On the Analyticity of the Bivariant JLO Cocycle

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Abstract. The goal of this note is to outline a proof that the JLO bivariant cocycle associated with a family of Dirac type operators over a smooth fibration, is entire for $C^{\ell+1} - C^\ell$ topologies, for any $\ell \geq 0$. This result is false if one uses the $C^\infty$ topologies, or when the base manifold is infinite dimensional.

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1. Preliminaries and notations

In this note we define a bivariant JLO cocycle in terms of which we can reformulate the local families index theorem [6, 5]. We prove that our bivariant cocycle is entire in the sense of the formalism introduced by Meyer [10].

Thus we shall consider a fibration $F \to M \xrightarrow{\pi} B$ of closed manifolds endowed with smooth metrics. As can be seen from the proofs of the present paper, the main result remains true in larger categories like manifolds with corners [9] or Heisenberg manifolds [12], but we will not give the details here. The dimension of the fibers is denoted by $p$ and the dimension of the base is $p'$. We assume for simplicity that the fibers of our fibration are odd dimensional, so $p$ is odd. The formulae in the even case are similar and are left as an exercise. We also fix a hermitian vector bundle $E \to M$ whose fibres are modules over the Clifford algebra of the fiberwise tangent bundle $T_v M$. There is a Clifford homomorphism of algebra bundles

$$c : Cl(T_v M \otimes \mathbb{C}) \to \text{End}(E)$$

with $c(\xi)^2 = |\xi|^2$ for $\xi \in T_v M$. We endow $E$ with a Clifford connection $\nabla^E$ and consider the fiberwise Dirac operator $D$ associated with this connection. $D$ can be regarded as a family of elliptic operators along the fibers parametrized by the elements of the base manifold $B$, i.e. $D = (D_b)_{b \in B}$ where $D_b : C^\infty(M_b, E|_{M_b}) \to C^\infty(M_b, E|_{M_b})$ is an essentially self adjoint operator [5]. We choose the horizontal distribution $H$ so that

$$TM = H \oplus T_v M$$

and for any $m \in M, \pi_* : H_m \to T_{\pi(m)} B$ is a linear isomorphism.

The dual vector bundle $\pi^* T^* B$ can be identified with a subbundle of $T^* M$ and $H$ allows us to define the restriction projection $q : T^* M \to \pi^* T^* B$ defined by $H$, which extends to the exterior powers and yields

$$q : C^\infty(M, E \otimes \Lambda^\ell T^* B) \to C^\infty(M, E \otimes \Lambda^\ell T^* B).$$

The space $C^\infty(M, E \otimes \Lambda^\ell T^* B)$ is clearly a module over the algebra $\Omega^* B$ of differential forms on the base manifold $B$.

For any integer $h \in \mathbb{Z}$, we denote by $\Psi^h(M|B, E)$ the space of 1-step polyhomogeneous classical pseudodifferential operators of order $h$, acting along the fibres of $\pi : M \to B$, see [14, 1]. The local coefficients of such operators are thus smooth in the base variables and the space $\Psi^h(M|B; E)$ is a module over the algebra $C^\infty(B)$ of smooth functions on $B$. We also set $\psi^h(M|B, E; \Lambda^\ell B) = \Psi^h(M|B, E) \otimes \Omega^* B$, for the space of order $h$ classical fiberwise pseudodifferential operators with coefficients in differential forms on the base $B$. See for instance [3]. In the sequel and as usual

$$\psi^\infty(M|B, E; \Lambda^\ell B) := \bigcup_{h \in \mathbb{Z}} \psi^h(M|B, E; \Lambda^\ell B) \text{ and } \psi^-\infty(M|B, E; \Lambda^\ell B) := \bigcap_{h \in \mathbb{Z}} \psi^h(M|B, E; \Lambda^\ell B).$$
Definition 1.1. The operator 
\[ \nabla := \varrho \circ \nabla^E : C^\infty(M, E \otimes \Lambda^*T^*B) \to C^\infty(M, E \otimes \Lambda T^*M) \to C^\infty(M, E \otimes \Lambda^*T^*B), \]
is called a quasi-connection.

