Regge-Like Mass Relation of Singly Heavy Hadrons

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We propose a general Regge-like mass relation to describe the spectra of singly heavy hadrons with the slope ratio \( \pi : 2 \) between radial and orbital Regge trajectories. The hadron mass shift \( M \rightarrow M - M_Q + k_0 \mu_Q \) with the heavy quark mass \( M_Q \) and the reduced mass of heavy-light systems \( \mu_Q \) are taken into account. The relation is successfully tested against the observed spin-averaged masses of singly heavy mesons and baryons in their radially and orbitally excited states. Some new predictions are made for more excited states.

**KEYWORDS:** Heavy hadrons, Excited spectra, QCD String

1. Introduction

During last ten years, the spectroscopy of heavy hadrons has been brought to attention due to the discovery of increasing number of excited heavy mesons and baryons by the experiments at BaBar, Belle, CLEO, and recently by the LHCb experiment [1]. Recent examples of these discoveries are heavy mesons \( B_J(5970)^{+0} \) [2], \( D(3000)^0 \) [3, 4], and heavy baryons \( \Sigma_b(6097)^{\pm}, \Xi_b(6227)^- \), the five \( \Omega_c \)'s, and the long-awaited doubly charmed baryon \( \Xi_{cc}^{++} \), observed mainly by LHCb experiment. The spectra of heavy baryons becomes of interest also because it may provide an analogue to understand the exotic states like tetraquark with open heavy flavors [5, 6]. Many analysis and calculations have been given with different theoretical approaches (see [7] for a review) while some of their properties still remain to be explored.

In this work, we use a general Regge-like mass relation to study the mass spectra of singly heavy mesons \((D/D_s\text{ and }B/B_s)\) and baryons \((\Lambda_{c/b}/\Xi_{c/b}\text{ and }\Sigma_{c/b}/\Xi_{c/b})\) in which it incorporates the short-distance QCD correction and flavor-dependent intercepts explicitly. This is done by assuming the slope ratio between radial and orbital Regge trajectories of singly heavy hadrons to be \( \pi : 2 \), and hadron mass to undergo a shift \( M \rightarrow M - M_Q + k_0 \mu_Q \) with the heavy quark mass \( M_Q \) and the reduced mass of heavy-light systems \( \mu_Q \). The relation is tested successfully against the observed excited spectra of the singly heavy mesons and baryons with reasonable trajectory parameters. Some predictions are made for the newly discovered states, \( D(3000)^0, \Sigma_b(6097)^{\pm} \) and \( \Xi_b(6227)^- \), and for some states with unknown quantum numbers, \( \Lambda_c(2765)^+, \Sigma_c(2800), \Xi_c(3055)^+ \) and \( \Xi_c(2930) \).

The notion of Regge trajectory for heavy hadrons is similar to the widely known formula for the light hadrons \( M^2 = \sigma L + d \), and can be stated as a mass relation [8–10]

\[
(M - M_Q)^2 = \alpha L + c,
\]

where \( L \) denotes the orbital angular momentum of the hadron and the slope for the heavy hadrons \( \alpha \) is nearly half of the slope for the light hadrons \( \sigma \) [9–13]. Eq. (1) indicates that the trajectory in \((M^2, L)\) plane that contains weakly nonlinear behaviors at low-\( L \) region can recover linearity by shifting the
static energy of the heavy quark in hadron mass: $M \rightarrow M - M_Q$. For more discussions, see Refs. [14, 15] and [16–18] among the others.

