A lattice Boltzmann model for the open channel flows described by the Saint-Venant equations

Zhonghua Yang, Fengpeng Bai and Ke Xiang

Article citation details
R. Soc. open sci. 6: 190439.
http://dx.doi.org/10.1098/rsos.190439

Review timeline
Original submission: 18 March 2019
Revised submission: 12 July 2019
Final acceptance: 4 October 2019

Review History
RSOS-190439.R0 (Original submission)

Review form: Reviewer 1 (Jingming Hou)

Is the manuscript scientifically sound in its present form?
Yes

Are the interpretations and conclusions justified by the results?
Yes

Is the language acceptable?
Yes

Is it clear how to access all supporting data?
Not Applicable

Do you have any ethical concerns with this paper?
No
Have you any concerns about statistical analyses in this paper?
No

Recommendation?
Accept with minor revision (please list in comments)

Comments to the Author(s)
Authors describe the method of simulation 1D open-channel flow by means of one of simplest LBM algorithms with the D1Q3 lattice arrangement. As a base, the system of Saint-Venant equations is used. The Saint-Venant equations are formulated in a conservative form with inclusion of two pressure terms. The idea to solve the conservative form of the Saint-Venant equations by using the LBM is by my opinion a step forward in attempt to develop robust 1D LBM solver for practical application.

Manuscript is in general correct and should be interesting for readers but it requires some minor corrections before publication:

1. The similar model has made appearance in <Study of the 1D lattice Boltzmann shallow water equation and its coupling to build a canal network. > Thang and Chopard et. al., J. of Comput. Phy., 229 (2010) 7373–7400. The authors should explain the difference compared with it.

2. In the second section, the governing equations had better be structured in non-dimensional form, since the original equation the LB method is trying to solve is in its dimensionless form.

3. The Chapman-Enskog expansion should be included to show exactly which governing equations can be recovered, if a brand new LB model is developed. However, considering the simplicity of the paper, the Chapman-Enskog expansion has better show to the reviewers and may not appeared in the paper.

4. Detailed description of obtaining of equilibrium distribution function in Sec. 3 may interrupt the train of thought of readers. I would recomend move everything between eq. (24) and (42) to an appendix.

5. Fig. 4 should be greater to improve its readability.

6. Some plots also require for improve their readability, e.g. Fig. 6 maight contain only one half of a channel while it is symmetric and then the vertical axis could have the scale from, let say, 0.4 to 0.6; Fig. 7b is also nearly empty, vertical scale 1.56-1.57 would improve it.

After these corrections I recommend the paper for publication.

Review form: Reviewer 2

Is the manuscript scientifically sound in its present form?
No

Are the interpretations and conclusions justified by the results?
No

Is the language acceptable?
Yes
Is it clear how to access all supporting data?
Yes

Do you have any ethical concerns with this paper?
No

Have you any concerns about statistical analyses in this paper?
I do not feel qualified to assess the statistics

Recommendation?
Reject

Comments to the Author(s)
The manuscript proposed the LBCSVE scheme for open channels flow. However, the paper is not clear about its originality and not convincing by presenting four cases of steady flow. Based on this, the reviewer do not recommend this manuscript to be published in current form.

Major concerns:
The originality of the LBCSVE should be discussed in the introduction. Why is this a new lattice Boltzmann method as stated in the abstract?
The advantage and disadvantage of the lattice Boltzmann method should be discussed.
The numerical test only include steady state solution. Does this imply the method only apply to steady state? If not, unsteady flow should be simulated.
In the numerical tests, numerical accuracy and errors should be reported, computational efficiency compared to other numerical methods (eg. Finite difference method, finite volume method) should be provided. The effects of lattice size should also be investigated.

Minor points:
Figure 9, there is no need to provide area A, since it’s related to surface elevation, results of flowrate should be given instead.
Figure 12, results of flowrate should be given.
There are some typos in the manuscript, for example m2/s, superscript should be used.

Review form: Reviewer 3

Is the manuscript scientifically sound in its present form?
Yes

Are the interpretations and conclusions justified by the results?
Yes

Is the language acceptable?
Yes

Is it clear how to access all supporting data?
Yes

Do you have any ethical concerns with this paper?
No
Have you any concerns about statistical analyses in this paper?
No

Recommendation?
Major revision is needed (please make suggestions in comments)

Comments to the Author(s)
The manuscript presents a lattice Boltzmann model for the Saint-Venant equations.

This topic is of potential interest but the actual contribution in the present form is still improvable. The English of the paper is good with a coherent structure. To my opinion the structure of the paper is the correct as the classic Introduction/methods/results and discussion/conclusion is followed.

