On Hořava-Lifshitz Cosmology

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Abstract

We discuss some aspects of the Horava-Lifshitz cosmology with different matter components considered as dominants at different stages of the cosmic evolution (each stage is represented by an equation of state pressure/density=constant). We compare cosmological solutions from this theory with their counterparts of General Relativity (Friedmann cosmology). At early times, the Horava-Lifshitz cosmology contains a curvature-dependent dominant term which is stiff matter-reminiscent and this fact motivates to discuss, in some detail, this term beside the usual stiff matter component (pressure=density) if we are thinking in the role that this fluid could have played early in the framework of the holographic cosmology. Nevertheless, we show that an early stiff matter component is of little relevance in Horava-Lifshitz cosmology.

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I. INTRODUCTION

Searching for a quantum theory of gravity has been a fruitful field for theoretical physics in the last century. In this sense, Hořava proposed in Ref. [1] a new quantizable theory of gravity, using the ideas from solid state physics. This theory was originally called Hořava-Lifshitz quantum gravity, that is a power-counting renormalizable theory with consistent ultraviolet (UV) behavior and, on the other hand, the theory has one fixed point in the infrared limit (IR) namely General Relativity (GR) [1–4]. Thus, this theory leads to a modification of the Einstein’s general relativity at high energies producing interesting features in cosmology. The first ideas about this subject have been presented in Refs. [5–7] where the cosmological consequences of Hořava-Lifshitz (HL) cosmology were studied in a vast detail, see also Refs. [8–25]. The new theory has virtues and defects, and its basis ideas are not free of controversy [26]. In this article we would like to address an interesting point concerning HL cosmology that is still an open problem. In particular, we are seeing a behavior of HL cosmology for different kinds of matter content of the universe such as dust, cosmological constant and phantom matter. The present paper is organized as follows. In Sec. I we discuss the HL cosmology for different kinds of the matter and also we present general settings and constrains in the theory. In Sec. II we analyze the role of stiff-like matter taken as the dominant at early times. Finally, in Sec. III we give some final remarks.

II. HOŘAVA-LIFSHITZ COSMOLOGY

In this section we would like to review the main results of Hl cosmology and its implications for different kind of matter. Also, we are doing the comparison between the behaviors of HL and GR cosmology. Let us first to introduce a cosmological model under consideration based on Ref. [27]. It is well-known that in Hořava-Lifshitz gravity we do not have the Hamiltonian constraint and therefore, from the formal cosmological point of view, also we do not have one Friedmann equation as in standard cosmology. In this theory the starting point is the dynamical equation for flat Friedmann-Robertson-Walker (FRW) spacetime

\[ \eta \left( 2\dot{H} + 3H^2 \right) = -p, \]  

(1)
where we introduced a parameter
\[
\eta = \frac{1}{2} (3\lambda - 1),
\] (2)
and \(\lambda\) represents a dimensionless constant. The value of \(\lambda\) is fixed by the diffeomorphism invariance of four-dimensional general relativity. At this point, we would like to emphasize that in HL cosmology any value of the constant \(\lambda\) is consistent with a foliation that preserves the diffeomorphism invariance. For example, it was shown in Ref. [28] that for \(1/3 < \lambda < 1\) (or equivalently \(0 < \eta < 1\)) the scalar graviton is a ghost and is ghosts-free if \(\lambda < 1/3\) (that is \(\eta < 0\)) and \(\lambda > 1\) (or \(\eta > 1\)). The field equation for matter field is described satisfies a non-conservation equation, and at high energy limit, or early times limit, we can write it in the following form
\[
\dot{\rho} + 3H(\rho + P) = -Q,
\] (3)
where \(Q\) represents the rate of energy non-conservation, and the low energy limit can be recovered only if \(Q \rightarrow 0\). We can conclude that the energy non-conservation is an effect of high-energy physics. Now, equations \([1]\) and \([3]\) contain enough information to describe a cosmological evolution of the spacetime, together with the equation of state (EoS) \(p = \omega \rho\). Indeed, using this dynamical equation plus the non-conservation equation we can obtain one first integral in the form
\[
3H^2 = \frac{1}{\eta} \rho + \frac{C(t)}{a^3},
\] (4)
and this result can be interpreted as a Friedmann equation in HL gravity where the integration "constant" is given by
\[
C(t) = C_0 + \frac{1}{\eta} \int_{t_0}^{t} d\tau a^3(\tau) Q(\tau).
\] (5)
In order to obtain a cosmological description from this kind of theories, we would like to discuss first some well known results from GR in the context of HL cosmology. The subscript 0 indicates quantities today \((t_0)\).

