**U(1)\(_{B_1+B_2-2L_1}\) mediation for the natural SUSY and the anomalous muon \(g-2\)**

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Abstract

We propose a U(1)' mediated supersymmetry (SUSY) breaking, in which U(1)' is identified with U(1)\(_{B_1+B_2-2L_1}\). The U(1)\(_{B_1+B_2-2L_1}\) gauge symmetry, which is anomaly-free with the field contents of the minimal supersymmetric standard model, assigns \(\pm 1/3\) charges to the first and second generations of the quarks, and \(\mp 2\) to the first generation of the leptons. As a result, the first two generations of squarks acquire masses of about 7 TeV, and the first generation of the sleptons do those of 40 TeV, respectively, in the presence of one or three pairs of extra vector-like matter \(\{5,\overline{5}\}\). Non-observation on extra colored particles below 1 TeV at the large hadron collider, and also the flavor violations such as \(\mu^- \rightarrow e^- \gamma\) are explained. By virtue of such a gauge symmetry, proton stability can be protected. The other squarks and sleptons as well as the gauginos can obtain masses of order \(10^{2-3}\) GeV through the conventional gravity or gauge mediated SUSY breaking mechanism. The relatively light smuon/sneutrino and the neutralino/chargino could be responsible for the \((g-2)_\mu\) deviated from the standard model prediction. The stop mass of \(\sim 500\) GeV relieves the fine-tuning problem in the Higgs sector. Two-loop effects by the relatively heavy sfermions can protect the smallness of the stop mass from the radiative correction by the heavy gluino \((\gtrsim 1\) TeV). Extra vector-like matter can enhance the radiative corrections to the Higgs mass up to 126 GeV, and induce the desired mixing among the chiral fermions after U(1)\(_{B_1+B_2-2L_1}\) breaking.

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I. INTRODUCTION

Recently, ATLAS and CMS have announced the discovery of the standard model (SM) Higgs(-like) boson in the 125 – 126 GeV invariant mass range [1, 2]. Thus, the SM seems to have been almost completely confirmed as the basic theory describing the nature. So far, any evidence beyond the SM including supersymmetry (SUSY) has not appeared yet at the large hadron collider (LHC). It implies that the theoretical puzzles raised in the SM such as the gauge hierarchy problem still remain unsolved [3].

In fact, 126 GeV is too large for the mass of Higgs boson in the minimal SUSY SM (MSSM), if we conservatively keep the original motivation of introducing SUSY at the EW scale. In the MSSM the observed 126 GeV Higgs mass requires a too heavy stop mass ($\tilde{m}_t \gtrsim$ a few TeV) for enhancing the radiative Higgs mass [4], which compels the soft parameters to be finely tuned to match the $Z$ boson mass at the minimum of the Higgs potential [3]. Thus, an excessively heavy stop mass spoils the status of SUSY as a solution of the fine-tuning problem associated with the SM Higgs boson mass.

For naturalness of the Higgs boson mass, thus, we need a relatively light stop. At the moment, fortunately the stop mass bound is much lower compared to those of the other squarks, $\tilde{m}_t \gtrsim 500$ GeV, which provides just $\Delta m^2_h|_{\text{top}} \gtrsim (71 \text{ GeV})^2$ through one-loop radiative correction together with the top quark. As a result, we need more ingredients beyond the MSSM for raising the Higgs mass except the stop: for explaining the 126 GeV Higgs mass, $\Delta m^2_h|_{\text{new}} \sim (89 – 51 \text{ GeV})^2$ for $\tan \beta = 2 – 50$ should be supplemented by extending the MSSM, if the stop mass is taken to be just around 500 GeV in order to minimize the fine-tuning in the Higgs sector, avoiding the experimental bound on it\(^1\) (see, for instance, Refs. [6–8]).

The fact that any SUSY particles have not been observed yet at the LHC means that the masses of new colored particles absent in the SM would be quite heavier than 1 TeV. Here we should note that such a bound is still applied only to the first and second generations of squarks and to the gluino: the constraint on the third generation is just around 500 GeV, as mentioned above. Fortunately, such heavy squark masses do not affect much the Higgs mass because of their small Yukawa couplings. Moreover, heavy masses of the first and second generations of squarks are helpful for avoiding the flavor problem expected in the gravity mediation scenario. If we insist on the stop mass being $\sim 500$ GeV at the electroweak (EW) scale, however, gluino heavier than 1 TeV at the EW scale would drive the squared mass of the stop negative at a higher energy scale through the renormalization group (RG) effects. It means that the stop becomes much heavier than 500 GeV at low energies, if we take it positive at high energy scales. To avoid color breaking at a higher energy scale, and keep the small stop mass at the EW scale, we need to somehow compensate the heavy gluino effect in the RG equation.

\(^1\) For heavy LSP ($\gtrsim 350$ GeV), $\tilde{m}_t$ is not constrained yet [2].
Brookhaven National Laboratory (BNL) reported a remarkable result on \((g-2)\) of the muon \((\mu)\) \[^9\] which is deviated from the SM prediction \[^10, 11\] by \(3.3\sigma - 3.6\sigma\),

\[
\Delta(g-2)_\mu = (g-2)_{\mu}^{\text{exp}} - (g-2)_\mu^{\text{SM}} = (26.1 \pm 8.0) \times 10^{-10}.
\] (1)

Even though the statistical significance is not strong enough in addition to the theoretical uncertainty on the hadronic effects, still this discrepancy may hint new physics beyond the SM. If it originates from SUSY, relatively light smuon/sneutrino and neutralino/chargino are needed to give this order of \((g-2)_{\mu}\) \[^12, 13\]. In contrast to the squarks, the mass bounds on sleptons at the LHC are less severe \[^14\]. As pointed out in Ref. \[^2\], moreover, the extra vector-like leptons lighter than \(\sim 500\) GeV is very helpful for explaining 126 GeV Higgs mass with \(\tilde{m}_t \sim 500\) GeV but without assuming a large mixing between the left- and the right-handed stops. Their order-one Yukawa coupling to the MSSM Higgs boson can enhance the radiative Higgs mass. The Landau-pole problem associated with the order-one Yukawa coupling can be avoided by introducing an extra gauge symmetry, under which only the new vector-like leptons are charged.

In view of the recent experimental data at the LHC, the “effective SUSY” (or “more minimal SUSY”) \[^15, 16\] and “split SUSY” \[^17\] scenarios look promising. According to the effective SUSY, the first two generations of superpartners are required to be about \(5-20\) TeV in order to avoid the SUSY flavor and SUSY CP problems, while the third ones and gauginos can be lighter than 1 TeV. However, the masses heavier than 22 TeV for the first two generations drive the stop mass squared negative at the EW scale through the two-loop RG effects, if the third ones are lighter than 4 TeV, and if such soft parameters are generated around the grand unification (GUT) scale \((\approx 2 \times 10^{16}\) GeV) \[^18\]. We note here that such a two-loop RG effect by heavy superpartners on lighter superpartners can be utilized to keep the stop mass squared positive at higher energy scales, when the gluino is also heavy enough \((\gtrsim 1\) TeV). Namely, it can compensate the heavy gluino effect on the stop mass we mentioned above, keeping the light stop mass at low energy. In this Letter, we will discuss this possibility.

