Convex Approximation Algorithm for AC/DC Distribution Network With Energy Router

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The optimal operation model of AC/DC distribution network with energy router (ER) is essentially a nonconvex nonlinear programming (NLP) problem. In order to improve the feasibility of solving the model, a convex approximation algorithm is proposed in this work. The steady-state model of ER is developed with considering the loss characteristics and multiport coordinated control strategy. It is embedded in the optimization formulations of AC/DC network as basic operating equations. Then, using second-order cone relaxation technology, the power flow equations of AC and DC distribution networks are convexly relaxed. On this basis, the highly nonlinear operating model of ER is linearized by introducing a successive approximation approach. Therefore, the original NLP problem is transformed into the convex programming problem and the solution efficiency is improved. Meanwhile, an iterative solution algorithm is developed to ensure the accuracy of the convex approximation approach. Simulation results verify the feasibility and efficiency of the proposed algorithm.

Keywords: AC/DC distribution network, energy router, steady-state model, second-order cone relaxation, successive approximation

INTRODUCTION

The development of renewable energy technology poses new challenges for flexible access and effective regulation of the power system (Guo et al., 2019). Energy router (ER) is also known as power electronic transformer (PET), and solid-state transformer. It has the functions of voltage conversion and power flow routing (Chen et al., 2020; Li et al., 2021; Liu et al., 2018). The AC/DC hybrid distribution network with ER (ER-based AC/DC HDN) can interconnect networks with multiple voltage levels and frequencies. It is conducive to the flexible access and consumption of distributed generations (DGs), and has attracted extensive attention (Zhang et al., 2020). However, flexible device access, AC/DC hybrid connection, three-phase asymmetrical, and active and reactive power coupling make the analysis and solution of ER-based AC/DC HDN face great challenges (Meng et al., 2018; Tang et al., 2021; Zhang et al., 2021).

The existing literatures have carried out research on the establishment of the ER steady-state model and its application in the distribution network. In (Miao et al., 2016), each port of ER is equivalent to the voltage source converter. On this basis, taking into account the loss characteristics of the converter, a power flow model for ER has been proposed in (Dong et al., 2019). Then, in order to reduce network losses, operating costs, and improve the voltage distribution, a multi-objective optimization model for AC network with ER was developed in (Miao et al., 2018). To take advantage of flexible regulation capabilities of ER, an optimization method for voltage imbalanced suppression
is studied in (Dong et al., 2018). The above literatures have improved the operating performance of the ER based distribution networks from various aspects. However, the coordination control capability among multport of ER needs to be further researched. Furthermore, the optimization model in the above literatures includes power flow constraints and highly nonlinear ER operation constraints. It is essentially a nonconvex nonlinear programming (NLP) problem, and obtaining the solution with an acceptable time is a great challenge (Deng et al., 2021)- (Soofi et al., 2021).

ER contains an intermediate DC/DC link. It isolates the distribution network into multiple AC and DC subnetworks with independent voltages and frequencies (Li et al., 2020). While operation, an appropriate coordinated control strategy is important to satisfy the power exchange demands of the ER and the subnetworks. For this issue, an optimal combination of operation strategy based on generalized droop control is introduced in (Rouzbehi et al., 2015). The switching of different operating modes (constant voltage control, constant power control, and droop control) of ER is determined by optimizing the droop coefficient. It is worth noting that the droop control strategy has been extensively used in islanded system. In (Yu et al., 2018), a peer-to-peer control strategy for islanded networks is developed. In order to be applicable to the asymmetric low voltage AC network, a three-phase droop control strategy has been studied in (Abdelaziz et al., 2013). On this basis, a secondary control strategy is introduced in (Allam et al., 2018) to improve the utilization efficiency of DGs. Applying these control methods to ER-based AC/DC HDN can improve the operating performance of the network, which has not been studied yet.

