An Analytical Model to Describe the Compression in Turbomolecular Pumps and Roots Blowers

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Abstract. An analytical model is presented, useful in practice, for calculating and analysing the compression curves of classical turbomolecular pumps, wide range turbomolecular pumps and Roots blowers. It is demonstrated that the model, primarily proposed for classical turbomolecular pumps, can be applied to wide range turbomolecular pumps and Roots blowers as well. The model is based on an ordinary differential equation for the pressure as a function of position inside the pump. Solving the differential equation makes it possible both to calculate the compression curves for a finite gas throughput ($Q > 0$) and for zero gas throughput. A hypothesis is posed holding that the compression curve for zero gas throughput can be derived from a compression curve for a finite gas throughput, e.g., $Q = 1$ sccm in the case of turbomolecular pumps. In the case of Roots blowers a proposal is made how to describe the backleakage phenomenon quantitatively. For each type of vacuum pump mentioned above the comparison with experimental data shows that the model provides an excellent qualitative and quantitative reproduction of the observed phenomena in the whole relevant pressure range, i.e., perfect agreement over more than five pressure decades is achieved.

1. Introduction

Pumping speed and compression are undoubtedly the most important characteristics of turbomolecular pumps and Roots blowers, since they determine in what applications and with which backing pumps turbomolecular pumps and Roots blowers can be used. In this article an analytical model is presented, useful in practice, for calculating and analysing the compression curves of classical turbomolecular pumps, wide range turbomolecular pumps and Roots blowers.

The compression $K$ of a turbomolecular pump or a Roots blower is defined as the ratio of foreline pressure $p_{FV}$ to high-vacuum pressure $p_{HV}$, i.e.,

$$K = \frac{p_{FV}}{p_{HV}} \quad (1.1)$$

The dependence of compression on foreline pressure $K(p_{FV})$ is referred to as compression curve. Accordingly, the graphs of “high-vacuum pressure as a function of foreline pressure” (“$p_{HV}$ versus $p_{FV}$”) and “compression as a function of foreline pressure” (“$K$ versus $p_{FV}$”) are the central subject of this article. The performance of any turbomolecular pump and any Roots blower can be fully and reliably evaluated with the aid of these graphs.
2. Compression in a turbomolecular pump without compound stage

2.1 The model

A turbomolecular pump without compound stage is often referred to as a “classical turbomolecular pump”. A classical turbomolecular pump is a bladed turbine that compresses a gas provided the mean free path of the gas molecules is larger than the spacing between rotor blades and adjacent stator blades.

Within the scope of the model presented in this article the turbomolecular pump (abbreviated as “TMP” hereafter) is taken as a “black box” fitted with an inlet flange (= high-vacuum flange) at position \( x = 0 \) and a foreline port at position \( x = L \). \( L \) is referred to as the effective length of the TMP. The gas passing through the black box is compressed from the high-vacuum pressure (= inlet pressure) \( p_{HV} = p(x = 0) \) to the foreline pressure (= backing pressure) \( p_{FV} = p(x = L) \).

The basic assumption is that the net gas throughput \( Q \) can be written as the difference of the maximum gas throughput in forward direction \( Q_{\text{max}} \) and the backstreaming \( Q_{\text{bs}} \):

\[
Q = p_1 \cdot S_{\text{max}} = Q_{\text{max}} - Q_{\text{bs}} = p \cdot S_{\text{max}} - r \frac{1}{\eta v} (p + p_2)(dp/dx)
\] (2.1)

Here the following assumptions have been made:

1. The net gas throughput \( Q \) can be written as the product of the pressure \( p_1 \) and the pumping speed \( S_{\text{max}} \). \( p_1 \) is the lowest high-vacuum pressure which can be attained with the TMP at the prescribed gas throughput \( Q \) \{ \( p_1 = p_1(Q) \) \}.

\( S_{\text{max}} \) is the maximum pumping speed at the inlet flange which can be attained with the TMP at the prescribed gas throughput \( Q \) \{ \( S_{\text{max}} = Q/p_1 = S(p_1) \) \}.

2. The maximum gas throughput in forward direction \( Q_{\text{max}} \) can be written as the product of the pressure \( p \) inside the TMP and \( S_{\text{max}} \) \{ \( p_{HV} \leq p(x) \leq p_{FV}; \ 0 \leq x \leq L \) \}.

