Quantum tunnelling for Hawking radiation from a dynamical Black Hole

Nairwita Mazumder
Ritabrata Biswas
Subenoy Chakraborty

Department of Mathematics, Jadavpur University, Kolkata-700 032, India.

Abstract

The paper deals with Hawking radiation related to non-static spherically symmetric black hole. Quantum corrections are incorporated using Hamilton-Jacobi method beyond semi-classical approximation. It is found that different order correction terms satisfy identical differential equation as the semiclassical action and are solved by a typical technique. It has been shown that with proper choice of the proportionality factors, one loop back reaction effect in the space time can be obtained. Finally, using the law of black hole mechanics, a general modified form of the black hole entropy is obtained considering modified Hawking temperature.

Keywords : Hawking Temperature, Tunnelling, Quantum Correction

Pacs no : 04.70.Dy, 04.60.Kz, 04.62.+v

1 Introduction

The classical idea of a black hole(BH) that nothing can escape from it was ruled out by Hawking [1] in 1974. Based on quantum field theory, he had shown (in semi classical approximation) that there is a continuous emission of radiation from a BH identical to black body radiation [2, 3] having temperature \( T = \frac{\kappa}{2\pi} \), \( \kappa \) being the surface gravity of the BH. Subsequently in the past decades other semiclassical methods were developed for BH radiation [4, 5, 6, 7]. In fact these studies of black hole radiation can be classified into two groups. The first approach developed by Parikh and Wilczek [8, 9] is based on the heuristic pictures of visualization of the source of radiation as tunnelling and is known as radial null geodesic method. The essence of this method is to calculate the imaginary part of the action for the s-wave emission using the radial null geodesic equation and then Hawking temperature is obtained by relating it to the Boltzmann factor for emission. The alternative way of looking into this aspect is known as complex paths method developed by Sriivasan et. al. [10, 11]. In this approach, the differential equation of the action \( S(r, t) \) of a classical scalar particle can be obtained by plugging the scalar wave function \( \phi(r, t) = \exp\left\{-\frac{i}{\hbar}S(r, t)\right\} \) into the Klein Gordon(KG) equation in a gravitational background. Then Hamilton-Jacobi method is employed to solve the differential equation for \( S \). Finally, Hawking temperature is obtained, using the "Principle of detailed balance" [10, 11, 12].

In this work we consider a general non-static metric for dynamical BH. Hamilton Jacobi (HJ) method is extended beyond semiclassical approximation to consider all the terms in the expansion of the one particle action. It is found that the higher order terms (quantum corrections) satisfy identical differential equations as the semiclassical action and the complicated terms are eliminated considering BH horizon as one way barrier. We derive the modified Hawking Temperature using both the above approaches and are found to be identical at the semiclassical level. Finally, modified form of the BH entropy with quantum correction has been evaluated.

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1 nairwita15@gmail.com
2 biswas.ritabrata@gmail.com
3 schakraborty@math.jdvu.ac.in
2 Method of radial null geodesic: A survey of earlier works

This section deals with a brief survey of the method of radial null geodesic method [8] considering the picture of Hawking radiation as quantum tunnelling. In a word, the method correlates the imaginary part of the action for the classically forbidden process of s-wave emission across the horizon with the Boltzmann factor for the black body radiation at the Hawking temperature. We start with a general class of non-static spherically symmetric BH metric of the form

\[ ds^2 = -A(r, t)dt^2 + \frac{dr^2}{B(r, t)} + r^2d\Omega^2 \]  

(1)

where the horizon \( r_h \) is located at \( A(r_h, t) = 0 = B(r_h, t) \) and the metric has a coordinate singularity at the horizon. To remove this coordinate singularity we make the following Painleve-type transformation of coordinates:

\[ dt \rightarrow dt - \sqrt{1 - \frac{B}{AB}} dr \]  

(2)

and as a result the metric (1) transforms to

\[ ds^2 = -Adt^2 + 2\sqrt{\frac{1}{B} - 1} dt dr + dr^2 + r^2d\Omega^2 \]  

(3)

This metric (i.e., the choice of coordinates) has some distinct features over the former one namely

(i) the metric is singularity free across the horizon
(ii) at any fixed time we have flat spatial geometry
(iii) both the metric has the same boundary geometry at any fixed radius

The radial null geodesic (characterised by \( ds^2 = 0 = d\Omega^2 \)) has the differential equation (using (3))

\[ \frac{dr}{dt} = \sqrt{\frac{A}{B}} \left[ \pm 1 - \sqrt{1 - B(r, t)} \right] \]  

