From Syntactic Theories to Interpreters: A Specification Language and Its Compilation

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Abstract. Recent years have seen an increasing need of high-level specification languages and tools generating code from specifications. In this paper, we introduce a specification language, SL, which is tailored to the writing of syntactic theories of language semantics. More specifically, the language supports specifying primitive notions such as dynamic constraints, contexts, axioms, and inference rules. We also introduce a system which generates interpreters from SL specifications. A prototype system is implemented and has been tested on a number of examples, including a syntactic theory for Verilog.

1 Introduction

Syntactic theories have been developed to reason about many aspects of modern programming languages[AB97, AFM+95, LS97, SS99, FLS99]. Having roots in the \(\lambda\)-calculus, these theories rely on transforming source programs to other source programs. Only the syntax of the programming language is relevant.

Experience shows that the development of such theories is error-prone. In order to ensure that the theories are sensible, many properties need to be checked. For example, we need to know if we have enough rules to rewrite a program to its value, if the type of a program is preserved during evaluation, and whether each program has a unique value. In many situations, the proofs of these properties do not require deep insight. In fact, many purported proofs suffer from being incomplete, and usually the missed case is the problematic one. Thus, we think that in order to rely on syntactic theories, it is of mandatory importance to design tools that support their development. The work described in this paper offers a first step towards that direction.

We introduce the specification language SL, which can directly reflect the primitive notions of syntactic theories such as evaluation contexts and dynamic constraints. An experimental system has been implemented that generates interpreters from SL specifications. Currently the generated interpreters are programs in CAML[Cam] which is a dialect in the ML family. Various examples have been tested, including the operational semantics of core-ML (lambda-calculus with built-in operations, store operations, and exception handling), type inference

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for core-ML, a syntactic theory for a store encapsulation language [SS99], and a syntactic theory for Verilog [FLS99].

The paper is organized as follows: Section 2 gives an overview of the SL system using the call-by-value \( \lambda \)-calculus as an example. Section 3 describes some issues in compiling SL programs, such as type checking for contexts, pattern-matching, and code generation. Section 4 presents related work and concludes the paper.

2 Overview of the SL System

We introduce the call-by-value \( \lambda \)-calculus [Plo75] and show how it is specified in the SL language. By convention, we call the SL language the meta-language, and call the call-by-value \( \lambda \)-calculus the object-language.

2.1 A Syntactic Theory for Call-by-value \( \lambda \)-calculus

The set of terms of the call-by-value lambda calculus is generated inductively over an infinite set of variables (ranged over by \( x, y \), etc): it includes \( \lambda \)-abstractions and applications:

\[
\begin{align*}
\text{Terms} & \quad M ::= x \mid \lambda x. M \mid MM
\end{align*}
\]

The semantics is based on the \( \beta_v \) reduction rule which requires a syntactic definition of the notion of value:

\[
\begin{align*}
\text{Values} & \quad V ::= \lambda x. M \\
\beta_v & \quad (\lambda x. M)V \rightarrow M[x := V]
\end{align*}
\]

where \( M[x := V] \) is the term resulting from substituting free occurrences of variable \( x \) with \( V \).

A call-by-value computation consists of successively applying the \( \beta_v \) reduction rule to a subterm. Positions of \( \beta_v \) redexes are restricted by an evaluation context which is defined as follows:

\[
\begin{align*}
H ::= \square \mid HM \mid VH
\end{align*}
\]

where \( \square \) represents a “hole”. If \( H \) is an evaluation context, then \( H[M] \) denotes the term that results from placing \( M \) in the hole of \( H \). The evaluation of a program is then defined by a stepping relation, denoted by \( \rightarrow \), given as follows:

\[
\frac{M \rightarrow M'}{H[M] \rightarrow H[M']}
\]
SIGNATURE:
type M = Var of string | Lam of string*M | App of M*M ;;
startfrom M ;;

SPECIFICATION:
#open "namesupply" ;;
let rec subst (t1, x, t2) =
  match t1 with
  | Var s -> if s = x then t2 else t1
  | Lam(s, t1') -> if s = x then t1
                  else let s' = freshname() in
                      Lam(s', subst(subst(t1', s, Var s'), x, t2))
  | App(t11, t12) -> App(subst(t11, x, t2), subst(t12, x, t2));;

dynamic V = Lam _ ;;

axiom betav: App(Lam(x, t1), (t2: V)) ==> subst(t1, x, t2);;

context H = BOX | App(H, _) | App(V, H);;

inference eval:
t1 ==> t2
-------------
(h:H) t1 |==> h t2 ;;

