Comprehensive SNN Compression Using ADMM Optimization and Activity Regularization

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Abstract—Spiking neural network (SNN) is an important family of models to emulate the brain, which has been widely adopted by neuromorphic platforms. In the meantime, it is well-known that the huge memory and compute costs of neural networks greatly hinder the execution with high efficiency, especially on edge devices. To this end, model compression is proposed as a promising technique to improve the running efficiency via parameter and operation reduction, which has been evidenced in deep learning. Therefore, it is interesting to investigate how much an SNN model can be compressed without compromising much functionality. However, this is quite challenging because SNNs usually behave distinctly from deep learning models. Specifically, i) the accuracy of spike-coded SNNs is usually sensitive to any network change; ii) the computation of SNNs is event-driven rather than static. The former difference demands an accurate compression methodology and the latter one produces an extra compression dimension on dynamic spikes.

In this work, we realize a comprehensive SNN compression through three steps. First, we formulate the connection pruning and the weight quantization as a supervised learning-based constrained optimization problem. Second, we combine the emerging spatio-temporal backpropagation (STBP) and the powerful alternating direction method of multipliers (ADMM) to solve the problem with minimum accuracy loss. Third, we further propose an activity regularization to reduce the spike events for fewer active operations. The connection pruning, weight quantization, and activity regularization can be used in either a single way for moderate compression or a joint way for aggressive compression. We define several quantitative metrics to evaluate the compression performance for SNNs and validate our methodology in pattern recognition tasks over MNIST, N-MNIST, and CIFAR10 datasets. Extensive comparisons between different compression strategies, the corresponding result analysis, and some interesting insights are provided. To our best knowledge, this is the first work that studies SNN compression in a comprehensive manner by exploiting all possible compression ways and achieves better results. Our work offers a promising solution to pursue ultra-efficient neuromorphic systems.

Keywords: SNN Compression, Connection Pruning, Weight Quantization, Activity Regularization, ADMM

I. INTRODUCTION

Neural networks, constructed by a plenty of nodes (neurons) and connections (synapses), are powerful in information representation, which has been evidenced in a wide spectrum of intelligent tasks such as visual or auditory recognition [1]–[4], language modelling [5], [6], medical diagnosis [7], [8], game playing [9], heuristic solution of hard computational problems [10], sparse coding [11], etc. The models include two categories: application-oriented artificial neural networks (ANNs) and neuroscience-oriented spiking neural networks (SNNs). The former process continuous signals layer by layer with nonlinear activation functions; while the latter memorize temporal information via neuronal dynamics and use binary spike signals (0-nothing or 1-event spike) for inter-neuron communication. The success of these models spurs numerous researchers to study domain-specific hardwares for ANNs and SNNs, termed as deep learning accelerators [12]–[14] and neuromorphic chips [15]–[17], respectively.

Whereas, the huge amount of parameters and operations in neural networks greatly limits the running performance and hinders the deployment on edge devices with tight resources. To solve this problem, various model compression technologies including low-rank decomposition [18], network pruning [19]–[22], and data quantization [23]–[26] have been proposed to shrink the model size, which is quite helpful in boosting the hardware performance [27]–[33]. Although this solution has become a promising way to reduce the memory and compute costs in deep learning, it has yet to be well studied in the neuromorphic computing domain. The underlying reason is because the behaviors of SNNs are quite different from those of ANNs. For example, i) the spike coding of SNNs makes the accuracy very sensitive to any network change, which demands an accurate compression methodology; ii) the processing of SNNs is event-driven with a dynamic rather than static execution pattern, which produces an extra compression dimension on dynamic spikes.

In fact, we find several previous work that tried tentative explorations on this topic. S. K. Esser et al. [3] adapted normal ANN models to their variants with ternary weights and binary activations, and then deployed them on the TrueNorth chip that only supports SNNs. A two-stage growing-pruning algorithm for compact fully-connected (FC) SNNs was verified on small-scale datasets [34]. Based on a single FC layer with spike timing dependent plasticity (STDP) learning rule, a soft-pruning method (setting part of weights to a lower bound during training) achieved 95.04% accuracy on MNIST dataset [35]. Similarly, on FC-based SNNs with STDP, both connection pruning and weight quantization were conducted and
validated on MNIST with 91.5% accuracy \[35\]. Combining an FC feature extraction layer with binary weights trained by stochastic STDP and an FC classification layer with 24-bit precision, A. Yousefzadeh et al. \[37\] presented 95.7% accuracy on MNIST. G. Srinivasan et al. \[38\] introduced residual paths into SNNs and combined spiking convolutional (Conv) layers with binary weight kernels trained by probabilistic STDP and non-spiking FC layers trained by conventional backpropagation (BP) algorithm, which demonstrated 98.54% accuracy on MNIST dataset but only 66.23% accuracy on CIFAR10 dataset. Unfortunately, these existing works on SNN compression did not either touch real SNNs (just ANN variants) or harness large-scale models with impressive performance.

Hence, we formally pose a question that \textit{how much an SNN model can be compressed without compromising much functionality}. We answer this question through three steps. (1) First, we formulate the connection pruning and the weight quantization as a constrained optimization problem based on supervised learning. (2) Second, we combine the emerging spatio-temporal backpropagation (STBP) supervised learning \[39, 40\] and the powerful alternating direction method of multipliers (ADMM) optimization tool \[41\] to solve the problem with minimum accuracy loss. (3) Third, we propose an activity regularization to reduce the number of spike events, leading to fewer active operations. These approaches can be flexibly used in a single or joint manner according to actual needs for compression performance. We comprehensively validate our methods in SNN-based pattern recognition tasks over MNIST, N-MNIST, and CIFAR10 datasets. Several quantitative metrics to evaluate the compression ratio are defined, based on which a variety of comparisons between different compression strategies and the in-depth result analysis are conducted. Our work can achieve aggressive compression ratio with advanced accuracy maintaining, which promises ultra-efficient neuromorphic systems.

For better readability, we briefly summarize our contributions as follows:

- We present the first work that investigates comprehensive and aggressive compression for SNNs by exploiting all possible compression ways and defining quantitative evaluation metrics.
- The effectiveness of the ADMM optimization tool is validated on SNNs to reduce the parameter memory space and baseline compute cost for the first time. Then, the activity regularization method is further proposed to reduce the number of active operations. All the proposed approaches can be flexibly used in either a single manner for moderate compression or a joint manner for aggressive compression.
- We demonstrate high compression performance in SNN-based pattern recognition tasks without compromising much accuracy. Rich contrast experiments, in-depth result analysis, and interesting insights are provided.

The rest of this paper is organized as follows: Section II introduces some preliminaries of the SNN model, the STBP learning algorithm, and the ADMM optimization approach; Section III systematically explains the possible compression ways, the proposed ADMM-based connection pruning and weight quantization, the activity regularization, their joint use, and the evaluation metrics; The experimental setup, experimental results, and in-depth analysis are provided in Section IV. Finally, Section V concludes and discusses the paper.

