Does Quartessence Ease Cosmic Tensions?

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Abstract

Tensions between cosmic microwave background observations and the growth of the large-scale structure at late times pose a serious challenge to the concordance $\Lambda$CDM cosmological model. State-of-the-art data from Planck predicts a higher rate of structure growth than what preferred by low-redshift observables. Such tension has hitherto eluded conclusive explanations in terms of straightforward modifications to $\Lambda$CDM, e.g. the inclusion of massive neutrinos or dynamical dark energy. Here, we investigate ‘quartessence’—a single dark component mimicking both dark matter and dark energy—whose non-vanishing sound speed inhibits structure growth at late times on scales smaller than its corresponding Jeans’ length. In principle, this could reconcile high- and low-redshift observations. We test this hypothesis with temperature and polarisation spectra from the latest Planck release, SDSS DR12 measurements of baryon acoustic oscillations and redshift-space distortions, and cosmic shear from KiDS. This the first time that a specific model of quartessence is applied to actual data. We show that, if we naively apply $\Lambda$CDM nonlinear prescription to quartessence, the combined data sets allow for tight constraints on the model parameters. Apparently, quartessence alleviates the tension between the total matter fraction and late-time structure clustering, although the tension is actually transferred from to the quartessence sound speed parameter. However, this strongly depends upon information from nonlinear scales. Indeed, if we relax this assumption, quartessence models appear still viable. For this reason, we argue that the nonlinear behaviour of quartessence deserves further investigation and may lead to a deeper understanding of the physics of the dark Universe.

1. Introduction

The current concordance cosmological model owes its name, $\Lambda$CDM, to the two most abundant constituents of the present-day Universe: the cosmological constant, $\Lambda$, and a (cold) dark matter component. The former, responsible for the late-time accelerated expansion of the cosmos, amounts to $\sim 70\%$ of the total energy budget; the latter, whose gravitational pull shaped the cosmic large-scale structure (LSS), constitutes more than $85\%$ of all the matter in the Universe, and roughly a quarter of its total content\textsuperscript{1}. Despite the success of the $\Lambda$CDM model, a comparison among recent data sets suggests that the agreement between theory and observations is not Adamantine. CMB data by Planck\textsuperscript{1} is in tension with low-redshift observations, e.g. galaxy clustering as inferred through redshift-space distortions (RSD)\textsuperscript{2}, weak gravitational lensing\textsuperscript{3,4}, and galaxy cluster counts\textsuperscript{5}. Summarising, if we extrapolate CMB data to late times following $\Lambda$CDM prescriptions, we will expect a higher rate of structure growth than what favoured by low redshift probes of the LSS\textsuperscript{6,7}. Notably, more recent LSS data, as cosmic shear from the Dark Energy Survey\textsuperscript{8}, seems to exhibit a lesser degree of discrepancy with CMB data, but this does not in itself solve the issue of the pre-existing tensions.

Various approaches to tackling this serious problem have been adopted, ranging from re-analyses of observational data in the attempt of assessing the effect of possible systematics\textsuperscript{e.g. 9,10}, to extensions or modifications of the $\Lambda$CDM model. Among the latter approach, there is the inclusion of a free parameter accounting for the total mass of neutrinos\textsuperscript{e.g. 11,12}. It is known from particle physics that neutrinos have small but non-negligible masses, as well as it is known, in cosmology, that the presence of massive neutrinos during the formation of the LSS causes a damping of matter fluctuations on scales smaller than the neutrino free-streaming length—which, in turn, is related to their masses. Thence the idea of reconciling CMB and LSS data by including massive neutrinos, whose effect will be negligible in the early Universe, although suppressing structure growth at low redshifts. Unfortunately, as shown by Joudaki et al.\textsuperscript{12}, this scenario turns out not to be viable, mostly because of degeneracies between neutrino masses and other cosmological parameters.

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Another possible route is that of dark energy and modified gravity theories [for comprehensive reviews see 13, 14]. As the underlying nature of dark matter and \( \Lambda \) remains utterly unknown, a plethora of models, either phenomenological or emanated from first principles, have been proposed as alternatives to \( \Lambda \)CDM. In literature, many of such extended theories have been considered over the last decade. In particular, let us mention the following, recent analyses:

- Using results from the latest Planck release [1], the Kilo-Degree Survey (KiDS) [3] and local \( H_0 \) estimates [15], Joukaki et al. [12] considered the curvature, constant-\( w \) dark energy, modified gravity models, and running of the spectral index. The authors found that the inclusion of a constant \( w \) parameter and/or curvature \( \Omega_K \neq 0 \) is enough to solve the tension, as these extended models are weakly favoured over \( \Lambda \)CDM.

