Heavy Quark Symmetry Violation in Semileptonic Decays of D Mesons

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Abstract

The decays of $D$ mesons to $K\ell\nu$ and $K^*\ell\nu$ final states exhibit significant deviations from the predictions of heavy-quark symmetry, as one might expect since the strange quark’s mass is of the same order as the QCD scale. Nonetheless, in order to understand where the most significant effects might lie for heavier systems (such as $B \to D\ell\nu$ and $B \to D^*\ell\nu$), the pattern of these deviations is analyzed from the standpoint of perturbative QCD and $\mathcal{O}(1/m_s)$ corrections. Two main effects are noted. First, the perturbative QCD corrections lead to an overall decrease of predicted rates, which can be understood in terms of production of excited kaonic states. Second, $\mathcal{O}(1/m_s)$ effects tend to cancel the perturbative QCD corrections in the case of $K\ell\nu$ decay, while they have minimal effect in $K^*\ell\nu$ decay.

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1 Introduction

In atomic physics, one can often separate the nuclear and electronic effects from one another. Similar progress has been made for strongly interacting particles within the past few years \[1,2,3,4\]. Particles containing a single heavy quark can be thought of as analogues of atoms for the strong interactions. The single heavy quark behaves as the nucleus, while the light quarks and gluons behave as the electron cloud. To a large extent the degrees of freedom of the heavy quark and those of the remaining matter can be discussed separately. The resulting description validates some (but not all) older quark-model results in a systematic way, and has come to be known as heavy quark symmetry. It represents a limit of quantum chromodynamics (QCD) for infinite mass \(m_Q\) of the heavy quark \(Q\).

The semileptonic decays of mesons containing a single heavy quark are an ideal laboratory for testing heavy quark symmetry. Both the leading-order results and the \(\mathcal{O}(1/m_Q)\) corrections to them have been fully worked out. In practice the symmetry is applicable only to the semileptonic decays \(B \to Dl\nu, B \to D^*l\nu\), and possibly to decays involving excited \(D\) mesons in the final state. Only the \(b\) quark (in the \(B\) meson) and the charmed quark (in the \(D\) and \(D^*\)) are heavy enough compared to the QCD scale of several hundred MeV
that one can begin to think of applying heavy quark symmetry. It is possible to describe six form factors (two for $B \to Dl\nu$ and four for $B \to D^*l\nu$) in terms of a single universal function $\xi(w)$ of the variable $w \equiv v \cdot v'$, where $v$ and $v'$ are the four-velocities of the initial and final states. We shall refer to $\xi$ as the Isgur-Wise function.

One would like very much to know the size of corrections to the universal Isgur-Wise behavior in $B$ semileptonic decays. One benefit of this information would be the improved ability to not only measure the $b \to c$ weak coupling accurately [7,8], but to be able to reliably estimate the theoretical error. However, up to now it has not been possible to estimate the actual magnitude of the corrections.

Early estimates of semileptonic decays were not restricted to cases in which heavy-quark symmetry was guaranteed to work. In particular, many attempts were made to describe the semileptonic decays of $D$ mesons within the framework of various quark models [9, 10, 11, 12, 13, 14]. The main shortcoming of these descriptions was that they predicted rates for $D \to K^*l\nu$ which were too large. An attempt was made [15] to isolate the source of the discrepancy by analysis of specific form factors, but the underlying physics remained elusive.

One should not expect heavy quark symmetry to apply to $D$ meson decays, since no quark in the final state is heavy enough compared to the QCD scale. Nonetheless, if regarded as a constituent of hadrons, the strange quark has an effective mass of about 1/2 GeV, which is large enough that one might at least expect some vestiges of heavy quark symmetry to apply. In the present paper we take this point of view. We compare the predictions of heavy quark symmetry for the decays $D \to Kl\nu$ and $D \to K^*l\nu$ with experiment, extracting the nonleading form factors which account for the corrections. From these we attempt to isolate the physics responsible for the violation of the symmetry. The question of whether heavy-quark symmetry applies in this case, and even in the case of $K^0\bar{\nu}$ decays, was raised some time ago [16, 17]. Others [18] have shown that the leading-order predictions of heavy-quark symmetry are not sufficient to explain the branching ratios in semileptonic $D$ decays.

In brief, we have found that both the perturbative QCD and $O(1/m_s)$ effects are important.

There is an overall suppression of decay rates to the $Kl\nu$ and $K^*l\nu$ final states due to perturbative QCD corrections [1, 3, 20, 21]. The $O(1/m_s)$ corrections add together such that they tend to cancel this suppression in the case of $Kl\nu$ decay. In the case of $K^*l\nu$ decay, however, the $O(1/m_s)$ corrections tend to cancel each other, leaving the net suppression from the perturbative QCD correction.

Since the total semileptonic decay rate for $D$ mesons is close to that expected for free charmed quarks, there must be important final states besides $Kl\nu$ and $K^*l\nu$. (We recognize that a recent search for such final states has yielded only upper limits [22], and that one reasonably successful description of the inclusive semileptonic rates in terms of known final states has appeared [23].) We are then led to conclude that if there are indeed “missing” final states, they are produced primarily at the expense of the $K^*l\nu$ channel, which would help us to identify their nature.

A previous analysis of $D$ semileptonic decays in the same spirit as ours has appeared [24]. Our analysis is somewhat more explicit about the sources of heavy quark symmetry breaking, and differs in conclusions.

We have organized this paper as follows. We begin by describing the predictions of heavy quark symmetry in Section 2. After a discussion of lifetimes and branching ratios in the
data and the leading-order heavy quark theory (Section 3), we use $B$ decays to determine the Isgur-Wise function $\xi$ and the $D \to K^*\ell\nu$ differential branching ratio to determine the size of the perturbative QCD corrections in $D$ decays (Section 4). We then apply the theory to $D$ decay form factors in Section 5. The implications of this analysis for $B$ decays is discussed in Section 6. Section 7 concludes.

2 Heavy Quark Symmetry

2.1 Leading-order Results

In this subsection we discuss both the formalism and leading-order results of heavy quark symmetry. In the following subsections we will discuss both perturbative QCD [$\mathcal{O}(\alpha_s)$] and finite-mass [$\mathcal{O}(1/m_Q)$] corrections.

It is most convenient to talk about heavy quark processes in terms of velocities instead of momenta. The most general velocity-dependent form factors for the decay of a pseudoscalar meson $H$ to a pseudoscalar (vector) meson $h$ ($h^*$) \[25\] are:

\[
\begin{align*}
\langle h(v')|V_\mu|H(v)\rangle &= \sqrt{m_H m_h} \left[ \xi_+(w)(v + v')_\mu + \xi_-(w)(v - v')_\mu \right], \\
\langle h^*(v', \varepsilon)|V_\mu|H(v)\rangle &= i \sqrt{m_H m_h} \xi_V(w) \varepsilon_\mu \varepsilon^{\alpha\beta} v'^\alpha v^\beta, \\
\langle h^*(v', \varepsilon)|A_\mu|H(v)\rangle &= \sqrt{m_H m_{h^*}} \left[ \xi_{A_1}(w)(w + 1) \varepsilon^\mu - \xi_{A_2}(w) \varepsilon^\mu \cdot vv - \xi_{A_3}(w) \varepsilon^\mu \cdot v'v' \right],
\end{align*}
\]

