An Advanced Fourier Descriptor Based on Centroid Contour Distances

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Abstract. In order to distinguish two shapes with similar centroid contour distances. This paper proposes a novel descriptor-AFD-CCD (advanced Fourier descriptor based on centroid contour distances), which is based on FD-CCD (Fourier descriptor based on centroid contour distance) descriptor. FD-CCD only provides the distance information from the centroid point to the contour point, and the lack of direction information affects the accuracy of matching. The algorithm uses circularity and rectangularity to supplement discriminability, with low computational complexity. Tests were performed on three datasets. The experimental results show that AFD-CCD has further improved the matching accuracy while ensuring that the matching speed does not decrease.

1. Introduction

Shape descriptor is an important research direction of object recognition, which is widely used in image analysis, machine vision and target recognition applications. While the target occupies too little of the whole image or the texture feature of the target is blurred, the shape descriptor can successfully identify the target through the shape features. It is worth mentioning that deep learning can also detect and identify targets perfectly before a large number of training datasets, but some image datasets are difficult to obtain. In this case, shape descriptor is a meaningful tool.

A number of shape descriptors have been proposed[1-5]. It can be roughly divided into four categories: global descriptors, local descriptors, multi-scale descriptors and multi-faceted descriptors. Shape descriptor is used to extract the shape feature of the object. FD (Fourier descriptor)[6], CCD (centroid contour distance), FD-CCD (FD based on CCD)[6], FPD (farthest point distance)[7], WD (wavelet descriptor)[8] are some classical descriptors. There are some improved descriptions FMSCCD (Fourier descriptor based on multiscale centroid distance)[9], and ASD&CCD (Angle scale descriptor & centroid distance)[10].

FD is a widely used fast descriptor. The coordinate of the moving point p on the contour is a function whose period is the perimeter of the shape boundary. This period function can be expressed by the Fourier series expansion. When describing the contour, Fourier descriptors are not sensitive to rotation, translation, and scale changes. CCD describes the contour feature of the object by using the vector composed of the distance from the shape centroid to the contour point. The principle of CCD method is simple and easy to understand. It is cited and improved by many researchers. FD-CCD combines the above. In El-ghazal’s paper, the FD-CCD performed best in the MPEG-7 CE1 Part B database.

ASD is an angle scale descriptor, which can calculate the feature vector of the angle sequence on different scales in multiple scales. Multi-scale can include local and global information. ASD&CCD adds angle information on the basis of CCD. Compared with CCD and FPD, ASD&CCD are more
accurate, but it runs slowly. FMSCCD also adds direction information on the basis of CCD. FMSCCD is a kind of multi-scale descriptor, which can solve the problem of noise sensitivity well, but the large amount of calculation is unavoidable. In [9], FMSCCD and FASD (Fourier angle scale descriptor) have the highest bulls-eye-test scores but the matching time on MPEG-7 CE1 Part B is longer than FD-CCD.

To sum up, descriptors with easy-to-understand principles and simple calculations can only represent the general characteristics of the shape but lacks detailed descriptions such as direction feature. There may be a problem of distinguishing two shapes with similar centroid contour distances, as Figure 1 shows. The AFDCCD proposed in this paper uses two simple features—circularity and rectangularity. This descriptor supplement discriminability without increasing the amount of calculation.

![Figure 1](image-url)

**Figure 1.** Two pairs of shapes, each pair of which shows two shapes with similar centroid contour distances.

### 2. Preliminary

CCD describes the distance from the centroid of the shape to the contour point. Combine these distances into a sequence. This sequence can concisely describe the shape feature of the target. Assume that the number of sampling points on the contour is \(N\). Equation (1) shows the coordinates of these sampling points. Equation (1) shows the coordinates of these sampling points.

\[
p_{\text{sam}} = \frac{1}{N} \sum_{i=1}^{N} p_{i} = \left(\frac{1}{N} \sum_{i=1}^{N} x_{i}, \frac{1}{N} \sum_{i=1}^{N} y_{i}\right),
\]

where \(p_{\text{sam}}\) is the centroid of the sampling point. \(p_{i} = (x_{i}, y_{i})\) is the \(i\)th contour point of a shape. Use Euclidean distance to describe the distance from the centroid point to the contour point. Euclidean distance is the true distance between two points in \(n\)-dimensional space. As shown in Equation (2).

\[
d_{\text{sam}}^{i} = \left((x_{i} - x_{k})^{2} + (y_{i} - y_{k})^{2}\right)^{1/2}, k = 1, 2, ..., N
\]

Where \(d_{\text{sam}}^{i}\) is the unnormalized Euclidean distance between the \(i\)th contour point and the centroid contour point \(p_{\text{sam}}\). Each shape and contour are not the same. The number of sampling points is also different. Normalization of Euclidean distance can reduce the interference caused by different number of sampling points. The normalization is shown in Equation (3).

