Radiative Penguin Decays at $e^+e^-$ Colliders

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In this review, the most recent results of the radiative decays $B \to X_s \gamma$, $B \to K^{(*)}\ell^+\ell^-$ and $B \to \pi/\eta\ell^+\ell^-$ at $e^+e^-$ colliders are discussed. The new, most precise $CP$ asymmetry measurements in $B \to X_s \gamma$ from BABAR are presented together with branching fractions and photon energy moments. For $B \to K^{(*)}\ell^+\ell^-$ modes, $B$ factory results on partial branching fractions, rate asymmetries and angular observables are combined with measurements from CDF and the LHC experiments. The first branching fraction upper limits for $B \to \eta\ell^+\ell^-$ are shown along with updated upper limits of $B \to \pi\ell^+\ell^-$ branching fractions.

PRESENTED AT

Flavor Physics and $CP$ Violation
Buzios, Rio, Brasil, May 19–24, 2013

\footnote{Work supported by the Norwegian Research Council.}
1 Introduction

The decays $B \to X_s (d) \gamma$ and $B \to X_s (d) \ell^+ \ell^-$, where $\ell^+ \ell^-$ is $e^+ e^-$ or $\mu^+ \mu^-$, are flavor-changing neutral-current processes that are forbidden in the Standard Model (SM) at tree level. They occur in higher-order processes and are described by an effective Hamiltonian that factorizes short-distance contributions in terms of scale-dependent Wilson coefficients $C_i (\mu)$ from long-distance effects expressed by local four-fermion operators $O_i$ that define hadronic matrix elements,

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_i C_i (\mu) O_i.$$  \hfill (1)

While Wilson coefficients are calculable perturbatively, the calculation of the hadronic matrix elements requires non-perturbative methods such as the heavy quark expansion [1, 2, 3, 4].

Figure 1 shows the lowest order diagrams. In the $B \to X_s (d) \gamma$ decay, the electromagnetic penguin loop dominates. The short-distance part is expressed by the effective Wilson coefficient $C^\text{eff}_7$. Through operator mixing at higher orders, the chromo-magnetic penguin enters whose short distance part is parameterized by $C^\text{eff}_8$. In $B \to X_s (d) \ell^+ \ell^-$ modes, the Z penguin and the $WW$ box diagram contribute in addition. Their short-distance parts are parametrized in terms of $C^\text{eff}_{9}$ (vector current part) and $C^\text{eff}_{10}$ (axial-vector current part). Physics beyond the SM introduces new loops and box diagrams with new particles (e.g. charged Higgs boson, supersymmetric particles) as shown in Fig. 2. Such contributions modify the Wilson coefficients and may introduce new diagrams with scalar and pseudoscalar current interactions and in turn new Wilson coefficients, $C_S$ and $C_P$. To determine $C^\text{eff}_7$, $C^\text{eff}_8$, $C^\text{eff}_9$, and $C^\text{eff}_{10}$ precisely, we need to measure many observables in several radiative decays. These rare decays can potentially probe new physics at a scale of a few TeV.

![Figure 1: Lowest-order diagrams for $B \to X_s (d) \gamma$ (left) and $B \to X_s (d) \ell^+ \ell^-$ (middle, right).](image-url)
fractions, photon energy spectra, photon energy moments and CP asymmetries from $e^+e^-$ colliders for fully inclusive and semi-inclusive $B \to X_s\gamma$ analyses. We determine the $b$ quark mass $m_b$ and its kinetic energy $\mu_b^2$ in the kinetic and shape function models. We review the status of branching fractions, rate asymmetries and angular observables for $B \to K^{(*)}\ell^+\ell^-$ modes. Finally, we present a new BABAR search for $B \to \pi\ell^+\ell^-$ modes and a first search for $B \to \eta\ell^+\ell^-$ modes. BABAR performs all analyses blinded.

2 Study of $B \to X_s\gamma$

In the SM, the $B \to X_s\gamma$ branching fraction is calculated at next-to-next-to-leading order (up to four loops) yielding $\mathcal{B}(B \to X_s\gamma) = (3.14 \pm 0.22) \times 10^{-4}$ for photon energies $E_\gamma^* > 1.6$ GeV \cite{5,6,7}. For larger minimum values of $E_\gamma^*$, the prediction depends on the shape of the $E_\gamma^*$ spectrum, which is modeled in terms of a shape function that depends on the Fermi motion of the $b$ quark inside the $B$ meson and thus on the $b$ quark mass. Since the shape function is expected to be similar to that used to determine the lepton-energy spectrum in $B \to X_u\ell\nu$, precision measurements of the $E_\gamma^*$ spectrum are helpful for the determination of $V_{ub}$. The measurement of $\mathcal{B}(B \to X_s\gamma)$ provides constraints on the charged Higgs mass $m_{H^\pm}$.

Experimentally, the challenge is to extract the $E_\gamma^*$ signal from photon background copiously produced in $\pi^0$ decays in $q\bar{q}$ continuum\cite{9} and $B\bar{B}$ processes that increases exponentially with smaller photon energy. We use three different strategies to suppress these backgrounds: i) an inclusive analysis with a lepton tag, ii) a semi-inclusive analysis and iii) an inclusive analysis with a fully reconstructed $B$ meson. Herein, we present results of the first two strategies.