(4)

where outgoing or ingoing geodesic is identified by the ‘+’ or ‘−’ sign within the square bracket in equation (4). In the present case we deal with the outgoing particles through the horizon (i.e., ‘+’ sign only) and according to Parikh and Wilczek [8] the imaginary part of the action is obtained as

\[ ImS = \text{Im} \int_{r_{in}}^{r_{out}} p_r dr = \text{Im} \int_{r_{in}}^{r_{out}} \int_{0}^{p_r} dp_r \int_{0}^{H} dH \} dr \]  

(5)

Note that in the last step of the above derivation we have used the Hamilton’s equation \( \dot{t} = \frac{dH}{dp_r} \), where \((r, p_r)\) are canonical pair. Further, it is to be mentioned that in quantum mechanics the action of a tunnelled particle in a potential barrier having energy larger than the energy of the particle will be imaginary as \( p_r = \sqrt{2m(E - V)} \). For the present non-static BH the mass of the BH is not constant and hence the \( dH' \) integration extends over all values of energy of outgoing particle, from zero to \( E(t) \) [13] (say). As we are dealing with tunnelling across the BH horizon so using Taylor series expansion about the horizon \( r_h \) we write

\[ A(r, t) \bigg|_t = \frac{\partial A(r, t)}{\partial r} \bigg|_t (r - r_h) + O((r - r_h)^2) \]  

(6)

\[ B(r, t) \bigg|_t = \frac{\partial B(r, t)}{\partial r} \bigg|_t (r - r_h) + O((r - r_h)^2) \]  

(7)

So in the neighbourhood of the horizon the geodesic equation (4) can be approximated as

\[ \frac{dr}{dt} \approx \frac{1}{2} \sqrt{\frac{A'}{B'}} (r - r_h) \]  

(8)

Substituting this value of \( \frac{dr}{dt} \) in the last step of equation (4) we have

\[ ImS = \frac{2\pi E(t)}{\sqrt{A(r_h, t)B'(r_h, t)}} \]  

(9)
where the choice of contour for \( r \)-integration is on the upper-half complex plane to avoid the coordinate singularity at \( r_h \). Thus the tunnelling probability is given by

\[
\Gamma \sim \exp \left\{ -\frac{2}{\hbar} \text{Im} S \right\} = \exp \left\{ -\frac{4\pi E(t)}{\hbar \sqrt{AB'}} \right\},
\]

which in turn equating with the Boltzmann factor \( \exp \left\{ -\frac{E(t)}{T} \right\} \), the expression for the Hawking temperature is

\[
T_H = \frac{\hbar \sqrt{A'(r_h, t)B'(r_h, t)}}{4\pi},
\]

From the above expression it is to be noted that \( T_H \) is time dependent.

Recently, a drawback of the above approach has been noted \([14, 15, 16]\). It has been shown that \( \Gamma \sim \exp \left\{ -\frac{2}{\hbar} \text{Im} S \right\} \) is not canonically invariant and hence is not a proper observable, it should be modified as \( \exp \left\{ -\frac{\text{Im} \oint p_r \, dr}{\hbar} \right\} \). The closed path goes across the horizon and back. For tunnelling across the ordinary barrier it is immaterial whether the particle goes from the left to the right or the reverse path. So in that case

\[
\oint p_r \, dr = 2 \int_{r_{in}}^{r_{out}} p_r \, dr
\]

and there is no problem of canonical invariance. But difficulty arises for black horizon which behaves as barrier for particles going from inside of the BH to outside but it does not act as a barrier for particles going from outside to the inside. So relation (12) is no longer valid. Also using the tunnelling probability as \( \Gamma \sim \exp \left\{ -\frac{1}{\hbar} \text{Im} \oint p_r \, dr \right\} \), there will be a problem of factor two in Hawking temperature \([15, 16]\).

Further the above analysis of tunnelling approach remain incomplete unless effects of self gravitation and back reaction are taken into account. But unfortunately, no general approach to account for the above effects are there in the literature – only few results are available for some known BH solutions \([17, 18, 19, 20]\).

Finally, it is worthy to mention that so far the above tunnelling approach is purely semiclassical in nature and quantum corrections are not included. Also this method is applicable for Painleve type coordinates only – one can not use the original metric co-ordinates to avoid horizon singularity. Lastly, the tunnelling approach is not applicable for massive particles \([21]\).

