Improved LuGre-based friction modeling of the electric linear load simulator

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Abstract—In order to fulfill the requirement that the electric linear load simulator needs to improve the loading accuracy by the compensation method based on the friction model in the active loading situation with small load, considering the influence of load on coulomb friction and maximum static friction, an improved LuGre friction model with load is proposed. At the same time, since the friction effect is highly nonlinear and determined by numerous parameters, the differential evolution algorithm is utilized to identify the static and dynamic parameters of the model in two steps offline. Finally, under the sinusoidal signal for force command, the experimental test results verify that the improved LuGre model with identified parameters can accurately describe the friction characteristics of the electric linear load simulator.

1. Introduction

The electric linear load simulator is mainly used in the occasion of active loading with small load, so the proportion of friction in the total load will be more prominent. Especially in the motion stages such as startup, low speed and speed reversal, the phenomena of startup dead zone, low-speed crawling and loading waveform distortion are caused due to the existence of nonlinear friction, which restrict the improvement of loading performance of the load simulator. It is a more effective way to make targeted compensation from the control point of view by model-based behavior prediction of friction\cite{1}. Therefore, it is necessary to study its friction characteristics and establish an accurate friction model.

At present, the description of friction is mainly divided into static or dynamic models, such as coulomb, viscous, Stribeck, Dahl, LuGre, and GMS friction model, etc. And a series of static and dynamic characteristics such as Stribeck effect, pre-slip displacement, stick-slip motion, friction memory, and variable critical friction can accurately be described by LuGre model, which have been widely used in the research of surface friction involving metal-to-metal contact\cite{2}. In addition, it is also worth to improve the model by combining the influence of factors on friction under realistic working conditions, such as temperature, load and position\cite{3}.

In the previous LuGre-based modeling research of load simulators, it is generally considered that friction is an inherent property, the parameters will not change after its own structure and installation are fixed, but under actual operating conditions, the simulator and the loaded mechanism move together, which lead to output load also acts on the simulator itself, the friction characteristics will change with the load. Therefore, in this paper, an improved LuGre friction model with the load is proposed, and its parameters are identified offline by the differential evolution algorithm. The accuracy of the model is verified by the comparison between simulation and experiment.
2. System Specification

The architecture and object of the electric linear load simulator and the loaded mechanism are shown in Fig. 1. The servo driver receives command signal to act on the permanent magnet synchronous motor, and then load from the output rod through motion conversion and power transmission by planetary roller screw is finally applied to the loaded mechanism through the force sensor.

![Fig. 1 The architecture and object of the electric linear load simulator and the loaded mechanism](image)

According to the system composition and principle, the kinetic equation is obtained as

\[ T_m = k_t i_q = J_m \omega + T_f + T_L \]  \( \text{(1)} \)

\[ \theta_s = \frac{2\pi}{l_s} x_s \]  \( \text{(2)} \)

\[ F_L = k_s T_L = k_p (x_s - x_p) \]  \( \text{(3)} \)

Where \( T_m \) is electromagnetic torque of motor; \( k_t \) is coefficient of torque; \( i_q \) is control current of servo driver; \( J_m \) is moment of inertia; \( \omega \) is angular velocity; \( T_f \) is friction torque; \( T_L \) is load torque; \( \theta_s \) is rotation angle of nut; \( l_s \) is lead; \( x_s \) is linear displacement; \( F_L \) is load force of output rod; \( k_p \) is elastic stiffness of force sensor; \( x_p \) is displacement of the loaded mechanism.

The LuGre friction model assumes that two objects are in contact with elastic bristles at the microscopic level. Because the deformation of each bristle is random at different stages, so the average deformation is used to represent, and give it stiffness and damping properties\(^4\). Therefore, its expressions are

\[ T_f = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \omega \]  \( \text{(4)} \)

\[ \dot{z} = \omega - \frac{\sigma_0}{g(\omega)} z \]  \( \text{(5)} \)

\[ g(\omega) = T_c + (T_s - T_c) e^{-(\omega/\omega_{st})^2} \]  \( \text{(6)} \)

Where \( z \) is average deformation of bristles; \( \sigma_0 \) and \( \sigma_1 \) are respectively coefficients of stiffness and damping; \( \sigma_2 \) is viscous friction coefficient; \( g(\omega) \) represents the Stribeck effect and is a positive monotonic decreasing function; \( T_c \) is coulomb friction; \( T_s \) is maximum static friction; \( \omega_{st} \) is Stribeck characteristic angular velocity.

