Abstract

We estimated the differential cross sections of the Higgs boson production in \( h \to ZZ^* \to 4\ell \) decay channel within the framework of \( k_T \)-factorization. Estimates are obtained using off-shell matrix element for \( g^* g^* \to h \to ZZ^* \to 4\ell \) process and CCFM evolution equations for transverse momentum dependent (or unintegrated) gluon distribution functions. We have compared our results with the latest experimental data for \( \sqrt{S} = 8 \text{ TeV} \) from the ATLAS and CMS collaborations at the LHC, whereas the predictions for \( \sqrt{S} = 13 \text{ TeV} \) in each case are given. Results are also compared with the estimates from collinear factorization formalism calculated up to next to next leading order plus next to next leading logarithm (NNLO+NNLL) obtained using HRes tool for the Higgs production in gluon-gluon fusion process. Our estimates are consistently close to NNLO+NNLL results obtained using HRes tool and also have an agreement with experimental data for \( \sqrt{S} = 8 \text{ TeV} \).

I. INTRODUCTION

The Higgs boson discovery at the LHC by ATLAS and CMS collaborations [1, 2] confirmed the last missing piece of the Standard Model (SM). The ATLAS and CMS collaborations have done an improved measurement of the Higgs boson mass considering invariant mass spectra of the decay channels \( h \to \gamma\gamma \) and \( h \to ZZ^* \to 4\ell \) [3, 4]. Further studies of spin and parity quantum number of the Higgs boson established that the discovered particle is a neutral scalar boson rather than a pseudoscalar with mass equal to 125.09 GeV [5–7]. Its coupling strength to vector boson and fermion is studied using analysis of various decay modes of the Higgs boson [4, 8–10]. Establishing various aspects of the Higgs boson properties and coupling strength enable us to study other aspects of it.

The dominant channel for the inclusive Higgs boson production at LHC is the gluon-gluon fusion [11–13]. The Higgs boson production at LHC can be effectively used to understand the gluon dynamics inside a proton. The gluon density \( x f_g(x, \mu_F^2) \) in a proton is a function of Bjorken variable \( x \) and hard scale \( \mu_F^2 \), described within the framework of the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation [14–17] where large logarithmic terms proportional to \( \ln \mu_F^2 \) are resummed up to all orders.

Factorization theorem in perturbative quantum chromodynamics (pQCD) allows us to write convolution of the matrix element of short distance process and the universal parton density functions to obtain inclusive cross section for given scattering process. This factorization theorem is based on the collinear approach governed by the longitudinal momentum fraction \( x \) and hard scale \( \mu_F^2 \), and it is called a collinear factorization. For inclusive Higgs boson production at LHC, the longitudinal momentum fraction of incident gluons is small. This domain of small \( x \) is still in the perturbative regime where it is expected that collinear factorization should break down because contribution from large logarithmic term proportional to \( 1/x \) becomes dominant [18–20]. The contribution from terms proportional to \( 1/x \) is taken into account in Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution equation [21–23]. The unintegrated parton densities (uPDFs) obeying BFKL evolution convoluted with an off-shell matrix element within generalized factorization is called \( k_T \)-factorization [24–28]. Evolution equation valid for both small and large \( x \) is given by the Ciafaloni-Catani-Fiorani-Marchesini (CCFM) evolution equation [29–32]. CCFM evolution is equivalent to BFKL evolution in the limit of very small \( x \), whereas similar to the DGLAP evolution for large \( x \) and high \( \mu_F^2 \).

The inclusive Higgs boson production within \( k_T \)-factorization approach together with CCFM evolution equations have been studied and demonstrated the importance of higher order corrections within the \( k_T \)-factorization approach [33]. The authors of Ref. [34] have shown that \( k_T \)-factorization gives description of experimental data from ATLAS experiment for differential cross section for the Higgs boson production in diphoton channel. They calculated a leading order (LO) matrix element for partonic subprocess \( gg \to h \to \gamma\gamma \) considering gluons to be off-shell in effective field theory. Recently ATLAS and CMS collaboration at LHC presented a measurement of fiducial differential cross section in the transverse momentum and rapidity of the Higgs boson decay into four-leptons [35, 36] for \( \sqrt{S} = 8 \text{ TeV} \).

