The relation between spatial reasoning and creativity in solving geometric problems

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Abstract. Spatial reasoning has been used in daily life, such as in building designs, determining routes and using maps. Spatial reasoning is also used in mathematics, especially in geometry. Therefore, this study aims to find out how spatial reasoning is used by students in solving geometry problems, especially in their creativity to transform 2-dimensional images into 3-dimensional images. Creativity in this study is focused on fluency, flexibility, and originality. This study is an explorative descriptive study through a qualitative approach. The subjects of the study are 11 third grade junior high school students. Data were collected by geometry problem-solving tests with two types of questions in the form of true or false statements and open-ended problems. The results showed that: (1) in terms of fluency, students gave an average of three correct answers out of five, (2) in terms of flexibility, there were five different types of answers, and (3) in terms of originality, there are two students who gave different answers than other students. The results also showed that students applied their spatial reasoning in the form of spatial visualization and mental rotation in their creativity to solve the problems.

1. Introduction

Spatial reasoning, whether we realize it or not, is used in daily life. For example, when we use or read a map to find a location, compile pieces of the puzzle to be complete, assemble parts of the lego into a shape, draw a house design, and determine the position of each room. All of these activities use spatial reasoning. Spatial ability is even referred to as one of the keys to success in science, technology, engineering and mathematics known as STEM [1].

Spatial reasoning is commonly used in mathematics, especially in geometry but not only in geometry but also in other fields of mathematics. For example in arithmetic, to understand the concept of 2 + 3, children will count it by imagining using concrete objects like they have two apples and then given three more apples so that the number of apples they have now is five. This mental process occurs in their minds.

The National Council of Teachers of Mathematics even states that mathematical reasoning is one of the five mathematical abilities students must possess, especially spatial reasoning which is very closely related to geometry [2]. Children use spatial skills in understanding the world, for example visualizing the relationships between objects around them and arranging it through spatial activities.

Embedding spatial skills in the mathematics curriculum can have a positive impact on the learning process and mathematics education [3]. Several studies show that students with good spatial reasoning...
also have better results in mathematical assessment [3]–[6] and their spatial reasoning will help them to understand the concepts of the material being taught [1].

There are three important components in spatial reasoning, namely spatial visualization, mental rotation and spatial orientation [6]. Spatial visualization is a mental ability to imagine and manipulate images using different perspectives [7]. This activity allows people to predict what will happen when they do visual activities such as folding and rearranging the pieces [8]. Mental rotation is a cognitive process in which a person imagines how 2-dimensional and 3-dimensional objects will appear after they have turned a point with a certain angle [4]. Spatial orientation is almost the same as mental rotation, objects are manipulated mentally but with a fixed observer's point of view. In other words, imagine how an object looks from a different perspective from the observer.

Preliminary research has been conducted by researchers [9], [10] by asking students to solve as many as possible 3-dimensional objects based on known information. Students gave average answers by drawing the same shape, which is a cube, a block, and a prism. The difference is only in the size of each edge that is drawn. Besides, variations in answers are obtained by rotating the image so that it appears in the form of a different 3-dimensional object image. This process shows that students use spatial reasoning in producing alternative solutions to solve the problems. Research conducted by Septia et al. [11] shows that mental rotation is the most aspect being used by students compared to other aspects of spatial reasoning to solve geometric problems.

In addition to spatial reasoning, creativity is also important [12], [13] and has even become a major concern in psychology, one of which focuses on the mental process of creating creative ideas [14]. In mathematics, creativity is also needed to process information that we already have to solve problems.

Guildford in his three-dimensional structure model states that aspects of creativity consist of fluency, flexibility, originality, and elaboration [15]. Fluency (the number of meaningful ideas) is related to the number of ideas that can be given in a short time. Flexibility (number of different response categories), namely the ability to think flexibly in seeing a variety of different perspectives. Originality (originality of the responses given) is the ability to think in unusual ways to generate new ideas and approaches used in problem-solving [16], [17]. Elaboration is the ability to imagine and explain something in detail [15].

The findings of the previous research on creativity between mathematical ability and creativity obtained that there is a positive correlation between mathematical ability and creativity [18]. The use of visualization provides important assistance for all types of problems, for example leading to the development of intuition and the ability to build new relationships and enable creative thinking. This is supported by several studies relating to creativity and spatial abilities. Cho and Suh [13] show that there is a positive correlation between spatial ability and creativity where the creative ideas that emerge are the result of the visual process. Object visualization and spatial visualization are related to scientific creativity [14].

Research on creativity and spatial reasoning has not been too much studied. One of them was done by Fiantika et al. [19] who analyzed students' spatial thinking processes in transforming 3-dimensional objects into 2-dimensional representations between female and male students. Their research shows that boy and girl have different ideas and their way of finding the fastest way to draw cube net using their representation in spatial thinking. However, how the spatial thinking process is used in the creativity of solving mathematical problems is not very clear. Therefore in this study, we try to see how students use their spatial reasoning (spatial visualization and mental rotation) in creativity to solving geometric problems, especially in terms of fluency, flexibility, and originality.

