Effect of refraction on dose reconstruction in optical-CT gel dosimetry

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Abstract. We address the problem of dose reconstruction based on limited experimentally accessible data due to the effect of refraction in optical-CT gel dosimetry. The refractive index mismatch between the components of the optical-CT scanner result in light scattering and ultimately in the inability to capture parts of the projection datasets. We determine the maximum loss of data and the corresponding refractive index mismatch for which accurate dose reconstruction in the central part of the phantom is still possible. Also, a mathematical formalism that indicates how exact reconstructions can be obtained using a priori knowledge of the optical attenuation coefficient of the gel is presented. This study establishes rigorous design principles for accurate 3D dose reconstruction.

1. Introduction

Advances in radiation therapy and in particular new radiotherapy procedures such as intensity modulated radiation therapy or high dose rate brachytherapy call for the development of new techniques for accurate 3D dose reconstruction [1]. In this context, optical-CT gel dosimetry has emerged as a promising candidate for 3D dose verification. Gel dosimeters utilize sensitive chemicals (gels), which, upon irradiation, polymerize as a function of absorbed dose and therefore have the capability to uniquely record dose distribution in three dimensions. In optical-CT gel dosimetry, dose distribution is obtained from the CT 3D reconstruction of the optical attenuation coefficient of a gel phantom [2,3].

In an optical-CT scanner for gel dosimetry [1], a laser beam illuminates a cylindrical gel phantom and a photo-detector detects the transmitted light on the opposite side of the phantom. The light source and photo-detector are mechanically coupled and translated synchronously laterally with respect to the sample. After each lateral scan, the gel phantom is rotated. The scan-rotate process is repeated until the phantom has been rotated through 180°. Multiple optical projections acquired this way enable the reconstruction of the optical attenuation coefficient of the gel in a 2D transverse slice. Image reconstruction in different transverse slices is obtained by moving the gel phantom vertically, and 3D reconstruction is performed slice-by-slice. Absorbed dose is then reconstructed from the attenuation coefficient assuming a linear dependence between the two.

To avoid artifacts resulting from the light refraction at the air-gel interface, the gel phantom is placed in a square tank containing a liquid having the same refractive index as the gel. However, in practice, the refractive indexes of the gel and liquid do not match perfectly, resulting in scattering of the laser beam at the gel-liquid interface and therefore in spatial and angular deflections of the beam.
passing through the gel phantom. This ultimately results in loss of data. For large enough deflections, corresponding to beams incident at positions close to the edge of the phantom, the detector fails to capture the transmitted light. Therefore, it is of interest to investigate how the inability to scan the lateral sides of the phantom influences the reconstruction of the optical attenuation coefficient (and then dose reconstruction) in the central portion of the phantom that is usually irradiated.

The purpose of this study is to investigate the effect of incomplete data on dose reconstruction, and to identify the optimal design for an accurate gel dosimeter. Numerical experiments are carried out for a series of model attenuation coefficients of the gel phantom and various degrees of data availability corresponding to situations where only projections through certain central sections of the sample are available. A mathematical study for dose reconstruction based on truncated data is also performed in the framework of the so-called interior problem of Radon transform inversion.

2. Dose reconstruction
Since in optical-CT gel dosimetry dose is determined from the optical attenuation coefficient of the gel phantom, the problem of dose reconstruction becomes the problem of reconstruction of the optical attenuation coefficient. Optical CT tomography is based on inverting the Radon transform $(\text{line integral})$ of the total attenuation coefficient $\mu(x,y)$, 

$$\Phi(t,\theta) = R\mu(t,\theta) = \int_{-\infty}^{\infty} \mu(t\cos\theta - s\sin\theta, t\sin\theta + s\cos\theta) ds,$$  

(1)

to obtain the attenuation coefficient in a given transverse slice from measured data $\Phi(t,\theta)$. Here $t$ and $\theta$ are the position of the incident laser beam with respect to the central axis and the rotation angle, respectively. The problem of image reconstruction in the case of limited data due to the refractive index mismatch consists on recovering $\mu(x,y)$ from its Radon transform $\Phi(t,\theta)$ known for all possible angles $\theta$ but limited range of distances from the origin $t$.

First we address the problem of image reconstruction based on limited data numerically. For this purpose, we consider a model $\mu_{\text{model}}(x,y)$ for the attenuation coefficient of the gel. The measured data is calculated as the Radon transform (1) of the model attenuation coefficient $\mu_{\text{model}}(x,y)$. We assume that there are available only optical projections for a central cylindrical volume of radius $L \leq R$, where $R$ is the radius of the gel cylinder. This is equivalent to setting $\Phi(t,\theta) = 0$ for $|t| > L$. Finally, using data calculated this way, the attenuation coefficient is obtained by inverting (1) numerically using a filtered back projection algorithm with the Shepp-Logan filter.

