Possible direct measurement of the expansion rate of the universe

Shi Q† and Tan Lu‡

Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
Joint Center for Particle, Nuclear Physics and Cosmology,
Nanjing University – Purple Mountain Observatory, Nanjing 210093, China and
Kavli Institute for Theoretical Physics China, CAS, Beijing 100190, China
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A new method is proposed for directly measuring the expansion rate of the universe through very precise measurement of the flux of extremely stable sources. The method is based on the definition of the luminosity distance and its change along the time due to cosmic expansion. It is argued that galaxies may be chosen as the targets of the observation to perform such measurements. Once the required precision of the flux is achieved, profiting from the abundance of galaxies in the universe, the method could be quite promising.

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I. INTRODUCTION

Measuring the expansion rate of the universe along the redshift has been one of the most important scientific objectives in cosmology since the discovery of the expansion. It is usually pursued by measuring the distances at different redshifts, which, in turn, could be done with the data of the standard candles like the type Ia supernovae (SNe Ia) [1], Gamma-ray bursts (GRBs) [2, 3] etc or the standard rulers from cosmic microwave background (CMB) [4] and baryon acoustic oscillations (BAO) [5]. In fact, it is the measurement of the luminosity distances of SNe Ia [6, 7] that leads to the discovery of the unexpected accelerating universe, which is attributed to the mysterious dark energy. The further study of the nature of the dark energy requires more precise expansion history of the universe. The ongoing joint dark energy mission (JDEM) [8] has aimed at such a precise measurement. Currently, the expansion rate of the universe is still mainly measured through the distance measurement. The distances depend on the expansion rate through an integration, so the extraction of the expansion rate from the distances involves differentials, which significantly affects the precision of the measurement of the expansion, not to mention that the derivation of the dark energy equation of state (EOS) from the expansion rate involves differentials again.

On the other hand, despite of the difficulties, some proposals have been presented for directly measuring the expansion rate of the universe, for example, through the measurement of radial BAO [9], the relative ages of passively evolving galaxies [10], the temperature and polarization anisotropies of the CMB [11], or the redshift drift [12, 13] (the so-called Sandage-Loeb test). In this paper, we propose another method to directly measure the expansion rate by measuring the changes of the flux of extremely stable sources.

II. METHODOLOGY AND DISCUSSION

Consider a source rest at comoving distance r, with a redshift of z, its luminosity distance to us is given by

\[ d_L = a(t_0)r(1 + z), \]

where \( a \) is the scale factor as a function of time and \( t_0 \) denotes the time of today. We assume the signal we observed at the time of \( t_0 \) is emitted by the source at \( t_{em} \). Due to the expansion of the universe, if we observe the source again after a time interval of \( \Delta t_0 \), i.e. at the time of \( t_0 + \Delta t_0 \) (the corresponding signal is emitted by the source at the time of \( t_{em} + \Delta t_{em} \) with \( \Delta t_{em} = \Delta t_0/(1 + z) \)), we will find its luminosity distance changed with a value (we only take into account the first order terms in this paper) of

\[ \Delta d_L = \Delta a(t_0)r(1 + z) + a(t_0)r\Delta z \]

\[ = d_L \frac{\Delta a(t_0)}{a(t_0)} + d_L \frac{\Delta z}{1 + z}, \]

(2)

where \( \Delta a(t_0) \) and \( \Delta z \) are the changes of \( a(t_0) \) and \( z \) in the time interval \( \Delta t_0 \) due to the expansion of the universe. We can rewrite Eq. (2) into

\[ \frac{\Delta d_L}{d_L} = \frac{\Delta a(t_0)}{a(t_0)} + \frac{\Delta z}{1 + z}. \]

(3)

For the expansion of the universe, we have

\[ \frac{\Delta a(t_0)}{a(t_0)} = \frac{\dot{a}(t_0)}{a(t_0)} \Delta t_0 = H_0 \Delta t_0 \]

