Detecting Sub-lunar Mass Compact Objects toward the Local Group Galaxies

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Abstract
By monitoring a large number of stars in the Local Group galaxies, we can detect nanolensing events by sub-lunar mass compact objects (SULCOs) such as primordial black holes (PBHs) and rogue (free-floating) dwarf planets in the Milky Way halo. In contrast to microlensing by stellar-mass objects, the finite-source size effect becomes important and the lensing time duration becomes shorter ($\sim 10^{1-4}$ s). Using stars with $V < 26$ in M33 as sources, for one-night observation, we would be able to detect $10^{3-4}$ nanolensing events caused by SULCOs in the Milky Way halo with a mass of $10^{-9} M_\odot$ to $10^{-7} M_\odot$ for sources with S/N $> 5$ if SULCOs constitute all the dark matter components. Moreover, we expect $10^{1-2}$ events in which bright blue stars with S/N $> 100$ are weakly amplified due to lensing by SULCOs with a mass range of $10^{-11} M_\odot$ to $10^{-9} M_\odot$. Thus the method would open a new window on SULCOs in the Milky Way halo that would otherwise not be observable.

Keywords: cosmology: theory - gravitational lensing - dark matter.

1. Introduction
Currently, there has not been stringent observational constraint on the abundance of Sub-Lunar mass Compact Objects (SULCOs) with a mass of $10^{-13} M_\odot \leq M \leq 10^{-9} M_\odot$ as the dark matter candidates. SULCOs can be either small planets, satellites, or primordial black holes (PBHs). Microlensing tests such as the MACHO and EROS collaborations ruled out the possibility that the compact objects with a mass of $10^{-7} M_\odot \leq M \leq 10^{-3} M_\odot$ constitute the Milky Way halo. On the other hand, femtolensing test of Gamma-ray bursts (GRBs) ruled out the mass range $10^{-16} M_\odot \leq M \leq 10^{-13} M_\odot$ assuming that the GRBs are at cosmological distance so that the angular size of the GRB source is sufficiently small. The SULCOs also induce picolensing of GRBs but the observational limit is very weak. Recent lensing analyses based on the time variability of stars in the Milky Way disk using the data from the Kepler satellite, have yielded a constraint on the mass range $2 \times 10^{-9} M_\odot \leq M \leq 10^{-7} M_\odot$. However, the constraint is
limited to local SULCOs at a distance < 4 kpc. Since the size of the Milky Way halo is much larger, it is important to constrain the abundance of SULCOs that reside at distance > 4 kpc as well. The neutron-star capture constraint for a mass range of $10^{-15} M_\odot \leq M \leq 10^{-9} M_\odot$ [7] depends on the assumption that the PBHs reside in globular clusters, thus the limit is uncertain [11].

SULCOs may be free-floating or rogue dwarf planets that have been ejected from developing or developed planetary systems [21, 25]. Although the mass scale of the detected rogue planets [24], typically a Jupiter mass scale, are much larger than the SULCO mass scale, a large number of rogue dwarf planets may reside in the Milky Way halo. For instance, the gravitational perturbation by passing stars may cause destruction of extrasolar planetary systems that may correspond to the Kuiper belt objects or the Oort cloud comets.

In this paper, we propose a method using gravitational lensing to constrain the abundance of SULCOs in the Milky Way halo: By monitoring a large number of individual bright stars in the Local Group galaxies such as M33 [1], we can obtain stringent constraint on the abundance of SULCOs as they induce amplification of the background source stars. The key factors are the source size and the scale of lensing time duration. In order to constrain compact objects with a sub-lunar mass via gravitational lensing, we need to have sources that are more distant than LMC or SMC for which the angular source size is smaller than the angular Einstein radius. Since the Einstein angular radius of SULCOs (typically $\lesssim 10^{-9}$ arcsec) is comparable to the radius of source stars, we need to consider the finite-source size effect [27]. Even if the angular source size is larger than the Einstein angular radius of SULCOs, we can still detect the weak amplification of source stars if they are bright enough. Note that the finite-source size effect and the feasibility of observation with short time duration have not been explored in Abe [1]. Moreover, the scale of lensing time duration becomes shorter as the lens mass decreases. Thus it is important to assess whether currently available telescopes can probe SULCOs with a reasonable observation time. In what follows, for simplicity, we assume that the surface brightness of source stars is constant (limb darkening is not taken into account) and circular.