\[
d_{\text{ccd}}^{i} = N^{1/2} d_{\text{sam}}^{i} \left(\sum_{j=1}^{N} d_{\text{sam}}^{j}\right)^{-1}, i = 1, 2, ..., N,
\]
The sequence \( D_{\text{ccd}} \{d_{\text{ccd}}^1, d_{\text{ccd}}^2, \ldots, d_{\text{ccd}}^N \} \) obtained from the above equation describes the shape feature. But when the starting position of the contour sampling point changes, the distance sequence will shift. Therefore, Equation (4) calculates all the centroid contour distances of the two shapes.

\[
\text{dis}_{\text{two-ccd}}(s_1, s_2) = \min_{0 \leq n \leq N} \left( \sum_{i=1}^{N} (d_{\text{ccd},s_i}^n - d_{\text{ccd},s_j}^n)^{2} \right)^{1/2}, \quad n \in \mathbb{Z}
\]  

(4)

When the shape contour is a closed curve, \( d_{\text{ccd}}^n = d_{\text{ccd}}^{n+1-N} \).

In some applications, a descriptor with faster speed and stronger anti-interference ability is required. Some scholars proposed FD-CCD. The FD-CCD has done a Fourier transform on the centroid contour distances and converted it to the frequency domain. Solve general convex optimization solution more efficiently. As shown in Equation (5).

\[
F_{\text{ccd}}(k) = \frac{1}{N} \sum_{i=0}^{N-1} d_{\text{ccd}}^{i} e^{-j2\pi k i/N}, \quad k = 0, 1, \ldots, N - 1
\]  

(5)

The distance between the two shapes is also converted to the frequency domain. As shown in Equation (6).

\[
\text{dis}_{\text{two-fc}}(s_1, s_2) = \sum_{k=0}^{K} |F_{\text{ccd}}^{s_1}(k) - F_{\text{ccd}}^{s_2}(k)|, \quad 0 \leq K \leq N
\]  

(6)

where \( K \) is the number of characteristic coefficients in the frequency domain. In this paper, \( K = 60 \) exists.

3. Advanced FD-CCD

In-depth study of CCD and FD-CCD will find that these two algorithms have the same weaknesses. FD-CCD is only slightly better in terms of anti-interference. But in the two pairs of images shown in Figure 1, neither of these two algorithms can distinguish well. Figure 2 and Figure 3 shows CCD feature vector curves of the two pair of shapes.

**Figure 2.** The solid line represents the CCD feature of shape (a), and the dashed line represents the CCD feature of shape (b).

**Figure 3.** The solid line represents the CCD feature of shape (c), and the dashed line represents the CCD feature of shape (d).

It can be seen intuitively from Figure 2 and Figure 3 that the feature vectors of these two types of shapes are very similar.

Figure 4 and Figure 5 shows FD-CCD feature vector curves of the two pair of shapes.
Although the FD-CCD performs Fourier transform on the distance, when the distance is normalized, the direction information of the contour point relative to the centroid point will be lost. This leads to the similarity of feature vectors.

The AFDCCD (advanced Fourier descriptor based on centroid contour distances) method is proposed on the basis of FD-CCD (Fourier descriptor based on CDD). AFDCC adds circularity and rectangularity to increase local differences information. The calculation method of circularity and rectangularity is shown in the following equation:

$$c = \frac{S_{oc}}{S_{nc}} = \frac{N_{elp}}{N_{mep}}, r = \frac{S_{oc}}{S_{nc}} = \frac{N_{elp}}{N_{mep}}$$

(7)

Finding the smallest circumcircle and the smallest circumscribed rectangle is the most critical step in calculating circularity and rectangularity. The method of finding the smallest circumcircle and smallest bounding rectangle will be introduced.

3.1 Minimum circumscribed circle
In 2000, Wang Wei et al. proposed an accurate and fast least circumscribed circle algorithm (DFAA). The main steps of the algorithm are:

- Step1: Choose 3 points: A, B, C, in the contour point set P.
- Step2: Construct the smallest enclosing circle D with these 3 points.
- Step3: Query the point V which is the farthest point from the center of D in the point set P. If V is within D, the algorithm terminates. Otherwise, continue to the next step.
- Step4: Select 3 points in \{A, B, C, V\}, construct the smallest enclosing circle \(D'\), which contain 4 points. Then turn to the step2. It is worth mentioning that this 3 points are selected from the boundary points as much as possible.

