Reliability-based analysis of highway geometric Elements: A systematic review

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Abstract: Conventional highway design approaches have primarily focused on the use of guidelines in the design of highways. These design guidelines provide nominal safety where conservative percentile values of the design inputs are used to account for the uncertainty associated with the inputs. Reliability-based analysis (RBA) been one of the elements of reliability, availability, maintainability, and safety (RAMS) has been identified as an effective method to account for the uncertainty in the design input and to assess the risk related to a particular design. RBA approaches have effectively been used for certain purposes in other disciplines. In highway geometric design literature, these methods were also investigated and showed promise. Given the compelling importance of RBA in highway design, this paper provides a systematic analysis and evaluation of RBA applications for ten highway geometric elements: stopping sight distance, passing sight distance, intersection sight distance, horizontal curve design, vertical curve design, number of freeway lanes, highway grade length, truck escape ramp, and design guide calibration. The review consists of four parts: the concept of RAMS, background on reliability theories, applications in highway geometric design, and guidelines for the use of reliability analysis. The literature review revealed that the application of reliability-based analysis in highway geometric design leads to significant improvements in traffic safety. It is our hope that this paper will serve as a source of

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PUBLIC INTEREST STATEMENT
A lot of challenges are encountered by highway engineers as a result of variability in drivers, roadways and vehicle characteristics and conditions. Current design guides provide a deterministic approach for design requirements; conservative percentile values were used for inconstant design inputs to account for uncertainty. These conservative percentile values are not based on safety thus leading to designs with unknown safety levels. An alternative approach to account for uncertainty in the geometric design process is reliability analysis. This paper presents a systematic review and appraisal of different reliability methods and software for it analysis. The review demonstrated an improvement in the design of highway geometric elements as it relate to safety in terms of reliability.
information on RBA for highway designers and practitioners, promoting its development and application in highway geometric design.

**Subjects:** Reliability & Risk Analysis; Civil, Environmental and Geotechnical Engineering; Transportation Engineering

**Keywords:** reliability-based; analysis; highway; geometric elements; design; systematic; review; applications

1. **Introduction**

Highway geometric design is concerned with the arrangement of roadway's physical characteristics according to standards and restrictions. The main aim of geometric design is to improve on efficiency and safety while reducing cost and environmental effects. Geometric design of highway includes designing roads to support wider community objectives, including providing access to workplaces, colleges, businesses, and homes, accommodating a variety of modes of transport such as walking, cycling, public transportation, and vehicles, and minimizing fuel use, pollution and environmental harm (FHWA (Federal Highway Administration), U. S Department of Transportation, 2017). Alignment, profile, and cross-section are the three main subdivision of geometric roadway design. Combined, they provide a three-dimensional layout for a roadway. The alignment is made up of horizontal tangents and curves. The profile is the vertical aspect of the road, including the crest and sag curves, as well as the straight grade lines connecting them. The arrangement and number of automobiles and bicycle lanes and sidewalks, along with their cross slope or banking is the cross-section (FHWA (Federal Highway Administration), U. S Department of Transportation, 2017).

The heterogeneity of drivers, vehicles, and roadway conditions and capabilities must be taken into account by highway designers when designing (Porter et al., 2012). As a result of intrinsic haphazardness and inadequate knowledge about design controls and inputs that affect design standards and decisions, variability and uncertainty are essential. In addition, since natural variability is correlated with the features and efficiency of vehicles, drivers, environment, and the traffic, uncertainty from intrinsic haphazardness in the design parameters cannot be completely removed. These “inputs” in architecture also amount to differences in time and space.

Uncertainty in the factors that influence design decisions have traditionally been discussed absolutely in civil engineering disciplines (Benjamin & Cornell, 1970). When the amount of uncertainty in parameters affecting the design is negligible, average values are used. Conservative values are used, as is often the case with highway geometric design, when the degree of variability is high (Musunuru & Porter, 2014). Variability and uncertainty have also been clearly discussed in many civil engineering design disciplines using probabilistic design methods. In the geometric design literature, these design methods were often explored using reliability principles, although it is not introduced yet in the U.S. and most African design practices. The application of this reliability methodology has demonstrated how to explicitly consider the range of expected designs as well as operational and/or safety performance to inform design decisions (Musunuru et al., 2019).

Recently, as an important tool to account for uncertainty in the geometric design process and to assess the risk associated with a specific design is reliability analysis which is an element of RAMS. In this approach, the design parameters are interpreted as random variables expressed in terms of their probability distributions as compared to single value estimates in the deterministic method (De Santos-Berbel et al., 2017). The aim of employing reliability theory to roadway geometric design is to build and encourage a more coherent and dependable design process (Mollashahi et al., 2017). The reliability theory can be applied to establish safety factors that combine the
uncertainty of supply and demand variables. The resulting safety factor is the probability of non-compliance ($P_{nc}$) which is correlated with an estimate of the probability that demand exceeds supply or that a specific design does not comply with standard requirements (Hussein et al., 2014). Reliability analysis depicts a complementary approach of estimating safety in forms of risk and not considered to be an alternative to the use of collision frequency to evaluate safety (Richl & Sayed, 2006).

A systematic review of diverse uses of reliability analysis to roadway geometric design criteria and decisions is presented in this paper. Applications of safety and reliability analysis to passing sight distance, stopping sight distance, intersection sight distance, vertical and horizontal curve design, design guide calibration, highway grade length, truck escape ramp, and basic number of lanes on freeways are also presented. The purpose of this paper is to locate, review and systematically combine published literature that specifically integrates analysis of uncertainty and reliability into standards and decisions for highway geometric design. The paper establishes other methods, appraising geometric design standards and decisions that critically take into account the variabilities in geometric design controls and “inputs” design and their impact on the degree of uncertainty in “outputs” design with regard to safety. The review included national and international research papers. These tools, attempted to indicate the various ways in which reliability has been used in the design of highway geometric elements, and to identify any research gaps that requires immediate attention.

The rest of the paper is structured as follows. Section 2 gives a brief concept on reliability, availability, maintainability, and safety (RAMS). The background theory on RBA methods in highway geometric design is reviewed in section 3. Some analytical methods are also explained, along with their advantages, disadvantages, assumptions, and application examples. Sections 4–5 presents the application of RBA to passing sight distance, stopping sight distance, intersection sight distance, vertical and horizontal curve design, number of freeway lanes, highway grade length, truck escape ramp, and design guide calibration with their respective limit-state functions. The features of various applications (organized from the previous to the latest) are outlined in table format. The table presents the geometric design area, analytical models used, and references. Software packages for reliability analysis are also presented in this section. Their features, limitations, main reliability methods, and applications examples are described. Section 6 presents the knowledge gained from the review, including the broad variety of analytical models used, the software packages used for reliability analysis, and the future applications of RBA in geometric element design. Section 7 presents concluding remarks.

2. The concept of RAMS
To ensure that engineering systems work safely for the planned service life, while adhering to risk-related safety criteria. In order to get the most out of the systems operation while minimizing the effect on different factors, design decisions must be made. RAMS are index for measuring the performance of this (Coit & Zio, 2019). RAMS engineering has thus emerged as a formal engineering discipline based on mathematical principles, especially those of probability theory and statistics, for systematically and rigorously analyzing functional problems in components and systems with the goal of producing a design that has the desired properties. It is an engineering discipline that focuses on studying and assessing the ability of processes, products, and services to perform the functions that they were designed to accomplish. For this, RAMS principles and methods have been developed and refined over time to determine the quality of design decisions with the goal of minimizing errors and unplanned downtime for purpose of safety or economics, inevitably leading to optimum machine safety and efficiency (Coit & Zio, 2019).
Modern engineering systems are becoming increasingly complex in order to meet the public high demands for high functionality, efficiency and reliability, and RAMS properties have become even more important in terms of design, maintenance and commercialization. RAMS focuses on the most important aspects of success and technicalities (Pratico & Giunta, 2018). The following basic RAMS elements were defined by (CENELEC (European Committee for Electrotechnical Standardization), 1999): (1) Reliability is the likelihood that an item will perform a required function under specified conditions for a specified time interval; (2) maintainability is the likelihood that a given active maintenance action, for an item under specified conditions of use will be completed within a specified time interval when performed under specified conditions and with specified procedures and resources; (3) availability refers to the possibility that a system or its components will be operational at any given time, t; and (4) safety refers to the state of a technological system’s freedom from reasonable risk of damage. During the lifecycle of a device, the obligation to not harm individuals, the environment, or any other properties is known as safety. Any RAMS analysis considers safety and availability as output, and any contradictions between safety and availability requirements will prevent the development of a dependable system (CENELEC (European Committee for Electrotechnical Standardization), 1999).