On the other hand, in the split SUSY all the superpartners of the SM chiral fermions are assumed to be heavy while the superpartners of bosonic particles in the SM, i.e. gauginos and Higgsinos remain relatively light. One such idea realizing the split SUSY is to introduce the so-called \(Z'\) mediated SUSY breaking \[^19\] or \(U(1)'\) mediation, in which a \(U(1)'\) gauge sector plays the role of the messenger sector for SUSY breaking in the visible sector. In \(U(1)'\) mediation, scalar components of chiral superfields charged under \(U(1)'\) acquire quite heavy masses of order \(100\) TeV at one-loop level, while the MSSM gauginos get masses of \(10^{2-3}\) GeV at two-loop level. Moreover, by employing a family dependent \(U(1)'\) charge assignment, one can achieve a hierarchical sfermion spectrum. Thus, e.g. the third family of squarks can be made the lightest \[^20\] or heaviest family \[^21\].

With the original forms of the effective SUSY and split SUSY, however, the discrepancy of \((g-2)_{\mu}\) observed at BNL from the MSSM prediction cannot be accommodated. In this
Letter, we attempt to obtain a more desirable spectrum on the superparticles by employing a proper $U(1)'$ mediation such that the $(g - 2)_{\mu}$ observed at BNL is explained in the SUSY framework. To be consistent with the LHC results, the first two generations of squarks should be made quite heavy while the second generation of sleptons and the stop need to remain light to explain $(g - 2)_{\mu}$ and to avoid the fine-tuning problem. Indeed, such a hierarchical spectrum of the superparticles is hard to obtain in the conventional SUSY breaking scenarios. In order to obtain such a spectrum, we need to find a desirable $U(1)'$ gauge symmetry useful for the $U(1)'$ mediation.

It is well-known that with the MSSM chiral matter contents, $U(1)_B$ and $U(1)_L$ are anomalous for each generation. However, their proper combinations could cancel all the gauge anomalies. For instance, $U(1)_{B-L}$ is anomaly-free for each generation. $U(1)_{B_i-B_j}$ and $U(1)_{L_i-L_j}$ ($i \neq j$) are also anomaly-free, where $i$ and $j$ denote generation numbers. Of course, all the linear combinations of $U(1)_{B-L}$, $U(1)_{B_i-B_j}$ and $U(1)_{L_i-L_j}$ are also anomaly-free. Hence, $U(1)_{B_i-L_j}$ ($i \neq j$) can be an anomaly-free gauge symmetry. Indeed, $U(1)_{(B_1+B_2-L_1-L_3)+B_3-L_3}$ was adopted for $U(1)'$ mediation [21] where the smuon/sneutrino can be light enough for an explanation of $(g - 2)_{\mu}$.

In this Letter, we propose a $U(1)'$ mediation with

$$U(1)_{B_1+B_2-2L_1},$$

(2)

which is a linear combination of $U(1)_{B_2-B_1}$ and $U(1)_{B_1-L_1}$. By virtue of this gauge symmetry, proton stability can be protected: $U(1)_{B_1+B_2-2L_1}$ disallows all the dimension 4 and 5 baryon number violating operators.\(^2\) Hence, the first two generations of squarks and the selectron are expected to be quite heavy through the $U(1)'$ mediation mechanism since they carry nonzero charges of $U(1)_{B_1+B_2-2L_1}$. On the other hand, the stops and smuons/sneutrino remain relatively light. Their masses could be obtained through the ordinary gravity or gauge mediation mechanism.

The paper is organized as follows. In Section II, we introduce the $U(1)_{B_1+B_2-2L_1}$ mediated SUSY breaking, and attempt to obtain the low energy spectrum of superparticles. We also discuss $(g - 2)_{\mu}$. In Section III, we propose a concrete $U(1)'$ mediation model. Section IV is a conclusion.

\(^2\) $U(1)_{B_1+B_2-L_1-L_3}$ is also a quite interesting gauge symmetry, in which the stop and the smuon/sneutrino can still be light. However, we note that the extra $U(1)$ gauge symmetry allows a heavy right-handed neutrino mass term in the superpotential only for one right-handed neutrino, unless they are broken at a high energy scale. For a successful seesaw mechanism, we intend to cheaply obtain heavy mass terms for at least two right-handed neutrinos, which should be neutral under the extra $U(1)$. It is another motivation to study $U(1)_{B_1+B_2-2L_1}$. 
II. SUPERPARTICLE SPECTRUM AND \((g - 2)_\mu\)

We suppose that the visible and the hidden sectors can communicate through a U(1) gauge interaction. The SUSY breaking in the hidden sector, which can be parametrized with a spurion field \(X = M + \theta^2 F\), is assumed to generate a mass of the gaugino of U(1)’ (\(\equiv M_{\tilde{Z}'}\)) at the scale \(\Lambda_S\) [19]:

\[ M_{\tilde{Z}'} \sim \frac{g_{\tilde{Z}'}^2 F}{16\pi^2 M}, \tag{3} \]

which plays the role of the order parameter of the SUSY breaking effects in the visible sector. The SUSY breaking in the U(1)’ gauge sector can be transferred to the visible sector [19], inducing the soft masses for the scalar components of the superfields carrying U(1)’ charges:

\[ \tilde{m}_f^2 \sim \frac{Q_f^2 g_{\tilde{Z}'}^2 M_{\tilde{Z}'}^2}{16\pi^2} \log \left( \frac{\Lambda_S}{M_{\tilde{Z}'}^2} \right), \tag{4} \]

where \(Q_f\) stands for the U(1)’ gauge charge. We note here that it can be designed to solve the flavor changing neutral coupling (FCNC) problem in the gravity mediation scenario. By assigning the same U(1)’ charges to the families needed to avoid the FCNC problem, we can get the degenerate and diagonal form of mass matrices for those families of superpartners. If \(\tilde{m}_f^2\) dominates the soft mass squared over the one induced by gravity mediation, it does not have to be much heavier than 1 TeV to avoid the FCNC. Unlike the original U(1)’ mediation of Ref. [19], thus, we assume that the soft scalar masses generated through this mechanism are heavy enough just to avoid the experimental bounds on superparticles at the LHC. For Eq. (4) we suppose that \(M_{\tilde{Z}'}\) is of order \(10^4\) GeV and \(\Lambda_S \sim 10^{16}\) GeV. Hence, for instance, \(M\) would be of order \(10^{14}\) GeV if \(F \sim (10^{10}\ \text{GeV})^2\), and \(M \sim 10^{10}\) GeV if \(F \sim (10^8\ \text{GeV})^2\).

