Is the lack of power anomaly in the CMB correlated with the orientation of the Galactic plane?

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Abstract. The lack of power at large angular scales in the CMB temperature anisotropy pattern is a feature known to depend on the size of the Galactic mask. Not only the large scale anisotropy power in the CMB is lower than the best-fit ΛCDM model predicts, but most of the power seems to be localised close to the Galactic plane, making high-Galactic latitude regions more anomalous. We assess how likely the latter behaviour is in a ΛCDM model by extracting simulations from the Planck 2018 fiducial model. By comparing the former to Planck data in different Galactic masks, we reproduce the anomaly found in previous works, at a statistical significance of ∼ 3σ. This result suggests the existence of a bizzare correlation between the particular orientation of the Galaxy and the lack of power anomaly. To test this hypothesis, we perform random rotations of the Planck 2018 data and compare these to similarly rotated ΛCDM realisations. We find that, among all possible rotations, the lower-tail probability of the observed high-Galactic latitude data variance is still low at the level of 2.8σ. Furthermore, the lowering trend of the variance when moving from low- to high-Galactic latitude is anomalous in the data at ∼ 3σ when comparing to ΛCDM rotated realisations. This shows that the lack of power at high Galactic latitude is substantially stable against the “look elsewhere” effect induced by random rotations of the Galaxy orientation. Moreover, this analysis turns out to be substantially stable if we employ, in place of generic ΛCDM simulations, a specific set whose variance is constrained to reproduce the observed data variance.

Keywords: CMB, CMB anomalies, data analysis
1 Introduction

The low-variance anomaly is a feature of the Cosmic Microwave Background (CMB) temperature anisotropy pattern present in both WMAP [1–3] and Planck data [4–6]. It shows up at large angular scales, where the instrumental noise is negligible, with a statistical significance around 2-3 σ C.L. depending on the estimator employed. This effect is correlated with other CMB anomalies, see e.g. [7–10], which are sensitive to the lack of power with respect to expectations of the ΛCDM model, see [11] for further details. For this reason, we will use the expressions lack-of-power and low-variance as synonyms.

A statistical fluke is of course the simplest explanation for this phenomenon. However, in this case, one has to accept to live in a rare ΛCDM realisation. In any case, there are at least three reasons why this anomaly is worth of further investigations [12]:

1. it is unlikely that the effect is due to an unaccounted instrumental systematics: both WMAP and Planck observe it with similar significance despite being two separate experiments with different data gathering schemes and scanning strategies;

2. it is not natural to attribute this effect to foreground residuals: the latter are not expected to be correlated to the CMB, so a foreground residual should increase and
not lower the total anisotropy power\(^1\). A similar argument would also apply to possible extensions of the ΛCDM as long as their source is statistically independent from the primary CMB anisotropy [13, 14].

3. it is suspiciously dependent on the Galactic mask: its statistical significance increases when only high Galactic regions are considered, which is usually a conservative choice in CMB data analysis [15]. It was also shown [16] that this effect was dominated by odd over even multipoles, see e.g. [17–19].

This paper wants to focus on the last item by estimating, from a statistical point of view, how likely is to find a CMB map of the ΛCDM model with such a behaviour between low- and high-Galactic latitudes. To perform this analysis we will use random rotations (see Appendix A) of simulated CMB maps in order to evaluate among all the possible orientations what is the probability of having most of the power at low-Galactic latitudes. The adopted estimator is the variance, \(V\), of the temperature anisotropies, \(\delta T(\hat{n})\),

\[
V \equiv \langle (\delta T(\hat{n}))^2 \rangle ,
\]

where \(\hat{n}\) is the unit-vector pointing a given direction of observation. \(V\) is built through the angular power spectrum (APS), \(C_\ell\):

\[
V = \sum_{\ell=2}^{\ell_{\text{max}}} \frac{2\ell + 1}{4\pi} C_\ell ,
\]

where the maximum multipole, \(\ell_{\text{max}}\), is set to 29 in the following since we want to be consistent with the maximum multipole considered in the Planck pixel-based low-\(\ell\) Likelihood functions [20]. However, the dependence of \(V\) upon \(\ell_{\text{max}}\) is very weak for \(\ell_{\text{max}} \gtrsim 10\) and therefore such a choice does not impact significantly on our results.

The paper is organised as follows: in Section 2 we describe the dataset we consider, and how we generate Monte Carlo simulations; in Section 3 we perform the analysis of the Planck 2018 dataset comparing the results with ΛCDM simulations. After recovering results in agreement with previous works, we consider random rotations of the data and the simulations, to assess the a posteriori choice of assuming a particular orientation for the Galactic plane; in Section 4 we repeat the same analyses focusing on a specific set of ΛCDM simulations that show the same low-variance of the observed map. Conclusions are drawn in Section 5.

2 Data set and simulations

2.1 CMB maps and masks

We use data products from the Planck 2018 data release, available in the Planck Legacy Archive\(^2\). In particular we employ the temperature Commander 2018 map [21] downgraded to HEALPix\(^3\) [23] resolution \(N_{\text{side}} = 16\) with a Gaussian beam with full width half maximum, FWHM, of 440 arcmin. The map is shown in the left panel of Fig. 1. As a consistency check we

\(^1\)Note also that typically (and in particular at large scales where this work is focused) the foreground mitigation is performed at the map level (in the harmonic or pixel space) and not at the \(C_\ell\) level.

