Robust Transit Network Design with Stochastic Demand Considering Development Density

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Abstract

This paper analyzes the influence of urban development density on transit network design with stochastic demand by considering two types of services, rapid transit services, such as rail, and flexible services, such as dial-a-ride shuttles. Rapid transit services operate on fixed routes and dedicated lanes, and with fixed schedules, whereas dial-a-ride services can make use of the existing road network, hence are much more economical to implement. It is obvious that the urban development densities to financially sustain these two service types are different. This study integrates these two service networks into one multi-modal network and then determines the optimal combination of these two service types under user equilibrium (UE) flows for a given urban density. Then we investigate the minimum or critical urban density required to financially sustain the rapid transit line(s). The approach of robust optimization is used to address the stochastic demands as captured in a polyhedral uncertainty set, which is then reformulated by its dual problem and incorporated accordingly. The UE principle is represented by a set of variational inequality (VI) constraints. Eventually, the whole problem is linearized and formulated as a mixed-integer linear program. A cutting constraint algorithm is adopted to address the computational difficulty arising from the VI constraints. The paper studies the implications of three different population distribution patterns, two CBD locations, and produces the resultant sequences of adding more rapid transit services as the population density increases.

Keywords: transit network design; robust; stochastic demand; population density

1. Introduction

The Transit Network Design Problem (TNDP) is to decide the locations of stations, route alignment as well as...
frequency to serve the travel demands between specific origin-destination (OD) pairs. Due to high construction and
operating costs of rapid transit line (RTL), some lines may face low passenger load, and some may even require
government subsidy for their operations. Indeed, the population density has a great influence on the sustainability of
RTL. The government thus must be prudent in developing RTL and the sequence in constructing different lines to
cope with population increases. For a newly developed region, the initial residential density may not be sufficient to
support a RTL. Even for regions with high population densities, the travel demand may fluctuate from day to day,
making it uneconomical to rely on RTL alone to serve the demand. Dial-a-ride (DAR) services, in contrast, are able
to utilize the existing road network, thus having relatively lower capital costs, mainly involving the procurement,
operations and maintenance of the vehicle fleet. Meanwhile, they have great flexibility to cater for demand
fluctuation. However, the congestion effect of dial-a-ride services cannot be neglected. Thus, it may not be
economical and environmentally efficient to rely heavily on dial-a-ride services for areas with a large population
producing relatively stable travel demands. The goal of this study is to find out the critical development density
when RTL is to be first built and the construction phases as the population density gradually increases.

The Network Design Problem (NDP) can be classified into discrete, continuous and mixed, as discussed in Yang
and Bell (1998). The Discrete NDP is concerned with the network topology itself (Wang et al., 2013, Gao et al.,
2005, Lai and Lo, 2004). Examples include scheduling or routing of a service network. The Continuous NDP takes
the network topology as given and is concerned with optimizing the network parameters. Examples include
enhancing the link capacity or setting the toll charges (Gao et al., 2004, Ekström et al., 2012). The Mixed NDP
combines the two types to simultaneously determine new links to be added and capacity increases of existing roads
(Luathep et al., 2011). The transit network design problem falls in the category of Mixed NDP which involves
determining discrete and continuous variables, namely, transit line alignment and frequency. Most existing studies
focus on the deterministic TNDP, assuming that the OD demand is fixed and known. It is typically formulated as a
mixed integer linear program (MILP) where the station selection, line alignment and frequencies are determined
simultaneously to achieve a certain objective, such as cost minimization or coverage maximization (Wan and Lo,
2009, Bruno et al., 1998).

The literature on NDP concerning uncertain demand can be classified into two categories. The first approach is
stochastic programming, which assumes known demand distributions and utilizes Monte-Carlo simulation to
decompose the random demands into a finite number of scenarios for approximating the cost expectation, and is
formulated as a MILP to minimize the total expected cost (Ruszczynski, 2008, Birge and Louveaux, 1988, Benders,
1962) and solved by a commercial software, such as CPLEX, or the L-shaped method. An and Lo (2014a, 2014b)
proposed an alternative method, namely, the service reliability (SR)-based approach, which separates the large-size
MILP into two phases for solution efficiency. The second approach is robust optimization, which focuses on the
min-max problem, namely, optimization of the worst case scenario. It requires that the network design solutions,
determined before the demand realization, are feasible for all demand realizations. The side effect is that the
solutions may be overly conservative. Some studies thus turned to refining the uncertainty set such that all the
realized demand within the set are satisfied while those outside are ignored. It is important to trade off the size of the
uncertainty set and the robustness level (Bertsimas and Sim, 2004, Ben-Tal and Nemirovski, 1999).

The aforementioned studies generally specified the OD demand to be satisfied, either deterministic or stochastic
(Ben-Tal et al., 2011, Wan and Lo, 2009), only a few traced back to the urban development density which generates
the OD demands in the first place. Samanta and Jha (2011) proposed a rail transit line model considering different
objectives such as ridership maximization or user cost minimization. Laporte et al. (2007, 2005) integrated trip
generation, trip distribution and mode choice into the transit network design problem to produce OD demands for
each transit mode. Quadrifoglio and Li (2010, 2009) investigated the feeder transit design problem to find the
critical population density for fixed and demand responsive services. Although these studies somewhat examined the
relationship between population density and OD demands on network design (Samanta and Jha, 2011, Laporte et al.,
2005, Li and Quadrifoglio, 2010), they did not consider the inherent OD demand fluctuation.

In addition to the challenge of including stochastic demand and development density simultaneously, this study
also incorporates user equilibrium (UE) passenger flows in a multi-modal transit network. The NDP with UE flows
is typically formulated as a bi-level problem, in which the upper level problem focuses on generating the optimal
network design; whereas the lower level represents travelers’ travel choices. This bi-level problem is typically non-
linear and non-convex. Some studies formulate the UE principle as variational inequality (VI) constraints, which
