Supersymmetric type-II seesaw model: LHC and lepton flavour violating phenomenology

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Abstract. We study the supersymmetric type-II seesaw model assuming minimal supergravity boundary conditions. We calculate branching ratios for lepton flavour violating (LFV) scalar tau decays, potentially observable at the LHC, as well as LFV decays at low energy, such as $l_i \rightarrow l_j + \gamma$ and compare their sensitivity to the unknown seesaw parameters. In the minimal case of only one triplet coupling to the standard model lepton doublets, ratios of LFV branching ratios can be related unambiguously to neutrino oscillation parameters. We also discuss how measurements of soft SUSY breaking parameters at the LHC can be used to indirectly extract information of the seesaw scale.

1. Introduction
The most popular realization of dimension-5 operator of Majorana neutrino mass is the exchange of a heavy fermionic singlet. This is the celebrated seesaw mechanism [2–4], which we will call seesaw type-I. The second possibility is the exchange of a scalar triplet [5, 6]. This is commonly known as seesaw type-II. The tree-level realizations of the seesaw, unfortunately, can not be put to the test in a direct way. Indirect inside into the high-energy world might be possible in supersymmetric versions of the seesaw. In the renormalization group equations for the soft SUSY breaking slepton mass parameters terms proportional to the neutrino Yukawa couplings appear. If the scale where the right-handed neutrinos and/or the triplet decouple is below the scale at which SUSY breaks, lepton flavour violating (LFV) entries in the Yukawa matrices then induce LFV off-diagonals in the slepton mass matrices. This effect potentially leads to large values for lepton flavour violating lepton decays, such as $\mu \rightarrow e + \gamma$, even if the soft masses are completely flavour blind at high scale, as was first pointed out for the case of seesaw type-I in [7]. It is maybe not surprising then that with the increasingly convincing experimental evidence for non-zero neutrino masses a number of articles have studied the prospects for observing LFV processes, both at low energies and at future colliders, within the supersymmetric seesaw [8–11].

While absolute values of LFV observables depend very strongly on the soft SUSY breaking parameters, we discuss how ratios of LFV branching ratios can be used to eliminate most of the dependence on the unknown SUSY spectrum. For example, ratios such as $\text{Br}(\tilde{\tau}_2 \rightarrow e + \chi^0_1)/\text{Br}(\tilde{\tau}_2 \rightarrow \mu + \chi^0_1)$ are constants for fixed neutrino parameters over large parts of the supersymmetric parameter space. Measurements of such ratios would allow to extract valuable

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1 This talk is based on the paper [1].
2 For the other related refs., see the refs. cited in [1].
information about the seesaw parameters: In the minimal type-II seesaw case these ratios can be calculated as function of measurable low-energy neutrino data. For the more involved case of the $15 + \overline{15}$ model this simple connection is lost in general, but relations to neutrino data can be (re-) established in some simple, extreme cases for the Yukawa matrix $Y_{15}$. We therefore study such ratios in some detail, first analytically then numerically.

The presence of new non-singlet states below the GUT scale does not only affect the running of gauge couplings but also the evolution of the soft SUSY breaking parameters. Measurements of soft SUSY masses at the LHC and at a possible ILC therefore contain indirect information about the physics at higher energy scales [11, 13]. From the different soft scalar and gaugino masses one can define certain “invariants”, i.e. parameter combinations which are nearly constant over large ranges of the mSugra parameter space [14], at least in leading order approximation. If the measured values of all the invariants depart from the mSugra expectation in a consistent way, one could gain some indirect estimate of the mass scale of the new particles, the scale of the seesaw type-II. We discuss first some leading order analytical approximation, before showing by numerical calculation the limitations of the simplified analytical approach. While the different invariants indeed contain useful information about the high energy physics, reliable quantitative conclusions about the mass scale of the $15$ require highly precise measurements of soft masses as well as a full numerical 2-loop analysis.

2. Setup: mSugra with seesaw type II
In this section, to set up the notation, we briefly recall the main features of the seesaw type-II and mSugra. We then outline a simple $SU(5)$ motivated model based on the work of [12].

