ON THE CONSISTENCY OF NEUTRON-STAR RADIUS MEASUREMENTS FROM THERMONUCLEAR BURSTS

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ABSTRACT

The radius of neutron stars can in principle be measured via the normalization of a blackbody fitted to the X-ray spectrum during thermonuclear (type-I) X-ray bursts, although few previous studies have addressed the reliability of such measurements. Here we examine the apparent radius in a homogeneous sample of long, mixed H/He bursts from the low-mass X-ray binaries GS 1826−24 and KS 1731−26. The measured blackbody normalization (proportional to the emitting area) in these bursts is constant over a period of up to 60 s in the burst tail, even though the flux (blackbody temperature) decreased by a factor of 60%−75% (30%−40%). The typical rms variation in the mean normalization from burst to burst was 3%−5%, although a variation of 17% was found between bursts observed from GS 1826−24 in two epochs. A comparison of the time-resolved spectroscopic measurements during bursts from the two epochs shows that the normalization evolves consistently through the burst rise and peak, but subsequently increases further in the earlier epoch bursts. The elevated normalization values may arise from a change in the anisotropy of the burst emission or alternatively variations in the spectral correction factor, $f_c$, of order 10%. Since burst samples observed from systems other than GS 1826−24 are more heterogeneous, we expect that systematic uncertainties of at least 10% are likely to apply generally to measurements of neutron-star radii, unless the effects described here can be corrected for.

Key words: stars: neutron – techniques: spectroscopic – X-rays: bursts – X-rays: individual (GS 1826–24, KS 1731-26)

Online-only material: color figure

1. INTRODUCTION

Interest has been raised in recent years in the prospects of inferring the neutron-star mass and radius from thermonuclear bursts. Such a possibility can provide stringent constraints on the neutron-star equation of state, which remains uncertain (e.g., Lattimer & Prakash 2007). From combining measurements of the neutron-star equation of state, which remains uncertain (e.g., Madej et al. 2004) and often neglect the variations in the predicted model curves cannot yet reproduce the range of predictions, in response to the changing $f_c$) as a function of flux (e.g., Suleimanov et al. 2011a), although the predicted model curves cannot yet reproduce the range of observed behavior.

Independent of the issues for measurement are uncertainties about precisely how the neutron star’s atmosphere affects the emerging radiation. Although the burst spectra are typically found observationally to be consistent with a blackbody (e.g., Swank et al. 1977; Kuulkers et al. 2002), scattering effects have long been understood to distort the spectrum sufficiently to bias the measured temperature (e.g., London et al. 1984, 1986). This distortion is usually parameterized via a spectral distortion factor $f_c = T_{bb}/T_{eff}$, where $T_{bb}$ is the measured blackbody (or color) temperature and $T_{eff}$ is the effective temperature of the atmosphere. Most recent work adopt a narrow range of $f_c = 1.3–1.4$ at burst luminosities well below the Eddington limit (e.g., Madej et al. 2004) and often neglect the variations in $f_c$ that may arise during the burst (e.g., Suleimanov et al. 2011b). A more rigorous approach involves fitting the observed variation in the blackbody normalization (in response to the changing $f_c$) as a function of flux (e.g., Suleimanov et al. 2011a), although the predicted model curves cannot yet reproduce the range of observed behavior.

Samples of bursts accumulated from individual sources can be extremely heterogeneous in their properties. From low duty-cycle observations featuring gaps due to Earth occultations and other conditions, it is usually impossible to be confident about the burst recurrence time, in which case the detailed ignition conditions, fuel composition, and even the completeness of thermonuclear burning are also uncertain. Under such conditions, it is difficult to disentangle the various systematic influences which might influence the normalization measurements (e.g., Güver et al. 2012). Here we investigate the intrinsic reproducibility of burst normalization measurements using a uniform, homogeneous sample of bursts from the LMXBs GS 1826−24 and KS 1731−26. We use Rossi X-ray Timing Explorer (RXTE)
Proportional Counter Array (PCA) data to test for intrinsic systematic effects which might influence burst normalization measurements beyond any additional effects which might arise from variations in the ignition conditions and fuel composition. In a related paper, Zamfir et al. (2012, hereafter Z12) used the same data to infer the mass and radius of GS 1826−24.

