What Is Wrong with Interpretation Q?:
A Case of Concrete Skeptic’s Alternative Interpretation of Algebra

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Abstract

Previous discussion of skepticism about meaning as it appears in Kripke’s Wittgenstein has not provided complete examples of an alternative interpretation of a language. Sakakibara recently gave an instance of a nonstandard interpretation of algebra called interpretation Q. The present paper compares interpretation Q and the standard interpretation C of algebra in three respects: complexity of meaning, engagement with reality, and convenience of use. Although both interpretations are equal in complexity and engage with reality properly, interpretation C is superior to interpretation Q because interpretation C alone allows digit-by-digit calculations, which justifies our asserting that interpretation C is the correct one. Since this way of reasoning does not make mention of the linguistic community we belong to, the present case study suggests that the link between language and community is not necessary.

1. Introduction

In Wittgenstein on Rules and Private Language, Kripke reconstructs part of Wittgenstein’s Philosophical Investigations and proposes that Wittgenstein, as he struck Kripke (henceforth KW), argues that there can be no such thing as meaning, so long as meaning is considered to be something that determines potentially infinite correct applications of a rule. KW then asserts that each application of a rule is an “unjustified leap in the dark” (Kripke 1982, p. 10).

KW leads us to imagine a situation where we had never added a number
equal to or greater than 57. Then, KW’s skeptic tell her interlocutor, “The
answer to ‘58 + 67?’ might be ‘5’ rather than ‘125,’ for the possibility that you
meant by ‘+’ the following quus function \( \oplus \) cannot be excluded, because all
existing evidence is consistent with the interpretation that ‘+’ means quus”
(Ibid., p. 7f).

\[
x \oplus y = \begin{cases} x + y & (x < 57 \text{ and } y < 57) \\ 5 & \text{(otherwise)} \end{cases}
\]

There have been extensive discussions on KW’s skepticism about meaning.
However, all those discussions are made without providing concrete examples
of alternative interpretations of the language. To be sure, KW illustrated the
quus function, yet, it is an incomplete example, which has provoked cascades of
objections.

For example, would not the sentence expressing the associative law, namely
“\( \forall x \forall y \forall z \ (x + y) + z = x + (y + z) \)” be false? KW proposes that the seeming
objection is accommodated if we reconsider the symbol “\( \forall x \)” to mean <for
every \( x \) that is less than some number \( h \)> (Ibid., p. 16f, footnote 12). He
suggests that the skeptic is able to adjust the candidate alternative
interpretations whenever an interlocutor produces new objections. However,
interpreting “\( \forall x \)” as a bounded universal, in turn, makes the following sentence
false: “\( \exists x (x \in \mathbb{N} \land \forall y (y \in \mathbb{N} \rightarrow y \leq x)) \)” where the symbol “\( \mathbb{N} \)” represents the set
of natural numbers (Sakakibara 2013, p. 279). The skeptic’s alternative
interpretations are like will o’ wisps in that they seems to exist from a distance,
but we cannot actually catch them (Tennant 1997, p. 101). There is no
guarantee that we will finally reach a stable alternative interpretation.

Recently, Sakakibara (2013) has demonstrated that an alternative
interpretation of algebra, named interpretation Q, can be derived from the
standard interpretation (hereafter interpretation C) of the language of algebra.
However, the kind of insight that we gain from concrete examples of alternative
interpretation has yet to be investigated. In this paper, it is proposed that the
comparison of interpretations C and Q reveals what is wrong with the latter.

The rest of this paper is organized as follows. In section 2, Sakakibara’s
interpretation Q is briefly reviewed. In section 3, interpretations C and Q are
compared from three viewpoints, and the superiority of interpretation C over Q
is identified. In section 4, I will review the debate surrounding the simplicity
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consideration and defend the position that the superiority of interpretation C justifies us in asserting that interpretation C is the correct one.

