Dark Matter with Variable Masses *

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Abstract

String effective theories contain a dilaton scalar field which couples to gravity, matter and radiation. In general, particle masses will have different dilaton couplings. We can always choose a conformal frame in which baryons have constant masses while (non–baryonic) dark matter have variable masses, in the context of a scalar–tensor gravity theory. We are interested in the phenomenology of this scenario. Dark matter with variable masses could have a measurable effect on the dynamical motion of the halo of spiral galaxies, which may affect cold dark matter models of galaxy formation. As a consequence of variable masses, the energy–momentum tensor is not conserved; there is a dissipative effect, due to the dilaton coupling, associated with a “dark entropy” production. In particular, if axions had variable masses they could be diluted away, thus opening the “axion window”. Assuming that dark matter with variable masses dominates the cosmological evolution during the matter era, it will affect the primordial nucleosynthesis predictions on the abundances of light elements. Furthermore, the dilaton also couples to radiation in the form of a variable gauge coupling. Experimental bounds will constrain the parameters of this model.

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1 Introduction

Most particle physicists believe that the theory of gravity at low energies (general relativity, scalar-tensor theories, etc.) is an effective approximation of some fundamental theory of quantum gravity at energies beyond the Planck scale ($M_{Pl} \sim 10^{19}$ GeV). Nowadays, the only reliable candidates for such a fundamental theory are superstrings [1]. String theory assumes that elementary particles are extended one–dimensional objects, as an alternative to the point–like description of quantum field theory. This simple assumption has very interesting consequences. In particular, in the low energy limit of strings moving in curved backgrounds we recover a generally covariant field theory. However, the string effective gravity theory is not precisely general relativity. In fact, all closed bosonic strings contain in their massless gravitational sector, apart from the graviton, a dilaton and an antisymmetric tensor. These fields will appear in the low energy effective theory from strings.

We can write the tree level heterotic string effective action in four dimensions, keeping only linear terms in the string tension $\alpha'$ and in the curvature $R$, as [1]

$$S = \frac{1}{\alpha'} \int d^4 x \sqrt{e^{-2\phi} G e^{2\phi}} \left( R + 4G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + \alpha' L_m \right)$$

$$L_m = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \sum_i \left( \partial_\mu C_i \partial^\mu C_i + m_i^2 (\phi) C_i^2 \right)$$

where $H_{\mu\nu\lambda}$ is the field strength of the antisymmetric tensor, $F_{\mu\nu}$ is the electromagnetic tensor and $C_i$ are the matter fields that appear in the compactification to four dimensions. The dilaton dependence of $m_i(\phi)$ will be assumed to be linear in the exponential

$$m_i(\phi) = m_i e^{\gamma_i \phi}$$

where $\gamma_i$ parametrizes our ignorance on the details of supersymmetry breaking in string theory and the low energy non–perturbative effects. In general, different particles will have different $\phi$–dependences [4]. We will study the phenomenological signatures of this effective action, the most characteristic property being the coupling of the dilaton to the gravitational, gauge and matter sectors. A conformal redefinition ($G_{\mu\nu} = e^{2\phi} \tilde{g}_{\mu\nu}$) allows us to rewrite
the action (1) in the Einstein frame as
\[ S = \int d^4x \sqrt{-g} \left( \bar{R} - \frac{1}{2} (\partial \phi)^2 + 16\pi e^{\beta I \phi} \mathcal{L}_{m_I} + 16\pi e^{\beta V \phi} \mathcal{L}_{m_V} \right) \] (4)

\[ \mathcal{L}_m = \frac{-1}{\sqrt{-g}} \sum_n m_n \int d\tau_n \left( -\bar{g}_{\mu\nu} (x_n) \frac{dx_n^\mu}{d\tau_n} \frac{dx_n^\nu}{d\tau_n} \right)^{1/2} \delta^{(4)}(x - x_n), \] (5)

