Research Article

Numerical Analysis of Flow Field and Heat Transfer of 2D Wavy Ducts and Optimization by Entropy Generation Minimization Method

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This article provided a research for the trend of heat transfer and flow field through a 2-dimensional wavy duct. To construct a grid mesh, the physical domain was transferred to the computational domain and finite volume scheme was used for discretizing the governing equations. Through the simulation, the flow regime stayed in laminar mode. Constant temperature boundary condition has been used for solid walls. Air was used as a working fluid. Existence of waves makes some phenomenon like flow separation. Effect of Reynolds number, wave width, and wave number has been analyzed and velocity distribution, heat transfer coefficient, and tangential stress were computed for different cases. The final results were compared with the same straight duct. The entropy generation minimization method has been used for better comparison between final results.

1. Introduction

For many industrial thermal systems, there is much interest in reducing fuel consumption and/or increasing of system efficiency. For gas turbine recuperators, an efficient heat exchanger is required to reduce the system size and increase the cycle efficiency. The heat exchangers generally contain flow channels with various cross-sectional shapes which are corrugated and wavy curved in the mainstream to enhance heat/mass transfer rates by generating secondary flow. The heat transfer and flow characteristics in the channel with wavy plates have been widely studied previously. Sunden and Trollheden [1] studied on the heat transfer and pressure drop in the corrugated channels and the smooth tubes under constant heat flux conditions. Nishimura and Matsune [2] simulated and visualized the dynamical behavior of vortices flow in channels. Fabbri and Rossi [3, 4] studied the laminar convective heat transfer in the smooth and corrugated channels. Cheng and Wang [5] studied on the forced convection of micropolar fluid flow over the wavy surfaces. Ergin et al. [6] numerically studied periodic flow through a corrugated duct. Vasudevaiah and Balamurugan [7] studied the heat transfer in a corrugated microchannel under constant heat flux conditions. Niceno and Nobile [8] on the two-dimensional fluid flow and heat transfer in the periodic and wavy channels. Ali and Hanaoka [9] considered effects of the operating parameters on laminar flow forced-convection heat transfer of air flowing in a channel having a V-corrugated upper plate. Zimmerer et al. [10] studied effects of the inclination angle, the wave length, the amplitude, and the shape of the corrugation on the heat and mass transfer of the heat exchanger. Guzman and Amon [11–13] performed numerical investigations for high transitional Reynolds numbers in converging-diverging (symmetric wavy wall) channels. Wang and Chen [14] determined the heat transfer rates flowing through a sinusoidally curved converging-diverging channel. Savino et al. [15] studied effect of aspect ratio on convection heat transfer enhancement in the wavy channels. Hossain and Islam [16] numerically studied on the fully unsteady fluid flow and heat transfer in sine shaped wavy channels. Metwally and Manglik [17] simulated the laminar periodically developed forced convection in sinusoidal corrugated-plate channels by using the control volume finite-difference method. Naphon
3. Governing Equations

The combination of continuity, Navier-Stokes and energy equations in 2-dimensional and steady form was used as the essential governing equations. These equations and their boundary conditions are listed through numbers of 1 to 6.

Continuity.

\[ \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0. \]  \hspace{1cm} (1)

Momentum Equation in x-Direction.

\[ \left[ \frac{\partial}{\partial x} (\rho uu) + \frac{\partial}{\partial y} (\rho vu) \right] = -\frac{\partial P}{\partial x} + \left[ \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) \right]. \]  \hspace{1cm} (2)

Momentum Equation in y-Direction.

\[ \left[ \frac{\partial}{\partial x} (\rho vv) + \frac{\partial}{\partial y} (\rho vv) \right] = -\frac{\partial P}{\partial y} + \left[ \frac{\partial}{\partial x} \left( \mu \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial y} \right) \right]. \]  \hspace{1cm} (3)

Energy Equation.

\[ \left[ \frac{\partial}{\partial x} \left( \rho C_p u T \right) + \frac{\partial}{\partial y} \left( \rho C_p v T \right) \right] = \left[ \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) \right] + \mu \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 \right]. \]  \hspace{1cm} (4)

Inlet Boundary Conditions.

\[ T_{\text{inlet}} = 300 \text{ K}, \]
\[ u = -6 u_{\text{inlet}} \left( \frac{y}{h_{\text{ave}}} \right)^2 - \left( \frac{y}{h_{\text{ave}}} \right), \]  \hspace{1cm} (5)
\[ v = 0, \]
\[ \text{Pr} = 0.7. \]

Wall Boundary Conditions.

\[ T_{\text{up}} = 400 \text{ K}, \]
\[ T_{\text{down}} = 400 \text{ K}, \]  \hspace{1cm} (6)
\[ u = v = 0. \]

