A METHOD TO SEARCH FOR CORRELATIONS OF ULTRA-HIGH ENERGY COSMIC-RAY MASSES WITH THE LARGE-SCALE STRUCTURES IN THE LOCAL GALAXY DENSITY FIELD

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ABSTRACT

One of the main goals of investigations using present and future giant extensive air shower (EAS) arrays is the mass composition of ultra-high energy cosmic rays (UHECRs). A new approach to the problem is presented, combining the analysis of arrival directions with the statistical test of the paired EAS samples. One of the ideas of the method is to search for possible correlations between UHECR masses and their separate sources; for instance, if there are two sources in different areas of the celestial sphere injecting different nuclei, but the fluxes are comparable so that arrival directions are isotropic, then the aim is to reveal a difference in the mass composition of cosmic-ray fluxes. The method is based on a non-parametric statistical test—the Wilcoxon signed-rank routine—which does not depend on the populations fitting any parameterized distributions. Two particular algorithms are proposed: first, using measurements of the depth of the EAS maximum position in the atmosphere; and second, relying on the age variance of air showers initiated by different primary particles. The formulated method is applied to the Yakutsk array data, in order to demonstrate the possibility of searching for a difference in average mass composition between the two UHECR sets, arriving particularly from the supergalactic plane and a complementary region.

Key words: cosmic rays – instrumentation: miscellaneous – methods: data analysis

Online-only material: color figures

1. INTRODUCTION

The origin of ultra-high energy cosmic rays (UHECRs) is a long-standing challenge for astrophysics. The energy spectrum of particles constituting cosmic rays (CRs) is measured up to and slightly above $10^{20}$ eV ($\approx 100$ EeV) (Abbasi et al. 2008; Salamida et al. 2011; Egorova et al. 2004), but no evidence has been revealed until now, either of the sources or the sort(s) of highest energy particles, owing mainly to the arrival direction distribution, which is nearly isotropic (Cronin 2005; Grieder 2010). However, some hints have been found recently of the possible correlation of UHECR arrival directions with nearby active galactic nuclei (AGNs) at energies $E > 56$ EeV (PAO Collaboration 2007).

Disputable estimates of the mass composition of the highest energy CRs were given, based on the average depth of EAS maximum, $x_m$, and variance, $\sigma(x_m)$, measured by the Pierre Auger Observatory (PAO), the High Resolution Fly’s Eye (HiRes), the Telescope Array (TA), and the Yakutsk array (UHECR composition Working Group report at CERN Conference, Bellido et al. 2012). One possible explanation of the diverging results may be the different average masses of UHECRs, observed in different fields of view of the instruments.

In this paper, another approach is used to search for the possible correlation of UHECR masses with their sources, based on extensive air shower (EAS) observables, namely $x_m$ and the shower age, $\tau$, varying with the mass of primary particles. Here, “age” means the stage of the cascade development at the detector level, $x_0: \tau = x_0 \sec \theta/x_m$, where $\theta$ is the inclination angle of the shower axis (Ivanov et al. 2011a, 2011b). This method is convenient for revealing different primary particles initiating EAS, rather than searching for excess flux from candidate sources of UHECRs.

The paper is structured as follows. The new method is described in Section 2, where two algorithms are formulated using the depth of EAS maximum and the shower age measurements, and a non-parametric statistical test is applied to distinguish a pair of samples. In Section 3, the method is tested and applied to the Yakutsk array data observed in the energy range above 1 EeV. Conclusions are given in the final section.

2. SEARCHING FOR A DIFFERENCE IN MASS COMPOSITION OF UHECRS ARRIVING FROM COMPLEMENTARY CELESTIAL REGIONS

The method is aimed at the possible differences in the masses of UHECRs arriving from different regions in the celestial sphere. For example, in models of CR acceleration by shocks in AGN relativistic jets, mechanisms are proposed where the maximal energy of CRs is proportional to the particle charge, and where protons could be accelerated up to energy $\sim 100$ EeV (Berezhko 2009) and iron nuclei to energy $\sim 3000$ EeV (in Cen A; Honda 2009). Observable particle energies should be reduced due to fragmentation and energy loss in the intergalactic medium and in the Galactic wind. However, in the Cen A case, the object is nearby ($<5$ Mpc), so that the energy is not crucially degraded.

