Proposal of an algorithmic methodology in GeoGebra for the teaching of the Riemann sum a tool for approximating definite integrals

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Abstract. Integral calculus is a fundamental subject that enables the teaching of branches of physics like, mechanics, thermodynamics, and electromagnetism to engineering students. However, the incapacity of understanding either the concept of the area under a curve or to develop an efficient and simple way to calculate integrals continues to be a major cause of the high failure rate of students in these subjects. This work was developed parting from the proposal of using Information and communication technologies, in combination with flowcharts, for helping students to solve integrals through Riemann sums. Even, a poll was used to recollect and do a qualitative evaluation on the degree of comprehension that students had on Riemann sums and their relation to integrals and to get feedback on their impression about the proposal of this work. The results are compared against traditional learning, based on memorizing the steps of the algorithm for each method and the representation of the convergence of successive roots by numerical tables.

1. Introduction

Information and communication technologies (ICT’s) are defined as those technological resources that allow us to digitalize information [1]. Among many of their advantages, ICT’s allow access to academic information by using most conventional computers or any intelligent mobile device [2]. With the surge of the digital era, ICT’s have become very popular among students, since there is evidence that their usage facilitates the understanding of digital didactic material through the student’s visual channel [3]. This success in facilitating learning has made different organizations to promote the usage of ICT’s in all educational levels [4–6].

Calculus is one of the most important subjects in several branches of science as well as just as developing the ability to solve problems through the usage of integrals is fundamental for students. Even so, the failure rate in students of integral calculus remains to be very high. The usage of ICT’s for helping students to improve their proficiency in solving integrals has been proposed in many ways: the design of a Matlab app for calculating the area under a curve [7]; the use of Python and for solving problems that involve revolution solids [8] and the use of excel to estimate the area from some functions [9], are just some examples. However many of these applications are often require students to invert a considerable amount of time in learning the required commands and to pay for a user license.

GeoGebra is free software that can be freely discharged into a computer or mobile device or accessed through the internet [10, 11]. This platform allows its users to create, share, and use
didactic digital resources for many academic subjects, including integral calculus [11]. Referring to integral calculus, GeoGebra has been proposed in the past as a tool for facilitating the teaching of Riemann sums: through visualizing the rectangles that could approximate a given area under the curve, and giving its approximate value [12]; practices in which students were instructed into coding apps that could calculate the area under a curve for lineal and non-linear functions [13] and, similarly, by teaching students how to solve contextual problems, related to branches of physics like mechanics and thermodynamics, that required Riemann sums and asking them to validate their results by hand calculation [14].

Even though the aforementioned proposals facilitate learning, concerning traditional teaching, they are often limited to the cases proposed by teachers. Even more, their effectivity is compromised by student’s ability to adequately use them without the constant supervision by the teacher [15]. This work proposes to alleviate the shortcomings of its predecessor by teaching students how to evaluate any defined integral, in both pure and contextual problems, taking advantage of the simplified language and structure used in GeoGebra. First, students learned how to evaluate the defined integral of a given function by approximating through Riemann sums (encouraging them on extracting the algorithm required to solve each particular problem and to propose a generalization); afterward, students applied this algorithm to context problems to validate its effectivity. Finally, the degree of satisfaction of the students was evaluated by employing a digital poll.

2. Methodology
The present study was carried on with the aid of a group of 24 students coursing their bachelor’s degree in industrial engineering in the Technological Institute of Colima (one of the 239 campuses of the National Technological Institute of México). Even when the selected group of students had already taken a course on integral calculus during their last year in high school, most manifested that they were not capable of explaining how Riemann sums can approximate the area under a curve.

The present work was developed in two phases: the first phase was dedicated to showing students how the area under a curve can be approximated by employing regular geometric figures of known area. Next, each student was taught how to estimate the area under a curve by employing the method of superior and inferior sum (by using millimeter paper, a ruler, and color pencils). When students became proficient in this method, the teacher presented its relation to the concept of summation and how, by increasing the number of rectangles, the difference between the Riemann sum to the left and to the right (both representing an approximation to the exact area) becomes progressively closer.

In the second phase, students learned how to develop a simple GeoGebra applet whose input was the function and the interval of integration, defined as the points whose coordinates are A=(a, 0) and B=(b, 0). Next, the program allowed to divide the region bounded between the points (a, f(a)) and (b, f(b)) in rectangles, whose number can be varied by employing a slider, flow chart in the Figure 1 shows this steps. These rectangles, whose purpose was to serve as a visual aid for representing Riemann sums, considered the approximation to the exact area by either the superior or inferior Riemann sum. The slider was introduced as a way in which students could conclude on how the approximations obtained to the Riemann sum approaches to the exact integral as the rectangle number increased (the Riemann summation to the left and to the right were calculated by employing the ”LeftSum” and ”RightSum” GeoGebra commands, whereas the exact integral was calculated with the aid of the ”Integral” command).

After the students completed the calculation of approximations for the exact integral of various linear and non-linear functions, they applied Riemann sums for solving the context problems proposed by [14] and were encouraged to propose multidisciplinary problems that whose solution could be proposed by the calculation of Riemann sums. A poll was developed,
by using the Google-Docs service, as a follow-up. This poll allowed students to anonymously evaluate some qualitative aspects of the practices involving Riemann sums (i.e., the student’s level of learning, their impressions about the utility of Riemann sums, and their degree of satisfaction on developing the skill of solving integrals by using GeoGebra and Riemann sums). Finally, to determine the effectiveness of this strategy, the student’s grades were compared to last year’s grades on that same course by applying a hypothesis test on the student’s average score and the proportion of successful students.