In supersymmetry at least two $SU(2)$ triplet states $T_{1,2}$ with opposite hypercharge are needed to cancel anomalies. Thus, the minimal SUSY potential including triplets can be written as

$$W = W_{\text{MSSM}} + \frac{1}{\sqrt{2}} (Y_T^i L_i T_1 L_j + \lambda_i H_1 T_1 H_2 + \lambda_2 H_2 T_2 H_2) + M_T T_1 T_2. \quad (1)$$

Here $T_1$ ($T_2$) are supermultiplets with hypercharge $Y = 1$ ($Y = -1$) and $H_{1,2}$ are the standard Higgs doublets with $Y = \mp 1/2$. The matrix $Y_T$ is complex symmetric, $\lambda_i$ are arbitrary constants and $M_T$ gives mass to the triplets, supposedly at a very high scale. Note that only $T_1$ couples to the SM leptons, thus in the minimal (supersymmetric) model with two triplets the only source of lepton flavour violation resides in the matrix $Y_T$.

Integrating out the heavy triplets at their mass scale the dimension-5 operator is generated and after electro-weak symmetry breaking the resulting neutrino mass matrix can be written as

$$m_\nu = v_2^2 \frac{\lambda_2}{2 M_T} Y_T. \quad (2)$$

where $v_2$ is the vacuum expectation value of Higgs doublet $H_2$ and we use the convention $\langle H_i \rangle = \frac{v_2}{\sqrt{2}}$. In the basis where the charged lepton masses are diagonal, eq. (2) is diagonalized by $\bar{m}_\nu = U^T \cdot m_\nu \cdot U$, where the neutrino mixing matrix $U$ is, in standard notation [15], given by

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{i \delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta} & -c_{13} e^{i \delta} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & -s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13} \end{pmatrix} \times \begin{pmatrix} e^{i \alpha_1/2} & 0 & 0 \\ 0 & e^{i \alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

Here $s_{ij} \equiv \sin \theta_{ij}$ ($c_{ij} = \cos \theta_{ij}$). For Majorana neutrinos, $U$ contains three phases: $\delta$ is the (Dirac-) CP violating phase, which appears in neutrino oscillations, and $\alpha_{1,2}$ are Majorana phases, which can only be observed in lepton number violating processes. Neutrino oscillation
experiments can be fitted with either a normal hierarchical spectrum (NH), or with inverted hierarchy (IH). If one does not insist in ordering the neutrino mass eigenstates \( m_{\nu_i}, i = 1, 2, 3 \) with respect to increasing mass, the matrix \( U \) can describe both possibilities without re-ordering of angles. In this convention, which we will use in the following, \( m_{\nu_{13}} \approx 0 \) (or \( m_{\nu_{23}} \approx 0 \)) corresponds to normal (inverse) hierarchy and \( s_{12}, s_{13} \) and \( s_{23} \) are the solar (\( s_\odot \)), reactor (\( s_R \)) and atmospheric angle (\( s_{\text{atm}} \)) for both type of spectra. Here we outline the basics of an \( SU(5) \) inspired model, which adds a pair of \( 15 \) and \( \overline{15} \) to the MSSM particle spectrum [12]. Our numerical calculations will all be based on this variant, since it allows to maintain gauge coupling unification for \( M_T \ll M_G \), as discussed in the introduction. Under \( SU(3) \times SU(2) \times U(1)_Y \) the \( 15 \) decomposes as

\[
15 = S + T + Z, \quad S \sim (6,1,-2/3), \quad T \sim (1,3,1), \quad Z \sim (3,2,1/6).
\]

\( T \) has the same quantum numbers as the triplet \( T_1 \) discussed above. The \( SU(5) \) invariant superpotential reads as

\[
W = \frac{1}{\sqrt{2}} Y_{15} S \cdot 15 \cdot 5 + \frac{1}{\sqrt{2}} \lambda_1 5_H \cdot 15 \cdot 5_H + \frac{1}{\sqrt{2}} \lambda_2 5_H \cdot \overline{15} \cdot 5_H + Y_{5} 10 \cdot 5 \cdot 5_H
\]

