Abstract: Comparison and analysis of the regularised zero forcing precoder, rapid numerical algorithms-based precoder and the truncated polynomial expansion-based precoder are done for massive multiple-input multiple-output wireless system for multicell system. The analysis was done for the imperfect channel covariance information. The achievable signal-to-interference-and-noise ratio, spectral efficiency and energy efficiency were investigated. The simulated outcome of the rapid numerical algorithms, regularised zero forcing and truncated polynomial expansion precoders for multicell massive MIMO system was analysed. The rapid numerical algorithms-based precoder gave the best performance followed by the regularised zero forcing precoder, and the truncated polynomial expansion-based precoder had the lowest performance for the multicell massive MIMO system. The increase in spectral efficiency per cell can be attributed to the fact that the pre-log factor reduces with the increased number of pilots. Also, this leads to increased instantaneous signal-to-interference-and-noise ratio as the channel estimates become better with reduced pilot contamination. Again, for truncated polynomial expansion precoding there is a reduction in spectral efficiency because improvement in approximation quality do not overshadow the reduction in pre-log factor. The performance is evaluated for uncoordinated and coordinated massive MIMO.

1 Introduction

Multiple-input multiple-output (MIMO) procedures have increased impressive consideration in current wireless communication because it can altogether enhance the capacity and steadfast quality of wireless systems [1]. The embodiment of downlink multiuser MIMO is precoding, which implies that the antenna arrays are utilised to coordinate every information flag spatially towards its expected user terminal (UT). However, the precoding plan in multiuser MIMO needs extremely precise momentary channel state information (CSI) which can be bulky to accomplish. In a multiuser MIMO, the channel fluctuates rapidly in the order of a ten of time and frequency. If coding is allowed atwhart coherent intervals of many channels, it has been demonstrated that for system consisting of $M$ antennas transmitting to $K$ UTs, there is a linear increase in capacity in relation with the minimum $\log(1+\text{SINR})$ [3, 4]. This capacity scaling can be applied to massive MIMO systems with a base station (BS) having $M$ transmitting antennas in communication with $K$ UTs in the case of time-division duplexing (TDD).

The throughput can greatly be enhanced if BS communicating with the UT can do so within the same time-frequency resources. This can be achieved but at a price of enhanced inter-user interference, hence eroding the system performance. To mitigate this, dirty paper coding (DPC) is employed, where the transmitter jointly encodes the data symbols for all users [5, 6]. The transmission scheme based on DPC comes with complexity that prohibits its practical realisation. This then necessitates the reliance on transmit-side preprocessing techniques called precoding schemes. It has been said that with massive MIMO, pilot contamination can be optimally mitigated with the simplest forms of precoding like eigen beamforming (BF) and matched filtering [7]. Later research have demonstrated that realiseable BS antennas demand included linear precoding procedures, for example, MMSE [8]. We cannot overlook the complexity of computing the minimum mean square error (MMSE) precoding in massive MIMO network system, since MMSE relies on inverting of substantial matrices [9]. To circumvent this, a class of rapid numerical algorithms (RNA) and truncated polynomial expansion (TPE) precoders were discussed in [10] for the single cell multi-user case.

The multicell TPE-based precoder is discussed in [11] and compared to regularised zero forcing (RZF) precoding. The paper [12] develops a multicell MRC-BSC and ZF-BSC precoding to improve system downlink performance, and their influence on the received signal-to-interference-plus-noise ratio (SINR) of users in other cells. This paper discusses multicell precoding using RNA-based, RZF and TPE-based precoders with and without BS cooperation. In our work, the proposed precoding schemes with and without BS cooperation are obtained by changing the matrix structure of single-cell RNA, RZF and TPE precoding, so they are both linear precoding and have low computational complexity. We consider the effects and analyse the received SINR of user in other cells. We use a similar analysis method as that employed in [12]. The SINR is used to calculate the capacity per cell for comparison purposes. We also look, at energy efficient based on these three precoding schemes for the case of uncoordinated and coordinated BSs and conclude by comparing the performance of these three precoders on overall. The channel model considered includes the imperfect covariance experienced by the BSs.

We begin by modulating the optimal multicell linear precoding and then tailor it to each of the multicell linear precoding schemes. The imperfect covariance channel is modelled before evaluating the SINR and data rate of imperfect channel covariance information is used to analyse the behaviour of the massive MIMO downlink network system over the modelled channels in multicell scenario.

Notation: lower-case and upper-case boldface letters denote vectors and matrices, respectively; $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^{-1}$ and tr$(\cdot)$ denote the transpose, conjugate transpose, matrix inversion and trace, respectively; $C$ denotes the set of complex numbers, $I_N$ is the $N \times N$ identity matrix.

2 System model

We examine a downlink multicell massive MIMO system with $L > 1$ cells and $K$ single antenna UTs in each cell that transmits signals to a BS in corresponding cell having $M$ antennas at the same time. The propagation matrix, $G$, with dimensions $M \times K$ for each cell, is obtained by multiplying an $M \times K$ matrix [4, 13, 14].
where we define $\mathcal{C}_m \in \mathbb{C}^{M \times k}$ as the transmit signal from the $t$th BS and $h_{t,j,k}^\dagger \in \mathbb{C}^{M \times 1}$ represents the channel vector from the $t$th BS to the $m$th UT in the $j$th cell, and $n_{j,m} \sim \mathcal{C}(0, \sigma^2)$ accounts for the additive white Gaussian noise that has a variance $\sigma^2$, at the receiver's input [11, 15–18].

