Non-Fermi liquid behavior in an extended Anderson model

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Abstract

An extended Anderson model, including screening channels (non-hybridizing, but interacting with the local orbit), is studied within the Anderson-Yuval approach, originally devised for the single-channel Kondo problem. By comparing the perturbation expansions of this model and a generalized resonant level model, the spin-spin correlation functions are calculated which show non-Fermi liquid exponent depending on the strength of the scattering potential. The relevance of this result to experiments in some heavy fermion systems is briefly discussed.

75.20.Hr, 75.30.Mb, 71.28.+d, 71.27.+a
I. INTRODUCTION

The Anderson model, originally proposed to explain the formation of the local magnetic moment in metals, has played a fundamental role in exploring the correlation effects in many-body systems. Various theoretical approaches, including the Hartree-Fock [1], scaling theory [2], numerical renormalization group [3], perturbation expansion [4], variational calculations [5], slave-boson mean field theory [6], large-N diagrammatic expansion [7], exact Bethe-Ansatz solution [8], and many others [9], have been applied to study this important issue. The general understanding has been that the Anderson model, in both electron-hole symmetric and asymmetric cases (including the valence fluctuation regime) exhibits a local Fermi liquid (FL) behavior, i.e., the impurity contribution to the specific heat and magnetic susceptibility is regular.

Recently, this "common understanding" has been questioned by studies of a generalized Anderson model, including the so-called screening channels [10–13]. In fact, it was realized long time ago by Anderson and Haldane that the original Anderson model is not complete to fulfil the local charge neutrality Friedel sum rule in the mixed-valence regime, i.e., to satisfy two different equations, corresponding to two different valence states by varying only one single parameter—the local state energy level [14]. In addition to the channel of conduction electrons which hybridize with the local state, "screening" channels have been included, which do not hybridize with the local state but are related to it via Coulomb interactions. Haldane has also presented a Hartree-Fock mean field theory for this extended model [14], which, unfortunately, missed the non-FL behavior contained in it.

Recently, Varma and collaborators [10,11] have revived interest to this model, considering it as a single site version of the three-band Hubbard-type model proposed to describe the cuprates [15]. The "screening" channels correspond to oxygen orbits, not mixing with copper due to symmetry. Their original motivation was to find a microscopic justification for the phenomenological marginal FL theory [16]. At first, some numerical evidence of non-FL behavior was provided by a Wilson renormalization group study [10]. Later, a strong
coupling limit Hamiltonian was derived which exhibits a quantum critical point separating the Kondo and empty-orbit regions \[11,12\]. The correlation functions around this critical point show non-FL behavior. However, the calculation of the correlation functions is very delicate due to the presence of the single occupancy constraint. We believe the correct result has been obtained in \[13\] which demonstrates a power law singularity for the impurity specific heat and magnetic susceptibility, namely, \( C_{\text{imp}}/C_0 \sim \chi_{\text{imp}}/\chi_0 \sim T^{-3/4} \) in the unitarity limit (phase shift = \( \pi/2 \)), in contrast to the logarithmic singularity anticipated earlier \[11\]. Whether this result is relevant for the high \( T_c \) materials, is an open question. However, some heavy fermion compounds show such type of singularities. In particular, the alloys \( UPd_xCu_{5-x} \) (\( x = 1, 1.5 \)) have been studied in detail, using both static and neutron scattering techniques \[17\]. The critical exponent extracted from the experimental data \[18,19\] is \( \Delta = 1/3 \), which is qualitatively consistent but quantitatively different from the theoretical result obtained for the generalized Anderson model \[13\].

The calculations of \[13\] have been carried out using the bosonization technique and the canonical transformations which provide values for the critical exponents only in the unitarity limit. In view of the importance of this issue and the need to obtain results away from the unitary limit \[20\] it is highly desirable to have another way to reconfirm and extend the previous result \[13\]. In this paper we apply the Anderson-Yuval (AY) approach \[21\] to consider the extended Anderson model.

