Compact Bilinear Pooling

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Abstract

Bilinear models has been shown to achieve impressive performance on a wide range of visual tasks, such as semantic segmentation, fine grained recognition and face recognition. However, bilinear features are high dimensional, typically on the order of hundreds to a few million, which makes them impractical for subsequent analysis. We propose two compact bilinear representations with the same discriminative power as the full bilinear representation but with only a few thousand dimensions. Our compact representations allow back-propagation of classification errors enabling an end-to-end optimization of the visual recognition system. The compact bilinear representations are derived through a novel kernelized analysis of bilinear pooling which provide insights into the discriminative power of bilinear pooling, and a platform for further research in compact pooling methods. Extensive experimentation illustrate the applicability of the proposed compact representations, for image classification and few-shot learning across several visual recognition tasks.

1. Introduction

Encoding and pooling of visual features is an integral part of semantic image analysis methods. Before the influential 2012 paper of Krizhevsky et al.\cite{16} rediscovering the models pioneered by\cite{18} and related efforts, such methods typically involved a series of independent steps: feature extraction, encoding, pooling & classification; each thoroughly investigated in numerous publications as the bag of visual words (BoVW) framework. Notable contributions include the HOG\cite{8}, and SIFT\cite{23} descriptors, improved fisher encoding\cite{25}, bilinear pooling\cite{3}, & spatial pyramid matching\cite{17}, each significantly improving the baseline recognition accuracy. Recent results have showed that end-to-end back-propagation of gradients in a convolutional neural network (CNN) enables joint optimization of the whole pipeline, resulting in significantly higher recognition accuracy. While the distinction of the steps is less clear in a CNN than in a BoVW pipeline, one can view the first several convolutional layers as a feature extractor and the later fully connected layers as a pooling and encoding mechanism. This has been explored recently in methods combining the feature extraction architecture of the CNN paradigm, with a pooling, encoding & classification steps from the BoVW paradigm\cite{22,7}.

In particular, the recent work of Lin et al., who used bilinear pooling to encode the activations of a deep convolutional network for fine-grained visual recognition\cite{22}. The proposed method is remarkably powerful but unwieldy due to the high dimensionality of the bilinear pooling; in their paper the encoded feature dimension, $d$, is more than 250,000. Such representation is impractical for several reasons: (1) if used with a standard one-vs-rest linear classifier for $k$ classes, the number of model parameters becomes $kd$, which for e.g. $k = 1000$ means $> 250$ million model parameters, (2) for retrieval or deployment scenarios which require features to be stored in a database, the storage becomes expensive; storing a millions samples requires 2TB of storage at double precision, (3) further processing such as spatial pyramid matching\cite{17}, or domain adapta-
tion\cite{10} often requires feature concatenation; again, strain-
ing memory and storage capacities, and (4) classifier reg-
ularization, in particular under few-shot learning scenarios
becomes challenging\cite{11}.

The main contribution of this work is a pair of bilinear
pooling methods, each able to reduce the feature dimension-
ality three orders of magnitude with little-to-no loss in per-
formance compared to a full bilinear pooling. The proposed
methods are motivated by a novel kernelized viewpoint of
bilinear pooling, and, critically, allow back-propagation for
end-to-end learning.

Our proposed compact bilinear methods rely on the exis-
tence of low dimensional feature maps for kernel functions.
Rahimi\cite{28} first proposed a method to find explicit fea-
ture maps for Gaussian and Laplacian kernels. This was
later extended for kernels commonly used in vision, such as
intersection kernel, $\chi^2$ kernel and the exponential $\chi^2$ ker-
nel\cite{34,24,35}. Bilinear features are closely related to
polymer kernels, as shown in Sec.\ref{sec:compbilinear} and we propose
new methods for compact bilinear features based on algo-
rithms for the polynomial kernel first proposed by Kar\cite{14}
and Pham\cite{26}; a key aspect of our contribution is that we
show how to back-propagate through such representations.

Our experiments indicate that compact bilinear pooling
is practical not only for fine-grained visual recognition, but
also for a wider range of tasks including few-shot learning.

