Giant Resonances in $^{40}\text{Ca}$ and $^{48}\text{Ca}$

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Abstract. We present results of fully self-consistent Hartree-Fock based random phase approximation calculations of the centroid energies $E_{\text{CEN}}$ of isoscalar ($T = 0$) and isovector ($T = 1$) giant resonances of multipolarities $L = 0\text{-}3$ in $^{40}\text{Ca}$ and $^{48}\text{Ca}$, using a wide range of commonly employed Skyrme type nucleon-nucleon effective interactions. We determined the sensitivities of $E_{\text{CEN}}$ and of the isotopic differences $E_{\text{CEN}}(^{48}\text{Ca}) - E_{\text{CEN}}(^{40}\text{Ca})$ to physical quantities, such as nuclear matter incompressibility coefficient, symmetry energy density and effective mass, associated with the Skyrme interactions. We compare the results with the available experimental data, in particular, with the recent data indicating that the ISGMR centroid energy in $^{48}\text{Ca}$ is somewhat higher than in $^{40}\text{Ca}$.

1. Introduction

The study of collective modes in nuclei has been the subject of extensive theoretical and experimental studies during several decades [1-3], since it contributes significantly to our understanding of bulk properties of nuclei, their non-equilibrium properties and properties of the nuclear force. Of particular interest is the equation of state (EOS), i.e. the binding energy per nucleon as a function of the neutrons and protons densities, of infinite nuclear matter (no Coulomb interaction). The EOS is an important ingredient in the study of properties of nuclei at and away from stability, structure and evolution of compact astrophysical objects, such as neutron stars and core-collapse supernovae, and of heavy-ion collisions (HIC) [4,5]. The saturation point of the equation of state (EOS) for the symmetric nuclear matter (NM), equal numbers of neutrons (N) and protons (Z), is well determined from the measured binding energies and central matter densities of nuclei, by extrapolation to infinite NM [1,2]. To extend our knowledge of the EOS beyond the saturation point of the symmetric NM (SNM), an accurate value of the NM incompressibility coefficient $K_{\text{NM}}$, which is directly related to the curvature of the EOS of SNM, is needed. An accurate knowledge of the density dependence of the symmetry energy, $E_{\text{sym}}$, is needed for the EOS of asymmetric NM.

There have been many attempts over the years to determine $K_{\text{NM}}$ and $E_{\text{sym}}(\rho)$ by considering physical quantities which are sensitive to the values of $K_{\text{NM}}$ and $E_{\text{sym}}(\rho)$ [3,4,6,7]. Here we consider the sensitivity of the centroid energies of the isoscalar and isovector giant resonances with multipolarities $L = 0\text{-}3$ of the isotopes $^{40}\text{Ca}$ and $^{48}\text{Ca}$ to bulk properties of NM, such as $K_{\text{NM}}$, $E_{\text{sym}}$ and the effective mass $m^*$. It is well known that the energies of the compression modes, the isoscalar giant monopole resonance (ISGMR) and isoscalar giant dipole resonance (ISGDR), are very sensitive to the value of $K_{\text{NM}}$ [1,8,9]. Also the energies of the isovector giant resonances, in particular, the isovector giant dipole resonance (IVGDR), are sensitive to the density dependence of $E_{\text{sym}}$ [10,11]. Furthermore, information on the density dependence of $E_{\text{sym}}$ can also be obtained by studying the isotopic dependence of strength functions, such as the difference between the strength functions of $^{40}\text{Ca}$ and $^{48}\text{Ca}$.
$^{48}\text{Ca}$ and between $^{112}\text{Sn}$ and $^{124}\text{Sn}$. We note that the value of the neutron-proton asymmetry parameter $\delta = (N-Z)/A$ increases from $^{40}\text{Ca}$ to $^{48}\text{Ca}$ by a value of 0.167 which is significantly larger than the change of 0.087 between $^{112}\text{Sn}$ and $^{124}\text{Sn}$.

