Renormalisation of the Higgs sector of the NMSSM. Application to Higgs decays

Fawzi Boudjema
LAPTh, Université Savoie Mont Blanc, CNRS, B.P.110, Annecy-le-Vieux F-74941, France
E-mail: boudjema@lapth.cnrs.fr

Abstract. I will first recollect some memories of my collaboration with Shimizu-sensei, a collaboration that mainly covered the topic of renormalisation and loop corrections. I will then describe the renormalisation of the Higgs sector of the NMSSM, this work could be seen as a natural continuation of the long collaboration I have had with the Minami-Tateya group. This study has been done with Geneviève Belanger, Guillaume Chalons and Vincent Bizouard. Several on-shell renormalisation schemes as well as a mixed on-shell DR scheme are analysed. Using SloopS, a code that can perform automatic computation of one-loop corrections, the one-loop electroweak corrections to the partial widths of the pseudoscalar and scalar Higgses into lighter Higgses and/or neutralino/charginos are computed.

1. Introduction
My first correspondence with Shimizu-sensei was around 1987 when I was graduating from Sussex University, a few months earlier than the time when Norman Dombey and I were preparing the Workshop Radiative Corrections: Results and Perspectives[1] in Brighton, UK, in July 1989. This conference, just at the start of LEP1, turned out to be the first in the series that has come to be known as RADCOR. The correspondence with Shimizu was about a work on precision calculations, \( e^+ e^- \rightarrow \gamma \gamma \)[2]. The first time we met was around 1993 in Annecy, shortly after I joined the CNRS with LAPTh. Our first collaboration was during the year-long Physics at LEP2 CERN Workshop in 1994-1996 when I was convening the Standard Model Processes working group[3]. Shimizu with his Minami-Tateya group had contributed in a major way and showed the prowess of the GRACE system. I was not part of the GRACE collaboration but its through the CERN Yellow Book (LEP2) that my long discussions with Shimizu-sensei started. In the next year or two that followed, we embarked on a common project, the automation of one-loop electroweak corrections. LEP2 had revealed the importance of four-body final states where many groups competed to provide results for many channels of interest at LEP2. With the concomitant interest in the physics of a high energy \( e^+ e^- \) linear collider, the project aimed at providing better precision for many processes at the Linear Collider. Many of these processes required the calculation of multi-leg processes. Prime among these processes were Higgs production mechanisms \( e^+ e^- \rightarrow \nu \bar{\nu} H, t\bar{t}H, e^+ e^- H, ZHH, \nu \bar{\nu} HH \) and .. 4-fermion process that are needed to go beyond the important resonant contributions from \( e^+ e^- \rightarrow W^+ W^- \). Before embarking on this (then) titanic programme, there was no full one-loop calculations to \( 2 \rightarrow 3 \) and \( 2 \rightarrow 4 \) processes. We wanted to push the theoretical challenge and set these processes as targets. From the end of 1997 the exchanges were intense, as were the visits with which my love for Japan
grew tremendously. The first concrete aspect aimed at finding a powerful internal check of the code. Of course we had to make sure that the renormalisation procedure was implemented correctly by checking for each process that the corrections were ultra-violet finite. But much more powerful was the check on gauge invariance and/or gauge-parameter independence on all physical amplitudes. The problem is that we wanted efficiency, however reverting to the usual \( \xi \) ’t Hooft-Feynman gauges would have been intractable. The “longitudinal” terms in the vector boson propagators (which are absent in the much used Feynman gauge) not only introduce a high (superficial) degree of divergence in the intermediate steps of the computation but, perhaps worse, generate loop integrals with very high tensorial structures. Reduction of these (very) high rank tensors on the basis of scalar integrals is a tremendous and expensive task. The idea therefore was to have the gauge parameter dependence (in the intermediate stages of the calculations) in each amplitude in a simple polynomial form while keeping the analytic structure of the loop integral evident with only the physical masses showing up in the corresponding diagrams. We used a non-linear gauge fixing[4, 5] with no less than 5 gauge parameters: \( \bar{\alpha}, \beta, \bar{\delta}, \bar{\kappa}, \bar{\epsilon} \). Denoting \( A,Z,W^\pm, H \) for, respectively, the photon, the Z boson, the W and the Higgs boson, this non-linear gauge-fixing writes

\[
\mathcal{L}_{GF} = -\frac{1}{\xi_W} \left( \partial_{\mu} - ie\bar{\alpha}A_\mu - ig\bar{\beta}Z_\mu \right) W^{\mu +} + \xi_W \frac{g}{2} \left( v + \delta h + H + i\bar{\kappa}h \right) \chi^+ |^2
\]

\[
-\frac{1}{2\xi_Z} \left( \partial_{\mu} + \xi_Z \frac{g}{2e} (v + i\bar{\epsilon}H) \right) \chi^2 - \frac{1}{2\xi_Y} \left( \partial \cdot A \right)^2
\]  

(1)

I retrieved an e-mail from Shimizu-sensei dated 06/10/1997, exactly 19 years ago to the day!, stating that this step of the set-up for the full one-loop computation had been completed and I was invited to KEK so that the implementation of the non-linear gauge be completed in the GRACE system in view of the evaluation of the full one-loop amplitudes in the electroweak theory. This non-linear gauge was implemented in GRACE-Loop together with the full set of counterterms. It took a couple of years for the system to be ready for the first challenging process \( e^+e^- \rightarrow \nu\bar{\nu}H \).

