Multi-Majoron Modes for
Neutrinoless Double-Beta Decay*

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Abstract

We construct two new classes of models for double beta decay, each of which leads to an electron spectrum which differs from the decays which are usually considered. One of the classes has a spectrum which has not been considered to date, and which is softer than the usual two-neutrino decay of the Standard Model. We construct illustrative models to show how other phenomenological bounds can be accommodated. We typically find that, although all other bounds can be satisfied, the predicted double-beta decay rate only in one class of these models is at best large enough to be just detectable in current experiments.

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1. Introduction

The purpose of this paper is to present two classes of double-beta ($\beta\beta$) decay, which have not been hitherto considered by workers in the field, and are potentially experimentally distinguishable from the presently-known kinds of decays. Besides presenting a general discussion of the phenomenology of $\beta\beta$ decay in the kinds of theories which we consider, we also explore the other kinds of experiments to which these models might be expected to contribute. In so doing we fill in the last missing items in the classification of $\beta\beta$ decay that was presented in ref. [1].

First, a brief motivation for this calculation.

The past ten years have seen a great deal of effort go into the experimental search for $\beta\beta$ decay, an extremely rare process in which two neutrons simultaneously decay into protons with the accompanied emission of two electrons. This process — when it occurs together with the emission of two antineutrinos ($\beta\beta_{2\nu}$) — is predicted to occur at second-order in the charged-current weak interactions in the Standard Model (SM). The experimental effort has borne fruit in recent years, with the unambiguous observation of this decay in a number of different nuclear isotopes [2].

Of course, part of the interest in these experiments was originally motivated by the possibility of observing other decays, besides the SM process. There are several other types of decay modes to which these experiments are sensitive, such as the lepton-number violating neutrinoless process ($\beta\beta_{0\nu}$), or the decay ($\beta\beta_{\varphi}$) into two electrons plus a neutral scalar, $\varphi$. Both of these reactions have been predicted by a number of plausible alternatives to the SM, starting with the simple Gelmini-Roncadelli (GR) model of lepton-number breaking [3], [4].

The experimental quantity that is used to distinguish such exotic decays from the run-of-the-mill SM events is the shape of the decay rate as a function of the energies, $\varepsilon_1$ and $\varepsilon_2$, of the two emitted electrons. This shape is simplest for $\beta\beta_{0\nu}$ decay, since in this case the rate is zero unless the sum of the two electron energies, $\varepsilon = \varepsilon_1 + \varepsilon_2$, takes on a specific value, $Q$. $Q$ represents the total energy that can be carried off by the two electrons, and is given in terms of the relevant masses — denoted respectively by $M$ and $M'$ for the parent and daughter nuclei — by $Q = M - M'$. In all of the decays of interest $Q \simeq 2$ MeV in size. Other decays (like $\beta\beta_{2\nu}$ and $\beta\beta_{\varphi}$), which involve other relativistic decay products in addition to the final electrons, instead predict a continuous decay distribution throughout
the entire interval $2m_e \leq \varepsilon \leq Q$.

The spectrum that is predicted by the various types of decays turns out, quite gener-
ally, to take a particularly simple form that is completely characterized by a single integer,
or ‘spectral index’, $n$. Explicitly:

$$\frac{d\Gamma}{d\varepsilon_1 d\varepsilon_2} = C(Q - \varepsilon_1 - \varepsilon_2)^n \left[ p_1\varepsilon_1 F(\varepsilon_1) \right] \left[ p_2\varepsilon_2 F(\varepsilon_2) \right],$$

where $C$ is independent of $\varepsilon_1$ and $\varepsilon_2$, and $p_i = |p_i|$, for $i = 1, 2$, represents the magnitude of the three-momentum of the corresponding electron. The quantity $F(\varepsilon_i)$ is the Fermi function which describes the spectral distortion due to the electric charge of the nucleus. The spectral shape which follows from eq. (1) is shown, for various choices of $n$, in Fig. (1).

It is ultimately the small size of the energy, $Q \sim 2$ MeV, in comparison with the typical momenta, $p_F \sim 60$ MeV, of the nucleons in the decaying nucleus that ensures that the electron spectrum takes such a simple form — i.e. one that is characterized simply by the integer $n$. This is because the small ratio of these two scales justifies keeping only the lowest term in an expansion of the decay rate in powers of the momenta of the final electrons (and/or scalars and neutrinos). The index, $n$, that is appropriate for any particular kind of decay is determined by the leading term in this expansion of the relevant decay rate. For example, for the SM process, $\beta\beta_{2\nu}$, the phase space volume element for the final neutrinos implies an index $n(SM) = 5$. For the scalar decay, $\beta\beta_{\phi}$, of the GR model, on the other hand, the phase space of the lone scalar implies $n(GR) = 1$. The difference between these predictions — c.f. Figure (1) — forms the basis for the experimental discrimination of these two models.

Two developments have recently provoked a theoretical re-evaluation [5], [6], [7] of the kinds of new physics to which the current experiments can be sensitive. The first of these was the advent of precision measurements of the properties of the $Z$ resonance at LEP [8], which has ruled out the original GR-type models for new physics. The other development has come from the $\beta\beta$ experiments themselves, starting with the appearance of several indications of an excess of electrons just below the endpoint in some of the experimental data for several types of decays [9]. Although the evidence for the excess has since diminished [10], the conclusion reached by the theoretical re-evaluation still stands: viable theories which can produce observable $\beta\beta$ decay are possible, and can be consistent with all other, non-$\beta\beta$, experimental limits, but only if the new physics has rather different properties than have previously been assumed based on experience with GR-type models.
One of the main surprises which arose from this theoretical re-examination has been the observation that new physics can be very generally divided into a small number of classes — depending on how it addresses a few basic issues — and that the experimental features that any model predicts for $\beta\beta$ decay is a robust indicator of the class to which it belongs [1]. (These issues and categories are summarized in Table I.\(^1\))

| $L_e$       | A New Scalar:                              | $\beta\beta_{0\nu}$ | Dominant   | Spectral Scalar Decay | Spectral Index |
|-------------|--------------------------------------------|-----------------------|------------|-----------------------|----------------|
| IA Broken   | Does Not Exist                             | Yes                   | None       | N.A.                  |                |
| IB Broken   | Is Not a Goldstone Boson                   | Yes                   | $\beta\beta_\phi$ | $n = 1$              |                |
| IC Broken   | Is a Goldstone Boson                       | Yes                   | $\beta\beta_\phi$ | $n = 1$              |                |
| ID Broken   | Is Not a Goldstone Boson                   | Yes                   | $\beta\beta_\phi\phi$ | $n = 3$             |                |
| IE Broken   | Is a Goldstone Boson                       | Yes                   | $\beta\beta_\phi\phi$ | $n = 3$             |                |
| IIA Unbroken| Does Not Exist                             | No                    | None       | N.A.                  |                |
| IIB Unbroken| Is Not a Goldstone Boson ($L_e = -2$)       | No                    | $\beta\beta_\phi$ | $n = 1$              |                |
| IIC Unbroken| Is Not a Goldstone Boson ($L_e = -1$)       | No                    | $\beta\beta_\phi\phi$ | $n = 3$             |                |
| IID Unbroken| Is a Goldstone boson ($L_e = -2$)           | No                    | $\beta\beta_\phi$ | $n = 3$              |                |
| IIE Unbroken| Is a Goldstone boson ($L_e = -1$)           | No                    | $\beta\beta_\phi\phi$ | $n = 7$             |                |

Table I

A list of alternatives for modelling double beta decay.

As is argued in ref. [1], the observed absence of $\beta\beta_{0\nu}$ decays at the endpoint of the electron spectrum strongly constrains the classes of models for which lepton number is broken (i.e. classes IA – IE). In particular, it implies that all of the scalar-emitting decays in these classes are essentially indistinguishable from their counterparts in the lepton-number preserving classes. That is to say, so long as $\beta\beta_{0\nu}$ decays are not observed, models

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\(^1\) The table presented here differs slightly from the table in ref. [1]. A trivial change is our introduction of new categories ID and IE, even though these are in practice indistinguishable from IIC. Also, the index listed here for category IIE reflects the results of the present paper, and differs from the preliminary guess for this value which was given in ref. [1].
in class IB and IC are at present experimentally indistinguishable from those in class IIB. Similarly, classes ID and IE cannot be distinguished from class IIC.

Interestingly, all of the models which had been considered previously, as well as most of the more recent proposals, fall into only a few of the several possible categories — cases IB and IC of Table I. The other, unorthodox, categories remain essentially unexplored. The two exceptions to this statement are case IID, which was discovered and studied in ref. [5], and case IE, which contains a supersymmetric model [11], in which two scalars are emitted in \( \beta \beta \) decay: \( \beta \beta_{\varphi \varphi} \). Unfortunately, this last model has since been ruled out by the measurements at LEP. Both of these new classes of models predict a \( \beta \beta_{\varphi} \) spectrum having \( n = 3 \). The resulting shape is intermediate between the usual SM and GR spectra, and can be distinguished from these in current experiments [12], [13].

The purpose of the present paper is to explore in detail those classes of theories of Table I which have not been considered to date. Keeping in mind the indistinguishability of classes IB, IC and IIB, as well as ID, IE and IIC, we see there are two types of models which we must here consider: those of classes IIC and IIE, both of which involve \( \beta \beta \) decay accompanied by the emission of \( \text{two} \), rather than just one, scalar particles. Two scalars must be emitted because of lepton-number conservation, and the charge assignments of the light particles. Models in these two categories differ from one another according to whether or not the light scalar particle that is emitted is a Goldstone boson.

