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The dynamics of hemispherical drop under the influence of an alternating electric field

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Abstract. The forced oscillations of incompressible fluid sessile drop under the alternating electric field are considered. Under equilibrium the drop has the hemispherical form with right contact angle. The drop is surrounded by an incompressible fluid with another density. The external uniform electric field acts as an external force that causes motion of the contact line. In order to describe this contact line motion the modified Hocking boundary condition is applied: the velocity of the contact line is proportional to the deviation of the contact angle and the speed of the fast relaxation processes, which frequency is proportional to twice the frequency of the electric field. It is shown that under uniform electric field the axisymmetric oscillation amplitude at the resonance frequency increases with the increase of the Hocking parameter.

1. Introduction
The behaviour of fluid drops under external electrical field continues to attract research efforts \cite{1,2}, in particular the dynamics and manipulation processes is electrowetting (EW) \cite{2,3} and electrowetting-on-dielectric (EWOD) \cite{4-6}. EWOD has been put into applications in various fields, such as digital (droplet) microfluidic devices for bioanalysis (lab-on-a-chip) \cite{7,8}, variable-focus liquid lenses \cite{9,10}, electronic display technology \cite{11,12} etc. The typical electrowetting devices are shown in Figure 1.

The Young–Lippmann equation is used most often to describe the varying contact angle $\theta$ of the drop under a direct current (DC) voltage \cite{2-6}:

$$\cos \theta = \cos \theta_0 + E_w, \quad E_w = 0.5CV^2\sigma_{cp}^{-1}, \quad C = \varepsilon_0 \varepsilon \sigma_{cp}^{-1}, \quad \cos \theta_0 = \sigma_{ic}^{-1} \left( \sigma_{ic} - \sigma_{cp} \right),$$

(1)

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where $E_w$ is EW number (it represents the ratio of the electrostatic energy to the liquid – surrounding fluid interfacial energy), $C$ is capacitance per unit area, $V$ is the value of the applied DC voltage, $\theta_0$ is the contact angle without applied voltage – equilibrium contact angle, which is defined by well-known Young’s equation, $\sigma$ is the interfacial tension between the drop of conducting fluid ($c$) and the isolating (surrounding) fluid ($i$) and the dielectric plate ($p$), $d$ is the thickness of the dielectric film, $\varepsilon_0$ and $\varepsilon$ are the permittivity of vacuum and the dielectric layer, respectively. For alternating current (AC) instead of $V^2$ in equation (1) the square value of the voltage, or effective voltage, $U^2$ is used [3]. Although Young–Lippmann equation (1) is widely accepted, different hypotheses have been advanced to derive it [2,6].

It should be possible to obtain zero contact angle (complete wetting and the contact angle does not change) by continuing to increase the applied voltage according to the equation (1), i.e. this equation is adequate for the voltage $V$ which is smaller than the critical voltage $V$. [2,3,6]. At $V \leq V_0$ the electrowetting contact angle does not increase with increasing of applied voltage, this effect is known as contact angle saturation. The mechanism of the contact angle saturation has not yet been elucidated and is the object of discuss [2].

We proposed in [13] another effective boundary condition on the basis of Hocking’s equation [14] for cylindrical drop:

$$\frac{\partial \zeta^*}{\partial t} = \pm \Lambda' \left( \frac{\partial \zeta^*}{\partial z} + A' \cos \left(2\omega' t'\right) \right),$$

(2)

where $\zeta^*$ is the deviation of the drop interface from the equilibrium position, $z^*$ is axial coordinate, $\Lambda'$ is a phenomenological constant (the so-called wetting parameter or Hocking parameter), having the dimension of velocity, $A'$ is effective amplitude, $\omega'$ is AC frequency. The second term in boundary condition (2) is external action and written similarly to the Young–Lippmann equation (1): $\cos \theta = E_w = \hat{E}_w V^2$ as $\cos \theta_0 = 0$ in our problem, therefore $\zeta_0 \sim \cot \theta = E_w / \sqrt{1 - E_w^2} = \hat{E}_w V^2 / \sqrt{1 - \hat{E}_w^2 V^2} \approx \hat{E}_w V^2 + O(V^4) \approx \hat{E}_w V^2 \sim V^2$. The special cases of boundary condition (2) are corresponding to the fixed contact line $\zeta^* = 0$ (the so-called pinned-end edge condition) and infinitely fast relaxation of the contact angle $\frac{\partial \zeta^*}{\partial z} \sim \cos \left(2\omega' t'\right)$ (the contact angle quickly adjusts to the slowly changing force). For example, the Hocking’s equation [14] was used in the studies of oscillations of a capillary bridge [15], sessile drop and bubble [16-18], “sandwiched” drop and bubble [19-21] and film interface [22].

