Two simple textures of the magic neutrino mass matrix

Kanwaljeet S. Channey \(^*\)\(^1,2\) and Sanjeev Kumar\(^†\)\(^1\)

\(^1\)Department of Physics and Astrophysics, University of Delhi, Delhi, 110007, India.
\(^2\)Department of Physics, University Institute of Sciences, Chandigarh University, Mohali, Punjab 140413, India.

Abstract

The Tri-Bimaximal (TBM) mixing predicts a vanishing \(\theta_{13}\). This can be attributed to the inherited \(\mu - \tau\) symmetry of TBM mixing. We break its \(\mu - \tau\) symmetry by adding a complex magic matrix with one variable to TBM neutrino mass matrix with one vanishing eigenvalue. We present two such textures and study their phenomenological implications.

In the Standard Model of electroweak and strong interactions, neutrino flavor states \(\nu_l\) \((l = e, \mu, \tau)\) are the states which form weak doublets with the corresponding charged lepton states \(l\):

\[
j^\mu = \bar{l}_\gamma^\mu (1 - \gamma_5)\nu_l.
\]

These neutrino states \(\nu_l\) are the coherent combinations of the neutrino mass states \(\nu_i\) \((i = 1, 2, 3)\) which are also the eigenstates of the Hamiltonian in vacuum. Neutrino flavor and mass states are related by the relation

\[
\nu_l = U_{\text{PMNS}}\nu_i,
\]

where \(U_{\text{PMNS}}\) is the unitary matrix called Pontecorvo-Maki-Nakagawa-Sakata mixing matrix. By convention, PMNS mixing matrix is defined as

\[
U_{\text{PMNS}} = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & -c_{13}s_{23} - s_{13}c_{23}s_{13}e^{i\delta} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{13}s_{23} - c_{13}c_{23}s_{13}e^{i\delta}
\end{pmatrix}
\]

where \(s_{ij} = \sin \theta_{ij}\), \(c_{ij} = \cos \theta_{ij}\) \((i, j = 1, 2, 3)\) and \(\delta\) is the Dirac type CP violating phase.

In the flavor basis, where lepton mass matrix is diagonal, the neutrino mass matrix \(M_{\nu}\) is related with the unitary mixing matrix and the complex neutrino masses \(m_d = \text{diag}(m_1, m_2 e^{2i\alpha}, m_3 e^{2i\beta})\) by the relation

\[
M_{\nu} = U_{\text{PMNS}}^{\dagger} m_d U_{\text{PMNS}},
\]

where \(\alpha\) and \(\beta\) are the Majorana phases. While analyzing the experimental results on neutrino mixing and mass matrices, we can often look for some particular features like equalities [1, 2], zeros [3, 4, 5, 6, 7, 8, 9, 10, 11, 12], hybrids of zeros and equalities [13, 14, 15, 16, 17], zero trace [18], or some other pattern [19, 20, 21, 22] in their elements.

Harrison, Perkins and Scott proposed one such type of mixing matrix [23] which had \(\nu_2\) trimaximally mixed and \(\nu_3\) bimaximally mixed. Hence, they named it Tri-Bimaximal (TBM) mixing. The TBM mixing matrix is

\[
U_{\text{TBM}} = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\
-\sqrt{\frac{1}{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\end{pmatrix}.
\]
The mass matrix $M_{TBM}$ corresponding to $U_{TBM}$ can be written by using Eq. (4)

$$M_{TBM} = \begin{pmatrix} a & b & b \\ b & a + d & b - d \\ b & b - d & a + d \end{pmatrix}. \quad (6)$$

TBM mixing matrix predicts $\sin^2 \theta_{13} = 0$, $\sin^2 \theta_{12} = \frac{1}{3}$ and $\sin^2 \theta_{23} = \frac{1}{2}$. Mixing angles $\theta_{12}$ and $\theta_{23}$ are in agreement at 3$\sigma$ with their experimental values, $\sin^2 \theta_{12} = 0.306^{+0.012}_{-0.012}$ and $\sin^2 \theta_{23} = 0.441^{+0.027}_{-0.021}$, provided by the latest global fit of the neutrino experimental data [21].

