Numerical studies on vortices in rotating dipolar Bose-Einstein condensates

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Abstract.

We study the formation and dynamics of vortices in quasi two-dimensional dipolar Bose-Einstein condensates (BECs). We solve the time dependent two-dimensional Gross-Pitaevskii equation by using a combined split-step Crank-Nicolson (SSCN) and Fast Fourier-Transform (FFT) based numerical scheme and investigate the vortex dynamics in dipolar BECs trapped with harmonic and optical lattice potentials. The consequence of dipole-dipole interaction on vortex nucleation and number of vortices has been analysed. We observe that the breaking of symmetry due to anisotropic dipolar interaction enormously increase the speed of creation of vortices.

1. Introduction

Bose-Einstein condensates provide an adaptable testing environment for superfluidity. An important property of a superfluid is the creation of quantized vortices. The concept of quantized vortices in superfluid was first developed theoretically by Onsager and Feynman for liquid helium [1, 2]. Their view on vortices received a strong support from the experiments of Hall and Vinen in superfluid helium II [3]. The first experiment directly aimed to detect the quantization of circulation is performed by Vinen in 1961 [4]. One of the most significant phenomena associated with BECs is the creation of vortices under rotation. The successful experimental realization of Bose-Einstein condensates of alkali atoms during last century has provided a pathway to observe the quantized vortices in a controllable quantum system [5, 6, 7, 8, 9, 10]. In BEC experiments vortices are nucleated with help of either by laser stirring or by rotating magnetic traps, and they are observed above certain critical rotational velocity. Experimentally a single vortex has been created in two-component BECs using phase engineering technique [5]. Multiply charged vortices has been studied and critical velocity also was calculated in stirred BECs of $^{87}$Rb atoms confined in an axis symmetric magnetic trap [6]. Later, formation of highly ordered triangular vortex lattices contained over more than 100 number of vortices has been observed in Na atoms [7]. Also, BECs in optical lattice under rotation have been shown to exhibit various interesting properties such as vortex nucleation, vortex structures, vortex structural phase transition, pinning effect of vortices with the peak of shallow OL [11, 12, 13].

Several numerical schemes have been proposed to study the properties of BECs including vortices [14]. For instance, finite element method has been used to study stationary state of vortices and to solve Hartree-Fock-Bogoliubov treatment of a vortex lattice in rapidly...
rotating BECs [15, 16]. Vortices in anisotropic traps has investigated by discrete variable approximation [17] and BEC vortices in OL has studied using two-parameter continuation algorithm with spectral collocation methods [18]. The numerical schemes used in earlier studies are mainly focussed on the alkali BECs with with local and isotropic interaction. The alkali metal atoms used in early BEC experiments have negligible dipole moment. However, most bosonic atoms and molecules have large dipole moments and BECs of $^{52}$Cr [19], $^{168}$Er [20] and $^{164}$Dy [21], with strong long-range dipolar interaction superposed, have been realized. The recent progress in the study of BECs of dipolar atoms with non-local and anisotropic interaction has exposed various fascinating physics due to the peculiar competition between short-range contact interaction and long-range dipolar interaction [22]. Due to the presence of anisotropic interaction in dipolar BECs one can expect the distinct feature during quantized circulation. Studies based on mean-field models revealed several interesting properties of dipolar BECs. For example, it has been shown that the anisotropic nature of dipolar BEC strongly influence the number, structure and stability of the vortices. The critical rotation frequency for vortex nucleation found to decrease as the strength of dipolar interaction increases [23, 24]. It has also been noted that the dipolar interaction increases the number of vortices while the contact interaction enhances the vortex stability [24].

In this paper, we present a combined split-step Crank-Nicolson and Fast Fourier-Transform (FFT) based numerical scheme to study the vortex dynamics in dipolar BECs trapped with harmonic and optical lattice potentials. In section 2 we introduce the theoretical model for dipolar Gross-Pitaevskii equation in rotating frame with different trapping potentials. We provide the numerical method for solving two-dimensional time dependent dipolar Gross-Pitaevskii equation with angular momentum in section 3. Numerical results and conclusions are given in sections 4 and 5, respectively.

