On cosmology of interacting varying polytropic dark fluids

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Abstract

In this paper a possibility of the accelerated expansion of the large scale universe with a varying polytropic fluid of a certain type is presented. About a special role of non-gravitational interactions between dark energy and dark matter, in particular, about a possibility of improvement and solution of problems arising in modern cosmology, has been discussed for a long time. This motivates us to consider new models, where non-gravitational interactions between varying polytropic fluid and cold dark matter are allowed. Mainly non-linear interactions of a specific type is considered, found in recent literature. In order to understand the behavior of suggested cosmological models, besides cosmographic analysis, Om analysis is applied. Moreover, with different datasets, including a strong gravitational lensing dataset, the observational constraints on the model parameters are obtained using $\chi^2$ analysis.

1 Introduction

Available observational data indicating an accelerated expanding flat universe suggests an existence of dark energy and dark matter, if the dynamics of the background is according to general relativity [1] (and references therien to follow how the accelerated expansion of the universe have been detected for the first time). However, recent research on this problem indicates alternative scenarios as well including modification of general relativity [2] - [10] and, for instance, gravitationally induced particle creation [11] - [17]. The aim of all developed approaches is to generate an appropriate negative pressure to cancel the attractive nature of gravity. It is clear, that the source of anti-gravity cannot be arbitrary and cannot destroy the recent universe and existing symmetries. Moreover, it should have appropriate properties not to alter the dynamics started from the birth of the universe. Therefore, suggested models should pass astrophysical and cosmological tests and the models of dark energy and dark matter should be constrained from available observational data. On the other hand, possible tension existing between different observational datasets from one hand side, and the technological limitations on the other hand, allows a scanning of the physics of our universe up to some redshifts. Incompleteness of this kind makes impossible to finalize our knowledge giving the final models for dark energy and dark matter. Therefore, a phenomenological approach to parameterize the darkness of the recent universe in a form of dark energy and dark matter in recent literature has been used frequently. In particular, recently, a phenomenological modification of polytropic dark fluid has been suggested and the study showed that the model with the following equation of state [18]

\[ P = -AH^{-k}\rho^u, \]  

(1)
where $A$, $k$ and $u$ are constants, while $H$ is the Hubble parameter, can be used as a source of anti-gravity. $A$, $k$ and $u$ parameters should be determined from the observational data. The interest towards to polytropic type fluids is related to their applications in astrophysics. On the other hand, in recent literature alternative models of dark fluids like Chaplygin gas and various viscous dark fluids, also able to solve the problem, systematically have been presented \[10\] - \[22\] (and references therein). In general, dark energy can be parameterized via the equation of state, which provides a functional dependency of the pressure from the energy density: the examples are Chaplygin gas and polytropic fluid with their different modifications. The second option includes a parameterization of the energy density of the source and the examples are ghost dark energy and generalized holographic dark energy models with Nojiri-Odintsov cut-offs (Ricci dark energy and other holographic dark energy models are particular examples of the holographic dark energy with Nojiri-Odintsov cut-offs) \[24\] - \[34\]. Finally, the third option describing dark energy can be a parameterization of the equation of state parameter of dark energy (see for instance \[35\]). It is well known that the simplest model of dark energy is the cosmological constant $\Lambda$, which provides results in well correspondence with available observational data. However, there are two main problems that $\Lambda$CDM faced and then the need of introducing of dynamical dark energy models raised. The first attempt to build a dynamical dark energy model has been based on the idea of varying cosmological constant $\Lambda(t)$. Various phenomenological models of $\Lambda(t)$ have been considered in literature successfully and there is a significant attempt to use of a false vacuum decay to construct models of $\Lambda(t)$. Sometimes in literature such models represented as the models of dark energy based on quantum theory (see for instance \[37\] - \[39\] and reference therein for more information). Other models of dynamical dark energy, besides mentioned dark fluid models, are quintessence, phantom, quintom and k-essence scalar field dark energy models among the others \[36\]. On the other, hand mentioned dark energy models are introduced by hand in to the dynamics of the background, therefore a direct modification of general relativity accounts as a straightforward way to explain the accelerated expansion of the large scale universe. Moreover, a modification of general relativity has proved to be useful also for the physics of the early universe including a possibility to explain the inflation \[40\] - \[41\] (and references therein). In this paper, we already had mentioned that general relativity will describe the background dynamics and it is known that with such models it is very useful to use the idea of non-gravitational interaction (we refer the readers to the cited papers for more information on this idea). Usually, it is accounted that non-gravitational interaction is a specific type of interaction which is deduced from the properties of dark energy and dark matter. Therefore, in cosmological models we usually consider interaction between dark energy and dark matter only. Non-gravitational interaction is not only a phenomenological assumption, it also allows to improve theoretical results, therefore there is also an increasing amount of interest towards this option/idea. It is believed, that with the understanding of the structures of dark energy and dark matter, this question will be understood as well. There are also some models of non-gravitational interactions which are already can be considered as classical ones. Moreover, there are new developments in this direction mainly in recent literature, again based on various phenomenological modifications. In this paper we will study new cosmological models involving new forms of non-linear non-gravitational interactions considered in recent literature and the main goal is to demonstrate the viability of these models to the problems of the accelerated expansion of the recent universe. Moreover, we will use $\Omega m$ analysis and the relative change of this parameter \[42\] 

$$\Delta \Omega m = 100 \times \left[ \frac{\Omega m_{\text{Model}}}{\Omega m_{\Lambda \text{CDM}}} - 1 \right]$$

(2)

to study the possible differences between the new models and $\Lambda$CDM model, for which $\Omega m = \Omega_{dm}^{(0)} = 0.27$. Moreover, $\chi^2$ analysis will be used in order to constrain the parameters of the models.