In the present paper, we consider the behavior of a sessile drop under AC. In order to describe the motion of the contact line the modified boundary condition (2) is used: the velocity of the contact line is proportional to the deviation of the contact angle and the speed of the fast relaxation processes, whose frequency is proportional to twice of the electric field frequency. Once again note, that natural and power “mechanical” oscillations of sessile drop with taking into account the contact angle dynamics were examined in [17]. In future we plan to compare the results with experimental data (for example, [6]).

![Figure 2. Problem geometry (1 – electrode, 2 – dielectric layer).](image-url)
2. Problem formulation

The problem formulation largely coincides with the articles [13]. Consider a hemispherical liquid drop of radius \( a \) sitting atop a solid substrate (Figure 2). The sessile drop and unbounded ambient fluid are characterized by densities \( \rho_j \) and kinematic viscosities \( \nu_j \) (\( j = i, e \), here and in the following, quantities with subscript \( i \) refer to the drop, and those with subscript \( e \) to the surrounding liquid). The equilibrium form of drop is hemispherical of radius \( R_0' \) and the equilibrium contact angle \( \theta_0 \) between the side drop surface and the solid plane is assumed to be right. We assume that, on the one hand, the AC frequency \( \omega' \) is large enough for the viscosity can be ignored – \( \delta^2 = \omega' (\nu')^{-1} R_0'^2 \), and, on the other hand, the oscillation frequency is small enough, so that we can use the incompressibility condition – \( \omega' R_0' \ll c \) (\( \delta \) is the boundary-layer thickness, \( c \) is the sound velocity). The characteristic amplitude of oscillations of the drop \( A' \) is small compared to the equilibrium radius \( R_0' \), \( \varepsilon = A' \left( R_0' \right)^{1/2} \leq 1 \).

We use \( \sigma^{-1/2} \rho_i R_0'^{1/2} \), \( R_0' \), \( \rho_i' \), \( A' \), \( A' \sigma (R_0')^{-2} \), \( A' \sigma^{-1/2} (\rho_i' R_0'^{1/2})^{-1/2} \) as the scales for the time, length, height, density, deviation of drop surface from its equilibrium position, pressure, and velocity potential, respectively. The axisymmetric problem is described by the following equations and boundary conditions to spherical coordinates \((r, \vartheta)\):

\[
\Delta \Phi_j = 0, \quad P_j = -\rho_j \frac{\partial \Phi_j}{\partial r}, \quad j = i, e, \quad (3)
\]

\[
r = 1: \left[ \frac{\partial \Phi_i}{\partial r} \right] = 0, \quad [P] = -(\Delta_s + 2)Z, \quad \frac{\partial Z}{\partial t} = \frac{\partial \Phi_i}{\partial r}, \quad (4)
\]

\[
q = \frac{\pi}{2}: \frac{\partial \Phi_i}{\partial q} = 0, \quad \frac{\partial Z}{\partial t} = -\lambda \left( \frac{\partial Z}{\partial q} + \alpha \cos(2\omega t) \right), \quad (5)
\]

\[
r = 1, q = \frac{\pi}{2}: \frac{\partial Z}{\partial t} = -\lambda \left( \frac{\partial Z}{\partial q} + \alpha \cos(2\omega t) \right), \quad (6)
\]

where \( \Phi \) is the potential of fluid velocity, \( P \) is the fluid pressure, \( Z \) is the interface deviation from equilibrium position, \( \rho_0 = 1, \rho_e = \rho \), the square brackets denote the jump in the quantity at the interface between the external liquid and the drop. The boundary-value problem (3)–(6) involves four parameters: the dimensionless density, the wetting parameter, the AC frequency and amplitude \( \rho = \rho_i' \left( \rho_i' \right)^{-1}, \lambda = \lambda' \left( \sigma^{-1} \rho_i' R_0'^{1/2} \right), \omega = \omega' \left( \sigma^{-1} \rho_i' R_0'^{1/2} \right)^{1/2}, a = 0.5 A' C \left( \sigma^{-1} \rho_i' R_0'^{1/2} \right)^{1/2} \).

