Abstract

We discuss that Schnabl’s solution is an off-shell extension of the boundary state describing a D-brane in the closed string sector. It gives the physical meaning of the gauge invariant overlaps for the solution in our previous paper and supports Ellwood’s recent proposal in the operator formalism.

In our previous paper [1], it was discussed that a class of gauge invariant observables gives the same values for the analytic solution [2] given by Schnabl as the ones for the numerical solution [3, 4, 5] in the level truncation in open string field theory [6]. It gives another interesting evidence that the numerical solution is gauge equivalent to the analytic solution.

The gauge invariant observables $O_V(\Psi)$ for an open string field $\Psi$ are called gauge invariant overlaps in [1] and are given by

$$O_V(\Psi) = \langle V(i) f_\mathcal{I}[\Psi] \rangle = \langle \mathcal{I} | V(i) | \Psi \rangle, \quad f_\mathcal{I}(z) = \frac{2z}{1-z^2},$$

where the CFT correlator is defined on an upper half plane. The on-shell closed string vertex operator $V(i)$ is inserted at the midpoint of the open string, and the conformal mapping $f_\mathcal{I}(z)$ plays the role of the identity state $\langle \mathcal{I} \rangle$, identifying the left half of the string with its right half. They were originally discussed in [7] to give the interaction of an on-shell closed string with open strings in the open string field theory.

For the analytic solution

$$|\Psi_{\lambda=1}\rangle = |\psi_0\rangle + \sum_{n=0}^{\infty} (|\psi_{n+1}\rangle - |\psi_n\rangle - \partial_r |\psi_r\rangle |r=n\rangle \equiv |\psi_0\rangle + |\chi\rangle,$$

Schnabl’s Solution and Boundary States in Open String Field Theory

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since it was shown [1, 8] that $O_V(\psi_r)$ doesn’t depend on $r$, one can see that

$$O_V(\Psi_{\lambda=1}) = \langle I | V(i) | \Psi_{\lambda=1} \rangle = \langle I | V(i) | \psi_0 \rangle.$$  

Therefore, it suggests that only the first term $|\psi_0\rangle$ contributes to the gauge invariant overlaps $O_V(\Psi_{\lambda=1})$.

In the paper [1], the gauge invariant overlaps were used to examine whether the numerical solution is gauge equivalent to the analytic solution, but their physical meaning wasn’t clear. Recently, Ellwood has made an interesting proposal [8] that the gauge invariant overlaps are related to the closed string tadpoles as

$$O_V(\Psi) = A_{\Psi}(V) - A_0(V),$$

where $A_{\Psi}(V)$ is the disk amplitude for a closed string vertex operator $V$ with the boundary condition of the CFT given by the open string field solution $\Psi$, and $A_0(V)$ is the usual disk amplitude in the perturbative vacuum.

In fact, for the analytic solution, he has shown that this is the case. Since, after tachyon condensation, there are no D-branes, no closed tadpoles are available, and thus $A_{\Psi}(V) = 0$. Therefore, the gauge invariant overlap $O_V(\Psi)$ for the solution gives the usual disk amplitude of the opposite sign with one closed string emitted. It suggests that the analytic solution given by Schnabl is closely related to the boundary state describing a D-brane in the closed string perturbation theory, as expected.

In this note, we will explicitly demonstrate this in the operator formalism of open string field theory by using the Shapiro-Thorn vertex $\langle \hat{\gamma}(1, 2) | \rangle$ [9], which maps an open string state to the corresponding closed string state. (For more detail, see appendix B in [1].) By using the open-closed string vertex $\langle \hat{\gamma}(1, 2) |$, as discussed in detail in [1], one may rewrite a gauge invariant overlap as

$$O_V(\psi) = \langle \hat{\gamma}(1, 2) | \phi_c \rangle_{1c} | \psi \rangle_2,$$

where $| \phi_c \rangle_{1c}$ is an on-shell state given by the vertex operator $V$ of the closed string $1_c$, and $| \psi \rangle_2$ is a state of the open string 2.

For the analytic solution, one has

$$O_V(\Psi_{\lambda=1}) = \langle \hat{\gamma}(1, 2) | \phi_c \rangle_{1c} | \Psi_{\lambda=1} \rangle_2 = \langle \hat{\gamma}(1, 2) | \phi_c \rangle_{1c} | \psi_0 \rangle_2 + \langle \hat{\gamma}(1, 2) | \phi_c \rangle_{1c} | \chi \rangle_2,$$

and as mentioned above, the second term on the right hand side is zero for the on-shell closed string state $| \phi_c \rangle_{1c}$. As for the first term on the right hand side, one can see that the open string tachyon state $| \psi_0 \rangle = (2/\pi) c_1 | 0 \rangle$ is transformed via the vertex $\langle \hat{\gamma}(1, 2) |$ into the boundary state $\langle B |$. In fact, one can obtain the relation

$$\langle \hat{\gamma}(1, 2) | \psi_0 \rangle_2 \mathcal{P}_{1c} = \frac{1}{2\pi} \langle B | c_0^-$$

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with the level matching projection $\mathcal{P}_c$ for the closed string $1_c$, where the boundary state is the usual one

$$
\langle B \mid = \langle 0 \mid c_{-1} \bar{c}_{-1} c_0^+ e^{-\sum_{n=1}^{\infty} \frac{1}{n} \alpha_n + c_n \bar{b}_n + \bar{c}_n b_n} \n$$
in the closed string perturbation theory. One thus finds that

$$
\mathcal{O}_V (\Psi_{\lambda=1}) = \frac{1}{2\pi} \langle B \mid c_0^- \mid \phi_c \rangle,
$$

which is in precise agreement with Ellwood’s result.

Since the second term $\langle \hat{\gamma}(1_c, 2) \mid \phi_c \rangle_{1_c} \mid \chi \rangle_{2}$ doesn’t necessarily vanish for off-shell closed string states, one may conclude that the transform of the analytic solution $\mid \Psi_{\lambda=1} \rangle$ via the Shapiro-Thorn vertex is an off-shell extension of the boundary state $\mid B \rangle$. Although it seems more elaborate to calculate $\langle \hat{\gamma}(1_c, 2) \mid \chi \rangle_{2}$, it would be interesting to find the relation of the off-shell boundary state with the equation of motion in closed string field theory \[10, 11, 12\].

Furthermore, given all the interactions between open strings and closed strings in the open-closed string field theory \[13\], one may raise a question whether the Schnabl solution is consistent with the equations of motion of the open-closed string field theory, even in the vanishing string coupling constant limit, but it is beyond the scope of this paper. However, if it is consistent, the relation of the transform of the analytic solution $\mid \Psi_{\lambda=1} \rangle$ via the Shapiro-Thorn vertex with the boundary state $\mid B \rangle$ could be clearer along with the interactions of the theory \[13\]. We think that our observation in this paper may serve as an encouraging step in the investigation.

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1 More recently, the idea of this paper is extended to the marginal solutions \[14\] and the rolling tachyon solution \[15\] by one of the authors \[16\]. It may suggest that our observation of this paper could be more generic.
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