General Relativistic Thick Disks in the Accelerating Expanding Universe Dominated By Dark Energy

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Abstract
In this article, we study the structure of the general relativistic thick disks in the accelerating expanding universe which is dynamically dominated by dark energy. By applying a conformal transformation on the metric of a thick disk in isotropic coordinates of a static universe, we study the disk in the Friedman-Robertson-Walker (FRW) spacetime of an expanding universe. Also, we investigate the structure of a thick disk in a dynamical state by solving Einstein equations in the presence of cosmological constant $\Lambda$. Moreover, modified mass and energy densities of the thick disk, as well as its radial, azimuthal, and vertical pressures, are examined as the functions of time. Furthermore, we check out energy conditions for the relativistic thick disks in the expanding universe.

Keywords General relativistic thick disks · Conformal transformation · Accelerating expanding universe · Dark energy

1 Introduction
Solving Einstein equations for thick disks has become a topic of interest in recent years. Indeed, thick disks are good approximations for various physical objects. Hence, in an accelerating expanding universe [1, 2], solutions of general relativistic disks may help us to understand the mainspring of the accelerating expansion.

Exact solutions of the Einstein equations are often studied in ideal physical systems with special geometrical properties. Under certain conditions, it is possible to obtain important results on the properties of large physical systems by using simple exact solutions. Since the natural form of an isolated self-gravitating fluid is in fact symmetric, the exact solutions with axial symmetry play a crucial role in astronomical applications of general relativity. In particular, disk-like formations of matter are of special interests, due to the fact that they can be utilized as the models of galaxies and integrated disks.

Additionally, various exact solution of Einstein equations were found for static thin disks with, or without, radial pressure. For instance, thin disks solutions without radial pressure are studied by Bonnor and Sackfield [3], and Morgan and Morgan [4]. Such solutions were also calculated by Morgan and Morgan for static thin disks in the presence of radial pressure [5]. Also, rotating thin disks are studied in depth by Bicak and Ledvinka [6, 7].

On the other hand, Cooperstock and Tieu introduced a general relativistic galactic dynamical model for galaxies as rotating axially symmetric fluid without radial pressure [8], which aroused the series of controversial publications discussing the results thereafter [9–15].

Moreover, to address the problem of galactic rotation curves, Ramos-Caro, Agón, and Pedraza introduced a method to obtain galactic disk models in the general relativity realm through the post-Newtonian approximation [16].
Although thin disks can be used to model galaxies in the first approximation, the thickness of the disks is needed to be taken into an account to examine more realistic models. Relativistic thick disks in some coordinates are studied in [17]. In all above-mentioned studies for disks, a method which is called “g-method” has been employed to solve Einstein equations, where the metric of the disk is first assumed and then it is used to calculate the source (energy-momentum tensor). This method is the opposite in approach for “t-method,” as the direct approach to use the source to solve Einstein equations.

Here, we try to find solutions for thick disks by using “displace, cut, fill and reflect” method, which is a more realistic model of galaxies based on thick disks solutions. This method, which is introduced by Vogt and Letelier [17], is a “g-method” solution, where first the metric is conjectured, then by using Einstein equations the elements of energy-momentum tensor are found [17–19]. One should notice that most of aforementioned studies have focused on thin or thick disks in the static spacetime. However, solving the problem in an expanding universe, although adding complexity, could offer a solution closer to reality to understand the leading mechanisms of those physical phenomena. For instance, Vaidya [20], and Pattel and Trivedi [21] computed the appropriate metric for Kerr black holes in FRW spacetime by applying Kerr-Schild transformations to FRW spacetime.

On the other side, although there are many alternative explanations to model accelerating expansion of the universe [22–27], in this article we consider “dark energy” as the source of the expansion of the universe to solve Einstein equations in cylindrically symmetric coordinates. Then, we utilize “displace, cut, fill and reflect” method to make static thick disk, and we apply conformal transformation to calculate the metric of the disk in the spacetime of an expanding FRW universe. Considering the effect of the universe expansion, we solve Einstein equations, including the cosmological constant, for a relativistic thick disk. It is worth noting that we use latest experimental data to estimate the cosmological constant.

