Solar proton burning, 
photon and anti–neutrino disintegration of the 
deuteron in the relativistic field theory model of the 
deuteron

A. N. Ivanov ∗‡, H. Oberhummer †, N. I. Troitskaya ‡, M. Faber §

Institut für Kernphysik, Technische Universität Wien, 
Wiedner Hauptstr. 8-10, A-1040 Vienna, Austria

Abstract

The relativistic field theory model of the deuteron (RFMD) is applied to the calculation of the astrophysical factor $S_{pp}(0)$ for the process of the solar proton burning $p + p \to D + e^+ + \nu_e$ and the cross sections for the disintegration of the deuteron by photons $\gamma + D \to n + p$ and anti–neutrinos $\bar{\nu}_e + D \to e^+ + n + n$. Our theoretical value of the astrophysical factor $S_{pp}(0) = 4.02 \times 10^{-25}$ MeV b agrees with the classical result obtained by Bahcall and Kamionkowski $S_{pp}(0) = 3.89 \times 10^{-25}$ MeV b in the potential model approach (PMA). The cross sections for the disintegration of the deuteron by photons and anti–neutrinos calculated near thresholds are in good agreement with the PMA. An extrapolation of the cross sections for energies far from thresholds is suggested and related to the inclusion of form factors describing spatial smearing of the deuteron and the NN system. The extrapolated cross section for the disintegration of the deuteron by anti–neutrinos agrees with that calculated in the PMA in the anti–neutrino energy region from threshold up to $E_{\bar{\nu}_e} = 10$ MeV. The extrapolated cross section averaged over the reactor anti–neutrino energy spectrum is obtained in agreement with the experimental data. It is shown that the RFMD enables to describe elastic low–energy NN scattering in accordance with low–energy nuclear phenomenology.

PACS: 11.10.Ef, 13.75.Cs, 14.20.Dh, 21.30.Fe, 25.40.Lw, 26.65.+t

Keywords: relativistic field theory, deuteron, proton–proton fusion, photo disintegration, anti–neutrino disintegration

∗E–mail: ivanov@kph.tuwien.ac.at, Tel.: +43–1–58801–14261, Fax: +43–1–5864203
†E–mail: ohu@kph.tuwien.ac.at, Tel.: +43–1–58801–14251, Fax: +43–1–5864203
‡Permanent Address: State Technical University, Department of Nuclear Physics, 195251 St. Petersburg, Russian Federation
§E–mail: faber@kph.tuwien.ac.at, Tel.: +43–1–58801–14261, Fax: +43–1–5864203
1 Introduction

The relativistic field theory model of the deuteron (RFMD) formulated in Refs. [1,2] has been applied to the calculation of the reaction rate of the neutron–proton radiative capture \( n + p \rightarrow D + \gamma \) and the astrophysical factor \( S_{pp}(0) \) of the solar proton burning \( p + p \rightarrow D + e^+ + \nu_e \) [2]. Some mistakes, which had been made for the first calculation [2], have been then partly, mainly for the reaction rate of the neutron–proton radiative capture, corrected in Ref. [3]. However, our conclusion concerning the value of the astrophysical factor \( S_{pp}(0) \) [3] is still erroneous.

In this paper we would like to amend our results obtained in Refs. [1–3] and to apply the RFMD to the calculation of the cross sections for the disintegration of the deuteron by photons \( \gamma + D \rightarrow n + p \) and anti–neutrinos \( \bar{\nu}_e + D \rightarrow e^+ + n + n \). The reaction of the disintegration of the deuteron by anti–neutrinos \( \bar{\nu}_e + D \rightarrow e^+ + n + n \) is caused by the charged weak current and valued, in the sense of charge independence of the weakinteraction strength, to be equivalent to the observation of the reaction of the solar proton burning \( p + p \rightarrow D + e^- + \nu_e \) in the terrestrial laboratories [4]. Experimentally the reaction of the disintegration of the deuteron by anti–neutrinos \( \bar{\nu}_e + D \rightarrow e^+ + n + n \) induces itself by reactor anti–neutrinos with an equilibrium energy spectrum [5,6]. Therefore, experimental data on the reaction \( \bar{\nu}_e + D \rightarrow e^+ + n + n \) are given in the form of the cross section averaged over the reactor anti–neutrino energy spectrum [4,7–9].

We show that all processes under consideration can be described in the RFMD in agreement with the potential model approach (PMA) in spite of completely different dynamics of strong low–energy nuclear interactions. Indeed, in the RFMD [1,2] the physical deuteron appears through long–wavelength vacuum fluctuations of the proton and the neutron field in the one–nucleon loop approximation. In terms of one–nucleon loop exchanges we describe in the RFMD a non–trivial wave function of the relative movement of the nucleons inside the physical deuteron. Therefore, the physical deuteron couples to nucleons and other particles only through one–nucleon loop exchanges.

In order to couple to the deuteron through the one–nucleon loop exchange the nucleons should pass through intermediate interactions providing low–energy transitions \( N + N \rightarrow N + N \). In a quantum field theory approach such interactions should be induced by meson exchanges. As the nucleons couple at low energies, the main contribution should come from the one–pion exchange. The contributions of heavier meson exchanges can be taken into account effectively by integrating them out.

Since in the reactions \( p + p \rightarrow D + e^+ + \nu_e \), \( n + p \rightarrow D + \gamma \), \( \gamma + D \rightarrow n + p \) and \( \bar{\nu}_e + D \rightarrow e^+ + n + n \) the nucleons couple in the \(^1S_0\)–state, the low–energy transitions \( N + N \rightarrow N + N \) can be described by the effective local four–nucleon interactions [1,2]:

\[
\mathcal{L}_{\text{eff}}^{\text{NN} \rightarrow \text{NN}}(x) = G_{\pi NN} \left\{ [\bar{n}(x)\gamma_\mu \gamma_5 p^\epsilon(x)] [\bar{p}(x)\gamma^\mu \gamma_5 n(x)] + \frac{1}{2} [\bar{n}(x)\gamma_\mu \gamma_5 n^\epsilon(x)] [\bar{n}^\epsilon(x)\gamma^\mu \gamma_5 n(x)] + \frac{1}{2} [\bar{p}(x)\gamma_\mu \gamma_5 p^\epsilon(x)] [\bar{p}^\epsilon(x)\gamma^\mu \gamma_5 p(x)] + (\gamma_\mu \gamma_5 \otimes \gamma^\mu \gamma_5 \otimes \gamma^\mu \gamma_5 \otimes \gamma^\mu \gamma_5), \right\},
\]

(1.1)

where \( n(x) \) and \( p(x) \) are the operators of the neutron and the proton interpolating fields, \( n^\epsilon(x) = C\bar{n}^T(x) \), etc., then \( C \) is a charge conjugation matrix and \( T \) is a transposition.
The effective coupling constant \( G_{\pi NN} \) is defined by

\[
G_{\pi NN} = \frac{g_{\pi NN}^2}{4M_\pi^2} - \frac{2\pi a_{np}}{M_N} = 3.27 \times 10^{-3} \text{ MeV}^{-2},
\]  

(1.2)

where \( g_{\pi NN} = 13.4 \) is the coupling constant of the \( \pi \)NN interaction, \( M_\pi = 135 \text{ MeV} \) is the pion mass, \( M_p = M_n = M_N = 940 \text{ MeV} \) is the mass of the proton and the neutron neglecting the electromagnetic mass difference, which is taken into account only for the calculation of the phase volumes of the final states of the reactions \( p + p \to D + e^+ + \nu_e \) and \( \bar{\nu}_e + D \to e^+ + n + n \), and \( a_{np} = (-23.748 \pm 0.010) \) fm is the S-wave scattering length of the np scattering in the \( ^1S_0 \) state [10].

The first term in the effective coupling constant \( G_{\pi NN} \) comes from the one-pion exchange for the squared momenta transfer \(-q^2\) much less than the squared pion mass \(-q^2 \ll M_\pi^2\) and the subsequent Fierz transformation of the nucleon fields [1,2]. We should emphasize that due to Fierz transformation the effective NN interaction caused by the one-pion exchange contains a few contributions with different spinorial structure, we have taken into account only those terms which contribute to the \( ^1S_0 \) state of the NN system. The second term is a phenomenological one representing a collective contribution caused by the integration over heavier meson fields like scalar mesons \( \sigma(700) \), \( a_0(980) \) and \( f_0(980) \), vector mesons \( \rho(770) \) and \( \omega(780) \) and so on. This term is taken in the form used in the Effective Field Theory (EFT) approach [11–13]. The effective interaction Eq. (1.1) is written in the isotopically invariant form, and the coupling constant \( G_{\pi NN} \) can be never equal zero at \( a_{np} \neq 0 \) due to negative value of \( a_{np} \) imposed by nuclear forces, i.e., \( a_{np} < 0 \) [14].

In the low-energy limit the effective local four-nucleon interaction Eq. (1.1) vanishes due to the reduction

\[
[\bar{N}(x)\gamma_\mu\gamma^5N^c(x)][\bar{N}^c(x)\gamma^\mu\gamma^5N(x)] \to -[\bar{N}(x)\gamma^5N^c(x)][\bar{N}^c(x)\gamma^5N(x)],
\]  

(1.3)

where \( N(x) \) is the neutron or the proton interpolating field. Such a vanishing of the one-pion exchange contribution to the NN potential is well-known in the EFT approach [11–13] and the PMA [14]. In power counting [11–13] the interaction induced by the one-pion exchange is of order \( O(k^2) \), where \( k \) is a relative momentum of the NN system. The former is due the Dirac matrix \( \gamma^5 \) which leads to the interaction between small components of the Dirac bispinors of the nucleon wave functions.

Thus, if either in the PMA or the EFT approach the effective local four-nucleon interaction Eq. (1.1) would be applied to the description of the deuteron coupled to the nucleons, the contribution would be scarcely significant. Therefore, both in the PMA and the EFT for the correct description of strong low-energy nuclear forces one needs to include an effective phenomenological NN potential, for instance, the Argonne \( \nu_{18} \) [15]. The one-pion exchange contribution is considered as a perturbation.

In the RFMD due to the one-nucleon loop exchange the contributions of the interactions \([\bar{N}(x)\gamma_\mu\gamma^5N^c(x)][\bar{N}^c(x)\gamma^\mu\gamma^5N(x)] \) and \([\bar{N}(x)\gamma^5N^c(x)][\bar{N}^c(x)\gamma^5N(x)] \), or shortly \( \gamma_\mu\gamma^5 \otimes \gamma^\mu\gamma^5 \) and \( \gamma^5 \otimes \gamma^5 \), to the amplitudes of nuclear processes are different and do not cancel each other in the low-energy limit. This is completely a peculiarity of one-nucleon loop diagrams related to one-fermion loop anomalies [16,17,2]. For instance, in the case of the neutron-proton radiative capture and the photomagnetic disintegration of
the deuteron the amplitudes of the processes are defined by the triangle one–nucleon loop diagrams with AVV (axial–vector–vector) and PVV (pseudoscalar–vector–vector) vertices [2] caused by $\gamma_\mu \gamma^5 \otimes \gamma^\mu \gamma^5$ and $\gamma^5 \otimes \gamma^5$ interactions, respectively. These diagrams are well–known in particle physics in connection with the Adler–Bell–Jackiw axial anomaly [16] which plays a dominant role for the processes of the decays $\pi^0 \to \gamma \gamma$, $\omega \to \pi^0 \gamma$ and so on [16]. The results of the calculation of these diagrams differ each other. Hence, they give different contributions to the amplitudes of the processes and do not cancel themselves in the low–energy limit.

Then, the amplitudes of the solar proton burning and the anti–neutrino disintegration of the deuteron are defined by the one–nucleon loop diagrams with AAV and APV vertices caused by $\gamma_\mu \gamma^5 \otimes \gamma^\mu \gamma^5$ and $\gamma^5 \otimes \gamma^5$ interactions, respectively. The contribution of the diagrams with APV vertices turns out to be divergent and, therefore, negligibly small compared with the contribution of the diagrams with AAV vertices [2], which contains non–trivial convergent part related to one–fermion loop anomalies [2,17].

As a result in the low–energy limit amplitudes of nuclear processes described by the RFMD contain only large components of Dirac bispinors of wave functions of nucleons. This provides an effective enhancement of the one–pion exchange for the description of strong low–energy interactions of the nucleons in the $^1S_0$–state. Thus, in the RFMD the dynamics of strong low–energy nuclear interactions caused by one–nucleon loop exchanges is the point of a dominant role of the one–pion exchange contribution to the effective low–energy interactions of nucleons in the $^1S_0$–state.

The paper is organized as follows. In Sect. 2 we calculate the astrophysical factor for the solar proton burning. In Sect. 3 we calculate the cross section for the photomagnetic disintegration of the deuteron near threshold and formulate the extrapolation procedure for the cross section for energies far from threshold. In Sect. 4 we compute the cross section for the anti–neutrino disintegration of the deuteron near threshold, extrapolate this cross section for energies far from threshold and average the extrapolated cross section over the reactor anti–neutrino energy spectrum. In Conclusion we discuss the obtained results and a justification of the RFMD. In Appendix A we adduce the detailed calculation of the amplitude of the solar proton burning. In Appendix B we show that in the RFMD one can describe low–energy elastic NN scattering with non–zero effective range in accordance with low–energy nuclear phenomenology.

## 2 Astrophyysical factor for the solar proton burning

In the RFMD the amplitude of the solar proton burning is defined by one–nucleon loop diagrams [2]. The detailed calculation of the amplitude of the solar proton burning is given in Appendix A. The result reads

\[
\langle iM(p + p \to D + e^+ + \nu_e) = C(\eta) G_V g_A M_N G_{\pi NN} \frac{3g_V}{4\pi^2} \times \epsilon^*_\mu(k_D) [\bar{u}(k_{\nu e}) \gamma^\mu (1 - \gamma^5) v(k_{e^+})] [\bar{u}^c(p_2) \gamma^5 u(p_1)], \quad (2.1)
\]

where $G_V = G_F \cos \vartheta_C$ with $G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}$ and $\vartheta_C$ are is the Fermi weak coupling constant and the Cabibbo angle $\cos \vartheta_C = 0.975$. Then $g_A = 1.260 \pm 0.012$ describes the renormalization of the weak axial hadron current by strong interactions.
\[ g_V \] is the phenomenological coupling constant of the RFMD related to the electric quadrupole moment of the deuteron: \( g_V^2 = 2 \pi^2 Q_D M_N^2 \) [2] with \( Q_D = 0.286 \text{fm}^2 \) [10], \( e_\mu(k_D) \) is a 4–vector of a polarization of the deuteron and \( \bar{u}(k_{\nu_e}), v(k_{e^+}), \bar{u}(p_2) \) and \( u(p_1) \) are the Dirac bispinors of neutrino, positron, and two protons, respectively. For the binding energy of the deuteron we use the value \( E_\text{D} = 2.225 \text{MeV} \) [10]. The Coulomb repulsion between protons is taken into account only in terms of the Gamow penetration factor [2,3] \( C(\eta) = \sqrt{2\pi \eta} \exp(-\pi \eta) \) depending on the relative velocity of the protons \( \eta \) as \( \eta = \alpha/v \) and \( \alpha = 1/137 \) is the fine structure constant.

