Status of the $B_{(s)}^0 - \bar{B}_{(s)}^0$ mixing from QCD spectral sum Rules

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Abstract

In this talk, I present new results obtained from QCD spectral sum rules (QSSR), on the bag constant parameters entering in the analysis of the $B_{(s)}^0 - \bar{B}_{(s)}^0$ mass-differences. Taking the average of the results from the Laplace and moment sum rules, one obtains to order $\alpha_s$: $f_B \sqrt{\hat{B}_B} \simeq (228 \pm 61)$ MeV, $f_B \sqrt{\hat{B}_{Bs}}/f_B \sqrt{\hat{B}_B} \simeq 1.18 \pm 0.03$, in units where $f_\pi = 130.7$ MeV. Combined with the experimental data on the mass-differences $\Delta M_{d,s}$, one obtains the constraint on the CKM weak mixing angle: $|V_{ts}/V_{td}|^2 \geq 20\,2(1.3)$. Alternatively, using the weak mixing angle from the analysis of the unitarity triangle and the data on $\Delta M_d$, one predicts $\Delta M_s = 18.3(2.1)$ ps$^{-1}$ in agreement with the present experimental lower bound and within the reach of Tevatron 2.

1 Introduction

$B_{(s)}^0$ and $\bar{B}_{(s)}^0$ are not eigenstates of the weak hamiltonian, such that their oscillation frequency is governed by their mass-difference $\Delta M_q$. The measurement by the UA1 collaboration of a large value of $\Delta M_d$ was the first indication of a heavy top-quark mass. In the SM, the mass-difference is approximately given by 3:

$$\Delta M_q \simeq \frac{G_F^2}{4\pi^2} M_W^2 |V_{tq} V_{tb}^*|^2 S_0 \left( \frac{m_t^2}{M_W^2} \right) \eta_B C_B(\nu) \frac{1}{2M_{B_q}} \langle \bar{B}_q^0 | O_q(\nu) | B_q^0 \rangle$$

(1)

where the $\Delta B = 2$ local operator $O_q$ is defined as:

$$O_q(x) \equiv (\bar{b}\gamma_\mu L q)(\bar{b}\gamma_\mu L q) ,$$

(2)

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with: \( L \equiv (1 - \gamma_5)/2 \) and \( q \equiv d, s, \) \( S_0, \eta_B \) and \( C_B(\nu) \) are short distance quantities and Wilson coefficients which are calculable perturbatively, while the matrix element \( \langle B_q^0|O_q|B_q^0 \rangle \) requires non-perturbative QCD calculations, and is usually parametrized for \( SU(N)_c \) colours as:

\[
\langle B_q^0|O_q|B_q^0 \rangle = N_c \left( 1 + \frac{1}{N_c} \right) f_{B_q}^2 M_{B_q}^2 B_{B_q} .
\]

\( f_{B_q} \) is the \( B_q \) decay constant normalized as \( f_\pi = 92.4 \text{ MeV} \), and \( B_{B_q} \) is the so-called bag parameter which is \( B_{B_q} \approx 1 \) if one uses a vacuum saturation of the matrix element. From Eq. \( \[6\] \), it is clear that the measurement of \( \Delta M_d \) provides the one of the CKM mixing angle \( |V_{td}| \) if one uses \( |V_{ts}| \approx 1 \).

One can also extract this quantity from the ratio:

\[
\frac{\Delta M_s}{\Delta M_d} = \frac{|V_{ts}|^2 M_{B_d} \langle B_0^0|O_s|B_d^0 \rangle}{|V_{td}|^2 M_{B_s} \langle B_0^0|O_d|B_d^0 \rangle} = \frac{|V_{ts}|^2 M_{B_d}}{M_{B_s}} \xi^2 ,
\]

since in the SM with three generations and unitarity constraints, \( |V_{ts}| \approx |V_{td}| \). Here:

\[
\xi \equiv \sqrt{\frac{g_s}{g_d}} \equiv \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}} .
\]

The great advantage of Eq. \( \[4\] \) compared with the former relation in Eq. \( \[3\] \) is that in the ratio, different systematics in the evaluation of the matrix element tends to cancel out, thus providing a more accurate prediction. However, unlike \( \Delta M_d = 0.473(17) \text{ ps}^{-1} \), which is measured with a good precision \( \[8\] \), the determination of \( \Delta M_s \) is an experimental challenge due to the rapid oscillation of the \( B^0_s-B^0_s \) system. At present, only a lower bound of \( 13.1 \text{ ps}^{-1} \) is available at the 95% CL from experiments \( \[8\] \), but this bound already provides a strong constraint on \( |V_{td}| \).

## 2 Two-point function sum rule

Ref. \( \[5\] \) has extended the analysis of the \( K^0-\bar{K}^0 \) systems of \( \[4\] \), using two-point correlator of the four-quark operators into the analysis of the quantity \( f_{B_s} \sqrt{B_{B_s}} \) which governs the \( B^0_s-B^0_s \) mass difference. The two-point correlator defined as:

\[
\psi_H(q^2) \equiv i \int d^4x \, e^{iqx} \langle 0|\mathcal{T} O_q(x) (O_q(0))^\dagger |0 \rangle ,
\]

is built from the \( \Delta B = 2 \) weak operator defined in Eq. \( \[2\] \). The corresponding Laplace (resp. moment) sum rules are:

\[
\mathcal{L}(\tau) = \int_{4M_{B_s}^2}^{\infty} dt \, e^{-\tau t} \text{Im} \psi_H(t) , \quad \mathcal{M}_n = \int_{4M_{B_s}^2}^{\infty} dt \, t^n \text{Im} \psi_H(t) ,
\]

