A two-dimensional finite element method to calculate the AC loss in superconducting cables, wires and coated conductors

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Abstract. In order to utilize HTS conductors in AC electrical devices, it is very important to be able to understand the characteristics of HTS materials in the AC electromagnetic conditions and give an accurate estimate of the AC loss. A numerical method is proposed in this paper to estimate the AC loss in superconducting conductors including MgB2 wires and YBCO coated conductors. This method is based on solving a set of partial differential equations in which the magnetic field is used as the state variable to get the current and electric field distributions in the cross sections of the conductors and hence the AC loss can be calculated. This method is used to model a single-element and a multi-element MgB2 wires. The results demonstrate that the multi-element MgB2 wire has a lower AC loss than a single-element one when carrying the same current. The model is also used to simulate YBCO coated conductors by simplifying the superconducting thin tape into a one-dimensional region where the thickness of the coated conductor can be ignored. The results show a good agreement with the measurement.

1. Introduction
Superconductors have huge potentials in the area of power generation and transmission. In the past, a great amount of research has focused on the fabrication and application of superconducting cables and wires using high-Tc and low-Tc superconducting material. The development of the materials and the increasing potential of industrial application of superconducting wires and cables require a fast, accurate and reliable analysis tool to calculate the electromagnetic behaviour in the conductors. AC loss, as one of the most important properties in superconducting power transmission, received significant concerns. In the past several years, a range of methods have been proposed to solve the AC loss problem in the superconducting tapes and cables. An analytical method was proposed by Norris [1] in 1969, to estimate the AC loss in a self-field problem. Brandt [2] proposed a theoretical solution of the AC loss for a superconducting strip in the perpendicular external magnetic field. Recently, finite element method (FEM) has been intensively applied to develop the numerical solutions of the AC loss problem in superconductors. Some 2D and 3D FEM models, based on the critical-state model, have been proposed to solve the AC loss in superconducting cables [3, 4].

In this paper, a numerical model is proposed to simulate the application of superconducting wires and tapes carrying transport current. Firstly, the model is used to estimate the AC loss in MgB2 wires

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in self-field condition. The basic parameters of the sample are determined by experiment. The parameters in the numerical model are selected to make the simulated $E$-$J$ curve match best to that obtained by measurement. After that, the model is used to calculate the current distribution and magnetic field distribution in the MgB$_2$ wires and hence predict the AC loss, which so far is still a challenge to be measured in experiment. After that, the model is applied to simulate the AC losses in YBCO coated conductors. Assumptions are made to simplify the two-dimensional superconducting thin tape with neglectable thickness to a one-dimensional region to save significant amount of computational time and CPU consumption.

2. Numerical method

The numerical solver of critical state used in this paper is based on a set of Maxwell’s equations using magnetic field $H$, and $H$, as the knowns. The equations are incorporated with a commercial finite element software Comsol Multiphysics to give fast and accurate solutions.

According to Faraday’s Law,

$$\nabla \times E = -\mu_0 \mu_r \frac{\partial H}{\partial t}$$  \hspace{1cm} (1)

The magnetic flux is flowing in the x-y plane and the magnetic field is used as the dependent variable $\mathbf{H} = [H_x, H_y]$ representing the components of the magnetic field in the x and y directions. The induced electric field and the current density only have the components in the z-direction. So $E = E_{sc,z}$ and $J = J_{sc,z}$. The electrical behaviour of the superconducting material is represented by the $E$-$J$ power law. In this case, it is expressed as:

$$E_{sc,z} = E_0 \left( \frac{J_{sc,z}}{J_c(B)} \right)^n$$  \hspace{1cm} (2)

where $J_c$ is the critical current density (defined with the standard $1\mu$V/cm electric field criterion). $E_0 = 1 \times 10^{-4}$ V/m and the value of $n$ is determined by the characteristics of the superconducting materials.

According to Ampere’s law:

$$\nabla \times H = J$$  \hspace{1cm} (3)

Since $J_{sc,z}$ only has the component in the z-direction, Ampere’s law in this case is expressed as:

$$J_{sc,z} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}$$  \hspace{1cm} (4)

Equations (1), (2) and (4) consist of the basic coupled equations of this numerical method, with some boundary conditions, they can be solved by many finite element software’s to calculate the current and field distributions in the superconductors.

