MEASUREMENT OF THE STRONG COUPLING $\alpha_S$ FROM THE 3-JET RATE IN $e^+e^-$ ANNIHILATION USING JADE DATA

S. KLUTH
Max-Planck-Institut für Physik, Föhringer Ring 6, 80805 Germany

We describe a measurement of the strong coupling $\alpha_S(m_{Z^0})$ from the 3-jet rate in hadronic final states of $e^+e^-$ annihilation recorded with the JADE detector at centre-of-mass energies of 14 to 44 GeV. The jets are reconstructed with the Durham jet clustering algorithm. The JADE 3-jet rate data are compared with QCD predictions in NNLO combined with resummed NNLA calculations. We find good agreement between the data and the prediction and extract

$$\alpha_S(m_{Z^0}) = 0.1199 \pm 0.0010(\text{stat.}) \pm 0.0021(\text{exp.}) \pm 0.0054(\text{had.}) \pm 0.0007(\text{theo.})$$

1 Introduction

We report on the first measurement of $\alpha_S(m_{Z^0})$ from the 3-jet rate with the Durham algorithm with matched NNLO+NLLA QCD calculations. The first measurement of $\alpha_S$ from $R_3$ with NNLO QCD calculations was shown in [1].

2 JADE Detector and Data

The JADE detector was a universal and hermetic detector covering a solid angle of almost $4\pi$. The interaction point was surrounded by a large tracking detector (jet chamber) of 1.6 m diameter and 2.4 m length inside a solenoid magnet coil with a magnetic field of 0.48 T. Outside of the magnetic coil was the electromagnetic calorimeter consisting of 2520 lead glass blocks in the barrel section and 96 lead glass blocks in each endcap with a total acceptance of 90% of $4\pi$. The measurement of hadronic final states relies mainly on these two detector systems. More details can be found e.g. in [2].

The data used in the analysis are from the JADE experiment which operated at the PETRA $e^+e^-$ collider at DESY in Hamburg, Germany, from 1979 to 1986. The main data samples were collected at centre-of-mass (cms) energies of 14, 22, 35, 38 and 44 GeV. The integrated luminosities range from about 1/pb at 14 and 22 GeV to about 100/pb at 35 GeV and correspond to sample sizes of $O(10^3)$ events at 14, 22, 38 and 44 GeV and $O(10^5)$ events at 35 GeV.

3 QCD Predictions

The Durham jet clustering algorithm defines $y_{ij} = 2 \min(E_i, E_j)^2(1 - \cos \theta_{ij})/s$ as distance in phase space between a pair of particles or jets $i$ and $j$ with energies $E_i, E_j$ and angle $\theta_{ij}$ between them. The pair with the smallest $y_{ij}$ is combined by adding their 4-vectors, the particles or jets $i, j$ are removed and the combined 4-vector is added. This procedure is repeated until all
$y_{ij} > y_{\text{cut}}$. The 3-jet rate for a given value of $y_{\text{cut}}$ at a CMS energy $Q = \sqrt{s}$ is defined as 

$$R_3(y_{\text{cut}}, Q) = \frac{N_{3-\text{jet}}(y_{\text{cut}}, Q)}{N(Q)},$$

where $N_{3-\text{jet}}$ is the number of 3-jet events and $N$ is the total number of events in the sample. The 3-jet rate is a measurement of $\sigma_{3-\text{jet}}(y_{\text{cut}}, Q)/\sigma_{\text{had}}(Q)$ where $\sigma_{3-\text{jet}}(y_{\text{cut}}, Q)$ is the exclusive 3-jet cross section and $\sigma_{\text{had}}(Q)$ is the total hadronic cross section.

The NNLO QCD prediction\cite{5,6} can be written as:

$$R_{3,\text{NNLO}}(y_{\text{cut}}, Q) = A(y_{\text{cut}})\hat{\alpha}_S(Q) + B(y_{\text{cut}})\hat{\alpha}^2_S(Q) + C(y_{\text{cut}})\hat{\alpha}^3_S(Q)$$

(1)

with $\hat{\alpha}_S(Q) = \alpha_S(Q)/(2\pi)$. The coefficient functions $A(y_{\text{cut}})$, $B(y_{\text{cut}})$ and $C(y_{\text{cut}})$ are obtained by numerical integration of the QCD matrix elements in LO, NLO or NNLO. The resummed NLLA calculations use an improved resummation scheme\cite{7} and are matched to the NNLO prediction\cite{1}.

Figure 1 (left) shows these QCD predictions as black band with renormalisation scale uncertainty defined by multiplying the renormalisation scale $\mu$ by a factor of 1/2 or 2. The other bands show NLO and NLO+NLLA predictions for comparison. The theoretical uncertainties of the NNLO+NLLA prediction are significantly smaller compared to the less advanced predictions.

4 Data Analysis

The data for the 3-jet rate $R_3$ are corrected for the effects of detector resolution and acceptance and for photon initial state radiation to the so-called hadron-level using samples of simulated events. The expected contributions from $e^+e^- \rightarrow b\bar{b}$ events are subtracted. The Monte Carlo generators PYTHIA 5.7, HERWIG 6.2 or ARIADNE 4.11 with parameter settings from OPAL are used to produce the simulated events together with a full simulation of the JADE detector. The corrected data for $R_3$ are well described by the simulations.

