Structures and productions of typical $sd$-shell hypernuclei in shell-model calculations

Atsushi Umeya$^1$, Toshio Motoba$^2$, Toru Harada$^2$

$^1$ Nishina Center for Accelerator-Based Science, RIKEN, Wako, Saitama 351-0198, Japan
$^2$ Research Center for Physics and Mathematics, Osaka Electro-Communication University, Neyagawa, Osaka, 572-8530, Japan

E-mail: aumeya@riken.jp

Abstract. As one of the typical $sd$-shell hypernuclei, the positive- and negative-parity energy levels of $^{20}_Λ$Ne are calculated to investigate structures of low-lying states within the shell model calculations. In addition to the conventional $NN$ effective interactions, the $ΛN$ G-matrix interactions derived from the Nijmegen NSC97f potentials are employed. The production cross sections of ($\pi^+, K^+$) and ($K^-, \pi^-$) reactions are estimated using the calculated shell-model wave functions.

1. Introduction

Detailed hypernuclear studies have been mainly focused on structures of $p$-shell systems. In addition to the ($\pi^+, K^+$) reaction experiments done for wide mass number region [1], the $γ$-ray measurements [2] from $p$-shell hypernuclei provide us with remarkable possibility of high-precision spectroscopic studies on theoretical side [3]. Recently, also the ($e, e' K^+$) reaction experiments done at Hall-A and Hall-C of JLab [4, 5] have been proved to be very fruitful in disclosing hypernuclear structure details within the energy resolution of few hundreds of keV.

These aspects encourage us to perform theoretical study of $sd$-shell hypernuclear structures, because even the Λ single-particle energies and its interplay with nuclear core excitations are not well known in these medium-mass systems. As a theoretical example of photoproduction study, one may refer to Ref. [6]. In general, there should be more bound states of Λ in $sd$-shell hypernuclei so that we can expect novel aspect of hyperon coupling with nuclear rotational excitations. We anticipate innovated ($K^-, \pi^-$) reaction experiments to be done at J-PARC, as well as ($e, e' K^+$) experiments at JLab.

As a starting typical example, in this article, we focus on a $^{20}_Λ$Ne hypernucleus whose nuclear core, $^{19}_{\Lambda}$Ne, has an interesting level structure in low-lying states. (i) In a naive configuration picture, it is expected that $|0d_{5/2}^20^+, 0d_{5/2}; J^\pi=\frac{5}{2}^+\rangle$ will be the ground state because the lowest $sd$-shell orbit is $0d_{5/2}$. However, actually, the ground state of $^{19}_{\Lambda}$Ne has $J^\pi=\frac{1}{2}^+$ and is mainly described by a $|0d_{5/2}^20^+, 1s_{1/2}; J^\pi=\frac{1}{2}^+\rangle$ basis state, because an energy difference between $1s_{1/2}$ and $0d_{5/2}$ orbits is not so large and a mixing of $1s_{1/2}$ leads to strong attraction due to an $NN$ interaction. (ii) Also, the negative-parity state with $J^\pi=\frac{1}{2}^-$ has a small excitation energy because of the strong and attractive four-body correlation which compensate the $1p-1h$ excitation energy creating a $p$-state hole in the $^{16}_{\Lambda}$O closed core. These states with $J^\pi=\frac{1}{2}^+, \frac{5}{2}^+$ and $\frac{1}{2}^-$ are in a narrow energy region of few hundreds of keV, and then it is interesting to investigate...
For the $\Lambda$ single-particle state $\Phi(19\text{Ne}; J^+; T)$ with a positive parity, the $19\text{Ne}$ nuclear core consisting of sixteen nucleons is assumed to be inert and three valence nucleons are distributed in the $sd$-shell $(0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$) orbits. In order to calculate energy levels of $19\text{Ne}$, $K_\Lambda = 1$ excitation is employed. Basis states of $19\text{Ne}$ are schematically shown in Fig. 1, and the positive- and negative-parity-wave functions are described, respectively, as

$$
\Phi(19\text{Ne}; J^+; T^+) = \sum_k c_k \left[ (0s)^4 (0p)^{12} (0d_{5/2} 1s_{1/2} 0d_{3/2})^3 \right]_{J^+ T^+} ,
$$