where $\varepsilon$ is the $h^*$'s polarization vector and $w \equiv v \cdot v' = (m_H^2 + m_{h^*}^2 - q^2)/(2m_H m_{h^*})$. Throughout this work we denote the mass of the heavy meson $H$ ($h$) by $m_H$ ($m_h$) and the mass of the corresponding heavy quark by $m_Q$ ($m_q$). The allowed kinematic range is given by

\[
1 \leq w \leq w_{max},
\]

where

\[
w_{max} \equiv \frac{1}{2} \left( \frac{z^{(*)}}{z^{(*)}} + \frac{1}{z^{(*)}} \right)
\]

and $z^{(*)} \equiv m_{h^{(*)}}/m_H$. Table I gives numerical values of $z^{(*)}$ and $w_{max}$ for $B$ and $D$ semileptonic decays.
In the absence of finite-mass and radiative corrections
\[ \xi_+ = \xi_0 = \xi_1 = \xi_2 \equiv \xi \] (4)
and
\[ \xi_- = \xi_3 \equiv 0, \] (5)
where \( \xi(w) \) is the Isgur-Wise function. This symmetry is broken by both radiative and finite-mass corrections.

The Isgur-Wise function satisfies the zero-recoil condition
\[ \xi(1) = 1. \] (6)
It has become standard in the literature to characterize its behavior for \( w \) near unity by
\[ \xi(w) = 1 - \rho^2 (w - 1). \] (7)
Common parameterizations of the Isgur-Wise function include the monopole, dipole, or more generally, the \( n \)-pole function
\[ \xi(w) = \frac{1}{\left(1 + \frac{\rho}{n}(w - 1)\right)^n}. \] (8)
Another possible form is an exponential
\[ \xi(w) = \exp\left[-\rho^2 (w - 1)\right]. \] (9)

Other common parameterizations \[25\] fall in between the exponential and monopole forms. Since Eq. (8) is the \( n \to \infty \) limit of Eq. (8), we use the monopole and exponential forms to represent a reasonable range of forms for the Isgur-Wise function.

### 2.2 Perturbative QCD Corrections

The perturbative QCD [i. e., \( \mathcal{O}(\alpha_s) \)] corrections to the heavy quark limit are calculable and, as such, have been extensively studied \[13,14,20,21\]. There are two approaches to calculating the perturbative QCD corrections in the effective theory. The two schemes involve different schemes for matching the effective theory on to the full theory. One approach is to assume the scales \( m_q \) and \( m_Q \) are well separated and then do matching both at \( m_q \) and \( m_Q \) \[13\]. A second approach is to do the matching at some intermediate scale \( \mu \) \[19\]. Hybrid approaches have also been developed \[20,21\].

For \( B \to D \) decays these corrections are well defined. For \( D \to K \) decays, however, the situation is somewhat ambiguous. Perturbation theory makes sense at the scale \( m_c \), but, whichever approach we use, we inevitably refer to the quantity \( \alpha_s(m_s) \). There are several ambiguities in using \( \alpha_s(m_s \approx 500 \text{ MeV}) \): There is a strong dependence on the precise value of \( m_s \). Even given a specific value of \( m_s \), there is a large experimental uncertainty in the precise value of \( \alpha_s(m_s) \). Finally, the value of \( \alpha_s(m_s) \) is large, so higher-order corrections can be substantial. We will have to be satisfied with finding the rough effects of perturbative QCD corrections for \( D \to K \) decays.
Table 2: Perturbative QCD correction common to all form factors in $D \to K^{(*)}$ decays.

| w  | $X_{\text{QCD}}$ |
|-----|-----------------|
| 1.00 | 1.269          |
| 1.20 | 1.227          |
| 1.40 | 1.189          |
| 1.60 | 1.155          |
| 1.80 | 1.124          |
| 2.00 | 1.096          |

Table 3: Perturbative QCD functions for $D \to K$ decays

| w  | $\tilde{\beta}_+$ | $\tilde{\beta}_-$ |
|-----|-------------------|-------------------|
| 1.00 | -0.982            | -0.260            |
| 1.20 | -1.068            | -0.254            |
| 1.40 | -1.153            | -0.248            |
| 1.60 | -1.238            | -0.241            |
| 1.80 | -1.320            | -0.235            |
| 2.00 | -1.401            | -0.229            |

In the spirit of looking for the rough effects of the perturbative QCD corrections, we choose to use the simplest set of radiative corrections that contain the basic physics of the process. Therefore, for this work we use the radiative corrections given in Ref. \[20\]. This work is a hybrid method where the matching is done at an intermediate scale and the leading logarithms are summed. The result is

$$\xi_i(w) = X_{\text{QCD}} \left[ c_i + \frac{\alpha_s(\mu)}{\pi} \tilde{\beta}_i(w) \right] \xi(w),$$

(10)

where

$$X_{\text{QCD}} = Z_{IR}(w) \left( \frac{\alpha_s(m_Q)}{\alpha_s(m_q)} \right)^{-6/(33-2n_f)},$$

(11)

and $c_i = \{1, 0, 1, 1, 0, 1\}$ for $i = \{+, -, V, A_1, A_2, A_3\}$. The mass scale $\mu$ is chosen at some intermediate scale between $m_q$ and $m_Q$. See Ref. \[20\] for the definitions of the functions $\tilde{\beta}_i$ and $Z_{IR}$. Tables \[2\,4\] give numerical values of the relevant functions.

We are now left with the ambiguity in choosing $\mu$, the scale at which to do the matching of the effective theory to the full theory. The depth of this problem is illustrated in Fig. \[1\] where we have plotted $\alpha_s$ over the range in question. For the $B \to D$ case Neubert \[20\,21\].
Table 4: Perturbative QCD functions for $D \to K^*$ decays

| $w$   | $\tilde{\beta}_V$ | $\tilde{\beta}_{A_1}$ | $\tilde{\beta}_{A_2}$ | $\tilde{\beta}_{A_3}$ |
|-------|-------------------|------------------------|------------------------|------------------------|
| 1.00  | -0.315            | -1.648                 | -1.198                 | -1.229                 |
| 1.05  | -0.333            | -1.644                 | -1.180                 | -1.231                 |
| 1.10  | -0.350            | -1.641                 | -1.162                 | -1.233                 |
| 1.15  | -0.368            | -1.638                 | -1.145                 | -1.237                 |
| 1.20  | -0.386            | -1.637                 | -1.129                 | -1.240                 |
| 1.25  | -0.404            | -1.636                 | -1.113                 | -1.245                 |
| 1.30  | -0.422            | -1.636                 | -1.098                 | -1.249                 |

has used the summation of the leading logs as a guide for picking the appropriate scale. To do this, notice that, under the renormalization group \cite{1,3,19},

$$
1 + \frac{\alpha_s(\mu)}{\pi} \ln \frac{\alpha_s(m_Q)}{\alpha_s(m_q)} \to \left(\frac{\alpha_s(m_Q)}{\alpha_s(m_q)}\right)^{-6/(33-2n_f)}.
$$

We can then use this to choose the scale $\mu$ such that the two expressions are equal. Following this prescription for $D \to K$ decays gives $\alpha_s(\mu) = 0.73$. The uncertainty in $\alpha_s(m_c)$ induces an uncertainty of 0.13 in this quantity. The uncertainty due to the other effects mentioned above is hard to estimate, but it is certainly sizable. Later we will use the data to estimate the value of $\alpha_s(\mu)$ appropriate for $D \to K$ decays. Note that the relevant term in Eq. \ref{eq:10} is $\alpha_s(\mu)/\pi$, so the corrections due to an $\alpha_s(\mu)$ of 0.73 are $0.73/3.14 \simeq 25\%$.

