A Nonlinear Dynamic Boiler Model accounting for Highly Variable Load Conditions

Diego S. Carrasco and Graham C. Goodwin

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Abstract

This paper describes a new nonlinear dynamic model for a natural circulation boiler. The model is based on physical principles, i.e. mass, energy and momentum balances. A systematic approach is followed leading to new insights into the physics of drum water level and downcomer mass flow. The final model captures fast dynamic responses that are necessary to describe the operation of a boiler under highly variable load conditions. New features of the model include (i) a multi-compartment model for the risers, (ii) a new model for drum water level, and (iii) a new dynamic model for the flow of water in the downcomers. Implications of the model for control system design are also explored.

1 Introduction

Dynamic models for Boilers have appeared in the literature for decades [2, 10, 12, 13, 14]. Early work focused on obtaining empirical models capable of describing the internal dynamics with a certain degree of accuracy [5, 6]. Throughout the years, the focus has shifted to develop models based on universal truths, i.e. first-principle models [4, 8, 12]. Models for boilers have also taken many different formats, e.g. linear/nonlinear [4, 17], high/low order [7], one/two fluid [1], lumped/distributed parameters [4, 16]. Some of these models have been developed based on physical principles and involve complex dynamics – see e.g. [1].

In the seminal work of [4] a model using mass and energy balances is derived. However, some simplifications are used when developing this model, namely (i) a steady state equation for the downcomer mass flow, and (ii) the assumption that steam quality varies linearly as a function of height in the risers. The more recent work described in [15] derives a first principles model using mass, energy and momentum conservation equations. However, other simplifications are used when developing this model, including the fact that pressure and other internal variables are reconstructed by first order filters. It is shown in the current paper that the simplifications and assumptions used in [4, 15] are not valid under rapidly changing load conditions.

The current paper resulted from a three year project carried out in collaboration with Wilmar Sugar at their Proserpine Mill in Queensland, Australia. Sugar mills burn sugar cane residue (called bagasse) to produce steam in a boiler. The generated steam is used for many purposes including (i) to power the factory, (ii) to crystallise sugar, and (iii) to co-generate electricity.

Unlike boilers used in conventional gas or coal-fired power stations, boilers in sugar mills are subject to large and rapidly changing loads, e.g. when cane crushers are started or stopped. In addition, the fuel (bagasse) has a highly variable calorific value due to the different moisture content in the original cane. The result of these two factors is that (i) boilers in sugar mills must necessarily cope with large and rapid load changes. These can be of the order of 50%, over the span of 6 – 8 seconds, and (ii) the high moisture content in the bagasse can make it very difficult to burn, thus impacting the furnace dynamics.
The situation described above provided motivation for the project. A new model was needed that would accurately describe the impact of rapid load changes on key process variables, especially drum water level (DWL). In the past, large load variations at the mill often resulted in large swings in drum water level, causing dangerous conditions for boiler operation. For example, low water level can damage the risers and high water level can damage downstream machinery.

Based on the above background, this paper presents a new nonlinear model for natural a circulation boiler based entirely on physical principles. The goal is to develop a model that (i) captures the internal fast dynamics needed to account for large and rapid load variations and fuel variability, and (ii) is simple enough to support basic controller design.

Physical modeling is typically based on a few universal truths. The principles used herein are:

- Conservation of mass
- Conservation of energy
- Conservation of momentum
- Constant volume of the drum, risers and downcomers.

The consequences of each of these concepts will be described in detail in the sequel. The key new features of the model described in this paper are:

- A multi-compartment model is developed for the risers. It will be shown that the spatially distributed nature of boiling water in the risers plays a crucial role in drum water level dynamics under fast and large load changes.
- A new model for drum water level is developed. The model gives rise to a key controller design insight, i.e. drum water level deviations are proportional to steam flow out of the boiler.
- A new model for downcomer water flow in a natural circulation boiler is described. It will be shown that a general momentum balance approach provides a link between flow and pressure derivatives. This link is of critical importance under fast transient responses.

Another aspect of the model development is that all assumptions are clearly and explicitly stated. Thus they can be readily assessed for their validity in specific cases.

The work presented here embellishes and extends the work in [4]. The pressure model turns out to be equivalent to that presented in [4]. The novelty of the current paper lies in (i) the three key features describes above, and (ii) the manner in which the equations are used to describe the model. It will be seen that, through simple algebraic manipulations, the final description of the model allows for easy understanding, and facilitates the design and (re)-tuning of controllers.

The remainder of the paper is organised as follows: In Section 2 the equations from mass balances in the drum and risers are introduced. In Section 3 equations resulting from the constant volume of the drum, risers and downcomers are presented. They are then used to derive further equations that are used in the sequel. In Section 4 the equations describing the energy balance in the drum and risers are introduced. In Section 5 a model for boiler pressure is derived. In Section 6 a model for drum water level is presented. It is shown that further equations are necessary. In Section 7 the equations corresponding to the spatial discretisation of the risers are derived. Also, an additional assumption of homogeneous mixing in the risers is introduced. In Section 8 an equation for the mass flow of water in the downcomers is derived from a general momentum balance analysis. In Section 9 a model for a superheater is derived.
based on constant volume, plus energy and mass balances. In Section 11, key consequences of the new model developed in the paper are discussed and simulations are presented highlighting the key new features of the model. In Section 12, the implications of the new model relative to the control of drum water level are explored. In Section 13, conclusions are drawn. For easy reference, a list of the variables used throughout the paper is presented in Table 1.

| Symbol | Description | See Eq. |
|--------|-------------|---------|
| $\alpha_i$ | Steam quality in riser section $i$ | (44) |
| $\delta$ | Drum water level deviations | (34) |
| $f_\ell$ | Mass flow of water converted into steam | (25) |
| $f_s$ | Steam mass flow from risers into drum | (46) and (39d) |
| $f_w$ | Water mass flow from risers into drum | (48) and (39e) |
| $f_w^*$ | Water mass flow from drum into risers | (57) |
| $h_s$ | Enthalpy of steam | Steam tables |
| $h_w$ | Enthalpy of water | Steam tables |
| $h_n$ | Enthalpy of feedwater | Assumed known |
| $M_{Ds}$ | Mass of steam in the drum (above water line) | (1) |
| $M_{ Dw}$ | Mass of water in the drum | (2) |
| $M_{Rs}$ | Mass of steam in the risers | (3) |
| $M_{ Rw}$ | Mass of water in the risers | (4) |
| $M_{BW}$ | Mass of steam below the water line | (31) |
| $P$ | Drum Pressure | (30) |
| $Q^T$ | Total heat flow | Control variable |
| $Q^H$ | Heat flow needed to raise water temperature | (21) |
| $Q^B$ | Heat flow used to turn water into steam | (27) |
| $q_f$ | Mass flow of feedwater | Control variable |
| $q_s$ | Mass flow of steam exiting the drum | Assumed known |
| $\rho_s$ | Steam density | Steam tables |
| $\rho_w$ | Water density | Steam tables |
| $V_{D}$ | Total volume of the drum | Assumed known |
| $V_{Ds}$ | Volume of steam in the drum | (5) |
| $V_{ Dw}$ | Volume of water in the drum | (5) |
| $V_{BW}$ | Volume of steam below the water line | (33) |
| $V_{R}$ | Total volume of the risers | Assumed known |
| $V_{Rs}$ | Volume of steam in the risers | (6) |
| $V_{ Rw}$ | Volume of water in the risers | (6) |

Table 1: List of Variables
2 Mass Balances

This section focuses on mass balance relationships. The development uses the schematic of the boiler shown in Fig. 1.

