Application of multidimensional interpolation on nonhomogeneous cancellous bone

Sikai Liu, MD, Sheng Li, MD, Ning Wei, MD, Wenli Chang, MD, Pan Hu, MD, Xiaodong Cheng, BS, Ling Wang, MD, Wei Chen, MD

Abstract

Bone, especially cancellous bone, has been demonstrated to be nonhomogeneous. When applied to bone study, it raises the following question: How should the material properties of the bone from the available experimental data be interpolated?

In this study, the finite element model of the femur has been built and the nonhomogeneous material properties of the femur have been assigned from the computed tomography (CT) data. These results have been applied to assess some common interpolation algorithms on the bone study, such as Linear Multivariate, Radial Basis, and Nearest Neighbor. It was found that among 3 tested algorithms, the RBAS algorithm has more points with errors from 0% to 15% than in the other 2 algorithms. When the supporting points jump from 160 to 288, the interpolation results significantly improve. When the finite element model reduces the element number from 38,230 to 13,424, all 3 algorithms have slightly better results.

The interpolation of bone material properties should use 2 different approaches. The bone interpolation should be applied only to the bone with uniform structure. For the area with dramatic change of structure, the material properties can be defined directly. Among 3 tested algorithms, the Radial Basis algorithm performs best in the statistic study and should be the first choice in the bone study. In addition, the Radial Basis algorithm can be introduced into other methods to smooth the distribution of material properties. Also, with more supporting points (experimental data), the interpolation error becomes less. The interpolation approach offers a significant advantage in the finite element analysis: only 1 material ID needs to define the material interpolated from experimental data, unlike the several hundred material IDs defined for the elements derived from CT data that take material inhomogeneity into account.

Abbreviations: CT = computed tomography, ID = identification, LMUL = linear multivariate, NNEI = nearest neighbor, RBAS = radial basis.

Keywords: computed tomography, finite element, multidimensional interpolation, nonhomogeneous bone

Editor: Goran Augustin.

Author summary

Why was this study done?

Accurate mechanical stresses of human bones help the design of prostheses and the evaluation of fracture risk. However, bone, especially cancellous bone, has been demonstrated to be nonhomogeneous. Thus, it is very important to determine the material properties of the bone from the available experimental data, which provides a base for obtaining accurate mechanical stresses of human bones.

What did the researchers do and find?

In this study, 3 common interpolation algorithms have been applied to interpolate the material properties of the femur. It was found that the Radial Basis algorithm, a global algorithm, performs best among 3 common interpolation algorithms and should be the first choice in the bone study.

What do these findings mean?

Compared to the traditional method, the interpolation approach provides accurate material properties of the femur, consequently accurate mechanical stresses that help the design of prostheses and the evaluation of fracture risk.

SL, SL, and NW contributed equally to this work.

Data sharing is not applicable to this article as no data sets were generated or analyzed during the present study.

This work was supported by the National Natural Science Foundation of China (grant no: 81401789), Natural Science Foundation of Hebei Province (CNI-Outstanding Youth Foundation (grant no: H2017260104), and Support Program for the Top Young Talents of Colleges and Universities of Hebei Province (grant no: BJ2016035). All authors have read and contributed to the submitted manuscript.

The authors have no conflicts of interest to disclose.

Department of Orthopedic Surgery, The Third Hospital of Hebei Medical University, Shijiazhuang, China.

Correspondence: Wei Chen, Department of Orthopedic Surgery, The Third Hospital of Hebei Medical University, No 139 Ziqiang Road, Qiaoxi District, Shijiazhuang 050051, China (e-mail: surgeonchenwei@126.com).

Copyright © 2018 the Author(s). Published by Wolters Kluwer Health, Inc. This is an open access article distributed under the terms of the Creative Commons Attribution-Non Commercial-No Derivatives License 4.0 (CCBY-NC-ND), where it is permissible to download and share the work provided it is properly cited. The work cannot be changed in any way or used commercially without permission from the journal.

