UWB/PDR integrated indoor positioning method based on robust adaptive square root cubature Kalman filter

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Abstract. Aiming at the problem that the ultra wide band (UWB) wireless signal is easy to be interfered, a robust adaptive square root cubature Kalman filter (RA-SRCKF) is proposed to be used in UWB/PDR integrated indoor positioning. RA-SRCKF is based on square root cubature Kalman filter (SRCKF) and its statistical characteristic of measurement noise is updated with the method of first estimating and then modifying. The experimental results show that the proposed RA-SRCKF can effectively improve the accuracy and stability of UWB/PDR integrated positioning.

Keywords: integrated positioning, ultra wide band, pedestrian dead reckoning, robust, adaptive.

1. Introduction
With the development of navigation and positioning technology, location-based services are playing an increasingly important role in people's life. Due to the influence of complex environment, commonly used satellite positioning systems cannot meet indoor positioning requirements [1]. Ultra wide band (UWB) can achieve centimeter level positioning accuracy in ideal indoor environment [2]. However, its signal transmission is vulnerable to non-line-of-sight (NLOS) interference, and positioning accuracy will decline sharply in complex indoor environment. Pedestrian dead reckoning (PDR) method can realize positioning without relying on external information, but its positioning error will accumulate over time [3]. Therefore, UWB/PDR integrated positioning can achieve higher positioning performance.

The data fusion algorithm is one of the key factors affecting the precision of integrated positioning system. The data fusion algorithms such as Kalman filter (KF), extended Kalman filter (EKF), unscented Kalman filter (UKF) and cubature Kalman filter (CKF) are commonly used. CKF has higher estimation accuracy. In order to solve the problem that the error covariance matrix in CKF is not positive definite which leads to the filtering stop, the square root cubature Kalman filter (SRCKF) algorithm was proposed in [4]. SRCKF has higher tracking accuracy than EKF and UKF, and SRCKF has faster computing speed than UKF [5]. In order to improve the estimation accuracy of CKF in the positioning application of the wireless sensor network, a third-order noise estimator was proposed in [6] which has higher estimation accuracy than the SAGE-HUSA method. Besides, a new method of robust cubature Kalman filtering is proposed in [7] which can effectively reduce the influence of abnormal measurements measurement on the filtering results. In order to improve the positioning accuracy of UWB/PDR integrated positioning system, this paper proposes a UWB/PDR integrated positioning
method based on robust adaptive square root cubature Kalman filter (RA-SRCKF). In this method, SRCKF is improved, and the statistical characteristics of measurement noise are estimated online first and then corrected by robust correction.

This paper’s structure is organized as follows. In section 2, the basic principle of UWB/PDR integrated positioning system and its system model are introduced. In Section 3, a new robust adaptive square root cubature Kalman filter is proposed. In Section 4, the method proposed based on RA-SRCKF is verified by experiment. Finally, the paper is concluded in Section 5.

2. Basic principle of UWB/PDR integrated positioning system and its mathematical model

2.1. Basic principle of UWB/PDR integrated positioning system

The principle block diagram of UWB/PDR integrated positioning system which includes UWB positioning system, PDR positioning system and data fusion algorithm is shown in Fig. 1. PDR positioning system is the main reference system and the UWB positioning result is the measurement information. The filtering algorithm fuses the UWB positioning result with the PDR positioning result and the PDR positioning result is modified by the fusion positioning result, so as to avoid the accumulation of PDR positioning error and to reduce the influence of UWB positioning error.

![Figure 1. The structure of the system.](image)

The UWB positioning equipment includes a tag carried by the pedestrian and three fixed base stations. When the pedestrian with the tag walks indoors, the distance from the tag to some base station can be obtained according to the signal transmission time between the tag and the base station and the pedestrian position can be obtained by solving the following equations

\[ d_i = \sqrt{(x_k - x_i)^2 + (y_k - y_i)^2}, \quad i = 1, 2, 3 \]  \hspace{1cm} (1)

Where \( d_i \) is the distance from the tag to the \( i \)th base station, \((x_k, y_k)\) is the pedestrian position and \((x_i, y_i)\) is the known base station position. The PDR positioning system calculates the step length and heading angle according to the acceleration and the angular rate measured by the inertial sensors (acceleration and gyroscope) carried by the pedestrian, and then calculates the pedestrian position with the following formula

\[
\begin{align*}
\dot{x}_k &= x_{k-1} + L_{k-1} \cdot \cos \alpha_{k-1} \\
\dot{y}_k &= y_{k-1} + L_{k-1} \cdot \sin \alpha_{k-1}
\end{align*}
\]  \hspace{1cm} (2)

Where \( \alpha_{k-1} \) is heading angle which is obtained by integrating the angular rate, \( L_{k-1} \) is the step length which is calculated from the maximum acceleration and the minimum acceleration in one step. The calculation formula of \( L_{k-1} \) is as follow

\[ L_{k-1} = K \sqrt{a_{\text{max}} - a_{\text{min}}} \]  \hspace{1cm} (3)

Where \( K \) is the empirical constant obtained with experiments and \( K = 0.45 \) in this paper.

