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Compression Behavior of Sheets Metals of Pure Titanium 2 and Ti6Al4V Alloy under High Temperature: Evaluation of the Tension–Compression Asymmetry

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Abstract: Determining the intrinsic indices of sheet metals under compression states at high temperatures is vital to accurately predict the behavior of the material in warm/hot forming processes. Nevertheless, the literature contains little previous experimental data in this regard due to the difficulty of carrying out specific test methodologies in sheet metals. The authors of the present manuscript previously developed an approach to evaluate the in-plane compression behavior under a wide range of test conditions, which was applied here to characterize pure titanium and Ti6Al4V alloy until 750 °C. This procedure allowed us to quantify the asymmetric and anisotropic tension–compression (T-C) response of the materials involved and their evolution with temperature and strain rate. The asymmetry detected at room temperature showed a higher compression response in all cases, mostly reaching differences of around 10%. For the lowest strain rate studied, the typical assumed symmetric T-C behavior was observed from 300 and 450 °C onwards, for the rolling and transverse direction, respectively. In addition, stepped compression tests led us to deduce the anisotropy indices, which were different from those found under tension, in contrast to the r-values applied by most authors. Using the experimental results, a factor related to the asymmetry found was proposed to formulate an extended constitutive model. The asymmetry and anisotropy data supplied for compression under warm/hot conditions are the main novelty of this research.

Keywords: sheet metal compression; tension-compression asymmetry; high temperature; compression anisotropy; titanium alloys

1. Introduction

Titanium alloy sheets are used in a large number of applications with high technological value within the aeronautical, biomedical, and energy sectors. This is due to their good resistance–density ratio, corrosion resistance, or biocompatibility [1–3]. However, due to their low Young’s modulus and high yield stress, forming these materials presents a high springback, which makes it difficult to obtain products with good dimensional accuracy. Moreover, most titanium alloys exhibit limited conformability at room temperature [4].

The deformation mechanisms of titanium alloys at room temperature and in general, of materials based on hcp-structures have been widely reported in the literature. This crystallographic structure is characterized by a highly anisotropic behavior due to its limited ability to accommodate deformations by sliding mechanisms. In this way, twinning takes place as an additional mechanism to achieve free deformation, with the main twinning systems being different for tensile and compression states [5]. The twinning polarity together with a strong texture, which is consequence of prior thermo-mechanical
processes, significantly influence the strain hardening due to crystallographic reorienta-
tion [6], the Hall–Petch effect, and the Basinski hardening mechanism [7]. Some authors
suggest that this phenomenon is one of the reasons for the tension–compression asymmetry
behavior observed [6,8]. However, this fact is controversial due to the contribution that
directional sliding mechanisms, and their interactions with the twinning, have on the
deformation [9]. For Ti6Al4V, the addition of Al causes the twinning not to be a major
deformation mechanism, so it is difficult to fully attribute to it the asymmetry phenomenon
of this material [10–12].

Twinning affects the deformation of titanium alloys, presenting several strain stages
according to the dominant deformation mechanisms. Nemat-Nasser et al. [13] found three
hardening stages for extruded pure titanium in compression tests. They correlated a high
temperature and/or a low strain rate with the non-existence of twinning mechanism in the
strain. The first stage is dominated by a prismatic and basal slip and is defined at the earlier
strain; the second stage responds to a significant twinning phenomenon and, finally when
this effect stabilizes, the third stage begins [14]. Therefore, a minimum quantity of previous
deformation is necessary for twinning nucleation [15,16]. At temperatures above 500 °C,
Zeng et al. [17] observed that the three-stage hardening of pure titanium disappears, and
the pyramidal slip <a + c> replaces twinning as the main compression deformation mecha-
nism. This is consistent with the importance of <a + c> slip in the titanium polycrystals
deforation at high temperature detected by Paton and Backofen [16]. However, under
tensile conditions, Roth et al. [9] described that the three-stage strain hardening of titanium
may be due to anisotropic nature of dislocation glide and not to twinning. Furthermore,
Nixon et al. [18] observed that this asymmetry is highly dependent on the sheet metal direc-
tion. The difference in the predominant twinning modes, {1012} for rolling direction (RD)
and {1122} for transverse direction (TD), and a greater concentration of poorly oriented
grains in one direction, which implies a greater hardening by reorientation, are the causes
of this variation [19]. In addition, twinning is much more significant for the transverse
direction, which causes the values of tension and compression obtained to be greater than
in the case of rolling direction [5,9,18]. No previous works have focused on evaluating the
asymmetry phenomenon at temperatures above room temperature, especially for titanium
sheet metal. Only for magnesium alloys a reference has been found [20], and even in this
study, the maximum temperature considered was 300 °C, which is technically affordable
without great complexity.