The first preliminary results[6] were shown at RADCOR 2002/LOOP and LEGS 2002 in September by Fujimoto-san in Kloster Banz (Germany). We were all a bit nervous, dreading to be scooped. From Germany Fujimoto was reporting by phone directly to Shimizu who in turn reported to me (from Japan!) by e-mail. In a message I recovered from my e-mail archive, the evening following Fujimoto presentation, Shimizu wrote “.... He (Fujimoto-san) said in last evening session Tarasov has given a talk on nu-nu-Higgs 1-loop calculations but without showing any numbers. ..... It seems that now the world competition is becoming very tough like in 1995 on W-pair for LEP-2. Sayonara, Shimizu Y ”. The mail came with a caring post-scriptum ”Though you will certainly get the true explanation tomorrow, I tried to tell my guess so that you can have a good sleep tonight!”

Our Physics Letters B[7] on \( \nu\bar{\nu}H \) appeared soon after. We were not scooped. At that point the collaboration was in full swing, with frequent visits to Japan and for our Minami-Tateya friends to France. The items on our list of priority processes were ticked one after the other[8, 9, 10, 11, 12, 13, 14]. This period corresponded with one of the happiest period in my life, my eldest daughter being born in 2001. The picture of Shimizu-sensei playing with Noor brings some sweet memories to me. At the same time as GRACE-1Loop was developing, Minami-Tateya was also developing the SUSY version and we exchanged also regarding this progress. Later on, it was decided that one should perform a full renormalisation of the MSSM and that the Annecy group should perform independent checks. In Annecy with the help of Andrei Semenov we decided to exploit LANHEP[16] as a model file builder to the FeynArts, FormCalc, LoopTools[17] packages in order to have an independent system which we called SloopS[18, 19, 20, 21]. In retrospect, if I go through my interaction with Shimizu-sensei, the first encounter was on a QED reaction controlled essentially with one parameter, \( \alpha \), then we
moved to a theory with a few parameters ($SU(2) \times U(1)$), specialising to multi-leg processes and then to a multi-parameter theory like the MSSM. Working with Shimizu-sensei, precision physics was an experience of beauty in different levels of complexity! Being in Japan, I can not help making the connection between our multi-leg calculations in multi-parameter theories with the multiplication of legs and arms within the multitude of statues in Sanjusangendo! Dealing with multi-parameter theories, eventually with multi-arm processes is not the same as what Gudrun referred to (for QCD N(N)LO processes), mass production of cookies! It’s rather recreating the subtlety in the difference contained in the 1000 figures demultiplied through arms (and legs) of the Sanjusangendo hall.

I will now report on one such theory that has even more parameters than the MSSM and the issues one has to face in renormalising this theory. The renormalisation of all the sectors of the NMSSM has been completed in SloopS[22, 23] here I will report on the Higgs sector. I will concentrate in particular on the definition/reconstruction of the parameters and the passage from the parameters at the Lagrangian level to the more physical parameters that may be chosen as inputs instead, namely the masses of the particles of the theory. The issue of mixing between the fields is critical, as is the issue of scheme and scale dependence.

2. The NMSSM
The NMSSM[24] provides a natural explanation for the scale of the higgsino parameter, $\mu$ by relating it to the vev of a scalar singlet, thus solving the little hierarchy problem of the MSSM. It makes it also much easier to produce a Higgs with a mass of 125GeV as observed at the LHC. For the MSSM such a mass requires large values in the stop sector which in turn creates an unbearable unease with naturalness. The NMSSM contains three Higgs superfields: two
$SU(2)_L$ doublets $\hat{H}_u$ and $\hat{H}_d$, as in the MSSM, and one additional gauge singlet $\hat{S}$

$$\hat{H}_u = \left( \begin{array}{c} \hat{H}_u^+ \\ H_u^0 \end{array} \right), \quad \hat{H}_d = \left( \begin{array}{c} \hat{H}_d^0 \\ H_d^+ \end{array} \right), \quad \hat{S}. \quad (2)$$

In the $\mathbb{Z}_3$ implementation that we will assume, the Higgs superpotential is made up of two operators, each one introduces two dimensionless couplings $\lambda$ and $\kappa$,

$$W_{Higgs} = -\lambda \hat{S} \hat{H}_d \cdot \hat{H}_u + \frac{1}{3} \kappa \hat{S}^3, \quad (3)$$

where $\hat{H}_d \cdot \hat{H}_u = \epsilon_{ab} \hat{H}_d^a \hat{H}_u^b$ and $\epsilon_{ab}$ is the two dimensional Levi-Civita symbol with $\epsilon_{12} = 1$. The two parameters $\lambda$ and $\kappa$ of the superpotential will, by construction, affect the phenomenology of both the Higgs and chargino/neutralino sector. $\lambda$ in particular is crucial, not only it is necessary to induce the $\mu$ term but it also gives rise to mixing in the neutralino sector as well as in the Higgs sector between the Higgs doublets and the new singlet. For the purpose of parameter-counting and of the renormalisation of the Higgs sector at one-loop there is no need to go over the Yukawa superpotential which can be found in [22]. However, we do need to clearly specify again the soft SUSY breaking Lagrangian, in particular the part relating to the Higgs sector,

$$-\mathcal{L}_{soft, scalar} = m_H^2 |H_u|^2 + m_H^2 |H_d|^2 + m_S^2 |S|^2$$

$$+ (\lambda A_H H_u \cdot H_d S + \frac{1}{3} \kappa A_S S^3 + h.c.) \quad (4)$$

The first two terms in the first line represent the soft mass terms for the Higgs doublets and the third, not present in the MSSM, of the singlet. The second line, not present in the MSSM, represents the NMSSM trilinear Higgs couplings $A_k, A_\lambda$. $A_\lambda$ affects the mixing between the Higgs doublets and the singlet, beside the mixing introduced by $\lambda$. This parameter plays an important role in the phenomenology of the Higgs sector in the NMSSM, note that it gives rise to a Higgs tri-linear coupling $H_u H_d S$. No source of CP violation is assumed.