(1)

$$
\Phi(19\text{Ne}; J^+; T^-) = \sum_k c_k \left[ (0s)^4 (0p)^{11} (0d_{5/2} 1s_{1/2} 0d_{3/2})^4 \right]_{J^+ T^-} .
$$

(2)

For the $\Lambda$ single-particle state $\phi^\Lambda(j)$, $0s$-shell $(0s_{1/2})$, $0p$-shell $(0p_{3/2}$ and $0p_{1/2})$, and $sd$-shell $(0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2})$ orbits are taken into consideration.

We adopt the Kuo $G$-matrix [7] for the $sd$-shell part of an $NN$ effective interaction, and the Millener-Kurath interaction [8] for the $p$-$sd$ cross-shell part. For the $\Lambda N$ effective interaction,
we try to adopt the Nijmegen soft-core model NSC97f [9]. For single-particle energies with respect to the $^{16}\text{O}$ core, we use rather standard values of $-21.86 \ (0p_{3/2})$, $-14.20 \ (0p_{1/2})$, $-4.14 \ (0d_{5/2})$, $-3.27 \ (1s_{1/2})$ and $+0.94 \ (0d_{3/2})$ MeV, which are obtained from the experimental energy levels of $^{17}\text{O}$ [10], except for the $0p_{1/2}$ orbit. For simplicity, we adjust the single-particle energy of $0p_{1/2}$ to reproduce the energy levels of the $^{19}\text{Ne}$ negative-parity states. Thus we take the satisfactory description with respect to the nuclear core excitations. This is a necessary option when we proceed next to couple the Λ particle to see what happens depending on the adopted ΛN interactions.

3. Numerical results of hypernuclear energy levels
Figure 2 shows the calculated energy levels of $^{20}\Lambda\text{Ne}$, together with the experimental and calculated levels of $^{19}\text{Ne}$. The calculation for $^{19}\text{Ne}$ well reproduces the experimental energy levels of three low-lying states with $J^{\pi} = 1^{+}, 5^{+}$ and $1^{−}$. The calculated excitation energies of $5^{+}$ and $1^{−}$ excited states are 0.36 and 0.43 MeV, respectively, which correspond to the experimental values, 0.24 and 0.28 MeV. However, the present shell model should be improved in order to describe the energies of the $3^{+}$ and $3^{−}$ states.

In the $^{20}\Lambda\text{Ne}$ hypernucleus, states in Fig. 1 are calculated in groups of $[\Phi(^{19}\text{Ne}, J_{c}^{+}T) \otimes \phi^{\Lambda}(0s)]$ or $[\Phi(^{19}\text{Ne}, J_{c}^{-}T) \otimes \phi^{\Lambda}(0s)]$. We are presently interested in the energy splittings and order of the “spin-doublet” states ($J = J_{c} \pm \frac{1}{2}$) which usually attributed to the weak coupling of Λ to a nuclear core state ($J_{c}$). The calculation using the NSC97f ΛN interaction leads to the $0^{+}$ ground state and the doublet energy splitting is 0.24 MeV. In the $p$-shell cases such as $^{12}\Lambda\text{C}$ and $^{16}\Lambda\text{O}$, it is known that the $J_{c}$ member comes lower in energy, so that spin-singlet interaction is more attractive than the spin-triplet one, and the energy splitting is generally very small. Therefore, one believes that the NSC97f interaction gives a plausible prediction for the low-lying energy
level sequence.

The Λ particle coupling to the \( J = \frac{3}{2}^+ \) core state \((E_{\text{cal}} = 0.36 \text{ MeV})\) leads to the 2\(^+\) and 3\(^+\) doublet in \(^{20}\Lambda\text{Ne}\). The energy of the 2\(^+\) member is slightly lower than that of the 3\(^+\) member, and the splitting energy of this doublet is smaller than that of the ground-state doublet. On the other hand, it is interesting to predict the negative-parity doublet of \(^{20}\Lambda\text{Ne}, 1^-\) and 0\(^-\) states, at \(E_x = 0.59 \text{ MeV}\) and 0.70 MeV, respectively. Both of these excitation energies are larger than the \( \frac{1}{2}^- \) nuclear core excitation energy.

