The effect of underexpanded jet flow conditions on the supersonic core length

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Abstract. The paper describes the results of measurements of the supersonic core length of underexpanded jets exhausted into a submerged space. Exhaustion of supersonic He, air, CO$_2$, Ar, and SF$_6$ jets into He, air, CO$_2$, Ar, and SF$_6$ at moderate Reynolds numbers for the laminar, transitional, and turbulent flow regimes is considered. These jets outflow from convergent nozzles with diameters of 0.52, 0.72 and 1.06 mm; the measurements are performed by a Pitot tube. The amplitude-frequency spectra of acoustic oscillations generated by the jets in the ambient space are measured. The experiments are performed in a low-pressure jet setup, which allows independently maintaining the Reynolds number of gas exhaustion from the nozzle and the jet pressure ratio. The jet setup is used to study exhaustion of a jet of one gas into the submerged space filled with another gas and the influence of the ratio of densities of these gases on the supersonic core length.

1. Introduction
Investigations of the mixing process in the case of exhaustion of supersonic micro-jets into a submerged space are of practical importance for local jet heating or cooling of the surface. Mixing of the gas jet with the ambient gas is the main process constricting the supersonic core length. This process is significantly affected by the parameters of gas exhaustion from the nozzle and by the ratio of the densities of gases in the jet and ambient space. The majority of experimental and theoretical investigations were performed for subsonic and isobaric supersonic two-dimensional mixing layers [1-8]. Nevertheless, it is not easy to extend these results to propagation of supersonic axisymmetric jets in the ambient space filled by a gas with different density. Supersonic axisymmetric underexpanded jets flowing out into a submerged space have some specific features associated with the existence of a shock wave structure in the supersonic flow region. Interaction of hydrodynamic disturbances in the mixing layer with the shock wave structure in the jet leads to global instability of the jet flow due to feedback during the transmission of acoustic perturbations to the nozzle over the ambient jet. For off-design jets and an ambient space with identical gas composition, this instability was intensely studied theoretically and experimentally for the purpose of acoustics of supersonic jet flows (see, e.g. [9, 10]). The flows of supersonic off-design jets in gas media with the density that differs from the jet gas density has not been adequately studied. The present paper describes an experimental study of the flow of supersonic underexpanded jets in different gases. The study was performed in a wide range of moderate Reynolds numbers of gas exhaustion from the nozzle, which covers the laminar, transitional, and turbulent jet flow regimes. The main attention is paid to the influence of the difference in the gas density in the jet and ambient medium on the supersonic core length of the underexpanded jet.
2. Experimental equipment and measurement technique

The experiments were performed in a low-pressure jet setup. The setup was a sealed chamber with a volume of 0.2 m³ evacuated by a high-efficiency vacuum pump. The orifice in the chamber wall was connected to the settling chamber having a convergent nozzle with a diameter \( d = 0.72 \) and 1.06 mm. The connection of the settling chamber with the sealed chamber allowed turning the settling chamber in the horizontal and vertical directions around the axes passing through the nozzle exit plane. As a result, the jet axis could be deflected in the horizontal and vertical directions for the direction to the Pitot tube input orifice to be adjusted without changing the nozzle exit position.

Such a scheme of jet flow organization was used in experiments with jets at “room” temperature. The test gases were air, He, Ar, CO₂ and sulfur hexafluoride. The gases were injected into the settling chamber through a low-flow-rate needle-shaped valve, which established the stagnation pressure of the jet \( P_0 \). The chamber where the jet exhausted was evacuated through a valve with a varied cross section to establish a prescribed pressure \( P_C \) in the chamber and to maintain a constant jet pressure ratio \( n \). The measurements were performed at the jet flow axis by a Pitot tube with an outer diameter of 0.4 mm and an inner diameter of 0.17 mm, which was mounted on a three-component traversing gear. The value of Pitot pressure was recorded with the use of a 12-bit ADC with a frequency of 20 Hz. The jet setup was equipped with a controlled system for gas injection into the chamber with a flow rate up to 1 liter per second at normal pressure for creating an ambient medium with a prescribed molecular weight. As a result, it was possible to generate and study various combinations of jet-ambient medium flows from the above-mentioned list of gases. The normalized supersonic core length \( Lc/d \) was determined as the distance from the nozzle exit to the point on the jet axis where the flow velocity is equal to the local velocity of sound. The possibility of using the Pitot tube for measuring the supersonic core length of the jets was investigated in [11].

