Implementations of Nonadiabatic Geometric Quantum Computation using NMR

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Recently, geometric phases, which is fault tolerant to certain errors intrinsically due to its geometric property, are getting considerable attention in quantum computing theoretically. So far, only one experiment about adiabatic geometric gate with NMR through Berry phase has been reported. However, there are two drawbacks in it. First, the adiabatic condition of Berry phase makes such gate very slowly. Second, the extra operation to eliminate the dynamic phase. As we know, geometric phase can exist both adiabatic(Berry phase) and nonadiabatic(Aharonov-Anandan phase). In this letter, we reports the first experimental realization of nonadiabatic geometric gate with NMR through conditional-AA phase. In our experiment the gates can be made faster and more easily, and the two drawbacks mentioned above are removed.

Quantum computers can perform certain tasks much more efficiently than classical Turing Machine [9]. It is well known that controlled two-qubit gate, combined with single qubit operations, is a universal gate for quantum computation [2]. This two-qubit gate preserves the target qubit for the controlling qubit in certain state, say, $|\uparrow\rangle$, and flips the target qubit for the controlling qubit in the other state, $|\downarrow\rangle$. Originally this has been achieved experimentally using dynamic method in different physical systems [3, 4, 5].

On the other hand, central to the experimental realization of quantum computer is the construction of fault-tolerant quantum logic gates [1]. In the quest for a low noise quantum computing device, geometric phases [1, 2] which is fault tolerant to certain errors intrinsically due to its geometric property, are getting considerable attention in quantum computing theoretically [8, 10, 11, 12, 13, 14, 15, 16, 17, 18]. Geometric logical gates based on geometric property, are getting considerable attention in quantum computing theoretically. So far, only a few experimental demonstrations. Here we show the realization of nonadiabatic two-qubit gate through conditional-AA phase of one two-level subsystem (qubit) controlled by the state of another qubit.

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FIG. 1: The path evolution of qubit $a$ when qubit $b$ is in $|\uparrow\rangle$ state. The first pulse (Fig.2) transformed the initial Hamiltonian $2\pi J J_{a}^z$ to $H = 2\pi J (I_{a}^x \cos \theta - I_{a}^y \sin \theta)$ which creates an evolution path on the geodesic circle ABC. After time $\tau = 1/(2J)$, the second pulse changes the Hamiltonian to the form $H = 2\pi J (-I_{a}^x \cos \theta - I_{a}^y \sin \theta)$ which creates an evolution path on the geodesic circle CDA. Again after time $\tau = 1/(2J)$, the third pulse restored the Hamiltonian to the initial form $2\pi J I_{a}^z$. Therefore, qubit $a$ undergoes a cyclic evolution through a slice circuit $C$ with angle $\theta$ in projective Hilbert (density operator) space, and then the AA geometrical phase is simply $\beta(C) = m \Omega$, where $m = \pm 1$ is the magnetic quantum number and $\Omega = 4\theta$ is the solid angle subtended by the slice circuit.
FIG. 2: Pulse sequence used to demonstrate controlled-AA phase of the state of qubit \( a \) or controlled two-qubit gate. The black boxes are pulses oscillating at frequency \( \omega_a = \omega_a - \pi J \), the flip angles are \(-\theta, 2\theta - \pi \) and \(-\pi - \theta\) from left to right which can be realized by choosing the different pulse duration \( t \) and pulse power \( P \). Here \( \theta \) was selected from 0 to \( \pi \) by \( \theta = \frac{4\pi}{16} \), \( n = \{0, 1, \ldots, 16\} \). All pulses that oscillated at frequency \( \omega_a - \pi J \) are hard pulses, each pulse duration is 5\( \mu \)s. Delay times are \( \tau = 1/2J \) between pair of pulses. Then the time of nonadiabatic controlled two-qubit gate is about 4.8\( \mu \)s. Note this gate time do not depend on the value of AA phase.

