Sum Rule for the Structure Functions of the Deuteron from the Current Algebra on the Null-Plane

Susumu Koretune

Department of Physics, Shimane University, Matsue 690-8504, Japan

(Received June 19, 2009; Revised August 12, 2009)

The fixed-mass sum rules for the deuteron target have been derived using the connected matrix element of the current anticommutation relation on the null-plane. From these sum rules, we obtain the relation between the pseudo-scalar meson deuteron total cross sections and the structure functions of the deuteron. We show that the nuclear effect on the mean hypercharge of the sea quark of the proton can be studied using this relation. Furthermore, we obtain the relation among the Born term, resonances, and non-resonant background in a small-$Q^2$ region, and as a new aspect of the spin 1 target, the explicit relation of the tensor structure function $b_2$ at a small or moderate $Q^2$ to that at a large $Q^2$.

Subject Index: 166, 167, 224, 225, 232

§1. Introduction

The current algebra based on canonical quantization at an equal-time gives us very general constraints. These constraints are essential ingredients in QCD. The sum rules in the current-hadron reaction in this formalism are called the fixed-mass sum rules because the mass of the current takes a fixed spacelike or null value. The Adler and Adler-Weisberger sum rules are typical examples of these sum rules. In the former case, the momentum transfer of a weak boson that couples to the hadronic weak current is fixed at a spacelike value; in the latter case, the square of the momentum of the pseudo-scalar meson related to the divergence of the axial-vector current is fixed at an off-shell value of 0. This algebra has been extended to the form based on the canonical quantization at an equal null-plane time. The superior points of the null-plane formalism over the equal-time formalism are as follows, (1) We need not take an infinite momentum frame. (2) Some sum rules in the equal-time formalism get corrections from bilocal currents, without which the sum rules were considered to be peculiar. (3) A technical point in dealing with some graphs contributing to the intermediate state. These were explained in Ref. 2). Apart from these facts, null-plane formalism involves further extension to the current anticommutation relation.,3,4) We briefly explain the fact in the following and technical aspects are summarized in Appendix A.

In the late 60’s, from the experimental finding of a parton at SLAC, the light-cone current algebra was proposed. This algebra was abstracted from the leading light-cone singularity of the current commutation relation in the free-quark model.5) Furthermore, since the leading singularity was mass-independent, it was suggested that the reasoning to reach this algebra can be extended to the current product if I am grateful to Prof. S. B. Gerasimov for informing me about the papers in Ref. 4).
we sacrifice the causal nature of the current commutation relation. However, this type of relation has been suggested but not considered further in Ref. 5). On the other hand, since the assumption for extracting the light-cone singularity was too restrictive, another method based on the canonical quantization on the null-plane was considered. 2) This algebra was a direct generalization of the equal-time formalism. The canonical quantization on the null-plane originated from Dirac 6) and was unrelated to the light-cone current algebra. However, similar bilocal quantities appeared in both methods. The bilocal quantities in the light-cone current algebra were regular operators where all singularities in the light-cone limit were taken out and hence were different from those in the null-plane formalism, where no such manipulation has been imposed. However, because of the similarity, the bilocal quantities in the light-cone current algebra and those in the null-plane formalism were often identified on the null-plane as a heuristic method for obtaining some physical insight. Among them the works in Ref. 4) is the first attempt to obtain some relations at the wrong signature point in this sense. Now, through the finding of the scaling violations that led to the QCD, it was recognized that the method of taking the leading light-cone singularity should be refined. In fact, the short distance expansion was taken first, and with the use of the dispersion relation, this expansion was analytically continued to the region near the light-cone. This light-cone expansion utilized the causality of the current commutation relation; hence, the moment sum rules obtained in this expansion were at alternate integers. Furthermore, each moment corresponded to the matrix element of the local operator obtained by the expansion of the bilocal operator, and it was found that because of the anomalous dimension we could not take out the light-cone singularity uniformly from each moment. Thus, the expansion by the singular coefficient function multiplied by the regular bilocal current in the light-cone current algebra had broken down. The relation at the missing integers was later shown to be obtained by the cut vertex formalism, 7) which suggested that these quantities are related to nonlocal quantities. A physical application of the light-cone expansion was restricted to the deep-inelastic region.

Now, through the study of the fixed-mass sum rules in the semi-inclusive lepton-hadron scatterings where one soft pion was observed, we encountered the current anticommutation relation on the null-plane. Since at that time we knew that the simple method of taking out the leading light-cone singularity was wrong and that the bilocal operator in this method should not be taken literally, we needed some methods of abstracting the current anticommutation relation on the null-plane. It was at this point where the Deser, Gilbert, and Sudarshan (hereafter called DGS) representation 8) played an important role. 3) Through this method, it became possible to consider the fixed-mass sum rules at the wrong signature point with the use of a connected matrix element of the current anticommutation relation between stable hadrons. Thus far, application of this method has been restricted to hadrons. However, as long as the s and u channels are disconnected and the target particle is stable, this method can be used. Now the sum rules from the current anticommutation relation gave us information on sea quarks in hadrons. A typical example of this fact can be seen in the modified Gottfried sum rule. 9)–12) Compared with the Adler sum rule obtained by the current commutation relation, the modified Gottfried sum


Sum Rule for the Structure Functions of the Deuteron

rule has the extra factor \((-1)^n\) from the contribution of the antiquark distributions.\(^{12} \)

Hence, the contribution of the sea quark distribution remains in the sum rule. Thus, the study of the sum rules can give us information on a hadronic vacuum. In other words, we can say that the sum rule controls how a quark-antiquark pair is produced or annihilated in hadrons. From this point of view, it is interesting to extend the method to a nuclear target since a nuclear effect can be studied in addition to effects of hadrons. In this study, as a first step in the application of the method to nuclear targets, the deuteron target case is considered. In §2, the kinematics of the spin 1 deuteron target is given. In §3, the sum rules are derived from the good-good component. In §4, the sum rules are transformed to various forms and the physical meanings of these forms are explained. The summary is given in §5.

\section{2. Kinematics}

The imaginary part of the forward reaction “current\((q) + \text{deuteron}(p) \rightarrow \text{current}(q) + \text{deuteron}(p)\)” is proportional to the total cross section of the inclusive reaction “current\((q) + \text{deuteron}(p) \rightarrow \text{anythings}(X)\)”, where \(q\) is the momentum of the current and \(p\) is that of the deuteron with its mass \(m_d\). This part is called the hadronic tensor and is expressed by assuming the completeness of the sum over \(X\) as

\[
W_{ab}^{\mu\nu}(p, q, E, E^*) = \frac{1}{2\pi} \int d^4x e^{iq\cdot x} \langle p, E|J_\mu^a(x), J_\nu^b(0)\rangle |p, E\rangle_c,
\]

where \(E\) is the polarization vector of the deuteron and the suffix \(c\) on the right-hand side of the equation means taking the connected part. Since the current is the induced hadronic current in the inclusive reaction “lepton + deuteron \(\rightarrow\) lepton + anythings\(\)”, the momentum \(q\) is the difference between the momentum of the initial lepton and that of the final lepton; hence, it takes a space-like value. We first discuss the conserved vector current \(J_\mu^a(x)\) where the suffix \(a\) denotes the flavor index. The generalization to the nonconserved and parity-violating case is given later in this section. Since the hadronic current is a color singlet, we ignore the color suffix in the quark field. Now, by requiring parity and time reversal invariance, we obtain\(^{13}\)

\[
W_{ab}^{\mu\nu}(p, q, E, E^*) = -F_1^{ab} G^{\mu\nu} + F_2^{ab} \frac{P_\mu P_\nu}{\nu} - b_1^{ab} r^{\mu\nu} + \frac{1}{6} b_2^{ab} (s^{\mu\nu} + t^{\mu\nu} + u^{\mu\nu})
\]

\[
+ \frac{1}{2} b_3^{ab} (s^{\mu\nu} - u^{\mu\nu}) + \frac{1}{2} b_4^{ab} (s^{\mu\nu} - t^{\mu\nu}) + \frac{i g_1^{ab}}{\nu} \epsilon^{\mu\nu\lambda\sigma} q_\lambda s_\sigma + \frac{i g_2^{ab}}{\nu^2} \epsilon^{\mu\nu\lambda\sigma} q_\lambda (\nu s_\sigma - s \cdot q P_\sigma),
\]

