Planck’s Scale Dissipative Effects in Atom Interferometry

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Abstract
Atom interferometers can be used to study phenomena leading to irreversibility and dissipation, induced by the dynamics of fundamental objects (strings and branes) at a large mass scale. Using an effective, but physically consistent description in terms of a master equation of Lindblad form, the modifications of the interferometric pattern induced by the new phenomena are analyzed in detail. We find that present experimental devices can in principle provide stringent bounds on the new effects.
1. INTRODUCTION

The evolution in time of a system $S$ immersed in a large environment $E$ can be obtained from the dynamics of the total system $S + E$ by eliminating (i.e. integrating over) the degrees of freedom of $E$. Since $S + E$ is closed, the total dynamics is unitary; this is no longer true for the evolution of the subsystem $S$ alone, which in general turns out to be very involved, developing non-linearities and memory effects. However, when the interaction between subsystem and environment is weak and there are no initial correlations between $S$ and $E$, the time evolution of $S$ can still be realized through linear maps on the states of $S$, satisfying basic physical requirements, like forward in time composition law (semigroup property), entropy increase (irreversibility) and complete positivity (that guarantees the physical consistency of the evolution in all situations). These one-parameter (=time) family of maps form a so called quantum dynamical semigroup [1-4] and are generated by a master equation of Lindblad form [5].

This description of the time-evolution of open systems is very general; it was originally developed in the framework of quantum optics [6-8], and subsequently used to model very different physical situations, from the study of various statistical systems [1-3], to the analysis of the interaction of a microsystem with a macroscopic measuring apparatus [9-11], to the description of the emergence of the classical world [12, 13] and of the so called dynamical reduction [14].

Master equation of Lindblad form can also be used to describe phenomena leading to irreversibility and dissipation at low energy generated by the dynamics of fundamental objects at a large scale, typically the Planck mass [15]. Indeed, the dynamics of extended objects, strings and branes, gives rise at low energies to a weakly coupled heat bath, and as a consequence to decoherence phenomena [16]. From a more phenomenological point of view, similar effects have also been described in the framework of quantum gravity [17]: due to the quantum fluctuations of the gravitational field and the possible generation of virtual black holes, spacetime should become “foamy” at scales comparable to the Planck length, leading to loss of quantum coherence [18-23]. Furthermore, dissipation and decoherence are also the natural outcome of the general dynamics in theories with large extra dimensions [24]: indeed, the possible energy leakage from the boundary of spacetime (our four-dimensional brane universe) into the bulk due to gravity effects would inevitably inject noise into the boundary, thus inducing irreversibility and dissipation at low energy.

Our present knowledge of string theory does not allow precise estimates of the magnitude of these non-standard effects. Nevertheless, dimensional arguments suggest that they must be very small, being suppressed by at least one inverse power of a large fundamental mass. Despite of this, they can be in the reach of various interferometric devices. Indeed, detailed investigations involving different elementary particle systems (neutral mesons [25, 26], neutrons [27], photons [28], and neutrinos [29]) have shown that present and future experiments might soon reach the sensitivity required to detect the new, non-standard phenomena.

Another physical situation in which phenomena leading to irreversibility and dissipation can be studied is provided by atom interferometers, where a beam of nearly monoenergetic atoms is coherently split into two components that are recombined at the exit of the apparatus [30-33]. The interferometric pattern observed at the end of the devise is
influenced by the action of external phenomena, produced e.g. by an external electric or magnetic field, or by earth gravity. The sophistication of present interferometric apparatus is so high that the theoretically predicted changes in the interferometric figure for some of these phenomena have been confirmed with high accuracy [30-32].

Irreversibility and dissipation also affect the propagation of the atoms in the interferometer; this leads to a deformation of the corresponding interferometric pattern at the exit of the apparatus. It turns out that these modifications are very distinctive of the dissipative phenomena, and cannot be mimicked by other physical effects, as the ones mentioned before.

