Steerability is a characteristic nonlocal trait of quantum states lying in between entanglement and Bell nonlocality. A given quantum state is considered to be steerable if it violates a suitably chosen steering inequality. A quantum state which otherwise satisfies a certain inequality can violate the inequality under a global change of basis i.e., if the state is transformed by a nonlocal unitary operation. Intriguingly there are states which preserve their non-violation (pertaining to the said inequality) under any global unitary operation. The present work explores the effect of global unitary operations on the steering ability of a quantum state which live in two qubits. We characterize such states in terms of a necessary and sufficient condition on their spectrum. Such states are also characterized in terms of some analytic characteristics of the set to which they belong. Looking back at steerability the present work also provides a relation between steerability and quantum teleportation together with the derivation of a result related to the optimal violation of steering inequality. An analytic estimation of the size of such non-violating states in terms of purity is also obtained. Interestingly the estimation in terms of purity also gives a necessary and sufficient condition in terms of Bloch parameters of the state. Illustrations from some signature class of quantum states further underscore our observations.

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I. INTRODUCTION

The ubiquitous roles of entanglement and non-locality[1–4] are exemplified in many quantum computation and information processing tasks[5–10]. These two inequivalent features have been the cornerstone of Quantum Mechanics.

In 1935 Einstein, Podolsky and Rosen contributed an argument claiming the incompleteness of Quantum Mechanics[11]. In the following year Schrödinger envisaged the celebrated concept of ‘steering’[12]. Recently, Wiseman et al. have developed this phenomena in the form of a task[13, 14]. They argued that steering refers to the scenario where one party usually called Alice, wishes to convince the other party (commonly referred to as Bob) that she can steer or construct the conditional state on Bob’s side by making measurements on her part. This kind of interpretation of the phenomena have induced great interest in foundational research in recent times[15–20].

Apart from the foundational interest, the study of steering also finds applications in one-sided device independent scenario where only one party trusts his/her quantum device but the other party’s device is untrusted. As a concrete example it has been shown that steering allows for secure quantum key distribution when one of the parties’ device cannot be trusted. One big advantage in this direction is that such scenarios are experimentally less demanding than fully device-independent protocols (where both of the parties distrust their devices) and, at the same time, require less assumptions than standard quantum cryptographic scenarios.

In 1964, Bell sought a way to demonstrate that certain correlations appearing in quantum mechanics are incompatible with the notions of locality and reality a.k.a. local-realism, through an inequality involving measurement statistics. In 1969, Clauser–Horne–Shimony–Holt (CHSH) proposed a set of simple Bell inequalities which are easy to realize experimentally. In the same spirit of Bell’s inequality in nonlocality, several steering inequalities (SIs) have been proposed, so that a violation of any such SI can render a state to be steerable. To test EPR steering Reid first proposed a testable formulation for continuous-variable systems based on position-momentum uncertainty relation [21] which was experimentally tested by Ou et al. [22]. Cavalcanti et al. developed a general construction of experimental EPR-steering criteria based on the assumption of existence of LHS(Local Hidden state) model [23]. Importantly this general construction is applicable to both discrete as well as continuous-variable observables and Reid’s criterion appears as a special case of this general formulation.

One of the most intriguing problem in quantum mechanics is to find the signature of entanglement[24]. The necessary and sufficient criteria for identifying entanglement in lower dimensions(2 ⊗ 2 and 2 ⊗ 3) [25, 26] through neg-
ativity of the partial transpose fails in higher dimensions due to the presence of PPT bound entangled state[27]. However an effective method can be provided for detection of entanglement via entanglement witness[26, 28, 29]. Entanglement witnesses $W_E$ are hermitian operators having at least one negative eigenvalue which satisfy the inequalities (i) $Tr(W_{E\text{sep}}) \geq 0, \forall$ separable states $g_{\text{sep}}$ and (ii) $Tr(W_{E\text{ent}}) < 0$ for at least one entangled state $g_{\text{ent}}$. The geometric form of the Hahn-Banach theorem states the existence of entanglement witnesses for detection of entanglement[26]. As entanglement witnesses are hermitian they have been conveniently used in experimental detection of entanglement [31, 32]. Method of constructing witness to detect entanglement has been further extended to detect useful resources for teleportation[33–35].

