Commensurability oscillations in the Hall resistance of unidirectional lateral superlattices

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We have observed commensurability oscillations (CO) in the Hall resistance \( R_{yz} \) of a unidirectional lateral superlattice (ULSL). The CO, having small amplitudes (\( \sim 1 \, \Omega \)) and being superposed on a roughly three orders of magnitude larger background, are obtained by directly detecting the difference in \( R_{yx} \) between the ULSL area and the adjacent unmodulated two-dimensional electron gas area and then extracting the odd part with respect to the magnetic field. The CO thus obtained are compared with a theoretical calculation and turn out to have the amplitude much smaller than the theoretical prediction. The implication of the smaller-than-predicted CO in \( R_{yx} \) on the thermoelectric power of ULSL is briefly discussed.

I. INTRODUCTION

Commensurability oscillations (CO), also known as Weiss oscillations, have been arguably one of the best-known magnetoresistance phenomena in mesoscopic systems since their discovery in 1989 [1, 2]. They were uncovered in a unidirectional lateral superlattice (ULSL), a two-dimensional electron gas (2DEG) subjected to a weak one-dimensional (1D) periodic modulation \( V(x) \) of the electrostatic potential. The most prominent oscillations were observed in the magnetoresistance \( R_{xx} \) along the modulation, with the minima taking place at the flat-band conditions,

\[
\frac{2R_c}{a} = n - \frac{1}{4}, \quad (n = 1, 2, 3, ...) \tag{1}
\]

where \( a \) is the period of \( V(x) \) and \( R_c = \hbar k_F/(e|B|) \) is the cyclotron radius, with \( k_F = \sqrt{2\pi n_e} \) the Fermi wavenumber, \( n_e \) the electron density, and \( e \) the elementary charge. We assume a sinusoidal modulation \( V(x) = V_0 \cos(2\pi x/a) \) throughout the paper. The magnetic field B is applied perpendicular (\( \parallel \) z axis) to the 2DEG plane (\( x-y \) plane), see Fig. 1. Oscillations were also observed in the transverse direction \( R_{yy} \), albeit with much smaller amplitudes and taking maxima instead of minima at Eq. 1. Soon after the discovery, a pictorial explanation invoking the \( E \times B \) drift velocity of semiclassical cyclotron orbits was presented [3], which captures the physics behind the dominant mechanism (band contribution, ascribed to the modulation of the Landau-band dispersion and hence of the group velocity) generating the oscillations in \( R_{xx} \). However, full understanding of CO in a ULSL, including the oscillations in \( R_{yy} \), requires quantum mechanical theories [4], [5], in which additional contribution due to the modulation of the density of states (collisional contribution) is implemented.

Although occasionally overlooked, the theories [4], [5] predict the presence of CO also in the Hall resistance \( R_{yx} \), resulting from the collisional contribution as is the case in \( R_{yy} \). To the knowledge of the present authors, however, an unambiguous experimental observation of CO in the Hall resistance of a ULSL has never been reported for nearly three decades after the theoretical predictions [3]. We surmise that the observation has been hampered mainly by two obstacles: the smallness of the amplitudes and unintentional mixing of the \( R_{xx} \) component into the measurement. First, the amplitude of the oscillatory part \( \delta R_{yz} \) is predicted to be of the order of \( 1 \, \Omega \), accounting for only \( \sim 0.1\% \) of the total \( R_{yx} \geq 1 \, k\Omega \gg R_{xx} \). The signal from \( \delta R_{yx} \) can thus readily be buried in the noise level for the measurement setup with sensitivity adjusted to measure \( R_{yx} \). Second, due to inevitable imperfectness of the Hall bar device, e.g., the misalignment of voltage probes, a small portion of \( R_{xx} \) can inadvertently mix into the measured \( R_{yx} \). The effect of the mixed \( R_{xx} \) is totally insignificant in the usual measurement of \( R_{yx} \), since \( R_{xx} \ll |R_{yx}| \) for \( B \geq 0.1 \, T \) in high-mobility 2DEGs. Focusing on the oscillatory parts, however, parasitic \( \delta R_{xx} \) component can easily outweigh the intrinsic \( \delta R_{yx} \), since the amplitudes of the former is about two orders of magnitude larger than the predicted amplitudes of the latter.

Here and in what follows, we denote the oscillatory part of a quantity \( X \) by \( \delta X \), and the difference in \( X \) with and without \( V(x) \) by \( \Delta X \). The latter can contain the nonoscillatory part induced by \( V(x) \) in addition to \( \Delta X \).

In the present study, we circumvent these problems by employing simple techniques: directly measuring the excess Hall resistivity \( \Delta R_{yx} \) attributable to \( V(x) \) and then extracting the antisymmetric part with respect to the magnetic field. The \( \delta R_{yx} \) thus obtained is compared with \( \delta R_{yx} \) numerically calculated from the formula for the conductivity \( \sigma_{yx} \) expressed in terms of summation over the Landau indices given in Ref. [5]. To gain transparent insight into the behavior of \( \sigma_{yx} \) and to efficiently extract the oscillatory part, we also deduce an analytic asymptotic expression that approximates the \( \sigma_{yx} \) quite well. We find that the observed \( \delta R_{yx} \) is much smaller than the theoretical prediction, even if we consider damping of the oscillations due to small angle scatterings neglected in the original theory.
The present study is partly motivated by rather counterintuitive isotropic behavior of the CO in the Seebeck coefficients (diagonal components of the thermopower tensor) of a ULSL predicted in Ref. [8], which we have recently noticed to be strongly related via the Mott tensor. Wiring for directly measuring the excess Hall resistance, \( \Delta R_{yx} = R_{yx,U} - R_{yx,P} \), introduced by \( V(x) \) is also shown.

