No-go theorem for static boson stars with negative cosmological constants

Yan Peng\textsuperscript{1}\textsuperscript{*}

\textsuperscript{1} School of Mathematical Sciences, Qufu Normal University, Qufu, Shandong 273165, China

Abstract

In a recent paper, Hod has proven no-go theorem for asymptotically flat static regular boson stars. In the present work, we extend discussions to the gravity with a negative cosmological constant. We consider a scalar field vanishing at infinity. In the asymptotically AdS background, we show that spherically symmetric regular boson stars cannot be constructed with self-gravitating static scalar fields, whose potential is positive semidefinite and increases with respect to its argument.

PACS numbers: 11.25.Tq, 04.70.Bw, 74.20.-z

\textsuperscript{*} yanpengphy@163.com
I. INTRODUCTION

There is accumulating evidence that fundamental scalar fields may exist in nature. Black hole no hair theorems, see e.g. [1–4], play an important role in the development of the black hole theory. Classical black hole no hair theorems state that asymptotically flat black holes cannot support static scalar fields outside the horizon, see recent progress [5]–[19] and reviews [20, 21].

In contrast, it has recently been shown that rotating black holes allow the existence of stationary massive scalar field hairs [22]–[33]. Similarly, no static scalar hair behaviors were also found in horizonless reflecting object backgrounds and rotating regular reflecting objects can support stationary scalar hairs [34]–[50]. A well known regular scalar configuration is the boson star, which may theoretically be described by either static or stationary scalar fields. It was found that stationary self-gravitating massive scalar fields can form the spatially regular boson star [51, 52].

Then whether static scalar fields can make boson stars is a question to be answered. In the flat spacetime, boson stars cannot be constructed with static scalar fields due to Derricks theorem [53]. Lately, Hod extended this no-go theorem for boson stars to the asymptotically flat curved spacetime, considering self-interaction static scalar fields possessing a positive semidefinite potential increasing as a function of its argument [54]. It was further shown that this intriguing no-go theorem also holds for static scalar fields nonminimally coupled to the asymptotically flat gravity [55]. On the other side, the AdS boundary could provide the confinement of the scalar field and usually makes the scalar field easier to condense [56–58]. So it is of some interest to extend the discussion of no-go theorem for static boson stars to the asymptotically AdS gravity.

In the following, we introduce the model of self-gravitating static scalar fields in the background with negative cosmological constants. We prove that static scalar fields cannot form spatially regular spherically symmetric boson stars in the asymptotically AdS gravity. We give conclusions in the last section.

II. NO-GO THEOREM FOR ASYMPTOTICALLY ADS STATIC BOSON STARS

We study the gravity system of static scalar field in the spacetime with negative cosmological constants. The Lagrangian density describing static scalar fields in the curved spacetime reads [59, 61]

$$\mathcal{L} = \frac{R - 2\Lambda}{16\pi G} - |\nabla_\alpha \psi|^2 - V(\psi^2).$$  (1)
R is the Ricci curvature and $\Lambda < 0$ is the negative cosmological constant. Hereafter we choose $G = 1$ for simplicity. We take static scalar fields only depending on the radial coordinate in the form $\psi = \psi(r)$. The scalar field self-interaction potential $V(\psi^2)$ satisfies relations

$$V(0) = 0 \quad \text{and} \quad \dot{V} = \frac{dV(\psi^2)}{d(\psi^2)} > 0. \quad (2)$$

It means the potential is positive semidefinite and increases as a function of its argument. And in the case of free scalar fields with mass $\mu$, there is $V(\psi^2) = \mu^2 \psi^2$, $V(0) = 0$ and $\dot{V} = \mu^2 > 0$.

The four dimensional spherically symmetric boson star metric reads \[ds^2 = -fe^{-\chi}dt^2 + \frac{dr^2}{f} + r^2(d\theta^2 + \sin^2 \theta d\phi^2).\] \(\chi\) and $f = 1 - \frac{2m(r,\Lambda)}{r}$ are functions depending on the radial coordinate $r$ and cosmological constants, where $m(r,\Lambda)$ is the effective mass \[68, 69\]. Angular coordinates are taken to be $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$.

