Performance Analysis of Amplify and Forward Cooperative Networks over Rayleigh and Nakagami-m Channels based Relaying Selection

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ABSTRACT

In this paper, we present a performance analysis of all relays for the AF cooperative networks of the half-access (DS-CDMA). In their work, they studied the performance and analysis of downlink multiuser relay networks using amplify and forward (AF) where the outage probability was evaluated at a high SNR [5]. The authors also show performance analysis of the diversity gain in wireless networks [1-3].

[4] The authors used amplify and forward (AF) relaying over Nakagami-m fading channels to derive a new lower bound and asymptotic expression for the outage probability of cooperative diversity in direct sequence code-division multiple access (DS-CDMA). In their work, they studied the performance and analysis of downlink multiuser relay networks using single-relay in amplify and forward (AF) where the outage probability was evaluated at a high SNR [5-6].

For example, [7-8] analyzed the performance of cooperative diversity wireless networks using amplify-and-forward relaying over independent, non-identical (i.n.d), Nakagami-m fading channels. The error rate and outage probability are determined using the MGF of the total SNR at the destination. Furthermore, closed form expressions for the CDF, probability density function (PDF) and MGF of the total SNR [9-10]. The authors also show performance analysis of the best relay selection scheme which has robustness based on selecting only the best single relay. Analytical expressions for the PDF, CDF and MGF of the SNR are used to derive the expressions for outage probability over Rayleigh Fading channels. In [11], a performance analysis has been performed for the lower bound SER performance of BPSK signal and for the outage capacity of AF cooperative diversity systems with best relay selection, and the SER expression given only for the BPSK signal and is involved with an integral requiring numerical calculations.

In this paper, we present a performance analysis of all relays for the AF cooperative networks of the half-hop and we derive new closed-form expressions for the cumulative density function (CDF), probability density function (PDF) of the end-to-end signal-to-noise ratio (SNR) of a dual-hop opportunistic AF relaying system, moment generating function (MGF), symbol error rate (SER) and outage probability ($P_{out}$) . We also compared performance study the effect of increasing the number of relays over Nakagami-m and Rayleigh fading channels we also analytical investigation of the
system performance and its numerical evaluation. Our analyses provide insights for the wireless cooperative system designers in order to fully exploit the advantages of the relay selection and the cooperative transmission paradigm.

**SYSTEM MODEL**

We consider a dual hop cooperative network model composed of a source, half duplex relay(s) ($1 \leq i \leq R$) and a destination, as shown in Fig. 1.

Assume that the channel state information is distinguished at the destination. We utilize time division multiplexing (TDM) to ensure the transmission from the source and the relay(s) occur in consecutive time slot. In the first time slot, the source transmits a signal $x$ to the relays. The received signal from the $i$th relay can be given as:

$$y_{SR} = h_{SR}x + \eta_{SR}$$  \hspace{1cm} (1)

where $h_{SR}$ is the channel gain between the source and the $i$th relay. $\eta_{SR} \sim CN(0,N_0)$ is the complex additive white Gaussian noise (AWGN) between the source and the $i$th relay while $N_0$ is the noise variance.

In the second time slot, the $i$th relay amplifies its received signal and forwards it to the destination. The destination receives the transmission as:

$$y_{RD} = G_i h_{RD} y_{SR} + \eta_{RD}$$  \hspace{1cm} (2)

where $h_{RD}$ is the channel gain between the $i$th relay and the destination and $\eta_{RD} \sim CN(0,N_0)$ is the complex additive white noise. Between the $R$ and $D$ by $G_i$ is the amplified gain expressed as:

$$G_i = E_s / (E_s |h_{SR}|^2 + N_o)$$ with $E_s$ as the average energy per symbol [12]. From (2), we can estimate the SNR at the destination as follows:

$$\gamma_D = \sum_{i=1}^{R} \frac{\gamma_{SR} \gamma_{RD}}{1 + \gamma_{SR} + \gamma_{RD}}$$  \hspace{1cm} (3)

where $\gamma_{SR} = |h_{SR}|^2 E_s / N_o$ and $\gamma_{RD} = |h_{RD}|^2 E_s / N_o$ are the instantaneous SNR of the $S-R_i$ and $R_i-D$ hops, respectively.

Since, solving the exact expressions in (3) can be complicated; we derive an upper bound from (3) as:
\[ \gamma_i^{up} = \min(\gamma_{SR}, \gamma_{RD}) \geq \frac{\gamma_{SR} \gamma_{RD}}{1 + \gamma_{SR} + \gamma_{RD}} \]  

(4)

Therefore, the upper bound for the equivalent SNR can be written as:

\[ \gamma_i^{up} \leq \sum_{i=1}^{R} \gamma_i^{up} \]  

(5)

The upper bound SNR given by (5) is more suitable for analysis and is shown to be quite accurate at medium and high SNR values [10].