In this work we use \( k_T \)-factorization approach to study recent ATLAS and CMS data of the measurement of fiducial differential cross section of the Higgs boson in the four-lepton decay channel. For this we have evaluated off-shell matrix element for the partonic subprocess \( g^* g^* \to h \to ZZ^* \to 4\ell, \ell = e, \mu \). We have convoluted off-shell matrix element of partonic subprocess with CCFM uPDFs to obtain differential cross section. The transverse
momentum of initial gluon is related to the transverse momentum of the Higgs boson and hence such study can be used to impose constraint on the unintegrated gluon density of the proton.

This article is organised as follows: We discuss in detail the formalism behind our study and the necessary expressions for further numerical analysis in Section II. In Section III we give the results of our numerical simulation. Here we also discuss the details of the analyses that has gone into the study. And finally we conclude and draw inferences from the analysis in Section IV.

II. Formalism

In the present section we briefly discuss the formalism we have used in our study. The details are in Appendices A and B. In the introduction we mentioned that, to explore the effects of $k_T$-factorization, we need to take the partons as off-shell. In calculating the off-shell matrix element for the process $g^* g^* \rightarrow h \rightarrow ZZ \rightarrow 4\ell$ (see Fig. 1), we have considered the effective field theory approach.

![Figure 1: Momentum assignment for the process $g^* g^* \rightarrow h \rightarrow ZZ \rightarrow 4\ell$.](image)

The effective Lagrangian in the large top quark mass limit $m_t \rightarrow \infty$ for the Higgs boson coupling to gluon is

$$\mathcal{L}_{gg} = \frac{\alpha_s}{12\pi} (\sqrt{2G_F})^{1/2} G^a_{\mu\nu} G^{a\mu\nu} h,$$

where $\alpha_s$ and $G_F$ are the strong and Fermi coupling constants respectively. $G^a_{\mu\nu}$ is the gluon field strength tensor and $h$ is the Higgs scalar field. The effective $gg$ triangle vertex (see Eq. (10)) thus becomes

$$T^{\mu\nu,ab}(k_1, k_2) = \delta^{ab} \frac{\alpha_s G_F}{3\pi} (\sqrt{2G_F})^{1/2} [k_2^\mu k_1^\nu - (k_1 \cdot k_2) g^{\mu\nu}].$$

The non-zero transverse momentum for an initial gluon lead to the corresponding polarization sum

$$\frac{\sum e^{\pm}_{k_1} e^{\mp}_{k_2}}{k_1 \pm \not{k_2}}.$$ (3)

Using Eqs. (2) and (3) we derived the off-shell matrix element for hard scattering process. The matrix element thus obtained is given by Eq. (23) as

$$|\mathcal{M}|^2 = \frac{2}{9\pi^2} \frac{m_Z^4}{v^4} \left[ s + (\sum_{i=1}^{2} k_{i\perp})^2 \right]^2 \cos^2 \varphi \times \left[ g_L^4 + g_R^4 \right] \left[ p_1 \cdot p_3 \right] \left[ p_2 \cdot p_4 \right] + 2g_L^2 g_R^2 \left[ p_1 \cdot p_4 \right] \left[ p_2 \cdot p_3 \right] \left[ \left( p_1 \cdot p_2 - m_Z^2 \right)^2 + m_Z^2 \Gamma_Z^2 \right] \left[ \left( p_2 \cdot p_4 - m_Z^2 \right)^2 + m_Z^2 \Gamma_Z^2 \right].$$ (4)

where

$$g_L = \frac{g_W}{\cos \theta_W} \left( -\frac{1}{2} + \sin^2 \theta_W \right),$$

$$g_R = \frac{g_W}{\cos \theta_W} \sin^2 \theta_W,$$

and $v = (\sqrt{2G_F})^{-1/2}$.