In the vicinity of the saturation density $\rho_0$ of symmetric NM, the EOS can be approximated by

$$E_0[\rho] = E[\rho_0] + \frac{1}{10} K_{NM} \left( \frac{\rho_0 - \rho}{\rho_0} \right)^2,$$

where $E_0[\rho]$ is the binding energy per nucleon and $K_{NM}$ is the incompressibility coefficient which is directly related to the curvature of the EOS, $K_{NM} = 9\rho_0^2 \frac{\partial^2 E_0}{\partial \rho^2} |_{\rho_0}$. Similarly, the EOS of asymmetric NM, with proton density $\rho_p$ and neutron density $\rho_n$, can be approximated by

$$E[\rho_p, \rho_n] = E_0[\rho] + E_{sym}[\rho] \left( \frac{\rho_0 - \rho_p}{\rho_0} \right)^2,$$

where $E_{sym}[\rho]$ is the symmetry energy at matter density $\rho$, approximated as

$$E_{sym}[\rho] = J + \frac{1}{3} L \left( \frac{\rho_0 - \rho}{\rho_0} \right) + \frac{1}{10} K_{sym} \left( \frac{\rho_0 - \rho}{\rho_0} \right)^2,$$

where $J = E_{sym}[\rho_0]$ is the symmetry energy at saturation density $\rho_0$, $L = 3\rho_0 \frac{\partial E_{sym}}{\partial \rho} |_{\rho_0}$, and $K_{sym} = 9\rho_0 \frac{\partial^2 E_{sym}}{\partial \rho^2} |_{\rho_0}$.

Very recently the giant resonance region from $9.5 \text{ MeV} < E_x < 40 \text{ MeV}$ in $^{48}\text{Ca}$ has been studied with inelastic scattering of 240 MeV $\alpha$ particles at small angles, including $0^\circ$. Close to 100% of the ISGMR ($E_0$), ISGDR ($E_1$) and isoscalar giant quadrupole resonance ($E_2$) strengths have been located between 9.5 and 40 MeV in $^{48}\text{Ca}$ [12]. A comparison with the available data for $^{40}\text{Ca}$ [13-15] was carried out in Ref. [12]. The ISGMR has been found, surprisingly, at somewhat higher energy in $^{48}\text{Ca}$ than in $^{40}\text{Ca}$. This led us to carry out extensive fully self-consistent calculations within the Hartree-Fock (HF) based random-phase-approximation (RPA) of the centroid energies of isoscalar and isovector giant resonances in $^{40}\text{Ca}$ and $^{48}\text{Ca}$, using 16 commonly employed Skyrme type effective nucleon-nucleon interactions.

In the next section we review the basic elements of the self-consistent Hartree-Fock (HF)-based random-phase approximation (RPA) theory for the strength functions of isoscalar ($T = 0$) and isovector ($T = 1$) giant resonances. Next, we present results of our calculations [16] for the strength functions S(E) and centroid energies $E_{CEN}$ obtained for giant resonances of $T = 0$, $I$ and multipolarities $L = 0$ - 3 in $^{40}\text{Ca}$ and $^{48}\text{Ca}$, using a wide range of Skyrme type nucleon-nucleon effective interactions. We consider, in particular the issue of self-consistency and investigate the sensitivities of $E_{CEN}$ and of the isotopic differences $E_{CEN}^{(48}\text{Ca}) - E_{CEN}^{(40}\text{Ca})$ to physical quantities, such as nuclear matter incompressibility coefficient, symmetry energy density and effective mass, associated with the effective nucleon-nucleon interactions and compare the results with available experimental data. In the last section, we discuss our results and present our conclusions.

