Nanoscale Waveguide Beam Splitter in Quantum Technologies

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Abstract—Studying beam splitters as sources of entangled photons is a relevant topic in modern physics. In most cases, in quantum optics the theories of waveguide beam splitters of large and small sizes have no difference. In this work, it is shown that a nanosized waveguide beam splitter has a significant difference from a similar device, but on a larger scale. The output wave function of the beam splitter’s ports is analyzed in the arbitrary case of photons entering the first and second ports. The results obtained are sensitive to the size of the waveguide beam splitter, the coupling parameter of two waveguides, the monochromatic photons case of a linear beam splitter and the non-monochromaticity grade for photons entering both input ports of the beam splitter. The results are very important for various quantum technologies (e.g. integrated circuits, optical quantum computers) which use this type of beam splitter and quantum metrology.

Keywords—beam splitter, nanosize, photons, wave function, non-monochromatic photons, reflection coefficient, transmission coefficient.

I. INTRODUCTION

It is common knowledge that modern quantum technologies [1,2] can not be imagined without a waveguide beam splitter (BS). This device has a great prospect of application due to its small size [3-8]. This, in turn, leads to the fact that at small sizes new phenomena can arise that are not inherent in similar devices, but on a large scale. It is usually considered that the main characteristics of a waveguide beam splitter are the reflection coefficients R and transmission coefficients T are constant values. This means that by setting these parameters one can always obtain the required characteristics at the output ports of the beam splitter. It was shown in [9,10] that using a beam splitter based on coupled waveguides, i.e. waveguide beam splitters Hong-Ou-Mandel (HOM) effect may not be performed even if R=T=1/2 and the photons used are identical. It was also pointed out in these papers [9,10] that in the main such changes in the example of the HOM effect appear for sufficiently small waveguide beam splitters.

Here we show that this problem is inherent not only for the HOM effect, but also for a waveguide beam splitter in general, which has sufficiently small dimensions, and this is micro and nanoscale. Undoubtedly, the results obtained are important primarily for quantum technologies, since the use of micro and nanosized beam splitters is one of the prospects that is already beginning to be realized today.

II. THEORY

For this purpose, let us examine a linear beam splitter (BS) that consists of two coupled waveguides. The spot where they are closest to each other is subjected to the superimposition of electromagnetic fields where the entanglement of photons takes place. Original photons end up on incoming channels and outgoing photons are recorded by detectors. Known transformations for a linear beam splitter with the key characteristics - reflection coefficient R and transmission coefficient T, are presented below. This work demonstrates that they are considered constant for the linear beam splitter in the monochromatic case and frequency-dependent in the non-monochromatic case. It has been shown in [9,10] that the reflection coefficient R and the transmittance T for a waveguide beam splitter are

\[ R = \frac{\sin^2 \left( \frac{\Omega t}{2(1+\varepsilon^2)} \right)}{1+\varepsilon^2}, \]

\[ T = 1 - R; \quad \varepsilon = \frac{\omega_2 - \omega_1}{\Delta} \]

where \( \Omega \) is a certain frequency characterizing the BS; \( t_{BS} = L/v \) is the time of interaction of photons in the BS (L is the length of the coupled region of the waveguide (see Fig. 1), v is the speed of wave propagation in the waveguide); \( \omega_1 \) and \( \omega_2 \) are characterizing photon oscillations on the input ports of the BS. It is frequently named frequencies.

Fig. 1. The waveguide beam splitter’s 3D visualization consists of input and output ports of the waveguides with a coupler in the middle of it. Both ports have their wave functions respectively.
In order to evaluate quantum entanglement the von Neumann entropy was selected. The main problem during the quantum entanglement measurements is the calculation of the Schmidt modes.

\[ S_N = -\sum_k A_k \ln(A_k); \]  

(2)

Using known transformations for a linear beam splitter we obtain an expression for the calculation of the Schmidt modes for monochromatic photons which looks the following way. It is clear that the expression depends solely on one parameter - the reflection coefficient. It is worth noting that the choice of quantum numbers of states must satisfy the condition \( s_1 + s_2 = k + p \).

It is an interesting fact that the level of coupling in the waveguides is determined by the parameter \( \Omega \). Varying the parameter \( \Omega \) can be used as an instrument for altering the level of coupling in the waveguides. There is the representation of the complete output wave function of the photons at the beam splitter

\[ \psi_{out} = \sum_{k=0}^{k+s_2} \int \phi(\omega_1, \omega_2) c_{k,p} |k, s_1 + s_2 - k\rangle d\omega_1 d\omega_2, \]  

(3)

where \( |k, s_1 + s_2 - k\rangle = |k\rangle |p\rangle \) is the state of the photons at the output ports of the BS, \( c_{k,p} \) these are some coefficients, see [9]. \( \phi(\omega_1, \omega_2) \) is the joint spectral amplitude (JSA) of the two-modes wavefunction

\[ \int |\phi(\omega_1, \omega_2)|^2 d\omega_1 d\omega_2 = 1 \]  

(4)

III. RESULTS AND DISCUSSION

In (3) the coefficient \( c_{k,p} \) depends on \( R \) and \( T \) [9,10]. Analysis and numerical calculations show that if we consider the photons to be monochromatic, the \( \psi_{out} \) wave function that describes the state of output photons of the beam splitter has the same form at small and large dimensions of the beam splitter. If one assumes that the incoming photons are non-monochromatic (this is usually the case), then at small and large dimensions of the beam splitter the wave functions of photons at the output ports have a different form.

