Radius stabilization in 5D SUGRA models on orbifold

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Abstract. We study a four-dimensional effective theory of the five-dimensional (5D) gauged supergravity with a universal hypermultiplet and perturbative superpotential terms at the orbifold fixed points. The class of models we consider includes the 5D heterotic M-theory and the supersymmetric Randall-Sundrum model as special limits of the gauging parameters. We analyse the vacuum structure of the models, especially the nature of the moduli stabilization, from the viewpoint of the effective theory.

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1 Introduction

The five-dimensional (5D) gauged supergravity compactified on an orbifold $S^1/Z_2$ includes interesting models, such as the low-energy effective theory of the strongly coupled heterotic string theory [1], or the supersymmetric (SUSY) extension of the Randall-Sundrum (RS) models [2]. Here we consider a certain class of models that is described as the 5D gauged supergravity with a universal hypermultiplet and superpotentials at the orbifold fixed points (boundaries). The hyperscalar manifold has an $SU(2,1)$ isometries and we gauge two directions among them by the graviphoton. This theory is reduced to the above two models when we take certain limits of the gauging parameters. Our purpose here is to investigate the vacuum structures for this class of models [3].

2 $N=1$ off-shell description of 5D action

For our purpose the off-shell (superconformal) description of the 5D supergravity [4] is useful because it enables us to treat the localized terms at the orbifold boundaries independently from the bulk action, and the isometries of the scalar manifold are linearized. Each 5D superconformal multiplet can be decomposed into $N=1$ multiplets. In our case, we have two compensator hypermultiplets $(\Phi^1, \Phi^2)$, $(\Phi^3, \Phi^4)$, one physical hypermultiplet $(\Phi^5, \Phi^6)$, and the graviphoton multiplet $(V, \Sigma)$ besides the 5D Weyl multiplet. Here $\Phi^a (a = 1,2,\cdots, 6)$ and $\Sigma$ are $N=1$ chiral multiplets and $V$ is an $N=1$ vector multiplet. The 5D off-shell action is expressed in terms of these $N=1$ multiplets and an $N=1$ general multiplet $V_E$ whose scalar component is $e^\phi$ [5]. Since $V_E$ has no kinetic term in the $N=1$ off-shell description, it can be integrated out and the 5D action is rewritten as [6]

$$
\mathcal{L} = -3e^{2\sigma} \int d^4\theta \mathcal{V} \left\{ d_a^b \Phi^b \left( e^{-2(\tilde{\alpha} \cdot T)\phi} \right)^a_c \Phi^c \right\}^{2/3}
$$

$$
- e^{3\sigma} \left[ \int d^2\theta \Phi^a d_a^b \rho_{bc} (\partial y - 2(\tilde{\alpha} \cdot T) \Sigma)^c_d \Phi^d + \text{h.c.} \right] + \sum_{\theta=0,\pi} \mathcal{L}^\theta \delta (y - \theta R) + \cdots,
$$

where $e^\sigma$ is the warp factor of the background metric, $d_a^b = \text{diag}(1_4,-1_2)$, $\rho_{ab} = \sigma_2 \otimes \mathbf{1}_3$, and $\mathcal{V} = -\partial y V + \Sigma^2$ is a gauge-invariant quantity. The ellipsis denotes terms irrelevant to the following discussion. The boundary Lagrangians $\mathcal{L}^\theta (\theta = 0,\pi)$ are written as

$$
\mathcal{L}^\theta = e^{3\sigma} \left[ \int d^2\theta \Phi^2 \Phi^3 P_\theta \left( \Phi^5 \Phi^6 \right) + \text{h.c.} \right],
$$

where $P_\theta (\theta = 0,\pi)$ are the boundary superpotentials.