The outline of the article is as follows. In Section 2, a brief explanation of the “displace, cut, fill and reflect” method is described. Section 3 includes the mechanism which we used to transform the metric in the static spacetime into a metric in the expanding spacetime by applying the conformal transformation. Particularly, in the Section 3.2, by utilizing conformal transformations on a static thick disk, we obtain the solution of thick disks in the expanding spacetime. Finally, considering the effect of cosmological constant, we express the effect of dark energy on the acceleration of the expanding universe and find the scale factor $a(t)$ in Section 3.3. Numerical solutions of the problem are presented in Section 3.4.

2 Constructing Thick Disks Using “Displace, Cut, Fill and Reflect” Method

2.1 Introducing the “Displace, Cut, Fill and Reflect” Method

Newtonian gravitational potential of a thin disk can be obtained via a simple procedure, called “displace, cut and reflect” method which is described as the following steps [17, 28].

First, we divide the space into two parts, a part without singularities (or sources), and a part containing them (Fig. 1). Second, we neglect the part of the space which contains singularity (Fig. 2).

In the third step, we use a surface to make the reflection of the non-singular part of the space. The result will be a space with a singularity of the form of delta function with support on $z = 0$ plane (Fig. 3).

To generate a thick disk, we need to expand the above-mentioned procedure. Essentially, we need to replace the surface of discontinuity with a thick shell such that the matter comprising the disk can be described with continuous functions. In this case, our method involves an additional step, where after ignoring the part of the space containing singularity, we put a thick shell bellow the plane and then we reflect the lower surface of this shell (Fig. 4). Hence, this method is referred as “displace, cut, fill and reflect.”

Thus, instead of plotting the initial space and the quarantined space at $z=0$, the transformation, $z \rightarrow h(z) + b$, transforms the initial space to the parts of $z = l$ and $z = -l$ which are bounded together. Here, $b$ is a constant and $h(z)$ is a function where its amount (or type) depends on the density and the stress of the disk, and can be determined by its metric. Also, the region $-l < z < l$, where the source density exists, is related to the internal solution, and the
external solution of a thick disk is obtained in the regions 
\( z > l \) and \( z < -l \).

In current manuscript, we start with the source of the Schwarzschild point. By using the above-mentioned method, we assume that the Schwarzschild solution of the external solution in \( z > l \), \( z < -l \) is equivalent to the solution corresponding to a thick disk. Then we find the source density and disk tension as the elements of the energy-momentum tensor. The key point here is to apply a descent \( h(z) \) to have the Schwazschild solution closer to the thick disk solution.

A subset of exact solutions to Einstein’s field equations for thick disks is obtained in Weyl coordinates which describe particle distributions with stable axial symmetry [17, 30–32]. These class of metrics have the following generic form,

\[
ds^2 = -e^{2\Phi} dt^2 + e^{-2\Phi} [e^{2\Lambda} (dr^2 + dz^2) + r^2 d\phi^2],
\]

where, \( \Phi \) and \( \Lambda \) are the functions of \( r \) and \( z \), related to each other via Weyl equations,

\[
\Phi_{,rr} + \Phi_{,zz} + \frac{\Phi_{,r}}{r} = 0,
\]

\[
\Lambda_{,r} = r (\Phi_r^2 - \Phi_z^2),
\]

\[
\Lambda_{,z} = 2 r \Phi_r \Phi_z.
\]

To obtain a thick disk, considering the solutions of Eq. 1, we apply the transformation \( z \rightarrow h(z) + b \), where \( b \) is a positive constant and \( h(z) \) is an even function of \( z \) and \( h(0) = 0 \). The parameter \( b \) controls the amount of matter which is needed to be concentrated in the proximity of the symmetry axis. Larger amount of \( b \) generates more fluid mass distribution, such that by increasing the value of \( b \), the rate of mass distribution from the center to the edge of the disk will decrease. Further, the function \( h(z) \) forces the mass distribution to be along the \( z \) axis. This method is equivalent to constructing a disk using the “displace, cut, fill and reflect” method, and we talk about the implementation of this method in depth.