As seen in Eq. (4), the RG running of \(\tilde{m}_f^2\) between the two energy scales \(\Lambda_S\) and \(M_{\tilde{Z}'}\) is dominated by the U(1)’ gaugino. Below the \(M_{\tilde{Z}'}\) scale, however, the U(1)’ gaugino is decoupled, and so the RG running of \(\tilde{m}_f^2\) is governed only by the RG equations of the relevant MSSM superfields and the U(1)’ gauge field. Accordingly, below the \(M_{\tilde{Z}'}\) scale the change of \(\tilde{m}_f^2\) would become much smaller.

The mass splitting between the bosonic and fermionic modes in the chiral matter sector as well as in the U(1)’ gauge sector generate also the MSSM gaugino masses at two-loop level [19]. Hence, the MSSM gaugino masses are much suppressed compared to the soft masses of the scalars with U(1)’ charges:

\[ \delta M_k \sim \frac{g_{\tilde{Z}'}^2 g_k^2}{(16\pi^2)^2} M_{\tilde{Z}'} \log \left( \frac{\Lambda_S}{M_{\tilde{Z}'}^2} \right) \sim 10 \ g_k^2 \ \text{GeV}, \tag{5} \]

where \(g_k\) \((k = 3, 2, 1)\) denotes the MSSM gauge couplings. These gaugino masses are too light, and so they are dominated by other mediation mechanisms of SUSY breaking so that they are of order \(10^{2-3}\) GeV. We will neglect the contributions by Eq. (5) to the MSSM gaugino masses.
Table I: Matter fields carrying $U(1)_{B_1 + B_2 - 2L_1}$ gauge charges. $\{N^{(t)}_{(H)}, N^{c(t)}_{(H)}\}$ are MSSM singlets. The scalar components of them acquire heavy masses of $7 - 40$ TeV depending on their charges (and the full matter content) through the $U(1)_{B_1 + B_2 - 2L_1}$ mediated SUSY breaking mechanism.

We identify $U(1)'$ with $U(1)_{B_1 + B_2 - 2L_1}$, which is anomaly-free. The charge assignments of $U(1)_{B_1 + B_2 - 2L_1}$ are presented in Table I. From Eq. (4) the masses of the first and second generations of squarks are in the range of $5 - 10$ TeV, and the masses of the first generation of the sleptons are of $\sim 30 - 60$ TeV. Since the third generation of the quarks, and the second and third generations of leptons are neutral under $U(1)_{B_1 + B_2 - 2L_1}$, their superpartners get the soft masses only from the ordinary gravity or gauge mediation. The MSSM gaugino masses are also dominated by those mechanisms. Thus, their masses of order $10^{2 - 3}$ GeV are expected.

The masses of $5 - 10$ TeV for the first two squark families and $500$ GeV for the third squark family satisfy the LHC mass bounds on the extra colored particles. Such a hierarchical mass spectrum is non-trivial to realize in the constrained MSSM (CMSSM). In our $U(1)'$ mediation, a serious fine-tuning problem in the Higgs sector can be avoided due to the relatively light stop. We will briefly discuss how to explain $126$ GeV Higgs mass later. The relatively light smuon/sneutrino and neutralino/chargino admit the possibility that $(g - 2)_{\mu}$ deviated from the SM prediction is supported from SUSY, as will be discussed later.

The SUSY FCNC problems are associated with the ratios of the off-diagonal to the diagonal components in the soft mass matrices. In the $U(1)'$ mediation, where the $U(1)'$ charges are given by $B_1 + B_2 - 2L_1$, SUSY breaking effects generates degenerate heavy soft masses ($5 - 10$ TeV) in the diagonal $(11)$ and $(22)$ components in the squark mass matrix. Since the masses induced by the $U(1)'$ mediation dominate gravity mediation for the soft masses, the FCNC in the quark sector can be sufficiently suppressed. Also, the $U(1)'$ mediation induces very heavy masses ($\sim 40$ TeV) to the first family of sleptons. Note that the $U(1)_{B_1 + B_2 - 2L_1}$ symmetry forbids the off-diagonal elements $M^2_{\text{lepton}(1i)}(i = 2, 3)$ in the slepton mass matrix. If $U(1)_{B_1 + B_2 - 2L_1}$ survives down to low energies, thus, the FCNC problem in the lepton sector, e.g. such as $\mu^- \rightarrow e^- \gamma$ and $\tau^- \rightarrow e^- \gamma$, could not be severe.$^3$

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$^3$ We suppose that the $(2, 3)$ and $(3, 2)$ elements in the slepton mass matrix are small enough for suppression of $\tau^- \rightarrow \mu^- \gamma$. Note, however, that the bound on the $\tau^- \rightarrow \mu^- \gamma$ decay is less severe, $\text{BR}(\tau^- \rightarrow \mu^- \gamma) < 4.4 \times 10^{-8}$ [14].
In supergravity models, the predicted mass relation for the MSSM gauginos is

\[
\frac{M_3(Q)}{g_3^2(Q)} = \frac{M_2(Q)}{g_2^2(Q)} = \frac{M_1(Q)}{g_1^2(Q)} = \frac{m_{1/2}}{g_U^2},
\]

where \(Q\) denotes the renormalization energy scale, \(m_{1/2}\) and \(g_U\) mean the gaugino mass and gauge coupling at the GUT scale. In Eq. (6), the MSSM gauge couplings are given by

\[
g_k^2(t) = \frac{g_U^2}{1 - \frac{g_U^2}{8\pi^2}b_k(t - t_0)} \quad \text{for} \quad k = 3, \ 2, \ 1,
\]

where \(t - t_0 \equiv \log(Q/M_{\text{GUT}})\), and \(b_k (k = 3, 2, 1)\) denotes the beta function coefficients of the gauge couplings for \(SU(3)_c\), \(SU(2)_L\) and \(U(1)_Y\). If there exist extra \(v\) pairs of \(\{|5, 5\}\), \(b_k\) are given by \(b_k = (-3 + v, 1 + v, 33/5 + v)\). The unified gauge coupling \(g_U^2\) is estimated as 0.52, 0.62, 0.82, and 1.18, for \(v = 0, 1, 2,\) and 3, respectively. Note that \(SU(3)_c\) becomes (approximately) conformal for \(v = 3\), because \(b_3\) vanishes at one-loop level. Since the low energy magnitudes of the MSSM gauge couplings are related to each other as \(g_3^2 : g_2^2 : g_1^2 \approx 6 : 2 : 1\), we have \(M_3 : M_2 : M_1 \approx 6 : 2 : 1\) at the TeV scale. For instance, thus, we have \(M_2 \approx 400\ \text{GeV}\) and \(M_1 \approx 200\ \text{GeV}\) for \(M_3 \approx 1.2\ \text{TeV}\). In the CMSSM \((v = 0)\), it can be achieved by setting \(m_{1/2} \approx 532\ \text{GeV}\) at the GUT scale. On the other hand, for \(v = 1\) or 3, which are the cases of our particular interest in this Letter, \(m_{1/2}\) should be taken as a larger value, 631 GeV \((v = 1)\) or 1.2 TeV \((v = 3)\) in order to get \(M_3 = 1.2\ \text{TeV}\) at the TeV scale.