\(^2\)https://www.cosmos.esa.int/web/planck/pla

\(^3\)http://healpix.sourceforge.net
also employ the SMICA temperature map [21], also downgraded from high resolution to $N_{\text{side}} = 16$. These CMB maps have been delivered already with a constrained CMB realisation along the Galactic plane. We have added to those maps a regularisation noise realisation with 2 $\mu$K rms, consistently considered in the extraction of the APS. This choice is consistent with the procedure adopted in [22]. We checked that such a noise has a negligible impact on our results. The maps have been masked with several Galactic masks, shown in the right panel of Fig. 1 and whose sky fractions are listed in Table 1. More specifically, the considered masks are the $N_{\text{side}} = 16$ confidence mask provided with the 2018 Commander solution [21], named Std 2018, and other four masks built extending the edges of the Likelihood 2015 standard mask [20] by 12, 18, 24 and 30 degrees, called respectively Ext$_{12}$, Ext$_{18}$, Ext$_{24}$ and Ext$_{30}$. This choice is done in order to make contact with previous works, i.e. [15, 16], and to compare the impact of the most recent Planck 2018 data with respect to that of the 2015 release, see Appendix B.

| Mask   | Sky Fraction [%] |
|--------|------------------|
| Std 2018 | 85.6            |
| Ext$_{12}$ | 70.8           |
| Ext$_{18}$ | 59.1            |
| Ext$_{24}$ | 48.7            |
| Ext$_{30}$ | 39.4            |

Table 1. Observed sky fractions for the masks shown in Figure 1.

2.2 Sets of simulations

We generate $10^5$ CMB temperature maps at HEALPix resolution $N_{\text{side}} = 16$ randomly extracted from the Planck 2018 best-fit model through the synfast function of healpy [23] with a Gaussian beam of 440 arcmin FWHM. To provide numerical regularisation, a different random noise realisation, with rms of 2 $\mu$K, is added to each of the CMB simulations, as done for the observed Commander and SMICA 2018 maps. This set is used to estimate the statistical significance of the low-variance in a $\Lambda$CDM framework. A subset of $10^3$ simulations of this set of $\Lambda$CDM realisations is referred to as ensemble 0. Another subset of $10^3$
simulations constrained to have variance $V$ close to the value observed by Commander 2018, $V_c = 2090.02 \, \mu K^2$ obtained with the Std 2018 mask, is called ensemble 1. More precisely a map $m_i$ with variance $V_i$ belongs to ensemble 1, if $V_c - 20 \, \mu K^2 \leq V_i \leq V_c + 20 \, \mu K^2$. The analysis of the stability of our results with respect to the choice of the threshold of 20 $\mu K^2$ is given in Appendix C. Note that in the case of SMICA the variance is also constrained in the same range which contains the value observed in the data ($V_s = 2085.57 \, \mu K^2$).

### 2.3 Angular power spectrum estimator

As anticipated in Section 1, we use the variance $V$ as estimator for the lack of power, built through Eq. (1.2). The $C_\ell$ are obtained with an optimal angular power spectrum estimator, namely BolPol [24], an implementation of the Quadratic Maximum Likelihood (QML) method [25, 26]. The choice of the QML algorithm minimises the introduction of extra statistical uncertainty in our analysis with respect to other, suboptimal, APS estimators [27]. For each of the simulated maps and for the various masks defined above, we have used the estimates of BolPol to build the variance, $V$. In Fig. 2 we show the APS of the Commander 2018 temperature map estimated with the five masks shown in Fig. 1 and whose sky fraction is reported in Table 1.

### 3 Analysis in $\Lambda$CDM framework

As already known in the literature, the observed value of $V$ is low and its statistical significance increases considering regions at high Galactic latitude, see e.g. [1–6]. Employing the Bolpol code to extract the TT APS for each of the $10^5$ $\Lambda$CDM simulations, we have built the probability distribution functions of $V$ for each of the five masks shown in Fig. 1. The MC distributions are displayed in Fig. 3 where they are compared to the corresponding Planck 2018 observed values shown as vertical bars.
Figure 3. Each panel shows the empirical distribution of $V$ in $\mu K^2$ expected in a ΛCDM model (blue) and computed through Eq. (1.2) for the masks listed in Table 1. We report also the even and odd splits of the variance, through Eq. 3.1 (orange and green, respectively). Vertical dashed and dotted bars correspond to the Planck 2018 Commander and SMICA CMB solutions, respectively.

In the same panels we provide also $V_+$ ($V_-$), shown in orange (green), defined as $V$ but where the sum in Eq. (1.2) is performed only over the even (odd) multipoles, i.e.

$$V_\pm = \ell_{\text{max}} \sum_{\ell=2}^{\ell_{\text{max}}} \left[ \frac{1 \pm (-1)\ell}{2} \right] \frac{2\ell + 1}{4\pi} C_\ell. \quad (3.1)$$

In addition, we display as vertical bars, with the same color convention, the corresponding observed Planck 2018 values for $V_\pm$.

