Field calculation of electric dipole excitation of a metafilm printed on a finite-thickness dielectric slab using the susceptibility homogenization method

A. Azari | Z.H. Firouzeh | A. Bakhtafruz

Abstract

Here, the authors calculate the fields due to the electric dipole that excites a homogenized metasurface which is printed on a finite thickness dielectric slab. The problem is solved by replacing the total parts below the periodic array which contain the dielectric slab and the half-space under it, with an effective homogenous half-space. Furthermore, the static interaction constants between dipoles which are located at two different materials’ interface are calculated. Then, with the help of the Lorentz method, the analytical formulae are presented to calculate the surface susceptibility densities of the homogenized metasurface located at two materials’ interface. These analytical formulae can be applied to arrays with both square and rectangular periodicity. An array of circular patches printed on dielectric slabs with different thicknesses is considered to verify the results. The results show good agreement compared to the fields calculated by commercial numerical software FEKO.

1 | INTRODUCTION

Metamaterials with simultaneous negative constitutive parameters were first introduced by Veselago [1] and realized by Pendry [2] and Smith et al. [3]. Because of the special features of these synthetic materials, they have attracted many researchers’ attention. A three-dimensional arrangement of inclusions are embedded in a host medium in a way that creates effective negative constitutive parameters is known as backward wave (BW) medium, double-negative (DNG) metamaterial, left-handed medium, and a few other names [4].

There are lots of methods to construct metamaterials. The most popular method is based on the natural crystal structure in which the polarizable particles are arranged in a periodic three-dimensional lattice suspended in free space or embedded in a host medium. If the dimensions of the particles and the array period are too small compared to the operating wavelength, the lattice can be seen as an effective homogenous medium with its equivalent constitutive parameters [5]. If the electric and magnetic polarizabilities of the individual particles are known, the equivalent parameters of the homogenous medium can be obtained by the Lorentz theory using the interaction constants between the array elements [6]. The electric and magnetic polarizabilities of some simple shapes such as magneto-dielectric spheres [7], [8], square metallic patches [9], circular metallic patches, elliptic disks, metallic spheres [6], and also, some complex shapes such as metallic ellipsoid, torus, and cross [10], [11] are available in the literature. If the polarizabilities of the individual structural components are not known, they can be retrieved from the scattering parameters, which are obtained with the help of simulation or measurement [5].

An artificial material sheet whose thickness and periodicity are electrically small with respect to the operating wavelength is called metasurface. In many applications, metasurfaces are preferable to bulk metamaterials because of their properties such as light weight, ease of fabrication, and low loss features. Efforts have not been successful in model metasurfaces with effective constitutive parameters similar to bulk metamaterials. Nevertheless, surface susceptibility densities can well model the metasurfaces [12]. There are two classes of metasurfaces: metafilms with isolated scatterers which have cermet topology
and metascreens which have fishnet topology [13]. Note that the authors focus only on the metafilm here.

In many electromagnetic problems, it is necessary to calculate the radiation fields due to finite electric sources that radiate in the vicinity of the periodic surface. This can be solved using one of the electric field integral equation or mixed potential integral equation (MPIE), if the electric field or potential Green’s functions are known, respectively. Although the array scanning method [14-18] is the most accurate way to calculate the potential Green’s functions for periodic surfaces located in multilayer media, it is very numerically demanding. If the periodic surface has periodicity much less than the wavelength in the surrounding medium, the surface susceptibility homogenization method can lead to a rapid and almost accurate way to analyse this problem [19], [20].

Dyadic electric field Green’s function for a metafilm that is suspended in free space, homogenized by its surface electric and magnetic susceptibilities, and excited by the electric dipole was obtained by Liang et al. [21]. Also, the potential Green’s functions for a dipole-excited planar metafilm which is located in free space were derived in [22] and [23], and then generalized for a planar metafilm printed at two half-space interface in [24]. However, in practical applications, metafilm is located on a finite thickness dielectric layer such as a dielectric slab or a microstrip structure, not on the dielectric half-space. The authors attempt to solve this practical problem by replacing all the parts below the periodic array (the dielectric slab and the half-space under it), with an effective homogenous half-space.