2.1 Fully Inclusive $B \to X_s\gamma$ Analysis

Using a sample of $384 \times 10^6$ $B\bar{B}$ events, BABAR measured total and partial branching fractions, photon energy moments and the $B \to X_{s+d}\gamma$ CP asymmetry in a fully inclusive analysis \cite{8,9}. To suppress $e^+e^- \to q\bar{q}$ continuum and $B\bar{B}$ backgrounds, we

\*q refers to $u,d,s$ and $c$
tag the recoiling $B$ meson in semileptonic decays and use optimized $\pi^0$ and $\eta$ vetoes, missing energy requirements and the output of two neural networks (NN). For a signal efficiency of 2.5%, the efficiency for accepting continuum ($B\bar{B}$) background is reduced to $5 \times 10^{-6}$ ($1.3 \times 10^{-4}$). We estimate the residual continuum background by studying data taken 40 MeV below the $\Upsilon(4S)$ peak. Figure 3 (left) shows the $B \to X_s\gamma$ partial branching fraction after background subtraction and corrections for efficiency, resolution effects and Doppler smearing. For comparison, we show the predicted $E^*_\gamma$ spectrum in the kinetic scheme [10, 11] using HFAG world averages [12] for the shape function parameters. For $E^*_\gamma > 1.8$ GeV, BABAR measures a total branching fraction of $\mathcal{B}(B \to X_S\gamma) = (3.21 \pm 0.15_{\text{stat}} \pm 0.29_{\text{sys}} \pm 0.08_{\text{model}}) \times 10^{-4}$, where uncertainties are statistical, systematic and from model dependence, respectively. This is in good agreement with previous measurements [13, 14, 15]. After extrapolation to $E_\gamma > 1.6$ GeV, the branching fraction increases to $\mathcal{B}(B \to X_S\gamma) = (3.31 \pm 0.16_{\text{stat}} \pm 0.30_{\text{sys}} \pm 0.09_{\text{model}}) \times 10^{-4}$, which is still in good agreement with the SM prediction. We use this result to constrain new physics in the type II two-Higgs doublet model [5, 16, 17] excluding $m_{H^\pm} < 327 \text{ GeV}/c^2$ at 95% confidence level (CL) independent of $\tan \beta$. Recent BABAR results on $\mathcal{B}(B \to D^{(*)}\tau\nu)$, however, are in conflict with both the SM and the type II Higgs doublet model at the $3\sigma$ level [18, 19].

For $E^*_\gamma > 1.8$ GeV, BABAR measured energy moments of $\langle E_\gamma \rangle = (2.267 \pm 0.019_{\text{stat}} \pm 0.032_{\text{sys}} \pm 0.003_{\text{mod}})$ GeV and $\langle (E_\gamma - \langle E_\gamma \rangle)^2 \rangle = (0.0484 \pm 0.0053_{\text{stat}} \pm 0.0077_{\text{sys}} \pm 0.0005_{\text{mod}})$ GeV$^2$ that are consistent with previous results [13, 14, 15], where uncertainties are statistical, systematic and from model dependence, respectively.

Figure 3: Partial branching fraction versus $E^*_\gamma$ measured in a fully inclusive analysis (left) and for the sum of exclusive modes (right). Error bars (left) show statistical and total uncertainties. The solid curve shows a prediction for the kinetic scheme with HFAG averages [12]. The vertical bar separates signal from the control region. Errors bars (right) show total uncertainties.
Table 1: Determination of $m_b$ and $\mu_b^2$ in the kinetic-scheme [10] and shape function scheme [21] using the semi-inclusive analysis in comparison to the world average [12].

|                | BABAR kinetic scheme | BABAR shape function scheme | world average kinetic scheme | world average shape function scheme |
|----------------|----------------------|-----------------------------|------------------------------|-------------------------------------|
| $m_b$ [GeV/c$^2$] | $4.568^{+0.038}_{-0.036}$ | $4.579^{+0.032}_{-0.029}$ | $4.560 \pm 0.023$ | $4.588 \pm 0.025$ |
| $\mu_b^2$ [GeV$^2$] | $0.450 \pm 0.054$ | $0.257^{+0.034}_{-0.039}$ | $0.453 \pm 0.036$ | $0.189^{+0.046}_{-0.057}$ |

2.2 Semi-Inclusive $B \to X_s \gamma$ Analysis

Using 471 $B\bar{B}$ events in a semi-inclusive analysis, we combine 38 exclusive $B \to X_s \gamma$ final states containing a $K^+\pi^0$ or $K^0\pi^0$ and up to four pions with at most two $\pi^0$s, $K^+K^-$ with up to one pion, or up to one $\eta$ with up to two pions [20]. We reconstruct the hadronic mass $m_{X_s}$ in 100 MeV/c$^2$ bins and calculate the photon energy by $E_{\gamma} = \frac{m_B^2 - m_{X_s}^2}{2m_B}$. Figure 3 (right) shows the partial branching fraction versus $E_{\gamma}^*$. Summing the partial branching fraction over all $m_{X_s}$ bins yields $B(B \to X_s\gamma) = (3.29 \pm 0.19_{\text{stat}} \pm 0.48_{\text{sys}}) \times 10^{-4}$ for $E_{\gamma} > 1.9$ GeV, which is in good agreement with the results of the inclusive analysis. We also measure the mean and variance of the photon energy spectrum, $\langle E_{\gamma} \rangle = (2.346 \pm 0.018^{+0.027}_{-0.022})$ GeV and $\langle (E_{\gamma} - \langle E_{\gamma} \rangle)^2 \rangle = (0.0211 \pm 0.0057^{+0.0055}_{-0.0069})$ GeV$^2$ for $E_{\gamma} > 1.9$ GeV. These results agree with the measurements of the inclusive analysis after increasing the minimum $E_{\gamma}^*$ selection to 1.9 GeV. Note that $\langle E_{\gamma} \rangle$ ($\langle (E_{\gamma} - \langle E_{\gamma} \rangle)^2 \rangle$) increases (decreases) with a larger minimum $E_{\gamma}^*$ selection.

