Gluonic effects in $\eta$ and $\eta'$ physics

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Abstract

We review the theory and phenomenology of the axial U(1) problem with emphasis on the role of gluonic degrees of freedom in $\eta$ and $\eta'$ production processes, especially the low-energy $pN \rightarrow pN\eta$ and $pN \rightarrow pN\eta'$ reactions.

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1 Introduction

\( \eta \) and \( \eta' \) physics together with polarised deep inelastic scattering provide complementary windows on the role of gluons in dynamical chiral symmetry breaking. Gluonic degrees of freedom play an important role in the physics of the flavour-singlet \( J^P = 1^+ \) channel [2, 3] through the QCD axial anomaly [3]. The most famous example is the \( U_A(1) \) problem: the masses of the \( \eta \) and \( \eta' \) mesons are much greater than the values they would have if these mesons were pure Goldstone bosons associated with spontaneously broken chiral symmetry [4, 5]. This extra mass is induced by non-perturbative gluon dynamics [6, 7, 8, 9] and the axial anomaly [10, 11] – for a recent discussion see [12].

For the first time since the discovery of QCD (and the \( U(1) \) problem) precise data are emerging on processes involving \( \eta' \) production and decays. There is presently a vigorous experimental programme to study the \( pp \to pp\eta \) and \( pp \to pp\eta' \) reactions close to threshold in low-energy proton-nucleon collisions at CELSIUS [13] and COSY [14]. New data on \( \eta' \) photoproduction, \( \gamma p \to \eta' \), are expected soon from Jefferson Laboratory [15] following earlier measurements at ELSA [16]. The light-mass “exotic” meson states with quantum numbers \( J^P C = 1^−+ \) observed at BNL [17] and CERN [18] in \( \pi^-p \) and \( \bar{p}N \) scattering were discovered in decays to \( \eta\pi \) and \( \eta'\pi \) suggesting a possible connection with axial \( U(1) \) dynamics. Further “exotic” studies are proposed in photoproduction experiments at Jefferson Laboratory. At higher energies anomalously large branching ratios have been observed by CLEO for \( B \)-meson decays to an \( \eta' \) plus additional hadrons [19] and for the \( D^+_s \to \eta'\rho^+ \) process. The \( B \) decay measurements have recently been confirmed in new, more precise, data from BABAR [21, 22] and BELLE [23]. The LEP data on \( \eta' \) production in hadronic jets is about 40% short of the predictions of the string fragmentation models employed in the JETSET and ARIADNE Monte-Carlos without an additional \( \eta' \) “suppression factor” [24]. First measurements of \( \eta' \to \gamma\gamma^* \) decays have been performed at CLEO [25]. The new WASA 4\pi detector [26] at CELSIUS will enable precision studies of \( \eta \) and \( \eta' \) decays. Data expected in the next few years provides an exciting new opportunity to study axial \( U(1) \) dynamics and to investigate the role of gluonic degrees of freedom in \( \eta \) and \( \eta' \) physics.

In this paper we focus primarily on \( \eta' \) production – especially in proton-nucleon collisions – together with a brief review of the axial \( U(1) \) problem in QCD. Subjects not covered here are \( \eta \) and \( \eta' \) decays and lattice calculations (covered in other contributions to this volume), the strong CP problem, and axial \( U(1) \) symmetry at finite temperature.

The role of gluonic degrees of freedom and OZI violation in the \( \eta' \)-nucleon system has been investigated through the flavour-singlet Goldberger-Treiman relation [27, 28], the low-energy \( pp \to pp\eta' \) reaction [29], \( \eta' \) photoproduction [30] and the decays of light-mass “exotic” mesons [31]. The flavour-singlet Goldberger-Treiman relation connects the flavour-singlet axial-charge \( g_A^{(0)} \) measured in polarised deep inelastic scattering with the \( \eta' \)-nucleon coupling constant \( g_{\eta'NN} \). Working in the chiral limit it reads

\[
Mg_A^{(0)} = \sqrt{\frac{3}{2}} F_0 \left( g_{\eta'NN} - g_{QNN} \right) ,
\]

where \( g_{\eta'NN} \) is the \( \eta' \)-nucleon coupling constant and \( g_{QNN} \) is an OZI violating
coupling which measures the one particle irreducible coupling of the topological charge density \( Q = \frac{a_s}{4\pi} G \tilde{G} \) to the nucleon. In Eq. (1) \( M \) is the nucleon mass and \( F_0 \) \((\sim 0.1\text{GeV})\) renormalises \[29\] the flavour-singlet decay constant. The coupling constant \( g_{QNN} \) is, in part, related \[27\] to the amount of spin carried by polarised gluons in a polarised proton. The large mass of the \( \eta' \) and the small value of \( g_A^{(0)} \)

\[
g_A^{(0)}|_{\text{pDIS}} = 0.2 - 0.35
\]

extracted from deep inelastic scattering \[32, 33\] (about a 50% OZI suppression) point to substantial violations of the OZI rule in the flavour-singlet \( J^P = 1^+ \) channel \[4\]. A large positive \( g_{QNN} \sim 2.45 \) is one possible explanation of the small value of \( g_A^{(0)} \)|_{pDIS}.

It is important to look for other observables which are sensitive to \( g_{QNN} \). OZI violation in the \( \eta' \)–nucleon system is a probe of the role of gluons in dynamical chiral symmetry breaking in low-energy QCD.

Working with the \( U_A(1) \)–extended chiral Lagrangian for low-energy QCD \[35, 36\] — see Section 3 below — one finds a gluon-induced contact interaction in the \( pp \to pp\eta' \) reaction close to threshold \[29\]:

\[
L_{\text{contact}} = -\frac{i}{F_0^2} g_{QNN} \tilde{m}_{\eta_0}^2 \mathcal{C} \eta_0 \left( \bar{p} \gamma_5 p \right) \left( \bar{p} p \right)
\]

Here \( \tilde{m}_{\eta_0} \) is the gluonic contribution to the mass of the singlet \( 0^- \) boson and \( \mathcal{C} \) is a second OZI violating coupling which also features in \( \eta'N \) scattering. The physical interpretation of the contact term (3) is a “short distance” \((\sim 0.2\text{fm})\) interaction where glue is excited in the interaction region of the proton-proton collision and then evolves to become an \( \eta' \) in the final state. This gluonic contribution to the cross-section for \( pp \to pp\eta' \) is extra to the contributions associated with meson exchange models \[37, 38, 39, 40\]. There is no reason, a priori, to expect it to be small.

