There has been recently a renewed interest, both theoretical and experimental, in the problem of nonlinear (NL) magnetization dynamics inside confined nanostructures [1]. NL phenomena are responsible for the creation of novel dynamical objects [2], analogs of dynamical solitons. They also set the figure of merit of spintronics devices, e.g. the spectral purity and tuning sensitivity of spin transfer torque nano-oscillators [1]. On the theoretical side, predictions on the amplitude of the NL coefficients have been found to be extremely difficult to compute beyond the uniformly magnetized ground state. The difficulty raises both from the magneto-dipolar field, which introduces a non-local interaction, and from the kinetic part of the effective field (or gauge field), which modifies the texture of the magnetic configuration. On the experimental side, the most promising findings have been discovered on non-uniform ground states, such as magnetic vortex existing in ferromagnetic nanodot. Vortices have stimulated the emergence of higher performance microwave oscillators using isolated [3] or dipolarly coupled [4] nanodot, or for future magnetic memories by allowing the resonant switching of the magnetic configuration [5].

Magnetic vortex corresponds to a curling in-plane magnetization spatial distribution leaving a nanometric in size core region (∼ the exchange length), where the magnetization is pointing out-of-plane. The lowest energy mode is a translational (or gyrotropic) mode of the vortex core position $X$, expressed here in reduced unit of the exchange length), where the magneto-static potential well $W^{(M)}$ in which the core evolves. For a circular nanodot, the magnetostatic energy is isotropic in the dot plane and it can be written as a series expansion of even powers of the dimensionless $X$ [6]:

$$W^{(M)} = W_0^{(M)} + \frac{1}{2} \kappa |X|^2 + \frac{1}{4} \kappa' |X|^4 + \mathcal{O}(|X|^6), \quad (1)$$

At the present, only the parabolicity of the confinement, $\kappa$, has been well characterized experimentally and the measured value is in agreement with theoretical predictions [7]. This is not the case for the higher order terms and there is no consensus yet on the order of magnitude or the sign of the anharmonic coefficient $\lambda \equiv \kappa''/\kappa$ afferent to the depolarisation effect of a displaced vortex inside a large planar circular nanodot. The asymptotic limit of large radius is the relevant aspect ratio to test the dipole dominating limit and the circular symmetry is necessary to avoid the additional complexity of non-isotropic potential found for example in square shaped elements [8].

Up to now, attempts to measure $\lambda$ in circular nanodot using large rf excitation have so far lead to inconsistent results between experiments [9] (red shift) and theory [10] (blue shift). The measurement of $\lambda$ through a variational approach, consisting in studying the small change of oscillation period when a large static displacement of the vortex core equilibrium position is produced, has so far failed too: this counterpart of large rf oscillation has mostly revealed the potential well inhomogeneities leading to pinning of the core [11, 12].

In this work, we report on an experimental measurement of $\lambda$ in a large planar circular nanodot using a Magnetic Resonance Force Microscope (MRFM). All the experimental measurements are performed on an individual nanodisk of thickness $t = 26.7$ nm thick and nominal radius $R = 300$ nm, patterned out of a single crystal FeV film. Only the perfect crystalline structure ensures an unpinned displacement of the vortex core throughout the sample volume. We rely here on the non-uniform stray field of the magnetic tip of the MRFM to displace the vortex core away from the nanodot center. The anharmonic coefficient is then inferred from the measurement of the relative variation of the eigen-frequency of the gyrotropic mode as a function of the tip displacement.

Probing the anharmonicity of the potential well for magnetic vortex core in a nanodot

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The magnetic vortex state is shown in a bi-variate colormap of thenanodot to preserve the vortex core. From there, lateral displacement of the magnetic tip produces a vector shift $X_0$ of the vortex core equilibrium position from the dot center. The process is driven by the growth of the in-plane domain parallel to the in-plane component of the tip stray field. We observe in FIG.2 that the eigen-frequency increases (blue shift) upon increasing $|X_0|$. Noting that the frequency shift is symmetric and isotropic (the signature that the intrinsic potential is being probed) we find that the vortex core dynamics can be tuned on a relative large range ($\sim 10\%$), hereby demonstrating that the magnetostatic potential must be anharmonic, since a purely parabolic shape would have lead to a frequency independent behavior on $X_0$.

