Method for correcting overlapping band phase difference of parallel ADC structure with mixer

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ABSTRACT: Various parallel ADC structures with mixers (such as DBI-ADC, ATI-ADC) are attractive structures for broadband high-speed data acquisition systems. However, these structures have inherent disadvantages: the phase difference between subbands in the vicinity of overlapping bands of adjacent subbands causes the system frequency response to deteriorate significantly. The original correction method relies on high-performance circuits and components or correction algorithms that consume a lot of digital resources. This paper proposes a new method of phase difference correction for overlapping frequency bands and proves its effectiveness through experimental results. This method uses the structural characteristics of the acquisition system itself, without additional resources, and ensures that the phase difference does not affect the system SNR.

KEYWORDS: Digital signal processing (DSP); Data acquisition circuits; Digital electronic circuits

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1 Introduction
Various parallel ADC structures are common methods used in high-speed data acquisition systems to break the performance limitations of single-chip ADC. In the commonly used parallel ADC structure, time-interleaving ADC (TI-ADC) can only increase the sampling rate of the system and cannot increase the bandwidth of the system [1]. Frequency-interleaving ADC (FI-ADC) can improve the sampling rate while solving the problem that the system bandwidth is limited by the Nyquist frequency of a single ADC. However, the bandwidth of the FI-ADC is still limited by the maximum input frequency of a single ADC, so it is not obvious to increase the bandwidth [2]. For acquisition systems that also require high bandwidth, some parallel ADC structures with mixers developed on FI-ADC appears.

Since these parallel ADC structures use analysis filter banks for subband decomposition, overlapping bands due to filter non-ideal truncation characteristics will inevitably occur between adjacent subbands. When adjacent sub-bands have different phase-frequency characteristics on overlapping bands, their superposition results will be affected to produce corresponding changes in frequency response characteristics.

In FI-ADC, through the orthogonal design of the analysis filter bank and the synthetic filter bank, the phase difference of the overlapping bands of the system can be suppressed well, so as not to cause impact [3]. However, for the parallel ADC structure using a mixer, the phase difference control is more difficult because the analog circuit is more complicated.

Currently, asynchronous time-interleaved ADC (ATI-ADC) and digital bandwidth interleaved ADC (DBI-ADC) are two practically used parallel ADC structures with mixers. ATI-ADC adopts a symmetrical structure similar to FI-ADC, which relies on a more complex and strict symmetrical design to achieve the effect of overlapping band phase difference processing similar to FI-ADC, but correspondingly has higher design and implementation difficulty [4]. This is also one of the reasons
why, besides ATI-ADC, other parallel ADC structures with mixers with symmetric structure have not been used in practice.

The asymmetrical design of DBI-ADC is relatively easy to implement on analog circuits and components, but sub-band 1 is significantly different in structure from other sub-bands so that there is at least one overlapping band with obvious phase difference in the system. To this end, LeCroy specially designed the corresponding overlapping band correction module and corresponding algorithm to solve it [5]. This consumes a considerable portion of digital system resources.

In this paper, a new overlapping band correction method is proposed for the DBI-ADC structure. The effectiveness of this method was verified with a 20 GSps, 5.5GHz DBI-ADC experimental prototype. In fact, this correction method can also be applied to the symmetric structure like ATI-ADC, which can greatly reduce its requirements for analog circuit design and manufacturing and component performance.

This correction method has the following significant characteristics:

1. No new analog circuits are added, and performance requirements for components are not increased.

2. Ensure that the system does not reduce the final corrected SNR due to the overlapping band phase difference.

3. By making full use of the structural characteristics of the DBI-ADC system, the correction work can be realized in FPGA, DSP, and other devices through a very simple digital calculation operation.

2 System model analysis

Based on the DBI-ADC structure model proposed by the reference [3], we will analyze the influence of the phase difference of the overlapping band between subbands on the system.

For convenience, we consider the frequency of LO in channel 1 is zero. Then taking the two-channel DBI system shown in figure 1 as an example, the channel sampling period is 2T. In this paper, it is believed that the initial phase of the analog LO and the initial phase of the digital LO have been corrected to be consistent, that is $\Phi_2 = \varphi_2$.