3. Monocore and multifilamentary MgB$_2$ wires

In this section, the numerical results of the electromagnetic properties of MgB$_2$ wires will be presented. The samples are fabricated by Hyper Tech Research, Inc., USA and the measurement and test were preformed at IRC in Superconductivity, Cambridge. Two samples will be investigated: sample 1 is a monofilamentary MgB$_2$ wire with sheaths of CuNi and sample 2 is a CuNi-sheathed multifilamentary wire with Cu stabilization. $E$-$J$ characteristics and quench properties of these MgB$_2$ straight wires were measured at 25K in self-field at 50Hz. The critical current density $J_c$ to be used in the numerical model is determined by the measured results. The $n$-value in $E$-$J$ power law is selected by making the calculated $E$-$J$ curve fit best to that obtained by experiment.

The geometry of the monocore MgB$_2$ model is shown in Figure 1. $E$-$J$ power law is used with $n=15$ and constant $J_c=1.37 \times 10^9$A/m is applied. The boundary condition between the superconducting wires and air is continuity:

$$n \times (H_x - H_z) = 0$$  \hspace{1cm} (5)

Two boundary conditions need to be applied to the outer boundary of the air subdomain. The first one is magnetic insulation as shown in (6):
Ampere’s law is used as the other boundary condition, which is the line integration of the tangential component of the magnetic field on the boundary equal to the forced transport current:

\[
I_{\text{transport}} = \oint_{\text{Boundary1}} \mathbf{H} \cdot d\mathbf{l}
\]

(7)

To model the multifilamentary MgB\(_2\) wires, some modifications have to be made upon the monocore model. The multifilamentary wire consists of 18 filaments. To optimize the FEM simulation, the mesh quality of each sub-element should be equally fine, which will lead to a huge number of elements in the finite element method. To save memory and CPU computational time, the method of symmetrical images can be used. The property of rotational symmetry allows the numerical model to be considered from 1/6 of the real geometry and only 3 filaments need to be calculated. As shown in Figure 2. The equations of the superconducting region and air region are exactly the same as those in the monocore MgB\(_2\) model. The critical current density is \(4.1 \times 10^9\) A/m and E-J power law is used with \(n=5\) in this case. A 50Hz sinusoidal current with a series of different amplitudes is applied to both of the two wires. Two cycles with 200 time steps per cycle are simulated. The amplitude of transport current \(I_0\) varies from 60A to 600A.

Figure 3 and 4 show the results of current distributions along the cross section of the MgB\(_2\) wires for monocore and multifilamentary respectively. Three rows in the figure represent the cases of \(I_0=100\)A, \(I_0=300\)A and \(I_0=600\)A, respectively. The results at three time steps of \(\omega t=0\), \(\omega t=\pi/2\) and \(\omega t=\pi\) are shown in three columns. The pictures in the same row demonstrate the evolution of the current in a cycle. When \(\omega t=0\) or \(\omega t=\pi\), the net current \(I_{\text{transport}}=I_0 \times \sin(\omega \times t)\) in the wires is equal to zero; at this moment, the cross section of the wire is half occupied by positive current and half occupied by negative current, thus giving a total zero net current. When \(\omega t=\pi/2\), the net current reaches the maximum negative value so that the negative current distributes almost uniformly along the cross section within the penetration depth. If we focus on every column in Figure 3, the procedure of current penetration can be clearly found. When the applied current is small (\(I_0=100\)A), the cross section of the superconducting wire is divided into three regions: a region with positive current, a region with reverse current and a current-free region. As the applied transport current increases, the central current-free region fades away gradually and eventually the whole cross section of the wire is occupied by uniform current (the moment of \(I_0=600\)A and \(\omega t=\pi/2\)).
It can be found in figure 4 that, for multifilamentary MgB$_2$ wire, the current not only tends to flow on the boundary of the whole wire but also has the tendency to flow on the boundary of every single filament which leads to the crescent shape of current distribution in every filament rather than a regular segment of a circle. Comparing with the results of monocore MgB$_2$ wire, the multifilamentary wire is more resistant to penetration even though the total cross-section area of superconducting material in multifilamentary wires is smaller than that in monocore wires. 600A of peak current can fully penetrate the monocore MgB$_2$ wire but can only penetrate 2/3 of the multifilamentary one. These results indicate that the multifilamentary wire has a larger current carrying capability than the monocore wire, which has been proved by experiment.

