Spin drag in an ultracold Fermi gas on the verge of a ferromagnetic instability

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Recent experiments [Jo et al., Science 325, 1521 (2009)] have presented evidence of ferromagnetic correlations in a two-component ultracold Fermi gas with strong repulsive interactions. Motivated by these experiments we consider spin drag, i.e., frictional drag due to scattering of particles with opposite spin, in such systems. We show that when the ferromagnetic state is approached from the normal side, the spin drag relaxation rate is strongly enhanced near the critical point. We also determine the temperature dependence of the spin diffusion constant. In a trapped gas the spin drag relaxation rate determines the damping of the spin dipole mode, which therefore provides a precursor signal of the ferromagnetic phase transition that may be used to experimentally determine the proximity to the ferromagnetic phase.

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Introduction. — Interest in electronic transport ranges from everyday applications to fundamental physics. One of the most interesting phenomena that spans this entire range, is the influence of a thermodynamic phase transition on the electrical conductivity. The most direct example is the phase transition from a normal conductor to a superconductor characterized by a vanishing resistivity. The applications of this phenomenon are ubiquitous and the basic physics that underlies the transition in superconductors, the Bose-Einstein condensation of fermionic pairs, has emerged in research fields from astroparticle physics [1] to cold-atom systems [2, 3].

A system in between these two temperature extremes, in which analogies of superconductivity have been predicted, is that of a two-dimensional (2D) electron-hole bilayer [4, 5]. In this case the pairs that condense are excitons formed by electrons from one layer with holes in the other. The relevant transport probe is in this case the Coulomb drag measurement [6]: a current $I$ is driven through one layer, known as the “active” layer, causing a voltage drop $V_D$ in the other. As the layers are separated by an essentially impenetrable tunnel barrier, the voltage drop is predominantly caused by Coulomb scattering, and the drag resistivity $\rho_D = V_D/I$ has the characteristic quadratic Fermi-liquid-like low-temperature dependence $\rho_D \propto T^2$. When the excitons undergo Bose-Einstein condensation, however, the drag resistivity is predicted to jump from the relatively small value proportional to $T^2$ to a value equal to the ordinary resistivity of the active layer [7]. Although conclusive evidence of exciton condensation is still lacking, two experimental groups [8, 9] have recently reported the observation of an upturn in the drag resistivity as the temperature is lowered. This upturn is interpreted as being due to strong pairing fluctuations that precede exciton condensation [10] and thus serves as a precursor signal for the transition, similar to the enhancement of the conductivity in superconductors due to superconducting fluctuations above but close to the critical temperature [11].

A closely related situation arises when the two layers of a 2D electron-electron bilayer are close enough to allow the establishment of interlayer coherence [12]. In this

FIG. 1: (Color online) Spin drag relaxation rate $1/\tau_{sd}(T)$ as a function of temperature $T$, for various values of the interaction parameter $k_Fa$. The Fermi energy is denoted by $\varepsilon_F = k_F T_F = h^2 k_F^2 / 2m$. Note that for $k_Fa > \pi/2$, the spin drag relaxation rate shows a distinctive upturn when the critical temperatures, indicated by thin vertical lines, are approached from above. In particular, we have $T_c \approx 0.43 T_F$ for $k_F a = 1.9$, $T_c \approx 0.36 T_F$ for $k_F a = 1.8$, and $T_c \approx 0.27 T_F$ for $k_F a = 1.7$. For $k_F a < \pi/2$ no ferromagnetism occurs within mean-field theory. In that case $1/\tau_{sd}(T)$ is smooth throughout the temperature range and exhibits the standard Fermi-liquid behavior $1/\tau_{sd}(T) \propto T^2$ for $T \to 0$. 

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case, the two layers in the system can be labelled “up” and “down” along a “z”-axis, so that the which-layer degree of freedom becomes a spin one-half pseudospin. Interlayer coherence in this language corresponds to pseudospin ferromagnetism with an easy x-y plane, since this orientation of the pseudospin describes a particle that is neither in the left nor in the right layer, but in a coherent superposition of the two. Furthermore, Coulomb drag becomes pseudospin drag, the mutual friction between two pseudospin states due to Coulomb scattering. This analogy prompted studies of spin drag, the frictional drag between electrons with opposite spin projection, in a single semiconductor [13]. While the realization of separate electric contacts to the two spin states remains an experimentally challenging problem, the spin drag is observed indirectly, by measuring different diffusion constants for charge and spin [14].

Because of the presence of other relaxation mechanisms, spin drag effects are usually not very large in semiconductors, and are even smaller in metals. This is completely different in cold atomic gases where scattering between different hyperfine spins is the only mechanism to relax spin currents, and was considered both for fermionic atoms [13] [16], and for bosonic ones [17]. It is the purpose of this Letter to point out that a particularly interesting situation occurs when spin drag is considered in a two-component Fermi gas that is close to a ferromagnetic instability [18] [22]. Based on the analogy between electron-hole bilayers and pseudospin ferromagnets we expect that the spin drag will be strongly enhanced as the ferromagnetic state is approached from the normal side. Because atoms are neutral, the relevant experimental quantity is the spin drag relaxation rate, which for instance determines the damping rate of the spin dipole mode in trapped cold-atom systems [16] and is thus accessible experimentally. Interestingly, an electronic analog of the spin dipole mode also exists [23].

