Quark Contributions to Baryon Magnetic Moments
in Full, Quenched and Partially Quenched QCD

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The chiral nonanalytic behaviour of quark-flavor contributions to the magnetic moments of octet baryons are determined in full, quenched and partially-quenched QCD, using an intuitive and efficient diagrammatic formulation of quenched and partially-quenched chiral perturbation theory. The technique provides a separation of quark-sector magnetic-moment contributions into direct sea-quark loop, valence-quark, indirect sea-quark loop and quenched valence contributions, the latter being the conventional view of the quenched approximation. Both meson and baryon mass violations of SU(3)-flavor symmetry are accounted for. Following a comprehensive examination of the individual quark-sector contributions to octet baryon magnetic moments, numerous opportunities to observe and test the underlying structure of baryons and the nature of chiral nonanalytic behavior in QCD and its quenched variants are discussed. In particular, the valence $u$-quark contribution to the proton magnetic moment provides the optimal opportunity to directly view nonanalytic behavior associated with the meson cloud of full QCD and the quenched meson cloud of quenched QCD. The $u$ quark in $\Sigma^+$ provides the best opportunity to display the artifacts of the quenched approximation.

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I. INTRODUCTION

One of the grand promises of lattice-regularized QCD is to provide ab initio predictions of hadronic observables with statistical and systematic uncertainties constrained to within 1%. To realize this goal, extrapolations are required. The finite lattice spacing must be extrapolated to the continuum limit. The impact of the finite volume of the periodic lattice must be understood and extrapolated to infinity. For computational reasons, the masses of the light $u$ and $d$ quarks must be extrapolated from rather large values to the point at which the physical hadron masses are reproduced.

The realization of chiral symmetry in the dynamically-broken Goldstone mode induces important nonanalytic behaviour in hadronic observables as a function of quark mass. This makes extrapolations of hadronic observables highly nontrivial.\cite{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}. Significant curvature is encountered as one approaches the chiral limit. Indeed there is some controversy on the optimal methodology for achieving systematic errors within the 1% bound.\cite{16}

The origin of significant nonanalytic behaviour lies in the dressings of hadrons by light pseudoscalar mesons. While many studies of chiral nonanalytic behaviour are formulated in the infinite-volume continuum limit, it is important to understand that the nonanalytic behaviour is intimately linked to the finite volume of the lattice. The momenta available to the hadrons participating in the loop integrals which give rise to the nonanalytic terms of the chiral expansion are modified in a finite volume.

For $P$-wave intermediate states there is a threshold effect\cite{17} where there is no strength in the momentum integral until the first nontrivial momentum $(2\pi/L)$ is reached. Even then the number of discrete momenta available to the integral is governed by the number of lattice sites $(L/a)$.

Systematic errors in the extrapolation of the finite lattice spacing to the continuum limit have been minimized through the development of improved actions\cite{17, 18, 19} displaying excellent scaling properties\cite{20, 21, 22}. As such, understanding the entangled properties of chiral nonanalytic behavior on a finite volume at a quantitative level with systematic errors at the level of 1% is a problem which remains at the forefront of lattice gauge theory.

For the foreseeable future, extrapolations will continue to be required to connect lattice simulation results to physical observables. Hence, the development of systematically accurate chiral extrapolation techniques is of central importance to the field\cite{17, 14, 23}.

Fortunately it is possible to test the development of chiral extrapolation techniques today. The key point is that one can probe the chiral regime of quenched or partially quenched QCD using fermion actions with improved\cite{17} or perfect\cite{18} chiral symmetry properties. Thus, one can confront the analytic techniques of chiral effective field theory with numerical simulation results and test the extent to which effective field theory reproduces the exact results of numerical simulations.

This investigation will establish the leading chiral nonanalytic behaviour of quark sector contributions to octet baryon magnetic moments in full, quenched and partially quenched QCD. As we will see, significant nonanalytic behavior remains for some quark-sector contributions to baryon magnetic moments making these observables ideal for the confrontation of effective field theory

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and lattice QCD.

Separation of the valence and sea-quark-loop contributions to the meson cloud of full QCD hadrons is a non-trivial task. Early calculations addressing the meson cloud of mesons employed a diagrammatic method \[24\]. The formal theory of quenched chiral perturbation theory (Q\(_\chi\)PT) was subsequently established in Ref. \[25\]. There, meson properties were examined in a graded-symmetry formulation where extra commuting ghost-quark fields are introduced to eliminate the dependence of the path integral on the fermion-matrix determinant. This approach was extended to the baryon sector in Ref. \[26\].

While the graded-symmetry formalism is essential to establishing the field theoretic properties of Q\(_\chi\)PT, it is desirable to formulate an efficient and perhaps more intuitive approach to the calculation of quenched chiral coefficients. Rather than introducing extra degrees of freedom to remove the effects of sea-quark loops, the approach described herein introduces a formalism for the identification and calculation of sea-quark-loop contributions to baryon properties, allowing the systematic separation of valence- and sea-quark contributions to baryon form factors in general. Upon removing the contributions of sea-quark-loops, one arrives at the conventional view of quenched chiral perturbation theory.

A brief account of these methods is published in Ref. \[27\]. Since this presentation, there has been a resurgence in quenched and partially-quenched \(\chi\)PT calculations of baryon magnetic moments \[28, 29\]. In particular, the magnetic moments of octet baryons have been examined \[28\] using the formal approach of Q\(_\chi\)PT \[29\]. There the leading-nonanalytic (LNA) behavior of the magnetic moment for each baryon of the octet is calculated. Upon summing the quark sector contributions obtained from the diagrammatic method presented here, one finds complete agreement between the diagrammatic and formal approaches. The approaches are equivalent.

In its standard implementation, the formal approach completely eliminates all sea-quark-loop contributions to quenched baryon moments. However, sea-quark loops do make a contribution to matrix elements in the quenched approximation. Insertion of the current in calculating the three-point correlation function provides pair(s) of quark-creation and annihilation operators. These can be contracted with the quark field operators of the hadron interpolating fields providing “connected insertions” of the current, or self-contracted to form a direct sea-quark-loop contribution or “disconnected insertion” of the current. The latter contributions to baryon electromagnetic form factors are under intense investigation in quenched simulations \[30, 31, 32, 33\]. Hence in formulating quenched chiral perturbation theory it is important to provide an opportunity to include these particular sea-quark-loop contributions. A more flexible approach to the calculation of quenched chiral coefficients is desirable.

Moreover, the present calculations \[28, 29, 30\] of chiral nonanalytic behavior in baryon magnetic moments focus on bulk baryon properties. The formalism presented here provides a method for the isolation of individual quark sector contributions \[35, 36, 37, 38, 39\] to form factors in full, quenched and partially-quenched QCD. Individual quark sector contributions to the nucleon magnetic moments are under experimental investigation \[11, 12, 13\] where the strange-quark contribution to the nucleon moment is of paramount interest. Understanding the manner in which quarks compose baryons is essential to a complete understanding of QCD.

In the process, we will see that it is possible to separate valence- and sea-quark contributions to baryon form factors in full QCD. This separation is of significant value as contributions from disconnected insertions of the current are difficult to determine in lattice QCD simulations. Moreover, chiral nonanalytic behaviour of quark sector contributions can be significantly enhanced when direct sea-quark-loop couplings to the current are removed.

In contrast to conventional calculations of chiral nonanalytic structure, SU\((3)\)-flavor violations in both meson and baryon masses are accounted for in the following. These are particularly important for \(K\)-meson dressings of hyperons where the baryon mass splitting can be positive or negative, suppressing or enhancing contributions depending on whether the intermediate baryon is heavier or lighter respectively. Incorporation of the baryon mass splittings significantly alters the functional structure and associated curvature of the nonanalytic terms. SU\((3)\)-flavor violations will also affect the analytic terms of the chiral expansion. The leading constant, the coefficient of \(m^2_q (\propto m_q)\) etc. will all exhibit a strangeness dependence due to symmetry breaking. Ultimately, these coefficients will be determined by matching the chiral expansion to lattice QCD simulation results \[13, 14\].

In principle, the axial couplings, \(D\) and \(F\), are defined and specified in the chiral SU\((3)\)-flavor symmetric limit. The mass dependence of the couplings is incorporated in the chiral expansion through higher-order terms. Upon formulating the chiral expansion in the SU\((2)\) limit with explicit and substantial SU\((3)\)-flavor symmetry breaking as is physically realized, one can break the symmetry of the axial couplings. However, previous examinations of the chiral corrections to baryon axial currents suggest that such phenomenological flavor-symmetry breaking is small \[14\].

Similarly, SU\((3)\) flavor-symmetry breaking is often incorporated by allowing the decay constant of the kaon to exceed that of the pion, \(f_K \simeq 1.2 f_\pi\). In the past, \(K\)-loop contributions were often found to be uncomfortably large when matching to phenomenology and the 30\% reduction obtained in using \(f_K\) in place of \(f_\pi\) proved to be helpful \[34\]. However, it is not yet clear whether this phenomenological suppression of \(K\)-loops through the use of \(f_K\) is necessary when regulators suppressing short distance physics are used in chiral effective field theory.

To make the numerical results of this study readily accessible and of the widest utility, the established tree-level axial couplings \(F = 0.50\) and \(D = 0.76\) are adopted with the SU\((3)\)-symmetric limit of \(f_\pi = f_K = 93\) MeV.
for the meson decay constants.
In summary, the purpose of this study is:

1. To provide the first calculation of the leading nonanalytic behavior of quark-flavor contributions to baryon magnetic moments in full QCD,
2. quenched QCD, and
3. partially-quenched QCD.
4. To separate valence- and sea-quark contributions to baryon form factors in full QCD at the individual quark-flavor level.
5. To extend previous baryon-moment calculations to include both meson and baryon mass splittings in SU(3)-flavor violations, as is done for items 1 through 4 above.
6. To identify quark-flavor channels displaying significant chiral nonanalytic behavior in the quenched approximation or revealing the artifacts of the quenched approximation.

7. To establish the diagrammatic method for calculating the chiral coefficients of quenched and partially-quenched QCD in the baryon sector. The method is rapid, intuitive and transparent, allowing complete flexibility in the consideration of quark contributions to baryon form factors.

It will become apparent that this technique and most of the results may be applied to other baryon form factor studies in general.

Sec. II presents the essential concepts for isolating and calculating sea-quark loop contributions to baryon properties and proves the technique via a consideration of baryon masses. The derivation of the quenched chiral coefficients for the quark-sector contributions to the quenched magnetic moments of octet baryons is described in Sec. III. The technique provides a separation of magnetic moment contributions into “total” full-QCD contributions, “direct sea-quark loop” and “valence” contributions of full-QCD. The latter are obtained by removing the direct-current coupling to sea-quark loops from the total contributions. Upon further removing “indirect sea-quark loop” contributions, one obtains the “quenched valence” contributions, the conventional view of the quenched approximation. We will use the quoted terms for reference to these contributions in the following. Sec. III also accounts for both baryon mass and meson mass violations of SU(3)-flavor symmetry. The quenched η′ gives rise to new nonanalytic behavior and this is briefly reviewed in Sec. III D.

A comprehensive examination of the individual quark-sector contributions to octet baryon magnetic moments is presented in Sec. IV. General expressions are accompanied by numerical evaluations to identify channels of particular interest. Partially-quenched results are presented in Sec. V. Sec. VI provides a summary of the highlights of the findings.