2. **Regge-like mass relation for excited system**

To explore the excited hadrons of the heavy-light (HL) system, we extend the Regge-like relation (1) to the general case that applies for the radially and angularly excited states, by choosing the slope between the heavy quark and the light (anti-)quark (diquark), the coefficient $k$ describes effectively the scale-dependent mass running of the light (anti-)quarks (diquarks) that may be relevant in the higher excitations. The ratio $\pi : 2$ stems from the fact that the energy cost for orbital motion of a string linked to quarks differs itself from that for radial motion [19]. The main proposal is then

$$M \rightarrow M - M_Q + k_s \mu_Q = M - M_Q + \frac{k_s k m M_Q}{k m + M_Q}.$$  \hspace{1cm} (2)

Here, $k_s$ denotes the linear factor for the short-distance Coulomb-like interaction $\sim a_s^2 \mu_Q$, $\mu_Q$ is the effective reduced mass of the heavy quark mass $M_Q$ and the light (anti-)quark (diquark) mass $m$:

$$\left( M - M_Q + \frac{k_s k m}{1 + k m / M_Q} \right)^2 = \pi b \left( L + \frac{\pi}{2} n \right) + \left( m + \frac{p_Q^2}{M_Q} \right)^2,$$ \hspace{1cm} (3)

where $L$ is the orbital angular momentum, $n$ denotes the radial quantum number, $b$ denotes the parameter of slope, and

$$P_Q \equiv M_Q \cdot v_Q \simeq M_Q \left( 1 - \frac{m_{bareQ}^2}{M_Q^2} \right)^{1/2}.$$ \hspace{1cm} (4)

with $m_{bareQ} = 1.275 \text{GeV}$ and $4.18 \text{GeV}$ as the bare masses of the heavy quark $Q = c, b$ respectively.

Alternatively, Eq. (3) takes the form

$$M_{nl} = M_Q - \frac{k_s k m}{1 + k m / M_Q} + \sqrt{\pi b \left( L + \frac{\pi}{2} n \right) + a_M},$$ \hspace{1cm} (5)

and it can be viewed as an extension of the mass relation suggested by Selem and Wilczek [14]

$$M - M_Q = \sqrt{\frac{\sigma L}{2} + \frac{1}{4} k \left( \frac{m_{bareQ}}{L} \right)^2},$$ \hspace{1cm} (6)

where $\sigma = 2 \alpha$ is the slope for light hadrons. Eq. (6) implies $(M - M_Q)^2 \propto (\sigma / 2) L$ when $L \rightarrow \infty$ or the mass of light quark $m \rightarrow 0$, as examined in Ref. [13]. It agrees with the relation (1) with $n = 0$ up to a mass shift $M \rightarrow M - O(M_Q)$. When $L$ is very large, Eq. (5) becomes $M_{nl} - M_Q \simeq \sqrt{\sigma L / 2} + a_M / \sqrt{2 \alpha L}$, quite similar to the Selem-Wilczek relation (6), whereas it avoids the singularity when $L \rightarrow 0$. For the classifications of heavy hadrons via (6) see Ref. [14], where a picture of QCD string (flux-tube) is used. For the derivation of (3) without the reduced mass term based on the rotating string model see Ref. [19] and [9, 10, 12]. Note that in the string picture, the full quantum treatment provides merely the "quantum defect" $\alpha(0)$ in $L$ [18].
Our approach for hadronic excitations is to extrapolate (1) to the lower-$L$ region by incorporating an extra binding effect between the heavy-quark and strange quark in it [20, 21], which relies on the empirical fact that the hadronic trajectories are nearly linear even for lower excitations, though the trajectory parameters may be flavor-dependent weakly. The feature of our proposal (3) lies in (i) the flavor-dependence enters via an extra (negative) binding energy and the intercept $a_0$, which dominate at the low-lying states; (ii) The slope ratio $(\pi : 2)$ between radial and angular trajectories is examined successfully and enables us to predict the quantum number of the newly-observed $D_s(3000)$ and many others; (iii) When removed the heavy quark mass, the hadron mass $M$ is the sum of the extra binding energy linear in $\mu_Q$ and the string energy tied to light (anti-)quark (diquark) $\sim \sqrt{n_\eta L}$; (iv) The coefficient $k$ accounts for the reduced (increased) dressing effect of the light (anti-)quark (diquark) for excitations of the HL systems.

