FROM CHIRAL LAGRANGIANS TO LANDAU FERMI LIQUID THEORY OF NUCLEAR MATTER

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Abstract

A simple relation between the effective parameters of chiral Lagrangians in medium as predicted by BR scaling and Landau Fermi liquid parameters is derived. This provides a link between an effective theory of QCD at mean-field level and many-body theory of nuclear matter. It connects in particular the scaling vector-meson mass probed by dileptons produced in heavy-ion collisions (e.g., CERES of CERN-SPS) to the scaling nucleon-mass relevant for low-energy spectroscopic properties, e.g., the nuclear gyromagnetic ratios $\delta g_i$ and the effective axial-vector constant $g_A^*$. 
1 Introduction

In recent publications [1], Li, Ko and Brown showed that the dilepton production data of CERES [2] and HELIOS-3 [3] can be simply and quantitatively understood if the mass of the vector mesons $\rho$ and $\omega$ scales in dense and/or hot medium according to the scaling (BR scaling) proposed by Brown and Rho [4]. That the vector mesons “shed” their masses as the density (or temperature) of the matter increases is expected in an intuitive interpretation of the interplay of the condensation of quark-antiquark pairs and the dynamical generation of light-quark hadron masses and is in fact corroborated by QCD sum rules [5, 6] and model calculations [7]. Thus, the dilepton data are consistent with the most conspicuous prediction of BR scaling.

The proposal of [4], however, goes further than this and makes a statement on the relation between the scaling of meson masses and that of baryon masses:

$$\frac{m^*_M}{m_M} \approx \sqrt{\frac{g_A m^*_B}{g^*_A m_B}} \approx \frac{f^*_\pi}{f_\pi} \equiv \Phi(\rho)$$  \hspace{1cm} (1)

where the subscript $M$ stands for light-quark non-Goldstone mesons, $B$ for light-quark baryons, $g_A$ the axial-current coupling constant and $f_\pi$ the pion decay constant. The star denotes an in-medium quantity. (Although temperature effects can also be discussed in a similar way, we will be primarily interested in density effects in this paper.)

Two important questions remained unanswered in these developments: Firstly, is there evidence that the baryon mass scaling and the meson mass scaling are related as implied by the chiral Lagrangian? Secondly, we know from the Walecka model of nuclear matter [9] that the “scalar mass” of the nucleon drops as a function of density and that this reduction of the nucleon mass has significant consequences on nuclear spectroscopy and the static properties of nuclei. The question is: Is BR scaling related to the “conventional” mechanism for the reduction of the nucleon mass in nuclear matter and if so, how does it manifest itself in low-energy nuclear properties?

The purpose of this paper is to show, based on recent work [10, 11], that the connection between the meson and baryon scalings can be made using the Landau-Migdal theory of nuclei and nuclear matter. A similar attempt was recently made by Brown [12]. Our starting point is the effective chiral Lagrangian used in [4] where the scale anomaly of QCD is incorporated and baryons arise as skyrmions. This theory is mapped onto an effective meson-baryon chiral Lagrangian. We establish the relation between chiral and Walecka mean fields in medium as suggested in [11] and then invoke the Galilei invariance argument of Landau, which relates the nucleon effective mass to the Landau Fermi liquid parameters. Thus, we establish a relation between the parameters in eq. (1) and the Landau parameters. We discuss how this relation can be tested with the effective $g^*_A$ and the gyromagnetic ratios $\delta g_l$ in nuclear matter. This then supplies an intriguing – and hitherto undiscovered – relation between the scaled masses, which may be reflected in the spectrum of dileptons produced in relativistic heavy-ion collisions, and low-energy spectroscopic information, $g^*_A$ and $\delta g_l$. It also supplies an albeit indirect and poorly understood connection between quantities figuring in chiral Lagrangians of QCD and those appearing in familiar many-body theory. This connection indicates that low-energy effective theories can provide important insight necessary to understand ultrarelativistic heavy-ion reactions in which QCD variables are relevant.

*This of course does not exclude other explanations based on different dynamical schemes with different Lagrangians such as, e.g., that in [8]. It should, however, be borne in mind that such explanations are not necessarily alternative ones; they may in fact be overlapping to a varying degree in physics.
In order to avoid unnecessary complications we shall use the nonrelativistic approach to Landau Fermi liquid theory, referring to results obtained in the relativistic formulation \[13, 14\] where appropriate. The latter approach is briefly discussed in the Appendix.