The novelty of the paper is not well described and presented. To the reader (including myself) it is hard to see what is effectively the novelty of the paper since all the methodologies, as far as I could understand, have already been presented in other research. Images are of very low quality and results not perceptible in the manuscript images.

To my opinion there are some flaws in the paper that have to be clarified regarding the novelty of the paper and as such, I cannot recommend that this paper is accepted for publication in its present form. As such my advice is to review.

Comments:
- Abstracts should not have acronyms
- P4 EQ5 - define alpha
- P5 L20 - the subscript should be alpha
- Eq 5 to Eq 24 and the whole methodology is similar to [18] (Van Thang, P.; Chopard, B.; Lefèvre, L. Study of the 1D lattice Boltzmann shallow water equation and its coupling to build a canal network. J. Comput. Phys. 2010, 229, 7373-7400. (doi: 10.1016/j.jcp.2010.06.022)). The only novelty I found relevant is the use of the variable A instead of h. Please state very explicitly what is the real novelty of this paper.
- P6 L11 - Zhou’s scheme for the force does not conserve mass locally. You should clearly state, or at least give an insight on why you have almost no errors since the scheme you used is not conservative.
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- P9 L35 - Why use v=6m/s? please explain the value of v used for this and other tests
- P11 L25 - m/s1/3. - fix units
- P13 Table A2 - Table is unnecessary
- references are formatted incorrectly. Some have an initial indentation

Decision letter (RSOS-190439.R0)

13-Jun-2019

Dear Dr Bai,

The editors assigned to your paper (“A lattice Boltzmann model for the open channel flows...
described by the Saint-Venant equations”) have now received comments from reviewers. We would like you to revise your paper in accordance with the referee and Associate Editor suggestions which can be found below (not including confidential reports to the Editor). Please note this decision does not guarantee eventual acceptance.

Please submit a copy of your revised paper before 06-Jul-2019. Please note that the revision deadline will expire at 00.00am on this date. If we do not hear from you within this time then it will be assumed that the paper has been withdrawn. In exceptional circumstances, extensions may be possible if agreed with the Editorial Office in advance. We do not allow multiple rounds of revision so we urge you to make every effort to fully address all of the comments at this stage. If deemed necessary by the Editors, your manuscript will be sent back to one or more of the original reviewers for assessment. If the original reviewers are not available, we may invite new reviewers.

To revise your manuscript, log into http://mc.manuscriptcentral.com/rsos and enter your Author Centre, where you will find your manuscript title listed under "Manuscripts with Decisions." Under "Actions," click on "Create a Revision." Your manuscript number has been appended to denote a revision. Revise your manuscript and upload a new version through your Author Centre.

When submitting your revised manuscript, you must respond to the comments made by the referees and upload a file "Response to Referees" in "Section 6 - File Upload". Please use this to document how you have responded to the comments, and the adjustments you have made. In order to expedite the processing of the revised manuscript, please be as specific as possible in your response.

In addition to addressing all of the reviewers’ and editor’s comments please also ensure that your revised manuscript contains the following sections as appropriate before the reference list:

• Ethics statement (if applicable)
If your study uses humans or animals please include details of the ethical approval received, including the name of the committee that granted approval. For human studies please also detail whether informed consent was obtained. For field studies on animals please include details of all permissions, licences and/or approvals granted to carry out the fieldwork.

• Data accessibility
It is a condition of publication that all supporting data are made available either as supplementary information or preferably in a suitable permanent repository. The data accessibility section should state where the article’s supporting data can be accessed. This section should also include details, where possible of where to access other relevant research materials such as statistical tools, protocols, software etc can be accessed. If the data have been deposited in an external repository this section should list the database, accession number and link to the DOI for all data from the article that have been made publicly available. Data sets that have been deposited in an external repository and have a DOI should also be appropriately cited in the manuscript and included in the reference list.

If you wish to submit your supporting data or code to Dryad (http://datadryad.org/), or modify your current submission to dryad, please use the following link:
http://datadryad.org/submit?journalID=RSOS&manu=RSOS-190439

• Competing interests
Please declare any financial or non-financial competing interests, or state that you have no competing interests.
• Authors’ contributions

All submissions, other than those with a single author, must include an Authors’ Contributions section which individually lists the specific contribution of each author. The list of Authors should meet all of the following criteria; 1) substantial contributions to conception and design, or acquisition of data, or analysis and interpretation of data; 2) drafting the article or revising it critically for important intellectual content; and 3) final approval of the version to be published.

All contributors who do not meet all of these criteria should be included in the acknowledgements.

We suggest the following format:
AB carried out the molecular lab work, participated in data analysis, carried out sequence alignments, participated in the design of the study and drafted the manuscript; CD carried out the statistical analyses; EF collected field data; GH conceived of the study, designed the study, coordinated the study and helped draft the manuscript. All authors gave final approval for publication.