3. The backstreaming \( Q_{\text{bs}} \) is proportional to \( p + p_2 \) times pressure gradient. \( p_2 \) is an empirical parameter which depends on the geometry inside the TMP. \( \eta v \) is the dynamic viscosity in the range of laminar viscous flow. The parameter \( p_2 \) and the backstreaming coefficient \( r \) may depend on \( Q \) \{ \( p_2 = p_2(Q), r = r(Q) \) \}.

The ansatz (2.1) results in the differential equation

\[
(p + p_2)(dp/dx) = (p - p_1)(C_2/L)
\] (2.2)
with the boundary condition \( p(x = L) = p_{FV} \).

The solution of this differential equation is

\[
p = p_1 + (p_{FV} - p_1) \exp(\beta (p_{FV} - p - C_2[1 - x/L]))
\] (2.3)

\[
\beta = 1/(p_1 + p_2) \quad \quad C_2 = \eta v \cdot S_{\text{max}} \cdot L/r
\]

Note that \( p_2 \) and \( C_2 \) are the crucial parameters of the model.

2.2 High-vacuum pressure as a function of foreline pressure

In the experiment we have direct access to the high-vacuum pressure \( p_{HV} \) and to the foreline pressure \( p_{FV} \) as well. Consequently, we are interested in the relationship between \( p_{HV} \) and \( p_{FV} \).
Having in mind that \( p(x = 0) = p_{HV} \) we obtain from (2.3):

\[
p_{HV} = p_1 + (p_{FV} - p_1) \exp(\beta(p_{FV} - p_{HV} - C_2))
\] (2.4)

As (2.4) cannot be solved for \( p_{HV} \) explicitly, the graphs of “\( p_{HV} \) versus \( p_{FV} \)” have to be calculated numerically.

Initially, the parameters \( p_1, p_2 \) and \( C_2 \) are unknown. They are determined by fitting the “\( p_{HV} \) versus \( p_{FV} \)” graphs to the experimental data.

![Diagram](image)

**Figure 1.** High-vacuum pressure as a function of foreline pressure, classical TMP, gas type: nitrogen, gas throughput in mbar l/s: a – 1.84E-02; b – 1.84E-01; c – 1.84; d – 3.68, experimental data: symbol †

Figure 1 exemplifies the graphs of “\( p_{HV} \) versus \( p_{FV} \)” and compares the mathematical result of the model, i.e., equation (2.4), with the data resulting from the experiment. The curves drawn in heavy lines are graphical representations of equation (2.4) for the individual measurement series. Each measurement series is characterized by a certain prescribed gas throughput \( Q \). The experimental data are indicated by the symbol †. It is evident that the measured dependence of the high-vacuum pressure on the foreline pressure is reproduced excellently by equation (2.4).

As an example: In the case of measurement series “a” from figure 1 with prescribed nitrogen gas throughput \( Q_a = 1.84E-02 \) mbar l/s, the value 8.00E-05 mbar was chosen for the parameter \( p_1 \). From that we know that the lowest high-vacuum pressure that can be attained with the TMP at a nitrogen gas throughput of 1.84E-02 mbar l/s is 8.00E-05 mbar.

For the parameters \( p_2 \) and \( C_2 \) the values 4.95E-02 mbar and 1.00 mbar respectively were chosen.
It is remarkable that the analytical model presented above provides an excellent qualitative and quantitative reproduction of the measured dependence of the high-vacuum pressure on the foreline pressure in the whole relevant pressure range, i.e., there is perfect agreement over more than five pressure decades: At low foreline pressures \((p_{FV} < 0.20 \, \text{mbar})\) \(p_{HV}\) attains the value \(8.00 \times 10^{-5} \, \text{mbar}\). For foreline pressures larger than \(0.50 \, \text{mbar}\) the high-vacuum pressure shoots up rapidly and approaches asymptotically the line \(p_{HV} = p_{FV}\) at high foreline pressures \((p_{FV} > 2 \, \text{mbar})\). Note that for all graphs shown in figure 1 one and the same \((p_2; C_2)\) parameter set has been used: \((p_2; C_2) = (4.95 \times 10^{-2} \, \text{mbar}; 1.00 \, \text{mbar})\). This means that the parameters \(p_2\) and \(C_2\) do not depend on the gas throughput \(Q\). Consequently, the quotient \(S_{\text{max}}/r\) does not depend on \(Q\), as we have to assume that \(\eta_{v}\) and \(L\) are constants which do not depend on \(Q\). These remarkable and very important results have been confirmed by all experiments and evaluations performed so far.