From a fit to the photon energy spectrum, we can extract the $b$ quark mass and its kinetic energy. Table 1 summarizes the results of $m_b$ and $\mu_b^2$ for fits to the $E_{\gamma}^*$ spectrum in the kinetic scheme [10] and shape function scheme [21].

Figure 4 shows a comparison of all $B \to X_s \gamma$ total branching fraction measurements after extrapolating them to a $E_{\gamma}^* > 1.6$ GeV selection. In addition, the HFAG average [12] and the SM prediction [7] are depicted. All $B(B \to X_s\gamma)$ measurements are in good agreement with each other and with the SM prediction.

2.3 Direct $CP$ Asymmetry

For the sum of exclusive modes, the direct $CP$ asymmetry is defined by

$$A_{\text{CP}}(B \to X_s\gamma) = \frac{\mathcal{B}(B \to \bar{X}_s\gamma) - \mathcal{B}(B \to X_s\gamma)}{\mathcal{B}(B \to \bar{X}_s\gamma) + \mathcal{B}(B \to X_s\gamma).}$$ (2)

The present world average of $A_{\text{CP}} = (-0.8 \pm 2.9)\%$ is in good agreement with the SM prediction of $-0.6% < A_{\text{CP}} < 2.8\%$ at 95% CL [21]. The presently large

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†Charge conjugation is implied throughout the article unless stated otherwise.
uncertainties still allow for new physics contributions, which modify $C_7^{\text{eff}}$. Particularly, the $CP$ asymmetry difference between $B^+$ and $B^0$ decays, $\Delta A_{CP}(B \to X_s^+\gamma) = A_{CP}(B^+ \to X_s^+\gamma) - A_{CP}(B^0 \to X_s^0\gamma)$, is very sensitive to new physics since it is caused by interference between the electromagnetic and the chromo-magnetic penguin diagrams in which the latter enters through higher-order corrections. Calculations yield [24]

$$\Delta A_{CP}(B \to X_s^+\gamma) \simeq 4\pi^2\alpha_s \frac{\overline{\Lambda}_{78}}{m_b} \Im \frac{C_8^{\text{eff}}}{C_7^{\text{eff}}} \simeq 0.12 \frac{\overline{\Lambda}_{78}}{100 \text{ MeV}} \Im \frac{C_8^{\text{eff}}}{C_7^{\text{eff}}},$$

where $\overline{\Lambda}_{78}$ is the hadronic matrix element of the $O_7 - O_8$ interference, predicted to lie in the range $17 \text{ MeV} < \overline{\Lambda}_{78} < 190 \text{ MeV}$. In the SM, $\Delta A_{CP}(B \to X_s^+\gamma)$ vanishes since $C_7^{\text{eff}}$ and $C_8^{\text{eff}}$ are real.

In a sample of 471 $B\bar{B}$ events, BABAR studied $A_{CP}$ and $\Delta A_{CP}$ in a semi-inclusive analysis using ten $B^+$ and six $B^0$ exclusive final states. We maximize the signal

$^1B^+ \to K_S^0\pi^+\gamma, K^+\pi^0\gamma, K^+\pi^+\pi^-\gamma, K_S^0\pi^-\pi^0\gamma, K^+\pi^0\pi^0\gamma, K^+\pi^+\pi^-\pi^0, K^+\pi^+\pi^-\pi^0, K_S^0\pi^+\pi^-\pi^0, K^+\pi^+\pi^-\pi^0, K^+\pi^+\pi^-\pi^0, K^+\pi^+\pi^-\pi^0, K^+\pi^+\pi^-\pi^0, K^+\pi^+\pi^-\pi^0, K^+\pi^+\pi^-\pi^0, K^+\pi^+\pi^-\pi^0$.

Figure 4: Comparison of $B(B \to X_s^+\gamma)$ measurements from BABAR [8, 20, 22], Belle [14, 23] and CLEO [15] to the SM prediction [7] after extrapolation to $E^*_\gamma > 1.6 \text{ GeV}$. 

$^{\text{‡}}$ We maximize the signal
extraction using a bagged decision tree with six input variables. This improves the efficiency considerably with respect to the standard $\Delta E = E^*_B - E^*_{\text{beam}}$ selection, where $E^*_B$ and $E^*_{\text{beam}}$ are the beam energy and $B$ meson energy in the center-of-mass frame, respectively. To remove continuum background, we train a separate bagged decision tree using event shape variables. We perform an $X_s$ mass-dependent optimization with loosely identified pions and kaons using the sensitivity $S/\sqrt{S+B}$ where $E^*_{\text{beam}}$ and $E^*_{\text{beam}}$ are the beam energy and $B$ meson energy in the center-of-mass frame, respectively. To remove continuum background, we train a separate bagged decision tree using event shape variables. We perform an $X_s$ mass-dependent optimization with loosely identified pions and kaons using the sensitivity $S/\sqrt{S+B}$ where $E^*_{\text{beam}}$ and $E^*_{\text{beam}}$ are the beam energy and $B$ meson energy in the center-of-mass frame, respectively.

