Non-Fermi Liquid behavior in Neutral Bilayer Graphene

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We calculate the density-density response function and electron self-energy for undoped bilayer graphene, within the Random Phase Approximation (RPA). We show that the quasiparticle decay rate scales linearly with the quasiparticle energy, and quasiparticle weight vanishes logarithmically in the low-energy limit, indicating non-Fermi liquid behavior. This is a consequence of the absence of a Fermi surface for neutral bilayer graphene and corresponding larger phase space available for scattering processes. Experimental consequences of our results as well as their differences from those of single-layer graphene are discussed.

Introduction — The isolation and subsequent identification of graphene, an atomically thin electron system, has led to intense experimental and theoretical interest$^1$. Recent experimental progress$^1$ has also led to techniques that enable isolation and study of systems with a small number of graphene layers, of particular importance is AB-Bernal stacked bilayer graphene, a system which shares some features both with graphene and two dimensional electron gas (2DEGs)$^2$, however at the same time different from both. Neglecting trigonal warping balanced bilayer graphene can be identified as a zero-gap semiconductor with quadratic dispersion; for undoped bilayer graphene the Fermi energy lies at the neutral Fermi point, described as the point where the degenerate particle-hole bands meet. Collectively these systems can be classified as chiral 2DEGs$^2$. Electron-electron interactions in chiral 2DEGs can lead to interesting quasiparticle properties, for example quasiparticle velocity enhancement in graphene due to the presence of unscreened Coulomb interactions. Most of the physics in this paper focuses on the difference in graphene and bilayer graphene’s chiral 2DEG.

In this paper we investigate the quasiparticle properties of undoped bilayer graphene due to electron-electron interactions. Short-ranged interactions for quadratic dispersion in two dimensions are marginal at the tree level while Coulomb interactions are relevant; this already points to the possibility of non-Fermi liquid behavior. Unlike neutral graphene where electron-electron interactions are unscreened, Coulomb interactions in bilayer graphene are screened due to the presence of a finite density of states, and dynamically generate a momentum scale $\lambda = me^2/\varepsilon$ proportional to the inverse Thomas-Fermi screening length $q_{TF} = 4\lambda \log[4]$. Below this scale this system resembles one with effective short-ranged interactions. Based on the scaling form of the density-density response function we demonstrate within the Random Phase Approximation (RPA) that there is no renormalization to the electron effective mass, while the imaginary part of the electron self-energy $\Im \Sigma \sim \omega$ below the screening scale. As a result the quasiparticle has a logarithmically vanishing spectral weight at low-energies, indicative of non-Fermi liquid behavior. This is very different from a 2DEG where the phase space available for scattering is limited by the energy thereby giving energy squared dependence for the quasiparticle decay rate, and the quasiparticle weight remains finite at the Fermi surface. We argue this non-Fermi liquid behavior is a consequence of the absence of a Fermi surface for neutral bilayer graphene and corresponding larger phase space available for scattering processes, which should be a robust result beyond RPA.

The long-wavelength behavior of the electron spectral function $A(k,\omega) = A(k, -\omega)$ is plotted in Fig. 1 which can be compared with ARPES measurements currently being employed to study the effects of interactions in graphene systems.$^9$

FIG. 1: (Color online) Intensity plot of electron spectral function at long-wavelengths and low-energies in units of $m/\lambda^2$. The solid line corresponds to the noninteracting dispersion. $m$ is the electron effective mass and $\lambda = me^2/\varepsilon$ is proportional to $q_{TF}$ the Thomas-Fermi screening wave-vector.