What is the phenomenology of this gluonic interaction?

Since glue is flavour-blind the contact interaction (3) has the same size in both the \( pp \to ppp\eta' \) and \( pn \to pm\eta' \) reactions. CELSIUS \[13\] have measured the ratio \( R_{\eta} = \sigma(pn \to pm\eta)/\sigma(pp \to ppp\eta) \) for quasifree \( \eta \) production from a deuteron target up to 100 MeV above threshold. They observed that \( R_{\eta} \) is approximately energy-independent \( \simeq 6.5 \) over the whole energy range — see Fig.1. The value of this ratio signifies a strong isovector exchange contribution to the \( \eta \) production mechanism \[13\]. This experiment should be repeated for \( \eta' \) production. The cross-section for \( pp \to ppp\eta' \) close to threshold has been measured at COSY \[14\]. Following the suggestion in \[29\] a new COSY-11, Uppsala University Collaboration \[41\] has been initiated to carry out the \( pn \to pm\eta' \) measurement. Further studies of \( \eta' \) production in proton-deuteron collisions will soon be possible using the ANKE detector at COSY \[12\]. The more important that the gluon-induced process (3) is in the \( pp \to ppp\eta' \) reaction the more one would expect \( R_{\eta'} = \sigma(pn \to pm\eta')/\sigma(pp \to ppp\eta') \) to approach unity near threshold after we correct for the final state interaction \[39, 43\] between the two outgoing nucleons. (After we turn on the quark masses, the small \( \eta - \eta' \) mixing angle \( \theta \simeq -18 \) degrees means that the gluonic effect (3) should be considerably bigger in \( \eta' \) production than \( \eta \) production.) \( \eta' \) phenomenology is characterised by large OZI violations. It is natural to expect large gluonic effects in the \( pp \to ppp\eta' \) process.
In Section 2 we give a brief Introduction to the U(1) problem. Section 3 introduces the chiral Lagrangian approach and Section 4 makes contact with the experimental data from CELSIUS and COSY. Section 5 gives a brief overview of data and recent theoretical ideas about possible OZI violation and gluonic effects in $\eta$ and $\eta'$ photoproduction and the structure of light-mass “exotic” mesons with $J^{PC} = 1^{-+}$. Section 6 reviews the status of heavy-quark meson decays into an $\eta'$ plus additional hadrons.

2 The U(1) problem

In classical field theory Noether’s theorem tells us that there is a conserved current associated with each global symmetry of the Lagrangian. The QCD Lagrangian

$$\mathcal{L}_{QCD} = \sum_q \bar{q}_L (i\dot{D} - g\dot{A}) q_L + \bar{q}_R (i\dot{D} - g\dot{A}) q_R - \sum_q m_q (\bar{q}_L q_R + \bar{q}_R q_L) - \frac{1}{2} G_{\mu\nu} G^{\mu\nu} \quad (4)$$

exhibits chiral symmetry for massless quarks: when the quark mass term is turned off the left- and right-handed quark fields do not couple in the Lagrangian and transform independently under chiral rotations.
Chiral $SU(2)_L \otimes SU(2)_R$

\[
\begin{pmatrix}
  u_L \\
  d_L 
\end{pmatrix} \mapsto e^{i \frac{\alpha}{2} \gamma_5} \begin{pmatrix}
  u_L \\
  d_L 
\end{pmatrix} ,
\begin{pmatrix}
  u_R \\
  d_R 
\end{pmatrix} \mapsto e^{i \frac{\beta}{2} \gamma_5} \begin{pmatrix}
  u_R \\
  d_R 
\end{pmatrix}
\]

is associated with the isotriplet axial-vector current $J^{(3)}_{\mu 5}$

\[
J^{(3)}_{\mu 5} = \left[ \bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d \right]
\]

which is partially conserved

\[
\partial^\mu J^{(3)}_{\mu 5} = 2m_u \bar{u} i \gamma_5 u - 2m_d \bar{d} i \gamma_5 d
\]

The absence of parity doublets in the hadron spectrum tells us that the near-chiral symmetry for light $u$ and $d$ quarks is spontaneously broken. Spontaneous chiral symmetry breaking is associated with a non-vanishing chiral condensate

\[
\langle \text{vac} | \bar{q} q | \text{vac} \rangle < 0
\]

The light-mass pion is identified as the corresponding Goldstone boson and the current $J^{(3)}_{\mu 5}$ is associated with the pion through PCAC

\[
\langle \text{vac} | J^{(3)}_{\mu 5}(z) | \pi(q) \rangle = -if_\pi q_\mu e^{-iq.z}
\]

Taking the divergence equation

\[
\langle \text{vac} | \partial^\mu J^{(3)}_{\mu 5}(z) | \pi(q) \rangle = -f_\pi m_\pi^2 e^{-iq.z}
\]

the pion mass-squared vanishes in the chiral limit as $m_\pi^2 \sim m_q$. This and PCAC [44] are the starting points for chiral perturbation theory [45].

The non-vanishing chiral condensate also spontaneously breaks the axial U(1) symmetry so, naively, one might expect an isosinglet pseudoscalar degenerate with the pion. The lightest mass isosinglet pseudoscalar is the $\eta$ meson which has a mass of 547 MeV.