In the next section, we will propose a concrete model with \(v = 3\).

We require \((500\ \text{GeV})^2 \lesssim \tilde{m}_t^2 \ll (1\ \text{TeV})^2\), which can be obtained from \(m_0^2 \lesssim (100\ \text{GeV})^2\) at the GUT scale in the CMSSM \((v = 0)\), if \(m_{1/2} \approx 300\ \text{GeV}\). In this case, the slepton masses of the second generation become much lighter than 500 GeV. In the case \(m_{1/2} \gtrsim 1\ \text{TeV}\), however, the stop mass squared rapidly decreases with energy, and becomes negative at some high energy scale. It means that the stop mass should be much heavier than 500 GeV, if it is taken to be positive at the GUT scale. We will see that such a heavy gluino effect on the RG equation could be compensated with the two-loop effects by very heavy superpartners, maintaining the positive squared mass of the stop all the way up to the GUT scale.

The relatively light smuon/sneutrino (the second generation of the sleptons) and neutralino/chargino could be responsible for the deviation of the muon \(g - 2\) from the SM prediction \([12, 21, 23]\). In the MSSM, the muon \(g - 2\) is contributed by the sneutrino/chargino loops and the smuon/neutralino loops. For a large \(\mu\), i.e. \(\mu^2 \gg \tilde{m}_{L,R}^2\), the contribution by the smuon/neutralino loops dominates that by the the sneutrino/chargino loops \([12, 21]\). If \(M_2, \mu \sim 500\ \text{GeV}\), and the sleptons’ masses are quite smaller than this value, i.e. \(\ll 500\ \text{GeV}\), however, the sneutrino/chargino loop becomes dominant \([12, 23]\):

\[
\Delta(g - 2)_\mu \approx \frac{g_2^2 m_\mu^2}{16\pi^2 M_2 \mu} \tan \beta \frac{I^+(x, y)}{20} \approx 1.9 \times 10^{-9} \left( \frac{\tan \beta}{20} \right) \left( \frac{400\ \text{GeV}}{M_2 \mu} \right) I^+(x, y),
\]

where \(m_\mu\) is the muon mass. The loop function \(I^+(x, y)\) is defined as

\[
I^+(x, y) \equiv xy \left[ \frac{13 - 7(x + y) + xy}{(x - 1)^2(y - 1)^2} \right] - \frac{(4 + 2x)\log x}{(x - y)(x - 1)^3} + \frac{(4 + 2y)\log y}{(x - y)(y - 1)^3}.
\]
where $x \equiv M_2^2/\tilde{m}_t^2$, and $y \equiv \mu^2/\tilde{m}_\mu^2$. Here, we assume $\tilde{m}_\mu^2, \tilde{m}_t^2 \gg M_W^2$ and set $\tilde{m}_\mu^2 \approx \tilde{m}_t^2$. For $2 < x, y < 5$, $I^+ (x, y)$ is in the range of $1.0 - 1.1$. Thus, $\Delta(g-2)_{\mu} \approx 2.6 \times 10^{-9}$ can be achieved for a large $\tan \beta \geq 20$ and a small $M_2 \mu \lesssim (400 \text{ GeV})^2$. Since an excessively large $\mu$ could make the fine-tuning in the Higgs sector more serious, we take the case that the sneutrino/chargino loop is dominant.

Now let us explore the conditions for $\tilde{m}_t^2 \approx (500 \text{ GeV})^2$ and $\tilde{m}_\mu^2 \approx (400 \text{ GeV})^2$ at the EW scale. First, we consider the RG behavior of the heavier soft mass squareds, namely, the mass squareds of the first two generations of squarks and the first generation of sleptons, which are charged under $U(1)_{B_1+B_2-2L_1}$. The RG evolution of such heavy soft mass squareds are mainly governed by the $U(1)_{B_1+B_2-2L_1}$ gaugino, because it is much heavier ($\sim 10^4 \text{ GeV}$) than other superpartners:

$$16\pi^2 \frac{d\tilde{m}_t^2}{dt} \approx -8Q_f^2 g_Z^2 (t) |M_{\tilde{Z}_t}(t)|^2.$$  \hspace{1cm} (10)

Here, $g_Z^2(t)$ and $M_{\tilde{Z}_t}(t)$ are given by $g_{Z0}^2/[1 - g_{Z0}^2/8\pi^2 b_Z(t-t_0)]$ and $M_{\tilde{Z}_0}/[1 - g_{Z0}^2/8\pi^2 b_Z(t-t_0)]$, respectively, where $g_{Z0}^2$ and $M_{\tilde{Z}_0}$ indicate the boundary values of $g_Z^2(t)$ and $M_{\tilde{Z}_t}(t)$ at the GUT scale, and $b_Z$ is the beta function coefficient of the $U(1)_{B_1+B_2-2L_1}$ gauge coupling. Eq. (10) is integrable. The analytic solution of Eq. (10) is

$$\tilde{m}_t^2(t) \approx \frac{2Q_f^2}{b_Z} M_{\tilde{Z}_0}^2 \left[ 1 - \frac{g_A^4(t)}{g_{Z0}^4} \right] \equiv Q_f^2 m_0^2 \left[ 1 - \frac{g_A^4(t)}{g_{Z0}^4} \right]. \hspace{1cm} (11)$$

For a large $b_Z$, $g_A^4(t)$ rapidly drops down, and so $\tilde{m}_t^2(t)$ quickly approaches a constant (\(= Q_f^2 m_0^2\)) as in the case considered in Ref. [18]. In our case, $b_Z = \frac{314}{9}$ by the charge assignment in Table I. Thus, we will ignore the second term of Eq. (11).