where \(\nu = p \cdot q, \kappa = 1 - m_d^2 q^2 / \nu^2\), \(P_\mu = p_\mu - (\nu / q^2) q_\mu\), \(G^{\mu\nu} = g^{\mu\nu} - (1 / q^2) q_\mu q_\nu\), \(s^{\mu\nu} = -(i / m_d^2) \epsilon^{\mu \alpha \beta \gamma} E^*_\alpha E_\beta P_\gamma\), \(E \cdot E^* = -m_d^2, p \cdot E = p \cdot E^* = 0\), and

\[
r^{\mu\nu} = \frac{1}{\nu^2} \left( q \cdot E^* q \cdot E - \frac{1}{3} \nu^2 \kappa \right) G^{\mu\nu},
\]

\[
s^{\mu\nu} = \frac{2}{\nu^2} \left( q \cdot E^* q \cdot E - \frac{1}{3} \frac{\nu^2}{\kappa} \right) \frac{P_\mu P_\nu}{\nu},
\]

\[
t^{\mu\nu} = \frac{1}{2\nu^2} \left( q \cdot E^* P_\mu \tilde{E}_\nu + q \cdot E \tilde{E}^* P_\mu + q \cdot E P_\mu \tilde{E}^* + q \cdot E P_\nu \tilde{E}^* + \frac{4\nu}{3} P_\mu P_\nu \right),
\]

\[
u^{\mu\nu} = \frac{1}{\nu} \left( \tilde{E}^* \mu \tilde{E}^* + \tilde{E}^* \nu \tilde{E}^* + \frac{2 m_d^2}{3} G^{\mu\nu} - \frac{2}{3} \frac{P_\mu P_\nu}{\nu} \right),
\]

\(13\)

\(12\)
with \( \tilde{E}^\mu = E^\mu - (q \cdot E/q^2)q^\mu \) and \( \tilde{E}^*^\mu = E^{*^\mu} - (q \cdot E^*/q^2)q^\mu \). A similar hadronic tensor, \( \tilde{W}^{\mu\nu}_{ab}(p,q,E,E^*) \), can be defined by the current anticommutation relation as

\[
\tilde{W}^{\mu\nu}_{ab}(p,q,E,E^*) = \frac{1}{4\pi} \int d^4xe^{iq \cdot x}\langle p,E|\{J^\mu_a(x),J^\nu_b(0)\}|p,E\rangle_c
\]

\[
= -\tilde{F}^{ab}\mu\nu + \tilde{F}^{ab}_{\nu \mu} - \tilde{b}^{ab}\mu\nu + \frac{1}{6}\tilde{b}^a\mu\nu + \frac{1}{2}\tilde{b}^a_{\mu\nu} + \frac{1}{2}\tilde{b}^a_{\mu\nu} - t^{\mu\nu} + \frac{1}{2}\tilde{b}^a_{\mu\nu} - u^{\mu\nu} + \frac{1}{2}\tilde{b}^a_{\mu\nu}.
\]

(4)

The structure functions defined by the current commutation relation and those of the current anticommutation relation are the same quantity in the s channel but are opposite in sign in the u channel. The crossing relation under \( \mu \leftrightarrow \nu \), \( a \leftrightarrow b \) and \( q \rightarrow -q \), are \( F^{ab}_{\mu\nu}(x,Q^2) = -F^{ba}_{\nu \mu}(x,Q^2) \), \( b^{ab}(+,-,Q^2) = b^{ba}(-,+), Q^2) \) = \( b^{ba}(x,Q^2), b^{b}\mu\nu(\mu\nu) = b^{b\mu\nu}(\mu\nu) \), \( g^{ab}(x,Q^2), g^{b\mu\nu}(\mu\nu) = g^{b\mu\nu}(\mu\nu) \), while the structure functions defined by the anticommutation relation are opposite in sign, where \( x = Q^2/2\nu \) with \( q^2 = -Q^2 \).

Now, we take the current as \( J^\mu_a(x) = : \hat{q}(x)\gamma^\mu \frac{\lambda_a}{2} q(x) : \) in the chiral \( SU(N) \times SU(N) \) model. On the null-plane \( x^+ = 0 \), the quark field is decomposed as \( q^{(\pm)}(x) = A^{\pm}q(x) \), where the projection operator is defined as \( A^{\pm} = \frac{1}{2}(1 \pm \gamma^\alpha z^\alpha) \) with \( z^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3) \) and the suffixes of internal symmetry are discarded since their inclusion does not affect the following discussion. Through the equation of motion, \( q^{(-)}(x) \) is related to \( q^{(+)}(x) \). Hence, \( q^{(-)}(x) \) depends on \( q^{(+)}(x) \), and the independent field on the null-plane is \( q^{(+)}(x) \); hence, the canonical quantization is given as \( \{q^{(+)}(x),q^{(+)}(0)\}|_{x^+ = 0} = \sqrt{2}A^+ \delta^{(2)}(\vec{x}) \delta(x^-) \). Since \( J^\mu_+ = : \tilde{q}(x)\gamma^\mu \frac{\lambda_a}{2} \tilde{q}(x) : = \sqrt{2} : q^{(+)}(x)\frac{\lambda_a}{2} q^{(+)}(x) : \), the current for \( \mu = + \) depends only on \( q^{(+)}(x) \), not on the equation of motion. In this sense the current commutation relation on the null-plane for \( \mu = \nu = + \) is called the good-good component. The current \( J^\mu_+ \) depends on one \( q^{(-)}(x) \) and is called the bad component. Thus, the good-good component is

\[
\langle p,E|J^\mu_+(x),J^\mu_+(0)|p,E\rangle_c|_{x^+ = 0}
\]

\[
= i\delta(x^-)\delta^2(\vec{x}) [d_{abc}\langle p,E|A^+_c(x^0)|p,E\rangle_c + f_{abc}\langle p,E|S^+_c(x^0)|p,E\rangle_c],
\]

where

\[
S^{\mu}_{a}(x|0) = \frac{1}{2} \left[ : \tilde{q}(x)\gamma^\mu \frac{\lambda_a}{2} q(0) : + : \tilde{q}(0)\gamma^\mu \frac{\lambda_a}{2} q(x) : \right],
\]

\[
A^{\mu}_{a}(x|0) = \frac{1}{2i} \left[ : \tilde{q}(x)\gamma^\mu \frac{\lambda_a}{2} q(0) : - : \tilde{q}(0)\gamma^\mu \frac{\lambda_a}{2} q(x) : \right],
\]

\[
\langle p,E|S^{\mu}_{a}(x|0)|p,E\rangle_c = p^{\mu}S_{a}(p \cdot x, x^2) + x^{\mu}S_{a}(p \cdot x, x^2)
\]

\[
+ p^{\mu} \left\{ (E^* \cdot x)(E \cdot x) - \frac{1}{3}(x \cdot p)^2 - m^2 x^2 \right\} S_{a}^{P}(p \cdot x, x^2)
\]

\[
+ x^{\mu} \left\{ (E^* \cdot x)(E \cdot x) - \frac{1}{3}(x \cdot p)^2 - m^2 x^2 \right\} S_{a}^{P}(p \cdot x, x^2)
\]

\[
+ \left\{ E^{\mu}(E^* \cdot x) + E^{*\mu}(E \cdot x) - \frac{2}{3}(x \cdot p)^{\mu} - m^2 x^2 \right\} S_{a}^{P}(p \cdot x, x^2),
\]
\[ 
\langle p, E | A_0^+(x|0) | p, E \rangle_c = p^\mu A_\mu(p \cdot x, x^2) + x^\mu \tilde{A}_\mu(p \cdot x, x^2) \\
+ p^\mu \left\{ (E^\nu \cdot x)(E \cdot x) - \frac{1}{3}((x \cdot p)^2 - m_d^2x^2) \right\} A_\nu^P(p \cdot x, x^2) \\
+ x^\mu \left\{ (E^\nu \cdot x)(E \cdot x) - \frac{1}{3}((x \cdot p)^2 - m_d^2x^2) \right\} \tilde{A}_\nu^P(p \cdot x, x^2) \\
+ \left\{ E^\mu(E^\nu \cdot x) + E^{\nu\mu}(E \cdot x) - \frac{2}{3}((x \cdot p)p^\mu - m_d^2x^\mu) \right\} \tilde{A}_\nu^P(p \cdot x, x^2). \tag{6}
\]