By selecting EASs with arrival directions in the vicinity of Cen A, in contrast with the complementary area where, presumably, protons dominate, one can reveal the fraction of heavy nuclei, if the model of Honda (2009) is applicable. An obstacle is deflections of particles in magnetic fields—it was shown that protons of GZK energy ($\sim 40$ EeV) are deflected a few degrees coming from any source within 100 Mpc (Sommers & Westerhoff 2009). Deflections of iron nuclei from Cen A would be greater by $\sim 30\%$, so that the energy of particles detected should be well above the GZK energy.

In the transition region between galactic and extragalactic components of CRs, this method can be used to verify the difference in mass composition of the two components, comparing, for example, equatorial and polar regions in galactic coordinates.

The method can be regarded as an extension of the matter tracer model proposed by Koers & Tinyakov (2009) for the...
case where different particles are supposed to be generated in UHECR sources.

Our first task is to test the null hypothesis; that is, there is no difference in mass composition between two regions. We have to compare two distributions of some EAS observables sensitive to the composition of the primaries in the given energy range, in order to decide whether or not there is a significant difference.

If the difference exceeds experimental errors, then the only cause should be the mass composition in the UHECR samples. So the next task would be to evaluate the most probable value and confidence interval of the mass difference. In this paper, we focus on the first task.

2.1. Non-parametric Statistical Test of Data Samples

The distributions of measured EAS parameters are not usually described by the normal distribution, and moreover, they have a specific form in each particular case, so that the general approach to the statistical test of the data samples is preferably non-parametric. The meaning of the term refers here to distribution-free methods, which do not rely on the assumption that the data are drawn from a known probability distribution. Non-parametric methods can be used, for instance, to study samples of EAS data where certain assumptions cannot be made about the original population. An additional advantage of this approach is that in the case when the use of parametric methods is justified, non-parametric methods are easier to use, and leave less room for improper use and misunderstanding.

In this paper, a Wilcoxon test for a pair of samples is used in the data analysis. This test is one of the widely known, non-parametric significance tests (open-access description is given by Lowry 1998). It is useful for deciding whether the two samples of observations belong to the same original distribution: the null hypothesis, \( H_0 \), is that the two samples are drawn from a single population.

Hereafter, we are going to consider pairs of matched samples, \( S \) and \( T \), containing an equal number of measurements, \( N \). In this case, the test is called the “Wilcoxon signed-rank test” (WSRT), in contrast to the case with independent samples, named the “Wilcoxon rank-sum test,” or Mann–Whitney U test. As an example of paired samples \( S \) and \( T \) in the case of CRs, we can consider two samples of \( x_m \) measurements in EAS detected in Summer and in Winter with the same array, with equal energies in the pairs of events. Independent samples are those measured in the same energy interval, but without equality in pairs.

The test procedure is as follows. Excluding all pairs in samples where observed values are equal, \( S_i = T_i \), we reduce \( N \) to the number of pairs with unequal measurements. We rank the differences \( |S_i - T_i| \) in ascending series, where rank is assigned as an item number in a series. The WSRT statistic, \( W \), is then a sum of signed ranks\(^2\): \( |W| \leq 0.5N(N + 1) \). Under \( H_0 \), the distribution of \( W \) is symmetric with \( W = 0 \) and \( \sigma_W = \sqrt{N(N + 1)(2N + 1)/6} \). There are tabulated values of the ratio \( z_0 = (W_0 - 0.5)/\sigma_W \) with which to compare the resultant statistic.

