Monopoles and fractional vortices in chiral superconductors

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We discuss two exotic objects which must be experimentally identified in chiral superfluids and superconductors. These are (i) the vortex with a fractional quantum number ($N = 1/2$ in chiral superfluids, and $N = 1/2$ and $N = 1/4$ in chiral superconductors), which plays the part of the Alice string in relativistic theories; and (ii) the hedgehog in the $\mathbf{I}$ field, which is the counterpart of the Dirac magnetic monopole. These objects of different dimensions are topologically connected. They form the combined object which is called a nexus in relativistic theories. In chiral superconductors the nexus has magnetic charge emanating radially from the hedgehog, while the half-quantum vortices play the part of the Dirac string. Each of them supplies the fractional magnetic flux to the hedgehog, representing $1/4$ of the "conventional" Dirac string. We discuss the topological interaction of the superconductor’s nexus with the ‘t Hooft-Polyakov magnetic monopole, which can exist in Grand Unified Theories. The monopole and the hedgehog with the same magnetic charge are topologically confined by a piece of the Abrikosov vortex. This makes the nexus a natural trap for the magnetic monopole. Other properties of half-quantum vortices and monopoles are discussed as well including fermion zero modes.

I. INTRODUCTION

Magnetic monopoles do not exist in classical electromagnetism. Maxwell equations show that the magnetic field is divergenceless, $\nabla \cdot \mathbf{B} = 0$, which implies that the magnetic flux through any closed surface is zero: $\oint_S d\mathbf{S} \cdot \mathbf{B} = 0$. If one tries to construct the monopole solution $\mathbf{B} = gr/r^3$, the condition that magnetic field is nondivergent requires that magnetic flux $\Phi = 4\pi g$ from the monopole must be accompanied by an equal singular flux supplied to the monopole by an attached Dirac string. Quantum electrodynamics, however, can be successfully modified to include magnetic monopoles. In 1931 Dirac showed that the string emanating from a magnetic monopole becomes invisible for electrons if the magnetic flux of the monopole is quantized in terms of the elementary magnetic flux [1]:

$$4\pi g = n\Phi_0 , \quad \Phi_0 = \frac{hc}{e} ,$$

where $e$ is the charge of the electron.

In 1974 it was shown by ‘t Hooft [2] and Polyakov [3], that a magnetic monopole with quantization of the magnetic charge according to Eq. (1) can really occur as a physical object if the $U(1)$ group of electromagnetism is a part of the higher symmetry group. The magnetic flux of a monopole in terms of the elementary magnetic flux coincides with the topological charge of the monopole: this is the quantity which remains constant under any smooth deformation of the quantum fields. Such monopoles can appear only in Grand Unified Theories, where all interactions are united by, say, the $SU(5)$ group.

FIG. 1. Dirac monopole, ‘t Hooft-Polyakov monopole, and electroweak monopole with physical Dirac string.
In the Standard Model of electroweak interactions such monopoles do not exist, but the combined objects monopole + string can be constructed without violating of the condition $\nabla \cdot B = 0$. Further, following the terminology of Ref. we shall call such combined object the nexus. In a nexus the magnetic monopole looks like a Dirac monopole but the Dirac string is physical and is represented by the cosmic string. An example is the electroweak monopole discussed for the Standard Model (see Review): the outgoing flux of the hypermagnetic field is compensated by the incoming hypercharge flux through the Z-string (Fig.1.)

In condensed matter there are also topological objects, which imitate magnetic monopoles. In chiral superconductors their structure is very similar to the nexus: it is the magnetic monopole combined either with two Abrikosov vortices, each carrying the flux $(1/2)\Phi_0$, or with 4 half-quantum vortices, each playing the part of 1/4 of the Dirac string. We also discuss the interaction of such topological defects in superconductors with the 't Hooft-Polyakov monopole. If the latter exists, then the nexus provides a natural topological trap for the magnetic monopole.

