PT-Symmetric Plasmonic Metamaterials

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We theoretically investigate the optical properties of parity-time (PT) symmetric three-dimensional metamaterials composed of strongly-coupled planar plasmonic waveguides. By tuning the loss-gain balance, we show how the initially isotropic material becomes both asymmetric and unidirectional. Investigation of the band structure near the material’s exceptional point reveals several intriguing optical properties, including double negative refraction, Bloch power oscillations, unidirectional invisibility, and reflection and transmission coefficients that are simultaneously equal to or greater than unity. The highly tunable optical dispersion of PT-symmetric metamaterials provides a foundation for designing an entirely new class of threedimensional bulk synthetic media, with applications ranging from lossless sub-diffraction-limited optical lenses to non-reciprocal nanophotonic devices.

Textbook conceptions of light-matter interactions have been challenged by two recent material advances — the development of metamaterials and the discovery of parity-time PT-symmetric media. Metamaterials allow considerable control over the electric and magnetic fields of light, so that permittivities, permeabilities, and refractive indices can be tuned throughout positive, negative, and near-zero values. Metamaterials have enabled negative refraction, optical lensing below the diffraction limit of light and invisibility cloaking \textsuperscript{[2–6]}. Complementarily, PT-symmetric media allow control over electromagnetic field distributions in loss and gain media, so that light propagation can be asymmetric and even unidirectional. PT-symmetric media have enabled loss-induced optical transparency, lossless Talbot revivals and unidirectional invisibility \textsuperscript{[10–21]}. Combined with non-linear media, they have also been suggested as optical diodes, insulators, circulators, and perfect cavity absorber-lasers \textsuperscript{[22–28]}.

While metamaterials rely on subwavelength engineered ‘building blocks’ to control electric and magnetic light-matter interactions, PT-symmetric media rely on judicious spatial arrangement of loss and gain media. Their unique asymmetric properties are based on a fundamental insight from quantum mechanics indicating that Hamiltonians need not be Hermitian to yield real eigenvalues and hence physical observables. Instead, the weaker condition of parity and time symmetry is sufficient to yield real eigenvalues below a certain threshold. Above this threshold, eigenvalues move into the complex plane and become complex conjugates of each other \textsuperscript{[29–33]}.

In the context of optics, PT-symmetric Hamiltonians arise from the duality between the quantum mechanical Schrodinger equation and the wave equation. Provided the refractive index profile satisfies \( n(x) = n(−x) \), light will propagate as if it experiences a PT-symmetric potential. Below the PT-symmetric exceptional point, the optical eigenvalues will be purely real; however, as the loss and gain of the material are increased beyond the exceptional point, the eigenvalues will become complex. In particular, certain eigenmodes will experience increased loss while other eigenmodes will exhibit strong optical gain. This behavior is at the core of the asymmetric and unidirectional optical properties observed in PT media to date.

While nearly all PT-symmetric media have been constructed from macroscopic (i.e., greater than wavelength-scale) elements, the optical Hamiltonian places no restrictions on the length scales over which the index profile can vary. This insight drives the question: can we create PT-symmetric metamaterials — i.e., bulk photonic media whose optical properties are determined both by their subwavelength building blocks and a judicious choice of their loss/gain profile? Such metamaterials would enable unprecedented control over electric and magnetic optical fields across wavelength and subwavelength scales, and may enable an entirely new class of bulk synthetic photonic media.

In this Letter, we investigate the emergent optical properties of bulk, three-dimensional PT-symmetric metamaterials. As a prototype metamaterial, we consider a multilayer stack of alternating layers of metal and dielectric. Both theoretical \textsuperscript{[34]} and experimental \textsuperscript{[6]} work has demonstrated the isotropic negative index response of this metamaterial, resulting in all-angle negative refraction and Veselago ‘perfect optical lensing.’ Its operation is based on the negative index plasmonic modes of its unit cell — a five-layer ‘metal-insulator-metal’ waveguide. By varying the thickness of the layers as well as the materials, the frequency of operation and the emergent bulk index of refraction can be precisely controlled throughout optical frequencies. While practical utilization of this negative index metamaterial has been limited by propagation and coupling losses, we will show that these losses could be overcome by subjecting the plasmonic modes to PT-symmetric potentials. Moreover, PT-symmetric potentials in this metamater-
rial can enable above-unity transmission and reflection, Bloch power oscillations, hyperbolic to elliptic dispersion transitions, and unidirectional invisibility.

