An Understanding of The Dark Matter in The Universe And The Variation of The Universal Gravitational Contant G With Time

D.N. Tripathy and Subodha Mishra

Institute of Physics, Sachivalaya Marg, Bhubaneswar-751005, INDIA

Abstract

Considering the fact that the present universe might have been formed out of a system of ficticious self-gravitating particles, fermionic in nature, each of mass \( m \), we are able to obtain a compact expression for the radius \( R_0 \) of the universe by using a model density distribution \( \rho(r) \) for the particles which is singular at the origin. This singularity in \( \rho(r) \) can be considered to be consistent with the so-called Big Bang theory of the universe. By assuming that Mach’s principle holds good in the evolution of the universe, we determine the number of particles, \( N \), of the universe and its \( R_0 \), which are obtained in terms of the mass \( m \) of the constituent particles and the Universal Gravitational constant \( G \) only. It is seen that for a mass of the constituent particles \( m \simeq 1.07 \times 10^{-35} \text{g} \) the age of the present universe, \( \tau_0 \), becomes \( \tau_0 \simeq 20 \times 10^9 \text{yr} \), or equivalently \( R_0 \simeq 1.9 \times 10^{28} \text{cm} \).

For this \( m \), the total number of particles constituting the present universe is found to be \( N \simeq 2.4 \times 10^{91} \) and its total mass \( (M \simeq 1.27916 \times 10^{23} M_\odot) \), \( M_\odot \) being the solar mass. All these numbers seem to be quantitatively agreeing with those evaluated from other theories. Using the present theory, we have also made an estimation of the variation of the universal gravitational constant \( G \) with time which gives \((\dot{G}/G) = -9.6 \times 10^{-11} \text{yr}^{-1}\). This is again in extremely good agreement with the results of some of the most recent calculations. Lastly, a plausible explanation for the Dark Matter present in today’s universe is given. Assuming neutrinoes to be one of the most possible candidate for the Dark Matter, we have estimated the ratio of the number of neutrinos to nucleons.
and the number of neutrinos per unit volume of the universe, which respectively gives 
\( \frac{N_\nu}{N_n} \sim 3 \times 10^9 \), \( \frac{N_\nu}{V} \sim 500 \). Both these numbers seem to be in agreement with the 
findings of the recent observations. The present calculation gives a mass for the neutrino 
to be \( m_\nu \sim 1.7 \times 10^{-32} g \) or equivalently, 8eV, which is the right order of magnitude, as 
speculated by several workers.
I. Introduction

The universe, as we know till today, consists of as many as $10^{11}$ galaxies. These galaxies are of course complicated structures, bound gravitationally, each consisting of upto $10^{11}$ stars as well as gas clouds. Each star is a nuclear power-house, and our sun is a typical star, situated about halfway out towards the edge of the disc of our galaxy called the Milky way. The Earth is one of the few planets bound to the sun through gravitational forces. There may well be other form of matter in the universe besides the galaxies we see. For instance, the galaxies that have ceased to radiate, black holes of all sizes and intergalactic dust and gas. However, firm experimental evidence for these is lacking.

An understanding of the universe has not only remained as a fascinating problem for scientists since a longtime, but also its birth has been a challenging problem for them. There is no satisfactory theory yet relating to the evolution of the universe. The evolution of the universe from nothing is what is known as the Big Bang Theory [1]. According to this theory, the universe started with a huge explosion from a superdense and a superhot stage. Mathematically, a superdense state corresponds to imagining of a singularity in the density distribution function at the origin of the co-ordinate system. The very concept of having an infinite density at the instant of the Big Bang is not to be taken too seriously. Because, as one looks backward in time with density going up and up, one is led far away from the condition under which the laws of physics, as we know them, were developed. Thus, it is quite likely that at some point, these laws become totally invalid and it is a matter of guess work to discuss what happened at still earlier times. Nonetheless, as the history of the universe is extrapolated, the density increases without limit for as long as the known laws of physics apply. Indeed, most cosmologists today are reasonably confident about our understanding of the history of the universe.
back to one micro second \( (10^{-6}\ \text{sec}) \) after the Big Bang. The goal of the cosmological research involving Grand Unified Theories (GUTs) \[2\] is to solidify our understanding back to \( 10^{-35} \) secs after the Big Bang. Having discussed the key features of the Big Bang, it is natural to ask now what evidence can be found to support it. Before going to discuss about this, one might mention here about the expansion of the physical universe which is being considered to be one of the greatest discoveries of the century. The concept of an expanding universe was gradually found to be accepted by means of a famous law known as Hubble’s law \[3\] which states that each distant galaxy is receding from us with a velocity \( v \) which, to a high degree of accuracy, is proportional to its distance \( d \). That is,

\[
v = H_0 d,
\]