The AY approach \[21\] was originally devised to study the single channel Kondo problem. The main idea of this approach is based on the time-dependent one-body formulation of the X-ray edge problem, developed by Nozières and De Dominicis (ND) \[22\]. Using the AY perturbation expansion Toulouse could map the Kondo problem onto a resonant level model from which he could derive the well-known strong-coupling Toulouse limit \[23\]. Later, the bosonization technique was used by Schlottmann \[24\] to calculate physical properties in this strong coupling limit, although bosonization was employed much earlier by Schotte to derive the AY perturbation expansion for the partition function \[25\]. The equivalence of this perturbation expansion with that of a resonant level model (at and away from the
Toulouse limit) was shown explicitly and made use of for studying the physical properties by Wiegmann and Finkel'stein [26]. Recently, Fabrizio, Gogolin and Nozières [27] have generalized the AY approach to consider the asymmetric two-channel Kondo problem and the FL-non-FL crossover within that model. They have mapped term-by-term the perturbation expansion of the two-channel Kondo problem to that of a generalized resonant level model. Under the scaling assumption these authors could provide an analytical description of the FL-non-FL crossover. In this paper we will follow their approach rather closely. Bosonization and AY approach are similar in the sense of scaling assumption, but the latter is not limited to the Born approximation and can be used away from the unitarity limit. Moreover, it is a ”brute force” partial resummation of diagrams without resorting to canonical transformations used in the bosonization approach.

We should mention that there is another generalization of the Anderson model considered by Si and Kotliar [28,29], who have included all possible density-density interactions of the hybridizing channel without invoking the screening channels. Using the renormalization group expansion they considered the weak coupling case [28], while the strong coupling limit was treated by bosonization [29]. In the final section we will discuss how their results are related to ours in the overlapping region.

The rest of the paper is organized as follows: The model is defined and the basic strategy to treat the model is given in Sec.II, while the perturbation expansion and the mapping onto a generalized resonant-level model is presented in Sec.III. Furthermore, the correlation functions are calculated in Sec. IV to be confronted with experimental results. Finally, some concluding remarks are made in Sec.V.

II. THE MODEL AND THE BASIC STRATEGY

The Hamiltonian we consider in this paper is given as a sum

\[ H_T = H + \bar{H}, \]

where the hybridizing part is
\[ H = H_0 + H_I + t \sum_\sigma (c_{\sigma}^+(0)d_\sigma + d_{\bar{\sigma}}^+c_\sigma(0)), \]

\[ H_0 = \sum_{\sigma,k} \epsilon_k c_{k\sigma}^+c_{k\sigma}, \]

\[ H_I = V_0 \sum_\sigma c_{\sigma}^+(0)c_\sigma(0)(n_\sigma - \frac{1}{2}), \]

while the screening part is

\[ \bar{H} = \sum_{l,k,\sigma} \epsilon_k c_{l\sigma}^+c_{l\sigma} + \sum_{l,\sigma} V_l c_{l\sigma}^+(0)c_{l\sigma}(0)(n - \frac{1}{2}). \]

Here \( c_{k\sigma}^+, c_{k\sigma} \) are conduction electron operators in the hybridizing channel, with \( c_{\sigma}^+(0), c_\sigma(0) \) as their Fourier transforms at the origin of the coordinates (where the impurity is located). Similarly, \( c_{l\sigma}^+, c_{l\sigma} \) are conduction electron operators in the screening channels.

\( n = \sum_\sigma n_\sigma, n_\sigma = d_{\sigma}^+d_\sigma \) is the local impurity operator. The Hubbard \( U \) on impurity itself has been taken as infinity, so the only allowed states are \(|0>, |\sigma>, \sigma = \uparrow, \downarrow\). \( V_0, V_l \) are the Coulomb interactions of the local electron with conduction electrons in the hybridizing and screening channels, respectively. The essential part of this Hamiltonian is the same as that in earlier papers [11–13]. We have not included here the anti-parallel spin Coulomb interactions \( V_0' \sum_\sigma c_{\sigma}^+c_\sigma(n_\sigma - \frac{1}{2}) \) and the spin-flip scattering \( V_\perp \sum_\sigma c_{\sigma}^+c_\sigma d_{\sigma}^+d_\sigma \) in the hybridizing channel, considered in [12,13]. As shown in those references, these terms do not affect essentially the behavior in the strong coupling limit. For simplicity, we have also taken the local level \( \epsilon_d \) at the Fermi level, since we are mainly interested in the critical behavior itself, rather than the level renormalization per se.