2. Theoretical Approach

In the numerical calculations of the properties of giant resonances in nuclei within the HF-based RPA theory, one starts by adopting an effective nucleon-nucleon interaction $V_{12}$, such as the Skyrme interaction, with parameters determined by a fit [17] of the HF predictions to experimental data on ground state properties, such as binding energies and radii, of a selected set of a wide range of nuclei. Then, the RPA equations are solved using the particle-hole (p-h) interaction deduced from $V_{12}$, by employing a certain numerical method [18,19,20], and the physical quantities of interest, such as the
strength functions $S(E)$ and transition densities, are calculated. We point out that in fully self-consistent RPA calculations, one should employ all the components of the p-h interactions obtained from the $V_{12}$ used in the HF calculations \[9,21,22\]. We have adopted the following form for the Skyrme interaction \[23\]:

$$V_{12} = t_0 (1 + x_0 P_{12}^0) \delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{2} t_0 (1 + x_1 P_{12}^0) \left[ \vec{k}_{12}^2 \delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2) \vec{k}_{12}^2 \right] + t_2 (1 + x_2 P_{12}^0) \vec{k}_{12}^2 \delta(\vec{r}_1 - \vec{r}_2) \vec{k}_{12} + \frac{1}{6} t_3 (1 + x_3 P_{12}^0) \rho^a \left( \frac{\vec{r}_1 + \vec{r}_2}{2} \right) \delta(\vec{r}_1 - \vec{r}_2) + \frac{i W_0^\ast \vec{k}_{12}}{2} \delta(\vec{r}_1 - \vec{r}_2) \left( \vec{\sigma}_1 + \vec{\sigma}_2 \right) \times \vec{k}_{12} \right),$$

where $t_i$, $x_i$, $\alpha$, and $W_0$ are the parameters of the interaction and $P_{12}^0$ is the spin exchange operator, $\vec{\sigma}_i$ is the Pauli spin operator, $\vec{k}_{12} = -i(\vec{r}_1 - \vec{r}_2)/2$, and $\vec{k}_{12} = -i(\vec{r}_1 - \vec{r}_2)/2$. Here, the right and left arrows indicate that the momentum operators act on the right and on the left, respectively.

In this work we employed the fully self-consistent method based on the Q-P representation described in detail in Refs. \[19,24\] and calculate the strength function

$$S(E) = \sum_j |\langle 0 | F_{Lj} | j \rangle|^2 \delta(E_j - E_0).$$

Here, $|0\rangle$ is the RPA ground state and the sum is over all RPA excited states $|j\rangle$ with the corresponding excitation energies $E_j$. We adopt the scattering operator

$$F_L = \sum_i f(r_i) Y_{L0}(i)$$

for isoscalar ($T = 0$) excitations and

$$F_L = \frac{2}{A} \sum_n f(\eta_n) Y_{L0}(n) - \frac{N}{A} \sum_p f(\eta_p) Y_{L0}(p)$$

for isovector ($T = 1$) excitations. In Eqs. (6) and (7) we use the operator $f(r) = r$, for the isovector dipole ($T = 1$, $L = 1$) and $f(r) = r^3 - (5/3)r^2$, for the isoscalar dipole ($T = 0$, $L = 1$), to eliminate possible contribution of the spurious state mixing \[9,21,22\]. For the isoscalar and isovector monopole ($L = 0$), quadrupole ($L = 2$) and octopole ($L = 3$) excitations we use the operators $f(r) = r^2$, $r^2$, and $r^3$, respectively. We then determine the energy moments of the strength function,

$$m_k = \int_0^\infty E^k S(E) dE \right)$$

The centroid energy, $E_{CEN}$, is then obtained from

$$E_{CEN} = \frac{m_1}{m_0}. \right)$$

We have carried out fully self-consistent HF-based RPA calculations of the isoscalar giant monopole resonance (ISGMR), dipole (ISGDR), quadrupole (ISGQR), and the octopole (ISGOR) strength functions, adopting the scattering operator of Eq. (6), and for the isovector giant monopole resonance (IVGMR), dipole (IVGDR), quadrupole (IVGQR) and octopole (IVGOR) strength functions, adopting the scattering operator of Eq. (7), for $^{40}$Ca and for $^{48}$Ca, using a wide range of Skyrme interactions. In the next section we present the results of our calculations and compare with available experimental data.
3. Results