Fig. 4 shows the three lower tail probabilities, henceforth LTP, for $V_+$ (orange), $V_-$ (green) and $V$ (blue) against the observed sky fraction of the five cases of Fig. 3. $V$ shows a monotonic behaviour: as one considers regions at higher and higher Galactic latitude the Planck observed values shift towards lower variances more rapidly than the increase of the width of the distribution due to sampling variance because of the smaller observed sky fraction considered. In other words, the observed values are more and more unlikely and for the extreme case, i.e. Ext$_{30}$ mask, we find a compatibility with ΛCDM model only at 0.3% C.L. for the Commander map and 0.5% for SMICA. This is dominated by $V_+$ which is constantly low, independently on the considered sky fraction. Indeed, for Commander, its LTP varies around 0.3 – 0.5%, for all the considered sky fractions lower than the Std 2018 one. For SMICA, instead, its LTP varies in a slightly higher but still low range [0.5%, 1.1%]. On the other hand, $V_-$ is more sensitive to the sky fraction, decreasing monotonically as one takes into account regions at higher and higher Galactic latitude. However, its LTP remains inside the 1 σ dispersion of the MC’s, reaching ~ 11% in the Ext$_{30}$ mask, independently from the employed CMB solution.
Figure 4. Lower tail probability of the Planck 2018 Commander and SMICA maps with respect to the $10^5 \Lambda$CDM simulations as a function of the sky fraction.

The fact that the LTP of $V$ decreases when using more aggressive masks suggests that the low power of the Planck data is somehow anisotropically distributed on the map. In other words, the increasing discrepancy of the data with respect to $\Lambda$CDM when we exclude from the analysis pixels around the Galactic plane, indicates a sort of “localisation” of most of the power around the Galactic plane itself.

Moreover, Fig. 3 and 4 show that, at large angular scales, such a low-Galactic-latitude power turns out to be dominated by the odd multipoles, see also [16].

3.1 Variance analyses including rotations

We now further investigate the dependency of $V$ with respect to the Galactic mask by implementing random rotations of the maps (see Appendix A for details which include the validation). This is performed in order to evaluate among all the possible orientations what is the probability of having most of the power at low Galactic latitude. The above procedure can be seen as a sort of look-elsewhere effect on the orientation of the mask. For computational reasons we reduce the number of MC simulations by considering the ensemble 0 made of $10^3$ maps generated from the Planck 2018 best-fit model. Note that $V$ is invariant under rotation of the input maps by construction only in the full sky case. In fact, when a mask is applied, the variance $V$ is not conserved under rotation for a single realisation but invariance is restored only on ensemble average. This effect is nicely captured already at the angular power spectrum level: in Fig. 5 each panel shows the average and the statistical uncertainty at $1\sigma$ of the TT spectra of $10^3$ random rotations of the Commander map for the various masks$^4$. Notice that the APS estimates obtained with the Std 2018 mask are recovered only on average (blue lines) in the other masks. Moreover, as expected, the standard deviation (blue region) increases as the mask gets larger, allowing less observed sky for the analysis. In addition, still in Fig. 5 we show the TT spectrum of the Commander 2018 map without any rotation (red symbols).

$^4$We obtain a similar behaviour for SMICA that is not shown here for sake of brevity.
We analyse random rotations of the ensemble 0 and corresponding observed data building two estimators, the LTP-estimator (Section 3.1.1) and the $r$-estimator (Section 3.1.2). With the former we investigate separately for each mask how anomalous is the particular orientation of the Galactic plane. With the latter we quantify the statistical significance of the lowering trend of $V$ with respect to its value in the Std 2018 mask with all the possible orientations.

3.1.1 LTP estimator

For each map $m_i$ belonging to ensemble 0 we build the histogram of $V_i$ obtained through $10^3$ random rotations of that map. Hence, we compute the LTP of that map $m_i$, denoted with LTP$_i$, with respect to the corresponding set of rotations. This can be repeated for $i = 1, \ldots, 10^3$, i.e. for all the maps of the ensemble 0 and for all the considered masks. Thus, for each mask, we obtain a MC of $10^3$ values of LTP representing the distribution of probabilities expected in a ΛCDM model. Since the variance does not depend on the orientation, the distribution of LTP is expected to be uniform, that is, each LTP is equiprobable. The empirical distribution of the LTP-estimator for each considered mask shown in Fig. 6 confirms our expectations. In the same Figure we also show the LTP obtained from Planck data as vertical bars, red for Commander and green for SMICA. The corresponding values are reported in left panel of Table 2. When we consider higher Galactic latitude, we find that the probability of observing a LTP with respect to its rotations lower than the corresponding LTP of Commander (SMICA) 2018 is anomalous at $\sim 2.8 \sigma$ ($\sim 2.5 \sigma$). Indeed, in the Ext$_{30}$
Figure 6. Histograms of the LTP of finding a rotated map of the ensemble 0 with $V^{\text{rot}} < V$, where $V$ is the variance of the corresponding unrotated map. Each panel shows the results obtained using a different mask. Red dashed and green dotted vertical bars are the LTP for Commander and SMICA respectively.