Before proceeding, let us briefly recall the basic strategy of ND [22] and AY [21] to see why their approach can be applied to our problem. The crucial term in the X-ray edge problem is \( V \sum c^+(0)c(0)dd^+, \) where \( d \) is the deep-level electron, while \( c^+(0), c(0) \) are the conduction electron operators at the origin. As it stands, this is a many-body problem. However, ND have realized that it can be converted into a time-dependent one-body problem, because the scattering potential is effective only after X-ray absorption or before X-ray emission (when \( dd^+ = 1 \)). Since the internal degrees of freedom for the local electron are not involved, its propagator in the standard many-body technique can be traced out, thus converting it
into a one-body problem. Moreover, instead of the usual Fourier representation of Green’s functions, ND solved the Dyson equation directly in the time domain. The integral equation then obtained turned out to be singular, of the Muskhelishvili-type [30] from which the power-law time asymptotic behavior is extracted with exponents expressed in terms of a phase shift \( \delta = \tan^{-1}(\pi \nu_0 V) \), where \( \nu_0 \) is the conduction electron density of states.

Soon afterwards AY [21] realized that the ND trick can be used to obtain the perturbation expansion for the Kondo problem with a Hamiltonian

\[
H_K = \frac{1}{2} J \sum \left[ c_{\uparrow}^+ c_{\downarrow} S^+ + c_{\downarrow}^+ c_{\uparrow} S^- + (c_{\uparrow}^+ c_{\uparrow} - c_{\downarrow}^+ c_{\downarrow}) S^z \right],
\]

where, \( S^+, S^- , S^z \) are local spin components, while \( c_{\uparrow}^+, c_{\downarrow}, \ldots \) are conduction electron operators at the origin. For any given sequence of spin flips at moments \( t_1, \ldots, t_n \) (from up to down), \( t_1', \ldots, t_n' \) (from down to up) in a perturbation expansion of the partition function, the asymptotic expression can be used if the time difference \( t_i - t_i' \), \( \ldots \), is much greater than the transient time \( t_0 \sim \) the inverse of the bandwidth. The exponents can again be expressed in terms of the phase shift \( \delta = \delta_+ - \delta_- = 2 \tan^{-1} \frac{\nu J}{\nu_0} \), due to the difference of scattering potential experienced by the up and down spins. By mapping this perturbation onto an expansion for some kind of resonant level model which can be solved exactly in limiting cases, the low energy physics can be extracted. This was the basic strategy of [26] and [27] and will be followed in this paper. Instead of \( | \uparrow > \) and \( | \downarrow > \) two states for the single channel Kondo problem, we have here three states \( | 0 >, | \uparrow >, \ldots, | \downarrow > \). Also, we have screening channels in addition to the hybridizing channels. Nevertheless, the above programme can be still implemented, as seen from the next Section.

III. PERTURBATION EXPANSION AND MAPPING ONTO A GENERALIZED RESONANT LEVEL MODEL

Like in the X-ray edge and Kondo problems, we are interested in the time evolution of the system described by

\[
F(\tau) = \langle 0 | e^{iH_T \tau} | 0 \rangle,
\]
where $H_T$ is the total Hamiltonian given by (1) and (2), while $|0\rangle$ is the ground state which is degenerate in the mixed valence regime with the local electron in one of the states $|\alpha\rangle$, $\alpha = 0, \uparrow$ and $\downarrow$. We consider the perturbation expansion in terms of the hybridization parameter $t$

$$F(\tau) = 0\langle e^{iH_T\tau}T\{e^{i\int_0^\tau dr' H_h(r')}\}|0\rangle = \sum_{n=0}^{\infty} \int_0^\tau d\tau_{2n} \int_0^{\tau_{2n}} d\tau_{2n-1} \cdots \int_0^{\tau_2} d\tau_1$$

$$< 0|e^{iH'(\tau-\tau_{2n})iH_h} \cdots e^{iH'(\tau_{2j+1}-\tau_{2j})iH_h} e^{iH'(\tau_{2j-1})iH_h} \cdots iH_h e^{iH'\tau_1} |0 >$$

with

$$H_h = t \sum_\sigma (c^{+}_\sigma(0)d_\sigma + d^{+}_\sigma c(0)),$$

$$H' = H_0 + H_I + \bar{H}.$$  (5)

