AFFINE ALGEBRAS, N=2 SUPERCONFORMAL
ALGEBRAS, AND GAUGED WZNW MODELS

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ABSTRACT

We find a canonical N=2 superconformal algebra (SCA) in the BRST complex associated to any affine Lie algebra \( \hat{h} \) with \( h \) semisimple. In contrast with the similar known results for the Virasoro, N=1 supervirasoro, and \( W_3 \) algebras, this SCA does not depend on the particular “matter” representation chosen. Therefore it follows that every gauged WZNW model with data \( (g \supset h, k) \) has an \( N=2 \) SCA with central charge \( c = 3 \dim h \) independent of the level \( k \). In particular, this associates to every embedding \( sl(2) \subset g \) a one-parameter family of \( c=9 \) \( N=2 \) supervirasoro algebras. As a by-product of the construction, one can deduce a new set of “master equations” for generalized \( N=2 \) supervirasoro constructions which is simpler than the one considered thus far.

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1 Introduction

For some time now it has been realized that to any conformal field theory with $N=2$ superconformal symmetry one can associate a cohomology theory which is intimately related to the topological conformal field theory one obtains after twisting\cite{1}. The converse statement—namely, that in any "reasonable" cohomology theory one can find an underlying $N=2$ SCA is not so firmly established, based mostly on indirect arguments and special examples taken from string theory. These examples suffer from one major drawback: the $N=2$ SCA is not at all evident and finding it requires a tedious and unenlightening albeit systematic procedure. Moreover the $N=2$ depends on the "matter" representation and hence the construction is not canonical. The purpose of this paper is to describe a natural $N=2$ SCA present in the BRST complex associated to any (untwisted) affine algebra $\hat{\mathfrak{h}}$ with $\mathfrak{h}$ semisimple. In particular, this means that every gauged WZNW model with semisimple gauged subgroup has an $N=2$ SCA whose central charge, as we will see, is fixed to $c = 3 \dim \mathfrak{h}$.

To place this result in context, let us first consider the $N=2$ supervirasoro algebra. It is generated by fields $J, G^\pm$ and $T$, where $T$ generates a Virasoro algebra relative to which $J$ and $G^\pm$ are primary fields of weights 1 and $3/2$ respectively. The remaining OPEs are given by

$$J(z)J(w) = \frac{c/3}{(z-w)^2} + \text{reg.},$$

$$J(z)G^\pm(w) = \pm \frac{G^\pm(w)}{z-w} + \text{reg.},$$

$$G^\pm(z)G^\pm(w) = \text{reg.},$$

and

$$G^\pm(z)G^\mp(w) = \frac{2c/3}{(z-w)^3} \pm \frac{2J(w)}{(z-w)^2} + \frac{2T(w) \pm J(w)}{z-w} + \text{reg.}.$$  \hspace{1cm} (1.4)

In particular, (1.3) implies that the charge $Q$ corresponding to $G^+(z)$ ($Q = G^+_{-1/2}$ in the Neveu-Schwarz sector and $Q = G^+_{-1}$ in the Ramond) squares to zero. The resulting cohomology defines the chiral ring of the theory.