Medicine (2018) 97:36(e12224)

Received: 17 March 2018 / Accepted: 13 August 2018

http://dx.doi.org/10.1097/MD.0000000000012224
1. Introduction

Clinical studies demonstrate that bone, especially cancellous bone, is nonhomogeneous. A biomechanical study adopts 2 approaches to take cancellous bone into account. The first approach is a very popular one. Numerous studies have researched how to assign bone material properties to each element of the finite element model of the bone from computed tomography (CT) data. However, due to the image quality, the material properties are neither very accurate nor continuous through the bone. Another approach requires the availability of the experimental data of material properties of the bone. The material properties of cancellous bone can be interpolated from the experimental data that are scattered data cross the cancellous bone. Some algorithms for multidimensional interpolation are available for this 3-dimensional (3D) interpolation problem, such as Linear Multivariate (LMUL), Radial Basis (RBAS), and Nearest Neighbor (NNEI). Although these algorithms have been studied intensively in other fields such as medical imaging and geologic modeling, few studies have been conducted in the bone research. In this study, these algorithms were applied for interpolating the material properties of the femur; the interpolation results were evaluated by using CT data to compare the material properties of the femur. The results may provide a guideline for the application of interpolation algorithms in bone study.

2. Methods

2.1. Multidimensional interpolation

The different approaches to the interpolation of scattered data are always divided into global methods and local methods. In the global methods, each interpolated value is affected by all the collection data, but in the local methods, it is only influenced by the values of the surrounding supporting points. The most widely used local method is the LMUL algorithm, while the simplest local method is the NNEI algorithm. The RBAS algorithm is the most popular global algorithm, especially useful for higher dimensional scatter data. These 3 algorithms are introduced briefly as follows.

2.1.1. LMUL algorithm. The LMUL algorithm is a local algorithm that uses supporting points around the query location to perform the interpolation process. This approach is very similar to that of the finite element method. The query point \( q \) is written as a linear combination of its \( n+1 \) nearest neighbors:

\[
q = a_1 x_1 + a_2 x_2 + \cdots + a_{n+1} x_{n+1}
\]

where \( \sum_{i=1}^{n+1} a_i = 1 \)

The \( a_i \) is a function of the coordinates of \( x_i \) of the query point. In the finite element method, the \( a_i \) is called shape function.

Like finite element method, the value of the query point \( f(q) \) is expressed as

\[
f(q) = a_1 f(x_1) + a_2 f(x_2) + \cdots + a_{n+1} f(x_{n+1})
\]

where \( f(x_i) \) is the dependent value of the supporting point \( x_i \).

The algorithm creates an \( n \)-dimensional rectangular bounding box (Fig. 1), and the above equations are applicable to the query point in the bounding box. The queries outside of the bounds are projected on the surface or edge of the bounds. One disadvantage of the LMUL algorithm is that sometimes the out of bounds interpolation results are not realistic enough to affect the application. To avoid this, this study proposed a modified LMUL algorithm that the linear interpolation is applied when the query point is within the bounds of the supporting points; when out of the bounds, the dependent value of the query point equals that of the supporting point closest to the query point.

2.1.2. RBAS algorithm. The RBAS interpolation is a global algorithm. Its interpolation requires solving the following radial-basis function

\[
F = \sum_{i=1}^{N} a_i \left[ \left( \sum_{j=1}^{O} q_j - x_j \right)^2 + c^2 \right]
\]

where \( N \) is the number of supporting points and \( O \) is the number of free variables (or the order of the interpolation). In
this study, \( O \) is 3. Input data are \((x_1, x_2, \ldots, x_N)\), where \( j \) varies from 1 to 3.

The unknown values are \( a_i \) (where \( i \) varies from 1 to \( N \)) and \( c \). \( q \) refers to the query point. The equation is evaluated for all supporting points given in the input to compute the \( a_i \) and \( c \) values.

2.1.3. NNEI algorithm. The NNEI algorithm searches the supporting point closest to the query point and returns the corresponding dependent value.\(^{[13]}\) It is expressed as

\[
x_0 - q = \min_{x \in X} (x - q)
\]

\[
f(q) = f(x_0)
\]

where \( X \) is the set of all supporting points, \( q \) is the query point, and \( x_0 \) is the closest supporting point to the query point.