- Pourtsidou and Tram [16] and An et al. [17] studied interacting dark matter and dark energy models, where they consider a non-gravitational coupling between cold dark matter and dark energy. They concluded that this model can reconcile the tension between CMB observations and structure growth inferred from cluster counts.

An interesting family of such unorthodox cosmologies treats \( \Lambda \) and dark matter as two faces of the same entity, a ‘dark component’ that both drives the current accelerated cosmic expansion and is responsible for the growth of the LSS. A large variety of these models—often called ‘unified dark matter’ or ‘quartessence’, in analogy to quintessence dark energy—are based on adiabatic fluids or on scalar field Lagrangians. On this topic, pioneering studies were made by e.g. Sahni and Wang [18], Kamenshchik et al. [19], Bilic et al. [20] and have been further developed in Refs [21–24]. For a recent review see Bertacca et al. [25] and subsequent works [26–31]. The distinctive feature of this class of models is the existence of pressure perturbations in the rest frame of the quartessence, effectively originating a Jeans’ length below which the growth of density inhomogeneities is impeded and the evolution of the gravitational potential is characterised by an oscillatory and decaying behaviour.

## 2. Quartessence models

In this work we consider a particular class of quartessence models where a classical scalar field \( \phi \) with non-canonical kinetic term accounts for both dark matter a cosmological constant term in the background. We investigate the family of scalar-field quartessence models described by the following Lagrangian [24, 25]

\[
\mathcal{L}_Q = f(\phi)g(X) - V(\phi),
\]

where \( g(X) \) is a Born-Infeld type kinetic term [32] and

\[
f(\phi) = \frac{\Lambda c_\infty}{1 - c_\infty^2} \cosh(\xi \phi) \\
\times \left[ \sinh(\xi \phi) \left[ 1 + (1 - c_\infty^2) \sinh^2(\xi \phi) \right] \right]^{-1}
\]

\[
V(\phi) = \frac{\Lambda}{1 - c_\infty^2} \left[ 1 + (1 - c_\infty^2) \sin^2(\xi \phi) \right]^{-1} \\
\times \left[ (1 - c_\infty^2) \sin^2(\xi \phi) + 2(1 - c_\infty^2) - 1 \right]
\]

with \( \xi = \sqrt{3/(4(1 - c_\infty^2))} \), and \( c_\infty \) a free parameter. This Lagrangian can be thought as a field theory generalisation of the Lagrangian of a relativistic particle [33, 35].

The most important thing to bear in mind here is that these models are indistinguishable from \( \Lambda \)CDM at background level. Furthermore, there is only one additional parameter with respect to \( \Lambda \)CDM, as in the case of massive neutrinos. This parameter, \( c_\infty \), is related to the effective sound speed of quartessence, \( c_s^2 \), and represents its asymptotic value at \( t \to \infty \). Specifically, the speed of sound evolves with redshift according to [24]

\[
c_s^2(z) = \frac{\Omega_\Lambda c_\infty^2}{\Omega_\Lambda + (1 - c_\infty^2) \Omega_{DM}(1 + z)^3}.
\]

Here, we ‘interpret’ \( \Omega_\Lambda \) and \( \Omega_{DM} \) as the present-day densities of an effective cosmological constant and a cold dark component, respectively. Note that this model recovers \( \Lambda \)CDM for \( c_\infty = 0 \). In other words, the \( \Lambda \)CDM parameter space is an hypersurface of the higher-dimensional parameter space of this family of quartessence models; we shall thus hereafter use the notation \( \Lambda \)CDM and \( \Lambda \)CDM+\( c_\infty \). Similarly, we shall later refer to a cosmological model with free neutrino masses as \( \Lambda \)CDM+\( \sum m_\nu \).

In quartessence, the evolution of the gravitational potential of the LSS is determined by the background and perturbation evolution of quartessence alone. This implies that the appearance of a sound speed significantly different from zero at late times will correspond to an effective Jeans scale. In turn, this will cause a strong evolution in time of the gravitational potential.