The channel vectors are assumed to be Rayleigh fading and modeled as

$$h_{t,j,m} \sim \mathcal{C}(0, \Phi_{t,j,m})$$

where $\Phi_{t,j,m}$ is the covariance matrix corresponding from the $t$th BS to the $m$th UT in the $j$th cell. The BSs use linear precoding with Gaussian codebooks in that the signal transmitted by the $j$th cell is represented by

$$u_j = \sum_{k=1}^K f_{j,m}^k x_j = F_j x_j$$

where $F_j = [f_{j,1}, \ldots, f_{j,K}] \in \mathbb{C}^{M \times K}$ is the precoding matrix and $x_j = [x_{j,1}, \ldots, x_{j,K}] \sim \mathcal{C}(0, I_k)$ is the information vector with data symbols from all the UTs in the $j$th cell. The average power to be transmitted at BS $j$ is constrained as $\frac{1}{K} \text{tr}(F_j F_j^\dagger) = P_j$. Thus, the received signal vector at $k$th UT is expressed as

$$y_{j,k} = \sum_{m=1}^L \sum_{k=1}^K h_{t,j,m}^\dagger f_{j,k} n_{k} + h_{j,k} n_{j,m}$$

This expression can be expanded to

$$y_{j,m} = h_{j,m}^\dagger f_{j,m}^\dagger n_{j,m} + \sum_{k=1}^K h_{j,k}^\dagger f_{j,k} n_{k} + n_{j,m}$$

A notable property of massive MIMO systems is channel hardening, which infers that the helpful channel $h_{j,m}^\dagger f_{j,m}^\dagger$ merges to its average value as $M \to \infty$. The signal at the received $m$th UT can be further broken down as [8, 19]

$$y_{j,m} = \mathbb{E}[h_{j,m}^\dagger f_{j,m}^\dagger n_{j,m}] + \sum_{k \neq m} h_{j,k}^\dagger f_{j,k} n_{k} + n_{j,m}$$

Furthermore, we expect that the channel gain $\mathbb{E}[h_{j,m}^\dagger f_{j,m}^\dagger]$ is perceived at the corresponding UT, alongside its fluctuation $\text{var}(h_{j,m}^\dagger f_{j,m}^\dagger)$, and the average sum interference power $\sum_{k \neq m} \mathbb{E}[h_{j,k}^\dagger f_{j,k} n_{k}]$ caused by concurrent transmission to different UTs in the same and different cells. By treating the interference between users from the same cell and the users from the other cell as well as the channel ambiguity as worst scenario Gaussian noise, the $m$th UT in $j$th cell can accomplish a spectral efficiency (SE) of

$$r_{j,m} = \log(1 + \text{SINR}_{j,m})$$

without the knowledge of the transitory values of $h_{j,k}^\dagger f_{j,k}$ of its channel [8, 19–21] and where

$$\text{SINR}_{j,m} = \frac{\mathbb{E}[h_{j,m}^\dagger f_{j,m}^\dagger]^2}{\sigma^2 + \sum_{k \neq m} \mathbb{E}[h_{j,k}^\dagger f_{j,k} n_{k}]}$$

### 2.1 Model of imperfect covariance channel information

If we assume that the transmitter has imperfect information concerning the instantaneous channel realisation $h_{j,k}$ of every UT. We can approximate the channel relating to $k$th UT in $j$th cell, by letting every BS to correlate the received signal with the pilot sequence of that user. This gives the processed received signal as

$$y_{j,m}^{\text{P}} = h_{j,k} + \sum_{k \neq m} h_{j,k} + \frac{1}{\rho_k} n_{j,m}$$

where $n_{j,m} \sim \mathcal{C}(0, \rho_k)$ and $\rho_k > 0$ gives the effective training SINR [8]. We use the MMSE estimate $\hat{h}_{j,k}^{\text{MMSE}}$ of $h_{j,k}$ as

$$\hat{h}_{j,k} = F_j S_j y_{j,k}$$

where $S_j = (1/\rho_k) + \sum_{k \neq m} \Phi_{j,k}^{-1}$ and $\Phi_{j,k}$ is the channel covariance matrix of vector $h_{j,k}$. The approximated channels from the $j$th BS to all UTs in $j$th cell are denoted

$$\hat{H}_{j,m} = [\hat{h}_{j,1}, \ldots, \hat{h}_{j,K}] \in \mathbb{C}^{M \times K}$$

Then we define $R_{\epsilon,j,k} = \Phi_{j,k}^{-1} S_j \Phi_{j,k}$ and we state here that $\hat{h}_{j,k} \sim \mathcal{C}(0, R_{\epsilon,j,k})$ is not dependent on the estimation error $\hat{h}_{j,k} = \hat{h}_{j,k} - h_{j,k}$ as MMSE approximation is used.