The experiments with heated jets were performed with the help of the nozzle block. It includes a metal case with a convergent nozzle of 0.52 mm in diameter and a tubular copper heat exchanger, external ohmic heater and a thermocouple for gas temperature monitoring. The nozzle block fixtures and the tubes for gas injection are made of a non-heat-conducting material. The nozzle block was mounted inside the chamber of the low-pressure jet setup. The maximum gas temperature that could be reached was 650 K. During the measurements, the Pitot tube moved in three directions in order to reach the maximum signal of the pressure sensor, corresponding to the pressure of the transition from the supersonic to subsonic flow.

In the present study the amplitude of pressure fluctuations and the frequency of acoustic radiation generated by the underexpanded jet were also measured. The intensity of this radiation serves as an indicator of jet flow stability. The measurements were performed in jets flowing out of the nozzle block. The piezo-ceramic sensor of pressure fluctuations \( P_C 132A \) with the maximum frequency of 1 MHz was moved along the jet at a distance of 10 mm from the jet axis. Acoustic fluctuations in the medium around the jet were measured at points of the sensor motion trajectory. The amplitude-frequency spectra were calculated by using the Fast Fourier Transform (FFT). In these experiments the influence of the medium density and jet temperature on the intensity and frequency of acoustic oscillations of pressure was studied.

3. Results

3.1. Supersonic core length

Figure 1 shows the effect of the Reynolds number \( Re_d \) on the normalized supersonic core length \( Lc/d \) of the air, SF₆ and He jets propagating in air, SF₆ and He media. \( Re_d \) was based on the nozzle diameter \( d \) and flow parameters at the sonic nozzle exit. The drastic decrease in the supersonic core length of the underexpanded air jet flowing out into air is related to the transition of the jet flow from the laminar to turbulent regime [12]. In the case of air jet exhaustion into the SF₆ medium, SF₆ jet exhaustion into the
air medium and He jet exhaustion into the air medium there is a large difference between the gas densities in the jet and ambient medium. It is seen that a large difference in density between the gas of the jet and the gas of the surrounding space leads to an increase in the Reynolds number of the laminar–turbulent transition and an increase in the length of the supersonic core.

Figure 2 shows the supersonic core length for the case of small differences in the gas densities in the jet and ambient space, in particular, air jet exhaustion into the air and CO₂ media and CO₂ jet exhaustion into the air and CO₂ media. It is seen that a small difference in the gas density of the jet and

the gas in the surrounding space does not lead to a significant change in the value of the Reynolds number of the laminar–turbulent transition or a change in the length of the supersonic core. The same results were obtained in experiments with Ar jet exhaustion into the air and Ar media. Moreover, no significant differences in the dependences of $L_c/d$ on $Re_d$ were found for the gas pair (Ar, CO₂) in the jet-ambient medium system within the experimental scatter of results.

