Probing measurement problem of quantum theory with an operational approach

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Exploiting the tension between the two dynamics of quantum theory (QT) or so called “measurement problem” of QT in the Wigner’s Friend thought experiment, we point out that the textbook QT leads to inconsistency in observed probabilities of measurement outcomes between two observers - Wigner, and his Student. To avoid such inconsistent predictions of QT, we hypothesize two distinct solutions inspired by the perspectives of Wigner and Student, which we term as “Absoluteness of measurement (AoM)” and “Non-absoluteness of measurement (NoM)”. We introduce an operational approach, first with one friend and then with two spatially separated friends, to test the validity of these two perceptions without assuming the details of the experiment. We show that the set of probabilities obtainable for NoM is strictly larger than the set obtainable for AoM. We characterize different interpretations of QT based on both the perceptions and whether they lead to consistent predictions or not.

I. INTRODUCTION

Quantum theory (QT) has been one of the most successful theories describing nature. Even with the counter-intuitive predictions, QT has always been validated by experiments till date, not just for microscopic systems, but even large macroscopic states \cite{1-5}, complex molecules \cite{6,7} and living systems \cite{8}. Despite of it, a long sought after question in physics is whether there exists some scale above which QT is not valid. From a theoretical perspective, the postulates of QT are valid for any system. As a consequence, QT in general can be regarded to be universally valid unless we find some experiment which refutes the predictions of QT at some scale.

Universal validity of QT has far reaching consequences, as was first observed by Schrodinger in 1935 \cite{9}. Famously known as the Schrodinger’s cat paradox, the thought experiment involves putting a cat into a state of superposition of two different macroscopic states, dead or alive. The existence of such a state, even if classically incomprehensible, does not contradict quantum laws of physics. However, such a scenario leads to some discrepancy for quantum theory, which is known as the “measurement problem” of QT. The issue is nicely illustrated by Wigner in 1967 \cite{10} using a thought experiment involving two different observers who give two different descriptions of the same physical process of measuring a superposed quantum state. First observer, named Friend measures a quantum system and the second observer, named Wigner describes the Friend along with the experiment he performs, that is, the Friend’s lab. The discrepancy inherently lies in the fact that QT allows Wigner to describe Friend’s measurement as a reversible or unitary process, while according to Friend the measurement changes the quantum state irreversibly. Although, such a discrepancy can be avoided if we do not associate quantum state with any underlying reality of physical systems and take the standpoint that quantum state only represents some knowledge about physical systems. This thought experiment is commonly referred to as Wigner’s Friend (WF) scenario, and has been rigorously dealt later.

One can comprehend that the discrepancy arises in the WF scenario because of the presence of two different dynamics within QT, one when systems are evolving with time and other when systems are measured. Even when such discrepancy, or, the so called measurement problem, itself is counter intuitive, it does not give rise to any disagreement at the empirical level. Moreover, “measurement problem” is a problem for orthodox interpretation of QT wherein the quantum state is considered to be the realist description of physical system. This leads to a significant question of whether the tension between the two dynamics of QT results to any observable inconsistency? In other words, what would be the operational version of the inconsistency arising in WF thought experiment? If any inconsistency persists at the empirical level, then it would be particularly interesting to find different solutions that can resolve those inconsistencies and probe whether those solutions can be distinguished operationally.

In this work, we first propose an extension of Wigner’s Friend scenario, named as “Wigner’s Friend and Student’ scenario, that gives rise to inconsistency at the empirical level between two super-observers Wigner and his Student, due to the two different dynamics of QT. To resolve this inconsistency, we hypothesize two distinct solutions inspired by the perspectives of Student and Wigner: (1) “Absoluteness of Measurement (AoM),” that the measurement is an absolute event and yields a single outcome, (2) “Non-absoluteness of measurement (NoM),” that physical systems always evolve via time-evolution with respect to an observer unless the observer observes an outcome. To test the validity of these two hypotheses in an operational way, we extend the thought experiment wherein Friend first performs a measurement on the quantum system depending on an input variable, then one of the super-observers, say, Wigner, applies a transformation on Friend’s lab, and subsequently, both the super-observers ask Friend to re-
veal the measurement outcome as well as the input variable. We consider two full sets of probabilities obtained in universal quantum theory imposing the hypotheses of AoM and NoM, without assuming the details of the experiment. We show that set of probabilities obtainable for AoM is a proper subset of the set of probabilities obtained for NoM. We then further extend our Wigner’s Friend scenario for two spatially separated Friends, and provide an empirical expression to single out theories that are incompatible with AoM. One may consider this expression to be the figure of merit of an operational task in which a super-observer is restricted to apply reversible transformations. Notably, the optimal value of the figure of merit in QT with AoM is the same as in classical theory. We also analyze whether different interpretations of QT are compatible with these two perceptions and provide consistent predictions or not. Finally, we point out that the no-go theorems of QT shown in [11, 14, 15] are particular instances of the violation of AoM. This provides conceptual insight into these no-go theorems that the hypothesis of AoM that is fundamentally incompatible with standard quantum theory.

II. OPERATIONAL CONSISTENCY IN QUANTUM THEORY

We first introduce an extended version of WF experiment wherein the tension between two different dynamics of QT leads to inconsistency at the empirical level. Subsequently, we propose two perspectives for applying QT so that it provides consistent predictions.

A. Wigner’s Friend and Student thought experiment

The modified Wigner’s Friend and Student (WFS) scenario [10] can be described as follows: The Friend, who is confined in an isolated lab, receives a quantum system \(Q_s\), performs a measurement on it and subsequently, observes a definite outcome. Let us term the isolated lab including Friend, measurement device and the environment inside it by ”Lab”. Wigner and his Student, on the other hand, describe the physical process of the combined system of Lab and \(Q_s\). A logical inconsistency regarding the state of the combined system arises if QT is supposed to be universal as defined below.

**Definition 1** (Universal Quantum Theory (UQT)). The textbook or standard quantum theory is applicable to all physical systems including macroscopic systems like an observer.