Here $T_k$ and $\Gamma_Z$ are the total decay widths of the Higgs boson and $Z$ boson respectively. $k_{i\perp}$'s are the intrinsic transverse momenta of the initial gluons. $\varphi$ is the azimuthal angle between $k_{1\perp}$ and $k_{2\perp}$. $m_t$ and $m_Z$ are the Higgs boson and $Z$ boson masses respectively. Partonic center of mass energy is denoted by $s$. $\theta_W$ and $g_W$ are weak mixing angle and coupling of weak interaction respectively.

Finally, we arrived at the hadronic cross section for the off-shell hard scattering amplitude of Eq. (4) within the framework of $k_T$-factorization as (see Eq. (36))

$$\sigma = \int \frac{2}{2\pi} \frac{d\varphi}{2\pi} \frac{d\Gamma}{2\pi} \left( \frac{v}{\sqrt{s}} \right)^2 \prod_{f=1}^{3} d^2 p_f \cdot dy_f \cdot dl_4 \left( \frac{\mathcal{M}^2}{2\pi^2} \right)^2.$$

with the longitudinal momentum fractions $x_1$ and $x_2$ of initial gluons to be

$$x_1 = \frac{4}{3} \frac{|p_1|}{\sqrt{s}} e^{y_f}, \quad x_2 = \frac{4}{3} \frac{|p_2|}{\sqrt{s}} e^{-y_f},$$

and the transverse momenta

$$\sum_{i=1}^{2} k_{i\perp} = \sum_{f=1}^{4} p_{f\perp}.$$ (7)

In the above expressions $\varphi$'s are the azimuthal angle of $k_{1\perp, 2\perp}$. $y$'s and $p$'s are rapidities and transverse momenta of the final state leptons. Hadronic center of mass energy is denoted by $S$.

III. Results and Discussion

With all the calculational tools at our disposal, we proceed to perform a parton level and leading order calculation using Eq. (5) together with off-shell hard scattering amplitude given in Eq. (4). We estimate cross section of the Higgs boson production as a function of transverse momentum $p_T$ and rapidity $y$ of the Higgs boson in the four-lepton decay channel. Estimates are obtained using CCFM A0 set of uPDFs [37] which is commonly used for such phenomenological studies.

Total decay width and mass of the Higgs boson is set to be equal to 4.0 MeV and 125.09 GeV respectively [7]. We have implemented experimental cuts on rapidity and transverse momentum of leptons used by ATLAS and CMS experiments in their measurements. For ATLAS experiment, absolute value of rapidity $|\eta| < 2.5$ and leading transverse momentum of lepton $p_T < 20$ GeV. The transverse momentum of sub-leading leptons $p_T < 15, 10, 7$ GeV. Similarly for CMS experiment, absolute values of rapidity $|\eta| < 2.5$ and ordered transverse momentum of leptons $p_T < 20, 10, 7, 7$ GeV. The cross section in Eq. (5) depend on renormalization and factorization scale $\mu_R$ and $\mu_F$. The scale uncertainty in cross
section is estimated by varying scale between $\mu_R = \mu_F = m_h/2$ and $\mu_R = \mu_F = 2m_h$.

We have also estimated a total inclusive cross section for the Higgs boson production with $p_T \to 0$ and averaging over an azimuthal angle of the Higgs boson. This result for inclusive cross section is equivalent to collinear factorization approach at LO using collinear parton densities. We have used Martin-Stirling-Thorn-Watt (MSTW) set [38] for collinear parton densities. We have also obtained results of total inclusive cross section for the Higgs production with $k_T$-factorization formalism using the CCFM A0 set of uPDFs. In Table 1 we have given our results for total inclusive cross section for the Higgs boson production in gluon-gluon fusion channel.

| $\sqrt{s}$ (TeV) | (Collinear factorization) | (Collinear factorization) |
|-----------------|---------------------------|---------------------------|
| 8               | 11.27                     | 6.11                      |
| 13              | 30.20                     | 17.21                     |

Table 1: Total inclusive cross section ($\sigma^{tot}$) for the Higgs boson production in gluon-gluon fusion channel.

cross section for the Higgs production using both collinear and $k_T$-factorization framework. Our results for total inclusive cross section obtained using collinear approach are consistent with the results obtained in ref. [39] at $\sqrt{s} = 8$ TeV. The results obtained with $k_T$-factorization is close to NLO results given in ref. [39] at $\sqrt{s} = 8$ TeV. The cross section estimates given here are for gluon fusion process only. The inclusive cross section for the Higgs boson production can be obtained using a hadron level Monte Carlo event generator called CASCADE [40]. CASCADE use CCFM evolution equation in the initial state with off-shell parton level matrix element.

