Spin-polarized vortices with reversed circulation

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We present the analysis of the structure of fermionic vortices with the spin-polarized core from a weak coupling limit to the unitary regime. We show the mechanism for the generation of the reversed circulation in the vortex core induced by an excess of majority spin particles. We introduce the classification of the polarized vortices based on the number of Fermi circles where the minigap vanishes. This provides a unique description of the vortex as one cannot smoothly map wave functions into one another corresponding to vortices differing by the number of Fermi circles. The effective mass of quasiparticles along the vortex core is analyzed and its role in the propagation of spin-polarization along the vortex line is discussed.

I. INTRODUCTION

Quantum vortices are one of the most prominent examples of topological excitations in superfluids [1, 2]. They occur both in bosonic systems, where $^4$He liquid below lambda point and atomic BECs are prime examples, as well as in fermionic systems including superfluid $^3$He, metallic superconductors or fermionic ultracold gases. They are also believed to exist in superfluid neutron matter forming neutron stars. Although the stability of the vortex originates from the topology of the order parameter, its properties vary significantly for fermionic and bosonic systems. Namely, in bosonic systems at low temperatures, the core of the vortex is essentially empty as the superfluid density reaches zero in the center of the vortex. The only particles that can reside there are those which form the thermal cloud vanishing at $T=0$ [3]. In the case of fermionic systems, the strength of the interparticle interaction to large extent defines the vortex core structures (see e.g. Refs. [4–7] discussing the vortex structures in $^3$He, II-type superconductors, fermionic ultracold gases and in multiply quantized vortices, respectively).

For dilute Fermi gases the interaction is parametrized via dimensionless quantity $ak_F$, where $a$ is $s$-wave scattering length and $k_F = (3\pi^2 n)^{1/3}$ is Fermi wave vector corresponding to the density $n$. If $ak_F$ is positive then bound states (dimers) are formed, and typical characteristics of bosonic systems are recovered, with the modification that bosons can split into two fermions, which may form a normal state occupying the center of the vortex. In the far BEC limit ($ak_F \rightarrow 0^+$) this would require significant excitation energy and therefore in practice is not expected to occur below the condensation temperature. The situation is different for the dimers that are getting weakly bound when approaching the unitary limit ($ak_F \rightarrow \pm \infty$) at which their binding energy eventually reaches zero. At a certain point, the first Andreev state appears inside the core and the density of normal fermions becomes nonzero in the core. As the strength of the interaction becomes weaker the system enters into the BCS regime ($ak_F \rightarrow 0^-$) where fermions with opposite spins form Cooper pairs. In this regime, the density of Andreev states increases, implying that density of matter in the core reaches a significant level, comparable with the bulk value [8–10].

Spin imbalance may serve as another degree of freedom affecting pairing properties in Fermi system. It also affects the structure of the vortex as the excess of unpaired fermions tend to accumulate at the core [11, 12]. In this paper, we investigate impact of the spin polarization on the structure of the vortex in weakly and strongly interacting Fermi superfluid. We find that the spin-imbalance affects the flow inside the vortex core, leading eventually...
to its inversion at sufficiently high imbalances. This peculiar phenomenon is presented in Fig. 1, where we show velocity fields as a function of distance from the core. The three cases correspond to different amounts of mismatch between chemical potentials of two spin components $\Delta \mu = \mu_\uparrow - \mu_\downarrow$. As we increase $\Delta \mu$, the velocity field in the core is suppressed, and eventually changes direction. In this letter we reveal the origin of the reversed circulation and discuss its consequences. The effect is relevant for ultracold atomic systems with spin imbalance at the BCS regime up to the unitary limit, where quantum vortices were already observed \cite{13,14} and numerically simulated \cite{15}, and also to neutron stars. Particularly for magnetars that are expected to generate magnetic field of the order or larger than $10^{16}\text{G}$ \cite{16,17}, which is sufficient to effectively spin polarize neutron matter inside vortex core \cite{18,19}.

II. BDG EQUATIONS FOR SPIN-IMBALANCED SYSTEM

Our studies rely on Bogoliubov-de Gennes (BdG) formalism. The explicit form of BdG equations for spin-imbalanced system reads (no spin-orbit coupling is considered):

$$\mathcal{H} \begin{pmatrix} u_{n,\uparrow}(r) \\ u_{n,\downarrow}(r) \\ v_{n,\uparrow}(r) \\ v_{n,\downarrow}(r) \end{pmatrix} = E_n \begin{pmatrix} u_{n,\uparrow}(r) \\ u_{n,\downarrow}(r) \\ v_{n,\uparrow}(r) \\ v_{n,\downarrow}(r) \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} h_\uparrow(r) - \mu_\uparrow & 0 & 0 & \Delta(r) \\ 0 & h_\downarrow(r) - \mu_\downarrow & -\Delta(r) & 0 \\ 0 & -\Delta^*(r) & h_\downarrow^*(r) + \mu_\uparrow & 0 \\ \Delta^*(r) & 0 & 0 & -h_\downarrow^*(r) + \mu_\downarrow \end{pmatrix}$$