To extract $A_{CP}$, we fit the beam energy-constrained mass $m_{ES} = \sqrt{E^*_{\text{beam}}^2 - p_B^2}$ simultaneously for $B$-tagged and $B$-tagged samples. After correcting the raw $A_{CP}$ for detector bias determined from the $m_{ES}$ sideband below the signal region, we measure $A_{CP}(B \rightarrow X_s \gamma) = (1.73 \pm 1.93_{\text{stat}} \pm 1.02_{\text{sys}})\%$, which agrees well with the SM prediction. This new measurement has the smallest uncertainty. From a simultaneous fit to $B^+$ and $B^0$ samples, we measure $\Delta A_{CP}(B \rightarrow X_s \gamma) = (4.97 \pm 3.90_{\text{stat}} \pm 1.45_{\text{sys}})\%$ from which we obtain the constraint $-1.64 < \mathcal{I} m(C^{eff}_8/C^{eff}_7) < 6.52$ at 90% CL. Note, this is the first $\Delta A_{CP}$ measurement and first constraint on $\mathcal{I} m(C^{eff}_8/C^{eff}_7)$.

Figure 5: The $\Delta \chi^2$ function versus $\mathcal{I} m(C^{eff}_8/C^{eff}_7)$ (left) and the dependence of $\bar{\Lambda}_{\bar{7}_8}$ on $\mathcal{I} m(C^{eff}_8/C^{eff}_7)$ (right). The blue dark-shaded (orange light-shaded) regions show the 68% (90%) CL intervals.

Figure 5 (left) show the $\Delta \chi^2$ of the simultaneous fit as a function of $\mathcal{I} m(C^{eff}_8/C^{eff}_7)$. Figure 5 (right) shows the constraints of $\bar{\Lambda}_{\bar{7}_8}$ as a function of $\mathcal{I} m(C^{eff}_8/C^{eff}_7)$. The shape of $\Delta \chi^2$ as a function of $\mathcal{I} m(C^{eff}_8/C^{eff}_7)$ is not parabolic indicating that the likelihood has a non-Gaussian shape. The reason is that $\Delta \chi^2$ is determined from all $^5p_B^2$ is the $B$ momentum in the center-of-mass frame.
possible values of $\Lambda_{78}$. In the region $0.2 < \Im(C_8^{eff}/C_7^{eff}) < 2.6$ a change in $\Im(C_8^{eff}/C_7^{eff})$ $\Delta \chi^2$ can be compensated by a change in $\Lambda_{78}$ leaving $\Delta \chi^2$ unchanged. For positive values larger (smaller) than 2.6 (0.2), $\Delta \chi^2$ increases slowly (rapidly), since $\Lambda_{78}$ remains nearly constant at the minimum value (increases rapidly). For negative $\Im(C_8^{eff}/C_7^{eff})$ values, $\Lambda_{78}$ starts to decrease again, which leads to a change in $\Delta \chi^2$ shape.

In the fully inclusive analysis, $A_{CP}$ involves contributions from $B \rightarrow X_s \gamma$ and $B \rightarrow X_d \gamma$ that cannot be separated on an event-by-event basis. Therefore, we define $A_{CP}$ here as

$$A_{CP}(B \rightarrow X_{s+d}\gamma) \equiv \frac{\mathcal{B}(\overline{B} \rightarrow X_{s+d}\gamma) - \mathcal{B}(\overline{B} \rightarrow X_{s}\gamma)}{\mathcal{B}(\overline{B} \rightarrow X_{s+d}\gamma) + \mathcal{B}(\overline{B} \rightarrow X_{s}\gamma)}.$$ (4)

We tag the $B$ flavor by the lepton charge. Using a sample $384 \times 10^6 \overline{B}B$ events, we measure $A_{CP}(B \rightarrow X_{s+d}\gamma) = 0.057 \pm 0.06_{\text{stat}} \pm 0.018_{\text{sys}}$ after correcting for charge bias and mistagging [8]. Figure 6 shows all $A_{CP}$ measurements from BABAR [8, 20, 22], Belle [25] and CLEO [15]. They all agree well with the SM prediction [27, 28].

![Figure 6: Summary of $A_{CP}$ measurements for $B \rightarrow X_{s+d}\gamma$ from semi-inclusive analyses (BABAR preliminary, Belle [25]) and for $B \rightarrow X_s\gamma$ from fully inclusive analyses (BABAR [8, 22] and CLEO [26]) in comparison to the SM prediction for $B \rightarrow X_s\gamma$ [24].](image-url)
Using 471 (657) $\times 10^6$ $B\bar{B}$ events, BABAR (Belle) reconstructs eight (ten) $B \rightarrow K^{(*)}\ell^+\ell^-$ final states consisting of $K^+, K^0_S, K^{+}\pi^-, K^0_S\pi^+, (K^+\pi^0)$ recoiling against $e^+e^-$ or $\mu^+\mu^-$. BABAR (Belle) selects $e^\pm$ with momenta $p_{e}>0.3$ (0.4) GeV/c. Both experiments select muons with $p_{\mu}>0.7$ GeV/c, require good particle identification for $e^\pm, \mu^\pm, \pi^\pm$ and $K^\pm$ and reconstruct $K^0_S$ in the $\pi^+\pi^-$ final state. To suppress combinatorial $q\bar{q}$ and $B\bar{B}$ backgrounds, BABAR uses eight boosted decision trees (BDT)\(^\dagger\) while Belle uses likelihood ratios. Both experiments select signal with $m_{ES}$ and $\Delta E$ and veto the $J/\psi$ and $\psi(2S)$ mass regions. The vetoed $J/\psi$ and $\psi(2S)$ samples and generated pseudo experiments are used to check the performance of the selection. To extract signal yields, both experiments perform one-dimensional fits of the $m_{ES}$ distributions for $B \rightarrow K\ell^+\ell^-$ modes and two-dimensional fits of the $m_{ES}$ and $m_{K\pi}$ mass distributions for $B \rightarrow K^*\ell^+\ell^-$ modes.