### 2 BR Scaling in Chiral Lagrangians

The BR scaling relation \([1]\) that relates the dropping of light-quark non-Goldstone-boson masses to that of the nucleon mass which in turn is related to that of the pion decay constant was first derived by incorporating the trace anomaly of QCD into an effective chiral Lagrangian. The basic idea can be summarized as follows. We wish to write an effective chiral Lagrangian which at mean-field level reproduces the quantum trace anomaly while including higher chiral order effects relevant for nuclear dynamics. To do this, we write the effective Lagrangian in two parts

\[
\mathcal{L} = \mathcal{L}_{\text{inv}} + \mathcal{L}_{\text{sb}}
\]

where \(\mathcal{L}_{\text{inv}}\) is the scale-invariant part and \(\mathcal{L}_{\text{sb}}\) the scale-breaking part of the effective Lagrangian. We introduce the *chiral-singlet* scalar field \(\chi\), as an interpolating field for \(\text{Tr} G^2\),

\[
\theta_\mu = \frac{\beta(g)}{2g} \text{Tr} G_{\mu\nu} G^{\mu\nu} \equiv \chi^4,
\]

where we have dropped the quark mass term (here we consider the chiral limit). The simplest possible invariant piece of the Lagrangian then takes the form

\[
\mathcal{L}_{\text{inv}} = \frac{f_\pi^2}{4} \left( \frac{\chi^2}{\chi_0^2} \right) \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{32g^2} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 + \cdots
\]

where \(\chi_0\) is a number which we define to be the expectation value of \(\chi\) in matter-free vacuum and the ellipsis stands for other-scale invariant terms including the kinetic energy term for the \(\chi\) field. Note that this is the simplest possible form based on the most economical assumption. One could perhaps write much more complicated and yet scale-invariant forms using the same set of fields but invoking different assumptions, and thus obtain a different type of scaling. Experiments will tell us which one is the right form.

As for the scale-breaking term \(\mathcal{L}_{\text{sb}}\), we assume that it contains just the terms needed to reproduce the full trace anomaly. We add other scale-invariant terms representing higher chiral order terms to assure the correct vacuum potential which we shall call \(V(\chi, U)\). Fortunately all we need to know about the potential \(V\) is that it contains a source for the \(\chi\) mass term and that, for a given density, it attains its minimum at \(\chi^* = \langle \chi \rangle^*\) in the sense of the Coleman-Weinberg mechanism \([15]\). (We will return later to what this quantity \(\chi^*\) represents physically.)

The fact that the vacuum expectation value is obtained by minimizing the potential, which contains a scale-breaking term, implies that we are treating the breaking of the scale invariance as a spontaneous symmetry breaking. It is well-known that the spontaneous breaking of the scale symmetry occurs only if it is explicitly broken, since otherwise the potential would be flat \([16]\). Given the ground state characterized by \(\chi^*\) which is fixed by the anomaly, we then shift the field in \(\mathcal{L}\)

\[
\chi(x) = \chi'(x) + \chi^*.
\]

After shifting, we still have the scale-invariant and scale-breaking pieces although the manifest invariance is lost as is the case with *all* spontaneously broken symmetries. The low-energy
physics for the scaling we are interested in is lodged in the former. Since the theory contains two parameters, $f_\pi$ and $g$, we define

$$f_\pi^* = f_\pi \frac{\chi^*}{\lambda_0},$$

$$g^* = g. \quad (6)$$

The second relation follows since the Skyrme quartic term in (4) is scale-invariant by itself. This allows us to redefine the parameters that appear in the chiral Lagrangian in terms of the “starred” parameters $f_\pi^*$ and $g^*$. Since the KSRF relation [17] is an exact low-energy theorem as shown by Harada, Kugo and Yamawaki [18], it is reasonable to assume that it holds also in medium. This leads to

$$m_V^*/m_V \approx \frac{f_\pi^* g^*}{f_\pi g} \approx \frac{f_\pi^*}{f_\pi} \equiv \Phi(\rho) < 1 \quad \text{for} \quad \rho \neq 0 \quad (7)$$

where the subscript $V$ stands for $\rho$ or $\omega$ meson. Similarly the mass of the scalar field is reduced

$$m_\sigma^*/m_\sigma \approx \Phi(\rho). \quad (8)$$

Here we denote the relevant scalar field by the usual notation $\sigma$ for reasons given below.

Now in order to find the scaling behavior of the nucleon mass, we use the fact that the nucleon arises as a soliton (skyrmion) from the effective chiral Lagrangian as in the free-space. The soliton mass goes like

$$m_S \sim f_\pi/g. \quad (9)$$

If one assumes that by the same token the coupling constant $g$ in the soliton sector is not modified in the medium, eq. (9) implies that the nucleon mass is also proportional to $f_\pi^*$,

$$m_N^*/m_N \sim \Phi(\rho). \quad (10)$$

However there is a caveat to this. When it comes to the nucleon effective mass, there is one important non-mean-field effect of short range that is known to be important. This is an intrinsically quantum effect that cannot be accounted for in low orders of the chiral expansion, namely the mechanism that quenches the axial-current coupling constant $g_A$ in nuclear matter. This effect is closely related to the Landau-Migdal interaction in the spin-isospin channel $g'_0$ (involving $\Delta$-hole excitations) as discussed in [13, 20]. The axial-vector coupling constant of the skyrmion is governed by coefficient $g$ of the Skyrme quartic term. This implies that in the baryon sector, the mean-field argument, which is valid in the mesonic sector, needs to be modified. This is reminiscent of the deviation in the nucleon electromagnetic form factor from the vector dominance model which works very well for non-anomalous processes involving mesons. These two phenomena may be related.