We now present results of our [16] fully self-consistent HF based RPA calculations of the centroid energies for the isoscalar and isovector giant resonances of multipolarities \( L = 0 - 3 \) in \(^{40}\)Ca and \(^{48}\)Ca obtained for the Skyrme interactions: SkP [25], SGII [26], SKM* [27], SV-min [28], SkO’ [29], SkMP [30], SLy6 [31], SLy4 [31], SLy5 [31], KDE0 [17], KDE0v1 [17], SV-bas [28], SKi4 [32], SKi3 [32], SKi5 [32], and SK255 [33]. For these interactions we show in Table 1 the associated properties of symmetric nuclear matter: total binding energy per nucleon \( E/A \) [MeV], \( \rho_0 \) [fm\(^{-3}\)], isoscalar effective mass \( m*/m \), \( K_{NM} \) [MeV], \( J \) [MeV], \( L \) [MeV], \( K_{sym} \) [MeV], and the IVGDR enhancement factor \( \kappa \) of the energy weighted sum rule (EWSR).

Table 1. Properties of NM are presented for the Skyrme interactions used in the calculations.

| SkP   | 15.93 | 0.162 | 1.00 | 200.8 | 32.98 | 45.21 | -266.60 | 0.30 |
|-------|-------|-------|------|-------|-------|-------|---------|-----|
| SGII  | 15.59 | 0.159 | 0.79 | 215.0 | 26.80 | 37.63 | -145.90 | 0.49 |
| SKM*  | 15.78 | 0.160 | 0.79 | 216.7 | 30.03 | 45.78 | -155.94 | 0.53 |
| SV-min| 15.91 | 0.161 | 0.95 | 222.0 | 30.01 | 44.76 | -156.57 | 0.08 |
| SkO’  | 15.75 | 0.160 | 0.90 | 222.3 | 31.95 | 68.93 | -78.82  | 0.15 |
| SkMP  | 15.56 | 0.157 | 0.65 | 230.9 | 29.88 | 70.31 | -49.82  | 0.71 |
| SLy6  | 15.92 | 0.159 | 0.69 | 229.8 | 31.96 | 47.44 | -112.71 | 0.25 |
| SLy4  | 15.97 | 0.160 | 0.70 | 229.9 | 32.00 | 45.96 | -119.73 | 0.25 |
| SLy5  | 15.98 | 0.160 | 0.70 | 229.9 | 32.03 | 48.27 | -112.76 | 0.25 |
| KDE0  | 16.11 | 0.161 | 0.72 | 228.8 | 33.00 | 45.22 | -144.78 | 0.30 |
| KDE0v1| 16.23 | 0.165 | 0.74 | 227.5 | 34.58 | 54.70 | -127.12 | 0.23 |
| SV-bas| 15.90 | 0.160 | 0.90 | 234.0 | 30.00 | 45.21 | -221.75 | 0.40 |
| SKi4  | 15.92 | 0.160 | 0.65 | 247.9 | 29.50 | 60.39 | -40.56  | 0.25 |
| SKi3  | 15.96 | 0.158 | 0.58 | 258.1 | 34.80 | 100.52| 73.04   | 0.25 |
| SKi5  | 15.83 | 0.156 | 0.58 | 255.7 | 36.70 | 129.33| 159.57  | 0.25 |
| SK255 | 16.33 | 0.157 | 0.80 | 255.0 | 37.40 | 95.00 | -58.33  | 0.54 |

The selected interactions have the following ranges of NM properties: \( E/A = 15.56 – 16.33 \) MeV, \( \rho_0 = 0.156 – 0.165 \) fm\(^{-3}\), \( K_{NM} = 201 – 258 \) MeV, \( J = 26.80 – 37.40 \) MeV, \( L = 37 – 129 \) MeV, \( K_{sym} = -267 +160 \) MeV, \( m*/m = 0.58 – 1.00\), \( \kappa = 0.08 – 0.71\). For the calculations of the centroid energies, we used the appropriate excitation energy ranges: ISGMR, 0 – 60 MeV, ISGDR, 20 – 60 MeV, ISGQR, 9 – 60 MeV, ISGOR, 20 – 60 MeV, IVGMR, 0 – 60 MeV, IVGDR, 0 – 60 MeV, IVGQR, 9 – 60 MeV, and IVGOR, 25 – 60 MeV.