The mass of steam and water in the drum and risers satisfies conservation equations. In particular, the time rate of change of mass contained in an open system is equal to the difference between mass inflows and outflows of the system. This leads to:

\[
\frac{dM_D}{dt} = f - q_s
\]  
\[
\frac{dM_{Dw}}{dt} = q_f + f_w - f_w^* 
\]  
\[
\frac{dM_{Rs}}{dt} = f_\ell - f_s 
\]  
\[
\frac{dM_{Rw}}{dt} = f_w^* - f_w - f_\ell 
\]

where \( M_D \), \( M_{Dw} \), \( M_R \), \( M_{Rw} \) denote the mass of steam in the drum, the mass of water in the drum, the...
mass of steam in the risers, and the mass of water in the risers, respectively, and where $f_s$, $q_s$, $q_f$, $f_w$, $f_w^*$, $f_t$ denote the mass flow of steam from the risers into the drum, the mass flow of steam out of the drum, the mass flow of feedwater into the drum, the mass flow of water from the top of the risers into the drum, the mass flow of water from the drum into the downcomers, and the mass flow of water converted into steam in the risers, respectively.

Remark 2.1. Note that $M_s^D$ is defined as the total mass of steam in the drum, i.e., it includes both the mass of steam above and below water.

Remark 2.2. The above equations are the backbone of the model. A departure from the work in [4] is that these equations will be used as shown, i.e., no further manipulation will be done. This is a core difference of the model proposed here. The reasons will be made clearer when the model for drum water level is derived.

Remark 2.3. The above equations consider the risers as one section. That is sufficient for the moment. However, in section 7 a multiple-compartment model for the risers will be introduced. This will be necessary to more accurately describe the drum water level dynamics.

3 Constant volume of Drum, Risers and Downcomers

The total volume of the boiler is fixed. Three different sections of the boiler are considered, i.e., drum, risers, and downcomers. This leads to the following equations:

\[
V^D = V_w^D + V_s^D \quad (5)
\]

\[
V^R = V_w^R + V_s^R \quad (6)
\]

\[
V^{DC} = V_w^{DC} \quad (7)
\]

where $V^D$, $V^R$, $V^{DC}$ denote the volume of the drum, the volume of the risers, and the volume of the downcomers, respectively. Also, $V_w^D$, $V_s^D$, $V_w^R$, $V_s^R$, $V_w^{DC}$ denote the volume of water in the drum, the volume of steam in the drum (both above and below the water line), the volume of water in the risers, the volume of steam in the risers, and the volume of water in the downcomers, respectively.

Remark 3.1. Note that the total volume of the boiler, $V^T$, is given by $V^T = V^D + V^R + V^{DC}$. "\]

Assumption A. Boiling of water only occurs in the risers.

3.1 Consequences of constant volume of the drum

Since the volume of the drum is constant, using (5) and taking its derivative leads to:

\[
0 = \frac{d}{dt} \left( V_w^D + V_s^D \right) = \frac{d}{dt} \left( \frac{M_w^D}{\rho_w} + \frac{M_s^D}{\rho_s} \right)
\]

where the fact that $V = M/\rho$ has been used, where $M$, $\rho$ denote mass and density, respectively. Expanding the derivative leads to:

\[
0 = \frac{\dot{M}_w^D}{\rho_w} - \frac{M_w^D}{\rho_w^2} \cdot \dot{\rho}_w + \frac{\dot{M}_s^D}{\rho_s} - \frac{M_s^D}{\rho_s^2} \cdot \dot{\rho}_s \quad (8)
\]
**Assumption B.** Water in the boiler is at its saturation temperature.

A consequence of Assumption B is that density is a function of pressure only. Thus,

$$\dot{\rho} = \frac{d\rho}{dt} = \frac{\partial \rho}{\partial P} \cdot \dot{P} \quad (9)$$

where $P$ denotes the pressure of the boiler. Using (9), and substituting the mass balance equations (2) and (1) into (8) leads to:

$$0 = \frac{q_f + f_w - f_w^*}{\rho_w} + \frac{f_s - q_s}{\rho_s} - \left( \frac{M_w^D}{\rho_w^2} \frac{\partial \rho_w}{\partial P} + \frac{M_s^D}{\rho_s^2} \frac{\partial \rho_s}{\partial P} \right) \cdot \dot{P}$$

The following variable is defined to simplify the expressions:

$$C_1 \equiv \frac{M_w^D}{\rho_w^2} \frac{\partial \rho_w}{\partial P} + \frac{M_s^D}{\rho_s^2} \frac{\partial \rho_s}{\partial P} \quad (10)$$

Then,

$$0 = \frac{q_f + f_w - f_w^*}{\rho_w} + \frac{f_s - q_s}{\rho_s} - C_1 \cdot \dot{P}$$

Rearranging the above equation leads to:

$$\dot{P} = C_1^{-1} \left( \frac{q_f - f_w^*}{\rho_w} - \frac{q_s}{\rho_s} + \frac{f_w}{\rho_w} + \frac{f_s}{\rho_s} \right) \quad (11)$$

This equation will form part of the model for boiler pressure developed in Section 5.

### 3.2 Consequences of constant volume of the risers

In the same fashion, since the volume of the risers is constant, using (6) and taking its derivative leads to:

$$0 = \frac{d}{dt} \left( V_w^R + V_s^R \right) = \frac{d}{dt} \left( \frac{M_w^R}{\rho_w} + \frac{M_s^R}{\rho_s} \right) = \frac{\dot{M}_w^R}{\rho_w} - \frac{\dot{M}_w^R}{\rho_w} \cdot \dot{P} + \frac{\dot{M}_s^R}{\rho_s} - \frac{\dot{M}_s^R}{\rho_s} \cdot \dot{P} = \frac{f_w^* - f_w - f_s^*}{\rho_w} + \frac{f_s - f_s^*}{\rho_s} - \left( \frac{M_w^R}{\rho_w^2} \frac{\partial \rho_w}{\partial P} + \frac{M_s^R}{\rho_s^2} \frac{\partial \rho_s}{\partial P} \right) \cdot \dot{P}$$

The following variable is introduced to simplify the expressions:

$$C_2 \equiv \frac{M_w^R}{\rho_w^2} \frac{\partial \rho_w}{\partial P} + \frac{M_s^R}{\rho_s^2} \frac{\partial \rho_s}{\partial P} \quad (12)$$

Then,

$$0 = \frac{f_w^*}{\rho_w} - \frac{f_w}{\rho_w} - \frac{f_s^*}{\rho_s} + \frac{f_s}{\rho_s} - C_2 \cdot \dot{P}$$

Rearranging the above equation leads to:

$$\frac{f_w}{\rho_w} + \frac{f_s}{\rho_s} = \frac{f_w^*}{\rho_w} - \frac{f_s^*}{\rho_s} + C_2 \cdot \dot{P} \quad (13)$$

This equation will also form part of the model for boiler pressure described in Section 5.
4 Energy Balances

The time rate of change of the energy contained in an open system is equal to the difference between energy inflows and outflows of the system. The energy contained in the drum is given by the energy contained in the masses of water and steam in the drum, plus the energy contained in the metal walls of the drum. The energy inflows and outflows are given by the energy carried by the masses flowing into and out of the drum, plus a component of heat from the furnace. The energy balance for the risers follows the same principle. The following energy balance equations result from these considerations:

\[
\frac{d}{dt} \left\{ M_s^D u_s + M_w^D u_w + M_m^D C_p T_m \right\} = (f_s - q_s) h_s + (f_w - f_w^* ) h_w + q_f h_w^f + Q^H
\] (14)

\[
\frac{d}{dt} \left\{ M_s^R u_s + M_w^R u_w + M_m^R C_p T_m \right\} = - f_s h_s + (f_w^* - f_w) h_w + Q^R
\] (15)

where \( u_s, u_w, M_s^D, M_w^R, C_p, T_m \) denote the internal energy of steam, internal energy of water, mass of metal in the drum, mass of metal in the risers, heat capacity of metal, and temperature of metal, respectively. Also \( h_s, h_w, h_w^f, Q^H, Q^R \) denote the enthalpy of steam, enthalpy of water, enthalpy of feedwater, heat flow used for heating the water to boiling temperature, and heat flow used for boiling water in the risers, respectively.