**PERFORMANCE ANALYSIS**

In order to find the expressions of the outage probability, and average SER, we need to derive the MGF, thus we first derive the CDF of \( \gamma_i^{up} \), and then the PDF as a derivative from the CDF. Finally, we derive the MGF of \( \gamma_i^{up} \) with the help of the inverse Laplace transform. The MGF of \( \gamma_i^{up} \) can be written as:

\[ M_{\gamma_i^{up}}(s) = \prod_{i=1}^{R} M_{\gamma_i}(s) \]  

(6)

the CDF from the source to the \( i^{th} \) relay and from the \( i^{th} \) relay to the destination respectively. Let the \( \gamma_i = c_i \gamma_i^{up} \), \( \gamma_{SR} = c_i \gamma_i^{up} \) where \( c_1 = c_{SR} |h_{SR}|^2 |P_i| \), \( c_2 = c_{RD} |h_{RD}|^2 |P_i| \), \( c_1 \) and \( c_2 \) are the energy consumed at the source and destination respectively. Now we consider \( c_1 = c_2 = c \). The expressions for the CDF and the PDF of the SNR of the \( S-R_i \) and \( R_i-D \) hops, AF systems the CDF and PDF for Nakagami-\( m \) fading channels can be expressed as:

\[ F_{\gamma_i^{up}}(\gamma) = \begin{cases} \frac{\Gamma(m, \frac{m}{c_i \gamma^{up}} \gamma)}{\Gamma(m)} - \frac{2\Gamma(m, \frac{m}{c_i \gamma^{up}} \gamma)}{\Gamma(m)} & \text{for } \gamma < \frac{c_i \gamma^{up}}{m} \\
+ \left(\frac{\Gamma(m, \frac{m}{c_i \gamma^{up}} \gamma)}{\Gamma(m)}\right)^2 & \text{for } \gamma \geq \frac{c_i \gamma^{up}}{m} 
\end{cases} \]  

(7)

\[ f_{\gamma_i^{up}}(\gamma) = \frac{2\gamma^{m-1} e^{-\gamma/c_i \gamma^{up}}}{\Gamma(m)} \frac{1}{1-\frac{\Gamma(m, \frac{m}{c_i \gamma^{up}} \gamma)}{\Gamma(m)}} \]  

(8)

In the Rayleigh fading channel the special case for the CDF and PDF of the SNR of the \( S-R_i \) and \( R_i-D \) hops, AF systems the expression to:

\[ F_{\gamma_i}(\gamma) = 1 - 2 \exp \left( -\frac{\gamma}{\gamma_i^{up}} \right) \times \frac{\gamma^{(\gamma+1)}}{(\gamma_i^{up})^2} K_1 \left( \frac{\gamma^{(\gamma+1)}}{(\gamma_i^{up})^2} \right) \]  

(9)

\[ f_{\gamma_i}(\gamma) = \frac{2}{(\gamma_i^{up})^2} e^{-\gamma/c_i \gamma^{up}} \times \sqrt{\gamma^{(\gamma+1)}} K_1 \left( \frac{\gamma^{(\gamma+1)}}{(\gamma_i^{up})^2} \right) + (2\gamma+1)K_0 \left( \frac{\gamma^{(\gamma+1)}}{(\gamma_i^{up})^2} \right) \]  

(10)

where \( \gamma_i \) \( i = 1, 2 \) represents the instantaneous end-to-end received SNR at the destination, \( \gamma_i \) in link, the N source-to-destination link SNRs, \( S-R_i \) and \( R_i-D \) hops, the CDF of \( \gamma_i \) can be written as:

\[ F_{\gamma_i}(\gamma) = \left[ F_{\gamma_i^{up}}(\gamma) \right]^N \]  

(11)

And the PDF of \( \gamma_i \) can be written as:
When the CDF and the PDF of the instantaneous end- to- end SNR, \( \gamma_i \), are known, different system performance metrics can be evaluated. The outage probability, \( P_{\text{out}} \), defined as the probability that the instantaneous end-to-end SNR falls below a certain threshold SNR, \( \alpha_{1/2} \). \( P_{\text{out}} \) can be written as:

\[
P_{\text{out}} = P \{ \gamma \leq \alpha_{1/2} \} = \int_0^{\alpha_{1/2}} f_{\gamma_i} (\gamma) \, d\gamma = F_{\gamma_i} (\alpha_{1/2})
\]