Generally, converting the original NLP problem into a convex programming problem can improve the efficiency of the solution. Using the second-order cone relaxation technology, an optimal operation model for AC distribution network is proposed in (Li et al., 2017), which improves the feasibility of the solution. In (Gan and Low, 2014) a second-order cone programming (SOCP) model for DC network is introduced. These researches provide a basis for convex programming modeling of AC/DC network. However, several limitations can be found. 1) The highly nonlinear loss characteristic equation of ER is a quadratic function involving current. The existing SOCP approach cannot effectively deal with it. 2) The square of the voltage in SOCP model is the optimization variable. The droop control strategy involves the first-order term of voltage, which cannot be transformed in the form of SOCP manner.

In order to improve the feasibility of solving the optimization model of ER-based AC/DC HDN, a convex approximation algorithm is proposed in this work. The main contributions are summarized as follows.

1) An optimal operation model for ER-based AC/DC HDN is devoted in this paper. The multiport coordinated control strategy of ER is considered. Thus, the power balanced of ET and the system can be guaranteed. On this basis, a hierarchical droop control strategy is introduced, which improves the utilization efficiency of DGs.

2) The second-order cone relaxation method is adopted to handle the nonlinear power flow equations. Then, combined with the linear approximation approach, the operation constraints of ER are successfully linearized. The original NLP problem is converted into a convex programming problem, which improves the efficiency of the solution. Furthermore, the successive iteration algorithm is proposed to ensure the accuracy of the solution.

### STEADY-STATE MODEL AND COORDINATED CONTROL STRATEGY OF ER

#### Steady-State Model of Multiport ER

The equivalent structure diagram of ER based on PET is shown in Figure 1. It includes M AC ports and N DC ports. Each port is composed of AC/DC or DC/DC converter, which is used to connect AC and DC networks, respectively.

While operation, the active power balanced of ER should be satisfied, as described in Eq. 1.

$$
\sum_{m=1}^{M} P_{ER,ac,m}^b + \sum_{n=1}^{N} P_{ER,dc,n}^b - P_{ER,loss} = 0 \tag{1}
$$

The power loss of each ER port $P_{loss,\ell}$ can be obtained by curve fitting. It is expressed as the quadratic function of the port current $I_{\ell,i}$ (Sun et al., 2019; Khan and Bhowmick, 2019).

$$
P_{loss,\ell,i} = a_{\ell,i} + b_{\ell,i} I_{\ell,i} + c_{\ell,i} (I_{\ell,i})^2 \tag{2}
$$

Then, the total loss of ER $P_{PET,loss}$ is the sum of the power loss of all AC ports and DC ports, which is shown in Eq. 3.

$$
P_{ER,loss} = \sum_{m=1}^{M} \sum_{i=a}^{c} P_{loss,ac,m,i} + \sum_{n=1}^{N} P_{loss,dc,n} \tag{3}
$$

#### Coordinated Control Strategy of Multiport ER

The AC and DC subnetworks are connected via ER coupling. Due to the intermediate DC/DC isolation link of ER, multiple AC subnetworks operate at different frequencies. To maintain the power balanced of ER, AC, and DC subnetworks, an appropriate coordinated control strategy is necessary. In (Rouzbehi et al., 2015), a generalized droop control strategy is proposed, which is the integration of constant voltage control, constant power control and droop control. The control mode is determined by the generalized droop coefficient. The simulation results show that the droop control strategy ($\alpha \neq 0, \beta \neq 0, \gamma \neq 0$) can effectively deal with the fluctuation of renewable energy. In this work, the secondary control layer is introduced. Then, the regulation performance of the controller can be improved, as shown in Figure 2. The droop control strategy is applied to the low voltage port of ER, as shown in Eqs 4–6. And the high voltage AC port of ER adopts constant voltage control mode to maintain the power balance.

$$
P_{ER,ac,\ell,i}^b = \phi_{ac,\ell,i} (\omega_0 + \delta \omega_{ac,\ell,i} - \omega_i) \tag{4}
$$
Objective Function