In above equations \( h_{\text{ave}} \) is the distance between the up and down plates and equal to the entrance diameter. To descritize the above equations finite volume method was used. Reynolds number, number and the width of waves were changed. For each case some graphs like Nusselt number, rate of heat transfer and stress distribution were sketched.
4. Results

Three characteristic parameters like Reynolds number, number of waves, and the width of the waves have been analyzed in this research. Figure 3 illustrates the velocity distribution along the duct for Reynolds number 300. After passing the flat part of the duct, the flow enters the wavy region and senses a sudden expansion which produces wakes in concave part of waves. Before releasing these wakes from the solid walls, they act as barriers between the wall and the flow stream; hence, the fluid could not sense the hot walls and the heat transfer mechanism will be so weak. In convex part of waves, the flow velocity increases; hence the rate of heat transfer will be raised in these parts (Figure 4). This graph is divided into 2 regions. In the first region, the shape of the duct is straight and the flow is fully developed; hence the trend of graph is approximately constant. In the second region, the heat transfer coefficient finds a wavy trend because of the shape of the duct. From this graph, we can judge that the average heat transfer coefficient in second region is nearly 1.5 times of first region which means better and more powerful heat transfer mechanism.

4.1. Effect of Reynolds Number. Reynolds number is one of the most important parameter through the flow field. By increasing the value of this parameter, the velocity gradient near the solid wall will be increased and causes the separation phenomenon happened with delay. The size of the produced wakes in concave parts of the waves will be much smaller by increasing the magnitude of the Reynolds number. This reduction in wake’s size increases the rate of heat transfer between the fluid stream and the walls of the duct. In this study, the flow field has been studied steadily and the vortexes were not allowed to enter the flow; hence the vortexes act as barriers near the wall. With increasing the Reynolds number, the size of these barriers will be reduced.
In Figure 5, the distribution of average Nusselt number along the wall of duct has been sketched for different Reynolds numbers. By increasing the Reynolds number, the value of dimensionless heat transfer coefficient will be raised which means better heat transfer mechanism. To complete this reason, the quantity of heat which transferred by fluid flow through the same duct has been illustrated for different Reynolds numbers in Figure 6. This figure has the same trend.

Distribution of tangential stress along the wall has been sketched in Figure 7. This graph has a raising trend which means by increasing the Reynolds number, the gradient of velocity near the wall will be increased so; the magnitude of stress will be also raised.

4.2. Effect of Wave Width. Analysis of effect of wave width is one of the most important parameters for improving the quality of heat transfer. Assume a sinus correlation like \( y = A \sin(y/(L \times b)) \) for duct geometry. By increasing the value of \( A \) which controls the width of the wave, the edges of wavy region find more sharp shape which causes the diameter of the duct in convex parts reduced. This reduction causes the velocity of the flow and also the rate of heat transfer increases considerably. Figure 8 illustrates the change of velocity profile in same place and same Reynolds number for different widths.

Figure 9 illustrates the trend of heat transfer coefficient along the wall of duct for different widths. This graph predicts that by increasing the width of the wave the magnitude of the heat transfer maximum points raised considerably.

4.3. Effect of Wave Number. Number of produced waves along the duct has been studied. Figure 11 illustrates the distribution of heat transfer coefficient for different wave number. This figure shows that if the flow senses more waves along its path, the value of heat transfer coefficient will be more significant.
5. Entropy Generation Minimization Method

One of the newest methods for comparing and optimizing the results is entropy generation method. This method based on the rate of entropy generation. By minimizing the value of this property, the apparatus will work better and more optimize. Term of entropy generation is a summation of conductive heat transfer irreversibility and viscous dissipation which is produced from motion of fluid. By using classic thermodynamic relations, this term can be found in dimensional form. The mass conservation principle, the first and second laws are used to derive the entropy generation term:

\[
\frac{\partial \rho}{\partial t} = -\rho \nabla \cdot V,
\]

\[
\rho \frac{\partial i}{\partial t} = -\nabla \cdot q - P \nabla \cdot V - w''', \tag{7}
\]

\[
\dot{s}_{\text{gen}} = \rho \frac{\partial s}{\partial t} + \nabla \cdot \left( \frac{q}{T} \right) \geq 0.
\]

Eliminating \( i \) and \( s \) between these laws and using the Gibbs equation

\[
\frac{\partial i}{\partial t} = T \frac{\partial s}{\partial t} + \frac{P}{\rho} \frac{\partial \rho}{\partial t}, \tag{8}
\]

yield the general two-term expression for the volumetric rate of entropy generation:

\[
\dot{s}_{\text{gen}} = -\frac{1}{T^2} q \cdot \nabla T - w''' \geq 0. \tag{9}
\]