Fig. 1. An SL Specification of a Simple CBV Language

2.2 Representation in SL

The specification of the call-by-value \( \lambda \)-calculus in SL is given in Fig. 1. The SIGNATURE part describes the abstract syntax of the language using CAML type definitions. In general, the SL type definitions may be polymorphic but are restricted to first-order type expressions (no function types). To account for cases in which the description needs more than one type, the type of programs in the object language is explicitly given by the startfrom phrase.

The SPECIFICATION part describes the semantics of the language. A dynamic definition defines a subset of a type with a semantic significance. The axioms are conditional rewriting rules. The optional conditions are CAML expressions following the keyword when. The meta-language also has a primitive notion of contexts with BOX as the empty context. Each inference rule has one premise clause and one conclusion clause, also with an optional condition expression. Axioms and inference rules use a richer notion of pattern-matching than the one used in most functional languages: they include dynamic constraints like in t2:V, context constraints and context fillings like in h:H and h t2. Meta-
operations of the semantics like substitution are written directly in CAML as auxiliary definitions.

### 2.3 Generating interpreters

The SL system is very domain-specific, targeting exactly the kind of semantic specifications based on syntactic theories. In addition, it performs basic checks to ensure the specifications are well-formed. Other than the basic syntactic checks, the SL system has a (meta-)type system that ensures that contexts are used appropriately, e.g., every context has one hole, and contexts are filled with expressions of the appropriate types, and both sides of each axiom have the same type. After performing these basic checks, the SL system compiles the specification into a non-deterministic automaton, which is then transliterated into CAML code; the code uses success continuations for encoding the sequencing of states; and exceptions with handlers for encoding the non-deterministic selection of a state.

Feeding the code in Fig. 1 to the SL system produces an interpreter. This interpreter can then be invoked on terms of the language to evaluate them by repeatedly decomposing them into evaluation contexts and redexes, and contracting the redexes, until an answer is reached. For example, if the input file contains:

```caml
App(Lam("y",Var "y"),App(Lam("x",Var "x"),Lam("z",Var "z")));
```

the generated interpreter produces:

```caml
App(Lam("y",Var "y"),App(Lam("x",Var "x"),Lam("z",Var "z")))

==>
by betav,eval
App(Lam("z",Var "z"))

==>
by betav,eval
Lam("z",Var "z")
```

Interpreters generated by the SL system preserve the semantics of object-languages in the sense that if a specification is non-deterministic, the generated interpreter evaluates input programs non-deterministically. The current version does not employ backtracking in evaluation.

### 3 Compiling SL specifications

The compilation of an SL specification includes the usual phases such as lexing, parsing, static checking, and code generation. For an object-language specified
by an SL program, a parser and a pretty-printer of the object-language are generated from the signature part, and a reduction machine based on pattern-matching automata is generated from the semantic rules. These parts work together as an interpreter for the object-language with the support of the SL runtime libraries.

Next, we introduce some issues in the SL compilation such as typing contexts, building automata, and transforming automata into CAML code.

### 3.1 Typing contexts

The type system for SL extends the type system for CAML. The extensions deal with dynamic definitions and contexts. Here, we only present the idea of typing contexts in a simply-typed framework. Typing dynamic definitions is similar.

First, we give definitions of meta-expressions, context expressions, and their types.

#### Expressions

\[ E ::= c_0 \mid c_1 \mid (E, E) \mid x \mid \lambda x.E \mid E E \mid N[E] \]

#### Context Expressions

\[ H ::= \square \mid N \mid c_1 H \mid (H, E) \mid (E, H) \mid N[H] \]

#### Context Definitions

\[ L ::= N = H \mid \cdots \mid H \]

#### Types

\[ T ::= a \mid T \cdot T \mid T \rightarrow T \]

#### Context Types

\[ U ::= T \circ \rightarrow T \]

We write \( x \) for variables, \( c_0 \) for nullary constructors, \( c_1 \) for constructors of arity one, \( a \) for constant types, and \( N \) for context names. Note that the symbol “\( = \)” and the symbol “\( | \)” in a context definition are symbols of the SL language.