We are now in a position to write the Higgs potential whose parameters will need to be renormalised. With $g, g'$ being respectively the $SU(2)$ and $U(1)$ gauge couplings and specifying the components of the doublets,

$$H_d = \left( \begin{array}{c} H_d^0 \\ H_d^+ \end{array} \right), \quad H_u = \left( \begin{array}{c} H_u^0 \\ H_u^+ \end{array} \right)$$

the potential writes

$$V_{Higgs} = |\lambda (H_u^+ H_d^- - H_u^0 H_d^0) + \kappa S^2|^2 + (m_H^2 + |\lambda S|^2) (|H_u^0|^2 + |H_u^+|^2)$$

$$+ (m_H^2 + |\lambda S|^2) (|H_d^0|^2 + |H_d^+|^2) + \frac{g^2 + g'^2}{8} (|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^+|^2)^2$$

$$+ \frac{g^2}{2} |H_u^+ H_d^*|^2 + H_u^0 H_d^- S|^2 + m_S^2 |S|^2 + (\lambda A_H H_u^+ H_d^- - H_u^0 H_d^0) S + \frac{1}{3} \kappa A_S S^3 + h.c.). \quad (5)$$

The physical scalar fields consist of 3 neutral CP-even Higgs bosons, $h_0^0, h_0^0, h_0^0$, 2 CP-odd Higgs bosons, $A_1^0, A_2^0$ and a charged Higgs boson, $H^\pm$. The NMSSM contains of course the SM. In particular the SM gauge parameters $g, g'$ and $v^2 = v_u^2 + v_d^2$ are traded for the input parameters
\( e, M_W, M_Z \). From the Higgs potential, equation (5), it is clear that the Higgs sector of the NMSSM depends on the parameters:

\[
t_{ij}, \lambda, \kappa, \mu, A_\lambda, A_\kappa, m_{H_d}, m_{H_u}, m_S.
\]

The first four are also involved in the characterisation of the neutralino/chargino sector. Alternatively the last three soft Higgs masses can be traded for the tadpoles of the neutral Higgs which need to be constrained to zero to impose that the potential is at its minimum such that the independent parameters can be taken to be

\[
t_{ij}, \lambda, \kappa(m_c), \mu, A_\lambda, A_\kappa, (T_{H_d}, T_{H_u}, T_S).
\]

The latter are therefore considered as physical observables. The other six parameters are not unambiguously defined in a simple mapping to an observable. We will have a lot to say about the choice and definitions of the input parameters that will construct the set of these six parameters. This issue is directly related to the renormalisation scheme.

3. Renormalisation

In a nutshell, renormalisation consists in adding (subtracting) the ultra-violet infinite parameters and introducing wave function renormalisation for all fields. Then we need to impose conditions that amount to a definition of the parameters. The procedure consists in the following steps

- From \( G_p \) = \( t_{ij}, \lambda, \kappa, \mu, A_\lambda, A_\kappa, (M_{H_d}^2, M_{H_u}^2, M_S^2 \rightarrow t_{H_d}, t_{H_u}, t_S), M_1, M_2(\text{and} g, g', \xi, \eta, \epsilon, \zeta, \kappa, \rho, \sigma, \eta, \xi, \zeta, \kappa, \rho, \sigma) \)

- shift all (independent basic Lagrangian) parameters: \( G_p \rightarrow G_p + \delta G_p \)

- this means that mass mixing will appear: non diagonal transition \( A_0^0 Z^0, A_0^0 G^i, h_1 h_2, \ldots \) \((\delta m_{h_1 h_2}, \delta m_{h_1 h_2}^G)\) are generated and diagonal masses are shifted \((\delta m_{h_1}^2)\).

- No need to apply shifts to the diagonalising matrices \((S_h, U(\beta), \cdots)\), these are renormalised (no shift), same with gauge-fixing (not physical)

\[
\begin{pmatrix}
G^+ \\
H^+
\end{pmatrix}_0 = U(\beta) \begin{pmatrix}
h_d^+ \\
h_u^+
\end{pmatrix}_0 \quad \text{implies also} \quad \begin{pmatrix}
G^+ \\
H^+
\end{pmatrix}_0 = U(\beta) \begin{pmatrix}
h_d^+ \\
h_u^+
\end{pmatrix}_0.
\]

- In any case field renormalisation (before or after rotation) still needed

\[
\begin{pmatrix}
h_1^0 \\
\bar{h}_1^0
\end{pmatrix}_0 = Z_S \begin{pmatrix}
h_1^0 \\
\bar{h}_1^0
\end{pmatrix}_0, \quad \begin{pmatrix}
A_1^0 \\
A_2^0 \\
C^0
\end{pmatrix}_0 = Z_P \begin{pmatrix}
A_1^0 \\
A_2^0 \\
C^0
\end{pmatrix}_0, \quad \begin{pmatrix}
G^+ \\
H^+
\end{pmatrix}_0 = Z_G \begin{pmatrix}
G^+ \\
H^+
\end{pmatrix}_0 (Z_S)_{ij} = 1_{ij} + \delta Z_{h_1, h_2}/2
\]

