Constraining the Standard Model in Motivic Quantum Gravity

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Abstract. A physical approach to a category of motives must account for the emergent nature of spacetime, where real and complex numbers play a secondary role to discrete operations in quantum computation. In quantum logic, the cardinality of a set is initially replaced by a dimension of a linear space, making contact with the increasing dimensions in an operad. The operad of associahedra governs tree level scattering, and is closely related to the permutohedra and cube tiles, where cube vertices can encode components of a spinor in higher dimensional octonionic approaches. A study of rest mass generation begins with the cosmological infrared scale, set by the neutrino masses, and its related see-saw mechanism. We employ the anyonic ribbon spectrum for Standard Model states, and consider its relation to magic star algebras, giving a context for the Koide rest mass phenomenology of charged leptons and quarks.

1. Overview
What is motivic quantum gravity? Algebraic geometers study periods associated to Feynman integrals in particle physics. Periods form a ring \( P \subset \mathbb{C} \), and we might construct this ring from smaller rings in the plane. As physicists, we seek canonical discrete structures for quantum gravity, rich enough to underlie a universal cohomology theory within which all physical amplitudes are computed. In particular, the lattice of \( e_8 \) is embedded in \( \mathbb{Z}^8/2 \), which in turn is mapped densely into \( \mathbb{C} \) using only the integers of the golden ring extension of \( \mathbb{Q} \), where \( \phi = (1 + \sqrt{5})/2 \) and \( \sigma = \sqrt{\phi + 2} \) define real numbers of the form

\[
a + b\phi + c\sigma + d\phi\sigma,
\]

for integral \( a, b, c, d \). The lattice defines the Penrose pentagonal tiling.

Thus our philosophy is not to begin with ordinary varieties or manifolds, but to consider the difficult question of their emergence from combinatorial data associated to quantum gravitational logic. Then any real or complex space, including spacetime or the full algebra of \( e_8 \), become secondary to the algorithms that generate them. Since quantum mechanics naturally defines a symmetric monoidal category \([2][3]\), one expects that its extension to gravity will also employ higher dimensional category theory in a fundamental way, combining generalized associahedra with canonical algebraic data.

Physically, the hypothesis of non local right handed neutrino states is capable of solving many cosmological conundrums. After the 2010 discovery \([4][5][6]\) of the exact correspondence between a neutrino rest mass and the present day temperature of the CMB, it was natural to consider a new IR scale defined by the overall neutrino scale at around 0.01 eV, as considered
in the neutrino condensate picture [7]. Successful predictions included a computation of the observable mass of our universe, tighter constraints on neutrino masses and an effective sterile mass for oscillation anomalies [8][9]. Connections to holography were discussed in [6]. The crucial mass photon relation comes from Wien’s law

\[ mc^2 = \beta T, \]  

now justified by quantum inertia [10][11][12][13]. Neutrino states fit into the triplet ribbon spectrum of [14][15] for the Standard Model. Branan showed in [16] how to extend Koide’s formula [17][18] for the low energy charged lepton masses to neutrino masses. An inverted see-saw relation for the neutrino scale \( m_\nu = 0.01 \) eV,

\[ m_H = \sqrt{m_\nu m_P} \]  
suggestively approximates the Higgs mass.

Neutrino states are also expected to define the vacuum in the octonionic algebras of [19][20][21], as outlined in the next section. Section 3 introduces operad polytopes and a few important categorical concepts. According to Vaughan Jones, to understand the Standard Model, you need to see three monads. A proper monad is an endofunctor that generalizes the idempotent rule \( P^2 = P \). Points are generically idempotents, either as objects in a Heyting algebra of open sets or as matrices in a Jordan algebra. A crucial monad in classical mathematics is the power set monad, and its simplest quantum analogue is determined by a symmetric monoidal category of vector spaces [22].