2.3 Finite-mass Corrections

The leading-order heavy quark results given in Eqs. \ref{eq:4} and \ref{eq:5} can be easily obtained using the trace formalism \cite{27}. In this formalism

$$
\langle h^{(*)}(v')|\Gamma|H(v)\rangle = -\sqrt{m_{h^{(*)}} m_H} \xi(w) \text{Tr}[\overline{h}^{(*)}\Gamma H],
$$

where

$$
h = \frac{1 + \not{\epsilon}}{2} \gamma_5, \hfill (14)$$

$$
h^* = \frac{1 + \not{\epsilon}}{2} \not{x}, \hfill (15)$$

$$
\overline{h} = \gamma_0 h^\dagger \gamma_0 \hfill (16)
$$

and $\epsilon$ is the vector meson’s polarization vector. $H$ is defined similarly. In meson decays the current $\Gamma$ can be either $V_\mu$ or $A_\mu$. This formalism is the most convenient for considering the leading-order finite mass corrections \cite{5,6}. There are subleading terms of $O(\Lambda/m_q)$
and $\mathcal{O}(\bar{\Lambda}/m_Q)$, where $Q$ is the heavy quark associated with the meson $H$.

To $\mathcal{O}(1/m_Q)$,

$$\bar{\Lambda} = m_H - m_Q = m_H - m_q.$$  

(17)

The subscripts $H$ and $h^{(s)}$ are barred to indicated that there is no distinction between the pseudoscalar and vector masses to this order. We take this average mass to be the spin average, i.e., $m_h = (3m_{h^s} + m_h)/4$. The quantity $\bar{\Lambda}$ is a fundamental parameter of QCD which is not calculable in perturbation theory. The experimental determination of this quantity is very important. Quark models suggest $\bar{\Lambda} = 300$ MeV. With this choice of $\bar{\Lambda}$, we have $m_b = (3m_{B^*} + m_B)/4 - \bar{\Lambda} = 5.02$ GeV, $m_c = (3m_{D^*} + m_D)/4 - \bar{\Lambda} = 1.67$ GeV and $m_s = (3m_{K^*} + m_K)/4 - \bar{\Lambda} = 0.49$ GeV. (A value $\bar{\Lambda} = 0.5$ GeV, obtained from QCD sum rules [28], would imply that the quark masses are lighter, and an expansion in powers of $\bar{\Lambda}/m_s$ would not be valid.) In general there are corrections to infinite-mass limit of both order $1/m_Q$ and $1/m_q$, but we keep only the dominant (i.e., $1/m_q$) terms. In this approximation Eq. (13) becomes [3]

$$
\langle h^{(s)}(v)|H(v)\rangle = \sqrt{m_{h^{(s)}}m_H}\xi(w)\text{Tr}[h^{(s)}\Gamma H]
+ \sqrt{m_{h^{(s)}}m_H} \frac{\bar{\Lambda}}{2m_q} \{\psi_1(w)\text{Tr}[h^{(s)}\Gamma H]
+ i\psi_2(w)\text{Tr}[\gamma_{\mu}\gamma_{\nu}h^{(s)}\sigma_{\mu\nu}\Gamma H] + \psi_3(w)\text{Tr}[\gamma_{\mu}h^{(s)}\gamma_{\nu}\Gamma H]
+ \left[\psi_+(w)(v_\mu + v'_\mu) + \xi(w)(v_\mu - v'_\mu)\right] \text{Tr}\left[h^{(s)}\gamma_{\mu}\Gamma H\right]
- \left[\psi_+(w)(1 + w) - \xi(w)(1 - w)\right] \text{Tr}\left[\gamma_{\mu}h^{(s)}\gamma_{\nu}\Gamma H\right]} \right).
$$

(18)

To this order there are four new undetermined functions $\psi_{1,2,3,+}$. These dimensionless functions are related to the dimensionful functions defined in Ref. [3] by

$$
\chi_{1,2,3} = \frac{\bar{\Lambda}}{2}\psi_{1,2,3}
$$

(19)

and

$$
\xi_{\text{Luke}}^{+} = \frac{\bar{\Lambda}}{2}\psi_{+}.
$$

(20)

The superscript “Luke” is to distinguish the $\xi_+$ used in Ref. [3], which is one of the four unknown functions introduced at this order, from our $\xi_+$, defined in Eq. (1). The functions $\psi_1$ and $\psi_3$ satisfy the constraint [3]

$$
\psi_1(w = 1) = \psi_3(w = 1) = 0.
$$

(21)

This constraint, which has come to be known as Luke’s theorem, has been shown to follow from the Ademollo-Gatto theorem [29].

Eqs. (13) and (18) can be understood schematically via Figs. 2 and 3. In Fig. 2 the decay of the meson $H$ is described by the decay of the heavy quark $Q$ via the current $\Gamma$. This is represented by the trace in Eq. (13). The light degrees of freedom (the dashed line) factor out of the trace to become the Isgur-Wise function $\xi$. At sub-leading order the current $\Gamma$ is modified and the heavy quark can interact with the light degrees of freedom. The interaction
of the heavy quark $q$ with the light degrees of freedom is represented schematically by Fig. 3. The heavy quark $q$ can interact in a spin-independent way yielding the same trace, but a new undetermined function ($\psi_1$) for the light degrees of freedom. There can be two different spin-dependent interactions. The first has the spin of the heavy quark ($\sigma^{\mu\nu}$) interacting with the light degrees of freedom and the velocity of the parent quark, yielding the function $\psi_2$. The factor $(1 + \psi')/2$ represents the propagation of the heavy quark. Note that

$$\frac{1 + \psi'}{2} h(\ast) = h(\ast).$$

The second spin-dependent interaction corresponds to the hyperfine interaction in the quark model. It is represented by $\psi_3$. The modification of the current $\Gamma$ produces two new trace structures, but can be shown to only require the combination of the original Isgur-Wise function and one new unknown function $\psi_+$. The net effect on the form factors is as follows:

$$\xi_+ = \xi + \frac{\Lambda}{2m_q} [\psi_1 - 2(w - 1)\psi_2 + 6\psi_3]$$

$$\xi_- = \frac{\Lambda}{2m_q} [(2 - w)\xi + (w + 1)\psi_+]$$

$$\xi_V = \xi + \frac{\Lambda}{2m_q} [\xi + \psi_1 - 2\psi_3]$$

$$\xi_{A_1} = \xi + \frac{\Lambda}{2m_q} \left[ \frac{w - 1}{w + 1} \xi + \psi_1 - 2\psi_3 \right]$$

$$\xi_{A_2} = \frac{\Lambda}{2m_q} [-\xi + 2\psi_2 + \psi_+]$$

$$\xi_{A_3} = \xi + \frac{\Lambda}{2m_q} [\psi_1 - 2\psi_2 - 2\psi_3 + \psi_+]$$

### 2.4 Momentum-dependent Form Factors

Experiments measure the momentum-dependent form factors defined by [cf. the velocity dependent form factors defined in Eq. (29)]