II. EXPERIMENTAL DETAILS AND RESULTS

Figure 1 illustrates the schematics of the Hall bar device used in the present study. The device contains a modulated area (ULSL) and a plain 2DEG (p2DEG) area in series, where the subscripts U and P represent ULSL and p2DEG areas, respectively. The device was fabricated from a conventional GaAs/AlGaAs 2DEG wafer having the mobility \( \mu = 70 \, \text{m}^2/\text{Vs} \) and the electron density \( n_e = 2.1 \times 10^{15} \, \text{m}^{-2} \). Modulation \( V(x) \) with the period \( a = 184 \, \text{nm} \) was introduced by placing a grating of negative-tone electron-beam resist on the surface of the ULSL area [11], exploiting the strain-induced piezoelectric effect [11, 12]. All the measurements in this study were performed at 4.2 K.

In Fig. 2(a), we plot \( R_{yx,U}/P \) and \( R_{xx,U}/P \) measured employing standard low-frequency (\( f = 73 \, \text{Hz} \)) ac lock-in technique with the current \( I = 100 \, \text{nA} \). \( R_{xx,U} \) exhibits prominent CO with the minima occurring at the positions given by Eq. (1). Small-amplitude oscillations observed at higher magnetic-field regions (\( |B| \gtrsim 0.5 \, \text{T} \)) both in \( R_{xx,U} \) and \( R_{xx,P} \) are the Shubnikov-de Haas (SdH) oscillations. On the other hand, \( R_{yx,U}/P \) appears as a featureless line in the plots. To extract the component deriving from \( V(x) \), we take the differences

\[
\Delta R_{yx} = R_{yx,U} - R_{yx,P} \quad \text{and} \quad \Delta R_{xx} = R_{xx,U} - R_{xx,P},
\]

and plot them in Fig. 2(b). Since the difference is large for \( R_{xx} \), \( \Delta R_{xx} \) can be obtained reliably by simply subtracting the two traces in Fig. 2(a) numerically. We can see that the SdH oscillations are partially canceled out in \( \Delta R_{xx} \) [13]. In \( R_{yx,U} \), by contrast, minuscule difference \( \sim \Omega \) observable in Fig. 2(a) needs to be drawn out from orders of magnitude larger \( \sim \Omega \) values. To do this with sufficient signal-to-noise (S/N) ratio, we collect the excess Hall resistance \( \Delta R_{yx} \) directly, employing the arrangement depicted in Fig. 1. The Hall voltages from ULSL and p2DEG areas are first amplified (\( \times 100 \)) by separate differential preamplifiers [14], and then their outputs are plugged into differential input of a lock-in amplifier [13]. The input voltage range of the lock-in amplifier can thus be adjusted to the minimum range that encompasses the small difference voltage, which serves to significantly improve the S/N ratio. As can be seen in Fig. 2(b), \( \Delta R_{yx,U} \) obtained by this method clearly shows oscillations corresponding to both CO and partially canceled SdH oscillations (or, more precisely, incipient quan-
tum Hall plateaus). In the present paper, we focus on the CO. A notable feature is the asymmetry between $B > 0$ and $B < 0$ regions. In both regions, maxima are observed roughly at the flat-band conditions Eq. (1). However, the amplitudes of the oscillations are much larger in $B > 0$. As mentioned earlier, the observed CO are considered to be composed of two components: intrinsic $\delta R_{xx}$ and parasitic $\delta R_{xx}$. Since $\delta R_{xx}$ is an odd function of $B$ while $\delta R_{xx}$ is an even function, the two components are superposed either destructively ($B < 0$ in the present case) or constructively ($B > 0$), depending on the sign of the magnetic field. This explains the observed asymmetry in the CO amplitudes. We have measured several ULSL devices in addition to the one shown in Fig. 2. Similar asymmetry was observed for all of them.