The paper is organised as follows: In section 2 a description of the background dynamics with the datasets in use are presented. Moreover, the description of $\Omega m$ analysis is also presented in section 2. In section 3 the best fit values of the parameters obtained during $\chi^2$ analysis for appropriate datasets are presented and appropriate key consequences are discussed for all models. Finally, discussion on obtained results and possible future extension of considered cosmological model are summarized in section 4.
2 Background dynamics and datasets

In case of interacting dark energy models we should take into account that the dynamics of the energy densities of dark energy and dark matter should be modified. In particular, the following differential equation should be considered

\[ \dot{\rho}_{dm} + 3H\rho_{dm} = Q, \tag{3} \]

and

\[ \dot{\rho}_{de} + 3H(\rho_{de} + P_{de}) = -Q, \tag{4} \]

where \( Q \) indicates non-gravitational interaction and dark matter is assumed to be cold with \( P = 0 \). Such representation directly depends on the assumption that the effective fluid in the universe will be described as follows

\[ P_{eff} = P_{de}, \tag{5} \]

\[ \rho_{eff} = \rho_{dm} + \rho_{de}. \tag{6} \]

On the other hand, in an isotropic and spatially homogeneous flat FRW universe, the Friedmann equations are as follows

\[ H^2 = \frac{8\pi G}{3}\rho_{eff}, \tag{7} \]

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_{eff} + 3P_{eff}), \tag{8} \]

and describe the background dynamics. To separate a physically reasonable solution in case of a phenomenological assumption it is necessary to constrain the parameters of the models. In this paper we will use the following datasets

1. The differential age of old galaxies, given by \( H(z) \).
2. The peak position of baryonic acoustic oscillations (BAO).
3. The SN Ia data.
4. Strong Gravitation Lensing data.

In the case of the Observed Hubble Data, one defines chi-square given by

\[ \chi^2_{OHD} = \sum \frac{(H(P, z) - H_{obs}(z))^2}{\sigma_{OHD}^2}, \tag{9} \]

where \( H_{obs}(z) \) is the observed Hubble parameter at redshift \( z \) and \( \sigma_{OHD} \) is the error associated with that particular observation, while \( H(P, z) \) is the Hubble parameter obtained from the model and \( P \) is the set of the parameters to be determined/constrained from the dataset. On the other hand, 7 measurements have been jointly used determining the BAO (Baryon Acoustic Oscillation) peak parameter to constrain the models by

\[ \chi^2_{BAO} = \sum \frac{(A(P, z) - A_{obs}(z))^2}{\sigma_{BAO}^2}, \tag{10} \]

where the theoretical value for the \( P \) set of the parameters \( A(P, z) \) is determined as

\[ A(P, z_1) = \sqrt{\Omega_m} \int_{z_1}^0 \frac{dz}{E(z)} \left( \int_{z_1}^0 \frac{dz}{E(z)} \right)^{2/3}, \tag{11} \]

with \( E(z) = H(z)/H_0 \) and \( H_0 \) is the value of the Hubble parameter at \( z = 0 \). For the Supernovae Data, \( \chi^2_\mu \) is defined as

\[ \chi^2_\mu = A - \frac{B^2}{C}, \tag{12} \]
where

\[ A = \sum \frac{\mu(P, z) - \mu_{\text{obs}}}{\sigma_{\mu}^2}, \]  
(13)

\[ B = \sum \frac{\mu(P, z) - \mu_{\text{obs}}}{\sigma_{\mu}^2}, \]  
(14)

and

\[ C = \sum \frac{1}{\sigma_{\mu}^2}. \]  
(15)

In the last 3 equations \( \sigma_{\mu} \) is the uncertainty in the distance modulus [43]. We will follow to the receipt of Ref. [44] and use the data presented there in order to obtain constraints on the parameters of the models from the strong gravitational lensing. To obtain appropriate constraints, we will look for the set of the values of the parameters of the models to minimize \( \chi^2 \) function defined as

\[ \chi^2 = \chi_{\text{OHD}}^2 + \chi_{\text{BAO}}^2 + \chi_{\mu}^2 + \chi_{\text{SGL}}^2, \]  
(16)

if we want to obtain the constraints using all datasets presented above.

3 Models and data fitting

Three different types of models will be analyzed in this paper involving different forms of non-gravitational interactions between dark energy and dark matter. The forms of non-gravitational interactions have been considered for the first time in Ref. [19]. The parameters of the models to be constrained using \( \chi^2 \) statistical tool are as follows \( P = \{H_0, \Omega_{\text{dm}}(0), A, b, u, k\} \). In order to simplify the discussion on the results of the fit to find the best fit values of the parameters and the discussion on a relative change of \( \Omega_m \) parameter, we organized appropriate subsections.

3.1 Models of the first type

In the case of the models consider in this subsection the following general form describing the non-gravitation interaction between dark energy and dark matter will be taken into account

\[ Q = 3Hb \left( \rho_{\text{de}} + \rho_{\text{dm}} + \frac{\rho_{\text{de}}^2}{\rho_{\text{de}} + \rho_{\text{dm}}} \right), \]  
(17)

where \( b \) is a constant, \( H \) is the Hubble parameter, while \( \rho_{\text{de}} \) and \( \rho_{\text{dm}} \) represent the energy density of the varying polytropic dark fluid under the consideration and the energy density of dark matter, respectively. However, before the presentation of the results associated to this model we will study other two models as well, where non-gravitational interactions between dark energy and dark matter are particular examples obtained from Eq. (17).