The most important issue in this homogenization method is the correct calculation of the surface susceptibility densities. The analytical and retrieval formulae to calculate these parameters for a metafilm located in free space were presented in [20] and [12], respectively. The electric and magnetic polarizabilities of a scatterer located at two different materials interface differ from those of a scatterer located in free space due to the presence of the induced bounded charges in the dielectric [25], which also result in different surface susceptibility densities. Just like bulk metamaterials, the surface susceptibilities of a two-dimensional periodic array can be obtained using the Lorentz method, if the interaction constants between elements are known. The static and dynamic interaction constants of a two-dimensional array which is located in free space were calculated in [6] and [26], respectively. However, as far as we know, the interaction constant between elements of a two-dimensional array which is located on two materials interface has not been calculated in the articles. Therefore, the authors also calculate the interaction constants corresponding to such an array elements. To achieve this aim, the static electric field and electric displacement density due to the electric dipole embedded at the interface of the two dielectrics obtained in [25] are employed.

The work of authors is organized as follows. After the introduction, the generalized sheet transition conditions (GSTCs) and Hertzian vector and scalar potential Green’s functions for a printed metafilm on a dielectric half-space are reviewed in Section 2. Then, the effective half-space in the finite thickness dielectric slab problem is characterized. In Section 3, the analytical formulae to calculate the surface susceptibility dyads corresponding to a metafilm which is located at two different materials interface are derived using the Lorentz method and using the interaction constants between the array elements. Numerical results are presented in Section 4. Finally, some remarks and conclusions are explained in Section 5.

2      THEORY

2.1      Generalized sheet transition conditions

A two-dimensional array made of isolated scatterers can be replaced by an effective surface with corresponding electric and magnetic surface susceptibility densities if the size of the particles and their periods are much smaller than the operating wavelength (λ/10 or less) [12]. Assuming the general form of anisotropic surface susceptibility densities, the discontinuity in the fields arising from them can be characterized by the generalized sheet transition conditions introduced by Holloway and Kuster [27] as:

\[
\vec{z} \times \vec{H} \big|_{z=0^+} = j \omega \varepsilon_0 \left( \vec{\chi}_{ES} \vec{E}_{av} \right)_t - \vec{z} \times \nabla \left( \vec{z} \cdot \vec{\chi}_{MS} \vec{H}_{av} \right) \quad (1)
\]

\[
\vec{z} \times \vec{E} \big|_{z=0^+} = - j \omega \mu_0 \left( \vec{\chi}_{MS} \vec{H}_{av} \right)_t - \vec{z} \times \nabla \left( \vec{z} \cdot \vec{\chi}_{ES} \vec{E}_{av} \right) \quad (2)
\]

where

\[
\vec{E}_{av} = \left( \vec{x} E_{av,x} + \vec{y} E_{av,y} + \frac{\vec{D}_{av,z}}{\varepsilon_0} \right)_{z=0}
\]

\[
\vec{H}_{av} = \left( \vec{x} H_{av,x} + \vec{y} H_{av,y} + \frac{\vec{B}_{av,z}}{\mu_0} \right)_{z=0}
\]

with the average field defined as:

\[
\vec{E}_{av} = \frac{1}{2} \left( \vec{E}^{0+} - \vec{E}^{0-} \right)
\]

and similar for the average of the \( \vec{D}, \vec{H}, \) and \( \vec{B} \) fields.