3 Hamilton-Jacobi Method : Quantum Corrections

We shall now follow the alternative approach as mentioned in the introduction, i.e., the HJ method to evaluate the imaginary part of the action and hence the Hawking temperature. We shall analyze beyond semiclassical approximation by incorporating possible quantum corrections. As this method is not affected by the coordinate singularity at the horizon so we shall use the general BH metric (1) for convenience.

In the back ground of the gravitational field described by the metric (1), massless scalar particles obey the Klein Gordan equation

\[
-\frac{\hbar^2}{\sqrt{-g}} \partial_{\mu} \left[ g^{\mu\nu} \sqrt{-g} \partial_{\nu} \right] \psi = 0
\]

For spherically symmetric BH as we are only considering radial trajectories so we shall consider \((t, r)\) -sector in the space time given by equation (11), i.e., we concentrate to two-dimensional BH problems. Using (11) the above Klein -Gordan equation becomes

\[
\frac{\partial^2 \psi}{\partial t^2} - \frac{1}{2AB} \frac{\partial}{\partial t} \left( AB \right) \frac{\partial \psi}{\partial t} - \frac{1}{2} \frac{\partial}{\partial r} \left( AB \right) \frac{\partial \psi}{\partial r} - AB \frac{\partial^2 \psi}{\partial r^2} = 0
\]

Using the standard ansatz for the semiclassical wave function namely

\[
\psi(r, t) = \exp \left\{ -\frac{i}{\hbar} S(r, t) \right\}
\]
the differential equation for the action $S$ is
\[ \left( \frac{\partial S}{\partial t} \right)^2 - AB \left( \frac{\partial S}{\partial r} \right)^2 + i\kappa \left[ \frac{\partial^2 S}{\partial t^2} - AB \frac{\partial^2 S}{\partial r^2} - \frac{1}{2AB} \frac{\partial (AB)}{\partial t} \frac{\partial S}{\partial t} - \frac{1}{2} \frac{\partial (AB)}{\partial r} \frac{\partial S}{\partial r} \right] = 0 \] (16)

To solve this partial differential equation we expand the action $S$ in powers of Planck’s constant $\hbar$ as
\[ S(r, t) = S_0(r, t) + \sum h^k S_k(r, t) \] (17)

with $\kappa$, a positive integer. Note that in the above expansion terms of the order of Planck’s constant and its higher powers are considered as quantum corrections over the semiclassical action $S_0$. Now substituting the ansatz (17) for $S$ into (16) and equating different powers of $\hbar$ on the both sides we obtain the following set of partial differential equations:

\[ h^0 : \left( \frac{\partial S_0}{\partial t} \right)^2 - AB \left( \frac{\partial S_0}{\partial r} \right)^2 = 0 \] (18)

\[ h^1 : \frac{\partial S_0}{\partial t} \frac{\partial S_1}{\partial t} - AB \frac{\partial S_0}{\partial r} \frac{\partial S_1}{\partial r} + i \frac{1}{2} \left[ \frac{\partial^2 S_0}{\partial t^2} - AB \frac{\partial^2 S_0}{\partial r^2} - \frac{1}{2AB} \frac{\partial (AB)}{\partial t} \frac{\partial S_0}{\partial t} - \frac{1}{2} \frac{\partial (AB)}{\partial r} \frac{\partial S_0}{\partial r} \right] = 0 \] (19)

\[ h^2 : \left( \frac{\partial S_1}{\partial t} \right)^2 + 2 \frac{\partial S_0}{\partial t} \frac{\partial S_2}{\partial t} - AB \left( \frac{\partial S_1}{\partial r} \right)^2 - 2AB \frac{\partial S_0}{\partial r} \frac{\partial S_2}{\partial r} + i \left[ \frac{\partial^2 S_1}{\partial t^2} - AB \frac{\partial^2 S_1}{\partial r^2} - \frac{1}{2AB} \frac{\partial (AB)}{\partial t} \frac{\partial S_1}{\partial t} - \frac{1}{2} \frac{\partial (AB)}{\partial r} \frac{\partial S_1}{\partial r} \right] = 0 \] (20)

and so on.

Apparently, different order partial differential equations are very complicated but fortunately there will be lots of simplifications if in the partial differential equation corresponding to $h^k$, all previous partial differential equations are used and finally we obtain identical partial differential equation, namely

\[ h^k : \frac{\partial S_k}{\partial t} = \pm \sqrt{A(r, t)B(r, t)} \frac{\partial S_k}{\partial r} \] (21)

for $k = 0, 1, 2, \ldots$.