The puzzle deepens when one considers $SU(3)$. Spontaneous chiral symmetry breaking suggests an octet of Goldstone bosons associated with chiral $SU(3)_L \otimes SU(3)_R$ plus a singlet boson associated with axial U(1) — each with mass $m_{\text{Goldstone}}^2 \sim m_q$. If the $\eta$ is associated with the octet boson then the Gell-Mann Okubo relation

\[
m_{\eta s}^2 = \frac{4}{3} m_K^2 - \frac{1}{3} m_\pi^2
\]

is satisfied to within a few percent. Extending the theory from $SU(3)$ to $SU(3)_L \otimes SU(3)_R \otimes U(1)$ the large strange quark mass induces considerable $\eta - \eta'$ mixing. Taking $m_{\text{Goldstone}}^2 \sim m_q$ the $\eta$ would be approximately an isosinglet light-quark state $(\frac{1}{\sqrt{2}} \bar{u} u + \bar{d} d)$ degenerate with the pion and the $\eta'$ would be approximately a strange quark state $|\bar{s} s\rangle$ with mass about $\sqrt{2m_K^2 - m_\pi^2}$. That is, the masses for the $\eta$ and $\eta'$ mesons with $\eta - \eta'$ mixing and without extra physical input come out about 300-400 MeV too small! This is the axial U(1) problem.
The extra physics which is needed to understand the U(1) problem are gluon topology and the QCD axial anomaly. The (gauge-invariantly renormalised) flavour-singlet axial-vector current in QCD satisfies the anomalous divergence equation

$$\partial_{\mu} J_{\mu 5} = \sum_{k=1}^{f} 2i \left[ m_k \bar{q}_k \gamma_5 q_k \right] + N_f \left[ \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} \right]$$  \hspace{1cm} (12)$$

where

$$J_{\mu 5} = \left[ \bar{u}_\mu \gamma_5 u + \bar{d}_\mu \gamma_5 d + \bar{s}_\mu \gamma_5 s \right]$$  \hspace{1cm} (13)$$

Here $N_f = 3$ is the number of light flavours, $G_{\mu\nu}$ is the gluon field tensor and $\tilde{G}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}$. The anomalous term $Q(z) \equiv \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}(z)$ is the topological charge density. Its integral over space $\int d^4 z \, Q = n$ measures the gluonic “winding number” \[46\], which is an integer for (anti-)instantons and which vanishes in perturbative QCD. The exact dynamical mechanism how (non-perturbative) gluonic degrees of freedom contribute to axial U(1) symmetry breaking through the anomaly is still hotly debated \[3, 46, 47, 48\]: suggestions include instantons \[6\] and possible connections with confinement \[49\]. Recent lattice investigations of the $U_A(1)$ problem are reported in \[12, 50, 51, 52\].

The role of instantons in $U_A(1)$ symmetry breaking is particularly interesting. Quark-instanton interactions flip chirality, thus connecting $q_L$ and $q_R$. Whether instantons spontaneously \[46\] or explicitly \[47\] break $U_A(1)$ symmetry depends on the role of zero-modes in the quark-instanton interaction and how one should include non-local structure into the local anomalous Ward identity, Eq. (12). Explicit symmetry breaking by instantons would generate an instanton induced contribution to the $\eta'$ mass whereas spontaneous symmetry breaking would not. $\nu p$ elastic scattering offers a possible tool to investigate this physics \[53\]. Gluon topology and $U_A(1)$ symmetry breaking have the potential to induce zero-mode contributions to the flavour-singlet axial-charge $g_A^{(0)}$ which are associated with Bjorken $x = 0$ in polarized deep inelastic scattering \[32\]. Topological $x = 0$ polarization is inaccessible to deep inelastic scattering experiments but could be measured through $\nu p$ elastic scattering. By flipping chirality quark-instanton interactions act to reduce the spin asymmetry measured in polarized deep inelastic scattering and the value of $g_A^{(0)}|_{\nu p}$ extracted from these experiments. Topological $x = 0$ polarization is natural \[24, 53\] in the spontaneous symmetry breaking scenario where any instanton induced suppression of $g_A^{(0)}|_{\nu p}$ is compensated by a shift of flavour-singlet axial-charge from partons carrying finite momentum $x > 0$ to a zero-mode at $x = 0$, whereas it is not generated by the explicit symmetry breaking scenario. A definitive measurement of $\nu p$ elastic scattering may be possible with the Mini-Boone set-up at FNAL \[54\]. Comparing the values of $g_A^{(0)}|_{\nu p}$ extracted from $\nu p$ elastic and polarized deep inelastic scattering would provide valuable information on $U_A(1)$ symmetry breaking in QCD.

Independent of the detailed QCD dynamics one can construct low-energy effective chiral Lagrangians which include the effect of the anomaly and axial U(1) symmetry, and use these Lagrangians to study low-energy processes involving the $\eta$ and $\eta'$. 
3 The low-energy effective Lagrangian

Starting in the meson sector, the building block for the $U_A(1)$-extended low-energy effective Lagrangian \cite{35,36} is

$$\mathcal{L}_m = \frac{F^2}{4} \text{Tr}(\partial^\mu U \partial_\mu U^\dagger) + \frac{F^2}{4} \text{Tr} \left[ \chi_0 \left( U + U^\dagger \right) \right] + \frac{1}{2} i Q \text{Tr} \left[ \log U - \log U^\dagger \right] + \frac{3}{m_{\eta_0}^2 F_0^2} Q^2.$$  

(14)

Here

$$U = \exp \left( i \phi \frac{F_\pi}{m_\pi} + i \sqrt{\frac{2}{3}} \frac{\eta_0}{F_0} \right)$$  

(15)

is the unitary meson matrix where $\phi = \sum_k \phi_k \lambda_k$ with $\phi_k$ denotes the octet of would-be Goldstone bosons ($\eta$, $K$, $\eta'$) associated with spontaneous chiral $SU(3)_L \otimes SU(3)_R$ breaking, $\eta_0$ is the singlet boson and $Q$ is the topological charge density; $\chi_0 = \text{diag}[m_\pi^2, m_\pi^2, (2m_K^2 - m_\pi^2)]$ is the meson mass matrix. The pion decay constant $F_\pi = 92.4 \text{MeV}$ and $F_0$ renormalises the flavour-singlet decay constant.