Then, for scales \( k \) smaller than the cosmological horizon and redshift \( z < z_{\text{rec}} \approx 1000 \), we have

\[
\delta(k, z) = T_Q(k, z) \delta_m(k, z),
\]

where \( \delta \) is the quartessence density contrast, \( \delta_m \) is the matter density perturbation in \( \Lambda \)CDM and \( T_Q \) is the transfer function for the quartessence component. We adopt the approximate functional form

\[
T_Q(g) = 2^{5/8} \Gamma \left( \frac{13}{8} \right) g^{-5/8} J_{5/8}(g),
\]

where \( g(X) \) is a Born-Infeld type kinetic term [32] and

\[
f(\phi) = \frac{\Lambda c_\infty}{1 - c_\infty^2} \cosh(\xi \phi) \\
\times \left[ \sinh(\xi \phi) \left[ 1 + (1 - c_\infty^2) \sinh^2(\xi \phi) \right] \right]^{-1}
\]

\[
V(\phi) = \frac{\Lambda}{1 - c_\infty^2} \left[ 1 + (1 - c_\infty^2) \sin^2(\xi \phi) \right]^{-1} \\
\times \left[ (1 - c_\infty^2) \sin^2(\xi \phi) + 2(1 - c_\infty^2) - 1 \right]
\]

with \( \xi = \sqrt{3/(4(1 - c_\infty^2))} \), and \( c_\infty \) a free parameter. This Lagrangian can be thought as a field theory generalisation of the Lagrangian of a relativistic particle [33, 35].

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Here, we ‘interpret’ \( \Omega_\Lambda \) and \( \Omega_{DM} \) as the present-day densities of an effective cosmological constant and a cold dark component, respectively. Note that this model recovers \( \Lambda \)CDM for \( c_\infty = 0 \). In other words, the \( \Lambda \)CDM parameter space is an hypersurface of the higher-dimensional parameter space of this family of quartessence models; we shall thus hereafter use the notation \( \Lambda \)CDM and \( \Lambda \)CDM+\( c_\infty \). Similarly, we shall later refer to a cosmological model with free neutrino masses as \( \Lambda \)CDM+\( \sum m_\nu \).

In quartessence, the evolution of the gravitational potential of the LSS is determined by the background and perturbation evolution of quartessence alone. This implies that the appearance of a sound speed significantly different from zero at late times will correspond to an effective Jeans length below which quartessence does clustering is inhibited [23]. To give a flavour of this effect, the quartessence sound speed of Eq. (4) with, say, \( c_\infty = 10^{-3} \), determines a typical Jeans length for perturbations \( \lambda_J = 16, 1.6 \) and \( 0.29 \, h^{-1}\text{Mpc} \) at \( z = 0, 1 \) and 2, respectively. This quantity is directly proportional to \( c_\infty \) [see 36 for further details on quartessence Jean’s length]. In turn, this will cause a strong evolution in time of the gravitational potential.

Then, for scales \( k \) smaller than the cosmological horizon and redshift \( z < z_{\text{rec}} \approx 1000 \), we have

\[
\delta(k, z) = T_Q(k, z) \delta_m(k, z),
\]

where \( \delta \) is the quartessence density contrast, \( \delta_m \) is the matter density perturbation in \( \Lambda \)CDM and \( T_Q \) is the transfer function for the quartessence component. We adopt the approximate functional form

\[
T_Q(g) = 2^{5/8} \Gamma \left( \frac{13}{8} \right) g^{-5/8} J_{5/8}(g),
\]
where \( q(k, \eta(z)) = k \int_{n_{rec}}^{\eta} d\eta' c_6(\eta') \) and \( \eta \) is the conformal time. Piattella and Bertacca \cite{38} proved that the error produced by this approximation is almost negligible.\(^1\)

It is important to notice that, albeit negligible at the time of recombination, the presence of quartessence will also impact observations of the CMB. Indeed, CMB photons are lensed by cosmological structures—an effect leading to a smoothing of the acoustic peaks of CMB spectra. Since the larger \( c_\infty \) the more suppressed the growth of matter perturbations, increasing \( c_\infty \) values therefore lead to lower amplitudes of the CMB lensing potential \cite{27, 37}. In turn, \( \Lambda CDM+c_\infty \) CMB spectra differ from the standard \( \Lambda CDM \) prediction at angular scales corresponding to the acoustic peaks, as shown in Fig. 1. This effect is described here for the first time, but let us remark that the impact of quartessence peculiar clustering on CMB lensing power spectrum had already been already investigated in Camera et al. \cite{37} see their Fig. 4).