2.2. Material properties of femur by CT

Because it has been verified the CT numbers have almost a linear correlation with the density of biological tissues, it is well known that CT images can not only provide accurate information of bone geometry, but also give information of the density of biological tissues.\(^{[14]}\) In addition, experiments have strongly established the relationship between the density and CT number.\(^{[14]}\) Finally, from the empirical relation between the Young modulus and the apparent bone density, the Young modulus for this element was obtained.\(^{[15,16]}\) Therefore, the nonhomogeneous material properties of the bone were assigned all over the bone, which were used for appraising the interpolation results (Fig. 3).

2.3. CT data and finite element mesh

The CT data set in standard DICOM formats was obtained after a CT scan of a 67-year-old woman’s femur. With the CT data, 3D geometry was created in Mimics as well as 3D finite element model in Integrated Computer Engineering and Manufacturing. The mesh is made of 3D 8-node structural elements, and composed of 38,230 elements and 42,281 nodes (Fig. 2A).

2.4. Procedure of material property assignment

For each element of the mesh, an average Hounsfield unit (HU) value was computed from HU values within a block that has the same center and volume as the element. Then, the apparent bone density of the element was calculated from the linear relation between the density and CT number.\(^{[14]}\) Finally, from the empirical relation between the Young modulus and the apparent bone density, the Young modulus for this element was obtained.\(^{[15,16]}\) Therefore, the nonhomogeneous material properties of the bone were assigned all over the bone, which were used for appraising the interpolation results (Fig. 3).

2.5. Interpolation process

One hundred sixty supporting points located at grid points of 2 blocks were designed to act as the experimental data (Fig. 4A). The elements within 2 blocks are regarded as in the bounds and those out of the blocks as out of the bounds. The dependent value of each supporting point was defined as the Young modulus, and equal to the Young modulus of the element where closest to the supporting point. The LMUL algorithm, RBAS algorithm, and NNEI algorithm were implemented in ANSYS170. The material of femur was defined with just 1 material ID using the Tersoff-Brenner (TB) field that contains all information of supporting points. The interpolation results, which are Young modulus of elements, compare against the
Young modulus at the corresponding elements by CT scan. The error of interpolation for each element is calculated by

\[ \text{Err} = \left| \frac{E_{\text{interpolation}} - E_{\text{CT}}}{E_{\text{CT}}} \right| \times 100\% \quad (6) \]

where \( E_{\text{interpolation}} \) is the Young modulus by the interpolation algorithm, and \( E_{\text{CT}} \) is the Young modulus by CT data.

To explore the factors that affect the interpolation, 2 more cases were studied. One was to increase the supporting points from 160 to 288 (Fig. 4B). Another case was to build a finite element model from the same CT data but to reduce the element numbers from 38,230 to 13,424 (Fig. 2B).

### 2.6. Ethical review

This study was approved by the Institutional Review Board of the Third Hospital of Hebei Medical University (IRB Reference no: Guo2017-001-1). Written consent was obtained from the participant.

### 3. Results

The Young modulus of the femur by 3 interpolation algorithms is plotted from Figures 5–7. Obviously, some of the Young modulus by LMUL algorithm is negative, which is not realistic. The interpolation results by the modified LMUL algorithm, which were much better than that by the LMUL algorithm, are
presented in Figure 8. Clearly, the Young modulus contour plot by the RBAS algorithm is continuous in both within the bounds and out of the bounds (Fig. 6), while the contour by the NNEI algorithm is composed of many segments, each indicating one of Young modulus (Fig. 7). For the modified LMUL algorithm, the contour plot is continuous within the bounds, but discontinuous out of the bounds (Fig. 8).

The error plots of the 3 interpolation algorithms are presented in Figures 9–11. The error results, which are summarized in Figure 12, show that the RBAS algorithm
has more points with errors from 0% to 5% than in the other 2 algorithms. The statistical results demonstrate that the RBAS algorithm has the best performance among the 3 algorithms.

Figures 13–15 showed the results of 3 algorithms with different supporting points and finite element models. These figures indicate that when the supporting points jump from 160 to 288, the interpolation results significantly improve, especially the error
range from 0% to 10%. When the finite element model reduces the element number from 38,230 to 13,424, all 3 algorithms have slightly better results.

4. Discussion

From CT data, not only the geometry of the femur and the finite element model were built, but also the nonhomogeneous material properties of the bone were assigned all over the bone. Although the material properties of the bone obtained by CT data are not very accurate, they reflect the basic features of the femur, and they can be used to evaluate interpolation results of 3 interpolation algorithms of the femur.