(5)

Here, \( \bar{5} = (d^c, L), 10 = (u^c, e^c, Q), 5_H = (t, H_2) \) and \( \bar{5}_H = (\bar{t}, H_1) \). Below the GUT scale in the \( SU(5) \)-broken phase the potential contains the terms

\[
\frac{1}{\sqrt{2}} (Y_T L T_1 L + Y_S d^c S d^c) + Y_Z d^c Z L + Y_d d^c Q H_1 + Y_u u^c Q H_2 + Y_e e^c L H_1
\]

(6)

\[
+ \frac{1}{\sqrt{2}} (\lambda_1 H_1 T_1 H_1 + \lambda_2 H_2 T_2 H_2) + M_T T_1 T_2 + M_Z Z_1 Z_2 + M_S S_1 S_2 + \mu H_1 H_2.
\]

The first term in eq. (6) is responsible for the generation of the neutrino masses in the same way as discussed for the triplet-only case in the previous subsection. \( Y_d, Y_u \) and \( Y_e \) generate quark and charged lepton masses in the usual manner. However, in addition there are the matrices \( Y_S \) and \( Y_Z \), which, in principle, are not determined by any low-energy data. In the calculation of LFV observables in supersymmetry both matrices, \( Y_T \) and \( Y_Z \), contribute. For the case of a complete \( 15 \), apart from threshold corrections, \( Y_T = Y_S = Y_Z \). One can recover the results for the simplest triplet-only model, as far as lepton flavour violation is concerned, by putting \( Y_S = Y_Z = 0 \).

3. Analytical results

In mSugra one has in total five parameters at the GUT scale [17]. These are usually chosen to be \( M_0 \), the common scalar mass, \( M_{1/2} \), the gaugino mass parameter, \( A_0 \), the common trilinear parameter, \( \tan \beta = \frac{v_u}{v_d} \) and the sign of \( \mu \). For the full set of RGEs for the \( 15 + \overline{15} \) see [12]. In the numerical calculation, presented in the next section, we solve the exact RGEs. However, the following approximative solutions are very helpful in gaining a qualitative understanding.

For the gaugino masses one finds

\[
M_i (m_{\text{SUSY}}) = \frac{\alpha_i (m_{\text{SUSY}})}{\alpha (M_G)} M_{1/2}.
\]

(7)

Eq. (7) implies that the ratio \( M_0 / M_1 \), which is measured at low-energies, has the usual mSugra value, but the relationship to \( M_{1/2} \) is changed. Neglecting the Yukawa couplings \( Y_{15} \), see below,
for the soft scalar mass parameters of the first two generations one obtains

\[ m_f^2 = M_0^2 + \frac{3}{2} \sum_{i=1}^{\tilde{f}_l} \left( \frac{\alpha_i(M_T)}{\alpha(M_G)} \right)^2 f_i + f'_i \left( \frac{M_1^2}{M_T^2} \right)^2, \tag{8} \]

\[ f_i = \frac{1}{b_i} \left( 1 - \left[ 1 + \frac{\alpha_i(M_T)}{4\pi b_i \log \frac{m_T^2}{m_Z^2}} \right]^{-2} \right), \]

\[ f'_i = \frac{1}{b_i + \Delta b_i} \left( 1 - \left[ 1 + \frac{\alpha_i(M_G)}{4\pi (b_i + \Delta b_i) \log \frac{M_G^2}{M_T^2}} \right]^{-2} \right). \tag{9} \]

The various coefficients \( c_i \) are given in the Table 1 of [1]. Individual SUSY masses depend strongly on the initial values for \( M_0 \) and \( M_{1/2} \). However, one can form different combinations, such as

\[ \frac{(m_L^2 - m_E^2)}{M_1^2} = \left( \frac{\alpha(M_G)}{\alpha_1(m_{\text{SUSY}})} \right)^2 \left( \frac{3}{2} \left[ \frac{\alpha_2(m_T)}{\alpha(m_G)} \right] f_2 + f'_2 \right) - \frac{9}{10} \left[ \frac{\alpha_1(m_T)}{\alpha(m_G)} \right]^2 f_1 + f'_1 \]

which, to first approximation, are constants over large regions of mSugra space. We will call such combinations “invariants”.