The cross section for the low–energy \( p + p \to D + e^+ + \nu_e \) reaction is defined

\[
\sigma(pp \to \text{De}^+\nu_e) = \frac{1}{v} \frac{1}{2E_1E_2} \int |\mathcal{M}(p + p \to D + e^+ + \nu_e)|^2
\times (2\pi)^4 \delta^4(k_D + k_{e^+} + k_{\nu_e} − p_1 − p_2) \frac{d^3k_D}{(2\pi)^32E_D} \frac{d^3k_{e^+}}{(2\pi)^32E_{e^+}} \frac{d^3k_{\nu_e}}{(2\pi)^32E_{\nu_e}}, \tag{2.2}
\]

where \( v \) is a relative velocity of the protons and \( E_i \) (\( i = 1, 2 \)) are the energies of the protons in the center of mass frame.

Then, \( |\mathcal{M}(p + p \to D + e^+ + \nu_e)|^2 \) is the squared amplitude averaged over polarizations of protons and summed over polarizations of final particles:

\[
|\mathcal{M}(p + p \to D + e^+ + \nu_e)|^2 = C^2(\eta) G^2 \sqrt{\pi} \frac{g^2_A M_N^4 G^2_{\pi NN}}{8\pi^2} \times \left( -g^\alpha\beta + \frac{k_D^\alpha k_D^\beta}{M_D^2} \right) \times \text{tr}\{(−m_e + \hat{k}_{e^+})\gamma_\alpha(1 − \gamma^5)\hat{k}_{\nu_e}\gamma_\beta(1 − \gamma^5)\} \times \frac{1}{4} \times \text{tr}\{(M_N − \hat{p}_2)\gamma^5(M_N + \hat{p}_1)\gamma^5\}, \tag{2.3}
\]

where \( m_e = 0.511 \text{MeV} \) is the mass of positron, and we have used the relation \( g^2_\pi/\pi^2 = 2Q_D M_N^2 \) [2].

In the low–energy limit the computation of the traces yields

\[
\left( -g^\alpha\beta + \frac{k_D^\alpha k_D^\beta}{M_D^2} \right) \times \text{tr}\{(−m_e + \hat{k}_{e^+})\gamma_\alpha(1 − \gamma^5)\hat{k}_{\nu_e}\gamma_\beta(1 − \gamma^5)\} =
\]

\[
24 \left( E_{e^+} + E_{\nu_e} − \frac{1}{3} \hat{k}_{e^+} \cdot \hat{k}_{\nu_e} \right),
\]

\[
\frac{1}{4} \times \text{tr}\{(M_N − \hat{p}_2)\gamma^5(M_N + \hat{p}_1)\gamma^5\} = 2 M_N^2,
\tag{2.4}
\]

where we have neglected the relative kinetic energy of the protons with respect to the mass of the proton.

Substituting Eq. (2.4) in Eq. (2.3) we get

\[
|\mathcal{M}(p + p \to D + e^+ + \nu_e)|^2 = C^2(\eta) G^2 \sqrt{\pi} \frac{g^2_A M_N^4 G^2_{\pi NN}}{8\pi^2} \frac{54Q_D}{512} \left( E_{e^+} + E_{\nu_e} − \frac{1}{3} \hat{k}_{e^+} \cdot \hat{k}_{\nu_e} \right). \tag{2.5}
\]

The integration over the phase volume of the final \( \text{De}^+\nu_e \)–state we perform in the non–relativistic limit

\[
\int \frac{d^3k_D}{(2\pi)^32E_D} \frac{d^3k_{e^+}}{(2\pi)^32E_{e^+}} \frac{d^3k_{\nu_e}}{(2\pi)^32E_{\nu_e}} (2\pi)^4 \delta^4(k_D + k_{\ell} − p_1 − p_2) \left( E_{e^+} + E_{\nu_e} − \frac{1}{3} \hat{k}_{e^+} \cdot \hat{k}_{\nu_e} \right)
\]

\[
= \frac{1}{32\pi^3M_N} \int_{m_e}^{W + T_{pp}} \sqrt{E_{e^+}^2 - m_e^2} \left( W + T_{pp} - E_{e^+} \right)^2 dE_{e^+} = \frac{(W + T_{pp})^5 f(\xi)}{960\pi^3M_N}, \tag{2.6}
\]

5
where $W = \varepsilon_D - (M_n - M_p) = (2.225 - 1.293)\text{ MeV} = 0.932\text{ MeV}$, $T_{pp} = M_N v^2/4$ is the kinetic energy of the relative movement of the protons, and $\xi = m_e/(W + T_{pp})$. The function $f(\xi)$ is defined by the integral
\begin{equation}
\frac{15}{2} \xi^4 \ln \left(1 + \frac{\sqrt{1 - \xi^2}}{\xi}\right)_{T_{pp}=0} = 0.222
\end{equation}
and normalized to unity at $\xi = 0$.

Thus, the cross section for the solar proton burning is given by
\begin{equation}
\sigma(\text{pp} \to \text{De}^+\nu_e) = e^{-\frac{2\pi\eta}{v^2}} \alpha \frac{9g_A^2G_V^2Q_D M_N^3}{320 \pi^4} G_{\pi NN}^2 (W + T_{pp})^5 f\left(\frac{m_e}{W + T_{pp}}\right) = \frac{S_{pp}(T_{pp})}{T_{pp}} e^{-\frac{2\pi\eta}{v^2}}. \tag{2.8}
\end{equation}

The astrophysical factor $S_{pp}(T_{pp})$ reads
\begin{equation}
S_{pp}(T_{pp}) = \alpha \frac{9g_A^2G_V^2Q_D M_N^4}{1280 \pi^4} G_{\pi NN}^2 (W + T_{pp})^5 f\left(\frac{m_e}{W + T_{pp}}\right). \tag{2.9}
\end{equation}

At zero kinetic energy of the protons $T_{pp} = 0$ the astrophysical factor $S_{pp}(0)$ is given by
\begin{equation}
S_{pp}(0) = \alpha \frac{9g_A^2G_V^2Q_D M_N^4}{1280 \pi^4} G_{\pi NN}^2 W^5 f\left(\frac{m_e}{W}\right) = 4.02 \times 10^{-25}\text{ MeV b.} \tag{2.10}
\end{equation}

The value $S_{pp}(0) = 4.02 \times 10^{-25}\text{ MeV b}$ agrees with the value $S_{pp}(0) = 3.89 \times 10^{-25}\text{ MeV b}$ obtained by Kamionkowski and Bahcall in the PMA [18] and $S_{pp}(0) = 4.05 \times 10^{-25}\text{ MeV b}$ having been calculated recently in the EFT approach [19].

Since due to charge independence of the weak interaction strength the reaction of the anti–neutrino disintegration of the deuteron $\bar{\nu}_e + D \to e^+ + n + n$ is valued as a terrestrial equivalent of the solar proton burning, for the justification of the validity of our result Eq. (2.10) we suggest to calculate the cross section for the disintegration of the deuteron by reactor anti–neutrinos $\bar{\nu}_e + D \to e^+ + n + n$. This calculation is carried out in Sect. 4. However, the RFMD with the effective local four–nucleon interaction Eq. (1.1) allows to calculate the cross section for the reaction $\bar{\nu}_e + D \to e^+ + n + n$ for anti–neutrino energies close to threshold. As it is shown in Sect. 4 this cross section agrees well with the result obtained in the PMA. For the comparison of our result with the experimental data represented in the form of the cross section averaged over the anti–neutrino energy spectrum we need to extrapolate the cross section calculated near threshold for the anti–neutrino energies far from threshold. In order to formulate such an extrapolation procedure we turn to the consideration of the photomagnetic disintegration of the deuteron $\gamma + D \to n + p$. 


3 Photomagnetic disintegration of the deuteron

First, let us consider the cross section for the photomagnetic disintegration of the deuteron $\gamma + D \rightarrow n + p$ near threshold. The cross section can be easily obtained by using the results of Ref. [2,3]. It reads

$$\sigma^{\gamma D}(\omega) = \sigma_0 \left( \frac{\omega}{\varepsilon_D} \right) k r_D, \quad (3.1)$$

where $k = \sqrt{M_N (\omega - \varepsilon_D)}$ is the relative momentum of the np system, $\omega$ is the energy of the photon and $r_D = 1/\sqrt{\varepsilon_D M_N} = 4.315$ fm is the radius of the deuteron, and $\sigma_0$ is given by

$$\sigma_0 = (\mu_p - \mu_n)^2 \frac{25Q_D^2}{192\pi^2} G_{\pi NN}^2 \varepsilon_D^{3/2} M_N^{5/2} = 5.9 \times 10^{-27} \text{ cm}^2. \quad (3.2)$$

The cross section $\sigma^{\gamma D}(\omega)$, calculated in the PMA near threshold has the same form as Eq. (3.1) but with $\sigma_0$ given by [14]

$$\sigma_0 = \frac{2\pi\alpha}{3M_N^2} (\mu_p - \mu_n)^2 \left( 1 - a_{np} \sqrt{\varepsilon_D M_N} \right)^2 = 6.3 \times 10^{-27} \text{ cm}^2. \quad (3.3)$$

It is seen that $\sigma_0$ defined by Eq. (3.2) and Eq. (3.3) agree within an accuracy better than 10%. The analogous agreement can be drawn out from the comparison of the reaction rates for the neutron–proton radiative capture for thermal neutrons [3,14]:

$$v \sigma^{np}(k) = \begin{cases} \frac{25Q_D^2}{64\pi^2} G_{\pi NN}^2 \varepsilon_D^{3/2} M_N = 2.0 \times 10^{-30} \text{ cm}^2, \\ (\mu_p - \mu_n)^2 \frac{2\pi\alpha}{M_N^2} \left( 1 - a_{np} \sqrt{\varepsilon_D M_N} \right)^2 \left( \frac{\varepsilon_D}{M_N} \right)^{3/2} = 2.2 \times 10^{-30} \text{ cm}^2, \end{cases} \quad (3.4)$$

where $v$ and $k$ are a relative velocity and a relative 3–momentum of the np system. Thus, near threshold the RFMD supplemented by the effective local four–nucleon interaction Eq. (1.1) provides a dynamics of strong low–energy nuclear forces describing the cross sections for the neutron–proton radiative capture and the photomagnetic disintegration of the deuteron in good agreement with the PMA.

Now let us proceed to the formulation of the extrapolation procedure. For this aim we suggest to consider the cross section for the photomagnetic disintegration of the deuteron calculated in the PMA for the photon energies far from threshold [14]

$$\sigma^{\gamma D}(\omega) = \sigma_0 \left( \frac{\omega}{\varepsilon_D} \right) \frac{k r_D}{1 + r_D^2 k^2} \frac{1}{1 + a_{np}^2 k^2} = \sigma_0 \left( \frac{\omega}{\varepsilon_D} \right) k r_D F_{np}^{\gamma D}(k^2), \quad (3.5)$$

where the function $F_{np}^{\gamma D}(k^2)$ is defined as

$$F_{np}^{\gamma D}(k^2) = \frac{1}{1 + r_D^2 k^2} \frac{1}{1 + a_{np}^2 k^2}. \quad (3.6)$$

The cross section Eq. (3.5) differs from the cross section Eq. (3.1) calculated near threshold by the factor $F_{np}^{\gamma D}(k^2)$, which can be considered as a form factor taking into account a
spatial smearing of the deuteron through the factor $1/(1 + r_D^2 k^2)$ and the np system through the factor $1/(1 + a_{np}^2 k^2)$.

In order not to come into the contradiction with the PMA we suggest to extrapolate the cross section Eq. (3.1) for the photon energies far from threshold by means of the form factor $F_{np}(k^2)$. This extrapolation assumes that the cross section for the photomagnetic disintegration of the deuteron calculated in the RFMD has the form Eq. (3.3) with $\sigma_0$ defined by Eq. (3.2). Such an extrapolation applied to the cross section for the anti–neutrino disintegration of the deuteron should lead to the appearance of the form factor

$$F_{nn}(k^2) = \frac{1}{1 + r_D^2 k^2 \left(1 + \frac{a_{nn}^2 k^2}{1 + a_{nn}^2 k^2}\right)}$$

(3.7)

describing a spatial smearing of the deuteron through the factor $1/(1 + r_D^2 k^2)$ and the nn system through the factor $1/(1 + a_{nn}^2 k^2)$, where $a_{nn} = -17$ fm is the S–wave scattering length of the nn scattering in the $^1S_0$–state [10].

We should accentuate that we have discussed the photomagnetic disintegration of the deuteron in order to draw out the hint for the formulation of the extrapolation procedure for the cross section for the disintegration of the deuteron by anti–neutrinos. We are not aiming here to compute a total cross section for the disintegration of the deuteron by photons. In fact, it is well known [14] that a photomagnetic part predominates only at small relative momenta of the np pair at $k r_D \ll 1$ and the photoelectric part becomes important at $k r_D \geq 1$, i.e., $\omega \geq \varepsilon_D = 4.45$ MeV.

Also the computation of the reaction rate for the neutron–proton radiative capture given by Eq. (3.4) can be considered as a lowest approximation. Certainly, the cross section $\sigma_{np}(k) = 276$ mb computed for thermal neutrons at laboratory velocities $v/c = 7.34 \cdot 10^{-6}$ (the absolute value is $v = 2.2 \cdot 10^5$ cm/sec) [3] agrees reasonably well with the experimental data $\sigma_{exp}^{np}(k) = (334.2 \pm 0.5)$ mb [20]. However, the central theoretical value is 20% smaller than the central experimental value. Such a problem has been solved for the first time within the PMA by Riska and Brown [21] who showed that the discrepancy of order 10% between the theoretical value of the cross section $\sigma_{np}(k) = (302.5 \pm 0.40)$ mb [14] calculated in the PMA and the experimental value $\sigma_{exp}^{np}(k) = (334.2 \pm 0.5)$ mb can be explained by exchange–current contributions. In the EFT approach and Chiral perturbation theory the same result has been obtained by Park et al. [22]. The investigation of such fine effects of the neutron–proton radiative capture and incorporation of Chiral perturbation theory into the RFMD is in the programme of further applications of the RFMD to the processes of low–energy interactions of the deuteron.

