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Two series expansions for the logarithm of the gamma function involving Stirling numbers and containing only rational coefficients for certain arguments related to $\pi^{-1}$. (English)

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Summary: In this paper, two new series for the logarithm of the $\Gamma$-function are presented and studied. Their polygamma analogs are also obtained and discussed. These series involve the Stirling numbers of the first kind and have the property to contain only rational coefficients for certain arguments related to $\pi^{-1}$. In particular, for any value of the form $\ln (\pi n + \alpha \pi^{-1})$ and $\Psi_k (\pi n + \alpha \pi^{-1})$, where $\Psi_k$ stands for the $k$th polygamma function, $\alpha$ is positive rational greater than $\frac{1}{2}$, $n$ is integer and $k$ is non-negative integer, these series have rational terms only. In the specified zones of convergence, derived series converge uniformly at the same rate as $\sum (n \ln^n n)^{-2}$, where $n = 1, 2, 3, \ldots$, depending on the order of the polygamma function.

Explicit expansions into the series with rational coefficients are given for the most attracting values, such as $\ln (\pi n^{-1})$, $\ln (2\pi n^{-1})$, $\ln (\pi n^{-1})$, $\Psi (\pi^{-1})$, $\Psi (\pi^{-1}) + \Psi (\pi^{-1})$ and $\Psi_k (\pi^{-1})$. Besides, in this article, the reader will also find a number of other series involving Stirling numbers, Gregory’s coefficients (logarithmic numbers, also known as Bernoulli numbers of the second kind), Cauchy numbers and generalized Bernoulli numbers. Finally, several estimations and full asymptotics for Gregory’s coefficients, for Cauchy numbers, for certain generalized Bernoulli numbers and for certain sums with the Stirling numbers are obtained. In particular, these include sharp bounds for Gregory’s coefficients and for the Cauchy numbers of the second kind.

MSC:

33B15 Gamma, beta and polygamma functions
11B73 Bell and Stirling numbers

Keywords:

gamma function; polygamma functions; Stirling numbers; factorial coefficients; Gregory’s coefficients; Cauchy numbers

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