BREAKDOWN OF DUALITY IN (0,2) SUPERSTRING MODELS

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ABSTRACT

After pointing out the role of the compactification lattice for spectrum calculations in orbifold models, I discuss modular discrete symmetry groups for $Z_N$ orbifolds. I consider the $Z_7$ orbifold as a nontrivial example of a (2,2) model and give the generators of the modular group for this case, which does not contain $[SL(2,\mathbb{Z})]^3$ as had been speculated. I also discuss how to treat cases where quantized Wilson lines are present. I consider in detail an example, demonstrating that quantized Wilson lines affect the modular group in a nontrivial manner. In particular, I show that it is possible for a Wilson line to break $SL(2,\mathbb{Z})$.

1. Introduction

The space of string vacua is locally parametrized by moduli, which are marginal deformations of the underlying conformal field theory (CFT). In the low energy effective theory the moduli correspond to vacuum expectation values of massless scalar fields that have flat potentials to all orders in perturbation theory. An intriguing feature of string compactifications is that the natural parametrization of the moduli space label the points in a redundant way, since all physical quantities are invariant under the action of some discrete group acting on the moduli.

The motivations for studying this modular discrete symmetry groups for compactifications of the heterotic string are manifold. The most obvious one is that its knowledge allows a restriction to a fundamental domain of moduli space, reducing the number of string vacua. Next, being an exact symmetry it should persist in any approximation, and in particular in the low energy effective supergravity theory. This constrains the superpotential, the gauge kinetic functions, etc. Assuming the exactness of this symmetry even after taking into account nonperturbative string effects, gives a powerful tool for the construction of nonperturbative potentials. They can fix the moduli vacuum expectation values (vev’s) and lift the vacuum degeneracy. Recent discussions of soft supersymmetry breaking terms and minimal string models were restricted to the modular group $SL(2,\mathbb{Z})$. It will be one of the main results of this talk that this group plays a much smaller role as a symmetry...
in moduli space as previously believed. Finally, modular symmetries may also play
an important role in the context of gauge coupling unification. Here threshold
corrections potentially explain the discrepancy between the reduced Planck scale
and the unification scale. Again, the relevant threshold terms have to transform
appropriately under the given symmetry group\(^2\).

I will discuss these modular symmetries for orbifold models. They represent
a simple construction and the orbifold CFT is exactly solvable. This implies that the
discrete symmetry groups are exactly derivable, as well, and that the phenomenol-
ogy of orbifold models can be pushed forward very far. At the same time “quasi re-
alistic” models with 3 fermion generations transforming under \(SU(3) \times SU(2) \times U(1)^n\)
belong to this class\(^3\). A natural hierarchy of fermion masses arises due to the fixed
point structure of the twisted sectors. Finally, exact deformations of fermionically
constructed models can be obtained via the \(Z_2 \times Z_2\) orbifold.

2. Symmetric \(Z_N\) Orbifolds

The possible point groups leading to \(D = 4, N = 1\) supersymmetric \(Z_N\) orbi-

folds are well known\(^4\):

\[
Z_3, \quad Z_4, \quad Z_6, \quad Z'_6, \quad Z_7, \quad Z_8, \quad Z'_8, \quad Z_{12}, \quad Z'_{12}.
\]

(1)

However, in order to find all the corresponding models a classification of the possible
compactification lattices is necessary. As an example of how the underlying lattice
affects basic properties of the model consider the \(Z_4\) orbifold. It can be realized e.g.
using three copies of the root lattice of \(SO(4)\) with a twist simultaneously acting as
90° rotations in two copies and as a reflection in the last one:

\[
\Lambda_1 = [SO(4)]^3 : \quad e_{1,3} \rightarrow e_{2,4} \quad e_{2,4} \rightarrow -e_{1,3} \quad e_{5,6} \rightarrow -e_{5,6}.
\]

Alternatively, one can utilize two copies of the root lattice of \(SU(4)\) with Coxeter
twists acting in them:

\[
\Lambda_2 = [SU(4)]^2 : \quad e_{1,4} \rightarrow e_{2,5} \quad e_{2,5} \rightarrow e_{3,6} \quad e_{3,6} \rightarrow -\sum_{i=1}^{3} e_{i,i+3}.
\]

(2)

It has been shown that for two models to be equivalent, there must be a matrix
\(M \in GL(d, Z)\) with respect to which the integer valued twist (lattice basis) matrices
\(\theta\) are similar\(^5\), i.e.

\[
\exists \quad M \in GL(d, Z) \quad \text{with} \quad \theta' = M \theta M^{-1}.
\]

(3)

For the case at hand such a matrix cannot be found. What does that mean for the
spectra?