\section{2. Related work}

Bilinear models were first introduced in Tenenbaum and
Freeman\cite{31} to separate style and content. Second order
pooling have since been considered for semantic segmen-
tation and fine grained recognition respectively, both using
hand-tuned features\cite{3}, and CNN features\cite{22}. Although
repeatedly shown to produce state-of-the-art results, it has
not been widely adopted in the literature; we believe this is
partly due to the prohibitively large dimensionality of the
extracted features.

Several other clustering methods have been considered
for visual recognition. Leung and Malik used vector quanti-
zation in the Bag of Visual Words (BoVW) framework\cite{19},
first used for texture classification, but later found to be use-
ful across many visual tasks. VLAD\cite{13} and Improved
Fisher Vector\cite{25} encoding improved over hard vector
quantization by including second order information in the
descriptors. Fisher vector has been recently used to
achieved start-of-art performances on many data-sets\cite{7}.

Reducing the number of parameters in CNN is impor-
tant for training large networks and for deployment (e.g.
on embedded systems). Deep Fried Convnets\cite{59} aims to re-
duce the number of parameters in the fully connected layer,
which usually accounts for 90\% of parameters. Several
other papers pursue similar goals, such as the Fast Circulant
Projection which uses a circular structure to reduce mem-

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|c|}
\hline
 & Compact & Highly & Flexible & End-to-end \\
& & discriminative & input size & learnable \\
\hline
Fully connected & ✓ & ✗ & ✗ & ✓ \\
Fisher Encoding & ✗ & ✓ & ✓ & ✗ \\
Bilinear pooling & ✗ & ✓ & ✓ & ✓ \\
Compact bilinear & ✓ & ✓ & ✓ & ✓ \\
\hline
\end{tabular}
\caption{High-level properties of pooling methods. Fully connected pooling, the ‘default’ method proposed by \cite{16}, is compact and can be learned end-to-end by back propagation, but it requires a fixed input image size and is less effective than other methods \cite{7,22}. Fisher encoding is more discriminative but high dimensional and can not be learned end-to-end \cite{7}. Bilinear pooling is discriminative and tune-
able but very high dimensional (> 250k) \cite{22}. Our compact bilinear methods are as effective as bilinear pooling, but compact (10k).

\section{3. Compact bilinear models}

Bilinear pooling\cite{22} or second order pooling\cite{3} forms
a global image descriptor by calculating:
\begin{equation}
B(\mathcal{X}) = \sum_{s \in \mathcal{S}} x_s x_s^T
\end{equation}

where $\mathcal{X} = (x_1, \ldots, x_{|\mathcal{S}|}, x_s \in \mathbb{R}^c)$ is a set of local
descriptors, and $\mathcal{S}$ is the set of spatial locations (combinations
of rows & columns). Local descriptors, $x_s$ are typically
extracted using SIFT\cite{23}, HOG\cite{8} or by a forward pass
through a CNN\cite{16}. As defined in \ref{eq:1}, $B(\mathcal{X})$ is a $c \times c$
matrix, but for the purpose of our analysis, we will view it
as a length $c^2$ vector.

\subsection{3.1. A kernelized view of bilinear pooling}

Image classification using bilinear descriptors is typi-
cally achieved using linear Support Vector Machines
(SVM) or logistic regression. These can both be viewed
as linear kernel machines, and we provide an analysis be-
low.\footnote{We ignore the normalization (signed square root and $\ell_2$ normalization) which is typically applied before classification}

Given two sets of local descriptors: $\mathcal{X}$ and $\mathcal{Y}$, a linear
kernel machine compares these as:

\[ \langle B(\mathcal{X}), B(\mathcal{Y}) \rangle = \left( \sum_{x \in S} \sum_{u \in \mathcal{U}} x_s^T y_u y_u^T \right) \]

\[ = \sum_{x \in S} \sum_{u \in \mathcal{U}} \langle x_s^T y_u \rangle \]

\[ = \sum_{x \in S} \sum_{u \in \mathcal{U}} (x_s, y_u)^2 \]  

(2)

From the last line in (2), it is clear that the bilinear descriptor contains that in second image and that the comparison operator is a second order polynomial kernel. Bilinear pooling thus gives a linear classifier the discriminative power of a second order kernel-machine, which may help explain the strong empirical performance observed in previous work \[22\,3\,7\,29\].

