On the Combined Effect of Directional Antennas and Imperfect Spectrum Sensing upon Ergodic Capacity of Cognitive Radio Systems

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Abstract—We consider a cognitive radio system, consisting of a primary transmitter (PU\textsubscript{tx}), a secondary transmitter (SU\textsubscript{tx}), and a secondary receiver (SU\textsubscript{rx}). The secondary users (SUs) are equipped with steerable directional antennas. We assume the SUs and primary users (PUs) coexist and the SU\textsubscript{tx} knows the geometry of network. We find the ergodic capacity of the channel between SU\textsubscript{tx} and SU\textsubscript{rx}, and study how spectrum sensing errors affect the capacity. In our system, the SU\textsubscript{tx} first senses the spectrum and then transmits data at two power levels, according to the result of sensing. The optimal SU\textsubscript{tx} transmit power levels and the optimal directions of SU\textsubscript{tx} transmit antenna and SU\textsubscript{rx} receive antenna are obtained by maximizing the ergodic capacity, subject to average transmit power and average interference power constraints. To study the effect of fading channel, we considered three scenarios: 1) when SU\textsubscript{tx} knows the geometry of network, and the statistics of the other two links, and, 3) when SU\textsubscript{tx} only knows the statistics of these three fading channels. For each scenario, we explore the optimal SU\textsubscript{tx} transmit power levels and the optimal directions of SU\textsubscript{tx} and SU\textsubscript{rx} antennas, such that the ergodic capacity is maximized, while the power constraints are satisfied.

I. INTRODUCTION

Cognitive radio (CR) systems can alleviate spectrum scarcity problem by allowing an unlicensed user to access licensed bands under the condition that its imposed interference on the licensed users are limited [1]. Optimizing the transmission strategy of secondary users (SUs) in the presence of a primary user (PU) has attracted much research interests in industry and academia [2]–[10], where most of these works assume the SUs are equipped with omni-directional antennas and the result of spectrum sensing is perfect. However, spectrum sensing methods are prone to errors and their false alarm and detection probabilities should be incorporated in the design and performance analysis. Different from the bulk of the literature, in this paper we assume the SUs and PUs can coexist, the SU\textsubscript{tx} knows the geometry of network. Also, SUs are equipped with steerable directional antennas and can use spatial spectrum holes [11]–[13] to increase spectrum utilization.

In this work, the SU transmitter (SU\textsubscript{tx}) first senses the spectrum and then adapts its transmit power, according to the result of spectrum sensing, i.e., SU\textsubscript{tx} transmits signal to secondary receiver (SU\textsubscript{rx}) with power levels \(P_0\) and \(P_1\) when spectrum is sensed idle and busy, respectively. To study the effect of fading channels, we consider three scenarios: 1) when SU\textsubscript{tx} has channel state information (CSI) of links between SU\textsubscript{tx} and PU\textsubscript{tx}, PU\textsubscript{tx} and SU\textsubscript{rx}, SU\textsubscript{tx} and SU\textsubscript{rx}, 2) when SU\textsubscript{tx} knows only the CSI of link between SU\textsubscript{tx} and SU\textsubscript{rx}, and the statistics of the other two links, and, 3) when SU\textsubscript{tx} only knows the statistics of these three fading channels. For each scenario, we establish the ergodic capacity of the channel between SU\textsubscript{tx} and SU\textsubscript{rx}, when spectrum sensing is imperfect and find the optimal directions of SU\textsubscript{tx} and SU\textsubscript{rx} antennas and optimal SU\textsubscript{tx} power levels such that the ergodic capacity is maximized, subject to average transmit power and average interference power constraints.

