Faraday waves over a permeable substrate

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We report on an experimental study of the Faraday instability for a fluid layer situated over a permeable rough substrate. A set of meshes and a flat solid bottom are used to study different substrate permeabilities. We observe that, in comparison with a flat solid bottom, the presence of a mesh leads to an increase of the critical acceleration of the instability. We interpret the results as a decrease of the effective thickness of the fluid layer, and we discuss its relation with the mesh properties.

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I. INTRODUCTION

The instability of a fluid layer in a vertical vibrated vessel was first studied by Faraday [1], and became one of the most referenced examples of non-linear dynamical systems. Above a critical acceleration $a_c$, a subharmonic instability develops in the form of standing waves at half the driving frequency. For inviscid fluids, the phenomenon relies in a competition between the destabilizing vibration and restoring gravity and surface tension forces [2]. By using a linear analysis, Kumar [3] extended the theory to viscous fluids, finding a damping coefficient that depends explicitly on the layer thickness as opposed to the inviscid case. Moreover, the critical acceleration, $a_c$, was shown to be extremely sensitive to this thickness $h$ [4]. For thin layers, as the thickness decreases, $a_c$ increases drastically. This strong dependence promoted analytical [5] and experimental studies for the case in which the thickness is not well defined, e.g. on a rough substrate [6]. In this latter study, the measured variations in the critical acceleration are successfully interpreted in terms of effective thickness of the fluid layer. Another example of ill-defined thickness corresponds to the case of a permeable substrate. In the present work, we report on the Faraday instability in a fluid layer situated over a mesh.

II. EXPERIMENTAL SETUP AND PROCEDURE

The principle of the experiment is to vertically vibrate a thin layer of fluid situated over a mesh, that is either coated or uncoated.

The vessel is made of Poly-methyl methacrylate (PMMA), with inner dimensions 62 mm (l) x 25 mm (w) x 30 mm (h) (Fig. 1). A mesh is firmly clamped, by means of a set of iron screws, between the upper edge of the vessel and a rectangular cover (PMMA, thickness 3 mm). We use meshes composed of interlacing iron wires, characterized by their diameter, $d$, and the distance between their centerlines, $L$ (Table I). Depending on the experimental set, the mesh is used as is (uncoated), or sealed by a paint (coated), to study a permeable or impermeable substrate. For comparison, we also report data for a flat solid bottom (PMMA plate of thickness 4 mm) that can replace the mesh. The upper surface of the plate coincides with the top plane of the meshes.

![FIG. 1. (Color online) Sketch of the experimental setup.](image)

The vessel is attached to the axis of an electromagnetic shaker (Bruel and Kjaer, Model V406). The shaker is fed by a sinusoidal current from a power generator (MAXD, Model 4210) driven by a function generator (GW-INSTEK, Model 8219A). The resulting acceleration $a$ is measured with an accelerometer (Analog Devices™, ADXL325) attached to the body of the vessel.
TABLE I. Characteristics of the meshes used in the experiments: \( L \) is the distance between wire centerlines, \( d \) the wire diameter, and \( s \) the solidity factor = \( \frac{1 - (\frac{L}{d})^2}{2} \).

| \( L \) (mm) | \( d \) (mm) | \( s \) |
|---|---|---|
| 1 | 1.77 | 0.35 | 0.35 |
| 2 | 1.06 | 0.35 | 0.55 |
| 3 | 0.31 | 0.20 | 0.87 |

and monitored with an oscilloscope (GRATTEN, Model GA1102CAL). The frequency \( f \) ranges from 28 to 42 Hz for an acceleration \( a \) up to 2 \( g \), where \( g \) is the acceleration of gravity.

The vessel is filled with the fluid under study (silicon oil, Sigma Aldrich) with kinematic viscosity \( \nu = 20 \) cSt, density \( \rho = 0.95 \text{ g/cm}^3 \), and surface tension \( T = 20.6 \) dyn/cm. Over the mesh, the supernatant fluid layer has a thickness of 2.1 mm. A digital camera (NIKON D90) with vertical optical images the free surface of the fluid. A beam splitter is used to achieve normal illumination.

