Isospin Recoupling and Bose-Einstein Pion Correlations in $\bar{N}N$ Annihilations

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Abstract

We study the effect of isospin projections on a two source picture of coherent pions coming from nucleon-antinucleon annihilation. We show that important Hanbury-Brown Twiss correlations among like charge pion pairs compared with unlike arise from these isospin projections.

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In a recent letter \[1\], we showed that intensity correlations among pions from $^\Lambda NN$ annihilation at rest can show strong Hanbury-Brown Twiss \[2\] (H-BT) correlations for like charged pion pairs without having to invoke any thermal or related random phase assumptions. Integration over the pion sources, experimental binning and isospin projections are enough to produce a relative enhancement between like pions pairs at small relative momentum compared with unlike pairs. In that discussion we treated isospin rather cursorily, averaging the isospin of the pion field independently at each field emission point. We reported there, without proof, that if more careful isospin projections are performed, the central results do not change. The purpose of this note is to present the calculation that substantiates that claim. We will assume two sources of pion radiation, each emitting an ensemble of pions restricted to a fixed isospin. Coupling the two isospins to a fixed total isospin we shall show that there is an enhancement at small relative momentum of two pions with like charge.

Our presentation will be in terms of coherent states and will use the isospin projection formalism we exploited before \[3, 4\]. We will also invoke results from our letter \[1\]. Hence this note will not stand by itself and the preceding papers should be consulted.

Consider the coherent state for pion radiation from two sources. The sources each have the same Fourier profile, $f(\vec{p})$, but are displaced in space and perhaps time from each other. One source emits pions in a field of isospin $I_1$ and the other in isospin $I_2$. These two are coupled up to the total isospin $I$ and z-component $M$ of the entire system. That state can be written \[4\]

$$|f, I_1, I_2; I, M > = \sum_{M_1, M_2} <IM|I_1M_1; I_2M_2 > \int d\tilde{T}_1 d\tilde{T}_2 Y_{I_1M_1}^*(\tilde{T}_1) Y_{I_2M_2}^*(\tilde{T}_2) \exp \left[ \int d^3p f(\vec{p}) \exp(i\phi_1(p)) \tilde{T}_1 \cdot \vec{a}^\dagger(\vec{p}) \right] \exp \left[ \int d^3p f(\vec{p}) \exp(i\phi_2(p)) \tilde{T}_2 \cdot \vec{a}^\dagger(\vec{p}) \right] |0 >$$

(1)

where the phases $\phi_i(p)$ ($i = 1, 2$) are

$$\phi_i(p) = \tilde{\phi}_i + \omega_i t_i - \vec{p} \cdot \vec{x}_i.$$  

(2)

and where $\vec{a}^\dagger(\vec{p})$ is the isovector creation operator of a pion of momentum $\vec{p}$. Here $\tilde{\phi}_i$ is a phase associated with site $i$. It is introduced to represent the
effect of the various averaging mechanisms discussed in [4]. As we explain
there, these averages have the same effect as random phases of thermal or
other origin, but arise in the purely coherent sources we consider. Rather
than repeating the discussion from [4] here, we introduce the phases \( \phi_i \) with
the property \( e^{i\phi_i}e^{-i\phi_j} = \delta_{ij} \). In all matrix elements we write, we un-
derstand this average to take place. This average replaces all the summing,
binning, etc. we discussed in [1]. It should be noted that the state we have
defined in (1) is not normalized, but that the normalization will cancel when
we take ratios.

Consider first the one pion spectrum. To save writing let us call the state
defined in (1) \( |f> \). The density of pions of charge type \( \mu \) and momentum \( \vec{k} \)
in that state is given by

\[
W_1(\vec{k}, \mu) = < f | a_\mu^\dagger(\vec{k})a_\mu(\vec{k}) | f > = |f(\vec{k})|^2 \sum_{(1)(2)} < IM|I_1M_1; I_2M_2 > < I'M'|I'_1M'_1; I'_2M'_2 > \\
\int d\hat{T}_1 d\hat{T}_2 d\hat{T}_1' d\hat{T}_2' \ Y_{I_1;M_1}(\hat{T}_1)Y_{I_2;M_2}(\hat{T}_2)Y_{I'_1;M'_1}(\hat{T}_1')Y_{I'_2;M'_2}(\hat{T}_2') \\
\exp \left[ N_0(\hat{T}_1 \cdot \hat{T}_1' + \hat{T}_2 \cdot \hat{T}_2') \right] \left[ T_{1\mu}^* T_{1\mu} + T_{2\mu}^* T_{2\mu} \right]
\]

(3)

where \( \sum_{(1)(2)} \) is over indices \( M_1, M_2, M'_1 \) and \( M'_2 \). The mean number of pi-
ons emitted by each source, \( N_0 \), is given by \( \int d^3k |f(\vec{k})|^2 \). In obtaining this
expression, we have used the phase averaging discussed above both in the
matrix element and in the exponent.

