Primordial Nucleosynthesis Constraints on $Z'$ Properties

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Abstract

In models involving new TeV-scale $Z'$ gauge bosons, the new $U(1)'$ symmetry often prevents the generation of Majorana masses needed for a conventional neutrino seesaw, leading to three superweakly interacting “right-handed” neutrinos $\nu_R$, the Dirac partners of the ordinary neutrinos. These can be produced prior to big bang nucleosynthesis by the $Z'$ interactions, leading to a faster expansion rate and too much $^4\text{He}$. We quantify the constraints on the $Z'$ properties from nucleosynthesis for $Z'$ couplings motivated by a class of $E_6$ models parametrized by an angle $\theta_{E6}$. The rate for the annihilation of three approximately massless right-handed neutrinos into other particle pairs through the $Z'$ channel is calculated. The decoupling temperature, which is higher than that of ordinary left-handed neutrinos due to the large $Z'$ mass, is evaluated, and the equivalent number of new doublet neutrinos $\Delta N_\nu$ is obtained numerically as a function of the $Z'$ mass and couplings for a variety of assumptions concerning the $Z-Z'$ mixing angle and the quark-hadron transition temperature $T_c$. Except near the values of $\theta_{E6}$ for which the $Z'$ decouples from the right-handed neutrinos, the $Z'$ mass and mixing constraints from nucleosynthesis are much more stringent than the existing laboratory limits from searches for direct production or from precision electroweak data, and are comparable to the ranges that may ultimately be probed at proposed colliders. For the case $T_c = 150$ MeV with the theoretically favored range of $Z-Z'$ mixings, $\Delta N_\nu \lesssim 0.3$ for $M_{Z'} \gtrsim 4.3$ TeV for any value of $\theta_{E6}$. Larger mixing or larger $T_c$ often lead to unacceptably large $\Delta N_\nu$ except near the $\nu_R$ decoupling limit.
1 Introduction

Additional heavy $Z'$ gauge bosons \cite{11} are predicted in many superstring \cite{2} and grand unified \cite{3} theories, and also in models of dynamical symmetry breaking \cite{4}. If present at a scale of a TeV or so they could provide a solution to the $\mu$ problem \cite{5} and other problems of the minimal supersymmetric standard model (MSSM) \cite{6}. Current limits from collider \cite{7, 8} and precision \cite{9} experiments are model dependent, but generally imply that $M_{Z'} > (500 \sim 800)$ GeV and that the $Z - Z'$ mixing angle is smaller than a few $\times 10^{-5}$. There are even hints of deviations in atomic parity violation \cite{10}, and the NuTeV experiment \cite{12}, which could be an early indication of a $Z'$ \cite{13}. A $Z'$ lighter than a TeV or so should be observable at Run II at the Tevatron. Future colliders should be able to observe a $Z'$ with mass up to around 5 TeV and perform diagnostics on the couplings up to a few TeV \cite{14}.

An electroweak or TeV-scale $Z'$ would have important implications for theories of neutrino mass. If the right-handed neutrinos carry a non-zero $U(1)'$ charge, then the $U(1)'$ symmetry forbids them from obtaining a Majorana mass much larger than the $U(1)'$-breaking scale, and in particular would forbid a conventional neutrino seesaw model \cite{15}. In this case, it might still be possible to generate small Majorana masses for the ordinary (active) neutrinos by some sort of TeV-scale seesaw mechanism in which there are additional mass suppressions \cite{16}. However, another possibility is that there are no Majorana mass terms, and that the neutrinos have Dirac masses which are small for some reason, such as higher dimensional operators \cite{17} or volume suppressions in theories with large extra dimensions \cite{18}. In this case, the model would contain three additional right-handed partners of the ordinary neutrinos, which would be almost massless. Such light Dirac neutrinos (i.e., with mass less than an eV or so) in the standard model or MSSM are essentially sterile, except for the tiny effects associated with their masses and Higgs couplings, which are much too small to produce them in significant numbers prior to nucleosynthesis or in a supernova. However, the superweak interactions of these states due to their coupling to a heavy $Z'$ (or a heavy $W'$ in the $SU(2)_L \times SU(2)_R \times U(1)$ extension of the standard model \cite{19}) might be sufficient to create them in large numbers in the early universe \cite{20, 21, 22} or in a supernova \cite{23}. In this paper, we consider the constraints following from big bang nucleosynthesis on $Z'$ properties in a class of $E_6$-motivated models.

It is well known that any new relativistic particle species that were present when the temperature $T$ was a few MeV would increase the expansion rate, leading to an earlier freeze-out of the neutron to proton ratio and therefore to a higher $^4He$ abundance \cite{24, 25}. Their contribution is usually parametrized by the number $\Delta N_{\nu}$ of additional neutrinos with full-strength weak interactions that would yield the same contribution to the energy density. The primordial $^4He$ abundance is still rather uncertain, but typical estimates of the upper limit on $\Delta N_{\nu}$ are in the range\footnote{The interpretation of these results is controversial. For recent discussion, see \cite{11}.}

\footnote{The limit can be weakened by invoking an excess of $\nu_\tau$ with respect to $\bar{\nu}_e$, which lowers the $n/p$ ratio.}
$\Delta N_\nu < (0.3 - 1)$ \[25, 26\]. Of course, the $Z$-width does not allow more than 3 light active neutrinos \[27\], so $\Delta N_\nu$ should be interpreted as an effective parameter describing degrees of freedom that do not couple with full strength to the $Z$.