In order to separate the two components, we take the even $\Delta R_{xx,even}$ and the odd $\Delta R_{xx,odd}$ parts of $\Delta R_{xx}$, $\Delta R_{xx,even}(B) = [\Delta R_{xx}(B) + \Delta R_{xx}(-B)]/2$ and $\Delta R_{xx,odd}(B) = [\Delta R_{xx}(B) - \Delta R_{xx}(-B)]/2$, corresponding to the parasitic and the intrinsic components, respectively, and plot them in Fig. 3. Noting that the Hall probes in both ULSL and p2DEG areas generally can pick up the corresponding parasitic $R_{xx,even}$ and the intrinsic $R_{xx,odd}$ components independently, possibly with differing weights, we can expect that the parasitic component can be expressed by the linear combination $\alpha R_{xx,even} + \beta R_{xx,odd}$ with small values of $\alpha$ and $\beta$. By properly selecting $\alpha$ and $\beta$, with special care to reproduce the oscillatory part due to the CO [see also Fig. 4(a)], fairly good agreement can be achieved between $\alpha R_{xx,even} + \beta R_{xx,odd}$ and the observed $\Delta R_{xx,even}$, supporting the interpretation on the origin of $\Delta R_{xx,even}$. This confirms that the remnant $\Delta R_{xx,odd}$ is the intrinsic CO in $R_{xx}$, the target we are seeking in the present study. In the plot of the odd part, we subtracted a linear term $\gamma B$, which is attributable to the small difference in the electron density, $\Delta n_e \approx -1.3 \times 10^{13} \text{ m}^{-2}$, between the ULSL and the p2DEG areas.
III. COMPARISON WITH THEORETICAL CALCULATIONS

A. Deducing superlattice parameters from commensurability oscillations in the magnetoresistance

The next step is to compare the observed $\delta R_{yx}$ with the theoretical prediction. Before discussing $\delta R_{yx}$, however, we briefly review well-documented behavior of $\delta R_{xx}$ from which we draw out parameters characterizing our ULSL. As mentioned earlier, two different mechanisms, the band and the collisional contributions, are responsible for CO. Asymptotic analytic expressions for the oscillatory parts of the conductivity, valid at low magnetic fields where large numbers of Landau levels are occupied, are given for the two contributions as,

$$\delta \sigma_{yy}^{\text{band}} = \frac{\sigma_0 V_0^2}{E_F \hbar \omega_c ak_F} A \left( \frac{\pi}{\mu_c B} \right) A \left( \frac{T}{T_a} \right) \sin r_c \quad (2)$$

and

$$\delta \sigma_{xx}^{\text{col}} = -\frac{3 \sigma_0 V_0^2 ak_F}{8 \pi^2 E_F \hbar \omega_c (\mu B)^2} A \left( \frac{\pi}{\mu_w B} \right) A \left( \frac{T}{T_a} \right) \sin r_c, \quad (3)$$

respectively, where $\sigma_0 = e n_e \mu$ is the conductivity at $B = 0$, $E_F$ the Fermi energy, $\omega_c = e B / m^*$ the cyclotron angular frequency with $m^*$ the effective mass, $r_c \equiv 4 \pi R_c / a$, $T_a \equiv \hbar \omega_c ak_F / (4 \pi^2 k_B)$, and $A(x) \equiv x / \sin x$. Although absent in the original theories in Refs. [8–11], an additional damping factor $A[\pi / (\mu_w B)]$ accounting for the effect of small angle scattering, with the value of $\mu_w$ close to the quantum mobility $\mu_q$, are contained in Eqs. (2) and (3) in addition to the thermal damping factor $A(T/T_a)$. More detailed discussion on the factor $A[\pi / (\mu_w B)]$ will be given below. The resistivity tensor $\rho_{ij}$ $(i,j = x,y)$ is obtained by inverting the conductivity tensor $\sigma_{ij}$: $\sigma_{xx} = \sigma_{xx}^{\text{col}} + \delta \sigma_{xx}^{\text{band}}$, $\sigma_{yy} = \sigma_{xx}^{\text{col}} + \delta \sigma_{xx}^{\text{band}}$, and $\sigma_{xy} = -\sigma_{yx} = \sigma_{xx}^{\text{col}} + \delta \sigma_{yx}$, where $\sigma_{xx}^{\text{col}} = \sigma_0 / (1 + \mu B^2)$ and $\sigma_{xx}^{\text{col}} = \sigma_0 / (1 + \mu_b B^2)$ are the semiclassical conductivities for a p2DEG. Noting that $\mu B \gg 1$ and $|\delta \sigma_{xx}^{\text{band}}| \gg |\delta \sigma_{xx}^{\text{col}}|$, and using the relation $R_{xx}/R_0 = \sigma_0 \rho_{xx}$ with $R_0$ representing $R_{xx}$ at $B = 0$, we obtain, to a good approximation,

$$\frac{\delta R_{xx}}{R_0} = (\mu B)^2 \frac{\delta \sigma_{xx}^{\text{band}}}{\sigma_0} \quad (4)$$

and likewise $R_{yx}/R_0 = (\mu B)^2 \delta \sigma_{xx}^{\text{col}} / \sigma_0$. Equation (4) has been shown to describe experimentally obtained CO extremely well [11]. This is confirmed in Fig. 3(a), which shows $\delta R_{xx}$ extracted from $R_{xx}$ in Fig. 2(a) by subtracting slowly varying background following the protocol detailed in Ref. [11], along with $\delta R_{xx}$ in Eq. (4) obtained by the fitting, employing $V_0$ as fitting parameters. The fitting yields $V_0 = 0.35$ meV and $\mu_w = 8.2$ m$^2$/Vs. The value of $\mu_w$ is close to $\mu_q = 8.6$ m$^2$/Vs deduced from the SdH oscillations in $R_{xx}$ plotted in Fig. 3(a).

