Uncertainty Analysis in Humidity Measurements by the Psychrometer Method

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Abstract: The most common and cheap indirect technique to measure relative humidity is by using psychrometer based on a dry and a wet temperature sensor. In this study, the measurement uncertainty of relative humidity was evaluated by this indirect method with some empirical equations for calculating relative humidity. Among the six equations tested, the Penman equation had the best predictive ability for the dry bulb temperature range of 15–50 °C. At a fixed dry bulb temperature, an increase in the wet bulb depression increased the error. A new equation for the psychrometer constant was established by regression analysis. This equation can be computed by using a calculator. The average predictive error of relative humidity was <0.1% by this new equation. The measurement uncertainty of the relative humidity affected by the accuracy of dry and wet bulb temperature and the numeric values of measurement uncertainty were evaluated for various conditions. The uncertainty of wet bulb temperature was the main factor on the RH measurement uncertainty.

Keywords: measurement uncertainty; psychrometer constant; dry bulb; wet bulb; humidity; regression analysis

1. Introduction

Humidity is an important factor not only in various industries [1], but also in indoor environmental quality [2,3], in environment control processing [3], and in the assessment of heat stress, health and productivity for workers [4,5]. It affects evaporation and disease development in plants and the quality of food, chemicals and pharmaceuticals [1]. Accurate and reliable measurement of humidity is a key point for civil building [2,6] and health risks [7]. Typically, the amount of vapor contained in a moist air sample is expressed in terms of relative humidity (RH) [3].

Many sensors have been developed to measure RH. Popular commercial devices are chilled mirror hygrometers, electrical sensors and dry and wet bulb psychrometers [1,8]. The mirror hygrometer is the most accurate and it is commonly used for calibration of other instruments. Limitations of this equipment are its expense, sensitivity to contaminants and the requirement for skilled staff for its maintenance. Two types of electrical humidity sensors are resistive and capacitive sensors. Both types feature a fast response, good stability and little hysteresis [9]. The capacitive polymer sensor has a wider measurement range than the resistive type, however, the sensing elements of capacitive plates are exposed to condensation with high humidity and are then damaged. The main disadvantages of electrical sensors are that they are sensitive to contaminants, affected by ambient temperature and feature nonlinear calibration curves [9,10]. With careful calibration, the measurement uncertainty of these electrical sensors is >1.3% RH [11].

Because of the low cost and ease of use, psychrometry has been a popular method for measuring RH for a long time. This device involves a pair of electrical thermometers for measuring dry and wet bulb temperatures. The wet bulb thermometer is enclosed with a wick material that is maintained.
under wet conditions with distilled water. According to ISO standard 7726 [12], the wet thermometer should be ventilated at a sufficient velocity, generally at least 4 m/s to 5 m/s. In this way, the wet bulb temperature is close to the thermodynamic wet bulb temperature. The RH of air is then calculated by the dry and wet bulb temperature.

To ensure the accuracy of any humidity measurement, the factors affecting performance need to be considered. These factors include the accuracy of the two thermometers, the wind speed passing over the thermometers, the maintenance of the wetted condition for the wick materials surrounding the wet bulb thermometer, the care in shielding both sensors from radiation [13] and the selection of calculation equations. The choice of thermometers and maintenance of wet bulb conditions are the basic needs for measurements. However, the effect of calculation equations on RH needs to be studied. The measurement uncertainty of RH by the psychrometer method needs to be evaluated.

The calculation of RH by using dry and wet bulb temperature can be traced with thermodynamic theory. Theoretical formulas were proposed by ASHRAE [14,15]. These equations are derived from thermodynamic reasoning involving complex iterative calculation and require computer software for their calculation. Singh et al. [16] proposed a numerical calculation of psychrometric properties with a calculator, but the calculation of RH with dry and wet bulb temperature was still complex. Bahadori e al. [17] proposed a predictive tool to estimate RH using dry and wet bulb temperature that could be easily applied by an engineer without extensive mathematical ability. However, the equations still needed to be solved by iterative calculation.

Harrison and Wood [18] evaluated the effect of wind speed passing over the wet bulb temperature on the error sources of the humidity measurement and recommended that 2 m wind speeds should be >3 m/s. Ustymczuk and Giner [19] studied the effect of the performance of temperature sensors on the RH error and found that error increased linearly with increasing RH and decreased exponentially with increasing dry bulb temperature. The atmospheric pressure had only a slight effect on error. Mathioulakis et al. [20] demonstrated an evaluation method to calculate measurement uncertainties with indirect humidity measurement. Because of the strong non-linear characteristic of these calculation equations, the authors suggested using the Monte Carlo simulation for evaluation. Some empirical equations have been proposed to simplify the calculation of RH with dry and wet bulb temperature [21–27].

The RH value calculated with dry and wet bulb temperatures is called the indirect measurement. The difference between the actual and indirect measurement RH is defined as the error. Error is an idealized parameter, and the quantifying factors that affect it are difficult to determine. The measurement uncertainty was defined first in the ISO Guide [28]. The evaluation method has been described in detail [28–30]. According the ISO GUM, the measurement uncertainty was divided into A (by statistical method) and B type (by other information). The difference in error and uncertainty was defined clearly. The advantages of measurement uncertainty included the identification of the dispersion of results, estimated with a statistical method and quantification of the contribution of the uncertainty sources.

In this study, the predictive performance of these empirical equations was compared. The psychrometer coefficient of the empirical equation was calculated by an inverse technique. The relationship between this coefficient and dry and wet bulb temperatures was then established by regression analysis. The validity of the new empirical equation is reported. The ISO GUM concept was used to study the effect of the uncertainty of dry bulb temperature ($T_d$) and wet bulb temperature ($T_w$) on the measurement uncertainty of RH.