In addition to the asymmetric tension–compression (T-C) character of the titanium
alloys, it is important to identify the anisotropy (r), especially for sheet metals whose indices
are strongly determined by the previous rolling procedure. This anisotropic behavior is due
to the limited capacity for accommodating the deformations and because, as mentioned,
the deformation mechanisms are different in tension and compression. Thus, r-values
in tension and compression need not necessarily be equal [21], as some authors assume
in their research [22]. Despite the expected importance of this aspect, no models have
been developed that include specific r-values for compression stresses. Moreover, very
few studies have obtained this parameter. For example, Zhou et al. [23] determined the
compression r-values for AZ31B magnesium sheet alloy and found that the values were
very different than the tensile ones, with these being higher.

The anisotropy and asymmetry existing in titanium alloys demand complex behavior
models to obtain accurate results in the forming analysis [24,25]. The definition of these
models would require experimental data correctly obtained for the different stress states.
Sheet metal compression is probably one of the least studied stress states. One of the reasons
for the lack of compressive experiments with sheet metals could be the complexity of the
methodology to be used, especially if temperature is involved. Two main methodological
lines can be distinguished. The first one was suggested by Boger et al. [26], who designed a
test system based on two lateral anti-buckling plates using a specific shape for the sample.
The second line was originally proposed by Kuwabara et al. [27], which used comb-shaped
clamps for achieving higher strains. Both guidelines have been followed by many authors for compression tests at room temperature.

Nevertheless, research on sheet metal compression at warm/high temperatures is insufficient. Piao et al. [28] modified Boger’s design, adding a heating system inside the lateral anti-buckling plates. They were able to evaluate the T-C behavior of AZ31B magnesium alloy, TWIP (Twinning-Induced Plasticity), and DP (Dual-Phase)-steels up to 250 °C, determining the Bauschinger effect on these materials. Lee et al. [29] developed a similar solution, but considering Kuwabara’s design, and validated the influence of T-C asymmetry on the springback phenomenon at a warm temperature (200 °C). Liu et al. [20] planned another alternative configuration consisting of employing an insulated cabinet to heat the sample during the compression test, reaching a temperature of 300 °C. They demonstrated that the existent T-C asymmetry at room temperature for the AZ31B magnesium alloy tended to disappear at higher temperatures. Running the test in a furnace makes it easier to control the temperature and simplifies the procedure. All these features, and the need to widen the field of temperatures involved in compression tests, led the authors of the present work to establish a relatively simple methodology for testing sheet metals at higher temperatures [30]. This procedure was validated at room temperature through the analysis of six different alloys. To this end, an inverse modeling approach was used that avoided the measurement of the material strains during the test, which is especially difficult at high temperature.

The present work consists of the application of the above-mentioned methodology to obtain, at several temperatures, the asymmetric tension–compression behavior of the most commonly used titanium alloys, commercial pure titanium grade 2 and Ti6Al4V. The influence of the strain rate on the results was also considered. In addition, the Lankford or anisotropy index was determined. The analysis and evaluation of both aspects, asymmetry and anisotropy, under high temperature compressive conditions, is a scientific and technological novelty in this field. Moreover, the results obtained under tension conditions are valuable, as they broaden and reinforce the existing knowledge in the area.

2. Materials and Methods
2.1. Materials

The tensile and compressive behavior of two titanium alloys was studied: commercial pure titanium grade 2 (Ticp2) (ASTM B265) and Ti6Al4V alloy (AMS 4911 N). Ticp2 was supplied as a cold rolled and annealed sheet and Ti6Al4V was supplied as a mill-annealed sheet. Both sheets were 0.8 mm thick. The chemical composition and the as-received micro-structures of both materials are shown in Table 1 and Figure 1. In Figure 1a, the microstructure of Ticp2 can be seen and consists of an α-phase with an average grain size of 40.7 µm. Figure 1b represents the biphasic α + β structure corresponding to the Ti6Al4V alloy. In this material, the α grains, with a mean size of 7.2 µm are embedded in intergranular fine β-phase. Some grade of grain deformation in the rolling plane can be appreciated properly for Ti6Al4V.

