Flow resistance over submerged vegetation in ecological channel

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Abstract. A discussion of flow resistance in ecological channel is a fundamental problem required by hydraulic engineers, but the well-established flow resistance mechanism is still not well discovered. This research studies on flow over submerged vegetation in ecological channel to explain mechanism of flow resistance on rough surfaces. With a detailed discussion, this research separates total resistance and unlike previous research, this study pays attention to the variation in vortices created by roughness, and new definitions of hydraulic radius and Reynolds Number are given. This study validated the proposed method can be used to simplify the friction factor calculations for hydraulic engineers.

1. Introduction

Artificial open channel are the major conveyance systems to deliver water into irrigation area. To improve the biological environment of traditional channels, some longitudinal vegetation is arranged into channel to create a proper ecological space, maintaining the biological diversity and enhance the ecological function of the irrigation channel [1, 2]. However, vegetation affect river hydraulic and its flow resistance plays an important role in an ecological channel. Flow over vegetation increases the flow resistance and further decrease the flow deliver capacity. Thus, it is important to determine the flow resistance in a channel partially covered with vegetation.

Flow over a channel partially covered with vegetation can be generalised as flow over roughness surfaces, and determination of flow resistance over roughness surfaces is a quite old topic [3]. Nikuradse [4] used two parameters to correlate the friction factor, \( \tau \), i.e., Reynolds number (Re = UR / \nu) and relative roughness, \( k_r / R \), in which \( U \) = mean velocity, \( R \) = hydraulic radius, \( \nu \) = kinematic viscosity, and equivalent roughness height \( k_e = d \). Schlichting’s [5] found that equivalent roughness height \( k_e = 0.627d \) when roughness packed tightly. However, Nikuradse’s experimental result of \( k_e = d \) cannot be replicated in many cases, e.g., for sediment transport, \( k_e = 2.5d \) is widely used [6]. In other words, Wooding et al [7] provided detailed experimental data to describe \( k_e \) varies with roughness concentration \( \lambda \), and the concept of \( k_s, D \) (pipe diameter) and \( \delta \) (boundary layer thickness) was introduced by Perry et al [8], who suggested that \( k_s = 0.02\delta \). For the dependence of \( k_s \) on \( \lambda \), many useful investigations have been done [9, 10].

Yang et al [11] proposed the flow resistance partitioning can be developed from the head loss (as shown in figure 1), that is
Figures 1. Combination of skin friction and form drag over a trough between roughness elements.

\[ h_L = h_L' + h_L'' \]  
where \( h_L \) = energy loss over the length of \( L \), \( h_L' \) = energy loss by skin friction over the length of \( L' \), and \( h_L'' \) = energy loss by the large eddy over the length of \( L'' \) (see figure 1). The energy slope can be determined accordingly as

\[ S = \frac{h_L}{L} = S' \frac{L'}{L} + S'' \frac{L'}{L} \]  
where \( S \) = energy slope, \( S' \) = energy slope associated with skin friction, and \( S'' \) = energy slope related with form drag. Alternatively, equation (2) can be rewritten as:

\[ \tau_0 = \tau' \frac{L'}{L} + \tau'' \frac{L'}{L} \]  

In an effort to continue this work, this paper will investigate flow resistance over 3-D roughness distributions by extending equation (3) from the 1-D roughness depicted in figure 1. The research objectives of the paper are tried to interpret and redefine hydraulic radius followed by a new definition, so that the influence of roughness on the friction factor in laminar flows can be described.

2. New definition of hydraulic radius and friction factor in laminar flow
The hydraulic radius is one of the most important parameters in hydraulics and fluid mechanics. The conventional hydraulic radius definition has no physical implications as it only measures the area and the length. The hydraulic radius can be redefined as follows:

\[ R = \frac{V}{A} \]  
where \( V \) = fluid volume and \( A \) = the wetted boundary area. The ratio of \( V/A \) represents the potential energy contained in \( V \) dissipated as heat at boundary \( A \). This can be demonstrated in a pipe flow, where the fluid volume in the shaded region is \( V \), and the momentum balance equation can be written as follows:

\[ \rho g VS = \tau_0 A \]  
where \( \rho \) = fluid density and \( g \) = gravitational acceleration. Equation (5) shows that \( \tau_0 = \rho g RS \), or \( R = V/A \). It is obvious that for a pipe flow, equation (5) generates the same hydraulic radius as the conventional definition, but equation (4) implies that \( R \) is a measure of skin friction that depends on the area of solid-liquid interface area \( A \).
Flow over rigid vegetation without sidewall effects presented in figure 2, modelled as submerged cylinders. The control volume can be calculated as $B \times L \times h$, and distance between submerged cylinders is $\Delta x$. If the number of vegetation in a flow control volume can be considered as $N$, the vegetation density ($\lambda$) per unit area could be:

$$\lambda = \left( \frac{\pi D^2 N}{4 BL} \right)$$

where $D$ denote the cylinder diameter of rigid vegetation. $\lambda$ can be defined in a similar manner for other shapes of roughness elements.

For strips and cubes:

$$\lambda = \left( \frac{L_1^2 N}{BL} \right)$$

where $L_1$ is the cross sectional length/width of the strip/square.

For 2D dunes:

$$\lambda = \left( \frac{NL_2 b}{BL} \right)$$

where $L_2$ = streamwise length of the dune, $b$= width of the dune.

3. Determination roughness concentration reynolds number in laminar flow

This new definition of the hydraulic radius is the same as the conventional hydraulic radius of open-channel flows without vegetation. Cheng [12] and Cheng et al [13] calculate flow resistance considered vegetation density and calculated the ratio of the cross-section area to the perimeter. Therefore, considering the aquatic plant as a cylinder, when interpreting the bed area for the form drag force, it is not consider the full contact area. Hence then the roughness concentration-related hydraulic radius for a cylindrical stem reasonably can be defined as:
\[ r' = \frac{\rho g V_{water} S}{A_{water}} = \frac{(1 - \lambda) BhL}{(1 - \lambda) BL + NhD} = \frac{(1 - \lambda) h}{1 + \lambda \left( \frac{4\lambda}{\pi D} h_v - 1 \right)} \]  

(9)

where \( V_{\text{water}} \) is potential energy volume in channel and \( A_{\text{water}} \) is potential energy dissipated on the area, \( h_v \) is the height of vegetation. It should consider the “\( r' \)” is the concentration length.

With the Roughness Concentration related hydraulic radius, the roughness concentration Reynolds number can be defined as follows.

\[ \text{Re}_r = \frac{U'_r r'}{\nu} \]  

(10)

where we name \( U'_r = g r' S \) as the “roughness concentration related friction velocity.

In comparison with the previous studies, the use of \( r' \) in Reynolds number is novel. As shown subsequently, \( r' \) performs much better than other length scales, such as \( h, D \) and \( L \) for collapsing total friction factor data from disparate sources.

4. Co-existence of form drag and skin friction for a flow with submerged vegetation simulated as cylinders

Figure 3 shows a two dimensional bed form, in which \( L \) = length between two adjacent vegetation zone; \( h_v \) = vegetation height; \( h \) = flow depth; \( L' \) = length of flow separation zone caused by vegetation; and \( L'' \) = skin friction length. The length \( L' \) is the resistance produced by the boundary surface to overcome the no slip condition which occurs as a result of the solid-water interface. Thus, the total energy loss over vegetation:

\[ F = F' \frac{L'}{L} + F'' \frac{L''}{L} \]  

(11)

Figure 3. Flow separation behind submerged vegetation, simulated as cylinders.

Equation (11) is very similar to the fundamental equation which was developed for the mathematical model for pipe friction equation (1). So that now we have confirmed the basis for the open channel flow with discrete roughness, it is quite admissible that, if used with the correct variables, equation (11) should work fine in open channel flows. After making a qualitative connection between the pipe friction and open channel friction, it was determined in the previous section that, the pipe friction analytical model developed by Yang et al [14] can be extended for channel friction with discrete roughness.
Channels with bluff bodies usually undergo a very high turbulent condition. Therefore it is assumed that, the following equation developed in pipe friction in the fully developed turbulent region will be the stepping stone for further study regarding channel friction. This has been derived based on equation (11):

\[ f = f' + \frac{L''}{L} (f'' - f') \]  

