Majorana Fermions: Direct Observation in $^3$He

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The Majorana fermions, which act as its own antiparticles, were suggested by Majorana in 1937. The short review of the history one can find in [1]. The Majorana impacted on diverse problems, from neutrino physics and dark matter searches to the fractional quantum Hall effect and superconductivity. Despite of this long history, the unambiguous observation of Majorana fermions remains an outstanding goal. No fundamental particles are known to be Majorana fermions, although there are speculations that the neutrino may be one. There is also theoretical suggestion that Majorana fermions may comprise a large fraction of cosmic Dark Matter [2]. There are also condensed matter systems, in which quasiparticles (QPs) may exhibit the properties of Majorana fermions. The topological media, which are gapped in the bulk, have topologically protected surface excitations with the properties of Majorana fermions [3]. This class of topological media includes topological insulators like superfluid $^3$He-B, thin film of $^3$He-A, Vortex lines in superfluids and superconductors [4], thin wires of superconductors and the quantum vacuum of the Standard Model of the Universe [5]. Graphene may belong to this class if a broad enough energy gap appears due to spin-orbit interaction (or due to spontaneously broken symmetry) [6]. Recent advances in the condensed-matter search for Majorana particles or QPs have convinced many in the field that this quest may soon bear fruit. In this letter we report the first direct observation of gapless Majorana QPs which appear as Andreev bound states on the surface of superfluid $^3$He-B [4]. We made the precise measurements of superfluid $^3$He-B heat capacity at the limit of extremely low temperatures. We have separated the heat capacity of bulk Bogolyubov QPs and the surface Majorana QPs by its different temperature dependence. We have found that at 0.12 mK the Majorana fermions constitute a part of about 30% of bulk $^3$He-B heat capacity at the conditions of our experiments.

The recent progress in condensed matter physics has focused on the "Majorana zero modes" i.e. emergent Majorana particles occurring at zero energy that have a remarkable property of being their own antiparticles. Mathematically, this property is expressed as an equality between the particle's creation and annihilation operators, $\gamma^+ = \gamma$. Majorana zero modes are believed to exhibit the so called non-Abelian exchange statistics [7], which may be used in future as a blocks of quantum memory, which avoid the problem of decoherence. There are many scenarios for experimental searching of Majorana signatures [8-12]. The high intensity of investigations focused on a superconducting thin wires. In this case the region for Majorana formation is situated near the ends of wires. It means that the energy of Majorana is exactly zero and they can not move. How to observe the not-moving zero energy state? This task looks very difficult to be performed. Indeed, there are an attempts to confirm the existence of Majorana by non-direct method, by observation of interference of Majorana states of both sides of the wire throw a superconducting bridge [8].

Recently, topological classification for generic symmetry classes of topological insulators and superconductors was given by Kitaev [13]. One example of a topological insulator is the time reversal invariant B-phase of superfluid $^3$He. The spin-triplet superfluid supports the existence of the Majorana QPs [14-15]. This follows from the particle-hole symmetry of Bogoliubov QPs, $\gamma_E^b = \gamma_E^c$. Consequently at zero energy they satisfy the Majorana condition $\gamma_0^+ = \gamma_0$. Zero energy bound states of the QPs in $^3$He-B appear when the underlying potential for the QPs changes its sign. This conditions take place at edges, surfaces or vortices with odd integer winding numbers, where the QPs form zero energy Andreev bound states (The unique QP state in which Andreev reflection [4,14] plays a fundamental role).

In this letter we report the results of successful observation of Majorana QPs in superfluid $^3$He-B. The superfluid energy gap of $^3$He-B is suppressed near the walls at a distance of the superfluid coherence length $\xi_0$, which is about 80 nm at zero bar. According to the theory [4], the zero energy quasiparticles and quasihalls banded at this region on a combined particles with the properties of Majorana fermions. In difference of superconducting wires, the Majorana QPs in $^3$He-B can move along the surface of the sample and consequently have a kinetic energy. They have a linear dispersion relation with a zero gap, referred to as the Majorana cone [10-11]. Arising from this specific dispersion relation, the Majorana QPs have a non-zero energy and consequently have a heat capacity. The suggestion about the experimental observation of additional Majorana heat capacity one can find in [16], and the first attempt of its realization in [18]. In the latter case the experiments was performed in superfluid $^3$He in a micropowder heat exchangers at a temperatures near $T_c$. The proper search for Majorana requests completely different conditions, which we have satisfied in the experiments and we describe below.