To reflect on the above fact, let us take an explicit example (see Figure 1). The observer Friend receives a quantum state \(|\psi\rangle \in \mathbb{C}^2\) and obtains an outcome after performing Pauli-Z measurement \((\sigma_z)\) on it. Although, the physical interaction between Lab and \(Q_s\) is certainly a measurement process with respect to Friend, universal QT allows Wigner and Student to describe the same physical interaction either by time-evolution dynamics according to Schrödinger equation, or, by post-measurement dynamics according to collapse postulate. Say, according to Wigner the combined state of Lab and \(Q_s\) evolves unitarily. Now, we know that if the initial state of \(Q_s\) is an eigenvector of \(\sigma_z\), that is, \(|0\rangle\) or \(|1\rangle\), then after the interaction Friend observes a definite outcome +1 or −1 and the state \(Q_s\) remains unchanged. This empirical fact implies that the unitary \(U\) describing the evolution of the combined system must satisfy the following conditions,

\[
U|0\rangle|f\rangle = |0\rangle|f_+\rangle, \quad U|1\rangle|f\rangle = |1\rangle|f_-\rangle,
\]

where \(|f\rangle\) is the initial state of Lab, and \(|f_\pm\rangle\) is the final state of the Lab such that the measurement device shows a definite outcome ±1 and Friend observes a definite outcome ±1. Besides, whenever Friend performs a measurement, she encodes the information that she has obtained a definite outcome on an ancillary system \(|\psi\rangle_{an} \in \mathbb{C}^2\) (this could be a classical system) and keeps outside his/her lab. Subsequently, the complete descriptions of Lab, \(Q_s\) and ancilla with respect to Wigner are

\[
\begin{align*}
U|0\rangle|f\rangle \otimes \sigma_z|0\rangle_{an} &= |0\rangle|f_+\rangle \otimes |1\rangle_{an}, \\
U|1\rangle|f\rangle \otimes \sigma_z|0\rangle_{an} &= |1\rangle|f_-\rangle \otimes |1\rangle_{an},
\end{align*}
\]

wherein Friend performs the unitary operation \(\sigma_z\) to encode the information of observing an outcome. On the other hand, Student or Friend describes the evolution of the combined system of Lab and \(Q_s\) (Friend including herself) according to the collapse postulate of UQT as

\[
|0\rangle|f\rangle \rightarrow |0\rangle|f_+\rangle, \quad |1\rangle|f\rangle \rightarrow |1\rangle|f_-\rangle,
\]

whereas the ancillary system evolves as

\[
\sigma_z|0\rangle_{an} = |1\rangle_{an}.
\]

Therefore, if the initial state of \(Q_s\) is \((|0\rangle + |1\rangle)/\sqrt{2}\), then it follows from (2) that the final state of the combined system of Lab and \(Q_s\), according to Wigner is given by,

\[
U\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|f\rangle = \frac{1}{\sqrt{2}}(|0\rangle|f_+\rangle + |1\rangle|f_-\rangle).
\]

While Student or Friend describes the following evolution of the combined system of Lab and \(Q_s\),

\[
\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|f\rangle \rightarrow \frac{1}{2}(|0\rangle|0\rangle \otimes |f_+\rangle|f_+\rangle + |1\rangle|1\rangle \otimes |f_-\rangle|f_-\rangle).
\]

Additionally, since the ancilla is not correlated with the outcome of the measurement, it evolves the same way as given in (4). The purpose of the ancillary system is for the friend to communicate outside the lab that the measurement has been performed and a definite outcome
has been observed. Moreover, if Wigner applies an operation on Lab to alter Friend’s observation, then, later on, by measuring the ancilla Friend can be sure that she indeed obtained an outcome before. To summarize, we note that the two observers Wigner and Student/Friend ascribe two different descriptions given by (5) and (6) for the same physical system comprising Lab and $Q_s$.

### B. Operational Consistency

Firstly, we note that according to Wigner the state of the combined system of Lab and $Q_s$ is given by (5), while the Student and Friend describes the combined system as given in (6). As a consequence, the quantum predictions of observed probabilities of measurements on the combined system are inconsistent. For instance, if the following observable is measured

$$\{|F\rangle\langle F|, 1 - |F\rangle\langle F|\}$$

where $|F\rangle = 1/(\sqrt{2})(|0\rangle|f_s\rangle + |1\rangle|f_s\rangle)$, the first outcome will be certain as per Wigner is concerned. On contrary, Student and Friend anticipate that both the outcomes are equally likely. Although, one may avoid such inconsistency between Wigner and Friend by postulating that an observer (in this case, Friend) cannot apply QT to describe herself. After-all, Friend cannot witness any measurement on herself. However, the discrepancy at the observable level between Wigner and Student is unavoidable in UQT. To avoid such an inconsistency, one can assume that quantum measurement with respect to any observer is an absolute fact so that every observer should infer the state to be (6).

**Definition 2** (Absoluteness of measurement (AoM)). An *any measurement performed by some observer yields a single outcome, and that measurement is an absolute event irrespective of other observers and processes.*

This essentially represents the viewpoint that if any observer performs a measurement on a quantum system, it qualifies as a measurement for every other observer. Equivalently, only the post-measurement dynamics is applicable to describe the system for all observers depending on his/her knowledge of the outcome. In contrast to that, one can also suppose that any observer who can describe Lab and $Q_s$ without the knowledge of the measurement outcome, describes it by some evolution specified by the theory. In case of UQT every observer, who doesn’t observe the outcome, infers the combined state of Lab and $Q_s$ as (5) via unitary time-evolution.

**Definition 3** (Non-absoluteness of measurement (NoM)). *Any physical system evolves via time-evolution with respect to an observer, unless that observer observers an outcome.*

This represents the viewpoint that even if Friend performed measurement on the quantum system, it did not qualify as a measurement for other observers unless they have the knowledge about the outcome of the measurement. In other words, the correct description of the quantum system along with Friend by other observers is given via time-evolution even if Friend describes the quantum system using post-measurement dynamics. Essentially, here Wigner represents the perspective of AoM and Student represents the perspective of NoM. Let us remark that these two notions depend on the assumption that Friend is an observer and can perform measurements on the quantum system.

For any physical theory, in order to avoid observable discrepancy among these two perspectives, we have two options - either all the observable probabilities should not depend upon whether a measurement with respect to an observer are considered to be absolute or not; or the status a measurement, whether it is absolute or not, is specified by the theory itself. In other words, any reasonable universal physical theory should satisfy the following notion of consistency.

**Definition 4** (Operationally Consistent theory for all Observers (OCO)). *A universal theory is ‘operationally consistent for all observers’ if its predictions of observable probabilities are unique for all observers irrespective of whether a measurement is considered to be absolute or not, whenever the status of measurement is not specified by the theory.*

Any theory with a unique dynamics, like classical theory, is always operationally consistent. Within QT, as the quantum state of the combined system of Lab and $Q_s$ is different depending on the perspective, we can conclude that the UQT is operationally inconsistent unless it specifies either AoM or NoM for every measurement. Let us stress here that for one observer even if a particular theory gives correct predictions, it might not be consistent for all observers. For instance, theories in which different observers can not account for each other observations are not OCO, even if they lead to correct predictions for a particular observer.

