Observable constraint on the frequency-mass relation of compact stars

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ABSTRACT
We study the relation between mass and limit rotation for various kinds of compact-star models. Under the considerations of Keplian and r-mode instability limits, we calculate tens of the limit frequency-mass relation curves. We present constraint of the observable rotation frequency on the frequency-mass relations. Along with the observable constraint on mass-radius relation, we may actually probe into the interior of compact stars. Our results show that the possible submillisecond pulsars would be serious to distinguish the compact stars contained quark matter from normal neutron stars. The recent discovery of burst oscillation at 1122Hz in the X-ray transient XTE J1739-285 provides us a challenge. As an example, the constraints at 1122Hz level allow the hybrid-star radii in the range $9 \leq R \leq 12$ km and the masses in the range $1.2 M_\odot \leq M \leq 2 M_\odot$.

Key words: dense matter — stars: evolution — stars: neutron — stars: rotation

1 INTRODUCTION

The interior of compact stars contains nuclear matter at very high densities that are a few times and even up to more than ten times the density of ordinary atomic nuclei. It provide a high-pressure environment in which numerous subatomic particles compete with each other. In early years, compact stars were regarded as the neutron stars which are constituted of neutrons and protons. However nuclear physical theory and experiments favor the presence of hyperons and the Bose condensates of pions and kaons in a few times the equilibrium density of nuclear matter. The quantum chromodynamics theory also predicts the occurrence of deconfinement phase transition in such high density. A large variety of particles other than just neutrons and protons could extensively exist inside various compact stars.

It has so far proved very difficult to find any effective method that could unambiguously distinguish them. Commonly ones believe that the maximum masses and typical radii of compact stars are the two most important properties which reflect rather difference among the equations of state(EOSs) of dense matter. The pioneers hereby presented the observable constraints on mass-radius relations. The maximum mass of compact stars has a limit of $3 M_\odot$, which is a consequence of general relativity(Rhoades Jr.& Ruffini 1974). The maximum mass is also controlled by the stiffness of dense matter EOS at densities in excess of a few times saturation density. The inclusion of non-nucleonic degrees of freedom at supra-nuclear densities generally implies a softening of the EOS. Some recent mass measurements, from timing of pulsars in binaries with white dwarf companions, observations of quasi-periodic oscillations and x-ray emission of accreting neutron stars, seem to suggest a maximum mass in the vicinity of $2 M_\odot$(Nice et al 2005) but the precisely known mass is only $1.44 M_\odot$(Taylor& Weissberg 1989), or about $1.68 M_\odot$ at 95 % confidence(Ransom et al, 2005). As for compact star radii, the measurements are far less precise than mass measurements. These masses and radii are unable to provide a good constraints on the compositions of dense matter and the structural properties of compact stars because most EOSs are allowed by the measurements.

Aside from direct mass and radius determinations, rapid rotation is another crucial property of compact stars. Measurements of pulsar spin frequency are extremely precise. Nearly two thousands of pulsar frequencies are well measured, tens of which have frequencies over 100Hz. The first millisecond pulsar spinning at 641Hz was discovered in 1982(Backer et al). And then a new one was reported during the next decade or so almost every year. The fastest presently known pulsar has 716Hz spin frequency which was reported in 2006(Jason et al). These are perhaps utilizable for constraining compact stars. The most well known attempt is the out of critical mass-radius relation by the measured large frequency assumed as Keplerian limit(Glendenning 1997; Lattimer& Prakash 2007). This way, displayed below, is not reliable to exclude the com-
positions of dense matter. The Keplerian frequency cannot reach in many situations. It is because other instabilities inside compact stars are even more efficient in determining the largest rotations. Some researches found the r-mode instability would play a key role for many cases. Immediately it has been realized that compact star spin evolution is also sensitive to the internal compositions which determine or not the dissipations of unstable modes are large or small. Ones were impelled to probe the interior of compact stars through the measured rotations restricting the instability windows. The efforts from this aspect is sometimes fruitless yet. In the circumstances, we think of the requirement of a new scheme for the stringent constraints on compact stars through pulsar’s frequencies. We suggest (1) the frequency-mass relations for various EOSs are constrained by the measured pulsar rotation frequencies, different from the conventional considerations of constraints on mass-radius relations. (2) Both Kepler and r-mode limit rotations are taken into account together.

We will below study various compact-star models, normal neutron star(NS), strange star(SS), hyperon star(HpS) and hybrid star(HbS, the inclusion of Maxwell(MC) and Gibbs(GC) constructions). We apply RMF(GlenDening 1997), APR(Akmal et al 1998) and BBG( Nicostra et al 2006) for NSs, MIT and eMIT(Schertle et al 1997) for SSs, RMF(Luckey et al 2000), BBG(Baldo et al 2000) for HpSs, RMF+eMIT(Pank & Zheng 2007), RMF+eMIT(Schertle et al 2000; Zheng et al 2007) and BBG+eMIT(Fahri & Jaffe 1984; Nicotra et al 2006)for HbSs. This paper is arranged follows.