### 3 Multi-cell linear precoding

#### 3.1 RZF precoding

From the notation in [11], the RZF precoding matrix for the BS in the $j$th cell is expressed as

$$\alpha_j^{RZF} = \sqrt{\theta_j} \left( \sum_{k=1}^K h_{j,k}^\dagger h_{j,k} + Z \right)^{-1} h_{j,k}$$

where $Z = \sum_{k=1}^K (\Phi_k - R_k) + (1/\rho_k) I_d$ and the scalar $\theta_j > 0$ is carefully chosen to satisfy the power constraint in the cell. We modify the coefficient of the precoding matrix by evoking norm minimisation scheme to improve the SINR performance of the transmission signal [22]. Then, the desired precoding matrix becomes

$$F_j^{RZF} = \alpha_j^{RZF}$$

#### 3.2 TPE precoding

Expanding on the idea of TPE, we presently give another class of low-complexity linear precoding plan for the multicell case. The Cayley-Hamilton hypothesis is used to specifically show that the inverse of a matrix $A$ of dimension $M$ can be composed as a weighted total of its first $M$ powers [9, 16]. If the $j$th BS utilises a truncation order $J$, then the proposed TPE precoding matrix is expressed as

$$\alpha_j^{TPE} = \sum_{m=0}^{J-1} \alpha_j^{RZF} \left( \sum_{k=1}^K h_{j,k}^\dagger h_{j,k} + Z \right)^{-1} h_{j,k}$$

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where \( \{ \theta_{0,j}, j = 0, \ldots, J - 1 \} \) are the \( J \) scalar coefficients that are employed by the \( j \)th BS and the normalisation by \( \sqrt{K} \) controls the energy of the precoding matrix. Then using the norm minimisation, the desired precoding matrix becomes

\[
F_{j}^{\text{PTE}} = \frac{\hat{a}_{j,m}}{\| \hat{a}_{j,m} \|}
\]  

(15)

For large-scale approximation of SINRs, it is shown that in the large-(\( M, K \)) regime, the SINR experienced by the \( m \)th UT served by the \( j \)th cell, can be estimated by a deterministic term, depending only on the channel statistics. Some extra notation are introduced to provide simpler representation of the SINR expression [16].

Then \( a_{j} = [a_{0,j}, \ldots, a_{J-1,j}] \) and let \( a_{j,m} \in \mathbb{C}^{J \times 1} \) and \( B_{j,m} \in \mathbb{C}^{J \times J} \) to be given by

\[
[a_{j,m}] = \left( \frac{\hat{h}_{j,m}^{H}V_{n,j}\hat{h}_{j,m}}{\sqrt{K}} \right), \quad n \in [0, J - 1]
\]  

(16)

\[
[B_{j,m}]_{p,p} = \frac{1}{K} \hat{h}_{j,m}^{H}V_{n+1,j}\hat{h}_{j,m} \quad p \in [0, J - 1]
\]  

(17)

where \( V_{n,j} = \{ \sum_{j=1}^{J} \sum_{k=1}^{L} \hat{h}_{j,k}^{H}(\hat{h}_{j,k})^{H} \}_{n}^{0} \). Then the SINR associated with the \( m \)th UT in the \( j \)th cell is

\[
\text{SINR}_{j,m}^{\text{PTE}} = \frac{\| \hat{a}_{j,m} \|^{2}}{\sum_{k=1}^{K} \| B_{j,m}a_{j,k} \|^{2} + \| \hat{a}_{j,m} \|^{2}}
\]  

(18)

### 3.3 RNA precoding

The ideal linear precoding is obscure in presence of imperfect CSI and involves wide-ranging optimisation procedures for the case of perfect CSI [23]. In this way, just heuristic precoding strategies are achievable in fading multi-cell networks. RNA is a state-of-the-art heuristic scheme with a simple closed-form precoding expression based on MMSE precoding [10].

It is stated in [10] that when the value of the power of the polynomial, \( J_{p} \), is low the performance of the TPE precoder is so poor, but as the value of \( J_{p} \) increases, its bit-rates improve and nears that of RNA precoder. This comes at the cost of increase in the hardware needed for the implementation of the TPE precoding. RNA precoding will have relatively the same complexity as TPE precoding at the specified hardware implementation while giving a better performance. This means that the increase in hardware to improve the performance of TPE precoder will suffice the RNA precoder to reduce its complexity and come as close to the complexity of the TPE precoder but with a better performance in terms of throughput [9, 10, 24]. Using the notation of [18], the RNA precoding matrix used by the BS in the \( j \)th cell is

\[
\hat{F}_{j}^{\text{RNA}} = \sqrt{\beta} \left( \sum_{k=1}^{K} \hat{h}_{j,k}^{H}(\hat{h}_{j,k})^{H} + Z_{j} \right)^{-1} \hat{h}_{j,k}
\]  

(19)

where \( Z = \sum_{k=1}^{K} \left( \Phi_{k} - R_{0} \right) + \left( 1/\theta_{0} \right) \mathbb{I}_{L} \) and the scalar \( \theta_{0} > 0 \) is carefully chosen to satisfy the power constraint in the cell. Then using the norm minimisation, the desired precoding matrix becomes

\[
F_{j}^{\text{RNA}} = \frac{\hat{a}_{j,m}^{\text{RNA}}}{\| \hat{a}_{j,m}^{\text{RNA}} \|}
\]  

(20)