We first examine models of category IIC to show that these can be altered to evade detection at LEP, and we then work through the completely new category, IIE, which predicts a spectral index, \( n = 7 \), that is completely different from all previous cases. Since this index is \textit{larger} than that for the SM decay — \( n(SM) = 5 \) — the resulting spectral shape is \textit{softer} than the presently-observed \( \beta \beta_{2\nu} \) spectra. This kind of softer spectrum is much more difficult to distinguish from the experimental background, and so we expect the experimental limit for the branching ratio into the new \( n = 7 \) decay mode to be comparatively weak.

On the theoretical side, we find that the nuclear form factors which appear in all of the new \( \beta \beta_{\varphi \varphi} \) decays are well understood, since they are the same as those that appear in the more well-known decay modes, such as for \( \beta \beta_{0\nu} \) and \( \beta \beta_{\varphi} \) in the GR model. We find that using these matrix elements to estimate the potential size of the \( \beta \beta_{\varphi \varphi} \) decay rate typically gives a result which can be close to the present experimental limit for the \( n = 3 \)
decays, provided we relax the conditions that are cast on the relevant masses by big bang nucleosynthesis constraints. The estimated rate for the \( n = 7 \) decays appears to be too small to be detectable.

This paper has the following organization. The next section computes the \( \beta\beta \phi\phi \) decay rate for the two classes of decays (IIC and IIE). We do so to verify their spectral index — which is their experimental signature — and to determine how the resulting rates depend on the couplings in underlying models. For class IIC (having \( n = 3 \)) we consider two types of decay mechanisms: those which proceed due to the exchange of new sterile neutrinos, and those which proceed through the interactions of new scalar particles. Representative models for decays in class IIC are then constructed in sections 3, and those for class IIE in section 4. Section 5 then summarizes the most important phenomenological bounds for these models. Our conclusions are finally summarized in section 6.

2. Two-Scalar \( \beta\beta \) Decays

Before constructing models in detail, we first pause here to record expressions for the decay rate into two electrons plus two scalars, \( \Gamma(\beta\beta\phi\phi) \). We do so in order to display how the \( \beta\beta\phi\phi \) rate depends on the various masses and couplings of the theory. We may then use this dependence to see what kinds of new particles can be used to generate an observable signal in \( \beta\beta \) experiments. We defer the discussion of any further phenomenology, which requires the construction of an explicit model, to the following sections.

We consider here three kinds of scenarios. First we consider two types of models for which \( \beta\beta\phi\phi \) decay is mediated by the admixture of ordinary neutrinos with new sterile fermions. The two kinds of such decays which we consider are those for which the scalar emission is either derivatively suppressed (i.e. case IIE, with \( n = 7 \)), or not (case IIC with \( n = 3 \)). Finally we consider the alternative for which \( \beta\beta\phi\phi \) decay occurs because of the interactions of new types of scalar particles. Such scalar-mediated decays can also arise with or without derivative suppression for the final scalar emission (classes IIE or IIC), but since the decay rate for the class IIE models are generally very small, we do not pursue them in detail for the scalar-mediated case.

2.1) Fermion-Mediated \( n = 3 \) Decays

Consider first a theory of neutrinos, \( \nu_i \) and \( N_a \), which respectively carry lepton number
$L_e(\nu_i) = +1$ and $L_e(N_a) = 0$, and which are coupled to a scalar, $\phi$, which has lepton number $L_e(\phi) = +1$. The most general renormalizable and $L_e$-conserving Yukawa couplings involving these fields is:

$$
\mathcal{L}_{\text{yuk}} = -\overline{\nu}_i (A_{ia} \gamma_L + B_{ia} \gamma_R) N_a \phi + \text{h.c.},
$$

(2)

where $A_{ia}$ and $B_{ia}$ represent arbitrary Yukawa-coupling matrices, and $\gamma_L$ and $\gamma_R$ denote the usual projections onto left- and right-handed spinors. We use majorana spinors here to represent the neutrinos, so the conjugate which appears in eq. (2) is $\overline{\nu}_i = \nu_i^\dagger C^{-1}$, where $C$ is the charge-conjugation matrix. Suppose also that a number of right-handed neutrinos are included in the theory, in order to form generic lepton number conserving masses, $m_{\nu_i}$, for the $L_e = 1$ states, and that the $L_e = 0$ neutrinos, $N_a$, have generic majorana masses, $m_{N_a}$.

As long as any of the $\nu_i$'s participate in the charged-current weak interactions, the couplings of eq. (2) generically produce $\beta\beta\phi\phi$ decay due to the Feynman graph of Fig. (2). (Two scalars must be emitted in this decay due to conservation of $L_e$.) Even though Fig. (2) arises at tree level, the four-momentum of the exchanged neutrino is not determined by energy and momentum conservation. This is because the amplitude for the decay of the two initial neutrinos must be convoluted with the wavefunction of the initial nucleus. The important momentum scale in the integration over the exchanged neutrino is therefore set by the scale of momenta to which the nucleons within the initial and final nuclei have access. Since this scale is much larger than the energy, $Q$, that is available for the final electrons and scalars, it is a good approximation to neglect the outgoing scalar and electron energies in the decay amplitude. Evaluating the graph in this approximation gives the following expression for the $\beta\beta\phi\phi$ decay rate:

$$
d\Gamma(\beta\beta\phi\phi) = \left(\frac{G_F \cos \theta_c}{8(2\pi)^5}\right)^4 |A(\beta\beta\phi\phi)|^2 (Q - \varepsilon_1 - \varepsilon_2)^3 \prod_{k=1}^2 p_k \varepsilon_k F(\varepsilon_k) \, d\varepsilon_k,
$$

(3)

where $G_F$ is Fermi’s constant, and $\theta_c$ is the Cabibbo angle. The quantity $A(\beta\beta\phi\phi)$ represents the integral:

$$
A(\beta\beta\phi\phi) = \left(\frac{2}{3\pi^2}\right)^{\frac{1}{2}} \sum_{ija} \int \frac{d^4\ell}{(2\pi)^4} \frac{V_{\nu_i} V_{\nu_j} N_{ija} W^\mu_\mu}{(\ell^2 + m_{\nu_i}^2 - i\varepsilon)(\ell^2 + m_{\nu_j}^2 - i\varepsilon)(\ell^2 + m_{N_a}^2 - i\varepsilon)},
$$

(4)
where $V_{e
u_i}$ is the Kobayashi-Maskawa-type matrix for the leptonic charged current. The factor, $N_{ija}$, in the numerator of eq. (4) denotes the following expression:

$$N_{ija} \equiv (-\ell^2) \left[ A_{ia}B_{ja}m_{\nu_i} + A_{ja}B_{ia}m_{\nu_j} + B_{ia}B_{ja}m_{N_a} \right] + A_{ia}A_{ja}m_{\nu_i}m_{\nu_j}m_{N_a}. \quad (5)$$

Finally, the tensor $W_{\mu\nu}$ is an independent function of the space- and time-like parts, $\ell^0$ and $|\ell|$, of $\ell_\mu$ in the rest frame of the decaying nucleus, which encodes the nuclear matrix element that is relevant to the decay [5]:

$$W_{\mu\nu} \equiv (2\pi)^3 \sqrt{\frac{EE'}{MM'}} \int d^4x \langle N'| T^\ast [J_\mu(x)J_\nu(0)] |N \rangle e^{i\ell \cdot x}. \quad (6)$$

Here $J_\mu = 2\pi\gamma_\mu \gamma_L d$ denotes the appropriate hadronic charged current. The prefactor involving the energy-to-mass ratio — $E/M$ and $E'/M'$, for the parent and daughter nuclei respectively — is required in order to ensure that $W_{\mu\nu}$ transforms under Lorentz transformations as a tensor. The factor of $(2\pi)^3$ is purely conventional.

There are several features in these expressions that are noteworthy.

1. Comparison of eqs. (3) and (1) shows that the spectral index for this decay is $n = 3$.

2. Eq. (4) depends on precisely the same combination of nuclear form factors, $W_{\mu\mu} = w_F - w_{GT}$, as do $\beta\beta_{2\nu}$ and $\beta\beta_{0\nu}$ decays, as well as $\beta\beta_\phi$ decay in GR-type models [5]. (For future use we express $W_{\mu\mu}$ in terms of the Fermi and Gamow-Teller matrix elements, $w_F = W_{00}$ and $w_{GT} = \sum_{i=1}^3 W_{ii}$. Ref. [5] gives $w_F$ and $w_{GT}$ as explicit expressions in terms of the nuclear matrix elements of various neutrino potentials.) Since these particular combinations of nuclear matrix elements have been well studied within the context of these other processes [14], [15], [16], there is relatively little uncertainty in the estimate for the total rate for this type of $\beta\beta_{\phi\phi}$ decay.

3. The differential decay distribution for this decay as a function of the opening angle, $\theta$, between the two electrons is exactly the same as it is for $\beta\beta_{2\nu}$, $\beta\beta_{0\nu}$ and $\beta\beta_\phi$ decays (of both the GR type and the new $\beta\beta_\phi$ decays of type IID). It is given explicitly (neglecting the mutual repulsion of the outgoing electrons) by

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta} = \frac{1}{2} (1 - v_1 v_2 \cos \theta), \quad (7)$$
where \( v_i = p_i/\varepsilon_i \), for \( i = 1, 2 \), is the speed of the corresponding electron in the nuclear rest frame.