Relative to the twisted energy-momentum tensor $T_{\text{twisted}} \equiv T + \frac{1}{2} \partial J$, the spectrum of the fields changes, since the conformal weight receives contributions from the charge. Hence positively (resp. negatively) charged fields decrease (resp. increase) their conformal weight by half of its charge, while neutral fields weigh the same. This means that $G^+$ is now a primary field of weight 1, whereas $G^-$ has weight 2. Moreover, $T_{\text{twisted}}(z) = \frac{1}{2} [Q, G^-(z)]$. This is very reminiscent of critical string theory, where the roles of $J, G^+, G^-$, and $T_{\text{twisted}}$ are played by the ghost number current, the BRST current, the antighost, and the total energy-momentum tensor, respectively. However the analogy seems to break down since, for example, the BRST current does not have a regular OPE with itself. The crucial observation is that the BRST current is defined up to a total derivative and that in some cases (for example, if there is a free boson in the theory) one can modify the BRST current in such a way that the singularities in the OPE with itself cancel. This also induces a modification in the ghost number current.
The resulting $N=2$ algebra was shown explicitly for the first time in the literature in [2] (see also [3]) for the $c=1$ noncritical string. In [4] this observation was generalized and argued to be a generic property of string theories: be it the “humble” string, the superstring, or the $W$-string. Explicit constructions of the $N=2$ algebra depend on the model under consideration and were given in [4] for the bosonic string (critical and noncritical), the NSR string (critical and noncritical)—where one actually has an $N=3$ SCA—and for the noncritical $W_3$-string, where one finds an extension of $N=2$ by an $N=2$ primary of weight 2 and charge 0. For the critical $W_3$ string with matter representation given by the Romans realization of $W_3$ in terms of a free boson and an underlying Virasoro algebra [5] one can also construct such an $N=2$ extended algebra [6] whose central charge is fixed to either of the two values $c = -18$ or $c = -\frac{15}{2}$. It is expected that this continues to be the case for other $W$-string theories built on other $W$-algebras; but proving it by the current means requires knowing (at least the existence of) the BRST charge for the relevant $W$-algebra, for which no general construction is known and hence must be done case by case. The computational complexity soon becomes forbidding (even to a computer) and so far the only other algebras whose BRST charge is known are: $W_4$ [7] [8], $WB_2$ [8] and some special quadratically nonlinear algebras [9]. One can certainly envision a few more algebras to be reached in the near future, but this is far from a general existence proof.

Since $W$-algebras are generically defined via the quantum Drinfel’d-Sokolov reduction [10], it is widely believed that the BRST charge and the underlying $N=2$ structure should also come induced from the analogous structures in the affine Lie algebra from which one reduces. The BRST charge for an (untwisted) affine algebra $\hat{h}$ is as old as semi-infinite cohomology itself. It corresponds to the differential in the subcomplex of the Feigin standard complex computing the semi-infinite cohomology of $\hat{h}$ relative to the center, and its definition goes back to Feigin [11]; although it is not inconceivable that it could be found already in the earlier physics literature.

In this paper we construct the underlying $N=2$ structure for the case of $h$ semisimple. No canonical construction seems to exist for $h$ abelian; although depending on the “matter” representation, an $N=2$ SCA sometimes exist. Since our construction is otherwise completely general, it means that this $N=2$ structure is present in any gauged WZNW model with semisimple gauged subgroup.

The plan of this paper is as follows. In section 2 we will review the construction of the BRST charge for an affine Lie algebra $\hat{h}$ with $h$ semisimple and define the other generators of the algebra. In a nutshell, the roles of $J$, $G^+$ and $G^-$ are played by the ghost number, BRST and antiBRST currents, respectively. In section 3 we prove that they correspond indeed to an $N=2$ SCA. In doing so we have found what we believe is the minimal data necessary to guarantee the existence of an $N=2$ supervirasoro algebra. The results of this section could thus be useful in the context of generalized $N=2$ supervirasoro constructions. In section 4 we apply this to the gauged WZNW model with data $(g \supset h, k)$. The construction yields for fixed $(g \supset h)$ a family of $N=2$ SCA’s with central charge $c = 3 \dim h$ indexed by the level $k$. Section 5 summarizes the results of the paper.
In this section we describe the BRST complex for an affine Lie algebra. Let us consider a semisimple Lie algebra \( h \) with a fixed invariant nondegenerate bilinear form \( \gamma \). Let us (reluctantly) introduce a basis \( \{ X_i \} \) for \( h \) relative to which the structure constants will be denoted by \( f_{ij}^k \) and such that the bilinear form \( \gamma \) has entries \( \gamma_{ij} \). We let \( \gamma^{ij} \) denote the inverse to \( \gamma_{ij} \), and \( c_h \) denote the eigenvalue of the Casimir element \( \gamma^{ij} X_i X_j \) on the adjoint representation; in other words,

\[
f_{ik}^l f_{jl}^k = c_h \gamma^{ij} .
\]  