In our study, the model attenuation coefficient is decomposed as $\mu_{\text{model}}(x,y) = \mu_0 + \delta(x,y)$, where $\mu_0$ is the optical attenuation coefficient of the non-irradiated gel, and $\delta(x,y)$ corresponds to the inhomogeneity produced by irradiation. We assume that the irradiated volume is a cylinder of radius $r < R$ having the same central axis as the gel phantom, and model $\delta\mu(x,y)$ as

$$\delta\mu(x,y) = \begin{cases} \mu_0, & (x^2 + y^2)^{1/2} \leq r \\ 0, & \text{otherwise} \end{cases}.$$  

(2)

Reconstructions of the total attenuation coefficient $\mu(x,y)$ are shown in figure 1 for $L/R = 100\%$, 85\% and 75\%. We note that image quality for the sides of the phantom depends on data availability. However, the image in the central part of the sample is little influenced by the loss of data. This can be also seen in figure 2, where we present cross sections through the reconstructed images. Also, figure 2 shows that $\mu(x,y)$ can be reconstructed with a relative error of less than 3\% in the central part of the sample even when only about 82.5\% of the projections are available, and this result does not depend on the size of the irradiated area (the graphs for different values $r$ of overlap).
Using geometrical optics considerations, we calculate the spatial deflection $\delta y$ of the laser beam at the detector position as a result of the refractive index mismatch $\delta n = n_g - n_l$, where $n_g$ and $n_l$ are the refractive indices of gel and liquid, respectively. This deflection is presented in figure 3 for $n_g = 1.3556$. Assuming a detector spatial width of 0.5 cm, we can estimate from figure 3 that more than 82.5% of data can be collected if the refractive index mismatch is less than 1.5%.

![Figure 1](image1.png)

**Figure 1:** Going from left to right, the figures show the model attenuation coefficient $\mu_{\text{model}}(x, y)$ and the reconstructed attenuation coefficient $\mu(x, y)$ for $L/R = 1, 0.85, \text{ and } 0.75$, for a sample where $r = R/2$.

Next, we address the problem of image reconstruction in the limited data case mathematically. It can be shown that the difference between any two solutions of this “interior problem” varies very little in the interior of the imaging domain [4]. For this purpose, assume that $\mu_1(x, y)$ and $\mu_2(x, y)$ are the true image function and the image function reconstructed from truncated data, respectively. If $u = \mu_1 - \mu_2$, then by the standard inversion formula of Radon transform one can obtain [4]

$$u(x, y) = \frac{1}{4\pi^2} \int_0^{2\pi} \int _{|t|+L} \frac{R u(t, \theta)}{(t-x \cos \theta - y \sin \theta)^2} dt \ d\theta$$

(3)

![Figure 2](image2.png)

**Figure 2:** a) The reconstructed attenuation coefficients $\mu(x, y)$ presented in Fig. 1 shown as cross sections along a line drawn through the center of the phantom in the directions parallel to the x-axis. b) The relative error, $\varepsilon = |\mu - \mu_{\text{model}}|/\mu_{\text{model}}$ for the central part of the sample for various sizes of the irradiated area.
Figure 3: Spatial deflection $\delta y$ of the laser beam caused by the refractive index mismatch $\delta n = n_g - n_i$ for (a) $\delta n > 0$ and (b) $\delta n < 0$, for the case where $n_g = 1.3556$, the tank side is 22 cm, and the detector is placed 2 cm away from the tank.

Using Cauchy-Schwarz inequality it follows that if $|(x, y)| \leq r < L$ then $|u(x, y) - u(0,0)| \leq C(L, r)\|\mathcal{R}u\|_{L_2}$, where $\|\mathcal{R}u\|_{L_2}$ is the $L_2$ norm of the function $\mathcal{R}u$. The value of $C(L, r)$ is small and depends on how much data is lost and how close to the center we want the estimate. For example, if $L = 0.8R$, and $r = 0.2R$ then $C(L, r) \approx 0.016$ [4]. On the other hand, using geometric considerations one can show [4] that $\|\mathcal{R}u\|_{L_2} \leq 2\sqrt{\pi(R^2 - L^2)^{1/4}}\|\mu_1\|_{L_2}$. The norm $\|\mu_1\|_{L_2}$ of the image function is a bounded quantity that can be estimated depending on the properties of the material. Since $L$ is close to $R$, and $C(L, r)$ is small, it follows that $u(x, y)$ is almost a constant, $u(x, y) = u(0,0) = c$ in the interior of the cylinder. Moreover, since certain parts of the image area are not irradiated, one may know ahead of time the values of $\mu_1(x_0, y_0)$ in that area. The values $\mu_2(x_0, y_0)$ will be reconstructed from truncated data. Hence one can compute the constant error $u(x_0, y_0) = \mu_1(x_0, y_0) - \mu_2(x_0, y_0) = c$. Finally, adding the computed constant $c$ to the reconstructed image $\mu_2$ we get $\mu(x, y) = \mu_2(x, y) + c$, which is a good approximation of the original image in the interior of the cylinder. The quantitative analysis of the relation between $L$ and $r$ to obtain the desired accuracy can be done numerically.

3. Conclusion
It was demonstrated numerically that the optical attenuation coefficient of the polymer gel and dose in an optical CT gel dosimeter can be reconstructed accurately in the central part of the dosimeter even if only incomplete experimental data are available. In particular, dose reconstruction with uncertainty less than 3% is possible even if only a central area of radius 82.5% of the radius of the gel cylinder is scanned. For usual gel dosimeters, this fraction of available data corresponds to a refractive index mismatch between the gel and liquid of 1.5%. It was also shown that a priori knowledge of the attenuation coefficient at a point in the sample enables an almost exact reconstruction of the attenuation coefficient at all points. This study provides a guideline for optimal design of optical-CT gel dosimeters.

4. References
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