ON THE GRAIN-MODIFIED MAGNETIC DIFFUSIVITIES IN PROTOPLANETARY DISKS

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ABSTRACT

Weakly ionized protoplanetary disks (PPDs) are subject to nonideal magnetohydrodynamic (MHD) effects, including ohmic resistivity, the Hall effect, and ambipolar diffusion (AD), and the resulting magnetic diffusivities ($\eta_O$, $\eta_H$, and $\eta_A$) largely control the disk gas dynamics. The presence of grains not only strongly reduces the disk ionization fraction, but also modifies the scalings of $\eta_H$ and $\eta_A$ with magnetic field strength. We analytically derive asymptotic expressions of $\eta_H$ and $\eta_A$ in both the strong and weak field limits and show that toward a strong field, $\eta_H$ can change sign (at a threshold field strength $B_{\text{th}}$), mimicking a flip of field polarity, and AD is substantially reduced. Applied to PPDs, we find that when small ~0.1 (0.01)$\mu$m grains are sufficiently abundant (mass ratio ~0.01 (10\textsuperscript{-4})), $\eta_H$ can change sign up to ~2–3 scale heights above the midplane at a modest field strength (plasma $\beta$ ~ 100) over a wide range of disk radii. The reduction of AD is also substantial toward the AD-dominated outer disk and may activate the magnetorotational instability. We further perform local nonideal MHD simulations of the inner disk (within 10 au) and show that, with sufficiently abundant small grains, the magnetic field amplification due to the Hall-shear instability saturates at a very low level near the threshold field strength $B_{\text{th}}$. Together with previous studies, we conclude by discussing the grain-abundance-dependent phenomenology of PPD gas dynamics.

Key words: accretion, accretion disks – magnetohydrodynamics (MHD) – protoplanetary disks

1. INTRODUCTION

The gas dynamics of weakly ionized protoplanetary disks (PPDs) is to a large extent controlled by its coupling with magnetic fields and is described by nonideal magnetohydrodynamic (MHD) effects, including ohmic resistivity, the Hall effect, and ambipolar diffusion (AD). The three effects dominate in different regions of PPDs and affect the gas dynamics in different ways. A constant theme of PPD research has been to estimate the strength of magnetic diffusivities and study their consequences for gas dynamics, especially on the level of turbulence and the efficiency of angular momentum transport (see Turner et al. 2014, p. 411 for recent review).

The strength of magnetic diffusivities largely depends on the gas density $\rho$, the level of ionization, and the magnetic field strength $B$. In the absence of grains, the ohmic, Hall, and ambipolar diffusivities can be expressed in particularly simple form (Salmeron & Wardle 2003):

$$\eta_O = \eta_\text{e}, \quad \eta_H = \frac{cB}{4\pi e n_e}, \quad \eta_A = \frac{B^2}{4\pi \gamma \rho \rho_i},$$

where $\rho_e \propto (n/n_e)$ is the resistivity due to electrons, $n_e$ and $n$ are the electron and neutral number densities, and $\rho_i$ is the ion density.

These expressions are complicated by the presence of dust grains. On the one hand, the grains can substantially reduce the level of ionization, hence boosting all three magnetic diffusivities proportionally. On the other hand, grains carry charge and result in a much more complex dependence of diffusivities on magnetic field strength (e.g., see Wardle 2007 and Bai 2014a), yet such complex dependence has not been systematically explored.

As an initial effort, Bai (2011b, hereafter B11) considered the limit when grains are tiny (approximately nanometer-sized polycyclic aromatic hydrocarbons (PAHs)) so that the conductivity of charged grains is similar to ions. He showed that when tiny grains are the dominant charge carrier, the Hall diffusivity diminishes and the AD is substantially reduced in the strong field limit, which may be important in the outer region of PPDs.

In this paper, we extend the work of B11 to more general situations with more typical grain sizes (submicron to micron) relevant to PPDs. We focus on the disk interior (rather than the surface), where grains are likely the dominant charge carrier and are mostly singly charged, and systematically explore how magnetic diffusivities depend on field strength. We hope to provide researchers with useful guidance for modeling nonideal MHD effects from ionization chemistry, which has been fruitful in recent years (e.g., Turner et al. 2007; Bai 2013, 2014b, 2015; Bai & Stone 2013; Lesur et al. 2014; Gressel et al. 2015; Simon et al. 2015).

This work is also motivated by the recent advances in understanding the gas dynamics of the Hall effect, where the Hall-shear instability may substantially amplify the midplane field strength in the inner region of PPDs (Kunz 2008; Bai 2014b; Lesur et al. 2014) when the background field is aligned with disk rotation. With strong field amplification, we discuss whether grain-modified diffusivities can feed back into the PPD gas dynamics.

We present our simplified model in Section 2, and its application to PPDs is discussed in Section 3. We perform MHD simulations of the inner disk and discuss the results in Section 4. In Section 5, we summarize and conclude.

2. A SIMPLIFIED MODEL FOR GRAIN-MODIFIED DIFFUSIVITIES

In a weakly ionized gas, the general expressions for the three magnetic diffusivities are (Wardle 2007; Bai 2011a)

$$\eta_O = \frac{c^2}{4\pi \sigma_0}, \quad \eta_H = \frac{c^2}{4\pi \sigma_H^2 + \sigma_P^2}, \quad \eta_A = \frac{c^2}{4\pi \sigma_H^2 + \sigma_P^2} - \eta_O,$$

where $\sigma_0$, $\sigma_H$, and $\sigma_P$ are the conductivity due to electrons, ions, and grains, respectively.
where \( \sigma_0, \sigma_H, \sigma_p \) are ohmic, Hall, and Pedersen conductivities, defined as

\[
\sigma_0 = \frac{e c}{B} \sum_{j} n_j |Z_j| \beta_j,
\]

\[
\sigma_H = \frac{e c}{B} \sum_{j} n_j Z_j \beta_j,
\]

\[
\sigma_p = \frac{e c}{B} \sum_{j} n_j |Z_j| \beta_j,
\]

(3)

where the summation goes over all charged species \( j \), with \( n_j \) and \( Z_j \) being the number density and charge of individual charged species, respectively. The Hall parameter \( \beta_j \) is given by

\[
\beta_j = \frac{|Z_j| e B}{m_j c} \frac{1}{\gamma_j \rho},
\]

