Analytical Method for Local Damage of Beams under Moving Vehicle Loading

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Abstract: In order to accurately identify damage of bridge under natural excitation, an analytical solution of local stiffness of simply supported beam structure is established by using structural mechanics method through establishing the moving load model of simply supported beam. On this basis, an accurate identification method of local damage of beam structure under vehicle load is proposed. Firstly, the local bending stiffness is described in terms of equal length intervals. Secondly, the corresponding loading methods are established for single concentrated load and two-axle vehicle respectively, and the local bending stiffness values of each interval are solved by analytic equation established by node displacement. Damage information can be identified by comparing the flexural stiffness values of simply supported beams before and after damage. Based on the discreteness of local bending stiffness of simply supported beams and the change of local bending stiffness caused by damage, the location and degree of local damage of beams can be accurately identified.

1. Introduction
Simply supported beams are widely used in practical engineering. It is of great engineering significance to discover the damage information of all kinds of simply supported beams in time and to ensure the safety of operation. Most of the existing damage identification methods assume that the structure has a uniform bending stiffness [1-4] under intact condition, and then the local stiffness changes represent the damage. However, for the actual simply supported beam structure, the initial stiffness of the beam is often discrete due to material characteristics, geometric size errors and other factors, which makes it difficult to directly adopt these methods. In addition, at present, many damage identification methods are based on structural dynamic performance index [5-10]. When applied, acceleration measurement points need to be densely distributed in the structure. The process is cumbersome, the cost is high and the accuracy is not easy to guarantee. In order to reduce the number of test points needed and improve the convenience of damage identification, another idea can be adopted: let the load (excitation) move and test the structural response at a fixed position. Research in this area can be divided into two categories.

One is based on structural dynamic response. For example, Hester et al. [11] analyzed the acceleration response data of bridge structures under moving loads by wavelet analysis to identify the damage; Li et al. [12] also introduced substructure method and modified Tikhonov regularization method to identify the damage based on the acceleration response data of bridge structures under moving loads; Zhang et al. [13] also measured various dynamic forces of bridge structures under moving loads. Response and Realization of single damage identification; Li et al. [14] carried out in-depth analysis of the interaction...
between mobile vehicles and bridge deck, according to the data collected by vehicles can be processed to obtain more accurate bridge frequency data, and then carry out damage identification. The second is based on structural static response (mainly displacement). For example, Khorram et al. [15] analyzed the mid-span deflection data of beam structures under moving loads by wavelet analysis to identify the location and damage degree of cracks; Yu et al. [16] introduced wavelet analysis and Exponent index to identify damage based on the deformation of bridge structures under moving loads; Zhao et al. [17] studied the rules of mid-span displacement varying with the location of moving loads for arch bridges. According to the law, a damage identification method is established, which is only suitable for single damage cases at present; Liu et al. [18] found that the static displacement influence line of simply supported beam under moving load can be treated as damage identification index, and it can identify multiple damage cases; Khorram et al. analyzed the displacement data of simply supported beam when moving load passes through the mid-span position by continuous wavelet analysis. A method for identifying multiple damages is established.

Generally speaking, damage identification of simply supported beams based on moving loads is completely feasible. However, the existing methods do not take into account the initial discreteness of the local stiffness of the beam sufficiently, and the specific mode and system of moving loading need to be further optimized in order to achieve more convenient and effective damage identification. For this reason, based on the idea of moving loading, this paper will design a reasonable loading mode and system for the simply supported beam structure in the linear elastic stage, and obtain the static structural response of the fixed measuring point. The purpose of this paper is to obtain the analytical solution of the local bending stiffness of the beam from the local bending stiffness change caused by damage. To achieve effective damage identification, analytical method for local damage of beams under moving vehicle loading was researched in this paper.

