Pair production of neutral Higgs bosons through noncommutative QED interactions at linear colliders

Harald Grosse
Institut für Theoretische Physik, Universität Wien, Boltzmannsgasse 5, A-1090 Wien, Austria

Yi Liao
Institut für Theoretische Physik, Universität Leipzig, Augustusplatz 10/11, D-04109 Leipzig, Germany

Abstract

We study the feasibility of detecting noncommutative (NC) QED through neutral Higgs boson ($H$) pair production at linear colliders (LC). This is based on the assumption that $H$ interacts directly with photon in NCQED as suggested by symmetry considerations and strongly hinted by our previous study on $\pi^0$-photon interactions. We find the following striking features as compared to the standard model (SM) result: (1) generally larger cross sections for an NC scale of order 1 TeV; (2) completely different dependence on initial beam polarizations; (3) distinct distributions in the polar and azimuthal angles; and (4) day-night asymmetry due to the Earth’s rotation. These will help to separate NC signals from those in the SM or other new physics at LC. We emphasize the importance of treating properly the Lorentz noninvariance problem and show how the impact of the Earth’s rotation can be used as an advantage for our purpose of searching for NC signals.

PACS: 12.60.-i, 02.40.Gh, 13.10.+q, 14.80.Cp
Keywords: noncommutative field theory, neutral Higgs boson, linear collider
Noncommutative (NC) field theories have recently received a lot of attention mainly because of their connection to string theories \[1\], but they are certainly interesting in their own right. A possible way to construct the NC version of a field theory from its ordinary commutative counterpart is by replacing the usual product of fields in the action with the \(\star\)-product of fields. The \(\star\)-product of the two fields \(\phi_1(x)\) and \(\phi_2(x)\) is defined as

\[
(\phi_1 \star \phi_2)(x) = \left[ \exp \left( i/2 \theta^{\mu\nu} \partial_\mu \partial_\nu \right) \phi_1(x) \phi_2(y) \right]_{y=x},
\]

where the \(\theta^{\mu\nu}\) is a real antisymmetric constant matrix that parametrizes the noncommutativity of spacetime,

\[
[x^\mu, x^\nu] = i\theta^{\mu\nu},
\]

and has dimensions of length squared.

NC quantum electrodynamics (NCQED) of photons and electrons is then given by the following Lagrangian \[2\]:

\[
\mathcal{L} = \frac{1}{4} F^{\mu\nu} \star F_{\mu\nu} + \bar{\psi} \star (\gamma^\mu iD_\mu - m)\psi,
\]

with \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu]_\star\) and \(D_\mu \psi = \partial_\mu \psi + ie A_\mu \star \psi\), where the Moyal brackets are defined as \([\phi_1, \phi_2]_\star = \phi_1 \star \phi_2 - \phi_2 \star \phi_1\). The action \(\int d^4x \mathcal{L}\) is invariant under the generalized \(U(1)\) gauge transformation

\[
A_\mu \rightarrow A'_\mu = U \star A_\mu \star U^{-1} + ie^{-1}U \star \partial_\mu U^{-1},
\]

\[
\psi \rightarrow \psi' = U \star \psi,
\]

with \(U(x) = (\exp[i\lambda(x)])_\star\), under which \(F_{\mu\nu}\) also undergoes a nontrivial transformation, \(F_{\mu\nu} \rightarrow F'_{\mu\nu} = U \star F_{\mu\nu} \star U^{-1}\). Note that the neutral photon interacts with itself due to the Moyal bracket term in \(F_{\mu\nu}\) as in the usual non-Abelian gauge theory.

NC field theories have rich phenomenological implications due to the appearance of new interactions and Lorentz noninvariance introduced by the constant \(\theta_{\mu\nu}\) matrix. Some aspects of NCQED have been explored recently. From the point of view of effective field theories new physics effects amounts to introduction of some high dimension operators made up of ordinary fields and proportional to \(\theta_{\mu\nu}\). Since the latter carries two negative units in mass the effects are suppressed by two factors of some energy scale \(\Lambda_{NC}\) at which the noncommutativity sets in. At low energies they could only be detectable with precisely measured quantities, e.g., in some atomic systems \[3\]. However, the suppression becomes less severe at high energy linear colliders (LC) if \(\Lambda_{NC}\) is not much larger than the
collider’s energy. Considering the connection to string theories and the possibility that gravitational and gauge interactions may unite at a scale of order 1 TeV in the framework of string theories, it is reasonable to expect that the NC effects may also enter into the game at a similar scale. Along this line of argument some authors have discussed the feasibility of detecting NC signals at future LC through corrections to standard model (SM) processes. Indeed they found that with a collider energy of $0.5 \sim 1.5$ TeV and an integrated luminosity of a few hundred fb$^{-1}$ it is possible to probe $\Lambda_{NC}$ up to a few TeV at 95% C.L. In this study we will work in the same spirit, but we will present some striking features of the process considered here which will be very helpful in distinguishing NC signals from those of the SM or other new physics. We will emphasize the important issue of Lorentz noninvariance in collider measurements and show how the impact of the Earth’s rotation can be used for our purpose of detecting NC signals.