• Acknowledgements

Please acknowledge anyone who contributed to the study but did not meet the authorship criteria.

• Funding statement

Please list the source of funding for each author.

Once again, thank you for submitting your manuscript to Royal Society Open Science and I look forward to receiving your revision. If you have any questions at all, please do not hesitate to get in touch.

Kind regards,
Alice Power
Editorial Coordinator
Royal Society Open Science
openscience@royalsociety.org

on behalf of Dr Mark Smith (Associate Editor) and Jon Blundy (Subject Editor)

openscience@royalsociety.org

Associate Editor’s comments (Dr Mark Smith):

My recommendation is that major revision is needed before this manuscript can be accepted for publication. In particular, I note that each of the reviewers has requested greater clarity on the novelty of the method used. I also suggest that these are described in greater detail which may help address the novelty issue.

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Authors describe the method of simulation 1D open-channel flow by means of one of simplest LBM algorithms with the D1Q3 lattice arrangement. As a base, the system of Saint-Venant equations is used. The Saint-Venant equations are formulated in a conservative form with inclusion of two pressure terms. The idea to solve the conservative form of the Saint-Venant equations by using the LBM is by my opinion a step forward in attempt to develop robust 1D LBM solver for practical application.

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After these corrections I recommend the paper for publication.
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Minor points:
Figure 9, there is no need to provide area A, since it’s related to surface elevation, results of flowrate should be given instead.
Figure 12, results of flowrate should be given.
There are some typos in the manuscript, for example m²/s, superscript should be used.

Reviewer: 3

Comments to the Author(s)
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The novelty of the paper is not well described and presented. To the reader (including myself) it is hard to see what is effectively the novelty of the paper since all the methodologies, as far as I could understand, have already been presented in other research. Images are of very low quality and results not perceptible in the manuscript images.

To my opinion there are some flaws in the paper that have to be clarified regarding the novelty of the paper and as such, I cannot recommend that this paper is accepted for publication in its present form. As such my advice is to review.

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- P10 L25 - m/s1/3. - fix units
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Author's Response to Decision Letter for (RSOS-190439.R0)

See Appendix A.

RSOS-190439.R1 (Revision)

Review form: Reviewer 2

Is the manuscript scientifically sound in its present form?
Yes

Are the interpretations and conclusions justified by the results?
Yes

Is the language acceptable?
Yes

Do you have any ethical concerns with this paper?
No

Have you any concerns about statistical analyses in this paper?
No

Recommendation?
Accept as is

Comments to the Author(s)
The reviewer (former #2) is satisfied with the modification to improve its quality and recommend the manuscript to be accepted.

Review form: Reviewer 3

Is the manuscript scientifically sound in its present form?
Yes

Are the interpretations and conclusions justified by the results?
Yes

Is the language acceptable?
Yes

Do you have any ethical concerns with this paper?
No
Have you any concerns about statistical analyses in this paper?
No

Recommendation?
Accept as is

Comments to the Author(s)
Dear Authors

Thank you for having incorporated the changes and the improvements suggested in the preceding round of the revision process. I do believe that the manuscript was greatly improved in terms of readability and scientific soundness. Overall I found the manuscript to be much easier to understand, with the aims, methodology, results and significance of the work more evident to the reader. The Authors are to be congratulated for the substantial revision of their manuscript.

Decision letter (RSOS-190439.R1)

04-Oct-2019

Dear Dr Bai,

I am pleased to inform you that your manuscript entitled "A lattice Boltzmann model for the open channel flows described by the Saint-Venant equations" is now accepted for publication in Royal Society Open Science.

You can expect to receive a proof of your article in the near future. Please contact the editorial office (openscience_proofs@royalsociety.org and openscience@royalsociety.org) to let us know if you are likely to be away from e-mail contact -- if you are going to be away, please nominate a co-author (if available) to manage the proofing process, and ensure they are copied into your email to the journal.

Due to rapid publication and an extremely tight schedule, if comments are not received, your paper may experience a delay in publication.

Royal Society Open Science operates under a continuous publication model (http://bit.ly/cpFAQ). Your article will be published straight into the next open issue and this will be the final version of the paper. As such, it can be cited immediately by other researchers. As the issue version of your paper will be the only version to be published I would advise you to check your proofs thoroughly as changes cannot be made once the paper is published.

On behalf of the Editors of Royal Society Open Science, we look forward to your continued contributions to the Journal.