The importance of discretizing spaces was understood a long time ago by Nikola Tesla [23], who famously said that the Universe comes down to the magic of 3, 6, 9. These numbers show up as follows. Take the Fibonacci sequence, associated to powers of \( \phi \). Take the decimal parity of each number in the sequence. For example, the parity of 13 is \( 1 + 3 = 4 \). As with binary codes, this defines a parity check bit. Now the entire Fibonacci sequence is a repeating list of 24 numbers,

\[ 1, 1, 2, 3, 5, 8, 4, 3, 7, 1, 8, 9, 8, 8, 7, 6, 4, 1, 5, 6, 2, 8, 1, 9, \]  

containing two copies of 3, 6 and 9, which label the vertices of a hexagon in Metatron’s cube. Decimal parity is analogous to the check bit in a classical binary code. We expect that quantum gravity employs not only qubits, but also qutrits and 10-dits, to make up the base 60 of the universal clock. Tesla’s code shows how the qutrit component separates from other numbers.

2. A topological particle spectrum

In the scheme of [14], a chiral set of massless Standard Model states is given as three strand ribbon diagrams. If we forget the dyonic braiding for the moment, states are still distinguished [15] by augmenting the underlying permutations in \( S_3 \). For this single generation, \( \nu \) and \( \bar{\nu} \) annihilate to a photon identity

\[ \gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]  

We approach electroweak symmetry breaking in reverse, allowing mass to emerge from some abstract entanglement network. If the basis neutrinos are

\[ \nu = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \bar{\nu} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \]  

(6)
the electromagnetic charge is a set of twists on the three ribbon strands, where distinct twists may be assigned to each strand. We represent this by replacing 1s by one of three phases: 1 for neutral, \( \omega \) for +1/3, or \( \bar{\omega} \) for −1/3. Then the charged leptons are

\[
e_L = \bar{\omega} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad e_R^+ = \omega \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},
\]

which indeed compose to the identity. Similarly,

\[
e_L^+ = \omega \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad e_R^- = \bar{\omega} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.
\]

For quarks, put the charges onto individual strands, as in the colored matrices

\[
u_L(1) = \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & \omega \\ 1 & 0 & 0 \end{pmatrix}, \quad \nu_L(2) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega \\ \omega & 0 & 0 \end{pmatrix}, \quad \nu_L(3) = \begin{pmatrix} 0 & \omega & 0 \\ 0 & 0 & 1 \\ \omega & 0 & 0 \end{pmatrix},
\]

for left handed up quarks. Leptons are circulants while quarks are not. The \( W^\pm \) bosons are given by

\[
W^- = \bar{\omega} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad W^+ = \omega \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\]

The tricky particle in this massless model is the \( Z \) boson, built from six remaining neutral boson matrices,

\[
\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \bar{\omega} \end{pmatrix}, \quad \begin{pmatrix} \bar{\omega} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

and their three conjugates. These matrices permit a natural difermionic supersymmetry [15] through the twisted Fourier transform \( \mathbf{F} \), defined on \( e_L^- \) by

\[
\mathbf{F}(e_L^-) = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \bar{\omega} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & \bar{\omega} & \omega \\ \bar{\omega} & \omega & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & \bar{\omega} & \omega \\ \bar{\omega} & \omega & 0 \end{pmatrix} = W^-.
\]

For right handed states we replace the mixed diagonal by its complex conjugate,

\[
\mathbf{F}(e_R^-) = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \bar{\omega} & 0 \\ 0 & 0 & \omega \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & \bar{\omega} & \omega \\ \bar{\omega} & \omega & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & \bar{\omega} & \omega \\ \bar{\omega} & \omega & 0 \end{pmatrix} = W^-.
\]

In this way, circulant leptons map to electroweak bosons

\[
e^\pm \mapsto W^\pm, \quad \nu, \bar{\nu} \mapsto \gamma.
\]

Since quarks are not circulant, the corresponding bosons are mixed phase non diagonals, presumably associated to the confinement of color.

The braid group \( B_3 \) extends the underlying permutations of \( S_3 \) and covers the modular group. In Fibonacci anyon categories, \( B_3 \) models \( SU(2) \) in the construction of gates for universality in
quantum computation [24]. It is also well known that the string net condensation of [25], using
modular tensor categories, can recover QED and QCD with an Anderson-Higgs mechanism.