### 4 Multi-cell linear precoding with coordination

In massive MIMO, the BSs are furnished with many antennas, this avails additional spatial dimensions which permit coordination of BF vectors athwart the BSs; this further improves the performance generally. This is possible due to the uplink-downlink duality in TDD massive MIMO systems. To enhance downlink attainable rate, the collaboration between BSs in various cells is evaluated, and it is expected that the \( j \)th BS knows the RNA, RZF or TPE precoding and channel approximation \( \hat{h}_{j,m} \) of every other single cell within the network. The new multi-cell linear precoding with BS cooperation can be expressed as follows [12]:

\[
\mathbf{Y}_{m} = \mathbf{h}_{j,m}^{H}\mathbf{f}_{m} + \sum_{k \neq m} \mathbf{h}_{k,m}^{H}\mathbf{f}_{k} + \mathbf{n}_{m}
\]  

(22)

These can be rewritten as in (5) to yield

\[
\mathbf{Y}_{m} = \mathbf{h}_{j,m}^{H}\mathbf{f}_{m} + \sum_{k \neq m} \mathbf{h}_{k,m}^{H}\mathbf{f}_{k} + \sum_{\ell \neq m} \mathbf{h}_{\ell,m}^{H}\mathbf{f}_{\ell} + \mathbf{n}_{m}
\]  

(22)

When compared with classical multi-cell precoding matrix in (5), the new multicell precoding matrix adds term \( \mathbf{h}_{j,m}^{H}\left[ \mathbf{diag}(\hat{h}_{j}) / \mathbf{diag}(\hat{h}_{j,m}) \right] \mathbf{f}_{\ell} + \mathbf{n}_{m} \) (which embodies BS cooperation). Through this altering of the matrix structure of classical multi-cell RNA and TPE precoding, the inter-cell interference can be scaled down, and the downlink achievable sum rate can be enhanced. Even though the cooperation between BSs will scale up the complexity of the system, the data required can be effectively obtained by the BS switch which is associated with these BSs.

If we assume the channel hardening property, then (26) becomes (see (23)). The second and the third terms on the RHS can be modified by use of the following identities:

\[
\mathbb{E}\left[ \mathbf{h}_{j,m}^{H}\mathbf{f}_{j,m} \mathbf{f}_{j,m}^{H} \right] = \mathbb{E}
\]

(24)

\[
\mathbb{E}\left[ \mathbf{h}_{j,m}^{H}\mathbf{f}_{j,m} \mathbf{f}_{j,m}^{H} \right] = \mathbb{E}
\]

(25)

\[
\mathbb{E}\left[ \mathbf{h}_{j,m}^{H}\mathbf{f}_{j,m} \mathbf{f}_{j,m}^{H} \right] = \mathbb{E}
\]  

(23)

\[
\mathbb{E}\left[ \mathbf{h}_{j,m}^{H}\mathbf{f}_{j,m} \mathbf{f}_{j,m}^{H} \right] = \mathbb{E}
\]

(20)
Using (25), the third term on the RHS of (23) can be written as

\[
\sum_{(\ell,k) \neq (j,m)} \mathbb{E} \left[ \left\| \frac{H_{\ell,j,m}^H - \frac{\text{diag}(h_{\ell,j,k})}{\text{diag}(h_{j,j,m})} f_{\ell,k}}{\text{diag}(h_{\ell,j,k})} f_{\ell,k} \right\| \right] = \sum_{(\ell,k) \neq (j,m)} \mathbb{E} \left[ \left\| \frac{H_{\ell,j,m}^H f_{\ell,k}}{\text{diag}(h_{\ell,j,k})} f_{\ell,k} \right\| \right] - \sum_{(\ell,k) \neq (j,m)} \mathbb{E} \left[ \left\| \frac{H_{\ell,j,m}^H f_{\ell,k}}{\text{diag}(h_{j,j,m})} f_{\ell,k} \right\| \right] = \sum_{(\ell,k) \neq (j,m)} \mathbb{E} \left[ \left\| \frac{H_{\ell,j,m}^H f_{\ell,k}}{\text{diag}(h_{\ell,j,k})} f_{\ell,k} \right\| \right] - \sum_{(\ell,k) \neq (j,m)} \mathbb{E} \left[ \left\| \frac{H_{\ell,j,m}^H f_{\ell,k}}{\text{diag}(h_{j,j,m})} f_{\ell,k} \right\| \right]
\]

(26)

Based on (26), we can rewrite (8) (see (27)), where the term \( Y_{cp} = \sum_{(\ell,k) \neq (j,m)} \mathbb{E} \left[ \left\| \frac{H_{\ell,j,m}^H (\text{diag}(h_{\ell,j,k})) / \text{diag}(h_{j,j,m}) f_{\ell,k} \right\| \right] \) represents the BSs cooperation component.

