The hadronization scheme for parton transport in relativistic heavy ion collisions is considered in detail. It is pointed out that the traditional scheme for particles being freezed out one by one leads to serious problem on unreasonable long lifetime for partons. A super-cooling of the parton system followed by a collective phase transition is implemented in a simple way. It turns out that the modified model with a global phase transition is able to reproduce the experimental longitudinal distributions of final state particles better than the original one does. The encouraging results indicate that a relevant parton transport model for relativistic heavy ion collision should take equilibrium phase transition into proper account.

PACS numbers: 25.75.-q, 12.38.Mh, 24.10.Lx

I. INTRODUCTION

The quark-gluon plasma (QGP) is expected to be formed in heavy-ion collisions at the Relativistic Heavy-Ion Collider (RHIC). So far very interesting experimental data have been collected [1]. There are strong evidences on the deconfinement of QCD vacuum and the appearance of a (locally) thermalized partonic system at the early stage of collision. Both theoretical and experimental investigations are stimulated on the evolution of the partonic system and the way it transforms to final state particles.

![FIG. 1: The different stages of heavy-ion collisions.](image)

The reaction process of relativistic heavy ion collision can be divided into several stages (see figure 1). At first the primary interaction creates many partons. These partons are certainly far from equilibrium and cascade through interaction among each other. If the colliding nuclei is heavy enough and the incident energy is high enough the partonic system is expected to reach equilibrium or local equilibrium, forming quark-gluon plasma. The system then expands and the temperature decreases, passing via a phase transition or crossover into the hadron gas stage. Hadrons continue to cascade until they freeze out from the collision region, forming final state particles.

At present there is no unique theory that can describe the reaction process as a whole. Different theoretical approaches are applied to different stages [2]. The primary interaction, creating many partons is often described by eikonalized parton model [3], Gribov-Regge theory [4] or parton saturation model [5]. Parton and/or hadron cascade is referred to as the solution of non-equilibrium transport equation [6] and is usually realized by Monte Carlo models [7] [8] [9]. The hydrodynamics [10] or thermal models [11] are employed to deal with the parton and/or hadron system in local or global equilibrium.

Usually, the hydrodynamics or thermal models take (local) equilibrium as model assumption and do not answer the question on how the system arrive at (local) equilibrium. The global properties of the system are the main issues considered in the model, while the detailed evolution of the constituents —— partons and/or hadrons is not taken into account.

On the contrary, the transport models follow the evolution of the partons and/or hadrons in detail through considering their interaction —— elastic and/or inelastic scattering, or cascade. Usually, the model allows the cascade continues to go on until the interaction ceases and then the parton or hadron freeze out from the system. A noticeable common property of such model is that, freeze out of parton or hadron occurs particle by particle. Each particle has its own freeze-out time. Such an approach is acceptable for a hadronic system, where hadrons after ceasing to interact with other hadrons fly away from the system freely toward the detectors, but will cause serious problem for a partonic system.

Partonic and hadronic systems are of different phases. In general, a phase is a portion of a system that is uniform and has a definite boundary [12]. In particular, the partonic and hadronic phases exist in different vacua —— the former is in perturbation QCD vacuum and the latter in physical vacuum. Therefore, the transition be-
tween partonic and hadronic phases should be a collective, thermal-equilibrium phenomenon, accompanied by a vacuum transform, which could not be realized in a particle-by-particle way.

In the transport models presently on market, the hadronization is realized parton-wise instead of collectively, and therefore, in these models there is only hadronization but no partonic to hadronic phase transition in the strict sense. To let a few partons live for a very long time is inconsistent with the general believe that the existing time of QGP is about 1-5 fm/c \cite{13} and hadron freeze out at about 20-40 fm/c \cite{14}. Even more seriously, when most of the partons have already hadronized, the system is dominated by hadrons and the corresponding vacuum is a physical one instead of a QCD perturbation vacuum. To let some partons survive and fly freely in such a circumstance is highly unphysical.

The aim of the present paper is to discuss this problem in detail. We will take as example a presently available model, that has parton transport implemented. The temperatures of the system at different time will be extracted using thermal-equilibrium transverse mass distribution. The partonic system will be allowed to somewhat super-cooled, i.e. the temperature is allowed to decrease to lower than the expected phase-transition temperature. At a certain point all the partons remaining in the system are forced to coalesce, forming hadrons. Then the hadrons start to cascade toward freeze out. Thus the difficulty of long-life parton is overcome in a simple manner. The phenomenological consequences of such an approach will be presented and compared with existing experimental data. The possible reason for the improvement of the present approach with global hadronization in comparison with the original model with long-life partons will be discussed.