The electric linear load simulator is hindered or promoted by the loaded mechanism during the movement process, i.e. aiding or opposing load. They cause change of the axial load and the normal load between the contact surfaces, which lead to the linear change of coulomb friction torque and maximum static friction torque, so a first-order linear function can be used to describe this relationship

\[ T_c = a_c T_L + b_c, T_s = a_s T_L + b_s \]  \( \text{(7)} \)

Where \( a_c(\delta), b_c(\delta) \) are slope and intercept of the first-order linear function.

So Eq.(6) is modified as
\[ g(\omega, T_L) = a_c |T_L| + b_c + [(a_s - a_c)|T_L| + b_s - b_c]e^{-(\omega/\omega_s)^2} \]  

Thus, the mathematical description of improved LuGre friction model is obtained by Eq.(4,5,8).

In order to analyse the influence of friction on force tracking performance, it is necessary to improve the force response and reduce speed fluctuation. Therefore, a cascade control method including an outer force loop and inner speed loop is used for the load simulator. In addition, the positional disturbance of the loaded mechanism is a measurable signal, which can be eliminated by using feedforward compensation. The block diagram is shown in fig.2 by Laplace transform.

![Fig. 2 The block diagram of closed-loop model of the electric linear load simulator](image)

Where \( F_{lc} \) is force command; \( K_{fp}, K_{op} \) are proportional gain of force closed loop and speed loop controller; \( G_{f\omega}(s) \) is feedforward compensation transfer function of the position disturbance of the loaded mechanism, its expression is deduced as

\[ G_{f\omega}(s) = \frac{k_s^2 J_m s^2 + k_s^2 k_t K_{op} s}{k_s k_t K_{fp}} \]  

(9)

3. Parameter Identification of Improved LuGre Model

The improved LuGre with load is a relatively complex eight-parameters model with highly nonlinear behavior. The differential evolution algorithm based on the inspiration of intelligent behavior of biological groups has a simple structure and is easy to be processed in parallel. Therefore, it is an ideal option for nonlinear parameter identification.

3.1 Differential evolution algorithm

The basic principle of the differential evolution algorithm is that the solution of each optimization problem is an individual in the search space. In iteration process, the vector difference of any two individuals in the group is weighted to scale, and then summed with the third individual to generate a mutant individual, and a new crossover individual is obtained by operation according to certain rules. If the new individual shows a better fitness than the old individual, it will be used as an superior individual to replace the old individual in the next generation, otherwise, the old individual will remain, as iterative operations, the global optimal individual will be gradually approached\(^3\).

Firstly, give a D-dimensional linear search space as \( X \in [X_{min}, X_{max}] \), and at the \( k \) iteration, the \( i \) individual is \( x_i^k \in X \). Set the corresponding population size \( S(1 \leq i \leq S) \) and the maximum number of iterations \( G(1 \leq k \leq G) \), then execute the following operations, including mutation, crossover and selection.

\[
\begin{align*}
    h_i^{k+1} &= x_{p1}^k + F_C(x_{p2}^k - x_{p3}^k) \\
    c_{i,j}^{k+1} &= \begin{cases} 
        h_{i,j}^{k+1} \quad rand[0,1] \leq F_Cr \\
        x_{i,j}^k \quad \text{otherwise}
    \end{cases} \\
    x_i^{k+1} &= \begin{cases} 
        c_i^{k} + f(c_i^{k+1}) \leq f(x_i^k) \\
        x_i^k + f(c_i^{k+1}) > f(x_i^k)
    \end{cases}
\end{align*}
\]  

(10) (11) (12)
Where $h_{k}^{k+1}$ is a new individual generated by mutation operation; $x_{p1}^{k}$, $x_{p2}^{k}$ and $x_{p3}^{k}$ are three different individuals randomly selected in the group; $F_{sc}$ is mutant factor; $F_{cr}$ is crossover factor; $J(x)$ is fitness function, to achieve "survival of the fittest" and further ensure that the group always evolves in the optimal direction.