where \( n \) labels a set of classical point particles with variable masses \( m_n(\phi) = e^{\beta \phi} m_n \) and we have assumed only two different dilaton couplings \((\beta_I, \beta_V)\) associated with baryonic (visible) and dark (invisible) matter sectors. Such a theory has been considered previously in refs.\[3, 2\] as a generalization of Jordan–Brans–Dicke theory of gravity \[4, 5\]. It has also been studied in the context of extended inflation in Ref.\[6\]. It explicitly violates the weak equivalence principle but is not ruled out by experiment. In fact, the weak equivalence principle has been tested only with ordinary (baryonic and leptonic) matter and energy \[5\], but we know that most of the matter in the universe is dark matter \[6\]. String theory gives no prediction on \((\beta_I, \beta_V)\) but suggests that this scenario may arise. We will study the physical consequences of this assumption and try to obtain as much phenomenological constraints as possible on the parameters of the model.

We know that physics cannot distinguish between conformal frames, therefore we are free to choose whatever masses are made constant. We choose constant visible masses for convenience since they give constant units of measure and also visible particles follow trajectories which are geodesics of the metric. We thus perform a conformal redefinition
\[ \bar{g}_{\mu\nu} = e^{-2\beta V \phi} g_{\mu\nu} = \Phi g_{\mu\nu} \] (6)

in order to leave constant masses for visible matter. The resulting theory has the form of a generalized Jordan–Brans–Dicke theory with variable masses for the dark matter sector \[2\]
\[ S = \int d^4x \sqrt{-\bar{g}} \left( \Phi R - \frac{\omega}{\Phi} (\partial \Phi)^2 + 16\pi \Phi^\sigma \mathcal{L}_{m_I} + 16\pi \mathcal{L}_{m_V} \right) \] (7)

\(^1\)We have redefined the scalar field \( \phi \) as twice the dilaton, for convenience, and considered only the matter fields as classical point particles, disregarding the antisymmetric field. From here on we will work in units of Planck’s mass, \( \alpha' = 16\pi \).

\(^2\)We restrict ourselves, for simplicity, to the case \( \beta_I, \beta_V > 0 \).
where the two parameters \((\omega, \sigma)\) are given by

\[
2\omega + 3 = \frac{1}{4\beta_v^2} \\
1 - 2\sigma = \frac{\beta_I}{\beta_V}.
\]  

(8)

We know that baryonic matter gives a very small contribution to the critical density of the universe [7]. We will assume that the cosmological evolution during the matter era is dominated by dark matter with variable masses. The gravitational equations of motion are then given by [8]

\[
R_{\mu\nu} = \frac{8\pi}{\Phi} \left( \frac{1}{2} g_{\mu\nu} T^\lambda_\lambda - T_{\mu\nu} \right) - \frac{\omega}{\Phi^2} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{\Phi} \left( D_\mu D_\nu \Phi + \frac{1}{2} g_{\mu\nu} D^2 \Phi \right) \\
(2\omega + 3) D^2 \Phi = 8\pi (1 - 2\sigma) T^\lambda_\lambda
\]  

(9)

with the energy–momentum conservation equation

\[
T^{\mu\nu} = \sigma \frac{\partial^\mu \Phi}{\Phi} T^\lambda_\lambda.
\]  

(11)

Note that during the radiation era the energy–momentum tensor is exactly conserved. In fact, in that era, the scalar field is constant, as we can see from eq.(10). The covariant non–conservation of the energy–momentum tensor just expresses the fact that there is an energy exchange between dark matter and the scalar, which gives a dissipative effect.