4 Anti–neutrino disintegration of the deuteron

The calculation of the amplitude of the anti–neutrino disintegration of the deuteron is analogous to the calculation of the amplitude of the solar proton burning. The effective Lagrangian responsible for the transition $\bar{\nu}_e + D \rightarrow e^+ + n + n$ is given by Eq. (A.49)

$$\mathcal{L}_{\bar{\nu}_e D \rightarrow e^+ nn}(x) = g_A G_{\pi NN} \frac{G_V}{\sqrt{2}} \frac{3g_N}{8\pi^2} D_{\mu \nu}(x) \left[\bar{n}(x)\gamma^\mu\gamma^5 n^e(x)\right] \left[\bar{\psi}_{\nu_e}(x)\gamma^\nu(1 - \gamma^5)\psi_e(x)\right].$$
The amplitude of the $\bar{\nu}_e + D \rightarrow e^+ + n + n$ process reads

$$i\mathcal{M}(\bar{\nu}_e + D \rightarrow e^+ + n + n) = -g_A M_N G_{\pi NN} \frac{G_V 3g_N}{\sqrt{2} 2\pi^2} \times \epsilon_\mu(Q) [\bar{v}(k_{\bar{\nu}_e}) \gamma^\mu (1 - \gamma^5) v(k_{e^+})] [\bar{u}(p_1) \gamma^5 u^c(p_2)], \quad (4.1)$$

where $\bar{v}(k_{\bar{\nu}_e})$, $v(k_{e^+})$, $\epsilon_\mu(Q)$ and $u^c(p_2)$ are the Dirac bispinors of the anti–neutrino, positron and neutrons, $\epsilon_\mu(Q)$ is the 4–vector of the polarization of the deuteron. We have taken into account that $\bar{u}(p_2) \gamma^5 u^c(p_1) = -\bar{u}(p_1) \gamma^5 u^c(p_2)$. The amplitude Eq. (4.1) squared, averaged over polarizations of the deuteron and summed over polarizations of the final particles reads

$$|\mathcal{M}(\bar{\nu}_e + D \rightarrow e^+ + n + n)|^2 = g_A^2 M_N^6 G_{\pi NN}^2 \frac{144 G_N^2 Q_D}{\pi^2} \left( E_{e^+} E_{\bar{\nu}_e} - \frac{1}{3} \vec{k}_{e^+} \cdot \vec{k}_{\bar{\nu}_e} \right). \quad (4.2)$$

Due to charge independence of the weak interaction strength the matrix element Eq. (4.2) is related to the matrix element of the solar proton burning Eq. (2.5) by the relation

$$|\mathcal{M}(\bar{\nu}_e + D \rightarrow e^+ + n + n)|^2 = \frac{8}{3} \times \frac{1}{C^2(\eta)} \times |\mathcal{M}(p + p \rightarrow D + e^+ + \nu_e)|^2, \quad (4.3)$$

where $8/3$ is a combinatorial factor. This relation means that the dynamics of strong low–energy nuclear forces governing the processes of the anti–neutrino disintegration of the deuteron $\bar{\nu}_e + D \rightarrow n + n + e^+$ and the solar proton burning $p + p \rightarrow D + e^+ + \nu_e$ should have the same origin.

The expression Eq. (4.3) extrapolated for energies far from threshold according to the procedure suggested in Sect. 3 reads

$$|\mathcal{M}(\bar{\nu}_e + D \rightarrow e^+ + n + n)|^2 = g_A^2 M_N^6 G_{\pi NN}^2 \frac{144 G_N^2 Q_D}{\pi^2} \left( E_{e^+} E_{\bar{\nu}_e} - \frac{1}{3} \vec{k}_{e^+} \cdot \vec{k}_{\bar{\nu}_e} \right) F_{nn}^D(k^2). \quad (4.4)$$

The form factor $F_{nn}^D(k^2)$, describing a spatial smearing of the deuteron and the nn system, is given by Eq. (3.7)

$$F_{nn}^D(k^2) = \frac{1}{1 + r_D^2 k^2} \frac{1}{1 + a_{nn}^2 k^2},$$

where $k = \sqrt{M_N T_{nn}}$ is the relative momentum and $T_{nn}$ is the kinetic energy of the relative movement of the nn system, and $a_{nn} = -17$ fm [10] is the $S$–wave scattering length of the low–energy elastic nn scattering in the $^1S_0$–state. A much more complicated extrapolation of the form factors of the amplitude of the disintegration of the deuteron by anti–neutrinos has been suggested by Mintz [23].

The cross section for the process $\bar{\nu}_e + D \rightarrow e^+ + n + n$ is defined by

$$\sigma^{\bar{\nu}_e D}(E_{\bar{\nu}_e}) = \frac{1}{4E_D E_{\bar{\nu}_e}} \int |\mathcal{M}(\bar{\nu}_e + D \rightarrow e^+ + n + n)|^2 \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3k_{e^+}}{(2\pi)^3 2E_{e^+}}, \quad (4.5)$$
where $E_D$, $E_{\bar{\nu}_e}$, $E_1$, $E_2$ and $E_{e^+}$ are the energies of the deuteron, the anti–neutrino, the neutrons and the positron. The integration over the phase volume of the $(ne^+)$–state we perform in the non–relativistic limit and in the rest frame of the deuteron,

$$\frac{1}{2} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{d^3k_{e^+}}{(2\pi)^3} \frac{(2\pi)^4}{E_{e^+}} \delta^{(4)}(Q + k_{\bar{\nu}_e} - p_1 - p_2 - k_{e^+})$$

$$\times \left( E_{e^+}E_{\bar{\nu}_e} - \frac{1}{3} \vec{k}_{e^+} \cdot \vec{k}_{\bar{\nu}_e} \right) \frac{F_{nn}(M_N T_{nn})}{1024 \pi^2}$$

$$= \frac{E_{\bar{\nu}_e} M_N^3 (E_{th})^{7/2}}{(2m_e)} \left( 2m_e \right)^{3/2} \frac{8}{\pi E_{th}^2} \int dT_{e^+} dT_{nn} \sqrt{T_{e^+} T_{nn}}$$

$$\times F_{nn}^0(M_N T_{nn}) \left( 1 + \frac{T_{e^+}}{m_e} \right) \sqrt{1 + \frac{T_{e^+}}{2m_e}} \delta (E_{\bar{\nu}_e} - E_{th} - T_{e^+} - T_{nn})$$

$$= \frac{E_{\bar{\nu}_e} M_N^3 (E_{th})^{7/2}}{(2m_e)} \left( 2m_e \right)^{3/2} \left( \frac{E_{\bar{\nu}_e}}{E_{th}} - 1 \right)^2 f \left( \frac{E_{\bar{\nu}_e}}{E_{th}} \right),$$

(4.6)

where $T_{e^+}$ and $m_e = 0.511 \text{ MeV}$ are the kinetic energy and the mass of the positron, $E_{th}$ is the anti–neutrino energy threshold of the reaction $\bar{\nu}_e + D \rightarrow e^+ + n + n$ and is given by $E_{th} = \varepsilon_D + m_e + (M_N - M_p) = (2.225 + 0.511 + 1.293) \text{ MeV} = 4.029 \text{ MeV}$. The function $f(y)$, where $y = E_{\bar{\nu}_e}/E_{th}$, is defined as

$$f(y) = \frac{8}{\pi} \int_0^1 dx \sqrt{x (1 - x) \frac{F_{nn}^0(M_N E_{th} (y - 1) x)}{T_{nn}}}$$

$$\times \left( 1 + \frac{E_{th}}{m_e} (y - 1)(1 - x) \right) \sqrt{1 + \frac{E_{th}}{2m_e} (y - 1)(1 - x)},$$

(4.7)

where we have changed the variable $T_{nn} = (E_{\bar{\nu}_e} - E_{th}) x$. The function $f(y)$ is normalized to unity at $y = 1$, i.e., at threshold $E_{\bar{\nu}_e} = E_{th}$. Thus, the cross section for the anti–neutrino disintegration of the deuteron reads

$$\sigma_{\bar{\nu}_e D}(E_{\bar{\nu}_e}) = \sigma_0 (y - 1)^2 f(y),$$

(4.8)

where $\sigma_0$ is defined by

$$\sigma_0 = Q_D G^2_{\pi NN} \frac{9g_{\pi NN}^2 G_{\nu N}^2 M_N^8}{512 \pi^4} \left( \frac{E_{th}}{M_N} \right)^{7/2} \left( 2m_e \right)^{3/2} = (4.53 \pm 0.86) \times 10^{-43} \text{ cm}^2.$$

(4.9)

Here ±0.86 describes the assumed theoretical uncertainty of the RFMD which is about 19%. The value $\sigma_0 = (4.53 \pm 0.86) \times 10^{-43} \text{ cm}^2$ agrees with the value $\sigma_0 = (4.68 \pm 1.14) \times 10^{-43} \text{ cm}^2$ obtained in the PMA [24,25] (see Fig. 7 of Ref. [4]).

The experimental data on the anti–neutrino disintegration of the deuteron are given in terms of the cross section averaged over the reactor anti–neutrino energy spectrum per anti–neutrino fission in the energy region of anti–neutrinos $E_{th} \leq E_{\bar{\nu}_e} \leq 10 \text{ MeV}$:

$$< \sigma_{\bar{\nu}_e D}(E_{\bar{\nu}_e}) >_{\text{exp}} = (1.5 \pm 0.4) \times 10^{-45} \text{ cm}^2/\bar{\nu}_e \text{ fission} [7],$$

$$< \sigma_{\bar{\nu}_e D}(E_{\bar{\nu}_e}) >_{\text{exp}} = (0.9 \pm 0.4) \times 10^{-45} \text{ cm}^2/\bar{\nu}_e \text{ fission} [8]$$

and

$$< \sigma_{\bar{\nu}_e D}(E_{\bar{\nu}_e}) >_{\text{exp}} = (1.84 \pm 0.04) \times 10^{-45} \text{ cm}^2/\bar{\nu}_e \text{ fission} [9].$$

\(^1\)This is the improved estimate of the theoretical uncertainty. The former was 30% [3].
The cross section \(<σ_{\bar{\nu}e}^D(E_{\bar{\nu}_e})>\), calculated in the RFMD, extrapolated and averaged over the reactor anti–neutrino Avignone–Greenwood energy spectrum [5,6] in the energy region \(E_{th} \leq E_{\bar{\nu}_e} \leq 10\) MeV, is given by
\[
<σ_{\bar{\nu}e}^D(E_{\bar{\nu}_e})> = \frac{a}{N_{\bar{\nu}_e}} \int dy e^{-by} \sigma_0 (y - 1)^2 f(y) = (2.10 \pm 0.40) \times 10^{-45} \text{cm}^2/\bar{\nu}_e \text{fission},
\]
where \(a = 17.8 E_{th} = 71.72\), \(b = 1.01 E_{th} = 4.07\), and \(N_{\bar{\nu}_e} = 6\) is the number of anti–neutrinos per fission [5,6]. The theoretical value Eq. (4.10) agrees well with the experimental values given by Reines et al. [7] and Russian groups [9], while the agreement with the value given by Reines et al. [8] is only qualitative.

Thus, the calculation of the cross section for the anti–neutrino disintegration of the deuteron in agreement with the PMA and the experimental data confirms our result obtained for the astrophysical factor \(S_{pp}(0) = 4.02 \times 10^{-25} \text{MeV b}\) for the solar proton burning.

5 Conclusion

We have shown that the RFMD supplemented by the local four–nucleon interaction Eq. (1.1) describes the processes of the solar proton burning \(p + p \rightarrow D + e^+ + \nu_e\), the neutron–proton radiative capture \(n + p \rightarrow D + \gamma\), the disintegration of the deuteron by photons \(\gamma + D \rightarrow n + p\) and anti–neutrinos \(\bar{\nu}_e + D \rightarrow e^+ + n + n\) in good agreement with the PMA. The astrophysical factor \(S_{pp}(0) = 4.02 \times 10^{-25} \text{MeV b}\) for the solar proton burning, the reaction rates for the neutron–proton radiative capture and the cross sections for the disintegration of the deuteron by photons and anti–neutrinos calculated near thresholds of the reactions agree with the results obtained in the PMA within an accuracy better than 10%.

In order to compare our result for the cross section for the disintegration of deuteron by anti–neutrinos with experimental data we suggested the procedure of the extrapolation of the cross section calculated near threshold to the energy region far from threshold. Of course, such an extrapolation is not unique. Therefore, we have suggested to formulate the extrapolation procedure fitting the cross section for the photomagnetic disintegration of the deuteron calculated in the PMA. This extrapolation assumes the multiplication of the cross section, calculated in the RFMD near threshold, by the form factor
\[
F_{NN}^{D}(k^2) = \frac{1}{1 + \rho_D^2 k^2} \frac{1}{1 + a_{NN}^2 k^2}
\]
describing a spatial smearing of the deuteron by a factor \(1/(1 + \rho_D^2 k^2)\) and the NN system by a factor \(1/(1 + a_{NN}^2 k^2)\), where \(k\) is a relative momentum of the NN system. A much more complicated extrapolation of the amplitude of the process \(\bar{\nu}_e + D \rightarrow e^+ + n + n\) has been suggested by Mintz [23].

The extrapolated cross section for the disintegration of the deuteron by antineutrinos averaged over the reactor anti–neutrino energy spectrum \(<σ_{\bar{\nu}e}^D(E_{\bar{\nu}_e})>= (2.10 \pm 0.40) \times 10^{-45} \text{cm}^2/\bar{\nu}_e \text{fission}\) agrees well with the experimental data [7,9].
The cross section for the disintegration of the deuteron by anti-neutrinos $\bar{\nu}_e + D \rightarrow e^+ + n + n$ in dependence on the anti-neutrino energy has been calculated recently in the PMA in Ref. [26]. Since near threshold our cross section agrees well with the PMA result, for the verification of the extrapolation procedure we can, say, compare the cross section at $E_{\bar{\nu}_e} = 10$ MeV:

$$\sigma^{\bar{\nu}_e D}(E_{\bar{\nu}_e})|_{E_{\bar{\nu}_e} = 10 \text{ MeV}} = 1.02 \times 10^{-42} \text{ cm}^2, \text{ RFMD},$$

$$\sigma^{\bar{\nu}_e D}(E_{\bar{\nu}_e})|_{E_{\bar{\nu}_e} = 10 \text{ MeV}} = 1.13 \times 10^{-42} \text{ cm}^2, \text{ PMA}. \quad (5.2)$$

The visible agreement with the PMA result can serve too as a confirmation of a validity of our extrapolation procedure.

The RFMD describing a relative movement of the nucleons inside the deuteron in terms of one-nucleon loop exchanges suggests a dynamics of strong low-energy nuclear forces completely different to the PMA and the EFT approach. The neutron–proton–deuteron vertices are point–like and defined by a phenomenological local conserving nucleon current $J^\mu(x) = -ig_V [\bar{n}(x)\gamma^\mu n^c(x) - \bar{p}(x)\gamma^\mu p^c(x)]$, i.e., $\partial_\mu J^\mu(x) = 0$, accounting for spinorial and isotopical properties of the deuteron, and $g_V$ is a dimensionless phenomenological coupling constant. The deuteron is represented by a local field operator $D_\mu(x)$ (or $D_\mu^\dagger(x)$), the action of which on a vacuum state annihilates (or creates) the deuteron. The low-energy parameters of the deuteron such as the binding energy $\varepsilon_D$, the electric quadrupole $Q_D$ and the anomalous magnetic dipole $\kappa_D$ moments are induced by vacuum fluctuations of the neutron and the proton fields in a quantum field theory way. The description of strong low-energy nuclear forces in terms of one-nucleon loop exchanges provides opportunity to convey in nuclear physics a huge experience of fermion anomalies [16,17,27] which had been stored in particle physics from the paper by Adler [16] concerning the derivation of the anomalous contribution to the axial Ward identity. In the area of low-energy interactions of low-lying mesons such an experience has been focused mainly upon the derivation of anomalous contributions [28–30] to Effective Chiral Lagrangians [31].

The main problem which we encounter for the practical realization of the derivation of effective Lagrangians of low-energy interactions of the deuteron through one-nucleon loop exchanges lies in the necessity to satisfy requirement of locality of these interactions related to the condition of microscopic causality in a quantum field theory approach [32]. Since in the RFMD one-nucleon loop diagrams are defined by the point–like vertices and the Green functions of free virtual nucleons with constant masses, there is only a naive way to satisfy requirement of locality of effective interactions through the formal application of the long–wavelength approximation to the computation of one–nucleon loop diagrams. This approximation implies the expansion of one–nucleon loop diagrams in powers of external momenta by keeping only the leading terms of the expansion. Of course, the application of such an approximation to the computation of one–nucleon loop diagrams, when on–mass shell the energy of the deuteron exceeds twice the masses of virtual nucleons, can seem rather unjustified.

