MAXIMAL COACTIONS

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Abstract. A coaction $\delta$ of a locally compact group $G$ on a $C^*$-algebra $A$ is maximal if a certain natural map from $A \times \hat{\delta} G \times \hat{\delta} G$ onto $A \otimes K(L^2(G))$ is an isomorphism. All dual coactions on full crossed products by group actions are maximal; a discrete coaction is maximal if and only if $A$ is the full cross-sectional algebra of the corresponding Fell bundle. For every nondegenerate coaction of $G$ on $A$, there is a maximal coaction of $G$ on an extension of $A$ such that the quotient map induces an isomorphism of the crossed products.

1. Introduction

One interesting thing about Katayama’s duality theorem for coactions, and more generally Mansfield’s imprimitivity theorem, is that it appears at first to fundamentally involve reduced crossed products by dual actions. This remains largely true even when the coactions are full (full coactions are defined using full group $C^*$-algebras; the original, spatially-defined coactions are now called reduced): in [2], the second two authors proved that for any nondegenerate normal full coaction $(A,G,\delta)$ and any closed normal subgroup $N$ of $G$, the reduced crossed product $A \times \hat{\delta} G \times \hat{\delta},r N$ is Rieffel-Morita equivalent to $A \times \delta| G/N$. On the other hand, the first two authors, together with Iain Raeburn, showed in [3] that for any dual coaction $\delta$, the full crossed product $A \times \delta G \times \hat{\delta}| N$ is naturally Rieffel-Morita equivalent to $A \times \delta| G/N$. An investigation of this phenomenon for arbitrary coactions of discrete groups led the first and third authors in [4] to conclude that in general, Mansfield imprimitivity holds for neither the full nor the reduced crossed product, but for an “intermediate” crossed product $A \times \delta G \times \hat{\delta},\mu N$. While that work relied heavily on the connection between discrete coactions and Fell bundles, it also revealed a general framework which provided the immediate impetus for the work which appears here. In particular, it motivated the definition and study of what we call maximal coactions.

Loosely speaking, the maximal coactions are those for which full-crossed product duality holds; more precisely, $(A,G,\delta)$ is maximal if a certain natural map from $A \times \delta G \times \hat{\delta} G$ onto $A \otimes K(L^2(G))$ is an isomorphism. Thus, the maximal coactions are those for which the universal properties of the full crossed product and the power of duality theory can simultaneously be engaged. Our main theorem (Theorem 3.3) says that for every nondegenerate coaction $(A,G,\delta)$, there is a maximal coaction on an extension of $A$ which has the same crossed product; we call this a maximalization of $\delta$, and show that it is unique up to an appropriate notion of isomorphism. Since there is always a coaction on a quotient of $A$ (the normalization of $A$) which has the same crossed product and for which reduced-crossed-product duality holds, we can conclude that in general (as for the discrete coactions in [4]), Katayama duality holds for some intermediate crossed product lying between the full and reduced crossed products by the dual action.

The paper is organized as follows: in Section 2, we prove a lemma on the naturality of normalizations which, surprisingly, forms an essential part of the proof of our main theorem. We also show that the normal coactions are precisely those for which reduced-crossed-product duality holds. In Section 3, we define maximality and maximalization, and prove our main theorem. The basic
ingredients of the proof are the observations that dual coactions are always maximal (as suggested by the results in [14]), and Rieffel-Morita equivalence preserves maximality. Section 4 details the natural relationship between maximalizations of discrete coactions and the maximal cross-sectional algebras of the corresponding Fell bundles.

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Notation. We adopt the conventions of [1, 15, 17] for coactions of groups on C*-algebras, and of [6, 8, 19] for Fell bundles. In particular, our coactions (A,G,δ) are full in the sense that they map A into M(A⊗C*(G)), the multiplier algebra of the minimal tensor product of A with the full group C*-algebra. Also, our coactions are assumed to be nondegenerate, in the usual sense that the closure of δ(A)(1⊗C*(G)) is A⊗C*(G). (When B is an algebra and X is a B-module, BX denotes the linear span of the set {bx | b ∈ B, x ∈ X}.) We use (jA, jC(G)) to denote the canonical covariant homomorphism of (A,C(G)) into M(A='+ δ G), and (iB, iC(G)) for the canonical covariant homomorphism of (B,G) into M(B×α G) for an action α of G on B. Additionally, we set kA = iA×G o jA, kC(G) = iA×G o jC(G), and kG = iG, which are maps of A, C0(G), and G, respectively, into M(A×δ G×≺ δ G). We use superscripts, as in kA, to distinguish between maps when coactions on more than one algebra are commingled.

If φ: A → B is a homomorphism and p ∈ M(A) is a projection, we write φ|p for the restriction of φ to the corner pAp. If δ is a coaction of G on A and p is δ-invariant in the sense that δ(p) = p⊗1, then we view δ|p as a coaction on pAp; δ|p will be nondegenerate when δ is.

For a locally compact group G, we denote by ρ and τ the left and right regular representations on L2(G), and by τ and σ the actions of G on C0(G) by left and right translation, respectively. M: C0(G) → B(L2(G)) denotes the representation by multiplication operators, and we use u for the canonical map of G into UM(C*(G)). By Σ we mean the flip map: A ⊗ B → B ⊗ A for any A and B.

2. Normal coactions

Let δ: A → M(A⊗C*(G)) be a coaction of a locally compact group G on A. Following [14] we define a *-homomorphism

$$Φ = π \times U: A \times δ G \times≺ δ G \to A \odot K(L^2(G)),$$

where the covariant homomorphism (π, U) of the dual system (A×δ G, G, δ) is defined by

$$π = (id_A \otimes λ) \circ δ \times (1 \otimes M), \quad U = 1 \otimes ρ.$$  (2.1)

As observed in [14], Corollary 2.6], Φ is always surjective, essentially because $M(C_0(G)) \rho(C^*(G)) = K(L^2(G))$. In what follows we shall refer to Φ as the canonical surjection of $A \times δ G \times≺ δ G$ onto $A \odot K(L^2(G))$.