(1)

where $\mu_{\sigma,\downarrow}$ are chemical potentials for two spin components. Single particle hamiltonian in the BdG approximation is defined as $h_\uparrow = h_\downarrow = -\frac{\hbar^2}{2m} \nabla^2$. The form of the Hamiltonian leading to the BdG equations reads:

$$\hat{H} = \int_\sigma \int d^3 r \hat{\psi}_\sigma^\dagger(r) \begin{pmatrix} -\frac{\hbar^2}{2m} \nabla^2 - \mu_\sigma & 0 \\ 0 & -\frac{\hbar^2}{2m} \nabla^2 - \mu_\sigma \end{pmatrix} \hat{\psi}_\sigma(r)$$

$$+ \frac{g}{2} \sum_{\sigma=\uparrow,\downarrow} \int d^3 r \hat{\psi}_\sigma^\dagger(r) \begin{pmatrix} \Delta(r) & 0 \\ -\Delta^*(r) & -\Delta^*(r) \end{pmatrix} \hat{\psi}_\sigma(r)$$

(2)

with coupling constant $g$. In the BdG equations one usually omit the mean-field term contributing to $h_\uparrow$ and $h_\downarrow$ and takes into account pairing contribution $\Delta_{\sigma,\downarrow}(r) = \Delta(r) = g \langle \hat{\psi}_\sigma^\dagger(\vec{r}) \hat{\psi}_\sigma(\vec{r}) \rangle$ only. Then the formalism is applicable to weakly interacting (BCS) regime. In more general case, the single particle hamiltonian $\hat{H}_\sigma$ explicitly depends on the spin state. For example, asymmetric superfluid local density approximation (ASLDA), that applies to the unitary Fermi gas (UFG), provides $h_\sigma = \nabla \frac{\hbar^2}{2m_\sigma^*} \nabla + U_\sigma(n, p)$, where $m_\sigma^*$ is an effective mass of particle with spin $\sigma = \{\uparrow, \downarrow\}$ that depends on local polarization $p(r) = \frac{n_\uparrow(r) - n_\downarrow(r)}{n_\uparrow(r) + n_\downarrow(r)}$ and $U$ is a mean field which depends on the polarization and the total density of particles $n(r) = n_\uparrow(r) + n_\downarrow(r)$. For explicit form of the ASLDA energy density functional and corresponding single particle hamiltonian see Ref. \cite{20}. The pairing gap is related to quasi-particle wave-functions:

$$\Delta(r) = -\frac{g_{\text{eff}}}{2} \sum_{0 < E_n < E_c} (u_{n,\uparrow}(r)v_{n,\downarrow}(r) - u_{n,\downarrow}(r)v_{n,\uparrow}(r))$$

(3)

where $g_{\text{eff}}$ is a regularized coupling constant and $E_c$ is cut-off energy scale, see \cite{20} for details of the regularization scheme. In the mean-field BdG approximation, the coupling constant is related to the scattering length (bare coupling constant is given by $g = 4\pi\hbar^2a/m$) whereas for ASLDA the coupling constant is fitted to the quantum Monte Carlo data. The densities $n_\sigma$ and currents $j_\sigma$ of spin components are constructed as:

$$n_\sigma(r) = \sum_{0 < E_n < E_c} |v_{n,\sigma}(r)|^2$$

(4)

$$j_\sigma(r) = \sum_{0 < E_n < E_c} \text{Im}[v_{n,\sigma}(r)\nabla v_{n,\sigma}^*(r)]$$

(5)

The BdG equations \cite{1} decouple into two independent sets:

$$\begin{pmatrix} \hat{h}_\uparrow(r) - \mu & -\Delta(r) \\ -\Delta^*(r) & \hat{h}_\downarrow(r) + \mu \end{pmatrix} \begin{pmatrix} u_{n,\uparrow}(r) \\ v_{n,\downarrow}(r) \end{pmatrix} = E_n \begin{pmatrix} u_{n,\uparrow}(r) \\ v_{n,\downarrow}(r) \end{pmatrix}$$