Considering the \( m \)th UT in the \( j \)th cell for the RZF precoding, then SINR based on (12) and (27) will be expressed as

\[
\text{SINR}_{j,m}^{\text{RZF}} = \frac{\mathbb{E} \left[ \left\| h_{j,j,m} RZF \right\| \right]}{\sigma^2 + r_{\text{RZF}}}
\]

(28)

where

\[
r_{\text{RZF}} = \mathbb{E} \left[ \left\| H_{j,j,m} RZF \right\| \right] - \mathbb{E} \left[ \left\| H_{j,j,m} RZF \right\| \right] + \sum_{\ell=1}^L \mathbb{E} \left[ \left\| H_{\ell,j,m}^H RZF \right\| \right] - Y_{cp}
\]

and

\[
Y_{cp}^{\text{RZF}} = \sum_{(\ell,k) \neq (j,m)} \left\| H_{\ell,j,m}^H f_{\ell,k} \right\| \]

(29)

Considering the \( m \)th UT in the \( j \)th cell for the TPE precoding, then SINR based on (18) and (27) will be expressed as

\[
\text{SINR}_{j,m}^{\text{TPE}} = \frac{\mathbb{E} \left[ \left\| a_{j,m} \right\| \right]}{\sigma^2 + r_{\text{TPE}}}
\]

(30)

where

\[
r_{\text{TPE}} = \mathbb{E} \left[ \left\| a_{j,m} \right\| \right] - \mathbb{E} \left[ \left\| a_{j,m} \right\| \right] + \sum_{\ell=1}^L \mathbb{E} \left[ \left\| B_{\ell,j,m} \right\| \right] - Y_{cp}
\]

(31)

and

\[
Y_{cp}^{\text{TPE}} = \sum_{(\ell,k) \neq (j,m)} \left\| H_{\ell,j,m}^H f_{\ell,k} \right\|
\]

(32)

From both the precoding schemes, the BSs cooperation term works to reduce the inter-cell interference hence improving the SINR and consequently the sum rate.

5 Energy efficiency (EE)

The EE of the various precoding schemes is analysed with reference to a pragmatic consumption model of the circuit power. Then the tradeoff between the EE and the SINR is analysed.

The EE is defined in [25, 26] as

\[
\text{EE} = \frac{\text{Throughput[bbs/s/cell]}}{\text{Power consumption[W/cell]}}
\]

(33)

and it is calculated in bits per joule. This can be viewed as a benefit-cost relationship, wherein the quality of service (throughput) is compared with the related costs (power consumption).

Realistic evaluation of EE requires that power consumption (PC) be calculated based on effective transmit power (ETP) and the CP needed in powering the cellular network [26]

\[
\text{PC} = \text{ETP} + \text{CP}
\]

(34)

The CP consumption model for a general BS \( j \) in massive MIMO system can be expressed as [27–32]

\[
\text{PC}_j = P_{\text{RX},j} + P_{\text{TX},j} + P_{\text{CE},j} + P_{\text{CD},j} + P_{\text{BB},j} + P_{\text{SP},j}
\]

(35)

where \( P_{\text{RX},j} \) is a fixed power quantity to account for the power requirement of control signalling and load-independent power consumed by the backhaul infrastructure and the baseband processors and the power due to economic expenses. Also, \( P_{\text{TX},j} \) represents the power utilised in the transceiver chains, \( P_{\text{CE},j} \) accounts for power consumed in channel estimation, \( P_{\text{CD},j} \) the power for channel encoding and decoding components, \( P_{\text{BB},j} \) accounts for power consumed in load-dependent backhaul signalling and \( P_{\text{SP},j} \) accounts for BS signal processing power.

The \( P_{\text{RX},j} \) can be expressed as follows:

\[
\text{SINR}_{j,m} = \frac{\mathbb{E} \left[ \left\| H_{j,j,m} RZF \right\| \right]}{\sigma^2 + \mathbb{E} \left[ \left\| H_{j,j,m} f_{\ell,k} \right\| \right] + \sum_{\ell=1}^L \mathbb{E} \left[ \left\| H_{\ell,j,m}^H f_{\ell,k} \right\| \right] - Y_{cp}}
\]

(36)
where $V$ is the quantity of traffic classes, $\mathcal{V}(\mathcal{V}, SE)$ is the actual chargeable information throughput corresponding to traffic class $\mathcal{V}$, $p_{BS}$ power expended by BSs during data transmission, $X_c$ is the cost of energy per joule and $C_{0}$ is the additional costs on top of the energy costs. The $P_{TC, j}$ for a cell $j$ can be expressed as [28, 29]

$$P_{TC, j} = M_{j}P_{BS, j} + P_{LO, j} + \frac{K_j P_{UT, j}}{\text{UT Circuit component}}$$

(36)

while the $P_{C, D, j}$ for cell $j$ is given as [30]

$$P_{C, D, j} = (P_{COD} + P_{DEC})CT_j$$

(37)

The channel estimation power $P_{CE, j}$ is then approximated according to the estimator employed

$$P_{CE, j} = \begin{cases} \frac{3B}{\tau_{BS} L_{BS}} K_j M_j f_p + M_j^2, & \text{with MMSE} \\ M_j f_p + M_j, & \text{With LS} \end{cases}$$

(39)

where $B$ is the bandwidth, $L_{BS}$ computational efficiency of BS, $\tau$ is the coherence time and $f_p$ is the length of the pilot sequence [34].