The layout of the paper is as the following. A short introduction to the AMPT model, used as an example for models with parton transport is given in Section II. A detailed analysis about the parton and hadron time evolution in AMPT is then presented in Section III. A super-cooling of the parton system followed by a collective hadronization scheme is proposed in Section IV together with the phenomenological consequences on the final hadron distribution and elliptic flow. Section V is conclusion and discussion.

II. A BRIEF INTRODUCTION TO AMPT

The AMPT model \cite{9} is based on non-equilibrium transport dynamics. It contains four main components: the initial conditions, partonic interactions, conversion from the partonic to the hadronic matter and hadronic interactions. The initial conditions, which includes the spacial and momentum distributions of minijet partons from hard processes and strings from soft processes, are obtained from the HIJING model in which eikonized parton model is employed. The time evolution of partons is then modeled by the ZPC \cite{15} parton cascade model. At present this model includes only parton-parton elastic scattering with cross section

$$\sigma_p \simeq \frac{9\pi\alpha^2}{2\mu^2},$$

where the screening mass $\mu$ is taken to be an input parameter of the model to obtain the desired total cross section. Two partons will undergo scattering when the closest distance between them is smaller than $\sqrt{\sigma/\pi}$.

There are two versions of AMPT model. In the default AMPT, after ceasing interactions minijet partons are combined with their parent strings to form excited strings, which are then converted to hadrons according to the Lund string fragmentation model. While in the AMPT with string melting, the strings in the initial conditions are melt to partons first and then interactions among all the partons are again modeled by ZPC. After partons stop interacting, a simple quark coalescence model is used to combine the two nearest partons into a meson and three nearest quarks (antiquarks) into a baryon (antibaryon). Scatterings among the resulting hadrons are described by a relativistic transport (ART) model \cite{10} which includes baryon-baryon, baryon-meson and meson-meson elastic and inelastic scatterings.

It turns out that the default AMPT (v1.11) is able to give a reasonable description on hadron rapidity distributions and transverse momentum spectra observed in heavy ion collisions at both SPS and RHIC. However, it fails to reproduce the experimental data about elliptic flow and two-pion correlation function. On the other hand, the AMPT model with string melting (v2.11) can well describe the elliptic flow and two-pion correlation function \cite{17,18} but agree badly with the hadron rapidity and transverse momentum spectra.

In the following we will utilize the AMPT with string melting — AMPT v2.11 to generate Au-Au central collision events at $\sqrt{s_{NN}} = 200$ GeV. The impact parameter is in the range $b \leq 3$ fm and the parton cross section is taken to be 10 mb.

III. THE TIME EVOLUTION OF PARTONS AND HADRONS IN AMPT

In AMPT v2.11, each initial parton has a formation time given by $t_f = E_H/m_{T,H}^2$ with $E_H, m_{T,H}$ the energy and transverse mass of its parent hadrons. After this formation time the partons start to scatter with each other and when a parton no longer scatters with any other parton, it will hadronize. Thus each parton has its own hadronization time.

In Fig. 2 are shown the percentages of parton and hadron, respectively, at different time after the collision. It can be seen from the figure that a few hadrons (about 4%) have already emerged at $t < 5$ fm/c. At this time, partons dominate, and the system as a whole is in the
deconfined phase, located in the perturbation QCD vacuum, with a few hadrons vaporized out.

As time increases, the number of partons decreases while that of hadrons increases. In this process a parton transforms to hadron when and only when it ceases to interact with other partons. Thus a part of parton survive up to an unreasonable long time, e.g. $t \sim 100$ fm/c. This is the difficulty common for this kind of models mentioned in the Introduction.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{The percentage of partons and hadrons, respectively, in AMPT v2.11 for $\sqrt{s_{NN}} = 200$ GeV Au-Au central collisions with $b \leq 3$ fm and parton cross section 10 mb.}
\end{figure}

For simplicity, we assume that the system arrives at equilibrium globally instead of locally and the equilibrium is examined at different time $t$ instead of at different intrinsic time $\tau$. Under this assumption the temperature of the system at different time is determined by thermodynamics.