2. Method
This research is a descriptive exploratory study with a qualitative approach. This study aims to find out how spatial reasoning is used by students in providing creative ideas to solve geometric problems. The subjects or participants of the study are 11 third grade junior high school students in Dompu, Indonesia.
The instrument used in this study is the geometry problem consisting of five questions of changing 2-dimensional objects into 3-dimensional objects or vice versa. The instrument is a modification of the 2006 PISA test. This instrument consists of two types of questions in the form of true or false statements consisting four questions to determine students' spatial reasoning and one open ended question to measure students' creativity in using spatial reasoning in providing as many answers as possible to solve the problems. Indicators of spatial reasoning can be seen in Table 1.

Table 1. Indicator of Spatial Reasoning

| Construct                | Spatial Reasoning Aspects                          | Spatial Reasoning Characteristics                      |
|--------------------------|--------------------------------------------------|------------------------------------------------------|
| Mental rotation          | Rotate 2D and 3D objects clockwise and counterclockwise | Rotating 2D and 3D objects both rotate clockwise and counterclockwise. |
| Spatial Visualization    | Symmetry, patterns, 2D and 3D forms and their relationship, reflection, symmetry | Visualize the results of folding or opening objects. Visually form and rearrange a form into another different form. Determine the symmetry of an object Reflecting on the object. |

Indicators of creativity consist of three aspects. There is fluency, flexibility, and originality. In this study, it will be seen how spatial reasoning is used by students in producing creative problem solving. Therefore, the fluency referred to in this study is the number of correct answers given by students using their spatial reasoning. Fluency is the number of meaningful ideas given by students. Flexibility is the number of different response categories and originality is the originality of responses given to generate new ideas and approaches in solving problems using their spatial reasoning [16], [17]. Data is collected by giving test instruments to students and then they are interviewed to find out what ideas or strategy they use to solve each problems. The data that has been collected were analyzed using descriptive methods by Miles and Huberman with five stages: 1) data collection, 2) transcribing the data that has been collected, 3) segmentation and categorization of the data, 4) describing the spatial reasoning of students in their creativity in solving problems and 5) make conclusions. [21].

3. Result and Discussion

The solution given by subjects in this study to the first problem can be seen in Table 2.

| Item | Q1  | Q2  | Q3  | Q4  |
|------|-----|-----|-----|-----|
| Correct Frequency  | 8   | 9   | 11  | 5   |
| Correct Percentage  | 72.73% | 81.82% | 100.00% | 45.45% |
| Wrong Frequency    | 3   | 2   | 0   | 6   |
| Wrong Percentage   | 27.27% | 18.18% | 0.00% | 54.55% |

From Table 2, it can be seen that from a total of eleven subjects, the most incorrect answer to question no. 4 while all subjects answered correctly to question no 3. Examples of questions in the form of true or false statements can be seen in Figure 1. The error in statement no. 4 is caused by students in imagining when they folding cube nets, not paying attention to overall the opposite side of the dots in the cube net where there is a side pairs that the opposite side when it folded, the number is not equal to seven. Examples of incorrect student answers can be seen in Figure 2 below.
Six students answered that the statement on question no 4 (in Figure 1) was true, that is, the picture of the cube net was correct. Based on the results of interviews, students use their spatial visualization abilities to determine the truth of the statement. They folded the picture of the cube net whether in accordance with the provisions or not. First, they fold the cube net, then use mental rotation to adjust the position of the dots on each square in the cube net (can be seen in Figure 2).

Students answer incorrect to question no 4 because their perceptions in determining pairs of the dots in opposite side in each other in figure 2 are wrong. This shows that the spatial visualization ability used by students is correct but the mental rotation is still incorrect.

Table 3. Types of error answers given by subject in problem 2

| Type of Error Answers                                      | S1 | S4 | S6 | S8 | S11 |
|------------------------------------------------------------|----|----|----|----|-----|
| Error answer in drawing cube nets                          | 1  | -  | 1  | 2  | 1   |
| The position of the dots on the cube is not by following the provisions | 2  | 1  | -  | -  | -   |

Table 3 shows that four students were an error in making cube nets. Most of the mistakes made by students are in determining the position of the dots on each square in the cube net to conform to the provisions. Based on the results of interviews that have been conducted, students still have difficulty in the process of imagining when folding images of the cube nets they made and imagining the position of the dots because they do not see it directly and practice using real objects. Next, we will explain how the relationship between spatial reasoning and student creativity in solving the problem.
3.1 Students’ Creativity In Term Of Fluency
Student creativity in terms of fluency can be seen through the spatial reasoning used by students in producing the number of correct answers. Students gave an average of three correct answers. Meanwhile, the most correct answers given by subject S3 with five correct answers. The answers given by students were mostly obtained by rotating the answers they had obtained previously. Rotations are carried clockwise or counterclockwise by rotating the image of the cube net and by rotating the position of the dots on the cube net to the answers they have obtained previously as can be seen in Figure 2 and Figure 3.

![Figure 3](image3.png)  
**Figure 3.** Example answers by S2. Subject S2 rotates the figure (i) from 180° clockwise then switches the position of the dots on each square in the cube net in figure (ii).

