Gluon contributions to the pion mass and light cone momentum fraction

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Introduction.— A striking feature of QCD is the large contribution of quarks to the mass and momentum of hadrons, so it is of fundamental interest to calculate the contributions of gluons from first principles using lattice QCD.

We calculate the matrix elements of the gluonic contributions to the energy-momentum tensor for a pion of mass 600 < M π < 1100 MeV in quenched lattice QCD. We find that gluons contribute (37 ± 8 ± 12)% of the pion’s light cone momentum. The bare matrix elements corresponding to the trace anomaly contribution to the pion mass are also obtained. The discretizations of the energy-momentum tensor we use have other promising applications, ranging from calculating the origin of hadron spin to QCD thermodynamics.

\begin{align}
\langle x \rangle_f (q^2) &= \sum_{f=u,d,...} \int_0^1 x dx \{ f(x, q^2) + f(x, q^2) \} \\
\langle x \rangle_g (q^2) &= \int_0^1 x dx g(x, q^2)
\end{align}

of the quark and gluon distribution functions \( f(x), \overline{f}(x) \) and \( g(x) \) acquire a precise field-theoretic meaning via the operator product expansion in QCD. They satisfy the well-known momentum sum rule (MSR) \( \langle x \rangle_f (q^2) + \langle x \rangle_g (q^2) = 1 \) and are related to the corresponding contributions to the energy-momentum tensor \( T_{\mu \nu} \) evaluated on the hadronic state. Separating the traceless part \( T_{\mu \nu}^{\text{traceless}} \) from the trace part \( S \) for gluons, denoted ‘g’, and quarks, denoted ‘f’, \( T_{\mu \nu} \) has the explicit form

\begin{align}
T_{\mu \nu} &= T_{\mu \nu}^{\text{traceless}} + \frac{1}{2} \delta_{\mu \nu} \langle S \rangle + \delta_{\mu \nu} \\
T_{\mu \nu}^{\text{traceless}} &= \frac{1}{4} \delta_{\mu \nu} E^a F^a_{\rho \sigma} - E^a_{\mu \rho} F^a_{\nu \sigma}, \\
S &= \beta(g)/(2g) \frac{F^a_{\rho \sigma} F^a_{\mu \nu}}{E_{\rho \sigma} E_{\mu \nu}}, \quad \delta_{\mu \nu} = [1 + \gamma_m(g)] \frac{\sum_f \bar{\psi}_f m\psi_f}{E_f^2 - p^2}
\end{align}

where \( \beta(g) \) is the beta-function, \( \gamma_m(g) \) is the anomalous dimension of the mass operator, and all expressions are written in Euclidean space. For an on-shell particle with four-momentum \( p = (i E_p, \mathbf{p}) \), \( E_p^2 = M^2 + \mathbf{p}^2 \), we have the relations

\begin{align}
\langle \Psi, \mathbf{p} | \hat{S} | \Psi, \mathbf{p} \rangle &= [E_p - \frac{1}{4} M^2 / E_p] \langle x \rangle_f + \langle x \rangle_g, \\
\langle \Psi, \mathbf{p} | \hat{S} | \Psi, \mathbf{p} \rangle &= (M^2 / E_p) b_{f,g}, \\
\langle x \rangle_f + \langle x \rangle_g &= b_f + b_g = 1,
\end{align}

where states are normalized according to \( \langle \mathbf{p} | \mathbf{p} \rangle = 1 \). We shall return to the renormalization of \( \langle x \rangle_f \) below.

Equation (4) shows that in the infinite momentum frame, where \( E_p \sim P \to \infty \), \( \langle x \rangle_f \) represents the momentum fraction arising from gluons, and calculating \( \langle x \rangle_f \) is the main goal of this work. In the rest frame, the gluon contribution of Eq. (4) to the hadron mass is \( \frac{1}{2} M (x) \). From Eq. (5) in the rest frame, the contribution of the trace anomaly \( S \) to the hadron mass is \( M (M) \), and in this work we perform the first step to calculate this matrix element as well.

Whereas non-singlet matrix elements can now be calculated to high precision in full QCD in the chiral regime \( \mu \ll M_{\pi} \), calculations of matrix elements of singlet operators are far less developed due to the computational challenges of calculating disconnected diagrams, which require all-to-all propagators, and matrix elements of gluon fields, which are notoriously noisy due to quantum fluctuations. The first attempt to calculate the quark momentum fraction was in the proton in [5], and was found to be numerically very challenging. In this exploratory study we treat the case of “heavy pions” with masses in the range 600 MeV < M π < 1000 MeV, where hadronic matrix elements in the quenched approximation, which neglects quark loops, are generally close to those in full QCD. The techniques developed here are applicable in full QCD calculations, and to the case of the proton.

