Relativity of Space-Time Geometry

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Abstract

We argue that space-time geometry is not absolute with respect to the frame of reference being used. The space-time metric differential form $ds$ in noninertial frames of reference (NIFR) is caused by the properties of the used frames in accordance with the Berkley - Leibnitz - Mach - Poincaré ideas about relativity of space and time. It is shown that the Sagnac effect and the existence of inertial forces in NIFR can be considered from this point of view.

1 Introduction

The geometrical properties of space-time can be described only by means of measuring instruments. At the same time, the description of the properties of measuring instruments, strictly speaking, requires knowledge of space-time geometry. One of the implications of it is that the geometrical properties of space and time have no experimentally verifiable significance by themselves but only within the aggregate “geometry + measuring instruments”. We got aware of it owing to Poincaré. It is a development of the idea going back to Berkley, Leibnitz and Mach. (Leibnitz, for example, considered that space and place are abstractions from relations of ordinary objects and should be analyzed in these terms).

If we proceed from the conception of relativity of space and time in Berkley - Leibnitz - Mach - Poincaré (BLMP) sense, we should assume that there is no way of quantitative description of physical phenomena other than attributing them to a certain frame of reference which in itself is a physical device for space and time measurements. But then the relativity of the geometrical properties of space and time mentioned above is nothing else but relativity of space-time geometry with respect to the frame of reference being used.
Thus, it should be assumed that the concept of frame of reference as a physical object whose properties are given and independent of the properties of space and time is approximate, and only the aggregate "frame of reference + space-time geometry" has a sense.

The Einstein theory of gravitation demonstrates relativity of space-time with respect to distribution of matter. However, space-time relativity with respect to measurement instruments hitherto has not been realized in physical theory. In this paper an attempt to show that there is also space-time relativity to measurement has been undertaken. (See also [5]).

In our analysis of the problem we start from the fact that an important distinction exists between a frame of reference (as a physical device) and a coordinate system (as a way of the space-time points parametrization). Any coordinate transformation in pseudo-Euclidean space-time (when the curvature tensor, certainly, remains equal to zero) does not mean yet a transition from an inertial frame of reference to the noninertial one.

At present we do not know how the space-time geometry in inertial frames of reference (IFR) is connected with the frames properties. Under the circumstances, we simply postulate (according to special relativity) that space-time in IFR is pseudo-Euclidean. Next, we find the space-time metric differential form in noninertial frames of reference (NIFR) from the viewpoint of an observer in a NIFR who proceeds from the relativity of space and time in the BLMP sense.

Then it appears that there are certain reasons to suppose that metric properties of the space-time in the NIFR do not have a physical meaning in themselves. The metric differential form $ds$ is completely conditioned by the properties of the frame being used as is to be expected according to the idea of relativity of space and time in the BLMP sense.

2 The Metric Form $ds$ in NIFR.

By a noninertial frame of reference (NIFR) we mean the frame, whose body of reference is formed by the point masses moving in the IFR under the effect of a given force field.

It would be a mistake to identify "a priori" the transition from an IFR to the NIFR with the transformation of coordinates related to the frames. If we act in such a way, we already assume that the properties of the space-time in both frames are identical. However, for an observer in the NIFR, who proceeds from the relativity of space and time in the BLM sense, space-time geometry is not given "a priori" and must be ascertained from the analysis of the experimental data.

We shall suppose that the reference body of the IFR or NIFR is formed by the identical point masses $m$. If the observer is at rest in one of the frames, his world line will coincide with the world line of some point of the reference
body. It is obvious to the observer in the IFR that the accelerations of the point masses forming the reference body are equal to zero. Certainly, this fact takes place also in relativistic sense. That is, if the differential metric form of space-time in the IFR is denoted by $d\eta$ and $\zeta_\alpha^0 = dx^\alpha/d\eta$ is the 4-velocity vector of the point masses forming the reference body, then the absolute derivative of the vector $\zeta_0^\alpha$ is equal to zero, i.e.

$$D\zeta_0^\alpha / d\eta = 0.$$  \hspace{1cm} (1)

(We mean that arbitrary coordinate system is used).