3. Methodology

3.1. Observables and data sets

As a result of the modified evolution of density fluctuations when \( c_\infty \neq 0 \), several cosmological observables differ from the \( \Lambda CDM \) expectation. For a start, we can look at galaxies’ peculiar velocities and gravitational lensing distortions induced by the intervening LSS on light emitted by distant sources. To find signatures of a non-vanishing \( c_\infty \), we therefore analyse currently available measurements of baryon acoustic oscillations (BAO) and RSD from the Sloan Digital Sky Survey (SDSS) DR12 \cite{39, 40}, and the correlation functions of weak lensing cosmic shear from KiDS \cite{41, 42}. We name the combination of BAO, RSD and cosmic shear data set as ‘LSS’\(^2\) (Note that in the DR12 SDSS data set the tension with \( Planck \) data reported by Macaulay et al. \cite{2} has disappeared.)

Thanks to the effect of quartessence on CMB lensing, described at the end of the previous section, we can also use CMB data to scrutinise the viability of \( \Lambda CDM+c_\infty \) models. Hence, we employ temperature and polarisation spectra from the latest \( Planck \) release \cite{41}. In particular, we employ the \texttt{plik} \( TT+TE+EE \) likelihood together with low-\( \ell \) polarisation data. Furthermore, to be consistent with the latest results from the \( Planck \) collaboration \cite{42}, we include a Gaussian prior on the optical depth to reionisation, \( \tau = 0.058 \pm 0.012 \).

On top of this CMB data, we also include in our analysis the CMB lensing power spectrum obtained through quadratic estimators by Ade et al. \cite{43}, when combining also with the LSS data set. However, as we shall see in Fig. 2 (left panel), the impact of this additional data set, although helping in constraining \( c_\infty \), does not affect significantly the results. For this reason, and given the slight discrepancies of the results inferred on the amplitude of lensing effect between the CMB spectra and the lensing quadratic estimator, we opt for not including it when estimating the tension between CMB and LSS, as well as when we calculate the model selection estimators.

3.2. Data analysis

Summarising, our aim is to state whether or not the non-standard clustering of quartessence models can ease the CMB-LSS tension. First, we proceed to quantify the constraints on \( c_\infty \) obtained from CMB and LSS data. For this purpose, we sample the standard \( \Lambda CDM \) 6 parameters space: the baryon and cold dark matter physical densities, \( \omega_b = \Omega_b h^2 \) and \( \omega_{DM} = \Omega_{DM} h^2 \); the sound horizon at the last scattering surface, \( \theta_s \); the amplitude and tilt of the primordial spectrum of scalar perturbations, \( A_s \) and \( n_s \); and the optical depth to recombination, \( \tau \). To these parameters, we add \( c_\infty \).

For comparison, we also analyse the case of massive neutrinos, \( \Lambda CDM+\sum m_\nu \), for which we sample the same cosmological parameter space adding one dimension related to a free value for the sum of neutrino masses, \( \sum m_\nu \) [cfr \cite{11, 44}]. The parameter space is sampled assuming flat priors for all the parameters and using Monte-Carlo Markov Chains (MCMC) and a Gelman-Rubin convergence diagnostic implemented in the publicly available code \texttt{CosmoMC} \cite{45, 46}. This is interfaced with a version of the public code \texttt{CAMB} \cite{17, 18}, modified in order to account for \( c_\infty \) by including the modifications shown in Eqs (3) - (9) in the dark matter transfer function of \texttt{CAMB}. To test the robustness of our results, in Appendix A we also explore the impact of our choice of sampling on \( c_\infty \). In particular, we make

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\(^1\) In particular, the absolute differences between the exact numerical result for \( T_Q \) and the fitting function in Eq. (6) is \( \leq 1\% \).

\(^2\) As a matter of fact, BAO is a ‘geometrical’ observable, as it only probes the background cosmic expansion. Nonetheless, we included it among the LSS data set, alongside the other low-\( z \) probes.
a change of variables, moving from \( c_\infty \) to \( \log_{10} c_\infty^2 \) and assuming, again, flat priors on all parameters.