Figure 1 illustrates the specific plasmonic metamaterial investigated in this paper, with the unit cell period indicated by $\Lambda$. Within each unit cell, the thicknesses of the metal $t_{m}$ and dielectric $t_{d}$ are deeply subwavelength, with $t_{m} = t_{d} = 30$ nm. We consider Ag as the metal, described by a lossless Drude model with dielectric constant $\varepsilon_{Ag} = 1 - (\omega/\omega_{p})^2$. The bulk plasma frequency of Ag, $\omega_p$, is assumed to be $8.85 \times 10^{15}$ s$^{-1}$. We consider the dielectric layers to be TiO$_2$ with $n = 3.2$. With these materials, the surface plasmon resonance, $\omega_{sp}$, occurs at 1.73 eV ($\omega_{sp}/\omega_p = 0.29$), and negative index modes are observed between this frequency and $\omega_p$. This particular materials combination was recently experimentally shown to exhibit all-angle negative refraction and Veselago lensing. Here, we theoretically investigate the evolution of the optical bands of this metamaterial upon varying the non-Hermiticity parameter, $k$, is identical for alternating dielectric layers.

Using the transfer matrix approach described in Ref. [36], we solve for the dispersion curves of the five-layer unit-cell plasmonic waveguide for transverse-magnetic (TM) polarized illumination. To determine the band diagrams of the periodic metamaterial, the wavevector along the $z$-direction is swept in the first Brillouin zone, $(0, \pi)$, and the characteristic equation is minimized to find the propagation constant along the $x$-direction at each frequency. The results are shown in panels (b)-(e) of Fig. 1 for $k = 0, 0.2, 0.3$ and 0.5, respectively. Note that the colormap indicates purely real values of $k_x$, corresponds to lossless propagation along the metamaterial. For a non-Hermiticity parameter $k$ = 0, four different branches are observed: two below $\omega_{sp}$ ($B_1$ and $B_2$) and two above ($B_3$ and $B_4$). Because all constituents are lossless, the wavevectors diverge at $\omega_{sp}$. $B_1$ and $B_2$ are characterized by positive slopes and hence positive refractive mode indices. In contrast, $B_3$ and $B_4$ are characterized by negative slopes and hence negative refractive mode indices.

When the non-Hermiticity parameter of the metamaterial is increased, the modes merge together at the exceptional points of the dispersion, denoted by black circles in panels (c)-(e). Beyond these exceptional points, the two distinguishable lossless modes below and above $\omega_{sp}$ (i.e., $B_1$ and $B_2$ or $B_3$ and $B_4$, respectively) evolve to a gain mode and a loss mode with the same phase velocity. Due to their complex wavevectors, we denote these modes as black, dashed lines in Fig. 1(c)-(e). To understand these loss and gain modes, note that the transfer matrix of the $PT$-symmetric metamaterial possesses the following symmetry property:

$$T(\omega, k_z, k_x^*)T^*(\omega, k_z, k_x) = I$$

where $I$ is the identity matrix. The Bloch modes of the metamaterial are eigenvalues of $T$ and satisfy:

$$[T(k_x) - e^{i\Lambda k_x} I] = 0$$

Taking the complex conjugate of Eq. (2) and using the symmetry property of Eq. (1) the following relation is ob-
tained:

\[ |T(k_x^*) - e^{iA k_z} I| = 0 \]  \hspace{1cm} (3)

Equation 3 means that if \( k_x \) (a complex number in general) admits a real solution for the Bloch wavevector, \( k_x^* \) is a solution for that Bloch mode as well. Accordingly, the bands have centro-symmetry in the complex \((k_x, k_z)\) plane. Also note that the loss and gain modes of Fig. 1 (c)-(e) conjoin at \( \omega_{sp} \); further, unlike the modes for a zero non-Hermiticity parameter, their wavevectors at \( \omega_{sp} \) remain finite.

While real periodic spatial refractive index profiles lead to the appearance of an infinite number of band gaps, complex periodic index profiles generally result in complex dispersion curves across the entire frequency range. Interestingly, if the refractive index profile satisfies the condition for PT symmetry \((n(z) = n^*(-z))\) real propagation constants and complete band gaps can exist provided \( k \leq k_{th} \). Here, \( k_{th} \) is the threshold value at which the Hamiltonian and the PT operator no longer commute, and consequently, real-valued solutions cease to be supported by the complex potential. Fig. 1 (c)-(e) illustrate this feature for increasing non-Hermiticity parameter. For example, for \( k = 0.2 \) and \( k = 0.3 \) purely real wavevectors and bandgaps are observed for all bands both above and below \( \omega_{sp} \). However, for \( k = 0.5 \), purely real eigenmodes below \( \omega_{sp} \) do not exist across visible and near-infrared frequencies. Further, the bandgap between \( B_3 \) and \( B_4 \) merges for large \( k_z \), and these bands only exist over a very limited wavevector and wavelength range.