The most general form of the gauging is parameterized by

$$
\tilde{\alpha} \cdot T \equiv \sum_{i=1}^{8} \tilde{\alpha}_i T^i,
$$

acting on $(\Phi^1, \Phi^3, \Phi^5)^t$ or $(\Phi^2, \Phi^4, \Phi^6)^t$, where $T^i (i = 1,2,\cdots, 8)$ are $3 \times 3$ matrix-valued generators of $SU(2,1)$ shown in the Appendix of Ref. [3]. The real coefficients $\tilde{\alpha}_i$ determine the gauging direction. Now we consider a case that two independent isometries are gauged by the graviphoton, that is, $\tilde{\alpha}_i$ are parametrized by two parameters $\alpha$ and $\beta$ as

$$
\tilde{\alpha}_3 = 2\beta, \quad \tilde{\alpha}_6 = \alpha, \quad \tilde{\alpha}_8 = \alpha + \beta,
$$

$$
\tilde{\alpha}_i = 0. \quad (i \neq 3,6,8)
$$

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$^1$ In Ref. [3], we also consider gauging of one more independent direction.
For simplicity we assume that the boundary superpotentials $P_\theta$ ($\theta = 0, \pi$) consist of only constant and tadpole terms for the universal hypermultiplet, i.e.,

$$P_\theta(Q) = w_\theta^{(0)} + w_\theta^{(1)} Q,$$

where $w_\theta^{(0)}$ and $w_\theta^{(1)}$ are constants.

### 3 4D effective action

We can derive the four-dimensional (4D) effective action by the off-shell dimensional reduction proposed by Ref. [6], which are based on the $N = 1$ superspace description [5] of the 5D off-shell supergravity and developed in subsequent studies [7]. This method enables us to derive the 4D effective action directly from the 5D off-shell supergravity action keeping the $N = 1$ off-shell structure. Note that only even multiplets under the orbifold $Z_2$-parity have zero-modes that appear in the effective theory. In our model the $Z_2$-even multiplets are $\Sigma, \Phi^2, \Phi^3$ and $\Phi^5$, and they appear in the 5D action only through the combinations of $\Sigma, \Phi^2 \Phi^3$ and $\Phi^5 / \Phi^3$, which have respectively the radion multiplet $T$, the 4D chiral compensator $\phi$ and the matter multiplet $H$ as the zero-modes. Following the procedure of Ref. [6], we obtain the 4D effective action as

$$S_{\text{eff}} = -3 \int d^4 \theta \ |\phi|^2 e^{-K/3} + \left\{ \int d^2 \theta \ \phi^3 W + \text{h.c.} \right\},$$

where the Kähler potential $K$ and the superpotential $W$ are given by

$$K = -3 \ln \left\{ \frac{1}{2\alpha} \mathcal{F}(S_0, S_\pi) \right\},$$

$$W = e^{-\frac{2}{3}qS_0} (a_0 + b_0 S_0) - e^{-\frac{2}{3}qS_\pi} (a_\pi + b_\pi S_\pi).$$

Here $q \equiv \beta / \alpha$ and

$$\mathcal{F}(S_0, S_\pi) = q^{-\frac{1}{2}} \left\{ \Gamma \left( \frac{4}{3}, q \text{Re} S_0 \right) - \Gamma \left( \frac{4}{3}, q \text{Re} S_\pi \right) \right\},$$

where $\Gamma(a, x) \equiv \int_x^\infty dt \ t^{a-1} e^{-t}$ is the incomplete gamma function. The chiral multiplets $S_\theta$ ($\theta = 0, \pi$) are defined from $H$ and $T$ as

$$S_0 \equiv \frac{1-H}{1+H}, \quad S_\pi \equiv S_0 + 2\pi \alpha T.$$

The parameters $a_\theta$ and $b_\theta$ in the superpotential $W$ are given by linear combinations of the constants in the boundary superpotentials (5) as

$$a_\theta = \frac{1}{8} \left( w^{(0)}_\theta + w^{(1)}_\theta \right), \quad b_\theta = \frac{1}{8} \left( w^{(0)}_\theta - w^{(1)}_\theta \right),$$

where $\theta = 0, \pi$.

