1/f noise in variable range hopping conduction.

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1/f noise induced by traps consisting of donors with no neighbors with close energies in their vicinity is studied. Such donors slowly exchange electrons with the rest of conducting media. It is shown that in the variable range hopping regime 1/f noise exponentially grows with the decreasing temperature. At high temperatures, when the variable range hopping crosses over to the nearest neighbor one, we predict a very weak temperature dependence in spite of the activation dependence of the conductivity.

I. INTRODUCTION

Low frequency 1/f noise was found in many conducting materials with the wide variety of transport mechanisms. Substantial attention was devoted to 1/f noise in the hopping conduction \cite{1,2,3,4,5,6,7,8,9,10,11}. One of the reasons is that 1/f noise limits performance of important devices working in the hopping regime, for example, thermistors for X-ray detection and other astrophysical applications \cite{5}. Understanding of the nature of 1/f noise can also shed light on the role of electron-electron interactions in the hopping transport.

In a lightly doped semiconductor at low temperatures electrons are localized on donors and the conductivity is due to hopping between donors. (For concreteness we talk about n-type moderately compensated semiconductor with concentration of donors \(N_D\).) There are two well known regimes of hopping transport. At relatively high temperatures electrons use almost all the donors for hopping. This regime is called the nearest neighbor hopping (NNH). In this case, the energy scatter of donors due to random potentials of charged donors and acceptors plays only a secondary role, leading to a fixed activation energy of conductivity. At low enough temperatures activation required for the use of the majority of donors becomes very costly and only donors within a narrow band of energies around the Fermi level participate in the conductivity. With decreasing temperature the width of this band shrinks and hops become longer. This explains the name ”variable range hopping” (VRH) of the regime.

Most of recent experiments deal with VRH and focus on temperature dependence of 1/f noise \cite{2,4,5,6,7,8,9,10,11,12,13}. Results seem to be quite controversial. For example, Lee \cite{4} found that in silicon 1/f noise amplitude decreases when temperature goes down, while McCammon \cite{5} observed exponential increase of 1/f noise with the decreasing temperature.

On the other hand, the original theory of 1/f noise in the hopping transport \cite{1,2,3} deals only with NNH and only with the case of relatively high temperatures when NNH conductivity is temperature independent. This theory is based on the idea of traps provided by rare isolated donors. Such a donor traps an electron from donors of the transport paths (conducting media) and releases it back, both with characteristic times, which are much larger than times of hops determining VRH conductivity. The resulting modulation of number of ”conducting” carriers at low frequency \(\omega\) leads to the spectral density of current noise \(I_C^2\) with a behavior close to \(1/\omega\) (or 1/f). This idea is similar to the McWorter’s explanation of 1/f noise in MOSFETs by electron exchange between two-dimensional electron gas and traps in the oxide, which have an extremely wide spectrum of times of tunneling to the interface \cite{12,13}. The difference is that in the MOSFET case traps are separated from the conducting two-dimensional gas by the interface, while in the case of the hopping transport traps are located inside pores scattered in the body of the conducting media of donors. This paper generalizes the idea of Refs. \cite{7,8} to the VRH conductivity (Sec. II) and to the temperature dependent NNH (Sec. III). We again identify those donors, which can work as traps with extremely wide spectrum of times of relaxation. Then we calculate the probability of a trap, which now becomes an exponentially decaying function of temperature. Integrating contributions of all traps to the noise we arrive at the conclusion that in the VRH case the spectral density of current noise \(I_C^2\) approximately obeys the Hooge law, Eq. (8), with the coefficient \(\alpha\), which is given by Eq. (1) and exponentially decreases with temperature. On the other hand, in the case of NNH conductivity, which grows exponentially with temperature with a constant activation energy, the Hooge’s \(\alpha\) is almost temperature independent. Thus, exponential temperature dependence of conductivity does not automatically lead to exponential dependence of 1/f noise.