B. Asymptotic analytic expressions for the Hall conductivity

Now we turn to the Hall component. We start with the expression of the Hall conductivity in a ULSL presented in Ref. [8],

$$\sigma_{yx} = -\frac{2e^2}{h} \sum_{N=0}^{\infty} (N+1) \int_0^1 f_{E_F}(E_N, \xi) - f_{E_F}(E_{N+1}, \xi) \frac{\pi N}{[1 + \lambda_N \cos (2\pi \xi)]^2} d\xi \quad (5)$$

FIG. 5. (a) The increment of the Hall conductivity due to $V(x)$, calculated with the exact [thick solid line, $\Delta \sigma_{yx}$ in Eq. (8)] and the approximate analytic [thin solid line, $\Delta \sigma_{yx}^{A(1)}$ in Eq. (13)] formulas. Three oscillatory parts calculated from exact (thick solid line, $\Delta \sigma_{yx}^{A(1)} = \sigma_{yx, \text{bg}}^{A(1)} + \delta \sigma_{yx}^{A(1)}$ in Eq. (10)), is also plotted (dashed line). (b) Oscillatory parts calculated from exact (thick solid line, $\Delta \sigma_{yx}^{A(1)} = \sigma_{yx, \text{bg}}^{A(1)} + \delta \sigma_{yx}^{A(1)}$) and approximate [thin solid line, $\delta \sigma_{yx}^{A(1)}$ in Eq. (15)] formulas. Three constituent terms of the oscillatory part are also plotted separately, with thin dashed, thin dotted, and thick dashed lines representing $\delta \sigma_{yx}^{A(1)}$, $\delta \sigma_{yx}^{A(21)}$, and $\delta \sigma_{yx}^{A(22)}$ in Eqs. (10c), (11c), and (12c), respectively. Vertical dotted lines indicate the locations of the $n$-th flat-band condition given by Eq. (1).
with

\[ E_{N,\xi} = E_N + V_0 \exp \left( -\frac{u}{2} \right) L_N(u) \cos(2\pi\xi) \] (6)

and

\[ \lambda_N = \frac{V_0}{\hbar \omega_c} \exp \left( -\frac{u}{2} \right) L_{N+1}(u), \] (7)

where \( f_{eq}(E) \) = \( \{1 + \exp[(E - E_F)/k_B T]\}^{-1} \) is the Fermi-Dirac distribution function, \( N \) is the Landau index, \( E_N \equiv (N + 1/2) \hbar \omega_c \), \( \xi \equiv x_0/a \) with \( x_0 \) the guiding center, \( u \equiv 2\pi^2 l^2 / a^2 \) with \( l = \sqrt{\hbar/(e|B|)} \) the magnetic length, and \( L_N(u) \) and \( L_{N+1}(u) \) are the Laguerre and the associated Laguerre polynomials. Note that Eq. (5) is valid only for \( B > 0 \). Since \( \sigma_{yx} \) is an antisymmetric function with respect to \( B \), \( \sigma_{yx} \) at \( B < 0 \) is obtained by inverting the sign of Eq. (5). The increment of \( \sigma_{yx} \) introduced by the modulation,

\[ \Delta \sigma_{yx} = \sigma_{yx}(V_0) - \sigma_{yx}(V_0 = 0), \] (8)

umerically calculated \cite{17} using Eq. (5) with the parameters in the present ULSL is plotted in Fig. 5(a).

Since the behavior of \( \sigma_{yx} \), notably the phase of the oscillations, are not readily perceived from Eq. (5), we deduce an asymptotic analytic expression, basically following the prescription taken for \( \sigma_{xx} \) and \( \sigma_{yy} \) in Ref. \cite{8}.

Via the deriving procedure detailed in the Appendix, we arrive at an approximate formula \( \sigma_{yx} \approx \sigma_{yx} \),

\[ \sigma_{yx} = \sigma_{yx}^{(1)} + \sigma_{yx}^{(21)} + \sigma_{yx}^{(22)}, \] (9)

with

\[ \sigma_{yx}^{(1)} = \frac{\nu e^2}{\hbar} + \Delta \sigma_{yx}^{(1)} = \frac{\nu e^2}{\hbar} + \sigma_{yx,bg} + \delta \sigma_{yx}^{(1)}, \] (10a)

\[ \sigma_{yx,bg} = \nu e^2 \left( \frac{3}{2} \right) \lambda c^2 \sin \left( r_c + \delta \phi \right), \] (10b)

\[ \delta \sigma_{yx}^{(1)} = -\nu e^2 \left( \frac{T_c}{T_a} \right) \frac{3}{2} \lambda c^2 \sin \left( r_c + \delta \phi - \frac{\pi}{a k_F} \right), \] (10c)

and

\[ \sigma_{yx}^{(21)} = \sigma_{yx,bg} + \delta \sigma_{yx}^{(2)} \] (11a)

\[ \delta \sigma_{yx}^{(2)} = -\text{sgn}(B) \frac{\nu e^2}{\hbar} \frac{1}{\pi} \cos \left( \frac{\pi}{a k_F} \right) \lambda c^2 \cos \left( \frac{\delta \phi}{2} + \frac{\pi}{a k_F} \right), \] (11b)

