have shown that each element \( \langle \bar{A}^1, \ldots, \bar{A}^H \rangle \in V \) has a corresponding value tuple for state variables \( S \) and time steps \( t \in \{2, \ldots, H+1\} \), we denote \( V' \subseteq V \) as the subset of feasible value tuples for action variables \( A \) and state variables \( S \) with respect to hard clause (9) (i.e., global function \( C = \text{true} \)) for time steps \( t \in \{1, \ldots, H\} \) and hard clause (12) (i.e., goal state function \( G = \text{true} \)) for time step \( t = H + 1 \).

**Base Case** (\( n = 1 \)): In the first iteration \( n = 1 \), Algorithm 1 either proves infeasibility of \( \tilde{\Pi} \) if and only if \( V' = \emptyset \), or finds values of action variables \( \pi = \langle \bar{A}^1, \ldots, \bar{A}^H \rangle \). If the planner returns infeasibility of \( \tilde{\Pi} \), Algorithm 1 terminates. Otherwise, values of action variables \( \pi \) are sequentially simulated for time steps \( t \in \{1, \ldots, H\} \) given the initial values of state variables \( \bar{S}_{t}^i \) using state transition function \( T \) and checked for its feasibility with respect to (i) global
function $C$ and (ii) goal state function $G$. If the domain simulator verifies all
the propagated values of state variables as feasible with respect to (i) and (ii),
Algorithm 1 terminates and returns $\pi$ as a feasible plan for the deterministic
factored planning problem $\Pi$. Otherwise, values of action variables $\pi$ are used
to generate a generalized landmark hard clause (or constraint) that is added
back to the planner, which only removes $\pi$ from the solution space $V'$ such that
$V' \leftarrow V' \setminus \pi$.

Induction Hypothesis ($n < i$): Assume that upto iteration $n < i$, Algorithm 1
removes exactly $n$ unique solutions from the solution space $V'$.

Induction Step ($n = i$): Let $n = i$ be the next iteration of Algorithm 1.
In iteration $n = i$, Algorithm 1 either proves infeasibility of $\tilde{\Pi}$ if and only if
$V' = \emptyset$, or finds a value tuple $\pi = \langle \bar{A}^1, \ldots, \bar{A}^H \rangle$ of action variables $A$ and time
steps $t \in \{1, \ldots, H\}$. If the planner returns infeasibility of $\tilde{\Pi}$, Algorithm 1
terminates. Otherwise, values of action variables $\pi$ are sequentially simulated
for time steps $t \in \{1, \ldots, H\}$ given the initial values of state variables $\bar{S}^{i,a}$ using
state transition function $T$ and checked for its feasibility with respect to (i)
global function $C$ and (ii) goal state function $G$. If the domain simulator verifies
all the propagated values of state variables as feasible with respect to (i) and (ii),
Algorithm 1 terminates and returns $\pi$ as a feasible plan for the deterministic
factored planning problem $\Pi$. Otherwise, values of action variables $\pi$ are used
to generate a generalized landmark hard clause (or constraint) that is added
back to the planner, which only removes $\pi$ from the solution space $V'$ such that
$V' \leftarrow V' \setminus \pi$.

By induction in at most $n = |A| \cdot m \cdot H$ iterations, Algorithm 1 either (i)
proves there does not exist values $\pi = \langle \bar{A}^1, \ldots, \bar{A}^H \rangle$ of action variables $A$ that
is both a solution to $\tilde{\Pi}$ and $\Pi$ by reducing $V'$ to an emptyset, or (ii) returns $\pi$ as
a solution to the deterministic factored planning problem $\Pi$ (i.e., $|V'| \geq 1$),
and terminates.
Appendix C. Online Repositories

The respective online repositories for FD-SAT-Plan+ and FD-BLP-Plan+ used to generate experiments in this article are the following:

- [https://github.com/saybuser/FD-SAT-Plan](https://github.com/saybuser/FD-SAT-Plan), and
- [https://github.com/saybuser/FD-BLP-Plan](https://github.com/saybuser/FD-BLP-Plan).

