Flood Flow Probability Distribution Model Selection on Niger/Benue River Basins in Nigeria

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Author’s contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

ABSTRACT

Flood frequency analysis is a crucial component of flood risk management which seeks to establish a quantile relationship between peak discharges and their exceedance (or non-exceedance) probabilities, for planning, design and management of infrastructure in river basins. This paper evaluates the performance of five probability distribution models using the method of moments for parameter estimation, with five GoF-tests and Q-Q plots for selection of best fit distribution. The probability distributions models employed are; Gumbel (EV1), 2-parameter lognormal (LN2), log Pearson type III (LP3), Pearson type III(PR3), and Generalised Extreme Value( GEV). The five statistical goodness of fit tests, namely; modified index of agreement (Dmod), relative root mean square error (RRMSE), Nash – Sutcliffe efficiency (NSE), Percent bias (PBIAS), ratio of RMSE and standard deviation of the measurement (RSR) were used to identify the most suitable distribution models. The study was conducted using annual maximum series of nine gauge stations in both Benue and Niger River Basins in Nigeria. The study reveals that GEV was the best fit distribution in six gauging stations, LP3 was best fit distribution in two gauging stations, and PR3 is best fit distribution in one gauging station. This study has provided a significant contribution to knowledge in the choice of distribution models for predicting extreme hydrological events for design of water infrastructure in Nigeria. It is recommended that GEV, PR3 and LP3 should be considered in the development of regional flood frequency using the existing hydrological map of Nigeria.
1. INTRODUCTION

Floods are the overflowing of the normal confines of a stream or other body of water or the accumulation of water over areas that are not normally submerged. Floods include river (fluvial) floods, flash floods, urban floods, pluvial floods, sewer floods, coastal floods, and glacial lake outburst floods [1]. The focus of this paper is fluvial flooding. Intense and long – lasting rainfall is the common cause of river (fluvial) floods in large river basins, for example, the Niger river basin, while floods in small basins are generated by short – duration, highly intense rainfall [2]. Floods are responsible for 20 – 30% of economic losses caused by natural hazards globally and they are also responsible for more than 50% of all fatalities due to natural disasters [1] and [3]. Similarly, [4] reported that the proportion of the world’s population living in flood – prone river basins has increased by 114% , while those living on cyclone – exposed coastlines have grown by 192% since the 1980s and flooding is the most frequent and greatest hazard for the 633 largest cities or urban agglomerations. Furthermore [5] and [6] reported that since 1990, the United States floods have caused more than 10,000 deaths and losses over US$470 billion. In Europe, 264 flood disasters were reported in the last 30 years, with the number increasing from 31 in the period of 1973 – 1982 to 179 during the last decade [7]. Also [3] reported that in 2010, a destructive flood in Pakistan affected up to 20 million people and left more than 1500 dead. The impacts of floods are expected to increase due to population growth, population migration to coastal areas, and climate change effects [8].

Furthermore, [3] reported that in 2012, “killer floods”, inducing more than 50 fatalities each, occurred in Madagascar, Niger and Nigeria in Africa; Bangladesh, China, India, North and South Korea, the Philippines and Russia in Asia; and Argentina, the United States and Haiti in the Americas. Similarly, [9] reported that the 2012 floods in Nigeria affected 7.7 million people, 363 fatalities were recorded and approximately 600,000 houses were damaged or destroyed. This disaster greatly worsened an already existing housing deficit thereby placing huge pressure on all levels of government to address the sharp increase in housing demand.

The application of probability distribution model to annual flood flow presupposes screening and application of non – parametric tests of randomness, independence, homogeneity and stationarity. It is only when empirical evidence was found to rule out the non – parametric tests, before the available data is considered fit for flood frequency analysis.

The two data types commonly used in flood frequency analysis; Annual Maximum Series (AMS) and Partial-duration series (POT). The AMS is used when only one damaging flood event per year is possible, while the POT is used when more than one damaging flood event per year is possible. The AMS is adopted in this study, because it is consistent with the occurrence of floods in Nigeria wherein one damaging fluvial flood event occurs annually [10].

The choice of suitable probability distribution model and parameter estimation method is crucial to a successful model selection exercise , because the parameters of the distribution and its uncertainty assessment are determined by the candidate distribution model.

