Non-Fermi-Liquid Scaling in Ce(Ru$_{0.5}$Rh$_{0.5}$)$_2$Si$_2$

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We study the temperature and field dependence of the magnetic and transport properties of the non-Fermi-liquid compound Ce(Ru$_{0.5}$Rh$_{0.5}$)$_2$Si$_2$. For fields $H \lesssim 0.1$ T the results suggest that the observed NFL behavior is disorder-driven. For higher fields, however, magnetic and transport properties are dominated by the coupling of the conduction electrons to critical spin-fluctuations. The temperature dependence of the susceptibility as well as the scaling properties of the magnetoresistance are in very good agreement with the predictions of recent dynamical mean-field theories of Kondo alloys close to a spin-glass quantum critical point.

The properties of a large class of f-electrons materials show striking departures from the predictions of standard Fermi-liquid theory at low temperature. Several mechanisms leading to non-Fermi-liquid (NFL) behavior have been proposed. In systems close to a quantum phase transition such as CeCu$_{6-x}$Au$_x$ or CeIn$_3$, NFL behavior is due to the coupling of the conduction electrons to critical spin fluctuations. Anomalous properties are observed when the system is driven through the quantum critical point (QCP) by alloying or by applying pressure. In other systems, such as UCu$_{5-x}$Pd$_x$, NFL properties are thought to be a consequence of the interplay of strong structural disorder and many body effects.

In this paper we report results of a study of the temperature and field dependence of the magnetic and transport properties of Ce(Ru$_{0.5}$Rh$_{0.5}$)$_2$Si$_2$. We found that this system exhibits different types of anomalies depending on the value of the applied field. At weak fields, we found signatures of disorder effects such as a diverging low-temperature susceptibility and an anomaly in the low-field magnetoresistance. Above 1kG, the $T$ and $H$-dependence of the susceptibility and the resistivity agrees with the predictions of recent mean-field theories of the spin-glass (SG) QCP. The magnetoresistance is found to exhibit universal scaling properties as predicted by the theory.

In the Ce(Ru$_{1-x}$Rh$_x$)$_2$Si$_2$ alloy series the Ce-sublattice is preserved and the hybridization between 4f and conduction electrons varies with the concentration of the ligand 4d atoms Ru and Rh. Pure CeRu$_2$Si$_2$ is a heavy fermion compound with a $\gamma$ of about 385 mJ/mol/K$^2$ and no long range magnetic order down to 20 mK. With substitution of Rh for Ru, a spin density wave (SDW) region appears between $x = 0.03$ and $x = 0.4$. While Ce(Ru$_{1-x}$Rh$_x$)$_2$Si$_2$ at $x = 0.03$ is a normal Fermi liquid, a NFL regime exists for $x = 0.4$ and 0.5. In pure CeRh$_2$Si$_2$, the 4f electrons are localized and the material is antiferromagnetic (AF) below $T_N = 35$ K. With increasing Ru substitution, $T_N$ decreases and eventually vanishes at a critical concentration $x_c \approx 0.5$. There is no direct evidence of long range AF order below $x = 0.7$. The admixture of Ru and Rh introduces magnetic frustration effects as it leads to a competition between widely different types of magnetic short-range order. A frustrated ground state of the SG type can not be excluded slightly above $x_c$. Recent $\mu$-SR studies showed that the $T$-dependence of the muon relaxation rate in the $x = 0.5$ alloy is similar to that observed in spin glasses. The muon depolarization rate decays exponentially, however, showing that the spin correlations are dynamic rather than static.

Samples of Ce(Ru$_{0.5}$Rh$_{0.5}$)$_2$Si$_2$ were prepared by arc-melting of the constituents in an argon atmosphere. The ingots were remelted several times to insure homogeneity. Single crystals oriented along $a$ and $c$-axis were grown by the Czochralsky method in a tri-arc furnace in argon atmosphere and parallelepiped-shaped samples of size $\approx 0.5 \times 0.5 \times 4$ mm$^3$ were obtained. The resistivity was measured by a standard ac method at 17 Hz in the range 16 mK - 4 K and in magnetic fields up to 5 T. The low-field susceptibility was measured in the dilution range $\approx 50$ mK - 4 K using a standard method at 130 Hz. The static magnetization was measured in the same range of temperature with a SQUID magnetometer in a dilution setup and, above 3 K, in a commercial SQUID magnetometer.