By substituting \((-w''')\) with \(\mu \phi\) and using Fourier law:

\[
q = -k \nabla T, \tag{10}
\]

the entropy generation equation will be derived:

\[
\dot{s}_{\text{gen}} = \frac{k}{T^2} \left( \nabla T \right)^2 + \frac{\mu}{T} \phi \geq 0 \tag{11}
\]

or

\[
\dot{s}_{\text{gen}} = \frac{k}{T^2} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{T} \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right]. \tag{12}
\]

For measuring the effect of heat transfer irreversibility and viscous dissipation in magnitude of entropy production, the Bejan number can be defined as

\[
\text{Be} = \frac{\left( k/T^2 \right) \left[ \left( \partial T/\partial x \right)^2 + \left( \partial T/\partial y \right)^2 \right]}{\dot{s}_{\text{gen}}}. \tag{13}
\]

Effect of Reynolds number, wave width, and wave number on the rate of entropy generation and Bejan number has been studied.

By increasing Reynolds number, the velocity and temperature gradients will be increased and the profiles of these
properties will be more flat; hence, the rate of entropy generation will be decreased. This trend has been illustrated in Figure 12.

Figure 13 shows the trend of average Bejan number versus Reynolds number. By increasing the Reynolds number, the value of Bejan number will be decreased.

Wave width is another parameter which affects the rate of entropy generation. Figure 14 illustrates the trend of total entropy generation versus wave width. By increasing the width of the wave, the rate of entropy generation or dissipation is increased.

By changing the $A$ coefficient, variation of Bejan number is negligible (Figure 15).

By increasing number of waves, the fluid flow senses more viscous dissipation and so the magnitude of entropy generation is raised. This trend is illustrated in Figure 16.

Changing of wave's number has a very small effect on the variation of Bejan number; hence, the distribution of this parameter has a constant trend (Figure 17).

6. Comparison with Flat Duct

For comparison, a flat duct with same length and width was selected. Same thermal boundary conditions were considered. Figure 18 shows the trend of Nusselt number for two ducts. In case of wavy duct, the value of averaged Nusselt number is more than the flat duct and this parameter increases with Reynolds number; hence, with producing waves in a duct the rate of heat transfer will be increased.

7. Conclusion

Numerical analysis of flow field and heat transfer of 2D wavy ducts and optimization by entropy generation method
has been studied in this research. The governing equations with their boundary conditions were discretized with finite-volume method. The results show that by increasing Reynolds number, the value of Nusselt number will be increased. Also, by making the waves to be sharper, the heat transfer mechanism will be more powerful and the average Nusselt number will be increased along the length of the duct. The third parameter which was studied is number of waves. With increasing the number of waves, the flow will be passed more concave and convex parts; hence, the rate of heat transfer will be increased. The obtained results were analyzed by entropy generation minimization method. The volumetric entropy generation rate was reduced with increasing the Reynolds number, but in the case of wave width and the number of waves, the value of this property will be increased. In all cases, the value of Bejan number is near one, which means through the laminar flow, the term of the viscous dissipation has a negligible effect.

**Nomenclature**

- **Be**: Bejan number
- **\( C_p \)**: Constant pressure specific heat (J/Kg K)
- **\( D_h \)**: Hydraulic diameter (m)
- **\( i \)**: Internal energy (J/Kg)
- **\( K \)**: Conductivity (W/m K)
- **\( L \)**: Length of the duct (m)
- **Nu**: Nusselt number
- **\( P \)**: Pressure (Pa)
- **Pr**: Prandtl number
- **\( q \)**: Heat flux (W/m²)
- **Re**: Reynolds number
- **\( s \)**: Entropy (J/Kg K)
- **\( \dot{s}_{gen} \)**: Volumetric entropy generation rate (W/m³ K)
- **\( T \)**: Temperature (K)
- **\( t \)**: Time (s)
- **\( u \)**: Velocity component along the wall (m/s)
- **\( V \)**: Velocity vector (m/s)
- **\( v \)**: Velocity component normal to the wall (m/s)
- **\( w'' \)**: Volume power density (W/m³)
- **\( x \)**: Coordinate component along the wall
- **\( y \)**: Coordinate component normal to the wall.

**Greek Letters**

- **\( \mu \)**: Absolute viscosity (Kg/m s)
- **\( \rho \)**: Density (Kg/m³)
- **\( \phi \)**: Viscous dissipation (J/Kg m³).

**Subscripts**

- **down**: Bottom wall
- **inlet**: Entrance of the wall
- **up**: Top wall.

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