Supersymmetry Tests from a Combined Analysis of Chargino, Neutralino, and Charged Higgs Boson Pair Production at a 1 TeV Linear Collider

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ABSTRACT

We consider the production of chargino, neutralino and charged Higgs boson pairs at future linear colliders for c.m. energies in the one TeV range within the MSSM. We compute the leading (double) and next-to-leading (linear) supersymmetric logarithmic terms of the so-called "Sudakov expansion" at one-loop level. We show that a combined analysis of the slopes of the chargino, neutralino, and charged Higgs boson pair production cross sections would offer a simple possibility for determining $\tan\beta$ for large ($\sim 10$) values, and the parameters $M_1$, $M_2$, and $\mu$. This test could provide a strong consistency check of the considered supersymmetric model.

1. Introduction

The Minimal Supersymmetric Standard Model (MSSM) looks still the most attractive extension of the Standard Model. In this model we have two charged bosons $H^\pm$, two charginos $\tilde{\chi}_i^\pm$, mixtures of the W-inos and charged Higgsinos, and four neutralinos $\tilde{\chi}_j^0$ which are mixtures of a B-ino, Z-ino and the two neutral Higgsinos. Realistic hope exists within the elementary particle physics community that at the upcoming experiments at Tevatron \cite{1} and LHC \cite{2} will finally reveal the existence of supersymmetric particles, via direct production of sparticle-antisparticle pairs. A logical and necessary next step then is to measure the properties of the underlying theory to get access to a more unified theory. For the MSSM case, a high luminosity linear $e^+e^-$ collider \cite{3} working in the TeV energy region will be the appropriate tool for that purpose. This machine should be sufficiently accurate to provide the same kind of consistency tests of the model that were achieved in the one and two hundred GeV region at LEP1 and LEP2 for the Standard Model, from detailed analyses of several independent one-loop virtual effects.

In this contribution we show how to get access to some of the parameters of the MSSM parameters. We will do a combined analysis of charged Higgs boson, chargino, and neutralino pair production at a linear $e^+e^-$ collider working at a CMS energy $\sqrt{s} \sim 1$ TeV. The production cross sections have been calculated at one-loop level in the "Sudakov" approximation \cite{4}.

2. One-loop results

If $\sqrt{s}$ is sufficiently larger than all the masses of the particles involved in the process, a logarithmic Sudakov expansion can be adopted. At a 1 TeV collider this is fullfilled for
a SUSY mass scale $M_{\text{SUSY}} \lesssim 350$ GeV. The universal, process independent part has the form $(a \ln s/M_W^2 - \ln^2 s/M_W^2)$. For the complete results for the production cross section $e^+ e^- \rightarrow H^+ H^-, \tilde{\chi}^+_i \tilde{\chi}^-_j, \tilde{\chi}^{0}_k \tilde{\chi}^{0}_l$ within this approximation we refer to [5,6,7].

The full amplitude $A$ can be decomposed into tree-level and one-loop part, $A = A^{\text{tree}} + A^{\text{1loop}}$, with the one-loop part consisting of three parts,

$$A^{\text{1loop}} = A^{\text{RG}} + A^{\text{nonuniv}} + A^{\text{univ}}.$$ (1)

The RG (renormalization group) contribution stems from the running behavior of the couplings. The non-universal, process dependent contribution is the scatter angle $\vartheta$ dependent part of the box graphs and the third term is the $\vartheta$ independent universal contribution.

The $\text{RG contribution} A^{\text{RG}}$ gives linear logs generated by the ”running” of $g$ and $g'$

$$A^{\text{RG}} = -\frac{1}{4\pi^2} \left( g^4 \tilde{\beta}_0 \frac{dA^{\text{tree}}}{dg^2} + g'^4 \tilde{\beta}'_0 \frac{dA^{\text{tree}}}{dg'^2} \right) \log \frac{s}{\mu^2}.$$ (2)

$\mu$ is the scale where the numerical values of $g$, and $g'$ are defined. The $\beta$-functions in the MSSM are: $\tilde{\beta}_0 = -1/4$ and $\tilde{\beta}'_0 = -11/4$.

The $\text{non universal contribution}$ can be written as $A^{\text{non univ}} = A^{\text{tree}} \cdot c^{\text{ang}}$. They are logarithmically dependent on $t/s = -(1 - \cos \vartheta)/2$ and $u/s = -(1 + \cos \vartheta)/2$.