Recall from [16, Definition 2.1] that a coaction (A,G,δ) is normal if the canonical map $j_A: A \to M(A \times δ G)$ is injective. If (A,G,δ) is not normal, there always exists a normal coaction $δ^n$ of G on $A^n = A/\ker j_A$ such that the quotient map of A onto $A^n$ is $δ - δ^n$ equivariant and induces an isomorphism of $A \times δ G$ onto $A^n \times≺ δ^n G$ ([16, Proposition 2.6]). We will call this coaction $δ^n$ the canonical normalization of δ. In general, we say that (B,G,ε) is a normalization of (A,G,δ) if ε is normal and there exists a $δ - ε$ equivariant surjection of A onto B which induces an isomorphism between $A \times δ G$ and $B \times ε G$. 
The following lemma implies that any two normalizations of a given coaction are isomorphic; we state it in somewhat greater generality for use in the proof of our main theorem.

**Lemma 2.1.** Let \((A,G,\delta)\) and \((C,G,\eta)\) be coactions, and let \(\vartheta: A \to C\) be a \(\delta - \eta\) equivariant homomorphism. If \((B,G,\varepsilon)\) is a normalization of \(\delta\) with associated equivariant surjection \(\psi: A \to B\), and if \((D,G,\zeta)\) is a normalization of \(\eta\) with associated equivariant surjection \(\omega: C \to D\), there exists a unique homomorphism \(\chi: B \to D\) such that the diagram

\[
\begin{array}{ccc}
A & \xrightarrow{\vartheta} & C \\
\downarrow{\psi} & & \downarrow{\omega} \\
B & \xrightarrow{\chi} & D
\end{array}
\]

commutes. The homomorphism \(\chi\) will be \(\varepsilon - \zeta\) equivariant; \(\chi\) will be an isomorphism if \(\vartheta\) is.

**Proof.** We first consider the case \(B = A^n\) and \(D = C^n\); we need to show that \(\ker \psi \subseteq \ker(\omega \circ \vartheta)\), and this is the same as showing \(\ker j_A \subseteq \ker(j_C \circ \vartheta)\). But since \(\vartheta\) is equivariant, the pair \((j_C \circ \vartheta, j_C^G)\) is covariant for \((A,G,\delta)\), whence \(j_C \circ \vartheta\) factors through \(j_A\) by the universal property of the crossed product. This proves the desired inclusion. Equivariance follows from a routine diagram chase and surjectivity of \(\psi\).

We next consider the case \(B = A^n, C = A\), and \(\vartheta = \text{id}\). Since \((D,G,\zeta)\) is assumed normal, we have \(D^n = D\), so the case proved above provides the equivariant homomorphism \(\chi: A^n \to D\), which is surjective because \(\omega\) is. Since \(\omega\) is equivariant, we have

\[(\omega \times G) \circ j_A = j_D \circ \omega,
\]

with \(\omega \times G\) bijective because \(\zeta\) is a normalization of \(\delta\), and \(j_D\) injective because \(\zeta\) is normal. Thus \(\ker \omega = \ker j_A = \ker \psi\), which implies that \(\chi\) is injective.

The general case is obtained by cobbling these special cases together. If \(\vartheta\) is an isomorphism, applying the above to \(\vartheta^{-1}\) (together with the surjectivity of \(\psi\) and \(\omega\)) shows that \(\chi\) is an isomorphism. \(\square\)

Katayama’s duality theorem ([11, Theorem 8]; see also [14, Corollary 2.6]) shows that crossed-product duality holds for any normal coaction. The following proposition provides a converse to this result: a coaction \((A,G,\delta)\) is normal if and only if Katayama duality holds in the sense that a certain natural map is an isomorphism of \(A \otimes K(L^2(G))\) onto \(A \times_\delta G \times_{\delta,r} G\).

**Proposition 2.2.** Let \((A,G,\delta)\) be a coaction, let \(\Lambda: A \times_\delta G \times_\delta G \to A \times_\delta G \times_{\delta,r} G\) be the regular representation, let \(\Phi: A \times_\delta G \times_\delta G \to A \otimes K(L^2(G))\) be the canonical surjection, and let \(\psi: A \to A^n\) denote the quotient map.

(i) There exists an isomorphism \(\Upsilon\) of \(A \times_\delta G \times_{\delta,r} G\) onto \(A^n \otimes K\) such that \((\psi \otimes \text{id}_K) \circ \Phi = \Upsilon \circ \Lambda.

(ii) \(\delta\) is normal if and only if the surjection

\[
\Psi = \Upsilon^{-1} \circ (\psi \otimes \text{id}) : A \otimes K \to A \times_\delta G \times_{\delta,r} G
\]

is an isomorphism.

The following diagram illustrates the proposition:

\[
\begin{array}{ccc}
A \times_\delta G \times_\delta G & \xrightarrow{\Phi} & A \otimes K(L^2(G)) \\
A \times_\delta G \times_{\delta,r} G & \xrightarrow{\psi \otimes \text{id}_K} & A^n \otimes K(L^2(G))
\end{array}
\]

\[
\Lambda \downarrow \quad \Upsilon^{-1} \circ (\psi \otimes \text{id}) \quad \Upsilon
\]