The non-Hermiticity parameter not only changes the propagation constant and bandgap of the metamaterial, but also the band curvature. Fig. 2 plots the equi-frequency contours of bands \( B_1-B_4 \) at wavelengths of \( \lambda = 954 \text{ nm} \) (\( \omega/\omega_p = 0.22 \)) for \( B_1 \) and \( B_2 \), \( \lambda = 604 \text{ nm} \) (\( \omega/\omega_p = 0.35 \)) for \( B_3 \), and \( \lambda = 445 \text{ nm} \) (\( \omega/\omega_p = 0.48 \)) for \( B_4 \). Since the metamaterial is isotropic in the \( xy \)-plane and the contours are centro-symmetric in the \((k_x, k_z)\) plane, a quadratic dispersion relation \((\frac{\omega_p}{\omega})^2 + (\frac{k_z}{k_0})^2 = k_x^2 \) can be used to model the bands. Here \( k_0 \) indicates the free-space wavevector. The fitted refractive mode indices are listed in Table I.

| \( k = 0 \) | \( k = 0.2 \) | \( k = 0.5 \) |
| \( (n_x^*, n_z^*) \) | \( (n_x^*, n_z^*) \) | \( (n_x^*, n_z^*) \) |
| \( B_1 \) | \( (95.86, 289.89) \) | \( (85.5, 53.56) \) | NA |
| \( B_2 \) | \( (54.14, -58.86) \) | \( (59.42, -24.83) \) | NA |
| \( B_3 \) | \( (-4.11, 1.27) \) | \( (-4.88, 1.21) \) | \( (11.55, 1.16) \) |
| \( B_4 \) | \( (1.9, 1.85) \) | \( (1.85, 1.45) \) | \( (1.02, 0.66) \) |

TABLE I. Effective refractive indices of the four bands based on a quadratic fit. The parameters are calculated for wavelengths of \( \lambda = 954 \text{ nm} \) \((B_1, B_2)\), 604 nm \((B_3)\) and 445 nm \((B_4)\).

As seen both in Fig. 2 and Table I for a non-Hermiticity parameter \( k = 0 \), bands \( B_1 \) and \( B_4 \) are elliptical (i.e., \( n_x^2 n_z^2 \geq 0 \)) while bands \( B_2 \) and \( B_3 \) are hyperbolic (i.e., \( n_x^2 n_z^2 \leq 0 \)). Moreover, \( B_4 \) is characterized by a nearly-perfect circular equi-frequency contour and almost equal values of effective refractive indices in both the \( x \) and \( z \)-directions. Accordingly, this metal-insulator-metal metamaterial is isotropic at \( \lambda = 445 \text{ nm} \), consistent with prior work [37, 41].

For increasing non-Hermiticity parameter, \( B_1 \) and \( B_4 \) remain elliptical while \( B_2 \) remains hyperbolic. Interestingly, however, band \( B_3 \) undergoes a hyperbolic to elliptical transition for \( k = 0.5 \). Such hyperbolic-to-elliptic transitions could enable dynamic tuning of Purcell enhancements for emitters near the metamaterial. Further, they could modulate Talbot revivals or the formation and resolution of images generated by hyperbolic metamaterial super-lenses [37, 41].

The results of Fig. 2 imply that with increasing non-Hermiticity parameter, the material can evolve from an isotropic metamaterial to an anisotropic one. Intriguingly, the structure can also become highly directional. This property cannot be derived from the band diagrams, but can be understood by considering the transfer matrix:

\[
T = \begin{bmatrix} a & b \\ c & a^* \end{bmatrix}
\]  \hspace{1cm} (4)

Here, the parameters \( a, b \) and \( c \) are related to the reflec-
tion and transmission coefficients $r$ and $t$ as $r_L = -\frac{a}{c}$, $r_R = \frac{a}{c}$ and $t_L = t_R = \frac{a}{c}$, where the subscripts L and R denote illumination from the left and right, respectively. As these equations indicate, an optical system composed of linear and reciprocal materials is non-directional provided the components are lossless. In other words, the transmitted and reflected powers $T = |t|^2$ and $R = |r|^2$ sum to unity and are independent of illumination direction, since $T_L = T_R = T$ and $T + R_R = 1 = T + R_L$, so $R_L = R_R$. When loss or gain is introduced into the system, the transmission coefficient remains the same for both directions of illumination. However the reflection coefficient need not be symmetric, as power can be attenuated or generated within the structure. The asymmetry is obtained at the price of losing propagating Bloch modes. However, as we will show, asymmetric responses can be obtained in a $PT$-symmetric potential where purely real bands exist as well.

