Do Majorana states really exist at normal metal-spin orbit coupled superconducting wire interfaces?

Subhajit Pal and Colin Benjamin
School of Physical Sciences, National Institute of Science Education & Research, HBNI, Jatni-752050, India

A popular evidence of the existence of Majorana bound states is a zero-bias quantized conductance peak which is robust to scattering by impurities, a consequence of its topological protection. In this work we examine the robustness of this zero-bias quantized conductance peak in a metal-topological superconductor junction in the vicinity of a spin flipper. Using BTK approach, we analytically calculate the differential charge conductance for metal-spin flipper-topological superconductor junctions with two distinct topological superconductors: (a) spin less p-wave superconductor and (b) spin-orbit-coupled s-wave superconducting wire in presence of a Zeeman field. We show that the zero-bias conductance peak remains quantized in presence of a spin flip scattering for metal-p-wave superconductor junction, while it loses its quantization when the p-wave superconductor is replaced by a spin orbit coupled superconducting wire.

**Introduction:** In recent years Majorana bound states\(^1\)\(^2\) (MBS) have been studied both theoretically and experimentally in many setups. Majorana zero modes are quasiparticle excitations within the superconductor at zero energy which are their own anti-particles. These Majorana zero modes can be observed at the boundary with a topological superconductor\(^2\). Possible signatures of MBS have been reportedly seen in nanowire-superconductor hybrid device\(^3\)\(^5\), topological insulator-superconductor heterostructure\(^6\) and atomic chains on a superconductor\(^7\). Further there have been reports of the generation of Majorana states at interfaces of normal metals and spin orbit coupled superconducting wire (SOCSW)\(^8\). Our main motivation in this work is to propose a reliable check on the existence of Majorana bound states at metal-topological superconductor interfaces. To do this we consider a spin flipper at a metal-topological superconductor interface, and see that the zero energy quantized conductance Majorana peak remains unchanged (robust) in presence of spin flip scattering for p-wave superconductor(pSc) but loses its quantization due to spin flip scattering in case of a SOCSW. Thus, the zero bias conductance does not take universal values when a spin flipper is placed at interface of a metal-SOC SW. This, begs the question do Majorana bound states really exist at metal-SOC SW interfaces?

The paper is organized as follows, we first present our model for spin-flip scattering at the interface with a pSc and then at the interface with a SOCSW by writing the Hamiltonian, wavefunctions and boundary conditions to determine the different scattering probabilities. Following this we discuss our results by comparing the zero-bias conductance in metal-SOC SW junction with that in metal-pSc junction in presence of spin flip scattering.

**Theory:** In this work we contrast the Majorana states arising at metal-pSc interfaces with those arising at metal-SOC SW interfaces. The exact setting we will use is shown in Figs. 1.2, it represents a spin flipper at \(x = 0\) while at \(x = a\) a \(\delta\)-like potential barrier separates normal metal from pSc or SOCSW. The regions I \((x < 0)\) and II \((0 < x < a)\) are metallic while \(x > a\) there is a pSc or SOCSW.

**Spin flipper in the vicinity of Metal-pSc junction:**

**Hamiltonian-** We consider a one-dimensional normal metal (NM)-normal metal (NM)-pSc junction wherein a spin flipper is embedded between two metallic regions at \(x = 0\). The interface at \(x = a\) (see Fig. 1) is modeled by a \(\delta\)-like potential barrier (strength \(Z\)) and the problem is solved using the BTK\(^9\) approach. The Hamiltonian for spin flip interaction, from Refs. \(^10\)\(^–\)\(^14\) is

\[
H_{\text{Spinflipper}} = -J\vec{s} \cdot \vec{S},
\]

\[
\vec{s} \cdot \vec{S} = s_x S_z + \frac{1}{2}(s^- S^+ + s^+ S^-),
\]

where \(J\) denotes the strength of exchange coupling between electron’s spin \(\vec{s}\) and spin \(\vec{S}\) (of spin flipper). \(S^\pm = S_x \pm iS_y\) are the spin raising and lowering operators for spin flipper while \(s^\pm = s_x \pm is_y\) are the spin raising and lowering operators for electron/hole with \(s_k = \frac{\hbar}{2} \begin{pmatrix} \sigma_k & 0 \\ 0 & -\sigma_k \end{pmatrix}\), \(k = x, y, z\), with \(\sigma_k\) being the usual Pauli matrices. The Bogoliubov-de Gennes (BdG) Hamiltonians for normal metal (NM) and p-wave superconductor (pSc), from \(^8\) are:

\[
H_{\text{NM}} = -(\hbar^2 \partial_x^2/2m^* - \mu_{\text{NM}})\tau_z,
\]

\[
H_{\text{pSc}} = -(\hbar^2 \partial_x^2/2m^* - \mu_{\text{pSc}})\tau_z - i\Delta_{\text{pSc}}\partial_x \tau_x,
\]

where \(\mu_{\text{NM}}\) and \(\mu_{\text{pSc}}\) are the respective chemical potentials, \(m^*\) the mass of electron, \(\Delta_{\text{pSc}} \geq 0\) is the p-wave pairing potential, \(\tau_\mu = \sigma_\mu \otimes I\), with \(I\) being \(2 \times 2\) identity matrix and \(\sigma_\mu\) being the Pauli matrices. For simplicity, we consider \(\hbar = \mu_{\text{NM}} = 2m^* = 1\). The energy spectrum are then \(\varepsilon_{\text{NM,}+}(k) = \pm(\sqrt{k^2 - 1})\) and \(\varepsilon_{\text{pSc,}+}(k) = \pm(\sqrt{k^2 - \mu_{\text{pSc}}^2} + (\Delta_{\text{pSc}}k)^2)\), respectively. In this work we only concentrate on the topological regime\(^8\), i.e., \(\mu_{\text{pSc}} > 0\). For \(\mu_{\text{pSc}} > \Delta_{\text{pSc}}^2/2\), the positive energy spectrum for pSc shows the characteristic “double-well” BCS structure with minimum values at \(\epsilon_1 = \Delta_{\text{pSc}}\sqrt{\mu_{\text{pSc}} - \Delta_{\text{pSc}}^2/4}\) at \(k = \pm\sqrt{\mu_{\text{pSc}} - \Delta_{\text{pSc}}^2/2}\).
FIG. 1: Normal metal (NM)-Normal metal (NM)-pSc junction with a spin flipper (with spin $S$, magnetic moment $m'$) at $x = 0$ and a $\delta$-like potential barrier (strength $Z$) at $x = a$. The scattering of an incident spin up electron is shown. Normal reflection, Andreev reflection and quasi-particle transmission into $p$-wave superconductor are represented.

Wavefunctions- The wave functions for the different regions of our system as shown in Fig. 1 can be written for a spin-up electron incident at $x = 0$ interface as-

$$
\psi_{NM}^I(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{i x S_{m'}} + r_{ee}^{+} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-i x S_{m'}} + r_{ee}^{+} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-i x S_{m'} + 1} + r_{eh}^{+} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{i x S_{m'}}, \text{for } x < 0, (3)
$$

$$
\psi_{NM}^{II}(x) = t_{ee}^{+} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{i(x-a) S_{m'} + 1} + r_{eh}^{+} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{i(x-a) S_{m'} + 1} + a_{eh}^{+} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} e^{-i S_{m'}}, \text{for } 0 < x < a, \quad (4)
$$

$$
\psi_{pSc}(x) = t_{ee}^{+} \begin{pmatrix} \eta_- \\ 0 \\ 1 \end{pmatrix} e^{i k_x S_{m'} + 1} + r_{eh}^{+} \begin{pmatrix} \eta_- \\ 0 \\ 0 \end{pmatrix} e^{i k_x S_{m'} + 1} + t_{eh}^{+} \begin{pmatrix} \eta_+ \\ 0 \\ 1 \end{pmatrix} e^{i k_x S_{m'}}, \text{for } x > a, \quad (5)
$$

where $\eta_{\pm} = \frac{E + k_x^2 - \mu_{pSc}}{\Delta_{pSc}}, k_x$ is the eigenspinor of spin flipper, with the $S_z$ operator acting as- $S_z S_{m'} = m' S_{m'}$, $m'$ is the magnetic moment of spin flipper. $r_{ee}^{+}, r_{eh}^{+}$, are the normal reflection amplitudes for no flip and spin flip, while $r_{eh}^{+}, r_{eh}^{+}$ are Andreev reflection amplitudes for no flip and spin flip respectively. Similarly, $t_{ee}^{+}, t_{ee}^{+}, t_{eh}^{+}, t_{eh}^{+}$ are the transmission amplitudes into the pSc. In Eqs. (3-5) we approximate the wave vector in normal metal by the Fermi wave vector $k_F = \sqrt{2 m^* \mu_{NM}} / \hbar = 1$ (since $\hbar = \mu_{NM} = 2 m^* = 1$) with $E << E_F$. The wave vector $k_x$ in $p$-wave superconductor are solutions of the equation mentioned below-