### III. AN OPERATIONAL APPROACH TO WITNESS CORRELATIONS THAT ARE INCOMPATIBLE WITH ABSOLUTENESS OF MEASUREMENTS

We address here the following question: In other words, is it possible to empirically test AoM def-(2) or NoM def-(3) without the knowledge of the underlying details of the experiment? Can we discriminate ‘UQT with AoM’ and ‘UQT with NoM’ in terms of accomplishing some operational tasks?

Let us first remark that if Wigner is allowed to perform an arbitrary operation on the combined system of Lab and $Q_s$, then Wigner can produce any empirical results, irrespective of the description of the combined system given by (5) or (6). For example, to achieve the
statistics of an arbitrary quantum state of the isolated lab and $Q_s$, Wigner can perform a measurement containing the projectors $\{\langle 0|0\rangle \otimes |f_+\rangle \langle f_-|, |1\rangle \langle 1| \otimes |f_-\rangle \langle f_-|\}$ and subsequently, applies a suitable unitary depending on the outcome. Therefore, in order to address the above question, Wigner should be restricted to apply certain operations. Let us consider that Wigner can apply only unitary transformation on the combined system. Specifically, Wigner performs a unitary transformation $U_w$ on the combined system in which every run of the experiment consists of the following steps:

- At $t_0 \rightarrow$ Friend and Wigner receive some random variable $x \in \{0, \ldots, n-1\}$ and $w \in \{0, \ldots, m-1\}$, respectively. So, in general $x$ takes $n$ possible values, while $w$ takes $m$ possible values. The initial state of Lab knowing the input $x$ is $|f^x\rangle$. After reading $x$, Friend keeps this random variable outside the lab. Along with Wigner, Student also knows the variable $w$.
- At $t_1 \rightarrow$ A preparation device prepares a quantum system $|\psi\rangle$ in the state $|\psi\rangle$ which enters the isolated lab of Friend.
- At $t_2 \rightarrow$ Depending on $x$, Friend performs a measurement, denoted by $\mathcal{A}_x$, on $Q_s$ and obtains an outcome. At this point, Friend encodes the information of observing a definite outcome on the ancilla as mentioned before. We consider the measurement $\mathcal{A}_x$ always to be $d$-outcome rank-one projective measurement defined by

$$\mathcal{A}_x := \{|\psi_x^1\rangle\langle \psi_x^1|\}_{i=0}^{d-1}.$$  

Here, we identify Friend’s measurement with overline. Note that, Wigner and Student do not know the value of $x$. But, they know the possible values of $x$ along with the complete description of $|f^x\rangle$ given every $x$. We denote $|f^x_i\rangle$ for the state of the lab such that

- At $t_3 \rightarrow$ Wigner performs a unitary transformation $U_w$ on the combined system depending on $w$. Throughout this article, we further assume that for $w=0$, Wigner does not do anything.
- At $t_4 \rightarrow$ Wigner and Student perform the following $(dn+1)$-outcome measurement on the combined system,

$$\Omega := \sum_{i=0}^{d-1} |F_i^x\rangle\langle F_i^x|$$

is the combined state of Lab and $Q_s$, $|F_i^x\rangle$ being the state of Lab observing the outcome $i$ after measuring $\mathcal{A}_x$, and $|\psi_x^i\rangle$ is given in (8). This measurement (9) is nothing but asking Friend about the input $x$ along with the measurement outcome she has obtained. It is also the most feasible measurement which Wigner or Student can perform. Also Wigner, Student and
Friend should agree on the value of input $x$ which was kept outside the lab. For that reason, an additional condition, that the $d$-dimensional subspace spanned by $\{ |F_i^x\rangle \}_{i=0}^{d-1}$ remains invariant under the application of some unitary for each $x$, is imposed on $U_w$. Apart from the value of $x$, note that, Wigner, Student and Friend observe the same outcome, say, $a \in \{0, \ldots, d - 1\}$. The dimension of the macroscopic system Lab is presumably larger than $d \cdot n$, and thus, an auxiliary outcome is taken into account to fulfill the completeness condition. However, the probability of observing this outcome is zero.

- At $t_5 \rightarrow$ Friend gets the value of $w$, matches the value of $x$ that was kept outside the lab, and measures the ancilla to ensure that she obtained a definite outcome before. Moreover, by measuring $Q_x$ in the basis (8), Friend can verify whether Wigner’s measurement was (9) or not, since the state of $Q_x$ should collapse to $|\psi_a^x\rangle$ for outcome $a$. Thus, if the $d$-dimensional subspace spanned by $\{ |F_i^x\rangle \}_{i=0}^{d-1}$ is not invariant after applying $U_w$ for some $x$, then Friend will detect a discrepancy.

Therefore, by the end of every run of the experiment, Friend, Student and Wigner agree upon the values of $x, w$ and register the same outcome $a$. This is repeated many times so that they obtain the empirical probability defined as

$$p(a|\overline{x, w}, \Omega) := p(a | x, w, \psi, \overline{x, w}, \Omega),$$

in which all the conditional variables are mentioned on the right-hand-side according to the sequence of time. Student agrees with Wigner about the empirical probabilities under the condition (1) which is specified below, assuming that Friend is unbiased and performs rank-one projective measurements.

**Condition 1.** Wigner applies some unitary transformation for $w \neq 0$.

In what follows, $\{ p(a|\overline{x, w}) \}$ denote the set of empirical probabilities on which Wigner and Student agree upon, under the condition (1) without assuming the exact form of the state $|\psi\rangle$, rank-one projective measurements $\overline{x}$, and the unitaries $U_w$. Depending on different perceptions of Wigner and Student, we can obtain two sets of empirical probabilities. The first set is given by,

**Definition 5.** [Quantum Correlations with absoluteness of measurement (QCAM)] Set of all observed probabilities $\{ p(a|\overline{x, w}) \}$ obtained in the WFS setup, employing universal QT with the additional condition that measurements are absolute according to definition (2).

and the second set is given by,

**Definition 6.** [Quantum Correlations with non-absoluteness of measurement (QCNM)] Set of all observed probabilities $\{ p(a|\overline{x, w}) \}$ obtained in the WFS setup, employing universal QT with the additional condition that measurements are not absolute according to definition (3).