3. Review of reliability analysis methods
Reliability mostly termed to the probability of safe performance over a given period of time, relates to the counterpart of the failure probability Equation (1). The word likelihood of failure in the reliability analysis reflects the possibility of an unwanted occurrence surpassing a given boundary. Researchers also suggested the use of probability of non-compliance (Prnc) in road design to define the likelihood of a design that does not meet requirements (Essa et al., 2016; Hussein et al., 2014; Navin, 1990; Richl & Sayed, 2006). The reliability is obtained as follows:

\[
\text{Reliability} = 1 - Prnc
\]  

(1)

Where \( Prnc \) = probability of non-compliance or system failure. The analysis consists of two components: the Limit State Function (LSF) that defines the failure mode, and random variables that explain the uncertainty. The first stage is identifying LSF, represented by \( g(X) \), which marks
non-compliance. The input vector of random variables is represented by $X$ (Haukaas, 2012c). The LSF is generally portrayed as a supply-and-demand equilibrium. The single-member system is the simplest in this regard, with variable $S$ representing the system supply (such as utmost power, friction resistance, and so on) and variable $D$ representing the system demand (such as the speed, traffic loading, and so on). This system is considered inefficient when demand exceeds supply, resulting in system failure (Mollashahi et al., 2017; Ranagnatan, 1999):

$$P_{nc} = P(S < D) = P(S - D < 0) = P\left(\frac{S}{D} < 1\right)$$

(2)

The safety margin ($Z$) is obtained as follows:

$$Z = S - D$$

(3)

Where $Z$ is a random variable. If $Z \geq 0$, the supply of the system is safe, otherwise it is non-compliance. The expected value and variance of $Z(E[Z]$ and $Var[Z]$) are obtained as follows:

$$E[Z] = E[S] - E[D]$$

(4)

$$Var[Z] = Var[S] + Var[D]$$

(5)

The ratio between the expected value of the safety margin $E[Z]$ and its standard deviation ($\sigma_Z = (Var[Z])^{1/2} = 1.5 \times 1.25 \times 10^{-2}$) is the reliability index $\beta$, obtained as follows:

$$\beta = \frac{E[Z]}{\sigma_Z}$$

(6)

Figure 1 shows the probability distribution of $Z$, and $P_{nc}$ is represented by the shaded area. A high value of $\beta$ shows that the $P_{nc}$ is small. Depending on the essence of the problem being studied, the limit state may be linear or non-linear. If it is non-linear, the probability of failure can be determined using different probability methods, as discussed below.

Several literature has explored the methods of determination of the failure probability in reliability analysis given due consideration to the LSF $g(X)$; LSF defines what constitutes a failure. It is the case for which the probability is sought. Depending on the nature of the problem being analyzed, the limit state can be linear (one for which the mean and standard deviation can be accurately computed) or non-linear (one in which the failure probability can be determined using different probability methods). Different reliability analysis methods can be used to calculate the probability of non-compliance; Analytical methods like the first-order reliability method (FORM), the first-order second-moment method (FOSM), the second-order reliability method (SORM), and the mean-value FOSM as well as sampling methods like Importance Sampling and Monte Carlo are among them.

The simplest reliability analysis method is the FOSM method. Its analysis is based on the output function’s first-order Taylor expansion at the mean values of the design inputs. While simple, this approach has several disadvantages. First, in FOSM the LSF is approximated at the mean. This leads to the invariance problem, in which different LSF formulations result in different non-compliance probabilities (Melchers, 1999). Furthermore, the FOSM approach is based on the assumption that there is a normal distribution of input parameters. Normal distribution emerges
in situations where the random variable is a sum of many underlying and independent variables. It is a two-parameter distribution in which the two parameters directly represent the standard deviation and the mean (Haukaas, 2019b). For random variables, the MVFOSM method considers only second-moment information along with a first-order approximation of the LSF on the mean of the random variables. This means that the limit-state surface, i.e., $g = 0$, for the linearized function and the actual function, is different for a nonlinear LSF. This approach uses the LSF’s mean and standard deviation to create a reliability measurement. The calculation is called the index of reliability and function as a surrogate for the likelihood of failure. For linear LSF, this reliability index is accurate but suffers from an invariance problem (Haukaas, 2012c, 2012b). The linearization of the non-linear LSF can result in an inaccurate calculation of the probability of non-compliance in this method and it also ignores information about the distribution type of the random variables.

The LSF $g(x)$ is first converted to the regular normal space, where the LSF is represented as $G(y)$, in the FORM method (Haukaas, 2011). The FORM method is defined by a linear approximation of the LSF at a point on the surface of the limit state where $g(x) = 0$, known as the design point ($y^*$), which is the closest point on the surface of the limit state to the origin of the regular normal space (Navin, 1991). This point is the failure domain with the highest probability density. This is where a substantial portion of the failure probability density is found (Haukaas, 2018). The design point, also known as the most possible point or beta point, is the optimal point for the linearization of the LSF and invariant of its formulation (Der Kiureghian, 2005). The limit-state surface, which distinguishes the failure domain from the safe domain at this point, was shared by all LSF equivalents. The FORM method uses the upgraded HassoferLind-Rackwitz-Fiessler algorithm (IHLRF), developed in the 1990s, to locate the design point by solving the following optimization problem:

$$y^* = \arg\min \{ || y \| / G(y) = 0 \}$$

(7)
where \( y^* \) is the design point coordinates and \( G(y) \) is the standard space in the LSF. The reliability index \( \beta \) specified the distance of the design point from the origin of the standard normal space.

\[
\beta = \| y^* \| \tag{8}
\]

The regular normal cumulative distribution function (CDF) can be used to estimate the probability of non-compliance as follows:

\[
P_{nc} = \Phi(-\beta) \tag{9}
\]

The FORM method has many advantages since it is performed in a transformed space, where all variables are translated from their original distributions to a standard normal distribution. Furthermore, in FORM analysis, the first-order linear expansion is performed at the design stage, which is more accurate than doing so at the mean value. Finally, the FORM approach helps to explore sensitivity to design inputs by the designer. It can be very useful to use the FORM analysis for limit state functions that are smooth and differentiable. Using a second-order function, the SORM method approximates the LSF. This fixes potential inaccuracies when the LSF is nonlinear, but compared to the FORM approach, more calculations are required. The FOSM method is simpler than the AFOSM method and extends the output function at the mean value of the random variables. The AFOSM method is a more precise method of reliability, using iterations to minimize errors in the FOSM method by extending the output function at the failure boundary design points. The Monte Carlo simulation provides a reliable method for estimating the probability of non-compliance, but it does not account for sensitivity to various design inputs, and sampling estimation of the sensitivities may be computationally costly. Importance Sampling can provide precise results than the Monte Carlo approach for the possibility of non-compliance using a smaller sample size since FORM analysis results is built upon it (Melchers, 1999).

If \( g(X) \) is nonlinear, the probability of failure can be determine using FORM or SORM method (Ranagnatian, 1999). To evaluate the index \( (\beta) \) of reliability, these methods normalize the nonlinear LSF. In other words, these methods linearize the LSF at the design point \( (X^*) \) and define \( \beta \) as the shortest distance between the coordinate system's and the design point at the normalized limit surface \( g(U) = 0 \), as shown in Figure 2. The LSF must be distinguishable in order to effectively implement these methods. The reliability index \( \beta \) is calculated using the correlation matrix between variables with non-normal distribution as follows (Low & Tang, 2007):

\[
\beta = \min_{x \in F} \left[ \frac{X_i - \mu_i}{\sigma_i} \right]^T \left[ R^{-1} \right]^{-1} \left[ \frac{X_i - \mu_i}{\sigma_i} \right] \tag{10}
\]

where: \( X_i \) = non-normal distribution random variables; \( \sigma_i \) and \( \mu_i \) = the standard deviations of \( X_i \) and mean values respectively; \( F \) = failure domain of \( g(X) = 0 \); and \( R \) = correlation matrix of \( X_i \). The total probability and the probability density ordinate of the equivalent normal distribution can be equated with those of the corresponding non-normal distribution to determine the values of \( \mu_i \) and \( \sigma_i \) in the equation (Low & Tang, 2007). The possibility of non-compliance that can be accomplished using the standard normal function of cumulative distribution (Echaveguren et al., 2005; Low & Tang, 2007) is obtained using:

\[
P_{nc} = \Phi(-\beta) \tag{11}
\]
| Method  | Consideration                                                                 | Merits                                         | Demerits                                                                 | Reference Geometric Design Applications                                                                 |
|--------|-------------------------------------------------------------------------------|-----------------------------------------------|--------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| FOSM   | Normal distribution of input parameters                                       | Simple to use                                  | Approximation of LSF at the mean; Ignores RV distributions               | (Hussain & Easa, 2015; Osama et al., 2016; Greto & Easa, 2019; Easa & Hussain, 2016; Hassein et al., 2015; Easa & Cheng, 2013; Easa, 2000) |
|        |                                                                               |                                               |                                                                          | Left turn SD at a signalized intersection; SD of permitted left turn; Length of TER; SD at stop-control intersection; Superelevation distribution on HC; Minimum PGI at intersections; Uncontrolled intersection |
| AFOSM  | Normal RV; Expands performance function at the design points                  | Normalize RV with a mean of zero and unit SDV  | Ignore RV distributions                                                 | (Greto & Easa, 2019; Easa, 1994; Faizi & Easa, 2018)                                                   |
|        |                                                                               |                                               |                                                                          | Length of TER; SD at railroad grade crossing; SSD at roundabouts                                           |
| FORM   | Linearized the LSF at the design point                                        | Considers RV distributions; Provides a good approximation if LSF is nearly linear | Higher computational cost; Non-convergence issue; Inaccurate estimate of the design point if LSF is nonlinear | (Alsaleh et al., 2020; Richl & Sayed, 2006; De Santos-Berbel et al., 2017; Shin & Lee, 2015; Dhahir & Hassan, 2016; Hussein et al., 2014; Wang et al., 2018; Llorca et al., 2014; De Santos-Berbel et al., 2017; Dhahir & Hassan, 2016; Cruz-Maraboli & Echaveguren, 2019; Ismail & Sayed, 2009; Echaveguren et al., 2005; Ibrahim et al., 2012; Dhahir & Hassan, 2019b) |
|        |                                                                               |                                               |                                                                          | Vehicle skidding and rollover on HC; Narrow median with tight HC; SD on highway alignments; Safety approach to HC design; Design of HC; Calibration of geometric design; Critical segment length of grade; PSD on two-lane rural roads; Middle ordinate; ASD on HC; HC on a two-lane rural highway; Rollover on HC; Crest VC; Restricted SD on HC; Framework for HC design |

(Continued)
Table 1. (Continued)

| Method         | Consideration                                                                 | Merits                                                                 | Demerits                                                                 | Reference Geometric Design Applications                          |
|----------------|-------------------------------------------------------------------------------|------------------------------------------------------------------------|------------------------------------------------------------------------|-------------------------------------------------------------------|
| MCS            | Transforms the integral function into the space of standard normal RV         | Suitable for solving complex functions; Samples of RV are generated from any distribution | Does not generate sensitivity to various design inputs; Requires large computations | (El-Khoury & Hobeika, 2007), (Sarhan & Hassan, 2008), Mollashahi et al., 2017; De Santos-Berbel & Castro, 2015; Musunuru & Porter, 2014; Rajbongshi & Kalita, 2018; Andrade-Cataño et al., 2020; Gargourm & El-Basyouny, 2020 | SD of straight highway segments; Insufficient SD on HC; Calibration of superelevation for HC; SSD and ASD on highway section; Vehicle density and levels of service; SSD on HC; HSD on sag curve; SSD on highway segment |
| Importance Sampling | Uses sampling distribution that is centered closer to the failure region | Gives accurate results for Pr | The auxiliary unit fraction is usually selected as the shifted standard normal distribution | (Osama et al., 2016) | SD of permitted left turn |

FOSM = first-order second-moment method, AFOSM = advanced first-order second-moment method, FORM = first-order reliability method, LSF = limit state function, MVFOSM = mean-value first-order second moment, SORM = second-order reliability method, MCS = Monte Carlo Simulation, P_{nc} = probability of noncompliance, RV = random variables, SD = sight distance, TER = truck escape ramp, HC = horizontal curve, SSD = stopping sight distance, PSD = passing sight distance, HSD = headlight sight distance, ASD = available sight distance, VC = vertical curve, PGI = pedestrian green interval, SDV = standard deviation

Easa (2000) performed the reliability of intersection sight distance for at-grade intersections in five different cases using the mean-value first-order second moment (MVFOSM) method based on AASHTO. The FORM was used by Richi & Sayed (2006) to assess the safety risk of narrow medians combined with tight horizontal curves. De Santos-Berbel et al. (2017) also used the FORM method to determine the level of risk related with insufficient sight distance for different available sight distance (ASD) modeling methods. Vehicle’s sideslip and rollover on horizontal curves was analyzed by Shin & Lee (2015) using the FORM reliability method. A probabilistic, safety definite approach of horizontal curve using the FORM method was developed by Dhahir & Hassan (2019a). Essa et al. (2016) applied the FORM method to the design of the horizontal curve. Hussein et al. (2014) used the FORM method to calibrate geometric design models to produce coherent level of risk.

The Monte Carlo Simulation (MCS) method has also been used in many studies. Distribution of passing sight distance on straight highway segments due to variations in the random variables was determined by (El-Khoury & Hobeika, 2007) using the MCS method. (Sarhan & Hassan, 2008) also applied this approach to evaluate the likelihood of risk in the design of horizontal curves overlapping with the flat grade, crest, and sag curves in excavated cross-sections of the road where the side slopes affect the sightline resulting in limited sight distances. (Mollashahi et al., 2017) used MCS techniques to calibrate super-elevation for the horizontal curve. (De Santos-Berbel & Castro, 2015) carried out a 3D full-length evaluation of SSD using the MCS method.
Porter, 2014) applied the MCS method to obtain the variability of vehicle density and levels of service (LOS) resulting from traffic-related variables.

In order to allow for the comparison and validation of a proposed reliability method, several researchers have used two different methods in their studies. (Alsaleh et al., 2020) computed the probability of non-compliance for passenger-car and heavy-truck vehicles using the FORM and MCS analysis methods. (Hussain & Easa, 2015) used the FOSM and Monte Carlo Simulation methods to determine the safety margin of left-turn sight distances at signalized intersections for a given probability of non-compliance based on various intersection and traffic characteristics. (Llorca et al., 2014) used the FORM method to evaluate the level of risk of passing sight distance (PSD) standards design and marking requirements on two-lane rural roads based on experimental data and compared the results with a Monte Carlo Simulation. The FOSM and Importance Sampling methods was used by (Osama et al., 2016) in their proposed reliability analysis model to assess the probability of a limited sight distance for permissible left turns in the presence of opposing left-turn vehicles. The FORM and MCS methods were used by (Wang et al., 2018) to define the proper important segment lengths of the near-maximum grade (NMG). (Himes & Donnell, 2014) developed a probabilistic approach to the design of horizontal curves using the FORM and SORM methods, and compared the results with the current design standards.

Table 1 shows a summary of the analytical methods used for the calculation of the probability of non-compliance. It is notable that the majority of the studies made use of the FORM and MCS methods. This may be due to their advantages over other methods. Section four provides details regarding the application of RBA in passing sight distance, stopping sight distance, intersection sight distance, vertical and horizontal curve design, number of freeway lanes, highway grade length, truck escape ramp, and design guide calibration. The LSF for each of the application areas are shown.

4. Applications in highway geometric design

4.1. General

The theory of reliability offers analytical tools for determining the variability of input parameters throughout the design process. Primarily the reliability analysis is used to evaluate the probability that the elements of a design are within acceptable limits. The probability of failure has been explored as an indicator in several civil engineering disciplines. However, according to (Ismail & Sayed, 2009), the probability of non-compliance (Pnc) is more appropriate for road safety applications since there is no physical failure in those systems.
The goal of applying reliability theory to highway geometric design is to establish and promote a more consistent and reliable design process. (Navin, 1990) pioneered the introduction of the limit-state concept to the field of highway geometric design, however, it has only been used in isolated conditions, such as horizontal curves. In 1991, he designated the probability of failure used in structural engineering as the probability of non-compliance to address designs that did not meet the standard requirements. Afterward, the application of reliability theory in the field of transportation engineering was reported—several times in open literature. Some specific areas for RBA application are discussed in the following sections.

4.2. Stopping sight distance (SSD)

Several researches have focused specifically on stopping sight distance. A plausible reason is that it is one of the main criteria for a design that is relevant to any location along a roadway. It influences the minimum dimensions of highway features, including the length of vertical curve and offsets to horizontal sightline obstructions (Sarhan & Hassan, 2008).

Reliability assessments consider two forms of stopping sight distance: the required stopping sight distance ($R_{SSD}$) and the available stopping sight distance ($A_{SSD}$) (Ismail & Sayed, 2009; Sarhan & Hassan, 2008). $R_{SSD}$ is the addition of two distances: (i) the distance that a vehicle traverses from the moment the driver sees an object in the roadway necessitating a stop to the instant the driver applies the brakes (i.e. brake reaction distance), and (ii) the distance required to stop a vehicle from the moment the driver applies the brakes (i.e. braking distance) (AASHTO, 2018). $R_{SSD}$ is therefore a function of several factors such as the design speed, driver, and vehicle factors, as illustrated in equation (12). In the equation, it is assumed that the driver is traveling at the designed speed.

\[
R_{SSD} = 0.278V_t + 0.039 \frac{V^2}{a}
\]  

(12)

Where $R_{SSD} =$ required stopping sight distance (m), $V =$ design speed (kmh$^{-1}$), $t =$ perception-reaction time (s), and $a =$ deceleration rate (ms$^{-2}$).