- As an example, the renormalised self-energies of the CP-even Higgs scalars can be cast in the form

\[
\begin{align*}
\hat{\Sigma}_{h_1 h_2}^0 (p^2) &= \Sigma_{h_1 h_2}^0 (p^2) - \delta m_{h_1 h_2}^2 + (p^2 - m_{h_1}^2) \delta Z_{h_1} \\
\hat{\Sigma}_{h_1 h_2}^0 (p^2) &= \Sigma_{h_1 h_2}^0 (p^2) - \delta m_{h_1 h_2}^2 + \frac{1}{2} (p^2 - m_{h_1}^2) \delta Z_{h_1 h_2} + \frac{1}{2} (p^2 - m_{h_2}^2) \delta Z_{h_2 h_1}
\end{align*}
\]

We then require that \( i) \) Mixing vanishes between physical states when these are on-shell, \textit{essentially} (to solve for \( \delta Z_{ij} \)’s); \( ii) \) the residue at the pole (mass) of the propagator is 1. The other conditions are set by using/choosing a minimum/sufficient set of physical masses as input.
parameters except $\alpha_{em}$ which defined like in QED from the Thomson limit. In this approach only two point-functions are needed. The big question is which minimum set of parameters to choose?

Any choice will require solving a coupled system of counterterms, in other words setting the passage from the underlying Lagrangian parameters to, if possible, physical parameters such as masses. Leaving aside the SM parameter,

$$
\begin{pmatrix}
\delta \text{input}_1 \\
\vdots \\
\delta \text{input}_s
\end{pmatrix}
= \mathcal{P}_{8, \text{param.}}
\begin{pmatrix}
\delta M_1 \\
\delta M_2 \\
\delta \kappa \\
\delta \mu \\
\delta \lambda \\
\delta t \beta \\
\delta A^p \\
\delta A^s
\end{pmatrix}
+ \mathcal{R}_{n, \text{residual}},
\tag{9}
$$

$\mathcal{R}_{n, \text{residual}}$ counterterms such as gauge couplings, etc (SM). Inverting the system means that the corresponding determinant($\rightarrow 1/\text{Det}(\mathcal{P}_{m, \text{param.}})$) better be not too small otherwise the finite part of the counterterms could introduce unnecessarily large radiative corrections (sic!). It is best to break up the system, into smaller subsystem $\mathcal{P}_{n, \text{param.}} = \mathcal{P}_{m, \text{param.}} \oplus \mathcal{P}_{p, \text{param.}} \oplus \cdots$, $m + p + \cdots = n$. At each step one should avoid a choice such that Det($\mathcal{P}_{m, \text{param.}}$) $\rightarrow 0$. For example picking up a wino-like neutralino to reconstruct $M_1$ would be a bad choice if the mixing between the bino and the wino is small. Since $t_3$ is central, the easiest set-up is to define $t_3$ in $\overline{\text{DR}}$ with $t_3$ extracted independently from the Higgs sector (through wave function renormalisation condition) $\delta t_3 / t_3 = \left\{ 1/2 (\delta Z_{H_0} - \delta Z_{H_0}) \right\}_\infty$, $\infty$ means we only keep the infinite part. In this mixed scheme, $\mu, M_2$ are easily constrained from the charginos masses. Note however that though algebraically this conversion is straightforward, it is not sure that the assignation of $\mu, M_2$ to the correct chargino mass will be correct. Again a knowledge of which is which is essential. One needs more than the masses. We are then left with having to define $M_1, \kappa, (\lambda)$ from neutralino masses again with the caveat that we somehow know the composition. This may not be a big hurdle, since if the masses are to be measured, this means they have been produced. If we know in which proportion this could give a valuable hint on the nature of the particle. $A_1, A_s (\lambda)$ can only be extracted from the Higgs $(A, h, H^+)$. If this were possible with $t_3$ from $\overline{\text{DR}}$ then the break-up corresponds to

$$
\mathcal{P}_8 = \mathcal{P}_{1, t_3} \oplus \mathcal{P}_{2, \chi^0_{1, 2}} \oplus \mathcal{P}_{3, \lambda^0} \oplus \mathcal{P}_{12, A^0, A^0} \text{ or } \mathcal{P}_8 = \mathcal{P}_{1, t_3} \oplus \mathcal{P}_{2, \chi^0_{1, 2}} \oplus \mathcal{P}_{3, \lambda^0} \oplus \mathcal{P}_{2, A^0, A^0} \text{ or } \mathcal{P}_{3, A^0, A^0} (\lambda^0)
$$

A complete OS scheme, based on the input provided by 8 masses would look like $\mathcal{P}_{8, \chi^0_{1, 2}, A^0_{1, 2, 3}, H^+}$, $A^0_{1, 2, 3}$. We have used different variants which nonetheless all take the chargino masses as input. i)OS$^{ij}_{jk}$A$_{1, 2}$H$^+$ with the masses of 3 $\chi^0$ preferably $\tilde{b}, \tilde{h}, \tilde{s}$-dominated; ii)OS$^{ij}_{h, s}$A$_{1, 2}$H$^+$ (only 2 neutralinos) and iii)OS$^{ij}_{h, s}$A$_{1, 2}$H$^+$ (only 1 neutralino, $\tilde{b})$.

