Logistic forecasting of GDP competitiveness

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The GDP growth of national economies is modelled by the logistic function. Applying it on the GDP data of the World Bank till the year 2020, we forecast the outcome of the competitive GDP growth of Japan, Germany, UK and India, all of whose current GDPs are very close to one another. Fulfilling one of the predictions, in 2022 the GDP of India has indeed overtaken the GDP of UK. Our overall forecast is that the GDP of India will be greater than that of the other three countries. We argue that when trade saturates, large and populous countries (like India) have the benefit of high domestic consumption to propel their GDP growth.

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I. INTRODUCTION

The logistic equation is a standard example of a first-order autonomous nonlinear dynamical system [1]. Introduced originally to study population dynamics [1, 2], it was later applied to multiple problems of socio-economic [2–5] and scientific interest [1]. This is because the growth of many natural systems is modelled quite accurately by the logistic equation, the growth of species being one of many such examples [2]. Hence, the logistic equation is organically compatible with natural evolution in a free and productive environment. This principle can be extended to the evolution of economic systems as well, a point of view that is supported by the generally successful logistic modelling of the GDP and trade dynamics of some leading national economies [5].

The GDP (an abbreviation of Gross Domestic Product) of a country is the market value of goods and services produced by the country in a year [6–8]. GDP thus quantifies the aggregate outcome of the economic activities of a country that are performed all round the year. As such, the GDP of a national economy is a dynamic quantity and its evolution (commonly implying growth) can be followed through time. To this very end, the logistic equation turns out to be a simple and convenient mathematical tool, as has been shown in an earlier study carried out on countries that are ranked high globally in terms of their national GDPs [5].

From a macroeconomic perspective, GDP is a standard yardstick with which the state of a national economy is gauged, and in a global comparison of national economies, the GDP of a country is a reliable point of reference. By this criterion, globally the top six economies pertain to USA, China, Japan, Germany, UK and India. At present these six countries account for nearly 60% of the global GDP and nearly 40% of the global trade. China, India and USA are the three most populous countries in the world, accounting for almost 40% of the world population. On the scale of strategic economic regions, the three most dominant economies in the North-Atlantic region are USA, Germany and UK. Likewise, the three most dominant economies in the Indo-Pacific region are China, Japan and India. All six countries are members of important economic blocs like G7 and BRICS. USA, Japan, Germany and UK belong to the former bloc, while China and India belong to the latter. Besides, all of these countries are the leading global representatives of three types of economic systems, namely, free economies (USA, Japan and Germany), controlled economies (China) and mixed economies (India). That only six countries should exert such an overarching influence on the global economy is compatible with the scale-free (power law) degree distribution of GDP [3, 9], because in scale-free distributions the disproportionate dominance of a few elements is a natural occurrence [10]. To this the global economic order can be no exception. Summing up these facts, we can now argue that our study on a restricted scale of six countries (which are global leaders in terms of their GDPs) adequately represents the essence of the GDP growth of more countries that can be studied on a larger scale.

Country-wise annual GDP data, on which we have based our modelling and analysis, have been collected from the World Bank website [11–16] up to the year 2020. The basic mathematical theory of the logistic equation and its application on the GDP data are laid out in Sec. II. The numerical and statistical analyses of the modelling are summarized in Table I. In Sec. III we consider the competitive GDP growth of Japan, Germany, UK and India. Extrapolating the theoretical logistic functions (all calibrated by the GDP data [13–16]) beyond 2020, we predict the specific years in which the GDP of one country will overtake the GDP of another. Three such overtakes are to occur in the future, one of which has already happened in 2022, in precise agreement with our forecast. In Sec. IV we remark on the impact of current geopolitics and adverse climatic events on GDP competitiveness.

II. LOGISTIC MODELLING OF GDP GROWTH

The GDPs of all the six countries in this study are measured in US dollars. Quantifying GDP by the variable \( G(t) \), in which \( t \) is time (measured in years), we set up a first-order autonomous dynamical system for \( G(t) \) as [1, 5]

\[
\frac{dG}{dt} = \gamma G \left( 1 - \frac{G}{k} \right).
\]