Another parametrization of the mixing matrix was given in Ref. [25]. In this parametrization, the mixing matrix was expressed as expansion in powers of the deviations of reactor, solar and atmospheric mixing angles from their TBM value. Let $r, s$, and $a$ are the real parameters which give deviations to the reactor, solar and atmospheric mixing angles from their TBM values:

$$\sin \theta_{13} = \frac{r}{\sqrt{2}}, \quad \sin \theta_{12} = \frac{1}{\sqrt{3}}(1 + s), \quad \sin \theta_{23} = \frac{1}{\sqrt{2}}(1 + a). \quad (7)$$

Since the parameters $r, s$, and $a$ are very small, we can expand the mixing matrix about $U_{TBM}$ in the powers of $r, s$, and $a$. We present here the mixing matrix to the first order in $r, s$, and $a$

$$U_p \approx \begin{pmatrix} \frac{\sqrt{2}}{3} (1 - \frac{1}{2}s) & \frac{1}{\sqrt{3}}(1 + s) & \frac{1}{\sqrt{2}}re^{i\delta} \\ -\frac{1}{3\sqrt{6}}(1 + s + a - re^{i\delta}) + \frac{1}{\sqrt{3}}(1 - \frac{1}{2}s - a - \frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1 + a) \end{pmatrix}. \quad (8)$$

The mixing angle $\theta_{13}$ is non zero as measured by the recent experiments: T2K [26], Daya Bay [27], RENO [28] and DOUBLE CHOOZ [29]. This leads to the realization that although TBM ansatz is ruled out by the experiments, it can still be used as leading order contribution to the neutrino mass matrix. We can add perturbations to $M_{TBM}$ so as to generate the non-zero $\theta_{13}$. TBM mass matrix obeys both the magic symmetry and the $\mu - \tau$ exchange symmetry. Magic symmetry means sum of the elements of each row and column of mass matrix remains the same, whereas $\mu - \tau$ exchange symmetry means that the neutrino mass matrix is invariant under the simultaneous interchange of its second and third ($\mu$-$\tau$) indices.

A neutrino mass matrix that is invariant under magic symmetry and $\mu - \tau$ exchange symmetry predicts maximal $\theta_{23}$ and vanishing $\theta_{13}$. These predictions are very close to the present neutrino oscillation data. This indicates that we can satisfy the present experimental data by introducing small perturbations to the magic mass matrix. Magic symmetry also provides sum rules between the mixing angles due to trimaximal structure of $\nu_2$ [30] which in return reduces the number of free parameters. These sum rules can be tested at the future neutrino oscillation experiments.

In the present paper, we propose two simple textures of $M_{\text{magic}}$ that break the $\mu - \tau$ symmetry of TBM neutrino mass matrix but preserve its magic symmetry. These textures can be written as

$$M_{\text{magic}}^i = M_{TBM} + M_i', \quad (i = a, b) \quad (9)$$

The $\mu - \tau$ breaking term $M_i'$ in these textures is function of only one complex variable $\eta = ze^{i\chi}$. To reduce the number of independent variables, in our study we have considered the $M_{TBM}$ with vanishing lowest eigenvalue. This assumption will lead to the condition that $b = a$ in the Eq. (6). Forms of $M_{TBM}$ and $M_i'$ ($i = a, b$) studied in the present work for normal hierarchy are:

$$M_{TBM} = \begin{pmatrix} a & a & a \\ a & a + d & a - d \\ a & a - d & a + d \end{pmatrix}, \quad M_a' = \begin{pmatrix} 0 & 0 & \eta \\ 0 & 0 & \eta \\ \eta & \eta & -\eta \end{pmatrix}, \quad M_b' = \begin{pmatrix} 0 & \eta & 0 \\ \eta & 0 & 0 \\ 0 & 0 & \eta \end{pmatrix}. \quad (10)$$

We then study the phenomenological implications for these textures of neutrino mass matrix.

While perturbing the TBM mass matrix by adding an extra matrix, we can break $M_{TBM}$ in such a way that out of the two symmetries that it possesses, we break only one. Since $\mu - \tau$ symmetry predicts vanishing $\theta_{13}$, preserving magic symmetry is a feasible choice.

If a transformation $G_j$ of the neutrino fields leaves the neutrino mass matrix unchanged such that

$$G_j^T M_{\nu} G_j = M_{\nu}, \quad (11)$$
Neutrino masses and Majorana phases can be calculated from $M$ mixing. One of the general form for $M$ is:

$$G_2 = \begin{pmatrix}
\frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\
-\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\
-\frac{2}{3} & -\frac{2}{3} & \frac{1}{3}
\end{pmatrix}.$$  \hspace{1cm} (12)

Therefore, a mass matrix $M_{\text{magic}}$ that preserves the magic symmetry will obey the relation $G_2^T M_{\text{magic}} G_2 = M_{\text{magic}}$. Mixing matrix corresponding to such mass matrices will have their middle column same as that of $U_{\text{TBM}}$ (trimaximal) and can be described in terms of two independent variables $\theta$ and $\phi$.