(4)

which describes the ratio between the gyroradius of the charged species and their collision frequency with the neutrals, where \( \gamma_j \equiv \langle \sigma v \rangle_j / (n_{\text{ne}} + n_j) \), with \( \langle \sigma v \rangle_j \) being the rate coefficient for collisional momentum transfer with the neutrals. The charged species \( j \) is strongly coupled to the neutrals if \( \beta_j \ll 1 \) (weak field), and it is strongly coupled to magnetic fields when \( \beta_j \gg 1 \) (strong field). The values of \( \langle \sigma v \rangle_j \) for electron, ion, and grain can be found in Equation (14)–(16) of (Bai 2011a, 2014a), giving the Hall parameter

\[
\beta_e \approx 2.1 \frac{B}{n_{\text{H}}} \min \{1, T_{100}^{-1/2}\},
\]

\[
\beta_i \approx 3.3 \times 10^{-3} \frac{B}{n_{\text{H}}} ,
\]

\[
\beta_g \approx (B/n_{\text{H}}) \min \{3.2 \times 10^{-3}|Z_g|^{-1}, 10^{-9} a_1^{-2} T_{100}^{-1/2}\},
\]

(5)

where \( B \) is the magnetic field measured in gauss, \( n_{\text{H}}=n_{\text{H}}/10^{15} \text{ cm}^3 \), \( Z_g \) is the grain charge, \( a_1 \) is the grain size in \( \mu \text{m} \), and the disk temperature \( T_{100}=T/100 \text{ K} \).

We treat grains less than a nanometer in size (i.e., PAHs) as “tiny grains.” In terms of collision rates with neutrals, charged tiny grains behave the same as typical ions. In this case, collisions are mediated by an \( r^{-4} \) potential resulting from ions interacting with an induced electric dipole in the neutrals, leading to a collision rate coefficient \( \langle \sigma v \rangle \) that is independent of temperature (e.g., Draine 2011). For larger-sized grains, the geometric cross section takes over, and hence the collision rate is proportional to \( a^2 T^{-1/2} \). The transition occurs at approximately nanometer size. B11 studied the tiny-grain limit, where \( \beta_g \gg \beta_i \approx \beta_g \). In this paper, we mainly consider grains that are larger, and hence \( \beta_g \gg \beta_i \gg \beta_e \).

As long as the ion mass is much larger than the neutral mass, different ion species have their Hall parameters very close to the \( \beta_i \) quoted above and can be treated as a single species. In the interior of PPDs where the ionization level is low, chemistry calculations show that (small) grains are largely singly charged until the ionization fraction exceeds the grain abundance toward the disk surface (e.g., see Figure 6 of Wardle 2007 and Figure 1 of Bai 2011a). We also assume single-sized grains for simplicity, which is typically adopted in PPD chemistry calculations. While a grain size distribution is expected in reality, we expect a single-size treatment to capture and better clarify the essence of their effects. Therefore, we consider only four types of charged species: electrons, ions, and positively and negatively charged grains, whose number densities are represented by \( n_e, n_i, n_{gr}^{+}, \) and \( n_{gr}^{-} \).

Under these conditions, the Hall and Pedersen conductivities are

\[
\sigma_H = -\frac{1}{1 + \beta_e^2} + \frac{n_i}{n_e} \frac{1}{1 + \beta_i^2} + \frac{n_{gr}^+ - n_{gr}^-}{n_e} \frac{1}{1 + \beta_g^2},
\]

\[
\sigma_p = \frac{\beta_e}{1 + \beta_e^2} + \frac{n_i}{n_e} \frac{\beta_i}{1 + \beta_i^2} + \frac{n_{gr}^+ - n_{gr}^-}{n_e} \frac{\beta_g}{1 + \beta_g^2}.
\]

(6)

Hereafter, we omit the prefactor \( e n_i c / B \) in conductivities and \( e B / 4 \pi n_e \) in diffusivities so as to make them dimensionless. The grain-free diffusivities (1) correspond to \( \eta_0 = 1/\beta_i, \eta_H = 1, \eta_H = \beta_i \), and hence the conventional ohmic regime applies in a weak field with \( \beta_e < 1 \), the AD regime applies in a strong field \( \beta_i > 1 \), and the Hall-dominated regime lies in between. For brevity, we further define \( n_{gr}^\pm = n_{gr}^+ + n_{gr}^- \).

Charge neutrality guarantees that only three of the four number densities are independent. With additional normalization by \( n_e \), only two of them serve as independent model parameters. We find it useful to choose the two ratios \( n_i / n_e \) and \( n_{gr}^\pm / n_e \) as model variables. Without grains, their values are obviously 1 and 0, while when grains become important charge carriers, \( n_i \gg n_e \), and \( n_{gr}^\pm > n_i \).

The Hall parameters \( \beta_e, \beta_i, \beta_g \) can be considered as proxies for magnetic field strength. According to Equation (5), we fix \( \beta_e = 10^{-3} \beta_i \) in our calculations. The ratio \( \beta_g / \beta_i \propto a^{-2} \) reflects grain size and defines our last (third) model parameter. For 0.1 \( \mu \text{m} \) sized grains, \( \beta_g \approx 10^{-4} \beta_i \), and only for \( a \approx 10^{-3} \mu \text{m} \) do we have \( \beta_i \approx \beta_g \).

2.1. Asymptotic Relations

We are mainly interested in reducing Equation (6) to sufficiently simple and intuitive expressions. The ohmic resistivity is the most straightforward and can be written as

\[
\eta_0 = \frac{1}{\beta_i (1 + \theta_1 + \theta_2)},
\]

(7)

where we have defined

\[
\theta_1 = \frac{n_i}{n_e} \frac{\beta_i}{\beta_e}, \quad \theta_2 = \frac{n_{gr}^\pm}{n_e} \frac{\beta_g}{\beta_e}.
\]

(8)

In general, ohmic resistivity results from electron conductivity. Contributions from the ions and grains become significant (\( \theta_{1,2} \gtrsim 1 \)) only when \( n_i / n_e > \beta_i / \beta_e \approx 1000 \) and \( n_{gr}^\pm / n_e > \beta_g / \beta_e \), both requiring the grain charge density to be overwhelmingly dominant.