$A_{SSD}$ is the distance along the roadway throughout which an object of a specified height is continuously visible to the driver. $A_{SSD}$ depends on the characteristics of the road geometry, vehicle, driver, and object, including cross-section elements, conditions of the roadside, horizontal and vertical alignments, height of the driver's eyes, and the height of object (Musunuru et al., 2019). Within the context of SSD, equation (13) can be used to determine the associated limit state function, in terms of $A_{SSD}$ and $R_{SSD}$.

\[
L_{SSD} = A_{SSD} - R_{SSD}
\]  

(13)

Where: $L_{SSD} =$ limit state function for SSD, $A_{SSD} =$ available SSD, and $R_{SSD} =$ required SSD.

The limit state function in terms of design speed, driver, and vehicle factors can then be written as follows:

\[
L_{SSD} = A_{SSD} - \left(0.278V_t + 0.039 \frac{V^2}{a}\right)
\]  

(14)

Rajbongshi & Kalita (2018) proposed a probabilistic method for the determination of the SSD, taking into account the variability of all input parameters for the horizontal curve design. It was observed that the SSD parameter follows a lognormal distribution with the coefficient of variance (COV) equal to 15.79 percent. The 98th percentile sight distance value (i.e. ASDcal with $Re =$ 98%) is much lower than the sight distance calculated based on the 98th percentile speed (i.e. ASD). The
authors concluded that the existing calculation approach for sight distance is an overestimated design concept that should be re-considered instead of introducing an excessively higher radius and/or middle ordinate. Reliability-based design charts have also been reported for both plain and hill areas taking into account the impact of lateral thrust. (Gargoum et al., 2018) also investigated the extent to which ASD on highways meets the needs of drivers with limited abilities. The authors developed algorithms to estimate ASD on seven different crash-prone segments in the Province of Alberta, Canada, and these were compared to the SSD requirements that integrate limitations in cognitive abilities on each highway. The results revealed that, a substantial portion of the analyzed segments do not meet the requirements (up to 20%). Similarly, when compared to the AASHTO SSD requirements, nearly 6% of the analyzed segments did not meet the standard. To assess the alignment of in-service highways, (De Santos-Berbel, 2017) developed a methodology for studying a three-dimensional sight distance for highway safety. The differences between the available and stopping sight distances were examined using different elevation models as inputs. The available sight distance values of the models used were found to be statistically significant. The results were used in conjunction with the reliability theory to establish the impact of available sight distance on highway safety. The author concluded that the developed models would help to establish a relationship between the collision frequency and the design variables, which could lead to a broader application of the reliability-based geometric design and the use of the reliability-based design in the conventional cost-benefit analysis.

De Santos-Berbel & Castro(2015) proposed a novel probabilistic method for estimating SSD. The equation provided by the German guidelines, which takes the effect of grade into account along the section traveled during the stopping maneuver, was adopted. Their simulation performed...
100,000 trials of emergency stops on the alignment of the real highway to test the reliability of the method, comparing the results to ASD. The results revealed that there were five clusters of 15 segments where the probability of hazard exceeded 38%. Three of these clusters were located within the horizontal curves of the minimum radius (450 m), two of them coinciding partially with crest curves (K = −6,000 m and K = −5,700 m). The authors concluded that areas in which the probability of hazard was higher were located within the zones where alignment elements have tighter parameters. (Ismail & Sayed, 2012) presented a probabilistic analysis methodology that enables the re-dimensioning of different geometric elements located on highway segments with restricted sight distance. It also provides a decision mechanism for the efficient use of the available right-of-way for new highway construction. While their previous work (Ismail & Sayed, 2010) presented a methodology for risk assessment, this work presents a methodology for risk-optimization for highway segments constructed in the restricted right-of-way.

De Santos-Berbel et al. (2017) employed reliability analysis to investigate the safety implications of modeling ASD. The analysis covered the ASD of 402 horizontal curves, located in twelve two-lane rural highways. The 3 ASD estimation methods used included 2D ASD, 3D digital terrain model (DTM) ASD, and 3D digital surface model (DSM) ASD. The ASD results obtained using the 2D and 3D methodologies were compared. The ASD comparison results showed a significant difference among the three ASD modeling methods in most cases. The authors emphasized the importance of using the 3D modeled sight distance, particularly the 3D DSM ASD, either in highway design or during service life as it is considered more realistic than the 2D modeled sight distance. (Castro et al., 2012) extended this work by analyzing the influence of DTM and DSM features on the outcome of sight distance calculation in highways. They used three different types of Digital Elevation Model (DEM): DTM01, DSM01 and DSM Mobile Mapping System (MMS). A comparative analysis of the results was carried out using both the Kolmogorov–Smirnov and Mann–Whitney–Wilcoxon tests to determine whether the three series of results statistically are from the same distribution. The authors concluded that, in sight distance studies of roads where it is essential to take into account roadside obstacles (such as trees or buildings), the use of a DSM is necessary if realistic results are to be achieved. (Andrade-Cataño et al., 2020) applied a probabilistic approach to evaluate the probability of non-compliance associated with sag curve designs and to assess the effects of headlight sight distance variables on the probability of non-compliance. A Monte Carlo Simulation was used to determine the risk level associated with 71,334 case studies generated by combining different values of the variables involved in headlight sight distance following Spanish standards. The result revealed that variables modeling headlight characteristics greatly affect the risk level.

4.3. Passing sight distance (PSD)
The design standards for PSD are intended to provide the passing driver with sufficient view ahead to determine that there are no potentially conflicting vehicles before beginning and completing a passing maneuver (El-Khoury & Hobeika, 2007). This reduces the risk of a collision with oncoming traffic (AASHTO, 2001). Design practices will be most effective when they anticipate traffic controls (i.e., passing and no-passing zone markings) that will be placed on the highways (AASHTO, 2018). Reliability assessments recognize two types of passing sight distance: required passing sight distance ($R_{PSD}$) and available passing sight distance ($A_{PSD}$).

$R_{PSD}$ is the sight distance required to safely pass a slower vehicle while an oncoming vehicle approaches in the opposing lane. $R_{PSD}$ models in (AASHTO, 2018) incorporate the interactions of three vehicles; the passing, passed and opposing vehicle. Many studies have verified that the passing sight distance values are consistent with field observations of passing maneuvers (Harwood et al., 2007). Two theoretical models for the sight distance needs of passing drivers have been used. Both models are based on the assumptions of passing drivers. The Glennon model
Glennon, 1998) assumes that the critical position occurs when the passing sight distance to complete the maneuver is equal to the sight distance needed to abort the maneuver. The model presented by (Hassan et al., 1996) assumes that the critical position occurs when the passing sight distances to complete or abort the maneuver are equal or when the passing and passed vehicles are abreast, whichever occurs first.

\[ R_{PSD} \] is comprised of four distances, as indicated in Figure 3 (El-Khoury & Hobeika, 2007). Contributing factors for the calculation of minimum \( R_{PSD} \) include driver and vehicle-related parameters such as perception-reaction time, acceleration, and deceleration rates when beginning or aborting the passing maneuver (Harwood et al., 2007).

\[ R_{PSD} = d_1 + d_2 + d_3 + d_4 \]  

(15)

Where \( R_{PSD} \) = required passing sight distance, \( d_1 \) = distance traveled during the perception and reaction times and initial acceleration to the point of encroachment on the left lane, \( d_2 \) = distance traveled while the passing vehicle occupies the left lane, \( d_3 \) = distance between the passing vehicle and opposing vehicle at the end of the passing maneuver (such as clearance distance), and \( d_4 \) = distance traveled by an opposing vehicle for two-thirds of the time the passing vehicle occupies the left lane, or 2/3 of \( d_2 \). Phase 1 represents the initial maneuvering stage. This phase includes a time for perception and reaction and an interval during which the driver brings the vehicle from the trailing speed to the point of encroachment on the left or passing lane. Phase 2 is the passing stage, which includes the clearance distance and the distance traversed by the opposing vehicle. As shown in Figure 3, the opposing vehicle appears at point Y when the passing vehicle reaches point X.

Available sight distance (\( A_{PSD} \)) is the length of the roadway ahead over which an object (opposing vehicle) would be visible to the driver. The available sight distance is dependent on the characteristics of the road geometry, vehicle, driver, and object, including cross-section elements, roadside conditions, horizontal alignments, vertical alignments, driver eye-height. It is also dependent on the characteristics of the objects to be seen, in this case, a certain portion of an opposing vehicle for a driver to recognize it as such. When estimating \( A_{PSD} \) in a geometric design, the driver eye height is assumed to be 1.07 m from the road surface and the opposing object height is also assumed to be 1.07 m (Musunuru et al., 2019). Applications of reliability analysis to the study of passing sight distance have received attention in the literature. Within the context of passing sight distance, the LSF that relates \( A_{PSD} \) to \( R_{PSD} \) is written as:

\[ L_{PSD} = (A_{PSD}) - (R_{PSD}) \]  

(16)

Where: \( L_{PSD} \) = limit state function for passing sight distance, \( A_{PSD} \) = available passing sight distance, and \( R_{PSD} \) = required passing sight distance.

