Peculiar seasoning in the neutrino day-night asymmetry: where and when to look for spices?

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16th Lomonosov Conference on Elementary Particle Physics
26 August, 2013
What is seasoning?

1. Culinary seasoning
2. Solar $\nu$ seasonal effects & other time-regular effects
   - $\sim 7\%$ flux variations due to seasonal variation of Sun-to-Earth distance
   - solar cycles (including those assumed to exist due to acoustic waves in the core) $\rightarrow$ neutrino flux variation
   - Solar $\nu$ oscillations inside the Earth $\rightarrow$ Day-Night Asymmetry and its bizarre time variations (OUR TALK!)
Why Solar $\nu$ Day-Night Asymmetry?

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![Image](image-url)
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N.B. Extraction of DNA needs a long-term observation $\rightarrow$ we inevitably face seasonal effects!
Oscillations in matter & Earth regeneration [1]

The theory of DNA is quite conventional,

\[ i\lambda \partial_x R(x, x_0) = H(x)R(x, x_0), \quad R(x_0, x_0) = 1; \]

\[ H(x) = \left( -\cos 2\theta_0 + \frac{2E V(x)}{\Delta m^2} \right) \sigma_1 + \sin 2\theta_0 \sigma_3, \]

\[ R_{f,f'}(x, x_0) \equiv \langle \nu_f(x) | \nu_{f'}(x_0) \rangle \] is the flavor evolution matrix \((f, f' = e, \mu)\)

\[ V(x) = \sqrt{2} G_F N_e(x) \] is the Wolfenstein potential
\[ \lambda = \Delta m^2 / 4E = \pi / l_{osc}, \quad l_{osc} \sim 20...300 \text{ km} \]
\[ \sin^2 2\theta_0 \approx 0.86, \Delta m^2 \approx 7.6 \times 10^{-5} \text{ eV}^2 [PDG2012] \]

\[ E \] is the \(\nu\) energy
\[ x \] goes along the \(\nu\) ray

\[ N_e(x) \] is the electron density
Oscillations in matter & Earth regeneration [2]

\[ i\lambda \partial_x R(x, x_0) = \left\{ \left( -\cos 2\theta_0 + \frac{2E V(x)}{\Delta m^2} \right) \sigma_1 + \sin 2\theta_0 \sigma_3 \right\} R(x, x_0) \]

- There are various approximate approaches to this equation which are relevant to the Earth regeneration effect for solar neutrinos [D’Olivo, 1992; Supanitsky, D’Olivo, Medina-Tanco, 2008; Lisi, Montanino, 1997; de Holanda, Wei Liao, Smirnov, 2004; Ioannisian, Smirnov, 2004; Blennow, Ohlsson, 2004; Aleshin, Kharlanov, Lobanov, 2013]

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These approaches give virtually the same results for solar neutrinos in the Earth, considered as a spherically-symmetric layered structure (PREM)
Oscillations in matter & Earth regeneration [3]

Day/night probabilities of observing $\nu_e$:

$P_e(\text{day}) = \frac{1}{2} + \frac{1}{2} \cos 2\theta_{\text{Sun}} \cos 2\theta_0,$

$P_e(\text{night}) = \frac{1}{2} + \frac{1}{2} \cos 2\theta_{\text{Sun}} \left\{ \cos 2\theta_n^- + 2 \sin 2\theta_0 \sum_{j=1}^{n-1} \Delta \theta_j \cos 2\Delta \psi_{n,j} \right\},$

$n$ is the number of crossed interfaces between the Earth’s layers

$\Delta \psi_{n,j} \approx \pi L_{n,j}/\ell_{\text{osc}}$ is the osc. phase diff. (detector–$j$th crossing pt.)

$\Delta \theta_j$ are jumps of the effective mixing angle

$\theta_n^-$ is the effective mixing angle under the detector
As time goes by...

\[ P_e(\text{night}) = \frac{1}{2} + \frac{1}{2} \cos 2\theta_{\text{Sun}} \left\{ \cos 2\theta_n - 2 \sin \theta_0 \sum_{j=1}^{n-1} \Delta \theta_j \cos 2\Delta \psi_{n,j} \right\} \]

- The number of crossed interfaces \( n \) changes, depending on the zenith angle \( \Theta_Z(t) \)
- The distance from the \( j \)th crossing pt. to the detector \( L_{n,j} = L_{n,j}(\Theta_Z(t)) \)
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In order to cope with such an object, we use the stationary phase approximation

\[
\int_{a}^{b} F(\tau) e^{i\lambda S(\tau)} d\tau = \sqrt{\frac{2\pi i}{\lambda S''(\tau_0)}} F(\tau_0) e^{i\lambda S(\tau_0)} + \frac{F(\tau_0) e^{i\lambda S(\tau_0)}}{i\lambda S'(\tau)} \bigg|_{a}^{b} + O(\lambda^{-3/2}), \quad \lambda \to +\infty,
\]

where \( F(\tau) \) and \( S(\tau) \) are smooth on \([a, b]\) and \( S'(\tau) = 0 \) only at \( \tau = \tau_0 \in (a, b) \).
It is easy to see that...