We take the cosmological constant term $-\frac{\Lambda}{8\pi G}$ in the lagrange density as effective matter fields. So both scalar fields and cosmological constants contribute to the total energy density $\rho = -T_{tt}$. There is the relation

$$\rho = \rho_1 + \rho_2, \quad (4)$$

where $\rho_1$ is the scalar field energy density and $\rho_2$ is the energy density of cosmological constants. The Einstein equations describing motions of the matter field and the background is $G^{\mu}_{\nu} = 8\pi T^{\mu}_{\nu}$. It yields metric equations

$$f' = -8\pi \rho + (1 - f)/r, \quad (5)$$

$$\chi' = -8\pi r(\rho + p)/f, \quad (6)$$

where $p = T^r_r$ is the radial pressure \[63, 64, 68, 69\].

Putting $f = 1 - \frac{2m(r,\Lambda)}{r}$ into (5), we obtain the equation

$$\frac{dm(r,\Lambda)}{dr} = 4\pi r^2 \rho, \quad (7)$$

which implies that \[68, 69\]

$$m(r,\Lambda) = \int_0^r 4\pi r'^2 \rho dr'. \quad (8)$$

The scalar field energy density reads

$$\rho_1(r,\Lambda) = f(\psi')^2 + V(\psi^2) \quad (9)$$
and the scalar field mass within a sphere with the radius \( r \) is given by

\[
m_1(r, \Lambda) = \int_0^r 4\pi r'^2 \rho_1(r', \Lambda) \, dr'.
\] (10)

Since (10) is a volume integral in the curved spacetime, the integral should depend on the geometry (3). In fact, \( \rho_1 \) depends on the metric function \( f \) according to relation (9). Within a sphere, \( m_1(r, \Lambda) \) is the mass corresponds to the energy density \( \rho_1 \), which is due to matter field terms \(-|\nabla_\alpha \psi|^2 - V(\psi^2)\) in the Lagrangian density (1).

The energy density corresponds to the cosmological constant is

\[
\rho_2(\Lambda) = \frac{1}{8\pi} \Lambda
\] (11)

and the cosmological constant effective mass within a radius \( r \) is

\[
m_2(r, \Lambda) = \int_0^r 4\pi r'^2 \rho_2(\Lambda) \, dr' = \frac{1}{6} \Lambda r^3.
\] (12)

According to (4), (10) and (12), we arrive at the relation

\[
m(r, \Lambda) = \int_0^r 4\pi r'^2 \rho \, dr' = \int_0^r 4\pi r'^2 (\rho_1 + \rho_2) \, dr' = m_1(r, \Lambda) + m_2(r, \Lambda) = m_1(r, \Lambda) + \frac{1}{6} \Lambda r^3.
\] (13)

With (13), the metric function \( f \) can be putted in the form \[70, 71\]

\[
g_{rr} = f = 1 - \frac{2m(r, \Lambda)}{r} = 1 - \frac{2(m_1(r, \Lambda) + m_2(r, \Lambda))}{r} = 1 - \frac{2m_1(r, \Lambda)}{r} - \frac{\Lambda}{3} r^2.
\] (14)

Since \( \rho_1 \geq 0 \), \( \frac{dm_1(r, \Lambda)}{dr} = 4\pi r^2 \rho_1 \geq 0 \) and \( m_1(r, \Lambda) \) is an increasing function of \( r \). We take the assumption that the total scalar field energy is finite, which means \( m_1(r, \Lambda) \) has an upper bound. Since \( m_1(r, \Lambda) \) is an increasing and upper bounded function, the limit value \( m_1(\infty, \Lambda) \) exists. As \( r \) approaching the infinity, the metric asymptotically goes to

\[
g_{rr} = f \to 1 - \frac{2m_1(\infty, \Lambda)}{r} - \frac{\Lambda}{3} r^2.
\] (15)

The usual Schwarzschild AdS background satisfies

\[
g_{rr} = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2,
\] (16)

where \( M \) is the mass of the spacetime \[71, 73\]. Comparing (15) and (16), we find that \( m_1(\infty, \Lambda) \) is the mass \( M \), for similar results in asymptotically flat spacetimes see \[74\].