In the following, we shall analyze in detail these modifications under the hypothesis that the generalized dynamics of the atoms inside the interferometer be generated by a master equation of Lindblad form. For sake of definiteness, we shall limit our discussion to three-grating atom interferometers in the Bragg regime, where the split and the recombination of the incident beam is realized by material gratings or laser standing waves. The approach has points in common with the one adopted in [27] to study similar effects in neutron interferometry [34]. The perturbative treatment adopted there is however inapplicable in the present case: this fact, together with the operational differences in the actual functioning of an atom interferometer require a completely different and independent analysis. As discussed in the final section, the outcome of our investigations is that atom interferometry experiments could provide the most accurate estimate of the non-standard, dissipative effects that can be induced by a fundamental dynamics at Planck’s scale.

2. MASTER EQUATION

The evolution of the atoms inside the interferometer can be analyzed using an abstract, two-dimensional Hilbert space. The states corresponding to the two split beams in the apparatus can be taken to be the basis states in this space. More in general, the quantum state of an atom travelling inside the interferometer will be a statistical mixture of the basis states, and therefore described by a density matrix \( \rho \), i.e. by a hermitian, positive defined operator with unit trace.\(^\dagger\) With respect to the chosen basis, one can then write:

\[
\rho = \begin{pmatrix} \rho_1 & \rho_3 \\ \rho_4 & \rho_2 \end{pmatrix}, \quad \rho_4 = \rho_3^*, \quad \rho_1 + \rho_2 = 1,
\]

where * signifies complex conjugation.

As explained in the introductory remarks, the starting point of our analysis is the assumption that the dynamics of the atoms inside the interferometer be generated by a master equation of Lindblad form\(\dagger\) [1-5]:

\[
\frac{\partial}{\partial t} \rho(t) = -i[H, \rho(t)] + D[\rho(t)].
\]

\(\dagger\) For earlier works on the use of the formalism of density matrices in atom interferometry, see [35-37, 33] and references therein.

\(\dagger\) An equation of this type has also been used to study decoherence effects in position space induced by the scattering of photons on the atoms inside the interferometer.
The first term in the r.h.s. represents the standard hamiltonian contribution. In the chosen basis, the effective hamiltonian can be written as:

\[ H = \left( \begin{array}{cc} E + \omega & 0 \\ 0 & E - \omega \end{array} \right), \quad (2.3) \]

where \( E \) is the energy of the atoms in the incident beam. On the other hand the splitting in energy \( 2\omega \) among the two internal beams is usually induced by the action of laboratory controlled effects, typically the presence of external fields. For open system though, even in absence of external fields, the quantity \( \omega \) is in general nonvanishing. Indeed, one can show that the weak interaction of the system with the external environment induces in general a hamiltonian contribution, giving rise to the “Lamb shift” term \( \omega \) in (2.3) [1-3, 16, 29].

Nevertheless, it is the additional piece \( D[\rho] \) in (2.2) that describes true mixing enhancing phenomena: in absence of it, the evolution of \( \rho \) would be unitary and reversible. It can be represented by a trace-preserving linear map acting on the three independent components of the density matrix in (2.1). Decomposing for convenience \( \rho_3 \) in its real and imaginary parts,

\[ \rho_3 = \rho_1^1 - i\rho_2^2, \quad (2.4a) \]

and introducing the combination

\[ \rho_1 - \rho_2 = 2\rho_3^1, \quad (2.4b) \]

one can then write \( D[\rho] \) as a \( 3 \times 3 \) real, symmetric matrix \( D \), acting on the real vector \( |\rho\rangle \) of components \( (\rho_1, \rho_2, \rho_3^3) \):

\[ D = -2 \begin{bmatrix} a & b & c \\ b & \alpha & \beta \\ c & \beta & \gamma \end{bmatrix}. \quad (2.5) \]