Formal text representations of all domains described and experimented in this article can be found in these repositories and read in by the respective planners.
Appendix  D.  Computational Results

Table D.4: Computational results including the runtimes and the total number of generalized landmark constraints generated for both FD-SAT-Plan+ and FD-BLP-Plan+ over all 27 instances within 1 hour time limit. For the instances that time out, secondary results on the solution quality of the returned plans (i.e., their duality gap) are provided.

| Instances     | Non-Incremental Runtimes | Incremental Runtimes | No. of Generalized Landmarks |
|---------------|--------------------------|----------------------|-----------------------------|
|               | FD-SAT-Plan+ | FD-BLP-Plan+ | FD-SAT-Plan+ | FD-BLP-Plan+ | FD-SAT-Plan+ | FD-BLP-Plan+ |
| Nav,3x3,4     | 3.15         | 1.41           | 3.15         | 1.41           | 0            | 0            |
| Nav,3x3,5     | 5.55         | 3.78           | 5.55         | 3.78           | 0            | 0            |
| Nav,3x3,6     | 9.19         | 17.82          | 9.19         | 17.82          | 0            | 0            |
| Nav,4x4,5     | 82.99        | 107.59         | 82.99        | 107.59         | 0            | 0            |
| Nav,4x4,6     | 190.77       | 159.42         | 190.77       | 159.42         | 0            | 0            |
| Nav,4x4,7     | 360.05       | 455.32         | 360.05       | 455.32         | 0            | 0            |
| Nav,5x5,8     | 1275.36      | 3600<,no sol.  | 1275.36      | 3600<,no sol.  | 0            | 0            |
| Nav,5x5,9     | 753.27       | 3600<,no sol.  | 753.27       | 3600<,no sol.  | 0            | 0            |
| Nav,5x5,10    | 2138.62      | 3600<,no sol.  | 2138.62      | 3600<,no sol.  | 0            | 0            |
| Inv,2,5       | 26.92        | 0.37           | 26.92        | 0.37           | 0            | 0            |
| Inv,2,6       | 33.25        | 0.45           | 33.25        | 0.45           | 0            | 0            |
| Inv,2,7       | 40.15        | 0.56           | 40.15        | 0.56           | 0            | 0            |
| Inv,4,6       | 63.18        | 0.51           | 63.18        | 0.51           | 0            | 0            |
| Inv,4,7       | 79.19        | 0.59           | 79.19        | 0.59           | 0            | 0            |
| Inv,4,8       | 86.57        | 0.74           | 170.56       | 1.49           | 1            | 1            |
| Sys,4,2       | 24.19        | 37.63          | 24.19        | 37.63          | 0            | 0            |
| Sys,4,3       | 619.57       | 3600<,100%     | 3600<,no sol.| 3600<,no sol.  | 6≤            | n/a          |
| Sys,4,4       | 1561.78      | 3600<,no sol.  | 3600<,no sol.| 3600<,no sol.  | 3≤            | n/a          |
| Sys,5,2       | 358.53       | 3600<,no sol.  | 358.53       | 3600<,no sol.  | 0            | n/a          |
| Sys,5,3       | 3600<,75%    | 3600<,no sol.  | 3600<,no sol.| 3600<,no sol.  | n/a           | n/a          |
| Sys,5,4       | 3600<,100%   | 3600<,no sol.  | 3600<,no sol.| 3600<,no sol.  | n/a           | n/a          |
| Cellda,x,10   | 197.16       | 106.93         | 592.44       | 469.52         | 2            | 2            |
| Cellda,x,11   | 219.72       | 403.24         | 835.36       | 1539.12        | 2            | 3            |
| Cellda,x,12   | 522.15       | 527.56         | 522.15       | 527.56         | 0            | 0            |
| Cellda,y,8    | 144.95       | 89.64          | 144.95       | 89.64          | 0            | 0            |
| Cellda,y,9    | 404.39       | 40.9           | 404.39       | 40.9           | 0            | 0            |
| Cellda,y,10   | 575.78       | 544.45         | 575.78       | 544.45         | 0            | 0            |
| Coverage      | 27/27        | 20/27          | 23/27        | 19/27          |
| Opt. Proved   | 25/27        | 19/27          | 23/27        | 19/27          |

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