Optical transition radiation in presence of acoustic waves for an oblique incidence

A R Mkrtchyan, V V Parazian and A A Saharian
Institute of Applied Problems in Physics, 25 Nersisyan Street, 0014 Yerevan, Armenia
E-mail: vparazian@gmail.com, saharian@ysu.am

Abstract. Forward transition radiation is considered in an ultrasonic superlattice excited in a finite thickness plate under oblique incidence of relativistic electrons. We investigate the influence of acoustic waves on both the intensity and polarization of the radiation. In the quasi-classical approximation, formula is derived for the spectral-angular distribution of the radiation intensity. It is shown that the acoustic waves generate new resonance peaks in the spectral and angular distributions. The heights and the location of the peaks can be controlled by choosing the parameters of the acoustic wave. The numerical examples are given for a plate of fused quartz.

1. Introduction
Transition radiation is produced when a uniformly moving charged particle crosses an interface between two media with different dielectric properties. Such radiation has a number of remarkable properties and at present it has found many important applications (see, for instance, Refs. [1]-[6]). In particular, the transition radiation is widely used for particle identification, for the measurement of transverse size, divergence and energy of electron and proton beams. An enhancement for the transition radiation intensity may be achieved by using the interference between the radiation emitted by many interfaces in a multilayer structure. From the point of view of controlling the parameters of various radiation processes in a medium, it is of interest to investigate the influence of external fields, such as acoustic waves, temperature gradient etc., on the corresponding characteristics.

In Refs. [7, 8] we have considered the X-ray and optical transition radiation from ultrarelativistic electrons in an ultrasonic superlattice excited in melted quartz plate. The radiation from a charged particle for a semi-infinite laminated medium has been recently discussed in [9]. In these references the transition radiation is considered under normal incidence of a charged particle upon the interface of the plate. In the present paper we generalize the corresponding results for oblique incidence. The angle between the particle velocity and the normal to the interface is an additional parameter which can be used for the control of angular-frequency distribution and the polarization of the radiation.

We have organized the paper as follows. In the next section the intensity of the radiation in forward direction is investigated for both parallel and perpendicular polarizations by using the quasi-classical approximation. In section 3, numerical examples are presented for the radiation intensity in the case of a plate made of fused quartz. The main results are summarized in section 4.
2. Spectral-angular distribution of the radiation

Consider the transition radiation from a charged particle in a plate of thickness \( l \) with dielectric permittivity \( \varepsilon_0 \) which is immersed into a homogeneous medium with permittivity \( \varepsilon_1 \). We assume that the \( z \) axis of the Cartesian coordinate system \((x, y, z)\) is directed along the normal to the plate and the boundaries of the plate are located at \( z = -l \) and \( z = 0 \). The trajectory of the particle with velocity \( \mathbf{v} \) is in the \((x, z)\) plane and forms with the \( z \) axis a given angle \( \alpha \). For the particle coordinates one has

\[
x(t) = v(t - t_0) \sin \alpha, \quad y(t) = 0, \quad z(t) = -l + v(t - t_0) \cos \alpha.
\]

Here we do not take multiple scattering effects into account (for the discussion of these effects see [1]). So, we can write \( \mathbf{v} = (v \sin \alpha, 0, v \cos \alpha) \). In the presence of longitudinal ultrasonic vibrations, excited in the plate along the normal to its surface, the dielectric permittivity can be written in the form

\[
\varepsilon(z) = \begin{cases} 
\varepsilon_0 + \Delta\varepsilon \cos (k_s z + \omega_s t + \phi), & -l \leq z \leq 0, \\
\varepsilon_1, & z < -l, \quad z > 0,
\end{cases}
\]

where \( \omega_s, k_s \) are the cyclic frequency and the wave number of the ultrasound, \( \phi \) is the initial phase. In the discussion below we will assume that during the transit time of the particle the dielectric permittivity in the superlattice is not notably changed. For relativistic particles and for the plate thickness \( l \lesssim 1 \) cm this condition is satisfied for \( \omega_s \ll 10^{12} \) Hz.

