Tau Neutrino Fluxes from Atmospheric Charm

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Abstract

We present an evaluation of the atmospheric tau neutrino flux in the energy range between $10^2$ and $10^6$ GeV. The main source of tau neutrinos is from charmed particle production and decay. The $\nu_{\tau}N \rightarrow \tau X$ event rate for a detector with a water equivalent volume of 1 km$^3$ is on the order of 60-100 events per year for $E_{\tau} > 100$ GeV, reducing to 18 events above 1 TeV. Event rates for atmospheric muon neutrino oscillations to tau neutrinos are also evaluated.

I. INTRODUCTION

Recent measurements of the atmospheric neutrino flux by the Super-Kamiokande Collaboration [1,2] show a deficit of muon neutrinos in comparison to theoretical predictions, while the measured electron neutrino flux is consistent with theory assuming that all neutrino masses vanish. Earlier, lower statistics experiments already showed inconsistencies with theoretical atmospheric flux predictions [3]. On the basis of event rates and the zenith angle dependence of the muon neutrino deficit, the Super-Kamiokande Collaboration has shown that their results could be explained by neutrino oscillations between $\nu_\mu$ and $\nu_{\tau}$ [2]. Oscillations imply at least one non-zero neutrino mass. Definitive evidence of massive neutrinos requires modifying the standard model of electroweak interactions.

The Super-Kamiokande Collaboration measures neutrino fluxes from observations of electrons and muons in neutrino nucleon interactions: $\nu_l + N \rightarrow l + X$. In view of the importance of the question of whether or not neutrinos have mass, one would like to see not just $\nu_\mu$ disappearance, but also the $\nu_\tau$ appearance coming from $\nu_\mu \rightarrow \nu_\tau$ oscillations. Oscillation sources of $\nu_\tau$’s include oscillations on the terrestrial scale from atmospheric $\nu_\mu$’s as well as oscillations over large astronomical distances of $\nu_\mu$’s produced in, for example, active galactic nuclei [4]. A background to the flux of neutrinos from $\nu_\mu \rightarrow \nu_\tau$ are tau neutrinos produced directly in the atmosphere.

Tau neutrinos are produced in the atmosphere by cosmic ray collisions with nuclei in the atmosphere, which produce charm quark pairs. A fraction of the time, the emerging hadrons are $D_s$’s, which have a leptonic decay channel $D_s \rightarrow \tau \nu_\tau$ with a branching ratio of a few percent. The subsequent $\tau$ decays also contribute to the atmospheric $\nu_{\tau}$ flux. Heavier mesons contribute to the flux of tau neutrinos, but as we show below, they are negligible compared to the $D_s$ contribution.
In this letter, we outline the procedure to calculate the atmospheric tau neutrino flux. The details of the method as applied to atmospheric electron neutrino, muon neutrino and muon fluxes from charm decay appear in Refs. [5] and [6]. We present our flux results for the neutrino energy range of $10^2 - 10^6$ GeV, followed by the resulting $\nu_\tau + N \rightarrow \tau + X$ event rates. For tau energies above 100 GeV, the rate is on the order of $60 - 100$ events per year per km$^3$ water equivalent volume. With a 1 TeV threshold, there are on the order of 20 events.

We also evaluate the expected event rate for the tau neutrino flux coming from $\nu_\mu \rightarrow \nu_\tau$ oscillations based on a range of parameters consistent with the Super-Kamiokande results [2]. For tau energies above a few hundred GeV, the atmospheric tau neutrino background flux from $D_s \rightarrow \nu_\tau \tau$ dominates the tau neutrino flux from atmospheric muon neutrino oscillations.