$$\langle h(p')|V_\mu|H(p)\rangle = f_+(q^2)(p + p')_\mu + f_-(q^2)(p - p')_\mu$$

$$\langle h^*(p', \varepsilon)|V_\mu|H(p)\rangle = \frac{i V(q^2)}{m_H + m_{H^*}} \epsilon_{\mu\nu\alpha\beta} \varepsilon^* \nu^\alpha v^\beta$$

$$\langle h^*(p', \varepsilon)|A_\mu|H(p)\rangle = (m_H + m_{H^*}) A_1(q^2) \varepsilon^* \mu - \frac{A_2(q^2)}{m_H + m_{H^*}} (\varepsilon \cdot p)(p + p)_\mu - \frac{A_3(q^2)}{m_H + m_{H^*}} (\varepsilon \cdot p)(p - p)_\mu.$$
The form factors $f_-$ and $A_3$ lead to contributions proportional to the electron mass and are thus unmeasurable. The four measurable form factors are related to the velocity-dependent form factors defined in Eq. (1) as follows:

$$f_+ = \frac{z + 1}{2\sqrt{z}} \left[ \xi_+ + \frac{z - 1}{z + 1} \xi_- \right],$$

$$A_1 = \sqrt{\frac{m_h m_H}{m_h + m_H}} (w + 1) \xi_{A_1},$$

$$A_2 = \frac{m_h + m_H}{2\sqrt{m_h m_H}} \left[ z^* \xi_{A_2} + \xi_{A_1} \right],$$

and

$$V = \frac{m_h + m_H}{2\sqrt{m_h m_H}} \xi_V.$$  

### 2.5 Heavy Quark Symmetry to $O(\alpha_s)$ and $O(1/m_q)$

Considering both subleading and perturbative QCD correction terms, Eq. (30) becomes

$$f_+ = \frac{z + 1}{2\sqrt{z}} \left\{ a_f X_{QCD} \xi + \frac{\Lambda}{2m_q} \left[ \frac{z - 1}{z + 1} (2 - w) \xi + \Psi_f \right] \right\},$$

where

$$a_f \equiv \left[ 1 + \frac{\alpha_s}{\pi} \left( \tilde{\beta}_+ + \frac{z - 1}{z + 1} \tilde{\beta}_- \right) \right]$$

and

$$\Psi_f \equiv \psi_1 - 2(w - 1)\psi_2 + 6\psi_3 + \frac{z - 1}{z + 1} (w + 1)\psi_+.$$ 

Proceeding similarly with the other form factors, we obtain

$$A_1 = \frac{\sqrt{m_h m_H}}{m_h^* + m_H} (w + 1) \left[ a_{A_1} X_{QCD} \xi + \frac{\Lambda}{2m_q} \left( \frac{w - 1}{w + 1} \xi + \Psi_{A_1} \right) \right],$$

where

$$a_{A_1} \equiv \left[ 1 + \frac{\alpha_s}{\pi} \beta_{A_1} \right]$$

and

$$\Psi_{A_1} \equiv \psi_1 - 2\psi_3 ;$$

$$A_2 = \frac{m_h^* + m_H}{2\sqrt{m_h^* m_H}} \left[ a_{A_2} X_{QCD} \xi + \frac{\Lambda}{2m_q} \left( \left( \frac{w - 1}{w + 1} - z^* \right) \xi + \Psi_{A_2} \right) \right],$$

where

$$a_{A_2} \equiv \left[ 1 + \frac{\alpha_s}{\pi} \left( z^* \beta_{A_2} + \bar{\beta}_{A_3} \right) \right]$$

and

$$\Psi_{A_2} \equiv \psi_1 + 2(z^* - 1)\psi_2 - 2\psi_3 + (z^* + 1)\psi_+.$$
### Table 5: Lifetimes, branching ratios (Ref. [31]), and decay rates of D mesons.

| Quantity                                      | $D^0$          | $D^+$          |
|-----------------------------------------------|----------------|----------------|
| Lifetime ($10^{-13}$ s)                       | $4.20 \pm 0.08$| $10.66 \pm 0.23$|
| $B(D \to Xe^+\nu_e)$ (%)                     | $7.7 \pm 1.2$  | $17.2 \pm 2.9$ |
| $\Gamma(D \to Xe^+\nu_e)$ ($10^{11}$ s$^{-1}$) | $1.83 \pm 0.29$| $1.61 \pm 0.27$|
| $\Gamma(D \to [\bar{K} + \bar{K}^*]e^+\nu_e)$ ($10^{11}$ s$^{-1}$) | $1.33 \pm 0.20$| $0.99 \pm 0.12$|
| $[\bar{K} + \bar{K}^*]/X$ ratio             | $0.73 \pm 0.15$| $0.62 \pm 0.13$|

and, finally,

$$V = \frac{m_{h^*} + m_{H}}{2\sqrt{m_{h^*}m_{H}}} \left[ a_V X_{\text{QCD}} \xi + \frac{\Lambda}{2m_q} (\xi + \Psi_V) \right],$$

(43)

where

$$a_V \equiv \left[ 1 + \frac{\alpha_s}{\pi} \beta_V \right]$$

(44)

and

$$\Psi_V \equiv \psi_1 - 2\psi_3.$$  

(45)

The functions $\Psi_{f,A_1,A_2,V}$ represent the contribution to the form factors due to the unknown functions $\psi_i$. Although there are four measured quantities and four unknown functions, there is still a relation among the form factors:

$$\Psi_{A_1} = \Psi_V.$$  

(46)

### 3 Lifetimes and branching ratios

In this section we shall discuss the total $D$ meson lifetimes, the inclusive semileptonic decay rates, and the rates for decays to exclusive final states. We shall also compare exclusive decay rates with results of specific models [9, 10, 11, 12, 13, 14, 15, 32], to show that many such models encounter problems in describing the data.

#### 3.1 A Brief Tour Through the Data

The charged and neutral $D$ mesons have different lifetimes and semileptonic branching ratios [31], as shown in Table 5. However, the semileptonic decay rates are compatible with one another. Their average is $\Gamma(D \to Xe^+\nu_e) = (1.71 \pm 0.20) \times 10^{11}$s$^{-1}$. We shall return to the last two lines of Table 5 presently.

The experimental data on specific semileptonic decay channels are summarized in Table 6. The data we use have been summarized in Ref. [32]. Original sources for the branching ratios are Refs. [23, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43].
Table 6: Semileptonic branching ratios and decay rates of $D$ mesons to $\bar{K}e^+\nu_e$ (Ref. [32]) and $K^*e^+\nu_e$.