Fig. 1 displays the resistivity $\rho$ measured in a field applied along the $c$-direction up to 5 T. The data correspond to a current flow in the $a$-direction. The behavior along the $c$-axis is similar but the resistivity is about four times smaller. The magnetoresistance is positive at low temperatures and changes sign at about 2.5 K in the $a$-direction and at about 1.5 K in the $c$-direction. In both cases the data at 5 T follow a $T^2$-law up to $T \approx 1$ K. The range of temperatures in which FL behavior is observed decreases with field and vanishes at $H = 0$. These results are qualitatively similar to those obtained in CeCu$_{6-x}$Au$_x$ and in the stoichiometric compound CeNi$_2$Ge$_2$. The resistivity of our samples is much higher, however, as a consequence of the high degree of
The temperature dependence of the zero-field resistivity is \( \delta \rho \propto T^{1.6} \) as shown in Fig. 2(a). The resistivity exponent is close but not identical (see below) to that expected for metallic antiferromagnets \(^4\) and spin-glasses \(^8\) at the QCP in the high resistivity limit \(^2\). The low-temperature magnetoresistance \((\rho(H) - \rho(0))/\rho(0)\) along the \( a \)-direction is plotted in Fig. 2(b) for \( H < 1 \) T. At high fields (not shown in the figure) it shows the classical \( H^2 \)-dependence due to the bending of the electron orbits. Below a few kG, however, the low-temperature resistance increases linearly with field as \( H \to 0 \).

The susceptibility \( \chi = M/H \) (\( M \) is the magnetization) is represented in Fig. 3 for \( H_{ac} = 1 \) G and \( H_{dc} = 0.01 \) T, 0.1 T, and 1 T for \( T < 10 \) K. At low temperatures \( \chi \) decreases strongly with increasing field between 1 G and 1 kG but weakly above 1 kG. Above 3 K the field has no effect on \( \chi \) up to 1 T as seen by comparison of the curves at 1 kG and 1 T in Fig. 3(b). At 1 G, \( \chi \) increases sharply with decreasing \( T \) below 2 K.

The divergence of \( \chi \) at low-fields as well as that of \( C/T \) suggests that, in this regime, NFL behavior may be driven by disorder \(^2\). The linear (rather than quadratic) rise of the low-field magnetoresistance at low temperatures reported here may be explained by Kondo-disorder effects \(^2\). Recent \( \mu \)SR experiments \(^2\) showed a sharp increase in the muon relaxation rate below 2 K that saturates at a \( T \)-independent value below 0.7 K. This behavior has been interpreted in terms of the Griffiths-phase scenario \(^3\) in which finite clusters carrying a magnetic moment fluctuate very slowly at low temperature due to quantum tunneling. Magnetic, NMR and specific heat experiments performed in a temperature range much higher than ours have also been interpreted in terms of this mechanism \(^24\).

For \( H \gtrsim 1 \) kG, \( \chi \) remains finite as \( T \to 0 \) and depends weakly on \( H \), suggesting that application of a moderate magnetic field quenches the mechanism that leads to the divergence observed at lower fields. Although a full understanding of this fact is still lacking, it should be noticed that in the Griffiths-phase model \(^8\) quantum fluctuations of the largest clusters are expected to be suppressed by a small magnetic field. Indeed, while the Zeeman energy of a cluster of size \( N \) grows as \( \sqrt{N} \), its tunneling energy vanishes exponentially with \( N \). In the following we concentrate on the physics above 1 kG.

The \( T \)-dependence of the susceptibility at 1 kG is still anomalous and \( \delta(\chi(T)) \) is approximately linear in \( T^{3/4} \) (cf. Fig. 3) except at the lowest temperatures, a point that will be discussed further below \(^2\). A \( T^{3/4} \) dependence of \( \chi \) as well as the value 3/2 for the resistivity exponent where predicted by recent dynamical mean field-theories (DMFT) of the spin-glass QCP \(^8\). In view of the important role that frustration is expected to play in this compound it is tempting to try to interpret our results in terms of the fully frustrated SG model. The latter describes conduction band electrons coupled to localized \( f \)-electron spins via a local Kondo coupling, \( J_k \). There is, in addition, a residual Ising-like interaction between the localized spins. The effects of the magnetic frustration introduced by disorder are incorporated by taking
random spin couplings $J_{ij}$ chosen from a symmetric distribution of width $\langle J_{ij}^2 \rangle = J^2$. Disorder in the Kondo temperature is not included in the model. From the Kondo-temperature distribution determined in Refs. \[24\] and \[25\] one can conclude that this effect should play a lesser role than frustration in the low-temperature range that interests us. The SG model \[10\] was investigated in the framework of dynamical mean-field theory \[27\]. It exhibits a zero-temperature SG transition when the typical exchange energy $J = J_c \sim T_K$, the Kondo temperature of the underlying Kondo lattice. Monte Carlo simulations \[10\] of this model showed that its critical properties are described by an effective strong-coupling theory closely related to other mean-field models \[8,9\]. The physical properties of the system depend on the effective distance to the QCP, $\Delta(T, H)$. It can be shown that this is

