Hall-potential distribution in AC quantum Hall effect

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Abstract. The distribution of the Hall voltage induced by low-frequency AC current is studied theoretically in the incoherent linear transport of quantum Hall systems with slowly-varying confining potential. It is shown that in the low-frequency limit the Hall potential has a steeper slope where the diagonal conductivity is smaller, while at higher frequencies it exhibits a peak or a dip where the Hall conductivity has a slope.

1. Introduction
In the quantum Hall effect [1, 2] observed in two-dimensional electron systems (2DES) in strong magnetic fields, the Hall voltage $V_H$ divided by the current $I$ is quantized as $V_H/I = h/(ie^2)$ with $i$ an integer. The distribution of this quantized Hall voltage along the width of the 2DES has been studied theoretically [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13] and experimentally [14], but has problems that remain to be solved. One of the problems is the distribution in the AC current. In this paper we study theoretically the Hall-voltage distribution in low-frequency AC current with the angular frequency $\omega$ in the incoherent linear transport in a system with slowly-varying confining potential. The Hall-voltage distribution in a uniform 2DES with sharp edges has been studied elsewhere [15].

2. Model and Equations
In this paper we consider the 2DES in the $xy$ plane with two boundaries which are both parallel to the $x$ axis. We assume that the conductivity and the current are uniform along $x$. From the equation of charge conservation, the Hall charge density $\rho(y, \omega)$ is given by

$$i\omega \rho(y, \omega) = -\nabla_y j_y(y, \omega).$$

(1)

We assume that the phase coherence length is shorter than the length scale of variations of the electric field induced by the current, and then we obtain the local response of the current density to the electric field:

$$j_y(y, \omega) = \sigma_{yx}(y, \omega) E_x(\omega) + \sigma_{yy}(y, \omega) E_y(y, \omega).$$

(2)

Here we have also assumed that $E_x$ has no dependence on $y$ since $\nabla_y E_x - \nabla_x E_y \approx 0$ when $\omega$ is small.

We restrict our discussion to the low-frequency region, where $\hbar \omega$ is much smaller than relevant energy scales. We expand $\sigma_{\alpha\beta}(\omega)$ ($\alpha, \beta = x, y$) in a power series of $\omega$. Since the real and
imaginary parts of $\sigma_{\alpha\beta}(\omega) = \sigma'_{\alpha\beta}(\omega) + i\sigma''_{\alpha\beta}(\omega)$ satisfy the following relation: $\sigma'_{\alpha\beta}(-\omega) = \sigma'_{\alpha\beta}(\omega)$ and $\sigma''_{\alpha\beta}(-\omega) = -\sigma''_{\alpha\beta}(\omega)$, we can write $\sigma_{\alpha\beta}(\omega)$, by retaining terms up to the first order of $\omega$, as

$$\sigma_{\alpha\beta}(\omega) = \sigma_{\alpha\beta}^0 + i\omega\chi_{\alpha\beta}^0,$$

(3)

where $\sigma_{\alpha\beta}^0$ is the DC conductivity and $\chi_{\alpha\beta}^0$ is the DC susceptibility.

We assume that the phase coherence length is shorter than the length scale of variations of the confining potential and define the local equilibrium chemical potential $\mu_{eq}$ as a function of $y$, which decreases as approaching the boundary. We describe this $y$ dependence by a simple function:

$$\mu_{eq}(y) = \mu_0, \quad (0 < y < W/2)$$

$$\mu_{eq}(y) = \mu_0 \left[ 1 - (y - W/2)^2/W_e^2 \right], \quad (y > W/2)$$

(4)

and $\mu_{eq}(-y) = \mu_{eq}(y)$.

