Quantum computation with quantum-dot spin qubits inside a cavity

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Universal set of quantum gates are realized from the conduction-band electron spin qubits of quantum dots embedded in a microcavity via two-channel Raman interaction. All of the gate operations are independent of the cavity mode states, i.e., insensitive to the thermal cavity field. Individual addressing and effective switch of the cavity mediated interaction are directly possible here. Meanwhile, gate operations also can be carried out in parallel. The simple realization of needed interaction for selective qubits makes current scenario more suitable for scalable quantum computation.

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Quantum computer can provide a possible alternative for resolving certain hard problems in comparison with classical computer with the help of the principle of coherent superposition and quantum entanglement.1 Solid state system has been generally accepted to be the most promising hardware for quantum computation since it can be easily integrated into large quantum networks. With the development of fabrication and manipulation technologies in semiconductor quantum dots, quantum computation based on this system has attracted much attention. In a quantum dot system, decoherence is still an important and challenging issue. However localized electron spin states have relatively long decoherence time, so it is more suitable as qubit. The realization of gate operations on arbitrary two qubits is another challenge in solid state system. In order to conquer this problem, Imamoglu and coworkers introduced the quantum dot cavity QED scheme2 where the cavity mode can be used as a data bus for long-distance information transfer and fast coupling of arbitrary two qubits. In addition, this setup can support parallel quantum logic gate operations. From then on, many schemes adopt quantum dots embedded in cavity have been presented.3, 4, 5, 6.

In this paper, we propose a scenario for realizing quantum computation via a two-channel Raman interaction of quantum dots embedded in a microcavity. Qubits are encoded on the conduction-band spins of semiconductor quantum dot. The valence-band state is used as an auxiliary state, which can be adiabatically eliminated. The decoherence time of qubits is long enough to complete indispensable gate operations. The two-channel Raman interaction model has been generally accepted as an alternative to the single-channel one in atomic cavity QED system7, 8, 9, 10, 11 as the easy realization of needed interactions. Therefore, it is very significant to generalized the two-channel Raman interaction model to quantum dot cavity QED system for solid quantum computation. In fact, in comparison with atomic cavity QED, quantum-dot cavity QED is more superior because quantum dots are always fixed in a cavity, thus the scale up of the solid nature system is quite straightforwardly. Meanwhile, individual addressing of quantum dot qubits, which is of great importance for scalable quantum computation, is directly possible taken into account the fact that quantum dot is generally fabricated as a mesoscopic quantum system.

We consider $N$ III-V semiconductor quantum dots embedded in a microcavity. All of the quantum dots are doped such that each quantum dot has a single conduction-band electron and a full valence band. Under the condition of quantum confinement, the conduction-band electron is always in the ground state orbital. The qubit is encoded on the conduction-band state $|\uparrow\rangle$ and $|\downarrow\rangle$ by a uniform magnetic field. The relevant energy levels of every quantum dot can be simulated as a three-level configuration as shown in Fig. 1. $\hbar\omega_1$, $\hbar\omega_2$ and $\hbar\omega_c$ are energies of the state $|\uparrow\rangle$, $|\downarrow\rangle$ and $|\psi\rangle$, respectively, and $\omega_1=\omega_2=\omega_3=\omega_c$. The frequencies of classical laser fields and $\omega_c$ is the frequency of the cavity field. $\Delta_1$, $\Delta_2$ and $\Delta$ are three detunings. Assuming that $\Delta_1=\omega_1-\omega_2=\omega_1-\omega_3-\Delta$ and $\Delta_2=\omega_2-\omega_3=\omega_1-\omega_2-\omega_3$, so we have $\omega_1+\Delta=\omega_2-\omega_3$ and $\omega_2+\omega_1=\omega_3-\omega_2$. Every quantum dot is off-resonant excited via two Raman channels by using classical laser fields and the microcavity. One channel consists of laser fields 1 and 3, the other consists of laser field 2 and the microcavity field. The total system contains $N$ quantum dots, a microcavity and $3N$ classical laser fields, the Hamiltonian of which can be described as (assuming $h=1$)

\begin{equation}
H = H_0 + H_{int},
\end{equation}

\begin{equation}
H_0 = \sum_{i=1}^{N}(\omega_1\sigma^i_1 + \omega_2\sigma^i_1 + \omega_3\sigma^i_1 + \omega_c\sigma^i_1) + \omega_c a^\dagger a,
\end{equation}

\begin{equation}
H_{int} = \sum_{i=1}^{N}\left[\Omega_1 e^{-i\omega_1 t} + \Omega_2 e^{-i\omega_2 t}\right]\sigma^i_1 \sigma^i_v + (\Omega_3 e^{-i\omega_3 t} + ga)\sigma^i_1 + H.c.,
\end{equation}

where $\Omega_j$ with $j=1, 2, 3$ are Rabi frequencies of classical fields, $g$ is coupling constant of the microcavity mode and each quantum dot with index number $i$, and $\sigma^i_{mn} = |m\rangle\langle n|$.
(m, n = 1, 1, v). In writing Eq. (12), we have assumed that \( \Omega_j = \Omega \) and \( g^i = g \).