Table 1. Chemical composition of the analyzed titanium alloys (wt.%).

| Material   | Ti      | V      | Al      | Fe   | C          | O         | N       | Y       | H       | O + H   |
|------------|---------|--------|---------|------|------------|-----------|---------|---------|---------|---------|
| Ticp2      | Remainder | -      | -       | 0.09–0.10 | 0.01 | 0.14–0.15 | <0.01   | -       | 0.002   | -       |
| Ti6Al4V    | Remainder | 4.01–3.97 | 6.19–6.16 | 0.18–0.16 | 0.017–0.014 | 0.20–0.19 | 0.008–0.007 | <0.0004 | -      | 0.21–0.20 |
2.2. Tensile Tests

Isothermal tensile tests were carried out using a universal testing machine, Servosis ME 401/10 +/- 100 kN (SERVOSIS Inc., Madrid, Spain), equipped with a furnace that permits temperatures of up to 1000 °C to be reached. These tests were conducted according to the procedure established in the EN ISO 6892-2 standard [31], using the test samples detailed in Figure 2a. Temperatures of 25, 300, 450, 600, and 750 °C and strain rates of 0.002, 0.02, and 0.2 s\(^{-1}\) were selected as experimental parameters. All the tests were performed for the main directions of the sheet, i.e., rolling direction (0° or RD) and transverse direction (90° or TD). To ensure a homogeneous temperature of the material, the samples were kept 10 min in the furnace before the test. Although no protection was applied to the samples, no severe oxidation indicating deterioration of the mechanical parameters was observed.

The engineering curves obtained were turned into true stress–strain curves up to the onset of the necking or the Ultimate Tensile Strength point (UTS), employing the standard procedure [30]. The homogeneous plastic strain zone was adjusted to a Swift function and extrapolated beyond the necking point until failure, as shown in Equation (1), where \(n\) is the strain hardening index, \(K\) is the strength coefficient and \(\varepsilon_0\) is the elastic strain.

\[
\sigma = K \left( \varepsilon_0 + \varepsilon_p \right)^n
\] (1)
In addition to the tensile tests, anisotropy was determined according to the EN ISO 10113 standard [32]. The specimens were deformed at a strain rate of 0.002 s\(^{-1}\) for all analyzed temperatures. To determine the anisotropy index during the uniform deformation range, interrupted tests were performed up to different strain values until that of the necking point was reached.

2.3. Compression Tests

The compression tests were carried out under the same conditions as the previous tensile tests to observe the possible T-C asymmetries. The sheet compression sample shown in Figure 2b was used. This test specimen permits work to be performed in a quasi-uniaxial compression state, avoiding buckling until high deformation values [30].

In view of the specimen dimensions, test speeds of 4, 40, and 400 mm/min were chosen, which represent approximate constant strain rates of 0.002, 0.02, and 0.2 s\(^{-1}\), which are similar to those defined in the tensile tests. For a strain rate of 0.002 s\(^{-1}\), additional tests were performed up to different plastic strain levels. This allowed the repeatability of the procedure to be quantified, as well as allowing the anisotropy values for compressive strains \((r_0\) and \(r_{90}\)) to be obtained.

Due to the non-uniform deformation observed in the compression samples, it was not possible to indirectly evaluate the thickness strains, as proposed by EN ISO 10113 standard [32]. This led to the necessity of determining this strain by direct measurement, using a micrometer with a standard uncertainty \(\leq 0.01\) mm.

To carry out the compression tests, comb-type dies, previously designed by the authors (Figure 3a,b), were used [30]. The dies were located in the furnace during the tests, which allowed temperatures of up to 800 °C to be reached (Figure 3c). This system simplifies the method used by other authors [28,29], as the furnace isolation makes any additional temperature control system unnecessary. Nevertheless, a setup operation was carried out, using external thermocouples, to check the temperature homogeneity in the chamber. Due to the mass involved, it was necessary to keep the assembly inside the furnace for 15–20 min in order for the sample to achieve a uniform temperature.

![Figure 3.](image-url)
The tightening torque of the main screws was sufficient to guarantee the joint of the dies under the temperature of the tests. Moreover, the goal of the lateral screws was to avoid elastic gaps in the test area (Figure 3a). In any event, the behavior of the system may be considered to have been similar in all cases, at least when temperature of the test was the same.

2.4. High-Temperature Lubricant Selection

The proposed compression tests require the reduction of the friction forces in the system, with it being necessary to use lubricants. However, the choice of lubricants is limited due to the need to work in a high-temperature range. Based on the experience of the use of solid lubricants in hot forming operations, and given their advantages, MoS$_2$ (molybdenum disulfide) and h-BN (hexagonal boron nitride) were selected [33].

In order to characterize the tribological behavior of these lubricants and to establish appropriate ranges of application, pin-on-disk tests were performed at different temperatures and velocities for the steel–titanium friction pair. Lubricants were applied in the same way in both the friction tests and the compression tests. To this end, the particles were dispersed in alcohol first, and the surfaces were primed. In this way, the evaporation of the solvent resulted in a uniform layer of MoS$_2$ or h-BN adhered on the surface. Friction tests without using lubricant were also carried out to quantify the improvement generated by the use of lubricants. The temperature values selected for these tests were 25, 300, 450, and 600 °C, and the velocity values were 6, 47, 94, 471, and 848 mm/min. For all tests, a load of 50 N was established, which resulted in a constant average pressure of 20–30 MPa. With this selection of variables, the conditions of the compression tests were reproduced.