$$U_{\text{TM}} = \begin{pmatrix}
\frac{\sqrt{\frac{2}{3}} \cos \theta}{\sqrt[4]{3}} & \frac{1}{\sqrt[4]{3}} & \frac{\sqrt{\frac{2}{3}} \sin \theta}{\sqrt[4]{3}} \\
\frac{e^{i \phi} \sin \theta - \cos \theta}{\sqrt[4]{2}} & \frac{1}{\sqrt[4]{3}} & \frac{-e^{i \phi} \cos \theta - \sin \theta}{\sqrt[4]{2}} \\
\frac{-e^{i \phi} \cos \theta + \sin \theta}{\sqrt[4]{2}} & \frac{1}{\sqrt[4]{3}} & \frac{e^{i \phi} \sin \theta + \cos \theta}{\sqrt[4]{2}}
\end{pmatrix}.$$  \hspace{1cm} (13)

This mixing matrix, has a trimaximally mixed column leading to its nomenclature as trimaximal mixing. One of the general form for $M_{\text{magic}}$ can be written as:

$$M_{\text{magic}} = \begin{pmatrix}
a & b & c \\
b & a + d & c - d \\
c - d & a + b - c + d
\end{pmatrix}.$$  \hspace{1cm} (14)

This mass matrix can be diagonalized by using the equation

$$M_d = U_{\text{TM}}^T M_{\text{magic}} U_{\text{TM}}.$$  \hspace{1cm} (15)

The rationale of choosing the form of $M_{\text{magic}}$ as given in Eq. \[15\] is that it reduces to $M_{\text{TBM}}$ (Eq. \[9\]) for $c = b$. It is the difference of $b$ and $c$ that breaks the $\mu - \tau$ symmetry of $M_{\text{TBM}}$. Therefore, to break $\mu - \tau$ exchange symmetry and to generate non-zero $\theta_{13}$, we can allow $b$ and $c$ to differ by a small amount ($\eta$).

The diagonal elements of $M_d$ will give us neutrino masses and Majorana phases, whereas the off-diagonal elements, when equated to zero, will give the variables $\theta$ and $\phi$ of $U_{\text{TM}}$ in terms of the parameters of $M_{\text{magic}}$. We can calculate the mixing angles in terms of $\theta$ and $\phi$ from the elements of $U = U_{\text{TM}}$ using the relations:

$$\sin^2 \theta_{12} = \frac{|U_{12}|^2}{1 - |U_{13}|^2}, \quad \sin^2 \theta_{23} = \frac{|U_{23}|^2}{1 - |U_{13}|^2}, \quad \sin^2 \theta_{13} = |U_{13}|^2.$$  \hspace{1cm} (16)

We can calculate the Dirac phase $\delta$ from the Jarlskog rephasing invariant measure of CP violation,

$$J = Im[U_{12} U_{23}^* U_{13}^* U_{22}],$$  \hspace{1cm} (17)

using the relation,

$$J = c_{12} s_{12} c_{23} s_{23} c_{13} s_{13} \sin \delta.$$  \hspace{1cm} (18)

Neutrino masses and Majorana phases can be calculated from $M_d$ using the following relations:

$$|m_1| = |[M_d]_{11}|, \quad |m_2| = |[M_d]_{22}|, \quad |m_3| = |[M_d]_{33}|,$$  \hspace{1cm} (19)

$$\alpha = \frac{1}{2} \arg \left( \frac{|M_d|_{22}}{|M_d|_{11}} \right), \quad \beta = \frac{1}{2} \arg \left( \frac{|M_d|_{33}}{|M_d|_{11}} \right).$$  \hspace{1cm} (20)

We can write $M_{\text{magic}}$ as the sum of $M_{\text{TBM}}$ and a $\mu - \tau$ symmetry breaking term $M'$:

$$M_{\text{magic}} = M_{\text{TBM}} + M',$$  \hspace{1cm} (21)

where $M'$ is also invariant under $G_2$. Looking at Eq. \[14\], we observe that we can write $M_{\text{magic}}$ as sum of four matrices:
The allowed parameter space for $a, b, z,$ and $\chi$ for the textures $M_{\text{magic}}^a$ and $M_{\text{magic}}^b$.

$$M_{\text{magic}} = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & d & -d \\ 0 & -d & d \end{pmatrix} + \begin{pmatrix} 0 & b & 0 \\ b & 0 & 0 \\ 0 & b & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & c \\ 0 & 0 & c \\ c & c & -c \end{pmatrix}. \quad (22)$$