(6)

$$\begin{pmatrix} \hat{h}_\downarrow(r) - \mu & -\Delta(r) \\ -\Delta^*(r) & \hat{h}_\uparrow(r) + \mu \end{pmatrix} \begin{pmatrix} v_{n,\uparrow}(r) \\ u_{n,\downarrow}(r) \end{pmatrix} = E_n \begin{pmatrix} v_{n,\uparrow}(r) \\ u_{n,\downarrow}(r) \end{pmatrix}$$

(7)

where $\mu = \frac{1}{2}(\mu_\uparrow + \mu_\downarrow)$ denotes mean chemical potential and $E_{n,\pm} = E_n \pm \frac{\Delta_\mp}{2}$ with $\Delta_\mp = \mu_\uparrow - \mu_\downarrow$. Solutions of equations \cite{6} and \cite{7} are connected via symmetry relation, namely if vector $\varphi_\pm = (u_{n,\uparrow}, v_{n,\downarrow})^T$ represents a solution of Eq. \cite{6} with eigenvalue $E_n$, then vector $\varphi_- = (v_{n,\uparrow}, u_{n,\downarrow})^T$ is a solution of Eq. \cite{7} with eigenvalue $-E_n$. In practice it is sufficient to solve equations \cite{6} only (for all quasiparticle energy states), and then solutions with positive quasiparticle energies contribute to the spin-down densities, whereas solutions with negative energies to the spin-up densities.

The equations were solved numerically for selected parameters presented in table \ref{table}. The calculations were executed on spatial 3D lattice $N_x \times N_y \times N_z$ with lattice spacing $\Delta x$. We considered straight vortex along $z$-direction, and thus by imposing the generic form of wave functions $\varphi(r) = \varphi(x, y) e^{ik_z z}$ the problem was effectively reduced to collections of 2D problems (parametrized by quantum number $k_z$). For calculations in BCS regime we have applied BdG approximation, while for calculations at the unitarity ASLDA functional has been employed \cite{20}. The
vortex solution was generated by imprinting technique, i.e. by imposing the particular structure of the order parameter of the form $\Delta(x,y) = |\Delta(\sqrt{x^2 + y^2})|e^{i\phi}$ with $\phi = \arctan(y/x)$. The calculations have been performed using W-SLDA Toolkit [15–21,22].

Fig. 2 presents cross sections through the vortex core along radial directions for the following quantities: spin-up $n_\uparrow(r)$ and spin-down $n_\downarrow(r)$ atoms density, strength of the order parameter $|\Delta(r)|$ and the velocity field $v(r) = |\mathbf{j}(r)|/n(r)$. Different lines correspond to different spin-imbalance populations measured by chemical potential difference $\Delta \mu = \mu_\uparrow - \mu_\downarrow$. The lattice formulation of the problem implies the usage of periodic boundary conditions. In order to remove the impact of periodicity on the results we placed the system in the potential well $\xi = \mu F/\hbar^2 \Delta$. Chemical potentials for individual spin components are indicated as $\mu_\uparrow$ and $\mu_\downarrow$.

| TABLE I. Characteristic parameters used in the numerical calculations. $N_\sigma$ stands for particle number of given spin $\sigma$. The Fermi wave vector and Fermi energy are related to density at large distance from the vortex core $n_\infty$ as follows $k_F = \sqrt{2m \mu F}/\hbar = (3\pi^2 n_\infty)^{1/3}$. The $\Delta_\infty$ stands for the pairing gap far from the vortex and defines the coherence length through relation $\xi = \varepsilon_F/k_F \Delta_\infty$. Chemical potentials for individual spin components are indicted as $\mu_\uparrow$ and $\mu_\downarrow$. |
|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$ [%] | BCS | UFG |
| Lattice | 0.0 | 0.5 | 1.0 | 0.0 | 0.5 | 1.0 | 0.0 | 0.5 | 1.0 |
| $k_F$ | 1.222 | 0.756 | 0.510 |
| $\Delta_\infty$ [eV] | 0.06 | 0.16 | 0.53 |
| $\varepsilon_F$ | 0.747 | 0.286 | 0.130 |
| $\xi$ [$\Delta$] | 13.7 | 8.5 | 3.7 |
| $ak_F$ | -0.61 | -0.84 | $\infty$ |
| $\mu_\uparrow$ [eV] | 1.031 | 1.077 | 1.089 | 0.279 | 0.294 | 0.299 | 0.014 | 0.026 | 0.027 |
| $\mu_\downarrow$ [eV] | 0.986 | 0.975 | 0.279 | 0.265 | 0.260 | 0.004 | 0.003 |

III. THE ORIGIN OF THE REVERSED CIRCULATION

The properties of polarized vortices are determined by the states in the cores. Their energies, for the unpolarized case, have been first estimated in Ref. [23]. In the BCS limit, due to the separation of scales related to pairing (coherence length $\xi$) and single particle motion (de Broglie wavelength $\lambda_B$), these states can be conveniently described in the Andreev approximation [24]. In the unitary regime despite the fact that chemical potential $\mu$ is of the same order as the pairing gap $\Delta$, as will be seen below, it can still provide useful qualitative relations.

