The condensation of monopoles (dual superconductivity) of QCD vacuum is reviewed. Direct evidence is produced that the system, in the confined phase, is a dual superconductor.

1. Introduction: statement of the problem

Dual superconductivity of the vacuum was advocated long ago as the mechanism for confinement of colour. Dual means that the role of electric and magnetic fields and charges are interchanged with respect to ordinary superconductors. The basic idea is that the chromoelectric field acting between a quark antiquark pair is channeled into an Abrikosov flux tube, by dual Meissner effect. The resulting static potential is proportional to the distance $R$

$$V(R) = \sigma R$$  \hspace{1cm} (1)

$\sigma$ is the string tension. Flux tubes are expected to behave as strings.

Numerical simulations of QCD on the lattice support this picture:

1) The interquark force at large distances obeys Eq. (1).

2) Flux tubes exist in field configurations produced by static $q\bar{q}$ pairs.

3) Higher modes of the string are visible.

Till recently, however, a convincing demonstration that the ground state of QCD behaves as a superconductor was still lacking. In the following I will analyse recent progress on this point. In particular I will present direct evidence of dual superconductivity of QCD vacuum, obtained by measuring on a lattice a disorder parameter.

Ordinary superconductivity is nothing but the spontaneous breaking (S.B.), à la Higgs, of the $U(1)$ symmetry related to charge conservation. A charged field

$$\Phi = \psi e^{i\theta q} \quad \psi = |\Phi|$$  \hspace{1cm} (2)

acquires a non vanishing vacuum expectation value (v.e.v.) $\langle \Phi \rangle$. As a consequence

(i) the photon acquires a mass $\mu$

$$\mu^2 = e^2 \langle \Phi \rangle^2$$  \hspace{1cm} (3)
the vacuum is not \( U(1) \) invariant, and has no definite charge: indeed if it were invariant the v.e.v. of any charged operator would vanish.

A well known consequence of the Higgs phenomenon is that the derivative of the angular variable \( \theta \) of Eq. (4) becomes the longitudinal component of the photon. Instead of \( A_\mu \) it proves convenient to use as a field variable \( \tilde{A}_\mu = A_\mu - \frac{1}{e} \partial_\mu \theta \) which is gauge invariant. In terms of \( \tilde{A}_\mu \) \( F_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu \); in particular \( \mathbf{H} = \nabla \wedge \tilde{\mathbf{A}} \). The equations of motion for a static configuration become

\[
\partial_i F_{ij} + \mu^2 \tilde{A}_j = 0 \tag{4}
\]

or

\[
\nabla \wedge \mathbf{H} = \mu^2 \tilde{\mathbf{A}} \tag{5}
\]

Taking the curl of both sides of Eq. (5) gives

\[
\nabla^2 \mathbf{H} + \mu^2 \mathbf{H} = 0 \tag{6}
\]

Eq. (5) means that a permanent current (London current)

\[
\mathbf{j} = \mu^2 \tilde{\mathbf{A}} \tag{7}
\]

is present in the superconductor, with \( \mathbf{E} = 0 \), or, since \( \rho \mathbf{j} = \mathbf{E} \), that \( \rho = 0 \).

Eq. (5) means that the magnetic field \( \mathbf{H} \) has a finite penetration depth, and this is nothing but Meissner effect. On a line around a flux tube at distance larger than the penetration depth \( \tilde{\mathbf{A}} = 0 \), \( \oint \tilde{\mathbf{A}} d\mathbf{x} = 0 \) or, by the definition of \( \tilde{A}_\mu \), \( \oint \mathbf{A} d\mathbf{x} = n\pi/q \), which is flux quantization. The key parameter in the game is \( \langle \Phi \rangle \). To detect superconductivity one can either look for permanent currents Eq. (7), i.e. demonstrate that \( \mu^2 \neq 0 \), or directly for the spontaneous breaking of \( U(1) \), i.e. look for a non vanishing v.e.v. of a charged operator.