Remark 4.1. Note that the total heat flow generated by the furnace, \( Q^T \), is given by \( Q^T = Q^H + Q^R \). □

Assumption C. The water in the drum is at its boiling temperature. □

4.1 Consequences energy balance in the drum

By definition, internal energy is related to enthalpy and pressure by \( u = h - P/\rho \). Substituting into the left hand side of (14) leads to:

\[
\frac{d}{dt} \left\{ M_s^D h_s + M_w^D h_w - \left( \frac{M_s^D}{\rho_s} + \frac{M_w}{\rho_w} \right) P + M_m^D C_p T_m \right\} = (f_s - q_s) h_s + (f_w - f_w^* ) h_w + q_f h_w^f + Q^H
\]

Noting that

\[
\frac{M_s^D}{\rho_s} + \frac{M_w}{\rho_w} = V_s^D + V_w^D = V^D
\]

then,

\[
\frac{d}{dt} \left\{ M_s^D h_s + M_w^D h_w - V^D P + M_m^D C_p T_m \right\} = (f_s - q_s) h_s + (f_w - f_w^* ) h_w + q_f h_w^f + Q^H
\]

Expanding the LHS, leads to:

\[
M_s^D h_s + M_s^D \dot{h} + M_w^D h_w + M_w^D \dot{h} - V^D \dot{P} + M_m^D C_p \dot{T}_m = (f_s - q_s) h_s + (f_w - f_w^* ) h_w + q_f h_w^f + Q^H
\] (16)

Since water is assumed to be at its saturation temperature, it follows that:

\[
\dot{h} = \frac{dh}{dt} = \frac{\partial h}{\partial P} \cdot \frac{dP}{dt} = \frac{\partial h}{\partial P} \cdot \dot{P}
\] (17)

Assumption D. The temperature of the metal, \( T_m \), is assumed to be the same as the saturation temperature, \( T_s \).
It then follows that:

\[ \dot{T}_m \approx \frac{dT_s}{dt} = \frac{\partial T_s}{\partial P} \cdot \frac{dP}{dt} = \frac{\partial T_s}{\partial P} \cdot \dot{P} \quad (18) \]

Substituting (17) and (18), as well as the mass balance equations (1) and (2), into (16) leads to:

\[
(f_s - q_s)h_s + (q_f + f_w - f_w^*)h_w + \left( M_s \frac{\partial h_s}{\partial P} + M_w \frac{\partial h_w}{\partial P} - V^D + M^D_m C_p \frac{\partial T_s}{\partial P} \right) \dot{P} \\
= (f_s - q_s)h_s + (f_w - f_w^*)h_w + q_f h_w^f + Q^H
\]

Cancelling the common terms on both sides leads to:

\[
q_f h_w + \left( M_s \frac{\partial h_s}{\partial P} + M_w \frac{\partial h_w}{\partial P} - V^D + M^D_m C_p \frac{\partial T_s}{\partial P} \right) \dot{P} = q_f h_w^f + Q^H
\]

The following variable is introduced to simplify the equations

\[
K_1 \equiv M_s \frac{\partial h_s}{\partial P} + M_w \frac{\partial h_w}{\partial P} - V^D + M^D_m C_p \frac{\partial T_s}{\partial P} \quad (19)
\]

Then,

\[
K_1 \dot{P} = q_f (h_w^f - h_w) + Q^H \quad (20)
\]

Rearranging the above equation leads to:

\[
Q^H = K_1 \dot{P} - q_f (h_w^f - h_w) \quad (21)
\]

This equation will form part of the model for boiler pressure described in Section 5.

4.2 Consequences energy balance in the risers

In the same fashion, substituting \( u = h - P/\rho \) into the left hand side of (15) leads to:

\[
\frac{d}{dt} \left[ M^R_s h_s + M^R_w h_w - \left( \frac{M^R_s}{\rho_s} + \frac{M^R_w}{\rho_w} \right) P + M^R_w C_p T_m \right] = -f_s h_s + (f_w^* - f_w) h_w + Q^B
\]

Again, noting that

\[
\frac{M^R_s}{\rho_s} + \frac{M^R_w}{\rho_w} = \rho_s^R + \rho_w^R = V^R
\]

Then,

\[
\frac{d}{dt} \left[ M^R_s h_s + M^R_w h_w - V^R P + M^R_w C_p T_m \right] = -f_s h_s + (f_w^* - f_w) h_w + Q^B
\]

Expanding the LHS leads to

\[
M^R_s h_s + M^R_s h_s + M^R_w h_w + M^R_w h_w - V^R P + M^R_w C_p T_m = -f_s h_s + (f_w^* - f_w) h_w + Q^B \quad (22)
\]
In view of Assumption B, equations (17) and (18) can be used here. In addition, substituting the mass balance equations, (3) and (4), into (22), leads to:

\[
(f_\ell - f_s)h_s + (f_w^* - f_w - f_\ell)h_w + \left(M_s^R \frac{\partial h_s}{\partial P} + M_w^R \frac{\partial h_w}{\partial P} - V_R + M_{m_c} C_p \frac{\partial T_s}{\partial P}\right) \dot{P} = -f_s h_s + (f_w^* - f_w - f_\ell)h_w + Q^B
\]

Cancelling the common terms on both sides leads to:

\[
f_\ell h_s - f_\ell h_w + \left(M_s^R \frac{\partial h_s}{\partial P} + M_w^R \frac{\partial h_w}{\partial P} - V_R + M_{m_c} C_p \frac{\partial T_s}{\partial P}\right) \dot{P} = Q^B
\]

The following variable is next defined to simplify the equations:

\[
K_2 \equiv M_s^R \frac{\partial h_s}{\partial P} + M_w^R \frac{\partial h_w}{\partial P} - V_R + M_{m_c} C_p \frac{\partial T_s}{\partial P}
\]

(23)

This leads to:

\[
(h_s - h_w)f_\ell + K_2 \dot{P} = Q^B
\]

(24)

Rearranging the above equation yields:

\[
f_\ell = (h_s - h_w)^{-1} \left(-K_2 \dot{P} + Q^B\right)
\]

(25)

This equation will also form part of the model for boiler pressure described in the next Section.

5 A Model for Boiler Pressure

So far, the model for the boiler dynamics comprises: (i) the four conservation of mass equations, (1)–(4), (ii) the two equations derived from the constant volume of the drum and risers, (11) and (13), and (iii) the two equations derived from energy balance in the drum and in the risers, (21) and (25). Here an expression for boiler pressure, $P$, will be derived based only on the latter four equations. Substituting (13) into (11) leads to:

\[
\dot{P} = C_1^{-1} \left(\frac{q_f - f_w^*}{\rho_w} - \frac{q_s}{\rho_s} + \frac{f_w^* - f_\ell}{\rho_w} + \frac{f_\ell}{\rho_s} - C_2 \cdot \dot{P}\right)
\]

Solving for $\dot{P}$ yields:

\[
(C_1 + C_2) \dot{P} = \left(\frac{q_f}{\rho_w} - \frac{q_s}{\rho_s} - \frac{1}{\rho_w} - \frac{1}{\rho_s}\right) f_\ell
\]

(26)

On the other hand, noting that

\[
Q^B = Q^T - Q^H
\]

(27)

where $Q^T$ is the total heat flow input, and substituting (21) into (25), it follows that:

\[
f_\ell = (h_s - h_w)^{-1} \left(-K_2 \dot{P} + Q^T - \left(K_1 \dot{P} - q_f (h_w - h_s)\right)\right)
\]

\[
= (h_s - h_w)^{-1} \left[\left(K_1 + K_2\right) \dot{P} + Q^T + q_f (h_w - h_s)\right]
\]

(28)
Substituting (28) into (26) leads to:

\[ (C_1 + C_2) \dot{P} = \left( \frac{q_f}{\rho_w} - \frac{q_s}{\rho_s} + \left( \frac{1}{\rho_w} - \frac{1}{\rho_s} \right) (h_s - h_w) \right) \left( (K_1 + K_2) \dot{P} + Q^T + q_f(h_w - h_s) \right) \]

The following variable is then introduced to simplify the equations:

\[ C_3 = \left( \frac{1}{\rho_w} - \frac{1}{\rho_s} \right) (h_s - h_w)^{-1} \] (29)

Then, rearranging the above equation leads to:

\[ (C_1 + C_2 - C_3(K_1 + K_2)) \dot{P} = \left( \frac{q_f}{\rho_w} - \frac{q_s}{\rho_s} - C_3 \left( Q^T + q_f(h_w - h_s) \right) \right) \]

Finally, solving for \( \dot{P} \) yields:

\[ \dot{P} = (C_1 + C_2 - C_3(K_1 + K_2))^{-1} \left( \frac{q_f}{\rho_w} - \frac{q_s}{\rho_s} - C_3 \left( Q^T + q_f(h_w - h_s) \right) \right) \] (30)

In summary, equation (30) is a differential equation for \( P \), equations (25) and (21) are algebraic equations for \( f \ell \) and \( Q_H \), respectively, and equations (1)–(4) are differential equations for the steam and water masses in the boiler. Together, they provide a complete model for boiler pressure.

