Two Squares of Opposition: for Analytic and Synthetic Propositions

Andrew Schumann

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Abstract

In the paper I prove that there are two squares of opposition. The unconventional one is built up for synthetic propositions. There $a, i$ are contrary, $a, o$ (resp. $e, i$) are contradictory, $e, o$ are subcontrary, $a, e$ (resp. $i, o$) are said to stand in the subalternation.

1 Introduction

Since Aristotle the best known logical pattern has been presented by square of opposition. However, since Kant and Rickert many philosophers have paid attention that this square does not satisfy synthetic propositions. In this paper we are proving that there are two squares of opposition under following assumptions:

- we have the Boolean complement;
- synthetic propositions cannot be reduced to manipulations with Venn diagrams, because they do not suppose including relations.

2 The short history of the square of opposition

Aristotle proposed the following four oppositions: contradiction, contrariety, relation, and privation (*Categories*, Chapter 10, *Metaphysics*, Book I) that semantically underlay the square of opposition:
We must next explain the various senses in which the term ‘opposite’ is used. Things are said to be opposed in four senses: (i) as correlatives to one another, (ii) as contraries to one another, (iii) as privatives to positives, (iv) as affirmatives to negatives (Categories, 10).

Aristotle himself described relations of the square of opposition to represent singular expressions (Prior Analytics, Chapter 46), see figure 1. He had an intuition that quantifiers (both universal and existential ones) satisfy the semantic relations of the square, too:

An affirmation is opposed to a denial in the sense which I denote by the term ‘contradictory’, when, while the subject remains the same, the affirmation is of universal character and the denial is not. The affirmation ‘every man is white’ is the contradictory of the denial ‘not every man is white’, or again, the proposition ‘no man is white’ is the contradictory of the proposition ‘some men are white’. But propositions are opposed as contraries when both the affirmation and the denial are universal, as in the sentences ‘every man is white’, ‘no man is white’, ‘every man is just’, ‘no man is just (On Interpretation, 7).

\[
\begin{array}{c|c}
S \text{ is } P & S \text{ isn’t not-} P \\
\hline
S \text{ is not-} P & S \text{ is not } P
\end{array}
\]

Figure 1: Aristotle’s square of opposition.

However, for the first time, Apuleius explicitly claimed that quantified propositions satisfy the square. He wrote a short book, the Peri Hermeneias
about logic that was used for centuries in teaching. This book is the most famous in the history of logic for including the first appearance of the square of opposition, the best known logical schema for the pedagogic purpose. He considered the four oppositions: contrary, subcontrary, contradictory, subalternation. First, he described that the two *incongruae* (contrary) propositions, on the left and right sides of the top of the square, never can be true at the same time and nonetheless are sometimes false at the same time. For example, when some pleasures are good, both universal propositions are false at the same time, since it is impossible that every pleasure is both a good and not a good. The two propositions along the bottom line (i.e. the mirror-image of the contrary) are called *subpares* (subcontrary). They are never false at the same time, but they can be true at the same time. Hence, to confirm that some pleasure is a good we cannot use an argument that some other pleasure is not a good. Further, we pair together the *alterutrum* (contradictory) propositions, if we add a negation to each of the pair of alternates, e.g. not every pleasure is a good means that some pleasure is not a good. Finally, the *subalternation* appears between universal and particular propositions, when the universal implies the particular, e.g. if every pleasure is a good, then some pleasure is a good.

The meaning of propositions that satisfy the square of opposition can be checked on Venn diagrams. Recall that a Venn diagram is an ellipse that designates an extent of a concept $A$, i.e. a class of all real things that are denoted by $A$. These things are called denotations. By assumption, all inner points of ellipse designate appropriate real things. For instance, ‘every man is mortal’ is a true proposition, because the Venn diagram of ‘man’ is included into the Venn diagram of ‘the mortal being’ (i.e. all denotations of ‘man’ occur among denotations of ‘the mortal being’).