There are two ways to write $M_{\text{magic}}$ as combination of $M_{\text{TBM}}$ and $M'$ as shown in Eq. [21]. First is by considering $b = a, c = a + \eta$

$$M_{\text{magic}}^a = \begin{pmatrix} a & a & a \\ a & a + d & a - d \\ a & a - d & a + d \end{pmatrix} + \begin{pmatrix} 0 & \eta & 0 \\ 0 & \eta & 0 \\ \eta & -\eta & 0 \end{pmatrix} \quad (23)$$

and second is by considering $b = a + \eta, c = a$

$$M_{\text{magic}}^b = \begin{pmatrix} a & a & a \\ a & a + d & a - d \\ a & a - d & a + d \end{pmatrix} + \begin{pmatrix} \eta & 0 \\ 0 & \eta \\ 0 & 0 \end{pmatrix}, \quad (24)$$

where $\eta = z e^{i \chi}$ is responsible for the breaking of $\mu - \tau$ symmetry. The assumption $a = b$ in $M_{\text{TBM}}$ results in vanishing lowest eigen value of $M_{\text{TBM}}$. We made this assumption to reduce the number of free parameters in $M_{\text{TBM}}$. This gives us the two textures studied in this paper given in Eq. [10] ($M_{\text{magic}}^a$ and $M_{\text{magic}}^b$).

We can diagonalize these mass matrices by using Eq. [15] and obtain our predictions from Eqs. [16, 20]. Equating the nondiagonal entry $[m_d]_{13}$ with zero for these textures will give us predictions for the variables $\theta$ and $\phi$ in terms of $a, d, z$ and $\chi$.

For texture $M_{\text{magic}}^a$, the nondiagonal entry $[m_d]_{13}$, $\theta$ and $\phi$ are as follows:

$$[m_d]_{13} = \frac{1}{4} e^{-2i\phi} \left( 2\sqrt{3}z \cos 2\theta e^{i(\chi + \phi)} - \frac{1}{4} e^{-2i\phi} \left( \sin 2\theta \left( 4d + e^{i\chi} z \left( -3 + e^{2i\phi} \right) \right) \right) \right), \quad (25)$$

$$\tan \phi = -\frac{d \tan \chi}{d - z \sec \chi}, \quad (26)$$

$$\tan 2\theta = -\frac{2\sqrt{3}z \cos (\phi - \chi)}{-4d \cos 2\phi - z (\cos \chi - 3 \cos (2\phi - \chi))}. \quad (27)$$
Figure 2: The correlations between atmospheric angle $\theta_{23}$ and CP violating phase $\delta$ for both the textures $M^a_{\text{magic}}$ and $M^b_{\text{magic}}$. Here dashed lines represent the 3$\sigma$ experimental range and solid lines represent the 1$\sigma$ experimental range.

For texture $M^b_{\text{magic}}$ we obtain the following relations for $[m_d]_{13}$, $\theta$ and $\phi$:

$$[m_d]_{13} = \frac{1}{4} e^{-2i\phi} \left( -\sin 2\theta \left( 4d + e^{i\chi} z \left( 1 + e^{2i\phi} \right) \right) \right)$$

$$- \frac{1}{4} e^{-2i\phi} \left( 2\sqrt{3}z \cos 2\theta e^{i(\chi+\phi)} \right),$$

$$\phi = -\chi,$$

$$\tan 2\theta = \frac{-\sqrt{3}z \cos(\phi - \chi) - 2d \cos 2\phi - z \cos \phi \cos(\phi - \chi)}{3 - 2 \sin^2 \theta - z \cos \phi \cos(\phi - \chi)}.$$

Corresponding mixing angles and Dirac type CP violating phase then can be calculated from these $\theta$ and $\phi$ by using the following relations

$$\sin^2 \theta_{12} = \frac{1}{3 - 2 \sin^2 \theta},$$

$$\sin^2 \theta_{23} = \frac{1}{2} \left( 1 + \sqrt{3} \sin 2\theta \cos \phi \right),$$

$$\sin^2 \theta_{13} = \frac{2 \sin^2 \theta}{3},$$

$$\csc^2 \delta = \csc^2 \phi - \frac{3 \sin^2 2\theta \cot^2 \phi}{(3 - 2 \sin^2 \theta)^2}.$$

Substituting the values of $\theta$ and $\phi$ in terms of $a$, $b$, $z$ and $\chi$ in $[m_d]_{11}$, $[m_d]_{22}$ and $[m_d]_{33}$, we can obtain the three neutrino masses ($m_1$, $m_2$ and $m_3$) and the Majorana phases ($\alpha$ and $\beta$) using Eqs. (19) and (20), where

Table 1: Experimental values of the oscillation parameters. Experimental bounds of $\theta_{23}$ are not used in the present Monte Carlo analysis.

| Parameters                  | 3$\sigma$ range |
|-----------------------------|-----------------|
| $\Delta m_{12}^2/10^{-5} eV^2$ | 7.03 $\rightarrow$ 8.09 |
| $\Delta m_{23}^2/10^{-3} eV^2$ | 2.407 $\rightarrow$ 2.643 |
| $\theta_{13}/^\circ$        | 7.99 $\rightarrow$ 8.90 |
| $\theta_{12}/^\circ$        | 31.38 $\rightarrow$ 35.99 |
| $\theta_{23}/^\circ$        | 38.4 $\rightarrow$ 52.8 |
Figure 3: Variation of $\beta$ with $\delta$ for both the textures $M^a_{\text{magic}}$ and $M^b_{\text{magic}}$.