2.2) Fermion-Mediated \( n = 7 \) Decays

We next consider the case of double scalar emission, but in the special case where the amplitude for emitting the scalars is suppressed by the scalar energy. This is the case when the scalars have purely derivative couplings, such as when the scalar is a Goldstone boson which carries an unbroken lepton number [5]. For such models the expression given in eq. (4) for the matrix element, \( A \), vanishes identically, and so it is necessary to work to higher order in the external scalar momenta in order to obtain a nonvanishing rate. Since it turns out that the first nonvanishing contribution to the amplitude arises at quadratic order in the scalar momenta, the calculation is quite cumbersome when performed directly with the couplings of eq. (2).

A better way to proceed is to perform a change of variables so that the derivative coupling nature of the Goldstone bosons is manifest from the outset. Once this has been done, the trilinear coupling to neutrinos of a Goldstone boson carrying \( L_e = 1 \), becomes:

\[
\mathcal{L}_{gb} = -i \bar{\nu}_i \gamma^\mu (X_{ia} \gamma_L + Y_{ia} \gamma_R) N_a \partial_\mu \phi + \text{h.c.},
\]

where the coefficients \( X_{ia} \) and \( Y_{ia} \) are computable [5] in terms of \( A_{ia} \) and \( B_{ia} \) of eq. (2), as well as the components of the broken-symmetry generator, \( Q \), for which \( \phi \) is the Goldstone boson.

The contribution to \( \beta \beta \varphi \varphi \) decay in these models arises from the same Feynman graph as before, Fig. (2), but with the interaction of eq. (8) at each vertex. Evaluation of this graph gives a fairly complicated expression for the decay rate, but this expression simplifies greatly in the special case of purely left-handed couplings, for which \( Y_{ia} = 0 \). Since this case is sufficient to analyse the models we consider in later sections, we just quote here the decay rate in the limit \( Y_{ia} = 0 \):

\[
d\Gamma(\beta \beta \varphi \varphi) = \frac{(G_F \cos \theta_c)^4}{8(2\pi)^5} \left| \tilde{A}(\beta \beta \varphi \varphi) \right|^2 (Q - \varepsilon_1 - \varepsilon_2)^7 \prod_{k=1}^2 p_k \varepsilon_k F(\varepsilon_k) \, d\varepsilon_k,
\]
where the new quantity, $\tilde{A}(\beta\beta\phi)$, represents the integral:

$$\tilde{A}(\beta\beta\phi) = \left(\frac{4}{105\pi^2}\right)^{\frac{1}{2}} \sum_{ija} \int \frac{d^4\ell}{(2\pi)^4} \frac{V_{e\nu_i}V_{e\nu_j}\tilde{N}_{ija}W_{\mu\mu}}{(\ell^2 + m_{\nu_i}^2 - i\epsilon)(\ell^2 + m_{\nu_j}^2 - i\epsilon)(\ell^2 + m_{N_a}^2 - i\epsilon)}. \quad (10)$$

Apart from its normalization, eq. (10), differs from eq. (4), obtained previously, only by the replacement $N_{ija} \rightarrow \tilde{N}_{ija}$, where:

$$\tilde{N}_{ija} \equiv (-\ell^2)X_{ia}X_{ja}m_{N_a}. \quad (11)$$

Once again there are several properties of the above expressions that bear emphasis.

1. The spectral index for this decay is $n = 7$. The four additional powers of $(Q - \epsilon)$, in comparison with eq. (3), arise because each scalar-emission vertex of eq. (8) is explicitly proportional to the scalar four-momentum.

2. The decay rate depends only on the form factor combination $W_{\mu\mu} = w_F - w_{GT}$, just as for the non-derivatively coupled calculation. (This is by contrast with the situation for $\beta\beta\phi$ decay, where the matrix elements for the derivatively-suppressed decay — class IID — differ from those for the emission of a generic scalar — class IIB.)

3. The differential decay distribution as a function of the electron opening angle remains exactly the same as for all of the other decays, as in eq. (7).

2.3) Scalar-Mediated $n = 3$ Decays

The previous calculations produced similar expressions for the decay amplitudes — c.f. eqs. (4) and (10) — largely because the same Feynman graph is responsible for it in both classes of models. In this section we explore an alternative class of models for which $\beta\beta\phi$ decay proceeds via a different process: the exchange of an exotic set of scalars.

Consider therefore supplementing the SM by a collection of new scalars, which we classify using their lepton and electric-charge quantum numbers, $L_e$ and $Q$. Besides the very light $(Q, L_e) = (0, +1)$ particle, $\varphi$, which is emitted in the decay, we imagine also adding two other new classes of fields, $\chi_i$ and $\Delta_a$, which respectively have the $(Q, L_e)$ assignments: $(+1, -1)$ and $(-2, +2)$. These quantum numbers permit the following types
of $L_e$-invariant trilinear-scalar, and Yukawa couplings:

$$\mathcal{L}_{\text{tri}} = -\frac{1}{2} \mu^{ij} \chi_i \chi_j \Delta_a + \text{h.c.}, \quad (12)$$

$$\mathcal{L}_{\text{yuk}} = -\frac{1}{2} \Sigma (h_L^a \gamma_L + h_R^a \gamma_R) e^c \Delta_a + \text{h.c.}, \quad (13)$$

in addition to the charged-current coupling

$$\mathcal{L}_{\text{sc}} = -i\lambda^i g W^\mu \left( \chi_i \partial_\mu \varphi - \partial_\mu \chi_i \varphi \right) + \text{h.c.}. \quad (14)$$

Here $e^c$ denotes the Dirac conjugate of the electron field, and $\lambda^i$ parameterizes the strength of the scalar charged-current interaction relative to the usual $SU_L(2)$ gauge coupling, $g$. For the sake of keeping our final expressions simple, we assume from here on that the index ‘$a$’ takes only one value, so that there is only one field, $\Delta$, having the quantum numbers $(Q, L_e) = (-2, +2)$. We therefore omit this index from the coupling parameters as these are defined above: $i.e.- \mu^{ij} \rightarrow \mu^{ij}$, and $h_{L,R}^a \rightarrow h_{L,R}$. We also denote the mass eigenvalues for $\chi_i$ and $\Delta$ respectively by $M_i$ and $M_\Delta$.

With these couplings and interactions $\beta\beta\varphi\varphi$ decay is mediated by the Feynman graph of Fig. (3). Evaluation of this graph gives the following decay rate:

$$d\Gamma_s(\beta\beta\varphi\varphi) = \frac{(G_F \cos \theta_c)^4}{8(2\pi)^5} |\mathcal{A}_s(\beta\beta\varphi\varphi)|^2 (Q - \epsilon_1 - \epsilon_2)^3 \prod_{k=1}^2 p_k \epsilon_k F(\epsilon_k) \, d\epsilon_k, \quad (15)$$

where the scalar-mediated decay amplitude, $\mathcal{A}_s(\beta\beta\varphi\varphi)$, is:

$$\mathcal{A}_s(\beta\beta\varphi\varphi) = \left( \frac{32}{3\pi^2} \right)^{\frac{1}{2}} \frac{h R}{M_\Delta^2} \sum_{ij} \int \frac{d^4\ell}{(2\pi)^4} \frac{\lambda^i \lambda^j \mu^{ij}}{(\ell^2 + M_i^2 - i\epsilon)(\ell^2 + M_j^2 - i\epsilon)} \left[ w_\epsilon \ell_0^2 + \frac{1}{3} w_\epsilon \Gamma^2 \right]. \quad (16)$$

In this expression $h = \sqrt{|h_L|^2 + |h_R|^2}$, and

$$R = \left( 1 + \frac{\xi m_e^2}{(h)^2 \epsilon_1 \epsilon_2} \right)^{\frac{1}{2}}, \quad (17)$$

with $\xi = 2 \text{Re} [h_L h_R^*]$ and $m_e$ denoting the electron mass.
These expressions differ from those for the fermion-mediated decays in a number of interesting ways.

1. The spectral index for this decay is determined by phase space, and is \( n = 3 \). This is because the assumed \( \varphi \) couplings are not derivatively suppressed. A class of similar \( n = 7 \) decays can be obtained by imposing the additional requirement of derivative coupling. We do not consider such decays explicitly here.

2. Once more the decay rate depends only on the familiar form factors, \( w_F \) and \( w_{GT} \), although this time they do not arise in the particular linear combination \( W^\mu_\mu = w_F - w_{GT} \).

3. If the \( \Delta \)-electron coupling has a right-handed component, \( h_R \neq 0 \), then the differential decay distribution as a function of the electron opening angle differs from that of all of the other decays we have considered. Assuming only one species of \( \Delta \), its explicit expression is,

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta} = \frac{1}{2} \left( 1 - \frac{v_1 v_2}{R^2} \cos \theta \right), \tag{18}
\]

so the deviation of the decay distribution from that of eq. (7) is controlled by the difference between \( R \) and unity.

We next turn to the construction of some underlying models for each of these types of decays.

3. Model-Building: The Case of Spectral Index \( n = 3 \).

We first consider models for which the dominant contribution to \( \beta \beta \) decay is through the emission of two scalars, but for which the emission of these scalars is not suppressed by powers of the scalar momentum, i.e. models of type IIC. As was mentioned earlier, and as is discussed in some detail in refs. [1] and [5], this also includes those models for which the light scalar is the Goldstone boson for lepton number (class IE), even though the couplings of such a scalar can always be put into the derivative form of eq. (8).\(^2\)

In the following sections we consider a few representative models which produce this kind of decay.