| Decay | Ref. | Branching ratio (%) | Rate ($10^{10}$ s$^{-1}$) |
|-------|------|---------------------|-----------------------------|
| $D^0 \to K^- e^+\nu_e$ | Mark III [33] | 3.4 ± 0.5 ± 0.4 | |
| | E691 [34] | 3.8 ± 0.5 ± 0.6 | |
| | CLEO [35] | 3.8 ± 0.3 ± 0.6 | |
| | ARGUS [36] | 3.9 ± 0.2 ± 0.7 | |
| | E653 [37] | 2.5 ± 0.4 ± 0.5 | |
| | Average | 3.3 ± 0.4 | 7.9 ± 1.0 |
| $D^+ \to \bar{K}^0 e^+\nu_e$ | Mark III [23] | 6.5 ± 1.6 ± 0.7 | |
| | E691 [38] | 6.1 ± 0.9 ± 1.6 | |
| | Average | 6.3 ± 1.3 | 5.8 ± 1.2 |
| $D \to \bar{K}e^+\nu_e$ | Average | 7.0 ± 0.8 | |
| $D^0 \to K^{*-} e^+\nu_e$ | Mark III [23] | 3.5 ± 1.5 | |
| | CLEO [35] | 1.7 ± 0.8 | |
| | Average | 2.2 ± 0.7 | 5.2 ± 1.7 |
| $D^+ \to \bar{K}^* e^+\nu_e$ | Mark III [23] | 4.2 ± 1.6 | |
| | E691 [38] | 4.4 ± 0.4 ± 0.8 | |
| | WA82 [40] | 5.6 ± 1.6 ± 0.9 | |
| | ARGUS [36] | 4.2 ± 0.6 ± 1.0 | |
| | E653 [37] | 3.25 ± 0.71 ± 0.75 | |
| | Average | 4.1 ± 0.5 | 3.9 ± 0.5 |
| $D \to K^* e^+\nu_e$ | Average | 4.0 ± 0.5 | |

$^a$ We became aware of the measurement of $BR(D^0 \to K^-\mu^+\nu_\mu) = 4.0 \pm 0.8 \pm 0.8\%$ after the completion of this work. This value is not included in the average.

$^b$ Based on quoted $B(D \to \bar{K}\pi)e^+\nu_e) = (4.4^{+1.9}_{-0.9} \pm 0.6)\%$
multiplied by resonant fraction $0.79^{+0.15}_{-0.03}$.

$^c$ Based on quoted ratio $B(D^0 \to K^{*-}e^+\nu_e)/B(D^0 \to K^-e^+\nu_e) = 0.51 \pm 0.18 \pm 0.06$ times our average for $B(D^0 \to K^-e^+\nu_e)$.

$^d$ Based on quoted $B(D \to \bar{K}\pi)e^+\nu_e) = (5.3^{+1.9}_{-1.1} \pm 0.6)\%$
multiplied by resonant fraction $0.79^{+0.15}_{-0.03}$.

$^e$ Based on decay $D^+ \to \bar{K}^*\mu^+\nu_\mu$. 

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The decay rates to exclusive final states for neutral and charged $D$ mesons are consistent with one another. The averages over charged and neutral decays for each channel are also quoted. If we sum over decays to $\bar{K}e^+\nu_e$ and $K^*e^+\nu_e$, we obtain the rates shown in the fourth row of Table 5. These rates fall short of the total semileptonic decay rates, though not with overwhelming statistical significance, as shown in the last row of Table 5. The average of the two numbers in the last row of Table 5 is $0.67\pm0.10$. The Cabibbo-suppressed exclusive final states should account for about $5\% \times (\text{relative phase space}) \approx 7\%$ of the shortfall, and the nonresonant fraction of the decay mode $D \to [\bar{K}\pi]e^+\nu_e$ should account for another $\approx 6\%$. This still leaves about $(20\pm10)\%$ of the inclusive semileptonic decays unaccounted for.

The ratio $\Gamma(D \to K^*e^+\nu_e)/\Gamma(D \to \bar{K}e^+\nu_e)$ is measured to be considerably smaller than that predicted in most models. Experimental values for this ratio are collected in Table 7. The predictions of a number of models for the ratio in Table 7 are reviewed in Ref. [32]. These predictions are typically 0.9 or higher [9, 10, 11, 12] except for the more recent lattice calculations [13,14], which are still quite uncertain. One can see a residue of the heavy-quark limit in these predictions. In the limit of very heavy initial and final quarks, a counting of spin degrees of freedom would then lead to a $Vl\nu/Pl\nu$ ratio of 3, where $V$ and $P$ stand for the ground state vector and pseudoscalar mesons [1]. This is not far from the actual situation in the decays $B \to D^*l\nu$ and $B \to Dl\nu$.

If the recoil of the final quark could be neglected, only the $P$ and $V$ mesons would be produced in the final state, so that $P$ would account for $1/4$ and $V$ for $3/4$ of the hadrons in the semileptonic final state [1]. By comparing the average inclusive semileptonic decay rate $\Gamma(D \to Xe^+\nu_e) = (1.71 \pm 0.20) \times 10^{11}s^{-1}$ with the averages in Table 5, we see that $K$ accounts for $(7.0 \pm 0.8)/(17.1 \pm 2.0) = 0.41 \pm 0.07$ of the $D$ semileptonic decay rate, while $K^*$ accounts for $(4.0 \pm 0.5)/(17.1 \pm 2.0) = 0.23 \pm 0.04$ of that rate. The discrepancy with respect to the heavy-quark limit thus is much more marked for the $K^*$ than for the $K$.

Our task is then to account for departures from the heavy-quark limit which lead to a modest enhancement of the rate for $D \to Kl\nu$ and a substantial suppression of that for $D \to K^*l\nu$. We have mentioned in the Introduction that a combination of two effects, perturbative QCD and $O(1/m_s)$, seems to be responsible. Our subsequent discussion attempts to put this claim on a quantitative footing.

Additional discrepancies with respect to the heavy-quark limit in charmed meson decays show up in the behavior of individual form factors [13]. These are discussed in Section 5.

### Table 7: Ratios $B(D \to K^*e^+\nu_e)/B(D \to \bar{K}e^+\nu_e)$.

| Ref.  | Ratio            |
|-------|------------------|
| Mark III [23] | $0.80^{+0.36}_{-0.26}$ |
| CLEO [33]   | $0.51 \pm 0.18 \pm 0.06$ |
| ARGUS [36]  | $0.55 \pm 0.08 \pm 0.10$ |
| E691 [38]   | $0.55 \pm 0.14$ |
| Average     | $0.57 \pm 0.08$ |
3.2 Comparison with Leading-order Heavy Quark Theory

It is convenient to have expressions for the widths in terms of the velocity-dependent form factors. Defining
\[ \Gamma_0 = \frac{G_F^2 |V_{Qq}|^2 m_H^5 z^3}{48 \pi^3}, \]
we find the following expressions for the branching ratios:

\[
\frac{d\Gamma(H \to h\ell\nu)}{dw} = \Gamma_0 (z + 1)^2 (w^2 - 1)^{3/2} \left[ \xi_+ (w) + \frac{z - 1}{z + 1} \xi_- (w) \right]^2.
\]

\[
\frac{d\Gamma_T(H \to h^*\ell\nu)}{dw} = 2\Gamma_0 (1 + z^*^2 - 2z^*w)(w + 1)(w^2 - 1)^{1/2} \times \\
\left[ (w - 1)\xi_T^2 (w) + (w + 1)\xi_A^2 (w) \right].
\]

\[
\frac{d\Gamma_L(H \to h^*\ell\nu)}{dw} = \Gamma_0 (w + 1)^2 (w^2 - 1)^{1/2} \times \\
\left\{ (w - z^*)\xi_A (w) - (w - 1) [z^*\xi_A^2 (w) + \xi_A^2 (w)] \right\}^2.
\]

These expressions agree with those found by Neubert in Ref. [8].

In order to illustrate the problems encountered by heavy quark symmetry for semileptonic decays, we adapt the above relations for the decays $D \to \bar{K}e^+\nu_e$ and $D \to K^*e^+\nu_e$. We use the relations (4) and (5) of Section 2.1, and predict rates as a function of $\alpha_s(\mu)$ as described in Section 2.2. The results are shown in Fig. 4.

