A Conjecture for Using Optical Methods for Affecting Superfluid Hydrodynamics

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Abstract

The relation between the macroscopic quantum coherent nature of superfluids and the coherent properties of optical interference patterns will be utilized to examine the optical properties of superfluid hydrodynamics. A Bragg pattern imposed on the superfluid (either holographically or using a phase mask) is expected to induce periodic variations in the local index of refraction of the normal and super fluid components. The altered optical properties can then be probed by a second coherent light source. In this manner, the behavior of the probe beam can be switched using the specific characteristics of the imposed pattern. Acoustic modes should also manifest measurable affects on incident coherent radiations.

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1 Introduction

There is considerable interest in developing optical phase masks with diffractive orders that can be tuned for varying applications without a need for material modifications or reconstruction. Such devices would not only be useful for writing Bragg patterns in optical fibers for communication purposes, but if the pattern is optically induced, would provide a mechanism for the optical switching of propagated modes. The development of such a device is explored in what follows. A more detailed description can be found in the literature\(^2\).

The imposed pattern induces spatial dependence of the dielectric constant of the form

\[
\epsilon = n_o^2 + \delta \epsilon \cos^2 \left( \frac{\pi y}{w} \right),
\]

(1.1)

where \(\delta \epsilon\) is dependent upon the field intensity of the pattern through induced temperature and density perturbations. The optical response of this pattern is then probed by directing a second coherent beam into this region, causing the probe beam to decompose into diffraction orders as illustrated in Figure 1:

As shown elsewhere, this response is effectively that of a tunable phase mask\(^3\),

![Figure 1: Probing Optical Density Pattern in Superfluid](image)

with the probe beam optically switched by the holographic pattern. The effect should manifest for any non-linear optical material or macroscopic quantum system, such as a superfluid or superconductor. In particular, the affects of such induced patterns on an optically thin superfluid will be examined in what follows.

2 Electrodynamic Equations

Maxwell’s equations in a spatially varying dielectric medium can be ex-

\(^2\)Lindesay, James V., Lyons, Donald R., and Quiett, Carranah J. “The Design of Fiber Optic Sensors for Measuring Hydrodynamic Parameters”, Trends in Electro-Optics Research, William T. Arkin, Ed., Nova Science Publishers, New York (ISBN 1-59454-498-0) (2006)

\(^3\)Lyons, Donald R and Lindesay, James V.. “Quantum Optical Methods of and Apparatuses for Writing Bragg Reflection Filters”, U.S. Patent 6,434,298 (Aug 2002)
pressed in the form
\[ \vec{\nabla}(\vec{E} \cdot \vec{\nabla} \log \epsilon) + \nabla^2 \vec{E} - \frac{\epsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0, \]
\[ -\vec{\nabla} \epsilon \times \vec{E} + \nabla^2 \vec{B} - \frac{\epsilon}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0. \]  

The time derivatives of the log of the dielectric constant are assumed small compared to the frequency of the electromagnetic fields. The polarization \( E_x \) of the probe field is chosen perpendicular to the induced variations in the dielectric \( \epsilon(y) \).

The probe field then reflects this periodic behavior
\[ E_x(y, z) = \sum_m G_m(z) e^{i2\pi m y} \]

Assuming a \( z \)-dependence of the form
\[ G_m(z; q) = A_m e^{iqz} + F_m e^{-iqz} \]
results in an eigenvalue equation for the propagation constants \( q^2 \), with degenerate eigenvectors \( A_m \) and \( F_m \). Substitution of the form in Eq. 1.1 gives the equation satisfied by the coefficients:
\[ \left\{ \epsilon_0 \left( \frac{\omega}{c} \right)^2 - \left( \frac{2\pi m w}{w} \right)^2 - q^2 \right\} A_m + \frac{\delta \epsilon}{4} \{ A_{m+1} + A_{m-1} \} = 0. \]

The unconstrained coefficients are chosen to satisfy the incoming and outgoing boundary conditions at the interphases.

To get a feel for the scale of the mode mixing in the probe beam, consider a pattern with spacing = 6\( \mu \)m imposed on a material with refractive index 1.75 inducing relative index variations of 0.5%. If a probe beam of wavelength 0.632\( \mu \)m is incident at the +1\textsuperscript{st} order angle 6.04637\( ^\circ \), the resulting orders can be numerically solved as a function of sample thickness. The amplitudes (assuming minimal beam absorption) are demonstrated in Figure 2. For pattern depths greater than about 50 \( \mu \)m there is significant mode mixing.