| Mask     | $V^{\text{rot}}_c < V_c$ | $V^{\text{rot}}_s < V_s$ |
|----------|--------------------------|--------------------------|
| Std 2018 | 22.7                     | 56.8                     |
| Ext$_{12}$ | 5.7                     | 3.5                     |
| Ext$_{18}$ | 4.8                     | 4.1                     |
| Ext$_{24}$ | 1.0                     | 1.7                     |
| Ext$_{30}$ | 0.7                     | 1.4                     |

Table 2. Left table: The probability of obtaining a value of the variance of the rotated Commander map (second row), $V^{\text{rot}}_c$, and rotated SMICA map (third row), $V^{\text{rot}}_s$, smaller than the unrotated one, $V_c$ and $V_s$ respectively. Right table: LTP of obtaining a simulation of the ensemble 0 with LTP lower than the one obtained with the Commander map, LTP$_c$, or SMICA map, LTP$_s$.

In this case, only 5 (13) out of $10^3$ maps of the ensemble 0 have a lower LTP than the Commander (SMICA) 2018 map, i.e. only in the 0.5% (1.3%) of the cases the anomaly associated to the power localisation around the Galactic plane is higher than data (see right panel of Tables 2).

3.1.2 $r$-estimator

We use here the $r$-estimator defined as

$$ r = \frac{V_{\text{std}} - V_{\text{mask}}}{\max_{j \in \text{rotations}} \left\{ V_{\text{std}}^{(j)} - V_{\text{mask}}^{(j)} \right\} } , $$  

(3.2)
where $V_{\text{std}}$ is the variance computed in the Std 2018 mask, while $V_{\text{mask}}$ is the variance computed in one of the other four extended masks. The numerator of Eq. (3.2) fixes the sign of the $r$-estimator as determined by the decrease ($r > 0$), or increase ($r < 0$), of the variance as we widen the Galactic mask. This behaviour is normalised by the denominator, which picks up the maximum decrease among all the rotations\(^5\). The $r$-estimator is therefore upper bounded by 1, but it can become lower than -1. In other words, the $r$-estimator represents the fractional change of $V$, computed in an extended mask with respect to the Std 2018 mask value, relative to the maximum decrease across rotations. For example, $r = 0.5$ means that, we are dealing with a map which, in a given mask, has a variance difference with respect to the standard mask equal to exactly half of the maximum difference which can be found among all rotations. In the left panel of Fig. 7 we show the $r$-estimator for all the considered cases. Dotted lines connect the MC values of $r$ represented with a plus symbol. Solid blue line connects the Commander 2018 values (dot symbols) and the solid green line connects the SMICA 2018 values (square symbols). For this estimator we consider the upper tail probability, UTP, defined as the fraction of simulations with larger values of $r$ than the observed one. They are shown in the right panel of Fig. 7 and quoted in Table 3. Notice that both Commander and SMICA present an increase of $r$ for higher and higher Galactic latitudes and in the Ext\(_{30}\) case, they are close to 1, being $r^c = 0.88$ for Commander and $r^s = 0.90$ for SMICA. This means that the observed maps in the Ext\(_{30}\) case are almost aligned to the direction which maximizes the lowering of $V$ obtainable through rotations. The probability corresponding to this event is 0.2% for both Commander and SMICA. This leads to an anomalous value of $r$ at a level of 3.1 $\sigma$.

4 Analysis of $\Lambda$CDM simulations with low variance

In this section we repeat the analysis performed in Section 3 but now considering simulated maps which have almost the same variance $V$ as the one observed by the CMB solutions (Commander and SMICA) of the Planck 2018 release. These are collected in the ensemble 1,\(^5\)

\(^5\)In the denominator of $r$ we include also the unrotated case, denoted here as the $0^{\text{th}}$ rotation.
Table 3. UTP of obtaining a simulation of the *ensemble* 0 with $r$ larger than the one obtained from the data. Second column shows the UTP for *Commander*, third column the UTP for *SMICA*.

| Mask   | $r^2 < r$ | $r^3 < r$ |
|--------|-----------|-----------|
| Ext12  | 10.9      | 7.5       |
| Ext18  | 3.9       | 2.0       |
| Ext24  | 0.9       | 1.5       |
| Ext30  | 0.2       | 0.2       |

Figure 8. Each panel shows the *Planck* 2018 best-fit model (black solid line) and the average APS of *ensemble* 1 (blue line), with its 1σ dispersion (blue region) for all the considered masks.

as described in Section 2. The aim of this analysis is to check whether the previous results still hold when the variance is constrained to be low also across the simulations. In other words we would like to exclude the possibility that the observed trend of a lowering variance when extending the Galactic mask, is connected to the low value of the variance measured in the Standard mask. In Fig. 8 we display the *Planck* 2018 best-fit model (black solid line) and the average of *ensemble* 1 (blue line), with its standard deviation (blue region) for all the considered masks. Notice the increase of the statistical uncertainty as the observed sky fraction decreases. This figure shows that *ensemble* 1 behaves differently from the fiducial power spectrum only at low-$\ell$. In other words, selecting a subset of ΛCDM realisations with low variance is in fact equivalent to choosing maps with suppressed $C_\ell$ at low multipoles.6

We evaluate the variance $V$ for each element of *ensemble* 1 and for each of the considered masks. Results are shown in Fig. 9 where each panel provides the histogram of $V$ for each

6 Note that we recover empirically the well-known correlation between low-$V$ and low-$C_2$ anomalies [11].
Figure 9. Histograms of the variances $V$ of the maps belonging to the ensemble 1 computed with the masks Std 2018, Ext$_{12}$, Ext$_{18}$, Ext$_{24}$ and Ext$_{30}$. The red dashed line identifies the variance of the Commander map, $V_c$. The green dashed line identifies the variance of the SMICA map, $V_s$.

