Abstract. The Tile Calorimeter (TileCal) is the central section of the hadronic calorimeter of ATLAS experiment of the Large Hadron Collider (LHC) and has about 10,000 electronic channels. An Optimal Filter (OF) has been used to estimate the energy sampled by the calorimeter and applies a Quality Factor (QF) for signal acceptance. An approach using Matched Filter (MF) has also been pursued. In order to cope with the luminosity rising foreseen for LHC operation upgrade, different algorithms have been developed. Currently, the OF measurement for signal acceptance is implemented through a chi-square test. At a low luminosity scenario, such QF measurement has been used as a way to describe how the acquired signal is compatible to the pulse shape pattern. However, at high-luminosity conditions, due to pile up, this QF acceptance is no longer possible when OF is employed, and the QF becomes a measurement to indicate whether the reconstructed signal suffers or not from pile up. Methods are being developed in order to recover the superimposed information, and the QF may be used again as signal acceptance criterion. In this work, a new QF measurement is introduced. It is based on divergence statistics, which measures the similarity of probability density functions.

1. Introduction
The Large Hadron Collider (LHC) collided protons with 8 TeV center-of-mass in 2012. The collision spacing was 50 ns and the mean number of interactions per crossing was 20 with the maximum value reaching 35. The planning for the next 10 years is to increase the number of colliding bunches. The center of mass energy will also increase the number of interactions [1]. The ATLAS detector [2] was designed to exploit the full potential of the LHC discoveries. As a general-purpose detector, ATLAS must be sensitive to a wide range of signatures left by particles.

In ATLAS, the calorimeter system plays a major role due to its excellent energy resolution and fast response. The system is split into electromagnetic [3] and hadronic [4] sections. The Tile Hadronic Calorimeter (TileCal) is the central hadronic calorimeter in ATLAS. It is designed to identify and measure the jets, tau’s, and contribute to the missing $E_T$. It is based on a sampling technique with scintillating tiles embedded in an absorber structure. The tiles are placed perpendicular to the beam axis for simplifying the mechanical construction and routing of the wavelength-shift (WLS) fibers for readout. It is mechanically divided into a central barrel and two extended barrels and each cylinder comprises 64 modules. Concerning the readout granularity, TileCal is divided into 3 radial layers with $\Delta\eta \times \Delta\phi = 0.1 \times 0.1$ (0.2 x 0.1 in outermost layer) resulting in approximately 5,000 double readout cells.
A particle traversing the calorimeter has part of its energy deposited in the scintillating tiles. This energy is converted into light that is transmitted by WLS fibers to photomultiplier tubes (PMT). The photomultipliers convert the light pulses into fast current signals, which are shaped into signals with full width at half maximum of 50 ns. The signals are sampled by two ADC converters operating at a frequency of 40 MHz, which is the nominal bunch crossing (BC) frequency of the LHC. The number of samples is programmable, and seven samples are being used for readout (see Figure 1). Such signal can be defined as

\[ s(t) = Ag(t_i - \tau) + ped \] (1)

where \( A \) is the signal amplitude, \( t_i \) is the time at which the sample \( i \) was acquired, \( ped \) is the pedestal level, \( \tau \) is the phase of the pulse and \( g \) is the expected pulse shape.

Figure 1. TileCal pulse shape generated at the front-end [7].

Most of energy reconstruction methods in calorimetry uses the knowledge of the pulse shape generated by the readout electronics and assume Gaussian noise characteristics for each channel [5]. Due to the increase in luminosity, the signal pile-up will deform the signal of interest. Such effect is highly dependent on the conditions of the LHC operation and reduce the efficiency of current energy estimation techniques, as the background becomes highly non-gaussian [6].

2. Signal Reconstruction

The baseline energy reconstruction method for TileCal is the Optimal Filter (OF) [7], but other methods have also been developed for offline data processing such as the Matched Filter (MF) [8]. However, both methods suffer from the increase of pile-up events. It has to be mentioned that the planned LHC luminosity increase in the future will require pile-up mitigation in order to preserve unbiased signal reconstruction performance. Therefore, a more sophisticated approach to deal with the pile-up is under development. Figure 2 exemplifies the considerable deterioration of the pulse shape for a signal under the presence of an out-of-time (neighbouring bunch crossings) signal at 50 ns.

2.1. Energy Estimation

The OF is based on a linear combination of the signal samples to obtain the pulse amplitude. The coefficients are chosen such that the noise impact on the resolution of the calorimeter is minimized [7]. The following equation describes how the OF reconstructs the signal:

\[ A = \sum_{i=1}^{7} a_i s_i, \quad \tau = \frac{1}{A} \sum_{i=1}^{7} b_i s_i \quad \text{and} \quad ped = \sum_{i=1}^{7} c_i s_i. \] (2)
Here, $s_i$ is the sample in the bunch-crossing $i$ and $a$, $b$, and $c$ are the filter coefficients.

The MF also uses the reference signal to develop a linear filter for TileCal. The coefficients are calculated using a set of training data to estimate the covariance of the noise and the baseline of the input signal [8]:

$$\hat{A} = \frac{\sum_{i=1}^{7} (s_i - \text{ped}) \cdot g_i}{\sum_{i=1}^{7} (g_i \cdot g_i)}$$

(3)

where $\text{ped}$ corresponds to the pedestal.

