Intermittent Pinning Synchronization of Memristor-Based Switching Networks With Multi-Links and Mixed Delays

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ABSTRACT In this paper, a new dynamical model of memristor-based switching networks (MSNs) with multi-links and mixed time-varying delays is proposed based on the real structure of neurons. We design a kind of switched intermittent pinning control law to realize the synchronization of the proposed MSNs which is flexible in lowering the control cost. With the key theory of the differential inclusions, some effective synchronization criteria are derived to ensure the asymptotic synchronization and the exponential synchronization between the drive and response networks. Finally, numerical simulations demonstrate the correctness of our results.

INDEX TERMS Memristor-based switching networks, multi-links, mixed time-varying delays, switched intermittent pinning control.

I. INTRODUCTION
In recent years, the research on synchronization of memristive neural networks (MNNs) has attracted plenty of attention since the component memristor of MNNs can help to emulate the behaviors and the functions of human brain [1]. Besides, the synchronization of MNNs has wide potential applications in engineering, such as the associative memory and the brain-like “neural” computer. The memristor is a vital term for the memristive neural networks, which was firstly proposed by Prof. Chua in 1971 [2]. Since 2008, the memristor has gotten much attention once the first real object of memristor was fabricated by the HP Lab [3]. As we know, it possesses an important function of memory which closely depends on its prominent feature that the value of memristor, namely memristance, relies on the polarity of the applied voltage and the quantity of the charge to pass through the circuit element. Therefore, the memristor is considered as a promising candidate to mimic the biological synapses of brain [4]. The conventional resistor is unable to perform the function of memory. So, the memristor is used to replace the conventional resistor to model the biological synapses in MNNs[5].

As a significant collective behavior, the synchronization has drawn more and more attention because of its important role in biology, secure communication, sociology and so on [6]–[9]. Therefore, the synchronization of memristive neural networks has drawn much attention [10]–[12]. Yang et al. [13] utilized delayed impulsive control to study the Pth moment exponential stochastic synchronization problem of MNNs with mixed delays. Wang et al. [14] was concerned in the adaptive synchronization of MNNs with time-varying delays. Guo et al. [15] studied the global exponential synchronization between two memristive recurrent neural networks with time delays via multiple coupling rules. Bao et al. [16] investigated the exponential synchronization of the coupled stochastic memristive neural networks with multiple time-varying delays.

It should be noticed that the connection between the neurons was considered as a simple link in the synchronization research of the existing memristive neural networks models [17]–[24]. However, with the help of the achievements...
of the neuron study, the synaptosomes (namely, the axon terminals) can contact dendrite, soma, etc. of other neurons, which forms different types of synapse [25]. In other words, there should be multiple contact connections between the neurons. We give a sketch map in FIGURE 1 which shows multiple connections between two neurons including different connection forms of synapses. The accomplishment of the excitability conduction from the presynaptic element to the postsynaptic element consists of the release and diffusion processes of the neurotransmitter. That means the signal conduction between neurons will result in time delay for different links. Apparently, the traditional memristive neural networks model with simple link cannot be sufficient to describe such complex structure.

Some related work and research basis are achieved. In Ref. [26], global exponential synchronization was studied via the controller with state or output coupling. In Ref. [27], finite-time synchronization of memristor-based chaotic neural networks with delays was investigated by using finite time stability theorem and designing a suitable controller. In Ref. [28], the function projective synchronization problem for memristor-based Cohen-Grossberg networks with time-delays was concerned. In Ref. [29], memristor-based neural networks with mixed delays were proposed and the finite-time synchronization criteria were obtained. In Ref. [30] pinning control was utilized to study the synchronization of MNNs with time-varying delays. In Ref. [31] and Ref. [32], the intermittent control law was introduced to investigate exponential synchronization of delayed memristor-based neural networks. However, even though some related investigations about synchronization of memristor-based neural networks were obtained, they considered the contact between two neurons to be a simple link. According to our learning, the traditional MNNs model with simple link cannot be sufficient to describe the real structure of neurons which is detailed stated in last paragraph. Moreover, the control law which can be flexible in lowering control cost is interesting and meaningful. Thus, it is necessary to improve the dynamical networks model and propose a new control policy to study its dynamical behaviors.

According to the above analyses, compared with the schemes in existing papers, the complex contacts between neurons and the conduction delays of multi-links need to be taken into full consideration in order to describe the working mechanism of neurons more closely. The main contributions of this paper are given as follows. First, we propose a novel model, namely the memristor-based switching networks with multi-links and mixed time-varying delays. The nonlinear dynamical behaviors of the proposed model are rich and more complex. Second, a new switched intermittent pinning control law is proposed which is more general control policy. It is flexible in lowering the control cost and applicable to wider application scenarios. Finally, we investigate the exponential synchronization of the proposed MSNs via the switched intermittent pinning control and the asymptotic synchronization via the adaptive switched intermittent pinning control.

The rest of this paper is organized as follows. In Section 2, the dynamical networks model and some preliminaries such as lemmas, assumptions are introduced. In Section 3, we investigate the asymptotic synchronization and the exponential synchronization of the MSNs with multi-links and mixed delays via switched intermittent pinning controller. In Section 4, numerical simulations are presented to demonstrate the correctness of our proposed results. In Section 5, some conclusions are given.

II. PRELIMINARIES

Compared with the traditional MNNs model with simple link, this paper proposes a new networks model with multiple links due to multiple contact connections between neurons in order to describe the complex structure of neurons. Meanwhile, this paper introduces mixed time-varying delays due to the release and diffusion processes of the neurotransmitter in order to reflect the work mechanism of neurons more closely. It is significant to firstly investigate the synchronization of this new networks model.

The model of memristor-based switching networks with multi-links and mixed time-varying delays is given as follows:

\[
x_i(t) = -c_i x_i(t) + \sum_{j=1}^{N} a_{ij} f_j^1(x_j(t)) + \sum_{j=1}^{N} b_{ij} f_j^2(x_j(t - \tau_1(t))) + \ldots + \sum_{j=1}^{N} b_{mij} f_j^2(x_j(t - \tau_m(t))) + \sum_{j=1}^{N} c_{1ij} f_j^3(x_j(s))ds
\]
that contains all the continuous functions which map from $\tau f$ positive; the initial conditions of system (1) are denoted by $t \text{e} [\tau f]$, respectively, the distributed delay and the discrete delay of the memristors $R_i$ where

$$
\begin{align*}
&= -c_i x(t) + \sum_{j=1}^{N} a_{ij}(x(t)) f_j^1(x_j(t)) \\
&+ \sum_{k=1}^{m} \sum_{j=1}^{N} b_{kij}(x(t)) f_j^2(x_j(t - \tau_k(t))) \\
&+ \sum_{k=1}^{m} \sum_{j=1}^{N} c_{kij}(x(t)) \int_{t-\delta_{k}(t)}^{t} f_j^3(x_j(s))ds \\
&+ I(t),
\end{align*}
$$

(1)

where $i \in \mathcal{Z} \triangle [1, \ldots , N]$ , and $N \geq 2$ is the node number in MSNs, $x_j(t)$ denotes the voltage of the capacitor $C_j$; $f_j^1(x_j(t))$ and $f_j^2(x_j(t - \tau_k(t)))$ are the feedback activation functions without and with time delays, respectively; $f_j^3(x_j(t))$ is the bounded feedback activation functions without time delays; $\delta_k(t)$ and $\tau_k(t)$ ($k = 1, \ldots , m$) are, respectively, the distributed delay and the discrete delay of the $k$th sub-network; $0 \leq \tau_k(t) \leq \tau_k, 0 \leq \delta_k(t) \leq \tau_k, \bar{\tau}(t) \leq \epsilon < 1$, all the constants $\tau_k, \tau_k, \epsilon$ and $\bar{\tau}$ are positive; the initial conditions of system (1) are denoted by $\varphi(s) = (\varphi_1(s), \varphi_2(s), \ldots, \varphi_N(s))^T \in C([-\tau_0, 0], \mathbb{R}^N)$, and $\tau_0 = \max(\tau_1, \tau_2, \ldots, \tau_k), C([-\tau_0, 0], \mathbb{R}^N)$ represents a Banach space that contains all the continuous functions which map from $[\tau_0, 0]$ into $\mathbb{R}^N$; $c_i$ is the self-inhibition of networks node; $I(t)$ is the external input; the parameters $a_{ij}(x(t))$, $b_{kij}(x(t))$ and $c_{kij}(x(t))$ are the memristor-based weights, and

$$
a_{ij}(x(t)) = \frac{M_{ij}}{C_i} \times \text{sgn}_{ij}, \quad b_{kij}(x(t)) = \frac{M_{kij}}{C_i} \times \text{sgn}_{ij},$$

$$
c_{kij}(x(t)) = \frac{M_{kij}}{C_i} \times \text{sgn}_{ij},$$

$$
\text{sgn}_{ij} = \begin{cases} 1 & i = j, \\ -1 & i \neq j, \end{cases}
$$

where $M_{ij}, M_{kij}$ and $M_{kij}$ denote the memductances of the memristors $R_{ij}, R_{kij}$ and $R_{kij}$, respectively. $R_{ij}$ denotes the memristor between $f_j^1(x_j(t))$ and $x_j(t)$. $R_{kij}$ denotes the memristor between $f_j^2(x_j(t - \tau_k(t)))$ and $x_j(t)$. $R_{kij}$ denotes the memristor between $f_j^3(x_j(t - \tau_k(t)))$ and $x_j(t)$.