Before providing the explicit examples of operational task to witness the difference between QCAM and QCNM, we first obtain a relation within the WFS setup that holds true for QCAM and will be used later in this manuscript.

**Result 1.** In QCAM, for any $d$-outcome rank-one projective measurement $\overline{A}$ given in (8), the following holds true for all $a = 0, \ldots, d - 1$,

$$p(a|\overline{A}, \Omega) \leq \max_i p(i|\overline{A}, \mathbb{1}),$$

where $U$ is any reversible process that can be applied by Wigner or Student on the combined system of $Q_x$ and Lab. Note that, we omit the random variable $x$ in the probability since this relation holds for every $x$.

**Proof.** According to universal quantum theory along with Absoluteness of Measurement (2) the combined state of $Q_x$ and Lab after time $t_3$,

$$\rho_L = \sum_{i=0}^{d-1} p_i |F_i\rangle \langle F_i|,$$

where $p_i$ is the probability of getting outcome $i$ with respect to Friend, and $|F_i\rangle$ is defined in (10). For any reversible process $U$, the left-hand-side of (11)

$$p(a|\overline{A}, \Omega) = \text{Tr} \left( U \rho_L U^\dagger |F_a\rangle \langle F_a| \right)$$

$$= \sum_{i=0}^{d-1} p_i \text{Tr} \left( U |F_i\rangle \langle F_i| U^\dagger |F_a\rangle \langle F_a| \right)$$

$$= \sum_{i=0}^{d-1} p_i q_i$$

for all $a = 0, \ldots, d - 1$. In the second line of the above equation, we denote the expression with trace by $q_i$ for each $i$. On the other hand, using the fact that $\sum_i |F_i\rangle \langle F_i| \leq \mathbb{1}$ we know

$$\sum_i U |F_i\rangle \langle F_i| U^\dagger \leq \mathbb{1}.$$  

Since $|F_a\rangle \langle F_a|$ is a rank-one projector, it follows from the above relation that

$$\sum_{i=0}^{d-1} q_i \leq 1.$$  

Due to facts that $p_i, q_i$ are all non-negative and satisfy the relations $\sum_i p_i = 1$ as well as Eq. (15), the right-hand-side of Eq. (13) is upper bounded by the maximum value of $p_i$, that is,

$$p(a|\overline{A}, \Omega) \leq \max_i p_i.$$  


Finally, by noting that \( p_i = p(i|A, 1) \), we obtain (11). □

Now, we show that every correlation achievable in universal QT assuming absoluteness of measurements (QCAM def-(5)) is a proper subset of correlations obtainable in universal QT assuming Non-absoluteness of measurement (QCNM def-(6)).

A. QCAM is a proper subset of QCNM

In what follows, we would first show that QCAM def-(5) is a subset of QCNM def-(6), where as discussed before the evolution is specified by unitary dynamics. For this, we show that there exist correlations within QCNM that can simulate all correlations within QCAM.

**Result 2. QCAM is a subset of QCNM, that is, QCAM \( \subseteq \) QCNM.**

**Proof.** We show that any probability \( p(a|A_x, U_w) \) that is obtained from arbitrary initial state \( |\psi\rangle \), rank-one measurement \( A_x \) and unitary \( U \), can also be contained within QCNM. Taking the general form of \( A_x \) given in (8), \( |\psi\rangle \) can be expressed as follows:

\[
|\psi\rangle = \sum_{i=0}^{d-1} \alpha_i^x |\psi_i^x\rangle
\]

so that \( \forall x, \sum_i |\alpha_i^x|^2 = 1 \). Within QCAM, the state after \( t_3 \) is given by

\[
\rho_L = \sum_{i=0}^{d-1} |\alpha_i^x|^2 U_w |F_i^x\rangle \langle F_i^x| (U_w)^\dagger
\]

where \( |F_i^x\rangle \) is defined in (10). As a consequence, the probability of obtaining outcome \( a \) when Friend performs the measurement \( A_x \),

\[
p(a|A_x, U_w) = \sum_{i=0}^{d-1} |\alpha_i^x|^2 \langle F_i^x| U_w |F_i^x\rangle|^2
\]

so that \( w = 0 \):

\[
\sum_{i=0}^{d-1} |\alpha_i^x|^2, \quad \text{if } w = 0
\]

is acting on \( d \)-dimensional subspace spanned by \( \{|F_i^x\rangle\}_{i=0}^{d-1} \). Let us consider a strategy in QCNM in which the initial state and the measurements are the same as before. For \( w = 0 \), the state after \( t_3 \) is

\[
|\psi_L\rangle = \sum_{i=0}^{d-1} \alpha_i^x |\psi_i^x\rangle.
\]

For \( w = 0 \), the probability of getting outcome \( a \) is \( |\alpha_a^x|^2 \), which is same as (19). While for \( w \neq 0 \), the applied unitary by Wigner is \( \bigoplus_x U_w \), so that

\[
\tilde{U}_{w} \left( \sum_{i=0}^{d-1} \alpha_i^x |F_i^x\rangle \right) = \sum_{i=0}^{d-1} \sqrt{\beta_{a,w,x}} |F_i^x\rangle
\]

where

\[
\beta_{a,w,x} = \sum_{i=0}^{d-1} |\alpha_i^x \langle F_i^x| U_w |F_i^x\rangle|^2.
\]

Finally, note that the transformed state at \( t_3 \) in the above Eq. (21) is such that the probability \( p(a|A_x, U_w) \) is exactly the same as obtained for QCAM in Eq. (19). This completes the proof. □

Let us now explore correlations that are not compatible with QCAM but exists in QCNM. Employing Result 1, we present a simple expression to distinguish between QCAM and QCNM. Consider the WFS scenario described above, where Friend does not receive any variable \( x \), and \( w \in \{0, 1\} \). Given any measurement \( A \) of the form (8) having two outcomes, that is, \( a \in \{0, 1\} \), Wigner seeks to maximize the following empirical expression over all possible preparations \( \rho \) and reversible transformations \( U \),

\[
T(q) = p(1|A, U) - |p(0|A, 1) - q| - |p(1|A, 1) - (1 - q)|
\]

which is a function of the variable \( q \in (0, 1) \).

