Minimal classical communication and measurement complexity for quantum information splitting

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Abstract

We present two quantum information splitting schemes using respectively tripartite GHZ and asymmetric W states as quantum channels. We show that if the secret state is chosen from a special ensemble and known to the sender (Alice), then she can split and distribute it to the receivers Bob and Charlie by performing only a single-qubit measurement and broadcasting a one-cbit message. It is clear that no other schemes could possibly achieve the same goal with simpler measurement and less classical communication. In comparison, existing schemes work for arbitrary quantum states which need not be known to Alice; however she is required to perform a two-qubit Bell measurement and communicate a two-cbit message. Hence there is a trade-off between flexibility and measurement complexity plus classical resource. In situations where our schemes are applicable, they will greatly reduce the measurement complexity and at the same time cut the communication overhead by one half.

1. Introduction

Understanding the minimal amount of resources required to implement a task is a fundamental issue in quantum information theory. It has been emphasized that, in addition to quantum resources, it is equally important to take into account the role of classical communication involved [1–3]. A prime example is provided by the novel scheme of quantum teleportation introduced by Bennett \textit{et al} [4] in 1993, which has attracted much attention over the years [5–15]. In this scheme, with the aid of an Einstein–Podolsky–Rosen (EPR) pair shared between two remote sites, an arbitrary unknown quantum state can be teleported from one site to the other without physically transmitting any particles between them. All the senders need to do is to perform a Bell-state measurement and publicly announce the outcome which is a message consisting of two classical bits (cbits). Upon receiving the sender’s message, the receiver can reconstruct the unknown state by executing an appropriate unitary operation on his share of the EPR pair. Quantum teleportation demonstrates the interchangeability of different types of resources [5]. Most importantly it shows that transmitting the information contained in one qubit consumes one entangled bit (ebit) plus two cbits from the sender to the receiver.

In 2000, Lo [1] considered a different but related task called ‘remote state preparation (RSP)’ in which a sender helps a receiver in a remote site to prepare a state chosen from a given ensemble. Such a task is clearly useful in distributed quantum information processing (QIP). The author studied the amount of classical communication cost needed and found that, for certain special ensembles of states, RSP requires a smaller amount of classical communication than teleportation. But he conjectured that for general states the classical communication costs of the two tasks would be equal. At about the same time, Pati [16] considered the minimum number of classical bits needed for RSP and also the complexity of the measurement involved. In 2001, Bennett \textit{et al} [17] proved that, to remotely prepare a large number \((n)\) of qubit states, the asymptotic...
classical communication cost is one ebit per qubit, which is only half that needed in teleportation. This cost can further be reduced when the goal is to transmit only part of a known entangled state. However, there exists no faithful RSP scheme for finite $n$ that always uses less classical communication than teleportation. The protocol of RSP has continued to attract attention from researchers in recent years [18–27].

As we saw, in RSP as well as teleportation, a prior established quantum channel of one ebit linking the two parties is mandatory. However, there is a key difference. Whereas by teleportation the sender can prepare an arbitrary unknown state in a remote site, the state to be prepared must be known to the sender in RSP. Due to this restriction, in RSP the sender only needs to perform a single-qubit measurement and sends a one-ebit message to the receiver. In contrast, the sender must perform a two-qubit Bell measurement and sends two ebits to the receiver in teleportation.

Quantum information splitting (QIS) or quantum state sharing is the quantum generalization of classical secret sharing. The first QIS protocols were proposed by Hillery et al. [28] and Karlsson et al. [29] in 1999. Since then many other protocols have been proposed, and the topic still attracts much attention today [30–41]. In QIS, a piece of quantum information (in the form of a quantum state) is divided and distributed to a number of receivers. Although it is possible to distribute quantum information by sending the qubits directly, for security reason and also to avoid decoherence effects in the physical channel, it is more desirable to do so via pre-established entanglement between the sender and the receivers. Actually in almost all existing schemes, the qubits are sent to individual receivers via teleportation.