2. OBSERVATIONS AND ANALYSIS

Few bursting sources exhibit trains of bursts with consistent light curves or recurrence times. The best-known example is GS 1826−24, so far unique for its consistently regular burst behavior and high degree of uniformity between successive burst light curves (e.g., Galloway et al. 2004). Comparison of the burst behavior and light curves suggests that the system accretes mixed helium and hydrogen at roughly solar mass fraction (Heger et al. 2007). We used observations of GS 1826−24 taken with the PCA (Jahoda et al. 1996) on board the RXTE, from the catalog of bursts detected over the mission lifetime (Galloway et al. 2008, hereafter G08). The flux−recurrence-time relationship for this sample has been extensively studied by Thompson et al. (2008). Optical photometry of the mass donor suggests an orbital period of 2.25 hr (Meshcheryakov et al. 2010). However, several alias peaks are present in the periodogram, and it is possible that one of these (particularly at 2.05 hr) represents the true orbital period.

We performed a search of the G08 sample for additional examples of regular, consistent bursts. Recurrence time provides the most obvious way to detect regular bursts, although for instruments in low-Earth orbit like RXTE, recurrence time measurement is confounded by regular interruptions due to occultations of the star by the Earth. An alternative approach is to test for consistency of the burst light curve, via commonly used parameters measuring the duration. The ratio \( \tau \) of the fluence \( E_{\text{b}} \) to the peak flux, as used by G08, provides a simple way of comparing light curves. A scatter plot of \( \tau \) against \( E_{\text{b}} \) for a source with consistent bursts will show strong clustering, and KS 1731−26 provides the next best example after GS 1826−24. Extensive observations of KS 1731−26 by the BeppoSAX/Wide-Field Camera (WFC) suggest that, at times, this system exhibits regular bursts more frequently than GS 1826−24 (see, e.g., Figure 3 from Cornelisse et al. 2003). Despite this, little attention has been directed at the burst behavior of this system. One reason is that, unlike GS 1826−24, KS 1731−26 exhibits both radius-expansion bursts and burst oscillations (Muno et al. 2000); previous analyses have focused largely on these phenomena. A series of RXTE observations of KS 1731−26 were made in 2000 August and September, detecting a total of 14 bursts with highly consistent light curves. WFC observations also made during this time show that the burst behavior was quite regular, with recurrence times of between 2 and 3 hr. An additional example, this time show that the burst behavior was quite regular, with consistent light curves. WFC observations also made during August and September, detecting a total of 14 bursts with highly consistent light curves. The best-known example is GS 1826−24, so far unique for its consistently regular burst behavior and high degree of uniformity between successive burst light curves (e.g., Galloway et al. 2004). Comparison of the burst behavior and light curves suggests that the system accretes mixed helium and hydrogen at roughly solar mass fraction (Heger et al. 2007). We used observations of GS 1826−24 taken with the PCA (Jahoda et al. 1996) on board the RXTE, from the catalog of bursts detected over the mission lifetime (Galloway et al. 2008, hereafter G08). The flux−recurrence-time relationship for this sample has been extensively studied by Thompson et al. (2008). Optical photometry of the mass donor suggests an orbital period of 2.25 hr (Meshcheryakov et al. 2010). However, several alias peaks are present in the periodogram, and it is possible that one of these (particularly at 2.05 hr) represents the true orbital period.

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Where not explicitly stated, the data analysis procedures are as in G08. Time-resolved spectra in the range 2−60 keV covering the burst duration were extracted on intervals as short as 0.25 s during the burst rise and peak, with the bin size increasing gradually into the burst tail to maintain roughly the same signal-to-noise level. A spectrum taken from a 16 s interval prior to the burst was adopted as the background. We re-fit the spectra over the energy range 2.5−20 keV using the revised PCA response matrices, v11.74 and adopted the recommended systematic error of 0.5%. The fitting was undertaken using XSPEC version 12.

In order to accommodate spectral bins with low count rates, we adopted Churazov weighting. No correction for instrumental deadtime was applied to the spectra.

We modeled the effects of interstellar absorption, using a multiplicative model component (wabs in XSPEC), with the column density \( n_{\text{H}} \) frozen at \( 4 \times 10^{21} \text{ cm}^{-2} \) (for GS 1826−24, e.g., in{tZand et al. 1999}) and \( 1.3 \times 10^{22} \text{ cm}^{-2} \) (for KS 1731−26, e.g., Cackett et al. 2006). In the original analysis carried out by G08, the neutral absorption was determined separately for each burst, from the mean value obtained for spectral fits carried out with the \( n_{\text{H}} \) value free to vary. This has a negligible effect on the fluxes, but can introduce spurious burst-to-burst variations in the blackbody normalization. We also computed averaged light curves of blackbody spectral parameters for subsets of bursts from GS 1826−24, following the procedure adopted by Galloway et al. (2004).