Remark 1: As for the proposed MSNs model, it can be found to be more general: 1) if we set the multiple links as a simple link, the MSNs model will be simplified to the traditional memristive neural networks model; 2) if we set the memristor-based weights to be constants, the MSNs model will be simplified to the traditional complex networks model.

According to the property of memristor, we apply the following conditions to $a_{ij}(x(t))$, $b_{kij}(x(t))$ and $c_{kij}(x(t))$

$$
a_{ij}(x(t)) = \begin{cases} \hat{a}_{ij} & |x(t)| \leq \Gamma_i, \\
\tilde{a}_{ij} & |x(t)| > \Gamma_i, \end{cases}$$

$$
b_{kij}(x(t)) = \begin{cases} \hat{b}_{kij} & |x(t)| \leq \Gamma_k, \\
\tilde{b}_{kij} & |x(t)| > \Gamma_k, \end{cases}$$

$$
c_{kij}(x(t)) = \begin{cases} \hat{c}_{kij} & |x(t)| \leq \Gamma_k, \\
\tilde{c}_{kij} & |x(t)| > \Gamma_k, \end{cases}$$

where the constants of $\Gamma_i$ and $\Gamma_k$ denote the switching jumps. All of notations $\hat{a}_{ij}, \tilde{a}_{ij}, \hat{b}_{kij}, \tilde{b}_{kij}, \hat{c}_{kij}$ and $\tilde{c}_{kij}$ denote constants.

For the state-dependent and discontinuous parameters $a_{ij}(x(t))$, $b_{kij}(x(t))$ and $c_{kij}(x(t))$, we introduce the set-valued maps and the differential inclusions theories [33], [34] in order to study the dynamical behaviors of MSNs. From the model (1), we can obtain:

$$
\dot{x}_i(t) \in -c_i x(t) + \sum_{j=1}^{N} \mathbb{C}[a_{ij}(x(t))|f_j^1(x_j(t))| + \sum_{k=1}^{m} \sum_{j=1}^{N} \mathbb{C}[b_{kij}(x(t))|f_j^2(x_j(t - \tau_k(t)))| + \sum_{k=1}^{m} \sum_{j=1}^{N} \mathbb{C}[c_{kij}(x(t))|f_j^3(x_j(t))|ds + I(t),
$$

(2)

where $t > 0, i = 1, 2, \ldots, N$; and

$$
\mathbb{C}[a_{ij}(x(t))] = \begin{cases} \hat{a}_{ij} & |x(t)| \leq \Gamma_i, \\
\tilde{a}_{ij} & |x(t)| > \Gamma_i, \end{cases}$$

$$
\mathbb{C}[b_{kij}(x(t))] = \begin{cases} \hat{b}_{kij} & |x(t)| \leq \Gamma_k, \\
\tilde{b}_{kij} & |x(t)| > \Gamma_k, \end{cases}$$

$$
\mathbb{C}[c_{kij}(x(t))] = \begin{cases} \hat{c}_{kij} & |x(t)| \leq \Gamma_k, \\
\tilde{c}_{kij} & |x(t)| > \Gamma_k, \end{cases}$$

$$
\mathbb{C} = \min\{\hat{a}_{ij}, \hat{b}_{kij}, \hat{c}_{kij}\}, \mathbb{C} = \max\{\tilde{a}_{ij}, \tilde{b}_{kij}, \tilde{c}_{kij}\}, \mathbb{C} = \min\{|\hat{a}_{ij}|, |\tilde{a}_{ij}|\}, \mathbb{C} = \max\{|\hat{b}_{kij}|, |\tilde{b}_{kij}|\}, \mathbb{C} = \max\{|\hat{c}_{kij}|, |\tilde{c}_{kij}|\}.
$$

In this paper, the system (1) is viewed as the drive system. Correspondingly, the response system is given as:

$$
\dot{y}_i(t) = -c_i y_i(t) + \sum_{j=1}^{N} \mathbb{C}[a_{ij}(y(t))|f_j^1(y_j(t))| + \sum_{j=1}^{N} \mathbb{C}[b_{kij}(y(t))|f_j^2(y_j(t - \tau_k(t)))| + \sum_{j=1}^{N} \mathbb{C}[c_{kij}(y(t))|f_j^3(y_j(s))|ds
$$

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where $i = 1, 2, \ldots, N$; $u_i(t)$ denotes the appropriate control law designed to get the synchronization target; the initial condition of system (3) is denoted by

$$
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$$

The explanation of other parameters shared by the drive and response systems is omitted in order to avoid repetition.

Similarly, the following differential inclusions can be obtained from the model of the response system (3):

$$
\dot{y}_i(t) \in -c_i y_i(t) + \sum_{j=1}^{N} \text{co}[a_{ij}(y_i(t))] f^1_j(y_j(t))
$$

$$
+ \sum_{k=1}^{m} \sum_{j=1}^{N} \text{co}[b_{kij}(y_i(t))] f^2_j(y_j(t) - \tau_k(t))
$$

$$
+ \sum_{k=1}^{m} \sum_{j=1}^{N} \text{co}[c_{kij}(y_i(t))] \int_{t-\tau_k(t)}^{t} f^3_j(y_j(s)) ds
$$

$$
+ I_i(t) + u_i(t), \quad t > 0,
$$

where $i = 1, 2, \ldots, N$. And

$$
\text{co}[a_{ij}(y_i(t))] = \begin{cases} 
\bar{a}_{ij} & |y_i(t)| \leq \Gamma_i, \\
\tilde{a}_{ij} & |y_i(t)| > \Gamma_i,
\end{cases}
$$

$$
\text{co}[b_{kij}(y_i(t))] = \begin{cases} 
\bar{b}_{kij} & |y_i(t)| \leq \Gamma_k, \\
\tilde{b}_{kij} & |y_i(t)| > \Gamma_k,
\end{cases}
$$

$$
\text{co}[c_{kij}(y_i(t))] = \begin{cases} 
\bar{c}_{kij} & |y_i(t)| \leq \Gamma_k, \\
\tilde{c}_{kij} & |y_i(t)| > \Gamma_k,
\end{cases}
$$

The error system is defined as $e_i(t) = y_i(t) - x_i(t)$, $i = 1, 2, \ldots, N$. Then we can get the following differential inclusions of the error system:

$$
\dot{e}_i(t) \in -c_i e_i(t)
$$

$$
+ \sum_{j=1}^{N} [\text{co}[a_{ij}(y_i(t))] f^1_j(y_j(t)) - \text{co}[a_{ij}(x_i(t))] f^1_j(x_j(t))]
$$

$$
+ \sum_{k=1}^{m} \sum_{j=1}^{N} [\text{co}[b_{kij}(y_i(t))] f^2_j(y_j(t) - \tau_k(t)) - \text{co}[b_{kij}(x_i(t))] f^2_j(x_j(t) - \tau_k(t))]
$$

$$
+ \sum_{k=1}^{m} \sum_{j=1}^{N} [\text{co}[c_{kij}(y_i(t))] \int_{t-\tau_k(t)}^{t} f^3_j(y_j(s)) ds + u_i(t), \quad t > 0,
$$

where $f^3_j(x_i(t)) = f^3_j(y_i(t)) - f^3_j(x_i(t))$.