2. Sakakibara’s interpretation Q

Interpretation Q is an instance of the alternative interpretations of algebra, which is proven to have features needed by KW’s skeptic. In this section, extracting from Sakakibara (2013), the derivation of interpretation Q from interpretation C is shown.

First of all, the operation called *Qfication* is defined. Let q be a bijective function in which \(q(x) = x\) holds as long as x is equal to or smaller than a certain fixed number, hereafter called “the bending point” of the function q. An instance of function q is defined as follows:

\[
q(x) = \begin{cases} 
  x & (x \leq 1000) \\
  2x - 1000 & (x > 1000) 
\end{cases} 
\]  

(2.1)

The bending point of q as defined in (2.1) is 1000. Because q is bijective, the inverse function \(q^{-1}\) exists. With function q, “bent” numerical functions and predicates are defined based on a systematic procedure called Qfication. The Qfication of an arbitrary n-ary numerical function f is written as Qf and formulated as follows:\(^1\)

\[
Qf(x_1, x_2, \ldots, x_n) \overset{\text{def}}{=} q(f(q^{-1}(x_1), q^{-1}(x_2), \ldots, q^{-1}(x_n))) 
\]  

(2.2)

For example,

\[
x \ Q+ \ y = q(q^{-1}(x) + q^{-1}(y)) \\
Q\sin(x) = q(\sin(q^{-1}(x))). 
\]

The Qfication of an n-ary predicate P is written as QP, and is formulated as follows:

\[
QP(x_1, x_2, \ldots, x_n) \overset{\text{def}}{=} P(q^{-1}(x_1), q^{-1}(x_2), \ldots, q^{-1}(x_n)) 
\]  

(2.3)

For example,

\[
x \ Q+ \ y \overset{\text{def}}{=} q^{-1}(x) < q^{-1}(y) \\
x \ Q\in \ A \overset{\text{def}}{=} q^{-1}(x) \in A. 
\]

Because Qfication does not alter the meaning of logical symbols or the equals
Employing the notion of Qfication, we can define interpretation Q as interpretation in which every function and predicate symbol denotes the Qfied version of what it denoted in interpretation C, and the interpretation of the notation of numbers is identical to that in interpretation C.

For example, in interpretation Q, "∀x(x > 2 → x + x < x × x)" means ∀x(x Q> 2 → x Q+ x Q< x Q× x). In fact, there are as many variations in interpretation Q as there are in function q. It is proved that a sentence of algebra containing no notations of numbers larger than the bending point is true in interpretation Q if and only if it is true in interpretation C, but when a sentence contains such notations, this correspondence is generally not sustained. For example, if q is defined as in (2.1), then “1016 + 12 = 1040” and “300 × 5 = 2000” are true in interpretation Q.

By solving (2.2) and (2.3) for f and P, respectively, we have the following:

\[ f(x_1, x_2, \cdots, x_n) = q^{-1}(Qf(q(x_1), q(x_2), \cdots, q(x_n))), \quad (2.4) \]
\[ P(x_1, x_2, \cdots, x_n) \overset{\text{def}}{=} QP(q(x_1),q(x_2), \cdots, q(x_n)). \quad (2.5) \]

(2.4) and (2.5) tell us that original functions and predicates are definable in terms of Qfied functions and predicates in combination with function q. Since q is proved to be definable from Qfied functions and predicates, Qfied functions and predicates are symmetrically interdefinable with the original functions and predicates except that the positions of q and q^{-1} are reversed.

If q is redefined and the bending point of function q is set as the largest number ever stated, then interpretation Q has the following features:

1. It systematically alters the meaning of those algebraic symbols that have been historically employed.
2. It preserves the truth value of every sentence that has been stated thus far.
3. It assigns abnormal truth values to some as yet unstated sentences.
4. It does not change the meaning of logical symbols, including the equals sign.
5. It is interdefinable in a symmetrical way with interpretation C.