\[ \delta \sigma_{yx}^{(22)} = \text{sgn}(B) \frac{\nu e^2}{\hbar} \frac{1}{\pi} \cos \left( \frac{\pi}{a k_F} \right) \lambda c^2 \cos \left( \frac{\delta \phi}{2} - \frac{\pi}{a k_F} \right), \] (11c)

where \( \nu = n_c e/(eB) \) is the Landau-level filling factor (\( \nu < 0 \) for \( B < 0 \) by the definition), \( \lambda_c = 2\sqrt{2/\pi} [V_0/(\hbar \omega_c)][\pi/(a k_F)] r_c^{-1/2} \), \( \delta \phi = 2 \cot^{-1} r_c \), and \( \text{sgn}(x) \) represents the sign of \( x \). By collecting the corresponding terms, we obtain the increment of the conductivity, the nonoscillatory background of the increment, and the oscillatory part,

\[ \Delta \sigma_{yx} = \Delta \sigma_{yx}^{(1)} + \sigma_{yx}^{(21)} + \sigma_{yx}^{(22)}, \] (13)

\[ \Delta \sigma_{yx,bg} = \sigma_{yx, bg} + \sigma_{yx, bg}^{(21)} + \sigma_{yx, bg}^{(22)}, \] (14)

and

\[ \delta \sigma_{yx} = \delta \sigma_{yx}^{(1)} + \delta \sigma_{yx}^{(21)} + \delta \sigma_{yx}^{(22)}, \] (15)

respectively. Figure 5(a) illustrates that the asymptotic analytic expression Eq. (13) reproduces Eq. (5) quite well, except for the small-amplitude oscillations at \( B \gtrsim 0.6 \) T resulting from the Landau quantization. This allows us to use the approximate background \( \Delta \sigma_{yx,bg}^{(1)} \) to extract the oscillatory part from \( \Delta \sigma_{yx} \) in Eq. (3). The oscillatory part thus obtained, \( \delta \sigma_{yx} = \Delta \sigma_{yx} - \Delta \sigma_{yx,bg}^{(1)} \), is plotted in Fig. 5(b) along with \( \delta \sigma_{yx}^{(21)} \) in Eq. (13).

The analytic expression lets us grasp the outline of the behavior of the oscillations. Since \( \delta \sigma_{yx}^{(1)} = \sigma_{yx}^{(21)} \), and \( \delta \sigma_{yx}^{(22)} \) all oscillate with different phases depending on \( B \) through \( \delta \phi \), the phase of the CO in \( \sigma_{yx} \) is expected to exhibit rather complicated behavior. We note, however, that \( \pi/(a k_F) = 0.148 \) in the present sample is generally small for experimentally achievable values of \( a \) and that \( \delta \phi \) is also small (\( \lesssim 0.4 \) in the magnetic-field range where CO is observed) and approaches 0 with decreasing \( B \). Furthermore, at low magnetic fields where
\( \nu \) is large, \( \delta \sigma_{yx}^{(22)} \) becomes much smaller than the other terms. The dominance of \( \delta \sigma_{yx}^{(1)} \) and \( \delta \sigma_{yx}^{(21)} \), combined with the smallness of \( \pi/(\akp \delta F) \) and \( \delta F \), indicates that the oscillation phase of \( \delta \sigma_{yx} \) is close to that of \( \delta \sigma_{xx}^{\text{col}} \) and thus takes maximum at the flat-band conditions Eq. (1), or equivalently, at \( \delta F = 2\pi n - \pi/2 \). The calculated \( \delta \sigma_{yx} \) plotted in Fig. 4(b) are seen to actually take maxima at the flat-band conditions at low magnetic fields. With the increase of the magnetic field, slight deviation of the relative importance of the third term \( \delta \sigma_{yx}^{(22)} \), whose oscillation phase differs from \( \delta \sigma_{yx}^{(21)} \) by \( \pi/2 \). Noting that \( (\akp/\pi) \sin [\pi/(\akp)] \sim 1 \), the two dominant terms are expected to have comparable oscillation amplitudes, which can also be confirmed in Fig. 4(b).

C. Comparison between experimental and calculated commensurability oscillations in the Hall resistance

We obtain the Hall resistivity \( \rho_{yx} (= R_{yx} \text{ in a 2DEG}) \) by inverting the conductivity tensor, and find that the oscillatory part of the Hall resistance \( R_{yx} \) is given, considering \( |\delta \sigma_{yx}^{(1)}| \ll |\sigma_{yx}| \), by

\[
\delta R_{yx} = \frac{1}{\sigma_0} \{[(\mu B)^2 - 1]|\delta \sigma_{yx}^{(1)} + \mu B(2\delta \sigma_{xx}^{\text{col}} + \delta \sigma_{yy}^{\text{band}})]\}.
\]

The oscillations are dominated by the first term. The second term is negligibly small since \( |\delta \sigma_{xx}^{\text{col}}| \ll |\delta \sigma_{yy}^{\text{band}}| \). The third term, having the phase roughly opposite to the first term, serves to reduce the oscillation amplitude. In Fig. 4(b), we compare the experimentally obtained oscillatory part \( \delta R_{yx,\text{odd}} \), extracted from \( \delta R_{yx,\text{odd}} \) shown in Fig. 3 by subtracting the slowly varying background \( \delta R_{yx} \) calculated by Eq. (16) using the sample parameters deduced above from the analysis of \( \delta R_{xx} \). The figure shows that the observed amplitude of the CO is much smaller than the theoretical prediction especially at lower magnetic fields, while the phase of the oscillations is roughly in agreement.