### II. Preliminaries

#### A. Spiking Neural Networks

In a neural network, neurons behave as the basic processing units which are wired by abundant synapses. Each synapse has a weight that affects the signal transfer efficacy. Figure 1 presents a typical spiking neuron, which is comprised of synapses, dendrites, soma, and axon. Dendrites integrate the weighted input spikes and the soma consequently conducts nonlinear transformation to produce the output spikes, then the axon transfers these output spikes to post-neurons. The neuronal behaviors can be described by the classic leaky integrate-and-fire (LIF) model \[42\] as follows:

\[
\begin{align*}
\tau \frac{du(t)}{dt} &= -[u(t) - u_{r1}] + \sum_j w_j \sum_{k \in [t - T_w, t]} K(t - t_k^j) \\
o(t) &= \begin{cases} 1 & \text{if } u(t) \geq u_{r2} \text{ and } u(t) \leq u_{r1} \\ 0 & \text{if } u(t) < u_{r2} \end{cases}
\end{align*}
\]

where \((t)\) denotes the time step, \(\tau\) is a time constant, \(u\) is the membrane potential of current neuron, and \(o\) is the output spike event. \(w_j\) is the synaptic weight from the \(j\)-th input neuron to the current neuron, and \(t_k^j\) is the time step when the \(k\)-th spike from the \(j\)-th input neuron occurs during the past integration time window of \(T_w\). \(K(\cdot)\) is a kernel function describing the temporal decay effect that a more recent spike should have a greater impact on the post-synaptic membrane potential. \(u_{r1}\) and \(u_{r2}\) are the resting potential and reset potential, respectively, and \(u_{rth}\) is a threshold that determines whether to fire a spike or not.

Figure 1: Illustration of a spiking neuron comprised of synapses, dendrites, soma, and axon.

According to Equation (1), SNNs have the following differences compared with ANNs: (1) each neuron has temporal dynamics, i.e. memorization of the historical states; (2) the multiplication operations during integration can be removed when \(T_w = 1\) owing to the binary spike inputs; (3) the network activities are very sparse because each neuron remains silent if the membrane potential does not exceed the firing threshold. In summary, the temporal memorization makes it well-suited for dynamic data with timing information, and the spike-driven paradigm with sparse activities enables power-efficient asynchronous circuit design.
B. STBP Supervised Learning

There exist three categories of learning algorithms for SNNs: unsupervised [43, 44], indirectly supervised [45–47], and directly supervised [39, 40]. Since SNN compression requires an accurate learning method and the ADMM optimization (to be shown later) relies on the supervised learning framework, we select an emerging directly supervised training algorithm, named spatio-temporal back-propagation (STBP) [39, 40]. We do not use the indirectly supervised training due to the complex model transformation between ANNs and SNNs.

STBP is based on an iterative version of the LIF model in Equation (1). Specifically, it yields

\[
\begin{align*}
u_{i,t,n+1}^t &= \exp^{-\frac{dt}{r_{\text{fire}}}} u_{i,t,n+1}^t (1 - o_{i,t,n+1}^t) + \sum_j w_{ij} o_{j,t,n}^t, \\
o_{i,t,n+1}^t &= \text{fire}(u_{i,t,n+1}^t - u_{th})
\end{align*}
\]

where \( dt \) is the length of simulation time step, \( o \) denotes the neuronal spike output, \( t \) and \( n \) are indices of time step and layer, respectively. \( \exp^{-\frac{dt}{r_{\text{fire}}}} \) reflects the leakage effect of the membrane potential. \( \text{fire}(\cdot) \) is a step function, which satisfies \( \text{fire}(x) = 1 \) when \( x \geq 0 \), otherwise \( \text{fire}(x) = 0 \). This iterative LIF format incorporates all behaviors including integration, fire, and reset in the original neuron model. For simplicity, here we set the parameters in Equation (1) with \( u_{r_1} = u_{r_2} = 0, T_w = 1, \) and \( K(\cdot) \equiv 1 \).

STBP uses rate coding to represent information, wherein the number of spikes matters. The loss function is given by

\[
L = \| Y^{\text{label}} - \frac{1}{T} \sum_{t=1}^{T} O_{t,N} \|_2^2.
\]

This loss function measures the discrepancy between the ground truth and the firing rate of the output layer (i.e. the \( N \)-th layer) during the given simulation time window \( T \).

Given Equation (2)–(3), the gradient propagation and parameter update in STBP can be derived as follows

\[
\begin{align*}
\frac{\partial L}{\partial o_{i,t,n}} &= \sum_j \frac{\partial L}{\partial o_{j,t,n}} \frac{\partial o_{j,t,n+1}^t}{\partial o_{i,t,n}} + \frac{\partial L}{\partial o_{i,t,n}} \frac{\partial o_{i,t,n+1}^t}{\partial o_{i,t,n}}, \\
\frac{\partial L}{\partial u_{i,t,n}} &= \frac{\partial L}{\partial o_{i,t,n}} \frac{\partial o_{i,t,n+1}^t}{\partial o_{i,t,n}} + \frac{\partial L}{\partial u_{i,t,n}} \frac{\partial o_{i,t,n+1}^t}{\partial u_{i,t,n}}, \\
\nabla u_{i,t,n} &= \sum_{t=1}^{T} \frac{\partial L}{\partial o_{i,t,n}} \frac{\partial o_{i,t,n+1}^t}{\partial u_{i,t,n}}.
\end{align*}
\]

Although \( \text{fire}(\cdot) \) is non-differentiable, the derivative approximation method can be used to calculate \( \frac{\partial u_{i,t,n}}{\partial u_{a}} \) [39]. Specifically, it is governed by \( \frac{\partial u_{i,t,n}}{\partial u_{a}} \approx \frac{1}{a} \text{sign}(u_{i,t,n} - u_{th}) \leq \frac{1}{a} \), where \( a \) is a hyper-parameter that determines the gradient width, and we define \( \text{sign}(x) = 1 \) if \( x > 0 \) and \( \text{sign}(x) = 0 \) otherwise.

C. ADMM Optimization Tool

ADMM is a classic and powerful tool to solve constrained optimization problems [41]. The main idea of ADMM is to decompose the original non-differentiable optimization problem to a differentiable sub-problem which can be solved by gradient descent and a non-differentiable sub-problem with an analytical or heuristic solution.

The basic problem of ADMM can be described as

\[
\min_{X, Z} f(X) + g(Z), \quad \text{s.t.} \quad AX + BZ = C
\]

where we assume \( X \in R^N, Z \in R^M, A \in R^{K \times N}, B \in R^{K \times M}, C \in R^K, f(\cdot) \) is the major cost function which is usually differentiable and \( g(\cdot) \) is an indicator of constraints which is usually non-differentiable. Then, the greedy optimization of its augmented Lagrangian [41], \( L_{\rho}(X, Z, Y) = f(X) + g(Z) + Y^T (AX + BZ - C) + \frac{\rho}{2} \| AX + BZ - C \|_2^2 \), can be iteratively calculated by

\[
\begin{align*}
X^{n+1} &= \arg \min_{X} L_{\rho}(X, Z^n, Y^n) \\
X^{n+1} &= \arg \min_{Z} L_{\rho}(X^{n+1}, Z, Y^n) \\
Y^{n+1} &= Y^n + \rho (AX^{n+1} + BZ^{n+1} - C)
\end{align*}
\]

where \( Y \) is the Lagrangian multipliers and \( \rho \) is a penalty parameter. The \( X \) minimization sub-problem is differentiable that is easy to solve via gradient descent. The \( Z \) minimization sub-problem is non-differentiable, but fortunately it can usually be solved analytically or heuristically.