II. QUENCHED BARYON MASSES

A. Formalism

The SU(3)-flavor invariant couplings are described in the standard notation by defining

\[
B = \left(\begin{array}{ccc}
\Sigma^0 & \Lambda & p \\
\Sigma^- & \Sigma^0 & n \\
\Xi^- & \Xi^0 & \frac{2\Lambda}{\sqrt{6}}
\end{array}\right),
\]

(1)

\[
P_{\text{oct}} = \left(\begin{array}{ccc}
\pi^0 & \eta & \pi^+ \\
\pi^- & \pi^0 & K^0 \\
K^- & K^0 & -2\eta
\end{array}\right),
\]

(2)

and

\[P_{\text{sin}} = \frac{1}{\sqrt{3}} \text{diag}(\eta', \eta', \eta').\]

(3)

The SU(3)-invariant combinations are

\[
[\overline{BBP}]_F = \text{Tr}(\overline{BBP}_{\text{oct}}B) - \text{Tr}(\overline{BBP}_{\text{oct}}),
\]

(4)

\[
[\overline{BBP}]_D = \text{Tr}(\overline{BBP}_{\text{oct}}B) + \text{Tr}(\overline{BBP}_{\text{oct}}),
\]

(5)

\[
[\overline{BBP}]_S = \text{Tr}(\overline{BBP})\text{Tr}(P_{\text{sin}}).
\]

(6)

The following calculations are simplified through the use of the corresponding interaction Lagrangians. The octet interaction Lagrangian is

\[
\mathcal{L}_{\text{int}}^{\text{oct}} = -f_{NN\eta}(\overline{N}T_{\eta}N)\cdot\pi + if_{\Sigma\Sigma\eta}(\overline{\Sigma}T\Sigma)\cdot\pi
- f_{\Lambda\Sigma\eta}(\overline{\Lambda}T\Sigma)\cdot\pi - f_{\Xi\Sigma\eta}(\overline{\Xi}T\Sigma)\cdot\pi
- f_{\Lambda\Lambda\eta}(\overline{\Lambda}T\Lambda)\eta
- f_{\Xi\Lambda\eta}(\overline{\Xi}T\Lambda)\eta
- f_{\Sigma\Lambda\eta}(\overline{\Sigma}T\Lambda)\eta
- f_{\Xi\Xi\eta}(\overline{\Xi}T\Xi)\eta
- f_{\Xi\Xi\eta}(\overline{\Xi}T\Xi)\eta,
\]

(7)

and the singlet interaction Lagrangian is

\[
\mathcal{L}_{\text{int}}^{\text{sin}} = -f_{NN\eta}(\overline{N}N)\eta' - f_{N\Lambda\eta}(\overline{\Lambda}N)\eta'
- f_{\Sigma\Sigma\eta}(\overline{\Sigma}\Sigma)\eta' - f_{\Xi\Xi\eta}(\overline{\Xi}\Xi)\eta',
\]

(8)

where

\[
N = \left(\begin{array}{c}
p \\
n
\end{array}\right), \quad \Xi = \left(\begin{array}{c}
\Xi^0 \\
\Xi^-
\end{array}\right),
\]

(9)

\[
K = \left(\begin{array}{c}
K^+ \\
K^0
\end{array}\right), \quad K_c = \left(\begin{array}{c}
K^0 \\
-K^-
\end{array}\right).
\]
The octet meson-baryon couplings are expressed in terms of the $F$ and $D$ coupling coefficients as follows:

\[
\begin{align*}
 f_{NN\pi} &= F + D, \\
 f_{NN\eta} &= \frac{1}{\sqrt{3}} (3F - D), \\
 f_{\Sigma K} &= \frac{1}{\sqrt{3}} (3F - D), \\
 f_{\Delta N} &= -\frac{2}{\sqrt{3}} D, \\
 f_{\Sigma N} &= \frac{\sqrt{3}}{3} D, \\
 f_{\Xi K} &= \frac{\sqrt{3}}{3} D, \\
 f_{\Xi\Xi} &= -\frac{1}{\sqrt{3}} (3F + D), \\
 f_{\Xi\Xi'} &= \frac{1}{\sqrt{3}} (3F - D), \\
 f_{\Sigma\Sigma'} &= -\frac{1}{\sqrt{3}} (3F + D),
\end{align*}
\]

and the singlet couplings satisfy

\[
f_{NN\eta'} = f_{\Lambda\Lambda'} = f_{\Sigma\Sigma'} = f_{\Xi\Xi'}. \tag{10}
\]

The light quark content of

\[
|\eta'\rangle = \frac{1}{\sqrt{3}} (|\bar{u}u\rangle + |\bar{d}d\rangle + |\bar{s}s\rangle),
\]

and

\[
|\eta\rangle = \frac{1}{\sqrt{6}} (|\bar{u}u\rangle + |\bar{d}d\rangle - 2 |\bar{s}s\rangle), \tag{12}
\]

mesons suggests

\[
f_{NN\eta'} = \sqrt{2} f_{NN\eta}. \tag{14}
\]

This relation between nucleon octet and singlet couplings is commonly used in Q\(\chi\)PT calculations to estimate \(\eta\)' couplings to octet baryons. In the following, numerical estimates are based on the tree-level axial couplings \(F = 0.50\) and \(D = 0.76\) with \(f_\pi = f_K = 93\) MeV.

The leading-nonanalytic term of the chiral expansion provides the dominant source of rapid variation in baryon magnetic moments. Taking the \(N\)-\(\Delta\) mass splitting to be of zeroth chiral order, the \(\Delta\) resonance provides the next-to-leading nonanalytic (NLNA) contribution. This contribution makes only a small correction to the proton magnetic moment obtained from the chiral extrapolation of lattice QCD simulation results. The \(N\)-\(\Delta\) mass splitting suppresses the nonanalytic curvature, and when combined with the analytic terms of the chiral expansion, only a small enhancement of the proton moment is observed. As the NLNA \(\Delta\) contribution is added, the proton moment increases from 2.61 to 2.66 \(\mu_N\). This 2% effect at the physical point is unlikely to be observed in lattice QCD simulation results. As the NLNA \(\Delta\) contribution is added, the proton moment increases from 2.61 to 2.66 \(\mu_N\). This 2% effect at the physical point is unlikely to be observed in lattice QCD simulation results for some time. As such, we do not encumber the reader with these small contributions.

### B. Baryon Mass

To calculate the quenched chiral coefficients, we begin by calculating the total full-QCD contribution in the limit where the \(\eta\) and \(\eta'\) mesons are taken to be degenerate with the pion. In the quenched approximation, quark loops which otherwise break this degeneracy are absent.

| TABLE I: Meson-cloud contributions of Fig. I in full QCD. \(\eta\) and \(\eta'\) masses are set degenerate with the pion in anticipation of quenching the theory. |
| --- |
| Fig. | Channel | Mass | Coupling | Coupling |
| --- | --- | --- | --- | --- |
| a,b,c | \(p\pi^0\) | \(N\pi\) | \(f_{NN\pi}^2\) | \((F + D)^2\) |
| a,b,c | \(p\eta\) | \(N\pi\) | \(f_{NN\eta}^2\) | \((F - D)^2/3\) |
| a,b,c | \(p\eta'\) | \(N\pi\) | \(f_{NN\eta'}^2\) | \(2(3F - D)^2/3\) |
| f,g | \(n\pi^+\) | \(N\pi\) | \(2f_{NN\pi}^2\) | \((F + D)^2\) |
| h | \(\Delta K^+\) | \(\Delta K\) | \(f_{\Lambda KK}^2\) | \((F + D)^2/3\) |
| h | \(\Sigma^0 K^+\) | \(\Sigma K\) | \(f_{\Sigma KK}^2\) | \((F - D)^2\) |
| i | \(\Sigma^+ K^0\) | \(\Sigma K\) | \(2f_{\Sigma\Sigma\pi}^2\) | \((2F)^2\) |

The quark flow diagrams of Fig. I illustrate the processes which give rise to the LNA behavior of proton observables. Table II summarizes the contributions of the \(\pi\)-, \(\eta\)-, \(\eta'\)- and \(K\)-cloud diagrams of Fig. I labeled by the corresponding quark-flow diagrams. Summation of these contributions and incorporation of the factors from the loop integral provides the LNA term proportional to \(m_\pi^2\) of

\[
- (3F^2 + D^2) \frac{m_\pi^2}{\beta_\pi f_\pi^2}. \tag{15}
\]

The separation of the meson cloud into valence and sea-quark contributions is shown in the quark-flow diagrams labeled by letters. To quench the theory, one must understand the chiral behavior of the valence-quark loops of Figs. I(a), (d) and (f) and the sea-quark loops of Figs. I(b), (c), (e), (g), (h) and (i) separately. If one can isolate the behavior of the diagrams involving a quark loop, then one can use the known LNA behavior of the full meson-based diagrams to extract the corresponding valence-loop contributions.

For example, Fig. I(b) involves a sea-quark contribution is shown in the quark-flow diagrams labeled by letters. To quench the theory, one must understand the chiral behavior of the valence-quark loops of Figs. I(a), (d) and (f) and the sea-quark loops of Figs. I(b), (c), (e), (g), (h) and (i) separately. If one can isolate the behavior of the diagrams involving a quark loop, then one can use the known LNA behavior of the full meson-based diagrams to extract the corresponding valence-loop contributions.
“strange” quark in this case is understood to have the same mass as the u-quark.

Similarly, the corresponding hadron diagram which gives rise to the LNA structure of Fig. (c) is therefore the $K^0$-loop diagram of Fig. (i), with the distinguishable “strange” quark mass set equal to the mass of the d-quark. That is, the intermediate $\Sigma$-baryon mass appearing in the $K^0$-loop diagram is degenerate with the nucleon. Similarly, the “kaon” mass is degenerate with the pion.

The sum of the first two lines of Table II provides the contribution of diagram Fig. (b). The third line provides the contribution of Fig. (c). Similar arguments allow one to establish the remaining loop contributions to the light-meson cloud. Summing the couplings of Table II indicates sea-quark-loops contribute a term

$$- (9F^2 - 6FD + 5D^2) \frac{m_\pi^3}{24\pi f_\pi^2},$$

(16)
to the LNA behavior of the nucleon such that the net quenched contribution proportional to $m_\pi^3$ is

$$- (3FD - D^2) \frac{m_\pi^3}{12\pi f_\pi^2},$$

(17)
in agreement with the formal approach of Labrenz and Sharpe [26]. We note that the kaon-cloud contributions of Fig. (h) and (i) are pure sea contributions and trivially vanish in subtracting sea-contributions from total contributions as outlined above.

III. BARYON MAGNETIC MOMENTS

A. Quark-Sector Contributions to the Proton

The LNA contribution to baryon magnetic moments proportional to $m_\pi$ or $m_K$ has its origin in couplings of the electromagnetic (EM) current to the meson propagating in the intermediate meson-baryon state. In order to pick out a particular quark-flavor contribution, one sets the electric charge for the quark of interest to one and the charge of all other flavors to zero.

Tables III through V report results for the u-quark in the proton. The total contributions are calculated in the standard way, but with charge assignments for the intermediate mesons (indicated in the Charge column) reflecting in this case $q_u = 1$ and $q_d = q_s = 0$. The extra baryon subscripts on the meson masses are a reminder of the baryons participating in the diagram to facilitate more accurate treatments of the loop integral in which baryon mass splittings are taken into account. The LNA...
with the following, the units of quark forms a meson composed with a sea-quark loop as illustrated in Fig. 1.

\[ \beta \chi \]

The “direct sea-quark-loop contributions” indicated in Table IV are contributions in which the EM current couples to a sea-quark loop, in this case a \( u \) quark. Using the techniques described in Sec. III one can calculate the contributions of these loops alone to the baryon magnetic moment. The Mass column of Table IV is a reminder that the mass of the “kaon” considered in determining the coupling is actually the pion mass for Figs. II b) and (e). These diagrams will contribute, even in the quenched approximation, when disconnected insertions of the EM current are included in simulations [31–33].

Subtraction of these sea-quark-loop contributions from the total contributions of III leaves a net valence contribution of \(-11.0 m_{\pi} - 0.15 m_{N\Sigma K} - 3.68 m_{N\Lambda K} \) in full QCD.

Table V focuses on diagrams in which the EM current couples to a valence quark in a meson composed with a sea-quark loop. These are the “indirect sea-quark loop” contributions. Subtracting off these couplings from the valence contribution provides the net quenched valence contribution of \(-3.33 m_{\pi} \).

Tables VI through VIII provide a similar analysis of the \( d \) quark in the proton, where \( q_u = q_d = 0 \). Subtraction of the direct sea-quark-loop contributions of Table VI from the total contributions of Table VII leaves a net valence contribution of \( 2.75 m_{\pi} - 0.29 m_{N\Sigma K} \). Further removal of the indirect sea-quark loops of Table VIII provides the final net \( d \)-quark quenched valence contribution to the proton magnetic moment of \(+3.33 m_{\pi} \).

Table IX describes the \( s \)-quark contributions to the proton magnetic moment, where \( q_s = 1 \) and \( q_u = q_d = 0 \). As there are no \( s \) valence quarks in the proton, the contributions are purely sea-quark-loop contributions. The net valence contribution is zero and there are no further quenching considerations.

Charge symmetry provides the quark-sector contributions to the neutron magnetic moment. For unit charge quarks, \( d_u = u_p, u_s = d_p \) and \( s_n = s_p \).