II. SYSTEM MODEL

Our CR system model is shown in Fig. 1. The SUs are equipped with steerable directional antennas. The orientation of PU\textsubscript{tx} and SU\textsubscript{tx} with respect to SU\textsubscript{tx} are denoted by \(\theta_{\text{p}}\) and \(\theta_{\text{s}}\), receptively, and the orientation of PU\textsubscript{rx} with respect to SU\textsubscript{tx} is denoted by \(\theta'_{\text{p}}\). The boresight of SU\textsubscript{tx} and SU\textsubscript{rx} antennas in their local coordination are denoted by \(\phi_{\text{a}}\) and \(\phi_{\text{r}}\), respectively. We assume \(\theta_{\text{p}}\), \(\theta_{\text{s}}\) and \(\theta'_{\text{p}}\) are known or can be estimated [14]. The antenna gain is given by \(A(\phi) = A_1 + A_0 \exp (-B(\frac{\phi}{\phi_{\text{b}}})^3)\) where \(B = \ln(2)\), \(\phi_{\text{b}}\) is the half-power beam-width, \(A_1\) and \(A_0\) are two constant parameters [12], [13]. Let \(d_{\text{ps}}, d_{\text{sp}}\) and \(d_{\text{ss}}\) be the distances between PU\textsubscript{tx} and SU\textsubscript{tx}, PU\textsubscript{tx} and SU\textsubscript{rx}, and SU\textsubscript{tx} and SU\textsubscript{rx}, respectively.
Spectrum sensing at the SU tx can be formulated as a binary hypothesis testing problem in which \( H_0 \) and \( H_1 \) with prior probabilities \( \pi_0 \) and \( \pi_1 \) denote the spectrum is truly idle and truly busy, respectively. When the spectrum is truly busy, the average transmit power of PUtx is \( P_t \), and we assume SUtx knows \( P_t \). Let \( \mathcal{H}_0 \) and \( \mathcal{H}_0 \) with probabilities \( \hat{P}_0 = \Pr\{\mathcal{H}_0\} \) and \( \hat{P}_1 = \Pr\{\mathcal{H}_1\} \) denote that the result of spectrum sensing is busy and idle, respectively. When the spectrum is sensed idle and busy, SUtx uses two power levels \( P_0 \) and \( P_1 \), respectively, to transmit signal to SUtx. The accuracy of spectrum sensing method is characterized by false alarm probability \( P_f = \Pr\{\mathcal{H}_1|\mathcal{H}_0\} \) and detection probability \( P_d = \Pr\{\mathcal{H}_0|\mathcal{H}_1\} \). We assume \( \pi_0 \), \( P_0 \), \( P_1 \) are known.

The fading from SUtx to SUrx, and PUtx to SUtx are denoted by \( g_{ss} \) and \( g_{ps} \), respectively, and \( g_{sp} \) is the fading from SUtx to PUtx. We assume \( g_{ss} \), \( g_{sp} \) and \( g_{sp} \) are three independent exponentially distributed random variables with means \( \gamma_{ss} \), \( \gamma_{ps} \) and \( \gamma_{sp} \), respectively. The path-loss is \( L = (d_0/d)^\nu \), where \( d_0 \) is the reference distance, \( d \) is the distance between users, and \( \nu \) is the path loss exponent. Our goal is to find the ergodic capacity of the channel between SUtx and SUrx, and explore the optimal SU transmit power levels and the optimal directions of SUtx and SUrx antennas, such that this capacity maximized, subject to average transmit power and average interference power constraints.

### III. Constrained Ergodic Capacity Maximization

When spectrum sensing is imperfect, depending on the true status of the PU and the spectrum sensing result, the ergodic capacity can be written as \( C = E_g\left\{ \sum_{i=0}^{1} \left( c_{i0,i} + \beta_i \right) \right\} \), where \( E_g\{\cdot\} \) is the expectation operator over random fading coefficients \( g_{ss}, g_{sp}, g_{sp} \) and \( c_{i,j} = \text{instantaneous capacity corresponding to} \mathcal{H}_i \text{ and} \mathcal{H}_j \text{ with probability} \alpha_i = \Pr\{\mathcal{H}_i, \mathcal{H}_j\} \text{ and} \beta_i = \Pr\{\mathcal{H}_i, \mathcal{H}_j\} \text{ for} i \in \{0, 1\} \), given as