Does this fact take place for an observer in NIFR? That is, if the differential metric form of space-time in the NIFR is denoted by $ds$, does the 4-velocity vector $\zeta^\alpha = dx^\alpha/ds$ of the point masses forming the reference body of this NIFR obey the equation

$$D\zeta^\alpha / ds = 0?$$  \hspace{1cm} (2)

The answer depends on whether space and time are absolute in Newtonian sense or they are relative in the BLMP sense.

If space and time are absolute, the point masses of the NIFR reference body are at relative rest. A notion of relative acceleration can be determined in a covariant way. This value is equal to zero. However, eq. (2), strictly speaking, are not satisfied.

If space and time are relative in the BLMP sense, then for observers in the IFR and NIFR the motion of the point masses forming the reference body (RB), which are kinematically equivalent, must be dynamically equivalent too (both in the nonrelativistic and relativistic sense). That is, if from the viewpoint of the observer in the IFR, the point masses forming the NIFR RB are at rest (are not subject to the influence of forces either), then from the viewpoint of the observer in the NIFR the point masses forming the RB of his frame are at rest too (are not subject to the influence of forces either). In other words, if for the observer in the IFR the world lines of the IFR RB points are, according to eq. (1), the geodesic lines, then for the observer in the NIFR the world lines of the NIFR RB points also are the geodesic lines in his space-time, which can be expressed by eq. (2). The differential equations of these world lines at the same times are the Lagrange equations of motion of the NIFR RB points. It is obvious that these equations are equations of the geodesic lines in space-time whose metric differential form is given by $ds = k \, dS$, where $S$ is the Lagrange action describing the motion of the identical material point masses $m$ forming NIFR RB, in the IFR, and $k$ is the constant. This constant $k = -(mc)^{-1}$, which can be seen from the analysis of the case when the frame of reference is inertial.

Thus, if we start from relativity of space and time in the BLM sense, then the differential metric form of space-time in the NIFR can be expected to have the following form

$$ds = -(mc)^{-1} \, dS,$$  \hspace{1cm} (3)
where $S$ is the Lagrangian action of the identical point masses $m$, forming the body of reference of the NIFR.

So, the properties of space-time in the NIFR are entirely determined by the properties of the used frame in accordance with the idea of relativity of space and time in the BLMP sense.

Let us consider two examples of the NIFR.

1. The motion of the point masses forming the body of reference is described in the Cartesian coordinates by the Lagrange function

\[ L = -mc^2 \left(1 - v^2/c^2\right)^{1/2} + mwx, \tag{4} \]

where $v$ is the speed of the point masses and $w$ is a constant. The points of the given frame move under the effect of a constant force along the axis $x$.

According to eq. (1), we have

\[ ds = d\eta - \left(\frac{wx}{c^2}\right)dx, \tag{5} \]

where $d\eta = \left(c^2dt^2 - dx^2 - dy^2 - dz^2\right)^{1/2}$.

The reference body of this NIFR can be realized by a system of non-interacting electric charges in a constant, homogeneous electric field.

2. The motion of the point masses forming the body of reference is described in Cartesian coordinates by the Lagrange function

\[ L = -mc^2 \left(1 - v^2/c^2\right)^{1/2} - \frac{m\Omega_0}{2}(\dot{x}y - x\dot{y}), \tag{6} \]

where $\dot{x} = dx/dt$, $\dot{y} = dy/dt$, $\Omega_0$ is a constant. The points of such a frame rotate in the plane $xy$ about the axis $z$ with the angular frequency

\[ \Omega = \Omega_0\left[1 + \frac{(\Omega_0r/c)^2}{2}\right]^{-1/2}, \tag{7} \]

where $r = (x^2 + y^2)^{1/2}$. The speed $v$ tends to $c$ when $r \to 0$. For the given NIFR

\[ ds = d\eta + \left(\frac{\Omega_0}{2c}\right) (ydx - xdy). \tag{8} \]