Several probability distribution models have been considered in different situations, for the probabilistic modelling of extreme events, such as GEV, LN2 and LN3, Gamma (Gam), EV1, LP3, PR3. Reviews of pertinent works include [11 - 21]. Assessment of cited literatures, show that there is no distribution that universally fits all the long - term series of flood data. Therefore, a number of commonly used distributions are evaluated, and then the best – fit distribution is selected amongst the candidate distributions. [22] presented the state-of-the-art review of current practice with regard to use of distribution types for frequency analyses on extremes of precipitation and floods. The Review reported six most frequently used distribution type for flood frequency analysis in the order of popularity as: EV1, LP3, LN2, P3, GEV and Gamma. Furthermore, [22] and [14] observed that there has been no consensus about a globally accepted probability distribution model for flood frequency analysis across various sites, thus the true model of the data at a site is unknown. Consequently, for flood frequency analysis to be of practical use, commonly used distributions have been evaluated to establish the best – fit distribution. [15] studied the selection of probability distribution for at-site flood frequency
Several methods have been developed to estimate the parameters of probability distribution functions. They are: (i) Methods of Moments (MoM), ( ii) The Maximum Likelihood Method (MLM) (iii) The Probability Weighted Moments Method (PWM) and method of L – moments, see [26] and [27] for details. A brief review by [28] and [29] revealed that PWM or L- moments is preferred to other parameter estimators while MLE are recommended for small sample sizes. Furthermore, MLE is generally best for fitting LN2 for sample sizes longer than 25 years. Also, MoM performs best for Log – normal distributions with low skewness coefficient. Similarly LP3 applies to hydrologic frequency analysis only when $\lambda > 1.0$ and $1/(\text{scale factor}) > 0.0$. [30] and [31], reported that the MoM was more suitable for data with lower skewness values and small sample sizes, whereas the method of L-moments was more suitable for data with higher skewness values and is appropriate for all sample sizes. In this study, MoM is used because of its simplicity and being relatively easy to apply by equating the sample moments with the moments of the population distribution functions.

The performances of fitted probability distribution models are compared using different accuracy measures, to identify the best – fit model among the employed probability distribution models. These measures may be categorised as follows; (i) graphical assessments (ii) statistical goodness –of – fit tests (iii) hypothesis-based goodness –of – fit tests and (iv) information –based criteria. The commonly used statistical indices are (i) Relative Root Mean Square error (RRMSE) (ii) Nash – Sutcliffe efficiency (NSE) (iii) Percent bias (PBIAS) (vi) ratio of the root mean square error to the standard deviation of measured data (RSR)[32-33]. Each method has its strengths and weaknesses when applied to model selection. Therefore the selection of the best efficiency measures should reflect the intended use of the model and should concern model quantities which are deemed relevant for the study at hand. [34] recommended calculating statistical goodness – of – fit for quantitative evaluation of the differences between observed and simulated discharges. The statistical goodness –of – fit tests and PPC test is adopted in this study. Flood flow model selection studies in Nigeria is still at infancy level as there is no established probability distribution model for accurate prediction of flood quantiles, while the country continues to suffer devastating floods. Besides the 2012 “killer floods” the frequency of flooding has been [9]. The objective of this paper is to evaluate the performances of five commonly used probability distribution models to find the best – fit distribution(s) that could be adopted in practice to accurately simulate or model flood flow in Nigeria. The introductory section presents a literature review and background information on flood frequency analysis, together with a statement of the problem. The study area, brief description of Benue and Niger river basins and data description are presented in section 2. Section 3 provides a synthesis of the methodology comprising the probability distribution functions used, parameter estimation method, goodness -of – fit tests, and procedure for estimation of flood quantiles. Section 4 contains results and discussion comprising derived flood quantiles, results of goodness –of – fit tests. The conclusion derived in this study and recommendations are presented in section 5. The results of this study will be useful in hydraulic engineering and design of infrastructure to control the devastating impacts of floods.

2. DATA DESCRIPTION AND STUDY AREA

The Niger River Basin covers a total area of approximately 2,156,000km$^2$, only about
1,270,000km² actively contributes to runoff and river discharge. The whole basin is spread over the territory of ten countries. Nigerian shares about 28.3% of the active Niger River Basin (424,500km²). The Niger Basin extends across 20 out of 36 states of Nigeria and comprises two main rivers: the Niger and Benue [35]. Table 1 shows the distribution of the gauging stations, geographical and background information and the descriptive statistics of both Niger and Benue river basins. The data length of the gauging station is 30 years each. For the Niger river basin the Coefficient of Variation (CV) ranges between 0.219 and 0.321 except at Asamabiri with 0.151. Thus the year-to-year variation of the AMS is moderately variable. The coefficient of skewness ranges between 0.162 and 0.480, all positive values, which implies non–normal probability distribution.