$$E^2 = (k^2 - \mu_{pSc})^2 + (\Delta_{pSc} k)^2. \quad (6)$$

The different solutions of Eq. (6) for various values of chemical potential $\mu_{pSc} > 0$ (topological regime) and energy $E$ are mentioned in Table I.

Boundary conditions- The boundary conditions at $x = 0$ are- $\psi_{NM}^I(x) = \psi_{NM}^{II}(x)$ (continuity of wavefunction) and, $2i \partial_x \tau_z \psi_{NM}^I(x) - 2i \partial_x \tau_z \psi_{NM}^{II}(x) = 2i J \hat{S} \tau_z S_{m'} \psi_{NM}^I(x)$ (discontinuity in first derivative). The boundary conditions at $x = a$ are- $\psi_{NM}^I(x) = \psi_{pSc}(x)$ (continuity of wavefunction) and, $(-2i \partial_x \tau_z + \Delta_{pSc} \tau_z) \psi_{pSc}(x) + 2i \partial_x \tau_z \psi_{NM}^I(x) = -2i Z \tau_z \psi_{NM}^I(x)$ (discontinuity in first derivative).
Further, the action of exchange operator on spin up electron gives-

$$\tilde{s}_z \tilde{S}_z \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \phi_m^S = \frac{m'}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \phi_m^S + \frac{F}{2} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \phi_{m'+1}^S, \tag{7}$$

where $F = \sqrt{(S - m')(S + m' + 1)}$ is the spin flip probability of spin flipper. Similarly, the action of exchange operator for spin down electron gives-

$$\tilde{s}_z \tilde{S}_z \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \phi_{m'+1}^S = -\frac{m' + 1}{2} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \phi_{m' + 1}^S + \frac{F}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \phi_m^S, \tag{8}$$

Further, the action of exchange operator on spin up holes gives-

$$\tilde{s}_z \tilde{S}_z \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \phi_{m'+1}^S = \frac{m' + 1}{2} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \phi_{m' + 1}^S - \frac{F}{2} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \phi_m^S, \tag{9}$$

and finally the action of exchange operator on spin down holes gives-

$$\tilde{s}_z \tilde{S}_z \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \phi_m^S = -\frac{m'}{2} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \phi_m^S - \frac{F}{2} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \phi_{m'+1}^S, \tag{10}$$

Using the above equations and solving the boundary conditions we obtain 16 equations. From these 16 equations we can compute the different scattering probabilities: $R_{ee}^{\uparrow\downarrow} = |r_{ee}^{\uparrow\downarrow}|^2$, $R_{ee}^{\downarrow\uparrow} = |r_{ee}^{\downarrow\uparrow}|^2$, $R_{eh}^{\uparrow\downarrow} = |r_{eh}^{\uparrow\downarrow}|^2$, $R_{eh}^{\downarrow\uparrow} = |r_{eh}^{\downarrow\uparrow}|^2$.

Similarly, if we consider a spin down electron incident from metallic region I, we get the different scattering probabilities as follows: $R_{ee}^{\uparrow\downarrow}, R_{ee}^{\downarrow\uparrow}, R_{eh}^{\uparrow\downarrow}, R_{eh}^{\downarrow\uparrow}$.

Spin flipper in the vicinity of metal-SOCSW junction: Next, we consider a spinful normal metal (NM)-spin flipper-normal metal (NM)-Insulator (I)-SOCSW junction with a spin flipper (spin $\hat{S}$, magnetic moment $m'$) at $x = 0$ and a $\delta$-like potential barrier (strength $Z$) at $x = a$.

The scattering of an incident spin-up electron is shown. Normal reflection, Andreev reflection and quasi particle transmission into SOCSW are shown.

Using the same convention as before, i.e., $V = 2m' = \mu_{NM} = 1$, the BdG Hamiltonians for normal metal (NM)-Normal metal (NM)-SOCSW junction as shown in Fig. 2. As done before, we model the metal superconductor interface as a $\delta$-like potential barrier with strength $Z$.

