Precision of exogenous post-stratification in small area estimation based on a continuous national forest inventory

| Journal:          | Canadian Journal of Forest Research |
|-------------------|-------------------------------------|
| Manuscript ID     | cjfr-2019-0139.R3                   |
| Manuscript Type:  | Article                             |
| Date Submitted by the Author: | 26-Nov-2019 |
| Complete List of Authors: | Haakana, Helena; Natural Resources Institute Finland, Bioeconomy and environment Heikkinen, Juha; Natural Resources Institute Finland, Economics and society Katila, Matti; Natural Resources Institute Finland Kangas, Annika; Natural Resources Institute Finland (Luke), Economics and society |
| Keyword:          | forest inventory, forest resources map, k-nearest neighbor, post-stratification, satellite images |
| Is the invited manuscript for consideration in a Special Issue?: | Not applicable (regular submission) |

https://mc06.manuscriptcentral.com/cjfr-pubs
Precision of exogenous post-stratification in small area estimation based on a continuous national forest inventory

Helena Haakana¹*, Juha Heikkinen¹, Matti Katila¹ and Annika Kangas²

¹ Natural Resources Institute Finland (Luke), PO Box 2, FI-00791 Helsinki, Finland
² Natural Resources Institute Finland (Luke), PO Box 68, FI-80101 Joensuu, Finland

* Corresponding author:
Email: helena.haakana@luke.fi
Tel: +358 29 532 2158, +358 50 391 2158
ORCID: 0000-0002-4830-800X
Abstract

National forest inventories (NFIs) are designed to provide accurate information on forest resources at the national and regional levels, but there is also a demand for such information at smaller spatial scales. Auxiliary data such as satellite imagery have been used to facilitate small area estimation. The commonly used method, k-nearest neighbour (k-NN), provides a model-based estimator for small areas, but a design-unbiased estimator for the mean square error is not available. Post-stratification (PS) is an alternative approach to using auxiliary information that allows for design-based variance estimation. In a case study using real inventory data of the Finnish NFI, we applied this method to the municipality level to explore the lower limit to the area for which the key forest parameters, forest area and growing stock volumes, can be estimated with sufficient precision. For post-stratification, we employed exogenous forest resources maps based on the previous NFI round. In the municipalities of the two study provinces, the relative standard errors of total volume estimates ranged from 2.3% to 26.9%. They were smaller than 10% for municipalities with an area of 390 km² or larger, corresponding to approximately 100 or more sample plots on forest land. We also demonstrated the usefulness of design-unbiased variance estimation in showing discrepancies between design-based PS and model-based k-NN estimates.

Keywords: forest inventory, forest resources map, k-nearest neighbour, post-stratification, satellite images
Introduction

In recent years, growing interest in forest ecosystem services, together with a boost in the bio-
economy, has increased the demand for accurate information on forest resources at smaller
spatial scales (Næsset et al. 2011, Goerndt et al. 2013, Hansen and Malmaeus 2016, European
Commission 2018, Kangas et al. 2018). Demand for forest statistics for small areas and
technological advances in the 1980s and 1990s led to the development of inventory applications
using satellite imagery and sample plot data of national forest inventories (NFIs) in small area
estimation. The development was primarily motivated by forest industries requiring more up-to-
date and localised information on forest resources for intensifying forest management, procuring
timber and planning new investments (Tomppo 1991, Tomppo et al. 1998). Forests also play an
important role in municipal land use planning by providing a means of livelihood, recreation and
well-being for residents, and in preserving natural environments. Forest resources data at the
municipality level enable quantitative impact assessments of land-use plans on forestry
(Kärkkäinen et al. 2017).

NFIs are designed for producing forest statistics for large areas (countries and counties or
provinces). As a result, sample sizes in small areas (or domains), such as municipalities, are
often too small for sufficiently precise estimation from field data only, that is, for direct domain
estimation (Rao 2003, p. 1). Model-based approaches relying on auxiliary data and a regression
model fitted to the training data from a larger region can overcome this problem by borrowing
strength from outside the area of interest, and therefore, they are commonly used for small area
estimation (Lappi 2001, Breidenbach and Astrup 2012, Breidenbach et al. 2016, McRoberts et al.
Auxiliary data used in forest inventory are typically remotely sensed data. The commonly applied k-nearest neighbour (k-NN) estimation method utilizes such data through a non-parametric regression model. The predicted value of a forest variable for a given target pixel is a weighted average of the variable’s measured values at the k sample plots with the most similar auxiliary data (Kilkki and Päivinen 1987, Tomppo 1996). Some of the k-nearest neighbours may be outside the target area, which implies that the design-unbiasedness of the resulting small-area estimator cannot be guaranteed and design-based assessment of the variance is impossible. Although Baffetta et al. (2009) presented a design-based, empirical-difference variance estimator for a k-NN model, and alternatives using model-based inference, bootstrap methods and jackknife methods have been presented by Magnussen et al. (2009), McRoberts et al. (2011) and Magnussen (2013), the usefulness of these variance estimators for a small-area estimation context is not known.

Auxiliary data can also be utilized in the design-based, model-assisted (MA) framework, which allows for design-unbiased variance estimation (Särndal et al. 1992). In forest inventory context, the most commonly used form of the MA approach has been post-stratification (PS), which leads to the regression estimator generated by a group-mean model (Särndal et al. 1992, pp. 264–269). PS using remotely sensed auxiliary information has been shown to improve the precision at the national and regional levels compared with estimation based on field sample plots only.
(McRoberts et al. 2002, Nilsson et al. 2003, 2005, McRoberts et al. 2006, Pulkkinen et al. 2018, Haakana et al. 2019). Accordingly, it is applied in operational NFIs in several countries, for example, France, Netherland, Slovakia, Sweden, Switzerland and the United States (Barrett et al. 2016, Pulkkinen et al. 2018).

The potential of PS in small area estimation has received less attention than the gain in efficiency in large-area inventories (McRoberts 2010, Haakana et al. 2019). In Sweden, forest statistics for small areas (sub-county level) are estimated using PS; full-coverage forest maps, estimated with k-NN, or other map products are used for stratification (Fridman et al. 2014). However, analysis of the precision of PS estimators in small areas is still lacking. Especially, information on the lower limit to the number of sample plots required for design-unbiased estimation in a typical NFI context would be widely useful for practitioners.

A necessary requirement for design-unbiased variance estimation in PS is that the assisting model is derived independently from the sample to be stratified (exogenous PS) (Särndal et al. 1992, p. 265); otherwise, the use of an internal model (endogenous PS) may lead to a serious underestimation of variances (Magnussen et al. 2015, Kangas et al. 2016). In some conditions, the efficiencies of exogenous and endogenous PS can be essentially equal (Breidt and Opsomer 2008, Dahlke et al. 2013, Tipton et al. 2013), but in more complex, real inventory settings, an endogenous PS underestimates the true variance (Tipton et al. 2013, Magnussen et al. 2015, Kangas et al. 2016).
In many NFIs, continuous field inventories and remotely sensed data acquired on a regular basis provide external information; data from a previous inventory can be used in fitting the model (Pulkkinen et al. 2018, Haakana et al. 2019). For example in Finland, forest resources maps based on NFI sample plot data, satellite images and numerical map data, extending over the whole country, are produced by the multi-source NFI (MS-NFI) method every two years (Tomppo et al. 2008, Mäkisara et al. 2016, 2019). The NFI field inventory in turn is a continuous inventory where one round is completed every five years.