In 1979, Steigman, Olive, and Schramm \[20, 21\] described the implications of a superweakly interacting light particle, such as a right-handed neutrino coupling to a heavy $Z'$. Because of their superweak interactions, such particles decoupled earlier than ordinary neutrinos. As the temperature dropped further, massive particles such as quarks, pions, and muons subsequently annihilated, reheating the ordinary neutrinos and other particles in equilibrium, but not the superweak particles. One must also take into account the transition between the quark-gluon phase and the hadron phase.

A simple estimate of the decoupling temperature is obtained as follows \[20, 21\]. Ordinary neutrinos have cross-sections $\sigma_W \propto G_W^2 T^2$, where $G_W$ is the Fermi constant, and interaction rates

$$\Gamma_W(T) = n \langle \sigma_W v \rangle \propto G_W^2 T^5,$$

where $n$ is the density of target particles. The Hubble expansion parameter varies as $H \propto T^2/M_P$, where $M_P$ is the Planck scale, so the decoupling temperature $T_d$ at which $\Gamma$ is equal to $H$ becomes

$$T_d \propto (G_W^2 M_P)^{-1/3}.$$

Putting in the coefficients, $T_d(\nu_L) \approx 1 \text{ MeV}$ for the ordinary neutrinos\[^3\]. Similarly, a superweakly interacting particle such as a right-handed neutrino with a cross-section $\sigma_{SW} \propto G_{SW}^2 T^2$, would decouple at

$$T_d(\nu_R) \sim \left( \frac{G_W}{G_{SW}} \right)^{2/3} T_d(\nu_L).$$

If in the specific model, the effective superweak coupling constant $G_{SW}$ is proportional to $M_{SW}^{-2}$, where $M_{SW}$ is the mass of a superweak gauge boson, the decoupling temperature can be written as

$$T_d(\nu_R) \sim \left( \frac{M_{SW}}{M_W} \right)^{4/3} T_d(\nu_L),$$

where $M_W$ is the $W$ mass. It is then straightforward to calculate the dilution by the subsequent quark-hadron transition and the annihilations of heavy particles, and the corresponding $\Delta N_\nu$ from the superweak particles.

Of course, the estimate in (4) is very rough. In particular, the detailed couplings of the $Z'$ to the $\nu_R$ and to all of the other relevant particles must be considered.

\[^3\]More detailed studies \[25\] obtain $T_d(\nu_L) \sim 3 \text{ MeV}$. We will obtain $T_d(\nu_R)$ by an explicit calculation, so the difference is irrelevant for our purposes.
for a precise estimate\textsuperscript{4} In this paper, we do this for a class of $Z'$ models with couplings motivated by $E_6$ grand unification \textsuperscript{32}. (The full structure of $E_6$ is not required.) We define the $U(1)'$ model in Section 2. The implications of superweakly coupled particles for nucleosynthesis and the uncertainties from the quark-hadron transition temperature $T_c$ are summarized in Section 3. Section 4 deals with the calculation of the decoupling temperature. We present our results and numerical analysis for $T_d$ and $\Delta N_\nu$ for three right-handed neutrinos as a function of the $Z'$ mass and couplings for various assumptions concerning the $Z - Z'$ mixing and $T_c$ in Section 5. The discussion and conclusion follows in Section 6.

2 $Z'$ in $E_6$-motivated models

A general model with an extra $Z'$ is characterized by the $Z'$ mass; the $Z - Z'$ mixing angle; the $U(1)'$ gauge coupling; the $U(1)'$ chiral charges for all of the fermions and scalars, which in general may be family non-universal, leading to flavor changing neutral currents \textsuperscript{33}; and an additional parameter associated with mixing between the $Z$ and $Z'$ kinetic terms \textsuperscript{34}. Furthermore, most concrete $Z'$ models involve additional particles with exotic standard model quantum numbers, which are required to prevent anomalies. It is difficult to work with the most general case, so many studies make use of the $U(1)'$ charges and exotic particle content associated with the $E_6$ model, as an example of a consistent anomaly-free construction\textsuperscript{5}. Explicit string constructions \textsuperscript{35} often lead to other patterns of couplings and exotics, but these are very model dependent.

$E_6$ actually yields two additional $U(1)'$ factors when broken to the standard model (or to $SU(5)$), i.e.,

$$E_6 \rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi.$$ \hfill (5)

It is usually assumed that only one linear combination survives to low energies, parametrized by a mixing angle $\theta_{E6}$. The resultant $U(1)'$ charge is then\textsuperscript{6}

$$Q = Q_\chi \cos \theta_{E6} + Q_\psi \sin \theta_{E6}.$$ \hfill (6)

A special case that is often considered is $U(1)_\eta$, which corresponds to $\theta_{E6} = 2\pi - \tan^{-1}\sqrt{\frac{5}{3}} = 1.71\pi$. We list the charges of $U(1)_\chi$ and $U(1)_\psi$ that we need in Table 1. The quantum numbers of the associated exotic particles are given in \textsuperscript{32}. It is

\textsuperscript{4}Detailed calculations were carried out in \textsuperscript{28} for the $\eta$ model (see Section 2), in \textsuperscript{29} for more general $E_6$ models, and in \textsuperscript{30} for a model with generators $T_{3R}$ and $B-L$. However, these studies considered only $\nu_R \leftrightarrow (e^+ e^-, \nu_L \nu_L^*)$. In the present paper we include the interactions with all of the particles in equilibrium at a given temperature. This leads to a lower $T_d(\nu_R)$ and more stringent limits. Constraints on extended technicolor models were considered in \textsuperscript{31}.