(4)

and since \( 1 + z = a(t_0)/a(t_{em}) \),

\[ \Delta z = \frac{\dot{a}(t_0)}{a(t_{em})} \Delta t_0 - \frac{a(t_0)}{a(t_{em})} \frac{\dot{a}(t_{em})}{a(t_{em})} \Delta t_{em} \]

\[ = \frac{a(t_0)}{a(t_{em})} \frac{\dot{a}(t_0)}{a(t_0)} \Delta t_0 - (1 + z)H(z) \frac{\Delta t_0}{1 + z} \]

\[ = [(1 + z)H_0 - H(z)] \Delta t_0, \]

(5)
where $H_0$ and $H(z)$ are the Hubble parameter of today and that at the redshift of $z$ correspondingly. Eq. (5) is in fact the core of the Sandage-Loeb test $^{12,13}$, i.e., if we manage to measure the redshift drift $\Delta z$, we obtain the Hubble parameter. Substitute Eq. (1) and Eq. (5) into Eq. (3), we have

$$\frac{\Delta d_L}{d_L} = \left[2H_0 - \frac{H(z)}{1 + z}\right]\Delta t_0.$$  

(6)

With this equation at hand, one may naturally think that, similarly as the Sandage-Loeb test, if the luminosity distances can be measured to a very high precision such that we could distinguish the small changes in the luminosity distances for a reasonable time duration, we could also immediately derive the corresponding Hubble parameters. Unfortunately, the distance measurement itself is a difficult task in astronomy, especially for cosmic distances. One of the most precise ways of measuring cosmic distances is through the observation of gravitational waves, for which a relative precision of about $10^{-3}$ is expected for the luminosity distance $^{[14]}$. From Figure 1 we can see that, even for such precise measurements, about $10^8$ years will be needed before we could tell the changes in the luminosity distances, which is obviously not feasible.

![Figure 1](https://via.placeholder.com/150)

**FIG. 1.** $\Delta d_L/d_L$ per year versus redshift $z$ for the flat $\Lambda$CDM cosmological model with $\Omega_{m,0} = 0.27$ and $H_0 = 70.5$ km s$^{-1}$ Mpc$^{-1}$.

Recall that the luminosity distance is defined through

$$F = \frac{L}{4\pi d_L^2},$$  

(7)

where $L$ is the luminosity of the source and $F$ is the observed flux, we have

$$\frac{\Delta F}{F} = \frac{\Delta L}{L} - 2\frac{\Delta d_L}{d_L},$$  

(8)

where $\Delta F$ and $\Delta L$ are the changes in $F$ and $L$ during the time interval $\Delta t_0$. So, if we point our telescope to some extremely stable source such that $\Delta L/L$ can be ignored in Eq. (8) and manage to measure the flux $F$ to a very high precision, we could measure $\Delta d_L/d_L$ indirectly through $\Delta F/F$. Thus, Eq. (6) could still be used to measure the expansion rate of the universe. Then the problem becomes whether there exist such extremely stable sources and whether we could manage to measure the flux $F$ to the precision needed.

For the requirement of extremely stable sources, we may consider objects which include lots of similar sources, so that we could statistically reduce $\Delta L/L$ to a very low level. For example, a galaxy includes lots of stars. During the time interval $\Delta t_0$, the luminosity of a star in the galaxy may increase or decrease. Let $L_i$ be the luminosity of the $i$th star in the galaxy, we may view $\Delta L_i/L_i$ as a random variable and, for a simple estimation, assume it follow the normal distribution $\mathcal{N}(0, \sigma^2)$, then, for the galaxy,

$$\frac{\Delta L}{L} = \sum_i \frac{\Delta L_i}{L_i} \sim \mathcal{N}(0, \frac{\sum_i L_i^2}{(\sum_i L_i)^2}\sigma^2)$$  