Note that the general electric and magnetic surface susceptibility dyads are:

\[
\vec{\chi}_{ES} = \chi^{xx}_{ES} \vec{a}_x \vec{a}_x + \chi^{yx}_{ES} \vec{a}_x \vec{a}_y + \chi^{zx}_{ES} \vec{a}_z \vec{a}_x + \chi^{zx}_{ES} \vec{a}_z \vec{a}_x + \chi^{yx}_{ES} \vec{a}_y \vec{a}_x + \chi^{yz}_{ES} \vec{a}_y \vec{a}_z + \chi^{zx}_{ES} \vec{a}_z \vec{a}_x + \chi^{xy}_{ES} \vec{a}_x \vec{a}_y
\]

\[
\vec{\chi}_{MS} = \chi^{xx}_{MS} \vec{a}_x \vec{a}_x + \chi^{yx}_{MS} \vec{a}_x \vec{a}_y + \chi^{zx}_{MS} \vec{a}_z \vec{a}_x + \chi^{zx}_{MS} \vec{a}_z \vec{a}_x + \chi^{yx}_{MS} \vec{a}_y \vec{a}_x + \chi^{yz}_{MS} \vec{a}_y \vec{a}_z + \chi^{zx}_{MS} \vec{a}_z \vec{a}_x + \chi^{xy}_{MS} \vec{a}_x \vec{a}_y
\]
Note that the authors focus on metafilms whose shape and arrangement of the structural components are in such a way that their surface susceptibility dyads are diagonal.

2.2 | Potential Green's functions

In many applications, there is a need to calculate the fields due to finite sources that illuminate the metafilm. The most common method is to use the integral equation technique if Green's functions are known. In practical applications, metafilm is often printed on a dielectric layer. The MPIE which uses both the magnetic vector and electric scalar potential Green's functions is a suitable integral equation form to solve such a problem in multilayered media. The Hertzian magnetic vector potential Green's function for a printed metafilm which is located at two dielectric half-spaces interface was presented in [24] for vertical electric dipole (VED) illumination as:

$$\tilde{\pi} = \begin{cases} 
\left( \frac{e^{-j_0 z}|z|_1}{2j_{k_s}(j_0e_1)} + A_{xz} e^{-j_0 z} \right) \hat{a}_z & z > 0 \\
B_{xz} e^{jk_s z} \hat{a}_z & z < 0 
\end{cases} \tag{8}$$

with coefficients:

$$A_{xz} = \frac{j_0 e_{1z} k_{1z} - e_{1z} k_{1z} + e_{2z} k_{2z}}{j_0 e_{1z} k_{1z} + e_{1z} k_{1z} + e_{2z} k_{2z}} \tag{9}$$

$$B_{xz} = \frac{2e_{1z} k_{1z}}{j_0 e_{1z} k_{1z} + e_{1z} k_{1z} + e_{2z} k_{2z}} \tag{10}$$

and for x-directed horizontal electric dipole (HED) illumination as:

$$\tilde{\pi} = \begin{cases} 
\left( \frac{e^{-j_0 z}|z|_2}{2j_{k_s}(j_0e_1)} + A_{xx} e^{-j_0 z} \right) \hat{a}_x & z > 0 \\
B_{xx} e^{jk_s z} \hat{a}_x + B_{xz} e^{j_0 z} \hat{a}_z & z < 0 
\end{cases} \tag{11}$$

with coefficients:

$$A_{xx} = -\frac{\chi_{xx} e_1 k_1^2 k_2^2 + \chi_{MS} e_1 k_1^2 k_2^2 + \chi_{xx} e_1 k_1^2 k_2^2 - j e_1 k_1^2 k_{2z}}{\chi_{xx} e_1 k_1^2 k_2^2 + \chi_{MS} e_1 k_1^2 k_2^2 + \chi_{xx} e_1 k_1^2 k_2^2 - j e_1 k_1^2 k_{2z}} \hat{a}_x \quad 0$$

$$A_{zz} = -\frac{\chi_{zz} e_1 k_1^2 k_2^2 + \chi_{MS} e_1 k_1^2 k_2^2 + \chi_{zz} e_1 k_1^2 k_2^2 - j e_1 k_1^2 k_{2z}}{\chi_{xx} e_1 k_1^2 k_2^2 + \chi_{MS} e_1 k_1^2 k_2^2 + \chi_{xx} e_1 k_1^2 k_2^2 - j e_1 k_1^2 k_{2z}} \hat{a}_z \quad 0$$

