Enhancement of coherence in qubits due to interaction with environment

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The influence of the initial preparation on dephasing in open quantum dynamics is studied using an exactly solvable model of a two-level system (qubit) interacting with a bosonic bath. It is found that for some classes of non-selective preparation measurements, qubit-bath correlations lead to a significant enhancement of coherence in the qubit at the initial stage of evolution. The time behavior of the qubit purity and entropy in the regime of enhancement of coherence is considered for different temperatures and coupling strengths.

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I. INTRODUCTION

An important point in the dynamics of open quantum systems is the initial state preparation. Due to unavoidable interactions, there generally exist initial correlations between a system and its environment. Thus any physical process of preparation of the initial state of the system will affect the state of the environment as well. The question then arises of how the preparation procedure and initial correlations influence the subsequent evolution of the open system. Different aspects of this question were discussed by many authors (see, e.g., Refs. [1–5] and references therein). Of special interest is the decoherence phenomenon (the environmentally induced destruction of quantum coherence). For instance, decoherence plays a crucial role in the dynamics of two-state systems (qubits) which are the elementary carries of quantum information [6, 7].

At first sight it is natural to expect that initial correlations between an open quantum system and an environment with a huge (or even infinite) number of degrees of freedom would increase the decoherence rate. This is indeed the case for some preparation procedures [8, 9] but is not true in general. We refer to the paper [8], where the authors demonstrated with examples of specific qubit-environment models that for some initial system-environment correlated states, the “purity” of the qubit is greater than in the case of initially uncorrelated states. Although the results of Ref. [8] rely on a somewhat artificial assumption that the environment is initially prepared in a pure quantum state, the fact that quantum coherence in open systems can be enhanced due to system-environment correlations seems as itself to deserve thorough studies. It could provide, for instance, a way for constructing dynamics of qubit registers with interesting and very promising properties.

In this paper we aim to examine in detail the connection between the initial preparation and the qubit dynamics. In particular, this allows us to establish the conditions for appreciable environment-induced enhancement of quantum coherence in a qubit. A somewhat unexpected result is that the purity of a qubit state can even increase with time. The principal difference between our analysis and that presented in Ref. [5] lies in the interpretation of initial states of the composite (qubit plus environment) system. As we have already mentioned, in Ref. [5] the initial states were taken in the form which could illustrate the role of qubit-environment correlations but, unfortunately, these states are very unlikely to be physically realizable. In the present paper we consider more realistic preparation procedures based on quantum measurements [10, 11].

The structure of the paper is as follows. In Sec. II we give a brief review of measurement schemes for open quantum systems and applications to qubits. In Sec. III we treat time evolution of a qubit coupled to a bosonic environment through a dephasing interaction. Exact expressions are given for the time-dependent elements of the qubit density matrix when the initial state is prepared by a selective or non-selective measurement. Our central goal in this section is to demonstrate that for a large class of initial states prepared by non-selective measurements, the coherences (off-diagonal elements of the qubit density matrix) increase with time at the initial stage of evolution. In Sec. IV we consider the time behavior of the qubit purity and entropy in the regime of enhancement of coherence. Conclusions are drawn in Sec. V.

II. SELECTIVE AND NON-SELECTIVE PREPARATION MEASUREMENTS

As an introduction to our subsequent development, we start with a brief discussion of a rather general quantum measurement scheme which can be used to construct statistical ensembles describing initial states of real open systems. We then apply this scheme to a qubit interacting with its environment.

Suppose that at all times $0 < t$ an open system $S$ is in thermal equilibrium with a heat bath $B$, and at time zero one makes a measurement on the system $S$ only. According to general principles of quantum measurement theory [10, 12], the state of the composite system $(S + B)$
after the measurement is described by the density matrix

$$\rho_{SB}(0) = \sum_m \Omega_m \rho_{eq} \Omega_m^\dagger,$$

(1)

where $\rho_{eq}$ is the equilibrium density matrix at temperature $T$. Operators $\Omega_m$ act in the Hilbert space of the system $S$ and correspond to possible outcomes $m$ of the measurement. In a particular case of a selective measurement, the system $S$ is prepared in some pure state $\lvert \psi \rangle$. Then the sum in Eq. (1) collapses into a single term, so that

$$\rho_{SB}(0) = \frac{1}{Z} P_\psi \rho_{eq} P_\psi,$$

(2)

where $P_\psi = \lvert \psi \rangle \langle \psi \rvert$ is the projector onto the quantum state $\lvert \psi \rangle$ and $Z$ is the normalization factor. In general, the density matrix (1) describes the resulting ensemble after a non-selective measurement in which the outcome $m$ may be viewed as a classical random number with the probability distribution

$$w(m) = \text{Tr} \{ F_m \rho_{eq} \}.$$

(3)

Positive operators $F_m = \Omega_m^\dagger \Omega_m$ are called the “effects”. Here and in what follows, $\text{Tr}$ denotes the trace over the Hilbert space of the composite ($S+B$) system, while the symbols $\text{Tr}_S$ and $\text{Tr}_B$ will be used to denote the partial traces over the Hilbert spaces of the system $S$ and the heat bath, respectively. In order that $w(m)$ be normalized to 1, the effects $F_m$ must satisfy the normalization condition (the resolution of the identity)

$$\sum_m F_m = \sum_m \Omega_m^\dagger \Omega_m = I,$$

(4)

where $I$ is the unit operator.

The precise form of $\Omega_m$ is determined by the details of the measuring device. We restrict ourselves to physical situations in which some observable $A$ with a discrete, non-degenerate spectrum $A_m$ is measured. Then we have [10, 11]

$$\Omega_m = U_m F_m^{1/2},$$

(5)

where $F_m^{1/2}$ is the square root of $F_m$, and the unitary operator $U_m$ describes the disturbance of the system $S$ by the measurement device. Formula (5) is applicable even to approximate measurements where the spectrum $A_m$ is measured with finite resolution [10, 11]. We will be content, however, with performing the analysis for the case of infinite resolution when the effects are written as

$$F_m = \lvert \psi_m \rangle \langle \psi_m \rvert$$

(6)

and Eq. (5) yields

$$\Omega_m = \lvert \varphi_m \rangle \langle \psi_m \rvert$$

(7)

with

$$\lvert \varphi_m \rangle = U_m \lvert \psi_m \rangle.$$

(8)

A few remarks are needed here. Whereas the states $\lvert \psi_m \rangle$ form an orthonormal basis, for the transformed states $\lvert \varphi_m \rangle$ this is true only if the unitary operators $U_m$ are identical for all outcomes $m$ (i.e., $U_m = U$). In such cases, since $\Omega_m \Omega_m^\dagger = \langle \varphi_m \rangle \langle \varphi_m \rvert$, we have, in addition to Eq. (4), another resolution of the identity

$$\sum_m \Omega_m \Omega_m^\dagger = I.$$

(9)

In other words, to the measurement scheme defined in terms of the effects (6) and the $\Omega$-operators (7) there corresponds the “dual scheme” characterized by

$$\tilde{F}_m = U F_m U^\dagger \equiv \langle \varphi_m \rangle \langle \varphi_m \rvert,$$

$$\tilde{\Omega}_m = \Omega_m^\dagger \equiv \langle \psi_m \rangle \langle \varphi_m \rvert.$$

(10)

If the form of $U_m$ depends on the outcome $m$, then the transformed states $\lvert \varphi_m \rangle$ are not orthogonal in general. Some of these states may even be identical.

