Radiative $\rho \to \eta\gamma$ decay in light cone QCD

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Abstract

The coupling constant of $\rho \to \eta\gamma$ decay is calculated in the framework of light cone QCD sum rules. A comparison of our prediction on the coupling constant with the result obtained from analysis of the experimental data is performed.

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1 Introduction

Radiative transition between vector (V) and pseudoscalar (P) mesons represent an important source of information on low energy hadron physics. These transitions are governed by magnetic dipole (M1) radiation of the photon and had played one of the central roles for checking the predictions of quark model and SU(3) symmetry, as well as it was very useful in the determination of the magnetic dipole moment of \( N^*(1535) \) in \( \gamma N \rightarrow \eta N \) process [3]. Recently the theoretical activity on the VP\( \gamma \) magnetic dipole transition have increased(see [1] and references therein), in particular, due to the fact that the analysis of the radiative \( V \rightarrow P\gamma \) decay with \( \eta \) and \( \eta' \) mesons in final state can provide insights to the long standing issue of the \( \eta \) and \( \eta' \) mixing (for review see [2] and references therein). The usual parametrization of \( \eta - \eta' \) mixing in the octet–singlet basis, which will be used in this work, is as follows: the current-particle matrix elements are defined as

\[
\langle 0 | J_{5\mu}^\alpha | P(p) \rangle = -i f_{5\mu}^\alpha p_\mu (\alpha = 8, 0; P = \eta, \eta')
\]

where \( J_{5\mu}^\alpha \), the \( SU(3)_F \) octet axial vector current, is given by

\[
J_{5\mu}^8 = \frac{1}{\sqrt{6}} (\overline{u} \gamma_\mu \gamma_5 u + \overline{d} \gamma_\mu \gamma_5 d - 2 \overline{s} \gamma_\mu \gamma_5 s)
\]

and \( J_{5\mu}^0 \), the \( SU(3)_F \) singlet current is given by:

\[
J_{5\mu}^0 = \frac{1}{\sqrt{3}} (\overline{u} \gamma_\mu \gamma_5 u + \overline{d} \gamma_\mu \gamma_5 d + \overline{s} \gamma_\mu \gamma_5 s)
\]

Two mixing angles \( \theta_8 \) and \( \theta_0 \) are required in order to consistently describe mixing [4]. Accordingly the couplings in Eq.(1) can be defined as follows

\[
\begin{align*}
&f_{8\eta} = f_8 \cos \theta_8, \quad f^0_{\eta} = -f_0 \sin \theta_0 \\
&f^s_{8\eta} = f_s \sin \theta_8, \quad f^s_{\eta'} = f_s \cos \theta_0
\end{align*}
\]

Alternatively, two independent axial vector currents with distinct flavors can be considered

\[
\begin{align*}
J_{5\mu}^q &= \frac{1}{\sqrt{2}} (\overline{u} \gamma_\mu \gamma_5 u + \overline{d} \gamma_\mu \gamma_5 d) \\
J_{5\mu}^s &= \overline{s} \gamma_\mu \gamma_5 s
\end{align*}
\]

and the couplings of these currents with \( \eta \) and \( \eta' \) mesons are defined similarly to Eq.(1) :

\[
\begin{align*}
&f^q_{\eta} = f_q \cos \varphi_q, \quad f^q_{\eta'} = f_q \sin \varphi_q \\
&f^s_{\eta} = -f_s \sin \varphi_s, \quad f^s_{\eta'} = f_s \cos \varphi_s
\end{align*}
\]

As we see that in each basis there are two angles and in [2] it was shown that, to a very good accuracy, the mixing can described in terms of single angle \( \varphi \) since \( |\varphi_s - \varphi_q|/|\varphi_s + \varphi_q| << 1 \) which is also confirmed by a QCD sum rules calculation [5].
In present work we will follow to the first approach, neglecting the mixing angles $\theta_0$ and $\theta_8$ due to their smallness and calculate the coupling constant of $\rho \to \eta \gamma$ decay in framework of light cone QCD sum rules (more about of light cone QCD sum rules and its applications can be found in [6] and [7]).