3.2. Compact bilinear pooling

In this section we define the proposed compact bilinear pooling methods summarized in Alg. \[3\] Let \( k(x, y) \) to denote the comparison kernel, i.e. the second order polynomial kernel. If we could find some low dimensional projection function \( \phi(x) \in \mathbb{R}^d \), where \( d \ll c^2 \), that satisfy \( \langle \phi(x), \phi(y) \rangle \approx k(x, y) \), then we could approximate the inner product of (2) by:

\[ \langle B(\mathcal{X}), B(\mathcal{Y}) \rangle = \sum_{s \in S} \sum_{u \in \mathcal{U}} (x_s, y_u)^2 \]

\[ \approx \sum_{s \in S} \sum_{u \in \mathcal{U}} \langle \phi(x), \phi(y) \rangle \]

\[ = \left( \sum_{s \in S} \phi(x_s) \right)^T \left( \sum_{u \in \mathcal{U}} \phi(y_u) \right) \]

\[ \equiv \langle C(\mathcal{X}), C(\mathcal{Y}) \rangle \]  

(3)

where

\[ C(\mathcal{X}) := \sum_{s \in S} \phi(x_s) \]  

(4)

is the compact bilinear feature. It is clear from this analysis that any low-dimensional approximation of the polynomial kernel can be used to towards our goal of creating a compact bilinear pooling method. We investigate two such approximations: Random Maclaurin (RM) \[14\] and Tensor Sketch (TS) \[26\], detailed in Alg. \[4\] and Alg. \[2\] respectively.

RM is an early approach developed to serve as a low dimensional explicit feature map to approximate the polynomial kernel \[14\]. The intuition is straight forward. If \( w_1, w_2 \in \mathbb{R}^c \) are two random \(-1,1\) vectors and \( \phi(x) = \langle w_1, x \rangle \langle w_2, x \rangle \), then for non-random \( x, y \in \mathbb{R}^c \), \( E[\phi(x)\phi(y)] = E[\langle w_1, x \rangle \langle w_1, y \rangle]^2 = (x, y)^2 \). Thus each projected entry in RM has an expectation of the quantity to be approximated. By using \( d \) entries in the output, the estimator variance could be brought down by a factor of \( \frac{1}{d} \).

TS uses sketching functions to improve the computational complexity during projection and tend to provide better approximations in practice \[26\].

3.2.1 Back propagation of compact bilinear pooling

Back propagation of gradients for end-to-end learning is a critical component of current state of the art visual recognition systems. This is true regardless of whether the pooling is fully connected layers or bilinear layer \[22\]. In this section, we derive back-propagation for the two compact bilinear pooling methods and show they're efficient both in computation and storage.

For RM, let \( L \) denote the loss function, \( s \) the spatial index, \( d \) the projected dimension, \( n \) the index of the training sample and \( y^n_u \in \mathbb{R} \) the output of the RM layer at dimension \( d \) for instance \( n \). Back propagation of RM pooling can then be written as:

\[ \frac{\partial L}{\partial x^n_s} = \sum_d \frac{\partial L}{\partial y^n_d} \sum_k (W_k(d), x^n_s) W_k(d) \]

\[ \frac{\partial L}{\partial W_k(d)} = \sum_n \frac{\partial L}{\partial y^n_d} \sum_s (W_k(d), x^n_s)x^n_s \]  

