Four Generation MSSM to Generate the Universe Matter-Antimatter Asymmetry

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Abstract. A four generation extension to the MSSM is considered in the context of the electroweak phase transition. This model could be consistent with the generation of the baryonic and leptonic asymmetry of the universe during such phase transition. The phase transition strength is briefly discussed for this model and one way to study the $CP$ violation during the electroweak phase transition is sketched.

1. Introduction
The electroweak phase transition is an important candidate to explain the baryonic and leptonic asymmetry of the universe. A strong phase transition is one of the necessary requirements to generate and maintain the baryonic asymmetry of the universe up to the present era.

The Standard Model does not provide an electroweak phase transition strong enough to generate and maintain this asymmetry up to present date. However it does have features that are necessary to generate this asymmetry. That is the reason why extensions to the Standard Model are considered as candidates to generate this asymmetry. The MSSM has been considered with this purpose [8, 9], however it requires the existence of a light stop quark, an scenario ruled out now. The introduction of a fourth generation within a supersymmetric framework strengthens the phase transition even beyond that of the MSSM since it introduces new bosons, the fourth generation superpartners, which strengthen the phase transition through their nonzero temperature contributions to the effective potential.

2. Effective potential and a phase transition
To study the v.e.v. of Higgs fields for a theory the Higgs effective potential can be introduced [1, 2]. The Higgs effective potential comes from considering the Higgs fields dependent part of the effective action when the classical fields, on which the effective action depends, are considered to be coordinate independent. Since Higgs v.e.v. are coordinate independent only coordinate independent classical fields need to be considered. Coordinate independent classical Higgs fields, on which the effective potential depends, will be denoted by $\phi$.

The effective potential can be calculated by adding the classical potential (the zero loop potential in the lagrangian) higher order loop terms. There is a diagrammatic formula to add this higher loop terms. Loop orders higher than two are usually not calculated for an effective potential. In this work we only use one loop corrections.
In general one loop Higgs contributions to the effective potential for a field with mass that depends on \( \phi \) are:

\[
N \left( \frac{m^4(\phi)}{64 \pi^2} \left[ \log \left( \frac{m^2(\phi)}{\Lambda^2} \right) - \frac{3}{2} \right] + \frac{T^4}{2 \pi^2} \int_0^\infty dx \ x^2 \log \left( 1 \pm e^{-\sqrt{x^2 + m^2(\phi)/T^2}} \right) \right). (1)
\]

The factor \( N \) is an integer that counts the particle’s spin and multiplicity due to internal degrees of freedom. \( N \) is set positive for bosons and negative for fermions. The sign on \( \pm \) also depends on the spin. It is + for fermions and – for bosons.

The Higgs effective potential is a real function of real variables, the coordinate independent classical fields. The effective potential minima determine the Higgs fields v.e.v.s. However the effective potential also depends on temperature and hence Higgs fields vacuum expectation values are temperature dependent. The temperature dependence for the effective potential can be separated in temperature independent and dependent terms, the first and second terms of (1) respectively. Temperature independent terms correspond to the zero temperature field theory Higgs effective potential. The temperature dependent terms vanish for \( T = 0 \).

Temperature corrections to the potential are such that even if the Higgs field has some non zero v.e.v.s at zero temperature there is a certain temperature that gives all the Higgs v.e.v.s equal to zero. At this temperature the Higgs mechanism is ruined and no particles acquire mass through it. Such temperature is called the critical temperature, \( T_c \).

If a system with dynamics like we have previously described goes from \( T > T_c \) to \( T < T_c \) then it could undergo a phase transition. An order parameter in this phase transition is a Higgs vacuum expectation value that is nonzero for temperatures below the critical value. If at the phase transition the change in the order parameter is continuous then the phase transition in second order, if the change is discontinuous then the phase transition is first order and the system is outside thermal equilibrium.

This type of phase transition is called electroweak phase transition since the Higgs mechanism breaks the \( SU(2)_L \times U(1)_Y \) gauge symmetry and gives rise to the electroweak interactions.