**Result 3.** Within QCAM, the maximum value of \( T(q) \) in Eq. (23) is bounded as,

\[
T(q) \leq \max\{q, (1 - q)\}.
\]

Moreover, there exists a realization in QCNM such that

\[
T(q) = 1 \quad \text{for any value of } q \in (0, 1).
\]

**Proof.** Let us take

\[
p(0|A, 1) = q + t
\]

for some \( t \in [-q, 1 - q] \). Using Result 1 and the normalization of probabilities, we see that the following holds within QCAM

\[
T(q) \leq \max\{q + t, 1 - q - t\} - |t| - |t|.
\]

It can be easily checked that the maximum value of the right-hand-side of the above equation is obtained for \( t = 0 \), and hence, (24) is true. The upper bound in QCAM is achieved when the system is prepared in the state,

\[
\rho = q |\psi_0\rangle \langle \psi_0| + (1 - q) |\psi_1\rangle \langle \psi_1|,
\]

where \( |\psi_i\rangle \) are eigenvectors of \( A \) as denoted in (8), and

\[
U = \begin{cases} I_x, & \text{if } q \leq \frac{1}{2} \\ \sigma_x, & \text{if } q > \frac{1}{2}. \end{cases}
\]
Note that the above realization is classical, and therefore, the upper bound in (24) also holds for classical theory.

We now state a realization within universal QT assuming Non-absoluteness of Measurement def-(3) that can attain the value $T(q) = 1$ (23) for any $q$. The quantum state is given by,

$$|\psi\rangle = \sqrt{q}|\psi_0\rangle + \sqrt{1-q}|\psi_1\rangle.$$ \hspace{1cm} (29)

As discussed above, employing universal QT with NoM, Wigner assigns the state $|\psi\rangle_L = \sqrt{q}|F_0\rangle + \sqrt{1-q}|F_1\rangle$ to the combined system of $Q_s$ and Lab. The following unitary by Wigner transforms this state to $|F_1\rangle$,

$$U = \begin{pmatrix} \sqrt{1-q} & -\sqrt{q} \\ \sqrt{q} & \sqrt{1-q} \end{pmatrix}$$

The above result can also be understood as an operational task where there is an advantage within universal QT with Non-absoluteness of measurement def-(3) than with absoluteness of measurements def-(2). In the later part of this manuscript, we would discuss the implications of this advantage towards different interpretations of QT. Now, we extend the WFS scenario to two spatially separated parties and show that there exist operational tasks which are possible in universal QT with NoM def-(3) but can never be performed in universal QT with AoM def-(2).

**FIG. 2. Activation of bipartite correlations.** Wigner’s Friend, named as, Alice (A) and another observer Bob (B) receive a system from the preparation device on which they perform measurements according to inputs $x, y$, respectively, in their spatially separated labs. Alice encodes the information about the fact that measurement has been performed by reversing the ancillary bit $|0\rangle_{an}$ to $|1\rangle_{an}$, which can be read by Wigner, Student and Alice at any further times. Alice and Bob are not allowed to communicate among each other. Wigner (W) or Student can apply reversible process on the Alice’s lab after her measurement is performed, and finally, Wigner or Student opens Alice’s lab to know the outcome of her measurement. By repeating this experiment many times, Wigner together with Bob observe the joint probabilities of the experiment.

**B. Activation of bipartite correlations that are incompatible with QCAM**

In general, we can extend the WFS setup involving single quantum systems to WFS setup involving multi-partite quantum systems. Let us consider the scenario illustrated in Fig. 2, wherein Wigner’s Friend, named as Alice, and another spatially separated observer, say, Bob, perform measurement on the subsystems of a bipartite quantum system. And the super-observer, Wigner or Student, can apply local reversible processes on Alice’s Lab. Each run of this two-party version of the WFS experiment comprises of the same steps as before on Alice’s side, along with the following additional steps on Bob’s side.

- At $t_0 \rightarrow$ Apart from Alice and Wigner, Bob also receives an input variables that is denoted by $y$.
- At $t_1 \rightarrow$ The preparation device prepares a bipartite system with one part of the system entering the Alice’s lab and the other part to Bob’s lab.
- At $t_2 \rightarrow$ Besides Alice’s measurement $\mathcal{A}_0$ (8), Bob per-
forms a measurement $B_y$ depending on $y$ that results an outcome $b$.

- At $t_3 \to$ Wigner applies some reversible operation denoted by $U_w$ on Alice’s Lab, each of which keeps the subspace spanned by $\{|E^x_i\}_{i=0}^{d-1}$ invariant. Wigner does not apply any operation for $w = 0$.

- At $t_4 \to$ Wigner and Student perform the measurements (9) on Alice’s Lab to obtain outcome $a$.

After repeating many times, Wigner and Bob communicate with each other to obtain the joint probability distribution of outcomes $a, b$ for different inputs $x, y, w$. For convenience, let us drop the terms $\overline{A}_x, B_y, \Omega$, and use the the following concise notation for these joint probabilities

$$t_0 \quad t_1 \quad t_2 \quad t_3 \quad t_4$$

$$p(a, b|x, y, U_w) := p(a, b | x, y, w, \psi, A_x, B_y, U_w, \Omega).$$

Since the operations by Wigner and Bob are spatially separated, the obtained joint probabilities should satisfy the notion of causality or the no-signalling conditions.

**Definition 7 (No-signalling conditions).** The local statistics of Alice as well as Wigner is independent of Bob’s measurement settings and vice versa.

$$\forall x, y, x’, y, w, \sum_b p(a, b|x, y, U_w) = \sum_b p(b, a|x, y’, U_w)$$

and

$$\forall x, x’, y, w, w’, \sum_a p(a, b|x, y, U_w) = \sum_a p(b, a|x’, y, U_{w'}).$$

As we would show that there exists some operational advantage if we consider universal QT with NoM def-(3) over AoM def-(2). The task involves Alice, Bob and Wigner as described above in which $x, y, a, b, w \in \{0, 1\}$. Consider the following two quantities denoted by $P_w$ that corresponds to $w = 0, 1$, respectively,

$$P_0 = \sum_{a,b,x,y} c_{a,b,x,y} p(a, b|x, y, 1),$$  

$$P_1 = \sum_{a,b,x,y} c_{a,b,x,y} p(a, b|x, y, U),$$

where

$$c_{a,b,x,y} = \begin{cases} 1, & \text{if } a \oplus b = x, y \\ 0, & \text{otherwise.} \end{cases}$$

Note that the expression (30) is nothing but the CHSH expression [16] written in the WFS scenario. The quantity $P_1$ captures predominantly the idea of the WFS scenario, while $P_0$ is needed to impose constraint on $P_1$. Now, we are ready to state the result whose explicit proof is provided in Appendix A.