2. Theoretical model

The dynamics of rotating BECs can be studied using mean field Gross-Pitaevskii (GP) equation [10]. A dipolar BEC with $N$ atoms, each of mass $m$ at absolute zero temperature in a rotating frame can be described by the Gross-Pitaevskii equation as [25]

$$i \frac{\partial \phi(r,t)}{\partial t} = \left[ -\frac{1}{2} \nabla^2 + V(r) + 4\pi a N|\phi(r,t)|^2 - \Omega L_z + N \int U_{dd}(r-r')|\phi(r',t)|^2 d^3r' \right] \phi(r,t),$$

(1)

where $V(r) = V_{ho}(r) + V_{OL}(\rho)$ is the confining axially symmetric harmonic potential and optical lattice potential, $\phi(r,t)$ the wave function at time $t$ with normalization $\int |\phi(r,t)|^2 d^3r = 1$, $a$ the atomic scattering length. The axial and radial trap frequencies of the harmonic potential, $V_{ho}(r)$, are $\Omega_z \omega$ and $\Omega_\rho \omega$, respectively, and are related with the trap aspect ratio, $\lambda = \Omega_z/\Omega_\rho$. In equation (1) length is measured in units of harmonic oscillator length $l = \sqrt{\hbar/m\omega}$, frequency in units of $\omega$, time $t$ in units of $\omega^{-1}$. $L_z = -i(x\partial_y - y\partial_x)$ corresponds to the $z$-component of the angular momentum due to the rotation of the dipolar BEC about $z$ axis with angular velocity $\Omega$. Here $\Omega$ is expressed in units of the radial trap frequency $\Omega_\rho \omega$. The integral term in equation (1) accounts for the dipole-dipole interaction with

$$U_{dd}(x) = a_{dd} \frac{1 - 3 \cos^2 \theta}{|x|^3},$$

(2)

where $x = r - r'$ determines the relative position of dipoles and $\theta$ is the angle between $x$ and the direction of polarization, $z$. The constant $a_{dd} = \mu_0 \mu^2 m/(12\pi \hbar^2)$ is a length characterizing the strength of dipolar interaction and, its experimental value for $^{52}$Cr, $^{168}$Er and $^{164}$Dy are $16a_0$. 


66a₀ and 131a₀, respectively, where a₀ is the Bohr radius [26]. ϱ corresponds to the magnetic dipole moment of a single atom and µ₀ the permeability of free space.

A two-dimensional optical lattice superimposed on a three-dimensional harmonic trap is given, in dimensionless form, as

\[ V(r) = \frac{1}{2} \Omega^2 \rho^2 + \frac{1}{2} \Omega^2 z^2 + V_0 \left[ \sin^2(kx) + \sin^2(ky) \right], \]  

(3)

where \( r = (\rho, z) \), with \( \rho \) the radial coordinate and \( z \) the axial coordinate, \( V_0 \) is the depth of the OL and \( k \) is the wave number. We consider the highly oblate dipolar BEC with trap aspect ratio \( \lambda = 50 \) for our present study. In strong axial traps the dynamics of the rotating dipolar BEC can effectively be studied in two-dimension by simply integrating out the \( z \) dependence in equation (1), as [27, 28, 29],

\[
i \frac{\partial \phi_{2D}(\rho, t)}{\partial t} = \left[ -\frac{\nabla^2}{2} + V_{2D} - \Omega L_z + \frac{4\pi a_N}{\sqrt{2\pi d_z}} |\phi_{2D}(\rho, t)|^2 \right. \\
\left. + \frac{4\pi a_{dd}N}{\sqrt{2\pi d_z}} \int \frac{d^2k_\rho}{(2\pi)^2} e^{ik_\rho \cdot \rho} \tilde{n}(k_\rho) \times h_{2D} \left( \frac{k_\rho dz}{\sqrt{2}} \right) \right] \phi_{2D}(\rho, t).
\]