We discuss the viscous dissipations and estimate that of the interface between pure quark matter and pure hadronic matter in section II. We investigate the constraints of the dissipations in various dense matter have been extensively studied. The considerations of the dissipation in various dense matter have been listed in Table 1. In addition, the surface rubbing of compact stars with solid crust has been also calculated(Bildsten & Ushomirsky 2000; Andersson et al, 2000). However, the bulk Gibbs calculation have been studied(Kokkotas & Stergioulas 1999). We then estimate the layer thickness inside the HbS and HbS. This paper is arranged follows as.

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### Table 1. Bulk viscosities in various dense matter. Compositions refers to the interacting components, n - neutrons, p - protons, e - electrons, H - hyperons and Q - quarks.

| Compositions | References | Adaptive models |
|--------------|------------|-----------------|
| npe          | Sawyer (1989) | NS, HpS, HbS    |
| npeH         | Jones (2001)  | HpS             |
| npeQ         | Drago et al (2005) | HbS         |
| npeHQ        | Pan & Zheng (2006) | HbS     |
| Q(u,d,s)     | Madsen (1992) | SS, HbS         |

hadron matter envelope needs to be estimated except two homogeneous neutral phases. We here have to reckon with the dissipation in the interface between pure quark matter and pure hadronic matter.

Let us assume that the HbS undergoes a radial pulsation $δn = Δn \sin Ω t$. During the course, the hadrons penetrate the interface to be transformed into quarks when $δn > 0$ in $[0, \frac{2π}{Ω}]$, while quarks confine into hadrons when $δn < 0$ in $[\frac{2π}{Ω}, τ]$. Using the standard definition of the bulk viscosity and the dissipation of the energy in the hydrodynamic motion due to the irreversibility of periodic compression-decompression process, we can write the energy dissipation rate per unit volume averaged over the pulsation period $τ$ as, $⟨\dot{ζ}⟩ = -ζ (⟨\dot{d}v^2⟩)^2$ and $⟨\dot{ζ}⟩ = -\frac{1}{τ} ∫_0^τ dt \frac{1}{τ^2} (\frac{δn}{n_h})^2$.

Supposing the baryon densities for the quark matter phase and hadron matter phase on both sides of the interface are $n_q$ and $n_h$, we can get

$$ζ (⟨\dot{d}v^2⟩)^2 = \frac{(n_q - n_h)^2 P_{tran}}{n_q n_h}$$

(1)

Thus the damping on pulsations due to confined-deconfined processes in the transition boundary layer is obtained as

$$\left(\frac{dE}{dt}\right)_{tran} = -\frac{(n_q - n_h)^2 P_{ tran} l_{ tran} 4π R_{ tran}^2}{n_q n_h},$$

(2)

where $P_{ tran}$, $l_{ tran}$ and $R_{ tran}$ represent the pressure in the interface, the layer thickness and the radius of quark matter core inside the MC HbS respectively. The height of the surface waves induced by the r-modes has been given by $δr_s ≈ 0.07α \left(\frac{R}{\kappa}\right)^2$ in dimensionless(Kokkotas & Stergioulas 1999). We then estimate the layer thickness inside the HbS by $l_{ tran}/R ≡ δ_r ≈ (δr_s) = \sim 10^{-3} α^2 \left(\frac{R}{\kappa}\right)^4$, where $R$ denotes the radius of the star. We immediately estimate $l_{ tran} ≈ 0.10 km$ for $R = 10 km$.

### 3 CONSTRAINTS ON FREQUENCY-MASS RELATIONS

For a uniform rigid sphere with mass $M$ and radius $R$, the mass-shedding limit is(Lattimer & Prakash 2004)

$$\frac{1}{ν_K} = 0.96 \left(\frac{R(10 km)}{M/M⊙}\right)^{3/2},$$

(3)

and the critical rotation frequency for a given stellar model can also be derived from following equations

$$\frac{1}{ν_e} + \frac{1}{ν_r} = 0, \quad ν_r = \min [ν(T)],$$

(4)
where $\tau_{sr} < 0$ is the characteristic time scale for energy loss due to the gravitational waves emission, $\tau_v$ denotes the damping timescales due to the shear, bulk viscosities and other rubbings. Surface rubbing is decisive for NSs, HpSs and HbSs, whereas $\frac{1}{\tau_v}$ can be neglected even for quark stars with maximal crust. The shear viscosity timescale for strange quark matter is calculated by (Heiselberg and Pethick 1993).

We here treat the bulk viscosity timescale with Lindblom et al method (1999). In order to constrain neutron-star matter EOS, as well-known, the traditional treatments of (3) and (4) are done in past works. From (3), a series of $M$, $R$ upper limits are often obtained if $\nu_K$ is surely given, such as 716Hz of the fastest rotation pulsar. The inferred $M - R$ curve can be used to confine the mass-radius relations of various compact stars and hence the EOSs. On the other hand, the equation (4) can give $r$-mode instability windows in temperature-frequency plane for the given compact stars. In comparison to the measured frequencies, the windows may pinpoint to the possible compact-star models.