We assume that the $y$ dependence of $\sigma_{yx}^0(y), \sigma_{yy}^0(y), \chi_{yx}^0(y)$ and $\chi_{yy}^0(y)$ originates from the $y$ dependence of $\mu_{eq}(y)$, that is $\sigma_{yx}^0(y) = \sigma_{yx}^0(\mu_{eq}(y))$ and so on. As for the $\mu_{eq}$ dependence of $\sigma_{yx}^0$ and $\sigma_{yy}^0$, we employ a model \cite{16} which retains the observed features of $\sigma_{yx}^0(B)$ and $\sigma_{yy}^0(B)$:

$$\sigma_{yx}^0(\mu_{eq}) = \frac{2e^2}{h} \frac{\sum N f_N}{N},$$

(5)

$$\sigma_{yy}^0(\mu_{eq}) = \frac{2e^2 D_0}{k_B T} \sum N (2N + 1) f_N (1 - f_N),$$

(6)

$$f_N = \{ 1 + \exp \left[ (\varepsilon_N - \mu_{eq})/(k_B T) \right] \}^{-1}$$

(7)

with $\varepsilon_N = \hbar\omega_c(N + 1/2)$, $N = 0, 1, 2, \ldots$ and $D_0$ is a constant. As for $\chi_{yx}^0(\mu_{eq})$ and $\chi_{yy}^0(\mu_{eq})$ we use simple formulas

$$\chi_{yx}^0(\mu_{eq}) = 0, \quad \chi_{yy}^0(\mu_{eq}) = e^2 \nu/(h\omega_c),$$

(8)

which are obtained by neglecting the Landau-level mixings and by neglecting the Landau-level broadening compared to $\hbar\omega_c$. In calculating the filling factor $\nu$, we use the following density of states:

$$D(\varepsilon) = 1/(2\pi l^2 \Gamma), \quad (|\varepsilon - \varepsilon_N| < \Gamma)$$

$$D(\varepsilon) = 0, \quad \text{(otherwise)}$$

(9)

Figure 1 shows the $\mu_{eq}$ dependence of $\sigma_{yx}^0, \sigma_{yy}^0$ and $\chi_{yx}^0$ given by eqs.(5), (6) and (8). Figure 2 presents the $y$ dependence of $\mu_{eq}, \sigma_{yx}^0, \sigma_{yy}^0$ and $\chi_{yy}^0$ for which the Hall potential is calculated in the following.

The electrostatic potential due to the Hall charge, $\phi(y)$, in a 2DES uniform along $x$ is given by

$$\phi(y) = \int_{-\infty}^{\infty} dy' K(y - y') \rho(y').$$

(10)

where $K(y - y')$ is the potential due to the unit line charge at a distance $|y - y'|$. Here and in the following the $\omega$ dependence of the variables and coefficients is not shown explicitly. We consider the two models of the electrostatic interaction. One is the long-range interaction with

$$K(y - y') = -\frac{2}{\varepsilon} \ln |y - y'|,$$

(11)
This is the potential in a dielectric material with the dielectric constant \( \varepsilon \). The other is the short-range interaction with

\[
K(y - y') = r_K \delta(y - y').
\]

In this model

\[
\phi(y) = r_K \rho(y).
\]

This model is valid when the range of \( K(y - y') \) is much shorter than the length scale of variation of \( \rho(y) \), \( L_\rho \). If we consider a 2DES with a gate electrode, this condition becomes \( d \ll L_\rho \) where \( d \) is the distance between the 2DES and the gate. In such system

\[
r_K = 4\pi d/\varepsilon.
\]

We calculate the Hall potential \( \phi(y) \) by solving eqs. (1) and (10). We consider a periodic array of infinitely-long 2DES strips. The centerline of the \( n \)th strip is at \( y = nW_p \) where \( n \) is the integer and \( W_p \) is the periodicity. The Hall potential satisfies \( \phi(-y) = -\phi(y) \) and \( \phi(y + W_p) = \phi(y) \), which lead to \( \phi(y) = 0 \) at \( y = \pm W_p/2 \). Then we expand \( \phi(y) \) in the Fourier series and solve for the Fourier coefficients. We have confirmed that \( \phi(y) \) within the 2DES has little dependence on \( W_p \) if the gap between 2DES strips is wide enough.