2.2. Mathematical model of UWB/PDR integrated positioning system

According to the basic principle of UWB/PDR integrated positioning shown in Figure 1, the state equation of the system is established as equation (4)
\[ X_k = AX_{k-1} + L_{k-1} \theta_{k-1} + W_{k-1} \]  \hspace{1cm} (4)

Where \( X_k = [x_k \ y_k]^T \) is the state vector, \( A \) is the Identity matrix, \( \theta_{k-1} = [\cos \alpha_{k-1} \ \sin \alpha_{k-1}]^T \), \( W_{k-1} \sim (0, Q_{k-1}) \) represents the system noise.

We choose the UWB location result as the measurement vector and the measurement equation of the system is as follows

\[ Z_k = HX_k + V_k \]  \hspace{1cm} (5)

Where \( H \) is the Identity matrix, \( V_k \sim (0, R_k) \) represents the measurement noise.

3. Design of robust adaptive SRCKF

3.1. Traditional SRCKF

The traditional SRCKF includes two stages. Initialize the system state and state covariance matrix as

\[ \hat{x}_0 = E[x_0] \]  \hspace{1cm} (6)

\[ P_0 = E\left((x_0 - \hat{x}_0) (x_0 - \hat{x}_0)^T\right) \]  \hspace{1cm} (7)

The steps of SRCKF are as follows[4]

Stage 1: state prediction

i. Decompose the state covariance matrix with Cholesky decomposition

\[ P_{k-1} = S_{k-1} S_{k-1}^T \]  \hspace{1cm} (8)

ii. Construct cubature points and propagate them with the state equation

\[ \xi_i = \sqrt{m/2} \times [1]\]  \hspace{1cm} (9)

\[ X_{i,k-1} = S_{k-1} \xi_i + x_{k-1} \]  \hspace{1cm} (10)

\[ X_k = AX_{i,k-1} + v_{k-1} \]  \hspace{1cm} (11)

Where \([1]_i \) is the \( i \)th column of the point set \([1]_i \), \( i = 1, 2, \ldots, m \), \( m = 2n \) is the number of cubature points, \( n \) is the dimension of the state vector.

iii. Calculate one-step prediction

\[ \hat{x}_k = \frac{1}{m} \sum_{i=1}^{m} X_k \]  \hspace{1cm} (12)

iv. Calculate the square root matrix of prediction error covariance matrix

\[ S_k = Tr(\{X_k S_{0,k-1}\}) \]  \hspace{1cm} (13)

Where \( Tr(\bullet) \) is represents QR decomposition, \( S_{0,k} \) is Square root matrix of \( Q_{k-1} \).

Stage 2: Measurement update

i. Construct cubature points and propagate them with the measurement equation

\[ X_{l,k} = S_k \xi_l + \hat{x}_k, \ i = 1, 2, \ldots, m \]  \hspace{1cm} (15)

\[ Y_{l,k} = HX_{l,k} + V_k, \ i = 1, 2, \ldots, m \]  \hspace{1cm} (16)

\[ \hat{z}_k = \frac{1}{m} \sum_{i=1}^{m} Y_{l,k} \]  \hspace{1cm} (17)

\[ S_{zz,k} = Tr(\{Y_k S_{R,k}\}) \]  \hspace{1cm} (19)

Where \( S_{R,k} \) is Square root matrix of \( R_k \).

iv. Computing the innovation covariance matrix

\[ P_{zz,k} = S_{zz,k} S_{zz,k}^T \]  \hspace{1cm} (20)

v. Calculation of cross covariance matrix

\[ X_k = \frac{1}{m} \left[ Y_{1,k} - \hat{z}_k Y_{2,k} - \hat{z}_k \cdots Y_{m,k} - \hat{z}_k \right] \]  \hspace{1cm} (18)

\[ P_{xz,k} = X_k Y_k^T \]  \hspace{1cm} (21)

vi. Calculate the filter gain and estimate the state

\[ K_k = (P_{xz,k}/S_{zz,k})S_{zz,k} \]  \hspace{1cm} (23)

\[ x_k = \hat{x}_k + K_k (z_k - \hat{z}_k) \]  \hspace{1cm} (24)
vii. Calculate the square root matrix of the estimated state covariance matrix
\[ S_k = \text{Triangularize} \left([X_k - K_k y_k K_k S_{R,k}]\right) \]  
(25)

3.2. Robust and adaptive method
In order to improve the estimation accuracy and reliability of SRCKF, a robust adaptive square root cubature Kalman filter algorithm (RA-SRCKF) is proposed by combining the noise estimation method in [6] and the robust filtering method in [9].

