Low-Lying States of $^6$He and $^6$Be in a Nodal Surface Structure Analysis

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Abstract

The low-lying states of the light nuclei $^6$He and $^6$Be are studied. Using the inherent nodal surface(INS) analysis approach, we deduce the quantum numbers and the spatial symmetries of the low-lying states with positive parity and negative parity of the two nuclei. The energy spectrum obtained agrees well with the experimental data.

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It is known that, when the number of nucleons $A$ in a nucleus is 5-10, the convergence of shell model calculations is usually poor, while in the cluster model it is complicated to include many different cluster configurations. Although many attempts have been made to investigate the nuclei with $A \approx 5-10$, no general approach has been established because of the complexity due to the many degrees of freedom, for instance, six-nucleon systems have 15 spatial degrees of freedom. Among these nuclei, $^6\text{He}$ has received considerable attention since studying such a nucleus at the neutron drip line can further refine our understanding of the nucleon-nucleon interaction. $^6\text{He}$, consisting of two protons and four neutrons, has a level structure which has been investigated over many years in a number of theoretical calculations. However, the existing literature concerns mainly the ground states and a few resonances\cite{1, 2, 3, 4, 5, 6}. Even now, little is known about the spectroscopy and energy level scheme of $^6\text{He}$ and $^6\text{Be}$. To shed more light on the situation, we make use of a new approach, namely the inherent nodal surface structure analysis approach, to study the effects of the inherent symmetries. It has been shown that, by investigating the nodal structure of the few-body wave functions, one can obtain certain important features of the wave functions and the energy spectra before actually solving the Schrödinger or Faddeev equation\cite{7, 8}. The properties of the 6-nucleon systems have been studied in this new approach, and the isospin $T = 0$ energy level scheme of $^6\text{Li}$ has been deduced\cite{8}, as an example. In this paper we will extend this approach to extract the qualitative characteristics of the low-lying states of $^6\text{He}$ and $^6\text{Be}$ with isospin $T = 1$.

It has been shown in Ref.\cite{7} that the low-lying states of $^4\text{He}$ are dominated by the component of total orbital angular momentum $L = 0$, while the resonances below the $2n+2p$ threshold are ruled with $L = 1$. The quite large excitation energies of the resonances ($\geq 20$ MeV) indicate that the increase of $L$ may lead to a great increase in energy. Furthermore, the internal wave-functions (the ones relative to a body-frame) of all the states below the $2n+2p$ threshold do not contain nodal surfaces. This fact implies that the excitation of internal oscillation takes a very large energy. Since 6-nucleon systems have comparable size and weight with the 4-nucleon ones, it would be reasonable to as-
sume that the $L = 0$ nodeless component will dominate the low-lying spectra of 6-nucleon systems. The success in describing the positive parity energy spectrum of $^6\text{Li} \otimes \otimes$ indicates that such an assumption is quite practical.

Let $\Psi$ be an eigenstate of a quantum system, $\mathcal{A}$ denote a geometric configuration, in some cases $\mathcal{A}$ may be invariant to a specific operation $\hat{O}$, we have then

$$\hat{O}\Psi(\mathcal{A}) = \Psi(\hat{O}\mathcal{A}) = \Psi(\mathcal{A}).$$

(1)

For example, when $\mathcal{A}$ is a regular octahedron (OCTA, see Fig. 1) for a 6-body system, $\mathcal{A}$ is invariant to a rotation about a 4-fold axis of the OCTA by $90^\circ$ together with a cyclic permutation of the particles 1, 2, 3 and 4. According to the representations of the operation on $\Psi$ (e.g., rotation, space inversion, and permutation), Eq. (1) can always be written in a matrix form and appears as a set of homogeneous linear equations. It is apparent that, whether there exists nonzero solution of $\Psi(\mathcal{A})$, in other word, whether the state $\Psi$ is accessible to the configuration $\mathcal{A}$, depends on the inherent symmetric property of the configuration. The symmetry imposes then a very strong constraint on the eigenstate so that the $\Psi$ may be zero at $\mathcal{A}$. It indicates that there may exist a specific kind nodal surface. Since such kind nodal surface is imposed by the intrinsic symmetry of the system (fixed at body-frames) and independent of the dynamical property at all, one usually refers it as inherent nodal surface (INS)\[7, 8, 9\].

