Kinetic gravity braiding and axion inflation

Debaprasad Maity\textsuperscript{a,b,}\textsuperscript{*}

\textsuperscript{a} Department of Physics and Center for Theoretical Sciences, National Taiwan University, Taipei 10617, Taiwan
\textsuperscript{b} Leung Center for Cosmology and Particle Astrophysics, National Taiwan University, Taipei 106, Taiwan

\textbf{A R T I C L E  I N F O}

Article history:
Received 26 December 2012
Received in revised form 4 February 2013
Accepted 12 February 2013
Available online 13 February 2013

Editor: T. Yanagida

\textbf{A B S T R A C T}

We constructed a new class of inflationary model with the higher derivative axion field which obeys constant shift symmetry. In the usual axion (natural) inflation, the axion decay constant is predicted to be in the super-Planckian regime which is believed to be incompatible with an effective field theory framework. With a novel mechanism originating from a higher derivative kinetic gravity braiding (KGB) of an axion field we found that there exists a huge parameter regime in our model where axion decay constant could be naturally sub-Planckian, thanks to the KGB which effectively reduces the Planck constant. This effectively reduced Planck scale provides us the mechanism of further lowering down the speed of an axion field rolling down its potential without introducing super-Planckian axion decay constant. We also find that with that wide range of parameter values, our model induces almost scale invariant power spectrum as observed in CMB experiments.

© 2013 Elsevier B.V. Open access under CC BY license

Inflation is an exponential expansion phase of our universe in its very early stage of evolution. Even though this is by far the only successful mechanism to solve several problems in standard Big-Bang model of the universe, we still do not have a fundamental theory which leads to such a mechanism. In order to realize such an exponential expansion, often a scalar field is invoked with an unnaturally flat potential which has already been proved to be very difficult to construct in the quantum field theory framework. However it has been well accepted that shift symmetry plays a very crucial role in the inflationary dynamics. It is this symmetry which keeps the potential sufficiently flat to realize inflation. In this respect usual Standard Model of particle physics could still be a natural framework to study inflation. A pseudo scalar field called axion may play an important role in this regard. This is a hypothetical field associated with the Peccei–Quinn symmetry which has been introduced to solve the strong CP problem in QCD in Standard Model of particle physics. This axion field obeys shift symmetry. By using this axionic shift symmetry a “natural” inflation has been proposed in [1]. In spite of its viability, observation suggests that axion decay constant should be $f \gtrsim 3M_p$. Question has been raised on this large axion decay constant. We also find that with that wide range of parameter values, our model induces almost scale invariant power spectrum as observed in CMB experiments.

\textsuperscript{*} Correspondence to: Department of Physics and Center for Theoretical Sciences, National Taiwan University, Taipei 10617, Taiwan.
E-mail address: debu.imsc@gmail.com.

\textsuperscript{[2]} One of the interesting properties of this scenario have been made [4]. Very recently some viable phenomenological extensions of this natural inflation with the sub-Planckian axion decay constant have also been proposed [5,6] which leads to the resurgence of interest in this subject. In this Letter we will construct another viable model of axion inflation which is relying on the higher derivative kinetic gravity braiding. There have been a lot of studies based on this kind of model in the context of inflation which goes by the name of G-inflation [7], and also in the context of dark energy mode building [8,9]. A very similar approach with a non-minimally coupled UV-protected inflationary model of axion field has also been proposed [6].

We will very closely follow those constructions in this Letter. The essential mechanism which has already been pointed out in [7] is that the kinetic braiding parameter is playing the role of flattening the potential in certain region of parameter space. We will see that in that range of parameter space we can make our axion decay constant $f$ to be sub-Planckian by appropriately choosing another sub-Planckian scale $s$ associated with the kinetic gravity braiding (KGB) of our model.

We start with the following action

$$L = \frac{M_p^2}{2}R - X - M(\phi)X\Box\phi - \Lambda^4\left(1 - \cos\left(\frac{\phi}{f}\right)\right)$$

(1)

where $X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ and $\Box = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu)$. $f$ is the axion decay constant. $\Lambda$ is related to the axionic shift symmetry breaking scale. We call the term associated with the higher derivative action as KGB following [8]. One of the interesting properties of this
higher derivative term is that it does not lead to any unwanted degrees of freedom.