However we below will see the traditional treatments are not always dependable. We will give up the previous idea. As we known, a compact star sequence can be constructed for a given EOS. Let the compact star sequence as the input of equations (3) and (4), we immediately obtain the corresponding frequency-mass relations, $\nu_K - M(R)$ and $\nu_R - M(R)$. Consequently we can find a genuine upper frequency from taking extreme value $\nu_C = \min(\nu_K, \nu_R)$ for a compact star with $M$ (and associated $R$). We impose the measured frequencies on the frequency-mass relations and hence tightly constrain the $\nu_C - M$ relations.

Fig 1 shows the $\nu - M$ relations on the basis of eqs (3) and (4) for typical NS, HpS, HbS (including MC and GC constructions) and SS sequences. They represent the stars which have the maximum rotation frequencies in respective models. Once the rapid rotations of pulsars are measured we can constrain the theoretical $\nu - M$ relations with the rotational frequencies. The observed millisecond pulsars are aimed at this issue. Figure 1 means that the observations only favor the models satisfying the condition $\nu_{obs} \leq \nu \leq \nu_C(M)$. The most models for HpSs and HbSs are compatible with the fastest rotating pulsar found yet, J1748-2446ad. The situation will be very different if higher frequency is regarded as the confined condition. The pulsar over 1000Hz frequency is perhaps decisive. XTE J1739-285 in which Kaaret reported the discovery of a burst oscillation at 1122Hz (Kaaret et al 2007) may be considered a candidate of such pulsar. Above 1122Hz, figure 1 shows that most models are ruled out. Some of only HbSs are possible. In comparison, we also have plotted the corresponding mass-radius relations in Fig. 2. Check Fig 2 against Fig 1, we find the constraints of observable masses on $M - R$ relations, even a supplement of traditional treatment of Keplerian limit (XTE J1739-285 in Fig2) to that, is rather wide or not always dependable. Fig 1 also hints that the traditional treatment of $r$-mode instability window plus Keplerian limit (defined as a fixed value such 1122Hz in Fig1) supports the models with limit frequencies above the fixed value but no available (for example, the dashed curves above 1122Hz but out of the dark green region).

![Figure 1](image1.png)

**Figure 1.** The constraint of the observable frequencies on frequency-mass relations for various models. The corresponding thick lines are the minimum frequencies for various compact star sequences that are taken in our calculations. The inserted figure here is a amplificatory image of the local. We find that most models are allowed by 716Hz rotation of J1748-2446ad but almost all the models couldn’t reach the limit by the 1122Hz of XTE J1739-285 expect the HbS model, which is denoted by the dark green region. The possible range of limit frequencies is so narrow that the possibility for the existence of compact stars with submillisecond period is small.

![Figure 2](image2.png)

**Figure 2.** The corresponding mass-radius relations of compact stars imposed by observations. The dark green contour represents our tight constraint of the masses and radii from Fig 1, which is evidently allowed by mass and causality limits. Although some of NSs, HpSs and SSs are also permitted by mass and simply traditional Keplerian limits denoted by the cyan district, which has been used in past works, they are excluded by the genuine limit frequency at 1122Hz.
4 CONCLUSIONS AND DISCUSSIONS

The EOS and composition of dense matter in the core of compact stars have attracted much attention owing to their importance for our understandings of compact stars. The researchers made great efforts to distinguish various dense matter suggested by nuclear physical theory. The most familiar treatment is the constraint of observable masses on the mass-radius relation of compact stars. We think it not to be always dependable. We present the constraint of observable rotation on the frequency-mass relation of compact stars and hence EOS of the dense matter. Under our considerations, both Keplerian and r-mode instability limits are involved. The constraint has been displayed to advantage, more stringent than that of masses. Especially, the constraint may bring us the decisive judgement if a submillisecond pulsar would be really confirmed in future. We have calculated tens of results but only the typical cases in which hadronic matter is in description of RMF are depicted in the figures. There are slight but nonessential changes when the other EOSs are applied. In addition, although magnetic fields can be important to suppress r-mode instabilities due to which the star may rapidly slow down other than the gravitational wave emission, their effect is probably negligible for frequencies exceeding 0.35 times the mass-shedding limit under the conditions that if the field $B$ is not larger than $10^{16}(\Omega/\Omega_K)$ Gauss, here $\Omega$ and $\Omega_K$ represent the angular velocities of the star and the mass-shedding limit respectively (Rezzolla et al 2000). This couldn’t do any influence on our conclusion about the star rotating above 1000 Hz at all, above which the effect of gravitational wave emission is important. In all, we argue that quark matter has perhaps been indicated by rotations in possible submillisecond pulsars.

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