Synthesis of an automatic temperature controller with sequential switching on of the correcting links

D Ya Antipin, D A Bondarenko and V I Vorobiev
Bryansk State Technical University, 7, Boulevard 50 let Oktyabrya, Bryansk, 241035, Russia

E-mail: dilekter@gmail.com

Abstract. The paper presents a method for the synthesis of an automatic control system regulator by means of integral assessment, which provides tuning to the technical optimum in the case when the object of control is a high-inertia process - control of the temperature of the traction motor. The proposed correction scheme assumes that a PI controller and a low-pass filter are connected in series together with the control object. The data obtained as a result of the modeling indicate satisfactory values of the quality criteria of the control process. It was found that overshoot and control time in a closed-loop automatic control system increases with an increase in the temperature setting, with an increase in the initial temperature, overshoot decreases, control time increases, with an increase in the load current of the traction motor, overshoot decreases and control time increases.

1. Introduction
The developed automatic control systems (ACS), in addition to realizing their direct functions, must also meet such requirements as ensuring stability and quality of control. The problem of designing such a system with predetermined properties is called a synthesis problem. When solving it, a part of the system is assumed to be already given and supplemented with the necessary correcting links or a regulator that ensure the fulfillment of the requirements for stability and quality of control [1-6].

At present, automatic PI (PID) – regulators, which are connected in series with the control object, are widely used [4-10].

With sequential correction, the block diagram of the control loop of the variable $x$ can be represented as shown in figure 1, consisting of a correcting link with a transfer function $W_{c \text{ seq}}(p)$ and an open uncorrected system with a transfer function $W_{op}(p)$.

![Sequential scheme of ACS correction](image)

Figure 1. Sequential scheme of ACS correction.
The transfer function of the open-loop corrected system takes the form

\[ W_{c_{\text{seq}}}(p) = W_{c_{\text{seq}}}(p)W_{op}(p) \]  

(1)

2. Methods of solution

According to the results of work [11], in the case when the object of control is a high-inertial process, for example, temperature control of a traction drive (TD), the transfer function can be reduced to the form [12]

\[ W_{c_{\text{op}}}(p) = \frac{1}{T_0 p (T_\mu p + 1)} \]  

(2)

The frequency characteristics of such an ACS are shown in figure 2.

![Figure 2](image)

**Figure 2.** Frequency characteristics of the corrected system with sequential correction.

The low- and mid-frequency asymptote of the LAFC has a slope of -20 dB/dec (straight line 1), and the phase margin \( \Delta \phi(\omega_c) \), determined from curve 2, depends on the degree of remoteness of the cutoff frequency \( \omega_c = 1/T_0 \) from the nearest coupling frequency \( \omega_{c_{\text{f}}} = 1/T_\mu + 1 \). The phase margin at the cutoff frequency will be:

\[ \Delta \phi(\omega_c) = -\pi + \frac{\pi}{2} + \sum_{i=1}^{m} \arg T_\mu \omega_c \]  

(3)

The angles \( \Delta \phi(\omega_c) \) in (3) are small, due to the fact that at the corresponding frequencies of the conjugation \( \arg T_\mu \omega_c = \arg t = \pi/4 \). Since \( T_\mu > T_\mu + 1 \) and \( \arg T_\mu \omega_c < \pi/4 \) can be taken approximately \( \arg T_\mu \omega_c \approx T_\mu \omega_c \).

Thus,

\[ \Delta \phi(\omega_c) = -\pi + \sum_{i=1}^{m} T_\mu \omega_c = \frac{\pi}{2} + T_\mu \omega_c \]  

(4)
where \( T_\mu = \sum_{i=1}^{m} T_i \) – the total uncompensated constant of the control loop, equivalent in terms of the phase margin loss at the cutoff frequency to all its real uncompensated inertia.

Corresponding (2) LAF of the control loop in the area of low and medium frequencies coincides with straight line 1 (figure 2), and in the region of high frequencies it is represented by asymptote 3, which has a slope of -40 dB/dec. The conjugation frequency for this asymptote \( 1/T_\mu \) is located closer to the cutoff frequency, which takes into account the determined (3) effect of all small constants on the dynamic properties of the control loop.

In this case, the transfer function of the closed control loop will have the form

\[
W_{ccl}(p) = \frac{1}{T_\mu p(T_\mu p + 1) + 1},
\]

and the roots of its characteristic equation are

\[
p_{1,2} = \frac{-1}{2T_\mu} \pm \frac{1}{4T_\mu^2} - \frac{1}{T_\mu T_0} = \frac{1}{T_0} \left( -\frac{a}{2} \pm \frac{a^2}{4} - a \right),
\]

where \( a = T_0 / T_\mu \) – ratio of control loop constants.