The bodies of reference of this frame can be realized by a system of non-interacting electric charges in a constant, homogeneous magnetic field. In the above NIFR $ds$ is of the form

\[ ds = F(x, dx), \tag{9} \]

where $F(x, dx) = d\eta + f_\alpha dx^\alpha$, $f_\alpha$ is a vector-function of $x$ and

\[ d\eta = \left[ -g_{\alpha\beta}(x)dx^\alpha dx^\beta \right]^{1/2} \]

is the differential metric form of pseudo-Euclidean space-time of the IFR in the used coordinate system. Therefore, generally speaking, the space-time in NIFR is Finslerian with the sign indefinite differential metric form, where $F$ is a homogeneous function of the first degree in $dx$. 

4
3 Sagnac effect

We shall show that metric differential form (3) does not contradict to experimental data. First consider the Sagnac effect.

The phase shift in the interference of two coherent light beams on a rotating frame was observed by Sagnac [8]. For a relativistic explanation of the effect it is postulated usually, that space-time in any frames of reference is pseudo-Euclidean [7], cite Anandsten. The motion in NIFR is considered as the relative one in absolute pseudo-Euclidean space-time.

However, for an isolated observer in the rotating frame, who proceeds from the notion of space and time relativity in the BLMP sense, the observed anisotropy in the time of light propagation (which contradicts from his viewpoint to the experiments of Michelson-Morley type) is not a trivial effect. It must have some "internal" physical explanation.

Consider a disk rotating with the constant angular velocity \( \Omega \) around the \( z \) axis. Let \( r \) and \( \theta \) be the coordinates, defined by the equations

\[
x = r \cos(\varphi), \quad y = r \sin(\varphi), \quad \varphi = \theta + \Omega t.
\]

(10)

In the coordinate system \((r, \theta, z, t)\) the disk points are at rest and the space-time metrical differential form \( ds \) in the rotating frame is of the form

\[
ds = d\eta + \left[ \frac{\Omega r^2}{2c^2} \right] d\theta + \left[ \frac{\Omega^2 r^2}{2c^2} \right] dx^0.
\]

(11)

where \( d\eta \) is the a pseudo-Euclidean metric form:

\[
d\eta^2 = \left[ 1 - \left( \frac{\Omega r^2}{c^2} \right)^2 \right] (dx^0)^2 - (dr)^2 - r^2 (d\theta)^2 - 2(r^2 \Omega/c) dx^0 dx^0 - dz^2.
\]

(12)

An ideal clock is a local periodic process measuring the length of its own world line \( \gamma \) to a certain scale. It follows from eq.(11) that in the coordinate system being used, the time element between two events in the same point, measured by an ideal clock on the rotating disk is given by

\[
dT = c^{-1} \sqrt{\left( g_{00} \right)^{1/2} + f_0} dx^0,
\]

(13)

where \( f_0 = \frac{\Omega^2 r^2}{2c^2} \).

Consider spatial and time measurements in the NIFR.

First we show how to find the spatial element and light velocity in the rotating frame provided space-time in the frame is pseudo-Euclidean, i.e. \( ds = d\eta \). (We proceed from the covariant method of the 3 + 1 decomposition of space-time in general relativity which goes back to Ulman, Pirani, Dehnen and other authors [6].)

In this case for a noninertial observer at rest the direction of time is given by the vector of 4-velocity \( \tau^\alpha = dx^\alpha/d\eta \) of the disk points, which satisfy the equation \( g_{\alpha\beta} \tau^\alpha \tau^\beta = 1 \).
The physical 3-space is orthogonal to the vector $\tau^\alpha$. Therefore, the arbitrary vector $\xi^\alpha$ in the point $x^\alpha$ can be represented as follows

$$\xi^\alpha = \xi^\alpha_{\text{sp}} + \lambda \tau^\alpha,$$

(14)

where $\xi^\alpha_{\text{sp}}$ are the spatial components. Using the orthogonality condition $\tau_\alpha \xi^\alpha = 0$, we find that: $\lambda = \tau_\alpha \xi^\alpha$. Therefore, $\xi^\alpha = h^\alpha_\beta \xi^\beta$, where $h^\alpha_\beta = \delta^\alpha_\beta - \tau^\alpha \tau^\beta$ is the operator of the spatial projection of a vector field. The spatial projection of the metrical tensor $g_{\alpha\beta}$ is $g_{\alpha\beta} = -h^\gamma_\alpha h^\delta_\beta g_{\gamma\delta}$.

