New methods for signal digital processing to determine the parameters of seismic wave propagation models

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Abstract. The methods based on the use of time-frequency representations are proposed. The useful signal is reconstructed from the amplitude peaks of time-frequency spectrograms obtained by S-transform. As a result of numerical experiments, it is shown that using narrow time-frequency intervals (about 10–20 time reports), it is possible to determine the parameters of a pulsed-type signal with substantially longer duration.

1. Introduction

Seismic wave propagation parameters include phase and group velocities, absorption coefficients, while dynamic elastic moduli characterize different physical and mechanical properties of rocks. The measurements are carried out both on samples (acoustic logging of drill cores) and directly in a rock massif (seismic studies). Determining these parameters (both in laboratory and in situ conditions) is largely challenged by the presence of different types of noise, interference of waves, and frequency dispersion.

S-transform (ST) is a method of time-frequency spectral analysis that uniquely combines the advantages of continuous wavelet transforms (CWT) and Fourier transforms. Like CWT, the S-transform (ST) technique provides adaptive time-frequency resolution. Unlike the CWT technique, S-transform maintains a direct connection with the Fourier spectrum, and the fast Fourier transform (FFT) method can therefore be effectively used for ST calculation. Besides, S-transform is shown to have absolutely referenced phase information and allows one to define the meaning of phase in a local spectrum setting. This property ensures its invertibility.

S-transform is widely utilized in the seismic data processing. Its applications related to the time-frequency filtering and noise suppression are considered in [1–3]. The approach proposed in [4] solves the problem of separating reflections from thin layers with the generalized S-transform. The method proposed in [5] is an automated analysis of passive microseismic data based on ST-filtration acting as suppression of the noise interference according to the threshold filter [6]. In [7], the modified ST is used for deconvolution of seismic signals in highly attenuating media. In [8], the ST analysis is considered within the seismic reflection technique. S-transform is also used for analysis of seismic surface wave propagation [9–11].

The standard S-transform filtration method is based on its invertibility, which implies inverse transform of the manipulated spectrum. The input time-frequency spectrum is multiplied with the adaptive time-frequency- window [12]. Since the inverse S-transform includes time averaging [1], the
standard filtration method is not optimal in a sense of time localization of signals. In other words, in the case of too narrow time window (e.g. the window does not contain the “totally useful” signal), the filtration output will have distorted waveforms.

In this work, we propose an alternative filtration method which is based on the reconstruction of a signal by S-transform ridges. As a rule, these ridges correspond to images of seismic "pulses", which are often of interest when the seismic data are processed. The proposed method for the signal reconstruction from S-transform ridges is therefore appropriate for such problems.

2. Synthetic data and numerical modeling results
S-transform of the \( h(t) \) signal is given by the following ratio

\[
S(\tau, f) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} h(t) \frac{f}{\sqrt{2\pi}} e^{-\frac{(t-\tau)^2}{2}} e^{-i2\pi ft} dt.
\]  

(1)

We can also derive the integral equation which expresses the S-transform ridge amplitudes in terms of Fourier amplitude spectrum \( H(f) \) of the signal \( h(t) \):

\[
S(\tau, f) = \int_{-\infty}^{\infty} H(\alpha + f) e^{\frac{2\pi^2 \alpha^2}{f^2}} e^{i2\pi \alpha t} d\alpha.
\]  

(2)

Write down the Fourier and ST spectra via amplitudes and phases:

\[
H(f) = A(f) e^{-i2\pi \varphi(f)},
\]

\[
S(\tau, f) = B(\tau, f) e^{-i2\pi \beta(\tau,f)}.
\]

S-transform ridge is the function for which \( B(\tau_R(\mathcal{R}), f) \geq B(\tau, f) \) is satisfied for any \( \tau \) and \( f \) (at least in some domain). Thus, we get two frequency functions: the phase and amplitude of the S-transform ridge:

\[
B_R(f) = B(\tau_R(f), f), \beta_R(f) = \beta(\tau_R(f), f).
\]  

(3)

The problem of the signal’s amplitude and phase reconstruction from the functions is stated in (3). Following [9, 13], we substitute the following approximation of the signal phase

\[
\varphi(\alpha + f) \sim \varphi(f) + \alpha \varphi'(f)
\]  

(4)

in equation (2). After making obvious computations, we obtain:

\[
\tau_R(f) = \varphi'(f), \varphi(f) = \beta_R(f).
\]  

(5)

Noteworthy is that approximation (4) becomes an exact equality in the case of symmetric signal (e.g. the Riker pulse). After reducing the phases in equation (2), we obtain the integral equation as follows

\[
B_R(f) = \int_{-\infty}^{\infty} A(\alpha + f) e^{\frac{2\pi^2 \alpha^2}{f^2}} d\alpha,
\]  

(6)
where the amplitude of signal $A$ is an unknown function. Solve equation (6) numerically. Assume that the unknown signal amplitude and the ST ridge amplitude are equal to zero beyond the limits of a certain frequency range:

$$A(f) = 0 \text{ and } B_{k}(f) = 0 \text{ for } f \notin (f_0, f_{N+1}).$$

The frequency is discretized within this interval:

$$f_j = f_0 + j\delta\alpha, \quad \delta\alpha = \frac{f_{N+1} - f_0}{N+1}, \quad j = 1 \ldots N.$$  

After applying the quadrature formulas to the integral from (6), we obtain a system of linear algebraic equation system (LAES):

$$Ma = b,$$  

where $M$ is $N \times N$ matrix with elements $M_{kj} = \frac{2\pi^2(k-j)^2}{k^2}\delta\alpha$; $a$ is the vector of unknowns: $a_j = A(f_j)$, the vector from the right side of the equation: $b_k = B_{k}(f_k)$.