- **Dust matter**: \(p = 0 \leftrightarrow \omega = 0\)

In this case from \([1]\) we find exactly the same solution for the Hubble parameter as in GR, i.e.,
\[
H(t) = H_0 \left[1 + \frac{3}{2} H_0 (t - t_0)\right]^{-1},
\] (6)
and for the cosmic scale factor reads

\[ a(t) = a_0 \left[ 1 + \frac{3}{2} H_0 (t - t_0) \right]^{2/3}. \]  

(7)

It is well known in GR, when we have \( \omega = 0 \), an evolution is described by an energy density proportional to \( a^{-3} \) (this result is obtained from the conservation energy density equation \( \dot{\rho} + 3H(\rho + P) = 0 \)) and the formal solution of the cosmological evolution in GR is identical to Eqs. (6) and (7), obtained for HL cosmology. Although the formal solutions for the Hubble parameter and the scale factor from HL and GR are identical in this case, the dynamical behavior of the energy density is absolutely different, because, in HL cosmology the energy density is not fixed by the non-conservation equation (3). Finally for the dust matter case, we would like to mention that (6) and (7) are also solutions of (1) if we set \( \rho = 0 \) and \( \eta \) is finite. Thus, these equations can also be seen as a self-decelerated evolution.

In a nutshell, both \( \omega = 0 \) and \( \rho = 0 \) lead to the same solutions described by Eqs. (6) and (7). Therefore, in case of dust matter we found that the effect of high energy physics is reflected in the fact that during the early stages, the rate of energy non-conservation is not constant \( Q(t) \neq 0 \), and we have an interchange of energy the between the matter of the universe and some unknown source.

- **Cosmological constant:** \( \omega = -1 \).

In this case, the conservation law

\[ \dot{\rho} = -Q, \]

therefore the behavior of cosmological constant at early universe would be different that the respective one in GR. Then, the interchange of energy between the matter (described in this case for the cosmological constant kind) and the unknown source gives a non constant behavior for the energy density for our cosmological constant type. We note that the GR limit is recovered if we take \( Q(a \to \infty) \to 0 \), i.e., \( \rho = \text{const.} \). Then the ”role” of the cosmological constant in HL gravity is different with the respective one in GR.

- **Phantom Evolution.**

In this case, from (1) we can write the following formal solution for the Hubble parameter

\[ H^2(a) = \frac{1}{a^3} \left( a_0^3 H_0^2 - \frac{\omega}{\eta} \int_{a_0}^{a} da a^2 \rho(a) \right), \]

(9)
and if we take one phantom Ansatz for the energy density given by \( \rho (a) = \rho_0 (a/a_0)^\beta \) with \( \beta > 1 \), we get the explicit form of the formal solution
\[
H^2(a) = H_0^2 \left( \frac{a_0}{a} \right)^3 \left[ 1 - \frac{\omega \rho_0/3H_0^2}{\eta (1 + \beta/3)} \left( \frac{a}{a_0} \right)^{\beta+3} - 1 \right],
\]
and the future behavior of the Hubble parameter reads as follow
\[
H^2(a \rightarrow \infty) = -\frac{\omega \rho_0/3}{\eta (1 + \beta/3)} \left( \frac{a_0}{a} \right)^\beta.
\]
One realistic model implies the following restriction for the barotropic index, \( \omega < 0 \). If we use the following setting
\[
|\omega| \rho_0/3H_0^2 = (1 + \beta/3) \eta,
\]
we obtain one phantom solution for the scale factor given by
\[
a(t) = a_0 \left( \frac{2}{\beta H_0} \right)^{2/\beta} (t_s - t)^{-2/\beta} \quad \text{and} \quad t_s = t_0 + \frac{2}{\beta H_0},
\]
It is worthwhile noticing that the solution (13) has a new region for the phantom evolution for the barotropic index \( \omega < 0 \) instead of the usual, more restrictive one \( \omega < -1 \) that appears in GR.