The power $P_{SP, j}$ used by BS $j$ in receive combining and transmit precoding can be computed based on computational complexity of the schemes used. $P_{SP, j}$ can be decomposed as [34]

$$P_{SP, j} = P_{SP, R.T, j} + P_{SP, C, j}$$

(40)

where $P_{SP, R.T, j}$ caters for overall power utilised in uplink (UL) reception and downlink (DL) transmission of information streams, $P_{SP, R.T, j}$ is power needed to compute the combining vector and $P_{SP, C, j}$ is power needed to compute the precoding vector at the $j$th BS. The $P_{SP, R.T, j}$ term is computed as

$$P_{SP, R.T, j} = \frac{3B}{\tau_{BS} L_{BS}} M_j K_j (\tau_c + \tau_d)$$

(41)

while the $P_{SP, C, j}$ is evaluated as

$$P_{SP, C, j} = \frac{4B}{\tau_{BS} L_{BS}} M_j K_j$$

(42)

The $P_{SP, C, j}$ power consumption is dependent on the preceding scheme used.

### 5.1 Computation complexity of receive combining

It is assumed that the complex divisions and multiplications have the greatest impact on complexity neglecting the additions and subtractions [34]. The complexity is particularly impacted by the precoding matrix for the various precoding schemes. To evaluate the complexity, the lemmas in the Appendix are evoked.

The combining complexity of the RZF precoding can thus be written as [34]

$$\frac{3K_j^2 M_j}{2} + \frac{3K_j M_j}{2} + \frac{K_j^2 - K_j}{3} + 7K_j$$

(43)

From (14), the combining complexity of the TPE precoder can be evaluated based on the Appendix. Then the multiplication in (14), $\sum_{k=1}^{K} h_{j,k} H_{j,k}^H$ gives a complexity of $((3K_j^2 + K_j)M_j)/2$. But after the multiplication, there is the power $n_j$, in this case it was set to 2 because a $J_0$ of 3 was used. Then this yielded a complexity of $K_j$. The normalisation of the precoding vector needed in the decoding unit costs $K_j$ divisions for every BS. Thus, the total complexity was computed as follows:

$$\sum_{j=0}^{J} \left( \frac{3M_j + M_j^2}{2} \right) K_j$$

(44)

From (19), the combining complexity of the RNA precoder can be evaluated based on the Appendix. It was pointed out in [10] that the consideration of the initial three terms gives the quickest convergence of the iterative process for finding the inverse in RNA. The term $Z_n$ is presumed to be available for free at $j$th BS. Since RNA uses intercell channel estimates for $\ell \neq j$ the complexity of computing these estimations is included. The multiplication complexity is expressed as

$$\sum_{\ell=0}^{J} \left( \frac{3M_{\ell} + M_{\ell}^2}{2} \right) K_{\ell}$$

Then to establish the complexity for the inversion, we follow the following procedure. From [10], the inverse for the first three terms ($P = 3$) in successive iterations can be written as

$$R_{x} = R_{x}(I + E_{\ell} + E_{\ell}^2)$$

(45)

$$R_{x} = R_{x}(I + E_{\ell} + E_{\ell}^2)$$

(46)

$$R_{x} = R_{x}(I + E_{\ell} + E_{\ell}^2)$$

(47)

And in general

$$R_{x} = R_{x}(I + (E_{\ell}^\infty) + (E_{\ell}^\infty - E_{\ell})^2)$$

(48)

From $R_{x}$ the number of operations count to calculate it is

$$N_i = 2M_j^2 - M_j + M_j^2 + 2M_j^2 + M_j^2 + 2M_j^2 - M_j^2 = 6M_j^2$$

(49)

From $R_{x}$ the number of operations count to calculate it is

$$N_i = 2M_j^2 - M_j + M_j^2 + 2M_j^2 - M_j^2 + M_j^2 + 2M_j^2 - M_j^2 = 6M_j^2$$

(50)

From $R_{x}$ the number of operations count to calculate it is

$$N_i = 2M_j^2 - M_j^2 + M_j^2 + 2M_j^2 - M_j^2 + M_j^2 + 2M_j^2 - M_j^2 = 6M_j^2$$

(51)
Thus, to perform \( \mathcal{X} \) iterations, the overall operations count for \( P = 3 \) can be expressed as

\[
N_1 = (\mathcal{X} - 1)(6M_j^3 - M_j^2) + 6M_j^2 = 6M_j^3\mathcal{X} - M_j^3(\mathcal{X} - 1)
\]  

(52)

Equation (52) gives us the inversion complexity of the RNA method. Then the multiplication after inversion gives \( M_j\) divisions for every BS. Thus, the complexity was computed as follows:

\[
\sum_{\ell = 0}^{L-1} \left( \frac{3M_j^3 + M_j}{K_j} \right) \tau_p + \sum_{\ell = 0}^{L-1} \left[ (6M_j^3 - M_j^2) + (M_j^3 - M_j^2(\mathcal{X} - 1)) \right] + 2M_j + M_j\tau_p
\]  

(53)

Each iteration is assumed to correspond to the coherence time instance. The increase in number of iterations reduces the complexity of inversion in RNA precoding by a factor of \( M_j^3(\mathcal{X} - 1) \).

