A SIMPLIFIED QUANTUM THEORETICAL DERIVATION OF THE UNRUH AND HAWKING TEMPERATURE

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Abstract

In this work we suggest a sufficiently simple for understanding “without knowing the details of the quantum gravity” and quite correct deduction of the Unruh temperature (but not whole Unruh radiation process!). Firstly, we shall directly apply usual consequences of the Unruh radiation and temperature at surface gravity of a large spherical physical system and we shall show that corresponding thermal energy can be formally quite correctly presented as the potential energy absolute value of the classical gravitational interaction between this large and a small quantum system with well-defined characteristics. Secondly, we shall inversely “postulate” small quantum system with necessary well-defined characteristics and then, after “supposition” on the equivalence between potential energy absolute value of its gravitational interaction with large system with thermal energy, we shall obtain exact value of the Unruh temperature. Moreover, by very simple and correct application of suggested formalism (with small quantum system) at thermodynamic laws, we shall successfully study other thermodynamic characteristics, especially entropy, characteristic for Unruh and Hawking radiation.

Key words: Unruh temperature, Hawking temperature, dark energy

1. Introduction

As it is well-known [1], [2] Unruh radiation represents a general quantum phenomena characteristic for all locally constantly accelerated systems (not only for the black-holes [3],[4]) on the one hand, and, on the other hand, according to the general relativistic equivalence principle there is no any difference between local acceleration and gravitational phenomena. However, deduction of the Unruh temperature (as well as Hawking black hole temperature and entropy [2],[3],[4],[5]) even in a simple way [1] is
mostly not so simple that it can be understand “without knowing the details of quantum gravity”, we paraphrase Fursaev [5].

In this work we shall suggest a sufficiently simple for understanding “without knowing the details of the quantum gravity” and quite correct deduction of the Unruh temperature (but not whole Unruh radiation process!). Firstly, we shall directly apply usual consequences of the Unruh radiation and temperature at surface gravity of a large spherical physical system and we shall show that corresponding thermal energy can be formally quite correctly presented as the potential energy absolute value of the classical gravitational interaction between this large and a small quantum system with well-defined characteristics. Secondly, we shall inversely “postulate” small quantum system with necessary well-defined characteristics and then, after “supposition” on the equivalence between potential energy absolute value of its gravitational interaction with large system with thermal energy, we shall obtain exact value of the Unruh temperature. Moreover, by very simple and correct application of suggested formalism (with small quantum system) at thermodynamic laws, we shall successfully study other thermodynamic characteristics, especially entropy, characteristic for Unruh and Hawking radiation.

2. Inverse deduction of the Unruh temperature

Consider a classical, large, spherical physical system with mass M, characteristic radius R and surface gravity, i.e. normal acceleration

\( a = \frac{GM}{R^2} \)

where G represents the Newtonian gravitational constant.

This acceleration quite generally corresponds to the Unruh temperature obtained at the well-known way [1], [2]

\( T = \frac{1}{k} \frac{\hbar}{(2\pi c)} a = \frac{1}{k} \frac{\hbar}{(2\pi c)} \left( \frac{GM}{R^2} \right) \)

or Unruh thermal energy.

\( kT = \frac{\hbar}{(2\pi c)} a = \frac{\hbar}{(2\pi c)} \left( \frac{GM}{R^2} \right) \)

where k represents the Boltzmann constant, \( \hbar \) - reduced Planck constant and c – speed of light.

Last expression can be simply formally transformed in

\( kT = G \left[ \frac{\hbar}{(2\pi c R)} \right] \frac{M}{R} = \frac{GmM}{R} \)

where

\( m = \frac{\hbar}{(2\pi c R)} \)

For

\( m \ll M \)

or, according to (5), for

\( 2\pi R = \frac{\hbar}{(mc)} \gg \frac{\hbar}{(Mc)} \)

m can be formally quantum theoretically interpreted as the mass of a small quantum system whose reduced Compton wavelength corresponds to the circumference of any great circle on the sphere corresponding to large system. But roughly classically small quantum system can be considered as a material point at the large system surface so that final right hand of (4) can be formally interpreted as the absolute value of the classical potential energy of gravitational interaction between large and small system

\( V = -\frac{GM}{R} \).
Now we shall suggest a very simple, formal, inverse deduction of the Unruh temperature (2).

Firstly, suppose formally that at the large system surface there is a quantum system with strictly defined mass by (5), so that this quantum system is small in sense of (6) or (7).

Secondly, suppose formally that usual gravitational interaction between small quantum system and large quantum system has such absolute value of the potential energy (8) that it is equivalent to some thermal energy (4).

Then, temperature corresponding to this thermal energy is formally identical to the Unruh temperature.

In further parts of this work we shall prove that suggested simple method for formal deduction of the Unruh temperature is not trivial.

3. Inversely defined Unruh temperature and entropy

Unruh radiation must satisfy thermodynamic law that, according to (2), implies

\[ \text{d}S = 1/T \text{d}(Mc^2) = k 2\pi [c^3/(hG)] R^2 \text{d}M/M = k 2\pi R^2 / L_P^2 \text{d}M/M \]

where \( S \) represents the entropy of the large system and \( L_P = [\hbar G/c^3]^{1/2} \) - the Planck length.

