HIGHER-DERIVATIVE TWO-DIMENSIONAL MASSIVE
FERMION THEORIES

L. V. Belvedere*, R. L. P. G. Amaral*, C. G. Carvalhaes** and N. A. Lemos*

* Instituto de Física, Universidade Federal Fluminense
Av. Litorânea, s/n, Boa Viagem, Niterói, CEP: 24210-340, RJ, Brasil.
** Instituto de Matemática e Estatística - Universidade do Estado do Rio de Janeiro
Rua São Francisco Xavier, 524, Maracanã, CEP: 20559-900, Rio de Janeiro, Brasil

(March 28, 2022)

Abstract

We consider the canonical quantization of a generalized two-dimensional massive fermion theory containing higher odd-order derivatives. The requirements of Lorentz invariance, hermiticity of the Hamiltonian and absence of tachyon excitations suffice to fix the mass term, which contains a derivative coupling. We show that the basic quantum excitations of a higher-derivative theory of order \(2N + 1\) consist of a physical usual massive fermion, quantized with positive metric, plus \(2N\) unphysical massless fermions, quantized with opposite metrics. The positive metric Hilbert subspace, which is isomorphic to the space of states of a massive free fermion theory, is selected by a subsidiary-like condition. Employing the standard bosonization scheme, the equivalent boson theory is derived. The results obtained are used as a guideline to discuss the solution of a theory including a current-current interaction.
I. INTRODUCTION

There is a continuing interest in quantum field theories defined by higher-derivative Lagrangians [1]. In spite of their possible shortcomings, such as ghost states and unitarity violation, field theories whose equations of motion are of order higher than the second are useful to regularize ultraviolet divergences [2], especially for supersymmetric gauge theories [3]. In certain higher-derivative string models, negative-norm states and unitarity violation can be avoided because it is possible to define a positive energy operator [4]. The appearance of curvature-squared terms as corrections to the Einstein-Hilbert Lagrangian in the effective action of superstring theories [5] is a further reason why higher-derivative field theories are worth investigating for their own sake, and as such they have been studied from several different points of view in the last few years.

Higher-derivative two-dimensional quantum field theories constitute a useful testing ground for powerful nonperturbative methods such as, for instance, Fujikawa’s technique in the context of path-integral quantization, or exact operator solutions by means of bosonization in the framework of standard canonical quantization [6]. Recently, a higher-derivative generalization of the two-dimensional free fermion theory [6,7] has been constructed and exactly solved by expressing the fermion fields of the model in terms of boson fields (“bosonization”). It turns out that the fermion fields that solve the higher-order equations of motion can be written in terms of usual Dirac fields, the so-called “infrafermions”. Some of these infrafermions, however, need to be quantized with a negative metric, giving rise to an indefinite-metric Hilbert space.

In this paper we study the effect of the inclusion of a mass term on the behavior of these generalized fermion theories. We find that the requirements of Lorentz invariance, absence of tachyon excitations and hermiticity fix the form of the mass term, which differs from the usual one by the appearance of derivative couplings. The model is solved exactly and it so happens that the higher-derivative fermion fields admit only nonlocal mappings from usual fermion fields. With the help of the standard bosonization technique [10,11], the solution is
expressed in terms of a sine-Gordon field and of two massless free scalar fields. These results are then employed to solve a theory with a current-current interaction.

The paper is organized as follows. In Section II we introduce the third order massive fermion quantum field theory. In Section III we discuss it by functional methods. Starting with a so-called interpolating partition function [12], and performing successive field transformations, the interpolating partition function is rewritten in terms of independent fields that satisfy equations of motion of order lower than the third. In this way we extract information regarding the physical content of the theory described by the original third-order partition function. Guided by these results, in Section IV the third-order massive fermion theory is quantized by the canonical procedure. The physical Hilbert space of positive defined norm, which is isomorphic to the Hilbert space of the free massive Fermi theory, is construct and the physical states are accomodated as equivalent classes. Section V is devoted to finding the equivalent boson theory. In Section VI a theory including a current-current interaction is discussed. Section VII is dedicated to general comments and conclusions.