II. SYMMETRY GROUPS

The similarity between the objects in Standard Model and in chiral superconductors stems from the similar symmetry breaking scheme. In the Standard Model the local electroweak symmetry group $SU(2)_W \times U(1)_Y$ at high energy is broken at low energy to the diagonal subgroup of the electromagnetism $U(1)_Q$, where $Q = Y - W_3$ is the electric charge. In amorphous chiral superconductors the relevant symmetry above the superconducting transition temperature $T_c$ is $SO(3)_L \times U(1)_Q$, where $SO(3)_L$ is a global group of the orbital rotations. Below $T_c$ the symmetry is broken to the diagonal subgroup $U(1)_Q - L_3$. In high energy physics such symmetry breaking of the global and local groups to the diagonal global subgroup is called semilocal and the corresponding topological defects are called semilocal strings [1]. So in chiral superconductors the strings are semilocal, while in chiral superfluids they are global, since both groups in $SO(3)_L \times U(1)$ are global there.

If one first neglects the difference between the global and local groups, the main difference between the symmetry breaking schemes in high energy physics and chiral superconductors is the discrete symmetry. It is the difference between $SU(2)$ and $SO(3) = SU(2)/Z_2$, and also one more discrete symmetry $Z_2$ which comes from the coupling with the spin degrees of freedom. This leads to the larger spectrum of the strings and nexuses in superconductors, as compared with the Standard Model.

III. FRACTIONAL VORTICES IN CHIRAL SUPERFLUIDS/SUPERCONDUCTORS

A. Order parameter in chiral superfluids/superconductors

The order parameter describing the vacuum manifold in a chiral $p$-wave superfluid (3He-A) is the so called gap function, which in the representation $S = 1$ ($S$ is the spin momentum of Cooper pairs) and $L = 1$ ($L$ is the orbital angular momentum of Cooper pairs) depends linearly on spin $\sigma$ and momentum $k$, viz.

$$\Delta(k, r) = A_{\alpha i}(r)\sigma_\alpha k_i , \quad A_{\alpha i} = \Delta a_i (\epsilon_i^{(1)} + i\epsilon_i^{(2)}) .$$

(2)

Here $\hat a$ is the unit vector of the spin-space anisotropy; $\hat e^{(1)}$ and $\hat e^{(2)}$ are mutually orthogonal unit vectors in the orbital space: they determine the superfluid velocity of the chiral condensate $v_s = \hat d \times \epsilon^{(1)} \nabla \epsilon^{(2)}$, where $2m$ is the mass of the Cooper pair; the orbital momentum vector is $l = \hat e^{(1)} \times \hat e^{(2)}$. The important discrete symmetry comes from the identification of the points $d, \hat e^{(1)} + i\hat e^{(2)}$ and $-d, -(\hat e^{(1)} + i\hat e^{(2)})$: they correspond to the same value of the order parameter in Eq. and are thus physically indistinguishable.

The same order parameter describes the chiral superconductor if the crystal lattice influence can be neglected, e.g. in an amorphous material. However, for crystals the symmetry group must take into account the underlying crystal symmetry, and the classification of the topological defects becomes different. It is believed that chiral superconductivity occurs in the tetragonal layered superconductor Sr$_2$RuO$_4$ [2]. The simplest representation of the order parameter, which reflects the underlying crystal structure, is

$$\Delta(k, r) = (d \cdot \sigma) (\sin k \cdot a(r) + i \sin k \cdot b(r)) e^{i\theta} ,$$

(3)

where $\theta$ is the phase of the order parameter; $a$ and $b$ are the elementary vectors of the crystal lattice. The order parameter is intrinsically complex which cannot be eliminated by a gauge transformation. This means that the time reversal symmetry is broken. Compare this with the structure of the nonchiral $d$-wave superconductor in layered cuprate oxides, where the order parameter is complex only because of its phase:

$$\Delta(k, r) = (\sin^2 k \cdot a(r) - \sin^2 k \cdot b(r)) e^{i\theta} .$$

(4)