This work takes the minimum active power loss as the objective function, shown as follows:

$$\min f = \sum_{i=1}^T \left( \sum_{\phi=\AC}^\phi \sum_{j=\AC}^j r_{\phi,\AC}^j (\hat{P}_{\phi,\AC})^j + \sum_{j=\AC}^j r_{\phi,\AC}^j (\hat{P}_{\phi,\AC})^j + P_{d,\AC}^i \right)$$

(7)

Constraints

The Distflow branch equations of AC/DC distribution networks include power flow constraints, branch voltage constraints, and current constraints (Zhang et al., 2018). It is relaxed by the cone relaxation technique, shown as follows:

$$\begin{align*}
\sum_{i=\AC}^i \left( p_{\phi,\AC}^i - r_{\phi,\AC}^i \hat{P}_{\phi,\AC}^i + r_{\phi,\AC}^i \hat{P}_{\phi,\AC}^i - p_{\phi,\AC}^i \right) &= \sum_{j=\AC}^j p_{\phi,\AC}^j \\
\sum_{i=\AC}^i \left( q_{\phi,\AC}^i - x_{\phi,\AC}^i \hat{Q}_{\phi,\AC}^i + x_{\phi,\AC}^i \hat{Q}_{\phi,\AC}^i - q_{\phi,\AC}^i \right) &= \sum_{j=\AC}^j q_{\phi,\AC}^j \\
\hat{V}_{\phi,\AC}^i &= \hat{V}_{\phi,\AC}^j - 2 r_{\phi,\AC}^i \hat{P}_{\phi,\AC}^i + 2 x_{\phi,\AC}^i \hat{Q}_{\phi,\AC}^i + \left( (r_{\phi,\AC}^i)^2 + (x_{\phi,\AC}^i)^2 \right) \hat{V}_{\phi,\AC}^j \\
2 r_{\phi,\AC}^i \hat{P}_{\phi,\AC}^i + 2 x_{\phi,\AC}^i \hat{Q}_{\phi,\AC}^i - \hat{V}_{\phi,\AC}^i &\leq \hat{V}_{\phi,\AC}^i + \hat{V}_{\phi,\AC}^j
\end{align*}$$

(8)

$$\begin{align*}
\sum_{i=\AC}^i \left( p_{\phi,\AC}^i - r_{\phi,\AC}^i \hat{P}_{\phi,\AC}^i + r_{\phi,\AC}^i \hat{P}_{\phi,\AC}^i - p_{\phi,\AC}^i \right) &= \sum_{j=\AC}^j p_{\phi,\AC}^j \\
\sum_{i=\AC}^i \left( q_{\phi,\AC}^i - x_{\phi,\AC}^i \hat{Q}_{\phi,\AC}^i + x_{\phi,\AC}^i \hat{Q}_{\phi,\AC}^i - q_{\phi,\AC}^i \right) &= \sum_{j=\AC}^j q_{\phi,\AC}^j \\
\hat{V}_{\phi,\AC}^i &= \hat{V}_{\phi,\AC}^j - 2 r_{\phi,\AC}^i \hat{P}_{\phi,\AC}^i + 2 x_{\phi,\AC}^i \hat{Q}_{\phi,\AC}^i + \left( (r_{\phi,\AC}^i)^2 + (x_{\phi,\AC}^i)^2 \right) \hat{V}_{\phi,\AC}^j \\
2 r_{\phi,\AC}^i \hat{P}_{\phi,\AC}^i + 2 x_{\phi,\AC}^i \hat{Q}_{\phi,\AC}^i - \hat{V}_{\phi,\AC}^i &\leq \hat{V}_{\phi,\AC}^i + \hat{V}_{\phi,\AC}^j
\end{align*}$$

(9)