![Figure 2. Compression as a function of foreline pressure, classical TMP, gas type: nitrogen, gas throughput in mbar l/s: a – 1.84E-02; b – 1.84E-01; c – 1.84; d – 3.68, experimental data: symbol †](image)

\(2.3 \, \text{Compression as a function of foreline pressure}\)

On the basis of the definition for the compression \(K\) given above the graphs of \(p_{HV} \text{ versus } p_{FV}\) shown in figure 1 can be transformed without difficulty into graphs of \(K \text{ versus } p_{FV}\). Figure 2 shows the result of that purely mathematical transformation. Note that the measurement series “a” from figure 1 is also identified by “a” again in figure 2, and correspondingly for measurement series “b” to “d”. The curves drawn in heavy lines are graphical representations of equation (1.1) with \(p_{HV} = p_{HV}(p_{FV})\) calculated from equation (2.4). Again, the experimental data are indicated by the symbol †.
In the case of measurement series “a” from figure 2 with prescribed nitrogen gas throughput $Q_a = 1.84 \times 10^{-2} \text{ mbar l/s}$, the compression curve $K(p_{\text{FV}})$ starts at $(p_1 = 8.00 \times 10^{-5} \text{ mbar}; K = 1)$ and runs linearly up to $p_{\text{FV}} = 0.20 \text{ mbar}$. At $p_{\text{FV}} = 0.46 \text{ mbar}$ the compression reaches its maximum value ($K = 5188$). For foreline pressures larger than $0.50 \text{ mbar}$ the compression curve drops rapidly and approaches asymptotically the value $K = 1$ at high foreline pressures ($p_{\text{FV}} > 2 \text{ mbar}$).

Since “a” is the measurement series with the lowest gas throughput and “d” is the one with the highest gas throughput, one fact becomes clear immediately: The compression of a TMP increases, if the gas throughput is reduced at a prescribed foreline pressure.

It is worth to remark that equation (2.4) can be simplified, if the compression is larger than 20 and/or $p_{\text{HV}}$ small compared to $C_2$. Then, $p_{\text{HV}}$ can be neglected in the argument of the exponential function on the right hand side of equation (2.4) and we obtain an analytical function describing the dependence of the high-vacuum pressure on the foreline pressure:

$$p_{\text{HV}} = p_{\text{HV}}(p_{\text{FV}}) = p_1 + (p_{\text{FV}} - p_1) \cdot \exp \left( \beta (p_{\text{FV}} - C_2) \right) \quad (2.5)$$

Using the “analytical approximation” (2.5) with the parameter set chosen for curve “a” ($p_1 = 8.00 \times 10^{-5} \text{ mbar}; p_2 = 4.95 \times 10^{-2} \text{ mbar}; C_2 = 1.00 \text{ mbar}$) the compression curve $K(p_{\text{FV}})$ was plotted in figure 2 as well. Note that this compression curve intersects the $K = 1$-axis at $C_2 = 1.00 \text{ mbar}$ and does not approach asymptotically the value $K = 1$ at high foreline pressures, but coincides with curve “a”, if the compression is larger than 20 and/or $p_{\text{HV}}$ small compared to $C_2$.

### 2.4 Zero-throughput compression

The **zero-throughput compression** $K_0$ of a TMP is defined as the pump’s compression at zero gas throughput. The problem raised by this definition is that the required **zero** gas throughput can never be attained. It is not sufficient to provide a blind flange on the measuring dome or on the TMP. The leak rate, outgassing and desorption from the walls can be made very very small, but they can never be reduced to zero. This is the fundamental problem of all attempts to measure true zero-throughput compression curves in an experiment.