In this approximation one decomposes the variation of $u$ and $v$ components of wave-functions (see Eq. 1) at the Fermi surface into rapidly oscillating parts associated with $k_F$ and smooth variations governed by the coherence length, i.e. $u(r) = e^{ik_F r}u(r)$ with $|k_F| = k_F$, and similarly for the $v$ component [25]. The Andreev approximation can be also used for studies of spin imbalance systems, providing the local polarization is relatively weak $\Delta \mu = \mu_\uparrow - \mu_\downarrow \ll \frac{1}{2}(\mu_\uparrow + \mu_\downarrow) \approx \varepsilon_F = k_F^2/2$. For the reasons presented in Sec. III we will focus only on one set of BdG equations, which in Andreev approximation describing states close to the Fermi surface acquire the form (we set $\hbar = m = 1$):

$$
\begin{pmatrix}
-ik_F \cdot \nabla & \Delta(r) \\
\Delta^*(r) & i k_F \cdot \nabla
\end{pmatrix}
\begin{pmatrix}
\hat{u}_{n,\uparrow}(r) \\
\hat{v}_{n,\uparrow}(r)
\end{pmatrix}
= \hat{E}_{n,+}
\begin{pmatrix}
\hat{u}_{n,\uparrow}(r) \\
\hat{v}_{n,\uparrow}(r)
\end{pmatrix}
,$$

where $\hat{E}_{n,+} = E_{n,+} + \frac{\Delta \mu}{\varepsilon_F}$. The second pair of equations for $\hat{u}_{n,\downarrow}(r)$ and $\hat{v}_{n,\downarrow}(r)$ has similar form and correspond to $\hat{E}_{n,-} = E_{n,-} - \frac{\Delta \mu}{\varepsilon_F}$. One may consider a schematic structure of a vortex core defined by the pairing field $\Delta(r)$ expressed in the polar variables $(\rho, \phi)$:

$$
\Delta(\rho, \phi) = |\Delta| e^{i\theta(\rho - r_v)}
$$

where $\theta$ is Heaviside step function. Ignoring for the moment the degree of freedom along the vortex axis (2D case), one may solve eqs. 8 and arrive at the quantization conditions associated with the trajectory of angular momentum $L_z$ (detailed derivation is provided in Appendix A):

$$
\frac{\hat{E}_{n,+}}{\varepsilon_F} k_F r_v \sqrt{1 - \left(\frac{L_z}{k_F r_v}\right)^2} + \arccos \left(\frac{-L_z}{k_F r_v}\right) = \arccos \left(\frac{E_{n,+}}{|\Delta|}\right) = \pi n,\quad (9)
$$

where $n \in \{0, \pm 1, \pm 2, \ldots\}$, $r_v$ denotes radius of the vortex core, and $|L_z| = \rho k_F$. Note that only the states with $n = 0$ correspond to core states, i.e. $E_{\pm,n=0} \lesssim |\Delta|$. The
limit $|\epsilon|/|\Delta| \ll 1$ can be quite accurately approximated by the expression:

$$E_{\pm,n=0,m} \approx -\frac{|\Delta|^2}{\epsilon F L_z} \frac{m \mp \Delta \mu}{2},$$

(10)

where $m$ is the magnetic quantum number associated with $L_z = \hbar m$, pointing along the vortex axis and $\xi = \epsilon F / |\Delta|$ is a coherence length. The energy of the first Andreev state in spin-symmetric case ($\Delta \mu = 0$), known as the minigap, is recovered when taking $r_v = \xi$: $E_0 = |\Delta|^2/2\epsilon F$. Since the vortex rotates counterclockwise (generating the flow with positive angular momentum along z-axis $L_z > 0$) for an unpolarized vortex ($\Delta \mu = 0$), the negative energies $E_{\pm,n=0,m}$ correspond to quasiparticles rotating in the same direction. In the case of nonzero spin imbalance, the two degenerate branches, corresponding to different spins, become shifted with respect to each other by the value $\Delta \mu$. Consequently, part of the branch of majority spin particles corresponds to states with the opposite value of $L_z$. The condition $E \approx 0$ sets the limit for the maximum value of the opposite angular momentum generated by the majority spin particle:

$$\max |m_{\text{opposite}}| \approx \frac{1}{2} \frac{\epsilon F L_v'}{\Delta |\Delta|^2} \left( \frac{r_v}{\xi} + 1 \right) \Delta \mu.$$

(11)

In Fig. 2(a) we present comparison of Eq. (10) originated from Andreev approximation and results of direct numerical solution of BdG equations (see sec. II) for the BCS regime. Clearly, the formula reveals satisfactory agreement with data when parameter $r_v$ is set to be approximately $\xi$.

