High coherent solid-state qubit from a pair of quantum dots

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In this letter we propose a scheme to build up high coherent solid-state quantum bit (qubit) from two coupled quantum dots. Quantum information is stored in electron-hole pair state with the electron and hole locating in different dots, and universal quantum gates involving any pair of qubits are realized by effective coupling interaction via virtually exchanging cavity photons.

To be scalable in quantum computation (QC), recently there have been considerable interest in the solid-state implementations\textsuperscript{1-10}. The main drawback in solid state systems is the severe decoherence. Thus in the solid-state based QC proposals, the spin (rather than charge) degree of freedom has been exploited for its relatively long decoherence time\textsuperscript{11}. On the other hand, some recent impressive experiments investigated the charge degree of freedom to realize coherent state operation and entanglement in quantum dots (QDs)\textsuperscript{12-16}, by making use of the ultrafast\textsuperscript{17} optical spectroscopy techniques. It is the ultrafast feature in these experiments to overcome the decoherence difficulty. In this letter, we propose a simple scheme to achieve low decoherence qubit from a pair of weakly coupled QDs.

The basic idea is from our recent work\textsuperscript{17}, by noticing that several shortcomings exist there and in some of other QC schemes based on QDs: (i) In each qubit (two coupled QDs), one and only one excess electron is required in the conduction band. This is a challenging task within current technology. (ii) The intersubband transition with THz lasers is currently not a mature technology. (ii) The intersubband transition with THz lasers is currently not a mature technology. (iii) The coupling between qubits is mediated by Coulomb interactions, which makes it very difficult to perform conditional gate operation between any pair of qubits. In this letter, we attempt to remove all these drawbacks based on an all-optical approach. The physical system we are concerned with is similar as that proposed by Imamoglu et al, i.e., many qubits (QDs) are located in an optical microcavity, and each qubit can be selectively performed by laser pulses due to the relatively large distance between qubits. However, in our structure, we suggest to use two weakly coupled QDs (rather than one QD) to construct a qubit as shown in Fig. 1, where only the relevant HOMO and LUMO states are plotted for clarity. In our present scheme, no excess electron in the conduction band is required: quantum information is stored in electron-hole pair state. Namely, logic $|1\rangle$ corresponds to electron-hole pair excitation with electron in state $|e\rangle$, and logic $|0\rangle$ corresponds to no electron-hole pair excitation with electron in state $|v\rangle$. Since we are exploiting the charge states for QC, the possible spin degeneracies (superposition and even decoherence of spin states) are irrelevant degrees of freedom.

A specific feature we would like to stress is that the energy level structure of the coupled QDs (nearly identical) under external electric field in Fig. 1 makes the phonon scattering induced decoherence on state $|e\rangle$ be negligible at low temperatures, since the electron-phonon scattering from $|e\rangle$ to $|\tilde{e}\rangle$ will be suppressed for a relatively large energy separation (e.g. $\sim 10$ meV) between $|e\rangle$ and $|\tilde{e}\rangle$. In the present QC scheme, $|\tilde{e}\rangle$ would play a role of virtually occupying intermediate state, thus its decoherence is insensitive to QC. Accordingly, the dominant decoherence source in our qubit structure stems...
from spontaneous emission of state $|e\rangle$, which would be much weaker than its counterpart in a single dot due to the hole largely locating in another dot. Regarding the external electric field, its thermal fluctuation (the Johnson noise) may cause additional dephasing. To avoid it, the electrodes that generate the electric field can be connected to a superconductor, which can remove the thermal fluctuations since there is no dissipation. In the following, we briefly show how to implement the universal quantum gates, then present more detailed analysis for decoherence versus operation time.

For single qubit operation, turning on a laser field with frequency $\omega_L$ on resonance with the energy difference between $|e\rangle$ and $|v\rangle$, qubit flipping takes place under the interaction Hamiltonian

$$H_I = \Omega_L \langle e | e \rangle e^{-i\omega_L t} + \text{H.c.},$$

where $\Omega_L$ is the Rabi frequency. Utilizing this interaction, arbitrary single-qubit operations can be performed. To realize the universal gates for quantum computation, two-bit operation such as the controlled-NOT (CNOT), or equivalently, conditional-phase-shift (CPS) gate, is necessary. To this end, we suggest to employ the cavity photon to virtually participate in qubit operation, which can effectively couple the two performed qubits together. Specifically, two lasers selectively act on two qubits (the $j$th and $k$th ones), both with frequency $\omega_L$ being off-resonance with the transition energy between $|e\rangle$ and $|v\rangle$, i.e., with detuning $\delta_1 = E_e - E_v - \omega_L$. To avoid real excitation of the cavity photon, $\delta_1$ should slightly differ from the detuning of the cavity phonon frequency with the level spacing between $|\tilde{e}\rangle$ and $|\tilde{e}\rangle$, namely, $\delta_1 \neq \delta_2 = E_e - E_v - \omega_C$. Under the laser pulse action, single qubit (e.g. the $j$th one) effective interaction Hamiltonian reads