(5)

where \( k = 1, 2, \hat{k} = 2, 1 \), and \( W_k(d) \) is row \( d \) of matrix \( W_k \). For TS, using the same notation,

\[ \frac{\partial L}{\partial x^n_s} = \sum_d \frac{\partial L}{\partial y^n_d} \sum_k T^k_d (x^n_s) \circ s_k \]

\[ \frac{\partial L}{\partial s_k} = \sum_{n,d} \frac{\partial L}{\partial y^n_d} \sum_s T^k_d (x^n_s) \circ x^n_s \]  

(6)

where \( T^k_d (x) \in \mathbb{R}^c \) and \( T^k_d (x) = \Psi(x, h_k, s_k) \). When \( d-h_k(c) \) is negative, it denotes the circular index \( (d-h_k(c)) + D \). \( D \) is the number of total projected dimension. Note that in TS, we could only get a gradient for \( s_k \). \( h_k \) is combinatorial, and thus we keep it fixed during back-prop.

The back-prop equation for RM can be conveniently written as a few matrix multiplications. It has the same computational and storage complexity as its forward pass, and can be calculated efficiently \[^2\]. Similarly, Equation \[6\] can also be expressed as a few FFT, IFFT and matrix multiplication operations. The computational and storage complexity of TS are also similar to its forward pass.

3.2.2 Some properties of compact bilinear pooling

Table \[1\] shows the comparison among bilinear and compact bilinear feature using RM and TS projections. Numbers indicated in brackets are the typical values when applying VGG-VD \[30\] with the selected pooling method on a 1000-class classification task. The output dimension of our

[^2]: We will release our code open source upon acceptance.
Table 1: Dimension, memory and computation comparison among bilinear and the proposed compact bilinear features. Parameters $c, d, h, w, k$ represent the number of channels before the pooling layer, the projected dimension of compact bilinear layer, the height and width of the previous layer and the number of classes respectively. Numbers in brackets indicate typical value when bilinear pooling is applied after the last convolutional layer of VGG-VD [30] model on a 1000-class classification task, i.e. $c = 512, d = 10,000, h = w = 13, k = 1000$. All data are stored in single precision.

**Algorithm 1 Random Maclaurin Projection**

Input: $x \in \mathbb{R}^c$

Output: feature map $\phi_{RM}(x) \in \mathbb{R}^d$, such that $\langle \phi_{RM}(x), \phi_{RM}(y) \rangle \approx \langle x, y \rangle^2$

1. Generate random but fixed $W_1, W_2 \in \mathbb{R}^{d \times c}$, where each entry is either $+1$ or $-1$ with equal probability.
2. Let $\phi_{RM}(x) \equiv \frac{1}{\sqrt{d}} (W_1 x) \circ (W_2 x)$, where $\circ$ denotes element-wise multiplication.

**Algorithm 2 Tensor Sketch Projection**

Input: $x \in \mathbb{R}^c$

Output: feature map $\phi_{TS}(x) \in \mathbb{R}^d$, such that $\langle \phi_{TS}(x), \phi_{TS}(y) \rangle \approx \langle x, y \rangle^2$

1. Generate random but fixed $h_k \in \mathbb{N}^c$ and $s_k \in \{+1, -1\}^c$ where $h_k(i)$ is uniformly drawn from $\{1, 2, \ldots, d\}$, $s_k(i)$ is uniformly drawn from $\{+1, -1\}$, and $k = 1, 2$.
2. Next, define sketch function $\Psi(x, h, s) = \{(Qx)_1, \ldots, (Qx)_d\}$, where $(Qx)_j = \sum_{t:h(t)=j} s(t)x_t$
3. Finally, define $\phi_{TS}(x) \equiv \text{FFT}^{-1}(\text{FFT}(\Psi(x, h_1, s_1)) \circ \text{FFT}(\Psi(x, h_2, s_2)))$, where the $\circ$ denotes element-wise multiplication.

**Algorithm 3 Compact Bilinear Pooling**

Input: $X = (x_1, \ldots, x_{|S|})$ a set of image local descriptors.

Output: the compact bilinear feature $C(X)$

1. $C(X) = \sum_{s \in S} \phi(x_s)$, where $\phi(\cdot)$ could either be $\phi_{RM}(\cdot)$, the Random Maclaurin Projection defined in Alg. 1 or $\phi_{TS}(\cdot)$, the Tensor Sketch Projection defined in Alg. 2.