The reversed total flow arises due to the cancellation effect of negative and positive contributions to angular momentum $L_z$ inside the core. To demonstrate this let us consider first the spin symmetric system. In panel b)
observed here is similar to an effect resulting with the reversion of a supercurrent in a controllable Josephson junctions [27, 29], which is due to the occupation pattern of Andreev states.

We note also that, qualitatively, the same effect of reversed circulations is observed in a strongly interacting regime, with the only difference that the density of Andreev states is lower in this case (see discussion of Fig. 4). The calculations for strong interactions (unitary regime) were carried out within ASLDA framework [20]. The ASLDA calculations were also conformed with experimental data [14] revealing remarkable agreement, and indicating that the vortices with polarized cores were already created in the laboratory [30]. We point out that the effect gets stronger as we tune the interaction strength towards the deep BCS regime. For example for $ak_F ≈−0.6$ the reversed flow in the core has the magnitude comparable to the maximum value of the current outside the core, see Fig. 3(d). One has to emphasize that increasing spin polarization even more may eventually lead to spatial modulation of the order parameter, even in the core, which represent a qualitatively different regime [31].

IV. FLAT BANDS AND EFFECTIVE MASS

The straight vortex admits the solution in the form of plane waves along the vortex line, which we choose to be the z-axis: $\varphi(x, y)e^{ik_zz}$. A peculiarity of Andreev reflection, however, leads to the significant suppression of the motion along the vortex core. In the pure Andreev scheme, the quasiparticle at the Fermi surface is reflected exactly backward and thus, except the case of a particle moving exactly in the direction of the vortex line, it will be localized not only within a plane perpendicular to the vortex core, but also along the vortex line. The manifestation of this behavior will result in almost flat bands $E(k_z) \propto \text{const}$ for $k_z \ll k_F$. This oversimplified result is however modified by the fact that the particles within a core are not exactly at the Fermi surface and, in particular, the departure from the Fermi energy by the value of the minigap $\Delta_0$ leads to a creeping motion along the vortex line. In this case the particle moving within the core are subject to the Andreev reflection law: $\sqrt{\varepsilon_F + E} \sin \alpha = \sqrt{\varepsilon_F - E} \sin \beta$, where $\alpha$ and $\beta$ are angles of trajectories for incident particle and reflected hole respectively [29, 32] and $E$ is the quasiparticle energy. Considering a series of Andreev reflections in a tube of radius $r_v = \xi$, we derive the relation between effective velocity of a particle/hole and momentum $k_z$ along the vortex line which reads (see Appendix A for details):

$$v_z = k_z \frac{\sqrt{k_p^2 - k_z^2} - \sqrt{k_h^2 - k_z^2}}{\sqrt{k_p^2 - k_z^2} + \sqrt{k_h^2 - k_z^2}},$$

(12)
the effective mass derived as $M_{\text{eff}} \approx \frac{1}{\epsilon} \frac{dE}{dk}$ and $k_p = \sqrt{2(\epsilon_F + E)}$ and $k_h = \sqrt{2(\epsilon_F - E)}$. The above formula estimates the relation for the effective mass of the particle along the vortex axis. Namely, considering the linear term in $\epsilon$ above formula estimates the relation for the effective mass of the particle along the vortex axis. Namely, considering the linear term in $\epsilon$ above formula estimates the relation for the effective mass of the particle along the vortex axis.

\[ \Delta \approx \frac{1}{2} \frac{dE}{dk} \left| k \right|_{k=0} \]

In the BCS limit, the effective mass is given as:

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FIG. 5. Quasiparticle energies of states corresponding to different \( m \)-values as a function of momentum along the vortex line \( k_x \). The results are obtained for spin-imbalanced system with \( \Delta\mu = 0.14 \varepsilon_F \) in the BCS regime \( ak_F \approx -0.84 \). The purple dots on \( E = 0 \) axis indicate positions of a level crossing, where the configuration changes by \( \Delta m = | 2m - 1 | \). For a better visibility, the negative energy states of \( E_n \)-branch are shown as positive and the energy levels are not plotted around the crossing point. The inset presents the classification of the polarized vortices based on the number of Fermi circles where the minigap vanishes.

the BCS regime.