\[
  c_{0,i} = \log_2 \left( 1 + \frac{g_{ss}L_{gs} G(\theta, \phi_x, \phi_y) P_t(g)}{\sigma_n^2} \right),
\]

\[
  c_{1,i} = \log_2 \left( 1 + \frac{g_{ss}L_{gs} G(\theta, \phi_x, \phi_y) P_t(g)}{\sigma_n^2 + P_t g_{ps} L_{ps} A(\theta_x - \theta_y)} \right).
\]

In (1) and (2), \( G(\theta, \phi_x, \phi_y) = A(\phi_x - \theta)A(\phi_y - \pi - \theta) \) is the product of SUtx and SUrx antennas’ gain and \( \sigma_n^2 \) is the variance of additive zero-mean Gaussian noise at SUtx. It is easy to verify

\[
  \alpha_0 = \pi_0 (1 - P_f), \quad \alpha_1 = \pi_0 P_f, \\
  \beta_0 = \pi_1 (1 - P_d), \quad \beta_1 = P_d.
\]

Note that the optimal antenna directions \( \phi_x \) and \( \phi_y \) are dropped parameter \( g \) and for simplicity, we are dropped parameter \( g \). Also, for simplicity of presentation, we drop the parameters \( \theta, \phi_x, \phi_y \) from \( G(\theta, \phi_x, \phi_y) \) and define \( a = g_{ss}L_{gs} G(\theta, \phi_x, \phi_y) \text{ and} \sigma_n^2 = P_t g_{ps} L_{ps} A(\phi_y - \theta)_y \). The term \( \sigma_n^2 \) captures the interference on SUtx due to PU activity. Then, we can rewrite (1) and (2) as

\[
  c_{0,i} = \log_2 \left( 1 + \frac{P_t(g)}{a \sigma_n^2} \right) \text{ and} \]

\[
  c_{1,i} = \log_2 \left( 1 + \frac{P_t(g)}{a \sigma_n^2 + a \sigma_n^2} \right),
\]

respectively.

Let \( I_{ss} \) indicate the maximum allowed interference power of PUtx and \( I_{av} \) denote the maximum allowed average transmit power of SUtx. To satisfy the average interference power constraint, we have

\[
  E_g \left\{ \left( \beta_0 P_0(g) + \beta_1 P_1(g) \right) g_{sp} L_{sp} A(\theta - \theta_p) \right\} \leq I_{av}.
\]

By defining \( \beta_0 = \beta_1 g_{sp} L_{sp} A(\theta - \theta_p) \), (3) can be written as

\[
  E_g \left\{ b_0 P_0(g) + b_1 P_1(g) \right\} \leq I_{av}.
\]

In (4), \( b_0 P_0(g) \) and \( b_1 P_1(g) \) denote the imposed interference to PUtx from SUtx when channel is sensed idle and busy, respectively. To satisfy the average transmit power constraint, we have

\[
  E_g \left\{ \hat{\pi}_0 P_0(g) + \hat{\pi}_1 P_1(g) \right\} \leq P_{av}.
\]

The problem we consider is maximizing the ergodic capacity \( C \) over \( P_0(g), P_1(g), \phi_x \) and \( \phi_y \) subject to constraints (4) and (5). The expression \( C \) is concave with respect to \( P_0(g), P_1(g) \) and \( \phi_x \). However, it is not concave with respect to \( \phi_y \). The optimal \( \phi_y \) can be obtained using one-dimensional search, i.e., we consider an initial value for \( \phi_y \) and find \( P_0(g), P_1(g) \) and \( \phi_y \). Then, we find the value of \( \phi_x \) which maximizes \( C \). Given \( \phi_y \), we can solve this problem using the Lagrange multipliers method to find \( P_0(g), P_1(g) \) and \( \phi_x \). The Lagrangian is given as