Kant first paid attention that there exists a true universal proposition like ‘all bodies are heavy’ such that Venn diagrams of its subject and predicate do not assume the including relation. So, the Venn diagram of ‘body’ just intersects the Venn diagram of ‘heavy’:

In all judgments in which the relation of a subject to the predicate is thought . . . , this relation is possible in two different ways. Either the predicate to the subject $A$, as something which is (covertly) contained in this concept $A$; or outside the concept $A$, although it does indeed stand in connection with it. In the one case I entitle the judgment analytic, in the other synthetic.
Analytic judgments (affirmative) are therefore those in which the connection of the predicate with the subject is thought through identity; those in which this connection is thought without identity should be entitled synthetic (…) If I say, for instance, ‘All bodies are extended’, this is an analytic judgment. For I do not require to go beyond the concept which I connect with ‘body’ in order to find extension as bound up with it … The judgment is therefore analytic. But when I say, ‘All bodies are heavy’, the predicate is something quite different from anything that I think in the mere concept of body in general; and the addition of such a predicate therefore yields a synthetic judgment.

Judgments of experience, as such, are one and all synthetic. For it would be absurd to found an analytic judgment on experience. Since, in framing the judgment, I must not go outside my concept, there is no need to appeal to the testimony of experience in its support.

Thus, in Kant’s opinion, only analytic judgments satisfy the square of opposition. For synthetic judgments Venn diagrams lose any sense and, as a result, we cannot apply the square for them. In Aristotelian logic there is the inverse relation between content (class of all connotations) and extent (class of all denotations) of a concept. By continuing the Kant’s ideas, Heinrich Rickert claimed that for synthetic judgments (propositions) there is the direct relation between content and extent of a concept. Therefore we cannot use Venn diagrams there at all.

Thus, according to Kant and Rickert, there are two logics (the Aristotelian for analytic propositions, where we can use Venn diagrams and the square of opposition, and the non-Aristotelian for synthetic propositions without Venn diagrams manipulations). This distinction entails another distinction (proposed first by Wilhelm Windelband and Heinrich Rickert) between two kinds of sciences: natural sciences (Naturwissenschaften) and cultural sciences (Geisteswissenschaften). In the first the Aristotelian logic is used by applying a nomothetic approach, in the second the non-Aristotelian by applying an idiographic approach. The idiographic approach is concerned with individual phenomena, as in biography and much of history, while its opposite, the nomothetic approach, aims to formulate laws as general propositions.

In the history as in an individualizing science (eine individualisierende Wissenschaft) we obtain the direct relation between content and extent of
historical concepts: the more general historical concept is more value relevant at the same time:

It obviously remains true that in characterizing one part of historical activity, we can speak of a re-creative understanding of the “spiritual” [der geistigen] world. But the concepts of understanding and re-creation are also too imprecise and general to provide a fully autonomous and exhaustive characterization of the nature of all historical representation. As regards understanding, it is important, first, that the object of understanding in history is always something more than merely real; namely, it is value relevant and meaningful. And second, to remain within the domain of history, the value relevant and the meaningful are comprehended not in a generalizing fashion but in an individualizing fashion, even though their content may be only relatively historical. Finally, even the concept of the “re-creation” of historical individuality acquires its precise significance for the theory of the historical sciences only on the basis of the concept of the individualizing understanding of meaning [10].

Any goodness will be crowned               Some goodness will be crowned

No goodness will be crowned               Some goodness will not be crowned

Figure 2: An example of the square of opposition for individualizing propositions.
Under these conditions, the general does not imply the particular. For instance, as we saw, we cannot differ the universal proposition ‘all bodies are heavy’ from the particular one ‘some bodies are heavy’ by Venn diagrams, because the extents of their concepts (the extents of ‘bodies’ and ‘heavy’) are just intersected.

Rickert did not think of creating a new square of opposition that may become suitable for describing semantic oppositions between synthetic (historical, individualizing) propositions. If we set up such a problem, we will start in distinguishing between general and particular synthetic propositions.

For analytic propositions while we move from the general (i.e. the concept with the larger extent and the smaller content) to the particular (i.e. the concept with the smaller extent and the larger content), we are losing definiteness and certainty. For synthetic propositions, the general and the particular are two different points of view, because both have different extents and different contents with the same certainty, which satisfy a direct relation between them.

In the Apuleian square of opposition there is a duality between the general and the particular. Indeed, for the general there is a contrary negation and for the particular a subcontrary negation, thereby the contrary negation tends to be maximized and the subcontrary negation tends to be minimized. In the new square of opposition (see figure 2) we could propose another duality that takes place between the affirmation and the negation. In this case the contrary negation holds between the general affirmative proposition and the particular affirmative proposition and the subcontrary negation between the general negative proposition and the particular negative proposition.

In next sections we are formally proving that there exist two squares of opposition and, correspondingly, two syllogistics (the first for analytic propositions and the second for synthetic propositions).