2. Theoretical Background

Equations for Determining Psychrometric Constant

The empirical equation for calculating RH with dry and wet bulb temperatures is as follows:

\[ P_w = P_{ws}(T_w) - A \times P \times (T_d - T_w) \] (1)
where $P_w$ is the partial pressure of water vapor in air in kPa, $P_{ws}(T_w)$ is the saturation vapor pressure of water at temperature $T_w$ in kPa, $T_d$ is the dry bulb temperature in °C, $T_w$ is the wet bulb temperature in °C, $P$ is the standard atmosphere pressure in kPa, and $A$ is the psychrometer coefficient in °C$^{-1}$·kPa$^{-1}$.

The difference between $T_d$ and $T_w$ is called wet bulb depression.

Relative humidity (RH) is calculated as follows:

$$RH = \frac{P_w}{P_{ws}(T_d)} \times 100\%$$

where RH is the relative humidity, and $P_{ws}(T_d)$ is the saturation vapor pressure of water at temperature $T_d$ in kPa.

For calculating $P_{ws}$, a simple equation was used with a the range of 0–100 °C [14]:

$$P_{ws} = 0.61078 \times \exp\left[\frac{17.2694T}{T + 237.3}\right]$$

where $T$ is the air temperature in °C.

The standard atmosphere pressure $P$ is considered in this study [6]:

$$P = 101.325 \text{ kPa}$$

Equation (1) then could be expressed as follows:

$$P_w = P_{ws}(T_w) - A_s \times (T_d - T_w)$$

where $A_s$ is the psychrometer constant in standard atmosphere pressure in °C$^{-1}$.

Some empirical equations have been proposed [21–27]. The Sensiron recommended the $A_s$ value in the range of $6.4 \times 10^{-4}$ to $6.8 \times 10^{-6}$ °C$^{-1}$ [12,21]. Other equations are listed as Table 1.

**Table 1.** Empirical equations used in this study.

|   | Equation                                      |
|---|-----------------------------------------------|
| 1 | Penman equation [22]                         |
|   | $P_w = P_{ws}(T_w) - 0.0664 \times (T_d - T_w)$ |
| 2 | Goff-Cratch equation [23]                     |
|   | $P_w = P_{ws}(T_w) - 0.067193 \times (T_d - T_w)$ |
| 3 | British United Turkeys (BUT) equation [24]    |
|   | $P_w = P_{ws}(T_w) - 0.066 \times (T_d - T_w)$ |
| 4 | Harrison equation [25]                        |
|   | $P_w = P_{ws}(T_w) - 0.067 \times (1 + 0.00115T_w) \times (T_d - T_w)$ |
| 5 | World meteorological Organisation (WMO) equation [26] |
|   | $P_w = P_{ws}(T_w) - 0.0662795 \times (1 + 0.000944T_w) \times (T_d - T_w)$ |
| 6 | Nevia et al. equation [27]                     |
|   | $P_w = P_{ws}(T_w) - 0.0647164 \times (1 + 0.00504T_w) \times (T_d - T_w)$ |

To evaluate the predictive performance of the above equations, the predictive error of empirical equations is defined as follows:

$$E = RH_{sta} - RH_{cal}$$

where $E$ is the predictive error of the empirical equation in a percentage, $RH_{sta}$ is the RH value calculated from the ASHRAE formula, and $RH_{cal}$ is the RH value calculated from these empirical equations.

Besides the minimum and maximum $E$ values, $E_{\text{min}}$ and $E_{\text{max}}$, a statistic $|E|_{\text{ave}}$ is defined as a criterion for evaluating the predictive ability:

$$|E|_{\text{ave}} = |E|/n$$

$$E_{\text{max}}$$ and $$E_{\text{min}}$$ are the maximum and minimum of $E$ values.

$$|E|_{\text{ave}}$$ is the average absolute error.
where \(|E|\) is the absolute value of \(E\), and \(n\) is the number of data.

To establish the new RH_{cal} equation, the psychrometer efficient at standard atmosphere is defined as \(A_s\). \(A_s\) was determined by rearranging Equations (1) and (2) as follows:

\[
A_s = \frac{P_{ws}(T_w) - P_w}{T_d - T_w} \tag{8}
\]

The calculation of \(A_s\) by Equation (8) was called the inverse technique. The evaluation of the measurement uncertainty is listed in Appendix A.

3. Materials and Methods

3.1. Equipment

The effect of air velocity on the measurement of wet bulb temperature was used as an example. The schematic of the experimental device is shown in Figure 1. The air was sucked into a wind tunnel by use of an adjustable fan. Two thermometers were used to measure the dry and wet bulb temperatures. The wet condition of the wet bulb thermometer was maintained with a wick and water reservoir. A resistant hygrometer served as the standard for RH measurement.

The air velocities were measured at several points to ensure the flow turbulence. Nets were used to filter particles and favoring the turbulence. The air velocity was adjusted by adjusting the fan speed. Air velocity was measured near the wet bulb thermometer by a hot-wired anemometer. A cotton wick 5 cm in length was attached to the wet bulb thermometer to maintain sufficient water to cool the sensor during aspiration. The measuring box was regularly maintained for each test. The wick must be clean and the de-ionized water was used as reservoir.

3.2. Sensors

The temperature was measured with use of the Sentron D9 temperature transmitter (Sentron Co., Taipei, Taiwan). This transmitter contains a Pt100 element. The error of this thermometer was 0.15 °C after calibration.