2. Lensing by SULCOs

2.1. Significance and amplification

If a source star is bright enough, the detectable lensing amplification can be weak: the relation between the signal-to-noise ratio for an unlensed source $\eta$ and that for an amplified event $\xi$ (an excess in flux) can be estimated as follows.

First, the signal-to-noise ratio $\eta$ is written in terms of photon counts for the signal $S$ and those for the sky background and the other noise sources $N$ during an exposure time $T_e$,  
\[
\eta = \frac{S}{\sqrt{S + N}}. 
\] (1)

If the source is amplified by an amplification factor $A$, then the photon counts for the signal during an exposure time $T_e$ is increased to $S + \Delta S = A S$. The
signal-to-noise ratio $\xi$ for an amplified event is then given by

$$\xi = \frac{\Delta S}{\sqrt{S + \Delta S + N}}$$  \hspace{1cm} (2)

Using equations (1) and (2), the amplification factor $A$ is given by

$$A(\eta, \xi, S) = 1 + \frac{\xi^2}{2S} + \xi \sqrt{\frac{1}{\eta^2} + \frac{\xi^2}{4S^2}}.$$  \hspace{1cm} (3)

In what follows, we assume that the photon counts of unlensed and lensed source are sufficiently large, i.e., $\eta^2 \ll S$ and $\xi^2 \ll S$. Then, equation (3) is reduced to a simpler form $A = 1 + \xi/\eta$. For instance, if we would like to observe an amplified event at $5\sigma$ and the significance of the unlensed source is $5\sigma$, then the lensing amplification should satisfy $A > 2$ and the event corresponds to a strong lensing. If the signal-to-noise ratio for an unlensed star is $\eta = 20$, then for detecting an event with $\xi = 5$, the lensing amplification should satisfy $A > 1.25$ and the event corresponds to a weak or strong lensing. If the significance of an unlensed star is $\ll 5\sigma$, then the amplification should satisfy $A \gg 2$, which corresponds to a 'very strong' lensing.

2.2. Maximum distance to lens

Since the distance to the source stars at galaxies in the Local Group is much smaller than the present Hubble length, we can neglect the effects of cosmological expansion. Hence, we omit a term "angular diameter" in the following.

If the distance to the lens is too large, the angular Einstein radius $\theta_E$ is so small that the amplification of an extended source cannot be observed. In what follows, we derive the maximum distance $D_{\text{max}}$ to the lens for which the amplification by SULCOs is detectable with a given signal-to-noise ratio $\xi_0$.

First, we consider the case in which the angular Einstein radius $\theta_E$ is equal to or smaller than the angular size of the unlensed source star $\theta_a$, i.e., $\epsilon \equiv \theta_E/\theta_a \leq 1$. In this case, we expect a weak amplification of a source star due to the finite-source size effect. The amplification factor $A$ is then approximately given by

$$A \approx \sqrt{1 + 4\epsilon^2},$$  \hspace{1cm} (4)

provided that a disk with radius $\theta_E$ centered at a point mass is totally contained in the source star (the angular source-lens separation is sufficiently smaller than the angular source radius). As shown in figure 1, equation (4) approximately holds even if $\epsilon \sim 1$. The error is typically less than a few percent.