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\(^2\) Briefly, with derivatively-coupled variables the dominant contribution to \( \beta \beta \) decay for small scalar momentum comes from graphs in which the scalar is emitted from the electron line, for which an infrared singularity compensates for the derivative coupling.
3.1) Decays Mediated by Sterile Neutrinos

A mechanism for producing $\beta\beta\phi\phi$ decay, without running into conflict with the LEP bounds, is to require the light scalar to be an electroweak singlet, which couples to the usual electroweak neutrinos through their mixing with various species of sterile neutrinos. This results in a variation of the old singlet-majoron model [17]. In this way an unacceptable contribution to the invisible $Z$ width can be avoided. A simple realization of this idea requires the introduction of three species of electroweak-singlet, left-handed neutrinos: $N_{\pm}$ and $N_0$, together with a single scalar field, $\phi$, carrying lepton number +1. The subscript $i=0,\pm$ on $N_i$ indicates the lepton number of the corresponding left-handed state.

Ignoring, for simplicity, all mixings with $\nu_\mu$ and $\nu_\tau$, we are led to the following, most general, lepton-number-invariant mass and Yukawa terms involving the new particles:

\[
\begin{align*}
\mathcal{L}_m &= -M(N_{\pm}\gamma_L N_-) - \frac{m}{2}(N_0\gamma_L N_0) + \text{h.c.}, \\
\mathcal{L}_y &= -\lambda(N_-\gamma_L L) H - g_-(N_-\gamma_L N_0) \phi - g_+(N_{\pm}\gamma_L N_0) \phi^* + \text{h.c.} \\
\end{align*}
\]

(19)

Here $L = (\nu_e e)$ and $H = (H_0^+)$ represent the usual SM lepton and Higgs doublets. (Where necessary the conjugate Higgs doublet is denoted by $\bar{H} = i\sigma_2 H^*$."

Finally, recall that our use of majorana spinors implies $\bar{N}_i = N_i C^{-1}$.

Although, in general, both $H^0$ and $\phi$ could acquire vacuum expectation values (vev’s): $\langle H^0 \rangle = v$ and $\langle \phi \rangle = u$, in practice it is a good approximation to work in the limit $u \approx 0$. This is because the absence of an observable $\beta\beta_{0\nu}$ decay signal implies the combination $g_i u$ must be small. On the other hand, the couplings $g_i$ themselves cannot be too small or else the $\beta\beta\phi\phi$ decay of interest here will itself be unobservable. As a result the vev $u$ must itself be constrained to be negligible in comparison to the other masses of the problem. We therefore neglect $u$ in all of what follows.\(^3\) We nevertheless assume that the scalar, $\phi$, is light enough to permit $\beta\beta\phi\phi$ decay to occur. This can be done by tuning the mass in the potential to be small, or by permitting $u \neq 0$ but with $u$ tuned to be small enough to not produce too much $\beta\beta_{0\nu}$ decay. Either choice involves a certain amount of arbitrariness.

The neutrino mass spectrum for this model greatly simplifies in the $u = 0$ limit. There is one $L_e = 0$ mass eigenstate, $N_0$, with mass $m$; there is a massless state, $\nu'_e = \nu_e c\theta - N_+ s\theta$;

\(^3\) Notice that, unlike the choice $u=0$, a very small but nonzero value for $u$ is quite difficult to make natural, even in the technical sense [5], [18].
and there is a massive, Dirac neutrino, $\nu_s = \gamma_\mu N'_+ + \gamma_\mu N'_-$, with $N'_+ = N_+ c_\theta + \nu_e s_\theta$ and having mass $M_s = \sqrt{M^2 + \lambda^2 v^2}$. Here $c_\theta$ and $s_\theta$ denote the cosine and sine of the mixing angle, $\theta$, which is defined by: $\tan \theta = \lambda v / M$.

The Yukawa couplings of these mass eigenstates take the form of eq. (2), with

$$A_{\nu'_e N_0} = 0, \quad A_{\nu_s N_0} = g_-, \quad B_{\nu'_e N_0} = -g_+ s_\theta, \quad B_{\nu_s N_0} = g_+ c_\theta. \quad (20)$$

Using these expressions in the general result, eq. (4), then gives:

$$A(\beta\beta\varphi\varphi) = \left(\frac{2}{3\pi^2}\right)^{\frac{1}{2}} M_s^2 s_\theta \int \frac{d^4 \ell}{(2\pi)^4} \frac{W^\mu_{\mu}(g^2_- m \ell^2 + 2g_+ g_- c_\theta M_s \ell^2 - g^2_+ c^2_\theta M_s^2 m)}{(\ell^2 - i\epsilon)(\ell^2 + M_s^2 - i\epsilon)^2 (\ell^2 + m^2 - i\epsilon)}. \quad (21)$$

This expression implies that the $\beta\beta\varphi\varphi$ decay rate is maximized for large couplings and mixing angles, and for sterile-neutrino masses in the vicinity of the nuclear-physics scale, $\sim (10 - 100)$ MeV, which defines the important integration region in eq. (21). In order to see how big this rate can get we therefore have to see how close we can get to this optimal range using phenomenologically acceptable values for the various parameters. The relevant constraints are summarized in section 5, and they suggest that the amplitude can be made biggest for the following values of masses and mixing angles:

$$M_s \sim 350 \text{ MeV}; \quad m \sim 1 \text{ MeV}; \quad \theta \sim 7.5 \times 10^{-2}; \quad g_+ \sim 0.2 \quad (22)$$

or, provided we relax the bound on $m$ coming from nucleosynthesis (as explained in section 5)

$$M_s \sim m \sim 350 \text{ MeV}; \quad \theta \sim 7.5 \times 10^{-2}; \quad g_+ \sim 1 \quad (23)$$

Interestingly, the coupling constant $g_-$ need not be large. Notice also that the smallness of $\theta$ implies $\lambda v \ll M_s \approx M$.

Using these values, and recalling that the integration momentum satisfies $\ell \lesssim 60$ MeV $\ll M_s$, we find eq. (21), to be given approximately by

$$A(\beta\beta\varphi\varphi) = \left(\frac{2}{3\pi^2}\right)^{\frac{1}{2}} g^2_+ s^2_\theta c^2_\theta m \int \frac{d^4 \ell}{(2\pi)^4} \frac{W^\mu_{\mu}}{(\ell^2 - i\epsilon)(\ell^2 + m^2 - i\epsilon)}. \quad (24)$$
For historical reasons, experimentalists conventionally state their results concerning the nonobservation of a scalar-emitting decay in terms of the old Gelmini-Roncadelli type $\beta\beta$ decay. The quantitative limit is expressed in terms of an upper bound to a dimensionless effective coupling constant, $g_{\text{eff}}$, which parametrizes the electron-neutrino/majoron interaction in the GR model:

$$L_{\text{phen}} = \frac{i g_{\text{eff}}}{2} \bar{\nu}_e \gamma_5 \nu_e \varphi.$$  \hspace{1cm} (25)

This interaction gives a $\beta\beta$ decay rate which can be obtained from eq. (3) by performing the replacement: $A(\beta\beta\varphi) (Q - \varepsilon_1 - \varepsilon_2)^3 \rightarrow A(\text{GR}) (Q - \varepsilon_1 - \varepsilon_2)$, with

$$A(\text{GR}) = 4i\sqrt{2} g_{\text{eff}} \int \frac{d^4 \ell}{(2\pi)^4} \frac{W^{\mu\mu}}{\ell^2 - i\epsilon}.$$  \hspace{1cm} (26)

Comparing this prediction with the results of current experiments gives an upper bound $g_{\text{eff}} \lesssim 10^{-4}$ [13].

To obtain a rough estimate of the sensitivity of current experiments to the $\beta\beta\varphi$ signal predicted by the model, we compare the $\beta\beta\varphi$ decay rate with the $\beta\beta$ rate which would be just detectable for the GR model, given the current experimental limit $g_{\text{eff}} \lesssim 10^{-4}$. For this kind of comparison the precision of a detailed matrix element analysis is unnecessary, so we follow Refs. [18] and [19] and make the following two approximations, which permit an analytical calculation of the various rates.

1. To perform the integral over $\ell$, we approximate the nuclear matrix elements by replacing the form factors, $w_F$ and $w_{GT}$ by step functions in energy and momentum: $w_i \approx w_i^0 \Theta(E_F - l_0) \Theta(p_F - |l|)$. $\beta\beta_{2\nu}$ decay rates are reproduced if we take $|w_F^0 - w_{GT}^0| \sim 4$ MeV$^{-1}$, and the present experimental sensitivity to an electron-neutrino majorana mass in $\beta\beta_{0\nu}$ decay requires a nuclear Fermi momentum of $p_F \sim 60$ MeV. The Fermi energy then becomes $E_F = p_F^2/2m_N \sim 2$ MeV. For example, with these approximations the decay amplitude for the GR model becomes $A(\text{GR}) \approx [8\sqrt{2}/(2\pi)^3] g_{\text{eff}} (w_F^0 - w_{GT}^0) p_F E_F$.

2. To perform the phase-space integrals, $P_n = \int d\varepsilon_1 d\varepsilon_2 \varepsilon_1 \varepsilon_2 (Q - \varepsilon_1 - \varepsilon_2)^n$, we neglect both the electron mass, $m_e$, and the Fermi functions, $F(\varepsilon_i)$. The integrals then become: $P_1 = Q^7/1, 260; P_3 = Q^9/15, 120; P_5 = Q^{11}/83, 160; \text{and } P_7 = Q^{13}/308, 880$.