During the electroweak phase transition, EWPT, two different phases of matter coexist, one where the Higgs mechanism is active, called broken phase, and one where it is not active, unbroken phase.

As the universe expands after the Big Bang its temperature decreases. The temperature right after the Big Bang is expected to be much higher than the critical temperature for a model which describes the dynamics of the universe so we think the universe underwent an electroweak phase transition.

3. Baryogenesis in the Standard Model and extensions

Four conditions are necessary to dynamically generate a baryonic asymmetry. They were stated by A.D. Sakharov in 1967 [3]. These conditions are that particle interactions have:

(i) Baryon and lepton number non conservation.
(ii) \( C \) violation.
(iii) \( CP \) violation.
(iv) Departure from thermal equilibrium.

The Standard Model of particle physics could have all these features.

Because of an anomaly called the Adler-Bell-Jackiw anomaly there is baryon and lepton number non conservation in the Standard Model. In fact the \( B + L \) current suffers the anomaly while \( B - L \) is still conserved [4]. However the rate for baryon and lepton number
non conservation in the Standard Model at this temperature is well below current experimental sensibilities.

Due to its chiral nature the Standard Model has explicit $C$ no conservation. 

$CP$ violating interactions have been observed and the SM provides an explanation to this measurements through the Cabbibo-Kobayashi-Maskawa matrix [5].

If the Standard Model parameters are such that it has a first order electroweak phase transition then the universe is out of thermal equilibrium during the electroweak phase transition.

The Standard Model could fulfill all the Sakharov conditions, however this conditions are necessary but not sufficient to dynamically generate the universe baryonic asymmetry.

Now let us look at how the baryonic asymmetry is thought to be generated during the electroweak transition. The Higgs field effective potential for the Standard Model depends on a single Higgs related variable, $\phi$. The universe reaches the critical temperature when the Higgs effective potential has two degenerate minima for $\phi$. Near the temperature $T_c$ regions in space where the Higgs mechanism is in action begin to form. This regions are called broken phase bubbles and they expand filling all the universe. The formation of this bubbles is somewhat similar to the formation of bubbles of steam on a liquid water container whose temperature is elevated until boiling temperature is reached.

**Figure 1.** Typical first order phase transition for an effective potential. Curves in red correspond to temperatures above $T_c$. The green curve corresponds to $T = T_c$. The blue curve corresponds to $T < T_c$.

However particles diffuse between bubbles and also interact with bubble walls. $CP$ violating interactions with the bubble walls give different diffusion properties for left and right quarks and leptons. This increases left quark and lepton densities on the unbroken phase.

Excess of left quarks and leptons on the unbroken phase enhances baryon and lepton number non conservation through processes called sphalerons. This processes are also supposed to have much bigger rates at higher temperatures, like that of the electroweak phase transition, than at the present temperature for the universe. Baryon and lepton number violation occurs on the unbroken phase where a baryonic and leptonic asymmetry is generated.

It is not sufficient to generate the correct baryonic asymmetry with this mechanism. It is also necessary that an asymmetry is not erased after the EWPT so that it persists to present time. For this to happen we demand that $B$ and $L$ violating processes are very suppressed inside the broken phase. This processes are such that

$$\frac{\phi_{\text{min}}(T_c)}{T_c} \geq 1$$

then the asymmetry is conserved inside the broken phase thus allowing it to survive up to now. This condition corresponds to demanding a large jump, bigger than the temperature, on the
Higgs v.e.v. at the phase transition. If there is such a jump we can also conclude that the phase transition is first order.

To generate a jump in the minimum of the effective potential it is necessary to have $T$ dependent odd powers of $\phi$. Otherwise such jump cannot be generated. If an expansion is performed on the temperature dependent contributions in (1) it is found that only bosons have such contributions [2]. Hence integer spin fields with significant couplings to the Higgs sector are needed to generate a first order phase transition.