The QCD Lagrangian is flavor blind in the \( SU(3) \)-flavor symmetry limit. This independence from quark flavor is manifest in Tables IV, VII and IX for the direct

\[ \frac{f_{\pi}}{8\pi f_{\pi}^2} m_{\pi} \equiv \chi m_{\pi} \]  

\[ \beta \]
TABLE VI: Determination of the total $d$-quark contribution to the proton magnetic moment as illustrated in Fig. I

| Diagram | Channel | Mass | Charge | Term | $\beta$ | $\chi$ |
|---------|---------|------|--------|------|--------|--------|
| $f_g$   | $n \pi^+$ | $N\pi$ | $-1$   | $-2f^2_{\Sigma N}\pi m_{\pi}$ | $(F + D)^2$ | +0.87  |
| $i$     | $\Sigma^+ K^0$ | $\Sigma K$ | $+1$   | $+2f^2_{\Sigma N}\pi m_{\Sigma K}$ | $-(D - F)^2$ | -0.29  |

TABLE VII: Determination of direct $d$-quark sea-quark loop contributions to the proton magnetic moment as illustrated in Fig. I

| Diagram | Channel | Mass | Charge | Term | $\beta$ | $\chi$ |
|---------|---------|------|--------|------|--------|--------|
| $c$     | $\Sigma^+ K^0$ | $N\pi$ | $-1$   | $-2f^2_{\Sigma N}\pi m_{\pi}$ | $(D - F)^2$ | +0.29  |
| $g$     | $\Lambda K^+$ | $N\pi$ | $-1$   | $-f^2_{\Sigma N}\pi m_{\pi}$ | $(3F + D)^2/6$ | +3.68  |
| $g$     | $\Sigma^0 K^+$ | $N\pi$ | $-1$   | $-f^2_{\Sigma N}\pi m_{\pi}$ | $(D - F)^2/2$ | +0.15  |

| Total   |         |      |        |      |        | +4.12  |

TABLE VIII: Indirect sea-quark loop contributions from $d$ valence quarks to the proton magnetic moment. Here the $d$-valence quark forms a meson composed with a sea-quark loop as illustrated in Fig. I

| Diagram | Channel | Mass | Charge | Term | $\beta$ | $\chi$ |
|---------|---------|------|--------|------|--------|--------|
| $c$     | $\Sigma^+ K^0$ | $N\pi$ | $+1$   | $2f^2_{\Sigma N}\pi m_{\pi}$ | $-(D - F)^2$ | -0.29  |
| $e$     | $\Sigma^+ K^0$ | $N\pi$ | $+1$   | $2f^2_{\Sigma N}\pi m_{\pi}$ | $-(D - F)^2$ | -0.29  |
| $i$     | $\Sigma^+ K^0$ | $\Sigma K$ | $+1$   | $+2f^2_{\Sigma N}\pi m_{\Sigma K}$ | $-(D - F)^2$ | -0.29  |

TABLE IX: Determination of the total $s$-quark contribution to the proton magnetic moment as illustrated in Fig. I

As there are no $s$ valence quarks in the proton, the contributions are purely sea-quark-loop contributions.

| Diagram | Channel | Mass | Charge | Term | $\beta$ | $\chi$ |
|---------|---------|------|--------|------|--------|--------|
| $h$     | $\Sigma^0 K^+$ | $\Sigma K$ | $-1$   | $-f^2_{\Sigma N}\pi m_{\Sigma K}$ | $(D - F)^2/2$ | 0.15   |
| $h$     | $\Lambda K^+$ | $\Lambda K$ | $-1$   | $-f^2_{\Sigma N}\pi m_{\Lambda K}$ | $(3F + D)^2/6$ | 3.68   |
| $i$     | $\Sigma^+ K^0$ | $\Sigma K$ | $-1$   | $-2f^2_{\Sigma N}\pi m_{\Sigma K}$ | $(D - F)^2$ | 0.29   |

sea-quark loop contributions to the proton form factor. In each case there are three channels for the coupling, $\beta$. Indeed the $u$ and $d$ direct sea-quark loop contributions are exactly equal. However $SU(3)$-flavor symmetry breaking due to the massive $s$ quark requires one to track the masses of intermediate mesons and baryons, and this introduces the $K$, $\Sigma$ and $\Lambda$ masses in Table IX.

$SU(3)$-flavor symmetry is also manifest in the indirect sea-quark loop contributions to the proton magnetic moment. For example, the $u$-quark indirect sea-quark loop result receives contributions from each of $u$, $d$ and $s$ quark loops in Figs. I(b), I(g) and I(h). Each of these contributions appearing in Table VIII are equal up to symmetry breaking in the meson and baryon masses. Similar results hold for the $d$-valence quark of the proton in Table VIII.

The flavor-blind nature of QCD makes it trivial to extend this calculation of quenched quark-sector magnetic moments to the partially-quenched theory. As new flavors are introduced through the use of dynamically generated gauge fields, one simply adds the direct and indirect sea-quark loop contributions evaluated here to the quenched results, keeping track of the meson mass of the valence-sea meson. The latter is simple to do as we have already isolated each valence quark flavor contribution to the baryon moment. This is described in further detail in Sec. VII.

B. Quark-Sector Contributions to $\Sigma^+$

Tables X through XII describe the various LNA contributions of the $u$ quark to the $\Sigma^+$ magnetic moment derived from Fig. 2. The total contribution of the $u$ quark alone is isolated by setting the charge of the $s$ and $d$ quarks to zero, and otherwise using standard techniques. The LNA behavior of the $u$-quark contribution to the $\Sigma^+$ magnetic moment is $-2.16 m_{\Sigma\pi} - 1.67 m_{\Sigma\pi} - 6.87 m_{\Sigma K}$. Sea-quark-loop contributions are isolated by using meson-baryon couplings where quark loops of $u$ and
s quark flavors (the valence flavors of $\Sigma^+$) are replaced by a $d$ quark. Table XI summarizes the direct sea-quark loop contributions. Subtracting these contributions leaves a net valence contribution of $-0.29 m_{\Lambda NK} - 4.32 m_{\Sigma \Sigma} - 3.33 m_{\Sigma \Lambda} - 6.87 m_{\Sigma \Xi}$ in full QCD. Table XII describes indirect sea-quark loop contributions from $u$ valence quarks in mesons formed with a sea-quark loop. Removing these contributions provides the net quenched $u$-valence contribution of $-0.29 m_{\Lambda NK} - 3.04 m_{\Sigma \Xi}$.

The $d$-quark contributions to the LNA behavior of the $\Sigma^+$ magnetic moment are pure sea in origin. Therefore the total contributions are the sea contributions such that the valence $d$-quark contributions vanish. Table XIII summarizes the contributions.

$s$-quark contributions to the $\Sigma^+$ magnetic moment are summarized in Tables XIV through XVI. Removal of the direct sea-quark loop contributions from the total contributions provides an $s$-valence contribution of $-0.29 m_{\Lambda NK} + 3.04 m_{\Sigma \Xi}$ in full QCD. Further removal of the indirect sea-quark loop contributions of Table XIV provides the net quenched $s$-valence contribution of $+0.29 m_{\Lambda NK} + 3.04 m_{\Sigma \Xi}$.

Charge symmetry provides the quark sector contributions to the $\Sigma^-$ baryon, while the $\Sigma^0$-baryon results are obtained from the isospin average of $\Sigma^+$ and $\Sigma^-$. The $SU(3)$-flavor symmetry of the direct sea-quark-loop contributions to the $\Sigma^+$ baryon magnetic moment is manifest throughout Tables XI, XIII and XV. However, the implementation of $SU(3)$-flavor breaking via the hadron masses hides the flavor symmetry in the results summarized in Sec. XV where both meson and baryon mass splittings are maintained. $SU(3)$-flavor breaking gives rise to very different behaviors for these contributions. This is particularly true in the common application of holding the strange-quark mass fixed while varying the light $u$ and $d$ masses. In this case the $\eta_s$ meson mass is constant.

C. $\Lambda$ and $\Xi$ Baryons

The derivation of the quark sector contributions to $\Xi$ baryons proceeds in precisely the same manner as that for the $\Sigma$ baryons. As there are no new concepts, derivation is left as an exercise for the interested reader. $\Xi$-baryon results are summarized in Sec. XV.

However, the flavor singlet structure of the $\Lambda$ baryon presents a problem to the approach described thus far. The necessary presence of $u$-, $d$- and $s$-quark flavors simultaneously, appears to require the introduction of a fourth quark flavor and its associated $SU(4)$ couplings to describe the disconnected sea-quark loop contributions.

Fortunately one can exploit the $SU(3)$-flavor symmetry relation among octet baryons. Such a relation is manifest in two- and three-point correlation functions for the $\Lambda$ [35]. Denoting the two-point correlation function for $\Sigma_0$ as $\Sigma_0(x)$, one has

$$\Lambda(x) = \frac{1}{3} \left( 2 \Sigma^0_0(x) + 2 \Sigma^0_0(x) - \Sigma^0_0(x) \right), \quad (19)$$

where $\Sigma^0_0(x)$ has symmetry between $u$ and $d$ quarks, $\Sigma^0_0(x)$ has symmetry between $s$ and $d$ quarks, and similarly $\Sigma^0_0(x)$ has symmetry between $u$ and $s$ quarks. Just as

$$\Sigma^0_0(x) = \frac{1}{2} \left( \Sigma^+(x) + \Sigma^-(x) \right), \quad (20)$$

FIG. 2: The pseudo-Goldstone meson cloud of $\Sigma^+$ and associated quark flow diagrams.
quark-flavor contributions can be resolved.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Diagram & Channel & Mass & Charge & Term & $\beta$ & $\chi$ \\
\hline
\hline
c & $\Lambda^0$ & $\pi^+$ & $\Sigma\pi$ & $+1$ & $f_{\Sigma\pi}^0$ & $-2F^2$ & -2.16 \\
\hline
c & $\Lambda^0$ & $\Lambda\pi$ & $+1$ & $f_{\Lambda\pi}^0$ & $-2D^2/3$ & -1.67 \\
\hline
g,h & $\Xi^0$ & $K^+$ & $\Xi K$ & $+1$ & $2f_{\Xi K}$ & $-(F+D)^2$ & -6.87 \\
\hline
\end{tabular}
\caption{Determination of the total $u$-quark contribution to the $\Sigma^+$ magnetic moment as illustrated in Fig. 2.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Diagram & Channel & Mass & Charge & Term & $\beta$ & $\chi$ \\
\hline
\hline
b & $\Sigma^0$ & $\pi^+$ & $\Sigma\pi$ & $-1$ & $-f_{\Sigma\pi}^0$ & $2F^2$ & +2.16 \\
\hline
b & $\Lambda^0$ & $\Lambda\pi$ & $+1$ & $f_{\Lambda\pi}^0$ & $-2D^2/3$ & -1.67 \\
\hline
c & $\Sigma^0$ & $\pi^+$ & $\Sigma\pi$ & $+1$ & $f_{\Sigma\pi}^0$ & $-2F^2$ & -2.16 \\
\hline
c & $\Lambda^0$ & $\Lambda\pi$ & $+1$ & $f_{\Lambda\pi}^0$ & $-2D^2/3$ & -1.67 \\
\hline
h & $\Sigma^0$ & $\Xi K$ & $+1$ & $f_{\Xi K}^0$ & $-2F^2$ & -2.16 \\
\hline
h & $\Lambda^0$ & $\Xi K$ & $+1$ & $f_{\Xi K}^0$ & $-2D^2/3$ & -1.67 \\
\hline
\end{tabular}
\caption{Determination of direct $u$-quark sea-quark loop contributions to the $\Sigma^+$ magnetic moment as illustrated in Fig. 2.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Diagram & Channel & Mass & Charge & Term & $\beta$ & $\chi$ \\
\hline
\hline
b & $\Sigma^0$ & $\pi^+$ & $\Sigma\pi$ & $+1$ & $f_{\Sigma\pi}^0$ & $-2F^2$ & -2.16 \\
\hline
b & $\Lambda^0$ & $\Lambda\pi$ & $+1$ & $f_{\Lambda\pi}^0$ & $-2D^2/3$ & -1.67 \\
\hline
c & $\Sigma^0$ & $\pi^+$ & $\Sigma\pi$ & $+1$ & $f_{\Sigma\pi}^0$ & $-2F^2$ & -2.16 \\
\hline
c & $\Lambda^0$ & $\Lambda\pi$ & $+1$ & $f_{\Lambda\pi}^0$ & $-2D^2/3$ & -1.67 \\
\hline
h & $\Sigma^0$ & $\Xi K$ & $+1$ & $f_{\Xi K}^0$ & $-2F^2$ & -2.16 \\
\hline
h & $\Lambda^0$ & $\Xi K$ & $+1$ & $f_{\Xi K}^0$ & $-2D^2/3$ & -1.67 \\
\hline
\end{tabular}
\caption{Indirect sea-quark loop contributions from $u$-valence quarks to the $\Sigma^+$ magnetic moment. Here the $u$-valence quark forms a meson composed with a sea-quark loop as illustrated in Fig. 2.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Diagram & Channel & Mass & Charge & Term & $\beta$ & $\chi$ \\
\hline
\hline
b & $\Sigma^0$ & $\pi^+$ & $\Sigma\pi$ & $-1$ & $-f_{\Sigma\pi}^0$ & $2F^2$ & +2.16 \\
\hline
b & $\Lambda^0$ & $\Lambda\pi$ & $+1$ & $f_{\Lambda\pi}^0$ & $-2D^2/3$ & -1.67 \\
\hline
c & $\Sigma^0$ & $\pi^+$ & $\Sigma\pi$ & $+1$ & $f_{\Sigma\pi}^0$ & $-2F^2$ & -2.16 \\
\hline
c & $\Lambda^0$ & $\Lambda\pi$ & $+1$ & $f_{\Lambda\pi}^0$ & $-2D^2/3$ & -1.67 \\
\hline
h & $\Sigma^0$ & $\Xi K$ & $+1$ & $f_{\Xi K}^0$ & $-2F^2$ & -2.16 \\
\hline
h & $\Lambda^0$ & $\Xi K$ & $+1$ & $f_{\Xi K}^0$ & $-2D^2/3$ & -1.67 \\
\hline
\end{tabular}
\caption{Determination of the total $d$-quark contribution to the $\Sigma^+$ magnetic moment as illustrated in Fig. 2. $d$-quark contributions are purely sea in origin such that the valence contribution vanishes.}
\end{table}

one also has

$$\Sigma_u^0(x) = \frac{1}{2} \left( n(x) + \Xi^0(x) \right), \quad (21)$$

and

$$\Sigma_d^0(x) = \frac{1}{2} \left( p(x) + \Xi^-(x) \right), \quad (22)$$

in the SU(3)-flavor limit, such that

$$\Lambda(x) = \frac{1}{3} \left( p(x) + n(x) + \Xi^0(x) + \Xi^-(x) \right) - \frac{1}{2} \left( \Sigma^+(x) + \Sigma^-(x) \right) \cdot (23)$$