Sec. IV is devoted to the discussion of the relationship between of spectral densities of fluctuations of the current and the electron concentration, which is used in Sec.II and Sec. III to calculate the noise spectral density. In Sec. V, we compare our theory with two previously published theoretical works \cite{9,10} on 1/f noise in VRH and with available experiments \cite{7,8,11}. We also make a comment about applicability of our theory to MOSFETs.
II. VARIABLE RANGE HOPPING

Due to the Coulomb interaction of localized electrons, the density of states of donor states, \( g(\varepsilon) \), of a three-dimensional lightly doped semiconductor is known [14,15] to have the Coulomb gap near the Fermi level,

\[
g(\varepsilon) = \frac{(3/\pi)\kappa^3}{e^6} \varepsilon^2.
\]  

(1)

Here \( \varepsilon \) is the energy of a localized electron on a donor calculated from the Fermi level, \( \kappa \) is the dielectric constant of the semiconductor and \( e \) is the proton charge. At low temperatures VRH conductivity \( \sigma(T) \) obeys the Efros-Shklovskii (ES) law [14,15]

\[
\sigma(T) = \sigma_0 \exp[-(T_0/T)^{1/2}],
\]  

(2)

where \( T_0 = \frac{Ce^2}{kB\kappa a} \), \( a \) is the localization length (Bohr radius) of an electron on a donor, and \( C \simeq 2.7 \).

The ES conductivity is determined only by donors with energies in the band \( \delta = \frac{e^2}{k_BT_0}a \) around the Fermi level (\( \delta \)-band). Similarly to Ref. [7,8] we are interested in a trap, i.e., in such a rare donor of the same band \( \delta \), which anomalously slowly exchanges an electron with the majority of donors of this conducting band. The rate of electron hops between two donors \( i \) and \( j \) is [3]

\[
\nu_{ij} = \nu_0 \exp\left(-\frac{2r_{ij}}{a} - \frac{\varepsilon_{ij}}{k_BT} \right),
\]  

(3)

where \( \varepsilon_i \) is the energy level of the donor \( i \), \( \varepsilon_{ij} = (|\varepsilon_i| + |\varepsilon_j| + |\varepsilon_i - \varepsilon_j|)/2 \). We want to find a trap with relaxation time larger than \( \nu^{-1} \), where \( \nu \) is so small that \( \ln(\nu_0/\nu) \gg (T_0/T)^{1/2} \). Such a trap is an isolated donor, for which all rates of transitions to the neighboring donors of the band of energies \( \delta \) are smaller than \( \nu \). Let us consider a donor \( i \) located in the center of a sphere of radius \( R(\nu) = (a/2)\ln(\nu_0/\nu) \), which contains no other donors \( j \) within \( \delta \)-band. Then there are no direct hops from our donor to the conducting \( \delta \)-band of donors. But this is not enough to have a trap we are looking for, because an electron can still escape from donor \( i \) to the conducting \( \delta \)-band through an intermediate donor \( j \), which has energy \( \varepsilon_j \) much larger than \( \delta \), but is located closer to \( i \), at the distance \( r_{ij} < R(\nu) \). In order to preserve the trap all such "dangerous" intermediate donors \( j \) near donor \( i \) should be eliminated or, in other words, all the remaining ones should satisfy inequality

\[
\frac{\varepsilon_{ij}}{k_BT} > \ln \frac{\nu_0}{\nu}.
\]  

(4)

This inequality means that the energy \( \varepsilon_j \) should be away from the energy band of the width \( \Delta = k_BT \ln(\nu_0/\nu) \gg \delta \) around the Fermi level (\( \Delta \)-band). The concentration of donors in \( \Delta \)-band is of the order of \( N(\Delta) \sim g(\Delta)\Delta = (\kappa\Delta/e^2)^{3/2} \). Therefore, the average number of \( \Delta \)-band donors in the sphere of radius \( R(\nu) \) equals

\[
M = \frac{(4\pi/3)R^3(\nu)N(\Delta)}{\omega}.
\]  

(5)

where \( B \) is a numerical factor. For a donor with the relaxation rate \( \nu \) equal the typical hopping frequency of conducting electrons, \( \nu_c \sim \nu_0 \exp[-(T_0/T)^{1/2}] \), we get \( W(\nu) \sim 1 \), confirming that this is a typical donor. Traps have \( \nu \ll \nu_c \) and, therefore, exponentially small probabilities. To find the coefficient \( B \) we calculated probability of a pore in four dimensional space of three coordinates and energy. This calculation gives \( B \simeq 0.3 \). Note that probability \( W(\nu) \) decreases with increasing temperature because at high temperatures the volume of the four dimensional pore from which one has to eliminate donors becomes larger.