From equations (3) and (4), we can calculate the minimum value for the size ratio $\epsilon_{\text{min}}$ as a function of a given signal-to-noise ratio $\xi_0$ for amplification and that for the source star $\eta$. Assuming $\eta^2 \ll S$ and $\xi^2 \ll S$, for a given $\eta$, the minimum of $\epsilon$ is given by

$$\epsilon_{\text{min}} = \frac{\xi_0}{2\eta} \sqrt{1 + \frac{2\eta}{\xi_0}}.$$  \hspace{1cm} (5)
Figure 1: Amplification by a point mass as a function of source radius. The surface brightness profile of a source is assumed to be a top-hat type. \(1/\epsilon = \theta_a/\theta_E\) is the source radius in unit of an Einstein radius. The approximated amplification factor in equation (4) is plotted as a blue solid curve. The exact amplifications for a circular top-hat sources centered at \((\theta_E/2, 0), (\theta_E, 0), (2\theta_E, 0)\) at the source plane are plotted as dashed, dot-dashed and dotted orange curves, respectively. A point mass is placed at the center of coordinates.

The angular Einstein radius \(\theta_E\) for a point mass \(M\) can be written in terms of the distances to the lens \(D_L\), to the source \(D_S\), and between the source and the lens \(D_{LS}\) as

\[
\theta_E = \sqrt{\frac{4GM}{c^2 D_{LS}}} D_L D_S,
\]

where \(c\) is the light velocity and \(G\) is the gravitational constant. Then the size ratio is given by

\[
\epsilon = 0.56 \times \left( \frac{D_S}{840 \text{ kpc}} \right)^{1/2} \left( \frac{a}{R_\odot} \right)^{-1} \left( \frac{M}{10^{-9} \text{M}_\odot} \right)^{1/2} \times \left( \frac{1-x}{x} \right)^{1/2}
\]

(7)

where \(a\) is the radius of the source star and \(x \equiv D_L/D_S\) is the normalized distance to the lens. For a given radius \(a\) and a mass \(M\), the size ratio \(\epsilon\) is a monotonically increasing function of \(D_S\) and a decreasing function of \(x\). For neighbouring lenses at \(x \ll 1\), we have \(x \propto (\epsilon a)^{-2}\). Using equation (6) and equation (7), we can calculate the maximum distance \(D_{L, \text{max}}\) (\(x_{\text{max}}\) if normalized by \(D_S\)) for which one can detect a lensing event for a given \(\xi_0\) and \(\eta\).

Second, we consider the case in which \(\theta_E\) is larger than \(\theta_a\), i.e., \(\epsilon > 1\). In this case, we expect a strong amplification of a source star if the source is
totally contained in a disk with $\theta_E$ (small source-lens separation). The mean amplification is $\langle A \rangle \gtrsim \sqrt{5} \approx 2.2$ if averaged over the source position inside $\theta_E$. For a given $\xi_0$, we can calculate $\eta$ that gives $A \gtrsim 2.2$. Then, equation (7) gives the range of $x$ for which $\epsilon > 1$ is satisfied. If the source star is not resolvable, i.e., $\eta \ll 5$, then the magnification factor should be large enough: $A \gg 2$. In this case (pixel lensing), the source star should be sufficiently close to the lens that $A > 1 + 5/\eta$.

2.3. Optical depth

First, we consider lensing events with weak amplification ($\epsilon \leq 1$) due to the finite-source size effect. For these events, the amplification factor is almost constant when a disk with a radius of $\theta_E$ centered at a point mass is totally contained in the source (small source-lens separation) but it suddenly drops to unity if the disk passes through the source star. Therefore, the lensing cross section (defined as the total area of a source that yields a constant amplification) is $\sim \pi(D_L \theta_E)^2$. Then the lensing optical depth (defined as the total area of sources that yields a constant amplification divided by the whole target area) for an extended source with $\epsilon \leq 1$ is approximately given by

$$\tau(\epsilon \leq 1) \approx \int_{D_L(\epsilon=1)}^{D_{L,max}} \rho(D_L) \frac{\pi}{M} \epsilon^{-1} \theta_E D_L^2 dD_L,$$

where $D_L$ is the distance to the lens and $\rho(D_L)$ denotes the density of dark matter at $D_L$. Although equation (8) gives a slightly larger value for $\epsilon \sim 1$, for the purpose of our analysis, such a small contribution (typically less than a few percent) is negligible. From equation (6) and equation (8), we have

$$\tau(\epsilon \leq 1) \approx \frac{4\pi G}{c^2} D_S^2 \int_{x(\epsilon=1)}^{x_{\text{max}}} \frac{\rho(x)x(1-x)}{\epsilon^2(x)} dx,$$

where $x_{\text{max}} = D_{L,max}/D_S$.