![Figure 1](image.png)

**Figure 1.** Four different “invariant” combinations of soft masses (left) versus the mass of the 15-plet, \( M_{15} = M_T \). The plot assumes that the Yukawa couplings \( Y_{15} \) are negligibly small. The calculation is at 1-loop order in the leading-log approximation.

Figure (1) shows four different invariants as a function of \( M_{15} = M_T \), calculated using eqs (7) - (8). For \( M_T = M_G \) one reaches the mSugra limit. For lower values of \( M_T \) one obtains a logarithmic dependence on the value of \( M_T \). If all the different invariants depart from their mSugra values in a consistent way, measurements of these parameter combinations can be used to obtain indirect information about the seesaw scale. In practice the “invariants” do depend on the SUSY spectrum and thus, indirectly still depend to some degree on the initial values of \( M_0 \) and \( M_{1/2} \). We will discuss this point in more details in the numerical section.

Here we also discuss the analytical results for flavour violating processes. We concentrate exclusively on the left-slepton sector. Taking into account the discussion given above this is expected to be a reasonable first approximation. The left-slepton mass matrix is diagonalized by a matrix \( R^l \), which in general can be written as a product of three Euler rotations. However, if the mixing between the different flavour eigenstates is sufficiently small, the three different angles can be estimated by the following simple formula

\[ \theta_{ij} \simeq \frac{(\Delta m^2_{L_{ij}})}{(\Delta m^2_{L_{1s}} - (\Delta m^2_{L_{3s}}))}. \]
are directly proportional to the squares of these mixing angles as long as all angles are small. Taking the ratio of two decays, for example,

\[
\frac{Br(\bar{\tau}_2 \rightarrow e + \chi_1^0)}{Br(\bar{\tau}_2 \rightarrow \mu + \chi_1^0)} \simeq \left(\frac{\theta_{e\mu}}{\theta_{\mu\mu}}\right)^2 \simeq \left(\frac{\Delta m^2_{L21}}{\Delta m^2_{L23}}\right)^2,
\]

one expects that (a) all the unknown SUSY mass parameters and (b) the denominators of approximate formula of \(\theta_{ij}\) cancel approximately. To calculate estimates for different ratios of branching ratios we define

\[
r_{ik}^{ij} = \frac{|(\Delta m^2_{L})_{ij}|}{|\Delta m^2_{L}|_{kl}}
\]

where the observable quantities are \((r_{ik}^{ij})^2\). Of course, only two of the three possible combinations that can be independent are.

We next derive some analytical formulas for \((r_{ik}^{ij})^2\) in terms of observable neutrino parameters. The neutrino Yukawa coupling \(Y_T\) can be written in terms of observable parameters

\[
Y_T = \frac{2M_T}{v^2\lambda_2}m_\nu = \frac{2M_T}{v^2\lambda_2}U^* \cdot \text{diag}(m_1, m_2, m_3) \cdot U^T.
\]

The running of the soft-SUSY breaking slepton mass matrix \((m^2_L)_{ij}\) is proportional to the parameter combination \((Y_T^*Y_T)_{ij}\). This combination can again be expressed in terms of low-energy neutrino observables times an unknown scale:

\[
(Y_T^*Y_T)_{ij} = \left(\frac{2M_T}{v^2\lambda_2}\right)^2 \left((U \cdot \text{diag}(m_1^2, m_2^2, m_3^2) \cdot U^T)_{ij}\right).
\]

