An adaptive routing strategy for packet delivery in complex networks

Huan Zhang,1 Zonghua Liu,1 Ming Tang,1 and P. M. Hui2

1Institute of theoretical physics and Department of Physics, East China Normal University, Shanghai, 200062, P. R. China
2Department of Physics, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong

(Dated: 25 June)

We present an efficient routing approach for delivering packets in complex networks. On delivering a message from a node to a destination, a node forwards the message to a neighbor by estimating the waiting time along the shortest path from each of its neighbors to the destination. This projected waiting time is dynamical in nature and the path through which a message is delivered would be adapted to the distribution of messages in the network. Implementing the approach on scale-free networks, we show that the present approach performs better than the shortest-path approach and another approach that takes into account of the waiting time only at the neighboring nodes. Key features in numerical results are explained by a mean field theory. The approach has the merit that messages are distributed among the nodes according to the capabilities of the nodes in handling messages.

PACS numbers: 89.75.Fb,89.20.-a,05.70.Jk

I. INTRODUCTION

The problem of traffic congestions in communication networks is undoubtedly an important issue. The problem is related to the geometry of the underlying network, the rate that messages are generated and delivered, and the routing strategy. Many studies have been focused on spatial structures such as regular lattices and the Cayley tree [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Random networks and scale-free (SF) networks have also been widely studied. The former is homogeneous with a Poisson degree distribution; while the latter typically exhibits a power-law degree distribution of the form $P(k) \sim k^{-\gamma}$ signifying the existence of nodes with large degrees. SF networks are found in many real-world networks, such as the Internet, World Wide Web (WWW), and metabolic network [11, 12, 13]. A standard model of SF networks is the Barabási and Albert (BA) model of growing networks with preferential attachments [14, 15]. The BA model gives a degree distribution of $P(k) \sim k^{-3}$ and is non-assortative [16, 17], i.e., the chance of two nodes being connected is independent of the degrees of the nodes concerned. While there are other variations on the BA models that give a degree exponent that deviates from 3 [18, 19, 20, 21, 22], the BA model still serves as the basic model for SF networks. In the present work, we study how a dynamical and adaptive routing strategy would enhance the performance in delivering messages over a BA scale-free network.

In communication models, the nodes are taken to be both hosts and routers, and the links serve as possible pathways through which messages or packets are forwarded to their destination. Early studies assumed a constant (degree-independent) packet generation rate $\lambda$ and a constant rate of delivering one packet per time step at each node. As $\lambda$ increases, the traffic goes from a free-flow phase to a congested or jamming phase. Obviously, such models are too simple for real-world networks. More realistic models should incorporate the fact that nodes with higher degrees would have higher capability of handling information packets and at the same time generate more packets. Models with degree-dependent packet-delivery rate of the form $(1+\beta k_i) \lambda_i$ [23, 24] and degree-dependent packet-generating rate [24] of the form $\lambda_i k_i$ have recently been proposed and studied, with routing strategy based on forwarding messages through the shortest path to their destination. Implementing this routing strategy in SF and random networks indicate that it is easier to lead to congestions in SF networks than in random networks [23, 24]. It is because the nodes with large degrees in SF networks are on many shortest-paths between two arbitrarily chosen nodes, i.e., large betweenness [25]. Many packets will be passing by and queuing up at these nodes enroute to their destination. In a random network, jamming is harder to occur as the packets tend to be distributed quite uniformly to each node.

A good routing algorithm is essential for sustaining the proper functioning of a network [26, 27, 28, 29]. The shortest-path routing approach is based on static information, i.e., once the network is constructed, the shortest-paths are fixed. To improve routing efficiency, Echenique et al. [30, 31] proposed an approach in which a node would choose a neighboring node to deliver a packet by considering the shortest-path from the neighboring node to the destination and the waiting time at the neighboring node. The waiting time depends on the number of packets in the queue at a neighboring node at the time of decision and thus corresponds to a dynamical or time-dependent information. This algorithm performs better than the shortest-path approach, as packets may be delivered not necessarily through the shortest-path and thus the loading at the higher degree nodes in a SF network is reduced. The approach has also been applied to networks with degree-dependent packet generation rate [32]. Recently, Wang et al. [33] proposed an algorithm that tends to spread the packets evenly to nodes by considering information on nearest neighbors. However, the delivering time turns out to be much longer than that in the shortest-path approach as the packets tend to wander around the network.