In RA-SRCKF measurement noise estimation includes the following two steps:
1. At the end of SRCKF described in 3.1, the improved sage-Husa method is used to estimate the measurement noise for the next iteration cycle, the estimation formulas are as follows[7]
\[ x_{i,k} = S_k x_{i,k} + x_{k,i} \]
\[ y_{i,k} = H x_{i,k} + V_{k,i} \]
\[ Z_k = \frac{1}{m} \sum_{i=1}^{m} y_{i,k} y_{i,k}^T - Z_k Z_k^T \]
(26-28)

\[ R_{k+1} = (1 - d_k) R_k + d_k \left[ \text{diag} \left( (\hat{Z}_k - Z_k) (\hat{Z}_k - Z_k)^T \right) + \frac{1}{m} \sum_{i=1}^{m} y_{i,k} y_{i,k}^T - Z_k Z_k^T \right] \]
(29)

\[ d_k = \frac{1 - b}{1 - b^{k+1}} \]
(30)

Where \( b \) is forgetting factor and \( b \) is set to 0.96 in this paper.

2. Between equation(18) and equation(19) in SRCKF, Huber method is used to calculate the robust factor \( \rho \) and \( \rho \) is used to modify the measurement noise estimated by equation (29)
\[ R_k = \rho R_k \]
(31)

where \( \rho = \text{diag}(\rho_1, \rho_2, \ldots, \rho_N) \), \( N \) is the dimension of measurement vector. When the measurement information is normal, let \( \rho_i = 1 \), otherwise modify \( R_k \) with \( \rho_i \). \( \rho_i (i = 1, 2, \ldots, N) \) are calculated as follows[9]
\[ \rho_i = \begin{cases} 
1 & \frac{1}{c} \left| \Delta \bar{v}_{i,k} \right| \leq C \\
\frac{\Delta \bar{v}_{i,k}}{\sigma_{i,k}} & \text{other} 
\end{cases} \]
(32)

\[ \Delta \bar{v}_{i,k} = \frac{\bar{v}_{i,k}}{\sigma_{i,k}} \]
(33)

Where \( C \) is a experience value, \( \bar{v}_{i,k} = (z_k - \hat{z}_k) i, \sigma_{i,k} = (P_{zz,k})_{ii} \).

4. Experiment
In order to verify the effectiveness of RA-SRCKF algorithm, we have conducted a semi physical simulation experiment in which bind the uwb-mini3s module designed by Yanchuang Internet of things Technology Co., Ltd and nine-aixs inertial measurement sensor(IMU) designed by Shenzhen Wit Intelligent Co. to the pedestrian's chest, collect the sensor data through serial port when the pedestrian walks along the predetermined route indoors, and realize the integrated positioning algorithm in MATLAB. The deployed indoor experimental environment is shown in Fig.2, sampling frequency of uwb-mini3s module is 3.5Hz and frequency sampling of IMU is 100Hz. Set the origin of the plane coordinate system to the location of base station 0(BS0). In this paper, root means square error (RMSE) is used to evaluate the performance of the algorithm
\[ \zeta_{\text{RMSE}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \hat{x}_i - x_i \right)^2 + \left( \hat{y}_i - y_i \right)^2} \]
(34)

Where \( (\hat{x}_i, \hat{y}_i) \) is the estimated location, and \( (x_i, y_i) \) is the true coordinate.
Set the initial value of measurement noise covariance as $R_0 = \text{diag}([10 10])$. KF, SRCKF, A-SRCKF and RA-SRCKF are used as data fusion algorithms in UWB/PDR integrated positioning, respectively. Positioning results is shown in figure.3, and the average RMSE is shown in Table.1.

As shown in Fig.3 and Table.1, the accuracy of UWB/PDR integrated positioning is higher than the single UWB or PDR positioning, the position error of SRCKF is smaller than KF which shows that the nonlinear Kalman filter is more suitable for UWB/PDR integrated positioning than Kalman filter, and the position error of RA-SRCKF is reduced by 30.4% and 10% compared with SRCKF and A-SRCKF, respectively. It shows that the combination of robust estimation and measurement noise estimation makes the accuracy and reliability of RA-SRCKF higher.

5. Conclusion
In order to improve the positioning accuracy of UWB / PDR combination, we proposed a UWB/PDR integrated positioning method based on RA-SRCKF in this paper. RA-SRCKF is based on SRCKF. In RA-SRCKF an improved sage Husa estimator is used to estimate the measurement noise covariance matrix to adapt to the change of UWB signal noise firstly and a robust factor is constructed to modify the statistical characteristics of measurement noise secondly, so as to eliminate the influence of abnormal UWB measurements on the filtering results. The experimental results show that RA-SRCKF has higher filtering performance than SRCKF and A-SRCKF and RA-SRCKF can effectively improve the accuracy and stability of UWB/PDR integrated positioning system.
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