Kind regards,
Anita Kristiansen
Royal Society Open Science Editorial Office
Royal Society Open Science
openscience@royalsociety.org
Associate Editor Comments to Author (Dr Mark Smith):

Comments to the Author:
Thank you for resubmitting "A lattice Boltzmann model for the open channel flows described by the Saint-Venant equations" to Royal Society Open Science. I have received 2 further reviews of your revised manuscript, which are included below and/or attached. As you can see, both reviewers are satisfied that you have addressed all points raised with the original submission and I am happy to recommend that we accept your contribution in its present form.

Reviewer comments to Author:

Reviewer: 2

Comments to the Author(s)
The reviewer (former #2) is satisfied with the modification to improve its quality and recommend the manuscript to be accepted.

Reviewer: 3

Comments to the Author(s)
Dear Authors

Thank you for having incorporated the changes and the improvements suggested in the preceding round of the revision process. I do believe that the manuscript was greatly improved in terms of readability and scientific soundness. Overall I found the manuscript to be much easier to understand, with the aims, methodology, results and significance of the work more evident to the reader. The Authors are to be congratulated for the substantial revision of their manuscript.

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**Appendix A**

**Response to Associate Editor:**

**Comments:** My recommendation is that major revision is needed before this manuscript can be accepted for publication. In particular, I note that each of the reviewers has requested greater clarity on the novelty of the method used. I also suggest that these are described in greater detail which may help address the novelty issue.

Each reviewer raises substantial areas for clarity and correction. I suggest these are addressed before the manuscript is returned.

**Response:** Thanks so much for giving us many opportunities to revise our manuscript. We appreciate the editor and reviewers very much for their constructive comments and suggestions. I have read the reviewer's suggestions and made corresponding revision one by one. We really carefully revise this paper marked with red color and hope to meet with your approval.
Response to Reviewer#1:

Comments to the Author(s)

Authors describe the method of simulation 1D open-channel flow by means of one of simplest LBM algorithms with the D1Q3 lattice arrangement. As a base, the system of Saint-Venant equations is used. The Saint-Venant equations are formulated in a conservative form with inclusion of two pressure terms. The idea to solve the conservative form of the Saint-Venant equations by using the LBM is by my opinion a step forward in attempt to develop robust 1D LBM solver for practical application.

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Reviewer#1, Concern # 1: The similar model has made appearance in <Study of the 1D lattice Boltzmann shallow water equation and its coupling to build a canal network. > Thang and Chopard et. al., J. of Comput. Phy., 229 (2010) 7373–7400. The authors should explain the difference compared with it.

Response: In Thang et. al’s work, the governing equations(Saint-Venant equations), are expressed as

\[
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0 \\
\frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2 + 0.5gh^2)}{\partial x} = F
\]

The model proposed by Thang et. al is applied to simulate a canal network without considering the variation of river width and the cross-section shape. So, the model cannot be used to simulate the real rivers.

In our work, the governing equations are

\[
\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} = s
\]
\[
\mathbf{u} = \begin{bmatrix} A \\ Q \end{bmatrix}; \quad \mathbf{f} = \begin{bmatrix} Q \\ \frac{Q^2}{A} + gL_i \end{bmatrix}; \quad \mathbf{s} = \begin{bmatrix} 0 \\ -gAS_f + g \frac{\partial I_i}{\partial x} \end{bmatrix}
\]

\[ I_i = \int_{h_i}^{h_c} (h_c - h) b(x, h) dh, \]

The primitive variables are the wetted cross-section area \( A \) and the discharge \( Q \). The Equations are the traditional and conservative forms\(^{[1-3]}\) and can be used in real rivers with arbitrary cross-section shapes. In order to calculate the hydrostatic pressure thrust \( I_1 \) which was a difficulty in simulating the real rivers, the Gauss–Legendre numerical integration method was used.

**Reviewer#1, Concern # 2:** In the second section, the governing equations had better be structured in non-dimensional form, since the original equation the LB method is trying to solve is in its dimensionless form.

**Response:** Thanks for the valuable suggestions. Originally, the LB method is do trying to solve is the dimensionless equation form. Recently, in the area of simulating the free-surface flows using LB method, many researchers have done a lot work and do not use the dimensionless form\(^{[4-10]}\). The form of governing equations in Thang et. al’s work is also not dimensionless form. For the LB model for free-surface flows used the dimensional form, two stability conditions must be satisfied:

1. The relaxation time \( \tau > 0.5 \)
2. The magnitude of the resultant velocity is smaller than the speed of calculated with the lattice speed:

\[ \frac{u^2}{v^2} < 1 \]

And also the celerity.

\[ \frac{gh}{v^2} < 1 \]

which \( u \) is the water velocity, \( h \) is the water depth, \( v \) is the lattice speed.

**Reviewer#1, Concern # 3:** The Chapman-Enskog expansion should be included to show exactly which governing equations can be recovered, if a brand new LB model
is developed. However, considering the simplicity of the paper, the Chapman-Enskog expansion has better show to the reviewers and may not appeared in the paper.