We also notice that if \( \eta \rightarrow \infty \), then \( \beta \rightarrow -3 \) and we can write
\[
H^2(a)_{\eta \rightarrow \infty} \rightarrow H_0^2 \left( \frac{a_0}{a} \right)^3,
\]
so that the phantom approach has no sense. According to
\[
\eta = |\omega| \left( \rho_0/3H_0^2 \right) (1 + \beta/3)^{-1} > 1,
\]
or
\[
|\omega| \left( \rho_0/3H_0^2 \right) > 1 + \beta/3,
\]
we conclude that the observational data do not allow fulfill the last inequality. On the other hand, if \( 0 < \eta < 1 \) (ghost scalar graviton) we have \( |\omega| \left( \rho_0/3H_0^2 \right) < 1 + \beta/3 \) and in this case we can have a phantom phase provided we can accept an existence of a ghost scalar graviton.

- **General Settings.** We are now interested in the limit where the matter sector is decoupled from the gravity sector, where dark matter as an integration constant dominated the evolution of the universe, that its \( \eta \rightarrow \infty \) (\( \lambda \rightarrow \infty \)). In this limit we can distinguish
several situations. For instance, if $p$ is finite and $\eta \to \infty$, from (11) we obtain exactly the same solution given by (6) and (7) and the Hubble parameter becomes in this limit
\[
3H^2 = \frac{C_0}{a^3},
\] (17)
thus, a dust-like evolution but $p$ and the barotropic index $\omega$ do not vanish. Also, we would like to notice that if $t \to \infty$ and $\eta$ is finite then
\[
3H^2(t \to \infty) \to \frac{1}{\eta}\rho(t \to \infty) + \frac{1}{\eta^3(t \to \infty)} \left[ C_0 + \frac{1}{\eta} \int_{t_0}^{\infty} d\tau a^3(\tau) Q(\tau) \right] = \frac{1}{\eta}\rho(t \to \infty),
\] (18)
if at least $Q(t)$ decreases as $a^{-4}(t)$ when $a(t \to \infty) \to \infty$. This condition on $Q$ is consistent with the recovering of local invariance for the matter sector, where it is demanded that $Q(a \to \infty) \to 0$ and $C(t \to \infty) \to C_0$ (see [28]). So, we have the following relation between observational parameters
\[
3H_0^2 = \frac{1}{\eta}\rho_0 + \frac{C_0}{a_0^3},
\] (19)
On the other hand, from (11) and (14) we can write
\[
\dot{H} = -\frac{1}{2} \frac{C_0}{a^3} - \frac{1}{2\eta} \left[ (1 + 3\omega) \rho + \frac{1}{a^3} \int_{t_0}^{t} d\tau a^3(\tau) Q(\tau) \right],
\] (20)
so that if we take the limit $\eta \to \infty$, and keep finite second term in (20), we obtain
\[
H(a) = H_0 \sqrt{1 + \frac{C_0a_0^{-3}}{3H_0^2} \left[ \left( \frac{a_0}{a} \right)^3 - 1 \right]}.
\] (21)
Furthermore, if $C_0a_0^{-3}/3H_0^2 < 1$, we find
\[
H(a \to \infty) \to H_0 \sqrt{1 - \frac{C_0a_0^{-3}}{3H_0^2}} < H_0.
\] (22)
To conclude, in case with $\eta \to \infty$ we find a de Sitter phase at late times. We also notice that this behavior can also be obtained if we choose $\omega = -1$ but keep $\eta$ finite, that is,
\[
\dot{H} = -\frac{1}{2} \frac{C(t)}{a^3} \to \dot{H}(t \to \infty) \to 0,
\] (23)
given that in this limit $C(t \to \infty) \to C_0$. In another words, taking either $\eta \to \infty$, for all $t$, and or $t \to \infty$ and $\eta$ finite, we arrive to a de Sitter phase at late times.

To complete this point we would like to comment about the acceleration of universe. By using the expression for $\dot{H}$, as well as (4), we can write
\[
\dot{H} + H^2 = -\frac{1}{6\eta} \left[ (1 + 3\omega) \rho + \eta \frac{C(t)}{a^3} \right],
\] (24)
and it is straightforward to check that \( \eta = 1 \) plus \( C(t) = 0 \) implies the standard expression for the acceleration in GR. Also, if \( \dot{H} + H^2 > 0 \) we must have \( \omega < -1/3 \), to avoid violating the weak energy condition (WEC) \( \rho > 0 \). Then, we can have quintessence, cosmological constant or phantom schemes. Now, if \( \dot{H} + H^2 < 0 \) then \( \omega > -1/3 \) the WEC is fulfilled, then we have an evolution driven by dark matter. Finally when \( \dot{H} + H^2 = 0 \) we obtain the solution

\[
H(t) = H_0 [1 + H_0 (t - t_0)]^{-1},
\]

i.e., \( \omega = -1/3 \) (string gas) as in GR. As a curiosity we notice that

\[
\left( \dot{H} + H^2 \right)_{\omega = -1/3} = \frac{1}{3} \left( \dot{H} \right)_{\omega = -1}.
\]