### Table 4

| Mask     | $V < V_c$ (%) | $V < V_s$ (%) |
|----------|---------------|---------------|
| Std 2018 | 50.7          | 41.5          |
| Ext$_{12}$ | 4.1          | 1.9           |
| Ext$_{18}$ | 2.3          | 1.1           |
| Ext$_{24}$ | 0.2          | 0.2           |
| Ext$_{30}$ | 0.3          | 0.4           |

The probability of obtaining a value for the variance $V$ smaller than that of Commander (second column), $V_c$, or SMICA (third column), $V_s$, for a map of the ensemble 1. Note that the difference between the Ext$_{24}$ and Ext$_{30}$ case is of the order of the numerical sensitivity of the ensemble 1, since it is made of $10^3$ simulations.

Dashed red line represents $V$ as measured from Commander, and the dashed green line stands for $V$ of SMICA. In the left panel of Fig. 10 we display the LTP of the Planck 2018 data in percentage as a function of the sky fraction. They are also reported in Table 4 for convenience. We find that the monotonic behaviour shown in Fig. 4 for the $10^5$ $\Lambda$CDM simulations is almost recovered for the ensemble 1: $V$ still decreases at high Galactic latitudes with a percentage of compatibility at the level of $0.3 - 0.4\%$ in the Ext$_{30}$ case. This means that a “low variance” model (low as the one observed by Planck) is not enough to explain this behaviour at high Galactic latitude. Notice also that this effect is largely dominated by

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Note that for the Commander case the difference between the two last cases, i.e. Ext$_{24}$ and Ext$_{30}$ case, is of the order of the numerical sensitivity of the ensemble 1, since it composed of $10^3$ simulations.
the quadrupole and the octupole. This is shown in the right panel of Fig. 10, where the LTP vs the observed sky fraction is shown when we exclude only the quadrupole (blue dashed lines) or both the quadrupole and the octupole (red dashed lines) in the computation of $V$.

4.1 Variance analyses including rotations

As for the ensemble 0 we now include random rotations in the analysis of the ensemble 1. We still use the LTP-estimator and the $r$-estimator defined above.

4.1.1 LTP estimator

For each map $m_i$ belonging to ensemble 1 and its rotations we obtain the MC of $10^3$ values of LTP$_i$. In Fig. 11 we show the histograms of such LTP$_i$ for each considered mask. The observed LTP (i.e. those obtained from Planck data and shown in left panel of Table 2) are also shown in the same figure as vertical bars, red for Commander and green for SMICA. Notice that, by construction, even in a $\Lambda$CDM model constrained to have a low-variance as ensemble 1, the variance does not depend on the orientation. Therefore the distribution of LTP is still uniform as it is found in the histograms of Fig. 11. In this case we find for ensemble 1 a very similar behaviour to ensemble 0. For Commander (SMICA) the LTP estimator gives a $\sim 2.8\sigma$ ($\sim 2.6\sigma$) anomaly at high Galactic latitude, see Table 5.

| Mask    | LTP$_{i} < $LTP$_{c}$ (%) | LTP$_{i} < $LTP$_{s}$ (%) |
|---------|-----------------------------|-----------------------------|
| Std 2018 | 21.6                        | 52.9                        |
| Ext$_{12}$ | 5.7                          | 3.2                         |
| Ext$_{18}$ | 4.5                          | 3.9                         |
| Ext$_{24}$ | 0.7                          | 1.3                         |
| Ext$_{30}$ | 0.5                          | 0.9                         |

Table 5. LTP of obtaining a simulation of the ensemble 1 with LTP lower than the one obtained with the Commander map, LTP$_{c}$, and SMICA map, LTP$_{s}$.
Figure 11. Histograms of the LTP of finding a rotated map of the ensemble 1 with $V^{\text{rot}} < V$, where $V$ is the variance of the corresponding unrotated map. Each panel shows the results obtained using a different mask. Red dashed and green dotted vertical bars are the LTP for Commander and SMICA respectively.