Compact bilinear feature is 2 orders of magnitude smaller than the bilinear feature dimension. In practice, the proposed compact representations achieve similar performance to the fully bilinear representation using only 2% of the bilinear feature dimension, suggesting a remarkable 98% redundancy in the bilinear representation.

The RM projection requires moderate amounts of parameter memory (i.e. the random generated but fixed matrix), while TS require almost no parameter memory. If a linear classifier is used after the pooling layer, i.e. a fully connected layer followed by a softmax loss, the number of classifier parameters increases linearly with the pooling output dimension and the number of classes. In the case mentioned above, classification parameters for bilinear pooling would require 1000MB of storage. Our compact bilinear method, on the other hand, requires far fewer parameters in the classification layer, potentially reducing the risk of over-fitting, and performing better in few shot learning scenarios [11], or domain adaptation [10] scenarios.

Computationally, Tensor Sketch is advantageous as it is linear in $d \log d + c$, whereas bilinear is quadratic in $c$, and Random Maclaurin is linear in $cd$ (Table 1).

### 3.3. Alternative dimension reduction methods

PCA, which is a commonly used dimensionality reduction method, is not a viable alternative in this scenario due to the high dimensionality of the bilinear feature. Solving a PCA usually involves operations on the order of $O(d^3)$, where $d$ is the feature dimension. This is impractical for the high dimensionality, $d = 262K$ used in bilinear pooling.

Lin et al. [22] circumvented these limitations by using PCA before forming the bilinear feature, reducing the bilinear feature dimension on CUB200 [38] from 262,000 to 33,000. While this is a substantial improvement, it still accounts for 12.6% of the original dimensionality. Moreover, the PCA reduction technique requires an expensive initial sweep over the whole dataset to get the principle components. In contrast, our proposed compact bilinear methods do not require any pre-training and can be as small as 4096 dimensions (Section 4.2).

Another alternative is to use a random projection method. Such method would require forming the whole bilinear feature and projecting it to lower dimensional using some random linear operator. Due to the Johnson-Lindenstrauss lemma [9], the random projection largely
preserves pairwise distances between the feature vectors. However, deploying this method requires constructing and storing both the bilinear feature and the fixed random projection matrix. For example, for VGG-VD, the projection matrix will have a shape of \(c^2 \times d\), where \(c\) and \(d\) are the number of channels in the previous layer and the projected dimension, as above. With \(d = 10,000\) and \(c = 512\), the projection matrix has 2.6 billion entries, making it impractical to store and work with. A classical dense random Gaussian matrix, with entries being i.i.d. \(N(0,1)\), would occupy 10.5GB of memory, which is too much for a high-end GPU such as K40. A sparse random projection matrix would improve the memory consumption to around 40MB[20], but would still require forming bilinear feature first. Furthermore, it requires sparse matrix operations on GPU, which are inevitably slower than dense matrix operations, such as the one used in RM (Alg. 1).

4. Experiments

In this section we detail three sets of experiments. First, in Sec. 4.1, we investigate some design-choices of the proposed pooling methods, notably the dimensionality, \(d\) but also whether to tune the projection parameters, \(W\). Second, in Sec. 4.3 we look at how bilinear pooling in general, and the proposed compact methods in particular, perform in comparison to state-of-the-art on three common computer vision benchmark data-sets. Third, in Sec. 4.4 we investigate a situation where a low-dimensional representation is mandated (\(d\)) and a set of random generated projection parameters. For notational convenience, we use \(W\) to refer to the projection parameters, although they’re generated and used differently (Algs. 1, 2).

Full Bilinear Pooling: Both VGG-M and VGG-D have 14 layers of VGG-M (\(\text{conv}_5 + \text{ReLU}\)) and the first 30 layers in VGG-D (\(\text{conv}_{5,3} + \text{ReLU}\)), as used in [22]. In addition to bilinear pooling, we also compare to fully connected layer and improved fisher vector encoding [25]. The latter one is known to outperform other clustering based coding methods[7], such as hard or soft vector quantization [19] and VLAD [13]. All experiments are performed using MatConvNet [33], and we use 448 × 448 input image size, except fully connected pooling as mentioned below.