$$B_{xx} = \frac{-2\mu_1 k_{2z}}{\chi_{xx} e_1 k_{2z} + k_{2z}^2 - j e_1 k_{2z} - j e_2 k_{2z}} \tag{13}$$

$$B_{xx} = \frac{-2\mu_1 k_{2z}}{\chi_{xx} e_1 k_{2z} + k_{2z}^2 - j e_1 k_{2z} - j e_2 k_{2z}} \tag{14}$$

$$B_{zz} = 2\frac{j_0 e_{1z} k_{1z}}{j_0 e_{1z} k_{1z} + e_{1z} k_{1z} + e_{2z} k_{2z}} \tag{15}$$

$$\nu_s = \frac{e^{-j_0 z} z}{2j_{k_s}(j_0e_1)}, \quad s = x, y, z \tag{16}$$

Equations (11)–(15) are valid for y-directed HED, if $k_s$ and $\nu_s$ are replaced by $k_y$ and $\nu_y$, respectively.

Also, the longitudinal and transverse scalar electric potentials were derived in [24], respectively as:

$$G^e_V = \begin{cases} 
\frac{\omega}{\mu_1 k_{2z}} \left( \frac{e^{-j_0 z}|z|_2}{2j_{k_s}(j_0e_1)} + A_{xx} e^{-j_0 z} \right) & z > 0 \\
\frac{\omega}{\mu_2 B_{xx} e^{j_0 z}} & z < 0 
\end{cases} \tag{17}$$

with coefficients which are explained in (9) and (10), and

$$G^e_V = \begin{cases} 
\frac{\omega}{\mu_1 k_{2z}} \left( \frac{e^{-j_0 z}|z|_2}{2j_{k_s}(j_0e_1)} + A_{xx} e^{-j_0 z} \right) & z > 0 \\
\frac{\omega}{\mu_2 B_{xx} e^{j_0 z}} \hat{a}_x + \frac{k_{2z}}{k_{2z}} B_{xx} e^{j_0 z} & z < 0 
\end{cases} \tag{18}$$

with coefficients of (12–15).

The Hertzian magnetic vector potentials in (8) and (11) can be used in (19) and (20) to calculate the fields due to VED and HED which incident into the metafilm.

$$\vec{E} = (k^2 + \nabla \left( k^2 + \nabla \right) \Pi) \tag{19}$$

$$\vec{H} = j_0 e \nabla \times \Pi \tag{20}$$

The formulae (8)–(18) can all be used in the MPIE formulation to calculate the fields due to any arbitrary finite source which is located in the vicinity of a planar metafilm printed on a dielectric half-space. As stated before, in practical applications, the metafilm is usually printed on a finite thickness dielectric slab. As shown in Figure 1, the total parts under
the periodic array, which contain the dielectric slab and the half-space under it, can be replaced by a homogenous half-space with effective relative dielectric constant $\varepsilon_{r,\text{eff}}$. Therefore, Green’s function formulae (8)–(18) can also be used in this case by replacing $\varepsilon_2 = \varepsilon_{r,\text{eff}}$.

2.3 Effective half-space characteristics

A rectangular array of scatterers which is located on a finite thickness dielectric slab is shown in Figure 1(a). As depicted in Figure 1(b), this structure can be replaced by a rectangular array of scatterers located on an infinite half-space with effective relative dielectric constant $\varepsilon_{r,\text{eff}}$.

The transmission line theory that can be used to calculate the effective dielectric constant is explained as follows [28]. The input impedance $Z_{in}$ which is seen from the array surface is equal to:

$$Z_{in} = Z_d \frac{\eta_0 + jZ_d \tan \beta b}{Z_d + \eta_0 \tan \beta b}$$

where $Z_d$ and $b$ are the characteristic impedance and thickness of the dielectric slab, respectively, $\eta_0$ is the characteristic impedance of the free space (under the dielectric slab) and $\beta$ is the wavenumber in the dielectric slab. Once the input impedance is determined, the effective relative dielectric constant is equal to:

$$\varepsilon_{r,\text{eff}} = \left( \frac{\eta_0}{Z_{in}} \right)^2$$

3 SURFACE SUSCEPTIBILITY DENSITY

The most important issue in the susceptibility homogenization method is the correct calculation of the electric and magnetic surface susceptibility densities. If the electric and magnetic polarizabilities of the individual scatterers and the interaction constant between dipole elements are known, the surface susceptibility densities of the periodic surface can be obtained by the Lorentz method [6].