2. Identification of Local Flexural Stiffness of Beam
To consider the discreteness of local bending stiffness, it is necessary to identify the real local stiffness. For this reason, the concept of "local" is quantified first, and the beam body can be divided into several equal-length intervals. The local bending stiffness can be examined in terms of intervals. If there are n intervals, the stiffness of intervals can be recorded as S1, S2, ..., Sn requires n independent equations to solve. For this reason, the moving load can be used for N times of loading, and the fixed structure response under each loading can be used to establish one equation, and then n equations can be obtained. Here, the mid-span deflection is selected as the fixed measurement object.

First, the simplest case of moving load - the movement of a single concentrated load was considered. The following is an example of a six-zone situation to illustrate the loading conditions and the data processing methods. The local bending stiffness of these six sections is recorded as S1~S6, respectively, as shown in Fig. 1.

Let the concentrated load P move on the beam and investigate the mid-span deflection response when it reaches each node:

$$\Delta_1 = F \left[ \frac{5l^3}{1776} S_1 + \frac{19l^3}{1944} S_2 + \frac{13l^3}{3888} S_3 + \frac{19l^3}{7776} S_4 + \frac{7l^3}{7776} S_5 + \frac{l^3}{7776} S_6 \right] \quad (1)$$

(2) When P reaches the second node, the corresponding moment diagram is shown in Fig. 2 (b). The mid-span deflection is denoted as 2 and the formula is (2):
\[ \Delta_i = F \left[ \frac{I^1}{1944 S_i} + \frac{7I^1}{1944 S_7} + \frac{37I^1}{1944 S_3} + \frac{19I^3}{1944 S_1} + \frac{7I^3}{1944 S_7} + \frac{I^3}{1944 S_3} \right] \] (2)

(3) When P reaches the third node, the corresponding moment diagram is shown in Fig. 2 (c). The mid-span deflection is denoted as 3 and the calculation formula is (3):
\[ \Delta_i = F \left[ \frac{I^1}{2592 S_i} + \frac{7I^1}{2592 S_7} + \frac{19I^1}{2592 S_3} + \frac{19I^3}{2592 S_1} + \frac{7I^3}{2592 S_7} + \frac{I^3}{2592 S_3} \right] \] (3)

(4) When P reaches the fourth node, the corresponding moment diagram is shown in Fig. 2 (d). The mid-span deflection is denoted as 4 and the calculation formula is (4):
\[ \Delta_i = F \left[ \frac{I^1}{3888 S_i} + \frac{7I^1}{3888 S_7} + \frac{19I^1}{3888 S_3} + \frac{37I^1}{3888 S_1} + \frac{7I^3}{3888 S_7} + \frac{I^3}{1944 S_3} \right] + \frac{5I^3}{1944 S_7} \] (4)

(5) When P reaches the fifth node, the corresponding moment diagram is shown in Fig. 2 (e). The mid-span deflection is denoted as 5 and the formula is (5):
\[ \Delta_i = F \left[ \frac{I^1}{7776 S_i} + \frac{7I^1}{7776 S_7} + \frac{19I^1}{7776 S_3} + \frac{13I^1}{7776 S_1} + \frac{5I^3}{7776 S_7} + \frac{5I^3}{7776 S_3} \right] \] (5)

In this way, five equations are established and one additional equation is needed. For this purpose, one additional load condition is added. The idea is to make two concentrated loads act on two joints simultaneously. These two joints can be selected from the five joints. The following examples are given:

(6) Two P acts on the second and fourth nodes simultaneously: the corresponding moment diagram is shown in Fig. 2 (P). At this time, the midspan deflection is denoted as 6, and the calculation formula is (6):
\[ \Delta_i = F \left[ \frac{I^1}{1296 S_i} + \frac{7I^1}{1296 S_7} + \frac{19I^1}{1296 S_3} + \frac{5I^3}{1296 S_1} + \frac{5I^3}{1296 S_7} + \frac{5I^3}{1296 S_3} \right] \] (6)

