A Method Reconstruct Ballistic Target Based on Compression Sensing

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Abstract: In order to solve the problem of large data volume and complex processing in the processing of ballistic target echo signal, compressing sensing is applied to the processing of ballistic target echo signal. Firstly, the mathematical model of the cone-cylinder ballistic target's precession is constructed, the signal processing when the radar radiates sparse Orthogonal Frequency Division Multiplexing-Linear Frequency (OFDM-LFM) signal is analyzed, and then match this process with compressed sensing decision branch; a new denoising recovery algorithm for signal-to-noise ratio(SNR) based on (SAMP) algorithm was proposed, and finally verify the effectiveness of the algorithm through simulation experiments.

1. Introduction
With the rapid development of ballistic technology, ballistic target has become a direct threat to national security. In recent years, the attack-defence confrontation of ballistic targets has become increasingly fierce, and the signal processing of ballistic targets has attracted more and more attention[1].

In the current era of big data, the signal becomes more and more complex, and the difficulty of data processing, transmission and storage is greatly increased. The traditional Nyquist sampling will face great obstacles [2]. Compressed Sensing [3-5] (CS), a new data processing method, breaks through these obstacles. CS theory points out that if the signal is sparse, a sparse representation of the signal can be projected from a high-dimensional space to a low-dimensional space using an observation matrix unrelated to the sparse dictionary basis, and then the high-dimensional original signal can be reconstructed from a small amount of data by the algorithm.

The use of Orthogonal Frequency Division Multiplexing [6] (OFDM), which is widely used in the field of communications, to modulate traditional LFM signals produces a new signalling system, the OFMD-LFM signal. The OFMD-LFM signal system combines the advantages of both OFDM and LFM signals. As a multi-carrier modulation system, it is often used in MIMO radar systems in radar; it simultaneously transmits multiple mutually orthogonal chirped subcarrier frequencies, each with a large time-width bandwidth product, which can be realized simultaneously.

In order to solve the above shortcomings, the OFDM-LFM signal system is first introduced into the MIMO radar system. Then, based on the current situation of complex and variable battlefield environment and serious electromagnetic interference, a compressed sensing method for low SNR is proposed.

2. The model of Ballistic target
In order to adapt to the signal form, MIMO radar system is adopted in this paper, including M...
transmitting units, which are denoted as $T_1, T_2, \ldots, T_M$ and $N$ receiving units, which are denoted as $R_1, R_2, \ldots, R_N$, respectively.

The geometry diagram of the precession of the cone-cylinder ballistic target of the MIMO radar is shown in Figure 1.

The distances from the center of mass $O$ to the $m$-th radar transmitting unit and the $n$-th radar receiving unit are $R_{T_m}$ and $R_{R_n}$, respectively.

The cone height is $h_1$, the cylinder height is $h_2$, the bottom radius is $r$, and the half cone angle is $\alpha$. The distance between the center of mass $O$ and $O'$ and the Z axis is the precession axis. The angle between the line of sight of the $m$-th radar transmitting unit and the cone rotating shaft is $\beta_{T_m}$, and the angle with the OZ is $\gamma_{T_m}$; the angle between the line of sight of the $n$-th radar receiving unit and the cone rotating shaft is $\beta_{R_n}$, and the angle with the OZ is $\gamma_{R_n}$. The precession axis is $OO'$, the cone angle is $\eta$, and the cone angle is $\omega_c$.

The top of cone-cylinder ballistic target is named A point. The plane defined by the direction of the line of sight of the radar and the plane of the axis of symmetry is the electromagnetic wave incident plane. The plane and the bottom surface of the cone intersect at B, C two points, and intersect with the bottom of the cylinder at D, E two points. These five points can be equivalent to the equivalent scattering point of the cone-cylinder ballistic target.

Through derivation, we can see that $\beta_{T_m}$ and $\beta_{R_n}$ satisfy the following equations respectively:

$$\cos \beta_{T_m} = A_{\gamma_{T_m}} \cos \omega t + A_{\eta \sin \omega t} + D_{\eta \cos \omega t}$$

$$\cos \beta_{R_n} = A_{\gamma_{R_n}} \cos \omega t + A_{\eta \sin \omega t} + D_{\eta \cos \omega t}$$

Among these,

$$\begin{align*}
A_{\gamma_{T_m}} &= \sin \gamma_{T_m} \sin \eta \sin \phi_0 \\
A_{\eta \sin \omega t} &= -\sin \gamma_{T_m} \sin \eta \cos \phi_0 \\
D_{\eta \cos \omega t} &= -\cos \gamma_{T_m} \cos \eta \\
A_{\gamma_{R_n}} &= \sin \gamma_{R_n} \sin \eta \cos \phi_0 \\
A_{\eta \sin \omega t} &= \sin \gamma_{R_n} \sin \eta \sin \phi_0 \\
D_{\eta \cos \omega t} &= \cos \gamma_{R_n} \cos \eta
\end{align*}$$