where \( H_0 \) is known as the Hubble constant, which is considered to be a constant presumably because, it remains approximately constant over the lifetime of an astronomer. However, value of \( H_0 \) changes as the universe evolves. Experimental evidence in favour of the expansion of the universe follows from the fact that light rays received from distant galaxies are red shifted. There is, of course, a great uncertainty in the measurement of the value of \( H_0 \). The inaccuracy is mostly due to the great difficulties in calculating the distances to the galaxies. However, as regards the redshifts are concerned, they can be determined very accurately from the Doppler shift of the spectral lines of the light rays coming from distant galaxies. An accurate measurement of \( H_0 \) has important consequences. In the context of Big Bang model, an erroneously high value for \( H_0 \) (the expansion rate) implies an erroneously low value for the age of the Universe. Leaving aside the fact that \( H_0 \) is not very accurately known, there are certain compelling reasons to suggest that the age of the universe should be around 20 billion years \[2\]. There is also an uncertainty in accurately measuring the mass density of the universe, which is
considered to be an important quantity in calculating the history of the universe. Because, it determines how fast does the cosmic expansion slow down under the influence of gravity. Having said so much regarding the expansion of the Universe, we are going to cite some of the evidences in support of the Big Bang theory. There are two significant observational evidences in favour of the Big Bang theory. The first one is the observation of the cosmic background radiation. The first observation of the cosmic background radiation in the microwave region by Penzias and Wilson [4] was the most important cosmological discovery since Hubble established the expansion of the universe. As we all know, all hot matter emits a glow, just like a glow of hot coals in a fire. Thus, an early universe would have been permeated by a glow of light emitted by the hot matter. As the universe expanded, the light would have redshifted. Besides, it also got cooled. Today, the universe would still be bathed by the radiation, a remnant of the intense heat of the Big Bang, now redshifted into the microwave part of the spectrum. This prediction was confirmed when Penzias and Wilson [4] discovered a background of microwave radiation with an effective temperature of \( \approx 2.70 K \). The characteristic of this radiation is that it is almost absolutely isotropic, meaning thereby that this radiation is not due to stars or galaxies. The only plausible explanation for the origin of the radiation is that the universe had perhaps passed through a state of very high density and high temperature in its early stage. The second important piece of evidence supporting the Big Bang theory is related to the calculation of what is called Big Bang nucleo-synthesis, which deals with the calculation of the rates of different nuclear reactions that took place in the early stage of the universe. Since it is assumed that the early universe was very hot, it is not possible to have stable nuclei formed at that stage. At about 2 minutes after the Big Bang, there were virtually no nuclei at all, because the temperature of the universe was then very high lying between \( 10^9 \sim 10^{10} K \). At that state, the universe was filled
with hot gas of photons, neutrinos with a much smaller density of protons, neutrons and electrons. As the universe cooled, protons and neutrons began to coalesce to form nuclei. From the nuclear reaction rates, one can calculate the expected abundances of the different kinds of nuclei that would have been formed. One finds that most of the matter in the universe would remain in the form of hydrogen. About 25% of the mass of the matter would have been converted to helium and trace amount of other nuclei would also have been produced. Most of the types of nuclei that we observe in the universe today were produced much later in the history of the universe, in the interior of stars and in supernova explosions. Hence, the lightest nuclei were produced primarily in the Big Bang and it is possible to compare the calculated abundances of these with direct observations. These calculations, depend on the density of protons and neutrons in the universe, a quantity which can be estimated only roughly by astronomical observations. It is interesting to see that there is a broad range of values for the density of protons and neutrons for which the calculated values for the abundances of light nuclei are in excellent agreement with the observed values [2]. If there was never a Big Bang, there would be no reason whatsoever to expect that helium-4 would be $10^8$ times as abundant as lithium-7. It might just as well have been the other way around. But, when calculated in the context of the Big Bang theory, the ratio works out just right.

So far we have been largely concerned with the observational informations about the universe such as the expansion of the universe, the cosmic background radiation and the primordial abundance of the elements. We now face the obvious question like whether the universe will go on expanding forever or will it eventually fall back onto itself? The answer to this question that is, the eventual fate of the universe depends on how much matter it contains on its present size and how fast it is expanding. The more matter there is, the greater are the gravitational forces holding the universe together and more
likely it is that the universe is finite and will eventually collapse back onto itself. The greater the present size of the universe, the farther are the galaxies and the rest of the matter are separated from each other, and the weaker is the gravitational attraction.

This gives rise to an increase of the likelihood that the universe is infinite and will expand forever. Finally, the greater the velocity of expansion, the harder it is for gravity to slow the expansion to zero and reverse it. Again this contributes towards the universe being infinite and ever expanding. Thus, one has to determine the relative contributions of each of the three factors such as the cosmic mass, size and expansion velocity to the evolution of the universe, and then to use them to decide, on theoretical grounds, what the density of the universe will be.