The paper is organized as follows: In section 2 we derive light cone QCD sum rules for $\rho \to \eta \gamma$ decay constant, section 3 is devoted to the numerical analysis of the sum rules and contain our conclusions.

2 Light Cone QCD Sum Rules for $\rho \to \eta \gamma$ coupling constant

In this section we calculate the coupling constant of $\rho \to \eta \gamma$ decay using light cone QCD sum rules method. In order to calculate this coupling constant we consider the following two point correlation function

$$\Pi_{\mu\nu} = \int d^4xe^{ipx} \langle 0 | T \left\{ J_\rho^\mu(x)J_\rho^\nu(0) \right\} | 0 \rangle \gamma$$

where $\gamma$ denotes the external electromagnetic field, and $J_\rho^\mu$ and $J_\rho^\nu$ are the interpolating currents with $\eta$ and $\rho$ meson quantum numbers. Here we would make the following remark. As we already noted that both of the mixing angles in Eq. (5) are small, in the following discussion we will neglect the mixing, i.e we take $J_\eta^\mu \equiv J_8^\mu$.

At the phenomenological level the Eq.(9) can be expressed as:

$$\Pi_{\mu\nu} = \sum \frac{\langle 0|J_\rho^\mu|\eta(p_2)\rangle \langle \eta(p_2)|\rho(p_1)\rangle \gamma \langle \rho(p_1)|J_\rho^\nu|0\rangle}{(p_2^2 - m_\eta^2)(p_1^2 - m_\rho^2)}$$

where $p_1 = p_2 + q$ and $q$ is the photon momentum. The matrix elements entering Eq.(10) are defined as

$$\langle 0|J_\rho^\mu|\rho \rangle = m_\rho f_\rho \varepsilon_\mu^\rho$$

$$\langle 0|J_8^\mu|\eta(p_2)\rangle = -if_8\varepsilon_\mu^\eta q_\mu F(q^2)$$

The remaining matrix element $\langle \eta(p_2)|\rho(p_1)\rangle \gamma$ which describes the M1 transition, can be parameterized as,

$$\langle \eta(p_2)|\rho(p_1)\rangle \gamma = e\epsilon_{\mu\nu\alpha\beta}\varepsilon_\mu^\rho p_1\varepsilon_\alpha^\gamma q_\beta F(q^2)$$

where $\varepsilon^\gamma$ is the photon polarization vector. Since the photon is real, we need the value of $F(q^2)$ only at the point $q^2 = 0$. We can use an alternative parametrization for the $\rho\eta\gamma$ vertex:

$$L_{int} = -\frac{e}{m_\rho}g_{\rho\gamma}\epsilon_{\mu\nu\alpha\beta} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) (\partial_\alpha A_\beta - \partial_\beta A_\alpha)$$

Comparing Eqns. (13) and (14) we see that

$$F(q^2 = 0) \equiv \frac{g_{\rho\gamma}}{m_\rho}$$
Using Eqns. (10) – (14), for the physical part of the sum rules we get

$$\Pi^{\mu\nu}_{ph} = \frac{g_{\rho\sigma\gamma} f_{\rho\nu} f_{\delta} \epsilon_{\mu\sigma\alpha\delta} \epsilon_{\nu\alpha\beta\gamma} \epsilon_{\alpha\beta} q^2}{(p^2 - m_\gamma^2)(p^2 - m_\rho^2)}$$  \hspace{1cm} (16)