Response: Thanks for the valuable suggestions. The recovery of shallow water equations which is the hyperbolic conservative form using the Chapman-Enskog expansion method have been carried out in many works\cite{4,9,11}. Due to the similar Chapman-Enskog analysis, we do not put the Chapman-Enskog expansion in our work. The following is the Chapman-Enskog expansion for our model.

The Saint-Venant equations can be derived from the Chapman-Enskog expansion.

Assuming $\Delta t$ is small and equal $\varepsilon$.

Eq.(10) in the manuscript can be expressed

$$f_\alpha(x + v_\alpha \varepsilon \Delta t + \varepsilon) - f_\alpha(x, t) = -\frac{1}{\tau}(f_\alpha - f^{eq}_\alpha) + w_a \frac{\varepsilon}{c_s^2} v_a F$$  \hspace{1cm} (R1)

The Eq. (R1) is taken a Taylor expansion to the left hand side of in the time and space

$$\varepsilon \left( \frac{\partial}{\partial t} + v_\alpha \frac{\partial}{\partial x} \right) f_\alpha + \frac{1}{2} \varepsilon^2 \left( \frac{\partial}{\partial t} + v_\alpha \frac{\partial}{\partial x} \right)^2 f_\alpha + o(\varepsilon^3)$$

$$= -\frac{1}{\tau}(f_\alpha - f^{eq}_\alpha) + w_a \frac{\varepsilon}{c_s^2} v_a F$$  \hspace{1cm} (R2)

Then, expanding the $f_\alpha$ around the $f^{eq}_\alpha$ gives

$$f_\alpha = f^{eq}_\alpha + \varepsilon f^1_\alpha + o(\varepsilon^2)$$  \hspace{1cm} (R3)

In which $f^{eq}_\alpha = f^{eq}_\alpha$. To order $\varepsilon$, Eq. (R2) becomes

$$\left( \frac{\partial}{\partial t} + v_\alpha \frac{\partial}{\partial x} \right) f^0_\alpha = -\frac{1}{\tau} f^1_\alpha + w_a \frac{1}{c_s^2} v_a F$$  \hspace{1cm} (R4)

And to order $\varepsilon^2$ is

$$\left( \frac{\partial}{\partial t} + v_\alpha \frac{\partial}{\partial x} \right) f^1_\alpha + \frac{1}{2} \left( \frac{\partial}{\partial t} + v_\alpha \frac{\partial}{\partial x} \right)^2 f^0_\alpha = 0$$  \hspace{1cm} (R5)

Substitution Eq.(R4) into Eq.(R5) leads to

$$\left( 1 - \frac{1}{2\tau} \left( \frac{\partial}{\partial t} + v_\alpha \frac{\partial}{\partial x} \right) \right) f^1_\alpha = -\frac{1}{2} \left( \frac{\partial}{\partial t} + v_\alpha \frac{\partial}{\partial x} \right) (w_a \frac{1}{c_s^2} v_a F)$$  \hspace{1cm} (R6)
Taking $\sum_a \ (\text{Eq.}(R4)+\varepsilon \ \text{Eq.(R6)})$ gives

$$\frac{\partial}{\partial t} \left( \sum_a f_a^0 \right) + \frac{\partial}{\partial x} \left( \sum_a v_a f_a^0 \right) = -\frac{1}{2c_s^2} \frac{\partial}{\partial x} \left( \sum_a w_a v_a v_a F \right) \tag{R7}$$

Considering the $f_a^0 = f_a^{eq}$ and evaluating the above equation using Eq.(5), Eq.(7) and Eqs.(14)-(17) in the manuscript gives the continuity equation

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{R8}$$

Then taking the $\sum_a v_a \ (\text{Eq.}(R4)+\varepsilon \ \text{Eq.(R6)})$ produces

$$\frac{\partial}{\partial t} \left( \sum_a v_a f_a^0 \right) + \frac{\partial}{\partial x} \left( \sum_a v_a v_a f_a^0 \right) + \varepsilon(1-\frac{1}{2\varepsilon}) \frac{\partial}{\partial x} \left( \sum_a w_a v_a f_a^1 \right) = -\frac{1}{2c_s^2} \frac{\partial}{\partial x} \left( \sum_a v_a \left( \frac{\partial}{\partial t} + v_a \frac{\partial}{\partial x} \right) (w_a v_a F) \right) \tag{R9}$$

Then, considering the $f_a^0 = f_a^{eq}$ and evaluating the above equation using Eq.(5), Eq.(7) and Eqs.(14)-(17) in the manuscript again, the momentum equation is obtained

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q^2}{A} + gl_1 \right) = F \tag{R10}$$

The kinematic viscosity is

$$\nu = \Delta tc_s^2(\tau - \frac{1}{2}) \tag{R11}$$

**Reviewer#1, Concern # 4:** Detailed description of obtaining of equilibrium distribution function in Sec. 3 may interrupt the train of thought of readers. I would recommend move everything between eq. (24) and (42) to an appendix.