3. Compressed sensing

For a discrete signal $x \in \mathbb{R}^N$ with a zero norm $K$, it can be referred to as a sparse signal with a sparsity of $K$. An orthogonal sparse dictionary base $a$ is constructed. The signal $x$ is expanded under
and the expansion coefficient vector is recorded as $\theta$.

$$ x = \sum_{i=1}^{N} \theta_{i} \psi_{i} = \Psi \theta $$

(5)

By constructing an observation vector $\Phi \in \mathbb{R}^{M \times N}$, the observed signal can be obtained. $y = \Phi x = \Phi \Psi \theta = T \theta$

(6)

Among them, $T = \Phi \Psi$ is denoted as CS information operator. Reconstructing the original signal $x$ from the observed signal $y$ using a corresponding algorithm essentially solves a zero norm problem,

$$\min \| \theta \|_{0} \quad s.t \quad y = \Phi \Psi \theta = T \theta$$

(7)

It can be seen that this is a $NP$-hard problem because of $M < N$.

Noise is inevitably introduced during signal observation and measurement. The basic principle of compressed sensing in the noise background is studied below.

In the noise background, you can rewrite (5) as:

$$ y = \Phi x + w = \Phi \Psi \theta + \Phi \Psi n + w = T \theta + e $$

(8)

Where $n$ is the measurement noise and $e = \Phi \Psi n + w$ is the total noise, and $e$ can be regarded as a zero-mean Gaussian process.

When noise is mixed into the signal, it will destroy the original sparseness of the signal and turn the sparse signal into an approximately sparse signal [8]. At this time, the measurement matrix is used to observe the signal (the observation process is similar to a whitening process), and the energy of the signal and the noise are aliased, so that the useful signal and the interference noise cannot be distinguished, ultimately, the influence of noise will be magnified $N/M$ times. This phenomenon is called the noise folding [9] (NF) effect.

4. Sparse OFDM-LFM Signal System

Suppose that one of the MIMO radar transceiver systems transmits P-cluster sparse OFDM-LFM pulse trains, each of which contains N sub-pulses, in which the expression of the i-path sub-pulse in the sparse OFDM-LFM pulse trains of cluster m is as follows:

$$ s(t_{i},i) = \text{rect}\left(\frac{t_{i}}{T}\right) \cdot \exp\left( j2\pi \left( f_{c} + \frac{q}{T} i + \frac{\mu}{2} t_{i}^{2} \right) \right) \quad i = 1,2,\ldots,N $$

(10)

Where $t_{k}$ is the fast time, $\mu$ is the frequency modulation slope, $q$ is an arbitrary natural number, and the frequency difference between the adjacent two subcarriers can be expressed as $\Delta f = q/T$. For OFDM signals, orthogonality is satisfied between signals between the same cluster of bursts.

$$ \int_{T/2}^{T/2} s(t_{k},i_{j}) \cdot s^{*}(t_{k},i_{g}) = \begin{cases} 1, & j = g \\ 0, & j \neq g \end{cases} \quad i = 1,2,\ldots,N $$

(11)

The echo signal of the target can be expressed as:

$$ s(t_{i},\delta) = \sigma \cdot \text{rect}\left(\frac{t_{i}-\delta}{T}\right) \cdot \exp\left( j2\pi \left( f_{c} + \frac{q}{T} i + \frac{\mu}{2} (t_{i} - \delta)^{2} \right) \right) \quad i = 1,2,\ldots,N $$

(12)

Where $\sigma$ is the electromagnetic scatter coefficient of the target and $\delta = 2R/c$ is the echo delay time of the target.

Set the reference signal as
\[
s(t_k, \delta_k) = \sigma \cdot \text{rect} \left( \frac{t_k - \delta_k}{T} \right) \cdot \exp \left\{ j2\pi \left[ f_c + \frac{q}{T} (t_k - \delta_k) + \frac{\mu}{2} (t_k - \delta_k)^2 \right] \right\} \quad i = 1, 2, \ldots N
\]

Where \( \delta_k = 2R_0/c \) is the echo delay time of the reference point.