3. RESULTS

We first explored a variety of different approaches to measure the blackbody normalization in the tail of the bursts. We used an event from GS 1826−24 on 2000 July 1 17:16:37 (no. 12 in G08). Four (0−3) of five Proportional Counter Units (PCUs) were functioning for this burst. We took the approach of iteratively determining the maximum extent in the tail (beginning from a few seconds after the burst start) over which a constant fit to the blackbody normalizations was an acceptable fit to below 3\( \sigma \) confidence. We refer throughout to the best-fit constant value as \( \langle K_{\text{bb}} \rangle \) to distinguish it from the individual \( K_{\text{bb}} \) values for each time-resolved spectrum.

We found a best-fit value of \( \langle K_{\text{bb}} \rangle = 103.3 \pm 0.7 \text{ (km/d10pc)}^2 \) over the interval 3.0−65.25 s (relative to the start of the burst; Figure 1). The reduced \( \chi^2/n_{\text{dof}} = \chi^2 \) was 1.45 for 95 degrees of freedom. Remarkably, this consistency was maintained despite the fitted blackbody temperature \( kT_{\text{bb}} \) falling from 2.32 to 1.55 keV.

In order to double-check our result, we carried out a similar procedure to determine the maximum extent over which the

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4 See http://www.universe.nasa.gov/xrays/programs/rxte/pca/doc/rmf/pcarmf-11.7
blackbody normalization was consistent with a constant value, but instead of simply averaging the fitted normalizations for each time bin, we performed a joint fit in XSPEC of the time-resolved spectra to an absorbed blackbody model, as used for the individual spectra. We froze the neutral column density at the same value as used for the individual spectral fits, $4 \times 10^{21}$ cm$^{-2}$. We linked the blackbody normalization for each of the individual spectra but allowed the blackbody temperatures to vary. With this treatment, the fitted $K_{\text{bb, joint}}$ was found over a shorter time range 3.25–50.75 s after the burst start, compared to the simple average.

We also explored the effect of the time binning approach on the results, by repeating the two analyses on data which used uniform 0.25 s bins throughout the burst. The bin sizes for the G08 data were 0.25 s from the start of the burst through to 14.5 s after the start, 0.5 s to 24.5 s, 1 s to 43.25 s, and 2 s through to 65.25 s. We found that the $(K_{\text{bb}})$ value obtained with uniform 0.25 s bins was identical to that with bins of variable size, although the extent of the constant fit was slightly less (extending to 50.75 s instead of 65.25 s; Table 1). Similarly, the joint fit to the uniform 0.25 s binned spectra gave a comparable result to the joint fit for variable bins and over roughly the same extent of the burst. The $\chi^2$ distribution for the 0.25 s binned data over the range 3.25–50.75 s after the burst start was similarly dispersive from the expected distribution assuming a statistically good fit,
but at a lower confidence level (Kolmogorov–Smirnov, K-S, test statistic of 0.12, equivalent to 2.5σ).

The RXTE PCUs are subject to a short (≈10 μs) interval of inactivity following the detection of each X-ray photon. This “deadtime” reduces the detected count rate below what is incident on the detector (by approximately 3% for an incident rate of 400 count s⁻¹ PCU⁻¹). For GS 1826–24, the bursts peak at ≈2500 count s⁻¹ PCU⁻¹, giving a peak deadtime rate of ≈6% (reducing in the burst tail to ≈4% after ≈60 s). Since the bursts reach approximately the same peak flux, the deadtime correction is also approximately the same and can be neglected for comparison purposes (we similarly neglect any issues related to the absolute PCA calibration). However, since the deadtime correction varies with time, it might also be expected to result in a slightly different evolution of the normalization throughout the burst. To test this possibility, we examined the duration of the constant interval for the burst analyzed in this section (no. 12 from G08), with and without the deadtime correction. Although we measured an increase in the average normalization with the deadtime correction, as expected, we found no change in the extent over which the normalization was found to be constant. Thus, we neglect deadtime corrections for the remainder of the analysis in this paper.

3.1. The Full Burst Sample from GS 1826–24

We analyzed 67 bursts observed from GS 1826–24 by RXTE between 1997 and 2007. This sample includes the set of 58 complete bursts in G08, excluding seven events for which the full light curve was not observed, or for which no appropriate data modes were available for the analysis, and one event which occurred during a slew and for which the burst flux could not be determined correctly.⁵ We also analyzed an additional nine bursts from observations in 2006 August, which were not part of the G08 sample.