We note that in all existing QIS schemes, the state to be shared need not be known to the sender. This is the reason why teleportation is used in the distribution process. However this may be a waste of resources in general. It is reasonable to assume that, in most situations, the sender in a QIS protocol is the owner of the secret information being distributed; hence he/she knows the secret. If this is the case, one could use a RSP (instead of teleportation) scheme to distribute the secret qubits. Clearly this will save the amount of classical communication needed, and at the same time reduced the complexity of the quantum measurement involved.

In this spirit, we consider in this paper two QIS schemes in which the sender knows exactly what the secret quantum information is. Specifically we study tripartite cases where the three legitimate parties share a GHZ or asymmetric W state. The secret quantum information is assumed to be chosen from an ensemble of states located on the equatorial or polar great circle on the Bloch sphere. They will be called ‘equatorial’ and ‘real’ states respectively in the following. Incidentally, the idea in this paper is important for it can be also applied to improve other QIS schemes to reduce the measurement complexity and meanwhile cut the communication overhead, provided that the sender knows the secret quantum information belongs to some special ensembles.

2. Two tripartite QIS schemes

Let the three legitimate parties be Alice, Bob and Charlie. Alice is the initial owner and sender of a qubit state which can be written as

$$|\xi\rangle = \alpha'|0\rangle + \beta|1\rangle. \quad (1)$$

where, without loss of generality, $\alpha'$ is taken to be real and $\beta$ complex, such that $|\alpha'|^2 + |\beta|^2 = 1$. Both $\alpha'$ and $\beta$ are known to Alice, but not to Bob and Charlie. $|\xi\rangle$ can be represented as a point on the Bloch sphere, which is specified by two real angular coordinates $\theta$ and $\phi$, such that $\alpha' = \cos(\theta/2)$ and $\beta = \sin(\theta/2)e^{i\phi}$. If $|\xi\rangle$ is chosen to be on the equatorial circle, then $\theta = \pi/2$ and accordingly $\alpha' = 1/\sqrt{2}$ and $\beta = e^{i\phi}/\sqrt{2}$ [16]; it is called an equatorial state. On the other hand, if the azimuthal angle $\phi = 0$ or $\pi$, then the parameters $\alpha'$ and $\beta$ are both real; it is called a real state. Now Alice wants to split the quantum information/state into two shares and distribute them to the receivers Bob and Charlie, such that the original information can be deterministically recovered only when the two receivers collaborate. Separately, each individual receiver should have no knowledge of the secret. In the following we will present two new schemes to achieve this goal. The quantum channel linking the three parties will be taken to be a GHZ state and asymmetric W state, respectively.

2.1. QIS with a shared GHZ state

In this subsection we present a QIS scheme when the shared quantum channel is a GHZ state. For short, this scheme will be referred to as an ZC1 scheme hereafter. In the beginning, Alice, Bob and Charlie own respectively qubits $a$, $b$ and $c$, which are in a GHZ state:

$$|\Psi\rangle_{abc} = \frac{1}{\sqrt{2}}(|000\rangle_{abc} + |111\rangle_{abc}). \quad (2)$$

Since Alice knows her quantum information exactly, she can perform a single-qubit measurement on her qubit $a$ in the basis $\{|\xi\rangle, |\xi^\perp\rangle\}$, where $|\xi^\perp\rangle = \beta^*|0\rangle - \alpha'|1\rangle$. $|\Psi\rangle_{abc}$ can be rewritten as

$$|\Psi\rangle_{abc} = \frac{1}{\sqrt{2}}(|\xi\rangle_{a}(\alpha'|0\rangle_{bc} + \beta^*|1\rangle_{bc})$$
$$+ |\xi^\perp\rangle_{a}(\beta|0\rangle_{bc} - \alpha'|1\rangle_{bc}). \quad (3)$$

If she gets $|\xi\rangle_{a}$, she publishes a single cbit ‘0’; otherwise, she publishes ‘1’. This classical bit informs Bob and Charlie the collapsed state of their two qubits after Alice’s measurement. 0 corresponds to the state $\alpha'|0\rangle_{bc} + \beta^*|1\rangle_{bc}$ while 1 to $\beta|0\rangle_{bc} - \alpha'|1\rangle_{bc}$.