We see that in the lowest order of the heavy-quark theory, in the absence of perturbative QCD corrections, the rate for the decay $D \to \bar{K}e^+\nu_e$ is not badly predicted, but that for $D \to K^*e^+\nu_e$ is about a factor of two above the experimental value. The $K^*/K$ ratio is predicted to lie in the range of 1.2 to 1.7 for the monopole form factor and 1.4 to 2.2 for the exponential form factor, in contrast to the experimental value of $0.57 \pm 0.08$. Perturbative QCD corrections lead to an overall decrease in the predicted rates and cannot account for the $K^*/K$ ratio. As we shall see, spin-dependent $O(1/m_s)$ corrections are needed for that purpose.

4 Extracting Parameters from the Data

4.1 Determining the Isgur-Wise Function from $B$ Decays.

We use both total and differential branching ratios in $B$ decay to determine $\xi(w)$. We have used the expressions (13)–(15) with $H = B^0, h, h^* = D^+, D^{*+}$; and $|V_{Qq}| = |V_{cb}| = 0.041$ [44] with the leading-order plus perturbative QCD form factors in Eq. (11) in comparison with the total branching ratios given in Ref. [44] to obtain the results shown in Table 8.

We can also use the $B \to D^*\ell\nu$ differential branching ratio to determine $\rho$. This process has been considered in detail by Neubert [8]. Our fit to the ARGUS data [13] is shown in
Table 8: $\rho$ values obtained from various fits.

| Decay       | Integrated Branching Ratios | Differential Branching Ratio | Average |
|-------------|-----------------------------|------------------------------|---------|
| $B \to D\nu$ | 1.02$^{+0.22}_{-0.19}$     | 0.97$\pm0.21$               | 1.00$\pm0.15$ |
| $B \to D^*\nu$ | 0.97$\pm0.21$             | 0.91$\pm0.17$               |         |
| $B \to D^*\nu$ | 1.11$\pm0.50$             | 1.02$^{+0.36}_{-0.42}$      |         |

Fig. 5. As can be seen in Table 8, our determination of $\rho$ is dominated by the integrated branching ratios. The plot of the differential branching ratio serves as a visual check of our results. We have chosen the ARGUS data as representative; other data exist [46]. The Isgur-Wise function can be easily extracted from the data by plotting the square root of the differential branching ratio divided by the the factors that multiply the Isgur-Wise function in the symmetry limit. This procedure can be improved by also dividing out the perturbative QCD corrections to the $\xi_{A_1}$ piece, which dominates the rate.

We then plot $|V_{cb}|((1/G) d\Gamma/dw)^{1/2}$, where

$$G \equiv |V_{q\bar{q}}|^2 \frac{G_F^2}{4\pi^3} m_H z^3 \sqrt{w^2 - 1} \left(1 + \frac{1}{w} - 2wz + z^2\right) + (1 - z^2) \right],$$

which amounts to taking the $m_{Q,q} \to \infty$ and $\alpha_s \to 0$ limit of Eqs. (49) and (50) and then subtracting the perturbative QCD correction to the dominant part of the rate. At this level of approximation, $[(1/G) d\Gamma/dw] = \xi(w)^2$. Furthermore, the zero recoil condition

$$\left(1 \frac{d\Gamma}{dw}\right)_{w=1} = 1$$

holds including corrections of $O(\alpha_s)$ and $O(1/m_q)$.

In Refs. [8,15] the quantity $[(1/G) d\Gamma/dw]^{1/2}$ was labeled simply $\xi(w)$, but we prefer the present notation, as it makes the assumptions involved explicit. Instead of trying to fit for $|V_{cb}|$, we have used a fixed value (0.041) because we are only interested in the overall shape of the form factor. In Ref. [8], values of $\rho = 1.14 \pm 0.23$ and $\rho = 1.07 \pm 0.22$ were obtained using the pole form $\xi(w) = [2/(w+1)]^{2\rho^2}$ and the exponential form Eq. (9), respectively. A monopole form (Eq. (8) with $n = 1$) was used in Ref. [17] to obtain $\rho = 1.26 \pm 0.19$.

Both methods show a systematic dependence of the fitted value for $\rho$ with the functional form. Fig. 5 shows that the two fitted functions look very similar with the different values of $\rho$. The precise value of $\rho$ is important for determining $|V_{cb}|$ by extrapolating to $w = 1$. We are not attempting to do this here, however. The functional dependence will have some
impact if we want to extrapolate beyond the end of the data \((w \sim 1.4)\). This is the case for
\(D \rightarrow Kl\nu\) decays, where \(w_{\text{max}} = 2.0\). To minimize the systematic error from the functional
forms, we will use \(\rho = 1.0\) with the monopole form and \(\rho = 0.93\) with the exponential
form.

Analyticity arguments suggest that the Isgur-Wise function should have a radius of con-
vergence of approximately unity, i. e., it should have a pole near \(w = -1\). This corresponds
to a threshold for particle production near \(q^2 = (m_b + m_c)^2\). The \(n\)-pole function has a pole
at \(w = 1 - n/\rho^2\). For \(n = 1\) and \(\rho \sim 1\) the pole is near \(w = 0\). The monopole description
can only be valid if there are a series of poles at \(w < -1\) which somehow conspire to mimic
a monopole description with a pole near zero. This seems unlikely, so we conclude the de-
pendence on the functional form of the Isgur-Wise function is even less than the difference
between the monopole and exponential forms. In the remaining plots we use the exponential
form with \(\rho = 0.93\).

### 4.2 Determining \(\alpha_s(\mu)\) from \(D\) decays

The condition in Eq. (52) has been used to determine the unknown parameter \(|V_{cb}|\) from \(B\)
decays. Although the corresponding parameter in \(D\) decays, \(|V_{cs}|\), is well known \textit{a priori}, we
have argued in Section 2.2 that the appropriate value of \(\alpha_s(\mu)\) is not. In this section we use
the \(D \rightarrow K^*l\nu\) differential width extrapolated to the point \(w = 1\) to determine \(\alpha_s(\mu)\). Not
only do \(1/m_s\) corrections vanish at this point, but the QCD corrections to the dominant \(\xi_{A_1}\n\) form factor are the largest of any of the QCD corrections, as can be seen from comparing
the values of \(\tilde{\beta}_{A_1}\) with the other \(\tilde{\beta}_i\)'s in Tables 3 and 4.

The published data \[42, 48, 49\] includes only raw data, i. e., including detector efficien-
cies, compared to Monte Carlo simulations used to determine the form factors. We have
turned this around to determine a net efficiency at each point and then determine the net
distributions. The experimentalists themselves could do a much better job of this, but our
method will have to suffice until they publish their own differential branching ratios.

In Fig. 6 we have plotted the average of the two available data sets along with our fit
used to extrapolate to \(w = 1\). The fitted function is an exponential. The data is very flat,
so the dependence of the extrapolation on the functional form is slight. The fit yields

\[
\left( a_{A_1} \frac{1}{G} \frac{d\Gamma}{dw} \right)_{w=1} = 0.457 \pm 0.024, \quad (53)
\]

which gives \(\alpha_s(\mu) = 1.03 \pm 0.11\). This value is considerably larger than the guess we had in
Section 2.2, but the uncertainties in that guess are very large. To get a consistent picture
of \(D\) decays using only first order symmetry breaking corrections we have to accept a large
value of \(\alpha_s(\mu)\). The corrections are then \(O(\alpha_s(\mu)/\pi) \approx 33\%\).