\[
\begin{array}{ccc}
A \times_\delta G \times_{\delta,r} G & \xrightarrow{\Psi} & A^n \otimes K(L^2(G))
\end{array}
\]
Proof. To prove part (i), it suffices to show that
\[
\ker \Lambda = \ker((\psi \otimes \id_K) \circ \Phi)
\]
in \(A \times \delta G \times \hat{G}\). Let \(W = M \otimes \id_G(w_G) \in UM(K(L^2(G)) \otimes C^*(G))\), where \(w_G\) denotes the canonical map \(s \mapsto u_s\) of \(G\) into \(UM(C^*(G))\), regarded as an element of \(UM(C_0(G) \otimes C^*(G))\). We claim that
\[
(2.3) \quad \Ad(1 \otimes W^*) \circ (\id_A \otimes \Sigma) \circ (\delta \otimes \id_K) \circ \Phi = (\pi \otimes 1) \times (U \otimes u)
\]
as maps from \(A \times \delta G \times \hat{G}\) to \(M(A \otimes K \otimes C^*(G))\), with \(\pi\) and \(U\) as in \((2.1)\). For this, first observe that
\[
(2.4) \quad \Sigma \circ (\id_G \otimes \lambda) \circ \delta_G = (\lambda \otimes \id_G) \circ \delta_G \quad \text{and} \quad \Ad W^* \circ (\lambda \otimes \id_G) \circ \delta_G = \lambda \otimes 1.
\]
To see the latter represent \(C^*(G)\) faithfully on a Hilbert space \(\mathcal{H}\) and compute for \(\xi \in L^2(G, \mathcal{H}) \cong L^2(G) \otimes \mathcal{H}\) and \(s, t \in G\):
\[
(W^*((\lambda \otimes \id_G) \circ \delta_G(s))W(t)) = (W^*((\lambda \otimes u_s)W(t)) = u_{t-1}((\lambda \otimes u_s)W(t)) = u_{t-1}u_s(W(t)) = u_{t-1}g^{-1}(g(s^{-1}) = ((\lambda \otimes 1)\xi)(t).
\]
Now we can verify that \((2.3)\) holds on generators: for \(a \in A\) we compute
\[
\Ad(1 \otimes W^*) \circ (\id_A \otimes \Sigma) \circ (\delta \otimes \id_K) \circ \Phi(k_A(a))
\]
while for \(f \in C_0(G)\) we have
\[
\Ad(1 \otimes W^*) \circ (\id_A \otimes \Sigma) \circ (\delta \otimes \id_K) \circ \Phi(k_{C(G)}(f))
\]
For \(s \in G\) and \(\xi \in L^2(G, \mathcal{H})\) we have
\[
(W^*(\rho_s \otimes 1)W(t)) = u_{t-1}((\rho_s \otimes 1)W(t)) = \Delta(s)^{1/2}u_{t-1}(W(t)) = \Delta(s)^{1/2}u_{t-1}(W(t)) = (\rho_s \otimes u_s)(\xi)(t),
\]
which implies
\[
\Ad(1 \otimes W^*) \circ (\id_A \otimes \Sigma) \circ (\delta \otimes \id_K) \circ \Phi (k_G(s)) \\
= \Ad(1 \otimes W^*) \circ (\id_A \otimes \Sigma) \circ (\delta \otimes \id_K)(1 \otimes \rho_s) \\
= \Ad(1 \otimes W^*)(1 \otimes \rho_s \otimes 1) \\
= 1 \otimes \rho_s \otimes u_s \\
= (\pi \otimes 1) \times (U \otimes u)(k_G(s)).
\]

Now applying \(\id_{A \otimes K} \otimes \lambda\) to both sides of (2.3) yields
\[
\Theta \circ ((\id_A \otimes \lambda) \circ \delta \otimes \id_K) \circ \Phi = (\pi \otimes 1) \times (U \otimes \lambda),
\]
where \(\Theta\) is the invertible map \(\Ad(1 \otimes (\id_K \otimes \lambda)(W^*))\circ(\id_A \otimes \Sigma)\). Since \((\pi \otimes 1) \times (U \otimes \lambda)\) is equivalent to the regular representation of \(A \times_\delta G \times_\delta G\) induced from \(\pi\), for (2.2) it is therefore enough to show that \(\ker((\id_A \otimes \lambda) \circ \delta \otimes \id_K) \circ \Phi = \ker(\psi \circ \id_K) \circ \Phi\). But \(\pi = (\id_A \otimes \lambda) \circ \delta \times (1 \otimes M)\) is a faithful representation of \(A \times_\delta G\), so we can identify the canonical map \(j_A : A \to M(A \times_\delta G)\) with \((\id_A \otimes \lambda) \circ \delta\). Thus, by definition, \(\ker \psi = \ker j_A = \ker(\id_A \otimes \lambda) \circ \delta\), so (since \(K\) is nuclear) we are done with part (i).

Since \(\delta\) is normal if and only if \(\psi\) is an isomorphism, part (ii) follows immediately from part (i). \(\square\)

3. Maximal coactions

**Definition 3.1.** Let \(\delta : A \to M(A \otimes C^*(G))\) be a coaction. We say that \(\delta\) is maximal if the canonical map \(\Phi : A \times_\delta G \times_\delta G \to A \otimes K(L^2(G))\) is an isomorphism. A maximal coaction \((B, G, \epsilon)\) is a normalization of \(\delta\) if there exists an \(\epsilon - \delta\) equivariant surjection \(\vartheta : B \to A\) such that the induced map \(\vartheta \times \delta : B \times_\epsilon G \to A \times_\delta G\) is an isomorphism.

It will follow from Theorem 3.3 that every coaction \((A, G, \delta)\) sits “between” a maximal coaction and a normal one, in the sense that there exists a maximal coaction \((A^m, G, \delta^m)\) and a normal coaction \((A^n, G, \delta^n)\), together with equivariant surjections: \(\varphi : A^m \to A\) and \(\psi : A \to A^n\).

**Examples 3.2.** If \(G\) is any locally compact group, \((C^*(G), G, \delta_G)\) is maximal, because \(\delta_G\) is the dual coaction on \(C^*(G) = C \times_\id G\), and dual coactions are always maximal (Proposition 3.4). The normalization of \(\delta_G\) is the coaction \(\delta^n_G\) of \(G\) on \(C^*_r(G)\) determined by \(\lambda(s) \mapsto \lambda(s) \otimes u(s)\) (see [15, Example 2.12]). Thus, if \(G\) is nonamenable, \(\delta_G\) is maximal but not normal, and \(\delta^n_G\) is normal but not maximal. See the discussion following [15, Proposition 3.12] for a coaction which is neither maximal nor normal.

**Theorem 3.3.** Every nondegenerate coaction \((A, G, \delta)\) has a maximalization. If \((B, G, \epsilon)\) and \((C, G, \eta)\) are two maximalizations of \((A, G, \delta)\) with canonical equivariant surjections \(\varphi : B \to A\) and \(\psi : C \to A\), then there exists a \(\epsilon - \eta\) equivariant isomorphism \(\chi \circ B\) onto \(C\) such that \(\vartheta \circ \chi = \varphi\).