In the present work, we propose an efficient routing strategy that is based on the projected waiting time along the shortest-
path from a neighboring node to the destination. The algorithm is implemented in BA scale-free networks, with degree-dependent packet generating and delivering rates. Results show that jamming is harder to occur using the present strategy, when compared with both the shortest-path approach and the Echenique’s approach. Key features observed in numerical results are explained within a mean field treatment. The present approach has the advantage of spreading the packets among the nodes according to the degrees of the nodes. In this way, every node can contribute to the packet delivery process.

The paper is organized as follows. The model, including the underlying network, the packet generation and delivery mechanisms, and routing strategy, is introduced in Sec.II. In Sec.III, we present numerical results and compared them with those of the other routing strategies. We also explain key features within a mean field theory. We summarize our results in Sec.IV.

II. MODEL

The underlying network structure is taken to be the Barabasi-Albert (BA) scale-free growing network with \( N \) nodes [15]. Starting with \( m_0 \) nodes, each new node entering the network is allowed to establish \( m \) new links to existing nodes. Preferential attachment whereby an existing node \( i \) with a higher degree \( k_i \) has a higher probability \( \Pi_i \sim k_i \) to attract a new link is imposed. The mean degree of the network is \( \langle k \rangle = 2m \) and the degree distribution \( P(k) \) follows a power-law behavior of the form \( P(k) \sim k^{-3} \).

The dynamics of packet generation and delivery is implemented as follows. Due to the inhomogeneous nature of the BA network, it is more natural to impose a packet generation rate that is proportional to the degree of a node. At each time step, a node \( i \) creates \( \lambda k_i \) new packets. The fractional part of \( \lambda k_i \) is implemented probabilistically. A destination is randomly assigned to each created packet. The newly created packets will be put in a queue at the node and delivery will be made on the first-in-first-out basis. The packets in the queue may consist of those which are created at previous time steps and received from neighboring nodes enroute to their destination. We also assume a packet delivery rate that is proportional to the degree of a node [24]. At each time step, a node \( i \) delivers at most \( (1 + \beta k_i) \) packets to its neighbors. The fractional part of \( (1 + \beta k_i) \) is implemented probabilistically. A larger \( \beta \) implies a higher packet-handling capability, but it would translate into higher cost or capital. Here, the parameters \( \lambda \) and \( \beta \) are taken to be node-independent. A packet is removed from the system upon arrival at its destination. For a given generation rate characterized by \( \lambda \), there exists a critical value of the delivery rate \( \beta_c \) such that for delivery rates \( \beta < \beta_c \), packets tend to accumulate in the network resulting in a jamming phase; while for \( \beta > \beta_c \), a non-jamming phase results as there are as many packets delivered to their destination as created. A better performance is thus characterized by a smaller value of \( \beta_c \).

The novel feature of the present work is the routing strategy or the selection of a neighbor in delivering a packet. The idea is to choose a neighbor that would give the shortest time, including waiting time, to deliver the packets along the shortest path from the chosen neighbor to the destination. Consider a packet with destination node \( j \) leaving node \( i \). Each of the \( k_i \) neighbors of node \( i \) has a shortest path to the destination node \( j \). The shortest path refers to the smallest number of links from a node to another. However, due to the possible accumulation of packets at each node, the number of time steps it takes to deliver the message may be different from the number of links along the shortest path. Consider a neighbor labelled \( \ell \) of the node \( i \). We label the shortest path from node \( \ell \) to \( j \) by \( \{SP : \ell, j\} \). Along this path, we evaluate the following quantity for the node \( \ell \):

\[
d(\ell) = \sum_{s \in \{SP : \ell, j\}} \frac{n_s}{1 + \beta k_s},
\]

where the sum is over the nodes along the shortest path \( \{SP : \ell, j\} \), excluding the destination. Here, \( n_s \) is the number of packets accumulated at node \( s \), at the moment of decision. Thus, \( d(\ell) \) is an estimate of the time that a packet would take to go from node \( \ell \) to the destination \( j \) through the shortest path. Node \( i \) would choose a neighboring node with the minimum \( d(\ell) \) to forward the packet, i.e., the selection is based on \( \min_{\ell \in \{i\}}\{d(\ell), \ell \in \{i\}\} \), where \( \{i\} \) is the set of \( k_i \) nodes consisting of the neighbors of node \( i \). This procedure is repeated for each node and each packet in every time step. For a network far from jamming, each node can handle all the packets in every time step. In this free-flow situation, the quantity \( d(\ell) \) simply measures the shortest path \( d_{\ell,j} \) from \( \ell \) to \( j \). When packets are queueing up at the nodes, however, a delivery mechanism based on \( d(\ell) \) takes into account of the queuing time and may not pass the packet to a neighboring node that is closest to the destination.