On the other hand, with the model presented in this article zero-throughput compression curves can be calculated without difficulty. It is easy to show that in the case $Q = 0$ the dependence of the high-vacuum pressure on the foreline pressure is given by

$$p_{\text{HV}} = p_{\text{FV}} \cdot \exp \left( \frac{p_{\text{FV}} - p_{\text{HV}} - C_2}{p_2} \right) \quad (2.6)$$

If $K_0$ is larger than 20 and/or $p_{\text{HV}}$ small compared to $C_2$, then we obtain analytical functions for $p_{\text{HV}}$ and $K_0$:

$$p_{\text{HV}} = p_{\text{HV}}(p_{\text{FV}}) = p_{\text{FV}} \cdot \exp \left( \frac{(p_{\text{FV}} - C_2)}{p_2} \right) \quad (2.7)$$

$$K_0 = K_0(p_{\text{FV}}) = \exp \left( - \frac{(p_{\text{FV}} - C_2)}{p_2} \right) \quad (2.8)$$

Furthermore, if the foreline pressure $p_{\text{FV}}$ is small compared to $C_2$, then the zero-throughput compression is a constant:

$$k_0 = \exp \left( \frac{C_2}{p_2} \right) \quad (2.9)$$

In the product brochures of TMP manufacturers $k_0$ is referred to as the **compression at zero throughput**. A striking result of the model presented in this article is that $k_0$ is just given by the exponential of the ratio of $C_2$ to $p_2$. 

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With figure 1 and figure 2 it has been demonstrated that there is strong evidence that the parameters \( p_2 \) and \( C_2 \) are constants provided the gas throughput \( Q \) is sufficiently small. Therefore, the hypothesis is posed holding that zero-throughput compression curves can be calculated with \( p_2 \) and \( C_2 \) determined from compression curves for finite gas throughput.

Figure 3. Compression as a function of foreline pressure, classical TMP, gas type: hydrogen, gas throughput in mbar·l/s: \( a \) – 1.84E-02; \( b \) – 1.84E-01; \( c \) – 9.20E-01; \( z \) – zero, experimental data: symbol †

Figure 3 exemplifies this procedure by presenting four compression curves of a classical TMP for hydrogen. Note again that for the graphs “a” to “c” shown in figure 3 as heavy lines one and the same \((p_2; C_2)\) parameter set has been used: \((p_2; C_2) = (6.20E-02 mbar; 0.40 mbar)\). With this set of parameters the zero-throughput compression curve – indicated as curve “z” – was calculated using equation (2.6). The experimental data for gas throughput “zero” are indicated by the symbol †. Comparing the “prediction” of the model with the experimental data we find a striking agreement. The general trend is that for the gases hydrogen and helium this high degree of agreement is confirmed by many experiments and evaluations. For the gases nitrogen and argon, however, the predicted compression for zero throughput is significantly higher than the one observed in the corresponding experiment. This phenomenon has to be examined in more detail.
3. Compression in a turbomolecular pump with compound stage
A turbomolecular pump with compound stage is often referred to as a “wide range turbomolecular pump”. In order to calculate and to analyse the compression curves of wide range TMPs it is proposed to use the model discussed above twice, i.e., it is proposed to use the same model and the same mathematical formalism for the turbo stage as well as for the compound stage. In this section it will be demonstrated that this procedure works perfectly.

The calculations are based on the assumption that the turbo stage compresses the gas from the high-vacuum pressure $p_{HV}$ to an intermediate pressure $p_{int}$ whereas the compound stage compresses the gas from that intermediate pressure to the foreline pressure $p_{FV}$. Consequently, we can use the following equations:

**Turbo stage:**

\[
\begin{align*}
K_1 &= \frac{p_{int}}{p_{HV}} \\
\beta_1 &= \frac{1}{(p_{11} + p_{21})} \\
C_{21} &= \left( \frac{\eta \cdot S_{max,1} \cdot L_1}{r_1} \right)
\end{align*}
\]  

**Compound stage:**

\[
\begin{align*}
K_2 &= \frac{p_{FV}}{p_{int}} \\
\beta_2 &= \frac{1}{(p_{12} + p_{22})} \\
C_{22} &= \left( \frac{\eta \cdot S_{max,2} \cdot L_2}{r_2} \right)
\end{align*}
\]  

**Total compression:**

\[
K = K_1 \cdot K_2 = \frac{p_{FV}}{p_{HV}}
\]
As an example, figure 4 presents two compression curves (“a”, “z”) of a wide range TMP for helium as well as the corresponding compression curves of the turbo stage (“aT”, “zT”) and the corresponding compression curves of the compound stage (“aC”, “zC”). Note that in the case of the turbo stage the foreline pressure is not \( p_{FV} \), but the foreline pressure is identical with the intermediate pressure \( p_{int} \). The measurement series “a” is characterized by a prescribed helium gas throughput \( Q_a = 1.84E-02 \) mbar l/s. The best fit to the experimental data indicated by the symbol \( \square \) is achieved by choosing the following parameters:

Turbo stage – \( p_{11} = 2.80E-05 \) mbar; \( p_{21} = 0.12 \) mbar; \( C_{21} = 1.13 \) mbar

Compound stage – \( p_{12} = 2.00E-03 \) mbar; \( p_{22} = 2.00 \) mbar; \( C_{22} = 9.90 \) mbar

It is evident that the measured dependence of the compression on the foreline pressure is excellently reproduced by the calculation in the whole relevant pressure range.