The expressions include context filling, and tuple expressions are represented as nested pairs. Context expressions are distinguished from expressions, for they always contains one hole. The type of a context expression has the form \( t_1 \circ \rightarrow t_2 \), where \( t_1 \) is the type for the hole and \( t_2 \) is the type of the whole expression if the hole is filled. For a context definition \( N = H_1 \mid \cdots \mid H_n \), each \( H_i \) should have the same type as the context type of \( N \).

The typing rules are given in Table 1. \( \Gamma \) is a basis for type checking, which contains type assignments for constructors, variables, and context names. It has properties such as weakening where it may have unused assignments, strengthening where unused assignments can be removed, permutation where the order of assignments is irrelevant, and contraction where assignments can be used more than once. The first part in the table is the set of rules for expressions. Most are standard except the rule for context filling which is similar to function application. The second part is the set of rules for context expressions. These rules express the “lifting” of the context type constructor \( \circ \rightarrow \) whenever possible, so that context expressions preserve context types. The rule for filling contexts with context expressions is similar to function composition. If a context \( N \) has type \( \tau_1 \circ \rightarrow \tau_2 \) and a context expression \( h \) has type \( \tau_0 \circ \rightarrow \tau_1 \), then the context expression \( N[h] \) has type \( \tau_0 \circ \rightarrow \tau_2 \).
3.2 Building pattern-matching automata

Pattern-matching is the crucial part in compiling an SL specification. A naive way is to check a list of patterns one by one. The obvious drawback of this approach is inefficiency. Tree-like automata [GS84] address the efficiency issue. The matching proceeds by making branches of different constructors and ascribing the list of patterns to those branches. This approach has the disadvantage of space explosion. The combination of tree automata with failures is the basis for the current implementations of most common functional languages [Mar94, Ler90]. The pattern-matching of the SL system follows this approach, but the support of semantic notions and the non-determinism of rewriting require extensions to the existing algorithms.

1. SL Patterns

The SL patterns include common patterns such as wildcard patterns, variable patterns, alternative patterns, type constraint patterns, alias patterns, and tuple patterns. The SL patterns are also enriched with dynamic constraint patterns and context filling patterns. The dynamic constraint (p :
dynamic name) and context filling \((p : context name) p_2\) require \(p\) to be a wildcard pattern or a variable pattern. The SL patterns are formally defined as follows:

**Patterns**
\[
P ::= _ | x | c_0 | c_1 P | P\backslash P | P \text{ as } x \\
(P : \text{type } \tau) | (P, \ldots, P) | \\
(P : \text{dynamic name}) |
\]

**Axioms**
\[
A ::= P \text{ when } E \implies E
\]

**Inference Rules**
\[
I ::= \frac{E \implies Q}{P \text{ when } E \implies E}
\]

**Dynamic Definitions**
\[
D ::= P
\]

**Context Definitions**
\[
H(x) ::= P
\]

where \(E\) denotes CAML expressions, and \(Q\) denotes restricted SL patterns which do not contain dynamic constraints and context fillings.

The SL patterns are used in the left-hand sides of axioms and of conclusion clauses of inference rules. Dynamic definitions can be considered as definitions of SL patterns, and context definitions are parametric patterns where the parameters denote the holes. Both definitions can be recursively defined. The dynamic definition, context definition, and rules in Fig. 1 can be represented in terms of SL patterns as follows:

\[
V = \text{Lam}_-
\]

\[
\beta_e : \text{App}(\text{Lam}(x, M), (v : V)) \implies M[x := v]
\]

\[
H(x) = x | \text{App}((h_1 : H)(x), \_ ) | \text{App}(v : V, (h_2 : H)(x))
\]

\[
eval : \frac{E \implies h \ t_2}{(h : H) \ t_1 \implies h \ t_2}
\]

where variables are introduced for dynamic constraint and context filling patterns.