Second, we consider lensing events with strong amplification ($\epsilon > 1$), in which the source size is sufficiently smaller than the Einstein radius and the source star is resolvable ($\eta \geq 5$). Then the lensing cross section for a point mass is $\sim \pi(D_L \theta_E)^2$ for which the mean amplification is $\langle A \rangle = \sqrt{5} \approx 2.2$. The lensing optical depth is given by

$$\tau(\epsilon > 1) \approx \frac{4\pi G}{c^2} D_S^2 \int_{0}^{x(\epsilon=1)} \rho(x)x(1-x) dx.$$

If the source star is unresolvable ($\eta \ll 5$), the cross section for a point mass should be much smaller than $\pi(D_L \theta_E)^2$, which depends on the detectable amplification factor $A = 1 + \xi/\eta$.

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1To be more precise, the cross section should be $\sim 5\pi(D_L \theta_E)^2/(1 + \xi/\eta)^2$. For $\eta = \xi = 5$, it becomes $\sim \pi(D_L \theta_E)^2$. 
2.4. Duration time

In terms of the transverse velocities \( v_O, v_L, \) and \( v_S \), of the observer, the lens at redshift \( z_L \), and the source at redshift \( z_S \) in the source plane, the effective transverse velocity in the source plane is given by

\[
v_{\text{eff}} = \frac{1}{1 + z_S} v_S - \frac{1}{1 + z_L} \frac{D_S}{D_L} v_L + \frac{1}{1 + z_L} \frac{D_{LS}}{D_L} v_O. \tag{11}
\]

For lenses \( D_L \ll D_S \) and \( z_O \sim z_L \sim 0 \), the root-mean-square of the effective velocity averaged over the lens objects at \( x \ll 1 \) is

\[
\sigma_{\text{eff}}(x) = \langle |v_{\text{eff}}(x)|^2 \rangle^{1/2} \approx x^{-1} \langle |v_L|^2 \rangle^{1/2}, \tag{12}
\]

where we assumed that \( v_S \sim v_L \sim v_O \). The average lensing duration time \( \Delta T(x) \) is equal to the average width of a disk (\( = \pi/2 \times \text{radius} \)) that gives a lensing cross section divided by the standard deviation of the effective transverse velocity \( \sigma_{\text{eff}} \),

\[
\Delta T(x) \approx \begin{cases} \pi a & \epsilon \leq 1 \\ \frac{\pi a_E(x)}{2\sigma_{\text{eff}}(x)} & \epsilon > 1, \end{cases} \tag{13}
\]

where \( a_E(x) = D_S \theta_E(x) \) is the Einstein radius\(^2\) of a lens at a distance \( x \) measured projected onto the source plane.

2.5. Event number

Let us consider an observation that consists of \( N_e \) consecutive shots with an exposure time \( \Delta T_e \). Then the differential effective optical depth for a spatial interval from \( x \) to \( x + dx \) is

\[
d\tau_{\text{eff}} \approx N_e \Delta T_e \Delta T_e^{-1}(x) \tau' dx,
\]

where \( \tau' \equiv d\tau/dx \). The condition for the detection of a lensing event \( \Delta T_e \leq \Delta T \) sets the lower limit for the distance to the lens \( x_{\text{min}} \). The effective lensing optical depth for such an observation is given by

\[
\tau_{\text{eff}} \approx \frac{8GD_S^2 N_e \Delta T_e}{c^2} \left[ \int_{x(\epsilon=1)}^{x_{\text{max}}} \frac{\rho(x)(1-x)\sigma_{\text{eff}}(x)}{ae^2(x)} dx + 2 \int_{x_{\text{min}}}^{x(\epsilon=1)} \frac{\rho(x)(1-x)\sigma_{\text{eff}}(x)}{aE(x)} dx \right]. \tag{15}
\]

