We make an extensive study of evolution of gravitational perturbations of D-dimensional black holes in Gauss-Bonnet theory. There is an instability at higher multi-poles $\ell$ and large Gauss-Bonnet coupling $\alpha$ for $D = 5, 6$, which is stabilized at higher $D$. Although small negative gap of the effective potential for scalar type of gravitational perturbations, exists for higher $D$ and whatever $\alpha$, it does not lead to any instability.

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\section{Introduction}

In recent years, higher dimensional black holes have been in the focus of high energy physics research. They are essential for our understanding of key moments of string theory, quantum gravity and brane-world scenarios. The most important classical property of black holes is evidently stability: unstable black holes simply cannot exist in our world. In addition, when considering higher dimensional black holes in the context of anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence, that is as a dual backgrounds rather than as real black holes, instability means a phase transition in the dual theory [1] being essential for understanding the field theory at finite temperature and high coupling. An opportunity of creating of mini black holes at particle collisions in Large Hadron Collider, according to Tev gravity extra dimensional scenarios, also gave a strong impetus to extensive studies of different properties of higher dimensional black holes, including quasinormal modes [3] and Hawking radiation [4].

The study of evolution of gravitational perturbations of the D-dimensional black holes in pure Einstein theory started from the work of Ishibashi and Kodama [2] who managed to reduce the cumbersome perturbation equations to the usual Schrodinger-like form. Yet the resulting effective potential of the wave equation is not always positive definite, so that stability is not taken for granted. In a few cases the technique of the so-called S-deformations, i.e. of deformations of the wave equation which does not touch stability properties, helped to transform a potential to a positive definite form, thereby proving the stability. In a general case, an extensive numerical investigation of evolution of gravitational perturbations allowed to prove stability for higher dimensional Reissner-Nordström-de Sitter black holes with arbitrary charge and A-term [5].

At the same time, the quantum gravity corrections to classical general relativity implies the existence of the so-called Gauss-Bonnet term in the dominant order correction, that is, the term in the Lagrangian squared in curvature. This term vanishes when $D = 4$. Therefore, the black holes in the Einstein-Gauss-Bonnet theory have attracted considerable interest recent years [6]. In particular, scalar field quasinormal modes of asymptotically flat Gauss-Bonnet black holes [9] and of asymptotically dS/AdS Gauss-Bonnet black holes were considered in [10]. The scalar quasinormal modes of Gauss-Bonnet black holes in the regime of high damping were considered in [11]. The scalar field, propagating in the background of a black hole, although gives the qualitative picture of evolution of perturbations, is not responsible for stability, so that perturbations of Einstein-Gauss-Bonnet equations must be considered instead [13], [14]. Thus, Dotti and Gleiser reduced the Einstein-Gauss-Bonnet perturbed equations to a wave like form with some effective potentials. They found an instability for the two particular cases: scalar type of gravitational perturbations for $D = 5$, and tensor type of gravitational perturbations for $D = 6$, leaving the analysis of stability of general $D$ open. The problem was that analytical treatment of stability is difficult in many cases: even if we have a wave like equation with a potential (extremely cumbersome in the Gauss-Bonnet case), negative gaps in potentials cannot be easily removed by the S-deformations, because one needs to know an ansatz that transforms the potential to a positive definite one.

The aim of our work is to perform a complete numerical analysis of the evolution of gravitational perturbations for D-dimensional Gauss-Bonnet black holes with $D = 5 - 11$, what is motivated by string theory and quantum gravity, and to determine the stability and instability regions for these black holes.

The paper is organized as follows: Sec. II introduces the metric and wave like equations for Gauss-Bonnet (GB) black holes. Sec. III describes the methods of time domain integration used here and shows all numer-
II. THE PERTURBATION WAVE EQUATIONS FOR GAUSS-BONNET BLACK HOLES

The Lagrangian of the Einstein-Gauss-Bonnet action is

$$I = \frac{1}{16\pi G_D} \int d^Dx \sqrt{-g}R + \alpha' \int d^Dx \sqrt{-g} (R_{abcd} R^{abcd} - 4 R_{cd} R^{cd} + R^2).$$

(1)

Here $\alpha'$ is a positive coupling constant.

The metric has the form,

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2 d\Omega_{D-2}^2,$$

(2)

$$f(r) = 1 + \frac{r^2}{\alpha(D-3)(D-4)} (1 - q(r)), \quad q(r) = \sqrt{1 + \frac{4\alpha(D-3)(D-4)\mu}{(D-2)r^{D-4}}},$$

where $\alpha = 16\pi G_D \alpha'$.