Figure 2: Differential cross section of the Higgs boson production as a function of transverse momentum ($p_T$) and rapidity (y) of the Higgs boson in four-lepton decay channel at $\sqrt{s} = 8$ TeV. Solid line (red) is a results obtained using $k_T$-factorization approach with CCFM unintegrated gluon densities and dashed lines corresponds to scale uncertainty in renormalization and factorization scale. Filled triangle and filled square points correspond to estimates obtained using HRes tool [41, 42] up to NNLO+NNLL accuracy and shaded region corresponds to scale uncertainty in renormalization and factorization scale. Experimental data points are from ATLAS [35]. The error bars on the data points shows total (statistical $\oplus$ systematic) uncertainty.

Figure 3: Differential cross section of the Higgs boson production as a function of transverse momentum ($p_T$) and rapidity (y) of the Higgs boson in four-lepton decay channel at $\sqrt{s} = 8$ TeV. Notation of all the histograms are same as in Fig. 2. Higher order pQCD predictions up to NNLO+NNLL accuracy are obtained using HRes tool [41, 42]. Experimental data points are from CMS [36].

Figure 4: Differential cross section of the Higgs boson production as a function of transverse momentum ($p_T$) and rapidity (y) of the Higgs boson in four-lepton decay channel at $\sqrt{s} = 13$ TeV. Notation of all the histograms are same as in Fig. 2. Higher order pQCD predictions up to NNLO+NNLL accuracy are obtained using HRes tool [41, 42].

Figure 5: Differential cross section of the Higgs boson production as a function of transverse momentum ($p_T$) and rapidity (y) of the Higgs boson in four-lepton decay channel at $\sqrt{s} = 13$ TeV. Notation of all the histograms are same as in Fig. 2. Higher order pQCD predictions up to NNLO+NNLL accuracy are obtained using HRes tool [41, 42].

We have presented our results in Figs. 2 to 5. Figs. 2 and 3 and Figs. 4 and 5 shows result of differential cross section for the Higgs boson production in four-lepton decay channel at $\sqrt{s} = 8$ TeV and $\sqrt{s} = 13$ TeV respectively. We have compared our estimates of differential cross section obtained using $k_T$-factorization approach calculated at $\sqrt{s} = 8$ TeV with experimental measurements from the ATLAS and CMS collaboration [35, 36]. The solid...
histogram corresponds to our results obtained using CCFM A0 set of uPDFs, whereas the upper and lower dashed histograms are obtained by varying renormalization and factorization scales as mentioned above. In addition to this, we have used set A0+ and set A0− rather than using A0 set for scale variation. Set A0+ and set A0− are obtained by changing scale in $\alpha_s$ in the off-shell matrix element (see Ref. [37] for more detail).

Our results are plotted against fixed order results for cross section calculated up to NLO+NLL and NNLO+NNLL using HRes tool [41, 42] within collinear factorization framework. Our results of both differential cross section in $p_T$ and $y$ using $k_T$-factorization framework including CCFM unintegrated PDFs are consistently close to NNLO+NNLL results at $\sqrt{S} = 8$ TeV and $\sqrt{S} = 13$ TeV. This can be explained considering the fact that the main part of higher order corrections included in the $k_T$-factorization approach [33, 43, 44]. For $p_T$ distribution we are using the convention that NLO+NLL and NNLO+NNLL results are labelled as LO+NLL and NLO+NNLL respectively. We have not normalized the HRes results with the SM inclusive cross section predictions currently available [45]. The contribution from vector boson fusion and associated production to the Higgs boson production are not included in our study. We find that the results obtained using $k_T$-factorization are reasonably well in agreement with the ATLAS and CMS data. Within $k_T$-factorization framework, transverse momentum of initial gluons contribute to transverse momentum of the final state. Hence the contribution of $k_T$ of initial gluons will have an effect on the $p_T$ distribution of the cross section.