$$\tilde{H}_I = \Omega_{\text{eff}}^{(j)} \langle e | j \rangle a^\dagger e^{-i\omega_{L} t} + \text{H.c.},$$

with the effective two-photon coupling coefficient $\Omega_{\text{eff}}^{(j)}(t) = \Omega_{L}^{(j)}(t) \delta_2 / (2 \delta_2)$, where $\Omega_{L}^{(j)}$ is the optical coupling strength between $|\tilde{e}\rangle$ and $|v\rangle$ due to the laser (cavity photon) field. To implement CNOT or CPS gate, we need to establish a near two-photon resonance condition between the control ($j$) and target ($k$) qubits, i.e., $\delta_2 = \delta - \delta_1$. Under this condition, and assuming that $\delta$ is larger than $\Omega_{\text{eff}}^{(j)}(t)$ and the cavity photon linewidth, the cavity-photon coordinate in Eq. (1) can be eliminated, and a two-bit effective coupling is established as (in interaction picture with respect to $H_0 = E_e |e\rangle \langle e| + E_v |v\rangle \langle v|$)

$$H_{\text{CPS}} = g_{\text{eff}} \left[ \sigma^+_j \sigma^{-}_k + \sigma^+_k \sigma^{-}_j \right],$$

where $g_{\text{eff}}(t) = \Omega_{\text{eff}}^{(j)}(t) \Omega_{\text{eff}}^{(k)}(t) \delta / \delta_2$. Here we have introduced the Pauli matrices to denote state transition, $\sigma^+_j = |e\rangle \langle v|$ and $\sigma^{-}_j = |v\rangle \langle e|$. Similarly, we can also introduce other Pauli matrices as follows: $\sigma^x = |e\rangle \langle e| - |v\rangle \langle v|$, $\sigma^y = -i |e\rangle \langle v| + i |v\rangle \langle e|$, and $\sigma^z = |e\rangle \langle e| + |v\rangle \langle v|$. In this way, the present system can be completely mapped to a spin model, for both the single qubit rotation [c.f. Eq. (1)], and two qubit coupling [c.f. Eq. (2)]. With this mapping, the gating technique based on spin interaction of the form $J S^+_j S^+_k$ can be straightforwardly adopted. Below we show that the nontrivial two-bit operation such as the CPS gate can be realized by combining $H_{\text{CPS}}$ with one-bit operations. Note that the interaction Hamiltonian of Eq. (3) defines a two-bit unitary evolution operator $U_{jk}(\phi) = \text{Exp}[i \int dt H_{\text{int}}(t)]$, where $\phi = \int dt g_{\text{eff}}(t)$. With this, the CPS gate can be implemented using the following pulse sequence:

$$U_{\text{CPS}} = e^{i\pi/4}e^{i\pi n_j \sigma^x_j /3}e^{i\pi n_k \sigma^x_k /3}U_{jk}(\pi/4) e^{-i\pi \sigma^x_j/2} \times U_{jk}(\pi/4)e^{-i\pi \sigma^x_j/2}e^{-i\pi \sigma^x_k/2}. $$

Here the vector Pauli operator $\sigma = (\sigma^x, \sigma^y, \sigma^z)$, unit vector $n_j = (1, 1, -1)/\sqrt{3}$, and $n_k = (1, -1, 1)/\sqrt{3}$. It can be straightforwardly shown that in the computational subspace $\{|rv\rangle_{jk}, |ve\rangle_{jk}, |er\rangle_{jk}, |ev\rangle_{jk}\}$, $U_{\text{CPS}}|ve\rangle_{jk} = e^{i\pi x} |ae\rangle_{jk}$, and other basis states are kept unchanged (with no phase shift). The more familiar CNOT gate is associated with the above CPS gate in terms of $U_{\text{CPS}} = U_{\text{CPS}}^{-1}U_{\text{CPS}}H_k$, where $H_k = \text{Exp}(i\pi \sigma^y_k /4)$ is the Hadamard gate acting on the $k$th qubit.

To carry out an analysis for QC operation, we need to specify the electronic states further. In two-level approximation, $|e\rangle$ and $|\tilde{e}\rangle$ in Fig. 1 are resulted from coupling of the two isolated dot states $|d\rangle$ and $|\tilde{d}\rangle$ with coupling strength $t$, and energy separation $\Delta = E_e - E_d$. (For the highest two valence band states, similar treatment can be done). As a result, the eigenstates $|e\rangle$ and $|\tilde{e}\rangle$ have eigenenergies $E_\tilde{e} = E_e + \delta$, and $E_e = E_d - \delta$, and energy separation $\Delta = E_\tilde{e} - E_d$.

$$|e\rangle = \sqrt{1 - \gamma |d\rangle + \sqrt{\gamma} |\tilde{d}\rangle},$$

$$|\tilde{e}\rangle = \sqrt{1 - \gamma |\tilde{d}\rangle - \sqrt{\gamma} |d\rangle},$$

where $\gamma = t^2 / (\Delta^2 + t^2)$. With this state nature in mind, we below estimate the decoherence and operation time in order.