The corresponding expression for the two pion spectrum, that is for the
probability of finding a pion of momentum \( \vec{p} \) and charge type \( \mu \) and another of
momentum \( \vec{q} \) and charge type \( \nu \) is given by

\[
W_2(\vec{p}, \mu; \vec{q}, \nu) = < f | a_\mu^\dagger(p)a_\mu(p)a_\nu^\dagger(q)a_\nu(q) | f > = |f(\vec{p})|^2 |f(\vec{q})|^2 \sum_{(1)(2)} < IM|I_1M_1; I_2M_2 > < I'M'|I'_1M'_1; I'_2M'_2 > \\
\int d\hat{T}_1 d\hat{T}_2 d\hat{T}_1' d\hat{T}_2' \ Y_{I_1;M_1}(\hat{T}_1)Y_{I_2;M_2}(\hat{T}_2)Y_{I'_1;M'_1}(\hat{T}_1')Y_{I'_2;M'_2}(\hat{T}_2') \\
\exp \left[ N_0(\hat{T}_1 \cdot \hat{T}_1' + \hat{T}_2 \cdot \hat{T}_2') \right] \left[ T_{1\mu}^* T_{1\nu}^* T_{1\mu} T_{1\nu} + (1 \leftrightarrow 2) \right] \\
+ T_{1\mu}^* T_{2\nu}^* T_{1\mu} T_{2\nu} + (1 \leftrightarrow 2)
\]

3
\[ + T_{1\mu}^* T_{2\nu}^* T_{2\mu}^* T_{1\nu}^* e^{-i\Delta E(t_1 - t_2) + i\vec{Q} \cdot (\vec{x}_1 - \vec{x}_2)} + (1 \leftrightarrow 2) \]  

(4)

where \( \Delta E = \omega_p - \omega_q \) and \( \vec{Q} = \vec{p} - \vec{q} \) and where again \( \vec{\phi} \) averaging has been used.

The integrals in the one and two pion densities can be done, but they are complicated. They can be greatly simplified by taking advantage of the fact that the mean number of pions, \( N_0 \) is large. If we include the normalization, we have the factor

\[ \exp \left( N_0 (\hat{T}_1^* \cdot \hat{T}_2 - 1) \right) \]  

(5)

in our integrals. For large \( N_0 \) this is well approximated by

\[ \delta(\hat{T}_1 - \hat{T}_2) \]  

(6)

with corrections of order \( 1/N_0 \). In this limit, only two integrals appear in our calculation of the pion spectra,

\[ A^{I_1; I_2}_{\mu\nu} = \int d\hat{T} \ Y^*_{I_1; M_1}(\hat{T})Y_{I_2; M_2}(\hat{T})T_{\mu}^* T_{\nu} \]  

(7)

and

\[ B^{I_1; I_2}_{\mu\nu} = \int d\hat{T} \ Y^*_{I_1; M_1}(\hat{T})Y_{I_2; M_2}(\hat{T})T_{\mu}^* T_{\nu} T_{\mu} T_{\nu}. \]  

(8)

A simple calculation yields

\[ A^{I_1; I_2}_{\mu\nu} = \sum_{LM} \frac{\sqrt{(2I_1 + 1)(2I_2 + 1)}}{2L + 1} \]  

\[ < L0|I_10; 10 > < L0|I_20; 10 > \]  

\[ < LM|I_1 M_1; 1\mu > < LM|I_2 M_2; 1\nu > \]  

(9)

and

\[ B^{I_1; I_2}_{\mu\nu} = \sum_{LMaab\beta} \frac{\sqrt{(2I_1 + 1)(2I_2 + 1)}}{2L + 1} \]  

\[ < a0|10; 10 > < b0|10; 10 > \]  

\[ < a\alpha|1\mu; 1\nu > < b\beta|1\mu; 1\nu > \]  

\[ < L0|a0; I_10 > < L0|b0; I_20 > \]  

\[ < LM|a\alpha; I_1 M_1 > < LM|b\beta; I_2 M_2 >. \]  

(10)
Note that in \( A \) and \( B \), \( I_1 \) and \( I_2 \) must have the same “parity,” that is they must be both even or both odd.