3.2 Static parameters identification
The static parameters identification mainly considers the situation under a stable speed. At this situation, the average deformation of bristles tends to a steady value, and then the friction torque $T_{fss}$ is:

$$T_{fss} = g(\omega, T_L)s gn(\omega) + \sigma \omega$$

(13)

The electric linear load simulator is loaded with a constant load force and the loaded mechanism runs at a constant speed. Since the load force is constant, the speeds of both load simulator and loaded mechanism can be considered the same. From Eq. (1), it can be known that the angular acceleration is zero at a constant speed:

$$T_{fss} = k_t i_q - T_L$$

(14)

Therefore, the friction torque data can be obtained indirectly by calculating the average value of the current $i_q$ within a constant speed stage. And then define the identified parameters, namely individual of group, as $I_p = [a_c, b_c, a_s, b_s, \sigma_2, \omega_{st}]$. The fitness function is the square loss of the $T_{fss}$ obtained from the experimental test and the $\hat{T}_{fss}$ obtained from the identified parameters. The smaller the value, the higher the fitness. Its expression is defined as

$$J = \frac{1}{2} \Sigma [T_{fss}(\omega, T_L) - \hat{T}_{fss}(\omega, T_L|I_p)]^2$$

(15)

3.3 Dynamic parameters identification
The dynamic parameters are mainly related to the average deformation of the bristles, which can not be directly measured. Generally, when the driving torque is less than the maximum static friction torque, the system is in the pre-slip stage. At the same time, due to the spring effect of the bristles, the system will produce a very small pre-slip angular displacement. Therefore, the friction dynamic parameters can be identified by using the driving torque input and the pre-slip angular displacement output. To avoid the influence of other factors, a current command with a slowly increasing amplitude is usually applied in the open-loop state, and the driving torque will gradually increase slowly until it exceeds the maximum static friction torque.

$$i_q = k_q t$$

(16)

Where $k_q$ is gradient coefficient of current command with respect to time.

Under the open-loop condition, the system is in no-load state, $T_L = 0$. Define the state variables $x_1 = z, x_2 = \theta_s, x_3 = \omega$, input $i_q$, output $y = \dot{\theta}_s$, the state space equation is obtained as

$$\begin{align*}
\dot{x}_1 &= x_3 - \frac{[\sigma_0]x_3}{g(x_3, 0)}x_1 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= \frac{(k_t/J_m)}{i_q} - \frac{(1/J_m)}{[\sigma_0 x_1 + \sigma_1 x_3 - \sigma_1 [\sigma_0 x_3]/g(x_3, 0)]}x_1 + \sigma x_3
\end{align*}$$

(17)

According to Eq.(17), the static parameters $b_c, b_s, \sigma_2, \omega_{st}$ can be identified. So the dynamic parameters expected to be identified can be defined as $I_d = [\sigma_0, \sigma_1]$. The fitness function becomes the square loss function of the pre-sliding displacement $y$ obtained by the experimental test at time $t_i$ and the pre-sliding displacement $\hat{y}$ obtained from the state equation with identified parameters:

$$J = \frac{1}{2} \Sigma [y(t_i) - \hat{y}(t_i|I_p, I_d)]^2$$

(18)

4. Experimental Results and Discussion
The experiments are carried out on the platform shown in Fig.1, and the system parameters involved are shown in Tab.1. The accuracy of the results is verified by simulation and experiment.
Tab. 1 Performance and control parameters of the electric linear load simulator

| Parameter | Value |
|-----------|-------|
| $k_t$ (N·m/A) | 1.65 |
| $J_m$ (kg·m²) | 2.6×10⁻³ |
| $l_s$ (m) | 2.54×10⁻³ |
| $k_p$ (N/m) | 4×10⁶ |
| $k_q$ (A/s) | 0.03 |
| $K_{FP}$ | 0.01 |
| $K_{UP}$ | 0.3 |

4.1 Results of parameter identification

Both the static and dynamic friction parameters are identified by the differential evolution algorithm. The upper and lower limits of the static parameters search space are $X_{pmax}=[1, 1, 1, 1, 1, 1]$ and $X_{pmin}=[0, 0, 0, 0, 0, 0]$. Similarly, the dynamic parameter search space are $X_{dmax}=[500, 5]$ and $X_{dmin}=[0.1, 0.1]$. In addition, the parameters of algorithm are set to $S=150$, $G=250$, $F_{sc}=0.95$, and $F_{cr}=0.85$. Results of the parameter identification are as follows in Tab.2.