$$\Delta = \Delta_0 + 2\sqrt{\Delta_0} \frac{T}{T_0} \left[ \sqrt{1 + \frac{T}{2\sqrt{3} T_0 \Delta_0}} - 1 \right], \tag{1}$$

where $\Delta_0 = (1 - J/J_c) + (H/H_0)^2$ and $H_0 = J_c/(g\mu_B)$ ($g$ is the gyromagnetic ratio of the Ce ion). The scale $T_0$ is proportional to $T_K$. Numerical simulations yield $J_c \sim 1.15 T_K$. The spin susceptibility is \[10\]

$$J_c \chi = \sqrt{1 + \Delta} - \sqrt{\Delta}. \tag{2}$$

The spectrum of magnetic excitation has a scaling form \[8,10\], $J_c \chi''(\omega) = \sqrt{\Delta} \Phi(\omega/J_c \Delta)$, where the universal scaling function $\Phi(x) = x/\sqrt{2} [(1 + x^2)^{3/2} + 1]^{-1/2}$. The temperature-dependent contribution to the resistivity is computed from $\delta \rho \propto 1/\tau$ with the inverse scattering time $\tau^{-1} \propto \int_0^\infty d\omega \chi''(\omega)/\sinh(\beta \omega)$, an expression valid in the dirty limit \[21\]. The resistivity has the scaling form

$$\rho(T, H) - \rho(0, H) \propto T^{3/2} \Psi \left( \frac{T}{T_0} \right), \tag{3}$$

with $\Psi(x) = x^{-1/2} \int_0^\infty du \Phi(u x)/\sinh u$. It follows that $\delta \chi \equiv \chi(0) - \chi(T) \propto T^{3/4}$ and $\delta \rho \propto T^{3/2}$ at the QCP. The resistivity exponent of the mean-field model coincides with that of the $d = 3$, $z = 2$ antiferromagnet \[33\]. The susceptibility exponent is specific to the SG model. Away from the QCP, normal Fermi-liquid behavior as $T \to 0$ is recovered with both $\delta \chi$ and $\delta \rho \propto T^2/\sqrt{\Delta_0}$ at sufficiently low $T$. The crossover between these limiting forms will be discussed below. The parameters of the theory can be determined from an analysis of the experimental data. At the critical concentration, $r = 1 - J/J_c$ vanishes. However, $\chi_c$ is not known accurately and a small but finite $r$ can not be excluded a priori. The characteristic field $H_0$ may be estimated from an extrapolation to $T = 0$ of the susceptibility per Ce atom. From the definitions above and Eq. \[2\] we estimate $H_0 = \mu_B / \chi(0) \approx 11$ T (we have assumed $g = 2$). Since $H_0$ is very large, the measuring field can be neglected in the analysis of the magnetization at 1 kG. A fit of $\chi$ using Eqs. \[4\] and \[6\] with $\Delta_0 = 0$ gives $T_0 \approx 20K$, which is slightly smaller than the Kondo temperature of the system estimated from the $T$-dependence of the resistivity \[11\].

![FIG. 3. Susceptibility $M/H$ measured in several magnetic fields along the $c$ direction, plotted as a function of $T^{3/4}$](image)

The solid curve is a fit of the data at $H = 1$ kG to the expressions in Eqs. \[4\] and \[6\]. Inset: The ac susceptibility at $H = 1$ G. The solid line is the fit mentioned in the text.