As shown in [4, 21], a more accurate expression, at least for densities up to $\rho \sim \rho_0$, is

$$m_N^*/m_N \approx \Phi(\rho) \sqrt{\frac{g_A^*}{g_A}}. \quad (11)$$

This relation will be used later to deduce a formula for $g_A^*$ in nuclear matter. Beyond $\rho = \rho_0$, we expect that $g_A^*$ remains constant ($g_A^* = 1$) and that $\Phi$ scaling takes over except near the chiral phase transition at which the coupling constant $g$ will fall according to the “vector limit.” [19]

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1. We will argue later that in the baryon sector there is an important radiative correction – absent in the meson sector – which modifies this scaling behavior.

2. This was derived using the scaling behavior of the Skyrme quartic Lagrangian and the relation between $g_A$ and the coefficient $g$. Although this relation is justified strictly at the large $N_c$ limit (where $N_c$ is the number of colors), we think that it is generic and will emerge in any chiral model that has the correct symmetries.
2.1 The meaning of \( \chi^* \)

The \( \chi \) field interpolating as \( \chi^4 \) for the dimension-4 field \( \text{Tr}G^2_{\mu\nu} \) may be dominated by a scalar glueball field, which perhaps could be identified with the \( f_J(1710) \) seen in lattice calculations [22]. However, for the scaling we are discussing which is an intrinsically low-energy property, this is too high in energy scale. In the effective Lagrangian (4), such a heavy degree of freedom should not appear explicitly. The only reasonable interpretation is that the \( \chi \) field has two components,

\[
\chi = \chi_h + \chi_l \quad \text{(12)}
\]

corresponding to high (h) and low (l) mass excitations, and that the high mass (glueball) component \( \chi_h \) is integrated out. The “vacuum” expectation value we are interested in is therefore \( \langle \chi_l \rangle^* \). The corresponding fluctuation must interpolate \( 2\pi, 4\pi \) etc. excitations as discussed in [13] and it is this field denoted by \( \sigma \) that becomes the dilaton degenerate with the pion at the chiral phase transition as suggested by Weinberg’s mended symmetry [23]. It is also this component which plays an essential role in the relation between chiral Lagrangians and the Walecka model [24, 11]. This procedure may also be justified by a phenomenological instanton model anchored in QCD [25].

For a more physical interpretation and a detailed discussion on the separation (12), see Adami and Brown [26]. A somewhat different separation is advocated by Furnstahl et al. in [24].

2.2 Baryon chiral Lagrangian

In order to make contact with many-body theory of nuclear matter, we reinterpret the BR scaling in terms of a baryon chiral Lagrangian in the relativistic baryon formalism. There is a problem with chiral counting in this formalism, but our argument will be made at mean-field order as in [11].

The Lagrangian contains the usual pionic piece \( \mathcal{L}_\pi \), the pion-baryon interaction \( \mathcal{L}_{N\pi} \) and the four-Fermi contact interactions

\[
\mathcal{L}_4 = \sum_\alpha \frac{C^2}{2} (\bar{N} \Gamma_\alpha N)(\bar{N} \Gamma^\alpha N) \quad \text{(13)}
\]

where the \( \Gamma^\alpha \)'s are Lorentz covariant quantities – including derivatives – that have the correct chiral properties. The leading chiral order four-Fermi contact interactions relevant for the scaling masses are of the form

\[
\mathcal{L}_4^{(\delta)} = \frac{C^2}{2} (\bar{N}N\bar{N}N) - \frac{C^2}{2} (\bar{N} \gamma_\mu N \bar{N} \gamma^\mu N). \quad \text{(14)}
\]

As indicated by our choice of notation, the first term can be thought of as arising when a massive isoscalar scalar meson (say, \( \sigma \)) is integrated out and similarly for the second term involving a massive isoscalar vector meson (say, \( \omega \)). Consequently, we can make the identification

\[
C^2_\sigma = \frac{g^2_\sigma}{m^2_\sigma}, \quad C^2_\omega = \frac{g^2_\omega}{m^2_\omega}. \quad \text{(15)}
\]

\[\text{As we know from the work of Gasser, Sainio and Svarc [27], the relativistic formulation of baryon chiral perturbation theory requires a special care in assuring a correct chiral counting. What we will find below is that in order to get to the correct formulation from the point of view of Landau Fermi liquid theory of normal nuclear matter and making contact with Walecka theory at mean-field order, it is essential to keep relativistic corrections from the start. This probably has to do with the presence of the Fermi sea in the effective chiral Lagrangian approach. This seems to suggest that the usual chiral counting valid in free space needs to be modified in medium.}\]
The four-Fermi interaction involving the $\rho$ meson quantum number will be introduced below, when we consider the electromagnetic currents. As is well known [24, 11], the first four-Fermi interaction in (14) shifts the nucleon mass in matter,

$$m_N^{\sigma} = m_N - C_\sigma^2 \langle \bar{N}N \rangle.$$  \hfill (16)