We compared the results of the joint fits $K_{bb,\text{joint}}$ with the average of the individual fits $\langle K_{bb} \rangle$ for the bursts from this sample to determine how the results from our sensitivity tests translated to a larger sample. Although in some cases the joint-fit durations were shorter than the constant-fit duration for the individual normalizations the median duration was 94% of the average interval. Furthermore, we found no systematic difference between the normalizations determined by the two methods and the agreement was at better than 2% for 76% of the measurements (maximum deviation was 8.5%). This compares favorably to the typical 1σ statistical uncertainty on the measurements, of 1%. Thus, we conclude that the parameter averages $\langle K_{bb} \rangle$ provide an unbiased measure of the blackbody normalization in the tail of these bursts and adopt that method for further analysis.

The $\langle K_{bb} \rangle$ for the bursts from GS 1826–24 determined from the individual spectral-fit parameters (using the spectra with variable bin sizes; see Section 3) is shown in Figure 3. The typical span of the constant fit was from <3 s after the burst start (52% of the bursts) to approximately 60 s. For one burst the normalization was found to be constant from 2 to 77 s after the start of the burst; the median duration was 48 s. Over the interval during which the blackbody normalization was constant, the blackbody temperature typically decreased from 2.3 to 1.6 keV, while the flux decreased from $\approx 3 \times 10^{-8}$ erg cm⁻² s⁻¹ to $\approx 5 \times 10^{-9}$ erg cm⁻² s⁻¹.

⁵ The excluded bursts are 8, 15, 21, 29, 39, 42, and 44 from Table 5 of G08.

Figure 3. Mean blackbody normalization values for 67 thermonuclear bursts observed by RXTE from GS 1826–24. The mean and standard deviation for the two observation epochs (1997–1998 and 2000–2007) are indicated. Note the marked discrepancy between the mean values for the two distributions; the K-S statistic indicates that they are discrepant at a significance of 1.3 × 10⁻⁶, equivalent to 4.7σ.

The $\langle K_{bb} \rangle$ values were not significantly correlated with either the duration over which the average was calculated nor the maximum time out to which the average was calculated. Interestingly, the earliest time to which the constant fits could be extended (≈1–3 s after the burst start) was within the ≈2 s burst rise. For several of the bursts, the time at which the normalization reaches the mean value corresponds approximately to a change in slope of the burst rise, seen in several of the bursts (e.g., Figure 1). This feature is similar to the “bump” seen in model light curves (e.g., Heger et al. 2007). Although in the simulations this feature must arise due to some variation in the rate of increase of thermonuclear energy production (as the one-dimensional models cannot account for lateral propagation), the coincidence of the observed change in slope with the normalization achieving its mean value suggests that this point marks where the effects of spreading cease. Further analysis of the detailed shape of the burst rises may help to clarify this situation.

We found significant variations in the mean normalization $\langle K_{bb} \rangle$, both between and within different observational epochs. For the bursts within each epoch (1997–1998 and 2000–2007), the normalization varied by 5% and 4%, respectively. This variation was highly significant; the $\chi^2$ values for constant fits were 218 (for 6 degrees of freedom) and 1007 (for 59 degrees of freedom), respectively. In terms of the inferred radius, this implies systematic uncertainties of order 2%–2.5%. Additionally, a much larger variation was measured between the two epochs. The first seven bursts, observed between 1997 November and 1998 June, had $\langle K_{bb} \rangle$ values substantially larger than the remaining bursts (observed from 2000 June onward). The mean and standard deviation for the normalizations of the 1997–1998 bursts was $118 ± 6$ (km/d₁₀₅kpc)² while for subsequent bursts was $101 ± 4$ (km/d₁₀₅kpc)². A two-sided K-S test confirms that these distributions are discrepant at the 1.3 × 10⁻⁶ significance level (equivalent to 4.7σ). This variation
was also noted by Galloway et al. (2004), who reported instead a correlation between the persistent flux and the blackbody normalization for a subset of the bursts analyzed here.