An analogous procedure to capture non-locality is through a Bell-CHSH witness [36]. For two qubits, a quantum states $\rho$ does not violate the Bell-CHSH inequality iff $M(\rho) \leq 1$, where $M(\rho)$ is defined as the sum of the two largest eigenvalues of the matrix $T_{\rho}^{T_1} T_{\rho}^{T_2}$, $T_\rho$ being the correlation matrix in the Hilbert-Schmidt representation of $\rho$. Thus, $M(\rho) > 1$ is a signature of the non-locality of the state[37].

Pertaining to separability of quantum states, questions have been raised on the characterization of absolutely separable[38, 39] and absolutely PPT states[40]. Precisely, a quantum state which is entangled(respectively PPT) in some basis might not be entangled(resp. PPT) in some other basis. This depends on the factorizability of the underlying Hilbert space. Thus, the characterization of states which remain separable(resp. PPT) under any factorization of the basis is pertinent [38–41]. Some of the authors have addressed the problem of a state being absolutely Bell-CHSH local [49, 50]. A state is termed as absolutely Bell-CHSH local if it remains local with respect to the CHSH inequality , under any global unitary operation [49, 50]. The effect of global unitary operations on the conditional entropy of a two-qubit system has also been probed recently [51].

In the present work, we address the question of non-violation of a steering inequality under any global unitary operation. This is understood that a pure product state can be converted to a maximally entangled state by a suitable global unitary operation which thereby can violate the steering inequality. However, if purity of the initial state goes below a certain extent, the state cannot be made to violate the steering inequality even by a global unitary operation. The characterization of such states forms one of the main constituents of the present submission. With some specific steering inequalities we have characterized such states which retain their non-violating character under the action of global unitary. We further find a criterion to determine whether a particular state exhibits absolute non-violation. This is done by the derivation of a result that the maximal steering violations w.r.t the inequality we have considered is attained at the respective Bell diagonal state for a fixed spectrum. The size of such states are estimated with illustrations to support our observations.

Starting with some prerequisites for our current work, we also observe some distinctive characteristics of a steerable state. While reviewing the basic concepts needed for our work we have derived some new results pertaining to the optimal violation of a steering inequality and the relation of steerability to teleportation.

The paper is arranged as follows: In section (II) we discuss some prerequisites needed for our study and also observe some distinctive traits of steerable states. In section (III) we derive the conditions for a state to be absolutely non-violating(w.r.t our chosen inequality) and find an estimation of the size of such states. Illustrations are provided in section (IV). Finally we conclude in section (V).

II. PREREQUISITES, FEW DEFINITIONS AND SOME NEW RESULTS

1. Notations and Definitions

At the outset we put together some notations and definitions to be followed in our analysis.$\mathcal{B}(X)$ denotes the set of bounded linear operators acting on X. The density matrices that we consider here, are operators acting on two qubits, i.e., $\rho \in \mathcal{B}(H_2 \otimes H_2)$. Q denotes the set of all density matrices.

In [23] authors have developed a series of steering inequalities to check whether a bipartite state is steerable when both the parties are allowed to perform $n$ measurements on his or her part, which is given by the following equation

$$F_n(\rho, \mu) = \frac{1}{\sqrt{n}} \left| \sum_{i=1}^{n} \langle A_i \otimes B_i \rangle \right| \leq 1,$$

The inequalities for $n = 2, 3$ are as given below:

$$F_2(\rho, \mu) = \frac{1}{\sqrt{2}} \left| \sum_{i=1}^{2} \langle A_i \otimes B_i \rangle \right| \leq 1,$$

$$F_3(\rho, \mu) = \frac{1}{\sqrt{3}} \left| \sum_{i=1}^{3} \langle A_i \otimes B_i \rangle \right| \leq 1,$$

where $A_i = \hat{u}_i \cdot \hat{s}$, $B_i = \hat{v}_i \cdot \hat{s}$, $\hat{s} = (s_1, s_2, s_3)$ is a vector composed of the Pauli matrices, $\hat{u}_i \in R^3$ are unit vectors, $\hat{v}_i \in R^3$ are orthonormal vectors, $\mu = \{\hat{u}_{i_1}, \cdots, \hat{u}_{i_n}, \hat{v}_{i_1}, \cdots, \hat{v}_{i_n}\}$ is the set of measurement directions, $\langle A_i \otimes B_i \rangle = Tr(\rho A_i \otimes B_i)$, and $\rho \in H_A \otimes H_B$ is some bipartite quantum state. Pertaining to inequality (1), one may construct steerability witnesses $S^W$ [23]. Precisely $Tr(S^W \chi) < 0$ for at least one steerable state $\chi$ and $Tr(S^W \rho) \geq 0$ for all unsteerable states $\rho$. 