3 Synthetic syllogistics

In Aristotle’s syllogistics, analytic propositions in Kant’s words are formalized. In such propositions, a subject is thought within a predicate, the more common concept, for example: ‘Socrates is a man’ or ‘All people are animals’. Therefore the predicate in formulas $\text{SaP}$ (‘every S is P’), $\text{SiP}$ (‘some S are P’), $\text{SeP}$ (‘no S is P’), $\text{SoP}$ (‘some S are not P’) may be replaced by nouns, but not by adjectives. In other words, we have the following gram-
mar: ‘noun 1 (subject) + is + noun 2 (predicate)’, and the concept of ‘noun 1’ is a kind (particular) of the concept of ‘noun 2’, i.e. ‘noun 2’ is thought as general for ‘noun 1’.

However, how far can we consider propositions like ‘Socrates is white’, ‘All bodies are heavy’ within a conventional syllogistics formalizing just analytic propositions? At the first blush, these troubles might be deleted if we transformed an appropriate adjective into a noun. For example, the proposition ‘Socrates is white’ may be converted to the proposition ‘Socrates is a white being’, and ‘All bodies are heavy’ to ‘All bodies are something heavy’. However, such a transformation does not solve our problem, because the predicate is not general for the subject still. Thus, Socrates’ whiteness is not his substantial attribute and the general of bodies is space, not weight. Any body is thought first as space entity.

For the first time, Aristotle noticed that there are propositions that have been called synthetic since Kant and these propositions cannot be used in syllogistics. Aristotle’s counterexample was as follows: “He who sits, writes and Socrates is sitting, then Socrates is writing” (Topics 10). This wrong syllogism was caused by using synthetic propositions. Kant’s key example of synthetic propositions: ‘All bodies are heavy’. They have the following grammar: ‘noun 1 (subject) + is + adjective (attribute)’. The connective ‘...is ...’ of synthetic propositions is understood in this paper as follows: $A$ is $B \equiv (\exists C (C \text{ is } A) \land \forall C \forall D ((C \text{ is } A \land D \text{ is } A) \Rightarrow C \text{ is } D) \land \forall C (C \text{ is } A \land C \text{ is } B))$. This understanding corresponds to the Kantian-Rickertian approach.

Assume that all the syllogistic synthetic propositions have the following meaning:

- ‘All $S$ are $P$’ (‘All bodies are heavy’): there exist $A$ such that $A$ is $S$ or for any $A$, $A$ is $S$ and $A$ is $P$;
- ‘Some $S$ are $P$’ (‘Socrates is white’): there exist $A$ such that ‘$A$ is $S$’ is false, but ‘$A$ is $P$’ is true;
- ‘No $S$ are $P$’ (‘No bodies are angels’): there exist $A$ such that ‘$A$ is $P$’ is false or ‘$A$ is $S$’ is true;
- ‘Some $S$ are not $P$’ (‘Socrates is not black’): for any $A$, ‘$A$ is $S$’ is false and there exist $A$ such that ‘$A$ is $P$’ is false or ‘$A$ is $S$’ is false.

Let us propose now the syllogistic system formalizing synthetic propositions. This system is said to be synthetic syllogistics, while we are assuming
that Aristotelian syllogistic is analytic. The basic logical connectives of synthetic syllogistic are as follows: \( a \) (‘every + noun + is + adjective’), \( i \) (‘some + noun + is + adjective’), \( e \) (‘no + noun + is + adjective’) and \( o \) (‘some + noun + is not + adjective’) that are defined in synthetic ontology in the following way:

\[
\begin{align*}
S_a P & := (\exists A (A \text{ is } S) \lor (\forall A (A \text{ is } P \land A \text{ is } S))); \\
S_i P & := \forall A (A \text{ is } P \land \neg (A \text{ is } S)); \\
S_o P & := \neg (\exists A (A \text{ is } S) \lor (\forall A (A \text{ is } P \land A \text{ is } S))), \text{ i.e. } (\forall A \neg (A \text{ is } S)) \land \exists A (\neg (A \text{ is } P) \lor \neg (A \text{ is } S))); \\
S_e P & := \neg \forall A (A \text{ is } P \land \neg (A \text{ is } S)), \text{ i.e. } \exists A (\neg (A \text{ is } P) \lor (A \text{ is } S)).
\end{align*}
\]