The particle trajectories corresponding to dark matter are given by

\[
\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \sigma \frac{\partial^\mu \Phi}{\Phi} = 0
\]  

(12)

and therefore do not follow geodesics, being modified by the term \(\sigma \frac{\partial^\mu \Phi}{\Phi}\). Note that this equation exactly corresponds to the geodesic of the metric \(\tilde{g}_{\mu\nu}(x) = \Phi(x)^{2\sigma} g_{\mu\nu}(x)\) since \(\tilde{\Gamma}^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda} - \frac{1}{2} \frac{\partial^\mu (\ln \Phi^{2\sigma})}{\Phi} g_{\nu\lambda}\). Of course, baryonic particles do follow geodesics of \(g_{\mu\nu}(x)\). To understand the physical significance of eq.(12), let us consider the acceleration of a non relativistic particle in a weak and static gravitational field \(g_{oo} \simeq -1 + \frac{2GM}{r}, g_{ij} \simeq \delta_{ij}, g_{oa} \simeq 0\) [9]

\[
\frac{d^2 x^i}{dt^2} + \sigma \frac{\dot{\Phi}}{\Phi} \frac{dx^i}{dt} \simeq - \frac{GM}{r^2} \frac{x^i}{r},
\]  

(13)

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which gives the Newtonian acceleration due to the gravitational attraction of a mass \( M \) plus a friction force due to the variation of mass. Eq. (13) exactly corresponds to Newton’s second law \( \frac{d}{dt}(mv^i) = F^i \), as one would expect from variable masses.

We will try to extract new phenomenological signals of this theory of gravity with matter coupled to a scalar. As we will show in the next section, the fact that dark matter with variable masses do not follow geodesics will affect the dynamical motion of the halo of spiral galaxies. At scales of super-clusters this effect could change the general picture of dark matter halos. On the other hand, the dissipative effect due to the scalar coupling could also give an important entropy production. If axions had variable masses this effect can be used to dilute their contribution to the critical energy density of the universe, therefore eluding the cosmological bounds on their mass.

2 Dark matter halo of spiral galaxies

From dynamical observations of spiral galaxies we know that there is a halo with great amounts of dark matter [7, 10]. If the dark matter in the halo were composed of particles with variable masses, it would have, in principle, a measurable effect on the dynamical motion of the halo. The measurements of the halo mass are obtained from Kepler’s third law \( v^2 = rg \) where \( g \) is the centripetal acceleration [1]. Dynamical observations show that the velocity of objects away from the disk of the galaxy remains constant for large distances, suggesting that there is dark matter with \( M(r) \propto r \) [7, 10].

The analysis of the post-Newtonian motion of the halo can be better described in standard coordinates [8]

\[
 ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2d\Omega^2 .
\] (14)

We have calculated the solutions to the equations of motion (9, 10) in the interior of the halo \( (r < R_{\text{halo}}) \) of a spiral galaxy with mass \( M(r) = M_{\text{halo}}\frac{r}{R_{\text{halo}}} \).
and negligible pressure

\[ B(r) = 1 + \frac{2GM(r)}{r} \ln \left( \frac{e^{-1} r}{R_{\text{halo}}} \right) + ... \]

\[ A(r) = 1 - \left( \frac{\omega + 1 + \sigma}{\omega + 2 - \sigma} \right) \frac{2GM(r)}{r} \ln \left( \frac{e^{-1} r}{R_{\text{halo}}} \right) + ... \]  \hspace{1cm} (15)

\[ \Phi(r) \simeq \left( \frac{e^{-1} r}{R_{\text{halo}}} \right) \frac{1 - 2\sigma}{\omega + 2 - \sigma} \frac{GM_{\text{halo}}}{R_{\text{halo}}} \],

continuously connected with the post–Newtonian solution for \( r \geq R_{\text{halo}} \).