The size of such states are estimated with illustrations to support our observations.
We denote by $L$ and $US$, the set of all states which do not violate Bell-CHSH and steering inequality (1) respectively. We denote respectively by $AL,AUS$ as the sets containing states which do not violate the Bell-CHSH inequality and the steering inequality (1) under any global unitary operation ($U$). One can easily verify that $AUS$ forms a non-empty subset of $US$, as $\frac{1}{4}(I \otimes I) \in AUS$. The set of states that do not violate inequality (3) under any global unitary operations will be denoted by $AUS_3$.

2. $AUS$ is convex and compact

A formal characterization follows below,

**Theorem 1.** $US$ is a convex and compact subset of $Q$.

**Proof.** First note that the statements below are equivalent:

(i) $\rho \in US$

(ii) $\forall$ steering operator $S, Tr(S\rho) \leq 1$

(iii) $\forall$ steering witness $SW, Tr(S^2\rho) \geq 0$

In view of the above, we can rewrite $US$ as, $US = \{\rho : Tr(S\rho) \geq 0, \forall S\}$. Now consider a function $f_1 : Q \rightarrow \mathbb{R}$, defined as

$$f_1(\chi) = Tr(SW_1^{\dagger}\chi)$$

where, $SW_1$ is a fixed steering witness. Let $US_1 = \{\chi_1 : Tr(SW_1^{\dagger}\chi_1) \geq 0\}$. $Tr(SW_1^{\dagger}\chi_1)$ will have a maximum value $d_1$ (say). Therefore, one may write $US_1 = f_1^{-1}[0,d_1]$. $f_1$ is a continuous function as $Tr$ is a continuous function. This in turn implies $US_1$ is a closed set. Continuing as above, one may define $US_i$ for a fixed $SW_i$. $US_i$ will be closed $\forall i$. Since, arbitrary intersection of closed sets is closed, $\bigcap_i US_i$ is closed. It is easy to see that $\bigcap_i US_i = US$. Hence, $US$ is closed. If we now take two arbitrary $\rho_1, \rho_2 \in US$, then $Tr(SW_1^{\dagger}(\lambda \rho_1 + (1-\lambda)\rho_2)) \geq 0$ for any $S^W_1, \lambda \in [0,1]$. This follows from the fact that $Tr(SW_1^{\dagger}\rho_i) \geq 0, i = 1,2$ for any $S^W_1$. Thus $US$ is convex. Since $Q$ is compact, $US$ being a closed subset of $Q$, is thus compact. Hence the theorem. 

This theorem facilitates the characterization of the set $AUS$ as stated in the theorem below:

**Theorem 2.** $AUS$ is a convex and compact subset of $US$.

**Proof.** We only show that $AUS$ is convex as the compactness follows from a retrace of the steps presented in [41]. Take two arbitrary $\sigma_1, \sigma_2 \in AUS$. One may rewrite $AUS = \{\sigma : Tr(S^W(U\sigma U^\dagger)) \geq 0, \forall S^W, U\}$. Therefore, for any $U, U[\lambda \sigma_1 + (1-\lambda)\sigma_2]U^\dagger = \lambda \sigma_1' + (1-\lambda)\sigma_2' \in AUS$. This follows, since $US$ is convex. $[\sigma_1' = U\sigma_1 U^\dagger]$.

Hence, the theorem. 