With these estimates, the effective equivalent GR coupling for which the $\beta\beta\varphi$ decay
rate equals the $\beta\beta_{\varphi\varphi}$ decay rate is
\[ g_{\text{eff}}(\beta\beta_{\varphi\varphi}) \sim (0.008) g_+ s_\theta^2 c_\theta^2 \left(\frac{Q}{p_F}\right) \approx 10^{-8} \ (10^{-6}). \] (27)

With eq.(22) (respectively (23)) in mind, we take $g_+ \sim 0.2$ (respectively $g_+ \sim 1$), $s_\theta \sim 0.1$, $Q \sim 2$ MeV, $m \sim 2$ MeV (respectively $m \sim 350$ MeV) and $p_F \sim 60$ MeV in obtaining this number. Although a coupling of $10^{-8}$ is probably hopeless to observe in present experiments (which can detect $g_{\text{eff}} \gtrsim 10^{-4}$), one at the level of $10^{-6}$ is close enough to observability to warrant a more detailed comparison of this type of model with the data.

### 3.2) Decays Mediated by Virtual Scalars

We next turn to models for which the $\beta\beta_{\varphi\varphi}$ decay is mediated by the self couplings in the scalar sector, without the need for any extra heavy sterile leptons. We argue in this section that $\beta\beta_{\varphi\varphi}$ decay in these models is always constrained by LEP data to be too small to be observed. To see how this works, consider an example of such a model in which the standard model is extended by the inclusion of: (i) a second SM Higgs doublet, $\chi$, which has non-zero lepton number $L = -1$; (ii) a singlet Higgs boson field, $\phi$, with lepton number $L = +1$, and (iii) a weak isotriplet scalar field, $\Delta$, with $L = -2$. These charge assignments permit the couplings considered in the previous section: the $\Delta$ Yukawa couplings contain a term of the form $L L \Delta$, which couples it to the electron, and the Higgs potential contains the following terms which are relevant to $\beta\beta_{\varphi\varphi}$ decay:
\[ V_{\text{tri}} = \mu_1 H^\dagger \chi \phi + \frac{\mu_2}{2} \chi^T \Delta^\dagger \chi + \text{h.c.}. \] (28)

Provided that the singlet field, $\phi$, is sufficiently light, the diagram of Fig. (3) then induces $\beta\beta_{\varphi\varphi}$ decay. In this model the extra scalar fields do not have nonzero vev’s so that lepton number remains a good symmetry and the usual neutrinos remain massless, as in the SM.

Neglecting $E^2_F u^0_F$ in comparison to $p_F^2 u^0_{GT}$, we find the approximate form for the amplitude of eq. (16):
\[ A_s(\beta\beta_{\varphi\varphi}) \sim \frac{1}{(2\pi)^3} \frac{2}{3} \sqrt{\frac{32}{3\pi^2}} \frac{s_\theta^2 h\mu_2 u^0_{GT} p_F E_F}{M^2_\Delta} F(p_F, M_{\chi}), \] (29)
where $s_\theta \propto \mu_1$ is the mixing angle which controls the strength of the mixing between the light mass eigenstate, $\varphi$, and the singlet field, $\phi$. $F(p_F, M_\chi)$ is the function obtained by performing the $\ell$ integration, which takes the values $F \sim 1$ for $M_\chi \lesssim p_F$ and $F \sim (p_F/M_\chi)^4$ for $M_\chi \gg p_F$. With these expressions we find:

$$g_{\text{eff}} s(\beta\beta \varphi\varphi) \approx (0.02) \frac{|w_0^{\ell\ell}|}{|w_0^0 - w_0^{GT}|} h s^2_\theta \left( \frac{\mu_2 Q}{M^2_\Delta} \right) F(p_F, M_\chi).$$  \hfill (30)

As usual, the rate is largest if all of the exotic scalars have couplings that are as large as possible, and masses that are in the range of $(10 - 100)$ MeV. This range is particularly dangerous for the phenomenology of the present model, however, since the new scalars couple to the photon and to the $Z$. Any such particles having masses $\lesssim 50$ GeV would contribute unacceptably to $e^+ e^-$ annihilation, and so are ruled out. The effective coupling, $g_{\text{eff}}$, for scalar-mediated $\beta\beta \varphi\varphi$ decay rate is therefore suppressed at least by the factors $(Q/M_\Delta) (p_F/M_\chi)^4 \sim 10^{-16}$, and so is much too small to be detectable.

Since the suppression factor due to the large scalar masses is so devastatingly small, this rules out observable scalar-mediated $\beta\beta \varphi\varphi$ decays in a much larger class of models than those we have directly considered. That is, one might entertain models for which scalar-mediated $\beta\beta \varphi\varphi$ decay does not involve the $W$ boson at all. After all, Fig. (3) works equally well if the longitudinal parts of the $W$ lines are replaced by yet another exotic scalar which couples directly to quarks. In this case the nuclear matrix elements can be larger since they can involve quark operators other than the electroweak charged current. Also, the Yukawa coupling of such a scalar to quarks could easily be of order $10^{-1}$ — a value which is much larger than the corresponding value of $10^{-4}$ for the SM Higgs. Nevertheless, none of these enhancements can overcome the suppression due to large scalar masses.

4. Model-Building: The Case of Spectral Index $n = 7$

We now turn to a model which produces $\beta\beta$ decay with spectral index $n = 7$, i.e. a model of type IIE. To do so we require a model in which lepton number is unbroken, and which contains a Goldstone boson, $\varphi$, that carries lepton number +1. This can only happen if the theory has a nonabelian flavour symmetry which acts on leptons, and which contains ordinary lepton number as a generator.

The simplest case is to take the lepton-number symmetry group to be $G = SU(2) \times U_L(1)$, with $G$ broken down to the $U_L(1)$ of lepton number. This can be done by working
with a variation of the model of ref. [5]. We therefore add the following electroweak-singlet, left-handed fermions to the standard model: \( N = \left( \frac{N_+}{N_0} \right) \), \( s_0 \) and \( s_- \), which transform under the flavour symmetry, \( G \), as: \( N \sim (2, \frac{1}{2}) \), \( s_0 \sim (1, 0) \) and \( s_- \sim (1, -1) \). If the unbroken lepton number is chosen to be \( L_e = T_3 + L' \), where \( L' \) is the generator of \( U_L(1) \), then the subscripts of \( N_+, N_0, s_0 \) and \( s_- \) give the corresponding particle’s lepton charge. In order to implement the symmetry breaking pattern \( G \to U_L(1) \), we also introduce the electroweak-singlet scalar field, \( \Phi = (\phi^+ + \phi^-) \), which transforms under \( G \) as: \( \Phi \sim (2, \frac{1}{2}) \).

For this model the most general renormalizable \( G \)-invariant mass and Yukawa interactions involving the new fields are

\[
L_m = -\frac{M}{2} (\bar{s}_0 \gamma_L s_0) + h.c.,
\]

\[
L_y = -\lambda (\bar{s}_- \gamma_L L) H - g_0 (\bar{N}_i \gamma_L s_-) \Phi_j e^{i\phi} + g_0 (\bar{N}_i \gamma_L s_0) \tilde{\Phi}_j e^{i\phi} + h.c.,
\]

(31)

If the scalar fields acquire the vev’s \( \langle H^0 \rangle = v \) and \( \langle \phi_0 \rangle = u \) (which we take, for simplicity, to be real), then \( G \) breaks to \( U_L(1) \) as required. If we also neglect mixings with \( \nu_{\mu} \) and \( \nu_{\tau} \), the following neutrino spectrum is produced. First, there is one massless \( L_e = +1 \) state, \( \nu'_e = \nu_e c\theta - N_+ s\theta \), where the mixing angle, \( \theta \), satisfies: \( \tan \theta = \lambda v/g_- u \). Then, there is a massive \( L_e = +1 \) Dirac state, \( \nu_s = \gamma_L N'_+ + \gamma_R s_- \), with \( N'_+ = N_+ c\theta + \nu_e s\theta \), and with mass \( M_s = \sqrt{\lambda^2 v^2 + g^2_0 u^2} \). There are also two majorana \( L_e = 0 \) states, \( \nu_\pm \), which respectively have masses \( M_{\pm} = \frac{1}{2} \left[ \sqrt{M^2 + 4g^2_0 u^2} \pm M \right] \). These mass eigenstates are related to \( N_0 \) and \( s_0 \) by:

\[
\begin{pmatrix}
\nu_- \\
\nu_+
\end{pmatrix} = \begin{pmatrix}
 ic\varphi & is\varphi \\
-8\varphi & - c\varphi
\end{pmatrix} \begin{pmatrix}
 N_0 \\
 s_0
\end{pmatrix},
\]

(32)

with \( \tan(2\varphi) = 2g_0 u/M \). The factors of \( i \) in this mixing matrix are due to the chiral rotation needed to ensure that both mass eigenvalues, \( M_{\pm} \), are positive.