![Figure 7](image_url)  

Figure 7: $dB/ds$ measurements for $B \rightarrow K\ell^+\ell^-$ (left) and $B \rightarrow K^*\ell^+\ell^-$ (right) from BABAR [29] (red squares), Belle [30] (green triangles) and a naive world average (black points) that is dominated by LHCb in comparison to the SM predictions [38] (grey curves). Vertical bands show the $J/\psi$ and $\psi(2S)$ vetoed regions.

### 3.1 $B \rightarrow K^{(*)}\ell^+\ell^-$ Rates and Rate Asymmetries

BABAR [29] and Belle [30] measured total and partial branching fractions $dB(B \rightarrow K^{(*)}\ell^+\ell^-)/ds$ in six $s=q^2=m^2_{\ell^+\ell^-}$ bins\(^\dagger\). Figure 7 (left) shows the BABAR and Belle $dB(B \rightarrow K\ell^+\ell^-)/ds$ measurements in comparison to a naive average that includes the $B \rightarrow K\ell^+\ell^-$ modes from BABAR [29] and Belle [30] and $B \rightarrow K\mu^+\mu^-$ modes from CDF [31] and LHCb [32]. Figure 7 (right) shows the corresponding $dB(B \rightarrow$\(\end{input}

\(^\dagger\)two BDTs are used to separate signal from $B\bar{B}$ and $q\bar{q}$ backgrounds, separately for $e^+e^-$ and $\mu^+\mu^-$ modes and separately for $s$ below and above the $J/\psi$ mass.

\(^\parallel\) $q$ is the momentum transfer and $m_{\ell^+\ell^-}$ is the dilepton mass.
\(K^*\ell^+\ell^-)/ds\) results. The average is calculated using \(B \to K^*\ell^+\ell^-\) modes from BABAR \cite{29} and Belle \cite{30}, \(B \to K^*\mu^+\mu^-\) modes from CDF \cite{31} and LHCb \cite{32,33} as well as the \(B \to K^{\ast 0}\mu^+\mu^-\) mode from CMS \cite{35}. Note that the average values are dominated by the LHCb result. All measurements agree well with the SM predictions that are calculated for low and high values of \(s\) \cite{36,37,38}. For low \(s\), the hadronic recoil is large and the \(K^\ast\) energy is much larger than the QCD scale \(\Lambda\) \((E_{K^\ast} \gg \Lambda)\). This region represents the perturbative regime in which QCD factorization yields reliable results \cite{39,40}. For high \(s \sim \mathcal{O}(m_b)\), the hadronic recoil becomes small and \(E_{K^\ast} \sim \Lambda\). This is the non-perturbative regime in which an operator product expansion in powers of \(1/m_b\) yields reliable results. The large uncertainties in the SM predictions result from the uncertainties in calculating the form factors of the hadronic matrix elements \cite{41}.

Figure 8: Branching fraction measurements for \(B \to K\ell^+\ell^-\) and \(B \to K^*\ell^+\ell^-\) in the low \(s\) region \((1 < s < 6 \text{ GeV}/c^2)\) from BABAR \cite{29}, Belle \cite{30}, CDF \cite{31}, LHCb \cite{32,34} and CMS \cite{35} in comparison to the SM predictions \cite{38}.

Figure 8 shows all \(B \to K\ell^+\ell^-\) and \(B \to K^*\ell^+\ell^-\) branching fractions measured in the low \(s\) region, \(1 < s < 6 \text{ GeV}/c^2\), in comparison to SM predictions \cite{38}. Figure 9 shows \(B \to K^{\ast 0}\ell^+\ell^-\) total branching fraction measurements in comparison to SM
predictions [42, 43]. All measurements show better agreement with the Ali model [42]. BABAR measured total branching fractions of $\mathcal{B}(B \rightarrow K\ell^+\ell^-) = (4.7 \pm 0.6_{\text{stat}} \pm 0.2_{\text{sys}}) \times 10^{-7}$ and $\mathcal{B}(B \rightarrow K^*\ell^+\ell^-) = (10.2^{+1.4}_{-1.3} \text{ stat} \pm 0.5 \text{ sys}) \times 10^{-7}$. Table 2 summarizes the BABAR total branching fraction and rate asymmetry measurements.