The above characterization enables one to formally define an operator($W^S$) which detects states that violate steering inequality under global unitary.

$$Tr(W^S\sigma) \geq 0, \forall \sigma \in AUS$$

($i$)$\exists \zeta \in US - AUS, Tr(W^S\zeta) < 0$

Consider $\zeta \in US - AUS$. There exists a unitary operator $U_\zeta$ such that $U_\zeta U_\zeta^\dagger$ violates steering inequality. Consider a steering witness $SW_\zeta$ that detects $U_\zeta U_\zeta^\dagger$, i.e., $Tr(S^W_\zeta U_\zeta U_\zeta^\dagger) < 0$. Using the cyclic property of the trace, one obtains $Tr(U_\zeta^\dagger S^W_\zeta U_\zeta) < 0$. We thus claim that

$$W^S = U_\zeta^\dagger S^W_\zeta U_\zeta$$

is our desired operator. To see that it satisfies inequality (1), we consider its action on a state $\sigma$ from $AUS$. We have $Tr(W^S\sigma) = Tr(U_\zeta^\dagger S^W_\zeta U_\zeta \sigma) = Tr(S^W_\zeta U_\zeta \sigma U_\zeta^\dagger)$. As $\sigma \in AUS$, and $S^W$ is a steering witness $Tr(S^W_\zeta U_\zeta \sigma U_\zeta^\dagger) \geq 0$. This implies that $W$ has a non-negative expectation value on all states $\sigma \in AUS$.

3. Steerability and Teleportation

In Hilbert-Schmidt representation any density matrix living in two qubits can be expressed as ,

$$\zeta = \frac{1}{4}(I \otimes I + \vec{a} \cdot \vec{s}_1 \otimes I + I \otimes \vec{b} \cdot \vec{s}_2 + \sum_{i,j} t_{ij} s_i \otimes s_j)$$

where, $\vec{a}, \vec{b}$, being the local bloch vectors and $T = [t_{ij}]$ is the correlation matrix.

In [42], a necessary and sufficient condition for a state to be useful for teleportation was derived in terms of bloch parameters. Namely, a state $\zeta$ is useful for teleportation iff [42],

$$N(\zeta) = Tr\sqrt{T^\dagger T} = \sum_{i=1}^{3} \sqrt{u_i} > 1$$

where $u_i$ are the eigenvalues of $T^\dagger T$.

In [43] the authors derived a steerability measure under 3 measurement settings for density matrices of the form,

$$\zeta' = \frac{1}{4}(I \otimes I + \vec{a}' \cdot \vec{s} \otimes I + I \otimes \vec{b}' \cdot \vec{s} + \sum_{i} c_i s_i \otimes s_i)$$

where the correlation matrix of $\zeta'$ is $T' = diag(c_1, c_2, c_3)$. The steerability measure was given by by $F_3(\zeta') = \sqrt{Tr(T'^\dagger T')}$ (subscript 3 denotes 3 measurement settings). Since the density matrices given in equations (8) and (10) are local unitary equivalent, we have,

$$F_3(\zeta') = \sqrt{Tr(T'^\dagger T')} = \sqrt{Tr(T^\dagger T)} = F_3(\zeta)$$

Now,

$$F_3(\zeta) = \sqrt{u_1 + u_2 + u_3} = \sqrt{N(\zeta)^2 - 2 \sum \sqrt{u_i} \sqrt{u_j} \leq N(\zeta)}$$
Theorem 4.

Therefore any quantum state which is 3-steerable i.e. $F_3(\zeta) > 1$, is useful for teleportation.

4. From joint measurability to steering inequality

Except from quantum entanglement, another necessary ingredient which is necessary for study of quantum nonlocality is the existence of incompatible set of measurements. In the simplest bipartite scenario Wolf et al. have shown that any set of two incompatible POVMs with binary outcomes can always lead to violation of the CHSH-Bell inequality [44]. But, recently in refs.[45, 46] the authors have proved that this result does not hold in the general scenario where numbers of POVMs and outcomes are arbitrary. However in this general settings the authors of [45, 46] have established a connection between measurement incomparability and a weaker form of quantum nonlocality i.e., EPR-Schrödinger steering. They have shown that for any set of incompatible POVMs (i.e. not jointly measurable), one can find an entangled state, such that the resulting statistics violate a steering inequality. Note that, it has been recently proved that the connection between measurement incomparability and steering holds for a more general class of tensor product theories rather than just Hilbert space quantum theory [47].