Due to the presence of $d^{+}_\sigma$, $d_\sigma$ in $H_h$, only even order terms are kept. A typical term will contain either spin up (down) operators only, or mixed. However, in view of the single occupancy constraint the spin up and down states can be connected only via the empty state. Therefore, these terms can be separated into up and down blocks. Using the relations

$$d^{+}_\sigma e^{iH'\tau_j}|\alpha\rangle = \begin{cases} e^{iH'_\alpha \tau_j}|\sigma\rangle, & \text{if } |\alpha\rangle = |0\rangle, \\ 0, & \text{if } |\alpha\rangle \neq |0\rangle, \end{cases}$$

$$d_\sigma e^{iH'\tau_j}|\alpha\rangle = \begin{cases} 0, & \text{if } |\alpha\rangle \neq |\sigma\rangle, \\ e^{iH'_\alpha \tau_j}|0\rangle, & \text{if } |\alpha\rangle = |\sigma\rangle \end{cases}$$

with $H'_\alpha \equiv <\alpha|H'|\alpha>$, we can trace out $d_\sigma$ operator. Since the Hamiltonian $H'_\alpha$ is different for the neighboring time moments $\tau_j$ and $\tau_{j+1}$ (or $\tau_{j-1}$), the calculation is similar to the X-ray edge or Kondo problem and we can apply AY approach to handle it. Explicitly, we fix $\tau_{1j}$ ($\tau'_{1j}$) as moments when a local spin-up electron is created (annihilated). Likewise $\tau_{2j}$ ($\tau'_{2j}$) for local spin-down electron. As said before, due to the single occupancy constraint, the only allowed sequences are: ($\tau_{1j}$, $\tau'_{1j}$), ($\tau_{2j}$, $\tau'_{2j}$), i.e., the up and down sequences do not intersect each other.
In the absence of the scattering potentials \( V_0 = V_\uparrow = 0 \), the propagator for the conduction electron in the local presentation \( G_\sigma (\tau) = \langle T c_\sigma (\tau) c_\sigma^+ (0) \rangle \) is

\[
G^{(0)} (\tau) = \frac{i \nu_0}{\tau - i \xi_0 \text{sgn} \tau},
\]

where \( \xi_0 \) is the bandwidth, serving as a cut-off. The contribution of \( 2n \) vertices connecting \( \tau_{\sigma_i}, \tau'_{\sigma_j} (\sigma = 1, 2) \) will be

\[
D_\sigma = \prod_{i<j} (\tau_{\sigma_i} - \tau_{\sigma_j}) \prod_{i<j} (\tau'_{\sigma_i} - \tau'_{\sigma_j}) \prod_{ij} (\tau_{\sigma_i} - \tau'_{\sigma_j}).
\]

In the absence of the scattering \( V_0 = V_\uparrow = 0 \), the contribution of separate terms in (4) will be

\[
U_0 = D_\uparrow D_\downarrow.
\]

Now we turn on the scattering potential. First consider the hybridizing channel. For the spin up conduction electron at moment \( \tau_1 \) a scattering potential is switched on to give a phase shift

\[
\delta = 2 \delta_0 = 2 \tan^{-1} \left( \frac{\pi \nu_0 V_0}{2} \right),
\]

while at moment \( \tau'_1 \) an opposite phase shift is produced. On the other hand, the spin down conduction electron does not experience any phase shift at these moments. Of course, at moments \( \tau_2 \) and \( \tau'_2 \) the situation is reversed. There are two types of contributions to the renormalization of the conduction electron propagation: from an open line \( U_L \) and closed loops (vacuum fluctuations). As shown earlier

\[
U_L = (D_\uparrow D_\downarrow)^{-2\delta/\pi},
\]

\[
U_c = (D_\uparrow D_\downarrow)^{4\delta/\pi^2}.
\]