Assuming the usual FRW Metric ansatz
\[ ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \]
we obtain the following Einstein equation for the scale factor \( a \)
\[ H^2 = -\frac{\dot{H}}{M^3}M^2(\phi) + \frac{X^2}{2}M'(\phi) + \frac{A^4}{M^2} \left( 1 - \cos \left( \frac{\phi}{f} \right) \right), \]
and for the axion field
\[ \frac{1}{\alpha^2} \frac{d}{dt} \left[ \alpha^2 \left( 1 - 3HM\dot{\phi} - 2M'X \right) \phi \right] = \partial^\mu \phi \partial_\mu (M'X) - \frac{A^4}{f} \sin \left( \frac{\phi}{f} \right). \]

where \( H = \dot{a}/a \) is the Hubble constant.

Following Ref. [7] if we consider slow roll condition, the scalar field equation turns out to be
\[ 3H\dot{\phi} \left( 1 - 3M(\phi)H\phi \right) + \frac{A^4}{f} \sin \left( \frac{\phi}{f} \right) = 0. \]
We assume that the inflation is driven by the KGB such that the function \( M(\phi) \) satisfies \( |M(\phi)H\phi| \gg 1 \). This condition will lead us to
\[ \tau = \frac{M(\phi)A^4}{f} \sin \left( \frac{\phi}{f} \right) \gg 1. \]

Once the above condition is satisfied, the expressions for slow roll parameters turn out to be
\[ \epsilon = \frac{M_p^2}{2f^2 \sqrt{\tau} (1 - \cos (\frac{\phi}{f}))^2}, \quad \eta = \frac{M_p^2}{2f \sqrt{\tau} (1 - \cos (\frac{\phi}{f}))}, \quad \alpha = \frac{M_p^2}{36 f} M' \left( \frac{4\epsilon^2}{\tau} \right)^{\frac{1}{2}}, \quad \beta = \frac{M_p^2}{M} \left( \frac{4\epsilon^2}{\tau} \right)^{\frac{1}{2}}. \]

As one can see from the above expressions for the slow roll parameters that KGB function \( M(\phi) \) flattens the axion potential in term of \( \tau \). As we will see this particular novel effect of KGB will help us to lower the axion decay constant \( f \) into the sub-Planckian regime. The condition equation (7) with \( \sin \left( \frac{\phi}{f} \right) \) function also tells us that in order to maintain those slow roll conditions inflation driven by KGB has to happen not very close to the maximum of the potential but little away from the maximum such that \( \sin \left( \frac{\phi}{f} \right) \simeq O(1) \). We will see in our subsequent analysis that this is indeed the case.

Keeping in mind the periodic nature of the potential we will study the following two different choices of braiding functions.

**Model-I**: For \( M(\phi) = \frac{1}{\tau} \), where for our subsequent discussion we fix \( s > 0 \) and call it as our new KGB scale, we get
\[ \tau_1 = \tau_0 \sin \left( \frac{\phi}{f} \right) \gg 1, \quad \alpha_1 = 0, \quad \beta_1 = 0, \]
where we define \( \tau_0 = \frac{A^4}{s^3 f} \). This also says that with this particular choice, inflation driven by KGB happens in region I of the potential as shown in Fig. 1. As one can see in this region of the potential speed of axion field \( \phi < 0 \). This could further be checked by doing perturbation analysis [7] that the solution in this region is also stable with the stability condition \( M\dot{\phi} \simeq \phi < 0 \).