Droop control characteristic constraints of DGs in the low voltage AC/DC networks are expressed as Eqs 10–12.
\[ P_{g,AC,j} = \sum_{g=\text{all}} P_{g,AC,j} = c_{ac}(\omega_0 + \delta \omega_{ac,j} - \omega_j) \] (10)

\[ Q_{g,AC,j} = \sum_{g=\text{all}} Q_{g,AC,j} = \xi_{ac}(V'_{j,\text{LAC}} + \delta V'_{j,\text{LAC}} - V'_{j,\text{LAC}}) \] (11)

\[ P_{g,DC,j} = \xi_{dc}(V'_{j,\text{LDC}} + \delta V'_{j,\text{LDC}} - V'_{j,\text{LDC}}) \] (12)

Operational constraints of the network are formulated as:

\[ I_j \leq \tilde{I}_j \leq I_j \] (13)

\[ E_{\text{SOC}} \leq \dot{E}_{\text{SOC}} \leq T_{\text{SOC}} \] (14)

\[ P_{\text{stor}} \leq P_{\text{stor}}^f \leq P_{\text{stor}} \] (15)

\[ \sum_{i=1}^{T} P_i^f = 0 \] (16)

\[ \|P_{\text{ER},ac}^d \|_{\Delta} \leq \omega_{\text{ER},ac} \] (17)

The allowable range of variables is shown in Eq. 13, in which \( I_j \in \{ V_{LAC,j}^f, V_{LDC,j}^f, \omega_j, P_{g,AC,j}^d, Q_{g,AC,j}^d, P_{g,DC,j}^d, Q_{g,AC,j}^d, P_{\text{ER},ac}^d \} \). The amount of charge of energy storage system (ESS) \( E_{\text{SOC}} \) is represented in Eq. 14. The charge and discharge power of ESS in a scheduling period is 0, as shown in Eq. 15. The capacity constraint of ER AC port is relaxed to the second-order cone form, as described in Eq. 17.

Eqs 7–17 together with the steady-state model of ER Eqs 1–6 form the original optimal operation model \( F_0 \). It is worth noting that the highly nonlinear ER loss characteristic constraint is involved in \( F_0 \), which is essentially an NLP problem. Accurate and efficient solving algorithm is very necessary.

**Successive Convex Approximation Conversion to NLP Problem**

*Linear Approximation*

In section **Optimal Operation Model for ER-Based AC/DC HDN**, the second-order cone relaxation technique is adopted to linearize the Distflow branch equations, as shown in Eqs 8, 9. The square of the branch current is introduced as the optimization variable. Thus, the constant term \( a_{ij} \) and second-order term \( c_{ij}I_j^2 \) in Eq. 2 are linear expressions. Given an initial value \( I_{ij}^{k-1} \), the first-order term \( b_{ij}I_j^{k-1} \) is approximately linearized by using Taylor expansion in this work.

\[ b_{ij}I_j^{k-1} = \frac{b_{ij}}{2} \left( \sqrt{I_j^{k-1}} \right)^2 + \frac{b_{ij}}{2} \left( \frac{I_j^{k-1}}{\sqrt{I_j^{k-1}}} \right)^2 = \frac{b_{ij}}{2} I_{ij}^{k-1} + \frac{b_{ij}}{2} I_{ij}^{k-1} \] (18)

Using Eq. 18, the ER loss characteristic constraint in Eq. 2 can be transformed into linear expression. Furthermore, the square of the node voltage is the optimization variable. The droop control constraints Eqs 5, 6, 11, 12 involve the first-order term of voltage, which are approximately linearized using Eq. 19.

\[ V_j^{k-1} = \sqrt{V_j^{k-1}}^{2} + \frac{(V_j^{k-1})^2}{2} = \frac{V_j^{k-1} + V_j^{k-1}}{2} \] (19)

where, \( V_j \in \{ V_{ER,ac}^d, V_{ER,dc}^d, V_{LAC,j}, V_{LDC,j} \} \). After relaxation and linearization, the objective function is linear and the constraints are linear or second-order cone expression. The original NLP problem \( F_0 \) is converted into an approximate convex programming problem \( F \), which can be efficiently solved.