\[
  L = E_g \left\{ \sum_{i=0}^{1} \left( \alpha_i c_{i0,i} + \beta_i c_{i1,i} \right) \right\} + \lambda \left( E_g \left\{ \hat{\pi}_0 P_0(g) + \hat{\pi}_1 P_1(g) \right\} - I_{av} \right) + \mu \left( E_g \left\{ b_0 P_0(g) + b_1 P_1(g) \right\} - P_{av} \right).
\]

where \( \lambda \) and \( \mu \) are nonnegative Lagrange multipliers. In the following subsections, we address this constrained maximization problem when 1) SUtx knows perfect CSI of \( g \), 2) when SUtx knows only \( g_{ss} \) and statistics of \( g_{ps} \) and \( g_{sp} \), 3) when SUtx only knows the statistics of \( g \).

### A. Perfect CSI for Three Fading Channels

In the first scenario, we assume \( \text{SUtx} \) has perfect knowledge of \( g_{ss}, g_{ps} \) and \( g_{sp} \) and it maximizes the capacity for each realization of fading coefficients. Taking the derivative of Lagrangian in (6) with respect to \( P_i(g) \) and equaling it to zero gives

\[
  \frac{\partial L}{\partial P_i(g)} = \frac{-a}{\sigma_n^2 \ln(2)} w_i(x, y) + \lambda \hat{\pi}_1 + \mu b_i = 0
\]

where \( y \equiv \sigma_n^2 / \sigma_p^2, x_i \equiv \alpha_i / \alpha_p P_i(g) \) and

\[
  w_i(x, y) = x \left( \frac{\alpha_i}{x + 1} + \frac{\beta_i y}{x y + x + y} \right).
\]

Also, \( x^{-1} \) and \( y^{-1} \) are the received signal-to-noise-ratio (SNR) and interference-to-noise-ratio (INR) at SUtx. By solving (7), the optimal transmit power levels can be written as

\[
  P_i(g) = \frac{F_i + \sqrt{\Delta_i}}{2}
\]

for \( i = 0, 1 \)

where \( [x]^+ \) denotes \( \max(x, 0) \) and

\[
  F_i = \frac{\hat{\pi}_i}{\ln(2) (\hat{\pi}_i + \mu b_i)} - \frac{2\sigma_n^2 + \sigma_p^2}{\alpha}
\]
\[ \Delta_i = P_i^2 - 4 \left( \frac{\sigma_\alpha^2 \sigma_\theta^2 \sigma_\phi^2}{\alpha} - \frac{\hat{\pi}_i \sigma_\theta^2 + \beta \sigma_\phi^2}{\ln(2) (\lambda \tilde{x}_i + \mu \bar{b}_i)} \right). \]

The Lagrange multipliers \( \lambda \) and \( \mu \) can be updated using the subgradient method as follows [5]

\[
\lambda^{(n+1)} = \left[ \lambda^{(n)} + t_0 \left( \mathbb{E}_g \left\{ \tilde{\pi}_0 P_0 \left( g \right) + \tilde{\pi}_1 P_1 \left( g \right) \right\} - \bar{P}_\lambda \right) \right]^{+} \quad (9a)
\]

\[
\mu^{(n+1)} = \left[ \mu^{(n)} + t_0 \left( \mathbb{E}_g \left\{ b_1 P_0 \left( g \right) + b_1 P_1 \left( g \right) \right\} - I_\mu \right) \right]^{+} \quad (9b)
\]

where \( t_0 \) is the step size and \( \lambda \) and \( \mu \) converge when for a small number \( \delta \) we get

\[
\lambda^{(n)} \left( \mathbb{E}_g \left\{ \tilde{\pi}_0 P_0 \left( g \right) + \tilde{\pi}_1 P_1 \left( g \right) \right\} - \bar{P}_\lambda \right) \leq \delta \quad (10a)
\]

\[
\mu^{(n)} \left( \mathbb{E}_g \left\{ b_1 P_0 \left( g \right) + b_1 P_1 \left( g \right) \right\} - I_\mu \right) \leq \delta. \quad (10b)
\]