Suppose the expansion of system is a slow and quasi-static process, then at each time step the system can be regarded as an equilibrium thermal system with temperature $T$. The invariant momentum distribution of particles emitted from the thermal system is

$$E(d^3n/d^4p) = \frac{dn}{dym_Td\phi} = \frac{gV}{(2\pi)^3}Ee^{-(E-\mu)/T}, \quad (2)$$

where $g$ is the particle spin-isospin degeneracy factor, $\mu$ is the chemical potential, $V$ is the system volume, $E$ is the energy of the particle, $y$ is rapidity and $m_T = \sqrt{m^2 + p_T^2}$ is the transverse mass. Integrating with respect to $y$ and $\phi$, we get the transverse mass distribution

$$\frac{dn}{m_Tdm_T} = \frac{gV}{2\pi^2}m_TK_1\left(\frac{m_T}{T}\right), \quad (3)$$

in which $K_1$ is the hyperbolic Bessel function of rank one. By fitting the transverse mass distribution, the temperature of the system can be extracted. As example, in Fig's. 3 are shown the transverse mass distributions for d-quarks at two different time $t = 0.5$ and 5 fm/c and the fit to Eq. (3). In the present work we assume the system to be static, and omit the effect of radial flow on the fitted temperature.

It can be seen from Fig's. 3 that the temperature of the parton system is about 180 MeV at $t = 0.5$ fm/c, slightly above the predicted critical temperature $T_c = 170$ MeV \cite{21}, while at $t = 5$ fm/c the temperature has already arrived at about 110 MeV, i.e. in the first few fm/c the temperature decreases rapidly and soon becomes lower than the expected phase transition temperature.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{The transverse mass distribution (histograms) for d-quarks at two different time $t = 0.5$ and 5 fm/c. The lines are the fit to Eq. (3).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{The temperature evolution extracted from parton and hadron transverse mass spectra in AMPT v2.11 with a collective phase transition implemented at $t = 5$ fm/c for $\sqrt{s_{NN}} = 200$ GeV Au-Au central collisions with $b \leq 3$ fm and parton cross section 10 mb.}
\end{figure}

IV. A HADRIZATION SCHEME WITH A SUPER-COOLING FOLLOWED BY A COLLECTIVE HADRONIZATION

We regard the temperature of parton system decreasing rapidly to lower than the critical temperature as a supercooling effect, i.e. after the formation of QGP, the system lowers its temperature by expanding and evaporating hadrons, arriving at a temperature lower than the phase transition temperature. Then at a certain point all the left partons are coalesced to hadrons. The duration of the supercooling state is taken as a model parameter and in the present work we take $t = 5$ fm/c (the corresponding parton temperature is about 110 MeV) to be the beginning of the phase transition. At this time, all the remaining partons stop to interact and start to
hadronize. The temperature of the resulting hadron system is higher than that of the parton system due to the release of latent heat, and then the temperature decreases again through expansion, cf. Fig. 4.

Thus we have implemented an equilibrium phase transition with supercooling to the transport model in a very simple way. Our purpose is to see how the phase transition affect the final state hadron distributions.

The rapidity distribution is

\[ \frac{dN}{dy} = \frac{1}{N_{\text{ev}}} \frac{\Delta n}{\Delta y}, \tag{4} \]

where \( N_{\text{ev}} \) is the number of events, \( \Delta y \) is the width of rapidity bin, \( \Delta n \) is the number of particles inside the rapidity bin. In Fig's. 5 are shown the rapidity distributions for charged particles, pions, kaons, net-protons, protons and anti-protons. The solid lines represent the results of AMPT v2.11 with parton-wise hadronization and the dotted lines are that with collective phase transition implemented in the above-mentioned way. Full circles are the AMPT v2.11 with parton-wise hadronization and the dashed lines are that with collective phase transition implemented. The impact parameter is \( b \leq 3 \) and parton cross section 10 mb. The dots are data from PHOBOS 6% and BRAHMS 5% central collisions.

In order to see why the model with collective phase transition can describe the experimental data better, the rapidity distributions of partons right before hadronization and the dashed lines are that with collective phase transition implemented. The impact parameter is \( b \leq 3 \) and parton cross section 10 mb. The dots are data from PHOBOS 6% and BRAHMS 5% central collisions.

V. CONCLUSION AND DISCUSSION

In the traditional transport models the particles in the system are freezed out one by one, each particle has its own freeze-out time. Extending such an approach to parton transport leads to serious problem on unreasonable long lifetime for partons.

In order to avoid this problem, we assume the system to have reached global equilibrium and extract temperature from the system by thermodynamics formula. As the decreasing of temperature the parton phase is allowed to hadronize as a whole after a supercooling stage. It turns out that the modified model with a global phase transition inherits the success of the original one in elliptic flow and is able to reproduce the experimental longitudinal distributions of final state particles better than the original one does.
of rapidity. This effect results in the hadron distribution in mid-rapidity being lower than that from the original model, and thus approaching the experimental data.