Hamiltonian- Using the same convention as before, i.e., $\hbar = 2m' = \mu_{NM} = 1$, the BdG Hamiltonians for normal metal (NM) and spin orbit coupled superconducting wire (SOC) can be written as [8]:

$$H_{NM} = (\partial_{x_1}^2 - 1)\tau_z, \tag{11a}$$

$$H_{SOCWS} = -\delta^2_{x_1} - i\beta\partial_{x_1}\tau_z = B_2\sigma_x + \Delta_0\tau_x, \tag{11b}$$

with $\beta$ being strength of spin orbit interaction, $B_2$ is Zeeman field, and $\Delta_0 \geq 0$ the proximity induced s-wave pairing potential. We also consider uniform electron masses throughout the system and fix chemical potential of the SOCSW and Zeeman interaction in the

### TABLE I: Solutions of Eq. (6) for various values of chemical potential $\mu_{pSc} > 0$ (topological regime) and energy $E$

| $\mu_{pSc}$ | $E$ | $k_+$ | $k_-$ |
|------------|-----|-------|-------|
| $0 < \mu_{pSc} \leq \Delta^2_{pSc}/4$ | $E \leq |e_1|$ | $i[(\Delta^2_{pSc}/2 - \mu_{pSc}) + \sqrt{E^2 - \epsilon_1^2}]^{1/2}$ | $i[(\Delta^2_{pSc}/2 - \mu_{pSc}) - \sqrt{E^2 - \epsilon_1^2}]^{1/2}$ |
| $\Delta^2_{pSc}/4 \leq \mu_{pSc} \leq \Delta^2_{pSc}/2$ | $E \leq |e_1|$ | $|e_1| \leq E \leq |e_2|$ | $i[(\Delta^2_{pSc}/2 - \mu_{pSc}) + \sqrt{E^2 - \epsilon_1^2}]^{1/2}$ | $-i[(\Delta^2_{pSc}/2 - \mu_{pSc}) - \sqrt{E^2 - \epsilon_1^2}]^{1/2}$ |
| $\mu_{pSc} \geq \Delta^2_{pSc}/2$ | $E \leq |e_1|$ | $|e_1| \leq E \leq |e_2|$ | $i[(\Delta^2_{pSc}/2 - \mu_{pSc}) + \sqrt{E^2 - \epsilon_1^2}]^{1/2}$ | $-i[(\Delta^2_{pSc}/2 - \mu_{pSc}) - \sqrt{E^2 - \epsilon_1^2}]^{1/2}$ |

where $\epsilon_1^2 = \mu_{pSc}^2 - \Delta_{pSc}^2/4$.
lead to be zero [8]. The positive branches of the energy spectrum [8] of SOCSW are given as \(\varepsilon_{\text{SOCSW},\pm}(k) = (k^2 + \beta^2 k^2 + \Delta_0^2 + B_2^2 + 2i(k^2\beta k^2 + B_2^2) + \Delta_0 B_2)^{1/2}\). We focus only on the topological regime \(B_2 > \Delta_0/2\) [13]. In the limit of strong spin orbit interaction (SOC) \(\beta \gg B_2, \Delta_0\), the energy spectrum of the SOCSW has two branches [8]—(i) interior branch, (ii) exterior branch. The BdG Hamiltonians for the interior and exterior branches of the SOCSW in the case of strong SOC can be written as [8].

\[
H^{(i)}_{\text{SOCSW}} = -i\beta \tau_z \partial_x + B_z \sigma_x + \Delta_0 \tau_x, \quad (12a)
H^{(e)}_{\text{SOCSW}} = -i\beta \tau_z \partial_x + \Delta_0 \tau_x, \quad (12b)
\]

where \(i, e\) denote the interior and exterior branches respectively.