(12)

where \( f = 8 \left( \frac{U_*}{V} \right)^2 \) = total friction factor, in which \( U_* \) = concentration relation friction velocity and \( V \) = average velocity of the channel. \( f'' \) = friction factor in the \( L'' \), \( f' \) = friction factor in the \( L' \). To make this an independent equation from the previous chapter, we have replaced the dimensionless eddy length \( \frac{L''}{L} \) with the dimensionless eddy volume \( \frac{V_{\text{eddy}}}{V_{\text{void}}} \) by simply multiplying by the flow area \( A \). This generates the following equation

\[ f = f' + \alpha (f'' - f') \]  

(13)

where \( \alpha = \frac{V_{\text{eddy}}}{V_{\text{void}}} \), \( V_{\text{eddy}} \) denote the eddy volume, and \( V_{\text{void}} \) denote the fluid volume between adjacent roughness.

5. Discussion

![Figure 4. Variation of friction factor in turbulent smooth flow (with Nikuradse’s database).](image)

Determination of the friction factor \( f \) in turbulent smooth flow is the first step, and it is quite important to find out the correct formulation of \( f' \) and \( f'' \). In order to account for the correct formulation of \( f' \), we have adopted the method developed by Yang and Dou [15] in pipe flows with the intermittent nature of turbulence. We have slightly modified their equation on turbulent smooth pipe flows with
our concentration related Reynolds number (Re\(\text{c}^\ast\)) to account for the vegetated flows, as follows. 

Figure 4 depicts the perfect agreement of this equation with the data available in literature.

\[
f' = 9.4/(2.5\ln\text{Re}_{\varepsilon}^{\ast} - \frac{87.8}{\text{Re}_{\varepsilon}^{\ast 0.35}} + 1.5)^2 \tag{14}
\]

![Figure 5. Classification of the change of flow patterns with the change of roughness density.](image)

When determining the friction factor in fully rough flow (the form drag friction factor), it is quite important to consider all the factors which would contribute to this friction force. As the relative void volume of the roughness elements, \(V_{\text{void}}/V_{\text{total}}\) is varied, so the nature of the flow at the boundary varies as well. Figure 5 shows different flow patterns with artificial strip roughness.

In smooth turbulent flow, the flow is free from any roughness elements, and the total friction force is skin friction force. When the roughness elements are introduced to the flow domain with a relatively large spacing between each, then the turbulent eddies start to form at the lee side of the bluff bodies gradually. Even though the skin friction force is dominant in the flow domain initially, the form drag friction force starts to take it over gradually. If both of these forces co-exist, flow would be in the transition region between turbulent smooth to turbulent fully rough flow. According to the above classification we name this as the "semi smooth/rough turbulent flow", where is fully functioning in this region. In this region, the total spacing between the roughness elements is not fully occupied by the turbulent eddies, in fact \(\frac{V_{\text{eddy}}}{V_{\text{void}}} = 1\). However when the total friction force is dominated by the form drag friction force, then we enter the region of “hyperl-turbulent flow”. This is the entire spacing between the roughness elements which is occupied by the turbulent eddies forming as a result of the form drag, where \(\frac{V_{\text{eddy}}}{V_{\text{void}}} = 1\). If the spacing between the elements is further reduced (\(\frac{V_{\text{void}}}{V_{\text{total}}}\) is reduced), then the flow domain attains a situation of “quasi smooth flow”. This illustrates how a trapped vortex shelters in the lee of the roughness elements, which we termed as “dead water”. If the entire spacing between adjacent elements is not occupied this trapped vortex, it is called the “semi quasi smooth flow”. The stable circulatory motion of the trapped vortex reduces the turbulent intensity of the eddy volume occurred as a result of the form drag. Higher the volume of dead water, lower the turbulent intensity, resulting in a decrease in the friction factor. The above phenomena will be formulated...
through the “eddy volume concept”.