**Model-II**: On the other hand if we consider \( M = \frac{1}{\tau^3} \sin \left( \frac{\phi}{f} \right) \), we get
\[ \tau_1 \approx \tau_0 \approx \tau_0 \approx \frac{A^4}{s^3 f} \gg 1 \quad \Rightarrow \quad s \ll \left( \frac{A^4}{\tau} \right)^{\frac{1}{3}}. \]

So we have enough region of the parameter space where the axion decay constant could be sub-Planckian along with the KGB scale. In the subsequent analysis we will derive this scale dependence more rigorously taking into account the dynamics of cosmological perturbations.
The region of parameter space could be constrained by the dynamics of fluctuations of the axion field which is directly related to the CMB power spectrum $P_R$ associated with curvature perturbation $\mathcal{R}$ and spectral index $n_s$. The expression for those quantities can be straightforwardly calculated as [7]

$$P_R = \frac{3\sqrt{6}}{64\pi^2} \frac{H^2}{M_\phi^2} e^{-\alpha}, \quad n_s = 1 - 6\epsilon + 3\eta + \frac{\alpha}{2}. \quad (11)$$

Now let us study the slow roll parameters with respect to both models that we have discussed above. For both the models the explicit form of the spectral index turns out be

$$n_I' \simeq 1 - \frac{3}{2} \frac{\sin(\phi_f/2)}{A(1 - \cos(\phi_f/2))^2} + \frac{3}{2} \frac{\cos(\phi_f/2) \cot(\phi_f/2)}{A(1 - \cos(\phi_f/2))^2},$$

$$n_I'' \simeq 1 - \frac{3}{2} \frac{\sin(\phi_f/2)}{A(1 - \cos(\phi_f/2))^2} \frac{\cos(\phi_f/2)}{2A(1 - \cos(\phi_f/2))}$$

$$- \frac{1}{2\sqrt{A}} \frac{\cot(\phi_f/2)}{\sqrt{2}\epsilon_f}.$$

For our future convenience we have defined a new constant $A = \sqrt{10} (f/M_p)^2$ in the above expressions. One can clearly see from the expression of $A$ that usual spectral tilt $n_s - 1$ of axion (natural) inflation is reduced by a factor of $\sqrt{10}$ or in other words it essentially suppresses the Planck scale. We will see that this effectively reduced Planck scale is playing the main role in bringing down the axion decay constant $f$ to be in the sub-Planckian regime.

In order to solve the homogeneity and flatness problem of the usual Big-Bang model, we need to have sufficient amount of inflation. This sufficient inflation is measured by the so-called e-folding number $\mathcal{N} = \int \frac{H dt}{H}$. From the current cosmological observations the constraint on $\mathcal{N} \approx 60$. So further constraint on our model parameters will come from this e-folding number. The analytic expressions for $\mathcal{N}$ for both the models under consideration turn out to be

$$\mathcal{N}_I = -A \left(2\sqrt{\sin x} + \sqrt{2}\sqrt{1 + \sin x} \frac{\cos x}{\sqrt{2} + \sin x}\right) \times \text{EllipticF} \left[\sin^{-1} \left(\cos \left(\frac{x}{2} - \sin \frac{x}{2} \frac{1}{2}\right)\right), 1\right]|_{x_1},$$

$$\mathcal{N}_{II} = A(x - \sin x)|_{x_1} x_2,$$

where we define $x = \phi_x/f$. In the above expressions, the upper limit on the axion field $x_2 = \phi_2/f$ will come from the slow roll parameter. As one can imagine that inflation ends when our slow roll parameter $\epsilon = 1$ which provides us the upper limit. Furthermore if we set $\mathcal{N} \approx 60$, we can constrain the parameter space of $(A, f)$. This in turn will constrain the value of the spectral index. In Table 1 we provide some possible numerical values of $A$ for which we found the values of $(\phi_1/f, \phi_2/f, n_s)$ for both the models. One can see that the values so obtained for the spectral index $n_s$ are close to the observed value from WMAP.