To illustrate this directional behavior, Fig. 3 plots plane-wave refraction of light from vacuum ($n = 1$) through a metamaterial composed of 10 unit cells. We consider TM-polarized illumination of wavelength $\lambda = 445$ nm impinging on the metamaterial at an angle of $\theta = 45^\circ$ in the $(x, z)$ plane. The colormap of Fig. 3 plots the $H_y$ component of the fields. The arrows of Fig. 3 indicate the direction of illumination, refraction and transmission, each determined by spatially-averaging the Poynting vector in each region.

For $k = 0$ (Fig. 3a), the power is negatively refracted with an angle of $\sim -32^\circ$. This result is in excellent agreement with our band structure calculations, which yield a refracted angle from Snell’s law of $\sim -31^\circ$. The refractive index, $n = -\sqrt{1.87} = -1.36$, at this non-Hermiticity value is independent of the illumination angle and direction. Indeed, for illumination in the $(x, z)$ plane, or an ‘endfire configuration’, the same refraction angle is observed (see Fig. 3b). The same refraction angle is also observed for illumination in all sides of the metamaterial (i.e., illumination from $\pm x$, $\pm y$, and $\pm z$).

Upon increasing the non-Hermiticity parameter of the metamaterial, the material becomes highly directional. Panels (b) and (c) of Fig. 3 illustrate the field profiles in a 10-layer $PT$-symmetric metamaterial when illumination is from the loss and gain side (i.e., illumination in the $+z$ or $-z$ directions, respectively). As a particular example, we consider $k = 0.445$. As seen, field profiles and refraction angles are completely different for illumination from $+z$ (loss side) versus $-z$ (gain side). Illumination from $+z$ yields negative refraction at an angle of $-81^\circ$ (see Fig. 3b). In contrast, illumination from $-z$ yields negative refraction at an angle of $\sim -43^\circ$ (Fig. 3c).

More intriguingly, this structure is characterized by tunable reflection and transmission coefficients that can range from zero to at or above unity. This characteristic is illustrated in the lower panels of Fig. 3 which plot the normalized-to-incidence power at each position along the direction of propagation. For example, for illumination from the $-z$ direction (panel (c)) power flows backward toward the source in the illumination region ($P_z = -1$), and away from the metamaterial on the transmission side ($P_z = +1$). Therefore, the metamaterial is completely transparent, in that the metamaterial can transmit all of the incident power, even though light is emitted back towards the source.

Moreover, for illumination from the $+z$ direction (panel (b)) this metamaterial can achieve unidirectional invisibility. As seen, the power is unity on both sides of the metamaterial, indicating complete suppression of reflection on the illumination side and complete trans-
power as light propagates through the array. For example, for \( k = 0.445 \), illumination at \( +45^\circ \) yields a refracted angle of \( -11^\circ \), while illumination at \( -45^\circ \) yields refraction along the metamaterial interface at an angle of \( -90^\circ \). This double refraction does not just manifest itself in the intensity of the transmitted beam, but also in the profile of the fields, as seen in the right panels of Fig. 4.

We now consider the effect of varying the non-Hermiticity parameter on the scattering properties of the metamaterial. As before, we consider TM-polarized illumination with a \( 45^\circ \) tilted planewave at \( \lambda = 445 \) nm. We limit our analysis to illumination along either \( +z \) (‘left-side illumination (L)’) or \( -z \) (‘right-side illumination (R)’) Based on Eq. 1, a generalized energy conservation formula can be derived as \( |T - 1| = \sqrt{R_L R_R} \), where \( T \) is the transmitted power and \( R_R \) and \( R_L \) are the reflected powers from the right and left sides, respectively.