The interaction Hamiltonian (12) can be rewritten, in the interaction picture with respect to (11), as

\[
H_I = \sum_{i=1}^{N} \left[ \Omega_2 \sigma_i^v e^{i\Delta_2 t} + (\Omega_1 \sigma_i^v + \Omega_3 \sigma_i^v) e^{i\Delta_1 t} + g a \sigma_i^v e^{i(\Delta_1 + \Delta_2) t} + H.c. \right].
\]

(2)

In the case of \( \Delta_1, \Delta_2 \gg \Omega, g \) and \( \Delta_1 - \Delta_2 \gg \{ \Delta, (\Delta_1 + \Delta_2) \Omega \omega_1, \Omega \omega_2, (2\Delta_1 + \Delta_2) \Omega \omega_1, (2\Delta_2 + \Delta_1) \Omega \omega_1 \} \), the valence-band state can be adiabatically eliminated [3]. We can then obtain an effective Hamiltonian by using rotating-wave approximation

\[
H_e^{(1)} = \sum_{i=1}^{N} \left[ \frac{\Omega_1 \Omega_2}{\Delta_2} (\sigma_{1i}^+ + \sigma_{1i}^-) + \frac{g \Omega_2}{2} \left( \frac{1}{\Delta_1} + \frac{1}{\Delta_1 + \Delta_2} \right) \left( a_i^+ e^{-i\Delta_1 t} + a_i^- e^{i\Delta_1 t} \right) \right],
\]

(3)

where we have neglected the ac-Stark energy shift, which can be easily compensated [12] by an addition laser field dispersively coupled to an energy level outside the qubit space in real experimental implementation.

For simplification of calculation, we choose a new computational basis \( | \pm \rangle^i = \frac{1}{\sqrt{2}} (| \uparrow \rangle^i \pm | \downarrow \rangle^i) \). We can rewrite the effective Hamiltonian [3] as

\[
H_e^{(2)} = \sum_{i=1}^{N} \left[ A \left( \frac{2S_i^+ - S_i^- + S_i^z}{4} e^{-i\Delta_1 t} + \frac{2S_i^+ - S_i^- - S_i^z}{4} a e^{i\Delta_1 t} \right) + B S_z^i \right],
\]

(4)

where \( A = \frac{g \Omega_2}{2} \left( \frac{1}{\Delta_1} + \frac{1}{\Delta_1 + \Delta_2} \right), B = \frac{2A \Omega_2}{\Delta_2} \), \( S_+ = | + \rangle \langle - |, S_+ = | - \rangle \langle + | \) and \( S_z = \frac{1}{2} (| + \rangle \langle + | - | - \rangle \langle - |) \).

Assume that \( B \gg \Delta_1, A \) and in the \( S_z \) framework \( H_0 = B S_z^i \), the Hamiltonian (4) can be reduced to

\[
H_e = \sum_{i=1}^{N} \left[ A \left( a_i^+ e^{-i\Delta_1 t} + a_i e^{i\Delta_1 t} \right) \right] S_z^i \]

(5)

For the implementation of quantum computation, the most important steps should be the realization of a set of universal quantum logical gates, *i.e.*, two-qubit logic gate, controlled-not gate or controlled phase shift, and arbitrary single-qubit rotations. Here we first introduce the scenario for implementing a controlled phase shift. We turn on three classical laser fields \( \omega_j \) on quantum dots \( m \) and \( n \), let quantum dot \( m \) interacts with \( n \) via the virtue excited cavity mode under the condition of \( \Delta_m = \Delta_n = \Delta \). The time evolution operator for this system can be expressed as this form

\[
U = e^{-i\alpha(t) (\sum_i S_i^z)^2} e^{-i\beta(t) (\sum_i S_i^z a e^{-i\gamma(t) (\sum_i S_i^z a^i)})},
\]

(6)

where \( l = m, n \). The coefficients \( \alpha(t), \beta(t) \) and \( \gamma(t) \) can be calculated by Schrödinger equation as [12, 14]

\[
\beta(t) = \int_{0}^{t} A e^{-\Delta t} dt = \frac{A}{2i\Delta} (e^{i\Delta t} - 1),
\]

(7a)

\[
\gamma(t) = \int_{0}^{t} A e^{-\Delta t} dt = -\frac{A}{2i\Delta} (e^{-i\Delta t} - 1),
\]