**Remark 5.1.** Note that, to evaluate (30), it is not necessary to know the values of \( f_s, f_w \) and \( f^*_w \). The variable \( C_3 \) is determined only by the current pressure, while \( K_1 + K_2 \) and \( C_1 + C_2 \) depend on pressure and the total mass of water and steam in the boiler, since \( \dot{M}_w^D + \dot{M}_w^R = q_f - f \ell \) and \( \dot{M}_s^D + \dot{M}_s^R = f \ell - q_s \). Pressure is a global property of the boiler, and therefore, it stands to reason, that it should not depend on internal properties of the boiler. □

**Remark 5.2.** On the other hand, unlike pressure, drum water level is not a global property of the boiler. It will be shown below that, in order to obtain a model for drum water level, it is necessary to know other internal quantities such as \( f_s, f_w \) and \( f^*_w \). □

### 6 A Model for Drum Water Level

Consider a section of the boiler drum as shown in Fig. 1, where \( L \) is the steady state (nominal) height of the water, and \( \delta \) denotes variations around \( L \). Note that, to determine the height of the water it is necessary to know, not only the mass of water in the drum, \( M_w^D \), but also the mass of steam below the water line, \( M_w^{BW} \). An expression for \( M_w^{BW} \) can be obtained by again applying the conservation of mass principle, i.e.:

\[ M_w^{BW}(t) = f_s(t) - f_s(t-a) \] (31)

where \( f_s(t) \), \( f_s(t-a) \) denote the mass flow of steam out of the risers, and a delayed version of \( f_s(t) \), respectively. The time delay \( a \) can be easily computed by noting that it is the time taken for a given mass to cover a certain distance, i.e.:

\[ a = \frac{\text{distance}}{\text{speed}} = \frac{L+\delta}{f_s(t)/(\rho_s A^R)} \] (32)

where \( A^R \) denotes the total area of the risers.
Assumption E. In steady state, the drum is half full of water, and the corresponding height of water is \( L \).

An immediate consequence of the above assumption is:

\[
V_w^D + V_s^{BW} = \dot{V}_w^D + \dot{V}_s^{BW} + \dot{V}_s^D = \frac{V^D}{2} + A^D \delta \tag{33}
\]

where \( A^D \) denotes the area at the centre line of the drum. Using the fact that \( V = M/\rho \), then equation (33) can be rewritten as:

\[
\frac{M_w^D}{\rho_w} + \frac{M_s^{BW}}{\rho_s} = \frac{V^D}{2} + A^D \delta
\]

Finally, the following expression for the drum water level deviation, \( \delta \), is obtained:

\[
\delta = \left( A^D \right)^{-1} \left( \frac{M_w^D}{\rho_w} + \frac{M_s^{BW}}{\rho_s} - \frac{V^D}{2} \right) \tag{34}
\]

Remark 6.1. Equations (34), (31) and (2) show that it is necessary to be able to independently describe \( f_s \), \( f_w \) in order to obtain drum water level.

Remark 6.2. Note that (11) and (13) are linearly dependent equations in \( f_s \) and \( f_w \). Therefore, with these two equations alone it is not possible to obtain independent expressions for \( f_s \) and \( f_w \). This problem will be resolved in Section 7.

Remark 6.3. An expression for \( f_w^* \) will be derived in Section 8 using conservation of momentum analysis in the downcomer-riser system.

7 Spatial Discretisation and Homogeneous Mixing in the Risers

In this section, the model of the risers will be expanded to account for the spatial distribution of the boiling process. An additional assumption will be introduced which allows separation of the expressions for \( f_s \) and \( f_w \) in the model.

7.1 Spatial discretisation

Consider a uniform subdivision of the volume of the risers into \( n \) sections. For each section, there are three core equations, describing mass balance, constant volume and energy balance. This leads to:

\[
\dot{M}_s^{R_i} = f_s^i + f_s^{i-1} - f_s^i
\]

\[
\dot{M}_w^{R_i} = f_w^{i-1} - f_w^i - f^i_{\ell}
\]

\[
V^{R_i} = V_w^{R_i} + V_s^{R_i}
\]

\[
\frac{d}{dt} \left\{ M_s^{R_i} u_s + M_w^{R_i} u_w + M_m^{R_i} C_p T_m \right\} = -f_s^i h_s + (f_w^{i-1} - f_w^i) h_w + Q_i^B
\]

where the superscript \( R_i \) denotes the \( i \)-th section of the risers, \( f_s^i \) denotes the flow of water converted into steam in section \( i \), and \( f_s^i, f_w^i \) denote the mass flow of steam and water entering section \( i \), respectively, \( f_s^{i-1}, f_w^{i-1} \) denote the mass flow of steam and water entering section \( i \), respectively, and \( Q_i^B \) denotes the heat flow directly affecting section \( i \), for \( i = 1, \ldots, n \).
**Assumption F.** The heat flow used for boiling water, $Q^B$, is distributed uniformly across the $n$ sections of the risers, i.e. $Q^B_i = Q^B/n$, $\forall i$. □

The following equations are immediate:

\[
\begin{align*}
  f_0^w &= f_0^s \\
  f_0^s &= 0 \\
  f_i^w &= f_w \\
  f_i^n &= f_s \\
  \sum_{i=1}^n f_i^f &= f_f \\
  \sum_{i=1}^n M_{R_i}^W &= M^W \\
  \sum_{i=1}^n M_{R_i}^S &= M^S 
\end{align*}
\]

To simplify the equations in the sequel, the following variables are defined:

\[
\begin{align*}
  C_i^2 &\triangleq \frac{M_{R_i}^W}{\rho_w} \frac{\partial \rho_w}{\partial P} + \frac{M_{R_i}^S}{\rho_s} \frac{\partial \rho_s}{\partial P} \\
  K_i^2 &\triangleq M_{R_i}^S \frac{\partial h_s}{\partial P} + M_{R_i}^W \frac{\partial h_w}{\partial P} - V_{R_i} + M_{m_i} C_p \frac{\partial T_s}{\partial P}
\end{align*}
\]

Then, using the same procedure as in Sections 3.2 and 4.2 for equations (37) and (38), it follows that:

\[
\begin{align*}
  f_i^w \frac{\rho_w}{\rho_w} + f_s^i \frac{\rho_s}{\rho_s} &= \frac{f_i^{w-1} - f_t^i}{\rho_w} + \frac{f_t^i + f_s^{i-1}}{\rho_s} - C_i^2 \dot{\rho} + Q_i^B \\
  f_t^i &= (h_s - h_w)^{-1} \left( -K_i^2 \dot{\rho} + Q_i^B \right)
\end{align*}
\]

Equations (42) and (43), together with (35) and (36), give a complete account of the dynamics of the $i$-th section of the risers. However, $f_i^s$ and $f_i^w$ are still linearly dependent. In the next subsection, an additional assumption is introduced which allows $f_i^s$ and $f_i^w$ to be separately described.