Calculations of the Hall and ambipolar diffusivities involve much more complex algebra, and sufficiently simple expressions can only be obtained in asymptotic regimes. In the weak field regime (equivalently, the ohmic-dominated regime) where \( \beta_e \ll \beta_i \ll \beta_g \ll 1 \), we Taylor expand the expressions for \( \sigma_H \) and \( \sigma_p \) in Equation (6) and keep the leading-order terms.4 The results are

\[
\sigma_H \approx \beta_e^2, \quad \sigma_p \approx \beta_i \left[ (1 + \theta_1 + \theta_2) - \beta_e^2 \right].
\]

(9)

4 We have also assumed \( n_i / n_e < 10^4 \), or \( \beta_i \theta_1 \ll \beta_e \), which is almost always the case from chemistry calculations.
Substituting them into Equation (2), we obtain

$$\eta_H \approx \frac{1}{(1 + \theta_1 + \theta_2)^2},$$

$$\eta_A \approx \frac{\beta_e (\theta_1 + \theta_2)}{(1 + \theta_1 + \theta_2)^2} = \frac{n_i + n_{gr}^+(\beta_g/\beta_i)}{n_e (1 + \theta_1 + \theta_2)^2}. \tag{10}$$

These expressions reduce exactly to Equation (15) of B11 ($\beta_g = \beta_e$) by replacing $\theta_1 + \theta_2$ by his $\theta$ and replacing $n_i + n_{gr}^+(\beta_g/\beta_i)$ by his $n_i \equiv n_i + n_{gr}^+$. As already discussed there, the ambipolar diffusivity is enhanced by the presence of charged grains, and the Hall diffusivity can be modestly reduced when $\theta_{1,2} \gtrsim 1$.

In the strong field limit where $1 \ll \beta_i \lesssim \beta_e \ll \beta_g$, we can reduce the Hall and Pedersen conductivities to

$$\sigma_H \approx \frac{\theta_1 \beta_e}{\beta_i} \left[ \frac{1}{\beta_i} - \frac{1}{\beta_g^2} \right] + \frac{1}{\beta_g^2},$$

$$\sigma_P \approx \beta_e \left[ \frac{\theta_1}{\beta_i^2} + \frac{\theta_2}{\beta_g^2} \right]. \tag{11}$$

The corresponding Hall and ambipolar diffusivities are

$$\eta_H \approx \frac{[\theta_1 \beta_e (\beta_g^2 - \beta_i^2) + \beta_i^3 \beta_g^2 \beta_e^2]}{\beta_i^2 (\beta_i^2 \beta_1 + \beta_i^2 \beta_2)^2},$$

$$\eta_A \approx \frac{\beta_e (\theta_1 \beta_g^2 + \theta_2 \beta_i^2)}{\beta_i (\theta_1 + \theta_2)^2}. \tag{12}$$

In the tiny grain limit of $\beta_g = \beta_e$, these relatively complex expressions can be further reduced to

$$\eta_H \approx \frac{\beta_i^2}{\beta_g^2 \theta_i^3} = \left( \frac{n_e}{\pi} \right)^2,$nii_i_i_i

$$\eta_A \approx \frac{\beta_e \beta_g^2 (\beta_1 + \beta_2)^2}{\beta_i^2 (\theta_1 + \theta_2)^2} = \frac{\beta_i n_e}{\pi}, \tag{13}$$

which agree exactly with Equation (16) of B11.

From Equation (12), we see that in the strong field regime, $\eta_H$ becomes negative when

$$\theta_1 \beta_e (\beta_g^2 - \beta_i^2) + \beta_i^3 \beta_g^2 \beta_e^2 < 0, \text{ or } \beta_g^2 < \beta_i^2 \left( 1 - \frac{n_e}{n_i} \right). \tag{14}$$

In general, the sign of $\eta_H$ depends on the relative mass (mobility) between the positive and negative charge carriers. In the absence of grains, with electrons much more mobile than ions, $\eta_H$ is positive. In the presence of tiny grains with $\beta_g = \beta_e$ as considered by B11, charged grains are as mobile as normal ions. Therefore, negative charge carriers are still effectively more mobile because of the presence of electrons, and hence $\eta_H \gtrsim 0$. Relation (14) states that, as long as grains are not too small (i.e., well above nanometer size so that $\beta_g \ll \beta_i$), $\eta_H$ can become negative whenever there are more ions than electrons. In other words, there are more negatively charged grains than positively charged ones, and hence negatively charged carriers are effectively less mobile. We will see in Section 3 that $n_i > n_e$ almost always holds from ionization chemistry calculations in PPDs. The main practical question then is whether the disk magnetic field can reach the level such that $\eta_H$ changes sign. We therefore define $B_{th}$ as the threshold field strength beyond which $\eta_H$ changes sign, and this is a main quantity of interest that we study in the rest of the paper.

### 2.2. Representative Cases

In Figure 1, we show the dimensionless ohmic, Hall, and the ambipolar diffusivities as a function of magnetic field strength (characterized by the electron Hall parameter), covering most of the parameter space relevant for grain size $a \gtrsim 0.01 \mu$m (see figure caption). We first confirm that $\eta_H \propto B$ and $\eta_A \propto B^2$ hold both in weak ($\beta_i \ll 1$) and strong ($\beta_g \gg 1$) field regimes as expected. In between, we see that both $\eta_H$ and $\eta_A$ exhibit complex variations with $B$, where $\eta_H$ changes sign in all cases, and $\eta_A$ is reduced in steps compared with the $B^2$ scaling.

The top two panels of Figure 1 correspond to cases where the grains are modestly important charge carriers, giving $n_i/n_e$ not far from order unity. We see that for a field strength with $\beta_i < 1$, $\eta_H$ and $\eta_A$ behave normally as in the conventional ohmic and Hall regimes. Sign change of $\eta_H$ and reduction of $\eta_A$ are both achieved toward a stronger field (in the conventional AD regime) with $\beta_i > 1$ and $\beta_g < 1$.

On the other hand, in the bottom two panels, assuming grains are the dominant charge carriers such that $n_i/n_e \gg 1$, we see that the magnetic diffusivities behave qualitatively differently. First, the threshold field strength $B_{th}$ for $\eta_H$ to change sign is much weaker, now in between $\beta_i = 1$ and $\beta_g = 1$. Similarly, the reduction of $\eta_A$ starts from close to $\beta_i = 1$. This trend continues as $n_i/n_e$ increases further, where $B_{th}$ reduces toward $\beta_i = 1$. Second, we see that for a field strength in between $\beta_i = 1$ and $\beta_g = 1$ (the conventional Hall regime), the Hall-dominated region narrows down significantly, and AD largely becomes the dominant nonideal MHD effect. The Hall diffusivity, after changing sign, dominates in the stronger field in between $\beta_i = 1$ and $\beta_g = 1$ (due to more significant reduction of AD), which is in the conventional AD-dominated regime. AD dominates again when the field is much stronger with $\beta_g > 1$.