Note that while the first three features are minimal requirements for KW's
alternative interpretations, the last two are extra features found in interpretation Q. Therefore, the demonstration of interpretation Q provides new information even to those who affirm the possibility of multiple interpretations, since alternative interpretations that do not disarrange the meaning of logical symbols and which are interdefinable with the standard interpretation have hitherto not been considered.\(^3\)

Sakakibara's interpretation Q makes it possible to reformulate the first step of the skeptical argument avoiding counterexamples. Suppose we had never added a number equal to or greater than 1000 and \(q\) is defined as in (2.1). Then, KW's skeptic whispers to her interlocutor, “the answer to ‘1016 + 12’ might be ‘1040’ rather than ‘1028’ because not only interpretation C but also interpretation Q is consistent with all existing linguistic evidence.” What can we say against such a challenge?

3. The superiority of interpretation C over interpretation Q

After discussing the underdetermination of meaning by linguistic precedents, KW treated other candidates, such as our dispositions and mental pictures, and concluded that they do not fix the meaning either (Kripke p. 22f, 41f). I abstain from delving into the details here because his argument holds true no less for interpretation Q than for quus, and the introduction of interpretation Q brings us no novel insight into this line of argument. In the following, I assume that our dispositions and mental pictures are useless for determining words' meaning.

KW's skeptic would insist that interpretations C and Q have exactly the same amount of empirical support, and hence that the facts cannot justify any judgment that either one is correct. However, that consequence is contrary to our gut feeling. We think that the meaning of “+” is plus and not Qplus. In addition, we feel that we are justified in thinking so. We feel that something is wrong with interpretation Q. This feeling indicates that there is something yet to be made explicit.

Because interpretations C and Q receive equal empirical support, I suspect that our gut feeling derives from the differences in the properties of interpretations C and Q themselves. Hence, it is worth checking if, when compared with interpretation C, interpretation Q contains any defects. Among the large amount of literature concerning KW's skepticism, just a few studies
discuss the properties of alternative interpretations of target languages. The main reason for this deficit is of course that no instance of a complete alternative interpretation has been available, and so all efforts to compare the properties of alternative interpretations were limited to the discussion of such impressive yet elusive partial examples as the quus function. Now, with interpretation Q, we have a complete example we can examine. The present paper makes the most of this advantage.

3.1 Complexities are equal

Let algebra C and algebra Q be languages of algebra that have meanings as attributed by interpretation C and Q, respectively. In other words, they comprise the same set of numbers, variables, logical connectives, operators, and predicates, with the latter two groups having different meanings (e.g., “+” stands for + in algebra C, whereas it stands for Q+ in algebra Q). Contrary to the first impression, they are symmetrically complex because they are symmetrically interdefinable.

In “The New Riddle of Induction,” Goodman emphasizes that “grue” and “bleen” are symmetrically interdefinable with “green” and “blue” and defends their evenness as concepts (Goodman 1983, pp. 72–81). Similarly, interdefinability between alternative interpretations reveals the following reciprocity: whereas for those who start with algebra C, algebra Q is more complex than algebra C because the former is defined in terms of the latter; those who start with algebra Q consider algebra C to be more complex than algebra Q because additional definitions are needed to make sense of algebra C. The complexities are relative to the point of departure.

Recently, Delancey (2007) argued that the quus function is more Kolmogorov-complex than the plus function. Kolmogorov proposed that the complexity of objects can be measured by seeing the length of the shortest possible description of the string in some fixed programming language such as BASIC (Li & Vitányi 2008). We can choose any programming language as a description language as long as it is a programming language of a universal computer. From the well-known fact that any universal computer can emulate any other universal computer, it is proved that choosing a different programming language will alter the complexity only up to some additive constant (Ibid., p. 3). In BASIC, the shortest program calculating the plus function is shorter than that
calculating the Q+ function.