3. Inviscid problem

In order to investigate the problem it is convenient to begin with a consideration of inviscid fluids, there is dissipation in this case. This damping is due to the interaction the contact line with solid plate [14-22]. Because external driving is periodic, we proceed to complex amplitudes by representing all the fields as \( F(r, t) = \text{Re} \left( f(r) e^{i\omega t} \right) \). We expand the amplitudes in series with respect to the Legendre polynomials (similarly [17]):

\[
\varphi_i(r, \vartheta) = 2i\omega \sum_{n=0}^\infty A_n P_n(\theta) r^{2n}, \quad \varphi_e(r, \vartheta) = 2i\omega \sum_{n=0}^\infty B_n P_n(\theta) r^{-2n-1}, \quad \zeta(\theta) = \sum_{n=0}^\infty C_n P_{2n}(\theta), \quad (7)
\]

where \( \theta = \cos \vartheta \). These solutions are written down to automatically satisfy the requirements of being regular as \( r \to 0 \) (drop) and \( r \to \infty \) (external fluid) and part of boundary conditions, as given first two relations in Eq. (4) and by Eq. (5). Substituting these relations into the dynamic boundary condition [see the final relation in 2 Eqs. (5)], we arrive at another representation for \( \zeta \).
\[ \zeta = \theta D - 4\omega^2 \sum_{n=0}^{\infty} 2n\Omega_n^2 A_n P_{2n}(\theta), \quad \Omega_n^2 = 2n(2n-1)(2n+1)(2n+2)(2n\rho+2n+1)^{-1} \]  

(8)

where \( D \) is a yet-to-be-determined amplitude of the contact angle oscillations and \( \Omega_n \) has the meaning of eigen frequencies associated with the even eigen modes for a spherical drop (so-called Rayleigh’s modes \([16,17]\)). Note that the term with \( n = 0 \) also contributes into this representation, as at \( n = 0 \) we have \( 2n\Omega_n^2 = -0.5 \). By comparing Eqs. (7) and (8) for \( \zeta \), we obtain the amplitudes

\[ A_n = \frac{D}{4\omega^2}, \quad A_n = \frac{\alpha_n \Omega_n^2 D}{2n(\Omega_n^2 - 4\omega^2)}, \quad B_n = -2n(2n+1)^{-1} A_n, \quad C_n = 2nA_n \]  

(9)

which are expressed in terms of coefficients \( \alpha_n \) defined as

\[ \alpha_n = -(4n+1)(2n-1)^{-1}(2n+2)^{-1}P_{2n}(0), \quad \theta = \sum_{n=0}^{\infty} \alpha_n P_{2n}(\theta). \]  

(10)

Finally, taking into account boundary condition (6), we figure out

\[ D = \left( 1 - \frac{i8\omega^3}{\lambda} \sum_{n=0}^{\infty} \frac{\alpha_n P_{2n}(0)}{\Omega_n^2 - 4\omega^2} \right)^{-1}. \]  

(11)

We note that as all the constants \( A_n \) are proportional to the same complex factor \( D \) (see (7) and (11)), we conclude that all the Rayleigh modes oscillate in phase, though with a certain phase shift relative to the “external force” exerted at the contact line. Thus, the shape oscillations can be represented as a standing wave, and therefore no traveling waves propagate along the surface in the system under consideration.

**Figure 3.** Response characteristics of drop oscillations. Amplitude of surface oscillation in the pole (a,d) and on the substrate (b,e) and contact line deviation (c,f) vs frequency \( (\alpha = 1) \).
\[ \lambda = 0.1 \text{ (solid line)}, \lambda = 1 \text{ (dashed line)}, \lambda = 10 \text{ (dotted line)}, \quad (a-c) \ \rho = 0.5, \quad (d-f) \ \rho = 1. \]

The dependence of the surface oscillation amplitude and the contact angle vs the frequency of the driving force is given in Figure 3 for different values of the Hocking parameter and of the density. Obviously, the larger the wetting parameter \( \lambda \), the lower the interaction of the contact line with the plate and larger the oscillation amplitude (Figure 3 a, d). Also, as it was mentioned in [16-21], at certain values of eigen frequencies the drop motion is independent of the wetting parameter: at any \( \lambda \) the contact line remains motionless. At such points the amplitude of the contact line goes to zero. This is so-called “antiresonance” frequencies. Values of these frequencies do not depend on \( \lambda \) and depend on the parameters of the drops (density \( \rho \) etc.). Note that the maximum deviation of the contact angle does not depend on the density (Figure 3 c, f).

![Figure 4](image)

**Figure 4.** Response characteristics of drop oscillations. Amplitude of surface oscillation in the pole (a) and on the substrate (b) and contact line deviation (c) vs frequency \( (\lambda = 1, \ \rho = 1) \).

\[ a = 0.5 \text{ (solid line)}, \ a = 1 \text{ (dashed line)}, \ a = 2 \text{ (dotted line)}. \]

Figure 4 shows response of the system for different values of the amplitude \( a \) for a given contrast of densities \( \rho = 1 \). The oscillations amplitude increases with growth of \( a \) (Figure 4 a, b) but the deviation of contact angle increases also (Figure 4 c).