It is well known that the scattering in a GaAs/AlGaAs 2DEG is predominantly caused by remote ionized donors, for which scattering angles are generally small \( [18] \). Although the momentum relaxation is not significant for small scattering angles, cyclotron orbits are disturbed regardless of the scattering angle and thus the CO amplitudes are severely diminished even by the small-angle scattering \( [11, 16] \). The damping of the CO is more prominent for lower magnetic fields where the circumference of the cyclotron orbit \( 2\pi R_c \) becomes large. The effect of small-angle scattering, which has not been considered thus far for \( \sigma_{yx}^{\text{col}} \), can be implemented by multiplying \( A[\pi/(\mu \omega B)] \), following the recipe applied for \( \delta \sigma_{xx}^{\text{band}} \) and \( \delta \sigma_{xy}^{\text{col}} \) described above. Figure 4(b) reveals, however, that the discrepancy between the amplitudes of the observed and the calculated CO is still large even with the inclusion of the effect of the small-angle scattering. Apparently, the theory overestimates the CO amplitudes in the Hall resistance, possibly because \( \delta R_{yx} \) is much more vulnerable to the scattering compared to \( \delta R_{xx} \) and thus its damping cannot be described by the factor \( A[\pi/(\mu \omega B)] \) with the same value of \( \mu \omega \). Note that the damping factor \( A[\pi/(\mu \omega B)] \) with \( \mu \omega \approx \mu_0 \) is firmly established theoretically \( [18] \) and experimentally \( [11] \) only for \( \delta \sigma_{yy}^{\text{band}} \) and thus may not be applicable to \( \delta \sigma_{xx}^{\text{col}} \) and \( \sigma_{yx}^{\text{A}} \) without modification \( [19] \). As demonstrated in Fig. 4(b), the heavy damping of \( \sigma_{yx}^{\text{A}} \) achieved by halving the \( \mu_0 \) roughly reproduces the experimental CO amplitudes, albeit without solid theoretical underpinnings.

IV. POSSIBLE ANISOTROPY IN THE SEEBECK COEFFICIENT

Finally, we briefly discuss the effect of \( \delta \sigma_{yx} \) on the CO of the Seebeck coefficients \( S_{xx} \) and \( S_{yy} \), the diagonal components of the thermopower tensor \( S_{ij} \). The theory \( [8] \) predicts that \( S_{xx} \) and \( S_{yy} \) are almost identical and thus the Seebeck coefficient accommodates CO isotropically. \( S_{ij} \) can be written as the product of the resistivity tensor \( \rho_{ij} \) and the thermoelectric conductivity tensor \( \sigma_{ij} \). The latter is related to the conductivity tensor by the Mott formula \( [21] \), \( \varepsilon_{ij} = -\text{LoTe}(\delta \sigma_{ij,T=0}/dE)\varepsilon_{i}E_{j} \) with \( L_{0} = \pi^{2}k_{B}^{2}/(3e^{2}) \) the Lorenz number, at low temperatures \( [21] \). We thus have \( \varepsilon_{xx} = \rho_{xx}\varepsilon_{xx} + \rho_{yx}\varepsilon_{yx} \) and \( S_{yy} = \rho_{yy}\varepsilon_{yy} - \rho_{yx}\varepsilon_{yx} \), where we made use of the relations \( \rho_{xy} = -\rho_{yx} \) and \( \varepsilon_{xy} = -\varepsilon_{yx} \). The corresponding oscillatory parts due to the CO are given, to a good approximation, by \( \delta S_{xx} \approx (\delta \varepsilon_{xx} + \mu B\delta \varepsilon_{yx})/\sigma_{0} \) and \( \delta S_{yy} \approx (\delta \varepsilon_{yy} + \mu B\delta \varepsilon_{yx})/\sigma_{0} \). Since \( \mu_{B} \gg 1 \) for a high-mobility 2DEG in the magnetic field range where CO can be observed, both \( \delta S_{xx} \) and \( \delta S_{yy} \) are dominated by the identical second term if the magnitude of \( \delta \varepsilon_{yx} \) is comparable to those of \( \delta \varepsilon_{xx} \) and \( \delta \varepsilon_{yy} \) as predicted in the theory \( [8] \), leading to the rather counterintuitive isotropic behavior \( \delta S_{xx} \approx \delta S_{yy} \). However, the small \( |\delta \varepsilon_{yx}| \) we have experimentally found in the present study, combined with the Mott formula, implies that \( |\delta \varepsilon_{yx}| \) is much smaller than the theoretical prediction. The resulting enhancement in the relative importance of the first terms can lead to anisotropic behavior \( \delta S_{xx} \neq \delta S_{yy} \). This, however, needs to be verified experimentally \( [23] \).