(4)

where \( V_{2D} = (x^2 + y^2)/2 + V_0 \left[ \sin^2(kx) + \sin^2(ky) \right] \) is a combined two-dimensional harmonic trap and optical lattice potential. In equation (4), \( \tilde{n}(k_\rho) = \int \exp(ik_\rho \cdot \rho) |\phi_{2D}(\rho)|^2 d\rho \). \( k_\rho \equiv (k_x, k_y), h_{2D}(\xi) = 2 - 3\sqrt{\pi} \exp(\xi^2) \text{erfc}(\xi) \), and the dipolar term is written in Fourier space. The lattice spacing \( (d_{\text{lat}}) \) and amplitude \( (V_0) \) of optical lattice potential can be varied by tuning the frequency and intensity of lasers. In the present study, the OL spacing is chosen as \( d_{\text{lat}} = \lambda_L/2 \approx 534 \text{nm} \), where \( \lambda_L = 1064 \text{nm} \) is the wavelength of the laser used in experiments [30]. The corresponding dimensionless parameters for OL spacing, \( d_{\text{lat}} = \pi/k = 0.534 \) and \( k = 1.87\pi \).

3. Numerical Method

The Crank-Nicolson method is a finite difference method used for numerically solving the heat equation and similar partial differential equations. The method was developed by J. Crank and P. Nicolson in the 20th century [31]. This method is a reliable and stable routine for numerically solving time-dependent Schrödinger-like equations.

We consider a split-step Crank-Nicolson based numerical scheme [32, 33] for studying rotational dynamics of dipolar BECs using quasi-2D GP equation with angular momentum.

\[
i \frac{\partial}{\partial t} \varphi(x, y; t) = \left[ -\frac{1}{2} \frac{\partial^2}{\partial \rho^2} + V_{2D} + \frac{4\pi a_N}{\sqrt{2\pi d_z}} |\varphi(x, y; t)|^2 - \Omega L_z \right. \\
\left. + \frac{4\pi a_{dd}N}{\sqrt{2\pi d_z}} \int \frac{d^2k_\rho}{(2\pi)^2} \exp(ik_\rho \cdot \rho) \tilde{n}(k_\rho) h_{2D} \left( \frac{k_\rho dz}{\sqrt{2}} \right) \right] \varphi(x, y; t),
\]

(5)

The Hamiltonian \( H \) can be conveniently broken into three pieces \( H = H_1 + H_2 + H_3 \), where

\[
H_1 = V_{2D} + \frac{4\pi a_N}{\sqrt{2\pi d_z}} |\varphi(x, y; t)|^2 + \frac{4\pi a_{dd}N}{\sqrt{2\pi d_z}} \int \frac{d^2k_\rho}{(2\pi)^2} \exp(ik_\rho \cdot \rho) \tilde{n}(k_\rho) h_{2D} \left( \frac{k_\rho dz}{\sqrt{2}} \right),
\]

(6)

\[
H_2 = -\frac{\partial^2}{\partial x^2} - i\Omega y \frac{\partial}{\partial x},
\]

(7)

\[
H_3 = -\frac{\partial^2}{\partial y^2} + i\Omega x \frac{\partial}{\partial y}.
\]