5.2 Computation of receive combining power

From the computational complexity of receive combining, the receive combining power for various precoding schemes can be derived. For the RZF precoding, the receive combining power can be evaluated as

\[
P_{\text{UL,C,J}}^{\text{RZF}} = \frac{3B}{\tau_cL_{\text{BS}}} \left( \frac{3K_jM_j}{2} + \frac{3K_jM_j}{2} + \frac{K_j}{3} + \frac{7}{3}K_j \right)
\]

(54)

Looking at the TPE precoding, the receive combining power can be expressed as

\[
P_{\text{UL,C,J}}^{\text{TPE}} = \frac{3B}{\tau_cL_{\text{BS}}} \left( \sum_{\ell = 0}^{L-1} \left( \frac{3K_j^\ell + K_jM_j}{2} \right) + \frac{7}{3}K_j \right)
\]

(55)

For the case of RNA precoding, the receive combining power can be evaluated according to the following expression:

\[
P_{\text{UL,C,J}}^{\text{RNA}} = \frac{3B}{\tau_cL_{\text{BS}}} \left( \sum_{\ell = 0}^{L-1} \left( \frac{3M_j^3 + M_j}{K_j} \right) \tau_p + \sum_{\ell = 0}^{L-1} \left( 6M_j^3 - M_j^2(\mathcal{X} - 1) \right) \right) + 2M_j + M_j\tau_p
\]

(56)

5.3 Energy efficiency and throughput

Based on the CP model developed, EE-CT analysis is performed to underline the importance of bandwidth in EE analysis. The EE in the \( j \)th cell is expressed as

\[
\text{EE} = \frac{\text{CT}_j}{\text{ETP}_j + \text{CP}_j}
\]

(57)

where \( \text{CT}_j \) is the capacity for \( j \)th cell, \( \text{ETP}_j \) is the ETP for \( j \)th cell and \( \text{CP}_j \) is the CP for \( j \)th cell. Then the \( \text{CT}_j \) is expressed as the

\[
\text{CT}_j = B \sum_{k=1}^{K_j} (\text{SINR}_{j,k}^{\text{UL}} + \text{SINR}_{j,k}^{\text{DL}})
\]

(58)

The ETP is then evaluated as follows in [34]

\[
\text{ETP}_j = \tau_u \sum_{k=1}^{K_j} \left( \frac{1}{\mu_{\text{UE},j,k}} p_{\text{RF}} + \frac{1}{\mu_{\text{BS},j,k}} \right) \sum_{k=1}^{K_j} p_{\text{RF}}
\]

(59)

where \( \tau_u \) is the uplink time and \( \tau_d \) is the downlink time, \( \mu_{\text{UE},j,k} \) is the power amplifier (PA) efficiency of the \( k \)th UT in the \( j \)th cell and \( \mu_{\text{BS},j,k} \) is the PA at the \( j \)th BS.

6 Results and discussion

This section provides the comparison and analysis of RZF precoding, TPE precoding and the RNA-based precoding. First, we look at the comparison and analysis in performance between RZF precoding, TPE precoding and the RNA-based precoding in multicell massive MIMO with 20 cells. This comparison is carried out for varying from \( M = 16 \) to \( 160 \), with a step of 16 and \( K = 10 \) massive MIMO system. For the TPE precoder, the power of the polynomial, \( J_{\text{C}} \), is chosen to be 3 in this work.

Fig. 1 compares the achievable SE per cell among RZF precoding, TPE precoding and the RNA-based precoding in the presence of different reuse factors. With a reuse factor of 2, the RNA-based precoding outperforms the RZF precoding and TPE precoding in the presence of different reuse factors. With a reuse factor of 4, the increment in the SE per cell for both the RNA-based precoding and the RZF precoder performs better than the TPE precoder and the RZF precoder under the same propagation condition.

From Figs. 2 and 3, we compare the RZF precoding, RNA-based precoding and the TPE precoding in the presence of different reuse factors. With a reuse factor of 2, the RNA-based precoding outperforms the RZF precoding and TPE precoding with the TPE precoding offering the lowest SE per cell. Furthermore, the RNA-based precoding and the RZF precoding exhibit an increase in SE per cell above that of a reuse factor of 1 but the TPE precoding exhibits a reduction in the SE per cell. This clearly shows that the TPE precoding can be applied for single cell case but its performance plummets in multicell massive MIMO system where practically reuse factors are introduced to optimally use the scarce radio resources. When the reuse factor is changed to 4, the increment in the SE per cell for both the RNA-based precoding and...
the RZF precoding increase, though that of the TPE precoder decreases further.

The increase in SE per cell can be attributed to the fact that the pre-log factor reduces with the increased number of pilots. Also, this leads to increased instantaneous SINR as the channel estimates become better with reduced pilot contamination. Again, for TPE precoding there is a reduction in SE because improvement in approximation quality do not overshadow the reduction in pre-log factor. This is a fact since the estimate is just evoked to enhance desired signal and not to mitigate interference. Thus, RNA based precoding has a good performance in presence of a high reuse factor as well as the RZF precoder and can render themselves easily for multicell massive MIMO systems. The performance of the precoding techniques for different reuse factors is tabulated in Table 1.

Fig. 4 shows the impact of coordination in the multicell massive MIMO system. With the reuse factor of 4 selected, it can be observed that the coordination within multicell massive MIMO further reduces the effect of intercell interference thereby improving the SE per cell in general. This enhancement in performance comes at the cost of increased complexity which simply points to increased hardware requirement.