With the parameters \( p_{21} = 0.12 \) mbar; \( C_{21} = 1.13 \) mbar; \( p_{22} = 2.00 \) mbar and \( C_{22} = 9.90 \) mbar the zero-throughput compression curves “z”, “zT” and “zC” can be calculated using the procedure presented in section 2. Obviously, in the pressure range from 1 to 10 mbar the measured values for the compression are higher – typically by a factor of two – than the values predicted by the calculation. Nevertheless, it is remarkable that the model predicts a reasonable zero-throughput compression \( k_0 \).

Again, the general trend is that with regard to the zero-throughput compression \( k_0 \) there is a high degree of agreement between experiment and calculation for the gases hydrogen and helium. For the gases nitrogen and argon, however, the predicted compression for zero throughput is significantly higher than the one observed in the corresponding experiment.

4. Compression in a Roots blower

4.1 The model

In section 2 an analytical model has been presented which can be used to calculate and to analyse the compression curves of TMPs. In this section it is demonstrated that the same model and the same mathematical formalism can be used to calculate and to analyse the compression curves of Roots blowers as well.

Again, within the scope of the model presented in this article the Roots blower is taken as a “black box” fitted with an inlet flange (= high-vacuum flange) at position \( x = 0 \) and a foreline port at position \( x = L \). \( L \) is referred to as the effective length of the Roots blower.

The gas passing through the black box is compressed from the high-vacuum pressure (= inlet pressure) \( p_{HV} = p(x = 0) \) to the foreline pressure (= backing pressure) \( p_{FV} = p(x = L) \).

The basic assumption is that the net gas throughput \( Q \) can be written as

\[
Q = p_1 S_{max} = Q_{max} - Q_{bs} - Q_{bl} = p_{HV} S_{max} = r(1/\eta_v)(p + p_2) + dp/dx - p_{FV} S_{bl}
\]

Here the following assumptions have been made:

1. As in section 2: The net gas throughput \( Q \) can be written as the product of the pressure \( p_1 \) and the pumping speed \( S_{max} \).

2. The maximum gas throughput in forward direction \( Q_{max} \) can be written as the product of the high-vacuum pressure \( p_{HV} \) and \( S_{max} \). This is the same ansatz as in the standard theory for Roots blowers.
3. As in section 2: The backstreaming \( Q_{bs} \) is proportional to \( p + p_2 \) times pressure gradient. \( p_2 \) is an empirical parameter which depends on the geometry inside the Roots blower. 

\( \eta \) is the dynamic viscosity in the range of laminar viscous flow. 
The parameter \( p_2 \) and the backstreaming coefficient \( r \) may depend on \( Q \) \( \{ p_2 = p_2(Q), r = r(Q) \} \). 

This ansatz is different from the term that appears in the standard theory for Roots blowers. In that theory a pressure-independent conductance \( C \) is used \( \{ Q_{bs} = C(p_{FV} - p_{HV}) \} \).

4. In a Roots blower the maximum gas throughput in forward direction is reduced by backleakage. 
Backleakage means that gas molecules are transported by adsorption and desorption processes from the foreline side of the Roots blower to its high-vacuum side. The backleakage \( Q_{bl} \) can be written as the product of the foreline pressure \( p_{FV} \) and the pumping speed \( S_{bl} \). This is the same ansatz as in the standard theory for Roots blowers.

At first, the backleakage term in (4.1) is neglected. This ansatz results in the differential equation

\[
\left( r/\eta \right) (p + p_2) \frac{dp}{dx} = \left( p_{HV} - p_1 \right) S_{max}
\]

with the boundary condition \( p(x = 0) = p_{HV} \).

The solution of this differential equation is

\[
p(x) = \left[ (p_{HV} + p_2)^2 + 2(p_{HV} - p_1)C_2^2(x/L) \right]^{1/2} - p_2
\]

\[
C_2 = \eta \nu S_{max} L/r
\]

Again, \( p_2 \) and \( C_2 \) are the crucial parameters of the model.