Without loss of generality, we will assume that the secret quantum information is to be reconstructed with qubit $c$ at Charlie’s site. In the case that Bob and Charlie get the state $\beta|0\rangle_{bc} - \alpha'|1\rangle_{bc}$ (the probability is 1/2), they can collaborate to recover the quantum information as follows. Bob performs a single-qubit measurement in the $X$ basis $|\pm x\rangle = 1/\sqrt{2}(|0\rangle \pm |1\rangle)$ on his qubit $b$ and informs Charlie of his result. Since,

$$\beta|0\rangle_{bc} - \alpha'|1\rangle_{bc} = \frac{1}{\sqrt{2}}(|+x\rangle_{b}(\sigma_z^b\sigma_z^c)|\xi\rangle_{c}$$
$$+ |-x\rangle_{b}(\sigma_z^c)|\xi^\perp\rangle_{c}, \quad (4)$$
where \( \sigma_x \equiv |11\rangle\langle 00 | - |00\rangle\langle 11 | \) and \( \sigma_z \equiv |00\rangle\langle 00 | - |11\rangle\langle 11 | \) are Pauli operators. It is easy to see that knowing Bob’s result, Charlie can recover the quantum information by performing an appropriate unitary operation on his qubit \( c \). Explicitly, corresponding to Bob’s outcomes \( |+\rangle_b \) and \( |-\rangle_b \), Charlie performs the unitary operation \( \sigma_x \sigma_z \) and \( \sigma_z \) respectively on his qubit \( c \). In this case, the quantum information is successfully shared and reconstructed.

Now if Bob and Charlie get the state \( \alpha'|00\rangle_{bc} + \beta'|11\rangle_{bc} \) instead (again with probability 1/2), then in general they cannot recover the quantum information faithfully by any means. However, if the secret quantum information is known to be chosen from a special ensemble of equatorial or real qubits states, then the reconstruction becomes possible. (i) As mentioned before, \( \theta = \pi/2 \) for equatorial states, such that \( |\xi\rangle = (|0\rangle + e^{i\theta}|1\rangle)/\sqrt{2} \) (i.e., \( \alpha' = 1/\sqrt{2} \) and \( \beta = e^{i\theta}/\sqrt{2} \)). In this case, the state \( \alpha'|00\rangle_{bc} + \beta'|11\rangle_{bc} \) becomes \((|0\rangle_{bc} + e^{-i\theta}|1\rangle_{bc})/\sqrt{2} \). Bob and Charlie can recover the quantum information from this state as follows. Bob carries out a single-qubit measurement in the X basis on his qubit \( b \) and tells Charlie his result. Since

\[
|00\rangle_{bc} + e^{-i\theta}|11\rangle_{bc} = e^{-i\theta}|+\rangle_b (\sigma_x \sigma_z)^{1/2}|\xi\rangle_c \\
+ |-\rangle_b (\sigma_x \sigma_z)^{1/2}|\xi\rangle_c, \tag{5}
\]

it is then clear that knowing Bob’s result, Charlie can recover the quantum information except for an unimportant overall phase factor by performing an appropriate unitary operation. Explicitly, corresponding to Bob’s measurement results \( |+\rangle_b \) and \( |-\rangle_b \), Charlie performs respectively the unitary operation \( \sigma_x \sigma_z \) and \( \sigma_z \) on qubit \( c \) in his possession. (ii) When Alice’s secret quantum information is a real qubit state (i.e., both \( \alpha' \) and \( \beta \) are real), then \( \alpha'|00\rangle_{bc} + \beta'|11\rangle_{bc} \) becomes \( \alpha'|00\rangle_{bc} + \beta|11\rangle_{bc} \), which is in essence the redundant state of the original quantum information. Since

\[
\alpha'|00\rangle_{bc} + \beta|11\rangle_{bc} = \frac{1}{\sqrt{2}}(|+\rangle_b |\xi\rangle_c + |-\rangle_b (\sigma_x \sigma_z)^{1/2}|\xi\rangle_c), \tag{6}
\]

Bob and Charlie can again reconstruct Alice’s secret quantum information, using the procedure described for case (i).