Inserting the convention for the matrix \(U\) from eq. (3) results in

\[
(Y_T^*Y_T)_{12} \propto c_{12}s_{12}c_{13}s_{23}(m_2^2 - m_1^2) - c_{13}s_{13}s_{23}e^{-i\delta}\{(m_3^2 - m_2^2) + c_{12}(m_2^2 - m_1^2)\},
\]

\[
(Y_T^*Y_T)_{13} \propto c_{12}s_{12}c_{13}s_{23}(m_1^2 - m_2^2) - c_{13}s_{13}s_{23}e^{-i\delta}\{(m_3^2 - m_2^2) + c_{12}(m_2^2 - m_1^2)\},
\]

\[
(Y_T^*Y_T)_{23} \propto s_{23}c_{23}(s_{12}^2 - c_{12}^2)(m_2^2 - m_1^2) + c_{13}\{(m_3^2 - m_2^2) + c_{12}(m_2^2 - m_1^2)\}
- \ s_{12}c_{12}s_{13}(c_{23}^2e^{-i\delta} - s_{23}^2e^{i\delta})(m_2^2 - m_1^2).
\]

Note, that the off-diagonals can be expressed as a function of mass squared differences only, i.e. there is no dependence on the overall neutrino mass scale. However, again note that eq. (13) depends on the energy scale, see the discussion below eq. (2) and in section (4). Also it is worth mentioning that with the convention of \(U\) from eq. (3) the Majorana phases cancel in eq. (13).

For the general mixing matrix one can derive \((r_{kl}^{ij})^2\) using eq. (13). For the currently allowed ranges of the neutrino parameters, the most important unknown turns out to be \(s_{13}\), as fig. (2) demonstrates. In this figure \((r_{kl}^{ij})^2\) are shown as function of \(s_{13}^2\) for \(\tan^2\theta_A = 1\) and \(\tan^2\theta_C = 1/2\), as well as for the \(\Delta m^2\) fixed at their best fit point values [18]. Currently \(s_{13}^2 \leq 0.05\) at 3 \(\sigma\) c.l. \(r_{kl}^{ij}\) strongly depend on the value of \(s_{13}\) and there exists a special value of \(s_{13}\), for which either \(r_{12}^{13}\) or \(r_{23}^{13}\) even vanish, due to a cancellation between the different terms in eq. (13). Note, however, that \(r_{12}^{13}\) and \(r_{23}^{13}\) can not vanish simultaneously. Note also, that for \(\tan^2\theta_A = 1\), \(r_{12}^{13}\) and \(r_{23}^{13}\) are symmetric under the exchange of \(\delta = 0 \leftrightarrow \delta = \pi\). Also, for non-zero values of \(s_{13}\) the results depend on the assumed hierarchy of the left neutrinos and the simultaneous exchange of the cases (normal hierarchy) NH \(\leftrightarrow\) IH (inverse hierarchy) and \((\delta = 0) \leftrightarrow (\delta = \pi)\) leads to the same values for the different \(r_{kl}^{ij}\) in case of \(\tan^2\theta_A = 1\).
Figure 2. Square ratios \((r_{13}^{12})^2\) (light blue line, full line), \((r_{12}^{23})^2\) (blue line, dashed line) and \((r_{12}^{13})^2\) (red line, dotted line) versus \(s_{13}^2\) for NH (upper panels), IH (lower panels) for \(\delta = 0\) (left panels) and \(\delta = \pi\) (right panels). The other light neutrino parameters have been fixed to their b.f.p. values. Note, that for \(\tan^2 \theta_A = 1\), \((r_{12}^{13})^2\) and \((r_{13}^{23})^2\) are symmetric under the exchange of \(\delta = 0 \leftrightarrow \delta = \pi\). Also the simultaneous exchange of NH \(\leftrightarrow\) IH and \((\delta = 0) \leftrightarrow (\delta = \pi)\) leads to the same values for the different \(r_{ik}^{ij}\), in case of \(\tan^2 \theta_A = 1\).