Of course it is essential to recover SU(3)-flavor violations. To do this one begins exactly as for the proton or $\Sigma^+$ described above by constructing the quark-flow diagrams describing the one-loop meson cloud of the $\Lambda$. The couplings of all sea-quark loop contributions can be related to the three quark-flow diagrams of Fig. 3. Unknown couplings $f_u^2$, $f_d^2$ and $f_s^2$ are introduced to describe the couplings of diagrams (a), (b) and (c) respectively. Our working approximation of exact $SU(2)$-isospin symmetry at the current-quark level provides $f_u^2 = f_d^2$ in $\Lambda$, leaving two parameters, $f_u^2$ and $f_s^2$, to be determined via the $SU(3)$ relation of Eq. (20). As both the light- and strange-quark contributions to the $\Lambda$ moment can be reolved, there are two $SU(3)$ relations to constrain the two parameters $f_u^2$ and $f_s^2$. This is particularly easy, when one recalls that the indirect sea-quark loop contribution from a $u$ or $s$ valence quark participating in a meson con-
TABLE XIV: Determination of the total s-quark contribution to the \( \Sigma^+ \) magnetic moment as illustrated in Fig. 2.

| Diagram | Channel | Mass | Charge | Term | \( \beta \) | \( \chi \) |
|---------|---------|------|--------|------|------------|--------|
| f, g, h | \( p\bar{K}^0 \) | \( NK \) | +1 | \( 2f^2_{\Sigma NK} m_{\Sigma NK} \) | \( -(D - F)^2 \) | −0.29 |
|         | \( \Xi^0 K^+ \) | \( \Xi K \) | −1 | \( -2f^2_{\Xi\Xi K} m_{\Xi\Xi K} \) | \( (F + D)^2 \) | +6.87 |

TABLE XV: Determination of direct s-quark sea-quark loop contributions to the \( \Sigma^+ \) magnetic moment as illustrated in Fig. 2. \( \eta_s \) denotes the s\( s \) \( \eta \) meson.

| Diagram | Channel | Mass | Charge | Term | \( \beta \) | \( \chi \) |
|---------|---------|------|--------|------|------------|--------|
| h       | \( \Sigma^0 \pi^+ \) | \( \Xi K \) | −1 | \( -f^2_{\Sigma\pi K} m_{\Sigma\pi K} \) | \( 2F^2 \) | +2.16 |
| h       | \( \Lambda \pi^+ \) | \( \Xi K \) | −1 | \( -f^2_{\Lambda\pi K} m_{\Lambda\pi K} \) | \( 2D^2/3 \) | +1.67 |
| i       | \( p\bar{K}^0 \) | \( \Sigma \eta_s \) | −1 | \( -2f^2_{\Sigma\eta_s K} m_{\Sigma\eta_s K} \) | \( (D - F)^2 \) | +0.29 |

TABLE XVI: Indirect sea-quark loop contributions from s valence quarks to the \( \Sigma^+ \) magnetic moment. Here the s-valence quark forms a meson composed with a sea-quark loop as illustrated in Fig. 2. \( \eta_s \) denotes the s\( s \) \( \eta \) meson.

| Diagram | Channel | Mass | Charge | Term | \( \beta \) | \( \chi \) |
|---------|---------|------|--------|------|------------|--------|
| e       | \( p\bar{K}^0 \) | \( NK \) | +1 | \( 2f^2_{\Sigma NK} m_{\Sigma NK} \) | \( -(D - F)^2 \) | −0.29 |
| f       | \( p\bar{K}^0 \) | \( NK \) | +1 | \( 2f^2_{\Sigma NK} m_{\Sigma NK} \) | \( -(D - F)^2 \) | −0.29 |
| i       | \( p\bar{K}^0 \) | \( \Sigma \eta_s \) | +1 | \( 2f^2_{\Sigma\eta_s K} m_{\Sigma\eta_s K} \) | \( -(D - F)^2 \) | −0.29 |

constructed with a sea-quark loop are proportional to either \( f^2_u \) or \( f^2_s \) alone. Results are summarized in Sec. IV.

D. Quenched Exotics

The double hair-pin graph of Fig. 4 associated with the quenched-\( \eta' \) meson gives rise to new singular \( \log(m_\pi) \) behavior in the chiral limit \[28\]. This logarithmic term provides a correction to the tree-level term. The contribution has its origin in the loop integral of Fig. 4(a)

\[
-i \frac{16\pi^2}{3} \int \frac{d^4q}{(2\pi)^4} \frac{q^2 - (v \cdot q)^2}{i[q^2 - (m_1^2 + i\epsilon)][q^2 - (m_2^2 + i\epsilon)]}.
\] (24)

For equal singlet-meson masses \( m_1 = m_2 \), as in the quark flow of Fig. 4(b), this integral provides the nonanalytic

FIG. 3: Key quark-flow diagrams for the \( \Lambda \) baryon. Diagrams (a), (b) and (c) are proportional to the introduced couplings \( f^2_u \), \( f^2_d \) and \( f^2_s \) respectively.

FIG. 4: Diagrams giving rise to the logarithmic divergence of a baryon magnetic moment in the quenched approximation. The cross on the meson propagator in (a) denotes the double hairpin graph of the quark-flow diagrams of (b) and (c).
behavior of
\[ \log \left( \frac{m^2}{\Lambda^2} \right). \] (25)

However, Fig. 4(c) indicates that the meson masses in the
double hair-pin graph need not be equal when considering
hyperon magnetic moments. Consider for example, \( \eta' (u\bar{u}) \) versus \( \eta'(s\bar{s}) \). When \( m_1 \neq m_2 \), one
finds a nonanalytic contribution of
\[ \frac{m_1^2 \log \left( \frac{m_1^2}{\Lambda^2} \right) - m_2^2 \log \left( \frac{m_2^2}{\Lambda^2} \right)}{m_1^2 - m_2^2}. \] (26)

Hence, for the hyperons, one must isolate the doubly-
and singly-represented quark sector couplings to \( \eta' \)
mesons. Consider for example, \( \eta' \) couplings for \( \Sigma^+ \) in-
volving \( u \) quarks. The transition \( \Sigma^+ \to \Sigma^+\pi^0 \) involves \( u \)
quarks alone at one loop and can be used to determine the \( \eta_u \)
coupling. The \( \Sigma^+\pi^0 \) coupling is \( f_{\Sigma\Sigma\pi} = 2 F \).

\[ |\pi^0\rangle = \frac{1}{\sqrt{2}} \left( |u\bar{u}\rangle - |d\bar{d}\rangle \right) \],
\[ |\eta\rangle = \frac{1}{\sqrt{6}} \left( |u\bar{u}\rangle + |d\bar{d}\rangle - 2 |s\bar{s}\rangle \right) \],
\[ |\eta'\rangle = \frac{1}{\sqrt{3}} \left( |u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle \right) \].

one has
\[ |u\bar{u}\rangle = \frac{1}{\sqrt{6}} \left( \sqrt{3} |\pi^0\rangle + |\eta\rangle + \sqrt{2} |\eta'\rangle \right). \] (27)

The \( \eta_u \) coupling is \( \sqrt{2/3} \) of the pion coupling to \( |u\bar{u}\rangle \);
i.e. \( 2\sqrt{2/3} F \).

For the singly represented quark sector, consider \( \Xi^0 \to \Xi^0\pi^0 \) involving \( u \)-quarks alone at one loop. Here the \( u \)-
quark coupling is \( f_{\Xi\Xi\pi} = (F - D) \), such that the \( \eta_u \)
coupling to \( \Xi \) baryons is \( \sqrt{2/3}(F - D) \).

To check this separation of quark sector contributions to \( \eta' \) contributions, consider the proton. Here the contribution
to intermediate \( \eta' \) states is
\[ \frac{2}{3} \left\{ 4 F^2 I(\eta_u, \eta_u) + 2 \cdot 2 F (F - D) \rangle I(\eta_u, \eta_d) \right. \\
\left. + (F - D)^2 I(\eta_d, \eta_d) \right\}, \] (28)

where \( I(\eta_u, \eta_d) \) denotes the loop integral of Fig. 4(a) for
quark flow diagram Fig. 4(c). For equal \( \eta_u \) and \( \eta_d \) masses
one recovers
\[ \frac{2}{3} (3F - D)^2 \log \left( \frac{m^2}{\Lambda^2} \right), \] (29)

where the leading factor is the standard \( NN\eta' \) coupling.

Double-hairpin \( \eta' \) contributions to \( \Sigma \) and \( \Xi \) baryons may
be obtained from Eq. (28) with the appropriate quark-
flavor assignments. For example, the factor multiplying
the tree level contribution to \( \Xi^- \)-baryon quark-sector
magnetic moments is
\[ 1 - \xi_0 \left[ \frac{2}{3} \left\{ 4 F^2 \log \left( \frac{m^2}{\Lambda^2} \right) \\
+ 2 \cdot 2 F (F - D) \rangle I(\eta_u, \eta_d) \right. \\
\left. + (F - D)^2 \log \left( \frac{m^2}{\Lambda^2} \right) \right\} \right], \] (30)

where remaining loop-integral factors have been incorpor-
ated in
\[ \xi_0 = \frac{M_0^2}{16 \pi^2 f^2}, \] (31)

with the double hair-pin interaction strength \( M_0 \sim 0.75 \)
GeV [16,17]. While this logarithmic divergence domi-
nates the chiral expansion near the chiral limit, applica-
tion of these results to the extrapolation of the quenched
proton magnetic moment [13] reveals that the curvature
associated with this term is small for \( m^2_\pi \geq 0.1 \) GeV².

IV. RESULTS

This approach allows one to separate an individual
quark-flavor contribution to a baryon form factor into
five categories, namely: “total” full-QCD contributions,
direct sea-quark loop,” and “valence” contributions of
full-QCD, obtained by removing the direct current cou-
ing to sea-quark loops from the total contributions.
Upon further removing “indirect sea-quark loop” con-
tributions, one obtains the “quenched valence” contribu-
tions. Tables XVII and XVIII report the axial couplings
for these quark-sector contributions to baryon magnetic
moments.

The LNA “direct sea-quark loop” contribution is rele-
ant to disconnected insertions of the EM current in ei-
ther full or quenched QCD, whereas the LNA “valence”
contribution is relevant to connected insertions of the EM
current only in full QCD. The final category of “quenched
valence” contributions is relevant to connected insertions
of the EM current in quenched QCD. The latter is com-
monly referred to as the quenched QCD result.

The channels denoted \( \Omega \) in Tables XVII XVIII
and in the following actually involve the propagation of an
octet \( sss \) baryon; i.e. the \( \Xi^- \) baryon with \( m_d = m_s \).
In separating valence and sea-quark loop contributions,
the cancellation of valence and sea-quark loop octet-
ssss-baryon contributions does not occur. Figure 5
provides quark flow diagrams for \( \Xi^0 \to \Omega^- K^+ \) which
illustrate this phenomenon.

In the \( SU(3) \)-flavor symmetry limit, Figs. 4(a) and (b)
are equivalent due to the flavor-blindness of QCD inter-
actions. Fig. 4(b) certainly has overlap with an octet-
\( \Xi^- \pi^+ \) intermediate state. Hence the diagram of Fig. 4(a)
also has an octet baryon propagating in the intermediate state. Of course, we know there is no \textit{sss} octet baryon and this problem is solved by the contribution of Fig. 5(c) which must be equal but opposite in sign to Fig. 5(a) when an octet baryon propagates in the intermediate state, thus eliminating the octet \textit{sss} baryon contribution in full QCD. In separating valence and sea contributions, each quark flow graph must be taken on its own such that octet baryons are not necessarily eliminated. Indeed, in quenched QCD, only Fig. 5(c) survives, and this quark flow graph has an \textit{sss}-octet baryon propagating in the intermediate state. To some extent this physics has already been seen in Figs. 1(d) plus (e) where only a decuplet baryon can contribute in full QCD, but octet baryons provide contributions in the process of separating valence and sea sectors.