Statistical results show <60% of interpolations for all 3 algorithms and have error <20%. The major errors occur at the femur body, where the material properties change from the cortical bone at the surface to the cancellous bone with a low Young modulus in a very short distance. This dramatic change of material properties causes the interpolation with high error. The interpolation error at other areas, such as the neck and head of the femur, is very low, usually below 15%. This is because these areas have a uniform structure by bone remodeling according to the stress generated within during activities. To avoid large interpolation errors, the bone interpolation should be limited to the bone with uniform structure. The material properties of the area with a dramatic change of structure can be defined directly.

The statistical results reveal that the RBAS algorithm has the best performance among the 3 algorithms. This is because the RBAS algorithm is a global method; the supporting points influence all interpolated values. Normally, the RBAS algorithm is limited to small data sets because it requires a huge computational effort. However, in the bone study, the data set is relatively small, around several hundred. Therefore, the RBAS algorithm should be the first choice in the bone interpolation. The LMUL algorithm is faster than other interpolation algorithms for a large set of data points and is highly accurate when provided...
with a sufficient distribution of points. However, it cannot guarantee the results out of the bounds. The proposed modified LMUL algorithm is a mixture of the NNEI and LMUL. The NNEI algorithm, which is the simplest algorithm, is adopted for very large scatter data sets but is not practical in bone study because the bone material properties change dramatically in some parts of the bone.

When the supporting points increase from 160 to 288, the interpolation results also significantly improve. This suggests that more supporting points have better interpolation results. Therefore, at the area where the material properties of the bone change greatly, more experimental data should be collected to provide enough information for interpolation. In addition, comparing the interpolation results from 2 finite element models based on the same CT data indicates that the interpolation results slightly depend on the query size (element number).

The bone material in this study is limited to isotropic, but the bone material is manifested to be anisotropic. However, if enough information is given for each supporting point, each component of the anisotropic material can be interpolated following the same procedure. The only difference is that the whole process requires more computational time.

One substantial advantage of the interpolation approach is that the material of cancellous bone is defined by one material ID in the finite element model; this differs from the approach by CT data, which sometimes needs several hundred material IDs. Moreover, the approach to assign bone material properties obtained from CT data has some drawbacks, including material properties that are not continuous. At this point, the RBAS algorithm can be introduced into this approach to improve it since it is a global method; its interpolation results are always smooth not only inside the bounds but also out of the bounds. With sufficient supporting points, the interpolation results can be both accurate and smooth.

The interpolation approach requires the available experimental data, which are time consuming to collect. However, once sufficient experimental data are provided, the nonhomogeneous material properties of the bone can be interpolated quickly and accurately. For example, the nonhomogeneous cancellous bone of the ankle joint can be interpolated from Jensen experimental data.

The finite element model of the femur has been built and the nonhomogeneous material properties of the femur have been assigned based on the CT data. These results, which reflect the basic features of the femur, have been used to evaluate the application of 3 interpolation algorithms on the bone study. Some of the findings are listed as follows:

1. To avoid large interpolation errors, the bone interpolation should only be applied to the bone with a uniform structure. The material properties can be defined directly for the area with a dramatic change of structure.
2. Among the 3 algorithms, the RBAS algorithm has the best performance as the statistical results demonstrate. Therefore, the RBAS algorithm should be the first choice in the bone study. In addition, the RBAS algorithm can be introduced into the approach that assigns bone material properties derived from CT data to improve the results.
3. The number of supporting points is a very important factor that influences the interpolation results. With more supporting points, the interpolation error becomes less.
4. One significant advantage of the interpolation approach in the finite element application is that the material is defined by only one material ID. For comparison, the approach by CT data sometimes needs several hundred material IDs.

One direct application of the new technology in both research and clinical practice is to obtain accurate mechanical stresses of human bones, which helps the design of prostheses and the evaluation of fracture risk. For example, after total hip replacement surgery, as the implant carries a part of the load, stresses in some regions of the remaining bone diminish, known as stress shielding. To alleviate this problem, the stress distribution of the bone with the prosthesis should be close to that of the healthy bone as much as possible, and the stress distribution of the bone can be determined by the new technology presented in this study.