The idea of the proof is as follows: if \((A^m, G, \delta^m)\) were a maximalization of \((A, G, \delta)\), then it would satisfy
\[
A \times_\delta G \times_\delta G \cong A^m \times_{\delta^m} G \times_{\delta^m} G \cong A^m \otimes K(L^2(G)).
\]
Thus \(A^m\) would be retrievable from \(A \times_\delta G \times_\delta G\) by taking a rank-one projection \(P\) in \(M(K)\) and then cutting down by the image \(p\) of \(1 \otimes P\) in \(M(A \times_\delta G \times_\delta G)\). A natural candidate for \(\delta^m\) would then be the restriction to \(A^m\) of the double-dual coaction \(\tilde{\delta}\) on \(A \times_\delta G \times_\delta G\), but a technicality
arises here because in general $p$ won’t be $\delta$-invariant; an adjustment by a one-cocycle makes up for this defect.

We begin by showing that dual coactions are always maximal. (It was shown in [13, Theorem 3.7] that $A \times_\delta G \times_\delta G \cong A \otimes \mathcal{K}(L^2(G))$ for any dual coaction $\delta$, but the canonical map $\Phi$ was not explicitly identified as the isomorphism.)

**Proposition 3.4.** Let $\beta: G \to \text{Aut} B$ be an action. Then the dual coaction

$$\overset{\sim}{\beta} = (i_B \otimes 1) \times (i_G \otimes \iota): B \times_\beta G \to M(B \times_\beta G \otimes C^*(G))$$

on the full crossed product $B \times_\beta G$ is maximal.

**Proof.** Let $\Phi: (B \times_\beta G) \times_\beta G \times_\beta G \to (B \times_\beta G) \otimes \mathcal{K}(L^2(G))$ be the canonical map for $\overset{\sim}{\beta}$. Since $(\text{id}_{B \times_\beta G} \otimes \lambda) \circ \overset{\sim}{\beta} = (i_B \otimes 1) \times (i_G \otimes \lambda)$, we have

$$\Phi = (i_B \otimes 1) \times (i_G \otimes \lambda) \times (1 \otimes M) \times (1 \otimes \rho).$$

We have to check that this map is injective.

For this recall first that the Imai-Takai duality theorem (see [18, Theorem 5.1]) provides an isomorphism $\Omega: B \times_\beta G \times_\beta G \to B \otimes \mathcal{K}$ which is given as the integrated form

$$\Omega = ((\text{id}_B \otimes M) \circ \overset{\sim}{\beta}) \times (1 \otimes \lambda) \times (1 \otimes M)$$

where for $b \in B$, $\overset{\sim}{\beta}(b) \in C^b(G, B) \subseteq M(B \otimes C_0(G))$ denotes the function $t \mapsto \beta_{t^{-1}}(b)$. Recall also that $\Omega$ transports the double dual action $\overset{\sim}{\beta}$ to the diagonal action $\beta \otimes \text{Ad} \rho$ of $G$ on $B \otimes \mathcal{K}$, so we get an isomorphism

$$\Omega \times G: B \times_\beta G \times_\beta G \times_\beta G \to (B \otimes \mathcal{K}) \times_\beta \text{Ad} \rho G.$$

Since $1 \otimes \rho$ implements an exterior equivalence between $\beta \otimes \text{id}$ and $\beta \otimes \text{Ad} \rho$, we also have an isomorphism

$$\Theta = (i_B \otimes \text{id}_\mathcal{K}) \times (i_G \otimes \rho): (B \otimes \mathcal{K}) \times_\beta \text{Ad} \rho G \to (B \times_\beta G) \otimes \mathcal{K}.$$

The composition $\Theta \circ (\Omega \times G)$ is thus an isomorphism between $B \times_\beta G \times_\beta G \times_\beta G$ and $(B \times_\beta G) \otimes \mathcal{K}$, and all we have to do is to check that this isomorphism has the same kernel as $\Phi$.

In order to do this, we fix a faithful representation $\sigma \times V$ of $B \times_\beta G$ on a Hilbert space $\mathcal{H}$, and define a unitary operator $R$ on $\mathcal{H} \otimes L^2(G) \cong L^2(G, \mathcal{H})$ by $(R\xi)(t) = V_t \xi(t)$ for $\xi \in L^2(G, \mathcal{H})$. We claim that

$$\text{Ad} R \circ (\sigma \times V \otimes \text{id}_\mathcal{K}) \circ \Theta \circ (\Omega \times G) = (\sigma \times V \otimes \text{id}_\mathcal{K}) \circ \Phi;$$

for this it is enough to check that both maps do the same on the generators of $B \times_\beta G \times_\beta G \times_\beta G$. For example, if $\ell_B: B \to M(B \times_\beta G \times_\beta G \times_\beta G)$ denotes the canonical imbedding, for all $b \in B$ we have

$$\text{Ad} R \circ (\sigma \times V \otimes \text{id}_\mathcal{K}) \circ \Theta \circ (\Omega \times G)(\ell_B(b)) = \text{Ad} R \circ (\sigma \otimes \text{id}_\mathcal{K}) \circ \Omega(k_B(b))$$

$$\begin{aligned}
&= \text{Ad} R \circ (\sigma \otimes M)(\tilde{\beta}(b)) \\
&\overset{(\ast)}{=} \sigma(b) \otimes 1 \\
&= (\sigma \times V \otimes \text{id}_\mathcal{K})(i_B(b) \otimes 1) \\
&= (\sigma \times V \otimes \text{id}_\mathcal{K}) \circ \Phi(\ell_B(b)),
\end{aligned}$$

where $\tilde{\beta}$ is the dual coaction of $\beta$ and $\ast$ means that $\tilde{\beta} = \sigma \otimes M(\tilde{\beta} \otimes M)$. Since $\sigma$ is faithful, it follows that $\ell_B(b)$ and $\sigma(b) \otimes 1$ are the same on the generators, and the proposition is proved.
where the starred equality follows from the calculation
\[(R(\sigma \otimes M)(\tilde{\beta}(b))R^*\delta)(t) = V_t\sigma(\tilde{\beta}_{t^{-1}}(b))V_t^*\delta(t) = \sigma(b)\delta(t)\]
for \(b \in B, \xi \in L^2(G, \mathcal{H}), \) and \(t \in G.\) We omit the easier computations on the other generators. \(\square\)

Let us say that two coactions \((A, G, \delta)\) and \((B, G, \epsilon)\) are Morita equivalent coactions — see \([9, \text{Proposition 2.3}]\). Exterior equivalent coactions have id\((C,G,\eta)\) and \((B,G,\epsilon)\) are Morita equivalent coactions. Then \(\delta\) is maximal if and only if \(\epsilon\) is.