Lemma 1.2. The quasi-connection \( \nabla \) increases the degree of the forms by one and satisfies the Leibniz rule 
\[ \nabla(\xi \omega) = (-1)^{\delta \omega} \xi d\omega + (\nabla \xi) \omega. \]
Moreover, the curvature operator \( \nabla^2 \) of \( \nabla \) is a fiberwise first order differential operator with coefficients in \( \Omega^2 B \), so \( \nabla^2 \in \psi^1(M|B, E; \Lambda^2 B) \).

This is a classical lemma that we proved in [2]. We quote another well known property of the quasi-connection \( \nabla \) for later use, namely that for any \( P \in \psi^h(M|B, E; \Lambda^k B) \) the commutator \( \partial(P) := [\nabla, P] \) is well defined and belongs to \( \psi^h(M|B, E; \Lambda^{k+1} B) \). In particular, the commutator operator \( \partial \) preserves \( \psi^{-\infty}(M|B, E; \Lambda^k B) \).

Our bivariant \((n,k)\)-cochains are linear maps \( f : B_n = C^\infty(M) \otimes_n \to L_k \) where \( L = \oplus_k L_k \) is one of the graded spaces 
\[ L = \Omega^r(B) \text{ and } L = L_{-\infty} = \psi^{-\infty}(M|B, E; \Lambda^* B). \]
We choose connections on these graded spaces 
\[ d_B = \text{de Rham differential on } B \text{ and } \partial := [\nabla, \cdot] \text{ on } L_{-\infty}. \]
Integration over the fibers with respect to the fiberwise volume form, composed with the pointwise trace, yields \( \tau : L_{-\infty} \to \Omega^r(B) \). Following [13] we introduce an extra Clifford variable \( \sigma \) having degree 1 and central in the graded sense (i.e. graded commuting with all the operators). Hence we replace for instance the algebra \( L_{-\infty} \) by its extension \( L_{-\infty}[\sigma] \). Recall that the fibers of our fibration are odd dimensional. We extend \( \tau \) and the commutator \( \partial = [\nabla, \cdot] \) into \( \partial : L_{-\infty}[\sigma] \to L_{-\infty}[\sigma] \) and \( \tau_{\sigma} : L_{-\infty}[\sigma] \to \Omega^r(B) \), by setting 
\[ \partial(\sigma S) := -\sigma \partial(S) \text{ and } \tau_{\sigma}(T + \sigma S) := \tau(S). \]

Lemma 1.3. [2] The map \( \tau_{\sigma} \) is a graded \( \Omega^r(B) \)-valued trace on \( L_{-\infty}[\sigma] \), which satisfies the relation \( \tau_{\sigma} \circ \partial + d_B \circ \tau_{\sigma} = 0. \)

The proof uses that \( \tau \) is a graded trace and that \( \tau \circ \partial = d_B \circ \tau \) [3]. We now consider the superconnection \( B_{\sigma} \) defined by \( B_{\sigma} = \nabla + \sigma D \), and write \( B_{\sigma}^2 = D^2 + X \) with \( X = \nabla^2 - \sigma \partial(D) \).

Definition 1.4. Following [5] we will use the notation \( e^{-uB_{\sigma}^2} \) to denote the semigroup, that is, the solution to the heat equation associated with \( B_{\sigma}^2 \), given by the following finite perturbative sum 
\[ e^{-uB_{\sigma}^2 \cdot} = \sum_{m \geq 0} (-u)^m \int \Delta(m) e^{-uv_2 \nabla^2} X e^{-uv_2 \nabla^2} \cdots X e^{-uv_m \nabla^2} dv_1 \cdots dv_m, \]
where \( \Delta(m) = \{(v_0, \cdots, v_m) \in \mathbb{R}_+^{m+1}, \sum v_j = 1\} \) is the m-simplex.