The six parameters \( a, b, c, \alpha, \beta \) and \( \gamma \), with \( a, \alpha \) and \( \gamma \) non negative, are not all independent; physical consistency of the full time evolution (i.e. the request of complete positivity, see [38, 39] for details) further imposes the following inequalities:

\[ \begin{align*}
2R & \equiv \alpha + \gamma - a \geq 0, & RS \geq & \beta^2, \\
2S & \equiv a + \gamma - \alpha \geq 0, & RT \geq & c^2, \\
2T & \equiv a + \alpha - \gamma \geq 0, & ST \geq & \beta^2, \\
RST & \geq 2bc\beta + R\beta^2 + Sc^2 + Tb^2.
\end{align*} \quad (2.6) \]

If one includes also the hamiltonian contribution and further recalls that \( \text{Tr}[\rho(t)] = 1 \), the evolution equation (2.2) can be rewritten as a diffusion equation for the 3-vector \( |\rho(t)\rangle \):

\[ \frac{\partial}{\partial t} |\rho(t)\rangle = -2\mathcal{H}|\rho(t)\rangle, \quad \mathcal{H} = \begin{bmatrix} a & b + \omega & c \\ b - \omega & \alpha & \beta \\ c & \beta & \gamma \end{bmatrix}. \quad (2.7) \]

[37]. Instead, the evolution equation (2.2) is written in “polarization” space, and, as explained below, represents the most general master equation compatible with basic physical requirements.
Its solution involves the exponentiation of the matrix $H$:

$$|\rho(t)\rangle = \mathcal{M}(t) |\rho(0)\rangle, \quad \mathcal{M}(t) = e^{-2Ht}, \quad (2.8)$$

where $|\rho(0)\rangle$ represents the initial state of the atoms entering the interferometer. It coincides with one of the following density matrices

$$\rho^{(1)} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \rho^{(2)} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad (2.9)$$

corresponding to the possible choices of orientation of the incident atomic beam with respect to the first diffracting grating. Both choices lead to the same final results; for definiteness, in the following we shall work with $\rho^{(1)}$, so that $|\rho(0)\rangle = (1/2, 0, 0)$.

### 3. OBSERVABLES

In the language of density matrices, physical observables are represented by suitable hermitian operators, whose mean values can be obtained by taking their trace with $\rho(t)$. In particular, the intensity pattern observed at the end of the interferometer is given by the mean value of the following projector operators [18, 27]:

$$\mathcal{O}_+ = \frac{1}{2} \begin{pmatrix} 1 & e^{-i\theta} \\ e^{i\theta} & 1 \end{pmatrix}, \quad \mathcal{O}_- = \frac{1}{2} \begin{pmatrix} 1 & e^{i(\theta+\pi)} \\ e^{-i(\theta+\pi)} & 1 \end{pmatrix}, \quad (3.1)$$

that correspond to the two possible exit beams in which an atom can be found while exiting the apparatus. In the standard situation, it is the phase $\theta$ that gives the modulation of the interferometric pattern. This is usually obtained by moving the transverse position of one of the gratings (or laser standing waves) responsible for the diffraction of the atom beam. Indeed, in an idealized situation one finds [30-32]:

$$\theta = \kappa (x_1 - 2x_2 + x_3), \quad (3.2)$$

where $x_i$, $i = 1, 2, 3$, represents the position, transverse with respect to the incident beam, of the $i$-th grating, while $\kappa$ is the wave vector of the diffracting lattice.

The intensity $I_\pm$ of the interference figure detected at the two possible exits is then given by:

$$I_\pm(t) = \langle \mathcal{O}_\pm \rangle = \text{Tr}[\mathcal{O}_\pm \rho(t)] = \frac{1}{2} + \mathcal{O}_\pm^1 \rho^1(t) + \mathcal{O}_\pm^2 \rho^2(t) + \mathcal{O}_\pm^3 \rho^3(t), \quad (3.3)$$

where definitions similar to the ones in (2.4) have been introduced also for the entries of the two matrices $\mathcal{O}_\pm$. Using (3.1), one finds:

$$I_\pm(t) = \frac{1}{2} \left\{ 1 \pm 2 \left[ \cos \theta \rho^1(t) + \sin \theta \rho^2(t) \right] \right\}. \quad (3.4)$$
Since an atom exiting the interferometer can only be found in one of the two exit beams, particle conservation requires \( I_+ (t) + I_- (t) = 1 \), which is clearly satisfied by (3.4).