For a scatterer that is totally or partially embedded in a material as shown in Figure 2, induced dipole moments due to the bounded charges and currents in the material can affect the polarizabilities of the element. If the scatterer is embedded symmetrically in two materials, the electric and magnetic polarizabilities change, respectively as [25]:

$$\alpha_{E,t} = \frac{\varepsilon_t}{\varepsilon_0} \alpha_{E,t,0} \alpha_{E,z} = \frac{\varepsilon_z}{\varepsilon_0} \alpha_{E,z,0}$$

$$\alpha_{M,t} = \frac{\mu_t}{\mu_0} \alpha_{M,t,0} \alpha_{M,z} = \frac{\mu_z}{\mu_0} \alpha_{M,z,0}$$

Indices 1 and 2 refer to the upper and lower materials, respectively.

Each scatterer in a rectangular periodic array which is located at two half-space interface can be replaced by its equivalent electric and magnetic dipole moments as shown in Figure 3(b). The intensity of each induced electric dipole moment in the array is related to the effective polarizing electric field $\vec{E}_e$ as:
FIGURE 3 (a) An array of isolated scatterers located at two materials’ interface and (b) scatterers replaced by their induced electric and magnetic dipole moments

\[ \vec{p} = e_0 \alpha \times \vec{E}_e \]  \hspace{1cm} (29)

where \( \vec{E}_e \) is equal to the external applied field \( E_0 \) plus the interaction field \( E_i \). The interaction field is the total electric field at the centre of the considered element due to whole induced dipoles in the array except the intended dipole and for \( y \)-directed induced dipoles, is equal to:

\[ \vec{E}_y = \hat{a}_y \frac{C_{e,yy} P_y}{E_0} \]  \hspace{1cm} (30)

where \( C \) is the interaction constant. According to the Lorentz theory, the equivalent electric and magnetic surface susceptibilities corresponding to an array of isolated particles are equal to [6]:

\[ \chi_{E,i} = \left( \frac{N \alpha_{E,ii}}{1 - \alpha_{E,ii} C_{e,ii}} \right) \frac{Na_E,ii}{\rho_{nm} \rho_{mn}} \]  \hspace{1cm} (31)

\[ \chi_{M,i} = \left( \frac{N \alpha_{M,ii}}{1 + \alpha_{M,ii} C_{m,ii}} \right) \frac{Na_M,ii}{\rho_{nm} \rho_{mn}} \]  \hspace{1cm} (32)

where \( N \) is the number of elements in the unit area \( (N = 1/ab) \). Since the polarizabilities of the particles may be different in various directions, the surface susceptibility densities are different in each direction; means that the effective surface shows degrees of anisotropy.

3.1 Interaction constant

The electrostatic displacement density and the electrostatic field due to one vertical and one horizontal electric dipoles which are located at two half-spaces interface have been given in (46) and (48) of the appendix, respectively. To calculate the interaction constant between the array elements, the fields due to whole array elements except one that is located at the origin must be calculated. For a rectangular array of dipole elements with periodicities of \( p_x = a \) and \( p_y = b \) that is located at two half-spaces interface, the distance vector from each element to the origin \( (x = y = z = 0) \) is:

\[ \vec{r} = (ma) \hat{a}_x + (nb) \hat{a}_y \]  \hspace{1cm} (33)

where \( (m, n = 0,1,2,3,\ldots) \) are the indices of the considered dipole. Note that \( m \) and \( n \) are not zero at the same time.