### 7.2 Homogeneous mixing in a section of the risers

Consider a section of the risers. Then, over an infinitesimal period of time $\Delta$, the mass of steam and water leaving the section are given by $f_i^s \Delta$ and $f_i^w \Delta$, respectively. The steam quality of each section is defined as:

\[
\alpha_i^i = \frac{M_{S_i}^{R_i}}{M_{R_i}^{S_i} + M_{R_i}^{W}}
\]

The following assumption is next introduced:

**Assumption G.** Perfect mixing of water and steam occurs in each section of the risers. □
An immediate consequence of the above assumption is that the mass of steam and water leaving a specific section over a period of time $\Delta$ must have the same ratio $\alpha^i$. Therefore,

$$\alpha^i = \frac{M^R_{si}}{M^R_{wi} + M^R_{wi}} = \frac{f^s_i \Delta}{f^s_i \Delta + f^w_i \Delta} \tag{45}$$

Solving for $f^s_i$ leads to:

$$f^s_i = \frac{M^R_{si}}{M^R_{wi} f^w_i} \tag{46}$$

Substituting (46) into (42) yields:

$$f^w_i \rho^w + \frac{1}{\rho_s} \frac{M^R_{si}}{M^R_{wi} f^w_i} f^w_i = \frac{f^{i-1}_w - f^i_w}{\rho_w} + \frac{f^i_w + f^{i-1}_w}{\rho_s} - C^j_2 \cdot \dot{P} \tag{47}$$

Solving for $f^w_i$ leads to:

$$f^w_i = \left( \frac{1}{\rho_w} + \frac{1}{\rho_s} \frac{M^R_{si}}{M^R_{wi}} \right)^{-1} \left( \frac{f^{i-1}_w - f^i_w}{\rho_w} + \frac{f^i_w + f^{i-1}_w}{\rho_s} - C^j_2 \cdot \dot{P} \right) \tag{48}$$

In summary, equations (46) and (48) provide a separate account of the mass flow of steam and water leaving the $i$-th section of the risers. Together with equations (43), (35) and (36) this constitutes a complete description of the dynamics of a section of the risers. Using equations (39), the model for the sections of the risers can be interfaced with the pressure and drum water level models presented in Sections 5 and 6.

Remark 7.1. Note that the concept of homogeneity of the steam-water mix is directly related to the concept that no slip occurs between the steam mass flow and the water mass flow – see [4]. Indeed, the no slip condition implies the linear speed of both steam and water masses leaving each section of the risers are the same, and therefore, the mass flows must be locked together. $\Box$

8 Water Flow in the Downcomers (Momentum Balance)

In order to obtain an expression for $f^w_i$, conservation of momentum is applied along the downcomers and risers. The fixed control volume is defined as the total volume of the downcomer-riser configuration as shown in Fig. 1. The control surface is defined as the surface of the control volume. A general expression for momentum balance is given by (see [3, Section 2.5]):

$$\frac{\partial}{\partial t} \iiint_V \rho \vec{v} dV + \iint_S (\rho \vec{v} \cdot d\vec{S}) = - \iint_S P dS + \iiint_V \rho f dV + F_{visc} \tag{49}$$

where the term A denotes the time rate change of the linear momentum of the contents of the control volume, the term B denotes the net flow of linear momentum out of the control surface by mass flow, the term C denotes the force exerted by pressure on the control surface, the term D denotes the body force acting on the control volume, and $F_{visc}$ denotes the viscous forces acting on the control surface.

Assumption H. The pressure dynamics are much slower than the momentum dynamics. $\Box$
A consequence of the above assumption is that density can be considered to be uniform across the volume of the downcomers/risers system. Therefore,

\[
\frac{\partial}{\partial t} \iiint_V \rho \vec{v} \, dV = \frac{\partial}{\partial t} \sum \left[ \frac{M_w^{DC} f_w^*}{\rho_w A^{DC}} + M_w^R f_w + M_s^R f_s \right]
\]

(50)

\[
\iiint_S (\rho \vec{v} \cdot dS) = k_w \left( \frac{(f_w^*)^2}{\rho_w A^{DC}} + k_w \frac{f_w^2}{\rho_w A^R} + k_s \frac{f_s^2}{\rho_s A^R} \right)
\]

(51)

\[
\iiint_S P dS = 0
\]

(52)

\[
\iiint_V \rho f dV = (M_w^{DC} - M_w^R - M_s^R) g
\]

(53)

\[
F_{\text{visc}} = 0
\]

(54)

Note that, since the downcomers contain only water, then

\[
\frac{M_w^{DC}}{\rho_w A^{DC}} = \frac{V_w^{DC}}{A^{DC}} = L^{DC}
\]

(55)

where \( L^{DC} \) is the length of the downcomers. Therefore, equation (49) can be written as:

\[
\frac{\partial}{\partial t} \left[ L^{DC} f_w^* + M_w^R \frac{f_w}{\rho_w A^R} + M_s^R \frac{f_s}{\rho_s A^R} \right] + k_w \left( \frac{(f_w^*)^2}{\rho_w A^{DC}} + k_w \frac{f_w^2}{\rho_w A^R} + k_s \frac{f_s^2}{\rho_s A^R} \right) = (M_w^{DC} - M_w^R - M_s^R) g
\]

(56)

Solving for \( f_w^* \) leads to:

\[
L^{DC} \frac{df_w^*}{dt} = -k_w \left( \frac{(f_w^*)^2}{\rho_w A^{DC}} - k_w \frac{f_w^2}{\rho_w A^R} - k_s \frac{f_s^2}{\rho_s A^R} + (M_w^{DC} - M_w^R - M_s^R) g \right)
\]

(57)

\[
-\frac{\partial}{\partial t} \left[ \frac{M_w^R f_w}{\rho_w A^R} + \frac{M_s^R f_s}{\rho_s A^R} \right]
\]

Remark 8.1. Note that (57) represents a significant departure from the equations used in [4]. □

9 Superheaters

A superheater is a heat exchanger used to convert saturated steam generated in a boiler into superheated steam by adding heat, thus drying the steam. Superheated steam is used in steam turbines to generate electricity. For the current purpose, the main difference between saturated and superheated steam is that, when considering saturated steam, it sufficed to use one state variable, namely the pressure of the water/steam mixture. This made it possible to unequivocally describe density, enthalpy, temperature, and other state variables, for both liquid and vapour phases. However, to describe the state of superheated steam it is necessary to consider two independent state variables. In the sequel, pressure and enthalpy will be used for this purpose.

Let the superheater have volume \( V^{SH} \) and a heat flow input \( Q^{SH} \). Then mass balance, energy balance and constant volume equations for a superheater can immediately be obtained as shown below.
9.1 Mass balance

\[ \dot{M}_{s}^{SH} = q_{s} - q_{s}^{SH} \]  \hspace{1cm} (58)

where \( M_{s}^{SH}, q_{s}, q_{s}^{SH} \) denote the mass of steam in the superheater, the steam mass flow out of the drum into the superheater, and the steam mass flow out of the superheater.