The isospin asymmetry is defined by

$$\frac{dA_I}{ds} \equiv \frac{d\mathcal{B}(B^0 \rightarrow K^{(*)0}\ell^+\ell^-)/ds - r_{\tau}d\mathcal{B}(B^+ \rightarrow K^{(*)+}\ell^+\ell^-)/ds}{d\mathcal{B}(B^0 \rightarrow K^{(*)0}\ell^+\ell^-)/ds + r_{\tau}d\mathcal{B}(B^+ \rightarrow K^{(*)+}\ell^+\ell^-)/ds},\tag{5}$$

where $r_{\tau} = \tau_{B^0}/\tau_{B^+}$ accounts for the different $B^0$ and $B^+$ lifetimes. In the SM, $A_I$ is expected to be at the order of $O(1\%)$ [44].
Table 2: BABAR results for $B \to K^{(*)}\ell^+\ell^-$ modes on total branching fractions, $CP$ asymmetries, lepton flavor ratios and isospin asymmetries. The first uncertainty is statistical, the second is systematic.

| Mode               | $B[10^{-7}]$ [GeV$^2$/c$^2$] | $A_{CP}$ [all s] | $R_{K^{(*)}}$ [s $> 0.1$ GeV$^2$/c$^4$] | $A_{I}$ [s $> 0.1$ GeV$^2$/c$^4$] | $0.1 \leq s \leq 8.12$ |
|--------------------|-------------------------------|-------------------|----------------------------------------|-----------------------------------|--------------------------|
| $K\ell^+\ell^-$   | $4.7 \pm 0.6 \pm 0.2$        | $-0.03 \pm 0.14 \pm 0.01$ | $1.00^{+0.31}_{-0.25} \pm 0.07$       | $-0.58^{+0.29}_{-0.37} \pm 0.02$ |
| $K^*\ell^+\ell^-$ | $10.2^{+1.4}_{-1.3} \pm 0.05$ | $0.03 \pm 0.13 \pm 0.01$ | $1.13^{+0.34}_{-0.26} \pm 0.10$       | $-0.25^{+0.17}_{-0.20} \pm 0.03$ |

Figure 10 shows BABAR and Belle isospin asymmetry measurements in six $s$ bins for $B \to K\ell^+\ell^-$ modes (left) and $B \to K^*\ell^+\ell^-$ modes (right) in comparison to a naive average over all experiments (BABAR [20], Belle [30], CDF [31], and LHCb [45]). The average points are dominated again by LHCb. At low $s$ ($1 < s < 6$ GeV$^2$/c$^2$), the naive average yields $A_{I}^{low}(B \to K\ell^+\ell^-) = -0.31 \pm 0.12$ and $A_{I}^{low}(B \to K^*\ell^+\ell^-) = -0.15 \pm 0.11$. For $B \to K\ell^+\ell^-$, consistency with the SM is at the $\sim 2.6\sigma$ level. For other $s$ values and for $B \to K^*\ell^+\ell^-$, the averaged data agree well with the SM prediction [44]. The BABAR measurements are listed in Table 2.

The $CP$ asymmetry is defined by

$$ A_{CP} = \frac{B(B \to \overline{K}^{(*)}\ell^+\ell^-) - B(B \to K^{(*)}\ell^+\ell^-)}{B(B \to \overline{K}^{(*)}\ell^+\ell^-) + B(B \to K^{(*)}\ell^+\ell^-)} . \quad (6) $$

In the SM, the $CP$ asymmetry is expected to be small, $A_{CP} = -0.01$ [47, 48]. The measurements from BABAR [8] (see Table 2), Belle [14] and LHCb [46] agree well with the SM prediction.

The lepton flavor ratios are defined by

$$ R_{K^{(*)}} = \frac{B(B \to K^{(*)}\mu^+\mu^-)}{B(B \to K^{(*)}e^+e^-)} . \quad (7) $$

In the SM for $s > 4m_{\mu}^2$, $R_{K^{(*)}}^{meas} \equiv 1$ [30]. For $s > 0.1$ GeV$^2$/c$^4$, BABAR [8] (see Table 2) and Belle [14] measure lepton flavor ratios that are consistent with unity and thus agree well with the SM prediction.

Except for $A_{I}(B \to K\ell^+\ell^-)$ at low $s$, all other measurements of branching fractions and rate asymmetries are in good agreement with the SM predictions.

### 3.2 $B \to K^{(*)}\ell^+\ell^-$ Angular Analyses

The $B \to K^{(*)}\ell^+\ell^-$ decay is characterized by three angles: $\theta_K$ is the angle between the $K$ and $B$ in the $K^*$ rest frame, $\theta_\ell$ is the angle between the $\ell^+$ and the $B$ in the

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**$m_{\mu}$** is the muon mass.
\[ \ell^+\ell^- \text{ rest frame and } \phi \text{ is the angle between the } K^* \text{ and } \ell^+\ell^- \text{ decay planes. The one-dimensional } \cos \theta_K \text{ and } \cos \theta_\ell \text{ projections depend on the } K^* \text{ longitudinal polarization } F_L \text{ and the lepton forward-backward asymmetry } A_{FB} \ [50, 51] \]

\[
W(\cos \theta_K) = \frac{3}{2} F_L \cos^2 \theta_K + \frac{3}{4} (1 - F_L) \sin^2 \theta_K, \\
W(\cos \theta_\ell) = \frac{3}{4} F_L \sin^2 \theta_\ell + \frac{3}{8} (1 - F_L)(1 + \cos^2 \theta_\ell) + A_{FB} \cos \theta_\ell. \tag{8}
\]

In the SM, \( A_{FB} \) and \( F_L \) are again calculated separately for the low \( s \) and high \( s \) regions.