5. Conclusion

The interpolation of bone material properties should use 2 different approaches. The bone interpolation should be only applied to the bone with uniform structure. For the area with dramatic change of structure, the material properties can be defined directly. Among 3 tested algorithms, the Radial Basis algorithm performs best in the statistic study and should be the first choice in the bone study. In addition, the Radial Basis algorithm can be introduced into other methods to smooth the distribution of material properties. Also, with more supporting points (experimental data), the interpolation error becomes less. The interpolation approach offers a significant advantage in the finite element analysis: only one material ID needs to define the material interpolated from experimental data, unlike the several hundred material IDs defined for the elements derived from CT data that take material in homogeneity into account.

Acknowledgment

The authors thank the participant who took part in this study and the information provided by all of the authors, for assistance with the calculations in this study.

Author contributions

WC and SKL designed the study; NW, SL, PH, XDC and WLC searched the relevant studies and did the analysis; SKL, NW, SL and WLC wrote the draft; WC and LW revised the manuscript. WC approved the final version of the manuscript.

Conceptualization: Sikai Liu, Wei Chen.
Data curation: Sikai Liu, Ning Wei, Wenli Chang, Pan Hu, Xiaodong Cheng.
Formal analysis: Sheng Li, Ning Wei.
Writing – original draft: Sikai Liu, Sheng Li, Ning Wei, Wenli Chang.
Writing – review & editing: Ling Wang, Wei Chen.

References

[1] Pal S. Design of Artificial Human Joints & Organs. 2014; Springer, Boston: 75–100.
[2] Sivalakumar V. Non-linear 3d finite element analysis of the femur bone. Int J Res Eng Technol 2013;2:266–75.
[3] Austman RL, Milner JS, Holdsworth DW, et al. The effect of the density-modulus relationship selected to apply material properties in a finite element model of long bone. J Biomech 2008;41:3171–6.
[4] Keyak JH, Rossi SA. Prediction of femoral fracture load using finite element models: an examination of stress- and strain-based failure theories. J Biomech 2000;33:209–14.
[5] Yang HS, Guo TT, Wu JH, et al. Inhomogeneous material property assignment and orientation definition of transverse isotropy of femur. J Biomed Sci Eng 2009;2:419–24.
[6] Cody DD, Hou FJ, Divine GW, et al. Femoral structure and stiffness in patients with femoral neck fracture. J Orthop Res 2000;18:443–8.
[7] Helgason B, Taddei F, Pålsson H. A modified method for assigning material properties to FE models of bones. Med Eng Phys 2008;30:444–53.
[8] Taddei F, Pancanti A, Viceconti M. An improved method for the automatic mapping of computed tomography numbers onto finite element models. Med Eng Phys 2004;26:61–9.
[9] Taddei F, Schileo E, Helgason B, et al. The material mapping strategy influences the accuracy of CT-based finite element models of bones: an evaluation against experimental measurements. Med Eng Phys 2007;29:973–9.
[10] Zannoni C, Mantovani R, Viceconti M. Material properties assignment to finite element models of bone structures: a new method. Med Eng Phys 1999;20:735–40.
[11] Cattaneo PM, Dalstra M, Frich LH. A three-dimensional finite element model from computed tomography data: a semi-automated method. Proc Inst Mech Eng HV 2011;203:103–13.
[12] Dalstra M, Huiskes R, Van EL. Development and validation of a three-dimensional finite element model of the pelvic bone. J Biomech Eng 1995;117:272–8.
[13] Amidror I. Scattered data interpolation methods for electronic imaging systems: a survey. J Elect Imag 2002;11:157–76.
[14] Rho JY, Hobatho MC, Ashman RB. Relations of mechanical properties to density and CT numbers in human bone. Med Eng Phys 1995;17:547–55.
[15] Carter DR, Hayes WC. The compressive behavior of bone as a two-phase porous structure. J Bone Joint Surg Am 1977;59:954–62.
[16] Wirtz D, Schiffers NT, Radermacher K, et al. Critical evaluation of known bone material properties to realize anisotropic FE-simulation of the proximal femur. J Biomech 2000;33:1325–30.
[17] Jensen NC, Hvid I, Kroner K. Strength pattern of cancellous bone at the ankle joint. Eng Med 1988;17:71–6.