Finally, by setting \( \dot{H} + H^2 = 0 \), we can write the following expression for the density of energy

\[
\rho(t) = \rho_0 (a/a_0)^{-3} + \frac{1}{\eta} \frac{1}{1 + 3\omega} \frac{1}{a^3} \int_{t_0}^{t} d\tau a^3(\tau) Q(\tau),
\]

where \( C_0 = |1 + 3\omega| \rho_0 a_0^{-3} \) and \( \omega < -1/3 \), and we find that \( \rho(t \rightarrow \infty) \rightarrow \rho_0 (a/a_0)^{-3} \), i.e., a dust-like behavior at late times is driven by a \( \omega \) that is dark energy-like.

### III. STIFF-LIKE MATTER AS THE DOMINANT TERM AT EARLY TIMES

When the higher curvature terms are included in the cosmological evolution the dynamic equation for the Hubble parameter is given by [27],

\[
\eta \left( 2\dot{H} + 3H^2 \right) = -\omega \rho + \frac{\alpha k^3}{a^6} + \frac{\alpha' k^2}{a^4} - \frac{k}{a^2} + \Lambda,
\]

where \( \alpha \) and \( \alpha' \) are constants and \( k \) is the spatial curvature. If we consider the early time limit, the dominant contribution is the term of the form \( \sim a^{-6} \), which could be interpreted as stiff matter like, and also would be explain by the supposition that stiff matter could be one important matter content at the very early universe [29, 30]. We write (28) in the form

\[
\eta \left( 2\dot{H} + 3H^2 \right) = -\omega \rho + \frac{\alpha k^3}{a^6} \left[ 1 + \frac{a^2}{\alpha} (\alpha'k - a^2) \right],
\]

and the incorporation of \( \Lambda \) will be discussed later. Therefore, we have a new equation to handle that is

\[
3\eta H^2 = \rho - \frac{\alpha k^3}{a^6}.
\]
Similarly as before, the parameter $\eta$ can take any value. The dynamical equation then can be written as

$$\eta \left( 2\dot{H} + 3H^2 \right) = -\omega \rho + \frac{\alpha k^3}{a^6},$$

(31)

The approximation at early times is justified if the scale factor is kept around $a^2 \sim |\alpha'|$, if $k = 1$ and for $k = -1$ we have a more restrictive condition: $a^2 (\alpha' + a^2) \ll \alpha$. From Eqs. (30) and (31) we obtain the conservation law (the term proportional to $C(t)/a^3$, i.e., proportional to $Q$, is not present here given that only we are holding the dominant contribution $\sim a^{-6}$)

$$\dot{\rho} + 3H (1 + \omega) \rho = 0,$$

(32)

as well as the equation

$$\dot{H} + 3H^2 = \frac{1}{2\eta} (1 - \omega) \rho,$$

(33)

and from these last equations we note that if we do $\omega = 1$, we obtain the usual solution as in GR, i.e. $\rho \sim a^{-6}$ and. We find the following solution for the Hubble parameter and the scale factor, respectively,

$$H(t) = H_0 \left[ 1 + 3H_0 (t - t_0) \right]^{-1},$$

(34)

and

$$a(t) = a_0 \left[ 1 + 3H_0 (t - t_0) \right]^{1/3}.$$

(35)

In order to see the implications of this result, we are comparing it to standard GR, and the equations $3H^2 = \rho$ and $\dot{\rho} + 6H \rho = 0$ ($\omega = 1$) that lead (34) and (35). Thus replacing (34) and (35) in (30), we obtain for $\omega = 1$

$$3\eta H^2 = \left( \rho_0 - \frac{\alpha k^3}{a_0^6} \right) \left( \frac{a_0}{a} \right)^6,$$