The intensity curves in (3.4) can be compared with the experiment, provided explicit expressions for the entries of the matrix \( \mathcal{M}(t) \) in (2.8) are given. Formally, this can be obtained by studying the eigenvalue problem for the \( 3 \times 3 \) matrix \( \mathcal{H} \) in (2.7):

\[
\mathcal{H}|v_k\rangle = \lambda_k |v_k\rangle , \quad k = 1, 2, 3 .
\]

The three eigenvalues \( \lambda_1, \lambda_2, \lambda_3 \) satisfy a cubic equation,

\[
\lambda^3 + r \lambda^2 + s \lambda + w = 0 ,
\]

with real coefficients:

\[
r = -(a + \alpha + \gamma), \quad s = a\alpha + a\gamma + \alpha\gamma - b^2 - c^2 - \beta^2 + \omega^2, \quad w = -\text{det} \mathcal{H}.
\]

It then follows that \( \lambda_k \) are either all real, or one is real and the remaining two are complex conjugate; further, in both situations, one can show that in presence of dissipation the three eigenvalues have always positive real parts [40].

Using the fact that the matrix \( \mathcal{H} \) itself obeys the equation (3.6), one finds:

\[
I_{\pm}(t) = \frac{1}{2} \left\{ 1 \pm \sum_{k=1}^{3} \frac{e^{-2\lambda_k t}}{3\lambda_k^2 + 2r\lambda_k + s} \left[ (\lambda_k^2 - (\alpha + \gamma)\lambda_k + \alpha\gamma - \beta^2) \cos \theta + (\omega - b)(\lambda_k - \gamma) - \beta c \right] \right\} .
\]

From this general expression, one sees that in presence of complex eigenvalues, a further harmonic modulation in time of the interference figures occurs, while exponential damping terms always prevail for long enough times. Further, note that in absence of dissipation, \( a = b = c = \alpha = \beta = \gamma = \omega = 0 \), and thus \( \lambda_k = 0 \), the expressions of \( I_{\pm} \) in (3.7) reduce to their standard, time-independent ones [30-32]:

\[
I_{\pm} = \frac{1}{2} \left\{ 1 \pm \cos \theta \right\} ;
\]

any deviation from this formula as described by (3.7) clearly signals the presence of dissipative phenomena in atom interferometry.

Although explicit expressions for the eigenvalues \( \lambda_k \) can always be found via Cardano’s formula [41], the form (3.7) of the intensities \( I_{\pm} \) is rather involved, and of limited use in practice. Having in mind possible comparison with experimental data, the study of suitable approximations of (3.7) might result appropriate.

In this respect, a useful working assumption is to take \( \gamma = 0; \)\(^\dagger\) in this case, the inequalities (2.6) further impose \( b = c = \beta = 0 \) and \( a = \alpha \). In this simplified situation,

\(^\dagger\) There are essentially two known ways of implementing the condition of weak interaction between subsystem and environment [1-3]: the singular coupling limit (in which the time-correlations in the environment are assumed to be much smaller than the typical time scale of the subsystem) and the weak coupling limit (in which it is the subsystem characteristic time scale that becomes large). One can check that the second situation leads precisely to the condition \( \gamma = 0 \) [29].
the formula in (3.7) reduces to:

\[ I_\pm(t) = \frac{1}{2} \left\{ 1 \pm e^{-2\alpha t} \cos(\theta - 2\omega t) \right\} . \]  

(3.9)

This is surely the most simple expression that the intensity probabilities \( I_\pm(t) \) take in presence of dissipative effects. It differs from the standard expression in (3.8) by the presence of an exponential damping factor and of an additional harmonic phase that accumulates in time.