![Figure 4](image4.png)  
**Figure 4.** Example answers by S10. Subject S10 switches the position of the dots on each square in the cube net in figure (i) to figure (ii) in the same net.

Based on Figure 3, it can be seen that the difference in students’ answers is in the position of the rotated nets that have been rotated and then change the position of the dots so that different answers are obtained from the other answers. Besides, there is also an answer obtained by changing the position of the dots while the shape of the net remains as shown in Figure 4. In the next process, they fold the cube nets that have been drawn whether they are by following the provisions of 1) forming a cube and, 2) the number of the dots on the opposite side of the cube is equal to seven. This process is done through the ability of spatial visualization in their minds. The process that has been carried out shows that, creativity of students in providing alternative answers in terms of fluency uses spatial reasoning in the form of spatial visualization and mental rotation abilities and the most dominant ability their used is mental rotation.

3.2 Students’ Creativity In term of Flexibility
Student creativity in terms of flexibility can be seen through the number of different ideas or ideas provided by students by using spatial reasoning correctly. The most dominant spatial reasoning used by students in producing answers is mental rotation. This ability is used by rotating the correct answers that have been generated.

In general, there are 5 answers to the form of cube nets given by students as shown in Figure 1 while the other answers are modifications of the general form by rotating (clockwise or counterclockwise) and shifting either on the side of the cube or in the position of the dots in the cube net. This activity involves the ability of mental rotation. Some alternative answers given by students can be seen in Figure 5.
However, some students are wrong in this spatial visualization process. Their error was to draw a cube net that was made wrong and if folded, the number of dots in the opposite side was not equal to seven. This can be seen in Figure 6.

Based on Figure 6, errors are caused because students try to shift the box in the cube nets in the previous drawing to another position to form new cube nets. But it was incorrect in the process of spatial visualization (when they folding the cube nets). This interpreted that students were less careful in checking the truth of the answers they gave. These results were in line with research conducted by Ma'rifatin, Amin and Siswono [5] and other researchers, Fauzi, Dirgeyase and Priyatno [22] where the errors experienced by students were in drawing objects 3-dimensions become 2-dimensional objects image or vice versa.

### 3.3 Students’ Creativity In term of Originality

Student creativity in terms of originality can be seen from new ideas provided by students who are different from ideas provided by other students by using their spatial reasoning. The originality of the answers given by students can be seen in Figure 7
There are two answers included in the originality category given by subject S2 which are different from the answers of other subjects as shown in Figure 6. Subject S4 also provides the same answer as Figure 6 in part (i). The difference is only in the position of the object drawn. We categorized the cube net image as originality because in addition to the two cube net images given by the subjects were different from the answers of other subjects, the cube net image they had never learned before (both in school and they learned it themselves) by them.

Students an average can solve the problems by imagining that they are folding the cube nets that they have comes-up in their minds. In general, students use two different strategies to solve the problem. The first strategy is to pay attention to several things: 1) imagine the image of a cube nets while folding it to form a cube, 2) determine the pair of the cube dots that if we add these dots then the result is equal to seven, and 3) determine the position of the dots to comply with the provisions given.

The second strategy is: 1) imagine the objects they know, for example, the eraser, like a cube (3-dimensional object) then they open each part to form cube nets in their mind (2 dimensions), 2) determine the pair of the cube dots which if we add these dots then the result is equal to seven, 3) determine the position of the dots on each square in the cube net to comply with the provisions given. This process indicates that students are conducting a spatial reasoning process. The difference between the two strategies is only in the first stages. After the nets cube was found, the next step is to provide cube numbering. In proving that the answers are correct, they refold the image that has been made by focusing that the number of cube dots on the opposite side is equal to seven. This shows the spatial visualization process.

In the process of giving another answer, students rotate the cube nets they have made before. Rotations are made to the sides of the cube nets that have been made so that new net shapes are obtained and different from the previous image.

Based on the results of the research described above, spatial reasoning in this case spatial visualization and mental rotation are used by students in providing creative ideas for solving geometrical problems in terms of fluency, flexibility and originality. These results are consistent with research conducted by Suh and Cho [13]. Visual processing is used to produce creative ideas.

4. Conclusion

Based on research that has been done, it can be concluded that student creativity are: (1) in terms of fluency, students on average can give three correct answers, (2) in terms of flexibility, there are five different cube net images that students can give, (3) in terms of originality, there were only two students who gave a picture of a cube nets that was different from other students. Whereas if we look at the relationship between creativity and spatial reasoning then: (1) students use spatial reasoning in producing creative problem-solving. Components of spatial reasoning used are spatial visualization and mental rotation. (2) spatial visualization is used by students to transform 3-dimensional objects into 2-dimensional objects image or vice versa when they open the cube into cube nets and fold the cube nets they have made into a cube again. (3) mental rotation is most often used to produce various alternative answers by rotating and shifting the sides of the cube and the dots to produce a different answer than before. Finally, we hope that the results of this study can contribute to the theory of spatial reasoning and creativity in solving geometry problems and also can expect to provide an overview of how spatial reasoning and creativity relate to problem-solving. Teachers are expected to help students in creating and developing spatial reasoning with their creativity.

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