Overall, we see that the ratio $n_i/n_e$ largely controls the dependence of $\eta_H$ and $\eta_A$ on magnetic field strength, especially toward a weaker field ($\beta_i \lesssim 1$). Other parameters (grain size and $n_{gr}/n_i$) mainly affect the detailed diffusivity behaviors toward a strong field with $\beta_i > 1$, leading to complex dependence before reaching the asymptotic regime (12) at $\beta_g > 1$. For instance, the Hall and ambipolar diffusivities decrease with increasing $n_{gr}/n_i$ (i.e., $\theta_2$) at the strong field regime $\beta_g > 1$, consistent with Equation (12). However, PPDs are unlikely to reach such a high level of magnetization (see next section), so we do not proceed with a more detailed discussion in the strong field regimes.

We note that the effect discussed in this paper is already present in the earlier work of Wardle (2007, see his Figures 8 and 11), although he did not explicitly discuss the dependence of diffusivities on field strength or the sign change in $\eta_H$, which was pointed out in Bai (2014a). The calculations in Bai (2011a), Mohanty et al. (2013), and Dzyurkevich et al. (2013) included the full dependence of diffusivities on field strength, though again they did not explicitly discuss the dependence, and they did not focus on the Hall effect. The full dependence of diffusivities on field strength was included in recent simulations by Gressel et al. (2015), though the Hall effect...
was not considered. Also, the grain abundance considered there is not sufficiently large for the effect discussed in this paper to be significant (see our empirical criterion in Section 3.2).

3. APPLICATION TO PPDs: IONIZATION CHEMISTRY

To test our analytical results, we conduct an ionization-recombination chemistry calculation using a complex reaction network developed in Bai & Goodman (2009) and Bai (2011a) that followed from Ilgner & Nelson (2006). There are 175 gas-phase species and over 2000 gas-phase reactions extracted from the latest version of the UMIST database (McElroy et al. 2013). The rate coefficients in the database are given as a function of temperature. In a change from our previous chemistry calculations, when the gas temperature lies outside the stated range of validity, we still use the given formula as if it remained valid (whereas originally we computed the coefficients using the upper or lower bound of the valid temperature range). For inner disk temperatures (≈100–300 K), about 10% of the reactions are affected, the majority of which are neutral–neutral reactions or reactions with a large activation energy. In the latter case, our original approach tends to overestimate the reaction rates at lower temperatures. Also, for many of these reactions, we note that the rate coefficients listed in the KIDA chemical database ( Wakelam et al. 2015) generally have different valid temperature ranges with similar coefficients. This change does not affect the level of ionization in the presence of grains (but leads to a higher ionization fraction in the grain-free case), while the composition of ion species is affected.

We include a single grain population with a maximum grain charge of ±2. Grains can possess a higher number of charges when the electron abundance well exceeds the grain abundance. This generally occurs at the disk surface ≳3 scale heights above the midplane. We are mainly interested in the disk interior up to ≈2–3 scale heights, where singly charged grains are dominant because of a very low level of ionization. The fiducial choices are grain size $a = 0.1 \mu m$ and dust-to-gas mass ratio of $f = 10^{-2}$. Although such grain size and abundance are likely exaggerated considering that substantial grain growth must have occurred in PPDs (e.g., Birnstiel et al. 2010), they are the best to illustrate the effects that we discuss in this paper. Given that large uncertainties remain in our understanding of the grain size distribution in PPDs (e.g., D’Alessio et al. 2006), we also consider a few other combinations of grain sizes and abundances to study parameter dependence. Note that for chemical purposes we are only concerned with the abundance of submicron-sized grains, which is the most relevant to ionization and charging.
We adopt the minimum-mass solar nebular (MMSN) disk model with surface density \( \Sigma = \frac{1700}{R_{\text{au}}^{3/2}} \text{g cm}^{-2} \), temperature \( T = \frac{280}{R_{\text{au}}^{1/2}} \text{K} \), assumed to be vertically isothermal, where \( R_{\text{au}} \) is the radius in units of au. We include cosmic-ray ionization, with an ionization rate of \( 10^{-17} \text{s}^{-1} \) attenuated exponentially with column density to the disk surface normalized by \( 96 \text{g cm}^{-2} \) (Umebayashi & Nakano 1981). For X-ray ionization, we use the fitting formula of Bai & Goodman (2009) to the Igea & Glassgold (1999) calculations assuming an X-ray temperature \( T_X = 5 \text{KeV} \) and an X-ray luminosity \( L_X = 10^{30} \text{erg s}^{-1} \). Ionization by radioactive decay is set to be a constant \( 7 \times 10^{-19} \text{s}^{-1} \), but the exact value matters little (Bai 2011a).

The relative importance of nonideal MHD effects in PPDs is conveniently described by the dimensionless Elsasser numbers, defined as

\[
\Lambda \equiv \frac{v_A^2}{\eta_\Omega \Omega_K}, \quad \chi \equiv \frac{v_A^2}{\eta_H \Omega_K}, \quad \Lambda_m \equiv \frac{v_A^2}{\eta_\Lambda \Omega_K},
\]

where \( v_A^2 = B^2 / 4 \pi \rho \) is the Alfvén speed, and \( \Omega_K \) is disk angular frequency. Nonideal MHD effects become dominant when the (absolute value of) respective Elsasser numbers become smaller than order unity. Note that, in our definition, \( \chi \) will change sign when \( \eta_H \) becomes negative.

3.1. Threshold Field Strength in PPDs

In Figure 2, we show the vertical profile of charged species abundances at 1 and 5 au using fiducial grain size \( a = 0.1 \mu m \) and mass ratio \( f = 10^{-2} \) in the top two panels. We see that grains are the dominant charge carriers within \( \sim 2-3 \) disk scale heights \( H \) about the disk midplane, leading to large ratios of \( n_g / n_e \). This situation is similar to the solid curves in the two bottom panels of Figure 1, and we have confirmed that our simple four-species model well captures the \( \eta_H \) and \( \eta_\Lambda \) behaviors computed from the full chemical abundances using Equation (3) as long as most grains are singly charged.

In the bottom panels of Figure 2, we further show the threshold field strength \( B_{\text{th}} \) (beyond which \( \eta_H \) changes sign) as
Figure 3. Threshold field strength $B_{th}$ (in gauss) beyond which the Hall coefficient changes sign, plotted as a function of disk radius $R$ from ionization chemistry calculations at the disk midplane (left) and at two scale heights above the midplane (right). Five combinations of grain sizes $a$ and dust-to-gas mass ratios $f$ are shown, as marked in the legend. To guide the eye, we also show field strengths that correspond to plasma $\beta = 1$ and 100 (blue dashed), as well as field strengths that give electron/ion Hall parameters $\beta_e = 1$ and $\beta_i = 1$ (gray dashed), and the solar nebular field strength inferred from the Semarkona meteorite at the expected radius of 2–3 au (black cross).