To justify our routing scheme, we will compare results with two other routing strategies widely studied in the literature. Using the same packet generating mechanism, the shortest-path approach selects a neighbor with the shortest path to the destination for forwarding a packet. Echenique et al. [30, 31] proposed an approach that takes into account of the waiting time. For a delivering rate of one packet per time step, they proposed to choose a neighbor that has a minimum value of \( h d_{\ell,j} + (1 - h) n_{\ell} \), where \( d_{\ell,j} \) is the shortest path length from node \( \ell \) to \( j \). The parameter \( h \) is a weighing factor, which can be taken as a variational parameter and \( h \approx 0.8 \) is found to give the best performance. The Echenique’s approach thus accounts for the waiting time only at the neighboring nodes. For a delivery rate of \( (1 + \beta k_i) \), a modified Echenique’s approach is to choose a neighboring node with a minimum value of \( \delta_{\ell} = h d_{\ell,j} + (1 - h) n_{\ell} / (1 + \beta k_{\ell}) \).

We have checked that for a given value of \( \lambda \), the smallest value of \( \beta_c \) is attained for values of \( h \approx 0.8 \). In what follows, we will use a value of \( h = 0.8 \) for the Echenique’s approach given by Eq.(2).
FIG. 1: (a) Numerical results for the average number of packets per node \( \langle n(t) \rangle \) and (b) the average delivering time \( \langle T \rangle \) as a function of time for \( \lambda = 0.02 \). Lines from top to bottom correspond to \( \beta = 0.04, 0.048 \) and 0.07.

### III. Results and Discussion

The different phases in a network can be illustrated by looking at the average number of packets per node at a given time \( \langle n(t) \rangle \) and the average time for a packet to remain in the network or the delivering time \( \langle T \rangle \). We take \( m_0 = 3 \) and \( m = 3 \) and construct a BA scale-free network of \( N = 1000 \) nodes. Figure 1 shows the results of \( \langle n(t) \rangle \) and \( \langle T \rangle \) as a function of time for a fixed value of \( \lambda = 0.02 \). As \( \beta \) increases, there are distinct behavior. For values of \( \beta \) smaller than some critical value \( \beta_c(\lambda) \), \( \langle n(t) \rangle \) grows almost linearly with time after the transient (see Fig 1(a)). This corresponds to a jamming phase. As \( \beta \) increases, the slope in the long time behavior decreases, indicating a slower accumulation of packets in the network as the ability of handling packets \( \beta \) increases. For \( \beta > \beta_c(\lambda) \), \( \langle n(t) \rangle \) becomes independent of time in the long time limit. This corresponds to a non-jamming phase. Similarly behavior is exhibited in \( \langle T \rangle \). In the jamming phase, \( \langle T \rangle \) increases with time monotonically, due to the increasing waiting time in the queues at intermediate nodes as a packet is forwarded to its destination. Fewer packets are delivered to their destination than generated. In the non-jamming phase, \( \langle T \rangle \) becomes independent of time in the long time limit. In this regime, further increasing \( \beta \) will lead to smaller \( \langle n(t) \rangle \) and shorter \( \langle T \rangle \) in the long time limit until these quantities saturate. This is possible since a non-jamming phase corresponds to the case in which all the packets at the nodes are forwarded every time step or steady queues of packets exist at the nodes. In both cases, the number of packets does not increase in the long time limit. The former case is the free-flow phase, while the latter is reminiscent of the synchronized phase in vehicular traffic flows \cite{34} in which the packets undergo a stop-and-go behavior. For \( \beta = 0.07 > \beta_c \), for example, \( \langle T \rangle \approx 9.5 \), which is somewhat larger than the average shortest distance or diameter \( D \approx 3.332 \) of the network. This indicates that, due to the routing strategy in forwarding a packet, the dynamics in the free-flow phase is different from that of the shortest-path approach.

The critical value \( \beta_c(\lambda) \) can be determined by considering the quantity

\[
\eta = \lim_{t \to \infty} \frac{1}{2m\lambda} \frac{\Delta n}{\Delta t},
\]

where \( \Delta n = n(t + \Delta t) - n(t) \) and the average is over all the nodes at a time \( t \). This quantity \( \eta \in [0, 1] \) is basically the slope of \( \langle n(t) \rangle \) in the long time limit. In the non-jamming phase, the slope vanishes and \( \eta = 0 \); while in the jamming phase, \( \eta > 0 \). Figure 2 shows \( \eta \) as a function of \( \beta \), for a fixed value of \( \lambda = 0.02 \). The critical value \( \beta_c \) can be identified as the value that separates the \( \eta = 0 \) and \( \eta \neq 0 \) behavior. We carried out similar calculations for different values of \( \lambda \) and determined \( \beta_c(\lambda) \). The results are shown in Fig 3(circles). We will explain the form of \( \beta_c(\lambda) \) using a mean field theory.