The procedure to control precisely the fluid level is based on an optical technique: we measure the average intensity of the light reflected by the free surface of the fluid at rest. When filling the vessel, we remark that the contact line is pinned to the upper edge of the cover, and that the surface can be either concave or convex. In practice, we adjust the fluid volume with a micropipette (by steps of 0.2 \( \mu \)l) in order to get a maximum in the reflected light, which corresponds to a flat free surface. From this reference situation, we remove 0.85 \( \mu \)l in order to avoid any overflowing during the experiments.

The vessel and the measurement procedure of \( a_c \) were designed after the work of Douady \[7\], that sought to optimize the detection of \( a_c \). In order to check the reliability of the experimental device, we measured the critical acceleration over a flat solid bottom in the same experimental conditions. By using a strobe light we verified that the instability was indeed in its subharmonic regime \[11\]. The transient onset of the instability was detected by using a checkpoint protocol, measurements were always performed for increasing \( a \) to avoid hysteresis effects, and repeatability was carefully verified. The agreement observed in Fig. 2 validates our experimental device and protocols. We estimate the error in \( a_c \) to be about 2\%, at maximum.

III. EXPERIMENTAL RESULTS

We perform the experiment consisting in vibrating a fluid layer situated over a mesh and report on the critical acceleration \( a_c \) in term of reduced acceleration \( \Gamma = \frac{a}{g} \). Fig. 3 shows the free surface of the fluid when vibrated at 42 Hz as observed by the camera for \( a = 0 \) (top), \( a < a_c \) (middle), \( a > a_c \) (bottom).

Fig. 4 shows the variation of \( \Gamma_c \) as a function of vibration frequency \( f \) for the coated meshes, while Fig. 5 shows the results obtained for the uncoated meshes.

In all cases, replacing the flat impermeable bottom by a mesh leads to an increase in \( \Gamma_c \). The results are puzzling at first sight. Indeed, if the fluid has the possibility to flow through the mesh, one may think that the fraction of fluid participating in the instability would increase, and thus \( \Gamma_c \) would decrease.

For the coated meshes, \( \Gamma_c \) increases as the distance between the wires, \( L \), increases (the solidity factor \( s \) decreases). Conversely, for the uncoated meshes, we observe the inverse trend. As \( s \) increases (\( L \) decreases), one might expect a smooth transition towards an impermeable substrate, because the mesh becomes denser, while Fig. 5 strongly contradicts this hypothesis. Finally, we mention that the wavelength of the Faraday waves, of about 1 cm, is not significantly altered by changing the substrate (Fig. 3).

IV. ANALYSIS

Coated and uncoated meshes were employed to differentiate two possible effects: permeability and roughness.

We observe in Fig. 4 that, for coated meshes (impermeable) larger \( L \) leads to larger \( \Gamma_c \). By using the results of Kumar \[3\] for viscous fluids, with our experimental parameters, we compute the effective layer thickness \( h_{\text{eff}} \) corresponding to the measured \( \Gamma_c \) (Fig. 6). For the coated mesh \( h_{\text{eff}} \) decreases linearly with \( L \), independently of \( d \).

The above behavior can be examined as follows. Several studies treated the slip boundary condition at the interface between a fluid and a rough solid surface composed of a periodical arrangement of grooves. The slip condition accounts for roughness of the surface. Hocking concluded that the nature of this condition depends much

FIG. 2. Critical acceleration \( a_c \) as a function of the driving frequency \( f \). Dashed line: results from the work of Douady \[7\]. Squares: this work. \([h = 2.25 \text{ mm}, \eta = 20 \text{ cSt}, \gamma = 20.6 \text{ dyn/cm and } \rho = 0.95 \text{ g/cm}^3]).\)
FIG. 3. Free surface of the fluid as observed by the camera. Top: Fluid at rest. Middle: Surface waves with vibration turned on (42 Hz, $a < a_c$). Bottom: Faraday waves (42 Hz, $a > a_c$). The texture of the mesh underneath is visible (bright points).