IV. CONCLUSIONS

In this letter we present our phenomenological study of the Higgs boson production in four lepton decay channel within the $k_T$-factorization framework. Here CCFM unintegrated parton densities were convoluted with the hard matrix element considering initial gluons to be off-shell. We present a comparison of our results with the experimental measurement conditions (i.e., the same $p_T$ and $y$ cuts were used for our estimates as given by the experimental results) for both the ATLAS and CMS at $\sqrt{S} = 8$ TeV and $\sqrt{S} = 13$ TeV respectively.

Further comparison of our results with the fixed order calculation of differential cross section within collinear factorization up to NLO+NLL and NNLO+NNLL obtained using HRes is presented. We have also estimated a total inclusive cross section for the Higgs boson production within both collinear factorization and $k_T$-factorization framework. Our results for differential cross section are close to NNLO+NNLL results obtained using HRes tool. The total inclusive cross section estimated using $k_T$-factorization is close to NLO results obtained using collinear factorization. The main reason for this behaviour is that the main part of higher order correction in collinear pQCD is included in the $k_T$-factorization. The higher order corrections within $k_T$-factorization at parton level would be an interesting study to see the additional effect. Considering the effect of transverse momentum of the initial gluon on the transverse momentum distribution of the final state, our study as well as further studies in this direction could impose constraint on uPDFs of gluons.

APPENDICES

A. AMPLITUDE OF $g^*g^* \rightarrow h \rightarrow ZZ^* \rightarrow 4\ell$

In this appendix we give details of our calculations which went into our analysis. Fig. 1 shows the assignment of the momenta. In the limit of large top quark mass $m_t \rightarrow \infty$ the effective Lagrangian for the Higgs boson coupling to gluons given in Eq. (1) can be written as

$$L_{ggh} = \frac{\alpha_S}{12\pi v} G_{\mu\nu} G^{\mu\nu h},$$

where $v = (\sqrt{2} G_F)^{-1/2}$. For $v$ the vacuum expectation value of the scalar field. The amplitude of the process $g^*g^* \rightarrow h \rightarrow ZZ^* \rightarrow \ell\ell\ell\ell'$ is

$$i\mathcal{M} = \epsilon^{\mu\nu} \epsilon_{\mu\nu} \epsilon_{\mu\nu} \epsilon_{\mu\nu} [i T_{ggg}^a(k_1, k_2)] [i T_{h \rightarrow ZZ^* \rightarrow \ell\ell\ell\ell'}(p_1, p_2, p_3, p_4)]^4 (\hat{s} - m_h^2)^2,$$

where

$$T_{ggg}^a(k_1, k_2) = \delta^{ab} \frac{\alpha_S}{3\pi v} \{ k_T^2 k_T^2 - (k_1 \cdot k_2) g^{\mu\nu} \},$$

$$T_{h \rightarrow ZZ^* \rightarrow \ell\ell\ell\ell'}(p_1, p_2, p_3, p_4) = -\frac{2 m_Z^2}{v} \bar{u}(p_1) \gamma^\mu [g_L^R + g_R^L] v(p_2) (p_1 + p_2)^2 - m_Z^2 \times \bar{u}(p_3) \gamma^\nu g_L^R v(p_4) (p_3 + p_4)^2 - m_Z^2,$$

where

$$g_L^{(i)} = \frac{g_W}{c_W} \left( -1 + s_W^2 \right), \quad g_R^{(i)} = \frac{g_W}{c_W} \frac{1}{s_W}, \quad i = 1, 2.$$
In the above \(k_{1\perp,2\perp}\) are vectors transverse to the momenta \(P_{1,2}\) of the incoming hadrons. It is convenient to take both \(P_{1,2}\) to be light-like (i.e. \(P_{1,2}^2 = 0\)). Because of this \(P_{1,2}\) are slightly different from the actual momenta of the incoming hadrons. Let us take \(P_{1,2}\) to be as follows

\[
P_{1,2} = \frac{\sqrt{S}}{2}(1, 0, \pm 1),
\]

where \(\sqrt{S}\) is the center of mass energy of the hadrons. In the high energy limit, introduction of strong ordering in longitudinal momenta gives

\[
k_1 = z_1 P_1 + k_{1\perp},
\]

\[
k_2 = z_2 P_2 + k_{2\perp}.
\]