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\( ^1 \Phi_i \) is the gravitational potential on a linearized scenario.

2.2 Static solution of Thick Disks from Schwarzschild Metric in Isotropic Coordinates

In cylindrical coordinates \((t, r, z, \phi)\) and in the static spacetime, the isotropic metric has the form of,

\[
ds^2 = -e^{2\nu} dt^2 + e^{-2\nu} (dr^2 + dz^2 + r^2 d\phi^2),
\]

where \( \mu \) and \( \nu \) are functions of \( r \) and \( z \). The Schwarzschild solution for such a metric has the form,

\[
\nu = \ln \left( \frac{2B - m}{2B + m} \right),
\]

\[
\mu = 2 \ln \left[ 1 + \frac{m}{2B} \right],
\]

where \( m > 0 \) and \( B^2 = r^2 + \phi^2 \). By applying the “displace, cut, fill and reflect” method on the above solutions, we obtain \( B^2 = r^2 + (h + b)^2 \) [29]. Energy-momentum tensor of such a configuration in the isotropic coordinates can be written in the form of orthonormal tetrads [38],

\[
T_{ij} = \sigma V_i V_j + p_r W_i W_j + p_\phi Y_i Y_j + p_\psi Z_i Z_j,
\]

with the components,

\[
V^i = e^{-\nu}(1, 0, 0, 0),
\]

\[
W^i = e^{-\mu}(0, 1, 0, 0),
\]

\[
Y^i = e^{-\mu}(0, 0, 1, 0),
\]

\[
Z^i = e^{-\mu}(0, 0, 0, 1).
\]

Hence, one can calculate the values of energy density as \( \sigma = -T_t^t \), radial pressure as \( P_r = T_r^r \) which is equal to the azimuthal pressure \( P_\phi = T_\phi^\phi \), and vertical pressure as \( P_z = T_z^z \); as well as the effective Newtonian density \( \phi = \sigma + P_r + P_\phi + P_\psi \) [17]. It is interesting to see that in isotropic coordinates, thick disks have equal radial and azimuthal pressures.

3 Thick Disks in an Expanding Universe Surrounded by Dark Energy

3.1 Static Solution of Thick Disks in Conformal Coordinates

In this part, by applying conformal transformation on static solution of thick disks in an expanding universe which is
surrounded by dark energy, we solve Einstein’s equations for thick gravitational disks.

Since the Universe is not static and experiences an accelerating phase, one can switch to FRW spacetime as the expanding universe by applying a suitable conformal transformation on static metrics \( g_{ij} \) as,

\[
\bar{g}_{ij} = \Omega^2(x^a)g_{ij},
\]

where \( \Omega(x^a) \) is a function of the coordinates and is referred to the conformal factor. The flat FRW metric is expressed as,

\[
ds^2 = -dt^2 + R^2(dr^2 + r^2d\Omega^2).
\]

Changing from cosmological time \( t_c \) to a new time coordinate \( t \) via,

\[
dt_c = R(t)dt,
\]

we will find,

\[
ds^2 = R^2(t)\left[-dt^2 + dr^2 + r^2d\Omega^2 \right].
\]

This is in fact the transformed Schwarzschild metric under conformal transformation with the conformal factor \( R(t) \). We notice that under conformal transformations, null geodesics are still conserved.

In the next section, we will apply the conformal transformation on the static metric of a thick disk to find the metric in an expanding universe.

### 3.2 The Effect of the Accelerating Expansion of the Universe on the Static Thick Disk’s Solution

As it is mentioned before, by using the “displace, cut, fill and reflect” method on the Schwarzschild metric, we find the metric of a static thick disk. Further, by applying a conformal transformation on static metric, the metric of a thick disk is obtained in FRW spacetime.