A different approximation of the general formula (3.7) can be obtained when the parameters \( a, b, c, \alpha, \beta \) and \( \gamma \) are small with respect of \( \omega \). This could happen when the interferometer is immersed in a strong external field, so that its contribution to the energy shift \( \omega \) due to its interaction with the atom beams largely overrides the one coming from the effects of a weakly coupled environment.\(^\dagger\) In this case, the additional piece \( D[\rho] \) in the evolution equation (2.2) can be treated as a perturbation. Using the solution of this equation expanded up to second order in the small parameters, from (3.4) one obtains:

\[ I_\pm(t) = \frac{1}{2} \left\{ 1 \pm e^{-(\alpha + \alpha)t} \left[ \cos 2\Omega t + \frac{\alpha - a}{2\Omega} \sin 2\Omega t - \frac{2\beta^2}{\Omega^2} \sin^2 \Omega t \right] \cos \theta \right. \\
+ \left. \left( \frac{b - \omega}{\Omega} \sin 2\Omega t - \frac{c\beta}{\Omega^2} \cos 2\Omega t \right) \sin \theta \right\} , \]  

(3.10)

where \( \Omega = \left[ \omega^2 - b^2 - c^2 - \beta^2 - (\alpha - a)^2 / 4 \right]^{1/2} \). In writing (3.10) we have reconstructed the exponential factor by putting together the terms linear and quadratic in \( t \); a similar treatment has allowed writing all harmonic pieces in terms of the frequency \( \Omega \). It is worth noting that for \( c = \beta = 0 \) the formula (3.10) gives the exact expression for the intensities \( I_\pm \): no approximation is involved. This is a consequence of the fact that for \( c = \beta = 0 \) the matrix \( \mathcal{H} \) in (2.7) becomes block diagonal, so that explicit, manageable expressions for its exponential \( \mathcal{M}(t) \), and therefore for \( I_\pm \), can be given. From this point of view, the validity of (3.10) goes beyond the second order approximation in which it has been derived: it can be considered as the expansion of the full expression (3.7) for \( I_\pm \) up to second order in \( c \) and \( \beta \).

\[ \dagger \]

In this respect, it should be noted that even in absence of any external field, a hierarchy between \( \omega \) and the other dissipative parameters \( a, b, c, \alpha, \beta \) and \( \gamma \) could be nevertheless generated by the interaction with the environment. For details, see [16] and the Appendix in [29].

4. INTERFERENCE PATTERN

The behaviour of the general expression (3.7) for \( I_\pm \) and of its special cases (3.9) and (3.10) crucially depend on the dissipative parameters: at least in principle, they can be used to obtain informations on their values from fits with the experimental data. The magnitude of \( a, b, c, \alpha, \beta, \gamma \) and \( \omega \) are nevertheless expected to be very small. In fact, for subsystems
in interactions with large environments, the effects leading to dissipation and decoherence can be roughly estimated to be proportional to the typical energy of the system, while suppressed by inverse powers of the characteristic energy scale of the environment \([1-3]\). In the case of non-standard phenomena induced by the dynamics of fundamental objects (strings, branes) at Planck’s mass \(M_P\), an upper bound on the magnitude of the dissipative parameters can be roughly evaluated to be of order \(M_A^2/M_P\), where \(M_A\) is the mass of the atoms used in the interferometer \([16, 29]\); in typical real situations, this ratio takes values between \(10^{-18}\) and \(10^{-15}\) GeV (or equivalently, between \(10^3\) and \(10^6\) KHz).

A further difficulty in comparing the theoretically predicted interference figures with the experimental data arises from the fact that the previously derived expressions for \(I_{\pm}\) hold in the case of an idealized interferometer, with perfectly monoenergetic atomic incident beams. In practice, the values of the atom momenta spread over a finite distribution. This fact, together with the inevitable imperfections in the construction of the actual interferometric apparatus, produce attenuation in the intensity of the signal.