Therefore using Eqs. (17) and (18) we get

\[
k_1 \cdot k_2 = z_1 z_2 P_1 \cdot P_2 + k_{1\perp} \cdot k_{2\perp},
\]

\[
\hat{s} = (k_1 + k_2)^2 = 2 z_1 z_2 P_1 \cdot P_2 - (k_{1\perp} + k_{2\perp})^2.
\]

In the last step we have used the definition of \(k_{1\perp,2\perp}\) introduced in Eq. (16).

With the help of the above expressions, we are now ready to calculate the gluon part of Eq. (15). Using the polarization sum for the off-shell gluons given by

\[
\sum e_{\mu}^{k_1} e_{\nu}^{k_2} \approx \frac{k_{1\perp} k_{2\perp}}{k_{\perp}^2},
\]

we can calculate

\[
\sum e_{\mu}^{k_1} e_{\nu}^{k_2} \sum e_{\mu}^{k_1} e_{\nu}^{k_2} \left[\frac{m_{k_1}^2 m_{k_2}^2}{(k_{1\perp} + k_{2\perp})^2} - (k_1 \cdot k_2) g^{\mu \nu}\right]
\]

\[
\times [k_{1\perp} k_{2\perp} - (k_1 \cdot k_2) g_{\perp \perp}] = \sqrt{\hat{s} + (k_{1\perp} + k_{2\perp})^2} \cos^2 \varphi.
\]

[Using Eq. (19) and then Eq. (20)]

Therefore putting the expression of Eq. (22) into Eq. (15) we get the total spin averaged squared amplitude of the process \(g^* g^* \rightarrow h \rightarrow ZZ^* \rightarrow \ell \ell' \ell'\) as

\[
|M|^2 = \frac{2 \alpha_s^2 m_1^2}{9 \pi^2} \frac{\hat{s}}{v^4} \left[\frac{\hat{s} + (k_{1\perp} + k_{2\perp})^2}{(\hat{s} - m_1^2)^2}\right] \cos^2 \varphi
\]

\[
\times \left[ g_{1}^2 + g_1^2 \right] |(p_1 \cdot p_3)(p_2 \cdot p_1) + 2 g_1^2 g_2^2 (p_1 \cdot p_4)(p_2 \cdot p_3)|
\]

\[
(2p_1 \cdot p_2 - m_2^2)^2 (2p_4 \cdot p_4 - m_2^2)^2.
\]

**B. Phase Space Calculation**

In this appendix we give details of the phase space calculations related to our analysis. Let us express the 4-momentum \(p\) in a 3-component vector in terms of the transverse momentum \(\mathbf{p}_T\) and the rapidity \(y\) as follows

\[
p = (E_T \cosh y, \mathbf{p}_T, E_T \sinh y),
\]

where the transverse energy \(E_T = \sqrt{p_T^2 + m^2}\) and \(m\) is the mass.

We can write the measure of the phase space integration as

\[
d^3 \mathbf{p}_T dE_T dy = d^2 \mathbf{p}_T dy.
\]

Let us further express the 4-momentum \(p\) in a 4-component vector in terms of \(p_T(= |\mathbf{p}_T|), y\) and the azimuthal angle \(\varphi\) as

\[
p = (E_T \cosh y, E_T \cos \varphi, E_T \sin \varphi, E_T \sinh y).
\]

Now the measure of integration over the phase space becomes

\[
d^3 \mathbf{p}_T = p_T dp_T d\varphi dy.
\]

The above can be easily understood if we express the 2-component vector \(\mathbf{p}_T\) in polar coordinates \((p_T \cos \varphi, p_T \sin \varphi)\). Then we can write \(d^2 \mathbf{p}_T = p_T dp_T d\varphi\).