(9)

and since $\sum_i L_i^2/(\sum_i L_i)^2$ has the order of $1/N$, where $N$ is the total number of stars in the galaxy, the standard deviation of $\Delta L/L$ for the galaxy is thus reduced by a factor of about $1/\sqrt{N}$ compared to that for a star. While considering the change in the luminosity, we first exclude galaxies involving violent astrophysical processes like stellar explosions, which usually could be easily identified. If we simply model the luminosity evolution of a steadily burning star to be of that it linearly increases to its maximum value and then linearly decreases, then $\Delta L/L$ for the star should have the order of the inverse of its lifetime. The lifetime of a star ranges from only a few million years (for the most massive) to trillions of years (for the least massive). Here, we conservatively set the $\Delta L/L$ for stars to be of the order of $10^{-6}$ per year which corresponds to stars with smallest lifetime. Typical galaxies consist of from $10^7$ to $10^{12}$ stars. If we set $N = 10^{10}$, the standard deviation of $\Delta L/L$ for the galaxy would be of the order of $10^{-11}$ per year. So the $\Delta L/L$ can hopefully be reduced to below the expected values of $\Delta d_L/d_L$. In practice, some selection criteria on the galaxies may be needed. The details on the selection are out of the scope of this short letter. But it is worth mentioning that one may concern the impact from the evolution of the observed galaxy beyond its stability. On this issue, first we should, of course, select passively evolving galaxies as our targets of observation basing our knowledge of galaxies themselves. Second, the $\Delta F/F$ caused by the cosmic expansion has a flat-line spectrum, while those caused by other astrophysical processes usually do not has such a character. With this, we could further exclude those galaxies whose evolution (including effects from star-formation, dust, etc.) dominates over the cosmic expansion on $\Delta F/F$.

For the measurement of the flux, cryogenic detectors usually can achieve very high precisions. See, for example $^{15,17}$, for cryogenic detectors. We can also improve
the precision by time integration, and/or fitting the flux observations along the time to a linear relation. Cross-correlation between observations of different bands may also help. The requirement on sensitivity of the instrument for the observation, when taking all the considerations into account, need further investigations.

Anyway, if we do manage to measure the flux to a very high precision, profiting from the abundance of galaxies in the universe, this method may be quite promising for measuring the expansion rate of the universe. Observation of galaxies at the same redshift may be averaged to increase the accuracy of the measurement of the expansion rate, or hopefully, if the accuracy achieved with single galaxies is good enough, we could even map the three-dimensional cosmic expansion, i.e. the cosmic expansion of the different directions in the sky along the redshift. Also note that $\Delta d_L/d_L$ has the same order from the redshift of zero to redshifts $z > 4$ (see Figure 1), where the dark energy plays its role in the cosmic expansion, so the method may track the whole dynamics of the dark energy.

Compared to the Sandage-Loeb test, though the derivation of our method is very similar to that of Sandage-Loeb test, the requirement on the instrument, the observational target, and the expected outcome are quite different. The Sandage-Loeb test requires precise measurement of the redshift, while our method requires precise measurement of the flux. The targets for the Sandage-Loeb test are quasars, while in our method, the targets are galaxies, which are much more abundant in our universe. Provided the measurement from single galaxies could achieve an accuracy good enough, our method would give more detailed information about the cosmic expansion across the sky. The redshift coverages of the two methods are also quite different. As shown in [13], the redshift coverage of the Sandage-Loeb test is roughly between 2 and 5, while our method could potentially cover the range from a redshift close to zero (see discussions in the next section) to redshifts $z > 4$. All these make our method a novel one from the Sandage-Loeb test despite the similar derivation.