2. Automata

An automaton consists of states. Some states are final. Matching a term consists of traversing the states until reaching a final one. States are inductively defined as follows:

**States**
\[
S ::= \text{branch} \ t, (test, S), \ldots, (test, S) | \text{accept } E | \\
S \cdot \cdot \cdot S | \text{fail} | \text{let } vars = f \ E \text{ in } S | \\
\text{if } E \text{ then } S \text{ else } S | \text{let } vars = vars \text{ in } S
\]

where \(vars\) denotes a variable or a tuple of variables.

The first four states are standard. \text{branch} \ t, (test_1, S_1), \ldots, (test_n, S_n)) is a branch-test state, where \(t\) is the term under test, and each \text{test}_i\) has the form \(c_0, c_1 x, \text{ or } \_ \_\) for otherwise. All tests are mutually disjoint. \text{accept } e\) is a final state, where \(e\) is a CAML expression representing the action after acceptance. A choice state \(S_1 \cdot \cdot \cdot S_n\) has alternatives \(S_1, \ldots, S_n\). When pattern-matching traverses this state, it non-deterministically chooses to enter an alternative. If a final state is reached, then traversing the choice state
is successful. Otherwise it backtracks to other alternatives. This semantics differs from the usual choice state whose alternatives are ordered (lexically). 

fail is a failure state. One new form, reference state let vars = f e in S₁, is added to support dynamic values and contexts. It calls a function matching the parameter as the corresponding dynamic value, context, or redex. If it succeeds, it continues in state S₁, failures in S₁ may cause backtracking to other possibilities in the function call f e.

States are also extended with conditional expressions and let variable bindings. The reason for the former extension is that the semantic rules are conditional. The latter extension is helpful for code generation. States are annotated with terms to be matched, but we made them implicit in our presentation.

3. Structures for pattern-matching

The SL compiler collects the inference rules and axioms with the same type together. The patterns of the rules form a vector which can be considered as a one-column matrix. Each rule contributes a row in the matrix. We introduce parameters bound to the terms matching the patterns, and we keep track of variable bindings in pattern-matching. There is also a state for each rule, indicating what to do when the patterns of the rule have been matched. The whole pattern-matching structure is represented as follows:

$$
\begin{bmatrix}
  t₁ & \cdots & tₙ \\
  p_{11} & \cdots & p_{1n} & s₁ \\
  \vdots & \ddots & \vdots \\
  p_{m1} & \cdots & p_{mn} & s_m \\
\end{bmatrix}
$$

The compilation of pattern-matching can be regarded as a function, C, which maps such a structure to a state.

The initialization of the pattern-matching sets the states in the structure.

- For an axiom p₁ when e_c => e_r, the corresponding state is:

  if e_c then accept e_r else fail

- For an inference rule \[ e₁ => p₁ \]

  \[ p₂ \text{ when } e_c => e₂ \]

  if e_c then let p₁ = rewrite₁ e₁ in accept e₂ else fail

where rewrite₁ is one-step rewriting function in the generated code.

4. Pattern-matching algorithm

The pattern-matching algorithm is a divide-and-conquer algorithm. It selects one column of the pattern matrix to work on according to certain criteria. Without loss of generality, we assume that the algorithm always chooses the first column. The algorithm repeats the following steps until the pattern matrix is empty:
(a) Preprocessing
This step canonicalizes the patterns in the first column. It removes the type constraints since the type information is not useful at the current stage. It binds variables in alias patterns to the corresponding parameters. It turns each alternative pattern into several rows having one alternative each. Formally, the preprocessing repeats the following simplifications.

\[
\begin{align*}
C \left( \begin{array}{cccc}
    t_1 & \cdots \\
    \vdots & \ddots \\
    \vdots & \vdots & \ddots \\
    \end{array} \right) \rightarrow C \left( \begin{array}{c}
    t_1 \\
    \vdots \\
    \vdots \\
\end{array} \right), \\
C \left( \begin{array}{cccc}
    p_{i1} \text{: } \text{type } \tau \cdots s_i \\
    \vdots & \ddots \\
    \vdots & \vdots & \ddots \\
    \end{array} \right) \rightarrow C \left( \begin{array}{c}
    p_{i1} \cdots s_i \\
    \vdots \\
    \vdots \\
\end{array} \right), \\
C \left( \begin{array}{cccc}
    t_1 & \cdots \\
    \vdots & \ddots \\
    \vdots & \vdots & \ddots \\
    \end{array} \right) \rightarrow C \left( \begin{array}{c}
    t_1 \\
    \vdots \\
    \vdots \\
\end{array} \right). \\
\end{align*}
\]

(b) Splitting the matrix
The algorithm splits the matrix horizontally, so that in each submatrix, all first-column patterns are in one of the following groups:
- variable group: wildcard patterns or variable patterns,
- tuple group: tuple patterns,
- constructor group: constructor patterns,
- dynamic constraint group: dynamic constraint patterns on the same dynamic definition,
- context filling group: context filling patterns on the same context definition.