The heat capacity of Majorana changes with cooling by
power-law, in particular with quadratic dependence on \( T \) for specular scattering of QPs. The Bogolyubov QPs in bulk \(^3\)He have a gap and consequently its heat capacity drops down exponentially with cooling. At some temperature the Bogolyubov QPs heat capacity can be lower than Majorana one. This temperature depends on the ratio between the volume and the surface of \(^3\)He sample. In the experiments, described in this letter, we have found the deviation of exponential law of QPs heat capacity, which corresponds well to an additional heat capacity due to zero gaped Majorana QPs. The very good agreement with a theoretical prediction supports our claim of a direct observation of zero gaped Majorana QPs.

In a Fig.1 the conditions of our experiments are schematically shown. We have performed the experiment at zero pressure at the temperatures down to about 0.11 mK. The energy gap of superfluid \(^3\)He at zero pressure is about 2 mK. At this conditions the gas of QPs is very diluted and consequently its heat capacity is very small. Near the wall the energy gap drops down to zero. This region allows existence of the Majorana QPs with zero energy gap. These QPs can move in 2D space along the surface. The heat capacity of bulk Bogolyubov QPs and Majorana QPs can be estimated by following way:

The Bogolyubov QPs heat capacity in superfluid \(^3\)He-B falls down with temperature [19]:

\[
C_{\text{bulk}} \sim V P_F^2 \left( \frac{\Delta}{kT} \right)^{3/2} \exp \left( - \frac{\Delta}{kT} \right)
\]

where \( P_F \) is the fermi momentum, \( \Delta \) - superfluid gap \( \simeq 2kT_c \) and \( V \) is the volume of the sample.

The Majorana heat capacity falls down by a power law, owing to the zero gap of these QPs [20]:

\[
C_{\text{maj}} \sim A \xi^3 P_F^2 \left( \frac{\Delta}{kT} \right)^{-2},
\]

where \( \xi \) is the \(^3\)He-B coherence length and \( A \) the surface area of the sample.

The ratio of these heat capacities, including the numerical factors, reads:

\[
\frac{C_{\text{maj}}}{C_{\text{bulk}}} = \frac{\pi^3}{8\sqrt{2}} \frac{\xi}{\lambda^3} \left( \frac{\Delta}{kT} \right)^{-7/2} \exp \left( \frac{\Delta}{kT} \right) = \frac{F}{\lambda}.
\]

where \( \lambda \) is a geometrical parameter of the experimental cell, the ratio between the volume and the surface area. The function \( F \), introduced by this equation, is very useful since may visualize the conditions of experiment, where Majorana heat capacity plays an important role. In a current experiments the parameter \( \lambda/\xi \) is about \( 10^{-4} \). The crossover of heat capacity between Majorana and Bogolyubov QPs should take place when \( F = 10^4 \), that is about 0.103 mK [21]. Our experimental results well correspond to this estimation.