4.1. Experimental details

We evaluate our design on two network structures: the M-net in [4] (VGG-M) and the D-net in [30] (VGG-D). We use the convolution layers of the each network as the local descriptor extractor. More precisely, in the notation of Sec. 3.2.1, \(x_s\) is the activation at each spatial location of the convolution layer output. Specifically, we retain the first 14 layers of VGG-M (\(\text{conv}_5 + \text{ReLU}\)) and the first 30 layers in VGG-D (\(\text{conv}_{5,3} + \text{ReLU}\)), as used in [22]. In addition to bilinear pooling, we also compare to fully connected layer and improved fisher vector encoding [25]. The latter one is known to outperform other clustering based coding methods[7], such as hard or soft vector quantization [19] and VLAD [13]. All experiments are performed using MatConvNet [33], and we use 448 × 448 input image size, except fully connected pooling as mentioned below.

4.1.1 Pooling Methods

Full Bilinear Pooling: Both VGG-M and VGG-D have 512 channels in the final convolutional layer, meaning that the bilinear feature dimension is \(512 \times 512 \approx 250K\). We use a symmetric underlying network structure, corresponding to the B-CNN[M,M] and B-CNN[D,D] configurations in [22]. We didn’t experiment with the asymmetric structure such as B-CNN[M, D] because it’s shown to have similar performance as the B-CNN[D,D] [22]. Before the final classification layer, we add an element-wise signed square root layer \((y = \text{sign}(x)\sqrt{|x|})\) and an instance-wise \(\ell_2\) normalization.

Improved Fisher Encoding: Similarly to bilinear pooling, fisher encoding [25] has recently been used as an encoding & pooling alternative to the fully connected layers [7]. Following [7], the activations of last convolutional layer (excluding ReLU) are used as input the encoding step, and the encoding uses 64 GMM components.

4.1.2 Learning Configuration

We first evaluate each method as a feature extractor. Using the forward-pass through the network, we train a linear classifier on the activations. We use \(\ell_2\) regularized logistic regression: \(\lambda||w||^2_2 + \sum_i l(y_i, w^T x_i)\) with \(\lambda = 0.001\) as we found that it slightly outperforms SVM.

Bilinear pooling allows for straight-forward end-to-end fine-tuning using back-propagation [22]. As shown in Sec. 3.2.1, so do the proposed compact pooling methods. For the fine-tuning experiments, we initialize the last layer using the weights of the trained logistic regression and attach a corresponding logistic loss. We then fine tune the
whole network until convergence using a constant small learning rate of $10^{-3}$, a weight decay of $5 \times 10^{-4}$, a batch size of 32 for VGG-M and 8 for VGG-D. In practice, convergence occurs in $< 100$ epochs. Note that for RM and TS, back-propagation can be used simply as a way to tune the deeper layers of the network (as it is used in full bilinear pooling), or to also tune the projection parameters, $W$. We investigate both options in Sec. 4.2. Fisher vector has an unsupervised dictionary learning phase, and it is unclear how to perform fine-tuning [7]. We therefore do not evaluate Fisher Vector under fine-tuning.

### 4.2. Configurations of compact pooling

Both RM and TS pooling have a user defined projection dimension $d$, and a set of projection parameters, $W$. To investigate the parameters of the proposed compact bilinear methods, we conducted extensive experiments on the CUB-200 [36] dataset. The CUB dataset contains 11,788 images of 200 bird species, with a fixed training and testing set split. We evaluate in the mode where part annotations are not provided at either training nor testing time, and use VGG-M for all experiments in this section.