Let us now apply the above general construction to a qubit. In the formal “spin” representation, the canonical orthonormal basis states of a qubit are

$$\lvert 0 \rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \lvert 1 \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

(11)

It is well known that all pure states $\langle \psi(a) \rangle \equiv \lvert a \rangle$ correspond to points of the sphere $\lvert a \rangle = 1$, where $a = (a_1, a_2, a_3) \in \mathbb{R}^3$. Normalized state vectors are given by (see, e.g., [12])

$$\lvert a \rangle = \begin{pmatrix} e^{-i \phi_a / 2} \cos(\theta_a / 2) \\ e^{i \phi_a / 2} \sin(\theta_a / 2) \end{pmatrix},$$

(12)

where $\phi_a$ and $\theta_a$ are the Euler angles of the unit vector $\hat{a}$ describing the direction of the “spin”. They satisfy $a_1 + ia_2 = \sin \theta_a e^{i \phi_a}$, $a_3 = \cos \theta_a$. The Euler angles corresponding to the state $\lvert - \hat{a} \rangle$ are

$$\theta_{-a} = \pi - \theta_a, \quad \phi_{-a} = \phi_a + \pi.$$

(13)

Using these relations together with Eq. (12) gives

$$\lvert - \hat{a} \rangle = \begin{pmatrix} -ie^{-i \phi_a / 2} \sin(\theta_a / 2) \\ ie^{i \phi_a / 2} \cos(\theta_a / 2) \end{pmatrix}.$$

(14)

Linear operators in the qubit’s Hilbert space can be represented as linear combinations of the unity operator and the Pauli matrices. For example, the operator

$$\sigma(\hat{a}) = \sigma_1 a_1 + \sigma_2 a_2 + \sigma_3 a_3$$

(15)

describes the spin component in the direction $\hat{a}$. The state vectors $\lvert \pm \hat{a} \rangle$ correspond to the eigenvalues $\pm 1$ of $\sigma(\hat{a})$ and form an orthonormal basis, i.e.,

$$\lvert \hat{a} \rangle \langle \hat{a} \rvert + \lvert - \hat{a} \rangle \langle - \hat{a} \rvert = I.$$

(16)
Note that the states $|\tilde{a}\rangle$ and $|-\tilde{a}\rangle$ are related to the canonical basis states (11) by

$$|\tilde{a}\rangle = U(\tilde{a})|1\rangle, \quad |-\tilde{a}\rangle = U(\tilde{a})|0\rangle$$

with the unitary operator

$$U(\tilde{a}) = \left( \begin{array}{cc} e^{-i\tilde{a}^2/2} \cos(\theta a/2) & -ie^{-i\tilde{a}^2/2} \sin(\theta a/2) \\ e^{i\tilde{a}^2/2} \sin(\theta a/2) & ie^{i\tilde{a}^2/2} \cos(\theta a/2) \end{array} \right).$$

(18)

Selective measurements (2) on a qubit can be characterized by the projectors $P(\tilde{a}) = |\tilde{a}\rangle\langle\tilde{a}|$ while the general non-selective measurement scheme (9–13) is associated with three states: $|\tilde{a}\rangle$, $|\tilde{b}_1\rangle$, and $|\tilde{b}_2\rangle$. The effects and the $\Omega$-operators are defined as

$$F_1 = |\tilde{a}\rangle\langle\tilde{a}|, \quad F_2 = |-\tilde{a}\rangle\langle-\tilde{a}|, \quad \Omega_1 = |\tilde{b}_1\rangle\langle\tilde{a}|, \quad \Omega_2 = |\tilde{b}_2\rangle\langle-\tilde{a}|.$$  

(19)

According to Eqs. (17), we have

$$\tilde{b}_1 = U(\tilde{a})|\tilde{a}\rangle, \quad \tilde{b}_2 = U(-\tilde{a})|\tilde{a}\rangle.$$  

(20)

where the unitary operator $U(\tilde{b}, \tilde{a})$ is expressed in terms of the operators (18):

$$U(\tilde{b}, \tilde{a}) = U(\tilde{b})U^1(\tilde{a}).$$  

(21)

Let us briefly consider some important special cases of the measurement scheme (19):

i) $\tilde{b}_1 = \tilde{a}$, $\tilde{b}_2 = -\tilde{a}$. This is the simplest scheme corresponding to $U_m = I$ in the general formula (8). Physically, here we are dealing with non-selective measurements where the measuring device does not disturb the basis states $|\tilde{a}\rangle$ and $|-\tilde{a}\rangle$. In this case the $\Omega$-operators coincide with the effects:

$$\Omega_1 = F_1 = |\tilde{a}\rangle\langle\tilde{a}|, \quad \Omega_2 = F_2 = |-\tilde{a}\rangle\langle-\tilde{a}|.$$  

(22)

Clearly, the same operators correspond to the dual measurement scheme (10).

ii) $\tilde{b}_1 = \tilde{b}$, $\tilde{b}_2 = -\tilde{b}$, where $\tilde{b}$ is an arbitrary unit vector. This case corresponds to $U_m = U \equiv U(\tilde{b}, \tilde{a})$ in the formula (8). It follows from Eqs. (19) that

$$F_1 = |\tilde{a}\rangle\langle\tilde{a}|, \quad F_2 = |-\tilde{a}\rangle\langle-\tilde{a}|, \quad \Omega_1 = |\tilde{b}\rangle\langle\tilde{a}|, \quad \Omega_2 = |-\tilde{b}\rangle\langle-\tilde{a}|.$$  

(23)

There exists the dual measurement scheme (10) with

$$\tilde{F}_1 = |\tilde{b}\rangle\langle\tilde{b}|, \quad \tilde{F}_2 = |-\tilde{b}\rangle\langle-\tilde{b}|, \quad \tilde{\Omega}_1 = |\tilde{a}\rangle\langle\tilde{b}|, \quad \tilde{\Omega}_2 = |-\tilde{a}\rangle\langle-\tilde{b}|.$$  