As we know, the conventional pinning control (CPC) is the technique that chooses some nodes to be controlled to realize the synchronization goal of the drive and response systems. In this paper, the control input $u_i(t)$ uses the switched intermittent pinning control (SIPC), which can be viewed as a kind of pinning control technique on the timeline. The control strategy of the switched intermittent pinning control technique is clearly shown in FIGURE 2.

![Sketch map of the switched intermittent pinning control](image_url)

**FIGURE 2.** Sketch map of the switched intermittent pinning control.

Let’s analyse the first period $T$. On the timeline, $n$ is the number of controlled nodes in the interval $[0, \eta)$ and the number of the uncontrolled nodes in the interval $[\eta, T)$. We can denote any node of the practical networks to be node 1 of the networks model. Therefore, without loss of generality, we pin the nodes from 1 to $n$ in the interval $[0, \eta)$, but pin the nodes from $n+1$ to $N$ in the interval $[\eta, T)$. We divide the continuous control time into infinite sequences $T$ of the recurrent time along the timeline. During each $T$, we set a threshold value $\eta$ (namely switchable control width) to completely switch
between the controlled nodes and the uncontrolled nodes, which is the core point of the switched intermittent pinning control technique.

Based on the control strategy of the switched intermittent pinning control, the mathematical model of the control law is given as follows:

\[
\begin{align*}
u_i(t) &= \begin{cases} 
    -r_1e_i(t) - r_2\text{sgn}(e_i(t)) - \hat{\xi}_i(t), & \text{case 1}, \\
    0, & \text{case 2}, \\
    -r_1e_i(t) - r_2\text{sgn}(e_i(t)) - \hat{\xi}_i(t), & \text{case 3}, \\
    0, & \text{case 4},
\end{cases}
\end{align*}
\]

where \(\hat{\xi}_i(t) = r_1[\sum_{j=n+1}^{N} |e_j(t)|][\sum_{j=1}^{n} \text{sgn}(e_j(t))]^{-1}, \hat{\xi}_i(t) = r_1[\sum_{j=1}^{n} |e_j(t)|][\sum_{j=n+1}^{N} \text{sgn}(e_j(t))]^{-1}; (l = 0, 1, 2, \ldots)\) represents a natural number; and the control input \(u_i(t)\) can be divided into the following four cases according to the time \(t\) and the node number \(i:\)

- \(\text{case 1}\) \(IT \leq t < IT + \eta, 1 \leq i \leq n,\)
- \(\text{case 2}\) \(IT \leq t < IT + \eta, n < i \leq N,\)
- \(\text{case 3}\) \(IT + \eta \leq t < (l + 1)T, 1 \leq i \leq n,\)
- \(\text{case 4}\) \(IT + \eta \leq t < (l + 1)T, n < i \leq N,\)

\(\text{sgn}(e_j(t)) = \begin{cases} 
    1 & e_j(t) \geq 0, \\
    -1 & e_j(t) < 0.
\end{cases}\)

**Remark 2:** Note that the terms \(\sum_{j=1}^{n} \text{sgn}(e_j(t))\) and \(\sum_{j=n+1}^{N} \text{sgn}(e_j(t))\) of (6) are required to be non-zero. Or the control law will be meaningless. Therefore, we always choose an odd number of nodes to be controlled (that means \(n\) is an odd number) for the networks whose total nodes number is an even number. As for the networks whose total nodes number is an odd number, we will artificially add a node (it is supposed to be existed in the system model) into the networks and appropriately set its connections with other nodes, which will make the sum of the total networks nodes \(N\) to be even. As we know, the synchronization of the networks is the ultimate goal. The addition of a node will not change this goal, i.e. the nodes marked from 1 to \(N\) in the drive-response systems will still be correspondingly synchronized under control law. Finally, the odd \(n\) and the even \(N\) will ensure the terms \(\sum_{j=1}^{n} \text{sgn}(e_j(t))\) and \(\sum_{j=n+1}^{N} \text{sgn}(e_j(t))\) to be non-zero and avoid \(u_i(t)\) to be meaningless.

**Remark 3:** The control cost of the SIPC is directly determined by the controlled node number and the corresponding control width, so we use the intermittent pinning control area \(SA\) in FIGURE 2 to measure the control cost. According to the sketch map, \(SA = P_1 + P_2 + P_3 + P_4 + \cdots\) The SIPC is closely related to the CPC and the intermittent control. Then, we analyze the control cost in the period \(T\) for these three control techniques (set the controlled node number \(n\) for the CPC and the control width \(\eta\) for the intermittent control) as follows: for the CPC, \(SA_1 = P_1 + U_2 = nT;\) for the intermittent control, \(SA_2 = P_1 + U_1 = n\eta;\) for the SIPC, \(SA_3 = P_1 + P_2 = n\eta + (N - n)(T - \eta);\)

1. Compared with \(SA_1\), we get the control cost differential \(\Delta SA_1 = SA_3 - SA_1 = P_2 - U_2 = (N - 2n)(T - \eta)\). Apparently, if \(\eta \leq 0.5T\), the control cost of the SIPC is lower than that of the CPC. This goal will be easy to obtain by adjusting \(n\). And if \(\eta\) further increases, \(\Delta SA_1\) will further decrease; moreover, when \(n(\eta \geq 0.5N)\) is constant, \(\Delta SA_1\) can further decrease by reducing the control width \(\eta\).

2. Compared with \(SA_2\), we get the control cost differential \(\Delta SA_2 = SA_3 - SA_2 = P_2 - U_1 = (T - 2\eta)(N - n)\). Apparently, if \(\eta \geq 0.5T\), the control cost of the SIPC is lower than that of the intermittent control. This goal will be easy to obtain by adjusting \(n\). And if \(\eta\) further increase, \(\Delta SA_2\) will further decrease; moreover, when \(\eta(\eta \geq 0.5T)\) is constant, \(\Delta SA_2\) can further decrease by reducing the controlled node number \(n\).

3. As for the \(SA_3 = \eta n + (N - n)(T - \eta)\), suppose that the control energy of synchronization needs to be saved as much as possible, it will be convenient to appropriately adjust \(n\) and \(\eta\) to achieve the applicable \(SA_3\) and limit the control cost. If we set \(n = N\) and \(\eta = T\), the SIPC transform to the full time and full nodes control technique.

Therefore, the switched intermittent pinning control is flexible in control and lower the control cost by adjusting parameters \(n\) and \(\eta\) to adapt different potential application requirements.

**Remark 4:** In this paper, we introduce switched intermittent pinning control for synchronization of MSNs. The proposed method has the following advantages: 1) from the view of theoretical basis, the proposed method is more general: if the assigned value of \(n\) is set to \(N\), the method will be intermittent control policy; if the assigned value of \(\eta\) is set to \(T\), the method will be pinning control policy. 2) from the view of application scenarios, the proposed method is more flexible and extensive: the controlled node number \(n\) and the switchable control width \(\eta\) which are closely linked with the control cost can be flexibly assigned suitable values so that the proposed method is able to meet wider application requirements.

Some important assumptions and lemmas are appropriately given for the research topic of this paper.

