Controlling spatiotemporal chaos and spiral turbulence in excitable media: A review

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Excitable media are a generic class of models used to simulate a wide variety of natural systems including cardiac tissue. Propagation of excitation waves in this medium results in the formation of characteristic patterns such as rotating spiral waves. Instabilities in these structures may lead to spatiotemporal chaos through spiral turbulence, which has been linked to clinically diagnosed conditions such as cardiac fibrillation. Usual methods for controlling such phenomena involve very large amplitude perturbations and have several drawbacks. There have been several recent attempts to develop low-amplitude control procedures for spatiotemporal chaos in excitable media which are reviewed in this paper. The control schemes have been broadly classified by us into three types: (i) global, (ii) non-global spatially-extended and (iii) local, depending on the way the control signal is applied, and we discuss the merits and drawbacks for each.

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I. INTRODUCTION

Excitable media denote a class of systems that share a set of features which make their dynamical behavior qualitatively similar. These features include (i) the existence of two characteristic dynamical states, comprising a stable resting state and a metastable excited state, (ii) a threshold value associated with one of the dynamical variables characterising the system, on exceeding which, the system switches from the resting state to the excited state, and (iii) a recovery period following an excitation, during which the response of the system to a supra-threshold stimulus is diminished, if not completely absent [1]. Natural systems which exhibit such features include, in biology, cells such as neurons, cardiac myocytes and pancreatic beta cells, all of which are vital to the function of a complex living organism. Other examples of dynamical phenomena associated with excitable media include cAMP waves observed during aggregation of slime mold, calcium waves observed in Xenopus oocytes, muscle contractions during childbirth in uterine tissue, chemical waves observed in the Belusov-Zhabotinsky reaction and concentration patterns in CO-oxidation reaction on Pt(110) surface. Excitation in such systems is observed as the characteristic action potential, where a variable associated with the system (e.g., membrane potential, in the case of biological cells) increases very fast from its resting value to the peak value corresponding to the excited state, followed by a slower process during which it gradually returns to the resting state.

The simplest model system capable of exhibiting all these features is the generic FitzHugh-Nagumo set of coupled differential equations:

\[ \frac{d e}{d t} = e(1-e)(e-b) - g, \quad \frac{d g}{d t} = \epsilon (ke - g), \]

which, having only two variables, is obviously incapable of exhibiting chaos. However, when several such sets are coupled together diffusively to simulate a spatially extended media (e.g., a piece of biological tissue made up of a large number of cells), the resulting high-dimensional dynamical system can display chaotic behavior. The genesis of this spatiotemporal chaos lies in the distinct property of interacting waves in excitable media, which mutually annihilate on colliding. This is a result of the fact that an excitation wavefront is followed by a region whose cells are all in the recovery period, and which, therefore, cannot be stimulated by another excitation wavefront, as for example when two waves cross each other [2]. Interaction between such waves result in the creation of spatial patterns, referred to variously as reentrant excitations (in 1D), vortices or spiral waves (in 2D) and scroll waves (in 3D), which form when an excitation wavefront is broken as the wave propagates across partially recovered tissue or encounters an inexitable obstacle [3]. The free ends of the wavefront gradually curl around to form spiral waves. Once formed, such waves become self-sustained sources of high-frequency excitation in the medium, and usually can only be terminated through external intervention. The existence of nonlinear properties of wave propagation in several excitable media can lead to complex non-chaotic spatiotemporal rhythms. Thus, spiral waves are associated with periodic as well as quasiperiodic patterns of temporal activity.

However, in this paper, we shall not be discussing the many schemes proposed to terminate single spiral waves, but instead, focus on the control of spatiotemporally chaotic patterns seen in excitable media (in 2 or 3 dimensions), that occur when under certain conditions, spiral or scroll waves become unstable and break up. Various mechanisms of such breakup have been identified [4], including meandering of the spiral focus. If the meandering is sufficiently high, the spiral wave can collide with itself and break up spontaneously, resulting in the creation of multiple smaller spirals (Fig. 1). The process continues until the spatial extent of the system is spanned by several coexisting spiral waves that activate different regions without any degree of coherence. This state of spiral turbulence marks the onset of spatiotemporal chaos, as
promises a safer treatment for people at risk from potentially fatal cardiac arrhythmias.