The AC losses for both monocore and multifilamentary MgB$_2$ wires are plotted together in log-log scale in Figure 5 for comparison. It can be clearly seen that the value of AC loss in multifilamentary wire is of the order of 1/10 of that in monocore wire. In addition, the gradient of the curve of monocore wire is larger than the gradient of the multifilamentary one, which means as the current increases, the AC loss in monocore wire increases faster than that of multifilamentary wire. Another characteristic that can be found in this figure is that both of these curves have an inflexion point which divides the curves into two sections with different values of gradient. These inflexion points
correspond to the full penetration currents of these two wires. When the applied current increases, the superconductor wires have the trend to either have a larger volume of current or larger magnitude of current. When that wire is fully penetrated (it has reached the maximum current carrying condition), there is no spare volume for new current to flow, so the magnitude of the current will increase significantly. According to $E$-$J$ power law, when current exceeds the critical value, the value of electric field will increase dramatically, as does the value of AC loss, which is proportional to the product of $J$ and $E$. These results indicate that the superconducting devices will suffer from a serious AC loss problem when it is used with a current much higher than the critical current.

4. AC loss in YBCO coated conductors

The numerical simulation of YBCO coated conductors is a real challenge because of the high geometric aspect ratio (typically 2000-10000) of YBCO tapes. To discretize this geometry, an extremely fine mesh quality with a huge number of elements is required to optimize the FEM simulation and this always leads to an enormous consumption of CPU time and memory. In this section, the numerical method proposed in this paper is going to be developed to calculate the AC loss in YBCO coated conductors in self field condition.

To model the YBCO thin tape, the thickness of the sample has been ignored, therefore the superconducting region is simplified from a two-dimensional (2D) cross section to a one-dimensional (1D) “line” region, as shown in Figure 6. The sample is infinitely long in the z-direction. The x-y plane is picked up as the plane of interest (plane $\alpha$) in the model. The magnetic fields ($H_x$ and $H_y$) are flowing parallel to the x-y plane, while the current ($J_z$) is perpendicular to the x-y plane. Because the thickness of the YBCO tape is numerically zero, an assumption can be drawn that the magnetic field in the x-direction has no influence on the current distribution of the YBCO tape, and hence the $H_x$ can be eliminated from (1) and (4). Concerning these particular conditions, (1) and (4) can be rewritten as:

$$\frac{\partial E_z}{\partial x} = -\mu_0 H_y \frac{\partial H_y}{\partial t}$$

$$\frac{\partial H_y}{\partial x} = J_z$$

Equations (9) and (10) coupled with (2) consist of the basic equations needed to be solved. To verify this numerical approach, a set of experiments have been performed in order to measure both the transport losses and the magnetization losses [5]. All the measurements presented here are carried out on the coated conductors provided by Superconductivity Technology Center of Los Alamos National Laboratory. The critical current density of the sample $J_c=2.3 \times 10^5$ A/m$^2$ was measured by the general four-terminal-electric method. The AC losses of the YBCO tape in self field was measured and the results will be shown together with the numerical results in the following paragraphs.

A series of 50Hz sinusoidal currents with amplitude up to 160A are applied to the tape in the model. Figure 7 shows the calculated profiles of the current distribution and the magnetic field distribution across the width (x-axis) of the sample when the magnitude of the transport current is equal to 90A. The moments of $ot = \pi / 4$ and $ot = \pi / 2$ are shown to illustrate the procession of the current and the magnetic field penetrating into the centre of the sample. According to the current profiles the maximum current is larger than the critical current density $J_c$. This is because the $E$-$J$ power law is used with the power index $n=21$, this finite number of the power index allows the current density to exceed the value of $J_c$. As the number of $n$ increases the value of the current density is clamped closer to $J_c$ and, eventually, when $n = \infty$ it corresponds to Brandt’s assumption [2].

The total transport losses of the sample in self field have been calculated with several different peak values of transport current up to 160A. The results, in Figure 8, are compared with the experimental results. Good agreement between the numerical and experimental results was found for AC transport currents. The transition of the AC loss in the superconducting tape is indicated by both the numerical calculations and experimental results when the losses increased at approximately 139A. This coincides with an increase of the non-linearity as the current approaches the critical current.
5. Conclusion

The electromagnetic properties of superconducting wires and tapes were investigated both by measurement and the numerical implementation of critical state models of superconductivity. A set of Maxwell’s equations with the unknown of magnetic field formed the basis of the models and the $E$–$J$ constitutive law together with H-formulations was used to calculate the current distribution and electromagnetic fields along the cross section of the superconductors, and then the AC losses of the wires were estimated. This numerical method was used to investigate a monocore MgB$_2$ wire and a multifilamentary MgB$_2$ wire under the same electromagnetic condition and it is shown that the multifilamentary MgB$_2$ wire exhibits less AC loss than the monocore wire when carrying the same mount of current. The H-formulations presented in this paper were also developed to solve the AC loss in YBCO coated conductors where assumptions have been drawn to simplify the thin tape as a one dimensional region to save computational time. The results demonstrated a good agreement with the measurement.

References

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