When we expand out the Lagrangian (14) the first term contains the kinetic energy term for the pseudoscalar mesons; the second term contains the meson mass terms before coupling to gluonic degrees of freedom. The $U_A(1)$ gluonic potential involving the topological charge density is constructed to reproduce the axial anomaly (12) in the divergence of the gauge-invariantly renormalised axial-vector current and to generate the gluonic contribution to the $\eta$ and $\eta'$ masses. The gluonic term $Q$ is treated as a background field with no kinetic term. It may be eliminated through its equation of motion

$$\frac{1}{2} i Q \text{Tr} \left[ \log U - \log U^\dagger \right] + \frac{3}{m_{\eta_0}^2 F_0^2} Q^2 \mapsto -\frac{1}{2} m_{\eta_0}^2 \eta_0^2$$  

(16)

making the gluonic mass term clear. After $Q$ is eliminated from the effective Lagrangian via (16), we expand $\mathcal{L}_m$ to $\mathcal{O}(p^2)$ in momentum keeping finite quark masses and obtain:

$$\mathcal{L}_m = \sum_k \frac{1}{2} \partial^\mu \phi_k \partial_\mu \phi_k + \frac{1}{2} \partial_\mu \eta_0 \partial^\mu \eta_0 \left( \frac{F_\pi}{F_0} \right)^2 - \frac{1}{2} m_{\eta_0}^2 \eta_0^2$$  

$$- \frac{1}{2} m_\pi^2 \left( 2\pi^+ \pi^- + \pi_0^2 \right) - m_K^2 \left( K^+ K^- + K^0 \bar{K}^0 \right) - \frac{1}{2} \left( \frac{4}{3} m_K^2 - \frac{1}{3} m_\pi^2 \right) \eta_8^2$$  

$$- \frac{1}{2} \left( \frac{2}{3} m_K^2 + \frac{1}{3} m_\pi^2 \right) \left( \frac{F_\pi}{F_0} \right)^2 \eta_0^2 + \frac{4}{3 \sqrt{2}} \left( m_K^2 - m_\pi^2 \right) \left( \frac{F_\pi}{F_0} \right) \eta_8 \eta_0 + ...$$  

(17)

\footnote{The Adler-Bardeen theorem \cite{53} provides a constraint on the low-energy effective Lagrangian. This theorem (in QCD) states that the coefficient of the anomaly on the right hand side of (12) is not renormalised to all orders in perturbation theory. Suppose we were to add an extra term

$$\mathcal{L}_{AB} = i Q \frac{3e}{4 m_{\eta_0}^2} \text{Tr} \left[ \chi_0^A U - \chi_0 U^\dagger \right]$$  

(18)

to the the Lagrangian. Then the coefficient of the topological charge density in the divergence of the flavour-singlet axial-vector current in the effective theory would receive a non-zero mass renormalisation $Q \mapsto Q \left[ 1 + 2 \frac{e}{m_{\eta_0}^2} (m_\pi^2 + 2m_K^2) \right]$ in violation of the Adler-Bardeen theorem.}
The value of \( F_0 \) is usually determined from the decay rate for \( \eta' \to 2\gamma \). In QCD one finds the relation \[ \frac{2\alpha}{\pi} = \sqrt{\frac{3}{2}} F_0 \left( g_{\eta'\gamma\gamma} - g_{Q\gamma\gamma} \right) \] (in the chiral limit) which is derived by coupling the effective Lagrangian (14) to photons. The observed decay rate \[ [57] \] is consistent \[ [58] \] with the OZI prediction for \( g_{\eta'\gamma\gamma} \) if \( F_0 \) and \( g_{Q\gamma\gamma} \) take their OZI values: \( F_0 \simeq F_\pi \) and \( g_{Q\gamma\gamma} = 0 \). Motivated by this observation it is common to take \( F_0 \simeq F_\pi \).

3.1 Glue and the \( \eta \) and \( \eta' \) masses

If we work in the approximation \( m_u = m_d \) and set \( F_0 = F_\pi \), then the \( \eta - \eta' \) mass matrix which follows from (17) becomes

\[
M_{\eta-\eta'}^2 = \begin{pmatrix}
\frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2 & -\frac{2}{3}\sqrt{2}(m_K^2 - m_\pi^2) \\
-\frac{2}{3}\sqrt{2}(m_K^2 - m_\pi^2) & \frac{2}{3}m_K^2 + \frac{1}{3}m_\pi^2 + \tilde{m}_{\eta_0}^2
\end{pmatrix}
\]

(20)

with \( \eta - \eta' \) mixing

\[
|\eta\rangle = \cos \theta |\eta_8\rangle - \sin \theta |\eta_0\rangle
\]

\[
|\eta'\rangle = \sin \theta |\eta_8\rangle + \cos \theta |\eta_0\rangle
\]

(21)

driven predominantly by the large strange-quark mass. The Gell-Mann Okubo mass formula (11) can be seen in the top left matrix element of the mass matrix (20). Diagonalising the \( \eta - \eta' \) mass matrix we obtain values for the \( \eta \) and \( \eta' \) masses:

\[
m_{\eta,\eta'}^2 = (m_K^2 + \tilde{m}_{\eta_0}^2/2) \pm \frac{1}{2} \sqrt{(2m_K^2 - 2m_\pi^2 - \frac{1}{3}\tilde{m}_{\eta_0}^2)^2 + \frac{8}{9}\tilde{m}_{\eta_0}^4}.
\]

(22)

If we turn off the gluon mixing term, then one finds \( m_{\eta'} = \sqrt{2m_K^2 - m_\pi^2} \) and \( m_\eta = m_\pi \). Without any extra input from glue, in the OZI limit, the \( \eta \) would be approximately an isosinglet light-quark state \((\frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d\rangle)\) degenerate with the pion and the \( \eta' \) would a strange-quark state \(|\bar{s}s\rangle\) — mirroring the isoscalar vector \( \omega \) and \( \phi \) mesons. Indeed, in an early paper \[ [4] \] Weinberg argued that the mass of the \( \eta \) would be less than \( \sqrt{3}m_\pi \) without any extra U(1) dynamics to further break the axial U(1) symmetry. Summing over the two eigenvalues in (22) yields \[ [35] \]

\[
m_{\eta}^2 + m_{\eta'}^2 = 2m_K^2 + \tilde{m}_{\eta_0}^2.
\]

(23)