The optimal \( \phi_\pi \) can be obtained by solving \( \partial \mathcal{L} / \partial \phi_\pi = 0 \). There is no closed form solution for \( \phi_\pi^{\text{opt}} \), but one can verify that when transmit power of PU\(_\text{tx} \) is zero (\( P_0 = 0 \)), \( \phi_\pi^{\text{opt}} = \pi + \theta \). We can reduce the computational complexity of one-dimensional search for finding \( \phi_\pi^{\text{opt}} \) by finding a narrower interval to which \( \phi_\pi^{\text{opt}} \) belongs to [13]. We define

\[
Z = \frac{I_{\bar{P}}}{\pi_1 g_{sp} A_0} - \frac{A_1}{A_0} \quad (11)
\]

If \( Z \) is greater than one, it means that PU\(_\text{tx} \) can tolerate an interference power that is larger than the interference power imposed by SU\(_\text{tx} \). Constraint (4) is loose and \( \phi_\pi^{\text{opt}} = \theta \). When \( 0 < Z \leq 1 \), we define \( \psi_\pi = \phi_\text{loss} \frac{1}{\pi_1} \ln(Z) \) and consider two cases. When \( \left| \theta - \theta \right| > \psi_\pi \), \( \phi_\pi^{\text{opt}} \) has to lie outside the shaded area shown in Fig. 2a. Since the unshaded area in Fig. 2a includes the line of sight (LOS) between SU\(_\text{tx} \) and SU\(_\text{rx} \), \( \phi_\pi^{\text{opt}} = \theta \). When \( \left| \theta - \theta \right| < \psi_\pi \), which is shown in Fig. 2b, \( \phi_\pi^{\text{opt}} \) lies in the

\[
\begin{align*}
\phi_\pi^{\text{opt}} &\in [\theta - \psi_\pi, \theta], \quad \text{if } \theta > \theta \\
\phi_\pi^{\text{opt}} &\in [\theta, \theta + \psi_\pi], \quad \text{if } \theta < \theta.
\end{align*}
\]

If \( Z \) is less than or equal to zero, we cannot find a narrower interval. Algorithm 1 summarizes our proposed approach to find the optimal solutions \( \phi_\pi^{\text{opt}} \), \( \phi_\pi^{\text{opt}} \), \( P_0^{\text{opt}} \) and \( P_1^{\text{opt}} \).

**Algorithm 1: Optimization Algorithm**

\[
k \leftarrow 0 \\
\phi_\pi^{(0)} = \pi + \theta \\
\text{repeat} \\
\lambda^{(0)} = \lambda_{\text{init}}, \mu^{(0)} = \mu_{\text{init}} \\
n \leftarrow 0 \\
\text{repeat} \\
\text{calculate } P_0^{(k)} \text{ and } P_1^{(k)} \text{ using (8).} \\
\text{update } \lambda \text{ and } \mu \text{ using (9).} \\
n \leftarrow n + 1 \\
\text{until } (10) \text{ is satisfied,} \\
\text{solve } \partial \mathcal{L} / \partial \phi_\pi = 0 \text{ and update } \phi_\pi^{(k+1)}.
\]

**B. Perfect CSI for \( g_{sp} \) and Statistical CSI for Other Channels**

For the second scenario, we assume that SUs cannot cooperate with PU\(_s \) and as a result, SU\(_\text{tx} \) and SU\(_\text{rx} \) cannot estimate the fading coefficients \( g_{sp} \) and \( g_{ps} \), respectively, and they only know the statistics of fading coefficients \( g_{sp} \) and \( g_{ps} \). On the other hand, we assume that SU\(_\text{tx} \) has perfect knowledge of fading coefficient \( g_{ss} \). Therefore, at first we take expectation with respect to \( g_{sp} \) and \( g_{ps} \) in ergodic capacity expression and then maximize the capacity. In this case the optimal transmit power levels and the optimal antenna directions are functions of \( g_{ss} \). The instantaneous capacity \( c_{1,i} \) is independent of \( g_{sp} \) and \( g_{ps} \), and \( \mathbb{E}_{g_{ps},g_{sp}} \{ c_{1,i} \} = c_{0,i} \). The expectation of \( c_{1,i} \) can be written as