The probability \( P_{\text{crit}}(z > z_0) \) is a measure of the significance of deviation from the expected value under \( H_0 \). In the following, \( P_{\text{crit}} = 0.01 \) is assumed as the critical value for rejecting the null hypothesis.

The \( \chi^2 \)-deviation from the observed numbers is 168.6 for the normal approximation in 17 intervals, and is 1426 for the lognormal approximation, while less than 32 expected at the significance level \( P_{\text{crit}} \) for good fit.

\(^1\) We do not consider the case of different CR energy spectra from different celestial regions.

\(^2\) Sign is +1 if \( S_i > T_i \), and is −1 if \( S_i < T_i \).
at minimal $\Delta x_m$. It means that the $t$-test is more efficient at small sample sizes, i.e., $N_{\text{min}}^{\text{Student}}$ can be reduced to $\sim 70\%$ of Wilcoxon test’s number with equal statistical power. However, in our case, asymptotic efficiencies of the tests, defined as the limit of the efficiency as the sample size grows, are equal. The ratio is valid in the case of normal distributions, where the $t$-test is applicable.

2.3. A Method Based on the Measurement of the Depth of EAS Maximum

In the UHECR domain, the shower maximum is directly observable with air fluorescence detectors (FDs). Collaborations working at HiRes (Abbasi et al. 2005, 2010), PAO (San Luis et al. 2011), and TA (Tsunesada for the TA Collaboration 2011; Tameda for the Telescope Array and HiRes Collaborations 2012) have measured $\overline{x}_m$ as a function of energy to estimate the mass composition of CRs. Future detectors, such as JEM-EUSO, Auger Next, etc., plan to use the fluorescence technique, including $x_m$ measurement, along with other developments. In all of these experiments, the proposed method can be applied to search for correlations of UHECR masses with their sources.

A relation between $x_m$ and CR mass $A$, is obvious in a superposition approximation, where an air shower, initiated by the nucleus of mass $A$, is treated as a superposition of $A$-nucleon-initiated showers of energy $E/A$. So the depth of a shower maximum is $x_m = x_{18} + D_{ER} \ln(E/A)$ (Linsley 1977), where $D_{ER}$ is the elongation rate, and $E$ is in EeV. Averaging it over the distribution of CR energy and mass, $f(E, A)$, we have

$$\overline{x}_m = x_{18} + D_{ER} \ln E - D_{ER} \ln A.$$  \hspace{1cm} (1)

Assuming $A$ as the slowly changing function of energy, we can use the relation

$$\Delta \overline{x}_m = D_{ER} \Delta \ln A$$  \hspace{1cm} (2)

in a narrow energy interval.

The accuracy of estimations (1) and (2) can be evaluated in comparison with the model simulation results. Monte Carlo codes such as CORSIKA (Heck et al. 1998), with implemented hadronic interaction models, give a more realistic description of the cascade than the superposition approximation. In Figure 3, a comparison of the results of CORSIKA simulations (QGSJetII, Sibyll2.1, and EPOSv1.99) are implemented; Kampert & Unger (2011) with Equation (1) is presented. The difference between the QGSJet results and that of Equation (1) is $4\%–25\%$ in the energy interval (0.1, 10) EeV; for Sibyll and EPOS models, the divergence is not greater than $2\%$.

Although the superposition approximation is not necessarily needed for our method to be applicable, it is convenient to accept it to simplify the conclusions. So we can use the results of the Sibyll2.1 or EPOSv1.99 models, together with Equations (1) and (2) within the intervals $E \in (0.1, 10)$ EeV, $A \in (1, 56)$, where the discrepancy is less than or equal to $2\%$.

A straightforward approach to estimating a difference in average mass composition is to compare the $\overline{x}_m$ of two samples of showers in the fixed energy interval. In the case of normal distributions with equal dispersions, this is the $t$-test mentioned above. If there is a statistically significant difference $\Delta x_m$, then one can certainly conclude that it is owed to the difference in the average masses of samples, $\Delta \ln A$.