The INS appears always at geometric configurations with certain geometric symmetry. For a 6-body system, the OCTA is the configuration with the strongest geometric symmetry. Let us assume that the six particles form an OCTA, $k'$ is a 4-fold axis of the OCTA, the particles 1, 2, 3 and 4 form a square surrounding $k'$, $R_{\delta}^{k'}$ denotes a rotation about $k'$ by an angle $\delta$ (in degree), $P(ijk)(P(ijkl))$ denotes a cyclic permutation of the particles i, j, k (i, j, k, l), the OCTA is evidently invariant to operations

$$\hat{O}_1 = P(1432)R_{90}^{k'},$$

(2)

$$\hat{O}_2 = P(253)P(146)R_{120}^{k'}. $$

(3)
Let $P_{ij}$ denote an interchange of the locations of particles $i$ and $j$, $\hat{I}$ stand for a space inversion, along the same way as discussed above, we know that the OCTA is also invariant to the operations

$$\hat{O}_3 = P_{14}P_{23}P_{56}R_{180}^\nu,$$  \hspace{1cm} (4)

$$\hat{O}_4 = P_{13}P_{24}P_{56}\hat{I},$$  \hspace{1cm} (5)

On the other hand, we can generally express an eigenstate of a 6-nucleon system with given total angular momentum $J$, parity $\pi$, and isospin $T$ as

$$\Psi = \sum_{L,S,\lambda} \Psi_{LS\lambda}$$  \hspace{1cm} (6)

where $S$ is the total spin, and the $\Psi_{LS\lambda}$ can be given as

$$\Psi_{LS\lambda} = \sum_{iMMS} C^i_{LM,SM}\tilde{F}_{LSM}\tilde{\chi}_{SM\lambda}^i,$$  \hspace{1cm} (7)

where $M$ is the $Z$-component of $L$, $F_{LSM}^\lambda$ is a function of the spatial coordinates, which is the $i^{th}$ basis function of the $\lambda$–representation of the permutation group $S_6$, $\tilde{\chi}_{SM}^\lambda$ is a basis function in the spin-isospin space with given $S$ and $T$ and belonging to $\tilde{\lambda}$, the conjugate representation of $\lambda$. Taking advantage of group theory, one has obtained the allowed $\lambda$, which depends on $S$ and $T$. The result in the case of $T = 1$ is listed in Table 1. We shall then figure out which components are favorite to forming bound states.

Table 1: The allowed representation $\lambda$ of the states with isospin $T = 1$

| $T$ | $S$ | $\lambda$ |
|-----|-----|-----------|
| 1   | 0   | $\{2^1\}, \{22\}, \{321\}, \{31^3\}, \{42\}$ |
| 1   | 1   | $\{1^6\}, \{2^1\}, \{221 1\}, \{31^3\}, \{232 1\}, \{33\}, \{41 1\}$ |
| 1   | 2   | $\{2^1\}, \{2 21 1\}, \{222\}, \{31^3\}, \{32 1\}$ |
| 1   | 3   | $\{221 1\}$ |
For the function $F^\lambda_{LSM}$ of the spatial coordinates, defining a body-frame, we have

$$F^\lambda_{LSM}(123456) = \sum_Q D^L_{QM}(-\gamma, -\beta, -\alpha) F^\lambda_{LSQ}(1'2'3'4'5'6'), \tag{8}$$

where $\alpha, \beta, \gamma$ are the Euler angles to specify the collective rotation, $D^L_{QM}$ is the well known Wigner function, $Q$ is the projection of $L$ along the $k'$-axis, the (123456) and (1'2'3'4'5'6') specify that the coordinates are relative to the laboratory frame and the body-frame, respectively.