The effect of a continuous change in the ratio of the gas density of the ambient space $\rho_a$ to the jet density $\rho_j$ on the normalized length of the supersonic core $L_c/d$ is demonstrated on the graphs in Fig. 3. In this experiment, the air jet propagated in a mixture of air and He with a variation in their fractions in the mixture, which allowed reducing smoothly the ratio of the density of surrounding medium to the jet density (Fig. 3a). A gradual increase in the ratio of the density of surrounding medium to the jet density was achieved by a variation of the fractions of air and SF₆ in the surrounding medium (Fig. 3b). It can be seen that a rapid increase in $L_c/d$ occurs for $1.0 < \rho_a/\rho_j < 0.4$, which agrees with the data in Fig. 1a. The ratio of the gas density in the ambient space of the jet to the gas density in the jet $\rho_a/\rho_j$ was determined with allowance for the increase in the gas density at the jet axis because of gas cooling.
3.2. Pressure fluctuations and frequency of acoustic radiation
As the supersonic core length and, hence, the mixing intensity, is determined by jet flow stability, it is necessary to find the relationship between the level of flow stability and the value of $\frac{\rho_a}{\rho_j}$. For this purpose we measured the characteristics of acoustic oscillations generated by the underexpanded jets in the ambient space, including the angular polar of acoustic radiation (Fig. 4), amplitude $p'_{ac}$ and frequency $f$ of acoustic oscillations as function of $\frac{\rho_a}{\rho_j}$ and stagnation temperature of the jet $T_0$.

Fig. 3 shows the amplitude of acoustic oscillations $p'_{ac}$ at the fundamental frequency of the underexpanded air jet versus $\frac{\rho_a}{\rho_j}$. It is seen that a decrease in the amplitude of pressure oscillations $p'_{ac}$ is observed as the ratio $\frac{\rho_a}{\rho_j}$ both decreases and increases. It should be noted that the frequency of oscillations $f$ increases in the case of jet exhaustion into the air-He mixture and decreases in the case of jet exhaustion into the air-SF$_6$ mixture. The detected trend was verified in experiments with heated air.
jets flowing out into air, because the ratio $\rho_a/\rho_j$ also increases if the jet is heated, and its density decreases.

4. Discussion
The results reported in the previous paragraph reveal a direct relationship between the suppression of generation of acoustic oscillations in the ambient medium by the underexpanded jet (enhancement of jet stability) and the increase in the supersonic core length. The acoustic oscillations can be suppressed both by changing the ratio of the density of the ambient medium around the underexpanded jet to the gas density in the jet by means of creating mixtures consisting of heavy and light gases and also by increasing the stagnation temperature. The most probable reason for underexpanded jet stability enhancement is suppression of global instability arising because of the presence of feedback in the underexpanded jet-ambient medium system. The theory of global instability of off-design jets containing a wave structure and screech tone generation was developed in [9, 10]. The screech tone is a product of interaction between downstream propagating instability waves that originate in the jet shear layer and the periodic shock cell structure. This interaction results in generation of strong acoustic waves, which propagate upstream (see Fig. 4) outside the supersonic region of the jet, perturbing the shear layer at the nozzle lip, thereby closing the feedback loop. The frequency of the oscillations at the fundamental frequency $f$ can then be estimated by accounting for the round-trip distance and the velocity of respective waves. The formula relating the oscillatory cycle period $t$ to the cell size of the wave structure $l$, velocity of sound in the ambient medium $c_0$, and velocity $u_c$ and Mach number $M_c$ of convection of perturbations in the shear layer of the jet was derived in [10]:

$$
t = \frac{l}{c_0} + \frac{1}{u_c} = \frac{l(l + M_c u_c)}{u_c}.
$$

In the shear layer of the jet, the Mach number of convection of perturbations is close to unity for the jet propagating in the same gas in the ambient medium. Then $c_0 \equiv u_c$, and the screech tone frequency $f$ can be obtained from the equation