For the radiation in the spectral range \( \omega \gg k_s c \), the presence of the small parameter \( k_s c/\omega \) allows us to employ the quasi-classical approximation for the investigation of the radiation field in the forward direction. The latter may be evaluated in a way similar to that discussed in [1] for the case of normal incidence (see also [10]). Considering the radiation in the region \( z > 0 \), we denote by \( \theta \) and \( \varphi \) the polar and azimuthal angles for the radiated photon with respect to the axis \( z \). For the corresponding wave vector one has

\[
k = \frac{\omega}{c} \sqrt{\varepsilon_1} (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta).
\]

Unlike to the case of the normal incidence, discussed in [7, 8], in the case of oblique incidence we have radiations with two different polarizations. For the first one the electric field is parallel to the observation plane (plane formed by the vectors \( \mathbf{k} \) and \( \mathbf{v} \)) and for the second one the electric field is perpendicular to the observation plane. These two polarizations are referred as parallel and perpendicular polarizations, respectively.

First we consider the spectral-angular density of the total radiation intensity averaged over the phase of particle flight into the plate. In the angular region \( \sin \theta < \sqrt{\varepsilon_0/\varepsilon_1} \), it is given by the expression

\[
I(\omega, \theta, \varphi) = \frac{dI(\omega, \theta, \varphi)}{d\omega d\theta d\varphi} = \frac{e^2 \sin^3 \theta \cos^2 \alpha}{\pi^2 c \sqrt{\varepsilon_1}} \sum_{m=-\infty}^{+\infty} J_m^2(a(\theta)) \times \left[ \frac{P(\theta, \varphi, \alpha)}{V(\theta, \varphi, \alpha)} - \frac{Q(\theta, \varphi, \alpha)}{U_m(\theta, \varphi, \alpha)} \left( \cos^{1/2} \theta \left( \varepsilon_0/\varepsilon_1 - \sin^2 \theta \right)^{1/2} \right) \right]^2 \\
\times \sin^2 \left( \frac{\omega \sqrt{\varepsilon_1}}{2c \cos \alpha} U_m(\theta, \varphi, \alpha) \right),
\]

where \( e \) is the charge of the particle, \( J_m(x) \) is the Bessel function, and

\[
a(\theta) = \frac{\omega \Delta \varepsilon / (2ck_s)}{\sqrt{\varepsilon_0 - \varepsilon_1 \sin^2 \theta}}.
\]
In (4) we have defined the functions

\[ U_m(\theta, \varphi, \alpha) = \frac{1}{\beta_1} - \sin \theta \cos \varphi \sin \alpha - \cos \alpha \sqrt{\varepsilon_0/\varepsilon_1 - \sin^2 \theta} - \frac{m k_0 c}{\omega \sqrt{\varepsilon_1}} \cos \alpha, \]

\[ V(\theta, \varphi, \alpha) = \frac{1}{\beta_1} - \sin \theta \cos \varphi \sin \alpha - \cos \theta \cos \alpha, \]  

with \( \beta_1 = v \sqrt{\varepsilon_1}/c \) and

\[ P(\theta, \varphi, \alpha) = (\sin \varphi, \cot \theta \tan \alpha - \cos \varphi, -\sin \varphi \tan \alpha), \]

\[ Q(\theta, \varphi, \alpha) = (\sqrt{\varepsilon_1/\varepsilon_0} \sin \varphi, \cot \theta \tan \alpha - \sqrt{\varepsilon_1/\varepsilon_0} \cos \varphi, -\sin \varphi \tan \alpha), \]  

In general, the dielectric permittivities \( \varepsilon_0 \) and \( \varepsilon_1 \) are functions of \( \omega \). In what follows we assume that \( \beta_1 < 1 \). For the case of normal incidence, \( \alpha = 0 \), the general formula (4) coincides with the result previously derived in [8].