The curve \( \beta_c(\lambda) \) can also be regarded as a phase boundary in the \( \lambda-\beta \) plane, separating the jamming phase below the curve and the non-jamming phase above the curve.

To show the superior performance of our routing strategy,
we also performed calculations using the shortest-path approach and the Echenique’s approach with $h = 0.8$ in Eq.(2). The same degree-dependent packet generating mechanism is used. Results of $\beta_c(\lambda)$ for these two models are also shown in Fig.4 for comparison. The present approach gives the best performance. For a given $\lambda$, we see the improvement in performance from the shortest-path approach through the Echenique’s approach to the present approach, signified by the drop of $\beta_c$. For the shortest-path approach, it has been shown [24] that $\beta_c(\lambda)$ follows the functional form of

$$\beta_c^{SP} = \alpha D (\lambda - \lambda_{min}^{SP}),$$

where $\alpha \approx 2$ and $\lambda_{min}^{SP} = 1/(\alpha Dk_{max})$ with $D$ being the diameter and $k_{max}$ the maximum degree of the network. For $\lambda < \lambda_{min}^{SP}$, $\beta_c^{SP} = 0$. With the present approach, $\beta_c(\lambda)$ follows a similar functional form, but with a higher value of $\lambda_{min}$ and a smaller prefactor that gives the slope. Both the present approach and the Echenique’s approach perform better than the shortest-path approach because packets are re-directed to other nodes when there are long queues at the hubs.

The better performance of the present approach is achieved by spreading the packets among the nodes so that the number of packets at a node is proportional to the degree $k$ of the node in the free-flow phase. We use a mean field approach to illustrate this point. Let $n_k$ be the average number of packets at the nodes with degree $k$. In the free-flow phase where $n_k < 1 + \beta k$, we have

$$\frac{dn_k(t)}{dt} = \lambda k - n_k(t) + k \sum_{k' = k_{min}}^{k_{max}} P(k'|k)n_{k'}(t) - \lambda(k).$$

The first and second terms denote the packets generated at the node and delivered to neighboring nodes, respectively. The third term accounts for the packets delivered into the node from its neighboring nodes. Here $P(k'|k)$ is the conditional probability that a node of degree $k$ has a neighbor of degree $k'$ and the sum runs from the minimum degree $k_{min}$ to $k_{max}$ in the network. In the free-flow regime, the packets that are removed upon arrival at their destination can be assumed to be $k$-independent and approximated by the term $\lambda(k)$. The non-assortative feature of BA networks gives $P(k'|k) = k'P(k'/k)$, where $P(k')$ is the degree distribution. After the transient behavior, $dn_k/dt = 0$ and we have

$$n_k = (\lambda + \frac{< n >}{< k >})k - \lambda(k),$$

where $< n > = \sum_{k' = k_{min}}^{k_{max}} P(k'|k)n_{k'}$ is the mean number of packets per node. Thus for $k > < k > = 6$, $n_k \sim k$ in the free-flow phase after the transient. Figure 4(a) shows the numerical results obtained by averaging the number of packets on the nodes with degree $k$ at different times (time $t = 100$, 200, 300 time steps) of a run. In the free-flow phase, $n_k \sim k$ and becomes time-independent after the transient, as shown in Fig.4(a) for the case of $\beta = 0.06$ and $\lambda = 0.02$. This behavior is consistent with that in Eq.(6).

For the jamming phase, numerical results (see Fig.4(b)) show that (i) $n_k \sim k$ at a fixed instant and (ii) $n_k$ increases with time for fixed value of $k$. This behavior can be understood provided that the packets are still distributed among the nodes in proportion to the degree $k$ of a node via our strategy. In this phase, the long time behavior is characterized by an increasing accumulation of packets and the delivery to destinations becomes negligible compared with packet generation. With $n_k > 1 + \beta k$ for all nodes and ignoring the removal of packets, Eq.(6) is modified to

$$\frac{dn_k(t)}{dt} = \lambda k - (1 + \beta k) + k \sum_{k' = k_{min}}^{k_{max}} P(k'|k) \frac{1 + \beta k'}{k'} = -1 + k(\lambda + \frac{1}{< k >}).$$

It follows that $n_k(t)$ increases with time $t$ as

$$n_k(t) = n_k(0) + (k(\lambda + \frac{1}{< k >}) - 1)t,$$

which describes very well the features in Fig.4(b). Thus, the present approach has the effect of reducing (increasing) the probability of passing packets to neighbors with high (low) degrees when there are long (no or short) queues, resulting in a distribution of packets according to the degrees of the nodes.