Thus quantum corrections satisfy same differential equation as the semiclassical action $S_0$. Hence the solutions will be very similar. To solve $S_0$ it is to be noted that due to non-static BHs the metric coefficients are functions of $r$ and $t$ and hence standard Hamilton-Jacobi method can not be applied, some generalization is needed. We start with a general ansatz [13]

\[ S_0(r, t) = \int_0^t \omega_0(t') dt + D_0(r, t) \] (22)

Here $\omega_0(t)$ behaves as the energy of the emitted particle and the justification of the choice of the integral is that outgoing particle should have time dependent continuum energy.

Now substituting the above ansatz for $S_0(r, t)$ into equation (18) and using the radial null geodesic in the usual metric from (11) namely

\[ \frac{dr}{dt} = \pm \sqrt{AB} \] (23)

we have,

\[ \frac{\partial D_0}{\partial r} + \frac{\partial D_0}{\partial t} \frac{dt}{dr} = \mp \omega_0(t) \frac{dt}{dr} \]

i.e.,

\[ \frac{dD_0}{dr} = \mp \frac{\omega_0(t)}{\sqrt{AB}} \]

\[ D_0 = \mp \omega_0(t) \int_0^r \frac{dr}{\sqrt{AB}} \] (24)

Hence the complete semiclassical action takes the form

\[ S_0(r, t) = \int_0^t \omega_0(t') dt' \mp \omega_0(t) \int_0^r \frac{dr}{\sqrt{AB}} \] (25)
Here the ′−′ (or ′+′) sign corresponds to outgoing (or ingoing) particle. As the solution (25) contains an arbitrary time dependent function \( \omega_0(t) \) so a general solution for \( S_k \) can be written as

\[
S_k(r, t) = \int_0^t \omega_k(t')dt' + \omega_0(t) \int_0^r \frac{dr}{\sqrt{AB}} , \quad k = 1, 2, 3, \ldots
\]

Thus from equation (15) using the solutions (25) and (26) into equation (17) the wave functions for outgoing and incoming scalar particle can be expressed as

\[
\psi_{\text{out}}(r, t) = \exp \left\{ -\frac{i}{\hbar} \left[ \left( \int_0^t \omega_0(t')dt' + \Sigma_k h^k \int_0^t \omega_k(t')dt' \right) - \left( \omega_0(t) + \Sigma_k h^k \omega_k(t) \right) \int_0^r \frac{dr}{\sqrt{AB}} \right] \right\}
\]

and

\[
\psi_{\text{in}}(r, t) = \exp \left\{ -\frac{i}{\hbar} \left[ \left( \int_0^t \omega_0(t')dt' + \Sigma_k h^k \int_0^t \omega_k(t')dt' \right) + \left( \omega_0(t) + \Sigma_k h^k \omega_k(t) \right) \int_0^r \frac{dr}{\sqrt{AB}} \right] \right\}
\]

respectively. Due to tunnelling across the horizon there will be a change of sign of the metric coefficients in the \((r, t)\)-part of the metric and as a result function of \( t \) coordinate has an imaginary part which will contribute to the probabilities. So we write

\[
P_{\text{in}} = |\psi_{\text{in}}(r, t)|^2 = \exp \left\{ \frac{2Im}{\hbar} \left[ \left( \int_0^t \omega_0(t')dt' + \Sigma_k h^k \int_0^t \omega_k(t')dt' \right) + \left( \omega_0(t) + \Sigma_k h^k \omega_k(t) \right) \int_0^r \frac{dr}{\sqrt{AB}} \right] \right\}
\]

and

\[
P_{\text{out}} = |\psi_{\text{out}}(r, t)|^2 = \exp \left\{ \frac{2Im}{\hbar} \left[ \left( \int_0^t \omega_0(t')dt' + \Sigma_k h^k \int_0^t \omega_k(t')dt' \right) - \left( \omega_0(t) + \Sigma_k h^k \omega_k(t) \right) \int_0^r \frac{dr}{\sqrt{AB}} \right] \right\}
\]