The spatial element $dl$ is produced by the spatial projections of the tensor $g_{\alpha\beta}$ and vector $dx^\alpha$:

$$dl = (g_{\alpha\beta} dx^\alpha dx^\beta)^{1/2}.$$

(15)

The time interval between the events in the points $x^\alpha$ and $x^\alpha + dx^\alpha$ is:

$$dT = c^{-1} \tau_\alpha dx^\alpha.$$  

(16)

Eqs. (14) for the vector $\xi^\alpha = dx^\alpha$ are

$$dx^\alpha = dx^\alpha_{\text{sp}} + c dT \tau^\alpha.$$  

(17)

In the used coordinate system $\tau^\alpha = \lambda_1 \delta^\alpha_0$, where $\lambda_1$ is $(g_{00})^{-1/2}$ which follows from the equality $g_{\alpha\beta} \tau^\alpha \tau^\beta = 1$ is $(g_{00})^{-1/2}$. Next, $\tau_\alpha = g_{\alpha\beta} (g_{00})^{-1/2}$, $dx^\alpha = dx^i$ and $g_{00} = 0$.

Therefore, $dl = (g_{ik} dx^i dx^k)^{1/2}$ and $dT = c^{-1}(g_{00})^{1/2} dx^0$. With the accuracy up to $v/c$, we have: $dl = [(dr)^2 + r^2 (d\theta)^2]^{1/2}$ and $dT = dt$.

The equation $d\eta = 0$ for light can be written as

$$g_{\alpha\beta} dx^\alpha dx^\beta + 2g_{\alpha\beta} c \tau_\alpha dx^\beta + c^2 g_{\alpha\beta} \tau^\alpha \tau^\beta = 0.$$  

(18)

In virtue of the equation $\tau_\alpha dx^\alpha = 0$ and eqs. (16) and (18), this equation can be written as $c^2 dr^2 - (dl)^2 = 0$. Therefore, the speed of light is $dl/dT = c$.

Let us return to the Finslerian space-time. For an observer in the rotating frame the time direction is given by the vector of 4-velocity $\zeta_\alpha = dx^\alpha / ds$ of the disk points, which satisfies the equation $F(x, \zeta) = 1$. We shall suppose that the spatial vector $dx^\alpha$ is orthogonal to the vector $\zeta_\alpha$ in the sense of the Finslerian metric:

$$\zeta_\alpha dx^\alpha = 0,$$

(19)

where $\zeta_\alpha = F(x, \zeta) \partial F(x, \zeta) / \partial \zeta.$

Then, $dx^\alpha = H^\alpha_\beta dx^\beta$, where the operator of the spatial projection

$$H^\alpha_\beta = \delta^\alpha_\beta - \zeta^\alpha \zeta_\beta.$$  

(20)

The spatial projection of the tensor $g_{\alpha\beta}$ is given by

$$g_{\alpha\beta} = -H^\gamma_\alpha H^\delta_\beta g_{\gamma\delta}.$$  

(21)
The time interval between events in the points \( x^\alpha \) and \( x^\alpha + dx^\alpha \) is of the form

\[
dT = c^{-1} \zeta_\alpha dx^\alpha. \tag{22}\]

The spatial element in NIFR is defined by virtue of the metric form (9) and the spatial projections of the tensor \( g_{\alpha\beta} \) and vector \( f_\alpha = g_{\alpha\beta} f^\beta \) as follows

\[
DL = (g_{\alpha\beta} dx^\alpha dx^\beta)^{1/2} + f_\alpha dx^\alpha. \tag{23}\]