As approaching the infinity, metric functions are characterized by

\[
\chi \to 0, \quad f \to -\frac{\Lambda}{3} r^2.
\] (17)
The finite energy density condition implies that \(m_1 \sim O(r^3)\) and \(m_2 \sim O(r^3)\), which are valid near the origin. And near the origin, asymptotically behaviors of functions are 

\[ \chi' \to 0, \quad f \to 1 + O(r^2). \] (18)

The scalar field equation is

\[ \psi'' + \left( \frac{2}{r} - \frac{\chi'}{2} + \frac{f'}{f} \right)\psi' - \frac{\dot{V}}{f} \psi = 0. \] (19)

Near the origin, the scalar field equation can be expressed as

\[ \psi'' + \frac{2}{r} \psi' - \frac{\dot{V}}{f} \psi = 0, \] (20)

which has a regular singular point \(r = 0\). According to Frobenius theorem \[75\], one solution behaves as

\[ \psi(r \to 0) \sim \frac{A}{r}, \] (21)

where \(A\) is a nonzero constant. Near the origin \(r = 0\), the finite mass condition \(M = m_1(\infty, \Lambda) = \int_0^\infty 4\pi r^2 \rho_1(r', \Lambda) dr' < \infty\) requires that \(r^2 \rho_1\) increases slower than \(1/r\) as \(r \to 0\). It yields the relation

\[ r^3 \rho_1(r, \Lambda) \to 0 \quad \text{for} \quad r \to 0. \] (22)

For the solution satisfying (21), there is \(\rho_1 \geq f(\psi')^2 \sim (\psi')^2 \sim \frac{A^2}{r^2}\) as \(r \to 0\), which is in contradiction with the relation (22). Here we use the finite mass condition to rule out the solution (21) while Hod used the finite energy density condition to rule out this type of solutions \[54\]. Around \(r = 0\), another physical solution of (20) can be expanded as \[54\]

\[ \psi(r) = a[A + \frac{1}{6} \dot{V}(a^2) \cdot r^2] + O(r^3), \] (23)

where \(a = \psi(0)\) is the value of the scalar field at the origin.

At the infinity, we impose the vanishing condition for the scalar field as

\[ \psi(\infty) = 0. \] (24)

As is well known, the AdS boundary usually provides the confinement of the system and serves as a box \[76-80\]. There is freedom in the choice of the field’s behavior at the box/AdS boundary, such as Dirichlet Boundary Conditions and Robin Boundary Conditions \[81, 82\]. So a more general infinity boundary condition compatible with finite energy may exist. However, the vanishing condition is essential in the present proof.
As such, the no-go theorem in this work applies to a scalar field vanishing at infinity, but the possible boson star with more generic boundary conditions is not excluded.

In the case of $a = \psi(0) = 0$, the scalar field must have one extremum point $r_{\text{peak}}$. At this extremum point, there are following characteristic relations

$$\{ \psi^2 > 0, \quad \psi' = 0 \quad \text{and} \quad \psi'' \leq 0 \} \quad \text{for} \quad r = r_{\text{peak}}.$$  \hfill (25)

At $r = r_{\text{peak}}$, we arrive at the inequality in the form

$$\psi'' + \left( \frac{2}{r} - \frac{\chi'}{2} + \frac{f'}{f} \right)\psi' - \frac{\dot{V}}{f} \psi^2 < 0.$$  \hfill (26)