3. Excited HL mesons

We use the observed spin-averaged masses [1] of the singly heavy mesons for mapping the parameters in (3). When averaging, the mass data of some peculiar mesons (e.g., $D_s^*(2317)$ and $D_s(2460)$) do not enter as their mass may be shifted due to the near-threshold effects (see [7] for recent review). The optimal values of the parameters are shown in Table I by which the ensuing trajectory parameters $(\alpha', a_0)$ in $L + n\pi/2 = \alpha'(M - M_Q + k_0\mu_Q)^2 + a_0$ can be obtained from (3) through

$$\alpha' = \frac{1}{\pi b}, \quad a_0 = \frac{1}{\pi b} \left( m + M_Q - m_{bare Q}^2 / M_Q \right)^2. \quad (7)$$

| Parameters | $M_c$ | $M_b$ | $m_c$ | $m_b$ | $b(D)$ | $b(D_s)$ | $b(B)$ | $b(B_s)$ | $\chi_{SM}$ |
|------------|-------|-------|-------|-------|--------|----------|--------|----------|-------------|
| This work  | 1.46  | 4.52  | 0.31  | 0.49  | 0.262  | 0.313    | 0.303  | 0.369    | 0.0117      |
| EFG [22]   | 1.55  | 4.88  | 0.33  | 0.5   | 0.64/0.58 | 0.68/0.64 | 1.25/1.21 | 1.28/1.23 |

Table II. The predicted masses (MeV) of the charmed and charmed strange mesons compared to the observed masses [1].

| $J^P$  | Meson | This work | EFG [22] | Exp. | Meson | This work | EFG [22] | Exp. |
|--------|-------|-----------|----------|------|-------|-----------|----------|------|
| (0,1)$^-$ | $D(1S)$ | 1967 | 1975 | 1975 | $D_s(1S)$ | 2071 | 2076 | 2076 |
| (0,1)$^+$ | $D(1P)$ | 2430 | 2449 | 2436(?) | $D_s(1P)$ | 2532 | 2558 | 2556(?) |
| (0,1)$^-$ | $D(2S)$ | 2624 | 2619 | 2619(?) | $D_s(2S)$ | 2733 | 2720 | 2708(?) |
| (1,2,3)$^+$ | $D(1D)$ | 2752 | 2834 | 2752(?) | $D_s(1D)$ | 2868 | 2950 | 2860(?) |
| (0,1)$^-$ | $D(2P)$ | 2907 | 2987 | 3214(?) | $D_s(2P)$ | 3032 | 3119 | 3044(?) |
| (0,1)$^-$ | $D(3S)$ | 3048 | 3088 | 3088 | $D_s(3S)$ | 3182 | 3236 | 3236 |
| (2,3,4)$^+$ | $D(1F)$ | 3014 | 3145 | 3145 | $D_s(1F)$ | 3146 | 3268 | 3268 |
| (1,2,3)$^-$ | $D(2D)$ | 3147 | 3293 | 3293 | $D_s(2D)$ | 3287 | 3436 | 3436 |

Our remarks: (i) The observed (spin-averaged) masses of the $D/D_s$ and $B/B_s$ mesons can be reasonably described by the mass relation (3), with the slope parameters depending weakly upon the flavors; (ii) The smaller predictions by (3) for the slope parameter $1/\alpha' \pi$ compared to that by
the hadron mass; (iii) Some predictions can be made for the mesonic configurations, for instance, the masses \[1\].

EFG \[22\] can be due to the nonlinearity of the \(M^2\) vs. \(L\) at low-\(L\) which was removed by shifting the hadron mass; (iii) Some predictions can be made for the mesonic configurations, for instance, the \(D(2P)\) has the mass of about 2907 MeV and the \(D(2D)\) of about 3147 MeV. The heavy quark masses \((M_{Q,b})\) determined here by the HL meson spectra are taken to be the known input for the HL baryons in Section 4.

4. Excited HL baryons

In the heavy quark-diquark picture, we apply Eq. (3) to the HL baryons \(Qqq\) \((Q = c, b)\), the heavy baryon \(\Lambda_Q\) and heavy strange baryon \(\Sigma_Q\) containing scalar (spin-0) diquark \([qq]\), and the heavy baryon \(\Sigma_Q\) and heavy strange baryon \(\Xi_Q\) containing vector (spin-1) diquark \([qq]\).