Our main findings are illustrated in Fig. 1. This plot shows the spin drag relaxation rate $1/\tau_{sd}(T)$ as a function of temperature, for various interaction strengths determined by the product of the Fermi wave vector $k_F$ and the scattering length $a$. The dramatic enhancement of the relaxation rate upon approaching the critical temperature for the ferromagnetic transition is clearly visible. One of our motivations for considering this effect is the recent observation of ferromagnetic correlations in a two-component Fermi gas with strong repulsive interactions [24]. The fact that spin-polarized domains were not directly observed adds to the theoretical interest [24] in this experiment. The enhancement of the spin drag relaxation rate as the ferromagnetic phase is approached serves as a precursor probe for ferromagnetism that is distinct from, and adds to, the experimental methods of Jo et al. [24], and is also interesting in its own right. In the following we present our calculations in detail, and present additional results and discussion.

**Spin drag relaxation rate.** We consider a 3D homogeneous gas of fermionic atoms of mass $m$, with two hyperfine states denoted by $|\uparrow\rangle$ and $|\downarrow\rangle$. The grand-canonical Hamiltonian with chemical potential $\mu$ is given by

$$\hat{H} = \int d^3x \sum_{\alpha \in \{\uparrow, \downarrow\}} \left( \frac{\hbar^2 \nabla^2}{2m} - \mu \right) \hat{\psi}_\alpha(x) + U \int d^3x \, \hat{\psi}_\uparrow(x) \hat{\psi}_\uparrow(x) \hat{\psi}_\downarrow(x) \hat{\psi}_\downarrow(x) ,$$

(1)

in terms of fermionic creation and annihilation operators $\hat{\psi}_\alpha(x)$ and $\hat{\psi}_\alpha(x)$, respectively. At low temperatures $s$-wave scattering, described by $U = 4\pi a \hbar^2 / m$, dominates, and we have therefore omitted other interaction terms from this Hamiltonian.

We first determine a frequency and momentum dependent scattering amplitude $A_{\uparrow \downarrow}(q, \omega)$ that takes into account many-body effects on the scattering of atoms with opposite spin. We use the common random-phase approximation that consists of summing all “bubble” diagram contribution to this effective interaction. This takes into account modifications of the interaction due to density and spin fluctuations in an approximate way. The latter are essential as the spin fluctuations are strongly enhanced close to the ferromagnetic phase transition. In terms of the noninteracting (Lindhard) response function at nonzero temperature

$$\chi_0(q, \omega) = 2 \int \frac{d^3k}{(2\pi)^3} \frac{N_{q+k} - N_k}{\varepsilon_{q+k} - \varepsilon_k - \hbar \omega - i0},$$

(2)

with $\varepsilon_k = \hbar^2 k^2 / 2m$ and $N_k = [e^{(\varepsilon_k - \mu)/k_BT} + 1]^{-1}$ the Fermi-Dirac distribution function, the scattering amplitude reads

$$A_{\uparrow \downarrow}(q, \omega) = U + \frac{U^2}{4} \chi_{nn}(q, \omega) - \frac{3U^2}{4} \chi_{S_s S_s (nn)}(q, \omega),$$

(3)

where $\chi_{S_s S_s (nn)}(q, \omega) = \chi_0(q, \omega) / [1 \pm U \chi_0(q, \omega)/2]$. In this notation $\chi_{nn}(q, \omega)$ is the density-density response function while $\chi_{S_s S_s (qq)}(q, \omega)$ describes the spin-spin response. The factor of three in the last term in the right-hand side of Eq. (3) comes about because longitudinal and transverse spin fluctuations are both taken into account.

Within Stoner mean-field theory (MFT), ferromagnetism occurs when $\chi_{S_s S_s (0,0)}$ diverges so that $1 + U \chi_0(0,0)/2 = 0$. This equation gives, together with the equation $n = 2 \int d^3q \, N_q / (2\pi)^3$ for the total density determining the chemical potential, the critical temperature $T_c$ as a function of $k_F a$. For $k_BT_c$ much smaller than the Fermi energy $\varepsilon_F \equiv \hbar^2 k_F^2 / 2m = \hbar^2 (3\pi^2 n)^{2/3} / 2m$, this gives a critical temperature

$$\frac{k_BT_c}{\varepsilon_F} \sim \frac{2 \sqrt{3}}{\pi} \sqrt{\frac{U \varepsilon_F}{2} - 1} = \frac{2 \sqrt{3}}{\pi} \sqrt{\frac{2k_F a}{\pi} - 1},$$

(4)
where \( \nu(\varepsilon_F) = mk_F/\pi^2\hbar^2 \) is the density of states at the Fermi level. Note that one needs \( k_F a > \pi/2 \) for the critical temperature to be nonzero, and that there is a quantum critical point when \( k_F a = \pi/2 \) [18].