Since the number of quasiparticle crossings is well defined for a polarized vortex, one can use the number of crossings through the \( E = 0 \) level to classify the vortices in spin-polarized Fermi systems. Namely, for spin-symmetric vortex, the number of crossings is zero. Polarizing the vortex is equivalent to introducing a series of crossing at the Fermi surface i.e. points for which minigap vanishes. As a consequence, the Fermi sphere will acquire a peculiar structure, consisting of rings which separate regions differing by a peculiar quasiparticle excitation pattern, see inset of Fig. 5 for illustration.

VI. SUMMARY

We have shown that polarized vortices in Fermi superfluid acquire a peculiar structure with a reversed circulation inside the core. Their structure admits the vanishing minigap with a characteristic pattern of single-quasiparticle level crossings at the Fermi surface. It is also predicted that the dynamics along the vortex line of spatially localized polarization inside the core will be suppressed. Bragg spectroscopy technique may provide experimental signatures of reversed flow \( [34, 35] \), see also Appendix C.

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Appendix A: Andreev states in the core of polarized vortex

The Andreev approximation assumes separation of two length scales: \( k_F^{-1} \ll \xi \) (\( \xi = \frac{\varepsilon_F}{k_F\hbar^2} \) being coherence length). It clearly holds in deep BCS regime and then also \( \mu \approx \varepsilon_F = \frac{\hbar^2k_F^2}{2m} \) is satisfied. The components of quasiparticle wave-functions attain generic form \( \varphi(r) = e^{ik_F\cdot r}\tilde{\varphi}(r) \). Action of the hamiltonian \( (\hat{h} - \mu)\varphi \) simplifies to:

\[
\left( \frac{\hbar^2}{2m} \nabla^2 - \mu \right) \varphi(r) \approx e^{ik_F\cdot r} \left( -\frac{i\hbar}{m} k_F \cdot \nabla \tilde{\varphi}(r) \right),
\]

where the term proportional to \( \nabla^2 \tilde{\varphi} \) is neglected, due to assumption of slow variation of the function \( \tilde{\varphi} \) over the length scale \( k_F^{-1} \). Inserting (A1) into (0) one arrives at Eq. 5 from the main paper (we set units: \( \hbar = m = 1 \)):

\[
\begin{pmatrix}
-ik_F \cdot \nabla \\
\Delta^*(r)
\end{pmatrix}
\begin{pmatrix}
\tilde{u}_{n,\uparrow}(r) \\
\tilde{v}_{n,\downarrow}(r)
\end{pmatrix}
= \tilde{E}_{n+} \begin{pmatrix}
\tilde{u}_{n,\uparrow}(r) \\
\tilde{v}_{n,\downarrow}(r)
\end{pmatrix},
\]

(A2)

where \( \tilde{E}_{n+} = E_{n+} + \frac{\Delta \mu}{2} \). The second pair of equations for \( \tilde{u}_{n,\downarrow}(r) \) and \( \tilde{v}_{n,\uparrow}(r) \) has similar form and correspond to \( \tilde{E}_{n-} = E_{n-} - \frac{\Delta \mu}{2} \):

\[
\begin{pmatrix}
-ik_F \cdot \nabla \\
-\Delta^*(r)
\end{pmatrix}
\begin{pmatrix}
\tilde{u}_{n,\downarrow}(r) \\
\tilde{v}_{n,\uparrow}(r)
\end{pmatrix}
= \tilde{E}_{n-} \begin{pmatrix}
\tilde{u}_{n,\downarrow}(r) \\
\tilde{v}_{n,\uparrow}(r)
\end{pmatrix},
\]

(A3)

In the case of the schematic vortex core structure in the form \( \Delta(r) = \Delta(\rho, \phi) = |\Delta|e^{i\phi} \rho - r_\rho \) (counterclockwise rotating vortex) one arrives at the quantization condition from eq. (A2) (see Fig. 6):

\[
\tilde{E}_{n+} = \varepsilon_F k_F r_v \sqrt{1 - \left( \frac{y_0}{r_v} \right)^2} + \arccos \left( \frac{y_0}{r_v} \right) - \pi n,
\]

(A4)
where $n = 0, \pm 1, \pm 2, \ldots$. Introducing the angular momentum component $L_z = -y_0 k_F$ one gets:

$$
\hat{E}_{n+} k_F r_v \sqrt{1 - \left( \frac{L_z}{k_F r_v} \right)^2} + \arccos \left( \frac{-L_z}{k_F r_v} \right) - \arccos \frac{\hat{E}_{n+}}{E} = \pi n. (A5)
$$

In the limit of $|y_0/r_v| \ll 1$ and $|\hat{E}_{n+}/\Delta| \ll 1$ the equation simplifies to:

$$
\hat{E}_{+} \approx -\frac{|\Delta|^2}{\epsilon_F \frac{r_v}{t} \left( \frac{\pi}{2} + 1 \right)} L_z, \quad (A6)
$$

where only the lowest energy branch corresponding to $n = 0$ is considered. Note that the minus sign appears as a result of counterclockwise rotation of the superflow.