4.1.2 $r$-estimator

We apply here the $r$-estimator to the ensemble 1 simulations. In Fig. 12 we show the results for all the considered cases. Dotted lines connect the MC values of $r$ represented with a plus symbol. Solid blue line connects the Commander values (dot symbols) and the solid green line connects the SMICA values (square symbols). The UTP are shown in the right panel of Fig. 12 and quoted in Table 6. At high Galactic latitude we find an anomalous value for $r$ at the level of $\sim 2.9 \sigma$ with a UTP of 0.4% for Commander and 0.3% for SMICA. In conclusions the results for the ensemble 1 are similar to those of ensemble 0 even when rotations are considered.

| Mask | $r^2 < r$ | $r^3 < r$ |
|------|-----------|-----------|
| Ext$_{12}$ | 4.0       | 5.3       |
| Ext$_{18}$ | 4.0       | 1.5       |
| Ext$_{24}$ | 0.5       | 0.7       |
| Ext$_{30}$ | 0.4       | 0.3       |

Table 6. UTP of obtaining a simulation with $r$ larger than the one obtained from the data. Second column shows the UTP for Commander, third column the UTP for SMICA.
5 Conclusions

In this paper we analysed the lack-of-power anomaly, a well known characteristic of the CMB temperature anisotropy pattern showing up at large angular scales. In particular, we focused on the intriguing fact that this feature is statistically more significant (at a $\sim 3\sigma$) when only high Galactic latitude data are taken into account. The latter observations suggests that most of the large scale anisotropy power happens to be mainly localised around the Galactic plane. This might sound bizzare because the early universe should not know anything about the “direction” of the disk of our Galaxy. To tackle the issue, we evaluated how often a $\Lambda$CDM realisation happens to have most of its power localised at low Galactic latitude.

To support the analysis, we generated a $\Lambda$CDM Monte Carlo set of $10^5$ CMB maps from the Planck 2018 best-fit model. By analysing this set, we first showed that the Planck 2018 data exhibits the same trend of decreasing CMB field variance while increasing the Galactic mask, which was found previously in the literature. We then proceeded to randomly rotate the simulated maps (denoted as ensemble 0), as well as the data, $10^3$ times. The rotated maps are employed to compute the empirical distribution function of two estimators, based on the CMB field variance (Section 3). With the LTP-estimator (Section 3.1.1) we test to what extent the low CMB anisotropy power in the data depends on the orientation of the Galactic plane. With the $r$-estimator (Section 3.1.2) we assess instead the behaviour against rotation of the decreasing trend of the CMB variance at increasing Galactic latitude. The introduction of random rotations is a key-element to evaluate whether the lack of power anomaly is indeed correlated with Galactic latitude.

To further investigate this behaviour we also selected from the $10^5$ $\Lambda$CDM simulations set a smaller set, of $10^3$ maps, which exhibits the same low-variance as the one observed in the Commander and SMICA 2018 maps. We called this set ensemble 1 and repeated the analyses performed on the ensemble 0.

We find that even when performing random rotations, our CMB sky is anomalous in power at about $2.8 - 2.5\sigma$ depending on the considered component separation method when employing the LTP estimator. Specifically, only 5 maps out of $10^3$ have a LTP at high Galactic latitude (in the Ext$_{30}$ mask) smaller than the Planck Commander data. For the

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**Figure 12.** Left panel: $r$-estimator computed with Eq. (3.2) versus the sky fraction. The coloured dotted lines stand for the $r$ value obtained from the ensemble 1. Blue and green solid lines stand for Commander and SMICA respectively. Right panel: UTP of obtaining a simulation with $r$ larger than the one obtained with Commander (blue line) or SMICA (green line) as a function of the sky fraction.
-estimator we evaluate that only the 0.2% of the maps show a larger value of $r$ between Std 2018 and Ext30 masks, again with respect to Commander. Results are substantially stable if we employ SMICA in place of Commander. Finally, using the low-variance constrained simulation of ensemble 1 yields similar results, showing that having a low-variance field in the first place is not enough to justify the observed trend with Galactic latitude.

In conclusion, the introduction of rotations do not spoil the lack of power anomaly at high Galactic latitude which turns out to be quite stable against the “look-elsewhere effect” spawned by random rotations of the reference frame.

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A Generating the rotations

Random rotations of temperature CMB maps are generated following an harmonic-based approach through a Python algorithm. We consider maps at HEALPix resolution $N_{\text{side}} = 16$ which are harmonic-expanded to obtain the initial $a^{lm}_{m'}$ coefficients. These coefficients are then rotated through the Wigner rotation matrices, $R(\theta, \varphi, \psi)$, whose rotations angles $(\theta, \varphi, \psi)$ (also known as Euler angles), are randomly extracted from uniform distributions. Technically this is performed thanks to the healpy subroutine rotate alm. After the rotation, the final map, or simply the rotated map, $m^R$ can be written as

$$\mathbf{m}^R = \sum_{\ell m} \left( \sum_{m'} R_{mm'}(\theta, \varphi, \psi) a^{lm}_{m'} \right) Y_{\ell m}(\theta, \varphi). \quad (A.1)$$

To validate the procedure which implements random rotations, we consider a map which is zero except for a spot of $9^\circ$, see Fig. 13. This is done simply setting to 1, nine neighboring pixels and then smoothing the map with a Gaussian beam with a FWHM= $9^\circ$. For convenience we call $m_0$ this initial map. Starting from $m_0$ we perform $N_{\text{rot}}$ rotations considering $m_{i-1}$ as the input for $i^{\text{th}}$ rotation, with $i = 1, \ldots, N_{\text{rot}}$. We then compute the following total map,

$$\mathbf{m}^{\text{tot}} = \sum_{i=0}^{N_{\text{rot}}} \mathbf{m}_i, \quad (A.2)$$

which is shown in Fig. 14, for $N_{\text{rot}} = 2, 50$ and 500. The idea is to use $\mathbf{m}^{\text{tot}}$ to test whether the set of considered rotations is able to “cover uniformly” all the possible directions. This is our requirement for validation which is quantified computing the APS of $\mathbf{m}^{\text{tot}}$ and comparing the monopole with higher order multipoles: when the former dominates over the latter we can safely state that the set of rotations is sufficiently populated to have its isotropic part.