Because of the breaking of time reversal symmetry in chiral crystalline superconductors, persistent electric current arises not only due to the phase coherence but also due to deformations of the crystal:

$$\mathbf{j} = \rho_s \left( v_s - \frac{e}{mc} \mathbf{A} \right) + K a_i \nabla b_i , \quad v_s = \frac{\hbar}{2m} \nabla \theta .$$

(5)

The parameter $K = 0$ in $d$-wave superconductors.
The symmetry breaking scheme $SO(3)_S \times SO(3)_L \times U(1)_N \to U(1)_S \times U(1)_{N-L_s} \times Z_2$, realized by the order parameter in Eq. (2), results in linear topological defects (vortices or strings) of group $Z_2$. Vortices are classified by the circulation quantum number $N = (2m/h) \oint d\mathbf{r} \cdot \mathbf{v}_s$ around the vortex core. Simplest realization of the $N$ vortex with integer $N$ is $\mathbf{e}^{(1)} + i\mathbf{e}^{(2)} = (\mathbf{x} + i\mathbf{y})e^{iN\phi}$, where $\phi$ is the azimuthal angle around the string. Vortices with even $N$ are topologically unstable and can be continuously transformed to a nonsingular configuration.

B. $N = 1/2$ and $N = 1/4$ vortices

Vortices with a half-integer $N$ result from the above identification of the points. They are combinations of the $\pi$-vortex and $\pi$-disclination in the $d$ field:

$$\mathbf{d} = \mathbf{x}\cos\frac{\phi}{2} + \mathbf{y}\sin\frac{\phi}{2}, \quad \mathbf{e}^{(1)} + i\mathbf{e}^{(2)} = e^{i\phi/2}(\mathbf{x} + i\mathbf{y}).$$

The $N = 1/2$ vortex is the counterpart of Alice strings considered in particle physics [3]: a particle which moves around an Alice string flips its charge. In $^3$He-A, the quasiparticle going around a $1/2$ vortex flips its $U(1)_S$ charge, that is, its spin. This is because the $d$-vector, which plays the role of the quantization axis for the spin of a quasiparticle, rotates by $\pi$ around the vortex, so that a quasiparticle adiabatically moving around the vortex insensibly finds its spin reversed with respect to the fixed environment. As a consequence, several phenomena (e.g., global Aharonov-Bohm effect) discussed in the particle physics literature have corresponding discussions in condensed matter literature (see [1],[2]) in particle physics).

In type II superconductors, vortices with $N$ circulation quanta carry a magnetic flux $\Phi_N = (N/2)\Phi_0$; the extra factor 1/2 comes from the Cooper pairing nature of superconductors. According to the London equations, screening of the electric current far from the vortex leads to the vector potential $\mathbf{A} = (mc/e)\mathbf{v}_s$ and to the magnetic flux $\int d\mathbf{S} \cdot \mathbf{B} = \frac{1}{2} \oint d\mathbf{r} \cdot \mathbf{A} = (mc/e) \oint d\mathbf{r} \cdot \mathbf{v}_s = (N/2)\Phi_0$. Therefore, the conventional $N = 1$ Abrikosov vortex in conventional superconductors carries $1/2\Phi_0$, while the $N = 1/2$ vortex carries 1/4 of the elementary magnetic flux $\Phi_0$. The vortex with $N = 1/2$ has been observed in high-temperature superconductors [13]: as predicted in [4], this vortex is attached to the tricrystal line, which is the junction of three grain boundaries (Fig. 2a).