II. TAU NEUTRINO FLUX CALCULATION

The main source of atmospheric tau neutrinos is the leptonic decay of the $D_s$: $D_s \rightarrow \tau \nu_\tau$, followed by $\tau \rightarrow \nu_\tau X$. For relativistic particles, a semianalytical, unidimensional approximate solution to cascade equations describing proton, meson and lepton fluxes is a reliable approximation [7,8,6]. The solutions rely on factorizing source terms in the cascade equations into factors which are weakly dependent on energy times the incident cosmic ray flux, here approximated by a proton flux. The source term for $p\text{Air} \rightarrow D_s$, for a $D_s$ of energy $E$ and column depth $X$ as measured from the top of the atmosphere is

$$S(p \rightarrow D_s) \simeq \frac{\phi_p(E, X)}{\lambda_p(E)} \int_E dE_p \frac{\phi_p(E_p, 0)}{\phi_p(E, 0)} \frac{\lambda(E)}{\lambda_p(E_p)} \frac{dn_{p \rightarrow D_s}}{dE}(E; E_p)$$

$$\equiv \frac{\phi_p(E, X)}{\lambda_p(E)} Z_{pD_s}(E).$$

Here $\phi_p(E, X)$ is the flux of cosmic ray protons at column depth $X$. At the top of the atmosphere ($X = 0$), following Ref. [3], we set

$$\phi_p(E, 0) = 1.7 \ (E/\text{GeV})^{-2.7} \ \text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{GeV}^{-1},$$

valid for energies values lower than $5 \cdot 10^6$ GeV. In Eq. (2.1), $\lambda_p$ is the proton interaction length and $dn/dE$ is the cross section normalized energy distribution of the $D_s$ emerging from the proton-Air collision. The quantity $Z_{pD_s}(E)$ is called a Z-moment. Generically, Z-moments describe sources of particles of energy $E$, whether by production, decay or energy loss through scattering. A complete discussion of the Z-moment method of solution appears in Refs. [7] and [8]. Its recent application to atmospheric muon, muon neutrino and electron neutrino fluxes from charm decays is found Refs. [3] and [4].

Solutions to the cascade equations in terms of Z-moments have two separate forms, depending on whether the decay lengths are short compared to the height of production (“low energy”) or long (“high energy”). For $\nu_\tau$'s from $D_s$ (and $\tau$) decays, we confine our attention to the neutrino energy range $10^2 - 10^6$ GeV. These neutrino energies are well below the critical energy of $\sim 10^8$ GeV, above which decay lengths of the relativistic $D_s$'s and $\tau$'s are longer than the vertical distance to height of production. Tau neutrinos are...
called “prompt” in the low energy regime. The approximate solution for the $\nu_\tau + \bar{\nu}_\tau$ flux, at the surface of the Earth, is

$$\phi_{\nu_\tau}(E) = \frac{Z_{pD_s}(E)Z_{D_s\nu_\tau}(E)}{1 - Z_{pp}(E)}\phi_p(E,0). \quad (2.3)$$

The prompt tau neutrino flux is isotropic. $Z_{pp}(E)$ accounts for the proton energy loss in proton-air collisions. For $Z_{pp}(E)$, we use the results obtained by Thunman et al. in their recent evaluation \[12\] using the Monte Carlo PYTHIA \[9\]. A similar, energy independent value was used in Ref. \[8\].

For $Z_{pD_s}(E)$, there are several approaches. Here we show the results from next-to-leading order (NLO) perturbative QCD production of charmed quark pairs, scaled by a factor of 0.13 to account for the fraction of $c \to D_s$ \[11\]. Details of the NLO calculation in the context of the prompt muon, muon neutrino and electron neutrino fluxes appear in Ref. \[5\]. A second evaluation relies on Thunman et al.’s \[6\] $Z_{pD_s}(E)$, rescaled by the ratio of $D_s$ to $D^0$ production taken to be 0.25.