### 3 Optical Properties of Quantum Fluids

Many of the hydrodynamic properties of liquid \(^4\)He can be understood in terms of a quantum two fluid model. The normal fluid has viscous flow and carries any entropy flux through the fluid, while below \( T_{\lambda} \cong 2.17^\circ K \) there is a superfluid component that behaves like a macroscopic quantum system exhibiting persistent non-viscous flow and quantization of circulation\(^4\). Helium

\(^4\)D.R. Tilley and J. Tilley, *Superfluidity and Superconductivity*, Adam Hilger, LTD, Bristol and Boston, 2\textsuperscript{nd} edition (1986)
forms a low density liquid \( (\rho \approx 0.15 \text{g/l}) \) with the refractive density smoothly modeled using the form\(^5\)

\[
n_{He} \cong 1.000 + 0.193 \rho / \text{g \cdot cm}^{-3}.
\] (3.1)

### 3.1 Two fluid hydrodynamics

The hydrodynamic flows of this system are described by two fluid equations\(^6\), which include the mass continuity equation

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho_s \vec{v}_s + \rho_n \vec{v}_n) = 0,
\] (3.2)

the entropy flux equation

\[
\frac{\partial (\rho \sigma)}{\partial t} + \nabla \cdot (\rho \sigma \vec{v}_n) = 0,
\] (3.3)

the superfluid Euler equation

\[
\rho \left( \frac{\partial \vec{v}_s}{\partial t} + (\vec{v}_s \cdot \nabla) \vec{v}_s \right) = -\nabla P + \rho \sigma \nabla T + \frac{\rho_n}{2} \nabla (\vec{v}_n - \vec{v}_s)^2,
\] (3.4)

and the Navier-Stokes equation

\[
\rho_s \left( \frac{\partial \vec{v}_s}{\partial t} + (\vec{v}_s \cdot \nabla) \vec{v}_s \right) + \rho_n \left( \frac{\partial \vec{v}_n}{\partial t} + (\vec{v}_n \cdot \nabla) \vec{v}_n \right) = -\nabla P + \eta \nabla^2 \vec{v}_n + \frac{\eta}{3} \nabla (\nabla \cdot \vec{v}_n).
\] (3.5)
Some relevant parameters of superfluid helium will be given for future reference. The speed of (first) sound (i.e. compressional waves) is given by \( u_1^2 \equiv \left( \frac{\partial P}{\partial \rho} \right)_\sigma \), with typical values of the order \( u_1 \sim 220 - 240 \text{ m/s} \). Temperature waves in superfluids propagate with the velocity of second sound given by \( (u_2)^2 = \frac{\rho_s}{\rho_n} \frac{T^2}{c_P} \). Typical values for this speed are \( u_2 \sim 20 \text{ m/s} \). The entropy per unit mass is about \( \sigma \approx 100 \text{ m}^2/\text{s}^2 \circ K \).

3.2 Standing waves in a bulk quantum fluid

The strategy of the present approach is to utilize the coherent nature of radiation from a laser to affect the coherent behavior of a macroscopic quantum system. Figure 3 represents an arrangement of a macroscopic loop which establishes a set of discrete valued properties within the quantum fluid. The region being probed could be excited by mechanical, thermal, or optical perturbations. Generally, the superfluid component is locally accelerated from regions of high chemical potential towards regions of lower chemical potential.

Assume that the speeds \( v_s, v_n \), the density and entropy perturbations \( \delta \rho, \delta \sigma \), and the pressure and temperature gradients are all first order small variations from equilibrium values. Stationary temperature variations will be imposed on the quantum fluid in the form of the real part of

\[
T(x, t) = T_o + \frac{\delta T}{2} \left( 1 + \cos \frac{2\pi x}{W} \right) e^{-i\omega t} \quad (3.6)
\]

The perturbative forms of Equations 3.2-3.5 are then given by

\[
-\omega \delta \rho + \frac{2\pi}{W} (\rho_s v_s + \rho_n v_n) = 0, \quad (3.7)
\]