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8The smoothing is applied in order to minimise aliasing effects when going from real to harmonic space and vice versa.

9In other words, we apply $N_{\text{rot}}$ times Eq. (A.1).
leading over accidental anisotropies. Note that, in turn, this procedure provides the minimum number of rotations which are needed to fulfill the requirement mentioned above. The left panel of Fig. 15 shows the behaviour of the lowest multipoles, namely the monopole $C_0$, the dipole $C_1$, the quadrupole $C_2$, and the octupole $C_3$, against the number of rotations. The monopole component increases its magnitude quadratically versus the number of rotations whereas low-$\ell$ components oscillate around a very slowly monotonic growth. We repeat this procedure 50 times and compute the mean distribution of the same first low-$\ell$ components, see right panel of Fig.15. The mean behaviour of the different components, and the hierarchy among the multipoles, is substantially unchanged with respect to what obtained with the single realisation. In particular the hierarchy among low-$\ell$ multipole components seems to become stable for $N_{\text{rot}} > 900$. Most importantly, we find that the magnitude of the ratio $C_0/C_1$ at $N_{\text{rot}} = 1000$ is of the order $10^3$; therefore we choose this threshold to define the minimal number of rotations needed to cover sufficiently homogeneously the whole sky.
Figure 15. Left panel: amplitude of the first low-$\ell$ components of the APS of the test map after each rotation. Right panel: the average over 50 repetitions of the machinery described in Sec. A. The filled regions correspond to the 1 $\sigma$ dispersion of $C_\ell$ components.

Figure 16. Std 2015 temperature mask.

B Comparison between 2018 and 2015 Planck release

In this section we consider the 2015 Planck data. This analysis is performed mainly because the Planck 2015 standard mask [20], henceforth called Std 2015, is smaller than the 2018 one. Its observed sky fraction is 93.6%, see Fig. 16, versus 85.6% of the Std 2018, see Fig. 1 and Table 1. Hence we employ here the Commander 2015 map used in [20] still at HEALPix resolution $N_{\text{side}} = 16$ and FWHM of 440 arcmin and consistently to what performed for the 2018 case, we added to this map a regularisation noise of 2 $\mu$K rms. The masks used during this analysis are the same listed in Table 1, with the exception of the Std 2018, which has been replaced with the Std 2015. Similarly to what performed in Section 2 for the generation of the ensemble 0, we build here a MC of $10^4$ maps using the Planck 2015 best-fit model. From these maps, we select a subset of $10^3$ maps with variance $V$ within 20 $\mu$K$^2$ from the value computed with the Commander 2015 map, i.e. $V_c = 2060.09$ $\mu$K$^2$. This set of simulations is called ensemble 1-2015. The behaviour of $V$ as a function of the masks obtained with the
Figure 17. Histograms of the variance $V$ of the maps belonging to *ensemble* 1-2015 computed for the masks Std 2015, Ext$_{12}$, Ext$_{18}$, Ext$_{24}$ and Ext$_{30}$. The red dashed line identifies the variance of the Commander 2015 map, $V_c$.

| Mask      | $V < V_c$ | $V_c^{\text{rot}} < V_c$ | LTP$_1 < \text{LTP}_c$ |
|-----------|-----------|--------------------------|-----------------------|
| Std 2015  | 47.2      | 64.2                     | 58.3                  |
| Ext$_{12}$| 7.2       | 11.1                     | 10.0                  |
| Ext$_{18}$| 0.8       | 2.9                      | 2.4                   |
| Ext$_{24}$| 0.4       | 1.6                      | 1.6                   |
| Ext$_{30}$| 0.3       | 0.5                      | 0.5                   |

Table 7. The probability of obtaining a value for the variance $V$ smaller than that of Commander 2015 for a map of the *ensemble* 1-2015 (first column). The probability of obtaining a value of the variance of the rotated Commander 2015 map, $V_c^{\text{rot}}$, smaller than the unrotated one (second column). LTP of obtaining a simulation with LTP$_1$ lower than the one obtained with the Commander 2015 map, LTP$_c$ (third column).

*ensemble* 1-2015 is shown in Fig. 17 and the corresponding LTP are reported in the first column of Table 7. We recover a similar monotonic behaviour as for the 2018 case. That is, in the Ext$_{30}$ case, the behaviour of the 2015 data is anomalous at $\sim 2.9\sigma$.