It is well known that \( e^{-tD^2} \) belongs to \( \psi^{-\infty}(M|B, E) \), therefore, \( e^{-uB_{\sigma}^2} \) belongs to \( \psi^{-\infty}(M|B, E; \Lambda^* B) \).

2. The main theorem

Let \( A_{i} \in \psi^\infty(M|B, E; \Lambda^* B)[\sigma] \) and with \( \Delta(n) \) being, as before, the n-simplex [8, 15] we set 
\[ \langle (A_0, \cdots, A_n) \rangle := \int_{\Delta(n)} \tau_{\sigma}(A_0 e^{-uB_{\sigma}^2} A_1 e^{-uB_{\sigma}^2} \cdots A_n e^{-uB_{\sigma}^2}) du_1 \cdots du_n \in \Omega^r(B). \]
It is proved in [2] that this formula defines a multilinear functional when the sum of the pseudodifferential orders of the \( A_j \)'s is \( \leq n \).

Definition 2.1. The bivariant JLO cochain is defined by the sequence \( (\psi_n) \) given for \( (f_0, \cdots, f_n) \in C^\infty(M)^{n+1} \) by the formula 
\[ \psi_n(f_0, \cdots, f_n) := \langle (f_0, [B_{\sigma}, f_1], \cdots, [B_{\sigma}, f_n]) \rangle. \]
For any $\ell \geq 0$ the semi-norms
\[
p_q(f) := \sup_{\|x_j\| \leq 1} \|X_1 \cdots X_q(f)\|_{\infty}, \quad 0 \leq q \leq \ell \text{ and } X_j \text{ are vector fields on } M,
\]
defines the $C^\ell$ topology on $C^\infty(M)$. We denote by $\Sigma_\ell$ the bornology which is given by the bounded sets for these semi-norms. We also denote by $\Sigma_\ell$ the corresponding bornology on $C^\infty(B)$. We introduce, $d_{X_j}$ to denote the operator $i_{X_j} \circ d_B$ and the bornology $\Sigma_\ell^0$ on the algebra $\Omega^*(B)$ given on $\Omega^k(B)$ by the bounded sets for the semi-norms
\[
p_q(\omega) := \sup_{\|y_j\| \leq 1} \frac{1}{2^k q} \|d_{X_{1}} \cdots d_{X_q}(i_{Z_1} \circ \cdots \circ i_{Z_k} \omega)\|, \quad 0 \leq q \leq \ell \text{ and } Y_j, Z_i \text{ vector fields on } B.
\]

We shall use the formalism of analytic cyclic homology for bornological algebras due to R. Meyer. Recall from [10] that the universal differential graded algebra $\Omega C^\infty(M)$ is endowed with the analytic bornology $\Sigma_{\ell,t}$ generated by the sets $<S>(dS)\infty$ where $S$ describes the bounded subsets of $C^\infty(M)$ for the bornology $\Sigma_{\ell,t}$ recalled above. We are now in position to state the main result. We restrict to the odd case for simplicity.

**Theorem 2.2.** For any $\ell \geq 0$, $\psi = (\psi_{2n+1})_{n \geq 0}$ is an analytic bivariant cyclic cocycle for the algebras $(C^\infty(M), \Sigma_{\ell+1})$ and $(C^\infty(B), \Sigma_{\ell})$. More precisely, $\psi = (\psi_{2n+1})_{n \geq 0}$ is a bounded cyclic cocycle from $(\Omega C^\infty(M), \Sigma_{\ell,t+1})$ to $(\Omega^* B, \Sigma_{\ell}^0)$.

