A hyperchaotic hyperjerk system with four nonlinearities, its dynamical analysis and circuit realization

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Abstract. A new four-dimensional hyperchaotic hyperjerk system with four nonlinearities is proposed in this paper. The dynamical properties of the new hyperjerk system are described in terms of phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc. Also, a detailed dynamical analysis of the new hyperjerk system has been carried out with bifurcation diagram and Lyapunov exponents. As an engineering application, an electronic circuit realization of the new hyperchaotic hyperjerk system is designed via MultiSIM to confirm the feasibility of the theoretical hyperchaotic hyperjerk model.

1. Introduction
Hyperjerk systems are special types of mechanical chaotic systems arising in chaos literature [1-2]. Chaotic systems are very useful in many applications in science and engineering such as weather systems [3-4], ecology [5], neurons [6-7], biology [8-10], cellular neural networks [11-12], chemical reactors [13-14], oscillators [15-20], robotics [21-24], encryption [25-30], finance systems [31-32], circuits [33-45], secure communication [46-50], etc.

In physics, a hyperjerk ODE can be written as the high-order dynamics

$$\frac{d^n x}{dt^n} = \varphi \left( x, \frac{dx}{dt}, \ldots, \frac{d^{n-1} x}{dt^{n-1}} \right), \quad (n \geq 4) \tag{1}$$

In (1), \(x(t)\) stands for the displacement, \(\frac{dx}{dt}\) the velocity, \(\frac{d^2 x}{dt^2}\) the acceleration, \(\frac{d^3 x}{dt^3}\) the jerk and higher-order derivatives are called as hyperjerk terms.

Thus, we call the ODE (1) as the hyperjerk differential equation.

For qualitative analysis, it is convenient to express the hyperjerk ODE (1) in a system form.

Using phase variables, we can express the hyperjerk differential equation (1) as follows:
Jerk systems are special cases of hyperjerk systems when \( n = 3 \). Thus, jerk systems can be described by the following general system of differential equations.

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
& \quad \vdots \\
\dot{x}_n &= \varphi(x_1, x_2, \ldots, x_n)
\end{align*}
\]  

(2)

Many jerk systems have been reported in the chaos literature [51-58]. Jerk systems have important applications in mechanical engineering [1-2]. Some famous jerk systems can be cited as Sprott systems [51], Li system [52], Elsonbaty system [53], Coulelet system [54], Kengne system [55], Vaidyanathan systems [56-60], etc.

In the literature, many hyperjerk systems have been reported by many scientists [61-68]. Some popular hyperjerk systems are Chlouverakis system [61], Munmuangsan system [62], Daltzis system [63], Wang system [64], Pham system [65], Vaidyanathan systems [66-70], etc.

In this research paper, we report the finding of a new hyperchaotic hyperjerk system with four nonlinearities. We describe the phase plots of the hyperjerk system and do a rigorous dynamic analysis by finding bifurcation diagrams, Lyapunov exponents, etc. Bifurcation analysis is very useful to understand the special properties of chaotic and hyperchaotic systems [71-76].

Section 2 describes the new hyperchaotic hyperjerk system, its phase plots and Lyapunov exponents. Section 3 describes the dynamic analysis of the new hyperchaotic hyperjerk system. Furthermore, an electronic circuit realization of the new chaotic system is presented in detail in Section 4. The circuit experimental results of the new hyperjerk system in Section 4 agreement with its numerical simulations via MATLAB obtained in Section 2. Section 5 draws the main conclusions.

2. A new hyperchaotic hyperjerk system

In this work, we report a new 4-D hyperjerk system given by the dynamics

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -x_1 - x_2 - ax_3 + b(|x_2| + |x_3|) - cx_4^2 x_4 - dx_2^2
\end{align*}
\]  

(4)

where \( x_1, x_2, x_3, x_4 \) are state variables and \( a, b, c, d \) are positive constants.

In this paper, we show that the hyperjerk system (1) is hyperchaotic for the parameter values

\[
a = 3.6, \quad b = 0.02, \quad c = 3, \quad d = 0.05
\]  

(5)

For numerical simulations, we take the initial values of the hyperjerk system (4) as \( X(0) = (0.1, 0.1, 0.1, 0.1) \).