The other solution corresponding to eq. (A2) with relation $E_{-} = -E_{+}$. Therefore the particle momentum is reversed $k_p \rightarrow -k_F$ and consequently $L_z \rightarrow -L_z$. As a result one arrives at $\hat{E}_{-} = \hat{E}_{+}$. In the case of spin-polarized system the solutions are shifted with respect to each other by $\Delta \mu$. Note that the energies corresponding to the highest angular momenta in the core are of the order of $|\Delta|$. Namely, for the maximum $L_z = \pm k_F r_v$ one gets $\hat{E}_{\pm} = \pm |\Delta|$, respectively.

FIG. 6. Schematic picture of the vortex core used for determination of states in Andreev approximation. The classical trajectory representing particle of momentum $k_F$ is denoted by red solid line, and reflected hole is shown as brown dashed line. Note that the angular momentum component $L_z$ corresponding to the trajectory is negative.

In order to extract the effective mass in the Andreev approximation one needs to consider particle/hole motion along the vortex line. Due to the properties of Andreev reflection the problem reduces to 2D problem, see Fig. 7. Contrary to the quantization condition which resulted from the assumption that the hole/particle is reflected exactly backward (which is true if the incoming particle/hole is exactly at the Fermi surface), here one needs to take into account more general case. Namely, as a result of momentum conservation along the vortex line the reflection law reads: $\sqrt{\epsilon_F + E} \sin \alpha = \sqrt{\epsilon_F - E} \sin \beta$, where $k_p = \sqrt{2(\epsilon_F + E)}$ and $k_h = \sqrt{2(\epsilon_F - E)}$, are particle and hole momenta, respectively. The effective velocity along the vortex line can be defined as $v = S/T$, where $S$ denotes the distance between two consecutive reflections where particle is converted into hole (see Fig. 7), and $T$ is the time interval between these reflections. Consequently one gets: $v = \sqrt{2(\epsilon_F + E)} \sin \alpha \sin(\beta - \alpha)/\sin(\beta + \alpha)$. Using the reflection law this relation can be rewritten as:

$$
v_z = k_z \sqrt{k_z^2 - k_p^2 - k_h^2} / \sqrt{k_p^2 + k_h^2 - k_z^2}, \quad (A7)
$$

where $k_z = k_p \sin \alpha = k_h \sin \beta$ is the momentum component along the vortex line. Note that the expression does not depend on the core radius and therefore in the Andreev approximation all bands originated from states $\Delta$ will have the same slope. Andreev approximation in practice is expected to work for small $|L_z| \ll k_F r_v$ and small $k_z \ll k_F$ (small angles of reflection) as is shown in the manuscript.

Appendix B: Wave packet excitation in the vortex core

Let us consider an unpolarized vortex of length $L$. The Hamiltonian describing the structure of the vortex core reads:

$$
\hat{H} = \frac{L}{2\pi} \int dk_z \sum_{m>0} \left[ E_{m\uparrow}(k_z) \alpha_{m\uparrow}^\dagger(k_z) \alpha_{m\uparrow}(k_z) + E_{m\downarrow}(k_z) \alpha_{m\downarrow}^\dagger(k_z) \alpha_{m\downarrow}(k_z) \right] \quad (B1)
$$
where for \( k_z/k_F \ll 1 \): \( E_{m+1}(k_z) \approx \Omega m + \frac{1}{2\pi N} k_z^2 \) with \( \Omega \) being proportionality coefficient between energy and quantum number \( m = L_z/h \) in Eq. (A6) and

\[
\alpha_{m+1}^\dagger(k_z) = \sqrt{\frac{1}{2\pi \sigma}} \int dk_z \exp \left( -\frac{(k_z - k_0)^2}{4\sigma^2} \right) \alpha_{m+1}^\dagger(k_z)|0\rangle
\]