(24)

iii) $\tilde{b}_1 = \tilde{b}$, $\tilde{b}_2 = \tilde{b}$ with an arbitrary unit vector $\tilde{b}$. This is an example of a non-selective measurement scheme for a qubit, which is described by Eqs. (6)–(8) with different unitary operators $U_m$. Recalling Eqs. (19), we write

$$F_1 = |\tilde{a}\rangle\langle\tilde{a}|, \quad F_2 = |-\tilde{a}\rangle\langle-\tilde{a}|, \quad \Omega_1 = |\tilde{b}\rangle\langle\tilde{a}|, \quad \Omega_2 = |\tilde{b}\rangle\langle-\tilde{a}|.$$  

(25)

It is easily verified that

$$|\tilde{b}\rangle = U_1|\tilde{a}\rangle, \quad |\tilde{\tilde{b}}\rangle = U_2|\tilde{a}\rangle,$$

(26)

where $U_1 \neq U_2$ and are given by

$$U_1 = U(\tilde{b}, \tilde{a}), \quad U_2 = U(-\tilde{b}, \tilde{a}).$$  

(27)

Note that in this case there is no dual measurement scheme.

We close this section with a remark about the above-discussed non-selective measurement schemes. Mathematically, all the schemes are generated by orthogonal resolutions of the identity (4) where the effects $F_m$ are projectors $(F_m^2 = F_m)$ and are given by Eq. (6). This is a natural generalization of the well-known von Neumann-Lüders projection postulate for ideal quantum measurements (see, e.g., Ref. [10]). One can, however, construct more general measurement schemes associated with the notion of the positive operator-valued measure (POVM) [13, 14]. A POVM is defined by $N$ positive operators $F_m$, which form the resolution of the identity, but in general $F_m^2 \neq F_m$. Usually the operators $F_m$ can be represented in the form (16) where the set $\{|\psi_p\rangle\}$ is overcomplete, i.e., the number of outcomes $N$ exceeds the rank of the density matrix of the system (for a qubit $N > 2$). In the present paper we will not consider such general situations and restrict ourselves to the von Neumann-Lüders projection measurements.

III. THE DEPHASING MODEL: DYNAMICS OF DECOHERENCE

A. Exact expressions for the coherences

Our central goal in this section is to demonstrate that the qubit dynamics with initial states prepared by non-selective measurements exhibits a number of physically interesting and even somewhat unexpected features when compared with the case of selective measurements. This is especially important when one is dealing with decoherence phenomena.

Our discussion will be based on the analysis of the simple dephasing model describing the main decoherence mechanism for certain types of system-environment interactions [4, 17, 18]. In this model, a two-state system (qubit) $S$ is coupled to a bath $B$ of harmonic oscillators. Using the “spin” representation for the qubit with the basis states (11), the total Hamiltonian in the Schrödinger picture is taken to be (in our units $\hbar = 1$)

$$H = HS + HB + H_{nt} = \frac{\omega_0}{2} \sigma_3 + \sum_k \omega_k \sigma^k_b \sigma^k_b + \sigma_3 \sum_k (g_k \sigma^k_b + g_k^* \sigma^k_b),$$  

(28)
where \( \omega_0 \) is the energy difference between the excited state \( |1\rangle \) and the ground state \( |0\rangle \) of the qubit. Bosonic creation and annihilation operators \( b_k^{\dagger} \) and \( b_k \) correspond to the \( k \)th bath mode with frequency \( \omega_k \), and \( g_k \) are the coupling constants.

Suppose that at time \( t = 0 \) the system is prepared in a pure quantum state \( |\psi\rangle \). Then, at time \( t \) the average value of a Heisenberg picture operator \( A(t) \) is given by

\[
\langle A(t) \rangle = \text{Tr} \left\{ \exp(iHt)A \exp(-iHt) \rho_{SB}(0) \right\}.
\]

Below, we choose to quote the initial density matrix \( \rho_{SB}(0) \). Then at time \( t \) the average value of a Heisenberg picture operator \( A(t) \) is given by

\[
\langle A(t) \rangle = \text{Tr} \left\{ \exp(iHt)A \exp(-iHt) \rho_{SB}(0) \right\}.
\]

The quantities of principal interest are the coherences \( \langle \sigma_\pm(t) \rangle \), where \( \sigma_\pm = (|1\rangle \langle 1| \pm |0\rangle \langle 0|) / 2 \). They are related directly to the off-diagonal elements of the reduced density matrix of the qubit:

\[
\langle \sigma_+(t) \rangle = \langle 0| \rho_S(t) |1\rangle, \quad \langle \sigma_-(t) \rangle = \langle 1| \rho_S(t) |0\rangle,
\]

where

\[
\rho_S(t) = \text{Tr}_B \left\{ \exp(-iHt) \rho_{SB}(0) \exp(iHt) \right\}.
\]

The dephasing model \([28]\) has two distinctive features. First, the operator \( \sigma_3 = \sigma_x \) commutes with the Hamiltonian and, consequently, the average populations of the canonical states \([17, 18]\) do not depend on time. Thus we have a unique situation where the system relaxation may be interpreted physically as “pure” decoherence and exchange of entropy \([17, 18]\) rather than dissipation of energy. Second, in this model the equations of motion for all relevant operators can be solved exactly \([4]\). This allows one to study the time evolution of the coherences for different initial conditions. Here we leave out many details for which we refer to Ref. \([4]\) and simply quote some important results.

\[
\langle \sigma_\pm(t) \rangle = \langle \sigma_\pm \rangle e^{\pm i\omega_0 t} e^{-\gamma(t)} \sum_m \left\{ \langle 0| \Omega_m^\dagger \sigma_\pm \Omega_m |0\rangle e^{\beta \omega_0 / 2 \mp i \Phi(t)} + \langle 1| \Omega_m^\dagger \sigma_\pm \Omega_m |1\rangle e^{\beta \omega_0 / 2 \mp i \Phi(t)} \right\} / \sum_m \left\{ \langle 0| \Omega_m^\dagger \Omega_m |0\rangle e^{\beta \omega_0 / 2} + \langle 1| \Omega_m^\dagger \Omega_m |1\rangle e^{-\beta \omega_0 / 2} \right\},
\]

with the initial coherences

\[
\langle \sigma_\pm \rangle = \frac{1}{2 \cosh(\beta \omega_0 / 2)} \sum_m \left\{ \langle 0| \Omega_m^\dagger \sigma_\pm \Omega_m |0\rangle e^{\beta \omega_0 / 2} + \langle 1| \Omega_m^\dagger \sigma_\pm \Omega_m |1\rangle e^{-\beta \omega_0 / 2} \right\}.
\]

