Nernst effect in the phase-fluctuating superconductor InO$_x$

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Abstract – We present a study of the Nernst effect in the amorphous 2D superconductor InO$_x$, whose low carrier density implies low phase rigidity and strong superconducting phase fluctuations. Instead of presenting the abrupt jump expected at a BCS transition, the Nernst signal evolves continuously through the superconducting transition as previously observed in underdoped cuprates. This contrasts with the case of Nb$_{0.15}$Si$_{0.85}$, where the Nernst signal due to vortices below $T_c$ and by Gaussian fluctuations above are clearly distinct. The behavior of the ghost critical field in InO$_x$ points to a correlation length which does not diverge at $T_c$, a temperature below which the amplitude fluctuations freeze, but phase fluctuations survive.

The last few years have witnessed the emergence of the Nernst effect as an important probe of Superconducting Fluctuations (SF), following the observation of an anomalous Nernst signal above $T_c$ in cuprates [1]. In amorphous superconducting thin films of Nb$_{0.15}$Si$_{0.85}$, a Nernst signal produced by Cooper-pair fluctuations could be detected in a wide temperature and field range [2,3]. Close to $T_c$, the magnitude of the Nernst coefficient found in this experiment was in very good agreement with the predictions of a theory by Ussishkin, Sondhi and Huse (USH) for the transverse thermoelectric response of the Gaussian fluctuations of the Superconducting Order Parameter (SOP) [4]. This is not the case of underdoped cuprates, where the Nernst signal does not follow the predictions of the USH theory [4] and phase fluctuations of the SOP are believed to play a major role.

To address this issue, new theories have been proposed for cases where the Nernst signal is only generated by phase fluctuations of the SOP [5] or by quantum fluctuations near a Superconductor-Insulator Transition (SIT) [6]. On the experimental side, recent measurements on organic quasi-2D superconductors [7] detected a finite Nernst signal above $T_c$ in a temperature range widening with the approach of the Mott insulator as in the case of cuprates [1]. However, since the Nernst response of normal electrons scales with their mobility [8], the normal-state Nernst response is not negligible in either cuprate or organic superconductors. This complicates any quantitative comparison of the measured Nernst signal with theoretical predictions.

In this letter, we report on the case of InO$_x$. Several factors make thin films of this system an appealing candidate for the study of the Nernst signal generated by superconducting phase fluctuations. First of all, due to its low carrier density, a poor superfluid stiffness and, consequently, strong phase fluctuations are expected [9]. Moreover, the normal state is a dirty metal, with a negligible Nernst response. This system is also believed to host a Kosterlitz-Thouless-Berezinskii (KTB) transition [10]. Finally, due to its large sheet resistance, quantum fluctuations of the phase of the SOP are expected to give rise to a SIT at zero-temperature.

According to our findings, the Nernst effect in this system shares common features with cuprates. In contrast with Nb$_{0.15}$Si$_{0.85}$, its temperature dependence does not follow the predictions of the USH theory. Moreover, both the field and temperature dependence of the Nernst signal in InO$_x$ indicate that the blurred transition reflects a regime of superconducting fluctuations whose Correlation Length (CL) does not diverge. Our analysis is based on the previous study of the Nernst data in Nb$_{0.15}$Si$_{0.85}$ [2,3], which established the link between the Nernst signal and the CL [3].

The 300 Å-thick amorphous InO$_x$ film used in this study is deposited on a glass substrate by e-gun evaporation.

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In contrast, only a continuous change is observed for the Nernst coefficient in Nb$_{0.15}$Si$_{0.85}$. The shaded region represents the temperature range corresponding to 0.1–0.9 of the volume shrinkage of the sample during annealing [11]. Using a one-heater-two-thermometers setup, four-point resistance, Hall effect and thermoelectric measurements are measured in a single cooldown. The as-prepared film has an insulating-like behavior down to the lowest measured temperature of 60 mK. After thermal annealing at 50 °C under vacuum as described elsewhere [12], the room temperature sheet resistance decreases by about 30% and a superconducting state appears. According to optical-absorption experiments, this drop of resistivity is the consequence of the volume shrinkage of the sample during annealing [11]. During all measurements, the film has been kept below liquid-nitrogen temperature to avoid aging effects.