In [11] it was shown that this shifted nucleon mass scales the same way as the vector and scalar mesons

$$\frac{m_V^{\sigma}}{m_V} \approx \frac{m_\omega^{\sigma}}{m_\omega} \approx \frac{m_N^{\sigma}}{m_N} \approx \Phi(\rho).$$  \hfill (17)

This relation was referred to in [4] as “universal scaling.” There are two points to note here: First as argued in [11], the vector-meson mass scaling applies also to the masses in (15). Thus, in medium the meson mass should be replaced by $m_\sigma^{\sigma,\omega}$. Consequently, the coupling strengths $C_\sigma$ and $C_\omega$ are density-dependent. Second, the scaling can be understood in terms of effects due to the four-Fermi interactions, which for nucleons on the Fermi surface correspond to the fixed-point interactions of Landau Fermi liquid theory according to Shankar and Polchinski [28]. We shall establish a direct connection to the Landau parameters of the quasiparticle-interaction.

3 Effective Nucleon Mass à la Landau

In the Landau-Migdal Fermi liquid theory of nuclear matter [29, 30], the interaction between two quasiparticles on the Fermi surface is of the form (neglecting tensor interactions)

$$F(\vec{p}, \vec{p}') = F(\cos \theta) + F'(\cos \theta)(\vec{\tau} \cdot \vec{\tau}') + G(\cos \theta) (\vec{\sigma} \cdot \vec{\sigma}') + G'(\cos \theta)(\vec{\tau} \cdot \vec{\tau}')(\vec{\sigma} \cdot \vec{\sigma}'),$$  \hfill (18)

where $\theta$ is the angle between $\vec{p}$ and $\vec{p}'$. The function $F(\cos \theta)$ can be expanded in Legendre polynomials,

$$F(\cos \theta) = \sum_l F_l P_l(\cos \theta),$$  \hfill (19)

with analogous expansions for the spin- and isospin-dependent interactions. The coefficients $F_l$ etc. are the Landau Fermi liquid parameters. Some of the parameters can be related to physical properties of the system. The relation between the effective mass and the Landau parameter $F_1$ (eq. (23)) is crucial for our discussion.

An important point of this paper is that one must distinguish between the effective mass $m_N^{\sigma}$, which is of the same form as Walecka’s effective mass, and the Landau effective mass, which is more directly related to nuclear observables. To see what the precise relation is, we include the non-local four-Fermi interaction due to the one-pion exchange term, $L_4^{(\pi)}$, shown in fig. [4].

The total four-Fermi interaction that enters in the renormalization-group flow consideration à la Shankar-Polchinski is then the sum

$$L_4 = L_4^{(\pi)} + L_4^{(\delta)}.$$  \hfill (20)

The point here is that the non-local one-pion-exchange term brings additional contributions to the effective nucleon mass on top of the universal scaling mass discussed above. We now compute the nucleon effective mass with the chiral Lagrangian and make contact with the results of Fermi liquid theory [10]. We start with the single-nucleon energy in the non-relativistic approximation

$$\epsilon(p) = \frac{p^2}{2m_N^2} + C_\omega^2 \langle N^\dagger N \rangle + \Sigma_{\pi}(p).$$  \hfill (21)

*We treat the scalar and vector fields self-consistently, as described in the Appendix, and the self-energy from the pion exchange graph as a perturbation.
where $\Sigma_\pi(p)$ is the self-energy from the pion exchange graph, shown in fig. 2. The self-energy contribution from the vector meson (second term on the right hand side of (21)) corresponds to the diagram shown in fig. 3. The Landau effective mass $m_L^*$ is related to the quasiparticle velocity at the Fermi surface

$$\frac{d}{dp} \epsilon(p)\big|_{p=p_F} = \frac{p_F}{m_L^*} = \frac{p_F}{m_N^*} + \frac{d}{dp} \Sigma_\pi(p)\big|_{p=p_F}. \quad (22)$$