In addition, by solving Einstein equations in the presence of cosmological constant and finding the energy-momentum tensor, one can compute other properties of the disk such as the energy density and mass density, together with the azimuthal, radial, and vertical pressures.

As we know, isotropic metric in the cylindrical coordinate system is described by (3) and (4). Utilizing the transformation \( z \rightarrow h(z) + b \), described in Section 2.2, we obtain,

\[
\nu = \ln\left(\frac{2\sqrt{r^2 + (h + b)^2} - m}{2\sqrt{r^2 + (h + b)^2} + m}\right),
\]

\[
\mu = 2\ln\left(1 + \frac{m}{2\sqrt{r^2 + (h + b)^2}}\right). \tag{10}
\]

The continuity of the metric function and its derivatives on \( z = \pm l \) is determined via the continuity of \( h(z) \) and \( h'(z) \) on \( z = \pm l \). Equation 10 determines the thick disk’s metric in the static spacetime that we mentioned in Section 2.

By applying the conformal transformation on variables (10), one can find the metric of thick disk in an expanding universe in the form of (3), with following variables,

\[
\nu = \ln\left(\frac{2Ba(t) - m}{2Ba(t) + m}\right), \tag{11}
\]

\[
\mu = 2\ln\left(1 + \frac{m}{2Ba(t)}\right). \tag{12}
\]

Substituting \( B = \sqrt{r^2 + (h + b)^2} \), we find,

\[
\nu = \ln\left(\frac{2a(t)\sqrt{r^2 + (h + b)^2} - m}{2a(t)\sqrt{r^2 + (h + b)^2} + m}\right), \tag{13}
\]

\[
\mu = 2\ln\sqrt{a(t)}\left(1 + \frac{m}{2a(t)\sqrt{r^2 + (h + b)^2}}\right), \tag{14}
\]

where \( a(t) \) is the scale factor, which will be studied in the the following section.

### 3.3 Evolution of the Scale Factor \( a(t) \)

According to the cosmological experimental data, our universe undergoes an accelerating expansion phase, which
should be taken into account for more realistic thick disk’s solutions. As a model, this effect is considered by adding the term containing cosmological constant to Einstein equations. The final effect appears in the function \(a(t)\) [33], which will be discussed in detail in current section.

Hence, to calculate the temporal evolution of \(a(t)\), we start with Einstein equations, containing cosmological constant \(\Lambda\) as,
\[
R_{ij} - \frac{1}{2} g_{ij} R - \Lambda g_{ij} = -8\pi G T_{ij}.
\]
(15)
The relation between the cosmological constant and energy density is,
\[
\Lambda = 8\pi G \rho_D,
\]
(16)
where \(\rho_D\) is the dark energy density and \(\Lambda = 2 \times 10^{-35} \text{m}^{-2}\) [37].

Given \(P = \sigma \rho\) as the equation of the state, with \(P\) as pressure and \(\rho\) as matter density [34], and using Einstein equations and FRW relations we find,
\[
H^2 + \frac{K}{a^2} = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3},
\]
(17)
\[
H^2 + \dot{H} = -\frac{4\pi G}{3} \rho (1 + 3\sigma) + \frac{\Lambda}{3},
\]
(18)
\[
\dot{\rho} + 3(1 + \sigma) \rho H = -\frac{\dot{\Lambda}}{8\pi G},
\]
(19)
where \(H = \dot{a}(t)/a(t)\) is the Hubble parameter, and \(K\) is the curvature constant with the probable values of \(-1, 0,\) and \(+1\) for open, flat, and close models of the universe.

