TWO-DIMENSIONAL MAGNETOHYDRODYNAMIC SIMULATIONS OF RELATIVISTIC MAGNETIC RECONNECTION

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ABSTRACT

It has been recognized that the magnetic reconnection process is of great importance in high-energy astrophysics. We develop a new two-dimensional relativistic resistive magnetohydrodynamic (R2MHD) code and carry out numerical simulations of magnetic reconnection. We find that the outflow velocity reaches the Alfvén velocity in the inflow region and that a higher Alfvén velocity provides a higher reconnection rate. We also find that Lorentz contraction plays an important role in enhancement of the reconnection rate.

Subject headings: magnetic fields — MHD — plasmas — pulsars: individual (Crab Nebula) — relativity

1. INTRODUCTION

Magnetic reconnection is widely recognized as a very important phenomenon in astrophysics. Over the last decade, it has been recognized that magnetic reconnection processes are very important in high-energy astrophysics. Dissipation of such superstrong magnetic fields may play an important role both in the global dynamics of a system and as a way to produce high-energy emission. Relativistic magnetic reconnection has been proposed as a source of high-energy emission (Lyubarskii 1996; Kirk et al. 2002) and as a solution to the σ-problem (Coroniti 1990; Lyubarsky & Kirk 2001; Kirk & Skjæraasen 2003; Lyubarsky 2003). Similar models have also been developed for cosmological gamma-ray bursts (Drenkhahn 2002; Drenkhahn & Spruit 2002; Lyutikov & Blackman 2001). Magnetic reconnection has been invoked as an explanation of the rapid variability observed in active galactic nuclei (Di Matteo 1998). Particle acceleration in the reconnection process has been proposed as operating in radio jets (Romanova & Lovelace 1992; Birk et al. 2001).

Due to the extreme complexity and richness of the possible effects arising in relativistic plasma physics, there is a strong interest in developing computer codes for relativistic magnetohydrodynamics (hereafter RMHD). Van Putten (1993) illustrated the implementation on the Riemann problem for MHD. Koide et al. (1996) then developed an RMHD code that has been extensively used in relativistic two-dimensional and three-dimensional jet simulations. Komissarov (1999) and others developed and tested a Godunov-type code that is a truly multidimensional scheme (Balsara 2001; Koldoba et al. 2002). Recently, Del Zanna et al. (2003) presented a third-order shock-capturing scheme for three-dimensional RMHD and validated it by several numerical tests. On the other hand, Koide et al. (1998, 1999) extended the code to general relativistic (GRMHD) effects and applied it to the jet formation mechanism. Gammie et al. (2003) and De Villiers & Hawley (2003) also developed GRMHD codes.

Despite the fact that magnetic reconnection is recognized as an important process in high-energy astrophysics, there are not a lot of theoretical studies. Blackman & Field (1994) considered kinematics of relativistic reconnection in the Sweet-Parker and Petschek configurations and concluded that due to the Lorentz contraction, the reconnection inflow is significantly enhanced and may approach the speed of light. Lyutikov & Uzdensky (2003) confirmed this conclusion for the Sweet-Parker case. Lyubarsky (2005) presented a generalization of Sweet-Parker and Petschek reconnection models to the relativistic case and argued that the reconnection inflow does not approach the speed of light. Particle acceleration in relativistic current sheets was studied both in the test particle approximation (Romanova & Lovelace 1992; Birk et al. 2001) and in two-dimensional PIC simulations (Zenitani & Hoshino 2001; Jaroschek et al. 2004). Furthermore, Zenitani & Hoshino (2005) studied three-dimensional PIC simulations and suggested the importance of the current-aligned magnetic field for studying the energetics of relativistic current sheets. Meanwhile, there are several RMHD simulations as we write; all of these codes, however, are applied to ideal MHD and take no account of resistivity.

In this Letter, we develop a new two-dimensional relativistic resistive MHD (R2MHD) code and carry out numerical simulations of two-dimensional relativistic magnetic reconnection.