The observed variation is unlikely to arise from any instrumental effect, as the PCA is precisely calibrated to maintain stable flux measurements for calibration sources over the entire mission lifetime. As we discuss below, the discrepancy is also unlikely to result from variations in the neutral column density of hydrogen as a function of epoch. Closer examination of the spectral variation in the two groups of bursts provides a possible explanation. In contrast to the example burst discussed in the previous section, and other bursts observed in 2000–2007, the constant-fit interval for the bursts observed in 1997–1998 began later than in the 2000–2007 bursts: typically 10 s after the burst start for the 1997–1998 bursts, compared to 3 s after the burst start for the other bursts. To put this another way, there was additional variation in the blackbody normalization during the burst rise for the 1997–1998 bursts that prevented extension of the constant interval to the same point as in the later bursts. This is illustrated in Figure 4, which compares the blackbody normalization averaged over the 1997–1998 bursts with that of the 2000–2007 bursts. Remarkably, however, the behavior of the normalization in the two groups of bursts is virtually identical during the burst rise; the discrepancy sets in from 10 s after the burst start, with the normalization of the 1997–1998 bursts gradually increasing over another ≈10 s to a higher level.

### 3.2. KS 1731–26

RXTE observations of KS 1731–26 in 2000 August–September detected 14 bursts, 8 in August and 6 in September (these are bursts 14–21 and 22–27 in G08, respectively). Although the RXTE/PCA observations were interrupted regularly due to the satellite orbit and other scheduled observations, the times of the detected bursts were consistent with a regular recurrence time. The shortest burst separation measured by RXTE during each month of data was ≈2.5 hr; the longer separations were consistent with integer multiples of this value, indicating regular bursts where intervening events were missed within data gaps. We measured the average recurrence times based on linear fits to the burst arrival times measured by RXTE as 2.577 ± 0.011 hr and 2.636 ± 0.003 hr for August and September, respectively. One further burst, observed on 1999 August 26 (no. 13 in G08), exhibited a light curve consistent with the 14 observed in 2000, and we included it in this sample.

The regular bursts featured similar long (∼5 s) rises and decays (∼60 s) as those typically observed from GS 1826–24 (Figure 5). The peak count rate was ∼2300 count s⁻¹ PCU⁻¹, so that deadtime corrections are comparable to those of GS 1826–24 (see Section 3). However, the recurrence time for the bursts from KS 1731–26, at ≈2.6 hr, was significantly shorter than has been observed from GS 1826–24 in years of RXTE observations (e.g., Thompson et al. 2008). Based on the assumed distances for the two sources (6 kpc for GS 1826–24 and 7.2 kpc for KS 1731–26; see G08), the bursts from KS 1731–26 reached significantly higher luminosities of \((1.8–1.9) \times 10^{38} \text{ erg s}^{-1}\), and the burst durations were also significantly shorter \((\tau = 24 \text{ s} \pm 10 \text{ s} \text{ for GS 1826–24})\). A shorter burst duration for KS 1731–26 suggests a smaller fraction of hydrogen in the burst fuel, although the shorter recurrence time also allows less hydrogen to be consumed by steady burning prior to the burst.

As with GS 1826–24, we measured the best-fit normalization \(K_{\text{bb}}\) from the time-resolved blackbody spectral fits over the longest possible time interval without exceeding the 3σ confidence limit. With the shorter bursts from KS 1731–26, the constant \(K_{\text{bb}}\) fits extended typically over the range 2–30 s after the burst start. Over this time interval, the blackbody temperature

Figure 4. Comparison of averaged blackbody normalization profiles for bursts from GS 1826–24 measured in 1997–1998 (nos. 1–5 of G08) and 2000–2007 (nos. 9, 10, 11, 12, 13, 16, 17, 19, 20). The vertical dashed lines indicate the time of maximum flux for each set of bursts. Note the agreement in the normalization throughout the burst rise and maximum, and the increasing discrepancy from 10 s after the burst start. The inset shows the corresponding variation of the averaged burst flux.

(A color version of this figure is available in the online journal.)

Figure 5. Example burst observed by RXTE from KS 1731–26 on 2000 September 29 14:08:35 UT (no. 24 in G08), illustrating the constancy of the blackbody normalization over a significant extent of the burst rise and tail. Panel descriptions are as for Figure 1. The reduced \(\chi^2 = \chi^2_0 \approx 1.39\) for the constant fit is 1.39, for \(v = 79\) degrees of freedom.
dropped (typically) from 2.5 to 1.8 keV, with the flux dropping from $3 \times 10^{-8}$ erg cm$^{-2}$ s$^{-1}$ to $7 \times 10^{-9}$ erg cm$^{-2}$ s$^{-1}$.