Now according to WMAP observations, considering the expression for Model-I, we know

$$P_I = \frac{A \sqrt{6}}{32\pi^2} \left(\frac{A}{M}\right)^4 \frac{(1 - \cos x_1)^3}{\sin x_1^2} \simeq 2.4 \times 10^{-9}. \quad (13)$$

For a fixed value of $A$ the above Eq. (13) can further provide us a constraint on the value of the axionic symmetry breaking scale $A$. As for example if we consider $A = 65$, from the above expression (13) we find $A_I = 5.08 \times 10^{-3}$ in Planck units for Model-I. With this value of $A_I$ one can choose one set of values for $(f_1, s_1) = (10^{-2}, 1.634 \times 10^{-5})$ in Planck units such that all the above bounds are satisfied with the cosmological observations. A similar estimate can be done for Model-II where we found $(A_{II}, f_2, s_2) = (4.99 \times 10^{-3}, 10^{-2}, 1.136 \times 10^{-5})$ in Planck units. So one can clearly see that axion decay constant $f$ as well as the KGB scaling parameter $s$ simultaneously could be several orders of magnitude lower than the Planck mass in order to meet observational constraints. In addition another interesting outcome of our construction is that we are getting sufficient amount of inflation with the value of axion field well below the Planck mass as well. For example with the above choices of parameters we have $\phi_1^f \approx \phi_2^f \approx 1.9 \times 10^{-2}$ in Planck units. Above estimation depends on particular choice of parameters $(f, s)$. In principle one has a large number of choices for $(f, s)$ as

$$f^3 = \frac{1}{A^2} \frac{M_p^2}{A^4}. \quad (14)$$

In order to totally fix our model parameters we need to have one more observable quantity which has non-trivial dependence on the axion decay constant $f$. For this $\phi$-gaussianity would be one of the interesting observables which we defer for our future study. However with the above derived constraint in what follows we will discuss about another cosmological observable quantity related to tensor perturbation. As one can see from the equation below, once we fix the value of $A$ and scalar spectral index $n_s$, it also fixes the tensor spectral index $n_T$ as well as tensor-to-scalar ratio as follows [7]

$$n_T = -\frac{1}{A} \frac{\sin(\phi_f/2)^3}{(1 - \cos(\phi_f/2))^2},$$

$$r = \frac{32\sqrt{6}}{9} \left[n_T^2, n_T^2\right] = (0.0757, 0.0773). \quad (15)$$

So, the value of tensor-to-scalar ratio $r$ for both the models is very small to be detectable in near future. We have also checked that as we increase the value of $A$, $r$ also increases. Important point to note is that perturbation in the tensor sector does not provide us further constraint on our model parameters. We, therefore, are left with one free parameter which can probably be fixed if we go beyond the linear cosmological perturbation theory.

In this Letter we have discussed a new model of axion inflation which includes a specific form of higher derivative terms consistent with the shift symmetry. Our model is strongly motivated by the recent studies on galileon scalar field theory first introduced in [10]. The specific form of the higher derivative term called kinetic gravity braiding is playing the crucial role in our model. Interesting point to note is that this particular form of higher derivative term has a property that it does not introduce any ghost which generally appears in a higher derivative theory. We have seen that this particular form of higher derivative term helps us to construct
a successful axion inflation model with sub-Planckian axion decay constant. One of the main problems in a standard axion (natural) inflation model is that the axion decay constant \( f \) turned out to be above the Planck scale in order to meet CMB observations. Throughout our current analysis we have shown that this problem can be easily circumvented by introducing a higher derivative so-called KGB term in the action for an axion field. This particular KGB term is playing the role in pushing the axion decay scale \( f \) into the sub-Planckian regime. The physical reason behind this mechanism is coming from the fact that KGB parameter effectively reduces the Planck constant which in turn makes the speed of the rolling axion field along its potential slower. According to our model we also find a huge parameter region where inflation driven by KGB occurs with almost scale invariant power spectrum which has already been observed in the WMAP experiment.