### 4 Heterotic M-theory limit

In the limit $\beta \to 0$, Eq.(7) becomes

$$K = -3 \ln \left[ \frac{3}{8\alpha} \left\{ \left( \text{Re} S_\pi \right)^{4/3} - \left( \text{Re} S_0 \right)^{4/3} \right\} \right],$$

$$W = b_0 (C + S_0 - r S_\pi),$$

where $C \equiv (a_0 - a_\pi) / b_0$ and $r \equiv b_\pi / b_0$. The above Kähler potential reproduces the known result, i.e., the 4D effective Kähler potential of the heterotic M-theory [1] when $\text{Re} S_0 \gg \pi \alpha \text{Re} T$.

The scalar potential $V$ is calculated as

$$V = \left( \frac{8\alpha}{3} \right)^{3/2} |b_0|^2 \left\{ \frac{|C - S_0 + r S_\pi|^2}{\left\{ \left( \text{Re} S_\pi \right)^{4/3} - \left( \text{Re} S_0 \right)^{4/3} \right\}^3} \right\} - \frac{3 |r|^2 \left( \text{Re} S_\pi \right)^{2/3} - \left( \text{Re} S_0 \right)^{2/3}}{\left\{ \left( \text{Re} S_\pi \right)^{4/3} - \left( \text{Re} S_0 \right)^{4/3} \right\}^2},$$

In this article we use the same symbols for the scalar fields as those for the chiral multiplets they belong to. From the SUSY preserving conditions: $D_{S_0} W = D_{S_\pi} W = 0$, we find a SUSY point,

$$\left( \text{Re} S_0, \text{Re} S_\pi \right) = \left( \frac{2\pi C}{r^4 - 1}, \frac{2\pi \text{Re} C}{r^4 - 1} \right),$$

$$\text{Im} C + \text{Im} S_0 - \pi \text{Im} S_\pi = 0.$$

From the second equation, we find a flat direction in the imaginary direction of $S_\theta$ ($\theta = 0, \pi$). The superpotential $W$ takes the nonzero value at this point. Thus the vacuum energy is negative, that is, the geometry is $\text{AdS}_4$. By evaluating the second derivatives of the potential (12), we can see that this SUSY point is a saddle point. Here we should note that SUSY points are always stable in a sense that they satisfy the Breitenlohner-Freedman bound [8]. In Sect. 6.2, we uplift the negative vacuum energy of the SUSY $\text{AdS}_4$ vacuum by a SUSY breaking vacuum energy in the hidden sector in order to obtain a SUSY breaking Minkowski vacuum, which is a candidate of our present universe. In general a SUSY saddle point remains to be a saddle point after the uplifting unless the uplifting potential is sufficiently steep, and it will not be stable any more. So we would like to look for a local minimum of the potential which is expected to be stable even after the uplifting.

### 5 SUSY Randall-Sundrum limit

In the limit $\alpha \to 0$, Eq.(7) becomes

$$K = -3 \ln \left( \frac{1 - |Q|^2}{2\beta} \right) - \ln(\text{Re} S_0),$$

$$W = (a_0 + b_0 S_0) - (a_\pi + b_\pi S_0) Q^2,$$

where $Q \equiv e^{-\beta \pi T}$ is a warp factor superfield. The above $K$ reproduces the radion Kähler potential of the
SUSY RS model [9]. Although $H$ is more conventional than $S_0$ for the SUSY RS model, we use the latter as a matter chiral multiplet because we will interpolate this model and the Heterotic M-theory limit discussed in the previous section. We can always translate $S_0$ to $H$ by the relation (9).

For simplicity, we assume in the following that the parameters satisfy a relation $a_0 b_\pi - b_0 a_\pi = 0$, that is,

$$a_0 b_\pi = c(b_0, b_\pi),$$

where $c$ is a constant. Then, from the SUSY conditions: $D_{S_0} W = D_H W = 0$, we find a SUSY point,

$$(S_0, \Omega) = (-c, r^{-1/3}),$$

for $Re c < 0$, and

$$(S_0, \Omega) = (\bar{c}, |r|^{-4/3} \bar{r}^{1/3}),$$

for $Re c > 0$. Here $r \equiv b_\pi / b_0$. When $Re c = 0$, $\Omega$ is undetermined by the SUSY conditions and has a flat direction. At the SUSY point (16) $W = 0$ and thus the vacuum energy vanishes, resulting a local Minkowski minimum. This corresponds to the SUSY Minkowski vacuum discussed in Ref. [10], in which the boundary superpotentials (5) consist of only the tadpole terms, i.e., $w_{\text{us}}^{(0)} = 0$ (or $c = -1$). On the other hand, the SUSY point (17) is a saddle point and $W$ does not vanish there.