However, in this connection we would like to recall that the analogous problem encounters itself for the derivation of Effective Chiral Lagrangians [28,31] within effective quark models motivated by QCD like the extended Nambu–Jona–Lasino (ENJL) model with chiral $U(3) \times U(3)$ symmetry [33–36]. Indeed, all phenomenological low–energy interactions predicted by Effective Chiral Lagrangians [28,31] for the nonet of low–lying
vector mesons ($\rho(770)$, $\omega(780)$ and so on) can be derived within the ENJL model by calculating one–constituent quark loop diagrams at leading order in the long–wavelength approximation. As has turned out the long–wavelength approximation works very good in spite of the fact that the constituent quark loop diagrams are defined by point–like vertices of quark–meson interactions and the Green functions of the free constituent quarks with constant masses $M_q \sim 330 \text{ MeV}$, and the masses of vector mesons exceed twice the constituent quark mass. A formal justification of the validity of this approximation can be given by attracting the Vector Dominance (VD) hypothesis [31,37] due to which the effective vertices of low–energy interactions of low–lying vector mesons should be smooth functions of squared 4–momenta of interacting mesons varying from on–mass shell to zero values. This allows to calculate the vertices of low–energy interactions of vector mesons keeping them off–mass shell around zero values of their squared momenta [37], and then having had kept the leading terms of the long–wavelength expansion to continue the resultant expression on–mass shell. Such a procedure describes perfectly well all phenomenological vertices of low–energy interactions of low–lying vector mesons predicted by Effective Chiral Lagrangians [31,34–37].

One cannot say exactly, whether we really have in the RFMD some kind of the VD hypothesis, i.e., smooth dependence of effective low–energy interactions of the deuteron coupled to other particles on squared 4–momenta of interacting external particles including the deuteron. However, the application of the long–wavelength approximation to the computation of one–nucleon loop diagrams leads eventually to effective local Lagrangians describing reasonably well a dynamics of strong low–energy nuclear interactions. The static parameters of the deuteron and amplitudes of strong low–energy interactions of the deuteron coupled to nucleons and other particles can be described in the RFMD in complete agreement with the philosophy and technique of the derivation of Effective Chiral Lagrangians within effective quark models motivated by QCD.

The agreement between the reaction rates for the neutron–proton radiative capture, which is the M1 transition, calculated in the RFMD and the PMA is not surprising. Indeed, it is known from particle physics that the radiative decays of pseudoscalar and vector mesons like $\pi^0 \rightarrow \gamma \gamma$, $\omega \rightarrow \pi^0 \gamma$ and so on, caused by the M1 transitions, can be computed both in the non–relativistic quark model [38] , which is some kind of the PMA, and in the Effective Chiral Lagrangian approach. In the non–relativistic quark model the matrix elements of these decays are given in terms of magnetic moments of constituent quarks proportional to $1/M_q$, whereas in the Effective Chiral Lagrangian approach they are defined by the axial anomaly and proportional to $1/F_\pi$, the inverse power of the PCAC constant $F_\pi = 92.4 \text{ MeV}$ [16,27,39]. Equating the matrix elements of these decays calculated in the non–relativistic quark model and in the Effective Chiral Lagrangian approach one can express a constituent quark mass in terms of the PCAC constant $F_\pi$ [39]. The estimated value of the constituent quark mass $M_q \simeq 400 \text{ MeV}$ is comparable with the values $M_q = 330 \div 380 \text{ MeV}$ accepted in the literature [38]. This testifies that both the non–relativistic quark model and the Effective Chiral Lagrangian approach describe equally well the dynamics of strong low–energy interactions of low–lying mesons even if for the decays caused by the M1 transitions. Referring to this example the agreement between the reaction rates for the neutron–proton radiative capture calculated in the RFMD and in the PMA, respectively, is understandable. The computation of the astrophysical factor for the solar proton burning and the disintegration of the deuteron
by anti–neutrinos and photons in agreement with the PMA has only confirmed a validity of dynamics of strong low–energy nuclear interactions suggested by the RFMD.

The only problem which is left to discuss concerns the Coulomb repulsion between protons in the process of the solar proton burning. As has been shown by Kamionkowski and Bahcall [18] the Coulomb repulsion plays an important role for calculation of the astrophysical factor $S_{pp}(0) = 3.89 \times 10^{-25}$ MeV b. In our case the Coulomb repulsion is taken into account in the form of the Gamow penetration factor $C(\eta)$ and, apart from weak interactions, only strong low–energy nuclear forces are responsible for the value of the astrophysical factor $S_{pp}(0) = 4.02 \times 10^{-25}$ MeV b. This discrepancy with the PMA, which can be attributed to the peculiarity of the model with a local four–nucleon interaction like Eq. (1.1) and a description of strong low–energy nuclear interactions through one–nucleon loop exchanges, we are planning to resolve in our further development of the RFMD.

Finally, we have also shown that in the RFMD with the effective local four–nucleon interaction Eq. (1.1) one can describe low–energy elastic NN scattering in terms of the $S$–wave scattering length $a_{NN}$ and the effective range $r_{NN}$ in spirit of the EFT approach [11–13] and in complete agreement with low–energy nuclear phenomenology.

Appendix A. Computation of the amplitude of the solar proton burning

In order to acquaint readers with the machinery of the RFMD we give below the detailed derivation of the amplitude of the solar proton burning $p + p \rightarrow D + e^+ + \nu_e$.

The process $p + p \rightarrow D + e^+ + \nu_e$ runs through the intermediate W–boson exchange, i.e., $p + p \rightarrow D + W^+ \rightarrow D + e^+ + \nu_e$. The RFMD defines the transition in terms of the following effective interactions [1,2] (see Eq. (1.1)):

$$L_{npD}(x) = -i g_V [\bar{p}(x)\gamma^\mu n(x) - \bar{n}(x)\gamma^\mu p(x)] D^\mu_p(x),$$
$$L_{npW}(x) = -g_W^2 \sqrt{2} \cos \vartheta_C \bar{n}(x)\gamma^\nu(1 - g_A\gamma^5)p(x) W^\nu_\nu(x).$$

(A.1)

For convenience, in the effective local four–nucleon Lagrangian Eq. (1.1) we have introduced the interaction over a radius–vector $\vec{\rho}$ of a relative movement of the protons with a $\delta$–function $\delta^{(3)}(\vec{\rho})$.

Then, the transition $W^+ \rightarrow e^+ + \nu_e$ is defined by the Lagrangian

$$L_{\nu_e e^+ W}(x) = -g_W \sqrt{2} \left[\bar{\nu}_e(x)\gamma^\nu(1 - \gamma^5)\nu_e(x)\right] W^{\nu+}_\nu(x).$$

(A.2)
The electroweak coupling constant $g_W$ is connected with the Fermi weak constant $G_F$ and the mass of the $W$-boson $M_W$ through the relation

$$\frac{g_W^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}. \quad (A.3)$$

In order not to deal with the intermediate coupling constant $g_W$ it is convenient to apply to the computation of the matrix element of the transition $p + p \rightarrow D + W^+$ the interaction

$$\mathcal{L}_{npW}(x) = [\bar{n}(x)\gamma^\nu (1 - g_A\gamma^5)p(x)] W_\nu^-(x), \quad (A.4)$$

and for the description of the subsequent weak transition $W^+ \rightarrow e^+ + \nu_e$ to replace the operator of the $W$-boson field by the operator of the leptonic weak current

$$W_\nu^-(x) \rightarrow -\frac{G_V}{\sqrt{2}} [\bar{\psi}_\nu(x)\gamma_\nu(1 - \gamma^5)\psi_e(x)]. \quad (A.5)$$

The $S$ matrix describing the transitions like $p + p \rightarrow D + W^+$ is defined

$$S = Te^i \int d^4x [\mathcal{L}_{npD}(x) + \mathcal{L}_{npW}(x) + \mathcal{L}_{pp\rightarrow pp}^{\text{eff}}(x) + \ldots], \quad (A.6)$$

where $T$ is the time–ordering operator and the ellipses denote the contribution of interactions irrelevant to the computation of the transition $p + p \rightarrow D + W^+$.

For the computation of the transition $p + p \rightarrow D + W^+$ we have to consider the third order term of the $S$ matrix which reads

$$S^{(3)} = \frac{i^3}{3!} \int d^4x_1d^4x_2d^4x_3 T([\mathcal{L}_{npD}(x_1) + \mathcal{L}_{npW}(x_1) + \mathcal{L}_{pp\rightarrow pp}^{\text{eff}}(x_1) + \ldots]
\times [\mathcal{L}_{npD}(x_2) + \mathcal{L}_{npW}(x_2) + \mathcal{L}_{pp\rightarrow pp}^{\text{eff}}(x_2) + \ldots]
\times [\mathcal{L}_{npD}(x_3) + \mathcal{L}_{npW}(x_3) + \mathcal{L}_{pp\rightarrow pp}^{\text{eff}}(x_3) + \ldots]) =
- i \int d^4x_1d^4x_2d^4x_3 T(\mathcal{L}_{pp\rightarrow pp}^{\text{eff}}(x_1)\mathcal{L}_{npD}(x_2)\mathcal{L}_{npW}(x_3)) + \ldots \quad (A.7)$$

The ellipses denote the terms which do not contribute to the matrix element of the transition $p + p \rightarrow D + W^+$ and the interaction $\mathcal{L}_{npW}(x)$ is given by Eq. (A.4). The $S$ matrix element $S_{pp\rightarrow DW^+}^{(3)}$ contributing to the transition $p + p \rightarrow D + W^+$ we determine as follows

$$S_{pp\rightarrow DW^+}^{(3)} = -i \int d^4x_1d^4x_2d^4x_3 T(\mathcal{L}_{pp\rightarrow pp}^{\text{eff}}(x_1)\mathcal{L}_{npD}(x_2)\mathcal{L}_{npW}(x_3)). \quad (A.8)$$

For the derivation of the effective Lagrangian $\mathcal{L}_{pp\rightarrow DW^+}(x)$ containing only the fields of the initial and the final particles we should make all necessary contractions of the operators of the proton and the neutron fields. These contractions we denote by the brackets as

$$<S_{pp\rightarrow DW^+}^{(3)}>= -i \int d^4x_1d^4x_2d^4x_3 <T(\mathcal{L}_{pp\rightarrow pp}^{\text{eff}}(x_1)\mathcal{L}_{npD}(x_2)\mathcal{L}_{npW}(x_3))>. \quad (A.9)$$

Now the effective Lagrangian $\mathcal{L}_{pp\rightarrow DW^+}(x)$ related to the $S$ matrix element $<S_{pp\rightarrow DW^+}^{(3)}>$ can be defined as

$$<S_{pp\rightarrow DW^+}^{(3)}>= i \int d^4x \mathcal{L}_{pp\rightarrow DW^+}(x) =$$
simplified as follows

\[ W - \text{boson coupled with the axial nucleon current.} \]

In terms of the operators of the interacting fields the effective Lagrangian \( \mathcal{L}_{pp \to DW}(x) \) reads

\[
\int d^4x \mathcal{L}_{pp \to DW}(x) = -\int d^4x_1d^4x_2d^4x_3 < T(\mathcal{L}_{eff}^{pp}(x_1)\mathcal{L}_{npD}(x_2)\mathcal{L}_{npW}(x_3)) > .
\]

(A.10)

Since \( p + p \to D + W^+ \) is the Gamow–Teller transition, we have taken into account the \( W \)--boson coupled with the axial nucleon current.

Due to the relation \( \bar{n}(x_3)\gamma^n p(x_2) = -\bar{p}(x_2)\gamma^n n(x_2) \) the r.h.s. of Eq. (A.11) can be simplified as follows

\[
\int d^4x \mathcal{L}_{pp \to DW}(x) = -\int d^4x_1d^4x_2d^4x_3 < T(\mathcal{L}_{eff}^{pp}(x_1)\mathcal{L}_{npD}(x_2)\mathcal{L}_{npW}(x_3)) > 
\]

\[
= G_{\pi NN} \times (-ig_\gamma) \times g_A \int d^4x_1d^4x_2d^4x_3 \int d^3 \rho \delta^{(3)}(\vec{\rho}) 
\times T(\bar{p}(t_1, \vec{x}_1 - \frac{1}{2} \vec{\rho}) \gamma_\alpha \gamma^5 p(t_1, \vec{x}_1 - \frac{1}{2} \vec{\rho})) D^\dagger_\mu(x_2) W^{-}_\nu(x_3)) 
\times < 0|T(\bar{p}(t_1, \vec{x}_1 - \frac{1}{2} \vec{\rho}) \gamma_\alpha \gamma^5 p(t_1, \vec{x}_1 - \frac{1}{2} \vec{\rho}))|\bar{p}(x_2)\gamma^n n(x_2)|\bar{n}(x_3)\gamma^n p(x_3))|0 > .
\]

(A.11)