The best value of the Hubble constant known at present is \( H_0 \approx 100h_0\text{km}\text{s}^{-1}\text{Mpc}^{-1} \), \( 0.5 \leq h_0 \leq 1 \) [5]. For this rate of expansion, Einstein’s General Theory of Relativity predicts that the universe is finite and closed [6] if the average density of matter \( \rho_0 \) in the Universe is greater than \( 5 \times 10^{-30} \text{g/cm}^3 \), it is infinite and open if the average density is less than this value; and it is infinite and flat if the average density is equal to it, where the quantity \( 5 \times 10^{-30} \text{g/cm}^3 \) is called the critical density, \( \rho_c \), of the universe and this is related to the Hubble constant \( H_0 \) as \( \rho_c = \left( \frac{3H_0^2}{4\pi G} \right) q \), [1], where \( q \) is the deceleration parameter.

The measurement of \( \rho_0 \) is an extremely difficult task, because the universe consists of all sorts of matter and all of them contribute to \( \rho_0 \). Considering the fact that stars form into galaxies, galaxies into clusters, and clusters into rich super clusters, the average density of matter in the universe is to be obtained by adding up the masses of galaxies of superclusters in our vicinity of the universe and dividing it by the corresponding volume. In other words, for some fixed regions of the sky, one counts the number of galaxies and from that the galaxies per volume, \( n_G \), is calculated. Determination of the mean mass of a galaxy, \( M_G \), is done following the method as used for the sun. For example, it is
known that the mass of the sun is measured by using the properties associated with the motion of the planets around the sun. Like the planets in the solar system, the stars in many galaxies revolve around the centre of the galaxy. Hence, once we know the rotating velocity of some of the stars and their distance from the galactic centre, the mean mass of a galaxy, $M_G$, can be determined. Here it is assumed that the mass of a galaxy is concentrated at its centre. The average density of matter in the universe determined by just adding the masses of all the galaxies in a large volume of space of the sky is found to be $\approx 3 \times 10^{-31} g/cm^3$ [7]. This is, what is called the density of the observed (luminous) matter in the universe, which is about 20 times too low to account for a closed universe.

The above value for the average density of the universe has been obtained by assuming that cosmic matter is mainly concentrated in galaxies. This need not be the case, because, the space between galaxies is not a vacuum, but contains gas or extinguished stars. Looking at Zwicky’s work [8], one also finds that the measurement of the mass of the cluster of galaxies can be done in two ways. One way is via, the luminosity and the other method is based on dynamics, that is, by measuring the relative velocities among the galaxies. Since there is a definite relation between the luminosity of a galaxy and its mass, one can deduce the mass of the galaxy from luminosity measurements. Then by adding the masses of the member galaxies, one obtains the total mass of the cluster. In the method based on measuring the relative velocities among the galaxies, the mean relative velocity of the whole cluster is determined by its mass. Turning the problem around, one can obtain the mass of the cluster from the velocity measurements. It has been seen that the mass found by these two methods differ greatly. For example, the dynamical masses of the coma cluster [8] is found to be 400 times the luminosity mass. This result can be interpreted by the fact that the main mass of the coma cluster is not contributed by the visible galaxies, but by a large amount of invisible matter within the
cluster. The mass measured from the luminosities includes the mass in the light emitting regions and does not include the mass of any matter that exists in regions not emitting light. This is what has been identified as the Dark Matter or the Missing Matter. The question is, just how much Dark Matter is there in the universe and this, when added to the visible mass, can it give rise to an average density exceeding the critical density $\rho_c$? That would decide whether the universe should be finite, closed and eventually collapsing. However, at present, the estimate of the average density of the universe is too uncertain to allow to draw a definite conclusion. Today, it is mostly accepted that the mass of the universe is mainly contributed by Dark Matter and more than 90 percent of the matter in the universe is invisible. There is some evidence [9] and it has been indeed the current fashion to believe that the Dark Matter is not the ordinary matter made of protons, neutrons and electrons, but rather some new stable exotic neutral elementary particles. Although we know a great deal about the behaviour of the dark matter we have no observational clues as to what it may be. Considerable ingenuity is being expended now on devices which will detect the Dark Matter directly if the particles interact with ordinary matter through weak nuclear force, as is expected for some candidates. In order to say something about the most probable candidate for the Dark Matter we introduce a parameter $\Omega$ which is defined as the ratio of current density $\rho_0$ of the Universe to the critical density $\rho_c$, $\Omega = (\rho_0/\rho_c)$. If $\Omega < 0.2$, we imagine of a universe in which the Dark Matter is of the normal kind, that is the types that make up galaxies, star and atoms etc. This is what is termed as baryonic. The limits imposed by nucleogenesis[2] tells that the Dark Matter cannot be baryonic. Also if $\Omega \approx 1$, then the Dark Matter may not be baryonic. Non-baryonic forms of matter that have been suggested include neutrinos, gravitinos, photinos, sneutrinos, axions, and magnetic monopoles etc. Out of these, neutrinos are considered to be one of the first appealing candidates for the Dark
Matter. Neutrinos are known to exist and they are usually assumed to have zero rest mass. The universe is assumed to contain a vast number of neutrinos. If a neutrino has a tiny rest mass, it can far exceed the mass of the baryonic component, and thus, becoming the dominant component of the cosmic mass. It is the rest mass of the neutrinos, however small that may be, that makes the universe finite and closed. A recent study by Schramm and Steigman [10] seems to suggest that the mass for the neutrino could be between $4$ to $20 \text{ eV}/c^2$. Further, if neutrinos have non-zero rest masses, then it is possible that there are neutrino oscillations [1] between the various types of neutrinos. These neutrinos do not participate in electromagnetic interactions. Since they interact very weakly with matter, they cannot be detected in today’s laboratory experiments. Such features match with the properties expected for Dark Matter.