Fig. 2 illustrates how the von Neumann entropy alters with \( R \) for different pairings of quantum numbers \( s_1 \) and \( s_2 \) for monochromatic photons where \( R \in (0,1) \).

![Image](image.png)

Fig. 2. The dependence of von Neumann entropy as a function of \( R \) for different pairings of quantum numbers \((s_1,s_2)\).

It should be mentioned that most dependences have two main maxima except in the case when one of the quantum numbers equals zero.

\[ |\psi(t)\rangle = \sum_k \sqrt{\lambda(t)} u_k(x_1,t) v_k(x_2,t), \]  

(5)

\[ \lambda_k(R) = |c_{k,s_1+s_2-k}|^2, \]  

(6)

For purposes of illustration, there is the von Neumann entropy formulation for different pairings of quantum numbers:

\[ s_1 = 0 \text{ and } s_2 = 2: \]

\[ S_N = -2(R-1)(R \ln 2 + \ln(1-R)) - 2R \ln R \]  

(6)

\[ s_1 = 1 \text{ and } s_2 = 1: \]

\[ S_N = -(1-2R)^2(2R-1)^2 + 4R(R-1)\ln(2R(1-R)) \]  

(7)

\[ s_1 = 1 \text{ and } s_2 = 3: \]

\[ S_N = 6(1-2R)^2(R-1)(R-1)\ln(-6(1-2R)^2(R-1)) - 4R^3(4R-1)^2 + 4R^3(4R-1)\ln(-4R^3(4R-1)) - R^2(3-4R)^2\ln(R^2(4R-3)^2) - (1-5R + 4R^2)^2\ln((1-5R+4R^2)^2) \]  

(6)

| \((s_1,s_2)\) | \(S_N\) | \(R\) |
|---|---|---|
| \((1,1)\) | 3 | 0.5 (1± 1/\sqrt{3}) |
| \((0,2)\) | 2.67 | 0.5 |
| \((1,2)\) | 4.4 | 1/3 and 2/3 |
| \((1,3)\) | 4 | 0.5 |

It would appear that the highest entanglement should occur around \( R = 1/2 \), therefore this is a really fascinating conclusion. As is clearly seen from Fig. 2, if we change the reflection coefficient \( R \), it significantly alters the value of quantum entanglement, but it is set to zero at \( R = 0; 1 \). Quantum entanglement increases with increasing \( s_1 \) and \( s_2 \), quantum numbers.

Assuming the implementation of such a beam splitter, let’s think about the quantum entanglement of photons. A useful method for evaluating quantum entanglement is the von Neumann entropy. [9]

\[ A_k = \int |\phi(\omega_1, \omega_2)|^2 |c_{k,s_1+s_2-k}|^2 d\omega_1 d\omega_2 \]  

(8)

For calculations, we will use the most well-known spectral function:

\[ \phi(\omega_1, \omega_2) = \phi_1(\omega_1) \phi_2(\omega_2); \]  

(9)

\[ \phi_1(\omega_1) = \frac{1}{\sqrt{\pi(2\sigma_1^2)}} e^{-\frac{(\omega_1 - \omega_0)^2}{4\sigma_1^2}}. \]  

(10)

It should be added that form (3) is the best-known function and corresponds to the distribution of photons in Fock states.

The results of the calculations are presented in Fig. 3.
If we consider optical photons, we get $\omega_0 \approx 10^{15} \text{rad/s}$ we obtain

$$\Omega \approx (10^{14} - 10^{17}) \text{rad/s} \quad (11)$$

The value of $\Omega$ is important since it determines on what scales certain quantum-entangled properties of beam splitters will manifest themselves.

The dependence types for monochromatic and non-monochromatic photons are significantly different, and quantum entanglement reaches its maximum values in the case of non-monochromatic photons. At high values of beam splitter parameter $\Omega_{\text{BS}}$ the right side of the chart shows that quantum entanglement takes the highest values when the $\sigma/\Omega$ parameter is roughly equal to one. In addition, at high values of beam splitter parameter $\Omega_{\text{BS}}$ it is possible to obtain simple analytical expressions for the von Neumann entropy depending solely on the $\sigma/\Omega$ parameter. The same patterns exist for other pairs of quantum numbers as well.

IV. CONCLUSION

The reflection coefficient $R$, which is the only parameter used to represent the quantum entanglement of two linked harmonic oscillators, was demonstrated; During we achieve extremely uncomplicated analytical formulas for $S_N$ as a measure of quantum entanglement, using only one characteristic (the reflection coefficient $R$); these formulas were developed without considering the environment, which has a considerable impact on quantum entanglement; it is shown that for certain values of $R$ and pairing of $s_1$ and $s_2$, quantum entanglement reaches their peak value and can be considered as large; for $R = 0$ and $R = 1$ quantum entanglement does not exist.

- Using the frequency dependence of the coefficients $R$ and $T$, the quantum entanglement of the waveguide beam splitter differs from the previously known theory
- For very large values of $\Omega$, it is possible to obtain analytical expressions
- Quantum entangled photons can be generated in good quantities using waveguide beam splitters.
- For non-monochromatic photons, the maximum value of quantum entanglement is accurately attained.
- At large $L >> c/\Omega$, the quantum entanglement does not depend on the length of the beam splitter and transforms into a constant value.
- When $L$ is of the $c/\Omega$ order, beam splitter length must be considered since it affects quantum entanglement. This length for optical photons is on the order of nanometers.

The developed hypothesis must be taken into consideration while employing nanosized beam splitters since the results differ from those previously known e.g. [10].

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