Objects with fractional flux below $\Phi_0/2$ are also possible [13]. They can arise if the time reversal symmetry is broken [14]. Such fractional flux can be trapped by the crystal loop, which forms the topological defect, a disclination: the orientation of the crystal lattice continuously changes by $\pi/2$ around the loop, Fig. 2b. The other topologically similar loop can be constructed by

![FIG. 2. (a) Experimentally observed fractional flux, topologically trapped by junctions of the grain boundaries in high-$T_c$ superconductors. (b) Fractional flux topologically trapped by crystal loop.](image-url)
The fractional flux $\Phi_0/12$ flux can be trapped if the underlying crystal lattice has hexagonal symmetry.

In $^3$He-B the experimentally identified nonaxisymmetric $N = 1$ vortex \[15\] can be considered as a pair of $N = 1/2$ vortices, connected by a wall \[19, 21\].

IV. NEXUS IN CHIRAL SUPERFLUIDS/SUPERCONDUCTORS

A. Nexus

The Z-string in the Standard Model, which has $N = 1$, is topologically unstable, since $N = 0 \ (mod \ 1)$. This means that the string may end at some point (Fig. \[1\]c). The end point, a hedgehog in the orientation of the weak isospin vector, $\mathbf{l} = \hat{r}$, looks like a Dirac monopole with the hypermagnetic flux $\Phi_0$ in Eq. (1), if the electric charge $e$ is substituted by the hypercharge $\bar{e}$. The same combined object of a string and hedgehog, the nexus, appears in $^3$He-A when the topologically unstable vortex with $N = 2$ ends at the hedgehog in the orbital momentum field, $\mathbf{l} = \hat{r}$ \[22, 23\]. In both cases the distribution of the vector potential $\mathbf{A}$ of the hypermagnetic field and of the superfluid velocity $\mathbf{v}_s$ field have the same structure, if one identifies $\mathbf{v}_s = (e/mc)\mathbf{A}$. Assuming that the Z-string of the Standard Model or its counterpart in the electrically neutral $^3$He-A, the $N = 2$ vortex, occupy the lower half-axis $z < 0$, one has

$$\mathbf{A} = \frac{hc}{2er} \sin \theta \delta(\rho) \Phi_0 \quad \text{and} \quad \mathbf{B} = \nabla \times \mathbf{A} = \mathbf{B}_{\text{mon}} + \mathbf{B}_{\text{string}}$$

$$\mathbf{B}_{\text{mon}} = \frac{hc}{2e} \frac{r}{r^2}, \quad \mathbf{B}_{\text{string}} = -\frac{hc}{e} \Theta(-z) \delta_2(\rho)$$

In amorphous chiral superconductors Eq. (8) describes the distribution of the electromagnetic vector potential far from the nexus. The regular part of the magnetic field, radially propagating from the hedgehog, corresponds to a monopole with elementary magnetic flux $\Phi_0 = hc/e$, while the singular part is concentrated in the core of the vortex, which supplies the flux to the monopole \[24\]. This is the doubly quantized Abrikosov vortex, which is terminating on the hedgehog.

Because of the discrete symmetry group, the nexus structures in $^3$He-A and in amorphous chiral superconductors are richer than in the Standard Model. The $N = 2$ vortex can split into two $N = 1$ Abrikosov vortices or into four $N = 1/2$ vortices (Fig. \[4\]), or into their combination, provided that the total topological charge $N = 0 \ (mod \ 2)$. So, in general, the superfluid velocity field in the $^3$He-A nexus and the vector potential in its superconducting counterpart obey

$$\mathbf{v}_s = \frac{e}{mc} \mathbf{A}, \quad \mathbf{A} = \sum_a \mathbf{A}^a$$

4
means that there must be four vortices with

If its radius is less than 10

quantum chromodynamics, where

relativistic

case

characterized by the winding numbers

or magnetic

field B in the superconducting counterpart, a chiral superconductor. The magnetic flux of the nexus \( \Phi_0 \) is supplied by 4 half-quantum vortices, each carrying the flux \( \Phi_0/4 \) to the

center of the droplet. The stability of the monopole in the
center of the droplet is supported by the foreign body in the

center, for example by a cluster of \(^4\)He liquid which provides

the radial boundary condition for the \( \mathbf{l} \)-vector.

where \( \mathbf{A}^a \) is the vector potential of the electromagnetic

field produced by the \( a \)-th string, \( i.e. \) the Abrikosov vortex

with the circulation number \( N_a \) terminating on the

monopole, provided that \( \sum_a N_a = 0 \) (mod 2).