Next, we discuss the RG behavior of the lighter soft mass squareds, namely, the third generation of squarks, and the second (and third) generation(s) of sleptons. We require $M_2 \approx 400 \text{ GeV}$ at the EW scale for explaining $(g-2)_{\mu}$. Accordingly, the gluino mass should be $1.2 \text{ TeV}$ at low energy, as discussed above. Since the mass of gluino \([M_3(t) = m_{1/2}^2 g_3^2(t)/g_1^2]\) as well as the masses of the first two generations of squarks \([\tilde{m}_{q_{1,2}}^2(t) \approx (1/3) m_0^2]\) and the first generation of sleptons \([\tilde{m}_{\tau_1}^2(t) \approx 2 m_0^2]\) are assumed to be quite heavy, their contributions to the RG equations are dominant over the Yukawa couplings’ contributions at low energies. Neglecting the $U(1)_Y$ and Yukawa couplings’ contributions, the RG equations for $\tilde{m}_t^2(t)$ and $\tilde{m}_\mu^2(t)$ are approximately given by

$$\frac{d\tilde{m}_t^2}{dt} \approx -\frac{32}{3} \frac{g_3^2}{16\pi^2} |M_3|^2 + \left\{ \frac{128}{3} \frac{g_3^2 Q_q^2}{(16\pi^2)^2} + \frac{g_2^4}{(16\pi^2)^2} \left( 18Q_q^2 + 3Q_7^2 \right) \right\} m_0^2,$$

$$\frac{d\tilde{m}_\mu^2}{dt} \approx -\frac{64}{3} \frac{g_2^2}{16\pi^2} |M_2|^2 + \frac{g_2^4}{(16\pi^2)^2} \left( 18Q_q^2 + 3Q_7^2 \right) m_0^2,$$

where the terms with $Q_q, Q_l$ correspond to the two-loop effects by the heavy superpartners [18]. $Q_q (= \pm \frac{1}{3})$ [$Q_l (= \mp 2)$] denotes the $U(1)_{B_1+B_2-2L_1}$ charge for the heavy generation(s) of the
FIG. 1: Low energy values of \((\tilde{m}_t, \tilde{m}_\mu)\) vs. \((m_{1/2}, m_0)\) for \(v = 1\) (a) and \(v = 3\) (b). The solid [dotted] lines are contour lines of the same masses of \(\tilde{m}_t\) [\(\tilde{m}_\mu\)] for various \((m_{1/2}, m_0)\). \(\tilde{m}_0^2\) is taken as \((1.2 \text{ TeV})^2\) in (a) and \((1.7 \text{ TeV})^2\) in (b), respectively. \(m_{1/2} \approx 0.63 \text{ TeV}\) in (a) and \(m_{1/2} \approx 1.2 \text{ TeV}\) in (b) provide \((M_3, M_2, M_1) \approx (1.2 \text{ TeV}, 0.4 \text{ TeV}, 0.2 \text{ TeV})\) at TeV scale.

squarks [sleptons]. Thus, the two generations of (s)quarks and one generation of (s)leptons make contributions to the two-loop effects in Eq. (12). Here, we inserted the expression \(\tilde{m}_f^2(\mu) \approx Q_f^2 m_0^2\) in Eq. (11). The above approximations are possible because \(m_0^2, M_3^2 \gg \tilde{m}_f^2\), and \(g_3^2\) is of order unity at low energies.

Using Eqs. (6) and (7), the above equations are integrable. Requiring \(\tilde{m}_f^2 \approx (500 \text{ GeV})^2\) \(\tilde{m}_\mu^2 \approx (400 \text{ GeV})^2\), and \(M_3 \approx 1.2 \text{ TeV}\) at TeV scale, thus, we have

\[
\begin{align*}
(500 \text{ GeV})^2 & \approx \tilde{m}_0^2 + C_1^q m_{1/2}^2 - C_2^q m_0^2, \\
(400 \text{ GeV})^2 & \approx \tilde{m}_0^2 + C_1^l m_{1/2}^2 - C_2^l m_0^2.
\end{align*}
\]

(13)

Here \(\tilde{m}_0^2\) indicates a common boundary value of \(\tilde{m}_f^2(t)\) and \(\tilde{m}_\mu^2(t)\) at the GUT scale, which could be generated e.g. through the gravity mediated SUSY breaking mechanism. As discussed above, the boundary value of the gluino mass, \(m_{1/2}\) is given by 631 GeV for \(v = 1\) and 1.2 TeV for \(v = 3\). The coefficients in Eq. (13) are roughly estimated as \([C_1^q, C_2^q, C_1^l, C_2^l] \approx [3.5, 0.007; 0.45, 0.004]\) and \([2.39, 0.015; 0.33, 0.008]\) for \(v = 1\) and 3, respectively. They yield \([\tilde{m}_0^2, m_0^2] \approx [(1.2 \text{ TeV})^2, (19.4 \text{ TeV})^2]\) for \(v = 1\), and \([(1.7 \text{ TeV})^2, (20.3 \text{ TeV})^2]\) for \(v = 3\). They imply that the squared masses for the first two generations of squarks and the first generation of sleptons are given by \([\tilde{m}_{q,1}^2, \tilde{m}_l^2] \approx [(6.5 \text{ TeV})^2, (38.7 \text{ TeV})^2]\) for \(v = 1\), and \([(6.8 \text{ TeV})^2, (40.5 \text{ TeV})^2]\) for \(v = 3\) at TeV scale. For the low energy values of \((\tilde{m}_t, \tilde{m}_\mu)\)
resulted from other boundary values of \((m_{1/2}, m_0)\), see Fig. [1] Since the contour lines in the \(v = 3\) case are denser than those in the \(v = 1\) case, \(\tilde{m}_t\) and \(\tilde{m}_\mu\) are more sensitive to the choices of \(m_{1/2}\) and \(m_0\) in the \(v = 3\) case.

So far we have ignored the Yukawa couplings. They might be important around the GUT scale, since the \(m_0^2\) terms in Eq. (12) or \(\tilde{m}_2^2(t)\) in Eq. (11) vanish at the boundary. However, even the top quark Yukawa coupling decreases with energy, eventually down to about 0.5 (0.2) for \(\tan\beta = 20\) and \(v = 1\) \((v = 3)\) at the GUT scale. Thus, the \(\tilde{m}_2^2\) \((m_0^2)\) turns out to be just a bit larger (smaller) than the above estimation, when the Yukawa couplings are also considered in the RG equations.

\([\tilde{m}_0^2, m_0^2]\) chosen in the above might determine also the soft mass squareds of the Higgs, \(\tilde{m}_{h_u}^2\) and \(\tilde{m}_{h_d}^2\), and so affect the Z boson mass at the minimum of the Higgs scalar potential. However, a mechanism introduced in the next section for explaining 126 GeV Higgs mass (with vector-like leptons \(\{\hat{L}, \hat{L}; \tilde{N}, \tilde{N}\}\)) are also involved there. Another free parameter, \(\mu\) should also be considered. Moreover, one can take other boundary values different from \(\tilde{m}_0^2\) for \(\tilde{m}_{h_u}^2(t)\) and \(\tilde{m}_{h_d}^2(t)\). In this Letter, we don’t discuss their RG behaviors, and the EW symmetry breaking in details.