For the spectral-angular density of the radiation with the perpendicular polarization we have the following expression:

\[ I_\perp (\omega, \theta, \varphi) = \frac{e^2 \sin^2(2\alpha)}{4\pi^2 c} \frac{(1 - \sqrt{\varepsilon_1/\varepsilon_0})^2}{\sqrt{\varepsilon_0 - \varepsilon_1 \sin^2 \theta}} \sin^3 \theta \cos \theta \sin^2 \varphi \]

\[ \times \sum_{m=\pm \infty} J_m^2(\alpha(\theta)) \sin^2 \left[ \frac{\omega \sqrt{\varepsilon_1}}{2c \cos \alpha} U_m(\theta, \varphi) \right]. \]  

The radiation intensity for the parallel polarization is found from the relation

\[ I_\parallel (\omega, \theta, \varphi) = I(\omega, \theta, \varphi) - I_\perp (\omega, \theta, \varphi), \]  

with the total intensity given by (4). In the case of normal incidence, the intensity for the perpendicular polarization vanishes, \( I_\perp (\omega, \theta, \varphi) = 0 \), and the radiation is polarized in the observation plane.

Various special cases of formulas (4) and (8) have been considered in the literature. In the case of the normal incidence we recover the results of Refs. [7] and [8] for the X-ray and optical transition radiations respectively. In the absence of the acoustic wave we have \( \Delta \varepsilon = 0 \) and in formulas (4) and (8) the \( m = 0 \) term contributes only. In this case we obtain the quasi-classical approximation for the radiation intensity in a finite thickness plate for oblique incidence. The corresponding exact expression for the radiation intensity in this problem is well known from the literature [11] (see also Refs. [1]-[6]). The features of the optical transition radiation in a finite thickness plate for an oblique incidence have been discussed in Refs. [12] on the base of Pafomov’s formulas. For a transparent material in the over-threshold case and under the condition \( \omega l/c > 1 \), the dominant contribution comes from the term in the exact formula with the resonant factor \( \sin^2(y)/y^2 \), with \( y \) given by the argument of the sin function in (4) with \( m = 0 \). Now, for simplicity considering the case of the radiation into the vacuum, it can be seen that for a relativistic particle with \( 1 - v/c \ll 1 \), the radiation intensity near the Cherenkov peaks is well approximated by the formulas obtained from (4) and (8) in the limit \( \Delta \varepsilon = 0 \).

As it is seen from formulas (4) and (8), the radiation intensities for both polarizations have peaks the angular location of which is determined from the equation

\[ U_m(\theta, \varphi, \alpha) = 0, \]  

with the function \( U_m(\theta, \varphi, \alpha) \) defined in (6). In the approximation under consideration the location of the peaks does not depend on the amplitude of the acoustic oscillations. In the
presence of acoustic waves we have a set of peaks specified by $m$. The angular location of the peak with $m = 0$ coincides with that for the peak in the absence of acoustic waves. The angular distance between the peaks induced by the acoustic waves is of the order $(\omega_s/\omega)(c/v_s)$. In order to present the condition (10) in a physically more transparent form, we introduce the wave vector $k_0$ for the photon inside the plate (in the absence of the acoustic wave):

$$k_0 = \frac{\omega}{c} \sqrt{\varepsilon_0 (\sin \theta_0 \cos \varphi, \sin \theta_0 \sin \varphi, \cos \theta_0)}, \quad (11)$$

where $\sin \theta_0 = \sqrt{\varepsilon_1/\varepsilon_0} \sin \theta$. In terms of this vector, the condition for the peaks is written in the form

$$k_{0n} \mathbf{v} = \omega', \quad k_{0n} \equiv k_0 + m \mathbf{k}_s, \quad (12)$$

with $\mathbf{k}_s = (0, 0, k_s)$. The latter is the condition for the Cherenkov radiation in the medium of the plate. Hence, we conclude that the peaks in the region $z > 0$, determined by the condition (10), correspond to the Cherenkov radiation emitted inside the plate and refracted from the boundary. We can have a situation where the Cherenkov radiation emitted inside the plate is completely reflected from the boundary and in the exterior region we have no peaks. In this case the equation (10) has no solutions. There are also cases then the Cherenkov radiation is confined inside the plate in the absence of the acoustic excitations and the peaks defined by (10) appear as a result of the influence of the acoustic waves. The location of the peaks can be controlled by choosing the incidence angle $\alpha$.

