Length sensing and control for Einstein Telescope
Low Frequency

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Abstract. In this paper we describe a feasible length sensing and control scheme for the low frequency interferometers of the Einstein Telescope (ET-LF) along with the techniques used to optimise several optical parameters, including the length of the recycling cavities and the modulation frequencies, using two numerical interferometer simulation packages: Optickle and Finesse. The investigations have suggested the use of certain combinations of sidebands to obtain independent information about the different degrees of freedom.

1. Introduction
The Einstein Telescope (ET) is a third-generation gravitational wave observatory with a target sensitivity of more than an order of magnitude higher than the current advanced detectors, advanced LIGO (aLIGO) [1] and advanced Virgo (AdV) [2]. ET will be located underground consisting of three nested detectors, each of which is composed of two interferometers. This xylophone configuration [3] allows the dedication of one interferometer exclusively for low frequency detection (ET-LF) and another one for high frequency detection (ET-HF). Each of the interferometers will adopt different techniques to be able to cope with different challenges. Every individual interferometer will have a dual-recycled Michelson topology with the inclusion of Fabry-Perot Arm cavities.

A control system keeps the interferometer’s lengths as close as possible to the operating point in order to prevent degradation of the strain sensitivity. The signal recycling cavity of ET-LF is designed to have a detuning of 25 Hz [4]. Neither aLIGO nor AdV have been operated with detuning yet and although GEO 600 [5] was operated in a detuned mode its topology (lacking arm cavities) was different. The high finesse of the arm cavities of ET-LF results in an extremely small linear locking range thus posing a challenge to the linear control aspects. In the case of ET-LF, five degrees of freedom (DoFs) must be sensed and controlled in order to bring the interferometer to a locked state and maintain it there. These are i) differential motion of the arm cavities (DARM), ii) common motion of the arm cavities (CARM), iii) differential motion of the central Michelson interferometer (MICH), iv) length change of the power recycling cavity (PRCL), and v) length change of the signal recycling cavity (SRCL). The gravitational wave signal appears only in the DARM channel however, the auxillary DoFs must also be controlled in order to keep the interferometer at its operating point.
Figure 1. ET-LF interferometer design showing different sensing ports and the length DoFs. The carrier (1550 nm) is highlighted in red and the first and second RF modulation sidebands are shown in blue and pink respectively.

2. Simulation Models
In order to calculate the control matrix and the quantum noise limited sensitivity of the interferometer, an interferometer model was developed with two optical simulation softwares called Optickle [6] and Finesse [7]. Both packages have advantages of their own and the two models are in good agreement with each other in terms of the quantum noise limited sensitivity. Most of the parameters required to specify the optical system for example mirror reflectivities, masses, laser power etc. have been taken from the ET design study [4] with the exception of some lengths and tunings which we optimised using the two packages. The details of the optimisations are described in the following sections.

3. Macroscopic lengths, frequencies, DC Readout
3.1. Sideband frequencies
Sidebands are a necessary part of the control of ET-LF. The sidebands serve as additional light fields which see different resonance conditions and this is an essential trick used to decouple error signals for different DoFs. We follow a similar control scheme as the one for aLIGO, using two sidebands which are resonant in one or both recycling cavities in order to distinguish the motion of different mirrors. The first modulation frequency of 11.36 MHz (f1) is resonant in the power recycling cavity only and the second modulation frequency of 56.81 MHz (f2) is resonant in both recycling cavities. Neither of the sidebands are resonant in the arm cavities. For feasibility reasons, the modulation frequencies are limited to the range 10-100 MHz.
3.2. Macroscopic lengths and DC Readout
In order to critically couple the second sideband into the signal recycling cavity, it is necessary to introduce an asymmetry in the arm lengths of the inner Michelson interferometer called Schnupp asymmetry [8]. A compromise was made between maximising the ratio between the second and first sideband powers in the SR cavity and minimising the reflected light power from the interferometer. The power of the f2 sidebands circulating in the signal recycling cavity is maximised by choosing a small asymmetry of 8 cm which came with the added benefit of low laser frequency noise coupling. The optimal asymmetry depends on the choice of SR and PR transmissivities. With an asymmetry of 8 cm the losses for the f2 sidebands in the PR cavity to the asymmetric port are low enough to achieve high sideband gain here; yet the coupling to the SR cavity is high enough to achieve a higher power for the f2 sidebands in the SR cavity than for the f1 sidebands. The SR cavity length is set to 311.585 m in order to maintain the resonance condition for f2. A small intentional offset of about 10 pm is added to each of the end mirrors of the arm cavities to enable DC readout of the gravitational wave signal [9]. DC readout allows the gravitational wave strain to be measured without being affected by the sideband imbalance which is a consequence of the detuning introduced on the signal recycling mirror and also yields lower shot noise.