The $D_s \to \nu_\tau$ decay $Z$-moments have several contributions. The most straightforward is the direct $D_s \to \nu_\tau$ in the two body decay, where \[7\]

$$Z_{D_s\nu_\tau}^{(2 \text{ body})}(E) = \int_0^{1-R_{D_s}} dx \frac{Z_{pD_s}(E/x)}{Z_{pD_s}(E)} \frac{\sigma_{pA}(E/x)}{\sigma_{pA}(E)} \frac{\phi_p(E/x,0)}{\phi_p(E,0)} \frac{B}{1 - R_{D_s}} \quad (2.4)$$

in terms of $x = E/E_{D_s}$, $R_{D_s} = m_\tau^2/m_{D_s}^2$ and $B = 0.043$, the branching ratio for $D_s \to \tau\nu_\tau$ \[11\]. We use the inelastic proton-air cross section

$$\sigma_{pA}(E) = (290 - 8.7 \ln(E/\text{GeV}) + 1.14 \ln^2(E/\text{GeV})) \text{ mb} \quad (2.5)$$

parameterized in Ref. \[12\].

For neutrinos produced in the chain decay $D_s \to \tau \to \nu_\tau$ the decay $Z$-moment is given by

$$Z_{D_s\nu_\tau}^{(\text{chain})}(E) = \int_0^1 dy \int_0^{1-R_{D_s}} \frac{dx}{x} \frac{Z_{pD_s}(E/(xy))}{Z_{pD_s}(E)} \frac{\sigma_{pA}(E/(xy))}{\sigma_{pA}(E)} \frac{\phi_p(E/(xy),0)}{\phi_p(E,0)} \left[ \frac{B}{1 - R_{D_s}} \right] \frac{dn_{\tau\to\nu_\tau}}{dy} \quad (2.6)$$

where $B$, $R_{D_s}$ and $\sigma_{pA}$ are the same as in the previous case while now $x = E_\tau/E_{D_s}$ and $y = E/E_\tau$. The $y$ distribution that appears in Eq. (2.6) can be generically written in the following way \[7\]

$$\frac{dn_{\tau\to\nu_\tau}}{dy} = B_\tau \left( g_0(y) - P_\tau(x)g_1(y) \right) \quad (2.7)$$

where $B_\tau$ is the branching ratio for $\tau \to \nu_\tau X$ and $P_\tau(x)$ is the tau polarization. We have evaluated the functions $g_0(y)$ and $g_1(y)$ for $\tau \to \nu_\tau \rho$, $\tau \to \nu_\tau \pi$, $\tau \to \nu_\tau a_1$ \[13\] and $\tau \to \nu_\tau \ell \nu_\ell$. They are collected in Table I, as are the branching fractions that we use. The purely leptonic decay also appears in Refs. \[7\],[8]. In terms of the energy of the parent $D_s$ ($E_{D_s}$), the tau polarization can be written as
\[ P_\tau = \frac{2R_{Ds}}{1 - R_{Ds}} \frac{E_{Ds}}{E_\tau} - \frac{(1 + R_{Ds})}{(1 - R_{Ds})}. \tag{2.8} \]

The total \( D_s \rightarrow \nu_\tau \) Z-moment is

\[ Z_{Ds\nu_\tau} = Z^{(2\text{ body})}_{Ds\nu_\tau} + Z^{(\text{chain})}_{Ds\nu_\tau}, \tag{2.9} \]

where \( Z^{(\text{chain})}_{Ds\nu_\tau} \) includes the sum over all the tau decay modes in Table I.

III. TAU NEUTRINO FLUX RESULTS

Using NLO perturbative QCD with \( m_c = 1.3 \text{ GeV} \), and factorization scale \( M \) equal to twice the renormalization scale \( m \) with the CTEQ3 parton distribution functions [14], we obtain \( Z_{pDs} \) between \( 10^{-5} \) and \( 10^{-4} \). The Thouman et al. PYTHIA results (TIG) [6] give instead \( Z_{pDs} \) between \( 3.5 \cdot 10^{-5} \) and \( 6.5 \cdot 10^{-5} \). The corresponding decay moments \( Z_{Ds\nu_\tau} \) are \( 10^{-3} - 5.5 \cdot 10^{-3} \) and \( 0.8 \cdot 10^{-3} - 4.8 \cdot 10^{-3} \), respectively, over the energy range \( 10^2 - 10^6 \) GeV.