In renormalisable theories, most loop calculations involving 1-2- and 3-point functions give rise to ultraviolet infinities. The latter require a regularisation. In dimensional reduction/regularisation these infinities are contained in the quantity $\hat{C}_{UV} = 2/\epsilon - \gamma_E + \ln(4\pi/\mu^2) = 2/\epsilon - \gamma_E + \ln(4\pi) + \ln(1/\mu^2)$ where the continuation to $n = 4 - \epsilon$ dimension require the introduction of the compensating regularisation scale $\bar{\mu}$. The definition of an underlying parameter $p_i$ at one-loop, say OS scheme, based on a physical observable calls for the corresponding counterterm $\delta p_i / p_i = \beta_p (C_{UV} + \ln(Q_{p_i} / \bar{\mu})) (\beta_p = \partial p_i / \partial \ln 1/\bar{\mu})$. $Q_{p_i}$ is a scheme-dependent an effective scale, which depends on the point where the parameter has been
defined and other masses that may appear at one-loop in the definition of the parameter. $\beta_{p_i}$ is a universal, scheme independent factor. To track down the influence of the scheme on the correction to an observable, it is useful to know the parametric dependence of the observable on the parameter $p_i$’s, $\partial O/\partial p_i = \kappa_{p_i}$. In our so-called $\overline{\text{DR}}$ scheme we will only keep $\beta_{p_i} C_{UV}$, the “finite part” is set to 0. In On-shell (OS) scheme the “finite part” is $\beta_{p_i} \ln (Q_p/\mu)$. Then the full one-loop correction to the observable $\delta O/O = \Delta(C_{UV} + \ln(\Delta/\mu)) + \sum_i \kappa_{p_i} \delta p_i/p_i$ leads to

$$\delta O^{\text{OS}}/O = \sum_i \beta_{p_i} \kappa_{p_i} \ln(Q_p/Q_{\Delta})$$

$$\delta O^{\text{DR}}/O = \ln(\mu/Q_{\Delta}) \sum_i \beta_{p_i} \kappa_{p_i}.$$  

A good choice of $\mu$ or $Q_p$ will minimize the correction. These simple observations show that one can have large corrections when/if (i) $p_i$ is large, and/or (ii) $\kappa_{p_i}$ is large and/or (iii) when there is a large mismatch/difference between the scales $Q_{\Delta}$ and $Q_p$.

4. Applications

We have considered two scenarios. We will not, in this short write-up, give the full details of both models by listing all their defining underlying parameters. The salient differences is that point A is MSSM-like with a small $\lambda = 0.1$ while point B has $\lambda = 0.67$. In order for A to reproduce a standard model-like Higgs with mass 125GeV, the SUSY scale set by the mass of the stops as well as, more crucially, $A_t$ need to be large. A large $t_\beta$ helps also. For example for point A, $A_t/A_\lambda \sim 27$ while for point B $A_t/A_\lambda \sim 2.5$. While a large $A_t$ helps obtain a large correction for the Higgs mass it also provides a very large $\beta$ function for $A_\lambda$. The latter is induced by an enhanced top Yukawa coupling modulated by $A_t/A_\lambda$ as seen from the RGE’s. With $h_t$ the Yukawa coupling, the dominant contributions to the running of the underlying couplings of the NMSSM can be cast as[24]

$$\frac{1}{h_t^2} \frac{d h_t^2}{d \tau} = \frac{1}{A_t} \frac{d A_t}{d \tau} = \frac{2}{\lambda^2} \frac{d \lambda^2}{d \tau} = \frac{2}{\mu^2} \frac{d \mu^2}{d \tau} = 6 h_t^2 \frac{d h_t^2}{d \lambda} = \frac{1}{A_\lambda} \frac{d A_\lambda}{d \tau} = 3 h_t^2 \frac{A_t}{A_\lambda},$$

with $\tau = \ln(\mu^2/16\pi^2)$. Numerically and with no approximation we find for the beta functions expressed in units of $10^{-3}$,

Point A: $\beta_\mu = -11.40, \beta_{t_\beta} = 16.9, \beta_\lambda = -11.65, \beta_{A_\lambda} = -0.76, \beta_{A_\lambda} = -109.4$ and

Point B: $\beta_\mu = -14.25, \beta_{t_\beta} = 17.63, \beta_\lambda = -20.45, \beta_{A_\lambda} = -18.57, \beta_{A_\lambda} = -122.7$

It is also useful to list the finite parts of the counterterms in some of the schemes for both points A and B.

**Point A** The finite parts computed at $\mu = Q_{\text{susy}} = 1117.25$GeV give

$$\frac{t_{134A_1A_2}}{t_{134A_1A_2}}^{\text{OS}_{\text{34th}A_1A_2H^+}}$$

$$(\delta \mu/\mu, \delta t_{t_\beta}/t_\beta, \delta \lambda/\lambda)_{\text{finite}} = (-2.42\%, 0, 62.26\%); (-1.57\%, -80.69\%,-7.88\%)$$

$$\frac{t_{134A_1A_2}}{t_{134A_1A_2}}^{\text{OS}_{\text{44th}A_1A_2H^+}}$$

$$(\delta \kappa/\kappa, \delta A_\lambda/\lambda, \delta A_\kappa)_{\text{finite}} = (64.01\%, -5.49\% , 0.66); (-6.01\%, 134\%, 0.66).$$