The $\text{universal contribution} A^{\text{univ}}$ can be of quadratic and of linear log type, produced by initial vertex, and final vertex and box diagrams, $A^{\text{univ}} = A^{\text{in}} + A^{\text{fin}}$.

In all three productions the correction to the initial $e^+ e^-$ lines can be written in terms of tree-level structures with the coefficients

$$c_{\alpha}^{\text{in}} = \frac{1}{16\pi^2} \left( g^2 I_{e\alpha} (I_{e\alpha} + 1) + g'^2 \frac{Y^2}{4} \right) \left( 2 \log \frac{s}{M_W^2} - \log^2 \frac{s}{M_W^2} \right), \quad \alpha = L, R.$$ (3)

The universal term $A^{\text{fin}}$ is different for $H^+ H^-, \tilde{\chi}^+_i \tilde{\chi}^-_j$, and $\tilde{\chi}^{0}_k \tilde{\chi}^{0}_l$ production. In general, the total amplitude can be written as a combination of a S-, T-, and U-term, with the coefficients

$$A_{ij}^{ab} \equiv \frac{e^2}{s} S_{ij}^{ab} + \frac{e^2}{u} U_{ij}^{ab} + \frac{e^2}{t} T_{ij}^{ab}, \quad a, b = L, R.$$ (4)

$\text{Charged Higgs boson pair production}$: the total relative one-loop correction $\Delta(q^2)$ can be written as

$$\Delta(q^2) = \frac{\sigma^{\text{tree+1loop}} - \sigma^{\text{tree}}}{\sigma^{\text{tree}}}.$$ (5)

In this case the one-loop Sudakov expansion is a very compact expression,

$$\Delta(q^2) = - \left( \frac{3\alpha}{4\pi s_W^2 M_W^2} \right) (m_t^2 \cot^2 \beta + m_b^2 \tan^2 \beta) \log \frac{q^2}{M_W^2} + \Delta_{\text{RG}}(q^2),$$ (6)
dependent only on one SUSY parameter, on $\tan \beta$.

**Chargino pair production:** $A^{\text{fin}}$ has many contribution. Only the Yukawa part is shown here,

$$S_{ij}^{\text{fin}} = \frac{\alpha}{4\pi} \sum_{k=1}^{2} S_{ik}^{\text{tree}} \cdot c_{kj}^{\text{Yuk}} + \ldots , \quad \text{with}$$

$$c_{kj}^{\text{Yuk}} = - \frac{3\alpha}{2s_w^2M_W^2} \left[ \frac{m_t^2}{\sin^2 \beta} Z_{2k}^* Z_{2j}^* \delta_{bL} + \frac{m_b^2}{\cos^2 \beta} Z_{2k}^* Z_{2j}^* \delta_{bR} \right] \log \frac{s}{M_W^2} . \quad (8)$$

**Neutralino pair production:** $A^{\text{fin}}$ has contributions from all three possible structures,

$$S_{ij}^{\text{fin}} = \frac{\alpha}{4\pi} \sum_{k=1}^{4} S_{ik}^{\text{fin gauge}} \cdot c_{kj}^{\text{Yuk}} , \quad X_{ij}^{\text{fin}} = \frac{\alpha}{4\pi} \sum_{k=1}^{4} X_{ik}^{\text{tree}} \cdot c_{kj}^{\text{Yuk}} , \quad (9)$$

with $X = U, T$, and symmetrized indizes $i$ and $j$. The coefficients are

$$c_{kj}^{\text{fin gauge}} = \frac{1 + 2c_w^2}{8s_w^2c_w^2} \left[ \left( Z_{4k}^* Z_{4j}^N + Z_{3k}^* Z_{3j}^N \right) \delta_{bL} + (\text{h.c.})\delta_{bR} \right] \left( 2\log \frac{s}{M_W^2} - \log^2 \frac{s}{M_W^2} \right) ,$$

$$c_{kj}^{\text{fin Yuk}} = - \frac{1}{s_w^2} \left[ Z_{2k}^* Z_{2j}^N \delta_{bL} + Z_{2k}^* Z_{2j}^N \delta_{bR} \right] \log^2 \frac{s}{M_W^2} ,$$

$$c_{kj}^{\text{Yuk}} = - \frac{3}{4s_w^2M_W^2} \left[ \left( \frac{m_t^2}{\sin^2 \beta} Z_{4k}^* Z_{4j}^N + \frac{m_b^2}{\cos^2 \beta} Z_{3k}^* Z_{3j}^N \right) P_L + (\text{h.c.})P_R \right] \log \frac{s}{M_W^2} .$$