However, the interdefinability between algebras C and Q reveals that no asymmetry exists between the two algebras in terms of Kolmogorov complexity. Let us imagine another programming language named BASIQ. In BASIQ, all operator signs mean a Qfied version of what they mean in BASIC, for example “+” means Q+. Since algebra Q is definable from algebra C, if a machine implementing BASIC can be assembled, so can be one implementing BASIQ. In addition, because algebra C is definable from algebra Q, BASIQ can emulate BASIC. Therefore, provided that BASIC is an implementable programming language of a universal computer, so is BASIQ. In BASIQ, it is obvious that the shortest program calculating the plus function is longer than that calculating the Q+ function. It was mentioned that the Kolmogorov complexity varies only up to some additive constant even if a different programming language is chosen. “Some additive constant” is fatal here, for interdefinability assures that if there is a programming language with which + is less Kolmogorov-complex than Q+, then there is another programming language with which that order is reversed.4

3.2 Both interpretations engage with reality

When function q is defined as in (2.1), “300 × 5 = 2000” is true in interpretation Q. In contrast, if we get 5 boxes each of which contain 300 apples, we have 1500 apples. Does this not indicate that Q+ fails to engage with reality?

It might be suggested that the meaning of “engage with” can be reinterpreted. This suggestion is unacceptable, however. The predicates such as “engage with” are those of metalanguage with which we specify interpretations of an object language and formulate skeptical arguments, and are located outside the scope of skeptical doubts. Whether the calculations engage with reality or not is not the same matter as whether the sentence of the object language “the calculations engage with reality” is true or not. No matter what people might say, it is a plain fact that 1500 is not equal to 2000. Moreover, miscalculations sometimes result in a matter of life and death.

Then, will not that incoherence be avoided by reinterpreting the meaning of some words in the object language? This is exactly the case in algebra Q. The key point to understand this is that the notion of number is also Qfied according
to the procedure of Qfication (Sakakibara 2013, p. 287). Let Num(S) represent
the number of elements (in other words, the cardinality) of set S. Then, QNum
(Qfied Num) is defined as follows:

\[
Q\text{Num}(S) \equiv q(\text{Num}(S)).
\]

Note that the notion of counting is transformed into Qcounting accordingly.

Imagine that there are 5 boxes, each of which contains 300 apples. Those
who adopt algebra C will say, “There are 5 boxes, each of which contains 300
apples. 300 multiplied by 5 is 1500. Putting those apples together and counting
them, I get 1500 apples in all, as expected by the calculation result.” On the
other hand, those who adopt algebra Q will say, “There are 5 boxes, each of
which contains 300 apples. 300 multiplied (meaning Qmultiplied) by 5 is 2000.
Putting those apples together and counting (meaning Qcounting) them, I get
2000 apples in all, as expected by the calculation result.” The trick at work here
is a formalized version of what KW suggested when he reinterpreted “counting”
as quouting (Kripke, p. 16). In conclusion, interpretation Q never departs from
reality because the Qfication of functions and predicates perfectly corresponds
with that of the notions of number and counting.

3.3. Digit-by-digit calculation is disallowed in algebra Q

When we are unable to keep the entire notation of large numbers in mind, we
are able to calculate digit by digit, checking one or two digits that constitute a
subpart of the notation of the number. Algebra C allows various types of digit-
by-digit calculations. For example, when two natural numbers are “…6” and
“…2,” (where “…” indicates some digit or digits occupying a higher notational
order, for instance if the numbers just given were in fact 26 and 12) then their
sum is “…8” irrespective of the upper digits.

In contrast, digit-by-digit calculations are disallowed in algebra Q: If q is
defined as in (2.1), then “…6 + …2 = …8” does not generally hold because
while the result of “16 + 12” is “28,” that of “1016 + 12” is “1040.” In algebra
Q, we must know what digits occupy higher notational places before we can
complete the calculation of the digit occupying a given notational place. An
omniscient god would see such a difference as valueless. Digit-by-digit
calculability, however, is an epistemic merit for all beings of limited cognitive
capacity, including human beings.