The scalar potential $V$ is calculated as

$$V = \frac{8 \beta^3 |b_0|^2}{(1 - |\Omega|^2)^2} \Re S_0 \left\{ \frac{|c - S_0|^2 |1 - r \Omega^3|^2}{1 - |\Omega|^2} - 3 |c + S_0|^2 (1 - |r|^2 |\Omega|^4) \right\}. \tag{18}$$

Now we focus on the SUSY minimum (16). We decompose the complex scalars into real ones as

$$S_0 = s + i \sigma, \quad \Omega = \omega e^{i \varphi}. \tag{19}$$

Evaluating the second derivatives of the scalar potential (18), we can see that the four real scalars $(s, \omega, \sigma, \varphi)$ do not mix with each other. Then after normalizing them canonically, the mass eigenvalues are found as

$$m_s^2 = m_\sigma^2 = 96 \beta^3 |b_0|^2 |\Re c| \left\{ \frac{|r|^{4/3} (1 + |r|^{-2/3})^2}{1 - |r|^{-2/3}} \right\}$$

$$m_\omega^2 = m_\varphi^2 = 48 \beta^3 |b_0|^2 |\Re c| \left\{ \frac{|r|^{2/3}}{1 - |r|^{-2/3}} \right\}. \tag{20}$$

We have assumed that $Re c < 0$ and $|r| > 1$.

6 Interpolation between the two models

6.1 Vacuum structure

In the vicinity of $\alpha = 0$, the Kähler and the superpotentials in (7) are expressed as

$$K = -3 \ln \left[ \frac{1}{\beta} \left( 1 - |\Omega|^2 + \frac{1 - |\Omega|^2 + |\Omega|^2 \ln |\Omega|^2}{3 q \Re S_0} \right) \right]$$

$$+ O \left( \frac{1}{q^2 (\Re S_0)^2} \right) \right] - \ln (\Re S_0),$$

$$W = b_0 \left\{ (c + S_0)(1 - r \Omega^3) + \frac{2 r}{q} \Omega^2 \ln \Omega \right\}. \tag{21}$$

Here $q |\Re c|$ is supposed to be large. From the SUSY conditions: $D_{S_0} W = D_H W = 0$, we find a SUSY point as

$$S_0 = -c \left\{ 1 - \frac{2}{3 q c} \left( \frac{1 - \ln r}{1 - |r|^{-2/3}} \right) + O \left( \frac{1}{q^2 (\Re c)^2} \right) \right\}$$

$$\Omega = r^{-1/3} \left\{ 1 - \frac{\ln r}{9 q \Re c} + O \left( \frac{1}{q^2 (\Re c)^2} \right) \right\}. \tag{22}$$

Since the SUSY point (16) is a local minimum of the potential, this SUSY point is also a local minimum when $q |\Re c| \gg 1$. Due to the correction from the SUSY RS limit, the superpotential $W$ does not vanish at this point,

$$W = -\frac{2 b_0}{3 q} \ln r \left\{ 1 + O \left( \frac{1}{q^2 (\Re c)^2} \right) \right\}, \tag{23}$$

and the vacuum energy is

$$V = -3 e^K |W|^2$$

$$= -\frac{32 \beta^3 |b_0|^2 \ln r^2}{3 q^2 |\Re c| (1 - |r|^{-2/3})^3} \left\{ 1 + O \left( \frac{1}{q \Re c} \right) \right\}. \tag{24}$$

Thus this is an AdS$_4$ SUSY vacuum.