One can take into account these spurious effects by modifying the previously derived intensity spectra with the introduction of suitable unknown parameters. To keep the discussion as simple as possible, we shall concentrate on the expression (3.9) for \(I_{\pm}\); similar arguments apply to the other formulas. By denoting with \(N_{\pm}\) the atom countings at the two exit beams of the interferometer, one generalizes the spectra in (3.9) as:

\[
N_{\pm}(t) = N_{\pm}^{(0)} \left\{ 1 \pm C_{\pm} e^{-2\alpha t} \cos \left( \theta - 2\omega t \right) \right\} .
\] (4.1)

The constants \(C_{\pm}\) are the fringe contrast and parametrize the intensity attenuation, while \(N_{\pm}^{(0)}\) are suitable normalization factors;\(^\dagger\) note that particle conservation now requires: \(N_{\pm}^{(0)} C_{\pm} = N_{\pm}^{(0)} C_{\mp}\). Clearly, the higher the fringe contrast, the more accurate the determination of the dissipative parameters \(\alpha\) and \(\omega\) from the experiment will be.

In order to fit actual experimental data with the expression (4.1), further elaborations are however needed. As mentioned before, the intensity spectra are reconstructed by counting the atoms at one of the exit beams as a function of the transverse position \(x\) of the final grating (or standing laser wave), with respect to a reference, initial situation. This means that the geometry of the two paths followed by the atoms inside the interferometer slightly changes as \(x\) varies;\(^\ddagger\) as a consequence, also the total evolution time \(t\), the time spent by the atoms inside the interferometer, changes with \(x\). Since the two path inside the apparatus are very close to each other (their actual separation is at most 100 \(\mu\)m), for small \(x\) one finds:

\[
t = t_0 + \frac{\varphi_1}{v} x ,
\] (4.2)

\(^\dagger\) In absence of dissipative effects, a theoretical estimate of \(C_{\pm}\) has been obtained using atom optics \([42, 43]\).

\(^\ddagger\) The situation is completely different in a neutron interferometer \([34]\): made of a silicon crystal, its geometry can not be varied. In this case, the interferometric spectra are obtained through a thin slab of material inserted transversally to the two beams inside the interferometer; a slight rotation of it produces a phase difference \(\omega\) between the two “optical” paths.
where $v$ is the average velocity of the atoms in the incident beam, while $\vartheta_1$ is the first order Bragg diffraction angle (typically of order $10^{-4}$ rad). On the other hand $t_0$ is the fixed time of flight of the atoms inside the interferometer when it is in its initial, reference status; it can be determined with high accuracy from the geometric specifications of the actual apparatus and can be modified only by changing the energy of the primary atom beam, or by modifying the longitudinal dimension of the interferometer.

The outcome of this discussion is that it should be possible to estimate the values of $\alpha$ and $\omega$ from the behaviour in $x$ of the expression in (4.1), and therefore from a fit with experimental data. Unfortunately, in the experimental set-ups so far constructed the dependence of $t$ on $x$ can hardly be seen: one finds that while $t_0$ is at most $10^{-3}$ sec, the quantity $\vartheta_1 x/v$ results at least ten orders of magnitude smaller, even for maximal values of the displacement $x$ (a few hundreds nm).

Therefore, as a good approximation one can safely take $t \simeq t_0$, and rewrite the interference pattern (4.1) as

$$N_\pm(x) = N_\pm^{(0)} \left\{ 1 \pm \left[ P_\pm \cos(\theta_0 + \kappa x) + Q_\pm \sin(\theta_0 + \kappa x) \right] \right\}, \quad (4.3)$$

where

$$P_\pm = C_\pm e^{-2\alpha t_0} \cos 2\omega t_0, \quad Q_\pm = C_\pm e^{-2\alpha t_0} \sin 2\omega t_0, \quad (4.4)$$

while $\theta_0$ is a fixed phase that is characteristic of each interferometer. A fit of (4.3) with experimental data will allow to determine the parameters $N_\pm^{(0)}$, $P_\pm$, $Q_\pm$ and $\theta_0$, and therefore to obtain informations on the dissipative parameters.