\[ P_e(\text{night}) = \frac{1}{2} + \frac{1}{2} \cos 2\theta_{\text{Sun}} \left\{ \cos 2\theta_n + 2 \sin 2\theta_0 \sum_{j=1}^{n-1} \Delta \theta_j \cos 2\Delta \psi_{n,j} \right\} \]

- Stationary points \( \Theta'_Z(t) = 0 \): midnights when integrating over the night and solstices when integrating over the seasons;
- Edge terms vanish
- We take the dependence \( \Theta_Z(t) \) from spherical astronomy
- The small parameter here is \( \ell_{osc}/L_{n,j} \)
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\[ \langle P_e(\text{night}) - P_e(\text{day}) \rangle_{\text{year}} \approx \frac{1}{2} \cos 2\theta_{\text{Sun}} (\cos 2\theta_n^- - \cos 2\theta_0) \]

\[ + \cos 2\theta_{\text{Sun}} \sin 2\theta_0 \sum_{j=1}^{n-1} \Delta \theta_j \sum_{s=\pm 1} \frac{\mathcal{O}(r_j - r_n \sin(\chi + s\varepsilon))}{2\pi \sqrt{\sin \varepsilon \cos \chi \sin(\chi + s\varepsilon)}} \frac{\sqrt{r_j^2/r_n^2 - \sin^2(\chi + s\varepsilon)}}{\lambda L_{n,j}^{\text{solstice}}} \times \cos\{2\Delta \psi_{n,j}^{\text{solstice}} + s'(s - 1)\pi/4\}, \]

\[ s' \equiv \text{sgn}\{L_{n,j}^{\text{solstice}} - r_n \cos(\chi + s\varepsilon)\}, \quad \Theta_Z = \pi - \chi - s\varepsilon \quad (\text{solstice } s = \pm 1); \quad \chi \text{ is the detector latitude, } \varepsilon = 23.5^\circ; \quad r_j \text{ are the radii of the Earth's shells.} \]
A test drive: analytics vs. numerics

\[ A_{dn} = \frac{2(P_e(\text{night}) - P_e(\text{day}))}{P_e(\text{night}) + P_e(\text{day})} \]

The constant vertical shift is due to the unaccounted fine structure of the crust under the detector. It is smooth enough and is season-independent, so we do not bother about it.
A test drive: different latitudes

\[ A_{dn} = \frac{2(P_e(\text{night}) - P_e(\text{day}))}{P_e(\text{night}) + P_e(\text{day})} \]

Indeed, the closer to the Tropic, the more vivid are the oscillatory contributions of the stationary points!

Gran Sasso Kamioka Tropic
(42.5N) (36.4N) (23.5N)
• The peak structure of $A_{dn}(E)$ is up to 10% in amplitude and comes from the vicinities of the two solstices.

• For off-tropic latitudes, it is strongly suppressed.
The Miracles of the Stationary points

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- For off-tropic latitudes, it is strongly suppressed.
- The contributions of the stationary points are localized, i.e. are not proportional to the observation time. In particular, their contributions to the half-year average $A_{dn}$ are two times larger!!! Thus, they can be extracted with a $\sqrt{2}$ times larger (statistical) efficiency.
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Moreover, it is wrong to say that the Earth’s core does not contribute to DNA since the Sun rarely descends low enough to shine through it. Rareness is not a measure for this localized contribution!
The tropical Sun

\[ A_{\text{dn}} = \frac{2(P_e(\text{night})-P_e(\text{day}))}{P_e(\text{night})+P_e(\text{day})} \]

The curves around \( \chi = 23.5^\circ \) exhibit the clean and vivid oscillations; the positions of the peaks are very sensitive to \( \Delta m^2 \) and \( \{r_j\} \).
Interference experiment in $\nu$ oscillations?
When and where to look for spices? (a conclusion)

- **When?** Near the **solstices**
- **Where?** Near the Tropics, the closer the better (Sao Paolo, 23°33’S)
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- **Why?** To extremely precisely determine the radii of the Earth’s shells and the solar neutrino mass-squared difference
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- **Why?** To extremely precisely determine the radii of the Earth’s shells and the solar neutrino mass-squared difference
- **Really?** In order to distinguish these effects, one needs within one order more detection events at the currently achieved energy resolution $\delta E \sim 0.5$ MeV, if one employ the adaptive recognition of wave-like patterns on the $A_{dn}(E)$ profile [to be published]
The numerical simulations were made using the Supercomputing cluster “Lomonosov” (MSU)

References
[1] S. S. Aleshin, O. G. Kharlanov, and A. E. Lobanov, Analytical treatment of long-term observations of the day-night asymmetry for solar neutrinos, Phys. Rev. D 87, 045025 (2013).
[2] O. G. Kharlanov, and A. E. Lobanov, Peculiar seasonal effects in the neutrino day-night asymmetry, submitted to Phys. Rev. D.
Thank you for your attention!