Modeling and Control of Discrete Event Systems under Joint Sensor-Actuator Cyber Attacks

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Abstract—In this paper, we investigate joint sensor-actuator cyber attacks in discrete event systems. We assume that attackers can attack some sensors and actuators at the same time by altering observations and control commands. Because of the nondeterminism in observation and control caused by cyber attacks, the behavior of the supervised system becomes nondeterministic and may deviate from the safety specification. We define the upper-bound on all possible languages that can be generated by the supervised system to investigate the safety supervisory control problem under cyber attacks. After introducing CA-controllability and CA-observability, we prove that the supervisory control problem under cyber attacks is solvable if and only if the given specification language is CA-controllable and CA-observable. Furthermore, we obtain methods to calculate the state estimates under sensor attacks and to synthesize a state-estimate-based supervisor to achieve a given safety specification under cyber attacks. We further show that of all the solutions, the proposed state-estimate-based supervisor is maximally-permissive.

Index Terms—Discrete event systems, supervisory control, cyber attacks, network security, resilient control

I. INTRODUCTION

In the past ten years and more, cyber-physical systems have become a hot research topic [1], [2]. For a typical cyber-physical system, the information is transmitted via wired/wireless networks and such networked systems are vulnerable to cyber attacks [3], [4]. Indeed, several attacks on networked systems have been reported in the literature [5]–[8]; the following examples are given in [5], [9]–[12]: (1) Stuxnet attack on an Iranian uranium enrichment plant in 2011; (2) RQ-170 attack in 2011, where US operators lost control of an RQ-170 unmanned aerial vehicle which subsequently landed in Iran; (3) a series of attacks on Ukrainian power distribution networks that caused outages as well as lasting damage in 2015; (4) the Maroochy water services attack in Queensland, Australia by a disgruntled employee in 2000; and (5) a Jeep hijacked remotely by attackers while driving at 70 mph on a highway in St. Louis, USA.

Cyber attacks may change the readings of sensors and hence change the observations of controllers which will cause the controllers to make incorrect decisions. They may also alter the actions of actuators and hence change the behavior of the system. How to protect the feedback control systems from cyber attacks in critical infrastructures has become an increasingly important problem [5], [13]–[15].

At the supervisory layer, a cyber-physical system can be abstracted as a discrete event system and the control objective is often to ensure that the supervised system is safe [16]–[18]. Within the discrete event system framework, cyber attacks are investigated extensively.

An early work is done in [19], which shows that cyber attacks can damage the security of the supervised system and proposes a method to estimate how much damage that cyber attacks might cause. In [20], [21], four types of cyber attacks are discussed: event insertion attacks, event deletion attacks, event enablement attacks and event disablement attacks. In these works, a module is embedded in the supervisor to detect cyber attacks. Once an attack is detected, the supervisor will disable all controllable events.

If cyber attacks cannot be detected in time, the above methods cannot ensure the safety/security of the supervised systems. Hence more work has been done to investigate deception attacks [22]–[27]. Some work considers how to synthesize deception attack strategies. In [23], the authors show how to synthesize a deterministic attack strategy. In [24], an equivalence relation is proposed to reduce the complexity of synthesizing a deception attack strategy. On the other hand, some work considers how to design a robust supervisor against deception attacks. In [25], the authors propose a supervisor against bounded sensor attacks. In [26], the supervisory control problem under deception attacks is solved and the necessary and sufficient conditions are derived. In [27], a robust supervisor is synthesized to mitigate the attacks on the sensor reading based on the solution of a partially observed supervisory control problem with arbitrary control patterns. Using game theory, [31] proposes a bipartite-graph-based method to solve the supervisory control problem for partially-observed discrete event systems under cyber attacks. All these works focus on sensor attacks.

For actuator attacks, [32] proposes a method to synthesize a deception attack strategy through subset construction. In [33], the synthesis problem of deception attack strategies is converted into a traditional supervisor synthesis problem. Based on the attack model in [32], an obfuscated supervisor is proposed in [34] to prevent the supervised system from entering unsafe states even though actuator attacks may occur using a technique of solving the boolean satisfiability problem.
In practice, sensors and actuators are usually distributed in the same network in a cyber-physical system. The attackers can invade the sensors to change their readings and invade the actuators to alter their actions at the same time. In this sense, we need to consider the impacts of joint sensor and actuator attacks on the supervised systems \([35]–[42]\). We call the joint sensor and actuator attacks as man-in-the-middle attacks. They assume sensor attacks can delete the current observed event or replace it with another observable event and the actuator attacks can enable a disabled event or disable an enabled event in the attackable event set. The control objective is to ensure that the system runs normally before attacks are detected and not enters unsafe states after attacks are detected. A security supervisor is synthesized to control the given system together with the normal supervisor. The security supervisor begins to work when attacks are detected. \(NA\)-security is proposed to ensure this control problem has solutions. \([39]\) uses the similar models for sensor attacks and actuator attacks. The control objective is to ensure the dynamics of the closed-loop control system is in a given range and can be solved with the range supervisory control framework. In \([40]\), sensor attacks are nondeterministic and the attacker can randomly choose one observation/control from a set to replace the current observation/control. In this case, the observation and control for an occurred string becomes nondeterministic and hence the dynamics of the closed-loop control system becomes nondeterministic. A supervisor is synthesized which ensures the closed-loop control system is deterministic (a string can either occur in all possible attacks or never occur in any possible attack), which may be hard to achieve. In \([41]\), \([42]\), the authors investigate how to find a powerful joint sensor and actuator attack policy, not how to synthesize a supervisor to tame attacks.

As shown in the above papers, the conventional supervisory control theory cannot handle cyber attacks in the sense that the supervisor designed cannot ensure safety of the supervised system. In order to design a robust supervisor that ensures safety of the supervised system under a large class of joint sensor-actuator cyber attacks, we develop a new supervisory control theory. The theory uses a new cyber attack model that is more general than the existing models in the sense that other models such as models for deletions, insertions, and replacements are special cases of the general model. (2) In order to deal with nondeterminism in a supervised system caused by nondeterministic attacks, an upper-bound language of the supervised system under cyber attacks is defined. The control objective is to ensure the safety of the supervised system by requiring that the upper-bound language is equal to or contained in a given specification language \(K\). (3) A new supervisory control theory is developed and necessary and sufficient conditions are derived for the existence of a supervisor under cyber attacks whose upper-bound language is equal to \(K\). The conditions are given in terms of two new concepts of \(CA\)-controllability and \(CA\)-observability. (4) Using the general model and the new theory, new methods are proposed to calculate state estimates and to design a robust supervisor that works for all cyber attacks allowed by the model.

The paper is organized as follows. Section II introduces discrete event systems and cyber attacks. Section III formally states the safety supervisory control problem under cyber attacks. Section IV proposes a method to calculate state estimates from observations under cyber attacks and a state-estimate-based supervisor is then proposed. Section V solves the supervisory control problem under cyber attacks. Section VI finds the maximally-permissive supervisor for the supervisory control problem under cyber attacks. In Section VII we extend the results to a more general case where sensor attack strategies are observation-based. Finally, we conclude the paper in Section VIII. Preliminary version of some results on the supervisory control problem under cyber attacks in this paper is presented in \([46]\). Compared with \([46]\), this paper contains proofs omitted in \([46]\) as well as detailed explanations and examples. We further investigate the maximally-permissive supervisor and observation-based sensor attacks which are not considered in \([46]\).

II. DISCRETE EVENT SYSTEMS UNDER CYBER ATTACKS

A discrete event system is modeled as a deterministic automaton \([16]\)

\[
G = (Q, \Sigma, \delta, q_0),
\]
where $Q$ is the set of states; $\Sigma$ is the set of events; $\delta : Q \times \Sigma \rightarrow Q$ is the (partial) transition function; and $q_0$ is the initial state. The set of all possible transitions is also denoted by $\delta$, that is, $\delta = \{(q, \sigma, q') : \delta(q, \sigma) = q'\}$.

We use $\Sigma^*$ to denote the set of all strings over $\Sigma$. The transition function $\delta$ can be extended to strings, that is, $\delta : Q \times \Sigma^* \rightarrow Q$, in the usual way. We use $\delta(q, s)$ to denote that $\delta(q, s)$ is defined. The language generated by $G$ is the set of all strings defined in $G$ from the initial state $q_0$, namely

$$L(G) = \{s \in \Sigma^* : \delta(q_0, s)\}.$$ 

In general, a language $K \subseteq \Sigma^*$ is a set of strings. For a string $s \in \Sigma^*$, the set of all prefixes of $s$ is denoted as $P_r(s) = \{s' \in \Sigma^* : \exists t \in \Sigma^*, s't = s\}$. The (prefix) closure of $K$, denoted as $\overline{K}$, is the set of prefixes of strings in $K$ as

$$\overline{K} = \{s' : (\exists s \in K)s' \in P_r(s)\}.$$ 

A language is (prefix) closed if it equals its prefix closure. By the definition, $L(G)$ is closed. For a string $s \in \Sigma^*$, we use $|s|$ to denote its length. For a set $x \subseteq Q$, we use $|x|$ to denote its cardinality (the number of its elements).