Through "Dechirp" processing:
\[
s_d(t_k) = s(t_k, \delta) s^\ast(t_k, \delta_0)
\]

After LPF filtering, the obtained result is obtained by performing FFT in the fast time domain and eliminating the RVP term:
\[
S_{\text{CRRP}}(f, i) = \sigma T \sin c \left( 2\pi T \left( f + \mu \cdot \Delta \delta \right) \right) \cdot \exp \left( -j2\pi \left( f_c + \frac{q}{T} i \right) \Delta \delta \right)
\]

Where \( \Delta \delta = 2(R - R_0)/c \)

Let \( f = -\mu \Delta \delta \), that is, take the peak point in the frequency domain, you can get
\[
S_{\text{CRRP}}(i) = \sigma T \cdot \exp \left( -j2\pi \left( f_c + \frac{q}{T} i \right) \Delta \delta \right)
\]

For \( S_{\text{CRRP}}(i) \), the N-point DFT for \( i \) is obtained.
\[
S_d(k) = \sigma x_1 \cdot \text{sinc} \left( x_2 \left( k + 2\pi \cdot \frac{q}{T} \cdot \Delta \delta \right) \right) \cdot \exp \left( -j2\pi f_c \cdot \Delta \delta \right)
\]

Where \( x_1, x_2 \) are constants.

This process can also be written as follows.
\[
S_{\text{CRRP}}(i) = D_N^{-1} S_d(k)
\]

Where
\[
D_N^{-1} = \begin{pmatrix}
1 & 1 & 1 & \ldots & 1 \\
1 & W_N^{-1} & W_N^{-2} & \ldots & W_N^{-(N-1)} \\
1 & W_N^{-2} & W_N^{-4} & \ldots & W_N^{2(N-1)} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & W_N^{-(N-1)} & W_N^{-2(N-1)} & \ldots & W_N^{-N-1}
\end{pmatrix}
\]

Where \( W_N = \exp \left( -j \frac{2\pi}{N} \right) \).

Comparing equation (18) with equation (5), it can be considered that \( D_N^{-1} \) is the sparse dictionary base \( \Psi \) of \( S_{\text{CRRP}}(i) \), and \( S_d(k) \) is the expansion coefficient vector \( \theta \).

The M-path sub-pulses are randomly selected in the N-way sub-pulses so that the adjacent sub-pulses are no longer strictly followed by \( \Delta f \) step, but are stepped by a random integer multiple of \( \Delta f \), and the first and last sub-pulses are kept unchanged.

This process can be described as:
\[
S_{\text{CRRP}}(m) = \Phi S_{\text{CRRP}}(i) \quad , m = 1, 2, \ldots M
\]

Where \( \Phi \in R^{M \times N} \) is the observation matrix.

5. Denoising algorithm based on adaptive threshold

The traditional algorithm gives a threshold \( \xi \), and when the residual is less than \( \xi \), the iteration is terminated. However, when noise is introduced into the radar signal, the situation becomes complicated, and the fixed threshold cannot assume the responsibility of eliminating the influence of noise.

This paper proposes an algorithm-based adaptive threshold denoising algorithm, which calculates
the ratio of the residual at the current time to the residual at the previous moment, the adjacent residual ratio \( s \), and then sets a new iteration termination condition.

This paper proposes an algorithm-based adaptive threshold denoising algorithm, which calculates the ratio of the residual at the current time to the residual at the previous moment, the adjacent residual ratio \( \tau_k = \frac{\| r_k \|}{\| r_{k-1} \|} \), and then sets a new iteration termination condition,

\[
\gamma_k = \frac{\| r_k - \tau_k r_{k-1} \|}{\| r_{k-1} \|} \leq \gamma 
\]

(21)

Where \( \gamma \) is a constant chosen between (0, 1).

The complete algorithm steps are as follows:

Step1: Initialization: initial residual \( r_0 = y \), initialization step size \( L = s \), initial-dimensional observation matrix \( \Phi \), \( M \times N \)-dimensional sparse dictionary base \( \Psi \), CS information operator \( T \), stage = 1, iterative index set \( \Lambda_0 = \emptyset \), Column Vector Set \( P_0 = \emptyset \), Candidate Set \( C_0 = \emptyset \), Thresholds of CS Information Operator Corresponding to Index Set \( \gamma \).

Step2: Calculating the inner product value \( \| r_{k-1} T \| \), and put the largest \( L \) column vector in Pro into the index set.

Step3: Combining \( \Lambda \) and \( P \) to get a new candidate set \( C_k \).

Step4: Estimating the signal \( x \) by least squares.

Step5: Update residual value: \( r_k = y - T \hat{x} \).

Step6: Calculate the termination iteration condition \( \gamma_k \)

Step7: If \( \gamma_k \leq \gamma \), the iteration is terminated, and the process proceeds to step (8). If \( \gamma_k > \gamma \), \( r_k > r_{k-1} \), update phase \( \text{stage} = \text{stage} + 1 \), update the length of the support set. If \( \gamma_k > \gamma \), \( r_k < r_{k-1} \), then update the residual value, and then calculate the \( \gamma_k \) value according to the formula (21).