Using Galilean invariance, Landau [29] derived a relation between the effective mass of the quasiparticles and the velocity dependence of the effective interaction described by the Fermi-liquid parameter $F_1$:

$$\frac{m_L^*}{m_N} = 1 + \frac{\tilde{F}_1}{3} = (1 - \frac{\tilde{F}_1}{3})^{-1}, \quad (23)$$

where $\tilde{F}_1 = (m_N/m_L^*)F_1$. The corresponding relation for relativistic systems follows from Lorentz invariance and has been derived by Baym and Chin [13] (see Appendix).
With the four-Fermi interaction (20), there are two distinct velocity-dependent terms in the quasi-particle interaction, namely the spatial part of the current-current interaction and the exchange (or Fock) term of the one-pion-exchange. In the non relativistic approximation, their contributions to \(\tilde{F}_1\) are (\(\tilde{F}_1 = \tilde{F}_1^\omega + \tilde{F}_1^\pi\))

\[
\tilde{F}_1^\omega = \frac{m_N}{m_L^*} F_1^\omega = -C_\omega \frac{2p_F^2}{\pi^2 m_{\pi}^2},
\]

\[
\tilde{F}_1^\pi = -3 \frac{m_N}{p_F} \frac{d}{dp} \Sigma_\pi(p)|_{p=p_F},
\]

respectively. The relativistic expression for \(\tilde{F}_1^\omega\) is given in the Appendix.

Using eq. (22) we find

\[
\left(\frac{m_L^*}{m_N}\right)^{-1} = \frac{m_N}{m_L^*} F_1^\omega + \frac{m_N}{p_F} \frac{d}{dp} \Sigma_\pi(p)|_{p=p_F} = 1 - \frac{1}{3} \tilde{F}_1^\omega,
\]

which implies that

\[
\frac{m_N}{m_L^*} = 1 - \frac{1}{3} \tilde{F}_1^\omega.
\]

This is an interesting relation between the \(\sigma\)-nucleon interaction (eq. (16)) and the \(\omega\)-nucleon coupling (eq. (24)). The \(\omega\)-exchange contribution to the Landau parameter \(F_1\) is due to the velocity-dependent part of the potential, \(\sim \vec{p}_1 \cdot \vec{p}_2 / m_{\pi}^2\). This is an \(\mathcal{O}(p^2)\) term, and consequently suppressed in naive chiral counting. Nonetheless it is this chirally non-leading term in the four-Fermi interaction (14) that appears on the same footing with the chirally leading terms in the \(\omega\) and \(\sigma\) tadpole graphs. This suggests a subtlety in the chiral counting in the presence of a Fermi sea.

The pion contribution to \(F_1\) can be evaluated explicitly [31]

\[
\frac{1}{3} \tilde{F}_1^\pi = -\frac{3 f_{\pi NN}^2 m_N}{8 \pi^2 p_F} \frac{m_{\pi}^2 + 2p_F^2}{2p_F^2} \ln \frac{m_{\pi}^2 + 4p_F^2}{m_F^2} - 2 \approx -0.153.
\]

Here \(f_{\pi NN} \approx 1\) is the non-relativistic \(\pi N\) coupling constant. The numerical value of \(\tilde{F}_1^\pi\) is obtained at nuclear matter density, where \(p_F \approx 2m_{\pi}\).

One of the important results of this paper is that eq. (27) relates the only unknown parameter \(\tilde{F}_1^\omega\) to the universal scaling factor \(\Phi\). Note that in the absence of the one-pion-exchange interaction – and in the nonrelativistic approximation – \(m_N^\sigma\) can be identified with the Landau...
effective mass $m^*_L$. In its presence, however, the two masses are different due to the pionic Fock term. We propose to identify the scaling nucleon mass defined in eq. (11) with the Landau effective mass:

$$m^*_L = m^*_N.$$  \hspace{1cm} (29)

We note that the Landau mass is defined at the Fermi surface, while the scaling mass refers to a nucleon propagating in a “vacuum” modified by the nuclear medium. Although the two definitions are closely related, their precise connection is not understood at present. Nevertheless, eq. (29) is expected to be a good approximation (see also section 5.2).

4 Nucleon Gyromagnetic Ratios in Nuclei

Given the effective Lagrangian with the BR scaling and its relation to Landau Fermi liquid theory, how can one describe nuclear magnetic moments and axial charge transitions? This is an important question because these nuclear processes are sensitive to both the scaling properties and exchange currents. Here we consider the gyromagnetic ratios $g_l^{(p,n)}$ of the proton and the neutron in heavy nuclei, deferring the issue of the nuclear axial-charge transitions [32] to a later publication [10]. We start with the Fermi liquid theory result for the gyromagnetic ratio.

4.1 Migdal’s formula for the gyromagnetic ratio

The response to a slowly-varying electromagnetic field of an odd nucleon with momentum $\vec{p}$ added to a closed Fermi sea can, in Landau theory, be represented by the current $\vec{J}$

$$\vec{J} = \frac{\vec{p}}{m_N} \left( \frac{1 + \tau_3}{2} + \frac{1}{6.1 + F_1/3} F_1 \right) \tau_3,$$  \hspace{1cm} (30)

where $m_N$ is the nucleon mass in medium-free space. The long-wavelength limit of the current is not unique. As discussed in the Appendix, the physically relevant one corresponds to the limit $q \to 0, \omega \to 0$ with $q/\omega \to 0$, where $(\omega, q)$ is the four-momentum transfer. The current (30) defines the gyromagnetic ratio

$$g_l = \frac{1 + \tau_3}{2} + \delta g_l,$$  \hspace{1cm} (31)

where

$$\delta g_l = \frac{1}{6.1 + F_1/3} F_1 \tau_3 = \frac{1}{6}(\tilde{F}_1' - \tilde{F}_1) \tau_3.$$  \hspace{1cm} (32)

