Anisotropic pressure of deconfined QCD matter in presence of strong magnetic field within one-loop approximation

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ABSTRACT: Considering the general structure of the two point functions of quarks and gluons, we compute the free energy and pressure of a strongly magnetized hot and dense QCD matter created in heavy-ion collisions. In presence of strong magnetic field we found that the deconfined QCD matter exhibits a paramagnetic nature. One gets different pressure in a direction parallel and perpendicular to magnetic field due to the magnetization acquired by the system. We obtain both longitudinal and transverse pressure, and magnetization of a hot deconfined QCD matter in presence of magnetic field. We have used hard thermal loop approximation (HTL) for heat bath. We obtained completely analytic expression for pressure and magnetization under certain approximation. Various divergences appearing in free energy are regulated using appropriate counter terms. The obtained anisotropic pressure may be useful for magnetohydrodynamics description of a hot and dense deconfined QCD matter produced in heavy-ion collisions.
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1 Introduction

Quantum Chromodynamics (QCD) is the theory of strong interaction which has two important features. One is the feeble interaction of quarks and gluons at high energy and the other one is the confinement in which the interaction strength becomes strong at low energy. A transition between these two phases, namely confined to deconfined state of hadronic matter known as quark-gluon plasma (QGP), is supposed to occur at around energy scale or temperature 160 MeV. In early universe such state of matter is presumed to be created after a few micro seconds of big-bang. It can also exist in the core of neutron stars as matter density is much higher than the normal nuclear matter density. Various high energy heavy-ion experiments are underway in laboratories (LHC at CERN and RHIC at BNL) to study the formation of QGP and its properties for unraveling the characteristics of the QCD phase diagram. Future experiments are also planned in FAIR at GSI and NICA at Dubna to explore high baryon density domain of the QCD phase diagram.

In recent years much attention has been paid in non-central heavy-ion collisions where a magnetic field as high as $(10-30) m^2_\pi$ can be generated in a direction perpendicular to the reaction plane. This magnetic field is primarily created when the spectators recede from each other. This magnetic field strength, however, also decreases very fast to $(1-2) m^2_\pi$ after a time scale of $(4-5) \text{fm/c}$. The presence of an external magnetic field introduces an extra energy scale in the system in addition to the scales $(gT$ and $T$, $g$ is the strong coupling) associated with a heat bath. One can work with two limiting cases: strong magnetic field limit ($eB > T^2$) and weak magnetic field limit ($eB < T^2$). We note that the presence of an anisotropic magnetic field in the medium requires an appropriate modification of the present theoretical tools to investigate various properties of QGP. In recent years a numerous activities are in progress such as magnetic catalysis [1–3], inverse magnetic catalysis [4–12] and chiral magnetic effect [13–15] at finite temperature, and chiral- and color-symmetry broken/restoration phase [16–20]. Also in progress the study related to the equation of state (EoS) in thermal perturbative QCD (pQCD) models [21, 22], holographic models [23, 24] and various thermodynamic properties [19, 20, 25, 26], refractive indices and decay constant of hadrons [27–34]; soft photon production from conformal anomaly [35, 36] in HIC; modification of dispersion properties in a magnetized hot QED [37, 38] and QCD [38–41] medium; various transport coefficients [42–44], properties of quarkonia [45, 46], synchrotron radiation [47], dilepton production from a hot magnetized QCD plasma [47–52] and in strongly coupled plasma in a strong magnetic field [53].

The equation of state (EoS) is an important quantity and of phenomenological importance for studying the hot and dense QCD matter, QGP, created in the relativistic heavy ion collisions. This is because the EoS determines the thermodynamic properties of a hot and dense medium. Also the time evolution of the hot and dense fireball is studied through hydrodynamic models which require an EoS of the deconfined QCD matter as an input. In absence of magnetic field the EoS has systematically been computed in Lattice QCD (LQCD) [54–56] and in Hard Thermal Loop perturbation theory (HTLpt) within two loop (next-to-leading order (NLO)) [57] and three loop (next-to-NLO (NNLO)) [58–64] at finite temperature and chemical potential. On the other hand the expansion dynamics of
a thermo-magnetic medium is governed by magneto-hydrodynamics [65, 66]. This requires thermomagnetic EoS as an input. In view of this, a systematic determination of EOS for magnetized hot QCD medium is of great importance. Some LQCD effort has been made in Ref. [67] which is limited to temperature range (100-300)MeV. Recently we have computed [21] the thermomagnetic EoS for hot magnetized QCD medium within the weak magnetic field and HTL approximation. Also using HTL approximation, some thermodynamic quantities in lowest Landau level (LLL) within the strong field approximation has been numerically computed in Ref. [22]. However, for gluonic case, this calculation assumes that the only effect of the magnetic field is to shift the Debye mass without any change in the general structure of two point functions at finite temperature. It has explicitly been shown [38–41] that the presence of an external magnetic field breaks the rotational symmetry and the situation is quite different than what has been assumed in Ref. [22]. This seeks an improvement of the general structure of two point functions used in Ref. [22].

In view of this we systematically compute the EoS within strong field approximation by exploiting the general structure of effective propagator of quarks and gluons in a thermo-magnetic QCD medium. In the strong field limit, the magnetic field pushes higher Landau levels (HLL) to infinity compared to the lowest Landau level (LLL) [48]. Thus we work with LLL approximation along with a scale hierarchy \( eB > T^2 > m_f^2 \), where \( m_f \) is the intrinsic mass scale associated with quarks. We also compute magnetization which indicates that the deconfied hot and dense QCD matter is paramagnetic in nature. We further note that, in presence of strong magnetic field, we take into account the anisotropy between longitudinal and transverse pressure which is created due to the fact that the system acquires a magnetization along the field direction and is likely to elongate more along the direction of magnetic field. We obtain completely analytic expression for anisotropic (both longitudinal and transverse) pressures and magnetization under certain approximation.

The paper has been organized as follows. In section 2 we describe the basic setup for the computation of the free energy in this manuscript. In section 3 we discuss the general structure of fermion self-energy in presence of strong magnetic field, the effective fermion propagator and associated form factors, and quark free energy in one loop upto \( O(g^4) \). The hard and soft contributions of gluon free energy upto \( O(g^4) \) are also calculated in section 4 within one-loop HTL approximation. In section 5 the pressure of an anisotropic system is discussed. We discuss our results in section 6. Finally, we conclude in section 7.

2 Setup

The total thermodynamic free energy upto one-loop order in HTLpt in presence of a background magnetic field, \( B \), can be written as

\[
F = F_q + F_g + F_0 + \Delta \mathcal{E}_T^0 + \Delta \mathcal{E}_T^B
\]

where \( F_q \) and \( F_g \) are, respectively, the quark and gluon part of the free energy which will be computed in presence of magnetic field with HTL approximation. \( F_0 \) is the tree level contribution due to the constant magnetic field, given as

\[
F_0 \rightarrow \frac{1}{2} B^2 + \Delta \mathcal{E}_0^B,
\]

\( -3 - \)
where $\Delta E_0^{B^2}$ is a counter term of $\mathcal{O}[(q_fB)^2]$ from vacuum as we will see later. The $\Delta E_T$ is a counter term independent of magnetic field (viz. $\mathcal{O}[(q_fB)^0T^4]$) as

$$\Delta E_T^0 = \Delta E_T^{HTL} + \Delta E_T,$$

(2.3)

where $\Delta E_T^{HTL}$ is the HTL counter term [63, 64]. The counter term $\Delta E_T$ arises due to the quark loop in gluonic two point function in presence of magnetic field but the field dependence explicitly gets cancelled from denominator and numerator as we will see later. Finally, the counter term $\Delta E_B^T$ is of order $\mathcal{O}[(q_fB)^2T^2]$ and $\mathcal{O}[(q_fB)^3/T^2]$.

The pressure of a system is defined as

$$P = -F.$$  

(2.4)

We also note the QCD Casimir numbers are $C_A = N_c$, $d_A = N_c^2 - 1$, $d_F = N_cN_f$ and $C_F = (N_c^2 - 1)/2N_c$ where $N_c$ is the number of color and $N_f$ is the number of quark flavour.

3 Quarks in a strong magnetic field

3.1 General structure of fermion self-energy in strong field approximation

It is established that the presence of a heat bath breaks the Lorentz(Boost) invariance whereas the presence of a magnetic field, $B$, breaks the rotational symmetry of the system. In such a situation one needs to construct a manifestly covariant structure of the self-energy. The presence of a heat bath introduces a four vector $u^\mu$, which is the velocity of the heat bath in addition to $P^\mu$, the four momentum of the external fermion. When the heat bath is further magnetized, one can construct another four vector $n^\mu$ by combining the electromagnetic field tensor $F^{\mu\nu}$ or its dual $\tilde{F}^{\mu\nu}$ with the fluid velocity $u^\mu$ as

$$n^\mu = \frac{1}{2B} \epsilon_{\mu\nu\rho\lambda} u^\nu F^{\rho\lambda} = \frac{1}{B} u^\nu \tilde{F}_{\nu\mu} = (0, 0, 0, 1).$$  

(3.1)

which is in the $z$-direction. This is because the electromagnetic field tensor

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -B & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$  

(3.2)

has been projected out along the four velocity which is in the rest frame of heat bath as $u^\mu = (1, 0, 0, 0)$. This also uniquely establishes a connection between the heat bath and the external magnetic field along the $z$-direction.

The fermion self-energy $\Sigma(P)$ is $4 \times 4$ matrix which can be constructed from Dirac $\gamma$-matrices\(^1\). The self-energy will also depend on four vectors $P^\mu$, $u^\mu$ and $n^\mu$. In presence of magnetic field one can generally define

$$P^\mu_\perp = P^\mu - (P \cdot u)u^\mu + (P \cdot n)n^\mu,$$

(3.3)

\(^1\) We note that the $\sigma^{\mu\nu}$ do not appear due to antisymmetric nature of it in any loop order of self-energy.
\[ P_\mu = (P \cdot u)u^\mu - (P \cdot n)n^\mu, \]  
\[ p_\perp^2 = (P \mu u^\mu)^2 - (P \mu n^\mu)^2 - P^\mu P_\mu \]  
\[ = p_1^2 + p_2^2 = -P_\perp^2. \]  
\[ (3.4) \]

Since we will be working in strong field approximation, we confine ourselves in the lowest Landau level (LLL). In the LLL the transverse component of the momentum, \( P_\perp = 0 \). Thus, \( P^\mu \) reduces to \( P_\mu \). \( P^\mu \) can be written as a linear combination of \( u^\mu \) and \( n^\mu \). In the chiral limit the general structure of fermion self-energy in lowest Landau level can be written as

\[ \Sigma(p_0, p_3) = a\hat{\gamma} + b\gamma^3\hat{n} + c\gamma^5\hat{\gamma} + d\gamma^5\gamma^3, \]  
\[ (3.6) \]

where \( \hat{\gamma} = \gamma_0 \) and \( \gamma^3 = \gamma^3n_3 = \gamma^3 \). Now, the various form factors can be obtained as

\[ a = \frac{1}{4} \text{Tr}[\Sigma\hat{\gamma}], \]  
\[ b = -\frac{1}{4} \text{Tr}[\Sigma\hat{n}], \]  
\[ c = \frac{1}{4} \text{Tr}[\gamma^5\Sigma\hat{\gamma}], \]  
\[ d = -\frac{1}{4} \text{Tr}[\gamma^5\Sigma\hat{n}]. \]  
\[ (3.7 - 3.10) \]