Recently, the relation of the ribbon spectrum to octonion algebras was clarified in [26], based
on the work of Furey [21]. We start with the 64 dimensions of the $\mathbb{C} \otimes \mathbb{O}$ ideal algebra [19][20][21],
which assigns $U(1)_Q$ and $SU(3)_C$ color charges to the quarks and leptons of the Standard Model.
Selecting one octonion unit for the lepton doublet, define the $\mathbb{C} \otimes \mathbb{O}$ idempotent
$\nu = \frac{1}{2}(-e_5 + ie_4)$, $\alpha_2 = \frac{1}{2}(-e_3 + ie_1)$, $\alpha_3 = \frac{1}{2}(-e_6 + ie_2)$. (15)

The Lie algebra generators $\Lambda_a$ for $SU(3)_C$ occur in three different ways. Taking $(I, \nu, \nu)$ in
$\mathbb{C} \otimes \mathbb{O}$, with $I = 1$,

$$\frac{1}{4}[\Lambda_a, \Lambda_b] = i \frac{1}{2} f_{abc} \Lambda_c$$

$$\frac{1}{4}[\Lambda_a \nu, \Lambda_b \nu] = i \frac{1}{2} f_{abc} \Lambda_c \nu$$

$$\frac{1}{4}[-\bar{\Lambda}_a \nu, -\bar{\Lambda}_b \nu] = -i \frac{1}{2} f_{abc} \bar{\Lambda}_c \nu.$$ (16)

Complex conjugation $i \mapsto -i$ sends particles to antiparticles. Charges for one generation [21]
come from the number operator

$$N = \sum_{j=1}^{3} \alpha_j^{\dagger} \alpha_j,$$ (17)

with values in $\{0, 1, 2, 3\}$. Writing out the $\alpha_j$ components of $N$, a set of eight charges on a three
qubit parity cube gives the set $\{\nu, d(3), \pi(3), e^-\}$ from

$$A \nu, \alpha_j A \nu, \alpha_j \alpha_k A \nu, \alpha_j \alpha_k \alpha_l A \nu,$$ (18)

where

$$A \equiv \alpha_1 \alpha_2 \alpha_3 = i \nu.$$ (19)

The three copies of 64 for the generations suggest a (massless form) of triality for $e_8$. Moving
in this direction, in (16) $I$ represents $LL^{-1}$, where $L$ and $R = L^{-1}$ are chiralities for the massless
neutrino $\nu$. Since ribbon diagrams will characterise chirality, we start with a basis for the Hopf
algebra $\mathbb{C}C_3$,

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \quad (20)$$

In every prime power dimension, the Fourier transform $F_{p^r}$ diagonalizes 1-circulants, and there
exists $p^r + 1$ mutually unbiased bases [27][28][29] generalizing the $2 \times 2$ Pauli matrices, providing
a canonical matrix representation for multiplication in finite fields.

Now let $\omega$ be the primitive cubed root of unity. The $3 \times 3$ Fourier transform

$$F_3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \bar{\omega} \\ 1 & \bar{\omega} & \omega \end{pmatrix}$$ (21)

diagonalizes 1-circulants, as in

$$\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} = F_3 \begin{pmatrix} r & \theta & \bar{\theta} \\ \bar{\theta} & r & \theta \\ \theta & \bar{\theta} & r \end{pmatrix} F_3^{\dagger}. \quad (22)$$
The circulant idempotents are

\[ I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad H = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega & \bar{\omega} \\ \omega & 1 & \bar{\omega} \\ \bar{\omega} & \omega & 1 \end{pmatrix}, \quad \overline{H} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \bar{\omega} & \omega \\ \bar{\omega} & 1 & \omega \\ \omega & \bar{\omega} & 1 \end{pmatrix}. \quad (23) \]

As Hermitian matrices, they belong to the Jordan algebra \( J_3(\mathbb{C}) \). We are particularly interested in idempotents for the exceptional algebra \( J_3(\mathbb{O}) \) over the octonions \[30][31]\, and its extensions by an arbitrary division algebra. Its off diagonal integral points are known to give the Leech lattice \[32][33].