**Proposition 3.5.** Suppose \((A, G, \delta)\) and \((B, G, \epsilon)\) are Morita equivalent coactions. Then \(\delta\) is maximal if and only if \(\epsilon\) is.

**Proof.** It suffices to consider the case where \(p \in M(B)\) is an \(\epsilon\)-invariant full projection, \(A = pBp,\) and \(\delta = \epsilon|_A;\) then the isomorphisms \(A \times_\delta G \times_\delta G \cong k_B(p)(B \times_\epsilon G \times_\epsilon G)k_B(p)\) and \(A \otimes K \cong (p \otimes 1)(B \otimes K)(p \otimes 1)\) transport the canonical surjection \(\Phi_A\) to the restriction of \(\Phi_B\) to \(A \times_\delta G \times_\delta G.\) Thus \(\ker \Phi_A\) is associated to \(\ker \Phi_B\) under the Rieffel correspondence set up by the full projection \(k_B(p),\) so \(\ker \Phi_A = \{0\}\) if and only if \(\ker \Phi_B = \{0\},\) which completes the proof. \(\square\)

Adapting the definition from \([12, \text{Definition 2.7}]\), where it appears for reduced coactions, we say that a unitary \(U\) in \(M(A \otimes C^*(G))\) is a 1-cocycle for a coaction \((A, G, \delta)\) if
\[
\begin{align*}
(\text{i}) & \quad \text{id} \otimes \delta_G(U) = (U \otimes 1)(\delta \otimes \text{id}(U)), \quad \text{and} \\
(\text{ii}) & \quad U\delta(A)U^* \left(1 \otimes C^*(G)\right) \subseteq A \otimes C^*(G).
\end{align*}
\]

Two coactions \(\delta\) and \(\epsilon\) of \(G\) on \(A\) are exterior equivalent if there exists a 1-cocycle \(U\) for \(\delta\) such that \(\epsilon = (\text{Ad} U) \circ \delta;\) in this case, \(\delta\) is nondegenerate if and only if \(\epsilon\) is (more generally, this is true for Morita equivalent coactions — see \([4, \text{Proposition 2.3}]\)). Exterior equivalent coactions have naturally isomorphic crossed products: to see this, realize \(A \times_\delta G\) and \(A \times_\epsilon G\) as subalgebras of \(M(A \otimes K(L^2(G)))\), and then argue exactly as in the proof of \([14, \text{Theorem 2.9}]\) (where the same result is proved for reduced coactions) that \(\text{id}_A \otimes \lambda(U)\) conjugates one to the other.

**Lemma 3.6.** \(V = (k_{C(G)} \otimes \text{id})(w_G)\) is a 1-cocycle for \(\tilde{\delta}\); hence \(\tilde{\delta} = \text{Ad}(V) \circ \tilde{\delta}\) is a (nondegenerate) coaction of \(G\) on \(A \times_\delta G \times_\delta G.\)

**Proof.** To verify condition (i), we have:
\[
\begin{align*}
\text{id} \otimes \delta_G(V) & = \text{id} \otimes \delta_G(k_{C(G)} \otimes \text{id}(w_G)) \\
& = k_{C(G)} \otimes \text{id} \otimes \text{id} \otimes \delta_G(w_G) \\
& = k_{C(G)} \otimes \text{id} \otimes \text{id} ((w_G)_{12}(w_G)_{13}),
\end{align*}
\]
using the identity \(\text{id} \otimes \delta_G(w_G) = (w_G)_{12}(w_G)_{13} \). Now from the definitions,
\[
\tilde{\delta} \otimes \text{id}(V) = ((k_A \otimes 1) \times (k_{C(G)} \otimes 1) \times (k_G \otimes u)) \otimes \text{id}(k_{C(G)} \otimes \text{id}(w_G)) = k_{C(G)} \otimes \text{id} \otimes \text{id} ((w_G)_{13}),
\]
and clearly \(V \otimes 1 = k_{C(G)} \otimes \text{id} \otimes \text{id} ((w_G)_{12}).\)

For condition (ii), note that for \(f \in C_0(G)\) and \(z \in C^*(G),\) the product \((f \otimes 1)w_G^*(1 \otimes z) \in M(C_0(G) \otimes C^*(G))\) is actually in \(C_0(G) \otimes C^*(G),\) since it corresponds to the function \(s \mapsto f(s)u_s^*z\) in \(C_0(G, C^*(G)).\) Thus \((C_0(G) \otimes 1)w_G^*(1 \otimes C^*(G)) \subseteq C_0(G) \otimes C^*(G).\) Now temporarily let \(B =\)
Lemma 3.8. Φ straightforward calculations verify that from that of (i) the same way the covariance of any homomorphism (id \ Ω, µ) of (C∗(G), G, δG) follows from that of (µ, U) for (C0(G), G, τ) (see [ES, Example 2.9(1)]).}

Nondegeneracy of ̂δ follows from that of ̂δ. □

Lemma 3.7. (i) The pair (kC(G), kG) is a covariant homomorphism of (C0(G), G, σ) into M(A × δ G × δ G).

(ii) ̂δ(x) = x ⊗ 1 for all x ∈ (kC(G) × kG)(C0(G) × σ G).

Proof. Assertion (i) is a straightforward consequence of the definition of ̂δ. It suffices to check (ii) on generators. For f ∈ C0(G),

\begin{align*}
\hat{\delta}(kC(G)(f)) & = V\hat{\delta}(kC(G)(f))V^* \\
& = kC(G) \otimes id(w_G)(kC(G)(f) \otimes 1)kC(G) \otimes id(w_G^*) \\
& = kC(G) \otimes id(w_G(f) \otimes 1)w_G^*) \\
& = kC(G)(f) \otimes 1,
\end{align*}

since C0(G) ⊗ 1 commutes with M(C0(G) ⊗ C∗(G)). For s ∈ G we have

\begin{align*}
\hat{\delta}(k_G(s)) & = k_G(s) \otimes id(w_G)(k_G(s) \otimes u(s))k_G(s) \otimes id(w_G^*) \\
& = k_G(s) \otimes id(w_G)k_G \otimes id(\delta_G(s))k_G(s) \otimes id(w_G^*) \\
& = k_G(s) \otimes 1.
\end{align*}

The identity at ̂δ, which almost says that the pair (kG, kC(G)) is covariant for (C∗(G), G, δG), follows from part (i) the same way the covariance of any homomorphism (U, µ) of (C∗(G), G, δG) follows from that of (µ, U) for (C0(G), G, τ) (see [ES, Example 2.9(1)]). □

For brevity, let δ ⊗ id denote the coaction (idA ⊗ Σ) ∘ (δ ⊗ idK) of G on A ⊗ K.