\textsuperscript{5}The full structure of $E_6$ grand unification is not required, and in fact the $E_6$ Yukawa coupling relations must not be respected in order to prevent rapid proton decay \textsuperscript{32}.

\textsuperscript{6}We ignore the possibility of kinetic mixing \textsuperscript{34}. 


Table 1: The (family-universal) charges of the $U(1)_\chi$ and the $U(1)_\psi$.

| Fields | $Q_\chi$ | $Q_\psi$ |
|--------|----------|----------|
| $u_L$  | $-1/2\sqrt{10}$ | $1/2\sqrt{6}$ |
| $u_R$  | $1/2\sqrt{10}$ | $-1/2\sqrt{6}$ |
| $d_L$  | $-1/2\sqrt{10}$ | $1/2\sqrt{6}$ |
| $d_R$  | $-3/2\sqrt{10}$ | $-1/2\sqrt{6}$ |
| $e_L$  | $3/2\sqrt{10}$ | $1/2\sqrt{6}$ |
| $e_R$  | $1/2\sqrt{10}$ | $-1/2\sqrt{6}$ |
| $\nu_L$ | $3/2\sqrt{10}$ | $1/2\sqrt{6}$ |
| $\nu_R$ | $5/2\sqrt{10}$ | $-1/2\sqrt{6}$ |

conventional to choose $\theta_{E6}$ to be in the range $(0, \pi)$, since the charges merely change sign for $\theta_{E6} \to \theta_{E6} + \pi$. With this convention one must allow both positive and negative values for the $Z - Z'$ mixing angle $\delta$. In this paper, we find it convenient to choose a different convention in which $\theta_{E6}$ varies from 0 to $2 \pi$, but for which $\delta \leq 0$. That is, the range $0 - \pi$ corresponds to the $E_6$ models with negative mixing, while $\pi - 2 \pi$ corresponds to positive mixing. The $\nu_R$ charge is nonzero, precluding an ordinary seesaw, except for $\theta_{E6} \sim 0.42 \pi$ and $1.42 \pi$. We will always assume that the neutrinos are Dirac and that the three right-handed neutrinos are therefore very light. (In fact, the non-zero Dirac masses play no role in the analysis.) There could be additional sterile states, such as the $SO(10)$-singlet states occurring in the 27-plet of $E_6$. If these involve nearly-massless fermions they could also contribute to the expansion rate prior to nucleosynthesis. We assume that these additional neutralinos acquire electroweak scale masses from the gauge symmetry breaking [1].

Let $Z$ and $Z'$ represent the Standard Model and $U(1)'$ gauge bosons, respectively, and $Z_{1,2}$ the mass eigenstate bosons, related by

$$
\begin{pmatrix}
Z_1 \\
Z_2
\end{pmatrix} =
\begin{pmatrix}
\cos \delta & -\sin \delta \\
\sin \delta & \cos \delta
\end{pmatrix}
\begin{pmatrix}
Z \\
Z'
\end{pmatrix},
$$

(7)

where $\delta$ is the $Z - Z'$ mixing angle. As stated in the Introduction, the limits on $M_{Z_2} \sim M_{Z'}$ depend on $\theta_{E6}$ and also on the masses of any exotics and superpartners to which the $Z'$ couples, but are typically in the range $M_{Z'} > (500-800)$ GeV. The limits on $\delta$ are correlated with those for $M_{Z'}$ and are asymmetric under $\delta \to -\delta$. However, for $M_{Z'} \sim 1$ TeV the constraints are less sensitive to $\theta_{E6}$ and are approximately
symmetric, with $|\delta| < 0.002$ giving a reasonable approximation for all $\theta_{E6}$. For larger $M_{Z'}$, there are two theoretical constraints on the mixing, corresponding to equations (6) and (5) of [36]. The first is a theoretical relation between the mass and mixing,

$$\delta = C \frac{g'_Z M_{Z_1}^2}{g_Z M_{Z_2}^2},$$

(8)

where $g_Z \equiv \sqrt{g_1^2 + g_2^2}$ and $g'_Z$ is the $U(1)'$ gauge coupling constant. The value of $g'_Z$ depends on the embedding and breaking of the underlying theory. We will choose $g'_Z = \sqrt{\frac{5}{3}} g_Z \sin \theta_W$, which corresponds to a unification of $g'_Z$ with the other gauge couplings for the exotic particle quantum numbers of supersymmetric $E_6$. In (8) $C$ depends on the charges of the scalar fields which lead to the mixing (see Table III of [36]). However, for the typical cases in which the mixing is induced by scalars in an $E_6$ 27 or $\overline{27}$-plet, it is a reasonable approximation to take $-1 < C < 1$ for all $\theta_{E6}$. (One can have a slightly more restrictive range for some $\theta_{E6}$.) The assumption $|C| < 1$ corresponds to $|\delta| < 0.0051/M_{Z_2}^2$, where $M_{Z_2}$ is in TeV. The second theoretical constraint is the requirement that the mixing should not change the mass of the lighter $Z$ more than is allowed by the data. It is equivalent to

$$|\delta| \sim \sqrt{\rho_0 - 1} \frac{M_{Z_1}}{M_{Z_2}},$$

(9)

where $M_{Z_1} = M_Z$, and the $\rho_0$ parameter, defined precisely in [37], should be exactly 1 in the standard model. The precision data imply $\rho_0 < 1.001$. Hence, $|\delta| < 0.0029/M_{Z_2}$, where $M_{Z_2}$ is again in TeV. We will consider the following cases:

(A0) $\delta = 0$ (no mixing)

(A1) $|\delta| < 0.0051/M_{Z_2}$ (theoretical mass – mixing relation)

(A2) $|\delta| < 0.0029/M_{Z_2}$ ($\rho_0$ constraint)

(A3) $|\delta| = 0.002$ (maximal mixing allowed for $M_{Z_2} \sim 1$ TeV).

(10)

A1 is more stringent than A2 and A3 in the large mass range, so we will mainly focus on A0 and A1.

The lagrangian for the massive neutral current coupling to fermion $f$ is [36]

$$-\mathcal{L}_{int} = g_Z Q_Z(f_L) \bar{f}_L \gamma^{\mu} f_L Z_\mu + g_Z Q_Z(f_R) \bar{f}_R \gamma^{\mu} f_R Z_\mu + g'_Z Q(f_L) \bar{f}_L \gamma^{\mu} f_L Z'_\mu + g'_Z Q(f_R) \bar{f}_R \gamma^{\mu} f_R Z'_\mu,$$

(11)

where

$$Q_Z(f_L) \equiv T_f^3 - q_f \sin^2 \theta_W,$$

$$Q_Z(f_R) \equiv -q_f \sin^2 \theta_W,$$

(12)

and $Q(f_{L,R})$ is given by (6). The annihilation cross-section through $Z'$ has both (light) $Z_1$ and (heavy) $Z_2$ contributions unless $\delta = 0$ and is calculated in Section 4.
3 Nucleosynthesis

As described in the Introduction, the observed $^4\text{He}$ abundance constrains the energy density at the time of Big Bang Nucleosynthesis [24], with most recent estimates [25, 26] of the number of equivalent new active neutrino types in the range $\Delta N_\nu < (0.3-1)$.

The contribution of new relativistic species can be written

$$\Delta N_\nu = \frac{8}{7} \sum_B g_B \left( \frac{T_B}{T_{BBN}} \right)^4 + \sum_F g_F \left( \frac{T_F}{T_{BBN}} \right)^4,$$

(13)

where $g_B$ and $g_F$ are degrees of freedom of new bosons (B) and new fermions (F), respectively, $T_{B,F}$ are their effective temperatures, and $T_{BBN} \sim 1$ MeV is the temperature at the time of the freeze-out of the neutron to proton ratio. In particular, the contribution of three types of right-handed neutrinos is

$$\Delta N_\nu = 3 \cdot 1 \cdot \left( \frac{T_{\nu_R}}{T_{BBN}} \right)^4 = 3 \left( \frac{g(T_{BBN})}{g(T_{d(\nu_R)})} \right)^{4/3},$$

(14)

where $T_d(\nu_R)$ is the decoupling temperature of the right-handed neutrinos. $g(T)$ is the effective number of degrees of freedom at temperature $T$. Neglecting finite mass corrections, it is given by $g_B(T) + \frac{7}{8} g_F(T)$, where $g_{B,F}(T)$ are the number of bosonic and fermionic relativistic degrees of freedom in equilibrium at temperature $T$ [20, 21]. In particular, $g(T_{BBN}) = 43/4$ from the three active neutrinos, $e^\pm$, and $\gamma$, and $g(T)$ increases (in this approximation) as a series of step functions at higher temperature as more particles are in equilibrium. The second equality in (14) comes from entropy conservation [20] in the heavy particle decouplings and quark-hadron transition subsequent to the $\nu_R$ decoupling. Therefore, the $\nu_R$ are not included in our definition of $g(T)$. (They will be included in the expansion rate formula prior to decoupling.)

In calculating $g(T)$ one must also take into account the QCD phase transition at temperature $T_c$. Above $T_c$ the $u$ and $d$ (and possibly $s$) quarks and the gluons were the relevant hadronic degrees of freedom, while below $T_c$ they are replaced by pions [20, 21]. The value of $T_c$ is poorly known, but is usually estimated to be in the range $(150-400)$ MeV [38]. This range is estimated in quark and hadron potential models as the temperature above which hadrons start to overlap (lower end) or as the temperature below which the quark gas in no longer ideal (upper end). A related uncertainty is whether to use current or constituent quark masses. At very high temperatures the quarks can be considered as asymptotically free and current masses are appropriate, while around $T_c$ constituent effects become important\(^7\). The range of estimates for $T_c$ is essentially unchanged if one simply fixes the quark masses at either value [38].

\(^7\)One can alternatively argue that the current masses are appropriate above a temperature $T_{chiral}$, above which chiral symmetry is restored, and constituent masses below $T_{chiral}$. One would expect $T_c$ and $T_{chiral}$ to be comparable, but their precise relation is uncertain.
Figure 1 shows the explicit values of $g(T)$ from the more detailed analysis of Ref. [39], which includes finite mass and other corrections, and uses the two values $T_c = 150$ MeV and 400 MeV. We will also use these values for our numerical analysis. The sharp increase in $g(T)$ above $T_c$ (because of the large number of quark and gluon degrees of freedom) is extremely important for relaxing the constraints on the $Z'$ mass.