\(^2\)To be more precise, \( \theta_E(x) \) should be replaced by \( \sqrt{5} \theta_E(x)/(1 + \xi/\eta) \).
Plugging equation (6) into equation (15), we have

\[
\tau_{\text{eff}} \approx N_e \Delta T_e \left[ \frac{2aD_S}{M} \int_{x(x=1)}^{x_{\text{max}}} \rho(x)x^2 \sigma_{\text{eff}}(x) dx + 4 \sqrt{\frac{GD_S^3}{Mc^2}} \int_{x_{\text{min}}}^{x_{x(\epsilon=1)}} \rho(x) \sqrt{x^3(1-x)} \sigma_{\text{eff}}(x) dx \right].
\]

(16)

Multiplying equation (16) by the total number of observable stars \(N_*\) for an exposure time \(\Delta T_e\) and a signal-to-noise ratio \(\eta\) for source stars, we obtain the total expected event number

\[
E(N_e, \Delta T_e, \eta, \xi_0) = N_* (\Delta T_e, \eta) \tau_{\text{eff}}(N_e, \Delta T_e, \eta, \xi_0).
\]

(17)

2.6. Order estimate

To probe SULCOs by monitoring a number of background stars, we take into account the finite-source size effect: the Einstein radius of a point mass should be sufficiently large with respect to the radius of the source star. In other words, the lens should be sufficiently close to us. However, in that case, the lensing duration time becomes small and thus the exposure time for each snapshot must be small. This means that the target source stars should be sufficiently bright.

Let us estimate the event rate of nanolensing more quantitatively. First, we need a number of bright stars at a distant place, i.e., large \(D_S\) in order to reduce the angular source size. For simplicity, we assume that the source radius \(a\) is constant (typically of the order of the solar radius), and the smallest size ratio is \(\epsilon = 1\). This corresponds to strong lensing events with an amplification factor \(A \gtrsim 2\) and \(\epsilon \geq 1\). We also fix \(D_S\) and assume that the lenses are sufficiently close to us, i.e., the density of dark matter \(\rho(x)\) is constant in \(x\), and the number of observable source is fixed. From equations (9), (13) and (14), we have the distance to the lens \(D_L \propto M\), the lensing duration time \(\Delta T \propto D_L \propto M\) and the optical depth \(\tau \propto D_L^2 \propto M^2\). Therefore, the event number is proportional to \(\tau/\Delta T \propto M\). Thus, the observable distance to a strong lensing event, the duration time, and the event number are all proportional to \(M\). In real setting, the number of available source stars depends on the exposure time \(\Delta T_e < \Delta T\). Therefore, the event number depends on \(M\) more strongly.

As a source galaxy, we adopt a spiral galaxy M33 at \(\sim 840\) kpc\[13\]. Although M33 is more compact and less massive than M31, the flux contribution from faint unresolved stars per pixel is expected to be smaller than M31 as M33 is a typical face-on galaxy with much diffuse spiral arm where bright A- and F-type main sequence stars are relatively abundant. Therefore, for the purpose of observing weak amplification of bright resolved stars, M33 would be much suitable. The luminosity function of the whole region of the disk and halo is known only at the upper end \((V < 19.2)\)[12]. To estimate the luminosity function of stars with \(V > 19.2\) in M33, we assume that it has the same profile of the local luminosity.
function of the Milky Way \[4\] except for the normalization. Note that the slope 0.677 at \(V = 21\) is consistent with the observed value 0.67 \(\pm\) 0.03 \[12\].

To normalize the luminosity function, we use 2112 blue stars \((U - V < 0\) and \(U - B < 0\)) and 389 red giants \((B - V > 1.8\)), which are complete to \(V = 19.5\) \[20\].

It turned out that the number of stars is \(6 \times 10^5\) for \(19.5 < V < 23\) and \(3 \times 10^7\) for \(19.5 < V < 26\).