The next step is to rotate fields to make the derivative couplings of the Goldstone bosons explicit (this is done in detail for a related model in ref. [5]). The appearance of the derivative coupling can be seen as follows: use exponential parameterization of the scalar fields that have non-zero vev so that the Goldstone field appears in the exponential. Then redefine the fermion fields so that the new fermion field is given by \( F' \equiv F e^{i\varphi/v} \) (in this case \( F' \) is the heavy neutral lepton). After this redefinition, the Yukawa interaction becomes completely independent of the Goldstone field, which reappears with a derivative coupling due to the fermion kinetic energy term. The final form of the interaction given in
eq.(8) then emerges on diagonalization of the fermion mass terms with coupling matrices $X_{ia}$ and $Y_{ia}$ given by $Y_{ia} = 0$, and

$$\begin{align*}
X_{\nu'\nu} &= i \frac{s_\theta c_\varphi}{u}, \\
X_{\nu'\nu} &= \frac{s_\theta s_\varphi}{u}, \\
X_{\nu\nu'} &= -i \frac{c_\theta c_\varphi}{u}, \\
X_{\nu\nu'} &= -\frac{c_\theta s_\varphi}{u}.
\end{align*}$$

(33)

With these expressions we may evaluate the $\beta\beta$ matrix element of eq. (10). It becomes:

$$\tilde{A}(\beta\beta) = \frac{2}{\sqrt{105} \pi} \frac{s_\theta^2 c_\theta^2 M_s^4}{u^2} \times \int \frac{d^4 \ell}{(2\pi)^4} \frac{W_\mu^\nu}{(\ell^2 - i\epsilon)(\ell^2 + M_s^2 - i\epsilon)^2} \left[ \frac{s_\varphi^2 M_+}{(\ell^2 + M_+^2 - i\epsilon)} - \frac{c_\varphi^2 M_-}{(\ell^2 + M_-^2 - i\epsilon)} \right].$$

(34)

We quantify the size of this rate in a similar way as already done for the model of section 3.1, namely by means of estimating the magnitude of $g_{\text{eff}}$. We therefore choose values for masses and mixing angles that are compatible with the phenomenological bounds of section 5, but which also yield a maximal rate. Specifically we take

$$M_s \gtrsim 350 \text{ MeV}; \quad M_+, M_- \sim \text{ a few MeV}; \quad \theta \sim 7.5 \times 10^{-2}; \quad g_- \sim 0.2.$$ 

(35)

The amplitude can then be written in the following approximate form:

$$\tilde{A}(\beta\beta) = \frac{2}{\sqrt{105} \pi} \frac{s_\theta^2 c_\theta^2 g^2 M_s}{M_s^2} \left( s_\varphi^2 M_- - c_\varphi^2 M_+ \right) \times \int \frac{d^4 \ell}{(2\pi)^4} \frac{W_\mu^\nu}{(\ell^2 - i\epsilon)(\ell^2 + M_+^2 - i\epsilon)(\ell^2 + M_-^2 - i\epsilon)}.$$ 

(36)

To see this keep in mind that in the limit of small $\theta$ we have $u \sim \frac{M_-}{g_-}$. Comparing this with the amplitude of the GR model gives an equivalent $g_{\text{eff}}$ of

$$g_{\text{eff}} \sim (0.0006) s_\theta^2 c_\theta^2 g^2 \left( \frac{Q^3}{M_s^2 p_F} \right) \sim 10^{-14}. $$

(37)

where the values of eq.(35) have been applied and we take the mixing angle to satisfy $\tan(\varphi) \sim 1/\sqrt{2}$ to maximize the rate (i.e. equivalently $M_+ \sim 2M_-).$ Specifically we take $M_+ \sim E_F \sim Q \sim 2 \text{ MeV}, g_- \sim 0.2, s_\theta \sim 0.1$ and $M_s \sim 6p_F \sim 360 \text{ MeV}.$
In the above we have chosen 0.2 as the maximal value for \( g_- \), as is naively required by kaon decay measurements (see section 5). It is probable that values as large as \( g_- \approx 1 \) would actually prove to be consistent with these measurements in a more careful treatment, however. This is because the scalar’s derivative coupling provides an additional suppression to its contribution to kaon decays. If so, then the upper bound on \( g_{\text{eff}} \) can be relaxed to around \( g_{\text{eff}} \lesssim 10^{-12} \).

It is clear that this rate is far too small to have any chance of being detected, largely due to the suppression by powers of \( Q \) that originate with the extremely soft electron spectrum. We consider this model therefore only as an example that such an exotic decay spectrum, with spectral index \( n = 7 \), can in principle exist. Given that this is also the kind of spectrum that is least well constrained experimentally, since cuts usually exclude the lowest-energy electrons which are usually the most contaminated by backgrounds, it would be very interesting to construct a model which predicts a detectable rate. We have so far been unable to find such a model.

5. Phenomenological Constraints

The previous sections have seen us use particular kinds of masses and couplings in order to maximize the \( \beta\beta\phi\phi \) decay rate. The present section is meant to justify the masses and couplings we have chosen. We do so by listing the main phenomenological constraints which the models we consider must confront. Since the models with the largest \( \beta\beta\phi\phi \) signals involve sterile neutrinos coupled to light scalars, we focus principally on the constraints on these.

5.1) Laboratory Limits

In most of the models considered in this paper the SM has been supplemented by a number of electroweak-singlet fermion and scalar fields. Of these, it is only the mixing of the sterile neutrinos with (in this case) the electron neutrino, \( \nu_e \), that is bounded experimentally. Since the sterile neutrinos in the models we consider dominantly decay into lighter neutrinos and scalars, all of whom are invisible to present day detectors, the only bounds which need be considered are those which do not assume a decay into a visible mode. The bounds that do apply can be classified into two subclasses according to whether they are due to low- or high-energy experiments [20].
At low energies we have those experiments which examine the decays of pions [21] and kaons [22] at rest, and search for nonstandard contributions to the decay rate and the outgoing electron spectrum. These experiments constrain the masses of sterile neutrinos in a model-independent way from a few MeV almost up to $\sim 100$ MeV for pion decays, and up to $\sim 350$ MeV for kaon decays. Mixing angles, $U_{ei}$, with the electron neutrino, $\nu_e$, are strongly constrained for sterile neutrinos with masses in this range. For example, upper bounds on $|U_{ei}|$ can be as low as $7 \times 10^{-4}$ for a mass of 50 MeV [21].

These same experiments, as well as measurements of the muon lifetime, also limit the possibility for the existence of weak decays into very light scalars in addition to neutrinos [23]. Specifically these limits come from precision measurements of the Michel parameter, $\rho$, in muon decay, and from using the measured decay spectra of the outgoing leptons to constrain decays such as $K \rightarrow \ell N \varphi$ or $\pi \rightarrow \ell N \varphi$. The failure to observe such decays can be expressed as an upper bound on a hypothetical dimensionless Yukawa coupling, $\tilde{g}_{\text{eff}}$, of an effective $\nu_e - N - \varphi$ interaction. In terms of this coupling, the current bounds are respectively $\tilde{g}_{\text{eff}} \lesssim 5.7 \times 10^{-2}$ (Muon decay), $\lesssim 1.6 \times 10^{-2}$ (Pion decay) and $\lesssim 1.3 \times 10^{-2}$ (Kaon decay) [23]. Of course, these bounds assume that the kinematics permit the emission of $N \varphi$ in these decays. For the models of interest here, the bound therefore applies if the $L_e = 0$ sterile fermions, $N_a$, are sufficiently light compared to the $\pi$, $K$ and/or $\mu$, as is certainly the case for $m_{N_a} \sim 1$ MeV. For $m_{N_a}$ in this mass range, the effective coupling that is then bounded in this analysis turns out to be $\tilde{g}_{\text{eff}} \approx s_\theta c_\theta g_+$. As a result, keeping in mind the bound on $\theta$ discussed below, $g_+$ must be smaller than 0.18.

It is clear however that provided $N_a$ is too heavy to be produced in these kinds of decays (i.e. $m_{N_a} \gtrsim 350$ MeV) then the bound described above no longer applies, and $g_+$ can be of order unity. We are led to consider $m_{N_a} \sim 1$ MeV because of the specific way we choose to avoid conflict with nucleosynthesis (see section 5.2). If another way to evade this bound should be found, or if we simply accept the comparatively large number of effective degrees of freedom contributed at this epoch, then $g_+$ can indeed be as large as $g_+ \sim 1$.

For masses heavier than 350 MeV the best bounds on sterile neutrinos usually come from beam dump experiments. However this type of experiment relies crucially on the assumption that the isosinglet leptons decay via their charged current weak interactions.

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4 These bounds can be even more stringent if additional model-dependent assumptions for the couplings involved are made.
The same is also true for bounds coming from searches on the $Z$-pole for exotic decays of the $Z$ boson, such as $Z \rightarrow N\bar{\nu} \rightarrow W^*e\bar{\nu}$. (Here $N$ denotes the isosinglet state, and $W^*$ denotes a virtual $W$ boson.) However, since such decays do not dominate in the models we consider — the sterile states instead prefer to decay into light scalars — these experiments need not be considered further here.

The best limits, for the mass range above $\sim 350$ MeV, therefore come from the reduction of the effective charged-current coupling of the electron, which is suppressed by the cosine of the mixing angle between $\nu_e$ and the heavy sterile state. This reduction potentially affects electroweak precision experiments by influencing the measured value of Fermi’s constant, $G_F$, that is inferred from muon decay. It also shows up as a failure of lepton universality in low-energy weak decays. This leads to the bound $|U_{ee}| < 7.5 \times 10^{-2}$ (2σ) for masses above $\sim 350$ MeV [24]. Another bound comes from the reduction of the neutral current coupling of the electron-neutrino, leading to a decreased number of light neutrino species as deduced from $Z$’s invisible width. However this bound is somewhat weaker than the aforementioned universality bound, and applies only to sterile neutrinos that are heavier than about 90 GeV, hence it is of no particular importance in our case.