**Wavefunctions**—The wavefunction in the normal metal regions I and II for spin up electron incident with energy \(E\) are already mentioned in Eqs. [3, 4]. The wavefunction in the SOCSW is sum of solutions for exterior as well as an interior branch (Eq. [12]) and in the topological regime can be written as [8].

\[
\psi_{\text{SOCSW}}(x) = t^{(i)}_{\text{NM}}(x) \psi^{(i)}_{\text{NM}}(x) + t^{(e)}_{\text{NM}}(x) \psi^{(e)}_{\text{NM}}(x)
\]

where the first two terms on the right-hand side denote contributions from the interior branches, while the other two terms give contributions from the exterior branches. \(t^{(i)}_{\text{NM}}(x)\) and \(t^{(e)}_{\text{NM}}(x)\) are the transmission amplitudes into SOCSW. In Eq. [13] for \(E > |\Delta_\lambda|\), \(u^2_\lambda = \frac{E+(E^2-\Delta_\lambda^2)^{1/2}}{2E}\) and \(u^2_\lambda + v^2_\lambda = 1\), while for \(0 \leq E < |\Delta_\lambda|\), \(u^2_\lambda = \frac{E+(E^2-\Delta_\lambda^2)^{1/2}}{2E}\) and \(u^2_\lambda + v^2_\lambda = \frac{E}{|\Delta_\lambda|}\), where \(\lambda = \pm, 0\), and \(\Delta_\pm = \Delta_0 \pm B_2\). The wave vectors in Eq. [13] are \(k^{(i)}_0 = \frac{(E^2-\Delta_\lambda^2)^{1/2}}{\beta}\) for the interior branch, and \(k^{(e)}_0 = \frac{(E^2-\Delta_\lambda^2)^{1/2}}{\beta}\) for the exterior branch.

**Boundary conditions**—The boundary conditions at \(x = 0\) are—\(\psi^{(i)}_{\text{NM}}(x) = \psi^{(e)}_{\text{NM}}(x)\) (continuity of wavefunction) and, \(2i\partial_x \tau_z \psi^{(i)}_{\text{NM}}(x) - 2i\partial_x \tau_z \psi^{(e)}_{\text{NM}}(x) = 2i JS \tau_z \psi^{(i)}_{\text{NM}}(x)\) (discontinuity in first derivative). The boundary conditions at \(x = a\) are—\(\psi^{(e)}_{\text{NM}}(x) = \psi^{(i)}_{\text{SOCSW}}(x)\) (continuity of wavefunction) and, \((-2i\partial_x \tau_z + \beta \tau_x \sigma_z) \psi^{(i)}_{\text{SOCSW}}(x) + 2i\partial_x \tau_z \psi^{(e)}_{\text{SOCSW}}(x) = -2iZ \tau_x \psi^{(i)}_{\text{SOCSW}}(x)\) (discontinuity in first derivative). Substituting the wavefunctions in the above boundary conditions we get 16 equations. Solving the 16 equations we get different normal and Andreev reflection probabilities: \(R_{\text{ee}}^{\uparrow\uparrow} = |r_{\text{ee}}^{\uparrow\uparrow}|^2\), \(R_{\text{ee}}^{\uparrow\downarrow} = |r_{\text{ee}}^{\uparrow\downarrow}|^2\), \(R_{\text{eh}}^{\uparrow\downarrow} = |r_{\text{eh}}^{\uparrow\downarrow}|^2\), \(R_{\text{eh}}^{\downarrow\uparrow} = |r_{\text{eh}}^{\downarrow\uparrow}|^2\). Similarly, if we consider a spin down electron incident from normal metal (region I), we can easily calculate the different reflection probabilities as: \(R_{\text{ee}}^{\downarrow\uparrow} = |r_{\text{ee}}^{\downarrow\uparrow}|^2\), \(R_{\text{ee}}^{\downarrow\downarrow} = |r_{\text{ee}}^{\downarrow\downarrow}|^2\), \(R_{\text{eh}}^{\downarrow\uparrow} = |r_{\text{eh}}^{\downarrow\uparrow}|^2\), \(R_{\text{eh}}^{\uparrow\downarrow} = |r_{\text{eh}}^{\uparrow\downarrow}|^2\). We do not repeat them here but in analogy to spin up case the wavefunctions can be easily written and solved for the aforementioned boundary conditions.

**Differential charge conductance**—Using the well established definitions as in Refs. [16,17], we calculate the net differential charge conductance as—

\[
G_\text{e} = G_\text{e}^\uparrow + G_\text{e}^\downarrow, \quad \text{with } G_\text{e}^\uparrow = G_0(1 + R_{\text{eh}}^{\uparrow\downarrow} - R_{\text{ee}}^{\uparrow\downarrow} - R_{\text{eh}}^{\downarrow\uparrow}) \quad \text{and } G_\text{e}^\downarrow = G_0(1 + R_{\text{eh}}^{\downarrow\uparrow} - R_{\text{ee}}^{\downarrow\uparrow} - R_{\text{eh}}^{\uparrow\downarrow})
\]

where \(G_0 = e^2/h\) and \(G_\text{e}^\pm\) the differential charge conductance when spin up electron is incident from region I, while \(G_\text{e}^\pm\) the differential charge conductance when spin down electron is incident from region I.