Substituting the physical values of \((m_K^2 + m_\eta^2)\) and \(m_K^2\) in Eq.(23) yields \( \tilde{m}_{\eta_0}^2 = 0.73\text{GeV}^2 \), which corresponds to \( m_\eta = 499\text{MeV} \) and \( m_{\eta'} = 984\text{MeV} \). The value \( \tilde{m}_{\eta_0}^2 = 0.73\text{GeV}^2 \) corresponds to an \( \eta - \eta' \) mixing angle \( \theta \simeq -18 \) degrees — which is within the range -17 to -20 degrees obtained from a study of various decay processes in \[ [58, 59] \]. The physical masses are \( m_\eta = 547\text{MeV} \) and \( m_{\eta'} = 958\text{MeV} \). Closer agreement with the physical masses can be obtained by taking \( F_0 \neq F_\pi \) and including.
higher-order mass terms in the chiral expansion. Two mixing angles \([6, 61]\) enter the \(\eta-\eta'\) system when one extends the theory and \(\mathcal{L}_m\) to \(O(p^4)\) in the meson momentum. (The two mixing angles are induced by \(F_\pi \neq F_K\) due to chiral corrections at \(O(p^4)\)\([45]\).)

An alternative formulation of \(\mathcal{L}_m\) has been developed by Leutwyler \([60]\) and Herrera-Siklody et al. \([62]\) within the framework of the large \(N_c\) approximation. Taking \(m_\eta^2 \sim 1/N_c\) in the chiral limit \([9]\) these authors make a systematic expansion in \(1/N_c = O(\delta)\), \(p = O(\sqrt{\delta})\) and \(m_q = O(\delta)\), where \(m_q\) represents the light quark masses. Although the two formalisms look rather different, the essential dynamical input and assumptions are the same and the final physical predictions should agree within the limitations of the assumptions. One finds that the number of terms expands rapidly as one goes to higher orders in the momentum: at \(O(p^4)\) the general \(U_A(1)\)-extended effective chiral Lagrangian contain altogether 57 potentials before coupling to baryons \([62]\).

### 3.2 OZI violation and the \(\eta'\)–nucleon interaction

The low-energy effective Lagrangian (14) is readily extended to include \(\eta\)–nucleon and \(\eta'\)–nucleon coupling. Working to \(O(p)\) in the meson momentum the chiral Lagrangian for meson-baryon coupling is

\[
\mathcal{L}_{\text{mb}} = \text{Tr} \overline{B}(i\gamma_\mu D^\mu - M_0)B
\]

\[
+ F \text{Tr}(\overline{B}\gamma_\mu \gamma_5[a^\mu, B]) + D \text{Tr}(\overline{B}\gamma_\mu \gamma_5\{a^\mu, B\})
\]

\[
+ \frac{i}{3} K \text{Tr}(\overline{B}\gamma_\mu \gamma_5 B) \text{Tr}(U^\dagger \partial^\mu U) - \frac{G_{QNN}}{2M_0} \partial^\mu Q \text{Tr}(\overline{B}\gamma_\mu \gamma_5 B) + \frac{C}{F_0^4} Q^2 \text{Tr}(\overline{B}B)
\]

(Eq.24)

Here

\[
B = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & p \\
\Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & n \\
\Xi^- & \Xi^0 & -\frac{2}{3} \Lambda
\end{pmatrix}
\]

(Eq.25)

denotes the baryon octet and \(M_0\) denotes the baryon mass in the chiral limit. In Eq.(24) \(D_\mu\) is the chiral covariant derivative and \(a^\mu = -\frac{1}{2F_\pi} \partial_\mu \phi - \frac{1}{2F_0} \sqrt{\frac{2}{3}} \partial_\mu \eta_0 + \ldots\) is the axial-vector current operator. The SU(3) couplings are \(F = 0.459 \pm 0.008\) and \(D = 0.798 \pm 0.008\) \([33]\). The Pauli-principle forbids any flavour-singlet \(J^P = 1^+\) ground-state baryon degenerate with the baryon octet \(B\). In general, one may expect OZI violation wherever a coupling involving the \(Q\)-field occurs.

Following Eq.(16), we eliminate \(Q\) from the total Lagrangian \(\mathcal{L} = \mathcal{L}_m + \mathcal{L}_{\text{mb}}\) through its equation of motion. The \(Q\) dependent terms in the effective Lagrangian become:

\[
\mathcal{L}_Q = \frac{1}{12} \tilde{m}_{\eta_0}^2 \left[ -6\eta_0^2 - \frac{\sqrt{6}}{M_0} G_{QNN} F_0 \partial^\mu \eta_0 \text{Tr}(\overline{B}\gamma_\mu \gamma_5 B) \right]
\]

\[
+ G_{QNN}^2 F_0^2 \left( \text{Tr}\overline{B}\gamma_5 B \right)^2 + 2 \frac{C}{F_0^4} \tilde{m}_{\eta_0}^2 \eta_0^2 \text{Tr}(\overline{B}B)
\]

(Eq.26)
This equation describes the gluonic contributions to the \( \eta \)-nucleon and \( \eta' \)-nucleon interactions. The term \(-\frac{\sqrt{3}}{3m_0} G_{QNN} F_0 \partial^\mu \eta_0 \operatorname{Tr}(B \gamma_\mu \gamma_5 B) \operatorname{Tr}(BB) + \ldots\) is a gluonic (OZI violating) contribution to the \( \eta' \)-nucleon coupling constant, which is \( g_{\eta NN} = \sqrt{\frac{3}{3m_0}} (2D + 2K + G_{QNN} F_0 \tilde{m}_{\eta_0}^2) \) in the notation of (24). The Lagrangian (26) has three contact terms associated with the gluonic potential in \( Q \). We recognise \( \mathcal{L}_{\text{contact}}^{(2)} = -\frac{\sqrt{3}}{12m_0} G_{QNN} \tilde{m}_{\eta_0}^2 \frac{1}{3} C \tilde{m}_{\eta_0}^2 \eta_0 \partial^\mu \operatorname{Tr}(B \gamma_\mu \gamma_5 B) \operatorname{Tr}(BB) \) as the gluonic contact term (3) in the low-energy \( pp \to pp\eta' \) reaction with \( g_{QNN} \equiv \sqrt{\frac{2}{3}} G_{QNN} F_0 \tilde{m}_{\eta_0}^2 \). The term \( \mathcal{L}_{\text{contact}}^{(3)} = \frac{1}{6\sqrt{3}} C \tilde{m}_{\eta_0}^4 \eta_0^2 \operatorname{Tr}(BB) \) is potentially important to \( \eta \)-nucleon and \( \eta' \)-nucleon scattering processes. The contact terms \( \mathcal{L}_{\text{contact}}^{(j)} \) are proportional to \( \tilde{m}_{\eta_0}^2 \) \( (j = 2) \) and \( \tilde{m}_{\eta_0}^4 \) \( (j = 3) \) which vanish in the formal OZI limit. Phenomenologically, the large masses of the \( \eta \) and \( \eta' \) mesons means that there is no reason, a priori, to expect the \( \mathcal{L}_{\text{contact}}^{(j)} \) to be small.