These bounds have immediate implications for the models considered here, such as those discussed in sections 3.1 and 4. Since these models have only one sterile state, the heavy Dirac neutrino $\nu_s$, that mixes with $\nu_e$, we need only constrain the masses and mixings of this one particle. To maximize the $\beta\beta$ rate we prefer a mixing angle, $\theta$, that is as large as possible, and so we choose $\nu_s$ to be heavy enough to evade the bounds from pion and kaon decay — $M_s \gtrsim 350$ MeV — and we take $\theta$ to saturate the universality bound — $\theta \lesssim 7.5 \times 10^{-2}$.

5.2) Nucleosynthesis

Standard big-bang cosmology very successfully explains the abundances of light elements produced during the nucleosynthesis epoch [25]. In fact, this success leaves little room for the existence of new species of particles at this critical time. Specifically if there were new particles present at a temperature of $\sim 0.1 - 2$ MeV their additional degrees of freedom would increase the expansion rate of the universe leading to an earlier freeze-out of neutron-proton converting interactions and thus to a larger amount of neutrons that eventually would be cooked into $^4He$. The bound on the number of additional degrees
of freedom is conventionally stated in terms of the maximal number of additional light neutrino species allowed at nucleosynthesis $\delta N_{\nu}$ [26]:

$$\delta N_{\nu} \lesssim 0.4$$

There are two simple ways to satisfy this bound when additional particle types are present in a theory. For particles that are heavier than about 1 MeV, the most straightforward way is to have them not be present at all during nucleosynthesis. This can be achieved by either having them decouple and subsequently decay sufficiently early — keeping in mind the possible contributions of the decay products — or by having them remain in thermal equilibrium as they become nonrelativistic, so that their abundance becomes suppressed by the Boltzmann factor, $e^{-m/T}$. For particles which are much lighter than 1 MeV, on the other hand, the contribution to $\delta N_{\nu}$ is strongly suppressed if they decouple before the QCD phase transition at $T \sim 200$ MeV. In this case they become sufficiently diluted compared to ordinary particles afterwards due to the reheating of the photon bath during this phase transition.

Unfortunately, these methods alone are not enough to avoid conflict of the models considered in this paper with nucleosynthesis. This is because the very light scalars, that are present in all of our models, couple through dimensionless couplings to ordinary neutrinos, such as $\nu_e$. As a result, they remain in equilibrium with these neutrinos down to temperatures that are well below 1 MeV. This makes a conflict with nucleosynthesis generic for models which can produce $\beta\beta$ and $\beta\beta\varphi\varphi$ decays.

Rather than therefore considering these models to be inevitably ruled out, we demonstrate here an existence proof that the nucleosynthesis bound can be avoided, given suitable masses for the sterile neutrinos. We propose this as an example of the kinds of information that could be inferred from cosmological bounds, should one of the exotic $\beta\beta$ decays we describe ever be detected experimentally. The loophole we describe follows its similar application in ref. [18], and is based on an observation of ref. [27].

Suppose, then, that a heavy neutrino state either decays [27] or annihilates [18], into particles that are in equilibrium with ordinary neutrinos, right after these neutrinos freeze out of chemical equilibrium with the photon bath (i.e. $T_\nu \sim 2.3$ MeV.) Provided this happens before the neutron-proton ratio freezes out (at $T_{n/p} \sim 0.7$ MeV), then the abundance of neutrinos will be increased relative to the standard cosmological scenario. This over-
abundance of neutrinos will act to suppress the neutron abundance at freezeout, and so decreases the predicted production of \(^4\)He. As a result it contributes to nucleosynthesis as a \textit{negative} contribution to \(\delta N_\nu\) [27].

Consider, therefore, a single majorana neutrino with a mass in the critical region (\textit{i.e.} \(\sim 1 - 2 \text{ MeV}\)) which is still in equilibrium with \(n_S\) species of real scalars in addition to the \(n_F = 3\) species of ordinary neutrinos, when it becomes nonrelativistic. Such a particle will annihilate out right during the critical epoch, thereby heating the neutrino-scalar bath above that of the photon bath. Then the magnitude of \(\delta N_\nu\) that results due to the above mechanism turns out to be [27], [18] \(\delta N_\nu = -4.6 \delta n\), where

\[
\delta n = \left(\frac{n}{n_o}\right) \left(\frac{E}{E_o}\right)^2 - 1 = \left(\frac{(n_F + 1) + \frac{4}{7}n_S}{n_F + \frac{4}{7}n_S}\right)^{5/3} - 1. \tag{39}
\]

Here \(n/n_o\) and \(E/E_o\) denote the ratios of the electron-neutrino number densities and energies after and before annihilation. The second expression is obtained from the first one using entropy conservation.

We can now see how the models described in previous sections can accommodate the nucleosynthesis bound. Take the model of section 3.1 as a representative case. This model contains a massive Dirac state, \(\nu_s\), which mixes appreciably with the electron neutrino, and so which must be heavier than \(\sim 350 \text{ MeV}\) to avoid the laboratory bounds described earlier. It also contains a massive majorana neutrino, \(N_0\), as well as a very light complex scalar.

Since the sterile neutrino is kept in thermal equilibrium with the photon bath by neutrino scattering \textit{via} scalar (\(\phi\)) exchange, it remains in equilibrium long after the temperature has dropped below its mass. Its abundance is therefore sufficiently suppressed to ensure that it never plays a role at nucleosynthesis. The same would be true for the majorana neutrino if it were heavy, however this would leave the light scalars in equilibrium at nucleosynthesis, contributing \(\delta N_\nu = \frac{8}{7}\). The contribution of these scalars can be very efficiently cancelled, however, if \(N_0\) should have a mass of around 1 MeV, since in this case it annihilates right in time to reheat the neutrino sector.\(^5\) In this case we can employ eq. (39) with \(n_F = 3\) and \(n_S = 2\), yielding \(\delta n = 0.43\) and so a total shift in the effective number of neutrino species of \((\delta N_\nu)_{\text{tot}} = \frac{8}{7} - 4.6(0.43) = -0.86\). Since \((\delta N_\nu)_{\text{tot}}\) rises to \(\frac{8}{7}\) as

\(^5\) \(N_0\) remains in equilibrium with \(\nu_e\) through scalar exchange.
$N_0$’s mass increases above $T_\nu \sim 2.3$ MeV, it is possible to find an effective number of light neutrinos which is compatible with observations. To summarize this paragraph: Conflict with nucleosynthesis can be avoided if $N_0$ has a mass in the range of a few MeV.

Following similar lines of reasoning one can also make the model of section 4 satisfy the nucleosynthesis bound. In this case the annihilation of just one majorana neutrino during the critical epoch is not enough though because we have twice as many light scalar degrees of freedom as in the model discussed above. Assuming however that the two majorana mass eigenstates $\nu_\pm$ are both not much heavier than a few MeV solves the problem. To make this point explicit we just state that if both $\nu_+$ and $\nu_-$ annihilated out right in the critical epoch then this would result in $(\delta N_\nu)_{\text{tot}} = -1$.

5.3) Other Constraints from Astrophysics and Cosmology

Other astrophysical bounds can constrain the properties of very weakly interacting particles, such as the sterile neutrinos and scalars we are entertaining. These turn out to not significantly constrain the models we consider. We illustrate this point in this section by examining these bounds for the model considered in section 3.1.

- (1) Stellar Evolution: Sterile particles produced in the core of stars can disrupt our understanding of stellar evolution since they can provide an extremely efficient additional cooling mechanism for a star. For the model of section 3.1, the sterile neutrinos $\nu_s$ and $N_0$ are simply too heavy to be produced in significant numbers inside the core of a star, since their masses are respectively taken to be $\sim 350$ MeV and $\sim 1$ MeV. They are therefore not constrained by stellar evolution. The same conclusion also holds for the light scalars of this model, although for different reasons. In this case the scalars are light enough to be produced, but they only couple appreciably to neutrinos. But since temperatures and densities inside a star are not sufficiently high to produce a thermal or degenerate population of neutrinos, scalars are not produced in significant quantities from the interior of the star. Direct couplings with other particles, such as electrons, do arise due to loop effects, but these are too small to be significant.\footnote{See, for example, the second Ref. of [5], where this issue is treated in more detail in a similar context.}

- (2) Supernovae: Populations of neutrinos are maintained, however, within the cores of supernovae, due to the much higher temperatures ($\sim 50 - 100$ MeV) and densities
that are found there. Both the scalar, \( \varphi \), and the sterile majorana neutrino, \( N_0 \), are therefore sufficiently light to be produced in significant numbers. However since both of these particles are also in thermal equilibrium with the ordinary SM neutrinos, they are trapped in the core of the supernova and so do not lead to premature cooling.

- (3) ‘Present-Day’ Cosmology: One last issue is the effect of \( \varphi \) on structure formation, and on the energy density of the universe observed today. Since \( \varphi \) would remain in thermal equilibrium with the electron neutrinos (through exchange of virtual \( N_0 \)) until a temperature in the region of 1 eV if it was light, it inevitably annihilates out to the neutrino sector once it becomes nonrelativistic. It will therefore never dominate the energy density of the universe and hence also not affect structure formation nor the age of the universe in any significant way. This in turn means that \( \varphi \) can have any mass below 1 MeV.