4.2 High-vacuum pressure as a function of foreline pressure

Again, we are interested in the relationship between \( p_{HV} \) and \( p_{FV} \).

Having in mind that \( p(x = L) = p_{FV} \) we obtain from (4.3):

\[
p_{FV} = \left[ (p_{HV} + p_2)^2 + 2(p_{HV} - p_1)C_2^2 \right]^{1/2} - p_2
\]

Equation (4.4) can be transformed into

\[
(p_{HV} + p_2 + C_2^2)^2 = (p_{FV} + p_2 + C_2^2)^2 - 2(p_{FV} - p_1)C_2
\]

or

\[
(p_{HV} + p_2 + C_2^2)^2 - (p_{FV} + p_2)^2 = 2(p_1 + p_2)C_2 + C_2^2
\]

Equation (4.6) is a striking result of the model presented in this article. The statement is: If there is a prescribed gas throughput \( Q > 0 \) in a Roots blower, then high-vacuum pressure \( p_{HV} \) and foreline pressure \( p_{FV} \) form a hyperbola!

Furthermore, from (4.6) we find that for large values of \( p_{FV} (p_{FV} >> C_2) \) the asymptotic behaviour is given by the linear relationship \( p_{HV} = p_{FV} - C_2 \).

4.3 Compression as a function of foreline pressure

Figure 5 exemplifies the graphs of “K versus \( p_{FV} \)” and compares the mathematical result of the model, i.e., equation (4.6), with the data resulting from the experiment: The curves drawn in heavy lines are graphical representations of equation (1.1) with \( p_{FV} = p_{HV}(p_{FV}) \) calculated from equation (4.6) for the individual measurement series. Again, each measurement series is characterized by a certain prescribed gas throughput \( Q \). The experimental data are indicated by the symbol \( \Box \).
It is remarkable that the analytical model presented above provides an excellent qualitative and quantitative reproduction of the measured dependence of the compression on the foreline pressure in the whole relevant pressure range, i.e., there is perfect agreement over more than three pressure decades.

As an example: In the case of measurement series “a” from figure 5 with prescribed air throughput $Q_a = 44$ mbar m$^3$/h, the parameter set $(p_1 = 0.19$ mbar, $p_2 = 6$ mbar, $C_2 = 192$ mbar) was chosen. The compression curve $K(p_{FV})$ starts at $(p_1 = 0.19$ mbar; $K = 1)$ and reaches its maximum $(K = 13.6)$ at $p_{FV} = 8.56$ mbar (symbol $\Delta$). For foreline pressures larger than 20 mbar the compression curve drops rapidly and approaches asymptotically the value $K = 1$ at high foreline pressures ($p_{FV} > 800$ mbar).

Note that for all graphs shown in figure 5 one and the same $(p_2; C_2)$ parameter set has been used: $(p_2; C_2) = (p_2 = 6$ mbar, $C_2 = 192$ mbar). This means that just as in the case of the TMP discussed above – the parameters $p_2$ and $C_2$ do not depend on the gas throughput $Q$. Consequently, the quotient $S_{max}/r$ does not depend on $Q$, as we have to assume that $\eta_v$ and $L$ are constants which do not depend on $Q$. These remarkable and very important results have been confirmed by all experiments and evaluations performed so far.

It is worth to remark that equation (4.6) can be used to calculate the foreline pressure $p_{FV,K_{max}}$ at which the compression curve $K(p_{FV})$ reaches its maximum. A straightforward calculation leads to the result:

$$p_{FV,K_{max}} = \frac{\sqrt{2 \cdot p_1 \cdot p_2 + (p_2 + C_2) \cdot 2 \cdot p_1 \cdot \psi}}{2 \cdot p_2 + C_2} = \frac{\sqrt{2 \cdot p_1 \cdot (2 \cdot p_2 + C_2)}}{2 \cdot (p_1 + p_2) + C_2}$$

(4.7)

For each measurement series shown in figure 5 the maximum is indicated by the symbol $\Delta$. 