Let Alice perform a measurement assemblage $\{A_{a|x}\}$ on her part of a bipartite shared quantum state $\rho_{AB}$. Upon performing measurement $x$, and obtaining outcome $a$, the (un-normalized) state held by Bob is given by $\sigma_{a|x} = \text{Tr}(A_{a|x} \otimes 1_{\rho_{AB}})$. The normalized state on Bob’s side is given by $\sigma_{a|x}/\text{Tr}(\sigma_{a|x})$. Also we have $\sum_a \sigma_{a|x} = \sum_a \sigma_{a|x'}$, for $x \neq x'$, which actually ensure no signaling from Alice to Bob. The state assemblage $\{\sigma_{a|x}\}$ is unsteerable iff it admits a decomposition of the form

$$\sigma_{a|x} = \pi(\lambda)p(a|x, \lambda)\sigma_{\lambda}, \; \forall \; a, x,$$  \hspace{1cm} (12)

where $\sum_{\lambda} \pi(\lambda) = 1$. Existence of such decomposition for state assemblage on Bob’s side ensures that the statistics obtained from the state $\rho_{AB}$ admit a combined LHV-LHS model of the form of Eq.(12). The authors in refs.[45, 46] have shown that the assemblage $\{\sigma_{a|x}\}$, with $\sigma_{a|x} = \text{Tr}(A_{a|x} \otimes 1_{\rho_{AB}})$, is unsteerable for any state $\rho_{AB}$ acting on $C^a \otimes C^d$ if and only if the set of POVMs $\{A_{a|x}\}$ acting on $C^d$ are jointly measurable. As a result we can say that

**Lemma 3.** The assemblage $\{\sigma_{a|x}\}$, with $\sigma_{a|x} = \text{Tr}_A(A_{a|x} \otimes 1_{\rho_{AB}})$ is unsteerable for any state $\rho_{AB}$ acting in $C^a \otimes C^d$ if and only if the set of POVMs $\{A_{a|x}\}$ acting on $C^d$ are jointly measurable.

We are now in a position to present the result, which is described in the following theorem.

**Theorem 4.** Consider a composite quantum system composed of two subsystem with state spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively. In two qubits, for any trio of dichotomic observables $A_1, A_2, A_3$ for the first system and three dichotomic observables $B_1, B_2, B_3$ for the second system and the joint state $\rho_{AB}$ acting on $\mathcal{H}_1 \otimes \mathcal{H}_2$, we have the following inequality:

$$F_3 \leq \frac{1}{\eta_{opt}},$$  \hspace{1cm} (13)

where $\eta_{opt}$ is the optimal unsharpness parameter that allows joint measurement for any three dichotomic quantum observables.$(F_3$ refers to the violation of inequality (3))

**Proof.** Let us consider two arbitrary dichotomic observables $\{A_{a|x}\}$ on Alice’s side, $x \in \{1,2,3\}$ and $a \in \{-1, +1\}$. These two observables in general may not allow joint measurement. However, introduction of unsharpness makes it possible to measure the unsharp versions of these two observables jointly. Let the optimal unsharpness be $\eta_{opt}$ which allows joint measurement for any two dichotomic observables.

Now according to lemma (3), as far as observables on Alice’s side are jointly measurable, they will not violate any steering inequality and hence the steering inequality

$$F_3(\rho, \mu) = \frac{1}{\sqrt{3}} \left| \sum_{i=1}^3 \langle A_i^a \otimes B_i \rangle \right| \leq 1,$$  \hspace{1cm} (14)

$$F_3(\rho, \mu) = \frac{1}{\sqrt{3}} \left| \sum_{i=1}^3 \eta(A_i^a \otimes B_i) \right| \leq 1,$$  \hspace{1cm} (15)

as,

$$\langle A_k^{(a)} B_j \rangle_{\rho_{AB}} = \eta(A_k B_j)_{\rho_{AB}}.$$  \hspace{1cm} (16)

The value of $\eta_{opt}$ in quantum theory is proved to be $1/\sqrt{3}$. Therefore the upper bound of the steering inequality (3) in quantum theory is $\sqrt{3}$.

\[\blacksquare\]

III. CHARACTERIZATION IN TERMS OF SPECTRUM

In this section we will derive the condition for absolute non-violation with respect to the inequality (3), where $F_3 > 1$ implies that the state is steerable in the 3 measurement scenario. We do not consider inequality (2) as under two measurement settings it is equivalent to the Bell-CHSH inequality [48], regarding which we have already derived results in [49, 50].