(7b)

\[
\alpha(t) = i \int_{0}^{t} \beta(t') \frac{A}{2} e^{-i\Delta t'} dt' = \frac{A^2}{4\Delta} \left[ \frac{t}{i} (e^{i\Delta t} - 1) \right].
\]

(7c)

Setting \( \Delta t = 2\pi / A \) results in \( \beta(t) = \gamma(t) = 0 \) and \( \alpha(t) = A^2 / 4\Delta t \), and thus the total evolution operator of the system becomes

\[
U_{mn} = e^{-iBt (\sum_i S_i^z)^2} e^{-i\frac{A^2}{4\Delta} (\sum_i S_i^z)^2},
\]

(8)

so that the state evolutions of \( | + + \rangle_{mn}, | + - \rangle_{mn}, | - + \rangle_{mn} \) and \( | - - \rangle_{mn} \) are

\[
| + + \rangle_{mn} \rightarrow e^{-i(Bt + \frac{1}{4A^2})} | + + \rangle_{mn},
\]

(9a)

\[
| + - \rangle_{mn} \rightarrow e^{-i(Bt - \frac{1}{4A^2})} | - - \rangle_{mn},
\]

(9b)

\[
| + - \rangle_{mn} \rightarrow | + - \rangle_{mn},
\]

(9c)

\[
| + + \rangle_{mn} \rightarrow | + + \rangle_{mn}.
\]

(9d)
If we parameterize the interaction time and Rabi frequencies as

\[ Bt = 2k\pi + \frac{\pi}{2}, \]  
\[ \frac{A^2}{4\Delta t} = \frac{\pi}{2}, \]

where \( k = 0, \pm1, \pm2, \cdots \), which correspond that all parameters should satisfy the conditions

\[ \Delta = \frac{g\Omega_2}{2} \left( \frac{1}{\Delta_1} + \frac{1}{\Delta_1 + \Delta} \right), \]

\[ \frac{4\pi\Omega_1\Omega_3}{\Delta\Delta_2} = 2k\pi + \frac{\pi}{2}, \]

then, the evolution of states of Eq. (9) becomes

\[ | + + \rangle_{mn} \rightarrow | + + \rangle_{mn}, \quad | - - \rangle_{mn} \rightarrow | - - \rangle_{mn}, \]

\[ | + - \rangle_{mn} \rightarrow | + - \rangle_{mn}, \quad | - + \rangle_{mn} \rightarrow | - + \rangle_{mn}. \]

Obviously, this is a standard controlled phase shift transformation under the basis \{\{|+\rangle, |\rangle\}. If we rotate the basis with an angle \( \theta = \pi/4 \), the controlled phase gate is then implemented in the qubit space.

Similarly, we can realized the controlled phase shift between arbitrary two spin qubits of the \( N \) quantum dots. The cavity mediated interaction can be implemented on selective qubits, this can be achieved by turning on/off the external driving lasers on certain qubits. In fact, the qubits interaction is mediated by virtual exchange of photons with the cavity, which requires the qubits are "degenerate" with each other \[15\]. Here, "degenerate" should means the same effective detuning \( \Delta \) for considered qubits. When the qubits are non-degenerate, similar to the argument in \[15\], the cavity mediated process can be effectively turned off as it does not conserve energy. Therefore, we also can carry out the controlled phase gate in parallel if we turn on the two-channel Raman resonant on different pair of qubits simultaneously with different pairs working in different detunings. Then, cross-talk of different pairs can be effective neglected provided that the difference of the working detunings is considerably large, thus results in parallel computation a natural merit in present scenario of quantum computation.

The cavity-state-free evolution \[10\] is achieved by periodical evolution \( (\Delta T = 2\pi) \) of a near-resonant driving with detuning \( \delta \) and periodicity \( T = 2\pi/\delta \) (in the rotating frame). Physically, after periodical evolution following the path \( \mathcal{L} = A \Delta \frac{1}{\Delta} \left( 1 - e^{i\Delta t} \right), \) the cavity state returns to its original phase space coordinates with an additional geometric phase \( \alpha(t) \) equivalent to the area enclosed by the trajectory \[14\]. So, the present gate operation is also of geometric nature, which is generally believed to be more robust against random operation errors.

Then we briefly introduce the single-qubit operations in this system. As single-qubit operations is much faster than that of the two-qubit case, we can simply consult the Raman process with laser fields 1 and 3. Laser field 2 is turn off now, thus the cavity field can be effectively eliminated. The net effect of the cavity field is an additional ac Stark shift, inversely proportional to \( \Delta_3 \). Considering the fact that \( \Delta_3 \) is the detuning of optical frequencies, this term can be safely neglected. In this case, the process only controlled by the single-channel Raman interaction, consists of laser fields 1 and 3, and the effective Hamiltonian is

\[ H' = \frac{\Omega_1\Omega_3}{\Delta_2} (\sigma_{1\downarrow} + \sigma_{1\uparrow}), \]

from which we can realize arbitrary single-qubit rotations by choosing appropriate interaction time. The process is equivalent to the single-channel Raman process with two classical laser fields of Ref. \[3\]. Single-qubit operations also can be implemented by using external magnetic fields with different directions \[16\].