This property is equivalent to what Tennant calls factorizability (Tennant
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1997, p. 154). He conceived of this notion as a way to capture what it means for us to have a grasp the meaning of a sentence that are too long to be surveyable. The crucial point is that one need not be able to check the entire sentence. All that is required for one to be regarded as grasping the meaning of a sentence is that he or she is able to check whichever appropriate small part of the sentence. Factorizability is essential to the explanation of how those with finite cognitive capacity are considered to understand potentially infinite applications of a rule.

Although Tennant’s example described the grasping of the meaning of a sentence, the same point is true for the process of addition. The process of adding two numbers is comprised of the iteration of small steps that can be performed with reference only to a small amount of information. For this characteristic, we can be said to know how to add two numbers, even if they are insurveylably large. The trouble with Q is precisely that it is not factorizable. The lack of factorizability not only makes Q less easy to perform than normal addition, but also threatens the sense with which finite beings grasp the meaning of “+.”

In the following, I will address some potential objections regarding digit-by-digit calculation and factorizability. First, it might be claimed that Qfied functions also permit digit-by-digit calculation within certain limits, such as “less than a trillion places.” If the bending point is so large that we can safely say that we will not reach the bending point in the future, is not such difference negligible? However, the existence of any bending point destroys digit-by-digit calculability. The existence of any bending point makes the ability to perform digit-by-digit calculation logically dependent on the ability to recognize which notational place one is currently handling, and whether one’s current place is smaller than that of the bending point or not. Counting very large numbers and comparing them is not a feasible task for those possessing finite cognitive capacity: We are thought to have the capacity to count and compare very large numbers only because those tasks themselves are factorizable in Tennant’s sense.

Second, the merit of digit-by-digit calculability is not limited to a positional notation system. Roman numerals, which represent an additive notation system, represent numbers by a series of numerical signs that when added together equal the represented number. When we add two numbers in Roman numerals, “...C... + ...C... = ...CC... (or ...CCC..., where carrying exists)” holds true
irrespective of other coexisting numerical signs.\(^6\)

Third, the lack of digit-by-digit calculability is not a problem that is resolved with the cessation of the performance of hand calculation and the use of, for instance, computers or calculators. Just the opposite is true: computers are more dependent on factorizability in the performance of calculations than human beings. Central processing units of computers repeat simple operations at a very high frequency, in which outputs are determined from a small amount of input. The swift performance of computer is dependent on the fact that each operations are carried out without referring to the information other than that limited input.

### 3.4. The significance of the comparison

The actual comparison between interpretations C and Q pinned down what seems to be the strongest of the objections to KW’s skeptical argument. The adoption of interpretation Q rather than the quus function immunizes KW’s skeptical argument against some frequently raised objections, such as that quus is far more complex than plus or that quus does not engage with reality. Although the impossibility of digit-by-digit calculation may also be among those objections, that point turned out to be exceptional because it alone remains effective against the skeptical argument employing interpretation Q as a competitor for the standard interpretation. To put it another way, \(\text{algebra C has the epistemic advantage of easier access to the algebraic truth.}\) The difference in the possibility of digit-by-digit calculability derives from the fact that the complete set of true sentences in algebra C differs from that in algebra Q, which is one of the minimal requirements of the skeptic’s alternative interpretation. Although this is a case study based on a single example, the point extracted here is a promising candidate of a general feature common to all alternative interpretations of algebra.

### 4. Simplicity consideration revisited

Based on the arguments in the previous sections, I propose that algebra C is more advantageous than algebra Q. I assert that the interlocutor can rebut the skeptic by saying, “You can tell I meant plus and not Qplus by ‘+,’ because if I had meant Qplus by ‘+,’ digit-by-digit calculations would not be allowed.”\(^7\) In other words, adopting interpretation C and answering “1028” to “1012 + 16 = ?”
is justified on the ground that interpretation Q is problematic and algebra C is better than algebra Q.

However, the idea that the adoption of a given interpretation can be justified based on the interpretation’s relative merit has met objections from several authors. Since those objections are recapitulations of KW’s treatment of simplicity considerations, I will address this issue by revisiting KW’s disapproval of simplicity considerations in the following.