For the smallest areas containing too few sample plots for reasonably precise estimation by any design-based method, a model-based approach is the only option. Even in those cases, design-unbiased estimators can be useful. For example, we can compute both the k-NN estimates and PS estimates with design-based confidence intervals (CIs) for the same areas of interest. The properties of the areas where the k-NN estimate is outside the CI of the PS estimator could then be examined more closely for identifying potential problems in k-NN estimation. To our knowledge, any such analysis has not been reported.

The primary objective of our study was to evaluate the precision of post-stratification based on an external model for small area estimation in the NFI context. Using the operational NFI in Finland as an example, we estimated the precision of PS estimates for growing stock volume and volumes by tree species, as well as the area of productive forest land and poorly productive forest land at the municipality level. The approach was direct domain estimation, that is, the estimates were based only on the sample plots within the small area (Rao 2003, p. 1).
additional objective was to demonstrate the use of PS estimates as a tool to identify potential bias of the k-NN estimator at small spatial scales.

Materials and methods

Study areas and field data

Municipalities in Finland are local-level, self-governing administrative units providing public services, and their areas vary depending on the population. The largest municipalities (maximum size about 17,000 km$^2$) are in North Finland and the smallest (about 10 km$^2$) are cities in southern Finland. The number of municipalities is currently 311.

We selected two distinct provinces, Kainuu in North-East Finland and Pirkanmaa in western South Finland as study areas (Fig. 1). The province of Kainuu consists of eight municipalities, the areas of which ranged from 898 km$^2$ to 5,858 km$^2$ (Table 1). In Pirkanmaa, the number of municipalities is 22 and their areas ranged from 104 km$^2$ to 1,532 km$^2$. The digital boundaries and land areas of the municipalities as of January 1, 2014 were employed in this study. Other land use including agriculture, infrastructure and waterbodies covered 15% of the total area in Kainuu and 34% in Pirkanmaa. In both provinces, forests were dominated by Scots pine (Pinus sylvestris L.) and Norway spruce (Picea abies (L.) Karst.) with a mixture of birch (Betula spp.) and other deciduous tree species, primarily aspen (Populus tremula L.) and alder (Alnus spp.). Pine accounted for the largest proportion of the total volume in Kainuu; spruce in Pirkanmaa, where there was also more variation in the distribution of tree species among municipalities (Table 1).
For estimation, we used the sample plot data from the eleventh NFI (NFI11), measured in 2009–2013 (Table 2). The sampling design of NFI11 was systematic cluster sampling, but the designs were slightly different in the Kainuu and Pirkanmaa regions (Fig. 2). The total number of sample plots was 5,467 in Kainuu and 4,614 in Pirkanmaa. For the sample plots, both forest stand-level and tree-level characteristics were measured. Trees belonging to a sample plot were selected by restricted angle count sampling. In North Finland, including Kainuu, the basal area factor (relascope factor) was 1.5 and the maximum radius of the plot was 12.45 m. In Pirkanmaa (South Finland), a basal area factor of 2.0 and a maximum radius of 12.52 m were applied. Every seventh tally tree was measured as a sample tree in more detail. Sample tree volumes were estimated using volume models (Laasasenaho 1982), and volumes for tally trees were estimated with the help of the most similar sample trees (Korhonen et al. 2017).

Auxiliary information and post-stratification

Auxiliary information for post-stratification was derived from the MS-NFI volume map based on Landsat satellite imagery from 2007, the digital map data and the sample plot data of the previous NFI (NFI10) carried out in 2005–2008 (Tomppo et al. 2012) (Table 2). To cover cloudy areas in the Landsat images, IRS and Spot 4/5 images from 2005 and 2006 were used (Tomppo et al. 2008). The pixel size of the satellite images was 20 m × 20 m, and the mean volume of the growing stock (m³/ha) for each image pixel on forestry land was estimated using the k-NN method (Tomppo et al. 2008). For the estimation, a forestry land mask excluding other land use classes, such as agricultural land, roads, built-up areas and waterbodies, was derived from the digital map data from the National Land Survey of Finland. For post-stratification, the MS-NFI volume map, including also other land use classes was compiled to cover the whole country.
MS-NFI volume maps are produced every two years, and the MS-NFI-2007 volume map was selected for PS because it was based on the sample plot data collected before the NFI11 field data used in the estimation.

Four volume strata plus three strata for other land uses were formed. The boundaries of the volume strata were determined separately for the two study areas using the method of Dalenius and Hodges (1959). In this approach, boundaries $y_i, i=0,\ldots,4$, are determined so that $y_0$ is the minimum and $y_4$ the maximum of predicted volumes $\hat{y}$, and the values $\int_{y_0}^{y_i} \int f(t) \, dt, i=1,\ldots,3$, divide the interval $[0, \int_{y_0}^{y_4} \int f(t) \, dt]$ into four equal sub-intervals, where $f$ is the probability density of the predictions (Dalenius and Hodges 1959, Cochran 1977, p.127). In Kainuu, the stratum boundaries, 34 m$^3$/ha, 73 m$^3$/ha and 122 m$^3$/ha, were based on the volume map of North Finland, excluding the three northernmost municipalities, covered largely by open fells (Fig. 1). North Finland comprises the three northernmost provinces (Fig. 1). In Pirkanmaa, post-stratification was applied based on the volume map of South Finland, excluding the Åland islands in South-West Finland (Fig. 1). The stratum boundaries were 68 m$^3$/ha, 130 m$^3$/ha and 202 m$^3$/ha. PS of a smaller area could result in more precise volume estimates for municipalities, but applying the same PS for the whole of South Finland and of North Finland is more operational (Haakana et al. 2019).

The selection of four volume strata was based on the previous use of satellite-derived data for PS (McRoberts et al. 2002, Nilsson et al. 2005, McRoberts 2010, Magnussen et al. 2015). Three strata for other land use classes (agricultural land, built-up area and waterbodies) were used because the forestry land mask was not reliable enough (Katila et al. 2000). Hence, all NFI

https://mc06.manuscriptcentral.com/cjfr-pubs
sample plots assessed as forest land in the field were included in the estimation, though some of them were classified as other land use according to the map data. Cochran (1977, pp. 132–134) recommended four to six strata, provided that the sample size is reasonably large (around 20) in every stratum. Westfall et al. (2011) suggested that the smallest within-stratum sample size should be at least 10 to obtain approximately unbiased variance estimator. The selection of four volume strata also supported having enough sample plots per stratum. If there was an empty stratum, as in one of the municipalities in Pirkanmaa (no sample plots in waterbodies), two strata of other land use (agriculture and waterbodies) were combined before estimation.

**Estimation of forest characteristics and sampling variances**

For forest areas and total volumes, post-stratification estimators (Cochran 1977, Haakana et al. 2019) were used. Forest areas and volumes and their variances were first estimated for each stratum in each municipality and then aggregated to the municipalities. Because the municipality areas from the map data differed slightly from the official area statistics provided by the National Land Survey of Finland, the sum of stratum areas within a municipality was calibrated to the official municipality area, the strata on land with the official land area and the stratum of waterbodies separately with the area of waterbodies within the municipality in question.