4.2 Chiral Lagrangian prediction

In this section we compute the gyromagnetic ratio using the chiral Lagrangian and demonstrate that Migdal’s result (32) is reproduced. The derivation will be made in terms of Feynman diagrams. The single-particle current $\vec{J}_1 = \vec{p}/m_N^2$ is given by the diagram shown in fig. 4 where the external nucleon lines are dressed by the scalar and vector fields. Note that it is the

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universally scaled mass $m_N^\sigma$ that enters, not the Landau mass. This leads to a gyromagnetic ratio

$$
(g_1)_{sp} = \frac{m_N}{m_N^\sigma} \frac{1 + \tau_3}{2}.
$$

(33)

The first correction to this is the contribution from short-ranged high-energy isoscalar vibrations as depicted in fig. 4, with the exchanged particle being an $\omega$ meson. This contribution has been computed by several authors \cite{34, 35}. In the nonrelativistic approximation one finds

$$
g_{\omega l} = -\frac{1}{6} C_2^\omega \frac{2p^3_F}{\pi^2} \frac{1}{m_N^\sigma} = \frac{1}{6} \tilde{F}_1^\omega.
$$

(34)

Now using (27), we obtain the second principal result of this paper,

$$
g_{\omega l} = \frac{1}{6} \tilde{F}_1^\omega = \frac{1}{2}(1 - \Phi(\rho)^{-1}).
$$

(35)

The corresponding contribution with a $\rho$ exchange in the graph yields an isovector term

$$
g_{\rho l} = -\frac{1}{6} C_2^\rho \frac{2p^3_F}{\pi^2} \frac{1}{m_N^\sigma} \tau_3 = \frac{1}{6} (\tilde{F}_1^\rho)' \tau_3
$$

(36)

where the constant $C_\rho$ is the coupling strength of the four-Fermi interaction

$$
\delta \mathcal{L} = -\frac{C_\rho^2}{2} (\bar{N}_\gamma \gamma^a N \bar{N}_\gamma \gamma^a N).
$$

(37)

In analogy with the isoscalar channel, we may consider this as arising when the $\rho$ is integrated out from the Lagrangian, and consequently identify

$$
C_\rho^2 = g_{\rho l}^2/m_{\rho}^2.
$$

(38)

Again in medium, $m_{\rho}$ should be replaced by $m_{\rho}^\star$. The results (34) and (36) can be interpreted in the language of chiral perturbation theory as arising from four-Fermi interaction counterterms in the presence of electromagnetic field, with the counter terms saturated by the $\omega$ and $\rho$ mesons respectively (see eq. (92) of \cite{36}).
The next correction is the pionic exchange current (known as Miyazawa term, see fig. 3) which yields \[ g_\pi l = \frac{1}{6}((\tilde{F}_1^\pi)' - \tilde{F}_1^\pi)\tau_3 = \frac{2}{9}\tilde{F}_1^\pi\tau_3, \] (39)

where the last equality follows from \((\tilde{F}_1^\pi)' = -(1/3)\tilde{F}_1^\pi\). Thus, the sum of all contributions is

\[
g_l = \frac{m_N}{m_N^*} \frac{1 + \tau_3}{2} + \frac{1}{6}(\tilde{F}_1^\omega + (\tilde{F}_1^\rho)'\tau_3) + \frac{1}{6}((\tilde{F}_1^\pi)' - \tilde{F}_1^\pi)\tau_3 = \frac{1 + \tau_3}{2} + \frac{1}{6}(\tilde{F}_1' - \tilde{F}_1)\tau_3, \] (40)

where eq. (27) was used with

\[
\tilde{F}_1 = \tilde{F}_1^\omega + \tilde{F}_1^\pi, \tag{41}
\]
\[
\tilde{F}_1' = (\tilde{F}_1^\pi)' + (\tilde{F}_1^\rho)' \tag{42}
\]

Thus, using our chiral Lagrangian we reproduce the Fermi-liquid theory result for \(\delta g_l\) \[ \delta g_l = \frac{1}{6}((\tilde{F}_1' - \tilde{F}_1)\tau_3 \] (43)

with \(\tilde{F}\) and \(\tilde{F}'\) in the theory given entirely by (41) and (42), respectively. Equation (40) shows that the isoscalar gyromagnetic ratio is not renormalized by the medium (other than binding effect implicit in the matrix elements) while the isovector one is. It should be emphasized that contrary to naive expectations, BR scaling is not in conflict with the observed nuclear magnetic moments. We will show below that the theory agrees quantitatively with experimental data.

