Beyond Boundary Conditions: General Cosmological Theories

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Time-symmetric cosmological theories, in which the initial and final states are arranged to have similar features or are independently fixed, have been quite extensively discussed in the literature. However, a more general and perhaps more useful possibility seems, surprisingly, never to have been considered. Quantum cosmology, like any inherently probabilistic dynamical theory, can be modified by conditioning on the occurrence of a sequence of cosmological events fixed a priori and independently of the hamiltonian and initial state. These more general theories provide a natural class of alternatives which should eventually be useful in quantifying how well the hypothesis that initial conditions suffice is supported by the data.

1 Introduction

Standard quantum theory is explicitly time asymmetric. The quantum state $|\psi(t)\rangle$ of a system at any time $t$ is defined entirely by past events. It carries all the information that there is about the probabilities of future measurement outcomes on the system, but not about the probabilities of past measurement outcomes: to estimate the latter, we need some information about the state of the system prior to the measurements.

The same is true of standard quantum cosmological models: the initial conditions are taken to be fixed and simple; the probability of any given cosmological event is then determined by the initial conditions and (in interpretations which assign probabilities to sequences of events) by earlier events. If standard quantum cosmology gives essentially the right picture, then the time asymmetry of quantum theory ultimately derives from this cosmological asymmetry.

The hypothesis that initial causes suffice is simple, natural, attractive, unifying and confirmed by the many successes of existing cosmological theory. Present data does not hint that there may be any need to go beyond it. So it looks perverse to spend serious effort building cosmological theories in which the hypothesis does not hold — unless these theories have some great, as yet unappreciated, advantage. It may perhaps seem perverse even to mention alternatives at all. Still, however sure we are of a physical principle, it’s always interesting to ask whether it absolutely has be true and how well it can be confirmed.

The most obvious alternative to the standard time asymmetric picture is
a time symmetric cosmology. Classical cosmologies with some form of time symmetry have been discussed by several authors.\textsuperscript{1-3} Gell-Mann and Hartle\textsuperscript{4} have recently shown how to define time-symmetric (or time-neutral) quantum cosmologies, in which the initial and final states are fixed independently of each other and of the hamiltonian. The hamiltonian continues to play a well-defined rôle, but the probabilities of cosmological events are no longer defined by the hamiltonian and initial state alone: they depend symmetrically on the fixed initial and final states.

2 Quantum Cosmologies Constrained by more than Boundary Conditions

Time-neutral cosmologies have some surprising features,\textsuperscript{5} but they are internally consistent theories which (in principle) make distinct empirical predictions, and have been proposed as foils against which to test the standard understanding of time asymmetry. However, it is hard to find examples of time-neutral cosmologies with any theoretical appeal at all except in closed universe scenarios in which there is recontraction to a final singularity. It is hard (though perhaps not impossible) to find tests that would allow us to distinguish time-asymmetric and time-neutral closed universe cosmologies. And the data do not currently seem to favour a closed universe. In short, there is currently really no very serious time-symmetric alternative to standard ideas — and unless one is found, the discussion may never really affect practical cosmology. Time (or CPT) symmetry in the boundary conditions is a natural idea in its way, but to implement it seems to require a very large step from our current picture. Time-neutral cosmologies may unfortunately thus be of limited use as foils — small perturbations of successful theories are generally more useful than large ones in this rôle.

Focussing on the boundary conditions, though, misses the fact that quantum cosmology — or the quantum theory of any closed system, or indeed any probabilistic dynamical theory — can be modified by imposing any of a large class of dynamical constraints. In essentially the same way as the predictions of a classical stochastic differential equation can be modified by restricting to the sub-ensemble of solutions which satisfy particular constraints at various times, a standard quantum cosmology can be modified so that sequences of events from some fixed sub-class of possibilities necessarily take place. Quite generally, given any initial state $|\psi(t_0)\rangle$ and hamiltonian $H$, we can, if we wish, define a theory by hypothesising that events corresponding to the projections $P_1, P_2, \ldots, P_n$ take place at times $t_1 < t_2 < \ldots < t_n$, for any $n \leq \infty$, and then calculate the probabilities of other events conditional on this hypothesis — so
long as the probability of the sequence of \( P_i \) occurring under free evolution of \( |\psi(0)\rangle \) by \( H \) would have been non-zero.

Now, if the present time \( t > t_n \), it is true that the probabilities of present events could equally well be calculated, for example, from the hypothesis that the initial state was

\[
|\psi'(t_n)\rangle = P_n \exp(iH(t_n-t_{n-1}))P_{n-1} \ldots P_1 \exp(iH(t_1-t_0))|\psi(t_0)\rangle
\]

at time \( t_n \). The idea here, though, is that \( |\psi(t_0)\rangle \) and the \( P_i \) should be relatively simple, and \( |\psi'(t_n)\rangle \) rather obviously more complicated and derivative, in the sense that its occurrence in a theory can only elegantly be explained via Eq. 1 and the originally stated hypothesis.

Instead of projections, of course, any of the more complicated notions of quantum event discussed, for example, in the consistent histories literature may be used. Of these, covariant notions of event defined via path integrals seem fundamentally the most satisfactory and best adapted to quantum cosmology. It would be possible, thus, for example, to define quantum cosmologies in which we stipulate in advance that, when the compact 3-metric has volume \( V_i \), the matter inhomogeneities are of scale \( \delta_i \), for some sequence \( V_1 < V_2 < \ldots \). There are, of course, infinitely many theories of this type, including quite simple ones. Since, apart from the imposed constraints, they preserve all of standard physics, and need not differ greatly from standard theories in their predictions, they seem ideally designed as foils against which to test the fundamental postulate that initial causes suffice.

Acknowledgments

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References

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