The target-polarization-dependent parts are defined so that their contributions vanish when the target polarizations are averaged. The right-hand side of Eq. (5) is equal to \(i \delta(x^-)\delta^2(\vec{x}^\perp)f_{abc}\langle p, E | J_a^+(0) | p, E \rangle_c \) because of the delta function constraint; however, we write the expression before this constraint is applied, since what the DGS representation tells us is that the term corresponding to this term remains in the anticommutation relation and that the other terms are zero on the null-plane.\(^3,9\)

Then the corresponding relation for the current anticommutation relation is

\[ 
\langle p, E | \{ J_a^+(x), J_b^+(0) \} | p, E \rangle_c |_{x^+ = 0} = \frac{1}{\pi} P\left( \frac{1}{x} \right) \delta^2(\vec{x}^\perp) \left[ d_{abc}\langle p, E | A_c^+(x|0) | p, E \rangle_c + f_{abc}\langle p, E | S_c^+(x|0) | p, E \rangle_c \right], \tag{7}
\]

where \( P \) means taking the principal value. Before going to a detailed derivation of the sum rule, we explain the hadronic tensor for the nonconserved and parity-violating currents including the cases of the weak-boson-mediated reactions. In such a general case, we have 36 independent helicity amplitudes since we have two types of helicity 0 state for the nonconserved current.\(^14\) Then, by the time reversal invariance, the independent amplitudes are reduced to 24, and by the parity invariance they are further reduced to 14. Among the 14 amplitudes, 6 correspond to the tensors which enter the hadronic tensor owing to the non-conservation of the currents. These tensors are

\[ 
p^\mu q^\nu + p'^\mu q'^\nu, \quad \left( q \cdot E^\nu q \cdot E - \frac{1}{3} \nu^2 \kappa \right)(p^\mu q'^\nu + p'^\mu q^\nu), \quad \left( q \cdot E^\nu q \cdot E - \frac{1}{3} \nu^2 \kappa \right) q^\mu q^\nu, \]

\[ 
q^\mu q'^\nu, \quad \left\{ q \cdot E^\nu q^\mu \tilde{E}^\nu + q \cdot E^\nu q^\mu \tilde{E}^\mu + q \cdot E q^\mu \tilde{E}^\nu + q \cdot E q^\nu \tilde{E}^\mu - \frac{2}{3} \nu(p^\mu q^\nu + p'^\mu q'^\nu) \right\} + \frac{4\nu^2}{3q^2}q^\mu q^\nu, \quad i\epsilon^{\mu\nu\alpha\beta}p_\alpha s_\beta. \tag{8}
\]

\section{3. Sum rules from the good-good component}

Now, with the use of Eqs. (5)–(7), the sum rule can be obtained by integrating \( W_{ab}^{++} \) and \( \tilde{W}_{ab}^{++} \) over \( q^- \) and assuming the interchange of the setting \( q^+ = 0 \) and \( \nu \) integration. For the polarization-averaged quantities, we obtain from the current commutation relation

\[ 
\int_0^1 \frac{dx}{x} F_2^{[ab]}(x, Q^2) = \frac{1}{4} f_{abc}\Gamma_c, \tag{9}
\]
where \( F_2^{(ab)} = (F_{2}^{ab} - F_{2}^{(a)})/2i \) and \( \langle p, E|J_{a}^{(b)}(0)|p, E \rangle = p^{\mu} \Gamma_{a} \), and from the current anticommutation relation
\[
\int_{0}^{1} \frac{dx}{x} F_{2}^{(ab)}(x, Q^2) = \frac{1}{4\pi} d_{abc} P \int_{-\infty}^{\infty} \frac{d\alpha}{\alpha} A_{c}(\alpha, 0), \quad (10)
\]
where \( F_2^{(ab)} = (F_{2}^{ab} + F_{2}^{(ab)})/2 \). These sum rules take the same form as that in the case of the nucleon target. Let us now derive the sum rules for the polarization-dependent part. We denote the helicity of the polarization vector as \( E_{h} \), and take \( p = (p^{0}, 0, 0, p^{3}) \), \( q = (q^{0}, q^{1}, q^{2}, q^{3}) \), \( \sqrt{2}E_{\pm} = m_{d}(0, \mp, -i, 0) \), \( E_{0} = (p^{0}, 0, 0, p^{3}) \). Then, by taking the polarization vector \( E_{\pm} \), we obtain from the commutation relation
\[
\int_{0}^{1} \frac{dx}{x} \left( (\kappa - 7)b_{2}^{(ab)}(x, Q^2) + 3(\kappa - 1)b_{3}^{(ab)}(x, Q^2) + 3(\kappa - 1)b_{4}^{(ab)}(x, Q^2) \right) = 0, \quad (11)
\]
and from the anticommutation relation
\[
\int_{0}^{1} \frac{dx}{x} \left( (\kappa - 7)b_{2}^{(ab)}(x, Q^2) + 3(\kappa - 1)b_{3}^{(ab)}(x, Q^2) + 3(\kappa - 1)b_{4}^{(ab)}(x, Q^2) \right) \\
= -\frac{3}{2\pi} d_{abc} \int_{-\infty}^{\infty} d\alpha \{\alpha A_{c}^{P}(\alpha, 0) + 2\tilde{A}_{c}^{P}(\alpha, 0)\}, \quad (12)
\]
where the symmetric and antisymmetric combinations of the spin-dependent structure functions are defined similarly as the structure function \( F_{2}^{(ab)} \). In the case of \( E_{0} \), we obtain no new sum rules. Now, as the transverse polarization vector, we take the cases \( E_{1} = m_{d}(0, 1, 0, 0) \) and \( E_{2} = m_{d}(0, 0, 1, 0) \), and \( p = (p^{0}, 0, 0, p^{3}) \), and \( q = (q^{0}, q^{1}, 0, q^{3}) \). Then, since \( E_{2} \cdot q = E_{2} \cdot q = 0 \), we obtain from the commutation relation
\[
\int_{0}^{1} \frac{dx}{x} \left( (\kappa + 2)b_{2}^{(ab)}(x, Q^2) + 3(\kappa - 1)b_{3}^{(ab)}(x, Q^2) + 3(\kappa - 1)b_{4}^{(ab)}(x, Q^2) \right) = 0, \quad (13)
\]
and from the anticommutation relation
\[
\int_{0}^{1} \frac{dx}{x} \left( (\kappa + 2)b_{2}^{(ab)}(x, Q^2) + 3(\kappa - 1)b_{3}^{(ab)}(x, Q^2) + 3(\kappa - 1)b_{4}^{(ab)}(x, Q^2) \right) \\
= \frac{3}{4\pi} d_{abc} \int_{-\infty}^{\infty} d\alpha \{\alpha A_{c}^{P}(\alpha, 0) + 2\tilde{A}_{c}^{P}(\alpha, 0)\}. \quad (14)
\]
Since \( \kappa - 1 = 4x^{2}m_{d}^{2}/Q^2 \), from Eqs. (11) and (13), we obtain
\[
\int_{0}^{1} \frac{dx}{x} b_{2}^{(ab)}(x, Q^2) = 0, \quad (15)
\]
and
\[
\int_{0}^{1} dx x \left( b_{2}^{(ab)}(x, Q^2) + 3b_{3}^{(ab)}(x, Q^2) + 3b_{4}^{(ab)}(x, Q^2) \right) = 0. \quad (16)
\]
Similarly, from Eqs. (12) and (14), we obtain
\[
\int_{0}^{1} \frac{dx}{x} b_{2}^{(ab)}(x, Q^2) = \frac{1}{4\pi} d_{abc} \int_{-\infty}^{\infty} d\alpha \{\alpha A_{c}^{P}(\alpha, 0) + 2\tilde{A}_{c}^{P}(\alpha, 0)\}, \quad (17)
\]
and
\[
\int_0^1 dx x \left( b_2^{(ab)}(x, Q^2) + 3b_3^{(ab)}(x, Q^2) + 3b_4^{(ab)}(x, Q^2) \right) = 0. \tag{18}
\]

The sum rules (10), (17), and (18) are for the symmetric combination; hence, they can be applied to the electromagnetic current.

