Analytical and numerical solution of the stress-strain state of sleeper foundations

Alexey Glushkov*1, Vyacheslav Glushkov1 and Ilya Glushkov2
1Volga State University of Technology, Yoshkar-Ola, Russian Federation
2Perm National Research Polytechnic University, Perm, Russian Federation

* Corresponding author: 256289@list.ru

Abstract. The article presents the results of numerical modelling of strip sleeper foundations. The analysis of the influence of the distance between the sleepers in the conditions of a two-dimensional problem. As a comparison, the results of the calculation of the analytical corner points method is presented. The results of the study of sleeper foundations with different distances between sleepers up to 3b are shown. The analytical solution shows the distribution of stress over depth when changing the size of sleepers, distances between sleepers. An analytical solution for the distribution of vertical displacements over depth is obtained. The results of the numerical solution show the components of stress and strain at different distances between sleepers, the dependence of precipitation on the applied pressure and the distance between sleepers. These studies are confirmed by the results of experiments of various authors. Designs of sleeper foundations are implemented in the construction of buildings and structures, which reduce the cost of the zero cycle, reduce the material consumption in comparison with standard solutions. The article is of interest to specialists in the field of soil mechanics, geotechnics, and foundation engineering.

1. Introduction
One of the most promising areas of research in the field of foundation construction is the development of effective structures of sleeper foundations that would provide increased load-bearing capacity of the foundation soil, simple and reliable foundation design, reducing the material consumption, the volume of formwork and earthworks when installing foundations [1].

These requirements are most met by sleeper foundations, the methods for determining the bearing capacity and settlements of which are not yet sufficiently developed by the standards [3-6]. The methods recommended by regulatory documents give results that differ significantly from experimental data [2-4].

The purpose of this work is to develop the optimal design of sleeper foundations, to find out the regularities and features of the joint work of the foundation and the ground base, to develop methods for calculating the sleeper foundation by limit states.

In modern conditions, there is a tendency to increase the load on the soil basis of buildings and structures. In this case, one of the ways to increase the load-bearing capacity and reduce the deformation of the base is the use of sleeper foundations. Joint deformation of the "sleeper foundation – soil basis" system, taking into account the redistribution of forces during the application of the load, has not been studied. There are no normative documents on the calculation of sleeper foundations taking into account the formation of an "arch effect" in the soil basis [1].
Sleeper foundations are made in the form of separate reinforced concrete blocks (sleepers) with a width of 30-50 cm, and in the form of a group of blocks connected to each other. They are laid on a ground base across the supporting walls of the building on a compacted sand preparation with a thickness of $\delta=15$ cm with $\gamma_d=16.0-18.5$ kN/m$^3$. Foundation beams and walls or basement panels of the building are installed on top of the sleeper foundations.

The construction of uninterrupted continuous strip foundations from foundation pads is impractical due to the irrational use of the material and significant deformations of the base. The works of Golubkov V. N., Tugayenko Yu. F., Fidarov M. I., Saurin A. N., Korpach O. I., Ilyichev V. A. and other scientists are devoted to the study of the interaction of sleeper foundations with the Foundation [2-5].

2. Methods

2.1. Using the angle point method

The base of sleeper foundations is proposed to be calculated using the analytical method of angular points, which can be used to predict the distribution of deformations and stresses in the base, taking into account the formation of the "arch effect" [6]. Sleeper foundations, taking into account the formation of the "arch effect", work together with the ground base in the conditions of the spatial stress-strain state [7].

Let's calculate the stress value using V. G. Korotkin's solution.

When $x=y=0$, the expression for $\sigma_z$ takes the form:

$$\sigma_z = \frac{2q}{\pi} \left[ \frac{ab}{z\sqrt{a^2 + b^2 + z^2}} + \frac{abz(a^2 + b^2 + 2z^2)}{(a^2 + z^2)(a^2 + b^2 + z^2)} \right].$$

The stresses for points located at depth $z$ on a straight line passing through one of the corner points of a rectangular loading platform take the form:

$$\sigma_x = \frac{q}{2\pi} \left[ \frac{\pi}{2} - \frac{4abz}{(4a^2 + z^2)\sqrt{4a^2 + 4b^2 + z^2}} \right] - \arctg \frac{4abz}{z\sqrt{4a^2 + 4b^2 + z^2}} + \frac{abz(a^2 + b^2 + 2z^2)}{(a^2 + z^2)(a^2 + b^2 + z^2)} \right],$$

$$\sigma_y = \frac{q}{2\pi} \left[ \frac{\pi}{2} - \frac{4abz}{(4b^2 + z^2)\sqrt{4a^2 + 4b^2 + z^2}} \right] - \arctg \frac{4abz}{z\sqrt{4a^2 + 4b^2 + z^2}} + \frac{abz(a^2 + b^2 + 2z^2)}{(a^2 + z^2)(a^2 + b^2 + z^2)} \right],$$

