Non-Life Insurance Pricing: Multi Agents Model

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Abstract

We use the maximum entropy principle for pricing the non-life insurance and recover the Bühlmann results for the economic premium principle. The concept of economic equilibrium is revised in this respect.

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I. INTRODUCTION

Recently the physicists are interested in a new branch of economics, the insurance market. The financial reaction of insurance company to variation in number of insurants was studied by author and his colleague [1] and subsequently a way for pricing the insurance premium was suggested by author on the basis of the equilibrium statistical mechanics [2, 3, 4, 5]. The insurer company interacts with it’s environment, rest of the financial market, by money exchanging. In the equilibrium state the probability for exchanging specified amount of money is similar to what we see in the canonical ensemble theory.

In this paper we proceed with the viewpoint of the latter reference for using the principles which are borrowed from statistical mechanics for premium calculation but restrict ourselves to a multi agents model for insurance market which was used previously in the actuary literature [6, 7].

II. BÜHLMANN ECONOMIC PREMIUM PRINCIPLE

The insurance companies and buyers of insurance are the typical economic agents in the financial market. They compete with each other to benefit more from their trade. The utility function demonstrates what an agent is interested for making specified amounts of profit. The common sense tell us, the agent’s utility function should depend on its financial status which is frequently described by its wealth, $u(W)$. It is assumed that the utility function has positive first derivative, $u'(W) > 0$, to guarantee that the agent is willing the profit and negative second derivative, $u''(W) < 0$, to restrict it’s avarice. The risk aversion parameter, $\beta(W) = -u''(W)/u'(W)$, is also involved in the utility function to scale the agent’s will in the market with respect to it’s wealth.

The equilibrium is attained when all agents are satisfied from their trade. In other word their utility functions should be maximum in the equilibrium state. This condition may be expressed as an average form because the risks in the market alter the agent’s wealth randomly.

$$\int_{\Omega} u_i(W_i(\omega))d\Pi(\omega) = \max.$$  \hspace{1cm} (1)

Where $\omega$ stands for an element of the risk’s probability space $\Omega$. The measure of the integral
demonstrates the weight for occurrence a random event (risk) thus we have,

\[ \int_{\Omega} d\Pi(\omega) = 1. \]  \hspace{1cm} (2)

The index \( i \) in the equation distinguishes the different agents.

Each agent in the market will be incurred \( X_i(\omega) \) if \( \omega \) is happening. He insured himself for the price \( < Y_i > \) and receives \( Y_i(\omega) \) upon occurrence of this event. The insurance price is given by,

\[ < Y_i > = \int_{\Omega} \varphi(\omega)Y_i(\omega)d\Pi(\omega). \]  \hspace{1cm} (3)

The function \( \varphi : \Omega \rightarrow \mathbb{R} \) is called price density. The agent’s wealth also varies due to this trading as follows,

\[ W_i(\omega) = W_{0i} - X_i(\omega) + Y_i(\omega) - < Y_i >. \]  \hspace{1cm} (4)

We suppose the market is a closed system hence the clearing condition satisfied.

\[ \sum_i Y_i(\omega) = 0. \]  \hspace{1cm} (5)

The sum is over all agents in the market. The above equation in addition to equation let us to find the price density \([6, 7]\),

\[ \varphi(\omega) = \frac{e^{\beta Z(\omega)}}{\int_{\Omega} e^{\beta Z(\omega)}d\Pi(\omega)}. \]  \hspace{1cm} (6)

Where \( Z(\omega) \) is aggregate loss in the market,

\[ Z(\omega) = \sum_i X_i(\omega). \]  \hspace{1cm} (7)

The coefficient \( \beta \) comes from combination of risk aversion parameter of different agents,

\[ \frac{1}{\beta} = \sum_i \frac{1}{\beta_i}. \]  \hspace{1cm} (8)

The equation firstly was derived by Bühlmann in his famous articles \([6, 7]\). In the following section we retrieve this result again based on the maximum entropy principle.

**III. THE MAXIMUM ENTROPY METHOD IN ECONOMICS**

The risks induce the random condition in the market even the agents had definite state at the first. The randomness in the market will be increased when the time goes forward.
Finally the market falls into a state with the most randomness. This is what we nominate as the equilibrium state. The consequence of randomness in the market is losing the information about the agents and their strategies for trading.