It is often common in the literature, for extensions of ΛCDM model affecting the growth of cosmic structures, to remove from the analysis scales where nonlinear effects are relevant [e.g. 12, 19], not to rely on nonlinear modelling based on ΛCDM numerical simulations. Here, however, we deal with \( c_\infty \) values that do not lead to significant deviations from the standard ΛCDM behaviour at the scales probed by the data [see also 27]. Therefore, we decide to include such scales, using the corrections to the linear power spectrum computed by the HMCODE [50].

4. Results and discussion

Table I shows the results obtained with different dataset combinations. As in the recent literature [4, 12], we also quote constraints on the derived parameter \( S_8 = \sigma_8 \sqrt{\Omega_m/0.3} \), which is the combination of \( \sigma_8 \), the rms mass fluctuations on a scale of \( 8 h^{-1} \) Mpc, and the total matter fraction \( \Omega_m = h^{-2}(\omega_0 + \omega_{\text{DM}}) \), mainly constrained by current weak lensing data. The latter is often used as a proxy to exemplify the CMB-LSS tension.

These results highlight how \( c_\infty \) is strongly constrained by LSS observables, whilst Planck data alone, which is only affected by quartessence through CMB lensing, allows for larger values of the parameter, as it can be seen in Figs 2 and 3. This translates into much broader bounds on \( S_8 \) from CMB data and with a lower mean value for this parameter combination with respect to the standard ΛCDM bound \( (S_8 = 0.719^{+0.15}_{-0.046}) \), a scenario opposite to that of ΛCDM + \( \sum m_\nu \), where the neutrino mass is strongly constrained by CMB.

We emphasise that, albeit the allowed range of values for quartessence speed-of-sound parameter \( c_\infty \) may at a first glance seem fine tuned, this is in fact not the case. Indeed, let us point out that, at linear order, it is possible to recast quartessence Lagrangians as effective axion-like Lagrangians in the Thomas-Fermi approximation [51], where the effective mass of the axion changes with the time (this mapping is currently under investigation and it is left to future work).

As a result of the peculiar quartessence dynamics, the CMB-LSS tension can be eased by quartessence with \( c_\infty \neq 0 \) due to its effect on \( S_8 \), as shown in Fig. 3. In the left panel, it is easy to see that larger values of \( c_\infty \) imply a smaller \( S_8 \), thus reconciling high- and low-redshift observations. This can be also appreciated by quantifying the \( S_8 \) tension with the estimator proposed by Refs [4, 12].

\[
T(S_8) = \frac{|S_8^{\text{CMB}} - S_8^{\text{LSS}}|}{\sqrt{\sigma^2(S_8^{\text{CMB}}) + \sigma^2(S_8^{\text{LSS}})}},
\]

where we remind the reader that by ‘CMB’ and ‘LSS’ we indicate that the parameter constraint is obtained from a Planck or KiDS+BAO+RSD analysis, respectively. For the ΛCDM + \( c_\infty \) model considered here, we obtain \( T(S_8) = 0.3 \), therefore quartessence strongly eases the 3.5σ tension we find when comparing CMB and LSS data in standard ΛCDM. Another way to see this is shown in Fig. 4 where the 1σ error intervals on \( S_8 \), marginalised over all the other parameters, are shown for ΛCDM, ΛCDM + \( c_\infty \) and ΛCDM + \( \sum m_\nu \), with red, green and blue lines for CMB, LSS and their combination, respectively. Our results on neutrinos slightly differ from those by Joudaki et al. [12], because we use more recent data compared to them. Similarly, more recent data sets, as weak lensing cosmic shear from the first year of data taking of the Dark Energy Survey, appear in better agreement with Planck, but nonetheless give \( T(S_8) = 2 \) [see 8, Table III].

Figure 5 shows the 2D joint marginal error contours for \( \Omega_m \) and \( \sigma_8 \) in standard ΛCDM and for both ΛCDM + \( c_\infty \) and ΛCDM + \( \sum m_\nu \). Interestingly, even though both \( \sum m_\nu \) and \( c_\infty \) are able to ease the \( \sigma_8 \) tension (although to a much different extent), their impact is substantially different, thus we find that a joint analysis ΛCDM + \( c_\infty \) + \( \sum m_\nu \) would not significantly change the constraints.