Now, since \( \langle p, E | [J_a^\pm(x), J_b^\pm(0)] | p, E \rangle_{c|x^+\to0} = \langle p, E | [J_a^\pm(0), J_b^\pm(x)] | p, E \rangle_{c|x^+\to0} \), we obtain the relation \( \langle p, E | [J_a^\pm(0), J_b^\pm(0)] | p, E \rangle_{c|x^+\to0} = \langle p, E | [J_a^\pm(x), J_b^\pm(0)] | p, E \rangle_{c|x^+\to0} \) with the use of the DGS representation.\(^3\),\(^9\) The hadronic tensor for the axial-vector current is a nonconserved one. Then the tensors given in Eq. (8) are proportional to \( q^+ \), we see that they do not affect the derivation of the sum rule in the above discussion. Thus, the sum rule (10) also holds in this case. Then, by using the PCAC relation, we can transform the sum rule (10) to the ones for the pseudo-scalar deuteron total cross section as in the nucleon case.\(^3\),\(^9\)

\section{4. Application}

Now, the sum rule (10) is the equality of the possible divergent quantity that definitely breaks the condition necessary to derive the sum rule. Including such a case, the importance of the regularization of possible divergent sum rules was explained in Ref. 15). Here, we follow the method in Ref. 16). We first derive the sum rule in the nonforward direction. Then, we see that the right-hand side of the sum rule given by the integral of the non-forward matrix element of the bilocal sum rule in the nonforward direction. Then, we see that the right-hand side of the sum rule is convergent. Next we change \( |t| \) to a smaller value, and subtract the pole singularity from both sides of the sum rule. From this, we obtain the condition that the residue of the pole is \( Q^2 \)-independent. After that, we can still take a smaller \( |t| \) in the subtracted sum rule; we finally obtain the relation at \( t = 0 \). A net result of this manipulation can be mimicked in the forward sum rule by changing the intercept of the slope parameter appropriately.\(^9\),\(^10\),\(^12\) Now we take the chiral \( SU(3) \times SU(3) \) flavor symmetry and obtain

\[
B_\pi + \frac{2f_\pi^2}{4\pi} \int_{\nu_0}^{\alpha} \frac{d\nu}{\nu} \left\{ \sigma_{\pi^+}^d(\nu) + \sigma_{\pi^-}^d(\nu) \right\} \\
= \left[ \frac{1}{2\pi} P \int_{\nu_0^K}^{\alpha} \frac{d\alpha}{\alpha} \left\{ \frac{2\sqrt{6}}{3} A_0(\alpha, 0) + \frac{2\sqrt{3}}{3} A_8(\alpha, 0) \right\} \right. \tag{19}
\]

\[
B_K + \frac{2f_K^2}{4\pi} \int_{\nu_0^K}^{\alpha} \frac{d\nu}{\nu} \left\{ \sigma_{K^+}^d(\nu) + \sigma_{K^-}^d(\nu) \right\} \\
= \left[ \frac{1}{2\pi} P \int_{\nu_0^K}^{\alpha} \frac{d\alpha}{\alpha} \left\{ \frac{2\sqrt{6}}{3} A_0(\alpha, 0) + A_3(\alpha, 0) - \frac{\sqrt{3}}{3} A_8(\alpha, 0) \right\} \right]. \tag{20}
\]
where $\sigma$ represents the off-shell $q^2 = 0$ total cross section of the reaction specified by its upper suffix and can be assumed to smoothly continue to the on-shell one, and $\nu_0^\pi = m_\pi m_d$ and $\nu_0^K = m_K m_d$. $f_\pi$ and $f_K$ are the pion and kaon decay constant, respectively. $B_\pi$ and $B_K$ is the contribution of the Born terms and the unphysical region below the threshold of the continuum contribution. In neutrino reactions, we obtain

$$
\int_0^1 \frac{dx}{2x} \left( F_2^{\pi d}(x, Q^2) + F_2^{\nu d}(x, Q^2) \right) = \frac{1}{2\pi} P \int_0^\infty \frac{\alpha}{\alpha}\left\{ \frac{2\sqrt{6}}{3} A_0(\alpha, 0) + \frac{2\sqrt{3}}{3} A_8(\alpha, 0) \right\},
$$

(21)

and in the electroproduction we obtain

$$
\int_0^1 \frac{dx}{x} F_2^{\pi d}(x, Q^2) = \frac{1}{18\pi} P \int_0^\infty \frac{\alpha}{\alpha}\left\{ 2\sqrt{6} A_0(\alpha, 0) + 3A_3(\alpha, 0) + \sqrt{3} A_8(\alpha, 0) \right\}. \quad (22)
$$

On the left-hand side of the sum rules (21) and (22), the contribution of the Born term is included but it can be neglected in the deep-inelastic region. We regularize the sum rules (19)–(22) by the method explained just before Eq. (19). In addition, here, we assume that $P \int \frac{d\alpha}{\alpha} A_3(\alpha, 0) = 0$ since the deuteron is an isosinglet. Note that this quantity corresponds to the difference between the mean $I_3$ of the quark and antiquark in the proton and that in the neutron in the deuteron; hence, it is zero under isospin symmetry. In this way, we obtain

$$
\int_0^1 \frac{dx}{x} \left\{ \left( F_2^{\pi d}(x, Q^2) + F_2^{\nu d}(x, Q^2) \right) - 3F_2^{\pi d}(x, Q^2) \right\} = \frac{I_d - I_K^d}{3}, \quad (23)
$$

where, by assuming the smooth extrapolation to the on-shell quantity, $I_\pi$ and $I_K$ are defined as

$$
I_\pi^d = B_\pi + \frac{2f_\pi^2}{\pi} \int_0^\infty \frac{d\nu}{\nu^2} \left[ (\nu^2 - m_\pi^2 m_d^2)^{1/2} \left\{ \sigma^{\pi^+ d}(\nu) + \sigma^{\pi^- d}(\nu) \right\} - \nu s_\pi^b \beta_{\pi d} \right]
$$

$$
+ \frac{2f_\pi^2 \beta_{\pi N}}{\pi} \ln \left[ \frac{1}{2\nu_0^\pi} \right], \quad (24)
$$

$$
I_K^d = B_K + \frac{2f_K^2}{\pi} \int_0^\infty \frac{d\nu}{\nu^2} \left[ (\nu^2 - m_K^2 m_d^2)^{1/2} \left\{ \sigma^{K^+ d}(\nu) + \sigma^{K^- d}(\nu) \right\} - \nu s_K^b \beta_{K d} \right]
$$

$$
+ \frac{2f_K^2 \beta_{K N}}{\pi} \ln \left[ \frac{1}{2\nu_0^K} \right], \quad (25)
$$

with the leading high-energy behavior being given by the soft Pomeron as

$$
\left\{ \sigma^{\pi^+ d}(\nu) + \sigma^{\pi^- d}(\nu) \right\} \sim \beta_{\pi d} s_\pi^{\alpha P(0)-1}, \quad \left\{ \sigma^{K^+ d}(\nu) + \sigma^{K^- d}(\nu) \right\} \sim \beta_{K d} s_K^{\alpha P(0)-1},
$$

(26)

where $\alpha P(0) = 1 + b$ with $b = 0.0808$, $s_\pi = m_\pi^2 + m_d^2 + 2\nu$, and $s_K = m_K^2 + m_d^2 + 2\nu$; as a result of the assumption that the divergence in the forward direction comes from the singlet, we obtain $f_\pi^2 \beta_{\pi d} = f_K^2 \beta_{K d}$, and the relation between the residue of
the Pomeron in the pion deuteron cross section and that of the structure function in the lepton-hadron scatterings. In terms of the sea quark distribution function $\lambda_i(x, Q^2)$ of the proton in the deuteron where $i = u, d, s$ specifies the quark, the sum rule (23) can be transformed as

$$\frac{1}{3}\int_0^1 dx \left\{ \lambda_u(x, Q^2) + \lambda_d(x, Q^2) - 2\lambda_s(x, Q^2) \right\} = \frac{1}{2} \left( -1 + \frac{I^d_{\pi} - I^d_{K}}{3} \right). \quad (27)$$

In Eq. (27), we have assumed the isospin symmetry of the quark distribution function and expressed the quark distribution function of the neutron in the deuteron by that of the proton in the deuteron. Furthermore, although we take $\lambda_i = \lambda_i$ for simplicity, what is really required in our formalism is the equality of the integrated quantity. Hence, they can take different values locally. The left-hand side of Eq. (27) is the modification of the sum rules (23) and (27). These sum rules are derived under the assumption where the Pomeron is a flavor singlet. The condition $f^2_{\pi}\beta_{\pi} = f^2_{K}\beta_{K}$ obtained under this assumption is violated phenomenologically. One way to account for this effect is explained in Ref. 10). However, it should be noted that the sum rules (23) and (27) correspond to the quantity related to the hypercharge and hence have a clear physical meaning. They show that the large symmetry restoration of the strange sea quark is necessary in the small-$x$ region. Since the strange sea quark distribution is suppressed above $x = 0.01$ markedly, this symmetry restoration itself is an interesting phenomenon. Hence, the sum rule (23) or (27) should be studied first by neglecting the symmetry breaking effect and taking the symmetry limit of sea quark distributions.\(^{18}\) We explain the possible symmetry breaking effects together with the symmetry relation in Appendix B.