The typical time duration scale \(\Delta T\) is given by the velocity of a lens at which the angular Einstein radius is equal to the source radius, i.e., \(\epsilon = 1\). For instance, for M33, equation \(7\) gives that the typical distance to the lens with a mass of \(M = 10^{-9} M_\odot\) is \(D_L = 60\) kpc assuming that the source radius is \(a = 2R_\odot\), i.e., the typical size of a A-type main-sequence star, which is common at the M33 disk. Then, assuming the standard deviation of velocity \(\sigma_{\text{eff}} = 200\) km/s for lenses that reside in the halo of the Milky Way, equation \(13\) gives \(\Delta T \sim 10^3\) s for \(M = 10^{-9} M_\odot\). Assuming the local dark matter density \(\rho_0 = 0.0079 M_\odot/\text{pc}^3\), and the dark matter density for the halo of the Milky Way \[2\] \[3\], \(\rho(r) = \rho_0 r / (r^2 + (5\text{kpc})^2)^{3/2}\), and the optical depth for an observation with exposure time \(\Delta T_e = \Delta T / 2 = 5 \times 10^2\) s is \(\tau = 3 \times 10^{-7}\).

For a 8 m class telescope, the V-band magnitude limit \((S/N=5)\) for an exposure time of \(5 \times 10^5\) s is typically \(V \sim 26\). Therefore, the expected lensing event number is \(\sim 10\) provided that point masses with \(M = 10^{-9} M_\odot\) constitute all the dark matter. For a total integration time \(\sim 7\) hours with a field-of-view of 1 deg\(^2\), we expect 100 – 1000 events. If the lenses have a mass \(M = 10^{-11} M_\odot\), the typical time duration is \(\sim 8\) s and the expected event number is \(< 10\) for a total integration time \(\sim 7\) hours. These numbers give approximate lower limits of event number as we did not take into account contributions from weakly amplified events \((\epsilon < 1)\) in this order estimate. It should be noted that the contribution from neighbouring unresolvable stars in each pixel is negligible because the surface brightness of disk stars in M33 is typically \(V \sim 26/\text{arcsec}^2\) \[18\] at angular distances \(\lesssim 1^\circ\), which is much fainter than that of sky.

### 2.7. Simulation of observation

In order to assess the feasibility of our method, we assume an observation of M33 with a wide field (1.5 degree in diameter) camera that is attached to a 8 m class telescope. For a limiting magnitude \(V = 26\), with a seeing of \(\sim 0.6\) arcsec, it can resolve \(\sim 10^7\) stars at angular distances \(\theta > 14\) arcmin from the center assuming that the surface brightness is proportional to \(\exp(-\theta)\). We also assume a relatively good condition, such that seeing is \(\sim 0.6\) arcsec and the sky is dark. We use the same Milky Way halo model used in subsection 2.6, and we also take into account the size of stars, which depends on the luminosity. We use the mass-radius relation for luminous main sequence A and B type stars \((2 M_\odot \lesssim M_* \lesssim 20 M_\odot)\), which yields \(a/a_\odot = -(V - 20) + 8\) \[6\]. As the number of bright red stars are smaller (about 1/5) than that of bright blue stars for

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\[3\] We omitted the contribution from SULCOs in the halo of M33 as the Einstein radii are much smaller than those in the halo of the Milky Way.
Figure 2: Expected event number as a function of SULCO mass for 7 hours observation of M33 using a wide field camera attached to a 8 m class telescope with (left) or without (right) the CCD readout time of ~ 30 s taken into account. The exposure time for each snapshot and the signal-to-noise ratio of the source stars are \((T_e, \eta) = (1 \text{ s}, 5)(\text{blue, full curve}), (60 \text{ s}, 5)(\text{red, dot-dashed}), \text{and} (60 \text{ s}, 100)(\text{red, dashed})\). For the low-end \((M < 10^{-10} M_\odot)\), the finite source-size effect becomes important.