Figure 1 compares the behavior of the Nernst signal $N = E_y/(-\nabla_x T)$ of InO$_x$ with previously published data on Nb$_{0.15}$Si$_{0.85}$ [2], measured in the linear regime at low field, in the vicinity of the superconducting transition. In the case of Nb$_{0.15}$Si$_{0.85}$ (fig. 1a), $N$ increases abruptly at the BCS superconducting transition. It was shown [2,3] that above $T_c$, the Nernst signal is generated by Cooper-pairs fluctuations, and below $T_c$, by well-defined vortices.

In contrast, in the case of Nb$_{0.15}$Si$_{0.85}$, $N(T)$ does not show any distinct anomaly at any temperature separating these two regimes, the Nernst signal evolves continuously across the superconducting transition. Remarkably, this behavior has been observed previously for the evolution of the Nernst signal through the superconducting transition of underdoped cuprates, as in La$_{1.94}$Sr$_{0.06}$CuO$_4$ [13].

As seen in the inset of the same figure, the signal is at least 100 times larger than the product of the Seebeck coefficient and the normal-state Hall angle. Since the latter has been observed previously for the evolution of the Nernst signal through the superconducting transition of underdoped cuprates, as in La$_{1.94}$Sr$_{0.06}$CuO$_4$ [13].

![Figure 1: Superconducting state as it manifests itself on resistance (left axis) and Nernst coefficient (right axis) in Nb$_{0.15}$Si$_{0.85}$ ref. [2] (panel a) and InO$_x$ (panel b). The shaded region represents the temperature range corresponding to 0.1–0.9 of $R_{\text{max}}$, where $T_c$ is expected. The inset of panel b compares the Nernst coefficient with $S \tan \Theta_H$. Note the sharp increase of the Nernst coefficient in Nb$_{0.15}$Si$_{0.85}$ at $T_c$, indicated as a vertical line. In contrast, only a continuous change is observed for InO$_x$.](image1)

![Figure 2: Transverse Peltier coefficient $\alpha_{xy}$ of InO$_x$ and Nb$_{0.15}$Si$_{0.85}$ (from [2]) vs. temperature for $B \rightarrow 0$, as a function of $T/T_{\text{mid}}$. $T_{\text{mid}}$ is the midpoint of resistive transition and correspond to the superconducting transition only in Nb$_{0.15}$Si$_{0.85}$. The shaded region represents the temperature range where $T_c$ is expected for InO$_x$. The theoretical prediction from the USH theory is represented by the continuous and dotted lines.](image2)
lower-temperature scale of the KBT type where phase coherence sets in. In that case, a unique superconducting transition temperature can be defined for the sample and a unique CL value exists within the sample.

This last scenario is made plausible because of the low carrier density in InO$_x$, whose value is comparable to the underdoped cuprates. The Hall coefficient measured in our film ($R_H = 6 \times 10^{-8}$ m$^3$ A$^{-1}$) is close to the one found in La$_{2-x}$Sr$_x$CuO$_4$ (LSCO) at $x=0.05$ [14] and yields a carrier density of $n = 10^{21}$ cm$^{-3}$. On the other hand, the Hall coefficient in Nb$_{0.15}$Si$_{0.85}$ is 80 times lower [2], implying a much higher carrier density. Since the superfluid stiffness is proportional to the superfluid density, superconductors such as InO$_x$ and La$_{1.94}$Sr$_{0.06}$CuO$_4$ are expected to display strong phase fluctuations [9].

Thus, these two scenarios differ fundamentally by the behavior of the CL across the superconducting transition. In the inhomogeneities scenario, a broad distribution of CL values is expected. In the phase fluctuations scenario, the superconducting CL does not diverge at $T_c$ but at the lower temperature $T_{KTB}$. We provide below an analysis of the magnetic-field dependence of the Nernst data showing that the CL does not diverge at $T_c$, implying that the broad transition observed in InO$_x$ reflects the persistence of phase fluctuations below $T_c$.