$$\sigma_z = \frac{q}{2\pi} \left[ \frac{4abz(4a^2 + 4b^2 + 2z^2)}{(4a^2 + z^2)(4b^2 + z^2)\sqrt{4a^2 + 4b^2 + z^2}} + \arctg \frac{4abz}{z\sqrt{4a^2 + 4b^2 + z^2}} \right] + \frac{abz(a^2 + b^2 + 2z^2)}{(a^2 + z^2)(a^2 + b^2 + z^2)} \right],$$

$$\tau_{xy} = \frac{qz^2}{\pi} \left[ \frac{a}{z^2\sqrt{4a^2 + z^2}} - \frac{1}{(4b^2 + z^2)\sqrt{4a^2 + 4b^2 + z^2}} \right].$$
When using the angle point method for point A vertical stresses:
\[ \sigma_{zA} = q(K_i - K_{II} + K_{III} - K_{IV} + K_v). \]

The vertical stresses at the angle of the loaded rectangle are determined by the formula:
\[ \sigma_z = K_c q, \]
where \( K_c \) is a table coefficient for determining compressive stresses along the vertical line passing through the corner point of the loaded rectangle.

The vertical movement of a point at depth \( z \) at the corners of a rectangular loaded platform with sides \( a \) and \( b \) is determined by the formula:
\[ \omega_{zA}(z) = \frac{aq}{2E} (1 - \nu^2) \left( A - \frac{1 - 2\nu}{1 - \nu} B \right), \]
where
\[ A = \frac{1}{\pi} \left( \ln \frac{\sqrt{1+m^2+n^2}+m}{\sqrt{1+m^2+n^2}-m} + m \ln \frac{\sqrt{1+m^2+n^2}+1}{\sqrt{1+m^2+n^2}+1} \right); \]
\[ B = \frac{n}{\pi} \arctg \left( \frac{m}{n\sqrt{1+m^2+n^2}} \right); \]
\[ m = \frac{b}{a}; n = \frac{z}{a}; \nu - \text{Poisson ratio}. \]

Using the superposition principle, we can determine the vertical movement of any point in the soil mass.

For the special case of vertical movements of the ground surface (\( z=0 \)), the values of the coefficient \( B \) in the equation are zero. Only the values of the coefficient \( A \) corresponding to \( n=0 \) make sense.

N. A. Tsytyovich in his works gives dimensionless values of the coefficient \( K_0 \) when applying a load over a rectangular area with a width of \( 2a \) and a length of \( 2b \). Using the values of the coefficient \( K_0 \), we can determine the corresponding vertical movements using the formula:
\[ \omega = \frac{2aq}{E} (1 - \nu^2) K_0. \]

2.2. Finite elements method

The finite element method was used to evaluate the deformed and stressed state of sleeper foundations, taking into account their joint work. The mutual influence was studied by changing the distance...
between adjacent of sleeper foundations $l$ in the range from 0 to $3b$ (where $b$ is the width of the sole of a separate sleeper foundations) [8].

To solve this problem, geotechnical complex Plaxis was used, it implements the finite element method numerically. Simultaneous consideration of the strength and deformation properties of the soil in the calculations of the stress-strain state of the base of the sleeper foundation is carried out in the solution of the plane problem of the finite element method. The soil in the pre-limit state is a continuous linearly deformed medium that passes with subsequent loading into the limit (plastic) state in accordance with the Morh-Coulomb yield criterion (strength). The calculation was performed using a step-by-step load application procedure. The boundary conditions assume the restriction of horizontal movements along the edges of the area and the absence of horizontal and vertical movements along the lower border.

When selecting the size of the region, the condition of attenuation to the minimum values of the value of deformations and stresses in the base was assumed.

The characteristics of the soil material in the base are taken according to the standard characteristics for soft plastic clay with $\gamma = 18$ kN/m$^3$, $\phi = 14^\circ$, $c = 41$ kPa, $E = 15$ MPa.

The conducted research allowed us to establish the main regularities of interaction of sleeper foundations with the basis in real conditions [9].
3. Results and discussion

Analysis of graphs $S=f(p)$ shows that increasing the distance between the sleeper foundations $l/b$ from 0 to 3 at $p=261$ kPa leads to a decrease in precipitation by 2.2 times.

When a load is applied to the sleeper foundation, a deformation zone is formed under the sole at the base. Settlements with increasing load on the sleeper foundation increase in proportion to the size of the deformation zone. The depth of the deformation zone under the sleeper foundation is less than under equivalent foundations with a solid sole, taking into account the "arch effect" [10-13].

Depending on the distance between the sleeper foundations ($l/b =0÷3$), different deformation zones are formed. At large distances between sleeper foundations $l>b$, excluding their mutual influence through the base, an independent deformation zone is formed under each foundation (Fig. 6).

