Resource Letter: $2(2S + 1)$- Component Model and Its Connection with Other Field Theories.

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This Resource Letter has been presented at the IF-UNAM SEMINAR (12-11-1993). In the Joos-Weinberg $2(2S + 1)$- component approach [1] to description of particles of spin $S = 1$, the wave function (WF) of vector boson is written as six component column. It satisfies the following motion equation:

$$\left[\gamma_{\mu\nu}p_\mu p_\nu + M^2\right]\Psi^{(S=1)} = 0,$$

(1)

where $\gamma_{\mu\nu}$'s are covariantly defined $6 \otimes 6$- matrices, $\mu, \nu = 1, \ldots, 4$.

The main results are [2]:

– the scalar Lagrangian of Weinberg’s theory (the case of massless $S = 1$ particles and a la Majorana interpretation of Weinberg’s WF)

$$L^W = \partial_\mu \bar{\Psi} \gamma^{\mu\nu} \partial_\nu \Psi$$

(2)

is shown to be equivalent to the Lagrangian of free massless skew-symmetric field $L^H = F_k F_k/8$ ($F_k = i\epsilon_{jmn} F_{jm,n}$), presented by Hayashi (1973), ref. [3]. It describes massless particles with the longitudinal physical components only. The transversal components are removed by means of the new ”gauge” transformation;

– the vector Lagrangian, proposed in [2a], gives the dynamical invariants which are equivalent to the ones found by Lipkin (1964) and Sudbery (1986), ref. [4]. The energy-momentum conservation is associated not with translational invariance but with invariance under duality rotations;

– since the result of the item (1) is in contradiction with Weinberg’s theorem about connection between ($A, B$)- representation of the Lorentz group and helicity of particle ($B - A = \lambda$) and, moreover, the Weinberg’s massless equations [1] admit the acausal ($E \neq \pm p$) solutions [5], the new interpretation of the Weinberg’s $S = 1$ spinor has been proposed. It is based on use of a pseudovector potential $\tilde{A}_k$, $\Psi = \text{col}(\tilde{A}_k + iA_k, \tilde{A}_k - iA_k)$;

– the interaction Hamiltonian (two $S = 1/2$ particles and one massless $S = 1$ particle), proposed by Marinov (1968), appears to lead to the equations which are analogous to the equations for the Dirac oscillator [7, 8]:

\[
\begin{align*}
(E^2 - m^2)\xi &= \left[\vec{p}^2 + m^2 \omega^2 \vec{r}^2 + 2iEm\omega(\vec{\sigma}\vec{r}) + 3m\omega + 4m\omega\vec{S}[\vec{r} \times \vec{p}]\right] \xi,

(E^2 - m^2)\eta &= \left[\vec{p}^2 + m^2 \omega^2 \vec{r}^2 + 2iEm\omega(\vec{\sigma}\vec{r}) - 3m\omega - 4m\omega\vec{S}[\vec{r} \times \vec{p}]\right] \eta.
\end{align*}
\]

(3)

Similarly to [8] these equations include the term $(\vec{\sigma}\vec{r})$ and are not invariant under parity. To keep parity conservation it is necessary to assume that $\omega$, the frequency, is a pseudoscalar quantity, what means complicated dispersion law. However, ”irregular” invariants (where upper and down components of bispinors have been mixed) for interaction between such types of fields were pointed out in ref [6] to be possible. Since vector and pseudovector WF’s could be expressed by using some combination of two bispinors, this fact gives the opportunity
to construct other invariant which leads to the Dirac oscillator equations, proposed in ref. [7].

The presented formalism has been used for calculation of the scattering amplitude for two gluon interaction. The remarkable fact is that the amplitude coincides with the amplitude in Lobachevsky space \( (p^2_0 - \vec{p}^2 = M^2) \) for interaction of two spinor particles except for obvious substitutions. Moreover, the relativistic partial-wave equations have been obtained for the singlet gluonium state, the triplet one and the 5-plet gluonium state in relativistic configurational representation, which is just generalization of \( x \)-representation. Shapiro plane-wave functions are used instead of Fourier transformation.

The relativistic analogue of the Shay-Good Hamiltonian has been also obtained in the Lobachevsky space. The new magnetic momentum vector has been defined [2f].

The conclusions are:
– searches of alternative formulations of vector boson theory do have definite reasons because of some shortcomings (in my opinion) of usual QFT (the problem of indefinite metrics, the nonrenormalizability of vector boson field theories with the presence of mass term in the Lagrangian, e.t.c.);
– the Weinberg’s \( 2(2S+1) \)-formalism, which is used in the presented work, is very similar to the standard Dirac’s approach to spinor particles and, therefore, seems to be convenient for practical calculations;
– interpretation of WF of massless \( S = 1 \) particle, which has been given by Weinberg [1b], is not sufficiently satisfactory;
– we have, in fact, a bivector form of interaction between spinor particle and Weinberg’s vector boson instead of a minimal form of interaction in the case of the new interpretation of the Weinberg’s WF;
– the estimations of the possible influence of the above-mentioned model on the experimental results deserve further elaboration.