Lemma 3.8. Φ is a ̂δ − δ ⊗ id equivariant surjection of A × δ G × δ G onto A ⊗ K.

Proof. We only need to check equivariance; that is, we need to show that (idA ⊗ Σ) ∘ (δ ⊗ idK) ∘ Φ = (Φ ⊗ id) ∘ ̂δ. But by [2.3], we have (idA ⊗ Σ) ∘ (δ ⊗ idK) ∘ Φ = Ad(1 ⊗ W) ∘ ((π ⊗ 1) \times (U ⊗ u)), and straightforward calculations verify that

\begin{align*}
Ad(1 \otimes W) \circ ((π \otimes 1) \times (U \otimes u)) & = Ad(1 \otimes W) \circ (Φ \otimes id) \circ ̂δ \\
& = (Φ \otimes id) \circ (AdV) \circ ̂δ \\
& = (Φ \otimes id) \circ ̂δ.
\end{align*}

□
Proof of Theorem 3.3. Fix, for the entire proof, a rank-one projection \( P \in \mathcal{K}(L^2(G)) \), let \( q = (M \times \rho)^{-1}(P) \in \mathcal{C}_0(G) \times \sigma \mathcal{G} \), and let \( p = (k_{C(G)} \times k_{G})(q) \), which is a \( \delta \)-invariant projection in \( M(A \times_{\delta} G \times_{\delta} G) \) by Lemma 3.7. We may therefore define a nondegenerate coaction \((A^m, G, \delta^m)\) by setting

\[
A^m = p(A \times_{\delta} G \times_{\delta} G)p \quad \text{and} \quad \delta^m = \delta|_p.
\]

Notice that, by the definition of \( \Phi \), we have

\[
(3.1) \quad \Phi(p) = \Phi \circ (k_{C(G)} \times k_{G})(q) = 1 \otimes (M \times \rho)(q) = 1 \otimes P.
\]

Now \( p \) is a full projection in \( M(A \times_{\delta} G \times_{\delta} G) \), since if we put \( C = k_{C(G)} \times k_{G}(C_0(G) \times \tau G) \), then clearly \( \mathcal{C}p\mathcal{C} = C \), whence

\[
\begin{align*}
(A \times_{\delta} G \times_{\delta} G)p(A \times_{\delta} G \times_{\delta} G) &= (A \times_{\delta} G \times_{\delta} G)\mathcal{C}p\mathcal{C}(A \times_{\delta} G \times_{\delta} G) \\
&= (A \times_{\delta} G \times_{\delta} G)\mathcal{C}(A \times_{\delta} G \times_{\delta} G) = A \times_{\delta} G \times_{\delta} G.
\end{align*}
\]

Thus \( \delta^m \) is Morita equivalent to \( \tilde{\delta} \), which is in turn Morita equivalent (in fact exterior equivalent) to \( \sigma \), which is maximal by Proposition 3.4. It follows from Proposition 3.3 that \( \delta^m \) is also maximal.

Now, identifying \((1 \otimes P)(A \otimes \mathcal{K})(1 \otimes P)\) with \( \mathcal{A} \), it follows from Lemma 3.8 that the restriction \( \Phi|_p \) of \( \Phi \) to \( A^m \) is a \( \delta^m - \delta \) equivariant surjection of \( A^m \) onto \( \mathcal{A} \). To prove that \( (A^m, \delta^m) \) is a maximalization of \((A, \delta)\), it remains to show that the integrated form \( \Phi|_p \times G \) is an isomorphism of \( A^m \times_{\delta m} G \) onto \( A \times_{\delta} G \).

For this, we first point out that if \( \epsilon \) and \( \text{Ad}U \circ \epsilon \) are exterior equivalent coactions of \( G \) on \( B \), and \((C,G,\eta)\) is a normalization of \( \epsilon \) with associated surjection \( \chi: B \to C \), then \((C,G,\text{Ad}V \circ \eta)\) is a normalization of \( \text{Ad}U \circ \epsilon \) with the same surjection, where \( V = \chi \otimes \text{id}(U) \). (This is a straightforward calculation.) In particular, since the dual coaction \( \tilde{\delta}_r \) of \( G \) on the reduced crossed product \( A \times_{\delta} G \times_{\tilde{\delta}_r} G \) is a normalization of \( \tilde{\delta} \) with associated surjection \( \Lambda \) ([16, Propositions 2.3 and 2.6]), \((A \times_{\delta} G \times_{\tilde{\delta}_r} G, G, \eta)\) is a normalization of \((A \times_{\delta} G \times_{\tilde{\delta}_r} G, \tilde{\delta})\) with the same surjection, for the appropriate choice of \( \eta \).

Moreover, it is easily seen that \((A^n \otimes \mathcal{K}, G, \delta^n \otimes \epsilon, \text{id})\) is a normalization of \((A \otimes \mathcal{K}, G, \delta \otimes \epsilon, \text{id})\), with surjection \( \psi \otimes \text{id}_K \). It follows that the isomorphism \( \mathcal{T}: A \times_{\delta} G \times_{\tilde{\delta}_r} G \to A^n \otimes \mathcal{K} \) of Proposition 2.2 is the unique homomorphism between normalizations provided by Lemma 2.1, and hence is \( \eta - \delta^n \otimes \epsilon, \text{id} \) equivariant.