**Point B** Here the finite parts” computed at $\bar{\mu} = Q_{\text{susy}} = 753.55$GeV

$$\frac{t_{123A_1A_2}}{t_{123A_1A_2}}^{\text{OS}_{\text{12th}A_1A_2H^+}}$$

$$(\delta \mu/\mu, \delta t_{t_\beta}/t_\beta, \delta \lambda/\lambda)_{\text{finite}} = (-1.04\%, 0, 3.71\%); (-1.63\%, 6.49\%, 5.94\%)$$

$$\frac{t_{123A_1A_2}}{t_{123A_1A_2}}^{\text{OS}_{\text{2th}A_1A_2H^+}}$$

$$(\delta \kappa/\kappa, \delta A_\lambda/\lambda, \delta A_\kappa)_{\text{finite}} = (3.25\%, 6.85\%, 10.84); (6.05\%, 3.40\%, 11.54).$$
All the schemes $t$ where $t_\beta$ is defined as $\overline{\text{DR}}$ and the other parameters are taken as $\text{OS}$ take the chargino masses as inputs. The other parameters are explicit if one note that $1, 2, 3$ stand for the neutralinos ordered by increasing mass. The $\beta$ functions as well as the finite parts of the counterterms give an insight on the value of the full one-loop corrections and more importantly on the differences between the results of the schemes. The parametric dependence is weighed by perturbing slightly around the values of the model. Fig. 2 shows that for Point A the dependence

![Figure 2](image_url)

**Figure 2.** Dependence on $A_\lambda$ of the square of the couplings, $G_{ijk}$, shown in the insert of the figures for Point A (left panel) and Point B (right panel). $G_{ijk}$ represents the coupling constant of the interaction between $ijk$. The variations shown do not take into account the changes in the decay due to phase space that can also change because some of the masses involved in the decay change when the underlying parameters change. $h_i$ stand for the CP-even Higgses. $O$ are for the neutralinos and $C$ the charginos.

one some of the underlying parameters for some of the chosen decays are practically flat or linear in contrast to Point B where the dependence is quite strong. For point B a small change in $A_\lambda$ can trigger a large difference with a highly non linear variation.

We have computed the full one-loop electroweak corrections to various Higgs decays either to other Higgses or to Higgs gauge bosons or to a pair of neutralinos/charginos. Table 1 is for Point A. To understand the scheme dependence of the majority of the most important decays of the different Higgses, it is important to keep in mind the singlet nature of the states involved. Recall that this scenario is characterised by very small mixing whereby the Higgs states $h_0^0, A_1^0$ and the neutralino $\chi^0_3$ are essentially singlet states. There are three classes of decays, those among the predominantly singlets ($h_0^0 \rightarrow A_1^0 A_1^0$), those not involving any of these states ($A_2^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$, $h_2^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ and $A_2^0 \rightarrow \chi^0_1 \chi^0_1$) with characteristics close to the MSSM and with very little dependence on $\lambda$. The last class involves both singlet and non singlet states. The latter decays are therefore sensitive to the mixing parameters due to the addition of the singlet in the NMSSM. For most decays this mixing parameter is essentially $\lambda$ while for $h_3 h_2 h_1$ $A_\lambda$ is crucial. These observations together with the numerical values of the $\beta$ functions and the values of the finite terms of the counterterms can explain the differences. The mixed scheme $t_{134A_1A_2}$ performs quite badly for almost all decays that involve singlet/non singlet states. The latter are very sensitive to $\lambda$ which is badly reconstructed in this scheme. This also explains the almost equal value of the correction in the many channels, $\sim 120\%$ which is mainly accounted for by
the finite part of $2 \delta \lambda$ in this scheme. The decay $h_0^0 \to h_1^0 h_2^0$ (and to some extent $A_0^0 \to Z h_2^0$) stands apart. It is the only channel where there is a large running, as testified by the comparison between the two $\overline{\text{DR}}$ schemes. In fact, this coupling depends on $A_\lambda$ whose running is quite well approximated by the large value of $\beta_{A_\lambda}$.

For point B, the value of $\lambda$ is large enough, this is reflected also in the fact the partial widths are for all channels an order of magnitude larger than for point A. A discussion in terms of almost pure or singlet states is not appropriate. For this more genuine NMSSM, the OS scheme performs quite well. It rests that the scheme and scale dependence for $h_0^0 \to h_1^0 h_2^0$ is quite large. This is due not so much to $\beta_{A_\lambda}$ but rather to the strong parametric dependence on $A_\lambda$ as seen in Fig. 2.

### 5. Summary and conclusions

Full renormalisation (all sectors) of the NMSSM at one-loop is now completed and implemented in SloopS. The set-up allows to choose between different on-shell schemes and also “mixed schemes” with one or several parameters through a $\overline{\text{DR}}$ subtraction. The on-shell schemes are at the moment all based on the usage of physical masses and hence the use of two-point functions in the definition of the parameters. Especially in scenarios where the singlet mixing parameter is small, these schemes are not optimal in extracting this crucial mixing parameter nor in extracting $t_\beta$. We must go beyond taking only masses as inputs. When new particles are discovered, not only their masses will be measured but the way they are produced, the strengths of their production and decays offer an important handle that may not need the reconstruction of the whole spectrum. This is technically much more challenging, but it is possible (at least in some manifestations). In the MSSM $A^0 \to \tau \bar{\tau}$ was shown to be an excellent input for $t_\beta$[19].