For $a > 0$, there are relations $\psi'(0) = 0$ and $\psi''(0) = \frac{1}{3} a V(a^2) > 0$ implying $\psi' > 0$ for $r > 0$ around the origin. Also considering $\psi(\infty) = 0$, the scalar field firstly increases to be more positive and finally approaches zero at the infinity. So we conclude that one extremum point $\tilde{r}_{\text{peak}}$ of the scalar field must exist. There are following characteristic relations

$$\{ \psi > 0, \quad \psi' = 0 \quad \text{and} \quad \psi'' \leq 0 \} \quad \text{for} \quad r = \tilde{r}_{\text{peak}}.$$  \hfill (27)

At $r = \tilde{r}_{\text{peak}}$, the characteristic inequality is

$$\psi'' + \left( \frac{2}{r} - \frac{\chi'}{2} + \frac{f'}{f} \right)\psi' - \frac{\dot{V}}{f} \psi < 0.$$  \hfill (28)

And in cases of $a < 0$, one can deduce the conclusion that one extremum point $\tilde{r}_{\text{peak}}$ exists. At this extremum point, there are following characteristic relations

$$\{ \psi < 0, \quad \psi' = 0 \quad \text{and} \quad \psi'' \geq 0 \} \quad \text{for} \quad r = \tilde{r}_{\text{peak}}.$$  \hfill (29)

At $r = \tilde{r}_{\text{peak}}$, there is the characteristic inequality

$$\psi'' + \left( \frac{2}{r} - \frac{\chi'}{2} + \frac{f'}{f} \right)\psi' - \frac{\dot{V}}{f} \psi > 0.$$  \hfill (30)

Relations (26), (28) and (30) are in contradiction with the scalar field equation (19). It means that asymptotically AdS spherically symmetric regular boson stars cannot be constructed with static scalar fields, whose potential is positive semidefinite and monotonically increases with respect to its argument.

III. CONCLUSIONS

We studied the gravity model of static massive scalar fields in the background of spherically symmetric gravity with negative cosmological constants. We considered self-gravitating scalar fields vanishing at infinity.
The scalar field potential is positive semidefinite and monotonically increases as a function of its argument. We obtained the scalar field characteristic relations (26), (28) and (30) at extremum points. However, these characteristic relations are in contradiction with the static scalar field equation (19), which means the existence of no-go theorem for static boson stars. In summary, we found that spherically symmetric regular boson stars cannot be made of static scalar fields in the asymptotically AdS background. We pointed out that the no-go theorem in this work applies to a scalar field vanishing at infinity. We plan to examine whether there is no-go theorem for more generic boundary conditions in the next work.

Acknowledgments

We would like to thank the anonymous referee for the constructive suggestions to improve the manuscript. This work was supported by the Shandong Provincial Natural Science Foundation of China under Grant No. ZR2018QA008. This work was also supported by a grant from Qufu Normal University of China under Grant No. xkjje201906.