A system of linear equations can be obtained by combining formula (1)–formula (6):
\[ \begin{align*}
\frac{I^1}{7776 S_i} + \frac{5I^1}{1944 S_7} + \frac{3I^1}{3888 S_3} + \frac{19I^1}{3888 S_1} + \frac{7I^1}{3888 S_7} + \frac{I^1}{3888 S_3} &= \Delta_i \\
\frac{I^1}{1944 S_i} + \frac{7I^1}{1944 S_7} + \frac{37I^1}{1944 S_3} + \frac{19I^3}{1944 S_1} + \frac{7I^3}{1944 S_7} + \frac{I^3}{1944 S_3} &= \Delta_i \\
\frac{I^1}{2592 S_i} + \frac{7I^1}{2592 S_7} + \frac{19I^1}{2592 S_3} + \frac{19I^3}{2592 S_1} + \frac{7I^3}{2592 S_7} + \frac{I^3}{2592 S_3} &= \Delta_i \\
\frac{I^1}{3888 S_i} + \frac{7I^1}{3888 S_7} + \frac{19I^1}{3888 S_3} + \frac{37I^1}{3888 S_1} + \frac{7I^3}{3888 S_7} + \frac{I^3}{1944 S_3} + \frac{5I^3}{1944 S_7} &= \Delta_i \\
\frac{I^1}{7776 S_i} + \frac{7I^1}{7776 S_7} + \frac{19I^1}{7776 S_3} + \frac{13I^1}{7776 S_1} + \frac{5I^3}{7776 S_7} + \frac{5I^3}{7776 S_3} &= \Delta_i \\
\frac{I^1}{1296 S_i} + \frac{7I^1}{1296 S_7} + \frac{19I^1}{1296 S_3} + \frac{5I^3}{1296 S_1} + \frac{5I^3}{1296 S_7} + \frac{5I^3}{1296 S_3} &= \Delta_i \\
\end{align*} \] (7)

According to the measured values of 1, 2, 3, 4, 5 and 6, S1, S2, S3, S4, S5 and S6 can be solved by formula (7). The case of any n intervals can be treated similarly: n-1 equation can be obtained according to the mid-span deflection value when P moves to n-1 node, and then n equations can be obtained by adding a case where two concentrated loads act simultaneously, and the local bending stiffness parameters S1–Sn can be obtained.

3. Method of identifying damage
When the simply supported beam is in service or considered intact, the initial local bending stiffness data can be obtained according to the above-mentioned operation. After a period of service, the above-mentioned loading and data processing process can be carried out again, and then the local bending stiffness data of the simply supported beam can be obtained. Comparing these two sets of data, if there is a change, the damage on the beam can be considered.

At the same time, the location (in terms of intervals) and extent (in terms of equivalent stiffness changes in intervals) of the damage can be obtained according to the variation of the data.
4. Description
(1) The size of the partition on the beam determines the precision of local bending stiffness identification and subsequent damage identification. The more the intervals are divided, the more detailed the damage location is, but the more loading and data processing are needed. Therefore, the size of the intervals should be determined according to the specific conditions of the simply supported beams studied and the available manpower and material resources.
(2) The moving loads should not be too large to avoid artificial damage to the beam.
(3) In the calculation of deflection, only bending deformation is considered, but shear deformation is neglected. First of all, simple supported beams do not belong to deep beams, and shear deformation is very small. In addition, the formula for calculating shear deformation is not very accurate. In order to avoid introducing unnecessary errors, only the formula for calculating bending deformation is used for theoretical derivation. This is equivalent to covering shear deformation with the calculation theory of bending deformation. Because the purpose of the study is damage identification, data comparison between the two states before and after damage is needed, it is reasonable to use such a theoretical model to deal with the two states before and after damage.

5. Conclusion
(1) Considering the discreteness of initial local bending stiffness, a damage identification method based on moving loading (including three different loading modes) is proposed for simply supported beams. The structural excitation is carried out by means of fixed measuring points and moving loads to establish the solution equation. The analytical solution of local bending stiffness of each section can be obtained, and then the location and degree of real damage can be determined.
(2) Based on the local bending stiffness data identified in the method, an accurate finite element model of the beam can be established to improve the accuracy of subsequent safety assessment and state prediction.

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