(36)

and for $k = 1$ we must to have $\rho_0 a_0^6 > \alpha$ and $\eta > 0$ (do not forget that if $0 < \eta < 1$ we have a ghost scalar graviton and If $\eta > 1$ we are ghosts-free). If $\rho_0 a_0^6 < \alpha$, then $\eta < 0$ and we are ghosts-free. For $k = -1$ we must have $\eta > 0$. We want now to discuss an early universe that contains a mixture of stiff matter-like and cosmological constant,

$$3\eta H^2 = \rho - \frac{\alpha k^3}{a^6} + \Lambda,$$

(37)

and

$$\eta \left( 2\dot{H} + 3H^2 \right) = -\omega \rho + \frac{\alpha k^3}{a^6} + \Lambda.$$
Then we obtain the $k$-independent equation

$$\dot{H} + 3H^2 = \frac{1}{2\eta} [(1 - \omega) \rho + 2\Lambda],$$

and for $\omega = 1$ and $\eta > 0$, the formal solution is

$$H(t) = \sqrt{\Lambda/3\eta} \left( \frac{1 + \Delta_0 \exp \left[ -2\sqrt{3\Lambda/\eta}(t - t_0) \right]}{1 - \Delta_0 \exp \left[ -2\sqrt{3\Lambda/\eta}(t - t_0) \right]} \right),$$

where we have denoted

$$\Delta_0 = \left( H_0 - \sqrt{\Lambda/3\eta} \right) \left( H_0 + \sqrt{\Lambda/3\eta} \right)^{-1},$$

and we observe that $\Delta_0 = 0 \rightarrow H = \sqrt{\Lambda/3\eta}$, i.e., an usual early de Sitter phase (old inflation-like). If $\Delta_0 \neq 0$, the solution (40) is a reasonable one solution for $t << t_0$ (very early times), i.e., $H(t << t_0) \rightarrow$constant and we note also that there is no singularity in $H(t)$ for $\Delta_0 > 0$. For completeness, with $\omega = 1$ and $\eta < 0$, the Eq. (39) has the solution

$$H(t) = \sqrt{\Lambda/3|\eta|} \tan \left[ \arctan \left( \frac{H_0}{\sqrt{\Lambda/3|\eta|}} \right) - 6\sqrt{\Lambda/3|\eta|}(t - t_0) \right],$$

and this solution will be reasonable only if $t << t_0$ (very early times and, for instance, $0 < \arctan \left( H_0/\sqrt{\Lambda/3|\eta|} \right) + 6\sqrt{\Lambda/3|\eta|}t_0 < \pi/2 \implies H(t) > 0$), i.e., $H(t << t_0) \rightarrow$constant. Thus, even the presence of $\omega = 1$, is a cosmological constant the dominant component at early times. In other words, even accepting the presence of stiff matter at early times, in HL cosmology this is a little relevant fact (nevertheless, a fluid for which $p = \rho$ could play a relevant role at early times if we are thinking in to build an holographic approach to cosmology, see [29] and [30]). Finally, we compare the deceleration parameter as given in GR and HL cosmology. This parameter is defined by $q = -\left(1 + \dot{H}/H^2\right)$ so that, in GR reads

$$q = \frac{1}{2} (1 + 3\omega) - \frac{1}{2} (1 + \omega) \frac{\Lambda}{H^2},$$

and in HL cosmology

$$q = \frac{1}{2} (1 + 3\omega) - (1 - \omega) \frac{\alpha k^3}{\eta H^2 a^6} - \frac{1}{2} (1 + \omega) \frac{\Lambda}{\eta H^2},$$

and for $\omega = 1$ we have $q = 2 - \Lambda/H^2$ and $q = 2 - \Lambda/\eta H^2$, respectively, and we note that when $\omega = 1$, the curvature term $\sim \alpha k^3$ disappears.
IV. FINAL REMARKS

We have studied some aspects of the HL cosmology where the emphasis has been put on discuss some cosmological solutions which are present too in GR, in particular, an evolution driven by dust is the same in both theories (flat case). A possible phantom stage has been discussed where we have found a condition less restrictive over the $\omega$-parameter ($\omega < 0$, not $\omega < -1$). At late times, the energy density exhibits a like dust behavior nevertheless the $\omega$-parameter satisfies the inequality $\omega < -1/3$. If we do infinite the parameter which preserves the diffeomorphism invariance in the present theory, a late de Sitter phase can be obtained without consider the usual scheme $\omega = -1$. The combined effect of a curvature-dependent term, which is stiff matter reminiscent and dominant at early times, beside the usual stiff matter component ($\omega = 1$, possible important role at early evolution) and a cosmological constant has been discussed and solutions has been found which exhibit an early de Sitter phase and this fact shows that in HL cosmology the role of stiff matter is of little relevance.