Making the necessary contractions we arrive at the expression

\[
\int d^4x \mathcal{L}_{pp \to DW}(x) = -\int d^4x_1d^4x_2d^4x_3 < T(\mathcal{L}_{eff}^{pp}(x_1)\mathcal{L}_{npD}(x_2)\mathcal{L}_{npW}(x_3)) > 
\]

\[
= 2 \times G_{\pi NN} \times (-ig_\gamma) \times g_A \int d^4x_1d^4x_2d^4x_3 \int d^3 \rho \delta^{(3)}(\vec{\rho})
\]
where the combinatorial factor 2 takes into account the fact that the protons are identical particles in the nucleon loop. Let us corroborate the appearance of the factor 2 by a direct calculation:

\[
\int d^4x \mathcal{L}_{pp \to DW^+}(x) = -\int d^4x_1 d^4x_2 d^4x_3 < T(\mathcal{L}_{\text{eff}}^{pp \to pp}(x_1) \mathcal{L}_{\text{npD}}(x_2) \mathcal{L}_{\text{npW}}(x_3)) >
\]

\[
= G_{\pi NN} \times (-ig_N) \times g_A \int d^4x_1 d^4x_2 d^4x_3 \int d^3\rho \delta^{(3)}(\vec{\rho})
\]

\[
\times \left[ \bar{\psi}(t_1, \vec{x}_1 + \frac{1}{2} \vec{\rho}) \gamma_\alpha \gamma_5 \gamma_\beta \gamma_5 p(t_1, \vec{x}_1 - \frac{1}{2} \vec{\rho}) \right] D^{\zeta}_{\mu}(x_2) W^{-}_\nu(x_3)
\]

\[
\times (-1) \text{tr}\{ \gamma_\alpha \gamma_5 (-i) S_F(t_1 - t_2, \vec{x}_1 - \vec{x}_2 - \frac{1}{2} \vec{\rho}) \gamma_\mu (-i) S_F(x_2 - x_3) \gamma_\nu \gamma_5
\]

\[
\times (-i) S_F(t_3 - t_1, \vec{x}_3 - \vec{x}_1 - \frac{1}{2} \vec{\rho}) \}
\]

\[
+ 2 \times G_{\pi NN} \times (-ig_N) \times g_A \int d^4x_1 d^4x_2 d^4x_3 \int d^3\rho \delta^{(3)}(\vec{\rho})
\]

\[
\times \left[ \bar{\psi}(t_1, \vec{x}_1 + \frac{1}{2} \vec{\rho}) \gamma_\alpha \gamma_5 \gamma_\beta \gamma_5 p(t_1, \vec{x}_1 - \frac{1}{2} \vec{\rho}) \right] D^{\zeta}_{\mu}(x_2) W^{-}_\nu(x_3)
\]

\[
\times (-1) \text{tr}\{ \gamma_\alpha \gamma_5 (-i) S_F(t_1 - t_2, \vec{x}_1 - \vec{x}_2 - \frac{1}{2} \vec{\rho}) \gamma_\mu (-i) S_F(x_2 - x_3) \gamma_\nu \gamma_5
\]

\[
\times (-i) S_F(t_3 - t_1, \vec{x}_3 - \vec{x}_1 - \frac{1}{2} \vec{\rho}) \}
\]

\[\text{(A.13)}\]
Here we have used the relation $C = -C^T$. Then, by applying the relation $C^T(\gamma^a\gamma^5)^T C = \gamma^a\gamma^5$ we obtain the following expression

$$\int d^4x \mathcal{L}_{pp \rightarrow DW^+}(x) = - \int d^4x_1d^4x_2d^4x_3 < T(\mathcal{L}_{eff}^{pp \rightarrow pp}(x_1)\mathcal{L}_{npD}(x_2)\mathcal{L}_{npW}(x_3)) >$$

$$= G_{\pi NN} \times (-ig_\pi) \times g_A \int d^4x_1d^4x_2d^4x_3 \int d^3\rho \delta^{(3)}(\rho)$$

$$\times \left\{ (\gamma^a\gamma^5(-i)S_F^c(t_1 - t_2, \vec{x}_1 - \vec{x}_2 + \vec{1}_2\rho)\gamma^a\gamma^5(-i)S_F(x_2 - x_3)\gamma^\nu\gamma^5 \right. \right.$$
\[ \times (-i)S_F(t_3 - t_1, \vec{x}_3 - \vec{x}_1 - \frac{1}{2}\vec{\rho}) \}
+ \text{T}([\vec{p}(t_1, \vec{x}_1 + \frac{1}{2}\vec{\rho}) \gamma_\alpha \gamma^5 p(t_1, \vec{x}_1 - 1/2\vec{\rho})] D_\mu^\dagger(x_2) W^-_\nu(x_3)) \times (-1) \text{tr}\{\gamma^\alpha\gamma^5(-i)S_F^c(t_1 - t_2, \vec{x}_1 - \vec{x}_2 + 1/2\vec{\rho})C\gamma^\mu(-i)S_F(x_2 - x_3)\gamma^\nu\gamma^5 \times (-i)S_F(t_3 - t_1, \vec{x}_3 + \vec{x}_1 + 1/2\vec{\rho})\} + (\gamma_\alpha\gamma^5 \otimes \gamma^\alpha\gamma^5 \rightarrow \gamma^5 \otimes \gamma^5). \quad (A.15) \]

Using the property of the operators
\[ [\vec{p}(t_1, \vec{x}_1 + \frac{1}{2}\vec{\rho}) \Gamma p(t_1, \vec{x}_1 - \frac{1}{2}\vec{\rho})] = [\vec{p}(t_1, \vec{x}_1 - \frac{1}{2}\vec{\rho}) \Gamma p(t_1, \vec{x}_1 + \frac{1}{2}\vec{\rho})] \quad (A.16) \]

for \( \Gamma = \gamma^\alpha\gamma^5 \) and \( \gamma^5 \), we get
\[ \int d^4x \mathcal{L}_{pp \rightarrow DW^+}(x) = - \int d^4x_1 d^4x_2 d^4x_3 <\text{T}(\mathcal{L}^{pp \rightarrow pp}_{\text{eff}}(x_1) \mathcal{L}_{npD}(x_2) \mathcal{L}_{npW}(x_3)) > \]
\[ = G_{\pi NN} \times (-ig_\nu) \times g_A \int d^4x_1 d^4x_2 d^4x_3 \int d^3\rho \delta^{(3)}(\vec{\rho}) \times \left\{ \text{T}([\vec{p}(t_1, \vec{x}_1 + \frac{1}{2}\vec{\rho}) \gamma_\alpha \gamma^5 p(t_1, \vec{x}_1 - \frac{1}{2}\vec{\rho})] D_\mu^\dagger(x_2) W^-_\nu(x_3)) \times (-1) \text{tr}\{\gamma^\alpha\gamma^5S_F^c(t_1 - t_2, \vec{x}_1 - \vec{x}_2 - 1/2\vec{\rho})C\gamma^\mu(-i)S_F(x_2 - x_3)\gamma^\nu\gamma^5 \times (-i)S_F(t_3 - t_1, \vec{x}_3 - \vec{x}_1 - 1/2\vec{\rho})\} \right. \]
\[ + \text{T}([\vec{p}(t_1, \vec{x}_1 - \frac{1}{2}\vec{\rho}) \gamma_\alpha \gamma^5 p(t_1, \vec{x}_1 + \frac{1}{2}\vec{\rho})] D_\mu^\dagger(x_2) W^-_\nu(x_3)) \times (-1) \text{tr}\{(-i)\gamma^\alpha\gamma^5S_F^c(t_1 - t_2, \vec{x}_1 - \vec{x}_2 + 1/2\vec{\rho})C\gamma^\mu(-i)S_F(x_2 - x_3)\gamma^\nu\gamma^5 \times (-i)S_F(t_3 - t_1, \vec{x}_3 + \vec{x}_1 + 1/2\vec{\rho})\} \right\} + (\gamma_\alpha\gamma^5 \otimes \gamma^\alpha\gamma^5 \rightarrow \gamma^5 \otimes \gamma^5). \quad (A.17) \]

Making a change of variables \( \vec{\rho} \rightarrow -\vec{\rho} \) in the last term, we arrive at the expression Eq. (A.13).

Then, \( S_F^c(x) \) and \( S_F(x) \) are the Green functions of the free anti–nucleon and nucleon field, respectively:
\[ S_F^c(x) = CS_F^c(-x)C^T = S_F(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik \cdot x}}{M_N - k}. \quad (A.18) \]

Passing to the momentum representation of the Green functions we get
\[ \int d^4x \mathcal{L}_{pp \rightarrow DW^+}(x) = \]
\[ = -ig_A G_{\pi NN} g_\nu \frac{g_\nu}{8\pi^2} \int d^4x_1 \int \frac{d^4x_2 d^4k_2}{(2\pi)^4} \frac{d^4x_3 d^4k_3}{(2\pi)^4} e^{-ik_2 \cdot (x_2 - x_1)} e^{-ik_3 \cdot (x_3 - x_1)} \]
the energies of the protons, the deuteron, positron and neutrino, and the final

\[ \text{where } \vec{q} = \vec{k}_1 + (\vec{k}_3 - \vec{k}_2)/2. \]

In order to obtain the effective Lagrangian describing the process \( p + p \rightarrow D + e^+ + \nu_e \) we have to replace the operator of the W–boson field by the operator of the leptonic weak current Eq. (A.5):

\[
\int d^4x \mathcal{L}_{pp \rightarrow De^+\nu_e}(x) =
\]

\[
= i g_A G_{\pi NN} \frac{G_V g_N}{\sqrt{2} 8\pi^2} \int d^4x \int \frac{d^4x_2 d^4k_2 d^4x_3 d^4k_3}{(2\pi)^4 (2\pi)^4} e^{-ik_2 \cdot (x_2 - x_1)} e^{-ik_3 \cdot (x_3 - x_1)}
\]

\[
\times \int d^3\rho \delta^{(3)}(\vec{\rho}) T([\bar{p}\vec{r}(t_1, \vec{x}_1 + 1)\vec{\rho} \alpha \gamma^5 p(t_1, \vec{x}_1 - 1)\vec{\rho}]) D^{\dagger}_\mu(x_2) [\bar{\nu}_\nu(x_3) \gamma_\nu(1 - \gamma^5) \nu_\nu(x_3)]
\]

\[
\times \int \frac{d^4k_1}{\pi^2 i} e^{i\vec{q} \cdot \vec{\rho}} \text{tr} \left\{ \frac{\gamma^5}{M_\nu - \vec{k}_1 + \vec{k}_2} \frac{\gamma^\mu}{M_\nu - \vec{k}_1} \frac{\gamma^\nu}{M_\nu - \vec{k}_1 - \vec{k}_3} \frac{1}{M_\nu - \vec{k}_1 - \vec{k}_3} \right\}.
\]

\[
(\text{A.19)}
\]

Now we are able to determine the matrix element of the process \( p + p \rightarrow D + e^+ + \nu_e \) as

\[
\int d^4x < D(k_\nu) e^+(k_\nu) \nu_e(k_\nu) | \mathcal{L}_{pp \rightarrow De^+\nu_e}(x) | p(p_1) p(p_2) > =
\]

\[
(2\pi)^4 \delta^{(4)}(k_D + k_\nu - p_1 - p_2) \frac{\mathcal{M}(p + p \rightarrow D + e^+ + \nu_e)}{\sqrt{2 E_1 V 2 E_2 V 2 E_D V 2 E_e+ V 2 E_{\nu_e} V}}.
\]

\[
(\text{A.21)}
\]

where \( k_\nu = k_{e+} + k_{\nu_e} \) is the 4–momentum of the leptonic pair, \( E_i (i = 1, 2, D, e, \nu_e) \) are the energies of the protons, the deuteron, positron and neutrino, \( V \) is the normalization volume.

Taking the r.h.s. of Eq. (A.20) between the wave functions of the initial \( | p(p_1) p(p_2) > \) and the final \( < D(k_\nu) e^+(k_\nu) \nu_e(k_\nu) | \) states we get

\[
(2\pi)^4 \delta^{(4)}(k_D + k_\nu - p_1 - p_2) \frac{\mathcal{M}(p + p \rightarrow D + e^+ + \nu_e)}{\sqrt{2 E_1 V 2 E_2 V 2 E_D V 2 E_e+ V 2 E_{\nu_e} V}} =
\]

\[
\text{(A.21)}
\]
where $t$ is symmetric under permutations of the protons: 
\[
\bar{\psi}_{\nu c}\gamma_\nu (1 - \gamma^5)\psi_{\nu c}(x_3)] |p(p_1)p(p_2) >
\]
\[
\times \left\{ \gamma^5 \frac{1}{M_N - k_1 + \vec{k}_2} \frac{1}{M_N - \vec{k}_2} \frac{1}{M_N - \vec{k}_1} \frac{1}{M_N - \vec{k}_1 - \vec{k}_3} \right\}.
\] (A.22)

Between the initial $|p(p_1)p(p_2) >$ and the final $< D(k_D)e^+(k_{e+})\nu_c(k_{\nu c})|$ states the matrix elements are defined
\[
< D(k_D)e^+(k_{e+})\nu_c(k_{\nu c})| T([\bar{p}\gamma^5 p(t_1, \vec{x}_1 + \frac{1}{2} \vec{\rho}) \gamma_\nu p(t_1, \vec{x}_1 - \frac{1}{2} \vec{\rho})]|D^\dagger_\mu(x_2)
\]
\[
\times \left[ \bar{\psi}_{\nu c}(x_3)\gamma_\nu (1 - \gamma^5)\psi_{\nu c}(x_3) \right] |p(p_1)p(p_2) > = [\bar{u}^c(p_2)\gamma_\alpha \gamma^5 u(p_1)] [\bar{u}(k_{\nu c})\gamma_\nu (1 - \gamma^5)v(k_{e+})]
\]
\[
\times e^*_\mu(k_D) \sqrt{2} \psi_{pp}(\vec{\rho}) \left| e^{-i(p_1 + p_2) \cdot \vec{x}_1} e^{ik_D \cdot \vec{x}_2} e^{ik_\ell \cdot \vec{x}_3} \right| \sqrt{2E_1V 2E_2V 2E_3V 2E_{\nu c}V}.
\] (A.23)

where $\psi_{pp}(\vec{\rho})$ is the wave function of the relative movement of the free protons in the $^1S_0$-state normalized to unit density [40]:
\[
\psi_{pp}(\vec{\rho}) = \frac{\sin k\rho}{k\rho},
\] (A.24)

where $k$ is a 3-momentum of a relative movement of the protons. Since the spatial part of the wave function of the protons is symmetric under permutations of the protons, so the spinorial part should be antisymmetric. In our approach the spinorial part of the wave function of the protons is described by $[\bar{u}^c(p_2)\gamma_\alpha \gamma^5 u(p_1)]$ and $[\bar{u}^c(p_2)\gamma^5 u(p_1)]$, antisymmetric under permutations of the protons: $[\bar{u}^c(p_2)\gamma_\alpha \gamma^5 u(p_1)] = -[\bar{u}^c(p_1)\gamma_\alpha \gamma^5 u(p_2)]$ and $[\bar{u}^c(p_2)\gamma^5 u(p_1)] = -[\bar{u}^c(p_1)\gamma^5 u(p_2)]$. 

21
Now let us discuss in details the computation of the matrix elements:

\[
< 0 | \tilde{p}^c(t_1, \bar{x}_1 + \frac{1}{2} \bar{\rho}) \Gamma p(t_1, \bar{x}_1 - \frac{1}{2} \bar{\rho}) | p(p_1) p(p_2) >, \tag{A.25}
\]

where we have denoted \( \Gamma = \gamma_5 \) or \( \gamma^5 \).

In the quantum field theory approach the wave function \( | p(p_1) p(p_2) > \) should be described in terms of the operators of the creation of the protons \( a^\dagger(p_1, \sigma_1) \) and \( a^\dagger(p_2, \sigma_2) \), where \( \vec{p}_i \) and \( \sigma_i \) (\( i = 1, 2 \)) are the 3–momenta and the polarizations of the protons. Therefore, \( | p(p_1) p(p_2) > \) reads

\[
| p(p_1) p(p_2) > = \frac{1}{\sqrt{2}} a^\dagger(\bar{p}_1, \sigma_1) a^\dagger(\bar{p}_2, \sigma_2) | 0 >. \tag{A.26}
\]

The wave function Eq. (A.26) is taken in the standard form [41]. It is antisymmetric under permutations of the protons due to the anti–commutation relation

\[
a^\dagger(\bar{p}_1, \sigma_1) a^\dagger(\bar{p}_2, \sigma_2) = -a^\dagger(\bar{p}_2, \sigma_2) a^\dagger(\bar{p}_1, \sigma_1)
\]

and normalized to unity. The factor \( 1/\sqrt{2} \) takes into account that the protons are correlated in the initial state.