4. Numerical results
In this section we present our numerical calculations. All results presented below have been obtained with the lepton flavour violating version of the program package SPheno [19]. Calculations are done for the 15-plet case, using the assumption \(Y_Z = Y_T\) at \(M_G\). Unless mentioned otherwise, we fit neutrino mass squared differences to their best fit values [18] and the angle to TBM values. Our numerical procedure is as follows. Inverting the seesaw equation, see eq. (2), one can get a first guess of the Yukawa couplings for any fixed values of the light neutrino masses (and angles) as a function of the corresponding triplet mass for any fixed value of \(\lambda_2\). This first guess will not give the correct Yukawa couplings, since the neutrino masses and mixing angles are measured at low energy, whereas for the calculation of \(m_\nu\) we need to insert the parameters at the high energy scale. However, we can use this first guess to run numerically the RGEs to obtain the exact neutrino masses and angles (at low energies) for these input parameters. The difference between the results obtained numerically and the input numbers can then be minimized in a simple iterative procedure until convergence is achieved. As long as neutrino Yukawas are not too close to one we reach convergence in a few steps. However, in seesaw type-II the Yukawas run stronger than in seesaw type-I, thus our initial guess can deviate sizeably from the exact Yukawas. Since neutrino data requires at least one neutrino mass to be larger than about 0.05 eV, we do not find any solutions for \(M_T > \lambda_2 \cdot 10^{15}\) GeV.
4.1. Numerical results for LFV

The analytical results presented in the previous section allow to estimate ratios of branching ratios for LFV decays. For absolute values of the branching ratios, as well as for cross-checking the reliability of the analytical estimates, one has to resort to a numerical calculation. Below we show results only for a few “standard” mSugra points, namely for SPS3 [20] and SPS1a’ [21]. However, we have checked with a number of other points that our results for ratios of branching ratios are generally valid.

Fig. (3) shows examples of LFV decays for the mSugra point SPS3 as a function of $M_T = M_{15}$ for two different values of $\lambda_2$. The upper plots show $\text{Br}(l_i \to l_j + \gamma)$, while the lower ones show $\text{Br}(\tau_2 \to e, \mu + \chi^0)$. We have also calculated $\text{Br}(l_i \to 3l_j)$, but these are not shown in the plots, because they follow very well the approximate relation [10,22]

$$\frac{\text{Br}(l_i \to 3l_j)}{\text{Br}(l_i \to l_j + \gamma)} \approx \frac{\alpha}{3\pi} \left( \log \left( \frac{m_{l_i}^2}{m_{l_j}^2} \right) \frac{11}{4} \right).$$

(14)

All LFV branching ratios show a very strong dependence on the value of $M_T$ and due to the stronger running of parameters in the seesaw type-II case, compared to the seesaw type-I, the dependence on the seesaw scale is stronger than in seesaw-I [23]. See also the discussion in section (3).

For the calculation shown in fig. (3), we have fitted the neutrino angles to exact tri-bimaximal values. One sees that, as long as the different LFV branching ratios are small, ratios of branching ratios are constants, which follow very well the analytical expectations. Currently the most important phenomenological constraints comes from the upper limit on $\text{Br}(\mu \to e + \gamma)$,
\( Br(\mu \to e + \gamma) \leq 1.2 \cdot 10^{-11} \) \[15\]. Note that the “dip” in \( Br(\mu \to e + \gamma) \) is due to a level-crossing of selectron and smuon mass eigenstates. For SPS3 one finds that this limit rules out \( Br(\tilde{\tau}_2 \to \mu + \chi_1^0) \) larger than a few percent, the exact number depending on the unknown parameter \( \lambda_2 \). Fig. (3) to the left (right) shows results for \( \lambda_2 = 0.05 \) (\( \lambda_2 = 0.5 \)). Recall that neutrino physics fixes only \( M_T/\lambda_2 \). However, note also that the upper limit on \( Br(\tilde{\tau}_2 \to \mu + \chi_1^0) \) depends only weakly on \( \lambda_2 \).