The Benue River Basin is a major tributary of the River Niger forming a confluence at Lokoja and it contributes more than the actual Niger River discharges at the confluence. It originates from the Adamawa Plateau in Cameroun and has a total length of about 1200km from origin to the confluence at Lokoja with about 4.4 percent (66,000km²) of the Benue Basin lies in Cameroun [35]. In terms of the descriptive statistics the coefficient of variation (CV) ranges between 0.194 and 0.313, which implies that the year-to-year variation of the AMS is also moderately variable. The coefficient of skewness (CS) ranges between 0.301 and 0.575, all positive values, which also implies non–normal probability distribution. The AMS data used in this study, was obtained from the Nigerian Inland Waterways Authority (NIWA), Lokoja Nigeria. The NIWA authority operates the river gauging stations in Nigeria.

3. METHODOLOGY

3.1 Probability Distribution Functions (PDFs)

The five PDFs employed in this study comprised; EVI, LN2, LP3, PR3 and GEV. Table 2 shows the probability distribution models, sample parameters and quantile estimators. Detailed procedures for flood frequency analysis may be found in [26,36] and [37].

3.2 Parameter Estimation Method

The central moments of a distribution are given by:
$$\mu_r = E(Q - \mu)^r = \int(Q - \mu)^r f_Q (Q)dQ \quad (1)$$

Variance: $$\sigma^2 = \mu_2$$

Skewness:
$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} \quad (2)$$

Fig. 1. Map of Nigeria showing the studied stations
Table 1. Characteristics of selected gauge stations

| S/N | Station | Latitude (N) | Longitude (E) | River | Catchment (Km²) | Annual Streamflow (Max. m³/s) | Annual Streamflow (Min. m³/s) | Co. of Var. (cv) | Skewness (cs) |
|-----|---------|--------------|---------------|-------|----------------|-----------------------------|-----------------------------|----------------|--------------|
| 1   | Asamabiri | 05°32' | 06°31' | Niger | 1,112,830 | 18671.41 | 12281.48 | 0.151 | 0.480 |
| 2   | Baro     | 08°35' | 06°23' | Niger | 729,510   | 8852.21  | 103.45  | 0.321 | 0.162 |
| 3   | Idah     | 07°06' | 06°43' | Niger | 1,105,780 | 26,760.24 | 826.32  | 0.252 | 0.174 |
| 4   | Lokoja   | 07°49' | 06°44' | Niger | 750,790   | 28,360   | 248.75  | 0.219 | 0.337 |
| 5   | Onitsha  | 06°10' | 06°45' | Niger | 1,125,170 | 26,607.53 | 426.84  | 0.237 | 0.164 |
| 6   | Ibi      | 08°11' | 09°45' | Benue | 275,370   | 12,454.94 | 12.68   | 0.251 | 0.575 |
| 7   | Makurdi  | 07°45' | 08°32' | Benue | 317,430   | 16,034.93 | 30.48   | 0.194 | 0.469 |
| 8   | Umaisha  | 08°00' | 07°14' | Benue | 343,210   | 18,408.97 | 7.71    | 0.254 | 0.349 |
| 9   | Yola     | 09°14' | 12°28' | Benue | 112,680   | 6641.30  | 8.93    | 0.313 | 0.301 |

Extracted from hydrological year books (1914 – 1989); NIWA, Lokoja, Nigeria
Table 2. Probability models, sample parameters and moments

| Probability Model       | Probability Density Function | Range                     | MoM                      | Quantile Function (Q_T)                                      |
|-------------------------|------------------------------|---------------------------|--------------------------|-------------------------------------------------------------|
| Lognormal (LN2)         | \( f(Q) = \frac{1}{Q\sigma_\nu \sqrt{2\pi}} \exp\left(-\frac{Q - \mu_\nu}{2\sigma_\nu^2}\right) \) | \(-\infty \leq Q \leq \infty\) | Indirect MoM, \( \sigma_\nu = \sqrt{\ln(C_y^2 + 1)} \) | \( Q_T = \mu_\nu + Z\sigma_\nu \)                             |
|                         | Q = \ln Q                   | (0 \leq x \leq \infty)   | \( \mu_\nu = \ln(Q) \) | \( Q_T = e^{(\mu_\nu + Z\sigma_\nu)} \)                   |
| Gumbel (EV1)            | \( f(Q) = \frac{1}{a} \exp\left(-\frac{Q - \beta}{a}\right) - \exp\left(-\frac{Q - \beta}{a}\right) \) | \(-\infty \leq Q \leq \infty\) | 0 \leq \beta \leq \infty | \( \mu = \beta + 0.5772 \alpha \) \( \sigma_\nu^2 = 1.645 \alpha^2 \) | \( Q_T = \beta + \alpha \ln(-\ln\left(\frac{1}{T}\right)) \) |
| Log Pearson (LP3)       | \( f(Q) = \frac{1}{a} \exp\left[-\frac{(y-a)^{1/\beta}}{Q^{(\alpha+1)}}\right] \) | \(-\infty \leq Q \leq \infty\) | 0 \leq Q \leq \infty | \( \mu_z = \mu + \alpha \beta \) | \( \sigma_\nu^2 = \alpha^2 \beta \) | \( Q_T = e^{\frac{\beta}{\alpha^2 \sigma_z^2}} \) |
| Pearson Type III (PR3)  | \( f(Q) = \frac{1}{a} \left(\frac{Q - \beta}{a}\right)^{1/\alpha - 1} \exp\left(-\frac{(Q - \beta)}{a}\right) \) | \( u < Q < \infty \) | \( \beta = \left(\frac{2}{\alpha^2 \sigma_z^2}\right)^{1/2} \) | \( Q_T = \alpha \beta + u + K_T \sqrt{\alpha^2 \beta} \) |
| Generalized Extreme Value (GEV) | \( f(Q) = \frac{1}{a} \left[1 - K \left(\frac{Q - \beta}{a}\right)^{1/\alpha}\right]^{1 - \frac{1}{\alpha}} \) | \( \beta + \alpha k > Q < \infty \) | Equations 31 – 33 | \( Q_T = \beta + \frac{\alpha}{K} \left[1 - \ln\left(\frac{1 - \frac{1}{T}}{\alpha^2 \beta}\right)\right] \) |