Stratum-specific sampling variances were estimated by using local quadratic forms of cluster-level residuals (Matérn 1960, p. 110, Tomppo et al. 2011). In the variance estimation, the correlation between the estimates from different strata was taken into account, due to the fact that sample plots within one cluster could belong to different strata (Appendix A). Sampling uncertainty was quantified through relative standard error (SE): $\sqrt{\text{Var}(\hat{Y})}/\hat{Y}$ where $\hat{Y}$ is a post-
stratified estimator of forest area (km$^2$), total volume (m$^3$) and volumes by tree species groups (m$^3$).

The area results are presented separately for productive and poorly productive forests. In Finnish NFI, forested land is divided into productive forest land (growth at least 1 m$^3$/ha/a) and poorly productive forest land (growth at least 0.1 m$^3$/ha/a). These categories are of importance because only their growing stock is included in the NFI volume estimates. In addition, separation of productive forest land is important because many forest statistics are calculated only for productive forests, and generally only productive forests are included in the analyses of future forest production possibilities.

MS-NFI estimates for comparison

PS estimates were compared to the MS-NFI results corresponding to the years 2011 (MS-NFI-2011) and 2013 (MS-NFI-2013) (Tomppo et al. 2014, Mäkisara et al. 2016). The time point of MS-NFI-2011 results matched best with the average measurement year of NFI11 field plots (the time point referred to by the PS estimates). The MS-NFI-2011 results for the municipalities in Kainuu and Pirkanmaa were based on Landsat 5 satellite images from 2010–2011 and NFI sample plots measured in 2007–2011 (Table 2) and the k-NN estimation where weights for image features and coarse scale forest variables were computed by means of a genetic algorithm (Tomppo et al. 2008, Tomppo et al. 2014). The sample plot data measured before 2011 was computationally updated to the reference year 2011 (Tomppo et al. 2014). In MS-NFI, the updating is controlled by the field data and image data (Tomppo et al. 2014). Large changes, e.g., regeneration felling, are detected by comparing the field plot data and the image data, and the
field plot data is changed accordingly. The key plot and stand level variables are updated using
growth models.

To give a broader view of the compatibility of MS-NFI and PS estimates, MS-NFI-2013 results
were included in the comparisons, although they refer to the situation two years later than do the
PS results (Table 2) (Mäkisara et al. 2016). The MS-NFI method, including data preparation,
parameter selection for k-NN and correction of map errors, is reported in detail in Tomppo et al.
(2008) and in MS-NFI working reports (Tomppo et al. 2012, 2014, Mäkisara et al. 2016).

As an operational validation procedure in MS-NFI, aggregates of municipality-level estimates
are compared with the field inventory estimates and their SEs in the same area (Katila et al.
2000, Katila and Tomppo 2002). SEs based on the field inventory are estimated for groups of
municipalities, generally for areas larger than 2,000 km² where the relative SE of area estimates
is not greater than 5% (Tomppo et al. 1998, Katila and Tomppo 2002, Mäkisara et al. 2016). The
criteria in selecting parameters for MS-NFI is that the estimates for groups of municipalities
should be within ±2 × SE of the field estimates (Katila and Tomppo 2002). The same measure
(±2 × SE) was used for approximating the CI of PS estimates in the comparison of PS and MS-
NFI estimates at the municipality level in this study.

Results

Post-stratified estimates

In estimating the area of productive forest, the relative SEs were small (1.5%–3.5%) for all
municipalities in Kainuu, but clearly larger (7.8%–30.4%) for areas of poorly productive forest
due to its small share (6.6% on average) of the total land area (Table 3, Fig. 3). In Pirkanmaa, the
relative SE of the productive forest area estimates varied from 1.9% to 21.4% (Table 3, Fig. 3).
In Pirkanmaa municipalities, the average share of poorly productive forest area was only 1.3%,
and its area could not be estimated reliably. Only for two large municipalities in northern
Pirkanmaa with the largest areas of poorly productive forest land, the relative SE of the area
estimate was slightly less than 30%.

The relative SEs of total volume estimates ranged from 2.3% to 5.2% for the municipalities in
Kainuu (Table 3, Fig. 3). The relative SEs were also moderate for the volumes by tree species,
with averages of 5.6% for pine, 12.5% for spruce and 10.0% for birch. For the volume of other
deciduous trees, the precision was poor, with an average SE of 29.1%. The municipalities in
Pirkanmaa were on average smaller in size than in Kainuu, and this was reflected in the
estimation results. In Pirkanmaa, the range of relative SEs of total volume estimates was 3.8%–
26.9%, and the average SEs of volume estimates by tree species were 19.9%, 16.1%, 20.2% and
49.2%, respectively (Table 3, Fig. 3). As in the case of the volume of other deciduous trees, very
large SEs of greater than 50% were also estimated for pine and birch volumes in some
municipalities.

The relative SEs of area and volume estimates obviously increased when the municipality area,
and consequently, the number of sample plots in the area, decreased. In the case of the total
forest area, the SE was less than 5% for all municipalities in Kainuu (the smallest being 898 km²
in size) and for most of the municipalities in Pirkanmaa (Fig. 4). The SE was greater than 10%
only for the smallest municipality with a total area of 104 km² and for another small municipality
(Lempäälä) with a large portion of land use other than forestry. Total volume could be estimated with a relative SE smaller than 10% for municipalities with an area of 390 km$^2$ or larger (Fig. 4). The estimates of pine, spruce and birch volumes were less precise, but SEs were mostly less than 15% in municipalities larger than 390 km$^2$ (Fig. 5).

Comparison of post-stratification and MS-NFI estimates

The MS-NFI-2011 estimates of productive forest and poorly productive forest areas, and hence, the estimates of total forest area, were compatible with the PS estimates in all municipalities in Kainuu and in most municipalities in Pirkanmaa. For one municipality in Pirkanmaa (Ruovesi), the MS-NFI 2011 estimate of productive forest area was smaller than the PS estimate by more than 2 × SE (Fig. 6). In Pirkanmaa, a comparison of the estimates of poorly productive forest area was not meaningful because the area could not be estimated reliably with PS.

The MS-NFI-2011 estimates of the total volume were within ±2 × SE of the PS estimates for all municipalities except one in Kainuu (Hyrynsalmi) (Fig. 7), and for all municipalities in Pirkanmaa. For the volumes by tree species, in general the MS-NFI-2011 estimates differed more from the PS estimates; in most cases, MS-NFI appeared to overestimate the pine volume and underestimate the volumes of other tree species relative to the PS estimates. In Kainuu, the largest differences (more than ±2 × SE) were in the two municipalities with the largest proportion of spruce (Ristijärvi and Sotkamo) and in the one with the largest proportion of pine (Kuhmo) (Fig. 8). Similar results were seen in Pirkanmaa; the MS-NFI-2011 estimate of pine volume was more than ±2 × SE larger than the PS estimate in five municipalities with a relatively small proportion of pine (Fig. 9), and the MS-NFI-2011 estimate of spruce volume was
more than ±2 × SE smaller than the PS estimate in one municipality with a large proportion of
spruce volume (Hämeenkyrö). The MS-NFI-2011 volume estimates for birch were within
±2 × SE of the PS estimates both in Kainuu and Pirkanmaa. In the case of volume of other
deciduous trees, the comparison was not meaningful because of the low precision of PS
estimates. In the cases of significant differences between PS and MS-NFI-2011 estimates, MS-
NFI-2013 estimates deviated from PS estimates to the same direction as MS-NFI-2011 estimates,
with the exception of the MS-NFI-2013 estimate of pine volume in Kuhmo (Fig. 8).