The vector field \( \zeta^\alpha \) can be written as

\[
\zeta^\alpha = \tau^\alpha (ds/d\eta)^{-1} = (1 + f_\beta \tau^\beta)^{-1} \tau^\alpha. \tag{24}\]

Therefore, with accuracy up to \( v/c \), the spatial element is given by

\[
DL = dl + f_i dx^i = dl(1 + f_i k^i), \tag{25}\]

where \( k^i \) is the unit vector of the direction in the 3-space. Then, from the equations \( g_{\alpha\beta} dx^\alpha dx^\beta = 0 \) and \( dx^\alpha = \overline{dx}^\alpha + c dT \zeta^\alpha \) we obtain

\[
g_{\alpha\beta} dx^\alpha dx^\beta + 2g_{\alpha\beta} dx^\alpha c \zeta^\alpha dT + c^2 (dT)^2 g_{\alpha\beta} d\zeta^\alpha d\zeta^\beta = 0. \tag{26}\]

The first term in the left-hand side of eq.(26), with accuracy up to \( v/c \), coincides with \(-d \ell^2\) and the third term - with \( c^2(dT)^2\). The second term is not zero since the orthogonality condition in the Finslerian space-time has the form (13), i.e.

\[
g_{\alpha\beta} \zeta^\alpha dx^\beta (g_{\alpha\beta} \zeta^\alpha \zeta^\beta)^{-1/2} + f_\alpha dx^\alpha = 0. \tag{27}\]

Therefore, with accuracy up to \( v/c \), the second term equals \(-2c dT f_i dx^i\). Putting \( v_p = dL/dT \) and \( v^i_p = \overline{dx}^i/dT = k^i v_p \), and using eq.(25) (where \( f_i k^i \) is of the order of \( v/c \)), equation (26), accurate up to \( v/c \), can be written as

\[
v^2_p + 2v_p c f_i k^i - c^2 = 0. \tag{28}\]

The solution of this equation, which coincides with the light velocity \( c \) in non-inertial frames, is given by

\[
v_p = c(1 - f_i k^i). \tag{29}\]

In virtue of equations (25) and (29) the time of light spread from the point \( x^i \) to \( x^i + dx^i \) is \( dL/v_p = c^{-1} dl(1 + 2 \ast f_i k^i) \). The unique nonzero component of the 3-vector \( f_i \) is \( \Omega r^2/(2c) \). (See eq.(11)). For this reason the difference in time interval between light propagation around the rotating disk in a clockwise and counterclockwise direction is \( 4\pi r^2 \Omega/c^2 \), which gives the Sagnac phase shift \( \frac{\pi}{2} \). Thus, the Sagnac effect for the isolated observer in the rotating frame can be treated as caused by the Finslerian metric of space-time in noninertial frames of reference, which conditions the anisotropy of the space element and the velocity of light.


4 Inertial Forces

Let us show that the existence of the inertial forces in NIFR can be interpreted as the exhibition of the Finslerian connection of space-time in such frames. According to our initial assumption in Section 2, the differential equations of motion in an IFR of the point masses, forming the reference body of the NIFR, are the geodesic lines of space-time in NIFR. These equations can be found from the variational principle $\delta \int ds = 0$.

The equations are of the form

$$du^\alpha /ds + G^\alpha (x, u) = 0,$$

where $u^\alpha$ is the 4-velocity of the point mass, the world line of which is $x^\alpha = x^\alpha(s)$, and

$$G^\alpha (x, u) = \Gamma^\alpha_{\beta\gamma} u^\beta u^\gamma + B^\alpha_{\beta} u^\beta + u^\alpha \beta d(\beta^{-1})/ds,$$

where $\beta = d\eta /ds = (g_{\alpha\beta}u^\alpha u^\beta)^{1/2}$, $B^\alpha_{\beta} = g^{\alpha\delta}B_{\delta\beta}$

and

$$B_{\delta\beta} = \partial f_{\beta}/\partial x^\delta - \partial f_{\delta}/\partial x^\beta.$$

