Critical current densities estimated from AC susceptibilities in proximity-induced superconducting matrix of multifilamentary wire

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Abstract. In a multifilamentary wire proximity-currents between filaments show a close resemblance with the inter-grain current in a high-$T_c$ superconductor. The critical current densities of the proximity-induced superconducting matrix $J_{cm}$ can be estimated from measured twist-pitch dependence of magnetization and have been shown to follow the well-known scaling law of the pinning strength. The grained Bean model is applied on the multifilamentary wire to obtain $J_{cm}$, where the filaments are immersed in the proximity-induced superconducting matrix. Difference of the superconducting characteristics of the filament, the matrix and the filament content factor give a variety of deformation on the AC susceptibility curves. The computed AC susceptibility curves of multifilamentary wires using the grained Bean model are favorably compared with the experimental results. The values of $J_{cm}$ estimated from the susceptibilities using the grained Bean model are comparable to those estimated from measured twist-pitch dependence of magnetization. The applicability of the grained Bean model on the multifilamentary wire is discussed in detail.

1. Introduction
It is necessary to enhance the inter-grain coupling in high-$T_c$ superconductors for fulfilling the demand of high critical current densities and low losses for industrial AC applications. It has been reported [1] that the AC losses in the multifilamentary wires increased with decreasing interfilamentary spacing $d_N$ and were noticeable when $d_N$ became comparable to or smaller than the coherence length $\xi_N$ of the matrix. It is widely known that this effect results from an excess magnetization caused by interfilamentary proximity coupling. In this paper the relation of the proximity effect and the AC susceptibilities in the multifilamentary wire is discussed. Field distribution and magnetization in the multifilamentary wire are calculated by Bean model [2]. Fourier integration of magnetization is carried out numerically and gives rise to the AC susceptibilities. Measured results of NbTi multifilamentary wires with CuMn matrix are compared with the simulated results to give the matrix characteristics.

2. Experimental
Sample specifications of multifilamentary wires are shown in table 1. The matrix is Cu-0.5wt%Mn for series A and B samples. Samples were manufactured by double stacking and are composed of 24186 NbTi filaments embedded in the Cu-0.5wt%Mn matrix with the volume...
ratio being 1.58. The values of $d_N$ and $d_f$ were calculated from the measured values of the wire diameter $D_w$. A commercial SQUID magnetometer was used to measure magnetic moments. The magnetization measurement was carried out in a temperature range 2 to 10 K, and the DC magnetic fields up to 1 T and the superposed AC magnetic fields of 0.05 mT were applied perpendicular to the wire axis. The superconducting transition temperatures $T_c$ of wires are 8.5 K, which were measured from temperature dependence of magnetization under zero-field cooling and field cooling conditions.

### Table 1. Sample specifications.

| Sample | $D_w$(mm) | $d_l$(µm) | $d_N$(µm) | $\ell_p$(mm) |
|--------|-----------|-----------|-----------|--------------|
| A      | 0.102     | 0.324     | 0.050     | 2.6          |
| B      | 0.145     | 0.461     | 0.072     | 2.2          |

3. Grained Bean model

Proximity-induced superconducting matrix with a penetration field $B_{pm}$ determines the magnetic fields at the filament surface with the penetration field $B_{pf}$. Field distribution $B_m$ outside the $i$-th filament at a filament position $X_i$ is given as $B_m = B_0 - B_{pm}(2X_i/D_w)$: outside the filament and field inside $B_i$ of $i$-th filament with the filament diameter $d_{fi}$ as $B_i = B_{m} - B_{pf}(x/d_{fi})$: inside the filament, where $x$ is a position inside the filament from the each filament surface [3]. In the case of uniform wire structure with constant filament size $d_f$ (= $d_{fi}$ for all $i$) and interfilamentary spacing $d_N$ (= $d_{Ni}$ for all $i$), the average of the magnetic flux density $< B >$ in the increasing period for both of the matrix and filaments with interfilamentary spacing $d_N$ is given by

$$<B> = B_0 - \frac{B_{pm}}{2} + \frac{f_l}{\ell_p} \sum_{i=1}^{n_s} M_{fi},$$

where $M_f$ is filament magnetization and $f_l$ (= $d_l/(d_N + d_l)$) is the filament content factor. Then the magnetization $M$ of the superconductor at the field $B_0$ for each magnetization processes is given by $M = <B> - B_0$.