Finally using the chirality projectors we can express the general structure of the fermion self-energy as

\[ \Sigma(P) = A P_R A + B P_L B \]  
\[ (3.11) \]

where

\[ A = (a + c)\hat{\gamma} + (b + d)\hat{n}, \]  
\[ B = (a - c)\hat{\gamma} + (b - d)\hat{n}, \]  
\[ P_R = \frac{1}{2}(1 + \gamma_5), \]  
\[ P_L = \frac{1}{2}(1 - \gamma_5). \]  
\[ (3.12 - 3.15) \]

### 3.2 One loop quark self-energy in presence of a strong magnetic field

Using the modified fermion propagator in strong field approximation one can straight way write down the quark self-energy in Feynman gauge from Fig. 1 as

\[ \Sigma(P) = -i g^2 C_F \int \frac{d^4 K}{(2\pi)^4} \gamma_\mu S(K)\gamma^\mu \Delta(K - P), \]  
\[ (3.16) \]

where the unmodified gluonic propagator is given as

\[ \Delta(K - P) = \frac{1}{(k_0 - p_0)^2 - (k - p)^2} = \frac{1}{(K - P)^2 - (K_\perp - p_\perp)^2}, \]  
\[ (3.17) \]

and the modified fermion propagator in LLL is given by

\[ iS(K) = i e^{-k^2/\eta B} \frac{K_\perp + m_f}{K_\perp^2 - m_f^2} (1 - i \gamma_1 \gamma_2). \]  
\[ (3.18) \]
Now the thermo-magnetic self-energy $\Sigma(P)$ can be written from Eq. (3.16) as
\[
\Sigma(P) = -ig^2 C_F \int \frac{d^4 K}{(2\pi)^4} e^{-k_\perp^2 /q_B} \gamma^\mu \frac{(K_\perp + m_f)}{(K_\perp^2 - m_f^2)} (1 - i\gamma_1 \gamma_2) \gamma^\mu \Delta(K - P)
\]
\[
= -ig^2 C_F \int \frac{d^4 K}{(2\pi)^4} e^{-k_\perp^2 /q_B} \gamma^\mu \bar{K}_\perp (1 - i\gamma_1 \gamma_2) \gamma^\mu \bar{\Delta}_\perp(K) \Delta(K - P)
\]
\[
= -2g^2 C_F \sum_{\{K\}} e^{-k_\perp^2 /q_B} [(1 + i\gamma_1 \gamma_2) \bar{K}_\perp] \bar{\Delta}_\perp(K) \Delta(K - P),
\]  
(3.19)

where
\[
\bar{\Delta}_\perp(K) = \frac{1}{k_0^2 - k_3^2}
\]
(3.20)

Also at finite temperature, the loop integration measure is replaced by
\[
\int \frac{d^4 K}{(2\pi)^4} \longrightarrow iT \sum_{\{k_0\}} \int \frac{d^3 k}{2\pi} \int \frac{dk_3}{2\pi} \int \frac{d^2 k_\perp}{(2\pi)^2}.
\]  
(3.21)

Now the expressions of form factors for a particular flavor $f$ become

\[a = \frac{1}{4} \text{Tr}[\Sigma_f] = -\frac{2g^2 C_F}{4} \sum_{\{K\}} e^{-\frac{k_\perp^2}{q_B}} \text{Tr} \left[(1 + i\gamma_1 \gamma_2) \bar{f}\gamma^\mu \bar{\Delta}_\perp(K) \Delta(K - P)\right]
\]
\[= -2g^2 C_F \sum_{\{K\}} e^{-\frac{k_\perp^2}{q_B}} k^0 \bar{\Delta}_\perp(K) \Delta(K - P),
\]  
(3.22)

\[b = \frac{1}{4} \text{Tr}[\Sigma_f] = \frac{2g^2 C_F}{4} \sum_{\{K\}} e^{-\frac{k_\perp^2}{q_B}} \text{Tr} \left[(1 + i\gamma_1 \gamma_2) \bar{f}\gamma^\mu \bar{\Delta}_\perp(K) \Delta(K - P)\right]
\]
\[= 2g^2 C_F \sum_{\{K\}} e^{-\frac{k_\perp^2}{q_B}} k^3 \bar{\Delta}_\perp(K) \Delta(K - P),
\]  
(3.23)

\[c = \frac{1}{4} \text{Tr}[\gamma_5 \Sigma_f] = -\frac{2g^2 C_F}{4} \sum_{\{K\}} e^{-\frac{k_\perp^2}{q_B}} \text{Tr} \left[\gamma_5 (1 + i\gamma_1 \gamma_2) \bar{f}\gamma^\mu \bar{\Delta}_\perp(K) \Delta(K - P)\right]
\]
\[ d = -\frac{1}{4} \text{Tr}\{\gamma_5 \Sigma \gamma_5 \} = 2g^2 C_F \sum_{\{K\}} \frac{\varepsilon_{K}^2}{\tau_{K}} \text{Tr} \left[ \gamma_5 (1 + i\gamma_1 \gamma_2) \right] \Delta_\sigma(K) \Delta(K - P) \]
\[ = 2g^2 C_F \sum_{\{K\}} e^{-\frac{\varepsilon_{K}^2}{\tau_{K}}} \Delta_\sigma(K) \Delta(K - P). \]  

From the above four expression it can be noted that \( b = -c \) and \( d = -a \). These form factors will be calculated in Appendix A.

### 3.3 Effective propagator and dispersion relation

Transverse momentum of fermion becomes zero i.e. \( P_\perp = 0 \) in LLL. Thus effective fermion propagator can be written using Dyson-Schwinger equation as,

\[ S_{\text{eff}}(P) = \frac{1}{P_0 + \Sigma}. \]  

Subsequently the inverse fermion propagator can be written as

\[ S_{\text{eff}}^{-1}(P) = P_0 + \Sigma \]
\[ = \mathcal{P}_R \mathcal{P}_L \mathcal{P}_L \mathcal{P}_R, \]

where

\[ \mathcal{L} = \mathcal{P} + (a + c)\mathcal{P}_{12} + (b + d)\mathcal{P}_{13}, \]
\[ \mathcal{R} = \mathcal{P} + (a - c)\mathcal{P}_{12} + (b - d)\mathcal{P}_{13}. \]

Now the effective propagator can be written as,

\[ S_{\text{eff}}(P) = \frac{1}{2} \left[ \mathcal{P}_R \mathcal{R} \mathcal{P}_L \mathcal{L} + \mathcal{P}_L \mathcal{L} \mathcal{P}_R \mathcal{R} \right]. \]

We have,

\[ L^2 = (p_0 + (a + c))^2 - (p_3 - (b + d))^2, \]
\[ R^2 = (p_0 + (a - c))^2 - (p_3 - (b - d))^2. \]

Now putting \( a = -d \) and \( b = -c \), one gets

\[ L^2 = p_0^2 - p_3^2 + 2(a - b)(p_0 - p_3) = (p_0 - p_3)(p_0 + p_3 + 2(a - b)), \]
\[ R^2 = p_0^2 - p_3^2 + 2(a + b)(p_0 + p_3) = (p_0 + p_3)(p_0 - p_3 + 2(a + b)). \]
Various discrete symmetries of the effective two point functions are discussed in details in Ref. [40]. The form factors are calculated in Appendix A and given as

\[
a = -d = -g^2 C_F \left[ \sum_f q_f B \frac{p_0}{p_0^2 - p_3^2} \ln 2 - \sum_f (q_f B)^2 \frac{\zeta'(-2)}{2 T^2} \left( p_0 (p_0^2 + p_3^2) \right) \right],
\]

(3.36)

\[
b = -c = -g^2 C_F \left[ \sum_f q_f B \frac{p_3}{p_0^2 - p_3^2} \ln 2 - \sum_f (q_f B) \frac{p_0^2 p_3}{T^2} \frac{\zeta'(-2)}{p_0^2 - p_3^2} \right].
\]

(3.37)

Magnetic mass is found by taking dynamic limit of \( R^2 \) and \( L^2 \) in Eq (3.31), i.e., \( R^2 |_{p \to 0, p_0 = 0} = L^2 |_{p \to 0, p_0 = 0} \) and is given by

\[
M_{sa}^2 = \frac{g^2 C_F}{4 \pi^2 T^2} \left( \sum_f (q_f B) T^2 \ln 4 - \sum_f (q_f B)^2 \zeta'(-2) \right).
\]

(3.38)

One can notice that the magnetic mass is dependent on both magnetic field and temperature.

Now we discuss the dispersion properties of a fermions in a hot magnetized medium. The dispersion curves are obtained by solving, \( L^2 = 0 \) and \( R^2 = 0 \) given in Eq (3.31), numerically. There are four modes, two comes from \( L^2 = 0 \) and two from \( R^2 = 0 \). In LLL only two modes are allowed [40]: one \( L \)-mode with energy \( \omega_L \) of a positively charged fermion having spin up and another one from \( R \)-mode with energy \( \omega_R \) of a negatively charged fermion having spin down. These two modes are plotted in Fig. 2. In LLL approximation transverse momentum of fermion becomes zero. Thus the dynamics of the system becomes two dimensional. At high \( p_3 \) both the mode of dispersion resembles free dispersion mode. We also note that the reflection symmetry is broken in presence of magnetic field [40].