In the Finslerian space-time a number of connections can be defined according to eq. (30) [7]. In particular, this equation can be interpreted in the sense that in the NIFR space-time the absolute derivative of a vector field $\xi^\alpha(x)$ along the world line $x^\alpha = x^\alpha(s)$ is of the form

$$D\xi^\alpha /ds = d\xi^\alpha /ds + G^\alpha_{\beta}(x, dx /ds)\xi^\beta,$$

where

$$G^\alpha_{\beta}(x, dx /ds) = \Gamma^\alpha_{\beta\gamma} dx^\gamma /ds + \beta d\beta^{-1} /ds.$$

Equation (32) defines a connection of Laugvitz type [7] in space-time of the NIFR, which is nonlinear relative to $dx^\alpha$. The change in the vector $\xi^\alpha$ because of an infinitesimal parallel transport is

$$d\xi^\alpha = -G^\alpha_{\beta}(x, dx)\xi^\beta,$$

Consider the motion of a particle of the mass $m$ in a NIFR unneffected by forces of any kind in the laboratory (inertial) frame of reference. The differential equations of motion of such a particle can be found from the variational principle $\delta \int d\eta = 0$. Since $ds = d\eta - f_\alpha dx^\alpha$, the equations of motion are

$$Du^\alpha /ds = B^\alpha_{\beta} u^\beta.$$

As an example, consider the nonrelativistic disk rotating in the $xy$ plane about the $z$ axis with the angular velocity $\Omega$. Under the circumstances the equations of motion (30) are

$$d\vec{v}/dt + \vec{\Omega} \times \vec{r} = 0,$$
where $\vec{v} = d\vec{r}/dt$, $\vec{r} = \{x, y, z\}$ and the coordinates origin coincides with the disk center. The absolute derivative of the vector $\vec{ξ}$ is given by
\[
\frac{D\vec{ξ}}{dt} = \frac{d\vec{ξ}}{dt} - \vec{Ω} \times \vec{ξ},
\] (36)
and the equations of motion of the considered particle in the NIFR are
\[
\frac{D\vec{v}}{dt} = -\vec{Ω} \times \vec{v}.
\] (37)
Next, for the 4-velocity $u^α$ we have
\[
u^α = \bar{u}^α + \lambda \zeta^α,
\] (38)
where $\lambda = \zeta^α u_α$, $\bar{u}^α$ is the velocity of the particle in the NIFR found with the help of measuring instruments. In the nonrelativistic limit eq.(38) is written in the form
\[
\vec{v} = \bar{v} + \vec{r},
\] (39)
where $\bar{v}$ is the relative velocity of the particle and $\vec{r}$ is the velocity of the disk point in the laboratory frame. Substituting (39) in (36), we find that
\[
\frac{D\vec{v}}{dt} = -\frac{D\vec{r}}{dt} - \vec{Ω} \times \vec{v} - \vec{Ω} \times \vec{r}.
\] (40)
The value $\frac{D\vec{v}}{dt}$ is an acceleration of the considered particle in the used NIFR found with the help of measuring instruments. The velocities field $\vec{r}$ of the disk point is given by $\vec{r} = \vec{Ω} \times \vec{r}$. Hence, along the particle path we have $\frac{d\vec{r}}{dt} = \vec{Ω} \times \vec{v}$ and
\[
\frac{D\vec{r}}{dt} = \frac{d\vec{r}}{dt} - \vec{Ω} \times \vec{r} = \vec{Ω} \times \vec{v}.
\] (41)
Thus, finally, we find from (37)
\[
\frac{mD\vec{v}}{dt} = -\frac{2m(\vec{Ω} \times \vec{v})}{2m} - m \vec{Ω} \times (\vec{Ω} \times \vec{r}).
\] (42)
We arrived at the nonrelativistic equations of motion of a point in a rotating frame. The right-hand of eq.(36) is the ordinary expression for the Coriolis forces and the centrifugal force in the rotating frames. (See also [13] and [14]).
Thus, in the nonrelativistic limit the Finslerian space-time in NIFR manifests itself in the structure of vector derivatives with respect to time $t$. It should be noted that eq.(36) is considered sometimes in classical dynamics nominally [12] just for the derivation of the inertial forces in NIFR’s.