This is similar to the other realization of the nexus in relativistic \( SU(n) \) quantum field theories, for example in

quantum chromodynamics, where \( n \) vortices of the group

\( Z_n \) meet at a center (nexus) provided the total flux of vortices adds to zero (mod \( n \)).

B. Nexus in a \(^3\)He droplet

A nexus can be the ground state of \(^3\)He-A in a droplet,

if its radius is less than 10 \( \mu m \). In this case the lowest

energy of the nexus occurs when all vortices terminating

on the monopole have the lowest circulation number: this

means that there must be four vortices with \( N_1 = N_2 =

N_3 = N_4 = 1/2 \).

According to Eq. (3) each half-quantum vortex is accom-

panied by a spin disclination. Assuming that the

\( \mathbf{d} \)-field is confined into a plane, the disclinations can be

characterized by the winding numbers \( \nu_a \) of the \( \mathbf{d} \) vector,

which have values \( \pm 1/2 \) in half-quantum vortices. The

corresponding spin-superfluid velocity \( \mathbf{v}_{sp} \) is

\[
\mathbf{v}_{sp} = \frac{2e}{mc} \sum_{a=1}^{4} \nu_a \mathbf{A}^a, \quad \sum_{a=1}^{4} \nu_a = 0 ,
\]

where the last condition means the absence of the

monopole in the spin sector of the order parameter. Thus we

have \( \nu_1 = \nu_2 = -\nu_3 = -\nu_4 = 1/2 \).

If \( \mathbf{l} \) is fixed, the energy of the nexus in the spherical

bubble of radius \( R \) is determined by the kinetic energy of

mass and spin superflow:

\[
\frac{1}{2} \int dV (\rho_s \mathbf{v}_s^2 + \rho_{sp} \mathbf{v}_{sp}^2) = \\
\frac{e^2}{2mc^2} \int dV (\rho_s + \rho_{sp}) [(\mathbf{A}^1 + \mathbf{A}^2)^2 + (\mathbf{A}^3 + \mathbf{A}^4)^2] \\
+ \frac{e^2}{m^2c^2} \int dV (\rho_s - \rho_{sp}) (\mathbf{A}^1 + \mathbf{A}^2)(\mathbf{A}^3 + \mathbf{A}^4) .
\]

In the simplest case, which occurs in the ideal Fermi gas, one has \( \rho_s = \rho_{sp} \) \(^{27}\). In this case the 1/2-vortices with positive spin-current circulation \( \nu \) do not interact

with 1/2-vortices of negative \( \nu \). The energy minimum occurs when the orientations of two positive-\( \nu \) vortices are opposite, so that these two \( \pm \) fractions of the Dirac strings form one line along the diameter (see Fig. 5). The same happens for the other fractions with negative \( \nu \). The mutual orientations of the two diameters is arbitrary in this limit. However, in real \(^3\)He-A, one has \( \rho_s < \rho_{sp} \) \(^{27}\). If \( \rho_{sp} \) is slightly smaller than \( \rho_s \), the positive-\( \nu \) and negative-\( \nu \) strings repel each other, so that the equilibrium angle between them is \( \pi/2 \). In the extreme case \( \rho_{sp} \ll \rho_s \), the ends of four half-quantum vortices form the vertices of a regular tetrahedron.

To fix the position of the nexus in the center of the

droplet, one must introduce a spherical body inside,

which will attach the nexus because of the normal bound-

ary conditions for the \( \mathbf{l} \)-vector. The body can be a

droplet of \(^4\)He immersed in the \(^3\)He liquid. For the mixed

\(^4\)He/\(^3\)He droplets, obtained via the nozzle beam expan-

sion of He gas, it is known that the \( \mathbf{l} \)-component of the mixture does form a cluster in the central region of the

\(^3\)He droplet \(^{28}\).