Discussion

This study explored the precision of post-stratification based on external auxiliary data in small
area estimation. Our case study represented a typical setting of a continuous NFI with a five-year
rotation, with a sampling intensity of one sample plot per 366 ha in the study provinces on
average, and Landsat satellite imagery and digital map data available for the regular production
of thematic maps. Using the volume map based on the previous NFI round as auxiliary data for
PS, forest area and total volume could be estimated precisely (with a maximum SE of 5%) for all
municipalities (with a minimum size of 898 km²) in Kainuu and for the largest municipalities in
Pirkanmaa. If the same precision of 5% SE for the total volume estimate was required in
Pirkanmaa, the minimum municipality area would need to be approximately 960 km². If a SE of
10% was allowed, total volume could be estimated for municipalities with areas of
approximately 390 km². Consequently, PS would be adequate for estimating results for 79% of
the municipalities, which would cover 96% of the total area in continental Finland. In the
smallest municipalities, model-based estimates are still the only option. Yet, PS’s ability to
provide estimates of standard error is helpful because the error estimates enable the informed use of the inventory results, as the precision requirements of different data users may vary.

In estimating volume by tree species, in general the SEs were larger, and were influenced by not only the forest area but also by the composition of tree species in the municipality. In Kainuu municipalities, forests were dominated by pine, and the precision of pine volume estimates was also better than that of other tree species in all municipalities. In Pirkanmaa, there was more variation between the municipalities; forests were mostly dominated by spruce, and only in the five northernmost municipalities did pine account for the largest share of the total volume. Again, the relative SE of the dominant tree species was smaller than that of other species. The SE level of 15%–20% in estimating tree species volumes other than other deciduous trees was obtained for municipalities larger than 390 km².

The minimum areas for which forest variables could be precisely estimated with PS are not directly comparable because the NFI sampling designs differed in Kainuu and Pirkanmaa provinces. One NFI11 sample plot represented approximately 415 ha in Kainuu and 317 ha in Pirkanmaa. Moreover, in Pirkanmaa the proportion of other land use area was larger, and the precision of area and volume estimates was influenced by the forest land area rather than the total municipality area. In Pirkanmaa, the approximate limits of the municipality areas, 390 km² and 960 km², for which the total volume could be estimated with the SE of 10% and 5%, respectively, corresponded to 100 and 182 sample plots on forest land. In Kainuu, the number of sample plots on forest land in the smallest municipality (898 km²) with the SE of 5% was 177.
It is obvious, that NFI sampling was not dense enough for PS estimation in the smallest municipalities. Following Cochran’s (1977) recommendation of 20 sample units per stratum and using 7 strata, a minimum of 140 sample plots per municipality would be needed. For example, in the smallest municipality in Pirkanmaa (104 km²), the total sample size was only 44, and the numbers of sample plots in the four volume strata were 6, 2, 7 and 4, and in the other land use strata (agricultural land, built-up area and waterbodies) they were 6, 1 and 18. According to Westfall et al. (2011), the within-stratum sample size should be at least 10 to obtain approximately unbiased estimator of variance. For smaller sample sizes, estimates of standard errors were found to be too small (Westfall et al. 2011). A smaller sample size could be compensated for by using fewer strata, or by combining two adjacent strata together, as was done in the case of an empty stratum.

Since the 1990s, municipal forest statistics in Finland have been produced by the MS-NFI method (Tomppo et al. 1998). While MS-NFI estimates have previously been validated by comparing them to NFI estimates calculated with field plots over groups of municipalities (Katila et al. 2000, Mäkisara et al. 2019), PS enables municipality-specific validation in the operational MS-NFI. MS-NFI estimates of pine and spruce volumes were found to differ significantly from the PS estimates in some municipalities, both in Kainuu and Pirkanmaa. This was related to the dominancy of tree species and possible overestimation of pine volume. In MS-NFI, sample plots outside the target areas are used. In Kainuu, forests are mostly pine-dominated which can therefore lead to underestimation of spruce in spruce-dominated municipalities. This indication of bias in the k-NN estimators of volume by tree species was discovered early on, and a method utilizing coarse-scale species-specific volume maps as additional image features was
developed to improve the k-NN estimators (Tomppo and Halme 2004). In the latest MS-NFI, the average values of the MS-NFI volume estimates within each satellite image area were calibrated to the field data-based averages of the corresponding area (Mäkisara et al. 2019).

In this study, the small-area estimation method used was direct domain estimation. Since direct design-based estimates are not sufficiently precise in all municipalities, model-based indirect estimates are still needed. Then, if both estimates are produced, the users would need to decide if they wanted to use design-based estimators with known (but possibly poor) precision, or model-based estimators with unknown bias but possibly better overall accuracy. In such a situation, an informed decision is not really possible. One option could be to make a composite estimate of the model-based (MS-NFI) estimate and PS estimate, with the greater weight given to PS, the larger the number of plots. However, also the composite would require information on the precision of the model-based estimates. An empirical best linear unbiased predictor (EBLUP) would be possible if the municipality effect could be estimated (Breidenbach and Astrup 2012, Breidenbach et al. 2018). Lappi (2001) used a calibration estimator with plots outside the municipality in question included from a fixed range. That would give the plots outside the area the greater weight, the smaller the municipality.

Continuous or repeated NFIs offer the possibility to obtain auxiliary data from the previous inventory round. In this study, we used the readily available MS-NFI maps based on the field data of the previous NFI for stratification in order to adhere to the key assumption of PS that auxiliary data should be external to the actual sample (Särndal et al. 1992). The auxiliary data was not completely independent, because one fifth of the sample plots used in deriving the MS-
NFI volume map were permanent, and hence, their remeasurements were included in the actual sample. Breidt and Opsomer (2008) questioned the necessity of auxiliary data to be external, but Magnussen et al. (2015) and Kangas et al. (2016) demonstrated that the use of an internal model (endogenous PS) can lead to a serious underestimation of sampling variance.

PS is a special case of model-assisted (MA) estimation, including only categorical explanatory variables (the group-mean model, Särndal et al. 1992, sec. 7.6). Regression estimators using continuous explanatory variables instead can be more efficient, especially when compared to PS with a small number of strata (McRoberts et al. 2017). However, with a sufficient number of strata, the precision of PS is comparable to that of the regression estimator (or the difference estimator when the model is known) (Myllymäki et al. 2017). The main reason for choosing PS in this study was to enhance the operational work in NFI and to provide design-based results for municipalities or other small areas. In addition, the compatibility of the results, which is very important in an operational NFI (e.g. tree-species level results adding up to total volume), is easy to achieve with PS. Future work in the development of the NFI system includes testing of a hybrid of PS (land-use strata) and regression estimation (within forest). Even then, it would be important for the operational NFI to ensure that the results can be produced more or less automatically.