4. The Control Scheme
4.1. Sensing matrix
As a first step in designing a control scheme for ET-LF, we used an extension of the standard Pound-Drever-Hall length-sensing scheme, in which the formerly chosen radio-frequency (RF) sidebands are imposed (modulation) on the input laser beam (carrier). Following the aLIGO approach [10], we calculated the error signals generated by the beat between the sidebands for the shorter degrees of freedom. The sensing matrix presented in Table 1 indicates the DC optical gain of different DoFs at various sensing ports.

4.2. Sensing Ports/Signal Extraction Ports
There are three main detection ports that we utilised for detecting the error signals: the reflection port (REFL), pick-off port in the PRC (POP) and the anti-symmetric port (AS). REFL is the light coming back to the laser i.e. the symmetric port pick off; POP is the reflection off the AR coating of the central beam splitter; the AS port is the transmission of the signal recycling mirror. The pick off ports are designed to attenuate the power such that none of the photodiodes see more than 100 mW of power.

4.3. The matrix itself
Most of the common arm motion couples into the bright port. Hence, we would expect the CARM and PRCL signal to show up there. However, since the arm cavity finesse is large, the CARM motion dominates the PRCL motion at REFL at f1 thereby allowing us to use this signal for the CARM DoF and PRCL at POP by demodulating at the difference between the two sideband frequencies (denoted by DIFF in Table 1). All the differential arm motion couples into the dark port and hence DARM and MICH error signals show up at the AS port. We use DC readout to read out DARM, and MICH can be read out at the AS port with the demodulation frequency being the sum of f1 and f2. With DARM and MICH controlled independently, SRCL can be controlled at the AS port (even though there is a strong coupling between SRCL, MICH and DARM) by demodulating at the difference between the sideband frequencies (f2-f1). As proposed for aLIGO, we use the double demodulation technique for the short degrees of freedom (MICH, PRCL, SRCL) to derive individual error signals as these signals are not affected by cavity motion owing to the fact that the sidebands are not resonant in the arm cavities.
### Table 1.
Preliminary DC Sensing matrix for detuned ET-LF obtained using FINESSE. Each row corresponds to the slope of the error signal around operating point. AS DC is the DC readout at the AS port. Other signal names consist of a signal port (REFL, POP or AS), the sideband frequency which is used for demodulation and in-phase demodulation is denoted by I and quadrature phase by Q. The preliminarily chosen ports are highlighted in bold.

| DoF       | CARM       | DARM       | MICH       | PRCL       | SRCL       |
|-----------|------------|------------|------------|------------|------------|
| REFLQ1    | 9.49e08    | 1.48e06    | 3.03e03    | -1.65e06   | 1.0e01     |
| ASDC      | 1.55e05    | 3.82e07    | 7.34e05    | -3.29e05   | -3.62e03   |
| ASSUMI    | -1.10e03   | -1.60e04   | -4.09e05   | 1.51e03    | -1.53e04   |
| POPDIFFQ  | -8.42e02   | -2.30e01   | -2.29e03   | -1.17e05   | 4.89e03    |
| ASDIFFI   | 6.09e03    | -2.89e03   | -2.06e04   | -9.66e04   | -1.07e04   |

5. Conclusion and Outlook
With the aid of the numerical simulation tools Finesse and Optickle, we derived a first control matrix on the way to a complete control scheme design for ET-LF. We are now working on a frequency dependent control matrix. Carrier independent error signals are obtained for the control of the shorter DoFs by double demodulation. Optimisation of the demodulation phase of the sensing photodiodes and characterisation of control loop properties in terms of stability and performance is in progress. Several other options such as amplitude modulation schemes, a single sideband approach and the use of an off-resonant sideband for the control of the shorter degrees of freedom are also under consideration.

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