The QCD phase transition does not occur instantaneously or at one temperature but rather smoothly (meaning both quarks and hadrons exist at the same temperature) for a period of time around $T_c$, as illustrated by the smooth curves in Figure 1. Risking a small inconsistency, we approximate our calculation of the interaction rate by simply switching from quarks to hadrons for temperatures below $T_c$. We will take the values $T_c = 150$ and 400 MeV to illustrate the range of hadronic uncertainties. Above $T_c$, the interaction rate depends in principle on the quark masses, especially for low $T_c$. However, we have found in practice that the results are almost identical for constituent and current masses, so we will mainly display them for the constituent case (both will be shown for the $\eta$ model).

The calculation of the right-handed neutrino decoupling temperature, $T_d(\nu_R)$ in terms of the $Z'$ parameters is discussed in the next section.

4 The expansion and interaction rates

A particle is decoupled from the background when its interaction rate drops below the expansion rate of the universe. In this section, we present the the cosmological expansion rate $H(T)$ along with the explicit form of the interaction rate $\Gamma(T)$ for $\nu_R\nu_R$ annihilating into all open channels\(^8\), and estimate the decoupling temperature $T_d$ of a right-handed neutrino by $\Gamma(T_d) \sim H(T_d)$.

The Hubble expansion parameter is given by

$$H(T) = \sqrt{\frac{8\pi G_N \rho(T)}{3}} = \sqrt{\frac{4\pi^3 G_N g'(T)}{45} T^2} \quad (15)$$

where $G_N = M_P^{-2}$ is the Newton constant and $\rho(T)$ is the energy density. We define $g'(T) = g(T) + \frac{21}{4}$, where the 21/4 reflects the 3 massless right-handed neutrinos.

The cross-section $\sigma_i(s) \equiv \sigma(\nu_R\nu_R \rightarrow f_if_i)$ for a massless right-handed neutrino pair to annihilate into a fermion pair through the $Z'$-channel is

$$\sigma_i(s) = N_C^2 \frac{s^{\beta_i}}{16\pi} \left\{ \left(1 + \frac{\beta_i^2}{3}\right) \left((G_{RL}^i)^2 + (G_{RR}^i)^2\right) + 2 \left(1 - \beta_i^2\right) G_{RL}^i G_{RR}^i \right\} \quad (16)$$

\(^8\)As long as equilibrium is maintained, the $\nu_R$ annihilation and production rates are the same. It is more convenient to estimate the annihilation rate of $\nu_R\nu_R$ into massive particles, because the final state mass effects are easily incorporated in the cross section formulae, whereas for the production rate one must explicitly consider the suppressed number density for the massive particles.
where \((s \ll M_{Z_1}^2, M_{Z_2}^2)\)

\[
G_{RX}^i = g_Z^2 Q(\nu_R)Q(f_{iX}) \left( \frac{\sin^2 \delta + \cos^2 \delta}{M_{Z_1}^2} \right) - g'_Z g Z(\nu_R)Q_Z(f_{iX}) \left( \frac{\sin \delta \cos \delta}{M_{Z_1}^2} + \frac{\sin \delta \cos \delta}{M_{Z_2}^2} \right),
\]

(17)

where \(X = L \text{ or } R, \quad \beta_i \equiv \sqrt{1 - 4m_{f_i}^2/s}\) is the relativistic velocity for the final particles, and \(N_C^i\) is the color factor of particle \(f_i\).

In the limit of no-mixing \((\delta = 0)\) and massless final particles \((\beta_i = 1)\), the cross-section simplifies to

\[
\sigma_i(s) \to N_C^i \frac{s}{12\pi} \left( \frac{g_Z^2}{M_{Z_1}^2} \right)^2 Q(\nu_R)^2 \left( Q(f_{iL})^2 + Q(f_{iR})^2 \right),
\]

(18)

consistent with the earlier estimate \(\sigma_{SW} \propto G_{SW}^2 T^2\) with \(G_{SW} \propto g_Z^2 M_{Z_1}^2\) and \(T \propto \sqrt{s}\).

For temperatures less than the quark-hadron transition temperature \(T_c = 150 - 400\) MeV, we replace the quark degrees of freedom with hadrons. The only relevant annihilation channels are into charged pions. We approximate the cross-section of \(\nu_R \nu_R\) annihilating into \(\pi^+ \pi^-\) by using the \(\rho\) dominance model \[40\].

\[
\sigma_{\pi}(s) \equiv \sigma(\nu_R \nu_R \to \pi^+ \pi^-) = \frac{s \beta^2}{96\pi} |F_{\pi}(s)|^2 \left( G_{RL}^u + G_{RL}^d + G_{RR}^u + G_{RR}^d \right)^2
\]

(19)

which is basically obtained by using \(Q(f_{iL}) = Q(u_L) + Q(d_L)\) and \(Q_Z(f_{iL}) = Q_Z(u_L) + Q_Z(d_L)\) for \(G_{RL}^i\) and likewise for \(G_{RR}^i\). The pion form factor\(^9\) is

\[
F_{\pi}(s) = \frac{m_{\rho}^2}{s - m_{\rho}^2 + im_{\rho} \Gamma_{\rho}},
\]

(20)

with \(m_{\rho} = 771\) MeV and \(\Gamma_{\rho} = 149\) MeV.