Where \( \beta, \alpha, \) and \( k \) are the location, scale, and shape parameters of the distributions.
Kurtosis:

\[ \beta_2 = \frac{\mu_4}{\mu_2^2} \]

The sample moment are calculated using Equations 1 and 2:

\[ \bar{Q} = n^{-1} \sum Q_i \]  
\[ m_r = n^{-1} \sum (Q_i - \bar{Q})^r \]

The moments for the selected distributions are shown in Table 2.

### 3.3 Goodness-of-fit-tests

The GoF measures are selected to give sound comparative evaluation study with quantities deemed relevant to objective estimates of the “closeness” of the simulated discharges to observed flood flow [34]. Details about the selected GoF measures may be found in [33,36,38-40].

#### 3.3.1 Nash-sutcliffe efficiency (NSE)

\[
\text{NSE} = 1 - \frac{\sum_{i=1}^{n} (Q_{i,\text{obs}} - Q_{i,\text{sim}})^2}{\sum_{i=1}^{n} (Q_{i,\text{obs}} - \bar{Q}_{\text{obs}})^2} \]  

\[
\text{..0} \leq \text{NSE} \leq 1.0 \]

\[
\text{..... (8)} \]

#### 3.3.2 RMSE – observation standard deviation ratio (RSR)

\[
\text{RSR} = \frac{\text{RMSE}}{STDEV_{\text{obs}}} = \frac{\sqrt{\sum_{i=1}^{n} (Q_{i,\text{obs}} - Q_{i,\text{sim}})^2}}{\sqrt{\sum_{i=1}^{n} (Q_{i,\text{obs}} - \bar{Q}_{\text{obs}})^2}} \]  

\[
\text{(9)} \]

#### 3.3.3 Modified index of agreement (Dmod)

\[
D \text{mod} = 1 - \frac{\sum_{i=1}^{n} \left| Q_{\text{sim},i} - Q_{\text{obs},i} \right|}{\sum_{i=1}^{n} \left( \left| Q_{\text{sim},i} - Q_{\text{obs},i} \right| + \left| Q_{\text{obs},i} - \bar{Q}_{\text{obs}} \right| \right)} \]  

\[
\text{(10)} \]

#### 3.3.4 Relative-root-mean-square error (RRMSE)

\[
\text{RRMS} = \left[ \frac{1}{n - m} \sum_{i=1}^{n} \left( \frac{Q_{\text{obs},i} - Q_{\text{sim},i}}{Q_{\text{obs},i}} \right)^2 \right]^{1/2} \]  

\[
\text{(11)} \]

#### 3.3.5 Percent bias (PBIAS)

\[
\text{PBIAS} = \left[ \frac{\sum_{i=1}^{n} (Q_{i,\text{obs}} - Q_{i,\text{sim}})^2 * 100}{\sqrt{\sum_{i=1}^{n} (Q_{i,\text{obs}})^2}} \right] \]  

\[
\text{(12)} \]

#### 3.3.6 Probability plot correlation coefficient (PPCC) test

The PPCC \( (r) \) statistic is calculated as:
The unbiased plotting position formulas have the general formula [41].

\[
p_i = \frac{i - a}{n + 1 - 2a}
\]  

Where “a” varies from 0 to 0.5; \(p_i\) is the plotting probability and \(i\) is the rank in ordered observation with \(i = 1\) for the smallest observation in data sample.