The spectral density of fluctuations of the concentration of "conducting" electrons, \( n^2_\omega \), can be evaluated by integration over all traps [13]

\[
n^2_\omega = \frac{N(\delta)}{V} \int_0^{\omega_0} \frac{dW}{d\nu} \frac{4\nu}{\omega^2 + \nu^2} d\nu.
\]  

(6)

Here \( N(\delta) \) is the concentration of donors in the \( \delta \)-band around the Fermi level and \( V \) is the sample volume.

According to the standard approximation is that relative fluctuations of the concentration of conducting electrons lead to fluctuations of conductivity, i.e.

\[
\frac{I^2}{I^2} \propto \frac{n^2_\omega}{N^2(\delta)}.
\]  

(7)

Here \( I^2 \) is the spectral density of current fluctuations and \( I \) is the average current through the sample. We will discuss justification of Eq. (7) in Sec. IV. Using Eq. (6) we can now estimate \( I^2/I^2 \). Let us start from the most interesting case when the exponential factor \( dW/d\nu \) in the integrand of Eq. (6) is a weaker function of \( \nu \), than \( \nu/(\omega^2 + \nu^2) \) and the integral is determined by \( \nu \sim \omega \). Then one can take exponential term of Eq. (6) out of the integral at the frequency \( \nu = \omega \). This leads to the Hooge law [17]

\[
\frac{I^2}{I^2} = \frac{\alpha(\omega, T)}{\omega N_{DV}V},
\]  

(8)

where \( N_{DV} \) is the total number of donors and \( \alpha \) is the Hooge’s coefficient. We obtain

\[
\alpha(\omega, T) \propto \exp[-B(T/T_0)^{3/2}] \ln \frac{\nu_0}{\omega}.
\]  

(9)

Here as everywhere below we are concentrated on exponential temperature dependence of noise and are not trying to calculate prefactor. Using Eq. (12) one can express \( \alpha(\omega, T) \) through \( \sigma(T) \):
\[
\ln \alpha(\omega, T) = -B \left( \frac{\ln(v_0/\omega)}{\ln(\sigma_0/\sigma)} \right)^6 .
\]

This simple relationship can be convenient for a direct experimental verification.

The above derivation of \( \alpha(\omega, T) \) is justified when the dependence of \( \alpha \) on \( \omega \) is weaker than \( 1/\omega \), i.e. when \( \omega \gg v_0 \exp[-(T_0/T)^{3/5}] \). On the other hand, our approach of traps is good only for frequencies \( \omega \) smaller than the frequency of transport hops \( \nu_c \), i.e. for \( \ln(v_0/\omega) \gg (T_0/T)^{1/2} \). Thus, the Hooge law is valid in the range of frequencies

\[
\exp[-(T_0/T)^{1/2}] \gg \omega/v_0 \gg \exp[-(T_0/T)^{3/5}] .
\]

In spite of close powers in the exponents of lower and upper limit this range can be quite broad. For example, even at a moderate \( (T_0/T)^{1/2} = 10 \) this range covers 2.5 decades, while at the very large \( (T_0/T)^{1/2} = 20 \) it reaches 7 decades.

One can approximately write \( I_2/I_0^2 \propto 1/\omega^\gamma \), where \( \gamma = 1 - 6B(T/T_0)^3 \ln^2(\nu_c/\omega) \). Thus, according to this theory \( \gamma \ll 1 \). In the range (11), the index \( \gamma \) is close to unity and approaches 1/2 at the low frequency limit of applicability of Eq. (8).

For a fixed \( \omega \) in the frequency range (11), Eq. (8) contains the main result of this paper - an exponential growth of the noise power with the decreasing temperature. The physics of this dependence is clear: the lower the temperature, the smaller is the width of the band of energies \( \Delta \), where one has to eliminate "dangerous" donors in order to get a trap for a given frequency, the smaller is the number of donors to be eliminated, the larger is the probability to find such a trap.

According to Eqs. (11) and (12) at smaller frequencies \( \omega \ll v_{min} = v_0 \exp[-(T_0/T)^{3/5}] \), the spectral density of noise becomes substantially weaker than \( 1/\omega^2 \) and eventually saturates at the level \( \sim \exp[(T_0/T)^{3/5}] \). To calculate behavior of \( 1/\omega \) noise in all range \( \omega < \nu_c \), more accurately one should similarly to (12) add the contribution of traps made of large finite clusters of donors and calculate the integral of Eq. (12) numerically.