The four dimensional Fourier transform is the eigenvectors of the chiral operator \( \gamma_5 \) in the Dirac representation,

\[ F_4 = F_2 \otimes F_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}. \quad (24) \]

As Alain Connes likes to say, our ignorance of numbers comes down to the interplay between multiplication and addition on the adelic line. Discreteness in the golden ring integers tells us to reconsider all valuations, which we do by introducing coordinates in terms of geometrical elements, as required in any motivic approach.

3. Operads, logic and the magic star
Scattering amplitudes using on-shell methods utilise combinatorial operads, most notably Stasheff’s associahedra \[34][35][22]. Word labels are given to finite rooted planar trees in the following way. Given \( l \) letters, the set of words of length \( n \) define the vertices of a subdivided cube. For example, when \( n = l = 3 \) we have a cube in dimension 3 such that each edge is cut in half, giving the vertices of a cuboctahedron. Words are noncommutative monomials. The commutative monomials define triangular simplices, sliced diagonally through the corresponding cube.

Postnikov \[36\] defines the associahedra and permutohedra in terms of these integral simplex coordinates, which we view as powers of prime components in the factorization of an integer. For the vertices of the permutohedron: write down all divisors of the number \( N = p_1^{n_1}p_2^{n_2}p_3^{n_3}\cdots p_n \) including 1. For the 24 permutations of \( S_4 \), we get

\[ 1, p_1, p_2, p_3, p_1^2, p_2^2, p_3^2, \quad (25) \]

\[ p_2p_3, p_1p_3, p_1p_2, p_3p_2^2, p_3p_1^2, p_2p_1^2, p_1p_2^2, p_2p_3^2, \]

\[ p_2p_3^3, p_2^2p_3^3, p_2^2p_3^2, p_1p_2p_3, \]

\[ p_1p_2^2p_3, p_1p_2p_3^2, p_1p_2^2p_3^2, p_1p_2p_3^3, p_1p_2p_3^2, p_1p_2p_3^3. \]

The vertices of the associahedron are obtained by looking at divisor classes: two divisors are equivalent if their set of powers is the same. In the case of \( S_4 \) we obtain the 14 vertex associahedron

\[ 1, p_1, p_1^2, p_3^3, p_1p_2, p_1^3p_2, p_1^3p_2^2, p_1^3p_2^3, p_1^2p_2^2, \quad (26) \]

\[ p_1p_2p_3, p_1^2p_2p_3, p_1^3p_2p_3, p_1^3p_2p_3^2, p_1^3p_2p_3^3, \]

alternatively written as

\[ 000, 100, 200, 300, 110, 210, 310, 320, 220, 111, 211, 221, 311, 321. \quad (27) \]
Finally, the vertices of Kapranov’s permutoassochiahedron [37] are pairs \((u,v)\), with \(u \in S_n\) and \(v \in\) the class for the associahedron. For example, write \((p_1^2)|q_1q_2\) to denote \(p_1^2 \in S_3\) and the class of \(p_1p_2\). The 120 vertex polytope contains 24 pentagons, much like the discrete Hopf fibration [38] for \(S^7 \to S^4\), related to the \(e_8\) lattice. Extending rooted trees in both the upwards and downwards directions (in a PRO) we obtain a \((3+3)\) dimensional catalog of the 240 roots of \(e_8\).

Consider now the magic star for \(e_8\). A basic parity cube for three qubits \((n = 3, l = 2)\) is inscribed inside larger cubes, starting with the cube with halved edges, for the \(d_3\) lattice. The magic star projection [39][40][41] of Lie algebra lattices contains a hexagon of edge centers on the cube, perpendicular to the three points on the diagonal, say \((0,0,0)\), \((1,1,1)\) and \((2,2,2)\). This diagonal collapses to the central \(e_8\) point in the magic plane.