As we mentioned, to solve Einstein equations for relativistic thick disk in FRW spacetime, one needs to consider the metric in an expanding spacetime and then solve Einstein equations in the presence of the cosmological constant. Hence, we need to compute the function \(a(t)\) under the condition where \(\Lambda \neq 0\). Here, we have \(\Lambda = 8\pi G \rho_D\) and we also assume the flat universe, i.e., \(K = 0\). Then, Eqs. 17 and 18 can be rewritten as,
\[
H^2 = \frac{8\pi G}{3} \rho (1 + \rho_D),
\]
(20)
\[
H^2 + \dot{H} = -\frac{4\pi G}{3} \left[ \rho (1 + 3\sigma) + 2\rho_D \right].
\]
(21)
To simplify the solutions, one can assume that the matter density is proportional to the dark energy density, \(\rho_D = f \rho\). Thus,
\[
H^2 = \frac{8\pi G}{3} \rho (1 + f),
\]
(22)
\[
H^2 + \dot{H} = -\frac{4\pi G}{3} \rho [1 + 3\sigma + 2f].
\]
(23)
Solving this equation, we obtain,
\[
a(t) = a_0(t/h_0) \left( \frac{1 + f}{1 + f_0} \right) .
\]
(24)

All the effects which are introduced by \(\Lambda \neq 0\) are contained in parameter \(f\). In fact, the effect of a non-zero cosmological constant leads to the usage of \(\Lambda = 8\pi G \rho_D\) and by making the assumption of \(\rho_D = f \rho\), we arrive to the above result. In Eq. 24, \(f\) denotes the effect of the cosmological constant. By substituting the obtained \(a(t)\) function in the metric (13), we solve Einstein equations in the presence of the cosmological constant, and by finding \(T_{ij}\), we calculate the values of \(\sigma, \alpha, P_r, P_\psi\), and \(P_z\), as effective Newtonian density, energy density, radial, azimuthal, and vertical pressures respectively.

### 3.4 Exact Solution to the Einstein Equations for Thick Disks in FRW Spacetime

The effect of the acceleration of universe expansion appear in the function \(a(t)\) in FRW spacetime of the form (24). Hence, by substituting \(a_0, f,\) and \(\sigma\), we arrive to the final solution. Based on [35], \(\sigma\) has the range of,
\[-1.33 < \sigma < -0.8.\]

Although \(f\) changes by the time evolution, we assumed a constant value of \(f \simeq 15\), since its variation rate is considered to be considerably slow and having a constant \(f\) does not affect our calculations. Hence, by considering \(f = \frac{\rho_D}{\rho}\), we conclude that the universe is filled by \(\%68.3\) of dark energy, \(\%4.9\) of ordinary matter, and \(\%26.8\) of dark matter. Additionally, \(a_0\) is a coefficient with the value of 1 [36, 37].

By solving Einstein Eq. 15, and calculating the elements of energy-momentum tensor, we obtain energy density and pressure components of a thick disk in FRW spacetime. Rescaling \(\sigma\) and \(P_1\) with respect to \(l\) as the half of the thickness of the disk, i.e., \(l^2 P_1 \rightarrow P_1\), and defining \(\xi = \sqrt{4r^2 + \psi^2}\), where \(\psi = (x^2 + 4)^2\), we obtain,
\[
\sigma = \frac{1}{4\xi^4 r^2 (\xi \sqrt{1 + i} + i^4 \xi^8 r^2 - 1)}(64r^8 (12r^{5/2} + \xi^3))
\]
\[+16r^6 i^{1/2} (3\xi^{1/2} \psi + 45\xi \sqrt{7} + 36t \psi + 80)\]
\[+4r^4 \sqrt{7} (3\xi^{1/2} \psi^2 + 90\xi t \psi^2 + 36t^2 \psi^2 + 75\xi \sqrt{7})\]
\[+160t \psi + 36) + r^2 (7\xi + 12r^{5/2} \psi^3 + 80r^{3/2} \psi^2)\]
\[+\xi (3z^{1/2} + 24z^{10} + 240z^8 + 1280z^6 + 3904z^4\]
\[+6848z^2 + 5376) + 45r^2 \psi^2 + 75\xi \psi + 36 \sqrt{7} \psi)\]
\[-16\xi r^2 \psi (z^2 - 4),\]
(25)
Fig. 5 Energy density Eq. (25) for a thick disk with parameters $\tilde{m} = 1, \tilde{b} = 2, \tilde{c} = 1$ in a constant time.