Since the $F^\lambda_{LSQ}$ spans a set of basis of the representation of the rotation group, space inversion group, and permutation group, the invariance of the OCTA to the operations $\hat{O}_1$ to $\hat{O}_4$ leads to four sets of equations. For example, from

$$\hat{O}_1 F^\lambda_{LSQ}(A) = F^\lambda_{LSQ}(\hat{O}_1 A) = F^\lambda_{LSQ}(A), \tag{9}$$

where $F^\lambda_{LSQ}(A)$ denotes that the coordinates in $F^\lambda_{LSQ}$ are given at an OCTA, for all $Q$ with $|Q| \leq L$ we have

$$\sum_{i'} [g^\lambda_{ii'}(P(1432)) e^{-i\pi Q} - \delta_{ii'}] F^\lambda_{LSQ}(A) = 0, \tag{10}$$

where $g^\lambda_{ii'}$ is the matrix element belonging to the representation $\lambda$, which can be fixed with the group theory method (see for example Ref.[11]). Similarly, for $\hat{O}_2$, $\hat{O}_3$ and $\hat{O}_4$, we have

$$\sum_{Q'Q''} [g^\lambda_{ii'}(P(253)P(146)) \sum_{Q''} D^L_{QQ''}(0, \theta, 0) e^{-i\pi Q''} \delta_{QQ''} F^\lambda_{LSQ''}(A) = 0, \tag{11}$$

$$\sum_{Q'Q''} [(-1)^L g^\lambda_{ii'}(P_{14}P_{23}P_{56}) \delta_{QQ'} - \delta_{ii'} \delta_{QQ''}] F^\lambda_{LSQ'}(A) = 0, \tag{12}$$

with $\bar{Q} = -Q$,

$$\sum_{i'} [g^\lambda_{ii'}(P_{13}P_{24}P_{56})(-1)^L - \delta_{ii'}] F^\lambda_{LSQ}(A) = 0. \tag{13}$$

Eqs. (10) to (13) are the equations that the $F^\lambda_{LSQ}(A)$ have to fulfill. In some cases there is one or more than one nonzero solution(s) to all these equations. In some other
cases, however, there are no nonzero solutions. In the latter case, the $\Psi_{LS}$ has to be zero at the OCTA configurations disregarding their size and orientation. Accordingly, an INS emerges and the OCTA is not accessible. Evidently, the solution of above equations depends on and only on $L$, $\pi$ and $\lambda$. Therefore the existence of the INS does not depend on the dynamics (e.g., not on the interaction, mass, etc.) at all.

Solving the sets of linear equations, we obtain the accessibility of the symmetry configurations of the OCTA with $L = 0$. The result is listed in the second row of Table 2, where the numbers in the blocks are the ones of the independent nonzero solutions.

Table 2: The accessibility of the OCTA and the C-PENTA to the $L^\pi = 0^+$ wavefunctions with different spatial permutation symmetries $\lambda$.

| $\lambda$ | $\{6\}$ | $\{51\}$ | $\{42\}$ | $\{411\}$ | $\{33\}$ | $\{321\}$ | $\{31^3\}$ | $\{222\}$ | $\{2211\}$ | $\{21^4\}$ | $\{1^6\}$ |
|-----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| OCTA      | 1       | 0       | 1       | 0       | 0       | 0       | 0       | 1       | 0       | 0       | 0       |
| C-PENTA   | 1       | 1       | 1       | 0       | 1       | 2       | 0       | 1       | 1       | 1       | 1       |

The INS existing at the OCTA may extend beyond the OCTA. For example, when the shape in Fig. 1 is prolonged along $k'$ (called a prolonged-octahedron and denoted as $B$), it is invariant to $\hat{O}_1$, $\hat{O}_3$, and $\hat{O}_4$, but not to $\hat{O}_2$. Hence, the $F_{LSQ}'(B)$ should fulfill Eqs.(10), (12) and (13). When nonzero common solutions of Eqs. (10) to (13) do not exist, the INS extends from the OCTA to the prolonged-octahedrons. In fact, an OCTA has many ways to deform, for instance, instead of a square, the particles 1, 2, 3 and 4 form a rectangle or form a diamond, and so on. Then, the INS at the OCTA has many possibilities to extend. How it extends is determined by the $(L^\pi \lambda)$ of the wavefunction. Thus, in the coordinate space, the OCTA is a source for the INS to emerge. For a wavefunction, if the OCTA is accessible, all the shapes in the neighborhood of the OCTA are also accessible. Therefore this wavefunction is inherent nodeless in this domain.