II. DESCRIPTION OF THE MODEL

In a previous work [11] we introduced a higher-derivative theory of a free massless fermion by means of the Lagrangian density

\[ \mathcal{L}^{(0)}(x) = -i\xi(x)(\bar{\psi}\gamma^5\psi)(x). \]  

Our conventions are:

\[ \psi = (\psi_1, \psi_2)^T, \quad \epsilon^{01} = g^{00} = -g^{11} = 1, \quad \bar{\psi} = \epsilon^{\mu\nu} \partial_\nu, \quad x^\pm = \frac{x^0 \pm x^1}{2}, \quad \partial_\pm = \partial_0 \pm \partial_1, \]

\[ \gamma^\mu \gamma^5 \equiv \epsilon^{\mu\nu} \gamma^\nu, \quad \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \]
This model was analyzed and it was found that its basic excitations consist of \((2N + 1)\) independent canonical Dirac fields (\textit{the infrafermions}), some quantized with positive metric and some with negative metric. These infrafermions were expressed in terms of scalar fields through the Mandelstam formula, so that the bosonization mechanism was generalized for higher derivative fields. As an application, the gauged version (higher-derivative \(QED_2\)) was exactly solved. In usual first-order fermionic theories the mass term appears as a nontrivial interaction term after bosonization. It is highly desirable to uncover the new features that arise from the introduction of a mass term for the higher-derivative fermion field. To this end we shall consider the Lagrangian density

\[
L_0(x) = -i\bar{\xi}(x)(\partial \partial^\dagger)N \partial\xi(x) + m\bar{\xi}(x)(\partial \partial)^N \xi(x), \quad (N \geq 0),
\]

where \(m\) is a parameter with dimension of mass.

In this Lagrangian density, the mass term is an even-order derivative coupling which prevents the appearance of tachyon excitations, preserves Lorentz invariance of \(L_0\) and provides a Hermitian Hamiltonian. Indeed, the Lagrangian density (2) is a generalization of the usual massive Lagrangian and is obtained from it through the transformation

\[
\psi \rightarrow \gamma^0 \partial \xi,
\]

\(\psi\) being the canonical massive free Dirac field. However, we remark that the transformation (3) cannot be seen as a solution to the model mapping the higher derivative field \(\xi\) to a lower derivative (infrafermion) field \(\psi\). It will be shown that the general quantum solution of the higher-derivative theory consist of a usual massive fermion, quantized with positive metric, plus \(2N\) usual massless fermions, quantized with opposite metrics. We will find the expression for the field \(\xi\) in terms of the infrafermions with canonical and functional integral methods. It means that (3) does not provide, upon inversion, a direct solution for the higher-derivative theory, since it does not allow the identification of the massless modes that contribute to the correlation functions of the phase space variables that define the quantum theory and that generate the complete space of states.
The complexity involved here is greater than in the massless case \cite{6}, where the spinor components decouple and are treated independently. Therefore, in order to avoid unnecessarily complicated expressions, instead of considering the fairly general form (1), we shall restrict ourselves to the third-order theory ($N = 1$). In this case, one can verify, by solving the equations of motion, that a generalization of (1) with mass term like $m\bar{\xi}\xi$ leads to tachyons, and no first-derivative mass term like $m\xi\frac{\partial}{\partial x} \xi$ respects Lorentz invariance, excluding non-local terms like $m\xi\sqrt{\partial\phi\phi} \xi$.

Using the light-cone variables, the Lagrangian density is written as

$$L_0(x) = -i\bar{\xi}_1(x) \frac{\partial^3}{\partial^3^*} \xi_1(x) - i\bar{\xi}_2(x) \frac{\partial^3}{\partial^3^*} \xi_2(x) + m\bar{\xi}_1(x) \frac{\partial}{\partial x} \xi_1(x) + m\bar{\xi}_2(x) \frac{\partial}{\partial x} \xi_2(x).$$

(4)

Following the discussion in \cite{7}, under a Lorentz transformation $x^+ \to \lambda x^+$ and $x^- \to \lambda^{-1} x^-$ we define $\xi_{(1,2)} \to \lambda^{\mp 3/2} \xi_{(1,2)}$, so as to ensure Lorentz invariance of the theory.