Baryon moments are constructed from the quark sector coefficients by multiplying the \( u \), \( d \) and \( s \) results by their appropriate charge factors and summing. For example, the proton moment is

\[
\mu_p = \frac{2}{3} u_p - \frac{1}{3} d_p - \frac{1}{3} s_p, \tag{32}
\]

and the neutron moment is

\[
\mu_n = -\frac{1}{3} u_p + \frac{2}{3} d_p - \frac{1}{3} s_p. \tag{33}
\]

Similarly, the \( \Sigma^+ \) moment is

\[
\mu_{\Sigma^+} = \frac{2}{3} u_{\Sigma^+} - \frac{1}{3} d_{\Sigma^+} - \frac{1}{3} s_{\Sigma^+}, \tag{34}
\]

and the \( \Sigma^- \) moment is

\[
\mu_{\Sigma^-} = -\frac{1}{3} u_{\Sigma^-} + \frac{2}{3} d_{\Sigma^-} - \frac{1}{3} s_{\Sigma^-}. \tag{35}
\]

Tables XXI and XXII report the axial couplings for the intermediate meson-baryon channels contributing to the nonanalytic behavior of baryon magnetic moments. We note that upon neglecting the baryon mass splittings, one recovers the full QCD results of Ref. 34 summarized in their Eqs. (A.2) and (A.4), and the quenched results of Ref. 28 summarized in their Table 2.

Table XXI reports values for the coefficient, \( \chi \), providing the LNA contribution to baryon magnetic moments (\( \chi m_\pi \) or \( \chi m_K \) or \( \chi m_\eta \), as appropriate) by quark sectors with each quark flavor normalized to unit charge. Charge symmetry provides the contributions for other baryons. Values are based on the tree-level axial couplings \( F = 0.50 \) and \( D = 0.76 \) with \( f_\pi = 93 \) MeV. Similar results for bulk baryon moments are provided in Table XXII. For convenience, values using the one-loop corrected values \( F = 0.40 \) and \( D = 0.61 \) are provided in Tables XXIII and XXIV respectively.

V. PARTIAL QUENCHING

A. Hadron Masses

The flavor-blind nature of QCD makes it trivial to extend this calculation of quenched baryon magnetic moments to the partially-quenched theory. As new flavors are introduced through the use of dynamically generated gauge fields, one simply adds the direct and indirect sea-quark loop contributions evaluated in Sec. III to the quenched results of Sec. IV. To incorporate hadron mass violations of \( SU(3) \)-flavor symmetry, one must track the meson mass of the valence-sea meson. As we have already isolated each valence quark flavor contribution to the baryon moment, the mass of the meson is identified by the valence- and sea-quark mass composing the meson.

It should be noted that the double hair-pin graph of the \( \eta' \) meson remains anomalous in the partially-quenched theory 19. However, the contribution of the \( \eta' \) propagator is suppressed by the difference in valence- and sea-quark masses.

B. Sea- and Ghost-Quark Electric Charge Assignments

There has been some discussion on the electric charge assignments that may be applied to the various quark sectors of partially-quenched effective field theory 29. In the conventional view of quenched chiral perturbation theory, the charges of the commuting ghost-quark fields are tied to the valence quark charges in order to eliminate both the direct and indirect sea-quark loop contributions of the valence sector. Similarly, for partially-quenched chiral perturbation theory, it is usually argued that the ghost quarks are identical to the valence quarks, except for their statistics.

However, it has been indicated that when the number of sea quarks matches the number of valence quarks, more general charge assignments are possible 29. The idea is that when the masses and charges of the sea-
TABLE XVII: Coefficients, $\beta$, providing the LNA contribution to nucleon, $\Sigma$ and $\Lambda$-baryon magnetic moments by quark sectors with quark charges normalized to unit charge. Intermediate (Int.) meson-baryon channels are indicated to allow for $SU(3)$-flavor breaking in both the meson and baryon masses. Total, direct sea-quark loop, valence, indirect sea-quark loop and quenched valence loop and quenched valence coefficients are indicated.

| $q$ | Int. | Total Quark Sector | Direct Sea-Quark Loop | Valence Sector | Indirect Loop | Quenched Valence |
|-----|------|--------------------|------------------------|---------------|---------------|-----------------|
| $u_p$ | $\Sigma\pi$ | $-2F^2$ | $2F^2$ | $-4F^2$ | $-4F^2$ | 0 |
|       | $\Lambda\pi$ | $-2D^2/3$ | $2D^2/3$ | $-4D^2/3$ | $-4D^2/3$ | 0 |
|       | $NK$ | 0 | $(D - F)^2$ | $-(D - F)^2$ | 0 | 0 |
|       | $\Xi K$ | $-2(D + F)^2$ | 0 | $-(D - F)^2$ | $-2(D - F)^2$ | $(D^2 + 6DF - 3F^2)/3$ |
| $d_p$ | $\Sigma\pi$ | 2$F^2$ | 0 | 0 | 0 | 0 |
|       | $\Lambda\pi$ | $2D^2/3$ | $2D^2/3$ | 0 | 0 | 0 |
|       | $NK$ | $(D - F)^2$ | $(D - F)^2$ | 0 | 0 | 0 |
|       | $\Xi K$ | $(D + F)^2$ | $2(D^2 + 3F^2)/3$ | $(D^2 + 6DF - 3F^2)/3$ | 0 | $(D^2 + 6DF - 3F^2)/3$ |
| $s_p$ | $\Lambda K$ | $(D + 3F)^2/6$ | $(D + 3F)^2/6$ | 0 | 0 | 0 |
|       | $\Sigma K$ | $3(D - F)^2/2$ | $3(D - F)^2/2$ | 0 | 0 | 0 |
| $u_{\Sigma^+}$ | $\Sigma\pi$ | $-2F^2$ | $2F^2$ | $-4F^2$ | $-4F^2$ | 0 |
|       | $\Lambda\pi$ | $-2D^2/3$ | $2D^2/3$ | $-4D^2/3$ | $-4D^2/3$ | 0 |
|       | $NK$ | 0 | $(D - F)^2$ | $-(D - F)^2$ | 0 | 0 |
|       | $\Xi K$ | $-2(D + F)^2$ | 0 | $-(D - F)^2$ | $-2(D - F)^2$ | $(D^2 + 6DF - 3F^2)/3$ |
| $d_{\Sigma^+}$ | $\Sigma\pi$ | $2F^2$ | $2F^2$ | 0 | 0 | 0 |
|       | $\Lambda\pi$ | $2D^2/3$ | $2D^2/3$ | 0 | 0 | 0 |
|       | $NK$ | $(D - F)^2$ | $(D - F)^2$ | 0 | 0 | 0 |
|       | $\Xi K$ | $(D + F)^2$ | $2(D^2 + 3F^2)/3$ | $(D^2 + 6DF - 3F^2)/3$ | 0 | $(D^2 + 6DF - 3F^2)/3$ |
| $s_{\Sigma^+}$ | $\Sigma\eta$ | 0 | $(D - F)^2$ | $-(D - F)^2$ | 0 | 0 |
|       | $NK$ | $-2(D - F)^2$ | 0 | $-(D - F)^2$ | $-2(D - F)^2$ | 0 |
|       | $\Xi K$ | $(D + F)^2$ | $2(D^2 + 3F^2)/3$ | $(D^2 + 6DF - 3F^2)/3$ | 0 | $(D^2 + 6DF - 3F^2)/3$ |
| $u_{\Sigma^0} | d_{\Sigma^0}$ | $\Sigma\pi$ | 0 | $2F^2$ | $-2F^2$ | 0 |
|       | $\Lambda\pi$ | 0 | $2D^2/3$ | $-2D^2/3$ | $-2D^2/3$ | 0 |
|       | $NK$ | $(D - F)^2/2$ | $(D - F)^2/2$ | 0 | 0 | 0 |
|       | $\Xi K$ | $-2(D + F)^2/2$ | 0 | $-(D - F)^2/2$ | $-(D - F)^2/2$ | $(D^2 + 6DF - 3F^2)/6$ |
TABLE XVIII: Coefficients, $\beta$, providing the LNA contribution to $\Xi$-baryon magnetic moments by quark sectors with quark charges normalized to unit charge. Intermediate (Int.) meson-baryon channels are indicated to allow for $SU(3)$-flavor breaking in both the meson and baryon masses. Total, direct sea-quark loop, valence, indirect sea-quark loop and quenched valence coefficients are indicated.

| $q$ | Int. | Total Quark Sector | Direct Sea-Quark Loop | Valence Sector | Indirect Loop | Quenched Valence |
|-----|------|--------------------|-----------------------|---------------|---------------|-----------------|
| $u_{\Xi}$ | $\Xi\pi$ | $-(D - F)^2$ | $(D - F)^2$ | $-2(D - F)^2$ | $-2(D - F)^2$ | 0 |
| | $\Lambda K$ | 0 | $(D - 3F)^2/6$ | $-(D - 3F)^2/6$ | 0 | $-(D - 3F)^2/6$ |
| | $\Sigma K$ | $(D + F)^2$ | $(D + F)^2/2$ | $(D + F)^2/2$ | 0 | $(D + F)^2/2$ |
| | $\Omega K$ | 0 | 0 | $-(D - F)^2$ | 0 | $(D - F)^2$ |

| $d_{\Xi}$ | $\Xi\pi$ | $(D - F)^2$ | $(D - F)^2$ | 0 | 0 | 0 |
| | $\Lambda K$ | $(D - 3F)^2/6$ | $(D - 3F)^2/6$ | 0 | 0 | 0 |
| | $\Sigma K$ | $(D + F)^2/2$ | $(D + F)^2/2$ | 0 | 0 | 0 |

| $s_{\Xi}$ | $\Lambda K$ | $-(D - 3F)^2/6$ | 0 | $-(D - 3F)^2/6$ | $-(D - 3F)^2/3$ | $(D - 3F)^2/6$ |
| | $\Sigma K$ | $-3(D + F)^2/2$ | 0 | $-3(D + F)^2/2$ | $-(D + F)^2$ | $-(D + F)^2/2$ |
| | $\Omega K$ | 0 | $(D - F)^2$ | $-(D - F)^2$ | 0 | $-(D - F)^2$ |
| | $\Sigma \eta_s$ | 0 | $2(D^2 + 3F^2)/3$ | $-2(D^2 + 3F^2)/3$ | $-2(D^2 + 3F^2)/3$ | 0 |

TABLE XIX: Coefficients, $\beta$, providing the LNA contribution to nucleon magnetic moments. Intermediate (Int.) meson-baryon channels are indicated to allow for $SU(3)$-flavor breaking in both the meson and baryon masses.

| Baryon | Int. | Total Quark Sector | Direct Sea-Quark Loop | Valence Sector | Indirect Loop | Quenched Valence |
|--------|------|--------------------|-----------------------|---------------|---------------|-----------------|
| $p$ | $N\pi$ | $-(D + F)^2$ | $(5D^2 - 6DF + 9F^2)/9$ | $2(-7D^2 - 6DF - 9F^2)/9$ | $-2(D + 3F)^2/9$ | $-4D^2/3$ |
| | $\Lambda K$ | $-(D + 3F)^2/18$ | $-(D + 3F)^2$ | $-(D + 3F)^2/9$ | $-(D + 3F)^2/9$ | 0 |
| | $\Sigma K$ | $-(D - F)^2/2$ | $-(D - F)^2$ | 0 | 0 | 0 |
| $n$ | $N\pi$ | $(D + F)^2$ | $(5D^2 - 6DF + 9F^2)/9$ | $4D(D + 6F)/9$ | $8D(-D + 3F)/9$ | $4D^2/3$ |
| | $\Lambda K$ | 0 | $-(D + 3F)^2/18$ | $(D + 3F)^2/18$ | $(D + 3F)^2/18$ | 0 |
| | $\Sigma K$ | $-(D - F)^2$ | $-(D - F)^2/2$ | $-(D - F)^2/2$ | $-(D - F)^2/2$ | 0 |
TABLE XX: Coefficients, $\beta$, providing the LNA contribution to $\Sigma$, $\Lambda$- and $\Xi$-baryon magnetic moments. Intermediate (Int.) meson-baryon channels are indicated to allow for $SU(3)$-flavor breaking in both the meson and baryon masses.