To have some simplification we shall now use the physical fact that all incoming particles certainly cross the horizon, i.e., \( P_{\text{in}} = 1 \). So from equation (29)

\[
Im \left( \int_0^t \omega_0(t')dt' + \Sigma_k h^k \int_0^t \omega_k(t')dt' \right) = -Im \left( \omega_0(t) + \Sigma_k h^k \omega_k(t) \right) \int_0^r \frac{dr}{\sqrt{AB}}
\]

and hence \( P_{\text{out}} \) simplifies to

\[
P_{\text{out}} = \exp \left\{ -\frac{2}{\hbar} \left( \omega_0(t) + \Sigma_k h^k \omega_k(t) \right) \int_0^r \frac{dr}{\sqrt{AB}} \right\}
\]

Then from the principle of ”detailed balance” [10] [11] [12] we write

\[
P_{\text{out}} = \exp \left\{ \left( -\frac{\omega_0(t)}{T_h} \right) \right\} P_{\text{in}} = \exp \left\{ \left( -\frac{\omega_0(t)}{T_h} \right) \right\}
\]

So comparing (32) and (33), the temperature of the BH is given by

\[
T_h = \frac{\hbar}{4} \left[ 1 + \Sigma_k h^k \frac{\omega_k(t)}{\omega_0(t)} \right]^{-1} \left[ Im \int_0^r \frac{dr}{\sqrt{AB}} \right]^{-1}
\]

where

\[
T_h = \frac{\hbar}{4} \left[ Im \int_0^r \frac{dr}{\sqrt{AB}} \right]^{-1}
\]

is the usual Hawking temperature of the BH. Thus due to quantum corrections the temperature of the BH is modified from the Hawking temperature and both the temperatures are functions of ′t′ and ′r′. Note that equation (35) is the standard expression for semiclassical Hawking temperature and it is valid for non-spherical metric also. However for spherical metric, one can use the Taylor series expansions (10) and (11) near the horizon and obtain \( T_H \) as given in equation (11) by performing the contour integration. The ambiguity of factor of two (as mentioned earlier) in the Hawking temperature does not arise here.
Further, one may note that solutions (25) or (26) are the unique solution to equations (18) or (21) except for a pre-multiplicative factor. This arbitrary multiplicative factor does not appear in the expression for Hawking temperature, only the particle energy (ω₀) or ω_k are re-scaled. As the quantum correction term contains ration of (ω_k/ω₀) so it does not involve the arbitrary multiplicative factor and hence unique.

To have some interpretation about the arbitrary functions ω_k(t) appear in the quantum correction terms we make use of dimensional analysis. As S₀ has the dimension h so the arbitrary function ω_k(t) has the dimension h^{-k}. In standard choice of units namely G = c = K_B = 1, h ∼ M_p^2 and so ω_k ∼ M^{-2k}, where M is the mass of the BH.

Similar to the Hawking temperature the surface gravity of the BH is modified due to quantum correction. If κ_c is the semiclassical surface gravity corresponding to Hawking temperature, i.e., κ_c = 2πT_H, then the quantum corrected surface gravity κ = 2πT_h is related to its semiclassical value by the relation

\[ \kappa = \kappa_c \left[ 1 + \sum_k h^k \omega_k(t) \right]^{-1} \]  

Moreover, based on the dimensional analysis if we choose for simplicity,

\[ \omega_k(t) = \frac{a^k \omega_0(t)}{M^{2k}} \quad \text{a is a dimensionless parameter.} \]  

then the expression (36) is simplified to

\[ \kappa = \kappa_0 \left( 1 - \frac{h a}{M^2} \right)^{-1} \]  

This is related to the one loop back reaction effects in the space time [6, 22] with the parameter 'a' corresponds to trace anomaly. Higher order loop corrections to the surface gravity can be obtained similarly by suitable choice of functions ω_k(t). For static BHs Banerjee et. al. [21] have studied these corrections in details. Lastly, it is worthy to mention that identical result for BH temperature may be obtained if we use the Painleve coordinate system as in the previous section.