The two-point function approach is very convenient due to its simple analytic properties which is not the case of approach based on three-point functions \( \[5\] \). However, it involves non-trivial QCD calculations which become technically complicated when one includes the contributions of radiative corrections due to non-factorizable diagrams. These perturbative radiative corrections due to factorizable and non-factorizable diagrams have been already computed in \( \[5\] \) (referred as NP), where it has been found that the factorizable corrections are large while the non-factorizable ones are negligibly small. NP analysis has confirmed the estimate in \( \[5\] \) from lowest order calculations, where under some assumptions on the contributions of higher mass resonances to the spectral function, the value of the bag parameter \( B_{B_s} \) has been found to be:

\[
B_{B_s}(4m_{B_s}^2) \approx (1 \pm 0.15) .
\]

\textsuperscript{2}For detailed criticisms, see \( \[5\] \).
This value is comparable with the one \( B_{B_d} = 1 \) from the vacuum saturation estimate, which is expected to be a quite good approximation due to the relative high-scale of the \( B \)-meson mass. Equivalently, the corresponding RGI quantity is:

\[
\hat{B}_{B_d} \simeq (1.5 \pm 0.2)
\]  

\( (9) \)

where we have used the relation:

\[
B_{B_d}(\nu) = \hat{B}_{B_d}\alpha_s \left( 1 - \left( \frac{5165}{12696} \right) \left( \frac{\alpha_s}{\pi} \right) \right),
\]

\( (10) \)

with \( \gamma_0 = 1 \) is the anomalous dimension of the operator \( O_q \) and \( \beta_1 = -23/6 \) for 5 flavours. The NLO corrections have been obtained in the \( \overline{MS} \) scheme \[3\]. We have also used, to this order, the value \[9, 7\]:

\[
\bar{m}_b(m_b) = (4.24 \pm 0.06) \text{ GeV},
\]

\( (11) \)

and \( \Lambda_5 = (250 \pm 50) \text{ MeV} \) \[10\]. In a forthcoming paper \[1\], we study (for the first time), from the QCD spectral sum rules (QSSR) method, the \( SU(3) \) breaking effects on the ratio: \( \xi \) defined previously in Eq. (5), where a similar analysis of the ratios of the decay constant \( s \) has given the values \[11\]:

\[
\hat{f}_{D_s}/f_{D} \simeq 1.15 \pm 0.04, \quad f_{B_s}/f_{B} \simeq 1.16 \pm 0.04.
\]

\( (12) \)

We also improve the previous result on \( B_{B_d} \) by the inclusion of the \( B^* - B^* \) resonances into the spectral function.

## Results and implications on \( |V_{ts}/V_{td}|^2 \) and \( \Delta M_s \)

We deduce by taking the average from the moments and Laplace sum rules results \[1\]:

\[
\xi \equiv \frac{f_{B_s} \sqrt{\hat{B}_{B_s}}}{f_B \sqrt{\hat{B}_B}} \simeq 1.18 \pm 0.03, \quad f_B \sqrt{\hat{B}_B} \simeq (228 \pm 61) \text{ MeV},
\]

\( (13) \)

in units where \( f_{\pi} = 130.7 \) MeV. As expected, we have smaller errors for the ratio \( \xi \) due to the cancellation of the systematics, while for \( f_B \sqrt{\hat{B}_B} \), the error comes mainly and equally from the pole mass \( M_b \) and the truncation of the PT series where we have estimated the strength of the \( \alpha_s^2 \) contribution assuming a geometric growth of the PT coefficients. These results can be compared with different lattice and phenomenological determinations given in \[12, 13\]. By comparing the ratio with the one of \( f_{B_s}/f_{B_d} \) in Eq. (12), one can conclude (to a good approximation) that:

\[
\hat{B}_{B_s} \approx \hat{B}_{B_d} \approx (1.41 \pm 0.33) \Rightarrow B_{B_d,s}(4m_b^2) \simeq (0.94 \pm 0.22) ,
\]

\( (14) \)

indicating a negligible \( SU(3) \) breaking for the bag parameter. For a consistency, we have used the estimate to order \( \alpha_s \) \[15\]:

\[
f_B \simeq (1.47 \pm 0.10)f_{\pi},
\]

\( (15) \)

and we have assumed that the error from \( f_B \) compensates the one in Eq. (13). The result is in excellent agreement with the previous result of \[8\] in Eqs (8) and (9). Using the experimental values:

\[
\Delta M_d = 0.472(17) \text{ ps}^{-1}, \quad \Delta M_s \geq 13.1 \text{ ps}^{-1} \text{ (95\% CL)},
\]

\( (16) \)

one can deduce from Eq. (8):

\[
\rho_{sd} \equiv \left| \frac{V_{ts}}{V_{td}} \right|^2 \geq 20.2(1.3),
\]

\( (17) \)

\footnote{One can notice that similar strengths of the \( SU(3) \) breakings have been obtained for the \( B \to K^* \gamma \) and \( B \to Kl\nu \) form factors \[14\].}
Alternatively, using: $\rho_{sd} \simeq 28.4(2.9)$ obtained by using the Wolfenstein parameters determined in [12], we deduce:

$$\Delta M_s \simeq 18.4(2.1) \text{ ps}^{-1},$$

in good agreement with the present experimental lower bound and within the reach of Tevatron run 2 experiments.

## 4 Conclusions

We have applied QCD spectral sum rules for extracting (for the first time) the $SU(3)$ breaking parameter in Eq. (13). The phenomenological consequences of our results for the $B_{d,s}^0 - \bar{B}_{d,s}^0$ mass-differences and CKM mixing angle have been discussed. An extension of this work to the study of the $B_{s,d}^0 - \bar{B}_{s,d}^0$ width difference is in progress [16].

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