The operators of the proton fields \( \tilde{p}^c(t_1, \bar{x}_1 + \frac{1}{2} \bar{\rho}) \) and \( p(t_1, \bar{x}_1 - \frac{1}{2} \bar{\rho}) \) we represent in terms of the plane–wave expansions

\[
\tilde{p}^c(t_1, \bar{x}_1 + \frac{1}{2} \bar{\rho}) = \sum_{\bar{q}_1, \alpha_1} \frac{1}{\sqrt{2E_{\bar{q}_1}V}} \left[ a(\bar{q}_1, \alpha_1) \tilde{c}(q_1) e^{-iE_{\bar{q}_1}t_1 + i\bar{q}_1 \cdot (x_1 + \bar{\rho}/2)}
\right.
\]

\[
+ b^\dagger(\bar{q}_1, \alpha_1) \tilde{c}(q_1) e^{iE_{\bar{q}_1}t_1 - i\bar{q}_1 \cdot (x_1 + \bar{\rho}/2)} \right],
\]

\[
p(t_1, \bar{x}_1 - \frac{1}{2} \bar{\rho}) = \sum_{\bar{q}_2, \alpha_2} \frac{1}{\sqrt{2E_{\bar{q}_2}V}} \left[ a(\bar{q}_2, \alpha_2) u(q_2) e^{-iE_{\bar{q}_2}t_1 + i\bar{q}_2 \cdot (x_1 - \bar{\rho}/2)}
\right.
\]

\[
+ b^\dagger(\bar{q}_2, \alpha_2) v(q_2) e^{iE_{\bar{q}_2}t_1 - i\bar{q}_2 \cdot (x_1 - \bar{\rho}/2)} \right], \tag{A.27}
\]

where \( a(q_1, \alpha_i) \) (\( i = 1, 2 \)) and \( b^\dagger(q_1, \alpha_i) \) (\( i = 1, 2 \)) are the operators of the annihilation and the creation of protons and anti-protons, respectively. The computation of the matrix element Eq. (A.25) runs the following way. Holding only the terms containing the operators of the annihilation of the protons we get

\[
< 0 | \tilde{p}^c(t_1, \bar{x}_1 + \frac{1}{2} \bar{\rho}) \Gamma p(t_1, \bar{x}_1 - \frac{1}{2} \bar{\rho}) | p(p_1) p(p_2) > =
\]

\[
= \sum_{\bar{q}_1, \alpha_1} \sum_{\bar{q}_2, \alpha_2} \frac{1}{\sqrt{2E_{\bar{q}_1}V}} \frac{1}{\sqrt{2E_{\bar{q}_2}V}} e^{-i(q_1 + q_2) \cdot x_1 + i(\bar{q}_1 - \bar{q}_2) \cdot \bar{\rho}/2}
\]

\[
\times [\tilde{c}(q_1) \Gamma u(q_2)] \frac{1}{\sqrt{2}} < 0 | a(\bar{q}_1, \alpha_1) a(\bar{q}_2, \alpha_2) a^\dagger(\bar{p}_1, \sigma_1) a^\dagger(\bar{p}_2, \sigma_2) | 0 >. \tag{A.28}
\]
The vacuum expectation value $< 0| a(\vec{q}_1, \alpha_1) a(\vec{q}_1, \alpha_1) a^\dagger(\vec{p}_1, \sigma_1) a^\dagger(\vec{p}_2, \sigma_2)|0 >$ reads:

$$< 0| a(\vec{q}_1, \alpha_1) a(\vec{q}_1, \alpha_1) a^\dagger(\vec{p}_1, \sigma_1) a^\dagger(\vec{p}_2, \sigma_2)|0 > = -\delta_{\vec{q}_1 \vec{p}_1} \delta_{\alpha_1 \sigma_1} \delta_{\vec{q}_2 \vec{p}_2} \delta_{\alpha_2 \sigma_2} + \delta_{\vec{q}_1 \vec{p}_1} \delta_{\alpha_2 \sigma_1} \delta_{\vec{q}_2 \vec{p}_2} \delta_{\alpha_1 \sigma_2},$$

(A.20)

where we have used the anti-commutation relations

$$a(\vec{q}, \alpha) a^\dagger(\vec{p}, \sigma) + a^\dagger(\vec{p}, \sigma) a(\vec{q}, \alpha) = \delta_{\vec{q} \vec{p}} \delta_{\alpha \sigma}$$

(A.30)

and the properties of the operators of the creation and the annihilation: $< 0| a^\dagger(\vec{p}, \sigma) = 0$ and $a(\vec{q}, \alpha)|0 > = 0$.

Substituting Eq. (A.29) in Eq. (A.28) and summing up the momenta and the spinorial indices we arrive at the expression

$$< 0| \vec{p}(t_1, \vec{x}_1 + \frac{1}{2} \vec{\rho}) \Gamma p(t_1, \vec{x}_1 - \frac{1}{2} \vec{\rho})| p(p_1)p(p_2) > = -\frac{e^{-i(p_1 + p_2) \cdot x_1}}{\sqrt{2E_1V 2E_2V}} \times \frac{1}{\sqrt{2}} \left( [\vec{u}^c(p_1) \Gamma u(p_2)] e^{i(\vec{p}_1 - \vec{p}_2) \cdot \vec{\rho}/2} - [\vec{u}^c(p_2) \Gamma u(p_1)] e^{-i(\vec{p}_1 - \vec{p}_2) \cdot \vec{\rho}/2} \right) =$$

$$= \frac{e^{-i(p_1 + p_2) \cdot x_1}}{\sqrt{2E_1V 2E_2V}} \sqrt{2} [\vec{u}^c(p_2) \Gamma u(p_1)] \frac{1}{2} \Bigg( e^{i(\vec{p}_1 - \vec{p}_2) \cdot \vec{\rho}/2} + e^{-i(\vec{p}_1 - \vec{p}_2) \cdot \vec{\rho}/2} \Bigg),$$

(A.31)

where the relation $[\vec{u}^c(p_1) \Gamma u(p_2)] = -[\vec{u}^c(p_2) \Gamma u(p_1)]$ has been used. The sum of the exponentials

$$\frac{1}{2} \left( e^{i(\vec{p}_1 - \vec{p}_2) \cdot \vec{\rho}/2} + e^{-i(\vec{p}_1 - \vec{p}_2) \cdot \vec{\rho}/2} \right)$$

(A.32)

describes the spatial part of the wave function of the relative movement of the free protons. This wave function is symmetric under permutations of the protons and normalized to unit density [40]. Since the protons should be in the $^1S_0$–state, expanding exponentials into spherical harmonics and keeping only the S–wave contribution we obtain [40]:

$$\frac{1}{2} \left( e^{i\vec{k} \cdot \vec{\rho}} + e^{-i\vec{k} \cdot \vec{\rho}} \right) = \frac{\sin k\rho}{k\rho} + \ldots,$$

(A.33)

where $\vec{k} = (\vec{p}_1 - \vec{p}_2)/2$ is the relative momentum of the protons. This completes the explanation of the derivation of the matrix elements in Eq. (A.23).

Substituting the matrix elements Eq. (A.23) in the r.h.s. of Eq. (A.22) we obtain the matrix element of the solar proton burning in the following form

$$(2\pi)^4δ^4(k_D + k_\ell - p_1 - p_2) i\mathcal{M}(p + p \rightarrow D + e^+ + \nu_e) =$$

$$= -\sqrt{2} C(\eta) g_S G_{\pi NN} \frac{G_V}{\sqrt{2}} g_\nu \frac{g_\nu}{8\pi^2} ([\vec{u}^c(p_2)\gamma_5 u(p_1)] [\bar{u}(k_{\nu_e})\gamma_\nu (1 - \gamma^5) v(k_{e^+})] e_\nu^*(k_D)$$

$$\times \int d^4x_1 \int \frac{d^4x_2 d^4k_2 d^4x_3 d^4k_3}{(2\pi)^4} e^{i(k_2 + k_3 - p_1 - p_2) \cdot x_1} e^{i(k_D - k_2) \cdot x_2} e^{i(k_\ell - k_3) \cdot x_3}$$

23
\[
\times \int d^3 \rho \delta^{(3)}(\vec{\rho}) \frac{\sin k \rho}{k \rho} \int \frac{d^4k_1}{\pi^2 i} e^{i \vec{q} \cdot \vec{p}_1} \{ \gamma^\mu \gamma^5 \frac{1}{M_N - k_1 + k_2} \gamma^\nu \gamma^5 \frac{1}{M_N - k_1 - k_3} \} \\
- \sqrt{2} C(\eta) g_A G_{\pi NN} \frac{G_V g_N}{\sqrt{2} 8 \pi^2 } [\bar{u}(p_2) \gamma^5 u(p_1)] [\bar{u}(k_{ve}) \gamma_{\mu}(1 - \gamma^5) v(k_{\nu})] e^*_\nu(k_D) \\
\times \int \frac{d^4x_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{d^4k_3}{(2\pi)^4} e^{i (k_2 + k_3 - p_1 - p_2) \cdot x_1} e^{i (k_D - k_2) \cdot x_2} e^{i (k_{\ell} - k_3) \cdot x_3} \\
\times \int d^3 \rho \delta^{(3)}(\vec{\rho}) \frac{\sin k \rho}{k \rho} \int \frac{d^4k_1}{\pi^2 i} e^{i \vec{q} \cdot \vec{p}_1} \{ \gamma^\mu \gamma^5 \frac{1}{M_N - k_1 + k_2} \gamma^\nu \gamma^5 \frac{1}{M_N - k_1 - k_3} \}, \tag{A.35}
\]

where we have appended the Gamow penetration factor \(C(\eta)\) taking into account the Coulomb repulsion between the protons [2].

Integrating over \(x_1, x_2, x_3\), \(k_2\) and \(k_3\) we obtain in the r.h.s. of Eq. (A.35) the \(\delta\)-function describing the 4-momentum conservation. Then, the matrix element of the \(p + p \rightarrow D + e^+ + \nu_e\) process becomes equal

\[
i M(p + p \rightarrow D + e^+ + \nu_e) = -\sqrt{2} C(\eta) g_A G_{\pi NN} \frac{G_V g_N}{\sqrt{2} 8 \pi^2 } \]

\[
\times [\bar{u}(p_2) \gamma_{\alpha} \gamma^5 u(p_1)] [\bar{u}(k_{ve}) \gamma_{\nu}(1 - \gamma^5) v(k_{\nu})] e^*_\nu(k_D) \\
\times \int \frac{d^4k_1}{\pi^2 i} \text{tr} \{ \gamma^\alpha \gamma^5 \frac{1}{M_N - k_1 + k_2} \gamma^\mu \frac{1}{M_N - k_1} \gamma^\nu \gamma^5 \frac{1}{M_N - k_1 - k_3} \} \]

\[
- \sqrt{2} C(\eta) g_A G_{\pi NN} \frac{G_V g_N}{\sqrt{2} 8 \pi^2 } [\bar{u}(p_2) \gamma^5 u(p_1)] [\bar{u}(k_{ve}) \gamma_{\nu}(1 - \gamma^5) v(k_{\nu})] e^*_\nu(k_D) \\
\times \int \frac{d^4k_1}{\pi^2 i} \text{tr} \{ \gamma^5 \frac{1}{M_N - k_1 + k_D} \gamma^\mu \frac{1}{M_N - k_1} \gamma^\nu \gamma^5 \frac{1}{M_N - k_1 - k_{\ell}} \}, \tag{A.36}
\]

where we have integrated over a relative radius-vector \(\vec{\rho}\) too. It is convenient to represent the matrix element Eq. (A.35) in terms of the structure functions \(J^{\alpha \nu}(k_D, k_{\ell}; Q)\) and \(J^{\mu \nu}(k_D, k_{\ell}; Q)\):

\[
i M(p + p \rightarrow D + e^+ + \nu_e) = \\
- C(\eta) G_V g_A G_{\pi NN} \frac{g_N}{8 \pi^2 } [\bar{u}(p_2) \gamma_{\alpha} \gamma^5 u(p_1)] [\bar{u}(k_{ve}) \gamma_{\nu}(1 - \gamma^5) v(k_{\nu})] e^*_\nu(k_D) J^{\alpha \nu}(k_D, k_{\ell}; Q) \\
- C(\eta) G_V g_A G_{\pi NN} \frac{g_N}{8 \pi^2 } [\bar{u}(p_2) \gamma^5 u(p_1)] [\bar{u}(k_{ve}) \gamma_{\nu}(1 - \gamma^5) v(k_{\nu})] e^*_\nu(k_D) J^{\mu \nu}(k_D, k_{\ell}; Q), \tag{A.37}
\]

where the structure functions \(J^{\alpha \nu}(k_D, k_{\ell}; Q)\) and \(J^{\mu \nu}(k_D, k_{\ell}; Q)\) are defined as [2]

\[
J^{\alpha \nu}(k_D, k_{\ell}; Q) = \\
= \int \frac{d^4k}{\pi^2 i} \text{tr} \{ \gamma^\alpha \gamma^5 \frac{1}{M_N - k - Q + k_D} \gamma^\mu \frac{1}{M_N - k - Q} \gamma^\nu \gamma^5 \frac{1}{M_N - k - Q - k_{\ell}} \}, \\
J^{\mu \nu}(k_D, k_{\ell}; Q) = \\
= \int \frac{d^4k}{\pi^2 i} \text{tr} \{ \gamma^5 \frac{1}{M_N - k - Q + k_D} \gamma^\mu \frac{1}{M_N - k - Q} \gamma^\nu \gamma^5 \frac{1}{M_N - k - Q - k_{\ell}} \}, \tag{A.38}
\]
We have introduced a 4–vector \( Q = a k_D + b k_\ell \) caused by an arbitrary shift of a virtual momentum with arbitrary parameters \( a \) and \( b \).

Thus, the problem of the computation of the matrix element of the \( p + p \rightarrow D + e^+ + \nu_e \) process reduces to the problem of the computation of the structure functions Eq. (A.38). Since the energy of the leptonic pair is small compared with the nucleon mass, we can set in the integrand \( k_\ell^\mu = 0 \) [2]. This gives

\[
\mathcal{J}^{\alpha\mu}(k_D, k_\ell; Q) = \int \frac{d^4k}{\pi^2i} \text{tr} \left\{ \gamma^\alpha \gamma^5 \frac{1}{M_N - \hat{k} - \hat{Q} + \hat{k}_D} \gamma^\mu \frac{1}{M_N - \hat{k} - \hat{Q}} \gamma^5 \frac{1}{M_N - \hat{k} - \hat{Q}} \right\},
\]

\[
\mathcal{J}^{\mu\nu}(k_D, k_\ell; Q) = \int \frac{d^4k}{\pi^2i} \text{tr} \left\{ \gamma^5 \frac{1}{M_N - \hat{k} - \hat{Q} + \hat{k}_D} \gamma^\mu \frac{1}{M_N - \hat{k} - \hat{Q}} \gamma^\nu \frac{1}{M_N - \hat{k} - \hat{Q}} \right\}, \tag{A.39}
\]

For the calculation of the momentum integrals we would follow the philosophy of the derivation of Effective Chiral Lagrangians within effective quark models motivated by QCD [33–36], in particularly, Chiral perturbation theory at the quark level (CHPT)\(_q\) [35] formulated on the basis of the ENJL model induced by the effective low–energy QCD with linearly rising confinement potential [42]. In (CHPT)\(_q\) all low–energy vertices of meson interactions are determined by constituent quark loop diagrams with point–like quark–meson vertices and the Green functions of the free constituent quarks with constant masses \( M_q = 330 \text{ MeV} \) [35]. To the computation of the momentum integrals one applies a generalized hypothesis of Vector Dominance [31,37] postulating a smooth dependence of low–energy vertices of hadron interactions on squared 4–momenta of interacting mesons. Due to this hypothesis one can hold all external particles off–mass shell at squared 4–momenta \( p^2 \) much less than \( M_q^2 \); i.e., \( M_q^2 \gg p^2 \). Then, after the computation of the momentum integrals at leading order in long–wavelength expansion, i.e., in powers of external momenta, the resultant expression should be continued on–mass shell of interacting particles. Within the framework of this procedure one can restore completely all variety of phenomenological vertices of low–energy meson interactions predicted by Effective Chiral Lagrangians [28,31,34–36]. It is important to emphasize that this procedure works good not only for light mesons like \( \pi \)–meson, which mass is less than the mass of constituent quarks, but for vector mesons like \( \rho(770), \omega(780) \) and so on, which masses are twice larger than the constituent quark mass. Since the former resembles the RFMD, where the mass of the deuteron amounts to twice the mass of virtual nucleons, we expect that the long–wavelength approximation should work in the RFMD as well as in the effective quark models with chiral \( U(3) \times U(3) \) symmetry applied to the derivation of Effective Chiral Lagrangians.