Considering the fact that the major constituents of the present universe are the neutrinos, which might have been formed out of some fictitious particles of certain mass that had filled the early universe at a time some two minutes after the Big Bang, we, in this paper, have tried to calculate the mass, mean density and radius of the universe by treating the universe just as a system made out of these particles which are self gravitating.. The present calculation is based on making an intuitive choice for the distribution of particles in the universe, characterized by a distribution function having a singularity at the origin. Such a form of single particle density seems to be consistent with the concept like the Big Bang theory of the Universe. The present calculation is the result of our earlier study [11] where such a form of singular density distribution has been used to calculate the binding energy of a system of self gravitating particles like the neutron stars and white dwarfs etc. having no source for nuclear power generation present at their cores. In all these cases our theory has proved to be of a great success due to the fact that it has correctly reproduced the results for the binding energies and
the radii of the neutron stars etc, that agree with those obtained by other workers. In this work, we have also been able to make an estimate of the critical mass of a neutron star beyond which the black hole formation should take place. In the present work, we assume the fictitious particles to be fermionic in nature, carrying a tiny mass \( m \), which interact among themselves through gravitational forces. Considering the fact that the present age of the universe is about 20 billion years \([2]\), we have been able to make an estimate for the radius of the universe by adjusting the mass of these fictitious particles. The age of the Universe is determined using the fact that the universe has been expanding with a speed roughly equal to the velocity of light \( 'c' \). With this, we not only arrive of a value of \( 10^{28} \) cm for the radius of the universe but also we obtain a value of its mass of about \( 10^{23} M_\odot \), \( M_\odot \) being the solar mass \((M_\odot = 2 \times 10^{33} g)\). All these numbers seem to be matching very nicely with the corresponding results known from other theories. Our estimated result for the average mass density of the present universe comes out to be \( \approx 10^{-29} g/cm^3 \), which is supposed to be the case as per expectation. In Sec.II of the paper the mathematical formalism for the present calculation is presented. In Sec.III all the results of the present theory have been derived. In Sec.IV the implication of having a large value for the average density of the universe is being discussed. Our calculated value for the mass of the particles responsible for the formation of the Dark Matter seems to be agreeing with the mass of neutrinos as speculated by several other workers \([1]\). In this section, we have also presented our estimated values for the ratio of the variation of the universal gravitational constant \( G \) with time to \( G \) itself, that is \((\frac{\Delta G}{G})\). This is again found to be in extremely good agreement with those obtained from some of the most recent calculations \([5]\).

II. Mathematical Derivation for the Radius of the Universe

If we visualize the present universe to be a system composed of some self grav-
itating fictitious particle, each of mass $m$, interacting through pair-wise gravitational interactions, then the Hamiltonian of the system can be written as

$$H = -\sum_{i=1}^{N} \left( \frac{\hbar^2}{2m} \right) \nabla_i^2 + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} v(|\vec{X}_i - \vec{X}_j|),$$  \hspace{1cm} (1)

where $i$ is not equal to $j$ and $v(|\vec{X}_i - \vec{X}_j|) = -\frac{g^2}{|\vec{X}_i - \vec{X}_j|}$, having $g^2 = Gm^2$, $G$ being Newton’s universal gravitational constant. The assumption that goes in while writing (1) is that there is no radiation source present at the centre of the universe. The particles which constitute the universe are some kind of fermions whose mass is estimated by using certain known results that are considered to be approximately true for the universe. As the universe evolves with time, it is these ficticious particles that might have, latter on, given rise to the production of heavy particles found in the present universe and to those known to constitute the the Dark Matter contained within it. In the Thomas-Fermi approximation [12], we evaluate the total kinetic energy of the system using the relation :

$$<KE> = \left( \frac{3\hbar^2}{10m} \right) (3\pi^2)^{2/3} \int d\vec{X} [\rho(\vec{X})]^{5/3},$$  \hspace{1cm} (2)

where $\rho(\vec{X})$ is the single particle distribution function which the constituent particle obey within the universe, such that

$$\int d\vec{X} \rho(\vec{X}) = N,$$  \hspace{1cm} (3)