**Response:** Thanks for this valuable suggestion. The detailed description of obtaining of equilibrium distribution function is moved to the Appendix 2.

**Reviewer#1, Concern # 5:** Fig. 4 should be greater to improve its readability.

**Response:** Thanks for this valuable suggestion. We have replaced the original Fig. 4 with a higher resolution figure.

**Reviewer#1, Concern # 6:** Some plots also require for improve their readability, e.g.
Fig. 6 might contain only one half of a channel while it is symmetric and then the vertical axis could have the scale from, let say, 0.4 to 0.6; Fig. 7b is also nearly empty, vertical scale 1.56-1.57 would improve it.

Response: Thanks for this valuable suggestion. For more readable, the vertical scale in Fig. 7b was set from 1.565 to 1.569. For Fig. 6, in order to show the channel characteristic of vertical contractions, we think it is better to display the two sides of the channel.

References:
[1] Franzini F, Soares-Frazão S. Efficiency and accuracy of Lateralized HLL, HLLS and Augmented Roe’s scheme with energy balance for river flows in irregular channels [J]. Applied Mathematical Modelling, 2016, 40(17–18):7427-7446.
[2] Zhang S, Duan J G. 1D finite volume model of unsteady flow over mobile bed[J]. Journal of Hydrology, 2011, 405(1–2):57-68.
[3] Chang T J, Kao H M, Chang K H, et al. Numerical simulation of shallow-water dam break flows in open channels using smoothed particle hydrodynamics[J]. Journal of Hydrology, 2011, 408(1):78-90.
[4] Frandsen J B. Free-surface lattice Boltzmann modeling in single phase flows[M]// Advanced Numerical Models For Simulating Tsunami Waves And Runup. 2008:163-219.
[5] Zhang J, Zhang Q, Qiao G. A lattice Boltzmann model for the non-equilibrium flocculation of cohesive sediments in turbulent flow[J]. Computers & Mathematics with Applications, 2014, 67(2):381-392.
[6] Rocca M L, Montessori A, Prestinini P, et al. A multispeed Discrete Boltzmann Model for transcritical 2D shallow water flows[J]. Journal of Computational Physics, 2015, 284:117-132.
[7] Liu H, Zhou J G, Burrows R. Lattice Boltzmann simulations of the transient shallow water flows[J]. Advances in Water Resources, 2010, 33(4):387-396.
[8] Xiong W, Zhang J. A two-dimensional lattice Boltzmann model for uniform channel flows[J]. Computers & Mathematics with Applications, 2011,
[9] Thang P V, Chopard B, Mendes E, et al. Study of the 1D lattice Boltzmann shallow water equation and its coupling to build a canal network[J]. Journal of Computational Physics, 2010, 229(19):7373-7400.

[10] Zhou J G, Haygarth P M, Withers P J, et al. Lattice Boltzmann method for the fractional advection-diffusion equation[J]. Physical Review E, 2016, 93(4-1):043310.

[11] Zhou J G. Lattice Boltzmann Methods for Shallow Water Flows [M]. 2003.
Response to Reviewer#2:

Comments to the Author(s)

The manuscript proposed the LBCSVE scheme for open channels flow. However, the paper is not clear about its originality and not convincing by presenting four cases of steady flow. Based on this, the reviewer do not recommend this manuscript to be published in current form.

Major concerns:

Reviewer#2, Concern # 1: The originality of the LBCSVE should be discussed in the introduction. Why is this a new lattice Boltzmann method as stated in the abstract?

Response: Thanks for this valuable suggestion. In the introduction, page 2, we add the following statement to prove the originality for the proposed LBCSVE:

Compared with the former LB models to solve the Saint–Venant equations [19, 20], the LBCSVE applied the conservative form of Saint–Venant equations for the first time and the Gauss–Legendre numerical integration method was used to solve the hydrostatic pressure thrust in the LB model first.

Reviewer#2, Concern # 2: The advantage and disadvantage of the lattice Boltzmann method should be discussed.

Response: Thanks for this valuable suggestion. We have add the discussions about the advantage and disadvantage of the lattice Boltzmann method in the introduction, which is:

The method is characterized by simple calculation, parallel process, easy implementation of boundary conditions and very efficient, flexible to simulate different flows within complex/varying geometries. It is these features that make the lattice Boltzmann method a very promising computational method in different areas.