In this paper, we have discussed most of the recent control methods that have been proposed for terminating spatiotemporal chaos in excitable media. These methods are also often applicable to the related class of systems known as oscillatory media, described by complex Landau-Ginzburg equation, which also exhibit spiral waves and spatiotemporal chaos through spiral breakup. We have broadly classified all control schemes into three types, depending on the nature of application of the control signal. If every region of the media is subjected to the signal (which, in general, can differ from region to region) it is termed as global control; on the other hand, if the control signal is applied only at a small, localised region from which its effects spread throughout the media, this is called local control. Between these two extremes lie control schemes where perturbations are applied simultaneously to a number of spatially distant regions. We have termed these methods as non-global, spatially extended control. While global control may be the easiest to understand, involving as it does the principle of synchronizing the activity of all regions, it is also the most difficult to implement in any practical situation. On the other hand, local control will be the easiest to implement (requiring a single control point) but hardest to achieve.

In the next section we describe a few of the more commonly used models for studying control of spatiotemporal chaos in excitable media. Section 3 discusses proposed methods of global control, while Section 4 discusses other spatially extended schemes. The next section deals with local control methods, and we conclude with a brief section containing general discussions about chaos control and its implications.

II. MODELS OF SPATIOTEMPORAL CHAOS IN EXCITABLE MEDIA

The generic Fitzhugh-Nagumo model for excitable media Eq. 1 exhibits a structure that is common to most models used in the papers discussed here. Typically, the dynamics is described by a fast variable, $e(x,t)$, and a slow variable, $g(x,t)$, the ratio of timescales being given by $\epsilon$. The resulting phase space behavior is shown in Fig. 2 (left). For biological cells, the fast variable is often associated with the transmembrane potential, while the slow (recovery) variable represents an effective membrane conductance that replaces the complexity of several different types of ion channels. For the spatially extended system, the fast variable of neighboring cells is coupled diffusively. There are several models belonging to this general class of excitable media which display breakup of spiral waves (in 2D) and scroll waves (in 3D), including the one proposed by Panfilov [3, 10]

$$\frac{\partial e}{\partial t} = \nabla^2 e - f(e) - g, \quad \frac{\partial g}{\partial t} = \epsilon(e, g)(ke - g). \quad (2)$$

Here, $f(e)$ is the function specifying the initiation of the action potential and is piecewise linear: $f(e) = C_1 e,$
and (ii) the Bär-Eiswirth model [13], which differs from the appropriate parameter values being given in Ref. [12], in 2D include (i) the Barkley model [11]:

$$f(e) = -C_2 e + a, \quad \text{for } e_1 < e \leq e_2,$$

and

$$f(e) = C_3(e - 1), \quad \text{for } e > e_2.$$  

The physically appropriate parameters given in Ref. [10] are

$$e_1 = 0.0026, \quad e_2 = 0.837, \quad C_1 = 20, \quad C_2 = 3, \quad C_3 = 15, \quad a = 0.06 \text{ and } k = 3.$$  

The function $f(e, g)$ determines the time scale for the dynamics of the recovery variable: $f(e, g) = \epsilon_1$ for $e < e_2$, $f(e, g) = \epsilon_2$ for $e > e_2$, and $f(e, g) = \epsilon_3$ for $e < e_1$ and $g < g_1$ with

$$g_1 = 1.8, \quad e_1 = 1/75, \quad e_2 = 1.0, \text{ and } \epsilon_3 = 0.3.$$  

Simpler variants that also display spiral wave breakup in 2D include (i) the Barkley model [11]:

$$\partial e/\partial t = \nabla^2 e + e^{-1}e(1-e)(e-g + b/a), \quad \partial g/\partial t = e-g, \quad (3)$$

the appropriate parameter values being given in Ref. [12], and (ii) the Bär-Eiswirth model [13], which differs from

(3) only in having $\partial g/\partial t = f(e) - g$, the functional form of $f(e)$ and parameter values being as in Ref. [14]. The Aliev-Panfilov model [15] is a modified form of the Panfilov model, that takes into account nonlinear effects such as the dependence of the action potential duration on the distance of the wavefront to the preceding waveback. It has been used for control in Refs. [16, 17].