The fitted $\langle K_{bb} \rangle$ values of the long bursts from KS 1731–26 exhibited significant variability; a fit with a constant model gave a $\chi^2 = 112.7$ for 14 degrees of freedom, indicating variability at the 8.5% level. The mean value was $80 \pm 2$ (km/d$_{10^{22}}$ cm$^2$)$^2$, with a standard deviation between the various measurements of 2.6%. The mean blackbody normalization of the burst on 1999 August 26, measured between 1.25 and 36 s after the burst start at 79.7 ± 0.8 (km/d$_{10^{22}}$ cm$^2$)$^2$, was fully consistent with the bursts observed in 2000 despite being observed a year earlier.

In an independent analysis of the bursts observed from KS 1731–26 with RXTE (of which the bursts we analyze here are a subset), Güver et al. (2012) measured a mean blackbody normalization of 88.4 ± 5.1 (km/d$_{10^{22}}$ cm$^2$)$^2$ and found weakly significant variation in the normalization as a function of the burst flux. Although this value is not significantly different from the normalization we derive from the subset of bursts analyzed here, we suggest that the difference arises from a systematic effect due to the value adopted for the neutral column density. For the analysis presented here, we assumed $n_{H} = 1.3 \times 10^{22}$ cm$^{-2}$ (from multi-epoch Chandra and XMM-Newton spectra of the source during quiescence; Cackett et al. 2006) while Güver et al. (2012) adopted a value of 2.98 ± 10$^{22}$ cm$^{-2}$, the mean of best-fit values from fits to the burst spectra measured by RXTE. For a simulated blackbody spectrum with known $kT$ and normalization, adopting a realistic model for the pre-burst emission as background, the fitted value of the normalization depends linearly on the assumed $n_{H}$ (Figure 6).

That is, over (under)-estimating the $n_{H}$ value assumed for the fits will have the effect of over (under)-estimating the $K_{bb}$. For the representative parameters chosen, the slope gives approximately 6.5 (km/d$_{10^{22}}$ cm$^2$)$^2$ for every additional 10$^{22}$ cm$^{-2}$ in column that is adopted. The difference between the adopted values for the two analyses could account for an offset in the normalizations of up to 11 (km/d$_{10^{22}}$ cm$^2$)$^2$, sufficient to explain the discrepancy.

**4. DISCUSSION**

We found significant variations in the blackbody normalization (K$_{bb}$) averaged over tens of seconds of the burst tail in a homogeneous sample of regular, consistent bursts from GS 1826–24 and KS 1731–26, both within and between observation epochs. We found that the variation within epochs was 4%–5% (2.6%) for GS 1826–24 (KS 1731–26), while the variation between epochs can be as large as 17% (for GS 1826–24). In terms of inferred radius, this corresponds to variations of 2%–2.5% (1.3%) and 8%. The variation within each epoch is within the range reported by Güver et al. (2012) in their more comprehensive study of bursts from several sources, although the 8% variation between epochs observed for GS 1826–24 is somewhat larger. We stress that this uncertainty is solely related to the measurement of the blackbody normalization in bursts and (as suggested by Z12) may be exceeded by other sources of uncertainty, for example the degree of anisotropy of burst emission.

The 1997–1998 bursts from GS 1826–24, which exhibited longer recurrence times and reached slightly higher fluxes, also exhibited larger mean normalizations (K$_{bb}$). Comparison of the normalization time series averaged over subsamples of bursts from the two epochs shows that the normalizations were essentially identical during the burst rise and through the burst peak, with deviations becoming apparent between 10–20 s after the burst start. The reproducibility of the K$_{bb}$ evolution throughout the burst rise and peak suggests that the system geometry, atmospheric composition, and temperature (and hence f$_{c}$) were essentially identical over that interval, and whatever physical condition gave rise to the elevated K$_{bb}$ in the 1997–1998 bursts set in after the burst peak. We consider three possible mechanisms to give rise to the variation.

First, it may be that the amount of neutral material close to the neutron star was reduced, so that the neutral column density $n_{H}$ decreased during the 1997–1998 bursts, leading to an overestimation of K$_{bb}$ (see, e.g., Figure 6). Simulations adopting the pre-burst emission from GS 1826–24 (as was done for KS 1731–26; see Section 3.2) indicate that in order to overestimate the $\langle K_{bb} \rangle$ by the required amount would necessitate decreasing the total $n_{H}$ value by $1.9 \times 10^{22}$ cm$^{-2}$. Since this value is almost five times the assumed line-of-sight value for GS 1826–24, we can rule out this mechanism.