(1)
the two competing economies in the Indo-Pacific region are the GDP growth of USA and China in Fig. 1. After China, on a scale of global competitiveness, we exclusively compare approximately a quarter of the GDPs of either USA or China. Hence, USA and China are the top two economies of the world, with their GDPs in the world. The smooth dotted curves model the GDP growth of USA, China, Japan, UK and India is 1960. For Germany, the initial condition of $G(0) = G_0$, is

$$G(t) = \frac{G_0 e^{\gamma t}}{1 + (G_0/k)(e^{\gamma t} - 1)},$$

which is the logistic function. The time scale that is implicit in Eq. (2) is $\gamma^{-1}$. On early time scales, when $t \ll \gamma^{-1}$, the growth of $G$ in Eq. (2) can be approximated to be exponential, i.e. $G \approx G_0 \exp(\gamma t)$. This gives $\ln G \sim \gamma t$, which is a linear relation on a linear-log plot. We interpret $\gamma \approx G/G$ as the relative growth rate in the early exponential regime. However, this exponential growth is not indefinite, and on time scales of $t \gg \gamma^{-1}$ (or $t \rightarrow \infty$) there is a convergence to $G = k$. Thus, according to Eq. (2), growth saturates to a finite limit on long time scales. The transition from the exponential regime to the saturation regime occurs when $t \sim \gamma^{-1}$.

Of the six countries in our study, the initial year of the GDP data for USA, China, Japan, UK and India is 1960. For Germany, the data begin from 1970. All data sets end either in 2019 or 2020. Hence, our study spans across six decades in all cases but one. USA, Germany and UK are the top three economies in the North-Atlantic region, and China, Japan and India are likewise in the Indo-Pacific region. Moreover, USA and China are the top two economies of the world, with their respective GDPs being of the order of 20 trillion US dollars. The GDPs of Japan, Germany, UK and India are each approximately a quarter of the GDPs of either USA or China. Hence, on a scale of global competitiveness, we exclusively compare the GDP growth of USA and China in Fig. 1 After China, the two competing economies in the Indo-Pacific region are Japan and India. Their GDP growth is compared jointly in Fig. 2 Similarly, after USA, the two competing economies in the North-Atlantic region are Germany and UK, whose GDP growth is compared together in Fig. 3. The early exponential growth of the GDP and its later convergence to a finite limit, as implied by Eq. (2), are modelled in all the linear-log plots in Figs. 1, 2, and 3. The uneven lines follow the movement of the real GDP data, available from the World Bank [11–16]. The smooth dotted curves theoretically model the real data with Eq. (2). The values of $\gamma$ (the relative annual growth rate of GDP in the early stage) and $k$ (the predicted maximum value of GDP), calibrated through the model fitting in all the cases, are to be found in Table I. The most convincing match of the GDP data with the logistic function is seen in Fig. 1 for USA. Consistent fitting of the GDP data with the logistic function is also seen for Japan, Germany, UK and India, for which Figs. 2 and 3 provide evidence. Similar consistency, however, is not observed in the model fitting of the GDP data for China, as we note from the lower plot in Fig. 1. These observations about the model-fitting of the GDP data are statistically summarized in Table I which sets down the mean $\mu$ and the standard deviation $\sigma$ of the yearly relative variations of the actual GDP data [11–16] about the theoretical logistic function. Going by the values in Table I we contend that the natural and balanced growth of the GDP of a country can be gauged from the closeness between the theoretical logistic function and the actual GDP data. In support of this view, the GDP growth of USA is a compelling example. Conditions that favour such a GDP growth are discussed in Sec. III.
TABLE I. Parameter values and statistical analyses of the logistic
modelling of the World Bank GDP data \([11–16]\) of the six countries
that are listed in the first column. The country-wise ranking is in
the order of decreasing GDP till the year 2020. For Germany, however, the GDP data \([13]\) start from 1970 \((t = 10 \text{ years})\) \([14]\). Till 1999-2000, both countries ran each
other very close in terms of their GDP growth. Thereafter, the GDP
of Germany has continuously led the GDP of UK. The beginning
of the trail for Germany is theoretically captured by the intersection
of the smooth dotted curves around the year 2000 (shown clearly in
Fig. 3). These two theoretical logistic curves model the GDP growth
of both countries according to Eq. (2), with the values of \(\gamma\) and \(k\) in Table I. In the case of Germany the theoretical logistic curve has
been extrapolated backward before 1970.

| Country | \(\gamma\) | \(k\) | \(\mu\) | \(\sigma\) |
|---------|---------|-------|-------|-------|
| USA     | 0.080   | 30.0  | 0.0492| 0.0873|
| China   | 0.095   | 80.0  | -0.3568| 0.2504|
| Japan   | 0.175   | 5.2   | -0.0833| 0.1395|
| Germany | 0.110   | 4.4   | 0.0489 | 0.1744|
| UK      | 0.105   | 3.0   | -0.1089| 0.1651|
| India   | 0.080   | 6.0   | -0.1359| 0.1743|

\* In 2022 India is in the fifth position and UK is in the sixth.