In the case of a selective measurement when the qubit is prepared in a pure quantum state \( |\psi\rangle = c_0 |0\rangle + c_1 |1\rangle \) with \( |c_0|^2 + |c_1|^2 = 1 \), the initial density matrix of the composite system is taken in the form \([4]\). Then, instead of Eq. \((37)\), we have

\[
\langle \sigma_\pm(t) \rangle = \langle \sigma_\pm \rangle e^{\pm i\omega_0 t} e^{-\gamma(t)}
\]

\[
\times \left\{ |c_0|^2 e^{2 \beta \omega_0 / 2 \mp i \Phi(t)} + |c_1|^2 e^{-2 \beta \omega_0 / 2 \mp i \Phi(t)} \right\} / \left\{ |c_0|^2 + |c_1|^2 \right\},
\]

where \( \langle \sigma_\pm \rangle = \langle \psi | \sigma_\pm | \psi \rangle \).

Formulas \((37)\) and \((39)\) contain two relevant functions. The so-called decoherence function \( \gamma(t) \) is defined as

\[
\gamma(t) = \int_0^\infty d\omega J(\omega) \coth(\beta \omega / 2) \frac{1 - \cos \omega t}{\omega^2},
\]

If the initial state is prepared by a non-selective measurement, then, taking the initial density matrix of the composite system in the form \([11]\), we get for the coherences \([50]\)

\[
\langle \sigma_\pm(t) \rangle = \frac{1}{Z} \sum_m \text{Tr} \left\{ \Omega_m^\dagger \sigma_\pm(t) \Omega_m e^{-\beta H} \right\},
\]

where \( \beta = 1 / k_B T \), and \( Z = \text{Tr} \left\{ e^{-\beta H} \right\} \) is the equilibrium partition function. The analogous formula for the case of a selective measurement [see Eq. \((22)\)] is evident.

As shown in Ref. \([4]\), the time-dependent qubit operators \( \sigma_\pm(t) \) in the dephasing model \([28]\) are given by

\[
\sigma_\pm(t) = \exp \{ \pm i \omega_0 t + R(t) \} \sigma_\pm
\]

with

\[
R(t) = \sum_k \left[ \alpha_k(t) b_k^\dagger - \alpha_k^*(t) b_k \right], \quad \alpha_k(t) = 2g_k \frac{1 - e^{i \omega_0 t}}{\omega_k}.
\]

Using the above expressions and the exact relations

\[
e^{-\beta H} |0\rangle = e^{i \omega_0 / 2 (e^{-\beta H} |0\rangle \otimes |0\rangle)} \otimes |0\rangle,
\]

\[
e^{-\beta H} |1\rangle = e^{-i \omega_0 / 2 (e^{-\beta H} |1\rangle \otimes |1\rangle)} \otimes |1\rangle,
\]

where

\[
H_B^{(\pm)} = \sum_k \omega_k b_k^\dagger b_k \pm \sum_k (g_k b_k^\dagger + g_k^* b_k),
\]

it is a straightforward matter to carry out the trace over the bath degrees of freedom in Eq. \((32)\). After some algebra (for details see Ref. \([4]\)), one obtains

\[
\langle \sigma_\pm(t) \rangle = \langle \sigma_\pm \rangle e^{\pm i\omega_0 t} e^{-\gamma(t)}
\]

\[
\times \left| c_0 |^2 e^{2 \beta \omega_0 / 2 \pm i \Phi(t)} + \left| c_1 |^2 e^{-2 \beta \omega_0 / 2 \mp i \Phi(t)} \right| / \left| c_0 |^2 + \left| c_1 |^2 \right|,
\]

where \( \langle \sigma_\pm \rangle = \langle \psi | \sigma_\pm | \psi \rangle \).
where the continuum limit of the bath modes is performed, and the spectral density $J(\omega)$ is introduced by the rule
\[
\sum_k 4|g_k|^2 f(\omega_k) = \int_0^\infty d\omega J(\omega) f(\omega).
\] (41)

It is clear that $\gamma(t)$ is precisely the quantity which determines the relaxation of the off-diagonals due to vacuum and thermal fluctuations in the bath [10]. The other function, $\Phi(t)$, is given by
\[
\Phi(t) = \int_0^\infty d\omega J(\omega) \frac{\sin \omega t}{\omega^2}.
\] (42)

As discussed in Ref. [4], this function accounts for the influence of initial qubit-environment correlations on the dynamics of decoherence. These correlations are inherited from the pre-measurement equilibrium state due to the presence of the interaction term in the total Hamiltonian [28]. Mathematically, it has the effect that the operators $H_B^I$ and $H_B$ [see Eqs. (35) and (36)] differ in the sign of the interaction term.

### B. Selective measurements

The result [39] for selective measurements has been analyzed in detail in Ref. [4], so that here we merely touch on some relevant points.

The expression [39] can be rewritten more transparently as
\[
\langle \sigma_\pm(t) \rangle = \langle \sigma_\pm \rangle \exp[\pm i(\omega_0 t + \gamma(t))] \exp[-\tilde{\gamma}(t)],
\] (43)
where
\[
\tilde{\gamma}(t) = \gamma(t) + \gamma_{\text{cor}}(t)
\] (44)
is the effective decoherence function including the correlation contribution
\[
\gamma_{\text{cor}}(t) = -\frac{1}{2} \ln \left[ 1 - \frac{(1 - \langle \sigma_3 \rangle^2) \sin^2 \Phi(t)}{\cosh(\beta \omega_0/2) - \langle \sigma_3 \rangle \sinh(\beta \omega_0/2)} \right],
\] (45)
whereas $\chi(t)$ is the time-dependent phase shift with
\[
\tan \chi(t) = \frac{\sinh(\beta \omega_0/2) - \langle \sigma_3 \rangle \cosh(\beta \omega_0/2)}{\cosh(\beta \omega_0/2) - \langle \sigma_3 \rangle \sinh(\beta \omega_0/2)} \tan \Phi(t).
\] (46)

In writing the above formulas we have used the obvious relations $\langle \sigma_3 \rangle = \langle \psi|\sigma_3|\psi \rangle = |c|^2 - |c_0|^2$.

It is important to note and easy to see from Eqs. (40) and (45) that both terms in the effective decoherence function (44) are always positive. Thus, in cases where the initial state is prepared by a selective measurement, initial qubit-bath correlations may be viewed as an additional source of decoherence.