Exact differential form of (9) can be approximately presented as the following finite difference form

\[ \Delta S = k 2\pi R^2 / L_P^2 \Delta M/M = k 2\pi R^2 / L_P^2 n m/M \]

for \( \Delta M = nm \) for \( n=1, 2, \ldots \). It, according to (5), turns out in

\[ \Delta S = n k R c^2 / (MG) \text{ for } n=1, 2, \ldots \]

Concrete forms of (9) and (11) depend of the concrete functional dependence between \( R \) and \( M \).

Suppose a linear functional dependence between \( R \) and \( M \), i.e.

\[ R = \alpha M \]

for a constant parameter \( \alpha \) with low value limit \( 2G/c^2 \) corresponding to the Schwarzschild black hole, i.e.

\[ \alpha \geq 2G/c^2 \]

Expression (12) introduced in (9) implies

\[ dS = k 2\pi \alpha^2 / L_P^2 \text{d}\alpha \text{d}M = k \pi / L_P^2 \text{d}(\alpha^2 M^2) = k \pi / L_P^2 \text{d}(R^2) = d(k \pi R^2 / L_P^2) \]

or, after integration,

\[ S = k \pi R^2 / L_P^2 = k 4\pi R^2 / (4L_P^2) = kA/(4L_P^2) \]

where \( A=4\pi R^2 \) represents the large system surface area. As it is not hard to see expression (15) is formally identical to the remarkable Bekenstein-Hawking entropy of the Schwarzschild black hole for horizon surface \( A \) even in case when is greater than \( \alpha \) low value limit \( 2G/c^2 \). It represents an interesting fact.

Expression (12) introduced in (11) implies

\[ \Delta S = n k \alpha c^2 / G \text{ for } n=1, 2, \ldots \]

As it is not hard to see only for \( \alpha \) low value limit \( 2G/c^2 \) this expression becomes formally identical to the remarkable Bekenstein quantization of the Schwarzschild black hole entropy

\[ \Delta S = n (2k) \text{ for } n=1, 2, \ldots \]

In all other, non-limit situations we obtain larger entropy difference than for Schwarzschild black hole. For this reason we can consider (16) as a generalized, \( \alpha \) value
dependent, Bekenstein quantization of the entropy characteristic for the linear dependence between $R$ and $M$.

Further, expression (12) introduced in (2), implies
\begin{equation}
T = \frac{1}{k} \frac{\hbar G}{(2\pi c)} \frac{1}{1/M} \frac{1}{\alpha^2}.
\end{equation}
As it is not hard to see only for $\alpha$ low value limit $2G/c^2$ this expression becomes formally identical to the remarkable Hawking temperature of the Schwarzschild black hole entropy
\begin{equation}
T = \frac{1}{k} \frac{\hbar c^3}{(8\pi G)} \frac{1}{1/M}.
\end{equation}
In all other, non-limit situations we obtain smaller temperature than Hawking Schwarzschild black hole temperature.

Suppose now that large system holds constant mass density which implies the following functional dependence between $R$ and $M$
\begin{equation}
R = \alpha M^{1/3}
\end{equation}
for a constant parameter $\alpha$. For example, such situation appears by FRW universe filled dominantly with vacuum (dark) energy.

Expression (20) introduced in (9) implies
\begin{equation}
ds = k \frac{2\pi \alpha^2}{L^2_p} M^{1/3} dM = k \frac{3\pi}{L^2_p} d(\alpha^2 M^{2/3}) = \frac{3\pi}{L^2_p} d(R^2)
\end{equation}
or, after integration,
\begin{equation}
S = k \frac{3\pi}{L^2_p} R^2 = k \frac{3(4\pi R^2)}{(4L_p^2)} = k\frac{A}{(4L_p^2)}.
\end{equation}
As it is not hard to see expression (22) is formally three times larger than the remarkable Bekenstein-Hawking entropy of the Schwarzschild black hole with the same horizon surface for arbitrary $\alpha$.

Expression (20) introduced in (11) implies
\begin{equation}
\Delta S = n k \alpha M^{2/3} c^2/G = n k \alpha^3 \frac{R^2 c^2}{G} \quad \text{for } n=1, 2, \ldots
\end{equation}
Obviously this expression is dependent not only of the parameter $\alpha$ but of $M$ or $R$ too.

Further, expression (20) introduced in (2), implies
\begin{equation}
T = \frac{1}{k} \frac{\hbar G}{(2\pi c)} \frac{M^{2/3}}{1/\alpha^{3/2}} = \frac{1}{k} \frac{\hbar G}{(2\pi c)} \frac{R^2}{1/\alpha^3}.
\end{equation}
Obviously this expression is dependent not only of the parameter $\alpha$ but of $M$ or $R$ too. Moreover, when $M$ or $R$ increases then according to (24) $T$ and $kT$ increase too. All this can be very interesting for cosmological problems (whose considerations go over basic intention of this work).

4. Conclusion

In conclusion we can shortly repeat and point out the following. In this work we suggest a sufficiently simple for understanding “without knowing the details of the quantum gravity” and quite correct deduction of the Unruh temperature (but not whole Unruh radiation process!). Firstly, we shall directly apply usual consequences of the Unruh radiation and temperature at surface gravity of a large spherical physical system and we shall show that corresponding thermal energy can be formally quite correctly presented as the potential energy absolute value of the classical gravitational interaction between this large and a small quantum system with well-defined characteristics.

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