Fig. 3 summarizes our results. As the projection dimension $d$ increases, the two compact bilinear methods reach the performance of the full bilinear pooling. When not fine-tuned, the error of TS with $d = 16K$ is 1.7% less than that of bilinear feature, while only using 6.1% of the original number of dimensions. When fine tuned, the performance gap disappears: TS with $d = 16K$ has an error rate of 22.66%, compared to 22.44% of bilinear pooling.

In lower dimension, RM outperforms TS, especially when tuning $W$. This may be because RM pooling has more parameters, which helps it retain discriminative (learning) power despite the low-dimensional output space (Table 1). Conversely, TS outperforms RM when $d > 2000$. This is consistent with the results of Pham & Pagh, who evaluated these projection methods on several smaller data-sets [26]. Note that these previous studies didn’t use pooling nor fine-tuning as part of their experimentation.

Fig. 3 also shows performances using extremely low dimensional representation, $d = 32, 128$ and 512. While the performance decreased significantly for the fixed representation, fine-tuning brought back much of the discriminative capability. For example, $d = 32$ achieved less than 50% error on the challenging 200-class fine grained classification task. Going up slightly, to 512 dimensions, it yields 25.54% error rate. This is only 3.1% drop in performance compared to the 250,000 dimensional bilinear feature. Such extremely compact but highly discriminative image feature representations are useful, for example, in image retrieval systems. For comparison, Wang et al. used a 4096 dimensional feature embedding in their recent retrieval system [37].

In conclusion, our experiments suggest that between 2000 and 8000 features dimension is appropriate. They also suggest that the projection parameters, $W$ should only be tuned when one using extremely low dimensional representations. Our experiments finally confirmed the importance of fine-tuning, emphasizing the critical importance of using projection methods which allow fine-tuning to be performed.

### 4.3. Evaluation across multiple data-sets

Bilinear pooling has been studied extensively. Carreira et al. used second order pooling to facilitate semantic segmentation [3]. Lin et al. used bilinear pooling for fine-grained visual classification [22], and Rowchowdhury used bilinear pooling for face verification [29]. These methods all achieved state-of-art on the respective tasks indicating the wide utility of bilinear pooling. In this section we show that the compact representations perform on par with bilinear pooling on three very different image classification tasks. Since the compact representation requires orders of magnitude less memory, this suggests that it is the preferable method for a wide array of visual recognition tasks.

Fully connected pooling, fisher vector encoding, bilinear pooling and the two compact bilinear pooling methods are compared on three visual recognition tasks: fine-grained visual categorization represented by CUB-200-2011 [36], scene recognition represented by the MIT indoor scene recognition dataset [27], and texture classification represented by the Describable Texture Dataset [6]. Sample figures are provided in Fig. 3 and dataset details are provided in Table 1. Guided by our results in Sec. 4.2 we use...
Table 2: Classification error of fully connected (FC), fisher vector, full bilinear (FB) and compact bilinear pooling methods, Random Maclaurin (RM) and Tensor Sketch (TS). For RM and TS we set the projection dimension, $d = 8192$ and we fix the projection parameters, $W$. The number before and after the slash represents the error without and with fine tuning respectively. Some fine tuning experiments diverged, when VGG-D is fine-tuned on MIT dataset. These are marked with an asterisk and we report the error rate at the 20th epoch.

$d = 8192$ dimensions and fix the projection parameters $W$.

| Data-set | # train img | # test img | # classes |
|----------|-------------|------------|-----------|
| CUB [36] | 5994        | 5794       | 200       |
| MIT [27] | 4017        | 1339       | 67        |
| DTD [6]  | 1880        | 3760       | 47        |

Table 3: Summary statistics of data-sets in Sec. 4.3

4.3.1 Bird species recognition

CUB is a fine-grained visual categorization dataset. Good performance on this dataset requires identification of overall bird shape, texture and colors, but also capacity to focus on subtle differences, such as the beak-shapes.

