Supernovae, Landau Levels, and Pulsar Kicks

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Abstract

We derive the energy asymmetry given the proto-neutron star during the time when the neutrino sphere is near the surface of the proto-neutron star, using the modified URCA process. The electrons produced with the anti-neutrinos are in Landau levels due to the strong magnetic field, and this leads to asymmetry in the neutrino momentum, and a pulsar kick. Our main prediction is that the large pulsar kicks start at about 10 s and last for about 10 s, with the corresponding neutrinos correlated in the direction of the magnetic field.

1 Outline

Supernova gravitational collapse of massive star often creates a pulsar. Many pulsars are moving with velocities much larger than stars in our galaxy: PULSAR KICK

I’ll review hydrodynamics of the collapse and neutrino emission \(0 < t < 10\) s: NO GO

I’ll review sterile neutrinos possibly seen at LSND (& MiniBooNe ?) giving the pulsar kicks.

Research by Ernest M. Henley, Mikkel B. Johnson and Leonard S. Kisslinger: We get pulsar kicks consistent with observations from asymmetry due to the electrons produced in the modified URCA process in strong magnetic fields being in Landau levels

2 Review Supernovae and Neutron Star Formation

A supernova is the gravitational collapse of a massive star (\(\geq 8\) sun masses):

1. Collapse to density \(> 10^{14} \text{ g cm}^{-3} > \) nuclear density
   Shocks, bounce, etc.

2. Protoneutron star formed \(\sim 0.01\) s.
   Neutrinos trapped in neutrinosphere. Radius of neutrinosphere \(\sim 40\) km.
3. From 0.1 to 10 sec neutrinosphere contracts from $\sim 40$ km to protostar radius $\sim 10$ km

Neutrinos carry gravitational energy from the emerging star.

4. From $\sim 10$ to $\sim 50$ sec n-n collisions dominate neutrino production and protoneutron star cooling.

The modified URCA process dominates energy emission by neutrinos.

\[ n + n \rightarrow n + p + e^- + \bar{\nu}_e \]

### 2.1 Pulsar Kicks

Resulting neutron stars (pulsar) have a bimodal velocity distribution

- \( 0 < v < 300 \text{km s}^{-1} \)
- \( 1000 < v < 1500 \text{km s}^{-1} \)

(see, e.g., B.M.S. Hansen and E.S. Phinney, [astro-ph/9708071](http://arxiv.org/abs/astro-ph/9708071))

I.e., pulsars with high luminosity tend to have velocities more than an order of magnitude greater than star velocities in our galaxy.
2.2 Attempts at Explanation

There have been many papers written on attempts to explain the pulsar kicks by processes taking place during the first 10 s, when most of the energy from the collapse is carried off by neutrino emission. The main attempts have been asymmetries in the hydrodynamic processes associated with the collapse, the scattering of neutrinos produced by the URCA process by nucleons polarized by the strong magnetic field within the neutrinosphere, and by sterile neutrino processes. We briefly review these attempts.

2.2.1 Asymmetries in core collapse

There have been many attempts to explain the pulsar kicks by the asymmetries in the initial collapse of the massive star. Recently authors of a calculation postulating a slow collapse (t \( \geq 1 s \)) claim that they can predict pulsar velocities greater than 500 km/s (L. Scheck et. al., P.R.L. 92 (2004) 011103). In a review by J.Murphy, A. Burrows, and A Heger, Astrophysics J.615 (2004) 615, a careful analysis of all hydrodynamic calculations shows that one cannot obtain a kick of more than about 200 km/s.

2.2.2 Processes involving standard neutrinos with a strong magnetic field

During the first 10 s, when the neutrinosphere has a radius starting about 40 km, the URCA process dominates neutrino emission (see O.F. Dorofeev et al, Sov. Astron. Lett. 11 (1985)),

\[
\begin{align*}
  n & \rightarrow p + e^- \bar{\nu} \\
  e^- + p & \rightarrow n + \nu_e \\
  e^+ + n & \rightarrow p + \bar{\nu}_e,
\end{align*}
\]

where the nucleons are polarized by the strong magnetic fields of the protoneutron star. A few years ago calculations of elastic neutrino scattering from the polarized nucleons ( C.J. Horowitz and J. Piekarewitz, N.P. A640 (1998) 281; C.J. Horowitz and G. Li, PRL 80 (1998) 3694) found this process to be promising to explain the pulsar kicks.