Another application of Eq. (22) is in considering the relations at the arbitrary two different $Q_1^2$ and $Q_2^2$ by separating out the Born term from $F_2^{ed}$:

$$\int_{x_0(Q_1^2)}^1 dx \frac{dx}{x} F_2^{ed}(x, Q_1^2) - \int_{x_0(Q_2^2)}^1 dx \frac{dx}{x} F_2^{ed}(x, Q_2^2) = B(Q_1^2, Q_2^2) + K^{ed}(Q_1^2, Q_2^2), \quad (28)$$

where the contribution of the Born term is given as

$$B(Q_1^2, Q_2^2) = \left[ G_C^2(Q_2^2) + \frac{8}{9} \eta_2 G_Q^2(Q_2^2) + \frac{2}{3} \eta_2 G_M^2(Q_2^2) \right] - \left[ G_C^2(Q_1^2) + \frac{8}{9} \eta_1 G_Q^2(Q_1^2) + \frac{2}{3} \eta_1 G_M^2(Q_1^2) \right] + \frac{2}{3} \eta_1 G_M^2(Q_1^2), \quad (29)$$

with $\eta_i = Q_i^2/4m_i^2$. $G_C, G_M$ and $G_Q$ are the charge, magnetic, and quadrupole
moments of the deuteron defined as
\[
\langle n, E'| J_{em}^\mu(0)| p, E \rangle = -\frac{1}{m_d^2} \left\{ \frac{G_1(Q^2)(E'^* \cdot E) - G_3(Q^2)(E'^* \cdot q)(E \cdot q)}{2m_d^2} \right\} (p + n)^\mu
\]
\[+ G_M(Q^2) \left\{ E^\mu(E'^* \cdot q) - E'^\mu(E \cdot q) \right\}, \tag{30} \]
with \( q = n - p \) for the electromagnetic current \( J_{em}^\mu(0) \), and \( G_1 \) and \( G_3 \) are related to \( G_C, G_M, \) and \( G_Q \) as \( G_C = G_1 + \frac{2}{3} \eta G_Q, G_Q = G_1 - G_M + (1 + \eta) G_3 \) with \( \eta = Q^2/4m_d^2 \). The derivation of the Born term is straightforward but tedious; hence, we give its sketch in Appendix C. \( K^{ed}(Q_1^2, Q_2^2) \) is defined as
\[
K^{ed}(Q_1^2, Q_2^2) = -\int_{x_c(Q_1^2)}^{x_c(Q_2^2)} \frac{dx}{x} F_2^{ed}(x, Q_1^2) + \int_{x_c(Q_2^2)}^{x_c(Q_2^2)} \frac{dx}{x} F_2^{ed}(x, Q_2^2), \tag{31} \]
where \( x_c(Q_2^2) = Q^2/2\nu_c(Q^2) \) with \( \nu_c(Q^2) = (W_c^2 - m_d^2 + Q^2)/2 \). Here we define \( W^2 = (p + q)^2 \) and \( W_c \) is the cutoff invariant mass \( W \). In Eq. (31), the integral over \( x \) should be taken after subtracting the small-\( x \) behavior of \( F_2^{ed}(x, Q_1^2) \) and \( F_2^{ed}(x, Q_2^2) \) by obtaining the condition under which the residue of the pole is \( Q^2 \) independent. It should be noted that, in this regularization, we need not consider the symmetry breaking effect of the Pomeron. In these sum rules, we take \( Q_1^2 \) to be fixed and investigate the \( Q_2^2 \) dependence of the sum rule. Furthermore, if we take \( Q_1^2 \) and \( Q_2^2 \) to be small such that \( K^{ed}(Q_1^2, Q_2^2) \) is negligible small, we have the relation that expresses the intimate relation among the Born term, resonances and non-resonant background.

The regularization of the sum rule (17) can be conducted similarly, and we can transform it to
\[
\int_{x_c(Q_1^2)}^{1} \frac{dx}{x} b_2^{ed}(x, Q_1^2) - \int_{x_c(Q_2^2)}^{1} \frac{dx}{x} b_2^{ed}(x, Q_2^2) = B_{b2}(Q_1^2, Q_2^2) + K_{b2}^{ed}((Q_1^2, Q_2^2), \tag{32} \]
where
\[
B_{b2}(Q_1^2, Q_2^2) = 4\eta_2 \left[ \frac{\eta_2}{1 + \eta_2} \left\{ G_C(Q_2^2) + \frac{\eta_2}{3} G_Q(Q_2^2) - G_M(Q_2^2) \right\} G_Q(Q_2^2) + \frac{1}{4} G_M^2(Q_2^2) \right] \]
\[\qquad\quad - 4\eta_1 \left[ \frac{\eta_1}{1 + \eta_1} \left\{ G_C(Q_1^2) + \frac{\eta_1}{3} G_Q(Q_1^2) - G_M(Q_1^2) \right\} G_Q(Q_1^2) + \frac{1}{4} G_M^2(Q_1^2) \right], \tag{33} \]
and
\[
K_{b2}^{ed}(Q_1^2, Q_2^2) = -\int_{0}^{x_c(Q_1^2)} \frac{dx}{x} b_2^{ed}(x, Q_1^2) + \int_{0}^{x_c(Q_2^2)} \frac{dx}{x} b_2^{ed}(x, Q_2^2). \tag{34} \]
In Eq. (34), the integral over \( x \) should be taken after subtracting the small-\( x \) behavior of \( b_2^{ed}(x, Q_1^2) \) and \( b_2^{ed}(x, Q_2^2) \). Now, we take \( Q_2^2 \) to be large by keeping \( Q_1^2 \) small or moderate. Then, since the Born term at a large \( Q_2^2 \) is negligible, we can neglect it in Eq. (33). When the integral is convergent, we can take \( x_c(Q_1^2) = x_c(Q_2^2) = 0 \); hence, \( K_{b2}^{ed}(Q_1^2, Q_2^2) = 0 \). Then the sum rule (32) relates the tensor polarization and the elastic form factors at a small or moderate \( Q_1^2 \) to the tensor polarization at a large
Q^2_2. In particular, if the Callan-Gross-like relation \( b^d_2 = 2x b^d_1 \) with the vanishing tensor polarization of the sea quark at a large \( Q^2_2 \) holds, the second term on the left-hand side of Eq. (32) is zero.\(^{19} \) In this case, the sum rule becomes the one at a small or moderate \( Q^2_1 \). Now the recent experiment at HERMES shows that\(^{20,21} \)

\[
\int_{0.002}^{0.85} \frac{dx}{x} b^d_2(x, Q^2 = 5 \text{ GeV}^2) > 0.
\]

Although there are unmeasured regions, the HERMES result possibly suggests the nonzero tensor polarization at \( Q^2 = 5 \text{ GeV}^2 \). Since the Born term at \( Q^2 = 5 \text{ GeV}^2 \) is negligible in Eq. (33), we can set \( B(Q^2_1, Q^2_2) = 0 \). Then the sum rule (32) shows that the nonzero polarization persists in the large-\( Q^2 \) region. When the integral over \( b^d_2(x, Q^2)/x \) diverges, since the main contribution comes from the small-\( x \) region and that, at a large \( Q^2 \), \( b^d_2(x, Q^2) \) behaves similarly as \( F^d_2(x, Q^2) \),\(^{13} \) we can expect \( K^d_2(Q^2_1, Q^2_2) > 0 \). Thus, the HERMES result is not contradictory with the zero tensor polarization of the sea quark at a large \( Q^2 \) in a regularized sense.

\[ \text{§5. Summary} \]

We have derived the sum rules for the structure functions of the deuteron from the current anticommutation relation on the null-plane. The sum rules correspond to those at the wrong signature point. As explained in Introduction, they give us information on the vacuum of the deuteron.