The friction coefficients obtained from these tests are shown in Figure 4. In view of the limited influence of velocity on the friction coefficient, this variable was removed to simplify the analysis. Therefore, the friction coefficient for each temperature was taken as the average value, regardless of the velocity considered.

Figure 4. Friction coefficients obtained: (a) Ticp2; (b) Ti6Al4V.
As can be seen, MoS$_2$ provides the best friction conditions up to 450 °C. Friction coefficients of around 0.2 in the case of Ticp2 and 0.1 in the case of Ti6Al4V were obtained, which means the friction force is reduced by about 65% and 70%, respectively. However, there is a marked change at 600 °C, which must be attributed to the decomposition and oxidation of MoS$_2$. This was confirmed by the Thermogravimetric and Differential Thermal Analysis, TG-DTA EXSTAR 6000 (Seiko Instruments Inc., Chiba, Japan), (Figure 5), which showed an exothermic degradation of MoS$_2$ at 450–500 °C. In the case of h-BN, the obtained values were not so good and presented higher dispersion, although they still represent an improvement of 5–30% in the case of Ticp2 and 20–50% for Ti6Al4V, depending on the temperature. However, as can be seen in Figure 5, this lubricant is an inert compound over the whole temperature range. This implies stable behavior, regardless of the working time at high temperature, which is not the case with MoS$_2$.

![Figure 5. Thermogravimetric and Differential Thermal Analysis (TG-DTA) analysis of the employed lubricants: MoS$_2$ and h-BN.](image)

In view of the results obtained, it was concluded that MoS$_2$ is the most suitable lubricant for application up to 450 °C. The stability of h-BN favors its use at higher temperatures. However, both lubricants were used at 600 °C to overlap their ranges of use.

2.5. Evaluation of the Friction Component in Compression Tests

Due to the characteristics of the compression system, it was necessary to control and evaluate the friction appearing during the test in the die–specimen–die contact, as the result of the specimen shortening in compression. In addition, some die–die sliding also occurs, which causes friction that must also be evaluated. Although some authors neglect the friction influence on the test [27], this is not possible at high temperatures, and it is important to take this phenomenon into account.

To determine the average friction force, specific tests were carried out, consisting of applying tensile forces to the compression die–specimen–die assembly at the same conditions as those in compression tests. From the results herein obtained, the typical tensile behavior of the material was subtracted, and the average friction force was determined, as shown in Figure 6a [30].
2.6. Determining the Compression True Stress–Strain Behavior

The compression engineering stress–strain curves were determined by subtracting the average friction force from the compression tests results (Figure 6b). Finally, the true compressive curves were obtained by using an inverse modeling method, since the non-uniform deformation produced in the samples prevents the application of the traditional equations. This methodology consists of an FEM iterative approach (Figure 7), with which the true stress–strain curve must be modified until the simulation-predicted engineering curve is equal to the experimental one [30].
The simulations were run using the explicit software Hypermesh + Radioss (Altair Engineering Inc., Troy, MI, USA). The model of the specimen consisted of 2D quad QEPH type elements of 1 mm in size, that is, a standard Radioss element improved and under-integrated with physical hourglass stabilization. As demonstrated [30], the simulations performed according to the Von Mises yield criterion were valid and simplified the procedure.

Initially, the material was modeled according to the Swift function obtained by the previous tensile tests for each temperature, as shown in Equation (1). In each iteration, the reference experimental engineering stress–strain curve was compared with those predicted by the FEM simulation (Figure 6d). For an elongation point, the experimental, \(\sigma_{\text{exp}}\), and simulated, \(\sigma_{\text{sim}}\), engineering stresses and the average simulated plastic strain, \(\varepsilon_{\text{pl}}\), in the cross-central section were obtained. For every plastic strain value, \(\varepsilon_{\text{pl}}\), the new compressive true stress value was calculated using Equation (2). By applying this procedure in all plastic strain points, \(i\), the true curve was corrected at each iteration, \(n\), as shown in Figure 6c.

\[
\sigma_{n+1}^i = \sigma_n^i \cdot \frac{\sigma_{\text{exp}}}{\sigma_{\text{sim}}} 
\]

Applying this procedure, in no case were more than three iterations needed to obtain good agreement between the experimental reference curve and that predicted by FEM simulation.