3.1.1 Case 1

The study shows, that when the interaction is defined in the following way

\[ Q = 3Hb \left( \rho_{\text{de}} + \frac{\rho_{\text{de}}^2}{\rho_{\text{de}} + \rho_{\text{dm}}} \right), \]  
(18)

then using all datasets described above give the results presented in Table [1]. The fit has been performed having the following priors on \( H_0 = 71.9, A \in [-2, 2] \) and \( b \in [-1, 1] \). The presented results in Table [1] are for \( \Omega_{\text{dm}}^{(0)} = 0.27, \Omega_{\text{dm}}^{(0)} = 0.28, \Omega_{\text{dm}}^{(0)} = 0.29, \Omega_{\text{dm}}^{(0)} = 0.30, \Omega_{\text{dm}}^{(0)} = 0.31 \) for \( u = 1, u = 1.25 \) and \( u = 1.5 \), respectively. Initial priors for \( u \) was \( u \in (0, 3) \), while for \( \Omega_{\text{DM}}^{(0)} \) was \( \Omega_{\text{DM}}^{(0)} \in [0.26, 0.32] \). The value of the Hubble parameter has been chosen according to the report of 2016 provided by Hubble Space Telescope mission [45]. From
The parameters of the models has not been affected, when we have considered the fit of theoretical model with observational data. We would like to mention that the best fit values of the instance, has been obtained for $\{A, b, u, k\}$.

Fig. (1) presents the graphical behavior of the deceleration parameter $q$ and $\Delta Om$. The behavior of the deceleration parameter indicates that considered model can explain the accelerated expansion of the universe. Moreover, a phase transition between decelerated expanding and accelerated expanding phases with different redshifts is on face. On the other hand, the graphical behavior of $\Delta Om$ presented on the right plot of Fig. (1) represents differences between the new model and $\Lambda$CDM standard cosmological model. In particular, the analysis shows that for $\Omega_{dm}^{(0)} = 0.27$ and $\Omega_{dm}^{(0)} = 0.28$ cases the relative change at $z = 0.0$ is about 0.1%. On the
other hand, the study showed that \( \{H_0, \Omega_{dm}^{(0)}, A, b, u, k\} = \{71.9, 0.30, 0.542, 0.072, 1.5, 0.914\} \) case satisfies to the known constraints from a modified two point \( Om \) analysis with the result from BOSS experiment for the Hubble parameter at \( z = 2.34 \) \[^{16}\]. In this case the present value of the equation of state parameter for considered varying polytropic dark fluid is \( \omega_{de} = -1.14 \) i.e. in this case the phantom line crossing is possible and \( \Delta Om \approx -24\% \). Moreover, if we will take into account constraint on \( \omega_{de} \) obtained from PLANCK 2015 experiment, then the applicability of this model will be under the doubt. On the other hand, the model with the following parameters \( \{H_0, \Omega_{dm}^{(0)}, A, b, u, k\} = \{71.9, 0.31, 0.517, 0.071, 1.25, 0.384\} \) with \( \omega_{de} = -1.022 \) will satisfy to the mentioned constraints (\( \Delta Om \approx 5.5\% \)). Having obtained results, we conclude that an additional data is needed in order to be able for any future conclusion.

### 3.1.2 Case 2

On the other hand, the study shows, that when the interaction \( Q \) has the following form

\[
Q = 3Hb \left( \rho_{dm} + \frac{\rho_{de}}{\rho_{de} + \rho_{dm}} \right),
\]

the, for instance, the minimum \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_{m} = 780.67 \) providing the best fit has been obtained with \( \{H_0, \Omega_{dm}^{(0)}, A, b, u, k\} = \{71.9, 0.28, 0.948, 0.047, 1.25, 0.516\} \). In Table 2 a comprehensive information is provided allowing to understand how \( \Omega_{dm}^{(0)} \) and \( u \) (both fixed in advance before the fit has been started) affect on the fit results with fixed value of the Hubble parameter \( H_0 = 71.9 \). On the other hand, imposing the constraints from a modified two point \( Om \) analysis, the result from BOSS experiment for the Hubble parameter at \( z = 2.34 \) and the constraints on the \( \omega_{de} \) from PLANCK 2015, only one option has been survived among presented in Table 2 \( \{H_0, \Omega_{dm}^{(0)}, A, b, u, k\} \) = \{71.9, 0.28, 0.689, 0.057, 1.0, -0.094\} with \( \omega_{de} = -1.03 \) and \( \Delta Om \approx 3.5 \) at \( z = 0 \). The transition redshift in this case is \( z_t \approx 0.9 \).

### 3.1.3 Case 3

The model considered in this subsection admits the interaction between dark energy and dark matter given by Eq. \[^{17}\]. Taking into account the same priors as it was in two other cases considered in \[^{3.1.1} \text{and} \ 3.1.2\] we found the best fit of theoretical results with \( \{H_0, \Omega_{dm}^{(0)}, A, b, u, k\} = \{71.9, 0.27, 0.569, 0.024, 1.5, 0.943\} \) giving