**Results & Analysis**—In Table II, we compare the zero-bias conductance of a NM-spin flipper-NM-I-SOCSW junction and NM-spin flipper-NM-I-Psc junction for both transparent \((Z = 0)\) and tunnel \((Z = 3)\) regimes. For spin flip case, \(F \neq 0\) as \(S \neq m'\), see Eqs. [7,10], implying there is finite possibility for spin-flipper to flip its own spin while interacting with an electron. On the other hand, for no flip case, \(F = 0\) as \(S = m'\). We take two different values of spin orbit coupling strength \(\beta\) \((\beta = 1\) and \(\beta = 2)\) for SOCSW in second and third column of Table II. We also take two different values of \(\mu_{\text{Psc}} = 0.01, 0.001\), for Psc in fourth and fifth column of Table II. For no flip \((F = 0)\) case we see that the normalized zero bias conductance \(G_\text{e}/G_0\) is quantized at 2 for NM-spin flipper-NM-I-SOCSW junction, while for NM-spin flipper-NM-I-Psc junction it is quantized at 4 regardless of other parameters like \(S, m', J, Z, \beta, \mu_{\text{Psc}}, \text{etc.}\). The reason for this is that the Andreev and normal reflection probabilities exactly cancel at zero bias in Eq. [14] for NM-spin flipper-NM-I-SOCSW junction while for NM-spin flipper-NM-I-Psc junction there is perfect Andreev reflection (i.e., normal reflection probabilities vanish) at zero bias and thus from Eq. [14] \(G_\text{e}/G_0\) is quantized at 4. In Table II, we have three non-zero values of spin flip probability \((F = 1, \sqrt{3}, 3)\) for transparent junction.
Thus, in a metal-pSc junction the “Majorana states” are not affected by presence of spin flipper, while for metal-SOCSW junction the “Majorana states” are affected by presence of spin flipper. This casts a huge doubt on the states observed in metal-SOCSW junction. Can they be really called Majorana states? since they are not protected against spin flip scattering. In Fig. 3 we plot the differential charge conductance as a function of energy $E$ for different values of interface barrier strength $Z$ in the topological regime. In Fig. 3(a) we concentrate on NM-spin flipper-NM-I-pSc junction and see that $G_c/G_0$ at $E = 0$ is quantized and independent of $Z$ and spin flip probability ($F$) of spin-flipper. Thus the topological character of the zero-bias conductance peak is evident, implying the existence of Majorana state in NM-spin flipper-NM-I-pSc junction. In Fig. 3(b) we do the same for NM-spin flipper-NM-I-SOCSW junction. We see that $G_c/G_0$ at $E = 0$ is not quantized and depends on interface transparency $Z$. Thus, in presence of spin flip scattering the topological character of zero-bias conductance peak seen in case of NM-spin flipper-NM-I-SOCSW junction is affected, suggesting the absence of Majorana states in such junctions. To conclude, we have shown that zero energy quantized conductance Majorana peaks remain unaffected, in presence of spin flipper, at metal-p-wave superconductor interfaces, while zero energy peak at metal-SOCSW interface loses its quantization in presence of spin flip scattering. Thus, it is not right to call

![Diagram](image-url)  
**FIG. 3:** Differential charge conductance as a function of energy $E$ for different values of interface barrier strength $Z$ in the topological regime, (a) for NM-spin flipper-NM-I-pSc and (b) for NM-spin flipper-NM-I-SOCSW junction. Parameters are $S = -m' = 3/2, F = \sqrt{3}, J = 1, \mu_{pSc} = 0.01, \Delta_{pSc} = 0.07, \alpha = 0, \Delta_0 = 0.001, \beta = 0.5, B_2 = 1.5\Delta_0$.