### 5 Form Factors

#### 5.1 Comparison with Experiment

All four \(D \rightarrow K\) form factors have been measured. Three groups \[33, 34, 35\] have measured
the \(q^2\) dependence of \(f_+\). They find that the data are consistent with the monopole vector
Table 9: Form factor measurements for $D \to K^* l \nu$.

| Experiment | $A_2(0)/A_1(0)$ | $V(0)/A_1(0)$ | $A_1(0)$ | $A_2(0)$ | $V(0)$ |
|------------|-----------------|---------------|---------|---------|-------|
| E691       | 0.0 ± 0.5 ± 0.2 | 2.0 ± 0.6 ± 0.3 | 0.46 ± 0.07 | 0.0 ± 0.2 | 0.9 ± 0.3 |
| E653       | 0.82 ± 0.23     | 2.0 ± 0.3 ± 0.2 | 0.49 ± 0.07 | 0.40 ± 0.14 | 0.99 ± 0.27 |
| Average    | 0.48 ± 0.05     | 0.27 ± 0.11    | 0.95 ± 0.20 |

dominance form

$$f_+(q^2) = \frac{f_+(0)}{1-q^2/m_f^2}.$$  \hspace{1cm} (54)

We use the world average value given by Stone [32]

$$m_f = 2.0^{+0.3}_{-0.2}.$$  \hspace{1cm} (55)

The normalization constant can be obtained from integrating the branching ratio. Here, too, we use the average value cited by Stone [32]:

$$f_+(0) = 0.69 \pm 0.04.$$  \hspace{1cm} (56)

The three measurable form factors in $D \to K^* l \nu$ semileptonic decay have also been measured [42, 48, 49]. Unfortunately, these measurements assume the $q^2$ dependences follow single-pole vector dominance so that

$$A_{1,2}(q^2) = \frac{A_{1,2}(0)}{1-q^2/m_A^2}$$ \hspace{1cm} (57)

and

$$V(q^2) = \frac{V(0)}{1-q^2/m_V^2}.$$  \hspace{1cm} (58)

where $m_A$ and $m_V$ are assumed to be the lowest mass $c\bar{s}$ states with $J^P = 1^+$ and $1^-$, i.e., $m_A = 2.5$ GeV and $m_V = 2.1$ GeV. The experimentalists then fit for the ratios $A_2(0)/A_1(0)$ and $V(0)/A_1(0)$. $A_2(0)$, $A_1(0)$ and $V(0)$ then may be determined from the total branching ratio, which is dominated by $A_1(0)$.

Table 9 displays the values obtained by the two experiments. The measurements of $V(0)$ and $A_1(0)$ are quite consistent with one another. The values of $A_2(0)$ differ from one another by slightly less than two standard deviations.

It should be pointed out that there is some confusion stemming from the way these values are reported. The numbers $V(0)$, $A_1(0)$ and $A_2(0)$ have been interpreted as measurements of the these form factors at a single point ($q^2 = 0$). In fact, both experiments do their fits to Eq. (57) and Eq. (58) \textit{over the entire range of $q^2$}. $V(0)$, $A_1(0)$ and $A_2(0)$ are merely the constants in Eq. (57) and Eq. (58).

In Fig. 7 we compare the measured form factors with our predictions from heavy quark symmetry. We see that the deviations from the leading-order heavy quark predictions are about as large as was expected: $\sim 2\Lambda/2m_s \sim 30\%$. The rates depend on the square of the form factors, however, so a deviation in the form factor of 25\% can lead to a difference in rates
by a factor of 2! This helps explain the discrepancies seen in Section 3. Note that, although the $D \to Kl\nu$ rate was closest to the leading-order prediction of heavy quark symmetry, it has the largest deviation in the form factors. The $1/m_s$ effects cancel the $\alpha_s$ effects for this decay, while they add coherently in $D \to K^*l\nu$ decay.

5.2 Determining $\Lambda$ from $V(w = 1)$

It follows from Eqs. (6), (21) and (43) that

$$V(w = 1) = \frac{m_{h^*} + m_H}{2\sqrt{m_{h^*}m_H}} \left( a_V X_{QCD} + \frac{\Lambda}{2m_q} \right). \tag{59}$$

A measurement of $V(w = 1)$ is the theoretically cleanest way to measure the fundamental quantity $\Lambda$. Even in $D$ decays, where the size of $\alpha_s(\mu)$ is in question, this relation is useful because the dependence on $\alpha_s(\mu)$ is relatively weak. Using the measured $V$ from the previous section, this leads to

$$\frac{\Lambda}{2m_s} = -0.05 \pm 0.239, \tag{60}$$

and

$$\Lambda = -44 \pm 297 \text{ MeV}. \tag{61}$$

This error is clearly too large to be truly useful. We have the further caveat that the extrapolation of $V(w)$ to $w = 1$ will only be reliable once the $w$-dependence of the form factors is measured. We hope this quantity will be measured with smaller uncertainties in the future.

The above measurement of $\Lambda$ is subject to the various uncertainties involved in applying heavy quark symmetry to the strange quark. It is much more important to measure $\Lambda/(2m_c)$, where heavy quark symmetry is on very sound theoretical ground. CLEO [50] has recently published a first measurement of $V(w = 1)$ in $B \to D^*l\nu$ decays. They obtain two different values from two different fits:

$$V(w = 1) = \begin{cases} 0.91 \pm 0.49 \pm 0.12 \text{ (fit a)} \\ 1.19 \pm 0.57 \pm 0.15 \text{ (fit b)} \end{cases}, \tag{62}$$

which we average to get $V(w = 1) = 1.05 \pm 0.5$. Using $m_H = 5.28 \text{ GeV}$, $m_{h^*} = 2.01 \text{ GeV}$, $X_{QCD} = 1.11$ and $a_V = 0.97$ in Eq. (59) leads to

$$\frac{\Lambda}{2m_c} = -0.14 \pm 0.45. \tag{63}$$

The uncertainties are so large that it doesn’t make sense to extract $\Lambda$. It is important that this quantity become better measured in the future.
5.3 Extracting the Subleading Effects From the Data

In Fig. 8 we have assumed that the differences between the predictions and the experiments shown in Fig. 7 are completely due to subleading terms. We have then subtracted out the $\xi$ dependent effects to obtain $\Psi_{f,A_1,A_2,V}$. This procedure depends on the value of $\Lambda$, so we have included results for both $\Lambda = 200$ and 400 MeV. The size of the experimental uncertainties depends on $\Lambda$ because we have scaled out $\Lambda/(2m_s)$.

Several things are apparent from Fig. 8: First, the dimensionless functions all come out to be $O(1)$. We also see that the prediction $\Psi_{A_1} = \Psi_V$ is better satisfied for the smaller value of $\Lambda$. This is consistent with the results in Section 5.2. We also see that the corrections to the pseudoscalar channel embodied by $\Psi_f$ are substantial and positive. The net effect of the corrections to the vector decays is small and negative.