We further plot the field strengths that correspond to $\beta_i = 1$ and $\beta_i = 1$ (pink), as well as the field strengths that give the plasma $\beta$, the ratio of gas pressure to magnetic pressure, to be 1, 100, and $10^4$.

We first focus on the fiducial case with $a = 0.1 \mu$m and $f = 0.01$ (blue lines). We see that within $\sim 2H$, $B_{th}$ is well below the $\beta_i = 1$ line, consistent with our expectations given $n_i \ll n_e$. A higher value of $n_i/n_e$ is achieved at 1 au, and we see that $B_{th}$ in this case is closer to the $\beta_i = 1$ line than in the 5 au case, consistent with the trend identified in the previous section.

Toward the surface, as the ionization fraction approaches and then exceeds the grain abundance, we see $\eta_{H}$ can still change sign but at a field strength well beyond $\beta_i = 1$, analogous to the top panels of Figure 1.5 We also note that the threshold field strength $B_{th}$ corresponds to only a modest level of magnetization, with $\beta \sim 100$ within $\pm 2H$ about the midplane, which is likely accessible in real PPDs (see the next subsection).

Using different grain sizes and abundances yield different threshold field strengths $B_{th}$. Reducing the dust-to-gas mass ratio by a factor of 100 to $f = 10^{-4}$ increases $B_{th}$ by about an order of magnitude, and we see that, at both 1 and 5 au, a field strength near or above the equipartition ($\beta = 1$) is needed to have $\eta_{H}$ change sign. This is much less likely to be achieved in PPDs, at least under standard disk models where the field strength well beyond the equipartition is sufficient to drive disk accretion at the expected accretion rate (e.g., Bai 2013). On the other hand, for the same dust-to-gas mass ratio $f = 10^{-4}$, reducing the grain size further to $0.01 \mu$m yields results similar to the fiducial case. This is because smaller grains have a higher total surface area or abundance at a given mass; both are favorable to making charged grains the dominant charge carriers.

3.2. An Empirical Criterion on Grain Size and Abundance

Extending the discussion to a broad range of disk radii, we show in Figure 3 the threshold field strength $B_{th}$ as a function of orbital radius $R$ and present the results at the disk midplane ($z = 0$) and at intermediate height ($z = 2H$). Moving toward larger disk radii, the ionization fraction increases systematically (due to reduced density and deeper penetration of external ionization), as can be traced from Figure 2. This makes $B_{th}$ shift upward relative to the $\beta_i = 1$ line, as discussed before. In the meantime, as the gas density/pressure drops, the field strength at fixed plasma $\beta$ also shifts upward relative to the $\beta_i = 1$ line, as is straightforward to show from their definitions ($B \propto \sqrt{\rho}$ versus $B \propto \rho$). As a result, we see from Figure 3 that the line showing $B_{th}$ as a function of $R$ is approximately parallel to the constant plasma $\beta$ line over a very wide range of radii.

In Figure 3, we have considered a broad collection of grain sizes and abundances. We are particularly interested in the question of in what combinations of grain sizes and abundances can $\eta_{H}$ change sign in the expected range of PPD magnetic field strength. We consider plasma $\beta = 100$ as a target field strength, which is easily achieved in either pure magnetorotational instability (MRI, Balbus & Hawley 1991) or turbulence (e.g., Davis et al. 2010), or in more realistic studies of PPD gas dynamics (e.g., Bai & Stone 2013; Bai 2014b, 2015; Lesur et al. 2014). On the other hand, a field strength above the equipartition is unlikely in the disk interior.
For reference, we also mark the possible field strength of the solar nebula inferred from the Semarkona meteorites (Fu et al. 2014), which likely corresponds to a midplane field strength at 2–3 au during the epoch of chondrule formation. Although large uncertainties remain, a field strength of up to $\beta = 100$ in an MMSN disk is reasonably consistent with the inferred nebular field strength.

From Figure 3, it is clear that small grains in large abundance are essential to make $B_{th}$ sufficiently weak (plasma $\beta \gtrsim 100$) to be potentially achieved in PPDs. In particular, the results from our fiducial choice of $a = 0.1 \mu m$ and $f = 10^{-2}$ are very similar to those obtained with the $a = 0.01 \mu m$, $f = 10^{-4}$ case, at both $z = 0$ and $z = 2H$. This might suggest an empirical criterion where $f/a^2$ should be at least order unity (with $a$ measured in $\mu m$) in order to achieve a sufficiently weak threshold field $B_{th}$. Note that a constant $f/a^2$ corresponds to a constant $na$ where $n$ is grain number density. Therefore, the controlling parameter is not the total grain surface area ($\propto na^2$) but lies in between total surface area and number density. We might further generalize this consideration into a distribution of grain sizes and define

$$G \equiv \int \frac{df}{(a/\mu m)^2} da.$$  \hspace{1cm} (16)

Our discussions above suggest $G \gtrsim 1$ to achieve sufficiently weak $B_{th}$.

We note that, when assuming the standard Mathis, Rumpl, & Nordsieck’s (MRN) size distribution for interstellar grains (Mathis et al. 1977), with $dn/da \propto a^{-3.5}$ between $a_{\text{min}} \approx 0.005 \mu m$ and $a_{\text{max}} \approx 0.25 \mu m$, we obtain $G \approx 22 \gg 1$. Therefore, while grain growth in PPDs must substantially reduce $G$ from the interstellar value, we do expect that grain-modified diffusivities are likely relevant at least in some stages of PPD evolution.

### 3.3. Diffusivities in the Outer Disk

Although $\eta_H$ can change sign essentially at all distances, it is mainly the inner disk ($\lesssim 10$ or at most a few tens of 10 au) that is the most relevant. Conventionally, $\beta_i > 1$ marks the dominance of AD over the Hall effect, which is achieved in lower-density regions toward the outer disk. The complex dependence of $\eta_L$ and $\eta_H$ on field strength in the presence of abundant small grains complicates the situation. In Figure 4, we show the AD and Hall Elsasser numbers $Am$ and $\chi$ as a function of magnetic field strength at 30 and 100 au, at both disk midplane and intermediate heights ($z = 2H$). We consider three combinations of grain size and abundance as before and discuss the results below.