Clearly, in the untwisted sector the lattice \(\Lambda\) (winding modes) and its dual
\(\Lambda^*\) (momenta) only affect the massive string modes. The untwisted massless states
are the same. Since there are no fixed tori in the first twisted sector, it is completely
independent of \(\Lambda\). The number of generations coming from this sector equals the
number of fixed points, which in turn can be determined with help of the Lefschetz fixed point theorem. In our case we find

\[ \text{Det}(1 - \theta) = 16. \]

(4)

The second twisted sector, however, *does* depend on \( \Lambda \) due to the complex plane which is left fixed under the twist action. The number of generations coming from such a sector is given by the number of fixed tori.

In the case of the \([SO(4)]^3\) lattice the fixed plane is orthogonal to the twisted directions. This implies that the Lefschetz fixed point theorem is again applicable if restricted to the twisted directions. One finds 16 fixed tori. A closer investigation of twist phases reveals that 10 of them correspond to generations and 6 to anti-generations. In contrast, in the \([SU(4)]^2\) model the Lefschetz theorem cannot be employed. Explicit construction of the fixed tori gives rise to four generations and no antigenerations.

The same result can be obtained by computing the one loop partition function. The relevant projections in the two cases are

\[ \Lambda_1 : \frac{1}{4}[16 + 4\Delta + 16\Delta^2 + 4\Delta^3], \]

(5)

\[ \Lambda_2 : \frac{1}{4}[4 + 4\Delta + 4\Delta^2 + 4\Delta^3], \]

(6)

with \( \Delta = +1 \) for generations and \( \Delta = -1 \) for antigenerations. The degeneracy factors in Eqs. (5) and (6) are different because the twist acts as a rotation in the even, selfdual lattice composed of winding and momentum states: In the orthogonal case of \( \Lambda_1 \) the invariant sublattice is even and selfdual itself and thus the volume factor which one encounters when performing modular world sheet transformations is trivially one. On the other hand, a nontrivial volume factor is found for \( \Lambda_2 \).

From Eqs. (5) and (6) it is also obvious, that the Euler number determining the number of net generations is independent of the underlying compactification lattice. This one indeed expects, since this number can be computed by a formula conjectured by Dixon, Harvey, Vafa and Witten\(^4\) and proved by Markushevich, Olshanetskii and Perelomov\(^7\), which only uses twist eigenvalues.

Because of their appealing phenomenology and a considerable confusion in the literature, it was worthwhile to construct all supersymmetric, symmetric \( Z_N \) orbifolds with \((2,2)\) world sheet supersymmetry\(^6\). The resulting 18 models have different massive and massless spectra, different modular symmetry groups and consequently different phenomenologies.

3. Duality and Discrete Symmetry Groups

3.1. Overview

To begin the discussion of discrete symmetries in moduli space, consider as a simple example the \( Z_3 \) orbifold in two dimensions. The twist acts in the root
lattice of $SU(3)$ and the scaling deformations thereof. There is also a continuous antisymmetric tensor field compatible with the twist:

$$g = R^2 \left( \begin{array}{cc} 2 & -1 \\ -1 & 2 \end{array} \right), \quad b_{ij} = \left( \begin{array}{cc} 0 & -b \\ b & 0 \end{array} \right). \quad (7)$$

The orbifold radius $R$ and the antisymmetric tensor parameter $b$, which are the real untwisted moduli of the model, can be combined to the complex parameter

$$\lambda = b + i \sqrt{\text{Det} g}, \quad (8)$$

which takes values in the complex upper half plane.