However, if one includes the neutrino absorption opacities modified by the strong magnetic fields, the asymmetric neutrino emission is reduced, and these processes with standard neutrinos cannot account for the pulsar kicks, D. Lai and Y-Z. Qian, astro-ph/9802345, ApJ.(1998). Note that although the neutrinosphere at 1 s is about 40 km, the mean free path of neutrinos is only or the order of 1 cm, and the asymmetric neutrinos are not emitted.

Mechanisms with standard neutrinos during the first 10 s have not been able to account for the large pulsar velocities.
2.2.3 Pulsar kicks from sterile neutrinos in a strong magnetic field

If sterile neutrinos exist they can be produced by oscillations in the protoneutron star matter. In the presence of the strong magnetic fields of the protoneutron star neutrino asymmetries will be produced. Since the opacities of sterile neutrinos are very small, this has been studied as a possible source of the pulsar kicks (A Kusenko and G. Segre, PRL 77 (1996) 4872; PL B396 (1997) 197).

Using an extension of usual concept of the CKM mixing matrix giving the weak interaction states in terms of the mass eigenstates, sterile/active neutrino mixing given by mixing angle $|\theta_m| << 1$:

$$|\nu_1 > = \cos \theta_m |\nu_e > - \sin \theta_m |\nu_s > \text{ trapped}$$
$$|\nu_2 > = \sin \theta_m |\nu_e > + \cos \theta_m |\nu_s > \text{ not trapped}$$

In a calculation that requires the consistency of sterile neutrinos and dark matter (G.A Fuller, A. Kusenko, I. Mocioiu and S. Pascoli, PR D68 (2003) 103002) the parameters needed to give the kick were found, as illustrated in the figure below.

![Graph showing the relationship between $m_S$ (KeV) and $\sin^2 \theta$](image)

One sees that a sterile neutrino mass of 1- 10 Kev is needed, which is not expected.

New data on the sterile neutrino’s mass and mixing angle is expected from MiniBoone. If the results are similar to those of LSND, with fits such as $\Delta m^2 = 0.2 eV^2 \text{ and } \sin^2 \theta = 0.068$ (From William Lewis, LANL) there is a possibility to get the pulsar kicks from sterile neutrinos.
3  Pulsar Kicks From the Modified URCA Process in a Strong Magnetic Field

In the later stage ($t \sim 10^8$ s) the Modified URCA process:

$$n + n \rightarrow n + p + e^- + \bar{\nu}_e$$

(1)

dominates the cooling of the neutron star.

**URCA DIAGRAMS**

The early work includes

J.N. Bahcall and R.A. Wolf, PRL 14 (1965) 343; PR 140 (1965) B1452;
E.G. Flowers, P.G. Sutherland and J.R. Bond, PRD 12 (1975) 315.
B.L. Friman and O.V. Maxwell, Astrophysical J. 232 (1979) 541, who used
pion exchange and short-range potential in detailed calculations.
Modified URCA ingredients: Nonrelativistic OPE interaction and Standard W-S Model:

\[ V_{OPE} = \left( \frac{f}{m_\pi} \right)^2 \sigma^{(1)} \cdot k \frac{-1}{k^2 + m_\pi^2} \sigma^{(2)} \cdot k_T^{(1)} \cdot \tau^{(2)}, \]

with \( f \simeq 1.0, \sigma, \tau \) the Pauli spin, isospin operators, and \((1,2)\) refer to the two nucleons at the pion vertices.

Weak axial n-p vertex in nonrelativistic Standard Weinberg-Salam model:

\[ W_A = -\frac{G}{\sqrt{2}} g_A \chi^\dagger \cdot \sigma \chi, \]

with \( G = \frac{10^{-5}}{M_P} \), \( g_A = 1.26 \), and the lepton current defined by

\[ l_\mu = \bar{u}(q^e)\gamma_\mu(1 - \gamma_5)u(q'^\nu). \]

We need the matrix element product

\[ \Sigma_{\text{lepton,nucleon spins}} |M|^2 \equiv \Sigma_{\text{lepton,nucleon spins}} |V_{\text{strong}} W_{\text{weak}}|^2. \]

In previous research the vector-axial vector product was used. However, we realize that in a strong magnetic field the axial-axial product, which is much larger, can be used, as the magnetic field provides the asymmetry.