**Result 4.** The following inequality holds true in QCAM,

$$P_S = P_1 - \left| P_0 - \frac{3}{4} \right| \leq \frac{3}{4}. \quad (33)$$

Moreover, there exists a realization in QCNM such that

$$P_S = \frac{1}{2} \left( 1 + \frac{1}{\sqrt{2}} \right). \quad (34)$$

For a note, we state the realisation within universal QT with Non-absoluteness of measurement def-(3) that can attain the value of $P_S$ given in (34). For this, the maximally entangled state $|\psi\rangle = (1/\sqrt{2})(|00\rangle + |11\rangle)_{AB}$ is prepared on which Alice and Bob choose the measurements

$$\overline{A}_0 = \sigma_z, \overline{A}_1 = \sigma_x, \quad B_0 = \sigma_z, B_1 = \sigma_x, \quad (35)$$

and Wigner performs the following unitaries,

$$U^x = \left( \begin{array}{cc} \cos(\frac{\pi}{8}) & -\sin(\frac{\pi}{8}) \\ \sin(\frac{\pi}{8}) & \cos(\frac{\pi}{8}) \end{array} \right) \quad \forall x = 0, 1. \quad (36)$$

In the Appendix, we give explicit steps to calculate $P_S$ using the above mentioned quantum realisation.

**IV. ANALYSIS WITHIN DIFFERENT INTERPRETATIONS OF QUANTUM THEORY**

Now, we analyse different interpretations of universal quantum theory and classify whether they reconcile with ‘UQT with AoM’ or ‘UQT with NoM’. The overall picture is summarised in Table I.

**Measurements are inherently classical.** Interpretations of QT like collapse theories [17–22], ETH approach [23] and CSM-ontology [24] to name a few fall under the category of interpretations in which all physical systems undergo a non-reversible dynamics subject to any measurement process. As a result, in such theories, macroscopic objects, like, measurement device, follow a different description than microscopic systems. These theories impose that the act of measurement physically changes the state of the system. As a consequence, they fall under the category of UQT with AoM, and do not show advantage in both the operational tasks presented before in this manuscript.

**Realist interpretations.** A realist model of UQT assumes the existence of some physical state that completely describes the objective reality of the corresponding physical system and predicts the outcome of all experiments. There are majorly two important class of realist models. One, where the quantum state is a part of the physical reality, which is commonly referred to as $\psi$-ontic models [13, 25, 26]. The other, where quantum state represents only information about some underlying physical reality, commonly referred to as $\psi$-epistemic models [27–31].
Any realist description of QT must satisfy some of very interesting no-go theorems [12, 32–35]. Moreover, there are absolute no-go theorems [34, 36–38] that constrain $\psi$-epistemic models to a certain extent.

Interestingly, the original Wigner’s Friend thought experiment is in itself a simple no-go theorem against all $\psi$-ontic models applicable to subsystems if universality of QT holds. In contrast, there exist two kinds of $\psi$-ontic explanations of quantum phenomena that are compatible with the notion of ‘UQT with NoM’. They are Bohmian mechanics and many-worlds or relative state formalism. According to Bohmian mechanics [25, 39–42], there is a unique description of the whole universe (which WFS experiment is a part of) which is compatible with the perspective of Wigner. While, in many-worlds interpretation and relative-state formalism of QT [43–47], measurements are not absolute facts in the sense that they do not yield single outcome. According to these two interpretations, the whole universe evolves via a unique reversible dynamics and thus, these two are compatible with NoM. As a consequence, we would see an advantage in both the aforementioned operational tasks.

Copenhagenish type interpretations. Now, we focus on Copenhagenish type interpretations of QT. For our analysis we refer to the lectures by Matt Leifer [48]. All such interpretations of QT are based on the following principles. First, universality of quantum predictions which imposes that every system is describable using QT. Secondly, anti-realism of physical theory that there is no objective description of reality. And thirdly, every measurement performed by an observer yields a single outcome. These principles imply an existence of a split known as the Heisenberg cut that separates the observer from the physical system which is being described by the observer. The placement of the Heisenberg cut is quite significant as it represents the boundary up-to which an observer can describe physical systems using QT and must invokes classical theory after this split. In the context of WFS scenario, we note that if the split is placed before the boundary of the isolated Lab, that is, if the split is placed on left side of the dotted line in Figure 1, then the description of the isolated Lab is unambiguous and compatible with AoM (2). On the contrary, if the split is placed after the boundary of the isolated Lab, that is, if the split is placed anywhere between the dotted line and the respective observer (Wigner or Student) in Figure 1, then the tension between the two dynamics of UQT persists, and as a result, we will arrive at operational inconsistency.

The Copenhagenish type interpretation of QT are broadly divided into two categories. First, Objective interpretations which include the original Copenhagen interpretation by Bohr [49], Quantum pragmatism [50] and information interpretation [51, 52]. Second, Perspectival interpretations which include Qbism [53, 54], Consistent histories [55] and Relational quantum mechanics [56]. The difference between these two types lies in the fact that, observing a measurement outcome by an observer (say Friend) is an objective fact in the former type, while it is considered as a subjective fact for that observer in later type. In Perspectival approach, the placement of the split is decided by the observer. Therefore, the operational consistency does not hold, however, this approach does not necessitate operational consistency. Moreover, depending on where the split is placed by an observer, the advantage in the operational tasks prevail. On the other hand, it is not fully clear whether the placement of the split depends on the observer or not in Objective Copenhagenish interpretations, and thus, there are ambiguities therein.

| Interpretations                                      | UQT with AoM | UQT with NoM | OCO | Advantage in operational tasks |
|-----------------------------------------------------|--------------|--------------|-----|--------------------------------|
| Spontaneous collapse theories [17–22],              | ✓            | X            | ✓   | X                              |
| ETH approach [23], CSM [24]                         |              |              |     |                                |
| Bohmian mechanics [25, 39–42],                      | X            | ✓            | ✓   | ✓                              |
| Many-world, Relative-state formalism [43–47]        | X            | X            | X   | ✓                              |
| Perspectival Copenhagenish [53–56]                  | X            | X            | X   | ✓                              |
| Objective Copenhagenish [50–52]                     | X            | ?            | ?   | ?                              |

TABLE I. Classification of interpretations of quantum theory based on - (i) whether they compatible with absoluteness of measurement (2) or non-absoluteness of measurement (3), (ii) whether they are OCO according to def-(4), and (iii) whether they provide advantage in the operational tasks presented in this manuscript. For Objective Copenhagenish type of interpretation, there are ambiguities with respect to such classification.