We use a controller, called supervisor, to control the plant so that some control objective is achieved. The supervisor can control some events and observe some others. The set of events that can be controlled/disabled, called controllable events, is denoted by $\Sigma_c = \Sigma - \Sigma_u$. The set of uncontrollable events, denoted by $\Sigma_u = \Sigma - \Sigma_c$, is the set of unobservable events. The set of observable events, denoted by $\Sigma_o = \Sigma - \Sigma_u$, is the set of observable events.

For a given string, its observation is described by the natural mapping $P : \Sigma^* \rightarrow \Sigma^*_o$ which is defined as

$$P(\varepsilon) = \varepsilon, \quad P(\sigma) = \begin{cases} \sigma & \text{if } \sigma \in \Sigma_o \\ \varepsilon & \text{if } \sigma \in \Sigma - \Sigma_o \end{cases}, \quad P(s\sigma) = P(s)P(\sigma), s \in \Sigma^*, \sigma \in \Sigma,$$

where $\varepsilon$ is the empty string.

As in [21], cyber attackers can change the readings of some related sensors. Hence, some observable events can be attacked. Denote the set of observable events and transitions that may be attacked by $\Sigma^a_o \subseteq \Sigma_o$ and $R^a = \{(q, \sigma, q) \in \delta : \sigma \in \Sigma^a_o\}$, respectively.

For a given transition $tr = (q, \sigma, q') \in R^a$, we assume that an attacker can change the observation from event $\sigma$ to any string in a language $A_{tr} \subseteq \Sigma^*$. $A_{tr}$ is determined based the knowledge of the attacker and the corresponding sensor. We denote the set of all such languages as $\mathcal{A}$

$$\mathcal{A} = \{A_{tr} : tr = (q, \sigma, q') \in R^a\}.$$ 

Sensor attack strategies are then described by a mapping from the set of transitions to be attacked to the set of languages $\mathcal{A}$

$$\pi : R^a \rightarrow \mathcal{A}.$$ 

where $\pi(tr) = A_{tr}$. Note that $\pi$ is transition-based. Note also that this general definition allows for nondeterministic attacks and includes the following special cases. (1) No attack: if $\sigma \in A_{tr}$, then $\sigma$ is altered to by $\sigma$ (no change), then there is no attack. (2) Deletion: if the empty string $\varepsilon \in A_{tr}$, then $\sigma$ is altered to $\varepsilon$, then $\sigma$ is replaced by $\varepsilon$. (3) Insertion: if $\sigma, \sigma' \in A_{tr}$, then $\sigma$ is replaced by $\sigma, \alpha$ is inserted.

If a string $s = \sigma_1\sigma_2...\sigma_s \in L(G)$ occurs in $G$, the set of possible strings after cyber attacks in observation channel, denoted by $\Theta^\pi(s)$, is obtained as follows. Denote $q_k = \delta(q_0, \sigma_1 \cdots \sigma_k), k = 1, 2, ..., |s|$, then

$$\Theta^\pi(s) = L_{1}L_{2}...L_{|s|},$$

where

$$L_k = \left\{ \begin{array}{ll} \{\sigma_k\} & \text{if } (q_{k-1}, \sigma_k, q_k) \notin \delta^a \\ A_{k-1, k, q_{k-1}, q_k} & \text{if } (q_{k-1}, \sigma_k, q_k) \in \delta^a \end{array} \right\}$$

Note that $\Theta^\pi(s)$ contains more than one string. Hence, $\Theta^\pi$ is a mapping from $L(G)$ to $2^{\Sigma^*}$:

$$\Theta^\pi : L(G) \rightarrow 2^{\Sigma^*}.$$ 

The observation under both partial observation and cyber attacks in observation channel is then given by

$$\Phi^\pi = P \circ \Theta^\pi,$$

where $\circ$ denotes composition (of functions). In other words, for $s \in L(G)$, $\Phi^\pi(s) = P(\Theta^\pi(s))$. Hence, $\Phi^\pi$ is a mapping from $L(G)$ to $2^{\Sigma^*}$:

$$\Phi^\pi : L(G) \rightarrow 2^{\Sigma^*}.$$ 

We extend $P$, $\Theta^\pi$, and $\Phi^\pi$ from strings $s$ to languages $L$ in the usual way

$$P(L) = \{t \in \Sigma^*_o : (\exists s \in L)t = P(s)\},$$

$$\Theta^\pi(L) = \{t \in \Sigma^* : (\exists s \in L)t = \Theta^\pi(s)\},$$

$$\Phi^\pi(L) = \{t \in \Sigma^*_o : (\exists s \in L)t = \Phi^\pi(s)\}.$$ 

After the occurrence of $s \in L(G)$, a supervisor $S$ observes one of the string $t \in \Phi^\pi(s)$. Based on $t$, $S$ enables a set of events, denoted by $S(t)$. Hence, $S$ is a mapping

$$S : \Phi^\pi(L(G)) \rightarrow \Gamma,$$

where $\Gamma = 2^\Sigma$ is the set of all possible controls. Note that $\Sigma_u \subseteq \Sigma_c$. Note that uncontrollable events are always permitted to occur.

We assume that the disablers/enablers of some controllable events can be altered by attackers in the control channel in the sense that an attacker can enable an event that is disabled by the supervisor or disable an event that is enabled by the supervisor. Denote the set of controllable events that may be attacked by $\Sigma^a_c \subseteq \Sigma_c$. Note that uncontrollable events are always permitted to occur and the attackers cannot disable them.

Under cyber attacks in the control channel, for a given control $\gamma \in \Gamma$, some events in $\Sigma^a_c$ can be added to it or removed from it by attackers. Hence the possible controls are

$$\Delta(\gamma) = \{\gamma_a \in 2^\Sigma : (\exists \gamma', \gamma'' \subseteq \Sigma^a_c)\gamma_a = (\gamma - \gamma') \cup \gamma''\}.$$
When the supervisor issues a control command $S(t)$ after observing $t \in \Phi^\pi(L(G))$, it may be altered under cyber attacks. We use $S^a(t)$ to denote the set of all possible control commands that may be received by the plant under cyber attacks, that is,

$$S^a(t) = \Delta(S(t)). \quad (4)$$

Let us use an example to illustrate how cyber attacks change the observations and controls.

**Example 1:** We consider a discrete event system $G$ shown in Fig. 1. We assume that all the events are controllable and observable. That is, $\Sigma_c = \Sigma_o = \Sigma$. Observations of events $\lambda$ and $\mu$ can be changed by attackers, that is, $\Sigma^a_o = \{\lambda, \mu\}$. Attackers can also enable/disable the occurrence of events $\alpha$ and $\beta$, that is, $\Sigma^a_c = \{\alpha, \beta\}$. The observations of transition $tr_1 = (2, \lambda, 3)$ under cyber attacks are described by $A_{tr_1} = \{\varepsilon, \lambda, \lambda \mu\}$. The observations of $tr_2 = (3, \mu, 1)$ under attacks are described by $A_{tr_2} = \{\mu, \beta\}$.

Fig. 1. A discrete event system $G$.

Let us consider the possible observations for string $s = \alpha \lambda$. By the definition, we have

$$\Theta^\pi(s) = \{\alpha\}A_{tr_1} = \{\alpha, \alpha \lambda, \alpha \lambda \mu\}.$$  

We then have $\Phi^\pi(s) = P(\Theta^\pi(s)) = \Theta^\pi(s)$ because all the events are observable.

Let us consider another string $s' = \alpha \lambda \mu$. Its observations are

$$\Phi^\pi(s') = \Theta^\pi(\alpha \lambda \mu) = \{\alpha\}A_{tr_1}A_{tr_2} = \{\alpha \mu, \alpha \beta, \alpha \lambda \mu, \alpha \lambda \beta, \alpha \lambda \mu \beta\}$$

If $\alpha \lambda \mu$ is observed, the supervisor issues a control $S(\alpha \lambda \mu) = \{\alpha, \lambda, \mu\}$. However, the actual control command received by $G$ is one of the following

$$\Delta(S(\alpha \lambda \mu)) = \{\{\alpha, \lambda, \mu\}, \{\alpha, \beta, \lambda, \mu\}, \{\lambda, \mu\}\}.$$  

Note that $\alpha$ is enabled by the supervisor without attack. However, if the control after attack is $\{\lambda, \mu\}$, then $\alpha$ is disabled.

**III. PROBLEM STATEMENT**

As discussed in the previous section, when we use a supervisor to control a discrete event system, both the observation and the control command may be altered due to cyber attacks as shown in Fig. 2.