4 Entropy Function and Quantum Correction

We shall now examine how the semiclassical Bekenstein-Hawking area law namely, \( S_{BH} = \frac{A}{4\hbar} \) (A = area of the horizon) is modified due to quantum corrections described in the previous section. The first law of the BH mechanics which is essentially the energy conservation relation, related the change of BH mass (M) to the change of its entropy (S_{BH}), electric charge (Q) and angular momentum (J) as

\[ dM = T_h dS_{BH} + \Phi dQ + \Omega dJ \]  

Here Ω is the angular velocity and Φ is the electro static potential. So for non-rotating uncharged BHs the entropy has the simple form

\[ S_{BH} = \int \frac{dM}{T_h} \]  

or using equation (34) for the \( T_h \) we get

\[ S_{BH} = \int \left[ 1 + \sum_k h^k \omega_k(t) \right] \frac{dM}{T_H} \]  

For the choice (37) corresponding to one loop back reaction effects we have from (11) the quantum corrected BH entropy as

\[ S_{BH} = \int \left[ 1 + \frac{a h}{M} + \frac{a^2 h^2}{M^2} + \ldots \right] \frac{dM}{T_H} \]  

The first term is the usual semi classical Bekenstein-Hawking entropy and the subsequent terms are the quantum corrections of different order. For static BHs Banerjee et. al. [21] have shown the corrections terms of which the leading one gives the standard logarithmic correction. On the other hand, for non-static BHs as the proportionality factors are time dependent and arbitrary (see equation(41)) so the leading order correction term may not be logarithmic. For future work, we shall attempt to determine physical interpretation of the arbitrary time dependent proportionality factors so that quantum corrections can be evaluated.
5 Summary:

The work is an attempt to study quantum corrections to Hawking radiation from a dynamical BH. At first radial null geodesic tunnelling approach has been used with Painleve-type choice of coordinate system to derive semi-classical Hawking temperature. Then a full quantum mechanical calculations have been performed writing action in a power series of the plank constant $\hbar$ to evaluate the quantum corrections to the hawking temperature. Subsequently quantum corrected surface gravity has been calculated and it is found that one loop back reaction effects in the space time can be obtained by suitable choice of the arbitrary functions and parameters. Finally, an expression for the quantum corrected entropy of the BH has been evaluated. It is found that due to presence of the arbitrary functions in the expression for entropy the leading order quantum correction may not be logarithmic in nature. For future work we shall try to find a solution of the partial differential in a more simpler form so that more physical interpretations can be done from the black hole parameters.

Acknowledgement:

RB and NM want to thank West Bengal State Govt. and CSIR, India respectively for awarding JRF. Authors are thankful to IUCAA, Pune for local hospitality and facilities for research works as this work was done during a visit there.

References

[1] Hawking, S. W. :- Commun. Math. Phys. 43, 199(1975); Nature 248 30(1974).
[2] Hartle, J.B., Hawking, S. W. :- Phys. Rev. D 13, 2188(1976).
[3] Bekenstein, J.D. :- Phys. Rev. D 9, 2188(1974).
[4] Damour, T. and Ruffini, R. :- Phys. Rev. D 14, 332(1976).
[5] Christensen, S. M., Fulling, S. A. :- Phys. Rev. D 15, 2088(1977).
[6] York, J.W. :- Phys. Rev. D 31, 755(1985).
[7] Zhao, Z. Zhu, J.Y. :- Acta. Phys. Sin. 48, 1555(1999).
[8] Parikh, M. K., Wilczek, F. :- Phys. Rev. Lett. 85, 5042(2000).
[9] Parikh, M. K. :- preprint 0402166[hep-th].
[10] Srinivasan, K., Padmanabhan, T. :- Phys. Rev. D 60, 024007(1999).
[11] Shankaranarayanan, S., Srinivasan, K., Padmanabhan, T. :- MPLA 16, 571(2001); Class. Quantum. Grav. 19, 2671(2002).
[12] Banerjee, R., Majhi, B.R., Samanta, S. :- arXiv : 0801.3583.
[13] Siahaan, H. M., Triyanta :- IJMPA 25, 145(2010).
[14] Chowdhury, B. D. :- Pramana 70, 593(2008).
[15] Akhmedov, E.T., Akhmedova, V., Singleton, D. :- Phys. Lett. B 642, 124(2006).
[16] Akhmedov, E.T., Pilling, T., Singleton, D. :- arXiv :0805.2653.
[17] Banerjee, R., Majhi, B.R. :- Phys. Lett. B 662, 62(2008).
[18] Li, H.-L., Yang, S.-Z. :- Euro. Phys. Lett. 79 20001(2007).
[19] Jiang, Q.-Q., Wu, S.-Q. :- Phys. Lett. B 635, 151(2006).
[20] Hu, Y.-P., Zhang, J.-Y., Zhao, Z. :- IJMPD 16, 847(2007).
[21] Banerjee, R., Majhi, B.R. :- JHEP 06, 095 (2008).
[22] Lousto, C.O, Sanchez, N.G. :- Phys. Lett. B 212,411(1988).