With \( a < 4 \) and a jump of the reference with zero initial conditions, the transition function is determined by the following equation

\[
h(t) = x \left[ 1 - e^{-2T_\mu} \left( \cos \frac{\sqrt{4a-a^2}}{2aT_\mu} t + \frac{a}{\sqrt{4a-a^2}} \sin \frac{\sqrt{4a-a^2}}{2aT_\mu} t \right) \right].
\]

The total uncompensated constant \( T_\mu \) completely determines the speed of the ACS in terms of the total time of the transient process \( t_p \). In accordance with (7), the free components of the transient process decay over time

\[
t_p = (3 - 4)2T_\mu = (6 - 8)T_\mu.
\]

The oscillation of such an open-loop linearized system is determined by the ratio of the loop constants \( a \), the same indicator determines the overshoot. Consequently, by selecting the ratio of constants, it is possible to provide the required dynamic performance at a speed limited by the level of the total uncompensated time constant \( T_\mu \).

Having given the required ratio of constants \( a \) and determined \( T_\mu \) by (4), we can write down the desired transfer function of the open control loop

\[
W_{op}(p) = \frac{1}{aT_\mu p(T_\mu p + 1)}.
\]

The transfer function of the open-loop corrected system is:

\[
W_{op}(p) = \frac{k_kk_k...k_k}{(T_\mu p + 1)\prod_{i=1}^{m}(T_i p + 1)}.
\]

The transfer function of the regulator in accordance with (1) is defined as
According to (1), we can conclude that the transfer functions of the regulator become more complicated as the number of uncompensated constants \( l \) increases. For \( l = 0 \) (all \( T_i \) are small) it takes the form

\[
W_c(p) = \frac{1}{T_p},
\]

where \( T_p = (c_1 c_2 \ldots c_n) a T_\mu \).

In this case, the controller is an integrator with an integration constant \( T_i \) (I-controller). For \( l = 1 \)

\[
W_c(p) = \frac{T_i p + 1}{T_p} = \frac{T_i}{T_p} + \frac{1}{T_p},
\]

that is, a proportional-integral controller (PI controller) is required. When \( l = 2 \), a proportional integro-differential controller (PID controller) is required, and with a further increase in \( l \) in its transfer function, a two-fold or greater differentiation of the input signal is required.

Based on the requirements of the necessary noise immunity of the circuit, only one signal differentiation is allowed. According to the same requirements, it is desirable to use controllers no more complicated than a PI controller [11].

Dynamic indicators of control quality with this approach to the synthesis of the ACS controller are determined by the ratio of constants \( a_l \). Figure 3 shows a number of dependencies \( h_1 = f(t) \) at different \( a_l \) values. If \( a_1 = 4 \), the transient function has aperiodic character, and the regulation time is \( t_r \approx t_{sp} = (6-8) T_\mu \). Decreasing \( a_1 \) to \( a_1 = 2 \) clearly increases the oscillation, overshoot appears, and the control time decreases. A further decrease of \( a_1 \) leads to a rapid increase in oscillation and overshoot, and the effect of reducing the control time \( t_r \) is gradually reduced.

![Figure 3. Transient function graphs for different \( a_l \).](image-url)
The curve corresponding to the value \( a_i = 2 \) (figure 3) provides the minimum regulation time \( t_p = 4,7T_\mu \) with practically negligible overshoot \( \Delta h_{t\text{max}} = 0,043 \). This setting is optimal for many control systems and is used as the main standard setting. It is called tuning to the technical optimum or modulus optimum.

When tuning the control loop to the technical optimum (\( a_i = 2 \)), its transfer function takes the form

\[
W_{c\text{,op}}(p) = \frac{1}{2T_\mu p(T_\mu p + 1)},
\]

(14)

The same for a closed control loop:

\[
W_{c\text{,cl}}(p) = \frac{1}{2T_\mu p(T_\mu p + 1) + 1},
\]

(15)

The logarithmic frequency characteristics of the control loop tuned to the technical optimum are shown in figure 4.