Our next task is the calculation of correlator Eq.(1) from the QCD side. The correlator receives both perturbative and non-perturbative contributions. In calculation of the non-perturbative contributions by the OPE on the light cone one needs to know the matrix elements of nonlocal operators between vacuum and the photon states; i.e. \( \langle \gamma(q)|q\Gamma_iq|0 \rangle \) where \( \Gamma_i \) is an arbitrary Dirac matrix. These matrix elements can be expressed in terms of photon wave functions with definite twist. In calculations we neglect twist 3 three particle photon wave functions since their contributions are small. Twist two and twist four photon wave functions are defined as \([8, 9, 10]\):

$$\langle \gamma(q)|q\sigma_{\alpha\beta}q|0 \rangle = i e_q (q q) \int_0^1 du e^{iuq x} \{ (\epsilon_{\alpha\beta} q_\beta - \epsilon_{\beta\alpha} q_\alpha) [\chi x(u) + x^2 (g_1(u) - g_2(u))] + [q x(\epsilon_{\alpha\beta} q_\beta - \epsilon_{\beta\alpha} q_\alpha) + \epsilon(\alpha x q_\beta - q_\alpha x_\beta)] g_2(u) \}$$  \hspace{1cm} (17)

$$\langle \gamma(q)|q\gamma_{\alpha\beta}q|0 \rangle = \frac{f}{4} e_q f_{\mu\alpha\beta\mu} \epsilon_\alpha q^2 x^2 \int_0^1 du e^{iuq x} \psi(u)$$  \hspace{1cm} (18)

where for simplicity we use \( \tilde{q}\Gamma q \) to denote \( J_{\gamma\mu}^s = \frac{1}{\sqrt{2}} (\tilde{u}\Gamma u + d\Gamma d - 2\tilde{s}\Gamma s) \).

In Eqs.(17) and (18) \( e_q \) is the corresponding quark charge, \( \chi \) is the magnetic susceptibility, \( \varphi(u) \), and \( \psi(u) \) are the leading twist two and \( g_1(u) \), and \( g_2(u) \) are the twist four photon wave functions.

After standard calculations we get the following expression for correlator from QCD side in the coordinate representation:

$$\Pi_{\mu\nu} = \frac{1}{\sqrt{12}} \frac{i}{16} (e_u - e_d) \int d^4 x e^{ip x} \{ [6(\epsilon x) \epsilon_{\alpha\beta\mu\nu} q_\alpha x_\beta - 6(q x) \epsilon_{\alpha\beta\nu\mu} \epsilon_{\alpha\beta} q_\beta - 12x_\nu \epsilon_{\alpha\beta\rho\mu} \epsilon_{\alpha\beta} q_\beta x_\rho] + f \pi^2 x^2 [x_\nu \epsilon_{\alpha\beta\rho\mu} \epsilon_{\alpha\beta} q_\beta x_\rho] + x_\mu \epsilon_{\alpha\beta\rho\mu} \epsilon_{\alpha\beta} q_\beta x_\rho \} \int_0^1 du e^{iuq x} \psi(u) / \pi^4 x^6$$  \hspace{1cm} (19)

The sum rules for \( g_{\rho\sigma\gamma} \) are obtained by equating the phenomenological and theoretical parts (in Eq.(19) it is necessary to perform Fourier transformation first) of the correlator.

Performing double Borel transformation on variables \( p_2^2 = p^2 \) and \( p_1^2 = (p + q)^2 \) on both sides of the correlation function in order to suppress the contributions of the continuum and higher states (procedure of subtraction in light cone sum rules one can find in \([11, 12]\)), and also to remove the subtraction terms in the dispersion relation, we obtain the following sum rules for \( g_{\rho\sigma\gamma} \) coupling constant

$$g_{\rho\sigma\gamma} = \frac{e^{i[m_\rho^2/M_1^2 + m_\gamma^2/M_2^2]}(e_u - e_d)}{2\sqrt{3} f_{\eta} f_{\rho}} \frac{1}{1 - u_0} \left\{ \frac{3}{2\pi^2} M^2 E_0(s_0/M^2) + f \psi(u_0) \right\}$$  \hspace{1cm} (20)

where \( E_0(s_0/M^2) = 1 - e^{-s_0/M^2} \) is the function used to subtract continuum, \( s_0 \) is the continuum and

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2}, M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}$$  \hspace{1cm} (21)
where $M_1^2$ and $M_2^2$ are the Borel mass parameters in $\rho$ and $\eta$ channels. Note that in Eq.(21) we take into account $F^u_\eta = F^d_\eta$. The masses of $\eta$ meson is close to the $\rho$ meson mass. For this reason, it is natural to set $M_1^2 = M_2^2 = 2M^2$ from which it follows that $u_0 = 1/2$.