9.2 Energy balance

\[ \frac{d}{dt} \left[ \bar{M}_{s}^{SH} \bar{u}_{s}^{SH} \right] = q_{s} h_{s} - q_{s}^{SH} \bar{h}_{s}^{SH} + Q^{SH} \]  \hspace{1cm} (59)

By definition \( u = h - P/\rho \), therefore,

\[ \frac{d}{dt} \left[ M_{s}^{SH} \bar{h}_{s}^{SH} - \frac{M_{s}^{SH}}{\rho_{s}^{SH}} P^{SH} \right] = q_{s} h_{s} - q_{s}^{SH} \bar{h}_{s}^{SH} + Q^{SH} \]

Noting that \( M_{s}^{SH}/\rho_{s}^{SH} = V^{SH} \), and expanding the derivative, leads to:

\[ M_{s}^{SH} \dot{h}_{s}^{SH} + M_{s}^{SH} \dot{\bar{h}}_{s}^{SH} - V^{SH} \dot{\bar{p}}^{SH} = q_{s} h_{s} - q_{s}^{SH} \bar{h}_{s}^{SH} + Q^{SH} \]

Finally, using equation (58) and cancelling the common terms yields:

\[ M_{s}^{SH} \dot{h}_{s}^{SH} - V^{SH} \dot{\bar{p}}^{SH} = q_{s} (h_{s} - \bar{h}_{s}^{SH}) + Q^{SH} \]  \hspace{1cm} (60)

9.3 Constant volume of the superheater

\[ \frac{d}{dt} \left[ V^{SH} \right] = 0 \]  \hspace{1cm} (61)

By definition we know that \( V = M/\rho \), thus

\[ \frac{d}{dt} \left[ V^{SH} \right] = \frac{d}{dt} \left[ \frac{M_{s}^{SH}}{\rho_{s}^{SH}} \right] = \frac{\dot{M}_{s}^{SH}}{\rho_{s}^{SH}} - \frac{M_{s}^{SH}}{\left(\rho_{s}^{SH}\right)^{2}} \dot{\rho}_{s}^{SH} \]  \hspace{1cm} (62)

However, as mentioned earlier, density of superheated steam is no longer a function of pressure only. Therefore, the time derivative of density must now be expanded as follows:

\[ \dot{\rho}_{s}^{SH} = \frac{\partial \rho_{s}^{SH}}{\partial h_{s}^{SH}} \dot{h}_{s}^{SH} + \frac{\partial \rho_{s}^{SH}}{\partial P^{SH}} \dot{P}^{SH} \]  \hspace{1cm} (63)

Substituting (62) and (63) into (61), and noting that \( M/\rho = V \) leads to:

\[ \frac{\dot{M}_{s}^{SH}}{\rho_{s}^{SH}} - V^{SH} \left( \frac{\partial \rho_{s}^{SH}}{\partial h_{s}^{SH}} \dot{h}_{s}^{SH} + \frac{\partial \rho_{s}^{SH}}{\partial P^{SH}} \dot{P}^{SH} \right) = 0 \]

Using (58) and reordering terms yields:

\[ V^{SH} \left( \frac{\partial \rho_{s}^{SH}}{\partial h_{s}^{SH}} \dot{h}_{s}^{SH} + \frac{\partial \rho_{s}^{SH}}{\partial P^{SH}} \dot{P}^{SH} \right) = q_{s} - q_{s}^{SH} \]  \hspace{1cm} (64)
9.4 A model for the superheater

Equations (58), (60) and (64) provide a complete model describing the dynamics of a superheater. To implement the model, equations (60) and (64) first have to be decoupled. From (64), it follows that:

\[ V_{SH} \dot{p}_{SH} = \left( \frac{\partial p_{SH}^s}{\partial p_{SH}} \right)^{-1} \left( \dot{q}_s - q_{s}^{SH} - V_{SH} \frac{\partial p_{SH}^s}{\partial h_{s}^{SH}} \right) \]

Substituting into (60) leads to:

\[ M_{s}^{SH} \dot{h}_{s}^{SH} + \left( \frac{\partial p_{s}^{SH}}{\partial P_{SH}} \right)^{-1} V_{SH} \frac{\partial p_{s}^{SH}}{\partial h_{s}^{SH}} \dot{h}_{s}^{SH} = q_{s}(h_{s} - h_{s}^{SH}) + Q_{SH} + \left( \frac{\partial p_{s}^{SH}}{\partial P_{SH}} \right)^{-1} \left( \dot{q}_s - q_{s}^{SH} \right) \]

Solving for \( \dot{h}_{s}^{SH} \) yields:

\[ \dot{h}_{s}^{SH} = \left( M_{s}^{SH} + \left( \frac{\partial p_{s}^{SH}}{\partial P_{SH}} \right)^{-1} V_{SH} \frac{\partial p_{s}^{SH}}{\partial h_{s}^{SH}} \right)^{-1} \left( q_{s}(h_{s} - h_{s}^{SH}) + Q_{SH} + \left( \frac{\partial p_{s}^{SH}}{\partial P_{SH}} \right)^{-1} \left( \dot{q}_s - q_{s}^{SH} \right) \right) \]

Then, from (60) it follows that:

\[ \dot{p}_{SH} = \left( V_{SH} \right)^{-1} \left( M_{s}^{SH} \dot{h}_{s}^{SH} - q_{s}(h_{s} - h_{s}^{SH}) - Q_{SH} \right) \]

In summary, the model for a superheater is given by the following equations:

\[ M_{s}^{SH} = q_{s} - q_{s}^{SH} \]  
\[ \dot{h}_{s}^{SH} = \left( M_{s}^{SH} + \left( \frac{\partial p_{s}^{SH}}{\partial P_{SH}} \right)^{-1} V_{SH} \frac{\partial p_{s}^{SH}}{\partial h_{s}^{SH}} \right)^{-1} \left( q_{s}(h_{s} - h_{s}^{SH}) + Q_{SH} + \left( \frac{\partial p_{s}^{SH}}{\partial P_{SH}} \right)^{-1} \left( \dot{q}_s - q_{s}^{SH} \right) \right) \]
\[ \dot{p}_{SH} = \left( V_{SH} \right)^{-1} \left( M_{s}^{SH} \dot{h}_{s}^{SH} - q_{s}(h_{s} - h_{s}^{SH}) - Q_{SH} \right) \]

10 Steam Receiver

A steam receiver is a metal vessel typically used to hold the steam generated by the boilers, and from where steam is drawn for the various load requirements.

The steam receiver model follows the same principles used to model a superheater. In particular, it is necessary to account for the use of superheated steam. The differences lie in that (i) the steam receiver will usually have multiple steam flow inputs, one from each connected boiler, and (ii) there is no heat input.

Let the steam receiver have volume \( V_{E} \). Then, mass balance, energy balance and constant volume equations can be derived as usual.

10.1 Mass balance

\[ M_{s}^{E} = \sum_{i} q_{i}^{E} - q_{s}^{E} \]  

where \( M_{s}^{E}, q_{i}^{E}, q_{s}^{E} \) denote the mass of steam in the steam receiver, the steam mass flow out of the boiler \( i \) and into the steam receiver, and the steam mass flow out of the steam receiver, i.e., to the loads.
10.2 Energy balance

\[ \frac{d}{dt} \left\{ M_s^E u_s^E \right\} = \sum_i q_i^i h_s^i - q_s^E h_s^E \]  

(69)

where \( h_s^i \) denotes the enthalpy of \( q_i^i \) from each boiler/superheater. By definition \( u = h - P/\rho \), therefore:

\[ \frac{d}{dt} \left\{ M_s^E h_s^E - \frac{M_s^E}{\rho_s^E} P^E \right\} = \sum_i q_i^i h_s^i - q_s^E h_s^E \]

Noting that \( M_s^E / \rho_s^E = V_E \), and expanding the derivative leads to:

\[ M_s^E h_s^E + M_s^E h_s^E - V_E \dot{P}^E = \sum_i q_i^i h_s^i - q_s^E h_s^E \]

Finally, using equation (68) and cancelling the common terms, it follows that:

\[ M_s^E h_s^E - V_E \dot{P}^E = \sum_i q_i^i (h_s^i - h_s^E) \]  

(70)

10.3 Constant volume of the steam receiver

\[ \frac{d}{dt} \left\{ V^E \right\} = 0 \]  

(71)

By definition \( V = M/\rho \), hence:

\[ \frac{d}{dt} \left\{ V^E \right\} = \frac{d}{dt} \left[ \frac{M_s^E}{\rho_s^E} \right] = \frac{\dot{M}_s^E}{\rho_s^E} - \frac{M_s^E}{(\rho_s^E)^2} \dot{\rho}_s^E \]

(72)