FIG. 4. Critical acceleration $\Gamma_c$ as a function of $f$ for the coated meshes. We observe a monotonic increase of $\Gamma_c$ with $f$ for a given mesh, while for fixed $f$, $\Gamma_c$ increases as $L$ decreases. For all meshes, $\Gamma_c$ is greater than that of the impermeable flat bottom [lines are only guides for the eye – $h = 2.1$ mm, $\eta = 20$ cSt, $\gamma = 20.6$ dyn/cm and $\rho = 0.95$ g/cm$^3$].

FIG. 5. Critical acceleration $\Gamma_c$ as a function of $f$ for the uncoated meshes. We observe a monotonic increase of $\Gamma_c$ with $f$ for a given mesh, while for fixed $f$, $\Gamma_c$ increases as $L$ decreases. For all meshes, $\Gamma_c$ is greater than that of the impermeable flat bottom [lines are only guides for the eye – $h = 2.1$ mm, $\eta = 20$ cSt, $\gamma = 20.6$ dyn/cm and $\rho = 0.95$ g/cm$^3$].

more on the distance between grooves than on their depth $d$, which is consistent with the fact that $\Gamma_c$ depends only on $L$. Moreover, Niavarani et al showed that inertial effects promote the formation of recirculation zones that, in turn, lead to an effective negative slip length $\delta$, again consistent with the observed increase of $\Gamma_c$ (decrease of the effective thickness $h_{eff}$) with $L$.

When the mesh is uncoated, the fluid can flow through the mesh, and one expects the effective thickness to increase with the permeability of the substrate.

In accordance, we observe in Fig. 4 that $h_{eff}$ increases ($\Gamma_c$ decreases) with the distance $L$. Even if this trend is compatible with the expectations, the case of the uncoated mesh remains far more complex to understand. For the thinnest mesh studied ($L = 0.31$ mm and $d = \ldots$)

FIG. 6. Effective thickness $h_{eff}$ vs. distance $L$ between the wires – Full symbols: coated mesh; Open symbols: uncoated mesh. The point at $L = 0$ corresponds to the flat impermeable bottom. We observe that, for the coated mesh, $h_{eff}$ decreases linearly with $L$, independently of $d$. [lines are only guides for the eye – $h = 2.1$ mm, $\eta = 20$ cSt, $\gamma = 20.6$ dyn/cm and $\rho = 0.95$ g/cm$^3$].
0.20 mm), \( h_{\text{eff}} \) is smaller if the mesh is uncoated. It is thus not evident that, qualitatively, the possible flow through the mesh leads to an increase in absolute the effective thickness. In addition, for the intermediate case (\( L = 1.06 \) mm and \( d = 0.35 \) mm), \( h_{\text{eff}} \) is similar for the coated and uncoated mesh. Let us finally comment that, within our experimental range of parameters, the smallest value of \( L \) approaches the characteristic viscous length \( l_v = \sqrt{\nu/f} \approx 0.7 \) mm. If the fluid flows more difficultly through the mesh as \( L \) decreases, then, at the chosen frequency, the thinnest mesh may be almost impermeable.

V. CONCLUSIONS

In conclusion, we studied the effects of the presence of a mesh as the substrate on the critical acceleration \( \Gamma_c \) that characterizes the onset of the Faraday instability. We observed that, in contradiction with our intuition, the presence of the mesh leads to an increase of \( \Gamma_c \). For coated meshes, the negative effective slip length derived from the existence of recirculation zones, as observed by other authors, might explain the decrease in the effective thickness of the fluid layer.

For uncoated meshes, we observe that \( \Gamma_c \) increases almost linearly with the spacing between the wires \( L \). However, for the smallest spacing, the coating of the mesh leads to a further increase of the effective fluid thickness. The effect is surprising, and still not well understood. A comprehensive explanation of this result would require a detailed description of the boundary conditions at the interface between the mesh and the fluid, which promotes further investigation on this topic.

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