$N$ being the total number of these particles in the universe. The total potential energy of the system in the Hartree-approximation is obtained as

$$<PE> = -\left( \frac{g^2}{2} \right) \int d\vec{X} d\vec{X}' \frac{1}{|\vec{X} - \vec{X}'|} \rho(\vec{X}) \rho(\vec{X}')$$  \hspace{1cm} (4)

Evaluation of the integrals shown in (2) and (4) is done using a single particle density distribution of the form:

$$\rho(\vec{X}) = A e^{-x},$$  \hspace{1cm} (5)
where \( x = (r/\lambda)^{1/2} \), \( r = |\vec{X}| \). Such a form of the single particle density for the ground state has been used by us before [11] while calculating the binding energy of a neutron star and its radius. Latter on [13], it was used to obtain a quantum mechanical derivation for the Schwarzschild radius of a blackhole, thereby indicating further justification of its use. After evaluating the integrals given in (2) and (4), the expression for total energy of the universe, \( E_0(\lambda) \), becomes

\[
E_0(\lambda) = \left( \frac{C}{\lambda^2} \right) - \left( \frac{D}{\lambda} \right),
\]

where

\[
C = \left( \frac{12}{25\pi} \right) \left( \frac{\hbar^2}{m} \right) \left( \frac{3\pi N}{16} \right)^{5/3},
\]

\[
D = \left( \frac{g^2 N^2}{16} \right).
\]

Now, minimising \( E_0(\lambda) \) with respect the \( \lambda \) and then evaluating it at \( \lambda = \lambda_0 \), where \( \lambda_0 \) is the value of \( \lambda \) at which the minimum occurs, the total binding energy of the universe, corresponding to its lowest energy state, becomes

\[
E_0 = -(0.015442)N^{7/3}\left( \frac{mg^4}{\hbar^2} \right).
\]

In view of our earlier work [13] concerning the derivation of the Schwarzschild radius, here also we identify \( 2\lambda_0 \) result with the radius \( R_0 \) of the universe, whose expression becomes

\[
R_0 = 2\lambda_0 = \left( \frac{\hbar^2}{mg^2} \right) \times (4.047528)/N^{1/3}
\]

III. Results of the Present Theory

We now try to recall the Mach’s principle [14], according to which all inertial forces are due to the distribution of matter in the Universe. This can be conveniently put forth in a mathematical form through the following relation:

\[
\frac{GM}{R_0c^2} \approx 1
\]
It has been categorically stated by Collins and Hawking [15] that Mach’s relation appears to hold good in the evolution of the universe. Furthermore, within the framework of the expanding universe, it also suggests that $G$ cannot be constant in time. Using (10) in (9), one arrives at

$$N = 2.8535954 \left( \frac{hc}{Gm^2} \right)^{3/2}$$  \hspace{1cm} (11)

Substituting the (11) in (9), the expression for $R_0$ becomes

$$R_0 = 2.8535954 \left( \frac{h}{mc} \right) \left( \frac{hc}{Gm^2} \right)^{1/2}$$  \hspace{1cm} (12)

Using (12), we have made an estimation for the radius of the present universe, $R_0$, by varying $m$, the mass of the fictitious particle and using the measured value for the gravitational constant $G$, that is by taking $G = 6.67 \times 10^{-8}$ dyn cm$^2$ g$^{-2}$. We have chosen a set of four values for $m$ and calculate $R_0$, from which the age of the Universe $\tau_0$ is estimated using the relation $R_0 \simeq c\tau_0$. All these are shown in Table-I of this paper for which $\tau_0$ has been considered to 20, 15, 10 and 5 billion years. For these $m$ values, the number of particles $N$ in the universe is calculated with the help of (11), which enables us to determine the total mass of the universe, $M_0$. These are also given in Table-I. Using the results for $M_0$ and $R_0$, the average mass density of the present universe is estimated. This is shown in Table-II of this paper.

We now try to calculate the variation of the gravitational constant $G$ with respect to time $G(t)$. For that, we, use the expression for $G$, as found from (12), which is obtained as

$$G = \frac{K}{R_0^2},$$  \hspace{1cm} (13)

where

$$K = (8.1430067) \left( \frac{h^3}{m^4c} \right)$$
Further, we assume that the above expression for $G$ is also valid for anytime 't' provided we take the value of $R_0$ at that time. Using (13), we therefore, find

$$\dot{G} = \left( \frac{\partial G}{\partial R_0} \right) \times \left( \frac{\partial R_0}{\partial t} \right) \simeq c \left( \frac{\partial G}{\partial R_0} \right) = -\frac{2cK}{R_0^3}$$

(14)