The results for the tau neutrino fluxes scaled by \( E^3 \) are shown in Fig. 1 where the solid line represents the flux obtained using NLO perturbative QCD, and the dashed line represents the flux obtained using the TIG \( D_s \) production Z-moment. In principle, one should also take into account the neutrinos produced by the semileptonic decay of the \( b \) quark \((b \rightarrow c\tau\nu_\tau)\) followed by the decay of the tau, but this contribution is less than a few percent of charmed meson source. The dotted line represents the flux from \( b \) quark production and decay in the limit of zero polarization for the tau.

The fact that the flux calculated using NLO perturbative QCD differs from the TIG PYTHIA based calculation [8] is apparently due to the inclusion of fragmentation effects in the PYTHIA Monte Carlo. A detailed comparison of the two approaches appears in Ref. [8]. The tau neutrino flux evaluated using NLO perturbative QCD can be parametrized as

\[ \log_{10}(E^3\phi_{\nu_\tau}(E)/(\text{GeV}^2/\text{cm}^2/\text{sr})) = -A + Bx - Cx^2 - Dx^3 \tag{3.1} \]

where \( x = \log_{10}(E/\text{GeV}) \), \( A = 7.08 \), \( B = 0.765 \), \( C = 0.00346 \) and \( D = 0.00349 \). The TIG rescaled result calculated here has \( A = 6.69 \), \( B = 1.05 \), \( C = 0.150 \) and \( D = -0.00820 \).

IV. DISCUSSION

Large underground experiments may detect \( \nu_\tau \)-nucleon charged current interactions \( \nu_\tau + N \rightarrow \tau + X \). In the absence of \( \nu_\mu \rightarrow \nu_\tau \) oscillations, event rates [15] for \( E_\tau > 100 \text{ GeV} \) and \( E_\tau > 1 \text{ TeV} \) are shown in Table II. We have summed over neutrino and antineutrinos and assumed that there is no attenuation of the \( \nu_\tau \) flux due to passage through the Earth at these energies [15]. Detection of atmospheric \( \nu_\tau \)'s in the absence of oscillations requires large detector volumes, on the order of 1 km\(^3\) water equivalent volume for \( \sim 60 - 100 \) events per year with a tau energy threshold of 100 GeV. Such large detector volumes for the detection of \( \nu_\mu \) charged current interactions are being considered [16].

In the case of neutrino oscillations, atmospheric muon neutrinos may oscillate as they travel to the detector. Even with a small fraction of \( \nu_\mu \rightarrow \nu_\tau \) conversions, the copious
production of $\nu_\mu$'s in the atmosphere make this a potentially large source of $\nu_\tau$'s. Again, assuming no attenuation due to neutrino interactions in the Earth, and approximating the height of production in the atmosphere by a constant 20 km [17], we have evaluated the probability of oscillation in terms of neutrino energy and zenith angle. Using the Bartol muon neutrino flux [18] from $\pi$ and $K$ decays extrapolated to $E = 10^6$ GeV, adding the NLO prompt $\nu_\mu$ flux [5] and integrating over zenith angle, Table III shows the event rates for two limiting values of $\Delta m^2$, the difference between the squares of the tau neutrino and muon neutrino masses, quoted by the Super-Kamiokande Collaboration [2]. We have set the oscillation mixing angle $\sin^2(2\theta) = 1$. For the larger mass difference, $\nu_\mu \rightarrow \nu_\tau$ events overwhelm the prompt $\nu_\tau$ events for $E > 100$ GeV, while for the lower mass difference the rates are comparable. Tau neutrino oscillations to $\nu_\mu$ in this parameter range have a negligible depletion of the prompt $\nu_\tau \rightarrow \tau$ event rate.