(8)
Essentially we split (5) into
\[ i \frac{\partial}{\partial t} \varphi(x, y; t) = \left[ \frac{4\pi a N}{\sqrt{2\pi d_x}} |\varphi(x, y; t)|^2 + \frac{4\pi a_d N}{\sqrt{2\pi d_z}} \times \int \frac{d^2 k_\rho}{(2\pi)^2} \exp(i\mathbf{k}_\rho \cdot \mathbf{r}) \tilde{n}(\mathbf{k}_\rho) h_{2D}(\frac{k_\rho d_z}{\sqrt{2}}) \right] \varphi(x, y; t) \equiv H_1 \varphi(x, y; t) \] (9)
\[ i \frac{\partial}{\partial t} \varphi(x, y; t) = -\frac{\partial^2}{\partial x^2} \varphi(x, y; t) - i\Omega y \frac{\partial}{\partial x} \varphi(x, y; t) \equiv H_2 \varphi(x, y; t) \] (10)
\[ i \frac{\partial}{\partial t} \varphi(x, y; t) = -\frac{\partial^2}{\partial y^2} \varphi(x, y; t) + i\Omega x \frac{\partial}{\partial y} \varphi(x, y; t) \equiv H_3 \varphi(x, y; t) \] (11)

In equation (6) the dipolar term is directly written in momentum space and the FFT algorithm is carried out in Cartesian coordinates.

We first solve equation (9) with an initial value \( \varphi(x, y; t_0) \) at \( t = t_0 + \Delta \), where \( \Delta \) is the time step. Then this intermediate solution is used as initial value to solve (10) and (11) yielding the final solution at \( t = t_0 + \Delta \) as \( \varphi(x, y; t_0 + \Delta) \). This procedure is repeated \( n \) times to get the final solution at a given time \( t_{\text{final}} = t_0 + n\Delta \). The time variable is discretized as \( t_n = n\Delta \).

The solution, say \( \varphi^n \), is advanced first over the time step \( \Delta \) at time \( t_n \) by solving the GP equation (9) with \( H = H_1 \) to produce an intermediate solution \( \varphi^{n+1/2} \). Here \( \varphi^n \) is the discretized wave function at time \( t_n \). As there is no derivative in \( H_1 \), this propagation is performed essentially exactly for small \( \Delta \) through the operation
\[ \varphi^{n+1/2} = \bigcirc_{\text{nd}}(H_1) \varphi^n \equiv e^{-i\Delta H_1} \varphi^n, \] (12)

where \( \bigcirc_{\text{nd}}(H_1) \) denotes time-evolution operation with \( H_1 \) and the suffix 'nd' denotes non-derivative.

Next we perform the time propagation corresponding to the operators \( H_2 \) and \( H_3 \) numerically by the semi-implicit Crank-Nicolson scheme [34]: To employ the Crank-Nicolson scheme the GP is carried out in Cartesian coordinates.

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\[ i(\varphi_{i,j}^{n+1} - \varphi_{i,j}^{n+1/2}) \Delta = \frac{1}{2h_x^2} \left[ (\varphi_{i+1,j}^{n+1} - 2\varphi_{i,j}^{n+1} + \varphi_{i-1,j}^{n+1}) + (\varphi_{i+1,j}^{n+1/2} - 2\varphi_{i,j}^{n+1/2} + \varphi_{i-1,j}^{n+1/2}) \right] \]
\[ -i\Omega y \frac{\Delta y}{4h_x} \left[ (\varphi_{i+1,j}^{n+1} - \varphi_{i-1,j}^{n+1}) + (\varphi_{i+1,j}^{n+1/2} - \varphi_{i-1,j}^{n+1/2}) \right], \] (13)

where for the spherically-symmetric \( \varphi_i^n = \varphi(x_i; t_n) \) refers to \( x_i = ih_x, \ i = 0, 1, 2, ..., N_x \) and \( h_x \) is the space step. For the discretization along \( x \), we choose \( x_i = -N_x h_x/2 + ih_x, \ i = 0, 1, 2, ..., N_x \).