Following this, we make an estimation of $(\dot{G}/G)$, which gives

$$\left( \frac{\dot{G}}{G} \right) = -9.6 \times 10^{-11} yr^{-1} \simeq -1 \times 10^{-10} yr^{-1}$$

(15)

The above value is in extremely good agreement with one of the recent estimates [1] and is also consistent with that of P.M. Muller and Hoyle and Narlikar [5]. This is roughly also the value reported by Van Flandern [16], following an analysis of the data relating to the effects of tidal friction on the elements and shape of the lunar orbit of the earth-moon system, within the framework of the Dirac cosmology. However Shapiro et al [17], have arrived at a limiting value for $|\dot{G}/G| \leq 4 \times 10^{-10} yr^{-1}$, based on a method which is used for monitoring the planets for a possible secular increase in their orbital periods by employing a radar reflection system between the Earth, Venus and Mercury. This very result obtained by Shapiro et al, is found to be almost ten times greater than the value predicted by Dicke from the Brans-Dicke scalar tensor theory [18]. This means that the Brans-Dicke theory gives a value for $|\dot{G}/G| \approx 4 \times 10^{-11} yr^{-1}$. In order to see whether we can at all reproduce some of these results from our theory, we have made an estimation of $(\dot{G}/G)$ by varying the mass $m$ of the constituent particles. These are shown in Table-II of the paper. From this, it can be seen that as $m$ increases, the age of universe, $\tau_0$, decreases. This amounts to also a further decrease in the value of $(\dot{G}/G)$. Accepting the fact that the age of the universe is around 20 billion years, one would find that the Brans-Dicke result would correspond to an age greater than twenty billion years.

IV. Discussion of Results and Conclusion
In arriving at the various results of the present theory, we have accepted the value for the age of the universe $\tau_0$ to be $\sim 20 \times 10^9\text{yr}$. The mass of the constituent particles corresponding to this is found to be $m \sim 1.07 \times 10^{-35}\text{g}$. It is also to be noted that in the present theory we have treated the constituent particles as fermions. Since we have mentioned that the neutrinos could be one of the most probable candidates for the Dark Matter present in today’s universe, it is therefore expected that the neutrinos are to be formed out of these fictitious particles whose masses are $m \simeq 1.07 \times 10^{-35}\text{g}$, or equivalently of energy $0.006\text{eV}$, instead of zero, as thought to be the case [7]. For a mass of $m \sim 1.07 \times 10^{-35}\text{g}$, the radius of the universe becomes $R_0 \sim 1.9 \times 10^{28}\text{cm}$, which is considered to be an acceptable result, and total mass of the universe $M_0$ becomes $M_0 \sim (1.3 \times 10^{23})M_{\odot}$, $M_{\odot}$ being the solar mass. If we assume the mass $M_0$ to be solely due to the nucleons that are there in the today’s universe, it would correspond to $\sim 1.5 \times 10^{80}$ nucleons to be present. This is nothing but the famous Eddington result [19]. Using this, the ratio of the number of fictitious particles constituting universe to the number of equivalent nucleons is found to be $\sim 1.6 \times 10^{11}$. This is of the right order of magnitude such as $10^{10}$, the value that has been speculated for the number of photons (neutrinos) per nucleon in the observed universe [2]. We would like to mention here that in arriving at the above value we have assumed that all the fictitious particles involved in the formation of the early universe have been subsequently converted to nucleons and heavy nuclei etc that are found in the today’s universe in the form of visible matter. Let us now look at our calculated value for the average mass density of the universe from Table-II corresponding to $m \sim 1.07 \times 10^{-35}\text{g}$ which gives $\sim 0.9 \times 10^{-29}\text{g/cm}^3$. This, we find to be almost thirty times larger then the observed mass density $,\rho_0$, of the present universe, $\rho_0 \sim 3 \times 10^{-31}\text{g/cm}^3$ where the latter is being thought to be arising solely due to nucleons and other heavy elements etc, present. Let us now assume that out of the
entire mass $M_0$ of the universe, some percentage of it has gone into the formation of the nucleons and other heavy elements present today and the remaining part has been lying in the form of Dark Matter in the present universe. This amounts to saying that the total number of fictitious particles responsible in the formation of the early universe have been subsequently converted to appear in the present universe as nucleons and other heavy nuclei and some other form of particles contributing to the Dark Matter of the universe. These particles could be most probably the neutrinos, as speculated by several workers. Let us accept the view that not more than three to ten percent of the entire mass of the universe is contributed by the nucleons and other heavy nuclei [7], the contribution from the heavy nuclei being negligibly small compared to that of the nucleons. This would therefore mean that the remaining ninety to ninety-seven percent of its total mass constitutes what is known as the Dark Matter of the universe. Having accepted this picture, we have calculated the ratio of the number of neutrinos to nucleons, $(N_\nu/N_n)$, and the neutrino number per unit volume of the universe, $(N_\nu/V)$, etc, $V$ being the volume of the universe, by varying the total mass of the existing nucleons so as to lie within three to ten percent. These calculations have been repeated for different values of the age of the universe kept within five to twenty billion years. These are exhibited in table-III as shown below.