The tree level coefficients can be found in [6,7]. Note, that the notation $Z^+ \equiv V, Z^- \equiv U$, $Z_N \equiv Z$, and e.g. $S_{ik}^{\text{fin Yuk}} \equiv S_{ik}^{\text{fin gauge}} c_{kj}^{\text{Yuk}}$ is used.

3. **Numerical results**

We show results for three scenarios, $S_1$, $S_2$, and $S_3$. The scenario $S_1$ is the Tesla benchmark point RR2[8] with the two lightest neutralinos being respectively 95% bino and 82% wino. The set $S_2$ is a mixed scenario with neutralinos having non negligible gaugino and Higgsino components; $\tilde{\chi}_1^0$ is 86% bino and 13% Higgsino, $\tilde{\chi}_2^0$ is 11% bino, 48% wino and 41% Higgsino. Finally, $S_3$ is a purely Higgsino one with the two lightest neutralinos being 92% and 98% Higgsino like. The values of the input parameters as well as the masses of the two charginos and of the two lightest neutralinos are summarized in Tab. (1). We have performed a standard $\chi^2$ analysis assuming in the various scenarios 10-12 experimental points at energies ranging from 700-850 GeV (depending on the scenario) up to 1200 GeV and assuming an experimental accuracy of 1% for the cross sections of chargino, and neutralino, and 2% for that of charged Higgs boson pair production.

All three figures have the same order of contour plots. The upper ones are the planes $(M_1, M_2)$, $(M_1, \mu)$, $(M_1, \tan \beta)$ and the lower ones the planes $(M_2, \mu)$, $(M_2, \tan \beta)$, and $(\mu, \tan \beta)$. The dashed lines denote the results based on the analysis in [6], where the neutralino channels are not yet included. In Fig. 1 we see that a combined analysis gives closed areas for all six planes, also for $(M_2, \mu)$, but the bounds on $(M_2, \tan \beta)$, and $(\mu, \tan \beta)$ practically do not improve. The results for $S_2$ are similar, but the improvement
Table 1: Input parameters and masses of charginos, and lightest neutralinos for the three input sets $S_1$, $S_2$, and $S_3$. All parameter are given in GeV, and $\tan \beta = 30$ in all three scenarios.

|       | $M_1$ | $M_2$ | $\mu$ | $\tilde{\chi}_1^\pm$ | $\tilde{\chi}_2^\pm$ | $\tilde{\chi}_1^0$ | $\tilde{\chi}_2^0$ |
|-------|-------|-------|-------|-----------------------|-----------------------|-------------------|-------------------|
| $S_1$ | 78    | 150   | 263   | 132                   | 295                   | 75                | 133               |
| $S_2$ | 100   | 200   | 200   | 149                   | 266                   | 92                | 153               |
| $S_3$ | 200   | 400   | 100   | 95                    | 417                   | 82                | 109               |

Figure 1: $S_1$ scenario, 1σ error bounds on the MSSM parameters $M_1$, $M_2$, $\mu$, and $\tan \beta$. In this and in the following figures the crosses denote the values of the parameters in the specific scenario.

Figure 2: $S_2$ scenario, 1σ error bounds on the MSSM parameters $M_1$, $M_2$, $\mu$, and $\tan \beta$. is better now, especially in the $(M_2, \mu)$ plane. In the Higgsino scenario $S_3$ the two light
neutralinos are quite insensitive to the Higgsino component. Therefore there is no upper bound on $M_1$, as can be seen in the first three figures. The error on $\tan \beta$ is reduced from $\sim 30\text{-}40\%$ in the first two figures to $\sim 10\%$.

From a mass measurement only, one cannot deduce a 1-1 correspondence between measured masses and the parameters $M_1$, $M_2$, $\mu$, and $\tan \beta$. Different sets of parameters can reproduce “essentially” the same masses \cite{9}. The $\chi^2$ analysis presented in this work helps to disentangle this ambiguity.

4. References

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