5 Experimental test

Consider an experimentally verifiable consequence of the above theory.
Let $p^\alpha = mc \, dx^\alpha / d\eta$ be 4-momentum of a particle in the IFR. Using 3 + 1 decomposition of space-time in the NIFR we have

$$p^\alpha = \overrightarrow{p}^\alpha + E \zeta^\alpha$$

(43)

where $\zeta^\alpha = dx^\alpha / ds$. From the viewpoint of an observer in the NIFR the spacial projection $\overrightarrow{p}^\alpha$ should be identified with the momentum $p^\alpha$, and the quantity $c E$ with the energy $E$ of the particle. It is obvious that $E = \zeta_\alpha p^\alpha$, where $\zeta_\alpha$ is defined in eq. (20) from Sec.2.

Therefore, the energy of the particle in the NIFR is

$$E = m Q c^2 \zeta_\alpha u^\alpha,$$

(44)

where $Q = ds / d\eta = F(x, dx / d\eta)$ and $u^\alpha = dx^\alpha / ds$ is the 4-velocity of the particle. For the particle at rest in the NIFR $u^\alpha = \zeta^\alpha$ and we obtain

$$E = m Q c^2$$

(45)

Thus, the inertial mass $m_n$ of the particle in the NIFR is given by

$$m_n = Q m$$

(46)

The quantity $m_n$ coincides with the proportionality factor between the momentum $\overrightarrow{p}^\alpha$ and the velocity $\overrightarrow{v}^\alpha = dx^\alpha / dT$ of a nonrelativistic particle in the NIFR.

Since $Q$ is the function of $x^\alpha$, the inertial mass in the NIFR is not a constant. For example, on the rotating disk we have

$$m_n = m / (1 - \Omega^2 r^2 / 2c^2),$$

(47)

where $\Omega$ is the rotation angular velocity and $r$ is the distance of the body from the disk center.

The difference between the inertial mass $m_e$ of a body on the Earth's equator and the mass $m_p$ of the same body on the pole is given by

$$(m_e - m_p) / m_p = 1.2 \cdot 10^{-12}$$

(48)

The dependence of the inertial mass of particles on the Earth's longitude can be observed by the Mössbauer effect. Indeed, the change $\Delta \lambda$ in the wave length $\lambda$ at the Compton scattering on particles of the masses $m$ is proportional to $m^{-1}$. If this value is measured for $\gamma$ quants with the help of the Mössbauer effect at a fixed scattering angle, then after transporting the measuring device from the longitude $\varphi$ to the longitude $\varphi_1$ we have

$$\frac{(\Delta \lambda)^{-1}_\varphi - (\Delta \lambda)^{-1}_{\varphi_1}}{(\Delta \lambda)^{-1}_\varphi} = K \left[ \cos(\varphi)^2 - \cos(\varphi_1)^2 \right],$$

(49)

where $K$ is a constant.
6 Conclusion

The above theory gives in some way the realization of the idea of space-time properties relativity in the BLMP sense. Starting from the given properties of space-time in the IFR, the theory demonstrates that space-time geometry in the NIFR is caused by the properties of the employed frames of reference.

The existence of inertial forces in NIFR obviously indicates that Newtonian inertia principle is violated in such frames. For this reason the Einstein’s general relativity principle (we mean his assumption that physical laws are identical in any frames of reference) is based on the interpretation of inertial forces as the "exterior" ones (in Mach sense). Our analysis of the problem shows the deep connection between the existence of inertial forces in NIFR and space-time geometry in such frames.

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