| \( \chi^2 \) | \( \Omega_{DM}^{(0)}(f) \) | \( H_0(f) \) | \( A \) | \( b \) | \( u(f) \) | \( k \) |
|-------|--------|--------|-----|-----|------|-----|
| 781.78 | 0.27   | 71.9   | 0.983 | 0.019 | 1.0  | -0.014 |
| 781.37 | 0.27   | 71.9   | 0.672 | 0.033 | 1.25 | 0.437 |
| 781.22 | 0.27   | 71.9   | 0.897 | 0.052 | 1.5  | 1.047 |
| 780.96 | 0.28   | 71.9   | 0.776 | 0.028 | 1.0  | -0.067 |
| 780.67 | 0.28   | 71.9   | 0.948 | 0.047 | 1.25 | 0.516 |
| 780.68 | 0.28   | 71.9   | 0.812 | 0.062 | 1.5  | 1.021 |
| 782.26 | 0.29   | 71.9   | 0.776 | 0.038 | 1.0  | -0.067 |
| 782.12 | 0.29   | 71.9   | 0.948 | 0.057 | 1.25 | 0.516 |
| 782.22 | 0.29   | 71.9   | 0.914 | 0.071 | 1.5  | 1.047 |
| 785.55 | 0.30   | 71.9   | 0.689 | 0.047 | 1.0  | -0.094 |
| 785.54 | 0.30   | 71.9   | 0.534 | 0.062 | 1.25 | 0.384 |
| 785.74 | 0.30   | 71.9   | 0.517 | 0.076 | 1.5  | 0.914 |
| 790.72 | 0.31   | 71.9   | 0.689 | 0.057 | 1.0  | -0.094 |
| 790.83 | 0.31   | 71.9   | 0.534 | 0.071 | 1.25 | 0.384 |
| 791.13 | 0.31   | 71.9   | 0.517 | 0.085 | 1.5  | 0.914 |
\( \chi^2 = \chi^2_{OHD} + \chi^2_{SGL} + \chi^2_{\mu} = 778.18 \). The constraining of the parameters of the model with \( \chi^2 = \chi^2_{OHD} + \chi^2_{SGL} + \chi^2_{\mu} + \chi^2_{BAO} \) reveals that the best fit does not affected. On the other, when we used only \( H(z) \) data, then the best fit has been obtained with \( \chi^2_{OHD} = 15.81 \) and \( \{ H_0, \Omega^{(0)}_{dm}, A, b, u, k \} = \{ 71.9, 0.3, 0.948, 0.037, 1.5, 1.021 \} \). The result presented here do not satisfy the constraints from the modified two point \( Om \) analysis, the result from BOSS experiment for the Hubble parameter at \( z = 2.34 \) and the constraints on the \( \omega_{de} \) from PLANCK 2015. Therefore, it is important to study the model in the light of strong gravitational lensing to improve the best fit values and see how the new results change the situation with mentioned constraints.

### 3.2 Models of the second type

The main model to be studied in this subsection admits the following form of non-gravitational interaction between varying polytropic dark energy and cold dark matter

\[
Q = 3Hb \left( \frac{\rho_{de} + \rho_{dm}}{\rho_{de} + \rho_{dm}} \right)
\]

where \( b \) is the parameter and should be determined from observational data under consideration. Before to present the main results obtained for this model, we will discuss other models of non-gravitational interactions, which are particular examples of more general form presented by Eq. (20).

#### 3.2.1 Case 1

The comparison of theoretical results with observational data reveals that when the non-gravitational interaction is given as follows

\[
Q = 3Hb \left( \frac{\rho_{de} + \rho_{dm}}{\rho_{de} + \rho_{dm}} \right)
\]

and when \( H_0 = 71.9, \Omega^{(0)}_{dm} = 0.27 \) and \( u = 1.5 \) are fixed, then with minimum \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_\mu = 781.30 \) the best fit will be obtained providing the following values \( A = 0.879, b = 0.052 \) and \( k = 1.044 \) for the rest of the parameters. On the other hand, the best fit values of the parameters had been obtained for \( H_0 = 71.9, \Omega^{(0)}_{dm} = 0.28 \) and \( u = 1.5 \) with \( A = 0.810, b = 0.062 \) and \( k = 1.025 \) giving the following minimum value \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_\mu = 780.71 \). Moreover, in case of \( H_0 = 71.9, \Omega^{(0)}_{dm} = 0.29 \) and \( u = 1.5 \) with \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_\mu = 782.23 \) providing the best fit has been found giving \( A, b \) and \( k \) to be 0.689, 0.071 and 0.987, respectively. Finally, the best fit values of the parameters of the model have been found when \( \Omega^{(0)}_{dm} = 0.30 \) and \( \Omega^{(0)}_{dm} = 0.31 \) have been fixed in advance giving \( \{ A, b, u, k \} = \{ 0.931, 0.071, 1.25, 0.516 \} \) and \( \{ A, b, u, k \} = \{ 0.931, 0.081, 1.25, 0.516 \} \), respectively (the parameter \( u \) and \( H_0 \) have been fixed in advance as well). On the other hand, a joint constraint from the modified two point \( Om \) analysis, the result from BOSS experiment for the Hubble parameter at \( z = 2.34 \) and the constraints on the \( \omega_{de} \) from PLANCK 2015 imply that \( \{ H_0, \Omega^{(0)}_{dm}, A, b, u, k \} = \{ 71.9, 0.30, 0.931, 0.071, 1.25, 0.516 \} \) with \( \omega_{de} = -1.047 \) is the best result among obtained once. In this case the transition redshift \( z_{tr} \approx 0.85 \) and \( \Delta Om = -1.0\% \).

#### 3.2.2 Case 2

When \( H_0 = 71.9, \Omega^{(0)}_{dm} = 0.27 \) and \( u = 1.5 \) are fixed in advance, then with minimum \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_\mu = 781.22 \) the best fit will be obtained when \( A = 0.707, b = 0.052 \) and \( k = 0.987 \) for the model described by the interaction of the following form

\[
Q = 3Hb \left( \frac{\rho_{de} + \rho_{dm}}{\rho_{de} + \rho_{dm}} \right)
\]

On the other hand, for the same model with fixed \( \Omega^{(0)}_{dm} = 0.28 \), the following result have been obtained \( \{ A, b, u, k \} = \{ 0.811, 0.047, 1.25, 0.478 \} \) giving \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_\mu = 780.65 \) minimal value. The results corresponding to fixed \( \Omega^{(0)}_{dm} = 0.29, 0.30, 0.31 \) with appropriate minimal value of \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_\mu \)
corresponding to the best fit of theoretical results with observational data when \( \{\Omega_{\text{DM}}^{(0)}, A, b, u, k\} = \{71.9, 0.30, 0.914, 0.052, 1.0, -0.031\} \). On the other hand, the bottom panel of Fig. (3) represents the graphical behavior of the deceleration parameter \( q \). The considered model is free from the cosmological coincidence problem. The form of non-gravitational interaction is given by Eq. (22).