**Assumption 1:** Assume that the feedback function \(f_j^3(s)\) is bounded, i.e. for the random number \(s \in R\), there exist positive constants \(\xi_j^*\) such that \(|f_j^3(s)| \leq \xi_j^*, j = 1, 2, \ldots, N.\)

**Assumption 2:** For \(x_1, x_2 \in R\), there exist positive constants \(\xi_j^1, \xi_j^2, \xi_j^3\) to ensure the feedback functions \(f_j^1, f_j^2, f_j^3\) to meet the inequalities as follows:

\[
\begin{align*}
|f_j^1(x_1) - f_j^1(x_2)| &\leq \xi_j^1 |x_1 - x_2|, \\
|f_j^2(x_1) - f_j^2(x_2)| &\leq \xi_j^2 |x_1 - x_2|, \\
|f_j^3(x_1) - f_j^3(x_2)| &\leq \xi_j^3 |x_1 - x_2|,
\end{align*}
\]

where \(j = 1, 2, \ldots, N.\)
Remark 5: In order to meet the necessity of the later proof, some notations are given in advance as follows:
\[ \xi_i = \max(\xi_1^*, \xi_2^*, \ldots, \xi_N^*), \quad \delta = \max(\tau_1', \tau_2', \ldots, \tau_m'), \quad \mathcal{C}_k = [c_{kj}]_{N \times N}, \quad \mathcal{C}_k = [c_{kj}]_{N \times N}. \]

Lemma 1: (see [30]) Assume that Assumption 2 holds, and the feedback functions \( f_j^1(\pm \Gamma_i^j) = f_j^2(\pm \Gamma_i^j) = 0 \), \( j = 1, 2, \ldots, N \). We get
\[ |\bar{c}_{kj}| \leq |\bar{c}_{kj}| |y_j(t) - x_j(t)| \leq (|\bar{c}_{kj}| - \bar{c}_{kj}) |y_j(t) - x_j(t)|, \quad j = 1, 2, \ldots, N. \]

Remark 6: In Ref. [30], pinning synchronization of the memristor-based neural networks with time-varying delays was investigated. And some significant results were obtained. The Ref. [35] focused on finite-time modified projective synchronization of memristor-based neural network with multi-links and leakage delay. In this paper, both the different kinds of contact connections between neurons and the corresponding conduction delays for different links are considered to establish the new dynamical networks model. With the corresponding switched intermittent pinning control law, the asymptotic synchronization and the exponential synchronization of MSNs are concerned.

III. MAIN RESULTS

In this section, we design appropriate switched intermittent pinning controllers to realize the pinning synchronization of the drive and response systems (1) and (3).

A. EXPONENTIAL SYNCHRONIZATION OF MSNS VIA SIPC

In this subsection, we consider the exponential synchronization of MSNs with multi-links and mixed time-varying delays via SIPC. We derive sufficient criteria to ensure the exponential synchronization of the drive system (1) and the response system (3) by adopting the controller (6). The definition of the exponentially synchronization is given as follows.

Definition 1: For any initial conditions, the following inequality is satisfied.
\[ \| e(t) \| \leq \theta \| \Phi - \Psi \| \exp(-\beta t). \]

where \( e(t) = (e_1(t), e_2(t), \ldots, e_N(t))^T; \Phi = (\phi_1(0), \phi_2(0), \ldots, \phi_N(0))^T; \Psi = (\varphi_1(0), \varphi_2(0), \ldots, \varphi_N(0))^T; \) and \( \beta > 0 \) is named the degree of exponential synchronization.

Then, the drive system (1) and the response system (3) are exponential synchronization.

Theorem 1: Assume that Assumption 1 and Assumption 2 hold, and the feedback functions \( f_j^1(\pm \Gamma_i^j) = f_j^2(\pm \Gamma_i^j) = 0, j = 1, 2, \ldots, N \). If the parameters \( r_1 \) and \( r_2 \) of the controller satisfy
\[ r_1 \geq \beta - c_j + \sum_{i=1}^{N} \hat{a}_{ij}\xi_i^j + \sum_{k=1}^{m} \sum_{i=1}^{N} \frac{\exp(\beta \tau_k) \hat{b}_{kj}\xi_i^j}{1 - \epsilon} + \sum_{k=1}^{m} \sum_{i=1}^{N} \hat{c}_{kj}\xi_i^j, \]
and
\[ r_2 \geq \sum_{k=1}^{m} \xi_i^j \delta \| \mathcal{C}_k - \mathcal{C}_k \|_\infty H, \]
where \( H = \max\left(\frac{\hat{N}}{r}, \frac{N}{N-r} \right) \), and \( \beta \) is a positive constant.

Then, the drive system (1) and the response system (3) can obtain the exponential synchronization under the control law (6).

Proof: The following model is the designed Lyapunov-Krasovskii function:
\[ V(t) = V_1(t) + V_2(t) + V_3(t), \]
where
\[ V_1(t) = \exp(\beta t) \sum_{i=1}^{N} \text{sgn}(e_i(t)) e_i(t), \]
\[ V_2(t) = \sum_{k=1}^{m} \sum_{i=1}^{N} \text{sgn}(e_i(t)) \int_{t-\tau_k}^{t} \exp(\beta |s|) |e_i(s)| ds, \]
\[ V_3(t) = \sum_{k=1}^{m} \sum_{i=1}^{N} \frac{\exp(\beta \tau_k) \hat{c}_{kj}\xi_i^j}{1 - \epsilon} \int_{t-\tau_k}^{t} \exp(\beta |s|) |e_i(s)| ds. \]

We calculate the derivative of \( V(t) \) with respect to time \( t \) along the error system (5) as follows:
\[ \dot{V}_2(t) \leq \sum_{k=1}^{m} \sum_{i=1}^{N} \sum_{j=1}^{N} \text{sgn}(e_i(t)) \hat{c}_{kj}\xi_i^j \exp(\beta |s|) |e_j(s)|, \]
\[ \dot{V}_3(t) \leq \sum_{k=1}^{m} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\exp(\beta \tau_k) \hat{c}_{kj}\xi_i^j}{1 - \epsilon} \int_{t-\tau_k}^{t} \exp(\beta |s|) |e_j(s)| ds. \]

As we know, the function \( \exp(\beta t) \) is strictly increasing in the time range \((-\infty, \infty) \). Therefore, \( \int_{t-\tau_k}^{t} \exp(\beta |s|) |e_j(s)| ds \geq \int_{t-\tau_k}^{t} \exp(\beta (t - \tau_k)) |e_j(s)| ds \). So \( \dot{V}_3(t) \) can be calculated as follows:
\[ \dot{V}_3(t) \leq \sum_{k=1}^{m} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\exp(\beta (t + \tau_k)) \hat{c}_{kj}\xi_i^j}{1 - \epsilon} |e_j(t)| \tau_k' \]

where
\[ = \sum_{k=1}^{m} \sum_{i=1}^{N} \sum_{j=1}^{N} \text{sgn}(e_i(t)) \hat{c}_{kj}\xi_i^j |e_j(t)| \tau_k' \]
\[ - \sum_{k=1}^{m} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\exp(\beta \tau_k) \hat{c}_{kj}\xi_i^j}{1 - \epsilon} \int_{t-\tau_k}^{t} \exp(\beta |s|) |e_j(s)| ds \]
\[ \leq \sum_{k=1}^{m} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\exp(\beta (t + \tau_k)) \hat{c}_{kj}\xi_i^j}{1 - \epsilon} |e_j(t)| \tau_k'. \]
Krasovskii function is decreasing for $t \in [0, +\infty)$, which means $V(t) \leq V(0)$. Therefore, we have
\[\exp(\beta t) \sum_{i=1}^{N} \sgn(e_i(t)) e_i(t) \leq V(t) \leq V(0).\]

So,
\[\| e(t) \|_2 \leq \| e(t) \|_1 \leq V(0) \exp(-\beta t).\]

It is easy to know that
\[V(0) = \sum_{i=1}^{N} |\phi_i(0) - \varphi_i(0)| + \sum_{k=1}^{m} \sum_{j=1}^{N} \exp(\beta \tau_k) \hat{\beta}_{kj} \xi_j^2 \int_{-\tau_k}^{0} \exp(\beta s) |\phi_j(s)| ds \]
\[+ \sum_{k=1}^{m} \sum_{i=1}^{N} \exp(\beta \tau_k) \hat{\beta}_{ki} \xi_i^3 \int_{-\tau_k}^{0} \exp(\beta s) |\phi_i(s)| ds \]
\[\leq \theta \sum_{i=1}^{N} |\phi_i(0) - \varphi_i(0)| = \theta \| \Phi - \Psi \|_1,\]
where $\theta (\theta > 1)$ is a positive constant. Therefore, we get
\[\| e(t) \|_2 \leq \theta \| \Phi - \Psi \|_1 \exp(-\beta t).\]

(2) For the period $IT + \eta \leq t < (l + 1)T$, we have
\[\sum_{i=1}^{N} \sgn(e_i(t)) u_i(t) = -r_1 \sum_{i=1}^{N} |e_i(t)| - (N - n)r_2.\]