In deriving (12) we have taken into account the fact that in the process \( |0> \rightarrow |\uparrow> \), nothing is changed for \( H'_\alpha \), with \( \alpha = 0, \downarrow \), so there are no crossing terms.
Next we consider the screening channels. For channel $l$ the conduction electron gets a phase shift

$$\delta_l = 2\delta(0)_l = 2\tan^{-1}\left(\frac{\pi\nu_0 V_l}{2}\right)$$

(13)

at both $\tau_{1i}$ and $\tau_{2i}$, and an opposite phase shift at $\tau'_{1i}$ and $\tau'_{2i}$. The screening electrons contribute only to the closed loops. However, as seen from (2), the screening channel electrons scatter on the total charge $n = \sum_\sigma n_\sigma$, so the process $|\uparrow\rangle \rightarrow |0\rangle$ will affect also spin down conduction electron in the screening channels. Therefore we will have crossing terms which are similar to the case of multichannel Kondo model [27] with the spin index replacing the channel index there. The final result is

$$U_{cl} = (D_\uparrow D_\downarrow F)^2(\frac{\delta_l}{\pi})^2$$

(14)

with the crossing term

$$F = \prod_{ij}(\tau_{1i} - \tau_{2j})(\tau'_{1i} - \tau'_{2j}) \prod_{ij}(\tau_{1i} - \tau'_{2j})(\tau'_{1i} - \tau_{2j})^2.$$  

(15)

Putting everything together, we find the total contribution of all channels to a given term is

$$U_T = U_0 U_{L} U_{c} \prod_{l=1}^{N} U_{cl} = (D_\uparrow D_\downarrow)^{(1 - \frac{\delta_l}{\pi})^2} (D_\uparrow D_\downarrow F)^2 \sum_{l=1}^{N} \frac{\delta_l}{\pi}^2.$$  

(16)

Following earlier treatments [26,27], we now consider a generalized resonant level model

$$H = H_0 + \lambda[(d_\uparrow^+ + d_\downarrow^+)\psi(0) + \psi^+(0)(d_\uparrow + d_\downarrow)] + V\psi^+(0)\psi(0)(n - \frac{1}{2}),$$

(17)

where $n = (d_\uparrow^+ + d_\downarrow^+)(d_\uparrow + d_\downarrow)$, with constraint $d_\uparrow^+ d_\uparrow + d_\downarrow^+ d_\downarrow \leq 1$, $\psi^+(0)$, $\psi(0)$ a spinless fermion. As before, we can expand the evolution operator in terms of $\lambda$. Again only even order terms survive and there are three types of terms (containing only $d_\uparrow^+$, $d_\uparrow$ or $d_\downarrow^+$, $d_\downarrow$, or mixed. Also, due to the single occupancy constraint any $|\sigma\rangle$ state can be created only from the empty state. We can then repeat the same procedure to derive contributions from the
open line and closed loops. However, there is one important difference, namely, the single type of spinless fermion $\psi(0)$ is coupled to both $d_\uparrow$ and $d_\downarrow$, so there are crossing terms even for the free propagator (where $V = 0$). When we switch on the scattering potential at moments $\tau_{\sigma_i}$ (the process $|0> \rightarrow |\sigma>\psi$ gets a phase shift

$$\delta' = 2 \tan^{-1}\left(\frac{\pi \nu_0 V}{2}\right)$$

(18)

and an opposite one at moments $\tau'_{\sigma_i}$ during the process $|\sigma> \rightarrow |0>$. Summing up the contributions from the free propagator, the open line and the closed loops, we find the $n$-th order term of $\lambda$ is given by

$$U' = (D_\uparrow D_\downarrow F)^{(1-\delta')/2}.$$  

(19)

If we take

$$\delta = \pi, \quad 2 \sum_{l=1}^{N} (\frac{\delta_l}{\pi})^2 = (1 - \frac{\delta'}{\pi})^2$$

(20)

the extended Anderson model $H_T$, given by (1) and (2), and the generalized resonant-level model (17) are equivalent to each other via a term-by-term mapping of the perturbation expansion. It is expected that they should contain the same low-energy physics. Of course, there is an underlying assumption that $V$ is the only scaling parameter for this universality class. A similar assumption was made for the single channel Kondo and the two-channel Kondo problem with channel asymmetry [27].