Analysis of the amplitude of the MRFM signal gives a hint on the amount of displacement $|X_0|$ achieved during a scan. The MRFM signal corresponds to the difference of vertical force $\Delta F_z$ acting on the cantilever when the vortex motion is excited. Defining $m = M/M_s$ the reduced magnetization vector, the gyromotion produces a diminution of the spontaneous magnetization along the local equilibrium direction $\Delta m_z = \frac{1}{2} [\partial_X m_x + j \partial_Y m_y]^2 X_0$, that mostly occurs outside the core region [20]. The generated force can be then calculated from the reaction force $-\Delta F_z = V M_s (g_{zz} \Delta m_z)$ acting on the nanodot due to the gradient tensor of the tip: $\hat{g} = \nabla H^{\text{tip}}$. Cartesian tensor notation is used here, with repeated indices being assumed summed. The chevron bracket indicates that the enclosed quantity is averaged over the volume $V$ of the nanodot. The red-blue colormap in FIG.2 codes the amplitude MRFM signal: red (blue) means attractive (repulsive) force. Small arrows at $\delta_x \approx \pm 1.7$ indicate the compensation point of the force: where the change of sign occurs. This distance is about twice smaller than the one required to change the sign of the force in the saturated state (contribution dominated by $g_{zz} \Delta m_z$). The difference is interpreted as due to the translation of the core position transversally to the tip position (along the $y$-axis). The growth of this domain generates a repulsive vertical force on the tip through the cross gradient term $g_{zz} \Delta m_z$. We shall thus use the position of the compensation point to calibrate precisely the amplitude of the displacement of the vortex core.

Our next step is to develop an analytical framework allowing the extraction of $\lambda$ from this variational study. The first stage of this analysis is to calculate the equilibrium position $X_0 = (X_0, Y_0)$ (here $X_0$ and $Y_0$ are the two in-plane cartesian coordinates) by minimizing the total energy $W = W^{(M)} + W^{(H)}$, the sum of $W^{(M)}$, the magnetostatic self-energy of the vortex ground state which confines the vortex core to the center of the nanodot and $W^{(H)} = -V M_s (m \cdot H)$, the Zeeman energy which represents the interaction with the external magnetic field $H = H_0 + H^{\text{tip}}$ and is responsible for the displacement $X_0$. The magnetic configuration inside the nanodot $m_x + j m_y = 2w/(1 + w^2)$ is conveniently described by a conformal mapping of the complex variable $w$ (* indicating the complex conjugate), which is a piece-wise function of the complex position $\bar{z} = (x + jy)/R$.
with \( w = f(\gamma)/| f(\gamma)| \) outside the vortex core region and \( w = f(\gamma) \) inside. The function \( f \) captures the texture of the spatial configuration. To calculate the new equilibrium position \( Z_0 = (X_0 + j Y_0) \), it is appropriate to describe the dot magnetization, \( \mathbf{m} \), by the rigid vortex model (RVM) \( [21] \), written as \( f(\gamma) = \pm j (\gamma - Z_0)/r_c \). Here \( r_c = R_c / R \) denotes the core radius \( R_c \) in reduced unit of \( R \) and the \( \pm \) sign depends on the chirality of the vortex. The static displacement \( X_0 \) is then obtained by minimizing the total energy \( W^{(M)} \) obtained by the RVM. In the RVM, the magnetostatic energy is generated by the surface magnetic charges \( \sigma \) located at the circumference of the disk (the volume charges \( \nabla \cdot M \) are absent). The confinement potential follows from the integral \( W^{(M)} = \frac{1}{2} \int d\phi \int d\phi' \sigma(\phi)\sigma(\phi')/\sqrt{2(1 - \cos(\phi - \phi'))} \) where the integration is taken over the disk periphery and \( \sigma \) is given by:

\[
\sigma(\phi) = +M_s \frac{-|X_0| \sin(\phi - \phi_0)}{\sqrt{1 - 2|X_0| \cos(\phi - \phi_0) + |X_0|^2}}, \tag{2}
\]

\( \phi_0 \) is the azimuthal direction of the vortex equilibrium position measured from the averaged in-plane bias field direction (here \( x \)-axis).