References

1. Weinberg S., "Phys. Rev." 133 (1964) B1318; ibid 134 (1964) B882; ibid 181 (1969) 1893.
2. Dvoeglazov V. V., Preprints IFUNAM FT-93-016, 019, 021, 024. Mexico. May-August 1993; Dvoeglazov V. V. and Skachkov N. B., JINR Communications P2-87-882. Dubna: JINR. December 1987; "Sov. J. Nucl. Phys." 48 (1988) 1065.
3. Hayashi K., "Phys. Lett." 44B (1973) 497.
4. Lipkin D. M., "J. Math. Phys." 5 (1964) 696; Sudbery A., "J. Phys. A" 19 (1986) L33.
5. Ahluwalia D. V. and Ernst D. J., "Mod. Phys. Lett." 7 (1992) 1967.
6. Marinov M. S., "Annals of Physics" 49, No. 3 (1968) 357.
7. Moshinsky M. and Szcepaniak A., "J. of Phys. A." 22 (1989) L817.
8. Dixit V. V., Santhanam T. S. and Thacker W. D., "J. Math. Phys." 33 (1992) 1114.
CLASSICAL WORKS

1. H. Joos, "Forts. Phys." 10 (1962) 65 (56 refs in SLAC database).

2. S. Weinberg, Feynman Rules for Any Spin. "Phys. Rev." 133, No. 5B (1964) B1318 (113 refs. in SLAC database); Feynman Rules for Any Spin. II. Massless Particles. Ibid 134, No. 4B (1964) B882 (79 refs in SLAC database); Feynman Rules for Any Spin. III. Ibid 181, No. 5 (1969) 1893 (22 refs. in SLAC database); The Quantum Theory of Massless Particles. In "Lectures on Particles and Field Theory. Vol. 2. Brandeis Summer Institute in Theoretical Physics.” (1964) 405.

3. D. L. Weaver, C. L. Hammer and R. H. Good, Jr., Description of a Particle with Arbitrary Mass and Spin. "Phys. Rev." 135 No. 1B (1964) B241 (15 refs in SLAC database).

4. A. O. Barut, I. Muzinich and D. N. Williams, Construction of Invariant Scattering Amplitudes for Arbitrary Spins and Analytic Continuation in Total Angular Momentum. "Phys. Rev." 130 No. 1 (1963) 442.

5. D. N. Williams, The Dirac Algebra for Any Spin. In "Lectures in Theoretical Physics. Vol. VII A - Lorentz Group. Summer Institute for Theoretical Physics, University of Colorado, Boulder.” (1964) 139.

6. A. Sankaranarayanan and R. H. Good, Jr., Spin-One Equation. "Nuovo Cimento" 36, No. 4 (1965) 1303; D. Shay, H. S. Song and R. H. Good, Jr., Spin Three-Halves Wave Equations. "Nuovo Cimento Suppl." 3, No. 3 (1965) 455; A. Sankaranarayanan, Covariant Polarization Theory of Spin-One Particles. "Nuovo Cimento" 38 No. 2 (1965) 889.

7. P. M. Mathews, Relativistic Schrödinger Equations for Particles of Arbitrary Spin. "Phys. Rev." 143, No. 4 (1966) 978; S. A. Williams, J. P. Draayer and T. A. Weber, Spin-Matrix Polynomial Development of the Hamiltonian for a Free Particle of Arbitrary Spin and Mass. "Phys.Rev." 152 (1966)1207

8. M. S. Marinov, Construction of Invariant Amplitudes for Interactions of Particles with Any Spin. "Annals of Physics" 49, No. 3 (1968) 357.

9. D. Shay and R. H. Good , Jr., Spin-One Particle in an External Electromagnetic Field. "Phys. Rev.”, 179, No. 5 (1969) 1410; L. D. Krase, Pao Lu and R. H. Good, Jr., Stationary States of a Spin-1 Particle in a Constant Magnetic Field. "Phys. Rev. D" 3, No.
10. R. H. Tucker and C. L. Hammer, *New Quantum Electrodynamics for Vector Mesons.* "Phys. Rev. D" 3, No. 10 (1971) 2448.

11. C. L. Hammer and R. H. Tucker, *A Method of Quantization for Relativistic Fields.* "J. of Math. Phys." 12, No. 7 (1971) 1327.

12. R. N. Faustov, *Relativistic Transformation of One-Particle Wave Functions.* Preprint ITF-71-117P (1971) Kiev (in Russian).