Figures 1-4 show the 2-D projections of the new hyperjerk system (4) in \( (x_1, x_2), (x_2, x_3), (x_3, x_4) \) and \( (x_1, x_4) \) coordinate planes, respectively.
Figure 1. 2-D plot of the hyperchaotic hyperjerk system (4) in the \((x_1, x_2)\) plane for 
\(X_0 = (0.1, 0.1, 0.1, 0.1)\) and \((a, b, c, d) = (3.6, 0.02, 3, 0.05)\)

Figure 2. 2-D plot of the hyperchaotic hyperjerk system (4) in the \((x_2, x_3)\) plane for 
\(X_0 = (0.1, 0.1, 0.1, 0.1)\) and \((a, b, c, d) = (3.6, 0.02, 3, 0.05)\)
Figure 3. 2-D plot of the hyperchaotic hyperjerk system (4) in the \((x_3, x_4)\) plane for 
\[X_0 = (0.1, 0.1, 0.1, 0.1)\] and \((a, b, c, d) = (3.6, 0.02, 3, 0.05)\)

Figure 4. 2-D plot of the hyperchaotic hyperjerk system (4) in the \((x_1, x_4)\) plane for 
\[X_0 = (0.1, 0.1, 0.1, 0.1)\] and \((a, b, c, d) = (3.6, 0.02, 3, 0.05)\)
For the rest of this section, we take the values of the parameters as in the hyperchaotic case (5), i.e. $(a, b, c, d) = (3.6, 0.02, 3, 0.05)$.

The equilibrium points of the new hyperchaotic hyperjerk system (4) are obtained by solving the system of equations

\begin{align*}
x_1 &= 0 \quad \text{(6a)} \\
x_2 &= 0 \quad \text{(6b)} \\
x_3 &= 0 \quad \text{(6c)} \\
-x_1 - x_2 - ax_3 + b(|x_2| + |x_3|) - cx_1^2x_4 - dx_2^2 &= 0 \quad \text{(6d)}
\end{align*}

From (6a), (6b) and (6c), we deduce that $x_1 = x_2 = x_3 = 0$.

Substituting these in (6d), we obtain $-x_1 = 0$. This gives $x_4 = 0$.

Hence, $E_0 = (0, 0, 0, 0)$ is the unique equilibrium of the chaotic jerk system (4).

The Jacobian matrix of the new hyperjerk system (4) at $E_0 = (0, 0, 0, 0)$ is obtained as

\[
J = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-1 & -1 & -3.6 & 0 \\
\end{bmatrix}
\]

The Jacobian $J$ has the spectral values $\lambda_{1,2} = 0.1604 \pm 1.8395i$, $\lambda_{3,4} = -0.1604 \pm 0.5172i$

This shows that the equilibrium point $E_0$ is a saddle-focus and unstable.

For the parameter values as in the hyperchaotic case (5) and the initial state $X_0 = (0.1, 0.1, 0.1, 0.1)$, the Lyapunov exponents of the new jerk system (4) are determined using Wolf’s algorithm as

\[
LE_1 = 0.1344, \quad LE_2 = 0.0411, \quad LE_3 = 0, \quad LE_4 = -1.2929
\]

The hyperjerk system (4) is hyperchaotic since it has 2 positive Lyapunov exponents. Thus, the system (4) exhibits a self-excited strange hyperchaotic attractor. Also, we note that the sum of the Lyapunov exponents in (8) is negative. This shows that the hyperjerk system (4) is dissipative.

The Kaplan-Yorke dimension of the hyperjerk system (4) is determined as

\[
D_{KY} = 3 + \frac{LE_1 + LE_2 + LE_3}{|LE_4|} = 3.1357,
\]

which indicates the high complexity of the hyperchaotic hyperjerk system (4).

3. Bifurcation Analysis for the New Hyperchaotic Hyperjerk System

In this section, we describe a bifurcation analysis for the new hyperjerk system (4) introduced in Section 2. Bifurcation analysis is an important topic for studying chaotic systems [71-76].

Here, we select $a$ and $c$ as the control parameters and fix others.

We fix $b = 0.02$, $c = 3$, $d = 0.05$, the initial condition $X_0 = (0.1, 0.1, 0.1, 0.1)$ and vary $a$ in the region of $[3.6, 4.6]$.

Obviously, from the bifurcation diagram and the Lyapunov exponents shown in Figure 5, one can get that the hyperjerk system (4) depicts hyperchaos in the region of $[3.6, 3.95]$; then the system gets into chaos and finally converts into periodic orbits. In addition, there is quasi-periodic behavior in the parameter range. Some sample results are shown in Figures 6-9.
Figure 5. Bifurcation diagram and Lyapunov exponents of the new hyperjerk system (4), where we fix $b = 0.02$, $c = 3$, $d = 0.05$ and the initial conditions $(0.1, 0.1, 0.1, 0.1)$

Figure 6. Phase plots of the new hyperjerk system (4), where we fix $b = 0.02$, $c = 3$, $d = 0.05$ and the initial conditions $(0.1, 0.1, 0.1, 0.1)$. When $a = 3.6$, the system (5) depicts hyperchaos.