The result of the splitting is a choice state.

\[
\begin{align*}
C \left( \begin{array}{cccc}
    t_1 & \cdots \\
    p_{i1} \cdots s_1 \\
    \vdots & \ddots \\
    p_{m1} \cdots s_m \\
    \end{array} \right) \rightarrow C \left( \begin{array}{cccc}
    t_1 & \cdots \\
    p_{i1} \cdots s_1 \\
    \vdots & \ddots \\
    p_{k_1} \cdots s_{k_1} \\
    \end{array} \right) \mid \cdots \mid C \left( \begin{array}{cccc}
    t_1 & \cdots \\
    p_{i(k+1)} \cdots s_{k+1} \\
    \vdots & \ddots \\
    p_{m} \cdots s_{m} \\
    \end{array} \right)
\end{align*}
\]

(c) Analyzing different cases
For each submatrix, the compilation function \( C \) is inductively defined as follows:
i. Base case:
When the pattern matrix is empty, pattern-matching is vacuously successful. The function $C$ creates a choice state.

$$C \left( \begin{array}{c} s_1 \\ \vdots \\ s_m \end{array} \right) \longrightarrow s_1 | \cdots | s_m$$

ii. Variable group:
Wildcard and variable patterns always match successfully. The function $C$ removes the column of patterns. For variable patterns, bindings are added for further access to the variables.

$$C \left( \begin{array}{c} t_1 \cdots \\ \vdots \\ \vdots \\ - \cdots s_i \\ \vdots \\ \vdots \\ x \cdots s_j \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{array} \right) \longrightarrow C \left( \begin{array}{c} t_2 \cdots \\ \vdots \\ \vdots \\ p_{ij} \cdots s_i \\ \vdots \\ \vdots \\ p_{ij} \cdots \text{let } x = t_1 \text{ in } s_j \\ \vdots \\ \vdots \\ \vdots \end{array} \right)$$

iii. Tuple group:
The function $C$ treats each component of the tuple as an individual pattern. It replaces the first column of patterns with columns of the component patterns, and introduce parameters for the components.

$$C \left( \begin{array}{c} t_1 \cdots \\ (p_{111}, \cdots, p_{11k}) \cdots s_1 \\ \vdots \\ (p_{m11}, \cdots, p_{m1k}) \cdots s_m \end{array} \right) \longrightarrow \text{let } (t_{11}, \cdots, t_{1k}) = t_1 \text{ in } C \left( \begin{array}{c} t_{11} \cdots t_{1k} \\ p_{111} \cdots p_{11k} \cdots s_1 \\ \vdots \\ p_{m11} \cdots p_{m1k} \cdots s_m \end{array} \right)$$

iv. Constructor group:
The function $C$ collects the rows which have the same constructor in the first column into a group of new pattern-matching structures, and it creates a branch-test state. The tests of the state are distinguished by having different constructors as roots. The corresponding actions for the tests are the results of compiling the new structures. In the new structures, the first column is removed for the constructors of zero arity, or it is replaced by the column of argument patterns for the constructors of non-zero arity.
v. Dynamic constraint group:
The function $C$ creates a reference state. The reference will initiate pattern-matching for the dynamic definition $D$ with the value of $t_1$. Its result is bound to a new parameter. The state in the `let` body is the result of compiling a structure consisting of the patterns without the constraint and the rest of the patterns.