\[
\Omega_{\text{DM}}^{(0)} = 0.27 \quad \text{and} \quad u = 1.5 \quad \text{fixed in advance are as follows:} \quad \{A, b, k\} = \{0.534, 0.028, 0.924\} \quad \text{giving} \quad \chi^2_{\text{OHD}} + \chi^2_{\text{BAO}} + \chi^2_{\text{SGL}} = 563.29. \]

\[
\Omega_{\text{DM}}^{(0)} = 0.29 \quad \text{and} \quad u = 1.5 \quad \text{fixed in advance the best fit values of the other parameters of the model are as follows:} \quad \{A, b, k\} = \{0.586, 0.043, 0.946\} \quad \text{giving} \quad \chi^2_{\text{OHD}} + \chi^2_{\text{BAO}} + \chi^2_{\mu} = 564.37. \]

\[
\Omega_{\text{DM}}^{(0)} = 0.30 \quad \text{and} \quad \Omega_{\text{DM}}^{(0)} = 0.31 \quad \text{fixed in advance the minimal values of} \quad \chi^2_{\text{OHD}} + \chi^2_{\text{BAO}} + \chi^2_{\mu} \quad \text{with} \quad 567.70 \quad \text{and} \quad 572.89 \quad \text{provided the best fit of theoretical results with observational data when} \quad \{A, b, u, k\} = \{0.931, 0.033, 1.0, -0.024\} \quad \text{and} \quad \{A, b, u, k\} = \{0.931, 0.038, 1.0, -0.024\}, \quad \text{respectively} \quad (u \quad \text{also had been fixed in advance).} \]

The results corresponding to \( \chi^2_{\text{OHD}} + \chi^2_{\text{BAO}} + \chi^2_{\text{SGL}} + \chi^2_{\mu} \) are presented in Table 3. On the left plot of Fig. (2) the graphical behavior of the deceleration parameter \( q \) is presented indicating the accelerated expansion of the large scale universe. On the other hand, the right plot represents the behavior of \( \Delta Om \) against the redshift \( z \). The considered model is free from the cosmological coincidence problem. The form of non-gravitational interaction is given by Eq. (22).

### Table 3: The best fit results for the model with \( Q = 3Hb \left( \rho_{\text{dm}} + \frac{\rho_{\text{de}}}{\rho_{\text{dm}}} \right) \) with \( \chi^2_{\text{OHD}} + \chi^2_{\text{BAO}} + \chi^2_{5}\).

| \( \chi^2 \) | \( \Omega_{\text{DM}}^{(0)}(f) \) | \( H_0(f) \) | \( A \) | \( b \) | \( u(f) \) | \( k \) |
|---|---|---|---|---|---|---|
| 782.16 | 0.29 | 71.9 | 0.502 | 0.051 | 1.25 | 0.365 |
| 785.56 | 0.30 | 71.9 | 0.914 | 0.052 | 1.0 | -0.031 |
| 790.79 | 0.31 | 71.9 | 0.517 | 0.037 | 1.0 | -0.162 |

### 3.2.3 Case 3

When the non-gravitational interaction is given by Eq. (20), then the best fit values of the parameters of the model with \( H_0 = 71.9, \Omega_{\text{DM}}^{(0)} = 0.27 \) and \( u = 1.5 \) fixed in advance are as follows: \( \{A, b, k\} = \{0.534, 0.028, 0.924\} \) giving \( \chi^2_{\text{OHD}} + \chi^2_{\text{BAO}} + \chi^2_{\mu} = 563.29 \). On the other hand, with \( H_0 = 71.9, \Omega_{\text{DM}}^{(0)} = 0.29 \) and \( u = 1.5 \) fixed in advance the best fit values of the other parameters of the model are as follows: \( \{A, b, k\} = \{0.586, 0.043, 0.946\} \) giving \( \chi^2_{\text{OHD}} + \chi^2_{\text{BAO}} + \chi^2_{\mu} = 564.37 \). Moreover, with \( \Omega_{\text{DM}}^{(0)} = 0.30 \) and \( \Omega_{\text{DM}}^{(0)} = 0.31 \) fixed in advance the minimal values of \( \chi^2_{\text{OHD}} + \chi^2_{\text{BAO}} + \chi^2_{\mu} \) with 567.70 and 572.89 provided the best fit of theoretical results with observational data when \( \{A, b, u, k\} = \{0.931, 0.033, 1.0, -0.024\} \) and \( \{A, b, u, k\} = \{0.931, 0.038, 1.0, -0.024\} \), respectively (\( u \) also had been fixed in advance). The results corresponding to \( \chi^2_{\text{OHD}} + \chi^2_{\text{BAO}} + \chi^2_{\text{SGL}} + \chi^2_{\mu} \) are presented in Table 4. The top panel of Fig. (2) represents the behavior of the deceleration parameter \( q \) and \( \Delta Om \) corresponding to the best fit values obtained for the parameters of the model using the data from the differential age of old galaxies, given by \( H(z) \), the peak position of baryonic acoustic oscillations (BAO) and the SN Ia data, when \( \Omega_{\text{DM}}^{(0)}, u \) and \( H_0 \) were fixed in advance. On the other hand the bottom panel of Fig. (3) represents the graphical behavior of the same parameters with the same parameters of the model fixed in advance, when together with mentioned
Table 4: The best fit results for the model with \( Q = 3Hb \left( \rho_{de} + \rho_{dm} + \frac{\rho_{dm}}{\rho_{de} + \rho_{dm}} \right) \) with \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{5GL} + \chi^2_{\mu} \). \( f \) means that the parameter has been fixed to the presented value in advance before the fit has been started.