III. SPIKING NEURAL NETWORK COMPRESSIO

In this section, we first give the possible compression ways, and then explain the proposed compression approaches, algorithms, and evaluation metrics in detail.

A. Possible Compression Ways

The compression of SNNs in this work targets the reduction of memory and computation in inference. Figure 2 illustrates the possible ways to compress an SNN model. On the memory side, synapses occupy the most storage space. There are usually two ways to reduce the synapse memory: the number of connections and the bitwidth of weights. On the compute side, although the connection pruning and the weight quantization can help reduce the amount of operations, there is an additional compression space on the dynamic spikes. As well known, the total number of operations for an SNN layer can be governed by \( N_{\text{ops}} \cdot R [50] \), where \( N_{\text{ops}} \) is the number of baseline operations and \( R \in [0, 1] \) is the average spike rate per neuron per time step that usually determines the active power of neuromorphic chips [16].

To realize a comprehensive SNN compression considering all above ways, we first try to combine the STBP supervised learning and the ADMM optimization tool for connection pruning and weight quantization to shrink memory and reduce \( N_{\text{ops}} \). The reason that we combine STBP and ADMM is two-fold: (1) ADMM recently shows an impressive compression ratio with good accuracy maintaining in the ANN domain [22], [51]–[54]; (2) ADMM requires a supervised learning framework, which excludes the conventional unsupervised learning algorithms for SNNs. Then, besides synapse compression, we additionally propose an activity regularization to reduce \( R \) for a further reduction of operations. We will explain these methods one by one in the rest subsections.

B. ADMM-based Connection Pruning

For the connection pruning, the ADMM problem in Equation (5) can be re-formulated as

\[
\min_{W \in \mathcal{P}} L = f(W)
\]
where $g(W) = 0$ if $W \in P$, $g(W) = +\infty$ otherwise. Second, it is further converted to

$$\min_{W, Z} L = f(W) + g(Z), \ s.t. \ W = Z. \quad (9)$$

Now the pruning problem is equivalent to the classic ADMM problem given in Equation (5).

With the constraint of $W = Z$, the augmented Lagrangian can be equivalently simplified to $L_\rho = f(W) + g(Z) + \rho/2 \|W - Z\|^2 + \frac{\rho}{2} \|W - Z + \frac{1}{\rho} Y\|^2 - \frac{\rho}{2} \|Y\|^2$ where $Y = Y/\rho$. In this way, the greedy minimization in Equation (6) can be re-written as

$$W^{n+1} = \arg \min_W L_\rho(W, Z^n, \tilde{Y}^n) \quad (10)$$

$$Z^{n+1} = \arg \min_Z L_\rho(W^{n+1}, Z, \tilde{Y}^n) \quad (10)$$

$$\tilde{Y}^{n+1} = \tilde{Y}^n + W^{n+1} - Z^{n+1}$$

Actually, the first sub-problem is $W^{n+1} = \arg \min_W f(W) + \frac{\rho}{2} \|W - Z^n + \tilde{Y}^n\|^2$ which is differentiable and can be directly solved by gradient descent. The second sub-problem is $Z^{n+1} = \arg \min_Z g(Z) + \frac{\rho}{2} \|W^{n+1} - Z + \tilde{Y}^n\|^2$, which is equivalent to

$$\arg \min_{Z \in P} \frac{\rho}{2} \|W^{n+1} - Z + \tilde{Y}^n\|^2. \quad (11)$$

The above sub-problem can be heuristically solved by keeping a fraction of elements in $(W^{n+1} + \tilde{Y}^n)$ with the largest magnitudes and setting the rest to zero. Given $W^{n+1}$ and $Z^{n+1}$, $\tilde{Y}$ can be updated according to $\tilde{Y} = \tilde{Y}^n + W^{n+1} - Z^{n+1}$.

**Algorithm 1**: ADMM-based Connection Pruning

where $f(W) = 0$ if $W \in P$, $f(W) = +\infty$ otherwise. Second, it is further converted to

$$\min_{W, Z} L = f(W) + g(Z), \ s.t. \ W = Z. \quad (9)$$

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The above sub-problem can be heuristically solved by keeping a fraction of elements in $(W^{n+1} + \tilde{Y}^n)$ with the largest magnitudes and setting the rest to zero. Given $W^{n+1}$ and $Z^{n+1}$, $\tilde{Y}$ can be updated according to $\tilde{Y} = \tilde{Y}^n + W^{n+1} - Z^{n+1}$. The overall training for ADMM-based connection pruning is provided in Algorithm 1. Note that the sparsification step when updating $Z$ is layer-wise rather than network-wise.

**C. ADMM-based Weight Quantization**

The overall framework of ADMM-based weight quantization is very similar to the ADMM-based connection pruning. The only difference is that the constraint on weights changes
Data: $b$ (weight bitwidth), $I$ (#quantization iterations)

Step I: ADMM Retraining for Quantization

Initialize $W_0^0$ with the pre-trained weights;
Initialize $\tilde{Y}^0 = 0$;
Initialize $Z_q^0 = Quan(W_0^0, b, I)$;

Data: $W_q^n$, $Z_q^n$, and $\tilde{Y}^n$ after the $n$-th iteration

Result: $W^{n+1}, Z_q^{n+1}$, and $\tilde{Y}^{n+1}$

1. Rewrite the loss function:
   \[
   L = f(W) + \frac{\rho}{2}\|W - Z_q^n + \tilde{Y}^n\|_2^2;
   \]
2. Update weights:
   \[
   W^{n+1} = \text{retrain the SNN model one more iteration;}
   \]
3. Update $Z_q^{n+1}$:
   \[
   Z_q^{n+1} = Quan(W^{n+1} + \tilde{Y}^{n}, b, I);
   \]
4. Update $\tilde{Y}^{n+1}$:
   \[
   \tilde{Y}^{n+1} = (\tilde{Y}^n + W^{n+1} - Z_q^{n+1});
   \]

Step II: Hard-Quantization Retraining

Initialize $W_0^0$ with the weights from Step I;
Initialize the loss function $L = f(W_q)$;

Data: $W_q^n$ after the $n$-th iteration

Result: $W_q^{n+1}, Z_q^{n+1}$

Update weights:
\[
W_q^{n+1} = \text{retrain the SNN model one more iteration;}
\]
\[
Z_q^{n+1} = Quan(W_q^{n+1}, b, I);
\]