When randomly selecting points \(A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)\), you may encounter the phenomenon of three points being collinear, two points overlapping, and three points overlapping. The calculation method of the circumscribed circle is as follows:

$$r = \frac{AB}{2}, a = \frac{x_1 + x_2}{2}, b = \frac{y_1 + y_2}{2}$$

(8)

where \(r\) is the radius of the circumscribed circle. \(a\) is the abscissa of the center of the circumscribed circle. \(b\) is the ordinate of the center of the circumscribed circle.

When the triangle composed of three points \(A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)\) is a right triangle, the calculation method of the circumscribed circle is as follows:
when $r$ is the radius of the circumcircle. $a$ is the abscissa of the center of the circumscribed circle. $b$ is the ordinate of the center of the circumscribed circle.

When the triangle composed of three points $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ is an obtuse triangle or an acute triangle. This case is the most general. The center of the circumscribed circle is the intersection of the perpendicular bisector of the triangle. The calculation method is as follows:

The calculation method of the circumscribed circle is as follows:

$$r = \frac{BC}{2}, a = \frac{x_2 + x_3}{2}, b = \frac{y_2 + y_3}{2}$$

(9)

Figure 6 is a schematic diagram of the minimum circumscribed circle.

![Figure 6](image)

**Figure 6.** Schematic diagram of a group of minimum circumscribed circles with similar centroid contour distances.

3.2. Minimum bounding rectangle

There are two types of circumscribed rectangles for general graphics, the smallest area circumscribed rectangle and the smallest perimeter circumscribed rectangle. In this article, we use the smallest area bounding rectangle. The following steps are the key to calculating the bounding rectangle.

- **Step1:** Use function to find the four points $x_{\text{min}}, x_{\text{max}}, y_{\text{min}}, y_{\text{max}}$ with the largest and smallest horizontal and vertical coordinates.
- **Step2:** Use these four points to construct the four tangents of the shape.
- **Step3:** If one (or two) lines coincide with one side, the area of the rectangle determined by the four lines is calculated and saved as the current minimum value. Otherwise, the current minimum value is defined as infinity.
- **Step4:** Rotate the lines clockwise until one of them coincides with an edge of the polygon.
- **Step5:** Calculate the area of the new rectangle and compare it with the current minimum. If it is less than the current minimum value, update and save the rectangle information that determines the minimum value.
- **Step6:** Repeat steps 4 and 5 until the angle of the line is rotated more than $90^\circ$.
- **Step7:** Output the minimum area of the bounding rectangle.

Figure 7 is a schematic diagram of the minimum bounding rectangle.
4. Experiment and Analysis
This article will be tested on the MPEG-7 CE1 Part B dataset. MPEG-7 CE1 Part B is an image template library used for image shape templates. The library contains template images of 1400 shapes (70 classes, each class containing 20 shapes), which can be used for the study of object shape features. Many papers[12-14] use this database for experiments. A part of the database is shown in Figure 8. Figure 9 and Figure 10 shows AFD-CCD feature vector curves of the two pair of shapes. As can be seen from the figure, after adding circularity and rectangularity, the feature vectors of the two groups of shapes are significantly different. The circularity of (a) and (b) in Figure 1 are 0.4292 and 0.2203 respectively. The rectangularity of (c) and (d) in Figure 1 are 0.4654 and 0.3948.

In order to make the effect of the algorithm more intuitive, four algorithms of CCD, FD-CCD, FPD, and AFD-CCD were tested in MATLAB, on a PC with i7-7700 CPU, 16GB RAM under Windows 7 system.

The bulls-eye-test scores and matching time of some state-of-the-art descriptors and AFD-CCD are shown in Table 1. From the bulls-eye-test score point of view, AFD-CCD is the highest (68.78%). The remaining algorithm’s bulls-eye-test scores are 67.63%, 68.32%, 64.45%. From the matching time point of view, FPD is fastest (3.1 ms). The matching time of our proposed algorithm is 3.8 ms and the remaining algorithm scores are 3.5 ms and 112.8 ms respectively. When the time is slow by 0.4, the bulls-eye-test score is increased by 4.32%. This is acceptable.
Figure 9. The solid line represents the AFD-CCD feature of shape (a), and the dashed line represents the AFD-CCD feature of shape (b).

Figure 10. The solid line represents the AFD-CCD feature of shape (c), and the dashed line represents the AFD-CCD feature of shape (d).

5. Conclusion
AFD-CCD is a descriptor with simple principle and strong robustness. It adds rectangularity and circularity to the feature vector of FD-CCD. As mentioned above, FD-CCD and CCD cannot distinguish the two groups of shapes as shown in Figure 1, but the AFD-CCD descriptor we proposed can be well distinguished in a matching time of 3.3ms. It is also because our method can distinguish the shapes similar to those in Figure 1, which makes the bull-eye-test score of MPEG-7 CE1 Part B dataset higher and improves the accuracy of target recognition.

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