The interior solutions (15) give a constant centrifugal velocity

\[ v^2 \simeq \frac{GM(r)}{r} = \frac{GM_{\text{halo}}}{R_{\text{halo}}} . \]  \hspace{1cm} (16)

Dark matter models of galaxy formation give generically a mass distribution \( \rho(r) = \rho_0 r^n \). If the halo were composed of particles with variable mass we would have \( \rho(r) \Phi^\sigma(r) = \frac{M_{\text{halo}}}{4\pi R_{\text{halo}}^2} \frac{1}{r^2} \), which corresponds to \( M(r) \propto r \). From the solutions in the interior of the halo (15) we obtain the relation

\[ n \simeq -2 + \frac{4\beta_I (\beta_I - \beta_V)}{1 + 4\beta_I \beta_V} \times 10^{-6} \frac{GM_{\text{halo}}}{R_{\text{halo}}} . \]  \hspace{1cm} (17)

Therefore, small deviations from \( r^{-2} \) in \( \rho(r) \) can be accounted for by variable masses for dark matter. At scales of spiral galaxies this effect is negligible but at larger scales, say superclusters, it could change the general picture of dark matter halos.

### 3 Dark entropy and axion dilution

Let us consider a perfect fluid composed of dark matter particles with variable masses. We can write the energy–momentum tensor and particle number current in the presence of a gravitational field as \( T^{\mu\nu} = p g^{\mu\nu} + (p + \rho) U^\mu U^\nu \) and \( N^\mu = n U^\mu \), where \( U^\mu \) is the local value of \( \frac{dx^\mu}{d\tau} \) for a comoving fluid element and \( n \) is the particle number density.

6
The general cosmological solutions to the equations of motion \((9, 1 0)\) in a Robertson–Walker frame, \(ds^2 = -dt^2 + a^2(t)dx^2\) (compatible with the properties of a perfect fluid) are \([8, 2]\)

\[
a(t) \sim t^p, \quad p = \frac{2(2\omega + 3) - 2(1 - 2\sigma)}{3(2\omega + 3) - 2(1 - 2\sigma) + (1 - 2\sigma)^2}
\]

\[
\Phi(t) \sim t^q, \quad q = \frac{4(1 - 2\sigma)}{3(2\omega + 3) - 2(1 - 2\sigma) + (1 - 2\sigma)^2}, \tag{18}
\]

while the energy and particle number conservation laws expressed in a Robertson–Walker frame give

\[
\frac{d}{dt}(\rho a^3) + p \frac{d}{dt}(a^3) = \frac{1}{m} \frac{dm}{dt}(\rho - 3p)a^3 \tag{19}
\]

\[
\frac{d}{dt}(na^3) = 0 . \tag{20}
\]

Non–relativistic particles in thermal equilibrium at a temperature \(T\) have negligible pressure and energy density given by

\[
\rho = n m = \frac{Nm}{a^3}, \tag{21}
\]

where \(N = na^3\) is the conserved number of dark matter particles in thermal equilibrium. During the matter era there is an entropy increase (for variable masses) that can be computed by comparing eq.\((19)\) with the second law of Thermodynamics \(dU + pdV = TdS\),

\[
TdS \simeq Ndm(\Phi) . \tag{22}
\]

The total entropy production per comoving volume from the time of equal matter and radiation energy density to now, due to the variable masses of dark matter, is given by

\[
\Delta S = \int_{t_{eq}}^{t_o} \frac{Ndm}{T} \simeq \frac{\sigma(1 - 2\sigma)}{2\omega + 3 - (1 - 2\sigma)^2} \frac{Nm(t_o)}{T_o} \equiv k(\omega, \sigma) \frac{Nm}{T_o} , \tag{23}
\]

where \(T_o\) is the dark matter temperature (approximately equal to the photon temperature) today and we have used the cosmological solutions \([18]\). This
should be considered as a new source of entropy, apart from the usual ones (cosmological phase transitions, galaxy formation, star evolution, etc.).