### C. Non-selective measurements

Now we turn to Eq. (37) and consider the general non-selective measurement scheme described by Eqs. (19). Using the representation [12], we obtain after some algebra the expression [39] in which the correlation part of the decoherence function is now given by
\[
\gamma_{\text{cor}}(t) = -\frac{1}{2} \ln \left\{ 1 + \left( \frac{N_1^2 + N_2^2}{D^2} - 1 \right) \sin^2 \Phi(t) + \frac{N_2^2}{D} \sin(2\Phi(t)) \right\},
\] (47)
where we have introduced
\[
N_1 = \left\{ e^{\beta \omega_0} \sin^4(\theta_a/2) - e^{-\beta \omega_0} \cos^4(\theta_a/2) \right\} \sin^2 \theta_1 + \left\{ e^{\beta \omega_0} \cos^4(\theta_a/2) - e^{-\beta \omega_0} \sin^4(\theta_a/2) \right\} \sin^2 \theta_2
\]
\[+ \sinh(\beta \omega_0) \sin^2 \theta_a \cos \Delta \phi \sin \theta_1 \sin \theta_2, \]
\[N_2 = 2 \cos \theta_a \sin \Delta \phi \sin \theta_1 \sin \theta_2, \]
\[D = \left\{ \frac{1}{2} \sin^2 \theta_a + e^{\beta \omega_0} \sin^4(\theta_a/2) + e^{-\beta \omega_0} \cos^4(\theta_a/2) \right\} \sin^2 \theta_1 + \left\{ \frac{1}{2} \sin^2 \theta_a + e^{\beta \omega_0} \cos^4(\theta_a/2) + e^{-\beta \omega_0} \sin^4(\theta_a/2) \right\} \sin^2 \theta_2
\]
\[+ \left\{ \cosh(\beta \omega_0) \sin^2 \theta_a + 2 \left[ \sin^4(\theta_a/2) + \cos^4(\theta_a/2) \right] \right\} \cos \Delta \phi \sin \theta_1 \sin \theta_2 \].

To simplify notation, we have written $\theta_1, \phi_1$ for $\theta_{b_1}, \phi_{b_1}$, and denoted $\Delta \phi = \phi_1 - \phi_2$. The expression for the phase shift $\chi(t)$ is
\[
\chi(t) = \arctan \left( \frac{N_1 \sin \Phi(t)}{D \cos \Phi(t) + N_2 \sin \Phi(t)} \right).
\] (49)

The initial coherences [38] can be directly evaluated to yield
\[
\langle \sigma_\pm \rangle = \left. \frac{e^{\pm i \phi_1}}{4 \cosh(\beta \omega_0/2)} \right. 
\]
\[\times \left\{ \sin \theta_1 \left[ e^{\beta \omega_0/2} \sin^2 \frac{\theta_a}{2} + e^{-\beta \omega_0/2} \cos^2 \frac{\theta_a}{2} \right]
\]
\[+ e^{\mp i \Delta \phi} \sin \theta_2 \left[ e^{\beta \omega_0/2} \cos^2 \frac{\theta_a}{2} + e^{-\beta \omega_0/2} \sin^2 \frac{\theta_a}{2} \right] \right\}.
\] (50)
Formulas \([44] - [49]\), together with Eqs. \([13]\) and \([14]\), determine the time-dependent coherences \(\langle \sigma_+ (t) \rangle\) or, what is the same, the off-diagonals of the qubit density matrix [see Eq. \([30]\)]. It is often convenient to use the representation of the qubit density matrix \(\rho_S(t)\) in terms of the Bloch vector \(\vec{v}(t) = \langle \vec{\sigma}(t) \rangle\) \([13, 20]\):

\[
\rho_S(t) = \frac{1}{2} \left[ 1 + \vec{\sigma} \cdot \vec{v}(t) \right].
\]

(51)

The magnitude of the Bloch vector satisfies \(0 \leq v(t) \leq 1\), and \(v = 1\) only if the qubit is in a pure quantum state. It is easy to check that

\[
v(t) = \left[ 4\langle \sigma_+ (t) \rangle \langle \sigma_- (t) \rangle + \langle \sigma_3 (t) \rangle^2 \right]^{1/2}.
\]

(52)

Since \(\sigma_3\) commutes with the Hamiltonian \([25]\), we have \(\langle \sigma_3 (t) \rangle = \langle \sigma_3 \rangle\). Taking also into account Eq. \([43]\), we obtain

\[
v(t) = \left[ 4\langle \sigma_+ \rangle \langle \sigma_- \rangle e^{-2\gamma(t)} + \langle \sigma_3 \rangle^2 \right]^{1/2}.
\]

(53)

Thus, to calculate \(v(t)\), all we need is the initial average \(\langle \sigma_3 \rangle\). For the general non-selective measurement scheme \([19]\), this average can be found by using the formula analogous to \([38]\). A straightforward algebra gives

\[
\langle \sigma_3 \rangle = \frac{1}{2 \cosh(\beta\omega_0/2)} \times \left\{ \cos \theta_1 \left[ e^{\beta\omega_0/2} \sin^2 \frac{\theta_2}{2} + e^{-\beta\omega_0/2} \cos^2 \frac{\theta_2}{2} \right] 
+ \cos \theta_2 \left[ e^{\beta\omega_0/2} \cos^2 \frac{\theta_2}{2} + e^{-\beta\omega_0/2} \sin^2 \frac{\theta_2}{2} \right] \right\}. \tag{54}
\]

D. Some special non-selective preparation measurements

To gain an insight into new features of the qubit dynamics in cases when the initial state is prepared by a non-selective measurement, we will apply the above general expressions to some special preparation schemes described in Sec. II.

We start with the scheme \([23]\) for which, according to relations \([13]\),

\[
\theta_1 + \theta_2 = \pi, \quad \sin \Delta_\phi = 0, \quad \cos \Delta_\phi = -1,
\]

(55)

where \(\theta_1 \equiv \theta_a\) and \(\phi_1 \equiv \phi_b\). Noting that in this case \(N_2 = 0\), Eq. \([44]\) is manipulated to the simple form

\[
\gamma_{cor}(t) = -\frac{1}{2} \ln \left[ 1 + \frac{\sin^2 \Phi(t)}{\sinh^2(\beta\omega_0/2)} \right]. \tag{56}
\]

For the phase shift \([49]\) we find

\[
\tan \chi(t) = \coth(\beta\omega_0/2) \tan \Phi(t). \tag{57}
\]