The above procedure results in a series of tridiagonal sets of equations (13) in \( \varphi_{i+1,j}^{n+1}, \varphi_{i,j}^{n+1}, \) and \( \varphi_{i-1,j}^{n+1} \) at time \( t_{n+1} \), which are solved using the proper boundary conditions. The tridiagonal equations emerging from (13) are written explicitly as [34]

\[ A_1^{-} \varphi_{i-1,j}^{n+1} + A_1^{0} \varphi_{i,j}^{n+1} + A_1^{+} \varphi_{i+1,j}^{n+1} = b_i, \] (14)

where

\[ b_i = \frac{i\Delta}{2h_x^2} (\varphi_{i+1,j}^{n+1/2} - 2\varphi_{i,j}^{n+1/2} + \varphi_{i-1,j}^{n+1/2}) - \frac{\Delta\Omega y h_y}{4h_x^2} (\varphi_{i+1,j}^{n+1/2} - \varphi_{i-1,j}^{n+1/2}) + \varphi_{i,j}^{n+1/2}, \] (15)
and $A_0^0 = 1 + i\Delta / h_x^2$, $A_i^- = -\Delta/(2h_x)(i/h_x + \Omega y_i/2)$, $A_i^+ = \Delta/(2h_x)(-i/h_x + \Omega y_i/2)$. All quantities in $b_i$ refer to time step $t_{n+1/2}$ and are considered known. The only unknowns in (14) are the wave forms $\varphi^{n+1}_{i+1,j}$ and $\varphi^{n+1}_{i,j}$ at time step $t_{n+1}$. To solve (14), we assume the one-term forward recursion relation

$$\varphi^{n+1}_{i+1,j} = \alpha_i \varphi^{n+1}_{i,j} + \beta_i,$$

where $\alpha_i$ and $\beta_i$ are coefficients to be determined. Substituting (16) in (14) we obtain

$$A_i^- \varphi^{n+1}_{i-1,j} + A_i^0 \varphi^{n+1}_{i,j} + A_i^+ (\alpha_i \varphi^{n+1}_{i,j} + \beta_i) = b_i,$$

which leads to the solution

$$\varphi^{n+1}_{i,j} = \gamma_i (A_i^- \varphi^{n+1}_{i-1,j} + A_i^+ \beta_i - b_i),$$

with

$$\gamma_i = -1/(A_i^0 + A_i^+ \alpha_i).$$

From (16) and (18) we obtain the following backward recursion relations for the coefficients $\alpha_i$ and $\beta_i$

$$\alpha_{i-1} = \gamma_i A_i^-, \quad \beta_{i-1} = \gamma_i (A_i^+ \beta_i - b_i).$$

We shall use the recursion relations (18), (19) and (20) in a backward sweep of the lattice to determine $\alpha_i$ and $\beta_i$ for $i$ running from $N_x - 2$ down to 0. The initial values chosen are $\alpha_{N_x-1} = 0$, $\beta_{N_x-1} = \varphi^{n+1}_{N_x}$. This ensures the correct value of $\varphi$ at the last lattice point. After determining the coefficients $\alpha_i$, $\beta_i$ and $\gamma_i$, we can use the recursion relation (16) from $i = 0$ to $N_x - 1$ to determine the solution for the entire space range using the starting value $\varphi^{n+1}_{0}(= 0)$ known from the boundary conditions. The value at the last lattice point is also taken to be known ($= 0$). Thus we have determined the solution by using two sets of recursion relations across the lattice each involving about $N_x$ operations. A similar discretization procedure is followed for $H_3$ also.

All the numerical simulations in this manuscript are carried out with $dx = dy = 0.2$ (space step) and $dt = 0.004$ (time step).

4. Vortices in dipolar BECs

In this section, we show the vortex nucleation in the dipolar BECs by numerical simulations of the time-dependent GP equation (4) using the numerical scheme discussed above. For this purpose, we fix the parameters at $N = 10^4$ atoms and $\Omega = 0.5$.