Employing (46), (48), and (33) for vertical and \( x \)-directed horizontal dipole arrays, respectively, give:

\[ D_z = \frac{\varepsilon_1}{4 \pi \varepsilon_0} P_{n} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{\left[ (ma)^2 + (nb)^2 \right]^{3/2}} \]  \hspace{1cm} (34)

\[ E_x = \frac{1}{4 \pi \varepsilon_0} P_{ex} \times \left\{ \begin{array}{c} \frac{1}{\left[ (ma)^2 + (nb)^2 \right]^{3/2}} \\
\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \end{array} \right\} \]  \hspace{1cm} (35)

The prime symbol in the series means that the term \( (m,n) = (0,0) \) corresponding to the dipole which is located at the origin must be extracted from the series. The two series in (34) and (35) can be converted to convergent series with modified Bessel function of the second kind, with the aid of the Poisson summation [6]. Using the recursive relations of Bessel functions [29], the interaction fields due to the dipole array elements at the origin are obtained. The resulting series converge rapidly and can be estimated with their first term.

\[ D_z = \frac{\varepsilon_1 P_{n}}{\varepsilon_0} \left[ -\frac{1.2}{\pi a^2} - \frac{1.2}{\pi b^2} + \frac{8\pi K_0(2\pi b/a)}{a^3} + \frac{8\pi K_0(2\pi a/b)}{b^3} \right] \]  \hspace{1cm} (36)

\[ E_x = \frac{\varepsilon_1 P_{ex}}{\varepsilon_0} \left[ \frac{1.2}{\pi a^3} - \frac{8\pi K_0(2\pi b/a)}{b^3} \right] \]  \hspace{1cm} (37)

And \( F_y(a,b) = F_x(b,a) \). Finally, According to (30), the interaction constants for a square array \( (a = b) \) are equal to:
4 | NUMERICAL RESULTS

This section considers a circular patch array with the periodicity of \( p = 3 \) mm and the radius of \( a = 0.4p \) which is placed on a dielectric slab with \( \varepsilon_r = 16 \) and varying thickness to validate the formulations. A circular patch with the radius of ‘\( a \)’ has analytical electric and magnetic polarizabilities as [6]:

\[
C_{e_x} = \frac{\varepsilon_1}{\varepsilon_0} \left[ \frac{2.4}{\pi a^3} + \frac{16\pi}{a^3}K_0(2\pi) \right]
\]

\[
C_{e_z(x,y)} = \frac{\varepsilon_0}{\varepsilon_0} \left[ \frac{1.2}{\pi a^3} - \frac{8\pi}{a^3}K_0(2\pi) \right]
\]

Using the duality principle [28], the magnetic interaction constants are equal to:

\[
C_{m,x} = \frac{\mu_1}{\mu_0} \left[ -\frac{2.4}{\pi a^3} + \frac{16\pi}{a^3}K_0(2\pi) \right]
\]

\[
C_{m_x(x,y)} = \frac{\mu_0}{\mu_0} \left[ \frac{1.2}{\pi a^3} - \frac{8\pi}{a^3}K_0(2\pi) \right]
\]

Using (42)–(44) and the interaction constants (38)–(41), the analytical formulae in (31)–(32) can be used to calculate the surface susceptibility density dyads. It is necessary to mention that (42)–(44) are the polarizabilities of a circular patch in free-space; therefore, formulae (23)–(24) must first be applied to them. Note that the scatterers are placed at the interface of free space and a dielectric material. This means that \( \varepsilon_r |_{r=1} = 1 \) and \( \varepsilon_r |_{r=2} = 16 \) must be employed in the formulæ.

The dielectric slab with a specified thickness is replaced by an effective half-space using (21) and (22). As the surface susceptibility dyads and the dielectric constant of the effective half-space are determined, the fields due to VED and HED which illuminate the circular patch array that is located on the dielectric slab can be calculated using (8)–(20).