Figure 7. Phase plots of the new hyperjerk system (4), where we fix $b = 0.02$, $c = 3$, $d = 0.05$ and the initial conditions $(0.1, 0.1, 0.1, 0.1)$. When $a = 4$, the system (4) depicts chaos.
Figure 8. Phase plots of the new hyperjerk system (4), where we fix $b = 0.02$, $c = 3$, $d = 0.05$ and the initial conditions $(0.1, 0.1, 0.1, 0.1)$. When $a = 4.25$, the system (4) depicts quasi-period motion.

Figure 9. Phase plots of the new hyperjerk system (4), where we fix $b = 0.02$, $c = 3$, $d = 0.05$ and the initial conditions $(0.1, 0.1, 0.1, 0.1)$. When $a = 4.6$, the system (4) depicts periodic orbit.

We fix $a = 3.6$, $b = 0.02$, $d = 0.05$, the initial conditions $(0.1, 0.1, 0.1, 0.1)$ and vary $c$ in the region of $[3, 6]$. The constant Lyapunov exponent behavior, meaning the values of the Lyapunov exponents keep invariable when the parameters vary in a certain range, has been reported in some chaotic systems [76]. From the Lyapunov exponent spectrum shown in Figure 10, one can see that the hyperjerk system (4) displays that all the values of the Lyapunov exponents are unchanged and moreover, the value of the maximum Lyapunov exponent keeps invariable and positive when the control parameter $c$ increases in the region of $[3, 6]$. That means the system exhibits robust chaos behavior, which is very important for real-world applications.
Figure 10. Bifurcation diagram and Lyapunov exponents of the new hyperjerk system (4), where we fix $a = 3.6$, $b = 0.02$, $d = 0.05$ and the initial conditions $(0.1, 0.1, 0.1, 0.1)$

4. Circuit Implementation of the New Hyperchaotic Hyperjerk System

In this section, we design an electronic circuit based on the hyperjerk system (4) in MultiSIM software. A circuit design containing three channels with respect to the variables $x_1, x_2, x_3, x_4$ of system (4) is given in Figure 11. The circuit includes simple electronic elements such as resistors, multipliers, capacitors, op-amps and diodes.

In this study, a linear scaling is considered as follows:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= \frac{x_4}{2} \\
\dot{x}_4 &= -2x_1 - 2x_2 - 2ax_3 + 2b(|x_2| + |x_3|) - 16cx_1x_4 - dx_2^2
\end{align*}
\]

By applying Kirchhoff’s laws to this circuit, its dynamics are presented by the following circuital equations:

\[
\begin{align*}
\dot{x}_1 &= \frac{1}{C_1R_1} x_2 \\
\dot{x}_2 &= \frac{1}{C_2R_2} x_3 \\
\dot{x}_3 &= \frac{1}{C_3R_3} x_4 \\
\dot{x}_4 &= -\frac{1}{C_4R_4} x_1 - \frac{1}{C_4R_5} x_2 - \frac{1}{C_4R_6} x_3 + \frac{1}{C_4R_7} |x_2| + \frac{1}{C_4R_8} |x_3| - \frac{1}{C_4R_9} x_1^4 x_4 - \frac{1}{C_4R_{10}} x_2^2
\end{align*}
\]

The values of components in Figure 11 are chosen to match the parameters of new hyperjerk system (4) as follows: $R_1 = R_2 = 400 \, k\Omega$, $R_3 = 800 \, k\Omega$, $R_4 = R_5 = 200 \, k\Omega$, $R_6 = 55.55 \, k\Omega$, $R_7 = R_8 = 10 \, M\Omega$, $R_9 = 8.33 \, k\Omega$, $R_{10} = 8 \, M\Omega$, $C_1 = C_2 = C_3 = C_4 = 1nF$. The circuit simulations of the phase plots are displayed in Figs 12 (a)-(d), which show the chaotic attractors in $x_1$-$x_2$ plane, $x_2$-$x_3$.
plane, $x_3-x_4$ plane and $x_1-x_4$ plane, respectively. As can be seen from the MultiSIM outputs in Figure 12 and numerical simulation in Figures 1-4, the results are similar.

5. Conclusions
A new four-dimensional hyperchaotic hyperjerk system with four nonlinearities was announced in this paper. The dynamical properties of the new hyperjerk system are described in terms of phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc. Also, a detailed dynamical analysis of the new hyperjerk system was done with bifurcation diagram and Lyapunov exponents. Furthermore, an electronic circuit realization of the new hyperchaotic hyperjerk system was carried out via MultiSIM to confirm the feasibility of the theoretical hyperchaotic hyperjerk model.
Fig. 11 Circuit design for the proposed new four-dimensional hyperchaotic hyperjerk system (4)

Fig. 12 MultiSIM chaotic attractors of the new four-dimensional hyperchaotic hyperjerk system (4)  
(a) $x_1$-$x_2$ plane, (b) $x_2$-$x_3$ plane, (c) $x_3$-$x_4$ plane and (d) $x_1$-$x_4$ plane.
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