III. FORECASTING GDP COMPETITIVENESS

From the GDP values in Figs. 1, 2 and 3 we realize that USA and China are at present the top two national economies of the world, both with an emphatic lead over the other four countries. Although it is unlikely that in the near future the GDP of Japan, Germany, UK or India may surpass the GDP of either USA or China, between USA and China themselves, the GDP gap is reducing progressively, as Fig. 1 shows. At this rate the GDP of China may surpass the GDP of USA. The year of this overtake can be identified as the year when the extrapolated logistic function of the China GDP crosses the extrapolated logistic function of the USA GDP. However, while the GDP growth of USA is modelled accurately by the logistic function (a claim supported by the clean fit of the upper plot in Fig. 1 and the low values of \(\mu\) and \(\sigma\) for USA in Table I), the same observation does not apply to China. The inadequacy of the logistic function to model the GDP growth of China is evident from the lack of closeness between the logistic function and the erratic GDP data in the lower plot in Fig. 1, as well as from the high values of \(\mu\) and \(\sigma\) for China in Table I.

That the logistic function falls short in modelling the GDP growth of China is known \([5]\). It has been argued that the logistic function properly models the GDP growth of countries that foster a democratic polity, are free of military conflicts on their borders, and promote free economic growth without excessive interference from the state \([5]\). The cumulative effect of these conditions is conducive to a natural development of material well-being. The absence of any one of the aforementioned conditions causes imbalance, as happens in the case of China. On the other hand, in the other five countries, all of the three foregoing conditions prevail in varying degrees, and as such, the logistic equation becomes effective in modelling the GDP growth of these countries \([5]\). This argument is substantiated by all the related plots in Figs. 1, 2 and 3 along with the corresponding values of \(\mu\) and \(\sigma\) in Table I.

Considering that China is an anomalous case in modelling the dynamics of GDP with the logistic equation, we make no further attempt to compare the logistic growth of the GDPs of USA and China for predicting the year in which the GDP of the latter will overtake the GDP of the former. Instead we study the competitiveness of the GDPs of the other five countries. However, the GDP of USA is so far ahead of the others that in the foreseeable future none of the GDPs of Japan, Germany, UK and India is likely to grow close enough to the GDP of USA. In that case, a study of the competitiveness of GDP growth is meaningful only among Japan, Germany, UK and India. Accordingly, it is for these four countries that we forecast the outcome of competitive GDP growth. Our method consists of extrapolating the theoretical logistic functions of Japan, Germany, UK and India beyond the year 2020 in a single graph, and noting the crossing points among the function curves. At the crossing points the GDP of one country overtakes the GDP of another. Since the four logistic functions have been calibrated with the GDP data available up to 2020 \([13–16]\), any crossing beyond this year enables us to forecast future GDP competitiveness among the four countries. The result of this whole exercise is to be seen in Fig. 4.

We first note that Fig. 4 has five crossing points. Of these, two occur in the years 1966 and 2000, in both of which the GDP of UK was successively overtaken by the GDPs of Japan and Germany. The actual GDP data \([13–15]\) do agree with these intersections, and thus confirm the fundamental soundness of the logistic modelling of GDP growth. While the crossings of 1966 and 2000 occur within the range of the
Forecasting the long term outcome of the GDP competitiveness among Japan, Germany, UK and India. The four theoretical logistic functions, pertaining to the aforementioned four countries, are calibrated with the annual GDP data of the World Bank till 2020 [13–15]. Two crossings of the theoretical functions occur before 2020, one in 1966, when Japan overtook UK, and the other in 2000, when Germany overtook UK. The years of these intersections are correctly borne out by the World Bank data [13–15]. The remaining three intersections are to occur after 2020, and hence, are predictive in nature. The first of these, in 2022, has already happened, when the GDP of India overtook the GDP of UK. Thereafter, the successive overtakes of the GDPs of Germany and Japan are predicted to occur in the years 2035 and 2047, respectively. In this plot, the relative positions of Japan and Germany remain qualitatively unchanged throughout. However, UK, which began ahead of the other three countries in 1960, brings up the rear of the group from 2022 onwards. In contrast, India, which started behind the others in 1960, is to lead the group from 2047 onwards.

Available GDP data, i.e. till the year 2020, there are three more crossing points beyond 2020. These are in the years 2022, 2035 and 2047, in all of which, the GDP of India is predicted to successively overtake the GDPs of UK, Germany and Japan. As it happens, fulfilling the first prediction precisely, in the year 2022 the GDP of India has indeed overtaken the GDP of UK. This certainly inspires confidence in the predictive power of the logistic modelling of GDP growth.