![Figure 4. LAFC when tuning to a technical optimum.](image)

According to the structural diagram of an open-loop ACS, we will supplement it with a series-connected correcting element (temperature controller) with a transfer function \( W_\rho(p) \) and a series-connected filter with a transfer function \( W_\mu(p) \). In this case, the transfer function of the filter will be equal to [11]

\[
W_\mu(p) = \frac{1}{(T_\mu p + 1)},
\]

(16)

where \( T_\mu \) – smallest uncompensated time constant.
3. Conclusions
The performed transformations make it possible to obtain the transfer function of the corrected open-loop system, taking into account the successively included correcting links

\[ W_{\text{cp}}(p) = W_{\mu}(p)W_{\text{op}}(p) = \frac{1}{2T_\mu p(T_\mu p + 1)}. \]  

(17)

From expression (17) we find the transfer function of the temperature controller \( W_\nu(p) \):

\[ W_\nu(p) = \frac{W_{\text{cp}}(p)}{W_\mu(p)W_{\text{op}}(p)} = \frac{(T_\mu p + 1)(T_{TD} p + 1)}{2T_\mu p(T_\mu p + 1)k_{cp}} = \frac{T_{TD}}{2k_{cp}T_\mu} + \frac{1}{2k_{cp}T_\mu p}, \]  

(18)

The resulting transfer function (18) of the temperature controller \( W_\nu(p) \) can be obtained by parallel connection of two links: proportional (with constant gain) and integral. This means that we can conclude that the obtained ACS temperature controller is a PI controller.

Let us verify the calculated temperature controller on a nonlinear temperature control system that takes into account the limitation on the rotation frequency of the cooling fan drive and the cooling intensity depending on the rotation frequency of the cooling fan shaft. The graph of the transient process of a closed ACS obtained in the Matlab Simulink software package is shown in figure 5.

![Figure 5](image)

**Figure 5.** Transient processes in a closed automated control system when using a proportional-integral temperature controller at different temperatures of \( \Delta \theta \) and different values of the load current of the traction motor: 1 – \( I_1 = 0.5 \); 2 – \( I_2 = 0.8 \); 3 – \( I_3 = 1.0 \).

The analysis of the data obtained made it possible to establish that when using a PI controller, which ensures the system is tuned to a technical optimum, overshoot and control time in a closed ACS increases with an increase in the temperature setting. With an increase in the initial temperature, overshoot decreases, and control time increases. With increasing load current of the traction motor, overshoot decreases and control time increases. The nature of the transient process is aperiodic, the error in the steady state is 0.
References

[1] Bucz S and Kozakova A 2018 Advanced Methods of PID Controller Tuning for Specified Performance (London: IntechOpen) p 218

[2] Zeng D, Zheng Y, Luo W, Hu Y, Cui Q, Li Q and Peng C 2019 Research on Improved Auto-Tuning of a PID Controller Based on Phase Angle Margin Energies 12 1704

[3] Ahmadi Dastjerdi A, Saikumar N and HosseinNia H 2018 Tuning guidelines for fractional order PID controllers: Rules of thumb Mechatronics 56 26-36

[4] Zhang J and Guo L 2019 Theory and Design of PID Controller for Nonlinear Uncertain Systems IEEE Control Systems Letters 3(3) 643-8

[5] Xingming X and Peng A 2012 Parameter Tuning for PID Controller Based on Grey Relational Analysis J. of Convergence Information Technology 7(8) 326-34

[6] Yu H 2014 Temperature control in PID controller by Labview (Sweden: University of Gävle) p 34

[7] Xie W, Wang J and Wang H 2019 PI Controller of Speed Regulation of Brushless DC Motor Based on Particle Swarm Optimization Algorithm with Improved Inertia Weights (Hindawi: Mathematical Problems in Engineering)

[8] Sabir A and Kassas M 2015 A novel and simple hybrid fuzzy/PI controller for brushless DC motor drives Automatika 56(4) 424-35

[9] Jin C, Ryu K and Sung S 2014 PID auto-tuning using new model reduction method and explicit PID tuning rule for a fractional order plus time delay model J. of Process Control 24(1) 113-28

[10] Kiree C, Kumpanya D, Tunyasrirut S and Puangdownreong D 2016 PSO-based optimal PI(D) controller design for brushless DC motor speed control with back EMF detection J. of Electrical Engineering & Technology 11(3) 715-23

[11] Antipin D, Bondarenko D and Vorobiev V 2019 Synthesis of the Automatic Temperature Control of the Locomotive Asynchronous Drive Engine International Multi-Conference on Industrial Engineering and Modern Technologies (FarEastCon) 18373348

[12] Antipin D, Bondarenko D and Vorobiev V 2018 Improving shunting-export locomotive towline when operating in strip mine based on its stationary thermal model IOP Conference Series: Earth and Environmental Science 194 5 052005