The interaction rate per \(\nu_R\) is

\[
\Gamma(T) = \sum_i \Gamma_i(T) = \sum_i \frac{n_{\nu_R}}{g_{\nu_R}} \left\langle \sigma v(\nu_R \nu_R \to f_{iL} \to f_{iR}, \pi^+ \pi^-) \right\rangle,
\]

(21)

where \(n_{\nu_R}\) is the number density of a single flavor of massless right-handed neutrinos plus antineutrinos, \(g_{\nu_R} = 2\) is the number of degrees of freedom, and \(\left\langle \sigma v \right\rangle\) is the thermal average of the cross-section times velocity.

We use the same masses (Table 2) used in the calculation \[21, 39\] of \(g(T)\) in Figure 1 except for the value \(m_b = 4200\) MeV of the \(b\) quark current mass \[27\]. We include the contributions of all particles up to the \(b\) quarks. The contributions

\(^9\)More complicated form factors are known to fit the experimental data better \[41\], but \[20\] is adequate for our purposes.
Table 2: The masses (in MeV) used for the numerical analysis.

| Quarks | Current (Constituent) masses | Others | Masses |
|--------|-----------------------------|--------|--------|
| u      | 4.2 (340)                   | ν      | 0      |
| d      | 7.5 (340)                   | e      | 0.511  |
| s      | 150 (540)                   | µ      | 105    |
| c      | 1150 (1500)                 | τ      | 1800   |
| b      | 4200 (4500)                 | π      | 137    |

from the top quark and heavy particles from new physics, such as squarks, sleptons, and exotics would only be relevant when the decoupling temperature is close to the electroweak scale or higher. This only occurs when \( \theta_{E6} \) is extremely close to the values for which the \( \nu_R \) decouples from the \( Z' \).

For a massless right-handed neutrino pair colliding with 4-momenta \( p^\mu \equiv (p, p) \) and \( k^\mu \equiv (k, k) \) with relative angle \( \theta \), the interaction rate per neutrino is

\[
\Gamma_i(T) = \frac{g_{\nu_R}}{n_{\nu_R}(T)} \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} f_\nu(p) f_\nu(k) \sigma_i(s) v_M
\]

\[
= \frac{g_{\nu_R}}{8\pi^4 n_{\nu_R}} \int_0^\infty p^2 dp \int_0^\infty k^2 dk \int_{-1}^1 d\cos \theta \frac{(1 - \cos \theta)}{(e^{k/T} + 1)(e^{p/T} + 1)} \sigma_i(s),
\]  

where \( f_\nu(k) = (e^{k/T} + 1)^{-1} \) is the Fermi-Dirac distribution with

\[
n_{\nu_R}(T) = g_{\nu_R} \int \frac{d^3k}{(2\pi)^3} f_\nu(k) = 2 \cdot \frac{3}{4\pi^2} \zeta(3) T^3,
\]  

\( v_M = p \cdot k/pk = 1 - \cos \theta \) is the Møller velocity, and \( s = 2pk(1 - \cos \theta) \) is the square of the center-of-mass energy.

A root-finding method was used to calculate the decoupling temperature, for which \( H = \Gamma \). A several percent error was allowed in the numerical result to calculate the roots efficiently. Finite temperature effects, such as changes in the phase space due to interactions with the thermal bath, can increase the ordinary neutrino decoupling temperature by several percent. Analogous effects for the \( \nu_R \) are too small to significantly affect our results.

5 Numerical results

In this section, we present the numerical results from the calculation. The marked points in Figures are the results of the actual calculation, while the curves inter-
latter.

Figures 2 and 3 show how the right-handed neutrino decoupling temperature $T_d$ and the equivalent number of extra neutrino species $\Delta N_\nu$ change with $M_{Z_2}$ for $\theta_{E6} = 2\pi - \tan^{-1}\sqrt{\frac{3}{5}} \sim 1.71\pi$ (the $\eta$ model) for constituent and current masses, respectively, for $T_c = 150$ and 400 MeV and the various assumptions concerning the $Z - Z'$ mixing listed in (10). The no-mixing curves (A0) exhibit an approximate $T_d \sim (M_{Z_2}/M_Z)^{4/3}$ dependence, in agreement with the simple estimate in the Introduction [20][21]. This is to be roughly expected because of the $M_{Z_2}^{-1}$ dependence of the cross section for no mixing, but is not exact because additional channels which affect both the expansion and interaction rates open up at higher temperatures. The no-mixing curves in Figures 2 and 3 are reasonably described by (1) for $T_d(\nu_L) \sim 3$ MeV for the $\eta$ model, but the coefficients in front of $(M_{Z_2}/M_Z)^{4/3}$ are strongly model dependent, as is apparent in Figures 4-5. $T_d$ is usually lower in the cases involving $Z - Z'$ mixing, because the $Z$ annihilation channel yields a contribution proportional to $\delta^2$ even for infinite $M_{Z_2}$. That is why the (theoretically unrealistic) curves A3 for fixed $|\delta| = 0.002$ are asymptotically flat for large $M_{Z_2}$. Case A1, in which $|\delta| \sim 0.0051/M_{Z_2}^2$, also has $T_d \sim (M_{Z_2}/M_Z)^{4/3}$, though with a smaller coefficient than for no mixing, while A2, with $|\delta| = 0.0029/M_{Z_2}$, has $T_d \sim (M_{Z_2}/M_Z)^{2/3}$. For case A1, $T_d$ is asymmetric under $\delta \rightarrow -\delta$ for all $M_{Z_2}$, as is apparent from (10) and (17). The difference vanishes asymptotically for A2 and A3, but even for $M_{Z_2} = 5$ TeV there is still a difference, especially for A2.