The limit state function in terms of the distances making up \( R_{PSD} \) can then be written as:

\[ L_{PSD} = (A_{PSD}) - (d_1 + d_2 + d_3 + d_4) \]  

(17)

Llorca et al., (2014) presented an application of reliability analysis to evaluate the risk associated with PSD standards in terms of the expected probability of non-compliance. A total of 1,098 passing maneuvers were observed on several two-lane highways in Spain. Two data collection methodologies were used: external observations and an instrumented vehicle. The most significant factors affecting PSD were impeding-vehicle speed, passing-vehicle
acceleration, and headways between the impeding and passing vehicles. The results revealed that the variables with a higher influence on the value of the probability of non-compliance were the acceleration and the initial speed of the passing vehicle as well as the headway h(t3) at the end of the maneuver. The authors concluded that the application of this tool to the evaluation of PSD may account for the existing variability and the risk of deviation from the design and marking standards. (El-Khoury & Hobeika, 2007) also applied reliability theory when analyzing the risk indices involved in PSD calculations on two-lane, two-way roads. The risk levels were identified for the various available values of the PSD distribution and the current PSD standards. The risk levels of the current PSD operational criteria fell within the acceptable risk range, but were lower than the limit value of that range. The authors concluded that the minimum PSD criteria can be reduced from the current values and remain within the acceptable risk level.

4.4. Intersection sight distance (ISD)
Basic methodologies for ISD design were developed by (AASHTO, 1990). The design variables of ISD include vehicle speed, perception-reaction time, longitudinal friction coefficient, and vehicle length. These variables are random and some of them are correlated as indicated by field measurements (Fitzpatrick, 1991; Lerner et al., 1995; Olson et al., 1984; Triggs & Harris, 1982).

As in the case of passing sight distance, the object to be seen by the driver in an intersection sight distance situation is another vehicle. Therefore, the design for intersection sight distance is based on the same object height used in the design for passing sight distance (1.07 m) (AASHTO, 2018). Several researchers have applied reliability analysis to ISD in various situations, either due to a vehicle approaching an intersection (Easa, 2000) or a left-turn movement at a signalized intersection (Hussain & Easa, 2015; Osama et al., 2016). For left-turn movements at an intersection, researchers have identified two types of intersection sight distance: required intersection sight distance (RISD) and available intersection sight distance (AISD).

RISD is the sight distance required by the driver of the left-turn vehicle to be able to see ahead of the opposing vehicle and complete the left-turn maneuver without any conflict. The required sight distance for a stopped left-turn vehicle from the major to the minor road (Figure 4) depends on the speed and time gap of the major-road (AASHTO, 2018). For a known design speed and time gap, RISD can be determined using the following equation:

$$R_{ISD} = 0.278V_{g}$$  \hspace{1cm} (18)

Where: \(R_{ISD}\) = required intersection sight distance (m), \(V\) = design speed of the major road (km/h), and \(T_{g}\) = time gap required for the left-turn vehicle to cross the roadway(s). The time gap is a function of the type of left-turn vehicle and the number of lanes to be crossed (AASHTO, 2018).

The AISD for a left-turn vehicle is the sight distance required to safely complete the left-turn maneuver. The sightline extends from the eye height of the left-turn vehicle driver to the center of the opposing vehicle on the through lane adjacent to the opposing left-turn lane. The actual point of conflict occurs at the intersection of the path of the left-turn vehicle and the centerline of the adjacent through lane of the major road. For simplicity, this conflict point is assumed to lie along the centerline of the receiving lane of the minor road to which the left-turn vehicle is moving (Hussain & Easa, 2015) (Figure 4).

The limit state function that relates \(A_{ISD}\) to \(R_{ISD}\) is written as:

$$L_{ISD} = A_{ISD} - R_{ISD}$$  \hspace{1cm} (19)
Where: \( L_{\text{ISO}} \) = limit state function for intersection sight distance, \( A_{\text{D}} \) = available intersection sight distance, and \( R_{\text{ISO}} \) = required intersection sight distance

The limit state in terms of the design speed and time gap is obtained as follows:

\[
L_{\text{ISO}} = (A_{\text{ISO}}) - (0.278VT_{p})
\]  
(20)

Easa (2000) developed a probabilistic model for the intersection sight distance, in which the design speed, perception-reaction time, and friction coefficient are the random variables. The proposed method uses the moments of probability distributions (i.e., mean and variance) of all of these random variables. The first-order probabilistic analysis was used to measure the randomness associated with these design variables in analyzing the design of the intergreen interval and sight distances at the intersections. This study was extended by (Easa & Hussain, 2016) with the development of an analytical model to determine the reliability level of the stop-control intersection sight distance. The model applies to any number of lanes of the minor and major roads with or without a major-road median. The proposed method is useful for the intersection sight distance design of a new intersection or a redesign of an existing intersection for the desired reliability level.

Osama et al., (2016) used a reliability analysis framework to evaluate the risk of limited sight distance for permitted left-turn movements due to the presence of opposing left-turn vehicles. Two signalized intersection approaches in the city of Surrey were used as case studies for the framework. The analysis revealed that one approach had a slight \( P_{\text{nc}} \) for the left-turn sight distance. The second approach showed a high \( P_{\text{nc}} \), which was attributed to the large left-turn offset. The reliability analysis also indicated that the \( P_{\text{nc}} \) increased at both approaches when the opposing vehicle was a bus compared to a passenger car. The sensitivity analysis revealed that the time gap had a higher impact on \( P_{\text{nc}} \) than the speed.

Hussain & Easa (2015) went even further and presented a reliability approach to determine the probability of non-compliance based on an offset value between the medians of the left-turn and opposing left-turn vehicles for a four-lane major road, a four-lane or two-lane minor road—with level terrain and an intersection angle of 90°, and unprotected (permitted) left-turn movements at signalized intersections. Their findings revealed that the deterministic method overestimates the values of the left-turn lane offset distance. For a lower probability of non-compliance, an increased offset between the left-turn lanes is required for an existing intersection. (Easa, 1994) presented a probabilistic method that accounts for the variability of the design variables and their correlations for the sight distance design at railroad crossings based on the advanced first-order second-moment reliability analysis. The probabilistic method was found to be logically correct. The proposed method is expected to result in safer operations at railroad grade crossings.

4.5. Horizontal curve design

The elements associated with the design of horizontal curves include the design speed, side friction, rate of superelevation, and horizontal curve radius (Musunuru et al., 2019). For a given design speed, the maximum rate of superelevation, and maximum side friction, the minimum horizontal curve radius is determined as follows:

\[
R_{\text{min}} = \frac{V_{0}^{2}}{15(0.01\varepsilon_{\text{max}} + f_{\text{max}})}
\]  
(21)

Where \( R_{\text{min}} \) = minimum horizontal curve radius (m), \( V_{0} \) = design speed (m/s), \( \varepsilon_{\text{max}} \) = maximum rate of superelevation (%), and \( f_{\text{max}} \) = maximum side friction factor.
The side friction factor represents the vehicle’s need for side friction, also called the side friction demand. It also represents the lateral acceleration \(q_f\) that acts on the vehicle. This acceleration can be computed as the product of the side friction demand factor \(f\) and the gravitational constant \(g\) (i.e., \(q_f = fg\)). The side friction factor at impending skid depends on several other factors, the most important being the speed of the vehicle, the type and condition of the roadway surface, and the type and condition of the vehicle tires (AASHTO, 2018).

The physical limiting conditions of side friction based on vehicle skidding without rollover (\(S_m\)) and with rollover (\(S_{swl}\)) have been analyzed from a reliability perspective. The required side-friction (\(R_{SF}\)) is the necessary side-friction to provide for enough centripetal acceleration for a vehicle to traverse a horizontal curve (Himes & Donnell, 2014). \(R_{SF}\) depends on the curve radius, speed, and design superelevation rate.

The available side-friction (\(A_{SF}\)) is the actual tire-pavement friction that is supplied for vehicles to traverse a horizontal curve. \(A_{SF}\) depends on the condition of the pavement, roadway geometry, weather, and weather-related roadway conditions. The limit state function for the side friction and horizontal curve design is obtained as follows:

\[
L_{SF} = (A_{SF}) - (R_{SF})
\]  

(22)

Where \(L_{SF}\) = limit state function for the side friction, \(A_{SF}\) = available side friction, and \(R_{SF}\) = required side friction.

The limit state function in terms of design speed, maximum available side friction (for skidding without rollover) and superelevation can be written as:

\[
L_s = R_s - \frac{V^2}{15(0.01e + A_{SFmax}(S))}
\]  

(23)

Where \(R_s\) = radius supplied (i.e., the existing horizontal curve radius (m)), \(V\) = vehicle speed (m/s), \(e\) = design superelevation rate for the horizontal curve (%), and \(A_{SFmax}(S)\) = maximum available side friction (for skidding without rollover).