\[ V < 19.5, \] we expect that the effect of red giants is negligible at the bright end. As shown in the right panel of figure 2, the event number for \((T_e, \eta) = (60 \text{ s}, 5)(\text{red, dot-dashed})\) is \( \sim 1000 \) for \( M = 10^{-9} M_\odot \) to \( M = 10^{-7} M_\odot \). If the exposure time is 1 s, the event number can be further increased to \( \sim 10000 \) but if one considers the CCD readout time of \( \sim 30 \text{ s} \), the event number is reduced by more than 10 times in real setting (the left panel of figure 2) For masses with \( M = 10^{-10} M_\odot \), the lensing effect becomes weak due to the finite source-size. However, if we restrict the source stars to only bright ones, this difficulty can be somehow avoided (though the CCD saturation inhibits the measurement of very bright stars \( V \lesssim 20.5 \)). For instance, for the parameters \((T_e, \eta) = (60 \text{ s}, 100)(\text{red, dashed})\), we expect \(10 - 100\) events during an observing time of 7 hours. This is due to the weak lensing effect: Even if the source size is much larger than the Einstein radius, the source can be slightly amplified by a lens. As one can see in figure 3, the maximum lensing time duration \( \Delta T(x_{\max}) \) and the maximum distance to the lens \( x_{\max} \) increase as the signal-to-noise ratio \( \eta \) increases as a result of small \( \epsilon_{\min} \). This boosts the chance of detection of amplified events caused by SULCOs. Of course, it is challenging to detect such a small change as other systematics such as intrinsic variability (flare of stars) or the CCD noises can easily hamper such detection. However, in principle, one can discriminate lensing events by looking into the “colourless” feature in the time variability and by comparing the measured flux to the templates of predicted light curves.

3. Conclusion and Discussion

In this paper, we have discussed that nanolensing events by dark sub-lunar mass compact objects (SULCOs) such as PBHs and free-floating dwarf planets can be detected by monitoring \(10^{5-7}\) stars in the Local Group galaxies such
Figure 3: The maximum lensing time duration (left) and the maximum distance to the lens (right) as a function of SULCO mass. The signal-to-noise ratios are 5, 20, 100 from the bottom to the top.

as M33. The typical lensing time scale is $\sim 100$ s. The exposure time for each snapshot must be $\sim 100$ s, which is the typical time scale of nanolensing variability for SULCOs with a mass of $\sim 10^{-9} M$. Using a 8 m class telescope, $10^{3-4}$ events per night would be detected if SULCOs constitute all the dark matter. Moreover, we expect $10^{1-2}$ events from weak amplification of very bright stars caused by SULCOs with a mass range of $10^{-11} M_{\odot}$ to $10^{-7} M_{\odot}$, though detection of such change may be a challenging task. Our method would provide a stringent constraint on the abundance of SULCOs at the distance $0.1 - 100$ kpc from us.

As a source galaxy, we have considered M33 as it has a relatively large number of blue main sequence stars and much diffuse spiral arms in comparison with M31. These features are important for observing weak amplification (which was not studied in Abe [1]) by SULCOs from time variability of very bright source stars. However, for the purpose of detecting nanolensing events, M31 would be much suitable as the number of available source stars is larger than M33 though the effect of blending due to neighbouring stars may be stronger as it is not face-on. For detecting SULCOs via weak amplification of source stars, other galaxies at a farther distance would be suitable if 30-m class telescope is available in the coming decade.

We have taken into account the finite source-size effect, which is important for estimating the weak amplification caused by SULCOs. However, the effect of spatial variability in the source brightness such as limb darkening has not been taken into account. Such an effect becomes important when the impact parameter of the source is comparable to the radius of the star. More precise treatment is necessary for the cases in which the angular Einstein radius is approximately equal to the angular source size.

A part of SULCOs may consist of free-floating dwarf planets. Detection of these objects in the intergalactic space is a challenging task. Like MACHOs, these small unbounded objects may constitute a large portion of baryonic masses in the disk or halo of our galaxy. Our method would open a new window on these small objects in the Milky Way halo that would otherwise not be observable.
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