**Successive Convex Approximation Algorithm**

The optimal values of branch current and node voltage \( (I_{ij}, V_j) \) in the original NLP problem \( F_0 \) cannot be obtained in advance. When the difference between the given initial values \( (I_{ij}, V_j) \) and the optimal values are large, high calculation error will occur in Eqs 18, 19. To improve the accuracy between \( F \) and \( F_0 \), an iterative calculation process is necessary. The successive convex approximation model is defined as \( F \) in this work.

The diagram of the successive convex approximation algorithm is shown in Figure 3.

The solution process is as follows:

1. **Step 1:** Given initial values \( (I_{ij}^{k-1}, V_j^{k-1}) \) to formulate and solve \( F_k \).
2. **Step 2:** The initial values are updated according to the current optimal values of \( F_k \) \( (I_{ij}^{k}, V_j^{k}) = \sqrt{I_{ij}^{k-1}}, V_j^{k} = \sqrt{V_j^{k-1}} \).
3. **Step 3:** If the difference between initial values in two adjacent iterations is less than the given threshold \( \varepsilon = 0.001 \), stop, and the convergence is obtained; otherwise, go back to step 1 based on the updated initial values \( (I_{ij}^{k}, V_j^{k}) \).

**CASE STUDIES**

In order to verify the effectiveness of the approach proposed in this work, the solver Cplex is used to solve the optimization program based on Matlab-Yalmip platform. The network topology of ER-based AC/DC HDN is shown in Figure 4. 10 kV high voltage AC network (Zhu, 2002), 380 V low voltage AC network, and 750 V low voltage DC network (Dong et al., 2019) are coupled through ER. The no load voltage in AC network is \( V_{LAC}^{k} = 1.05 \text{p.u.} \), and the allowable voltage range is set as \([0.95, 1.05]\text{p.u.} \). The no load voltage in DC network is \( V_{LDC}^{k} = 1.02 \text{p.u.} \), and the allowable voltage range is set as \([0.95, 1.02]\text{p.u.} \). The no load frequency is \( \omega_0 = 1.004 \text{p.u.} \), and the allowable frequency range is set as \([0.996, 1.004]\text{p.u.} \). Fitting coefficients of ER loss model are set as \( a_{ac,j} = 0.001, b_{ac,j} = 0.005, c_{ac,j} = 0.02 \). The given initial values are \( I_{ij}^{k-1} = 0.01 \), and \( V_j^{k-1} = 1 \).

**Results of Optimal Power Flow**

The original NLP problem \( F_0 \) and the successive convex approximation problem \( F \) are solved by the solver Ipopt and Cplex, respectively. The results of the optimal power flow in a period are shown as follows.

Table 1 shows the active power loss and solution time of model \( F_0 \) and \( F \). The active power loss in each network is
roughly the same, and the relative error of the total loss is only 0.67%. Thus, the accuracy of the proposed algorithm is verified. Furthermore, compared with the original NLP problem $F_0$ with solving time of 80.8 s, that is 17.3 s in the proposed model $\tilde{F}_s$. Calculation efficiency increased by 78.6%.

To verify the effectiveness of the introduced secondary control strategy, three cases are studied here.

Case I: Droop control strategy without secondary layer is adopted by ER and DGs;
Case II: Droop control strategy without secondary layer is adopted by ER and secondary droop control strategy is adopted by DGs;
Case III: Secondary droop control strategy is adopted by ER and DGs.

Active power output by DGs and ER in the low voltage network and the total power loss are demonstrated in Figure 5. Droop control characteristic is shifted by introducing the secondary droop control layer. The regulation performance of the controller is improved. The active power output of DGs is increased, and the active power injected by ER is reduced. Thus, the utilization efficiency of DGs in Case III is 55.4 and 19.7% higher than Case I and Case II, respectively. Compared with Case I and Case II, the total power loss in Case III is reduced by 29.6 and 15.5%, respectively.