Specifically, cyber attacks alter the observation of string $s$ from $P(s)$ to one in $\Phi^\pi(s)$. They also alter the control command from $S(t)(=\gamma)$ to one in $S^a(t) = \Delta(S(t))$ for a given observation $t$.

![Fig. 2. The closed-loop control framework under cyber attacks.](image)

We denote the supervised system under attacks as $S^a/G$. Because of the attacks, the language generated by the supervised system, denoted by $L(S^a/G)$, is nondeterministic, and hence may not be unique. We consider the upper bound of all languages that can be generated by the supervised system $S^a/G$. We call the upper bound *large language*, which is given below.

**Definition 1:** The large language generated by the supervised system $L(S^a/G)$, denoted by $L_a(S^a/G)$, is defined recursively as follows.

1) The empty string belongs to $L_a(S^a/G)$. That is, 

$$\varepsilon \in L_a(S^a/G).$$

2) If $s$ belongs to $L_a(S^a/G)$, then for any $\sigma \in \Sigma$, $s \sigma$ belongs to $L_a(S^a/G)$ if and only if $s \sigma$ is allowed in $L(G)$ and $\sigma$ is uncontrollable or enabled by $S^a$ in some situations. That is, for any $s \in L_a(S^a/G)$ and $\sigma \in \Sigma$,

$$s \sigma \in L_a(S^a/G) \iff \exists s \sigma \in L(G) \land \sigma \in \Sigma_{uc} \lor \exists t \in \Phi^\pi(s)(\exists \gamma_a \in S^a(t)) \in \gamma_a). \quad (5)$$

**Remark 1:** Note that $L_a(S^a/G)$ may include some strings which can not be generated by the closed-loop system. However, $L_a(S^a/G) = K$ ensures the closed-loop system is always safe.

Our goal is to control the system $G$ to be safe under cyber attacks. The safety specification is described by a specification language $K$ generated by a sub-automaton of $G$ as

$$H = (Q_H, \Sigma, \delta_H, q_0),$$

where $Q_H \subseteq Q$ and $\delta_H = \delta|_{Q_H \times \Sigma} \subseteq \delta$. Intuitively, $Q_H$ is the set of safe states and $Q - Q_H$ is the set of unsafe states.

To achieve this goal, a supervisor must be designed such that the large language of the supervised system $L_a(S^a/G)$ is equal to or contained in the specification language $K$. To design such a resilient supervisor, we first need to find the existence condition for a supervisor $S$ such that $L_a(S^a/G) = K$. If the existence condition is not satisfied, we can then find a sublanguage of $K$ that satisfies the existence condition. Therefore, we want to solve the following control problem.

**Safety Supervisory Control Problem for Discrete Event Systems under Cyber Attacks (SCPDES-CA):** Consider a discrete event system $G$ under cyber attacks in the observation channel described by $\Phi^\pi$, and in the control channel described by $\Delta$. For a non-empty closed specification language $K \subseteq L(G)$ modeled as a sub-automaton $H \subseteq G$, find a supervisor $S$ such that $L_a(S^a/G) = K$. 

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**Note:** The image is not provided, so the diagram cannot be included. However, the text describes the interactions between the Plant, Supervisor, Sensor, Actuator, and Attackers as demonstrated in the diagram. The text explains how cyber attacks alter the observations and control commands under the control of a supervisor, and how the existence condition for a supervisor is determined to ensure system safety.
To solve SCPDES-CA, we first investigate how to estimate states and then consider state-estimate-based supervisory control.

IV. STATE ESTIMATION AND STATE-ESTIMATE-BASED SUPERVISORS

After observing a string $t \in \Phi^*(L(G))$, the set of possible states that $G$ may be in is called the state estimate and is defined as

$$SE_G^x(t) = \{ q \in Q : (\exists s \in L(G))$$
$$t \in \Phi^*(s) \land \delta(q_0, s) = q \}.$$  

 Remark 2: Note that $\Phi^*(s)$ is not prefix-closed. If $t \in \Phi^*(s) - \Phi^*(s)$ is observed, it means that more events will be observed before any new event occurs in $G$. We can wait for more observations before making a control decision. Hence, there is no harm to let $SE_G^x(t) = \emptyset$ for $t \in \Phi^*(s) - \Phi^*(s)$ as in the above definition.

To obtain state estimate $SE_G^x(t)$, we do the following. For each transition $tr \in \delta^a$, let us assume $A_{tr}$ is marked by an automaton $F_{tr}$. In other words, $A_{tr} = L_m(F_{tr})$ for some $F_{tr} = (Q_{tr}, \Sigma, \delta_{tr}, q_{0, tr}, Q_{m, tr})$.

We replace each transition $tr = (\sigma, q, q') \in \delta^a$ by $(q, F_{tr}, q')$ as follows.

$$G_{tr\rightarrow(q,F_{tr},q')} = (Q \cup Q_{tr}, \Sigma, \delta_{tr\rightarrow(q,F_{tr},q')}, q_0)$$

where $\delta_{tr\rightarrow(q,F_{tr},q')} = (\delta \setminus \{ (q, \sigma, q') \}) \cup \delta_{tr} \cup \{ (q, \varepsilon, q_{0, tr}) \} \cup \{ (q_{m, tr}, \varepsilon, q') : q_{m, tr} \in Q_{m, tr} \}$.

In other words, $G_{tr\rightarrow(q,F_{tr},q')}$ contains all transitions in $G$ and $F_{tr}$ except $(q, \sigma, q')$, plus the $\varepsilon$-transitions from $q$ to the initial state of $F_{tr}$ and from marked states of $F_{tr}$ to $q'$.

Denote the automaton after replacing all transitions subject to attacks as

$$G^a = (Q^a, \Sigma, \delta^a, q_0, Q^a_m) = (Q \cup \hat{Q}, \Sigma, \delta^a, q_0, Q)$$

where $\hat{Q}$ is the set of states added during the replacement and $Q^a_m$ is the set of marked states. Note that $G^a$ is a nondeterministic automaton, that is, $\delta^a$ is a mapping $\delta^a : Q^a \times \Sigma \rightarrow 2^{Q^a}$.

To model the partial observation, we replace unobservable transitions by $\varepsilon$-transitions and denote the resulting automaton as

$$G^\varepsilon = (Q \cup \hat{Q}, \Sigma, \delta^\varepsilon, q_0, Q)$$

where $\delta^\varepsilon = \{ (q, \sigma, q') : (q, \sigma, q') \in \delta^a \land \sigma \in \Sigma_a \} \cup \{ (q, \varepsilon, q') : (q, \varepsilon, q') \in \delta^a \land \varepsilon \notin \Sigma_o \}$. $G^\varepsilon$ is also a nondeterministic automaton.

$G^\varepsilon$ can be used to calculate $\Phi^*(L(G))$ as shown in the following theorem. Proofs of all the results are in the appendices.

**Theorem 1**: The set of strings that can be observed under partial observation and cyber attacks in observation channel, $\Phi^*(L(G))$, is given by

$$\Phi^*(L(G)) = L_m(G^\varepsilon).$$

where $L_m(G^\varepsilon) = \{ s \in \Sigma^*_o : \delta^\varepsilon(s, q_0, s) \cap Q \neq \emptyset \}$ is the language marked by $G^\varepsilon$.

To obtain the state estimates, we convert $G^\varepsilon$ to a deterministic automaton $G^\varepsilon_{obs}$, called CA-observer, using operator $OBS$ as follows.

$$G^\varepsilon_{obs} = OBS(G^\varepsilon) = (X, \Sigma_o, \xi, x_0, X_m)$$

$$= Ac(2^{Q \cup \hat{Q}}, \Sigma_o, \xi, UR(q_0), X_m),$$

where $Ac(.)$ denotes the accessible part; $UR(.)$ is the unobservable reach defined, for $x \subseteq \hat{Q} \cup \hat{Q}$, as

$$UR(x) = \{ q \in \hat{Q} \cup \hat{Q} : (\exists q' \in x)q \in \delta^\varepsilon(q', \varepsilon) \}.$$

The transition function $\xi$ is defined, for $x \in X$ and $\sigma \in \Sigma_o$ as

$$\xi(x, \sigma) = UR(\{ q \in \hat{Q} \cup \hat{Q} : (\exists q' \in x)q \in \delta^\varepsilon(q', \sigma) \}).$$

The marked states are defined as

$$X_m = \{ x \in X : x \cap Q \neq \emptyset \}.$$}

It is well-known that $L(G^\varepsilon_{obs}) = L(G^\varepsilon_{obs})$ and $L_m(G^\varepsilon_{obs}) = L_m(G^\varepsilon_{obs})$. Hence, $\Phi^*(L(G)) = L_m(G^\varepsilon_{obs})$.

With observer $G^\varepsilon_{obs}$, we can calculate the estimate for every observation in $\Phi^*(L(G))$ as follows.