In Equation 14; when \(a = 0.375\); Blom formula was used for LN2 distribution, when \(a = 0.44\), Gringorten formula was used for Gumbel (EVI) distribution; for \(a = 0.4\), Cunnane formula, was used for LP3 and GEV distributions. The unbiased plotting formula for PR3 are Cunnane and IN-NA and Ngugen formulas; [42]. Where \(C_s\) is coefficient of skewness, \(N\) is sample size and \(i\) is rank with \(i = 1\) indicating the smallest sample number.

### 3.3.7 Plotting position formulae

The unbiased plotting position formulae have the general formula [41].

\[
r = \frac{\sum_{m=1}^{N} (Q_{obs} - \overline{Q}_{obs})(q_m - \overline{q})}{\sum_{m=1}^{N} (Q_{obs} - \overline{Q}_{obs})^2 \sum_{i=1}^{N} (q_m - \overline{q})^2}
\]  

### 3.4 Flood Quantile Estimation

The estimation of flood quantiles and related equations for various life expectancies of civil engineering systems can be found in the following texts [26, 36, 37, 43] and [44]. The exceedance probability is \(P(Q>T) = 1/T\). The cumulative probability of non – exceedance is \(F(Q_T)\) is given by;

\[
F(Q_T) = P(Q_T < Q) = 1 - P(Q_T > Q) = (1-1/T)
\]  

Equation 15 is the basis for estimating the magnitude of a flood, \(Q_T\) given its exceedance or non – exceedance probabilities.

The quantile estimate for T years is calculated by substituting the value of \(F = (1-1/T)\) into the quantile functions in Table 2.

### 3.5 Uncertainty Assessment

The confidence interval specify the probability that the quantiles estimates lie within the upper and lower confidence interval coefficients; \(K_{U,T,\beta}\) and \(K_{L,T,\beta}\) using the non – central t distribution. Confidence limits are computed as follows;

\[
U_{T,\beta}(Q) = \overline{Q} + K_{U,T,\beta} * \sigma
\]

\[
L_{T,\beta}(Q) = \overline{Q} + K_{L,T,\beta} * \sigma
\]

Where \(\overline{Q}\), and \(\sigma\) are the log base – 10 mean and standard deviation, \(U_{T,\beta}(Q)\) and \(L_{T,\beta}(Q)\) are the upper and lower limits respectively. More details may be found in [26], p200.

### 4. RESULTS AND DISCUSSION

#### 4.1 Quantile Estimates

The estimated parameters for each distribution across the hydrological stations and the quantile relations expressed in the form of \(Q_T – T\) relationships are presented in Table 3. The quantile relations for the probability models were derived via unbiased position formulas given in subsection 3.3.7 using the cumulative probability of non – exceedance. The predictive performances of the probability distribution models were evaluated using the statistical performance evaluation criteria, stated in section 3.3. These indices are recommended standardized guidelines for judging model performance and comparing various models [45] and [32]. A ranking scheme was devised to rank the distributions based on their test values. Ranking scores are assigned to each distribution according to the optimal value of the statistical criteria. For example, the distribution with the lowest RRMSE, RSR or PBIAS values close to zero, and highest NSE, Dmod, and total accuracy of 1.0 is given a rank of 5. Accordingly, for each criterion, the overall ranks associated with each distribution is computed by summing the individual ranks obtained for each study station. The highest score implies the best – fit distribution. Using this ranking scheme, it was plausible to find the best distribution for each station [32]. Tables 4 and 5 show the ranking scores of probability distributions models for both Niger and Benue river basins.

Columns 9 and 10 of Table 4 present the PPCC calculated test statistics and critical test statistics, at the 5% significance level of probability distributions across the study stations. The critical test statistics were obtained from various approximating equations, see [36], pp. 299 -300).
Table 4 also shows that the 2-parameter distributions; LN2 and EV1 performed better than their 3-parameter counterpart; LP3, PR3, GEV. The decision on the 3-parameter distributions just satisfactory, as the PPCC calculated test statistics are very close to their critical values. Figs. 2 – 4 show the graphical plots of observed and simulated discharges of Lokoja station, while Figs. 5 and 6 show the corresponding plots for Umaisha station. Due to lack of space, only the plots for GEV, PR3 and LP3 for Lokoja and Umaisha stations are displayed. Figs. 9 – 15 show the 95% confidence interval for GEV, PR3 and LP3 distributions. The total ranked scores presented in Column 8 of both in Tables 4 and 5 are graphical displayed in Fig. 7 for Niger river basin and Fig. 8 for Benue river basin. Figs. 7 and 8 both indicate that GEV is best – fit – distribution, seconded by PR3 and thirdly LP3 for both Niger and Benue river basins.