| Baryon | Total Quark Sector | Direct Sea-Quark Loop | Valence Sector | Indirect Loop | Quenched Valence |
|--------|---------------------|-----------------------|----------------|----------------|-----------------|
| $\Sigma^+$ | $\Sigma\pi$ | $-2 F^2$ | $2 F^2/3$ | $-8 F^2/3$ | $-8 F^2/3$ | 0 |
| | $\Lambda\pi$ | $-2 D^2/3$ | $2 D^2/9$ | $-8 D^2/9$ | $-8 D^2/9$ | 0 |
| | $N K$ | $(D - F)^2/3$ | $-(D - F)^2/3$ | $2 (D - F)^2/3$ | $-(D - F)^2$ | |
| | $\Xi K$ | $-(D + F)^2$ | $-2 (D^2 + 3 F^2)/9$ | $-7 D^2 - 18 D F - 3 F^2)/9$ | $-4 (D^2 + 3 F^2)/9$ | $-(D^2 + 6 D F - 3 F^2)/3$ |
| | $\Sigma \eta_s$ | 0 | $-(D - F)^2/3$ | $(D - F)^2/3$ | $(D - F)^2/3$ | 0 |
| $\Sigma^0$ | $\Sigma\pi$ | 0 | $2 F^2/3$ | $-2 F^2/3$ | $-2 F^2/3$ | 0 |
| | $\Lambda\pi$ | 0 | $2 D^2/3$ | $-2 D^2/9$ | $-2 D^2/9$ | 0 |
| | $N K$ | $(D - F)^2/2$ | $(D - F)^2/3$ | $(D - F)^2/6$ | $2 (D - F)^2/3$ | $-(D - F)^2/2$ |
| | $\Xi K$ | $-(D + F)^2/2$ | $-2 (D^2 + 3 F^2)/9$ | $-7 D^2 - 18 D F - 3 F^2)/18$ | $-4 (D^2 + 3 F^2)/9$ | $-(D^2 + 6 D F - 3 F^2)/6$ |
| | $\Sigma \eta_s$ | 0 | $-(D - F)^2/3$ | $(D - F)^2/3$ | $(D - F)^2/3$ | 0 |
| $\Sigma^-$ | $\Sigma\pi$ | $2 F^2$ | $2 F^2/3$ | $4 F^2/3$ | $4 F^2/3$ | 0 |
| | $\Lambda\pi$ | $2 D^2/3$ | $2 D^2/9$ | $4 D^2/9$ | $4 D^2/9$ | 0 |
| | $N K$ | $(D - F)^2$ | $(D - F)^2/3$ | $(D - F)^2/6$ | $2 (D - F)^2/3$ | $-(D - F)^2$ |
| | $\Xi K$ | 0 | $-2 (D^2 + 3 F^2)/9$ | $2 (D^2 + 3 F^2)/9$ | $2 (D^2 + 3 F^2)/9$ | 0 |
| | $\Sigma \eta_s$ | 0 | $-(D - F)^2/3$ | $(D - F)^2/3$ | $(D - F)^2/3$ | 0 |
| $\Lambda$ | $\Sigma\pi$ | 0 | $2 D^2/9$ | $-2 D^2/9$ | $-2 D^2/9$ | 0 |
| | $\Lambda\eta_t$ | 0 | $2 (2 D - 3 F)^2/27$ | $-2 (2 D - 3 F)^2/27$ | $-2 (2 D - 3 F)^2/27$ | 0 |
| | $N K$ | $(D + 3 F)^2/6$ | $(D + 3 F)^2/27$ | $7 (D + 3 F)^2/54$ | $2 (D + 3 F)^2/27$ | $(D + 3 F)^2/18$ |
| | $\Xi K$ | $-(D - 3 F)^2/6$ | $2 (-7 D^2 + 12 D F - 9 F^2)/27$ | $(19 D^2 + 6 D F - 45 F^2)/54$ | $-7 D^2 + 12 D F - 9 F^2)/27$ | $(11 D^2 - 6 D F - 9 F^2)/18$ |
| | $\Lambda \eta_s$ | 0 | $-(D - F)^2/27$ | $(D + 3 F)^2/27$ | $(D + 3 F)^2/27$ | 0 |
| $\Xi^0$ | $\Xi\pi$ | $-(D - F)^2$ | $(D - F)^2/3$ | $-4 (D - F)^2/3$ | $-4 (D - F)^2/3$ | 0 |
| | $\Lambda K$ | 0 | $(D - 3 F)^2/18$ | $-(D - 3 F)^2/18$ | $(D - 3 F)^2/27$ | $(D - 3 F)^2/9$ |
| | $\Sigma K$ | $(D + F)^2$ | $(D + F)^2/6$ | $5 (D + F)^2/6$ | $(D + F)^2/3$ | $(D + F)^2/2$ |
| | $\Omega K$ | 0 | $-(D - F)^2/3$ | $(D - F)^2/3$ | $-2 (D - F)^2/3$ | $(D - F)^2$ |
| | $\xi \eta_s$ | 0 | $2 (D^2 + 3 F^2)/9$ | $2 (D^2 + 3 F^2)/9$ | $2 (D^2 + 3 F^2)/9$ | 0 |
| $\Xi^-$ | $\Xi\pi$ | $(D - F)^2$ | $(D - F)^2/3$ | $2 (D - F)^2/3$ | $2 (D - F)^2/3$ | 0 |
| | $\Lambda K$ | $(D - 3 F)^2/6$ | $(D - 3 F)^2/18$ | $(D - 3 F)^2/27$ | $(D - 3 F)^2/9$ | $(D - 3 F)^2/9$ |
| | $\Sigma K$ | $(D + F)^2/6$ | $(D + F)^2/3$ | $(D + F)^2/3$ | $(D + F)^2/3$ | 0 |
| | $\Omega K$ | 0 | $-(D - F)^2/3$ | $(D - F)^2/3$ | $(D - F)^2/3$ | 0 |
| | $\xi \eta_s$ | 0 | $2 (D^2 + 3 F^2)/9$ | $2 (D^2 + 3 F^2)/9$ | $2 (D^2 + 3 F^2)/9$ | 0 |
TABLE XXI: Coefficients, $\chi_i$, providing the LNA contribution to baryon magnetic moments by quark sectors with quark charges normalized to unit charge. Intermediate (Int.) meson-baryon channels are indicated to allow for $SU(3)$-flavor breaking in both the meson and baryon masses. Total, direct sea-quark loop (Direct Loop), Valence, indirect sea-quark loop (Indirect Loop) and Quenched Valence coefficients are indicated. The axial couplings take the tree-level values $F = 0.50$ and $D = 0.76$ with $f_\pi = 93$ MeV. Note $\epsilon = 0.0004$.

| $q$ | Int. | Total | Direct Loop | Valence | Indirect Loop | Quenched Valence |
|-----|------|-------|-------------|---------|---------------|------------------|
| $u_p$ | $N\pi$ | $-6.87$ | $+4.12$ | $-11.0$ | $-7.65$ | $-3.33$ |
|      | $\Lambda K$ | $-3.68$ | $0$ | $-3.68$ | $-3.68$ | $0$ |
|      | $\Sigma K$ | $-0.15$ | $0$ | $-0.15$ | $-0.15$ | $0$ |
| $d_p$ | $N\pi$ | $+6.87$ | $+4.12$ | $+2.75$ | $-0.59$ | $+3.33$ |
|      | $\Sigma K$ | $-0.29$ | $0$ | $-0.29$ | $-0.29$ | $0$ |
|      | $\Lambda K$ | $+3.68$ | $+3.68$ | $0$ | $0$ | $0$ |
|      | $\Sigma K$ | $+0.44$ | $+0.44$ | $0$ | $0$ | $0$ |
| $s_p$ | $\Sigma\pi$ | $-2.16$ | $+2.16$ | $-4.32$ | $-4.32$ | $0$ |
|      | $\Lambda\pi$ | $-1.67$ | $+1.67$ | $-3.33$ | $-3.33$ | $0$ |
|      | $NK$ | $0$ | $+0.29$ | $-0.29$ | $0$ | $-0.29$ |
|      | $\Xi K$ | $-6.87$ | $0$ | $-6.87$ | $-3.83$ | $-3.04$ |
| $u_{\Sigma^+}$ | $\Sigma\pi$ | $+2.16$ | $+2.16$ | $0$ | $0$ | $0$ |
|      | $\Lambda\pi$ | $+1.67$ | $+1.67$ | $0$ | $0$ | $0$ |
|      | $NK$ | $+0.29$ | $+0.29$ | $0$ | $0$ | $0$ |
| $d_{\Sigma^+}$ | $\Sigma\pi$ | $+2.16$ | $+2.16$ | $0$ | $0$ | $0$ |
|      | $\Lambda\pi$ | $+1.67$ | $+1.67$ | $0$ | $0$ | $0$ |
|      | $NK$ | $+0.29$ | $+0.29$ | $0$ | $0$ | $0$ |
| $s_{\Sigma^+}$ | $NK$ | $-0.29$ | $0$ | $-0.29$ | $-0.59$ | $+0.29$ |
|      | $\Xi K$ | $+6.87$ | $+3.83$ | $+3.04$ | $0$ | $+3.04$ |
|      | $\Sigma\eta_b$ | $0$ | $+0.29$ | $-0.29$ | $0$ | $0$ |
| $u_{\Sigma^0}$ | $\Sigma\pi$ | $0$ | $+2.16$ | $-2.16$ | $-2.16$ | $0$ |
|      | $\Lambda\pi$ | $0$ | $+1.67$ | $-1.67$ | $-1.67$ | $0$ |
|      | $NK$ | $+0.15$ | $+0.29$ | $-0.15$ | $0$ | $-0.15$ |
| $d_{\Sigma^0}$ | $\Sigma\pi$ | $-3.43$ | $0$ | $-3.43$ | $-1.91$ | $-1.52$ |

| $u_{\Lambda} \mid d_{\Lambda}$ | $\Sigma\pi$ | $0$ | $+1.67$ | $-1.67$ | $-1.67$ | $0$ |
| $\Lambda\eta_b$ | $0$ | $\epsilon$ | $-\epsilon$ | $-\epsilon$ | $0$ | $0$ |
| $NK$ | $+3.68$ | $+2.45$ | $+1.23$ | $0$ | $+1.23$ |
| $\Xi K$ | $-0.40$ | $0$ | $-0.40$ | $-0.83$ | $+0.44$ |
| $s_{\Lambda}$ | $\Lambda\eta_b$ | $0$ | $+2.45$ | $-2.45$ | $-2.45$ | $0$ |
| $NK$ | $-7.36$ | $0$ | $-7.36$ | $-4.91$ | $-2.45$ |
| $\Xi K$ | $+0.79$ | $+1.67$ | $-0.88$ | $0$ | $-0.88$ |

| $u_{\Sigma^0}$ | $\Sigma\pi$ | $-0.29$ | $+0.29$ | $-0.59$ | $-0.59$ | $0$ |
|      | $\Lambda K$ | $0$ | $+0.40$ | $-0.40$ | $0$ | $-0.40$ |
|      | $\Sigma K$ | $+6.87$ | $+3.43$ | $+3.43$ | $0$ | $+3.43$ |
| $d_{\Sigma^0}$ | $\Xi\pi$ | $+0.29$ | $+0.29$ | $0$ | $0$ | $0$ |
|      | $\Lambda K$ | $+0.40$ | $+0.40$ | $0$ | $0$ | $0$ |
|      | $\Sigma K$ | $+3.43$ | $+3.43$ | $0$ | $0$ | $0$ |
| $s_{\Sigma^0}$ | $\Lambda K$ | $-0.40$ | $0$ | $-0.40$ | $-0.79$ | $+0.40$ |
|      | $\Sigma K$ | $-10.3$ | $0$ | $-10.3$ | $-6.87$ | $-3.43$ |
|      | $\Omega K$ | $0$ | $+0.29$ | $-0.29$ | $0$ | $-0.29$ |
|      | $\Xi\eta_b$ | $0$ | $+3.83$ | $-3.83$ | $-3.83$ | $0$ |
TABLE XXII: Coefficients, $\chi$, providing the LNA contribution to baryon magnetic moments. Intermediate (Int.) meson-baryon channels are indicated to allow for $SU(3)$-flavor breaking in both the meson and baryon masses. Total, direct sea-quark loop (Direct Loop), Valence, indirect sea-quark loop (Indirect Loop) and Quenched Valence coefficients are indicated. The axial couplings take the tree-level values $F = 0.50$ and $D = 0.76$ with $f_\pi = 93$ MeV. Note $\epsilon = 0.0001$.