Our results indicate that bilinear and compact bilinear pooling outperforms fully connected and fisher vector by a large margin, both with and without fine-tuning (Table 2). Among the compact bilinear methods, TS consistently outperformed MS. For the larger VGG-D network, bilinear pooling achieved 19.90% error rate before fine tuning, while RM and TS achieved 21.83% and 20.50% respectively. This is a modest 1.93% and 0.6% performance loss considering the huge reduction in feature dimension (from 250k to 8192). Notably, this difference disappeared after fine-tuning when the bilinear pooling methods all reached an error rate of 16.0%. This is, to the best of our knowledge, the state of the art performance on this dataset without part annotation \([15, 22]\). The story is similar for the smaller VGG-M network: TS is more favorable than RM and the performance gap between compact full bilinear shrinks to 0.5% after fine tuning.

4.3.2 Indoor scene recognition

Scene recognition is quite different from fine-grained visual categorization, requiring localization and classification of several discriminative and non-salient objects. As shown in the second row of Fig. 4, the intra-class variation can be quite large.

As expected, and previously observed \([7]\), improved Fisher vector encoding outperformed fully connected pooling by 6.87% on the MIT scene data-set (Table 2). More surprising, bilinear pooling outperformed Fisher vector by 3.03%. Even though bilinear pooling was proposed for object-centric tasks, such as fine grained visual recognition, this experiment thus suggests that is it appropriate also for scene recognition. Compact TS performs slightly worse (0.94%) than full bilinear pooling, but 2.09% better than Fisher vector. This is notable, since the high dimensional fisher vector is the used in the current state-of-art on this dataset \([7]\).

Surprisingly, fine-tuning negatively impacts the error rates of the full and compact bilinear methods, about 2%.

Figure 4: Samples images from the three datasets examined in Sec. 4.3. Each row contains samples from indigo bunting in CUB, jewelery shop in MIT, and honey comb in DTD.
4.3.3 Texture classification

Texture classification is similar to scene recognition in that it requires attention to small features which can occur anywhere in the image plane. Our results confirm this, and we see similar trends as on the MIT data-set (Table 2).

We have modeled bilinear pooling in a kernelized framework and suggested two compact representations, both of which allow back-propagation of gradients for end-to-end optimization of the classification pipeline. We conclude by summarizing the properties of our proposed method.

Compact representation: Our experiments indicate that compact bilinear pooling can be used in place of full bilinear pooling: an 8K dimensional compact TS feature reaches the same performance as bilinear pooling indicating a remarkable 96.5% compression of the full bilinear 262K feature dimension. 8K is a significant reduction also compared to Fisher encoding. In our experiment, with VGG-M or VGG-D networks and 64 Gaussian mixture models, the Fisher feature dimension is around 65K; an order magnitude larger than our compact bilinear feature.

No encoder pre-training: Unlike Fisher encoding, which require learning a Gaussian mixture model on each dataset before encoding, compact bilinear pooling doesn’t require any encoding phase. The weights in the TS method are only a few kilo-bytes and are random generated. The weights can also be fine-tuned along with the rest of the network parameters.

Compact network size: The VGG-D network has 528 megabytes of parameters. Among all parameters, around 90% of them are in the three fully connected layers (fc6, fc7, fc8) at the end of the network. If we replace the 2 fully connected layers (fc6, fc7) with 8192-dimensional TS pooling layer, retaining the 1000-way classification, the final classification layer only requires 31 MB of storage for a total of 87 MB. In comparison, bilinear pooling requires 1000 MB in the classification layer and thus 1056 MB in total. Fisher vector requires 250 MB in the classification layer and 306 MB in total. This difference is critical e.g. for deployment on embedded systems.

Future work: We provide a kernelized perspective of bilinear pooling, showing that it is equivalent to using a pairwise polynomial kernel to compare local descriptors, followed by a summing operation to aggregate the scores. Noting this connection to kernel methods, it would be interesting to explore alternatives to the polynomial kernel for visual tasks. Moreover, since our representation can be extremely compact with retained discriminative power, it could be applied to image retrieval, where storage and indexing are central issues. Further, the compact representation makes it suitable in situations which require further processing: e.g. part-based models [2][12], conditional random fields, multi-scale analysis, spatial pyramid pooling or hidden Markov models.
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