Second, it is possible that the degree of anisotropy of the burst emission changed in response to a variation in the accretion geometry (e.g., Fujimoto 1988), perhaps triggered by the burst. Such a change might be expected to be reflected in the X-ray spectrum. The broadband ($\approx$ 1–100 keV) spectral distribution of the persistent emission in GS 1826–24 has been well studied by Thompson et al. (2008), in the context of determining the best possible estimate of the bolometric flux (and hence the accretion rate). Those authors found substantial changes in the spectrum with epoch, most notably during 2003 April, when evidence for an additional soft ($< 1$ keV) component was found. However, the mean normalization from the six bursts observed in 2003 April was (102 ± 4) (km/d$_{10^{22}}$ cm$^2$)$^2$ (see Figure 3), fully consistent with the mean from the other bursts from 2000 onward (but excluding the 2003 April bursts) of (101 ± 4) (km/d$_{10^{22}}$ cm$^2$)$^2$. Conversely,
there is no evidence for a substantially different X-ray spectrum during the 1997–1998 observations (see Thompson et al. 2008; Figure 4) The lack of correspondence between the broadband spectral shape and the blackbody normalization from the bursts seems to contraindicate an influence of the burst anisotropy on the normalization, although cannot rule it out. On the other hand, the variations in the persistent X-ray spectrum might be transient, present only when the elevated $K_{bb}$ values were measured, beginning 10 s after the burst start. Such variations might be expected to lead to incorrect pre-burst emission subtraction later in the burst, although this is not observed consistently.

Third, we consider the possibility that the $f_c$ value changes to produce the variation in the measured $K_{bb}$ after the peak in the 1997–1998 bursts from GS 1826–24. Modeling studies indicate that $f_c$ depends on fixed parameters such as the neutron-star surface gravity, but also the composition and effective temperature of the scattering atmosphere (e.g., Madej et al. 2004; Suleimanov et al. 2011b). There is evidence in other sources that radius-expansion bursts can remove the outer, H-rich layers of the photosphere, leading to a change in the atmospheric composition during the burst (Galloway et al. 2006); no such effects have been suggested from non-radius-expansion bursts. Nevertheless, interpreting the maximum variation in $\langle K_{bb} \rangle$ in the 1997–1998 bursts, compared to the mean for the 2000–2007 bursts, as a variation in $f_c$ implies a maximum variation of 12% (8% in the mean). This result suggests that the assumption that $f_c$ is constant during bursts is not always true. While Suleimanov et al. (2011b) predict patterns of variation of $f_c$ with burst flux, the epoch-to-epoch differences in the burst light curves in GS 1826–24 are relatively subtle and do not appear sufficient to give rise to the inferred variation in $f_c$.

It is possible that the peak blackbody flux (or temperature) serves as the discriminant which results in elevated blackbody normalizations later in the burst. The 1997–1998 bursts reached maximum fluxes about 8% higher on average than for the 2000–2007 bursts. The discrepancy in the maximum temperature reached was proportionally smaller. The samples of bursts studied here were deliberately selected for the consistency of their light curves and regularity of their recurrence times. For other samples of bursts, which are typically much more heterogeneous, systematic errors in measurements of the blackbody radius of order $\geq 8\%$ should be assumed, unless the variation in $f_c$ can be modeled.

The mean blackbody normalizations measured during the later (2000–2007) bursts from GS 1826–24, and the regular bursts from KS 1731–26, were consistent with a constant value over several tens of seconds. This constancy was observed independent of the specific method for determining the mean value, although the choice of method can introduce small biases. In particular, comparisons of joint fits to the spectra with averages of the fitted normalizations show that in general the former method arrives at shorter durations for the constant normalization, likely because the spectra are not (en masse) statistically consistent with blackbodies. Trends in the blackbody normalization late in the burst tail, coupled with these marginally deviant spectra, likely give rise to systematic errors of a few percent between the two methods. The choice of time binning strategy does not have as large an effect.