**Conclusion**

Our results suggest that PS based on a five years old MS-NFI volume map results in a SE less than 5% for forest area and total volume estimates in small areas with at least approximately 200
sample plots on forest land, and a SE less than 10% in areas with at least 100 forest plots in
South Finland. In addition to the NFI sampling intensity, the proportion of other land use area
and tree-species structure of the growing stock in the municipality in question affected the
precision. To meet the precision requirements, especially in estimating volumes by tree species,
smaller municipalities should be combined to larger estimation units; alternatively, a new kind of
composite estimators should be developed. It is also advisable to combine small categories and
present results only for combinations, for instance, as in our case, a volume estimate of all
deciduous tree species together.

The practical advantage of PS is that it enables design-based variance estimation for target areas
of different sizes. PS estimates with design-based CIs can be used to validate model-based small-
area estimates. PS also avoids the bias that is possible in k-NN when sample plots outside of the
target area are used. NFI sampling designed for national and regional (provincial) information
needs restricts the use of PS for the smallest municipalities, where model-based estimation is still
needed.
Acknowledgements

We want to thank the NFI staff of the Natural Resources Institute Finland (until 1 January 2015 the Finnish Forest Research Institute) who have contributed to the NFI and MS-NFI materials applied in this study. The study was carried out in the project “NFI2020” supported financially by the Ministry of Agriculture and Forestry of Finland as a part of the government’s key project “Wood on the move and new products from forests”. We would also like to thank the anonymous referees for their valuable comments on the manuscript.
References

Baffetta, F., Fattorini, L., Franceschi, S., and Corona, P. 2009. Design-based approach to k-nearest neighbours technique for coupling field and remotely sensed data in forest surveys. Remote Sens Environ 113(3): 463–475. doi:10.1016/j.rse.2008.06.014.

Barrett, F., McRoberts, R.E., Tomppo, E., Cienciala, E. and Waser, L.T. 2016. A questionnaire-based review of the operational use of remotely sensed data by national inventories. Remote Sens. Environ. 174: 279–289. doi:10.1016/j.rse.2015.08.029.

Breidenbach, J. and Astrup, R. 2012. Small area estimation of forest attributes in the Norwegian National Forest Inventory. Eur. J. Forest Res. 131(4): 1255–1267. doi:10.1007/s10342-012-0596-7.

Breidenbach, J., McRoberts, R. E., and Astrup, R. 2016. Empirical coverage of model-based variance estimators for remote sensing assisted estimation of stand-level timber volume. Remote Sens. Environ. 173: 274–281.

Breidenbach, J., Magnussen, S., Rahlf, J., and Astrup, R. 2018. Unit-level and area-level small area estimation under heteroscedasticity using digital areal photogrammetry data. Remote Sens. of Environ. 212: 199–211. doi:10.1016/j.rse.2018.04.028.
Breidt, F.J., and Opsomer, J.D. 2008. Endogenous post-stratification in surveys: classification with a sample fitted model. Ann. Statist. 36(1): 403–427. doi:10.1214/009053607000000703.

Cochran, W.G. 1977. Sampling techniques. John Wiley and Sons, New York, 428 p.

Dahlke, M., Breidt, F.J., Opsomer, J.D., and Van Keilegom, I. 2013. Nonparametric endogenous post-stratification estimation. Statistica Sinica 23:189–211. doi:10.5705/ss.2011.272.

Dalenius, T., and Hodges, J.L. Jr. 1959. Minimum variance stratification. J. Am. Stat. Assoc. 54(1): 88–101. doi:10.1080/01621459.1959.10501501.

European Commission. 2018. A sustainable bioeconomy for Europe: strengthening the connection between economy, society and the environment. Updated Bioeconomy Strategy. Directorate – General for Research and Innovation, Unit F – Bioeconomy. doi:10.2777/478385. https://ec.europa.eu/research/bioeconomy/pdf/ec_bioeconomy_strategy_2018.pdf [accessed 26 June 2019].

Fridman, J., Holm, S., Nilsson, M., Nilsson, P., Ringvall, A.H., and Ståhl, G. 2014. Adapting National Forest Inventories to changing requirements – the case of the Swedish National Forest Inventory at the turn of the 20th century. Silva Fenn 48 no. 3 article 1095. 29p. doi.org/10.14214/sf.1095.
Goerndt, M.E., Monleon, V.J., and Temesgen, H. 2013. Small-Area Estimation of County-Level Forest Attributes Using Ground Data and Remote Sensed Auxiliary Information. Forest Science 59(5): 536–548.

Haakana, H., Heikkinen, J., Katila, M., and Kangas, A. 2019. Efficiency of post-stratification for a large-scale forest inventory – case Finnish NFI. Annals of Forest Science 79:9. doi.org/10.1007/s13595-018-0795-6.

Hansen, K., and Malmaeus, M. 2016. Ecosystem services in Swedish forests. Scand. J. For. Res. 31(6): 626–640. doi:10.1080/02827581.2016.1164888.

Kangas, A., Myllymäki, M., Gobakken, T., and Næsset, E. 2016. Model-assisted forest inventory with parametric, semi-parametric, and non-parametric models. Can. J. For. Res. 46(6): 855–868. doi:10.1139/cjfr-2015-0504.

Kangas, A., Astrup, R., Breidenbach, J., Fridman, J., Gobakken, T., Korhonen, Kari T., Maltamo, M., Nilsson, M., Nord-Larsen, T., Næsset, E., Olsson, H. 2018. Remote sensing and forest inventories in Nordic countries – roadmap for the future. Scand. J. For. Res. 33(4): 397–412. doi:10.1080/02827581.2017.1416666.

Kärkkäinen, L., Haakana, H., Hirvelä, H., and Packalen, T. 2017. Using a decision support system to study impacts of land use policies on wood procurement possibilities of the sawmill
industry — a case study at regional and municipal levels. Forest Policy and Economics. In Press, Corrected Proof. Available online 8 October 2017. doi.org/10.1016/j.forpol.2017.10.002.

Katila, M., and Tomppo, E. 2002. Stratification by ancillary data in multisource forest inventories employing k-nearest-neighbour estimation. Can. J. For. Res. 32: 1548–1561. doi:10.1139/x02-047.

Katila, M., Heikkinen, J., and Tomppo, E. 2000. Calibration of small-area estimates for map errors in multisource forest inventory. Can. J. For. Res. 30: 1329–1339. doi:10.1139/x99-234.

Kilkki, P., and Päivinen, R. 1987. Reference sample plots to combine field measurements and satellite data in forest inventory. In Remote sensing—aided forest inventory. Proceedings from seminars organised by SNS, Hyytiälä, Finland, Dec. 10-12, 1986. University of Helsinki, Department of Forest Mensuration and Management, Research Notes 19: 209–215.

Korhonen, K.T., Ihalainen, A., Ahola, A., Heikkinen, J., Henttonen, H.M., Hotanen, J-P., Nevalainen, S., Pitkänen, J., Strandström, M., and Viiri, H. 2017. Suomen metsät 2009–2013 ja niiden kehitys 1921–2013. Luonnonvara- ja biotalouden tutkimus 59/2017. Luonnonvarakeskus, Helsinki. 86 p. http://urn.fi/URN:ISBN:978-952-326-467-0 [accessed 26 March 2019].