All the preceding models tend to disregard several complex features of actual biological cells, e.g., the different types of ion channels that allow passage of electrically charged ions across the cellular membrane. There exists a class of models inspired by the Hodgkin-Huxley formulation describing action potential generation in the squid giant axon, which explicitly takes such details into account. While the simple models described above do reproduce generic features of several excitable media seen in nature, the more realistic models describe many properties of specific systems, e.g., cardiac tissue. The general form of such models are described by a partial differential equation for the transmembrane potential $V$,

$$\frac{\partial V}{\partial t} + \frac{I_{ion}}{C} = D \nabla^2 V,$$

where $C$ is the membrane capacitance density and $D$ is the diffusion constant, which, if the medium is isotropic, is a scalar. $I_{ion}$ is the instantaneous total ionic-current-density, and different realistic models essentially differ in its formulation. For example, in the Luo-Rudy I model [19] of guinea pig ventricular cells, $I_{ion}$ is assumed to be composed of six different ionic current densities, which are themselves determined by several time-dependent ion-channel gating variables whose time-evolution is governed by ordinary differential equations of the form:

$$\frac{d\xi}{dt} = \frac{\xi_{\infty} - \xi}{\tau_{\xi}}. \quad (5)$$

Here, $\xi_{\infty} = \alpha\xi/(\alpha\xi + \beta\xi)$ is the steady state value of $\xi$ and $\tau_{\xi} = 1/(\alpha\xi + \beta\xi)$ is its time constant. The voltage-dependent rate constants, $\alpha\xi$ and $\beta\xi$, are complicated functions of $V$ obtained by fitting experimental data.

### III. GLOBAL CONTROL

The first attempt at controlling chaotic activity in excitable media dates back almost to the beginning of the field of chaos control itself, when proportional perturbation feedback (PPF) control was used to stabilize cardiac arrhythmia in a piece of tissue from rabbit heart [20]. This method applied small electrical stimuli, at intervals calculated using a feedback protocol, to stabilize an unstable periodic rhythm. Unlike in the original proposal for controlling chaos [21], where the location of the stable manifold of the desired unstable periodic orbit (UPO) was moved using small perturbations, in the PPF method it is the state of the system that is moved onto the stable manifold. However, it has been later pointed out that
PPF does not necessarily require the existence of UPOs (and, by extension, deterministic chaos) and can be used even in systems with stochastic dynamics. Later, PPF method was used to control atrial fibrillation in human heart. However, the effectiveness of such control in suppressing spatiotemporal chaos, when applied only at a local region, has been questioned, especially as other experimental attempts in feedback control have not been able to terminate fibrillation by applying control stimuli at a single spatial location.

More successful, at least in numerical simulations, have been schemes where control stimuli is applied throughout the system. Such global control schemes either apply small perturbations to the dynamical variables (e or g) or one of the parameters (usually the excitation threshold). The general scheme involves introducing an external control signal A into the model equations, e.g., in the Panfilov model (Eq. 2):

$$\partial e/\partial t = \nabla^2 e - f(e) - g + A,$$

for a control duration \(\tau\). If \(A\) is a small, positive perturbation, added to the fast variable, the result is an effective reduction of the threshold (Fig. 2 bottom), thereby making simultaneous excitation of different regions more likely. In general, \(A\) can be periodic, consisting of a sequence of pulses. Fig. 3 shows the results of applying a pulse of fixed amplitude but varying durations. While in general, increasing the amplitude, or the duration, increases the likelihood of suppressing spatiotemporal chaos, it is not a simple, monotonic relationship. Depending on the initial state at which the control signal is applied, even a high amplitude (or long duration) control signal may not be able to uniformly excite all regions simultaneously. As a result, when the control signal is withdrawn, the inhomogeneous activation results in a few regions becoming active again and restarting the spatiotemporal chaotic behavior.

Most global control schemes are variations or modifications of the above scheme. Osipov and Collins have shown that a low-amplitude signal used to change the value of the slow variable at the front and back of an excitation wave can result in different wavefront and waveback velocities which destabilizes the traveling wave, eventually terminating all activity, and, hence, spatiotemporal chaos. Gray has investigated the termination of spiral wave breakup by using both short and long-duration pulses applied on the fast variable, in 2D and 3D systems. This study concluded that while short duration pulses affected only the fast variable, long duration pulses affected both fast and slow variables and that the latter is more efficient, i.e., uses less power, in terminating spatiotemporal chaos. The external control signal can also be periodic \([A = F\sin(\omega t)]\), in which case the critical amplitude \(F_c\) required for terminating activity has been found to be a function of the signal frequency \(\omega\).