We let \( \hat{h} \) denote the (untwisted) affine algebra associated to \( h \). It is spanned by currents \( \{ J_i(z) \} \) obeying the following OPE:

\[
J_i(z) J_j(w) = \frac{-c_h \gamma_{ij}}{(z-w)^2} + \frac{f_{ij}^k J_k(w)}{z-w} + \text{reg.} .
\]  

We have fixed the level to \(-c_h\) for reasons that are standard and which will become apparent presently. The other ingredients are the (fermionic) ghost fields \( (b_i, c_i) \) with the standard OPE:

\[
b_i(z)c_j(w) = \frac{\delta^j_i}{z-w} + \text{reg.} .
\]  

One can build \( \hat{h} \) currents out of ghost bilinears \( J_{gh}^{ih}(z) \equiv f_{ij}^k b_k c^j(z) \). The ghost currents satisfy the OPE

\[
J_{gh}^{ih}(z) J_{gh}^{ij}(w) = \frac{c_h \gamma_{ij}}{(z-w)^2} + \frac{f_{ij}^k J_{gh}^{kh}(w)}{z-w} + \text{reg.} .
\]  

Notice that the total currents \( J_{tot}^i = J_i + J_{gh}^{ih} \) represent \( \hat{h} \) at level 0. This fact is equivalent (see, for example, \([13]\)) to the “nilpotency” of the BRST charge \( Q \), which is the charge associated to the BRST current

\[
G^+(z) \equiv \frac{2i}{\sqrt{c_h}} \left( c^i J_i(z) + \frac{1}{2} c^i J_{gh}^{ih}(z) \right) ,
\]  

where we have chosen a convenient normalization. The equation \( Q^2 = 0 \) is equivalent to the fact that the first order pole in the OPE \( G^+(z) G^+(w) \) is a total derivative. Here, however, we find the stronger result that the OPE \( G^+(z) G^+(w) \) is regular. This is to be contrasted with the BRST current in (W-)string theory in which one finds singularities in its OPE with itself. Depending on the matter content of the string theory into consideration, these singularities can be cancelled by modifying the BRST current with total derivative pieces. But this makes the construction of the associated \( N=2 \) SCA rather noncanonical.

\footnote{Products of fields at the same point are assumed to be normal-ordered according to the point-spittling convention and, moreover, normal-ordering associates to the left (see, for example, \([12]\)).}
Let us now introduce the ghost number current
\[ J(z) \equiv c^i b_i(z) . \] (2.6)
A quick calculation shows that it obeys the following OPE
\[ J(z) G^+ (w) = \frac{G^+(w)}{z - w} + \text{reg.} . \] (2.7)
We now introduce an automorphism \( \pi \) of our operator product algebra:
\[ \pi(b_i) = \gamma_{ij} c^j \quad \pi(c^i) = \gamma_{ij} b_j \quad \pi(J_i) = J_i . \] (2.8)
It is an involutive automorphism that fixes the ghost currents \( \pi(J_{gh_i}) = J_{gh_i} \). Since it is essentially ghost conjugation, it obeys \( \pi(J) = -J \). Let us define \( G^- \equiv \pi(G^+) \); or, explicitly,
\[ G^-(z) \equiv \frac{2i}{\sqrt{c_h}} \gamma^{ij} \left( b_i J_j(z) + \frac{1}{2} b_i J_{gh_j}(z) \right) . \] (2.9)
Because \( \pi \) is an automorphism, we immediately have that
\[ J(z) G^-(w) = \frac{-G^-(w)}{z - w} + \text{reg.} , \] (2.10)
and also
\[ G^-(z) G^-(w) = \text{reg.} . \] (2.11)
A calculation now shows that
\[ G^+(z) G^-(w) = \frac{2c/3}{(z - w)^3} + \frac{2J(w)}{(z - w)^2} \frac{2T(w) + \partial J(w)}{z - w} + \text{reg.} , \] (2.12)
where \( c = 3 \dim h \) and \( T \) is found to be
\[ T = -\frac{1}{c_h} \gamma^{ij} \left( 2J_i J_j + J_{gh_i} J_{gh_j} + J_{gh_i} J_j + J_{gh_i} J_{gh_j} \right) + \frac{1}{2} \left( \partial b_i c^i - b_i \partial c^i \right) . \] (2.13)
We will see in the next section that these results together with the associativity of the underlying operator algebra, already imply that \( (J, G^\pm, T) \) generate an \( N=2 \) SCA with central charge \( c = 3 \dim h \).