The QCD predictions have to be corrected for effects of the transition from the partons (quarks and gluons) of the theory to the particles of the hadronic final state. These so-called hadronisation corrections are taken from the samples of simulated events by comparing $R_3$ values after the parton shower has stopped (parton-level) and the hadron-level consisting of all particles.
with a lifetime larger than 300 ps. OPAL has compared for the observable $y_{23}$ the parton-level predictions of the theory and the simulation and found agreement within the differences between the three simulations. Thus it is justified to use the simulations to derive the hadronisation corrections, since the hadronisation systematic uncertainty evaluated by comparing the three simulations covers any discrepancies.

The theory is compared with the data using a $\chi^2$-fit with $\alpha_S$ as a free parameter. The statistical correlations between the data points for $R_3(y_{\text{cut}})$ are taken into account. Only data points within a restricted range of $y_{\text{cut}}$ are used in the fits to ensure that the experimental and hadronisation corrections are under control and that the QCD predictions are reliable.

Several sources of systematic uncertainty are investigated. Experimental uncertainties are evaluated by repeating the analysis with different event selection cuts, reconstruction calibration versions, corrections for experimental effects, and with different fit ranges. The experimental uncertainties are dominated by the different detector calibrations and the detector corrections based on PYTHIA or HERWIG. Hadronisation uncertainties are estimated by changing the Monte Carlo generator for hadronisation corrections from PYTHIA to HERWIG or ARIADNE. The differences between PYTHIA and HERWIG determine this uncertainty. Theoretical systematic uncertainties are found by repeating the fits with the renormalisation scale factor $x_\mu = Q/\mu$ changed from $x_\mu = 1$ to 0.5 or 2.

## Results

The fit of the NNLO+NLLA QCD prediction to the 3-jet rate data at $\sqrt{s} = 35$ GeV is shown in figure 1(right). The fitted prediction agrees well with the data corrected to the hadron-level within the fit range. The extrapolation to the other data points also gives a good description of the data. For this fit based on statistical errors we find $\chi^2$/d.o.f. = 1.2. The fits at the other cms energies are similar with $1.2 < \chi^2$/d.o.f. < 3.8 except at $\sqrt{s} = 14$ GeV where we have $\chi^2$/d.o.f. = 6.3. At the lowest cms energy the hadronisation corrections are significantly larger compared to the other cms energies. The individual fit results for $\alpha_S$ are shown in figure 2(left) as a function of the cms energy where they were obtained.

The individual results for $\alpha_S$ are evolved to $\alpha_S(m_{Z^0})$ using the 3-loop evolution equations. Then they are combined into a single value taking account of correlated experiential, hadronisation and theory uncertainties as described in 1. The result from $\sqrt{s} = 14$ GeV is excluded from the combined value since it has a much larger value of $\chi^2$/d.o.f. and larger hadronisation corrections compared to the other results. The combined value is

$$\alpha_S(m_{Z^0}) = 0.1199 \pm 0.0010(\text{stat.}) \pm 0.0021(\text{exp.}) \pm 0.0051(\text{had.}) \pm 0.0007(\text{theo.}) . \tag{2}$$

The errors are dominated by the hadronisation correction uncertainties.

As a cross check the analysis is repeated with NNLO QCD predictions using the same fit ranges with $x_\mu = 1$. We find larger values of $\chi^2$/d.o.f., a less satisfactory description of the $R_3$ data and larger uncertainties from variations of the fit ranges compared to the NNLO+NLLA fits. The NNLO predictions do not reproduce the slope of the $R_3(y_{\text{cut}})$ data as well as the NNLO+NLLA predictions. A similar observation can be made in the analysis of 2.

In figure 2(right) the result of this analysis is compared with other measurements of $\alpha_S(m_{Z^0})$ using the 3-jet or 4-jet rate based on the Durham algorithm. The JADE measurement with $y_{23}$ is highly correlated with our measurement using $R_3$ and the good agreement of the results is a strong consistency check. The agreement with the other results and with the world average value is also satisfactory within the uncertainties.

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*The distribution of $y_{ij}$ values for which events change from 2 jets to 3 jets.*
Figure 2: (left) Results for $\alpha_S$ from the JADE energy points are shown. The lines give the prediction from the 3-loop QCD evolution with uncertainties for the value of $\alpha_S(m_{Z^0})$ as indicated on the figure. (right) The result for $\alpha_S(m_{Z^0})$ from this analysis (solid point) is compared with results from \cite{9,10,8,2} (solid triangles) and the current world average value \cite{11,12}.

6 Conclusion

We have shown the first measurement of $\alpha_S(m_{Z^0})$ using the 3-jet rate with the Durham algorithm and matched NNLO+NLLA QCD calculations and data from the JADE experiment. The agreement between data and the NNLO+NLLA QCD prediction is improved compared to less advanced predictions. The errors are dominated by the hadronisation correction uncertainties as expected at the low cms energies of the JADE experiment. However, the data of the JADE experiment at comparatively small cms energies can now be analysed with rather good precision thanks to the progress in perturbative QCD calculations and Monte Carlo simulations made since the data were recorded. Our analysis provides an independent and strong cross check on those recent QCD calculations made for the LHC which have related Feynman diagrams or share calculation techniques.

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