Then,
\[\hat{V}_1(t) \leq \exp(\beta t) \sum_{i=1}^{N} |e_i(t)| \leq \sigma \sum_{i=1}^{N} |e_i(t)| + \sum_{i=1}^{N} \hat{\alpha}_{ij} \xi_j^1 \int_{-1}^{t} |e_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{kj} \xi_j^2| \int_{-\tau_k}^{0} |\phi_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{ki} \xi_i^3| \int_{-\tau_k}^{0} |\phi_i(s)| ds \]
\[\leq \exp(\beta t) \sum_{i=1}^{N} |e_i(t)| \leq \sigma \sum_{i=1}^{N} |e_i(t)| + \sum_{i=1}^{N} \hat{\alpha}_{ij} \xi_j^1 \int_{-1}^{t} |e_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{kj} \xi_j^2| \int_{-\tau_k}^{0} |\phi_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{ki} \xi_i^3| \int_{-\tau_k}^{0} |\phi_i(s)| ds \]
\[\leq \exp(\beta t) \sum_{i=1}^{N} |e_i(t)| \leq \sigma \sum_{i=1}^{N} |e_i(t)| + \sum_{i=1}^{N} \hat{\alpha}_{ij} \xi_j^1 \int_{-1}^{t} |e_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{kj} \xi_j^2| \int_{-\tau_k}^{0} |\phi_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{ki} \xi_i^3| \int_{-\tau_k}^{0} |\phi_i(s)| ds \]
\[\leq \exp(\beta t) \sum_{i=1}^{N} |e_i(t)| \leq \sigma \sum_{i=1}^{N} |e_i(t)| + \sum_{i=1}^{N} \hat{\alpha}_{ij} \xi_j^1 \int_{-1}^{t} |e_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{kj} \xi_j^2| \int_{-\tau_k}^{0} |\phi_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{ki} \xi_i^3| \int_{-\tau_k}^{0} |\phi_i(s)| ds \]
\[\leq \exp(\beta t) \sum_{i=1}^{N} |e_i(t)| \leq \sigma \sum_{i=1}^{N} |e_i(t)| + \sum_{i=1}^{N} \hat{\alpha}_{ij} \xi_j^1 \int_{-1}^{t} |e_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{kj} \xi_j^2| \int_{-\tau_k}^{0} |\phi_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{ki} \xi_i^3| \int_{-\tau_k}^{0} |\phi_i(s)| ds \]
\[\leq \exp(\beta t) \sum_{i=1}^{N} |e_i(t)| \leq \sigma \sum_{i=1}^{N} |e_i(t)| + \sum_{i=1}^{N} \hat{\alpha}_{ij} \xi_j^1 \int_{-1}^{t} |e_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{kj} \xi_j^2| \int_{-\tau_k}^{0} |\phi_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{ki} \xi_i^3| \int_{-\tau_k}^{0} |\phi_i(s)| ds \]
\[\leq \exp(\beta t) \sum_{i=1}^{N} |e_i(t)| \leq \sigma \sum_{i=1}^{N} |e_i(t)| + \sum_{i=1}^{N} \hat{\alpha}_{ij} \xi_j^1 \int_{-1}^{t} |e_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{kj} \xi_j^2| \int_{-\tau_k}^{0} |\phi_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{ki} \xi_i^3| \int_{-\tau_k}^{0} |\phi_i(s)| ds \]
\[\leq \exp(\beta t) \sum_{i=1}^{N} |e_i(t)| \leq \sigma \sum_{i=1}^{N} |e_i(t)| + \sum_{i=1}^{N} \hat{\alpha}_{ij} \xi_j^1 \int_{-1}^{t} |e_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{kj} \xi_j^2| \int_{-\tau_k}^{0} |\phi_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{ki} \xi_i^3| \int_{-\tau_k}^{0} |\phi_i(s)| ds \]
\[\leq \exp(\beta t) \sum_{i=1}^{N} |e_i(t)| \leq \sigma \sum_{i=1}^{N} |e_i(t)| + \sum_{i=1}^{N} \hat{\alpha}_{ij} \xi_j^1 \int_{-1}^{t} |e_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{kj} \xi_j^2| \int_{-\tau_k}^{0} |\phi_j(s)| ds \]
\[+ \sum_{k=1}^{m} N \xi^3 \int_{-\tau_k}^{0} |\hat{\beta}_{ki} \xi_i^3| \int_{-\tau_k}^{0} |\phi_i(s)| ds \]
Similarly, we can also get the following conclusion for $IT + \eta \leq t < (l + 1)T$:
\[
\| e(t) \|_2 \leq \theta \| \Phi - \Psi \|_1 \exp(-\beta t).
\]

Therefore, according to Definition 1, the drive and response systems (1) and (3) can achieve the exponential synchronization under control law (6).

The proof is completed here.

If we design the control law as follows:
\[
u_i(t) = \begin{cases} 
\hat{R}(t) - r_1 \sum_{j=1}^{N} |e_j(t)\alpha_j(t)|E(t), & \text{case 1,} \\
0, & \text{case 2,} \\
0, & \text{case 3,} \\
\hat{R}(t) - r_1 \sum_{j=1}^{n} |e_j(t)\alpha_j(t)|E'(t), & \text{case 4,}
\end{cases} (14)
\]

where
\[
\hat{R}(t) = -r_1[\alpha_i(t)|e_i(t)| - r_2 \text{sgn}(e_i(t))]; E(t) = \left[ \sum_{j=1}^{n} \text{sgn}(e_j(t)) \right]^{-1}; E'(t) = \left[ \sum_{j=1}^{n} \text{sgn}(e_j(t)) \right]^{-1}; |\alpha_i(t)| \geq 1, i = 1, 2, \ldots, N; \text{ and the control input } u_i(t) \text{ is divided into the same four cases of those of (6) according to the factors of the time } t \text{ and the node number } i.
\]

Then we can obtain the following corollaries.

**Corollary 1:** If Assumption 1 and Assumption 2 hold, the feedback functions $f_j^1(\pm \Gamma_j^k) = f_j^2(\pm \Gamma_j^k) = 0, j = 1, 2, \ldots, N$, and other parameters satisfy the same conditions given in Theorem 1. Then, the drive system (1) and the response system (3) can obtain the exponential synchronization under the control law (14).

**Corollary 2:** Assume that Assumption 1 and Assumption 2 hold, and the feedback functions $f_j^1(\pm \Gamma_j^k) = f_j^2(\pm \Gamma_j^k) = 0, j = 1, 2, \ldots, N$. If the parameters $r_1$ and $r_2$ of the controller satisfy
\[
r_1 \geq -c_j + \sum_{i=1}^{N} \tilde{a}_{ij} \xi_j^1 + \sum_{k=1}^{m} \sum_{i=1}^{N} \tilde{b}_{kij} \xi_j^2 + \sum_{k=1}^{m} \sum_{i=1}^{N} \tilde{c}_{kij} \xi_j^3 \tau_j,
\]
and
\[
r_2 \geq \sum_{k=1}^{m} \xi \hat{\delta} \| \mathcal{T}_k - \mathcal{C}_k \|_{\infty} H,
\]

where $H = \max\{N, \frac{N}{\eta}, \frac{N}{\tau - \eta}\}$.

Then, the drive system (1) and the response system (3) can obtain the asymptotic synchronization under the control law (6).

**Remark 7:** In this paper, both Theorem 1 and Corollary 2 adopt the control law (6) which means the expression forms of the controllers are the same. But the limited conditions of the parameters of the controllers for Theorem 1 and Corollary 2 are different, which means different values of the parameters in the same controller expressions can change the synchronization type to be obtained for the drive and response systems. Therefore, the parameters of the controller are related to not only the control cost but also the obtained synchronization type.

**Asymptotic Synchronization of MSNs via Adaptive SIPC:**

In this subsection, we are concerned with the asymptotic synchronization of MSNs with multi-links and mixed time-varying delays via adaptive switched intermittent pinning control (ASIPC).

The mathematical model of the ASIPC is given as follows:
\[
u_i(t) = \begin{cases} 
-r_1 \sum_{j=1}^{n} |e_j(t)\alpha_j(t)|E(t), & \text{case 1,} \\
0, & \text{case 2,} \\
0, & \text{case 3,} \\
-r_1 \sum_{j=1}^{n} |e_j(t)\alpha_j(t)|E'(t), & \text{case 4,}
\end{cases} (15)
\]

where $\Delta \hat{R}(t) = -r_1 \sum_{j=1}^{n} \text{sgn}(e_j(t)), \Delta r_1(t) = r_1(t) - r_1^s, r_1(t)$ is the estimations of the bound $r_1^s$; the constat $r_2$ $\geq 0; E(t) = \left[ \sum_{j=1}^{n} \text{sgn}(e_j(t)) \right]^{-1}; E'(t) = \left[ \sum_{j=1}^{n} \text{sgn}(e_j(t)) \right]^{-1};$ and the control input $u_i(t)$ is divided into the same four cases as those of (6) according to the factors of the time $t$ and the node number $i$.