Another shape with a strong geometric symmetry is the regular pentagon pyramid (PENTA, see Fig. 2). In an extreme case, the PENTA can be C-PENTA, which corre-
sponds to that with \( h = 0 \) in Fig. 2. Let \( k' \) be the 5-fold axis, the C-PENTA is invariant to (i) a rotation about \( k' \) by \( \frac{2\pi}{5} \) together with a cyclic permutation of the five particles of the pentagon, (ii) a rotation about \( k' \) by \( \pi \) together with a space inversion, (iii) a rotation about \( i' \) by \( \pi \) together with \( P_{14}P_{23} \) (here \( i' \) is the axis vertical to \( k' \) and connecting \( O \) and particle 5). These invariances lead to constraints embodied in a set of homogeneous equations, and therefore the accessibility of the C-PENTA can be identified as given in the third row of Table 2.

In addition to the OCTA, the C-PENTA is another source where the INS may emerge and extend to its neighborhood; e.g., extend to the pentagon-pyramid as shown in Fig. 2 with \( h \neq 0 \). There are also other sources. For example, the one at triangular-prism and that at the regular hexagons. However, among the 15 bonds, 12 can be optimized at an OCTA, 10 at a pentagon-pyramid, 9 at a triangular-prism and 6 at a hexagon. Therefore in the neighborhood of the hexagon (and also other regular shapes) the total potential energy is considerably higher. Since the wavefunctions of the low-lying states are mainly distributed in the domain with a lower potential energy, we shall concentrate only in the domains surrounding the OCTA and the C-PENTA.

In most cases, the ground state of a nucleus obeys the condition \( T = T_3 \). Thus, we can only consider \( T = 1 \) instead of \( T_3 = 1 \), if we constrict our discussion in the low-lying states.

Referring to Table 2, one can find that, when a \( \Psi_{L_S\lambda} \) has \( (L^\pi\lambda) = (0^+\{6\}), (0^+\{4 2\}), \) or \( (0^+\{2 2 2\}) \), it can access both the OCTA and the C-PENTA. These and only these wavefunctions are inherent-nodeless in the two most important domains, and they should be the dominant components for the low-lying states. All the other \( L = 0 \) components must contain at least one INS. Table 1 shows that the \( (0^+\{6\}) \) component can not be contained in any \( T = 1 \) state. Then, we obtain that the \( (0^+\{4 2\}) \) component is accessible to \([T, S] = [1, 0] \) state, and the \( (0^+\{2 2 2\}) \) component is allowed to \([T, S] = [1, 0] \) and \([1, 2] \) states.

Since the state with \([T, S] = [1, 0] \) is accessible to both the \{4 2\} and \{2 2 2\} compo-
ponents, two $J^\pi = 0^+$ partner-states would be generated. Each of them is mainly a specific mixture of the \{42\} and \{2\,2\,2\} components. When $[T, S] = [1, 2]$, there is only one accessibility. We have then only one $J^\pi = 2^+$ state. Therefore, we predict that the low-lying $T = 1$ positive parity spectrum of a 6-nucleon system involves totally three states. Two of them have total angular momentum $J^\pi = 0^+$, and another one has $J^\pi = 2^+$. All the quantum numbers of these states are listed in Table 3.

According to experiment data, besides positive parity states, some low-lying resonances with negative parity have also been observed in light nuclei. Therefore, in addition to the $L = 0$ case discussed above, we have to study the case of $L = 1$.