The \( z \)-component of the electric fields due to a VED which is placed at \( (x', y', z') = (0, 0, 4)\) mm, over the circular periodic patch array with mentioned dimensions and printed
TABLE 1 Comparison of the computation time and the amount of the used RAM to obtain the fields due to the VED near the metafilm

| Method               | Computation time (s) | RAM          |
|----------------------|----------------------|--------------|
| Full-wave FEKO       | 6400                 | 16 GB        |
| This Paper           | 17                   | 101 MB       |

on a dielectric slab with \( \varepsilon_r = 16 \) and thicknesses of \( h = 2 \) and \( 10 \) mm as well as the corresponding relative errors are shown in Figures 4 and 5, respectively. Also, the \( y \)-component of the electric fields due to a HED which illuminates the intended metafilm and the corresponding relative errors are depicted in Figure 6. The observation points are on the \( x \)-axis (i.e. \( y = 0 \)) and at different heights, and the operating frequency is 10 GHz. The results show good agreements compared with the fields calculated by commercial numerical software FEKO. It should be noted that it is necessary to import a full structure into the commercial numerical software FEKO. However, due to limited resources, the array must be truncated. Here, a \( 4 \times 4 \) (41 \times 41 elements) have been inserted into the software. A computer with core i7, 2.4 GHz CPU, and 32 GB RAM was used to compute the results. As depicted in Table 1, the computation time and the amount of memory usage for the homogenization method is significantly less than the commercial numerical software FEKO. Also, the relative errors of the fields are calculated as:

\[
RE = \left| \frac{E_{\text{Homogenized}} - E_{\text{Full Wave}}}{E_{\text{Full Wave}}} \right| \times 100 \tag{45}
\]

5 | CONCLUSION

Note that the practical problem of finite sources radiation in the vicinity of a printed metafilm on a finite thickness dielectric slab is considered. The problem is solved by replacing the total parts below the periodic array which contain the dielectric slab and the half-space under it, with a homogenous effective half-space. The most important issue in the susceptibility homogenization method is the exact calculation of the electric and magnetic surface susceptibility densities. To achieve this aim, the static electric interaction constants between the elements of the periodic horizontal and VEDs which are located at two different dielectrics’ interfaces are calculated using the static electric fields which have been obtained by Mohamed et al. [25]. Also, the magnetic interaction constants are calculated using the duality principle. Afterward, using the Lorentz method, the analytical formulae are presented to calculate the surface susceptibility densities of the homogenized meta-surfaces located at two materials’ interface. These formulations can be used for metafilms with both square and rectangular periodicity. Once the surface susceptibility densities and the effective dielectric constant of the effective half-space are determined, the Hertzian magnetic vector potentials for a metafilm printed at two half-spaces interface can be used to calculate the fields due to VED and HED which illuminate the metafilm in this case. An array of circular patches which are printed on a dielectric slab with varying thickness is considered to verify the formulations. Results show good agreement compared to the fields calculated by the commercial numerical software FEKO. It can also be seen that the homogenization method is significantly efficient than the numerical solution.

ACKNOWLEDGMENT

The authors state that they have not received any funds for their research.

CONFLICT OF INTEREST

The authors state that they do not have any conflict of interest.

ORCID

Z.H. Firoozeh  
https://orcid.org/0000-0002-0064-0150

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APPENDIX

The static electric field and electric displacement density due to vertical and horizontal electric dipole which are located at the origin and embedded in the interface of two dielectrics were obtained in [25] as:

\[
\vec{D} = \varepsilon_0 \frac{3(\vec{P}_n \cdot \hat{a}_r) \hat{a}_r - \vec{P}_n}{4\pi r^3}, \quad \vec{P}_n = \hat{a}_r p_{nz}
\]  

(46)

\[
\vec{E} = \frac{3(\vec{P}_e \cdot \hat{a}_r) \hat{a}_r - \vec{P}_e}{4\pi r^3}
\]  

(47)

\[
\vec{P}_e = p_{ex} \hat{a}_x + p_{ey} \hat{a}_y
\]  

(48)

where \(\vec{P}_n\) and \(\vec{P}_e\) are the vertical and horizontal electric dipoles, respectively, \(\hat{a}_r\) is the unit normal vector points to the observation point, \(r\) is the distance of the observation point from the dipole and \(e_\perp\) and \(e_\parallel\) are introduced in (25) and (27).