13. Wu-yang Tsai and A. Yildiz, *Motion of Charged Particles in a Homogeneous Magnetic Field.* "Phys. Rev. D" 4, No. 12 (1971) 3643; T. Goldman and Wu-yang Tsai, *Motion of Charged Particles in a Homogeneous Magnetic Field. II.* "Phys. Rev. D" 4, No. 12 (1971) 3648.

14. Yu. V. Novozhilov, *Introduction to Elementary Particle Theory.* Pergamon Press, Oxford, 1975, section 3.3.

**RECENT WORKS**

1. H. M. Ruck and W. Greiner, *A Study of the Electromagnetic Interaction Given by Relativistic Spin-1 Wave Equations in Elastic Scattering of Polarized Spin-1 Nuclei or Mesons.* "J. of Phys. G: Nucl. Phys." 3, No. 5 (1977) 657.

2. F. D. Santos, *Relativistic Spin-1 Dynamics and Deuteron-Nucleus Elastic Scattering.* "Phys. Letters B" 175, No. 2 (1986) 110; F. D. Santos and H. van Dam, *Relativistic Dynamics of Spin-One Particles and Deuteron-Nucleus Scattering.* "Phys. Rev. C" 34, No. 1 (1991) 250.

3. L. Lukaszuk and L. Szimanowski, *Tensor and Bispinor Representation of Massless Fields.* "Phys. Rev. D" 36 (1987) 2440.

4. R. H. Good, Jr., *Relativistic Wave Equations for Particles in Electromagnetic Fields.* "Annals of Physics" 196, No. 1 (1989) 1.
5. V. K. Mishra et al., Implications of Various Spin-One Relativistic Wave Equations for Intermediate-Energy Deuteron-Nucleus Scattering. "Phys. Rev. C" 43, No. 2 (1991) 801.

6. L. P. Horwitz and N. Shnerb On the Group Theory of the Polarization States of a Gauge Field. Preprint IASSNS-92-56 (Sept. 1992) Tel Aviv.

7. V. V. Dvoeglazov and N. B. Skachkov, Three-Dimensional Covariant Equation for a Composite System Formed with a Fermion and a Boson. JINR Communications P2-84-199 (Apr.1984) Dubna (in Russian); Gluonium Mass Spectrum in the Quasipotential Approach. JINR Communications R2-87-882 (Dec. 1987) Dubna (in Russian); Relativistic Parametrization of the Hamiltonian for Vector-Particle Interaction with the External Electromagnetic Field. "Yadernaya Fiz.", 48 , No. 6(12) (1988) 1770 (in Russian), "Sov. Journal of Nucl. Phys.", 48 (1988)1065 (in English); V. V. Dvoeglazov, S. V. Khudyakov and S. B. Solganik Relativistic Covariant Equal-Time Equation For Quark-Diquark System. Preprint IFUNAM FT-93-024 (hep-ph/9308305) (Aug. 1993) Mexico, submitted to "Revista Mexicana de Física" (in English).

8. D. V. Ahluwalia and D. J. Ernst, Weinberg Equations for Arbitrary Spin: Kinematic Acausality but Causal Propagators. "Phys. Rev. C" 45 (1992) 3010; Paradoxial Kinematics Acausality in Weinberg’s Equations for Massless Particles of Arbitrary Spin. "Mod. Phys. Letters A" 7, No. 22 (1992) 1967; New Arbitrary-Spin wave Equations for \((j,0) \oplus (0,j)\) Matter Fields without Kinematical Acausality and Constraints. "Phys. Letters B" 287 (1992) 18; \((j,0) \oplus (0,j)\) Covariant Spinors and Causal Propagators Based on Weinberg Formalism. "Int. J. Mod. Phys. E" 2, No. 2 (1993) 397; D. V. Ahluwalia, Interpolating Dirac Spinors between Instant and Light Front Forms. "Phys. Letters B" 277 (1992) 243; D. V. Ahluwalia and T. Goldman, Space-Time Symmetries and Vortices in the Cosmos. "Mod. Phys. Letters A" 8, No. 28 (1993) 2623; D. V. Ahluwalia and M. Sawicki, Front Form Spinors in Weinberg-Soper Formalism and Melosh Transformations for any Spin. "Phys. Rev. D" 47, No. 11 (1993) 5161; Natural Hadronic Degrees of Freedom for an effective QCD Action in the Front Form. Preprint LA-UR-92-3133 (hep-ph/9210204) (1992) Los Alamos, submitted in "Phys. Rev. D"; D. V. Ahluwalia, T. Goldman and M. B. Johnson, A Bargmann -Wightman- Wigner Type Quantum Field Theory. Preprint LA-UR-92-3726-REV (hep-ph/9304243) (1993) Los Alamos; Majorana-Like \((j,0) \oplus (0,j)\) Representation Spaces: Construction and Physical Interpretation. Preprint LA-UR-93-2645 (hep-th/9307118) (July1993); Los Alamos.