We denote by $\theta = \theta^{(m)}(\alpha, \varphi)$ the location of the peaks with respect to the polar angle $\theta$, for given values of $\alpha$ and $\varphi$. For the radiation intensity at the peaks we get the expression

$$I(\omega, \theta^{(m)}, \varphi) = \frac{e^2 \varepsilon_1 l_2^2 \vartriangle \omega^2 \sin^3 \theta^{(m)} \cos \theta^{(m)}}{4 \pi^2 c^3 (\varepsilon_0 - \varepsilon_1 \sin^2 \theta^{(m)})^{1/2}} \times Q^2(\theta^{(m)}, \varphi, \alpha) J_m^2 \left( \frac{\omega \Delta \varepsilon/(2c_k)}{\sqrt{\varepsilon_0 - \varepsilon_1 \sin^2 \theta^{(m)}}} \right). \quad (13)$$

The relative contribution of the component with the perpendicular polarization is given by

$$\frac{I_\perp(\omega, \theta^{(m)}, \varphi)}{I(\omega, \theta^{(m)}, \varphi)} = \left(1 - \frac{\varepsilon_1}{\varepsilon_0} \right) \frac{\sin^2 \varphi \sin^2 \alpha}{Q^2(\theta^{(m)}, \varphi, \alpha)}. \quad (14)$$

In the absence of the ultrasonic vibrations the location of the peak is given by $\theta^{(0)}$. The ultrasound reduces the height of this peak by the factor $J_m^2 \left( a(\theta^{(0)}) \right)$. In particular, for a given radiation frequency, the frequency or the amplitude of the ultrasound can be tuned to eliminate this peak.

3. Radiation in a plate of fused quartz

In this section we consider the optical transition radiation in a plate of fused quartz. For the velocity of longitudinal ultrasonic vibrations one has $v_s \approx 5.6 \times 10^3$ cm/s. For the dielectric permittivity of fused quartz we use the Sellmeier dispersion formula

$$\varepsilon_0 = 1 + \sum_{i=1}^{3} \frac{a_i \lambda^2}{\lambda^2 - l_i^2} \quad (15)$$

with the parameters $a_1 = 0.6961663$, $a_2 = 0.4079426$, $a_3 = 0.8974794$, $l_1 = 0.0684043$, $l_2 = 0.1162414$, $l_3 = 9.896161$. In (15), $\lambda$ is the wavelength of the radiation measured in
micrometers. Formula (15) well describes the dispersion properties of fused quartz in the range $0.2 \mu m \leq \lambda \leq 6.7 \mu m$. In this spectral range fused quartz is very weakly absorbing.

In figures below we plot the spectral-angular density of the total radiation intensity in the forward direction, $I(\omega, \theta, \phi)/\hbar$, and the spectral-angular density for the component with perpendicular polarization, $I_\perp(\omega, \theta, \phi)/\hbar$, for electrons with the energy 2 MeV and for the plate thickness $l = 1$ cm. For the oscillation amplitude we have taken the value $\Delta n/n_0 = 0.05$, where $n_0$ is the number of electrons per unit volume for fused quartz.

In accordance with general features described above, the presence of the acoustic wave leads to the appearance of new peaks in both angular and spectral distributions of the radiation intensity. The height of the peaks can be tuned by choosing the parameters of the acoustic wave. In particular, the peak in the radiation intensity which is present in the absence of the acoustic wave is reduced by the factor $J_0^2(a)$. This peak can be completely removed by taking the parameters of the acoustic wave in such a way to have $a = j_{0,s}$, $s = 1, 2, 3, \ldots$, where $j_{0,s}$ are the zeroes of the function $J_0(z)$.