5. DISCUSSION

We shall now briefly report on the results of a $\chi^2$ fit of the formula (4.3) with recently published data from two experiments [44, 45]. The analysis that follows is of limited quantitative meaning: direct access to the data and a careful study of systematic errors are needed in order to obtain precise determination of the dissipative effects; nevertheless, it will provide a rough estimate about the sensitivity of present atom interferometers to the parameters $\alpha$ and $\omega$.

The atom interferometers used in the two experiments are particularly sensitive devices, reaching a very high fringe contrast; this is obtained by using neon, respectively lithium, atoms as “matter waves” and laser standing light as diffracting device. Both in [44] and [45], only data from one of the two exit beams are reported, the ones corresponding to the lower sign in (4.3). Since as mentioned before $t_0$ is known with high precision, from the ratio $P_-/Q_-$ one can immediately obtain an estimate for the parameter $\omega$. Using the data from the first experiment, from our fit we find $\omega = (0.7 \pm 0.2) \times 10^{-21}$ GeV, where the quoted error is only statistical.

On the other hand, the determination of $\alpha$ is subordinated to the estimate of the fringe contrast $C_-$. This would not be necessary if the parameters $P_-$ and $Q_-$ can be measured for two different values of the flight time $t_0$. As mentioned before, this can be obtained either by changing the average velocity of the incoming atoms, or by varying the
dimensions of the interferometer. In lacking of this extra information, we shall obtain an estimate for \( C_- \) using directly the data.

In absence of dissipative effects, \( \alpha = \omega = 0 \), the constant \( C_- \) can be obtained from the maximum, \( N^{(\text{max})} \), and the minimum, \( N^{(\text{min})} \), atom counts in the experimental interference figure: \( C_- = (N^{(\text{max})} - N^{(\text{min})})/(N^{(\text{max})} + N^{(\text{min})}) \). Although this formula is only approximately valid for nonvanishing \( \alpha \) and \( \omega \), in practice the systematic error that one makes in adopting it can be estimated to be at the end much smaller than the pure experimental uncertainty. Using the rough experimental data, one then deduces: \( C_- \approx 62\% \). With this value, one finally gets: \( \alpha = (0.1 \pm 0.1) \times 10^{-22} \text{ GeV} \), which is compatible with zero.

The accuracy in the determination of \( \alpha \) and \( \omega \) improves using the data from the most recent experiment, thanks to the higher fringe contrast (of about 74\%) and the increase in the number of experimental points. In fact, the same procedure adopted before now gives the following estimates: \( \alpha = (0.3 \pm 0.1) \times 10^{-23} \text{ GeV} \) and \( \omega = (0.20 \pm 0.01) \times 10^{-21} \text{ GeV} \).\( ^{\dagger} \) Note that these values are perfectly compatible with the ones previously determined in the case of the neon interferometer. As explained before, the values of the dissipative parameters should be proportional to the square of the mass of the atoms in the incident beams. Therefore, the values of \( \alpha \) and \( \omega \) determined with the data from the lithium beam should result smaller than those obtained from the first experiment.

In conclusion, the results of our discussion show that atom interferometers are potentially very sensitive to the presence of phenomena leading to dissipation and decoherence. Although the performed error analysis has been limited to statistical uncertainties, the derived estimates seem to indicate nonvanishing values for \( \alpha \) and \( \omega \), of magnitude compatible with an origin from a fundamental dynamics at a very large mass scale. As already remarked, direct access to the rough experimental data and more complete \( \chi^2 \) fits are needed in order to claim the presence of dissipative effects. We nevertheless hope that our preliminary analysis will stimulate further, more accurate investigations.

\( ^{\dagger} \) We remark that as before the quoted errors are purely statistical; a thorough analysis of the full experimental data, that takes into account also the systematics, would likely worsen the estimated errors, in particular that on \( \omega \).
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