Algorithm 2: ADMM-based Weight Quantization

from the sparse one to a quantized one. Hence, Equation (7) can be re-written as

\[
\min\limits_{W \in Q} L = f(W)
\]  (12)

where $Q$ is a set of discrete levels, e.g. $Q = \{0, \pm 2^0, \pm 2^1, \pm 2^2, \ldots\}$, and $\alpha$ is a scaling factor that can be independent between layers. Similarly, now Equation (11) should be

\[
\arg\min\limits_{Z \in Q} \frac{\rho}{2}\|W^{n+1} - Z + \tilde{Y}^n\|_2^2
\]  (13)

which is equivalent to

\[
\arg\min\limits_{Z, \alpha} \frac{\rho}{2}\|V - \alpha \tilde{Z}\|_2^2
\]  (14)

where $V = W^{n+1} + \tilde{Y}^n$, $\alpha \tilde{Z} = Z$, and $Z \subset \{0, \pm 2^0, \pm 2^1, \pm 2^2, \ldots\}$. This sub-problem can also be heuristically solved by an iterative quantization [51], i.e. iteratively fixing $\alpha$ and the quantized vector $\tilde{Z}$ to convert the bivariate optimization to two iterative univariate optimizations. Specifically, with $\alpha$ fixed, the quantized vector $\tilde{Z}$ is actually the projection of $\frac{V^T}{\|Z\|^2}$ onto \{0, $\pm 2^0, \pm 2^1, \pm 2^2, \ldots\}$, which can be simply obtained by approaching the closest discrete level of each element; with $\tilde{Z}$ fixed, $\alpha$ can be easily calculated by $\alpha = \frac{V^T \tilde{Z}}{\|\tilde{Z}\|^2}$. In practice, we find this iterative minimization converges very fast (e.g. in three iterations). The overall training for ADMM-based weight quantization is given in Algorithm 3 where the quantization function ($Quan(\cdot)$) is additionally given in Algorithm 3. Note that the quantization step when updating $Z$ is layer-wise too, which might cause different $\alpha$ values across layers.

Data: $V, b$ (weight bitwidth), $I$ (#quantization iterations)

Result: $Z$

Define a discrete space $Q = \{0, \pm 2^0, \pm 2^1, \ldots, \pm 2^b-1\}$;
Initialize $\alpha = 1$;

for $i = 0 : I - 1$
\begin{itemize}
  \item Update $\tilde{Z}$:
    \[
    Z \leftarrow \text{project each element in } V \text{ to its nearest discrete level in } Q;
    \]
  \item Update $\alpha$:
    \[
    \alpha \leftarrow V^T \tilde{Z};
    \]
end

\[
Z \leftarrow \alpha \tilde{Z};
\]

Algorithm 3: Quantization Function - $Quan(\cdot)$

Figure 3: Weight space evolution during ADMM training. At each iteration, $Z$ is obtained by constraining $W$ into a constrained space and $W$ gradually approaches $Z$.

Figure 3 presents the evolution of the weight space during ADMM training. In fact, $Z$ strictly satisfies the constraints (sparse or quantized) at each iteration by solving Equation (11) or (13), respectively. Moreover, $W$ gradually approaches $Z$ by minimizing the $L2$-norm regularizer, i.e. $\frac{\rho}{2}\|W - Z^n + \tilde{Y}^n\|_2^2$, in the first sub-problem of ADMM. The auxiliary variable $\tilde{Y}$ tends to be zero (we omit it from Figure 3 for simplicity). To evidence the above prediction, we visualize the distributions of $W$, $Z$, and $\tilde{Y}$ at different stages during the entire ADMM retraining process, as depicted in Figure 4.

Here we take the weight quantization as an example and set $\rho = 0.1$. Compared to the hard pruning [19], [55] or quantization [25], [56], ADMM-based compression is able to achieve better convergence due to the multivariable optimization.

D. Activity Regularization

As aforementioned in Section III-A, the compute cost of SNNs is jointly determined by the baseline operations and the average spike rate during runtime. Hence, besides the connection pruning and the weight quantization, there is an
ory and compute costs. On the memory side, we just count the extra opportunity to reduce the compute cost by activity regularization. To this end, we tune the loss function to

\[ L = L_{\text{normal}} + \lambda R \]  

(15)

where \( L_{\text{normal}} \) is the vanilla loss function in Equation (5), \( R \) is the mentioned average spike rate per neuron per time step, and \( \lambda \) is a penalty coefficient. The reason that we use the average spike rate rather than the total number of spikes is to unify the exploration of \( \lambda \) setting across different networks. By introducing the above regularization item, we can further sparsify the firing activities of an SNN model, resulting in decreased active operations.

E. Compression Strategy: Single-way or Joint-way

Based on the ADMM-based connection pruning and weight quantization, as well as the activity regularization, we propose two categories of compression strategy: single-way and joint-way. Specifically, i) single-way compression individually applies connection pruning, weight quantization, or activity regularization; ii) joint-way compression jointly applies connection pruning, weight quantization, and activity regularization, including “Pruning & Regularization”, “Quantization & Regularization”, “Pruning & Quantization”, and “Pruning & Quantization & Regularization”. Compared to the single-way compression, the joint-way compression can usually achieve a more aggressive overall compression ratio by exploiting multiple information ways.

For “Pruning & Regularization” and “Quantization & Regularization”, we introduce the activity regularization item \( \lambda R \) into the loss functions in Algorithm 1 and Algorithm 2 respectively. For “Pruning & Quantization”, we merge both the connection pruning and the weight quantization, as presented in Algorithm 3. For “Pruning & Quantization & Regularization”, we further incorporate the activity regularization item into the loss functions in Algorithm 4.

F. Quantitative Evaluation Metrics for SNN Compression

The compression ratio can be reflected by the reduced memory and compute costs. On the memory side, we just count the required storage space for weight parameters since they occupy the most memory space. On the compute side, we just count the required addition operations because the multiplications can be removed from SNNs with binary spike representation. The connection pruning reduces the number of parameters and baseline operations, thus lowering both the memory and compute costs; the weight quantization reduces the bitwidth of parameters and the basic cost of each addition operation, thus also lowering both the memory and compute costs; the activity regularization reduces the number of dynamic spikes, thus mainly lowering the compute cost.