Since the number of signal events in each \( s \) bin is small, BABAR and Belle analyze one-dimensional angular distributions. Using 471 \( B\overline{B} \) events, BABAR reconstructs six \( B \to K^*\ell^+\ell^- \) final states with \( K^* \to K^+\pi^-, K^0\pi^+, K^+\pi^0 \). The event selection is similar to that for rate asymmetries. BABAR extracts \( F_L \) and \( A_{FB} \) by performing a profile likelihood scan. Using 657 \( B\overline{B} \) events, Belle performs a fit to the one-dimensional angular distributions.

![Figure 11: BABAR preliminary measurements (red squares) and Belle results (green triangles) for \( A_{FB} \) (left) and \( F_L \) (right) for \( B \to K^*\ell^+\ell^- \) modes in comparison to the naive world average over all experiments (black points) that is dominated by LHCb, the SM prediction (shaded curves) and a model in which the sign of Wilson coefficient \( C_{eff}^7 \) is flipped (blue solid curve). Vertical bands show the \( J/\psi \) and \( \psi(2S) \) vetoed regions.](image)

Figure 11 (left) shows \( A_{FB} \) measurements in six \( s \) bins from BABAR (preliminary) and Belle [30] in comparison to a naive average over the \( B \to K^*\ell^+\ell^- \) results from BABAR and Belle [30], \( B \to K^*\mu^+\mu^- \) results from CDF [31] and \( B \to K^{*0}\mu^+\mu^- \) results from LHCb [34], CMS [35] and ATLAS [49]. The average values are dominated again by the LHCb measurements. In addition, predictions are shown for the SM and for a model in which the sign of Wilson coefficient \( C_{eff}^7 \) is flipped with respect to the expected value in the SM [12, 17, 50, 53]. The large uncertainties in the SM predictions result from uncertainties in the form factor calculations. While the BABAR
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uncertainty is statistical, the second is systematic.

Table 3: BABAR measurements of the lepton forward-backward asymmetry and K∗
longitudinal polarization for B → K∗+ℓ− modes in the low s region. The first
uncertainty is statistical, the second is systematic.

| Mode | s [GeV²/c⁴] | A_{FB} | F_L |
|------|-------------|--------|-----|
| B+ℓ− | s ≤ 6.0    | 1.0    | 1.0 |
| K∗+ℓ− | 0.26±0.27 ± 0.07 | 0.25±0.09 ± 0.03 |

measurements are consistent with both the SM and the flipped-sign C_{eff} model, the
world average values agree well with the SM prediction. In the low s region, the
world average yields A_{FB}(B → K∗+ℓ−) = −0.074^{+0.047}_{-0.048}, which agrees well with
the SM prediction of A_{FB}^{SM}(B → K∗+ℓ−) = −0.0494^{+0.0281}_{-0.0252} (for BABAR results see
Table 3).

Figure 11 (right) shows F_L measurements from BABAR (preliminary) and Belle [30]
in six s bins in comparison to a naive average using B → K∗+ℓ− results from
BABAR and Belle [30], B → K∗μ⁺μ− results from CDF [31] and B → K⁺⁰μ⁺μ−
results from LHCb [34], CMS [32] and ATLAS [40]. The naive average values are
again dominated by the LHCb results. The figure also shows the SM prediction [38]
and the prediction of the flipped-sign C_{eff} model [42, 47, 50]. All results are con-
sistent with the SM prediction, though the BABAR results fit better to the flipped-
sign C_{eff} model. In the low s region (1 < s < 6 GeV²/c⁴), the world average yields
F_L(B → K∗+ℓ−) = 0.523^{±0.047}, which is consistent with the SM prediction of
F_{L}^{SM}(B → K∗+ℓ−) = 0.735^{±0.06} [38, 47, 50, 52, 53] (for BABAR results see Table 3).

4 Search for B → πℓ⁺ℓ− and B → ηℓ⁺ℓ− Decays

In the SM in lowest order, B → X_ℓℓ⁰ modes are also mediated by the electromag-
netic penguin, Z penguin and WW box diagrams. However, they are suppressed by
|V_{td}/V_{ts}|² ∼ 0.04 with respect to the corresponding B → X_ℓ⁺ℓ− modes. In exten-
sions of the SM, rates may increase significantly [54]. Using 471 × 10⁶ B̅B events,
BABAR recently updated the search for B → πℓ⁺ℓ− modes and performed the first
search for B → ηℓ⁺ℓ− modes. The SM predictions lie in the range B(B → πℓ⁺ℓ−) =
(1.96 − 3.30) × 10⁻⁸ and B(B → ηℓ⁺ℓ−) = (2.5 − 3.7) × 10⁻⁸ where the large uncer-
tainties result from uncertainties in the B → π form factor calculations [54, 55, 56]
and from a lack of knowledge of B → η form factors [57].

BABAR fully reconstructs four B → πℓ⁺ℓ− and four B → ηℓ⁺ℓ− final states by
selecting π⁺, π⁰, η → γγ and η → π⁺π⁻π⁰ recoiling against e⁺e⁻ or μ⁺μ⁻ [58].
We select leptons with p_ℓ > 0.3 GeV/c, recover losses due to bremsstrahlung for
e⁺, remove γ → e⁺e⁻ decays and require good particle identification for e±, μ± and
We select photons with $E_\gamma > 50$ MeV and impose a $\pi^0$ mass constraint of $115 < m_{\gamma\gamma} < 150$ MeV/c$^2$ and an $\eta$ mass constraint of $500$ (535) $< m_{\gamma\gamma}$ $(m_{3\pi}) < 575$ (565) MeV/c$^2$. In addition, we require $(E_{1,\gamma} - E_{2,\gamma})/(E_{1,\gamma} + E_{2,\gamma}) < 0.8$ for the $\eta \to \gamma\gamma$ final states to remove asymmetric $q\bar{q}$ background that peaks near one. We veto $J/\psi$ and $\psi(2S)$ mass regions and use four NNs to suppress combinatorial $B\bar{B}$ and $q\bar{q}$ continuum backgrounds, separately for $e^+e^-$ modes and for $\mu^+\mu^-$ modes. The NNs for suppressing $B\bar{B}$ background uses 15 (14) input distributions for $e^+e^-$ ($\mu^+\mu^-$) modes, while those for suppressing $q\bar{q}$ continuum use 16 input distributions for both modes. For validations, we use pseudo-experiments and the vetoed $J/\psi$ and $\psi(2S)$ samples. The selection criteria used by Belle are given in [59].