It is well-known that absolute values of LFV branching ratios depend very strongly on the SUSY spectrum, for example \( Br(\mu \to e + \gamma) \propto 1/m_{\chi}^2 \) \[9\]. Since both left-sleptons as well as (lightest) neutralino and chargino are approximately a factor of two heavier for SPS3 than for SPS1a’, one expects that \( Br(\mu \to e + \gamma) \) gives a strong constraint on the observability of LFV at the LHC for SPS1a’. This is confirmed numerically, as shown in fig. (4), which shows \( Br(\tilde{l}_i \to l_j + \gamma) \) and \( Br(\tilde{\tau}_2 \to e, \mu + \chi_1^0) \) as function of \( M_T = M_{15} \) for the example of \( \lambda_2 = 0.5 \). Given the current limit on \( Br(\mu \to e + \gamma) \) one expects \( Br(\tilde{\tau}_2 \to \mu + \chi_1^0) \propto (\text{few}) \cdot 10^{-4} \). Note that again we have fitted neutrino angles to tri-bimaximal values in this calculation and that ratios of LFV branching ratios follow closely the analytical expressions.

4.2. Sparticles Masses and seesaw scale

As discussed in the analytic section, the running of soft parameters allows, in principle, an indirect determination of the seesaw scale. In this section we discuss numerical results for the running of the “invariants” defined above. Although below we show plots only for the combination \( (m_L^2 - m_E^2)/M_T^2 \) we have checked numerically that all invariants shown in fig. (1) behave qualitatively in the same way.

Fig. (5) shows \( (m_L^2 - m_E^2)/M_T^2 \) as a function of \( M_T = M_{15} \) for SPS1a’ and SPS3 comparing different calculations. This plot assumes that the Yukawas of the 15-plet are negligibly small, i.e. neutrino mass are not correctly fitted in this calculation. The black line is the analytical calculation based on 1-loop RGEs and the leading-log approximation with an assumed \( m_{\text{SUSY}} = 1 \) TeV. The dotted lines are the numerically exact results for this invariant using 1-loop RGEs, while the full lines are the exact results using 2-loop RGEs. Obviously the “invariant” does depend to a certain degree on the mSugra point, as already pointed out in section (3). However, we also find a considerable upward shift of \( (m_L^2 - m_E^2)/M_T^2 \), when going from the 1-loop to the 2-loop calculation. Since the dependence of \( (m_L^2 - m_E^2)/M_T^2 \) on the value of \( M_T \) is only logarithmic, even such a moderate change in the invariant is important, if one wants to extract an indirect estimate on \( M_T \) from such a measurement. Note that for the point SPS1a’ the calculation stops at \( M_{15} \sim 10^{11.6} \) GeV, the lowest value of \( M_{15} \) for which correct electro-weak
Man an arbitrary choice of $\lambda$, highly precise measurements will be necessary, especially if $M_T$ at 2-loop order will be necessary. Also note that, due to the logarithmic dependence on $\lambda$, symmetry breaking occurs.

We have checked by an exact numerical calculation that the other invariants shown in section (3) suffer from similar changes when going from 1-loop order to 2-loop. In other words, if one wants to learn about the seesaw scale from measurements of the soft masses, a careful analysis at 2-loop order will be necessary. Also note that, due to the logarithmic dependence on $M_T$, highly precise measurements will be necessary, especially if $M_T$ is large, say $M_T \geq 10^{12-13}$ GeV.

Fig. (6) shows $(m_L^2 - m_E^2)/M_T^2$ calculated with Yukawa couplings fitted to neutrino data, for an arbitrary choice of $\lambda_2 = 0.5$. The calculation uses 2-loop RGEs and results are shown again for the mSugra standard points SPS1a' and SPS3. For $M_T$ low, say $M_T \leq 10^{13}$ GeV or so in this example, Yukawa couplings which explain current neutrino data are too small to induce any significant effect in the determination of $(m_L^2 - m_E^2)/M_T^2$.