Figure 3a shows that, for each temperature, the Nernst signal $N(B)$ peaks with a maximum at a temperature-dependent magnetic field scale $B^*(T)$. This peak can be clearly observed in $N(B)$ down to a temperature of 0.9 K. As discussed in previous studies on Nb$_{0.15}$Si$_{0.85}$ [23], at any temperature and magnetic field, $\alpha_{xy}/B$ depends only on the size of SF. At zero magnetic field, this size is set by the CL. At high field, this size is set by the magnetic length $l_B = (\hbar/2eB)^{1/2}$ when it becomes shorter than the zero-field CL. Thus, this coefficient acquires a characteristic field-temperature dependence that is observed in Nb$_{0.15}$Si$_{0.85}$ [3] and in InO$_x$ as shown in fig. 3c. $\alpha_{xy}/B$ is field-independent at low magnetic field, however, at high magnetic field, all the data evolve toward a single curve weakly dependent on temperature. This crossover is responsible for the peak observed at $B^*(T)$ in the field dependence of the Nernst signal $N(B)$ (see arrows in fig. 3a).

The temperature dependence of $B^*(T)$ is presented in fig. 4b for InO$_x$ and fig. 4a for Nb$_{0.15}$Si$_{0.85}$ [3]. In both systems, above $T_c$, $B^* \propto \ln T$ as expected for the field scale, $\Phi_0/2\pi \xi^2$, set by the BCS CL, $\xi = \xi_0 e^{-\varepsilon/2}$ where $\varepsilon = k_B T/T_c$. Since this field scale mirrors the evolution of the upper critical field below $T_c$, it was dubbed “the Ghost Critical Field” (GCF) by Kapitulnik and co-workers [15]. In the case of Nb$_{0.15}$Si$_{0.85}$, $\xi_0$ and $T_c$ could be independently determined and compared with the GCF obtained from the Nernst data. This result has shown that measuring the SF contributions to the Nernst response of a superconductor provides a direct probe of this field scale. For InO$_x$, on the other hand, we set $T_c$ and $\xi_0$ such that the GCF line fits $B^*(T)$. The thick line fig. 4b is a fit using $\xi_0 = 8.4 \pm 0.2$ nm and $T_c = 1.2$ K. This value of $T_c$ corresponds to the mid-point of the resistive transition, as seen fig. 1b. A similar conclusion on the position of $T_c$ was drawn by a recent study on InO$_x$ [10].

Thus, the interpretation of the field position of the Nernst peak as the GCF appears straightforward. According to our analysis, this field scale reflects the CL, no matter the precise nature of SF, Gaussian or phase-only. This recently received some theoretical support. Functional forms for the field dependence compatible with a maximum at the GCF have been predicted by a theory expanding the USH theory to finite field [16] and by a recent theory of the Nernst effect in the vicinity of a STT [6].

As seen in fig. 3a, the Nernst peak remains narrow down to 0.9 K. If the broad superconducting transition, spread on a temperature interval $\delta T = 0.7$ K, were due to a large distribution of $T_c$, this would imply a large distribution of CL, from $+\infty$ to $\xi = \xi_0 \ln (1 + \delta T/T_c)^{-1/2}$,
corresponding to GCF values between 0 and 3 T. Obviously, this should significantly broaden the Nernst peak beyond what is observed experimentally at 0.9 K. Actually, inset of fig. 3a shows that, in InO$_x$, the width of the Nernst peak normalized by $B^*(T)$ is similar to the corresponding value in Nb$_{0.15}$Si$_{0.85}$, which has a well-defined superconducting transition. Thus, the observation of a narrow Nernst peak in $N(B)$ down to 0.9 K shows that the CL remains a well-defined quantity and that the width of the superconducting transition cannot be explained by sample inhomogeneities and $T_c$ variations alone.

So what is the origin of the broad transition observed in InO$_x$? A comparison of the temperature dependence of the GCF from InO$_x$ (fig. 4b) with the one from Nb$_{0.15}$Si$_{0.85}$ (fig. 4a) points to a plausible answer. In Nb$_{0.15}$Si$_{0.85}$, the GCF vanishes and at $T_c$ (reflecting the divergence of the CL) mirrors the behavior of $B_{c2}(T)$ below $T_c$. One striking observation of this work is the breakdown of this picture in InO$_x$. $B^*(T)$ keeps decreasing down to 0.9 K, well below our estimate of $T_c = 1.2$ K. This indicates that the CL does not diverge and that no true phase transition occurs at $T_c$, the temperature associated with the formation of Cooper pairs. An identical conclusion can be drawn from the temperature dependence of $\alpha_{xy}/B$ where no abrupt change is observed upon crossing $T_c$. This leads us to conclude that the wide superconducting transition reflects the presence of an intermediate fluctuation region between $T_c$, where amplitude fluctuations freeze, and a lower temperature, $T_{KTB}$, where phase coherence should be established. Such a region of phase-only fluctuations in InO$_x$ was recently inferred from high-frequency conductivity measurements [10].