### Results of Day-Ahead Optimal Scheduling

As demonstrated in Figure 6, the power output of DGs and main grid changes with the load curve. The ESS is charged at 1:00–10:00 and 15:00–17:00 during the low load period, and discharged at 12:00–14:00 and 19:00–23:00 during the peak load period, as shown in Figure 7. Compared with before

| Model | Active power loss in different networks (kW) | Total loss (kW) | Solution time (s) |
|-------|-------------------------------------------|----------------|------------------|
|       | 10 kV-AC | 380 V-AC | 750 V-DC | ER |
| $F_0$ | 65.4      | 40.4      | 5.1      | 22.8    | 133.7 | 80.8 |
| $F_\tilde{s}$ | 64.6      | 41.4      | 5.1      | 21.7    | 132.8 | 17.3 |
optimization, the total power loss after optimization has been reduced by 27.2%. Meanwhile, the active power loss of ER decreased from 331.9 to 300.3 kWh. The operating efficiency of the network is improved.

The node voltage is typically around 1p.u. To verify the convergence of the proposed successive convex approximation algorithm, various initial current values are analyzed here. As demonstrated in Figure 8, the convergence is obtained in all scenes. The solution time varies from 323.4 to 792.1 s, which is more than 5 h in the original NLP model. The number of iterations varies from 6 to 10. Furthermore, the relaxation error of branch current is 1e-6 order of magnitude, as depicted in Figure 9. These results reveal that the proposed approach in this work has good convergence and accuracy while improving the efficiency of the calculation.

CONCLUSION

The multi-period optimal operation model for ER-based AC/DC HDN is proposed in this work. The multiport coordinated control strategy of ER is considered in this paper. The introduction of a secondary droop control strategy improves the performance of the controller, which is conducive to enhance the utilization efficiency of DGs. On this basis, the operational efficiency of the network can be improved by coordinating and optimizing the active power output of DGs and ER. The second-order cone relaxation technique and
successive linear approximation approach are adopted to converter the original NLP problem into a convex programming model. The proposed successive convex approximation algorithm has good convergence and accuracy. Ensure that the optimal solution of ER-based AC/DC HDN is obtained within an acceptable time.

**DATA AVAILABILITY STATEMENT**

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding authors.

**REFERENCES**

Abdelaziz, M. M. A., Farag, H. E., El-Saadany, E. F., and Mohamed, Y. A.-R. I. (2013). A Novel and Generalized Three-Phase Power Flow Algorithm for Islanded Microgrids Using a newton Trust Region Method. *IEEE Trans. Power Syst.*, 28, 190–201. doi:10.1109/tpwrs.2012.2195785

Allam, M. A., Hamad, A. A., Kazerni, M., and El-Saadany, E. F. (2018). A Novel Dynamic Power Routing Scheme to Maximize Loadability of Islanded Hybrid AC/DC Microgrids under Unbalanced AC Loading. *IEEE Trans. Smart Grid*, 9, 5798–5809. doi:10.1109/tsg.2016.2697360

Chen, Y., Wang, P., Elasser, Y., and Chen, M. (2020). Multicell Reconfigurable Multi-Input Multi-Output Energy Router Architecture. *IEEE Trans. Power Electron.*, 35, 13210–13224. doi:10.1109/tpe.2020.2996199

Deng, Y., Zhang, Y., Luo, F., and Mu, Y. (2021). Operational Planning of Centralized Charging Stations Utilizing Second-Life Battery Energy Storage Systems. *IEEE Trans. Sustain. Energ.*, 12, 387–399. doi:10.1109/tse.2020.3001015

Dong, L., Zhang, T., Pu, T., and Sun, Y. (2019). A Decentralized Optimal Operation of AC/DC Hybrid Microgrids Equipped with Power Electronic Transformer. *IEEE Access*, 7, 157946–157959. doi:10.1109/access.2019.2949378

Dong, L., Zhang, T., Pu, T., Chen, N. S., and Sun, Y. Y. (2018). “Optimal Scheduling of Asymmetrical AC/DC Micro-grid Based on PET,” in International Conference on Power System Technology, Guangzhou, China, November 6-8, 2018, 1757–1763.