![Figure 1. Multi-channel DBI system model.](image-url)
Definitions $|A_1(\omega)|$ and $|A_2(\omega)|$ are the amplitude frequency response of channel 1,2 at digital frequency $\omega$, $\varphi_1(\omega)$ and $\varphi_2(\omega)$ are the phase frequency response of channel 1,2 at $\omega$. There is $\varphi_2(\omega) = \varphi_1(\omega) + \Delta \varphi(\omega)$.

Assuming the input signal spectrum is $X(\omega)$, the output signals’ spectrum of the two channel are given by:

$$Y_1(\omega) = |A_1(\omega)|e^{j\varphi_1(\omega)}X\left(\frac{\omega - \pi k}{T}\right) + E_1(\omega)$$

$$Y_2(\omega) = |A_2(\omega)|e^{j[\varphi_1(\omega) + \Delta \varphi(\omega)]}X\left(\frac{\omega - \pi k}{T}\right) + E_2(\omega)$$

(2.1)

where, $E_1$ and $E_2$ are the sum of various errors including aliasing error, image frequency, and random noise in the two channels, and they are considered to be Gaussian white noises.

Define the system bandwidth as $[0, \omega_c]$ and the system overlap band as $[\omega_1, \omega_2]$, the system output spectrum without any correction $Y(\omega)$ can be written as:

$$Y(\omega) \approx \begin{cases} 
Y_1(\omega), & \omega \in [0, \omega_1] \\
Y_1(\omega) + Y_2(\omega), & \omega \in [\omega_1, \omega_2] \\
Y_2(\omega), & \omega \in [\omega_2, \omega_c]
\end{cases}$$

(2.2)

According to the theory of FI-ADC, assuming the output signal is only a constant multiple of the time delay and amplitude of the input signal, it is considered that the output signal achieves a perfect reconstruction (PR) of the input signal. If there is a correction filter $H_c(\omega)$ to enable the system to achieve PR, the final output signal spectrum $Z(\omega)$ and $H_c(\omega)$ can be described as:

$$Z(\omega) = H_c(\omega)Y(\omega) = Ce^{-j\omega d}X\left(\frac{\omega - \pi k}{T}\right), \quad \omega \in [0, \omega_c]$$

(2.3)

where $d$ is a constant group delay.

In the actual system, due to the measurement error of the system, the design error of the correction filter and other various errors, equation (2.3) cannot be fully realized, that is, the system can only achieve almost PR.

Assuming $C = 1$, the aliasing error and image frequency in the system have been suppressed. Then a correction filter $H_s(\omega)$ that can achieve almost PR is given by:

$$H_s(\omega) = H_c(\omega) = \begin{cases} 
e^{-j\omega d} & \omega \in [0, \omega_1] \\
|A_1(\omega)|e^{j\varphi_1(\omega)} & \omega \in [\omega_1, \omega_2] \\
|A_1(\omega)|e^{j\varphi_1(\omega)} + |A_2(\omega)|e^{j[\varphi_1(\omega) + \Delta \varphi(\omega)]} & \omega \in [\omega_2, \omega_c]
\end{cases}$$

(2.4)

The method of suppressing aliasing error and image frequency and the design method of $H_s(\omega)$ can be found in [6–8], which will not be detailed in this article.

The real output after system correction $Z_s(\omega)$ is:

$$Z_s(\omega) = \begin{cases} 
H_s(\omega)Y_1(\omega), & \omega \in [0, \omega_1] \\
H_s(\omega)[Y_1(\omega) + Y_2(\omega)], & \omega \in [\omega_1, \omega_2] \\
H_s(\omega)Y_2(\omega), & \omega \in [\omega_2, \omega_c]
\end{cases}$$

(2.5)
It can be seen from equation (2.5) that the correction filter $H_c(\omega)$ will scale the error term in the system. In non-overlapping bands, the scaling factor is only determined by the frequency response of a single channel, which can be solved by improving the design of the channel. In the overlapping band of the system, it is determined by the vector sum of the frequency response of two adjacent sub-bands. At this time, the phase difference of the adjacent sub-bands in the overlapping band will have an important influence on the vector sum.