By evaluating the determinants of the sets of homogeneous linear equations, the inherent nodeless components of a nucleus with 6 nucleons and $L^\pi = 1^-$ are identified as the ones holding orbital symmetry

$$\lambda \in \{\{51\}, \{4\,1\,1\}, \{3\,3\}, \{3\,2\,1\}, \{2\,2\,1\,1\}\}.$$  \hspace{1cm} (14)

Since these states have angular momentum $L = 1$, the total angular momentum $J$, which is formed by the coupling of $S$ and $L$, have always three choices if $S \neq 0$. From Table 1 one knows that the \{51\} component is not allowed to the $T = 1$ states. Then, there exist three groups of P-wave states each with spin $S = 0, 1, 2$, respectively. Their quantum numbers and orbital symmetries are listed in Table 3, too.

It has been well known that the states having two or more components will split due to the coupling among them. For the positive parity states, one can obtain that, owing to the interference between the \{42\} and \{2\,2\,2\} components, there would be a large energy gap between the two $J^\pi = 0^+$ partner-states, so that the lower one becomes the ground state, while another has an energy higher than the first excited state $2^+$. Recalling the energy spectrum of $^6$Li whose gap between the ground state and its partner is 5.65 MeV and considering the charge independent characteristic of nuclear force and the similarity of the nuclei $^6$He and $^6$Li, we expect that the $0^+_2$ state of $^6$He would have an energy $E_x = 5.6$ MeV. Meanwhile, the expected $J^\pi = 2^+$ ($[T, S] = [1, 2]$) state is the first excited state at $E_x = 1.797$ MeV. For the negative parity states (P-wave resonance), the $J^\pi = 1^-$
Table 3: Prediction of the quantum numbers of low-lying states of the $T = 1$ six-nucleon systems and the energies of the states of $^6$He in experiments (the data marked with a, b, c are taken from Refs. [12], [13], [14], respectively).

| $T$ | $S$ | $J$ | $\pi$ | $L$ | $\lambda$ | $E_x$/MeV | $\Gamma$/MeV |
|-----|-----|-----|-------|-----|----------|-----------|------------|
| 1   | 0   | 0   | +     | 0   | $\{42\}$, $\{222\}$ | 0         |            |
| 1   | 2   | 2   | +     | 0   | $\{222\}$ | 1.797$^a$ |            |
| 1   | 0   | 0   | +     | 0   | $\{42\}$, $\{222\}$ | 5.6 ± 0.6$^b$ | 10.9 ± 1.9$^b$ |
| 1   | 2   | 2   | +     | 0   |            | 5.6 ± 0.6$^b$ | 10.9 ± 1.9$^b$ |
| 1   | 0   | 1   | −     | 1   | $\{321\}$ | 4 ± 1$^c$ | 4 ± 1$^c$ |
| 1   | 1   | 0, 1, 2 | − | 1 | $\{411\}$, $\{33\}$, $\{321\}$, $\{2211\}$ | 14.6 ± 0.7$^b$ | 7.4 ± 1.0$^b$ |
| 1   | 2   | 1, 2, 3 | − | 1 | $\{321\}$, $\{2211\}$ | 14.6 ± 0.7$^b$ | 7.4 ± 1.0$^b$ |