Fig. 6 Radial pressures Eq. (26) for a thick disk with parameters $\tilde{m} = 1, \tilde{b} = 2, \tilde{c} = 1$ in a constant time.

Fig. 7 Azimuthal pressure Eq. (27) for a thick disk with parameters $\tilde{m} = 1, \tilde{b} = 2, \tilde{c} = 1$ in a constant time.

Fig. 8 Vertical pressure Eq. (28) for a thick disk with parameters $\tilde{m} = 1, \tilde{b} = 2, \tilde{c} = 1$ in a constant time.
Fig. 9 Mass density Eq. (29) for a thick disk with parameters \( \tilde{m} = 1, \tilde{b} = 2, \tilde{c} = 1 \) in a constant time. Some levels curves of the density are displayed on the right graph.

Fig. 10 Weak energy condition in \( z = 0 \)

Fig. 11 Weak energy condition in \( r = 10 \)

Fig. 12 Strong energy condition in \( z = 0 \)
Fig. 13 Strong energy condition in $r = 10$

Fig. 14 Dominant energy condition $|\frac{\rho}{\sigma}| < 1$ in $r = 10$

Fig. 15 Dominant energy condition $|\frac{\rho}{\sigma}| < 1$ in $r = 10$

Fig. 16 Dominant energy condition $|\frac{\rho}{\sigma}| < 1$ in $r = 10$
shows how the disk is expected to be spread out. The effective Newtonian density is also defined as,

\[
P_r = \frac{-1}{4\xi^2 r^2(\xi \sqrt{t} + 1)^4(\xi^2 t - 1)}(\xi^4 t - 1)(192r^8 (4r^{5/2} + \xi t^3)) + 16r^6 t^2(15\xi - 16\xi t^2 + 9\xi t^2 \psi + 36\sqrt{\psi})
\]

\[-4r^4(-36r^{5/2}\psi^2 - 64r^{7/2}(3\xi^4 + 30\xi^2 + 56)) + 48\xi r^4 \psi - \xi r^2(9\xi^8 + 144\xi^6 + 864\xi^4 + 2304\xi^2)
\]

\[+ 2320 - 30\xi^2 r^2 + 15\xi t + 12\sqrt{t}) + r^2(-3\xi + 12\xi^{5/2}\psi^3 + 64r^{7/2}\psi(3\xi^4 + 24\xi^2 + 64)
\]

\[-48\xi r^2 \psi^2 + \xi r^4(3\xi^4 + 72\xi^2 + 720\xi + 3840\xi + 11360\xi^4 + 16768\xi^2 + 9216) + 15\xi t^2 \psi^2 - 15\xi t \psi
\]

\[-12\sqrt{\psi}) - 4r^3(\xi t^3 + 8\sqrt{\psi}(3\xi^2 - 4) - \xi(\psi^2 + 32\xi^2 - 16))). \quad (26)
\]

\[
P_\psi = \frac{1}{4r^2 r^2(4r^2 + \psi)^{5/2}(\xi^2 t^2 + 1)^4(\xi^2 t - 1)}(64r^8(12r^{5/2} + \xi t^3)) + 16r^6 t^2(3\xi^4 + 12\xi^2 + 16\xi t^2 + 45\xi \sqrt{\psi} + 36\psi + 80)
\]

\[+ 4r^4(36r^{5/2}\psi^2 + 160r^{3/2}\psi + 48\xi r^4 \psi + \xi^3(3\xi^4 + 48\xi^6 + 288\xi^4 + 768\xi^2 + 752) + 75\xi t
\]

\[+ 90\xi^2 \psi + 36\sqrt{\psi}) + r^2(7\xi + 12\psi^3 + 80r^{1/2}\psi^2 + 48\xi r^4 \psi + \xi^3(12 + 24r^4 + 240r^8 + 1280r^6 + 3872r^2 + 6592\xi^2 + 4864) + 45r^2 \psi^2 + 75\xi t \psi
\]