Gluonic \( U_A(1) \) degrees of freedom induce several “\( \eta' \)-nucleon coupling constants”. The three couplings \( (g_{\eta NN}, G_{QNN} \text{ and } C) \) are each potentially important in the theoretical description the \( \eta' \)-nucleon and \( \eta' \)-two-nucleon systems. Different combinations of these coupling constants are relevant to different \( \eta' \) production processes and to the flavour-singlet Goldberger-Treiman relation. Testing the sensitivity of \( \eta' \)-nucleon interactions to the gluonic terms in the effective chiral Lagrangian for low-energy QCD will teach us about the role of gluons in chiral dynamics.

4 Proton-proton collisions

How important is the contact interaction \( \mathcal{L}_{\text{contact}}^{(2)} \) in the \( pp \to pp\eta' \) reaction?

The T-matrix for \( \eta' \) production in proton-proton collisions, \( p_1(\vec{p}) + p_2(-\vec{p}) \to p + p + \eta' \), at threshold in the centre of mass frame is

\[
T_{\text{th}}^{\text{cm}}(pp \to pp\eta') = A\left[ i(\vec{\sigma}_1 - \vec{\sigma}_2) + \vec{\sigma}_1 \times \vec{\sigma}_2 \right] \vec{p} \tag{27}
\]

where \( A \) is the (complex) threshold amplitude for \( \eta' \) production. Measurements of the total cross-section for \( pp \to pp\eta' \) have been published by COSY [14] and SATURNE [64] between 1.5 and 24 MeV above threshold – see Fig.2.

The energy dependence of the data are well described by phase space plus proton-proton final state interaction (neglecting any \( \eta' \)-\( p \) FSI). Using the model of Bernard et al. [13] treating the \( pp \) final state interaction in effective range approximation one finds a good fit to the measured total cross-section data with

\[
|A| = 0.21 \text{ fm}^4. \tag{28}
\]

The present (total cross-section only) data on \( pp \to pp\eta' \) is insufficient to distinguish between possible production mechanisms involving the (short-range) gluonic contact term (3) and the long-range contributions associated with meson exchange...
models. Long-range meson exchange contributions to $A$ involve the exchange of a $\pi^0$, $\eta$, $\omega$ or $\rho^0$ between the two protons and the emission of an $\eta'$ from one of the two protons. This process involves $g_{\eta^0NN}$. The contact term (3) involves the excitation of gluonic degrees of freedom in the interaction region, is isotropic and involves the product of $G_{QNN}$ and the second gluonic coupling $C$. In their analysis of the SATURNE data on $pp \rightarrow ppp\eta'$ Hibou et al. [64] found that a one-pion exchange model adjusted to fit the S-wave contribution to the $pp \rightarrow ppp\eta'$ cross-section near threshold yields predictions about 30% below the measured $pp \rightarrow ppp\eta'$ total cross-section. The gluonic contact term (3) is a candidate for additional, potentially important, short range interaction.

To estimate how strong the contact term must be in order to make an important contribution to the measured $pp \rightarrow ppp\eta'$ cross-section, let us consider the extreme scenario where the value of $|A|$ in Eq.(28) is saturated by the contact term (3). If we take the estimate $g_{QNN} \sim 2.45$ (or equivalently $G_{QNN} \sim +60\text{GeV}^{-3}$) suggested by the polarised deep inelastic scattering and the flavour-singlet Goldberger-Treiman relation below Eq.(2), then we need $C \sim 1.8\text{GeV}^{-3}$ to saturate $|A|$. The OZI violating parameter $C \sim 1.8\text{GeV}^{-3}$ seems reasonable compared with $G_{QNN} \sim 60\text{GeV}^{-3}$.

To help resolve the different production mechanisms it will be important to test the isospin dependence of the $pN \rightarrow pN\eta'$ process through quasi-free production from the deuteron [29, 11] and to make a partial wave analysis of the $\eta'$ production process, following the work pioneered by CELSIUS for $\eta$ production [66]. Here, it is interesting to note that the recent higher-energy ($p_{\text{beam}} = 3.7\text{GeV}$) measurement of the $pp \rightarrow ppp\eta'$ cross-section by the DISTO collaboration [67] suggests isotropic $\eta'$

Figure 2: The COSY and SATURNE data on $pp \rightarrow ppp\eta'$
production at this energy.

In high energy experiments central production of $\eta$ and $\eta'$ mesons in proton-proton collisions at 450 GeV has been studied by the WA102 Collaboration at CERN \[68\]. At this energy the production cross-section is greatest when the azimuthal angle between the $p_T$ vectors of the two protons is 90 degrees. This result has been interpreted \[69\] in terms of pomeron-pomeron fusion as evidence that the pomeron transforms as a non-conserved vector current.