Another noteworthy aspect of Fig. 4 is that in the year 1960 (at \( t = 0 \) in the graph) it shows India to have the lowest GDP among the four national economies that we compare. The explanation for this lies in the history of the latter half of the twentieth century. In the years following the Second World War, which ended in 1945, it became a policy imperative for USA (mainly due to the Cold War against the erstwhile Soviet Union) to aid and expedite the economic revival of both war-ravaged Japan and Western Europe (the latter under the Marshall Plan). Guided by USA thus, Japan, Germany (then West Germany) and UK had achieved political peace and economic prosperity by 1960. In contrast, during the same period, India, freed from colonial rule about a decade earlier, did not experience the advantages that regenerated the economies of Japan, Germany and UK. Two factors, more than any other, impeded the GDP growth of India. The first is government policies in economic matters, and the second is a series of wars in which India was embroiled in the initial three decades of its sovereign existence. Unsurprisingly then, the GDP growth of India is seen to trail those of the other countries in Fig. 4 from 1960 to 2020. And yet by 2047, the GDP of India is projected in Fig. 4 to lead the GDPs of the other three countries. This will be possible only because India has maintained a steady GDP growth rate over an extended duration. One reason for this sustained growth rate is that India is a country of subcontinental proportions with a large population, unlike Japan, Germany and UK. Now, it is known that the GDP of a national economy is scaled as a function of its trade by a power law, \( G \sim T^{\alpha} \), in which \( T \) is the trade volume and \( \alpha (>0) \) is the power-law exponent [5]. For all the countries that we study here, the power-law scaling is known to hold true over at least two orders of magnitude [5]. What is more, the exponent \( \alpha \) distinguishes the economies of large countries (with large areas and populations) from the economies of small ones (with small areas and populations) [5]. In the former type, which includes India (as well as USA and China), \( \alpha \) has a relatively low value [5]. In the latter type, which includes Japan, Germany and UK, \( \alpha \) has a higher value [5].

We now explain how the distinction between the two types of national economies can cause a difference in trading patterns, with a concomitant effect on the GDP growth. A country with a large population has the advantage of a proportionately large domestic consumption of its own products, which in turn makes a proportionately greater contribution to the GDP, as compared to the contribution from trade. Consequently, the contribution of trade to the overall GDP growth reduces, a condition that is reflected by a lower value of \( \alpha \). The same feature is also known for USA and China, which like India, are geographically extended countries with large populations [5]. Countries with small populations, on the other hand, are bereft of the means of the high domestic consumption that attends a large population, and thus they have to rely more on trade with other countries to enhance their GDPs [17]. This condition is reflected by a higher value of \( \alpha \), as is known to happen in the case of Japan, Germany and UK [5]. Now, the growth of trade also saturates according to the logistic function [5], and since trade is highly correlated with GDP [5], a saturation of trade implies a corresponding saturation of the GDP. Therefore, countries that depend more on external trade than on domestic consumption will see their GDP growth saturate when their trade saturates due to market-driven inhibitors. This is what we see for Japan, Germany and UK in Fig. 4 whereas the GDP of India, despite its slow start, will outpace the GDP of the other three countries on the strength of its large volume of domestic consumption.

IV. CONCLUSIONS AND REMARKS

The suitability of the logistic equation to model the dynamics of GDP and trade has been established already [5]. In the present study we proceed further to show that the logistic equation is also effective in forecasting the outcome of GDP competitiveness among some leading national economies.
Our logistic forecasting method has been vindicated both retrospectively and for future times. In the former case, it has correctly estimated the years when the GDPs of Japan and Germany overtook the GDP of UK (1966 and 2000, respectively). In the latter case, looking forward in time, the logistic method has also been successful in forecasting 2022 as the year in which the GDP of India is to overtake the GDP of UK. What now remains to be seen is the fulfillment of the forecast that the GDP of India will overtake the GDPs of Germany and Japan in 2035 and 2047, respectively.

All that said, we have to remember that the subject of our present study is the dynamics of social systems (national economies). Hence, it must depend on real socio-economic data. For instance, the logistic functions in Fig. 4 which are at the core of our forecasts, have all been calibrated with the GDP data of the World Bank till 2020 [11–16]. However, unforeseen natural, social and political events can compromise our forecasts by recalibrating the parameters of the logistic equation. To understand what such events may be like, we consider some contemporary examples. Even in the first quarter of 2020 no one anticipated that within two years the global economy was to receive three major shocks. These are, first, Covid-19, which became a global pandemic by the middle of 2020 and whose aftereffects are felt even in late 2022. The second shock has been the war in Ukraine, which broke out in early 2022, even before the world economy could fully recover from the damage it had suffered due to the Covid-19 pandemic. The Ukraine war, by now protracted beyond all expectations, has disrupted global supply chains and energy markets, the severity of which is not yet fully understood. The third shock has been droughts in Europe and China, which will affect crucial sectors like power and agriculture. Considering that Europe and China belong to two major economic regions of the world, the former in the North-Atlantic and the latter in the Indo-Pacific, adverse climatic events in these regions will have an adverse impact on production globally.

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