The formation of eddies behind bluff bodies is mainly influenced by the hydrodynamic parameters prevailing in the flow. Thus, we propose the following relationship:

\[ L^w = C_D \frac{V^2}{2g} \]  \tag{15}

Thus the new model parameter \( \frac{V_{eddy}}{V_{total}} \) can be evaluated as follows.

\[ \frac{V_{eddy}}{V_{total}} = \left( \frac{A_r C_D V^2}{2g} \right) \frac{1}{B \times L \times h} \] \tag{16}

It is assumed that, in vegetated flows the earliest flow region which can be achieved, even with very small vegetation density is the transitional region. As the stems possess a reasonable height, most of the time, eddies start to form behind the stems. Therefore even with a small vegetation density, it is highly unlikely a smooth flow is achieved. In order to outline the behaviour of \( \frac{V_{eddy}}{V_{total}} \) with \( \frac{V_{void}}{V_{total}} \), equation (17) is presented with the experimental data by Knight [16]. Figure 6 can be demonstrated in two parts, first range is \( 0.003 < \frac{V_{void}}{V_{total}} < 0.014 \), the behaviour of \( \frac{V_{eddy}}{V_{total}} \) depended only on \( \frac{V_{void}}{V_{total}} \), thus we call it as the "fully developed region". It was quite clear that despite of the increase in Reynolds Number, \( \frac{V_{eddy}}{V_{total}} \) starts to fall down after the maximum of 0.014 is achieved. In this region, Note that the maximum \( \frac{V_{eddy}}{V_{total}} = 0.014 \) is achieved, when \( \frac{V_{void}}{V_{total}} \) is also equal to 0.014, and \( \frac{V_{eddy}}{V_{void}} = 1 \). The second part is \( 0.014 < \frac{V_{void}}{V_{total}} < 0.065 \), in this region \( \frac{V_{eddy}}{V_{total}} \) is influenced both by the increase in Reynolds Number and increase in vegetation density according to experimental data, and \( \frac{V_{eddy}}{V_{void}} < 1 \). After confirming the behavior \( \frac{V_{eddy}}{V_{total}} \) Vs \( \frac{V_{void}}{V_{total}} \) with experimental data, we now propose the following equation to determine \( f'' \):

\[ f'' = \left( 2 \log \left[ 2.9 \frac{V_{total}}{V_{eddy}} \right] \right)^2 \] \tag{17}

By substituting equations (14) and (17), to equation (13), the final equation for the submerged vegetation flow resistance model is derived as follows:

\[ f = \left[ 9.4 \left( \frac{2.5 \ln Re_*^{'} - \frac{87.8}{Re_*^{'} + 1.5} \right) + \frac{V_{eddy}}{V_{void}} \left( 2 \log \left[ 2.9 \frac{V_{total}}{V_{eddy}} \right] \right)^2 \right] - \left( 9.4 \left( \frac{2.5 \ln Re_*^{'} - \frac{87.8}{Re_*^{'} + 1.5} \right) \right)^2 \] \tag{18}
We checked the accuracy of the developed friction factor model equation (18) in comparison to available models in literature in figure 7, with regards to vegetated flows, and observation of research results developed by Dunn et al [17], Nezu and Sanjou [18], Cheng [19] appear to be less accurate, and its accuracy is close to that of the formula by Stone and Shen [20].
This study demonstrates that the concept of hydraulic radius is useful in unifying experimental data on flow resistance in vegetated open channel flows for various roughness element configurations. By taking the responsibility to eliminate the ambiguity prevailing in submerged vegetated flows in the determination of a proper length scale, it demonstrates the derivation of a new length parameter called “roughness concentration related hydraulic radius”, followed by a new definition for the Reynolds number, as roughness concentration related Reynolds number. These two parameters form the fundamental basis of a theoretical model of flow resistance in submerged vegetation flows. Further, it derives a new formula to determine the friction factor in vegetated flows, using the concept of eddy volume concentration. This model made a useful prediction on flow resistance in an ecological channel.

6. Conclusions
It investigated flow resistance over submerged fixed roughness elements in vegetated flows, and the following conclusions can be summarised:

- The hydraulic radius in vegetated flows denoted the ratio of the water volume to wetted contacted area over roughness. This study found that Nikuradse’s conclusion remains valid if the hydraulic radius is defined in terms of fluid volume to solid-liquid contact area. The roughness surface area should be included in the calculation.
- If there are no roughness elements in the boundary, no flow separation occurs, thus the total resistance is equal to the skin friction. When the roughness elements are gradually introduced into the boundary, then partial separation occurs and the length of the separation region gradually increases with the increase in Reynolds Number as well as increase in relative roughness height. Therefore at this stage the total friction factor depends on both the skin friction and form drag.
- This study has linked these phenomena with flow separation and provided a general theoretical equation to calculate friction factor in flow over vegetation. This new model made a perfect prediction on flow resistance in an ecological channel.

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