In the above analyses, we have only considered the ideal condition, for example, we have assumed the source rest in the comoving reference frame and the photons propagate freely from source to the observer. This is of course not the case of that in the real universe. For the very high precision required by the method, impacts from the peculiar velocity of the source and from the gravitational lensing should be investigated seriously. In the next section, we showed that the impacts from the peculiar velocity can be safely ignored in a wide redshift range. For the gravitational lensing, since we have so many galaxies in our universe, we can expect that its impacts could, at least, be eliminated by an average over the galaxies at the same redshift. We leave it to the future studies whether the gravitational lensing will affect the measurement of single galaxies in our method.

### III. PECULIAR VELOCITY

In this section, we studied the impacts from the peculiar velocity of the source on our method.

First, consider a isotropic point source and an observer nearby rest in a local inertial reference frame. Assume the source have a velocity of $\mathbf{v}$ with respect to the observer. Imagine a spherical coordinate with the source as the origin and the direction of $\mathbf{v}$ as the zenith direction. Say, the observer have an inclination angle $\alpha$ in the coordinate. The momenta of the photons received by the observer would have the same inclination angle $\alpha$.

But, to the source, as a result of the aberration of light caused by the motion of the source, the momenta have a different inclination angle $\bar{\alpha}$, which relates $\alpha$ through

$$\tan \frac{\alpha}{2} = \sqrt{\frac{1 - \beta}{1 + \beta}} \tan \frac{\bar{\alpha}}{2}, \quad (10)$$

where $\beta = v/c$. So, the corresponding solid angles around above mentioned momenta have different magnitudes to the observer and to the source. The ratio between them is

$$\frac{d\Omega}{d\bar{\Omega}} = \frac{\sin \alpha d\alpha}{\sin \bar{\alpha} d\bar{\alpha}} = J(\alpha, \beta). \quad (11)$$

In addition to this, there is also the Doppler effect

$$1 + z_d = (1 - \beta \cos \alpha) \gamma, \quad (12)$$

where $z_d$ is the Doppler redshift and $\gamma = 1/\sqrt{1 - \beta^2}$. Taking into account both the aberration of light and the Doppler effect, compared to the case of a rest source, the flux observed by the observer is increased by a factor of

$$\frac{1}{(1 + z_d)^2} \frac{1}{J(\alpha, \beta)} = \frac{1}{[(1 - \beta \cos \alpha)\gamma]^2 J(\alpha, \beta)}. \quad (13)$$

Next, consider the cosmological case of a isotropic point source at comoving distance $r$, with a luminosity of $L$, a peculiar velocity of $v$, and a cosmological redshift of $z_c$, and the direction of $\mathbf{v}$ is at an angle of $\theta$ relative to the line of sight (from the observer to the source). The photons emitted by the source first experience an aberration and the Doppler effect due to the velocity of the source, then the cosmological redshift before they reach us. As a result, the observed flux becomes

$$F = \frac{1}{4\pi a_0^2 r^2 (1 + z_c)^2 (1 + z_d)^2 J(\pi - \theta, \beta)} \frac{1}{1} \frac{1}{J(\pi - \theta, \beta)} L, \quad (14)$$

where $a_0 = a(t_0)$. So, when taking into account the peculiar velocity of the source, the corrected luminosity distance is given by