The implicit trajectory of \( X_0 \) is shown in FIG. 3a, for three different heights \( \delta_z \) around the nominal value. As expected the in-plane components of the tip magnetic field displace the vortex core mainly along the \( y \)-axis. The displacement along \( x \)-axis is approximately twice smaller. The resulting displacement distance \( |X_0| \) as a function of \( \delta_z \) is shown in FIG. 3b. We use here a skewed scale on the abcisse to show the behavior when \( \delta_z \gg 1 \). We have also calculated the corresponding dipolar force produced on the tip. The result is coded in the colormap using the same convention as in FIG. 2. We have placed small arrows at the compensation points. Since decreasing the scan height increases the amplitude of \( |X_0| \), we find that the position of the arrows sensitively depends on \( \delta_z \). Varying \( \delta_z \) in the experimental error bars \([2.6, 3.0]\) displaces the compensation point by \( \pm 0.3 \cdot R \) (or \( \pm 100 \text{ nm} \)) around the mean value \( \delta_z = 1.8 \), in agreement with the experimental data. We shall use this marker to evaluate the uncertainty window of \( |X_0| \) in our experiment.

The second stage of this analysis is to perform a linearization of the vortex equation of motion to a cyclic excitation field. The instantaneous response \( X = (X, Y) \) is decomposed into the static component \( X_0 \), calculated previously, and a dynamic component \( \xi = X - X_0 \) representing the small oscillating deviation of the vortex core position from its equilibrium \( [23] \). In the dynamical case, the dipolar pinning imposes a precession node at the dot circumference \( [20] \). It implies that the dynamical magnetization comes from the variation, \( \partial_j \mathbf{m} + j \partial_j \mathbf{m} \), of a magnetic configuration that has no radial component at the dot border. Therefore, to calculate the frequency of the small dynamic vortex displacement \( \xi \), it is appropriate to use the surface charges free model or two vortex ansatz (TVA) written as \( f(\gamma) = \mp j \frac{1}{2} (\gamma - Z)/(\gamma Z^* - 1)/(1 + |Z|^2) \) \( [24] \) with \( Z = (X + j Y) \). In our notation, the dampingless Thiele equation simply writes \( G \times \xi = \partial W/\partial \xi \), where \( W \) is the total energy and \( |G| = 2\pi M_i \omega /\gamma \) is the gyromagnetic ratio. Linearization around \( X_0 \) yields the gyrotropic angular frequency

\[
\omega^2 = \frac{K_{xx} K_{yy} - K_{xy}^2}{G^2}, \quad \text{with } K_{ij} = \frac{\partial^2 W}{\partial \xi_i \partial \xi_j} \bigg|_{X = X_0} \tag{3}
\]

being the stiffness of the vortex core to small displacements in both the \( i \) and \( j \) directions. Distinction between different cartesian directions is necessary once the trajectory becomes elliptical. This is precisely, what occurs when \( X_0 \gg \xi \): the amplitude of the \( \xi \)-component along \( X_0 \) differs from the amplitude of the \( \xi \)-component perpendicular to \( X_0 \) (short axis of the ellipse is along the radial direction). The degree of ellipticity is determined by the anharmonic contribution \( \lambda |X_0|^2 \). This is in contrast to the opposite limit \( X_0 \ll \xi \), where the trajectory corresponds to a large amplitude circular vortex core motion around the nanotip center \( [6] \).