However, as mentioned earlier, the density of superheated steam is considered here a function of pressure and enthalpy, therefore the time derivative must be expanded as follows:

\[ \dot{\rho}_s^E = \frac{\partial \rho_s^E}{\partial h_s^E} \dot{h}_s^E + \frac{\partial \rho_s^E}{\partial P^E} \dot{P}^E \]

(73)

Substituting (72) and (73) into (71), and noting that \( M/\rho = V \), leads to:

\[ \frac{\dot{M}_s^E}{\rho_s^E} = \frac{V^E}{\rho_s^E} \left( \frac{\partial \rho_s^E}{\partial h_s^E} \dot{h}_s^E + \frac{\partial \rho_s^E}{\partial P^E} \dot{P}^E \right) = 0 \]

Using (58) and reordering terms yields:

\[ V^E \left( \frac{\partial \rho_s^E}{\partial h_s^E} \dot{h}_s^E + \frac{\partial \rho_s^E}{\partial P^E} \dot{P}^E \right) = \sum_i q_i^i - q_s^E \]  

(74)
10.4 A model for the steam receiver

Equations (68), (70) and (74) provide a complete model describing the dynamics of a steam receiver. To implement the model, equations (70) and (74) first have to be decoupled. From (74), it follows that:

$$V^E \dot{p}^E = \left( \frac{\partial \rho_s^E}{\partial P^E} \right)^{-1} \left( \sum_i q_s^i - q_s^E - V^E \frac{\partial \rho_s^E}{\partial h_s^E} \right)$$

Substituting into (70) leads to:

$$M_s^E \dot{h}_s^E + \left( \frac{\partial \rho_s^E}{\partial P^E} \right)^{-1} V^E \frac{\partial \rho_s^E}{\partial h_s^E} h_s^E = \sum_i q_s^i (h_s^i - h_s^E) + \left( \frac{\partial \rho_s^E}{\partial P^E} \right)^{-1} \left( \sum_i q_s^i - q_s^E \right)$$

Solving for $\dot{h}_s^E$ yields:

$$\dot{h}_s^E = \left( M_s^E + \left( \frac{\partial \rho_s^E}{\partial P^E} \right)^{-1} V^E \frac{\partial \rho_s^E}{\partial h_s^E} \right)^{-1} \left( \sum_i q_s^i (h_s^i - h_s^E) + \left( \frac{\partial \rho_s^E}{\partial P^E} \right)^{-1} \left( \sum_i q_s^i - q_s^E \right) \right)$$

Then, from (70) it follows that:

$$\dot{p}^E = \left( V^E \right)^{-1} \left( M_s^E \dot{h}_s^E - \sum_i q_s^i (h_s^i - h_s^E) \right)$$

In summary, the model for a steam receiver is given by the following equations:

$$M_s^E = \sum_i q_s^i - q_s^E$$

$$\dot{h}_s^E = \left( M_s^E + \left( \frac{\partial \rho_s^E}{\partial P^E} \right)^{-1} V^E \frac{\partial \rho_s^E}{\partial h_s^E} \right)^{-1} \left( \sum_i q_s^i (h_s^i - h_s^E) + \left( \frac{\partial \rho_s^E}{\partial P^E} \right)^{-1} \left( \sum_i q_s^i - q_s^E \right) \right)$$

$$\dot{p}^E = \left( V^E \right)^{-1} \left( M_s^E \dot{h}_s^E - \sum_i q_s^i (h_s^i - h_s^E) \right)$$

11 Key consequences of the new model

This section summarises and illustrates the main consequences of the model derived in this paper. In particular, three key points are made, namely: (i) drum water level is proportional to steam flow out of the boiler, (ii) spatial discretisation of the risers is necessary for fast transient dynamic modelling, and (iii) the relationship between downcomer mass flow and pressure derivatives leads to a model that can describe fast transients in the drum water level responses.

In the sequel, the simulations and data presented correspond to Boiler 1 at Proserpine Mill. Boiler 1
does not have a superheater. The details of the physical parameters used in the simulation are as follows:

\[
A^R = 1.5 \text{[m}^2]\text{]} \quad (78a)
\]
\[
V^R = 10.5 \text{[m}^3]\text{]} \quad (78b)
\]
\[
A^D = 11.1 \text{[m}^2]\text{]} \quad (78c)
\]
\[
V^D = 12 \text{[m}^3]\text{]} \quad (78d)
\]
\[
L^{DC} = 7 \text{[m]} \quad (78e)
\]
\[
A^{DC} = 1.5 \text{[m}^2]\text{]} \quad (78f)
\]
\[
V^{DC} = 10.5 \text{[m}^3]\text{]} \quad (78g)
\]
\[
M^D = 7400 [\text{kg}] \quad (78h)
\]
\[
M^R = 40700 [\text{kg}] \quad (78i)
\]
\[
P_0 = 1.65 \cdot 10^6 [\text{Pa}] \quad (78j)
\]

**Remark 11.1.** It is very important to note that the simulation parameters have all been obtained from physical properties of the boiler and its associated datasheets. No estimation of parameters has been performed. This avoids the problem of overfitting due to the presence of many degrees of freedom [11].

### 11.1 Drum Water Level proportional to Steam Flow

One advantage of having a physical model is that particular occurrences observed in real life can be substantiated by using the model. Fig 2 shows real data from Boiler 1 at Proserpine Mill for a 30 [min] period. It can be seen that positive changes in Steam Flow are correlated with positive changes in Drum Water Level and vice versa. It is hypothesised that this is a general fact that can be explained by the model. In the following, the model presented in this paper will be used to show that this hypothesis is, in fact, true. First it will be proven that the derivative of pressure is proportional to steam flow, then it will be proven that drum water level deviations are proportional to the derivative of pressure. Combining these two observations leads to the final conclusion that drum water level is indeed proportional to steam flow.

![Figure 2: Drum Water Level proportional to Steam Flow](image-url)
11.1.1 Derivative of Pressure is proportional to Steam Flow

Consider equation (30). It can be seen that equation (30) can be rewritten as:

\[
\dot{P} = -\lambda_1(P, M_s, M_w, V^T) \cdot q_s + \beta(P, M_s, M_w, V^T, q_f, Q^T)
\]  

(79)

where \(\lambda(\cdot), \beta(\cdot)\) are nonlinear functions. Therefore \(\dot{P}\) is proportional to \(q_s\).

**Remark 11.2.** Consider the following quantities for Boiler 1 evaluated at the nominal operating point:

\[
\begin{align*}
\frac{1}{\rho_w} &= 1.16 \cdot 10^{-3} \\
\frac{1}{\rho_s} &= 1.12 \cdot 10^{-1} \\
C_3 &= -6.16 \cdot 10^{-8}
\end{align*}
\]

(80) \hspace{1cm} (81) \hspace{1cm} (82)

It can be seen that, in equation (30), the coefficient multiplying \(q_s\) is at least 100 times larger than the others. This implies that \(q_s\) is the main factor affecting pressure changes. \(\square\)

11.1.2 Drum Water Level deviations are proportional to Derivative of Pressure

Next, consider equation (34) for the drum water level deviations \(\delta\) and equation (31) for the mass of steam below the water line \(M_{BW}^s\). It can be seen that \(\delta\) is proportional to \(M_{BW}^s\). Using Laplace transforms and a Padé approximation for the time delay, equation (31) leads to:

\[
s \cdot M_{BW}^s(s) = F_s(s) - e^{-as} F_s(s)
\]

\[
= \left(1 - \frac{2}{2 + as}\right) F_s(s)
\]

\[
= \frac{2as}{2 + as} F_s(s)
\]

where \(s\) is the Laplace Transform variable. Cancelling the \(s\) (derivative) on both sides of the above equation leads to:

\[
M_{BW}^s(s) = \frac{2a}{2 + as} F_s(s)
\]

(83)

Using the inverse Laplace transform yields:

\[
M_{BW}^s(t) = 2 \cdot \int_0^t f_s(\tau) \cdot e^{-\frac{2a}{2 + as}(t - \tau)} d\tau
\]

(84)

Because of the convolution with the exponential decay, the above equation can also be written as:

\[
M_{BW}^s(t) = 2f_s(t) + \lambda_2(f_s(t - \tau), \tau), \quad 0 \leq \tau < t
\]

(85)

where \(\lambda_2(\cdot)\) denotes the tail of the convolution integral. Hence, any change in \(f_s(t)\) will appear over a short interval in \(M_{BW}^s(t)\), i.e. they are proportional. Finally, consider equations (46) and (48) for the top section of the risers, i.e. \(i = n\). Then it can be seen that \(f_s(t)\) is proportional to \(f_w(t)\), and that \(f_w(t)\) is proportional to \(\dot{P}\).