Here we propose several metrics to quantitatively evaluate the compression ratio of SNNs. For the memory compression, we define the following percentage of residual memory:

\[ R_{\text{mem}} = (1 - s) \cdot b / B \]  

(16)

where \( s \in [0, 1] \) is the connection sparsity, \( B \) and \( b \) are

Data: \( s \) (connection sparsity), \( b \) (weight bitwidth), \( I \) (#quantization iterations)

**Step I: ADMM Retraining for Pruning**

Initialize \( W^0 \) with the pre-trained weights;
Generate sparse weights \( W_p \) by retraining the SNN model with Algorithm 1;
Generate a binary mask \( M_p \) in which 1s and 0s denote the remained and pruned weights in \( W_p \), respectively;

**Step II: ADMM Retraining for Pruning & Quantization**

Initialize \( W^0 \) with the weights from Step I;
Initialize \( \tilde{Y}^0 = 0 \);
Initialize \( Z_{pq}^0 = \text{Quan}(W^0, b, I) \);

Data: \( W^0, Z_{pq}^0 \) and \( \tilde{Y}^n \) after the \( n \)-th iteration

Result: \( W^{n+1}, Z_{pq}^{n+1} \), and \( Y^{n+1} \);

1. Rewrite the loss function:
   \[ L \leftarrow f(W) + \frac{\lambda}{2} \| W - Z_{pq}^0 + \tilde{Y}^n \|_2^2; \]
2. Update weights:
   \[ W^{n+1} \leftarrow \text{retrain the SNN model one more iteration (update only the non-zero weights according to } M_p); \]
3. Update \( Z_{pq}^{n+1} \):
   \[ Z_{pq}^{n+1} \leftarrow \text{Quan}(W^{n+1} + \tilde{Y}^n, b, I); \]
4. Update \( \tilde{Y}^{n+1} \):
   \[ \tilde{Y}^{n+1} \leftarrow (\tilde{Y}^n + W^{n+1} - Z_{pq}^{n+1}); \]

**Step III: Hard-Pruning-Quantization Retraining**

Initialize \( W^0_{pq} \) with the weights from Step II;
Initialize the loss function \( L = f(W_{pq}); \)

Data: \( W_{pq}^n \) after the \( n \)-th iteration

Result: \( W_{pq}^{n+1} \)

Update weights:
   \[ W_{pq}^{n+1} \leftarrow \text{retrain the SNN model one more iteration (update only the non-zero weights according to } M_p); \]
   \[ W_{pq}^{n+1} \leftarrow \text{Quan}(W_{pq}^{n+1}, b, I); \]

**Algorithm 4: ADMM-based Pruning & Quantization**

Initialize \( W^0 \) with the pre-trained weights;
Generate sparse weights \( W_p \) by retraining the SNN model with Algorithm 1;
Generate a binary mask \( M_p \) in which 1s and 0s denote the remained and pruned weights in \( W_p \), respectively;

**Step I: ADMM Retraining for Pruning**

Initialize \( W^0 \) with the pre-trained weights;
Generate sparse weights \( W_p \) by retraining the SNN model with Algorithm 1;
Generate a binary mask \( M_p \) in which 1s and 0s denote the remained and pruned weights in \( W_p \), respectively;

**Step II: ADMM Retraining for Pruning & Quantization**

Initialize \( W^0 \) with the weights from Step I;
Initialize \( \tilde{Y}^0 = 0 \);
Initialize \( Z_{pq}^0 = \text{Quan}(W^0, b, I) \);

Data: \( W^0, Z_{pq}^0 \) and \( \tilde{Y}^n \) after the \( n \)-th iteration

Result: \( W^{n+1}, Z_{pq}^{n+1} \), and \( Y^{n+1} \);

1. Rewrite the loss function:
   \[ L \leftarrow f(W) + \frac{\lambda}{2} \| W - Z_{pq}^0 + \tilde{Y}^n \|_2^2; \]
2. Update weights:
   \[ W^{n+1} \leftarrow \text{retrain the SNN model one more iteration (update only the non-zero weights according to } M_p); \]
3. Update \( Z_{pq}^{n+1} \):
   \[ Z_{pq}^{n+1} \leftarrow \text{Quan}(W^{n+1} + \tilde{Y}^n, b, I); \]
4. Update \( \tilde{Y}^{n+1} \):
   \[ \tilde{Y}^{n+1} \leftarrow (\tilde{Y}^n + W^{n+1} - Z_{pq}^{n+1}); \]

**Step III: Hard-Pruning-Quantization Retraining**

Initialize \( W^0_{pq} \) with the weights from Step II;
Initialize the loss function \( L = f(W_{pq}); \)

Data: \( W_{pq}^n \) after the \( n \)-th iteration

Result: \( W_{pq}^{n+1} \)

Update weights:
   \[ W_{pq}^{n+1} \leftarrow \text{retrain the SNN model one more iteration (update only the non-zero weights according to } M_p); \]
   \[ W_{pq}^{n+1} \leftarrow \text{Quan}(W_{pq}^{n+1}, b, I); \]

**Algorithm 4: ADMM-based Pruning & Quantization**

Initialize \( W^0 \) with the pre-trained weights;
Generate sparse weights \( W_p \) by retraining the SNN model with Algorithm 1;
Generate a binary mask \( M_p \) in which 1s and 0s denote the remained and pruned weights in \( W_p \), respectively;
the weight bitwidth of the original model and the compressed model, respectively. Since the operation compression is related to the dynamic spikes, next, we define the percentage of residual spikes as

\[ R_s = r/R \]  

(17)

if we can reduce the average spike rate from \( R \) to \( r \). Based on the mentioned rule in Section III-A that the total number of operations in an SNN model is calculated by multiplying the number of baseline operations and the average spike rate, we define the percentage of residual operations as

\[ R_{ops} \approx (1-s) \cdot \frac{b}{B} \cdot \frac{r}{R} = R_{mem} \cdot R_s. \]  

(18)

Note that above equation is just a coarse estimation because the impact of bitwidth on the operation cost is not linear. For example, an FP32 (i.e. 32-bit floating point) dendrite integration is not strictly 4x costly than the INT8 (i.e. 8-bit integer) one.

### IV. EXPERIMENTAL RESULTS

#### A. Experimental Setup

We validate our compression methodology on various datasets, including the static image datasets (e.g. MNIST and CIFAR10) and the event-drive N-MNIST, and then observe their impact on the accuracy and summarize the extent to which an SNN model can be compressed with negligible functionality degradation. For MNIST and N-MNIST, we use the classic LeNet-5 structure; while for CIFAR10, we use a convolution with stride of 2 to replace the pooling operation and then design a 10-layer spiking convolutional neural network (CNN) with the structure of Input-128C3S1-256C3S2-256C3S1-512C3S2-512C3S1-1024C3S2-1024FC-512FC-10. We take the Bernoulli sampling to convert the raw pixel intensity to a spike train on MNIST; while on CIFAR10, inspired by [40], we use an encoding layer to convert the normalized image input into spike trains to improve the baseline accuracy. The programming environment for our experiments is Pytorch. We omit “INT” and only remain the bitwidth for simplicity in the results with weight quantization.

#### B. Single-way Compression

In this subsection, we analyze the results from single-way compression, i.e. applying connection pruning, weight quantization, and activity regularization individually.

**Connection Pruning.** As shown in Figure 5, the number of disconnected synapses dramatically increases after connection pruning, and the overall percentage of pruned connections grows accordingly as the sparsity becomes higher. More specifically, Table II shows the model accuracy under different pruning ratio (i.e. connection sparsity). Overall, a <40-50% pruning ratio causes negligible accuracy loss or even better accuracy due to the alleviation of over-fitting, while an over 60% sparsity would cause obvious accuracy degradation that even reaches 2-4% at 75% sparsity. The accuracy loss on N-MNIST is more severe than that on MNIST, especially in the low-sparsity region. This reflects the accuracy sensitivity to the connection pruning on N-MNIST with natural sparse features. The results on CIFAR10 suffer from a larger fluctuation is due to the increasing difficulty and model size.