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[1] P. Horava, Phys. Lett. B 694, 172 (2010) [arXiv:0811.2217 [hep-th]].
[2] P. Horava, JHEP 0903, 020 (2009) [arXiv:0812.4287 [hep-th]].
[3] P. Horava, Phys. Rev. Lett. 102, 161301 (2009) [arXiv:0902.3657 [hep-th]].
[4] P. Horava, Phys. Rev. D 79, 084008 (2009) [arXiv:0901.3775 [hep-th]].
[5] G. Calcagni, JHEP 0909, 112 (2009) [arXiv:0904.0829 [hep-th]].
[6] E. Kiritsis and G. Kofinas, Nucl. Phys. B 821, 467 (2009) [arXiv:0904.1334 [hep-th]].
[7] T. Takahashi and J. Soda, Phys. Rev. Lett. 102, 231301 (2009) [arXiv:0904.0554 [hep-th]].
[8] S. 'i. Nojiri and S. D. Odintsov, Phys. Rept. 505, 59 (2011) [arXiv:1011.0544 [gr-qc]].
[9] T. Clifton, P. G. Ferreira, A. Padilla and C. Skordis, Phys. Rept. 513, 1 (2012) [arXiv:1106.2476 [astro-ph.CO]].

[10] R. Brandenberger, Phys. Rev. D 80, 043516 (2009) [arXiv:0904.2835 [hep-th]].

[11] S. Mukohyama, JCAP 0906, 001 (2009) [arXiv:0904.2190 [hep-th]].

[12] R.-G. Cai, B. Hu and H.-B. Zhang, Phys. Rev. D 80, 041501 (2009) [arXiv:0905.0255 [hep-th]].

[13] E. N. Saridakis, Eur. Phys. J. C 67, 229 (2010) [arXiv:0905.3532 [hep-th]].

[14] S. Mukohyama, K. Nakayama, F. Takahashi and S. Yokoyama, Phys. Lett. B 679, 6 (2009) [arXiv:0905.0055 [hep-th]].

[15] S. Mukohyama, Phys. Rev. D 80, 064005 (2009) [arXiv:0905.3563 [hep-th]].

[16] Y.-F. Cai and E. N. Saridakis, JCAP 0910, 020 (2009) [arXiv:0906.1789 [hep-th]].

[17] A. Wang, D. Wands and R. Maartens, JCAP 1003, 013 (2010) [arXiv:0909.5167 [hep-th]].

[18] G. Leon and E. N. Saridakis, JCAP 0911, 006 (2009) [arXiv:0909.3571 [hep-th]].

[19] M. Minamitsuji, Phys. Lett. B 684, 194 (2010) [arXiv:0905.3892 [astro-ph.CO]].

[20] S. Carloni, E. Elizalde and P. J. Silva, Class. Quant. Grav. 27, 045004 (2010) [arXiv:0909.2219 [hep-th]].

[21] X. Gao, Y. Wang, W. Xue and R. Brandenberger, JCAP 1002, 020 (2010) [arXiv:0911.3196 [hep-th]].

[22] T. Kobayashi, Y. Urakawa and M. Yamaguchi, JCAP 1004, 025 (2010) [arXiv:1002.3101 [hep-th]].

[23] K.-i. Maeda, Y. Misonoh and T. Kobayashi, Phys. Rev. D 82, 064024 (2010) [arXiv:1006.2739 [hep-th]].

[24] E. N. Saridakis, Int. J. Mod. Phys. D 20, 1485 (2011) [arXiv:1101.0300 [astro-ph.CO]].

[25] O. Bertolami and C. A. D. Zarro, Phys. Rev. D 84, 044042 (2011) [arXiv:1106.0126 [hep-th]].

[26] M. Henneaux, A. Kleinschmidt and G. Lucena Gomez, Phys. Rev. D 81, 064002 (2010) [arXiv:0912.0399 [hep-th]].

[27] S. Mukohyama, Class. Quant. Grav. 27, 223101 (2010) [arXiv:1007.5199 [hep-th]].

[28] A. E. Gumrukcuoglu and S. Mukohyama, Phys. Rev. D 83, 124033 (2011) [arXiv:1104.2087 [hep-th]].

[29] T. Banks and W. Fischler, hep-th/0412097.

[30] T. Banks, J. Phys. A 42, 304002 (2009) [arXiv:0809.3951 [hep-th]].