V. DISCUSSIONS

In recent times, there has been a renewed interest in WF scenario due to a no-go theorem put forward by Frauchiger and Renner [11], where it is shown that QT is incompatible with three assumptions, namely univer-
sality, consistency and single-outcome. Any interpretation of QT compatible with all three assumptions cannot be true. For showing the contradiction a scenario consisting of four different observers was constructed where two observers behave like Friends and the other two behave like Wigner from the original Wigner’s Friend scenario. As shown in [11], the observers predict different results for the same experiment if the assumptions of universality, consistency and single-outcome holds. A similar no-go theorem exploiting Bell inequalities [12, 13] was proposed by Brukner [14] imposing the assumption that measurement outcomes are objective fact. Contrary to Frauchiger-Renner approach, the scenario involved two Friends who are spatially separated with each receiving a qubit on which two local measurements are performed and two super-observers who have access to their labs each. This is later extended to more number of measurements performed by the Friends and is experimentally implemented [15]. One can also understand the no-go theorems from Brukner [14] and later in [15], as an operational task which is performed better by some interpretations of QT than others.

Interestingly, we observe that the no-go theorems presented by Frauchiger and Renner in [11] as well as Brukner [14] and later by K. W. Bong et al. [15] can well be understood in a simplified way from the WFS scenario presented in this manuscript. Let us first consider the version presented by Frauchiger and Renner, where the assumption of universality of QT and single-outcome holds, along with the assumption of consistency, that is, if the Friend finds an outcome, then Wigner or Student should also conclude the existence of the same outcome. As a consequence, Absoluteness of Measurements def-(2) implies single-outcome and consistency. Similarly, in the Brukner’s or K. W. Bong et al.’s version, absoluteness of measurement outcomes are assumed for proving the no-go theorems. Again, Absoluteness of Measurement def-(2) certainly implies that the outcome of that measurement is also an objective or absolute fact irrespective of other observers. Thus, any violation of notion in [11, 14, 15] is also a violation of UQT with AoM def-(2) but the inverse might not be true. It is noteworthy that UQT with AoM is compatible with the point of view that Friend’s Lab cannot be treated as isolated system due to decoherence [57].

Another interesting consequence of our work is towards some interpretations of quantum theory. For all the other interpretations apart from copenhagenish type interpretations for which the provided tasks points to operational inconsistency of the interpretations, our results provide an experimental way to refute some of the interpretations of quantum theory. Any advantage in the presented operational tasks would rules out universal quantum theory with AoM def-(2) and collapse theories.

Further, we provided an operational framework to explore interesting correlations where macroscopic systems like observers or classical registers are treated as physical systems governed by universal physical laws. An important premise in our approach is that any physical system can be described according to quantum theory. We believe that this assumption can be relaxed and the theorems provided in this manuscript can be extended for any general probabilistic theory with certain properties. Our framework can be used to create more exotic scenarios where the distinctions among different interpretations of quantum theory become more relevant. Also, it would be interesting to see if the results presented in this manuscript hold true even for any reversible quantum channel instead of unitary transformations applied by Wigner. Also, it might be interesting from foundational perspective, to find necessary and sufficient conditions that fully characterize quantum correlation with absoluteness of measurement.

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Moreover, there exists a realization in QCNM such that

\[ P_S = P_1 - \left| P_0 - \frac{3}{4} \right| \leq \frac{3}{4}, \quad (A1) \]

Moreover, there exists a realization in QCNM such that

\[ P_S = \frac{1}{2} \left( 1 + \frac{1}{\sqrt{2}} \right). \quad (A2) \]

Proof. As discussed before, let us say the reversible process \( U = \bigoplus_U U^x \) acts on lab as

\[ U^x |F_0^x\rangle = a_{0,0}^x |F_0^x\rangle + a_{0,1}^x |F_1^x\rangle, \quad \text{and} \]

\[ U^x |F_1^x\rangle = a_{1,0}^x |F_0^x\rangle + a_{1,1}^x |F_1^x\rangle \]

(A3)

where \(|F_i^x\rangle = |\psi_i^x\rangle |f_i^x\rangle\) represent the state of the respective labs when Alice observes an outcome \( i \in \{0,1\} \) after performing the measurement \( A_x \), and

\[ |a_{x,0}^a|^2 + |a_{x,1}^a|^2 = 1 \quad \forall x, a = 0, 1. \quad (A4) \]

The fact that \(|F_0^x\rangle, |F_1^x\rangle\) are orthonormal, implies \( |a_{0,0}^x| a_{0,1}^x, i.e., \[ |a_{0,0}^x|^2 + |a_{0,1}^x|^2 = |a_{1,0}^x|^2 + |a_{1,1}^x|^2 = 1, \quad \forall x = 0, 1. \quad (A5) \]

On the other hand, using the no signalling conditions (7) for any joint probability, we have

\[ p(a, b | x, y, U) = p(b | B_y) p(a | x, U, y, b) \]

\[ = p(b | B_y) \left( |a_{x,0}^b|^2 p(a | x, 1, y, b) + |a_{x,1}^b|^2 p(a | 1, x, 1, y, b) \right). \quad (A6) \]
Thus, we can write the joint probability distribution in presence of super-observer who can access Alice’s lab as,

\[ p(a, b| x, y, U) = |a_x^a|^2 p(a, b| x, y, 1) + |a_x^b|^2 p(a \oplus 1, b| x, y, 1). \] (A7)

Using the above, the expression of \( P_1 \) in Eq. (31) can now be simplified as,

\[ P_1 = \sum_{a,b,x,y} \epsilon_{a,b,x,y} \left( |a_x^a|^2 p(a, b| x, y, 1) + |a_x^b|^2 p(a \oplus 1, b| x, y, 1) \right). \] (A8)

Expanding the above quantity, we find

\[
P_1 = \frac{1}{4} \left( |a_{0,0}^0|^2 p(0,0|0,0,1) + |a_{1,1}^0|^2 p(1,1|0,0,1) + |a_{0,1}^0|^2 p(1,0|0,0,1) + |a_{0,1}^0|^2 p(0,1|0,0,1) + |a_{0,0}^1|^2 p(0,0|1,1,1) + |a_{1,1}^1|^2 p(1,1|1,1,1) + |a_{0,1}^1|^2 p(1,0|1,1,1) + |a_{0,1}^1|^2 p(0,1|1,1,1) \right). \] (A9)