**Figure 5:** Effect of connection pruning on LeNet-5, (a) visualization of 800 randomly selected connections, where white pixels denote pruned connections; (b) weight value distribution before and after pruning with 75% sparsity.

#### Table II: Accuracy under different connection sparsity.

| Dataset | Sparsity (s) | Acc. (%) | Acc. Loss (%) |
|---------|--------------|----------|---------------|
| MNIST   | 0%           | 99.07    | 0.00          |
|         | 25%          | 99.19    | 0.12          |
|         | 40%          | 99.08    | 0.01          |
|         | 50%          | 99.10    | 0.03          |
|         | 60%          | 98.64    | -0.43         |
|         | 75%          | 96.84    | -2.23         |
| N-MNIST | 0%           | 98.95    | 0.00          |
|         | 25%          | 98.72    | -0.23         |
|         | 40%          | 98.59    | -0.36         |
|         | 50%          | 98.34    | -0.61         |
|         | 75%          | 96.83    | -2.12         |
| CIFAR10 | 0%           | 89.33    | 0.00          |
|         | 25%          | 89.8     | 0.27          |
|         | 40%          | 89.75    | 0.18          |
|         | 50%          | 89.15    | -0.38         |
|         | 60%          | 88.35    | -1.18         |
|         | 75%          | 86.0     | -3.53         |
Weight Quantization. Before discussing the results, we clarify that $b$ bits correspond to $2b + 1$ rather than $2^b$ discrete levels here, according to our definition of the quantized space in Algorithm 3. Figure 6 evidences that the number of weight levels can be significantly reduced after applying the weight quantization. Note that the number of discrete levels in the network is more than 5 at $b = 2$ due to the mentioned layer-wise quantization with different $\alpha$ across layers. Moreover, Table III presents the accuracy results under different weight bitwidth. On all datasets, we observe negligible accuracy loss when $b \geq 4$. The accuracy loss is still very small ($\leq 0.52\%$) even if under the aggressive compression with $b = 1$, which reflects the effectiveness of our ADMM-based weight quantization. The accuracy loss on MNIST is constantly smaller than others due to the simplicity of this task.

Activity Regularization. Different from the compression of synapses in previous connection pruning and weight quantization, the activity regularization reduces the number of dynamic spikes thus decreasing the number of active operations. The total number of spike events and the average spike rate can be greatly decreased by using this regularization (see Figure 7). Table IV further lists the accuracy results under different average spike rate, which is realized by adjusting $\lambda$. A larger $\lambda$ leads to a more aggressive regularization, i.e. lower spike rate. From MNIST to N-MNIST and CIFAR10, we observe a gradually weakened robustness to the activity regularization. Interestingly, we also find that their baseline average spike rate under $\lambda = 0$ also gradually decreases, which indicates that a higher baseline spike rate would have more space for activity regularization without compromising much accuracy. This is straightforward to understand because a higher baseline spike rate usually has a stronger capability for initial information representation.

C. Joint-way Compression

In this subsection, we analyze the results from joint-way compression, i.e. simultaneously applying two or three methods among connection pruning, weight quantization, and activity regularization. Table V and VI provide the accuracy results of the two-way compression and the three-way compression, respectively. Based on these two tables, we summarize several interesting observations as below.

Contribution to $R_{mem}$ and $R_s$. The weight quantization contributes most to the reduction of memory (reflected by $R_{mem}$) compared to the connection pruning. For example, an aggressive 75% connection sparsity (i.e. $R_{mem}=25\%$) just corresponds to a slight 8-bit weight quantization at the same level of memory compression ratio. Note that, as aforementioned, $b$-bit weights in this work have $2b + 1$ discrete levels, which is actually more aggressive than the standard definition of $2^b$.

![Figure 6](image1.png)

Figure 6: Weight distribution of LeNet-5 before and after weight quantization under $b = 2$.

**Table III: Accuracy under different weight bitwidth.**

| Dataset | Bitwidth ($b$) | Acc. (%) | Acc. Loss (%) |
|---------|----------------|----------|---------------|
| MNIST   | 32 (FP)        | 99.07    | 0.00          |
|         | 4              | 99.10    | 0.03          |
|         | 3              | 99.04    | -0.03         |
|         | 2              | 98.93    | -0.14         |
|         | 1              | 98.85    | -0.22         |
| N-MNIST | 32 (FP)        | 98.95    | 0.00          |
|         | 4              | 98.67    | -0.28         |
|         | 3              | 98.65    | -0.30         |
|         | 2              | 98.58    | -0.37         |
|         | 1              | 98.54    | -0.41         |
| CIFAR10 | 32 (FP)        | 89.53    | 0.00          |
|         | 4              | 89.40    | -0.13         |
|         | 3              | 89.32    | -0.21         |
|         | 2              | 89.23    | -0.30         |
|         | 1              | 89.01    | -0.52         |

**Table IV: Accuracy under different spike rate.**

| Dataset | $\lambda$ | Avg. Spike Rate ($r$) | Acc. (%) | Acc. Loss (%) |
|---------|------------|-----------------------|----------|---------------|
| MNIST   | 0.001      | 0.22                  | 99.07    | 0.00          |
|         | 0.01       | 0.12                  | 99.11    | 0.04          |
|         | 0.1        | 0.06                  | 98.54    | -0.53         |
| N-MNIST | 0.001      | 0.17                  | 98.56    | -0.39         |
|         | 0.01       | 0.13                  | 98.53    | -0.42         |
|         | 0.1        | 0.06                  | 98.23    | -0.72         |
| CIFAR10 | 0.001      | 0.11                  | 89.51    | -0.02         |
|         | 0.01       | 0.08                  | 87.62    | -1.91         |
|         | 0.03       | 0.03                  | 81.01    | -8.52         |

**Table V: Accuracy results of the two-way compression.**

| Dataset | $R_{mem}$ | $R_s$ | Acc. (%) | Acc. Loss (%) |
|---------|-----------|-------|----------|---------------|
| MNIST   | 0.25      | 0.20  | 99.07    | 0.00          |
|         | 0.01      | 0.00  | 99.07    | 0.00          |
| N-MNIST | 0.30      | 0.25  | 99.07    | 0.00          |
| CIFAR10 | 0.40      | 0.30  | 99.07    | 0.00          |

**Table VI: Accuracy results of the three-way compression.**

| Dataset | $R_{mem}$ | $R_s$ | $R_{act}$ | Acc. (%) | Acc. Loss (%) |
|---------|-----------|-------|-----------|----------|---------------|
| MNIST   | 0.25      | 0.20  | 0.30      | 99.07    | 0.00          |
|         | 0.01      | 0.00  | 0.00      | 99.07    | 0.00          |
| N-MNIST | 0.30      | 0.25  | 0.40      | 99.07    | 0.00          |
| CIFAR10 | 0.40      | 0.30  | 0.50      | 99.07    | 0.00          |
Table V: Accuracy on MNIST when jointly applying two compression methods.