We analyze the matrix elements of the Λ-nuclear interaction for large configurations of the nuclear-core states, at \( \Lambda = 0 \text{Ne}\). The energy of the 2\(^+\) \(^{20}\Lambda\text{Ne}\) is split by 6 MeV, and 3\(^+\) \(^{20}\Lambda\text{Ne}\) is split by 0.36 MeV leads to the 2\(^+\) \(^{20}\Lambda\text{Ne}\) eigenstates.

### Table 1. Wave functions calculated for the low-lying “spin-doublet” states of \(^{20}\Lambda\text{Ne}\). The amplitudes are expressed in terms of the nuclear-core configurations, \( \phi_N \), coupled with the 0\(s_{1/2}\)-state Λ particle.

| \( \psi_N \) | \(^{20}\Lambda\text{Ne}(J = 0^+_1)\) | \(^{20}\Lambda\text{Ne}(J = 1^+_1)\) |
|----------------|-----------------|-----------------|
| \([0d_{5/2}^N]_0(1s_{1/2}^N)\]_{1/2} | \(\sqrt{0.3744}\) | \(\sqrt{0.3558}\) |
| \([0d_{5/2}^N]_0(1s_{1/2}^N)\]_{1/2} | \(-\sqrt{0.1357}\) | \(-\sqrt{0.1471}\) |
| \(0d_{5/2}^N\) | \(-\sqrt{0.1153}\) | \(-\sqrt{0.1207}\) |
| \([0d_{5/2}^N](1s_{1/2}^N)(0d_{3/2}^N)\]_{21/2} | \(\sqrt{0.1161}\) | \(\sqrt{0.1224}\) |
| \(\psi_N\) | \(^{20}\Lambda\text{Ne}(J = 2^+_1)\) | \(^{20}\Lambda\text{Ne}(J = 3^+_1)\) |
| \(0d_{5/2}^N\) | \(-\sqrt{0.3663}\) | \(-\sqrt{0.3735}\) |
| \(\psi_N\) | \(^{20}\Lambda\text{Ne}(J = 0^-_1)\) | \(^{20}\Lambda\text{Ne}(J = 1^-_1)\) |
| \([0p_{1/2}^N]^{-1}(0d_{3/2}^N)\]_{1/2} | \(-\sqrt{0.1332}\) | \(-\sqrt{0.1332}\) |
| \([0p_{1/2}^N]^{-1}(0d_{3/2}^N)(1s_{1/2}^N)\]_{1/2} | \(+\sqrt{0.1618}\) | \(+\sqrt{0.1614}\) |

### Table 2. Matrix elements \( \langle \psi_N, 0s_{1/2}^N | V | \psi_N, 0s_{1/2}^N \rangle \) of the Λ-nuclear interaction in unit of MeV. \( \psi_N \) denotes a nuclear configuration that is included in the low-lying \(^{20}\Lambda\text{Ne}\) eigenstates.