![Figure 5](image)

**Figure 5.** The contact angle \( \Theta \) vs square root of amplitude \( a \) for three different values of the Hocking parameter \( \lambda \) and of the oscillation frequency \( \omega \) \((\rho = 1)\).

\[ (a) \ \lambda = 0.1, \ (b) \ \lambda = 1, \ (c) \ \lambda = 10, \quad \omega = 0.5 \text{ (solid line)}, \ \omega = 1 \text{ (dashed line)}, \ \omega = 1.5 \text{ (dotted line)}. \]

The contact angle versus square root of amplitude \( a \) (i.e. proportional to AC potential \( V \)) is given in Figure 5 for different values of the Hocking parameter \( \lambda \) and AC frequency \( \omega \). The responses
obtained in qualitative agreement with the experimental data. However the maximum deviation of contact angle tends to \( \pi/2 \), i.e. \( \theta \to 0 \) or \( \theta \to \pi \) whereas contact angle is finite in experiments.

Different wetting situations are addressed by changing the values of the parameter \( \lambda \) and range between two limiting cases: i) for \( \lambda = 0 \), the contact line motion is completely uncoupled from the external field – the contact line remains fixed and the droplet overall is motionless; ii) the opposite limit, \( \lambda \gg 1 \), corresponds to infinitely fast relaxation of the contact angle, when the latter quickly adjusts to the slowly changing force. In both these situations, there is no dissipation at the contact line. Apparently the fluid viscosity must be taken into account. It is giving a finite amplitude for both limit cases of Hocking boundary especially in limiting case \( \lambda \gg 1 \).

4. Conclusions
The dependences of surface amplitude and contact angle from oscillation amplitude and frequency are obtained for Hocking parameter and density. It is shown that the curve of contact angle \( \theta \) vs square root of amplitude obtained in qualitative agreement with the experimental data. However the maximum deviation of contact angle tends to \( \pi/2 \), i.e. \( \theta \to 0 \) or \( \theta \to \pi \) whereas contact angle is finite in experiments. Consequently, there are three ways of further development of problem solution: 1) fluid viscosity (see for example [23]), 2) contact angle hysteresis (see for example [24, 25]), 3) arbitrary equilibrium contact angle [20]. Both of these cases limit the amplitude of oscillation and may limit the change of the contact angle.

5. References
[1] Melcher J R and Taylor G I 1969 Annual Review of Fluid Mechanics 1 111
[2] Mugele F and Baret J-C 2005 J. Phys.: Condens. Matter. 17 705
[3] Chen L and Bonaccurso E 2014 Adv. Colloid Interface Sci. 210 2
[4] Berge B 1993 Comptes Rendus Acad. Sci. II 317 157
[5] Zhao Y-P and Wang Y 2013 Rev. Adhesion Adhesives 1 114
[6] Chevalliot S, Kuiper S and Heikenfeld J 2012 J. of Adhesion Sci. Tech. 26 1909–30
[7] Hua Z, Rouse J L, Eckhardt A E, Srinivasan V, Pamula V K, Schell W A, Benton J L, Mitchell T G and Pollack M G 2010 Anal Chem 82 2310–16
[8] Li J, Wang Y, Chen H and Wan J 2014 Lab Chip 14 4334–37
[9] Kuiper S and Hendriks B H W 2004 Appl. Phys. Lett. 85, 1128–30
[10] Li C and Jiang H 2014 Micromachines 5 432–41
[11] Hayes R A and Feenstra B J 2003 Nature 425 383–5
[12] Roques-Carmes T, Hayes R A, Feenstra B J and Schlangen L J M 2004 J. Appl. Phys 95 4389–96
[13] Alabuzhev A A and Kaysina M I 2016 J. Phys.: Conf. Ser. 681 012043
[14] Hocking L M 1987 J. Fluid Mech. 179 253–66
[15] Borkar A and Tsamopoulus J 1991 Phys. Fluids A 3 2866–74
[16] Lyubimov D V, Lyubimova T P and Shklyaev S V 2004 Fluid Dynamics 39 851–62
[17] Lyubimov D V, Lyubimova T P and Shklyaev S V 2006 Phys. Fluids 18 012101
[18] Shklyaev S and Straube A V 2008 Phys. Fluids 20 052102
[19] Alabuzhev A A and Lyubimov D V 2007 J. Appl. Mech. Tech. Phys. 48 686–93
[20] Alabuzhev A A and Lyubimov D V 2012 J. Appl. Mech. Tech. Phys. 53 9–19
[21] Alabuzhev A A 2016 Appl. Mech. Tech. Phys. 53 9–19
[22] Perlin M, Schultz W W and Liu Z 2004 Wave Motion 40 41–56
[23] Bostwick J B and Steen P H 2016 Soft Matter 12 8919–26
[24] Fayzrakhmanova I S and Straube A V 2009 Phys. Fluids 21 072104
[25] Fayzrakhmanova I S, Straube A V and Shklyaev S 2011 Phys. Fluids 23 102105