V. SUMMARY

To summarize, we have experimentally captured the CO in \( R_{yx} \) of a ULSL, theoretically predicted some 30 years ago, by employing the measurement arrangement designed to efficiently pick out the extra component of \( R_{yx} \) introduced by \( \varepsilon(x) \) and further by eliminating the parasitic component due to an unintentionally mixed \( R_{xx} \) distinguishable as an even function of the magnetic field.
The amplitude of the CO thus observed is found to be much smaller than the theoretical prediction. We have also deduced an asymptotic analytic expression for CO in the Hall conductivity $\Delta \sigma_{yx}$ to facilitate the comparison between the theory and the experiment and to clarify the oscillation phase of $\Delta \sigma_{yx}$. The smallness of $\Delta \sigma_{yx}$ demonstrated in the present experiment suggests the possibility of considerable anisotropy in the Seebeck coefficient, contrary to the theoretical prediction.

\[
\sigma_{yx} \simeq \frac{2e^2}{h} \sum_{N=0}^{\infty} (N + 1) \int_0^1 d\xi \left\{ f_{E_F}(E_N) - f_{E_F}(E_{N+1}) + \left[ \frac{df_{E_F}(E_N)}{dE} L_N(u) - \frac{df_{E_F}(E_{N+1})}{dE} L_{N+1}(u) \right] V_0 e^{-\varphi} \cos(2\pi \xi) \right\}
\times \left[ 1 - 2\lambda_N \cos(2\pi \xi) + 3\lambda_N^2 \cos^2(2\pi \xi) \right]
\equiv \sigma_{yx}^{(1)} + \sigma_{yx}^{(2)}
\]

with

\[
\sigma_{yx}^{(1)} = \frac{2e^2}{h} \sum_{N=0}^{\infty} (N + 1) \left[ f_{E_F}(E_N) - f_{E_F}(E_{N+1}) \right] \left( 1 + \frac{3}{2} \lambda_N^2 \right)
\]

and

\[
\sigma_{yx}^{(2)} = \frac{2e^2}{h} \sum_{N=0}^{\infty} (N + 1) \left[ \frac{df_{E_F}(E_N)}{dE} L_N(u) + \frac{df_{E_F}(E_{N+1})}{dE} L_{N+1}(u) \right] V_0 e^{-\varphi} \lambda_N,
\]

where we performed the integration with respect to $\xi$. In the asymptotic limit of many filled Landau levels ($N \gg 1$), we can make the replacements, $e^{-u/2L_N(u)} \to (\pi^2 Nu)^{-1/4} \cos(2\sqrt{Nu} \pi/4)$ and $e^{-u/2L_N^{-1}(u)} \to u(4\sqrt{\pi})^{-1}(Nu)^{-5/4} \left[ 4\sqrt{Nu} \cos \left( 2\sqrt{Nu} \pi/4 \right) - \sin \left( 2\sqrt{Nu} \pi/4 \right) \right]$, and take the continuum limit, $E_N \to E$, $\sum_N \to \int_{h\omega_c/2}^{\infty} dE/(h\omega_c)$ at low temperatures. We further make an approximation $f_{E_F}(E_N) - f_{E_F}(E_{N+1}) \approx h\omega_c(-\partial f/\partial E)|_{E_{Fy}}$. By performing the energy integral, we get

\[
\sigma_{yx}^{(1)} \simeq \frac{e^2}{h} \left[ 1 + \frac{3}{2} \lambda_c^2 - A \left( \frac{T}{T_{ah}} \right) \frac{3}{2} \lambda_c^2 \sin(r_c + \delta_F) \right]
\]

and $\sigma_{yx}^{(2)} = \sigma_{yx}^{(2a)} + \sigma_{yx}^{(2b)}$ with

\[
\sigma_{yx}^{(2a)} \simeq (\nu + 1) \frac{e^2}{h} \sqrt{\frac{2}{\pi}} \frac{V_0}{\hbar \omega_c} \frac{\lambda_c}{\sqrt{r_c}} \left[ A \left( \frac{T}{T_{ah}} \right) \cos \left( \frac{r_c + r_c}{2} + \frac{\delta_F}{2} \right) - A \left( \frac{T}{T_{ah}} \right) \sin \left( \frac{\delta_F}{2} + \frac{r_c - r_c}{2} \right) \right]
\]

and

\[
\sigma_{yx}^{(2b)} \simeq (\nu - 1) \frac{e^2}{h} \sqrt{\frac{2}{\pi}} \frac{V_0}{\hbar \omega_c} \frac{\lambda_c}{\sqrt{r_c}} \left[ A \left( \frac{T}{T_{ah}} \right) \cos \left( \frac{r_c - r_c}{2} - \frac{1}{\sqrt{1 + r_c^2}} \right) \right]
\]