We first see that $Am$ is constant in both weak and strong field regimes, as expected. In the presence of abundant small grains ($G \gtrsim 1$, black and red lines), the difference in $Am$ between the two regimes can reach several orders of magnitude. In between at intermediate field strength, we have $\eta_L \approx \text{constant or}$

![Figure 4: Ambipolar Elsasser number $Am$ (solid) and Hall Elsasser number $\chi$ (dashed) as a function of field strength (in gauss) from ionization chemistry calculations at 30 au (left) and 100 au (right). The top and bottom panels show the results from calculations at the disk midplane and at two scale heights above the midplane, respectively. Three combinations of grain sizes $a$ and dust-to-gas mass ratios $f$ are shown, as marked in the legends. We also mark the field strength corresponding to plasma $\beta = 1$, as well as field strengths that give ion Hall parameters $\beta_i = 1$, in vertical dash-dotted lines.](image-url)
\[ Am \propto B^2 \] (see also Figure 1). Therefore, AD behaves the same way as ohmic resistivity (except for its anisotropy). These trends are the same as those discussed in B11 in the presence of tiny \((a \sim \text{nm})\) grains. If we expect field strengths in the outer PPDs to correspond to a plasma \(\beta \sim 1 - 10^4\), then roughly speaking, the midplane region up to \(\sim 100\) au and an upper layer \((z = 2 H)\) up to \(\sim 30\) au would lie in this intermediate regime when \(G \gtrsim 1\).

With fewer small grains \((G \ll 1, \text{blue lines in Figure 4})\), on the other hand, a reduction of AD occurs at much stronger fields. For both 30 and 100 au, both at midplane and a few scale heights above, such a reduction requires a field strength above the equipartition level and thus is unlikely to be relevant. In fact, for our choice of \(a = 0.1\ \mu\text{m}\) and \(f = 10^{-4}\), the ionization fraction at the disk midplane already exceeds the grain abundance, so the role of grains is only minor. In this case, we see that \(Am \sim 1\) is a good proxy over a wide range of disk radii and vertical locations, consistent with previous findings (Bai 2011a, 2011b; Perez-Becker & Chiang 2011).

We also see that, in the presence of abundant small grains, a substantial reduction of AD toward a strong field \((\beta > 1)\) is accompanied by a sign change of \(\eta_H\), and the Hall effect can dominate over AD (in the conventional AD-dominated regime). This is consistent with the features seen in Figure 1 (e.g., bottom panels). Therefore, the Hall effect may play a more important role than commonly assumed.

The AD Elsasser number \(Am\) directly controls the level of the MRI turbulence, and \(Am \sim 1\) is generally considered to be sufficiently strong to substantially damp the MRI (Bai & Stone 2011; Simon et al. 2013a, 2013b). Further introducing a modestly strong Hall term would enhance the level of turbulence if the net vertical field is aligned with the disk rotation \(B_0 \cdot \Omega > 0\), while reducing it for opposite polarity (Sano & Stone 2002; Kunz & Lesur 2013; Bai 2015; Simon et al. 2015). The grain-modified magnetic diffusivities in the outer disk studied here may lead to two interesting effects when \(G \gtrsim 1\). First, the fact that AD behaves similar to ohmic resistivity at intermediate field strength may lead to cycles of growth and decay of the MRI, as observed in resistive MRI simulations (Simon et al. 2011). Second, the sign change of \(\eta_H\) means that the enhancement of the MRI by the Hall effect would occur at antialigned field polarity rather than aligned polarity. These effects deserve further investigation via three-dimensional MHD simulations.

4. MHD SIMULATIONS OF THE INNER PPD

4.1. Simulation Setup and Parameters

To further study the effect of grain-modified magnetic diffusivities on PPD gas dynamics, we follow Bai (2014b, hereafter B14) and conduct local stratified shearing-box MHD simulations that include all three nonideal MHD terms. Note that B14 simply assumed \(\eta_H \propto B\) and \(\eta_A \propto B^2\) in the calculations and used a 2D diffusivity table showing the proportional coefficients as a function of density and ionization rate to compute the diffusivities. To incorporate the complex dependence of \(\eta_H\) and \(\eta_A\) on magnetic field strength, we extend the diffusivity table to 3D, which explicitly accounts for this dependence.

Here we mainly focus on the inner disk region \((R < 10\) au\), where it suffices to conduct quasi-1D simulations (as verified in Bai 2015) because the MRI is largely suppressed. Quasi-1D means that the simulation box is largely 1D in vertical, but allows for four cells in the radial and azimuthal dimensions, which facilitates the simulations to relax to a steady state. The simulations use an isothermal equation of state, and the simulation box extends from \(-8 H\) to \(8 H\) in \(z\), with a resolution of 18 cells per \(H\). We consider examples at two radii \(R = 1\) and 5 au, using diffusivity tables with a fixed grain size \(a = 0.1\ \mu\text{m}\) but two different abundances \(f = 10^{-2}\) and \(10^{-4}\).

An artificial boost of ionization rate beyond column densities of \(\sim 0.03\ \text{g cm}^{-2}\) is included to mimic the effect of far ultraviolet (FUV) ionization, which brings the gas to the ideal MHD regime. All simulations are conducted with a net vertical magnetic field \(B_{0z}\) threading the disk, and the field strength is parameterized by the plasma \(\beta_0\), defined as the ratio of midplane gas pressure to magnetic pressure of the net vertical field. We consider \(\beta_0 = 10^{2}\) and \(10^{3}\), as adopted in most previous works, chosen to yield an accretion rate in the expected range for the MMSN disk model. In code units, we have a midplane density \(n_{\text{mid}} = 1\), isothermal sound speed \(c_s = 1\), Keplerian frequency \(\Omega_K = 1\), and magnetic permeability \(\mu_m = 1\).

It is well known that the effect of the Hall term depends on the polarity of the background vertical field \(B_{0z}\) threading the disk (e.g., Wardle 1999; Balbus & Terquem 2001; Wardle & Salmeron 2012). Of particular interest is the case where \(B_0 \cdot \Omega > 0\), where a background field is being aligned with the rotation axis. A linear analysis and numerical simulations have shown that, in this case, the horizontal components of the magnetic field can be strongly amplified due to the Hall-shear instability (HSI, Kunz 2008; Bai 2014b; Lesur et al. 2014). Because both radial and azimuthal components are amplified, the Maxwell stress \((-B_0 B_\phi)\) is strongly enhanced, which likely plays a nonnegligible role in disk angular momentum transport. On the other hand, when \(B_0 \cdot \Omega < 0\), the opposite occurs, and the horizontal field is reduced to close to zero near the midplane region (Bai 2014b, 2015). Because the interesting effects we have discussed in this paper operate when a magnetic field is amplified, we consider only aligned polarity \(B_0 \cdot \Omega > 0\) in our simulations.