Table III. The masses (MeV) of the bottomed and bottomed strange mesons compared to the observed masses [1].

| \(J^P\) | Meson | This work | EFG [22] | Exp. | Meson | This work | EFG [22] | Exp. |
|-------|-------|-----------|-----------|------|-------|-----------|-----------|------|
| (0, 1)\(^-\) | \(B(1S)\) | 5324 | 5315 | 5313 | \(B_s(1S)\) | 5413 | 5404 | 5403 |
| (0, 1, 2)\(^+\) | \(B(1P)\) | 5730 | 5745 | 5730(?) | \(B_s(1P)\) | 5840 | 5844 | 5840(?) |
| (0, 1)\(^-\) | \(B(2S)\) | 5916 | 5902 | 5944(?) | \(B_s(2S)\) | 6039 | 5988 |
| (1, 2, 3)\(^-\) | \(B(1D)\) | 6042 | 6106 | 6174 | \(B_s(1D)\) | 6174 | 6200 |
| (0, 1, 2)\(^+\) | \(B(2P)\) | 6196 | 6249 | 6341 | \(B_s(2P)\) | 6341 | 6343 |
| (0, 1)\(^-\) | \(B(3S)\) | 6338 | 6385 | 6495 | \(B_s(3S)\) | 6495 | 6473 |
| (2, 3, 4)\(^+\) | \(B(1F)\) | 6304 | 6398 | 6458 | \(B_s(1F)\) | 6458 | 6488 |
| (1, 2, 3)\(^-\) | \(B(2D)\) | 6439 | 6540 | 6604 | \(B_s(2D)\) | 6604 | 6636 |

Table IV. The effective masses (GeV) of the heavy quarks \(M_Q\) and the scalar diquarks \(m\), the tension \(a\) (GeV\(^2\)), and \(k_s = 0.214, k = 1.22\).

| Parameters | \(M_c\) | \(M_b\) | \(m_{(qq)}\) | \(m_{(gg)}\) | \(a(\Lambda_c)\) | \(a(\Xi_c)\) | \(a(\Lambda_b)\) | \(a(\Xi_b)\) | \(\chi_{SM}\) |
|-----------|--------|--------|-------------|----------|----------------|-------------|----------------|-------------|-------------|
| This work | 1.46   | 4.52   | 0.58        | 0.79     | 0.229          | 0.280       | 0.259          | 0.25        | 0.0020      |
| EFG [24]  | 1.55   | 4.88   | 0.71        | 0.948    | 0.43           | 0.46        | 0.90           | 0.91        |             |

Table V. The same with Table IV, but with the vector diquarks and \(k_s = 0.214, k = 0.4\).

| Parameters | \(M_c\) | \(M_b\) | \(m_{(qq)}\) | \(m_{(gg)}\) | \(a(\Sigma_c)\) | \(a(\Xi'_c)\) | \(a(\Sigma_b)\) | \(a(\Xi'_b)\) | \(\chi_{SM}\) |
|-----------|--------|--------|-------------|----------|----------------|-------------|----------------|-------------|-------------|
| This work | –      | –      | 0.73        | 0.88     | 0.251          | 0.258       | 0.245          | 0.258       | 0.0083      |
| EFG [24]  | –      | –      | 0.91        | 1.07     | 0.44           | 0.46        | 0.83           | 0.86        |             |

One sees from Tables IV and V that the diquark masses determined are close to that in the quark model [24]. Moreover, the slopes here become more flocked together around 0.24 compared with the plots in Refs. [13] and [24]. The ensuing predictions by (3) are listed in Tables VI and VII. The predicted masses for singly charmed baryons are

\[
\Lambda_c : 1.36 + \sqrt{0.86 + 0.72(L + n\pi/2)} \quad ; \quad \Xi_c : 1.34 + \sqrt{1.29 + 0.88(L + n\pi/2)}
\]

\[
\Sigma_c : 1.41 + \sqrt{1.15 + 0.79(L + n\pi/2)} \quad ; \quad \Xi'_c : 1.40 + \sqrt{1.50 + 0.81(L + n\pi/2)}
\]
Table VI. The predicted masses (MeV) of the heavy baryons compared to the observed masses [1] and other prediction cited.