The form of \( T \) may seem at first strange, but let us notice some things. The last term is nothing but the energy-momentum tensor for free fermions \( b_i \) and \( c^i \). In terms of the total currents \( J_{\text{tot}}^i \), we can rewrite the rest of \( T \) as follows
\[ -\frac{1}{c_h} \gamma^{ij} (J_{\text{tot}}^i J_{\text{tot}}^j + J_i J_j) \] (2.14)
The first term is—up to a sign—the Sugawara tensor for the total currents which represent \( \hat{h} \) at level zero; whereas the second term is proportional to what would be the Sugawara tensor for the \( J_i \), only that these currents are at the critical level, where the Sugawara construction breaks down. One should stress that whereas the last term of (2.13) generates a Virasoro algebra on its own, (2.14) does not. It is nevertheless interesting to remark that energy-momentum tensors of this form have appeared in the literature [14] as generalized Virasoro constructions based on affine Lie superalgebras. In fact, what appears in [14] is a one-parameter family of such tensors. Perhaps “affine Virasoro space” at the critical level deserves a closer study.
§3 When do we have an $N=2$ SCA?

In this section we prove a result, which may be of independent interest in the context of generalized $N=2$ constructions. It concerns the minimal data necessary to guarantee that we have an $N=2$ SCA. We tacitly assume the associativity of the operator product expansion. Suppose we have fields $G^\pm$ satisfying (1.3) and

$$G^+(z)G^-(w) = \frac{2c/3}{(z-w)^3} + \frac{2J(w)}{(z-w)^2} + \frac{2T(w) + \partial J(w)}{z-w} + \text{reg.}, \quad (3.1)$$

which defines $c$, $J$, and $T$. Then if, in addition, (1.2) is also obeyed, then $(J,G^\pm,T)$ satisfies an $N=2$ SCA with central charge $c$. In particular, $T$ is guaranteed to satisfy a Virasoro algebra with central charge $c$.

The proof exploits the associativity of the operator product expansion to compute the remaining OPEs and verify that they are indeed the ones of an $N=2$ SCA. It will prove convenient to introduce the following notation. If $A$ and $B$ are any two local fields, their OPE defines a family of bilinears $[\ ]_\ell$ by

$$A(z)B(w) = \sum_{\ell} \frac{[AB]_\ell(w)}{(z-w)\ell}. \quad (3.2)$$

The associativity of the operator product expansion implies certain well-known identities between nested bilinears. In particular, a very useful one is

$$[A[BC]_n]_m = (-)^{|A|}[B[AC]_m]_n + \sum_{\ell=0}^{m-1} \binom{m-1}{\ell} [AB]_{m-\ell}C]_{n+\ell}, \quad (3.3)$$

which is valid for all $A$, $B$, $C$ and all positive $n$, $m$. There are similar identities for all $m$ and $n$ but we shall not need them. Putting $m = 1$ in (3.3), we find that for all $A$ the operation $B \mapsto [AB]_1$ is a superderivation over the operator product.