In the area of simulating the open channel flows described by Saint–Venant equations, the lattice Boltzmann method is suitable for subcritical flows which are the most scenarios in coastal areas, estuaries and rivers. It suffer from a numerical instability when the LB method is used to solve the supercritical flows.
Reviewer#2, Concern # 3: The numerical test only include steady state solution. Does this imply the method only apply to steady state? If not, unsteady flow should be simulated.

Response: The LBCSVE method definitely can be used both in steady and unsteady flow. In the 4.1 section, the first test’ Tidal flow over a regular bed’ was an unsteady flow case. The inlet water level boundary was sine function and changed with time. The comparison of the analytical solution and numerical water and velocity in the computational area was given in Figure 5.

Reviewer#2, Concern # 4: In the numerical tests, numerical accuracy and errors should be reported, computational efficiency compared to other numerical methods (eg. Finite difference method, finite volume method) should be provided. The effects of lattice size should also be investigated.

Response: Thanks for this valuable suggestion. In section 4.1, the absolute errors of water level $z$ and velocity $u$ are shown in Figure 5c. For the numerical models to simulate one dimensional river flows, computational time is not the restricted condition and almost every successful model can obtain the computed results. For example, for the section 4.1 Tidal flow over a regular bed, with a laptop configured by 1.8GHz CPU, 16G RAM and Intel Core i7, 21.32 seconds was consumed for the proposed LBCSVE model and 22.52 seconds was used for the Godunov-type scheme proposed by the authors[1]. We think that the computational time and efficiency was not the point of concern when modeling the one dimensional river flows. For every test in the manuscript, the lattice size independence was carried out to find the proper lattice size.

Minor points:

Reviewer#2, Concern # 5: Figure 9, there is no need to provide area A, since it’s realted to surface elevation, results of flowrate should be given instead.

Response: Thanks for this valuable suggestion. We have replaced the area A by the flow rate for the Figure 9b.

Reviewer#2, Concern # 6: Figure 12, results of flowrate should be given.

Response: Thanks for this valuable suggestion. The flow rate was added in the Figure
Reviewer#2, Concern # 7: There are some typos in the manuscript, for example m2/s, superscript should be used.

Response: Thanks for this valuable suggestion. We have checked the manuscript carefully and revised the similar errors, such as '$1.566 \text{ m}^2/\text{s}$' in section 4.2 and '$0.03 \text{ m}/\text{s}^{1/3}$' in section 4.4.
Response to Reviewer#3:

Comments to the Author(s)

The manuscript presents a lattice Boltzmann model for the Saint-Venant equations.

This topic is of potential interest but the actual contribution in the present form is still improvable. The English of the paper is good with a coherent structure. To my opinion the structure of the paper is the correct as the classic Introduction/methods/results and discussion/conclusion is followed.

The novelty of the paper is not well described and presented. To the reader (including myself) it is hard to see what is effectively the novelty of the paper since all the methodologies, as far as I could understand, have already been presented in other research. Images are of very low quality and results not perceptible in the manuscript images.

To my opinion there are some flaws in the paper that have to be clarified regarding the novelty of the paper and as such, I cannot recommend that this paper is accepted for publication in its present form. As such my advice is to review.

Comments:

- Reviewer#3, Concern # 1: Abstracts should not have acronyms
  Response: Thanks for this valuable suggestion. The ‘LBCSVE’ was replaced by the ‘proposed model’ in the abstract.

- Reviewer#3, Concern # 2: P4 EQ5 - define alpha
  Response: Thanks for this valuable suggestion. We have added the definition of alpha in equation 5, which is:
  \[
  \alpha
  \]
  where \( \alpha \) is the link in a lattice.

- Reviewer#3, Concern # 3: P5 L20 - the subscript should be alpha
  Response: Thanks for this valuable suggestion. We have changed the Equation 20 as:
\[ F_a = -gA(x)S_f(x) \]

**Reviewer#3, Concern # 4:** Eq 5 to Eq 24 and the whole methodology is similar to [18] (Van Thang, P.; Chopard, B.; Lefèvre, L. Study of the 1D lattice Boltzmann shallow water equation and its coupling to build a canal network. J. Comput. Phys. 2010, 229, 7373-7400. (doi: 10.1016/j.jcp.2010.06.022)). The only novelty I found relevant is the use of the variable A instead of h. Please state very explicitly what is the real novelty of this paper.

**Response:** In Thang et. al’s work, the governing equations (Saint-Venant equations), are expressed as

\[
\begin{align*}
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} &= 0 \\
\frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2 + 0.5gh^2)}{\partial x} &= F
\end{align*}
\]

The model proposed by Thang et. al is applied to simulate a canal network without considering the variation of river width and the cross-section shape. So, the model cannot be used to simulate the real rivers.