In terms of the quantities \( A \) and \( B \), we can write

\[
W_1(\vec{k}, \mu) = |f(k)|^2 \sum_{(1)(2)} < IM | I_1 M_1; I_2 M_2 > < I'M' | I_1'M_1'; I_2'M_2' >
\]

\[
\left[ A_{\mu\mu}^{I_1 M_1; I_1'M_1'} \delta_{I_2 M_2} \delta_{I_2'M_2'} + (1 \leftrightarrow 2) \right]
\]

(11)

and

\[
W_2(\vec{p}, \mu; \vec{q}, \nu) = |f(p)|^2 |f(q)|^2 \sum_{(1)(2)} < IM | I_1 M_1; I_2 M_2 > < I'M' | I_1'M_1'; I_2'M_2' >
\]

\[
\left[ B_{\mu\nu}^{I_1 M_1; I_1'M_1'} \delta_{I_2 M_2} \delta_{I_2'M_2'} + (1 \leftrightarrow 2) \right]
\]

(12)

We now explore the consequences of these formulae for pion correlations. To simplify matters and to conform to our physical prejudices, we limit all isospins to be less than or equal to 1. That is we assume there are no isotensor or higher waves. We are interested in the two pion correlations, \( W_{22} \). For diagonal elements \((I = I')\), we can write

\[
W_2 = C + D \cos[\Delta E(t_1 - t_2) - \vec{Q} \cdot (\vec{x}_1 - \vec{x}_2)]
\]

(13)

For Hanbury-Brown Twiss correlations, it is the ratio \( D/C \) that counts. In Table 1 we show the values of \( C \) and \( D \) for various choices of \( I, M, I_1, \) and \( I_2 \) for different charges of the two pions. Since both \( C \) and \( D \) have the property \((-\mu, -\nu) = (\nu, \mu) = (\mu, \nu)\), we show only one case for each \( \mu\nu \). We see that in many cases, \( D = 0 \) for unlike charges while it is not 0 for like charges, leading to H-BT correlations between like pairs. In those cases in which \( D \) is not zero, the ratio \( D/C \) is considerably larger for the like charge case than for the unlike (there are even some unlike cases where the ratio is negative)

\footnote{The one-pion spectrum (11) can also be calculated. In the case of \( I = I' = 0 \), we have \( W_1(+) = W_1(-) = W_1(0) = 2/3 \left| f(k) \right|^2 \).}

\footnote{Actually, it is \( W_2/|f(p)|^2 |f(q)|^2 \) which is shown in (13). However, the overall factor cancels when ratios are taken, see (17).}
again leading to important H-BT correlations. Thus we see that, as asserted in [1], the effect of isospin projections is to favor H-BT correlations among like pion pairs.

For annihilation from the $\bar{p}p$ system, the initial state is not a pure isospin state but rather a superposition of the $I = 0, M = 0$ and the $I = 1, M = 0$ states. In the absence of initial state interactions this state is $|\bar{p}p\rangle = 1/\sqrt{2}(|I = 1, M = 0\rangle - |I = 0, M = 0\rangle)$. The two-pion correlation function from this $\bar{p}p$ initial state under the 2-source assumption is then, schematically

$$W_2(\bar{p}p) = \frac{1}{2}(\langle 1|W|1\rangle + \langle 0|W|0\rangle - \langle 1|W|0\rangle - \langle 0|W|1\rangle) \quad (14)$$

This involves off-diagonal terms in $I$ and $I'$. Their contribution to $W_2$ may be written

$$W_2 = C + iD \sin[\Delta E(t_1 - t_2) - \vec{Q} \cdot (\vec{x}_1 - \vec{x}_2)] \quad (15)$$

The relevant $C$ and $D$ are shown in Table 2. Note that $C = 0$ and the coefficients $D$ have the symmetry:

$$D(\mu, \nu) = -D(\nu, \mu) = -D(-\mu, -\nu) \quad (16)$$

One observes that the symmetry properties of the off-diagonal elements are such that they cancel each other in (14). However, they will not cancel if the initial state distortions make the relative phase between the isospin 0 and 1 amplitudes complex.