where $\sigma_{yx}^{(2a)} (\sigma_{yx}^{(2b)})$ derives from the first (second) term in Eq. (A.3), $\lambda_c \equiv 4\sqrt{2/\pi} V_0/(\hbar \omega_c) \sqrt{2} \sqrt{(1 + r_c^2)/r_c}$, $T_{ah} \equiv T_{ah}(r_c^2 + 1)/(r_c^2 - 1)$, $r_c^2 \equiv r_c \sqrt{1 \pm \nu^{-1}}$, $\delta_F \equiv 2 \cos^2 r_c^2$, $T_{ah}^* \equiv T_{ah} \cdot 2\sqrt{1 - \nu^{-2}(r_c^2 + 1)/\sqrt{1 + \nu^{-2}(r_c^2 + 1) + \sqrt{1 - \nu^{-2}(r_c^2 + 1)}}}$, $T_{ah}^* \equiv T_{ah} \cdot \sqrt{1 - \nu^{-2}(r_c^2 + 1)/\sqrt{1 + \nu^{-2}(r_c^2 + 1)}}$, and $\delta_F^* \equiv T_{ah}^* 2\sqrt{1 - \nu^{-2}(r_c^2 + 1)/\sqrt{1 + \nu^{-2}(r_c^2 + 1)}}$.

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APPENDIX: DERIVATION OF THE ASYMPTOTIC ANALYTIC EXPRESSIONS

In this Appendix, we describe the derivation of the asymptotic analytic expression $\sigma_{yx}^A$ given by Eq. (4) from $\sigma_{yx}$ in Eq. (3). Noting that $\lambda_N \lesssim 0.1$ for practical values of $V_0$, $a$, and $B$, we obtain, up to $O(\lambda_N^2)$,
\[ \sqrt{1 - \nu^{-1}(r_c^2 - 1)}. \]

Noting that \(\nu, r_c, r_c^\pm \gg 1\) in the range of the magnetic field where CO is observed, we may neglect the difference between \(T_a\) and \(T_{aH}\), \(T_{aH} \approx T_a\) to a good approximation. We can also make an approximation \(\lambda_c \approx 4\sqrt{2\pi/|V_0|}(\hbar\omega_c) |ur_c|^{-3/2}\) to attain the definition of \(\lambda_c\) presented in the main text. With these approximations, Eq. (A.4) becomes equivalent to Eq. (10a) in the main text. It can also readily be found that \(\lambda(T/T_{aH}) \approx 1\) at the cryogenic temperatures where CO is observed. After approximating \(\lambda_c/\sqrt{r_c}\) in Eq. (A.5) by \(\lambda_c/\sqrt{r_c}\) for the sake of simplicity, we expand \((\nu+1)\) and \((\nu-1)\) in Eqs. (A.5) and (A.6), respectively. Then we collect the terms containing (not containing) the factor \(\nu\), which yields Eq. (11a) [Eq. (12a)] by further using the approximations \(r_c^\pm \approx r_c(1 + \nu^{-1}/2)\) and \(\delta_{xy}^\pm \approx \delta_{B} \mp \nu^{-1}r_c/(1 + r_c^2)\) within the cos and sin terms. The factor \(\lambda_c(B)\) is incorporated in Eqs. (12b) and (12c) to ensure the antisymmetry with respect to \(B\). [See the caveat to Eq. (5) in the main text.]

We note in passing that the expression \(\delta \sigma_{y|x} (2e^2/h)(N \mp 1)(1 + 3\lambda N^2/2)\) presented just below Eq. (28) in \([8, 24]\) corresponds to the low temperature limit of \(\delta \sigma_{y|x} \) in the present study. As mentioned in the main text, \(\delta \sigma_{y|x} \) only accounts for roughly half of the CO in \(\sigma_{y|x} \) [see also Fig. 5(a)].

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[22] We can readily show numerically that \(\delta \sigma_{y|x} \approx \rho_{xx, bg} \delta \varepsilon_{xx} - \rho_{xx, bg} \delta \varepsilon_{yx} + \varepsilon_{xx, bg} \delta \rho_{xx} - \varepsilon_{yx, bg} \delta \rho_{yx} \approx \rho_{xx, bg} \delta \varepsilon_{xx} - \rho_{xx, bg} \delta \varepsilon_{yx} + \delta \varepsilon_{yy} \approx \rho_{xx, bg} \delta \varepsilon_{yy} - \varepsilon_{xx, bg} \delta \rho_{xx} - \varepsilon_{yx, bg} \delta \rho_{yx} \approx \rho_{xx, bg} \delta \varepsilon_{yy} - \varepsilon_{xx, bg} \delta \rho_{xx} - \varepsilon_{yx, bg} \delta \rho_{yx} \), where the subscript “bg” signifies the non-oscillatory background. With \(\rho_{xx, bg} = 1/\sigma_0\) and \(\rho_{yx, bg} = -\mu B/\sigma_0\), we arrive at the approximate equations presented here.

[23] Measurements of \(S_{y|x}\) in a ULSL have been reported in [27], but to the knowledge of the present authors, no attempt to measure \(S_{yy}\) has been made thus far. We have also made measurements of the thermopower in ULSLs, and have found it difficult to obtain \(S_{y|x}\) and \(S_{yy}\) correctly, owing to the dominance of the \(S_{yy}\) component and the tilting of the temperature gradient caused by the magnetic field.[11][28].

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