Duality symmetry is the statement that the exchange of winding and momentum quantum numbers,

$$n \rightarrow W m, \quad m \rightarrow W^{-1T} n, \quad (9)$$

and a simultaneous transformation of the background,

$$g \pm b \rightarrow W^{-1T} \frac{1}{g \pm b} W^{-1}, \quad (10)$$

lead to the same physical theory. Here the matrix $W \in GL(2, \mathbb{Z})$ has to satisfy the condition\(^8\)

$$\theta W = W \theta^{-1T}. \quad (11)$$

The transformation of the complex modulus $\lambda$ takes the particularly simple form of an inversion,

$$\lambda \rightarrow -1/\lambda. \quad (12)$$

Inclusion of the *axionic* shift symmetry,

$$b \rightarrow b + 1 \quad \Rightarrow \quad \lambda \rightarrow \lambda + 1, \quad (13)$$

completes the holomorphic symmetry group\(^9\) $PSL(2, \mathbb{Z})$. As shown by Lauer, Mas and Nilles\(^10\) these symmetries are also respected by the correlation functions. Finally, there is a nonholomorphic generator\(^11\),

$$b \rightarrow -b \quad \Rightarrow \quad \lambda \rightarrow -\bar{\lambda}, \quad (14)$$

which induces an exchange of particles and antiparticles in the twisted sectors.

The results of this example can be summarized in compact form by noting that the global structure of the moduli space is given by

$$\mathcal{M}_{T_2/Z_3} = \frac{SU(1,1)}{U(1)} \times \frac{SU(1,1; \mathbb{Z}) \times \mathbb{Z}_2}{SU(1,1; \mathbb{Z}) \times \mathbb{Z}_2}. \quad (15)$$

Written this way, for the holomorphic part one can state the result in saying that the discrete symmetry group is given by the maximal discrete subgroup of the group characterizing the local structure of the moduli space.
The question arises whether this is a general feature. Consider the local structures of the moduli spaces of the $Z_N$ orbifolds:

\[ \mathcal{M}_{Z_3} = \frac{SU(3,3)}{SU(3) \times SU(3) \times U(1)} \]  

\[ \mathcal{M}_{Z_{4,6}} = \frac{SU(2,2)}{SU(2) \times SU(2) \times U(1)} \times \left[ \frac{SU(1,1)}{U(1)} \right]_n \quad n = 1, 2 \]  

\[ \mathcal{M}_{Z_{N \geq 7}} = \left[ \frac{SU(1,1)}{U(1)} \right]_n \quad n = 3, 4. \]  

It has often been assumed that, in fact, the maximal discrete subgroups of the above local structures describe the modular symmetries. Also, extended versions for cases including Wilson lines leading to (0,2) models with broken $E_6 \times E_8$ gauge symmetry are used. For instance, for the $Z_3$ orbifold one would expect the group $SU(3,3; \mathbb{Z})$, and, similarly, $[SU(1,1; \mathbb{Z})]^3 \cong [PSL(2, \mathbb{Z})]^3$ for $Z_7$. Actually, this hypothesis has been confirmed for the $Z_3$ case without Wilson lines. Let us now turn to the more complicated case of the $Z_7$ orbifold.

### 3.2. Z_7 Orbifold

It is defined as the Coxeter twist

\[ e_i \rightarrow e_{i+1} \quad i = 1, \ldots, 5 \quad e_6 \rightarrow - \sum_{i=1}^{6} e_i \]  

in the $SU(7)$ root lattice and the three metrical deformations thereof. Analogously, three independent parameters of the antisymmetric tensor background are compatible with this twist. Again drastic simplifications occur when using complex moduli, e.g.

\[ t_1 = -i \tan \left( \frac{\pi}{7} \right) [g_1 + \frac{s_1}{s_3} g_2 + \frac{s_2}{s_3} g_3] + b_1 + \frac{s_2}{s_1} b_2 + \frac{s_3}{s_1} b_3, \]  

\[ t_2 = -i \tan \left( \frac{2\pi}{7} \right) [g_1 + \frac{s_1}{s_2} g_2 + \frac{s_2}{s_2} g_3] + b_1 - \frac{s_3}{s_2} b_2 - \frac{s_1}{s_2} b_3, \]  

\[ t_3 = -i \tan \left( \frac{3\pi}{7} \right) [g_1 + \frac{s_1}{s_2} g_2 + \frac{s_2}{s_2} g_3] + b_1 - \frac{s_1}{s_3} b_2 + \frac{s_2}{s_3} b_3, \]

with the abbreviations $s_k := \sin \frac{2\pi k}{7}$ and $c_k := \cos \frac{2\pi k}{7}$. These moduli are normalized in such a way that one of the axionic shift symmetries ($b_1 \rightarrow b_1 + 1$) takes a simple form,

\[ T : (t_1, t_2, t_3) \rightarrow (t_1 + 1, t_2 + 1, t_3 + 1). \]  