| λ  | Sparsity (s) | Bitwidth (b) | Avg. Spike Rate (r) | $R_{mem}$ (%) | $R_{ops}$ (%) | Acc. (%) | Acc. Loss (%) |
|----|-------------|-------------|---------------------|--------------|--------------|----------|---------------|
| 0  | 0%          | 32 (FP)     | 0.32                | 100.00%      | 100.00%      | 99.07    | 0.00          |
| 0.001 | 25%       | 32 (FP)     | 0.19                | 75.00%       | 43.14%       | 99.11    | 0.04          |
| 0.001 | 50%       | 32 (FP)     | 0.18                | 50.00%       | 28.30%       | 98.97    | -0.10         |
| 0.001 | 75%       | 32 (FP)     | 0.15                | 25.00%       | 11.74%       | 95.30    | -3.77         |
| 0.001 | 25%       | 32 (FP)     | 0.12                | 75.00%       | 27.52%       | 99.22    | 0.15          |
| 0.001 | 50%       | 32 (FP)     | 0.12                | 50.00%       | 18.19%       | 99.13    | 0.06          |
| 0.001 | 75%       | 32 (FP)     | 0.09                | 25.00%       | 11.74%       | 95.39    | -3.68         |
| 0.01  | 25%       | 32 (FP)     | 0.12                | 75.00%       | 7.15%        | 95.39    | -3.68         |
| 0.01  | 50%       | 32 (FP)     | 0.12                | 50.00%       | 3.13%        | 95.39    | -3.68         |
| 0.01  | 75%       | 32 (FP)     | 0.09                | 25.00%       | 1.56%        | 95.39    | -3.68         |
| 0.1   | 25%       | 32 (FP)     | 0.06                | 75.00%       | 1.56%        | 95.39    | -3.68         |
| 0.1   | 50%       | 32 (FP)     | 0.06                | 50.00%       | 0.78%        | 95.39    | -5.55         |
| 0.1   | 75%       | 32 (FP)     | 0.06                | 25.00%       | 0.45%        | 95.39    | -5.55         |

Note 1: The compression ratio in the parentheses is the reciprocal of $R_{mem}$ or $R_{ops}$.

Trade-off between $R_{mem}$ and $Rs$. The compression ratios of synapse memory and dynamic spikes actually behave as a trade-off. A too aggressive spike compression baseline (e.g. under $\lambda < 0.1$) will cause a large accuracy loss when $R_{mem}$ slightly decreases; a too aggressive memory compression baseline (e.g. $R_{mem} < 5\%$) will also cause a significant accuracy loss when $Rs$ slightly decreases. It is challenging to aggressively compress both the synapse memory and dynamic spikes without compromising accuracy. Figure 8 evidences this trade-off by visualizing all the joint-way compression results from Table V and VI on the $R_{mem}$-$Rs$ plane. Furthermore, since we have the $R_{ops}$ metric that takes both $R_{mem}$ and $Rs$ into account (see Equation [18]), we further visualize the relationship between $R_{ops}$ and accuracy, which is depicted in Figure 9. It can be seen that the accuracy is positively correlated to $R_{ops}$, i.e. a lower $R_{ops}$ is prone to cause a lower accuracy. However, as aforementioned in Section III-F, $R_{ops} \approx (1-s) \cdot \frac{b}{B} \cdot \frac{r}{R} = R_{mem} \cdot Rs$ is just a coarse estimation because the impact of bitwidth on operation cost is not linear. Therefore, the monotonic relationship between $R_{ops}$ and accuracy is perturbed to some extent if the weight quantization is applied.

**Joint-way Compression v.s. Single-way Compression.** We recommend to gently compress multi-way information rather than to aggressively compress only single-way information. Specifically, an aggressive compression in one way (e.g. $\geq 75\%$ connection sparsity, 1-bit weight bitwidth, or $\lambda \geq 0.1$ activity regularization) is easy to cause the accuracy collapse. In contrast, a gentle compression in each of the multiple ways is able to produce a better overall compression ratio while paying smaller accuracy loss. For example, the accuracy loss is only $-0.26\%$ on MNIST when concurrently applying 25%
Table VI: Accuracy on MNIST and NMNIST when applying all three compression methods.

| Dataset     | λ   | Sparsity (s) | Bitwidth (b) | Avg. Spike Rate (r) | \( R_{\text{mem}} \) (%) | \( R_{\text{ops}} \) (%) | Acc. (%) | Acc. Loss (%) |
|-------------|-----|--------------|--------------|---------------------|---------------------------|---------------------------|----------|--------------|
| MNIST       | 0.001 | 25% | 3 | 0.19 | 70.03 (14.22x) | 4.06 (24.63x) | 98.97 | -0.1 |
|            | 0.001 | 25% | 1 | 0.19 | 2.34 (42.74x) | 1.34 (74.63x) | 99.04 | -0.03 |
|            | 0.001 | 75% | 3 | 0.16 | 2.34 (42.74x) | 1.16 (86.21x) | 97.25 | -1.82 |
|            | 0.001 | 75% | 1 | 0.17 | 0.78 (128.21x) | 0.41 (243.90x) | 97.16 | -1.91 |
|            | 0.01  | 25% | 3 | 0.12 | 7.03 (14.22x) | 2.61 (38.31x) | 99.11 | 0.04 |
|            | 0.01  | 75% | 3 | 0.11 | 2.34 (42.74x) | 0.75 (133.33x) | 94.92 | -4.15 |
|            | 0.01  | 75% | 1 | 0.07 | 2.34 (42.74x) | 0.49 (204.08x) | 94.54 | -4.43 |
|            | 0.01  | 75% | 1 | 0.06 | 0.78 (128.21x) | 0.13 (769.23x) | 74.98 | -24.09 |
| NMNIST     | 0.001 | 25% | 3 | 0.03 | 7.03 (14.22x) | 0.97 (103.14x) | 98.73 | -0.22 |
|            | 0.001 | 25% | 1 | 0.03 | 2.34 (42.74x) | 0.32 (312.50x) | 98.66 | -0.29 |
|            | 0.001 | 75% | 3 | 0.03 | 2.34 (42.74x) | 0.32 (312.50x) | 98.66 | -0.29 |
|            | 0.001 | 75% | 1 | 0.02 | 7.03 (14.22x) | 0.1 (1000.00x) | 97.19 | -1.76 |
|            | 0.1   | 25% | 3 | 0.01 | 7.03 (14.22x) | 0.42 (238.10x) | 98.43 | -0.52 |
|            | 0.1   | 25% | 1 | 0.01 | 2.34 (42.74x) | 0.14 (714.29x) | 98.37 | -0.58 |
|            | 0.1   | 75% | 3 | 0.01 | 2.34 (42.74x) | 0.12 (833.33x) | 96.74 | -2.21 |
|            | 0.1   | 75% | 1 | 0.01 | 0.78 (128.21x) | 0.04 (2500.00x) | 96.87 | -2.08 |

Note 1: The compression ratio in the parentheses is the reciprocal of \( R_{\text{mem}} \) or \( R_{\text{ops}} \).

