Dynamical Chiral Symmetry Breaking in Landau gauge QCD

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Abstract. We summarise results for the propagators of Landau gauge QCD from the Green’s functions approach and lattice calculations. The nonperturbative solutions for the ghost, gluon and quark propagators from a coupled set of Dyson-Schwinger equations agree almost quantitatively with corresponding lattice results. Similar unquenching effects are found in both approaches. The dynamically generated quark masses are close to ‘phenomenological’ values. The chiral condensate is found to be large.

Keywords: Confinement, dynamical chiral symmetry breaking, gluon propagator, quark propagator, Dyson-Schwinger equations

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The infrared behaviour of the propagators of Landau gauge QCD has been investigated extensively over the past years in lattice Monte Carlo simulations and the continuum Green’s functions approach. Lattice simulations are the only ab initio method known so far and are by now precise enough to pin down these propagators accurately in a large momentum range centered around 1 GeV. In the deep infrared, however, lattice results are inevitably plagued by finite volume effects. In the continuum formulation of QCD the Dyson-Schwinger equations (DSEs) provide a tool complementary to lattice simulations. They can be solved analytically in the infrared. Furthermore numerical solutions over the whole momentum range are available by now. The truncation assumptions necessary to close the DSEs can be checked in the momentum regions where lattice results are available. In general, results from DSEs have the potential to provide a successful description of hadrons in terms of quarks and gluons, see [1,2,3] and references therein.

The ghost, gluon and quark propagators, \( D_G(p) \), \( D_{\mu\nu}(p) \) and \( S(p) \), in Euclidean momentum space can be generically written as

\[
D_G(p) = -\frac{G(p^2)}{p^2}, \tag{1}
\]

\[
D_{\mu\nu}(p) = \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}, \tag{2}
\]

\[
S(p) = \frac{1}{-i\not{p} A(p^2) + B(p^2)} = \frac{Z_G(p^2)}{-i\not{p} + M(p^2)}. \tag{3}
\]
Here we have chosen Landau gauge which is a fixed point under renormalization. The Dyson-Schwinger equations for the ghost and gluon dressing functions, \( G(p^2) \) and \( Z(p^2) \), have been investigated in refs. [5, 6]. They can be solved analytically in the infrared and one finds simple power laws,

\[
Z(p^2) \sim (p^2)^{2\kappa}, \\
G(p^2) \sim (p^2)^{-\kappa},
\]

for the gluon and ghost dressing function with exponents related to each other. The relations (4) can be determined from the ghost-DSE alone and are independent of the truncation scheme. The exponent \( \kappa \) is an irrational number and depends only slightly on the dressing of the ghost-gluon vertex [7, 8]. With a bare vertex one obtains \( \kappa = (93 - \sqrt{1201})/98 \approx 0.595 \). Recently these results have been confirmed independently in studies of the exact renormalisation group equation [9].

The dynamical generation of quark masses can be studied in the Dyson-Schwinger equation for the quark propagator. It is a genuinely non-perturbative phenomenon and requires a careful treatment of the quark-gluon interaction. In ref. [10] we demonstrated that sizeable nontrivial Dirac-structures in the quark-gluon vertex are necessary to generate dynamical quark masses of the order of 300-400 MeV. Our results for the quenched quark mass function \( M(p^2) \) and the wave function \( Z_Q(p^2) \) are compared to the quenched lattice results of refs. [11] in fig. 1. The overall qualitative and quantitative agreement between both approaches is very good. The DSE results are within the bounds given by the two different formulations of fermions on the lattice.

Including the backreaction of the quark-propagator on the ghost and gluon system leads to a coupled set of three Dyson-Schwinger equations for the propagators of QCD. These equations have been solved in [10] and allowed a prediction of possible effects of unquenching QCD on the propagators. As can be seen from fig. 1 including \( N_f = 3 \) chiral quarks in the gluon DSE hardly changes the results for the quark propagator. The

FIGURE 1. Left: The quenched and unquenched quark mass function \( M(p^2) \) and the wave function \( Z_Q(p^2) \) from the DSE approach [10] compared to results from quenched lattice calculations [11]. Right: The quenched and unquenched gluon dressing function from the DSE approach [10] compared to results from unquenched lattice calculations [13].
chiral condensate is nearly unaffected. It will be interesting to compare these results to unquenched lattice calculations when available.

Unquenched lattice results for the gluon propagator including the effects of two light (up-) and one heavy (strange-) quark have been published recently \[13\] and are compared to the corresponding results from our DSE-approach in fig. 1. The screening effect from the quark loop is clearly visible in the lattice results for momenta \(p\) larger than \(p = 0.5\ \text{GeV}\): the gluonic self interaction becomes less important in this region and the gluon dressing increases. This effect can also be seen in the DSE-approach. In the quenched case there is a discrepancy between the DSE-result and the lattice data, which can be traced back to the fact that not all effects from the gluonic self interaction are accounted for in the DSE truncation. When this part of the gluon interaction becomes less dominant in the unquenched case, both the lattice and the DSE-approach agree very well on a quantitative level, provided similar bare quark masses are taken into account.

In the chiral limit the screening effect of the quark loop becomes even stronger as can be seen from the DSE-results in fig. 1. This is expected as the energy needed to create a quark pair out of the vacuum becomes smaller with decreasing bare quark mass.

Both, the lattice calculations and the Green’s functions approach agree in the fact that unquenching does not affect the extreme infrared of the ghost and gluon propagators. Again, this is easily explained from dynamical chiral symmetry breaking: there is not enough energy to generate a quark pair from the vacuum below a certain threshold. Then the quark degrees of freedom decouple from the Yang-Mills sector of the theory.

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