Define $|A(\omega)|e^{j\varphi(\omega)}$ to be the vector sum of $|A_1(\omega)|e^{j\varphi_1(\omega)}$ and $|A_2(\omega)|e^{j[\varphi_1(\omega)+\Delta\varphi(\omega)]}$. where:

$$|A(\omega)| = \sqrt{|A_1(\omega)|^2 + |A_2(\omega)|^2 + 2|A_1(\omega)||A_2(\omega)|\cos\Delta\varphi(\omega)} \quad \omega \in [\omega_1, \omega_2]$$

(2.6)

Then:

$$|Z(\omega)| = 1 + \frac{E_1(\omega) + E_2(\omega)}{|A(\omega)|} \quad \omega \in [\omega_1, \omega_2]$$

(2.7)

According to equation (2.6) and (2.7), the smaller $|A(\omega)|$ is, the error term in the system after correction will be enlarged accordingly, making the system signal to noise ratio (SNR) lower. In extreme cases, if $|A_1(\omega)| \approx |A_2(\omega)|$ and $\Delta\varphi(\omega) \approx \pi$, then $|A(\omega)| \approx 0$, the correction filter $H_c(\omega)$ will be impossible to design and implement.

In summary, before designing the correction filter $H_c(\omega)$ in the DBI-ADC system, it is necessary to correct the phase difference of the overlapping bands between the sub-bands to avoid extreme situations ($|A(\omega)| \approx 0$) and as much as possible not to reduce the system SNR.

To facilitate analysis, we believe that the amplitude gain of the system before correction in the entire frequency band (including most overlapping bands) is approximately 0 dB.

Based on the this assumptions, the corrected system should meet:

$$\frac{E_1(\omega) + E_2(\omega)}{|A(\omega)|} \leq \frac{E_1(\omega) + E_2(\omega)}{\max(|A_1(\omega)|, |A_2(\omega)|)} \quad \omega \in [\omega_1, \omega_2]$$

(2.8)

Simplifying equation (2.8) we can get:

$$\Delta\varphi(\omega) \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \quad \omega \in [\omega_1, \omega_2]$$

(2.9)

Therefore, the goal of the correction of the phase difference of the overlap band of the DBI-ADC system is to make it satisfy the equation (2.9).

### 3 Principle of phase difference correction of overlapping bands

#### 3.1 Correction principle

The overlapping band $[\omega_1, \omega_2]$ between the two subbands of the DBI-ADC system is defined as the frequency band whose amplitude is affected by any subband by more than $\lambda$, which is:

$$\left|\frac{|A(\omega)| - \max(|A_1(\omega)|, |A_2(\omega)|)}{|A(\omega)|}\right| > \lambda \quad \omega \in [\omega_1, \omega_2]$$

(3.1)

When $\Delta\varphi(\omega) \approx \pi$, the left side of equation (3.1) gets the maximum value, and it can be obtained that the amplitude difference between subbands $\Lambda$ should satisfy:

$$\Lambda = 20\log_{10}\frac{\max(|A_1(\omega)|, |A_2(\omega)|)}{\min(|A_1(\omega)|, |A_2(\omega)|)} < 20\log_{10}\frac{(1 + \lambda)}{\lambda}$$

(3.2)
Equation (3.1) and (3.2) show that when \( \lambda \) takes a certain value, the steeper the frequency response of the adjacent sub-band in the overlapping band of the DBI-ADC system, the smaller the overlapping band range. That is, improving the performance of the system equivalent analysis filter can reduce the frequency band affected by the phase difference of the overlapping bands.

Assume that the group delay in channel 1 is \( \tau_1(\omega) \) and it in channel 2 is \( \tau_2(\omega) \).

In a discrete system after \( N \)-point sampling, \( \Delta \omega = \frac{2\pi}{N} \) is the frequency resolution, \( \omega_1 = \frac{2\pi n_1}{N} \), \( \omega_2 = \frac{2\pi n_2}{N} \). For any \( \omega_x \in (\omega_1, \omega_2) \), the phase difference of the two channels at any frequency in the overlapping band should meet:

\[
\Delta \varphi(\omega) = \Delta \varphi(\omega_1) - \sum_{\omega=\omega_1}^{\omega_x} \Delta \omega [\tau_2(\omega) - \tau_1(\omega)] \tag{3.3}
\]

Equation (3.3) shows that in the frequency band \( [\omega_1, \omega_x] \) if the group delay difference \( \Delta \tau(\omega) \) is smaller, the change in the phase difference \( (\Delta \varphi(\omega_x) - \Delta \varphi(\omega_1)) \) is smaller. The condition to keep the phase difference change in the entire band below \( \pi \) can be described as:

\[
0 < \max [\Delta \varphi(\omega_x) - \Delta \varphi(\omega_1)] < \pi \tag{3.4}
\]

Substituting equation (3.3) into equation (3.4), it can be obtained that in the corrected system the following condition is met:

\[
0 < \max \left\{ \sum_{\omega=\omega_1}^{\omega_x} [\Delta \tau(\omega) - d_x] - \sum_{\omega=\omega_1}^{\omega_x2} [\Delta \tau(\omega) - d_x] \right\} < \frac{\pi}{\Delta \omega} \tag{3.5}
\]

where \( d_x \) is the correction delay between the two channels, used to reduce their group delay difference, thereby reducing their phase difference.