$$
C \left( \begin{array}{c} t_1 \\
\vdots \\
c_0 \\
\vdots \\
c_1 \ p'_1 \cdots \ s_j \\
\vdots
\end{array} \right) \longrightarrow \text{branch} \quad \begin{array}{c}
(c_0, \ C \left( \begin{array}{c} t_2 \\
\vdots \\
p_{12} \cdots s_{12} \\
\vdots \\
(p_{22} \cdots s_{22})
\end{array} \right)) \\
t_1, \\
\vdots \\
\text{let} \ c_1 \ t'_1 = t_1 \ \text{in} \\
\begin{array}{c}
(t'_1 \\
\vdots \\
p_{11} \cdots s_1 \\
\vdots \\
p_{m1} \cdots s_m)
\end{array}
\end{array}
$$

Pattern-matching for dynamic definitions will be presented later.

vi. Context filling group:
Similar to the dynamic constraint group, the function $C$ creates a reference state. The reference will initiate pattern-matching for the context definition $H$ with the value of $t_1$. Its result is bound to a pair of parameters which represent the context and the corresponding hole occurring in $t_1$. The state of the `let` body is the result of compiling a structure consisting of the context patterns, the hole patterns, as well as the rest of the patterns.

$$
C \left( \begin{array}{c} t_1 \\
\vdots \\
(p'_{11} : H) p'_{11} \cdots s_1 \\
\vdots \\
(p'_{m1} : H) p'_{m1} \cdots s_m
\end{array} \right) \longrightarrow \text{let} \ (t'_1, t''_1) = \text{match} H \ t_1 \ \text{in} \\
\begin{array}{c}
(t'_1 \ t''_1 \\
\vdots \\
p_{11} \ p_{11} \cdots s_1 \\
\vdots \\
p'_{m1} \ p'_{m1} \cdots s_m)
\end{array}
$$

Pattern matching for context definitions will be presented next.
5. Matching dynamic definitions and context definitions

Pattern-matching for dynamic definitions and context definitions uses the same form of structures and uses the same algorithm. Each definition corresponds to a structure which has only one pattern, one parameter and one state. The pattern comes from the definition. Assume $t$ is the term to be matched, the state is initialized as follows:

- For a dynamic definition, the state is $\text{accept } t$.
- For a context definition, the state is $\text{accept } (\lambda x. \text{body}, x)$, where $x$ is the parameter for the context and the $\text{body}$ is a placeholder. When the base case in the algorithm is encountered, the SL compiler sets the contents of $\text{body}$ by reconstructing the term $t$ with the variable $x$. Each $\text{body}$ may have different content if the final state is copied. The reconstruction retrieves the bindings along the path from the start state to the final state.

In other words, matching a context definition returns a pair with a constructing function and a term filling the hole. Applying the function to the term results in the term $t$.

The state starting pattern-matching for a definition can be referred to by the name of the definition. For example, the dynamic definition and the context definition in Fig. 1 are associated with the following states:

$$\text{match}_D t = \text{branch}(t, (\text{Lam } t', \text{accept}(t)))$$

$$\text{match}_H t = \text{let } x = t \text{ in accept}(\lambda x. \text{body}_1, x) \mid \text{branch}(t, \text{App } t', \text{let } (t_1, t_2) = t' \text{ in} \text{match}_H t_1 \text{ in} \text{let } h_1 = t_1' \text{ in} \text{let } x = t_1' \text{ in accept}(\lambda x. \text{body}_2, x) \mid \text{match}_D t_1 \text{ in let } v = t_1' \text{ in} \text{let } (t_2', t_2'') = \text{match}_H t_2 \text{ in} \text{let } h_2 = t_2' \text{ in} \text{let } x = t_2'' \text{ in accept}(\lambda x. \text{body}_3, x)))$$

where $\text{body}_1$, $\text{body}_2$ and $\text{body}_3$ are as follows:

$$\text{body}_1: \text{let } t = x \text{ in } t$$

$$\text{body}_2: \text{let } t_1'' = x \text{ in let } t_1' = h_1 \text{ in let } t_1 = t_1' t_1'' \text{ in} \text{let } t' = (t_1, t_2) \text{ in let } t = \text{App } t' \text{ in } t$$

$$\text{body}_3: \text{let } t_2'' = x \text{ in let } t_2' = h_2 \text{ in let } t_2 = t_2' t_2'' \text{ in} \text{let } t_1' = v \text{ in let } t_1 = t_1' \text{ in let } t' = (t_1, t_2) \text{ in} \text{let } t = \text{App } t' \text{ in } t$$