Another motivation derives from a recent work in which we showed how a simple and reasonable generalization of the anomalous $\pi^0$-photon interaction can lead to the three photon decay of the $\pi^0$ in NCQED. The idea has got some support from analysis of anomalies in NC gauge theories. We found that for the consideration to be physically self-consistent it is mandatory to treat the electrically neutral photon and $\pi^0$ on the same footing; namely the $\pi^0$ field must also undergo the same nontrivial transformation under $U(1)$ as the photon as if they were in the adjoint representation of an effectively non-Abelian gauge theory,

$$\phi^0 \rightarrow \phi^{0*} = U \star \phi^0 \star U^{-1}, \quad (5)$$

where $\phi^0$ stands for the $\pi^0$ field, so that neutral particles also participate in electromagnetic interactions,

$$\mathcal{L}_{\phi^0} = \frac{1}{2} D_\mu \phi^0 \star D^\mu \phi^0, \quad (6)$$

with $D_\mu \phi^0 = \partial_\mu \phi^0 + ie[A_\mu, \phi^0]^*$. This is reminiscent of the wisdom in the usual quantum field theory that one must keep all possible interactions that are consistent with symmetries for the theory to be renormalizable. Indeed it is far from clear at the moment how to extend the electroweak SM to NC spacetime consistently although there are already theoretical efforts and even phenomenological analysis on flavor physics in this direction. However we believe that the impressive lesson learnt from $\pi^0$ should be general enough to be applicable to other neutral particles in the SM if it permits any kind of generalization to the NC case. Then the nice feature of uniformness and completeness
among neutral particles concerning their electromagnetic interactions can be preserved in the generalized SM. This is especially true of the Higgs boson which triggers the electroweak symmetry breakdown to the electromagnetic $U(1)$. Then Eqs. (5) and (6) apply equally well to the Higgs field $H$.

A direct result of interactions (3) and (6) with $\phi^0$ now denoting the neutral Higgs $H$ with mass $m_H$ is the occurrence of the following process at the tree level:

$$e^-(k_1, \lambda_1) + e^+(k_2, \lambda_2) \rightarrow H(p_1) + H(p_2),$$  \hspace{1cm} (7)

where $k_i(p_i)$ are incoming (outgoing) momenta, and $\lambda_i = \pm 1$ are initial state helicities. The process proceeds through the $s$-channel exchange of photon, whose amplitude is given by

$$A_{\lambda_1 \lambda_2} = i e^{-(i/2) k_1 \theta k_2} 4 e^2 s \sin \left( \frac{1}{2} p_1 \theta p_2 \right) \bar{u}_1 P_1 u,$$  \hspace{1cm} (8)

where $u, v$ are spinors, $P_{1,2} = (1 \pm \lambda_{1,2} \gamma^5)/2$, and $\sqrt{s}$ is the center-of-mass (c.m.) energy. We have used the abbreviation $p \theta q = \theta_{\mu \nu} p^\mu q^\nu$. Let us first work in the c.m. frame and denote as $(\theta, \phi)$ the polar and azimuthal angles of the Higgs boson. (This $\theta$ should not be confused with the NC parameter $\theta_{\mu \nu}$.)

Due to momentum conservation and antisymmetry of $\theta_{\mu \nu}$, only the components $\theta^{0i} \equiv \langle \vec{\theta} \rangle^i$ can contribute. Without loss of generality we assume that $\vec{\theta}$ lies in the $xz$ plane and deviates from the $z$ axis (i.e. the $e^-$ beam direction) by an angle $\gamma \in [0, \pi]$. For the parameters to be considered later, it is appropriate to use $\sin(p_1 \theta p_2/2) \approx p_1 \theta p_2/2$. The differential cross sections with polarized or unpolarized beams are

$$\left[ \frac{d\sigma}{d\Omega} \right]_{\lambda_1 \lambda_2} = (1 - \lambda_1 \lambda_2) \left[ \frac{d\sigma}{d\Omega} \right]_{\text{unpol}},$$