The main question in the economics is how we can calculate the probability for the market to have a specified amount of money. In other word we look for the probability of happening an event, namely $\omega$, when the market is in equilibrium. As is seen in the eq.3 the insurance price is defined in respect to this probability function, $\varphi(\omega)$, which is called the price density in the actuary terminology.

The maximum entropy principle appears as the best way when we make inference about an unknown distribution based only on the incomplete information [8]. We adopt this method for calculation the mentioned probability density.

The entropy functional can be written as [8],

$$H[\varphi] = -\int_{\Omega} \varphi(\omega) \ln \varphi(\omega) d\Pi(\omega).$$

The price density should satisfies in normalization condition.

$$\int_{\Omega} \varphi(\omega) d\Pi(\omega) = 1.$$ 

The wealth of market is defined as the sum of the agents wealth.

$$W(\omega) = \sum_i W_i(\omega).$$

We assume that the average of market’s wealth is constant.

$$< W > = \int_{\Omega} \varphi(\omega) W(\omega) d\Pi(\omega) = \text{Const.}$$

In the equilibrium the entropy eq.9 has maximum value and the constrains eqs 10 and 12 should also be satisfied simultaneously. This mathematical problem can be solved immediately by the method of Lagrange’s multipliers.

$$\delta H[\varphi] + \lambda \delta \int_{\Omega} \varphi(\omega) d\Pi(\omega) + \beta \delta < W > = 0.$$ 

The canonical distribution is the solution to above equation as we have seen before for special cases that are given in every statistical mechanics textbooks [9].

$$\varphi(\omega) = \frac{e^{-\beta W(\omega)}}{\int_{\Omega} e^{-\beta W(\omega)} d\Pi(\omega)}.$$
The above result is derived previously for agent-environment model of financial market\(^5\).

The eq\(^14\) may be applied to all economic systems. The parameter \(\beta\) is positive to ensure that extreme values for market’s wealth have small probability.

In the case of insurance the market’s wealth is given as

\[
W(\omega) = W_0 - Z(\omega) = \sum_i W_{0i} - \sum_i X_i(\omega). \quad (15)
\]

Since the market’s initial wealth is constant then we obtain the same form for the price density as what is seen in the eq\(^6\). It is worth to mention that the total risk must be lesser than the market’s initial wealth.

The premium that the \(i\)-th agent pays is calculated by using the eq\(^6\)

\[
p_i = \frac{\int_\Omega X_i(\omega)e^{\beta Z(\omega)}d\Pi(\omega)}{\int_\Omega e^{\beta Z(\omega)}d\Pi(\omega)}. \quad (16)
\]

If the risk function for different agents have no correlation and dependency then the Esscher principle is obtained \(^6, 7\).

\[
p_i = \frac{\int_\Omega X_i(\omega)e^{\beta X_i(\omega)}d\Pi_i(\omega)\int_\Omega e^{\beta(Z(\omega) - X_i(\omega))}d\Pi_{\text{other}}(\omega)}{\int_\Omega e^{\beta X_i(\omega)}d\Pi_i(\omega)\int_\Omega e^{\beta(Z(\omega) - X_i(\omega))}d\Pi_{\text{other}}(\omega)}
= \frac{\int_\Omega X_i(\omega)e^{\beta X_i(\omega)}d\Pi_i(\omega)}{\int_\Omega e^{\beta X_i(\omega)}d\Pi_i(\omega)}. \quad (17)
\]

Where the \(d\Pi_i(\omega)\) and \(d\Pi_{\text{other}}(\omega)\) demonstrate the weight for the \(i\)-th agent risks and remaining part of the market. The parameter \(\beta\) has important roll in price density, it can be calculated on basis of the method that is introduced in previous works \(^3, 5\), but our intuition from similar case in thermal physics tell us \(^9\),

\[
\beta \approx \frac{1}{< W >}. \quad (18)
\]

This result shows that the wealthier market offers low price and the prices also depends on the size of risks in the market.

The adopted way for premium calculation is more general and independent of the market’s models. It also enable us to apply easily any other constraint which are in the market.

The method of maximum entropy may be generalized for local finite market via Tsallis definition of entropy. This case is under investigation.
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