Actually, this approach is inefficient because of $\overline{x}_m$ rising with energy, and mass-dependent rms deviation $\sigma(x_m)$. The most stringent restriction is caused by energy dependence: the two samples should be within a narrow energy interval, where few showers are detected with present-day arrays. For instance, the maximum number of EAS events detected in the $\lg E$ interval of width 0.2, where $x_m$ observations are available, is 1287 for the PAO data (San Luis et al. 2011, $\lg E \in (18.0, 18.2)$), 171 for the HiRes data (Abbasi et al. 2005, 2010; $\lg E \in (18.0, 18.2)$), and 68 for the TA data (Tsunesada for the TA Collaboration 2011; Tameda for the Telescope Array and HiRes Collaborations 2012; $\lg E \in (18.6, 18.8)$).

In Table 1, the estimations of the majorant resolution are given for models and data sets. Here, the $t$-test is assumed to be applicable and the mass dependence of the $x_m$ dispersion

![Figure 2](image1.png)  

Figure 2. Comparison of the efficiency of Wilcoxon and Student tests. $N_{\text{min}}$ is the minimal sample size required to reject the null hypothesis, when an alternative hypothesis is true with a given difference, $\Delta x_m$, in paired EAS samples. (A color version of this figure is available in the online journal.)

![Figure 3](image2.png)  

Figure 3. Relation of elongation rates in EAS initiated by nuclei. CORSIKA simulation results (taken from the review of Kampert & Unger 2011) for models are shown by three points calculated at $E = 0.1/10$ EeV. Superposition approximation results in $|\partial x_m/\partial \ln E| = |\partial \overline{x}_m/\partial \ln A|$.

(A color version of this figure is available in the online journal.)

| Detector | $\sigma(x_m)$ (g cm$^{-2}$) | $\Delta x_m$ (g cm$^{-2}$) | $\Delta \ln A$ |
|----------|-----------------------------|-----------------------------|----------------|
| PAO      | 56                          | 7.2                         | 0.29           |
| HiRes    | 52                          | 20.0                        | 0.81           |
| TA       | 52                          | 32.1                        | 1.30           |
| SiByll2.1| 56                          | 7.2                         | 0.29           |
| EPOSv1.99| 52                          | 20.0                        | 0.81           |

Table 1  

Maximal Resolution of the Differences in Average $x_m$ and UHECR Mass, $\Delta \ln A$, with Student’s $t$-test, Using Available Data from EAS Arrays

In Table 1, the estimations of the majorant resolution are given for models and data sets. Here, the $t$-test is assumed to be applicable and the mass dependence of the $x_m$ dispersion...
is neglected, so the real resolution should be worse. The maximal number of events detected in \( \Delta \lg E = 0.2 \) intervals are divided equally between samples; \( \Delta x_m \) resolvable by the \( t \)-test is calculated as in Figure 2. In order to estimate \( \Delta \ln A \), Equation (2) is used.

Wilcoxon test results are comparable or slightly weak. The experimental values of \( \partial x_m / \partial \ln E \) are influenced by the possible changes of mass composition with energy; so one has to use the model simulation of \( D_{\text{UR}} \) with fixed mass in order to estimate the resolution.

Assessment of the resolution is ambiguous. In this paper, we assume that the resolution is sufficient if 10% of the flux of \( Fe \) nuclei is resolved in the background consisting of protons. In this context, the maximal resolution in \( x_m \) is comparable to the experimental uncertainties, and the resolution of the average mass differences is insufficient or hardly sufficient in the PAO case. Only future arrays with considerably larger aperture may improve the efficiency of this method.