\[
[m_d]_{11} = \frac{1}{2} e^{-2i\phi} \left( \sin^2 \theta \left(-4d + 3e^{i\chi}z \right) \right)
- \frac{1}{2} e^{-2i\phi} \left( 2\sqrt{3}z \cos \theta e^{i(x+\phi)} + z \cos^2 \theta e^{i(x+2\phi)} \right),
\]

\[ [m_d]_{22} = 3a + e^{i\chi}z, \]

and

\[
[m_d]_{33} = \frac{1}{2} e^{-2i\phi} \left( \cos^2 \theta \left(4d - 3e^{i\chi}z \right) \right)
+ \frac{1}{2} e^{-2i\phi} \left( z \sin \theta e^{i(x+\phi)} \left(2\sqrt{3} \cos \theta - e^{i\phi} \sin \theta \right) \right). \]  

Similarly, for texture $M^b_{\text{magic}}$, we have

\[
[m_d]_{11} = \frac{1}{2} e^{-2i\phi} \left( \sin^2 \theta \left(4d + e^{i\chi}z \right) + \sqrt{3}z \sin 2\theta e^{i(x+\phi)} \right)
\]

\[ + \frac{1}{2} e^{-2i\phi} \left( z \cos^2 \theta \left(-e^{i(x+2\phi)} \right) \right), \]  

\[ [m_d]_{22} = 3a + e^{i\chi}z, \]

and

\[
[m_d]_{33} = \frac{1}{2} e^{-2i\phi} \left( \cos^2 \theta \left(4d + e^{i\chi}z \right) \right)
- \frac{1}{2} e^{-2i\phi} \left( z \sin \theta e^{i(x+\phi)} \left(2\sqrt{3} \cos \theta + e^{i\phi} \sin \theta \right) \right). \]

These relations give us the three neutrino masses and two Majorana phases from Eqs. (19, 20).

| Parameters | $M^a_{\text{magic}}$ Allowed 3\(\sigma\) range | $M^b_{\text{magic}}$ Allowed 3\(\sigma\) range |
|------------|---------------------------------|---------------------------------|
| $a$        | $[-0.008,0.008]$                | $[-0.008,0.008]$                |
| $d$        | $[0.016,0.034]\cup[-0.034,-0.016]$ | $[0.021,0.027]\cup[-0.027,-0.021]$ |
| $z$        | $[0.009,0.013]$                 | $[0.009,0.013]$                 |
Current neutrino experiments cannot observe the three neutrino masses directly. $\beta$- decay experiments \[31\] are sensitive to the effective neutrino mass $m_\beta$ given as
\[
m_\beta^2 = m_1^2 |U_{e1}|^2 + m_2^2 |U_{e2}|^2 + m_3^2 |U_{e3}|^2.
\] (46)

The effective neutrino mass $m_{\beta\beta}$ given as
\[
m_{\beta\beta} = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2|.
\] (47)
can be measured in the neutrino-less double $\beta$-decay experiments \[32\].

We can obtain the parameters $r, s,$ and $a$ defined in Eq. (7) in terms of $\theta$ and $\phi$ by comparing the values of mixing angles given in Eq. (32-35) with Eq. (7). These relations are given as follows
\[
r = \frac{2}{\sqrt{3}} \sin \theta,
\] (48)
\[
s = \frac{1}{\sqrt{1 - \frac{2}{3} \sin^2 \theta}} - 1,
\] (49)
\[
a = \sqrt{1 + \left( \frac{\sqrt{3} \sin 2\theta \cos \phi}{3 - 2 \sin^2 \theta} \right)} - 1.
\] (50)