Lattice formulation.— We use the Wilson gluon action

\( \frac{1}{g_0^2} \sum_{x, \mu \neq 0} \text{Tr} \{ 1 - P_{\mu \nu}(x) \} \), where \( P_{\mu \nu} \) is the plaquette, and the Wilson fermion action [6] at an inverse coupling \( 6 / g_0^2 \equiv \beta = 6.0 \), corresponding to a lattice spacing \( a = 0.093 \) fm for \( r_0 = 0.5 \) fm [7]. There are two distinct ways [8] to discretize the Euclidean gluon energy operator \( \hat{T}_{00} = \frac{1}{2} (-E^a \cdot E^a + B^a \cdot B^a) \) and the trace anomaly \( S = \frac{\beta(g)}{g} (E^a \cdot E^a + B^a \cdot B^a) \) on a hypercubic lattice.

The first, denoted ‘bp’ for bare-plaquette, uses a sum of bare plaquettes \( P_{\mu \nu} \) around a body-centered point \( x_0 = x + \frac{1}{2} a \sum_{\mu} \mu \), which, when summed over a time slice,
yields
\[ a^3 \sum_{x} T_{00}^{bp}(x) = \frac{2 a^3 V g_{00} Z_{c}(g_{0})}{\kappa_0} \sum_{x} \text{Re Tr} \left[ \sum_{k} P_{0k}(x) - \sum_{k<l} \frac{1}{2} \left[ P_{kl}(x) + P_{kl}(x + a \hat{0}) \right] \right], \]
\[ a^3 \sum_{x} S_{00}^{bp}(x) = \frac{2 a^3 V g_{00}^{2}}{2 \kappa_0} \sum_{x} \text{Re Tr} \times \left[ \sum_{k} (1 - P_{0k}(x)) + \sum_{k<l} (1 - \frac{1}{2} [P_{kl}(x) + P_{kl}(x + a \hat{0})]) \right]. \]

The other form, denoted ‘bc’ for bare clover, is
\[ T_{00}^{bc}(x) = \frac{\chi_{b}^{bc}(g_{0}) Z_{c}(g_{0})}{g_{00}^{2}} \text{Re Tr} \sum_{k} \left[ \tilde{F}_{0k}^2 - \sum_{k<l} \left( \tilde{F}_{kl}^2 \right) \right], \]
\[ S_{00}^{bc}(x) = \frac{\chi_{s}^{bc}(g_{0})}{g_{00}^{2}} \sum_{x} \text{Tr} \sum_{k} \left[ \tilde{F}_{0k}^2 + \sum_{k<l} \left( \tilde{F}_{kl}^2 \right) \right], \]

where \( \tilde{F}_{\mu \nu}(x) \) is the clover-shaped discretization of the field-strength tensor (see (14)). This form allows for the discretizations of off-diagonal elements of \( T_{\mu \nu} \) as well. Each of the normalization factors \( Z_{c}(g_{0}) \), \( \chi_{b}^{bc}(g_{0}) \) and \( \chi_{s}^{bc}(g_{0}) \) in Eq. (7S) is of the form \( 1 + O(g_{0}^{2}) \).

An additional freedom in discretization is local smoothing of the fields by replacing each link in Eqs. (7S) by a sum of a connected product of links joining the same two lattice points. This only changes the fields by higher dimension operators, and HYP smearing \([11]\) is particularly suited for this application because it preserves the symmetry between all Euclidean directions and is localized within a single hypercube. We use the original HYP-smearing parameters \([11]\), and project onto SU(3) as in \([12]\).

Our criteria for the choice of the discretization are to maximize the signal-to-noise ratio, minimize cutoff effects, and preserve locality as much as possible. The noisiest quantity we calculate is \( \chi \) on \( a \hat{0} \), of the operators \( \langle O \rangle^2 / \langle O \rangle^2 - 1 \), of the operators \( O = \sum_{x} \left( \langle \sigma(x) \rangle - \langle \sigma(0) \rangle \right) \) on a \( 6 \times 16^3 \) lattice at \( \beta = 6.0 \) for different discretizations described in the text. Right: The normalization \( \chi_{s}(g_{0}, a/L_{0}) \) (top) and \( \chi_{s}(g_{0}, a/L_{0}) \) (bottom) of the operator relative to the bare plaquette, determined on the same lattice.