4. Proximity coupling in multifilamentary wire

The proximity-effected magnetization $M_p$ assisted by $J_{cm}$ is expressed as [1]

$$M_p \approx \mu_0 \lambda_p \lambda_m J_{cm} \ell_p / \pi^2,$$

where $\lambda_p$ denotes the volume fraction of the superconducting matrix in the multifilamentary region and $\lambda_m$ is the volume fraction of the multifilamentary region in the wire. Thus $J_{cm}$ values can be estimated from the slope of the $\ell_p$ dependence of the magnetization. Field dependence of $J_{cm}$ estimated from the $\ell_p$ dependence of magnetization is approximately expressed by the scaling law of the pinning effect as $J_{cm} = J_0(B/B_{c2p})^{-\gamma}[1-(B/B_{c2p})]^{\delta}$ [4], where $\gamma$ and $\delta$ are adjustable pinning parameters and $B_{c2p}$ is the upper critical field of the weakly superconducting matrix. Temperature dependence of $B_{c2p}$ is shown to be $B_{c2p} = B_{c2N}\exp(-K_Nd_N)$; $K_N = T^{1/2}/\beta$ where $\beta (K^{1/2}/\mu m)$ in the low-temperature range of $T < 4$ K is 0.126 and 0.172 for series A and B samples, respectively. In the high-temperature range of $T > 4$ K, $\beta$ is 0.062 for series A and 0.086 for series B samples, respectively [4].

2
Figure 1. Temperature dependence of AC susceptibilities for (a) sample A with twist-pitch $\ell_p = 11.7$ mm and (b) sample B with $\ell_p = 5.7$ mm at various fields of 0.1 – 0.8 T.

Figure 2. Numerically computed AC susceptibilities for (a) $d_N = 0.050 \, \mu$m and (b) $d_N = 0.072 \, \mu$m at various fields of 0.1 – 0.8 T as a function of temperature.

5. Results and discussion
The fundamental AC susceptibilities $\chi'$ and $\chi''$ under an AC field, denoted as $\chi'$ and $\chi''$ hereafter, are derived from Fourier integrals of the magnetization. Temperature dependence of $\chi'$ and $\chi''$ is obtained by introducing the temperature variation of the penetration fields of $B_{pf}$ and $B_{pm}$. If the $B_{pf}$ has a usual parabolic dependence of the form as $B_{pf} = B_{pf}(0)\left\{1 - (T/T_{cf})^2\right\}^{n_f}$, where over the temperature $T_{cf}$ the pinning effect for the fluxoid motion disappears in the filament. The temperature variation of $B_{pm}$ is obtained from the relation of $B_{pm} = \mu_0 D_w J_{cm}/2$.

Temperature dependence of the AC susceptibilities for sample A with twist-pitch $\ell_p = 11.7$ mm and sample B with $\ell_p = 5.7$ mm at various fields of 0.1 – 0.8 T is shown in figure 1 (a) and (b). With lowering temperature, a diamagnetic transition takes place, accompanied by a lower peak due to the filament and a higher peak due to the proximity-induced superconducting matrix in $\chi''$. With increasing magnetic field, the coupling peaks for the sample B shift to lower temperatures than those for the sample A. At low field of 0.1 T, a third diamagnetic step, together with another peak, occurs at lower temperatures. This peak is sensitive to magnetic fields and moves to lower temperatures. This type of peak structure is often observed in high-$T_c$...
The AC susceptibilities are numerically computed at various fields of $0.1 - 0.8 \text{T}$ and plotted in figure 2 (a) and (b) for $d_N = 0.050 \text{\mu m}$ and $d_N = 0.072 \text{\mu m}$, respectively. The two peaks at higher temperatures in the $\chi''$ curve is specifically indicated to originate in the cooperative work of the filament and the matrix. These curves are favorably compared with the experimental results in figure 1.

Field dependence of $J_{cm}$ is shown in figure 3. Symbols of open circle and solid circle present $J_{cm}$ estimated from the $\ell_p$ dependence of the magnetization and that obtained from simulation using the grained Bean model, respectively. $J_{cm}$ values estimated from AC susceptibilities are $5 - 7$ times for sample A larger than those estimated from measured twist-pitch dependence of magnetization and 2 times for sample B.

![Figure 3](image-url)

**Figure 3.** Field dependence of $J_{cm}$ for (a) sample A and (b) sample B. Symbols of open circle and solid circle present $J_{cm}$ estimated from the twist-pitch dependent magnetization and that obtained from the AC susceptibility, respectively.

6. Conclusions
AC susceptibilities of NbTi multifilamentary wires with CuMn matrix were measured using a SQUID magnetometer at temperatures of $2 - 10 \text{K}$, magnetic fields $B_{dc}$ up to $1 \text{T}$. Texture of multifilamentary wire is simulated by the grained Bean model, where the superconducting regions are divided two parts: filaments and proximity-induced superconducting matrix. The critical current densities of the proximity-induced superconducting matrix $J_{cm}$ were estimated from the susceptibilities numerically analyzed using the grained Bean model. These values estimated from twist-pitch dependent magnetization and AC susceptibility with the grained Bean model agree, however, they contain $2 - 7$ times difference.

References
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