In QCD the dual situation is expected to occur. The disorder parameter is the v.e.v. of an operator with non zero magnetic charge, and the London current is a magnetic current. The strategy of detecting dual superconductivity by looking for persistent currents will be reviewed by D. Haymaker in his talk to this conference. I will instead present a direct determination of the disorder parameter \( \langle \Phi \rangle \).

2. Monopoles in gauge theories

Monopoles as solitons in gauge theories are related to the elements of the first homotopy group of the gauge group \( \Pi_1(SU(N)) = \{1\} \). In order to have monopoles the symmetry has to reduce to some non simply connected group. In a theory with \( SU(2) \) gauge group coupled to a scalar field in the adjoint representation of \( \Phi \), when the Higgs phenomenon reduces the symmetry from \( SU(2) \) to \( U(1) \) monopoles...
do exist as stable static solutions. The relevant degrees of freedom are described
by the gauge invariant field strength
\[ f_{\mu\nu} = G_{\mu\nu} \cdot \Phi - \frac{1}{g} \cdot \left( D_\mu \Phi \wedge D_\nu \Phi \right) \]
(8)
\[ \Phi = \frac{\Phi}{|\Phi|} \]
is the colour direction of the Higgs field. At large distances the field \( f_{\mu\nu} \)
of a monopole configuration is the field of a Dirac monopole of magnetic charge 2.
One can define a gauge field
\[ a_\mu = \vec{A}_\mu \cdot \hat{\Phi} \]
(9)
Contrary to \( f_{\mu\nu} \) \( a_\mu \) is not gauge invariant, since \( \vec{A}_\mu \) is not gauge covariant. In general
\[ f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - \frac{1}{g} \hat{\Phi} \left( \partial_\mu \Phi \wedge \partial_\nu \Phi \right) \]
(10)
After a gauge rotation which brings \( \hat{\Phi} \) in a given colour direction \( (\hat{\Phi})^a = \delta_3^a \), the last term in Eq.(10) vanishes and
\[ f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \]
(11)
Such a gauge rotation is called an abelian projection: in a gauge defined by this
procedure the \( U(1) \) degrees of freedom relevant to the definition of monopoles coincide
with a subgroup of the gauge group. \( a_\nu \) and \( f_{\mu\nu} \) are formally identical to the fields
of a \( U(1) \) gauge theory. We notice for further reference that also the commutation
relations between \( f_{0i} \) and \( a_i \) are identical to those of a \( U(1) \) theory.
To define the monopoles which produce, by condensation in the vacuum, dual superconduc-
tivity and confinement, the relevant degrees of freedom have to be selected
by an abelian projection. A few different abelian projections have been proposed
in the literature as candidates for this purpose and will be discussed in detail in
what follows.
We conclude this section by noticing that, whatever the relevant abelian projection,
the problem is always reduced to detect dual superconductivity of a \( U(1) \) system.

3. Detecting dual superconductivity in \( U(1) \) gauge theory

I will sketch the construction of the creation operator for a monopole, whose
v.e.v. will be used as a disorder parameter for dual superconductivity. Let \( \Pi_i(x, t) = F_{0i}(x, t) \) be the usual conjugate momenta to the field variables \( A_i(x, t) \). The operator
\[ \mu(y, t) = \exp \left( i \int d^3x \Pi(x, t) \frac{1}{e} b(x - y) \right) \]
(12)
creates a monopole of magnetic charge \( m \) in the site \( y \) at time \( t \), if \( \frac{1}{e} b(x - y) \) is the classical vector potential produced by such a monopole, with the Dirac string subtracted. Putting the string along the direction \( n \)

\[
b(r) = \frac{m}{2} \frac{n \wedge r}{r(r - nr)}
\]  

(13)