Consider a unified constraint on the entire overlapping band, rewriting equation (3.5) as

\[
|\Delta \tau(\omega_x) - d_x| < \frac{\pi}{2(\omega_2 - \omega_1)} \tag{3.6}
\]

Where \( \Delta \tilde{\tau}(\omega_x) \) is the average value of the group delay difference from \( \omega_1 \) to \( \omega_x \).

When equation (3.6) is established, it can ensure that the phase difference change of the overlapping zone is less than \( \pi \).

From the definition of group delay, it can be seen that minimizing the change in phase difference is equivalent to minimizing the group delay difference. Define the function GD, which is the sum of squares of the system group delay difference after correction, it is given by:

\[
GD = \sum_{\omega=\omega_1}^{\omega_2} [\tau_2(\omega) - \tau_1(\omega) - d_x]^2 \tag{3.7}
\]

\( d_x \) should minimize the \( GD \) after correction, it means \( \frac{dGD}{dd_x} = 0 \). Solving this system of equations gives the least squares solution of \( d_x \) as:

\[
d_x = \frac{\Delta \omega \sum_{\omega=\omega_1}^{\omega_2} [\tau_2(\omega) - \tau_1(\omega)]}{\omega_2 - \omega_1 + 1} \tag{3.8}
\]
Since the DBI-ADC structure is usually used in high-system bandwidth applications, $\omega_2 - \omega_1$ is usually smaller. If necessary, it can be further reduced by using a steeper analysis filter. By adjusting the group delay of the channel, the requirement of equation (3.6) can be easily achieved.

For example, in ref. [9], the overlap band is approximately 5.8 ~ 6.2 GHz, the system sampling rate is 60GSa/s. $\pi \frac{\pi}{\omega_2 - \omega_1} = 75\pi \approx 236$. The experimental system designed in this paper also has this feature, and the details will be analyzed in section 4.

After the adjustment of the average group delay difference, the phase difference in the entire overlapping band will not exceed $\pi$, which means that if the phase difference at the center point $\Delta \varphi(\omega_p)$ of the overlapping band is controlled at about $0 \text{rad}$, the entire overlapping band will meet equation (2.9). This central point $(\omega_p)$ should have the following characteristics:

$$\Delta \varphi(\omega_p) = \frac{\max[\Delta \varphi(\omega)] + \min[\Delta \varphi(\omega)]}{2} \quad \omega_p \in (\omega_1, \omega_2) \quad (3.9)$$

By shifting the phase of the sampled data $\varphi_x$, the corrected center point phase can be zero. there is $\varphi_x = \Delta \varphi(\omega_p)$. As shown in figure 2, the phase item $\varphi_{x2}$ of the LO in channel 2 is the corrected phase shift value.

From the discussion in this section, we can see that after the delay $d_x$ and the phase shift $\varphi_d$, the system should be able to meet the correction requirements proposed by equation (2.9), that is, to theoretically achieve the correction requirements for the phase difference of the overlapping band of the two-channels DBI-ADC system. In section 3.2, the implementable correction model and method are proposed and extended to multi-channel systems.

### 3.2 Correction model and method

![Figure 2](image)

*Figure 2. DBI-ADC system overlapping band phase difference correction model.*

As discussed in section 3.1, for the two-channels DBI-ADC system, the key to the implementation of the correction method proposed in this paper is a delay and a phase shift. Taking into account the basic theory of simultaneous acquisition with multiple ADCs [1], and the structural characteristics of the parallel ADC sampling system with mixer [4, 5], we can get two important conclusions:

1. In a multi-ADC acquisition system, lost point synchronization is the basic ADC correction step, and the delay operation of this correction method can be approximated by dropping a positive integer number of sampling points.

2. In the parallel ADC structure with mixers, there must be digital mixing in the digital part, and the phase shift operation can be realized by changing the initial phase of the digital local oscillator.
Therefore, we can get the two-channels DBI-ADC system overlap band phase difference correction model shown in figure 2.