Figure 5 plots the reflection and transmission coefficients as a function of \( k \). As seen in Fig. 5, for \( k = 0.035 \), the transmitted power equals unity independent of the number of layers. Correspondingly at this point \( R_R \), shown in panel (b), vanishes for any number of layers. This property manifests itself as a peak in Fig. 4, where the quotient of effective reflection coefficients is plotted. Importantly, for \( k = 0.035 \), this PT-symmetric metamaterial is still isotropic, characterized by circular equifrequency contours. Therefore, this PT-symmetric optical potential could enable lossless and far-field Vesalago lensing.

For larger non-Hermiticity parameters (\( k > 0.035 \)), \( T \) exceeds unity. Interestingly, while the transmitted power varies with \( k \) and the number of unit cells, it never drops below 1 up through \( k = 0.445 \). At this non-Hermiticity value, the reflected power from the left/loss side \( (R_L) \) vanishes, as shown in Fig. 4. For larger \( k \), \( T \) remains at or below unity. Non-Hermiticity parameters \( k \) above 0.63 yield purely imaginary \( k_z \), so no propagation is allowed through the metamaterial. This property is accompanied by a rapid drop in \( T \) and strong increase in the reflectance for any number of layers.

While for left- (or loss-)side illumination the reflection coefficient is always less than unity, for right- (or gain)-side illumination the reflected power can exceed 1. This behavior is not monotonic however, depending strongly on the number of unit-cell layers and \( k \). Indeed, only at \( k = 0 \) and \( k = 0.12 \) are the reflected powers from the left and right identical and equal to unity. For \( k > 0.12 \), the ratio \( R_L/R_R \) remains below unity (Fig. 5).

Table II summarizes some of the interesting scattering properties of the 10-unit-cell metamaterial extracted from Fig. 4. For \( k = 0 \), the metamaterial is Hermitian and lossless. With increasing PT-symmetric potentials, unusual features such as above unity transmission \( (k = 0.26) \), above unity reflection \( (k = 0.445) \) and uni-

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\[ \phi_1 = 2m\pi \]
Hermiticity parameters increased the transmission of the material. Small but non-zero non-Hermiticity parameters increased the transmission of the material. The band curvatures, band gaps and effective refractive potentials, we demonstrated the broad tunability by varying the illumination wavelength is \( \lambda = 445 \) nm and \( \theta = 45^\circ \). The inset in panel (b) and (d) show the behavior of the reflected and transmitted powers around \( k = 0.035 \) where \( R_R \) vanished and \( T = 1 \) independent of the number of periods.

In conclusion, we have introduced the concept of a PT-symmetric metamaterial. The original lossless metamaterial, composed of a periodically-stacked 5-layer plasmonic waveguide, was designed to behave as an isotropic, three-dimensional negative refractive index material. By subjecting the plasmonic modes to PT-symmetric optical potentials, we demonstrated the broad tunability of the band curvatures, band gaps and effective refractive indices of the material. Small but non-zero non-Hermiticity parameters increased the transmission of the isotropic negative index metamaterial to unity. Larger non-Hermiticity parameters morphed the material from isotropic to anisotropic and directional. The highly unusual optical properties of PT-symmetric metamaterials could be used to devise an entirely new class of bulk synthetic media, ranging from lossless Veselago lenses to unidirectional metamaterial-based invisibility cloaks and new non-reciprocal nanophotonic devices.

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### TABLE II. Transmitted and reflected powers from left and right side

| # of periods | \( k = 0 \) | \( k = 0.035 \) | \( k = 0.26 \) | \( k = 0.445 \) | \( k = 0.5 \) |
|-------------|---------------|---------------|---------------|---------------|---------------|
| \( R_L \)   | 0.0080        | 0.03           | 0.0982        | 0              | \( 5 \times 10^{-4} \) |
| \( R_R \)   | 0.0080        | 0              | 0.8397        | 2              | 0.1219        |
| \( T_L \)   | 0.9920        | 1              | 1.2872        | 1              | 0.9922        |
| \( T_R \)   | 0.7633        | 0.7609         | 0.6502        | 0.4683         | 0.3948        |

Fig. 5. Reflected and transmitted powers of the metamaterial with increasing \( k \) and varying numbers of periods. (a) Reflected power upon illumination from the left (loss) side of the metamaterial; (b) reflected power upon illumination from the gain (right) right side; (c) Quotient of reflected powers; (d) Transmitted power. In all panels, the incident angle is \( \theta = 45^\circ \) and the illumination wavelength is \( \lambda = 445 \) nm. The inset in panel (b) and (d) show the behavior of the reflected and transmitted powers around \( k = 0.035 \) where \( R_R \) vanished and \( T = 1 \) independent of the number of periods.

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