### 3. Calculated Results

We use the following dimensionless variable:

\[
\tilde{y} = y/l_U,
\]

with a unit

\[
l_U = 2\chi_{yy}^0/\varepsilon = \nu_b l^2/(\pi a_B^*),
\]

where \( \chi_{yy}^0 \) is the value of \( \chi_{yy} \) in the bulk region \(-W/2 < y < W/2\) and \( a_B^* = h^2/\varepsilon m^* \) is the effective Bohr radius. When we use \( B = 5T \), \( \nu_b = 4 \) and the value of \( m \) and \( \varepsilon \) of GaAs, we have
l_U ~ l, while l_U(\propto B^{-2}) becomes larger at smaller B. We introduce a normalized Hall potential

$$\phi = \phi_{xy}^{0b}/(E_x\sigma_{xy}^{0b}l_U),$$

(17)

and a normalized angular frequency

$$\tilde{\omega} = \omega\chi_{yy}/\sigma_{yy}^{0b}.$$  (18)

Figures 3 and 4 present the Hall potential $\phi(y)$ as a function of $y$ in the long-range interaction eq.(11). In the low-frequency limit, $\phi(y)$ shows a steeper slope where $\sigma_{yy}^{0}$ is smaller and a plateau where $\sigma_{yy}^{0}$ is larger. At higher frequencies, on the other hand, $\phi(y)$ exhibits a peak (or a dip) where $\sigma_{yy}^{0}$ has a slope. These features are also seen in $\phi(y)$ calculated in the short-range interaction eq.(12) as shown in Figs.5 and 6.

![Figure 3. Hall potential $\phi(y)$ in the long-range interaction eq.(11) at lower frequencies. $W_p = W_p/l_U = 800$.](image1)

![Figure 4. Hall potential $\phi(y)$ in the long-range interaction eq.(11) at higher frequencies. $W_p = W_p/l_U = 800$.](image2)

Mechanisms of such features are understood from the equation for $\phi(y)$ in the short-range interaction. Equations (1) and (13) give $i\omega\phi(y) = \nabla_y D(y)\nabla_y\phi(y) + r_K\nabla_y\nabla\omega\chi_{yy}(y)\nabla_y\phi(y) - r_K(\nabla_y\chi_{yy}(y))\nabla_y\phi(y)$, where $D(y) = r_K\sigma_{yy}^{0}(y)$ is a diffusion coefficient, the second term $r_K\nabla_y\nabla\omega\chi_{yy}(y)\nabla_y\phi(y)$ represents a screening due to the polarization charge, and the third term $-r_K(\nabla_y\chi_{yy}(y))\nabla_y\phi(y)$ is the source term. When $\tilde{\omega} \ll 1$, the second term is negligible compared to the first term. Because of this first term, $\phi(y)$ is flat where $D(y)$ is larger, while it shows a slope where $D(y)$ is smaller. When $\tilde{\omega} \gg 1$, on the other hand, the first term is negligible. In this case, the third term, or the slope of $\sigma_{yy}^{0}(y)$, determines the spatial dependence of $\phi(y)$. In the limit of $\tilde{\omega} \rightarrow \infty$, $\phi(y)\omega/(iE_x)$ approaches a real value independent of $\omega$, which is the case also in the long-range interaction.

4. Conclusions and Discussion

We have studied the Hall potential $\phi(y)$ as a function of $y$ (in the width direction) in quantum Hall systems in the case of low-$\omega$ AC current in the incoherent linear transport where the dynamics of the local Hall-charge density is determined by the complex diagonal conductivity.
\[ \sigma_{yy} = \sigma_{yy}^0 + i\omega\chi_{yy}^0 \] as well as the Hall conductivity \( \sigma_{yx}^0 \). We have considered the case of slowly-varying confining potential, in which \( \sigma_{yy}^0(y), \chi_{yy}^0(y) \) and \( \sigma_{yx}^0(y) \) vary with \( y \). We have found the following conclusions both in the long-range interaction and in the short-range interaction. In the low-frequency limit, in the region of smaller \( \sigma_{yy}^0(y) \), the Hall potential \( \phi(y) \) shows a steeper slope and the current is concentrated. This has been shown before in the DC current \([11]\). At higher frequencies, on the other hand, \( \phi(y) \) exhibits a peak or a dip where \( \sigma_{yx}^0(y) \) has a slope.

We have also calculated the frequency dependence of the magnetoresistance \( R_{xx}(\omega) = V_x(\omega)/I_x(\omega) \). With increasing \( \omega \), \( |R_{xx}| \) exhibits a stepwise increase and the phase delay of \( I_x \) relative to \( V_x \) increases from 0 to \( \pi/2 \).

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