### 3.4 One loop quark free energy in presence of a strongly magnetized medium

The quark free energy can be written as,

\[
F_q = -d_F \sum_{\{p_0\}} \frac{d^3 p}{(2\pi)^3} \ln \left( \det [S_{eff}^{-1}(p_0, p_3)] \right).
\]

(3.39)

Effective fermion self-energy can be written as,

\[
S_{eff}^{-1} = P_\mu + \Sigma = (p_0 + a) \gamma^\mu + (b - p_3) \gamma^\mu + c \gamma_5 \gamma^\mu + d \gamma_5 \gamma^\mu
\]

\[= (p_0 + a) \gamma^0 + (b - p_3) \gamma^3 + c \gamma_5 \gamma^0 + d \gamma_5 \gamma^3.
\]

(3.40)

Now we evaluate the determinant as,

\[
\det [S_{eff}^{-1}] = \left( (b + c - p_3)^2 - (a + d + p_0)^2 \right) \left( (-b + c + p_3)^2 - (a - d + p_0)^2 \right)
\]
Figure 2: Dispersion relation of fermion in presence of strong magnetic field

$$
\frac{eB=15m_{\pi}^2}{T=0.2\text{GeV}}
$$

$$
\omega = \omega_L = \omega_R = \text{Free}
$$

$$
\begin{align*}
F_q &= -d_F \sum_{\{p_0\}} \left( \frac{d^3p}{(2\pi)^3} \ln \left[ P_i^4 \left( 1 + \frac{4a^2 - 4b^2 + 4ap_0 + 4bp_3}{P_i^2} \right) \right] \right) \\
&= -2d_F \sum_{\{p_0\}} \left( \frac{d^3p}{(2\pi)^3} \ln P_i^2 \right) - d_F \sum_{\{p_0\}} \left( \frac{d^3p}{(2\pi)^3} \ln \left[ 1 + \frac{4a^2 - 4b^2 + 4ap_0 + 4bp_3}{P_i^2} \right] \right) \\
&= F_{q_{\text{ideal}}} + F'_q,
\end{align*}
$$

(3.42)

where free energy of free quarks [25]

$$
F_{q_{\text{ideal}}} = -2d_F \sum_{\{p_0\}} \left( \frac{d^3p}{(2\pi)^3} \ln P_i^2 \right) = -2d_F \sum_{f} \frac{q_f B T^2}{(2\pi)^2} \sum_{\{p_0\}} dp_3 \ln P_i^2
$$

$$
= -d_F \sum_{f} \frac{q_f B T^2}{12},
$$

(3.43)

and

$$
F'_q = -d_F \sum_{\{p_0\}} \left( \frac{d^3p}{(2\pi)^3} \ln \left[ 1 + \frac{4a^2 - 4b^2 + 4ap_0 + 4bp_3}{P_i^2} \right] \right)
$$

$$
= -d_F \sum_{\{p_0\}} \left( \frac{d^3p}{(2\pi)^3} \left[ \frac{4 (ap_0 + bp_3)}{P_i^2} + 4 \frac{a^2 P^2 - b^2 P^2 - 2a^2 P_0^2 - 2b^2 P_3^2 - 4ab P_0 P_3}{P_i^4} \right] \right)
$$
where we have kept terms up to $O(g^4)$ to obtain analytic expression of free energy. The expansion made above is valid for $g^2(qfB/T^2) < 1$, which can be realized as $(qfB)/T^2 \gtrsim 1$ and $g \ll 1$.

As in strong field approximation, fermion is considered to be in LLL. So Eq. (3.44) becomes,

$$F'_q = -dF \sum_f \frac{q_f B T^2}{12} - 4dF \sum_f \frac{(q_f B)^2 g^2 C_F}{(2\pi)^2} \frac{\Lambda}{4\pi T} \left( 3f(2 - \frac{\ln \Lambda}{\ln \pi}) + 4 \ln 2 - q_f B \frac{\zeta'(\frac{-2}{T^2})}{T^2} \right)$$

$$+ \frac{1}{24576} \left[ 1288 \ln 2 (3\gamma_E + 4\ln 2 - \ln \pi) + \frac{256\zeta[3]}{\pi^4 T^2} (2\pi^4 T^2 - 3g^2 C_F(q_f B) \ln 2$$

$$+ 3\pi^2 (q_f B)(2 + 3\gamma_E + 4\ln 2 - \ln \pi) - \frac{8g^2 C_F(q_f B)^2 \zeta[3]^2(4 + 105 \ln 2)}{\pi^6 T^4}$$

$$+ \frac{7245g^2 C_F(q_f B)^3 \zeta[3]^3}{\pi^8 T^6} \right]$$

$$= -dF \sum_f \frac{q_f B T^2}{12} - 4dF \sum_f \frac{(q_f B)^2 g^2 C_F}{(2\pi)^2} \frac{\Lambda}{4\pi T} \left[ 1288 \ln 2 \left( \frac{3\gamma_E}{4\pi T} - 2 \ln \Lambda + 4 - \ln \pi \right)$$

$$+ \frac{256\zeta[3]}{\pi^4 T^2} \left( - 3C_F g^2 q_f B \ln 2$$

$$+ 6\pi^2 q_f B \ln \Lambda + \frac{7245g^2 C_F(q_f B)^3 \zeta[3]^3}{\pi^8 T^6} \right)$$

$$\times \zeta[3]^2(4 + 105 \ln 2)$$

$$+ \frac{256\zeta[3]}{\pi^4 T^2} \left( - 3C_F g^2 q_f B \ln 2$$

$$+ 6\pi^2 q_f B \ln \Lambda + 3\pi^2 (q_f B)(2 + 3\gamma_E + 4 - \ln \pi) + 2\pi^4 T^2 \right)$$

$$- \frac{8g^2 C_F(q_f B)^2}{\pi^6 T^4}$$

$$\times \zeta[3]^2(4 + 105 \ln 2)$$

$$+ \frac{7245g^2 C_F(q_f B)^3 \zeta[3]^3}{\pi^8 T^6} \right],$$

where $\Lambda = \Lambda/2\pi T$. The quark free energy has $O[(q_f B)^2 / \epsilon]$ and $O[(q_f B)^3 / T^2 \epsilon]$ divergences.

4 Gluons in a strong magnetic field

4.1 General structure of gauge boson free energy

The partition function for a gluon can generally be written in Euclidean space [21] as

$$Z_g = Z Z^{\text{ghost}}, \quad Z = N \prod_{n,p} \sqrt{\frac{(2\pi)^D}{\det D_{\mu \nu}^E}}, \quad Z^{\text{ghost}} = \prod_{n,p} P_E^2$$

$$\text{(4.1)}$$
where the product over \( p \) is for the spatial momentum whereas that over \( n \) is for the discrete Bosonic Matsubara frequencies \((\omega_n = 2\pi n \beta; \, n = 0, 1, 2, \cdots)\) due to Euclidean time, \( D^{-1}_{\mu\nu,E} \) is the inverse gauge boson propagator in Euclidean space with \( P^2_E = \omega_n^2 + p^2 \) is the square of four momentum. \( N_\xi = 1/(2\pi \xi)^{D/2} \) is the normalization originates from the introduction of Gaussian integral at each location of position while averaging over the gauge condition function with a width \( \xi \), the gauge fixing parameter. Gluon free energy can now be writte [21] as

\[
F_g = -d_A \frac{T}{V} \ln Z_g = d_A \left[ \frac{1}{2} \sum_{P^2_E} \ln \left( \det \left( D^{-1}_{\mu\nu,E}(P^2_E) \right) \right) - \sum_{P^2_E} \ln P^2_E \right]. \tag{4.2}
\]

We note that the presence of the normalization factor \( N_\xi \) eliminates the gauge dependence explicitly.

For ideal case \( \det \left( D^{-1}_{\mu\nu,E}(P) \right) = (P^2_E)^4/\xi \) and hence the free energy for \( d_A \) massless spin one gluons yields as

\[
F_{g}^{\text{ideal}} = d_A \sum_{P^2_E} \ln P^2_E = d_A \sum_{P} \ln (-P^2) = -d_A \frac{\pi^2 T^4}{45}, \tag{4.3}
\]

where \( P \) is four-momentum in Minkowski space and can be written as \( P^2 = p^2_0 - p^2 \).

In presence of thermal background medium [38, 68, 69] one can have

\[
\det \left( D^{-1}_{\mu\nu,E}(P^2) \right) = \frac{P^2_E}{\xi} \left( P^2_E + \Pi_T \right)^2 \left( P^2_E + \Pi_L \right), \tag{4.4}
\]

which has four eigenvalues. Those are, respectively, \( P^2_E/\xi, (P^2_E + \Pi_T) \) and two fold degenerate \( (P^2_E + \Pi_T) \) where \( \Pi_T \) and \( \Pi_L \), respectively, are the transverse and longitudinal part of the gluon self-energy in heat bath. Also we considered \( D = 4 \), and the spatial dimension, \( d = 3 \) throughout this manuscript\(^2\). From now on, we use Minkowski momentum \( P \). Eventually the free energy becomes [21]

\[
F_{g}^{\text{th}} = \frac{1}{2} \left[ \sum_{P} \ln (-P^2) + 2 \sum_{P} \ln (-P^2 + \Pi_T) + \sum_{P} \ln (-P^2 + \Pi_L) \right] - \sum_{P} \ln (-P^2), \tag{4.5}
\]

\[
= \sum_{P} \ln (-P^2 + \Pi_T) + \frac{1}{2} \sum_{P} \ln \left( 1 - \frac{\Pi_L}{P^2} \right) \tag{4.6}
\]

Also, \( F_{g}^{L} \) and \( F_{g}^{T} \) are, respectively, the longitudinal and transverse part of the gluon free energy. Using general structure of two point functions of gauge boson, both of them are evaluated in Refs. [38, 68, 69].

\(^2\) We will also use \( d = 3 - 2\epsilon \) for dimensional regularization.
Now, the general structure of inverse propagator of a gauge boson in presence of a hot magnetized medium is computed in Ref. [38] as

\[(D_{\mu\nu})^{-1} = \frac{P^2}{\xi} \eta_{\mu\nu} + (P_m^2 - b) B_{\mu\nu} + (P_m^2 - c) R_{\mu\nu} + (P_m^2 - d) Q_{\mu\nu},\] (4.7)

where

\[P_m^2 = \frac{P^2 \xi - 1}{\xi} \] (4.8)

and \(b, c, d\) are the form factors corresponding to the three projection tensors \(B^{\mu\nu}, R^{\mu\nu}\) and \(Q^{\mu\nu}\) respectively for gauge boson self-energy [38]. The determinant of inverse of the gauge boson propagator can be evaluated from Eq. (4.7) as

\[\det \left( D_{\mu\nu,E}^{-1}(P) \right) = -\frac{P^2}{\xi} (-P^2 + b) (-P^2 + c) (-P^2 + d),\] (4.9)

which has four eigenvalues: \(-P^2/\xi\), \((-P^2 + b)\), \((-P^2 + c)\) and \((-P^2 + d)\). We note here that one has two distinct transverse modes coming from \((-P^2 + c) = 0\) and \((-P^2 + d) = 0\) respectively, in a thermo-magnetic medium instead of a two fold degenerate transverse mode \((-P^2 + \Pi_T) = 0\) in thermal medium in Eq. (4.4).

Using Eq. (4.9) in Eq. (4.2), the one-loop gluon free energy for hot magnetized medium is given [38] by

\[F_g = d_A \left[ \mathcal{F}_g^1 + \mathcal{F}_g^2 + \mathcal{F}_g^3 \right],\] (4.10)

where

\[\mathcal{F}_g^1 = \frac{1}{2} \sum_P \ln \left( 1 - \frac{b}{P^2} \right),\] (4.11a)

\[\mathcal{F}_g^2 = \frac{1}{2} \sum_P \ln \left( -P^2 + c \right),\] (4.11b)

\[\mathcal{F}_g^3 = \frac{1}{2} \sum_P \ln \left( -P^2 + d \right).\] (4.11c)

The various structure functions are obtained in Ref. [38] in both strong and weak field approximation. In the following sections we obtain the gluon free energy in strong field approximation.

### 4.2 Gluon Free Energy in a strongly magnetized hot and dense medium

Within the strong field approximation \((m_f^2 < T^2 < q_f B)\) the expressions for the different contributions of the gluon free energy in LLL can now be obtained from Eqs. (4.10). Combining Eqs. (4.11a), (4.11b), (4.11c) with Eq. (4.10) the total one-loop free energy expanded upto \(O[g^4]\) is given by

\[F_g \approx d_A \left[ \sum_P \ln \left( -P^2 \right) - \frac{b + c + d}{2P^2} - \frac{b^2 + c^2 + d^2}{4P^4} \right],\] (4.12)
In strong field limit Eq. (4.12) becomes

\[ F_g = d_A \left[ \sum_p \ln(-P^2) - \frac{C_A g^2 T^2}{6} \sum_p \frac{1}{P^2} \right] - \left( \frac{C_A g^2 T^2}{3} \right)^2 \sum_p \frac{1}{2P^2} \left\{ \frac{3T_p^2}{2p^4} + \frac{1}{2p^2} - \frac{3T_p^2}{p^4} - \frac{1}{p^2 P^2} \right\} \]

where the expansion is made to obtain analytical expression for free energy which is valid for \( g^2 (q_f B / T^2) < 1 \), and it can be realized as \( (q_f B) / T^2 \gg 1 \) and \( g << 1 \).