We now apply this entropy production mechanism to the dilution of axions \cite{11}, thus opening the so-called “axion window” \cite{14}. Axions are very good candidates for the dark matter of the universe \cite{10}. They are non-relativistic particles that couple very weakly to matter and radiation. Astrophysical and cosmological constraints bound their mass to be in the range $10^{-3} - 10^{-5}$ eV $\lesssim m_a \lesssim 10^{-2} - 10^{-3}$ eV \cite{7}. The lower bounds come from the cosmological constraint $\Omega_a h^2 \lesssim 1$, where the axion contribution to the critical density is estimated as \cite{4}

$$ \Omega_a h^2 \simeq \left( \frac{m_a}{10^{-3} - 10^{-5}} \text{eV} \right)^{-1.18} . \quad (24) $$

and $h$ is Hubble’s constant in units of 100 km s$^{-1}$ Mpc$^{-1}$ \cite{14}. In the above estimation it was assumed that there has been no significant entropy production at later stages of the evolution of the universe. If, on the other hand, the entropy per comoving volume $S$ is increased by a factor $\gamma$ since the time of axion production, then $\Omega_a h^2$ is reduced by the same factor \cite{7}. There has been several attempts to open the “axion window”, the most important one being the use of inflation \cite{12}. Here we present an alternative mechanism for axion dilution.

If axions had variable masses their energy density would be diluted by a factor $\gamma = \frac{\Delta S}{S}$. Using the fact that baryons are also non-relativistic, $\rho_B = n_B m_B$, and their contribution to the critical density is $\Omega_B h^2 \sim 10^{-2}$ \cite{14} we find, see eq.(23),

$$ \gamma \simeq 10^2 k(\omega, \sigma) \frac{\eta N_e}{S} \frac{m_B}{T_0} , \quad (25) $$

where $\eta = n_B/n_\gamma \sim 4 \times 10^{-10}$ is the baryon to photon ratio, and the total entropy per comoving volume of the universe is related to the number of photons by $S \simeq 7N_\gamma$ \cite{7}. Note that the entropy production is precisely proportional to the axion energy density (24) and thus to the mass of the axion. Therefore, the fraction of critical density contributed by axions with variable masses does not depend on $m_a$,

$$ \Omega_a h^2_{\text{now}} \simeq \frac{\Omega_a h^2}{\gamma} \simeq 7 \times 10^{-2} k^{-1}(\omega, \sigma) \frac{T_0}{\eta m_B} . \quad (26) $$

\footnote{For a detailed discussion of axions in the early universe see Ref.\cite{14}.}
Imposing the bound $\Omega_{\omega} h^2_{\text{now}} \lesssim 1$, we find no constraint on the axion mass but only on the parameters of our model (4)

$$\frac{2\beta_I \beta_V - 2\beta_I^2}{1 - 4\beta_I^2} \gtrsim 7 \times 10^{-2} \frac{T_o}{\eta m_B},$$

(27)

where $m_B \sim 1$ GeV is the proton mass and $T_o \sim 3$ K is the photon temperature today. Variable masses of axions could be an important alternative mechanism to inflation as a source of axion dilution. It is important to know all possible sources of axion dilution since there is a proposal of an experimental search for dark matter axions [13] in the 0.6–26 $\mu$eV mass range that would not detect an axion with very small mass, on the other hand allowed by these processes.

4 Experimental bounds

As we have seen, the model has very original physical features but we must constrain its parameters in order to appreciate its quantitative relevance. Most of the bounds are cosmological. Damour et al. [3] gave bounds on the parameters of the action (4) from radar time–delay measurements, the age of the universe and the time variation of $G$. Visible baryonic matter dominates our solar system and therefore the $\omega$ parameter can be constrained by post–Newtonian experiments. In particular, the Viking experiments [14] give the bound $2\omega + 3 > 500$ (95% c.l.), which imply, see eq.(8),

$$\beta_V < 0.022.$$  

(28)

Cosmological observations give $H_o t_o > 0.4$ [15], where $H_o$ is the Hubble constant and $t_o$ is the age of the Universe. This bound imposes a constraint on $\beta_I$ which is almost independent of $\beta_V$, see eqs.(18, 8, 28), and gives (see also [3])

$$\beta_I < 0.674.$$  

(29)

Since we are assuming that dark matter dominates the cosmological evolution during the matter era, it will also be constrained by bounds from primordial nucleosynthesis [16]. A recent analysis of these bounds was made in Ref.[17] for an ordinary Jordan–Brans–Dicke theory. We have generalized
this result to a theory with $\Phi$-dependent masses, see also Ref. [18], by using the cosmological solutions (18) and the fact that $G \sim \frac{1}{\Phi}$ and $aT \sim$ constant.