We analyzed the $T$- and $H$-dependence of the resistivity using Eqs. \[4\] and \[6\]. The condition that all the data in Fig. \[3\] collapse into a single scaling curve leaves little freedom in the choice of the parameters. In particular, we found it impossible to scale all the curves with $r = 0$. A scaling plot of the resistivity along the $a$-axis is shown in Fig. \[4\]. The data points are the values of the scaled resistance $(\rho(T, H) - \rho(0, H)) \times T^{-3/2}$ plotted as the reduced variable $t/\Delta$ (at $T/T_0$) for $T \leq 0.9$ K and $H \leq 5$ T. The values of the parameters are $r = 7 \times 10^{-3}$, $T_0 = 20$ K and $H_0 = 13$ T. The value of $r$ measures the distance to the true QCP, $r = \delta J/J_c$ giving $\delta J \approx 0.2$ K, a very small energy compared to the other energy scales present in the problem. The characteristic field determined from this analysis is close to the theoretical estimate given above. The solid line in Fig. \[3\] is the theoretical scaling function $\Psi(x)$. There are no adjustable parameters other than an amplitude that fixes the vertical scale. The agreement between theory and experiment is very good except for the data for $H = 0$ (the empty squares in Fig. \[3\]) which lie slightly above the scaling curve. The slope of the curve, that measures the effective resistivity exponent, is correctly reproduced by the theory. We can now understand that the deviation of the resistivity exponent (cf. Fig. \[3\]) with respect to its value at the QCP (1.5) is due the small but finite value of $r$. The effective exponent only reaches 1.5 for $t/\Delta \to \infty$, i.e. for $r = 0$. We ascribe the excess amplitude for $H = 0$, represented by the vertical shift, to additional scattering processes...
dues to disorder. We can compare $A$, the amplitude of the $T^2$ term in the resistivity in the FL region, with the theoretical prediction, $A \propto 1/\Delta_0$. The inset in Fig. 1 shows the field dependence of $A$ as determined from the initial slope $d\rho/dT^2$ of the resistivity and the theoretical prediction. There is good agreement.

The susceptibility can also be calculated and compared with the data. The solid line in Fig. 4 is the theoretical result for $H = 1$ kG. The data (and the theoretical curve) deviate from a pure $T^{3/2}$ law as $T \to 0$. This is due to the finite value of $r$ which results in normal FL behavior below a crossover temperature $T_{cL} \sim T_0 \Delta_0$. $T_{cL}$ increases with field and can be estimated as $\approx 0.25$ K for $H=1$ T. The crossover to $T^2$ behavior in $\chi(H=1T)$ can be seen in Fig. 1. The low-field results can be described by adding to the dynamical mean-field result an additional diverging contribution. The presence of a paramagnetic phase giving rise to a $T^{-1}$ divergence of $\chi$ was suggested in Ref. 23. However, to suppress such a contribution at 3 K by a field as small as 1 kG one would need that the impurities carry huge moments ($> 3 \mu_B$). Furthermore, this hypothesis would not explain the divergence of $\gamma$ [16]. The inset in Fig. 2 shows a fit of the ac data for $H=1$ G to the expression $\chi(T) = \chi_{MF}(T) + aT^{-0.8}$. This can be interpreted in terms of the Griffiths-phase model 7 that predicts a power-law divergence $\delta \chi \propto T^{-1+\lambda}$ with an exponent $\lambda$ that vanishes at the QCP. The value $\lambda = 0.2$ that comes out of our analysis is consistent with this picture for a system close to the QCP. We also found that the $\gamma$ data [13] can be accurately described by the analogous expression $\gamma(T) = \gamma_{MF}(T) + aT^{-0.8}$ where $\gamma_{MF}(T) \propto 1 - b\sqrt{T}$ is the DMFT prediction 11 for $C/T$. The equality of the exponents describing the divergent parts of $\chi$ and $C/T$ is one of the predictions of the Griffith's phase model 7.

In summary, we have shown that the NFL properties of Ce(Ru$_{0.5}$Rh$_{0.5}$)$_2$Si$_2$ are determined by disorder and proximity to a QCP. The effects of the two mechanisms can be disentangled by applying a small magnetic field. At low fields, disorder effects dominate. Above 1kG, however, the temperature and field dependence of the susceptibility are well described by the dynamical mean-field theory of the spin glass QCP. The $T$- and $H$-dependent resistivity is found to obey a universal scaling law.

This work was supported in part by the Grant-in-Aid for Scientific Research (B) and the Monbusho International Scientific Research Program. One of us (DRG) thanks the Newton Institute for its kind hospitality and A. H. Castro Neto for useful correspondence.

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