$$\left[ \frac{d\sigma}{d\Omega} \right]_{\text{unpol}} = \frac{\alpha^2 \beta^5}{64 s} (s|\vec{\theta}|)^2 (s \gamma s s c_\phi + c_\gamma c_\theta)^2 s_\theta^2,$$  \hspace{1cm} (9)

where the factor $1/2!$ for identical particles has been included, $\beta = \sqrt{1 - 4 m_H^2/s}$ is the Higgs boson velocity, and $c_\theta = \cos \theta$, $s_\theta = \sin \theta$, etc. The factors $\beta^3 s_\theta^2$ are due to the scalar nature of the Higgs boson and phase space while the additional factor $\beta^2$ and the one in brackets are peculiar to the NCQED interaction $\mathcal{L}_{\phi \phi}$. The nontrivial azimuthal angle dependence arises because rotational invariance is broken by the preferred direction of $\vec{\theta}$.

The total cross sections are

$$\sigma_{\lambda_1 \lambda_2} = (1 - \lambda_1 \lambda_2) \sigma_{\text{unpol}},$$

$$\sigma_{\text{unpol}} = \sigma_0 \left( 1 - c_\gamma^2/2 \right),$$

$$\sigma_0 = \frac{\pi \alpha^2}{60} \beta^5 s|\vec{\theta}|^2.$$

\hspace{1cm} (10)
The proportionality to $s$ causes no problem with unitarity since it arises from an approximation which does not hold at very high energy.

One might expect to use Eqs. (9) and 10 for numerical analysis as was done in the literature [1]. However, the above results are not directly applicable to a practical collider experiment. This is important because it would result in an incorrect interpretation of data. Furthermore, as shown below this would also cause an unnecessary loss of information specific to NC signals. Since $\theta_{\mu\nu}$ is not a Lorentz tensor and is given in some a priori frame, it should change from one frame to another differently from a tensor. For a practical collider experiment it takes a much longer time than a day to collect data so we may expect important impacts from the Earth’s rotation on data analysis. In the case considered here, the particles involved move much faster than the Earth’s rotation, therefore we can safely ignore the change in magnitude of $\vec{\theta}$ in the local c.m. frame. But we must take into account the change of its direction relative to the local frame, or, to put it more correctly, the rotation of our local frame as the Earth rotates.

Let us denote as $\rho \in [0, \pi]$ the angle between $\vec{\theta}$ and the Earth’s rotation axis $\vec{R}$, which is fixed to good precision. The location of the lab is described in terms of two angles: the latitude $\sigma \in [-\pi/2, \pi/2]$ with positive (negative) $\sigma$ denoting northern (southern) hemisphere, and the Earth’s rotation angle (longitude) $\omega \in [0, 2\pi)$ measured relative to the plane spanned by $\vec{R}$ and $\vec{\theta}$. Suppose the collider beam has an angle $\delta \in [0, 2\pi)$ from the local longitudinal direction. The angles $\sigma$ and $\delta$ are fixed for a given collider. Then, we have

$$c_\gamma = -s_\rho (c_\delta s_\omega + s_\delta s_\sigma c_\omega) + c_\rho s_\delta c_\sigma.$$  

Upon considering the Earth’s rotation we may have two types of distributions, one in the local angles $\theta$ and $\phi$, and the other in $\omega$ of the Earth’s rotation. For the former we merely have to average over the rotation and find for the unpolarized cross section,

$$\frac{4\pi}{\sigma_0} \frac{d\sigma}{d\Omega} = f(\theta, \phi),$$  

$$f(\theta, \phi) = \frac{15}{4} \left( s_\gamma s_\theta c_\phi^2 + c_\gamma c_\theta^2 + s_\gamma s_\theta s_2 \theta \phi \right) s_\theta^2,$$

where $s_{2\theta} = \sin(2\theta)$, etc. This amounts to analyzing data as is usually done. To better describe the impact of the Earth’s rotation we define the following day-night asymmetry
as a periodic function of $\omega$ or time $t$,
\[
A_{DN}(\omega_a, \omega_b) = \frac{\int_{\omega_a}^{\omega_b} d\omega - \int_{\omega_a}^{\omega_a+\pi} d\omega}{\int_{\omega_a}^{\omega_a+\pi} d\omega + \int_{\omega_a}^{\omega_b+\pi} d\omega} \sigma(\omega) = \frac{N(\omega_b) - N(\omega_a)}{D(\omega_b) - D(\omega_a)},
\]
(13)

where $\sigma(\omega)$ is given in Eq. (10) and
\[
N(x) = (-c_3 c_x + s_3 s_\sigma s_x) s_{2p} s_3 c_\sigma,
\]
\[
D(x) = -x c_3^2 s_3^2 c_\sigma^2 + 2x - s_\rho^2/4 \left[ 2x (s_3^2 s_\sigma^2 + c_3^2) + (s_3^2 s_\sigma^2 - c_3^2) s_{2x} - s_{2s} s_\sigma c_{2x} \right].
\]
(14)