### III. THE MODEL

Unlike the first and second generations of \((s)\)quarks and the first generation of \((s)\)leptons, the other MSSM superfields do not carry \(U(1)_{B_1+B_2-2L_1}\) gauge charges. Hence the mixing between the first (last) two and the third (first) generations in the quark (lepton) sector is impossible, if \(U(1)_{B_1+B_2-2L_1}\) remains unbroken. In this section, we will describe how the desired mixing in the quark and lepton sectors can be induced.

We introduce a global \(U(1)_{PQ}\) symmetry. The global charges for the superfields neutral under \(U(1)_{B_1+B_2-2L_1}\) are listed in Table II. The renormalizable superpotential consistent with the gauged \(U(1)_{B_1+B_2-2L_1}\) and the global \(U(1)_{PQ}\) symmetries is

\[
W = \sum_{i,j=1,2} (y_{ij}^u q_i h_u u_j^c + y_{ij}^d q_i h_d d_j^c) + \sum_{i,j=2,3} \left( y_{ij}^u l_i h_u \nu_j^c + y_{ij}^d l_i h_d \nu_j^c + \frac{1}{2} M^{ij} \nu_i^c \nu_j^c \right) \\
+ y_{e3} q_3 h_u \nu_3^c + y_{e3} q_3 h_d \nu_3^c + y_{e1} l_1 h_d \nu_1^c + \mu h_u h_d,
\]

where the “\(y\)”s denote the dimensionless Yukawa coupling constants, and “\(M^{ij}\)” \((= M^{ji})\) means the Majorana masses for the two right-handed neutrinos neutral under \(U(1)_{B_1+B_2-2L_1}\). The Majorana mass terms of the right-handed neutrinos and the MSSM \(\mu\) term break \(U(1)_{PQ}\) explicitly. We will explain later how they can be generated with the desired sizes. Note that there are no Dirac and Majorana mass terms for \(\nu_i^c\). Only with the two heavy right-handed neutrinos, however, the seesaw mechanism and leptogenesis (through the CP violating phase of heavy neutrinos) are still possible [24].

In order to break \(U(1)_{B_1+B_2-2L_1}\), we introduce the MSSM singlets \(\{N_H, N_H^c; N_H^c, N_H^c\}\),
to be supplemented with a mass term of order EW scale. Their gauge and global quantum expectation values (VEVs), breaking $U(1)$ whose gauge and global quantum numbers are displayed in Table I. They can develop SUSY breaking.

TABLE II: Matter fields

| Superfields | $q_3$ | $u_3^c$ | $d_3^c$ | $l_{2,3}$ | $\nu_{2,3}^c$ | $e_{2,3}$ | $h_{u,d}$ | $D$ | $D^c$ | $L$ | $L^c$ | $P$ | $Q$ |
|-------------|------|--------|--------|----------|-------------|---------|--------|-----|------|-----|-----|-----|-----|
| $U(1)_{PQ}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{3}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | -1 | $\frac{3}{2}$ | $\frac{1}{2}$ | 1 | 1 | 1 | -1 |

whose gauge and global quantum numbers are displayed in Table I. They can develop vacuum expectation values (VEVs), breaking $U(1)_{B_1+B_2-2L_1}$, as will be explained below. For successful mixing in the lepton and quark sectors, we need the MSSM singlets and color triplets, \{$N, N^c; D, D^c$\}, where $D^c$ is assumed to have the same SM gauge quantum numbers with $d^c_i$. For the gauge coupling unification, vector-like lepton doublets, \{$L, L^c$\} need to be supplemented with a mass term of order EW scale. Their gauge and global quantum numbers are presented in Tables I and II. Then, the following superpotential is also allowed:

\[
W_{\text{mix}} = (y_u^i N_H \nu_i^c N + \mu_N N N^c + y_l \nu_i^c h_u l_1) + (y_d^i N_H d_i^c D + \mu_D D D^c + y_q D^c h_d q_3) + \mu_{N_H} N_H N_H^c + \mu_{L_L} L_L L^c \tag{15}
\]

where $i=2,3$ and $j=1,2$. The sizable Yukawa couplings $y_u^i$ and $y_d^i$ can drive the soft mass squareds of $\tilde{N}_H$ and $\tilde{N}_H^c$ negative at the EW scale. Then the TeV scale VEVs, $\langle \tilde{N}_H \rangle$ and $\langle \tilde{N}_H^c \rangle$ can be generated, breaking $U(1)_{B_1+B_2-2L_1}$ completely. After decoupling the \{N, N$^c$\} and \{D, D$^c$\} below the $\mu_N$ or $\mu_D$ scale, one can get the mixing terms of the Dirac neutrinos and d-type quarks, whose effective Yukawa couplings are estimated as $y_u^i y_H^i \langle \tilde{N}_H \rangle / \mu_N$, and $y_d^i y_q \langle \tilde{N}_H^c \rangle / \mu_D$, respectively. They fill the $(i,1)$ components of the mass matrices of Dirac neutrinos (± $M_{D_L}$), and the $(j,3)$ of the d-type quarks’ mass matrix (± $M_d$), respectively. The mass matrix for observed light neutrinos, $[M_{\nu}]^{pq} = ([M_{\nu}])^{pq} = -[M_{D_L} M^{-1} M_{D_R}]^{pq}$, where $p, q = 1, 2, 3$ can be obtained after integrating out \{N, N$^c$\} and $\nu_i^c$. The matrices diagonalizing $M_{\nu}$ and $M_{d_L} M_{d_R}, U_{L}^{(d)}$ are general enough to accommodate the PMNS and CKM matrices, respectively. Note that in contrast to the neutrinos and d-type quarks, the mass matrices of charged leptons and u-type quarks still remain block-diagonal.

The $U(1)_{PQ}$ breaking terms of $\mu_{L_L}, M_{Q}$ in Eq. (15) as well as $\mu$ in Eq. (14) can be replaced by the nonrenormalizable terms with $P^2/M_P or Q^2/M_P$, which respect the $U(1)_{PQ}$ symmetry (for a recent discussion, see Ref. 25). Here $M_P$ denotes the reduced Planck mass ($\approx 2.4 \times 10^{18}$ GeV). The superfields $P$ and $Q$ get VEVs of order $10^{10}$ GeV, breaking the $U(1)_{PQ}$ into the discrete $Z_2$ symmetry, e.g. through the superpotential $W \supset k \Sigma (PQ - M_l^2)$, where $\Sigma$ is a superfield, $k$ a dimensionless coupling, and $M_l$ indicates a mass parameter of order $10^{10}$ GeV. Thus, $\langle P^2 \rangle/M_P$ and $\langle Q^2 \rangle/M_P$ can be of order $10^{2-3}$ GeV. The

\footnote{The presence of superheavy lepton doublets \{L_G, L_G^c\} carrying $\mp 2 \left( \pm \frac{3}{2} \right)$ charges of U(1)$_{B_1+B_2-2L_1}$
remaining $Z_2$ symmetry is identified with the matter parity in the MSSM, which forbids the $R$-parity violating terms. The Majorana masses $M^j$ in Eq. (14) also can be replaced by $P$.