In order to measure all the quantities in terms of the black hole horizon $r_0$ radius we parameterize the black hole mass as

$$\mu = \frac{(D-2)r_0^{D-3}}{4} \left( 2 + \frac{\alpha(D-3)(D-4)}{r_0^2} \right).$$

(3)

As was shown in [13], [14], the gravitational perturbations of a Gauss-Bonnet black hole can be decoupled from their angular part and reduced to the wave-like equation of the form

$$\left( \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r^2} + V(r) \right) \Psi(t, r) = 0, \quad dr_* = \frac{dr}{f(r)}.$$  

(4)

with the effective potentials which have very cumbersome form [13], [14]. After some algebra, we managed to simplify the potentials for the tensor, vector and scalar types of the gravitational perturbation respectively:

$$V_t(r) = f(r) \frac{\chi}{r^2} \left( 3 - \frac{B(r)}{A(r)} \right) + \frac{1}{\sqrt{r^{D-2}A(r)q(r)}} \frac{d^2}{dr_*^2} \sqrt{r^{D-2}A(r)q(r)},$$

(5)

$$V_v(r) = f(r) \frac{(D-2)c}{r^2} A(r) + \sqrt{r^{D-2}A(r)q(r)} \frac{d^2}{dr_*^2} \frac{1}{\sqrt{r^{D-2}A(r)q(r)}},$$

(6)

$$V_s(r) = \frac{f(r)U(r)}{64r^2(D-3)^2A(r)^2q(r)^8(4c q(r) + (D-1)R(q(r)^2 - 1))^2}.$$  

(7)

We used the following dimensionless quantities

$$A(r) = \frac{1}{q(r)^2} \left( \frac{1}{2} + \frac{1}{D-3} \right) + \left( \frac{1}{2} - \frac{1}{D-3} \right),$$

$$B(r) = A(r)^2 \left( 1 + \frac{1}{D-4} \right) + \left( 1 - \frac{1}{D-4} \right),$$

$$R = \frac{r^2}{\alpha(D-3)(D-4)}.$$
We study the ringing of GB black hole using a numerical characteristic integration method [15], that uses the light-cone variables \( u = t - r_* \) and \( v = t + r_* \). In the characteristic initial value problem, initial data are specified on the two null surfaces \( u = u_0 \) and \( v = v_0 \). The discretization scheme we used, is

\[
\Psi(N) = \Psi(W) + \Psi(E) - \Psi(S) - \Delta^2 \frac{V(W)\Psi(W) + V(E)\Psi(E)}{8} + O(\Delta^4),
\]

where we have used the following definitions for the points: \( N = (u + \Delta, v + \Delta) \), \( W = (u + \Delta, v) \), \( E = (u, v + \Delta) \) and \( S = (u, v) \).

To see the correct time-domain profile at late time we need to calculate precisely the values of the effective potential which are used in (8). In order to do this we must integrate numerically the equation for the tortoise coordinate and then solve it with respect to the radial coordinate with high accuracy (we worked with a precision of \( \sim 2^{-90} \)). We used simple Runge-Kutta method for the integration. Since it interpolates the function by a cubic spline at each step we are able to find analytically \( r(r_*) \) at each step. The final C++ program that finds the time-domain profiles with arbitrary precision is available from the last author upon request.

Let us note that for \( D > 4 \) black holes in ordinary Einstein gravity the tensor type of gravitational perturbations is governed by the same wave equation as test scalar field. Therefore both types of perturbations produce the same quasinormal mode spectrum, that is they are isospectral. This coincidence is remarkable, but does not take place for Gauss-Bonnet black holes. Indeed, if we compare the tensor quasi-normal modes in Table I in this paper with test scalar field quasinormal modes of [8, 10], we can see that quasinormal modes as well as effective potentials are quite different.
The points $\ell = 16, 20, 32, 40, 50, 64$ were fit by the line $\alpha = 2.627\ell^{-1} + 1.005$. The theoretical result is $\alpha_t \approx 1.006$.

The points $\ell = 16, 20, 32, 40, 50$ were fit by the line $\alpha = 0.517\ell^{-1} + 0.209$. The theoretical result is $\alpha_t \approx 0.207$.