We derived Eq. (8) for a semiconductor in which the concentration \( N_D \) is substantially smaller than the critical concentration of the metal-insulator transition, \( N_{MI} \). In this case, we can use Eq. (8) for the density of states \( g(\varepsilon) \). In experiments VRH is typically measured at \( 0.25N_{MI} \leq N_D \leq N_{MI} \) because of a huge resistance of lighter doped samples. We believe that Eqs. (8) and (12) still work in the range \( 0.25N_{MI} \leq N_D \leq 0.5N_{MI} \).

We do not exactly know how to generalize Eq. (8) for the critical vicinity of the metal-insulator transition, where \( N_{MI} - N_D \ll N_D \). An additional source of the traps may be related to the spacial fluctuations of diverging at the transition localization length \( \xi \). Anomalously small \( \xi \) in a vicinity of some donor can create a trap. A simple conjecture is that Eqs. (8) and (12) remain valid at \( N_{MI} - N_D \ll N_D \) with the same \( T_0 = 2.7e^2/k_B\kappa \xi \) as in the ES law near the transition. Anyway an experimental study of this range of concentration can shed some light on the metal-insulator transition.

Until now we have been dealing with VRH conductivity in a doped crystalline moderately compensated semiconductor, where conductivity obeys Eq. (12). In systems with a weaker role of the electron-electron interaction such as amorphous semiconductors or two-dimensional systems screened by a parallel metallic gate, VRH can take place in the larger band of energies than the width of the Coulomb gap and, therefore, one can observe Mott law

\[
\sigma(T) = \sigma_0 \exp[-(T_d/T)^{1/(d+1)}],
\]

where \( d = 2, 3 \) is the dimensionality of the system, \( T_d = \beta_d/(g\alpha^4) \), \( g \) is the bare (unperturbed by the Coulomb interaction) \( d \)-dimensional density of states in the vicinity of the Fermi level and \( \beta_d \) are numerical constants [15].

Repeating the derivation of Eq. (3) we get the Hooge law with

\[
\alpha(\omega, T) \sim \exp[-B_d(T/T_d) \ln^{d+1}(v_0/\omega)] ,
\]

or

\[
\ln \alpha(\omega, T) = -B_d \left( \frac{\ln(v_0/\omega)}{\ln(\sigma_0/\sigma)} \right)^{d+1} ,
\]

where \( B_d \) are numerical coefficients. The physics of the origin of the exponential temperature dependence is exactly the same as above: the higher the temperature, the more difficult is to find a trap for a given small frequency \( \nu \).

### III. TEMPERATURE DEPENDENT NEAREST NEIGHBOR HOPPING

Eqs. (8) is valid when temperature is so small that the energy width \( \Delta \) of the band, where all "dangerous" donors are eliminated, is smaller than the width of the donor band, \( A = e^2N_{D}^{1/3}/\kappa \), i.e. at \( k_BT\ln(v_0/\nu) \ll e^2N_{D}^{1/3}/\kappa \). In the opposite case, \( \Delta \gg A \), or

\[
T \gg T_1 = \frac{e^2N_{D}^{1/3}}{k_B\kappa \ln(v_0/\nu)},
\]

the concentration of donors in the \( \Delta \)-band

\[
N(\Delta) = \int_{-\Delta}^{\Delta} g(\varepsilon) d\varepsilon \approx N_D[1 - (A/\Delta)^3]
\]