\[
\mathbb{E}_{g_{ps},g_{sp}} \{ c_{1,i} \} = \mathbb{E}_{g_{ps}} \left[ \log_2 \left( 1 + \frac{g_{ss} L_1 g_{ps} P_1 g_{ss}}{\sigma_\alpha^2 + P_\gamma g_{ps} L_1 A (\phi_\gamma - \theta')} \right) \right] \\
= \frac{1}{\ln(2)} \left[ \ln \left( 1 + \frac{1}{x_i} \right) + T(\bar{y}) - T(\bar{y} + \frac{\bar{y}}{x_i}) \right] \quad (12)
\]

where \( T(z) = e^z \text{Ei}(-z) \) and \( \text{Ei}(z) = -\int_{-\infty}^{\infty} e^{-t} \frac{t^{-1}}{t} dt \) is the exponential integration [15]. In (12), \( x_i = \sigma_\alpha^2 / a P_1(g_{ss}) \), \( \bar{y} = \sigma_\alpha^2 / a^2 \) and \( \sigma_\alpha^2 = \mathbb{E}_{g_{ps},g_{sp}} \{ \sigma_\phi^2 \} = P_k \gamma g_{ps} L_{ps} A (\phi_\gamma - \theta') \). Finally, the ergodic capacity in this scenario is

\[
C = \mathbb{E}_{g_{ps}} \left\{ \sum_{i=0}^{1} \left[ \tilde{\pi}_1 \log_2 \left( 1 + \frac{1}{x_i} \right) + \frac{\beta_i}{\ln(2)} \left( T(\bar{y}) - T(\bar{y} + \frac{\bar{y}}{x_i}) \right) \right] \right\}
\]

Moreover, the constraints in (4) and (5) can be written as

\[
\mathbb{E}_{g_{sp}} \{ \bar{b}_1 P_0(g_{ss}) + \bar{b}_1 P_1(g_{ss}) \} \leq I_{\bar{P}} \quad (13a)
\]

\[
\mathbb{E}_{g_{ps},g_{sp}} \{ \bar{\pi}_1 P_0(g_{ss}) + \bar{\pi}_1 P_1(g_{ss}) \} \leq P_\mu \quad (13b)
\]

where \( \bar{b}_1 = \mathbb{E}_{g_{sp}} \{ b_1 \} = \beta_1 \gamma g_{sp} L_{sp} A (\phi_\theta - \theta) \). The optimal transmit power levels \( P_1(g_{ss}) \) can be obtained by solving the following equation

\[
\frac{\partial \mathcal{L}}{\partial P_1(g_{ss})} = -\frac{a}{\sigma_\phi^2} \frac{\partial}{\partial (x_i)} f_i(x_i, \bar{y}) + \lambda \tilde{x}_i + \mu \bar{b}_1 = 0
\]

where

\[
f_i(x_i, \bar{y}) = \frac{\alpha_i x_i}{x_i + 1} - \beta_i \bar{y} T(\bar{y} + \frac{\bar{y}}{x_i}).
\]
This equation has no closed form solution and has to be solved numerically. Furthermore, the parameter $Z$ in (11) for this scenario is modified to

$$Z = \frac{I_w}{\pi_1 \gamma_p A_0 P_w} - \frac{A_1}{A_0}$$  \hspace{1cm} (14)

Algorithm 1 can be used for this scenario with some modifications.

C. Statistical CSI for All Fading Channels

In the third scenario we assume that SU$_{tx}$ cannot estimate $g_{ss}$ and it knows only the statistical CSI of all fading channels.