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**Figure 5.** Compression as a function of foreline pressure,
Roots blower system: RUVAC WA 251 + ScrewLine SP 630 (50 Hz), gas type: air,
gas throughput in mbar m³/h - from left to right: 44; 159; 376; 607; 989; 1967,
experimental data: symbol □, K-axis in linear scaling
4.4 Zero-throughput compression

The zero-throughput compression curve can be calculated without difficulty by setting $p_1$ equal to zero in equation (4.6). Furthermore, it is assumed that – just as in the case of the TMP discussed above – those values for $p_2$ and $C_2$ can be used which were chosen to fit the compression curves at finite gas throughput shown in figure 5. The result of the calculation is the curve “z” shown in figure 6. Obviously, this curve deviates significantly from the experimental data at foreline pressures below 1 mbar. The observed deviation indicates that the backleakage term has to be taken into account. This can be done in the way that in the equations (4.2) to (4.6) $p_1$ is replaced by $p_1 + \left[ \frac{p_{FV} \cdot S_{bl}(p_{FV})}{S_{max}} \right]$. Furthermore, an assumption has to be made how the pumping speed $S_{bl}$ depends on the foreline pressure $p_{FV}$. The ansatz proposed in this article reads:

$$S_{bl}(p_{FV}) = S_{bl,0} \cdot \left[ \frac{p_{bl}}{p_{bl} + p_{FV}} \right]$$

(4.8)

Initially, the empirical parameters $p_{bl}$ and $S_{bl,0}$ are unknown. They are determined by fitting the “$K_0$ versus $p_{FV}$” graph to the experimental data.

Note that the backleakage term $Q_{bl} = p_{FV} \cdot S_{bl}(p_{FV})$ is proportional to $p_{FV}$ in the case $p_{FV} \ll p_{bl}$ ($Q_{bl} = p_{FV} \cdot S_{bl,0}$) and independent of $p_{FV}$ in the case $p_{FV} \gg p_{bl}$ ($Q_{bl} = p_{bl} \cdot S_{bl,0}$).

Using (4.6) and (4.8) with the parameters $p_{bl} = 0.165$ mbar and $S_{bl,0}/S_{max} = 0.02$ the curve “zbl” shown in figure 6 was calculated. This curve is the zero-throughput compression curve taking the
The backleakage term into account. It is remarkable that the curve “zbl” provides an excellent quantitative reproduction of the measured dependence of the zero-throughput compression on the foreline pressure in the whole relevant pressure range, i.e., there is perfect agreement over four pressure decades.

The best fit to the experimental data is achieved by choosing the following set of parameters:

\[ p_1 = 3.50 \times 10^{-5} \text{ mbar}; \ p_2 = 6 \text{ mbar}; \ C_2 = 192 \text{ mbar}; \ p_{bl} = 0.165 \text{ mbar}; \ S_{bl,0}/S_{max} = 0.02. \]

In this case, we obtain the curve “zbl1” shown in figure 6. The fact that the value for \( p_1 \) is very small, but not zero, indicates that in the experiment \( Q \) was not zero, but finite.

It is worth to remark that the curve “zbl” is approaching the value

\[ k_0, bl = \frac{p_2 + C_2}{p_2 + C_2 \left( S_{bl,0}/S_{max} \right)}, \] (4.9)

if the foreline pressure \( p_{FV} \) becomes very small.

If we wouldn’t take the backleakage into account, then we would obtain

\[ k_0 = \frac{p_2 + C_2}{p_2} = 1 + \frac{C_2}{p_2} \] (4.10)

With the parameter set given above we find \( k_{0, bl} \approx 20 \) and \( k_0 = 33 \) (see figure 6), i.e., the zero-throughput compression is significantly reduced by the backleakage.

Note that in general, the backleakage term can be neglected in equation (4.1), if the gas throughput is finite. Only in the case of very small or “zero” gas throughput the backleakage has to be taken into account.

5. Conclusions

It has been demonstrated that one and the same mathematical formalism can be used to describe the compression in turbomolecular pumps and Roots blowers. This is a striking result, because the pump mechanism in a turbomolecular pump differs significantly from the pump mechanism in a Roots blower.

Furthermore, it is encouraging to see that only three parameters are required to describe the performance of a turbo stage, a compound stage or a Roots blower stage. Nevertheless, as the values of the parameters \( p_2 \) and \( C_2 \) have to be determined by fitting the calculated graphs “K versus \( p_{FV} \)” to the experimental data, the verification of the proposed model is an ongoing task.

Encouraged by the currently available results the next stage of the investigations will deal with the question, how the parameters \( p_2 \) and \( C_2 \) depend on the gas type.

6. References

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