is close to \( N_D \) and only weakly depends on \( \Delta \). Here we used the fact that as shown in Chapter 3 of Ref. [13] the large energy tails of the density of states of
the classical donor band behave as \( g(\varepsilon) \sim N_D A^3/|\varepsilon|^4 \) at \( |\varepsilon| \gg A \). Now we can use Eq. (3) with probability \( W(\nu) = \exp[-(\pi/6)N(\Delta)\alpha^3 \ln^3(\nu_0/\nu)] \) which can be rewritten as
\[
W(\nu) = \exp[-(\pi/6)N_D a^3 \ln^3(\nu_0/\nu)] \exp(T_c/T)^3.
\] (17)
Here \( T_c \approx e^2aN_D^{2/3}/k_B \) is the critical temperature of the transition from NNH to VRH \[13\]. In the range of frequencies
\[
\exp[-(T_0/T)^{1/2}] \gg \omega/\nu_0 \gg \exp[-(N_D a^3)^{-1/2}],
\] (18)
where \( dW/d\nu \) is weaker function of \( \nu \) than \( 4\nu/(\omega^2 + \nu^2) \). we again obtain the Hooge law with
\[
\alpha(\omega, T) = \exp[-(\pi/6)N_D a^3 \ln^3(\nu_0/\omega)] \exp(T_c/T)^3.
\] (19)
Eq. (19) is valid both for NNH (at \( T > T_c \)) and for VRH (at \( T_1 < T < T_c \)). First exponential factor of Eq. (19) originates from the probability of a pore free of all donors. This factor is temperature independent and coincides with the value of \( \alpha \) obtained for the high temperature limit of NNH \[7,8\]. Second, temperature dependent exponential factor of the right hand side of Eq. (19) provides only a relatively small correction to \( \ln \alpha(\omega, T) \), which grows as \( 1/T^3 \) with the decreasing temperature. In NNH range, when \( T \gg T_c \), the relative temperature dependent correction to \( \alpha(\omega, T) \) is much smaller than unity. At the transition point from NNH to VRH, i.e. at \( T \sim T_c \), this correction reaches 100\%. At \( T \ll T_c \) it becomes exponentially large and provides the smooth crossover to Eq. (1), when \( T \) approaches \( T_1 = e^2aN_D^{1/3}/k_B \ln(\nu_0/\nu) \). However, as we already mentioned above, because of huge resistances an experimental observation of VRH is not possible, when \( N_D a^3 \ll 1 \). Therefore, inequality (18) is very difficult to realize and the use of Eq. (19) for VRH regime is limited. But Eq. (19) is definitely important for the temperature dependence of noise of NNH, which takes place at \( N_D a^3 \ll 1 \).

To conclude this section we would like to repeat that for NNH temperature dependence of \( \alpha(\omega, T) \) is negligible in all range of temperatures \( A \gg T \gg T_c \), where conductivity itself grows with decreasing temperature with activation energy of the order of \( A \) \[3\].

**IV. RELATIONSHIP BETWEEN FLUCTUATIONS OF THE CURRENT AND OF THE CONCENTRATION OF CONDUCTING ELECTRONS**

Now we would like to return to Eq. (7), which establishes relation between fluctuations of concentration of conducting electrons (which are not trapped) \( n \) and fluctuations of current and discuss its validity. In MOSFETs, where most of electrons are free, fluctuations of the concentration of free electrons obviously lead to proportional fluctuations of the conductivity \[12,13\] and at a fixed electric field \( I^2/T^2 = n^2/\nu \).

For NNH such proportionality between fluctuations of \( I \) and \( n \) is obvious for strongly compensated samples, where electrons play the role of carriers, or for a weakly compensated semiconductor, where the role of carriers is played by a small concentration of holes (empty donors). However, even in this case there could be such an intermediate compensation ratio \( K = N_A/N_D \), where the coefficient in linear proportionality between fluctuations of \( I \) and \( n \) vanishes. (Here \( N_A \) is the concentration of acceptors). In VRH conductivity connection between fluctuations of \( I \) and \( n \) is even less trivial.

Let us consider relation of \( I \) and \( n \) for a simpler example when concentration, \( n \), of electrons in a two-dimensional hopping system is varied due to a change of the gate voltage. Any variation of \( n \) leads to a shift of the Fermi level. The Coulomb gap, however, moves together with the Fermi level and, in the first approximation, it remains unchanged at small energies \( \delta \), which are important for VRH. It is not obvious then whether Eq. (7) is valid.

In the first subsection of this section we show that in the generic case of asymmetric with respect of Fermi level bare (disorder related) density of states one can justify Eq. (7), because in this case there is a nonzero derivative \( d\sigma/dn \) for a sample of infinite size. In the second subsection we introduce more general mechanism of fluctuation of conductivity, which exists even in the case when \( d\sigma/dn = 0 \) for infinite sample, but still can be evaluated in the way it was done in Secs. II and III.