Solving the integral equation is an ill-defined problem, inasmuch as matrix $M$ is ill-conditioned. Following [14], we apply the truncated singular value decomposition (TSVD) when solving the system of equations (8).

To demonstrate the capabilities of the proposed S-transform method, a broadband seismic “pulse” was generated (Figure 1a) along with the second signal, which is a “noise” (Figure 1b). Their summation is shown in Figure 1c, while their amplitudes are illustrated by Figure 1d and Figure 1e, respectively.

The S-transform ridge on the amplitude ST spectrum (Figure 1f) corresponds to the image of the first signal. The purpose of filtration is the signal reconstruction from this ridge within the selected frequency range from 25 to 65 Hz (the peak is marked with a black band in Figure 1f).

**Figure 1.** Synthetic data and ST-spectra.
For solving the linear system (8) using the TSVD method, 20 higher-order singular vectors were utilized. The condition number of the corresponding low-rank matrix is about 11.6. The signal reconstruction result is shown in Figure 2.

![Figure 2. Synthetic data and ST-spectra.](image)

The reconstructed signal (shown in circles) matches well with the original reconstructed signal (a solid black line). Other data which are also shown here illustrate the filtration (gray color).

3. Conclusions

The proposed S-transform method provides a framework for data-adaptive filters which take advantage of time-frequency localized spectra. The useful signal is derived from the peaks of amplitude S-transform spectrum. The phase and amplitude Fourier spectrum of such a signal is reconstructed from the enhanced ST peak in the isolated frequency range. The method is suitable for extracting broadband seismic “pulse” signals, which is often necessary for the seismic data processing in the presence of noise and interference. The method’s efficiency is demonstrated on synthetic data.

An increase in crack opening above this value leads to a slight decrease in filtration resistance of the drainage zone. Hydraulic fracturing of gassy coal seams without proppant is effective at low compression pressures (no more than 3–4 MPa) the cracks are exposed to, however, under high rock pressures its efficiency is low. Improving the in-mine hydraulic fracturing efficiency in coal seams appears to be most promising with development of the technique for layer-wise longitudinal hydraulic fractures and their filling with proppant.

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References

[1] Stockwell RG, Mansinha L and Lowe RP 1996 Localization of the complex spectrum: The S transform *IEEE Transactions on Signal Processing* Vol 44 No 4 pp 998–1001

[2] Pinnegar CR and Mansinha L 2003 The S-transform with windows of arbitrary and varying shape *Geophysics* Vol 68 No 1 pp 381–385

[3] Pinnegar CR and Eaton DW 2003 Application of the S transform to prestack noise attenuation filtering *Journal of Geophysical Research: Solid Earth* Vol 108 No B9

[4] Askari R and Siahkoohi HR 2008 Ground roll attenuation using the S and xfk transforms *Geophysical Prospecting* Vol 56 No 1 pp 105–114

[5] Tselentis GA, Martakis N, Paraskevopoulos P, Lois A and Sokos E 2012 Strategy for automated analysis of passive microseismic data based on S-transform, Otsu’s thresholding, and higher
order statistics Geophysics Vol 77 No 6 pp KS43–KS54
[6] Otsu N 1979 A threshold selection method from gray-level histograms IEEE Transactions on Systems, Man, and Cybernetics Vol 9 No 1 pp 62–66
[7] Djeffal A, Pennington W and Askari R 2016 Enhancement of Margrave deconvolution of seismic signals in highly attenuating media using the modified S-transform SEG Technical Program Expanded Abstracts 2016 Society of Exploration Geophysicists pp 5198–5202
[8] Li D, Castagna J and Goloshubin G 2016 Investigation of generalized S-transform analysis windows for time-frequency analysis of seismic reflection dataInvestigation of GST analysis windows Geophysics Vol 81 No 3 pp V235–V247
[9] Askari R and Ferguson RJ Dispersion and the dissipative characteristics of surface waves in the generalized S-transform domainDispersion and dissipation of surface waves Geophysics Vol 77 No 1 pp V11–V20
[10] Askari R and Hejazi SH 2015 Estimation of surface-wave group velocity using slant stack in the generalized S-transform domainSurface-wave group velocity estimation Geophysics Vol 80 No 4 pp EN83–EN92
[11] Serdyukov AS, Yablokov AV, Duchkov AA, Azarov AV and Baranov VD 2019 Slant f-k transform of multichannel seismic surface wave data Geophysics Vol 84 No 1 pp A19–A24
[12] Schimmel M and Gallart J 2005 The inverse S-transform in filters with time-frequency localization IEEE Transactions on Signal Processing Vol 53 No 11 pp 4417–4422
[13] Kulesh M et al 2005 Modeling of wave dispersion using continuous wavelet transforms Pure and Applied Geophysics Vol 162 No 5 pp 843–855
[14] Hansen PC 1990 Truncated singular value decomposition solutions to discrete ill-posed problems with ill-determined numerical rank SIAM Journal on Scientific and Statistical Computing Vol 11 No 3 pp 503–518