5 FSI, photoproduction and exotic mesons

5.1 Photoproduction

$\eta$ photoproduction and electroproduction has been studied extensively in experiments at Jefferson Laboratory \[70, 71\], GRAAL \[72\] and MAMI \[73\] up to centre of mass energy $W \sim 1.65$GeV and up to $Q^2 = 3.6$GeV$^2$. JLab data on $\eta'$ photoproduction will soon be available \[74\]. $\eta$ photoproduction close to threshold is characterised by the s-wave resonance $N^*(1535)$ which decays strongly (about 45\% \[71\]) to $\eta N$. P-wave contributions are observed at photon energies greater than 900MeV. The $N^*(1535)$ contribution to the $(\eta)$ electroproduction cross-section was observed to fall away more slowly with increasing $Q^2$ than other resonance contributions. Further evidence for $\eta N$ resonance contributions has been observed in $pn \rightarrow \eta d$ production experiments at CELSIUS \[74\]. $\eta$ meson production from nuclei has also been studied in heavy-ion collisions \[75\] and in photoproduction experiments \[76, 77\], where some evidence for the broadening of the s-wave $N^*(1535)$ resonance in nuclei was reported in \[77\].

The $\eta$ and $\eta'$ photoproduction processes have recently been investigated within a coupled channels model of final state interaction using the Lippmann-Schwinger equation with potentials derived from the $U_A(1)$-extended chiral Lagrangian \[20\]. The shape of the $\eta$ photoproduction cross-section close to threshold predicted by the model was found to be quite sensitive to the OZI violating coupling $C$ in Eq.(24) after $\eta - \eta'$ mixing.

5.2 Light mass “exotic” resonances

Recent experiments at BNL \[17\] and CERN \[18\] have revealed evidence for QCD “exotic” meson states with quantum numbers $J^{PC} = 1^{-+}$. These mesons are particularly interesting because the quantum numbers $J^{PC} = 1^{-+}$ are inconsistent with a simple quark-antiquark bound state suggesting a possible “valence” gluonic component – for example through coupling to the operator $[\bar{q}\gamma_\mu q G^{\mu\nu}]$. Two such exotics, denoted $\pi_1$, have been observed through $\pi^- p \rightarrow \pi_1 p$ at BNL \[17\]: with masses 1400 MeV (in decays to $\eta \pi$) and 1600 MeV (in decays to $\eta' \pi$ and $\rho \pi$). The $\pi_1(1400)$ state has also been observed in $\bar{p}N$ processes by the Crystal Barrel Collaboration at CERN \[18\]. These states are considerably lighter than the predictions (about 1900 MeV) of quenched lattice QCD \[73\] and QCD inspired models \[79\] for the lowest mass $q\bar{q}g$ state with quantum numbers $J^{PC} = 1^{-+}$. While chiral corrections may bring the lattice predictions down by about 100 MeV \[80\] these results suggest
that, perhaps, the "exotic" states observed by the experimentalists might involve significant meson-meson bound state contributions.

The decays of the light mass exotics to $\eta$ or $\eta'$ mesons plus a pion may hint at a possible connection to axial U(1) dynamics. This idea has recently been investigated in a model of final state interaction in $\eta\pi$ and $\eta'\pi$ scattering using coupled channels and the Bethe-Salpeter equation following the approach in [31]. In this calculation the meson-meson scattering potentials were derived from the mesonic chiral Lagrangian working to $O(p^2)$ in the meson momenta. Fourth order terms in the meson fields are induced by the first two terms in (14) and also by the OZI violating interaction $\lambda Q^2 \text{Tr} \partial_\mu U \partial^\mu U^\dagger$ [81]. A simple estimate for $\lambda$ can be obtained from the decay $\eta' \to \eta\pi\pi$ yielding two possible solutions with different signs. These two different solutions yield two different dynamically generated resonance structures when substituted into the Bethe-Salpeter equation. The positive-sign solution for $\lambda$ was found to dynamically generate a scalar resonance with mass $\sim 1300$ MeV and width $\sim 200$ MeV (and no p-wave resonance). This scalar resonance is a possible candidate for the $a_0(1450)$. The negative-sign solution for $\lambda$ produced a p-wave resonance with exotic quantum numbers $J^{PC} = 1^{--}$, mass $\sim 1400$ MeV and width $\sim 300$ MeV – close to the observed exotics – and no s-wave resonance. (The width of the $\pi_1(1400)$ state measured in decays to $\eta\pi$ is $385 \pm 40$ MeV; the width of the $\pi_1(1600)$ measured in decays to $\eta'\pi$ is $340 \pm 64$ MeV.) These calculations clearly illustrate the possibility to describe light-mass exotics as hadronic resonances in the $\eta'\pi$ and $\eta\pi$ systems. The topological charge density mediates the coupling of the light-mass exotic state to the $\eta\pi$ and $\eta'\pi$ channels in the model [31].

6 J/Ψ, D_s and B decays

$\eta'$ production in heavy-quark meson decays has also been measured, where the strikingly large branching ratios have been observed for $D_s$ and $B$ decays to an $\eta'$ plus additional hadrons. We give a brief overview of this data and its theoretical interpretation.

6.1 J/Ψ → η'γ decays

This decay violates OZI since it proceeds from a $\bar{c}c$ state (the J/Ψ) to a light-quark state (the $\eta'$) and necessarily proceeds through a gluonic intermediate state. One finds [59]

$$R_{J/\Psi} = \frac{\Gamma(J/\Psi \to \eta'\gamma)}{\Gamma(J/\Psi \to \eta\gamma)}$$

$$= \left| \frac{\langle \text{vac} | \frac{\alpha_s}{4\pi} G \tilde{G} | \eta' \rangle}{\langle \text{vac} | \frac{\alpha_s}{4\pi} GG | \eta \rangle} \right|^2 \frac{(1 - \frac{m_{\eta'}}{m_{J/\Psi}})^3}{(1 - \frac{m_{\eta}}{m_{J/\Psi}})^3} = 5.0 \pm 0.6$$

corresponding to a mixing angle of $\theta = -17.3 \pm 1.3$ degrees.
6.2 $D_s^+ \to \eta'\rho^+$ decays

$D_s$ decays have been measured by the CLEO Collaboration [20] who observed that

$$\frac{\Gamma(D_s^+ \to \eta'\rho^+)}{\Gamma(D_s^+ \to \eta\ell^+\nu)} = 12.0 \pm 4.3$$

exceeds

$$\frac{\Gamma(D_s^+ \to \eta\rho^+)}{\Gamma(D_s^+ \to \eta\ell^+\nu)} = 4.4 \pm 1.2$$

The branching ratio for $D_s^+ \to \eta'\rho^+$ is much larger than the value predicted by factorisation arguments involving $c \to sW^-$, $W^- \to ud$ with the strange quark hadronizing with the antistrange quark from the original $D_s^+$ to produce the $\eta'$ and the $ud$ combination hadronizing to produce the $\rho^+$ outside the original $D_s^+$ wavefunction. Ball et al. [59] have argued that $\eta'$ production in this process could be enhanced by annihilation of the $c\bar{s}$ in $D_s$ into a $W^+$ and two gluons with the two gluons subsequently hadronizing to produce the $\eta'$.