Now the various structure functions involved therein are obtained in Ref. [38]. For simplicity we consider \( m_f = 0 \) with hierarchy of scales as \( m_f^2 \sim m_{th}^2 \sim g^2 T^2 \sim q_f B \) and write down those structure functions [38] as

\[ \sum \int \frac{P}{P} = \frac{C_A g^2 T^2}{3u^2} \left[ 1 - T_P(p_0, p) \right] - \sum_f \frac{g^2 q_f B}{4\pi^2 u^2} e^{-p_f^2 / 2q_f B} \frac{p_f^2}{p_0^2 - p_3^2}, \]  

\[ c = \frac{C_A g^2 T^2}{3} \frac{1}{2} \left[ \frac{p_0}{p^2} - \frac{p^2}{p^2} T_P(p_0, p) \right], \]

\[ d = \frac{C_A g^2 T^2}{3} \frac{1}{2} \left[ \frac{p_0}{p^2} - \frac{p^2}{p^2} T_P(p_0, p) \right] + \sum_f \frac{g^2 q_f B}{4\pi^2 u^2} e^{-p_f^2 / 2q_f B} \frac{p_f^2}{p_0^2 - p_3^2}, \]

where \( u^2 = -p^2 / P^2, T_P(p_0, p) = \frac{p_0}{2p} \ln \frac{p_0 + p}{p_0 - p} \).

Now we write down various terms in Eq. (4.12) as

\[ \sum \int \frac{b + c + d}{2P^2} = \frac{C_A g^2 T^2}{6} \sum \int \frac{1}{P^2} \]

\[ \sum \int \frac{b^2 + c^2 + d^2}{4P^4} = \left( \frac{C_A g^2 T^2}{3} \right)^2 \sum \int \left\{ \frac{3T_p^2}{2p^4} + \frac{1}{2p^2} + \frac{1}{2p^2} - \frac{3T_p^2}{p^4} - \frac{T_p}{p^2 P^2} \right\} \]

\[ + \frac{1}{2} \sum_{f_1, f_2} \left( \frac{g^2 B}{4\pi^2} \right)^2 q_{f_1} q_{f_2} \sum_p e^{-\frac{p_f^2}{2T_p^2}} e^{-\frac{p_f^2}{2T_p^2}} \frac{p_f^2}{p^4 (p_0^2 - p_3^2)^2} \]

\[ + \sum_f \frac{g^2 q_f B C_A g^2 T^2}{4\pi^2} \left( \frac{p_f^2}{4 (p_0^2 - p_3^2)} \right)^2 \left( \frac{3T_p^2}{p^4} - \frac{3}{p^4} - \frac{1}{p^2 P^2} \right) \]

In strong field limit Eq. (4.12) becomes

\[ F_g = d_A \left[ \sum_p \ln(-P^2) - \frac{C_A g^2 T^2}{6} \sum_p \frac{1}{P^2} \right] - \left( \frac{C_A g^2 T^2}{3} \right)^2 \sum_p \frac{1}{2P^2} \left\{ \frac{3T_p^2}{2p^4} + \frac{1}{2p^4} \right\} \]
where the Debye mass in strong field [21] is given by

\[
(m_D^s)^2 = \frac{g^2 N_c T^2}{3} + \sum_f g^2 q_f B \frac{1}{4\pi^2}.
\]
The total gluonic contribution becomes

\[ F_g = F_g^{\text{hard}} + F_g^{\text{soft}}. \]  

(4.19)

5 Anisotropic pressure of deconfined QCD matter in a strong magnetic field

5.1 Renormalized free energy in a strong field approximation

Combining Eq. (2.1) and Eq. (4.19) one-loop free energy of deconfined QCD matter in the presence of a strong magnetic field can be written as

\[ F = F_q + F_g^{\text{hard}} + F_g^{\text{soft}} + F_0 + \Delta E_T^0 + \Delta E_B^0, \]  

(5.1)

which has \( O[1/\epsilon] \) divergences in various order \((q_f B)^2\). The \( O[(q_f B)^2] \) divergences present in the free energy are regulated by redefining the tree level free energy \( B^2/2 \) as

\[ F_0 = \frac{B^2}{2} \rightarrow \frac{B^2}{2} + 4d_F \sum_f \frac{(q_f B)^2}{(2\pi)^2} \frac{g^2 C_F \ln 2}{4\pi^2} \frac{1}{2\epsilon} + \frac{d_A}{(4\pi)^2} \sum_{f_1, f_2} \frac{g^4 q_{f_1} B q_{f_2} B}{64\pi^4 \epsilon} \]  

\[ \rightarrow \frac{B^2}{2} + \Delta E_B^0. \]  

(5.2)

Now the other divergences of \( O[(q_f B)^2 T^4], O[T^2(q_f B)] \) and \( O[(q_f B)^2/2 T^2] \) are renormalized by adding suitable counter terms as follows: the \( O[(q_f B)^2 T^4] \) divergences are regulated through counter terms as

\[ \Delta E_T^0 = \Delta E_T^{\text{HTL}} + \Delta E_T \]  

\[ = d_A \frac{m_D^4}{128\pi^2 \epsilon} \frac{d_A}{(4\pi)^2} \left[ \frac{g^4 T^4}{96\epsilon} \sum_{f_1, f_2} q_{f_1} B q_{f_2} B + \frac{N_f g^4 T^4}{96\epsilon} + \frac{C_A N_f g^4 T^4}{36\epsilon} \right]. \]  

(5.3)

where \( m_D \) is the Debye screening mass in HTL approximation. Now the \( O[T^2(q_f B)] \) and \( O[(q_f B)^2/2 T^2] \) divergences are regulated through counter terms

\[ \Delta E_B^0 = -4d_F \sum_f \frac{(q_f B)^3}{(2\pi)^2} \frac{g^2 C_F}{4\pi^2} \frac{\zeta(-2)}{8T^2 \epsilon} - \frac{d_A}{(4\pi)^2} \left[ \frac{N_f g^4 T^2}{32\pi^2} \sum_f q_f B \right. \]  

\[ - \sum_f \frac{1}{4\pi^2} \frac{C_A g^4 T^2 q_f B}{6} (1 + \ln 2). \]  

(5.4)

Now using Eqs. (3.46), (4.16), (4.17), (5.2), (5.3) and (5.4) in Eq. (5.1), one obtains renormalized one loop quark-gluon free energy in presence of strong magnetic field as

\[ F = F_q^r + F_g^r + \frac{B^2}{2}, \]  

(5.5)
where renormalized quark free energy $F^r_q$ is given by

$$
F^r_q = -d_F \sum \frac{q_f B T^2}{12} - 4d_F \sum \frac{(q_f B)^2 g^2 C_F}{4\pi^2} \left[ \frac{1}{24576} \left( 12288 \ln 2 (3\gamma_E + 2 \ln \hat{\Lambda} + \ln 16\pi) + 256 \zeta[3](4 + 105 \ln 2) \right) - 2 \ln \hat{\Lambda} + \frac{1}{2} \right] 
$$

and the renormalized total gluon free energy containing both hard and soft contributions is given as

$$
F^r_g = \frac{d_A (4\pi)^2}{(4\pi)^2} \left[ -\frac{16\pi^4 T^4}{45} + \frac{2C_A g^2 \pi^2 T^4}{9} + \frac{1}{12} \left( \frac{C_A g^2 T^2}{3} \right)^2 \left( 8 - 3\gamma_E - \pi^2 + 4 \ln 2 \right) 
- 3 \ln \hat{\Lambda} + \frac{N_f \pi^2 T^2}{2} \left( \frac{g^2}{4\pi^2} \right)^2 \sum_{f} q_f B \left( \frac{2\zeta'(\zeta(-1)}{\zeta(-1)} - 1 - \ln \hat{\Lambda} \right) 
+ \left( N_f^2 + \sum_{f_1, f_2} \frac{q_{f_1} B q_{f_2} B}{g^4 T^4} \frac{g^4 T^4}{32} \left( \frac{2}{3} \ln \hat{\Lambda} + \gamma_E + \ln 2 \right) - \frac{C_A N_f g T^4}{36} \left( 1 - \frac{2\zeta'(\zeta(-1)}{\zeta(-1)} \right) 
- \frac{1}{2} \left( \frac{g^2}{4\pi^2} \right)^2 \sum_{f_1, f_2} q_{f_1} B q_{f_2} B \left( \ln \hat{\Lambda} + \gamma_E + \ln 2 \right) - \frac{C_A g T^2 q_f B}{144\pi^2} \left( \pi^2 - 4 + 12 \ln \hat{\Lambda} + 2 \ln 2 \right) \left( 6\gamma_E + 4 + 3 \ln 2 \right) 
- \frac{1}{2} \ln \hat{\Lambda} + 12\gamma_E \right) 
- \frac{d_A (m^2_q)}{12\pi} \right].
$$

### 5.2 Longitudinal and transverse pressure

In thermal background one can calculate QCD pressure from free energy of the system and the pressure is isotropic. Now in presence of thermo-magnetic background, one has another extensive parameter as external magnetic field $B$. In this case free energy can be written as

$$
F(T, V, B) = E^\text{total} - TS - eB \cdot M,
$$

where $M$ is the magnetization. Free energy density in a finite spatial volume $V$ is given by

$$
F = \frac{F}{V} = \epsilon^{\text{total}} - TS - eB \cdot M,
$$

where $\epsilon^{\text{total}}$ is total the energy density and the entropy density is given by

$$
s = -\frac{\partial F}{\partial T},
$$
and magnetization per unit volume is given by

\[ M = -\frac{\partial F}{\partial (eB)} \quad (5.10) \]

and total energy density \( \epsilon_{\text{total}} = \epsilon + \epsilon_{\text{field}} \). \( \epsilon \) is the energy density of the medium and \( \epsilon_{\text{field}} = eB \cdot M \). In presence of strong magnetic field the space becomes anisotropic and one gets different pressure \([70]\) for direction parallel and perpendicular to the magnetic field. Longitudinal and transverse pressures are given as

\[ P_z = -F, \quad P_\perp = -F - eB \cdot M = P_z - eB \cdot M. \quad (5.11) \]

### 5.3 Pressure of ideal quark and gluon gas in a strong magnetic field

The free energy of an ideal quark-gluon gas in absence of magnetic field is given as

\[ F_{\text{ideal}}^T = -d_F \frac{7\pi^2 T^4}{180} - d_A \frac{\pi^2 T^4}{45}, \quad (5.12) \]

and the corresponding pressure reads as

\[ P_{T}^i \equiv P_{T}^{\text{ideal}} = d_F \frac{7\pi^2 T^4}{180} + d_A \frac{\pi^2 T^4}{45} \equiv (P_{q}^{T})^i + (P_{g}^{T})^i. \quad (5.13) \]

