Notes on Evanescent Wave Bragg-Reflection Waveguides

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ABSTRACT

We investigate an extended version of the Bragg reflection waveguide (BRW) with air gaps as one of the layers. This design has the potential of drastically simplifying the epitaxial structure for integrated nonlinear optical elements at the expense of more complicated structuring. This approach would afford much more flexibility for designing and varying BRW structures. Here, we discuss an extension of the established theory for BRW slabs and report our results of applying Marcatili’s method for rectangular waveguides to the BRW case. With this analytic approach we can estimate the effective index of the modes orders of magnitude faster than with full numerical techniques, such as finite-difference time-domain (FDTD) or finite elements. Initial results are mixed; while phase-matched designs have been found, they currently have no significant advantage over other schemes.

Keywords: Bragg-reflection waveguides, nonlinear optics, parametric down-conversion

1. INTRODUCTION

Nonlinear optical elements are interesting building blocks for a wide range of applications, ranging from coherent sources for broadband spectroscopy via classical frequency conversion processes to the generation of entangled photon pairs with parametric down conversion. For practical purposes, miniaturization and integration on semiconductor platforms seems imperative - with the (aluminium) gallium arsenide (AlGaAs) platform as a possible choice. These materials have extraordinarily high nonlinear coefficients and refractive indices, which increase efficiency and decrease device size, respectively. However, owing to the cubic lattice structure, no kind of natural birefringence exists in GaAs, which is commonly exploited for phase matching. Thus, quasi-phase-matching or modal phase matching has to be employed. In the case of Bragg-reflection waveguides (BRW) [1], modal phase matching between the fundamental mode and the second harmonic mode is achieved with distributed Bragg reflectors enclosing a core [2, 3]. In a simple picture one can assume that the fundamental mode is confined by total internal reflection of the core, while the second harmonic mode is defined by the reflection of the distributed Bragg reflector (DBR) stacks. This allows different (almost independent) tuning of the modes [4].

In this paper we investigate a different - more generalized - way of designing the distributed Bragg reflector (DBR) stacks. Usually, the DBR structure is grown vertically by alternating layers of AlGaAs with different aluminum content and thus different refractive indices. This means that it is fixed for an entire wafer, which may conflict with other structures and devices one may want to integrate on the chip. Horizontal confinement is achieved by total internal reflection at the sidewalls of etched ridges. In this article in contrast, we propose a simple two-layer wafer, where the Bragg structure is achieved by etching thin trenches (figure (1)) on the sides of a core region. Here, vertical confinement is provided by total internal reflection while the horizontal confinement is provided by the DBR stack. Such a structure would afford significantly more flexibility in accommodating nonlinear interaction with other behavior or functionality. For example, the DBR could be introduced adiabatically with the goal of transferring the mode of a relatively large waveguide into the desired mode in the DBR region. Electrical current paths for integrated pump lasers would not have to traverse the relatively thick DBR. Finally, a single epitaxially grown wafer could yield many substantially different devices thus enabling a shorter design cycle and a wider range of adaptation to varying wavelengths or other characteristics of the desired optical processes.

Since in this approach one of the Bragg layers is simply air (or some kind of filling material with a low refractive index) the field becomes evanescent in these layers. Similar structures have already been studied in different contexts: in general as photonic crystals [5], as waveguide grating reflectors [6] or more specifically in photon tunneling experiments [7].
The analytical theory of BRWs is well established[1, 2, 4]. However, it fails if any of the material refractive indices are smaller than the effective index of the mode involved. The evanescent field components require special care as several simplifying assumptions break down. In the following we will first lay out the fundamentals of the evanescent BRW and then show how Marcatili’s method can be applied for a very efficient solution of the problem at hand. Finally we will draw our conclusions as to the usefulness of the concept regarding practical devices.

2. SLAB WAVEGUIDE EFFECTIVE INDEX CALCULATION FOR THE EVANESCENT CASE

In order to calculate the effective index of a mode, Maxwell’s equations have to be solved. Bragg reflection waveguides possess symmetries that allow to derive an exact analytical expression for the modes [1] if only one dimension is considered and the Bragg stack is assumed to be infinitely extended.

The basic formalism is already described in [8] and follows [4]. We carried out calculations starting with Maxwell’s equations to verify that there are no hidden assumptions that invalidate the treatment of evanescent fields. These evanescent fields mean that the wavevector is complex valued in certain parts of the structure. This prohibits an easy analytical solution for the dispersion relation; it has to be found numerically. Nevertheless, using analytic formulations as far as practical reduces the numerical workload to just a couple of equations instead of dealing with huge matrices, such as in the case of a gridded, full-vectorial, 2D eigenmode calculation.

In general, the eigenvalue equation (1) determines the dispersion of a mode specified by the boundary conditions between the core wavefunction $f_c$ (left hand side) and the wavefunction in the Bragg stack (right hand side). This (standard) approach yields two equations with two unknown factors for the continuity of the function and the derivative, respectively. Dividing these equations eliminates these constants and results in the following form:

$$
\frac{1}{k_c} f_c(k_c t_c/2) \left[ \frac{\partial f_c}{\partial x} \bigg|_{k_c t_c/2} \right]^{-1} = \frac{i \exp[iK\Lambda] - A + B}{k_1 \exp[iK\Lambda] - A - B}
$$