The first and the other generations of sleptons are not mixed even in the mass eigen basis, because their masses are quite hierarchical $[(40 \text{ TeV})^2 - (O(0.1) \text{ TeV})^2]$ and the $U(1)_{B_1+B_2-2L_1}$ breaking scale is low enough in this model. Moreover, the mass matrix of the charged leptons also remains block-diagonal, as mentioned above. Accordingly, $\mu^- \rightarrow e^- \gamma$ cannot arise through mediation by superparticles. The relatively light (\tilde{\nu}_\mu, \tilde{\nu}_L) and $\tilde{\nu}_{\mu R}$ can be responsible for the deviation of $(g-2)_\mu$, as discussed above. Large mixing among the left-handed neutrinos doesn’t affect it.

Since the charge assignment of $U(1)_{B_1+B_2-2L_1}$ is not universal, the $U(1)_{B_1+B_2-2L_1}$ gauge boson could give rise to lepton flavor violations. As seen in Eq. (11), however, the gauge coupling $g_Z$ is quite small at low energy, because $U(1)_{B_1+B_2-2L_1}$ survives down to low energy. Other FCNC effects by such a gauge boson can also be adequately suppressed, if the $U(1)_{B_1+B_2-2L_1}$ breaking scale is above a few TeV [8].

In the squark mass matrix, the diagonal components, (1,1) and (2,2) are degenerate with a squared mass of $(7 \text{ TeV})^2$, while the off-diagonal components, (j, 3) and (3, j), where $j = 1, 2$, remain zero due to $U(1)_{B_1+B_2-2L_1}$. The (1,2), (2,1), and (3,3) can be filled dominantly by the gravity mediation effect, which are quite suppressed compared to the (1,1) and (2,2) components. After diagonalization in the fermionic quarks sector, (1,1), (2,2), and (j,3), (3,j) can be induced also by the mixing effect. The (1,2) and (2,1) components affect e.g. $K$-$\bar{K}$ mixing. The amplitude of $K$-$\bar{K}$ mixing by the squark mixing is roughly estimated as [28]

\[
\mathcal{M}_{K\bar{K}} \approx \frac{4\alpha_2^2}{m_q^2} \left( \frac{\Delta \tilde{m}_q^2}{\tilde{m}_q^2} \right)^2,
\]

(16)

where $\tilde{m}_q^2 \approx (7 \text{ TeV})^2$, and $\Delta \tilde{m}_q^2$ denotes the off-diagonal component of the squark mass matrix. Since the SM still explains well the observed data, Eq. (16) should be smaller than the SM prediction, $\mathcal{M}_{K\bar{K}}^{\text{SM}} \approx \alpha_2^2 \sin^2 \theta_c \cos^2 \theta_c (m_c^2/M_W^4)$, where $\theta_c$ stands for the Cabibbo mixing angle. The condition $\mathcal{M}_{K\bar{K}} \ll \mathcal{M}_{K\bar{K}}^{\text{SM}}$ yields

\[
\left( \frac{\Delta \tilde{m}_q^2}{\tilde{m}_q^2} \right) \ll 5.6 \times 10^{-2} \times \left( \frac{\tilde{m}_q}{7 \text{ TeV}} \right).
\]

(17)

Note that when $\tilde{m}_q = 500 \text{ GeV}$, this estimation on $\Delta \tilde{m}_q^2/\tilde{m}_q^2 \approx \delta_{12}^4$ provides slightly stronger constraint ($\ll 4 \times 10^{-3}$) than those of Ref. [29]. In order to suppress the squark mixing effect, hence, the off-diagonal element (1,2) of the squark mass matrix should be much smaller than 5.6 percent of the diagonal one [i.e. $\ll (1.6 \text{ TeV})^2$] when $\tilde{m}_q = 7 \text{ TeV}$. If

[U(1)$_{PQ}$] could open the possibility that an extremely small Dirac neutrino mass ($= y_1 y_2 \langle P \rangle / M_G$) is naturally generated e.g. from the superpotential, $W \supset y_1 P L_G + M_G L_G L_G + y_2 L h_u \nu^c_1$, where $y_{1,2} \sim 10^{-3}$ and $M_G \sim 10^{16} \text{ GeV}$.
the mixings among the d-type quarks are given fully by the CKM (or a similar order mixing matrix) and the elements induced by the gravity mediation is smaller than \((1 \text{ TeV})^2\), this constraint can be satisfied.

Finally, let us discuss the Higgs sector. The simplest way to raise the Higgs mass in the framework of the MSSM is to consider the large A-term \([3, 30]\), assuming \((A_t - \mu \cot \beta)^2/\tilde{m}_t \approx 6\), where \(A_t\) indicates the A-term coefficient corresponding to the top quark Yukawa coupling. However, the relation \((A_t - \mu \cot \beta)^2/\tilde{m}_t^2 \approx 6\) would be a fine-tuning relation. Even in this case, we need one pair of \(\{5, 5\}\) (i.e. \(v = 1\)) to induce the full mixing among the three generations of the SM chiral fermions as discussed above.

Actually, the Higgs mass could easily be raised at tree level by promoting the MSSM \(\mu\) term to the renormalizable superpotential \(\lambda S h_u h_d\), 'a la the next to MSSM, introducing a new singlet \(S\) together with a new dimensionless coupling \(\lambda\). In fact, this approach was taken in the original suggestion of the \(U(1)'\) mediation \([19]\). For maintaining the perturbativity of the model all the way up to the GUT scale, however, \(\lambda\) and \(\tan \beta\) should be small enough: \(0.6 \lesssim \lambda \lesssim 0.7\) and \(1 \lesssim \tan \beta \lesssim 3\) \([31]\). As seen in the above, however, a large \(\tan \beta\) is needed to explain the central value of the observed \((g - 2)\mu\).

In this Letter, we will consider another method to raise the Higgs mass. As pointed out in Ref. \([7]\), introduction of new vector-like leptons \(\{\hat{L}, \hat{L}^c; \hat{N}, \hat{N}^c\}\) together with a new gauge symmetry is very helpful for raising the radiative Higgs mass, where the new vector-like leptons are charged under a new gauge symmetry: all the superfields (except \(\{L, L^c\}\)) in Tables I and II are regarded as being neutral.\(^6\) Due to the new gauge symmetry, we can take a relatively large Yukawa couplings between the new leptons and the Higgs field, avoiding a blowup of the Yukawa coupling below the GUT scale. Of course one can identify the new gauge symmetry with \(U(1)_{B_1 + B_2 - 2L_1}\). In this case, however, the charges of the new vector-like leptons should be small enough, say \(\pm \frac{1}{40}\) (rather than \(\pm 2\)) so that their soft masses result in about 500 GeV. Alternatively, one can introduce a new gauge symmetry such as \(SU(2)\), which is not related to a mediation mechanism. Then, \(\{\hat{L}, \hat{L}^c; \hat{N}, \hat{N}^c\}\) are regarded as the \(SU(2)\) doublets. Here, we comment on the latter case.