$$d_{L,p} = a_0 r (1 + z_c) (1 + z_d) \sqrt{J(\pi - \theta, \beta)} = a_0 r (1 + z_c) (1 + \beta \cos \theta) \gamma \sqrt{J(\pi - \theta, \beta)}. \quad (15)$$
It is easy to check that, under the condition of $\beta \ll 1$, Eq. (10) and Eq. (11) reduce to
\[
\dot{\alpha} = \alpha + \beta \sin \alpha, \\
J(\alpha, \beta) = \frac{1}{(1 + \beta \cos \alpha)^2},
\]
and Eq. (15) reduces to
\[
d_{L_p} = a_0 r (1 + z_c) (1 + z_d) / (1 - \beta \cos \theta) \\
= a_0 r (1 + z_c) (1 + \beta \cos \theta) \gamma / (1 - \beta \cos \theta).
\]
Further ignore the change in the velocity of the source during the observation, we have
\[
\frac{\Delta d_{L_p}}{d_{L_p}} = \frac{\Delta a_0}{a_0} + \frac{\Delta r}{r} + \frac{\Delta z_c}{1 + z_c}.
\]
Comparing the right hand side of this equation with that of Eq. (3), we can see that the first term is unchanged. The second term is newly introduced corresponding to the distance the source moved through during the observation. While the last term, at first glance, has the same form as the last term of Eq. (3), it actually includes two parts, which may be called the time part and the space part. The time part arises from that the redshift of a rest source changes along the time due to the cosmic expansion, as was shown in Eq. (5). The space part is in fact that the source has moved to a different location during the observation due to its peculiar velocity and sources at different comoving distances have different redshifts. It is given by
\[
\left[ \frac{\Delta z_c}{1 + z_c} \right]_p = \frac{H(z_c)}{c (1 + z_c)} \frac{a_0 \Delta r}{\sqrt{1 - kr^2}}
\]
where the subscript $p$ denotes the redshift drift caused by the peculiar velocity, i.e., the space part. So, the impact of the peculiar velocity on the change of the luminosity distance is
\[
\frac{\Delta d_{L_p}}{d_{L_p}} - \frac{\Delta d_L}{d_L} = \left[ 1 + \frac{H(z_c)}{c(1 + z_c^2)} \frac{d_L}{\sqrt{1 - kr^2}} \right] \frac{\Delta r}{r},
\]
where
\[
\frac{\Delta r}{r} = \frac{(1 + z_c)^2 \sqrt{1 - kr^2} a(t_{em}) \Delta r}{d_L} \frac{d_L}{\sqrt{1 - kr^2}} \\
= \frac{(1 + z_c)^2 \sqrt{1 - kr^2} v \cos \theta \Delta t_{em}}{d_L} \\
= \frac{(1 + z_c) \sqrt{1 - kr^2} v \cos \theta \Delta t_0}
\]
and
\[
\sqrt{1 - kr^2} = \sqrt{1 + \Omega_{k,0} \frac{H_0^2}{c^2} \frac{d_L^2}{(1 + z_c)^2}}.
\]
We plot $\left( \frac{\Delta d_{L_p}}{d_{L_p}} - \frac{\Delta d_L}{d_L} \right) / \frac{\Delta d_L}{d_L}$ along the redshift in Figure 2 from which we can see that, from very low redshift to very high redshift, the impact of the peculiar velocity is very small and can be safely ignored.

![Figure 2](image-url)

**FIG. 2.** $\left( \frac{\Delta d_{L_p}}{d_{L_p}} - \frac{\Delta d_L}{d_L} \right) / \frac{\Delta d_L}{d_L}$ versus redshift $z$ for a peculiar velocity of $v = 1000$ km s$^{-1}$ and the flat $\Lambda$CDM cosmological model with $\Omega_{m,0} = 0.27$ and $H_0 = 70.5$ km s$^{-1}$ Mpc$^{-1}$.

**IV. SUMMARY**

In summary, basing on the definition of the luminosity distance and its change along the time due to cosmic expansion, we discussed the possibility of directly measuring the expansion rate of the universe through very precise measurement of the flux. Extremely stable sources are needed for the method. We argued galaxies may be chosen as the targets of the observation. Since composed of many stars, their luminosity is very stable. Those involving violent astrophysical processes during the observation could be easily identified and excluded. Furthermore, the relative flux change caused by the cosmic expansion has a flat-line spectrum, while those caused by other astrophysical processes usually do not has such a character. With this, we could further exclude those galaxies whose evolution dominates over the cosmic expansion on the flux change. We also showed that the peculiar velocity can be safely ignored for a wide redshift range in our method. Once the required precision of the flux is achieved, profiting from the abundance of galaxies in the universe, the method could be quite promising.

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