**Theorem 2**: After observing $t \in \Phi^*(L(G)) = L_m(G^\varepsilon_{obs})$, the state estimate $SE_G^x(t)$ is given by

$$SE_G^x(t) = \xi(x_0, t) \cap Q.$$

**Remark 3**: Theorem considers $t \in \Phi^*(L(G))$ only. For $t \in \Phi^*(L(G)) - \Phi^*(L(G))$, we do not care what $SE_G^x(t)$ is. To be consistent, we can define $SE_G^x(t)$ as $SE_G^x(t) = \xi(x_0, t) \cap Q \neq \emptyset$ with no impact on control, because the control decision issued for $t \in \Phi^*(L(G)) - \Phi^*(L(G))$ does not change the dynamics of the closed-loop system.

The procedure of calculating the state estimates under the observation $\Phi^*$ is presented by Algorithm 1.

**Algorithm 1 Calculate State Estimates Under the Observation $\Phi^*$**

**Require**: $G, A, t \in \Phi^*(L(G))$.

**Ensure**: $SE_G^x(t)$.

1. Construct automata $F_{tr} = (Q_{tr}, \Sigma, \delta_{tr}, q_{0, tr}, Q_{m, tr})$ as $A_{tr} \in A$.
2. Construct automata $G^a$ by replacing all $tr \in \sigma^a$ in $G$ with $F_{tr}$, shown as Equation (7) and (8).
3. Replace all unobservable transitions with $\varepsilon$ in $G^a$ and construct $G^\varepsilon$ as Equation (9).
4. Construct CA-observer as Equation (10), the transition function of which is $\xi$.
5. Output $SE_G^x(t) = \xi(x_0, t) \cap Q$.

Let us use an example to illustrate how to calculate state estimates for observations.

**Example 2**: Let us continue Example 1 with discrete event system $G$ shown in Fig. 1. The two automata $F_{tr_1}$ and $F_{tr_2}$ corresponding to $A_{tr_1}$ and $A_{tr_2}$ for $tr_1 = (2, \lambda, 3)$ and $tr_2 = (3, \mu, 1)$ are shown in Fig. 1.

We replace transitions $tr_1$ and $tr_2$ with the corresponding automata $F_{tr_1}$ and $F_{tr_2}$ to obtain $G^a$, which is shown in Fig. 2. Since all events are observable, $G^\varepsilon = G^a$. 

}\]
We then construct the observer $G_{\text{obs}}^\circ$ for automaton $G^\circ$, which is shown in Fig. 4.

Using the observer $G_{\text{obs}}^\circ$, we can calculate the state estimate for any observation $t \in \Phi^\circ(L(G))$. For example, for $t = \alpha \lambda \mu$, we have

$$\xi(t) = \{1, 3, B, D, E\}.$$ 

Hence

$$SE_H^\circ(t) = \xi(x_0, t) \cap Q = \{1, 3, B, D, E\} \cap Q = \{1, 3\}.$$ 

Using the method of calculating state estimates proposed above, we can construct a state-estimate-based supervisor $S_{CA}$, called CA-supervisor, as follows.

For the specification automaton $H$, after observing a string $t \in \Phi^\circ(L(H))$, the state estimate of close-loop system is $SE_H^\circ(t)$ and defined as

$$SE_H^\circ(t) = \{q \in Q_H : (\exists s \in L(H)) t \in \Phi^\circ(s) \land \delta_H(q_0, s) = q\}.$$ 

(11)

To ensure that the supervised system will not enter unsafe states in $Q - Q_H$, the set of events that need to be disabled is

$$\eta(SE_H^\circ(t)) = \{\sigma \in \Sigma : (\exists q \in SE_H^\circ(t)) \delta(q, \sigma) \in Q - Q_H\}.$$ 

(12)

Since a supervisor cannot disable uncontrollable events, $S_{CA}(t)$ is defined as

$$S_{CA}(t) = \begin{cases} \{\Sigma - \eta(SE_H^\circ(t))\} \cup \Sigma_{uc} & \text{if } t \in \Phi^\circ(L(H)) \\ \Sigma_{uc} & \text{others} \end{cases}$$ 

(13)

Remark 4: Constructing $S_{CA}$ needs to calculate the state estimate for every observable string which can be implemented online with polynomial complexity or offline with exponential complexity since we need to construct the observer for the specification automaton $H$ and sensor attack automata.

In the next section, we show that $S_{CA}$ defined above is indeed a solution to the supervisory control problem under cyber attacks when some necessary and sufficient conditions are satisfied.

V. SOLUTIONS TO THE SUPERVISORY CONTROL PROBLEM UNDER CYBER ATTACKS

Let us now solve SCPDES-CA. We first extend controllability and observability to capture the impacts of cyber attacks on the control channel and observation channel.

We call the extended controllability CA-controllability, which is defined as follows.

**Definition 2**: A closed language $K \subseteq L(G)$ is CA-controllable with respect to $L(G)$, $\Sigma_{uc}$, and $\Sigma_a^\circ$ if

$$K(\Sigma_{uc} \cup \Sigma_a^\circ) \cap L(G) \subseteq K.$$ 

In other words, $K$ is CA-controllable with respect to $L(G)$, $\Sigma_{uc}$ and $\Sigma_a^\circ$ if and only if $K$ is controllable with respect to $L(G)$ and $\Sigma_a^\circ$ and $\Sigma_{uc}$. Intuitively, this is because attackers can enable events in $\Sigma_a^\circ$ and hence makes them uncontrollable as far as the large language is concerned.

We call the extended observability CA-observability, which is defined as follows.

**Definition 3**: A closed language $K \subseteq L(G)$ is CA-observable with respect to $L(G)$, $\Sigma_{uc}$, $\Sigma_a^\circ$ and $\Phi^\circ$ if

$$(\forall s \in \Sigma^\circ)(\forall \sigma \in \Sigma)(s \sigma \in K) \Rightarrow (\exists t \in \Phi^\circ(s))(\forall s' \in (\Phi^\circ)^{-1}(t))$$ 

$$\{s' \in K \land s' \sigma \in L(G) \Rightarrow s' \sigma \in K\},$$ 

(14)

where $(\Phi^\circ)^{-1}$ is the inverse mapping of $\Phi^\circ$: for $t \in \Phi^\circ(L(G))$,

$$(\Phi^\circ)^{-1}(t) = \{s' \in L(G) : t \in \Phi^\circ(s')\}.$$ 

With CA-controllability and CA-observability defined, we have the following theorem.

**Theorem 3**: Consider a discrete event system $G$ under cyber attacks. For a nonempty closed language $K \subseteq L(G)$, SCPDES-CA is solvable, that is, there exists a supervisor $S$ such that $L_a(S^\circ/G) = K$ if and only if $K$ is CA-controllable with respect to $L(G)$, $\Sigma_{uc}$, and $\Sigma_a^\circ$ and CA-observable with respect to $L(G)$, $\Sigma_{uc}$, $\Sigma_a^\circ$ and $\Phi^\circ$. Furthermore, if SCPDES-CA is solvable, then CA-supervisor $S_{CA}$ defined in Equation (13) is a valid solution, that is,

$$L_a(S_{CA}^\circ/G) = K.$$ 

Let us use an example to illustrate these results.
Example 3: Let us again consider the discrete event system given in Example[1]. The specification language $K$ is described by a sub-automaton $H$ shown in Fig. 6. Consider the following two cases.

Fig. 6. Automaton $H$ for the specification language $K$

Case 1: $\Sigma_o = \Sigma_c = \Sigma$, $\Sigma^a_o = \{\lambda, \mu\}$ and $\Sigma^a_c = \{\alpha, \beta\}$. In this case, $K$ is not CA-controllable because

$$\exists s \in \Sigma_o^*(\exists \sigma = \alpha \in \Sigma)$$

$$s \in K \land \sigma \in \Sigma_{uc} \cup \Sigma^a_c \land \sigma \in L(G) \land s \sigma \not\in K.$$ 

By Theorem 3, no supervisor exists.

Case 2: $\Sigma_o = \Sigma_c = \Sigma$, $\Sigma^a_o = \{\lambda, \mu\}$ and $\Sigma^a_c = \{\beta\}$. In this case, $K$ is CA-controllable because

$$\forall s \in \Sigma^* \land \sigma \in \Sigma)$$

$$s \in K \land \sigma \in L(G) \land s \sigma \not\in K.$$ 

Let us show that $K$ is also CA-observable, that is, Equation (14) is satisfied. Clearly, $s \sigma \in K$ if and only if (1) $s \sigma = (\alpha \lambda \mu)^n \alpha$, or (2) $s \sigma = (\alpha \lambda \mu)^n \alpha \lambda$, or (3) $s \sigma = (\alpha \lambda \mu)^n$, for $n = 1, 2, ...$.