6.2 Uplifting

From the AdS$_4$ SUSY vacuum such as (22), we can obtain a SUSY breaking Minkowski minimum by introducing a sequestered SUSY-breaking sector just like the KKLT model [11]. Following the KKLT model, the uplifting potential $U$ is assumed as [12]

$$U = \int d^4 \bar{\phi} \left( \bar{\phi} \phi \right)^n \kappa \theta^2 \bar{\theta}^2 = \kappa e^{n K/3}$$

$$= \frac{\kappa (2 \beta)^n}{(\Re S_0)^n (1 - |\Omega|^2)^n} \left\{ 1 + O \left( \frac{1}{q \Re c} \right) \right\}, \tag{25}$$

where $\kappa$ is a constant. The typical value of $n$ for the sequestered SUSY breaking source is given by $n = 2$. The total scalar potential is then given by $V_{\text{tot}} \equiv V + U$. If we choose $\kappa$ as

$$\kappa = \frac{4 (2 \beta)^3}{3 q^2 |\Re c|^{1 - n/3} (1 - |r|^{-2/3})^{3-n}} \left\{ 1 + O \left( \frac{1}{q \Re c} \right) \right\}, \tag{26}$$

then the minimum value of the total potential $V_{\text{tot}}$ vanishes, and we can obtain a SUSY breaking Minkowski.
vacuum. At this minimum, we can evaluate the order parameter of the SUSY breaking. Following Ref. [12], we define the anomaly/modulus ratio of SUSY breaking as

$$\alpha_{A/M} = \frac{1}{\ln(M_P^2/m_{3/2}^2)} \cdot \frac{F^\phi / \phi}{F^T / (T + T)}$$ (27)

where $F^\phi$ and $F^T$ are the F-terms of $\phi$ and $T$ respectively. Then we find that

$$\alpha_{A/M} = \frac{q \Re c}{\ln(M_P^2/m_{3/2}^2)} \left\{ \frac{6}{n} \ln r \right\} \left[ \frac{1}{n} |r|^{2/3} (1 + |r|^{-2/3})^2 \times (1 - |r|^{-2/3}) + O \left( \frac{1}{q \Re c} \right) \right\}. (28)$$

Since $|r| = e^{3\pi \beta \Re Te}$ from (16), we can see that $\alpha_{A/M} \gg 1$ unless $\beta$ is small. Thus the anomaly mediation tends to dominate in this model. However, for small values of $\beta$, the parameter $|r|$ is allowed to be of $\mathcal{O}(1)$ and the modulus mediated contribution can be comparable to that of the anomaly mediation. For example, $|r| = e^{3\pi \beta \Re Te} (1)$ when $n = 2, r = 2, q \Re c = -8, \ln(M_P^2/m_{3/2}^2) = 4\pi^2$. In this case, the mirage mediation is realized. Finally note that the moduli masses, which are given by (20) at the leading of the $(q \Re c)^{-1}$ expansion, are much larger than the gravitino mass,

$$m_{5/2}^2 = e^K |W|^2 = \frac{32\beta^3 |b_0 \ln r|^2}{9q^2 |\Re c| (1 - |r|^{-2/3})^3} \left( 1 + O \left( \frac{1}{q \Re c} \right) \right). (29)$$

7 Summary

We studied the 4D effective theory of the 5D gauged supergravity on an orbifold with a universal multiplet and boundary superpotentials. We analysed a class of models obtained by gauging two independent isometries on the scalar manifold. It includes the 5D heterotic M-theory and the SUSY RS model as special limits of the gauging parameters. We have investigated the vacuum structure of this class of models and the nature of modulus stabilization assuming perturbative superpotentials at the orbifold boundaries.

In the heterotic M-theory limit, the SUSY point is a saddle point of the potential. In the SUSY RS limit, on the other hand, the SUSY point is a local minimum with vanishing vacuum energy when the parameters satisfy the relation (15) with $\Re c < 0$. These SUSY points in the two different limits continuously transit to each other by changing the ratio $q = \beta/\alpha$ [3]. When $|q \Re c| \gg 1$, there is a SUSY AdS$_4$ vacuum which is a good candidate for the KKL T-type uplifting. Thus we studied the uplifting of this vacuum and find that the mirage mediation can be realized for small values of $\beta$, while the effect of the anomaly mediation is dominant for $\beta \sim \mathcal{O}(1)$. The moduli are much heavier than the gravitino in either case.

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