I would like to end with a personal note. Last time I saw Shimizu-sensei was in February 2015. The last dinner I had with him was in a Fugu restaurant. As always with him, un

---

**Table 1. Point A.** Corrections to the partial decay widths of Higgs bosons in other Higgs bosons and/or neutralinos, charginos and gauge bosons. The column TL represents the tree-level (in units of GeV/100) partial width. The other columns give the percentage relative full one-loop correction, in the mixed scheme where only $t_\beta$ is taken $\overline{\text{DR}}$ ($t_{134A_1A_2}$), the full OS scheme and full $\overline{\text{DR}}$. For $t_{134A_1A_2}$ and $\overline{\text{DR}}$ the scale $\tilde{\mu}$ is taken at the mass of the decaying Higgs, for $\overline{\text{DR}} Q_{\text{SUSY}}$ the scale at the SUSY scale.

| Decays | TL | $t_{134A_1A_2}$ | OS$_{34b2A_1A_2}h^+H^+$ | DR | DR| $Q_{\text{SUSY}}$ |
|--------|----|-----------------|------------------------|----|----|----------------|
| $h_2^0 \to A_1^0 A_1^0$ | $4.79 \times 10^{-2}$ | 128% | -12% | 0.4% | -0.4% |
| $h_0^0 \to h_1^0 h_2^0$ | 2.21 | 116% | 79% | 52% | -1.7% |
| $h_2^0 \to \tilde{\chi}_1^{0+}$ | 3.52 | 122% | -3% | 2% | 0.3% |
| $h_2^0 \to \tilde{\chi}_1^{0}$ | 3.38 | 126% | -35% | 3% | 1.1% |
| $h_2^0 \to \tilde{\chi}_1^{0+}$ | 4.55 | 1% | -11% | -9% | -7.4% |
| $A_2^0 \to Zh_2^0$ | 1.86 | 120% | 80% | -56% | -14.5% |
| $A_2^0 \to \tilde{\chi}_{1+}^{0+}$ | 3.30 | 28% | 13% | 0.3% | -1.6% |
| $A_2^0 \to \tilde{\chi}_{1+}^{0+}$ | 2.44 | 130% | -31% | 8% | 6.2% |
| $A_2^0 \to \tilde{\chi}_{1+}^{0+}$ | 3.02 | 122% | -5% | -0.4% | -1.9% |
| $A_2^0 \to \tilde{\chi}_{1+}^{0+}$ | 5.51 | -10% | -1.5% | -6% | -8% |
| $H^+ \to W^+ h_2^0$ | 2.01 | 119% | 79% | -56% | -16% |
| $H^+ \to \tilde{\chi}_{1+}^{0+}$ | 6.40 | 125% | -18% | 3% | 1.1% |
Table 2. Point B. Corrections to the partial decay widths of Higgs bosons in other Higgs bosons and/or neutralinos, charginos and gauge bosons. TL represents the partial decay widths (in GeV). The percentage relative full one-loop corrections are given as percentage relative corrections in the OS scheme, namely, $\text{OS}_{12h_2A_1A_2H^+}$ as well as a full $\text{DR}$ at the SUSY scale and the mass of the decaying Higgs.

| TL | OS | DR| $\text{DR}_{\text{Q SUSY}}$ |
|----|----|---|------------------------------|
| $h_0^0 \rightarrow \chi_1^0\chi_2^0$ | 0.727 | 14% | 5% | 3% |
| $h_0^0 \rightarrow A_1^0Z$ | 0.613 | 3% | -3% | -8% |
| $h_0^0 \rightarrow h_1^0h_1^0$ | 0.341 | -25% | -106% | -50% |
| $h_0^0 \rightarrow h_2^0h_2^0$ | 0.514 | 6% | 13% | -28% |
| $A_2^0 \rightarrow \chi_1^0\chi_3^0$ | 1.523 | 7% | 2% | 1% |
| $A_2^0 \rightarrow \chi_1^0\chi_1^0$ | 0.723 | 32% | 2% | 2% |
| $A_2^0 \rightarrow Zh_0^0$ | 0.638 | 12% | -16% | -9% |
| $A_2^0 \rightarrow A_1^0h_1^0$ | 0.415 | -0.3% | -32% | -17% |
| $H^+ \rightarrow \chi_1^+\chi_2^0$ | 1.056 | 6% | 10% | 8% |
| $H^+ \rightarrow W^+h_1^0$ | 0.609 | 11% | -18% | -10% |
| $H^+ \rightarrow W^+A_1^0$ | 0.603 | 2% | -3% | -9% |
| $H^+ \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^0$ | 0.561 | 21% | 9% | 9% |

fin gourmet, the meal was excellent as were all the subtleties about the dishes. The next morning, in an email, he told me he enjoyed having dinner with me (and with Kato-san and Kuroda-san) but he thought it was the last time we saw each other. I did not believe it, but he was right. Back to France, we exchanged regularly. Most of exchanges were about food and pottery. His celadon gifts to me are priceless. We did discuss physics and of all the exchanges the one which moved me the most was about the visit of Nakawaza-san to the hospital when we told me again about the 2-loop electroweak calculations project.