Since above quantity is linear function of the 8 variables \( \{ |a_x^a|^2, |a_x^b|^2 \} \) satisfying (A4) and (A5), it suffices to consider the extremal values of these variables to obtain its upper bound. It can be checked that there are four extremal values of these variables as follows

\[
(i) \quad |a_{0,0}^0|^2 = |a_{1,1}^0|^2 = |a_{0,1}^0|^2 = |a_{1,1}^1|^2 = 1, \\
|a_{0,0}^1|^2 = |a_{0,1}^1|^2 = |a_{1,1}^0|^2 = |a_{1,1}^1|^2 = 0;
(ii) \quad |a_{0,0}^0|^2 = |a_{0,1}^0|^2 = |a_{1,1}^0|^2 = |a_{1,1}^1|^2 = 1, \\
|a_{0,0}^1|^2 = |a_{0,1}^1|^2 = |a_{1,1}^0|^2 = |a_{1,1}^1|^2 = 0;
(iii) \quad |a_{0,0}^0|^2 = |a_{0,1}^0|^2 = |a_{1,1}^0|^2 = |a_{1,1}^1|^2 = 1, \\
|a_{0,0}^1|^2 = |a_{0,1}^1|^2 = |a_{1,1}^0|^2 = |a_{1,1}^1|^2 = 0;
(iv) \quad |a_{0,0}^0|^2 = |a_{0,1}^0|^2 = |a_{1,1}^0|^2 = |a_{1,1}^1|^2 = 1, \\
|a_{0,0}^1|^2 = |a_{0,1}^1|^2 = |a_{1,1}^0|^2 = |a_{1,1}^1|^2 = 0. \] (A10)

These 4 extremal point correspond to four different CHSH expressions under relabelling of inputs, and accordingly,

\[ P_1 \leq \frac{1}{4} \max \{ \sum_{a,b,x,y} p(a \oplus b = x \cdot y|x, y, 1), \\
\sum_{a,b,x,y} p(a \oplus b = (x \oplus 1) \cdot (y \oplus 1)|x, y, 1), \\
\sum_{a,b,x,y} p(a \oplus b = (x \oplus 1) \cdot y|x, y, 1), \\
\sum_{a,b,x,y} p(a \oplus b = x \cdot (y \oplus 1)|x, y, 1) \}. \] (A11)

Note that the first CHSH expression is same as \( P_0 \), while the second CHSH expression is nothing but \( 1 - P_0 \). While, the last two CHSH expressions are bounded by \( 3/2 - P_0 \). For instant, the third expression

\[ \frac{1}{4} \sum_{a,b,x,y} p(a \oplus b = (x \oplus 1) \cdot y|x, y, 1) \]

\[ = \frac{1}{4} \left[ 2 + 2(p(0,0|0,0,1) + p(1,1|0,0,1) + p(0,0|1,0,1) + p(1,1|1,0,1)) \right] - P_0 \]

\[ \leq \frac{3}{2} - P_0, \] (A12)

where we use the fact that \( p(0,0|0,0,1) + p(1,1|0,0,1) \leq 1, p(0,0|1,0,1) + p(1,1|1,0,1) \leq 1 \). Thus, for any quantum strategy, if the value of \( P_0 = q \) then the values of all CHSH expressions appearing in (A11) is bounded by \( 3/2 - q \). Using this fact we see that the figure of merit (A1) within QCAM,

\[ P \leq \max \{ q, 3/2 - q \} - |q - 3/4| = 3/4 \] (A13)
for any value of \( q \in [0, 1] \).

We now show that there exists QCNM that achieves the value of \( P \) given by (A2). For this, the maximally entangled state \( |\psi\rangle = (1/\sqrt{2}) \left( |00\rangle + |11\rangle \right)_{AB} \) is prepared on which Alice and Bob choose the measurements

\[
\mathcal{A}_0 = \sigma_z, \mathcal{A}_1 = \sigma_x, \quad B_0 = \sigma_z, B_1 = \sigma_x.
\]  

(A14)

Now, the description of the joint system with respect to super-observer assuming NoM def-(3) is determined by the unitary evolution after the measurements are done. Let us first look at the combined state of Alice’s Lab and the system on Bob’s side when the measurements

\[
A_0 = \sigma_z, \quad A_1 = \sigma_x, \quad B_0 = \sigma_z, \quad B_1 = \sigma_x.
\]

(A14)

\[
|\Psi\rangle_0 = \frac{1}{\sqrt{2}} \left( |0\rangle |f_0^0\rangle |0\rangle + |1\rangle |f_0^1\rangle |1\rangle \right)_{LB}, \quad \text{ and } \quad |\Psi\rangle_1 = \frac{1}{\sqrt{2}} \left( |+\rangle |f_1^0\rangle |+\rangle + |-\rangle |f_1^1\rangle |-\rangle \right)_{LB}.
\]

(A15)

(A16)

It can be readily verified from the above expressions that,

\[
\forall x, y = 0, 1, \quad p(0, 0|x, y, 1) = p(1, 1|x, y, 1) = \frac{1}{2},
\]

(A17)

and thus, \( P_0 = 3/4 \). For \( w = 1 \), say, the super-observer performs the reversible operation \( U = U^0 \oplus U^1 \) where

\[
U^x = \begin{pmatrix}
\cos \left( \frac{\pi}{8} \right) & -\sin \left( \frac{\pi}{8} \right) \\
\sin \left( \frac{\pi}{8} \right) & \cos \left( \frac{\pi}{8} \right)
\end{pmatrix}
\]

(A18)

written in the basis \( \{|0\rangle |f_0^x\rangle, |1\rangle |f_1^x\rangle \} \). The joint probabilities after the unitary is applied should be evaluated on the combined state \( U^x |\Psi\rangle_x \), wherein \( |\Psi\rangle_x \) is given in (A15)-(A16). It follows from some simple calculations that the joint probabilities are

\[
p(0, 0|0, 0, UI) + p(1, 1|0, 0, UI) = \cos^2 \left( \frac{\pi}{8} \right),
\]

\[
p(0, 0|0, 1, UI) + p(1, 1|0, 1, UI) = \frac{1}{2} \left( 1 + \sin \left( \frac{\pi}{4} \right) \right),
\]

\[
p(0, 0|1, 0, UI) + p(1, 1|1, 0, UI) = \frac{1}{2} \left( 1 + \sin \left( \frac{\pi}{4} \right) \right),
\]

\[
p(0, 0|1, 1, UI) + p(1, 1|1, 1, UI) = \cos^2 \left( \frac{\pi}{8} \right),
\]

(A19)

such that Eq.(A2) holds.