The free energy of a ideal quark-gluon gas in presence of magnetic field is given by

\[ F_{T}^\text{ideal} = F_{T}^q + F_{T}^g = -d_F \sum_f \langle q_f B \rangle \frac{T^2}{12} - d_A \frac{\pi^2 T^4}{45}. \quad (5.14) \]

As seen the quarks are affected by the magnetic field whereas the electric charge less gluons are not affected by the magnetic field. The quark contribution in presence of strong magnetic field makes the ideal quark-gluon gas pressure anisotropic. The ideal longitudinal pressure is given by

\[ P_{z}^i \equiv P_{z}^{\text{ideal}} = -F_{z}^{\text{ideal}} = d_F \sum_f \langle q_f B \rangle \frac{T^2}{12} + d_A \frac{\pi^2 T^4}{45} \equiv (P_{z}^{q})^i + (P_{z}^{g})^i. \quad (5.15) \]

Magnetization of the ideal quark-gluon gas is calculated using Eq. (5.10) as

\[ M^{\text{ideal}} = -\frac{\partial F^{\text{ideal}}}{\partial (eB)} = d_F \sum_f \langle q_f T^2 \rangle \frac{12}{12}. \quad (5.16) \]
As found the magnetization of a ideal quark-gluon gas in LLL in presence of strong magnetic field is independent of magnetic field. In LLL positive charge particles with spin up align along the magnetic field direction whereas negative charge particles with spin down align opposite to the magnetic field direction. Because of this the system remains in minimum free energy configuration with respect to $eB$. Now even if one increases magnetic field the spin alignment does not change, thus for given a $T$ the ideal quark-gluon gas acquires a constant magnetization. However, if one increases $T$ the spin alignment in LLL again does not change but the increased thermal motion along the field direction can result in an increase of magnetization.

Now the ideal transverse pressure of the quark-gluon gas can be written using Eq. (5.11) as

$$ P_{\perp} = P_{\perp}^{\text{ideal}} = d_A \frac{\pi^2 T^4}{45} \quad (5.17) $$

We note that the transverse pressure of ideal magnetized quark-gluon gas is independent of the magnetic field and is same as the ideal gluon pressure. As discussed above the gluons are not affected by magnetic field and contribute to this isotropic pressure. On the other hand quarks have momenta only along $z$ direction in LLL, and contribute only to the longitudinal pressure only.

6 Results

We use one-loop running coupling constant which evolves on both momentum transfer and the magnetic field [71] as

$$ \alpha_s(\Lambda^2, |eB|) = \frac{\alpha_s(\Lambda^2)}{1 + b_1 \alpha_s(\Lambda^2) \ln \left( \frac{\Lambda^2 + |eB|}{\Lambda^2} \right)} \quad (6.1) $$

in the strong magnetic field domain $|eB| > \Lambda^2$. The one-loop running coupling in absence of magnetic field at renormalization scale is given as

$$ \alpha_s(\Lambda^2) = \frac{1}{b_1 \ln \left( \frac{\Lambda^2}{\Lambda_{\text{MS}}^2} \right)} \quad (6.2) $$

where $b_1 = (11N_c - 2N_f)/12\pi$, $\Lambda_{\text{MS}} = 176$ MeV [72] at $\alpha_s(1.5\text{GeV}) = 0.326$ for $N_f = 3$. The renormalization scale is chosen as $\Lambda = 2\pi T$. The renormalization scale can be varied by a factor of 2 with respect to its central value. Furthermore, we are interested in thermomagnetic correction here and hence we will drop the tree level vacuum contribution $B^2/2$ from our discussion. We also note that some orders of coupling $g$ are not complete in one loop HTL calculation. To have a complete picture of pressure upto a specific order of $g$, one needs to perform higher loop order calculation. However, as a first effort we confine ourselves in one loop calculation here.
For ideal quark-gluon gas the gluons remain unaffected but quarks are strongly affected in presence of magnetic field. So in Fig. 4 we display a variation of ideal quark pressure with \((P_T^q)^i\) in Eq. (5.13) and without \((P_T^q)^i\) in Eq. (5.15) magnetic field as a function of temperature. The ideal quark pressure, \((P_T^q)^i\), in presence of magnetic field is proportional to \((eB)T^2\) whereas that in absence of magnetic field, \((P_T^q)^i\), is proportional to \(T^4\). For a given magnetic field, \(T^2\) dominates at low T whereas \(T^4\) dominates at high T and thus a crossing takes place at an intermediate temperature as seen in Fig. 4. Also the ideal longitudinal pressure increases linearly with the increase of magnetic field as can be seen from Eq. (5.15).

The left panel of Fig. 5 displays a comparison of 1-loop longitudinal pressure (solid curve) and ideal pressure (dashed curve) with temperature for different values of field strength whereas the right panel displays the same but with the strength of magnetic field for different temperatures. In both cases 1-loop pressure increases with the increase in temperature and field strength, respectively. However, the 1-loop interacting pressure is higher than that of ideal one in both panels. This enhancement can be understood as follows: in 1-loop order both effective quark two-point function and effective gluon two point function containing quark loop are strongly affected in presence of magnetic field, which contribute to the additional pressure compared to ideal case. For a given magnetic field this enhancement is stronger in the temperature domain (300-500) MeV as can be seen from the scaled pressure with ideal one \((P_z/P_z^i)\) in the left panel of Fig. 6. However, this enhancement gradually decreases with increase of temperature and approaches to ideal value at high temperature. For a given temperature the ratio \((P_z/P_z^i)\) increases with the increase of magnetic field strength as found in the right panel of Fig. 6. This is because \(P_z^i\) has linear dependence on \(eB\) whereas \(P_z\) has a higher power dependence on \(eB\).
\[ \Lambda = 2 \pi T \]

\[ |eB| = 20 \text{ m} \pi \]
\[ |eB| = 25 \text{ m} \pi \]
\[ |eB| = 30 \text{ m} \pi \]

\[ T = 0.3 \text{ GeV} \]
\[ T = 0.2 \text{ GeV} \]
\[ T = 0.25 \text{ GeV} \]
\[ T = 0.3 \text{ GeV} \]

\[ \frac{P_L}{P_L^i} \]

\[ \frac{P_L}{P_L^i} \]

\[ \Lambda = 2 \pi T \]

\[ |eB| = 20 \text{ m} \pi \]
\[ |eB| = 25 \text{ m} \pi \]
\[ |eB| = 30 \text{ m} \pi \]

\[ T = 0.3 \text{ GeV} \]
\[ T = 0.2 \text{ GeV} \]
\[ T = 0.25 \text{ GeV} \]
\[ T = 0.3 \text{ GeV} \]

\[ |eB| \]

\[ |eB| \]

\[ |eB| \]

\[ T = 0.2 \text{ GeV} \]
\[ T = 0.25 \text{ GeV} \]
\[ T = 0.3 \text{ GeV} \]

\[ \frac{P_L}{P_L^i} \]

\[ \frac{P_L}{P_L^i} \]

\[ \Lambda = 2 \pi T \]

**Figure 5:** Variation of one-loop longitudinal pressure as a function of temperature for different value of magnetic field (left panel) and as a function of magnetic field at different temperature (right panel) for \( N_f = 3 \) and the central value of the renormalization scale, \( \Lambda = 2 \pi T \). Dashed curves represent ideal longitudinal pressure.

**Figure 6:** Variation of the one-loop longitudinal pressure scaled with ideal longitudinal pressure as a function of temperature (left panel) for different values of magnetic field and as a function of magnetic field (right panel) for different temperatures with \( N_f = 3 \).

The magnetization of an ideal quark-gluon gas in presence of magnetic field has already been discussed in subsection 5.3. Now the magnetization of an interacting quark-gluon system is calculated using Eq. (5.10) and it is proportional to \([a T^2 + b(eB) + c(eB)^2/T^2 + d(eB)^3/T^4 + f(eB)^4/T^6]\) which is plotted in Fig. 7. So for a given value of \( eB \), at low \( T \) limit \( 1/T^n \) with \( n = 2, 4, 6 \) terms dominate but restricted by the scale \( gT \) whereas at high \( T \), \( T^2 \) terms dominate which is seen from the left panel. In contrast to ideal quark-gluon gas the magnetization of an interacting quark-gluon system increases with the
Figure 7: Variation of magnetization with temperature for different magnetic fields (left panel) and with magnetic field for various temperatures (right panel).

magnetic field which is displayed in the right panel. This trend is in agreement with LQCD results [73]. In strong magnetic field approximation $g^2 T^2 < T^2 < cB$, the magnetization acquires positive values with range $0 < M < 1$. So the deconfined QCD matter in presence of strong magnetic field within one-loop HTL approximation shows paramagnetic nature (i.e., magnetization is parallel to the field direction) [73]. Since the magnetization in strong field limit increases with the magnetic field, it causes an increase in pressure of the system along the field direction, i.e., longitudinal direction. This in turn also strongly affects the transverse pressure as we would see below.

One loop transverse pressure is calculated using Eq. (5.11). It is evident from Eq. (5.11) and from the left panel of Fig. 8 that the one loop transverse pressure increases with temperature and shows similar nature as longitudinal pressure (left panel of Fig. 5) but lower in magnitude. Dashed lines represent transverse ideal pressure which is independent of magnetic field as given in Eq. (5.17). For a given high value of magnetic field the pressure starts with a lower value than that of ideal gas particularly at low $T$ and then a crossing takes place. This could also be understood from the right panel. In the right panel the transverse pressure is displayed as a function of magnetic field for two different temperatures. Also the dashed lines here represent the ideal transverse pressure which is independent of magnetic field. The transverse pressure for interacting case is given in Eq. (5.11) as $P_\perp = P_z - eB \cdot M$. Now for a given temperature its variation is very slow (or almost remain unaltered) with lower value of the magnetic field because there is a competition between $P_z$ and $eBM$. Since the magnetization $M$ increases steadily with magnetic field (right panel of Fig. 7) the transverse pressure, $P_\perp$ tends to decrease, falls

---

3Even if the fermions are in LLL, the magnetization increases with magnetic field due to interactions.
\[ \Lambda = 2\pi T \]
\[ |e_B| = 20 m \pi \frac{\Lambda}{T} ; T = 0.3 \text{ GeV} \]
\[ |e_B| = 30 m \pi \frac{\Lambda}{T} ; T = 0.4 \text{ GeV} \]

Figure 8: Variation of the one-loop transverse pressure as a function of temperature for various magnetic fields (left panel) and as a function of magnetic field for different temperatures (right panel). Dashed curves represent ideal transverse pressure.

below ideal gas value and may even go to negative values for low \( T \) at large value of magnetic field. This is an indication that the system may shrink in the transverse direction \([73]\).