In Figure 1 (on the left) we compare the experimental data of the ISGMR centroid energies of \(^{40}\)Ca (a), \(^{48}\)Ca (b), and the energy difference between \(^{48}\)Ca and \(^{40}\)Ca (c), shown as the regions between the dashed lines, with the results of fully self consistent HF based RPA calculations (full circles), using the Skyrme interactions of Table 1. The results obtained with violation of self-consistency, by the neglecting the Coulomb and the spin orbit particle-hole interactions in the RPA calculations, are shown in (d). The HF-RPA results are plotted as a function of \( K_{NM} \). The energies shown were obtained using the experimental excitation energy range of 0 – 60 MeV. A strong correlation between \( E_{CEN} \) of \(^{40}\)Ca and \( E_{CEN} \) of \(^{48}\)Ca can be seen with \( K_{NM} \). The ISGMR centroid energies for \(^{48}\)Ca are all higher than the experimental value 19.18 +/- 0.37 MeV. The \(^{48}\)Ca ISGMR centroid energies are more consistent with the experimental value 19.88 +/- 0.16 MeV. We find that for all Skyrme interactions used in our theoretical calculations [16] the ISGMR centroid energy in \(^{48}\)Ca is lower than in \(^{40}\)Ca, in disagreement with experimental data. In Figure 1 we also show (on the right)
Figure 1. Comparison of experimental data (region between the dashed lines) with HF-RPA results in $^{40}$Ca and $^{48}$Ca for the centroid energies of the ISGMR (left) and ISGDR (right), plotted as a function of $K_{NM}$.

the results for the ISGDR. The HF-RPA energies for the ISGDR are higher than the experimental values by 3 – 6 MeV. We also note that a correlation is found between the ISGDR energy of $^{40}$Ca and $^{48}$Ca with both $K_{NM}$ and $m/m^*$ [16].

In Figure 2 we compare our [16] HF based RPA results (full circles) of the ISGQR (left) and ISGOR (right) centroid energies $E_{\text{CEN}}$, of $^{40}$Ca (a), $^{48}$Ca (b), and the energy difference between $^{48}$Ca and $^{40}$Ca (c) using various Skyrme type interactions having effective mass $m/m^* = 1.00 – 1.73$. The energies shown were calculated for the ISGQR over the excitation energy range of 9 – 60 MeV. A strong correlation exists between the ISGQR and ISGOR of $^{40}$Ca and $^{48}$Ca with $m/m^*$ as can be seen in Figure 2, with interactions having $m^*/m = 0.65 – 0.8$ reproducing the experimental data of the ISGQR.

To investigate the dependence of the energy difference between the ISGMR in $^{40}$Ca and $^{48}$Ca on the symmetry energy density, we show in Figure 3 our results [16] of fully self-consistent HF based RPA calculations (full circles), using the Skyrme interactions having nuclear matter symmetry energy coefficient $J = 26.80 – 36.70$ MeV on the left and having the value of $L = 37.63 – 129.33$ MeV on the right. The energies shown were calculated over the experimental excitation energy range of 0 – 60 MeV. An agreement with experimental data is obtained for several interactions. A correlation can be seen between $E_{\text{CEN}}$ of $^{40}$Ca and $E_{\text{CEN}}$ of $^{48}$Ca and $L$. However, only a very weak correlation is found between $\Delta E_{\text{CEN}}$ and $J$ or $L$ [16].
Figure 2. Comparison of experimental data (region between the dashed lines) with HF-RPA results in $^{40}\text{Ca}$ and $^{48}\text{Ca}$ for the centroid energies of the ISGQR (left) and ISGOR (right), plotted as a function of $m/m^*$. 