It now follows that \( \Phi \times G \) is an isomorphism, since by naturality it is the composition of isomorphisms

\[
(A \times_{\delta} G \times_{\tilde{\delta}_r} G) \times_{\tilde{\delta}_r} G \cong (A \times_{\delta} G \times_{\tilde{\delta}_r} G) \times_{\eta} G \cong (A^n \otimes \mathcal{K}) \times_{\delta^n \otimes \epsilon, \text{id}} G \cong (A \otimes \mathcal{K}) \times_{\delta \otimes \epsilon, \text{id}} G.
\]
Since \( \Phi \times G \) takes the image \( \ell(p) \) of \( p \) in \( M(\mathcal{A} \times_\delta \mathcal{B} \times_\delta \mathcal{G} \times_\delta \mathcal{G}) \) to \( j_{\mathcal{A} \otimes \mathcal{K}}(1 \otimes \mathcal{P}) \in M((A \otimes \mathcal{K}) \times_{\delta \otimes \cdot} \mathcal{I} \mathcal{D} \mathcal{G}, G) \), naturality (again) implies that \( \Phi|_p \times G \) is the composition of isomorphisms

\[
A^m \times_{\delta^m} G = p(A \times G \times G)p \times_{\delta|_p} G \\
\cong \ell(p)(A \times G \times G \times_\delta G)\ell(p) \\
(\Phi \times G)|_{\ell(p)} \\
\cong j_{\mathcal{A} \otimes \mathcal{K}}(1 \otimes \mathcal{P})(A \otimes \mathcal{K}) \times_{\delta \otimes \cdot} \mathcal{I} \mathcal{D} \mathcal{G}, G \\
\cong (1 \otimes \mathcal{P})(A \otimes \mathcal{K})(1 \otimes \mathcal{P}) \times_{(\delta \otimes \cdot)_{\mathcal{I} \mathcal{D} \mathcal{G}, \mathcal{K}}}(1 \otimes \mathcal{P}) \times_{\mathcal{K}} G \\
\cong A \times_\delta G.
\]

This completes the proof that \( \delta^m \) is a maximalization of \( \delta \).

For the last statement of the theorem, consider the diagram

\[
\begin{array}{ccc}
\mathcal{B} \times_\epsilon \mathcal{G} \times_\epsilon \mathcal{G}, \hat{\epsilon} & \xrightarrow{\varphi \times_\mathcal{G} \mathcal{G}} & A \times_\delta \mathcal{G} \times_\delta \mathcal{G}, \hat{\delta} & \xleftarrow{\vartheta \times_\mathcal{G} \mathcal{G}} & C \times_\eta \mathcal{G} \times_\eta \mathcal{G}, \eta \\
\Phi_B & & \Phi_A & & \Phi_C \\
\mathcal{B} \otimes \mathcal{K}, \epsilon \otimes_\mathcal{K} \mathcal{I} & \xleftarrow{\hat{\varphi} \otimes \mathcal{I}} & A \otimes_\mathcal{K}, \delta \otimes_\mathcal{K} \mathcal{I} & \xrightarrow{\hat{\vartheta} \otimes \mathcal{I}} & C \otimes_\mathcal{K}, \eta \otimes_\mathcal{K} \mathcal{I} \\
\end{array}
\]

Since \( \epsilon \) and \( \eta \) are assumed to be maximal, the outer two vertical arrows are equivariant isomorphisms. Since \( \epsilon \) and \( \eta \) are maximalizations of \( \delta \), the upper two horizontal arrows are also equivariant isomorphisms. Thus, an isomorphism \( \Xi: \mathcal{B} \otimes \mathcal{K} \to \mathcal{C} \otimes \mathcal{K} \) can be defined so that the outer rectangle commutes equivariantly. The two inner quadrilaterals commute equivariantly by straightforward calculation, and therefore the lower triangle does as well.

Now, (3.1) (applied to \( B \)) says that \( \Phi_B(p_B) = 1_B \otimes \mathcal{P} \), where \( p_B = (k^B_{\mathcal{C}(G)} \times k^B_{\mathcal{G}})(q) \), and moreover,

\[
p_A = (k^A_{\mathcal{C}(G)} \times k^A_{\mathcal{G}})(q) = (\varphi \times G \times G) \circ (1_{\mathcal{C}(G)} \times k^B_{\mathcal{G}})(q) = (\varphi \times G \times G)(p_B).
\]

Combining this with similar calculations for \( \Phi_C \) and \( \vartheta \), we can conclude that \( \Xi(1_B \otimes \mathcal{P}) = 1_C \otimes \mathcal{P} \). This, and the equivariant commutativity of the lower triangle, shows that \( \chi = \Xi|_{1_B \otimes \mathcal{P}} \) is an \( \epsilon - \eta \) equivariant isomorphism of \( B \) onto \( C \) such that \( \vartheta \circ \chi = \varphi \).

We remark that although any two maximalizations of \( \delta \) are isomorphic, we don’t yet know how to define a canonical maximalization. The construction of the maximalization \((A^m, G, \delta^m)\) in the proof of Theorem 3.3 depended on the arbitrary choice of a rank-one projection in \( \mathcal{K}(L^2(G)) \).

Nonetheless, we feel that the basic approach of the proof will be useful in greater generality; specifically, for Hopf \( C^* \)-algebras. We have chosen not to treat Hopf algebras here because amenability considerations are likely to muddy the waters significantly.

Also, we point out that while our results allow us to define an “intermediate” crossed product “\( \times_\mu \)” for dual actions such that \( A \times_\delta G \times_\delta \mu G \) is always naturally isomorphic to \( A \otimes \mathcal{K} \), this construction is not well-defined on isomorphism classes of dual actions. For instance, let \((B, G, \beta)\) be any action for which the regular representation is not faithful, and let \( \hat{\beta}_r \) be the dual coaction on the reduced crossed product \( B \times_{\beta, r} G \). Then

\[
(B \times_\beta G \times_\delta G, \hat{\beta}) \cong (B \times_{\beta, r} G \times_\delta G, G, \hat{\beta}_r)
\]

because \( \hat{\beta}_r \) is a normalization of \( \hat{\beta} \) ([16, Corollary 3.4 and Proposition 3.8]). However,

\[
(B \times_\beta G \times_\delta G) \times_\delta G \cong (B \times_\beta G) \otimes \mathcal{K},
\]
while \[(B \times_{\overline{\beta_r}} G \times_{\beta_r} G) \times_{\overline{\beta_r},\mu} G \cong (B \times_{\beta_r} G) \otimes K.\]

So \(\hat{\beta}\) and \(\hat{\beta}_r\) are isomorphic actions with nonisomorphic intermediate crossed products.