3. Results and Discussion

3.1. Asymmetric Tension–Compression Behaviour Depending on Temperature

According to the previous considerations, the compression behavior of Ticp2 and Ti6Al4V was evaluated at a range of temperatures of 25–750 °C and a strain rate value of 0.002 s\(^{-1}\), as shown in Figures 8 and 9, respectively. These figures depict, for the corresponding material, the experimental true tensile and compressive behavior. The compressive results are expressed by an average curve and a deviation band. Regarding the tensile curves, two different shapes from the necking point on can be pointed out: an extrapolation of the curve by applying the Swift function, and the other by considering the traditional engineering true transformation (Figure 8a). As is widely known, this last conversion is typically meaningless, as it is only valid until the necking point. Nevertheless, this part of the graphic results is relevant if the necking point cannot easily be located, as sometimes occurs at a high temperature [34,35].

Related to the compression behavior, the mean curve was obtained from the different tests carried out under the same conditions (between five and seven tests). The band represents the standard deviation involved for each point of the mean curve. Logically, the bandwidth is an index of the repeatability or the typical uncertainty of the methodology used, and it is quantified in Figure 10. As can be seen, the absolute average deviation is considerably similar for all temperatures, which means that the uncertainty of the experimental methodology is independent of that variable. Nevertheless, the results are more precise for low temperatures if the deviation is considered in relative terms of the total stress involved in each case.

According to the maximum temperature of use of MoS\(_2\), the tests at 600 °C were conducted with both lubricants (MoS\(_2\) and h-BN) (Figure 8g,h and Figure 9g,h). The corrected results are similar for both cases as the effect of friction is properly removed.
Figure 8. Tension and compression mechanical curves of Ticp2 at 0.002 s$^{-1}$ and temperatures of 25, 300, 450, 600, and 750 °C. (a,c,e,g,i) Mechanical behavior in rolling direction (0°); (b,d,f,h,j) Mechanical behavior in transverse direction (90°).
Figure 9. Tension and compression behavior of Ti6Al4V at 0.002 s$^{-1}$ and temperatures of 25, 300, 450, 600, and 750 °C. 
(a, c, e, g, i) Mechanical behavior in rolling direction (0°); (b, d, f, h, j) Mechanical behavior in transverse direction (90°).
The position of the curves establishes whether there is a symmetric or asymmetric response under tensile and compressive stress states. Thus, for the Ticp2 alloy, the asymmetric behavior is mainly observed for the rolling direction at room temperature (Figure 8a). For the transversal direction, the asymmetry remains until 450 °C. The same tendency can be seen for the Ti6Al4V alloy. These results are consistent with the signs detected by Odenberger et al. [36]. In the case of room temperature, the asymmetry detected in the transverse direction is around 10%, which is similar to previous results [37]. From the detailed analysis of the Ticp2 curves, it can be noted that the yield stress is slightly higher in tension than in compression, but immediately afterwards, the strain hardening promotes higher stress for compression in both directions (up to 450 °C). These results are similar to those obtained by Kuwabara et al. [38] for room temperature. For Ti6Al4V, the compression curves are located over the tensile ones when asymmetry appears (Figure 9a,b,d,f). It can be noted that the distance between the tension and compression curves in Figure 9 should be neglected based on the very low absolute values involved. In addition, this last statement is based, as well, on the tendency observed at lower temperatures for 90° and even for the total range of temperatures for 0°. Thus, the difference found in Figure 9 might be the result of an experimental error and not due to an asymmetric behavior of the material. In fact, it was relatively difficult to reach a uniform temperature in the sample during the test at such high temperature.

These results are consistent with those obtained by Battaini et al. [15], which demonstrated the contribution of twinning in the total strain undergone by pure titanium under compression as a function of the temperature. They observed that the contribution of twinning decreased rapidly with the temperature in the rolling direction. However, for the transversal direction of the plate, this contribution continues until higher temperatures (300–400 °C). In view of these results, a direct relationship can be established between T-C asymmetric behavior and the twinning density in the material.

Experimentally, from the friction forces and considering the coefficients of friction involved in the compressive tests, it could be inferred that the lateral pressure applied to the samples was low compared to the principal compressive stress. The biaxial influence was calculated and is shown in Figure 11; the maximum error incurred if biaxial correction is not considered is less than 1%. Accordingly, the equivalent stress can be considered a pure uniaxial compression; that is, no biaxial correction is necessary at any temperature [30,39,40].

**Figure 10.** Average relative and absolute typical uncertainty as a function of the temperature. (a) Ticp-2; (b) Ti6Al4V.
3.2. Asymmetric Tension–Compression Behavior Depending on Temperature and Strain Rates: Preliminary Studies

Figures 12 and 13 depict the influence of the strain rate on the asymmetric compression–tensile behavior at different temperatures. These figures show that the higher the strain rate, the more conservative the asymmetric behavior, i.e., the asymmetry effect is prolonged to higher temperatures. For example, this effect can be clearly observed in Figures 12h and 13h, where the symmetry only exists at the lowest strain rate. For greater clarification, Figure 14 depicts the asymmetry tendency with temperature and strain rate. Specifically, the values for 0.002 s⁻¹, which were previously analyzed in detail (Section 3.1), were extracted from the correlation surfaces. Again, these results might be related to the twin density existing in the material. Nemat-Nasser et al. [13] described the contribution of twinning as a function of the strain rate for different temperatures in compression tests of extruded pure titanium and found a direct correlation with the strain rate, that is, the higher the strain rate, the higher the contribution of twinning.
Figure 12. Tension and compression behavior of Ti62 at temperatures of 25, 300, 450, 600, and 750 °C, and strain rates of 0.002, 0.02, and 0.2 s⁻¹. (a,c,e,g,i) Mechanical behavior in rolling direction (0°); (b,d,f,h,j) Mechanical behavior in transverse direction (90°).