Another approach should be used that is applicable to all sorts of \( x_m \) distributions with energy- and mass-dependent parameters. WSRT is a promising method which can be adapted to the case. In order to use WSRT for the analysis of EASs arriving from different celestial regions, we have to compare the rank sum of \( x_m \) in the series of events. Due to the limited number of events available at the highest energies of interest, we should extend the boundaries of the energy interval from which to select showers.

To do so, the paired samples can be used in which every EAS from one sample has its counterpart in another sample with the same or closest energy. In this case, the two samples have the same \( x_m \) if the original mass compositions are identical, in spite of energy-dependent \( \overline{x_m} \) and \( \overline{A} \). On the other hand, if there is a difference in the average mass of the original populations, then \( \Delta x_m \) should exist and is approximated by Equation (1).

With a known efficiency of WSRT for resolving \( \Delta x_m \) (Figure 2) and model simulations of \( D_{\text{UR}} \), we can estimate \( \Delta \ln A \) for the observational number of showers. Namely, simulation results with the CORSIKA code of \( \partial x_m / \partial \ln A \) for Sibyll (25, 24, and 23 g cm\(^{-2}\)) and EPOS (27, 26, and 26 g cm\(^{-2}\)) models at energies 1, 10, and 55 EeV (Kampert & Unger 2011) are used to relate \( \Delta x_m \) and \( \Delta \ln A \) in Equation (2).

In Table 2, the results for the PAO FD real data (San Luis et al. 2011) and the estimated number of UHECRs (\( E > 55 \) EeV) to be detected using JEM-EUSO during five years of orbiting the earth onboard the ISS (Adams et al. 2012) are shown. Duty cycle 0.19 and cloud impact 0.7 factors are accepted for the JEM-EUSO exposure. The resolution in \( x_m \) is assumed to be 120 g cm\(^{-2}\).

The number of events is only sufficient in the PAO FD case for the threshold energy 1 EeV. The data of JEM-EUSO will not be convenient for this kind of analysis, owing mainly to poor resolution in the depth of the shower maximum.

To increase the number of events under analysis at the highest energies, the data of the surface detectors (SD) can be used. This possibility will be discussed in the next section.

### 2.4. A Method Based on the Age Variance of Air Showers Initiated by Different Primary Particles

There are some shower parameters measured by the SDs of the EAS arrays which are sensitive to the nuclear composition of the primary particle: muon content, shower front curvature, etc. It seems that one of the most appropriate parameters is the shower age. It is related explicitly to \( x_m \) and hence to the primary mass, and can be estimated in each shower using the universal relation with lateral distribution (LD) parameters, measured by the SDs, in particular, the slope, \( \eta_s \), of the charged particles LD (Ivanov et al. 2011a, 2011b). Only electromagnetic component detectors are needed in the surface stations in this case, so the method can be applied to arrays with no muonic and other component detectors.

As in the previous section, with two paired samples of EAS ages, one can apply WSRT in order to decide whether there is an appreciable difference in the mean age of the showers in samples \( S \) and \( T \), or whether these samples are drawn from the same distribution (the null hypothesis). To do so, one has to compare the ranks in samples of ages. By consulting with statistical tables about the deviation of the statistic from the expected value, one can accept or reject the null hypothesis at the significance level specified.

Using the age variance instead of \( \Delta x_m \) in EASs initiated by different primary particles, one has an additional variable to fix: the zenith angle. So the paired samples should be selected with close or equal energies and zenith angles. This can be done as follows: initially, one has two subsets of EASs that have arrived from different sources, and presumably, with different masses. For each shower from the minor subset, a counterpart shower should be found in another subset with equal or closest \( \theta \) and \( E \), and then ages of these EASs should be collected in corresponding samples \( S \) and \( T \). Selected showers should be removed from subsets. By repeating operations until the end of the minor subset, one assembles a pair of samples of size \( N \) that are congruous, on average, at \( N \gg 1 \), to the circumstances of zenith angle and energy.