The RH was measured using a TH T-B121 resistive transmitter (Shinyei Kalsha, Tokyo, Japan). The error of this RH sensor was 0.5% RH after calibration with several saturated salt solutions.

The air velocity passing over the wet-bulb thermometer was detected by use of the KANOMAX Hot-wired 6004 Anemometer (Kanomax USA, Andover, NJ, USA). The error was ±5% according to the manufacturer’s specifications.

Figure 1. Experimental setup used for measurements (the figure is not to real scale).

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3.3. Experimental Method

The experiment was performed in the laboratory. During the test, the air velocity was adjusted from 0 to 5 m/s. At each air velocity, the reading values of \( T_d \), \( T_w \), RH and wind velocity were recorded by use of a data logger (Delta-T, Cambridge, UK). The sampling frequency was 1 s until the reading values of \( T_w \) were stable. There were three measurements for each wind velocity. The actual \( T_w \) value was calculated with the measurement values of \( T_d \) and RH.

4. Results

4.1. Effect of Air Velocity on \( T_w \)

The effect of the air velocity on \( T_w \) measurement is shown in Figure 2. With 1.0 m/s, the deviation between reading values and actual values calculated by RH measurement of \( T_w \) was close to 0.8 °C.

![Figure 2. Cont.](image-url)
Figure 2. Wet bulb temperature readings with time. (a) wind velocity 1 m/s; (b) wind velocity 2 m/s; (c) wind velocity 3 m/s; (d) wind velocity 4 m/s; (e) wind velocity 5 m/s.

The result could be explained by the lower air velocity passing the wet-bulb thermometer, the difference being due to the fact that $T_w$ is not a thermodynamic quantity but only an indicator of the thermodynamic wet bulb temperature [15]. On increasing the air velocity to 2.0 m/s, the difference ranged from 0.3 °C to 0.4 °C. If the air velocity was >3.0 m/s, the measurement $T_w$ was close to the actual value. The result was similar to findings by Harrison and Wood [18].

The error sources of the $T_w$ measurement include the thermometer performance and the velocity of the air passing the wet bulb thermometer. The combined errors this study ranged from 0.15 °C to 0.9 °C. If other factors were involved, the errors of $T_w$ measurement may range from 0.2 °C to 1.0 °C. According to the study of Barber and Gu [31] ±0.5 °C was a common error for an aspirated psychrometer.

4.2. Comparison of Predictive Performance of Six Empirical Equations

The criteria for comparing the predictive performance of six empirical equations are given in Table 2. The $|\vartriangle|_{ave}$ was used to evaluate predictive performance. $E_{max}$ and $E_{min}$ show range of errors. From numerical values in Table 2, the Neiva et al. equation had the largest values for $E_{max}$, $E_{min}$ and $|\vartriangle|_{ave}$, so it was not adequate for RH calculation.
The result could be explained by the lower air velocity passing the wet-bulb thermometer, the difference being due to the fact that Tw is not a thermodynamic quantity but only an indicator of the thermodynamic wet bulb temperature [15]. On increasing the air velocity to 2.0 m/s, the difference ranged from 0.3 °C to 0.4 °C. If the air velocity was >3.0 m/s, the measurement Tw was close to the actual value. The result was similar to findings by Harrison and Wood [18].

The error sources of the Tw measurement include the thermometer performance and the velocity of the air passing the wet bulb thermometer. The combined errors in this study ranged from 0.15 °C to 0.9 °C. If other factors were involved, the errors of Tw measurement may range from 0.2 °C to 1.0 °C. According to the study of Barber and Gu [31] ±0.5 °C was a common error for an aspirated psychrometer.