The Standard Model of particles only satisfies this condition for currently ruled out Higgs mass values. For the currently experimentally allowed Higgs mass values [6]. The current lower experimental bound for the Higgs mass is at about 115 GeV. However for the electroweak phase transition within the Standard Model dynamics to fulfill condition (2) the Higgs mass has to be below 80 GeV.

One possible solution to this problem is to use an extension to the Standard Model. The minimal supersymmetric extension to the Standard Model is a viable candidate.

In supersymmetric versions of the Standard Model the Higgs sector is extended. Instead of one Higgs doublet there are two Higgs doubles, $H_1$ and $H_2$, each with a component which acquires a non-zero v.e.v.. After the electroweak symmetry breaking the MSSM has five Higgs particles. Two charged particles and three neutral, one of these three is $CP$ odd and the other two are $CP$ even. In this work we assume that only one linear combination of two $CP$ even particles is light and all the other Higgs particles have much higher mass. The linear combination that remains light is:

$$h = \text{Re}(H_1^0) \cos \beta + \text{Re}(H_2^0) \sin \beta.$$  \hfill (3)

Where $\tan \beta = v_2/v_1$ and $v_i/\sqrt{2}$ is the v.e.v. for the neutral component of $H_i$.

In this work we use the four generation Minimal Supersymmetric Standard Model. The Higgs and gauge sectors are the same for this model and the conventional MSSM. Only the lepton and quark sectors are extended. The fourth generation includes one left handed lepton doublet; a right handed charged lepton; a left handed quark doublet and two right handed quarks copies of the up and down right quarks.

Not all particles in the four generation MSSM contribute to the effective potential.

(i) From the Higgs sector only the scalar linear combination $h$ is relevant.

(ii) The gauge sector only contributes with the $W^\pm$ and $Z$ boson.

(iii) Quarks and leptons from the first and second generation play an unimportant role due to their small couplings with the Higgs sector compared to the top quark and the quarks and leptons from the fourth generation. From the third generation only the top quark has an important coupling. Quarks and leptons belonging to the fourth generation all have couplings similar to that of the top quark and hence have to be included. For the quark and lepton sector both fermions and their supersymmetric partners have important contributions.

We calculate the effective potential and find values for some of the 4gMSSM free parameters that are relevant to the phase transition that satisfy condition (2).

4. $CP$ violation and the bubble wall

In case the phase transition in strong enough diffusion equations between the bubble walls for quarks and leptons still have to be solved to see if left quark and lepton densities are such that the correct baryonic and leptonic asymmetry is generated by $B + L$ violating processes in the unbroken phase.

Thermal Quantum Field Theory is used to compute transport equations between the two sides of the bubble wall.
For a generic scalar, \( \varphi(x) \), we have the current:

\[
\langle J^\mu_\varphi(x) \rangle_\beta = i(\varphi^\dagger(x)(\partial^\mu_\varphi(x)) - (\partial^\mu_\varphi(x))^\dagger(\varphi(x)))_\beta \\
= -((\partial^\mu_\varphi - \partial^\mu_\varphi^\dagger))G^\varphi(x,y)|_{y=x}
\]  

(4)

Taking \( \langle J^\mu_\varphi(x) \rangle_\beta \) for the up type s-quarks and expanding the Green functions and the self energy in terms of the free Green functions, \( G_0 \), and the Lagrangian density, \( \mathcal{L}_{\text{MSSM}} \) we can write:

\[
\langle \partial_\mu J^\mu_\varphi(x) \rangle_\beta = \sum_{i,k=1}^{n_g} \sum_{B=L,R} \int d^4y \{ (g(x)^{AB}_{ik}) G^\varphi_0(x,y)_B^k \\
\cdot g(y)^{BA}_{ki} G^\varphi_0(y,x)_i^A - g(x)^{AB}_{ik} G^\varphi_0(x,y)_k^B g(y)^{BA}_{ki} G^\varphi_0(y,x)_i^A \\
+ g(y)^{AB}_{ik} G^\varphi_0(y,x)_k^B g(x)^{BA}_{ki} G^\varphi_0^\dagger(x,y)_i^A - g(y)^{AB}_{ik} G^\varphi_0(x,y)_k^B g(x)^{BA}_{ki} G^\varphi_0^\dagger(y,x)_i^A \\
\cdot g(x)^{AB}_{ki} G^\varphi_0^\dagger(x,y)_i^A \theta(x^0,y^0) \}.
\]  