| \( \chi^2 \) | \( \Omega^{(i)}_{DM} \) | \( H_0(f) \) | \( A \) | \( b \) | \( u(f) \) | \( k \) |
|---|---|---|---|---|---|---|
| 781.79 | 0.27 | 71.9 | 0.983 | 0.014 | 1.0 | -0.012 |
| 782.35 | 0.29 | 71.9 | 0.655 | 0.024 | 1.0 | -0.106 |
| 785.62 | 0.30 | 71.9 | 0.983 | 0.033 | 1.0 | -0.012 |
| 790.78 | 0.31 | 71.9 | 0.983 | 0.038 | 1.0 | -0.012 |

observational datasets the strong gravitational lensing data has been used. After imposing the constraints as has been discussed for the other cases we found the following picture

1. in case of the analysis with \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{\mu} \) we should take \( \{ A, b, k \} = \{ 0.931, 0.033, 1.0, -0.024 \} \) for the candidate supported from considered constraints

2. in case of the analysis with \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{5GL} + \chi^2_{\mu} \) the results presented in Table 4 for \( \Omega^{(i)}_{DM} = 0.29, 0.30, 0.31 \) are the candidates supported from considered constraints. However, only the result corresponding to \( \Omega^{(i)}_{DM} = 0.30 \) will be accounted as a candidate providing the best fit.

3.3 Models of the third type

The third model of this paper admits the following form of non-gravitational interaction

\[
Q = 3Hb \left( \rho_{de} + \rho_{dm} + \frac{\rho_{dm}^2}{\rho_{de} + \rho_{dm}} \right) ,
\]

which gives the best fit of theoretical results with observational data when \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{\mu} = 563.25 \) and \( H_0 = 71.9, \Omega^{(i)}_{DM} = 0.27, u = 1.5 \) are fixed in advance and the rest of the parameters of the model are defined as follows: \( \{ A, b, k \} = \{ 0.534, 0.028, 0.924 \} \). In this case we see that presented result coincidence with the result obtained for the model, when the non-gravitational interaction is given by Eq. (20). The consideration of the cases with \( u = 1.0 \) and \( u = 1.25 \) (with fixed \( H_0 = 71.9 \) and \( \Omega^{(i)}_{DM} = 0.27 \)) provided the best fit when \( \{ A, b, k \} = \{ 0.534, 0.028, 0.924 \} \) and \( \{ A, b, k \} = \{ 0.931, 0.014, -0.024 \} \), respectively. On the other hand, with \( u = 1.0 \) (with fixed \( H_0 = 71.9 \) and \( \Omega^{(i)}_{DM} = 0.29 \)) the best fit has been found with \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{\mu} = 564.25 \) when \( \{ A, b, k \} = \{ 0.776, 0.024, -0.068 \} \), while with fixed \( H_0 = 71.9, \Omega^{(i)}_{DM} = 0.30 \) and \( u = 1.0 \) the best fit has been found when \( \{ A, b, k \} = \{ 0.707, 0.029, -0.09 \} \). The last state is described by \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{\mu} = 567.61 \). For this model, the study of the question how the strong gravitational lensing data will affect on the best fit values of the parameters has been left as a topic of another study. Preliminary study presented here, showed that the best fit result also satisfying to the constraints imposed from BOSS and PLANCK 2015 experiments, should be accounted the result corresponding to the fixed \( \Omega^{(i)}_{DM} = 0.30 \) case with \( z_{tr} \approx 0.85, q \approx -0.58 \) and \( \Delta Om = 1\% \) at \( z = 0.0 \). Moreover, as can be seen from the right plot of Fig. 4 \( \Delta Om \) is an increasing function from the redshift.

3.3.1 Case 1

In this section we will present the results of the fit for the phenomenological model, when the non-gravitational interaction is given in the following way

\[
Q = 3Hb \left( \rho_{de} + \frac{\rho_{dm}^2}{\rho_{de} + \rho_{dm}} \right) ,
\]
Figure 3: The graphical behavior of the deceleration parameter $q$ and $\Delta Om$, Eq. (2), against the redshift $z$. The considered model is free from the cosmological coincidence problem. The form of non-gravitational interaction is given by Eq. (20). The top panel represents the result corresponding to the analysis with $\chi_{OHD}^2 + \chi_{BAO}^2 + \chi_{\mu}^2$, while the bottom panel represents the results for the analysis with $\chi_{OHD}^2 + \chi_{BAO}^2 + \chi_{SGL}^2 + \chi_{\mu}^2$.

Figure 4: The graphical behavior of the deceleration parameter $q$ and $\Delta Om$, Eq. (2), against the redshift $z$. The considered model is free from the cosmological coincidence problem. The form of non-gravitational interaction is given by Eq. (23). The presented result corresponds to the analysis with $\chi_{OHD}^2 + \chi_{BAO}^2 + \chi_{\mu}^2$. 

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Table 5: The best fit results for the model with $Q = 3Hb\left(\rho_{dc} + \frac{\rho_{dm}}{\rho_{dc}+\rho_{dm}}\right)$ with $\chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_{\mu}$. $f$ means that the parameter has been fixed to the presented value in advance before the fit has been started.