Thus, for the computation of the momentum integrals we assume that the deuteron is off–mass shell and \( M_N \gg \sqrt{k_D^2} \). Then, we expand the integrand of the structure functions Eq. (A.39) in powers of \( k_D \) keeping only the leading contributions. The result of the computation we continue on–mass shell of the deuteron \( k_D^2 \rightarrow M_B^2 \) [2].

Keeping the leading terms of the expansion in powers of \( k_D \) we get [2]:

\[
\mathcal{J}^{\alpha\mu}(k_D, k_\ell; Q) = 3 \left( k_D^\alpha g^{\nu\mu} - k_D^\nu g^{\alpha\mu} \right) + \frac{1}{9} (1 + 2a) \left( k_D^\alpha g^{\nu\mu} + k_D^\nu g^{\alpha\mu} \right),
\]
\[ J^{\mu\nu}(k_D, k^\ell; Q) = \gamma^{\mu\nu} 4 M_N J_2(M_N), \] (A.40)

where due to the relation \( k_D \cdot e^*_D(k_D) = 0 \) the terms proportional to \( k^\ell_D \) have been dropped out. Then, \( J_2(M_N) \) is a logarithmically divergent integral defined in the RFMD in terms of the cut–off \( \Lambda_D = 68.452 \text{ MeV} \) such as \( \Lambda_D \ll M_N \) [1,2]:

\[
J_2(M_N) = \int \frac{d^4 k}{\pi^2} \frac{1}{(M_N^2 - k^2)^2} = 2 \int_0^{\Lambda_D} \frac{d|k| |k|^2}{(M_N^2 + |k|^2)^{1/2}} = \frac{2}{3} \left( \frac{\Lambda_D}{M_N} \right)^3 \ll 1. \quad (A.41)
\]

The cut–off \( \Lambda_D \) restricts 3–momenta of the virtual nucleon fluctuations forming the physical deuteron [1,2]. Due to the uncertainty relation \( \Delta r \Lambda_D \geq 1/2 \) the spatial region of virtual nucleon fluctuations forming the physical deuteron is defined by \( \Delta r \geq 1.44 \text{ fm} \). This agrees with the range of nuclear forces (NF) caused by the one–pion exchange with the mass \( M_\pi = 135 \text{ MeV} \): \( r_{NF} = 1/M_\pi = 1.46 \text{ fm} \) [14].

After the continuation of the results of the calculation of the structure functions on–mass shell of the deuteron the contribution of \( J^{\mu\nu}(k_D, k^\ell; Q) \) can be neglected relative to the contribution of \( J^{\alpha\nu}(k_D, k^\ell; Q) \). The contribution of the structure function \( J^{\alpha\nu}(k_D, k^\ell; Q) \) does not depend on the mass of virtual nucleons and according to Ref. [17] can be valued as the anomaly of the AAV one–nucleon triangle diagram. The ambiguity of the calculation of \( J^{\alpha\mu}(k_D, k^\ell; Q) \) caused by the dependence on an arbitrary shift of a virtual momentum can be fixed by requirement of gauge invariance of the amplitude of the process \( p + p \rightarrow D + e^+ + \nu_e \) under gauge transformations of the deuteron field \( e^{\mu}(k_D) \rightarrow e^{\mu}(k_D) + \lambda k_D^\mu \), where \( \lambda \) is an arbitrary parameter. This gives \( a = -1/2 \) and the structure function in the form [2]:

\[
J^{\alpha\mu}(k_D, k^\ell; Q) = 3 (k_D^\alpha g^{\nu\mu} - k_D^\nu g^{\alpha\mu}). \quad (A.42)
\]

The attraction of requirement of gauge invariance in order to remove ambiguities of the structure function \( J^{\alpha\nu}(k_D, k^\ell; Q) \) and to fix the contribution of the anomaly is in complete agreement with the derivation of the Adler–Bell–Jackiw axial anomaly performed in terms of one–fermion loop diagrams [16].

Since we strive to draw a similarity between the RFMD and effective quark models motivated by QCD applied to the derivation of Effective Chiral Lagrangians, requirement of gauge invariance under gauge transformations of the deuteron field

\[
D_\mu(x) \rightarrow D_\mu(x) + \partial_\mu f(x), \quad (A.43)
\]

where \( f(x) \) is a gauge function, can be justified by referring to a dynamics of vector meson fields in these effective quark models [34–36]. The effective Lagrangian of the physical deuteron field \( D_\mu(x) \), which we apply to the calculation of one–nucleon loop diagrams describing effective low–energy interactions of the deuteron coupled to nucleons and other particles, reads [1,2]:

\[
\mathcal{L}(x) = -\frac{1}{2} D^\dagger_{\mu\nu}(x) D^{\mu\nu}(x) + M_D^2 D^\dagger_{\mu}(x) D^{\mu}(x) - ig_{\nu}[\bar{p}(x)\gamma^\mu n^c(x) - \bar{n}(x)\gamma^\mu p^c(x)] D^{\mu}(x)
\]

\[
-ig_{\nu}[\bar{p}^c(x)\gamma^\mu n(x) - \bar{n}^c(x)\gamma^\mu p(x)] D^\dagger_{\mu}(x) + \bar{p}(x)(i\gamma^\mu \partial_\mu - M_N)p(x) + \bar{n}(x)(i\gamma^\mu \partial_\mu - M_N)n(x), \quad (A.44)
\]
where \( D_{\mu\nu}(x) = \partial_\mu D_\nu(x) - \partial_\nu D_\mu(x) \). Since the deuteron field \( D_\mu(x) \) couples to the conserved nucleon current \( J^\mu(x) = -ig_\nu [\bar{p}_\mu(x)\gamma^\mu n(x) - \bar{n}(x)\gamma^\mu p(x)] \), i.e., \( \partial_\mu J^\mu(x) = 0 \), invariance of the effective Lagrangian Eq. (A.44) under gauge transformations of the deuteron field Eq. (A.43) is violated only by the mass term. The same problem encounters itself for description of dynamics of vector meson fields \( (\rho(770), \omega(780) \) and so on) in effective quark models with chiral \( U(3) \times U(3) \) symmetry [34–36]. Since these mesons are massive, the kinetic Lagrangians of vector meson fields are not invariant under gauge transformations of these fields. Nevertheless, for the derivation of effective low–energy interactions of vector mesons coupled to other particles requirement of gauge invariance turns out to be very important. For example, by virtue of requirement of gauge invariance under gauge transformations of \( \rho(770) \) and \( \omega(780) \) meson fields one can fix unambiguously the coupling constant of the \( \omega p \pi \) interaction defined by the Adler–Bell–Jackiw axial anomaly [16] that plays an important role for the correct description of the \( \omega \rightarrow \pi^+ \pi^- \pi^0 \) decay and many other low–energy processes.

Substituting the structure function Eq. (A.42) in Eq. (A.37) we obtain the matrix element of the solar proton burning in the following relativistically invariant form

\[
iM(p + p \rightarrow D + e^+ + \nu_e) = -C(\eta) G_V g_A G_{\pi NN} \frac{3g_V}{8\pi^2} \times \left(k^\mu_0 g^\mu - k^\mu_0 g^\mu \right) [\bar{u}(p_2)\gamma_\alpha \gamma_5 u(p_1)] [\bar{u}(k_{e+})] \gamma_\mu (1 - \gamma^5) v(k_{e+}) e^*_\mu(k_D). \tag{A.45}
\]

In the low–energy limit due to the low–energy reduction

\[
[\bar{u}(p_2)\gamma_\alpha \gamma_5 u(p_1)] \rightarrow -g_{\alpha\beta} [\bar{u}(p_2)\gamma_5 u(p_1)] \tag{A.46}
\]

the matrix element of the solar proton burning can be brought up to the form

\[
iM(p + p \rightarrow D + e^+ + \nu_e) = C(\eta) G_V g_A M_N G_{\pi NN} \frac{3g_V}{4\pi^2} \times \left[k^\mu_0 \gamma^\mu \right] [\bar{u}(k_{e+})] \gamma_\mu (1 - \gamma^5) v(k_{e+}) e^*_\mu(k_D), \tag{A.47}
\]

where we have set \( k^\mu_0 = M_D \simeq 2M_N \) valid on–mass shell of the deuteron. This completes the calculation of the matrix element of the solar proton burning.

Omitting the Gamow penetration factor \( C(\eta) \) the residual part of the matrix element of the solar proton burning Eq. (A.45) can be defined by the effective Lagrangian

\[
L_{pp\rightarrow De^+\nu_e}(x) = g_A G_{\pi NN} \frac{G_V 3g_V}{\sqrt{2} 8\pi^2} D^{i\mu}(x) [\bar{p}^i(x)\gamma^\mu \gamma^5 p(x)] [\bar{\psi}_{\nu_e}(x)\gamma^\nu (1 - \gamma^5) \psi_e(x)]. \tag{A.48}
\]

This Lagrangian is local in accordance with the condition of microscopic causality [32].

The calculation of the matrix element of the disintegration of the deuteron by anti–neutrinos \( \bar{\nu}_e + D \rightarrow e^+ + n + n \) can be carried out by an analogous way and defined by the same structure functions. The effective Lagrangian describing the matrix element of the transition \( \bar{\nu}_e + D \rightarrow e^+ + n + n \) can be written as follows

\[
L_{\bar{\nu}_e D \rightarrow e^+ nn}(x) = g_A G_{\pi NN} \frac{G_V 3g_V}{\sqrt{2} 8\pi^2} D_{\mu\nu}(x) [\bar{n}(x)\gamma^\mu \gamma^5 n^c(x)] [\bar{\psi}_{\nu_e}(x)\gamma^\nu (1 - \gamma^5) \psi_e(x)]. \tag{A.49}
\]

The effective Lagrangians Eq. (A.48) and Eq. (A.49) testify distinctly that the processes of the solar proton burning and the disintegration of the deuteron by anti–neutrinos are governed by the same dynamics of strong low–energy nuclear interactions in agreement with charge independence of the weak interaction strength.
Appendix B. Low–energy elastic NN scattering in the RFMD

In this Appendix we show how in the RFMD one can describe a phenomenological amplitude of elastic NN scattering by using the effective four–nucleon interaction Eq. (1.1). For simplicity we suggest to consider the elastic low–energy np scattering. In the RFMD the amplitude of the elastic low–energy np scattering can be written as follows

\[ \mathcal{M}(np \rightarrow np)(k) = -A(k) \frac{4\pi}{M_N} [\bar{u}(p'_2)\gamma^5 u(p'_1)] [\bar{u}(p_1)\gamma^5 u(p_2)], \]  

(B.1)

since at low energies the interaction \( \gamma^\mu \gamma^5 \otimes \gamma^\mu \gamma^5 \) reduces to \( \gamma^\mu \gamma^5 \otimes \gamma^5 \), then \( p_i \) and \( p'_i \) (i=1,2) are 4–momenta of the proton and the neutron in the initial and final states and \( k \) is a relative 3–momentum of the np system. The phenomenological amplitude of the low–energy elastic np scattering \( A(k)_{ph} \) reads [12,14]

\[ A(k)_{ph} = \frac{1}{-\frac{1}{a_{np}} + \frac{1}{2} r_{np} k^2 - i k}, \]  

(B.2)

where \( a_{np} \) and \( r_{np} = (2.75 \pm 0.05) \text{ fm} \) are the S–wave scattering length and the effective range of the np–scattering in the \(^1S_0\)–state [10]. At \( k \rightarrow 0 \) we get \( A(0)_{ph} = -a_{np} \) which gives the cross section equal \( \sigma(np \rightarrow np) = 4\pi a_{np}^2 \).

In the RFMD due to the low–energy reduction

\[ [\bar{n}(x)\gamma_\alpha \gamma^5 p^\alpha(x)] [\bar{p}^\alpha(x)\gamma^\alpha \gamma^5 n(x)] \rightarrow [\bar{n}(x)\gamma^5 p^\alpha(x)] [\bar{p}^\alpha(x)\gamma^5 n(x)] \]  

(B.3)

the np scattering runs through the one–nucleon loop exchange. Using the effective interaction Eq. (1.1) we can write down the effective Lagrangian for the low–energy elastic np scattering:

\[ \int d^4 x \mathcal{L}_{\text{eff}}^{np\rightarrow np}(x)_{\text{scattering}} = -\frac{G_{\pi NN}^2}{16\pi^2} \int d^4 x \int d^4 x_1 d^4 k_1 \frac{1}{(2\pi)^4} e^{-ik_1 \cdot (x - x_1)} \]

\[ \times \{ [\bar{n}(x)\gamma_\alpha \gamma^5 p^\alpha(x)][\bar{p}^\alpha(x_1)\gamma_\beta \gamma^5 n(x_1)] \mathcal{J}^{\alpha\beta}(k_1) + [\bar{n}(x)\gamma^5 p^\alpha(x)][\bar{p}^\alpha(x_1)\gamma^5 n(x_1)] \mathcal{J}(k_1) \]

\[ + [\bar{n}(x)\gamma_\alpha \gamma^5 p^\alpha(x)][\bar{p}^\alpha(x_1)\gamma_\beta \gamma^5 n(x_1)] \mathcal{J}(k_1) + [\bar{n}(x)\gamma^5 p^\alpha(x)][\bar{p}^\alpha(x_1)\gamma_\beta \gamma^5 n(x_1)] \mathcal{J}^{\alpha\beta}(k_1) \}, \]  

(B.4)

where \( \mathcal{J}^{\alpha\beta}(k_1), \mathcal{J}(k_1), \mathcal{J}(k_1) \) and \( \mathcal{J}^{\alpha}(k_1) \) are the structure functions defined by the momentum integrals:

\[ \mathcal{J}^{\alpha\beta}(k_1) = \int \frac{d^4 q}{\pi^2 i} \text{tr} \left\{ \frac{1}{M_N - \hat{q}} \gamma^\alpha \gamma^5 \frac{1}{M_N - \hat{q} - \hat{k}_1} \gamma^\beta \gamma^5 \right\}, \]

\[ \mathcal{J}(k_1) = \int \frac{d^4 q}{\pi^2 i} \text{tr} \left\{ \frac{1}{M_N - \hat{q}} \gamma^5 \frac{1}{M_N - \hat{q} - \hat{k}_1} \gamma^5 \right\}, \]

\[ \mathcal{J}(k_1) = \int \frac{d^4 q}{\pi^2 i} \text{tr} \left\{ \frac{1}{M_N - \hat{q}} \gamma^\alpha \gamma^5 \frac{1}{M_N - \hat{q} - \hat{k}_1} \gamma^5 \right\}. \]
\[ \mathcal{J}^\alpha(k_1) = \int \frac{d^4q}{\pi^2i} \text{tr} \left\{ \frac{1}{M_N - \hat{q} - k_1} \frac{1}{M_N - \hat{q} - \hat{P}} \gamma^\alpha \gamma^5 \right\}. \] (B.5)

The amplitude of the low–energy elastic np scattering defined by the effective Lagrangian Eq. (B.4) reads

\[ \mathcal{M}(np \to np) = -\frac{G_{\pi NN}^2}{16\pi^2} \times \{ [\bar{u}(p'_1)\gamma_\alpha \gamma^5 u^c(p'_1)] [\bar{u}(p_1)\gamma_\beta \gamma^5 u(p_2)] J^{\alpha\beta}(P) + [\bar{u}(p'_1)\gamma_\alpha \gamma^5 u^c(p'_1)] [\bar{u}(p_1)\gamma_\beta \gamma^5 u(p_2)] J^\alpha(P) \}
\]

\[ + [\bar{u}(p'_2)\gamma_\alpha \gamma^5 u^c(p'_2)] [\bar{u}(p_2)\gamma_\beta \gamma^5 u(p_1)] J^{\alpha\beta}(P) + [\bar{u}(p'_2)\gamma_\alpha \gamma^5 u^c(p'_2)] [\bar{u}^c(p_2)\gamma_\gamma \gamma^5 u(p_1)] J^\alpha(P), \] (B.6)

where \( P = p_1 + p_2 = p'_1 + p'_2 \) and in the center of mass frame \( P^\mu = (2\sqrt{k^2 + M_N^2}, 0) \).