We also would like to stress upon the fact that in the linear regime of cosmological perturbation theory, we could not constrain all our model parameters. What we infer from our analysis is that once we fix the value of our combined parameter \( \Lambda \) from the observed spectral index \( n_s \geq 0.964 \) and axion shift symmetry breaking scale \( \Lambda \) from the observed amount of primordial fluctuations \( \sqrt{f R} \approx 10^{-5} \), we can fix only the ratio \( s/f \) from Eq. (14) of two scales. So we can clearly see a huge range of values for \( f/s \) where both are sub-Planckian. Interestingly enough we obtain sufficient inflation with the axion field value well below the Planck constant. These are our main results in this Letter. In order to further constrain the parameters we need to go beyond the linear regime like non-gaussianity would be one of such effects. We defer this analysis for our future study. Another interesting effect that could potentially constrain our model is reheating after the inflation. During our study we overlooked Ref. [11], in which a similar mechanism is coming from the fact that KGB parameter effectively could potentially constrain our model is reheating after the inflation, which may ruin our conclusion. One of our concerns is that the conclusion derived in [11] is based on a particular choice of KGB function. It would be interesting to check for the other choices such as the function we have studied in the current Letter. Detailed study of reheating is beyond the scope of our current Letter. There also exist several other possible mechanisms of reheating which are worth studying in this scenario if this model is severely constrained by itself from reheating. Two of the scenarios are curvaton [12] and modulated reheating [13]. In both scenarios, there exist additional light degrees of freedom in addition to the inflaton, which play important role in the reheating process after the inflation. We keep this for our future studies in detail.

Acknowledgements

I particularly thank our “Sting Cosmology Group” members for various stimulating discussions related to this subject.

References

[1] K. Freese, et al., Phys. Rev. Lett. 65 (1990) 3233;
K. Freese, W.H. Kinney, Phys. Rev. D 70 (2004) 083512.
[2] T. Banks, et al., JCAP 0306 (2003) 001.
[3] R. Kallosh, et al., Phys. Rev. D 52 (1995) 912.
[4] M. Kawasaki, et al., Phys. Rev. Lett. 85 (2000) 3572;
N. Arkani-Hamed, et al., Phys. Rev. Lett. 90 (2003) 221302;
N. Arkani-Hamed, et al., JCAP 0307 (2003) 003;
D.E. Kaplan, N. Weiner, JCAP 0402 (2004) 005;
H. Firouzjahi, S.J.H. Tye, Phys. Lett. B 584 (2004) 147;
J.P. Hsu, R. Kallosh, JHEP 0404 (2004) 042.
[5] J.E. Kim, et al., JCAP 0501 (2005) 005;
N. Barnaby, M. Peloso, Phys. Rev. Lett. 106 (2011) 181301;
E. Silverstein, A. Westphal, Phys. Rev. D 78 (2008) 106003;
P. Adshead, M. Wyman, Phys. Rev. Lett. 108 (2012) 261302.
[6] C. Germani, A. Kehagias, Phys. Rev. Lett. 106 (2011) 161302.
[7] T. Kobayashi, et al., Phys. Rev. Lett. 105 (2010) 231302;
K. Kamada, et al., Phys. Rev. D 83 (2011) 083515;
T. Kobayashi, et al., Phys. Rev. D 83 (2011) 103524;
T. Kobayashi, et al., Progr. Theoret. Phys. 126 (2011) 511;
K. Kamada, et al., Phys. Rev. D 86 (2012) 023504.
[8] C. Deffayet, et al., JCAP 1010 (2010) 026.
[9] A.D. Felice, S. Tsujikawa, Phys. Rev. Lett. 105 (2010) 111301;
R. Gannouji, M. Sami, Phys. Rev. D 82 (2010) 024011.
[10] A. Nicolis, et al., Phys. Rev. D 79 (2009) 064036.
[11] J. Ohashi, S. Tsujikawa, arXiv:1207.4879 [gr-qc].
[12] S. Mollerach, Phys. Rev. D 42 (1990) 313;
A.D. Linde, V.F. Mukhanov, Phys. Rev. D 56 (1997) 535;
T. Moroi, T. Takahashi, Phys. Lett. B 522 (2001) 215;
T. Moroi, T. Takahashi, Phys. Lett. B 539 (2002) 303 (Erratum);
D.H. Lyth, D. Wands, Phys. Lett. B 524 (2002) 5;
K. Enqvist, M.S. Sloth, Nuclear Phys. B 626 (2002) 395.
[13] G. Dvali, A. Gruzinov, M. Zaldarriaga, Phys. Rev. D 69 (2004) 023505;
G. Dvali, A. Gruzinov, M. Zaldarriaga, Phys. Rev. D 69 (2004) 083505;
L. Hofman, arXiv:astro-ph/0306141.