The stochastic characteristics of horizontal curve related design variables have been investigated in multiple studies using experiments and field evaluations (Felipe, 1996; Ismail & Sayed, 2009; You et al., 2012; Zheng, 1997) and have been incorporated into the reliability analysis of horizontal alignment designs. (Dhahir & Hassan, Dhahir and Hassan, 2019a) developed a probabilistic, safety explicit approach of horizontal curve design using reliability analysis for four design criteria: vehicle stability, driver comfort, sight distance, and vehicle rollover. The results revealed that considering the curve radius (\(R\)) in the regression analysis for all models added more value to the models. They inferred that the expected safety performance of horizontal curves is more sensitive to other criteria that are not directly considered in the design model. (Cruz-Maraboli & Echaveguren, 2019) also developed an analytical model for the estimation of the rollover probability of heavy vehicles on horizontal curves. The model was obtained using the FORM and Gompertz logistic growth models. The minimum radii needed to obtain a zero rollover probability were compared with the minimum radii provided by the Chilean geometrical design standards. The results revealed that the minimum radii recommended by the standard to avoid sliding do not prevent rollover. (Shin & Lee, 2015) used first-order reliability techniques to analyze and optimize the minimum radii of roadway horizontal curves. Their analysis was based on vehicle dynamics and their applications mainly focused on exit ramps and interchanges. The work investigated the
probability of rollover and sideslip for the minimum radius provided by (AASHTO, 2018), using the FORM and limit state functions, respectively. Their results revealed that the probability of sideslip drastically decreases as the friction coefficient increases. The authors deduced that rollover is highly sensitive to vehicle speed and steering angle, whereas sideslip is vulnerable to changes in the tyre–road friction coefficient, and that the probabilities of both accidents can be decreased by increasing the superelevation.

Alsaleh et al., (2020) assessed the safety performance of a horizontal curve using the insufficient sight distance, vehicle skidding, and vehicle rollover as modes of non-compliance for passenger-cars and heavy-trucks on the Sea-to-Sky highway in British Columbia, Canada. The study provides a calibrated design chart that accommodates heavy-trucks on horizontal curves with sharp radii. (You et al., 2012) also applied reliability analysis in the design of horizontal curves. In their study, the risks associated with the design were based on failure modes of vehicle skidding and rollover in the performance functions of cars and trucks, respectively. Their study considered the vehicle speed, friction coefficient, and radius to be random variables, and superelevation and vehicle parameters to be deterministic. They took into account all of the possible combinations of the design variables and calculated the probability of vehicle skidding or rollover. Their results qualitatively revealed that trucks are more vulnerable to skidding than cars, and are more likely to rollover than to skid on dry pavement. (Jesna & Anjaneyulu, 2016) carried out a reliability analysis for a single horizontal curve using data from 118 horizontal curves in Kerala. The analysis was done using the sight distance, superelevation, and extra widening as the geometric characteristics that affect the safety of a horizontal curve. Geometric data, speed data, and crash data were used for the analysis. The reliability index was calculated for each of the geometric design parameters. A safety evaluation criterion was developed for horizontal curves with the reliability indices as a measure of reliability.

Dhahir & Hassan (2016) also adopted a probabilistic approach while considering two criteria: to overcome the shortcomings of the current horizontal curve design procedures: vehicle dynamic stability and driver comfort. They used reliability analysis to provide a quantitative evaluation for the design in terms of the probability of failure (POF), probability of non-compliance (PNC), and reliability index (θ). The authors inferred that these reliability measures can be considered as surrogate measures of safety, where collisions are expected to increase with the increase of POF for vehicle dynamic stability or the increase of PNC for driver comfort.

Mollashahi et al., (2017) assessed the safety margins obtained from the application of superelevation in a horizontal-curve design. They used geometric design guides and a reliability index to determine uncertainties in the design parameters. Their results revealed that the adjusted superelevation values are generally greater than those derived from the current standards and concluded that the adjusted design charts can help designers select an appropriate superelevation value for a particular horizontal curve for highways that have geometric constraints. The adjusted design charts can also help designers predict the consequences and safety margins associated with the selection of superelevation alternatives required to approve geometric designs that involve violations of the standard requirements due to environmental constraints. (Ecchaveguren et al., 2005) developed a methodology to determine the margin of safety of an existing horizontal curve. The methodology was based on the reliability theory by which the reliability of operational conditions can be analyzed by using a reliability index as a margin of safety. A case study for light vehicles was evaluated to determine high impact variables over reliability, such as, macrotecture, skid resistance, curve radius, and superelevation. The authors integrated texture, skid resistance, the radius of curvature, and superelevation into one index. Their results demonstrated that curve radius, skid resistance, and macrotecture are variables with
a high impact on the failure probability and concluded that superelevation has little effect on the failure probability.

4.6. Vertical curve design

To maintain road visibility, crest vertical curves are designed to provide adequate sight distance to allow enough time for braking action. These are designed as a transition between two different vertical grades when the value of the ongoing grade is lower than the value of the incoming grade (Musunuru et al., 2019).

The available stopping sight distance on crest vertical curves is limited by the curve itself. To provide adequate available stopping sight distance, the appropriate length of the curve for a given grade needs to be determined. The general equations relating the length of the crest vertical curve, grade break, driver eye height, object height, and available sight distance are as follows:

\[
\text{When } S \leq L = \frac{AS^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2}
\] (24)
Table 3. Characteristics of software for reliability analysis

| Software Package | Features                                                                 | Limitations                                                                                   | Main Reliability Method | Selected References                                                                 |
|------------------|---------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|--------------------------|------------------------------------------------------------------------------------|
| Rt “risk tools”  | - Object-oriented                                                        | - To make predictions, requires the use of variety of probabilistic models, especially in situations with multiple hazards | FORM                     | (Andrade-Cataño et al., 2020; De Santos-Berbel et al., 2017; De Santos-Berbel, 2017; Osama et al., 2016; Dhahir & Hassan, Dhahir and Hassan, 2019a, Dhahir & Hassan, 2016, Dhahir & Hassan, 2019b; Hussein et al., 2014; Ibrahim et al., 2012). |
|                  | - Allows steady expansion of analysis options and as new information becomes accessible, the model library will be updated. |                                                                                               |                          |                                                                                     |
|                  | - Performs reliability and optimizations analysis with various models.    |                                                                                               |                          |                                                                                     |
| Spreadsheet (MS Excel) | - Help for complex mathematical equations and formulas is built-in         | - Unfriendliness of alpha-numeric cell addresses                                            | MCS                     | (Mollashahi et al., 2017; Essa & Cheng, 2013).                                   |
|                  | - Ability to make graphs and customize data storage.                      | - User bias                                                                                    |                          |                                                                                     |
|                  |                                                                           | - Syntax skill is required                                                                     |                          |                                                                                     |
|                  |                                                                           | - Lacks security                                                                               |                          |                                                                                     |
|                  |                                                                           | - Capacity limit of rows and columns                                                          |                          |                                                                                     |
| MATLAB           | - Contains a large library of built-in functions                          | - Packaging tools for add-ons and MATLAB compiler are not supported                          | MCS, FORM, FOSM         | [Alsaleh et al., 2020; Dhahir & Hassan, Dhahir and Hassan, 2019a; Gargoum et al., 2018; Rajbanghi & Kalita, 2018; De Santos-Berbel, 2017; Essa et al., 2016; De Santos-Berbel & Castro, 2015; Wang et al., 2018; Ismail & Sayed, 2009; Sarhan & Hassan, 2008; Hassein et al., 2015; Gargoum & El-Basyouny, 2020; Dhahir & Hassan, 2019b]. |
|                  | - Visualization, application creation, and numeric computation in a high-level language | - Cross-compiling of other language is difficult                                             |                          |                                                                                     |
|                  | - Supports an iterative environment                                       | - Expensive                                                                                    |                          |                                                                                     |
|                  | - Interface helps users with development tools for optimizing performance and improving code maintainability | - Window specific research and not suitable for development activities                      |                          |                                                                                     |
|                  | - Can collect data from spreadsheets and conduct site-by-site reliability analysis. |                                                                                               |                          |                                                                                     |
| MINITAB          | - User-friendly                                                          | - Limited range of functions                                                                  | MCS                     | (Musunuru & Porter, 2014).                                                        |
|                  | - Goodness-of-fit statistics                                              | - Weak mathematical features                                                                  |                          |                                                                                     |
|                  | - Statistical tools                                                       |                                                                                               |                          |                                                                                     |
|                  | - Seamless data manipulation                                             |                                                                                               |                          |                                                                                     |
|                  | - Brainstorming tools                                                     |                                                                                               |                          |                                                                                     |
|                  | - Graphical analysis etc.                                                 |                                                                                               |                          |                                                                                     |
Table 3. (Continued)

| Software Package          | Features                                      | Limitations                          | Main Reliability Method | Selected References               |
|---------------------------|-----------------------------------------------|--------------------------------------|--------------------------|-----------------------------------|
| Reliability Analysis      | - Allows calculation of POF for each type of  | - More efforts are required in its   | MCS, FORM                | (Richl & Sayed, 2006; Echaveguren |
| Software (RELAN)          | failure as well as in the overall system      | application                         |                          | et al., 2005)                    |
|                           | - User-written FORTRAN subroutine.            | - Limited to wood structures         |                          |                                   |

FORM = first order reliability method, MCS = Monte Carlo Simulation, FOSM = first-order second-moment method, MVFOSM = mean-value first-order second-moment, AFOSM = advanced first-order second-moment, POF = probability of failure

\[ L = 2S \left( \frac{\sqrt{h_1} + \sqrt{h_2}}{A} \right)^2 \]

When \( S > L \), \( L = 2S \left( \frac{\sqrt{h_1} + \sqrt{h_2}}{A} \right)^2 \)  \( \text{(25)} \)

Where \( L \) = length of the vertical curve (m), \( S \) = minimum available stopping sight distance (m), \( A \) = algebraic difference in grades (%), \( h_1 \) = height of the eye above the roadway surface (m), and \( h_2 \) = height of the object above the roadway surface (m).