KW discourages opponents from exploiting the difference in the simplicity of alternative interpretations for overcoming underdetermination. Although it has already been shown in section 3 that algebras C and Q are symmetrical about simplicity, KW’s position deserves special attention because it applies to any attempt to overcome underdetermination by making reference to the properties of alternative interpretations. KW writes:

Now Wittgenstein’s skeptic argues that he knows of no fact about an individual that could constitute his state of meaning plus rather than quus. Against this claim simplicity considerations are irrelevant. Simplicity considerations would have been relevant against a skeptic who argued that the indirectness of our access to the facts of meaning and intention prevents us ever from knowing whether we mean plus or quus. But such merely epistemological skepticism is not in question (Kripke, p. 39).

KW draws a clear distinction between the ontology and the epistemology of meaning, and stresses that the simplicity consideration is irrelevant for the ontology of meaning. However, this point merits further explanation.

After Quine’s epoch-making book Word and Object, interpretation (or translation) theories of language have often been contrasted with scientific theories (Quine 1960). Simplicity considerations are well understood in the philosophy of science. For example, van Fraassen argues that consistency with observable data does not uniquely determine the values of unobservable theoretical entities (van Fraassen 1980, pp. 59–69). KW insists that more than one interpretation of a given word can be consistent with all physical and mental facts; van Fraassen similarly insists that more than one model of a theory that describes unobservable objects can be consistent with all observable facts. If we were to take the relative simplicity of theories into consideration,
we might single out one among its equally empirically adequate rivals, but van Fraassen refuses to do so, writing that it is “surely absurd to think that the world is more likely to be simple than complicated” (Ibid., p. 167). Similar treatment of the simplicity consideration is also found in Quine (1986, p. 155). We prefer simpler hypotheses because they are more convenient, not because they are more probable. The simplicity consideration should therefore be disregarded inasmuch as the target theories are, or at least might be, about the realities of the world.

Nevertheless, the dismissal of simplicity considerations applies only to a certain kind of realism, where any mention of values is renounced as subjective projection. Natural science is one of the fields where such realistic stance might be defended. Languages, however, depend inherently on our practice of them, and languages are, in a sense, the tools of thought and communication. This provides us with a practical reason to adopt better interpretations and attribute better meaning. Though it may seem paradoxical, simplicity and practicality, participate in the determination of meaning because the statements concerning the meanings of words are not about the value-unencumbered reality of the world.

What KW actually questions is whether there are objective facts that justify determined linguistic behaviors. If no such adequate justification exists, judging certain usages of words as correct is arbitrary. However, there is no reason to confine the grounds for justification to objective facts. Since the judgments about the correctness of instances of word usage are made at least partly on practical grounds, values, if they are relative to our point of view as human beings, make up part of the judgment that an instance of usage is correct. The dichotomy between objectivity and arbitrariness has a gap; there is the third option that statements about linguistic meaning may be justified by values defined in terms of the nature of human beings.

Allow me to restate the difference between my position and the position supported by KW. After the presentation of the skeptical argument, KW arrives at what he calls “the skeptical solution” of the paradox of rule-following, which contends that language does not exist without communal linguistic practice. He makes much of community because, for him, indicative utterances that appear to state facts about meaning should, in reality, be understood as speech acts approving or disapproving the designated person as a reliable member of the
linguistic community (p. 91f). It is therefore apparent that understanding such utterances about meaning presupposes the existence of a linguistic community, the membership of which is relevant to the function of those utterances.

In contrast, although I have conceded that assertions about meaning are not justified without reference to the values of our linguistic practice, I have not conceded that linguistic practice is necessarily *communal*, for the distinction between the correct and incorrect usage of words can be made on the grounds of considerations about the goodness of rival interpretations, which every member of humanity has to take account of. It is generally understood that utterances such as “He means plus by ‘+’” have the conversational implication of approving the referred-to person as a reliable member of the linguistic community. The point here is that the meaning of such utterance is not exhausted by such implication.