Before proceeding with the final derivation, we prove the following two lemmas which are analogous to the proofs used in [52] with respect to the Bell operator. Pertaining to inequality (1), we denote $\mathcal{G} = \frac{1}{\sqrt{n}} \left| \sum_{i=1}^n \langle A_i \otimes B_i \rangle \right|$
and by $F_n$ the corresponding violation.

**Lemma 5.** $\mathfrak{S}$ is diagonal in the Bell-basis

*Proof. Let $X = s_j \otimes I, (j = 1, 2, 3)$. We find $\langle A_i \otimes B_i \rangle_{X_j} = 0$. Similarly letting $Y = I \otimes s_j$, we observe $\langle A_i \otimes B_i \rangle_{Y_j} = 0$. ■

We now use Theorem 4 from [52] in the steering scenario.

**Lemma 6.** For any given spectrum of the density matrix, the respective Bell diagonal state $\zeta_B$ maximizes the steering violation. In other words $F_n(\zeta_B) \geq F_n(U\zeta_BU^\dagger)$.

*Proof. Retracing the steps of the proof in [52] we observe $\text{Tr}(U\zeta_BU^\dagger) \leq \text{Tr}(\zeta_B\mathfrak{S})$. ■

Therefore, the Bell diagonal states are optimal with respect to global unitary operations under steering scenarios 1.

The criteria now follows as,

**Theorem 7.** A state $\sigma \in \text{AUS}_3$ iff $3\text{Tr}(\sigma^2) - 2(x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4) \leq 1$. Here $x_i$ are the eigenvalues of $\sigma$.

*Proof. Any state can be brought to the respective Bell-diagonal state by a global unitary. Now, $F_3(\zeta_B) = 3(x_1^2 + x_2^2 + x_3^2 + x_4^2) - 2(x_1x_2x_1x_4 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4)$.

Hence the theorem. ■

Similar to the case of absolutely separable states [53, 54] an estimation of the size of the set $\text{AUS}_3$ can be obtained as below:

**Corollary 8.** (Estimation of the size of $\text{AUS}_3$) A state $\sigma \in \text{AUS}_3$ iff its purity is less than or equal to 1/2.

*Proof. The criterion in theorem (7) can be recasted in the form $4\text{Tr}(\sigma^2) \leq 2$. This also gives an estimation of the size of the ball around the maximally mixed state in which all the states $\in \text{AUS}_3$. In the Frobenius norm the ball is expressed as $||\sigma - I/4|| \leq 1/2$. ■

The above condition guarantees that absolute non-violation ($\text{AUS}_3$) can be also be verified by a necessary and sufficient condition in terms of local parameters. Consider the state $\sigma$ Hilbert-Schmidt form (8). Then the purity of any state $\sigma$ can be expressed as,

$$\text{Tr}(\sigma^2) = \frac{1}{4}(1 + ||\mathfrak{e}||^2 + ||\mathfrak{f}||^2 + ||G||^2) \quad (17)$$

where $\mathfrak{e}$, $\mathfrak{f}$ are the local bloch vectors of $\sigma$ and $G$ its correlation matrix. Therefore, $\frac{1}{4}(1 + ||\mathfrak{e}||^2 + ||\mathfrak{f}||^2 + ||G||^2) \leq 1/2$, if and only if $\sigma \in \text{AUS}_3$. Hence, 15 measurements are required to verify that a state $\in \text{AUS}_3$. This is in contrast to the fact that in order to check unsteerable w.r.t inequality (3) only 9 measurements are required.

We also note the following theorem pertaining to the unsteerability of the reduced subsystems of a three qubit pure state.

**Theorem 9.** The reduced state $\sigma_{AB}$ of any pure three-qubit state $|\psi\rangle_{ABC}$ belongs to $\text{AUS}_3$ if and only if $\sigma_C$ is the maximally-mixed state.