Due to the low–energy reduction Eq. (B.3) the amplitude Eq. (B.6) reduces to the form

\[ \mathcal{M}(np \to np) = -\frac{G_{\pi NN}^2}{16\pi^2} \times \{ -J^{00}(P) + J(P) + J^0(P) - \bar{J}^0(P) \}, \] (B.7)

where the structure functions \( J^{00}(P), J(P), J^0(P) \) and \( \bar{J}^0(P) \) are given by Eq. (B.5) with the change \( k_1 \to P \).

The integrals over \( q \) are both quadratically and logarithmically divergent. In the RFMD they are regularized by a cut–off \( \Lambda_D \ll M_N \) [2]. Then, calculating the integrals over \( q \) we have to take into account that quadratically divergent integrals regularized by a cut–off are defined ambiguously with respect to the shift of virtual momenta. Indeed, it is well known [17,2] that

\[ \int \frac{d^4q}{\pi^2i} \text{tr} \left\{ \frac{1}{M_N - \hat{q} - \hat{Q}} \frac{1}{M_N - \hat{q} - \hat{P}} \gamma^\alpha \gamma^5 \right\} = \int \frac{d^4q}{\pi^2i} \text{tr} \left\{ \frac{1}{M_N - \hat{q}} \frac{1}{M_N - \hat{P}} \gamma^\alpha \gamma^5 \right\} + 2 \left[ Q^\alpha (Q + P)^\beta + Q^\beta (Q + P)^\alpha - Q \cdot (Q + P) g^{\alpha\beta} \right], \]

\[ \int \frac{d^4q}{\pi^2i} \text{tr} \left\{ \frac{1}{M_N - \hat{q} - \hat{Q}} \frac{1}{M_N - \hat{q} - \hat{P}} \gamma^5 \right\} = \int \frac{d^4q}{\pi^2i} \text{tr} \left\{ \frac{1}{M_N - \hat{q}} \frac{1}{M_N - \hat{P}} \gamma^5 \right\} - 2 Q \cdot (Q + P), \] (B.8)

where a 4–momentum \( Q \) defines an arbitrary shift of a virtual momentum \( q \to q + Q \). The most general form of \( Q \) reads : \( Q = \xi P + N \), where \( \xi \) is an arbitrary parameter and \( N \) is an arbitrary 4–momentum orthogonal to \( P \), i.e., \( P \cdot N = 0 \). As \( P \) is a time–like vector, \( P^2 > 0 \), so \( N \) is a space–like, \( N^2 < 0 \). In the center of mass frame we have \( N^\mu = (0, \vec{N}) \).

The result of the calculation of the structure function \( -J^{00}(P) + J(P) \) can be given in the following general form:

\[ \frac{G_{\pi NN}^2}{16\pi^2} \left[ - J^{00}(P) + J(P) \right] = \frac{4\pi}{M_N} (C_1 + C_2 k^2), \] (B.9)

where \( C_i \) \((i=1,2)\) are arbitrary constants containing all uncertainties induced by shifts of virtual momenta. One should take into account that virtual momenta in the momentum
integrals defining the structure functions $J^{\alpha\beta}(P)$ and $J(P)$ can be shifted independently. The structure functions $J^0(P)$ and $J^0(P)$ are logarithmically divergent and, therefore, do not depend on the shift of virtual momentum. The contribution of these structure functions can be absorbed by the constants $C_1$ and $C_2$:

$$\frac{G^2_{\pi NN}}{16\pi^2} \left[ -J^{00}(P) + J(P) + J^0(P) - J^0(P) \right] = \frac{4\pi}{M_N}(C_1 + C_2 k^2), \quad (B.10)$$

Inserting Eq. (B.10) in the r.h.s. of Eq. (B.7) we obtain the amplitude of the low–energy elastic np scattering in terms of the constants $C_1$ and $C_2$:

$$\mathcal{M}(np \rightarrow np) = -\frac{4\pi}{M_N}(C_1 + C_2 k^2) \left[ \bar{u}(p'_2)\gamma^5 u^c(p'_1) \right] \left[ \bar{u}^c(p_1)\gamma^5 u(p_2) \right]. \quad (B.11)$$

Neglecting the terms of order $O(k^4)$ as it is accepted for the description of low–energy elastic np scattering [12], we arrive at the expression

$$\mathcal{M}(np \rightarrow np) = -\frac{4\pi}{M_N} \frac{1}{C_1 - \frac{C_2}{C_1^2} k^2} \left[ \bar{u}(p'_2)\gamma^5 u^c(p'_1) \right] \left[ \bar{u}^c(p_1)\gamma^5 u(p_2) \right]. \quad (B.12)$$

Matching Eq. (B.12) with Eq. (B.1) we obtain $A(k)_{\text{RFMD}}$ in the form:

$$A(k)_{\text{RFMD}} = \frac{1}{\frac{1}{C_1} - \frac{C_2}{C_1^2} k^2}. \quad (B.13)$$

Following [12] we can set

$$C_1 = -a_{np}, \quad C_2 = \frac{1}{2} r_{np} a_{np}^2. \quad (B.14)$$

This yields the amplitude of the low–energy elastic np scattering in the form

$$A(k)_{\text{RFMD}} = \frac{1}{-\frac{1}{a_{np}} + \frac{1}{2} r_{np} k^2}. \quad (B.15)$$

Up to the imaginary part $-ik$ which can be appended to the r.h.s. of Eq. (B.15) due to unitarity the amplitude $A(k)_{\text{RFMD}}$ coincides with the phenomenological amplitude Eq. (B.2). Thus, we have shown that in the RFMD with the local four–nucleon interaction Eq. (1.1) one can describe, in spirit of the EFT approach [11–13] and in agreement with low–energy nuclear phenomenology, the amplitude of the low–energy elastic np scattering in terms of the S–wave scattering length $a_{np}$ and the effective range $r_{np}$. This confutes the statement by Bahcall and Kamionkowski [43] that the effective local four–nucleon interaction Eq. (1.1) leads in the RFMD to the zero effective range for elastic NN scattering, i.e., $r_{NN} = 0$. This statement [43] is also confuted by our results on the astrophysical factor for the solar proton burning, the reaction rate for the neutron–proton radiative capture and the cross sections for the low–energy disintegration of the deuteron by photons and anti–neutrinos obtained in good agreement with the PMA.
References

[1] A. N. Ivanov, N. I. Troitskaya, M. Faber and H. Oberhummer, Phys. Lett. B361 (1995) 74.

[2] A. N. Ivanov, N. I. Troitskaya, M. Faber and H. Oberhummer, Nucl. Phys. A617 (1997) 414 and references therein.

[3] A. N. Ivanov, N. I. Troitskaya, M. Faber and H. Oberhummer, Nucl. Phys. A625 (1997) 896 (Erratum).

[4] T. L. Jenkins, F. E. Kinard and F. Reines, Phys. Rev. 185 (1969) 1599.

[5] F. T. Avignone and Z. D. Greenwood, Phys. Rev. D17 (1978) 154.

[6] H. C. Lee, Nucl. Phys. A294 (1978) 473; Phys. Lett. B87 (1979) 18.

[7] E. Paierb, H. S. Gurr, J. Lathrop, F. Reines and H. W. Sobel, Phys. Rev. Lett. 43 (1979) 96.

[8] F. Reines, H. W. Sobel and E. Paierb, Phys. Rev. Lett. 45 (1980) 1307.

[9] G. S. Vidiakin et al., JETP lett. 51 (1990) 245; A. G. Vershinsky et al., JETP lett. 51 (1990) 82.

[10] M. M. Nagels et al., Nucl. Phys. B147 (1979) 253.

[11] S. Weinberg, Phys. Lett. B251 (1990) 288; Nucl. Phys. B363 (1991) 3; Phys. Lett. B295 (1992) 114.

[12] D. R. Kaplan, M. J. Savage and M. B. Wise, Nucl. Phys. B478 (1996) 629 and references therein; S. R. Beane, T. D. Cohen and D. R. Phillips, Nucl. Phys. A632 (1998) 445.

[13] T.–S. Park, K. Kubodera, D.–P. Min and M. Rho, The Power of Effective Field Theories in Nuclei: The Deuteron, NN Scattering and Electroweak Processes, Preprint KIAS–P98015, nucl–th/9807054, July 1998.

[14] W. F. Hornyak, in NUCLEAR STRUCTURE, Academic Press, New York, 1975 and references therein.

[15] R. B. Wiringa, V. G. J. Stoks and R. Schiavilla, Phys. Rev. C51 (1995) 38 and references therein.

[16] S. L. Adler, Phys. Rev. 177 (1969) 2726; J. S. Bell and R. Jackiw, Nuovo Cim. A60 (1969) 47; R. Jackiw, in LECTURES ON CURRENT ALGEBRA AND ITS APPLICATIONS, Princeton University Press, Princeton–New Jersey, 1972, pp. 137–149.

[17] I. S. Gertsein and R. Jackiw, Phys. Rev. 181 (1969) 1955.

[18] M. Kamionkowski and J. N. Bahcall, ApJ. 420 (1994) 884.
[19] T.–S. Park, D.–P. Min and M. Rho, Phys. Rev. Lett. 74 (1995) 4153; Nucl. Phys. A596 (1996) 515; and Effective Field Theory for Low–Energy Two–Nucleon Systems, hep–ph/9711463 and references therein.

[20] A. E. Cox, A. R. Wynchank and C. H. Collie, Nucl. Phys. 74 (1965) 497 and references therein.

[21] D. O. Riska and G. E. Brown, Phys. Lett. B38 (1972) 193 and references therein.

[22] T.–S. Park, D.–P. Min and M. Rho, Phys. Rev. Lett. 74 (1995) 4153; Nucl. Phys. A596 (1996) 515.

[23] S. L. Mintz, Phys. Rev. C23 (1981) 421, ibid. C24 (1981) 1799.

[24] J. Weneser, Phys. Rev. 105 (1957) 1335.

[25] T. Ahrens and T. P. Lang, Phys. Rev. C3 (1971) 979; J. Hošek and E. Truhlik, Phys. Rev. C23 (1981) 665; F. T. Avignone, Phys. Rev. D24 (1981) 778.

[26] S. Ying, W. C. Haxton and E. M. Henley, Phys. Rev. C45 (1992) 1982; M. Doi and K. Kubodera, Phys. Rev. C45 (1992) 1988.

[27] R. A. Berthmann, in Anomalies in Quantum Field Theory, Oxford Scince Publications, Clarendon Press–Oxford, 1996.

[28] J. Wess and B. Zumino, Phys. Lett. B37 (1971) 95.

[29] E. Witten, Nucl. Phys. B159 (1979) 269; Nucl. Phys. B223 (1983) 422, 433.

[30] G. Veneziano, Nucl. Phys. B159 (1979) 213.

[31] S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. 41 (1969) 531 and references therein.

[32] N. N. Bogoliubov and D. V. Shirkov, in INTRODUCTION TO THE THEORY OF QUANTIZED FIELDS, Interscience Publisher, Inc. New York, 1959, pp.569–575.

[33] T. Eguchi, Phys. Rev. D14 (1976) 2755; K. Kikkawa, Progr. Theor. Phys. 56 (1976) 947; H. Kleinert, Proc. of Int. Summer School of Subnuclear Physics, Erice 1976, Ed. A. Zichichi, p.289.

[34] A. Dhar and S. R. Wadia, Phys. Rev. Lett. 52 (1984) 959; A. Dhar, R. Shankar and S. R. Wadia, Phys. Rev. D31 (1985) 3256; D. Ebert and H. Reinhardt, Nucl. Phys. B271 (1986) 188; M. Wakamatsu, Ann. Phys. 193 (1989) 287.

[35] A. N. Ivanov, M. Nagy and N. I. Troitskaya, Int. J. Mod. Phys. A7 (1992) 7305; A. N. Ivanov, Int. J. Mod. Phys. A8 (1993) 853; A. N. Ivanov, N. I. Troitskaya and M. Nagy, Int. J. Mod. Phys. A8 (1993) 2027; ibid. A8 (1993) 3425; Phys. Lett. B308 (1993) 111; ibid. B295 (1992) 308; A. N. Ivanov and N. I. Troitskaya, Nuovo Cim. A108 (1995) 555.
[36] J. Bijnens, C. Bruno and E. de Rafael E., Nucl. Phys. B390 (1993) 501 and references therein; J. Bijnens, E. de Rafael and H. Zheng, Z. Phys. C62 (1994) 437 and references therein.

[37] A. Bramon and F. J. Yndurain, Phys. Lett. B80 (1979) 239.

[38] J. J. Kokkedee, in THE QUARK MODEL, Benjamin, New York, 1979; F. E. Close, in AN INTRODUCTION TO QUARKS AND PARTONS, Academic Press, New York, 1979.

[39] A. N. Ivanov and V. M. Shechter, Sov. J. Nucl. Phys., 31 (1980) 275 and references therein.

[40] H. A. Bethe and C. L. Critchfield, Phys. Rev. 54 (1939) 248.

[41] M. L. Goldberger and K. M. Watson, in COLLISION THEORY, John Wiley & Sons, Inc., New York–London–Sydney, 1964.

[42] A. N. Ivanov, N. I. Troitskaya, M. Faber, M. Schaler and M. Nagy, Nuovo Cim. A107 (1994) 1667, Phys. Lett. B336 (1994) 555; A. N. Ivanov, N. I. Troitskaya and M. Faber, Nuovo Cim. A108 (1995) 613.

[43] J. N. Bahcall and M. Kamionkowski, Nucl. Phys. A625 (1997) 893.