In [2], we also prove that the Connes HKR map $\chi : \Omega C^\infty(B) \to \Omega(B)$, is bounded when we endow $\Omega C^\infty(B)$ with the analytic bornology associated with $\Sigma_{\ell}$ and $\Omega(B)$ with $\Sigma_{\ell}^0$. Thus, Connes’ HKR isomorphism holds in the analytic setting with respect to these bornologies. The reader may therefore rephrase Theorem 2.2 in the language of analytic universal completions as in [10].

**Remark 2.3.** Even for a trivial fibration and under the assumption that $\nabla^2 = 0$, the bivariant JLO cocycle is not analytic in general with respect to the bornology associated with the Frechet $C^\infty$-topology on $\Omega^*(B)$.

3. **Main steps of the proof**

The proof is lengthy and so we only explain here the main steps, referring to [2] for the details. Recall that we restrict to the odd case again. We first prove the easy part of the theorem, namely the algebraic cyclic cocycle condition, see also [15, 11].

**Lemma 3.1.** The sequence $\psi = (\psi_{2n+1})_{n \geq 0}$ is a cyclic cocycle for the algebras $(C^\infty(M), C^\infty(B))$, i.e. it maps the universal differential algebra of $C^\infty(M)$ to the graded Grassmann algebra $\Omega^*(B)$ and satisfies the cocycle cocycle relation
\[
\psi \circ (b + B) + d_B \circ \psi = 0.
\]
Moreover, its components of odd form degree are trivial.

This lemma is a consequence of a bivariant generalization of Quillen’s cochain formalism [13] but we gave another direct proof in [2].

Now, we concentrate on the heart of the theorem, that is the analyticity of the JLO cocycle. The proof is first reduced to the proof of Proposition 3.2 below. Let us introduce some notations. We set $J = \{(1,0,0); (0,1,0); (0,0,1)\}$. For $\alpha \in J$ we denote by $\alpha^{(j)}$ the $j$-th component of $\alpha$, $j = 1, 2, 3$. We shall write $b^{\alpha^{(j)}}$ in a given expression to mean that we take $b$ into account only when $\alpha^{(j)} = 1$. For instance
\[
(a_0, \cdots, a_k, b^{\alpha^{(j)}}, a_{k+1}, \cdots, a_n),
\]
equals the $(n + 2)$-tuple $(a_0, \cdots, a_k, b, a_{k+1}, \cdots, a_n)$ when $\alpha^{(j)} = 1$ and the $(n + 1)$-tuple $(a_0, \cdots, a_n)$ when $\alpha^{(j)} = 0$. For $\alpha \in J$, we set
\[
X_{\alpha}^n(b) := [\nabla, b]^{\alpha^{(1)}}[\nabla, D]^{\alpha^{(2)}}\nabla^{2\alpha^{(3)}}.
\]
For any \( m \geq 0, n = (n_0, \ldots, n_m) \in \mathbb{N}^{m+1} \) and \( \alpha = (\alpha_1, \ldots, \alpha_m) \in \mathcal{F}^m \), we define an \( \sum_{j=0}^m n_j + \sum_{i=1}^m \alpha_i \) cochain \( \phi_{\alpha, n}^m \) with values in \( m + \sum_{i=1}^m \alpha_i(\mathfrak{h}) \) differential forms on \( B \), by the formula

\[
\phi_{\alpha, n}(f_0, \ldots, f_{n_0}; g_1^{(1)}, \ldots, f_{n_0+1}; \ldots; f_{n_0+n_1}; \ldots, g_m^{(1)}, f_{n_0+n_1+1}; \ldots, f_{n_0+\ldots+n_m}) :=
\]

\[
< f_0, \sigma[D, f_1], \ldots, \sigma[D, f_{n_0}], X^{\alpha_1}(g_1), \sigma[D, f_{n_0+1}], \ldots, \sigma[D, f_{n_0+n_1}],
\]

\[
\ldots, X^{\alpha_m}(g_m), \sigma[D, f_{n_0+\ldots+n_m+1}], \ldots, \sigma[D, f_{n_0+\ldots+n_m}] > .
\]

We first show that the JLO cocycle can be expanded as a finite sum of cochains of the form \( \phi_{\alpha, n}^m \). Then we need to prove the following

**Proposition 3.2.** The bihomogeneous family \( (\phi_{\alpha, n}^m)_{m, n, \alpha} \) is a bounded morphism from \( (\Omega C^\infty(M), \Sigma_{\alpha, \ell+1}) \) to \( (\Omega^*_s(B), \Sigma''_s) \).