The idea developed above is similar to Humphrey’s position (Humphrey 1999). He grants that simplicity considerations are not relevant to arguments for the straight solution of meaning skepticism; he argues, instead, that they can be incorporated into the skeptical solution of meaning skepticism (Ibid., p. 50). Whereas simplicity of interpretation does not comprise the *truth condition* of statements about meaning, it does comprise the *assertability condition* of such statements. In other words, if one of the hypotheses is simpler than the other, we can be justified in *asserting* that one means the former by that word. This is a kind of skeptical solution to KW’s skepticism about meaning, for it denies that utterances about meaning state matters of fact. Yet, this is not the same as KW’s skeptical solution in that the assertability condition is “obviously unaffected by the number of people whose talk or rule following is being assessed” (Humphrey 1999, p. 45). Humphrey’s skeptical solution does not implicate a community view of language. Though algebras C and Q do not differ in their simplicity, the fact that algebra C allows digit-by-digit calculation in a way that algebra Q does not undertakes a similar role to that of simplicity.

5. Conclusion

In this paper, the standard interpretation C of the language of algebra was contrasted with interpretation Q, a nonstandard interpretation of the language of algebra recently derived by Sakakibara. Instances of previous word usage are insufficient to single out the correct interpretation, for both C and Q tally
equally with the precedents.

At this point, we focused on the properties of the interpretations themselves. We found that algebra C and algebra Q are equally complex and both engage with reality properly, but algebra C is more convenient, since it allows digit-by-digit calculation while algebra Q does not. This is an epistemic merit for all beings of limited cognitive capacity, including human beings.

KW himself argues against trying to use the properties of different interpretations, such as simplicity, as a way for determining which is correct. This would be reasonable if the meanings of words belonged to the reality of the world that excludes any kind of value. However, language is practical, and we are justified in asserting some interpretation is correct based on the values we determine to be good relative to the function language serves. Since the distinction between the correct and incorrect usage of words is made on the grounds of the relative merits of rival interpretations, linguistic practice is not necessarily dependent on the existence of a linguistic community.

Because the present argument is derived solely from the consideration of concrete examples, it should be acknowledged that it does not provide a conclusive demonstration, and we should be cautious about the generalizability of its conclusions. Nevertheless, I believe that the present result, in one way, makes a case against Krepke’s reconstruction of Wittgenstein. We often discover the reason that we should follow one kind of rule rather than another, as far as the alternatives are explicitly defined, but this in no way means that the problem of rule following tackled by Wittgenstein is misplaced. I suppose that the deeper insight into rule following is related to the fact that one cannot make someone a rule follower just by rational persuasion alone. Disentangling the interrelation between rationality and becoming a rule follower requires a whole set of separate discussions.

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Notes

1. Q cannot be equal to the quus function ⊕, however q is defined. This is proven
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by the following *reductio ad absurdum*:

Let us assume that $Q^+$ is equal to $\oplus$ if $q$ is defined appropriately,

$$x \oplus y = q(q^{-1}(x) + q^{-1}(y)). \quad (2.6)$$

Taking the both sides of (2.6) as the argument of $q^{-1}$,

$$q^{-1}(x \oplus y) = q^{-1}(x) + q^{-1}(y). \quad (2.7)$$

Substituting $x = 57, y = 68$ in (2.7),

$$q^{-1}(57) + q^{-1}(68). \quad (2.8)$$

Substituting $x = 57, y = 69$ in (2.7),

$$q^{-1}(57) + q^{-1}(69). \quad (2.9)$$

From (2.8) and (2.9),

$$q^{-1}(68) = q^{-1}(69). \quad (2.10)$$

Taking the both sides of (2.10) as the argument of $q$,

$$68 = 69.$$

This leads to a contradiction. Therefore, it remains unknown whether a consistent alternative interpretation in which “$\oplus$” is reinterpreted as a quus function can be constructed.