The predicted mass fraction $Y_p$ of primordial $^4He$ in this theory is correctly parametrized in the allowed region of observable parameters by [19, 17]

$$Y_p = 0.228 + 0.010 \ln \eta_{10} + 0.012(N_\nu - 3) + 0.185 \left( \frac{\tau_n - 889.6}{889.6} \right) + 0.327 \log \xi$$

(30)

where $\eta_{10}$ is the baryon to photon ratio in units of $10^{-10}$, $N_\nu$ the number of light neutrino species at nucleosynthesis and $\tau_n$ is the neutron lifetime $\tau_n = 889.6 \pm 5.8 \text{ s (2}\sigma) [20]$. $\xi$ is the ratio of the Hubble parameter at nucleosynthesis to its general relativity value and can be shown to be given in Jordan–Brans–Dicke theory with variable masses by, see eq.(18),

$$\xi^2 \equiv \frac{G_{\text{rad}}}{G_o} = \left( \frac{T_{\text{eq}}}{T_o} \right)^p = \left( 2 \times 10^4 \Omega_o h^2 \frac{1 - 2\sigma}{\omega + 1 + \sigma} \right),$$

(31)

where $\Omega_o$ is the observed ratio of the total energy density of the universe to the critical density. Present observations allow the range $0.008 < \Omega_o h^2 < 4.0 [21].$

Consistency, within two standard deviations, of the observational bounds on the primordial abundances of $D + ^3He$ and $^7Li$ and the corresponding predictions of GR for $N_\nu = 3$, forces $\eta_{10}$ to lie in the range $2.8 \leq \eta_{10} \leq 4.0 [13]$. The lower bound on $\eta_{10}$ comes from the upper observational limit on $D + ^3He$ and is the relevant one for our purposes.

Using the conservative 2$\sigma$ estimation of the observational value of $Y_p$ [19, 22] (see however Ref. [23])

$$Y_p = 0.23 \pm 0.01 \quad (2\sigma)$$

(32)

we obtain a bound on $(\omega, \sigma)$ from primordial nucleosynthesis, for $\Omega h^2 = 0.25$ and $N_\nu = 3$ [17, 2]

$$\frac{\omega + 1 + \sigma}{1 - 2\sigma} > 380 \quad \text{ (95\% c.l.)}$$

(33)
which constrains the parameters of our theory as

\[ \beta_I \beta_V < 3 \times 10^{-4} \]  \hspace{1cm} (34)

Suppose that string theory or any other fundamental theory of gravity fixes the coupling \( \beta_I \sim \mathcal{O}(1) \). Then the constraint (34) would dramatically improve the bound on \( \omega \) to \( 2\omega + 3 > 2 \times 10^6 \).

On the other hand, if axions had variable masses and constituted the dark matter of our universe, their contribution to the critical density would also impose a bound on the parameters of our model, see eq.(27). Using the previous bounds on \( \beta_I \beta_V \) (34) we estimate

\[ \beta_I < 0.0165 \]  \hspace{1cm} (35)

which improves significantly the bound on \( \beta_I \), see eq.(29).