Indeed \( \mu \) as defined by Eq.(12) is the operator which adds to any field configuration the field of the monopole, in the same ad the translation operator adds \( a \) to the position \( q \):

\[
\mu(y, t) \text{ carries magnetic charge } m. \quad \text{By use of the canonical commutation relation }\lbrack \Pi_i(x, t, A_j(y, t)) = -i \delta_{ij} \delta^3(x - y), \text{ and of the definition of the magnetic charge operator}
\]

\[
Q = \int d^3 x \nabla \cdot H = \int d^3 x \nabla \cdot (\nabla \wedge A) \quad (14)
\]

\[
[Q, \mu(y, t)] = \int d^3 x \frac{1}{e} \nabla \cdot (\nabla \wedge b) \mu(y, t) = \frac{2\pi m}{e} \mu(y, t)
\]  

(15)

We will use the v.e.v. \( \langle \mu \rangle \) as disorder parameter for dual superconductivity. Our construction is inspired by the classical work of ref.21 and by its application to monopole condensation of ref.22. In ref.22 condensation of monopoles is proved, in the infinite volume limit, for a specific form of the action, the Villain action. Our construction coincides with ref.22 for that case, but can be used for any form of the action, and for finite volumes. The infinite volume limit can be reached by finite size analysis. I refer to ref.12 for the details of the construction which I will summarize as follows.

i) \( \langle \mu \rangle \) can be determined either by the cluster property from the correlation of a monopole and an antimonopole at large distance \( d \)

\[
\langle \mu(d, 0) \bar{\mu}(0, 0) \rangle \bigg|_{d \to \infty} \simeq \langle \mu \rangle^2
\]  

(16)

or directly. It is known that, for Wilson action on lattice, electric charge is confined for \( \beta < \beta_c \) (\( \beta = 1/e^2, \beta_c \simeq 1.01 \)); for \( \beta > \beta_c \) the system is made of free photons. We expect \( \langle \mu \rangle_{V \to \infty} \neq 0 \) for \( \beta < \beta_c \) and \( \langle \mu \rangle_{V \to \infty} = 0 \) for \( \beta > \beta_c \). Of course \( \langle \mu \rangle \) being an analytic function of \( \beta \) for finite volume, it can be identically zero for \( \beta > \beta_c \) only in the thermodynamic limit \( V \to \infty \).

ii) Instead of \( \langle \mu \bar{\mu} \rangle \) itself it proves convenient to use the quantity

\[
\rho = \frac{1}{2} \ln \langle \mu(d, 0) \bar{\mu}(0, 0) \rangle
\]  

(17)

\( \rho \) has less fluctuations than \( \langle \mu \rangle \) itself, and is independent on the boundary conditions.
iii) If $\langle \mu \rangle$ tends to zero as $\beta \to \beta_c$

$$\langle \mu \rangle \simeq (\beta - \beta_c)^\delta$$  \hspace{1cm} (18)

then, from the definition Eq.(17)

$$\rho \simeq \frac{\delta}{\beta - \beta_c}$$ \hspace{1cm} (19)

Eq.(19) can be translated in terms of correlation length $\xi$ and of the critical index $\nu$ by use of the relation

$$\xi^{-1} \simeq (\beta - \beta_c)^\nu$$ \hspace{1cm} (20)

If $\xi \gg a$ ($a =$ lattice spacing) and $L \gg a$, then $\langle \mu \rangle$ is approximatively independent of $a$ (finite size scaling):

$$\langle \mu \rangle \simeq L^{-\delta/\nu} \Phi\left(\frac{L}{\xi}\right)$$ \hspace{1cm} (21)

$\Phi$ is an analytic function at finite volume, and Eq.(21) tends to Eq.(18) as $V \to \infty$. The Eq.(21) implies

$$\rho L^{-1/\nu} = f((\beta - \beta_c)L^{1/\nu})$$ \hspace{1cm} (22)