For (1) $s \sigma = (\alpha \lambda \mu)^n \alpha$, we have

$$s \sigma = (\alpha \lambda \mu)^n \alpha$$

$$\Rightarrow s = (\alpha \lambda \mu)^n \land \sigma = \alpha$$

$$\Rightarrow s = (\alpha \lambda \mu)^n = \sigma = \alpha = (\exists t = (\alpha \lambda \mu)^n \in \Phi^*(s))$$

$$\forall s' = (\alpha \lambda \mu)^n \in (\Phi^*)^{-1}(t))$$

$$s' = (\alpha \lambda \mu)^n \in K$$

$$\land s' \sigma = (\alpha \lambda \mu)^n \alpha \in L(G) \Rightarrow s' \sigma = (\alpha \lambda \mu)^n \alpha \in K.$$ 

Hence, Equation (14) is satisfied. Similarly, we can show that, for (2) $s \sigma = (\alpha \lambda \mu)^n \alpha \lambda$, and (3) $s \sigma = (\alpha \lambda \mu)^n$, Equation (14) is also satisfied. Therefore, $K$ is CA-observable.

By Theorem 3, CA-supervisor $S_{CA}$ defined in Equation (13) is a valid supervisor such that

$$L_o(S^o_{CA}/G) = K.$$ 

The observer $H^o_{obs}$ can be calculated in the same way as $G^o_{obs}$ and is shown in Fig. 7.

Fig. 7. Observer $H^o_{obs}$

VI. Maximally-permissive supervisor

Unlike the traditional supervisory control problem, the supervisor that solves SCPDES-CA is not unique.

Note that if a supervisor $S$ is a solution to SCPDES-CA, then the large language $L_o(S^o/G)$ equals to $K$ which is generated by $H$. Hence, the supervisor $S$ will never observe any string $t \in \Sigma_o^* - \Phi^*(L(H))$. Therefore, the control command $S(t)$ for $t \in \Sigma_o^* - \Phi^*(L(H))$ is inconsequential in the sense that it will not alter the large language. Hence, we only need to consider supervisors defined as $S : \Phi^*(L(H)) \rightarrow \Gamma$.

Given a finite or infinite set of supervisors $\{S_j : j = 1, 2, 3, ...\}$, their union, denoted as $\cup S_j$, is defined as

$$\forall t \in \Phi^*(L(H)) \cup S_j(t) = S_1(t) \cup S_2(t) \cup S_3(t) \cup ...$$ (15)

From the above definition, it is clear that

$$\forall t \in \Phi^*(L(H)) \cup S_j(t) \subseteq L_o(\cup S_j/G).$$ (16)

Denote the set of all supervisors that solve SCPDES-CA as

$$\Omega = \{S : L_o(S^o/G) = K\}$$ (17)

The following proposition says that the set $\Omega$ is closed under arbitrary union.

Proposition 1: If supervisors $\{S_j, j = 1, 2, 3, ...\}$ are solutions to SCPDES-CA, then their union is also a solution to SCPDES-CA, that is, for any set of supervisors $\{S_j, j = 1, 2, 3, ...\}$,

$$\forall t \in \Phi^*(L(H)) \cup S_j(t) \subseteq \cup S_j(t).$$ (18)

Given supervisors $S_1$ and $S_2$, we say $S_1$ is less permissive than $S_2$, denoted as $S_1 \leq S_2$, if $\forall t \in \Phi^*(L(H))S_1(t) \subseteq S_2(t)$. We say $S_1$ is strictly less permissive than $S_2$, denoted as $S_1 < S_2$, if $S_1 \leq S_2 \land \exists t \in \Phi^*(L(H))S_1(t) \subset S_2(t)$.

By Proposition 1, we conclude that there exists a unique supervisor in $\Omega$, denoted by $S^o$, that is more permissive than any other supervisor in $\Omega$, that is

$$\forall S_j \in \Omega \Rightarrow S_j \leq S^o.$$ (19)

$S^o$ is called the maximally-permissive supervisor. It can be obtained as follows. Enumerate all supervisors in $\Omega$ as

$$\Omega = \{S_j, j = 1, 2, 3, ...\}.$$ (20)

Then

$$S^o = \cup S_j.$$
By Proposition 1, \(S^o = \cup S_j \in \Omega\) is well defined and unique. Furthermore, by Equation (16),
\[
(\forall S_j \in \Omega) L_a((S^o_j \cap G) \subseteq L_a((\cup S_j^o) / G).
\]

Calculating the maximally-permissive supervisor that solves SCPDES-CA using \(S^o = \cup S_j\) is not practical because it requires to find all \(S_j \in \Omega\). This is however unnecessary because the following theorem says that the CA-supervisor \(S_{CA}\) defined in the previous section is the maximally-permissive supervisor.

**Theorem 4:** Assume that SCPDES-CA has a solution, that is, \(\Omega \neq \emptyset\). Then \(S_{CA}\) given in Equation (13) is the maximally-permissive supervisor that solves SCPDES-CA, that is,
\[
S_{CA} = S^o.
\]

**Example 4:** Let us continue Example 3. We assume \(\Sigma = \{\beta\}\). From Example 3 we know SCPDES-CA has solutions. Hence the supervisor \(S_{CA}\) is a solution.

In fact, we can find another valid supervisor \(S_1\) which is defined as follows.

For any observable string \(t\) which drives \(H^o\) to state \(\{2,3,A,B,D\}\), let \(S_1(t) = \{\lambda,\mu,\beta\}\).

For any observable string \(t\) which drives \(H^o\) to state \(\{3,B,C,D\}\), let \(S_1(t) = \{\beta,\mu\}\).

For the other observable strings \(t\), let \(S_1(t) = \{\alpha,\mu,\beta\}\).

It is not difficult to verify that \(L_a((S_1)^o / G) = K\) and \(S_1 < S_{CA}\).

**VII. OBSERVATION-BASED SENSOR ATTACKS**

In the previous sections, we have investigated transition-based sensor attack strategies. In practice, however, an attacker can make different sensor attack decisions for different observations. Hence, in this section, we extend our results to this general case by considering observation-based sensor attack strategies of the form
\[
\varpi : P(L(G)) \times \Sigma^o_\varpi \to 2^{\Sigma^a_\varpi}
\]
That is, after an observable string \(t\) is observed, the attacker changes an attacked event \(\sigma \in \Sigma^o_\varpi\) into a string \(t'\) in \(\varpi(t,\sigma)\) which is a set of observable strings (a language). The supervisor will see \(t'\), instead of \(\sigma\). In other words, we extend sensor attack strategies from \(\pi(tr)\) to \(\varpi(t,\sigma)\).

Since the number of observable strings is infinite, if a sensor attack strategy can be implemented by an attacker finitely, it must be described by a structure with finite states which models all possible situations for which different attack decisions are assigned. Without loss of generality, we assume such a structure is an automaton defined as
\[
SA = (Z, \Sigma_\varpi, \delta_{SA}, z_0)
\]
where \(Z\) is a finite set of states and \(\delta_{SA} : Z \times \Sigma_\varpi \to Z\) is the transition function. We require \(P(L(G)) \subseteq L(SA)\).

A sensor attack strategy is then defined on the state set \(Z\) as
\[
\omega : Z \times \Sigma^a_\varpi \to 2^{\Sigma^a_\varpi}.
\]
Since the sensor attack strategy does not change the observations of events in \(\Sigma_\varpi - \Sigma^a_\varpi\), for any state \(z \in Z\) and event \(\sigma \in \Sigma_\varpi - \Sigma^a_\varpi\), we let \(\omega(z,\sigma) = \{\sigma\}\). We also let \(\omega(z,\varepsilon) = \{\varepsilon\}\). From now on, we will consider the sensor attack strategy \(\omega\) instead of \(\varpi\).

Under the sensor attack strategy \(\omega\), the possible observations for an observable string \(t \in P(L(G))\), denoted as \(\Phi^\omega(t)\), can be calculated recursively as
\[
\Phi^\omega(\varepsilon) = \{\varepsilon\}
\]
\[
\Phi^\omega(t\sigma) = \Phi^\omega(t)\omega(\delta_{SA}(z_0, t), \sigma).
\]

After the occurrence of \(s \in L(G)\), a supervisor \(S\) under observation-based sensor attack observes one of the string \(t \in \Phi^\omega(P(s))\). Hence, \(S\) is a mapping
\[
S : \Phi^\omega(P(L(G))) \to 2^\Sigma.
\]

The behavior of supervised system under observation-based sensor attack is also described by the large language \(L_a(S(G))\), whose definition is similar to Definition 4 with \(\Phi^\varepsilon(s)\) replaced by \(\Phi^\omega(P(s))\).

The control problem we want to solve in this section is as follows.