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| Temp. (°C) | Criteria | A_s (This Study) °C⁻¹ | Penman | BUT | Goff-Cratch | Harrison | WMO | Neiva et al. |
|------------|----------|-----------------------|--------|-----|-------------|----------|-----|--------------|
| 15         | Emin     | −0.0332               | 0.0278 | 0.0336 | 0.0510     | 0.2783   | 0.0508 | 0.1171       |
|            | Emax     | 0.0476                | 0.5321 | 0.6259 | 0.1041     | 0.9483   | 0.6808 | 0.8323       |
|            | $|E_{ave}|$ | 0.0988               | 0.2261 | 0.2691 | 0.3966     | 0.6492   | 0.3331 | 0.5575       |
| 20         | Emin     | −0.00823              | 0.1999 | 0.0115 | 0.0370     | 0.2353   | 0.0435 | 0.1200       |
|            | Emax     | 0.0300                | 0.4577 | 0.2866 | 0.7969     | 0.9548   | 0.6735 | 1.3476       |
|            | $|E_{ave}|$ | 0.0079               | 0.1917 | 0.1947 | 0.3443     | 0.6330   | 0.3332 | 0.7268       |
| 25         | Emin     | −0.0090               | 0.0164 | 0.0195 | 0.0289     | 0.0557   | 0.0387 | 0.1161       |
|            | Emax     | 0.0056                | 0.3528 | 0.4223 | 0.6282     | 0.9358   | 0.6150 | 1.3540       |
|            | $|E_{ave}|$ | 0.0448               | 0.1531 | 0.1841 | 0.2739     | 0.4612   | 0.3091 | 0.8033       |
| 27.5       | Emin     | −0.0018               | 0.0156 | 0.0183 | 0.0264     | 0.2815   | 0.0370 | 0.1126       |
|            | Emax     | 0.0051                | 0.3161 | 0.3187 | 0.5645     | 0.9617   | 0.5918 | 1.4235       |
|            | $|E_{ave}|$ | 0.0029               | 0.1418 | 0.1696 | 0.2523     | 0.6542   | 0.3016 | 0.8282       |
| 30         | Emin     | −0.0423               | 0.0153 | 0.0176 | 0.0246     | 0.2474   | 0.0356 | 0.1080       |
|            | Emax     | 0.0054                | 0.3132 | 0.3745 | 0.5562     | 0.9635   | 0.6021 | 1.4964       |
|            | $|E_{ave}|$ | 0.0058               | 0.1422 | 0.1693 | 0.2494     | 0.6368   | 0.3205 | 0.8774       |
| 32.5       | Emin     | −0.0069               | 0.0153 | 0.0174 | 0.0234     | 0.2603   | 0.0346 | 0.1048       |
|            | Emax     | 0.0056                | 0.2670 | 0.3423 | 0.5060     | 0.9651   | 0.5818 | 1.5331       |
|            | $|E_{ave}|$ | 0.0024               | 0.1384 | 0.1628 | 0.2349     | 0.6418   | 0.3068 | 0.8004       |
| 35         | Emin     | −0.0015               | 0.0156 | 0.0174 | 0.0267     | 0.2715   | 0.0338 | 0.1008       |
|            | Emax     | 0.0057                | 0.2679 | 0.3119 | 0.4654     | 0.9665   | 0.5649 | 1.5543       |
|            | $|E_{ave}|$ | 0.00216             | 0.1576 | 0.1576 | 0.2224     | 0.6474   | 0.3015 | 0.8851       |
| 40         | Emin     | −0.0030               | 0.0170 | 0.0183 | 0.0223     | 0.0411   | 0.0329 | 0.0929       |
|            | Emax     | 0.0057                | 0.2470 | 0.2877 | 0.4083     | 0.7609   | 0.5406 | 1.5631       |
|            | $|E_{ave}|$ | 0.0024               | 0.1388 | 0.1566 | 0.2095     | 0.4017   | 0.3015 | 0.8751       |
| 45         | Emin     | −0.0020               | 0.0188 | 0.0199 | 0.0230     | 0.0399   | 0.0327 | 0.0858       |
|            | Emax     | 0.0034                | 0.2444 | 0.2778 | 0.3768     | 0.7177   | 0.5271 | 1.5428       |
|            | $|E_{ave}|$ | 0.0011               | 0.1511 | 0.1656 | 0.2088     | 0.3910   | 0.3016 | 0.8592       |
| 50         | Emin     | −0.0076               | 0.0210 | 0.0218 | 0.0224     | 0.0389   | 0.0331 | 0.0797       |
|            | Emax     | 0.0077                | 0.2568 | 0.2589 | 0.3725     | 0.7041   | 0.5313 | 1.5342       |
|            | $|E_{ave}|$ | 0.0046               | 0.1719 | 0.1848 | 0.2232     | 0.4015   | 0.3190 | 0.8706       |

A_s values for three equations, Penman, BUT and Goff-Cratch, were constant. The criteria for predictive errors was higher for the Goff-Cratch equation than for the Penman and BUT equations. In the case of the Penman and BUT equations, the A_s value was 0.664 and 0.666 °C⁻¹, respectively. Numeric values for A_s for the two equations were close. However, the criteria for predictive performance differed. The Penman equation had better performance than the BUT equation. The result indicated the sensitivity of the A_s value for the predictive performance of RH equations.
The $A_s$ for three equations, Harrison, WMO and Neiva et al., all involved a linear relationship with $T_w$ value. $E_{\text{min}}$ was larger for the WMO than the Harrison equation for $T_d < 40 ^\circ C$, and $|E_{\text{ave}}|$ values were smaller for the WMO than the Harrison equation for all 10 dry bulb temperatures. The Penman equation had the smallest values for $E_{\text{min}}$, $E_{\text{max}}$ and $|E_{\text{ave}}|$. Therefore, the Penman equation had the best predictive performance among the six empirical equations.

The error distribution of the Penman equation with 10 dry bulb temperatures is shown in Figures 3 and 4. Error increased with decreasing wet bulb temperature at fixed dry bulb temperature. The data distribution of errors was curved. When the wet bulb temperature was close to the dry bulb temperature, that is, when RH increased to saturation, the predicted error decreased.

The Penman equation had better predictive ability at high than low RH. The limitation of electrical sensors is poor performance with high RH. The psychrometer method can be used for high RH measurement. The error distribution for the WMO equations under dry bulb temperatures is shown in Figures 5 and 6. When the wet bulb temperature was near the dry bulb temperature, errors decreased. The error distribution patterns were similar to those for the Penman equation.
4.3. Development of a New \( A_s \) Equation

\( A_s \) values at fixed dry bulb temperature and different wet bulb temperature were calculated by the inverse technique from Equation (14). The relationship between \( A_s \) and wet bulb temperatures under 10 dry bulb temperatures is illustrated in Figures 7 and 8. Figure 7 shows that \( A_s \) was nearly constant for dry bulb temperatures <30 °C. \( A_s \) was close to 0.0654 °C\(^{-1}\). The data distribution for \( A_s \) with \( T_d > 30 \) °C in Figure 7 presents a clear curve shape. \( A_s \) strongly depended on the wet bulb temperature, \( T_w \), and weakly on dry bulb temperature, \( T_d \). The relationship for \( A_s \) and the two temperatures were evaluated by regression analysis:

\[
A_s = 0.0654 \text{ °C}^{-1}, \quad T_d < 30 \text{ °C} \tag{9}
\]

\[
A_s = 0.0637485 + 0.000187508 T_w - 4.376670 \times 10^{-6} T_w^2 - 1.21851 \times 10^{-5} T_d \tag{10}
\]

\[
R^2 = 0.99483, \quad s = 4.73762 \times 10^{-5}, \quad T_d > 30 \text{ °C},
\]