Prolonged by the nuclear spins of the \(^1H\) and \(^{13}C\) atoms in a Carbon-13 labeled chloroform molecule, the single \(^1H\) nucleus was used as target qubit \( a \), while the \(^{13}C\) nucleus was used as controlled qubit \( b \). This describes the spin state aligned with (against) an external equilibrium one, while pure \( 00 \) state must be prepared in the \( z \) direction for \( a \) and \( b \). The reduced Hamiltonian for this two-spin system is, to an excellent approximation, given by \( H = \omega_a I_z^a + \omega_b I_z^b + 2\pi J I_z^a I_z^b \), where the first two terms describe the free precession of spin \( a \) \((^1H)\) and \( b \) \((^{13}C)\) about \( B_0 \) with frequencies \( \omega_a/2\pi \approx 500\text{MHz} \) and \( \omega_b/2\pi \approx 125\text{MHz} \). \( I_z^a \) and \( I_z^b \) are the angular momentum operator in the \( z \) direction for \( a \) and \( b \).

From the whole process described above, the Hamiltonian of qubit \( a \) has experienced a cyclic evolution because of the cyclic property of the pulse sequence. Correspondingly, an evolution path of ABCDA on the Bloch sphere is produced for qubit \( a \), that is \( \frac{\sqrt{2}}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \rightarrow \frac{\sqrt{2}}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) \rightarrow \frac{2\pi - i\theta}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \). Obviously the angle \( \theta \) is the geometric parameter describing the behavior of the Hamiltonian evolution. This geometric parameter also describes the phase difference between the initial and final state. Note that the dynamic phases appearing in two steps of procession cancel out each other. The geometric AA phase is equal to \( \beta(C) = -2\theta \). So far we have considered the case in which the state of qubit \( b \) is \( |\uparrow\rangle \). If qubit \( b \) is in \( |\downarrow\rangle \), the Hamiltonian of qubit \( a \) is zero and nothing will happen to it. In other words, qubit \( a \) will preserve itself in this case. Therefore, the time evolution operator of the pulse sequence (Fig.2) with the property \( U(2\theta) |\pm\rangle_a |\uparrow\rangle_b = e^{\pm i\beta(C)} |\pm\rangle_a |\uparrow\rangle_b \) and \( U(2\theta) |\pm\rangle_a |\downarrow\rangle_b = |\pm\rangle_a |\downarrow\rangle_b \), where \( |\pm\rangle \) corresponds to point \( A \) and \( C \), respectively, in the Bloch sphere. Hence we can regard qubit \( b \) as a controlling qubit and qubit \( a \) as the target qubit; controlled-AA phase of qubit \( a \) is produced depended on the state of qubit \( b \). In the basis of \( |\uparrow\rangle \) and \( |\downarrow\rangle \), the unitary operator that describes this
circle evolution is

\[
\begin{pmatrix}
\cos(\beta(C)) & i\sin(\beta(C)) & 0 & 0 \\
i\sin(\beta(C)) & \cos(\beta(C)) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]

In particular, when \(|\beta(C)| = \pi/2\), this gate is just the C-not gate.

In order to measure the overall AA phase \(\beta(C)\) of qubit \(a\) we also apply a 90\(^\circ\) pulse, oscillating at frequency \(\omega_a\) along y-axes, to transform qubit \(b\) in a coherent superposition of states \(|\psi(0)\rangle_b = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)_b\) before the cyclic evolution. Therefore after this cyclic evolution the final state is \(\frac{1}{\sqrt{2}}|+\rangle_a \left( |\uparrow\rangle_a + e^{-i\beta(C)}|\downarrow\rangle_a \right)_b\). Here, the unobservable AA phase \(\beta(C) = 2\theta = \frac{1}{2}\Omega\) of qubit \(a\) transfers to inner phase of qubit \(b\) (13C in our experiments) which can be observed with NMR. To do so, we use the signal of initial \(\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)_b\) state as a reference one, compared with the signal of the final \(\frac{1}{\sqrt{2}}(|\uparrow\rangle_a + e^{-i\beta(C)}|\downarrow\rangle_a)|\downarrow\rangle_b\) state; both of them are in-phase doublet but out of phase by \(\beta(C)\). Experimentally, we apply additional phasing factor \(\beta(C)\) to obtain absorptive lineshape after Fourier transformation.

All experiments are performed at room temperature and pressure on Bruker Avance DMX-500 spectrometer in Laboratory of Structure Biology, University of Science and Technology of China. The experimental results are shown in Fig.3.