The proof of this proposition is tedious and is carried out for \( m = 0 \) and \( s = 0 \) first, then for \( m = 0 \) and general \( s \), and eventually for the general case. The relation \( d_A \circ \tau_h + \tau_h \circ \partial_Y = 0 \) is used to reduce to simpler estimates. In order to prove the method we prove and use Lemmas 3.3, 3.4, 3.5 and Proposition 3.6 below.

For a vector field \( Y \) on \( B \), we denote by \( \tilde{Y} \) the horizontal (i.e. \( H \) valued) vector field on \( M \) satisfying \( \tilde{\pi} \tilde{Y} = Y \). We denote for any horizontal vector field \( Z \) on \( M \) by \( \nabla_Z \) the composition \( i_Z \circ \nabla \) where \( i_Z \) is contraction by \( Z \). As usual, for \( P \in \psi^h(M|B; E; \Lambda^k B) \) we also denote by \( \partial_Z(P) \) the element \( [\nabla_Z, P] \) of \( \psi^h(M|B; E; \Lambda^k B) \). When \( h \leq 0 \), we set

\[
\|P\| := \sup_{b \in B} \|P_b\|
\]

where the norm \( \|P_b\| \) is the operator norm on the \( L^2 \) sections. In general, if \( P \in \psi^h(M|B; E; \Lambda^k B) \) for \( h \leq 0 \) and \( k \geq 0 \) we define the uniform norm of \( P \) by the same expression, except that now \( \|P_b\| \) is obtained by taking the supremum over \( k \)-multivectors \( Z \in \Lambda^k(T_b B) \) of norm \( \leq 1 \), of the operator norms \( \|i_Z P_b\| \).

**Lemma 3.3.** [2] For any \( q \geq 0 \), we have

\[
\sup_{\|Y_1\| \leq 1, \ldots, \|Y_q\| \leq 1} \| (I + D^2)^{-1/2} [\partial_{Y_1} \cdots \partial_{Y_q}] (D^2) (I + D^2)^{-1/2} \| < +\infty,
\]

\[
\sup_{\|Y_1\| \leq 1, \ldots, \|Y_q\| \leq 1} \text{Max} \left( \| (I + D^2)^{-1/2} [\partial_{Y_1} \cdots \partial_{Y_q}] (D) \|, \| (I + D^2)^{-1/2} [\partial_{Y_1} \cdots \partial_{Y_q}] (D) \| \right) < +\infty
\]

where the \( Y_i \)'s are vector fields on \( B \).

**Proof.** The operator \( [\partial_{Y_1} \cdots \partial_{Y_q}] (D^2) \) is a second order vertical differential operator, with smooth coefficients. Therefore, the norms of all of the operators involved are well defined. The method of proof of this lemma is to use compactness to reduce to estimates in terms of local coordinates where the proof may be seen to be straightforward.

The following two lemmas are proved by similar methods in [2].

**Lemma 3.4.** For any \( q \geq 0 \), we have

\[
\sup_{\|Y_1\| \leq 1, \ldots, \|Y_q\| \leq 1} \| (I + D^2)^{-1/2} [\partial_{Y_1} \cdots \partial_{Y_q}] (D^2) (I + D^2)^{-1/2} \| < +\infty,
\]

where \( Y_1, Z_2, Y_1, \ldots, Y_q \) are vector fields on \( B \) that we view through their horizontal lifts.