We also make use of the fact that in the strong B field with the electrons in Landau levels, \( u(q_e) \rightarrow \psi_{\text{Landau}}(q_e) \) one can get a large pulsar kick without n-polarization, greatly simplifying the calculation. The stationary wave function of an electron in a strong magnetic field is (J.J. Matese and R.F. O’Connel, Phys. Rev. 180, 1289 (1969))

\[ \tilde{B} = B \hat{z} \]

\[ \psi_{p,\text{Landau},n=0}^{\text{Landau}}(q_{\perp},p_{\parallel},\phi) = i(\sqrt{\gamma})^{-1} \left( q_e^e \right)^2 \gamma \psi_{\text{Dirac},-}(q^e) \delta(q^e - p_e), \]

where \( \gamma = Bm_e^2/2B_c \), with the critical magnetic field \( B_c \simeq 4 \times 10^{13} G \). I.e., in a strong magnetic field the electron moves as a plane wave in the z direction, with energy of Landau levels, and is fully negatively polarized in the lowest Landau level.

The leptonic traces needed are

\[ \text{Tr}(\text{lepton})[l_i^1 l_j] = \text{Tr}[l_i^1 l_j](\alpha) + \text{Tr}[l_i^1 l_j](\beta) + \text{Tr}[l_i^1 l_j](\gamma) \]

\[ \text{Tr}[l^1 4 \gamma_i l_j](\alpha) = -4\gamma^{-1} g_{ij} q^e \cdot q'^\nu(1 + (1 - \delta_{i3})) \]

\[ \text{Tr}[l_i^1 l_j](\beta) = i4\gamma^{-1} \epsilon_{ijk} q'^\nu q_0^e \]

\[ \text{Tr}[l_i^1 l_j](\gamma) = 4\gamma^{-1} (q_i^e q_j^e + q_i^e q_j^e). \]
Our notation for the matrix element products is

\[ k = p_1 - p_3, \quad p = p_2 - p_1, \quad k^e = k + p \]

\[ A = \left( \frac{f}{m_\pi} \right)^2 \frac{G}{\sqrt{2}} g_A \left( \frac{1}{\omega (k^2 + m_\pi^2)} \right) \]

\[ R(k) = \frac{k^2 + m_\pi^2}{(k^e)^2 + m_\pi^2}. \quad (6) \]

### 3.1 Neutrino emissivity

Defining the Axial Product Matrix Element, \( M_{AA} \equiv |M_A|^2 \):

\[ M_{AA} = |Tr(\text{lepton})[l_i l_j][W_A(V_{OPE} + \text{exchange})]_{ij}|^2, \quad (7) \]

the general form of neutrino emissivity with A-A process is

\[ e^\nu = (2\pi)^4 \int d^3p^i \frac{d^3q^e}{(2\pi)^3} \frac{d^3q^{e*}}{(2\pi)^3} \delta(E_{\text{final}} - E_{\text{initial}}) \delta(\vec{p}^e - \vec{p}^{e*}) M_{AA} F, \quad (8) \]

where \( F \) is the product of the initial and final Fermi-Dirac functions corresponding to the temperature and density of the medium. \( \int d^3q^e \) omitted in present work.

The nucleons and the electrons are in thermal equilibrium. A crucial observation of this work is that the time scale of the strong interaction in the modified URCA process is short compared to the time scale for the electron, which is that of electromagnetic interaction. Therefore the proton quickly reaches thermal equilibrium, while the process of the electron state transforming to the \( n=0 \) Landau state does not interfere with the proton reaching its Fermi momentum. Therefore we can use the values for the nucleon and electron momenta in the matter of the protoneutron star, as derived in the Friman-Maxwell paper.

The \( \int d^3q^e \), involving a Landau level, makes use of the momentum space representation, Eq. (4), and the integral over the transverse momentum direction is thereby given as

\[ \int dq^e e^{-q^e_1^2/2\lambda} = 2\lambda \cdot (9) \]

From this and Eq. (5) one finds that the factors of \( \gamma \) from the Landau level electrons cancel.

Recognizing that the nucleons are in thermal equilibrium, with Fermi momentum \( p_F \), the angular integrals can be done using the the pion exchange momentum in the direct diagram, as an independent vector by inserting:

\[ \int d^3k \delta(\vec{k} - \vec{p}_1 + \vec{p}_3) = I. \quad (10) \]

For the D-D diagram the only angular integral needed is \( d\Omega_k \), but for the E-E and D-E terms two angular integrals are needed. We use \( d\Omega_{k^e} \), which is almost independent, and also \( d\Omega_p \) as a check. The \( d\Omega_{q^e} \) is modified by a factor from the standard result.
With no nucleon polarization the calculation is greatly simplified, and we find

\[
\int \int M_{A-A} = 1.38 \times 10^4 A^2 q^e q^\nu p_F^4 \ (k, k^e) \text{ formalism}
\]

\[
\int \int M_{A-A} = 3.8 \times 10^4 A^2 q^e q^\nu p_F^4 \ (k, p) \text{ formalism}
\]  \hspace{1cm} (11)