7 Conclusion

We consider the deconfined QCD matter in presence of strong background magnetic field within HTL approximation. Quarks are directly affected by the external magnetic field. In strong field approximation we assume the quarks are in the lowest Landau level. Gluons are affected through the quark loop in gluon two-point function. Hard and soft contribution of quark-gluon free energy are calculated within one-loop HTL approximation. Various divergent terms are eliminated by choosing appropriate counter terms in \( \overline{MS} \) renormalization scheme. In presence of strong magnetic field the hot QCD matter acquires paramagnetic nature. The presence of magnetization makes the system anisotropic and one gets different pressure in a direction parallel and perpendicular to the magnetic field. Both longitudinal (along magnetic field direction) and transverse (perpendicular to the magnetic field direction) pressure are evaluated completely analytically by calculating magnetization of the system. Various thermodynamic properties can be studied using the obtained free energy here. Moreover, the anisotropic pressure obtained here may be useful for magnetohydrodynamics description and analysis of elliptic flow of hot and dense deconfined QCD matter created in heavy-ion collisions. Finally we note that since we have considered one-loop HTL perturbation theory upto \( \mathcal{O}[g^4] \), \( \mathcal{O}[g^2] \) and \( \mathcal{O}[g^4] \) are incomplete. The present result could be improved by going to higher loop order.
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A Calculation of form factors

Before computing the quark form factors, we first decompose the transverse and the longitudinal part from the expression of the self-energy in Eq. (3.19). Considering the transverse part of the fermionic momenta to be relatively weaker in the strong field limit, we make an assumption that \((K - P)_\perp < (K - P)_z\). Then we can write down the gluonic propagator as

\[
\Delta(K - P) = \frac{1}{(K - P)_z^2 - (k - p)_z^2} = \frac{1}{(K - P)_z^2} \left[ 1 - \frac{(k - p)_z^2}{(K - P)_z^2} \right]^{-1}
\]

\[
\approx \frac{1}{(K - P)_z^2} \left( 1 + \frac{(k - p)_z^2}{(K - P)_z^2} \right)
\]

\[
= \Delta(K - P) + (k - p)_z^2 \Delta^2(K - P). \tag{A.1}
\]

A.1 Calculation of quark form factor \(a\)

We calculate the form factor \(a\) by using Eq. (3.22),(A.1) as

\[
a = \frac{1}{4} \text{Tr}[\Sigma_i] = -2g^2C_F \int \frac{d^3 k}{(2\pi)^3} e^{-\frac{k^2}{4\beta^2}} \left[ \frac{k_0}{K^2(K - P)^2} + \frac{k_0}{(k - p)_z^2} \right] \frac{1}{K^2(K - P)^4}
\]

\[
= -2g^2C_F \int \frac{d^3 k}{(2\pi)^3} e^{-\frac{k^2}{4\beta^2}} \left[ T_2 + (k - p)_z^2 T_4 \right]
\]

\[
= -2g^2C_F \int_{-\infty}^{\infty} \frac{d k_3}{2\pi} \left[ \frac{q_f B}{4\pi} T_2 + \frac{q_f B}{4\pi} (p_3^2 + q_f B) T_4 \right]
\]

\[
= -\frac{g^2C_F(q_f B)}{4\pi^2} \int_{-\infty}^{\infty} dk_3 \left[ T_2 + (p_3^2 + q_f B) T_4 \right], \tag{A.2}
\]

where

\[
T_2 = \sum \frac{k_0}{K^2(K - P)^2},
\]

\[
T_4 = \sum \frac{k_0}{K^2(K - P)^4} = -\frac{1}{2k_3} \frac{\partial T_2}{\partial p_3}. \tag{A.3}
\]

Here we also note that in LLL, \(p_\perp = 0\). Now

\[
T_2 = -\frac{1}{4q_3} \left[ (1 + n_B(q_3) - n_F(k_3)) \left( \frac{1}{p_0 + k_3 + q_3} + \frac{1}{p_0 - k_3 - q_3} \right) \right]
\]

\[-23-\]
Similarly, one can calculate the quark form factor $b$

\begin{align}
A.2 \text{ Calculation of quark form factor } b
\end{align}

Similarly, one can calculate $b$ from Eq. (3.23) and (A.1) as

\begin{align}
b &= \frac{1}{4} \text{Tr} \left[ \Sigma f \right] = 2g^2 C_F \sum_{\{K\}} e^{-\frac{k}{\sqrt{s_j}P}} \left[ \frac{k_3}{K_n(K-P)^2_n} + \frac{(k-p)_3}{K_n(K-P)^4_n} \right] \\
&= 2g^2 C_F \int \frac{d^4k}{(2\pi)^4} e^{-\frac{k^2}{\sqrt{s_j}P}} k_3 \left[ T_1 + (k-p)_3 T_3 \right] \\
&= 2g^2 C_F \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{k_3}{T_1 + qfB T_3} \\
&= \frac{g^2 C_F(qfB)}{4\pi^2} \int_{-\infty}^{\infty} \frac{dk}{2\pi} k_3 [T_1 + qfB T_3], \tag{A.8}
\end{align}

where

\begin{align}
T_1 &= \sum_{\{K\}} \frac{1}{K_n(K-P)^2_n}.
\end{align}
Now
\[
T_1 = \frac{1}{4k_3q_3} \left[ (1 + n_B(q_3) - n_F(k_3)) \left( \frac{1}{p_0 + k_3 + q_3} - \frac{1}{p_0 - k_3 - q_3} \right) + (n_B(q_3) + n_F(k_3)) \left( \frac{1}{p_0 + k_3 - q_3} - \frac{1}{p_0 - k_3 + q_3} \right) \right]
\]
\[
\approx \frac{1}{4k_3^2} \left[ \left( n_B(k_3) - p_3 \frac{\partial n_B(k_3)}{\partial k_3} - n_F(k_3) \right) \frac{1}{k_3} + \left( n_B(k_3) - p_3 \frac{\partial n_B(k_3)}{\partial k_3} + n_F(k_3) \right) \left( \frac{-2p_3}{p_0^2 - p_3^2} \right) \right].
\] (A.10)

Let, \( \int_{-\infty}^{\infty} dk_3 \ k_3 T_1 = I_1 \) with
\[
I_1 = \int_{-\infty}^{\infty} dk_3 \frac{1}{4k_3} \left[ \left( n_B(k_3) - p_3 \frac{\partial n_B(k_3)}{\partial k_3} - n_F(k_3) \right) \frac{1}{k_3} + \left( n_B(k_3) - p_3 \frac{\partial n_B(k_3)}{\partial k_3} + n_F(k_3) \right) \left( \frac{-2p_3}{p_0^2 - p_3^2} \right) \right].
\] (A.12)

Fermion part of \( I_1 \) can be written as,
\[
- \int_{-\infty}^{\infty} dk_3 \frac{n_F(k_3)}{4k_3} \left( \frac{1}{k_3} + \frac{2p_3}{p_0^2 - p_3^2} \right) \approx - \int_{0}^{\infty} dk_3 \frac{1}{4} \left( \frac{2n_F - 2p_3}{k_3} \right) \left( \frac{2p_3}{p_0^2 - p_3^2} \right)
\]
\[
= \frac{p_3}{p_0^2 - p_3^2} \left[ \frac{1}{4} \gamma_E + \frac{1}{2} \ln \frac{2}{\pi} \right] + \mathcal{O}[\epsilon].
\] (A.13)

Bosonic part of \( I_1 \) is given as,
\[
\int_{-\infty}^{\infty} dk_3 \frac{1}{4k_3} \left( n_B(k_3) - p_3 \frac{\partial n_B}{\partial k_3} \right) \left( \frac{1}{k_3} - \frac{2p_3}{p_0^2 - p_3^2} \right)
\]
\[
\approx \frac{1}{4} \left[ -2 \int_{0}^{\infty} dk_3 \frac{n_B(k_3)}{k_3} \frac{2p_3}{k_3} - 2 \int_{0}^{\infty} dk_3 \frac{p_3 \frac{\partial n_B}{\partial k_3}}{k_3} \right]
\]
\[
= \frac{1}{4} \left[ -2 \int_{0}^{\infty} dk_3 \frac{n_B(k_3)}{k_3} \frac{2p_3}{k_3} - 2p_3 \frac{\partial}{\partial \beta} \int_{0}^{\infty} dk_3 \frac{n_B(k_3)}{k_3} \right]
\]
\[
= \frac{p_3}{p_0^2 - p_3^2} \left( -\frac{1}{4} \gamma_E + \frac{1}{2} \ln 2\pi \right) - p_3 \gamma \left( -\frac{2}{2T^2} \right) + \mathcal{O}[\epsilon].
\] (A.14)
After combining (A.13) and (A.14), $I_1$ can be written as

$$I_1 = \frac{p_3}{p_0^3 - p_3^3} \ln 2 - \frac{p_3 \zeta\('[2]}{2T^2},$$

(A.15)

$$\int_{-\infty}^{\infty} dk_3 \ k_3 T_3 = -\frac{1}{2} \frac{\partial}{\partial p_3} \int dk_3 \ T_1$$

$$\approx -\frac{1}{2} \frac{\partial}{\partial p_3} \left[ -\frac{1}{2} \int_{0}^{\infty} dk_3 \ \frac{n_F(k_3)}{k_3^3} + \frac{1}{2} \int_{0}^{\infty} dk_3 \ \frac{n_B(k_3)}{k_3^3} \right]$$

$$+ \frac{p_3}{T(p_0^3 - p_3^3)} \frac{\partial}{\partial \beta} \int_{0}^{\infty} dk_3 \ \frac{n_B(k_3)}{k_3^3}$$

$$= -\frac{\zeta([-2]}{T^2} \frac{p_3^2 p_3^3}{(p_0^3 - p_3^3)^2}.$$ (A.16)

So,

$$b = -c = \frac{g^2 C_F(q_f B)}{4\pi^2} \left[ \frac{p_3}{p_0^3 - p_3^3} \ln 2 - \frac{p_3}{2T^2} \zeta\('[2]) - q_f B \frac{\zeta\('[2]}{T^2} \frac{p_3^2 p_3^3}{(p_0^3 - p_3^3)^2} \right].$$ (A.17)

**B One-loop sum-integrals for quark free energy**

We write the form factors $a$ and $b$ as,

$$a = c_1 \left[ \frac{p_0}{p_0^3 - p_3^3} c_2 - d_1 \frac{p_0(p_0^3 + p_3^3)}{(p_0^3 - p_3^3)^2} \right]$$

$$= c_1 \left[ \frac{p_0}{p_0^2} c_2 - d_1 \left( \frac{p_0}{p_0^2} + \frac{2p_0 p_3}{p_0^3} \right) \right],$$

$$b = -c_1 \left[ \frac{p_3}{p_0^3 - p_3^3} c_2 - p_0 d_2 - 2d_1 \frac{p_0^2 p_3}{(p_0^3 - p_3^3)^2} \right]$$

$$= -c_1 \left[ \frac{p_3}{p_0^2} c_2 - p_0 d_2 - 2d_1 \left( \frac{p_3}{p_0^2} + \frac{p_3^3}{p_0^3} \right) \right],$$ (B.1)

where

$$c_1 = -\frac{g^2 C_F(q_f B)}{4\pi^2}, \quad c_2 = \ln 2, \quad d_1 = q_f B \frac{\zeta\('[2]}{2T^2}, \quad d_2 = \frac{1}{2T^2} \zeta\('[2).$$ (B.2)