**Supervisory Control Problem for Discrete Event Systems under Observation-Based Cyber Attacks (SCPDES-OBCA):** Consider a discrete event system \(G\) under cyber attacks in the observation channel described by \(\Phi^\omega\), and in the control channel described by \(\Delta\). For a non-empty closed specification language \(K \subseteq L(G)\) modeled as a sub-automaton \(H \subseteq G\), find a supervisor \(S\) such that \(L_a(Sa / G) = K\).

The approach for solving SCPDES-OBCA is similar to that for SCPDES-CA. We first need to calculate state estimates under observation-based sensor attack. For an observation \(t \in \Phi^\omega(P(L(G)))\), the state estimate is defined as
\[
SE^\omega_C(t) = \{q \in Q : (\exists s \in L(G)) t \in \Phi^\omega(P(s)) \land \delta(q_0, s) = q\}.
\]

To calculate \(SE^\omega_C(t)\), we first convert the problem of finding state estimates under observation-based sensor attack to the problem of finding state estimates under transition-based sensor attack. We then use the results of Section IV to find state estimates \(SE^\omega_S(t)\) under observation-based sensor attack.

The following procedure can be used for the conversion.

1. Take the parallel composition of \(G\) and \(SA\) to obtain an augmented automaton as
   \[
   \tilde{G} = G|SA = (Y, \Sigma, \tilde{\delta}, y_0) = Ac(Q \times Z, \Sigma, \tilde{\delta}, (q_0, z_0)).
   \]
2. Define \(\tilde{\delta}^a\) as
   \[
   \tilde{\delta}^a = \{(y, \sigma, y_j) \in \tilde{\delta} : \sigma \in \Sigma^a_\varpi\}.
   \]
3. For a transition \(t\sigma = (q_0, y, y_j)\), let \(A_{t\sigma} = \omega(z, \sigma, \sigma)\).
4. Define a transition-based sensor attack strategy \(\tilde{\pi}\) as follows. For a transition \(t\sigma = (q_i, y, y_j) \in \tilde{\delta}^a\),
   \[
   \tilde{\pi}(t\sigma) = A_{t\sigma}.
   \]
   Note that since \(P(L(G)) \subseteq L(SA)\), \(\tilde{G}\) and \(G\) generate the same language: \(L(\tilde{G}) = L(G)\).
Since \( \tilde{\pi} \) is transition-based, we can use the results in Section II to calculate \( \tilde{\Theta}(s) \) and \( \tilde{\Phi}(s) \) for any string \( s \in L(\tilde{G}) \) as follows.

Let \( s = \sigma_1 \sigma_2 \cdots \sigma_{|s|} \in L(\tilde{G}) \) and \( y_k = \tilde{\delta}(y_0, \sigma_1 \cdots \sigma_k), k = 1, 2, \ldots, |s| \). We have

\[
\tilde{\Theta}(s) = L_1L_2 \cdots L_{|s|},
\]

where

\[
L_k = \{ \sigma_k \mid \text{if } \tilde{\tau}_k = (y_{k-1}, \sigma_k, y_k) \notin \tilde{\delta}^a \}
\]

and

\[
\tilde{\Phi}(s) = P(\tilde{\Theta}(s)) = P(L_1)P(L_2) \cdots P(L_k). \tag{20}
\]

The following proposition shows the equivalence of the observation-based sensor attack \( \omega \) and the transition-based sensor attack \( \tilde{\pi} \).

**Proposition 2**: Sensor attack strategies \( \tilde{\pi} \) and \( \omega \) result in the same observations, that is,

\[
(\forall s \in L(G)) \tilde{\Phi}(s) = \Phi(2)(P(s)).
\]

Proposition 2 allows us to convert the problem of finding state estimates under observation-based sensor attack \( \omega \) to the problem of finding state estimates under transition-based sensor attack \( \tilde{\pi} \), which is defined as

\[
SE^\pi_G(t) = \{ y = (q, z) \in Y : (\exists s \in L(\tilde{G})) t \in \tilde{\Phi}(s) \land \tilde{\delta}(y_0, s) = y \}. \tag{21}
\]

\( SE^\pi_G(t) \) can be calculated using the methods in Section IV. After \( SE^\pi_G(t) \) is calculated, the corresponding state estimate in \( G \) is given by

\[
R(SE^\pi_G(t)) = \{ q \in Q : (\exists z \in Z)(q, z) \in SE^\pi_G(t) \}. \tag{22}
\]

The following theorem says that \( R(SE^\pi_G(t)) \) is the desired state estimate.

**Theorem 5**: For any observation \( t \in \Phi(2)(P(L(G))) \), its state estimate in \( G \) is given by

\[
SE^\pi_G(t) = R(SE^\pi_G(t)).
\]

Using the state estimate method proposed above, we derive a state-estimate-based supervisor as follows. (1) Construct the parallel composition \( \tilde{H} = H \parallel SA \). Since \( \tilde{H} \) is a sub-automaton of \( G \), \( \tilde{H} \) is a sub-automaton of \( \tilde{G} \). (2) Calculate state estimates \( SE^\pi_G(t) \) and \( SE^\pi_H(t) = R(SE^\pi_H(t)) \) in the same way as \( SE^\pi_G(t) \) and \( SE^\pi_H(t) = R(SE^\pi_H(t)) \) (replacing \( G \) by \( H \)). (3) The state-estimate-based supervisor \( \tilde{S}_{CA} \) is given by

\[
\tilde{S}_{CA}(t) = \begin{cases} 
(\Sigma - \eta(SE^\pi_H(t)) \cup \Sigma_{uc}) & \text{if } t \in \Phi(2)(P(L(H))) \\
\Sigma_{uc} & \text{others}
\end{cases} \tag{23}
\]

where

\[
\eta(SE^\pi_H(t)) = \{ \sigma \in \Sigma : (\exists q \in SE^\pi_H(t) )\delta(q, \sigma) \in Q - Q_H \}.
\]

We can now state the following necessary and sufficient condition for solving SCPDES-OBCA.

**Theorem 6**: Consider a discrete event system \( G \) under cyber attacks in the observation channel described by \( \tilde{\Phi} \), and in the control channel described by \( \Delta \). For a non-empty closed specification language \( K \subseteq L(G) \) modeled as a sub-automaton \( H \subseteq G \), SCPDES-OBCA is solvable, that is, there exists a supervisor \( S \) such that \( L_a(S^\pi/G) = K \) and only if \( K \) is CA-controllable with respect to \( L(G), \Sigma_{uc}, \Sigma_{\alpha} \) and \( \Sigma_{\alpha} \) and CA-observable with respect to \( L(G), \Sigma_{\alpha}, \Sigma_{\alpha} \) and \( \Phi(2) \). Furthermore, if SCPDES-OBCA is solvable, then CA-supervisor \( \tilde{S}_{CA} \) defined in Equation (23) is a valid solution, that is,

\[
L_a(S_{CA}^\pi/G) = K.
\]

Note that in Theorem 6 CA-controllability is identical to CA-controllability in Theorem 3 while CA-observability is with respect to \( \Phi(2) \) rather than \( \tilde{\Phi}(2) \).

**Example 5**: Let us again consider the discrete event system \( G \) given in Example 1 and the control specification automaton \( H \) in Example 3. We assume all events are controllable and observable; and \( \Sigma_{\alpha} = \{ \lambda, \mu \} \) and \( \Sigma_{\alpha} = \{ \beta \} \).

![Fig. 8. Automaton SA](image)

The observation-based sensor attack strategy is given by \( SA \) shown in Fig. 8 and the sensor attack strategy \( \omega \) is defined as

\[
\omega(z_2, \lambda) = \{ \varepsilon, \lambda, \lambda \mu \}
\]

\[
\omega(z_3, \mu) = \{ \mu, \beta \}.
\]

We construct \( \tilde{H} \) as shown in Fig. 9.

![Fig. 9. Automaton \( \tilde{H} \)](image)

With \( \omega \), we define the transition-based sensor attack strategy \( \tilde{\pi} \) as follows.

\[
\tilde{\pi}(2, z_2, \lambda, (3, z_3)) = \omega(z_2, \lambda)
\]

\[
\tilde{\pi}(3, z_3, \mu, (1, z_5)) = \omega(z_3, \mu).
\]

Under transition-based sensor attack strategy \( \tilde{\pi} \), we calculate \( \tilde{H}^\circ \) and \( \tilde{H}_{obs}^\circ \) as shown in Fig. 10 and Fig. 11 respectively.