Table 3. Distribution parameter and quantile relations

| Station | Distribution Parameters | Q_T – T Models |
|---------|-------------------------|----------------|
| Lokoja  | EV 1 146.77 3343.02 –  | Q_T = 14677.07 + 3343.02 Y_T |
|         | LN 2 9.69 0.26 –        | L_{NQT} = 9.69 + 0.26 Z_T |
|         | LP 3 9.749 0.0899 8.806 | Q_T = 9.68 + 0.281 K_T |
|         | PR 3 105.47 417.55 – 27431.25 | Q_T = 16606 + 4288.15 K_T |
|         | GEV 15211.62 4421.78 0.353 | Q_T = 15211.62 + 12909.76 \{1 – LN T – 170.353\} |
| Baro    | EV 1 4343.91 1272.03 –  | Q_T = 8434.91 + 1272.05 Y_T |
|         | LN 2 8.49 0.32 –        | L_{NQT} = 8.48 + 0.32 K_T |
|         | LP 3 2.64 0.24 7.85     | Q_T = 8.47 + 0.383 K_T |
|         | PR 3 152.35 132.18 – 15059.25 | Q_T = 5058.03 + 1631.48 K_T |
|         | GEV 4538.50 1674.12 0.3313 | Q_T = 4538.50 + 5052.68 \{1 – Ln T – 170.331\} |
| Idah    | EV 1 14480.30 3209.98 –  | Q_T = 14480.30 + 3209.98 Y_T |
|         | LN 2 9.67 0.25 –        | L_{NQT} = 9.67 + 0.25 Z_T |
|         | LP 3 6.31 0.110 8.97    | Q_T = 9.67 + 0.28 K_T |
|         | PR 3 132.82 357.23 – 31113.95 | Q_T = 16332.85 + 4116.94 K_T |
|         | GEV 14979.09 4231.93 0.335 | Q_T = 14979.09 + 12623.09 \\{1 – [ln(T – 1)]^{-0.335}\} |
| Onitsha | EV 1 15077.08 3126.02 –  | Q_T = 15077.08 + 3126.02 Y_T |
|         | LN 2 9.71 0.24 –        | L_{NQT} = 9.71 + 0.24 Z_T |
|         | LP 3 41.87 0.038 8.12   | Q_T = 9.71 + 0.25 K_T |
|         | PR 3 148.29 329.24 – 31940.62 | Q_T = 16881.17 + 4009.25 K_T |
|         | GEV 15364.91 3883.13 0.227 | Q_T = 15364.91 + 17147.41 \\{1 – [ln(T – 1)]^{-0.227}\} |
| Asamab | EV 1 14347.69 1812.51 –  | Q_T = 14347.69 + 1812.51 Y_T |
|         | LN 2 9.63 0.15 –        | L_{NQT} = 9.63 + 0.15 Z_T |
|         | LP 3 3.70 0.084 9.32    | Q_T = 9.63 + 0.16 K_T |
|         | PR 3 17.33 558.46 5717.31 | Q_T = 15393.73 + 2324.63 K_T |
|         | GEV 14755.66 5584.09 0.444 | Q_T = 14755.66 + 5584.09 \{1 – Ln T – 170.444\} |
4.2 Results