2. SIMULATION MODEL

The RMHD basic equations are written as follows:

\[
\frac{\partial \mathbf{B}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{B} = 0, \quad (1)
\]

\[
\frac{\partial \mathbf{E}}{\partial t} - c \mathbf{v} \times \mathbf{B} = -4\pi j, \quad (5)
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{\rho}, \quad (2)
\]

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (3)
\]

\[
\frac{\partial \mathbf{E}}{\partial t} - c \mathbf{v} \times \mathbf{B} = -4\pi j, \quad (5)
\]

where \( c, P, v, B, E, \) and \( j \) are the light speed, proper gas pressure, velocity, magnetic field, electric field, and current...
fairly general conditions has the MHD form (Blackman & Field 1993; Lyutikov & Uzdensky 2003)

\[ D = \gamma \rho, \]

\[ R = \gamma^2 (e + P) \frac{v}{c^2} + \frac{E \times B}{4\pi c}, \]

\[ \epsilon = \gamma^2 \left( e + P - P - Dc^2 + \frac{B^2 + E^2}{8\pi} \right). \]

Furthermore, Ohm’s law for a relativistic pair plasma under fairly general conditions has the MHD form (Blackman & Field 1993; Lyutikov & Uzdensky 2003)

\[ \gamma \left( E + \frac{v}{c} \times B \right) = \eta \left[ j + \gamma^2 \left( j \frac{v}{c} - \rho \frac{e}{c} \right) \frac{v}{c} \right], \]

where \( \eta \) is the resistivity and \( \rho \) is the electron mass density. The second term on the right-hand side of equation (9) is the convection current.

When we solve ideal RMHD equations, it is possible to eliminate the electric field using \( E = - (\psi c) \times B \). Therefore, past RMHD simulations solved equations (1)–(4) and evaluated only \( D, R, \epsilon, \) and \( B \) directly at each step from the equations. In our code, we take into consideration the effect of resistivity, so we are forced to take another way. We cannot eliminate the electric field; therefore we also solve equation (5) to evaluate \( E \) at each time step. Next, from \( D, R, \epsilon, B, \) and \( E \) obtained by solving equations (1)–(5), we calculate \( \gamma \). For this purpose, we solve an equation for the unknown variable \( \gamma \),

\[ \left\{ \frac{c[\Gamma (\gamma^2 - 1) + 1]}{\Gamma (Dc^2 + \epsilon - P_{\text{em}} \gamma^2 - (\Gamma - 1) \gamma Dc^2)} \right\}^2 \times |R - S|^2 = \frac{1 - \frac{1}{\gamma^2}}{\gamma^2}, \]

where \( S = (E \times B)/(4\pi c) \) and \( P_{\text{em}} = (B^2 + E^2)/(8\pi) \). This equation is obtained by vanishing \( v, \rho, \) and \( P \) using equations (6)–(8). We solve this equation at each cell using the Newton-Raphson iteration method, so that we obtain \( \gamma \). We calculate \( \nu \) and \( P \) after the iteration, and then we get \( \rho \) and \( j \) from equations (6) and (9), respectively.

We assume that the evolution is two-dimensional. We take a rectangular computation box with two-dimensional Cartesian coordinates in the \( x \)-\( y \) plane. The medium is assumed to be an inviscid perfect gas. The \( z \)-components of magnetic field \( B_z \), velocity \( v_z \), and partial derivative \( \partial \partial z \) are neglected. The electric field \( E_z \), and current density \( j_z \), however, are included, and there are no \( x \)- and \( y \)-components of these variables according to equations (4) and (5). Therefore, Ohm’s law (eq. [9]) only has the \( z \)-component, and we can neglect the convection current term. An anomalous resistivity model is assumed, as described below.

The region of the computation box for this study is \( -52.2L \leq x \leq 52.2L, -151.2L \leq y \leq 151.2L \), where \( L = 1 \) is the thickness of the initial current sheet. Nonuniform grids are used for both the \( x \)- and \( y \)-directions. The number of grid points is 200 \( \times 416 \). The minimum grid sizes are \( \Delta x = 0.02 \) and \( \Delta y = 0.05 \), which are concentrated near the neutral point.