From the above table we find that the calculation done by assuming that only three percent of the total mass of the universe constitutes the observed density of the universe, is the most appropriate one. Because, for this case, the average density of the nucleons (which constitute the observable matter of the universe) comes out to be $\rho_0 \sim 3 \times 10^{-31} \, g/cm^3$, which is considered to be the most accepted value. This corresponds to an age of twenty billion years for the universe, and for this the total nucleon number of the universe becomes $\sim 4.6 \times 10^{78}$, which again nicely agrees with the most speculated value
This being so, the remaining ninety seven percent of the total mass of the universe is considered to be solely due to the Dark Matter present within it. Accepting the fact that this mass is generated by the neutrinos present in the universe, we have made an estimation of the ratio \( \frac{N_\nu}{N_n} \) and \( \frac{N_\nu}{V} \) by varying the mass of the neutrinos. From table-III, one can see that for a neutrino mass of \( m_\nu \sim 1.73 \times 10^{-32} \) g, one arrives at \( \frac{N_\nu}{N_n} \approx 3.1 \times 10^9 \) and \( \frac{N_\nu}{V} \approx 500 \). Both these numbers show fantastic agreement with the findings of the recent observation [7]. A mass of \( m_\nu = 1.73 \times 10^{-32} g \) corresponds to an energy of \( 8eV \), which is again found to be the right order of magnitude for the neutrino mass as speculated by recent measurements by several workers.

Considering the value for the average mass density of the present universe (shown in Table-II) which comes out to be \( \rho_U \approx 9 \times 10^{-30} g/cm^3 \), one can clearly see that it is obviously much larger than the observed mass density \( \rho_0 \) (\( \rho_0 \approx 3 \times 10^{-31} g/cm^3 \)). Since this also exceeds the critical density \( \rho_c \) (\( \rho_c \approx 5 \times 10^{-30} g/cm^3 \)) we would expect the present universe to be close one. Of course, the results of this calculation should not be taken up conclusively, since a thorough analysis of this is needed to make a definite statement. A rough estimate of the Hubble constant \( H_0 \) from the present theory \( (H \approx \frac{1}{\tau_0}) \) gives a value of \( 48km/sec/Mpc \) [Table-II]. This is also in good agreement within the range of speculated value for \( H_0 \). As we have seen, a value of \( 48km/sec/Mpc \) for \( H_0 \) corresponds to an age \( 20 \times 10^9 \) yrs for the present universe, and a \( \rho_c \approx 5 \times 10^{-30} g/cm^3 \) \( [\rho_c = \left( \frac{3H_0^2}{8\pi G} \right)] \). Since the present theory gives rise to \( \approx 500 \) neutrinos per \( cm^3 \), this is to be equal to the number of photons per \( cm^3 \), in view of the common belief that there are an equal number of photons and neutrinos in the present universe [3].

To conclude, we have shown that the results of the present calculation are obtained by choosing a singular form of single particle density for the particles constituting the universe and a singular density is consistent with the idea relating to the Big Bang theory.
Since the standard Big Bang theory so far has not succeeded to explain for the ratio of $\approx 10^9$ for (number of photons/number of nuclei) [2] and here we do reproduce the above number correctly, the present work seems to be justifying the so called Big Bang theory of the universe.

Although the General Theory of Relativity assumes that $G$ has to remain constant, there seems to be some compelling reason to think that $G$ must vary with time, hence with the age of the universe. From the present theory we find that $G \alpha \frac{1}{\tau^2}$ (vide Eq.(13)), $\tau$ being the age of the universe, whereas it is usually assumed that $G$ is inversely proportional to $\tau$ [1]. It would be therefore interesting to see what kind of new features it can exhibit if $G$ or the laws of nature change with time.

In making an estimation of the Hubble constant $H_0$ we have approximated the velocity of expansion of the universe to be equal to the velocity of light ‘c’. Since after accounting for the presence of the Dark Matter in the universe, the actual particle density of the universe might exceed the critical density $\rho_c$, one would expect the present universe to show a contraction after certain stage. A contraction of the Universe would mean a gradual decrease in the velocity of expansion. It is therefore necessary to have an estimate of the deceleration parameter $q$ associated with the contraction. For that the present theory needs to be carefully studied.
References

[1] G. Contopoulos and D. Kotsakis, Cosmology, (Springer-Verlag, Heidelberg, 1987)

[2] Alan H. Guth, in Bubbles, voids and bumps in time: the new cosmology ed. James Cornell (Cambridge University Press, Cambridge, 1989)

[3] Fang Li Zhi and Li Shu Xian, Creation of the Universe, (World Scientific, Singapore, 1989)