Gan, L., and Low, S. H. (2014). Optimal Power Flow in Direct Current Networks. *IEEE Trans. Power Syst.*, 29, 2892–2904. doi:10.1109/tpwrs.2014.2313514

Guo, H., Wang, F., Zhang, L., and Luo, J. (2019). A Hierarchical Optimization Strategy of the Energy Router-Based Energy Internet. *IEEE Trans. Power Syst.*, 34, 4177–4185. doi:10.1109/tpwrs.2019.2907323

Ji, H., Li, P., Wang, C., Song, G., Zhao, J., Su, H., et al. (2017). A Strengthened SOCP-Based Approach for Evaluating the Distributed Generation Hosting Capacity with Soft Open Points. *Energ. Proc.*, 142, 1947–1952. doi:10.1016/j.egypro.2017.12.394

Khan, S., and Bhowmick, S. (2019). A Generalized Power-Flow Model of VSC-Based Hybrid AC-DC Systems Integrated with Offshore Wind Farms. *IEEE Trans. Sustain. Energ.*, 10, 1775–1783. doi:10.1109/tse.2018.2870914

Li, K., Wen, W., Zhao, Z., Yuan, L., Cai, W., Mo, X., et al. (2021). Design and Implementation of Four-Port Megawatt-Level High-Frequency-Bus Based Power Electronic Transformer. *IEEE Trans. Power Electron.*, 36, 6429–6442. doi:10.1109/tpe.2020.3036249

Li, P., Sheng, W., Duan, Q., Li, Z., Zhu, C., and Zhang, X. (2020). A Lyapunov Optimization-Based Energy Management Strategy for Energy Hub with Energy Router. *IEEE Trans. Smart Grid*, 11, 4860–4870. doi:10.1109/tsg.2020.2968747

Liu, C., Li, X. J., Zhi, Y. M., and Cai, G. W. (2018). New Breed of Solid-State Transformer Mainly Combining Hybrid Cascaded Multilevel Converter with Resonant DC-DC Converters. *Appl. Energ.*, 210, 724–736. doi:10.1016/j.apenergy.2017.08.052

Meng, F., Chowdhury, B., and Chamanamcha, M. (2018). Three-phase Optimal Power Flow for Market-Based Control and Optimization of Distributed Generations. *IEEE Trans. Smart Grid*, 9, 3691–3700. doi:10.1109/tsg.2016.2638963

Miao, J. Q., Zhang, N., and Kang, C. Q. (2016). “Generalized Steady-State Model for Energy Router with Applications in Power Flow Calculation,” in IEEE Power and Energy Society General Meeting, Boston, MA, USA, July 17-21, 2016, 1–5.

Miao, J., Zhang, N., Kang, C., Wang, J., Wang, Y., and Xia, Q. (2018). Steady-State Power Flow Model of Energy Router Embedded AC Network and its Application in Optimizing Power System Operation. *IEEE Trans. Smart Grid*, 9, 4828–4837. doi:10.1109/tsg.2016.2672821

Rouzbahri, K., Miranian, A., Candela, J. L., Luna, A., and Rodriguez, P. (2015). A Generalized Voltage Droop Strategy for Control of Multiterminal DC Grids. *IEEE Trans. Ind. Applicat.*, 51, 607–618. doi:10.1109/tia.2014.2332814

Soofi, A. F., Manshadi, S. D., Liu, G., and Dai, R. (2021). A SOCP Relaxation for Cycle Constraints in the Optimal Power Flow Problem. *IEEE Trans. Smart Grid*, 12, 1663–1673. doi:10.1109/tsg.2020.3023890