We perform a Monte Carlo analysis for these two textures by generating the variables $a, d, z$ and $\chi$ using uniform random distributions. The parameter space of these variables is restricted by imposing the experimental constraints \[33\] on $\Delta m_{12}^2 = m_2^2 - m_1^2$, $\Delta m_{23}^2 = m_3^2 - m_2^2$, $\theta_{12}$ and $\theta_{23}$ at 3$\sigma$ C. L. These ranges are shown in Tab. 1. The allowed parameter space for $a, d, z$ and $\chi$ is displayed in Fig. 1 and the allowed ranges can be read from Tab. 2. The correlation plots between $\theta_{23}$ and $\delta$ are presented in Fig. 2 for both the textures. Here, $\theta_{23}$ is well within the experimental range at 3$\sigma$ ([38.4, 52.8], shown as horizontal dashed lines) for both the textures. However, for 1$\sigma$ range of $\theta_{23}$, we find that only allowed ranges for $\delta$ are [215 $-$ 251] and [287 $-$ 312] ruling out $\delta = [250 $-$ 286]$. At 3$\sigma$, for both the textures, $\delta$ should be either 90$^o$ or 270$^o$ for a maximal $\theta_{23}$. The value of $\delta$ shifts towards 0$^o$ or 180$^o$ when $\theta_{23}$ takes its extreme values around 38$^o$ or 51$^o$. This feature is testable at the experiments like NO$\nu$A \[34\] and T2K \[26\]. The degeneracy in the Fig. 2 is because of the contributions of positive and negative values of the parameters $a$ and $d$. The allowed values of these parameters can be positive or negative and contribute to the different branches of this figure.

The correlation between the phases $\delta$ and $\beta$ is displayed in Fig. 3 and that between $\alpha$ and $\beta$ is shown in Fig. 4. Variation of $m_\beta$ and $m_{\beta\beta}$ with CP violating phase $\delta$ is presented in Fig. 5 and 6 respectively.

Figure 4: Correlation between majorana phases $\alpha$ and $\beta$ for both the textures $M^a_{\text{magic}}$ and $M^b_{\text{magic}}$. 
From Fig. 5, 6 and 7, it is clear that our predictions for $m_\beta$ and $m_{\beta\beta}$ are very small as compared to the sensitivities of the near future $\beta$-decay experiments like KATRIN [31, 35], Project 8 [36] and double $\beta$-decay experiment EXO-200 [37], KamLAND-Zen [38]. If any of these experiments will be successful in measuring the $m_\beta$ or $m_{\beta\beta}$, our textures will be ruled out. Correlations between the mixing angles for fixed values of $\delta (0, 45, 90, 135, 180, 225, 270, 315, 360)$ are given in the Fig. 9. Here the plots for $(\delta, \delta + 180)$ are identical. Similarly, the plots for $(\delta + 45, \delta + 135, \delta + 225, \delta + 315)$ and $(\delta + 90, \delta + 270)$ are same. The solid lines represent the 1σ experimental range and dashed lines represents the 3σ experimental range.

We have constructed the textures $M^a_{\text{magic}}$ and $M^b_{\text{magic}}$ for normal hierarchy with vanishing lowest eigenvalue of $M_{\text{TBM}}$. We can obtain similar textures for neutrino masses with inverted hierarchy which can also be written as $M'_{\text{magic}} = M_{\text{TBM}} + M'_i$. Assuming that $M_{\text{TBM}}$ has lowest vanishing eigenvalue, the forms of $M_{\text{TBM}}$ and $M'_i$ for the inverted hierarchy case are as follows:

\[
M_{\text{TBM}} = \begin{pmatrix}
    a & a + 2d & a + 2d \\
    a + 2d & a + d & a + d \\
    a + 2d & a + d & a + d
\end{pmatrix}, \quad M'_a = \begin{pmatrix}
    0 & 0 & \eta \\
    0 & 0 & \eta \\
    \eta & \eta & -\eta
\end{pmatrix}, \quad M'_b = \begin{pmatrix}
    0 & \eta & 0 \\
    \eta & 0 & 0 \\
    0 & 0 & \eta
\end{pmatrix}.
\] (52)

However, as shown in the Fig. 8, we find that the experimental ranges for the mixing angle $\theta_{13}$ and the ratio $\Delta m^2_{12}/|\Delta m^2_{23}|$ cannot be satisfied simultaneously for the inverted hierarchy case. Thus, the corresponding textures with inverted hierarchy are ruled out.

In one of our previous study [39], we had demonstrated the idea that viable textures $TM_1$ and $TM_2$ of the neutrino mass matrix can be created which are modifications of the mass matrix corresponding to TBM mixing. We did not provide any rationale to these textures. However in the present work, we
propose a systematic method to modify the neutrino mass matrix corresponding to the TBM mixing $M_{TBM}$. We modify the $M_{TBM}$ by breaking $\mu - \tau$ symmetry but preserving the magic symmetry. We generate two such textures $M_{magic}^a$ and $M_{magic}^b$. The texture $M_2$ of our previous study and the texture $M_{magic}^b$ of our present study are related by $\mu - \tau$ exchange symmetry resulting in identical predictions for these textures. The another texture $M_1$ of the previous study did not preserve either $\mu - \tau$ symmetry or the magic symmetry. Similarly the texture $M_{magic}^a$ of our current study is different from any of the previously proposed textures as can be seen by comparing the Fig. 1-5, 6 and 7 of the present manuscript.