We run the simulations following the procedures of Bai (2014b). We first turn off the Hall effect during the initial evolution to time \(t = 120\Omega^{-1}\). Then we turn on the Hall effect and run the simulations to \(t = 240\Omega^{-1}\), where a steady-state magnetic configuration fully saturates. In all cases, the saturated field geometry obeys “odd-\(\zeta\)” symmetry (e.g., Figure 9 of Bai & Stone 2013), where the horizontal components of the magnetic field remain the same sign across the disk and reach a maximum around the disk midplane. While this symmetry is unphysical for disk wind launching, it is likely an artifact of a shearing box, and here we set this issue aside. Because magnetic diffusivities in the disk midplane region are always excessively large at the inner disk, a diffusivity \(\eta_{\text{cap}}\) for each nonideal MHD term is implemented to prevent a prohibitively small numerical time step. Most previous works used \(\eta_{\text{cap}} \sim 10H^2\Omega\). We use \(\eta_{\text{cap}} = 10H^2\Omega\) in the runs up to \(t = 240\Omega^{-1}\) as described before, but then enlarge \(\eta_{\text{cap}}\) to 100\(H^2\Omega\) and run for another \(\Delta t = 60\Omega^{-1}\), which appears sufficient for the system to relax to a new state. We do observe

\footnote{Whether a physical “even-\(\zeta\)” symmetry can be obtained in a shearing box also depends on the diffusivity profiles. With \(a = 0.1\ \mu\text{m}\) and \(f = 10^{-2}\), Bai (2014b, 2015) obtained a physical wind geometry at \(R = 5\) au with \(\beta_0 = 10^2\). We can reproduce this result, but we fail to obtain physical symmetry with \(f = 10^{-2}\).}
changes in $B_h$ at midplane by up to 30% after enlarging the diffusivity cap, while typically the changes are within 10% and are much smaller for midplane $B_\phi$.

In parallel to simulations that take into account the full dependence of $\eta_H$ and $\eta_a$ on $B$ (referred to as “full”), we also conduct simulations using a 2D diffusivity table assuming $\eta_H \propto B$ and $\eta_a \propto B^2$, taking the proportional constants from the weak field limit of the full dependence (referred to as “simple”), and compare the difference between the two sets of runs.

Table 1 lists all our quasi-1D simulations. We also show four main simulation diagnostics at saturation: $\beta_{mid}$ is the plasma $\beta$ at disk midplane (based on total field strength), $\alpha_{Max}$ is the vertically integrated dimensionless Shakura–Sunyaev $\alpha$ based on Maxwell stress $\int_{-z_h}^{z_h} (-B_R B_\phi) dz / \int_{-z_h}^{z_h} \rho c_s^2 dz$. $T^{\alpha}_{mid}$ is proportional to the torque exerted at the wind base $z_B$, and $M_w = \rho c_s$ is the mass loss rate measured at the top of the simulation box (single-sided). The wind base $z_B$ is defined as the location where rotation transitions from sub-Keplerian to super-Keplerian (in practice, we choose it to be the first minimum of $|\psi_0 - \psi_k|$ as one varies $z$ from boundary toward midplane, as discussed in Bai et al. 2014). All aspects of these diagnostic quantities have been discussed in the literature, and this table mainly serves as a reference.

### 4.2. Simulation Results

In Figure 5, we show the simulation results with $a = 0.1 \mu$m, $f = 0.01$, and $\beta_0 = 10^3$ at 1 au and 5 au, respectively. The top panels show the vertical profiles of magnetic field strength in code units, where plasma $\beta = 1$ at midplane corresponds to $B = \sqrt{2}$. As usual, the toroidal field is the dominant component, and it maintains a relatively flat profile across the disk midplane because of the excessively low ionization level there (lack of charge carriers cannot support the current). We also mark the threshold field strength $B_{th}$ at which $\eta_H$ changes sign, which is computed based on the 3D diffusivity table directly. We have verified that the line shapes follow exactly the corresponding curves shown in Figure 2 in logarithmic scale. In a linear scale, the minimum of $B_{th}$ at $z = 2 - 3 H$ is the most prominent feature, and we see that the saturated field strength from our simulations (dominated by a toroidal field) matches closely the minimum value of $B_{th}$. On the other hand, simulations using a 2D diffusivity table assuming $\eta_H \propto B$ yield a stronger field around the disk midplane as a result of the HSI. Because the only differences between the two simulations are in the diffusivity tables, this comparison thus demonstrates that $B_{th}$ provides an approximate upper limit to which the magnetic field strength can grow, resulting in prematuresed saturation of the HSI.

The bottom panels of Figure 5 show the Elsasser number profiles for the three nonideal MHD effects. We first see that the locations where $B > B_{th}$ in the top panels have the negative Hall Elsasser number $\gamma$ seen in the bottom panels. In other regions where $\eta_H$ is positive, we also see that both $\eta_H$ and $\eta_a$ are reduced compared to the “simple” case using 2D diffusivity tables, which is due to (1) a weaker magnetic field strength and (2) reduction of $\eta_H$ and $\eta_a$ toward a stronger field.

We also performed simulations with reduced grain abundance ($a = 0.1 \mu$m and $f = 10^{-4}$). In this case, however, the threshold field strength is much stronger, and we find that simulations using a full diffusivity dependence on $B$ yield results identical to using a 2D diffusivity table assuming $\eta_H \propto B$, $\eta_a \propto B^2$ scalings. The latter was adopted in Bai et al. (2014b), and hence his results were unaffected. This is because, with a reduced grain abundance, the threshold field strength is much stronger (as seen in Figure 2) and exceeds the strength that can be attained from field amplification by the HSI.

Increasing the net vertical magnetic flux naturally leads to stronger wind, with larger wind torque and higher mass outflow rate, as listed in Table 1. Other than the wind being stronger, we find that the general phenomenologies discussed above hold exactly the same way where field amplification by the HSI saturates at weaker field strength when full dependence of diffusivities on $B$ is considered.

Field amplification by the HSI always weakens with increasing grain abundance as a result of enhanced resistive dissipation. This is already evident by comparing the effectively grain-free simulations of Lesur et al. (2014) and the simulations of Bai et al. (2014b) with $a = 0.1 \mu$m and $f = 10^{-4}$, where the former showed an extreme level of field amplification around the disk midplane that approaches equipartition field strength, and the latter only found a modest level of amplification. The trend continues when increasing the dust-to-gas ratio to $f = 10^{-2}$, and assuming simple $\eta_H \propto B$, $\eta_a \propto B^2$ dependence, the midplane field strength is already reduced by a factor of about three. Our simulations show that a further reduction is achieved when a full dependence of diffusivities on $B$ is considered. This further reduction in field strength is better viewed when plotting the Maxwell stress $(-B_R B_\phi)$ profiles,
which depend quadratically on field strength. This is shown in Figure 6, and the vertically integrated values $\alpha^{\text{Max}}$ are listed in Table 1.