For $B \to \pi^+\ell^+\ell^-$ and $B \to \pi^0\ell^+\ell^-$, BABAR performs simultaneous unbinned maximum likelihood fits to the $m_{ES}$ and $\Delta E$ distributions for $e^+e^-$ and $\mu^+\mu^-$ modes separately. We include the $B \to K^+\ell^+\ell^-$ mode in the fit to extract the peaking background contribution in the $B \to \pi^+\ell^+\ell^-$ modes by reconstructing the $K^+$ as a $\pi^+$. We use the vetoed $J/\psi$ and $\psi(2S)$ samples to validate the fit and check the peaking $B \to K^+\ell^+\ell^-$ contribution.

For the $B \to \eta\ell^+\ell^-$, we perform simultaneous unbinned maximum likelihood fits to the $m_{ES}$ and $\Delta E$ distributions, again for $e^+e^-$ and $\mu^+\mu^-$ modes separately. We use the vetoed $J/\psi$ and $\psi(2S)$ samples to validate the fits. In addition, we perform fits for the isospin-averaged modes $B \to \pi e^+e^-$ and $B \to \pi \mu^+\mu^-$, lepton-flavor averaged modes $B^+ \to \pi^+\ell^+\ell^-$, $B^0 \to \pi^0\ell^+\ell^-$ and $B^0 \to \eta\ell^+\ell^-$ and both isospin and lepton-flavor averaged modes $B \to \pi\ell^+\ell^-$. Similar to Belle, we see no signals in any of these modes and set branching fraction upper limits at 90% CL. Recently, LHCb observed the $B \to \pi^+\ell^+\ell^-$ decay and measured a branching fraction of $B(B \to \pi^+\ell^+\ell^-) = (2.4 \pm 0.6 \pm 0.1) \times 10^{-8}$ [60]. Figure 12 shows the preliminary BABAR branching fraction upper limits in comparison to those from Belle [59] and the $B \to \pi^+\ell^+\ell^-$ measurement from LHCb [60]. BABAR sets the best branching fraction upper limit for $B^0 \to \pi^0\ell^+\ell^-$ and presents the first results for $B \to \eta\ell^+\ell^-$ modes. Note that the present branching fraction upper limits lie within a factor of two to three of the SM predictions.

5 Conclusion

The BABAR $B \to X_s\gamma$ measurements of branching fractions, photon energy moments, $m_b$, $\mu_s^2$ are in good agreement with the SM predictions. The $B(B \to X_s\gamma)$ measurement provides a constraint on the charged Higgs mass of $M_{H^\pm} > 327$ GeV/c$^2$ at 95% CL independent of tan $\beta$. The new $A_{CP}(B \to X_s\gamma)$ measurement is the most precise result and agrees well with the SM prediction. The BABAR $\Delta A_{CP}(B \to X_s\gamma)$ measurement is the first one and provides the first constraint on $\text{Im}(C_8^{eff}/C_7^{eff})$. BABAR updated branching fraction upper limits for $B \to \pi^+\ell^-$ and presented the
Figure 12: Branching fraction upper limits at 90% CL for $B \to \pi \ell^+ \ell^-$ and $B \to \eta \ell^+ \ell^-$ modes from BABAR (preliminary) and Belle [59] and the measurement of $B \to \pi \ell^+ \ell^-$ by LHCb [60].

For $B \to K^{(*)} \ell^+ \ell^-$, the measurements of branching fractions, isospin asymmetries, lepton flavor ratios, $CP$ asymmetries, $K^*$ longitudinal polarization and lepton forward-backward asymmetry averaged over all experiments agree with the SM predictions. The largest deviation from the SM prediction is less than 3$\sigma$ and results from the isospin asymmetry of $B \to K \ell^+ \ell^-$ in the low $s$ region. To look for more deviations from the SM, the precision of all measurements needs to be improved significantly. Such improvements are expected to come from LHCb and Belle II. Furthermore, large data samples in these experiments will permit studies of the full angular distribution in $B \to K^{(*)} \ell^+ \ell^-$, which is described by 12 observables [61, 62]. Some the new observables have a higher discrimination power between the SM and new physics effects $A_{FB}$ and $F_L$. 

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ACKNOWLEDGEMENTS

I would like to thank members of the BABAR collaboration for giving me the opportunity to present these results. In particular, I would like to thank Piti Ongmongkolkul, Bill Dunwoodie, David Hitlin and Jack Ritchie for their fruitful suggestions. Furthermore, I would like to thank Gudrun Hiller and Aoife Bharucha for supplying Mathematica files with the SM predictions for $B \to K^{(*)}\ell^+\ell^-$ modes.

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