This magic plane is tiled by a tetractys simplex for 3 qutrits, since the triangles defining the star of David contain two interior points along each edge. The associahedra sit inside simplices of \(S\) \[\begin{array}{c}
\text{vertices of } S \\
\text{or by permutohedra. The parity cube is derived from the permutohedron as follows. The 24}
\end{array}\]

\[\text{larger copies of the magic star. Three dimensional space is regularly tiled either by parity cubes}
\]

\[\text{or by permutahedra. The parity cube is derived from the permutahedron as follows. The 24}
\]

\[\text{vertices of } S_4 \text{ have integral coordinates, namely the permutations of } (1,2,3,4) \text{ in } \mathbb{R}^3. \text{ Each}
\]

\[\text{s } \in \text{ S4 is assigned a signature, which lists the shifts between entries in } s. \text{ For example, } (2,3,1,4)
\]

\[\text{has signature } + - +. \text{ The eight signature classes for } S_4 \text{ define the vertices of the parity cube in}
\]

\[\text{three dimensions. The product of two signature classes for } S_n \text{ is given by the product in the group Hopf}
\]

\[\text{algebra } 
\]

\[CS_n, \text{ producing the descent Hopf algebra of Solomon [43]. For example,}
\]

\[\begin{array}{c}
\text{(- + +)(- - +) = (+ - -) + (+ - +) + (- - -).}
\end{array}\]  \(\text{(28)}\)

In any dimension, this is the Hopf algebra needed to construct Jordan pairs [44][45][46] for the magic star.

In the Jordan pair picture, points and higher dimensional objects are idempotents. Similarly, from a categorical perspective, open sets are idempotents in a Heyting algebra [47], which is a not necessarily distributive poset lattice, with 0, 1 and an implication map \(x \Rightarrow y\). Objects in the lattice satisfy

\[\begin{array}{c}
x \lor x = x, \quad x \land x = x
\end{array}\]  \(\text{(29)}\)

and

\[\begin{array}{c}
x \land (y \lor x) = x = (x \land y) \lor x.
\end{array}\]  \(\text{(30)}\)

Implication satisfies

\[\begin{array}{c}
(x \Rightarrow y) \land x = x \land y
\end{array}\]  \(\text{(31)}\)

and the higher distributivity rule

\[\begin{array}{c}
x \Rightarrow (y \land z) = (x \Rightarrow y) \land (x \Rightarrow z).
\end{array}\]  \(\text{(32)}\)
A lattice is non distributive if it contains a pentagon. Vector spaces for quantum logic are non distributive under the closure of union operation.

The loop structure on octonions in the $\mathbb{O}$ lattice weakens both commutativity and associativity, effectively introducing braiding and fusion rules for a category.

The duality of product and coproduct appears on the fundamental cubes. The $\alpha_i$ labels of (17) give the parity cube as a three element power set lattice. Conversely, when we consider instead a quantum configuration of three non parallel lines in a plane, the source (the empty set $\emptyset$) becomes the whole plane, while the target three point set is now the empty intersection of the three lines.

4. Masses and Mixings

If there are three rest mass particle states, and mass is energy, then the limit of the uncertainty principle requires three time lines. In the neutrino CMB correspondence, temperatures correspond to past, present and future [6]. The breaking of the massless $3 \times 3$ matrix spectrum requires a braiding, associated to dyonic states, and a quantum neutrino mass.

A rest mass operator $\sqrt{M}$ obeying the Koide relation [16][17][18] is a circulant of the form (22) plus a scale factor $\mu$. The diagonal of eigenvalues under $F_3$ gives a sum of three idempotents for $\mathbb{J}_3(\mathbb{C})$, where we recall that $\mathbb{C}$ is densely filled by $\mathbb{Z}_8/2$ using the quasilattice of [1]. The determinant of $\sqrt{M}$ [42] satisfies

$$1 + \sqrt{\lambda_1 \lambda_2 \lambda_3} \equiv r \cos(3\theta),$$

and this quantity goes to zero when one eigenvalue of $\sqrt{M}$ is negative, which occurs for Brannen’s extension [16] of the Koide rule to the neutrinos. This justifies the choice $r = \sqrt{2}$ for the leptons, since it is centered around zero for the basic arithmetic phase $\theta = \pm \pi/12$. This rule provides a second constraint on the mass triplet, after the Koide relation itself. For neutrinos, it takes the form

$$\frac{\sqrt{m_1 m_2 m_3}}{(\sqrt{m_1^2 + m_2^2} - \sqrt{m_3^2})^3} = \frac{1}{27}.$$  