Table 4 shows the performance ranking of the five distributions across the hydrological stations. The best fit distribution is identified based on the total score obtained using the GOF tests. Column 8 of Table 4 and column 9 of Table 5 show the total scores for each distribution across the nine hydrological stations. On assessment of the total scores in Table 4 shows that GEV is best-fit distribution for Lokoja, Baro and Asamabiri stations with a total score of 25 each. LP3 is best-fit distribution in Idah and Onitsha stations scoring 19 and 20 respectively. In terms of PPCC Gof test, the 2-parameter distributions have larger margins between the calculated test statistics and critical values than the 3-parameter distributions. Consequently, the decision to reject the null hypothesis for EV1 and LN2 is unquestionable but just satisfactory for GEV. Table 5 shows that GEV is the best fit distribution for Umaisha, Makurdi and Yola hydrological stations with total scores of 29 and 27 respectively, while PR3 is best fit distribution for Ibi station (1 station). The results also show that no single probability distribution model emerged the best fit distribution for Idah and Onitsha (Total 2), and PR3 is best fit distribution for Ibi station (1 station). The results also show that no single probability distribution model emerged the best fit distribution for Idah and Onitsha (Total 2), and PR3 is best fit distribution for Ibi station (1 station). The results also show that no single probability distribution model emerged the best fit distribution for Idah and Onitsha (Total 2), and PR3 is best fit distribution for Ibi station (1 station). The results also show that no single probability distribution model emerged the best fit distribution for Idah and Onitsha (Total 2), and PR3 is best fit distribution for Ibi station (1 station).
| Station    | PDF  | dmod | RRMSE | NSE  | PBIAS | RSR  | Total Score | PPCC$_{crit}$ | PPCC$_{cal}$ | Decision |
|------------|------|------|-------|------|-------|------|-------------|---------------|--------------|----------|
| Lokoja     | EV1  | 2    | 2     | 2    | 2     | 2    | 10          | 0.9218        | 0.9826       | Reject   |
|            | LN2  | 3    | 3     | 3    | 3     | 3    | 15          | 0.9345        | 0.9965       | Reject   |
|            | LP3  | 1    | 1     | 1    | 1     | 1    | 5           | 0.9844        | 0.9910       | Ha       |
|            | PR3  | 4    | 4     | 4    | 4     | 4    | 20          | 0.9738        | 0.9880       | Ha       |
|            | GEV  | 5    | 5     | 5    | 5     | 5    | 25          | 0.9295        | 0.9815       | Reject   |
|            | EV1  | 1    | 1     | 2    | 1     | 1    | 6           | 0.9325        | 0.9995       | Reject   |
|            | LN2  | 2    | 2     | 3    | 2     | 2    | 11          | 0.9203        | 0.9901       | Reject   |
| Baro       | LP3  | 3    | 3     | 4    | 3     | 3    | 16          | 0.9899        | 0.9865       | Ha       |
|            | PR3  | 4    | 4     | 5    | 4     | 4    | 21          | 0.9900        | 0.9780       | Ha       |
|            | GEV  | 5    | 5     | 5    | 5     | 5    | 25          | 0.9245        | 0.9655       | Reject   |
|            | EV1  | 4    | 5     | 4    | 1     | 1    | 15          | 0.9552        | 0.9838       | Reject   |
|            | LN2  | 5    | 4     | 3    | 1     | 3    | 16          | 0.9680        | 0.9839       | Reject   |
| Idah       | LP3  | 3    | 3     | 5    | 3     | 5    | 19          | 0.9845        | 0.9841       | Ha       |
|            | PR3  | 1    | 1     | 4    | 5     | 4    | 15          | 0.9765        | 0.9839       | Ha       |
|            | GEV  | 2    | 2     | 2    | 4     | 2    | 12          | 0.9675        | 0.9900       | Reject   |
|            | EV1  | 5    | 5     | 4    | 1     | 4    | 19          | 0.9526        | 0.9924       | Reject   |
|            | LN2  | 5    | 4     | 3    | 2     | 3    | 17          | 0.9741        | 0.9925       | Ha       |
| Onitsha    | LP3  | 4    | 3     | 5    | 3     | 5    | 20          | 0.9736        | 0.9937       | Ha       |
|            | PR3  | 1    | 1     | 2    | 4     | 2    | 10          | 0.9748        | 0.9924       | Ha       |
|            | GEV  | 3    | 2     | 1    | 5     | 1    | 12          | 0.9720        | 0.9938       | Reject   |
|            | EV1  | 1    | 1     | 1    | 2     | 1    | 6           | 0.9658        | 0.9680       | Ha       |
|            | LN2  | 2    | 2     | 2    | 3     | 2    | 11          | 0.9613        | 0.9678       | Ha       |
| Asamabiri  | LP3  | 2    | 2     | 4    | 1     | 4    | 17          | 0.9735        | 0.9636       | Ha       |
|            | PR3  | 3    | 3     | 3    | 4     | 3    | 16          | 0.9672        | 0.9681       | Ha       |
|            | GEV  | 5    | 5     | 5    | 5     | 5    | 25          | 0.9462        | 0.9786       | Reject   |
Table 5. Evaluation of probability distribution models