**Theorem 2:** Assume that Assumption 1 and Assumption 2 hold, and the feedback functions $f_j^1(\pm \Gamma_j^k) = f_j^2(\pm \Gamma_j^k) = 0, j = 1, 2, \ldots, N$. If the following conditions hold,
\[
r_{1j}^s \geq -c_j + \sum_{i=1}^{N} \tilde{a}_{ij} \xi_j^1 + \sum_{k=1}^{m} \sum_{i=1}^{N} \tilde{b}_{kij} \xi_j^2 + \sum_{k=1}^{m} \sum_{i=1}^{N} \tilde{c}_{kij} \xi_j^3 \tau_j,
\]
and
\[
r_{2j}^s \geq \sum_{k=1}^{m} N \xi \hat{\delta} \| \mathcal{T}_k - \mathcal{C}_k \|_{\infty} H,
\]

where $r_{2j}^s = \min\{r_{2j}, \frac{N}{\eta}, \frac{N}{\tau - \eta}\}$.

Then, the drive system (1) and the response system (3) can obtain the adaptive synchronization under the control law (15).

**Proof:** The following model is the designed Lyapunov-Krasovskii function:
\[
V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t),
\]

where
\[
V_1(t) = \sum_{i=1}^{N} \text{sgn}(e_i(t)) e_i(t),
\]
\[
V_2(t) = \sum_{k=1}^{m} \sum_{i=1}^{N} \sum_{j=1}^{N} \tilde{b}_{kij} \xi_j^2 \int_{t_{-\tau(k)}}^{t} |e_j(s)| ds,
\]
\[
V_3(t) = \sum_{k=1}^{m} \sum_{i=1}^{N} \sum_{j=1}^{N} \tilde{c}_{kij} \xi_j^3 \int_{t_{-\tau(k)}}^{t} |e_j(l)| dldl,
\]
\[
V_4(t) = \frac{1}{2m} \sum_{i=1}^{N} (r_{1i}(t) - r_{1i}^s)^2.
\]
We respectively calculate the derivatives of \( V_1(t) \), \( V_2(t) \), \( V_3(t) \) and \( V_4(t) \) with respect to time \( t \) along the error system (5):

1. During the period \( IT \leq t < IT + \eta \) for the natural number \( l = 0, 1, 2, \ldots \), we get

\[
\dot{V}_1(t) \\
\leq \sum_{i=1}^{n} \text{sgn}(e_i(t))u_i(t) - \sum_{i=1}^{N} c_i|e_i(t)| \\
+ \sum_{i=1}^{N} \sum_{j=1}^{N} \hat{a}_{ij}\xi_j^1|e_j(t)| \\
+ \sum_{i=1}^{m} \sum_{j=1}^{N} \hat{b}_{ij}\xi_j^2|e_j(t) - \tau_k(t)| \\
+ \sum_{k=1}^{m} \sum_{i=1}^{N} \xi_k\int_{t-\beta_k(t)}^{t} |e_i(s)| ds \\
+ \sum_{k=1}^{m} \sum_{i=1}^{N} (\xi_k - \xi_{kij})|sgn(e_i(t))| \int_{t-\beta_k(t)}^{t} \xi_k^* ds \\
\leq - \sum_{j=1}^{N} c_j|e_j(t)| + \sum_{j=1}^{N} \hat{a}_{ij}\xi_j^1|e_j(t)| \\
+ \sum_{k=1}^{m} \sum_{i=1}^{N} \hat{b}_{ij}\xi_j^2|e_j(t) - \tau_k(t)| \\
+ \sum_{k=1}^{m} \sum_{i=1}^{N} \xi_k\int_{t-\beta_k(t)}^{t} |e_i(s)| ds \\
+ \sum_{k=1}^{m} N\xi \delta \| \mathcal{C}_k - \mathcal{C}_k \|_\infty + \sum_{j=1}^{N} r_{ij}^*|e_j(t)| - \sum_{i=1}^{n} r_{2i}. \quad (16)
\]

We get \( \dot{V}_2(t) \) as follows:

\[
\dot{V}_2(t) \\
\leq \sum_{k=1}^{m} \sum_{i=1}^{N} \hat{b}_{ij}\xi_j^2 \frac{1}{1-\epsilon}|e_j(t)| - \sum_{k=1}^{m} \sum_{i=1}^{N} \hat{b}_{ij}\xi_j^2|e_j(t) - \tau_k(t)|.
\]

\( \dot{V}_3(t) \) and \( \dot{V}_4(t) \) can be calculated as follows:

\[
\dot{V}_3(t) = \sum_{k=1}^{m} \sum_{i=1}^{N} \sum_{j=1}^{N} \hat{c}_{ij}\xi_j^3|e_j(t)| \tau_k^r \\
- \sum_{k=1}^{m} \sum_{i=1}^{N} \sum_{j=1}^{N} \hat{c}_{ij}\xi_j^3 \int_{t-\tau_k}^{t} |e_i(s)| ds.
\]

\[
\dot{V}_4(t) = \sum_{i=1}^{n} (r_{1i}(t)|e_i(t)| - 2 r_{1i}^*|e_i(t)|).
\]

So,

\[
\dot{V}(t) \\
\leq \sum_{j=1}^{N} |e_j(t)|[-r_{ij}^* - c_j + \sum_{i=1}^{N} \hat{a}_{ij}\xi_j^1 + \sum_{k=1}^{m} \hat{b}_{ij}\xi_j^2 \frac{1}{1-\epsilon}] \\
+ \sum_{k=1}^{m} \sum_{i=1}^{N} \hat{c}_{ij}\xi_j^3 \tau_k^r + \sum_{k=1}^{m} N\xi \delta \| \mathcal{C}_k - \mathcal{C}_k \|_\infty - \sum_{i=1}^{n} r_{2i}. \quad (17)
\]

2. During the period \( IT + \eta \leq t < (l+1)T \) for the natural number \( l = 0, 1, 2, \ldots \), we get

\[
\sum_{i=1}^{N} \text{sgn}(e_i(t))u_i(t) = - \sum_{i=1}^{N} (r_{1i}(t) - r_{1i}^*)|e_i(t)| - \sum_{i=n+1}^{N} r_{2i}.
\]

Then,

\[
\dot{V}(t) \\
\leq - \sum_{j=1}^{N} c_j|e_j(t)| + \sum_{j=1}^{N} \hat{a}_{ij}\xi_j^1|e_j(t)| \\
+ \sum_{k=1}^{m} \sum_{i=1}^{N} \hat{b}_{ij}\xi_j^2|e_j(t) - \tau_k(t)| \\
+ \sum_{k=1}^{m} \sum_{i=1}^{N} \hat{c}_{ij}\xi_j^3 \int_{t-\tau_k}^{t} |e_i(s)| ds \\
+ \sum_{k=1}^{m} N\xi \delta \| \mathcal{C}_k - \mathcal{C}_k \|_\infty - \sum_{j=1}^{N} r_{ij}^*|e_j(t)| - \sum_{i=n+1}^{N} r_{2i}. \quad (18)
\]

Therefore,

\[
\dot{V}(t) \\
\leq \sum_{j=1}^{N} |e_j(t)|[-r_{ij}^* - c_j + \sum_{i=1}^{N} \hat{a}_{ij}\xi_j^1 + \sum_{k=1}^{m} \hat{b}_{ij}\xi_j^2 \frac{1}{1-\epsilon}] \\
+ \sum_{k=1}^{m} \sum_{i=1}^{N} \hat{c}_{ij}\xi_j^3 \tau_k^r + \sum_{k=1}^{m} N\xi \delta \| \mathcal{C}_k - \mathcal{C}_k \|_\infty - \sum_{i=n+1}^{N} r_{2i}. \quad (19)
\]

When the conditions given in Theorem 2 are satisfied, \( \dot{V}(t) \) gets 0. Therefore, the synchronization between the drive system (1) and the response system (3) will be achieved adaptive synchronization via the control law (15).

The proof is completed here.