Figure 3. Comparison of experimental data (region between the dashed lines) with HF-RPA results in $^{40}\text{Ca}$ and $^{48}\text{Ca}$ for the centroid energies of the ISGMR plotted as a function of $J$ (left) and $L$ (right).
Figure 4. Comparison of experimental data (region between the dashed lines) with HF-RPA results in $^{40}\text{Ca}$ and $^{48}\text{Ca}$ for the centroid energies of the IVGDR as a function of $J$ (left) and $L$ (right).

In Figure 4 we present a comparison of our [16] HF based RPA results (full circles) of the IVGDR centroid energies $E_{\text{CEN}}$ of $^{40}\text{Ca}$ (a), $^{48}\text{Ca}$ (b), and the energy difference between $^{48}\text{Ca}$ and $^{40}\text{Ca}$ (c) using various Skyrme type interactions having the value $J = 26.80 - 36.70$ MeV on the left and having the values of $L = 37.63 - 129.33$ MeV on the right. The energies shown were calculated over the experimental excitation energy range of $0 - 60$ MeV. A weak correlation between $E_{\text{CEN}}$ of $^{40}\text{Ca}$ and $E_{\text{CEN}}$ of $^{48}\text{Ca}$ can be seen with $J$ or $L$ [16].

To investigate further the correlations between the centroid energies of the giant resonances and NM properties associated with the Skyrme interactions, we show in Tables 2 and 3 the Pearson correlation coefficients among the various NM properties and spin-orbit strength $W_0$ with the calculated [16] $E_{\text{CEN}}$ of the isoscalar and isovector giant resonances, respectively. As shown in Table 3, only weak correlations exist between the $E_{\text{CEN}}$ of the IVGDR of $^{40}\text{Ca}$ or $^{48}\text{Ca}$ with $J$, $L$ and $K_{\text{sym}}$, and a medium correlation is found between the difference of the $E_{\text{CEN}}$ of the $^{48}\text{Ca}$ and $^{40}\text{Ca}$ IVGMRs, IVDGRs, and IVGQRs and $W_0$.

4. Conclusions
We have presented results of our fully self-consistent HF-RPA calculations [16], using 16 commonly employed Skyrme type interactions, for the centroid energies of isoscalar and isovector giant resonances of multipolarities $L = 0 - 3$ in $^{40}\text{Ca}$ and $^{48}\text{Ca}$ and compared with available experimental data. In particular we have discussed the sensitivity of the $E_{\text{CEN}}$ of the giant resonances to various properties of NM.

- We point out that for all the Skyrme interactions used in our calculation, the centroid energies of the ISGMR in $^{48}\text{Ca}$ is lower than that in $^{40}\text{Ca}$ in disagreement with experimental data.
- The calculated centroid energies of the ISGDR in $^{40}\text{Ca}$ and $^{48}\text{Ca}$ are consistently higher than the experimental data by about $3 - 6$ MeV.
- We point out the weak isotopic dependence of the ISGMR and IVGDR centroid energies on the density dependence of the symmetry energy.
- We also note the weak correlation between the energies of the IVGDR of $^{40}\text{Ca}$ (or $^{48}\text{Ca}$) and the density dependence of the symmetry energy.
- We have demonstrated the strong correlation of the $E_{\text{CEN}}$ of the compression modes with $K_{\text{NM}}$ and the strong correlations of the $E_{\text{CEN}}$ of the ISGQR and the ISGOR with $m/m^*$.

**Table 2.** Pearson correlation coefficients among the various NM properties and spin-orbit strength $W_0$ with the centroid energies of the isoscalar T0 giant resonances of multipolarities $L=0–3$.