For $E_\tau > 1$ TeV, because prompt fluxes decrease less rapidly with energy than the muon neutrino flux coupled with oscillations, the prompt $\nu_\tau$'s dominate the event rate. The cross over to prompt $\nu_\tau$ domination of the flux occurs at a few hundred GeV. Another distinction between prompt and oscillation tau neutrinos is the angular distribution. The prompt source is isotropic, while the oscillation to $\nu_\tau$'s will be strongly angular dependent, with the most $\nu_\tau$'s traveling upward because of the long path length.

We have concentrated here on atmospheric sources of prompt $\nu_\tau$'s and atmospheric $\nu_\mu \rightarrow \nu_\tau$. There are many flux models of astrophysical sources of $\nu_\mu$'s [4]. If neutrinos have mass, because of the large astronomical distance scales, on the order of half of the $\nu_\mu$'s would oscillate to $\nu_\tau$'s. The detectability of $\nu_\tau \rightarrow \tau$ conversions in the PeV energy range has been explored in the literature [19]. It is clear from our calculation that the atmospheric tau neutrino flux is negligible at PeV energies.

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TABLE I. Functions $g_0$ and $g_1$ in the tau neutrino energy distribution from $\tau$ decays, in terms of $y = E/E_\tau$ and $r_i = m_i^2/m_\tau^2$ and relative branching ratios.

| Process | $B_\tau$ | $g_0$ | $g_1$ |
|---------|----------|-------|-------|
| $\tau \to \nu_\tau \mu \nu_\mu$ | 0.18 | $\frac{5}{3} - 3y^2 + \frac{4}{3}y^3$ | $\frac{1}{3} - 3y^2 + \frac{8}{3}y^3$ |
| $\tau \to \nu_\tau \pi$ | 0.12 | $\frac{1}{1-r_\pi}$ | $-\frac{2y-1+r_\pi}{(1-r_\pi)^2}$ |
| $\tau \to \nu_\tau \rho$ | 0.26 | $\frac{1}{1-r_\rho}$ | $-\left(\frac{2y-1+r_\rho}{(1-r_\rho)^2}\right)\left(\frac{1-2r_\rho}{1+2r_\rho}\right)$ |
| $\tau \to \nu_\tau a_1$ | 0.13 | $\frac{1}{1-r_{a_1}}$ | $-\left(\frac{2y-1+r_{a_1}}{(1-r_{a_1})^2}\right)\left(\frac{1-2r_{a_1}}{1+2r_{a_1}}\right)$ |

TABLE II. Charged current event rate per year per km$^3$ water equivalent volume from the prompt $\nu_\tau + \bar{\nu}_\tau$ flux.

| Threshold | NLO QCD | TIG |
|-----------|---------|-----|
| 100 GeV   | 58      | 98  |
| 1 TeV     | 18      | 18  |

TABLE III. Charged current event rate per year per km$^3$ water equivalent volume from $\nu_\mu + \bar{\nu}_\mu \to \nu_\tau + \bar{\nu}_\tau$ oscillations, assuming $\sin^2(2\theta) = 1$.

| Threshold | $\Delta m^2 = 5 \cdot 10^{-4}$ eV$^2$ | $\Delta m^2 = 6 \cdot 10^{-3}$ eV$^2$ |
|-----------|-----------------------------------|-----------------------------------|
| 100 GeV   | 71                                | 9100                              |
| 1 TeV     | 0.036                             | 5.2                               |
FIG. 1. Tau neutrino plus antineutrino fluxes from $D_s$ using NLO perturbative QCD (solid line) and the TIG rescaled $Z$-moment (dashed line). The tau neutrino plus antineutrino fluxes from $b$ quark NLO perturbative QCD production and decay with the tau polarization equal to zero is shown by the dotted line.