Laasasenaho, J. 1982. Taper curves and volume functions for pine, spruce and birch. Communicationes Instituti Forestalis Fenniae 108. 74 pp.
Lappi, J. 2001. Forest inventory of small areas combining the calibration estimator and a spatial model. Can. J. For. Res. 31: 1551–1560. doi.org/10.1139/cjfr-31-9-1551.

Magnussen, S. 2013. An assessment of three variance estimators for the k-nearest neighbour technique. Silva Fenn 47 no. 1 article id 925. 19 p. doi.org/10.14214/sf.925.

Magnussen, S., McRoberts, R.E., and Tomppo, E. 2009. Model-based mean square error estimators for k-nearest neighbor predictions and applications using remotely sensed data for forest inventories. Remote Sensing of Environment 113(3):476-488.

Magnussen, S., Andersen, H-E., and Mundhenk, P. 2015. A Second Look at Endogenous Poststratification. For. Sci. 61(4): 624–634. doi.org/10.5849/forsci.14-183.

Mäkisara, K., Katila, M., Peräsaari, J., and Tomppo, E. 2016. The Multi-Source National Forest Inventory of Finland – methods and results 2013. Natural resources and bioeconomy studies 10/2016. Natural Resources Institute Finland, Helsinki. 215 p. http://urn.fi/URN:ISBN:978-952-326-186-0 [accessed 26 March 2019].

Mäkisara, K., Katila, M., and Peräsaari, J. 2019. The Multi-Source National Forest Inventory of Finland – methods and results 2015. Natural resources and bioeconomy studies 8/2019. Natural Resources Institute Finland, Helsinki. 57 p. http://urn.fi/URN:ISBN:978-952-326-711-4 [accessed 26 March 2019].
Matérn, B. 1960. Spatial variation. Meddelanden från Statens Skogsforskningsinstitut 49.5, 144 p. Also appeared as number 36 of Lecture Notes in Statistics. Springer-Verlag, New York, 1986.

McRoberts, R.E. 2010. Probability- and model-based approaches to inference for proportion forest using satellite imagery as ancillary data. Remote Sens. Environ. 114: 1017–1025. doi.org/10.1016/j.rse.2009.12.013.

McRoberts, R.E., Nelson, M.D., and Wendt, D.G. 2002. Stratified estimation of forest area using satellite imagery, inventory data, and the $k$-Nearest Neighbors technique. Remote Sens. Environ. 82: 457–468.

McRoberts, R.E., Holden, G.R., Nelson, M.D., Liknes, G.C., and Gormanson, D.D. 2006. Using satellite imagery as ancillary data for increasing the precision of estimates for the Forest Inventory and Analysis program of the USDA Forest Service. Can. J. For. Res. 36: 2968–2980. doi.org/10.1139/X05-222.

McRoberts, R.E., Magnussen, S., Tomppo, E., and Chirici, G. 2011. Parametric, bootstrap, and jackknife variance estimators for the $k$-Nearest Neighbors technique with illustrations using forest inventory and satellite image data. Remote Sensing of Environment 115(12): 3165–3174. doi.org/10.1016/j.rse.2011.07.002.
McRoberts, R.E., Chen, Q., and Walters, B.F. 2017. Multivariate inference for forest inventories using auxiliary airborne laser scanning data. For. Ecol. Manag. 401: 295–303. doi.org/10.1016/j.foreco.2017.07.017.

Myllymäki, M., Gobakken, T., Næsset, E., and Kangas, A. 2017. The efficiency of post-stratification compared to model-assisted estimation. Can. J. For. Res. 47(4): 515–526. doi.org/10.1139/cjfr-2016-0383.

Næsset, E., Gobakken, T., Solberg, S., Gregoire, T.G., Nelson, R., Ståhl, G., and Weydahl, D. 2011. Model-assisted regional forest biomass estimation using LiDAR and InSAR as auxiliary data: A case study from a boreal forest area. Remote sens. environ. 115: 3599–3614. doi.org/10.1016/j.rse.2011.08.021.

Nilsson, M., Folving, S., Kennedy, P., Puualainen, J., Chirici, G., Corona, P., Marchetti, M., Olsson, H., Ricotta, C., Ringvall, A., Ståhl, G., and Tomppo, E. 2003. Combining remote sensing and field data for deriving unbiased estimates of forest parameters over larger regions. In: Corona, P., Köhl, M., and Marchetti, M. (eds.). Advances in forest inventory for sustainable forest management and biodiversity monitoring. Forestry Sciences, vol 76, Springer, Dordrecht, pp 19–32. doi.org/10.1007/978-94-017-0649-0_2.

Nilsson, M., Holm, S., Reese, H., Wallerman, J., and Engberg, J. 2005. Improved forest statistics from the Swedish National Forest Inventory by combining field data and optical satellite data.
using post-stratification. In: Proceedings of ForestSat 2005 in Borås May 31 - June 3. Olsson, H.
(ed.). National Board of Forestry May 2005, Skogsstyrelsens förlag, Jönköping, pp 22–26.

Pulkkinen, M., Ginzler, C., Traub, B., and Lanz A. 2018. Stereo-imagery-based post-
stratification by regression-tree modelling in Swiss National Forest Inventory. Remote Sens.
Environ. 213: 182–194. doi:10.1016/j.rse.2018.04.052.

Rao, J.N.K. 2003. Small Area Estimation. John Wiley & Sons, New York. 313 p.
doi:10.1002/0471722189.

Särndal, C-E., Swensson, B., and Wretman, J. 1992. Model assisted survey sampling. Springer-
Verlag, New York. 694 p.

Tipton, J., Opsomer, J., and Moisen, G. 2013. Properties of Endogenous Post-Stratified
Estimation using remote sensing data. Remote Sens. Environ. 139: 130–137.
doi:10.1016/j.rse.2013.07.035.

Tomppo, E. 1991. Satellite Image-Based National Forest Inventory of Finland. In: Proceedings
of the symposium on Global and Environmental Monitoring, Techniques and Impacts,
September 17–21, 1990 Victoria, British Columbia Canada. International Archives of
Photogrammetry and Remote Sensing, Vol 28, Part 7-1, pp. 419–424.
Tomppo, E. 1996. Multi-source National Forest Inventory of Finland. In: Päivinen, R., Vanclay, J., and Miina, S. (eds.). New Thrusts in Forest Inventory. Proceedings of the subject group S4.02-00 ‘Forest Resource Inventory and Monitoring’ and subject group S4.12-00 ‘Remote Sensing Technology’, vol. 1. IUFRO XX World Congress, 6-12 Aug. 1995, Tampere, Finland. European Forest Institute, Joensuu, Finland. p. 27-41.

Tomppo, E., and Halme, M. 2004. Using coarse scale forest variables as ancillary information and weighting of variables in k-NN estimation: a genetic algorithm approach. Remote Sens. Environ. 92: 1–20. doi.org/10.1016/j.rse.2004.04.003.

Tomppo, E., Katila, M., Moilanen, J., Mäkelä, H., and Peräsaari J. 1998. Kunnittaiset metsävaratiedot 1990-94 [Forest resources by municipalities 1990-94]. Metsätieteen aikakauskirja - Folia Forestalia 4B/1998: 619–839. [In Finnish].

Tomppo, E., Haakana, M., Katila, M., and Peräsaari, J. 2008. Multi-source national forest inventory - Methods and applications. Managing Forest Ecosystems 18. Springer. 373 p. doi.org/10.1007/978-1-4020-8713-4.