A rough estimate of $\beta_c(\lambda)$ can be obtained by equating $n_k$ in the free-flow phase to $(1 + \beta k)$. In particular, taking $n_{k_{max}} = 1 + \beta_c k_{max}$, we get from Eq.(6) that

$$\beta_c = \lambda - \lambda \frac{< k >}{k_{max}} + \frac{< n >}{< k >} - \frac{1}{k_{max}} = \lambda - \frac{< k >}{k_{max}} + (D - 1) \sum \lambda k_i - \frac{1}{k_{max}}$$

$$= \left(D - \frac{< k >}{k_{max}}\right) \left(\lambda - \frac{1}{k_{max}}\right) \approx D \left(\lambda - \frac{1}{Dk_{max}}\right),$$

where $D$ is the dimensionality of the network.
where $D$ is the average number of nodes that a packet passes through from its origin to the destination, which is the diameter of the network in the free-flow phase. The last line is valid for $k_{\text{max}} \gg \langle k \rangle$. Comparing with Eq. (3) for the shortest-path approach, we note that $\lambda_{\text{min}} = 1/(Dk_{\text{max}}) > \lambda_{\text{SP}}^{\text{min}}$, and the prefactor $D$, which gives the slope in Fig. 3, is smaller than that in the shortest-path approach. These features are consistent with numerical results. In particular, for $N = 1000$ nodes, we found that $D \approx 3.332$ and $k_{\text{max}} \approx 85$, giving $\lambda_{\text{min}} \approx 0.007$, which is in reasonable agreement with numerical results in Fig. 3.

IV. SUMMARY

In summary, we have proposed an efficient routing strategy on forwarding packets in a scale-free network. The strategy accounts not only for the physical separation from the destination but also on the waiting time along possible paths. We showed that our strategy performs better than both the shortest-path approach and the Echenique’s approach. Analytically, we construct a mean field treatment which gives results in agreement with observed features in numerical results. Our routing strategy has the merit of distributing the packets among the nodes according to the degree, and hence handling capability, of the nodes. Although our discussion was carried out on BA networks, we believe that our approach is also applicable in other spatial structures.

We end by comparing the three different routing strategies in more general terms. The shortest-path approach depends entirely on geometrical information that is static. Once the origin and the destination of a packet is known, the shortest-path is fixed. This strategy is non-adaptive, i.e., it will not be change with time. The Echenique’s approach considers both geometrical and local dynamical information. By considering the waiting time at a neighboring node, a packet from a node $i$ to a destination $j$ will not always follow the same path. Thus, the Echenique’s approach is a strategy that is adaptive, i.e., a decision based on the current situation. The present strategy, like the Echenique’s approach, is also adaptive and makes use of global information in which all the waiting times along a path are taken into consideration. We see that by allowing for adaptive strategies and taking more information into consideration, a better performance results. This line of thought is in accordance with that in complex adaptive systems whereby active agents may adapt, interact, and learn from past experience. It should be, however, noted that it pays to be better. The shortest-path approach does not require update of the routing strategy. The Echenique’s approach and the present approach require continuing update of the number of packets accumulated at the nodes. Such updating plays the role of a cost, with the payoff being the better performance. Practical implementation would have to consider the balance between the cost and the payoff.

This work was supported by the NNSF of China under Grant No. 10475027 and No. 10635040, by the PPS under Grant No. 05PJ14036, and by SPS under Grant No. 05SSG27. P.M.H. acknowledges the support from the Research Grants Council of the Hong Kong SAR Government under grant number CUHK-401005.

Email: zhluy@phy.ecnu.edu.cn Suggested Referees:
[31] P. Echenique, J. Gomez-Gardenes, Y. Moreno, Europhys. Lett. 71 (2005) 325.
[32] Z. Chen, X. Wang, Phys. Rev. E 73 (2006) 036107.
[33] W. Wang, B. Wang, C. Yin, Y. Xie, T. Zhou, Phys. Rev. E 73 (2006) 026111.

[34] D. Chowdhury, L. Santen, A. Schadschneider, Phys. Rep. 329 (2000) 199.
[35] N.F. Johnson, P. Jefferies, P.M. Hui, Financial Market Complexity (Oxford University Press, Oxford 2003).