With the temperature dependence of the CL determined above, we find that USH formula [4], when $B \to 0$:

$$\alpha_{xy}^sc = \frac{1}{12\pi} \frac{k_B e \xi^2}{\hbar l^2}$$  \hspace{1cm} (1)

is close to the measured $\alpha_{xy}$, as seen fig. 2. However, $\alpha_{xy}$ decreases with temperature much faster than the power $T^{-1}$ predicted by the USH theory. Deviations from USH theory are indeed expected. Close to $T_c$, phase-only fluctuations models may be more relevant [5], while far above $T_c$ the USH model is not supposed to be valid.

Upon cooling, phase fluctuations are expected to disappear at KT B transition where the vortex and antivortex bind together. Since the vortices are a major source of the Nernst signal, the latter should be strongly affected by KT B transition. Below 0.9 K, the overall magnitude of the Nernst signal decreases and the field dependence $N(B)$ displays a broad maximum with complicated but reproducible substructures, fig. 3b. The reduced amplitude points to a reduced vortex mobility below 0.9 K, a possible signature of the KT B transition, whereas the observed substructures may be related to the bosonic properties of the magnetic-field–induced insulator. Indeed, as previously observed, at low temperature, the magnetoresistance increases steeply above the magnetic-field–induced SIT, (see fig. 5), followed by a negative magnetoresistance. This negative magnetoresistance was interpreted as a possible signature of the pair breaking effect of magnetic field on localized Cooper pair [17,18].

In contrast to the SIT observed in Nb$_{0.15}$Si$_{0.85}$ [19], MoGe [20], or Bi [21] thin films, the crossing-point $B_{SIT}(T)$, reported on the phase diagram fig. 4b, is temperature dependent. This behavior has been discussed previously for this compound [22] and remains yet to be understood.

To summarize, measuring the Nernst signal and resistivity in InO$_x$, we found that the transverse Peltier coefficient evolves continuously across the superconducting transition. Furthermore, we find that the GCF keeps decreasing in the temperature range where the Cooper-pair formation is expected to occur. This indicates that no true phase transition occurs at $T_c$ and implies the existence of a regime of phase-only fluctuations of the SOP. The similarity between the temperature dependence of the Nernst

Fig. 4: a) Phase diagram showing the GCF $B^*$ in Nb$_{0.15}$Si$_{0.85}$, taken from data shown in ref. [3]. The line below $T_c$ represents $H_{c2}(T)$. b) Phase diagram representing the critical field of SIT, $B_{SIT}$, and the field position $B^*$ of the Nernst maximum in $N(B)$. The thick line is an adjustment to the GCF $\Phi_0/2\pi \xi^2$.

\[ \text{Fig. 4: a) Phase diagram showing the GCF } B^* \text{ in Nb}_{0.15}\text{Si}_{0.85}, \text{ taken from data shown in ref. [3]. The line below } T_c \text{ represents } H_{c2}(T). \text{ b) Phase diagram representing the critical field of SIT, } B_{SIT}, \text{ and the field position } B^* \text{ of the Nernst maximum in } N(B). \text{ The thick line is an adjustment to the GCF } \Phi_0/2\pi \xi^2. \]
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Fig. 5: Magnetoresistance of $\text{InO}_x$ measured for several temperatures. The inset focuses on the low-field part showing the crossing between adjacent isotherms, the signature of the magnetic-field–induced SIT.

signal measured in $\text{InO}_x$ and the underdoped cuprates is additional support for the existence of a regime of phase-only fluctuations in the latter system. Such a similarity between $\text{InO}_x$ and underdoped cuprates has been noticed before where it was suggested that the underdoped cuprates may also host a magnetic-field–induced superconductor-insulator transition [23].

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