The elliptic flows in parton cascade models are built-up very early [17] [23] (less than 5 fm/c), thus the truncation of parton transport at 5 fm/c in the present work does not affect the elliptic flow.

We have proposed a super-cooling followed by a global hadronization as a prototype of the thermal-equilibrium phase transition from parton transport to hadron gas. Though our method for the implementation of phase transition seems to be very crude comparing to the real process in relativistic heavy ion collisions, the encouraging results indicate that a relevant parton transport model for relativistic heavy ion collision should take equilibrium phase transition into proper account.

FIG. 7: The rapidity distribution of partons right before hadronization in AMPT v2.11 with parton-wise hadronization (solid line) and with global phase transition (dashed line.)

[1] J. Adams et al. (STAR Collab.), Nucl. Phys. A757, 102 (2005); K. Adcox et al. (PHENIX Collab.), Nucl. Phys. A757, 184 (2005).
[2] K. Werner, J. Phys. G: Nucl. Part. Phys. 27, 625-634 (2001).
[3] X.-N. Wang, Phys. Rev. D43 104 (1991); M. Gyulassy and X.N. Wang, Comput. Phys. Commun. 83, 307 (1994).
[4] K. Werner, Phys. Rep. 232, 87 (1993).
[5] L. McLerran and R. Venugopalan, Phys. Rev. D49, 2233 (1994); K. J. Eskola, K. Kajantie, P. V. Ruuskanen and K. Tuominen, Phys. Lett. B543, 208 (2002).
[6] H. Sorge, H. Stocker and W. Greiner, Nucl. Phys. A498, 567c (1989); Y. Pang, T. J. Schlagel and S. H. Kahana, Phys. Rev. Lett. 68, 2743 (1992).
[7] H. Sorge, Phys. Rev. C52, 3291 (1995).
[8] S. A. Bass et al, Prog. Part. Nucl. Phys. 41, 225-370 (1998), also see nucl-th/9803035.
[9] Zi-Wei Lin, Che Ming Ko, Bao-An Li and Bin Zhang and Subrata Pal, Phys. Rev. C72, 064901 (2005). Z.W. Lin et al, Phys. Rev. C64, 0111902 (2001). B. Zhang et al, Phys. Rev. C61, 067901 (2000).
[10] C.M. Hung and E. V. Shuryak, Phys. Rev. Lett. 75, 4003 (1995);
P. F. Kolb, U. W. Heinz, P. Huovinen, K. J. Eskola and K. Tuominen, Nucl. Phys. A696, 197 (2001).
[11] P. Braun-Munzinger, J. Stachel, J. P. Wessels and N. Xu, Phys. Lett. B344, 43 (1995); F. Becattini, J. Cleymans, A. Keranen, E. Suhonen and K. Redlich, Phys. Rev. C64, 024901 (2001).
[12] C. Kittel, Thermal physics, John Wiley & Sons, New York, 1969.
[13] T. S. Biro, E. van Dorn, M. H. Thoma, B. Müller and X.-N. Wang, Phys. Rev. C48, 1275 (1993).
[14] L. Csemai and J. I. Kapusta, Phys. Rev. D46, 1379 (1992); Phys. Rev. Lett. 69, 737 (1992).
[15] B. Zhang, Comput. Phys. Commun. 109, 193 (1998).
[16] B.A. Li and C.M. Ko, Phys. Rev. C52, 2037 (1995).
[17] Zi-Wei Lin and C. M. Ko, Phys. Rev. C65 034904 (2002).
[18] Zi-Wei Lin, C.M. Ko and Subrata Pal, Phys. Rev. Lett. 89, 152301 (2002).
[19] Ekkard Schmedermann, Josef Sollfrank, and Ulrich Heinz, Phys. Rev. C48, 2462 (1993).
[20] K. Geiger, Nucl.Phys.A566, 253c (1994).
[21] F. Karsch, Lecture Notes in Physics 583, 209 (2002). E. V. Shuryak, Phys. Rev. Lett. 68, 3270 (1992).
[22] B.B. Back et. al. (PHOBOS Collaboration), Phys. Rev. C72, 051901(R) (2005).
[23] Bin Zhang, Miklos Gyulassy and Che Ming Ko, Phys. Lett. B455 45-48, (1999).