To calculate the different tensor elements of the stiffness \( K_{ij} = K_{ij}^{(M)} + K_{ij}^{(H)} \) one must decompose it in two contributions corresponding respectively to the magnetostatic and Zeeman energies. The first order value of the TVA magnetostatic stiffness, \( \kappa \), has been already expressed analytically \( [20] \). The analytical expression of the anharmonic correction is obtained by inserting Eq. (1) in Eq. (3) and it leads to a simplified expression

\[
(K_{ij}^{(M)}) = \kappa (\delta_{ij} + \lambda |X|^2 \delta_{ij} + 2\lambda X_i X_j) \bigg|_{X = X_0}. \tag{4}
\]

It turns out that
the Zeeman stiffness can be neglected. Indeed, it can be shown that the tip stray field produces no Zeeman stiffness along the diagonal elements ($K_{ii}^{(H)} = 0$). Only the cross-terms $K_{xy}^{(H)} \neq 0$ are non-vanishing but they represent a negligible correction (< 3%). We thus find that at $X_0 = 0$ and $H_z = 0$, Eq.(3) simplifies to the well known expression $\omega(0,0) = \kappa/G$ [24]. At $X_0 = 0$ and $H_z \neq 0$, the stiffness of the magnetostatic potential is renormalized by the in-plane magnetization projection of the cone state and one obtains $\omega(0, H_z)/\omega(0,0) = 1 + H_z/(4\pi M_s)$ [17]. In the general case $X_0 \neq 0$ and $H_z \neq 0$, the relative frequency shift reduces to the following analytical expression:

$$\frac{\omega(X_0, H_z)}{\omega(0, H_z)} = 1 + 2\lambda |X_0|^2 + O(|X|^4). \quad (4)$$

Notice that the prefactor of 2 multiplying $\lambda$ is specific to the limit $X_0 \gg \xi$.

The next step is to plot in FIG.4a, the experimental data extracted from FIG.2 renormalized by the predicted dependence of $\omega(0, H_z)$, as a function of the calculated $|X_0|$ during a lateral scan of the tip at fixed $\delta_z = 2.8$. Fitting the data of FIG.4a with a parabola (solid line) yields an average curvature $\lambda = 0.5$. We have plotted in FIG.4b, the experimentally measured relative frequency normalized by $\omega(0, H_0)$. The latter quantity is inferred experimentally by studying the decay of $\omega$ upon increasing $\delta_z$, while keeping the tip on the symmetry axis ($\delta_x = \delta_y = 0$): a fit of the decay behavior yields the asymptotic value $\omega(0, H_0)$. In FIG.4b, the data point are colored according to the colormap associated with the amplitude of the force. For comparison, we have also plotted the predicted variation of $\omega$ by Eq.(4) as a function of $\delta_z$, for two values of $\lambda$. Setting $\lambda = 0$ in Eq.(4), would have produced the usual bell-shaped curve [15], which corresponds to a diminution of $\omega(0, H_z)$ when the tip moves away from the nanodot axis. The behavior for $\lambda = 0.5$ is in excellent agreement with the experimental data, both in the amplitude of the NL frequency shift and in the position of the compensation point of the force.

We have then repeated the analysis by varying $\delta_z$ in the experimental error bar range: ±0.2 around the nominal value. Fit of the data by a parabola would lead to larger (smaller) values of $\lambda$ depending if the amplitude of the shift decreases (increases). This procedure yields an uncertainty window of 30% for the determination of $\lambda$, shown as a shaded area in FIG.4a. Our fitting analysis did not account for higher order corrections in Eq.(4). In FIG.4b, the curvature increases with the displacement distance. Inclusion in the fit of terms in $|X|^4$ would have decrease the value of $\lambda$ by about one standard deviation. As an additional check, we have performed a simulation of the expected $\omega(X_0, H_z)$ for our nanodot using a mesh-size of 2.3 nm and a GPU-accelerated micromagnetic code [27]. The result is shown as crosses in FIG.3.
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