IV. CORRELATION FUNCTIONS AND PHYSICAL IMPLICATIONS

Before proceeding we compare first the result of the preceding Section with earlier calculations using bosonization [12,13]. In the previous work a model very similar to (1), (2) was considered with additional opposite spin Coulomb interaction $V'_0 \sum_\sigma c^+_\sigma c_\sigma (n_\sigma - \frac{1}{2})$ and spin-flip scattering $V_\perp \sum_\sigma c^+_\sigma c_\sigma d^+_\sigma d_\sigma$ in the hybridizing channel (see Eq. (6) in [13]). It was shown there by using the canonical transformation that the strong coupling Toulouse limit is reached for $V_0 \rightarrow \infty, V'_0 \rightarrow 0, \tilde{V}_s = \sqrt{2N_s}V_s \rightarrow \infty$, where $V_s$ is the Coulomb potential.
in the screening channel and $N_s$ is the number of channels. As follows from (11), (13), (18) and (20), we have the same Toulouse limit here. In fact, $\delta_0 = \tan^{-1}(\pi \nu_0 V/2) = \frac{\pi}{2}$, $\delta' = 0$, $2 \sum_{l=1}^{N_s} (\delta_l \pi)^2 = 1$, meaning that the unitarity limit is reached in both hybridizing and screening channels. The earlier calculations correspond to the Born approximation of our present result summed to infinite orders. This reconfirms the consistency of bosonization and ND approach. However, here we have obtained the mapping to the generalized resonant-level model in a broader regime, namely, the unitarity limit should be reached in the hybridizing channel $(\delta = \pi)$, but not necessarily in the screening channels $(\delta' \neq 0)$. Since the Toulouse limit is materialized at $V_0' = 0$, the opposite spin scattering is not essential. As seen in [13], the effect of the $V_\perp$ term is reflected only in the energy difference of $\alpha$ and $\beta$ particles ($\alpha = \frac{1}{\sqrt{2}}(d_\uparrow + d_\downarrow)$, $\beta = \frac{1}{\sqrt{2}}(d_\uparrow - d_\downarrow)$) which we will include in our following discussion.

Now we study the physical properties of the generalized resonant-level model (17) rewritten as

$$H = H_0 + H_h + H_I,$$

$$H_0 = \sum_k \epsilon_k \psi_k^+ \psi_k + \epsilon_\alpha \alpha^+ \alpha + \epsilon_\beta \beta^+ \beta,$$

$$H_h = \lambda (\alpha^+ \psi(0) + \psi^+(0) \alpha),$$

$$H_I = V \psi^+(0) \psi(0) (\alpha^+ \alpha - \frac{1}{2})$$

which should still satisfy the single occupancy constraint $\alpha^+ \alpha + \beta^+ \beta \leq 1$. First we calculate the scattering amplitude

$$S(t) = \langle |e^{iHt}| \rangle = exp\{C(t)\},$$

$$C(t) = T\{exp[i \int_0^t d\tau (H_h(\tau) + H_I(\tau))]\} > c.$$  

(22)

There are contributions from the spinless fermion $\psi$ and the $\alpha$ particle, both from the closed loops. Neglecting the energy level renormalization factors, the long time asymptotics are given by

$$C(t) \sim -\left(\frac{\delta_1}{\pi}\right)^2 \ln t - \left(\frac{\delta_2}{\pi}\right)^2 \ln t,$$

$$S(t) \sim t^{-\left(\frac{\delta_1}{\pi}\right)^2 - \left(\frac{\delta_2}{\pi}\right)^2} (1 + e^{-i\epsilon_\alpha t}).$$  

(23)
where

\[ \delta_1 = \tan^{-1}(\pi \rho V), \quad \delta_2 = \tan^{-1}\left(\frac{\pi \rho \lambda^2}{\epsilon_\alpha}\right), \]

with \( \rho \) as the density of states for the spinless fermion \( \psi \). Here we have taken the ground state energy \( E_0 = 0 \) and \( \delta_2 = \frac{\pi}{2} \), if \( \epsilon_\alpha = 0 \). There are crossing terms coming from \( H_h \) and \( H_I \) of Eq. (21), but they do not contribute to power law singularities in the correlation functions.