Based on \( \tilde{H}_{obs}^\circ \), we can obtain the CA-supervisor \( \tilde{S}_{CA} \). For example, we can calculate the state estimate for observation \( t = \alpha \) as

\[
SE^{\omega}_{\tilde{H}}(\alpha) = R(SE^{\pi}_{\tilde{H}}(\alpha)) = R((2, z_2, (3, z_3))) = \{2, 3\}
\]

and then the control as

\[
\tilde{S}_{CA}(\alpha) = \{ \beta, \lambda, \mu \}.
\]
B. CA-observability, we successfully solve the supervisory controllability and observability into CA-controllability and discrete event system under joint sensor and actuator attacks to

\[ L = k \]

\[ \delta = \left( C, s \right) \]

\[ \pi = \left( m, \mu, \lambda \right) \]

\[ (1, z_1) \]

\[ (2, z_2) \]

\[ (1, z_5) \]

\[ (3, z_3) \]

\[ (1, z_5), (3, z_3), B, D, E \]

\[ (1, z_5), E \]

\[ (3, z_3), B, D, E \]

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\[ (1, z_5) \]

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\[ (2, z_2), (3, z_3) \]

\[ \varepsilon \]

\[ \beta, \mu \]

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(since \( s \sigma \in K \Rightarrow s \in K \land s \sigma \in L(G) \))

(\(\Leftarrow\)) We have
\[
s \in K \land s \sigma \in L(G)
\land (\exists t \in \Phi^+(s))(\exists \gamma \in (\Phi^+)^{-1}(t))
\Rightarrow (s' \in K \land s' \sigma \in L(G) \Rightarrow s' \sigma \in K)
\Rightarrow s \in K \land s \sigma \in L(G)
\land (s \sigma \in K \Rightarrow s \sigma \in L(G))
\Rightarrow s \sigma \in K
\]
(26)

Next, we show that in the definition of large language
\(L_a(S^*/G)\), Equation (5) can be re-written as
\[
s \sigma \in L_a(S^*/G)
\Leftrightarrow (\exists t \in \Phi^+(s))(\exists \gamma \in (\Phi^+)^{-1}(t))
\Leftrightarrow s \sigma \in L(G) \land (\exists \gamma \in \Sigma_{uc})
\land (\exists t \in \Phi^+(s))(\exists \gamma \in (\Phi^+)^{-1}(t))
\land (s' \in K \land s' \sigma \in L(G) \Rightarrow s' \sigma \in K)
\Rightarrow s \sigma \in K
\]

This is because
\[
(\exists \gamma \in (\Phi^+)^{-1}(t))(\exists \gamma \in (\Phi^+)^{-1}(t))
\Leftrightarrow (\exists \gamma \in \Delta(S(t)))
\Leftrightarrow (\exists \gamma \in \Delta(S(t)))
\land (\exists \gamma \in (\Phi^+)^{-1}(t))
\Rightarrow s \sigma \in S(t) \land (\exists \gamma \in (\Phi^+)^{-1}(t))
\]
(27)

(ONLY IF) Assume that there exists a CA-supervisor \(S\) such that \(L_a(S^*/G) = K\). We want to prove that \(K\) is CA-controllable and CA-observable.

For CA-controllability, it can be re-written as
\[
(\forall s \in \Sigma^*)(\forall s \in \Sigma) s \in K \land s \sigma \in \Sigma_{uc} \land s \sigma \in L(G)
\Rightarrow s \sigma \in K.
\]

We prove the above as follows. For all \(s \in \Sigma^*\) and \(\sigma \in \Sigma\), we have
\[
s \in K \land s \sigma \in L(G)
\Rightarrow s \sigma \in L_a(S^*/G) \land s \sigma \in L(G) \land (\exists \gamma \in \Sigma_{uc} \land s \sigma \in L(G))
\Rightarrow s \sigma \in K (\because L_a(S^*/G) = K).
\]

We prove CA-observability by contradiction. Suppose \(K\) is CA-controllable with respect to \(L(G), \Sigma_{uc}, \Sigma^c\), but not CA-observable with respect to \(L(G), \Sigma_a, \Sigma_c, \Phi^+\). By the definitions of CA-observability, we have
\[
(\exists s \in \Sigma^*)(\exists \sigma \in \Sigma) s \sigma \in K \land (\forall t \in \Phi^+(s))
\Rightarrow (\exists s' \in (\Phi^+)^{-1}(t)) s' \in K \land s' \sigma \in L(G) \land s' \sigma \in \Sigma_{uc} \land \Sigma_c
\Rightarrow (\exists \gamma \in \Delta(S(t)))
\Rightarrow s \sigma \in K (\because L_a(S^*/G) = K).
\]

We prove CA-observability by contradiction. Suppose \(K\) is CA-controllable with respect to \(L(G), \Sigma_{uc}, \Sigma^c\), but not CA-observable with respect to \(L(G), \Sigma_a, \Sigma_c, \Phi^+\). By the definitions of CA-observability, we have

Let us consider two possible cases.

**Case 1:** $(\exists t' \in \Phi^*(s)) \sigma \in S(t')$. In this case, by the derivation above,

$$\begin{align*}
(\exists s \in \Sigma^*)(\exists \sigma \in \Sigma) s \sigma &\in K \land (\exists t' \in \Phi^*(s)) \sigma \in S(t') \\
(\forall t \in \Phi^*(s))(\exists s' \in \Sigma^*) t \in \Phi^*(s') \\
s' \in K \land s' \sigma \in L(G) \land s' \sigma \notin K \\
\Rightarrow (\exists s' \in \Sigma^*)(\exists \sigma \in \Sigma) s' \sigma \notin K \land s' \sigma \in L(G) \\
(\forall t' \in \Phi^*(s)) t \in \Phi^*(s') \\
s' \in K \land s' \sigma \in L(G) \land s' \sigma \notin K \\
(\exists s' \in \Sigma^*)(\exists \sigma \in \Sigma) s' \sigma \notin K \land s' \sigma \in L(S^o/G) \\
\Rightarrow (\exists s' \in \Sigma^*)(\exists \sigma \in \Sigma) s' \sigma \notin K \land s' \sigma \in L(S^o/G) \\
(\because L_a(S^o/G) = K) \\
\Rightarrow (\exists s' \in \Sigma^*)(\exists \sigma \in \Sigma) s' \sigma \notin K \land s' \sigma \in L(S^o/G) \\
(\text{by Equation (27)}).
\end{align*}$$

which contradicts the assumption that $L_a(S^o/G) = K$.  

**Case 2:** $(\forall t' \in \Phi^*(s)) \sigma \notin S(t')$. In this case, we have

$$\begin{align*}
(\exists s \in \Sigma^*)(\exists \sigma \in \Sigma) s \sigma &\in K \land (\forall t' \in \Phi^*(s)) \sigma \notin S(t') \\
(\forall t \in \Phi^*(s))(\exists s' \in \Sigma^*) t \in \Phi^*(s') \\
s' \in K \land s' \sigma \in L(G) \land s' \sigma \notin K \\
(\exists s \in \Sigma^*)(\exists \sigma \in \Sigma) s \sigma \in K \\
(\forall t' \in \Phi^*(s)) \sigma \notin S(t') \land \sigma \notin \Sigma_{uc} \cup \Sigma_c \\
(\because K \text{ is CA-controllable}) \\
(\exists s \in \Sigma^*)(\exists \sigma \in \Sigma) s \sigma \in K \\
\neg(\sigma \in \Sigma_{uc} \cup \Sigma_c \cup (\exists t' \in \Phi^*(s)) \sigma \in S(t')) \\
\Rightarrow (\exists s \in \Sigma^*)(\exists \sigma \in \Sigma) s \sigma \in K \land s \sigma \in L_a(S^o/G) \\
(\text{by Equation (27)}).
\end{align*}$$

which contradicts the assumption that $L_a(S^o/G) = K$. Hence, $K$ is CA-observable with respect to $L(G)$.

**APPENDIX D**

**PROOF OF PROPOSITION 1**

We prove this by contradiction. Suppose $\neg((\forall j)S_j \in \Omega) \Rightarrow \cup S_j \in \Omega)$. Then

$$\begin{align*}
(\forall j)S_j \in \Omega \land \cup S_j &\notin \Omega \\
\Rightarrow (\forall j)L_a(S^o_j/G) &\neq K \land L_a(S^o/G) \\
(\text{by Equation (17)}).
\end{align*}$$

**APPENDIX E**

**PROOF OF THEOREM 2**

We prove the theorem by contradiction. Since $\Omega \neq \emptyset$, $S_{CA} \in \Omega$. Suppose $S_{CA} = S^o$ is not true, then

$$S_{CA} \neq S^o$$

$$S_{CA} < S^o$$

(by the definition of the large language)

$$S_{CA} \in \Omega$$

(by Equation (13))

which contradicts the assumption that $L_a(S^o/G) = K$. Hence, $K$ is CA-observable with respect to $L(G)$.

We hence obtain a contradiction.
t \in \Phi^\pi(s) \land (\exists \gamma_a = S^\pi(t) \in (S^\pi)^a(t)) \sigma \in \gamma_a \\
\land \sigma \in L(G) \land \sigma \notin L(H) \\
\Rightarrow (\exists \sigma \in \Phi^\pi(L(H))) (\exists \sigma \in \Sigma_a) (\exists s \in L_a((S^\pi)^a/G)) \\
\Rightarrow \sigma \in L_a((S^\pi)^a/G) \land \sigma \notin L(H) \\
(by \ the \ definition \ of \ the \ large \ language) \\
\Rightarrow L_a((S^\pi)^a/G) \neq L(H) = K, \\
which \ contradicts \ the \ fact \ L_a((S^\pi)^a/G) = K.