![Flow chart to determine the approximate and exact area.](image1)

**Figure 1.** Flow chart to determine the approximate and exact area.

### 3. Results

Starting from the steps described in the methods section, Figure 2(a) shows the applet’s input, next to the graphical representation of the exact integral shown in Figure 2(b). This exact integral is then approximated by the left and right Riemann sums in Figure 2(c) and Figure 2(d) with five rectangles. To evaluate if there was some statistical evidence on the effectivity of encouraging students to solve definite integrals by employing Riemann sums and their GeoGebra applet, two tests were proposed. In these tests, the average score of the group and the proportion of those students that were successful at scoring at least 70 points in the exam about Riemann sums as compared to the data from the control group reported by [14]. The results of these comparisons are shown in Table 1. The questions employed in the poll designed to recollect the qualitative aspects of the investigation were:

- “In my opinion, the use of GeoGebra to calculate the Riemann sum to the left or the right is”: The purpose of this open question was to recollect the student’s opinion on the usefulness of calculating an approximation to the exact integral using the Riemann sum to the left or the right constructed by the GeoGebra applet. Approximately 77% of the students considered their applet as an effective tool to facilitate their learning on Riemann sums; whereas 23% expressed that developing this skill employing GeoGebra was “useful but boring”. No opinions in which the GeoGebra applet was considered as useless were found.

- “When I use the GeoGebra applet, I consider that”: In this case, the purpose was to evaluate the usefulness of employing programming for plotting the graph and approximating the area by Riemann sums to the left or to the right. Approximately 94% of the students consider the proposed activities were useful to develop their proficiency in visualizing and calculating Riemann sums. The remaining students considered that the use of the applet was not necessary, since they declared to understand how to calculate Riemann sums. As with the previous questions, no answers expressing the futility of the GeoGebra applet.

- “Calculating the area by using the GeoGebra applet is”: This question was included to evaluate if students considered that using the GeoGebra applet makes it easier to calculate
the area under a curve. 71% of the students expressed their preference for relying on the GeoGebra applet. In contrast, 29% considered that it was easier to make the calculations by hand since they were not attracted by programming or considered themselves incapable of following the algorithm’s steps related to programming.

- “I consider that learning to calculate Riemann sums is”: This question was planned to evaluate student’s perceptions related to the basis of integral calculus and its importance in the long run. Similar to the other questions, no students express that the usage of the GeoGebra applet was useless. However, 19% of the students labeled creating the applet and using it to calculate the area under a curve as ”useful but boring”. In contrast, 81% of the students indicated that the applet helped them to remember more easily the required steps to calculate the area under a curve using the Riemann sum to the left or the right. Also, this subgroup commented that being able to increase the number of rectangles used to approximate the area under a curve helped them to understand how the associated error depends on it.

![Figure 2](image.png)

**Figure 2.** (a) Construction of the applet developed by students to (b) calculate the exact integral for the function $y = x^2 + 2$ a the approximations represented by the Riemann (c) sums to the left and (d) to the right.

| Group | Control | Test (this work) |
|-------|---------|-----------------|
| Students | 39 | 24 |
| Mean score | 62.560 | 84.38 |
| Std. deviation | 30.130 | 17.22 |
| t-score | -3.230 | |
| P-value | 0.002 | |

**Table 1.** Results from the comparison between this work’s proposal (group) and the control group reported by [14]. The data shown on the left was used to test the statistical significance of the difference in mean scores for both groups. The data shown on the right was used to test the statistical significance for the difference in proportional success rate.

4. Conclusions
This work reports a strategy developed to facilitate the competence of engineering students on calculating definite integrals by developing an algorithmic methodology that uses Geogebra to approximate the exact integral through the left and right Riemann sums. A series of practices were designed to be done by students during the second unit of the integral calculus course.
Evidence was found employing two hypothesis tests that evaluated the average score from that unit’s exam and the proportion of students that were successful at obtaining a minimal approbation score of 70 points over 100. When compared to the control group of a similar investigation carried out last year in the same institution, the results of this work experimental group show evidence of a significant difference in the average score and in the proportion of approved students.

A poll was developed to register some qualitative aspects of the study. The qualitative results showed that the students consider that the use of GeoGebra is a good guide to learn to estimate the value of the area under the curve and how employing this digital tool allows them to identify the difference in the development of the left and right Riemann sums. Whereas, when calculating the definite integral; students generally understood the importance of the Fundamental Theorem of Calculus and its relation to Riemann sums. In addition to this, the students were able to identify, pose and solve applied problems from different areas of engineering.

A problem that was observed during the practices was the exhaustion of some students who were able to estimate the area under the curve employing Riemann sums in less time compared to other students. Therefore, it is of interest to determine the ideal number of problems to be developed so that the majority of students acquire the ability to distinguish, develop and estimate the area under the curve.

As future work, it is proposed to determine how many exercises can be considered sufficient to avoid the student’s fatigue, identify attractive strategies for students to relate symbol-colors-commands and the design of practices based on the teaching of programming and application of algorithms for solving problems applied in different areas of engineering.

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