Even if SU$_{tx}$ can estimate $g_{ss}$, when we maximize the capacity for each realization of $g_{ss}$, the optimal $\psi_r$ and $\phi_r$ will be a function of $g_{ss}$ and as a result they may change very fast in a fast fading environment. In some cases where antennas are steered mechanically, their rotation speeds are limited and cannot adapt themselves according to channel variations.

Thus, in this scenario we wish the optimal directions to be independent of the realizations of fading coefficients. Hence, we take expectation with respect to all fading coefficients independent of the realizations of fading coefficients. Hence, we can write $E_g \{c_{1,i}\} = -U(\bar{x}_i, \bar{y})/\ln(2)$ where

$$U(\bar{x}_i, \bar{y}) = \begin{cases} \frac{-\bar{y}}{\bar{x}_i} [T(\bar{y}) - T(\bar{x}_i)], & \text{if } \bar{x}_i \neq \bar{y} \\ \bar{x}_i T(\bar{x}_i) - 1, & \text{if } \bar{x}_i = \bar{y} \end{cases}$$

The ergodic capacity is

$$C = \frac{-1}{\ln(2)} \frac{1}{\sum_{i=0}^{1} \alpha_i T(\bar{x}_i) + \beta_i U(\bar{x}_i, \bar{y})}$$

and the constraints in (4) and (5) can be written as

$$b_i P_0 + b_1 P_1 \leq I_w \hspace{1cm} (15a)$$

$$\bar{\pi}_0 P_0 + \bar{\pi}_1 P_1 \leq P_{av} \hspace{1cm} (15b)$$

The optimal transmit power levels can be obtained by solving the following equation numerically

$$\frac{\partial L}{\partial P_i} = \frac{-\bar{a}}{\sigma^2_0 \ln(2)} h_i(\bar{x}_i, \bar{y}) + \lambda \bar{\pi}_1 + \mu b_i = 0$$

where

$$h_i(\bar{x}_i, \bar{y}) = \bar{x}_i^2 \left( \alpha_i \frac{\partial T(\bar{x}_i)}{\partial x} + \beta_i \frac{\partial U(\bar{x}_i, \bar{y})}{\partial x} \right).$$

Algorithm 1 can be used for this scenario.

IV. NUMERICAL RESULTS

We numerically show the effect of using directional antennas on the ergodic capacity of the considered CR system when spectrum sensing is imperfect. Assume $\sigma^2_0 = 1$, $\phi_{MB} = 45^\circ$, $A_0 = 9.8$, $A_1 = 0.2$, $\gamma_{ss} = \gamma_p = \gamma_{ps} = 1$, $\pi_1 = 0.3$, $\theta_p = 90^\circ$ and $\theta_{sp} = 130^\circ$. For fair comparisons, we consider a fixed spectrum sensing method with $P_{av} = 0.9$ and $P_1 = 0.1$.

Suppose $C_{Dir}^{opt}$ denote the optimal capacity when we use directional antennas. Fig. 3 shows $C_{opt}$ versus $\theta$ for $P_p = 0.4, 3$ watts for all three scenarios when $P_{av} = 12$ dB. When $\theta$ increases from $0^\circ$ to $40^\circ$, SU$_{rx}$ receives less interference from PU$_{tx}$ and SU$_{tx}$ can increase transmit power and as a result the capacity increases. However, when $\theta$ increases from $40^\circ$ to $80^\circ$, SU$_{tx}$ imposes more interference on PU$_{tx}$ and the optimal capacity decreases. Furthermore, we observe that the capacity for scenario 3 is always smaller than that of scenarios 1 and 2. Increasing $P_p$ doesn’t have any impact on constraints, however, the capacity expression depends on $P_p$ and as it can be seen in Fig. 3, increasing $P_p$ decreases the capacity. Fig. 4 shows the optimal capacity for all three scenarios when $P_{av} = 15$ dB. Comparing Figs. 3 and 4, we can see that when the maximum allowed average transmit power of SU$_{tx}$ ($P_{av}$) increases, the capacity increases as well, provided that the constraint (4) is not violated. Fig. 5 which plots $C_{Dir}^{opt}$ versus $P_{av}$ when $\theta = 50^\circ$ and $I_{av} = 0$ dB also shows the similar fact.