\section*{Appendix F}

\textbf{PROOF OF PROPOSITION}\[2]\[2]

\textbf{From the definition of parallel composition, we know that, for any string } s \text{ in } L(G), \\
\delta(y_0, s) = \delta((q_0, z_0), s) = (\delta(q_0, s), \delta_{SA}(z_0, P(s))). \quad (28)

\textbf{Let } s = \sigma_1 \sigma_2 \cdots \sigma_s \in L(G) \text{ and } y_k = (q_k, z_k) = \\
\delta(y_k, \sigma_1 \cdots \sigma_k), k = 1, 2, \ldots, |s|. \text{ Then,} \\
\Phi^\omega(P(s)) = \Phi^\omega(P(\sigma_1)P(\sigma_2) \cdots P(\sigma_s)) \\
= \omega(z_0, P(\sigma_1))\omega(\delta_{SA}(z_0, P(\sigma_1)), P(\sigma_2)) \cdots \omega(\delta_{SA}(z_0, P(\sigma_1 \cdots \sigma_{s-1})), P(\sigma_s)) \\
(29)

\textbf{On the other hand, for each event } \sigma_i, \text{ let us calculate } P(L_i) \text{ as follows.} \\
\textbf{Case 1. } \sigma_i \in \Sigma_u. \\
P(L_i) = P(\{\sigma_i\}) = \{\varepsilon\} \\
= \omega(\delta_{SA}(z_0, P(\sigma_1 \cdots \sigma_{i-1})), P(\sigma_i)) \\
(\text{since } \omega(z, \varepsilon) = \{\varepsilon\}) \\
\textbf{Case 2. } \sigma_i \in \Sigma_a - \Sigma_o. \\
P(L_i) = P(\{\sigma_i\}) = \{\sigma_i\} \\
= \omega(\delta_{SA}(z_0, P(\sigma_1 \cdots \sigma_{i-1})), P(\sigma_i)) \\
(\text{since } \omega(z, \sigma) = \{\sigma\}) \\
\textbf{Case 3. } \sigma_i \in \Sigma_o. \\
P(L_i) = P(\tilde{\pi}((q_{i-1}, z_{i-1}), \sigma_i, (q_i, z_i))) \\
= \tilde{\pi}((q_{i-1}, z_{i-1}), \sigma_i, (q_i, z_i)) \\
= \omega(z_{i-1}, \sigma_i) \\
(by \ the \ definition \ of \ \tilde{\pi}) \\
= \omega(\delta_{SA}(z_0, P(\sigma_1 \cdots \sigma_{i-1})), \sigma_i) \\
(by \ Equation \ (28)) \\
= \omega(\delta_{SA}(z_0, P(\sigma_1 \cdots \sigma_{i-1})), P(\sigma_i)) \\
\text{Hence, for any event } \sigma_i \text{ in } s, \text{ we always have} \\
P(L_i) = \omega(\delta_{SA}(z_0, P(\sigma_1 \cdots \sigma_{i-1})), P(\sigma_i)). \\
\text{Therefore, by Equations } (20) \text{ and } (29), \text{ we have} \\
\tilde{\Phi}^\pi(s) = \Phi^\omega(P(s)).

\section*{Appendix G}

\textbf{PROOF OF THEOREM}\[5]\[5]

\textbf{For any state } q \in Q, \text{ we have} \\
q \in R(SE^\pi_t(t)) \\
\iff (\exists z \in Z)(q, z) \in SE^\pi_t(t) \\
(by \ Equation \ (22)) \\
\iff (\exists s \in L(G)) t \in \Phi^\pi(s) \land (\exists z \in Z)(q, z) = \tilde{\delta}(y_0, s) \\
(by \ Equation \ (21)) \\
\iff (\exists s \in L(G)) t \in \Phi^\pi(s) \land (\exists z \in Z) \\
(q, z) = (\delta(q_0, s), \delta_{SA}(z_0, P(s))) \\
(by \ Equation \ (23)) \\
\iff (\exists s \in L(G)) t \in \Phi^\pi(s) \land q = \delta(q_0, s) \\
(by \ Proposition \ 2) \\
\iff q \in SE^\pi_t(t) \\
(by \ Equation \ (19)).

\section*{Appendix H}

\textbf{PROOF OF THEOREM}\[6]\[6]

\textbf{The proof is similar to the proof of Theorem } [3] \text{ with } \Phi^\pi \\
\text{and } S_{CA} \text{ replaced by } \Phi^\omega \text{ and } \tilde{S}_{CA}, \text{ respectively.}

\section*{Appendix I}

\textbf{EXPLANATIONS OF ALL SYMBOLS}

Explanations of all symbols is shown in Table [11]

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**TABLE I MEANING OF ALL SYMBOLS**

| Symbol   | Meaning                                                                 | Symbol   | Meaning                                                                 |
|----------|-------------------------------------------------------------------------|----------|-------------------------------------------------------------------------|
| $G$      | a deterministic automaton                                              | $Q$      | a set of states                                                          |
| $q$      | a state in $Q$                                                          | $\Sigma$ | a set of events                                                          |
| $\sigma$ | an event in $\Sigma$                                                   | $\delta$ | a (partial) transition function                                          |
| $q_0$    | the initial state in $G$                                                | $\Sigma_r$ | a set of all string over $\Sigma$                                       |
| $K$      | a specification language                                               | $s$ or $t$ | a string in $\Sigma^*$                                                  |
| $s'$     | the prefix of $s$                                                      | $K$      | the prefix of $K$                                                        |
| $L(G)$   | the language of $G$                                                    | $|s|$     | the length of $s$                                                        |
| $|x|$     | the cardinality of set $x$                                             | $\Sigma_o$ | the set of controllable events                                           |
| $\Sigma_{uc}$ | the set of uncontrollable events                                      | $\Sigma_o$ | the set of observable events                                            |
| $\Sigma_{uo}$ | the set of unobservable events                                        | $\delta_o$ | the set of unobservable transitions                                      |
| $\delta$ | the set of attacked transitions                                         | $\Sigma^o$ | the set of attacked observable events                                     |
| $A_{tr}$ | the subset of $\Sigma^*_{obs}$                                         | $\delta^a$ | a transition in $\delta^a$                                              |
| $\pi$    | the mapping from $\delta^a$ to $A$                                      | $\sigma$ | the set of all possible controls                                         |
| $\Theta^a$ | the mapping from $L(G)$ to $2\Sigma^*$                               | $\Phi^a$ | the mapping consisted of $P$ and $\Theta^a$                              |
| $S$      | an automaton                                                            | $\Sigma^o$ | the set of all possible actuators                                        |
| $\gamma$ | the set of attacked controllable events                                 | $S^a/G$  | the supervised system under actuator attacks                             |
| $\alpha$, $\beta$, $\lambda$, $\mu$ | events is examples                                                   | $\gamma_0$ | a possible control in $\Delta(\gamma)$                                  |
| $L_{n}(S^a/G)$ | the large language generated by $S^a/G$ | $\gamma_0$ | the set of state estimations of $G$ after observing $t \in \Phi^a(L(G))$ |
| $H$      | a sub-automaton of $G$                                                 | $S_{E_{\Phi^0}(t)}(G)$ | the automaton after replacing all transitions subject to attacks         |
| $F_{tr}$ | an automaton marking $A_{tr}$                                          | $G_{2}^o$ | the automaton after replacing unobservable transitions by $\varepsilon$-transitions in $G^o$ |
| $Q$      | the set of states added during the replacement                          | $\varphi^a$ | the set of all possible controls                                         |
| $G_{obs}$ | the set of observable transitions                                       | $\Theta^a$ | the mapping of possible observations under $t \in \Phi^a(L(G))$          |
| $U_{R}(\cdot)$ | the unobservable reach                     | $\Theta_S$ | the set of all possible states                                          |
| $X_{m}$  | the set of marked states of $G_{obs}$                                  | $S_{\Theta_S}$ | the set of all possible states                                          |
| $(\Phi^a)^{-1}$ | the inverse mapping of $\Phi^a$                                        | $S_{\Theta_S}$ | the set of all possible states                                          |
| $\Omega$ | the set of all supervisors that solve SCPDES-CA                        | $S_{\Theta_S}$ | the set of all possible states                                          |
| $\omega$ | the mapping of observation-based sensor attack                          | $S_{\Theta_S}$ | the set of all possible states                                          |
| $G$      | $G/S_{\Theta}$ and all symbols with $\sim$ is w.r.t $G$               | $S_{\Theta_S}(t)$ | the set of all possible states                                          |

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