And the integrated asymmetry is simply
\[
A_{DN}(0, \pi) = \frac{s_{2p} s_3 c_\sigma}{2\pi \left[ 1 - \frac{1}{4} \left( s_\rho^2 (c_3^2 + s_3^2 s_\sigma^2) + 2c_3^2 s_3^2 s_\sigma^2 \right) \right]}.
\]
(15)

Now let us examine the numerical significance of the above results. The scale of cross section is set by $\sigma_0$ which is plotted in Fig. 1 as a function of $m_H$ at $\sqrt{s} = 0.5, 1, 1.5$ TeV and for $\Lambda_{NC} = |\vec{\theta}|^{-1/2} = 1$ TeV. This should be contrasted with the SM result \[11\] which is $0.1 \sim 0.2$ fb for $m_H < 2m_W$ at $\sqrt{s} = 0.5$ TeV and for the whole mass range shown at higher $\sqrt{s}$. We see that with $\sqrt{s} = 1$ TeV or higher the NC signal dominates over the SM background for an intermediate-mass Higgs boson. Since such processes will be searched for only after the Higgs boson has been found in its main production channels, the relatively low cross section can be compensated for by some knowledge of Higgs properties and by a high luminosity feasible at future LC. If the beams are properly polarized, the situation can even be better. Since NCQED interactions conserve helicity, we expect equal contributions from left-handed (LH) and right-handed (RH) polarized electron beams. For example, with RH electron and LH positron beams we have a cross section twice as large as the unpolarized one [see Eq. (10)]. In the SM, the same process is overwhelmingly dominated by one-loop $W^\pm$ boxes so that the cross section for RH electron and LH positron beams is smaller by at least one order of magnitude than in the oppositely polarized case. Thus with suitably polarized beams one can earn a signal over background ratio of a few tens even before a cutoff is imposed. This already makes the process considered here much more advantageous than those considered so far.

Some knowledge of Higgs properties beforehand also helps to reconstruct the final state of the process and to analyze the distribution of primary Higgs bosons. In Fig. 2(a) \[2(b)] we plot the distribution $f(\theta, \phi)$ in the local angle $\theta$ ($\phi$) at a specified value of $\phi = \pi/4$
(θ = π/4) after averaging over the Earth’s rotation. Note that \( f(\theta, \phi) \) depends only on orientation parameters. We consider three sets of them for illustration: (1) \( \rho = \pi/2, \delta = 0 \) and \( \sigma \) free; (2) \( \rho = 0, \delta \) and \( \sigma \) free but \( s_\delta c_\sigma = 1/\sqrt{2} \); and (3) \( \rho = \delta = \sigma = \pi/4 \). Also shown is the distribution further averaged over \( \phi (\theta) \) for the parameter set (3). In practice \( \sigma \) and \( \delta \) are known for a given collider; therefore we can fit the two distributions for just one angle \( \rho \), which will balance the relative rareness of data in determining the relative direction of \( \vec{\theta} \) to \( \vec{R} \). The distribution can also be easily discriminated from the SM one which follows approximately the \( \sim \sin^2 \theta \) law [11]. Even if the NC signal accidentally shares a similar \( \theta \) dependence (after averaging over \( \phi \)) with some other new physics signals, they can still be discriminated by \( \phi \) dependence since the latter are trivial in \( \phi \) dependence due to Lorentz invariance.

The above feature is further strengthened by the day-night asymmetry, shown in Fig. 3 as histograms binned per half an hour for two sets of orientation parameters: the above case (3), and (4) \( \rho = \pi/4, \delta = 3\pi/4 \) and \( \sigma = 0 \). Note that there is no asymmetry at \( \rho = 0, \pi/2, \pi, \) or \( \delta = 0, \pi, \) or \( \sigma = \pm \pi/2 \) and additionally at \( \delta = \pi/2, 3\pi/2 \) for the integrated asymmetry. For most of these unfortunate orientations we can still observe the periodic variation of the cross section with the Earth’s rotation. Since this asymmetry or periodic variation arises from Lorentz noninvariance in NCQED, it may be readily separated from the null results in ordinary theories like the SM and beyond.