Let $C_{Omn}^{opt}$ denote the capacity when SU$_{tx}$ and SU$_{rx}$ have omni-directional antennas and only transmit power levels $P_0$ and $P_1$ are optimized subject to constraints (4) and (5). Note that $P_{P0}^{opt}$ and $P_{P1}^{opt}$ are constant for all $\theta$ when SUs use omni-directional antennas and $C_{Omn}^{opt}$ is independent of $\theta$. Furthermore, let $C_{LOS}^{opt}$ be the capacity when directional antennas of SU$_{tx}$ and SU$_{rx}$ are exactly pointed at each other ($\phi_1 = \theta, \phi_2 = \pi + \theta$) and only $P_0$ and $P_1$ are optimized subject to constraints (4) and (5). We compare $C_{Dir}^{opt}$, $C_{Omn}^{opt}$ and $C_{LOS}^{opt}$.

We define three capacity ratios $\Gamma_{20} = C_{Dir}^{opt}/C_{Omn}^{opt}$, $\Gamma_{20} = C_{LOS}^{opt}/C_{Omn}^{opt}$ and $\Gamma_{20} = C_{LOS}^{opt}/C_{Dir}^{opt}$. Fig. 6 plots $\Gamma_{20}$ and $\Gamma_{20}$ versus $\theta$ when $P_{av} = 12, 15$ dB. We observe that when $\theta = \theta_p$, $C_{Dir}^{opt} \approx C_{LOS}^{opt}$ and as $|\theta - \theta_p|$ increases, the capacity gain increases. When PU$_{tx}$ and SU$_{rx}$ are close, using directional
antennas does not enhance the ergodic capacity (with respect to using omni-directional antennas). The capacity gain in Fig. 6 finally saturates, since the direction of SU$_{rx}$ goes sufficiently away from PU$_{tx}$ and directional antenna of SU$_{tx}$ reduces the interference imposed on PU$_{rx}$. In addition, we can see that when $P_{av}$ of SU$_{tx}$ increases, the ergodic capacity increases, while constraints (4) and (5) still hold true.

The effect of optimizing the orientation of directional antennas on ergodic capacity is illustrated in Fig. 7, where the capacity gain $\Gamma_{D2L}$ versus $\theta$ is plotted for $I_{av} = 12, 15$ dB. We note that when we optimize the angles $\phi_{tx}$ and $\phi_{rx}$, SU$_{tx}$ can use more power for transmission (i.e., use higher power levels $P_{tx}$ and $P_{rx}$) without violating constraints (4) and (5) and, hence, the capacity increases.

V. CONCLUSION

In this paper, we considered a CR system, where the SUs are equipped with steerable directional antennas. The SU$_{tx}$ first senses the spectrum (with error) and then transmits data at two power levels, according to the result of sensing. The optimal SU$_{tx}$ transmit power levels and the optimal directions of SU$_{tx}$ transmit antenna and SU$_{rx}$ receive antenna are obtained by maximizing the ergodic capacity, subject to average transmit power and average interference power constraints. To study the effect of fading channels, we considered three scenarios: 1) when SU$_{tx}$ knows fading channels between SU$_{tx}$ and PU$_{rx}$, PU$_{tx}$ and SU$_{rx}$, SU$_{tx}$ and SU$_{rx}$, 2) when SU$_{tx}$ knows only the channel between SU$_{tx}$ and SU$_{rx}$, and statistics of the other two channels, and, 3) when SU$_{tx}$ only knows the statistics of these three fading channels. Through simulations, we showed that directional antennas significantly enhance the ergodic capacity, without violating the power constraints.

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