For concreteness, we can attempt to estimate the individual functions $\psi_i$ under certain assumptions. First, we choose the smaller value of $\Lambda$ in Fig. 8. Using the definition of $\Psi_f$ [Eq. (36)] and $\psi_1(1) = \psi_3(1) = 0$ (Luke’s theorem), we obtain

$$\Psi_f(1) = -1.15 \psi_+(1)$$

Comparing with the light band in the plot of $\Psi_f$ in Fig. 8, we obtain $\psi_+(1) \approx -1.5$. Similarly, we use

$$\Psi_{A_2}(1) = -1.04 \psi_2 + 1.48 \psi_+$$

to obtain $\psi_2(1) \approx -2$. To get information about $\psi_1$ and $\psi_3$ we need to go to another kinematical point. It is best to use the minimum recoil point of the pseudoscalar decay, since it has the larger kinematic range and the kinematic dependence of the pseudoscalar form factor ($f$) has been measured. Using Eq. (36) once again, we have

$$\Psi_f(2) = \psi_1(2) - 2\psi_2(2) + 6\psi_3(2) - 1.7\psi_+(2).$$

Since $\Psi_{A_1}$ is consistent with zero, we set $\psi_1(w) = 2\psi_3(w)$. If we also assume $\psi_+(w)$ and $\psi_2(w)$ are roughly constant in $w$, we can then obtain $\psi_3(2) \approx -0.5$ and $\psi_1(2) \approx -1$. These estimates are very rough, owing to both experimental uncertainties and our assumptions.

6 Implications for B Decays

The overall QCD decrease of semileptonic decay rates into pseudoscalar and vector mesons found for $D$ meson decays implies that for $B$ meson decays similar, but slightly weaker, effects are to be expected. In view of the ambiguity reflected in Fig. 8, we can make only qualitative statements.

While the $O(1/m_q)$ effects discussed above are expected to reduce the ratio $\Gamma(B \to D^*l\nu)/\Gamma(B \to Dl\nu)$ from its infinite-mass limit 3 of 3, the lowest-order prediction of this ratio exceeds 3 slightly for reasonable slopes of the Isgur-Wise function [47]. The experimental branching ratios [44] are $B(B \to D^*l\nu) = (4.4 \pm 0.3 \pm 0.6)$% and $B(B \to Dl\nu) = (1.6 \pm 0.3 \pm 0.2)$%; their ratio is $2.75 \pm 0.75$. The error is too large to reflect the presence of any of the effects mentioned here.

The sum of the semileptonic $B$ branching ratios to $D$ and $D^*$, $(6.0 \pm 0.8)$%, is only 1/2 to 2/3 of the inclusive semileptonic branching ratio [44], $B(B \to Xl\nu) \approx 11$%. The remaining
1/2 to 1/3 of the semileptonic decays are expected to involve excited \(D\) mesons or extra pions. A non-trivial Isgur-Wise function (one with \(\rho > 0\)) is sufficient to reflect the presence of such excitations \[17\]; one does not require \(\mathcal{O}(1/m)\) effects to understand them.

Perhaps the most pressing question about heavy quark symmetry and \(B\) decays is how much we can learn about \(V_{cb}\). Our analysis suggests that measuring \(V_{cb}\) by extrapolating the \(B \to Dl\nu\) spectrum to the normalization point would be unreliable because of the substantial \(\Lambda/m_c\) effect at that point. Fortunately, extrapolating the \(B \to D^*l\nu\) spectrum, as was done in Ref. \[8\], avoids all complications due to \(\Lambda/m_c\) effects. The question in this case, then, is the size of the \((\Lambda/m_c)^2\) corrections. We have shown in Section 5.2 how the quantity \(\Lambda/(2m_c)\) can be extracted from the data. Although the error we obtained is very large, the results favor a small value of \(\Lambda\), which would be good news for an accurate determination of \(V_{cb}\) if more accurate measurements confirm that \(\Lambda\) is indeed small.

7 Conclusions

We have applied a broken version of heavy quark symmetry to the semileptonic decays \(D \to Kl\nu\) and \(D \to K^*l\nu\), in order to study the deviations from the symmetry in an environment where they are particularly prominent. The price that we pay is the possible loss of validity of the symmetry. We have to regard the strange quark as heavy, which we can do if we regard it as having a constituent-quark mass of order 1/2 GeV.

We have been able to identify several physical sources of the apparent deviations from heavy-quark symmetry (and of the shortcomings of early models for \(D\) semileptonic decays).

First, perturbative QCD effects lead to an overall reduction in predicted \(D \to Kl\nu\) and \(D \to K^*l\nu\) rates with respect to the lowest-order theory. One expects the remainder of the semileptonic decays to show up in excited kaons or \(K + n\pi\) final states. No evidence for these final states exists at present.

Second, \(\mathcal{O}(1/m_s)\) effects give different contributions to the pseudoscalar and vector final states. These effects add to cancel the perturbative QCD effects in the pseudoscalar final state. In the vector final state, however, these effects cancel one another, leaving the overall suppression due to the perturbative QCD effects untouched. This explains the suppression of the vector final state relative to the pseudoscalar final state.

We find that it is instructive to view the strange quark, under some circumstances, as “heavy,” which allows us to gain a qualitative understanding of effects in \(D\) decays which are still too small to show up in the decays \(B \to Dl\nu\) and \(B \to D^*l\nu\). A study of these decays with increased precision would provide an excellent check of our conclusions.

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Figure Captions

Figure 1: Variation of $\alpha_s$ over the region of interest for $D$ decays based on $\alpha_s(m_c) = 0.276 \pm 0.014$ The solid line represents the central value, while the dashed lines represent the one standard deviation uncertainties.

Figure 2: Diagram for heavy meson decays at leading order.

Figure 3: Subleading diagram where the heavy quark interacts with the light degrees of freedom.

Figure 4: Predictions of heavy-quark symmetry for decay rates of $D \to \bar{K}e^+\nu_e$ (a,c) and $D \to \bar{K}^*e^+\nu_e$ (b,d). Predictions without QCD corrections (i.e., $\alpha_s(\mu) = 0$ and $X_{QCD} = 1$) are shown in (a,b) as functions of $\rho$ for monopole (solid lines) and exponential (dashed lines) form factors. The shaded bands correspond to the experimental values with one standard deviation uncertainties. The ranges of $\rho$ allowed by the fits to $B$ decays in Section 4.1 are shown for monopole (circles) and exponential (diamonds) form factors. Predictions for central values of $\rho$ are shown as functions of $\alpha_s(\mu)$ (holding $X_{QCD}$ fixed) in (c,d), where the bars indicate the range of $\alpha_s(\mu)$ obtained in Section 4.2.

Figure 5: Data taken from Ref. [45]. The solid and dashed lines are fits to the exponential and monopole forms, respectively.

Figure 6: Differential $D \to K^*$ distribution from Refs. [48] and [49]. The line is a fit used to extrapolate to $w = 1$. The method of extracting this plot from the published data is described in the text.

Figure 7: Comparison of the leading order heavy quark predictions with data for the four measured $D$ meson semileptonic decay form factors. The solid and dotted lines correspond to the central value of $\alpha_s(\mu)$ obtained in Section 4.2 and the one standard deviation uncertainties, respectively. The shaded bands correspond to the experimental values with one standard deviation uncertainties.
Figure 8: Values of form factors associated with subleading operators obtained from experiment. The light and dark regions correspond to taking $\Lambda = 200$ and 400 MeV, respectively. The light regions have been extended horizontally for clarity in some cases.