$\frac{m\omega}{\lambda^2}$

$\frac{k}{\lambda}$

Bilayer Graphene Effective Model — The low energy
properties of Bernal stacked bilayer graphene can be ade-
quately described by quasiparticles with parabolic dis-
\[ \mathcal{H} = \sum_{\mathbf{k},\alpha} \frac{\tilde{E}_{\mathbf{k}}^2}{2m} \hat{n}_{\mathbf{k}}^{\alpha,\dagger} \tau^z \otimes (\hat{\sigma} \cdot \hat{n}_{\mathbf{k}}) \hat{n}_{\mathbf{k}}^{\alpha} \]  
where the Pauli matrix \( \tau^z \) acts on the two-degenerate (K and K') valleys, \( \hat{n}_{\mathbf{k}} \) is two-dimensional envelope function momentum measured from the two nodal points K and K', \( \sigma^1 \) and \( \sigma^2 \) are Pauli matrices that act on bilayer graphene’s pseudospin (layer) degrees of freedom, and \( \alpha = \{1, 2\} \) accounts for the spin degrees of freedom. The chirality of bilayer graphene’s chiral 2DEG is captured by the unit vector \( \hat{n}_{\mathbf{k}} = (\cos 2\varphi_{\mathbf{k}}^q, \sin 2\varphi_{\mathbf{k}}^q) \) where \( \varphi_{\mathbf{k}}^q = \tan^{-1}(b_{\mathbf{k}}/x_{\mathbf{k}}) \). In Eq. (11) the field operator \( \hat{\varphi}_{\mathbf{k}}^{\alpha,\dagger} = \sum_{s, s'} \hat{\varphi}_{\mathbf{k} + s}^{\alpha,s} \mathbf{S}_{s,s'}^{\alpha,\dagger} \) is a four-component spinor where the low energy sites are the top (t) and bottom (b) layer sites without a near-neighbor in the opposite layer. The effective mass is determined by \( m = \gamma_1/2v_t \sim 0.054m_e \) where \( v \) is the single-layer Dirac velocity and \( \gamma_1 \sim 0.4eV \) is the inter-layer hopping amplitude.

The interaction contribution to the bilayer graphene’s hamiltonian is layer-dependent:
\[ \mathcal{V} = \frac{1}{2\mathcal{L}^2} \sum_q \sum_{\alpha,\alpha',\alpha''} \left[ v_+(q) \rho_{-q}^{\alpha} \rho_{-q}^{\alpha'} + 2v_-(q) \mathcal{S}_{q,\alpha,\alpha'}^{z} \mathcal{S}_{q,\alpha',\alpha}^{z} \right], \]

valid for \( \Omega \geq 0 \). It is important to note that the above expression is a function of a single scaling variable \( y = m\Omega/q^2 \), contrary to the case of any two-dimensional system with a Fermi wavevector \( k_F \) where it is a function of two variables namely \( m\Omega/k_F^2 \) and \( q/k_F \). This scaling behavior is a consequence of the absence of a Fermi surface for neutral bilayer graphene.

The imaginary part of the density-density response function \( \Im \chi_{\rho\rho}(q, \Omega) = 0 \) for \( \Omega > q^2/(4\mathcal{L}) \), defines the edge of the particle-hole continuum. For interband excitations this is just given by the minimum and maximum values of \( \Omega = \epsilon_{\mathbf{k}+\hat{q}} - \epsilon_{\mathbf{k}} \). The minimum energy for a particle-hole pair is attained for \( \theta_{\mathbf{k}+\hat{q}} = \pi \), this can be seen by completing the square and writing
\[ \min(\Omega) = \frac{1}{m}(k - q/2)^2 + q^2/4 \]. To include the different contribution from switch to a parabolic dispersion.

Within the Random Phase Approximation (RPA) \[ v_+^{\text{RPA}}(q, \Omega) = \frac{v_+(q)}{1 + v_+(q)\chi_{\rho\rho}^{0}(q, \Omega)}, \]
we recover the static screening \( \Omega \to 0 \) limit of \( v_+^{\text{RPA}}(q, \Omega) \) form of Ref. [13] with the Thomas-Fermi wavevector \( q_{TF} = g(mn_c^2)/e \log[4] \). Due to the positive definite value of \( \Re \chi_{\rho\rho}^{0}(q, \Omega) \), bilayer graphene at the neutrality point excludes any plasma excitations. The absence of a plasmon mode is not unique for neutral bilayer graphene and is phenomenologically similar.
to the case of neutral single-layer graphene. In neutral single-layer graphene the density-density response function vanishes inside the particle-hole continuum thereby excluding plasmon excitations, technically different from the case of bilayer graphene. Using the continuity equation we can relate the optical conductivity to the density-density response function of a Fermi liquid is encoded in the imaginary part of the electron self-energy with Coulomb interaction yields 1/\tau_{\pi, \vec{k}} = 0.1076\hbar^{2}e^{2}/\epsilon_{F}. The linear energy behavior only depends on the fact that \Phi is a function of a single scaling variable y = m\Omega/\epsilon_{F}, which is a consequence of the scale invariance of neutral bilayer graphene, and is independent of the detailed behavior of \Phi. Bare Coulomb interactions would give a different result, however screening dynamically generates a new scale q_{TF}, below which interactions effectively behave as short ranged. Based on the effects of screening and independence of the inverse lifetime on the function \Phi we anticipate non-Fermi liquid behavior in the long-wavelength limit.