[4] A.A. Penzias and R.W. Wilson, Astrophys. Jour. 142, (1965), 419

[5] J.V. Narlikar, Introduction To Cosmology, (Cambridge University Press, London, 1993)

[6] G.S. Kutter, The universe and life, (Jones and Bartlett, USA, 1987)

[7] I. Novikov, Black holes and the Universe, (Cambridge University Press, Cambridge, 1990)

[8] F. Zwicky, Helv. Phys. Act. 6, (1933), 10

[9] James E. Gunn in Bubbles, voids and bumps in time: the new cosmology ed. James Cornell (Cambridge University Press, Cambridge, 1989)

[10] D.N. Schramm, Phys. Today, 36, (1983), 27. G. Steigman, Ann. Rev. Astron. Astrophys., 14, (1976), 339

[11] D.N. Tripathy and Subodha Mishra, Astr-Ph/9612097

[12] L.D. Landau and E.M. Lifshitz, Quantum Mecanics, (Pergamon Press, Oxford, 1965)

[13] D.N. Tripathy and Subodha Mishra, to be published.

[14] P.S. Wesson, Cosmology and Geophysics, (Adam Hilger Ltd, Bristol, 1978)

[15] C.B. Collins and S.W. Hawking, Astrophys. J. 180 (1973), 317.

[16] T.C. Van Flandern, Mon. Not. R. Astron. Soc., 170, (1975), 333.

[17] I.I. Shapiro, BAAS, 8, (1976), 308.

[18] C. Brans and R.H. Dicke, Phys. Rev, 124, (1961), 125

[19] E.R. Harrison, Cosmology, (Cambridge: Cambridge University Press, 1981)
| $m \times 10^{-35} g$ | $R \times 10^{28} cm$ | $N \times 10^{91}$ | $M \times 10^{46} gm$ | $\tau_0 \times 10^9 yr$ |
|---------------------|----------------------|-------------------|------------------------|---------------------|
| 1.07299             | 1.896                | 2.38429           | 2.55832                | 20                  |
| 1.23891             | 1.422                | 1.54865           | 1.91875                | 15                  |
| 1.51744             | 0.948                | 0.84297           | 1.27916                | 10                  |
| 2.14598             | 0.474                | 0.29804           | 0.639588               | 5                   |
| $M \times 10^{56} g$ | $R \times 10^{28} cm$ | $\rho \times 10^{-29} g/cm^3$ | $\dot{G} \times 10^{-25} cm^3 g^{-1} s^{-2}$ | $(G/G)yr^{-1}$ | $H_0 km/sec/Mpc$ |
|----------------------|----------------------|-----------------|-----------------|-----------------|----------------|
| 2.55832              | 1.896                | 0.89609         | -2.11076        | $-1 \times 10^{-10}$ |
| 1.91875              | 1.422                | 1.59305         | -2.81437        | $-1.3 \times 10^{-10}$ |
| 1.27916              | 0.948                | 3.58436         | -4.22149        | $-1.9 \times 10^{-10}$ |
| 0.63959              | 0.474                | 1.43376         | -8.44304        | $-4 \times 10^{-10}$ |
| $M_n \times 10^{56} g$ | $N_n \times 10^{78}$ | $N_\nu / N_n \times 10^9$ | $N_\nu \times 10^{88}$ | $N_\nu / V$ | $M_\nu \times 10^{46} g$ | $m_\nu \times 10^{-32} g$ |
|----------------|-----------------|----------------|----------------|----------------|----------------|----------------|
| 0.03 (2.55832) | 4.5875          | 3.344          | 1.53390        | 535            | 2.48157        | 1.6176         |
| 0.03 (2.55832) | 4.5875          | 3.125          | 1.43359        | 500            | 2.48157        | 1.6176         |
| 0.03 (2.55832) | 4.5875          | 2.813          | 1.29020        | 450            | 2.48157        | 1.6176         |
| 0.03 (2.55832) | 4.5875          | 1.000          | 0.45875        | 160            | 2.48157        | 5.4090         |
| $\tau_0 \times 10^9$ yr | $M_n \times 10^{36}$ g | $N_n \times 10^{78}$ | $N_\nu/N_n \times 10^9$ | $N_\nu \times 10^{38}$ | $N_\nu/V$ | $M_\nu \times 10^{36}$ g |
|------------------------|----------------------|---------------------|-------------------------|-----------------------|----------|--------------------------|
| 20                     | 0.03 (2.55832)       | 4.587500            | 3.125                   | 1.43359               | 500      | 2.48157                  |
| 15                     | 0.03 (1.91875)       | 3.440675            | 3.125                   | 1.07521               | 892      | 1.86119                  |
| 10                     | 0.03 (1.27916)       | 2.293772            | 3.125                   | 0.71680               | 2008     | 1.24079                  |
| 5                      | 0.03 (0.639598)      | 1.146900            | 3.125                   | 0.35841               | 8034     | 0.62040                  |