As have mentioned previously, the qubit decoherence rate ($\tau_d$) is characterized by the spontaneous emission rate of $|e\rangle$, which is given by Fermi golden rule as $1/\tau_d = \frac{2}{\hbar} \sum |M_{ee}(q)|^2 \delta(E_e - E_v - \hbar \omega_q)$, where $\omega_q$ is the emitted photon frequency, and $M_{ee}(q) = \langle v|H_{\text{ep}}(q)|e\rangle$ is the perturbative matrix element. For logic gate operation, the time scale ($\tau_G$) is determined by the optical coupling $H^{(L)}_I$ between $|e\rangle$ and $|v\rangle$ via the external laser field, and $H^{(C)}_I$ between $|e\rangle$ and $|\tilde{e}\rangle$ via the cavity photon. In terms of matrix element of the interaction Hamiltonian, the coupling strengths can be expressed as $\Omega_{L,C} = \langle e|H^{(L,C)}_I |e\rangle$. Due to the spatial separation of state $|e\rangle$ from $|v\rangle$ and $|\tilde{e}\rangle$ as shown in Eq. (4), compared to the corresponding counterparts in single dot,
the spontaneous emission rate will be reduced by a factor $\gamma$, while $\Omega_L$ and $\Omega_C$ will be reduced only by $\sqrt{\tau}$. As a consequence, the gate ratio $\rho = \tau_d/\tau_G$ will be enhanced by a factor $\sim 1/\sqrt{\tau}$. Similar conclusion has been quantitatively demonstrated by numerical calculation in Ref. 1.

Further, as an order of magnitude estimate, assume $t = 0.01$ meV, and $\Delta = E_c - E_e = 10$ meV. Accordingly, the spatial separation factor $\gamma = 10^{-6}$. For the intra-dot interband coupling due to the laser pulse, we assume $\Omega_L = 0.1$ meV; and for the intra-dot state coupling with the cavity photon, the typical value of $\Omega_C = 300$ MHz is adopted. To avoid real occupation on the state $|\tilde{e}\rangle$, detuning $\delta_1 = 1$ meV is assumed between the laser frequency and the energy difference between $|\tilde{e}\rangle$ and $|v\rangle$. With these parameters, the characteristic time for single qubit rotations is given by $\tau_G^{(1)} = \pi/\Omega_L$, where $\Omega_L \sim \sqrt{\tau}\Omega_L = 10^{-4}$ meV, implying a time scale of hundreds of nanoseconds. For two qubit operation, the characteristic time $\tau_G^{(2)}$ is dominantly determined by the two-bit joint evolution [c.f. Eq. (4)]. $\tau_G^{(2)} \sim \pi/\gamma_{eff}$. Assuming $\delta = \Omega_{eff}/3$, we estimate $\Omega_{eff} = \Omega^{(j)} = \Omega^{(k)} \Omega_L/\delta_1 \approx 30$ KH, and $\gamma_{eff} \approx 10$ KH. Accordingly, the two-bit gate such as Eq. (4) can be accomplished within time scale of $10^{-3}$ sec.

For the spontaneous emission, due to the CQED effect and the possible dark state feature in certain symmetrical QDs,[1] the intra-dot interband radiative lifetime can be regarded longer than tens to hundreds of microsecond. Therefore, the qubit decoherence time can be as long as tens of second (note that $\tau_d \sim \tilde{\tau}_d/\gamma \sim 10^8 \times \tilde{\tau}_d$), owing to the spatial separation of the qubit states. Within this time scale, the single bit rotation can be performed as high as $10^8$ times, and the two-bit CNOT gate can be performed about $10^4$ times. We notice that the operation quality presented here, say, the decoherence time and computing speed, is comparable to the well-known ion-trap QC model.[4] As an interesting comparison, under the same interaction and relaxation strengths as assumed above, if the qubit is constructed from single quantum dot, the coherent operations can be only, $\sim 10^9$ for one-bit rotations, and $\sim 10$ for the two-bit CNOT gate.

In summary, we have proposed a scheme to build up high coherent solid-state qubit from a pair of quantum dots, and to implement the universal quantum gates by coupling qubits via virtually exchanging cavity photon. The central challenge to realize the proposed QC scheme is the development of few-mode THz cavities with extremely low loss. An attractive candidate is the dielectric cavities, which is currently an intensive research field.

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