A relation between shower age and LD slope can be measured experimentally by PAO and TA fluorescence and SDs working together in the same showers. Meanwhile, an estimation can be used which was derived by Ivanov et al. (2011a, 2011b) using the CORSIKA code with implemented SIBYLL2.1/UrQMD models.

A minimal mass difference, resolvable by applying WSRT to age samples, is limited by uncertainty in the age estimation and zenith angle. By neglecting the uncertainty of the \( \theta \) measurement in PAO data (\( d \cos \theta / \cos \theta < 0.01 \)), and using the number of EAS events detected during the period between 2004 January 1 and 2010 December 31 (Salamida et al. 2011), we have obtained the estimation of the minimal difference in average mass, resolvable by applying WSRT to PAO SD data (Table 3). Here, the minimal difference in shower age resolvable by the method is used, which in turn is related to the difference in \( x_m \): \( |\Delta \theta / \theta| = |\Delta x_m / x_m| \) for a fixed zenith angle. \( \Delta x_m \) is calculated by applying the test for a given number of EAS events in samples

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**Table 2**

| Experiment          | \( N_{\text{obs}} \) | \( \Delta \ln A \) |
|---------------------|-----------------------|---------------------|
|                      | Sibyll2.1  | EPOSv1.99         |
| PAO (\( E > 1 \) EeV) | 6744          | 0.13               | 0.12         |
| PAO (\( E > 10 \) EeV) | 339            | 0.61               | 0.56         |
| JEM-EUSO (\( E > 55 \) EeV) | Tilted mode | 1800               | 0.59         |

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4 For the fixed zenith angle.
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Table 3
Estimation of the Minimal Difference in the Shower Age, \( \Delta \tau \), and Average Mass, \( \Delta \ln A \), of UHECRs Observed by PAO Surface Detectors, Resolvable by WSRT

| \( E_{\text{obs}}, \text{EeV} \) | \( N_{\text{obs}} \) | \( \Delta \tau \) | \( \Delta \ln A \) |
|----------------|----------|----------------|----------------|
| 3              | 63,376   | 0.0015         | 0.04           | 0.03           |
| 10             | 4,790    | 0.0061         | 0.16           | 0.15           |
| 25             | 608      | 0.0102         | 0.28           | 0.26           |

\( \sim N_{\text{obs}}/2 \); average values of \( x_m \) are given by the approximation of the PAO data (San Luis et al. 2011).

A comparison of Tables 2 and 3 shows that, in general, the number of events detected with PAO SDs is sufficient (contrary to FDs with reduced duty cycle) to apply WSRT to the shower age and mass variance of EAS primaries in the energy range above 10 EeV, and may be applicable at energies above 25 EeV. Concerning future arrays, large apertures, sufficient resolution in \( x_m \) and/or \( \tau \), and the presence of the SDs would be essential conditions for the applicability of the method.

3. APPLICATION OF THE METHOD TO ANALYSIS OF THE YAKUTSK ARRAY DATA

In this section, the trial run of the method is realized with the Yakutsk array data.

3.1. The Yakutsk Array

The Yakutsk array detects EAS of CRs in the energy interval from 1 PeV to 100 EeV. The array is located at 61°7' N, 129°4'E, 105 m above sea level (1020 g cm\(^{-2}\)). It consists of 71 surface and 6 underground scintillation detectors of charged particles (electrons and muons), and 49 detectors of the air Cherenkov light. The total area covered by detectors with 500 m separation is \( \sim 10 \) km\(^2\). The array has been operating since 1970, and approximately \( 10^6 \) showers of the primary energy above about 10\(^15\) eV have been detected. The highest energy event \((E \sim 100 \text{ EeV})\) detected was 7.05.1989, with an axis within the array area, at zenith angle \( \theta = 59^\circ \). A more extended description of the array and results obtained can be found in Egorova et al. (2004), Ivanov et al. (2009), and Ivanov (2010).