In our heat capacity measurements we have used a bolometer which consists of a closed copper box with a small orifice. The box has a form of cylinder of 6 mm diameter and 5 mm height. The diameter of the orifice is about 0.2 mm. The top and bottom of the cell made from copper foils and cylinder made by turning lathe. Its volume is about 0.13 cm\(^3\). The bolometer is situated inside the chamber of nuclear demagnetization refrigerator filled up by superfluid \(^3\)He at extremely low temperatures. The temperature inside the bolometer is determined by heat leak and cooling by flow of QPs out of it. Owing to Kapitza resistance, the thermal conductivity between the \(^3\)He and the walls of the bolometer can be considered as an absolute zero. We was able to monitor the temperature (density of Bogolyubov QPs) by using the vibrating wire resonator (VWR) techniques first described in [23]. The broadening of VWR resonance is determined by the friction with the Bogolyubov QPs. (See the description of VWR in the section METHODS.) We have found that the main source of heat leak to the cell is a cosmic rays events, mainly muons [22], which deposit an about 6 MeV of energy in our cell per hour. That is about \( 25 \times 10^{-14} \) watt per cm\(^3\) of \(^3\)He. We are able to see the each event as a prompt heating of \(^3\)He in the cell as shown in Fig. 2. An energy deposition by an incident particle scattering creates a cloud of QPs in the cell, which comes to internal thermal equilibrium via collisions with the cell wall within about 1 ms. The density of Bogolyubov QPs in the bolometer drops down with time scale \( \tau_c \) of about few seconds, as shown in Fig. 2. This time scale determines by ratio of the volume and surface of the bolometer orifice. If we take into account only the Bogolyubov QPs, than \( \tau_c \) changes slowly with cooling since the heat capacity and flow out are both proportional to the density of QPs. Contrary the time constant changes as soon as an additional heat capacity due to Majorana QPs appears. By observation of temperature dependance of \( \tau_c \) with cooling we have made a first confirmation of.
existence of Majorana in superfluid $^3$He [21]. In this letter we report the direct measurements of additional heat capacity due to Majorana QPs by a classical method. That is the measurements of temperature jump versus heating by the calibrated pulse.

For the energy deposition to the bolometer we have used the second VWR, installed inside the bolometer. It is impossible to use the usual thermal heating pulse due to huge Kapitza resistance. But we may use the heating pulse made by the vibrating wire, which moves with a supercritical velocity. In this case the wire creates the QPs directly to superfluid $^3$He with the momentum about $P_F$. By integrating of voltage and current throw the VWR we are able to calculate the energy, deposited to QPs.

After a heating pulse the temperature inside the box suddenly rises, and then goes back to its initial temperature by thermalisation via the orifice. For calculation the form of the thermalisation the excitations in superfluid $^3$He can be considered as a ballistic gas of quasiparticles of the same energy. In this case the outgoing heat flux is proportional to the number of QPs leaving the box per unit of time. This outgoing stream is proportional to the difference of quasiparticle densities inside ($n_{\text{box}}$) and outside ($n_{\text{out}}$) the box:

$$\Delta n = \frac{1}{\tau_b} (n_{\text{box}} - n_{\text{out}}) \Delta t \quad (4)$$

After the heating event the quasiparticle density drops by ex-
have used the surface of the walls as a fitting parameter and found that it is about 6 times bigger than the geometrical surface, in a reasonable agreement with the possible roughness. There is also a problem of solid layer of $^3$He on the surface of the bolometer. In our case the surface was covered by $^4$He at the moment of condensation of $^3$He. Owing to the small surface to volume ratio it is enough to use the 0.1% of $^4$He for removing completely the solid $^3$He inside the bolometer. It was shown in [25] that the solid fraction of $^3$He gives a different time constant for bolometer temperature recovery owing the non-perfect thermal contact with liquid.

We can draw in conclusions that the existence of Majorana is confirmed by a very direct method, the measurements of Majorana heat capacity.

**METHODS** The VWR is the thin semi-loupe of superconducting NbTi filaments bent into an approximately semi-circular shape of a few mm, with both ends firmly fixed [24]. The VWR is driven by a Laplace force imposed by an AC current close to its mechanical resonance frequency, and oscillates perpendicularly to its main plane with an rms velocity $v$. The motion is damped by frictional forces of total amplitude $F(v)$ mainly due to momentum transfer to the QPs of the surrounding superfluid and proportional to the density of QPs, which depends on the temperature of superfluid. It means that VWR plays a role of very sensitive thermometer. If one deposit the energy to the bolometer, the temperature raises, the density of QPs increases and the damping force increases. If one measures the amplitude of VWR oscillation at a constant frequency he may monitor the temperature of the bolometer [22].

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