4.1. Dynamics of a rotating dipolar BEC in a harmonic trap ($V_0 = 0$)

First, we consider the harmonically trapped dipolar BECs with different dipolar strengths. Initially we prepare the ground state wavefunction by solving (4) numerically using imaginary time propagation in the presence of harmonic trap but without angular momentum ($\Omega = 0$). The ground state is then evolved in real time propagation by including angular momentum ($\Omega \neq 0$). In addition, a phenomenological dissipation is included to facilitate the smooth vortex formation [37, 38, 39]. The dissipation is introduced by replacing ‘$i$’ with ‘$(i - \gamma)$’ in the time dependent equation (4), where $\gamma \sim 10^{-5}$ accounts for the strength of dissipation.

The condensate begins to rotate after applying the rotation. During the rotation the condensate elongates and quadrupole shape deformation takes place. The boundary surfaces
Figure 1. Contour plots of the densities, $|\phi_{2D}(\rho, t)|^2$, showing the development of vortices in a dipolar BEC $a = 0$, $N = 10000$ and $\lambda = 50$ after the trapping potential begins to rotate with $\Omega = 0.5$ for $a_{dd} = 16 a_0$: (a) $t = 0$ (b) $t = 62000 \omega^{-1}$, (c) $t = 160000 \omega^{-1}$, for $a_{dd} = 66 a_0$ (e) $t = 0$, (f) $t = 15000 \omega^{-1}$ (g) $t = 70000 \omega^{-1}$, and for $a_{dd} = 131 a_0$ (i) $t = 0$, (j) $t = 14000 \omega^{-1}$ (k) $t = 58000 \omega^{-1}$. The phase distributions of condensate wave function $\phi_{2D}(\rho, t)$ are shown in (d), (h) and (l).

becomes unstable and ripples are formed and the surface ripples initiate the vortex nucleation. The nucleated vortices enter into the condensate and reach a stable configuration as time progress. In the following we compare the time taken for the creation of vortex or vortices and number of vortices corresponding for different dipolar strengths.

Figures 1(a)-(c) show the development of vortices for $a_{dd} = 16 a_0$ ($^{52}$Cr BEC). In this case the surface ripples are formed at $t \sim 62000 \omega^{-1}$. Then it attains an equilibrium state of 4 vortices at $t \sim 160000 \omega^{-1}$ as shown in figure 1(c). The corresponding phase pattern of final wavefunction is shown in figure 1(d).

Figures 1(e)-(g) show the development of vortices in the case of $a_{dd} = 66 a_0$ ($^{168}$Er BEC). Here the surface ripples are formed at time $t \sim 15000 \omega^{-1}$, which is much faster when compared to that of $^{52}$Cr BEC. Further, about 13 vortices form a stable configuration as shown in figure 1(g) and figure 1(h) shows the corresponding phase profile.

We have also shown the snapshots of vortices for the case of $^{164}$Dy BEC ($a_{dd} = 131 a_0$) in figure 1(i)-(k). The surface ripples are formed more rapidly, at $t \sim 14000 \omega^{-1}$ as shown in
figure 1(j), and a very stable pattern with 20 vortices, as seen in figure 1(k), are created for $t = 58000 \omega^{-1}$. Figure 1(l) depicts the phase profile of the corresponding wave function.

Thus the above observation clearly indicates that the time taken for the creation of steady vortex pattern in dipolar BECs decreases considerably with the increase of dipolar strength. Also, number of vortices is significant with respect to dipolar strengths. Next we study the effect of optical lattice on the vortices in dipolar BEC.

4.2. Dynamics of a rotating dipolar BEC in an optical lattice

In order to see the effect of optical lattice, a square optical lattice is introduced ($V_0 \neq 0$) in addition to harmonic trapping potential. In experiments the alkali BEC is loaded into a static square optical lattice and then studied by applying rotation [11]. The presence of OL found to suppress the shape deformation and quadrupole oscillations in conventional BECs [40]. Optical lattice also helps to produce the vortices rapidly in BECs. In figure 2 we show the time development of the vortices from the contours of the density profiles $|\phi_{2D}|^2$ by fixing the OL depth at $V_0 = 10$. The dipole-dipole interaction normally ease the vortex nucleation due