The predictive errors for this new \( A_s \) equation are in Table 1. Three criteria, \( E_{min} \), \( E_{max} \) and \( | \varepsilon |_{ave} \), were lower for the new \( A_s \) equation than other empirical equations. At \( T_d = 15 \) °C, \( | \varepsilon |_{ave} \) for the new \( A_s \), Penman and WMO equations was 0.0988%, 0.2261% and 0.3331% respectively. At \( T_d = 30 \) °C, \( | \varepsilon |_{ave} \) for the above three equations was 0.0058%, 0.1422% and 0.3016% respectively. The predictive errors of the new \( A_s \) equation improved significantly.
The new \( A_s \) equation was incorporated into Equation (1). Predictive errors for this new \( A_s \) equation are in Table 1. Three criteria, \( E_{\text{min}} \), \( E_{\text{max}} \) and \( |E|_{\text{ave}} \), were lower for the new \( A_s \) equation than other empirical equations. At \( T_d = 15^\circ \text{C} \), \( |E|_{\text{ave}} \) for the new \( A_s \), Penman and WMO equations was 0.0988%, 0.2261% and 0.3331%, respectively. At \( T_d = 30^\circ \text{C} \), \( |E|_{\text{ave}} \) for the above three equations was 0.0058%, 0.1422% and 0.3016%, respectively. At \( T_d = 50^\circ \text{C} \), \( |E|_{\text{ave}} \) for the above three equations was 0.00458%, 0.1719% and 0.3190%, respectively. The predictive errors of the new \( A_s \) equation improved significantly.

The error distribution for the new \( A_s \) equation for different dry bulb temperatures is shown in Figures 9 and 10. With \( T_d < 25^\circ \text{C} \), larger errors were found at the lower range of \( T_w \). With \( T_d > 30^\circ \text{C} \), error distributions were curved. With a more complex form of the \( A_s \) model, for example, when higher order polynomial equations were used, the error distribution of the curve shapes could be improved. However, the new \( A_s \) equation, Equation (10), could be easily computed with a calculator. The predictive value of errors was <0.1% RH. This error could be acceptable in term of practical application for humidity measurement [1,8], so Equation (10) is recommended as the adequate \( A_s \) equation.
Simoes-Moreire found that most of the empirical equations for the psychrometer coefficient of A were presented in a fixed range or as a constant [32]. Some researchers have proposed a linear relationship for the A value and wet bulb temperature [25–27]. However, the linear Tw model for As did not improve the predicted values of RH. The new As model proposed in this study can significantly improve the predictive ability for RH measurement.

4.4. Measurement Uncertainty of Humidity Calculated by Td and Tw Values

The measurement uncertainty of humidity of a direct method has been investigated [11]. The evaluation of measurement uncertainty was studied with the new As equation developed in this study.

The typical uncertainty of a dry bulb thermometer, u(Td), carefully calibrated was 0.15 °C [33]. The estimated uncertainty of a wet bulb thermometer, u(Tw), ranged from 0.15 °C to 1.0 °C.
The combined uncertainty of the RH value, $u(\text{RH})$, evaluated by Equations (A5)–(A12) for four dry bulb temperatures with two uncertainties $u(T_d)$, 0.15 °C and 0.3 °C in different wet bulb temperatures, is in Figures 11–18. The results of the calculation of $u(\text{RH})$ with other $u(T_d)$, 0.1 and 0.5, are available in supplemental information.

![u(RH) at T_d=15 °C, u(T_d)=0.15 °C](image1)

**Figure 11.** Uncertainties of relative humidity calculated with Equation (9) at $T_d = 15$ °C, $u(T_d) = 0.15$ °C, $T_w = 6$–14 °C and $u(T_w) = 0.1$–1 °C.

![u(RH) at T_d=15 °C, u(T_d)=0.3 °C](image2)

**Figure 12.** Uncertainties of relative humidity calculated with Equation (9) at $T_d = 15$ °C, $u(T_d) = 0.30$ °C, $T_w = 6$–14 °C and $u(T_w) = 0.1$–1 °C.

Figure 11 show that with increased uncertainty of the wet bulb thermometer, $u(T_w)$ enhanced the combined uncertainties of $u(\text{RH})$. At the same $u(T_w)$, higher wet bulb temperature induced larger $u(\text{RH})$ value.
With the smallest \( u(T_w) \) value, 0.1 °C, the combined uncertainty of \( u(RH) \) was 4.35% and 4.93% for the \( T_w \) at 6 °C and 14 °C, respectively. If the \( u(T_w) \) value was 0.5 °C, the \( u(RH) \) was 5.83% and 6.83% for the \( T_w \) at 6 °C and 14 °C, respectively. The uncertainty of \( u(T_w) \) affected the \( u(RH) \) value significantly. With the largest \( u(T_w) \), 1.0 °C, the largest uncertainty was 8.63% and 10.98%, respectively.

**Figure 13.** Uncertainties of relative humidity calculated with Equation (10) at \( T_d = 20 \) °C, \( u(T_d) = 0.15 \) °C, \( T_w = 11\sim19 \) °C and \( u(T_w) = 0.1\sim1 \) °C.