The search for Higgs bosons and NC gauge theories are important topics that attract a lot of attention. We attempted here to connect them through an analysis of neutral Higgs pair production at LC. The result is quite encouraging. We identified the salient features of the process which proves to be much more advantageous than others considered so far in that one can have a good signal-background (S/B) ratio with unpolarized beams and an even excellent S/B with suitably polarized beams, and is thus unique in search for NC signals at LC. We have described for the first time how a practical measurement is affected by the Earth’s rotation and how this impact may be used as an advantage in discriminating NC signals from those in the ordinary commutative theories like SM and other new physics.

We are grateful to Klaus Sibold for many encouraging and helpful discussions and for carefully reading the manuscript. H.G. enjoyed the stay at ITP, Universität Leipzig where part of work was done.
References

[1] A. Connes, M. R. Douglas and A. Schwarz, J. High Energy Phys. 02, 003 (1998); M. R. Douglas and C. Hull, *ibid.* 02, 008 (1998); N. Seiberg and E. Witten, *ibid.* 09, 032 (1999).

[2] C. P. Martin and D. Sanchez-Ruiz, Phys. Rev. Lett. 83, 476 (1999); T. Krajewski and R. Wulkenhaar, Int. J. Mod. Phys. A 15, 1011 (2000); M. M. Sheikh-Jabbari, J. High Energy Phys. 06, 015 (1999); M. Hayakawa, Phys. Lett. B 478, 394 (2000); [hep-th/9912167]; A. Matusis, L. Susskind and N. Toumbas, J. High Energy Phys. 12, 002 (2000).

[3] I. Mocioiu, M. Pospelov and R. Roiban, Phys. Lett. B 489, 390 (2000); M. Chaichian, M. M. Sheikh-Jabbari and A. Tureanu, Phys. Rev. Lett. 86, 2716 (2001).

[4] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 429, 263 (1998); Phys. Rev. D 59, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 436, 257 (1998).

[5] H. Arfaei, M. H. Yavartanoo, [hep-th/0010244]; J. L. Hewett, F. J. Petriello and T. G. Rizzo, [hep-ph/0010354]; P. Mathews, Phys. Rev. D 63, 075007 (2001); S. Baek, D. K. Ghosh, X.-G. He, W.-Y. P. Hwang, *ibid.* 64, 056001 (2001).

[6] H. Grosse, Y. Liao, [hep-ph/0104260].

[7] F. Ardalan, N. Sadooghi, Int. J. Mod. Phys. A 16, 3151 (2001); [hep-th/0009233]; J. M. Gracia-Bondia and C. P. Martin, Phys. Lett. B 479, 321 (2000); C. P. Martin, [hep-th/0008126].

[8] E. Langmann, J. Geom. Phys. 22, 259 (1997); L. Bonora, M. Schnabl and A. Tomasiello, Phys. Lett. B 485, 311 (2000).

[9] See, for example, B. A. Campbell and K. Kaminsky, Nucl. Phys. B581, 240 (2000); F. J. Petriello, *ibid.* B601, 169 (2001).

[10] I. Hinchliffe and N. Kersting, [hep-ph/0104137].

[11] K. J. F. Gaemers and F. Hoogeveen, Z. Phys. C 26, 249 (1984); A. Djouadi, V. Driesen and C. Junger, Phys. Rev. D 54, 759 (1996).
Figure Captions

Fig. 1. The cross section $\sigma_0$ as a function of $m_H$ at $\sqrt{s} = 0.5$ (dotted), 1.0 (solid) and 1.5 (dashed) TeV respectively.

Fig. 2. The distribution $f(\theta, \phi)$ as a function of $\cos \theta$ at $\phi = \pi/4$ (panel a) and as a function of $\phi$ at $\theta = \pi/4$ (panel b). The solid, dotted and short-dashed curves are for the parameter sets (1), (2) and (3) respectively. Also shown (long-dashed) in the panel a (b) is the distribution further averaged over $\phi (\theta)$ for the parameter set (3).

Fig. 3. Histograms of the day-night asymmetry $A_{DN}$ as a function of time $t$. The solid and dotted curves are for the parameter sets (3) and (4) respectively.
Figure 1
Figure 2
\[ A_{DN}(0, \pi) = \begin{cases} 
+0.133 \text{ (solid)} \\
-0.196 \text{ (dotted)} 
\end{cases} \]