In summary, Drum Water Level deviations are (approximately) proportional to the Derivative of Pressure.
11.1.3 Drum Water Level deviations are proportional to Steam Flow

The two facts established in the previous subsections have a major consequence, namely *Drum Water Level is proportional to Steam Flow*. This provides a physical explanation to the experimental results shown earlier in Fig.2.

11.2 Alpha is not a linear function of height in the risers under transient conditions

A common assumption in the literature is that the (mass) steam quality increases linearly with height in the risers at all times – see e.g. [4]. It will be shown below that, under transient conditions, such an assumption is not valid and in fact, leads to large errors.

Let \( h = 7 \) [m] be the height of the risers, and let the risers be divided in 7 sections. Let \( \alpha_k, \ k = 1, \ldots, 7 \) denote the (mass) steam quality in each of the \( k \) sections, where \( \alpha_1 \) corresponds to the section at the bottom of the risers and \( \alpha_7 \) to the section at the top.

Define the ratio \( \bar{\alpha}_k = \alpha_k / (k \ast \alpha_1) \) for \( k = 1, \ldots, 7 \). If the assumption that \( \alpha_k \) is linear with height, i.e. \( h/k \), were to be valid, then \( \bar{\alpha} \) would be equal to 1, \( \forall k \), and for all times.

The full model described in this paper was used to simulate the boiler response to the steam flow profile shown in Fig.4. Fig.3 shows \( \bar{\alpha}_k \) for \( k = 1, \ldots, 7 \), for the first 300 [s]. It can be seen that \( \bar{\alpha}_k = 1, \ k = 1, \ldots, 7 \) does not hold under transient conditions, although it does hold in steady state. Under transient conditions the discrepancy gets larger the further one moves up the risers. Indeed, at the top of the risers, there is an error of almost 50% at \( t = 210 \) [s] in the maximum steam quality predicted. Furthermore, the transient response persists for more than 20 [s] after a load change.

![Figure 3: Mass Steam Quality Ratio](image)

This is an important conclusion because the steam quality at the top of the risers is the main driving factor in the amount of water and steam entering the drum, and thus, it has a major impact on drum water level. A significant transient response such as the ones shown in Fig.3 cannot be ignored if the goal is to capture large and fast drum water level excursions.
11.3 DWL Simulation – Tracking fast changes

In this Section, the importance of equation (57) in describing fast Drum Water Level excursions will be illustrated. In particular, the contribution of a pressure derivative term is examined. To this end, first consider (57) and the expansion of the last term on the right hand side, i.e.:

\[ L DC \frac{d}{dt} \left\{ f_w^* \right\} = -k_w \frac{(f_w^*)^2}{\rho_w A_{DC}} - k_w \frac{f_w^2}{\rho_w A_R} - k_s \frac{f_s^2}{\rho_s A_R} + (M_w^{DC} - M_w^R - M_w^R)g + \Omega \]  

(86)

\[ \Omega = \frac{1}{A_R} \left\{ M_w \frac{f_w}{\rho_w} + M_w \frac{f_w}{\rho_w} - M_w \frac{f_w}{(\rho_w)^2} \cdot \frac{d\rho_w}{dP} \cdot \dot{P} + M_s \frac{f_s}{\rho_s} + M_s \frac{f_s}{\rho_s} - M_s \frac{f_s}{(\rho_s)^2} \cdot \frac{d\rho_s}{dP} \cdot \dot{P} \right\} \]  

(87)

Note the dependence of \( df_w^*/dt \) on \( \dot{P} \). The complete model, using the parameters shown in (78), will be used to simulate the Drum Water level response to a steam flow dataset obtained from Boiler 1 at Proserpine Mill. Fig. 4 shows this specific steam flow profile. It can be seen that large steam flow variations occur in a matter of seconds. In particular, at \( t = 200 \text{[s]} \) there is a 50\% spike in demand which occurs over a period of 10 [s].

![Figure 4: Steam Flow Dataset for simulation](image)

Fig. 5 shows a comparison between the real Drum Water Level response from Boiler 1 at Proserpine Mill and that predicted by the complete model using (86) with \( \Omega \) as in (87) and using \( \Omega = 0 \). The case \( \Omega = 0 \) is the result of a simple momentum balance approach. It can be seen that although not perfect, the model with \( \Omega \neq 0 \) performs significantly better at tracking the spike at \( t = 200 \text{[s]} \).

12 Implications for Boiler Control

This section briefly explores the impact that the new model has on boiler control architecture and tuning. A key observation is the one summarised in subsection 11.1.3 namely that drum water level deviations are proportional to steam flow.
This key observation has important implications for boiler control. This is because, if large Steam Flow fluctuations can be prevented from reaching the boiler, then the deviations in Drum Water Level can be greatly reduced, and consequently the number of boiler trips due to high or low water level can be significantly reduced. Indeed, controlling Drum Water Level under highly variable load conditions has been the main concern at Proserpine Mill. The insight provided in this section has proven crucial when developing a new controller.

In particular, the above observations imply that it is highly desirable to prevent large and rapid steam flow variations from reaching the boiler. To this end, two additional facts need to be taken into consideration, namely (i) the steam flow from the boiler into the steam receiver is proportional to the pressure difference, $\Delta P$, and is also dependant on the opening of the steam flow regulating valve, and (ii) the drum water level controller, which corrects drum water level perturbations by regulating the feedwater flow, is necessarily slow acting, e.g. ten times slower than the perturbations. With these two facts in mind, two scenarios arise:

1. When there is a sudden load increase, then pressure in the steam receiver will decrease. In turn, this means that $\Delta P$ will increase and thus the steam flow out of the boiler will also increase. An appropriate control response under these conditions is to quickly reduce the opening of the steam flow valve. Therefore the steam flow valve controller time constant must be of the same order as the time constant of the steam flow perturbations. The tradeoff associated with this is that there will be greater deviations in the steam receiver pressure.

2. When there is a sudden load decrease, then pressure in the steam receiver will increase. In turn, this means that $\Delta P$ will decrease (possibly to zero) and thus the steam flow out of the boiler will also decrease. Unlike the previous scenario, the opening of the control valve is ineffective as an appropriate control response, since no matter how open the valve is, the flow of steam is limited by $\Delta P$. Hence another approach is needed. One option is to use a let-down valve to release steam either to the atmosphere or other independent machinery. Two considerations must be made, namely (i) the letdown valve must be located as close as possible either to the source of the steam.
flow perturbation or to the steam receiver, and (ii) the time constant of the letdown valve controller must be of the same order as the time constant of the steam flow perturbations.

In conjunction, the two strategies mentioned above provide a viable strategy for reducing drum water level excursions due to steam flow variations.

13 Conclusions

This paper has described a new model for a Boiler operating under highly variable loads. The model is based on first principles. Significant departures have been made from the assumptions previously used in the literature. New features of the model include (i) a multi-compartment model for the risers, (ii) a new model for drum water level, and (iii) a new dynamic model for the flow of water in the downcomers. Simulations have been presented which (i) confirm the validity of the new model, and (ii) highlight the advantages of the new model under rapid load changing conditions. Implications of the model for boiler control have also been described with special emphasis on reducing drum water level excursions under large and rapid steam flow changes.

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