The results \([54] and \([57]\) have several notable properties. First, they are universal in the sense that they do not depend on the qubit states \(|a\rangle\) and \(|b\rangle\) in Eqs. \([23]\) describing this type of non-selective measurements. In particular, the same expressions for \(\gamma_{cor}(t)\) and \(\chi(t)\) hold for the simplest scheme \([22]\) where \(|b\rangle = |\bar{a}\rangle\), i.e., the measuring device does not disturb the basis states. Next, we note that the function \([56]\) satisfies \(\gamma_{cor}(t) \leq 0\) at all times \(t\). In other words, we have an enhancement of coherence in the qubit caused by initial qubit-bath correlations! Moreover, it is seen from Eq. \([56]\), that the \(|\gamma_{cor}(t)|\) grows with temperature, so that in the temperature range \(\beta\omega_0 < 1\) the effective decoherence function \([44]\) may even become negative, at least during the initial stage of the system’s evolution. To illustrate this point, we have evaluated the reduced coherence

\[
\langle |\sigma(t)|^2/|\sigma| \rangle \equiv \langle |\sigma_\pm (t)|^2/|\sigma_\pm| \rangle = \exp \left[ \frac{-\gamma(t)}{2} \right]. \tag{58}
\]

using Eqs. \([40], [44],\) and \([30]\). The bath spectral density was taken in the form

\[
J(\omega) = \lambda_s \omega_c^{-s} \omega^s e^{-\omega/\omega_c}, \tag{59}
\]

which is most commonly used in the theory of spin-boson systems \([10, 15, 16, 21]\). Here \(\omega_c\) stands for some “cutoff” frequency, and \(\lambda_s\) is a dimensionless coupling constant. The “dynamical part” \([40]\) of the decoherence function and the function \(\Phi(t)\), Eq. \([12]\), have been studied in detail in Ref. \([1]\) for the sub-Ohmic (\(0 < s < 1\), Ohmic (\(s = 1\)), and super-Ohmic (\(s > 1\)) cases. Here we shall restrict ourselves to the most prominent Ohmic case where

\[
\Phi(t) = \lambda \arctan(\omega_c t), \quad \lambda \equiv \lambda_1. \tag{60}
\]

and the time behavior of the correlation term \([50]\) is very sensitive to the value of the coupling constant \(\lambda\).

Figure \([1]\) shows the time dependence of the reduced coherence in the temperature range \(\beta\omega_0 < 1\). At the initial stage of evolution (\(\omega_c t \lesssim 1\) the correlation effects dominate, so that the decoherence function \([44]\) takes negative values. At times \(\omega_c t \gg 1\) the correlation effects are suppressed due to vacuum and thermal fluctuations contributing to the “dynamical part” \([40]\) of the decoherence function. Using the explicit expressions for the function \([42]\) in the super-Ohmic (\(s > 1\)) and sub-Ohmic (\(0 < s < 1\)) cases (see Ref. \([1]\)), it can be shown that a similar time behavior of the reduced coherence is expected in these coupling regimes.

It is worthwhile remarking that even for a moderate qubit-bath coupling (\(\lambda \approx 1\)), the maximum value of the coherences \(|\langle \sigma_\pm (t) \rangle|\) may be by one order of magnitude larger than the initial value \(|\langle \sigma_\pm \rangle|\). This fact appears at first rather strange and even paradoxical. In particular, it seems likely that for some initial conditions, the magnitude of the Bloch vector \([55]\) may exceed the unity. This is, of course, not so for the following reason. The point is that the initial averages \([50]\) and \([54]\) depend on temperature. For the preparation procedure under
The initial averages (50) and (54) now take the form
\[ \langle \sigma_\pm \rangle = -\frac{1}{2} e^{\pm i \phi} \tan(\beta \omega_0/2) \cos \theta_b \sin \theta_b, \]
\[ \langle \sigma_3 \rangle = -\tan(\beta \omega_0/2) \cos \theta_b \cos \theta_b. \]

In the temperature range \( \beta \omega_0 \ll 1 \), we have \( |\langle \sigma_+ \rangle| \ll 1 \) and \( |\langle \sigma_3 \rangle| \ll 1 \). Recalling Eqs. (53) and (56), one can show that, at all times, \( v(t) \leq 1 \), as it must be. Other physical questions related to the enhancement of coherence for this preparation scheme will be discussed in Sec. IV.

Now we shall consider the dynamics of decoherence in the case of another notable non-selective preparation scheme described by Eqs. (23). This scheme corresponds to the following choice in Eqs. (23):
\[ \theta_1 = \theta_2 \equiv \theta_b, \quad \sin \Delta \phi = 0, \quad \cos \Delta \phi = 1. \]

From the second of Eqs. (28), it is clear that \( N_2 \) is again zero. Then, after some simple algebra Eqs. (47) and (49) give
\[ \gamma_{\text{cor}}(t) = -\frac{1}{2} \ln \left( 1 - \frac{\sin^2 \Phi(t)}{\cosh^2(\beta \omega_0/2)} \right), \]
\[ \tan \chi(t) = \tanh(\beta \omega_0/2) \tan \Phi(t). \]

The initial averages (50) and (54) now take the form
\[ \langle \sigma_\pm \rangle = \frac{1}{2} e^{\pm i \phi} \sin \theta_b, \quad \langle \sigma_3 \rangle = \cos \theta_b. \]

Similar to formulas (50) and (54), the results (63) and (64) are universal in the sense that they do not depend on the qubit states \( |\tilde{a}\rangle \) and \( |\tilde{b}\rangle \) in Eqs. (23). Another distinctive property of Eq. (63) is that \( \gamma_{\text{cor}}(t) \geq 0 \) at all times. Physically, in this case the initial qubit-bath correlations lead to additional decoherence. It is also interesting to note that expressions (63) and (64) are identical to expressions (59) and (60) for the selective measurement with \( \langle \sigma_a \rangle = 0 \), i.e., with equal populations of the basis states (11). This is not accidental; for a discussion see Appendix A.

A comparison of Eqs. (55) and (62), together with the fact that the Euler angles \( \theta_1 \) and \( \theta_2 \) enter the functions (48) only through \( \sin \theta_1 \) and \( \sin \theta_2 \), suggests that \( \Delta \phi = \phi_1 - \phi_2 \) is a key quantity determining the main qualitative features of the system’s evolution. To illustrate this point, suppose that the system is initially prepared by the non-selective measurement (19) with the Euler angles of the qubit states \( |\tilde{b}_1\rangle \) and \( |\tilde{b}_2\rangle \) satisfying
\[ \theta_1 = \theta_2, \quad \sin \Delta \phi = 0, \quad \cos \Delta \phi = -1. \]

The corresponding initial averages (50) and (54) are
\[ \langle \sigma_\pm \rangle = -\frac{1}{2} e^{\pm i \phi} \tan(\beta \omega_0/2) \cos \theta_b \sin \theta_1, \]
\[ \langle \sigma_3 \rangle = \cos \theta_1. \]

We mention that formulas (66) differ from Eqs. (62) only in that \( \cos \Delta \phi \) is now of opposite sign. Nevertheless, it is evident that we obtain for \( \gamma_{\text{cor}}(t) \) the result (53) which corresponds to the entirely different evolution of the coherences \( \langle \sigma_\pm(t) \rangle \).