Hence, given a quantum channel of a GHZ state shared among the three parties, and if the quantum information (known to Alice) is chosen from a special ensemble of equatorial or real states, then our new QIS protocol requires only a single-qubit measurement and one cbit of classical communication.

Lastly we examine the case where the shared three-qubit state is not maximally entangled. Let the shared state \( |\Psi\rangle_{abc} \) be given by

\[
|\Psi\rangle_{abc} = a|000\rangle_{abc} + b|111\rangle_{abc}, \tag{7}
\]

where \( a \) and \( b \) are real (if not, they can be made real by a simple unitary rotation) and \( a \neq b \). Intuitively, as for teleportation, one would expect that in this case Bob and Charlie can still recover the secret qubit, but with a probability less than unity. Indeed, following [42], it is straightforward to show that the recovery probability is 2\( a^2b^2 \).

### 2.2. QIS with a shared asymmetric W state

Here we present another new QIS scheme when the shared quantum channel is an asymmetric W state. It will be referred to as the ZC2 scheme hereafter. As before, Alice, Bob and Charlie respectively own qubits \( a, b, \) and \( c \); and the three qubits are in an asymmetric W state given by

\[
|\Phi\rangle_{abc} = \frac{1}{\sqrt{2}}|001\rangle_{abc} + \frac{1}{\sqrt{2}}|100\rangle_{abc}. \tag{8}
\]

It is easy to show that this state can be rewritten as

\[
|\Phi\rangle_{abc} = \frac{1}{\sqrt{2}}|\xi\rangle_a \left[ \sqrt{2} (|01\rangle_{bc} + |10\rangle_{bc}) + \beta^*|00\rangle_{bc} \right] \\
- \frac{1}{\sqrt{2}}|\xi\rangle_a \Omega_{bc} \left[ |0\rangle_b (\sigma_z^c)^{1/2}|\xi\rangle_c \right], \tag{9}
\]

where

\[
\Omega = |00\rangle\langle 01| + |11\rangle\langle 10| + \frac{1}{\sqrt{2}}(|01\rangle\langle 01| + |10\rangle\langle 10|). \tag{10}
\]

From this equation, we see that, if Alice performs a single-qubit measurement on qubit \( a \) in the basis \( \{ |\xi\rangle, |\xi^*\rangle \} \), then the state of qubits \( b \) and \( c \) will collapse to either \( \alpha'/\sqrt{2}(|01\rangle_{bc} \pm |10\rangle_{bc}) + \beta^*|00\rangle_{bc} \) or \( \Omega_{bc} (|0\rangle_b (\sigma_z^c)^{1/2}|\xi\rangle_c) \). Bob and Charlie know what the state is if Alice announces her measurement result by broadcasting a one-cbit message as described earlier. In the case of \( \Omega_{bc} \), Bob and Charlie can collaborate and recover the quantum information by first performing the unitary operation \( \Omega \) on their qubits \( b \) and \( c \). Subsequently Charlie applies the unitary operation \( \sigma_z^c \) on his qubit \( c \) and obtains \( |\xi\rangle_c \). On the other hand, if the collapsed state is \( \alpha'/\sqrt{2}(|01\rangle_{bc} \pm |10\rangle_{bc}) + \beta^*|00\rangle_{bc} \), then in general, Bob and Charlie cannot work together to recover the quantum information, because the general complex coefficient \( \beta^* \) cannot be converted into \( \beta \) via an unitary operations. However, as before, if the secret quantum information is chosen from a special ensemble of equatorial or real states, then recovery again becomes possible. (I) If \( |\xi\rangle \) is an equatorial state, i.e., \( \alpha' = 1/\sqrt{2} \) and \( \beta = e^{i\theta}/\sqrt{2} \), then \( \alpha'/\sqrt{2}(|01\rangle_{bc} + |10\rangle_{bc}) + \beta^*|00\rangle_{bc} \) becomes \( 1/\sqrt{2}(|01\rangle_{bc} + |10\rangle_{bc}) + e^{-i\theta}/\sqrt{2}|00\rangle_{bc} \). Bob and Charlie can reconstruct \( |\xi\rangle \) at Charlie’s site by performing the joint unitary operation \( \Omega \) on their qubits \( b \) and \( c \). The resultant state is given by \( 1/\sqrt{2}e^{-i\theta}|0\rangle_b (e^{i\theta}|1\rangle_c + |0\rangle_c) = 1/\sqrt{2}e^{-i\theta}|0\rangle_b |\xi\rangle_c \), indicating that Charlie now holds the secret quantum information in his hand. (II) If \( |\xi\rangle \) is a real state, i.e., both \( \alpha' \) and \( \beta \) are real, then \( \alpha'/\sqrt{2}(|01\rangle_{bc} + |10\rangle_{bc}) + \beta^*|00\rangle_{bc} \) becomes \( \alpha'/\sqrt{2}(|01\rangle_{bc} + |10\rangle_{bc}) + \beta|00\rangle_{bc} \). Bob and Charlie can recover the quantum information by first performing the unitary operation \( \Omega \) on their qubits \( b \) and \( c \) together. Then Charlie can recover the quantum information by applying the unitary operation \( \sigma_z \) on his qubit \( c \).