### TABLE I: Fundamental QNMs for GB black hole perturbation of tensor type ($\ell = 2$). $\alpha$ and frequencies are measured in units of the horizon radius.

| $\ell$ | $D = 5$       | $D = 6$       | $D = 7$       | $D = 8$       | $D = 9$       | $D = 10$      | $D = 11$      |
|--------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 8      | 1.5898 - 0.3406i | 2.0112 - 0.4521i | 2.3697 - 0.5541i | 2.7006 - 0.6492i | 3.0212 - 0.7342i | 3.3386 - 0.8092i | 3.6565 - 0.8753i |
| 10     | 1.6435 - 0.3289i | 1.9557 - 0.4269i | 2.2166 - 0.5246i | 2.4761 - 0.6105i | 2.7428 - 0.6827i | 3.0167 - 0.7429i | 3.2970 - 0.7936i |
| 12     | 1.6766 - 0.3157i | 1.8881 - 0.4086i | 2.0901 - 0.5034i | 2.3143 - 0.5820i | 2.5541 - 0.6456i | 2.8045 - 0.6958i | 3.0633 - 0.7340i |
| 16     | 1.6946 - 0.3014i | 1.8217 - 0.3942i | 1.9874 - 0.4862i | 2.1907 - 0.5599i | 2.4126 - 0.6175i | 2.6448 - 0.6581i | 2.8886 - 0.6816i |
| 20     | 1.7017 - 0.2870i | 1.7603 - 0.3823i | 1.9024 - 0.4718i | 2.0919 - 0.5423i | 2.2990 - 0.5953i | 2.5154 - 0.6242i | 2.7514 - 0.6299i |
| 24     | 1.7012 - 0.2731i | 1.7046 - 0.3720i | 1.8308 - 0.4594i | 2.0100 - 0.5280i | 2.2031 - 0.5771i | 2.4066 - 0.5902i | 2.6438 - 0.5788i |
| 32     | 1.6956 - 0.2600i | 1.6541 - 0.3630i | 1.7693 - 0.4486i | 1.9403 - 0.5163i | 2.1190 - 0.5612i | 2.3149 - 0.5544i | 2.5596 - 0.5312i |
| 40     | 1.6863 - 0.2478i | 1.6083 - 0.3549i | 1.7158 - 0.4390i | 1.8799 - 0.5068i | 2.0427 - 0.5461i | 2.2389 - 0.5176i | 2.4927 - 0.4894i |
| 50     | 1.6745 - 0.2366i | 1.5667 - 0.3476i | 1.6685 - 0.4303i | 1.8265 - 0.4991i | 1.9716 - 0.5299i | 2.1767 - 0.4821i | 2.4378 - 0.4532i |
| 64     | 1.6611 - 0.2262i | 1.5286 - 0.3409i | 1.6264 - 0.4225i | 1.7790 - 0.4931i | 1.9054 - 0.5095i | 2.1255 - 0.4497i | 2.3915 - 0.4219i |
Here we shall consider \( \omega = \omega_{Re} - i \omega_{Im} \), and the \( \omega \) is chosen so that positive \( \omega_{Im} \) corresponds to a damped mode. On the tables I–III one can see fundamental quasinormal modes obtained by time-domain integration. We see that imaginary part of \( \omega \), which is proportional to the damping rate, is always decreasing, when \( \alpha \) is increasing, for all three types of gravitational perturbations and all \( D \). In other words the less \( D \) and the stronger the Gauss-Bonnet coupling \( \alpha \), the slower decay of the perturbations is. In contrast to the imaginary part, the real oscillation frequency \( \omega_{Re} \) does not behave uniform: \( \omega_{Re} \) decreases as \( \alpha \) grows for most cases of tensor and vector modes. The behavior of scalar mode is different: here we have two competing for the domination modes (see Fig. 6, for \( D = 10 \)) at different stages of the quasinormal ringing. Thus for \( D = 7 \), for example, at some values of \( \alpha \) the two modes with close imaginary parts appear, what makes the picture of time evolution more complicated. Then, at some larger \( \alpha \) the dominance takes another mode. This superposition of modes, also with competing excitation coefficients of the particular modes, makes dependence of the fundamental scalar type QNMs on \( \alpha \) and \( D \) non-monotonic.

To check that our computation scheme is working properly and gives no numerical error, we repeated the integration with much smaller step and higher accuracy of all incoming data. The picture of the evolution does not change, what means high stability and accuracy of the integration. Another check may be going to particular known limits, such as pure Schwarzschild case \( \alpha = 0 \). Then we must get pure Schwarzschild QNMs in that limit. Again the dominance of the two modes at different stages of the ringing complicate the picture. Indeed, for instance for

\[
D = 10 \text{ and } \alpha = 0 \quad \text{from } \mathbb{R} \quad \text{we have } \omega = 2.45 - 0.98 i,
\]

what is not a fundamental mode for the whole stage of ringing but rather for the first period. Indeed, we can see the approaching of our GB QNMs to the pure Schwarzschild ones in the following data:

| \( \alpha \) | \( \omega_0 \) | \( \omega_1 \) |
|-----|-----|-----|
| 0.00001 | 1.2347 - 0.9328 i | 2.4576 - 0.9873 i |
| 0.001 | 1.2298 - 0.93 i | 2.4580 - 0.9867 i |
| 0.01 | 1.1834 - 0.9116 i | 2.4615 - 0.9807 i |
| 0.1 | 0.8960 - 0.7825 i | 2.4387 - 0.9336 i |

Therefore we can conclude that there is a kind of conceptual gap of what one can consider as a fundamental mode of the quasinormal ringing. Probably, the better choice would be to consider the last stage of ringing, immediately before the tail stage as the one where the fundamental quasinormal modes must be defined.