| Baryon | Channel | Total | Direct Loop | Valence | Indirect Loop | Quenched Valence |
|--------|---------|-------|-------------|---------|---------------|------------------|
| $p$    | $N\pi$  | -6.87 | +1.37       | -8.24   | -4.91         | -3.33            |
|        | $\Lambda K$ | -3.68 | -1.23       | -2.45   | -2.45         | 0                |
|        | $\Sigma K$ | -0.15 | -0.15       | 0       | 0             | 0                |
| $n$    | $N\pi$  | +6.87 | +1.37       | +5.49   | +2.16         | +3.33            |
|        | $\Lambda K$ | 0     | -1.23       | +1.23   | +1.23         | 0                |
|        | $\Sigma K$ | -0.29 | -0.15       | -0.15   | -0.15         | 0                |
| $\Sigma^+$ | $\Sigma\pi$ | -2.16 | +0.72       | -2.88   | -2.88         | 0                |
|        | $\Lambda \pi$ | -1.67 | +0.56       | -2.22   | -2.22         | 0                |
|        | $NK$    | 0     | +0.10       | -0.10   | +0.20         | -0.29            |
|        | $\Xi K$ | -6.87 | -1.28       | -5.59   | -2.55         | -3.04            |
|        | $\Sigma\eta_b$ | 0     | -0.10       | +0.10   | +0.10         | 0                |
| $\Sigma^0$ | $\Sigma\pi$ | 0     | +0.72       | -0.72   | -0.72         | 0                |
|        | $\Lambda \pi$ | 0     | +0.56       | -0.56   | -0.56         | 0                |
|        | $NK$    | +0.15 | +0.10       | +0.05   | +0.20         | -0.15            |
|        | $\Xi K$ | -3.43 | -1.28       | -2.16   | -0.64         | -1.52            |
|        | $\Sigma\eta_b$ | 0     | -0.10       | +0.10   | +0.10         | 0                |
| $\Sigma^-$ | $\Sigma\pi$ | +2.16 | +0.72       | +1.44   | +1.44         | 0                |
|        | $\Lambda \pi$ | +1.67 | +0.56       | +1.11   | +1.11         | 0                |
|        | $NK$    | +0.29 | +0.10       | +0.20   | +0.20         | 0                |
|        | $\Xi K$ | 0     | -1.28       | +1.28   | +1.28         | 0                |
|        | $\Sigma\eta_b$ | 0     | -0.10       | +0.10   | +0.10         | 0                |
| $\Lambda$ | $\Sigma\pi$ | 0     | +0.56       | -0.56   | -0.56         | 0                |
|        | $\Lambda \eta_b$ | 0     | $\epsilon$ | $-\epsilon$ | $-\epsilon$ | 0                |
|        | $NK$    | +3.68 | +0.82       | +2.86   | +1.64         | +1.23            |
|        | $\Xi K$ | -0.40 | -0.56       | +0.16   | -0.28         | +0.44            |
|        | $\Lambda \eta_b$ | 0     | -0.82       | +0.82   | +0.82         | 0                |
| $\Xi^0$ | $\Xi\pi$ | -0.29 | +0.10       | -0.39   | -0.39         | 0                |
|        | $\Lambda K$ | 0     | +0.13       | -0.13   | +0.26         | -0.40            |
|        | $\Sigma K$ | +6.87 | +1.14       | +5.72   | +2.29         | +3.43            |
|        | $\Omega K$ | 0     | -0.10       | +0.10   | -0.20         | +0.29            |
|        | $\Xi\eta_b$ | 0     | -1.28       | +1.28   | +1.28         | 0                |
| $\Xi^-$ | $\Xi\pi$ | +0.29 | +0.10       | +0.20   | +0.20         | 0                |
|        | $\Lambda K$ | +0.40 | +0.13       | +0.26   | +0.26         | 0                |
|        | $\Sigma K$ | +3.43 | +1.14       | +2.29   | +2.29         | 0                |
|        | $\Omega K$ | 0     | -0.10       | +0.10   | +0.10         | 0                |
|        | $\Xi\eta_b$ | 0     | -1.28       | +1.28   | +1.28         | 0                |
ghost-quarks match, these contributions cancel leaving the theory of full QCD. In this case the charges of the sea and ghost quarks need not be related to the the valence quarks. However, it is essential that the quark masses of the valence and ghost sectors match, such that the indirect sea-quark loop contributions of the valence sector continue to be quenched.

We have already argued in the Introduction that it is important to provide an opportunity to include disconnected insertions of the electromagnetic current in the quenched approximation. These insertions can be calculated in the quenched approximation and give rise to direct sea-quark loop contributions. It is now clear that this goal can be realized in the formal theory of quenched chiral perturbation theory by assigning neutral electric charges to the ghost-quark fields. Indirect sea-quark loop contributions are removed while leaving direct sea-quark loop contributions from the valence sector unaltered.

C. Examples

Consider for example the quark sector contributions to the quark sector contributions to a baryon magnetic moment in a partially quenched theory with two degenerate light quarks and one heavy sea quark, labeled $u'$, $d'$ and $s'$. Electric charge assignments are $q_u, q_d, q_s$ for the valence sector of the theory and $q_u', q_d', q_s'$ for the ghost- and sea-quark sectors.

1. Proton Magnetic Moment

The quenched quark-sector results for the proton are complemented by direct sea-quark loop contributions from the valence- and ghost-quark sectors plus both direct and indirect contributions from the sea-quark sector. As discussed in Sec. III A such loop contributions are flavor blind and the couplings are easily extracted from Tables XI, XII, or XIII for the direct contributions and Table XIV for the indirect contribution. For simplicity, we will suppress baryon mass splittings in the following. However, they may be introduced in a transparent manner.

For the $u$-quark sector in the proton, one has

$$ u_p = \xi \left\{ -q_u \frac{4}{3} D^2 m_\pi + q_u \frac{1}{3} (5 D^2 - 6 D F + 9 F^2) m_\pi ight. $$
$$ + q_u' \frac{1}{3} (5 D^2 - 6 D F + 9 F^2) (\bar{m}_\pi - m_\pi) $$
$$ - q_u \frac{4}{3} (D^2 + 3 F^2) \bar{m}_\pi $$
$$ \left. - q_u \frac{2}{3} (D^2 + 3 F^2) \bar{m}_K \right\}, \tag{36} $$

where $\xi \equiv m_N/(8\pi f_\pi^2)$, $\bar{m}_\pi$ denotes a $\pi$-meson composed of a light-valence and light-sea quark, and $\bar{m}_K$ denotes a $K$-meson composed of a light-valence and heavy-sea quark. The second term is a direct $u$ sea-quark loop contribution associated with the valence sector, canceled by the ghost-quark contribution in the third term when $q_u' = q_u$. The third term also includes the direct $u'$ sea-quark loop contribution associated with the sea-quark sector and originates from Table XV for Figs. (b), (e) and (h). The last two terms are indirect $u$ sea-quark loop contributions and originate from Table XIV for diagrams (b), (g) and (h) respectively.

Similarly

$$ d_p = \xi \left\{ + q_d \frac{4}{3} D^2 m_\pi + q_d \frac{1}{3} (5 D^2 - 6 D F + 9 F^2) m_\pi ight. $$
$$ + q_d' \frac{1}{3} (5 D^2 - 6 D F + 9 F^2) (\bar{m}_\pi - m_\pi) $$
$$ - q_d \frac{2}{3} (D - F)^2 \bar{m}_\pi - q_d (D - F)^2 \bar{m}_K \right\}, \tag{37} $$

and

$$ s_p = \xi \left\{ q_s \frac{1}{3} (5 D^2 - 6 D F + 9 F^2) m_K \right. $$
$$ + q_s' \frac{1}{3} (5 D^2 - 6 D F + 9 F^2) (\bar{m}_K - m_K) \right\}. \tag{38} $$

The LNA behavior of the proton magnetic moment in the partially quenched theory is

$$ \mu_p = \xi \left\{ -\frac{4}{3} D^2 m_\pi ight. $$
$$ + \frac{1}{9} (5 D^2 - 6 D F + 9 F^2) (m_\pi - m_K) $$
$$ + (q_u' + q_d') \frac{1}{3} (5 D^2 - 6 D F + 9 F^2) (\bar{m}_\pi - m_\pi) $$
$$ + q_u \frac{4}{3} (5 D^2 - 6 D F + 9 F^2) (\bar{m}_K - m_K) $$
$$ - \frac{2}{9} (D + 3 F^2) \bar{m}_\pi - \frac{1}{9} (D + 3 F^2) \bar{m}_K \right\}. \tag{39} $$

We note that this final expression agrees with that of Eq. (48) in Ref. 29.

2. $\Sigma^+$ Magnetic Moment

To clearly establish the method for constructing partially-quenched chiral coefficients, we consider the $\Sigma^+$ hyperon. The direct and indirect sea-quark loop contributions are flavor blind and the couplings are easily extracted from Tables XI, XII, or XIII for the direct contributions and Table XIV for the indirect contribution. For the $u$-quark sector in $\Sigma^+$, one has

$$ u_{\Sigma^+} = \xi \left\{ -q_u \frac{4}{3} D^2 m_K ight. $$
$$ + q_u \frac{2}{3} (D^2 + 3 F^2) m_\pi + q_u (D - F)^2 m_K $$
$$ + q_u \frac{2}{3} (D^2 + 3 F^2) (\bar{m}_\pi - m_\pi) \right\}.$$
where $\tilde{m}_K$ denotes a $K$-meson composed of a light-valence and heavy-sea quark and $\tilde{m}_K$ denotes a $K$-meson composed of a strange-valence and light-sea quark. The second and third terms are direct $u$ sea-quark loop contributions associated with the valence sector, canceled by the ghost-quark contribution in the fourth and fifth terms when $q'_u = q_u$. The fourth and fifth terms also include the direct $u'$ sea-quark loop contribution associated with the sea-quark sector and originate from Table X for Figs. 2(b) and (c). The last two terms are indirect $u$ sea-quark loop contributions and originate from Table X for Figs. 2(b) + (c) and (h) respectively. Similarly

$$s_{\Sigma^+} = \xi \left\{ +q_s \frac{4}{3} (D^2 - F^2) \tilde{m}_K + q_s (D - F)^2 m_{\eta}, ight.$$  

$$+q_s \frac{2}{3} (D^2 + 3 F^2) m_K + q_s (D - F)^2 m_{\eta},$$  

$$+q_s \frac{2}{3} (D^2 + 3 F^2) (\tilde{m}_K - m_K)$$  

$$+q_s (D - F)^2 (\tilde{m}_K - m_K),$$  

$$-q_s 2 (D - F)^2 \tilde{m}_K - q_s (D - F)^2 \tilde{m}_{\eta},$$  

$$\right\} \right.$$

where $\tilde{m}_{\eta,\pi}$ denotes an $s\pi'$ $\eta$-meson composed of a strange-valence and anti-heavy sea-quark, and

$$d_{\Sigma^+} = \xi \left\{ +q_d \frac{2}{3} (D^2 + 3 F^2) m_{\pi} + q_d (D - F)^2 m_K,$$  

$$+q_d \frac{2}{3} (D^2 + 3 F^2) (\tilde{m}_\pi - m_{\pi})$$  

$$+q_d (D - F)^2 (\tilde{m}_K - m_K),$$  

$$\right\}$$

Thus, the LNA behavior of the $\Sigma^+$ magnetic moment in the partially quenched theory is

$$\mu_{\Sigma^+} = \xi \left\{ \frac{4}{3} D^2 m_K,$$  

$$+\frac{2}{9} (D^2 + 3 F^2) m_{\pi} + \frac{1}{3} (D - F)^2 m_K,$$  

$$-\frac{2}{9} (D^2 + 3 F^2) m_K - \frac{1}{3} (D - F)^2 m_{\pi},$$  

$$+ (q_u' + q_d') \frac{2}{3} (D^2 + 3 F^2) (\tilde{m}_\pi - m_{\pi})$$  

$$+ (q_u' + q_d') (D - F)^2 (\tilde{m}_K - m_K)$$  

$$+q_s \frac{2}{3} (D^2 + 3 F^2) (\tilde{m}_K - m_K)$$  

$$+q'_s (D - F)^2 (\tilde{m}_K - m_{\eta}),$$  

$$\frac{1}{3} (D - F)^2 (2 \tilde{m}_K + \tilde{m}_{\eta}),$$  

$$-\frac{4}{9} (D^2 + 3 F^2) (2 \tilde{m}_\pi + \tilde{m}_K),$$  

$$\right\} \right.$$

again in agreement with Eq. (51) of Ref. 20.

Partially-quenched results may be similarly obtained for the remainder of the quark-sector contributions to octet baryon magnetic moments using the approach described here in detail. Since the results require specific knowledge of the number and nature of dynamical flavors, we defer writing further specific results.

VI. SUMMARY

The diagrammatic method for separating valence and sea-quark-loop contributions to the meson cloud of hadrons provides a transparent approach to the calculation of quenched chiral coefficients. The origin of chiral nonanalytic structure is obvious, and facilitates the incorporation of the correct nonanalytic structure matching today’s numerical simulations. In the process, the coefficients for partially-quenched QCD are derived; no new calculations are required.

The valence sector of full QCD contains the largest coefficients for the leading nonanalytic behavior of magnetic moments. The $u$-quark contribution to the proton magnetic moment has a coefficient of $-1.10$ for the rapidly varying pion-cloud contribution, which is complemented further by the kaon cloud. These are connected insertions of the electromagnetic current in full QCD and should reveal significant curvature in the approach to the chiral limit. It is also encouraging to note that the $u$-quark sector is known to have relatively small statistical uncertainties in the quenched approximation compared to that for the $d$ quark.

The coefficients of the leading nonanalytic terms of full QCD change significantly upon quenching. Some channels still hold excellent promise for revealing the nonanalytic behavior of meson-cloud physics even in the quenched approximation. For example, the $u$ or $d$-quark in the proton have large coefficients for the nonanalytic term proportional to $m_{\pi}$, with opposite signs respectively. Similarly, both the proton and neutron magnetic moments have large coefficients surviving in quenched QCD. Because the $u$-quark in the proton has significantly smaller statistical errors than that for the $d$ quark in the proton, the $u$-quark contribution to the proton magnetic moment provides the optimal opportunity to directly view nonanalytic behavior associated with the quenched meson cloud of baryons in the quenched approximation. Figure 6 illustrates the anticipated curvature associated with the term $-(4/3) D^2 m_N m_{\pi}/(8 \pi f_\pi^2)$ surviving in the quenched approximation.