Tomppo, E., Heikkinen, J., Henttonen, H.M., Ihalainen, A., Katila, M., Mäkelä, H., Tuomainen, T., and Vainikainen, N. 2011. Designing and conducting a forest inventory - case: 9th National Forest Inventory of Finland. Managing Forest Ecosystems 21, Springer. 270 p. doi.org/10.1007/978-94-007-1652-0.
Tompson, E., Katila, M., Mäkisara, K., and Peräsaari, J. 2012. The Multi-source National Forest Inventory of Finland – methods and results 2007. Working Papers of the Finnish Forest Research Institute 227. 233 p. http://www.metla.fi/julkaisut/workingpapers/2012/mwp227.htm [accessed 11 January 2019].

Tompson, E., Katila, M., Mäkisara, K., and Peräsaari, J. 2014. The Multi-source National Forest Inventory of Finland – methods and results 2011. Working Papers of the Finnish Forest Research Institute 319. 224 p. http://www.metla.fi/julkaisut/workingpapers/2014/mwp319.htm [accessed 11 January 2019].

Westfall, J.A., Patterson, P.L., and Coulston, J.W. 2011. Post-stratified estimation: within-strata and total sample size recommendations. Can. J. For. Res. 41: 1130–1139. doi.org/10.1139/x11-031.
Table 1. Statistics of the estimates based on PS for the municipalities in the provinces of Kainuu (N = 8) and Pirkanmaa (N = 22).

| Province  | Variable                  | Mean | Std Dev | Min | Max | Sum  |
|-----------|---------------------------|------|---------|-----|-----|------|
| Kainuu    | Total area (km²)          | 2,835| 1,881   | 898 | 5,858| 22,687|
|           | Land area (km²)           | 2,522| 1,691   | 835 | 5,282| 20,178|
|           | Forest area (km²)         | 2,219| 1,479   | 748 | 4,572| 17,756|
|           | Total volume (1000 m³)    | 20,463| 12,686 | 6,670| 40,579| 163,705|
|           | Mean volume (m³/ha)       | 94   | 10      | 80  | 106 | -    |
|           | Proportion of pine (%)    | 58   | 7       | 48  | 65  | -    |
|           | Proportion of spruce (%)  | 23   | 5       | 18  | 30  | -    |
|           | Proportion of birch (%)   | 17   | 4       | 12  | 21  | -    |
|           | Proportion of other deciduous (%) | 2 | 1 | 1 | 3 | - |

|         | Total area (km²)          | 664 | 385     | 104 | 1,532| 14,613|
|         | Land area (km²)           | 570 | 340     | 80  | 1,425| 12,548|
|         | Forest area (km²)         | 422 | 277     | 51  | 970  | 9,292 |
|         | Total volume (1000 m³)    | 6,250| 3,854 | 736 | 14,265| 137,506|
|         | Mean volume (m³/ha)       | 151 | 21      | 114 | 200  | -    |
|         | Proportion of pine (%)    | 33   | 13      | 14  | 60   | -    |
|         | Proportion of spruce (%)  | 45   | 10      | 22  | 60   | -    |
|         | Proportion of birch (%)   | 17   | 4       | 12  | 27   | -    |
|         | Proportion of other deciduous (%) | 4 | 2 | 1 | 9 | - |
Table 2. Time points of NFI and MS-NFI data used in post-stratification and in comparison of PS and MS-NFI estimates.

| Estimation                      | Satellite images | NFI round | Field measurements | Reference year |
|---------------------------------|------------------|-----------|--------------------|----------------|
| PS, strata based on:            |                  |           |                    |                |
| MS-NFI 2007                     | 2005 – 2007      | NFI10     | 2005 – 2008        | 2007           |
| MS-NFI 2011                     | 2010 – 2011      | NFI11/10  | 2007 – 2011        | 2011           |
| MS-NFI 2013                     | 2012 – 2014      | NFI11     | 2009 – 2013        | 2013           |
Table 3. Statistics of relative standard errors (SE) of forest variables estimated with post-stratification for the municipalities in the provinces of Kainuu (N=8) and Pirkanmaa (N=22; N=11 in estimating the area of poorly productive forest).

| Province  | Variable                                | SE (%) |   |   |
|-----------|-----------------------------------------|--------|---|---|
|           | Mean                                   | Min    | Max |
| Kainuu    | Productive forest area (km$^2$)         | 2.26   | 1.52 | 3.50 |
|           | Poorly productive forest area (km$^2$)  | 19.06  | 7.81 | 30.37 |
|           | Total forest area (km$^2$)              | 1.67   | 1.15 | 2.55 |
|           | Total volume (1000 m$^3$)               | 3.73   | 2.29 | 5.20 |
|           | Pine volume (1000 m$^3$)                | 5.60   | 2.80 | 11.29 |
|           | Spruce volume (1000 m$^3$)              | 12.52  | 7.42 | 18.03 |
|           | Birch volume (1000 m$^3$)               | 9.99   | 5.94 | 19.24 |
|           | Volume of other deciduous (1000 m$^3$)  | 29.13  | 15.48 | 42.47 |
| Pirkanmaa | Productive forest area (km$^2$)         | 5.59   | 1.85 | 21.35 |
|           | Poorly productive forest area (km$^2$)  | 63.96  | 27.58 | 113.16 |
|           | Total forest area (km$^2$)              | 5.28   | 1.85 | 21.35 |
|           | Total volume (1000 m$^3$)               | 9.85   | 3.81 | 26.94 |
|           | Pine volume (1000 m$^3$)                | 19.92  | 7.05 | 97.73 |
|           | Spruce volume (1000 m$^3$)              | 16.08  | 7.06 | 43.86 |
|           | Birch volume (1000 m$^3$)               | 20.21  | 8.50 | 55.74 |
|           | Volume of other deciduous (1000 m$^3$)  | 49.18  | 23.89 | 277.83 |
Figure captions

Fig. 1. Locations of the study areas, the provinces of Kainuu (1) and Pirkanmaa (2) in grey on the map of municipalities in Finland, the borders of all provinces in Finland with a thicker line. Digital map data: National Land Survey of Finland, 2018.

Fig. 2. NFI sampling designs in Pirkanmaa (upper) and Kainuu regions (lower).

Fig. 3. Relative standard errors (SE) of the estimated forest variables, derived with post-stratification for the municipalities in Kainuu (upper) and Pirkanmaa (lower) provinces. In Pirkanmaa, y-axis maximum limited to 100%.

Fig. 4. Relative standard errors (SE) of the estimates of forest area and total volume, derived with post-stratification by the sizes of the municipalities in Kainuu and Pirkanmaa provinces.

Fig. 5. Relative standard errors (SE) of the volume estimates by tree species, derived with post-stratification by the sizes of the municipalities in Kainuu and Pirkanmaa provinces.

Fig. 6. Estimated areas of productive forest land, derived with post-stratification (PS) and MS-NFI by the municipalities in Pirkanmaa province, and confidence intervals (2 × standard error) for PS estimates.
**Fig. 7.** Estimated total volumes on forest land, derived with post-stratification (PS) and MS-NFI by the municipalities in Kainuu province, and confidence intervals (2 × standard error) for PS estimates.