Hence again, we have shown that, to split a qubit chosen from a special ensemble (equatorial or real) and known to Alice, she only needs to perform a single-qubit measurement and publish one cbit, provided that the three parties share an asymmetric tripartite W state.

Finally we examine the case where the shared three-qubit state is not maximally entangled. Following the above
procedure, we found that if the shared state $|\Phi\rangle_{abc}$ is of the form given by

$$|\Phi\rangle_{abc} = a|001\rangle_{abc} + b|010\rangle_{abc} + c|100\rangle_{abc},$$  \hspace{1cm} (11)$$

where $a \neq b \neq c$, then it is in general not possible for Bob and Charlie to recover the original secret qubit. In the special case of $a = b \neq c$, our procedure works if $c = \sqrt{2}a$, which however just brings $|\Phi\rangle_{abc}$ back to the original asymmetric W state $|\Phi\rangle_{abc}$.

### 3. Comparisons and discussions

In the last section we have proposed two QIS schemes, ZC1 and ZC2, using respectively a tripartite GHZ state and an asymmetric W state as the quantum channel. Here we compare them with two other QIS schemes, the HBB scheme [28] and the Zheng scheme [39], using respectively the same quantum channels as ZC1 and ZC2.

As we shall see, in ordinary QIS schemes such as those of HBB and Zheng, Alice must make a Bell state measurement and send two cbits to Bob and Charlie. In the schemes we have proposed, however, Alice needs only to make a single-qubit measurement and announce a one-bit classical information. With these simplifications, our schemes are subjected to two restrictions: (1) Alice must know the qubit being sent, and (2) the secret qubit must be a member of a special ensemble which is public knowledge. In most practical situations, the sender Alice is the owner of the secret qubit, so the first restriction is not a trade-off in most cases. With regard to the second restriction, the secret information in our schemes must be in the form of either an equatorial or real qubit, which is characterized by a single parameter, $\phi$ or $\theta$. In contrast, ordinary QIS schemes work for an arbitrary secret qubit characterized by two parameters $\phi$ and $\theta$. This loss of freedom is a trade-off that we have to make in our schemes.

Finally, we also discuss the case of Alice withholding the classical information as a control [43].

#### 3.1. Comparison of the ZC1 scheme with the HBB scheme

In 1999, Hillery, Bûzek and Berthiaume [28] first presented a QIS scheme (HBB scheme) using GHZ states as the quantum channel. In the HBB QIS scheme, Alice’s secret quantum information to be shared between Bob and Charlie is given by

$$|u\rangle_x = \alpha|0\rangle_x + \beta|1\rangle_x,$$  \hspace{1cm} (12)$$

where $\alpha$ and $\beta$ are complex and satisfy $|\alpha|^2 + |\beta|^2 = 1$. In addition, Alice owns another qubit $a$ which forms a GHZ state with Bob’s qubit $b$ and Charlie’s qubit $c$. The combined four-qubit state is given by