**Figure 14.** Uncertainties of relative humidity calculated with Equation (10) at \( T_d = 20 \) °C, \( u(T_d) = 0.3 \) °C, \( T_w = 11\sim19 \) °C and \( u(T_w) = 0.1\sim1 \) °C.
The contributions of \( u(T_w) \) are the performance of the sensor and the measurement technique for the wet bulb condition. If the uncertainty of wet bulb temperature was >0.5 °C, the \( u(RH) \) ranged from 5% to 11%. The measurement error of RH with the psychrometer was obvious. A similar result could be found for \( u(T_d) = 0.3 °C \). At the dry bulb temperature of 20 °C, the distribution of \( u(RH) \) in different wet bulb temperatures, \( u(T_d) \) and \( u(T_w) \) values was similar to the results at 15 °C (Figures 13 and 14). However, the numeric values of \( u(RH) \) were lower than the results at 15 °C. If the \( u(T_d) \) and \( u(T_w) = 0.15 °C \), the \( u(RH) \) was 3.24% and 3.87%, respectively. With the \( u(T_w) = 0.5 °C \), the \( u(RH) \) ranged from 4.49% to 5.16% for different \( T_w \) values. At the worst conditions of \( u(T_w) = 1.0 °C \), the \( u(RH) \) values ranged from 7.22% to 9.40%.

The \( u(RH) \) values for the \( T_d \) at 30 °C in different \( T_w \), \( u(T_d) \) and \( u(T_w) \) values are in Figures 15 and 16. With \( u(T_d) \) and \( u(T_w) = 0.15 °C \), the results for \( u(RH) \) ranged from 0.99% to 1.47%. With \( u(T_w) = 0.5 °C \), the \( u(RH) \) ranged from 2.64% to 3.64%. At the worst conditions of \( u(T_w) = 1.0 °C \),
the u(RH) ranged from 5.18% to 7.07%. The RH values calculated by the new psychrometric equation with high dry bulb temperature had smaller u(RH) values.

The distribution of u(RH) of 40 °C T_d in different u(T_d), u(T_w) and T_w conditions are in Figures 17 and 18. Nine T_w were considered. With u(T_d) = 0.15 °C, the u(RH) ranged from 1.63% to 3.11% with u(T_w) = 0.5 °C, and from 2.97% to 6.02% with u(T_w) = 1.0 °C.

The u(RH) values at higher T_d, 30 °C and 40 °C, were less than at T_d 15 °C and 20 °C. The result confirmed that the new psychrometric A_s equation was adequate for RH measurement in high temperature.

**Figure 17.** Uncertainties of relative humidity calculated with Equation (10) at T_d = 40 °C, u(T_d) = 0.15 °C, T_w = 21–39 °C and u(T_w) = 0.1–1 °C.

**Figure 18.** Uncertainties of relative humidity calculated with Equation (10) at T_d = 40 °C, u(T_d) = 0.3 °C, T_w = 21–39 °C and u(T_w) = 0.1–1 °C.
5. Conclusions

This work investigated some empirical equations for calculating the relative humidity (RH) from indirect measurement of dry and aspirated wet bulb temperatures. The standard value for RH was obtained from the equations reported in the ASHRAE Handbook. The Penman equation had the best predictive ability among the six equations tested with dry bulb temperature ranging from 15 °C to 50 °C. Some equations with a linear relationship of psychrometer coefficients and wet bulb temperature did not have good predictive ability. At a fixed dry bulb temperature, the increase in wet bulb depression increased the errors. The psychrometer method is adequate for measuring high RH.

A relationship between the psychrometer constant $A_s$ and dry and wet bulb temperature was established by regression analysis. The new $A_s$ equation included the polynomial form of $T_w$. With this new $A_s$ equation, the average predictive error was <0.1% RH.

The measurement uncertainty of RH calculated from dry and wet bulb temperature with this new $A_s$ equation was evaluated. At the $T_d$ values of 10 °C and 25 °C, the combined uncertainty of RH values ranged from 3.2% to 4.0% with $u(T_w) = 0.15$ °C, 4.69% to 7.46% with $u(T_w) = 0.5$ °C and 6.5% to 11.0% with $u(T_w) = 1.0$ °C. At the $T_d$ values of 30 °C and 40 °C, the combined uncertainty of RH values ranged from 0.52% to 2.31% with $u(T_w) = 0.15$ °C, 1.72% to 4.06% with $u(T_w) = 0.5$ °C and 2.97% to 7.29% with $u(T_w) = 1.0$ °C.

The uncertainty of $T_w$ had a significant effect on the combined uncertainty of RH. The uncertainty sources of $T_w$ were performance of the thermometer and maintenance of wet bulb conditions. A quantification method was provided to evaluate measurement errors in calculating RH with dry and wet bulb temperatures.

Supplementary Materials: The following are available online at http://www.mdpi.com/1424-8220/17/2/368/s1, Figures S1–S8.