Now one can write the following frequency sum as,

$$\sum_{\{p_0\}} \frac{a p_0}{p_0^3} = c_1 \sum_{\{p_0\}} \left[ (c_2 - d_1) \frac{1}{p_0^2} + (c_2 - d_1) \frac{p_3^2}{p_0^3} - 2d_1 \left( \frac{p_0^3}{p_0^2} + \frac{p_3^3}{p_0^3} \right) \right],$$ (B.3)

$$\sum_{\{p_0\}} \frac{b p_3}{p_0^3} = -c_1 \sum_{\{p_0\}} \left[ (c_2 - 2d_1) \frac{p_3^2}{p_0^3} - d_2 \frac{p_3^2}{p_0^3} - 2d_1 \frac{p_3^4}{p_0^3} \right],$$ (B.4)

$$\sum_{\{p_0\}} \frac{a^2}{p_0^3} = c_1 \sum_{\{p_0\}} \left[ \frac{p_0^2}{p_0^3} (\ln 2)^2 + d_1 \frac{p_0^2(p_0^3 + p_3^3)^2}{p_0^{10}} - 2d_1 \frac{p_0^2(p_0^3 + p_3^3)^2}{p_0^3} \ln 2 \right].$$

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So, $F'_{q}$ in Eq. 3.44 becomes

$$F'_{q} = -4dF \sum \frac{g_f B}{(2\pi)^2} \sum_{\{p_0\}} dP_{3} \left[ \sum_{\{p_0\}} \frac{aibp_0 p_3}{P_{10}^4} \right]$$

(B.10)

$$= -4dF \left[ c_1 (c_2 - d_1) \sum_{\{p_0\}} \frac{1}{P_{10}^2} - c_1 (d_1 - 6c_1 c_2 d_2) + 8c_1 d_1 d_2 \sum_{\{p_0\}} \frac{p_3}{P_{10}^2} + 8c_2 d_2 (c_2 - 3d_1) \sum_{\{p_0\}} \frac{p_3^2}{P_{10}^6} - 16c_1 d_1 d_2 \sum_{\{p_0\}} \frac{p_6}{P_{10}^8} + c_1 d_2 \sum_{\{p_0\}} \frac{p_3}{P_{10}^2} - c_1^2 (c_2 - d_1)^2 \sum_{\{p_0\}} \frac{1}{P_{10}^2} - c_1^2 (8c_2^2 - 26c_2 d_1 + 19d_1^2) \right]$$
Thus using Eq. (B.13) in Eq. (B.12), one can write

\[ \sum_{\{p_0\}} \frac{p_3^2}{P_3^2} - 2c_1^2 (2c_2 - 11d_1)(2c_2 - 3d_1) \sum_{\{p_0\}} \frac{p_3^4}{P_3^4} - 16c_1^2d_1 (-2c_2 + 5d_1) \times \sum_{\{p_0\}} \frac{p_3^6}{P_3^6} - 32c_1^2d_1^2 \sum_{\{p_0\}} \frac{p_3^8}{P_3^8} \].

(B.11)

Now we calculate the frequency sums as

\[
\begin{align*}
\sum_{\{p_0\}} \frac{1}{P_4^4} &= \frac{1}{2p_3} \frac{\partial}{\partial p_3} \sum_{\{p_0\}} \frac{1}{P_2^2}, \\
\sum_{\{p_0\}} \frac{1}{P_6^6} &= \frac{1}{4p_3} \frac{\partial}{\partial p_3} \sum_{\{p_0\}} \frac{1}{P_4^4}, \\
\sum_{\{p_0\}} \frac{1}{P_8^8} &= \frac{1}{6p_3} \frac{\partial}{\partial p_3} \sum_{\{p_0\}} \frac{1}{P_6^6}, \\
\sum_{\{p_0\}} \frac{1}{P_{10}^{10}} &= \frac{1}{8p_3} \frac{\partial}{\partial p_3} \sum_{\{p_0\}} \frac{1}{P_8^8}.
\end{align*}
\]

(B.12)

One can calculate

\[
\sum_{\{p_0\}} \frac{1}{P_2^2} = -\frac{1}{2p_3} \left( 1 - 2n_F(p_3) \right),
\]

(B.13)

Thus using Eq. (B.13) in Eq. (B.12), one can write

\[
\sum_{\{p_0\}} \frac{1}{P_4^4} = \frac{1}{2p_3} \frac{\partial}{\partial p_3} \left( \sum_{\{p_0\}} \frac{1}{P_2^2} \right) \approx \frac{1}{2p_3} \frac{\partial}{\partial p_3} \left[ \frac{n_F(p_3)}{p_3} \right] - \frac{1}{2p_3} \frac{\partial}{\partial p_3} \left[ \frac{n_F(p_3)}{p_3} \right] - \frac{\partial}{\partial p_3} \left[ \frac{\beta \partial n_F(p_3)}{p_3} \right] - \frac{\partial}{\partial p_3} \left[ \frac{n_F(p_3)}{p_3} \right]
\]

(B.14)

Now we perform the sum-integrals in Eq. (B.11) as

\[
\sum_{\{p_0\}} \frac{p_3^2}{P_3^2} = \left( \frac{e^{\gamma} \Lambda^2}{4\pi} \right)^2 \int_{-\infty}^{\infty} d^{1-2\epsilon}p_3 \left( \frac{\partial}{\partial p_3} \right)^2 \frac{n_F(p_3)}{2p_3^3} \\
\approx 2 \left( \frac{e^{\gamma} \Lambda^2}{4\pi} \right)^2 \int_{0}^{\infty} d^{1-2\epsilon}p_3 \left( \frac{\partial}{\partial p_3} \right)^2 \frac{n_F(p_3)}{2p_3^3} \\
\approx \left( \frac{\Lambda}{4\pi T} \right)^2 \left[ 7 \frac{\zeta(-2)}{2} + O(\epsilon) \right],
\]

(B.15)

\[
\sum_{\{p_0\}} \frac{p_3^4}{P_3^4} = \left( \frac{e^{\gamma} \Lambda^2}{4\pi} \right)^2 \int_{-\infty}^{\infty} d^{1-2\epsilon}p_3 \left( \frac{\partial^2}{\partial p_3^2} - 3\frac{\partial}{\partial p_3} + 3 \right) \frac{n_F(p_3)}{8p_3^3} \\
\approx \left( \frac{\Lambda}{4\pi T} \right)^2 \left[ \frac{\zeta(-2)}{8} + O(\epsilon) \right],
\]

(B.16)

\[
\sum_{\{p_0\}} \frac{p_3^4}{P_3^4} = \left( \frac{e^{\gamma} \Lambda^2}{4\pi} \right)^2 \int_{-\infty}^{\infty} d^{1-2\epsilon}p_3 \left( \frac{\partial^3}{\partial p_3^3} - 6\frac{\partial^2}{\partial p_3^2} + 15\frac{\partial}{\partial p_3} - 15 \right) \frac{n_F(p_3)}{48p_3^3}
\]
Similarly one can calculate

\[
\sum_{\{p_0\}} \frac{p_5}{p_{10}}^1 = \left( \frac{\Lambda}{4\pi T} \right)^{2e} \epsilon \int_{-\infty}^{\infty} d^{1-2e} p_3 \left( \frac{7}{16} + O(\epsilon) \right),
\]

\[
\sum_{\{p_0\}} \frac{p_4}{p_{10}}^3 = \left( \frac{\Lambda}{4\pi T} \right)^{2e} \epsilon \int_{-\infty}^{\infty} d^{1-2e} p_3 \left( \frac{1}{2} - \frac{1}{2} \left( 3\gamma_E + 4\ln 2 - \ln \pi \right) + O(\epsilon) \right),
\]

\[
\sum_{\{p_0\}} \frac{p_3}{p_{10}}^5 = \left( \frac{\Lambda}{4\pi T} \right)^{2e} \epsilon \int_{-\infty}^{\infty} d^{1-2e} p_3 \left( \frac{1}{4} + \frac{1}{4} \left( -2 + 3\gamma_E + 4\ln 2 - \ln \pi \right) + O(\epsilon) \right),
\]

\[
\sum_{\{p_0\}} \frac{p_2}{p_{10}}^7 = \left( \frac{\Lambda}{4\pi T} \right)^{2e} \epsilon \int_{-\infty}^{\infty} d^{1-2e} p_3 \left( \frac{3}{16} - \frac{3}{16} \left( -\frac{8}{3} + 3\gamma_E + 4\ln 2 - \ln \pi \right) + O(\epsilon) \right),
\]

\[
\sum_{\{p_0\}} \frac{p_1}{p_{10}}^9 = \left( \frac{\Lambda}{4\pi T} \right)^{2e} \epsilon \int_{-\infty}^{\infty} d^{1-2e} p_3 \left( \frac{5}{32} + \frac{5}{32} \left( -\frac{46}{15} + 3\gamma_E + 4\ln 2 - \ln \pi \right) + O(\epsilon) \right),
\]

\[
\sum_{\{p_0\}} \frac{p_0}{p_{10}}^{10} = \left( \frac{\Lambda}{4\pi T} \right)^{2e} \epsilon \int_{-\infty}^{\infty} d^{1-2e} p_3 \left( \frac{\pi^2 T^2}{6} + O(\epsilon) \right).
\]
Using the above sum-integrals in Eq. (B.11) $F'_q$ upto $O(g^4)$ can be written as,

\[
F'_q = -4d_F \sum_f \frac{(q_f B)^2 g^2 C_F}{(2\pi)^4} \left( \frac{\Lambda}{4\pi T} \right)^{2e} \left[ \frac{1}{8\epsilon} \left( 4\ln 2 - q_f B \zeta'(-2) \right) + \frac{1}{24576} \right. \\
\times \left\{ 12288 \ln 2(3\gamma_E + 4\ln 2 - \ln \pi) + \frac{256\zeta[3]}{\pi^4 T^2} (2\pi^4 T^2 - 3g^2 C_F(q_f B) \ln 2 \\
+ 3\pi^2(q_f B)(2 + 3\gamma_E + 4\ln 2 - \ln \pi) - \frac{8g^2 C_F}{\pi^6 T^4} (q_f B)^2 \zeta[3]^2 (4 + 105 \ln 2) \\
+ \frac{7245g^2 C_F}{\pi^8 T^6} (q_f B)^3 \zeta[3]^3 \right\}. \tag{B.26}
\]