Our next step is to use the scattering amplitude \( A_{\uparrow\downarrow}(q, \omega) \) in Eq. (3) in the well-known expression for the spin drag relaxation rate \( 1/\tau_{sd} \), following from Boltzmann theory [13, 16]. This yields the result

\[
\frac{1}{\tau_{sd}(T)} = \frac{\hbar^2}{4\pi m k_B T} \int \frac{d^3q}{(2\pi)^3} \frac{q^2}{3} \times \int_{-\infty}^{+\infty} \frac{d\omega}{\pi} |A_{\uparrow\downarrow}(q, \omega)|^2 \left[ \frac{3m\chi(0,q,\omega)}{\sinh^2[\omega/(2k_B T)]} \right],
\]

which, together with Eq. (3), is the central result of this Letter and will be evaluated next. Before proceeding we note that our result is similar to the theory of spin diffusion in liquid He\(^3\) [26], although the ferromagnetic phase transition was not considered in this context.

Results. — In Figs. 1 and 2 we present the results of a numerical evaluation of Eq. (5), both as a function of temperature for various interaction strengths \( k_F a \), and as a function of interaction strength for various temperatures. In experiments both dependencies can be explored using a Feshbach resonance to tune the interaction strength [27]. Both figures clearly show the strong enhancement of the spin drag relaxation rate as the ferromagnetic state is approached. The precise form of the enhancement is understood by keeping only the most divergent term in the scattering amplitude in Eq. (3) so that

\[
A_{\uparrow\downarrow}(q, \omega) \simeq -\frac{3U^2 \nu(\varepsilon_F)}{3} \frac{q^2}{k_F^2} - i \frac{6\pi m \omega}{\hbar k_F q} + 4\alpha(T),
\]

with \( \alpha(T) = 1 + U\chi(0,0)/2 \simeq 1 - U\nu(\varepsilon_F) + \pi(T/T_F)^{1/2} \simeq \pi/2 \). Using these results in Eq. (5), and expanding \( 1/\sinh^2(x) \approx 1/x^2 \), both the frequency integral and the momentum integral can be performed analytically if we use a cut-off of \( 2k_F \) on the momentum integration that diverges because of the expansion of \( 1/\sinh^2(x) \). Ultimately we find in this manner that \( 1/\tau_{sd}(T) = 1/\tau_{sd}(T_c) \propto (T - T_c) \ln(T - T_c) \) for \( T < T_c \), which indeed accurately describes our numerical results near the critical temperature.

We also consider the spin diffusion constant, which from the Einstein relation is given by \( D_s(T) = \sigma_s(T)/\chi_{S_z S_z}(0,0) \) and the “spin conductivity” \( \sigma_s(T) = n\tau_{sd}(T)/m \) [28]. In Fig. 3 we show this constant as a function of temperature for various values of the interaction strength. Near the critical temperature the spin diffusion constant vanishes as \( D_s(T) \propto (T - T_c)^{\kappa} \) with an exponent \( \kappa = 1 \), because \( \tau_{sd}(T_c) \) remains finite and \( \chi_{S_z S_z}(0,0) \) diverges as \( 1/(T - T_c) \) within our random-phase approximation.

At this point it is important to realize that from the point of view of critical dynamics our findings are mean-field like. If the spin dynamics can be effectively described by an isotropic Heisenberg ferromagnet (model J.
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dimensionality, and the implications of critical and quantum temperatures. We expect that these modifications will not

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approximations are generally valid for trapped Fermi systems,

considered a homogeneous system, local-density approx-

ations will take place well outside the critical region where critical

fluctuations can be neglected and our random-phase approximation is appropriate.

Discussion and conclusions — As we have mentioned in the introduction, the spin drag relaxation rate can be
determined from the damping of the spin dipole mode [16–23] in a trapped gas. Since the Fermi energy is

usually much larger than the level splitting in the trap, our results show that the spin dipole mode is typically

strongly overdamped, which makes the experiment more challenging. Nevertheless, such a measurement, as well

as measurements of the spin diffusion constant as a function of temperature, gives information on the proximity

of the ferromagnetic phase transition. Although we have considered a homogeneous system, local-density approx-

imations are generally valid for trapped Fermi systems, and in determining the damping of the spin dipole mode of a

trapped two-component gas the homogeneous density should in first approximation be taken as the central
density of the atomic cloud.

Within our present approach, we consider the transition to ferromagnetism within MFT, which predicts it to

be continuous. One interesting aspect is that taking

into account correlation effects beyond MFT [22] results in: (i) an increase in the critical temperature for a given

value of \( k_B T \) and (ii) a change in the character of the transition from second to first order at very low

temperatures. We expect that these modifications will not

qualitatively affect the upturn of \( 1/\tau_{sd} \), as long as one remains outside the critical-fluctuation region.

In future work we intend to explore the effects of lower dimensionality, and the implications of critical and quantum

fluctuations on the exponent \( \kappa \) that determines the behavior of the spin diffusion constant. Further-

more, as our approach is distinct from the work of Hu [10], who considered the upturn of the Coulomb drag

resistivity as one approaches the exciton-condensed state in bilayers, we intend to study this system as well with

our present approach.

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