Another essential and distinctive feature of the Gauss-Bonnet quasinormal ringing is that instability occurs at higher multipole numbers \( \ell \), while lowest \( \ell \) are stable! This is indeed remarkable as, naïvely, one would expect that if the lowest multipole is stable, then higher multipoles just raise up the pick of the potential barrier, so that higher multipoles should stabilize the potential. Yet,
FIG. 3: Potential and profile for GB black hole perturbation of scalar type \((D = 6, \ell = 2, \alpha = 0.3)\). The negative gap does not lead to instability. It causes exponentially damping “tails” to appear just after the initial outburst. Therefore we are unable to see QN ringing.

FIG. 4: Time-domain profiles for the “region of the irregular QN-ringing” of the gravitational perturbation of scalar type \(D = 6, \ell = 2, \alpha = 0.15, 0.20, 0.25, 0.30, 0.35, 0.40\) (plots from left to right). For \(\alpha = 0.15\) (first plot) we see usual decaying oscillations. For \(\alpha = 0.20\) two concurrent modes with the same damping rate. As \(\alpha\) increases we observe exponentially damping tails, that do not oscillate. At higher \(\alpha\) we see the oscillation behavior again, but the frequency of oscillation for \(\alpha = 0.40\) (last plot) differs significantly from that for \(\alpha = 0.15\).

for Gauss-Bonnet black holes, higher multipoles also increase the negative gap near the black hole horizon \([13], [14]\), allowing existence of bound states in the gap. This instability at higher multipoles seems to be intrinsic to Gauss-Bonnet theories, because similar instability was found some time ago in \([17]\) for Gauss-Bonnet cosmologies.

One can see the evolution of instability in time domain in Fig. 5. The larger \(\ell\), at the earlier times instability growth occurs, and the stronger the growth rate is (Fig. 5). In addition, the higher \(\ell\), the smaller threshold value \(\alpha\) at which instability happens. At \(\alpha\) smaller than the critical value, black holes are stable. Therefore it is very important not to be limited by small \(\ell\) but to see the regime high multipoles, in order to determine the threshold \(\alpha\) with good accuracy. Analytical estimations of \([14]\) are compared here with our numerical results in Fig.1, 2. There one can see that the estimation of \([14]\), which gives an expression for the threshold value \(\alpha \approx A\ell^{-1}+B\), valid for not very large \(\ell\), while at larger \(\ell\) instability occurs at smaller \(\alpha\). This decreases the minimal value of \(\alpha\), when Gauss-Bonnet black holes become unstable. Essential point is that the above described instability exists only for \(D = 5\) and 6, while at higher \(D\) the black holes are stable.

It should be explained here that in Table III we have two values of \(\alpha\) which do not correspond to instabilities, yet do not have any definite quasinormal modes. The point is that for scalar type of gravitational perturbations, in addition to the negative gap which may deepen
and then some other mode start dominating. We stress that this rather odd picture, in no way can be suspected as an instability, because this “transition” behavior happen also for some parameters, for test scalar field, when stability can be proved analytically. This kind of negative gap at \( D > 5 \), is similar to that for the Reissner-Nordström black holes, which is not deep enough and do not produce the instability.

IV. DISCUSSION

In this paper the numerical integration in time domain was done for the gravitational perturbations of black holes in \( D = 5 - 11 \) Gauss-Bonnet theories. The instability occurs only for \( D = 5 \) and \( D = 6 \) cases at some large values of \( \alpha \). Higher \( D \) stabilize the perturbations. The instability starts after some period of quasinormal ringing and at earlier time for larger multipoles \( \ell \). Apparently the instability at large \( \alpha \) is a physically expected result: Gauss-Bonnet black holes is inspired by a one-loop string theory approximation, so that GB theory is valid only as soon as \( \alpha \) is small enough. Otherwise one needs to take into consideration higher order corrections.

It is interesting to generalize the present work to the case of charged Gauss-Bonnet black holes and asymptotically de-Sitter black holes, because we know that \( \Lambda \)-term and the black hole charge give also negative gap to the effective potential, so that if taking into account all these factors, \( \alpha \)-coupling, and \( \Lambda \) and or charge, the parameters range of instability might be increased.

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