From the spin-independent part, we obtain the sum rule for the mean hypercharge of the sea quark of the proton in the deuteron. Furthermore, in the small-\( Q^2 \) region, we obtain the relation among the Born term, resonances, and nonresonant background. From the spin-dependent part, we obtain the relation between the tensor polarization at a small or moderate \( Q^2 \) and that at a large \( Q^2 \).

Now, the application of these sum rules in other forms such as those in photoproduction is possible as in the nucleon target case. Furthermore, although only the sum rules from the good-good component are discussed, the same method can be applied to the good-bad component. In this case, we obtain the sum rules for the spin-dependent structure functions \( g^d_1 \) and \( g^d_2 \). These sum rules take the same form as those in the nucleon target case.\(^{22} \)

\[ \text{Appendix A} \]

---

\[ \text{Current Anticommutation Relation on the Null-Plane through DGS Representation} \]

---

Let us consider the DGS representation of the connected matrix element of the current commutation relation on the null-plane between stable hadrons.\(^8 \)

\[ W_{ab}(p \cdot q, q^2) = \int d^4x \exp(iq \cdot x) \langle p | [J_a(x), J_b(0)] | p \rangle_c \]

\[ \text{Downloaded from https://academic.oup.com/ptp/article-abstract/122/5/1151/1938670} \]

by guest

on 30 July 2018
\[ \int d^4x \exp(iq \cdot x) \int_0^{\infty} d\lambda^2 \int_{-1}^{1} d\beta \exp((i\beta p \cdot x)h_{ab}(\lambda^2, \beta)i\Delta(x, \lambda^2) \]

\[ = (2\pi) \int_0^{\infty} d\lambda^2 \int_{-1}^{1} d\beta \delta((q + \beta p)^2 - \lambda^2)\epsilon(q^0 + \beta p^0)h_{ab}(\lambda^2, \beta), \quad (A.1) \]

where \( W_{ab}(p, q, q^2) \) can be expressed as

\[ W_{ab}(p, q, q^2) = \sum_n (2\pi)^4 \delta^4(p + q - n)\langle p|J_a(0)|n\rangle\langle n|J_b(0)|p\rangle \]

\[ - \sum_n (2\pi)^4 \delta^4(p - q - n)\langle p|J_b(0)|n\rangle\langle n|J_a(0)|p\rangle. \quad (A.2) \]

We denote the lowest mass in the \( s \) channel continuum as \( M_s \) and that in the \( u \) channel as \( M_u \). In the rest frame, \( p = (m, \vec{0}) \), since the first term in Eq. (A.2) is restricted to \( m + q^0 = n^0 \), \( q^0 \) satisfies \( q^0 \geq M_s - m \). Similarly, since the second term is restricted to \( m - q^0 = n^0 \), \( q^0 \) satisfies \( q^0 \leq m - M_u \). Hence, the first and second terms in Eq. (A.2) are disconnected as far as \( m < (M_s + M_u)/2 \).

Now, in the DGS representation, \( h_{ab}(\lambda^2, \beta) \) is not zero only in the shaded region in Fig. 1. The integration path is \( \sigma = 2\beta p \cdot q + q^2 \), where \( \sigma = \lambda^2 - \beta^2 m^2 \). In the rest frame, we see that the point in the integration path where the sign changes through the factor \( \epsilon(p \cdot q + \beta m^2) \) lies always in the region \( \sigma < -\beta^2 m^2 \). In the \( s \) channel, since \( p \cdot q > 0 \), the slope of the integration path is positive. Thus, only the region \( \epsilon(p \cdot q + \beta m^2) = 1 \) contributes to the integral; hence, in the \( s \) channel, we obtain

\[ (2\pi) \int_0^{\infty} d\lambda^2 \int_{-1}^{1} d\beta \delta((q + \beta p)^2 - \lambda^2)h_{ab}(\lambda^2, \beta)\theta(q^0 + \beta p^0) \]

\[ = \sum_n (2\pi)^4 \delta^4(p + q - n)\langle p|J_a(0)|n\rangle\langle n|J_b(0)|p\rangle. \quad (A.3) \]

Similarly, in the \( u \) channel, we obtain

\[ (2\pi) \int_0^{\infty} d\lambda^2 \int_{-1}^{1} d\beta \delta((q + \beta p)^2 - \lambda^2)h_{ab}(\lambda^2, \beta)\theta(-(q^0 + \beta p^0)) \]

Fig. 1. Support property of the spectral function \( h_{ab}(\lambda^2, \beta) \) in the \((\beta, \sigma)\) plane. It is not zero only in the shaded region; below the parabola \( \sigma = -\beta^2 m^2 \), it is zero by causality.
\[
\sum_n (2\pi)^4 \delta^4(p - q - n) \langle p | J_b(0) | n \rangle \langle n | J_a(0) | p \rangle. \tag{A-4}
\]

By combining these two relations, we obtain the DGS representation of the current anticommutation relation as

\[
\widetilde{W}_{ab}(p \cdot q, q^2) = \int d^4x \exp(iq \cdot x) \langle p | \{ J_a(x), J_b(0) \} | p \rangle_c
\]
\[
= \int d^4x \exp(iq \cdot x) \int_0^{\infty} d\lambda^2 \int_{-1}^1 d\beta \exp(i\beta p \cdot x) h_{ab}(\lambda^2, \beta) \Delta^{(1)}(x, \lambda^2)
\]
\[
= (2\pi) \int_0^{\infty} d\lambda^2 \int_{-1}^1 d\beta \delta((q + \beta p)^2 - \lambda^2) h_{ab}(\lambda^2, \beta). \tag{A-5}
\]

The null-plane restriction of the current commutation relation or the anticommutation relation can be obtained by the integration of \(W_{ab}\) and \(\tilde{W}_{ab}\) with respect to \(q^-\).

Now we take the scalar current \(J_a(x) = :\phi^+(x)\tau_a \phi(x) :\) where

\[
[\phi^+(x), \phi(0)]|_{x^+ = 0} = i\Delta(x) \tag{A-6}
\]

with \(\Delta(x) = -\epsilon(x^-)\delta(\vec{x}^\perp)/4\) at \(x^+ = 0\). Using this relation, the current commutation relation at \(x^+ = 0\) becomes

\[
\langle p | [J_a(x), J_b(0)] | p \rangle_c|_{x^+ = 0} = i\Delta(x) \langle p | :\phi^+(x)\tau_a \tau_b \phi(0) : + : \phi^+(0)\tau_b \tau_a \phi(x) : | p \rangle_c. \tag{A-7}
\]

From Eq. (A.1) restricted at the null-plane, we have

\[
\langle p | [J_a(x), J_b(0)] | p \rangle_c|_{x^+ = 0} = \int_0^{\infty} d\lambda^2 \int_{-1}^1 d\beta \exp(i\beta p \cdot x) h_{ab}(\lambda^2, \beta) i\Delta(x, \lambda^2). \tag{A-8}
\]

Since \(\Delta(x, \lambda^2) = \Delta(x) = -\epsilon(x^-)\delta(\vec{x}^\perp)/4\) at \(x^+ = 0\), we obtain the relation

\[
\langle p | :\phi^+(x)\tau_a \tau_b \phi(0) : + : \phi^+(0)\tau_b \tau_a \phi(x) : | p \rangle_c = \int_0^{\infty} d\lambda^2 \int_{-1}^1 d\beta \exp(i\beta p \cdot x) h_{ab}(\lambda^2, \beta). \tag{A-9}
\]