Therefore, we have observed controlled-AA phase or implemented a nonadiabatic two-qubit gate. Note that though the observation of AA phase has been done in NMR with a three-level system, it cannot be used to implement universal two-qubit gate.

Our experiment resolves two drawbacks of the adiabatic geometric computation, namely the slow evolution and the need of refocusing to eliminate the dynamical phases. Let us now compare the gate time of this nonadiabatic geometric gate to that of adiabatic geometric gate and dynamic gate. Since the gate time is limited directly by the strength of coupling constant \(J\) of the sample. Two experiments we selected to compare used the same sample as ours\[6\], that is, Carbon-13 labelled chloroform sample. In our experiment, it took about 4.8ms to realize the gate, slightly longer than the time used to realize dynamic two-qubit gate (about 2.4ms)\[7\], yet much shorter than the time it took to realize the adiabatic geometric two-qubit gate (about 120ms)\[6\]. As the adiabatic geometric gate operates for a significantly longer time, it is much more severely affected by decoherence. This has serious implications for the physical realization of adiabatic geometric quantum computation. On the other hand, since the state is always perpendicular to the effective magnetic field, there is no dynamical phase accumulation during the evolution, hence the resulted phase factor after cyclic evolution was pure geometric phase. Although this nonadiabatic geometric gate is experimentally realized in the NMR system, the basic idea is general, and could be applied in other physical systems. We believe our experiment has led the idea of geometric quantum computation much more practical than before.

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[1] D. Deutsch, Proc. R. Soc. London A 400, 97 (1985).
[2] Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. A. Smolin, and H. Weinfurter, Phys. Rev. A 52, 124 (1995).
[3] C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. 75, 4714 (1995).
[4] Q. A. Turchette, C. J. Hood, W. Lange, H. Mabuchi and H. J. Kimble, Phys. Rev. Lett. 75, 4710 (1995).
[5] N. A. Gershenfeld and I. L. Chuang, Science 275, 350 (1997).
[6] M. Steane, Nature (London) 399, 124 (1999).
[7] M. V. Berry, Proc. R. Soc. A. 392, 45 (1984).
[8] Y. Aharonov and J. Anandan, Phys. Rev. Lett. 58, 1593 (1987).
[9] J.A.Jones, V.Vedral, A. Ekert, and G. Castagnoli, Nature (London) 403, 869 (2000).
[10] G. Fald, R. Fazio, G.M.Palma, G. Slewert and V.Vedral, Nature (London) 407, 355 (2000).
[11] L.-M. Duan, J. I. Cirac and P. Zoller, Science 292, 1695 (2001).
[12] T. Pellizzari, S. A. Gardiner, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 75, 3788 (1995).
[13] D. V. Aervin, Solid State Commun. 105, 659 (1998).
[14] P. Zanardi, Physical Review A 63, 012301 (2001).
[15] J. Pachos and S. Chountasis, Physical Review A 62, 052318 (2000).
[16] J. Pachos and M. Rasetti, Phys. Lett. A 264, 94 (1999).
[17] Pachos, P. Zanardi and M. Rasetti, Physical Review A 61, 010305(R) (2000).
[18] D. Ellinas and J. Pachos, Physical Review A 64, 022310 (2001).
[19] W. Xiangbin and M. Keiji, Physical Review Letter 87, 097901 (2001).
[20] C. Wellard, L. C. L. Hollenberg, and H. C. Pauli, Physical Review A 65, 032303 (2002).
[21] W. Xiangbin and M. Keiji, J. Phys. A: Math. Gen. 34, L631 (2001).
[22] W. Xiangbin and M. Keiji, quant-ph/0104127.
[23] Xin-Qi Li et al., quant-ph/0204028.
[24] D. Suter, K. T. Mueller, and A. Pines, Phys. Rev. Lett. 60, 1218 (1988).
[25] D. G. Cory, M. D. Price, and T. F. Havel, Physica D 120, 82 (1998).
[26] E. Knill, I. Chuang, and R. Laflamme, Phys. Rev. A 57, 3348 (1997).
[27] L. M. K. Vandersypen, C. S. Yannoni, M. H. Sherwood, and I. L. Chuang, Phys. Rev. Lett. 83, 3085 (1999).