3.3 Transforming automata into Caml code

The transformation of states in the pattern-matching automata into CAML code is described in Fig. 2. Each state corresponds to a function with implicit parameters for the terms to be matched and a continuation expressing the remaining
pattern-matching work. The initial continuation for rewriting is the identity function. State functions may raise an exception when matching of the state fails. The branch-test state corresponds to the \texttt{match \cdots with \cdots} construct in Caml. If the branch tests do not cover all constructors of the same type, a default test associated with a failure state is added. For a final state, the success continuation is consumed by applying it to the expression. For a choice state, a random alternative is tried. Failure exceptions may be caught and then other alternatives can be tried. The failure state just raises a failure exception. For a reference state, it calls the matching function with the continuation accepting the result, then it continues with the state in the body of the \texttt{let}. The conditional expressions and variable bindings in states are considered atomic with respect to continuation passing and exception handling. Transforming a conditional state is thus transforming both branch states, and transforming a \texttt{let} state is transforming the \texttt{in} part.

\begin{verbatim}
[ branch (t, (c_0, S_0), \ldots, (c_1, S_1)) ] k =
  match t with
  c_0 -> [ S_0 ] k
  \vdots
  c_1 \to [ S_1 ] k
  _ -> raise failure
[ accept e ] k = k(e)
[S_1|\ldots|S_m] k =
  try [ S_i ] k
  with failure ->
  [ S_1|\ldots|S_{i-1} | S_{i+1}|\ldots|S_m ] k
[ fail ] k = raise failure
[ let vars = f e in S ] k =
  f e (\lambda t. let vars = t in [ S ] k)
[ if e_c then S_1 else S_2 ] k =
  if e_c then [ S_1 ] k else [ S_2 ] k
[ let vars_1 = vars_2 in S ] k =
  let vars_1 = vars_2 in [ S ] k
\end{verbatim}

\textbf{Fig. 2.} Generating Caml code

4 Summary and Discussion

The SL system uses the first-order types of functional languages for specifying abstract syntax. We have not followed the approach of \textit{higher-order abstract syntax (HOAS)} [PE88]. The HOAS representation would allow the variables of the object language to be represented as meta-variables, and hence alleviates
the need for explicitly reasoning about variable renaming and substitution of variables. However, higher-order abstract syntax interacts poorly with the inductive reasoning techniques needed to reason about the properties of semantic specifications [DPS97].

The SL system uses conditional rewriting rules for specifying semantic rules. In other words, the system employs rewriting semantics (a.k.a. reduction semantics). Some systems [HM92, CDD+88] express rules in natural semantics [CDD+85]. Natural semantics is well adapted to describing static behaviors such as typing. For transformational behaviors, such as dynamic semantics, rewriting semantics proves to be more modular [WF94], and should therefore be more tractable when it comes to expressing full-size programming languages. Another advantage of rewriting semantics is that it allows one to observe intermediate states of reductions, so that it is more suitable for non-terminating object-systems.

ELAN [ELA] is a system also based on first-order rewriting semantics. It has more general application areas than the SL system. It supports many-to-one associative-commutative (AC) patterns and provides an efficient algorithm [MK98]. Its strategy language gives flexible control over non-deterministic reductions. The Stratego [VB98] system has more generic strategy specifications. There are also general-purpose tools that can be used for manipulating formal semantics, such as Coq [Coq], Isabelle [Isa] and Twelf [Twe]. Compared to those systems, the novelty of the SL system is that it directly supports the specification of the semantic notions such as dynamic values and evaluation contexts, and it automatically generates executable interpreters. Associative-commutative patterns can be represented by contexts, but the current SL system does not optimize matching for efficiency.

Fahndrich and Boyland [FB97] investigate abstract patterns which are even more general than contexts in the SL system. They build automata for patterns and check the properties of the patterns such as exhaustiveness and non-overlapping, but they did not provide a computational view of pattern-matching.

The current prototype of the SL system constitutes a first step towards designing a specification language for syntactic theories and implementing a system generating interpreters from specifications. It also provides an extension to pattern-matching techniques. A number of interesting examples have been tested in the prototype system. Follow-up work is underway to allow more expressive specifications, to optimize the compilation of the SL language, and to automatically prove properties of syntactic theories.

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