Taking the two-channel DBI-ADC system as an example, the correction method is as follows:

1. Test the frequency response of the DBI-ADC system, determine the effect parameter $\lambda$ of the overlap band, calculate the corresponding $\Lambda$ and draw the amplitude-frequency response curve, and determine the range of the system overlap band ($\omega_1$ and $\omega_2$).

2. According to equation (3.8), calculate the correction delay $d_x$ between channels. Assuming $d$ is positive, it means $d_{x1} = d_x$ and $d_{x2} = 0$, and when it is negative, it means $d_{x1} = 0$ and $d_{x2} = d_x$.

3. Delay the sampling data of the two channels by $d_{x1}$, $d_{x2}$ respectively.

4. According to equation (3.9) and $\varphi_x = \Delta \varphi(\omega_p)$, calculate the phase $\varphi_x$.

5. Increase the initial phase of local oscillator by $\varphi_{x2} = \varphi_x$.

For the M-channels DBI-ADC system, the correction method can be generalized by the two-channel DBI-ADC system, the difference is that the delay $d_{xn}$ and phase $\varphi_{xn}$ need to be modified as follows: defining $d_{xnm}$ is the delay difference calculated by equation (3.8) between adjacent channels $n$ and $m$, $d_{xn}$ is the delay of channels $n$ used for correction, $\varphi_{xnm}$ is the phase by equation (3.9) between adjacent channels $n$ and $m$, $\varphi_{xn}$ is the phase of channels $n$ used for correction,

$$
\begin{align*}
 d_{x(n+1)} &= \min[0, d_{x12}, d_{x23}, \ldots, d_{x(M-1)M}] & n < M \\
 d_{xM} &= -\min[0, d_{x12}, d_{x23}, \ldots, d_{x(M-1)M}] & n = M \\
 \varphi_{x1} &= 0 & n = 1 \\
 \varphi_{xn} &= \sum_{n=2}^{M} \varphi_{x(n-1)n} & 1 < n \leq M
\end{align*}
$$

(3.10)

4 Experiment

In this section, a DBI-ADC system is designed to verify the DBI-ADC system overlapping band phase difference correction method proposed in this paper. It is a two-channel DBI-ADC system, which already has an actual test platform, verified by processing the measured data.

This system has a sampling rate of 20GSa/s and a bandwidth of 5.5GHz. Its first subband is about 0.33GHz, and its second subband is about 3.3~5.5GHz. LO frequency is 6GHz. The digital part of this system is implemented in FPGA.

In this system, $\lambda = 0.1$, $\Lambda \approx 21dB$. As shown in figure 3, the part between the two vertical lines is the overlapping band of the system, which is 3.33~3.68GHz. \( \frac{\pi}{2(\omega_2 - \omega_1)} = 28.5\pi \approx 90 \)

According to equation (3.8) and (3.9), we can get $d = 329.08$(point), $\varphi_x = -0.679\pi$(rad). To facilitate implementation, $d = 329$ is used. After calibration, $|\Delta \tau(\omega_x) - d_x| \approx 25$, meet the requirements of equation (3.6).

During the corrections process, figure 4 shows the variation of the amplitude-frequency response and phase difference of the system overlapping band.
It can be seen that after delay correction, the phase difference does not appear phase wrapping, and the fluctuation of amplitude-frequency response becomes flat obviously.

After phase shift correction, the phase difference is limited to \([-0.1\pi, 0.1\pi]\). The amplitude-frequency response has been substantially improved, the curve is smoother, and there are no deep pits compared to the uncorrected response. This indicates that if the amplitude-frequency response is subsequently corrected to 0 dB, the corrected system noise will be relatively smaller than before the correction. That is, for the same amplitude-frequency flatness correction, the system SNR after the overlap band correction is higher. These show that the correction method has achieved the expected correction goal.
5 Conclusion

This paper introduces and demonstrates a new DBI-ADC system correction method of phase difference in the overlapping band, that measures the overall frequency response of the system and estimate the range of overlapping bands. This method is simple and convenient to achieve correction by using delay and phase shift. In digital systems, it can be achieved by simply dropping points and changing the initial phase of the digital local oscillator, without the need to design a complex IIR filter. The difficulty of the system design is reduced, and the system does not reduce the SNR due to the phase difference of overlapping bands. The experiment results verify the effectiveness of this method.

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