| \( \psi_N \) | \(^{20}\Lambda\text{Ne}(J = 0^+_1)\) | \(^{20}\Lambda\text{Ne}(J = 1^+_1)\) |
|----------------|-----------------|-----------------|
| \([0d_{5/2}^N]_0(1s_{1/2}^N)\]_{1/2} | \(-2.89\) | \(-2.44\) |
| \([0d_{5/2}^N]_0(1s_{1/2}^N)\]_{1/2} | \(-2.45\) | \(-2.58\) |
| \(0d_{5/2}^N\) | \(-2.51\) | \(-2.50\) |
| \([0d_{5/2}^N](1s_{1/2}^N)(0d_{3/2}^N)\]_{21/2} | \(-1.79\) | \(-1.82\) |
| \(\psi_N\) | \(^{20}\Lambda\text{Ne}(J = 2^+_1)\) | \(^{20}\Lambda\text{Ne}(J = 3^+_1)\) |
| \(0d_{5/2}^N\) | \(-2.52\) | \(-2.49\) |
| \(\psi_N\) | \(^{20}\Lambda\text{Ne}(J = 0^-_1)\) | \(^{20}\Lambda\text{Ne}(J = 1^-_1)\) |
| \(0d_{5/2}^N\) | \(-3.34\) | \(-3.34\) |
| \([0d_{5/2}^N](1s_{1/2}^N)\]_{1/2} | \(-3.43\) | \(-3.43\) |
| \([0p_{1/2}^N]^{-1}\) | \(+1.01\) | \(+0.70\) |
$^{20}_{\Lambda}\text{Ne}$ states,

$$
\Psi^{(20}_{\Lambda}\text{Ne}) = \sum_k c_k \left[ \{(^{16}\text{O core}) \otimes \psi_N \}_J^l \otimes 0s_{1/2} \right],
$$

where $\psi_N$ consists of three valence nucleons for positive-parity states, and of four valence nucleons and a single-hole for negative-parity states. Table 1 shows amplitudes of the large configurations, where $(j)^l_J$ denotes a configuration with the coupled angular momentum $J$ that consists of $n$ valence nucleons in a $j$ orbit, and $(0p_{1/2})^{-1}$ denotes a single-hole configuration in the $0p_{1/2}$ orbit. The nuclear core state with positive-parity in $^{20}_{\Lambda}\text{Ne}$ mainly consists of $(0d_{5/2})^3$ and $(0d_{5/2})^2(1s_{1/2})$ configurations. On the other hand, the nuclear core state with negative-parity is described as a product of a $(sd)^4$ valence-particle configuration and a $(0p_{1/2})^{-1}$ hole configuration. Table 2 shows values of matrix elements of the $\Lambda$-nuclear interaction for these configurations. We find that the $\Lambda-(sd)^4$ interaction in the negative-parity state is more attractive than the $\Lambda-(sd)^3$ interaction, while the $\Lambda-(0p_{1/2})^{-1}$ interaction is repulsive. As a result, the strength of the $\Lambda-^{19}\text{Ne}(1^-)$ interaction is slightly weaker than that of the $\Lambda-^{19}\text{Ne}(\frac{1}{2}^+)$ interaction.

4. DWIA cross sections for light $sd$-shell hypernuclear production

Figure 3 shows production cross sections of $(\pi^+, K^+)$ and $(K^-, \pi^-)$ reactions in DWIA calculations. In general, in $(\pi^+, K^+)$ reactions at forward angles, natural-parity stretched states are most probably to be populated. In the case of $^{20}\text{Ne}$ $(\pi^+, K^+)$ reaction at $p_\pi = 1.05$ GeV/$c$ and $\theta^{\text{lab}} = 3^\circ$ in the left panel of Fig. 3, the 2$^+$, 3$^-$ and 4$^+$ states are populated at the first, second and third peaks, respectively. On the other hand, in in-flight $(K^-, \pi^-)$ reactions at forward angles, the substitutional states are dominantly produced via the $L \simeq 0$ angular momentum transfer. In the case of $^{20}\text{Ne}$ $(K^-, \pi^-)$ reaction at $p_K = 0.80$ GeV/$c$ and $\theta^{\text{lab}} = 5^\circ$ in the right panel of Fig. 3, the 1$^-$ and 0$^+$ states are dominant at the second and third peaks. The level structure of $^{20}_{\Lambda}\text{Ne}$ low-lying states may not be determined from comparison of the first peaks in $(\pi^+, K^+)$ and $(K^-, \pi^-)$ cross sections because the difference of positions between these peaks are very small. On the other hand, the positions of the second and third peaks in the $(\pi^+, K^+)$ production are different from those in the $(K^-, \pi^-)$ production. If the differences are measured, the order of $J = 3^-$ and 1$^-$ states with $0p_{\Lambda}$ and of $J = 4^+$ and 0$^+$ states with $0d_{\Lambda}$ can be determined.