4. Discrete coactions

For a coaction \(\delta\) of a discrete group \(G\), we can use the results of [19] to show that the dual coaction \(\delta^m\) of \(G\) on the maximal cross-sectional algebra \(C^*(A)\) of the corresponding Fell bundle \(A\) is a maximalization of \(\delta\), and it is the unique maximalization with the same underlying Fell bundle.

In preparation for this, we prove the following technical lemma, an easy modification of [19, Lemma 2.5], to streamline the task of verifying that a linear map is a right-Hilbert bimodule homomorphism. The essential idea is that a linear map between Hilbert modules which preserves inner products is automatically a module homomorphism. Let \(C\) and \(D\) be \(C^*\)-algebras, let \(Z\) be a (right) Hilbert \(D\)-module, and suppose \(C\) is represented by adjointable operators on \(Z\). If \(Z\) is full as a Hilbert \(D\)-module and the action of \(C\) on \(Z\) is nondegenerate, we say \(Z\) is a right-Hilbert \(C - D\) bimodule.

We use the notation \(CZD\) to indicate the coefficient algebras. If \(C_0 \subseteq C\) and \(D_0 \subseteq D\) are dense *-subalgebras and \(Z_0\) is a dense linear subspace of \(Z\) such that \(C_0Z_0 \cup Z_0D_0 \subseteq Z_0\) and \(\langle Z_0, Z_0D \rangle \subseteq D_0\), we say \(CZD\) is the completion of the right-pre-Hilbert bimodule \(C_0(Z_0)D_0\).

**Lemma 4.1.** Suppose \(CZD\) and \(EWF\) are right-Hilbert bimodules such that \(CZD\) is the completion of a right-pre-Hilbert bimodule \(C_0(Z_0)D_0\), and suppose we are given homomorphisms \(\varphi: C \to E\) and \(\psi: D \to F\) and a linear map \(\vartheta: Z_0 \to W\) with dense range such that for all \(c \in C_0\) and \(z, w \in Z_0\) we have

1. \(\vartheta(cz) = \varphi(c)\vartheta(z)\), and
2. \(\langle \vartheta(z), \vartheta(w) \rangle_F = \psi(\langle z, w \rangle_D)\).

Then \(\vartheta\) extends uniquely to a right-Hilbert bimodule homomorphism of \(CZD\) onto \(EWF\). Moreover, if \(Z\) and \(W\) are actually imprimitivity bimodules, this extension of \(\vartheta\) is an imprimitivity bimodule homomorphism.

**Proof.** The argument of [19, Lemma 2.5] shows that \(\vartheta\) is bounded and preserves the right module actions, which implies that \(\vartheta\) extends uniquely to a right-Hilbert bimodule homomorphism from \(CZD\) to \(EWF\). Since the range of \(\vartheta\) is dense and Hilbert module homomorphisms have closed range, the extension of \(\vartheta\) is onto, giving the first assertion. Moreover, when \(Z\) and \(W\) are actually imprimitivity bimodules, the density of \(\vartheta(Z_0)\) together with the compatibility of the left and right inner products in both \(Z\) and \(W\) imply that \(\vartheta\) preserves the left inner products. \(\Box\)

**Proposition 4.2.** Let \(\delta: A \to A \otimes C^*(G)\) be a coaction of a discrete group \(G\) and let \(A\) be the corresponding Fell bundle over \(G\). Then \(\delta\) is maximal if and only if \(A = C^*(A)\). In general, the dual coaction \(\delta^m: C^*(A) \to C^*(A) \otimes C^*(G)\) is a maximalization of \(\delta\).

**Proof.** Let \(X\) be the \(C^*(A) \times_{\delta^m} G \times_{\delta^m} G - C^*(A)\) imprimitivity bimodule of [4, Theorem 3.1] (for the case \(H = G\)), which is a completion of the sectional algebra \(\Gamma_c(A \times G)\) of the Fell bundle \(A \times G\). Using the isomorphism \(C^*(A) \times_{\delta^m} G \cong A \times G\) of [4, Lemma 2.1], \(X\) becomes an \(A \times G \times G - C^*(A)\) imprimitivity bimodule, with left action and right inner-product given, for \((a_r, s, t) \in \Gamma_c(A \times G)\) \subseteq X, by

\[(a_r, s, t) \cdot (a_u, v) = (a_r a_u, vt^{-1})\]  
if \(st = uv\) (and 0 otherwise), and
\[
((a_s, t), (a_u, v))_{C^*(A)} = a_s^* a_u\]  
if \(st = uv\) (and 0 otherwise).
Let $\psi$ denote the unique homomorphism of $C^*(A)$ onto $A$ which extends the identity map on $\Gamma_c(A)$. We must show that the canonical surjection $\Phi: A \times_\delta \hat{G} \to A \otimes K$ is an isomorphism if and only if $\psi$ is; for this, it suffices to construct an imprimitivity bimodule homomorphism of $X$ onto the $A \otimes K - A$ imprimitivity bimodule $A \otimes \ell^2(G)$ with coefficient homomorphisms $\psi$ and $\Phi$. For $a_s \in A$ and $t \in G$, define

$$\Theta(a_s, t) = a_s \otimes \chi_{st}.$$  

Then $\Theta$ extends uniquely to a linear map from $\Gamma_c(A \times G)$ to $A \otimes \ell^2(G)$ with dense range. Since $\Phi(a_s, t, r) = a_s \otimes \lambda_s \mu_{Xr} \rho_r$, a routine computation with generators shows that

$$\Theta(c \cdot z) = \Phi(c) \cdot \Theta(z) \quad \text{and} \quad \langle \Theta(z), \Theta(w) \rangle_A = \psi(\langle z, w \rangle_{C^*(A)})$$

for $c \in \Gamma_c(A \times G \times G) \subset A \times G \times G$ and $z, w \in \Gamma_c(A \times G) \subset X$. Thus by Lemma 4.1 the triple $(\Phi, \Theta, \psi)$ extends uniquely to an imprimitivity bimodule homomorphism, as desired.

The above arguments show that $\delta$ is maximal if and only if $A = C^*(A)$. The remaining part follows from [4, Lemma 2.1].

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