Other schemes have proposed applying perturbations to the parameter controlling the excitation threshold, \(b\). Applying a control pulse on this parameter \((b = b_f, \text{ during duration of control pulse})\) has been shown to cause splitting of an excitation wave into a pair of forward and backward moving waves. Splitting of a spiral wave causes the two newly created spirals to annihilate each other on collision. For a spatiotemporally chaotic state, a sequence of such pulses may cause termination of all excitation, there being an optimal time interval between pulses that results in fastest control. Another control scheme that also applies perturbation to the threshold parameter is the uniform periodic forcing method suggested by Alonso et al. for controlling scroll wave turbulence in three-dimensional excitable media. Such turbulence results from negative tension between scroll wave filaments, i.e., the line joining the phase singularities about which the scroll wave rotates. In this control method, the threshold is varied in periodic manner \([b = b_0 + b_f\cos(\omega t)]\) and the result depends on the relation between the control frequency \(\omega\) and the spiral rotation frequency. If the former is higher than the latter, sufficiently strong forcing is seen to eliminate turbulence; otherwise, turbulence suppression is not achieved. The mechanism underlying termination has been suggested to be the effective increase of filament tension due to rapid forcing, such that, the originally negative tension between scroll wave filaments is changed to positive tension. This results in expanding scroll wave filaments to instead shrink and collapse, eliminating spatiotemporal chaotic activity. In a variant method, the threshold parameter has been perturbed by spatially uncorrelated

FIG. 3: Global control of the 2-dimensional Panfilov model with \(L = 256\) starting from a spatiotemporally chaotic state (top left). Pseudo-gray-scale plots of excitability \(e\) show the result of applying a pulse of amplitude \(A = 0.833\) between \(t = 11\) ms and 27.5 ms (top centre) that eventually leads to elimination of all activity (top right). Applying the pulse between \(t = 11\) ms and 33 ms (bottom left) results in some regions becoming active again after the control pulse ends (bottom centre) eventually reinitiating spiral waves (bottom right).
FIG. 4: Spatiotemporal chaos (top row) and its control (bottom row) in the 2-dimensional Luo-Rudy I model with \( L = 90 \) mm. Pseudo-gray-scale plots of the transmembrane potential \( V \) show the evolution of spiral turbulence at times \( T = 30 \) ms, 90 ms, 150 ms and 210 ms. Control is achieved by applying an external current density \( I = 150 \mu A/cm^2 \) for \( \tau = 2.5 \) msec over a square mesh with each block of linear dimension \( L/K = 1.35 \) cm. Within 210 msec of applying control, most of the simulation domain has reached a transmembrane potential close to the resting state value; moreover, the entire domain is much below the excitation threshold. The corresponding uncontrolled case shows spatiotemporal chaos across the entire domain.

Gaussian noise, rather than a periodic signal, which also results in suppression of scroll wave turbulence \[27\].

As already mentioned, global control, although easy to understand, is difficult to achieve in experimental systems. A few cases in which such control could be implemented include the case of eliminating spiral wave patterns in populations of the Dictyostelium amoebae by spraying a fine mist of cAMP onto the agar surface over which the amoebae cells grow \[28\]. Another experimental system where global control has been implemented is the photosensitive Belousov-Zhabotinsky reaction, where a light pulse shining over the entire system is used as a control signal \[29\]. Indeed, conventional defibrillation can be thought of as a kind of global control, where a large amplitude control signal is used to synchronize the phase of activity at all points by either exciting a previously unexcited region (advancing the phase) or slowing the recovery of an already excited region (delaying the phase) \[30\].

IV. NON-GLOBAL SPATIALLY EXTENDED CONTROL

The control methods discussed so far apply control signal to all points in the system. As the chaotic activity is spatially extended, one may naively expect that any control scheme also has to be global. However, we will now discuss some schemes that, while being spatially extended, do not require the application of control stimuli at all points of the system.

A. Applying control over a mesh

The control method of Sinha et al \[31\] involving suprathreshold stimulation along a grid of points is based on the observation that spatiotemporal chaos in excitable media is a long-lived transient that lasts long enough to establish a non-equilibrium statistical steady state displaying spiral turbulence. The lifetime of this transient, \( \tau_L \), increases rapidly with linear size of the system, \( L \), e.g., increasing from 850 ms to 3200 ms as \( L \) increases from 100 to 128 in the two-dimensional Panfilov model. This accords with the well-known observation that small mammals do not get life-threatening VF spontaneously whereas large mammals do \[32\] and has been experimentally verified by trying to initiate VF in swine ventricular tissue while gradually reducing its mass \[33\]. A related observation is that non-conducting boundaries tend to absorb spiral excitations, which results in spiral waves not lasting for appreciable periods in small systems.