6. Conclusions

In this paper we investigate a novel class of neutrinoless double-beta decays, in which two (rather than one or no) scalars are emitted in addition to the two observed electrons. We do so to provide sample models of the two remaining classes of spectra for \( \beta\beta \) decays, which had not been hitherto explored. We regard it as useful to construct representative models for each class, since such models are required in order to determine the implications of other experiments, at different energies, for \( \beta\beta \) decay.

We have obtained the following results:

- 1: We have found that two-scalar decays always lead to a \( \beta\beta \) electron sum-energy spectrum whose spectral index is either \( n = 3 \) or \( n = 7 \). The \( n = 3 \) spectrum is intermediate between the SM shape (for which \( n = 5 \)), and the traditional scalar-emitting shape \( (n = 1) \), such as is found in the Gelmini-Roncadelli model. The \( n = 3 \) spectrum is identical to a recently-discovered class of models [5] for which only a single scalar is emitted during the decay.

- 2: The \( n = 7 \) spectrum is completely new, and is softer than the observed SM \( \beta\beta_{2\nu} \) spectrum. As a result, it is more difficult to experimentally extract, since most backgrounds tend to dominantly produce low-energy electrons. It is intriguing that such an exotic spectrum could in principle be hiding undetected even now in the present \( \beta\beta_{2\nu} \) data!
3: Viable models can be constructed which predict an $n = 3$ $\beta\beta$ decay rate which is reasonably close to the present experimental limit. In the viable models with detectable rates the decay is mediated by the exchange of a number of sterile neutrinos, whose masses must lie in the (1-100) MeV range. Their couplings must also be as large as is permitted. The requirement of such large couplings to obtain observable signals suggests that an observed $\beta\beta\varphi\varphi$ signal should be accompanied by the appearance of new physics in other experiments, such as in violations of lepton universality.

A potential problem with this model is that it predicts an excess number of degrees of freedom at nucleosynthesis that is equivalent to $\frac{8}{7}$ of a neutrino, in comparison with the presently-quoted bound of 0.4. A way of evading this nucleosynthesis bound is described in section 5, but the required couplings and masses for this scenario would predict a $\beta\beta\varphi\varphi$ rate which is two orders of magnitude smaller, and so out of reach of current experiments. We do not regard this as sufficiently worrying to preclude searching for this decay in current experiments, since an experimental signal would likely prompt more imaginative approaches to understanding nucleosynthesis in these models.

4: Because of its soft spectrum, we were unable to construct viable models of the $n = 7$ $\beta\beta\varphi\varphi$ decay, for which the predicted rate is large enough to be detectable. Thus, although the existence of an $n = 7$ decay mode is experimentally tantalizing, it is unlikely to be discovered in $\beta\beta$ decay experiments for the foreseeable future.

5: For the purposes of comparison we also examined a class of models for which $\beta\beta\varphi\varphi$ arises due to the mutual interactions amongst new particles in the scalar sector, rather than from new sterile leptons. Although the decay rate that is predicted for these models is also too small to be seen, the decays nevertheless exhibit novel features. One such is a nonstandard angular distribution of the two electrons as a function of the opening angle between them, similar to what arises in models with right-handed currents.

All of these results serve to underline the theoretical understanding which has emerged over the past few years. Scalar-emitting $\beta\beta$ decays can reasonably be expected to be observed, although the properties of the phenomenologically viable models which can do so are very different than what would be expected based on intuition that is based on the original GR-type models which originally motivated these experiments.
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7. References

[1] C.P. Burgess and J.M. Cline, in the proceedings of The 1st International Conference on Nonaccelerator Physics, Bangalore, January 1994, (World Scientific, Singapore).

[2] The experimental situation has recently been thoroughly reviewed in: M. Moe, Int. J. Mod. Phys. **E2** (1993) 507; M. Moe and P. Vogel, Ann. Rev. of Nucl. and Part. Sc. (to appear).

[3] G.B. Gelmini and M. Roncadelli, *Phys. Lett.* **99B** (1981) 411.

[4] H.M. Georgi, S.L. Glashow and S. Nussinov, *Nucl. Phys.* **B193** (1981)) 297.

[5] C.P. Burgess and J.M. Cline, *Phys. Lett.* **298B** (1993) 141; *Phys. Rev.* **D49** (1994) 5925.

[6] Z.G. Berezhiani, A.Yu. Smirnov and J.W.F. Valle, *Phys. Lett.* **291B** (1992) 99.

[7] C.D. Carone, *Phys. Lett.* **308B** (1993) 85.

[8] D. Schaile, in the proceedings of the XXVIIth International Conference on High Energy Physics, Glasgow, July 1994.

[9] F.T. Avignone III *et.al.*, in *Neutrino Masses and Neutrino Astrophysics*, proceedings of the IV Telemark Conference, Ashland, Wisconsin, 1987, edited by V. Barger, F. Halzen, M. Marshak and K. Olive (World Scientific, Sinagpore, 1987), p. 248;
M. Moe, M. Nelson, M. Vient and S. Elliott, *Nucl. Phys.* (Proc. Suppl.) **B31** (1993).

[10] See, for example, M. Moe, ref. [2].

[11] R. Mohapatra and E. Takasugi, *Phys. Lett.* **211B** (1988) 192.

[12] Kai Zuber, in Relativistic Astrophysics and Particle Cosmology, ed. by C.W. Akerlof and M.A. Srednicki, Annals of the New York Academy of Sciences, Vol. 688 (New York, 1993).

[13] See e.g. M. Beck *et.al.*, *Phys. Rev. Lett.* **70** (1993) 2853 and references therein, or
J.-L. Vuilleumier et al., *Nucl. Phys. (Proc. Suppl.)* **B31** (1993).

[14] A. Halprin, P. Minkowski, H. Primakoff and P. Rosen, *Phys. Rev.* **D13** (1976) 2567.

[15] W. Haxton and G. Stephenson, *Prog. in Particle and Nucl. Physics* **12**, 409 (1984); J. Vergados, *Phys. Rep.* **133**, 1 (1986); M. Doi, T. Kotani, H. Nishiura and E. Takasugi, *Prog. Theor. Phys., Prog. Theor. Phys. Suppl.* **83**, 1 (1985).

[16] H. Klapdor-Kleingrothaus, K. Muto and A. Staudt, *Europhys. Lett.* **13**, 31 (1990); M. Hirsch et. al., *Zeit. Phys. A* **345**, 163 (1994); H. Klapdor-Kleingrothaus, *Prog. Part. Nucl. Phys.* **32**, 261 (1994) for a review.

[17] Y. Chikashige, R.N. Mohapatra and R.D. Peccei, *Phys. Rev. Lett.* **45** (1980) 1926.

[18] C.P. Burgess and O. Hernández, *Phys. Rev.* **D48** (1993) 4326.

[19] P. Bamert, C.P. Burgess and R.N. Mohapatra, preprint McGill-94/37, NEIP-94-007, UMD-PP-95-11; *Nucl. Phys. B* (to appear), hep-ph/9408367.

[20] For a review on the different types of experiments constraining heavy isosinglet leptons see e.g. F.J. Gilman, *Comments Nucl. Part. Phys.* **16** (1986) 231 and references therein.

[21] D.I. Britton et al., *Phys. Rev.* **D46** (1992) R885; *Phys. Rev. Lett.* **68** (1992) 3000.

[22] R. Shrock, *Phys. Lett.* **96B** (1980) 159; T. Yamazaki et al., in Proc. XIth Intern. Conf. on Neutrino Physics and Astrophysics, K. Kleinknecht and E.A. Paschos (World Scientific, Singapore, 1984) p. 183.

[23] V. Barger, W. Y. Keung and S. Pakvasa, *Phys. Rev.* **D25** (1982) 907; T. Goldman, E. Kolb and G. Stephenson, *Phys. Rev.* **D26** (1982) 2503; A. Santamaria, J. Bernabeu and A. Pich, *Phys. Rev.* **D36** (1987) 1408; C. E. Picciotto et al., *Phys. Rev.* **D37** (1988) 1131.

[24] W. Marciano and A. Sirlin, *Phys. Rev. Lett.* **71** (1993) 3629; C.P. Burgess, S. Godfrey, H. König, D. London and I. Maksymyky, *Phys. Rev.* **D49**
(1994) 6115.

[25] T. Walker et al., Ap. J. 376, 51(1991).

[26] C. Copi, D. Schramm and M. Turner, preprint FERMILAB-Pub-94/174-A, astro-ph/9407006, (1994).

[27] K. Enqvist, K. Kainulainen and M. Thomson, Phys. Rev. Lett. 68 (1992) 744; Nucl. Phys. B373 (1992) 498.
8. Figure Captions

(1) The $\beta\beta$ spectrum as a function of the two electrons’ total kinetic energy for various choices of the ‘spectral index’ $n$. $n = 1$ corresponds to the dotted line, $n = 3$ is the dashed line, $n = 5$ is the solid line and $n = 7$ is the dash-dotted line. All four curves have been arbitrarily assigned the same maximal value for purposes of comparison.

(2) The Feynman graph representing the leading order contribution to the two-scalar $\beta\beta_{\phi\phi}$ decay through sterile lepton exchange.

(3) The Feynman graph contributing to $\beta\beta_{\phi\phi}$ decay purely due to the self-interactions of the Higgs sector.
This figure "fig1-1.png" is available in "png" format from:

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Figure 1

Electron Spectrum

\[ dN / dE \]

\[ E = E_1 + E_2 \]

n=1  n=3  n=5  n=7
Figure 2
Figure 3