| \(J^P\) | Baryon | This work | EFG [24] | Exp. | Baryon | This work | EFG [24] | Exp. |
|---|---|---|---|---|---|---|---|---|
| \(\frac{1}{2}^+\) | \(\Lambda_c(1S)\) | 2284 | 2286 | 2286 | \(\Xi_c(1S)\) | 2470 | 2476 | 2471 |
| \(\frac{1}{2}, \frac{3}{2}\) | \(\Lambda_c(1P)\) | 2613 | 2617 | 2616 | \(\Xi_c(1P)\) | 2808 | 2810 | 2811 |
| \(\frac{1}{2}^-\) | \(\Lambda_c(2S)\) | 2767 | 2769 | 2767 (?) | \(\Xi_c(2S)\) | 2970 | 2959 | 2968 (?) |
| \(\frac{1}{2}, \frac{3}{2}\) | \(\Lambda_c(1D)\) | 2873 | 2878 | 2871 | \(\Xi_c(1D)\) | 3082 | 3069 | 3070 (?) |
| \(\frac{1}{2}^-\) | \(\Lambda_c(2P)\) | 3002 | 2998 | 2940 (?) | \(\Xi_c(2P)\) | 3220 | 3194 | 3194 |
| \(\frac{3}{2}^-\) | \(\Lambda_b(3S)\) | 3123 | 3130 | 3349 | \(\Xi_b(3S)\) | 3233 | 3323 | 3323 |
| \(\frac{3}{2}, \frac{5}{2}\) | \(\Lambda_b(1F)\) | 3094 | 3086 | 3318 | \(\Xi_b(1F)\) | 3286 | 3286 | 3286 |
| \(\frac{3}{2}^-\) | \(\Lambda_b(2D)\) | 3208 | 3201 | 3441 | \(\Xi_b(2D)\) | 3399 | 3399 | 3399 |

| \(\frac{3}{2}^-\) | \(\Lambda_b(1S)\) | 5623 | 5620 | 5620 | \(\Xi_b(1S)\) | 5792 | 5803 | 5795 |
| \(\frac{3}{2}, \frac{5}{2}\) | \(\Lambda_b(1P)\) | 5917 | 5938 | 5917 | \(\Xi_b(1P)\) | 6043 | 6127 | 6127 |
| \(\frac{3}{2}^-\) | \(\Lambda_b(2S)\) | 6062 | 6089 | \(\Xi_b(2S)\) | 6171 | 6266 | 6266 |
| \(\frac{3}{2}, \frac{5}{2}\) | \(\Lambda_b(1D)\) | 6163 | 6194 | \(\Xi_b(1D)\) | 6261 | 6370 | 6370 |

Table VII. The predicted masses (MeV) of the heavy baryons compared to the observed masses [1] and other prediction cited.