Sun, Y., Hu, L., Malik, S. M., Chen, N., and Pu, T. (2019). Unified Method for Optimal Power Flow of AC/DC System with PET Based on Extended branch Model. *J. Eng.*, 2019, 1942–1946. doi:10.1049/joe.2018.8707

Tang, K., Dong, S., Ma, X., Lv, L., and Song, Y. (2021). Chance-constrained Optimal Power Flow of Integrated Transmission and Distribution Networks with Limited Information Interaction. *IEEE Trans. Smart Grid*, 12, 821–833. doi:10.1109/tsg.2020.3021829

Yu, M., Huang, W., Tai, N., Zheng, X., Wu, P., and Chen, W. (2018). Transient Stability Mechanism of Grid-Connected Inverter-Interfaced Distributed Generators Using Droop Control Strategy. *Appl. Energ.*, 210, 737–747. doi:10.1016/j.apenergy.2017.08.014

Zhang, H., Moura, S. J., Hu, Z., Qi, W., and Song, Y. (2018). A Second-Order Cone Programming Model for Planning PEV Fast-Charging Stations. *IEEE Trans. Power Syst.*, 33, 2763–2777. doi:10.1109/tpwrs.2017.2754940

Zhang, S., Pei, W., Xiao, H., Yang, Y., Ye, H., and Kong, L. (2020). Enhancing the Survival Time of Multiple Islanding Microgrids through Composable Modular Energy Router after Natural Disasters. *Appl. Energ.*, 270, 115138. doi:10.1016/j.apenergy.2020.115138

Zhang, Y., Meng, X., Shotorbani, A. M., and Wang, L. (2021). Minimization of AC-DC Grid Transmission Loss and DC Voltage Deviation Using Adaptive Droop Control and Improved AC-DC Power Flow Algorithm. *IEEE Trans. Power Syst.*, 36, 744–756. doi:10.1109/tpwrs.2020.3020039

Zhu, J. Z. (2002). Optimal Reconfiguration of Electrical Distribution Network Using the Refined Genetic Algorithm. *Electric Power Syst. Res.*, 62, 37–42. doi:10.1016/s0378-7796(02)00041-x

**AUTHOR CONTRIBUTIONS**

TZ: conceptualization, writing-review and editing. YM: formal analysis, writing-review and editing. YS: data curation, and software. KY: data curation, and review. HJ: methodology, software, and supervision.

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NOMENCLATURE

Sets
Ψ_{AC} Set of the branches in AC network
Ψ_{DC} Set of the branches in DC network
ϕ Three-phase system ϕ ∈ {a, b, c}

Variables
P^{\phi, i}_{PET, ac} Active power injected into the AC port of ER
Q^{\phi, i}_{PET, ac} Reactive power injected into the AC port of ER
P^{\phi, j}_{PET, dc} Active power injected into the DC port of ER
Q^{\phi, j}_{PET, dc} Reactive power injected into the DC port of ER
P^{g, i}_{AC} Active power flow of branch in AC network
Q^{g, i}_{AC} Reactive power flow of branch in AC network
P^{g, j}_{AC} Active power output of DG in AC network
Q^{g, j}_{AC} Reactive power output of DG in AC network
\hat{V}^{\phi, i}_{AC} The square of the voltage in AC network
\hat{I}^{\phi, i}_{AC} The square of the current in AC network
\hat{V}^{\phi, j}_{DC} The square of the voltage in DC network
\hat{I}^{\phi, j}_{DC} The square of the current in DC network
\omega^f The frequency in low voltage AC network
\delta V Voltage bias of the secondary control
\delta \omega Frequency bias of the secondary control

Parameters
a, b, c The fitting coefficient of ER loss model
ζ, ξ, ζ The droop coefficient of the controller
r^{\phi, i}_{AC} Resistance of branch in AC network
x^{\phi, i}_{AC} Reactance of branch in AC network
r^{\phi, j}_{AC} Resistance of branch in DC network