In our work, the governing equations are

\[
\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}}{\partial x} = \mathbf{s}
\]

\[
\mathbf{u} = \begin{bmatrix} A \\ Q \end{bmatrix}; \quad \mathbf{f} = \begin{bmatrix} Q \\ \frac{Q^2}{A} + gI_1 \end{bmatrix}; \quad \mathbf{s} = \begin{bmatrix} 0 \\ -gAS_f + g \frac{\partial I_1}{\partial x} \end{bmatrix}
\]

\[
I_1 = \int_0^{h_i} (h - h_i)b(x, h)dh_i
\]

The primitive variables are the wetted cross-section area \( A \) and the discharge \( Q \). The Equations are the traditional and conservative forms and can be used in real rivers with arbitrary cross-section shapes. In order to calculate the hydrostatic pressure thrust \( I_1 \) which was a difficulty in simulating the real rivers, the Gauss–Legendre numerical integration method was used.

In the introduction, page 2, we add the following statement to prove the originality for the proposed LBCSVE:

**Compared with the former LB models to solve the Saint–Venant equations [19, 20],**
the LBCSVE applied the conservative form of Saint–Venant equations for the first time and the Gauss–Legendre numerical integration method was used to solve the hydrostatic pressure thrust in the LB model first.

- **Reviewer#3, Concern # 5**: P6 L11 - Zhou’s scheme for the force does not conserve mass locally. You should clearly state, or at least give an insight on why you have almost no errors since the scheme you used is not conservative.

**Response:** Using the Chapman-Enskog procedure, Zhou has verified that (1) the lattice Boltzmann equation with the centred scheme for the force term can generate the second-order accurate macroscopic continuity and momentum equations in time and space; (2) the averaged force acting on the particles during streaming can be best represented only with the centred scheme; hence the centred scheme is the correct choice for determining the force term in the lattice Boltzmann equation; (3) the centred scheme can satisfy the N-property. The four tests in the manuscript are all subcritical flows. Both the conservative form and not conservative form, very small errors is produced when simulating the subcritical flows.

- **Reviewer#3, Concern # 6**: P8 L53 - Please compute an absolute error - L2-norm or something similar is advised. Also, for each test the computational time and the machine used should be stated so that there is a comparison of efficiency between your methodology and others.

**Response:** Thanks for this valuable suggestion. In section 4.1, the absolute errors of water level $z$ and velocity $u$ are shown in Figure 5c. For the numerical models to simulate one dimensional river flows, computational time is not the restricted condition and almost every successful model can obtain the computed results. For example, for the section 4.1 Tidal flow over a regular bed, with a laptop configured by 1.8GHz CPU, 16G RAM and Intel Core i7, 21.32 seconds was consumed for the proposed LBCSVE model and 22.52 seconds was used for the Godunov-type scheme proposed by the authors\(^1\). We think that the computational time and efficiency was not the point of concern when modeling the one dimensional river flows.

- **Reviewer#3, Concern # 7**: P9 L35 - Why use $v=6m/s$? please explain the value of $v$ used for this and other tests
\[ v = \frac{\Delta x}{\Delta t} \] denotes the velocity along a lattice link, with \( \Delta x \) being the lattice and \( \Delta t \) being the time step. In the lattice Boltzmann method, the \( v \) plays a key role for the numerical stability. In simulating the river flows, the stable conditions include :(1) the resultant velocity is smaller than the speed of the sound \( (u_1 u_2 < v^2) \); (2) the wave speed in shallow water is smaller than the speed of the sound \( (gh < v^2) \). The lattice velocity \( v \) is determined by the lattice size \( \Delta x \) and time step \( \Delta t \). Moreover, it is must meet the stable conditions (1) and (2). For a test, the lattice velocity \( v \) varied due to the difference of lattice size \( \Delta x \) and time step \( \Delta t \).

- **Reviewer#3, Concern #8:** P11 L25 - m/s1/3. - fix units

**Response:** Thanks for this valuable suggestion. We have checked the manuscript carefully and revised the similar errors.

- **Reviewer#3, Concern #9:** P13 Table A2 - Table is unnecessary

**Response:** Thanks for this valuable suggestion. The Table A2 was deleted.

- **Reviewer#3, Concern #10:** references are formatted incorrectly. Some have an initial indentation

**Response:** Thanks for this valuable suggestion. We have checked the references carefully and reset the format.

**References:**

[1] Zhu Z, Yang Z, Bai F, et al. A New Well-Balanced Reconstruction Technique for the Numerical Simulation of Shallow Water Flows with Wet/Dry Fronts and Complex Topography[J]. Water, 2018, 10(11): 1661.