Finally, we explore another way to quantify these result, namely we perform a model selection analysis by computing the deviance information criterion (DIC) [52], in order to assess which of the considered models is favoured by the data. For a given model, the DIC is defined as

\[
\text{DIC} = \chi^2_{\text{eff}}(\hat{\vartheta}) + 2p_D,
\]

with \( \chi^2_{\text{eff}}(\hat{\vartheta}) = -2 \ln L(\hat{\vartheta}) \), \( \hat{\vartheta} \) the parameters vector at the maximum likelihood and

\[
p_D = \chi^2_{\text{eff}}(\hat{\vartheta}) - \chi^2_{\text{eff}}(\bar{\vartheta}),
\]

where an average taken over the posterior distribution is implied by the bar. The term \( p_D \) accounts for the complexity of the model, balancing the improvement brought on the goodness of fit, \( \chi^2_{\text{eff}} \), by the introduction of additional parameters.

We compute \( \Delta \text{DIC} \) for the two extended models, using the DIC of ΛCDM as a reference; given the definition of Eq. [5], a negative (positive) value of \( \Delta \text{DIC} \) will show that the extended model is favoured (disfavoured) by the data over ΛCDM. Combining CMB and LSS data sets we find that in ΛCDM + \( c_\infty \) \( \Delta \text{DIC} = -0.21 \); although this results is better than what found for ΛCDM + \( \sum m_\nu \) (\( \Delta \text{DIC} \approx 1 \) when combining CMB and LSS), no preference for quartessence over the standard ΛCDM cosmology is found. To state this, we assume thresholds of \(-5 \) and \(-10 \) for moderate and strong preference of the extended model over ΛCDM, as discussed in Joudaki et al. [12].

The poor evidence in favour of quartessence is due to the fact that, whilst CMB results leave quite some freedom for the \( c_\infty \) parameter, thus enabling to ease the tension, the LSS data set tightly constrains this parameter.
Table 1: Marginalised values and 1-σ errors for $S_8$ and 2-σ upper bound on $c_\infty$ and $\sum m_\nu$ for Planck, LSS and Planck+lens+LSS.

|                      | Planck    | LSS       | Planck+LSS | Planck+lens+LSS |
|----------------------|-----------|-----------|------------|-----------------|
| $\Lambda$CDM        |           |           |            |                 |
| $S_8$                | 0.850 ± 0.017 | 0.741 ± 0.026 | 0.817 ± 0.012 | 0.815 ± 0.010  |
| $\Lambda$CDM+$c_\infty$ |           |           |            |                 |
| $c_\infty$           | $< 5 \times 10^{-3}$ | $< 0.6 \times 10^{-3}$ | $< 0.5 \times 10^{-3}$ | $< 0.5 \times 10^{-3}$ |
| $S_8$                | 0.719$^{+0.15}_{-0.046}$ | 0.736$^{+0.029}_{-0.026}$ | 0.814 ± 0.012 | 0.810$^{+0.012}_{-0.010}$ |
| $\Lambda$CDM+$\sum m_\nu$ |           |           |            |                 |
| $\sum m_\nu$ [eV]   | $< 0.59$ | $< 2.9$  | $< 0.21$ | $< 0.21$        |
| $S_8$                | 0.837$^{+0.021}_{-0.019}$ | 0.743 ± 0.027 | 0.809 ± 0.014 | 0.811 ± 0.011 |

Figure 2: Posterior distributions for $c_\infty$ (left panel) and $\sum m_\nu$ (right panel), showing constraints from Planck (red, solid lines), LSS observables (green, dashed lines) and the combination of the two (blue, dot-dashed lines).

Figure 3: 2D contour plots showing the degeneracy between $S_8$ and $c_\infty$ or $\sum m_\nu$ (left or right panel, respectively), for the various combinations of data sets examined, with darker (lighter) areas depicting 68.3% (95.5%) joint marginal bounds.
(with constraining power coming mostly from nonlinear scales). Hence, when combining the two data-sets to evaluate the DIC, the LSS dominates and does not allow $c_{\infty}$ to assume the values able to ease the tension. As a result, the DIC does not significantly improve with respect to $\Lambda$CDM. Therefore, we conclude that the effect of the $\Lambda$CDM+$c_{\infty}$ model on the growth of structure decreases significantly the tension on the $S_8$ parameter: $0.3\sigma$ for $\Lambda$CDM+$c_{\infty}$ vs $3.4\sigma$ for $\Lambda$CDM. However, the comparison of the two models, quantified through the DIC, highlights that the available data does not favour quartessence over $\Lambda$CDM in a statistically significant manner.