The essential idea of the control scheme is that a domain can be divided into electrically disconnected regions by creating boundaries composed of recovering cells between them. These boundaries can be created by triggering excitation across a thin strip. For two-dimensional media, the simulation domain (of size \( L \times L \)) is divided into \( K^2 \) smaller blocks by a network of lines with the block size \( (L/K \times L/K) \) small enough so that spiral waves cannot form. For control in a 3D system, the mesh is used only on one of the faces of the simulation box. Control is achieved by applying a suprathreshold stimulation via the mesh for a duration \( \tau \). A network of excited and subthreshold stimulation along a grid of points is based on the observation that spatiotemporal chaos in excitable tissue while gradually reducing its mass \[33\].
fixed at a constant value $L/K$ for the duration of control. The network effectively simulates non-conducting boundary conditions (for the block bounded by the mesh) for the duration of its recovery period, in so far as it absorbs spirals formed inside this block. Note that $\tau$ need not be large at all because the individual blocks into which the mesh divides the system (of linear size $L/K$) are so small that they do not sustain long spatiotemporally chaotic transients. Nor does $K$, which is related to the mesh density, have to be very large since the transient lifetime, $\tau_L$, decreases rapidly with decreasing $L$. The method has been applied to multiple excitable models, including the Panfilov and Luo-Rudy models (Fig. 4).

An alternative method [17] for controlling spiral turbulence that also uses a grid of control points has been demonstrated for the Aliev-Panfilov model. Two layers of excitable media are considered, where the first layer represents the two-dimensional excitable media exhibiting spatiotemporal chaos that is to be controlled, and the second layer is a grid structure also made up of excitable media. The two layers are coupled using the fast variable but with asymmetric coupling constants, with excitation pulses travelling $\sqrt{D}$ 4 times faster in the second layer compared to the first. As the second layer consists only of grid lines, it is incapable of exhibiting chaotic behavior in the uncoupled state. If the coupling from the second layer to the first layer is sufficiently stronger than the other way round, the stable dynamics of the second layer (manifested as a single rotating spiral) overcomes the spiral chaos in the first layer, and drives it to an ordered state characterized by mutually synchronized spiral waves.

**B. Applying control over an array of points**

An alternative method of spatially extended control is to apply perturbations at a series of points arranged in a regular array. Rappel et al. [34] had proposed using such an arrangement for applying a time-delayed feedback control scheme. However, this scheme does not control spatiotemporal chaos and is outside the scope of this review.

More recently, the authors [35] have used an array of control points to terminate spatiotemporal chaos in the Panfilov model. Fig. 5 shows the result of applying a spatially non-uniform control scheme, which simulates an excitation wave traveling over the system, with the same wavefront velocity as in the actual medium. The control points are placed distance $d$ apart along a regular array. At certain times, the control points at one corner of the system is stimulated, followed by the successive stimulation of the neighboring control points, such that a wave of stimulation is seen to move radially away from the site of original stimulation. This process is repeated after suitable intervals. Note that, simulating a traveling wave using the array is found to be more effective at controlling spatiotemporal chaos than the simultaneous activation of all control points. Using a traveling wave allows the control signal to engage all high-frequency sources of excitation in the spiral turbulence regime, ultimately resulting in complete elimination of chaos. If, however, the control had only been applied locally the resulting wave could only have interacted with neighboring spiral waves and the effects of such control would not have been felt throughout the system. The efficacy of the control scheme depends upon the spacing between the control points, as well as the number of simulated traveling waves. Traveling waves have previously been used in Ref. [36] to control spatiotemporal chaos, although in the global control context with a spatiotemporally periodic signal being applied continuously for a certain duration, over the entire system.

**V. LOCAL CONTROL OF SPATIOTEMPORAL CHAOS**

We now turn to the possibility of controlling spatiotemporal chaos by applying control at only a small localized region of the spatially extended system. Virtually all the proposed local control methods use overdrive pacing, generating a series of waves with frequency higher than any of the existing excitations in the spiral turbulent state. As low-frequency activity is progressively invaded by faster excitation, the waves generated by the control stimulation gradually sweep the chaotic activity to the system boundary where they are absorbed. Although we cannot speak of a single frequency source in the case of chaos, the relevant timescale is that of the
spiral waves and is related to the recovery period of the medium. Control is manifested as a gradually growing region in which the waves generated by the control signal dominate, until the region expands to encompass the entire system. The time required to achieve termination depends on the frequency difference between the control stimulation and that of the chaotic activity, with control being achieved faster when this difference is greater.