We design a control law as follows:

\[
u_i(t) = \begin{cases} 
\dot{\Delta}R(t) - [\sum_{j=n+1}^{N} \Delta\hat{r}_{ij}(t)|e_j(t)||E(t)|, & \text{case 1}, \\
0, & \text{case 2}, \\
0, & \text{case 3}, \\
-\dot{\Delta}R(t) - [\sum_{j=1}^{n} \Delta\hat{r}_{1j}(t)|e_j(t)||E'(t)|, & \text{case 4}, \\
\end{cases}
\]

\[
\dot{r}_{1i}(t) = m_1|e_i(t)|
\]

where \( \Delta\hat{R}(t) = \Delta\hat{r}_{ij}(t)e_i(t) + r_{2i}\beta_i(t)\text{sgn}(e_i(t)) \), \( \Delta\hat{r}_{1j}(t) = r_{1j}(t) - |\alpha_i(t)||r_{1j}^*| \), \( r_{1j}(t) \) is the estimations of the bound \( r_{1j}^* \); the constat \( r_{2i} \geq 0; 0 < |\alpha_i(t)| \leq 1; |\beta_i(t)| \geq 1, i = 1, 2, \ldots, N; r_{1i}^* \geq 0; E(t) = \sum_{j=1}^{N} \text{sgn}(e_j(t))|E(t)|^{-1}; E'(t) = \sum_{j=1}^{N} \text{sgn}(e_j(t))|E(t)|^{-1}; \) the control input \( u_i(t) \) is divided into the same four cases as those of (6) according to the factors of the time \( t \) and the node number \( i \).
Corollary 3: If Assumption 1 and Assumption 2 hold, the feedback functions \( f_j^1(\pm \Gamma_1^j) = f_j^2(\pm \Gamma_1^j) = 0, j = 1, 2, \ldots, N \), and other parameters satisfy the same conditions given in Theorem 2, then, the drive system (1) and the response system (3) can obtain the adaptive synchronization under the control law (20).

Corollary 4: If the constant \( r_{2i} \) of the controller (15) is replaced by a state-dependent parameter \( r_{2i}(e_i(t)) \), where
\[
\begin{align*}
r_{2i}(e_i(t)) = & \begin{cases} r_{2i}^* & |e_i(t)| \geq \bar{F}, \\ r_{2i}^{**} & |e_i(t)| < \bar{F}. \end{cases}
\end{align*}
\]
the constants \( r_{2i}^*, r_{2i}^{**} \) and \( \bar{F} \) are positive; and other parameters satisfy the same conditions give in Theorem 2.

Then, the drive system (1) and the response system (3) can obtain the adaptive synchronization under this controller with switchable parameter \( r_{2i}(e_i(t)) \).

IV. NUMERICAL SIMULATIONS
In this section, we consider memristor-based switching networks with 2-links and mixed delays as follows:
\[
\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{2} a_{ij}(x_i(t)) f_j^1(x_j(t)) + \sum_{k=1,j=1}^{2} b_{kij}(x_i(t)) f_j^2(x_j(t) - \tau_k(t)) + \sum_{k=1,j=1}^{2} c_{kij}(x_i(t)) \int_{t-k(t)}^{t} f_j^3(x_j(s)) ds + I_i(t), \quad i = 1, 2,
\]
(21)
where \( c_1 = 1, \) and \( c_2 = 1.5. \) We have
\[
\begin{align*}
\hat{A} = [\hat{a}_{ij}]_{2\times2} \quad &\hat{A} = [\hat{a}_{ij}]_{2\times2} = \begin{bmatrix} -0.7 & 1.5 \\ 0.5 & 1.6 \end{bmatrix}, &\hat{B}_1 = [\hat{b}_{1ij}]_{2\times2} = \begin{bmatrix} 1.4 & -1 \\ 0.8 & -1.2 \end{bmatrix}, &\hat{B}_2 = [\hat{b}_{2ij}]_{2\times2} = \begin{bmatrix} 0.9 & -0.6 \\ 1.8 & 0.8 \end{bmatrix}, \\
\hat{C}_1 = [\hat{c}_{1ij}]_{2\times2} \quad &\hat{C}_1 = [\hat{c}_{1ij}]_{2\times2} = \begin{bmatrix} 1.2 & 0.8 \\ 0.4 & -1.6 \end{bmatrix}, &\hat{C}_2 = [\hat{c}_{2ij}]_{2\times2} = \begin{bmatrix} 0.6 & -1.4 \\ 1.4 & 0.7 \end{bmatrix}, & \hat{\mathcal{C}}_1 - \mathcal{C}_1 = \begin{bmatrix} 0.4 & 0.8 \\ 1 & 0.7 \end{bmatrix}, & \hat{\mathcal{C}}_2 - \mathcal{C}_2 = \begin{bmatrix} 0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix}.
\end{align*}
\]

The discrete time-varying delays \( \tau_1(t) = \tau_2(t) = \frac{t^\epsilon}{\Gamma_1^0} \), so \( \tau_1 = \tau_2 = 1, \epsilon = 0.25. \) The distributed time-varying delays are \( \delta_1 = \delta_2 = 0.5(1 + \sin t) \), so we can calculate that \( \tau_1' = \tau_2' = 1, \delta = 1. I_1(t) = 1.5 \sin t, \) and \( I_2(t) = 1.5 \cos t. \) The feedback activation functions \( f_j^1(\ell) = f_j^2(\ell) = \tanh(\ell) - 1, \) \( f_j^3(\ell) = \frac{[\ell + 1][|\ell - 1|]}{2}, j = 1, 2. \) Obviously, \( \xi^1_j = \xi^3_j = 1, \xi^2_j = 1, \xi^1 = 1, \xi^2 = 1, j = 1, 2. \) The initial values of the drive system (21) are \( x(t) = (10.5, -1.2)^T, t \in [-1, 0]. \) \( \| \bar{\mathcal{C}}_1 - \mathcal{C}_1 \|_\infty = 1.7, \) \( \| \bar{\mathcal{C}}_2 - \mathcal{C}_2 \|_\infty = 1.4. \) As for the intermittent control, we choose parameters \( T = 0.002, \eta = 0.001 \) and \( n = 1. \)

The corresponding response system is given as follows:
\[
\begin{align*}
\dot{y}_i(t) = & -c_i y_i(t) + \sum_{j=1}^{2} a_{ij}(y_i(t)) f_j^1(y_j(t)) + \sum_{k=1,j=1}^{2} b_{kij}(y_i(t)) f_j^2(y_j(t) - \tau_k(t)) + \sum_{k=1,j=1}^{2} c_{kij}(y_i(t)) \int_{t-k(t)}^{t} f_j^3(y_j(s)) ds + I_i(t) + u_i(t), \quad i = 1, 2,
\end{align*}
\]
(22)
where the initial values of (22) are \( y(t) = (10.2, -8.1)^T, t \in [-1, 0]. \)
When the controller is not applied to the drive system, the plots of the time responses of state variables as well as the phase curve of the error system for the drive system (21) and the response system (22) are shown in FIGURE 3.

(1) According to the conditions of Theorem 1, we choose $\beta = 1, r_1 = 38, r_2 = 13.8$. With the controller (6), the drive-response systems (21) and (22) can be exponentially synchronized. The plots of the time responses of state variables as well as the phase curve of the error system for the drive system (21) and the response system (22) with controller (6) are shown in FIGURE 4.

(2) According to the conditions of Theorem 2, we choose $m_1 = 10.5, r^+_1 = r^+_2 = 25.5, r_1 = r_{21} = 10.8$. With the controller (15), the drive-response systems (21) and (22) can obtain adaptive synchronization. The plots of the time responses of state variables as well as the phase curve of the error system for the drive system (21) and the response system (22) with controller (15) are shown in FIGURE 5.

In order to compare four sufficient synchronization criteria of Corollary 1, 2, 3, 4 in both theory and numerical simulations, we accomplish their numerical simulations shown in FIGURE 6 as follows.

(a) Numerical simulation for Corollary 1. We set $\alpha_i(t) = \sin(t) + 2, i = 1, 2$; the other same parameters are assigned the same values which are used in Theorem 1.

(b) Numerical simulation for Corollary 2. We set $r_1 = 15$; the other same parameters are assigned the same values which are used in Theorem 1.