By using this relation, the equation restricted at the null-plane obtained from Eq. (A-5) becomes

\[
\langle p | [J_a(x), J_b(0)] | p \rangle_c|_{x^+ = 0}
\]
\[
= \int_0^{\infty} d\lambda^2 \int_{-1}^1 d\beta \exp(i\beta p \cdot x) h_{ab}(\lambda^2, \beta) \Delta^{(1)}(x, \lambda^2)
\]
\[
= \Delta^{(1)}(x) \langle p | :\phi^+(x)\tau_a \tau_b \phi(0) : + : \phi^+(0)\tau_b \tau_a \phi(x) : | p \rangle_c, \tag{A-10}
\]

where we use the fact that \(\Delta^{(1)}(x, \lambda^2)\) at \(x^+ = 0\) is also independent of the mass \(\lambda^2\) and is given as \(\Delta^{(1)}(x, \lambda^2) = \Delta^{(1)}(x) = -\ln |x^-|\delta(\vec{x}^\perp)/2\pi\). The net result is that the current anticommutation relation on the null-plane can be obtained from the current commutation relation on the null-plane simply by changing \(i\Delta(x, \lambda^2)\) to \(\Delta^{(1)}(x, \lambda^2)\).
at \( x^+ = 0 \). More rigorous reasoning can be made by taking the Fourier transform and considering the conditions necessary to restrict \( W_{ab} \) or \( \tilde{W}_{ab} \) to the null-plane such as

\[
\lim_{\Lambda \to \infty} \int_{-\infty}^{\infty} dq^- \exp \left( -(q^-)^2 / \Lambda^2 \right) H_{ab}(p \cdot q, q^2),
\]

where \( H_{ab} \) is \( W_{ab} \) or \( \tilde{W}_{ab} \). This results in the condition for \( h_{ab} \) and is known to be equivalent to the superconvergence relation required to obtain the fixed-mass sum rule; in the null-plane formalism. It is equivalent to the interchange for setting \( q^+ = 0 \) and the integral. It is at this point where the difference between the connected matrix element of the stable hadron of the current commutation relation and that of the current anticommutation relation appears.

### Appendix B

**SU(3) Symmetry Breaking Effect on Symmetry Relation**

Here we consider the sum rules for SU(3). The regularization of the sum rules (19)–(22) can be conducted as explained in the paragraph before Eq. (19). The detailed method is given, for example, in Ref. 12). We first summarize the result. We assume the leading high-energy behavior is given by the soft Pomeron as

\[
\{ F_2^{\bar{d}d} + F_2^{vd} \} \sim \left( Q_0^2/Q^2 \right)^{\alpha_P(0)-1} \beta_{vd}(Q^2, 1 - \alpha_P(0))(2\nu)^{\alpha_P(0)-1},
\]

where \( Q_0^2 = 1 \) GeV\(^2\) and

\[
F_2^{ed} \sim \left( Q_0^2/Q^2 \right)^{\alpha_P(0)-1} \beta_{ed}(Q^2, 1 - \alpha_P(0))(2\nu)^{\alpha_P(0)-1}.
\]

We expand \( \beta_{ld} \) with \( l = e \) or \( \nu \) as

\[
\beta_{ld}^0 - (\epsilon - b)\beta_{ld}^1 + O((\epsilon - b)^2),
\]

where the intercept of the Pomeron is set as \( \alpha_P(0) = 1 + b - \epsilon \) and \( \epsilon \) approaches \( b \) from above. The parameter \( \epsilon \) goes to 0 finally after taking out the pole terms from both sides of the sum rule. This change in the parameter mimics the \( -t \) in the nonforward sum rules. Then by assuming that the Pomeron is a flavor singlet and that it comes from the term \( A_0(\alpha, 0) \) being flavor singlet, we obtain the relations \( \beta_{vd}^0 = 6 \beta_{ed}^0 \) and \( \beta_{vd}^1 = 6 \beta_{ed}^1 \) from the sum rules (21) and (22). Furthermore, from the sum rules (19) and (21), we obtain the condition \( \pi \beta_{\nu d}^0 = 4 f_2^2 \beta_{\pi d} \), and from the sum rules (19) and (20), the condition \( f_{\pi}^2 \beta_{\pi d} = f_{K}^2 \beta_{K d} \). The regularization-dependent terms in the sum rules are related to each other by these relations, and we obtain the relations independent of the regularization. In this way, we obtain the relation (23) and

\[
C_d = \frac{1}{9} (2I_\pi + I_K),
\]

where

\[
C_d = \int_0^1 dx \left\{ \frac{F_2^{ed} - \beta_{ed}^0 x^{-b}}{x} - \beta_{ed}^1 \right\}.
\]
Now the condition \( f_\pi^2 \beta_{\pi d} = f_K^2 \beta_{K d} \) is violated about 20% phenomenologically. Hence, the sum rules (23) and (27) still diverge if we use the phenomenological value. To remedy this, we consider the mixing of the singlet and octet as

\[
\tilde{A}_0(\alpha, 0) = A_0(\alpha, 0) \cos \theta + A_8(\alpha, 0) \sin \theta,
\]
\[
\tilde{A}_8(\alpha, 0) = -A_0(\alpha, 0) \sin \theta + A_8(\alpha, 0) \cos \theta,
\]
and assume that the contribution of the Pomeron is given by the term \( \tilde{A}_0(\alpha, 0) \).

Thus the residue of the Pomeron has an \( SU(3) \)-symmetry-breaking piece. Then, by rewriting the sum rules (19)–(22) with the use of Eq. (B.6) and by regularizing them, we obtain

\[
\frac{f_\pi^2 \beta_{\pi d}}{2(\sqrt{2} \cos \theta + \sin \theta)} = \frac{f_K^2 \beta_{K d}}{2\sqrt{2} \cos \theta - \sin \theta},
\]

and

\[
\beta_{\nu d}^i = \frac{12(\sqrt{2} \cos \theta + \sin \theta)}{2\sqrt{2} \cos \theta + \sin \theta} \cdot \beta_{\nu d}^i,
\]  

where \( i = 0, 1 \). The relation between \( \beta_{\nu d}^0 \) and \( \beta_{\nu d}^1 \) is the same as that before the mixing. In the case of the nucleon target, a relation similar to Eq. (B.7) was derived, and it was found that the relation is satisfied phenomenologically at about \( \theta \sim -13^\circ \).

In the deuteron case, the relation (B.7) seems to be satisfied well phenomenologically at about the same angle because the large constant term at a high energy in the cross-section formula of the Particle data group\(^{23}\) satisfies the relation (B.7) at this angle.

Now, by this mixing, we find that the relation (B.4) also holds and that the sum rule (23) can be rewritten as

\[
\int_0^1 \frac{dx}{x} \left\{ \left( F_{2v}^{\nu d}(x, Q^2) + F_{2d}^{\nu d}(x, Q^2) \right) - \frac{6(\sqrt{2} \cos \theta + \sin \theta)}{2\sqrt{2} \cos \theta + \sin \theta} F_{2d}^{\nu d}(x, Q^2) \right\} = \frac{I_{\pi}^d - I_{K}^d}{3} - \frac{3 \sin \theta}{2\sqrt{2} \cos \theta + \sin \theta} \cdot C_d.
\]  

Then, the sum rule (27) becomes

\[
(2\sqrt{2} \cos \theta - \sin \theta)I_{\pi} - 2(\sqrt{2} \cos \theta + \sin \theta)I_{K} = 6\sqrt{2}(\cos \theta - \sqrt{2} \sin \theta)
\]
\[
+ \int_0^1 dx \left\{ 4\sqrt{2}(\cos \theta - \sqrt{2} \sin \theta)(\lambda_u + \lambda_d) - 8(\sqrt{2} \cos \theta + \sin \theta)\lambda_s \right\}.
\]

Since the strange sea quark is suppressed above \( x = 0.01 \) markedly, the large symmetry restoration of the sea quark exists in a small-\( x \) region,\(^{18}\) and that to satisfy Eq. (B.10) the small-\( x \) limit of the strange sea quark distribution must be larger than that of the \( u \)- or \( d \)-type sea quark.