Figure 13. Cont.
Figure 13. Tension and compression behavior of Ti6Al4V at temperatures of 25, 300, 450, 600, and 750 °C, and strain rates of 0.002, 0.02, and 0.2 s⁻¹. (a,c,e,g,i) Mechanical behavior in rolling direction (0°); (b,d,f,h,j) Mechanical behavior in transverse direction (90°).

Figure 14. Average asymmetry as a function of the temperature and strain-rate. (a) Ticp2; (b) Ti6Al4V; (c,d) Evolution of the average asymmetry for a strain rate of 0.002 s⁻¹.
3.3. Analysis and Evaluation of the Anisotropy

The values of anisotropy for Ticp2 and Ti6Al4V were experimentally obtained for tensile and compressive stresses at different temperatures (Figure 15).

![Figure 15](image.png)

**Figure 15.** r-values obtained at 0.002 s$^{-1}$ under tension (T) and compression (C) conditions. (a) Ticp2 0°; (b) Ticp2 90°; (c) Ti6Al4V 0°; and (d) Ti6Al4V 90°.

The tensile r-values for Ticp2 depend on the plastic strain at all analyzed temperatures, especially for the transverse direction, as can be observed in Figure 15a,b (blue surfaces). In this direction, the anisotropy is very high at the onset of the plastic period with a decreasing tendency to reach a stable value at high strain values, near the necking point. This transient evolution is less severe in the rolling direction (Figure 15a). Although there are no references in the literature for tensile anisotropy strain evolution at high temperatures, these results are consistent with those found by other authors at room temperature [41]. Moreover, although some dispersion data can be found in the literature [22,42–44], the results for Ticp2 obtained herein were higher than those reported by other authors. This discrepancy of the results might be explained by the use of different criteria for expressing the values, that is, whether the stable anisotropy value or the average value in a range of strains is considered. Nevertheless, other aspects can affect the anisotropy, such as the specific rolling route or rolling program followed by the material sheet. Some authors underline the significant differences this aspect can generate [44,45].

Furthermore, the anisotropy results obtained under the tensile condition for Ti6Al4V are typically similar to those found in the literature, as shown in Figure 16 [46–50]. There are a limited number of previous studies at warm/high temperatures [46,49,50], and the values found present the same tendency as those obtained in the present work (Figure 15c,d). As can be appreciated, no transient anisotropy strain evolution phenomenon appears.
Figure 16. Ti6Al4V r-values obtained in this paper compared to those obtained by previous authors. Due to the non-existence of the necking phenomenon in compression tests, the anisotropy could be evaluated over a large range of strains, as observed in the red surfaces of Figure 15. The r-values are more homogeneous at each temperature condition for the two materials experimented.

In view of the anisotropy evolution, the r-values were extracted at a strain reference of 0.1. These are shown in Figure 17, where the evolution of stable and representative r-values can be clearly visualized with the temperature and direction under tension and compression states. Drawing on the above discussion, it can be stated that the anisotropy values under tension condition can be extrapolated to the compressive states for temperatures above 450 °C. However, for lower temperatures, the expected behavior of the sheet will be conditioned by the stress state. This difference in behavior must be justified based on the combination of different existing deformation mechanisms as a function of the direction in the sheet and the direction of the applied loads. This causes the material to flow in a preferential way, originating differences in the width strain and in the thickness strain depending on if the stress state is tension or compression. Therefore, the anisotropy index will be different [23]. The highest values of compression anisotropy obtained for Ti6Al4V are consistent with the highest anisotropy detected by Tuninetti et al. [37].