In an amorphous \( p \)-wave superconductor, but with

preserved layered structure, such a nexus will be formed in a

spherical shell. In the crystalline \( \text{Sr}_2\text{RuO}_4 \) superconduc-
tor the spin-orbit coupling between the spin vector \( \mathbf{d} \)

and the crystal lattice seems to align the \( \mathbf{d} \) vector along \( \mathbf{l} \) \(^{29}\).

In this case the half quantum vortices are energetically

unfavorable and, instead of 4 half-quantum vortices, one

would have 2 singly quantized vortices in the spherical

shell.

Nexuses of this kind can be formed also in the so called

ferromagnetic Bose condensate in optical traps. Such a

condensate is described by a vector or spinor chiral order

parameter \(^{30}\).
living in the effective gauge and gravity fields, produced by the bosonic collective modes of the superfluid vacuum (see Review [38]). In particular, the superfluid velocity acts on quasiparticles in the same way as the metric element \( g^{0i} = -v_i^s \) acts on a relativistic particle in Einstein’s theory. This element \( g = -g^{0i} \) plays the part of the vector potential of the gravimagnetic field \( B_g = \nabla \times g \).

For the nexus in Fig. 5 the \( l \) vector, the superfluid velocity \( v_s \), and its "gravimagnetic field", i.e. vorticity \( B_g \), on the A-phase side are

\[
\hat{\mathbf{l}} = \hat{\mathbf{r}} , \quad v_s = \frac{\hbar}{2m_3} \frac{1 - \cos \theta}{r \sin \theta} \phi , \quad B_g = \nabla \times v_s = \frac{\hbar}{2m_3} \frac{r}{r^3} .
\]

On the B-phase side one has

\[
B_g = \nabla \times v_s = \frac{\pi \hbar}{m_3} \delta_2(\rho) .
\]

The gravimagnetic flux propagates along the vortex in the B phase towards the nexus (boojum) and then radially and divergencelessly from the boojum into the A phase. This is the analog of the gravimagnetic monopole discussed in [37].

V. TOPOLOGICAL INTERACTION OF MAGNETIC MONOPOLES WITH CHIRAL SUPERCONDUCTORS

Since the 't Hooft-Polyakov magnetic monopole, which can exist in GUT, and the monopole part of the nexus in chiral superconductors have the same magnetic and topological charges, there is a topological interaction between them. First, let us recall what happens when the magnetic monopole enters a conventional superconductor: because of the Meissner effect – expulsion of the magnetic field from the superconductor – the magnetic field from the monopole will be concentrated in two flux tubes of Abrikosov vortices with the total winding number \( N = 2 \) (Fig. 6). In a chiral amorphous superconductor these can form 4 flux tubes, represented by half-quantum Abrikosov vortices (Fig. 6b).

However, the most interesting situation occurs if one takes into account that in a chiral superconductor the Meissner effect is not complete because of the \( l \) texture. As we discussed above the magnetic flux is not necessarily concentrated in the tubes, but can propagate radially from the hedgehog (Fig. 6c). If now the 't Hooft-Polyakov magnetic monopole enters the core of the hedgehog in Fig. 6, which has the same magnetic charge, their strings, i.e. Abrikosov vortices carried by the monopole and Abrikosov vortices attached to the nexus, will annihilate each other. What is left is the combined point defect: hedgehog + magnetic monopole without any attached strings (Fig. 6d). This means that

C. Nexus with fractional magnetic flux

A nexus with fractional magnetic charge can be constructed using geometry with several condensates. Fig. 5 shows the nexus pinned by the interface between superfluid \(^3\)He-A and the nonchiral superfluid \(^3\)He-B. Due to the tangential boundary condition for the \( l \) vector at the interface, the nexus covers only half of the unit sphere. For the superconducting analogs such a nexus accounts for 1/2 of the magnetic charge of the Dirac monopole, whose Dirac string is the B-phase vortex. In superconductors such a nexus accounts for 1/2 of the magnetic charge of the Dirac monopole, whose flux is supported by a single \( N = 1 \) vortex on the B-phase side.