**A. Asymmetry of the density of states**

In a doped \( n \)-type semiconductor at a generic compensation ratio \( K \) the density of states at large \( \varepsilon \) is asymmetric with respect to the Fermi level (see plots in Chapter 14 of Ref. \[15\]). In spite of tendency of Coulomb gap to be symmetric, the density of states, \( g(\varepsilon) \), at \( \varepsilon \sim \delta \), is sensitive to behavior of the density of states at larger energies. This happens because the Coulomb gap should crossover in its “shoulders” to the bare density of states of the classical impurity band \[16\]. Using the self-consistent equation for the one-electron density of states derived by Efros \[17\] one can show that the absolute value of nonuniversal (depending on position of Fermi level) correction to the density states at small \( \varepsilon \) is proportional to \( \kappa^3\varepsilon^3/e^6A \), where \( A \approx e^2N_D^{1/3}/\kappa \) is the energy width of the classical donor band. In other words, the relative asymmetric correction to Eq. (7) at \( \varepsilon \sim \delta \) is proportional to \( \delta/A \).

Our final goal is to evaluate fluctuations of the hopping conductivity. In VRH regime the conductivity
fluctuates because \( g(\delta) \) fluctuates following the Fermi level. It is easy to show that relative fluctuations of the conductivity are larger than fluctuations of \( g(\delta) \) by \((T_0/T)^{1/2} \sim T_0/\delta\). Thus, as in a free electron gas, the fluctuations of conductivity are proportional to fluctuations of the concentration of conducting electrons with the temperature independent coefficient of proportionality \((\delta/A)(T_0/\delta) = T_0/A\).

Now we can return to slow traps immersed in the conducting media, which work similarly to a gate. While the number of trapped electrons is slowly changing with time, the conducting media arrives at the quasi-equilibrium at much shorter time scale and develops the quasi-equilibrium Fermi level tracking fluctuations of the electron concentration. In turn, the position of the quasi-Fermi level affects the density of states relevant to the conductivity and, therefore, the hopping conductivity itself.

Without Coulomb interactions fluctuations of the concentration are uniform over the conducting media. Coulomb interactions makes these fluctuations localized, because of screening of trapped electrons. When an electron is trapped, a hole remaining in the conducting media is localized close to the trap. In other words, fluctuations of the concentration of conducting electrons do not propagate to the bulk of the conducting media. In this sense, traps are equivalent to small gates separated from each other by large distances. They locally perturb the concentration of conducting electrons. This perturbation, of course, changes conductivity only near traps. Now we have to discuss these changes and the way how they affect the total current through the sample.

We start from an estimate of the screening radius of a spherical trap with radius \( R = (a/2)\ln(v_0/\nu) \). In other words, we want to find the radius of the concentric sphere, where the hole left behind by trapped electron is localized. In the Coulomb gap, the density of states at the Fermi level is very small at small temperatures. One can find substituting \( k_B T \) for \( \varepsilon \) into Eq. (4) so that linear screening Debye radius \( R_s \) which describes screening of very small charge is very large, \( R_s \sim a(T_0/T) \). On the other hand, one electron charge \( -e \) of the trap is large enough to produce additional nonlinear screening. Nonlinear screening radius \( R_N \) can be found from the condition that potential of trapped electron \( -e/\kappa R \) can attract a positive charge to one of donors inside the sphere with radius \( R_N \). This condition can be written as

\[
\frac{4\pi}{3} (r_N^3 - R^3) \int_0^{e/\kappa R} 2g(\varepsilon)d\varepsilon \sim 1 ,
\]

This gives \( R_N - R \sim R \). To understand changes of the macroscopic conductivity we should compare \( R_N \sim R \) with the characteristic distance \( L \) at which VRH conductivity self averages, or in other words, becomes uniform. This is the characteristic period of the critical subnetwork (percolation infinite cluster) of the Miller-Abrahams resistor network, which carries all the current \( \Omega \). It is known to be of the order of \( L = (a/2)(T_0/T)^{1+\nu_1}/2 \approx a(T_0/T)^{0.95} \). (Here we used the fact that exponent of the correlation length of the percolation theory, \( \nu_1 = 0.9 \simeq 1 \).) The length \( L \) is of course much larger than the characteristic length of a hop \( r_h = (a/2)(T_0/T)^{1/2} \). It is interesting that \( L \) is only somewhat smaller than the linear screening radius \( R_s \).