| $m^*/m$ | $K_{\text{NM}}$ | $J$ | $L$ | $K_{\text{sym}}$ | $K$ | $W_0$ |
|---------|-----------------|-----|-----|-----------------|-----|-------|
| L0 T0 Ca 40 $E_{\text{CEN}}$ | -0.71 | 0.95 | 0.39 | 0.76 | 0.86 | -0.02 |
| L0 T0 Ca 48 $E_{\text{CEN}}$ | -0.85 | 0.85 | 0.29 | 0.73 | 0.87 | 0.16 |
| L0 T0 $\Delta E_{\text{CEN}}$ | -0.59 | 0.13 | -0.07 | 0.21 | 0.36 | 0.41 |
| L1 T0 Ca 40 $E_{\text{CEN}}$ | -0.86 | 0.88 | 0.35 | 0.77 | 0.92 | 0.05 |
| L1 T0 Ca 48 $E_{\text{CEN}}$ | -0.94 | 0.82 | 0.25 | 0.58 | 0.82 | 0.03 |
| L1 T0 $\Delta E_{\text{CEN}}$ | -0.71 | 0.32 | -0.07 | -0.10 | 0.24 | -0.02 |
| L2 T0 Ca 40 $E_{\text{CEN}}$ | -0.96 | 0.76 | 0.29 | 0.61 | 0.85 | -0.04 |
| L2 T0 Ca 48 $E_{\text{CEN}}$ | -0.98 | 0.68 | 0.25 | 0.54 | 0.80 | 0.03 |
| L2 T0 $\Delta E_{\text{CEN}}$ | -0.56 | -0.04 | -0.10 | -0.03 | 0.18 | 0.32 |
| L3 T0 Ca 40 $E_{\text{CEN}}$ | -0.96 | 0.77 | 0.30 | 0.58 | 0.83 | -0.03 |
| L3 T0 Ca 48 $E_{\text{CEN}}$ | -0.98 | 0.68 | 0.20 | 0.51 | 0.79 | 0.06 |
| L3 T0 $\Delta E_{\text{CEN}}$ | -0.25 | -0.17 | -0.32 | -0.13 | 0.01 | 0.35 |

**Table 3.** Pearson correlation coefficients among the various NM properties and spin-orbit strength $W_0$ with the centroid energies of the isovector T1 giant resonances of multipolarity $L=0–3$.

| $m^*/m$ | $K_{\text{NM}}$ | $J$ | $L$ | $K_{\text{sym}}$ | $K$ | $W_0$ |
|---------|-----------------|-----|-----|-----------------|-----|-------|
| L0 T1 Ca 40 $E_{\text{CEN}}$ | -0.43 | 0.69 | 0.09 | 0.48 | 0.48 | 0.43 |
| L0 T1 Ca 48 $E_{\text{CEN}}$ | -0.56 | 0.58 | -0.03 | 0.46 | 0.52 | 0.63 |
| L0 T1 $\Delta E_{\text{CEN}}$ | -0.36 | -0.06 | -0.21 | 0.09 | 0.20 | 0.51 |
| L1 T1 Ca 40 $E_{\text{CEN}}$ | -0.03 | 0.08 | -0.27 | -0.30 | -0.32 | 0.58 |
| L1 T1 Ca 48 $E_{\text{CEN}}$ | -0.17 | 0.07 | -0.32 | -0.24 | -0.21 | 0.70 |
| L1 T1 $\Delta E_{\text{CEN}}$ | -0.42 | 0.00 | -0.20 | 0.11 | 0.24 | 0.47 |
| L2 T1 Ca 40 $E_{\text{CEN}}$ | -0.68 | 0.51 | -0.01 | 0.20 | 0.33 | 0.60 |
| L2 T1 Ca 48 $E_{\text{CEN}}$ | -0.70 | 0.39 | -0.14 | 0.14 | 0.31 | 0.64 |
| L2 T1 $\Delta E_{\text{CEN}}$ | -0.47 | -0.03 | -0.37 | -0.06 | 0.14 | 0.47 |
| L3 T1 Ca 40 $E_{\text{CEN}}$ | -0.71 | 0.58 | 0.13 | 0.38 | 0.53 | 0.40 |
| L3 T1 Ca 48 $E_{\text{CEN}}$ | -0.73 | 0.53 | 0.01 | 0.26 | 0.43 | 0.48 |
| L3 T1 $\Delta E_{\text{CEN}}$ | 0.30 | -0.38 | -0.33 | -0.45 | -0.45 | -0.02 |
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