The retarded quasiparticle self-energy within RPA can be written as:

\begin{equation}
\Sigma_{\pi ret}(\vec{k}, \omega) = \Sigma_{\pi eff}(\vec{k}, \omega) + \Sigma_{\pi line}(\vec{k}, \omega),
\end{equation}

following the the line and residue decomposition of Quinn and Ferrell. The quasiparticle self-energy within RPA remains diagonal in the particle-hole basis. It can be shown that the line contribution is purely real and does not contribute to the imaginary part of \Im\Sigma_{\pi ret} which for (\omega > 0):

\begin{equation}
\Im\Sigma_{\pi ret}(k, \omega) = \sum_{q \neq 0} \int \frac{d^{2}q}{(2\pi)^{2}} v_{+}(|\vec{k} - \vec{q}|) \left( 1 + s \cos(2\theta_{q,\omega}) \right)
\end{equation}

\begin{equation}
\Im \left[ \frac{1}{\epsilon(|\vec{k} - \vec{q}|, \omega - \epsilon(q))} \right] [\Theta(\omega - \epsilon(q))],
\end{equation}

where \epsilon(q, \Omega) = 1 + v_{+}(q)\chi_{rh}(q, \Omega). In the above expression we have neglected the contribution of \psi_{-} as it is logarithmically suppressed once screening effects are accounted for within RPA. Dimensional analysis of (9) implies that \Im\Sigma_{\pi}(k, \omega) = |k|^2 f(m\omega/|k|^2, k/\Lambda_{TF}), where f is a two variable function. Interactions introduce an inverse length scale \lambda = me^{2}/\epsilon which turns out to be of the same order of magnitude as the bandwidth cutoff \kappa_{c}. In the long-wavelength limit we find that:

\begin{equation}
\Im \Sigma_{\pi ret}(k, \omega = 0, \omega = 0) = \frac{|k|^2}{g2\pi n} h_{+}(\frac{m\omega}{k^2}) \Theta(4m\omega - k^2).
\end{equation}

The scaling function \h_{+}(m\omega/k^2) was numerically attained and is plotted in Fig.2. The theta function in (10) comes from the particle-hole continuum and is independent of the nature of interactions. Using the symmetry relations \Im\Sigma_{\pi ret}(\vec{k}, -\omega) = \Im\Sigma_{\pi ret}(\vec{k}, \omega), where \delta = -s it is clear to see that \Im\Sigma_{\pi ret}(\vec{k}, -\omega) gives a similar expressions as (10) with a different scaling function \h_{-}(m\omega/k^2) plotted in the inset of Fig.2. The residue contribution to the real part of the retarded electron self-energy \Re\Sigma_{\pi ret}
yields a similar expression, however the contribution due to $\Sigma^\text{line}_{\text{Dirac}}$ is more singular thereby dominating in the long-wavelength limit. We find that ($\omega > 0$):

$$\text{Re} \Sigma^{\text{ret}}_{s}(\vec{k}, \omega) = \frac{2k^2}{g^2 \pi m} (\log \left| \frac{\lambda}{k_0^2} \right|^2 - 4\omega \left( \log \left( \frac{\lambda}{\sqrt{m^* k}} \right) \right)^2 + ...,$$

where "..." represent the subleading terms. The expression for $\omega < 0$ can be attained by exploiting the symmetry relations $\text{Re} \Sigma^{\text{ret}}_{s}(\vec{k}, -\omega) = -\text{Re} \Sigma^{\text{ret}}_{s}(\vec{k}, \omega)$. The quasiparticle spectral weight:

$$\lim_{k \rightarrow 0} Z_+ = \frac{1}{1 - \partial_\omega \text{Re} \Sigma^{\text{ret}}_{s}(\vec{k}, \omega)} \sim \frac{g^2 n^2}{4} \left( \log \left[ \frac{2m^*}{m^* k} \right] \right)^2,$$

vanishes logarithmically, whereas the effective mass $m^*$

$$\frac{m_+}{m^*} = \frac{1 + m_0 \partial_\omega \text{Re} \Sigma^{\text{ret}}_{s}(\vec{k}, \omega)}{1 - \partial_\omega \text{Re} \Sigma^{\text{ret}}_{s}(\vec{k}, \omega)} \rightarrow 1,$$

remains finite and is not renormalized by interactions. The long-wavelength behavior of the spectral function $A(\vec{k}, \omega) = A_+(\vec{k}, \omega) + A_-(\vec{k}, \omega)$ plotted in Fig. 1 with:

$$A_s(\vec{k}, \omega) = \frac{1}{\pi \omega - \epsilon_k - \text{Re} \Sigma^{\text{ret}}_{s}(\vec{k}, \omega)^2 + \text{Im} \Sigma^{\text{ret}}_{s}(\vec{k}, \omega)^2},$$

was calculated from the leading order behavior of the electron self-energy and neglecting regular contributions. Symmetry relations for the electron self-energy dictate that $A(\vec{k}, \omega) = A(\vec{k}, -\omega)$.

The linear dependence of the $\text{Im} \Sigma$ on the quasiparticle energy predicted above is different form the case of neutral single layer graphene. In neutral single layer graphene due to the lack of screening associated with the Dirac point the Fermi velocity develops a logarithmic enhancement. For neutral graphene this logarithmic velocity enhancement implies that $\text{Im} \Sigma(\omega) \sim \omega / (\log \omega)^2$ which is smaller than $\text{Im} \Sigma(\omega) \sim \omega$. In contrast to single layer graphene interactions in bilayer graphene are screened, as we have shown and has been pointed out in the literature with Thomas-Fermi screening, that the quasiparticle dispersion in bilayer remains quadratic. Most of our analysis of quasiparticle properties in neutral bilayer graphene has relied on the fact that $\lambda_{\rho_0}(q, \Omega) \propto \omega^0$ (i.e. it has a scaling behavior after sending the bandwidth $k_c \rightarrow \infty$). In the next section we identify this regime of non-Fermi liquid behavior at finite temperature.

**Regime of Non-Fermi liquid behavior** — Weak interlayer hopping leads to trigonal warping of the band structure in bilayer graphene. The temperature associated with this effect can be estimated by calculating the energy scale at which trigonal warping effects compete with the quadratic dispersion kept within our model. Using the bare parameters of graphite $\gamma_3 \sim 0.1 \gamma_{\rho_0}$ we can estimate that the temperature below which the trigonal warping effect becomes relevant is $T_1 \sim 40K$. Below this scale the electron dispersion crosses over from quadratic to linear, and the system behaves like single-layer graphene, and our results no longer apply. Recently there has been discussions of interaction-driven spontaneous symmetry breaking in neutral bilayer graphene in the absence of trigonal warping, due to the marginal relevance of weak short-range repulsive interactions. Since the interaction is only marginally relevant, the transition temperature $T_c$ into the possible broken symmetry phases are exponentially small. The non-Fermi liquid behavior discussed here thus applies to temperatures above the higher of $T_c$ and $T_1$. Our analysis can be extended to finite temperatures, again dimensional analysis dictates that the polarization function have the scaling form $\Phi(m \Omega / k_B T, m \Omega / q^2)$. From a simple scaling analysis of the electron self-energy we conjecture that for temperatures $T > \max(T_1, T_c)$:

$$\text{Im} \Sigma \sim \begin{cases} \frac{\omega}{k_B T} & \omega > k_B T \\ k_B T & \omega > k_B T \end{cases}$$

indicative of non-Fermi liquid behavior. In this paper we have analyzed the regime $k < q_{TF}$, in the opposite regime $k > q_{TF}$ one can show that the non-Fermi behavior becomes even more pronounced as screening effect is less significant.

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