**Appendix C**

**Born Term Contributions**

The Born term contribution to Eq. (1) is given as

\[
\frac{1}{2} \delta(2\nu - Q^2)B^{\mu \nu},
\]
with
\[
B^{\mu\nu} = \langle p, E | J_{em}^\mu(0) | n, E' \rangle \langle n, E' | J_{em}^\nu(0) | p, E \rangle
= \frac{1}{m_d^4} \sum_\lambda \left\{ \left\{ G_1(Q^2)(E^* \cdot E') - G_3(Q^2) \frac{(E^* \cdot q)(E' \cdot q)}{2m_d^2} \right\}(p + n)\mu
+ G_M(Q^2)\left\{ - E^{\mu}(E^* \cdot q) + E^{*\mu}(E' \cdot q) \right\} \right\}
\times \left\{ G_1(Q^2)(E'^* \cdot E) - G_3(Q^2) \frac{(E'^* \cdot q)(E \cdot q)}{2m_d^2} \right\}(p + n)\nu
+ G_M(Q^2)\left\{ E^{\nu}(E'^* \cdot q) - E'^{*\nu}(E \cdot q) \right\},
\right. \tag{C.2}
\]
where \( n = p + q \) and \( \lambda \) is the polarization of \( E' \). Here, we denote it as \( E'(n, \lambda) \). Then using
\[
\sum_\lambda E^{*\mu}(n, \lambda)E^{*\nu}(n, \lambda) = n^{\mu}n^{\nu} - m_d^2g^{\mu\nu}, \tag{C.3}
\]
with \( E' \cdot n = E'^* \cdot n = 0 \) and \( E' \cdot E'^* = -m_d^2 \), we take the product on the right-hand side of Eq. (C.2). Then we classify the product into symmetric and antisymmetric terms under the interchange of \( E \) and \( E^* \). We first calculate the symmetric ones and take the polarization average of the initial deuteron and obtain the Born term contributions to \( F_1 \) and \( F_2 \) as
\[
F_1 = \delta(2\nu - Q^2)\frac{Q^2}{3}(\eta + 1)G_M^2, \tag{C.4}
\]
and
\[
F_2 = \delta(2\nu - Q^2)Q^2\left( G_C^2 + \frac{8}{9}\eta^2G_Q^2 + \frac{2}{3}\eta G_M^2 \right). \tag{C.5}
\]
The rest of the symmetric terms contribute to \( b_1 \sim b_4 \). By noting that the polarization-averaged parts are subtracted, and that \( g^{\mu\nu} \) is only in the tensor \( G^{\mu\nu} \) we find the contribution to the sum of \( b_1 \) and \( b_2 - 3b_3 \) with an appropriate coefficient. Furthermore, since \( E^{\mu}E'^{\nu} + E'^{\mu}E^{\nu} \) is only in the tensor \( u^{\mu\nu} \), we obtain the contribution to \( b_2 - 3b_3 \). Hence, we can separate the contribution to \( b_1 \) and \( b_2 - 3b_3 \). A similar consideration can be made to the coefficient of \( t^{\mu\nu} \), which gives the contribution to \( b_2 - 3b_4 \), and \( p^{\mu}p^{\nu} \), which gives the contribution to \( b_2 + 3b_3 + 3b_4 \). Thus, we obtain
\[
b_1 = \delta(2\nu - Q^2)\frac{Q^2}{2}\eta G_M^2, \tag{C.6}
\]
\[
b_2 = \delta(2\nu - Q^2)4Q^2\eta^2\left( \frac{1}{1 + \eta} \left( G_C + \frac{\eta}{3}G_Q - G_M \right)G_Q + \frac{1}{4\eta}G_M^2 \right), \tag{C.7}
\]
\[
b_3 = \delta(2\nu - Q^2)4Q^2\eta^2\left( \frac{1}{3(1 + \eta)} \left( G_C + \frac{\eta}{3}G_Q - G_M \right)G_Q - \frac{3\eta + 2}{12\eta}G_M^2 \right), \tag{C.8}
\]
\[
b_4 = \delta(2\nu - Q^2)4Q^2\eta^2\left( \frac{1}{3(1 + \eta)} \left( G_C + \frac{\eta}{3}G_Q - G_M \right)G_Q + \frac{1 + 6\eta}{12\eta}G_M^2 + G_QG_M \right). \tag{C.9}
\]
Now the antisymmetric parts under the interchange of $E$ and $E^*$ give the contribution to $g_1$ and $g_2$. In this case, we first note the identity
\[ a^\mu \epsilon^{\nu \rho \sigma} q_\rho s_\sigma p_\beta - a^\nu \epsilon^{\mu \rho \sigma} q_\rho s_\sigma p_\beta = -(a \cdot q) \epsilon^{\nu \rho \sigma} s_\sigma p_\beta - (a \cdot s) \epsilon^{\mu \nu \rho} p_\alpha q_\beta + (a \cdot p) \epsilon^{\mu \nu \sigma} s_\alpha q_\beta. \quad (C.10) \]

Since $s^\mu = -(i/m_a^2) \epsilon^{\mu \alpha \beta \gamma} E_\alpha E_\beta p_\gamma$, we have
\[ \epsilon^{\nu \rho \sigma} q_\rho s_\sigma p_\beta = i(E^{*\nu}(q \cdot E) - E^\nu(q \cdot E^*)). \quad (C.11) \]

Thus, we obtain
\[ i \left( a^\mu E^{*\nu}(q \cdot E) - E^\nu(q \cdot E^*) \right) - a^\nu (E^{*\mu}(q \cdot E) - E^\mu(q \cdot E^*)) \]
\[ = -(a \cdot q) \epsilon^{\mu \nu \rho} s_\alpha p_\rho - (a \cdot s) \epsilon^{\mu \nu \rho} p_\alpha q_\beta + (a \cdot p) \epsilon^{\mu \nu \sigma} s_\alpha q_\beta. \quad (C.12) \]

In the case of $a = 2p + q$, $a \cdot q = 2p \cdot q + q^2 = 0$ for the Born term. Another useful relation is
\[ \epsilon^{\mu \nu \rho} s_\alpha p_\rho = i(E^{*\mu} E^\nu - E^{*\nu} E^\mu). \quad (C.13) \]

Using this, we obtain
\[ g_1 = \delta(2\nu - Q^2) \frac{Q^2}{2} G_M \left( G_C + \frac{\eta}{3} G_Q + \frac{\eta}{2} G_M \right), \quad (C.14) \]
\[ g_2 = \delta(2\nu - Q^2) \frac{Q^2 \eta}{2} G_M \left( G_C + \frac{\eta}{3} G_Q - \frac{1}{2} G_M \right). \quad (C.15) \]

References

1) S. L. Adler and R. F. Dashen, Current Algebras and Applications to Particle Physics (W. A. Benjamin, New York, 1968).
2) D. A. Dicus, R. Jackiw and V. L. Teplitze, Phys. Rev. D 4 (1971), 1733.
3) S. Koretune, Phys. Rev. D 21 (1980), 820; Prog. Theor. Phys. 72 (1984), 821.
4) S. B. Gerasimov and J. Moulin, JINR, E2-7247, Dubna, 1973.
5) J. Moulin, E2-7638, Dubna, 1973.
6) P. A. M. Dirac, Rev. Mod. Phys. 21 (1949), 392.
7) A. H. Mueller, Phys. Rev. D 18 (1978), 3705.
8) S. Deser, W. Gilbert and E. C. G. Sudarshan, Phys. Rev. 115 (1959), 731.
9) S. Koretune, Phys. Rev. D 47 (1993), 2690.
10) S. Koretune, Phys. Rev. D 52 (1995), 44.
11) S. Koretune, Prog. Theor. Phys. 98 (1997), 749.
12) S. Koretune, Nucl. Phys. B 526 (1998), 445.
13) P. Hoobdhoy, R. L. Jaffe and A. Manohar, Nucl. Phys. B 312 (1989), 571.
14) X. Ji, Nucl. Phys. B 402 (1993), 217.
15) D. J. Broadhurst, J. F. Gunion and R. L. Jaffe, Ann. of Phys. 81 (1973), 88.
16) S. P. Deulawis, Nucl. Phys. B 43 (1972), 579.
17) A. Dommachi and P. V. Landschoff, Phys. Lett. B 296 (1992), 227.
18) S. Koretune, Phys. Rev. D 68 (2003), 054011.
19) F. E. Close and S. Kumano, Phys. Rev. D 42 (1990), 2377.
20) HERMES Collaboration, Phys. Rev. Lett. 95 (2005), 242001.
21) C. Riedl, DESY-THESIS-2005-027.
22) S. Koretune, Phys. Rev. C 72 (2005), 045205 [Errata; 74 (2006), 059901].
23) S. Koretune and H. Kurokawa, Phys. Rev. C 75 (2007), 025204.
24) C. Amsler et al. (Particle Data Group), Phys. Lett. B 667 (2008), 1.