5 Test of the Scaling
Figure 6: The pion-exchange current contribution to the nucleon current in matter.

5.1 QCD sum rules

It is possible to extract the scaling factor $\Phi(\rho)$ from QCD sum rules – as well as from an in-medium Gell-Mann-Oakes-Renner relation \cite{11} – and compare with our theory. In particular, the key information is available from the calculations of the masses of the $\rho$ meson \cite{5,6} and the nucleon \cite{37,38} in medium. In their recent work, Jin and collaborators find (for $\rho = \rho_0$) \cite{6,38}

$$\frac{m^*_\rho}{m_\rho} = 0.78 \pm 0.08,$$

$$\frac{m^*_N}{m_N} = 0.67 \pm 0.05.$$  \hspace{1cm} (44)

We identify the $\rho$-meson scaling with the universal scaling factor,

$$\Phi(\rho_0) = 0.78.$$  \hspace{1cm} (46)

This is tantalizingly close to the result that follows from the GMOR relation in medium \cite{19,39}

$$\Phi^2(\rho_0) \approx \frac{m^*_\pi^2}{m^2_\pi} \left(1 - \frac{\Sigma_{\pi N} \rho_0}{f^2_\pi m^2_\pi} + \cdots \right) \approx 0.6,$$  \hspace{1cm} (47)

where the pion-nucleon sigma term $\Sigma_{\pi N} \approx 45$ MeV is used.

5.2 Prediction

Our theory has only one quantity that is not fixed by the theory, namely the scaling factor $\Phi(\rho)$ ($\tilde{F}_1^\pi$ is of course fixed for any density by the chiral Lagrangian.). Since this is given by QCD sum rules for $\rho = \rho_0$, we use this information to make quantitative prediction.

The first quantity is the Landau effective mass of the nucleon \cite{26},

$$\frac{m^*_N}{m_N} = \Phi \left(1 + \frac{1}{3} F_1^\pi \right)$$  \hspace{1cm} (48)

$$= \left(\Phi^{-1} - \frac{1}{3} \tilde{F}_1^\pi \right)^{-1} = (1/0.78 + 0.153)^{-1} = 0.69(7)$$  \hspace{1cm} (49)

where we used \cite{28} and \cite{46}. The agreement with the QCD sum-rule result \cite{43} is both surprising and intriguing since as mentioned above, the Landau mass is “measured” at the Fermi
momentum $p = p_f$ while the QCD sum-rule mass is defined in the rest frame, so the direct connection remains to be established.

The next quantity of interest is the axial-vector coupling constant in medium, $g^*_A$, which can be obtained from the Landau mass (26) and the chiral mass (11) as

$$\frac{g^*_A}{g_A} = \left( 1 + \frac{1}{3} F_1^\pi \right)^2 = \left( 1 - \frac{1}{3} \Phi \tilde{F}_1^\pi \right)^{-2},$$

which at $\rho = \rho_0$ gives

$$g^*_A = 1.0(0).$$

This agrees well with the observations in heavy nuclei [40]. Again this is an intriguing result. While it is not understood how this relation is related to the old one in terms of the Landau-Migdal parameter $g'_0$ in $NN \leftrightarrow N\Delta$ channel [20], it is clearly a short-distance effect in the “pionic channel” involving the factor $\Phi$. This supports the argument [32] that the renormalization of the axial-vector coupling constant in medium cannot be described in low-order chiral perturbation theory.

Finally, the correction to the single-particle gyromagnetic ratio can be rewritten as

$$\delta g_l = \frac{4}{9} \left[ \Phi^{-1} - 1 - \frac{1}{2} \tilde{F}_1^\pi \right] \tau_3$$

where we have used (39) and the nonet relation $C_\rho^2 = C_\omega^2 / 9$. At $\rho = \rho_0$, we find

$$\delta g_l = 0.22(7) \tau_3.$$ 

This is in agreement with the result [41] for protons extracted from the dipole sum rule in $^{209}$Bi using the Fujita-Hirata relation [42]:

$$\delta g_l^{proton} = \kappa/2 = 0.23 \pm 0.03.$$

Here $\kappa$ is the enhancement factor in the giant dipole sum rule. Given that this is extracted from the sum rule in the giant dipole resonance region, this is a bulk property, so our theory is directly relevant.