One quasiparticle excitation within a band formed by quantum number \( m \) can be constructed in the standard way:

\[
|k_0m \uparrow\downarrow\rangle = \frac{1}{\sqrt{2\pi \sigma}} \int dk_z \exp \left( -\frac{(k_z - k_0)^2}{4\sigma^2} \right) \alpha_{m+1}^\dagger(k_z)|0\rangle
\]

and clearly \( \langle k_0m \uparrow \downarrow | k_0m \uparrow \downarrow \rangle = 1 \). The wave packet excitation change the spin polarization by unity, since eg. \( \langle k_0m \uparrow | (N^\uparrow - N^\downarrow)k_0m \uparrow \rangle - \langle 0| (N^\uparrow - N^\downarrow)|0\rangle = 1 \), where \( N^\uparrow, N^\downarrow \) are particle number operators for spin-up and spin-down particle, respectively. The evolution of this wave packet: \( |k_0m \uparrow\downarrow,t\rangle = \exp(-i\hat{H}t)|k_0m \uparrow\downarrow\rangle \) gives rise to the relations:

\[
\langle z \rangle = \langle k_0m \uparrow\downarrow | z |k_0m \uparrow\downarrow \rangle = \frac{k_0}{M_{\text{eff}}} t
\]

\[
\sqrt{\langle (z - \frac{k_0}{M_{\text{eff}}}t)^2 \rangle} = \frac{1}{2\sigma} \sqrt{2\frac{\sigma^2}{M_{\text{eff}}}t^2} + 1 \approx \frac{\sigma}{M_{\text{eff}}} t
\]

for long times: \( t \gg \frac{M_{\text{eff}}}{2\sigma} \).

### Appendix C: Impact of reversed circulation on Bragg scattering

Reversed circulation is manifested as a change in the collective motion of atoms in a condensate. Bragg spectroscopy can be a promising tool for the investigation of this effect. Below we present qualitative arguments supporting the design of the Bragg scattering experiment, omitting the issue if current experimental capabilities allow sufficiently accurate measurements.

Bragg scattering experiments were successfully employed to investigate fermionic condensates [36, 37] as well as to probe quantum vortices in BEC [38, 39]. In a typical setup of the experiment two laser beams (having certain frequency difference \( \omega \)) are generated, crossing each other inside the atomic cloud. They produce a standing wave moving in the laboratory frame and thus inducing Bragg scattering of the atomic cloud. Namely, crossing laser beams form an effective optical potential \( V_{\text{op}} \propto \cos(q \cdot r - \omega t) \) acting on a gas [40, 41]. As a result, energy \( \hbar \omega \) and momentum \( \mathbf{q} \) are transferred to an atom through the two-photon scattering process.

The resonant Bragg scattering occurs under condition:

\[
\hbar \omega = \frac{\hbar^2 q^2}{2m} + \mathbf{q} \cdot \mathbf{v},
\]

where \( \mathbf{v} \) denotes velocity of an atom. In the above expression we assumed that the dispersion relation for an atom in the cloud is the same as for non-interacting particle (see e.g. [35, 37, 38]), although more realistic expression can be employed as well. The second term is crucial in this case as it makes Bragg scattering process sensitive to local atomic velocity. In the case of ultracold Fermi gas with vortex line, we define the velocity field through ratio of the probability current and the density \( \mathbf{v}(r) = \mathbf{j}(r)/\rho(\mathbf{r}) \), which corresponds to expectation value of single atom velocity. Note that Bragg scattering process selects in this case group of atoms from a particular part of the system where the condition holds:

\[
\hbar(\omega - \delta(r)) = \frac{\hbar^2 q^2}{2m}.
\]

with \( \hbar \delta(r) = \mathbf{q} \cdot \mathbf{v}(r)/\hbar \). The quantity \( \delta(r) \) is shown in Fig. 8. for vortex with and without reversed flow. The figure reveals qualitative and quantitative changes of resonant frequency distribution due to the reversed circulation.

As an experimental signal one can use density distribution of scattered atoms [36, 38, 39]. Due to the sensitivity of Bragg scattering process on the local flow velocity the presence of reversed circulation should induce a significant modification in the density distribution. Consequently we expect that the density distributions corresponding to spin unpolarized and polarized vortex can be distinguished. We emphasize that more refined study of Bragg scattering intensity and the density distribution evolution for given \( \mathbf{q}, \omega \) is required in order to settle if such measurements are feasible.

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FIG. 8. Color maps are showing the relative change in Bragg scattering resonance frequency distribution $\delta(r)/\delta_{\text{max}}$ due to velocity of atoms for spin unpolarized (BCS, $P=0\%$) and spin polarized systems (BCS, $P=0.5\%$), in panels (a) and (b), respectively. In this particular setup we chose $q$ to be aligned along $x$-axis of the system (while vortex is oriented along $z$-axis). The quantity is normalized to its maximal value $\delta_{\text{max}}$ for unpolarized case. Vector field related to the vortex $v(r)$ is indicated by arrows. In the insets we show corresponding velocity profiles as a function of distance from the vortex core obtained numerically (solid line). The ideal quantum vortex velocity profile $v(r) \sim 1/r$ is marked by dashed line.

[10] a) BCS $0\%$, $ak_F = -0.61$

[11] b) BCS $0.5\%$, $ak_F = -0.61$

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