The result for the energy integral over the Fermi distributions is that of Friman-Maxwell:

\[
I = 5.68 \times 10^3 (kT)^8 \hspace{1cm} (12)
\]

Using the standard values for the nucleon Fermi momenta, and \( q^e = 85 MeV, q^\nu = 4.7kT \) we find the emissivity

\[
\epsilon^{AS}(k, k^e) \simeq 1.18 \times 10^{24} \left( \frac{T}{10^9 K} \right)^7 \text{ erg cm}^{-3}\text{s}^{-1}
\]

\[
\epsilon^{AS}(k, p) \simeq 2.96 \times 10^{24} \left( \frac{T}{10^9 K} \right)^7 \text{ erg cm}^{-3}\text{s}^{-1},
\]  \hspace{1cm} (13)

in the two formalisms.

During \( 10s < t < 30s, \ 10^9 K < T < 10^{11} K \) approximate period and temperature, the mean free path of the neutrinos emitted is or the order of a few cm. Thus the modified URCA asymmetric emission takes place when the neutrinosphere is just within the neutron star, as depicted. Therefore, the asymmetric energy emitted in the 10-20 s period is

\[
E^{AS}(k, k^e) \simeq 1.21 \times 10^{40} \left( \frac{T}{10^9 K} \right)^7 (R_{ns}^3 - R_{\nu}^3) \text{ ergs}
\]

\[
E^{AS}(k, p) \simeq 2.74 \times E^{AS}(k, k^e),
\]  \hspace{1cm} (14)

with a factor of .52 for the neutrinos that miss the protoneutron star.
3.1.1 Radius of neutrinosphere during modified URCA emission

The final step in our derivation is to estimate the volume in which the neutrino emission takes place with the modified URCA process in a strong magnetic field, which means finding the radius of the neutrinosphere during the period of emission.

Our starting point is the energy-momentum tensor, \( T^{\mu \nu} \) for the neutrinos with a distribution function \( f_\nu(x, k, t) \) for each type of neutrino, giving an energy density, \( U \) and momentum density \( F^i \)

\[
U = T^{00} = \int \frac{d^3k}{(2\pi)^3} k_0 f_\nu, \\
F^i = T^{0i} = \int \frac{d^3k}{(2\pi)^3} k^i f_\nu. \tag{15}
\]

Using the Conservation of momentum, \( \partial_\nu T^{0\nu} = 0 \) gives t-dependence:

\[
\partial_t U + \Delta \cdot F = 0.
\]

Based on these equations, and using the Spherical Eddington Model for neutrino atmosphere (Schindler-Shapiro, ApJ 259 (1982) 311; Janka-Raffelt, PR D59 (1998) 023005, Barkovich-Olivo-Montemayor, hep-ph/0503113) with techniques that can be found in Barkovich et al, we find for the radius of the neutrinosphere during the period of emission:

\[
R_\nu \approx 9.96 \text{ km} \tag{16}
\]

when when the temperature is in the range \( T \approx 10^{10} K \) giving

\[
E^{AS} \approx 1.45 \times 10^{42} \text{ ergs} \left(\frac{T}{10^9}\right)^7, \tag{17}
\]

and a pulsar velocity \( v = 1000 \text{ km/s} \) for an expected \( T=10^{10} \text{ K} \). Our results are as shown:

![Graph showing pulsar velocity vs T assuming constant emission volume](image-url)
4 CONCLUSIONS

At the time the radius of the neutrinosphere is that of the neutron star the modified URCA process dominates the energy emission.

At this time the electrons produced in the modified URCA process are in Landau levels, causing asymmetry of all neutrinos emitted.

When the radius of the neutrinosphere is about 9.96 km vs the neutron star radius of 10 km, the neutron star receives a kick of 1000 km and more, consistent with observation.

WE PREDICT A STRONG CORRELATION BETWEEN THE PULSAR’S VELOCITY AND LUMINOSCITY FOR LARGE PULSAR KICKS, AS OBSERVED (see Hansen et al).

WE PREDICT THE BIG KICK STARTS AT ABOUT 10 s, CONSISTENT WITH AN ANALYSIS OF SN1987 (K. Hirata et al, P.R.D38, 448 (1988)).

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