Observationally, the charged lepton phase $\theta$ is very close to $2/9$, while the active neutrino triplet fits oscillation data with a phase of $\theta = 2/9 + \pi/12$ [9]. Similarly, quark rest mass triplets are obtained with phases $2/27$ and $4/27$. The charged lepton scale $\mu$ is a simple multiple of the proton mass [16] and the right handed neutrino phase is $2/9 - \pi/12$ [6].

Although a better clarification of mixing angles relies on further details in motivic quantum gravity, we outline here a useful quark lepton complementarity model. The first Euler angle in the CKM matrix [48][49] is the Cabibbo angle $\delta_{12}$, approximated by the rule

$$\delta_{12} + \delta_{13} = \frac{\pi}{4} - \arctan \frac{1}{\phi} = 13.28^\circ.$$  

The other two irrationals in the golden ring give

$$\delta_{23} = \frac{\pi}{6} - \arctan \frac{1}{\sqrt{\phi + 2}} = 2.3^\circ, \quad \frac{\pi}{2} - \arctan \frac{1}{\phi \sqrt{\phi + 2}} = 72^\circ,$$

where $72^\circ$ is an angle in the Penrose tiling [1][50][51]. The phases $\pi/6$ and $\pi/4$ define the tribimaximal matrix [52], which is a planar approximation to the PMNS neutrino mixing matrix [53][54]. For the small quark phase $\delta_{13} \simeq 0.2^\circ$ we look to the breaking of tribimaximal mixing in the neutrino sector. The small mixing angle $8.5^\circ$ is close to $4/27$, which is obtained as two thirds of $2/9$ from a triality action on the complex phase $X$ in

$$\begin{pmatrix}
a & X & \overline{X} \\
\overline{X} & a & X \\
X & \overline{X} & a
\end{pmatrix}$$
in $J_3(\mathbb{C})$ [42]. With this deformation, the CKM matrix $\delta_{12} = 13.01^\circ$ and $\delta_{13}$ is close to 0.27. Since Euler angles are expressed as circulants in the Hopf algebra $\mathbb{C}S_3$, complex phases are automatically included and the CKM and PMNS matrices exhibit maximal CP violation, in line with observations.

In the full theory, the phases $\pi/6$ and $\pi/4$ are of course associated to automorphic forms, which we construct from the combinatorics of higher dimensional discrete geometries. Here, for instance, the allowed dimensions of matrix components in the $3 \times 3$ $T$-algebras of [41] determine the restricted Euler factors, such as

$$\begin{pmatrix} a & b & 0 \\ b & a & 0 \\ 0 & 0 & a+b \end{pmatrix},$$

(38)

of our circulant mixing matrices.

Acknowledgments

The author thanks Michael Rios, Piero Truini, Alessio Marrani and Ray Aschheim for numerous discussions and also for their participation in Group32. Thanks also to the team at Quantum Gravity Research in Los Angeles.