| $\chi^2$ | $\Omega_{DM}^{(0)}(f)$ | $H_0(f)$ | $A$ | $b$ | $u(f)$ | $k$ |
|----------|----------------|---------|----|----|------|----|
| 782.11   | 0.29           | 71.9    | 0.896 | 0.057 | 1.25 | 0.505 |
| 785.56   | 0.30           | 71.9    | 0.638 | 0.047 | 1.0  | -0.112|
| 790.73   | 0.31           | 71.9    | 0.638 | 0.057 | 1.0  | -0.012|

which is a particular case of more general form given by Eq. (23). First of all we would like to present the results according to $\chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{\mu}$ constrain and compare them with the results obtained from $\chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_{\mu}$ constrain. For instance, the study shows that when $H_0 = 71.9$, $\Omega^{(0)}_{DM} = 0.27$ and $u = 1.5$ the minimal $\chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{\mu} = 563.24$ will be obtained providing the best fit. In this case for the rest parameters we obtained $\{A,b,u,k\} = \{0.586,0.047,0.946\}$. On the other hand, when $\Omega^{(0)}_{DM} = 0.28$, then the best fit with $\{A,b,u,k\} = \{0.896,0.047,1.25,0.505\}$ ($\chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} = 562.70$) will be obtained. Moreover, the study showed that the model with $\Omega^{(0)}_{DM} = 0.29$, $\Omega^{(0)}_{DM} = 0.30$ and $\Omega^{(0)}_{DM} = 0.31$ provides the best fit when $\{A,b,u,k\} = \{0.586,0.059,1.25,0.505\}$ ($\chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{\mu} = 564.20$), $\{A,b,u,k\} = \{0.638,0.047,1.0,-0.112\}$ ($\chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} = 567.63$) and $\{A,b,u,k\} = \{0.638,0.057,1.0,-0.112\}$ ($\chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{\mu} = 572.83$), respectively. Now, including strong gravitational lensing data, we obtained the following results. In particular, when $\Omega^{(0)}_{DM} = 0.27$ the best fit will be obtained when $\{A,b,u,k\} = \{0.586,0.047,1.5,0.946\}$ ($\chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_{\mu} = 781.21$) i.e. the best fit values of the parameters will not be affected. On the other hand, when $\Omega^{(0)}_{DM} = 0.28$, then the consideration of strong gravitational lensing data will significantly affect on the best fit values of the parameters - $\{A,b,u,k\} = \{0.586,0.057,1.5,0.946\}$ ($\chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_{\mu} = 780.64$). The results for $\Omega^{(0)}_{DM} = 0.29,0.30,0.31$ are presented in Table 5 and it can be seen, that including of strong gravitational lensing data under the consideration will not affect the best fit values of the parameters. Future constraints mentioned earlier in this paper, support the case with $\Omega^{(0)}_{DM} = 0.30$ to be the candidate for the best fit. On the other hand, the results corresponding to $\Omega^{(0)}_{DM} = 0.31$ also can be counted to satisfy to imposed constraints. The left plot of Fig. 4 indicates how the relational change $\Delta Om$ evolves with the evolution of the universe.

Figure 5: The graphical behavior of the deceleration parameter $q$ and $\Delta Om$, Eq. (2), against the redshift $z$. The considered model is free from the cosmological coincidence problem. The form of non-gravitational interaction is given by Eq. (24). The presented result corresponds to the analysis with $\chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{\mu}$. 
The last model studied in this work admits the following form of non-gravitational interaction

\[ Q = 3H_0 \left( \rho_{dm} + \frac{\rho_{dm}^2}{\rho_{de} + \rho_{dm}} \right). \]  

(25)

During the study of the model, when in addition to \( \chi^2 \) analysis with \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_{\mu} \), we applied the constraints from BOSS and PLANCK 2015 experiments, and take into account the constraints from modified two-point \( \Omega_{m} \) analysis gives us the best fit values of the parameters of the model as follows:

\[ \omega_{de} \approx -1.039 \text{ at } z = 0 \text{ with } \{ H_0, \Omega_{dm}^{(0)}, A, b, u, k \} = \{ 71.9, 0.30, 0.638, 0.047, 1.0, -0.112 \}. \]

The graphical behavior of the deceleration parameter is presented on the left plot of Fig. (6), while the graphical behavior of \( \Delta \Omega_{m} \) is presented on the right plot. Both clearly indicates how the mentioned parameters evolve during the evolution of the universe, moreover, it is possible also to estimate the present day values of them very easily. During the study of the behavior of the equation of state parameter of considered polytropic fluid for all models we found two possibilities. In particular we observed that for some models (the difference between the models is the form of non-gravitational interaction) \( \omega_{de} > 0 \) at high redshifts and there is a phase transition to a phantom dark fluid state satisfying to the constraints on \( \omega_{de} \) according to PLANCK 2015 experiment. However, there is also possibility to have a phantom - phantom transitions also providing the accelerated expansion of the universe. We would like to mention that mentioned phantom - phantom transitions have been observed firstly in scope of generalized holographic dark energy models with Nojiri-Odintsov cut-offs.

Table 6: The best fit results for the model with \( Q = 3H_0 \left( \rho_{dm} + \frac{\rho_{dm}^2}{\rho_{de} + \rho_{dm}} \right) \) with \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_{\mu} \). f means that the parameter has been fixed to the presented value in advance before the fit has been started.

| \( \chi^2 \) | \( \Omega_{dm}^{(0)}(f) \) | \( H_0(f) \) | \( A \) | \( b \) | \( u(f) \) | \( k \) |
|-------|-----------------|--------|-----|-----|------|-----|
| 782.10 | 0.29            | 71.9   | 0.948 | 0.038 | 1.0  | -0.024 |
| 785.51 | 0.30            | 71.9   | 0.586 | 0.043 | 1.0  | -0.134 |
| 790.75 | 0.31            | 71.9   | 0.534 | 0.047 | 1.0  | -0.156 |

3.3.2 Case 2

The last model studied in this work admits the following form of non-gravitational interaction

\[ Q = 3H_0 \left( \rho_{dm} + \frac{\rho_{dm}^2}{\rho_{de} + \rho_{dm}} \right). \]  

(25)