Next we discuss the feasibility of the current scenario. From Eq. (11), we can obtain \( g = \frac{16}{k\pi - 1} \) meV where \( k = 5 \) can be deserved in terms of the current experimental parameter \( g \sim 0.5 \) meV \[2\]. We also can obtain \( A = 0.1g, B = 0.4 \) meV and \( \Delta = 0.1g \). Choosing the typical parameters to satisfy \( \Omega_j = 1 \) meV, \( \Delta_2 = 5 \) meV \[6\] and \( \Delta_1 = 10 \) meV, we will obtain \( \Delta_2 - \Delta_2 \sim 5 \) meV, which satisfies the first approximate condition \( \Delta_1 - \Delta_2 \gg \frac{\Delta_1 + \Delta_2}{2\Delta_1\Delta_2} \left( \sigma_{1\downarrow} + \sigma_{1\uparrow} \right), \sigma_{3\downarrow} \gg \frac{2\Delta_1 + \Delta_3}{2\Delta_1(\Delta_1 + \Delta_3)} \). The second approximate condition \( B \gg \Delta_1 - \Delta_1 \), \( A \) can be satisfied automatically because \( B = 0.4 \) meV is larger than \( A = \Delta = 0.1g \sim 0.05 \) meV.

The time required to complete a single-qubit rotation is about \( t_\tau \sim 10 \) ps under above mentioned conditions, which is the same as the single-channel Raman interaction process. The implementation of two-qubit gate operation need about \( t_\tau \sim 100 \) ps, which is close to that in the case of sing-channel Raman interaction process \[2\], where one needs twice single-channel interactions on the two qubits and some single-qubit rotations for realizing a controlled phase shift. However, in our scheme, we only needs one two-channel Raman interaction on the two qubits without the help of single-qubit rotations. Thus the two-channel Raman interaction process is simpler for scalable quantum computation than single-channel Raman interaction process in the quantum-dot spin system.

Decoherence is a main obstacle in quantum information processing, thus we should consider the relative magnitude of the decoherence rates as compared to the gate-operation time. The coherent time of conduction-band electrons is about \( 1 \) \( \mu \)s in doped quantum well and bulk semiconductors \[2\]. Recent experiment indicated that the spin coherent time can reach \( 1.2 \) \( \mu \)s by using spin-echo technology \[17\]. Obviously the gate-operation time is much less than spin coherent time. Generally, in cavity QED schemes one should consider the cavity
decay factor and thermal field, which may introduce a decoherence mechanism. In our scheme, the cavity mode is only virtually excited during the interaction, but the effective decoherence time will still be on the order of 1 ns with the cavity lifetime $\Gamma \sim 10$ ps \([2]\). Situation can still be better, we can embed the quantum dots in a microdisk (or microchip) structure \([3]\) to enhance the couple of quantum dots and a single photon. Meanwhile, we can improve the cavity quality factor by using high-Q whispering gallery mode of a silica microsphere \([19]\) as well as photonic-crystal microcavity mode \([20]\). Furthermore, the photon-number-dependent parts in the evolution operator are canceled, thus our scheme is insensitive to the thermal field. In addition, one can carry out the single- or two-qubit operations in parallel, which can reduce the total operation time comparing with the sequential individual gate operation method.

Addressing and capture for particles are another issues for quantum information processing. In current scheme, all of the quantum dots are trapped in a microcavity, the positions of quantum dots are fixed, so one does not need capture the quantum dots. We only need consider the selective addressing problem, which has been successfully demonstrated in experiments. In the case that the number of quantum dots is small, every quantum dot can be addressed selectively by a laser field from a fiber tip (near-field technology) \([2]\). In the case of scalable quantum computation, we should consult to the switch on/off technology, which has already been considered above.

In conclusion, we have presented a scenario for realizing quantum computation with quantum dots spins and microcavity by using a two-channel Raman resonant interaction. The two-channel Raman resonant interaction model is more convenient than the previous single-channel Raman process \([2]\). The gate operations do not depend on the state of cavity mode, \textit{i.e.}, insensitive to the thermal field, and the acquired phase is of geometric nature. The effective switch method presented makes the selective qubits interaction and parallel computation are both possible, which is very important for scalable quantum computation. Detail discussions show that the present set-up is also in the reach of current technology.

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