![Figure 2](image_url)

**Figure 2.** Contour plots of the densities, $|\phi_{2D}(\rho,t)|^2$, showing the development of vortices in a dipolar BEC loaded in OL with amplitude $V_0 = 10$, $a = 0$, $N = 10000$ and $\lambda = 50$ after the trapping potential begins to rotate with $\Omega = 0.5$ for $a_{dd} = 16 a_0$: (a) $t = 0$ (b) $t = 40000 \omega^{-1}$, (c) $t = 60000 \omega^{-1}$, for $a_{dd} = 66 a_0$ (e) $t = 0$, (f) $t = 8000 \omega^{-1}$ (g) $t = 24000 \omega^{-1}$ and for $a_{dd} = 131 a_0$ (i) $t = 0$, (j) $t = 900 \omega^{-1}$ (k) $t = 1000 \omega^{-1}$. The phase distributions of condensate wave function $\phi_{2D}(\rho,t)$ are shown in (d) and (h).
Table 1. The approximate time ($t_{vor}$) for the creation of equilibrium vortices and the number of vortices ($N_v$) in the presence and absence of OL for different dipolar BECs.

| BEC     | $t_{vor}$ ($\omega^{-1}$) | $N_v$ |
|---------|---------------------------|-------|
| $^{52}$Cr | $V_0 = 0$ | $V_0 = 10$ | $V_0 = 0$ | $V_0 = 10$ |
|         | 160 000 | 60 000 | 4 | 4 |
| $^{168}$Er | 70 000 | 24 000 | 13 | 13 |
| $^{164}$Dy | 58 000 | — | 20 | — |

to symmetry breaking. In addition, the presence of OL provides a supplementary symmetry breaking due to which the speed of creation of vortices has enhanced considerably. Figures 2(a)-(c) show the development of vortices for $a_{dd} = 16 a_0$ ($^{52}$Cr BEC). In this case, the surface ripples are formed at $t \sim 40 000 \omega^{-1}$ and attain an equilibrium state of vortices pinned within the peaks of OL. About 4 vortices form a stable pattern as shown in figure 2(c). The corresponding phase pattern of final wavefunction is shown in figure 2(d). Figures 2(e)-(g) show the development of vortices in the case of $a_{dd} = 66 a_0$ ($^{168}$Er BEC). Here the surface ripples are formed at time $t \sim 8000 \omega^{-1}$, which is much faster when compared to that of $^{52}$Cr BEC discussed above. Further, about 13 vortices form a stable pinned configuration as shown in figure 2(g) and figure 2(h) shows the corresponding phase profile. We have also shown the development of vortices for the case of $^{164}$Dy atoms in figures 2(i)-(k), where the vortices are created very rapidly at $t \sim 900 \omega^{-1}$. However, the strong dipolar interaction destabilized the condensate. Actually a pure dipolar BEC of with larger dipolar strength, when the contact interaction is made zero, collapses in OL during rotation.

The above observation clearly indicates that the time taken for the creation of steady vortex pattern in dipolar BECs in OL decreases considerably with the increase of dipolar strength. The numerical results are summarized and given in table 1.

5. Conclusion
We have studied numerically the influence of dipole-dipole interaction on the formation of vortices in a rotating dipolar Bose-Einstein condensate of $^{52}$Cr, $^{168}$Er and $^{164}$Dy atoms in harmonic and optical lattice potential. We have presented a combined split-step Crank-Nicolson and FFT based numerical scheme to solve the time-dependent quasi-two dimensional Gross-Pitaevskii equation. Using the above scheme we have analysed the dynamics of the creation of vortices in dipolar BECs. The time taken for the creation of vortices in dipolar BECs has been calculated. The presence of dipolar interaction and optical lattice potential greatly enhance the fast creation of ordered vortices in a dipolar BEC. The number of vortices increases significantly with respect to dipolar strength.

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