The superpotential of the Higgs sector with such fields is given by

\[
W_{\text{Higgs}} = y_h \hat{L} h_u \hat{N}^c + y'_h \hat{L}^c h_d \hat{N} + \mu_L \hat{L} \hat{L}^c + \mu_N \hat{N} \hat{N}^c + \mu_H \hat{N}_H \hat{N}_H^c,
\]

where \(y_h^{(l)}\) and \(\mu_{L,N,H}\) (\(\mu_L \gtrsim \mu_N\)) are dimensionless and dimensionful parameters. \(\{\hat{N}_H, \hat{N}_H^c\}\) are also \(SU(2)\) doublets, but neutral under the SM gauge symmetry. They are the spontaneous breaking sector of \(SU(2)\): their negative soft mass squareds are assumed, breaking \(SU(2)\) above TeV scale. For the gauge coupling unification, two pairs of \(\{D', D''\}\) need to be accompanied with their relatively heavy mass terms (\(\gtrsim\) a few TeV), even if we do not

\(^5\) The error is still 3.3 \(\sigma - 3.6\) \(\sigma\), and it is premature to exclude all the NMSSM models, not accommodating the BNL \((g - 2)_\mu\).

\(^6\) In principle, \(\{L, L^c\}\) can be identified with \(\{\hat{L}, \hat{L}^c\}\).
They are neutral under SU(2). Thus, we have three pairs of \{5,\overline{5}\} in total (i.e. \(v = 3\)), including one pair of \{5,\overline{5}\} introduced in Table II for the mixing of the SM chiral fermions.

Since \{\hat{L},\hat{N}^c\} couple to the Higgs \(h_u\), they make contributions to the radiative Higgs mass (\(\equiv \Delta m_h^2\)) as well as the renormalization of the soft mass squared of \(h_u\) (\(\equiv \Delta m_{\tilde{Q}}^2\)) [7]:

\[
\Delta m_h^2|_{\text{new}} \approx n_c \frac{|y_h|^4}{4\pi^2} v_h^2 \sin^4 \beta \log \left( \frac{M^2 + \tilde{m}_l^2}{M^2} \right),
\]

\[
\Delta m_{\tilde{Q}}^2|_{\text{new}} \approx n_c \frac{|y_h|^2}{8\pi^2} \left[ f_Q(M^2 + \tilde{m}_l^2) - f_Q(M^2) \right]_{Q=M_G},
\]

where \(n_c = 2\) for the SU(2) doublets, \(y_h (\approx 174\ \text{GeV})\) indicates the Higgs VEV, and \(f_Q(m^2)\) is defined as \(f_Q(m^2) \equiv m^2\{\log(\frac{m^2}{M^2}) - 1\}\). \(\tilde{m}_l^2\) denotes the soft mass squared of the extra vector-like leptons, and \(M^2\) is the mass squared of a fermionic component, \(|\hat{L}|^2 + |\hat{N}|^2\) \(\approx \tilde{\mu}_l^2 + |y_h|^2 v_h^2 \sin^2 \beta\). Note that \(\Delta m_h^2|_{\text{new}}\) is proportional to \(|y_h|^4 \times v_h^2 \sin^4 \beta\), while \(\Delta m_{\tilde{Q}}^2|_{\text{new}}\) is to \(|y_h|^2 \times (M^2 + \tilde{m}_l^2, M^2)\). Similarly, one could consider the radiative Higgs mass by \{\hat{L}^c, \hat{N}\} via the \(y_h^c\) coupling in Eq. [18]. However, its contribution would be proportional to \(\cos^4 \beta\), and so it is relatively suppressed for \(\tan \beta \gtrsim 1\). \(\Delta m_{\tilde{Q}}^2|_{\text{new}}\) is eventually associated with the fine-tuning issue, because it affects determination of the Z boson mass. The *quartic* power of \(y_h\) in \(\Delta m_h^2|_{\text{new}}\) makes the radiative Higgs mass very efficiently raised. In order to raise the radiative Higgs mass, but holding the fine-tuning, hence, a larger \(y_h\) and smaller masses \{\(\tilde{m}_l^2, |\hat{\mu}_l|^2\}\} need to be taken. Since the experimental bound is not severe yet, relatively light vector-like leptons are still conceivable.

As shown in Ref. [7], in the case of three pairs of \{5,\overline{5}\}, the *maximally allowed* \(y_h\) at the EW scale can reach 1.78 for \(\tan \beta \gtrsim 2\), which makes 126 GeV Higgs mass easily explained, avoiding the Landau-pole problem. For the masses of the new vector-like leptons heavier than 470 GeV (440 GeV) with \(\tan \beta = 10\) (50), the oblique parameters (\(\Delta S, \Delta T\)) induced by the new vector-like leptons become inside the 1σ band. Only if \(\hat{\mu}_L \gtrsim \hat{\mu}_N\), the charged components of \{\(\hat{L}, \hat{L}^c\)\} produced in the collider can immediately decay into \{\(\hat{N}, \hat{N}^c\)\}, the neutralino, and SM fermions.

**IV. CONCLUSION**

We have constructed the U(1)\(_{B_1+B_2-2L_1}\) mediated SUSY breaking model, in which the first two generations of squarks (\(\approx 7\ \text{TeV}\)) and the first generation of sleptons (\(\approx 40\ \text{TeV}\)) can be made quite heavier than the other SUSY particles. Hence, non-observation of SUSY particles at the LHC and FCNC associated with the electron such as \(\mu^- \rightarrow e^- \gamma\) can easily be understood in this framework. The discrepancy of \((g - 2)_\mu\) can be explained with the relatively light smuon/sneutrino and chargino/neutralino (\(\approx 400\ \text{GeV}\)). The fine-tuning in the Higgs sector associated with the stop can be relieved, since the stop mass is relatively light (\(\gtrsim 500\ \text{GeV}\)) in this model. Two-loop effects by the heavy sfermions can protect the
small stop and smuon/sneutrino masses against the quantum correction by the heavy gluino ($\approx 1.2$ TeV). By introducing extra vector-like matter, the radiative corrections to the Higgs mass can be enhanced up to 126 GeV, and the desired mixings among the SM chiral fermions can be generated after $U(1)_{B_1+B_2-2L_1}$ breaking.

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