Furthermore, string theory predicts a scalar coupling of the dilaton to the electromagnetic sector given by \( \alpha = \frac{e^2}{4\pi} \propto e^\phi \) (at tree level). Therefore, a variation of the electromagnetic coupling constant is related to the corresponding variation of Newton’s constant by

\[ \frac{\delta \alpha}{\alpha} = \sqrt{2\omega + 3} \frac{\delta G}{G} . \]  \hspace{1cm} (36)

There are bounds on the variation of \( \alpha \) which are extraordinarily strong [24]. For example, Dyson gives a bound from the nuclear stability of the \( \beta^- \)–isotopes \( ^{187}\text{Re} \) and \( ^{187}\text{Os} \), \( |\frac{\delta \alpha}{\alpha}| < 2.5 \times 10^{-5} \) since the formation of the Earth. This bound translates into the parameters of our theory as

\[ \beta_I \beta_V < 1.7 \times 10^{-8} \]  \hspace{1cm} (37)

For a scenario in which all the masses have the same \( \Phi \)-dependence (\( \beta_I = \beta_V \)), this imposes a strong bound on \( \omega \)

\[ |2\omega + 3| > 1.4 \times 10^8 \]  \hspace{1cm} (38)

As we can see, these bounds are much greater than any other bound from nucleosynthesis or post–Newtonian experiments. However, in string theory there are other scalar fields, the moduli, that parametrize the size and shape of compactified space and share many properties with the dilaton. In particular, in some string models the gauge coupling is a combination of the
dilaton and moduli which could be fixed by some mechanism, say gaugino condensation \cite{25}, without fixing the dilaton. Therefore the last bound on the parameters from the constancy of gauge couplings is a model dependent constraint.

5 Conclusions

We have analyzed the phenomenology of a cosmological scenario in which a dilaton scalar field couples differently to dark matter than to visible matter. String theory gives no theoretical prediction on the value of the dilaton coupling, it just makes it plausible that this scenario may arise. This kind of dilaton coupling violates the weak equivalence principle but is not ruled out by experiment.

We study the physical consequences of such a simple assumption and constrain the parameters of the model by experiment. Dark matter particles with variable masses do not follow geodesics in the conformal frame of visible matter with constant masses, and therefore may have a measurable effect in the dynamical motion of the halo of spiral galaxies. At scales of superclusters this effect could change the general picture of dark matter halos.

As a consequence of variable masses, the energy–momentum tensor of dark matter is not conserved. There is a “dark entropy” production associated with this dissipative effect. Axions are very good candidates for the cold dark matter of our universe. Their masses are constrained by astrophysics and cosmology to lie in a very narrow range, the so-called “axion window”. If axions had variable masses their contribution to the critical energy density would be diluted and therefore open the axion window. At the same time the cosmological constraint on the axion energy density imposes a relatively strong bound on the parameters of the model.

We assume that dark matter with variable masses dominates the cosmological evolution during the matter era. The age of the universe gives a bound on their dilaton coupling, while the Viking experiment of radar time–delay bounds the coupling to visible matter. If dark matter had variable masses, Newton’s constant would vary with time. In particular, it would be different at the time of primordial nucleosynthesis. Observational bounds on the mass fraction of primordial $^4$He, $^3$He + D and $^7$Li also constrain the parameters of the model.
In string theory the dilaton also couples to the electromagnetic sector in the form of a variable gauge coupling. There are bounds on the variation of the electromagnetic coupling that are extraordinarily strong and therefore reduce significantly our parameter space. However, as mentioned above, these bounds are very model dependent. In some string scenarios the gauge coupling is constant without fixing the dilaton.

Let us briefly summarize the cosmological bounds on the parameters of the model, coming from radar time–delay experiments, primordial nucleosynthesis and the contribution of axions with variable masses to the critical density of the universe

\[
\begin{align*}
\beta_V &< 0.022 \\
\beta_I \beta_V &< 3 \times 10^{-4} \\
\beta_I &< 0.017,
\end{align*}
\]

which gives a rather small parameter space.

Finally, we think that this scenario for dark matter may be interesting in the future, where models of structure formation will have to take into account the recently observed anisotropy of the cosmic background radiation. On the other hand, if dark matter were composed of axions with variable masses, the lower bound on their mass can be relaxed and may not be observed in the recently proposed experimental search for axions.
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