During the study of the model, when in addition to \( \chi^2 \) analysis with \( \chi^2_{OHD} + \chi^2_{BAO} + \chi^2_{SGL} + \chi^2_{\mu} \), we applied the constraints from BOSS and PLANCK 2015 experiments, and take into account the constraints from modified two-point \( \Omega_{m} \) analysis gives us the best fit values of the parameters of the model as follows:

\[ \omega_{de} \approx -1.039 \text{ at } z = 0 \text{ with } \{ H_0, \Omega_{dm}^{(0)}, A, b, u, k \} = \{ 71.9, 0.30, 0.638, 0.047, 1.0, -0.112 \}. \]

The graphical behavior of the deceleration parameter is presented on the left plot of Fig. (6), while the graphical behavior of \( \Delta \Omega_{m} \) is presented on the right plot. Both clearly indicates how the mentioned parameters evolve during the evolution of the universe, moreover, it is possible also to estimate the present day values of them very easily. During the study of the behavior of the equation of state parameter of considered polytropic fluid for all models we found two possibilities. In particular we observed that for some models (the difference between the models is the form of non-gravitational interaction) \( \omega_{de} > 0 \) at high redshifts and there is a phase transition to a phantom dark fluid state satisfying to the constraints on \( \omega_{de} \) according to PLANCK 2015 experiment. However, there is also possibility to have a phantom - phantom transitions also providing the accelerated expansion of the universe. We would like to mention that mentioned phantom - phantom transitions have been observed firstly in scope of generalized holographic dark energy models with Nojiri-Odintsov cut-offs.

Figure 6: The graphical behavior of the deceleration parameter \( q \) and \( \Delta \Omega_{m} \), Eq. (25), against the redshift \( z \). The considered model is free from the cosmological coincidence problem. The form of non-gravitational interaction is given by Eq. (25).
4 Discussion

Available observational data suggests to include dark energy and dark matter in general relativity to explain the accelerated expansion of the universe. On the other hand, it is possible to introduce non-gravitationally interacting dark energy and dark matter to solve the problems of the large scale universe. The duality is on face with its problematic consequences. From one hand side, seems that it will be easy to parameterize the dark side of the universe and solve the problems, but from the other hand side, it appears that existing tension between observational datasets makes additional complexity. Therefore, there is ongoing active research in order to find a solution involving many phenomenological assumptions. In particular, there are different phenomenological assumptions concerning to the form of dark energy and non-gravitational interaction.

Motivated by recent developments, in this paper we considered new cosmological models involving new interacting varying polytropic gas models. In order to obtain the best fit values of the parameters of the models we used $\chi^2$ analysis involving the differential age of old galaxies, given by $H(z)$, the peak position of baryonic acoustic oscillations known as BAO data, the SN Ia data and strong gravitation lensing data. To simplify the analysis we fixed the values of some of the parameters before the fit has been start and kept them frozen in future, we involved constraints on the equation of state parameter of dark fluid $\omega_{de}$ from PLANCK 2015 experiment, then we took into account reported value for the Hubble parameter at $z = 2.34$ from BOSS experiment. Moreover, we used constraints obtained from a modified two-point $Om$ analysis giving $Om h^2(z_1; z_2) = 0.124 \pm 0.045$, $Om h^2(z_1; z_3) = 0.122 \pm 0.01$ and $Om h^2(z_2; z_3) = 0.122 \pm 0.012$ for $z_1 = 0$, $z_2 = 0.57$ and $z_3 = 2.34$, respectively [47]. Mentioned additional constraints allowed us to establish the best fit values of the parameters of the models with fixed $H_0$, $\omega$ and $\Omega^{(0)}_{DM}$. The study shows, that considered models, which are differ from each other by the form of non-linear non-gravitational interactions between dark energy and dark matter can explain the accelerated expansion. It is possible to explain the phase transition between decelerated expanding and accelerated expanding phases during the evolution of the universe. Moreover, the study of the relative change of $Om$ parameter shows clear difference between new models and $\Lambda$CDM standard model of cosmology.

The main interesting result has been observed during the study of the equation of state parameter of polytropic fluid. In particular, the study shows that considered models when non-gravitational interactions are given by Eq. (18), Eq. (19), Eq. (17), Eq. (21), respectively, then quintessence - phantom transition for the equation of state parameter of dark energy will be observed. On the other hand, when we consider non-gravitational interactions given by Eq. (22), Eq. (20), Eq. (23), Eq. (25), then phantom - phantom phases unification will be observed. However, when the interaction, for instance, is given by Eq. (19), then the results corresponding to $\Omega^{(0)}_{DM} = 0.31$ satisfying the considered constraints, provides dark energy with phantom - phantom transition. On the other hand, when the interaction is given by Eq. (19), then we will observe also quintessence - phantom transition. In summary - we need more observational data in order to be able to choose the best model of interacting varying polytropic dark fluid model from the models considered in this paper. It can be done, for instance, involving constraints provided by the study of the structure formation. Moreover, this will allow to define which one of mentioned transitions for dark energy is the realistic scenario, because from the study of the deceleration parameter and the equation of state parameter this question cannot be answered. If it will be found that a model with phantom - phantom transition will be the best model among considered models, then it is necessary to find Nojiri-Odintsov holographic dark energy representation of the model and study the application of the model to the inflationary expansion phase of the universe.

In future research reconstruction of modified theories of gravity for considered models should be performed and since we saw a non-unique imprint of the type of non-gravitational interaction on the behavior of the equation of state parameter of dark fluid, then it is necessary to determine the type of future singularities which will provide additional sources to extend applied constraints used in this paper. Mentioned possibilities and tasks are the subject of additional research and will be reported in another paper.

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Author Contributions:

Martiros Khurshudyan designed the main subject of this research and mainly wrote the manuscript. Asatur Khurshudyan commented on the manuscript at all stages with performing part of the analysis. All authors equally promoted the research and discussed the results. All authors have read and approved the final manuscript.

Conflicts of Interest:

The authors declare no conflict of interest.

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