C HTL One-loop sum-integrals for gluon free energy

\[
\sum \frac{1}{p^2} = -\frac{T^2}{12} \left( \frac{\Lambda}{4\pi T} \right)^{2e} \left[ 1 + 2\epsilon \left( 1 + \frac{\zeta'(-1)}{\zeta(-1)} \right) \right] + O(\epsilon)^2, \tag{C.1}
\]

\[
\sum \frac{1}{p^2 p^2} = -\frac{2}{(4\pi)^2} \left( \frac{\Lambda}{4\pi T} \right)^{2e} \left[ \frac{1}{e} + 2\gamma_E + 2 + \epsilon \left( 4 + 4\gamma_E + \frac{\pi^2}{4} - 4\gamma_1 \right) \right] \\
+ O(\epsilon)^2, \tag{C.2}
\]

\[
\sum \frac{T_p}{p^2 p^2} = \sum_p \left< \frac{1 - c^{2e+1}}{1 - c^2} \right>_e \frac{1}{p^2 p^2} \\
= -\frac{2}{(4\pi)^2} \left( \frac{\Lambda}{4\pi T} \right)^{2e} \left[ \ln 2 + \left( \frac{\pi^2}{6} - (2 - \ln 2) \ln 2 \right) \epsilon \right] \left[ \frac{1}{\epsilon} + 2\gamma_E + 2 \\
+ \epsilon \left( 4 + 4\gamma_E + \frac{\pi^2}{4} - 4\gamma_1 \right) \right] \\
= -\frac{2}{(4\pi)^2} \left( \frac{\Lambda}{4\pi T} \right)^{2e} \left[ \frac{1}{\epsilon} \ln 2 + \frac{\pi^2}{6} + (\ln 2)^2 + 2\gamma_E \ln 2 \right], \tag{C.4}
\]

\[
\sum \frac{T_p}{p^4} = \sum_p \left< c^{2e+1} \right>_e \frac{1}{p^4 p^2} \\
= -\frac{2}{(4\pi)^2} \left( \frac{\Lambda}{4\pi T} \right)^{2e} \left\{ \frac{1}{2} + (-1 + \ln 2) \epsilon \right\} \left[ \frac{1}{\epsilon} + 2\gamma_E + 2 \\
+ \epsilon \left( 4 + 4\gamma_E + \frac{\pi^2}{4} - 4\gamma_1 \right) \right] \\
= -\frac{2}{(4\pi)^2} \left( \frac{\Lambda}{4\pi T} \right)^{2e} \left[ \frac{1}{2\epsilon} + (\gamma_E + \ln 2) \right], \tag{C.5}
\]
\[
\sum_p \frac{T_p^2}{p^2} = \sum_p \left( \frac{2x+3}{c_1^2 - c_2^2} \right) \frac{1}{c_1^2 c_2^2} = -\frac{2}{(4\pi)^2} \left( \frac{\Lambda}{4\pi T} \right)^{2x} \left( \frac{1}{3} (1 + 2 \ln 2) + \frac{2}{9} (-5 + 2 (5 + 3 \ln 2)) \right) 
\times \left[ \frac{1}{\epsilon} + 2\gamma_E + 2 + \epsilon \left( 4 + 4\gamma_E + \frac{\pi^2}{4} - 4\gamma_1 \right) \right] 
\approx -\frac{2}{(4\pi)^2} \left( \frac{\Lambda}{4\pi T} \right)^{2x} \left[ \frac{1}{3\epsilon} (1 + 2 \ln 2) + \frac{2}{9} (2 + 3 (\ln 2)^2 + \gamma_E (3 + 6 \ln 2) + 11 \ln 2) \right]. 
\]

(C.6)

\[
\sum_p e^{-P_i^2/2qfB} \frac{p_i^2}{p_i^2 P^2 (p_i^2 - p_3^2)} = \sum_p e^{-P_i^2/2qfB} \frac{p_i^2}{p_i^2 (p_i^2 - p_3^2)} \left\{ \frac{1}{P^2} - \frac{1}{p_i^2 - p_3^2} \right\} 
= \sum_p e^{-P_i^2/2qfB} \frac{p_i^2}{p_i^2 (p_i^2 - p_3^2) P^2} - \sum_p e^{-P_i^2/2qfB} \frac{p_i^2}{p_i^2 (p_i^2 - p_3^2) (p_i^2 - p_3^2)} 
= \left( \frac{e^{\gamma_E} \Lambda^2}{4\pi} \right) \epsilon \int \frac{d^3-2 \epsilon}{(2\pi)^{3-2 \epsilon}} \left[ - e^{-P_i^2/2qfB} \frac{p_i^3}{p_i^3 (p_i^2 - p_3^2)} n_B(p) + e^{-P_i^2/2qfB} \frac{p_i^3}{p_i^2 (p_i^2 - p_3^2)} n_B(p) \right] 
\approx \left( \frac{\Lambda}{4\pi T} \right)^{2x} \frac{1}{(4\pi)^2 9qfB} \left\{ \frac{1}{\epsilon} \left( 18qfB - 3\pi^2 T^2 - 18qfB \ln 2 \right) + \left\{ -3qfB \right\} 
\left( -12 + \pi^2 + 6(\ln 2)^2 + 6\gamma_E (-2 + 2 \ln 2) \right) - \pi^2 T^2 \left( -8 + 6 \ln 2 + 6 \left( 1 + \frac{\zeta'(-1)}{\zeta(-1)} \right) \right) \right\} 
+ O \left( \frac{1}{(qfB)^2} \right). 
\]

(C.7)
\[
\left\{ -18 \left( q_{f_1} B \right)^2 \left( (q_{f_2} B)^2 \ln 4 + 12 q_{f_2} B T^2 \zeta'[2] + 60 T^4 \zeta'[4] \right) + 12 q_{f_1} B q_{f_2} B T^2 (q_{f_2} B \zeta'[2] + 10 T^2 \zeta'[4]) + 60 (q_{f_2} B)^2 T^4 \zeta'[4] \right) + 12 \gamma_E ((q_{f_2} B)^2 - 3 (q_{f_2} B)^2 + 3 \pi^2 q_{f_2} B T^2 + \pi^4 T^4) + \pi^2 q_{f_1} B q_{f_2} B T^2 (3 q_{f_2} B + 2 \pi^2 T^2) + \pi^4 (q_{f_2} B)^2 T^4 \\
+ \pi^4 T^4 (12 \ln (4 \pi) - 25) (q_{f_1} B + q_{f_2} B)^2 + 18 \pi^2 q_{f_1} B q_{f_2} B T^2 (-3 + 2 \ln (4 \pi)) \right) \right] + O \left[ \frac{1}{(q_f B)^3} \right]. 
\]

(C.8)

\[
\sum_p e^{-p_+^2 / 2 q_f B} \frac{p_3^2}{p^4 (p_0^2 - p_3^2)} = - \left( \frac{e^{\gamma_E} \Lambda^2}{4\pi} \right)^\epsilon \int \frac{d^3-2 \epsilon}{4\pi} e^{-p_+^2 / 2 q_f B} \left( \frac{p_3 n_B(p_3) - p_3 c^2 n_B(p_3)}{p^4} - \frac{p_3 c n_B(p_c)}{p^3 \left( p_0^2 - p^2 \right)} \right) \right)e \\
\approx - \left( \frac{\Lambda}{4\pi T} \right)^{2\epsilon} \frac{1}{(4\pi)^2} \left[ \frac{1}{\epsilon} \left( 1 - \frac{\pi^2 T^2}{3 q_f B} \right) + \frac{\pi^2 T^2}{3 q_f B} \left( 2 \left( 1 + \frac{\zeta'[1]}{\zeta[1]} \right) - 3 + 2 \ln 2 \right) + 6 \gamma_E + 6 \ln 2 \right] + O \left[ \frac{1}{(q_f B)^2} \right]. 
\]

(C.9)

\[
\sum_p e^{-p_+^2 / 2 q_f B} \frac{p_3 T_p}{p^4 (p_0^2 - p_3^2)} = \left( \frac{e^{\gamma_E} \Lambda^2}{4\pi} \right)^\epsilon \int \frac{d^3-2 \epsilon}{4\pi} e^{-p_+^2 / 2 q_f B} \left( - \frac{p_3 n_B(p_3)}{p^4} - \frac{p_3 c^2 n_B(p_3)}{p^2 \left( p_0^2 - p^2 \right)} + \frac{p_3 c n_B(p_c)}{p^3 \left( p_0^2 - p^2 \right)} \right) \right)e \\
\approx \left( \frac{\Lambda}{4\pi T} \right)^{2\epsilon} \frac{1}{(4\pi)^2} \left[ \frac{1}{\epsilon} \left( \frac{1}{3} - \frac{2 \pi^2 T^2}{9 q_f B} \right) + \frac{1}{27 q_f B} \left\{ \pi^2 T^2 \left( -12 \left( 1 + \frac{\zeta'[1]}{\zeta[1]} \right) + 17 + 12 \ln 2 \right) \\
- 3 q_f B \left( -8 + 12 \ln 2 \right)^2 + 18 \ln 2 + 6 \gamma_E (4 \ln 2 - 1) + 8 \ln 2 \right\} \right] + O \left[ \frac{1}{(q_f B)^2} \right]. 
\]

(C.10)

Using Eqs. (C.1)-(C.10) in Eq. (4.15) one can get expression for gluon free energy in strongly magnetized medium computed up to \( O[g^4] \) as

\[
F_g = \left. \frac{d_A}{(4\pi)^2} \right\{ \frac{1}{\epsilon} \left[ - \frac{1}{8} \left( \frac{C_A g^2 T^2}{3} \right)^2 + \sum_{f_1, f_2} \frac{g^4 T^4}{192(q_{f_1} B)(q_{f_2} B)} ( (q_{f_1} B)^2 + (q_{f_2} B)^2 ) \right] \right\} \\
+ \frac{N_f^2 g^4 T^4}{96} + \frac{C_A N_f g^4 T^4}{36} - \sum_{f_1, f_2} \frac{g^4(q_{f_1} B)(q_{f_2} B)}{64 \pi^4} + \sum_{f_1, f_2} \frac{g^4 T^4}{64 \pi^4} (q_{f_1} B + q_{f_2} B) \\
- \frac{1}{12} \left[ 8 - 3 \gamma_E - \pi^2 + 7 \ln 2 - 3 \ln \Lambda \right] + \frac{1}{2} \sum_{f_1, f_2} \frac{g^2}{4 \pi^2} \left[ \frac{1}{2} \pi^2 T^2 \left( 2 \left( 1 + \frac{\zeta'[1]}{\zeta[1]} \right) \\
- 3 + 2 \ln 2 \right) (q_{f_1} B + q_{f_2} B) + (2 \ln 2 - 2 \ln \Lambda) \right] \left( - \frac{\pi^4 T^4}{6(q_{f_1} B)(q_{f_2} B)} (q_{f_1} B)^2 \right) \right\}.
\]
\begin{align}
+ (q_f^2 B)^2 & - \frac{1}{2} \pi^2 T^2 (q_f^1 B + q_f^2 B) + \frac{(q_f^1 B)(q_f^2 B)}{2} - \frac{\pi^4 T^4}{3} (q_f^1 B)(q_f^2 B) \\
\times \left( (q_f^1 B)^2 + (q_f^2 B)^2 \right) + \frac{\pi^4 T^4}{36 q_f^1 B q_f^2 B} (-25 + 12\gamma_E + 12 \ln(4\pi)) (q_f^1 B)^2 \\
+ (q_f^2 B)^2 & - q_f^1 B q_f^2 B (\gamma_E + \ln 2) - 60 T^4 \zeta'[4] + \frac{1}{18} \pi^4 T^4 (-25 + 12\gamma_E) \\
+ 12 \ln(4\pi) \right) \\
- 8\pi^2 \ln \frac{\Lambda}{2} & + \frac{1}{3} \left( 4(3 + 3 \ln 2) \ln \frac{\Lambda}{2} + \pi^2 - 4 - 6(\ln 2)^2 - 6\gamma_E(2 \ln 2 - 2) \\
- 8 \ln 2 \right) \right] .
\end{align}

(C.11)

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