In conclusion, we have investigated two simple textures of the neutrino mass matrix with magic symmetry. These textures can be written as combination of TBM mass matrix with a vanishing eigenvalue and a simple perturbation matrix with one complex parameter preserving the magic symmetry. These textures have four real parameters: $a$, $d$, $z$, and $\chi$. We find the allowed ranges for these parameters and present the resulting correlations between $\theta_{23}$ and $\delta$. We find that $\delta$ should be around $90^\circ$ or $270^\circ$ for maximal $\theta_{23}$ mixing for both textures. When $\theta_{23}$ is around $40^\circ$ or $50^\circ$, $\delta$ can take values near $0^\circ$ or $180^\circ$. Such correlations are generic features of magic symmetry and are testable at future neutrino experiments like NOvA and T2K. Our textures have definite predictions for $m_\beta$ [31, 35, 36, 40, 41] and $m_{\beta\beta}$ [42, 43, 44] which can be tested at $\beta$-decay and neutrino-less double $\beta$-decay experiments.

![Figure 7](image1.png)

Figure 7: Variation of $m_{\beta\beta}$ with $m_1$ for both the textures $M_{magic}^a$ and $M_{magic}^b$.

![Figure 8](image2.png)

Figure 8: Variation of $\Delta m^2_{21}$ with $\theta_{13}$ for both the textures $M_{magic}^a$ and $M_{magic}^b$ assuming inverted hierarchy of neutrino masses.
Figure 9: Corrections between the mixing angles corresponding a fixed value of $\delta$ for both $M^{a}_{\text{magic}}$ (first column) and $M^{b}_{\text{magic}}$ (second column). Here solid lines represent the 1$\sigma$ experimental range and dashed lines represent the 3$\sigma$ experimental range.
References

[1] S. Dev, R. R. Gautam, L. Singh, Neutrino Mass Matrices with Two Equalities Between the Elements or Cofactors, Physical Review D 87 (7) (2013) 073011.

[2] J. Han, R. Wang, W. Wang, X.-N. Wei, Neutrino mass matrices with one texture equality and one vanishing neutrino mass.

[3] P. H. Frampton, S. L. Glashow, D. Marfatia, Zeroes of the Neutrino Mass Matrix, Physics Letters B 536 (1-2) (2002) 79–82.

[4] W. Grimus, A. S. Joshipura, L. Lavoura, M. Tanimoto, Symmetry realization of texture zeros, European Physical Journal C 36 (2) (2004) 227–232.

[5] A. Merle, W. Rodejohann, Elements of the neutrino mass matrix: Allowed ranges and implications of texture zeros, Physical Review D 73 (7) (2006) 1–11.

[6] S. Dev, S. Kumar, S. Verma, S. Gupta, Phenomenology of two-texture zero neutrino mass matrices, Physical Review D 76 (1) (2007) 013002.

[7] S. Dev, S. Kumar, S. Verma, S. Gupta, Phenomenological implications of a class of neutrino mass matrices, Nuclear Physics B 784 (1-2) (2007) 103–117.

[8] H. Fritzsch, Z. Z. Xing, S. Zhou, Two-zero textures of the Majorana neutrino mass matrix and current experimental tests, Journal of High Energy Physics 2011 (9) (2011) 83.

[9] W. Grimus, P. O. Ludl, Two-parameter neutrino mass matrices with two texture zeros, Journal of Physics G: Nuclear and Particle Physics 40 (5) (2013) 055003.

[10] G. Blankenburg, D. Meloni, Fine-tuning and naturalness issues in the two-zero neutrino mass textures, Nuclear Physics B 867 (3) (2013) 749–762.

[11] J. Liao, D. Marfatia, K. Whisnant, Texture and cofactor zeros of the neutrino mass matrix, Journal of High Energy Physics 2014 (9) (2014) 13.

[12] B. Dziewit, J. Holeczek, M. Richter, S. Zajac, M. Zralek, Texture zeros in neutrino mass matrix, Physics of Atomic Nuclei 80 (2) (2017) 353–357.

[13] S. Kaneko, H. Sawanaka, M. Tanimoto, Hybrid Textures of Neutrinos, Journal of High Energy Physics 2005 (08) (2005) 073–073.

[14] S. Goswami, S. Khan, A. Watanabe, Hybrid textures in minimal seesaw mass matrices, Physics Letters B 693 (3) (2008) 249–254.

[15] S. Dev, S. Verma, S. Gupta, Phenomenological Analysis of Hybrid Textures of Neutrinos, Physics Letters B 687 (1) (2009) 53–60.

[16] S. Dev, S. Gupta, R. R. Gautam, Parallel hybrid textures of lepton mass matrices, Physical Review D 82 (7) (2010) 073015.