D. Gravimagnetic monopole

In addition to the symmetry breaking scheme there is another level of analogies between superfluids/superconductors and quantum vacuum. They are related to the behavior of quasiparticles in both systems. In chiral superfluids quasiparticles behave as chiral fermions...
the monopole destroys the topological connection of the hedgehog and Abrikosov vortices, instead one has topological confinement between the monopole and hedgehog. The core of the hedgehog represents the natural trap for one magnetic monopole: if one tries to separate the monopole from the hedgehog, one must create the piece(s) of the Abrikosov vortex(ices) which connect the hedgehog and the monopole (Fig. 6).

VI. DISCUSSION: FERMIONS IN THE PRESENCE OF TOPOLOGICAL DEFECTS

Fermions in topologically nontrivial environments behave in a curious way, especially in the presence of such exotic objects as fractional vortices and monopoles discussed in this paper. In the presence of a monopole the quantum statistics can change, for example, the isospin degrees of freedom are transformed to spin degrees [8]. There are also the so-called fermion zero modes: the bound states at monopole or vortex, which have exactly zero energy. For example the $N = 1/2$ vortex in a two-dimensional chiral superconductor, which contains only one layer, has one fermionic state with exactly zero energy [39]. Since the zero-energy level can be either filled or empty, there is a fractional entropy $(1/2) \ln 2$ per layer related to the vortex. The factor $(1/2)$ appears because in superconductors the particle excitation coincides with its antiparticle (hole), i.e., the quasiparticle is a Majorana fermion (Nice discussion of Majorana fermions in chiral superconductors can be found in [60]). Also the spin of the vortex in a chiral superconductor can be fractional. According to [41], the $N = 1$ vortex in a chiral superconductor must have a spin $S = 1/4$ (per layer); this implies the spin $S = 1/8$ per layer for $N = 1/2$ vortex. Similarly there can be an anomalous fractional electric charge of the $N = 1/2$ vortex, which is $1/2$ of the fractional charge $e/4$ discussed for the $N = 1$ vortex [42]. There is still some work to be done to elucidate the problem with the fractional charge, spin and statistics, related to the topological defects in chiral superconductors.

FIG. 6. Hedgehogs and magnetic monopoles in superconductors. (a) ‘t Hooft-Polyakov magnetic monopole in conventional superconductor. Magnetic flux of the monopole is concentrated in Abrikosov vortices because of the Meissner effect; (b) Magnetic monopole in a chiral superconductor with uniform $\Phi$ vector. As distinct from the monopole inside the conventional superconductors, the magnetic flux $\Phi_0$ of the monopole can be carried away from the monopole by 4 half-quantum vortices. (c) Nexus: hedgehog with two Abrikosov vortices emanating from the core. Magnetic flux $\Phi_0$ enters the core of the hedgehog along two Abrikosov vortices with $N = 1$ (or 4 vortices with $N = 1/2$) and then flows out radially along the lines of the $l$-vector field. (d) ‘t Hooft-Polyakov magnetic monopole + hedgehog in a chiral superconductor. The Abrikosov vortices attached to the ‘t Hooft-Polyakov magnetic monopole annihilate the Abrikosov vortices attached to the hedgehog. Magnetic field of the monopole penetrates radially into the bulk chiral superconductor along the lines of the $l$-vector field. (e) Topological confinement of the ‘t Hooft-Polyakov magnetic monopole and hedgehog by Abrikosov strings in a chiral superconductor. For simplicity a single Abrikosov string with $N = 2$ is depicted.

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