Figure 17. Tensile and compressive anisotropy r-values in the rolling (0°) and transverse (90°) directions as a function of temperature for $\epsilon = 0.1$. (a) Ticp2 0.002 s$^{-1}$; (b) Ti6Al4V 0.002 s$^{-1}$.
4. Modeling the Experimental Results

Traditional mechanical behavior models consider tensile and compression states separately, defining the corresponding equations and applying them by means of the equivalent stress–strain criterion employed. Eventually, it may be useful to have a complete behavior model, especially for the programming of analytical procedures in applications, such as bending, where the stress system is typically a tensile–compression one [51,52]. Moreover, this proposal would simplify the material modeling with complex flow surfaces or loci dependent of the plastic work [21]. Thus, using the experimental data, a mechanical model for the tensile–compression field of the material is proposed. This model is based on a modifier-factor, $\zeta$, which permits the asymmetric behaviour observed for the titanium alloys to be taken into account. The $\zeta$-factor is valid for all typical behavior equations to which it can be applied, such as the Swift or Fields–Backofen functions, for example, and it turns the tensile model into the expected compression one.

For every temperature and strain rate, it was observed that the asymmetry obtained in tension–compression showed a linear increment in stress, $\sigma_a$, with the strain in all cases (Figure 18a), which can be established according to Equation (3), where $K_a$ and $\sigma_{a0}$ are the coefficients of the linear evolution of the asymmetry. To apply the indicated correction Equation (3) only for the compression strains, the parameter $f_{c-t}$ had to be considered, as shown in Equation (4). The non-dimensional exponent $Q$ in Equation (4) adopts a high value to turn $f_{c-t}$ into 0 or 1 if the strain is positive or negative, respectively. That is, $f_{c-t}$ behaves, in practice, similarly to the typical Heaviside step function (Figure 18b). In this way, $\zeta$ results from the product of $f_{c-t}$ and $\sigma_a$.

$$\sigma_a = K_a \varepsilon + \sigma_{a0}$$  \hspace{1cm} (3)

$$f_{c-t} = \frac{1}{1 + e^{Q \varepsilon}}$$  \hspace{1cm} (4)

Potential functions, which are traditionally considered to define mechanical models, do not allow negative input values to be applied. For this reason, a new function parameter, $\chi$, was defined, Equation (5), which allows us to transform the tensile function into a negative stress–strain curve (Figure 18b), which is typically located in the Cartesian third quadrant.

$$\chi = \frac{2}{\pi} \arctg (Q \varepsilon)$$  \hspace{1cm} (5)

Figure 18. Tension–compression fitting by the modified Swift function, as shown in Equation (8). Case: T1cp2 90° 25 °C 0.002 s$^{-1}$. (a) Tension-Compression difference and fitting according to equation 3; (b) Definition of switching functions (Equations (4) and (5)); (c) Agreement between the proposed model (Equation (7)) and experimental data.
Thus, the T-C model is written in Equation (6) for a general behavior function, $\sigma(\varepsilon)$, or in more detail, in Equation (7), if a Swift type of function is selected. Finally, the proposed model is shown in Figure 18c.

$$\sigma_t - c(\varepsilon) = \zeta + \chi \cdot \sigma(\varepsilon) \quad (6)$$

$$\sigma_t - c = \frac{1}{1 + e^{Q\varepsilon}} (K_a \varepsilon + \sigma_{a0}) + \frac{2}{\pi} \arctan (Q\varepsilon) K |\varepsilon_0 + \varepsilon|^n \quad (7)$$

The temperature and strain rate should be included in the general function in Equation (6). For this, the $\zeta$-factor must be expressed as a function of those variables. Thus, $K_a$ and $\sigma_{a0}$ should be correlated with temperature and strain rate (Figure 19). In this way, Equations (8)–(10) depict the functions obtained, where $T$ is the temperature (in K), $\alpha_{ij}$ are functions of the strain rate, and $A_{ij}$ and $B_{ij}$ are constants.

$$\sigma_{a0} = a_{a1} e^{\alpha_{a2} T} \quad (8)$$

$$K_a = a_{k1} e^{\alpha_{k2} T} \quad (9)$$

$$\alpha_{ij} = A_{ij} \ln \dot{\varepsilon} + B_{ij} \quad (10)$$

Finally, the global asymmetry-modifier factor, $\zeta$, is expressed according to Equation (11).

$$\zeta = f_{c-1} \left[ a_{a1} e^{\alpha_{a2} T} \varepsilon + a_{k1} e^{\alpha_{k2} T} \right] \quad (11)$$

Nevertheless, to model the material behavior at any temperature and strain rate, it is necessary to use an equivalent tensile stress–strain function including these variables, $\sigma(\varepsilon, T, \dot{\varepsilon})$. For this purpose and taking into account estimations carried out with different constitutive models [53], a modified Fields–Backofen function was finally selected, Equations (12)–(15), where $C_i, D_i$, and $F_i$ are constants. The parameters of this function were estimated by using a pre-adjustment procedure based on the progressive evaluation of the different variables involved in the law. Once the pre-model was obtained, an accurate fit was carried out by using an iterative process to minimize the global quadratic error implied.