Unlike OF and MF, signal deconvolution techniques estimate the amplitude for each BC simultaneously [9].

2.2. Quality Factor

These energy reconstruction methods use a $\chi^2$ to evaluate the goodness of the reconstructed pulse, but they have shown to be highly sensitive to the effect of pile-up. The $\chi^2$ is calculated according to

$$QF = \sqrt{\sum_{i=1}^{7} (s_i - \hat{a}_0 h_{0,i} - \sum_{j=1}^{N} \hat{a}_j h_{j,i})^2}$$

(4)

where $\hat{a}$ corresponds to the amplitude of each signal present in a given readout window, and $h$ is a matrix that contains the expected signals.

2.3. Divergence as Quality Factor

A new way to measure the quality factor is the use of divergence statistic, which is considered as a measurement of similarity between two probability distributions [10]. For this, the shaped TileCal pulse is considered as a distribution and its divergence is measured with respect to the one obtained from the signal sampled by the electronics chain. In this way, the quality factor qualifies a given signal if the measured divergence is low. The simplest way to use diverge as quality factor is by defining cuts in order to validate signals (physics of interest) against background (minimum bias or corrupted events). As the statistical fluctuations in the standard Tilecal shaped pulse are small, higher values of the divergence measurement indicate signal deterioration. These small statistical fluctuations of the Tilecal pulse are used to calibrate the divergence measurement, when a valid pulse is present.
2.3.1. KL Divergence  

The Kullback-Leibler divergence, also known as KL divergence, uses mutual information for the calculation of this quasi-distance [11]. Considering two probability density functions $\hat{p}_A$ (as observed) and $\hat{p}_B$ (theoretical expectation), we can define the Kullback-Leibler divergence as shown by Equation (5).

$$D_{KL}(\hat{p}_A, \hat{p}_B) = \int_{-\infty}^{+\infty} \hat{p}_A(\xi) \log \frac{\hat{p}_A(\xi)}{\hat{p}_B(\xi)} d\xi$$  \hspace{1cm} (5)

2.3.2. JS Divergence  

The Jensen-Shannon divergence (JS) as defined by Equation (6), is a symmetric and smoother version of the Kullback-Leibler divergence [12]. Moreover, the square root of JS divergence is a metric:

$$D_{JS}(\hat{p}_A, \hat{p}_B) = \frac{D_{KL}(\hat{p}_A, \hat{p}_M) + D_{KL}(\hat{p}_B, \hat{p}_M)}{2}$$  \hspace{1cm} (6)

where $\hat{p}_M = \frac{\hat{p}_A + \hat{p}_B}{2}$.

3. Results  

The methods described above are applied for signal reconstruction using ATLAS Monte Carlo simulation [13] under high luminosity conditions, where the averaged number of interactions per bunch crossing reaches up to 40. The only information needed to compute the proposed quality factor is the standard TileCal pulse shape and the signal baseline (computed through special calibration runs).

If the reconstructed amplitude values are evaluated through their respective quality factors, a large difference between the quality factor values (for different reconstruction methods, OF and deconvolution techniques) is expected, even if both methods use the $\chi^2$ computation. Due to the out-of-time pile-up, large QF values are expected when the central bunch crossing is estimated with OF, so that QF becomes an useful measurement for pile-up detection, but loses efficiency as a measurement of signal reconstruction quality. For a simulation of a high occupancy TileCal cell (located in the A layer with $|\eta| = 1.2$ [4]) and LHC operating at 25 ns of bunch spacing with $\mu > 40$, the results show that the QF through $\chi^2$ is not able to validate signals that are affected by high pile-up. It discards approximately 15% of the events in the data set that have $\chi^2 > 15$. When the original signal samples are better recovered, signal reconstruction produces low QF values. In this case, QF recovers its capacity as an efficient measurement of signal reconstruction quality in such high pile-up conditions.

Because the divergence is measurement of similarity, it can not be compared in terms of absolute values with the $\chi^2$, which is currently used in ATLAS calorimeter system. However, divergence measurements allow the distinction between signals and background and, as a result, it can be applied in harsh pile-up environments. The divergence measurement is more sensitive to deviations from a valid pulse, as it considers a density estimation. For the same simulation mentioned previously (also the same TileCal cell), divergence measurement presents lower values than usual $\chi^2$ for the whole dinamic range, and it is able to validate approximately 95% of the events (JS divergence $< 0.4$).

4. Conclusions  

Currently, the Optimal Filter is used for reconstruction of sampled signals in TileCal. However, due to the increase of luminosity in the near future, the number of pile-up events will increase substantially. In TileCal, the quality factor was conceived as a measurement of the adherence of an acquired signal to the reference shaped pulse. However, for high luminosity conditions this measurement becomes more of a pile-up indicator. New reconstruction methods are...
being developed for optimal reconstruction under severe pile-up conditions, which allow signal
reconstruction quality evaluation.

The quality factor based on the $\chi^2$ from independent amplitude estimates is supposed to
present lower values than those obtained by the OF. On the other hand, a quality factor through
divergence statistics shows to be a more appropriate measurement to distinguish the physics
of interest from background for pile-up under non-Gaussian background that is introduced by
the pile-up. The application of the proposed quality factor is under evaluation in TileCal, and
studies considering higher pile-up conditions are on the way.

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