Ismail & Sayed (2009) presented a general framework for the development of probabilistic highway design criteria that deal with the uncertainty associated with the design inputs. Their study focused on modifying typically used design models by adding calibration factors that would yield consistent \( P_{ve} \) values to the design of the crest vertical curve. The mathematical form of the calibration factors was constructed so that it compensates for the pre-calibration distribution of the design safety levels.

4.7. Number of freeway lanes

The basic number of lanes is “a minimum number of lanes designated and maintained over a significant length of a route, regardless of the changes in traffic volume and lane balance needs”. It is the constant number of lanes assigned to a route, exclusive of auxiliary lanes (AASHTO, 2018; Leisch & Mason, 2005). Decisions regarding the basic number of lanes are often driven by design LOS criteria, which can vary by agency and across area type.

The traffic density of a facility directly depends on the daily traffic of the design year, the percentage of daily traffic in the design hour, the directional distribution, the percentage of heavy vehicles, and the free-flow speed. The demand volume and the estimated average speed are used to determine the density of the traffic stream, as seen below:

\[ K = \frac{V}{S} = \frac{(K_{30} \times AADT \times D)}{(5 \times PHF \times N \times f_{HV})} \]

Where \( K \) = density (vpm), \( V \) = demand volume (vph), \( S \) = average speed (mph), \( K_{30} \) = proportion of the design year traffic in the 30th highest hour of the design year (whether the 30th highest hourly volume is used often depends on the area type), \( AADT \) = annual average daily traffic (veh/day), \( D \) = directional distribution, \( PHF \) = peak hour factor, \( N \) = number of lanes in analysis direction, and \( f_{HV} \) = adjusted factor for the presence of heavy vehicles.
There is variability in the estimate of the traffic-related design input parameters that influences
decision regarding the number of lanes due to uncertainty in traffic growth calculations, road and
vehicle characteristics, and driver characteristics. The predicted density \( P_o \) during the design hour
is used to determine a LOS, which is compared to the design LOS criterion for a specific area type
(urban/rural) where the basic freeway segment is located.

With the maximum design density \( D_o \) determined, to attain a selected design LOS, the LSF can
be written as:

\[
L_D = D_D - P_D \tag{27}
\]

Where \( L_D = \) limit state function for density, \( D_D = \) maximum design density to achieve
a selected design LOS (veh/mi/ln), and \( P_D = \) predicted density in the design hour (veh/mi/ln)

This limit state function can also be expressed in terms of the parameters for density written as:

\[
L_D = D_D - \frac{(K_{30}X_{AADTxD})}{(S\times PHFxNxf_{HV})} \tag{28}
\]

Musunuru & Porter (2014) used reliability analysis to estimate the probability distribution of
operational performance that might result from the basic number of lanes decisions made to
achieve a design LOS on a freeway. They demonstrated this concept using data from Interstate
I-15 and Interstate I-80 in Utah. Their results indicated that uncertainty in input variables has
a significant effect on the probability distribution of the operational performance on a freeway. The
uncertainty was attributed to the aleatory variability (i.e., natural randomness) in the input vari-
ables. They proposed that designers can use this method to consider uncertainty explicitly in
vehicle density and LOS (i.e., operational performance).

### 4.8. Highway grade length

Wang et al., (2018) presented a probabilistic method to specify the proper critical segment lengths of
the Near-Maximum Grade (NMG) with full consideration of expressway safety. A long section with
NMG on the Nanjing-Hangzhou Expressway was chosen as the study area and the crash data, road
geometry, and vehicle operational performance were all carefully recorded. Their results revealed
that China’s regulated values for grade lengths are slightly higher for trucks, which can be partially
attributed to poor dynamic performance including climbing and braking. The authors developed a set
of reliable limiting values for NMG section lengths and concluded that it provides more suitable
parameters for highway design to achieve enhanced operational safety performance.

The climbing distance of the vehicle on a grade is obtained as follows (Zhang, 2009):

\[
L = D(X_i/V, P, f, i) = \frac{\delta (V_2^2 - V_1^2)}{254 (P_1 + P_2 - f - i)} \tag{29}
\]

Where \( L = \) length of the vehicle climbing distance, \( L = D(X_i); D(X_i) \) demand function, \( X_i = \) set of
variables concerning grade length, \( \delta = \) correction coefficient of the rotating mass, \( V_1 = \) speed at
the center point of the first half of the vertical curve, \( V_2 = \) speed at the center point of the vertical
curve corresponding to VPI, \( P_1, P_2 = \) dynamic factors corresponding to \( V_1 \) and \( V_2 \) respectively, \( f = \)
rolling resistance coefficient and \( i = \) longitudinal gradient.
If the random variables in Equation 26 (demanded values $S$) are assumed to follow a normal distribution and the variables are independent of each other, and the value of the supply function $(D(X_i))$ is derived from the specification area set of constant value the limit-state equation of grade length $Z = (\mu, \sigma)$ can be written as:

$$Z(S, D) = S - L = D - \frac{\delta (V_2^2 - V_1^2)}{254 (R_1 + G_2 - f - i)}$$ (30)

**4.9. Truck escape ramp (TER)**

The truck escape ramp (TER) is a kind of supplementary lane used in extreme situations by unrestrained vehicles that are out of control. The ramp is designed to allow an unrestrained vehicle to stop, contributing to roadway safety. The design of TERs depends on several criteria such as the speed of the unrestrained vehicle that enters the ramp, the rolling resistance, and the ramp grade. The rolling resistance represents the energy the truck tires lose while the tire is rolling and is created by the friction between the truck tire tread and the TER surface (German Formula Auto Repair, 2019). According to the highway design guide provided by the (Transportation Association of Canada, 2017), the basic categories of TER include: gravity, sand pile, and arrestor bed.

The safety margins (SM) for a Truck Escape Ramp are as follows:

$$SM_1 = L_{\text{supply}} - L_{\text{demand}}$$ (31)

Where $L_{\text{supply}}$ = length of the TER to be supplied on the ramp (m) and $L_{\text{demand}}$ = demanded length of the TER (m).

The limit state function can be expressed in terms of the parameters of the length of the truck escape ramp, obtained as follows:

$$SM_1 = L_{\text{supply}} - \frac{V_1^2}{254 (R_1 + G_2)}$$ (32)

Where $V_1$ = vehicle entering speed in (km/hr), $R_1$ = rolling resistance, and $G_2$ = percent grade in %

Greto & Easa (2019) developed a reliability model for the design of a TER that allows designers to determine the required ramp length for a given probability of failure, $P_f$. Two reliability methods were applied to analyze various TER designs: FOSM and AFOSSM. These methods depend on the distribution of the element random variables. Each method was used to apply a TER with one-grade and two-grades. The AFOSSM method was used to establish design graphs for the required length of the TER. The application of the proposed method was demonstrated using an actual TER in the United States, while considering a speculative design situation. The results revealed that factors such as a change in grade, entering vehicle speed and rolling resistance contribute to the final design length for a given TER.

**4.10. Design guide calibration**

Hussein et al., (2014) presented an application of reliability analysis for the calibration of geometric design models to yield consistent and adequate safety levels. Their main assumption was that the design safety level associated with standard design outputs should be consistent and close to a predefined level. Their results revealed that the current design guides are conservative, especially
for high speeds and sharp curves. They concluded that significant reductions in the current design requirements could be obtained at reasonable risk levels. The authors proposed that their calibrated charts could help designers assess the safety implications of deviating from geometric design standards and quantify the safety level built-in design values that are deemed acceptable. (Ismail & Sayed, 2009) also presented an application of the calibration framework to the standard design model of crest vertical curves and constructed calibrated design charts to yield a consistent design safety level. They concluded that the proposed calibration method can be applied to all geometric design models.

5. Guidelines for the implementation of reliability analysis

5.1. Analytical models used
In geometric design research, reliability analysis has been used to determine design standards and measure the likelihood that design decisions would consistently work as expected. The random input variables were supposed to obey selected probability density distributions, but in the majority of the studies examined, no empirical data were used to produce these distributions. Real-world data can be used to verify the usefulness of the reliability approach (Musunuru et al., 2019). Table 2 provides a summary of the design criteria that were included in the reliability research reviewed in this section.