[1] J. D. Bekenstein, Transcendence of the law of baryon-number conservation in black hole physics, Phys. Rev. Lett. 28, 452 (1972).
[2] J. E. Chase, Event horizons in Static Scalar-Vacuum Space-Times, Commun. Math. Phys. 19, 276 (1970).
[3] C. Teitelboim, Nonmeasurability of the baryon number of a black-hole, Lett. Nuovo Cimento 3, 326 (1972).
[4] R. Ruffini and J. A. Wheeler, Introducing the black hole, Phys. Today 24, 30 (1971).
[5] S. Hod, Stationary resonances of rapidly-rotating Kerr black holes, The Euro. Phys. Journal C 73, 2378 (2013).
[6] S. Hod, The superradiant instability regime of the spinning Kerr black hole, Phys. Lett. B 758, 181 (2016).
[7] Carlos Herdeiro, Vanush Patryan, Eugen Radu, D.H. Tchrakian, Reissner-Nordström black holes with non-Abelian hair, Phys. Lett. B 772 (2017) 63-69.
[8] Mauricio Richartz, Carlos A. R. Herdeiro, Emanuele Berti, Synchronous frequencies of extremal Kerr black holes: resonances, scattering and stability, Phys. Rev. D 96, 044034 (2017).
[9] Yves Brihaye, Carlos Herdeiro, Eugen Radu, D.H. Tchrakian, Skyrmions, Skyrme stars and black holes with Skyrme hair in five spacetime dimension, JHEP 1711(2017)037.
[10] C. L. Benone, L. C. B. Crispino, C. Herdeiro, and E. Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90, 104024 (2014).
[11] C. Herdeiro, E. Radu, and H. Runarsson, Non-linear QQ-clouds around Kerr black holes, Phys. Lett. B 739, 302 (2014).
[12] Elizabeth Winstanley, Classical Yang-Mills black hole hair in anti-de Sitter space, Lect. Notes Phys. 769:49-87, 2009.
[13] Yan Peng, Hair mass bound in the black hole with non-zero cosmological constants, Physical Review D 98, 104041 (2018).
[14] Yan Peng, Hair distributions in noncommutative Einstein-Born-Infeld black holes, Nucl. Phys. B 941 (2019) 1-10.
[15] Yves Brihaye, Thomas Delplace, Carlos Herdeiro, Eugen Radu, An analytic effective model for hairy black holes, Phys. Lett. B 782 (2018) 124-130.
[16] Marek Rogatko, Uniqueness of higher-dimensional phantom field wormholes, Phys. Rev. D 97 (2018) no. 2, 024001.
[17] J. C. Degollado and C. A. R. Herdeiro, Stationary scalar configurations around extremal charged black holes, Gen. Rel. Grav. 45, 2483 (2013).
[18] P. V. P. Cunha, C. A. R. Herdeiro, E. Radu, and H. F. Runarsson, Shadows of Kerr black holes with scalar hair, Phys. Rev. Lett. 115, 211102 (2015).
[19] Y. Brihaye, C. Herdeiro, and E. Radu, Inside black holes with synchronized hair, Phys. Lett. B 760, 279 (2016).
[20] S. Hod, Black hole hair: 25-years after, arXiv:gr-qc/9605059.
[21] Carlos A. R. Herdeiro, Eugen Radu, Asymptotically flat black holes with scalar hair: a review, Int. J. Mod. Phys. D 24 (2015) no. 15, 1542014.
[22] Shahar Hod, Stationary Scalar Clouds Around Rotating Black Holes, Phys. Rev. D 86 (2012) 104026.
[23] Carlos A. R. Herdeiro, Eugen Radu, Kerr black holes with scalar hair, Phys. Rev. Lett. 112(2014)221101.
[24] Carlos Herdeiro, Eugen Radu, Construction and physical properties of Kerr black holes with scalar hair, Class. Quant. Grav. 32(2015)no.14,144001.
[25] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[26] Shahar Hod, Kerr-Newman black holes with stationary charged scalar clouds, Phys. Rev. D 90(2014)no.2,024051.
[27] Shahar Hod, The large-mass limit of cloudy black holes, Class. Quant. Grav. 32(2015)no.13,134002.
[28] Pedro V. P. Cunha, Carlos A. R. Herdeiro, Eugen Radu, Helgi F. Runarsson, Shadows of Kerr black holes with scalar hair, Phys. Rev. Lett. 115(2015)no.21,211102.
[29] Bogdan Ganchev, Jorge E. Santos, Scalar Hairy Black Holes in Four Dimensions are Unstable, Phys. Rev. Lett. 120(2018)no.17,171101.
[30] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[31] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[32] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[33] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[34] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[35] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[36] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[37] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[38] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[39] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[40] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[41] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[42] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[43] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[44] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[45] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[46] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[47] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[48] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[49] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[50] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[51] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[52] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[53] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[54] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[55] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[56] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[57] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[58] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[59] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[60] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[61] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[62] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[63] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[64] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[65] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[66] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[67] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[68] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[69] Carolina L. Benone, Luís C.B. Crispino, Carlos Herdeiro, Eugen Radu, Kerr-Newman scalar clouds, Phys. Rev. D 90(2014)no.10,104024.
[61] Bartłomiej Kiczek, Marek Rogatko, Karol I. Wysokinski, DC SQUID as a sensitive detector of dark matter, arXiv:1904.00653[hep-th].
[62] D. Núñez, H. Quevedo, and D. Sudarsky,Black Holes Have No Short Hair, Phys. Rev. Lett. 76, 571 (1996).
[63] S. Hod,Hairy Black Holes and Null Circular Geodesics,Phys. Rev. D 84, 124030 (2011) [arXiv:1112.3286 [gr-qc]].
[64] Yan Peng, Upper bound on the radii of regular ultra-compact star photonspheres, Phys. Lett. B 790(2019)396-399.
[65] Pallab Basu, Pankaj Chaturvedi, Prasanta Samantray, Chaotic dynamics of strings in charged black hole backgrounds, Phys. Rev. D 95(2017)066014.
[66] Bartłomiej Kiczek, Marek Rogatko,Ultra-compact spherically symmetric dark matter charged star objects, arXiv:1904.07232[gr-qc].
[67] Peng Wang, Houwen Wu, Haitang Yang, Thermodynamics and Phase Transition of a Nonlinear Electrodynamics Black Hole in a Cavity, arXiv:1901.06216[gr-qc].
[68] Shahar Hod,Analytic study of self-gravitating polytropic spheres with light rings, Eur. Phys. J. C 78(2018)no.5.417.
[69] Shahar Hod, Bounds on the mass-to-radius ratio for non-compact field configurations, Class. Quant. Grav. 24(2007)6019-6024.
[70] Elizabeth Winstanley, Classical Yang-Mills black hole hair in anti-de Sitter space, Lect. Notes Phys.769:49-87,2009, arXiv:0801.0527[gr-qc].
[71] Zdeněk Stuchlík, Stanislav Hledík, Jan Novotný, General relativistic polytropes with a repulsive cosmological constant, Phys. Rev. D 94(2016)103513.
[72] M. Giammatteo, Jiliang Jing, Dirac quasinormal frequencies in Schwarzschild-AdS space-time,Phys. Rev. D 71(2005)024007.
[73] Zhiying Zhu, Shao-Jun Zhang, C. E. Pellicer, Bin Wang, Elcio Abdalla, Stability of Reissner-Nordström black hole in de Sitter background under charged scalar perturbation, Phys. Rev. D 90(2014)044042.
[74] Shahar Hod,Upper bound on the radii of black-hole photonspheres, Physics Letters B 727(2013)345.
[75] Arfken, G. “Series Solutions–Frobenius Method,” 8.5 in Mathematical Methods for Physicists, 3rd ed. Orlando, FL: Academic Press, pp.454-467,1985.
[76] Nicolas Sanchis-Gual,Juan Carlos Degollado,Pedro J. Montero,Jos A. Font,Carlos Herdeiro,Explosion and final state of an unstable Reissner-Nordström black hole, Phys. Rev. Lett. 116(2016)141101.
[77] Pablo Bosch, Stephen R. Green, and Luis Lehner, Nonlinear Evolution and Final Fate of Charged Anti-de Sitter Black Hole Superradiant Instability, Phys. Rev. Lett. 116(2016)141102.
[78] Pallab Basu, Chethan Krishnan, P. N. Bala Subramanian,Hairy Black Holes in a Box, JHEP 1611(2016)041.
[79] Yan Peng, Studies of a general flat space/boson star transition model in a box through a language similar to holographic superconductors, JHEP 1707(2017)042.
[80] Yan Peng, Bin Wang, Yunqi Liu,On the thermodynamics of the black hole and hairy black hole transitions in the asymptotically flat spacetime with a box, Eur. Phys. J. C 78(2018) no.3.176.
[81] Hugo R. C. Ferreira, Carlos A. R. Herdeiro, Superradiant instabilities in the Kerr-mirror and Kerr-AdS black holes with Robin boundary conditions, Phys. Rev. D 97(2018)no.8.084003.
[82] Claudio Dappiaggi, Hugo R. C. Ferreira, Carlos A. R. Herdeiro, Superradiance in the BTZ black hole with Robin boundary conditions, Phys. Lett. B 778(2018)146-154.