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Nomenclature

| Symbol | Description |
|--------|-------------|
| $A$ | psychrometric coefficient $^\circ$C$^{-1}$·kPa$^{-1}$ |
| $A_s$ | psychrometric constant incorporated air atmosphere term, $^\circ$C$^{-1}$ |
| $e$ | vapor pressure, kPa |
| $e_s$ | saturated vapor pressure, kPa |
| $E$ | error of predictive performance, % |
| $E_{\text{max}}$ | maximum error of predictive performance, % |
| $E_{\text{min}}$ | minima error of predictive performance, % |
| $|E|$ | absolute error of predictive performance, % |
| $|E|_{\text{ave}}$ | average of $|E|$ |
| $n$ | number of data |
| $P$ | atmosphere air pressure, kPa |
| $\text{RH}_{\text{cal}}$ | calculated RH value from empirical equation |
| $\text{RH}_{\text{sta}}$ | standard RH value from ASHRAE Handbook |
| $T$ | temperature of air, $^\circ$C |
| $T_d$ | dry bulb temperature, $^\circ$C |
| $T_w$ | wet bulb temperature, $^\circ$C |
| $u(\text{RH})$ | uncertainty of relative humidity |
| $u(T_d)$ | uncertainty of dry bulb temperature |
| $u(T_w)$ | uncertainty of wet bulb temperature |
Appendix A. Evaluation of the Measurement Uncertainty

The steps to evaluate uncertainty are as follows [28–30]:

1. Model the measurement
   \( y \) is not measured directly and is determined from \( K \) quantities \( x_1, x_2, \ldots, x_K \)
   The functional relationship is as follows:
   \[
   y = f(x_1, x_2, \ldots, x_K) \quad (A1)
   \]

2. Ensure the uncertainty source \( x_i \) and calculate the estimated values of \( x_i \)
   \[
   u_c^2[y] = \sum_{i=1}^{n} \left( \frac{\partial y}{\partial x_i} \right)^2 u^2(x_i) \quad (A2)
   \]
   where \( u_c(y) \) is the combined uncertainty and \( y \) is the output quantity.

3. Evaluate the uncertainty classified as A and B types.

4. Estimate the covariance of each \( x_i \).

5. Calculate the sensitivity coefficient,
   \[
   c_i = \frac{\partial f}{\partial x_i} \quad (A3)
   \]

6. Calculate the combining uncertainty and effective degree of freedom.

7. Determine a coverage factor and expanded uncertainty.

8. Report the uncertainty.

The uncertainty of RH value that calculated from \( T_d \) and \( T_w \) values.

\[
RH = \frac{P_w}{P_{ws}(T_d)} - \frac{P_{ws}(T_d) - A_s(T_d - T_w)}{P_{ws}(T_d)} \quad (A4)
\]

By Equation (A2), the uncertainty of RH can be calculated follows:

\[
u_c^2[RH] = \left( \frac{\partial RH}{\partial T_d} \right)^2 u^2(T_d) + \left( \frac{\partial RH}{\partial T_w} \right)^2 u^2(T_w) \quad (A5)
\]

\[
\frac{\partial P_{ws}(T_d)}{\partial T_d} = 2502.99 \exp \left( \frac{17.2694 T_d}{T_d + 237.3} \right) \left( \frac{1}{T_d + 237.3} \right)^2 \quad (A6)
\]

\[
\frac{\partial P_{ws}(T_w)}{\partial T_w} = 2502.99 \exp \left( \frac{17.2694 T_w}{T_w + 237.3} \right) \left( \frac{1}{T_w + 237.3} \right)^2 \quad (A7)
\]

If \( A_s \) is a constant,

\[
\frac{\partial RH}{\partial T_w} = \frac{1}{P_{ws}(T_d)} \left[ \frac{\partial P_{ws}(T_w)}{\partial T_w} + A_s \right] = \frac{1}{P_{ws}(T_d)} \left[ \frac{\partial P_{ws}(T_w)}{\partial T_w} + A_s \right] \quad (A8)
\]

\[
\frac{\partial RH}{\partial T_d} = \frac{1}{P_{ws}(T_d)} \left[ -A_s P_{ws}(T_d) - \left( P_{ws}(T_w) - A_s(T_d - T_w) \right) \frac{\partial P_{ws}(T_d)}{\partial T_d} \right] \quad (A9)
\]

If \( A_s = C_0 + C_1 T_w + C_2 T_w^2 + C_3 T_d \),

\[
RH = \frac{1}{P_{ws}(T_d)} \left[ P_{ws}(T_w) - \left( C_0 + C_1 T_w + C_2 T_w^2 + C_3 T_d \right) (T_d - T_w) \right] \quad (A10)
\]
\[
\frac{\partial RH}{\partial T_w} = \frac{1}{P_{wT}(T_d)} \left( \frac{\partial P_{wT}(T_w)}{\partial T_w} - 2(C_2T_w + C_1)(T_d - T_w) + C_2T_w^2 + C_1T_d + C_1T_w + C_0 \right) \\
\frac{\partial RH}{\partial T_d} = \frac{1}{P_{wT}(T_d)} \left[ \left( \frac{C_3}{P_{wT}(T_d)} \right) - \frac{\partial P_{wT}(T_w)}{\partial T_d} \left( - \left( C_0 + C_1T_w + C_2T_w^2 + C_3T_d \right) \right) \right]
\]