state with $[T, S] = [1, 0]$ and orbital symmetry $\{321\}$ would be the lowest one. The resonance with $J^\pi = 2^−$, $1^−$, $0^−$ and $[T, S] = [1, 1]$ and those with $J^\pi = 3^−$, $2^−$, $1^−$ and $[T, S] = [1, 2]$ are the second and the third set of resonant states, respectively. In experiments, the lowest $1^−$ resonance once was assigned as a component of the resonance at $E_x = 5.6$ MeV with a width $\Gamma = 10.9$ MeV in the $^6$Li($^7$Li, $^7$Be)$^6$He reaction$^{13}$. The subsequent similar experiment provided a clue that it would be the one at $E_x = 4 ± 1$ MeV with $\Gamma = 4 ± 1$ MeV$^{14}$. While the recent $^6$Li(t, $^3$He)$^6$He experiment$^{15}$ indicated that the broad resonance at $E_x \approx 5.6$ MeV involves at least three Gaussians with energy 4.4 ± 0.1 MeV, 7.7 ± 0.2 MeV, 9.9 ± 0.4 MeV, respectively. And each of these three peaks contains the $1^-_1$ and other components. In some detail, the other constituents of this broad resonance must involve a $2^+$ state$^{13, 15}$. Even though the present approach can not give this state naturally since it is not a nodeless accessible one (then we leave the configuration $\lambda$ in Table 3 blank), we can assign it as the one dominated by the PENTA-accessible but OCTA-inaccessible S-wave components $\{321\}$ and $\{2211\}$, even $\{21^4\}$. In a short word, the resonance centred at 5.6 MeV contains at least the $0^+_2$, $1^-_1$ and $2^+_2$ states. As to the
$1^-_2$ and $2^-_1$ states, according to the $^6$Li($^7$Li, $^7$Be)$^6$He experiment\cite{13}, they correspond to the resonance at $E_x = 14.6 \pm 0.7$ MeV with $\Gamma = 7.4 \pm 1.0$ MeV. If the possible splitting of this resonance into two or three components in the range $E_x = 13 \sim 18$ MeV proposed in Ref.\cite{13} is confirmed, the $J^\pi = 0^-$ resonance may also be included. In addition, the $3^-$ and other $2^-, 1^-$ resonance may be the constituents of that at $E_x \approx 23.3$ MeV reported in Ref.\cite{13}. Summarizing these experimental correspondences, we list the data in Table 3 and illustrate the energy spectrum in Fig. 3. From Table 3 and Fig. 3, one can recognize that the energy spectrum of $^6$He obtained in the present analysis agrees very well with experimental data (for a recent compilation, see Ref.\cite{16}).

Since $^6$Be and $^6$He are mirror nuclei, each of which has $A = 6$ and $|T_3| = 1$, we can extend the discussion above to induce the quantum numbers of the low-lying states of $^6$Be and obtain the same result as for $^6$He. Unfortunately, only two low-lying states have been observed in experiments\cite{16}. For comparison we illustrate these states in Fig. 3, too.

In summary, using the inherent nodal surface structure analysis, we have determined the quantum numbers of the low-lying states of 6-nucleon systems with isospin $T = 1$. The orbital symmetries \{4 2\} and \{2 2 2\} are found to be the important components for the S-wave states and the \{3 2 1\}, \{2 2 1 1\} symmetries for the P-wave states. The energy spectra of $^6$He and $^6$Be obtained in the present analysis agree very well with the experimental data, except for the $2^+_2$ state. In fact, although the $2^+_2$ state does not appear as a low-lying one in our result, we haven’t excluded its possibility to be resonance at higher energy. On the other hand, a shell model calculation\cite{5} once predicted that the second $0^+$ state of $^6$He is at 12 MeV. However, in our analysis, this state must be much lower, at least as low as $1^-_1$ and $2^+_2$ states, which is quite consistent with the available experimental data at present. The present result provides then further evidence for that the inherent nodal surface analysis is a quite powerful approach for few-body problems.

It is evident that, although our analysis is simply based on the INS analysis, the obtained energy level structure agrees quite well with the experimental data. It indicates that the INS embodies the basic and intrinsic properties of the system (e.g., symmetry,
configuration’s structure) and with which one can discuss both positive and negative parity states simultaneously. Thus, it can help us to make reasonable choice between dynamical models. Meanwhile, the inherent nodeless wave functions are the most important building blocks of the low-lying states. The identification of these favorite components plays then a key role in understanding the low-lying energy spectrum. However, it is remarkable that the INS analysis can not give accurate numerical results directly. To get numerical results one must implement dynamical calculations. Then combining dynamical calculation and the INS analysis is the efficient way to investigate few-body problems.

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Figure 1: The sketch of regular octahedron
Figure 2: The sketch of regular pentagon pyramid
Figure 3: The energy level scheme of $^6$He and $^6$Be