Below we consider two different cases, \( R > L \) and \( R < \text{both} \). In the first case, \( R_N > R_s \) and, therefore, screening is linear \( (R_N \text{ plays no role}) \). The hole is not bound to a particular donor but freely travels in the spherical layer (atmosphere) between \( R \) and \( R + R_s \). In this case, one can use macroscopic approach for the role of traps, namely one can say that the concentration of electrons (and the local VRH conductivity) slightly varies in spherical atmospheres around all traps. It is known that in a slightly inhomogeneous media in the first approximation the sample conductivity is equal to the average value of the local conductivity. Therefore, the change of the sample conductivity reflects fluctuations of the total number of conducting electrons and Eq. (6) is valid.

However, the case \( R > L \) can be realized only at very low frequencies \( \nu \) and we have to consider the second case, \( R < L, R_s \). In this case, the simple macroscopic approach based on fluctuations of concentration does not work, because traps perturb the critical network locally, at the distances much smaller than the scale of self-averaging of the VRH conductivity. We show below that for \( R < L, R_s \) no noise exists even for the perfect symmetry of the density of states with respect of the Fermi level.

### B. Effect of the trap potentials

In the case \( R < L, R_s \) nonlinear screening is important because \( R_N < R_s \). Therefore, the hole is bound to the trapped electron and is localized on a particular donor. In other words, electron travels forth and back between this donor and the trap creating the dipole potential \( \phi(R,t) \), which modulates conductances \( G_{ij} \) of neighboring donors of the Miller-Abrahams resistor network \( \Omega \). In order to calculate the effect of this potential let us divide the sample in cubic blocks with linear size \( L \) each and consider for simplicity such frequencies \( \nu \), when \( R < L \), but is still comparable to \( L \), say \( R \sim L/5 \). Only very small fraction of these cubes of the order of \( (dW/d\nu)/\nu \) has one trap inside. When an electron is captured or released, the dipole potential \( \phi(R,t) \) changes energy of critical resistors which determine conductance of two halves of the block on both sides of the trap by energy \( e\phi(R,t) = e^2/\kappa R \sim k_B T \). Positive and negative potentials of the dipole applied to two different halves
of the cube, produce different effects because two halves have different random conductivities (conductivity self averages only at a distance larger than $L$). As a result the conductance of such cube changes by 100%. Thus, due to the trapping and detrapping of an electron the conductance of the cube with the trap inside fluctuates roughly speaking by factor two. Macroscopic conductivity then fluctuates according to Eqs. (8) and (9).

V. COMPARISON WITH OTHER THEORETICAL MODELS AND EXPERIMENT

Let us start from two theoretical papers, which aim at the theory of $1/f$ noise in the VRH conduction. Kozub \cite{Kozub} suggested seemingly different mechanism related to pairs of donors slowly exchanging electron with each other and modulating transport on current paths by their potential. To produce effect at a very small frequency, two donors should be far from each other. But they also should not be "shortened" by a chain of other donors providing faster exchange between them. In other words, they should be isolated from the conducting media. One could think that each of two donor should have a pore similar to ones discussed in our paper around it. In this case, for a given frequency a pore for a "modulator" pair would have much smaller probability than spherical traps we study and "modulators" would produce a negligible noise on the background of the noise created by traps.

Actually, a pair is not "shortened" even if only one of its donors has a spherical pore around it or, in other words, this donor is exactly a trap we study above. The second donor is the donor of the $\theta$-band closest to the pore. So one can say that we presented here a quantitative development of the Kozub's qualitative idea. Indeed, the language of the second half of Sec. IV is very close to that of Ref. \cite{Kozub}.

In another paper, Shlengel and Yu \cite{Shlengel} apparently deal with the same mechanism as discussed in our paper. They did not study analytical asymptotic dependencies, but evaluated expression for $I_0^2/I^2$ numerically and obtained results dramatically different from ours. They found stronger than $1/f$ growth of $I_0^2/I^2$ at small frequencies and a weak growth of the noise with temperature. I have no explanation for their numerical results.