The decoupling temperature is slightly lower for $T_c = 400$ MeV than for 150 MeV, provided it is in the range for which the two curves in Figure 1 differ. Both the expansion and annihilation rates are smaller for $T_c = 400$ MeV, but the effect on the expansion rate is more important because of the gluonic degrees of freedom. Similarly, $T_d$ is smaller for current quark masses than for constituent masses, provided $T_d > T_c$, because of the larger annihilation rate$^{11}$.

The $\Delta N_\nu$ curves change rapidly when $T_d$ reaches the quark-hadron phase transition temperature $T_c$, where $g(T)$ changes significantly. That is why $\Delta N_\nu$ is so much larger for $T_c = 400$ MeV than for 150 MeV. For the no-mixing case, the difference is significant for $M_{Z_2} \approx 4$ TeV, and it persists to even higher masses for the mixing cases (and to infinite mass for maximal mixing). The only significant difference between the constituent and current quark masses is in the maximal mixing case with $T_c = 150$ MeV. That is because $T_d$ is very close to $T_c$, and even a small change in $T_d$ leads to a significant change in $g(T)$, as can be seen in Figure 1.

It is apparent from Figures 2 and 3 that the $\eta$ model leads to a significant $\Delta N_\nu$ for all of the cases and parameter ranges considered. Even the very conservative constraint $\Delta N_\nu < 1$ implies $M_{Z_2} > 1.5 - 2.2$ TeV for $T_c = 150$ MeV, or, limiting ourselves to the most realistic cases A0 and A1, $M_{Z_2} > 1.5 - 1.9$ TeV. For $T_c = 400$

$^{10}$The coefficient is smaller for most but not all values of $\theta_{E6}$.

$^{11}$The difference between current and constituent masses would be reduced if their effects in the annihilation rate were properly correlated with those in the expansion rate. However, as described in Section 3, the effect on $g(T)$ is small compared with the uncertainty from $T_c$, and will be neglected.
Many theories beyond the standard model predict the existence of additional $Z'$ gauge bosons at the TeV scale. The associated $U(1)'$ gauge symmetry often prevents the large Majorana masses needed for an ordinary neutrino seesaw model. One possibility is that the neutrino masses are Dirac and small. In that case, there is a possibility
of producing the sterile “right-handed” neutrino partners $\nu_R$ via $Z'$ interactions prior to nucleosynthesis [20][21], leading to a faster expansion and additional $^4He$.

We have studied the right-handed neutrino decoupling temperature $T_d$ in a class of $E_6$-motivated $U(1)'$ models as a function of the $Z'$ mass and couplings (determined by an angle $\theta_{E6}$) for a variety of assumptions concerning the $Z - Z'$ mixing angle $\delta$, the quark-hadron transition temperature $T_c$, and the nature (constituent or current) of the quark masses. We have taken all relevant channels (quark, gluon, lepton, and hadron) into account, not only in the expansion rate $H(T)$ and entropy, but also in the rate $\Gamma(T)$ for a massless right-handed neutrino pair to annihilate into a fermion or pion pair via the ordinary or heavy but also in the rate $\Gamma(T)$ for a massless right-handed neutrino pair to annihilate into a fermion or pion pair via the ordinary or heavy $Z$ bosons. We therefore obtain a larger annihilation rate, and thus a lower decoupling temperature and more stringent constraints, than earlier calculations, which only included annihilation into $e^+e^-$ and $\nu_L\bar{\nu}_L$.

From the decoupling temperature and entropy conservation as quarks and gluons are confined or as various heavy particle types decouple and annihilate, one can obtain the number of right-handed neutrinos at nucleosynthesis, expressed in terms of the equivalent number $\Delta N_\nu$ of new ordinary neutrino species, for various sets of model parameters $M_{Z_2}$, $\delta$, $\theta_{E6}$, and $T_c$. Most recent studies of the primordial abundances obtain upper limits on $\Delta N_\nu$ in the range $(0.3-1)$ [25][26]. As can be seen in Figures 4-5, this implies rather stringent constraints on the $Z'$ parameters for most values of $\theta_{E6}$. For $T_c = 150$ MeV, the constraint $\Delta N_\nu < 0.3(1)$ is satisfied for all $\theta_{E6}$ for $M_{Z_2}$ $\gtrsim 3.8(2.2)$ TeV for no $Z-Z'$ mixing, and for $M_{Z_2}$ $\gtrsim 4.3(2.4)$ TeV allowing the range of mixing angles $\delta$ obtained approximately when one assumes that the scalar fields responsible for the mixing are contained in the 27 or 27-plet of $E_6$ (case A1 in [10]). For $T_c = 400$ MeV the constraints are much stronger, $M_{Z_2}$ $\gtrsim 6.1(5.1)$ TeV for $\Delta N_\nu < 0.3(1)$. The strong dependence on $T_c$ is due to the large increase in the number of degrees of freedom for temperatures $\gtrsim T_c$ (Figure 11), so that the number density of $\nu_R$ is strongly diluted for $T_d \gtrsim T_c$. The constraints are strongest for $\theta_{E6}$ close to 0 or $\pi$, i.e., near the $\chi$ model, which corresponds to $SO(10) \rightarrow SU(5) \times U(1)_\chi$, and are very weak near the $\psi$ model corresponding to $E_6 \rightarrow SO(10) \times U(1)_\psi$, $\theta_{E6} = \pi/2$. They disappear entirely at the values $\theta_{E6} = 0.42\pi$ and $1.42\pi$, for which the $\nu_R$ decouple from the $Z'$. The often considered $\eta$ model, $\theta_{E6} = 2\pi - \tan^{-1}\sqrt{2} = 1.71\pi$ (or $0.71\pi$ for $-Z_\eta$) is somewhere in between, with the constraints shown in more detail in Figures 2 and 3.