| \(J^P\) | Baryon | This work | EFG [24] | Exp. | Baryon | This work | EFG [24] | Exp. |
|---|---|---|---|---|---|---|---|---|
| \(\frac{1}{2}, \frac{3}{2}\) | \(\Sigma_c(1S)\) | 2480 | 2494 | 2497 | \(\Xi_c(1S)\) | 2624 | 2626 | 2624 |
| \(\frac{1}{2}, \frac{3}{2}\) | \(\Sigma_c(1P)\) | 2801 | 2780 | 2806 (?) | \(\Xi_c(1P)\) | 2919 | 2919 | 2931 (?) |
| \(\frac{1}{2}, \frac{3}{2}\) | \(\Sigma_c(2S)\) | 2954 | 2924 | \(\Xi_c(2S)\) | 3064 | 3012 | \(\Xi_c(2S)\) |
| \(\frac{1}{2}, \frac{3}{2}\) | \(\Sigma_c(1D)\) | 3060 | 3029 | \(\Xi_c(1D)\) | 3165 | 3157 | 3123 (?) |
| \(\frac{1}{2}, \frac{3}{2}\) | \(\Sigma_c(2P)\) | 3192 | 3158 | \(\Xi_c(2P)\) | 3292 | 3300 | 3300 |
| \(\frac{3}{2}, \frac{5}{2}\) | \(\Sigma_c(3S)\) | 3314 | 3286 | \(\Xi_c(3S)\) | 3410 | 3390 | 3390 |
| \(\frac{3}{2}, \frac{5}{2}\) | \(\Sigma_c(1F)\) | 3284 | 3245 | \(\Xi_c(1F)\) | 3381 | 3385 | 3385 |
| \(\frac{3}{2}, \frac{5}{2}\) | \(\Sigma_c(2D)\) | 3401 | 3356 | \(\Xi_c(2D)\) | 3495 | 3496 | 3496 |

As seen here, a good agreement is achieved between the predictions and the observed data. The same is true for the predictions 2801 MeV and 2919 MeV for the 1P-wave \(\Sigma_c\) and \(\Xi_c\) baryon, considering that the mass spin splits have been ignored. The mass formula for the singly bottom baryons are

\[
\Lambda_b : 4.39 + \sqrt{1.52 + 0.81(L + n\pi/2)} \quad ; \quad \Xi_b : 4.35 + \sqrt{2.08 + 0.79(L + n\pi/2)}
\]

\[
\Sigma_b : 4.46 + \sqrt{1.90 + 0.77(L + n\pi/2)} \quad ; \quad \Xi_b' : 4.45 + \sqrt{2.35 + 0.81(L + n\pi/2)}
\]

Based on the computations in Tables VI and VII, we predict

\[
\Lambda_c(2765)^+ : 2S \\
\Xi_c(3055)^+ : 1D \\
\Sigma_c(2800), \Xi_c(2930), \Sigma_b(6097)^+, \Xi_b(6227)^- : 1P
\]

Our remarks: (i) The masses of the scalar and vector diquarks are slightly smaller than that predicted by other quark models. They differ 150 MeV and 90 MeV for scalar and vector diquarks re-
pectively; (ii) The coefficient \( k \) becomes larger (smaller) for scalar (vector) diquark since the diquark mass, estimated roughly by \( m(\Lambda_c) - M_c(m(\Sigma_c)/3 + 2m(\Sigma^+)) - M_c \), scales like \( 2m_q - 3a_{qq}/m_q^2(2m_q + a_{qq}/m_q^2) \) in which the strength \( a_{qq} \) of quark pair in diquark becomes smaller (larger) when \( L \) increases as it is proportional to the overlap of the relative wave function of diquark; (iii) Once determined, the diquark mass can be used to compute the masses of the bottom baryons (partners \( bqq \) and \( bqs \)) for which the less data available.

5. Summary

We proposed a general Regge-like mass relation to study the excited spectra of singly heavy mesons (\( D/D_s \) and \( B/B_s \)) and baryons (\( \Lambda_{c,b}/\Xi_{c,b} \) and \( \Sigma_{c,b}/\Xi_{c,b}' \)) in which the slope ratio between the radial and orbital Regge trajectories is \( \pi/2 \) and an extra binding effect of heavy quark and flavored light quarks has been taken into account. We test the relation against the observed spin-averaged data of the singly heavy mesons and baryons in their radially and angularly excited states and find that the agreement is remarkable for the reasonable values of \( e/2 \) and an extra binding effect of heavy quark and flavored light quarks has been taken into account. We test the relation against the observed spin-averaged data of the singly heavy mesons and baryons in their radially and angularly excited states and find that the agreement is remarkable for the reasonable values of effective masses of heavy quarks and diquarks.

Some new predictions are made for the experimentally observed states with unknown quantum numbers, awaiting for the forthcoming Belle II and LHCb experiments to test.

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