However, for larger values of $M_T$ sizeable differences between fig. (5) and fig. (6) show up. First of all, for negligibly small Yukawas the calculation can vary $M_T$ freely up to the GUT scale. If instead we insist to fit neutrino masses, such large values for $M_T$ are not allowed. The downward turn in $(m_L^2 - m_E^2)/M_T^2$ is due to Yukawas, which if larger than $\mathcal{O}(0.1)$ contribute sizeable in the running of the soft parameters. In the example shown in this figure $\lambda_2 = 0.5$ has been chosen. For smaller values of $\lambda_2$ again for fixed values of the Yukawa couplings to fit neutrino masses a lower $M_T$ is required. Correspondingly, for smaller $\lambda_2$ the effect of the Yukawas is seen for smaller values of $M_T$.

It is also found that slepton mass parameters of the first and second generation run differently for large values of $M_T$, see fig. (6). This difference can be traced to the fact that we have fitted neutrino angles to take exact TBM values. In this limit, $m_{L_1}^2 \propto \Delta m_{3\odot}^2$ while $m_{L_2}^2 \propto \Delta m_{\text{Atm}}^2$. Thus, at the largest values of $M_T$ sizeable mass differences between 1st and 2nd generation sleptons show up. This difference is expected to be smaller for non-zero values of $s_{13}$. Note, however, that for the example points shown in fig. (6), there is the upper limit on $M_T$ from $Br(\mu \rightarrow e + \gamma)$, discussed in the last subsection. For SPS1a' $M_T \sim 1.5 \cdot 10^{13}$ GeV, for SPS3 $M_T \sim 6 \cdot 10^{13}$ GeV. This limits the range of $M_T$ where differences between 1st and 2nd generation slepton masses might be observable. We mention that a recent paper [24] claims

**Figure 5.** (left) "Invariant" $(m_L^2 - m_E^2)/M_T^2$, calculated with negligibly small Yukawa couplings for two mSugra standard points. The figure shows a comparison of different calculation. The curve labeled “Analytic” uses the formulas presented in the previous section. 1-loop and 2-loop stand for exactly solved numerical calculations using 1-loop and 2-loop RGEs. Note the significant shift when going from 1-loop order to 2-loop order.

**Figure 6.** (right) "Invariant" $(m_L^2 - m_E^2)/M_T^2$ calculated with Yukawa couplings fitted to neutrino data, for an arbitrary choice of $\lambda_2 = 0.5$. The calculation uses 2-loop RGEs. Results are shown for SPS1a’ and SPS3. Neutrino angles are assumed to have exact TBM values.
that mass differences between smuons and selectrons can be measured very accurately, even at the LHC. Depending on the mSugra point \((m_{\tilde{\mu}}^2 - m_E^2)/(m_{\tilde{\mu}}^2 + m_E^2)\) as small as \(O(10^{-4})\) might be measurable [24] provided the leptons have sufficient energy to pass the experimental cuts.

**Figure 7.** Branching ratios for LFV decays versus \((m_{\tilde{\tau}}^2 - m_{\tilde{\tau}}^2)/M_T^2\) for SPS3 for two different values of \(\lambda_2\). Measuring both types of observables allow in principle to disentangle \(\lambda_2\) and \(M_T\).

All observables discussed so far are sensitive only to a combination of \(M_T\) and \(\lambda_2\). If, however, both LFV decays as well as \((m_{\tilde{\chi}_1^2} - m_{\tilde{\chi}_2^2})/M_T^2\) could be measured in the future, one could disentangle the two parameters, in principle, by combining both measurements. This is demonstrated in fig. (7), which shows LFV decays, \(Br(\mu \rightarrow e + \gamma)\) and \(Br(\tilde{\tau}_2 \rightarrow e, \mu + \chi^0_1)\) versus \((m_{\tilde{\tau}}^2 - m_{\tilde{\tau}}^2)/M_T^2\), for two different values of \(\lambda_2\). Note again that the “dip” in \(Br(\mu \rightarrow e + \gamma)\) is due to a level-crossing of selectron and smuon mass eigenstates. However, again we warn that a full 2-loop calculation is needed, before any quantitative conclusions could be drawn from such a measurement.

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