So much about my 2015 trip to Japan. The last trip of Shimizu to France, Paris and Annecy, was wonderful for my family and myself. I was told by a dear common friend that his dream and one of his wishes had been to take his dearest friends to see Paris. These friends are always welcome to France and I very much hope you will visit. And when you come we will go to Minami...in Annecy! Minami is a new, small, genuine Japanese restaurant that opened about 4 or 5 years ago. How wonderful that I can get back to the days of Minami-Tateya by walking 2 minutes from my home and step in Minami like Shimizu did in January 2013.
Acknowledgments

I first of all would like to thank Shimizu-sensei for the intense scientific collaboration I enjoyed with him and the Minami-Tateya Group. On a personal level he has helped me go through some very difficult times and shared with me some wonderful moments, not to mention introducing me to so many subtleties of Japanese culture. I would also like to thank all those who have been by his side during the last months, days and hours and took much of their time and energy to comfort him and who were kind enough to report to me from time to time. I also thank the large circle of Minami-Tateya for making this gathering, the trip to Hakone and the rest place of Shimizu-sensei possible and enjoyable. I can not be thankful enough for allowing me to meet some of Shimizu’s friends I had not known before and with whom I promise to keep in touch!

Thank you!

References

[1] Dombey N and Boudjema F 1990 NATO Sci. Ser. B 233 pp.1–578
[2] Fujimoto J, Igarashi M and Shimizu Y 1987 Prog. Theor. Phys. 77 118
[3] Boudjema F et al. 1996 Standard model processes Workshop on Physics at LEP2, v.1, G. Altarelli (ed.), S. Torbijn (ed.) and F. Zwirner (ed.) pp 207–248 [207(1996)] (Preprint hep-ph/9601224) URL https://inspirehep.net/record/415134/files/arXiv:hep-ph_9601224.pdf
[4] Boudjema F and Chopin E 1996 Z. Phys. C73 85–110 (Preprint hep-ph/9507396)
[5] Belanger G, Boudjema F, Fujimoto J, Ishikawa T, Kaneko T, Kato K, Lafage V, Nakazawa N and Shimizu Y 1999 Implementation of the nonlinear gauge into GRACE 6th International Workshop on New Computing Techniques in Physics Research: Software Engineering, Artificial Intelligence Neural Nets, Genetic Algorithms, Symbolic Algebra, Automatic Calculation (AIHENP 99) Heraklion, Crete, Greece, April 12-16, 1999 (Preprint hep-ph/9907406) URL http://alice.cern.ch/format/showfull?sysnb=0320104
[6] Belanger G, Boudjema F, Fujimoto J, Ishikawa T, Kaneko T, Kato K and Shimizu Y 2003 Nucl. Phys. Proc. Suppl. 116 353–357 [.353(2002)] (Preprint hep-ph/0211268)
[7] Belanger G, Boudjema F, Fujimoto J, Ishikawa T, Kaneko T, Kato K and Shimizu Y 2003 Phys. Lett. B559 252–262 (Preprint hep-ph/0212261)
[8] Belanger G, Boudjema F, Fujimoto J, Ishikawa T, Kaneko T, Kato K, Shimizu Y and Yasui Y 2003 Phys. Lett. B571 163–172 (Preprint hep-ph/0307029)
[9] Belanger G, Boudjema F, Fujimoto J, Ishikawa T, Kaneko T, Kurihara Y, Kato K and Shimizu Y 2003 Phys. Lett. B576 152–164 (Preprint hep-ph/0309010)
[10] Boudjema F, Fujimoto J, Ishikawa T, Kaneko T, Kato K, Kurihara Y, Shimizu Y, Yamashita S and Yasui Y 2004 Nucl. Instrum. Meth. A534 334–338 (Preprint hep-ph/0404098)
[11] Boudjema F, Fujimoto J, Ishikawa T, Kaneko T, Kato K, Kurihara Y, Shimizu Y, Yamashita S and Yasui Y 2004 Nucl. Phys. Proc. Suppl. 135 323–327 [.323(2004)] (Preprint hep-ph/0407079)
[12] Boudjema F, Fujimoto J, Ishikawa T, Kato K, Kurihara Y, Shimizu Y, Kotera K and Yasui Y 2005 eConf C050318 0601 (Preprint hep-ph/0510184)
[13] Kato K, Boudjema F, Fujimoto J, Ishikawa T, Kaneko T, Kurihara Y, Shimizu Y and Yasui Y 2006 PoS HEP2005 312
[14] Belanger G, Boudjema F, Fujimoto J, Ishikawa T, Kaneko T, Kato K and Shimizu Y 2006 Phys. Rept. 430 117–209 (Preprint hep-ph/0308080)
[15] Semenov A 2009 Comput. Phys. Commun. 180 431–454 (Preprint 0805.0555)
[16] Hahn T 2000 Nucl. Phys. Proc. Suppl. 89 231–236 (Preprint hep-ph/0005029)
[17] Baro N, Boudjema F and Semenov A 2008 Phys. Lett. B660 550–560 (Preprint 0710.1921)
[18] Baro N, Boudjema F and Semenov A 2008 Phys. Rev. D78 115003 (Preprint 0807.4668)
[19] Baro N and Boudjema F 2009 Phys.Rev. D80 076010 (Preprint 0906.1668)
[20] Baro N, Boudjema F, Chalons G and Hao S 2010 Phys. Rev. D81 015005 (Preprint 0910.3293)
[21] Belanger G, Bizouard V, Boudjema F and Chalons G 2016 Phys. Rev. D93 115031 (Preprint 1602.05495)
[22] Bizouard V 2015 Precision calculations in the Next-to-Minimal Supersymmetric Standard Model Theses Universite Grenoble Alpes (Preprint https://hal.archives-ouvertes.fr/tel-01447488)
[23] Ellwanger U, Hugonie C and Teixeira A M 2010 Phys. Rept. 496 1–77 (Preprint 0910.1785)