There are other interesting opportunities. Consider for example the $s$-quark contribution to the quenched $\Lambda$ magnetic moment. Table XIX indicates that the coefficient of the $NK$ contribution to the $\Lambda$ magnetic moment is large. Because the nucleon is significantly lighter than the $\Lambda$, the $NK$ loop can contribute enhanced nonlinear behavior. Hence, there is a prediction of significant cur-
TABLE XXIII: One-loop corrected coefficients, $\chi$, providing the LNA contribution to baryon magnetic moments by quark sectors with quark charges normalized to unit charge. Total, direct sea-quark loop (Direct Loop), Valence, indirect sea-quark loop (Indirect Loop) and Quenched Valence coefficients are indicated. Here, the axial couplings take the one-loop corrected values $F = 0.40$ and $D = 0.61$ with $f_\pi = 93$ MeV. Note $\epsilon = 0.0004$.

| $q$   | Int. | Total | Direct Loop | Valence | Indirect Loop | Quenched Valence |
|-------|------|-------|-------------|---------|---------------|------------------|
| $u_p$ | $N\pi$ | $-4.41$ | $+2.65$ | $-7.06$ | $-4.91$ | $-2.15$ |
|       | $\Lambda K$ | $-2.36$ | $0$ | $-2.36$ | $-2.36$ | $0$ |
|       | $\Sigma K$ | $-0.10$ | $0$ | $-0.10$ | $-0.10$ | $0$ |
| $d_p$ | $N\pi$ | $+4.41$ | $+2.65$ | $+1.76$ | $-0.38$ | $+2.15$ |
|       | $\Sigma K$ | $-0.19$ | $0$ | $-0.19$ | $-0.19$ | $0$ |
| $\xi_p$ | $\Lambda K$ | $+2.36$ | $+2.36$ | $0$ | $0$ | $0$ |
|       | $\Sigma K$ | $+0.29$ | $+0.29$ | $0$ | $0$ | $0$ |

| $u_{\Sigma^+}$ | $\Sigma \pi$ | $-1.38$ | $+1.38$ | $-2.77$ | $-2.77$ | $0$ |
|                | $\Lambda \pi$ | $-1.07$ | $+1.07$ | $-2.15$ | $-2.15$ | $0$ |
|                | $NK$ | $0$ | $+0.19$ | $-0.19$ | $0$ | $-0.19$ |
|                | $\Xi K$ | $-4.41$ | $0$ | $-4.41$ | $-2.46$ | $-1.95$ |
| $d_{\Sigma^+}$ | $\Sigma \pi$ | $+1.38$ | $+1.38$ | $0$ | $0$ | $0$ |
|                | $\Lambda \pi$ | $+1.07$ | $+1.07$ | $0$ | $0$ | $0$ |
|                | $NK$ | $0$ | $+0.19$ | $0$ | $0$ | $0$ |
|                | $\Xi K$ | $-4.41$ | $+2.46$ | $+1.95$ | $0$ | $+1.95$ |
| $s_{\Sigma^+}$ | $NK$ | $-0.19$ | $0$ | $-0.19$ | $-0.38$ | $+0.19$ |
|                | $\Xi K$ | $+4.41$ | $+2.46$ | $+1.95$ | $0$ | $+1.95$ |
|                | $\Sigma \eta_\pi$ | $0$ | $+0.19$ | $-0.19$ | $-0.19$ | $0$ |
| $u_{\Sigma^0}$ | $\Sigma \pi$ | $0$ | $+1.38$ | $-1.38$ | $-1.38$ | $0$ |
|                | $\Lambda \pi$ | $0$ | $+1.07$ | $-1.07$ | $-1.07$ | $0$ |
|                | $NK$ | $+0.10$ | $+0.19$ | $0$ | $-0.10$ | $-0.10$ |
|                | $\Xi K$ | $-2.21$ | $0$ | $-2.21$ | $-1.23$ | $-0.98$ |
| $u_{\Omega}$ | $\Sigma \pi$ | $0$ | $+1.07$ | $-1.07$ | $-1.07$ | $0$ |
|                | $\Lambda \pi$ | $0$ | $\epsilon$ | $-\epsilon$ | $-\epsilon$ | $0$ |
|                | $NK$ | $+2.36$ | $+1.57$ | $+0.79$ | $0$ | $+0.79$ |
|                | $\Xi K$ | $-0.25$ | $0$ | $-0.25$ | $-0.54$ | $+0.29$ |
| $s_{\Omega}$ | $\Lambda \eta_\pi$ | $0$ | $+1.57$ | $-1.57$ | $-1.57$ | $0$ |
|                | $NK$ | $-4.72$ | $0$ | $-4.72$ | $-3.15$ | $-1.57$ |
|                | $\Xi K$ | $+0.50$ | $+1.07$ | $+0.57$ | $0$ | $-0.57$ |
| $u_{\eta_\pi}$ | $\Xi \pi$ | $-0.19$ | $+0.19$ | $-0.38$ | $-0.38$ | $0$ |
|                | $\Lambda K$ | $0$ | $+0.25$ | $-0.25$ | $0$ | $-0.25$ |
|                | $\Sigma K$ | $+4.41$ | $+2.21$ | $+2.21$ | $0$ | $+2.21$ |
|                | $\Omega K$ | $0$ | $0$ | $-0.19$ | $-0.19$ | $0$ |
| $d_{\eta_\pi}$ | $\Xi \pi$ | $+0.19$ | $+0.19$ | $0$ | $0$ | $0$ |
|                | $\Lambda K$ | $+0.25$ | $+0.25$ | $0$ | $0$ | $0$ |
|                | $\Sigma K$ | $+2.21$ | $+2.21$ | $0$ | $0$ | $0$ |
| $s_{\eta_\pi}$ | $\Lambda K$ | $-0.25$ | $0$ | $-0.25$ | $-0.50$ | $+0.25$ |
|                | $\Sigma K$ | $-6.62$ | $0$ | $-6.62$ | $-4.41$ | $-2.21$ |
|                | $\Omega K$ | $0$ | $+0.19$ | $-0.19$ | $0$ | $-0.19$ |
|                | $\Xi \eta_\pi$ | $0$ | $+2.46$ | $-2.46$ | $-2.46$ | $0$ |
TABLE XXIV: One-loop corrected coefficients, $\chi$, providing the LNA contribution to baryon magnetic moments. Total, direct sea-quark loop (Direct Loop), Valence, indirect sea-quark loop (Indirect Loop) and Quenched Valence coefficients are indicated. Here, the axial couplings take the one-loop corrected values $F = 0.40$ and $D = 0.61$ with $f_s = 93$ MeV. Note $\epsilon = 0.000128$.

| Baryon | Channel | Total | Direct Loop | Valence | Indirect Loop | Quenched Valence |
|--------|---------|-------|-------------|---------|---------------|-----------------|
| $p$    | $N\pi$  | -4.41 | +0.88       | -5.29   | -3.15         | -2.15           |
|        | $\Lambda K$ | -2.36 | -0.79       | -1.57   | -1.57         | 0               |
|        | $\Sigma K$  | -0.10 | -0.10       | 0       | 0             | 0               |
| $n$    | $N\pi$  | +4.41 | +0.88       | +3.53   | +1.38         | +2.15           |
|        | $\Lambda K$ | 0     | -0.79       | +0.79   | +0.79         | 0               |
|        | $\Sigma K$  | -0.19 | -0.10       | -0.10   | -0.10         | 0               |
| $\Sigma^+$ | $\Sigma\pi$ | -1.38 | +0.46       | -1.85   | -1.85         | 0               |
|        | $\Lambda\pi$ | -1.07 | +0.36       | -1.43   | -1.43         | 0               |
|        | $\Lambda K$ | 0     | +0.06       | -0.06   | +0.13         | -0.19           |
|        | $\Xi K$    | -4.41 | -0.82       | -3.59   | -1.64         | -1.95           |
|        | $\Sigma\eta$ | 0     | -0.06       | +0.06   | +0.06         | 0               |
| $\Sigma^0$ | $\Sigma\pi$ | 0     | +0.46       | -0.46   | -0.46         | 0               |
|        | $\Lambda\pi$ | 0     | +0.36       | -0.36   | -0.36         | 0               |
|        | $\Lambda K$ | +0.10 | +0.06       | +0.03   | +0.13         | -0.10           |
|        | $\Xi K$    | -2.21 | -0.82       | -1.39   | -0.41         | -0.98           |
|        | $\Sigma\eta$ | 0     | -0.06       | +0.06   | +0.06         | 0               |
| $\Sigma^-$ | $\Sigma\pi$ | +1.38 | +0.46       | +0.92   | +0.92         | 0               |
|        | $\Lambda\pi$ | +1.07 | +0.36       | +0.72   | +0.72         | 0               |
|        | $\Lambda K$ | +0.19 | +0.06       | +0.13   | +0.13         | 0               |
|        | $\Xi K$    | 0     | -0.82       | +0.82   | +0.82         | 0               |
|        | $\Sigma\eta$ | 0     | -0.06       | +0.06   | +0.06         | 0               |
| $\Lambda$ | $\Sigma\pi$ | 0     | +0.36       | -0.36   | -0.36         | 0               |
|        | $\Lambda\eta$ | 0     | $\epsilon$ | $-\epsilon$ | $-\epsilon$     | 0               |
|        | $\Lambda K$ | +2.36 | +0.53       | +1.84   | +1.05         | +0.79           |
|        | $\Xi K$    | -0.25 | -0.36       | +0.11   | -0.18         | +0.29           |
|        | $\Lambda\eta$ | 0     | -0.53       | +0.53   | +0.53         | 0               |
| $\Xi^0$ | $\Xi\pi$  | -0.19 | +0.06       | -0.25   | -0.25         | 0               |
|        | $\Lambda K$ | 0     | +0.08       | -0.08   | +0.17         | -0.25           |
|        | $\Xi K$    | +4.41 | +0.74       | +3.68   | +1.47         | +2.21           |
|        | $\Omega K$ | 0     | -0.06       | +0.06   | -0.13         | +0.19           |
|        | $\Xi\eta$ | 0     | -0.82       | +0.82   | +0.82         | 0               |
| $\Xi^-$ | $\Xi\pi$  | +0.19 | +0.06       | +0.13   | +0.13         | 0               |
|        | $\Lambda K$ | +0.25 | +0.08       | +0.17   | +0.17         | 0               |
|        | $\Xi K$    | +2.21 | +0.74       | +1.47   | +1.47         | 0               |
|        | $\Omega K$ | 0     | -0.06       | +0.06   | +0.06         | 0               |
|        | $\Xi\eta$ | 0     | -0.82       | +0.82   | +0.82         | 0               |
vature in the extrapolation of the $s$-quark contribution, even when the mass of the strange quark is held fixed as is commonly done in lattice QCD simulations. As such, the effect is purely environmental as $m_s$ changes. The effect arises from the extrapolation of light $u$ and $d$ quarks in the $\Lambda$.

The $s$-quark in $\Xi$ provides another opportunity to observe a purely environmental effect in quenched QCD. Here the coefficient of the $\Sigma K$ channel is very large, again predicting curvature in the $s$-quark contribution to the magnetic moment, even when the $s$-quark mass is held fixed. The mass of $\Sigma$ is less than the mass of $\Xi$ allowing the kaon to provide enhanced nonlinear behavior.

A few channels hold potential for revealing *quenched artifacts* in quenched simulations. Despite the prediction of curvature for both the $s$ and $d$-quark sectors of $\Xi$—in both full and quenched QCD, the extrapolation of the total $\Xi$-baryon magnetic moment receives no leading nonanalytic contribution from neither the $\pi$- nor the $K$-meson cloud in the quenched approximation. Similar results hold for the $\Sigma^-$ magnetic moment.

It is particularly difficult to directly determine the loop contribution to baryon magnetic moments in numerical simulations. As such, it is of particular interest to compare the coefficients of the valence quark contributions in full QCD (column “Valence” of Tables XXI and XXIII) to that for the valence quark contributions of quenched QCD (column “Quenched Valence” of Tables XXI and XXIII). Here, the $\nu$-quark in $\Sigma^+$ stands out with the significant curvature of the $\Sigma^+$ and $\Lambda^+$ channels completely suppressed from $-3.33$ to $0$ respectively. Only the $\Xi K$ channel has a significant coupling for the $\nu$ quark in the quenched $\Sigma^+$, but curvature in this channel is suppressed by the large excitation energy required to form the intermediate state. The $\nu$ quark in the proton is also worthy of note, with the coefficient of the rapidly-varying $\pi N$ channel dropping significantly from $-11.0$ in full QCD to $-3.33$ in quenched QCD and the kaon contribution vanishing completely.

In summary, this study of quark-sector contributions to baryon magnetic moments in quenched, partially-quenched and full QCD indicates there are numerous opportunities to observe and understand the underlying structure of baryons and the nature of chiral nonanalytic behavior in QCD and its quenched variants. Numerical simulations of the observables discussed herein are currently in production on the Australian Partnership for Advanced Computing (APAC) National Facility using FLIC fermions which provide efficient access to the light quark-mass regime. It will be interesting to confront these predictions with numerical simulation results.

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