Direct comparison with magnetic moment measurements is difficult since BR scaling is expected to quench the tensor force which is crucial for the calculation of contributions from high-excitation states needed to extract the $\delta g_l$. Calculations with this effect taken into account are not available at present. Modulo this caveat, our prediction (53) compares well with Yamazaki’s analysis [43] of magnetic moments in the $^{208}$Pb region

$$\delta g_l^{proton} \approx 0.33,$$

$$\delta g_l^{neutron} \approx -0.22$$

and also with the result of Arima et al. [44, 43]

$$\delta g_l \approx 0.25 \tau_3.$$ 

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We dedicate this article to Gerry Brown on the occasion of his 70th birthday. Much of the ideas discussed here came from discussions with him and were put into a precise form while one of the authors (MR) was visiting GSI as a “Humboldtpreisträger.” MR wishes to thank Wolfgang Nörenberg and the members of the Theory Group at GSI for hospitality and the Humboldt Foundation for support.
Appendix

In this appendix, we collect some results of Landau Fermi liquid theory for relativistic systems. The results are illustrated for a system with scalar and vector interactions, where the self-consistent mean-field approximation is tractable. In this approximation, the model is identical to the Walecka model as noted in [11]. To obtain the nonrelativistic results given in the main text, the long-range nonlocal pion exchange contribution has to be introduced as a perturbative correction.

The Landau effective mass is related to the quasiparticle velocity at the Fermi surface, i.e.,

\[ v_F = \frac{d\epsilon(p)}{dp} \bigg|_{p=p_F} = \frac{p_F}{m_L^*}. \]  

(A.1)

As shown by Baym and Chin [13], Lorentz invariance implies a relation between the effective mass and the quasiparticle interaction, analogous to the one derived by Landau for non-relativistic systems

\[ \frac{m_L^*}{\mu} = 1 + \frac{F_1}{3} = (1 - \frac{\bar{F}_1}{3})^{-1} \]  

(A.2)

where \( \bar{F}_1 = (\mu/m_L^*)F_1 \) and \( \mu \) is the baryon chemical potential. In the non-relativistic limit \( \mu = m_N \), and Landau’s result (23) is recovered.

In the mean-field approximation to the scalar-vector model, the single-particle energy is

\[ \epsilon(p) = \sqrt{p^2 + (m_N^*)^2 + C_\omega^2 \langle N^\dagger N \rangle}, \]

where \( m_N^* \) is determined by solving equation (16) and the last term is due to the time component of the vector interaction. By computing the quasiparticle velocity at the Fermi surface, one finds the Landau effective mass in this model,

\[ m_L^* = \sqrt{p_F^2 + (m_N^*)^2}. \]

On the other hand, the only velocity-dependent contribution to the quasiparticle interaction is due to the spatial part of the vector interaction. One finds [34]

\[ \bar{F}_1^\omega = -C_\omega^2 \frac{2p_F^3}{\pi^2 \sqrt{p_F^2 + (m_N^*)^2}}, \]  

(A.3)

i.e., eq. (24) with the appropriate \( m_L^* \). It is easy to check that this model satisfies the relativistic effective mass relation (A.2).

In an interacting system, the current \( \vec{J}_p \) carried by a quasiparticle with momentum \( \vec{p} \) is not necessarily equal to its velocity \( \vec{v}_p \). The difference between the quasiparticle velocity and the current is the so-called backflow current [45]. Diagrammatically the backflow current is associated with a polarization of the nuclear medium of the particle-hole type. The particle-hole pairs contribute to the current in the long-wave-length limit for \( \omega/q \to 0 \) and vanish in the opposite limit \( q/\omega \to 0 \). In the former limit the current is determined by the Ward identities to be equal to the quasiparticle velocity [46]. Physically, this limit corresponds to a localized quasiparticle excitation [15], while the gyromagnetic ratio corresponds to the opposite limit, i.e., to a homogeneous quasiparticle excitation.

The contribution of the particle-hole pairs to the current may be computed within Landau Fermi-liquid theory (see e.g. [30, 45, 46]). One then finds the total current in a relativistic

**Another argument for why this is the physically relevant limit is that it guarantees that the total charge of the system remains constant [14].**
system, e.g. by subtracting the particle-hole contribution from the quasiparticle velocity,

\[ \vec{J} = \begin{pmatrix} \vec{p}_m \star L \\ (1 + \frac{F_1}{3}) \frac{1 + \tau_3}{2} + \frac{1}{6} (F'_1 - F_1) \tau_3 \end{pmatrix} \]  

(A.4)

\[ \begin{pmatrix} \vec{p} \\ \frac{1 + \tau_3}{2} + \frac{1}{6} \frac{F'_1 - F_1}{1 + F_1/3} \tau_3 \end{pmatrix}. \]

This is the relativistic generalization of (30).

In the scalar-vector model, the quasiparticle velocity is

\[ v_F = \frac{p_F}{m^*_L} = \frac{p_F}{\sqrt{p_F^2 + (m_N^*)^2}}. \]

The isoscalar part of the current is renormalized by the velocity-dependent part of the interaction \( F_1 \), which corresponds to the spatial part of the vector interaction. Diagrammatically, this contribution to the current corresponds to summing up the diagrams shown in fig. 3. In this model, the isovector part of the interaction vanishes, so \( F'_1 = 0 \) and the isovector current is equal to the quasiparticle velocity. A vector-isovector interaction, like that generated by the exchange of a \( \rho \) meson, as well as the pion exchange gives rise to a non-vanishing \( F'_1 \).

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