### TABLE II: Comparison of differential charge conductance at zero bias ($E = 0$) in the topological regime between NM-spin flipper-NM-I-SOCSW and NM-spin flipper-NM-I-p-wave junction

| Parameters| NM-spin flipper-NM-I-SOCSW| NM-spin flipper-NM-I-p-wave |
|---------|-----------------------------|-------------------------------|
| $Z = 0, E = 0, \alpha = 0, \Delta_0 = 0.001, B_2 = 1.5\Delta_0$| $G_c/G_0$ for $\beta = 1$| $G_c/G_0$ for $\beta = 2$ | $G_c/G_0$ ($\mu_{pSc} = 0.001$) |
| No flip ($F = 0, S = m'$) | 2 | 2 | 4 |
| Flip ($F = 1, S = -m' = 1/2$) | $800 + 272J^+ + 50J^-$ | $2048 + 128J^+ + 50J^-$ | 4 |
| Flip ($F = \sqrt{3}, S = -m' = 3/2$) | $800 + 816J^+ + 64\sqrt{3}J^- + 442J^-$ | $2048 + 384J^+ + 128\sqrt{3}J^- + 442J^-$ | 4 |
| Flip ($F = 3, S = -m' = 9/2$) | $800 + 6288J^+ + 768\sqrt{3}J^- + 17170J^-$ | $2048 + 7296J^+ + 1536\sqrt{3}J^- + 17170J^-$ | 4 |

| Parameters| NM-spin flipper-NM-I-SOCSW| NM-spin flipper-NM-I-p-wave |
|---------|-----------------------------|-------------------------------|
| $Z = 3, E = 0, \alpha = 0, \Delta_0 = 0.001, B_2 = 1.5\Delta_0$| $G_c/G_0$ ($\mu_{pSc} = 0.001$) |
| No flip ($F = 0, S = m'$) | 2 | 2 | 4 |
| Flip ($F = 1, S = -m' = 1/2$) | $53792 + 15724J^+ + 1904J^- + 480J^+ + 50J^-$ | $20996 + 1788J^+ + 1242J^- + 338J^+ + 29J^-$ | 4 |
| Flip ($F = 3, S = -m' = 9/2$) | $53792 + 47212J^+ + 42576J^- + 3604J^+ + 17170J^-$ | $20996 + 8448J^+ + 5604J^- + 8404J^+ + 1024J^-$ | 4 |
the states at the interface of metal-SOCSW junction as “Majorana states”.

*colin.nano@gmail.com*

[1] C. W. J. Beenakker, Annu. Rev. Con. Mat. Phys. 4, 113 (2013).
[2] A. Y. Kitaev, Phys. Usp. 44, 131 (2001).
[3] M. T. Deng, C. L. Yu, G. Y. Huang, M. Larsson, P. Caroff, and H. Q. Xu, Nano Lett. 12, 6414 (2012).
[4] A. Das, Y. Ronen, Y. Most, Y. Oreg, M. Heiblum, and H. Shtrikman, Nat. Phys. 8, 887 (2012).
[5] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, and E. P. A. M. Bakkers, and L. P. Kouwenhoven, Science 336, 1003 (2012).
[6] Jin-Peng Xu, et. al., Phys. Rev. Lett. 114, 017001 (2015).
[7] S. N. Perge, et. al., Science 346, 602 (2014).
[8] F. Setiawan, P. M. R. Brydon, Jay D. Sau, and S. Das Sarma, Phys. Rev. B 91, 214513 (2015).
[9] G. E. Blonder, M. Tinkham & T. M. Klapwijk, Phys. Rev. B 25, 4515 (1982).
[10] O. L. T. de Menezes and J. S. Helman, American Journal of Physics 53, 1100 (1985).
[11] H.D. Liu, X. X. Yi, Phys. Rev. A 84, 022114 (2011).
[12] G. Cordourier-Maruri, Y. Omar, R. de Coss, and S. Bose, Phys. Rev. B 89, 075426 (2014).
[13] F. Ciccarello, G. M. Palma, and M. Zarcone, Phys. Rev. B 75, 205415 (2007).
[14] S. Pal and C. Benjamin, Scientific Reports 8: 11949 (2018).
[15] J. D. Sau, S. Tewari, R. M. Lutchyn, T. D. Stanescu, and S. DasSarma, Phys. Rev. B 82, 214509 (2010).
[16] Q. Cheng and B. Jin, Physica B 426, 42 (2013).
[17] S. Kashiwaya, Y. Tanaka, N. Yoshida, and M. R. Beasley, Phys. Rev. B 60, 3572 (1999).