To get a sense of the size of the H-BT correlations let us write the ratio of like to unlike charge pions two particle pion spectra as

$$W_2(\rho, +; q, +) = 1 + H \cos[\Delta E(t_1 - t_2) - \vec{Q} \cdot (\vec{x}_1 - \vec{x}_2)] \quad (17)$$

where the size of $H$ controls the size of the H-BT correlations. We can calculate $H$ for some typical situations from Table 1. (For those cases in which $D$ is not zero for the unlike charges, it is always small compared with $C$ and we expand up from the denominator to obtain $H$.) For $I = I'$ the values of $H$ are given in Table 3. We see that for all cases, $H$ is of reasonable size and for some cases it is quite large. Thus the isospin projections lead to important H-BT correlations among the pions.
In summary, we have seen that for two sources the effects of isospin projections when combined with the other averaging mechanisms we discussed in [1] give strong H-BT enhancements among like charge pions from annihilation compared with unlike charge. For one of the cases we discuss here ($I = 0$ with $I_1 = I_2 = 0$) the argument is the same as the isospin averaging of [1]. If one sums over all isospins and uses completeness, the result for $W_2$ again reduces to the isospin averaging of [1]. That it holds for individual isospins is a happy but not obvious fact.

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Table 1: Diagonal matrix element for two pion spectrum.

| $I$ | $M$ | $I_1$ | $I_2$ | $\mu$ | $\nu$ | $C$  | $D$  |
|-----|-----|-------|-------|-------|-------|------|------|
| 0   | 0   | 0     | 0     | +     | +     | 22/45| 2/9  |
|     |     |       |       | +     | -     | 22/45| 0    |
|     |     |       |       | +     | 0     | 16/45| 0    |
|     |     |       |       | 0     | 0     | 28/45| 2/9  |
| 1   | 0   | 1     | 0     | +     | +     | 34/105| 2/15 |
|     |     |       |       | +     | -     | 34/105| 0    |
|     |     |       |       | +     | 0     | 44/105| 0    |
|     |     |       |       | 0     | 0     | 36/35| 2/5  |
| 1   | 1   | 1     | 0     | +     | +     | 4/7 | 4/15 |
|     |     |       |       | +     | -     | 4/7 | 0    |
|     |     |       |       | +     | 0     | 34/105| 0    |
|     |     |       |       | 0     | 0     | 44/105| 2/15 |
| 0   | 0   | 1     | 1     | +     | +     | 38/75| 6/25 |
|     |     |       |       | +     | -     | 38/75| 8/75 |
|     |     |       |       | +     | 0     | 8/25| 2/75 |
|     |     |       |       | 0     | 0     | 52/75| 22/75 |
| 1   | 0   | 1     | 1     | +     | +     | 116/175| 8/25 |
|     |     |       |       | +     | -     | 116/175| -4/25 |
|     |     |       |       | +     | 0     | 48/175| 0    |
|     |     |       |       | 0     | 0     | 44/175| 2/25 |
| 1   | 1   | 1     | 1     | +     | +     | 68/175| 4/25 |
|     |     |       |       | +     | -     | 68/175| 0    |
|     |     |       |       | +     | 0     | 74/175| -1/25 |
|     |     |       |       | 0     | 0     | 132/175| 6/25 |
Table 2: Off-diagonal matrix elements of two pion spectrum.

| $I$ | $M$ | $I'$ | $M'$ | $I_1$ | $I_2$ | $\mu$ | $\nu$ | $C$    | $D$         |
|-----|-----|------|------|-------|-------|--------|--------|--------|-------------|
| 1   | 0   | 0    | 0    | 1     | 1     | +      | -      | 0      | $4\sqrt{6/75}$ |
|     |     |      |      |       |       |        |        | + 0    | $\sqrt{6/75}$  |

Table 3: Coefficient $H$ of the Bose-Einstein term calculated from Table 1.

| $I$ | $M$ | $I_1$ | $I_2$ | $H$   |
|-----|-----|-------|-------|-------|
| 0   | 0   | 0     | 0     | $5/11$ |
| 1   | 0   | 1     | 0     | $7/17$ |
| 1   | 1   | 1     | 0     | $7/15$ |
| 0   | 0   | 1     | 1     | $5/19$ |
| 1   | 0   | 1     | 1     | $21/29$ |
| 1   | 1   | 1     | 1     | $7/17$ |