Stamp et al. [37] has looked at the possibility of using low-amplitude, high-frequency pacing using a series of pulses to terminate spiral turbulence. However, using a series of pulses (having various waveform shapes) has met with only limited success in suppressing spatiotemporal chaos. By contrast, a periodic stimulation protocol [38] has successfully controlled chaos in the 2D Panfilov model, as well as other models [39]. The key mechanism underlying such control is the periodic alternation between positive and negative stimulation. A more general control scheme proposed in Ref. [40] uses biphasic pacing, i.e., applying a series of positive and negative pulses, that shortens the recovery period around the region of control stimulation, and thus allows the generation of very high-frequency waves than would have been possible using positive stimulation alone. A simple argument shows why a negative rectangular pulse decreases the recovery period for an excitable system. The stimulation vertically displaces the e-nullcline and therefore, the maximum value of $q$ that can be attained is reduced. Consequently, the system will recover faster from the recovery period (Fig. 2 bottom).

To understand how negative stimulation affects the response behavior of the spatially extended system, we can use pacing response diagrams (Fig. 6 left) indicating the relation between the control stimulation frequency $f$ and the effective frequency $f_{\text{eff}}$, measured by applying a series of pulses at one site and then recording the number of pulses that reach another site located at a distance without being blocked by a region in the recovery period. Depending on the relative value of $f^{-1}$ and the recovery period, we observe instances of $n : m$ response, i.e., $m$ responses evoked by $n$ stimuli. If, for any range of $f$, the corresponding $f_{\text{eff}}$ is significantly higher than the effective frequency of spatiotemporal chaos, then termination of spiral turbulence is possible. As shown in Ref. [40], there are indeed ranges of stimulation frequencies that give rise to effective frequencies that dominate chaotic activity. As a result, the periodic waves emerging from the stimulation region gradually impose control over the regions exhibiting chaos (Fig. 6). Note that, there is a tradeoff involved here. If $f_{\text{eff}}$ is only slightly higher than the chaos frequency, control takes too long. On the other hand, if it is too high the waves suffer conduction block at inhomogeneities produced by chaotic activity which reduces the effective frequency, and therefore, control fails.

Recently, another local control scheme has been proposed [41] that periodically perturbs the model parameter governing the threshold. In fact, it is the local control analog of the global control scheme proposed by Alonso et al. [12] discussed in section III. As in the other methods discussed here, the local stimulation generates high-frequency waves that propagate into the medium and suppress spiral or scroll waves. Unlike the global control scheme, $b_f >> b_0$, so that the threshold can be negative for a part of the time. This means that the regions in resting state can become spontaneously excited, which allow very high-frequency waves to be generated.

VI. DISCUSSION

Most of the methods proposed for controlling spatiotemporal chaos in excitable media involve applying perturbations either globally or over a spatially extended system of control points covering a significant proportion of the entire system. However, in most practical situations this may not be a feasible option, either for issues of implementation, or because of the high power for the control signal that such methods would need. Moreover,
if one is using such methods in the clinical context, e.g., terminating fibrillation, a local control scheme has the advantage that it can be readily implemented with existing hardware of the Implantable Cardioverter Defibrillator (ICD). This is a device implanted into patients at high risk from fibrillation that monitors the heart rhythm and applies electrical treatment when necessary through electrodes placed on the heart wall. A low-energy control method involving ICDs should therefore aim towards achieving control of spatiotemporal chaos by applying small perturbations from a few local sources.

However, the problem with most local control schemes proposed so far is that they use very high-frequency waves to overdrive chaos. Such waves are themselves unstable and may break during propagation, resulting in reinitiation of spiral waves after the original chaotic activity has been terminated. The problem is compounded by the existence of inhomogeneities in real excitable media. Recently, Shajahan et al. [42] have found complicated dependence of spatiotemporal chaos on the presence of non-conducting regions and other types of inhomogeneities in an excitable system. Such inhomogeneities make the proposed local control schemes more vulnerable, as it is known that high-frequency pacing interacting with, e.g., non-conducting obstacles, results in wave breaks and subsequent genesis of spatiotemporal chaos [43].

The search is still on for a control algorithm for terminating spatiotemporal chaos in excitable media, that can be implemented using low power, or, that need be applied in only a small, local region of the system, and which will yet be robust, capable of terminating spiral turbulence without the control stimulation itself breaking up subsequently. The payoffs for coming up with such a method are enormous, as the potential benefits include an efficient device for cardiac defibrillation.

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