This result might seem puzzling at a first glance, given the significant reduction of the tension on $S_8$. However, assessing the tension focusing only on a single parameter might be misleading as, moving from $\Lambda$CDM to $\Lambda$CDM+$c_{\infty}$, the tension on $S_8$ might have been moved in other parts of the parameter space. This can be quantified exploiting the DIC estimator to assess the concordance of the two different data sets [see 12],

$$2\log I = \mathcal{G}(\text{CMB, LSS}),$$

with

$$\mathcal{G}(\text{CMB, LSS}) = \text{DIC}_{\text{CMB,LSS}} - \text{DIC}_{\text{CMB}} - \text{DIC}_{\text{LSS}}.$$  \hspace{1cm} \text{(11)}$$

This quantity estimates the concordance of the data sets, evaluating, through the DIC estimator, the ratio between the evidence of the two data sets being described by the same set of parameters, and the evidence of the two being described by different parameter sets. We find $\Delta \log I \approx 0.5$, meaning a not substantial evidence in favour of $\Lambda$CDM+$c_{\infty}$. This implies a negligible difference between $\Lambda$CDM and $\Lambda$CDM+$c_{\infty}$. Hence, we can conclude that widening the parameter space from $\Lambda$CDM to $\Lambda$CDM+$c_{\infty}$ does not significantly improve the concordance of CMB and LSS data sets, despite an apparent easing of the tension.

Let us emphasise that these conclusions depend on the estimators used to quantify the tension, namely $T(S_8)$, and to compare the models, viz. $\Delta$DIC. Our results justify further investigations of quartessence models, which we leave for future work, in order to design dependence using other tension estimators [e.g. 53, 54] and more refined model comparison techniques, such as the computation of the Bayesian evidence for the extended model [55]. In any case, we believe that the reconciliation of early- and late-Universe observations attained by quartessence models makes them worth deeper investigations, in particular for what concerns their nonlinear behaviour, either with dedicated numerical $N$-body simulations or through nonlinear perturbation theory approaches.

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### Appendix A. Effects of parameter sampling

Throughout the paper, we have assumed in our MCMCs a linear sampling for $c_{\infty}$ with a flat prior over the whole allowed range. However, as shown also in Fig. 1 this parameter is in principle spanning several orders of magnitude; therefore, one could also think to sample $\log_{10} c_{\infty}^2$ rather than $c_{\infty}$, allowing the MCMC to probe lower values of this parameter better. Such values we expect to be favoured by the data.

Choosing a flat prior on $\log_{10} c_{\infty}^2$ would however bias the results towards low values of $c_{\infty}$. Indeed, if one computes the marginalised posterior probability on a parameter $\theta$ of a $d$-dimensional set $\Theta$, $P(\theta)$, as a function of the likelihood $\mathcal{L}(\Theta)$ and priors $\Pi(\Theta)$, this will read

$$P(\theta) \propto \int d^{d-1}\theta \mathcal{L}(\Theta)\Pi(\Theta).$$ \hspace{1cm} \text{(A.1)}$$

The quantity $\mathcal{G}$ is supported by the Deutsche Forschungsgemeinschaft through the Transregio 33, The Dark Universe.
Therefore, changing parameterisation $\theta \to \zeta(\theta)$ implies that a prior that was flat on $\theta$ will not be so for $\zeta$ in general. In turn, the priors in Eq. (A.1) will change according to

$$\Pi(\zeta) = \Pi(\theta) \frac{d\theta}{d\zeta}. \quad (A.2)$$

As shown in [56], a logarithmic sampling produces a negative tilt of the prior on the physical parameter, in our case effectively down-weighting large $c_\infty$ values. This is indeed what we find if we re-analyse our cases sampling $\log_{10} c_\infty^2$ rather than $c_\infty$, as shown in Fig. A.6.

Given this result, we chose in this paper to sample the linear $c_\infty$ parameter in order not to bias our results towards $\Lambda$CDM. However, as it is particularly evident in the $\text{Planck}$ alone case, the difference between the two choices can reach pretty significant levels and solving this issue will require more sensitive data, able to constrain better this parameter and to limit the range allowed for it.

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