It would be instructive to look at the last example from another point of view. Let us write the post-measurement qubit state \( |\tilde{b}_1\rangle \) in the canonical basis (11):
\[ |\tilde{b}_1\rangle = c_0|0\rangle + c_1|1\rangle, \]
where the amplitudes \( c_0 \) and \( c_1 \) can be expressed in terms of the Euler angles by using Eq. (12). Then using Eqs. (66) leads to the following representation for the state \( |\tilde{b}_2\rangle \):
\[ |\tilde{b}_2\rangle = i(c_0|0\rangle - c_1|1\rangle). \]

There is no new physics in the appearance of \( i \), but the additional phase shift between the basis states, as compared to Eq. (65), radically influences the qubit’s dynamics.

Thus far we have been concerned with special types of non-selective measurement schemes which lead to physically important features of decoherence. To give a comprehensive review of all possible regimes of evolution, one should appeal to Eq. (47). In general, the quantities (47)–(49) are rather complicated functions of the polar angles \( \theta_a, \theta_1, \theta_2 \), and the difference \( \Delta \phi = \phi_1 - \phi_2 \) of the azimuthal angles. In addition, they depend on temperature and the qubit energy \( \omega_0 \). This makes a detailed analysis of \( \gamma_{\text{cor}}(t) \) rather cumbersome for our discussion. Nevertheless, we can formulate a simple sufficient condition that the initial preparation of the system by a general non-selective measurement (19) leads to enhancement of
coherence in the qubit. Notice that for all qubit states \(|\hat{\sigma}_1\rangle\) and \(|\hat{\sigma}_2\rangle\) with \(\sin \Delta_\phi = 0\), we have \(N_2 = 0\) and, consequently, the term with \(\sin(2\Phi(t))\) in Eq. (47) vanishes. Then we arrive at the conclusion that \(\gamma_{\text{cor}}(t) \leq 0\) at all times if

\[
N_1^2 > D^2, \quad \sin \Delta_\phi = 0. \tag{70}
\]

Clearly, the second condition implies \(\cos \Delta_\phi = \pm 1\). We will not give here a somewhat lengthy formal analysis of Eqs. (70) since it does not add anything substantially new to the results of the above discussion. We only note that the enhancement of coherence takes place for the post-measurement states \(|\hat{\sigma}_1\rangle\) and \(|\hat{\sigma}_2\rangle\) with \(\cos \Delta_\phi = -1\) and \(\theta_1 \pm \theta_2 \approx \pi\) or \(\theta_1 \approx \theta_2\). In other words, the post-measurement states should be close to the states in the measurement schemes (55) or (66).

IV. THE PURITY AND ENTROPY OF THE QUBIT

If at time \(t\) the effective decoherence function (44) is negative, the state of the qubit is “less mixed” than initially. This property can be characterized quantitatively by the von-Neumann-Shannon information entropy

\[
S(t) = -\text{Tr}_S \{ \hat{\sigma}_S(t) \ln \hat{\sigma}_S(t) \}. \tag{71}
\]

Using Eq. (51), the information entropy of a qubit can be expressed in terms of the Bloch vector magnitude [4]:

\[
S(t) = \ln 2 - \frac{1}{2} (1 + v) \ln (1 + v) - \frac{1}{2} (1 - v) \ln (1 - v). \tag{72}
\]

Since \(0 \leq v \leq 1\), we have \(0 \leq S \leq \ln 2\) with \(S = 0\) for a pure quantum state \((v = 1)\).

Another measure of the “mixedness” (or the lack of information about a system) is the so-called purity of the system’s state [3, 22]:

\[
\mathcal{P}(t) = \text{Tr}_S \{ |\hat{\sigma}_S(t)\rangle \langle \hat{\sigma}_S(t) | \}. \tag{73}
\]

Again using Eq. (51), we obtain for a qubit

\[
\mathcal{P}(t) = \frac{1}{2} (1 + v^2). \tag{74}
\]

Obviously \(1/2 \leq \mathcal{P} \leq 1\) with \(\mathcal{P} = 1\) for a pure state.

In principle, either \(S(t)\) or \(\mathcal{P}(t)\) may be used to measure the degree of coherence in a qubit. In both cases the key quantity is the magnitude of the Bloch vector (33). Here we shall discuss the time behavior of the purity and entropy in the non-selective measurement schemes (55) and (66) for which the enhancement of coherence is most pronounced.

Figures 2 and 3 illustrate the evolution of the qubit entropy (72) and purity (74) at a fixed temperature for different values of the coupling constant.

It is seen that with increasing the coupling constant the maximum purity becomes larger and is shifted to smaller \(t\). For large values of the coupling constant, there appear oscillations in both the purity and entropy (cf. Fig. 1). According to Eqs. (61) and (67), the initial coherences \(|\langle \sigma_z \rangle|\) are identical in the preparation measurement schemes described by Eqs. (55) and (66). Note also that the time dependence of the decoherence function \(\tilde{\gamma}(t)\) is the same in both cases. The corresponding pairs of lines in Figs. 2 and 3 are therefore just shifted vertically from each other because of the different values of \(\langle \sigma_z \rangle\). Expressions (61) and (67) for \(\langle \sigma_z \rangle\) show that, at all tem-

![FIG. 2: Time evolution of the qubit entropy \(S(t)\) in the Ohmic case for different values of the coupling constant: \(\lambda = 2\) (B and E), \(\lambda = 4\) (C and F), \(\lambda = 6\) (D and G). Filled symbols correspond to the preparation measurement described by Eqs. (55), half-filled – to the measurement described by Eqs. (66). Other parameter values: \(\beta \omega_0 = 1\), \(\omega_0/\omega_c = 0.1\), \(\theta_a = 0\), \(\theta_1 = \pi/4\).](image)

![FIG. 3: Time evolution of the qubit purity \(\mathcal{P}(t)\) in the Ohmic case for different values of the coupling constant. The symbols and the system parameters are the same as in Fig. 2.](image)
temperatures, the preparation scheme (55) leads to less pure states of the qubit as compared to the scheme (66).

Figures 4 and 5 display the evolution of the qubit purity and entropy at different temperatures but for a fixed value of the coupling constant.