At distances $l<b$ between individual foundations, a deformation zone is first formed under each sleeper foundation, and then a general deformation zone is formed as the load increases. Integration of individual deformation zones and further development of the general deformation zone with increasing load under all sleeper foundations occurs smoothly [14, 15].

![Figure 6. Vertical displacements $Uz$ in the basis of the sleeper foundations.](image-url)
Figure 7. Horizontal displacements $U_x$ in the basis of the sleeper foundations.

Figure 8. Vertical stresses $U_z$ displacements in the basis of the sleeper foundations.
Figure 9. Plastic deformation zones in the basis of the sleeper foundations.

4. Conclusions
The use of non-linear calculation methods makes it possible to design sleeper foundations based on the ultimate deformations.

Even better technical and economic indicators are characterized by scoring wedge-shaped sleeper foundations due to the compaction of the soil when driving these foundations and the inclusion of a larger volume of soil in the work, the load-bearing capacity increases and precipitation decreases.

The results of numerical studies are in good agreement with the results of experimental studies performed by professor Golubkov V. N. The data of Golubkov's experiments confirm the positive effect of sleeper foundations sliding, taking into account the formation of an "arch effect" in the core. An increase in the \( l/b \) ratio leads to an increase in the load-bearing capacity and a decrease in the precipitation of sleeper foundations.

The results of studies of joint work of sleeper foundations with the base and the study of patterns of formation of the deformation zone allowed us to apply sleeper foundations in the experimental construction of two nine-story residential buildings in Odessa [7]. As the foundations of the blocks were used sleeper size 200×2500 mm with the distance between ties \( l=350 \) mm. At the top of the established Foundation beam with a width of 400-600 mm. At the base of the sleeper foundations overlain by loess loam with a capacity of 9-14 m, which have been experienced soaking. Made on sleeper foundations, the building is normally operated, which allowed to reduce the cost of the zero cycle by 2.0-2.5 times, compared with prismatic piles and a monolithic reinforced concrete grillage.

References
[1] Vyacheslavovich G A and Adolfovich B L 2016 Influence of the form and size of the isolated foundations on the stress-strain state of the soil base J. Appl. Eng. Sci. 14
[2] Tugaenko Y F and Kushchak S I 1986 Deformations in the bases of sleeper foundations Soil Mech. Found. Eng. 23 42–5
[3] Saurin A and Korpach A 2012 Technology features of the tie elements distribution design in the conditions of a new construction Bull. Civ. Eng. 5 98–102
[4] Saurin A and Korpach A 2011 Specialized installation for strengthening the “base-foundation” system in emergency buildings with a sleeper distributor Bull. Civ. Eng. 4 69–73
[5] Saurin A, Il’ichev V, Zhadanovskij B, Kuznetsova E, Saurin J, Red’kina J and Miljutin D 2009 Method of foundation bed construction by horizontal reinforcing with precast concrete components
[6] Fidarov M I 1986 Design and Construction of Intermittent Foundations (Moscow)
[7] Golubkov V N, Taugaenko Y F, Kolesnikov I L and Kokorzhickij K M 1976 Sleeper and wedge-shaped sleeper scoring foundations (Kiev)
[8] Bogomolov A, Ponomaryov A and Bogomolova O 2017 Assessment of slope stability on the basis of soil mass stress state analysis ICSMGE 2017 - 19th International Conference on Soil Mechanics and Geotechnical Engineering vol 2017-Septe (19th ICSMGE Secretariat) pp 2095–8
[9] Bogomolov A, Ponomaryov A and Bogomolova O 2017 Assessment of slope stability on the basis of soil mass stress state analysis ICSMGE 2017 - 19th International Conference on Soil Mechanics and Geotechnical Engineering vol 2017-Septe (19th ICSMGE Secretariat) pp 2095–8
[10] Ponomaryov A B and Zakharov A V. 2016 Numerical Simulation of the Process of the Geothermal Low-potential Ground Energy Extraction in the Perm Region (Russia) Procedia Engineering vol 150 (Elsevier Ltd) pp 2272–7
[11] Alfano G and Crisfield M A 2001 Finite element interface models for the delamination analysis of laminated composites: Mechanical and computational issues Int. J. Numer. Methods Eng. 50 1701–36
[12] Lachler A, Vermeer P A and Wehnert M 2007 Assessment of diaphragm wall stability and deformation XIV European Conference on Soil Mechanics and Geotechnical Engineering (Madrid) pp 1055–60
[13] Wit de J C W M and Lengkeek H J 2002 Full scale test on environmental impact of diaphragm wall trench installation in Amsterdam - final results International Symposium on geotechnical Aspects of Underground Construction in Soft Ground (Toulouse)
[14] Verruijt A 2006 Offshore soil mechanics (Delft: Delft University of technology)
[15] Bauer M A, Dmitrienko V A and Kapustin A I 2016 Assessment of Deformations of Earth’s Surface at Mine Construction on Sub-Soils Procedia Engineering vol 150 (Elsevier Ltd) pp 2278–86