2. While $q$ must be bijective, it need not be monotonous. Qification does not change the meaning of the smaller-than sign as far as $q$ is monotonous. However, the following is also a valid definition of $q$.

$$q(x) = \begin{cases} x & (x \leq 1000) \\ 2500 - x & (1000 < x < 1500) \\ x & (x \geq 1500) \end{cases}$$

With this definition of $q$, “$1300 < 1200$” is true in interpretation Q.

3. Although KW’s skeptical argument about meaning and Quine’s argument for the indeterminacy of translation have notable similarities, Quine but not KW thought that the structure of first order logic is a necessary component of any language (Quine 1960). Therefore, supporters of Quine would not allow KW’s maneuvers on the ground that they distort the meaning of logical symbols. Interpretation Q is advantageous in this regard as an example, since it does not manipulate the meaning of logical symbols and hence may be allowed as valid from a Quinean point of view.

4. Delancey also realizes that Kolmogorov complexity is reversible relative to the programming language chosen as the reference and writes, “[b]ut implementations are physical facts, and so we can in principle discover the concrete Kolmogorov complexity of some implementation” (Delancey 2007, p. 239). However, BASIC and BASIQ are both implementable. In addition, if there are physical facts that determine the programming language which is actually implemented in one’s brain, they are also the facts that determine whether “$+$” means $+$ or $Q^+$. Such facts are just what we have been seeking in the first place and are precisely those whose existence we cast doubt on. I agree with Delancey that the quus function
and BASIQ is queer compared with the plus function and BASIC. However, the notion of Kolmogorov complexity fails to deepen our insight into the queerness of quus function and BASIQ.

5. The sentences of algebra include some that express the legitimacy of digit-by-digit calculation in interpretation C, such as “for every natural number x, y, and n, if the n-th digit of x is 6 and the n-th digit of y is 2, then the n-th digit of x + y is 8 or 9.” Such sentences are also true in interpretation Q, for they do not contain notations of numbers larger than the bending point (e.g., 1000). This may seem to contradict the claim that digit-by-digit calculations are illegitimate in algebra Q. The apparent contradiction vanishes when we notice that “n-th digit of x” is a kind of functional expression and should be Qfied in interpretation Q. Let Ndigit(x, n) be the function that returns the n-th digit of x. From (2.2),

$$QNdigit(x, n) \equiv q(Ndigit(q^{-1}(x), q^{-1}(n)))$$

Since “n-th digit of x” has different meanings in algebras C and Q, the above sentence does not express the legitimacy of digit-by-digit calculations in interpretation Q. Whether or not digit-by-digit calculations are legitimate in algebra Q is one thing, and whether the sentence expressing the legitimacy of digit-by-digit calculation in interpretation C is also true in interpretation Q is another. This is because human beings are unable to recognize the n-th Qdigit of x just by seeing the n-th symbol from the right in the notation of number x. For instance, the first Qdigit of 504 is 4, whereas the first Qdigit of 1004 is 2. If you see the notation of a number that reads “…4,” it is uncertain whether the first Qdigit of that number is 4 until you finish checking the upper digits of that number.

6. Here, “XC,” “CD,” “CM,” “CCC,” etc. are treated as chunks. Therefore, in roman numerals, one must check a few consecutive numerical signs to see how those signs divide into chunks.

7. It is a fact that we have performed digit-by-digit calculations. If interpretation Q is correct, we have been doing so unrightfully, since although digit-by-digit calculations are generally disallowed, we have been fortunately avoiding inconsistency because the numbers we have calculated happen to be less than the bending point. Interpretation Q makes our success in daily calculation unnecessarily vulnerable to epistemic luck. The queerness of this explanation seems to provide another reason for rejecting interpretation Q.

8. For example, see Kusch (2006 p. 128f) and Miller (2002 p. 9f).

9. Steven Davies points out the contrast between “being a member of some social community” and “being a member of human beings” (Davies 1988, p. 62). He also writes: “It would be inhuman (and not merely anti-social) to establish and follow rules of quaddition rather than some version of the rule of addition” (Ibid, p. 64).
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