FORM = first order reliability method, MCS = Monte Carlo Simulation, FOSM = first-order second-moment method, MVFOSM = mean-value first-order second-moment, AFOSM = advanced first-order second-moment

As noted, most of the applications are based on the geometric design guidelines provided by AASHTO (AASHTO, 2018) and (Transportation Association of Canada, 2017). The majority of the studies focused on stopping sight distance and horizontal curve design. This is possibly due to their importance in ensuring safety on roads. Several analytical tools were used, as shown in Table 2. The most common tools included FORM, MCS, and FOSM. This is likely due to their simplicity, accuracy, and efficiency. For instance, in terms of the number of iterations needed to achieve the probability of failure, FORM is more effective than sampling (Haukaas, 2012c, 2012b, 2019a). The features of MCS and FOSM are detailed in Table 1. (Easa & Yan, 2019) presented a cutting-edge research on Performance-Based Analysis (PBA) in Civil Engineering. Their work also examined various PBA analytical tools and the characteristics of highway transportation applications.

5.2. Software for reliability analysis
The reliability approach has become increasingly common in the estimation of complex highway geometric infrastructures, leading to a need for computer programming to solve associated complex computing tasks. Data analysis software enables ease of analysis, saving time and improving validity. For these reasons, various software packages for RBA in highway geometric design have been made available. A brief description of the software packages is provided below and their characteristics are listed in Table 3.

Rt “risk tools” is a computer program developed by (Mahsuli & Haukaas, 2013) at the University of British Columbia in Vancouver, Canada, for system reliability and optimization analysis. It is a framework for multi-model probabilistic analysis. Its unique features include: a thorough parameterization, efficient and versatile communication within the context of models, direct differentiation process computation of response sensitivities, and the implementation of a time parameter that facilitates the modeling of time-varying phenomena (Mahsuli & Haukaas, 2013). The spreadsheet is quantitative modeling software. It is a computer program that organizes, analyzes, and stores data in tabular format. It was created as a computerized version of accounting worksheets
on paper. The software works by reading data from a table’s cells (Spreadsheet, 2020). This software can be used to create budgets, produce graphs, chart. Furthermore, it is used for storing and sorting data. It is also used to forecast future performance. MATLAB (Matrix Laboratory) is a high-level programming language that includes an immersive environment that is primarily used for numerical computing, programming, and visualization. The plotting of functions and data, the design of user interfaces, and matrix manipulations are its primary functions. It can also communicate with other programming languages such as C, C++, Fortran, and Java. It is used to analyze data, construct models and applications, and build algorithms in some cases (Introduction to MATLAB, 2020). MINITAB is statistical program that can help you create distributions for design inputs with a lot of variation. It includes goodness-of-fit statistics to aid in the identification of the best-fitting distributions (Musunuru & Porter, 2014). It is a multi-purpose software kit with a user-friendly interface. It is well adopted for educational purposes and can also be used to analyze research data as a primary tool (Minitab review, 2020).

De Santos-Berbel et al.,(2017) and (Osama et al., 2016) used Rt software to determine the probability of non-compliance (Pnc) value of the LSF. (Dhahir & Hassan, Dhahir and Hassan, 2019a) developed a MATLAB code to determine the probability of failure. (Essa et al., 2016) used the Rt software to evaluate the probability of non-compliance value for each limit state and the MATLAB script to automate the iteration process. (Hussein et al., 2014) used the Rt software (Mahsuli & Haukaas, 2013) to determine the probability of non-compliance for the middle ordinate. (Wang et al., 2018) and (Andrade-Cataño et al., 2020) used the MATLAB software for the computation of Pnc. (Musunuru & Porter, 2014) also used the MINITAB software to run a Monte Carlo simulation in their study.

In summary, these software programs have been developed by designers to assess the probability of non-compliance for highway geometric design elements for any given scenario. The design process would be performance-based, resulting in likely benefits in cost through more informed decision-making. Table 3 provides a summary of the different software used by researchers in previous studies.

6. Discussion
A broad collection of RBA applications in highway geometric design was presented in this review. Current design manuals, according to some scholars, take a deterministic approach to design criteria, accounting for uncertainty with conservative percentile value for unknown design inputs. These conservative percentile values bare not based on protection, resulting in designs with uncertain level of safety (Ismail & Sayed, 2009). Geometric design elements depend on many parameters including speed, acceleration, perception-reaction time, and deceleration. Since these parameters are stochastic and display high variability among drivers, traffic conditions, and geographic locations, knowledge of them is limited. The problem with the conventional method of selecting conservative percentiles is that it can lead to overly conservative design requirements. Furthermore, the deterministic approach gives little insight into the possible implications of any deviation from these design criteria (Llorca et al., 2014).

Reliability analysis is an alternative approach that can be used to consider any uncertainty that can arise during the geometric design phase. This method is based on the limit state design procedure and considers design parameters as stochastic variables with probability distributions rather than single values, accounting for their variability. The use of reliability analysis in highway geometric design elements has been documented by a number of researchers. Their results revealed that the design parameters that have an effect on the design requirements and decisions in highway geometric design have variability and uncertainty in both time and space. The application of these reliability methodologies has demonstrated how to explicitly consider the range of expected designs and operational and/or safety performance when making design decisions. RBA
analysis involves the development of a limit state function between the supply and demand. Various analytical tools are used to solve the limit state functions, including FORM, FOSM, AFOSM, MVFOSM, and MCS. The method used depends on the accuracy required.

Since the reliability approach has become more common in the estimation of complex transportation infrastructures, a need for computer programming to solve complex computing tasks has emerged. Data analysis software enables ease of analysis, time-saving, improved validity, ability to handle large data, and flexibility. Various software packages for RBA in highway geometric design have therefore been made available. This software helps reduce the cumbersome nature of manual analysis and offers the possibility of carrying out reliability analysis. Some of the software is available to users at no cost, such as the Rt “risk tools” (Chehade & Younes, 2020).

RBA is based on the end objectives rather than how they are accomplished. As a result, RBA is thought to be a more cost-effective strategy than conventional approaches (Task Committee on Performance-Based Design, 2018) due to their confidence and reliability in desired performance, freedom of choice of performance level by designers or analysts, and promotion of research and innovation. Some of the advantages of RBA in highway geometric design include lower cost, accident reduction, safety improvement, and a reduction of potential hazards.

According to the research papers reviewed, several countries have applied RBA in various civil engineering fields including highway geometric design. North American countries like Canada and the United States are the driving force behind the promotion and establishment of specifications/codes for the use of PBA/RBA (Eosa & Yan, 2019). Asian countries such as Japan, China, India, and Iran have recently begun to concentrate on this topic. Australian, European, and African countries have fewer publications in this area. This may be due to challenges involved in RBA application including a lack of decision tools, a lack of proficiency, inadequate knowledge on RBA, government policy, and a lack of data. Concerted efforts are required to address these challenges.

The resulting performance-based process for establishing design criteria would allow designers to consider and balance driver and vehicle operating characteristics, safety, design, and construction costs in any given situation.

7. Concluding remarks
This paper reviewed the application of reliability-based analysis to geometric design of highway. With differences in the design input parameters, the probability of non-compliance was calculated for selected design standards, including passing sight distance, stopping sight distance, intersection sight distance, vertical and horizontal curve design, number of lanes on freeways, highway grade length, truck escape ramp, and design guide calibration. The review revealed that RBA implementation in sight distance analysis has been more advanced than other elements of geometric design. The review also revealed that most reliability analysis applications in geometric design have only concentrated on evaluating one design element, with the likelihood of non-compliance representing one failure mode and recommendations for future highway design. It is important to apply system reliability analysis to highway elements that are subjected to multiple failure modes, as well as to establish more reliable operating speed prediction models that integrate various geometric elements and take into account all highway classes and their effects on traffic accidents.

Our understanding of the probability distributions of random input parameters such as braking deceleration, operating speed, and side friction factor is still limited and requires further investigation. Furthermore, code calibration must take into account all potential non-compliance modes of highway geometric elements (e.g., restricted sight distance, vehicle skidding, rolling over, etc.) as
well as the relationship between non-compliance probabilities and other protection measures such as accident data or traffic conflicts.

The methodologies described in this paper will provide highway engineers and designers with more knowledge to increase their faith in the reliability of design output in the presence of practical and context-specific variables. The inherent risk of design specifications has long been related to the likelihood of non-compliance. Many studies have proven that reliability analysis is a useful method for accounting for uncertainty in the geometric design process and assessing the risk of a particular design.

When choosing a software package for their analysis, researchers and practitioners are strongly advised to consider the suitability of the package, its features and limitations, the computer skills required, and the validity of the analysis for their research and practice. The literature reviewed in this paper included peer-reviewed journal articles, postgraduate theses, conference proceedings, technical reports, design standards, textbooks, online information, and association bulletins. The wide variety of sources is believed to provide an accurate picture of the current RBA applications in highway geometric design. This review will serve as a good source of information and references for highway designers and practitioners.

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