V. CONCLUSIONS

In this paper we have examined the properties of the reduced qubit dynamics in cases where the initial state of the composite system (qubit plus environment) is prepared through non-selective quantum measurements. Our main result is that for some preparation schemes the interplay of the measurement process and qubit-environment correlations can lead to a significant enhancement of coherence in the qubit during the initial stage of evolution. The non-trivial feature of this effect is that, in general, a non-selective measurement produces a mixed state of the composite system. Then the purity of the qubit’s state grows while its entropy decreases with time until thermal fluctuations suppress this environmentally induced “purification” process. It deserves to be pointed out that the temperature dependence of the purity growth is determined by several factors. First, the initial coherences (i.e., the off-diagonal elements of the qubit density matrix) generated by a non-selective measurement decrease dramatically with increasing temperature. Second, the destructive effect of thermal fluctuations in the bath also becomes important just at high temperatures. But surprisingly, the dynamical enhancement of coherence due to the qubit-bath correlations grows with temperature, so that the maximum of the coherences $|\langle \sigma^\pm(t) \rangle |$ may be much larger than the initial values $|\langle \sigma^\pm \rangle |$ (see Sec. III). Due to the interplay of the above factors, the resulting purity of the qubit states decreases with temperature, but not so rapidly as one might expect from intuitive considerations.

Summarizing, if the initial state is prepared by a non-selective measurement, it is possible to achieve the environmentally induced purity growth in a qubit during some time interval by setting the measurement device in a proper way. This effect may be of interest in view of its connection with problems of measurement-based quantum control in open quantum systems. We refer, e.g., to the recent paper [23] where the dephasing model (28) was used to study the influence of system-environment correlations on the so-called quantum Zeno and anti-Zeno effects in repeated selective measurements on single-qubit and many-qubit systems.

As a final remark we wish to emphasize that our study of the environmentally induced purity growth does not claim to be complete even on the single-qubit level. Here we will touch briefly on two open questions which deserve further investigation. First, the influence of measurement noise on the so-called quantum Zeno and anti-Zeno effects in repeated measurements on single-qubit and many-qubit systems.
on the assumption that initial correlations between the qubit and the environment are inherited from the pre-measurement equilibrium state due to the presence of the interaction term in the total Hamiltonian $H$. This assumption is adequate in describing many real situations and is commonly accepted in the theory of open quantum systems \[24\], \[26\]. Nevertheless, one may imagine a variety of different correlated initial states by replacing $\rho_{\text{eq}}$ in Eq. (1) by some nonequilibrium premeasurement density matrix $\rho'$. One should note, however, that within such a general formulation of the problem the trace over the bath degrees of freedom in $\langle \sigma_{\pm}(t) \rangle$ cannot be carried out without a detailed information about physically reasonable forms of $\rho'$. The second important point is that the model \[28\] describes only the dephasing mechanism of decoherence. Although this mechanism can dominate in real physical systems \[27\], \[28\], in general the population decay should be taken into account. However, it is a challenging problem because in this case the model is no longer exactly solvable.

**Appendix A: Preparation of pure qubit states through non-selective measurements**

Here we briefly discuss the connection between the non-selective preparation scheme \[25\] [see also Eqs. (62)] and selective measurements. Using the explicit expressions for the $\Omega$-operators, the initial density matrix (11) of the composite system is written as

$$\rho_{SB}(0) = |\vec{b}\rangle\langle\vec{b}| \otimes \rho_B(\vec{a})$$

(B.1)

with the bath density matrix

$$\rho_B(\vec{a}) = \langle \vec{a}| \rho_{eq}| \vec{a}\rangle + \langle -\vec{a}| \rho_{eq}| -\vec{a}\rangle.$$  

(B.2)

Formula (B.1) shows that after a non-selective measurement of this type, the qubit is prepared in a pure state $|\vec{b}\rangle$. Note also that the qubit and the bath are completely uncoupled since the bath state does not depend on the qubit state, and vice versa. Although the bath density matrix (B.2) formally depends on the basis state $|\vec{a}\rangle$ determining the effects $F_i$ in Eqs. (25), it is easy to see that $\rho_B(\vec{a})$ in fact is independent of $|\vec{a}\rangle$. Indeed, since the states $|\vec{a}\rangle$ and $| -\vec{a}\rangle$ form an orthonormal basis, Eq. (B.2) may be rewritten as $\rho_B(\vec{a}) = \text{Tr}_S \rho_{eq}$. If so, the trace can now be calculated with any other orthonormal basis. In particular, it can be done with the canonical states (11), so that the initial bath density matrix takes a universal form

$$\rho_B = \langle 0| \rho_{eq}| 0\rangle + \langle 1| \rho_{eq}| 1\rangle.$$  

(B.3)

Let us now assume that the qubit is initially prepared in some pure state $|\psi\rangle$ through a selective measurement. Then the initial state of the composite system is given by formula (2) or, what is the same, by

$$\rho_{SB}(0) = |\psi\rangle\langle\psi| \otimes \rho_B(\psi)$$

with the initial density matrix of the bath

$$\rho_B(\psi) = \frac{|\psi\rangle\langle \rho_{eq}| \psi\rangle}{\text{Tr}_B \langle \psi| \rho_{eq}| \psi\rangle}.$$  

(B.4)

In this case the bath carries information on the qubit state since its density matrix depends on $|\psi\rangle$ through the interaction term in the Hamiltonian. Consequently, the product (B.4) should be interpreted as a correlated state of the composite system (4). For the dephasing model (28), the bath density matrix (B.5) can be written in a more transparent form. To do this, we note that the equilibrium density matrix $\rho_{eq}$ of the composite system is diagonal with respect to the canonical qubit states (11). Then, writing $|\psi\rangle$ as a decomposition $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$, we obtain for the density matrix (B.5):

$$\rho_B(\psi) = \frac{|c_0|^2 \langle 0| \rho_{eq}| 0\rangle + |c_1|^2 \langle 1| \rho_{eq}| 1\rangle}{|c_0|^2 \text{Tr}_B \langle 0| \rho_{eq}| 0\rangle + |c_1|^2 \text{Tr}_B \langle 1| \rho_{eq}| 1\rangle}.$$  

(B.6)

Since, in general, the bath density matrices (B.5) and (B.6) differ from each other, the time evolution of the composite system depends on the type of the preparation measurement. Suppose, however, that the qubit is prepared selectively in a pure state $|\psi\rangle$ with equal populations of the canonical basis states, i.e., with $\langle \sigma_3 \rangle \equiv \langle \psi| \sigma_3 |\psi\rangle = 0$. In this case we have $|c_0|^2 = |c_1|^2$, so that the density matrices (B.3) and (B.6) coincide. This is the reason why the decoherence dynamics for the non-selective measurement scheme (25) is identical to the decoherence dynamics for a selective measurement with $\langle \sigma_3 \rangle = 0$.

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