References

1. Wernecke, R.; Wernecke, J. Industrial Moisture and Humidity Measurement: A Practical Guide; Wiley: Hoboken, NJ, USA, 2014.
2. EN 15251. Indoor Environmental Input Parameters for Design and Assessment of Energy Performance of Buildings Addressing Indoor Air Quality, Thermal Environment, Lighting and Acoustics; European Committee for Standardization: Brussels, Belgium, 2007.
3. ASHRAE. The ASHRAE Guide for Buildings in Hot and Humid Climates; ASHRAE: Atlanta, GA, USA, 2009.
4. D’Ambrosio Alfano, F.; Malchaire, J.; Palella, B.I.; Riccio, G. The WBGT index revisited after 60 years of use. *Ann. Occup. Hyg.* 2014, 58, 955–970. [PubMed]
5. Kjellstrom, T.; Holmér, I.; Lemke, B. Workplace heat stress, health and productivity an increasing challenge for low and middle-income countries during climate change. *Glob. Health Action* 2009, 2, 1–6. [CrossRef] [PubMed]
6. Dell’Isola, M.; Frattolillo, A.; Palella, B.I.; Riccio, G. Influence of measurement uncertainties on the thermal environment assessment. *Int. J. Thermophys.* 2012, 33, 1616–1632. [CrossRef]
7. D’Ambrosio Alfano, F.; Palella, B.I.; Riccio, G. The role of measurement accuracy on the heat stress assessment according to ISO 7933: 2004. *WIT Trans. Biomed. Health Sci.Eng.* 2007, 11, 115–124.
8. Wiederhold, P.R. *Water Vapor Measurement*; Marcel Dekker, Inc.: New York, NY, USA, 1997.
9. Pallas-Areny, R.; Webster, J.G. Sensors and Signal Conditioning; John Wiley & Sons: New York, NY, USA, 2001.
10. Fleming, R.J. A note on temperature and relative humidity corrections for humidity sensors. *J. Atmos. Ocean. Technol.* 1998, 15, 1511–1515. [CrossRef] [PubMed]
11. Lu, T.; Chen, C. Uncertainty evaluation of humidity sensors calibrated by saturated salt solutions. *Measurement* 2007, 40, 591–599. [CrossRef]
12. ASHRAE. *Ergonomics of the Thermal Environment: Instruments for Measuring Physical Quantities*; ISO: Geneva, Switzerland, 2002.
13. D’Ambrosio Alfano, F.; Palella, B.I.; Riccio, G. On the problems related to natural wet bulb temperature indirect evaluation for the assessment of hot thermal environments by means of WBGT. *Ann. Occup. Hyg.* 2012, 56, 1063–1079. [PubMed]
14. ASHRAE. *Brochure on Psychrometry*; ASHRAE: Atlanta, GA, USA, 1977.
15. ASHRAE. *ASHRAE Handbook: Fundamentals*; ASHRAE: Atlanta, GA, USA, 2013.
16. Singh, A.K.; Singh, H.; Singh, S.P.; Sawhney, R.L. Numerical calculation of psychrometric properties on a calculator. *Build. Environ.* 2002, 37, 415–419. [CrossRef]
17. Bahadori, A.; Zahedi, G.; Zendehboudi, S.; Hooman, K. Simple predictive tool to estimate relative humidity using wet bulb depression and dry bulb temperature. *Appl. Therm. Eng.* 2013, 50, 511–515. [CrossRef]
18. Harrison, R.G.; Wood, C.R. Ventilation effects on humidity measurements in thermometer screens. *Q. R. Meteorol. Soc.* 2012, 138, 1114–120. [CrossRef]
19. Ustymczuk, A.; Giner, S.A. Relative humidity errors when measuring dry and wet bulb temperatures. *Biosyst. Eng.* 2012, 110, 106–111. [CrossRef]
20. Mathioulakis, E.; Panaras, G.; Belessiotis, V. Estimation of uncertainties in indirect humidity measurements. *Energy Build.* 2011, 43, 2806–2812. [CrossRef]
21. The Sensirion Company. *Introduction to Humidity—Basic Principles on Physics of Water Vapor*; The Sensirion Company: Staefa, Switzerland, 2009.
22. Penman, H.L. *Humidity*; Reinhold Publishing Co.: New York, NY, USA, 1958.
23. Goff, J.A.; Gratch, S. Thermodynamic properties of moist air. *Trans. ASHVE* 1945, 51, 125–164.
24. British United Turkeys Ltd. *Measurement of Incubation Humidity*; British United Turkeys Ltd.: Cheshire, UK, 2005.
25. Harrison, L.D. Fundamental Concepts and Definitions Relating to Humidity. In *Humidity and Moisture*; Wexler, A., Ed.; Reinhold Publishing Co.: New York, NY, USA, 1963; Volume 3.

26. WMO. *Guide to Meteorological Instruments and Methods of Observation*; WMO-No. 8; World Meteorological Organisation: Geneva, Switzerland, 2006.

27. De, B.; Neiva, A.C.; dos Reis, E.; Sanchez, C.G. Calibration and Validation of a New Aspirated Psychrometer for Technological Development of Humidifier. In Proceedings of the XVIII IMEKO World Congress, Rio de Janeiro, Brazil, 17–22 September 2006.

28. ISO/IEC 98–3. *Uncertainty of Measurement—Part 3: Guide to the Expression of Uncertainty in Measurement*; ISO Edition: Geneva, Switzerland, 2010.

29. National Aeronautics and Space Administration. *Measurement Uncertainty Analysis Principles and Methods, NASA Measurement Quality Assurance Handbook—Annex 3*; National Aeronautics and Space Administration: Washington, DC, USA, 2010.

30. Grabe, M. *Measurement Uncertainties in Science and Technology*, 2nd ed.; Springer: Heidelberg, Germany, 2005.

31. Barber, E.M.; Gu, D. Performance of an aspirated psychrometer and three hygrometers in livestock barns. *Appl. Eng. Agric.* 1988, 5, 595–599. [CrossRef]

32. Simoes-Moreira, J.R. A thermodynamic formulation of the psychrometer constant. *Meas. Sci. Technol.* 1999, 10, 302–311. [CrossRef]

33. Chen, C. Evaluation of measurement uncertainty for thermometers with calibration equations. *Accredit. Qual. Assur.* 2006, 11, 75–82. [CrossRef]