Step8: Output: Reconstructed signal \( \hat{x} \).

6. Simulation Analysis

Let the MIMO radar system have 3 transmitting arrays and 3 receiving arrays, and the coordinates of the transmitting arrays are (185000,0,0), (30000,0,0), (-60000,0,0); the coordinates of the receiving arrays are (0,190000,0), (0,170000,0), (0,150000,0).

Subpulse Width of Signal \( T_p = 1\text{ns}, \ q = 1 \), frequency difference between adjacent subcarriers \( \Delta f = q/T_p \), frequency modulation rate \( \mu = 1 \times 10^8 \), each cluster contains 64 sub-pulse echoes.

Synthetic bandwidth \( B = N\Delta f = 64\text{GHz} \), carrier frequency \( f_c = 12\text{GHz} \). Signal length is 1 second.

Cone height \( h_1 = 2 \) meter, cylinder height \( h_2 = 1.5 \), \( a = -0.5 \) meters, Cone bottom radius \( r = 0.5 \) meters, Half cone angle \( \alpha = 14 \) degrees, precession angle \( \eta = 20 \) degrees, The angle between the projection of the initial position of the spin axis in the plane XOY and the X axis is \( \phi_0 = 40 \) degrees.

Assume that translational compensation has been performed.

32 sub-pulses are randomly selected from the 64 sub-pulses of the (1, 1) transceiver element, and the rate of reduction is 50%. First simulate with a signal-to-noise ratio of 10dB.
Fig. 2 and Fig. 3 are high resolution range images and ISAR images of the original radar echo signals, respectively; Fig. 4 is a two-dimensional ISAR image of the signal reconstructed by the OMP algorithm; Fig. 5 is reconstructed by using the denoising algorithm proposed in this paper. The ISAR image of the signal. It can be seen that in the case of high SNR, both the OMP algorithm and the denoising algorithm can reconstruct the original signal better and obtain a clearer two-dimensional ISAR image.

The simulation is performed below with a signal-to-noise ratio of -10 db.
Fig. 8 ISAR image of the reconstructed signal by OMP algorithm

Fig. 9 ISAR image of the reconstructed signal by denoising algorithm

Fig. 6 and Fig. 7 are high resolution range images and ISAR images of the original radar echo signals, respectively; Fig. 8 is a two-dimensional ISAR image of the signals reconstructed by the OMP algorithm; Fig. 9 is respectively using the denoising algorithm proposed in this paper. A two-dimensional ISAR image of the constructed signal. It can be seen that in the case of lower signal-to-noise ratio, the OMP algorithm has been seriously interfered by noise, and it is impossible to distinguish the useful signal and noise. The obtained two-dimensional ISAR is disorganized. However, the denoising algorithm proposed in this paper can suppress it better. Noise interference, more accurate extraction of useful signals, the two equivalent scattering points of the ballistic target can still be seen from the two-dimensional ISAR image.

Next, the normalized mean square error of the OMP algorithm and the denoising algorithm under the same rate reduction and different SNR conditions are compared. The normalized mean square error is defined as: 

$$MSE = E \left[ \sum (X - \hat{X})^2 \right] / E \left[ \sum |X|^2 \right].$$

In the signal-to-noise ratio (-10dB, 10dB) between 2dB intervals, 100 times of Monte Carlo simulation under each SNR condition, the noise uses Gaussian white noise, the simulation results are shown below.

Fig. 10 Comparison between denoising algorithm and OMP algorithm

It can be seen that the reconstruction performance of OMP algorithm and the denoising algorithm proposed in this paper is ideal in the case of high signal-to-noise ratio. But when the signal-to-noise ratio
is low, especially when the signal-to-noise ratio is less than 0 db, the reconstruction effect of OMP algorithm decreases sharply, and there will be many wrong sparse solutions, while the denoising algorithm can still restore useful information in the signal.

7. Conclusion
In this paper, the conical cylindrical ballistic target is mathematically modeled, and the compressed sensing theory of noisy signals is analyzed. Then the sparse OFDM-LFM signal system is analyzed, and the signal system is deduced in the echo processing process. The sparse dictionary base and observation matrix corresponding to the compressed sensing are obtained from the specific process. Then an algorithm-based adaptive threshold denoising algorithm is proposed for the signal folding effect in compressed sensing. Finally, the simulation results show that the algorithm has better performance. Anti-noise performance, can extract useful information in the original signal at a lower signal to noise ratio.

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