*Proof. Consider that $l = ||\mathfrak{1}||$, where $\mathfrak{1}$ is the bloch vector for $\sigma_C$. Then the eigenvalues of $\sigma_{AB}$ are $\{(1 + l)/2, (1 - l)/2, 0, 0\}$. Hence, in view of theorem (7), $\sigma_{AB} \in \text{AUS}_3$ iff $l$ vanishes. Therefore, as a consequence if all the local bloch vectors pertaining to subsystems $A, B, C$ are zero, then $\sigma_{AB}, \sigma_{BC}, \sigma_{AC} \in \text{AUS}_3$. ■

It is also of interest to also note that if Alice, Bob and Charlie share a pure 3-qubit state, with nonzero local bloch vectors, then any two of the parties can 'cheat' and collaborate, via application of of some unitary, to be able to violate the steering inequality.

**IV. ILLUSTRATIONS**

We now provide some illustrations on the application of our criterion.

(i) Absolutely Separable states - Any absolutely separable state will $\in \text{AUS}_3$. This is because, absolutely separable states preserve their separability under global unitary operation. In set theoretic language, if we denote the set containing the absolutely separable states by $\text{AS}$, then $\text{AS}$ forms a subset of $\text{AUS}_3$.

(ii) Any pure product state cannot be in $\text{AUS}_3$ as they can always be converted to a pure entangled state by some global unitary.

(iii) Werner states- The Werner state is given as [55], $\sigma_{\text{wcr}} = |\psi^{-}\rangle\langle\psi^{-}| + \frac{1}{4}I(|\psi^{-}\rangle = \frac{10}{\sqrt{2}})$. The state is absolutely separable for $p \leq 1/3$, hence $\in \text{AUS}_3$ in that range. The eigenvalues are $\{(1 + 3p)/4, (1 - p)/4, (1 - p)/4\}$. For $p \leq 1/\sqrt{3}$ it is in $\in \text{AUS}_3$. The range is depicted in Fig (1).

(iv) Gisin states- Gisin states were proposed in [56]. Let $|\psi_\theta\rangle = \sin\theta|01\rangle + \cos\theta|10\rangle$ and $\sigma_{\text{mix}} = \frac{3}{2}|00\rangle\langle00| + $
Steering

In the initial part of our investigation, we probe non-violation of steering inequalities under global unitary action. Relations are derived between steerability and teleportation. We also analyse the optimal violation of steering inequalities under $3$ measurement setting for two qubit systems. Such states are characterized in terms of their spectrum and the size of the set to which they belong is also estimated. Interestingly we find the necessary and sufficient conditions for absolute non-violation w.r.t the inequality in terms of bloch parameters of the state. Illustrations are provided to gain hands on insight to the impact of our work.

Our work also opens future directions of research. In our work we have provided criterion for absoluteness of non-violation of a $3$-setting steering inequality by two-qubit states. There are steerable states which satisfy our criterion for absoluteness in terms of non-violation of the steering inequality. This implies that the proposed criterion does not imply that the given two-qubit state is also absolutely unsteerable in the $3$-setting scenario. Therefore, one might be interested to probe absoluteness in terms of local hidden variable–local hidden state models for the measurement correlations arising from the given steering scenario. The extension of our work to other important steering inequalities in both bipartite and multipartite systems will be another area of useful investigation.

V. CONCLUSION

Steerability is a distinct notion of nonlocality weaker than Bell nonlocality but stronger than entanglement. Although envisaged by Schrödinger, it was recently recasted in the form of a task in [58]. In the initial part of our present contribution we revisit steering inequalities and observe some typical features of steerability related to information processing tasks.

The main focus of our work is to probe non-violation of some steering inequalities under global unitary action. In our work we have provided criterion for absoluteness of non-violation of a $3$-setting steering inequality by two-qubit states. There are steerable states which satisfy our criterion for absoluteness in terms of non-violation of the steering inequality. This implies that the proposed criterion does not imply that the given two-qubit state is also absolutely unsteerable in the $3$-setting scenario. Therefore, one might be interested to probe absoluteness in terms of local hidden variable–local hidden state models for the measurement correlations arising from the given steering scenario. The extension of our work to other important steering inequalities in both bipartite and multipartite systems will be another area of useful investigation.

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