3.2. Slope of the Lateral Distribution Function of Charged Particles in EAS

An air shower cascading higher in the atmosphere ("old" shower, \( \tau > 1 \)) has a broad and flat LD of secondary particles at the observational level, while a "young" one (\( \tau < 1 \)) has a steep LD. It was shown previously that the LD parameters (slope of the Cherenkov light and charged particles LDs, etc.) can be used as indicators of the shower age (Dyakonov et al. 1979; Schmidt et al. 2008; Ivanov et al. 2011a, 2011b).

In this work, the LD slope of charged particles detected with scintillators of the Yakutsk array is used to estimate the shower age in the given energy and zenith angle intervals. Cherenkov light data are not used because of the small sample size of showers having this kind of signal detected. The same reason concerns the array’s muon detectors data.

The data set used to analyze the slope parameter consists of EAS events collected during the period 1974–2004, with energies from 1 to 100 EeV, zenith angles \( \theta < 50^\circ \), and axes within the array area. Inclined events beyond \( 50^\circ \) are rejected because of a substantial fraction of muons in the distribution of charged particles measured. In order to estimate the LD slope of each shower in a set, additional selection criteria were applied: (1) at least four stations in the core distance interval \( r \in (200, 1000) \text{ m} \) should have particle density above a threshold and (2) the slope calculated using the least-squares method should be in the interval \( \eta \in (−8, 0) \). The total number of events that survived after rejections is 19,600.

A distribution of the slopes in the narrow interval of energy and zenith angle is illustrated in Figure 4. Normal approximation is rejected by Pearson’s \( \chi^2 \) test because of the test-statistic equal to \( \chi^2 = 1697.8 \), while 23.2 is expected at the significance level \( P_{\text{crit}} \) with 10 degrees of freedom.

The same procedure as in Section 2.4 is used to apply the Wilcoxon test to paired samples, except for the shower age replaced by the LD slope here. Sample S consists of showers with arrival directions in the supergalactic (SG) "pancake" or "dumbbell" structure around the SG plane (Lahav et al. 2000). Sample T consists of all other showers. A hypothesis tested is that UHECR sources in SG pancake emit nuclei, whereas from other (distant) sources protons arrive. Our task is to ascertain if there is an appreciable difference between the average mass of UHECRs in the two samples.

In order to reveal the reliability bounds of WSRT in our particular case, we have applied a procedure to a pair of EAS event samples selected in the narrow primary energy bin \( E \sim 1 \text{ EeV} \), with arrival directions within the whole sky, but in adjacent zenith angle intervals, so that paired showers are of the same energy, and with a given difference in sec \( \theta \). This results in the same \( x_m \) but different ages of paired EAS. Due to the universality of EAS, the average shower age is connected with the LD slope (Ivanov et al. 2011a, 2011b).

Shifting adjacent zenith angle intervals, we can assign the difference in average LD slopes of samples. Then, we have to determine the minimal subsample size, \( N_{\text{min}} \), sufficient to reject the null hypothesis that there is no difference in our source samples of slopes, by applying the Wilcoxon test to paired subsamples. It is the efficiency of WSRT in the case of paired LD samples. In Figure 5, this limit is shown as a function of the given difference in average slopes. Experimental errors in charged particle densities measured by scintillators result in

\( 5 \) Where CRs are presumably of Galactic origin and homogeneous in composition as a result of confinement in magnetic fields.
in uncertainties of differences, $\Delta \eta$, and in the dispersion of points in the plot. The distribution of the signed rank sum in the case of a null hypothesis is calculated by extracting two paired subsamples of size $N_{min}$ from a single sample of LD slopes.

Although in Optical Redshift Survey and IRAS 1.2 Jy redshift survey data the density contrast in the local galaxy density field is aligned along SG axes with radius $40 \ h^{-1}$ Mpc and a thickness of $20 \ h^{-1}$ Mpc (Lahav et al. 2000), we have used three variants of the pancake angular boundaries in SG latitudes: $b_{SG} < 15^\circ/30^\circ/45^\circ$ in order to not miss the possible correlation of UHECRs with the SG plane proposed by Stanev (2008).