Now let us formulate axioms of synthetic syllogistics:

\[
\begin{align*}
S_a P & \Rightarrow S_e P; \\
S_o P & \Rightarrow P o S; \\
(M a P \land S a M) & \Rightarrow S a P; \\
(M a P \land S e M) & \Rightarrow S e P.
\end{align*}
\]

In synthetic syllogistics we have a novel square of opposition that we call the synthetic square of opposition (see figure 3), where the following theorems are inferred: \( S a P \Rightarrow \neg (S o P), \neg (S o P) \Rightarrow S a P, S i P \Rightarrow \neg (S c P), \neg (S c P) \Rightarrow S i P, S c P \Rightarrow \neg (S i P), \neg (S i P) \Rightarrow S c P, S o P \Rightarrow \neg (S a P), \neg (S a P) \Rightarrow S o P, \\
S a P \Rightarrow \neg (S i P), S i P \Rightarrow \neg (S a P), \neg (S c P) \Rightarrow S o P, \neg (S o P) \Rightarrow S c P, S a P \Rightarrow S c P, S i P \Rightarrow S o P, S c P \lor S i P, \neg (S c P \land S i P), S a P \lor S o P, \neg (S a P \land S o P), \\
\neg (S a P \land S i P), S c P \lor S o P.
\]

4 Non-Archimedean models of Aristotelian syllogistics and synthetic syllogistics

Suppose \( B \) is a complete Boolean algebra with the bottom element 0 and the top element 1 such that the cardinality of its domain \( |B| \) is an infinite
Figure 3: The synthetic square of opposition (for synthetic syllogistics, where synthetic propositions are formalized).

number. Build up the set $B^B$ of all functions $f: B \to B$. The set of all complements for finite subsets of $B$ is a filter and it is called a Frechét filter, it is denoted by $\mathcal{U}$. Further, define a new relation $\approx$ on the set $B^B$ by

$$f \approx g = \{ a \in B : f(a) = g(a) \} \in \mathcal{U}.$$ 

It is easily proved that the relation $\approx$ is an equivalence. For each $f \in B^B$ let $[f]$ denote the equivalence class of $f$ under $\approx$. The ultrapower $B^B/\mathcal{U}$ is then defined to be the set of all equivalence classes $[f]$ as $f$ ranges over $B^B$. This ultrapower is called a nonstandard (or non-Archimedean) extension of Boolean algebra $B$, for more details see [8] and [12]. It is denoted by $^*B$.

There exist two groups of members of $^*B$: (1) functions that are constant, e.g. $f(a) = m \in B$ on the set $\mathcal{U}$, a constant function $[f = m]$ is denoted by $^*m$, (2) functions that are not constant. The set of all constant functions of $^*B$ is called standard set and it is denoted by $^*^0B$. The members of $^*^0B$ are called standard. It is readily seen that $B$ and $^*^0B$ are isomorphic.

We can extend the usual partial order structure on $B$ to a partial order structure on $^*^0B$:

1. for any members $x, y \in B$ we have $x \leq y$ in $B$ iff $^*x \leq ^*y$ in $^*^0B$,

2. each member $^*x \in ^*^0B \setminus \{ ^*0 \}$ (i.e. that is not a bottom element $^*0$ of $^*^0B$) is greater than any number $[f] \in ^*B \setminus ^*^0B$, i.e. $^*x > [f]$ for any $x \in B$, where $[f]$ is not constant function,

3. $^*0$ is the bottom element of $^*B$.

Notice that under these conditions, there exist the top element $^*1 \in ^*B$ such that $1 \in B$ and the bottom element $^*0 \in ^*B$ such that $0 \in B$. 

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The ordering conditions mentioned above have the following informal sense: (1) the sets $\circ B$ and $B$ have isomorphic order structure; (2) the set $\ast B \setminus \{\ast 0\}$ contains actual infinities that are less than any member of $\circ B \setminus \{\ast 0\}$. These members are called Boolean infinitesimals.

Introduce three operations ‘sup’, ‘inf’, ‘¬’ in the partial order structure of $\ast B$:

$$\inf([f], [g]) = \inf(f, g);$$
$$\sup([f], [g]) = \sup(f, g);$$
$$\neg[f] = \neg f.$$ 

This means that a nonstandard extension $\ast B$ of a Boolean algebra $B$ preserves the least upper bound ‘sup’, the greatest lower bound ‘inf’, and the complement ‘¬’ of $B$.

Consider the member $[h]$ of $\ast B$ such that $\{a \in B: h(a) = f(\neg a)\} \in U$. Denote $[h]$ by $[f\neg]$. Then we see that $\inf([f], [f\neg]) \geq \ast 0$ and $\sup([f], [f\neg]) \leq \ast 1$. Really, we have several cases.