| Station | PDF | Dmod | NSE | PBIAS | Tot. Accuracy | RSR | RRMSE | Total Score |
|---------|-----|------|-----|-------|---------------|-----|-------|-------------|
| Umaisha |     |      |     |       |               |     |       |             |
| EV1     | 1   | 2    | 2   | 1     | 1             | 2   | 2     | 9           |
| LN2     | 2   | 3    | 3   | 2     | 2             | 3   | 3     | 15          |
| LP3     | 3   | 3    | 1   | 3     | 3             | 4   | 4     | 17          |
| PR3     | 4   | 4    | 4   | 4     | 4             | 4   | 4     | 25          |
| GEV     | 5   | 5    | 4   | 5     | 5             | 5   | 5     | 29          |
| EV1     | 2   | 2    | 2   | 1     | 1             | 1   | 1     | 9           |
| LN2     | 4   | 4    | 4   | 4     | 5             | 5   | 5     | 26          |
| Makurdi |     |      |     |       |               |     |       |             |
| LP3     | 3   | 2    | 1   | 2     | 3             | 2   | 1     | 13          |
| PR3     | 4   | 4    | 3   | 3     | 4             | 3   | 2     | 21          |
| GEV     | 5   | 5    | 5   | 5     | 5             | 4   | 4     | 29          |
| EV1     | 1   | 1    | 3   | 1     | 1             | 1   | 1     | 8           |
| LN2     | 2   | 2    | 4   | 2     | 2             | 2   | 2     | 14          |
| Ibi     |     |      |     |       |               |     |       |             |
| LP3     | 3   | 3    | 1   | 3     | 3             | 3   | 3     | 16          |
| PR3     | 5   | 5    | 2   | 5     | 5             | 5   | 5     | 27          |
| GEV     | 4   | 4    | 5   | 4     | 4             | 4   | 4     | 25          |
| EV1     | 2   | 2    | 3   | 2     | 2             | 2   | 2     | 13          |
| LN2     | 1   | 1    | 1   | 1     | 1             | 1   | 1     | 6           |
| Yola    |     |      |     |       |               |     |       |             |
| LP3     | 5   | 3    | 2   | 4     | 4             | 5   | 5     | 23          |
| PR3     | 3   | 4    | 5   | 3     | 3             | 3   | 3     | 21          |
| GEV     | 4   | 5    | 4   | 5     | 5             | 4   | 4     | 27          |

Fig. 2. Observed and simulated discharge (lokoja)

Fig. 3. Observed and simulated discharge (lokoja)
Fig. 4. Observed and simulated discharge (lokoja)

Fig. 5. Observed and simulated discharges (Umaisha)

Fig. 6. Observed and simulated discharges (Umaisha)
Fig. 7. Total scores all distribution on river Niger

Fig. 8. Total scores all distribution on river Benue

Fig. 9. 95% confidence band for GEV at Lokoja Station
Fig. 10. 95% confidence band for GEV at Umaisha Station

Fig. 11. 95% confidence band for GEV at Baro Station

Fig. 12. 95% confidence band for GEV at Yola Station
Fig. 13. 95% confidence band for PR3 at Ibi Station

Fig. 14. 95% confidence band for LP3 at Idah Station

Fig. 15. 95% confidence band for LP3 at Onitsha

4.3 Discussion

The discussion is presented in the following order:

4.3.1 Comparison with previous studies in Nigeria

The hydrological stations evaluated have not been systematically studied. However, two previous studies worthy of mention in the Niger river basin, are [46], who conducted flood frequency analysis of Niger River at Shintaku and found LP3, the best fit distribution [47] conducted a similar study at Agenebode using LN2, LP3 and EV1 distributions and found LN2, the best – fit distribution. This study agrees with [46] who found LP3, the best – fit distribution for Shintaku gauging station. But disagrees with [47] who found LN2, the best – fit model for Agenebode.
4.3.2 Suitability of selected distributions

The best-fit distributions found in this study are GEV, PR3 and LP3 distributions. The choice of GEV distribution agrees with [48] who reported that GEV has a convincing relevance to the peak of floods, as most other probability distributions are not true depictions of flood peaks from the theoretical cause – effect standpoint. The GEV distribution is used as standard probability distribution model in 1 country. PR3 is a standard probability model in 7 countries while LP3 in 7 countries. The optimum distribution found in this study are in line with global practice for utility of probability distributions according to [22,16] and [11].

5. CONCLUSION

This paper presents the evaluation of probability distribution models of flood flow in Nigeria, comprising Niger and Benue river basins, EV1, LN2, LP3, PR3 and GEV. The performances of the five probability distribution models are compared using the GOF tests, namely Dmod, RRMSE, NSE, RSR and PPCC to identify the best-fit probability distribution model with MoM for parameter estimation. The study was conducted using AMS of five hydrological stations in Niger river basin and four hydrological stations in Benue river basin. The study found GEV is best – fit distribution for Lokoja, Baro, Asamabiri, Umaisha, Makurdi, and Yola (total 6), LP3 is best – fit distribution for Idah and Onitshe (total 2), and PR3 is best – fit distribution for Ibi station (1 station). The study recommends the development of a regional GEV, PR3 and LP3 distributions using the existing hydrological map of Nigeria which had demarcated the country into eight homogenous hydrological regions. The results of this study would be useful for at – site flood frequency analysis on other stations in the lower Niger river basin in Nigeria.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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