The results are shown in Figure 6. In all cases, we cannot reject the null hypothesis: there is no appreciable difference in LD slopes of EAS samples. Only at $E \sim 20 \ EeV$, $b_{SG} < 30^\circ$ is there the minimum of a probability, but it is greater than $P_{crit}$.

In Table 4, the minimal differences in EAS age-related parameters are given, resolvable by WSRT, with the sample sizes determined by the threshold energy. Simulation results concerning relations among the LD slope and the shower age and $x_m$, derived with the Sibyll model, are used in this case (Ivanov et al. 2011a, 2011b). A conclusion to be drawn is that the Yakutsk array data are too scanty to distinguish a possible difference in the mass composition of CRs, either at a threshold energy of $10 \ EeV$ or $26 \ EeV$, corresponding to C and Fe, respectively. Instead, the data are sufficient and can be used to search for variations in the composition of galactic CR at energies below $1 \ EeV$.

## 4. CONCLUSIONS

A new method has been developed combining the analysis of UHECR arrival directions with the statistical test of paired EAS samples. It is applicable in cases when sky regions can be separated, where CR mass compositions are presumably different. The most straightforward application of the method lies at high energies ($E > 10 \ EeV$) where magnetic deflections are smallest and are not expected to completely isotropize the CR sky distribution.

The method is based on a non-parametric statistical test, WSRT, which does not depend on the populations fitting any parameterized distributions. Two algorithms are proposed: first, using measurements of the depth of the EAS maximum position in the atmosphere; and second, relying on the age variance of air showers initiated by different primary particles.

The efficiency of WSRT is estimated with the distributions of shower maximum and the slope of LD in EAS. It is shown that the efficiency of Wilcoxon and Student tests are approximately equal in the case of a normal distribution of the variable.

It is also shown that the amount of data concerning $x_m$ measurements with present-day EAS arrays, and even those planned for the JEM-EUSO telescope, is not sufficient to distinguish a 10% flux of iron nuclei from the proton background at the significance level 0.01. However, measurements of EAS parameters related to the shower age with the SDs, specifically in the PAO experiment, provide the bulk of the data able to reveal such a flux.

A trial run of the test is performed with the Yakutsk array data in order to search for a possible difference in the mass of CRs arriving from the SG plane, and a complementary region. No significant difference is found in the LD slope parameter of the two subsets of EAS events detected with different threshold energies.

Using the correlation between the LD slope and the shower age, and the maximum position provided by the Sibyll model simulations of EAS initiated by different primary particles, estimations are given of minimal differences in these parameters, as well as the average mass of CRs, resolvable by the WSRT. The number of EAS events detected with the Yakutsk array is found to be insufficient to indicate any difference in masses of

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**Table 4**

| $E_{thr}$, $EeV$ | $N$ | $\Delta \eta$ | $\Delta \tau$ | $\Delta \ln A$ |
|-----------------|-----|---------------|---------------|---------------|
| 1.0             | 9180| 0.014         | 0.016         | 0.33          |
| 10.0            | 114 | 0.120         | 0.133         | 3.10          |
| 26.0            | 17  | 0.340         | 0.378         | 9.16          |

Note. $N$ is the number of CRs arrived at $|b_{SG}| < 30^\circ$; differences: in LD slope, $\Delta \eta$, in the shower age, $\Delta \tau$, in average CR mass, $\Delta \ln A$.

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6 Here, the rigidity $1 \ EeV$ is assumed as a very minimal limit to charged particles not confined in the Galaxy (Hörandel 2008).
CRs, in the energy range above 1 EeV. Instead, it can be used to search for variations of galactic CR composition below this energy.

The method developed is quite general and can be applied at other energies and to the various EAS parameters measured at existing and future observatories.

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