We take into account now the rotations applied to *ensemble* 1-2015. The results for the LTP-estimator and $r$-estimator are shown in Fig. 18 and Fig. 19. For the LTP-estimator we find in the Ext$_{30}$ mask a LTP of 0.5% which is in line with the 2018 analysis. All the LTP for this estimator are reported in Table 7. On the other hand, the $r$-estimator gives $r^c = 0.80$ for the Ext$_{30}$ mask with a p-value of 1.2%, see Table 8. While the general behaviour of $r$ across the mask is recovered here, the probability at high Galactic latitude is slightly higher.
Table 8. UTP of obtaining a simulation of the ensemble 1 - 2015 with $r > r^c$ larger than the one obtained with the Commander 2015 map.

| Mask  | UTP [%] |
|-------|---------|
| Ext$_{12}$ | 12.1 |
| Ext$_{18}$ | 2.3 |
| Ext$_{24}$ | 1.4 |
| Ext$_{30}$ | 1.2 |

Figure 18. LTP of finding a rotated map of the ensemble 1 - 2015 with $V^{\text{rot}} < V$, where $V$ is the variance of the corresponding unrotated map. Each panel shows the results obtained using a different mask. The dashed vertical bars are the LTP of Commander 2015.

C Dependence on threshold

In this section we study the impact on our results of the threshold of $V$ we choose to select the maps of the ensemble 1 from the 10$^5$ ΛCDM simulations. Specifically, in addition to the threshold of 20 $\mu$K$^2$ used in Section 2, we choose two other thresholds at 10 $\mu$K$^2$ and 30 $\mu$K$^2$. These will define two new subsets of 10$^3$ CMB temperature maps which have a variance $V$ close to the value observed by Commander 2018. We refer to these two additional subsets as ensemble 2 (E2) and ensemble 3 (E3), respectively.

Therefore, we repeat on E2 and E3, the same analysis previously performed on ensemble 1 for both the considered estimators, focusing on the mask Ext$_{30}$. We start building the distribution of $V$, see Fig. 20, where the left panel refers to E2 while the right one to E3. The LTP of Commander 2018 are LTP$_{E2}(V_c < V_i) = 0.2\%$ and LTP$_{E3}(V_c < V_i) = 0.7\%$, which are consistent with what obtained with ensemble 1.
Figure 19. Left panel: r-estimator computed with Eq. (3.2) versus sky fraction. Right panel: UTP of obtaining a simulation with r larger than the one obtained with Commander 2015 as a function of the sky fraction.

Figure 20. Variance distribution of the ensemble 2 (left panel) and ensemble 3 (right panel) for the Ext30 mask. Red dashed line corresponds to the variance of the Commander 2018 map.

As done for the ensemble 1, we can apply random rotations to E2 and E3 and build the LTP-estimator and the r-estimator in the Ext30 case. The distributions of the former are shown in the left panels of Fig. 21 and Fig. 22 for the E2 and E3 case respectively. The vertical dashed bars stand for the Commander 2018 values of the estimator, see again Table 5. The LTP of the LTP-estimator, turn out to be 0.3% and 0.2% for E2 and E3 respectively. In the right panels of Fig. 21 and Fig. 22 we show the r-estimator for the E2 and E3. We find UTP_{E2}(r_c < r_i)=0.4% and UTP_{E3}(r_c < r_i)=0.5% for E2 and E3 respectively. We conclude that our results are stable with respect to the choice of the threshold which defines the set of constrained realisations.
Figure 21. Left panel: distribution of probability of observing, in a ΛCDM model with low variance, a lower value with respect to $V_c$ due to random rotations of ensemble 2. Right panel: $r$-estimator computed with Eq. (3.2) for the ensemble 2. Both the results have been obtained using the Ext$_{30}$ mask.

Figure 22. The same as Fig. 21 but for ensemble 3.

References

[1] C. Monteserin, R. B. B. Barreiro, P. Vielva, E. Martinez-Gonzalez, M. P. Hobson and A. N. Lasenby, Mon. Not. Roy. Astron. Soc. 387 (2008) 209 doi:10.1111/j.1365-2966.2008.13149.x [arXiv:0706.4289 [astro-ph]].

[2] M. Cruz, P. Vielva, E. Martinez-Gonzalez and R. B. Barreiro, Mon. Not. Roy. Astron. Soc. 412 (2011) 2383 doi:10.1111/j.1365-2966.2010.18067.x [arXiv:1005.1264 [astro-ph.CO]].

[3] A. Gruppuso, P. Natoli, F. Paci, F. Finelli, D. Molinari, A. De Rosa and N. Mandolesi, JCAP 1307 (2013) 047 doi:10.1088/1475-7516/2013/07/047 [arXiv:1304.5493 [astro-ph.CO]].

[4] Planck Collaboration XXIII, Astron. Astrophys. 571 (2014) A23 doi:10.1051/0004-6361/201321534 [arXiv:1303.5083 [astro-ph.CO]].

[5] Planck Collaboration XVI, Astron. Astrophys. 594 (2016) A16 doi:10.1051/0004-6361/201526681 [arXiv:1506.07135 [astro-ph.CO]].

[6] Planck Collaboration VII, arXiv:1906.02552 [astro-ph.CO].