Similarly, a matrix $W$ (cf. Eqs. (9) and (10)) can be found such that a $PSL(2, \mathbb{Z})$ structure arises,

\[ S : (t_1, t_2, t_3) \rightarrow (-1/t_1, -1/t_2, -1/t_3). \]

However, no transformation changing only one individual complex modulus can be found, no matter which definition for the $t_i$ one uses. This already reveals that
there is no simple $[SL(2, \mathbb{Z})]^3$ symmetry which one would expect on the basis of the aforementioned conjectures. In contrast, a rich structure arises. It is best described by introducing another type of symmetry generators:

Transforming quantum numbers and background according to

$$n \rightarrow Vn, \quad m \rightarrow V^{-1}Tm,$$

and

$$g \pm b \rightarrow V^{-1}T(g \pm b)V^{-1},$$

respectively, gives rise to a symmetry, presupposing the matrix $V \in GL(2, \mathbb{Z})$ satisfies

$$\theta V = V\theta^p.$$  \hfill (27)

$p$ is an integer which is in general allowed to be greater than one\textsuperscript{15}. However, $p$ is required\textsuperscript{11} to have no common divisor with the twist order $N$. The significance of transformations with $p > 1$ is that they correspond to nonholomorphic transformations (cf. Eq. (14)).

For the $Z_7$ case we found the transformation

$$R : (t_1, t_2, t_3) \rightarrow (-2c_1\bar{t}_2, -2c_2t_3, -2c_3t_1).$$ \hfill (28)

$R^3$ yields just complex conjugation. It is interesting that the inclusion of nonholomorphic transformations actually reduces the number of symmetry generators since it relates holomorphic ones. For instance the shift symmetries $T_{2(3)} := b_{2(3)} \rightarrow b_{2(3)} + 1$ can be generated like $(T = T_1)$,

$$RT_1R^{-1} =: T_3, \quad RT_3R^{-1} =: T_2.$$ \hfill (29, 30)

Similarly $(S = S_1)$,

$$S_2 := R^2S_1R^{-2}, \quad S_3 := RS_1R^{-1}.$$ \hfill (31, 32)

This way more $PSL(2, \mathbb{Z})$ subgroups arise, and in fact, there is an infinity of them. However, no pair of these subgroups mutually commute, again showing that there is no $[SL(2, \mathbb{Z})]^3$.

The three generators $R$, $S$ and $T$ are sufficient to generate the whole symmetry group. There are many relations between them, e.g.

$$S^2 = (ST)^3 = R^6 = 1$$

$$(TR)^6 = (TR^3)^2 = (SR^3)^2 = 1$$

$$(SRTR^{-1})^7 = 1$$

etc.
It is interesting to note that whereas duality is known to relate small and large radii, the combination \( SR^2SR^{-2} \) yields a rescaling symmetry:

\[
(t_1, t_2, t_3) \rightarrow \left( \frac{1}{4c_1^2} t_1, \frac{1}{4c_2^2} t_2, \frac{1}{4c_3^2} t_3 \right).
\] (34)

It is of infinite order, transforming two of the moduli to smaller and one of them to larger values.

All holomorphic transformations are of the form

\[
t_k \rightarrow \frac{a_k + b_k t_k}{c_k + d_k t_k}.
\] (35)

The 12 integers appearing in Eq. (35) satisfy three nonlinear relations, generalizing the determinant condition \( ad - bc = 1 \) in the case of \( SL(2, \mathbb{Z}) \). They enter the matrix representation of the symmetry group at hand\(^{14}\).

We checked that the symmetries are respected by the correlation functions, which are now available with their complete moduli dependence\(^{10,16}\).