### Table I: Left: the relative variance, \( \langle O^2 \rangle / \langle O \rangle^2 - 1 \), of the operators \( O = \sum_{x} \left( \langle \sigma(x) \rangle - \langle \sigma(0) \rangle \right) \) on a \( 6 \times 16^3 \) lattice at \( \beta = 6.0 \) for different discretizations described in the text. Right: the normalization \( \chi_{s}(g_{0}, a/L_{0}) \) (top) and \( \chi_{s}(g_{0}, a/L_{0}) \) (bottom) of the operator relative to the bare plaquette, determined on the same lattice.

| Operator Type | Relative Variance | Normalization |
|---------------|-------------------|---------------|
| bare plaquette | 26.4(71) 0.6518(43) | 0.54(20) 1.05(39) |
| hyp-plaquette  | 3.85(11) 0.3049(41) | 0.2975(72) 1.180(39) |
| hyp-clover     | 2.64(12) 0.474(13)  | 0.9951(77) 4.062(30) |

### FIG. 1: A study of cutoff effects: the normalization \( \chi_{s}(g_{0}, a/L_{0}) \) of three discretizations of \( T_{00} \) relative to the one based on the bare plaquette as a function of \( L_{0}/a \).

**Wilson gauge action \( \beta=6.0, (L_{0}/a) x16\)³**

**The gluon momentum fraction in the pion.** — We consider a triplet of Wilson quarks, labeled \( u, d, s \), with periodic boundary conditions in all directions and with common \( \kappa = 0.1515, 0.1530 \) and 0.1550 corresponding to pion masses approximately 1060, 890 and 620 MeV on lattices \( 32 \times 12^3 \), \( 32 \times 16^3 \), \( 48 \times 16^3 \) and \( 24^4 \). To calculate the gluonic momentum fraction in the pion, we define the
where, disregarding disconnected diagrams, \( \langle x \rangle_{g,\text{bare}} \) has been computed in [22] at the same bare parameters \((\beta = 6, \kappa = 0.1530)\). The factor \( Z_{T}(g_0) \) is the fermion analog of \( Z_{g}(g_0) \), see Eq. (7).
Finally, our result for the glue momentum fraction in a (heavy) pion is compatible with phenomenological determinations \cite{23, 24}, $(x)_{g}^{M_S} = 0.38(5)$ at $Q^2 = 4\text{GeV}^2$, based on Drell-Yan, prompt photoproduction, and the model assumption that sea quarks carry 10-20\% of the momentum. The agreement suggests a mild quark-mass dependence, but only a calculation in full QCD and at smaller masses can substantiate this. Our result at $Q^2 = 4\text{GeV}^2$ lies clearly below the $N_f = 3$ asymptotic glue momentum fraction of 0.64. The fact that our result and the valance quark momentum fraction, computed in \cite{20}, add up to 0.99(8)(12) suggests that the omitted disconnected diagrams are small.

Discussion of $b_g$.— In a chirally symmetric formulation of massless QCD, the trace anomaly is the only contribution to $S(x)$, and its matrix elements are renormalization group invariant. With Wilson fermions however, the absence of chiral symmetry implies that the trace anomaly acquires a linearly divergent contribution from the operator $\bar{\psi}\psi$. Thus our matrix elements $b_g^{(\text{bare})}$ should be regarded as intermediate results. The coefficient of the counterterm, as well as its disconnected diagrams, will have to be computed before we can quote a physical value for $b_g$ in the pion. Not surprisingly, $b_g^{(\text{bare})}$ shows a strong quark-mass dependence, since the missing disconnected diagrams are suppressed by $1/m$. We note that $b_g^{(\text{bare})} \sim 0.9(1)$ at the largest mass is of the same order of magnitude as Ji’s phenomenological estimate of $b_g$ in the proton \cite{1}, 0.85(5).

Conclusion.— We have computed the glue momentum fraction $\langle x \rangle_g$ in a pion of mass $0.6\text{GeV} < M_\pi < 1.06\text{GeV}$ using quenched lattice QCD simulations. We find 37(8)(12)\% at $T = 2\text{GeV}$, a result compatible with phenomenological determinations \cite{23, 24}.

Although it appears difficult to achieve precision at the percent level, the present method is applicable to full QCD with dynamical quarks. Presently the larger uncertainty comes from the normalization of the quark contribution to the renormalized $\langle x \rangle_g$, and could be reduced significantly by a one-loop calculation.

We also evaluated the bare trace anomaly contribution to the pion’s mass in the same framework. The counterterm remains to be calculated, but it will ultimately be preferable to use chiral fermions to avoid mixing with the lower dimensional fermion operator.

Finally, we remark that the freedom of choosing a numerically advantageous discretization of $T_{\mu\nu}$ has not been fully exploited in previous lattice simulations. The improvement that was essential in the present computation of the pion momentum fraction can be carried over to fully dynamical calculations and the exploration of other observables, such as the gluon contribution to the nucleon spin. It is also particularly promising for thermodynamic studies of pressure, energy density and transport coefficients.

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