The $sd$-shell nuclear targets used so far in the actual experiments are quite limited: The $^{28}\text{Si}$ nucleus is the favorite target often used in the $(\pi^+, K^+)$ reactions [11] and in the recent $(e,e'K^+)$ reaction [5]. In fact, theoretical predictions for their reactions have been made [6, 12]. Here, however, we focus our attention to the light $sd$-shell hypernuclei for which it seems easier to discuss hypernuclear structures in close connection with properties of $\Lambda N$ interactions. Only one example exploited so far in the light $sd$-shell region is the $^{18}\text{O}(K^-, \pi^-)^{16}\text{O}$ reaction experiment [13], and its DWIA analysis was performed long ago [14]. Although it may be difficult to use $^{20}\text{Ne}$ as an actual target, we are now calculating the DWIA production cross sections on the basis of the similar theoretical prescriptions, so that we can have a concrete idea to be utilized in possible J-PARC projects for light $sd$-shell hypernuclear production.

5. Summary and outlook

We have calculated the energy levels of $^{20}_{\Lambda}\text{Ne}$ hypernucleus by using the multi-configuration shell model with the Nijmegen NSC97f potentials for the $\Lambda N$ effective interaction. The calculation has predicted properties of the $0^+$ ground state. The calculated wave functions have been used for the estimates of cross sections of $(\pi^+, K^+)$ and $(K^-, \pi^-)$ productions. The level scheme of $^{20}_{\Lambda}\text{Ne}$ can be determined through the differences of peak positions between these production cross sections.
Figure 3. Production cross sections of (+,K+) and (K−,π−) reactions in DWIA calculations. Production cross sections of $^{20}$Ne ($\pi^+,K^+$) (left) and $^{20}$Ne ($K^−,\pi^−$) (right) reactions in DWIA calculations.

Acknowledgments
The authors are grateful to Y. Yamamoto for providing the YNG-type effective AN interaction derived from Nijmegen NSC97f potentials. This work is supported by Grant-in-Aid for Scientific Research C (No. 21540284).

References
[1] Hashimoto O and Tamura H 2006 Prog. Part. Nucl. Phys. 57 564, and references therein
[2] Tamura H 2009 Nucl. Phys. A827 153c, and references therein
[3] Millener D J 2008 Nucl. Phys. A804 84, and references therein
[4] LeRose J J et al. 2008 Nucl. Phys. A804 116; Garibaldi F et al. 2010 Proc. of the Sendai International Symposium on Strangeness in Nuclear and Hadronic Systems (World Scientific) p 195
[5] Hashimoto O et al. 2008 Nucl. Phys. A804 125; Fujii Y et al. 2010 Proc. of the Sendai International Symposium on Strangeness in Nuclear and Hadronic Systems (World Scientific) p 188
[6] Motoba T, Bydžovský P, Sotona M, Itonaga K, Ogawa K and Hashimoto O 2010 Proc. of the Sendai International Symposium on Strangeness in Nuclear and Hadronic Systems (World Scientific) p 178
[7] Kuo T S 1967 Nucl. Phys. A103 71
[8] Millener D J and Kurath D 1975 Nucl. Phys. A255 315
[9] Rijken Th A, Stoks V G J and Yamamoto Y 1999 Phys. Rev. C 59 21
[10] Firestone R B and Shirley V S 1996 Table of Isotopes 8th ed. (John Wiley and Sons, Inc., New York)
[11] Hasegawa T et al. 1996 Phys. Rev. C 53 1210; Hotchi H et al. 2001 Phys. Rev. C 64 044302
[12] Motoba T, Bando H, Wünsch R and Žofka J 1988 Phys. Rev. C 38 1322
[13] May M et al. 1981 Phys. Rev. Lett. 47 1106
[14] Yamada T, Motoba T, Ikeda K and Bando H 1985 Prog. Theor. Phys. Suppl. 81 104