3 Numerical Analysis

In this section we present our numerical result on $g_{\rho \eta \gamma}$ coupling constant. From sum rules Eq.(20) we see that for estimating $g_{\rho \eta \gamma}$ coupling constant first of all one needs to know the photon wave function $\psi(u)$. It was shown in [9, 10] that the photon wave function do not deviate remarkably from its asymptotic form which is given by $\psi(u) = 1$ [9, 8]. The values of the other constants appearing in the sum rules are: $m_\rho = 0.77$ GeV, $m_\eta = 0.55$ GeV, $f = 0.028$ GeV$^2$. Leptonic decay constant of $\rho$ meson $f_\rho = 0.15$ GeV follows from experimental result of the $\rho \to e^+e^-$ decay, $\Gamma(\rho \to e^+e^-) = (6.85 \pm 0,11)$ KeV[13]. More recent analysis shows that the coupling of $\eta$ meson with the octet axial vector current is $f_8^\eta = 0.159$ GeV [2] and this result we will use in our analysis.

In Fig. 1 we present the dependence of the coupling constant $g_{\rho \eta \gamma}$ on the Borel parameter $M^2$ at three different values of the continuum threshold: $s_0 = 1.4$ GeV$^2$, 1.6 GeV$^2$, 1.8 GeV$^2$. Since the Borel mass $M^2$ is an auxiliary parameter and the physical quantities should not depend on it, we must look for the region where $g_{\rho \eta \gamma}$ is practically independent of $M^2$. We obtain that this condition is satisfied when $1$ GeV$^2 \leq M^2 \leq 1.4$ GeV$^2$. From this figure we also obtained that the variation of $s_0$ from $s_0 = 1.4$ GeV$^2$ to $s_0 = 1.8$ GeV$^2$ causes a change on the result on $g_{\rho \eta \gamma}$ of about 10%. Therefore one can say that the result $g_{\rho \eta \gamma}$ is insensitive to $s_0$ and $M^2$. Our final prediction on the coupling constant is

$$g_{\rho \eta \gamma} = (1.4 \pm 0.2)$$ \hspace{1cm} (22)

where the error is attributed to the variation of $s_0$, $M^2$ and neglected twist three photon wave functions.

At the end we would like to compare our prediction on $g_{\rho \eta \gamma}$ with experimental result. The decay width of the $\rho \to \eta \gamma$ decay is given by

$$\Gamma(\rho \to \eta \gamma) = \frac{\alpha g^2_{\rho \eta \gamma}}{24} m_\rho (1 - m_\eta^2/m_\rho^2)^3$$ \hspace{1cm} (23)

The experimental value is $\Gamma(\rho^0 \to \eta \gamma) = (57 \pm 10)$ KeV[13]. Using this value of $\Gamma(\rho^0 \to \eta \gamma)$, the $g_{\rho \eta \gamma}$ coupling constant is obtained from Eq.(23) as:

$$g_{\rho \eta \gamma} = (1.42 \pm 0.12)$$ \hspace{1cm} (24)

which is very close to the sum rule prediction.

Finally we note that the coupling constant $g_{\omega \eta \gamma}$ can be obtained from $g_{\rho \eta \gamma}$ with the help of the relation $g_{\rho \eta \gamma} = 3g_{\omega \eta \gamma}$.
Figure 1: The dependence of the $g_{\rho\eta\gamma}$ coupling constant on the Borel parameter $M^2$ at three different values of the continuum threshold $s_0 = 1.4 \text{ GeV}^2$, $s_0 = 1.6 \text{ GeV}^2$ and $s_0 = 1.8 \text{ GeV}^2$. 
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