The jigsaw puzzle of scalar mesons

M. Boglione

Dipartimento di Fisica Teorica, Università di Torino, via P. Giuria 1, 10125 Italy

Abstract. This is a brief overview of light scalar meson spectroscopy, addressing longstanding problems, recent developments and future perspectives. In particular, a new comprehensive data analysis is introduced which will help to unravel the structure of the $f_0(980)$.

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We all know that the quark model works well for most mesons: nice nonet structures arise when all possible combinations of $q\bar{q}$ pairs are ordered according to their isospin and strangeness. Then, by exploiting the mass and decay properties of the physical mesons delivered by experiments, we can find a slot for each candidate in the nonet. This game can be safely played for vector and tensor mesons and, to some extent, for pseudoscalar mesons, if the appropriate mixing angles are taken into account. Consider, for instance, the $\omega(782)$ and the $\phi(1020)$: experiments tell us that the $\omega(782)$ decays mostly into pions and is lighter than the $\phi(1020)$ which, on the contrary, decays into $K\bar{K}$ 85\% of the time. It’s mass being close to that of the $\rho(770)$ provides clear indication that the $\omega(782)$ is the $I=0$ non-strange candidate, whereas $\phi(1020)$ is undoubtedly the $I=0 s\bar{s}$ member of the vector nonet. Similarly for the tensors $f_2$ and $f'_2$.

For scalar mesons this does not work. The quark model fails inexorably: first of all, experiments detect many more physical scalar resonances than can fit in a nonet. Secondly, their decay properties are mostly unknown, so there is little guide to their classification, thirdly their spectra cannot be approximated by Breit-Wigner shapes, because they overlap and interfere with each other, some of them being very broad. Therefore, the classical methods of analysing data cannot be applied.

How can we try and disentangle such a complicated picture? *Unitarity* comes to our rescue. Indeed, this property, which follows from conservation of probability, has to be fulfilled whatever the quantum numbers of the $q\bar{q}$ pair, and give very useful constraints for our analyses. Unitarity requires the $T$ matrix for each partial wave to satisfy $\text{Im}T = \rho |T|^2$, where $\rho$ is the phase space matrix. This relation constrains the imaginary part of $(1/T)$ to be $\text{Im}(1/T) = -\rho$, in the simplest case, leaving $\text{Re}(1/T)$ unconstrained. By parametrizing $\text{Re}(1/T)$ by a real matrix $1/K$, one obtains $T = \frac{K}{1 - i\rho K}$, which is the usual $K$-matrix representation. If there is only one channel, like in $\pi\pi \rightarrow \pi\pi$ scattering below $K\bar{K}$ threshold, and only one narrow resonance, this resonance will appear like a single pole in the $K$ amplitude, $K = \frac{s}{M^2 - s - i\rho}$, and the $T$ amplitude can be approximated by $T = \frac{s}{M^2 - s - i\rho}$. The pole of $K$ gives the “bare state” and $T$ has a Breit-Wigner form. This simple picture works only for narrow and well separated resonances, where coupling to hadronic loops has little effect. For the scalar sector, where resonances
FIGURE 1. Coupled channel unitarity constrains the amplitudes $F(\phi \to \gamma\pi\pi)$ and $F(\gamma\gamma \to \pi\pi)$ in terms of hadronic amplitudes corresponding to final state interactions, $\pi\pi \to \pi\pi$ and $KK \to \pi\pi$ for a given $I, J$. For $\phi$-decay, the photon is assumed to be a spectator.

are broad (i.e. their poles are located very far from the real $s$-axis, where experiments happen), interfering and overlapping (i.e. their spectra are not made of nicely separated peaks), this simple interpretation breaks down.