The light speed \( c \) is taken to be unity. The initial proper density outside the current sheet is given as \( \rho = \rho_0 = 1 \) in nondimensional units. We set the initial proper gas pressure outside the current sheet as \( P = P_0 = 1 \), and the initial temperature \( T = P/\rho = \rho_0/\rho_0 = 1 \) is uniform everywhere. The magnitude of the magnetic field \( B_n = (8\pi P_0/\beta)^{1/2} \) is prescribed by proper gas pressure \( P_0 \) and the plasma \( \beta \). We study several values for \( \beta \), but it is \( \beta = 0.1 \) in the typical case. We take a relativistic Harris model as the initial current sheet configuration (Kirk & Skjæraasen 2003), so we give the initial conditions as \( B_z = B_0 \tan (2x), P = 1 + [\beta \cosh^2 (2x)]^{-1}, \rho = 1 + [\beta \cosh^2 (2x)]^{-1}, v_i = 0, \) and \( E_z = \eta (\partial B_z/\partial x) \). Since there is no vertical magnetic field \( B_z \), we can write \( E = E_x \hat{x} \) by using equations (4) and (5). Here \( \eta \) is the resistivity, which is defined as

\[ \eta(x, y) = \frac{\eta_0 + \eta_0 (2(r/r_e)^3 - 3(r/r_e)^2 + 1)}{r \leq r_e, r > r_e}, \]

where \( \eta_0 = 5.0 \times 10^{-3} \) is a uniform resistivity in the computation box, \( \eta_0 = 0.3 \) is the amplitude of the anomalous resistivity, \( r = (x^2 + y^2)^{1/2} \) is the distance from the center of the spot (the origin), and \( r_e = 0.8 \) is the radius of the spot.

3. RESULTS AND DISCUSSION

Figure 1 shows the density distribution of the typical case (\( \beta = 0.1 \)). Because of the enhanced resistivity around the origin, magnetic reconnection starts at this point. This point evolves to become an X-type neutral point. The reconnect field lines together with the frozen-in plasma are ejected from this X-point toward the positive and negative \( y \)-directions because of the tension force of the reconnect field lines. The velocity of the reconnection outflow \( V_{\text{out}} \approx 0.9 \) is approximately the Alfvén speed of the inflow region (\( C_{\text{Alfvén}} = 0.894 \)). To complement these outflows, inflows take place from positive and negative \( x \)-directions of the current sheet. At the boundary between this inflow and the outflow, a shock is formed, emanating from the neutral point.

Figure 2 shows one-dimensional plots of various physical variables at \( t = 100 \), when the distribution becomes nearly steady state, and along \( y = 10 \), which is well upstream of the plasmoid ejected in the positive \( y \)-direction. At \( x \approx \pm 0.5 \), there are strong jumps for several variables. The value of the current density \( j \) becomes large, and the \( y \)-component of magnetic field \( B_y \) becomes weak at these jumps. Therefore, we can say that these jumps are the slow-mode MHD shocks. We also checked these jump conditions using the arranged model of Lyubarsky (2005), shown by the dotted and dashed lines. From Figures 1 and 2 we obtain \( \tan \theta \approx 0.21 \), where \( \theta \) is the angle between the magnetic field and the shock plane, and this value is close to the inflow velocity at the slow shock (e.g., Fig. 2e). According to the model of Lyubarsky (2005), the inflow velocity \( v_i \) \( \sim \tan \theta \) in the highly relativistic regime, and our results support this model.