$$
f = \frac{1}{t} = \frac{u_c}{l(l + M_c u_c)} \approx \frac{c_0}{2l}.
$$

The wavelength $\lambda$ in the ambient medium can be obtained from the formula

$$
\lambda = \frac{c_0}{f} \approx 2l.
$$

The Schlieren-visualization of the flow field and acoustic waves in the ambient space of air jets, flowing out into the air medium, shows that the screech tone wavelength is indeed close to the doubled length of the wave structure cells in the underexpanded jet. In this case, the interference of acoustic waves forms the angular polar of the intensity of the pressure pulsations which has the form $\cos \varphi$ (see Fig. 4). Though the oscillations can be also generated at the harmonic frequency, the oscillations at the fundamental frequency dominate because they are more unstable in the shear layer. Another reason for domination of oscillations at the fundamental frequency is the interference of acoustic waves generated on the cells of the wave structure of the jet that formed the angular distribution of the acoustic pressure oscillations as shown in Fig. 4. The waves propagate toward the nozzle close the feedback loop. The efficiency of transfer of pressure fluctuations toward the nozzle determines the feedback efficiency in the course of global instability of the underexpanded jet and depends on the velocity of sound $c_0$ in the ambient space. As $c_0$ increases (decreases), the frequency $f$ also increases (decreases). The sound velocity $c_0$ in the ambient space depends on the composition of gases in this region. The sound velocity
in a mixture of two gases with the densities $\rho_1$ and $\rho_2$ is calculated by the Wood formula ($\alpha_1$ and $\alpha_2$ are the volume fractions of the gases in the mixture):

$$c_0 = \left(\frac{1}{\alpha_1\rho_1 + \alpha_2\rho_2}\left(\frac{\alpha_1}{\rho_1c_1^2} + \frac{\alpha_2}{\rho_2c_2^2}\right)\right)^{1/2}$$

Fig. 5 shows the velocity of sound in a mixture of gases as a function for the ratio of the air density to the air-He and air-SF$_6$ mixture density. A strong change in the value of $c_0$ from the ratio $\rho_{\text{mix}}/\rho_{\text{air}}$ can be seen. As a result, the feedback loop of the mechanism of global instability of the underexpanded air jet is violated, the screech tone frequency becomes different, and the direction diagram of acoustic perturbations generated by the jet changes. As a consequence, the jet becomes stable to the development of perturbations in the shear layer, the mixing process becomes less intense, and the supersonic core length increases.

**Conclusion**

The supersonic core length in supersonic axisymmetric underexpanded air, He, Ar, CO$_2$ and SF$_6$ jets exhausting into motionless mixtures of air, He, Ar, CO$_2$ and SF$_6$ was measured in the range of moderate Reynolds numbers of exhaustion. Experiments with a heated air jet flowing out into the air medium were performed as well.

The supersonic core length of the jet was obtained as a function of the Reynolds number of exhaustion and the gas composition of the ambient medium. Data on the intensity and frequency of acoustic radiation generated by the jet and on the influence of the density ratio of the gases in the jet and ambient medium were obtained.

It was demonstrated that the supersonic off-design jet becomes more stable and the supersonic core length becomes greater as the difference in the gas densities in the jet and ambient medium increases, regardless of the sign of the density gradient in the underexpanded jet-ambient medium system.

It was found that underexpanded jet stabilization and, hence, the increase in the supersonic core length are related to suppression of the mechanism of global instability for the flow in a gas medium whose velocity of sound differs from the velocity of disturbances convection in the jet shear layer. The velocity of sound in the ambient medium is determined by the density and normalized volumes of its components.

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**References**

[1] Brown G L and Roshko A 1974 *J. Fluid Mech.* 64 775–816
[2] Bogdanoff D W 1984 *AIAA J.* 22 1550–5
[3] Dimotakis P E 1986 *AIAA J.* 24 1791–6
[4] Koochesfahani M M and Frieler C E 1989 *AIAA J.* 27 1735–40
[5] Hegde U G and Zinn B T 1990 *AIAA J.* 28 1389–96
[6] Soterion M C and Ghoniem A 1995 *Phys. Fluids* 7 2036–51
[7] Peroomian O 1996 *Phys. Fluids* 8 225–40
[8] Peroomian O 1996 *Phys. Fluids* 8 241–7
[9] Powell A 1953 *Acoustica* 3 233–43
[10] Tam C K. W, Seiner J M and Yu J C 1986 *J. Sound Vibration* 110 309–21
[11] Mironov S G, Aniskin V M, Korotayeva T A and Tstryulnikov I S 2019 *Micromachines* 10 (4) 235–48
[12] Aniskin V M, Mironov S G, Maslov A A and Tstryulnikov I S 2015 *Microfluid Nanofluid* 19 621–34