Fig. 1 shows how similarly coupled-channel unitarity constrains the partial wave amplitudes $F$ corresponding to two different processes $\phi \to \gamma\pi\pi$ and $\gamma\gamma \to \pi\pi$; scalar meson resonances are produced in the final state interactions $\pi\pi \to \pi\pi$ and $KK \to \pi\pi$ and are embodied as poles in the $I = J = 0$ hadronic amplitudes, $T$. The general solution of the unitarity requirement for the $F$’s is given by a linear combination of the $T$’s, where the coefficients $\alpha_i(s)$ are real functions of $s$, simple polynomials apart from some factors as explained in [1]. Notice that unitarity requires consistency between reactions, in that the same strong interaction amplitudes $T$, combined and weighted using appropriate $\alpha_i$ coefficients, form the amplitudes corresponding to different reactions. The $\alpha$-vector formulation embodies universality, demanding that poles of the $S$ matrix transmit to all processes with the same quantum numbers in exactly the same position. This indeed makes the determination of the $F$ amplitudes very sensitive to the details of the $T$’s.

Recently, M.R. Pennington and I made an analysis [1] of $\phi \to \gamma\pi\pi$ experimental data [2] based on the coupled channel unitarity constrains of Fig. 1 and showed that huge differences arise in the determination of the relevant couplings and the $\phi \to f_0(980)$ branching ratio due to different choices of underlying amplitudes $T$. We chose an old set of hadronic amplitudes called ReVAMP, determined as in [3] and a recent one, obtained by Anisovich and Sarantsev in [4] fitting a much larger amount of data. In the first set of amplitudes, the $f_0(980)$ appears as a narrow resonance, lighter than the $\phi(1020)$. In the second case the $f_0(980)$ is a much broader object, heavier than the $\phi(1020)$. Since the decay rate distribution depends crucially on the cube of the photon momentum, i.e. $(m^2_\phi - s)^3$, and since the $f_0(980)$ is so close to the end of phase space, it turns out that the determination of the couplings and branching ratio is extremely sensitive to the...
exact position of the \( f_0(980) \) pole in the \( T \)’s. The fit clearly favours the ReVAMP set of amplitudes, which give an excellent quality of results with constant \( \alpha_i(s) \) (3 parameter fit), confirming that the \( \pi\pi \) final state interactions in this particular process are consistent with those of the processes exploited to determine the ReVAMP amplitudes. Indeed, when the new, high statistics, KLOE data will be released, we will have the chance to test this consistency further.

While for decays like \( \phi \to \pi\pi X \) we have to assume \( X \) is a spectator to apply unitarity as in Fig.1, for \( \gamma\gamma \to \pi\pi \) scattering unitarity and universality apply with no assumptions. A few years ago, M.R. Pennington and I analysed \( \gamma\gamma \to \pi\pi \) world data \[[5]\] to determine the radiative widths of scalar mesons. The underlying hadronic amplitudes we used were the same ReVAMP set described above. We found two classes of solutions, delivered by fits equally good in quality and giving comparable scalar widths: one where the \( f_0(980) \) showed up as a peak, and the other where the \( f_0(980) \) showed up as a dip. Shortly, new very high statistics data from BELLE and BaBar will be available: they will allow us a global reanalysis to discern between the two solutions and to test the \( T \) underlying hadronic amplitudes.

For these re-analysis, we are considering a different parametrization for the \( T \)’s. In fact, the simplest solution to the unitarity requirement, as shown above, violates left hand cut analyticity: each \( \rho \) matrix element is singular at \( s \to \infty \), which constrains the \( T \)’s in an artificial and unnecessary way. To avoid this, we perform new fits \[[6]\] that include recent experimental data in addition to those used for the original ReVAMP analysis, in which \( \text{Im}(1/T) \) is given by the Chew-Mandelstam function, which is not affected by that flaw.

Concluding, the main message of this talk is the following: unitarity and analyticity give powerful constraints and must be at the very basis of any data analysis. Unitarity requires consistency among different reactions, so that analysing data where final state interactions are important only makes sense if it is done in a global and comprehensive way. It’s like a big jigsaw puzzle game: you have to take care of combining appropriately all the single pieces before the total picture is revealed.

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