Overall, our studies show that the full dependence on field strength needs to be taken into account when small grains are abundant (or, effectively, $G_1 < 1$ defined in (16)). Reducing the grain abundance leads to a larger threshold field strength $B_{\text{th}}$ and in the meantime a higher level of field amplification by the HSI. It turns out that the former increases much faster, and the effect studied in this paper becomes essentially irrelevant when $G_1 = 1$.

5. SUMMARY AND DISCUSSION

We have conducted a comprehensive study on the dependence of magnetic diffusivities on magnetic field strength in the presence of charged grains with application to PPDs. Ohmic resistivity $\eta_\Omega$ is always independent of field strength. The Hall and ambipolar diffusivities $\eta_H$ and $\eta_A$ obey simple asymptotic relations $\eta_H \propto B$ and $\eta_A \propto B^2$ in both the weak field regime where $\beta_e < 1$ and the very strong field regime where $\beta_e > 1$, while showing complex dependence in between (see Figure 1). In particular, when relation (14) is satisfied (easily achieved when $n_e$ falls below $n_i$ due to the presence of charged grains), $\eta_H$ changes sign toward a strong field, accompanied by a reduction of AD. The threshold field strength $B_{\text{th}}$ beyond which $\eta_H$ changes sign largely depends on the ratio $n_e/n_i$, and it is weaker when $n_e/n_i$ is smaller (e.g., charged grains are more dominant). Similarly, more abundant charged grains lead to a stronger reduction of $\eta_A/B^2$ between the weak and strong field regimes.

Applying this to the standard MMSN model of PPDs with ionization chemistry, we find that in order for the threshold field $B_{\text{th}}$ to be well below the equipartition level, a large number of small grains are required, as summarized in the empirical condition $G > 1$ (defined in Equation (16)), which is applicable to a wide range of disk radii. When the same

Figure 5. Top: vertical profiles for three components of magnetic field strength (in code units) from simulations at 1 au (left) and 5 au (right) in a steady state, where the equipartition field strength at midplane corresponds to $B = \sqrt{2}$. We have chosen grain size $a = 0.1\, \mu\text{m}$ and dust-to-gas mass ratio $f = 0.01$, with weak net vertical field $\beta_0 = 10^3$. Dashed lines correspond to simulations using a 2D diffusivity table assuming $\eta_H \propto B$ and $\eta_A \propto B^2$ in the weak field limit, while solid lines (except the cyan line) correspond to simulations using a 3D diffusivity table accounting for the full dependence of $\eta_H$ and $\eta_A$ on $B$. Cyan solid lines mark the threshold field strength above which $\eta_H$ changes sign. Bottom: vertical profiles of ohmic, Hall, and AD Elsasser numbers from simulations using the full dependence of $\eta_H$ and $\eta_A$ on $B$ at 1 au (left) and 5 au (right).
condition is satisfied, AD is also reduced substantially and may promote the action of MRI in the outer PPDs.

We further conducted quasi-1D shearing-box simulations for the inner region of PPDs, including all three nonideal MHD effects with the net vertical field aligned with the rotation. We show that when small grains are abundant, magnetic field amplification due to the Hall-shear instability saturates at close to the minimum value of $B_{\text{th}}$. Therefore, besides the fact that abundant small grains limit the magnetic field growth by enhancing resistivity, they also affect the gas dynamics in a more complex way by modifying the field-strength dependence of magnetic diffusivities, especially by enabling $\eta_H$ to change sign toward a strong field. On the other hand, with substantial grain growth (so that $G \ll 1$), the threshold field strength becomes too strong to be accessible via internal MHD processes in PPDs, and hence it suffices to simply adopt $\eta_H \propto B$ and $\eta_\alpha \propto B^2$ in the weak field limit.

Our work helps clarify how the presence of grains affects the ionization chemistry and magnetic diffusivities in PPDs and hence the general disk dynamics. This is particularly relevant to the Hall effect: inclusion of the Hall effect not only makes PPD gas dynamics dependent on the polarity of the net vertical magnetic field, but also leads to a more sensitive dependence on grain abundance, as compared to the Hall-free case of Bai & Stone (2013). In combination with other recent studies, we can identify three regimes in the gas dynamics of inner PPDs ($\lesssim 10 \sim 15$ au) depending on the grain abundance.

1. Grain-free ($G \sim 0$): very strong magnetic field amplification by the HSI when $\Omega \cdot B > 0$ (Bai 2014b; Lesur et al. 2014), and bursty behavior at $\sim 5 \sim 10$ au when $\Omega \cdot B < 0$ (Simon et al. 2015).

2. Modest grain abundance ($G < 1$): modest magnetic field amplification by the HSI when $\Omega \cdot B > 0$, while the horizontal field is reduced to close to zero when $\Omega \cdot B < 0$ (Bai 2014b, 2015).

3. High grain abundance ($G \gtrsim 1$): the HSI saturates due to the sign change of $\eta_H$ at threshold field strength $B_{\text{th}}$ when $\Omega \cdot B > 0$.

Exploration of the Hall effect in PPDs is still at an early stage. There are still pressing issues, especially concerning the symmetry of the disk wind, the direction of magnetic flux transport, wind kinematics, and so on. The complexity introduced by grains further adds to the richness of the overall subject of PPD gas dynamics. Conversely, it is well known that the grain size and spatial distribution in PPDs depend on the level of turbulence and global structure of the disk and evolve with time (e.g., Birnstiel et al. 2010). The two interrelated aspects are inherent in PPD dynamics and further call for more in-depth investigations in the future.

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Figure 6. Vertical profiles of Maxwell stress from simulations at 1 au (left) and 5 au (right). We have chosen grain size $a = 0.1 \mu m$ and dust-to-gas mass ratio $f = 10^{-4}$ (blue) and $f = 0.01$ (black and red), with a weak net vertical field $\beta_0 = 10^3$. Black lines correspond to simulations assuming $\eta_H \propto B$ and $\eta_\alpha \propto B^2$ in the weak field limit, while red lines correspond to simulations using the full dependence of $\eta_H$ and $\eta_\alpha$ on $B$. 

1AU

5AU

Maxwell Stress

$Z/H$

$B_{\text{th}}$

$B_{\text{th}}$

$B_{\text{th}}$

$B_{\text{th}}$

$B_{\text{th}}$

$B_{\text{th}}$

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