(5)

The \( g^{AB}_{ij} \) come from \( \mathcal{L}_{\text{bilinear}} \), the part of \( \mathcal{L}_{\text{MSSM}} \) which is bilinear in the up s-quarks. Upper indices, \( A, B \), in \( g^{AB}_{ij} \) distinguish \( L \) and \( R \) s-quarks. Lower indices, \( i, j \), are generation indices.

\[
\mathcal{L}_{\text{BL}} = \tilde{U}_{L,i} g_{ij}^{LL} \tilde{U}_{L,j} + \tilde{U}_{R,i} g_{ij}^{RR} \tilde{U}_{R,j} + \tilde{U}_{L,i} g_{ij}^{LR} \tilde{U}_{R,j} + \tilde{U}_{R,i} g_{ij}^{RL} \tilde{U}_{L,j}.
\]  

(6)

More specifically:

\[
g(x)^{LL}_{ij} = [M^2_Q]_{ij} + v_2^2(x)[h_u^*h_u^T]_{ij} + \delta_{ij} D_2^R(x) \\
g(x)^{RR}_{ij} = [M^2_U]_{ij} + v_2^2(x)[h_u^*h_u^T]_{ij} + \delta_{ij} D_2^R(x) \\
g(x)^{LR}_{ij} = -v_1(x) \mu [h_u^*]_{ij} + v_2(x)[(Y_u)^*]_{ij} \\
g(x)^{RL}_{ij} = (g(x)^{RL})^* \]

(7)

- \([M^2_Q]_{ij}, [M^2_U]_{ij}\) are Soft breaking terms.
- \([h_u^*h_u^T]_{ij}\) are scalar Yukawa mass terms.
- \([(Y_u)^*]_{ij}\) is a soft trilinear term.
- \(\mu\) is the Higgs bilinear term coefficient.

Higgs v.e.v.’s \( v_1, v_2 \) have a coordinate dependence since there are two regions in space, the broken \( (v_1 \neq 0, v_2 \neq 0) \) and unbroken \( (v_1 = 0, v_2 = 0) \) phases.

Equation (5) is a \( g^2 \) order expression for the up, both left and right, s-quark asymmetry. It has contributions from diagrams of the following kind: Equation (5) is the Quantum Boltzmann equation for the up s-quark particle density asymmetry. It is an integral equation. It comes from the \( g^2 \) approximation. Eq. (5) and (7) can be used to get:

\[
\langle \partial_\mu J^\mu_\varphi(x) \rangle_\beta = 4 \int_{\mathbb{R}^4} d^4y \{ \theta(x^0,y^0)(v_1(x)v_2(y) - v_2(x)v_1(y)) \\
\{ \Im[G^\varphi_0^\dagger(x,y)_B^k (G^\varphi_0^\dagger(y,x)_i^A)\} \Im[\mu (Y_u)_{ik}^T (h_u)_{ki}^*] \}
\]  

(8)

The form of free propagators \( G_0 \) is known from Thermal Quantum Field Theory [7].
The $x$ coordinate will be considered as the coordinate in the rest frame for the bubble wall. Higgs v.e.v. $v_1$ and $v_2$ have an $x$ coordinate dependence since there it has a different value depending on whether it is inside or outside the bubble.

The bubble wall will be considered flat in the calculation of diffusion equations.

So far this work has already been performed for three generations in the light s-top scenario [8]. We intend to repeat the calculation for the four generation case.

5. Conclusions

It is important to find a convenient basis for the Yukawa matrices which allows us to clearly visualize the $CP$ violation sources. Down s-quarks could also play a role, Quantum Boltzmann equations for theme also should be included.

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