This constancy of the blackbody normalization was maintained despite significant decreases in the blackbody temperature, of (typically) 30%, and decreases in the flux of 60%–75% over the same time interval. Such a lack of variation in the blackbody normalization implies constraints on the relative degree of spectral distortion over this temperature range, suggesting one of two situations. Either any variation in the spectral distortion factor $f_c$ as a result of the varying effective temperature (as indicated by the decreasing blackbody temperature) is exactly balanced by some other variation, e.g., a change in the emitting radius as the burst flux decreases, or the color temperature correction $f_c$ is also constant throughout the interval in which $K_{bb}$ is constant. The former explanation is rather contrived, particularly considering that in the burst tail the burning front is expected to have already spread to the entire surface area of the neutron star, and no further increase (or decrease) in burning area is expected. Thus, these measurements suggest that for the range of effective temperatures spanned in the burst tails, the color temperature correction $f_c$ is roughly constant. This conclusion is difficult to reconcile with the predictions of atmosphere models (e.g., Suleimanov et al. 2011b), which indicate significant variations in $f_c$ over most of the flux range spanned during the burst. The distance for GS 1826–24 is thought to be $\approx 6\ kpc$ (Heger et al. 2007), at which an Eddington-limited burst would be expected to reach $F/F_{\text{Edd}} = 3.7 \times 10^{-3} (6.3 \times 10^{-8})$ erg cm$^{-2}$ s$^{-1}$ for an H-rich (pure He) atmosphere (G08). This would imply that the range of flux over which the burst normalization is found to be constant is 0.16–0.75 (0.10–0.46) $F/F_{\text{Edd}}$. The greatest variation in the normalization predicted by Suleimanov et al. (2011a) is outside these ranges, although significant variations would yet be expected (particularly for the solar metallicity models; see also Z12).

The $\approx 3\%$–5% fractional variation in $\langle K_{bb} \rangle$ observed from GS 1826–24 and KS 1731–26 within each observational epoch was smaller than that seen in two radius-expansion bursts observed from EXO 1745–248, of 25% (Özel et al. 2009). This observation perhaps suggests an explanation of the variation. EXO 1745–248 exhibited during its 2000 outburst an initial period of strong variability, reminiscent of dipping behavior observed in high-inclination systems (e.g., Galloway et al. 2008). No such variability has ever been observed from GS 1826–24 or KS 1731–26, suggesting that perhaps the inclination is lower in those systems than in EXO 1745–248. If system inclination is the main factor in determining the variation in apparent blackbody normalization, a possible explanation is the reprocessing of some fraction of the burst flux off an accretion disk whose projected area varies with time. In 4U 1728–34, the timescale inferred for the variation was several tens of days (Galloway et al. 2003), much longer than the expected orbital period of this system (e.g., Galloway et al. 2010). However, in GS 1826–24 and KS 1731–26, the variation is on a much shorter timescale, comparable to the recurrence time of the bursts themselves (hours). For comparison, the discrepant bursts from EXO 1745–248 were separated by 8.5 days. We note that such an explanation fails to account for the factor of $\approx 2$ difference in normalization in the bursts from 4U 1724–307 (Suleimanov et al. 2011a), which does not show dips and thus is unlikely to be at high inclination. However, those bursts also exhibited markedly different timescales, indicative of a varying accumulated fuel reservoir at ignition; we suggest instead that different physical conditions (temperature, composition) gave rise to the difference in the measured blackbody normalization.

The shorter timescale of variation in $\langle K_{bb} \rangle$ for GS 1826–24 and KS 1731–26 suggests that there may be an orbital component of the variation. The specific value of the blackbody normalization depends upon the assumed neutral column...
density, as illustrated in Figure 6. Thus, an orbital variation in the line-of-sight column density, perhaps arising from cool clouds of material above the point of contact of the accretion stream with the disk, will manifest as a variation in the blackbody normalization on the same timescale. In order to test this hypothesis, we calculated a Lomb-normalized periodogram on the blackbody normalization measurements as a function of time. Recall that the orbital period in both sources is unknown, but for GS 1826−24 it is thought to be around 2 hr; based on the typical orbital periods for other burst sources, we searched a frequency range of 0.5–24 hr. For GS 1826−24, we divided the measurements from the two epochs through by the appropriate mean and found a peak Lomb power of 10.48; for KS 1731−26, we found a peak Lomb power of 5.29. Neither of these detections is significant. For KS 1731−26, the known sensitivity of the normalization measurement to discrepancies between the assumed and true value of $n_{\text{H}}$ indicates that a variation of $0.3 \times 10^{22}$ cm$^{-2}$ in $n_{\text{H}}$ could account for the variation in the measured blackbody normalization. For GS 1826−24, the measurements are slightly more sensitive to discrepancies in $n_{\text{H}}$, so that a variation in $n_{\text{H}}$ of $0.45 \times 10^{22}$ cm$^{-2}$ could account for the variation in the blackbody normalization within each epoch. If orbital or longer-timescale variations in $n_{\text{H}}$ are driving the variation in the blackbody normalization, it may be possible to verify through time-resolved high-spectral-resolution measurements of absorption edges in the X-ray spectra.

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