We begin by computing $J(z)J(w)$. From (3.1) we see that $J = \frac{1}{2}[G^+G^-]_2$. Therefore,

$$[JJ]_n = \frac{1}{2}[J[G^+G^-]_2]_n$$

$$= \frac{1}{2}[G^+[JG^-]_n]_2 + \frac{1}{2} \sum_{\ell=0}^{n-1} \binom{n-1}{\ell} [[AB]_{n-\ell}C]_{2+\ell} \quad \text{by (3.3)}$$

$$= -\frac{1}{2} \delta_{n1}[G^+G^-]_2 + \frac{1}{2}[G^+G^-]_{n+1},$$

from which we read off

$$J(z)J(w) = \frac{c/3}{(z-w)^2} + \text{reg.}, \quad (3.4)$$

agreeing with (1.1).
We now compute \( T(z)J(w) \). From (3.1) we find that
\[
[TJ]_n = \frac{1}{2}[J[G^+G^-]_1]_n - \frac{1}{2}[J\partial J]_n
\]
from where we can read off
\[
T(z)J(w) = \frac{J(w)}{(z-w)^2} + \frac{\partial J(w)}{z-w} + \text{reg.} .
\]  

(3.5)

Let us now compute \( T(z)G^\pm(w) \). We shall do \( T(z)G^+(w) \) first. The result for \( T(z)G^-(w) \) will then follow after applying the involutive automorphism \( J \mapsto -J, G^\pm \mapsto G^\mp, T \mapsto T \) of the abstract algebra defined by (1.2), (1.3), and (3.1). Substituting for \( T \) as before, we obtain
\[
[G^+T]_n = \frac{1}{2}[G^+[G^+G^-]_1]_n - \frac{1}{2}[G^+\partial J]_n
\]
From this we read that \( [G^+T]_2 = \frac{3}{2}G^+ \) and \( [G^+T]_1 = \frac{1}{2}\partial G^+ \) are the only nonzero terms in the singular part of the OPE. Therefore, and after applying the automorphism,
\[
T(z)G^\pm(w) = \frac{\frac{3}{2}G^\pm(w)}{(z-w)^2} + \frac{\partial G^\pm(w)}{z-w} + \text{reg.} .
\]  

(3.6)

Finally we compute \( T(z)T(w) \). Substituting for \( T \) once, we again find
\[
[TT]_n = \frac{1}{2}[T[G^+G^-]_1]_n - \frac{1}{2}[T\partial J]_n
\]
which expands to the expected
\[
T(z)T(w) = \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} + \text{reg.} .
\]  

(3.7)

We now briefly comment on how this result can be helpful in the search for generalized \( N=2 \) supervirasoro constructions. Generalized \( N=2 \) supervirasoro constructions starting from Lie algebraic data have not been subject to the same intensive study as its \( N=0 \) and \( N=1 \) counterparts. The only reference seems to be an appendix of [15] where a set of “master equations” are written down which are sufficient for the construction of an \( N=2 \) supervirasoro algebra out of a Kač–Todorov algebra with data \((\mathfrak{g},k)\). As a result of trying
to construct an $N=2$ algebra with $N=1$ ingredients, one finds that no natural Sugawara-type solution exists. Instead one finds only the Kazama-Suzuki construction. This is not surprising and perhaps one should attempt the construction from an $N=2$ extension of an affine Lie algebra, of the type considered by Hull and Spence [16]. Work on this is in progress [17]. Nevertheless, in order to make contact with the existing literature, we shall present a simplified set of $N=2$ master equations on a Kač–Todorov algebra. There is no reason to expect that the $N=2$ SCA under construction should preserve the $N=1$ present in the Kač–Todorov algebra, so we shall not insist in working in $N=1$ superfields, but in components.