$$|\Lambda\rangle_{xabc} = |u\rangle_x \otimes |\Psi\rangle_{abc},$$  \hspace{1cm} (13)$$

which can be rewritten as

$$|\Lambda\rangle_{xabc} = \frac{1}{2}(|\psi^+\rangle_{xaxb} (\alpha|00\rangle_{bc} + \beta|11\rangle_{bc})$$

$$+ |\psi^-\rangle_{xaxb} (\alpha|00\rangle_{bc} - \beta|11\rangle_{bc})$$

$$+ |\phi^+\rangle_{xaxb} (\beta|00\rangle_{bc} + \alpha|11\rangle_{bc})$$

$$+ |\phi^-\rangle_{xaxb} (\beta|00\rangle_{bc} - \alpha|11\rangle_{bc})|u\rangle_x.$$

Hence again Alice can split the quantum information and distribute the shares to Bob and Charlie by performing a two-qubit Bell measurement and publishing the outcome in the form of a two cbits message. With this message, Bob can obtain the secret information shared by Alice and Charlie. This loss of freedom is a trade-off that we have to make in our schemes.

#### 3.2. Comparison of the ZC2 scheme with the Zheng scheme

In 2006, Zheng [39] presented a QIS scheme (Zheng scheme) using an asymmetric W state as shown in the equation (8). This scheme also works for arbitrary secret quantum information as shown in equation (12). In this case, the joint state of Alice’s two qubits $x$ and $a$, Bob’s qubit $b$ and Charlie’s qubit $c$ is given by

$$\Gamma_{xabc} = |u\rangle_x|\Phi\rangle_{abc}$$

$$= (|\alpha|0\rangle_x + \beta|1\rangle_x) \left(\frac{1}{2}|001\rangle_{abc} + \frac{1}{2}|010\rangle_{abc} + \frac{1}{2}\sqrt{6}|100\rangle_{abc}\right),$$  \hspace{1cm} (16)$$

It can also be rewritten as

$$\Gamma_{xabc} = \frac{1}{2}|\psi^+\rangle_{xaxb} \Omega_{0b} (\alpha|0\rangle_{ac} + \beta|1\rangle_{ac})$$

$$+ \frac{1}{2}|\psi^-\rangle_{xaxb} \Omega_{0b} (\alpha|0\rangle_{ac} - \beta|1\rangle_{ac})$$

$$+ \frac{1}{2}|\phi^+\rangle_{xaxb} \Omega_{0b} (\alpha|0\rangle_{ac} + \beta|1\rangle_{ac})$$

$$+ \frac{1}{2}|\phi^-\rangle_{xaxb} \Omega_{0b} (\alpha|0\rangle_{ac} - \beta|1\rangle_{ac}).$$  \hspace{1cm} (17)$$

Therefore, Alice can split the quantum information and have it shared by Bob and Charlie by first performing a two-qubit Bell measurement on qubits $x$ and $a$, and then announcing the outcome as a two cbits message. Knowing Alice’s outcome, Bob and Charlie can easily recover the quantum information via a similar procedure as already described.

Note that, since the state $|\alpha\rangle_x$ in equation (12) is arbitrary, the HBB scheme can be used for splitting any single-qubit quantum information including the equatorial and real states. However, independent of the nature of the quantum information to be shared, the HBB requires the sender Alice to perform a Bell measurement and publish two cbits. It is clear that, in the case of equatorial or real states, this is a waste of resources. As we have demonstrated in subsection 2.1, given the same GHZ channel, an equatorial or real qubit can be split and shared using only a single-qubit measurement and one cbits of classical communication. Hence, for these special qubit states, our ZC1 scheme not only decreases the measurement complexity, it also reduces the required classical communication by one half.
and Charlie can then collaborate and reconstruct the quantum information as described previously.