For lattices of different size $L$ the quantity $\rho L^{-1/\nu}$ must be a universal function of the scaled variable $(\beta - \beta_c)L^{1/\nu}$. The limit $L \to \infty$ is thus extracted and the exponents $\delta$ and $\nu$ can be determined. Typical data for $\rho(d)$ are shown in fig.1.
The scaling (Eq.(22)) is demonstrated in fig.2, where $L^{1/\nu} \rho^{-1}$ is plotted versus $L^{1/\nu}(\beta_c - \beta)$ for different lattice sizes. Data for periodic b.c. are well described by

$$\langle \mu \rangle \simeq L^{-\delta/\nu} \left[ (\beta_c - \beta)L^{1/\nu} + v_0 \right]^{\Delta/2}$$

(23)

A best fit gives $\delta = 2.0 \pm .2$, $\beta_c = 1.0111(1)$, $1/\nu = 3.97 \pm .40$, $v_0 \sim v_1 \sim 1$. For $\beta < \beta_c$ vacuum is a dual superconductor.

4. Dual superconductivity in $SU(2)$ gauge theory

We have applied the construction described in sect.3 to detect dual superconductivity in $SU(2)$ gauge theory. We have probed condensation of the monopoles defined by two different abelian projections:

(a) The abelian projection defined by diagonalizing as effective Higgs field $\hat{\Phi}$ the Polyakov line.

(b) The abelian projection defined by diagonalizing a component (say $F_{12}$) of the field strength $F_{0i}$.

For the projection (a) the relevant abelian field strength $F_{0i}$ (Eq.(8)) is simply $F_{0i} = \hat{\Phi}^a G_{0i}^a$, since $D_0 \hat{\Phi} = 0$. The operator $\mu$ [Eq.(12)] is constructed in terms of $\Pi_i = F_{0i}$ and the analysis of the $U(1)$ model is repeated. A typical behaviour is shown in Fig.3, where $\rho$ is plotted vs $\beta$.

Fig.3

$$L^{1/\nu} \rho \text{ vs } (\beta/\beta_c - 1)L^{1/\nu}$$

Fig.4
The simulation is done on asymmetric lattices \( N_t = 4, 6, N_s = 16, 20 \). A clear signal is visible at the deconfining temperature. A finite size analysis confirms that condensation survives the limit \( V \to \infty \) (fig.4). The best fit gives:

\[
\nu \simeq 0.65 \quad \delta = 1.3 \pm 0.1 \quad \Delta \beta_c \equiv \beta_c(N_T = 6) - \beta_c(N_T = 4) = 0.048 \pm 0.002
\]

to be compared to \( \Delta \beta_c = 0.07 \) predicted by two loop asymptotic scaling. For the abelian projection (b) no signal is observed. There is no correlation between the condensation of monopoles defined by this projection and deconfinement.

5. Concluding remarks

(i) We have demonstrated that the abelian projection which diagonalizes the Polyakov line defines monopoles condensing in QCD vacuum. The dual \( U(1) \) corresponding to their charge is spontaneously broken and the QCD vacuum is a dual superconductor. Recent observations that the abelian string tension in this projection is almost equal the usual string tension support our conclusion.

(ii) Most of the work done in the literature on the role of monopoles in confinement consists in correlating confinement to the density of monopoles or of monopoles world lines, as suggested by the pioneering work of ref. on \( U(1) \): a good review is contained in ref. Of course the density of monopoles is not a disorder parameter for dual superconductivity, in the sense described in sect. 1, in the same way as the density of electrons or of Cooper pairs is not for ordinary superconductors. In fact the density of monopoles, contrary to \( \mu \) (Eq. (15)), commutes with the monopole charge \( M \), and cannot signal condensation.

(iii) Most of the work done in the literature has been done with the so called “maximal abelian” projection. The monopoles defined by this projection seem to be relevant to confinement, as evidenced also by the detection of persistent currents. The maximal abelian gauge presents less lattice artifacts than others. We plan to investigate also this projection by our method: a problem with computing power comes from the fact that the gauge is defined by a maximization which has to be repeated at each updating step in the computation of \( \rho \).

In conclusion we have produced conclusive and direct evidence that

(i) QCD vacuum is a dual superconductor.

(ii) not all the abelian projections are equally good to define the monopoles relevant to confinement.
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