(c) Numerical simulation for Corollary 3. We set $\alpha_i(t) = \cos(t), \beta_i(t) = \sin(t) + 2, i = 1, 2$; the other same parameters are assigned the same values which are used in Theorem 2.

(d) Numerical simulation for Corollary 4. We set $r_{2i}^* = 10.8, r_{1i}^* = 12.8, i = 1, 2, \tilde{F} = 1$; the other same parameters are assigned the same values which are used in Theorem 2.

Strategy Illustrating for the Case ($N$ is odd):

In this subsection, in order to illustrate our approach to deal with the case ($N$ is odd), we construct MSNs with 2-links which consists of 3 nodes.

\[
\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{3} a_{ij}(x_i(t)) f_1^{(j)}(x_j(t)) + \sum_{k=1}^{2} \sum_{j=1}^{3} b_{kij}(x_i(t)) f_2^{(j)}(x_j(t - \tau_k(t))) + \sum_{k=1}^{2} \sum_{j=1}^{3} c_{kij}(x_i(t)) \int_{t-H_k(t)}^{t} f_3^{(j)}(x_j(s)) ds + I_i(t),
\]

where $i = 1, 2, 3, c_1 = 1, c_2 = 1.5, c_3 = 1$ and $c_4 = 1.5$. 

\[\text{(23)}\]
non-zero. However, the node numbers \( N = 3 \) in drive and response systems are odd, which may result in the problem that the terms to be zero. So we artificially add a node to the systems (23) and (24), which makes the node numbers of (23) and (24) be even. The scale of the systems is extended to 4. Still, our goal is to make the original three nodes in (23) correspondingly synchronize with the original three nodes in (24).

In order to obtain the synchronization of the extended systems, we appropriately set the connections of the added node with other three nodes. The connection parameters are given as follows:

\[
A = \begin{bmatrix}
-0.7 & 1.5 & 0.5 & 0.9 \\
0.5 & 1.6 & -1.7 & 2.2 \\
-0.3 & 1 & 0.8 & -0.7 \\
0.9 & 1.5 & -0.5 & 1.4 \\
\end{bmatrix}
\]

\[
\dot{A} = \begin{bmatrix}
-0.6 & 0.9 & 0.8 & 0.5 \\
0.4 & 1.1 & -1 & 0.8 \\
-0.5 & 1.5 & 0.3 & -0.6 \\
0.8 & 0.7 & -0.7 & 1.2 \\
\end{bmatrix}
\]

\[
B_1 = \begin{bmatrix}
1.4 & -1 & 2.4 & 0.6 \\
0.8 & -1.2 & 0.4 & -1.8 \\
-0.6 & 0.5 & -1.5 & 0.5 \\
2 & 1.4 & -0.9 & 1.6 \\
\end{bmatrix}
\]

\[
\dot{B}_1 = \begin{bmatrix}
0.5 & -1.2 & 2.2 & 0.8 \\
1.7 & -0.6 & 1.2 & -1.4 \\
-2.2 & 0.8 & -0.9 & 1 \\
0.8 & 1.2 & -0.6 & 0.4 \\
\end{bmatrix}
\]

\[
B_1 = \begin{bmatrix}
0.9 & -0.6 & 1.4 & 2 \\
1.8 & 0.8 & -1.2 & -1.8 \\
-0.8 & -1.2 & 0.5 & 0.5 \\
-1 & 0.7 & 0.6 & 1.6 \\
\end{bmatrix}
\]

\[
\dot{B}_2 = \begin{bmatrix}
1.2 & -0.6 & 1 & 1.6 \\
-1.5 & -1.8 & 1.4 & 0.8 \\
-0.7 & 1.4 & 0.9 & 1.8 \\
\end{bmatrix}
\]

\[
C_1 = \begin{bmatrix}
1.2 & 0.8 & -1.2 & 1.5 \\
0.4 & -1.6 & 0.6 & -1.8 \\
-0.6 & 1 & -1.5 & 0.8 \\
-1.4 & -2.2 & 0.9 & 1.4 \\
\end{bmatrix}
\]

\[
\dot{C}_1 = \begin{bmatrix}
0.8 & 1.6 & -1.6 & 1.8 \\
1.4 & -0.6 & 0.7 & -1.2 \\
-0.8 & 2 & -1.2 & 1.8 \\
-1.6 & -0.8 & 0.4 & 1.6 \\
\end{bmatrix}
\]

\[
C_2 = \begin{bmatrix}
0.6 & -1.4 & 0.8 & 1 \\
1.4 & 0.7 & -1.2 & -1.6 \\
-1.8 & 1.4 & 0.6 & -0.8 \\
0.8 & -2.6 & 0.4 & 1.8 \\
\end{bmatrix}
\]

\[
\dot{C}_2 = \begin{bmatrix}
1.4 & -0.8 & 1.4 & 1.6 \\
0.8 & 1.5 & -0.6 & -1.2 \\
-1.2 & 1 & 2 & -1 \\
1.6 & -1.6 & 1.6 & 1.4 \\
\end{bmatrix}
\]

The corresponding response system is given as follows:

\[
\dot{y}_i(t) = -c_i y_i(t) + \sum_{j=1}^{3} a_{ij} (y_i(t)) f_j^1(y_j(t))
+ \sum_{k=1}^{2} \sum_{j=1}^{3} b_{ijk} (y_i(t)) f_j^2(y_j(t - \tau_k(t)))
+ \sum_{k=1}^{2} \sum_{j=1}^{3} c_{ijk} (y_i(t)) \int_{t-\delta_k(t)}^{t} f_j^3(y_j(s))ds + I_i(t) + u_i(t).
\]

where \( i = 1, 2, 3, c_1 = 1 \).

\textbf{Remark 8}: Combining the analyses in Remark 2, we need to ensure the terms \( \sum_{j=1}^{n} \text{sgn}(e_j(t)) \) and \( \sum_{j=n+1}^{N} \text{sgn}(e_j(t)) \) to be

\textbf{FIGURE 6}. (a), (b), (c) and (d) are the curves of the error system between the systems (21) and (22) for Corollary 1, 2, 3, 4, respectively.
and (24) without and with controller (6), respectively.

The phase curve of the error system for the drive system (23) and the response system (24) without and with controller (6) are shown in FIGURE 7.

The discrete time-varying delays are same with those in (21). The distributed time-varying delays are \( \delta_1 = \delta_2 = 0.25(1 + \sin t) \), so we can calculate that \( \tau'_1 = \tau'_2 = 0.5, \delta = 0.5, I_1(t) = 1.5 \sin t, I_2(t) = -1.5 \cos t, I_3(t) = \sin t, \) and \( I_4(t) = -1.5 \cos t \). The feedback activation functions \( f^j_1(\ell) = f^j_2(\ell) = f^j_3(\ell) = \tanh(|\ell| - 1) \), \( j = 1, 2, 3, 4 \). Obviously, \( \xi^1_j = \xi^2_j = \xi^3_j = 1, \xi^*_j = 1, \xi = 1 \). The initial values of (23) and (24) are \( x(t) = (-2.5, -5.2, -4.9, 2.8)^T, y(t) = (3.2, -10.1, -0.8, -1.5)^T, t \in [-1, 0] \). As for the intermittent control, we choose parameters \( T = 0.002, \eta = 0.001 \) and \( n = 3 \).

According to the conditions of Corollary 2, we choose \( r_1 = 26, r_2 = 12.5 \). With the controller (6), the drive-response systems (23) and (24) can be asymptotically synchronized. The phase curve of the error system for the drive system (23) and the response system (24) without and with controller (6) are shown in FIGURE 7.

According to the simulation results, the extended systems obtain the asymptotical synchronization. So, the original three nodes in (23) and (24) are correspondingly synchronized.

V. CONCLUSION

Based on the real structure of neurons, this paper proposes a new networks model, i.e. the MSNs with multi-links and mixed delays. Further, this paper firstly investigates the exponential synchronization and the adaptive synchronization for this new networks model. Then, the switched intermittent pinning control law is appropriately designed to stabilize the error system between the drive system and the response system, which is quite convenient and flexible. Some sufficient conditions are obtained to ensure the synchronization of MSNs with multi-links and mixed delays. Finally, numerical simulations are given to testify the correctness of our results. As an interesting and significant problem, we will continue to focus on the dynamical networks from the view of bionics and their dynamical behaviors, such as finite-time synchronization and fix-time synchronization.

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