Let us now compare our predictions with experiments on crystalline semiconductors concentrating mainly on the temperature dependence. In the range of VRH exponential growth of $1/f$ noise with decreasing temperature following from Eq. (8) agrees qualitatively with $1/f$ noise measurements for implantation doped silicon samples \cite{Kozub}, but disagrees with results on bulk silicon crystals \cite{Kozub}. On the other hand, in agreement with Eq. (10), $1/f$ noise is almost temperature independent for NNH in germanium \cite{Kozub}. $1/f$ noise in the wide range of temperatures including the transition between VRH and NNH was also studied in two-dimensional n-GaAs channels \cite{Kozub}. In agreement with our theory it was observed that $I_0^2/I^2$ sharply decreases with increasing temperature in VRH range, while in NNH range the temperature dependence becomes weak.

Frequency dependencies are more problematic for the theory, because deviations in the direction of smaller power or saturation at small frequencies were not seen in experimental works. This can be explained for samples with very large resistances. When $(T_0/T)^{1/2}$ is very large the frequency of saturation can become too small to be measured. Indeed, at $v_0 = 10^{12}$Hz and $(T_0/T)^{1/2} = 20$, we find that $\nu_c = 2.5 \times 10^3$Hz and $\nu_{\min} = 2.5 \times 10^{-4}$Hz. On the other hand, the absence of a tendency to saturation in moderately resistive samples with $(T_0/T)^{1/2} \sim 10$ may signal that the actual mechanism of $1/f$ noise is different from the one considered here. Indeed, there are many different pseudoground states separated by large potential barriers from each other in the Coulomb glass formed by a frozen distribution of interacting electrons on random donors \cite{Kozub}. Different pseudoground states (valleys) can have different conductivities and random walk of the system through a sequence of these states may lead to $1/f$ noise \cite{Kozub}. Logically it is possible that this noise can be larger than the one we estimated, but nobody was able to evaluate it.

Until now we have dealt with the hopping transport, where both roles of conducting media and traps are played by localized states of impurities. There are also systems where current is carried by free electrons, while localized states play the role of traps only. The best example of such a system is a MOSFET, where a free two-dimensional gas exchanges electrons with localized states (traps) of the oxide. McWorter's theory \cite{McWorter} assumes that density of states of traps is so small that any localized state of the oxide, which is close to the interface and close in energy to the Fermi level plays the role of the trap. These traps are assumed to be uniformly distributed with respect to the distance from the interface and, thus, lead to $1/f$ noise.

McWorter’s theory \cite{McWorter} neglects the possibility that near a trap, which exchanges electrons with the conducting channel, other localized states can be located. They can provide alternative hopping paths with exponentially smaller exchange time and "shunt" the trap. However, for small enough frequencies and a large enough density of localized states in oxide these paths always exist. Then most effective traps consist of a localized state surrounded by a spherical pore, which touches the interface of the channel. As in the case of VRH, all localized states within energy band $\Delta$ should be excluded from the pore, while localized states with higher energies can stay there. Therefore, one can say that the pore is arranged in the four-dimensional coordinate-energy space.

The theory of this paper can be directly used to calculate the probability of such traps and therefore the frequency dependence of spectral density of noise. We
assume that in the gap of the oxide the bare density of states, \( g \), is constant. The Coulomb interaction forms the Coulomb gap of the width \( E_{CG} \sim e^3 g^{1/2}/\kappa^{3/2} \) in the vicinity of the Fermi level. One has to compare the energy ”width” of the trap \( \Delta \) and \( E_{CG} \). If \( \Delta < E_{CG} \) one can use Eq. (9). On the other hand, when \( \Delta \geq E_{CG} \) Eq. (9) crosses over to Eq. (13).

In conclusion, this paper explores the role of isolated donors as traps which modulate the concentration of conducting electrons and create noise of the hopping conductivity. It is shown that in the wide range of frequencies traps lead to approximately \( 1/f \) behavior of noise, with the crossover to saturation at extremely small frequencies. In the nearest neighbor hopping range, in spite of the activation temperature dependency of the conductivity, the relative intensity of \( 1/f \) noise is almost temperature independent (see Eqs. (3) and (13)). In the variable range hopping regime the intensity of \( 1/f \) noise grows exponentially with the decreasing temperature according to Eqs. (8) and (9).

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