Tab. 2 Results of parameter identification of the improved LuGre friction model

| Parameter | Value | Value | Value | Value | Value | Value |
|-----------|-------|-------|-------|-------|-------|-------|
| $a_c$ (N·m) | 0.3197 | 0.3036 | 0.2564 | 0.5098 | 0.0204 | 0.3449 |
| $b_c$ (N·m) | 0.0204 | 0.0204 | 0.0204 | 0.0204 | 0.0204 | 0.0204 |
| $a_s$ (N·m) | 0.0204 | 0.0204 | 0.0204 | 0.0204 | 0.0204 | 0.0204 |
| $b_s$ (N·m) | 0.5098 | 0.5098 | 0.5098 | 0.5098 | 0.5098 | 0.5098 |
| $\sigma_2$ (N·m·s/rad) | 351.03 | 351.03 | 351.03 | 351.03 | 351.03 | 351.03 |
| $\omega_{st}$ (rad/s) | 0.9014 | 0.9014 | 0.9014 | 0.9014 | 0.9014 | 0.9014 |
| $\sigma_0$ (N·m/rad) | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| $\sigma_1$ (N·m·s/rad) | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |

The three-dimensional surface of the improved LuGre model is redrawn according to the static parameters identified, as shown in fig.3(a), which can characterize the relationship between friction torque, speed and load torque. The fitted surface is in good agreement with the actual test data, illustrating that the result is highly reliable. At the same time, when the load torque section is given, it intersects with the surface to become Strubeck curve. While the speed section is given, the friction torque presents a linear relationship with the load torque. As for dynamic parameters, the identification simulation results and experimental test results of the pre-slip angular displacement are plotted as shown in Fig.3(b), which are basically consistent.

4.2 Experimental verification

Since the parameters can not be obtained directly through experiments, as long as the system output under identified parameters is consistent with the output from actual experimental measurement, the accuracy of parameters can be verified, and it will illustrate the validity of the friction model.

Therefore, sinusoidal signal for load force command is set as $F_{Lc}=-600\sin(\pi t)+800$ N, and the loaded mechanism is respectively experienced a stationary motion command ($x_{p0}=0$) and a S-curve motion command ($x_{p1}$, displacement is 10mm, velocity is 2mm/s, acceleration is 3mm/s², and duration is 6s). The results are shown in Fig.4, simulation of the load force output response is in good agreement with experimental results, and the maximum error does not exceed 15%, which can accurately describe the friction characteristics of the system.

When the loaded mechanism is stationary, the output rod needs to move in a reverse direction when the load force changes at peak value, causing speed to pass through the zero point. Therefore, stagnation occurs due to the influence of friction, resulting in the phenomenon of “flat top” of the load force. While the loaded mechanism moves in an S-curve, the output rod is forced to follow the
movement. Furthermore, additional speed is required to maintain the force tracking, which makes the friction magnitude greater along the moving direction. This friction difference causes that an global offset exists along the opposite direction in the actual load force curve in Fig.4 (b).

![Image](image_url)

(a) Under the conditions of static \( x_{p0}=0 \)  
(b) Under the conditions of S-curve motion \( x_{p1} \)

Fig. 4 Simulation and experimental results of load force output response under different position disturbance conditions

5. Conclusion

Based on the results and discussions presented above, the conclusions are obtained as below:

1) Based on the working condition of the electric linear load simulator and the loaded mechanism, considering the influence of load on its own friction, an improved LuGre friction model is introduced, and the experimental platform is used to respectively obtain the experimental data at constant speed and pre-slip stage. And then, the differential evolution algorithm was used to complete the static and dynamic parameters identification.

2) A closed-loop model of the system with friction is built. In the situation of different position disturbance of the loaded mechanism, the simulation results of the force tracking are compared with the experimental results under sinusoidal force command, which verifies that the restatic friction parameters are accurately identified. Consequently, the friction characteristics of the electric linear load simulator can be well reflected by the improved LuGre friction model.

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