$$\sigma = K \varepsilon^n \dot{\varepsilon}^m \quad (12)$$

$$K = C_1 e^{C_2 T} \quad (13)$$

$$n = D_1 + D_2 \ln (\dot{\varepsilon}) + D_3 e^{D_4 T} \quad (14)$$

$$m = F_1 e^{F_2 T} \quad (15)$$

Finally, the model proposed for the two materials under study is shown in Equation (16), and the values of the coefficients involved are shown in Table 2.

$$\sigma_t - c(\varepsilon, T, \dot{\varepsilon}) = f_{c-1} \left[ a_{a1} e^{\alpha_{a2} T} \varepsilon + a_{k1} e^{\alpha_{k2} T} \right] + \chi K \varepsilon^n \dot{\varepsilon}^m \quad (16)$$
Figure 19. Development of the tension–compression (T-C) asymmetry model as a function of temperature and strain rate. Case: Ticp2 90°.
Table 2. Material parameters of the asymmetric constitutive model for Ticp2 and Ti6Al4V.

| Material | Direction | \( A_{a1} \) (MPa) | \( B_{a1} \) (MPa) | \( A_{a2} \) (K\(^{-1}\)) | \( B_{a2} \) (K\(^{-1}\)) | \( A_{d1} \) (MPa) | \( B_{d1} \) (MPa) | \( A_{d2} \) (MPa) | \( B_{d2} \) (MPa) | \( Q \) |
|----------|-----------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| Ticp2    | 0\(^\circ\) | 1.428               | 8.786               | 0.050               | -2.13 \times 10^{-8} | 1.553               | 115167.7            | -1.25 \times 10^{-5} | -0.017              | 10,000              |
|          | 90\(^\circ\) | 1.428               | 8.786               | 0.050               | -2.13 \times 10^{-8} | 1.553               | 114321.7            | -1.25 \times 10^{-5} | -0.018              | 10,000              |
| Ti6Al4V  | 0\(^\circ\) | 1.428               | 8.786               | 0.050               | -3.84 \times 10^{-8} | 2.389               | 2491.76             | -1.92 \times 10^{-4} | -0.005              | 10,000              |
|          | 90\(^\circ\) | 49.667              | 45.919              | 8.59 \times 10^{-4} | 1.80 \times 10^{-3}  | -255.98             | 7.94                | -0.082              | -0.507              | 10,000              |

The experimental and modeled data are compared in Figure 20 for Ticp2 and Ti6Al4V and for the transversal direction. A reasonable agreement can be appreciated between them for all the ranges of temperature and strain rates implemented. In fact, the lack of fitting detected in particular cases of the compression functions is directly derived from the deficiency existent in the standard constitutive model selected (first Cartesian quadrant). Moreover, the modified constitutive model, based on the Fields–Backofen model performed, yielded better results than those obtained by traditional proposals. However, the minor deficiencies detected in the tensile adjustments would be corrected by applying specifically developed models that are capable of adopting the marked thermal softening of this kind of material [53]. In sum, the agreement of the proposed model Equation (16) is highly satisfactory, as it allows the asymmetry existent in compression to be reproduced.

Figure 20. Cont.
5. Conclusions

The results obtained in this research validate the method previously proposed by the authors, demonstrating the possibility of testing sheet metals under in-plane compressive stress and high temperatures (up to 750 °C). Under these test conditions, T-C asymmetry and compression anisotropy for Ticp2 and Ti6Al4V were determined, being the primary novelty of this work. In more detail, the following aspects can be underlined:

- The T-C asymmetry observed allows us to affirm that this phenomenon decreases with temperature and depends on the rolling direction. In the transverse direction, the asymmetry remains until higher temperatures (450 °C) are reached compared to the rolling direction. The strain rate seems to delay this effect. In addition, the asymmetry value obtained herein and the evolution of twinning density for pure titanium, as detected by other authors, are parallel phenomena that seem to behave in the same way as a function of the temperature and the strain rate. This statement might provide some light about the causes of asymmetry. Nevertheless, this point should be confirmed later on.

- The anisotropy for the mentioned conditions was determined, and it has been proven that under a compression state, the values are clearly different from those obtained by uniaxial tensile tests. In a deeper analysis, no pronounced anisotropy transient behavior with the strain has been found in compression, with the evolution of this parameter being more uniform than those obtained in some cases under tension. Nevertheless, for the highest temperatures, the r-values tend to be similar for both stress states.

- In view of the evolution of the anisotropy and asymmetry phenomena, it can be established that the traditionally assumed symmetrical behavior would not be considered for temperatures lower than 450 °C.

- An asymmetry-modifier parameter was defined to extend the traditional material constitutive models that permit the material behavior to be predicted for the whole field of tension–compression strains. The model was tested with an adapted Fields–Backofen equation, and a good agreement with the experimental results was obtained for all test conditions.

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