In the case of a simple step index core $f_c$ takes the form of either sin or cos, depending on whether we are interested in symmetric or antisymmetric modes. The left hand side then reduces to a tan or cot function; using the form of equation (1) assures proper factors in the case of higher order modes. $t_c$ is the core thickness and $k_i$ is the wavevector in layer $i$ (see also eq. [5]). The right hand side derives from the eigenvalues of a single Bragg stack. $A, B$ (transfer matrix entries) and $\exp[iK\Lambda]$ (Bloch-wave eigenvalues) are properties of the Bragg stack, with
\[
\exp[iKA] = \frac{1}{2}(A + D) \pm \sqrt{\frac{1}{4}(A + D)^2 - 1}
\] (2)

\[
A_{\text{TE}} = e^{ik_1a} \left[ \cos k_2b + \frac{i}{2} \left( \frac{k_2}{k_1} + \frac{k_1}{k_2} \right) \sin k_2b \right]
\] (3)

\[
B_{\text{TE}} = e^{-ik_1a} \left[ \frac{i}{2} \left( \frac{k_2}{k_1} - \frac{k_1}{k_2} \right) \sin k_2b \right]
\] (4)

being the coefficients for a TE mode (for the TM analogue see Refs. [4, 8]). \(a\) and \(b\) are the thickness of the two layers of the Bragg stacks with \(\Lambda = a + b\). The wavevectors in the individual layers (with vacuum wavevector \(k_0 = 2\pi/\lambda\)) are given by

\[
k_i = k_0 \sqrt{n_i^2 - n_{\text{eff}}^2}
\] (5)

With equations (2) to (5) the dispersion relation (1) is now solely a function of the effective refractive index of the mode \(n_{\text{eff}}\). Additional assumptions, such as \(n_i > n_{\text{eff}}\), lead to an analytical expression for \(n_{\text{eff}}\) [4]. In general, however, one of the \(k_i\) can be complex, so we have to find numerical solutions in the complex domain (\(n_{\text{eff}} \in \mathbb{C}\)). This is a two dimensional root-finding problem, but since we are only interested in low-loss solutions we can effectively restrict the search to the region where \(\text{Im } n_{\text{eff}} \approx 0\). Additionally, both branches of (2) have to be taken into account for finding the proper solution.

Figure (2) shows the graphical solutions for the refractive indices \(n_1 = 1\), \(n_2 = n_c = 3.3\), core thickness \(t_c = 0.5\mu m\), layer thicknesses \(a = b = 0.1\mu m\) at a wavelength \(\lambda = 0.775\mu m\). There are two physical solutions for the effective mode index at \(n_{\text{eff}} = 3.0196 - 9 \cdot 10^{-14}i\) (TE\(_2\) mode, with two peaks) and \(n_{\text{eff}} = 1.979 - 2 \cdot 10^{-14}i\) (TE\(_4\) mode). In comparison, a commercial-grade mode solver [9] yields \(n_{\text{eff}} = 3.0192 - 9 \cdot 10^{-16}i\) and \(n_{\text{eff}} = 1.991 + 9 \cdot 10^{-17}i\), showing excellent agreement. The inconsistent signs of the imaginary parts are due to limited machine precision.

3. MARCATILI METHOD FOR (EVANESCENT WAVE) BRAGG REFLECTION WAVEGUIDES

Having solved the slab waveguide, Marcatili’s method [10] for estimating the modes of rectangular waveguides can directly be employed for the full 2D problem. In this approximation it is assumed that both directions (e.g. \(x\) and \(y\) for a \(z\)-propagating mode) are independent slab waveguides. Both slab waveguides (here, 1D-BRW for \(x\); asymmetric, dielectric waveguide for \(y\)) are solved and then are joined by the approximate dispersion relation (see [6])

\[
\beta^2 = \beta_x^2 + \beta_y^2 - \beta_c^2
\] (6)

with \(\beta = n_{\text{eff}}k_0\), the desired propagation constant. \(\beta_x\) and \(\beta_y\) are the propagation constants of the respective slabs (\(\beta_i = n_{\text{eff},i}k_0\)), \(\beta_c = n_c k_0\) is the propagation constant of a plane wave in a bulk of core material.

Using the previous example and the structure from figure (1a), we introduce the slab height \(h\), the substrate refractive index \(n_S\) and the “background index” \(n_{bg} = 1\) (air). In such a configuration the substrate plays an important role: if the effective index of a mode cannot be less than the refractive index of the substrate. Here, a substrate index of \(n_S = 3\) and the vertical confinement - which further reduces the effective index - is already enough to push the TE\(_2\) found in the 1D case into the cutoff region (figure (2a)). As an example, Marcatili’s 2D method predicts an effective index of \(n_{\text{eff}} = 3.211 + 9 \cdot 10^{-13}i\) for the TE\(_1\) mode for an evanescent wave BRW with a height of \(h = 1\mu m\), while the commercial grade mode solver gives \(n_{\text{eff}} = 3.202 + 10^{-17}i\). In this case, a larger deviation than for the slab is expected as Marcatili’s method commonly underestimates the propagation index [6].
4. DISCUSSION AND SUMMARY

One advantage of these semi-analytical models is computational speed as only a few equations have to be solved. This allows many more structures to be analyzed for feasibility than with “numerically exact” mode solvers. Results for the intended structures etched structures (such as (1a)) are mixed: for the 1-D slab waveguide, configurations that phase-match a fundamental mode at \( \lambda = 1.55 \mu m \) with a second harmonic (Bragg) mode at \( \lambda = 0.775 \mu m \) can be found easily. With the added vertical confinement of a realistic 2D structure, however, it is much more difficult as the substrate imposes a very restrictive limit on the possible mode indices. At present state there is no significant advantage in terms of tunability, easiness in fabrication or nonlinear overlap compared to, for example, form-birefringence phase-matched nano-waveguides [11]. Some of these limitations are related to the extreme refractive index difference between GaAs and air and the relatively high substrate refractive index. Having a filling material - for example, a polymer [12] - for the trenches with a refractive index of > 2 would relax the requirements.

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