\footnote{Lyons, Donald R. “Apparatus for and methods of sensing evanescent events in a fluid field”, U.S. Patent 6,650,799 (Nov 2003) and U.S. Patent 6,915,028 (Jul 2005)}
\[-\omega (\sigma \delta \rho + \rho \sigma \delta \sigma) + \frac{2\pi}{W} (\rho \sigma v_n) = 0,\]  
(3.8)

\[-\omega \rho v_s = -\frac{2\pi}{W} \delta P + \rho \sigma \frac{2\pi}{W} \delta T + \rho_n \frac{2\pi}{W} (v_n - v_s)^2,\]  
(3.9)

\[\omega (\rho_s v_s + \rho_n v_n) = \frac{2\pi}{W} \delta P - i\eta \left( \left( \frac{2\pi}{W} \right)^2 v_n + \frac{1}{3} \omega \left( \frac{2\pi}{W} \right) \frac{\sigma \delta \rho + \rho \delta \sigma}{\rho \sigma} \right).\]  
(3.10)

The static limit \(\omega = 0\) gives the condition \(\delta P = \frac{1}{2} \rho \sigma \delta T\) known as the fountain effect. Using the thermodynamic Maxwell relation \((\frac{\partial P}{\partial \sigma})_\rho = \rho^2 \left( \frac{\partial T}{\partial \rho} \right)_\sigma = -\frac{\rho \beta \sigma}{\rho \sigma}\), the density perturbations can be expressed in terms of the speed of first sound and adiabatic thermal expansion coefficient using \(\delta \rho = \frac{\sigma \left( \frac{2\pi}{W} \delta T + \frac{\rho \beta \sigma}{\rho \sigma} \right)}{\omega^2}\). The density variations are therefore expected to satisfy

\[\delta \rho = \left( \frac{2\pi}{W} \right)^4 \frac{\sigma^2}{\omega^2} \left( \frac{\rho}{\rho_n} \right) \left( \frac{\rho}{\rho_n} \right) \frac{\delta T}{2 \left( \frac{2\pi}{W} \delta T - \omega^2 \right)},\]  
(3.11)

The scale of the variations are seen to be significant for low frequencies and a small coefficient of volumetric expansion. The isobaric expansion coefficient for superfluid helium changes significantly, as plotted in Figure 4. For temperatures

\[
\begin{align*}
Volumetric\ Expansion / \ deg\ Kelvin\ vs\ T
\end{align*}
\]

Figure 4: Coefficient of Isobaric Volumetric Expansion for Liquid Helium

\[T \sim 1.5^\circ K\] where \(\beta \sim 0.01/\circ K\), and a typical optical pattern spacing of \(W \approx 10^{-6} m\), the density perturbations for frequencies \(f = 2\pi \omega\) are of the order

\[\frac{\delta \rho}{\rho} \sim \begin{cases} 
-70 \frac{\delta T}{\pi K} \left( \frac{M_{Hz}}{f} \right)^2 & f << \frac{\pi}{W} \\
4 \times 10^5 \frac{\delta T}{\pi K} \left( \frac{M_{Hz}}{f} \right)^4 & f >> \frac{\pi}{W} 
\end{cases}\]  
(3.12)
This effect is considerably enhanced for slightly lower temperatures where the volumetric expansion coefficient becomes vanishingly small. Equation (3.1) then relates the dimensionless relative index variation to liquid helium density variations, which for low frequencies is of the order

\[ \frac{\delta n}{n} \approx 0.029 \frac{\delta \rho}{\rho}. \tag{3.13} \]

Since there is considerable variability with temperature of the normal fluid density (which decreases to zero at absolute zero) and the coefficient of volumetric expansion, one expects to be able to arrange conditions such that there are measurable effects upon the optical properties a superfluid due to an imposed time varying pattern.

Coherent light should also modify the material properties of the quantum fluid, changing its mechanical and thermal states. In particular, we expect quantized resonant responses of superfluid systems to coherent perturbative influences when configured as in Figure 3. Since as previously mentioned the speed of second sound is of order \( u_2 \sim 20 \text{ m/s} \), there would be micron scale temperature variations for frequencies of the order 200KHz, with lower frequencies requiring patterns of wider spacing. There should be resonant thermal wave effects for such patterns. Standing acoustic waves of micron scale wavelengths should immediately have measurable effects on the probe beam, assuming the wave pattern can be appropriately stabilized.

### 4 Conclusion

The two fluid model suggests that an optical interference pattern placed on a superfluid should induce local variations in the fluid’s hydrodynamic parameters. A second coherent light source can be used to probe those variations through diffractive effects. Calculations have been presented that suggest the scale of hydrodynamic variations that can be induced, and the potential measurement of those variations. Any measured diffractive effect on a probe beam due to the presence of an imposed pattern would demonstrate an optical switching of that probe beam. It is likewise suggested that coherent mechanical perturbations of appropriate scale in the superfluid should have a measurable impact on the properties of the probe beam.

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