Fig. 5 shows the total CP against the number of BS antennas. The CP was computed as per the values in Tables 2 and 3. It can be observed that as the number of BS antennas increase, the total CP increases. Which is expected since the increase in BS antennas simply means more circuit to consume power. However, the RNA based precoding has a high CP consumption as compared to TPE precoding and RZF precoding which have equal CP.

under consideration while holding the BS antennas and the UTs constant. This observation points to the fact that coordination in multicell massive MIMO further reduces the effect of intercell interference thereby improving the SE per cell in general. This enhancement in performance comes at the cost of increased complexity which simply points to increased hardware requirement.

Fig. 5 shows the total CP against the number of BS antennas. The CP was computed as per the values in Tables 2 and 3. It can be observed that as the number of BS antennas increase, the total CP increases. Which is expected since the increase in BS antennas simply means more circuit to consume power. However, the RNA based precoding has a high CP consumption as compared to TPE precoding and RZF precoding which have equal CP.

Figs. 6 and 7 depict how the EE varies with the throughput. From the two figures, it can be observed that the EE increase with an increase in throughput up to a certain point then the EE starts decreasing with increasing throughput. The maximum EE achievable serves as an indicator on the optimal throughput that will deliver maximum EE. Consequently, this throughput corresponds to the number of BS antennas to achieve the desired EE. Thus, it can be stated that the use of all the BS antennas may not be optimal. This then brings in the idea of antenna selection method within the BS in massive MIMO system. This can even be
The paper gives the performance analysis and comparison of the RNA-based precoder, RZF precoder and the TPE precoder for multicell downlink massive MIMO system. The performance of the three precoding schemes in terms of SE and EE for imperfect CSI is studied. The SE and EE were derived theoretically for each precoding scheme under similar assumptions and for the wireless massive MIMO system.

From the simulation and the theoretical results, RNA-based precoding has higher SE and EE followed by the RZF precoding and the TPE precoder has the lowest. But when the BS antennas are increased, TPE precoder SE and EE nears that of the RZF precoder, when the reuse factor of 1 is used. With a higher reuse factor of 4, the SE and EE performance of TPE precoder is poor and cannot compare to the RNA based precoder and that of the RZF precoder. Thus, RZF precoder and RNA-based precoder have good performance at both lower and higher reuse factor as compared to TPE-precoder which only has improved performance at high number of BS antennas and a lower reuse factor of 1.

The CP consumed by the RNA based precoder is the highest but both RZF and TPE precoders have the same CP consumption. Though consuming the same CP as the RZF precoding, TPE precoder has poor SE and EE. This points out that TPE precoder is an inefficient technique to be realised in multicell massive MIMO systems. The RNA based precoder has high CP, but its performance is superior and can justify its implementation in multicell massive MIMO systems.

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Appendix

9.1 Matrix analysis

9.1.1 Computational complexity of matrix operations: Elementary linear algebra operations take same characteristic form and productively renders themselves for execution in any hardware formation. However, the computational complexity poses a challenge when vast matrices need to be processed after every other millisecond. The definite complexity of a matrix processing depends emphatically on the hardware realisation, as well as the bit width and the information type. Here, we give first-order approximations by checking the quantity of complex multiplications and divisions that are required, whereas the complexity resulting from additions/subtractions is dismissed since these tasks are a lot less demanding to realise in hardware.

Lemma A.1: Assume we have matrices \( H \in \mathbb{C}^{K_1 \times K_2} \) and \( G \in \mathbb{C}^{K_2 \times K_3} \). Then for matrix–matrix multiplication \( HG \) we need \( K_1 K_2 K_3 \) complex multiplications. However, to multiply \( HH^H \) we only need \( ((K_1^2 + K_2)/2)K_3 \) complex multiplications, when Hermitian symmetry is applied.

Proof: There is a total of \( K_1 K_3 \) elements in \( HG \) and the calculation of every element needs \( K_2 \) multiplications (the \( H \) elements in a given row are multiplied by the \( G \) elements in a corresponding column). For the case when \( G = HH^H \), we use Hermitian symmetry to set the number of elements computed to \( (K_1^2 + K_3)/2 \), which is a representation of the leading diagonal and half of the elements from the other-diagonal.

To realise a competent hardware implementation in relation to computation and memory utilisation, the \( LDL^H \) decomposition is evoked when multiplying the inverse of a matrix with another matrix [35]. \( L \) represents a lower triangular matrix having ones on the leading diagonal and where \( D \) is a diagonal matrix with elements from the leading diagonal.

Lemma A.2: Assume a Hermitian positive semi-definite matrix \( H \in \mathbb{C}^{K_1 \times K_1} \) and the matrix \( G \in \mathbb{C}^{K_1 \times K_2} \). The decomposition of \( H \) using \( LDL^H \) can be realised with \( (K_1^2 + K_1)/3 \) complex multiplications. If we know the \( LDL^H \) decomposition of \( H \), then we can compute \( H^{-1} G \) with \( K_1 K_2 \) complex multiplications and \( K_1 \) complex divisions.

Proof: The algorithms to compute \( LDL^H \) decomposition efficiently is investigated in [35, 36], and in Table 1 of [36] the quantity of multiplications is given. \( H^{-1} G \) is calculated by evaluating \( K_1 \) linear systems of equations. If we know the decomposition of \( LDL^H \), we can exploit it to resolve the systems of equations using the forward-backward substitution [37] that involves \( K_1 \) multiplications for each system. In addition, \( K_1 \) divisions are needed to compute \( D^{-1} \).