1. **Case 1.** The members $\neg[f]$ and $[f\neg]$ are incompatible. Then $\inf([f], [f\neg]) \geq \ast 0$ and $\sup([f], [f\neg]) \leq \ast 1$.

2. **Case 2.** Suppose $\neg[f] \geq [f\neg]$. In this case $\inf([f], [f\neg]) = \ast 0$ and $\sup([f], [f\neg]) \leq \ast 1$.

3. **Case 3.** Suppose $\neg[f] \leq [f\neg]$. In this case $\inf([f], [f\neg]) \geq \ast 0$ and $\sup([f], [f\neg]) = \ast 1$.

4. **Case 4.** The members $[f]$ and $\neg[f\neg]$ are incompatible. Then $\inf(\neg[f], \neg[f\neg]) \geq \ast 0$ and $\sup(\neg[f], \neg[f\neg]) \leq \ast 1$.

5. **Case 5.** Suppose $\neg[f\neg] \geq [f]$. In this case $\inf(\neg[f], \neg[f\neg]) \geq \ast 0$ and $\sup(\neg[f], \neg[f\neg]) = \ast 1$.

6. **Case 6.** Suppose $\neg[f\neg] \leq [f]$. In this case $\inf(\neg[f], \neg[f\neg]) = \ast 0$ and $\sup(\neg[f], \neg[f\neg]) \leq \ast 1$.

7. **Case 7.** The members $[f\neg]$ and $\neg[f]$ are incompatible. Then $\inf([f], [f\neg]) \geq \ast 0$ and $\sup([f], [f\neg]) \leq \ast 1$.

8. **Case 8.** Suppose $\neg[f] \geq \neg[f\neg]$. In this case $\inf([f], [f\neg]) = \ast 0$ and $\sup([f], [f\neg]) \leq \ast 1$. 

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9. **Case 9.** Suppose \(\neg[f] \leq \neg[f\neg]\). In this case \(\inf([f], \neg[f\neg]) \geq *0\) and \(\sup([f], \neg[f\neg]) = *1\).

10. **Case 10.** The members \([f]\) and \([f\neg]\) are incompatible. Then \(\inf(\neg[f], [f\neg]) \geq *0\) and \(\sup(\neg[f], [f\neg]) \leq *1\),

11. **Case 11.** Suppose \([f\neg] \geq [f]\). In this case \(\inf(\neg[f\neg], [f\neg]) \geq *0\) and \(\sup(\neg[f], [f\neg]) = *1\).

12. **Case 12.** Suppose \([f\neg] \leq [f]\). In this case \(\inf(\neg[f\neg], [f\neg]) = *0\) and \(\sup(\neg[f], [f\neg]) \leq *1\).

**Definition 1** Now define hyperrational valued matrix logic \(\mathcal{M}_B\) as the ordered system \(\langle *B, \{*1\}, \neg, \Rightarrow, \lor, \land \rangle\), where

1. \(*B\) is the set of truth values,
2. \(\{*1\}\) is the set of designated truth values,
3. for all \([x] \in *B\), \(\neg[x] = *1 - [x]\),
4. for all \([x], [y] \in *B\), \([x] \Rightarrow [y] = *1 - \sup([x], [y]) + [y]\),
5. for all \([x], [y] \in *B\), \([x] \land [y] = \inf([x], [y])\),
6. for all \([x], [y] \in *B\), \([x] \lor [y] = \sup([x], [y])\).

**Proposition 1** In \(\mathcal{M}_B\) there are only two squares of opposition.

**Proof.** We have just eight cases: (1) \([f] \leq [f\neg]\), (2) \([f] \leq \neg[f\neg]\), (3) \([f\neg] \leq [f]\), (4) \([f\neg] \leq \neg[f]\), (5) \(\neg[f\neg] \leq [f]\), (6) \(\neg[f\neg] \leq \neg[f]\), (7) \(\neg[f] \leq [f\neg]\), (8) \(\neg[f] \leq \neg[f\neg]\). Taking into account that couples \([f]\) and \(\neg[f][f\neg]\) (\(\neg[f\neg]\) and \(\neg[f\neg]\)) are contradictory, we can claim that there exist two squares of opposition:

- if \([f\neg] \leq [f]\) (resp. \(\neg[f] \leq \neg[f\neg]\)), we have the conventional square of opposition (see figure 4); if \(\neg[f\neg] \leq [f]\) (resp. \(\neg[f] \leq \neg[f\neg]\)), we have its dual without changing meaning;

- if \([f] \leq \neg[f\neg]\) (resp. \([f\neg] \leq \neg[f]\)), we have the synthetic square of opposition (see figure 5); if \(\neg[f\neg] \leq [f]\) (resp. \(\neg[f] \leq \neg[f\neg]\)), we have its dual without changing meaning. \(\square\)
Figure 4: In case \([f\neg] \leq \neg[f]\), the square of oppositions for any members \([f]\), \([f\neg]\), \(\neg[f]\), \(\neg[f\neg]\) of \(*B\) holds true, i.e. \([f]\), \([f\neg]\) are contrary, \([f]\), \(\neg[f]\) (resp. \(\neg[f\neg]\), \(\neg[f\neg]\)) are contradictory, \(\neg[f\neg]\), \(\neg[f]\) are subcontrary, \([f]\), \(\neg[f\neg]\) (resp. \(\neg[f\neg]\), \(\neg[f]\)) are said to stand in the subalternation.

Now we can build models for atomic syllogistic formulas (i.e. syllogistic formulas without propositional connectives) due to algebra \(\mathfrak{M}_B\).

**Definition 2** A structure \(\mathfrak{B} = \langle O, I, \hat{\alpha}, \hat{\epsilon}, \hat{i}, \hat{o}, \hat{\alpha}, \hat{\epsilon}, \hat{i}, \hat{o}\rangle\) is a non-Archimedean syllogistic model iff:

1. \(O\) is a restriction of the set \(\mathfrak{M}_B\) to an appropriate square (triangle) of opposition (thereby the conventional square of opposition should hold true for Aristotelian syllogistics and the synthetic square of opposition holds for synthetic syllogistics).

2. \(I\) is a mapping that associates a class of equivalence \([f] \in O\) with each atomic syllogistic formula \(S \odot P\), where \(\odot \in \{a, e, i, o, \acute{a}, \acute{e}, \acute{i}, \acute{o}\}\), so that \(I(S \odot P) = |S|\odot|P|\), where \(\odot \in \{\hat{\alpha}, \hat{\epsilon}, \hat{i}, \hat{o}, \hat{\alpha}, \hat{\epsilon}, \hat{i}, \hat{o}\}\) and
   - \(|S|\hat{\alpha}|P| = [f]\) (resp. \(|S|\hat{\alpha}|P| = \neg[f\neg]\));
   - \(|S|\hat{\epsilon}|P| = [f\neg]\) (resp. \(|S|\hat{\epsilon}|P| = \neg[f]\));
   - \(|S|\hat{i}|P| = \neg[f\neg]\) (resp. \(|S|\hat{i}|P| = [f]\));
   - \(|S|\hat{o}|P| = \neg[f]\) (resp. \(|S|\hat{o}|P| = [f\neg]\));
Figure 5: In case $\neg [f] \leq [f]$, the synthetic square of oppositions for any members $[f], [f], \neg[f], \neg[f]$ of $^*B$ holds true, i.e. $[f], \neg[f]$ are contrary, $[f], \neg[f]$ (resp. $\neg[f], [f]$) are contradictory, $\neg[f], [f]$ are subcontrary, $[f], [f]$ (resp. $\neg[f], \neg[f]$) are said to stand in the subalternation.

We now give the truth conditions of Boolean combinations of atomic syllogistic formulas in a non-Archimedean syllogistic model:

**Definition 3**

- $B \models \neg \phi$ iff $B \not\models \phi$
- $B \models \phi \land \psi$ iff $B \models \phi$ and $B \models \psi$
- $B \models \phi \lor \psi$ iff $B \models \phi$ or $B \models \psi$
- $B \models \phi \Rightarrow \psi$ iff $B \models \neg \phi$ or $B \models \psi$

5 Conclusion

In this paper using non-Archimedean models I have just proved that there are only two squares of opposition if we assume Boolean algebra as the basis of an appropriate non-Archimedean extension. The conventional square of opposition may be aimed for getting analytic syllogistics (Aristotelian syllogistics) and the new one for getting synthetic syllogistics (syllogistics, proposed in this paper).
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Andrew Schumann
Department of Philosophy and Science Methodology,
Belarusian State University, Minsk, Belarus
e-mail: Andrew.Schumann@gmail.com