We derive the MGF of \( \gamma^\text{up}_D \), using the PDF of \( \gamma^\text{up}_i \) in (9) as follows:

\[
M_{\gamma_i} (s) = \int_0^\infty e^{-s\gamma} f_{\gamma_i} (\gamma) \, d\gamma
\]

We get the \( M_{\gamma_i} (s) \) by solving the integral, the equation in (15) is attained, with \( sF_i (\cdot; \cdot; \cdot; \cdot) \) symbolizing the Gauss hypergeometric function [13, eq (9.100)], as follows:

\[
M_{\gamma_i} (s) = \prod_{i=1}^{g} \left( (m-1)! \left( \frac{cT_c}{m} - S \right)^m - \frac{1}{\Gamma(m)} \frac{(m/cT_c)^m \Gamma(m)}{m(2m/cT_c + s)^m} F_i \left( 1, 2m, m+1, \frac{m}{2m+cT_c S} \right) \right)
\]

Let the average SER for M-PSK signals can be written as

\[
P_{\text{SER}} = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_{\gamma^\text{up}_i} (s) \left( g_{M-\text{PSK}} / \sin^2 \theta \right) \, d\theta
\]

where \( g_{M-\text{PSK}} = \sin^2 (\pi/M) \)

For the upper bound expression, the upper bound value of the SER is obtained by using the maximum value of the integral:

\[
P_{\text{SER}} \leq \left( 1 - \frac{1}{M} \right) M_{\gamma^\text{up}_i} (s) (g_{M-\text{PSK}})
\]

**NUMERICAL RESULTS**

In this section, we provide numerical results for the PDF, CDF, outage probability \( (p_{\text{out}}) \) and SER expressions. Our results are obtained from different values of M-PSK modulation index, \( m \) and \( R \). We validate and compare our derived expressions using Monte Carlo simulations under Nakagami- \( m \) and Rayleigh fading channels. The results obtained using the exact analysis of dual-hop AF relaying with end-to-end SNR selection are presented. Recall that no exact solutions for such systems, in the case of general fading environments, are known and only performance bounds have been reported in the literature.

Fig. 2 shows the Outage probability as a function of the average SNR and threshold, for different values of \( R \) versus the link average SNR, of dual-hop AF systems. Identical Rayleigh, Nakagami- \( m \) fading links with \( m = 2 \) and assumed for the case of \( R = 1, 2, 4 \) available relays. By using (14) the outage probability versus SNR with Binary phase shift keying (BPSK) is assumed. The figures show precise agreement between the results obtained from the analytical solution and simulation results at all values of SNR, with the exact agreement between the results obtained from the analytical solution and simulation results at all values of SNR.

Fig. 3 compares between Rayleigh and Nakagami- \( m \) fading for different numbers of relays ( \( R = 1, 2, 4 \) ). We can see that \( n \) increasing the number of relays significantly decreases the SER. increasing the number of relays significantly decreases the SER.

In Fig. 4 and Fig. 5 we compare the PDF and CDF values respectively for both theoretical and simulation . The results are obtained from different values of \( m \) parameter over Nakagami- \( m \). The figures reveal that if \( m \) increases both the PDF and the CDF shift to the right direction. Moreover, the peak value of the PDF becomes smaller which means that it will be more evenly distributed. The result also shows that \( (\gamma^\text{up}_D/m) \) is enhanced and the overall SNR improves as the number relays increases.
Fig 2: Outage probability with versus threshold. Identical Rayleigh and Nakagami-m fading channels. Different numbers of relays are assumed, \( R = 1, 2, 4 \).

Fig 3: Comparison of SER with versus average SNR of 8PSK for Rayleigh and Nakagami-m and Different numbers of relays are assumed, \((R=1,2,4)\).
Fig. 4. CDF of Nakagami-m fading channels with different parameter $m=0.6,2,6$.

Fig. 5. PDF of Nakagami-m fading channels with different parameter $m=1,2,3$. 
CONCLUSION

In this paper, we have derived exact closed-form expressions for the PDF, CDF and the MGF for the channels state information operates in a half–duplex mode, we had also derived and analyzed indeed closed-form expressions for the SER and P_{out} over Nakagami-m and Rayleigh fading channels and We compared between the cumulative density function (CDF) and probability density function (PDF) for theoretical and simulation links in which the best relay scheme is adopted for different SNRs that analyzed the performances of AF cooperative relaying in wireless communication systems with mean channel gains over the Nakagami-m fading channel in high SNR. We also provided Numerical results to shown the performance of SER for different M-PSK modulation signals improvement of our scheme compared Nakagami-m with Rayleigh fading channels, for a different number of relays and m parameters.

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