[17] R. Kalita, D. Borah, Hybrid Textures of Neutrino Mass Matrix under the Lamppost of Latest Neutrino and Cosmology Data, International Journal of Modern Physics A 31 (06) (2015) 1650008.

[18] X. G. He, A. Zee, Neutrino masses with 'zero sum' condition: m(μ(1)) + m(μ(2)) + m(μ(3)) = 0, Physical Review D 68 (3) (2003) 037302.

[19] L. Lavoura, Zeros of the inverted neutrino mass matrix, Physics Letters B 609 (3-4) (2005) 317–322.

[20] E. I. Lashin, N. Chamoun, Zero minors of the neutrino mass matrix, Physical Review D 78 (7) (2007) 073002.

[21] E. I. Lashin, N. Chamoun, One vanishing minor in the neutrino mass matrix, Physical Review D 80 (9) (2009) 093004.
[22] W. Wang, Neutrino mass textures with one vanishing minor and two equal cofactors, The European Physical Journal C 73 (9) (2013) 2551.
[23] P. Harrison, D. Perkins, W. Scott, Tri-bimaximal mixing and the neutrino oscillation data, Physics Letters B 530 (1-4) (2002) 167–173.
[24] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, T. Schwetz, Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity, Journal of High Energy Physics 2017 (1) (2016) 30.
[25] S. F. King, Parameterizing the lepton mixing matrix in terms of deviations from tri-bimaximal mixing, Physics Letters B 659 (1-2) (2008) 244–251.
[26] K. Abe, et al., Indication of Electron Neutrino Appearance from an Accelerator-produced Off-axis Muon Neutrino Beam, Phys. Rev. Lett. 107 (2011) 041801. arXiv:1106.2822
[27] F. P. An, et al., Observation of electron-antineutrino disappearance at Daya Bay, Phys. Rev. Lett. 108 (2012) 171803. arXiv:1203.1669.
[28] J. K. Ahn, et al., Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment, Phys. Rev. Lett. 108 (2012) 191802. arXiv:1204.0626
[29] Y. Abe, et al., Indication of Reactor $\bar{\nu}_e$ Disappearance in the Double Chooz Experiment, Phys. Rev. Lett. 108 (2012) 131801. arXiv:1112.6353
[30] C. Lam, Magic neutrino mass matrix and the BjorkenHarrisonScott parameterization, Physics Letters B 640 (5-6) (2006) 260–262.
[31] G. Drexlin, V. Hannen, S. Mertens, C. Weinheimer, Current Direct Neutrino Mass Experiments, Adv. High Energy Phys. 2013 (2013) 1–39.
[32] W. Rodejohann, Neutrinoless double beta decay and neutrino physics, J. Phys. G 39 (12) (2012) 124008.
[33] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, T. Schwetz, Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity, Journal of High Energy Physics 2017 (1) (2017) 87.
[34] P. Adamson, et al., First measurement of electron neutrino appearance in NOvA, Phys. Rev. Lett. 116 (15) (2016) 151806. arXiv:1601.05022
[35] M. Arenz, et al., First transmission of electrons and ions through the KATRIN beamline. arXiv:1802.04167
[36] A. A. Esfahani, et al., Determining the neutrino mass with cyclotron radiation emission spectrosopyProject 8, Journal of Physics G: Nuclear and Particle Physics 44 (5) (2017) 054004.
[37] J. B. Albert, et al., First search for Lorentz and CPT violation in double beta decay with EXO-200, Phys. Rev. D 93 (7) (2016) 072001.
[38] A. Gando, et al., Search for Majorana Neutrinos Near the Inverted Mass Hierarchy Region with KamLAND-Zen, Phys. Rev. Lett. 117 (8) (2016) 082503.
[39] K. S. Channey, S. Kumar, Phenomenological implications of two simple modifications to Tri-Bimaximal mixing, Modern Physics Letters A 32 (26) (2017) 1750137. arXiv:1708.03473
[40] D. Abdurashitov, et al., The current status of “Troitsk nu-mass” experiment in search for sterile neutrino, Journal of Instrumentation 10 (10) (2015) T10005–T10005.
[41] F. Fraenkle, Status of the neutrino mass experiments KATRIN and Project 8, PoS EPS-HEP2015 (2015) 084.
[42] A. Gando, et al., Search for Majorana Neutrinos Near the Inverted Mass Hierarchy Region with KamLAND-Zen, Physical Review Letters 117 (8) (2016) 082503.
[43] Ebert, et al., Results of a search for neutrinoless double-$\beta$ decay using the cobra demonstrator, Physical Review C 94 (2) (2016) 024603.

[44] M. Agostini, et al., Background-free search for neutrinoless double-$\beta$ decay of 76Ge with GERDA, Nature 544 (7648) (2017) 47–52.