\[+ 36\sqrt{\psi}) + 4r^3(\psi^3 - \psi \xi^2 + 12))) \quad (27)
\]

\[
P_\zeta = \frac{1}{4r^2 r^2(\xi \sqrt{t} + 1)^4(\xi^2 t^2 + 1)}(64r^8(12r^{5/2} + \xi t^3)) + 16r^6 t^2(3\xi^4 + 12\xi^2 + 16\xi t^2 + 45\xi \sqrt{\psi} + 36\psi + 80)
\]

\[+ 4r^4(36r^{5/2}\psi^2 + 160\psi^{3/2} + 48\xi r^4 \psi + \xi^3(3\xi^4 + 48\xi^6 + 288\xi^4 + 768\xi^2 + 752) + 75\xi t
\]

\[+ 90\xi^2 \psi + 36\sqrt{\psi}) + r^2(7\xi + 12\psi^3 + 80r^{1/2}\psi^2 + 48\xi r^4 \psi + \xi^3(12 + 24r^4 + 240r^8 + 1280r^6 + 3872r^2 + 6592\xi^2 + 4864) + 45r^2 \psi^2 + 75\xi t \psi
\]

\[+ 36\sqrt{\psi}) + 4r^3(\psi^3 - \psi \xi^2 + 12))). \quad (28)
\]

The effective Newtonian density is also defined as,

\[
\varrho = \sigma + P_r + P_\psi + P_\zeta. \quad (29)
\]

The corresponding source graphs and level curves of the relations 25 to 29 are depicted in diagrams 5 to 9 (Figs. 5, 6, 7, 8 and 9). As we see, these curves indicate certain features of the disk, e.g., they show that the density increases towards the disk’s center. Also, our results show that the energy and mass density, radial, azimuthal, and vertical pressures are decreased by getting away from the center of the disk, in accordance with the expected physical properties which shows how the disk is expected to be spread out.

Also, we investigate dominated energy conditions as,

(I) weak energy condition \(\sigma \geq 0\),

(II) strong energy condition \(\varrho = \sigma + P_r + P_\psi + P_\zeta \geq 0\),

(III) dominant energy conditions \(\frac{P_r}{\varrho} \leq 1, \frac{P_\psi}{\varrho} \leq 1, \frac{P_\zeta}{\varrho} \leq 1\),

in Figs. 10, 11, 12, 13, 14, 15 and 16, where we see that relativistic thick disks in FRW spacetime satisfy all above-mentioned energy conditions.

4 Discussion

We know that the real universe is in an accelerating expansion phase. Dark energy, which is modeled as a type of energy with negative pressure that acts as a repulsive force, is regarded as one of the several scenarios providing an explanation for the accelerating expansion of the universe. Geometrically, the accelerating expansion is considered as an additional component that is added to determine the curvature of the Universe. Dynamically, the negative pressure counteracts gravity and makes the accelerating expansion and affects the scale factor \(a(t)\). Anyway, the expansion of the universe affects the metric near the cosmological objects and consequently modifies their energy-momentum tensor (density of matter) [39–41].

In this article, we modeled the effect of the accelerating expanding universes, which is under the influence of dark energy, on the galaxies with the help of cylindrical symmetric thick disks.

The metric which is used in this article is for the cosmological outer solutions. Imposing boundary conditions, the cosmological expansion effects are found in the form of the terms of mass density and energy density and vertical, radial, and azimuthal pressures. Although these effects can be regarded negligible according to the amount of mass, radius, and the thickness of the disk, they are emerged in the form of the next-order correction terms.

Anyway, these results inspire some phenomenological researches for future works, such as investigating the effect of such a dark energy for another coordinate systems, or entering the effect of the dark energy in other terms more than the cosmological constant. Particularly, and in the case of doing an enough precise measurements on experimental data, such an investigations, can pave the way for better understanding of the properties of the universe accelerating expansion mechanism, and also the properties of dark energy.

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