**Fig. 8.** Estimated pine volumes on forest land, derived with post-stratification (PS) and MS-NFI by the municipalities in Kainuu province, and confidence intervals (2 × standard error) for PS estimates.

**Fig. 9.** Estimated pine volumes on forest land, derived with post-stratification (PS) and MS-NFI by the municipalities in Pirkanmaa province, and confidence intervals (2 × standard error) for PS estimates.
Appendix A. Post-stratified estimators and their sampling variance

For simplicity, we present the detailed formulae for the post-stratified estimators and their variances for the area of forest land. The estimators are presented for one arbitrary municipality; the numbers and the sums are taken over all sample plots within that municipality, and also the \( A_h \)'s are the stratum areas within it. The volume estimators were analogous to the area estimators, as explained in the end of this appendix. The motivation behind the basic formulae is discussed by Tomppo et al. (2011, sec. 3.5), for example.

Let \( x_{c,h} \) denote the number of those sample plots of cluster \( c \) that belong to stratum \( h \), and let \( y_{c,h} \) denote the number of forest land plots among them. The proportion of forest land within stratum \( h \) was estimated by

\[
\hat{p}_h = \frac{\sum_c y_{c,h}}{\sum_c x_{c,h}},
\]

and the sampling variance of \( \hat{p}_h \) by

\[
\hat{\text{Var}}(\hat{p}_h) = \frac{\sum_g \left( \sum_{c \in g} b_c z_{c,h} \right)^2}{\left( \sum_g x_{c,h} \right)^2},
\]

where \( z_{c,h} = y_{c,h} - \hat{p}_h x_{c,h} \) is a cluster-level residual, \( g \) is a group of clusters close to each other, and the weights \( b_c \) were determined so that each local quadratic form \( \left( \sum_{c \in g} b_c z_{c,h} \right)^2 \) is an unbiased estimator of the variance of residuals (Matérn 1960, p. 110).

Groups \( g \) either contained four temporary clusters forming a square or five clusters including a similar square of four temporary clusters and one permanent cluster in the center of the square.
(cf. Fig. 2). In groups of four clusters, weights $b_c$ were $\sqrt{5}/4$ for the SW and NE corner clusters and $-\sqrt{5}/4$ for the other two. In groups of five clusters, the permanent cluster in the center received weight $b_c = 1$ and all temporary ones in the corners weight $b_c = -1/4$. Since $\sum_{c\in g} b_c = 0$, this weighting scheme ensures that $E(\sum_{c\in g} b_c z_{c,h}) = 0$ under the assumption of stationary residuals $z_{c,h}$. It also leads to an unbiased variance estimator in the special case of uncorrelated residuals. To see this, let $\sigma^2 = \text{Var}(z_{c,h})$ (all variances are equal under the stationarity assumption), and let $n_c$ and $n_g$ denote the number of clusters and groups, respectively. Then

$$E\left[\left(\sum_{c\in g} b_c z_{c,h}\right)^2\right] = \text{Var}\left(\sum_{c\in g} b_c z_{c,h}\right) = \sum_{c\in g} b_c^2 \text{Var}(z_{c,h}) = \sigma^2 \sum_{c\in g} b_c^2 = \frac{5\sigma^2}{4},$$

so that

$$E\left[\sum_g \left(\sum_{c\in g} b_c z_{c,h}\right)^2\right] = \frac{5n_g \sigma^2}{4} = n_c \sigma^2 = E\left[\sum_c z_{c,h}^2\right] = \text{Var}\left(\sum_c z_{c,h}^2\right),$$

since $n_g/n_c = 4/5$. Finally, the chosen weighting scheme gives the same weight $\sum_{c\in g} b_c^2 = 5/4$ to both kinds of groups in the variance estimator.

Between-stratum covariance of $\hat{p}_h$ and $\hat{p}_h'$ was estimated by

$$\text{Cov}(\hat{p}_h, \hat{p}_h') = \frac{\sum_g \left(\sum_{c\in g} b_c z_{c,h}\right) \left(\sum_{c\in g} b_c z_{c,h'}\right)}{\left(\sum_g x_{c,h}\right) \left(\sum_g x_{c,h'}\right)},$$

and the sampling variance of the municipality-level estimator $\hat{Y} = \sum_h A_h \hat{p}_h$ of forest land area was obtained as

$$\text{Var}(\hat{Y}) = \sum_h \sum_{h'} A_h A_{h'} \text{Cov}(\hat{p}_h, \hat{p}_{h'}),$$

where $\text{Cov}(\hat{p}_h, \hat{p}_h) = \text{Var}(\hat{p}_h)$.
The equations for the tree volumes are similar to the equations for the forest area. The only difference is that $y_{c,h}$ is now the sum of volumes of trees in those sample plots of cluster $c$ that belong to stratum $h$.

References

Matérn, B. 1960. Spatial variation. Meddelanden från Statens Skogsforskningsinstitut 49.5, 144 p. Also appeared as number 36 of Lecture Notes in Statistics. Springer-Verlag, New York, 1986.

Tomppo, E., Heikkinen, J., Henttonen, H.M., Ihalainen, A., Katila, M., Mäkelä, H., Tuomainen, T., and Vainikainen, N. 2011. Designing and conducting a forest inventory - case: 9th National Forest Inventory of Finland. Managing Forest Ecosystems 21, Springer. 270 p. doi.org/10.1007/978-94-007-1652-0.
Fig. 1. Locations of the study areas, the provinces of Kainuu (1) and Pirkanmaa (2) in grey on the map of municipalities in Finland, the borders of all provinces in Finland with a thicker line. Digital map data: National Land Survey of Finland, 2018.

210x297mm (600 x 600 DPI)
Fig. 2. NFI sampling designs in Pirkanmaa (upper) and Kainuu regions (lower).
Fig. 3. Relative standard errors (SE) of the estimated forest variables, derived with post-stratification for the municipalities in Kainuu (upper) and Pirkanmaa (lower) provinces. In Pirkanmaa, y-axis maximum limited to 100%.

233x274mm (300 x 300 DPI)
Fig. 4. Relative standard errors (SE) of the estimates of forest area and total volume, derived with post-stratification by the sizes of the municipalities in Kainuu and Pirkanmaa provinces.
Fig. 5. Relative standard errors (SE) of the volume estimates by tree species, derived with post-stratification by the sizes of the municipalities in Kainuu and Pirkanmaa provinces.

233x137mm (300 x 300 DPI)
Fig. 6. Estimated areas of productive forest land, derived with post-stratification (PS) and MS-NFI by the municipalities in Pirkanmaa province, and confidence intervals (2 × standard error) for PS estimates.
Fig. 7. Estimated total volumes on forest land, derived with post-stratification (PS) and MS-NFI by the municipalities in Kainuu province, and confidence intervals (2 × standard error) for PS estimates.
Fig. 8. Estimated pine volumes on forest land, derived with post-stratification (PS) and MS-NFI by the municipalities in Kainuu province, and confidence intervals (2 × standard error) for PS estimates.
Fig. 9. Estimated pine volumes on forest land, derived with post-stratification (PS) and MS-NFI by the municipalities in Pirkanmaa province, and confidence intervals (2 × standard error) for PS estimates.

Method
- MS-NFI 2013
- MS-NFI 2011
- PS

233x152mm (300 x 300 DPI)