We denote for \( s \geq 0 \) by \( \|f\|_s \) the \( C^s \) norm sup\( \|z_{i-1} \| \leq 1, \|z_i\| \leq 1, \|Y_1\| \leq 1, \ldots, \|Y_q\| \leq 1 \) \( \| (I + D^2)^{-1/2} [\partial_{Y_1} \cdots \partial_{Y_q}] (D) \| \). For any vector fields \( Y_j \) of norm \( \leq 1 \).

**Lemma 3.5.** For any \( s \geq 0 \), there exists a constant \( C_s \geq 0 \) such that

\[
\|[\partial_{Y_1} \cdots \partial_{Y_q}] (f)\| \leq C_s \|f\|_s \quad \text{and} \quad \|[\partial_{Y_1} \cdots \partial_{Y_q}] (D, f)\| \leq C_s \|f\|_{s+1},
\]

for any \( f \in C^\infty(M) \) and any vector fields \( Y_j \) of norm \( \leq 1 \).

For \( A_i \in \psi^h(M|B; E; \Lambda^k B)[\sigma] \) such that the sum of the pseudodifferential orders is \( \leq n \), we define [8, 15]

\[
\langle A_0, \ldots, A_n \rangle := \int_{\Delta(n)} \tau_\sigma (A_0 e^{-u_0 D^2} A_1 e^{-u_1 D^2} \cdots A_n e^{-u_n D^2}) du_1 \cdots du_n \in \Omega^* (B).
\]

Using a fiberwise Holder inequality, we prove the following proposition, which extends results of Getzler-Szenes [8] to families:
Proposition 3.6. For any $\epsilon \in [0, 1/2]$, for any $A_0, \ldots, A_N \in \psi^0(M|B, E)$ and any $B_{j_0}, \ldots, B_{j_p} \in \psi^2(M|B, E)$ with $p < N$ and $0 \leq j_0 < \cdots < j_p \leq N$, the following estimate holds
\[
\|\langle A_0, \ldots, A_{j_0}, B_{j_0}, A_{j_0+1}, \ldots, A_{j_1}, B_{j_1}, \ldots, A_{j_p}, B_{j_p}, A_{j_p+1}, \ldots, A_N \rangle\| \leq \frac{\tau(\epsilon^{-1}(1-\epsilon)D^2)^{p+1}p!}{\epsilon^{p+1}N!} \times \Pi_{i=0}^N \|A_i\left\| (I + D^2)^{-1/2}B_{j_i}(I + D^2)^{-1/2}\right\|
\]

A consequence of Theorem 2.2 is the next corollary which is a consequence of the boundedness of Connes’ HKR map $\chi$. Recall the definition of entire cyclic cohomology for Banach algebras from [7].

Corollary 3.7. For any homology class $z$ of the base manifold $B$ of degree $N \in \{0, \cdots, \dim(B)\}$, there exists a closed de Rham current $C \in z$ and $\ell \geq 0$ such that the sequence
\[
\psi^C = \langle C, \psi_n \rangle_{n-N \in 2\mathbb{Z}+1},
\]
is an entire cyclic cocycle over the Banach algebra $C^{\ell+1}(M)$.

Theorem 2.2 implies that if $C$ is continuous with respect to the $C^\ell$ topology, then the sequence $\psi^C$ is an analytic cyclic cocycle over the algebra $C^\infty(M)$ with endowed with the bornology $\Sigma_{\ell+1}$, and hence over its Banach completion $C^{\ell+1}(M)$. Using the comparison of the analytic theory with Connes’ entire theory as carried out in [10], we deduce that $\psi^C$ also defines an entire cyclic cocycle over the Banach algebra $C^{\ell+1}(M)$. Now, that any $z$ can be represented by such $C$ is classical.

The other main consequence of Theorem 2.2 is of course that the Chern character of the Toeplitz index can be expressed as a cohomology class on $B$ of a convergent series in a $C^\ell$ topology of $\Omega^*(B)$ [2].

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