Figure 9: Relationship between \( R_{\text{ops}} \) and accuracy on MNIST. Abbreviations: P-connection pruning, Q-weight quantization, A-activity regularization.

The connection pruning, 1-bit weight quantization, and \( \lambda = 0.01 \) activity regularization. In this case, the overall compression ratio actually reaches as aggressive as \( R_{\text{mem}} = 2.34 \% \) (i.e. 42.74x compression) and \( R_{\text{ops}} = 0.91 \% \) (i.e. 109.89x compression). If we expect the same \( R_{\text{ops}} \) using the single-way compression, the accuracy would drop dramatically. Figure 9 reflects this guidance too, where the joint-way compression can reduce more \( R_{\text{ops}} \) with the same level of accuracy loss.

Accuracy Tolerance to Weight Quantization. Recalling Table II and III we observe that SNN models usually present a better accuracy tolerance to the weight quantization than the connection pruning. For example, the accuracy loss at 75% connection sparsity on MNIST and CIFAR10 could reach 2-4%, while the loss at 1-bit weight quantization is only <0.55%. We use Figure 10 to further evidence this speculation. It can be seen that, under the same weight compression ratio, we find the “aggressive quantization & slight pruning” schemes are able to maintain the accuracy better than the “slight quantization & aggressive pruning” schemes.

Robustness on N-MNIST Dataset. From Table VI we find that the SNNs on N-MNIST present a more graceful accuracy loss against the joint compression than other datasets we used, especially in the cases of aggressive compression.

Figure 10: Accuracy on MNIST under different weight compression ratio and strategy. Abbreviation: S-sparsity.
For instance, the accuracy loss is only about 2% even if an extremely aggressive compression of $R_{\text{mem}} = 0.78\%$ (i.e. 128.21x compression) and $R_{\text{ops}} = 0.04\%$ (i.e. 2500x compression) is applied. By contrast, this degree of compression on MNIST will cause >20% accuracy loss. Recalling our observations in the single-way compression, SNNs on N-MNIST are more prone to accuracy degradation than the ones on MNIST under low compression ratios. Considering these together, we expect that the underlying reason lies in the sparse features within these event-driven datasets (e.g. N-MNIST), where the information is heavily scattered along the temporal dimension. This temporal scattering of information causes accuracy degradation when the model meets any compression (even if the compression is slight) due to the sensitivity of intrinsic sparse features, while significantly reducing the accuracy drop when facing an aggressive compression owing to the lower data requirement to represent sparse features.

**D. Comparison with Prior Work**

Table VII compares our results with other existing works that touch SNN compression. Among the listed prior work, note that the recent ReStoCNet \[38\] is not a pure SNN model where the FC layers use non-spiking neurons and do not apply any compression technique. This specialized network construction significantly contributes to the accuracy maintaining. A fair comparison should take both the compression ratio and recognition accuracy into account. In this sense, our approach is able to achieve a much higher overall compression performance, owing to the accurate STBP learning and the powerful ADMM optimization.

| Net Structure       | Sparsity (%) | Bitwidth | Acc. (%) |
|---------------------|--------------|----------|----------|
| Spiking DBN          | 4-layer MLP  | 0%       | 91.35    |
| Pruning & Quantization | 2-layer MLP  | 92%      | ternary  | 91.5     |
| Soft-Pruning         | 3-layer MLP  | 75%      | 32 (FP)  | 94.05    |
| Stochastic-STDP      | 3-layer MLP  | 0%       | binary\(^2\) | 95.7   |
| NomAD                | 3-layer CNN  | 0%       | 3        | 97.31    |
| ReStoCNet            | 5-layer CNN  | 0%       | binary\(^4\) | 98.54   |
| ReStoCNet (CIFAR10)  | 5-layer CNN  | 0%       | binary\(^4\) | 66.23   |
| This work            | 5-layer CNN  | 0%       | 99.04    |
| This work            | 5-layer CNN  | 25%      | 1         | 98.81\(^5\) |
| This work (CIFAR10)  | 5-layer CNN  | 50%      | 1         | 68.52    |

Note\(^1\): There is an extra inhibitory layer without compression.
Note\(^2\): The last layer uses 24-bit weight precision.
Note\(^3\): The FC layers use non-spiking neurons.
Note\(^4\): The weights in FC layers are in full precision.
Note\(^5\): Additional spike compression ($\lambda = 0.01\%$) is applied.
Note\(^6\): For fair comparison, we use the same network structure as \[38\] and compress only the Conv layer too. Differently, the neurons in our network are all spiking neurons.

**V. Conclusion and Discussion**

In this paper, we combine STBP and ADMM to compress SNN models in two aspects: connection pruning and weight quantization, which greatly shrinks the memory space and baseline operations. Furthermore, we propose activity regularization to lower the number of dynamic spikes, which reduces the active operations. The three compression approaches can be used in a single paradigm or a joint paradigm according to actual needs. Our solution is the first work that investigates the SNN compression problem in a comprehensive manner by exploiting all possible compression ways and defining quantitative evaluation metrics. We demonstrate much better compression performance than prior work.

Through extensive contrast experiments along with in-depth analysis, we observe several interesting insights for SNN compression. First, the weight quantization contributes most to the memory reduction (i.e. $R_{\text{mem}}$) while the activity regularization contributes most to the spike reduction (i.e. $R_{\text{s}}$). Second, there is a trade-off between $R_{\text{mem}}$ and $R_{\text{s}}$, and $R_{\text{ops}}$ representing the overall compression ratio could approximately reflect the accuracy after compression. Third, the gentle compression of multi-way information usually pays less accuracy loss than the aggressive compression of only single-way information. Therefore, we recommend the joint-way compression if we expect a better overall compression performance. Fourth, we observe that SNN models show a good tolerance to the weight quantization. Finally, the accuracy drop of SNNs on event-driven datasets (e.g. N-MNIST) is higher than that on static image datasets (e.g. MNIST) under low compression ratios but quite graceful when coming to aggressive compression. These observations will be important to determine the best compression strategy in real-world applications with SNNs.

Although we provide a powerful solution for comprehensive SNN compression, there are still several issues that deserve investigations in future work. We focus more on presenting our methodology and just give limited testing results due to the tight budgets on page and time. This is acceptable for a starting work to study comprehensive SNN compression; whereas, in order to thoroughly understand the joint-way compression and mine more insights, a wider range of experiments (e.g. with different settings of compression hyper-parameters, on different benchmarking datasets, using more intuitive visualizations, etc.) are highly demanded. Reinforcement learning might be a promising tool to search the optimal configuration \[21\], \[26\] if substantial resources are available. For simplicity, we just focus on the element-wise sparsity with an irregular pattern that impedes efficient running due to the large indexing overhead. The structured sparsity \[52\] seems helpful to optimize the execution performance. At last, incorporating the hardware architecture constraints into the design of compression algorithm should be considered to achieve practical saving of latency and energy on neuromorphic devices.

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