In figure 1, for separate values of the angle $\alpha$ (numbers near the curves), we display the radiation intensity, defined by (4), as a function of the cyclic frequency for the fixed value the polar angle $\theta = 1$ (left panel) and as a function of $\theta$ for the fixed value of $\omega = 3.23 \times 10^{14}$ Hz (left panel). The graphs in figure 1 are plotted for the frequency of acoustic wave $\nu_s = 1.5$ MHz and for the azimuthal angle $\varphi = 0.1$. Similar graphs for the radiation with the perpendicular polarization are displayed in figure 2 for the same values of the parameters. In figure 3 we show the spectral distribution for the total intensity for different values of the acoustic wave frequency $\nu_s$ (numbers near the curves). The graphs are plotted for $\theta = 1$, $\varphi = 0.1$, $\alpha = 0.1$.

![Figure 1](image_url)

**Figure 1.** The spectral and angular distributions of the radiation intensity for different values of the incidence angle $\alpha$ (numbers near the curves). The values of the parameters are as follows: $\varphi = 0.1$, $\nu_s = 1.5$ MHz. For the left panel $\theta = 1$ and for the right one $\omega = 3.23 \times 10^{14}$ Hz.

4. Conclusion
In the present paper we have studied the influence of acoustic waves, excited in a plate, on the spectral and angular characteristics of the transition radiation for the oblique incidence of a charged particle. For the optical transition radiation, the presence of the small parameter $\lambda/\lambda_0$, with $\lambda$ and $\lambda_0$ being the wavelengths for the radiation and acoustic wave, respectively, allows us to apply the quasi-classical approximation for the evaluation of the radiation intensity in the forward direction. The spectral-angular distribution is described by formula (4) for the total...
radiated energy and by (8) for the component with perpendicular polarization. The radiation intensities for both polarizations have strong peaks with the angular location determined from the condition (10). These peaks correspond to the Cherenkov radiation emitted inside the plate and refracted from the boundary. In the presence of acoustic waves we have a set of peaks with the angular separation of the order \((\omega_s/\omega)(c/v_s)\). The numerical examples for the optical transition radiation in a plate of fused quartz show that the acoustic waves allow to control the both angular and spectral parameters of the radiation. In particular, new resonance peaks appear in the spectral-angular distribution of the radiation intensity.

**Acknowledgment**

The authors are grateful to Professors Babken Khachatryan and Levon Grigoryan for valuable discussions and suggestions. A.R.M and V.V.P. acknowledge the Organizers of the RREPS11 Conference for a support.
References

[1] Ter-Mikaelian M L 1972 High Energy Electromagnetic Processes in Condensed Media (New York: Wiley Interscience)
[2] Gharibian G M and Yan S 1983 Rentgenovskoye Perekhodnoye Izluchenie (Yerevan: Izdatelstvo AN Arm. SSR)
[3] Ginzburg V L and Tsytovich V N 1990 Transition Radiation and Transition Scattering (Bristol: Adam Hilger)
[4] Baier V N, Katkov V M and Strakhovenko V M 1998 Electromagnetic Processes at High Energies in Oriented Single Crystals (Singapore: World Scientific)
[5] Rullhusen P, Artru X and Dhez P 1998 Novel Radiation Sources Using Relativistic Electrons (Singapore: World Scientific)
[6] Potylitsin A P 2011 Electromagnetic Radiation of Electrons in Periodic Structures (Berlin: Springer)
[7] Grigoryan I Sh, Mkrtchyan A H and Saharian A A 1998 Nucl. Instr. Meth. B 145 197
[8] Mkrtchyan A R, Parazian V V and Saharian A A 2010 Mod. Phys. Lett. B 24 2693
[9] Grigoryan I Sh, Mkrtchyan A R, Khachatryan H F, Arzumanyan S R and Wagner W 2010 J. Phys.: Conf. Series 236 012012
[10] Khachatryan B V, 2004 Doctor of Sciences Thesis (Yerevan)
[11] Pafomov V E 1969 Proc. FIAN SSSR, Nuclear Physics and Particle Interaction with Matter 44 90 (Moscow: Nauka)
[12] Ružička J and Mehes J 1986 Nucl. Instr. Meth. A 250 491; Hrmo A and Ruzicka J 2000 Nucl. Instrum. Meth. A 451 506