4. Discrete Background Fields and (0,2) Models

4.1. Orbicircle \( \otimes \) Circle

In this section I describe how discrete background fields break the modular symmetry group down to a subgroup. The simplest example is the product theory of an orbicircle (a circle with its \( Z_2 \) symmetry divided out) with a circle. The twist matrix is given by

\[
\theta = \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix},
\] (36)

giving rise to the metric

\[
g = \begin{pmatrix}
R_{C}^{2} & 0 \\
0 & R_{O}^{2}
\end{pmatrix},
\] (37)

characterized by the orbifold radius \( R_{O} \) and the circle radius \( R_{C} \). In this example the antisymmetric tensor field does not possess a continuous parameter and, in contrast to the two-dimensional \( Z_3 \) orbifold discussed earlier, does not correspond to a modulus. It only takes discrete values,

\[
b_{ij} = \begin{pmatrix}
0 & k \\
-k & 0
\end{pmatrix}, \quad k \in \frac{1}{2}\mathbb{Z}.
\] (38)

Two different cases have to be distinguished. In the first, one has \( k \in \mathbb{Z} \cong 0 \). This corresponds to a proper direct product theory of the orbicircle and the circle. Thus it is clear that the symmetry transformations consist of the two independent duality involutions

\[
R_{C} \rightarrow 1/R_{C} \quad \text{and/or} \quad R_{O} \rightarrow 1/R_{O},
\] (39)
giving rise to $Z_2 \times Z_2$.

The second case has $k \in \mathbb{Z} + 1/2$. The nontrivial antisymmetric tensor field has two effects. Its presence only allows for a simultaneous inversion of the radii,

$$R_C \to 1/2R_C \quad \text{and} \quad R_O \to 1/2R_O,$$

and changes the self-dual point. In other words, it breaks the $Z_2 \times Z_2$ symmetry down to $Z_2$.

### 4.2. $Z_3$ Orbifold $\otimes$ Gauge Lattice

As an example with more phenomenological applicability consider the $Z_3$ orbifold with a quantized Wilson line turned on. In complete analogy to the toy example in the preceding subsection, we have the combination of an orbifold with a torus connected through a discrete background field. The presence of quantized Wilson lines is phenomenologically highly desired. They lift the degeneracy of the twisted sectors’ fixed points, thereby reducing the number of fermion generations from 36 to a smaller number and three generation models can be found\(^5\). At the same time the $E_6 \times E_8$ gauge symmetry is broken to realistic gauge groups. The lesson to be learned here is that they also break the discrete symmetry group in moduli space. Most surprisingly, they break duality symmetry. The $Z_2$ duality generator has to be replaced by an infinite order generator acting on the background as

$$A : g + b - \Delta_{1/3} \to W^{-1T} \frac{1}{9(g + b + \Delta_{1/3})} W^{-1},$$

where

$$\Delta_{1/3} := \frac{1}{3} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$  

(42)

In order to show that the remaining symmetry is in fact a subgroup of $SL(2, \mathbb{Z})$, I introduce the rescaled complex modulus

$$\lambda' = \frac{1}{3} \lambda = \frac{1}{3}(b + i\sqrt{\det g}).$$

(43)

The $SL(2, \mathbb{Z})$ transformations w.r.t. $\lambda'$ are defined as

$$\sigma : \quad \lambda' \to -\frac{1}{\lambda'},$$

(44)

$$\tau : \quad \lambda' \to \lambda' + 1.$$  

(45)

Now one can write

$$A := \tau \sigma \tau,$$

(46)

$$B := \tau^3,$$

(47)

where $B$ corresponds to the axionic shift symmetry. The $SL(2, \mathbb{Z})$ relations $S^2 = (ST)^3 = 1$ have to be replaced by $(AB)^9 = 1$ with no order two relation.
Thus quantized Wilson lines break $SL(2, \mathbb{Z})$ to a subgroup\textsuperscript{11,14}. On the other hand, the “canonical” duality transformation is not modular and leads to an asymmetric orbifold\textsuperscript{5}.

5. Conclusions and Outlook

It has become clear that only in a very limited number of cases the modular group $SL(2, \mathbb{Z})$ is realized as a symmetry in moduli space. This has to be compared with the result of an investigation of the mirror manifold of the quintic threefold\textsuperscript{17}. As in our case, there is a shift symmetry but no order two generator (duality), which in their case is replaced by an order five generator. The phenomenological investigations using $SL(2, \mathbb{Z})$ or the “maximally discrete symmetry hypothesis” have to be generalized to other groups.

The classification of (2,2) orbifolds necessarily involved the construction of all possible compactification lattices, since they can affect the massless spectra. The list of symmetric orbifolds is now complete and the one of asymmetric orbifolds (point groups and lattices) is under way.

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