As in the HBB scheme, this scheme works for arbitrary qubit states, and it requires the sender Alice to perform a Bell state measurement and broadcast the outcome as a two-qubit message. In comparison, our new scheme ZC2 works for equatorial and real states only. This restriction is however compensated by the fact that Alice needs to make only a single-qubit measurement, and broadcast a one-cbit message. Hence, in cases where the secret state is chosen from the ensemble of equatorial or real states, and it is known to Alice, the ZC2 scheme is more preferable than the Zheng scheme.

3.3. Controlled quantum information splitting

In ZC1, if the secret quantum information is an equatorial state, then the secret resides in the phase $\phi$. After Alice announces her one-cbit message, Bob or Charlie has absolutely no information of the secret if they do not collaborate. This can be seen from the fact that the density matrices of the qubits in their hands are both $1/2$ which is independent of $\phi$. In all other cases discussed, the density matrices of Bob’s and Charlie’s qubits are not $1/2$. That means each of them has some information of the secret even if they do not collaborate. This is undesirable in a QIS scheme.

This drawback can be avoided if Alice delays her classical communication. A similar scenario was first proposed by Cheung in 2006 [43]. The point is, in ordinary QIS schemes, the sender of the secret has no control over the use of the secret state after it is distributed. That is, the receivers can come together and reconstructed it any time they want. In a controlled quantum information splitting (or secret sharing) scheme, by withholding the classical communication, the sender (or controller) can decide when the secret information is to be recovered by the receivers. Before that, the receivers have absolutely no knowledge of the secret information, and it is impossible for them to recover the secret even if they want to.

Therefore, except for the case of ZC1 with equatorial qubits, Alice should not announce the one-cbit message right after her measurement. Instead, she should withhold the information until the time when she wants Bob and Charlie to reconstructed the secret state. Then it is clear that Bob and Charlie separately knows nothing about the secret before they start the reconstruction process. It is important to note that here the delay of the classical communication is not merely an option, but is necessary to protect the secret information from leaking to Bob and Charlie separately.

4. Summary and conclusion

To summarize, we have proposed two QIS schemes (ZC1 and ZC2) using respectively GHZ and asymmetric W states as quantum channels. The scheme works for qubits chosen from a special ensemble of equatorial or real states, moreover the state should be known to the sender Alice. In these new schemes, Alice is only required to perform a single-qubit measurement and broadcast a one-cbit message. Clearly no other scheme can possibly achieve the same goal with simpler measurement and less classical communication. In the existing QIS schemes (HBB [28] and Zheng [39]) using the same quantum channels, Alice must make a two-qubit joint measurement and announce a two-cbit message. In return for the more complex measurement and greater amount of classical communication, these schemes work for arbitrary quantum states which need not be known to the sender Alice. However it is reasonable to assume that, in most practical situations, Alice is the owner of the secret and so she knows what the secret is. Furthermore if the secret state belongs to the special ensemble of equatorial or real states, then our schemes are clearly more desirable.

With regard to experimental implementations, the main difficulty lies in the establishment of a three-qubit entangled state among three remote sites. While GHZ states have been observed in laboratories [44–46], creating and maintaining remote entanglements is still something to be desired. This obstacle is however common to our schemes as well as others. Nevertheless, as we saw, our schemes require only single-qubit measurements which, with present date technologies, are immensely simpler and more efficient to perform than two-qubits ones. In this regard, our schemes are relatively much easier to realize experimentally. Note also that, in order to be able to recover the secret qubit, in general Bob and Charlie must keep their qubits entangled after Alice’s measurement. This is however not necessarily if the initial state is a GHZ state. In this case, since Bob needs to make a measurement in the $\{ |\pm x \rangle \}$ basis independent of Alice’s outcomes, he could do it anytime but announces his result only when he wants Charlie to recover the secret qubit. Then there is no need to maintain the entanglement between Bob and Charlie for an indefinite length of time.

Finally we discussed the case of Alice withholding the classical information as a control [43]. For ZC1 with equatorial states, this is an option which allows Alice to decide if and when Bob and Charlie should start the reconstruction process. For all other cases, however, delaying the announcement of the classical information is also necessary to ensure that, before Bob and Charlie start to work together to reconstruct the secret qubit, each of them has no knowledge of the secret whatsoever.

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