Now let us turn our attention to the phase space integration of a 4-body final state process in a hadron collider. The hadronic cross section in \(k_T\)-factorization approach for the off-shell hard scattering amplitude of Appendix A is (see Eq. 2.1 of Ref. [46])

\[
\sigma = \int \frac{1}{x_1 x_2 S} \prod_{i=1}^2 \frac{dx_i}{x_i} f_g(x_i, k_{1\perp}^2, \mu_F^2) \frac{d^2 k_{1\perp}}{\pi} \times \frac{4}{(2\pi)^3 2 p_0} |\mathcal{M}|^2 \times (2\pi)^4 \delta(4) \left( \sum_{i=1}^n k_i - \sum_{j=1}^m p_j \right).
\]

Introducing the azimuthal angles \(\varphi_{1,2}\) of the off-shell gluons into Eq. (28) we can write the gluon phase space as

\[
f \int d^2 k_{1\perp} = \int d k_{1\perp} d k_{2\perp} d \varphi = \int d k_{1\perp} \frac{d \varphi_{1,2}}{2 \pi}.
\]

Let us write the 4-momenta of the partons, using Eq. (18), as

\[
k_1 = \left( x_1 \frac{\sqrt{s}}{2}, k_{1\perp}, x_1 \frac{\sqrt{s}}{2} \right),
\]

\[
k_2 = \left( x_2 \frac{\sqrt{s}}{2}, k_{2\perp} - x_2 \frac{\sqrt{s}}{2} \right).
\]

Here \(z_1 \rightarrow x_1\) and \(z_2 \rightarrow x_2\). Also the 4-momenta of the leptons are

\[
p_f = (|\mathbf{p}_f| \cosh y_f, \mathbf{p}_f, |\mathbf{p}_f| \sinh y_f).
\]

Using the above designation of the gluon and lepton momenta we can simplify the delta function of Eq. (28) as

\[
\delta^{(4)} \left( \sum_{i=1}^n k_i - \sum_{j=1}^m p_j \right) = \frac{1}{S} \delta^{(2)} \left( \sum_{i=1}^n k_{i\perp} - \sum_{j=1}^m p_{j\perp} \right)
\]

\[
\times \delta \left( x_1 - \sum_{f=1}^m |\mathbf{p}_f| \cosh y_f \right) \delta \left( x_2 - \sum_{f=1}^m |\mathbf{p}_f| \cosh y_f \right).
\]
Putting the results from Eqs. (29) and (32) into Eq. (28) and integrating over $x_1$ and $x_2$ we get
\[
\sigma = \int \frac{2}{x_1^2 S} f_d(x_i, k_{i \perp}^2, \mu_F^2) d k_{i \perp}^2 \frac{d \phi_i}{2\pi} \times \prod_{f=1}^{4} d^2 p_{f \perp} dy_f \frac{|M|^2}{2^{12} \pi^8} \delta^{(2)} \left( \sum_{i=1}^{2} k_{i \perp} - \sum_{f=1}^{4} p_{f \perp} \right). \tag{33}
\]
where
\[
x_1 = \sum_{f=1}^{4} \frac{|p_{f \perp}|}{\sqrt{S}} e^{x_f}, \quad x_2 = \sum_{f=1}^{4} \frac{|p_{f \perp}|}{\sqrt{S}} e^{-x_f},
\]
\[
\sum_{i=1}^{2} k_{i \perp} = \sum_{f=1}^{4} p_{f \perp}. \tag{34}
\]
Now integration over $p_{\perp}$ in Eq. (33) gives us
\[
\sigma = \int \frac{2}{x_1^2 S} f_d(x_i, k_{i \perp}^2, \mu_F^2) d k_{i \perp}^2 \frac{d \phi_i}{2\pi} \prod_{f=1}^{4} d^2 p_{f \perp} dy_f |M|^2 = \frac{\pi}{2^{12} \pi^8}. \tag{36}
\]
In the last step above we have used $d^2 p_{f \perp} = 2\pi p_{f \perp} dp_{f \perp} = \pi dp_{f \perp}^2, (f = 1, 2, 3)$.

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