Except near the $\nu_R$ decoupling angles, the $Z'$ mass and mixing constraints from nucleosynthesis are much more stringent than the existing laboratory limits from searches for direct production or from precision electroweak data, and are comparable to the ranges that may ultimately be probed at proposed colliders. They are qualitatively similar to the limits from energy emission from Supernova 1987A [23], but somewhat more stringent for $\Delta N_\nu < 0.3$, and have entirely different theoretical and systematic uncertainties.

There are several ways to evade the nucleosynthesis constraints on an extra $Z'$. One possibility is to generate small Majorana neutrino masses for the ordinary
neutrinos by invoking an extended seesaw model [16], in which the extra sterile neutrinos are typically at the TeV scale. Another possibility is that the $\nu_R$ decouple from the $Z'$, in which case the constraints disappear. This can in fact occur naturally in classes of models in which one combination of the $\chi$ and $\psi$ charges is broken at a large scale associated with an $F$ and $D$-flat direction [14], leaving a light $Z'$ which decouples from the $\nu_R$. Yet another possibility is to weaken the observational constraint on $\Delta N_{\nu}$ by allowing a large excess of $\nu_e$ with respect to $\bar{\nu}_e$. This would, however, require a somewhat fine-tuned cancellation between the effects of the $\nu_R$ and the $\nu_e - \bar{\nu}_e$ asymmetry.

Similar constraints on the $W'$ and $Z'$ properties in $SU(2)_L \times SU(2)_R \times U(1)$ models [19] are under investigation [46].

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12A large Majorana mass for the $\nu_R$ may still be forbidden in the model.

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Figure 1: The effective number of degrees of freedom as a function of temperature for the quark-hadron transition temperature $T_c = 150$ MeV and 400 MeV, from [39]. $g(T)$ does not include contributions from the three right-handed neutrinos, which are added separately in the expansion rate formula.
Figure 2: The decoupling temperature $T_d$ (top) and the equivalent number of extra neutrinos $\Delta N_\nu$ (bottom) for the $\eta$ model as a function of the $Z_2$ mass $M_{Z_2}$ for constituent quark masses, for a quark-hadron transition temperature $T_c = 150$ MeV (circles) and 400 MeV (crosses). The left two figures are for the cases A0 and A3 defined in (10), i.e., the solid, dashed and dotted lines represent zero-mixing ($\delta = 0$), and positive and negative maximal-mixing ($\delta = \pm 0.002$), respectively. The $T_c = 150$ MeV case has higher $T_d$ and lower $\Delta N_\nu$ for the same $M_{Z_2}$ than $T_c = 400$ MeV. The right figures are for the intermediate mixing assumptions A1 and A2. The solid and dash-dot curves are for the mass-mixing relations $\delta = \pm 0.0051/M_{Z_2}^2$, while the dashed and dotted curves are for the $\rho_0$ constraints $\delta = \pm 0.0029/M_{Z_2}$. 
Figure 3: Same as Figure 2 except that current quark masses are used. The upper graphs share most features with the constituent mass case except that $T_d$ can be slightly lower when $T_d > T_c$. The only significant change in $\Delta N_\nu$ is for the $T_c = 150$ MeV maximal mixing case (see text).
Figure 4: $T_d$ (top) and $\Delta N_\nu$ (middle) for $M_{Z_2} = 500, 1000, 1500, 2000, 2500, 3500, 4000$, and $5000$ GeV, for $T_c = 150$ MeV and constituent masses. Larger $M_{Z_2}$ corresponds to higher $T_d$ and smaller $\Delta N_\nu$. The graphs on the left are for no mixing (case A0 in (10)), while the right-hand graphs are for the mass-mixing relation $|\delta| < 0.0051/M_{Z_2}^2$ (case A1). The bottom graphs are $M_{Z_2}$ corresponding to $\Delta N_\nu = 0.3, 0.5, 1.0$ and $1.2$, with larger $\Delta N_\nu$ corresponding to smaller $M_{Z_2}$.
Figure 5: Same as Figure 4 except $T_c = 400$ MeV. $T_d$ is slightly smaller (for $T_d > 150$ MeV) for fixed $M_{Z_2}$ and $\theta_{E6}$, while $\Delta N_\nu$ and the bound on $M_{Z_2}$ for fixed $\Delta N_\nu$ are increased.