The Kač–Todorov algebra associated to the data $(g,k)$ is the extension of \( \hat{g} \) at level $k$ (represented by the currents $J_i(z)$) by dim $g$ free fermions ($\psi_i(z)$) transforming in the adjoint representation of $g$. The algebra is defined by the following OPE’s:

\[
J_i(z)J_j(w) = \frac{kg_{ij}}{(z-w)^2} + \frac{f_{ij}^k J_k(w)}{z-w} + \text{reg.}, \\
J_i(z)\psi_j(w) = \frac{f_{ij}^k \psi_k(w)}{z-w} + \text{reg.}, \\
\psi_i(z)\psi_j(w) = \frac{kg_{ij}}{z-w} + \text{reg.}.
\]

According to general decoupling arguments (see [18] generalizing the classical result of Goddard-Schwimmer [19]), one can decouple the fermions by shifting the currents $J_i \rightarrow J_i - \frac{1}{2k} f_{ij}^k g^{kj} \psi_j \psi_l$. Whereas decoupled fermions may simplify some of the calculations, notice that the shift is illegal for $k = 0$. Since we expect that something should happen there, we will leave the fermions coupled.

According to the results in this section, one need only write the most general $G^\pm$:

\[
G^\pm = A^{ij}_\pm \psi_i J_j + B^{ijk}_\pm \psi_i \psi_j \psi_k + C^i_\pm \partial \psi_i , \quad (3.8)
\]

where $B^{ijk}_\pm$ is totally antisymmetric and $A^{ij}_\pm$ is general. The first equations come from imposing (1.3). Computing $G^+(z)G^-(w)$ one finds that the central charge is given simply by $c = \frac{3}{2}[G^+G^-]_3$, whereas $J$ is given by $J = \frac{1}{2}[G^+G^-]_2$. The final set of equations comes from demanding (1.2). From the results of this section this is sufficient to guarantee an $N=2$ supervirasoro algebra. This not being the main point of the paper, we will refrain from writing the equations down explicitly. The interested reader should find it a good exercise to reproduce them.

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Gauged WZNW models provide a very large class of examples to which the construction in Section 2 applies. We briefly review the algebraic setup of the WZNW model and we refer the reader to [20] for more details. Algebraically, the gauged WZW model consists of three independent theories coupled by constraints which are treated à la BRST. Consider a pair of finite-dimensional semisimple Lie algebras \( h \subset g \). For ease of exposition we treat the case where the index of embedding is 1. Clearly the construction goes through in the general case. Let us consider their affine counterparts \( \hat{h} \subset \hat{g} \). We again choose a basis \( \{ X_i \} \) for \( h \) which we complete to a basis \( \{ X_a \} \) for \( g \). We consider an invariant non-degenerate symmetric bilinear form \( \gamma \) on \( g \). Relative to the basis \( \{ X_a \} \), \( \gamma \) has entries \( \gamma_{ab} \) and its restriction to \( h \) has entries \( \gamma_{ij} \). Consider then the algebra of currents of \( g \) at level \( k \):

\[
\hat{J}_a(z)\hat{J}_b(w) = \frac{k\gamma_{ab}}{(z-w)^2} + \frac{f_{ab}^c\hat{J}_c(w)}{z-w} + \text{reg.} .
\] (4.1)

This is our first theory: it corresponds to a (nongauged) WZNW model with algebra \( g \) at level \( k \). The second ingredient is another WZNW model with algebra \( h \) at level \( -k - c_h \) and described by currents

\[
\tilde{J}_i(z)\tilde{J}_j(w) = \frac{(k - c_h)\gamma_{ij}}{(z-w)^2} + \frac{f_{ij}^k\tilde{J}_k(w)}{z-w} + \text{reg.} .
\] (4.2)

The third and final theory are the ghosts \( (b_i, c^i) \) obeying (2.3). The constraints are then simply the currents \( J_i = \hat{J}_i + \tilde{J}_i \) which precisely satisfy (2.2). The data \( (J_i, b_i, c^i) \) is precisely the data that we need in order for the construction in Section 2 to work. Thus we conclude that in every gauged WZNW with data \( (g \supset h, k) \) with \( h \) semisimple, there is a canonical \( N=2 \) superconformal algebra with \( c = 3 \dim h \). Notice that this is independent of the level \( k \), hence in effect it gives a one-parameter family of \( N=2 \) SCA’s associated to every \( g \supset h \).