6.3 $B \to \eta'X$ decays

Strikingly large branching ratios for $B$ decays into an $\eta'$ and additional hadrons have been observed at the $B$-factories CLEO [18, 83], BABAR [21, 22] and BELLE [23]. For semi-inclusive decays CLEO found $B(B \to \eta'X_s) = (6.2 \pm 1.6 \pm 1.3)x10^{-4}$ where $X_s$ denotes a final state consisting of one charged kaon and up to four pions with the constraint $2.0 < p_{\eta'} < 2.7$GeV [19]. This result has recently been confirmed, with higher precision, by BABAR: $B(B \to \eta'X_s) = (6.8^{+0.7}_{-1.0}(\text{stat.}) \pm 1.0(\text{syst.})^{+0.9}_{-1.5})x10^{-4}$. The $M_X$ spectrum in these $B \to \eta'X_s$ measurement was observed to peak about 2 GeV [19, 22]. CLEO also found an upper limit on the branching ratio for the corresponding $B \to \eta X_s$ decay: $B(B \to \eta X_s) < 4.4x10^{-4}$ — consistent with the theoretical expectation that this rate should be suppressed relative to the $B \to \eta'X$ process by $\tan^2 \theta_W \sim 0.1$ if the decay into the $\eta'$ proceeds through its singlet component (possibly through a gluonic intermediate state — see below). Exclusive two-particle $B \to \eta'K$ and $B \to \eta K^*$ decays have also been observed with branching ratios listed in Table 1.

| Decay mode | CLEO (x 10^{-6}) | BABAR (x 10^{-6}) | BELLE (x 10^{-6}) |
|------------|----------------|-----------------|----------------|---|
| $B^+ \to \eta'K^+$ | $80^{+10}_{-9} \pm 7$ | $70 \pm 8 \pm 5$ | $79^{+14}_{-14} \pm 9$ |
| $B^0 \to \eta'K^0$ | $89^{+13}_{-16} \pm 9$ | $42^{+13}_{-11} \pm 4$ | $55^{+16}_{-16} \pm 8$ |
| $B^+ \to \eta K^+$ | $< 6.9$ | $< 35$ | $< 35$ |
| $B^0 \to \eta K^0$ | $< 9.3$ | $< 35$ | $< 35$ |
| $B^+ \to \eta K^{*+}$ | $26.4^{+9.6}_{-8.2} \pm 3.3$ | $19.8^{+6.5}_{-5.6} \pm 1.7$ | $19.8^{+6.5}_{-5.6} \pm 1.7$ |
| $B^0 \to \eta K^{*0}$ | $13.8^{+8.8}_{-4.9} \pm 1.6$ | $22.1^{+11.3}_{-9.2} \pm 3.3$ | $22.1^{+11.3}_{-9.2} \pm 3.3$ |

Table 1: Measured branching ratios for exclusive 2-particle $B$ decays involving an $\eta'$. 

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A number of different theoretical explanations have been proposed to explain the large branching ratio for $B \to \eta'X$. These include

(a) conventional $b \to q\bar{q}$ operators with constructive interference between the $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ components of the $\eta'$ \[84]\;

(b) $b \to c\bar{c}s$ decays enhanced by a large intrinsic charm component in the $\eta'$ wavefunction \[85]\;

(c) $b \to sg^*$ (generally involving the gluon being radiated from the virtual quark in a penguin diagram) followed by $g^* \to q\bar{q}'$ from the QCD axial anomaly \[86, 87, 88]\;.

The observed branching ratio is larger than what is expected from scenario (a). Furthermore, scenarios (a) and (b) will give an $X_s$ mass distribution peaked near 1.5GeV whereas only scenario (c) gives a three-body $gs\bar{q}X_s$ mass spectrum that peaks above 2GeV. The large intrinsic charm component in the $\eta'$ wavefunction proposal assumes a large decay constant $f_{\eta'} \simeq 50 - 180\text{MeV}$ which appears to be disfavoured by phenomenology \[89]\; and recent theoretical calculations \[90]\;.

For the exclusive channels a large ratio of $\mathcal{B}(B \to \eta'K, \eta'K^*)$ to $\mathcal{B}(B \to \eta K, \eta'K^*)$ was predicted qualitatively \[84]\; in terms of the interference of the two internal gluonic penguin diagrams ($b \to \bar{s}g^*$, $g^* \to q\bar{q}$ with the $q\bar{q}$ pair from the gluon hadronizing together with the $s$ quark from the $b$ decay or $u$ quark from the $B^+$ wavefunction to produce the $\eta'$ and the $(K, K^*)$) constructive for $\eta'K$ and $\eta'K^*$ and destructive for $\eta K$ and $\eta'K^*$. Most detailed calculations \[91, 92]\; predict a large branching ratio for $B \to \eta'K$ (but smaller than the observed signal) but no enhancement for $B \to \eta K^*$. More recent calculations \[93, 94]\; show that the expectations for $B \to \eta K^*$ can readily be enhanced: the effective Hamiltonian calculations accomplish this by increasing the relevant form-factor or decreasing the strange quark mass. We refer to \[95]\; for a pedagogical review of factorization issues in these two-body decays. The penguin calculations fall somewhat short of explaining the large rate for $B \to \eta'K$ suggesting that the solution may involve contributions which are unique to the $\eta'$.

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