References

[1] Battaglia F and Prato E 2010 Commun. Math. Phys. 299 577
[2] Coecke B and Abramsky S 2004 Proc. 19th Conf. on Logic in Computer Science (Turku) (IEEE) p 415
[3] Coecke B, Heunen C and Kissinger A 2016 Quant. Info. Process. 15 5179
[4] Dungworth G 2010 Galaxy Zoo forums Preprint Classified or Deleted
[5] Sheppeard M D 2010 Arcadian functor Preprint kea-monad.blogspot.com
[6] Sheppeard M D 2017 The algebra of non local neutrino gravity Preprint vixra:1712.0076
[7] Dvali G and Funcke L 2016 Phys. Rev. D 93 113002
[8] Dungworth G and Sheppeard M D 2017 Non local mirror neutrinos with $R=ct$ Preprint vixra:1711.0119
[9] de Salas P F, Forero D V, Ternes C A, Tortola M and Valle J W F 2018 Status of neutrino oscillations Preprint arXiv:1708.01186
[10] McCulloch M E 2012 Astrophys. and Space Science 342 575
[11] Gine J 2012 Mod. Phys. A 27 1250208
[12] McCulloch M E and Gine J 2017 Mod. Phys. A 32 1750148
[13] Gine J 2011 Int. J. Theor. Phys. 50 607
[14] Bilson-Thompson S O 2005 A topological model of composite preons Preprint hep-ph/0503213
[15] Sheppeard M D 2010 Quark lepton braids and heterotic supersymmetry Preprint vixra:1004.0083
[16] Brannen C A 2006 Lepton masses Preprint www.brannenworks.com
[17] Koide Y 1983 Phys. Rev. D 28 252
[18] Koide Y 1983 Phys. Lett. B 120 161
[19] Furey C 2014 JHEP 10 046
[20] Furey C 2012 Phys. Rev. D 86 025024
[21] Furey C 2015 Phys. Lett. B 742 195
[22] Sheppeard M D 2007 Ghon phenomenology and a linear topos PhD thesis University of Canterbury
[23] Tesla N 2015 The True Wireless (USA: Simon and Schuster)
[24] Freedman M, Larsen M and Wang Z 2002 Commun. Math. Phys. 227 605
[25] Levin M A and Wen X G 2005 Phys. Rev. B 71 045110
[26] Gresnigt N G 2018 Phys. Lett. B 783 212
[27] Schwinger J 1960 Proc. Nat. Acad. Sci. USA 46 570
[28] Wootters W K and Fields B D 1989 Ann. Phys. 191 363
[29] Combesure M 2009 J. Math. Phys. 50 032104
[30] Baez J 2002 Bull. Amer. Math. Soc. 39 145
[31] McCrimmon K 2003 A Taste of Jordan Algebras (Berlin: Springer)
[32] Wilson R A 2009 J. Alg. 322 2186
[33] Rios M 2013 Preprint arXiv:1307.1554
[34] Stasheff J D 1963 Trans. Amer. Math. Soc. 108 293
[35] Brown F C S 2009 Annal. Scient. de l’Ecole Normale Superieure 42 371
[36] Postnikov A 2009 Int. Math. Res. Not. 2009 1026
[37] Kapranov M 1993 J. Pure App. Alg. 85 119
[38] Sadoc J F and Mosseri R 1993 J. Non-crystalline Solids 153 247
[39] Truini P 2012 Pacific J. Math. 260 227
[40] Marrani A and Truini P 2016 P-adic Numbers, Ultrametric Anal. and App. 8 68
[41] Truini P, Rios M and Marrani A 2017 The magic star of exceptional periodicity Preprint arXiv:1711.07881
[42] Sheppeard M D 2017 Lepton mass phases and the CKM matrix Preprint vixra:1711.0336
[43] Loday J L and Ronco M O 1998 Adv. Math. 139 293
[44] Truini P and Biedenharn L C 1982 J. Math. Phys. 23 1327
[45] Faulkner J R 2000 J. Alg. 232 152
[46] Faulkner J R 2004 J. Alg. 279 91
[47] MacLane S and Moerdijk I 1994 Sheaves and Geometry in Logic (New York: Springer)
[48] Cabibbo N 1963 Phys. Rev. Lett. 10 531
[49] Kobayashi M and Maskawa T 1973 Prog. Theor. Phys. 49 652
[50] Irwin K, Amaral M M, Aschheim R and Fang P 2016 Proc. 4th Int. Conf. on the Nature and Ontology of Spacetime (Varna) (Minkowski Institute) p 117
[51] Aschheim A and Rios M 2018 Preprint in this volume
[52] Harrison P F, Perkins D H and Scott W G 2002 Phys. Lett. B 530 167
[53] Pontecorvo B 1958 Sov. Phys. JETP 7 172
[54] Maki Z, Nakagawa M and Sakata S 1962 Prog. Theor. Phys. 28 870