Using the asymptotic form for \( S(t) \), we can calculate the propagator

\[ < \beta(t)\beta^+(0) > \sim e^{-i\epsilon_\beta t}(t)^{-\left(\frac{\delta_1}{\pi}\right)^2-\left(\frac{\delta_2}{\pi}\right)^2} \]

and the spin-spin correlation function

\[ M(t) \sim < S^z(t)S^z(0) > \sim \cos[(\epsilon_\alpha - \epsilon_\beta)t](t)^{-\left(\frac{\delta_1}{\pi}\right)^2-\left(\frac{\delta_2}{\pi}\right)^2}, \]

where \( S^z = \frac{1}{2}(d^+_\uparrow d^+\downarrow - d^+_\downarrow d^+\uparrow) = \frac{1}{2}(\alpha^+ \beta + \beta^+ \alpha) \). Assuming \( \delta_1 = 0, \delta_2 = \frac{\pi}{2} \), we recover the previous result, i.e., \( M(t) \sim t^{-\frac{1}{4}} \). This is a nice and independent check of the correctness of calculations in [13]. Moreover, here we have also obtained result away from the unitarity limit \( \delta_1 = 0 \). Of course, the asymptotic form is valid only for small \( \delta_1 \), because the model (21) cannot be solved exactly.

As mentioned in the Introduction, the neutron scattering data as well as the static measurements in \( UPd_xCu_{5-x} \) (\( x = 1, 1.5 \)) show a power law behavior of the spin-spin correlation function and the impurity contribution to the specific heat and magnetic susceptibility [17, 19]. Using the conformal invariance for the impurity problem it has been argued that the available data are consistent with a critical exponent [13] \( \Delta = \frac{1}{3} \), while the value following from the bosonization calculation [13] was \( \Delta_B = \frac{2}{3} \) which is rather big compared with the experimental value. The deviation from the unitarity limit \( \delta_1 = 0 \) will reduce this value. To fit the data we need to assume \( \delta_1 = \pi \sqrt{5/12} \) which is still rather big to justify the applicability of our asymptotic expansion. Nevertheless, the correction is in the right direction.
V. CONCLUDING REMARKS

Using the ND and AY perturbation expansion and mapping onto a generalized resonant-level model we have reconfirmed and extended earlier results on non-FL behavior in an extended Anderson model with additional screening channels. The extension to regions away from the unitarity limit of screening electron scattering improves the agreement with experiment. However, in view of its physical implications this issue should be further studied using other techniques.

As mentioned in the Introduction, the Anderson model has been also generalized in a different way [28,29]. Without going into detailed comparison we briefly comment on the study of the Toulouse limit in that model [29]. Those authors have correctly pointed out the change of sign for the $V_{\perp}$ term upon bosonization. However, the correlation functions were calculated there using the mean field approximation in handling the constraint which missed the non-FL exponent. Of course, the physical consequences depend strongly on the positions of the renormalized levels $\epsilon_\alpha$ and $\epsilon_\beta$ (see eq. (26)). If the difference is big, the fast oscillation will suppress the power law component in the frequency response. Since the theoretical calculation of level renormalization is very difficult, we may count on experimental indication which seems to show the existence of remaining degeneracy. The situation here is similar to the two channel Kondo case [27]. There the decoupling of $d^\dagger + d$ from the conduction electron was a signature of non-FL behavior which becomes more apparent in the Majorana fermion formulation [31]. Here the $\beta$ particle decouples, giving rise to a X-ray edge like singularity and a residual entropy, as in the case of two-channel Kondo. Of course, this analogy is more mathematical than physical. The difference of non-FL behavior in the multi-channel Kondo model and the extended Anderson model in the mixed valence regime, and their possible connections have to be further explored. A detailed comparison of the scaling theory with numerical Wilson RG studies, as well as a stability analysis of the strong coupling fixed point in the extended Anderson remain outstanding issues.

Finally, we would like to thank G.M. Zhang for an earlier collaboration on this project.
and M. Fabrizio for helpful discussions. Mobility within Europe involved in this research project was partly sponsored by EEC, through contract ERB CHR XCT 940438.
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