We next studied the dependency on the initial plasma \( \beta \). Figure 3 shows \( a \) inflow velocity at \( x = 4 \) and \( y = 0 \), \( b \) maximum inflow velocity, \( c \) outflow velocity, and \( d \) outflow 4-velocity as function of time for \( \beta = 0.1, 0.2, 0.5, \) and 1.0 (\( C_{\text{Alfvén}} = 0.894, 0.816, 0.667, \) and 0.535, respectively). Velocities are normalized by \( C_{\text{Alfvén}} \) in Figures 3a, 3b, and 3c, and time is normalized by Alfvén transit time \( t_o = L/C_{\text{Alfvén}} \) in all figures. The maximum inflow velocity shown in Figure 3b is almost
Fig. 1.—Two-dimensional density distributions of typical model ($\beta = 0.1$) at $t = 50$ and $t = 100$. Solid lines show magnetic field lines, and arrows show velocity vectors.

Fig. 2.—One-dimensional distributions of various physical quantities of typical model ($\beta = 0.1$) parallel to the $x$-axis across $y = 0$. Displayed variables are (a) proper density $\rho$, (b) proper gas pressure $P$, (c) temperature $T$, (d) current density $j_x$, (e) $x$-component of velocity $v_x$, (f) $y$-component of magnetic field $B_y$, and (h) $y$-component of magnetic field $B_y$. Dashed lines show calculated downstream values, using upstream values shown by dotted lines based on the analytical jump conditions.

Fig. 3.—Plot of time dependence of (a) inflow velocity at $(x, y) = (4, 0)$, (b) maximum inflow velocity [almost same as the velocity at the edge of the anomalous resistivity spot $(x, y) \approx (\pm 0.8, 0)$], (c) outflow velocity, and (d) outflow 4-velocity for several values of initial plasma $\beta$. Each velocity in (a), (b), and (c) is normalized with Alfvén velocity outside the current sheet. Time is normalized with Alfvén transit time. Solid line, $\beta = 0.1$; dotted line, $\beta = 0.2$; dot-dashed line, $\beta = 0.5$; dashed line, $\beta = 1.0$.

the same as the inflow velocity at the edge of anomalous resistivity spot ($x \approx \pm 0.8$ and $y = 0$). Each line shows the case for a different value of $\beta$. The outflow velocity reaches $C_{A0}$ in all the cases. However, we obtain higher inflow velocity with lower $\beta$ (higher $C_{A0}$). This means higher $C_{A0}$ causes higher reconnection rate $v_{in}/C_{A0}$. In other words, reconnection rate is higher at the relativistic regime.

For a steady state reconnection, we can also express the reconnection rate by using the conservation of mass at the steady state, $\nabla \cdot (Dv) = \nabla \cdot (\gamma \rho v) = 0$, so that we obtain the following equation:

$$\frac{v_{in}}{v_{out}} \approx \frac{\delta \rho_{out}}{\delta \rho_{in}} \gamma_{out},$$  

where $\rho_{in}$ and $\rho_{out}$ are proper density of the inflow and the outflow region, and $\gamma_{out}$ is the Lorentz factor of the outflow velocity; $\delta$ and $d$ are evaluated by $\delta d = \tan \theta$, where $\theta$ is the angle between the $y$-axis and the slow-shock plane. We set the Lorentz factor of the inflow velocity $\gamma_{in} \sim 1$. Figure 4 shows the dependency of $\delta d$, $\rho_{out}/\rho_{in}$, $(\delta d)(\rho_{out}/\rho_{in})$, and $\gamma_{out}$ on initial plasma $\beta$ under the relativistic regime ($P_0 = 1.0$) and under the nonrelativistic regime ($P_0 = 10^{-3}$). From these panels, we can see the similar behaviors of ratios $\delta d/\rho_{out}/\rho_{in}$ in both regimes. Furthermore, $(\delta d)(\rho_{out}/\rho_{in}) \sim 0.11-0.14$ under the rel-
Fig. 4.—Left: Plot of plasma β dependence of \( \delta d/\delta r \) and \( \gamma_{\text{out}} \) for relativistic reconnection. Right: Plot of plasma β dependence of \( \delta d/\delta r \) and \( \gamma_{\text{out}} \) for nonrelativistic reconnection (\( \beta = 10^{-3} \)).

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