In the case of \( h \) some real form of \( sl(2) \), we find an infinite series of one-parameter families of \( c=9 \) theories, each family associated to an embedding \( sl(2) \subset g \). This same data seems to comprise the “moduli space” of \( W \)-algebras. Both this fact and the fact that \( c=9 \) is the phenomenologically interesting central charge in string theory are very intriguing and certainly deserve further study.

Finally, one should hasten to add that the \( N=2 \) SCA does not descend to the physical states; that is, it is not a symmetry of the coset model to which the gauged WZNW is equivalent. In fact, one of the generators is the BRST current which is BRST exact: \( G^+ = -[Q, J] \). For the same reason, neither does the twisted energy-momentum tensor \( T_{\text{twisted}} = T + \frac{1}{2} \partial J = [Q, G^-] \) descend. This is not surprising since not every coset theory has \( N=2 \) superconformal symmetry: it seems, in fact, that in the context of supersymmetric gauged WZNW models at least, it requires \( G/H \) to admit a Kähler structure [21] [22]. Nevertheless, it may be possible, via the identification between the chiral ring of the \( N=2 \) SCA and the physical states in the gauged WZNW model (equivalently, the coset model), that results in one area can help understand the other.
To recapitulate, we have shown the existence of a natural $N=2$ supervirasoro algebra in the BRST complex associated to any (untwisted) affine algebra $\hat{\mathfrak{h}}$ with semisimple $\mathfrak{h}$. Unlike the previous cases of $N=2$ SCA’s found in BRST complexes, this one is canonical and does not depend on the “matter” representation of the current algebra. Therefore this construction applies to any gauged WZNW model with data $(\mathfrak{h} \subset \mathfrak{g}, k)$ with $\mathfrak{h}$ semisimple.

The central charge of the $N=2$ SCA is equal to $3 \dim \mathfrak{h}$ which is independent of the level $k$. For $\mathfrak{h} = \mathfrak{sl}(2)$, this construction associates a one-parameter ($k$) family of $c=9$ $N=2$ SCA’s to any embedding $\mathfrak{sl}(2) \in \mathfrak{g}$—a highly intriguing fact since it seems to connect the “moduli space” of $\mathcal{W}$-algebras to the phenomenologically interesting string theories with $c=9$ $N=2$ superconformal symmetry.

As a more technical by-product of this construction, we have given what we think is the minimal data required to guarantee an $N=2$ supervirasoro algebra. This should simplify the search of generalized $N=2$ supervirasoro constructions since the set of “master equations” that need to be satisfied is now somewhat simpler than was thought in the past.

Finally, we should mention two immediate extensions of the results in this paper, both in supersymmetric directions. One is to consider also $\mathfrak{h}$ a semisimple Lie superalgebra with nondegenerate Cartan-Killing form. The other extension keeps $\mathfrak{h}$ a semisimple Lie algebra, but considers instead of $\hat{\mathfrak{h}}$, the associated Kac-Todorov algebra. This will be reported in [23].

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Note added:

After submission of this paper, Ezra Getzler informed me that this construction is a special case of a method by which one may assign an $N=2$ SCA to any Manin triple [24]. Getzler’s construction, which seems to be implicit in [25], has as two extreme special cases the Kazama-Suzuki models on the one hand, and the present construction $(G/G)$ on the other. I am grateful to him for his correspondence and for sending me his preprint [24] where, in addition, one can also find the results of our Section 3.
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