### III. FUNCTIONAL APPROACH

In order to obtain some insight on the general solution for the higher-derivative field $\xi$ in terms of infrafermions, we carry out a functional treatment. Our starting point is the partition function

$$Z_0 = \int D\bar{\xi}D\xi \exp \left\{ i \int d^2x \left[ -i\bar{\xi}(x)\frac{\partial}{\partial x} \phi \xi(x) + m\bar{\xi}(x)\frac{\partial}{\partial x} \phi \xi(x) \right] \right\}.$$

(5)

In order to display a systematic functional procedure to decouple the higher-derivative theory by lowering the derivative order, we follow the procedure of Ref. \cite{12} and introduce an "interpolating" partition function $Z_I$, defined by

$$Z_I = \int D\bar{\xi}D\xi D\bar{\chi}D\chi \exp \left\{ i \int d^2x \left[ -i\bar{\xi}(x)\frac{\partial}{\partial x} \phi \chi(x) + m\bar{\xi}(x)\frac{\partial}{\partial x} \phi \chi(x) \right] \right\}.$$

(6)

The partition function $Z_I$ describes an enlarged theory and is connected to $Z_0$ by a weight factor corresponding to the partition function of an extra ghost degree of freedom. This can be seen by performing in Eq. (5) the shift

\[5\]
\[ \chi = \chi' + \gamma^0 \partial \xi, \quad (7) \]

which yields

\[ Z_i = Z_0 \times \left( \int D\chi' D\chi e^{i \int d^2x ( -i \nabla \partial \chi')} \right). \quad (8) \]

The additional degree of freedom is associated with a free massless fermion field quantized with negative metric.

After having displayed the connection between the interpolating partition function \( Z_i \) and the partition function \( Z_0 \), describing the original higher-derivative theory, we can use \( Z_i \) as a starting point to decouple the third-order theory by the successive lowering of the derivative order.

First we shall reduce the order of the third-order derivative term. To begin with, we diagonalize the partition function \( Z_i \) introducing the transformation

\[ \partial \xi = \partial \xi' + \frac{i}{m} \gamma^0 \partial \chi, \quad (9) \]

In this way we reduce \( Z_i \), as given by (6), to a partition function of a theory with a lowered derivative of second order,

\[ Z_i = \int D\xi' D\xi D\chi D\chi \exp \left\{ i \int d^2x \left[ -i \nabla \partial \chi - m \nabla \partial \chi + m \nabla \partial \chi \right] \right\}. \quad (10) \]

The next step is the reduction of \( Z_i \) to a first order theory. Following the same procedure, in order to obtain a further reduction of (10), we introduce a second interpolating partition function \( Z_i' \), defined by

\[ Z_i' = \int D\xi' D\xi D\chi D\chi D\psi D\psi \exp \left\{ i \int d^2x \left[ -i \nabla \partial \chi - m \nabla \partial \psi - i \nabla \partial \chi - i \nabla \partial \psi + m \nabla \partial \chi \right] \right\}. \quad (11) \]

\[ \text{This can be seen as} \]

\[ \xi = \xi' + \frac{i}{m} \frac{1}{\Box} \gamma^0 \partial \chi. \]

In spite of nonlocality it is clear that there appears no Jacobian in this transformation.
The connection between the partition functions $Z'_I$ and $Z_I$ can be displayed by performing in (11) the transformation

$$\psi = \psi' - (i/m)\partial \chi,$$

which yields

$$Z'_I = Z_I \int D\psi' D\psi e^{i \int d^2 x (-m\overline{\psi}' \psi')}.$$  \hspace{1cm} (13)

Since $\psi'$ is not a dynamical field, the partition functions $Z'_I$, given by (11), and $Z_I$, given by (6), have the same number of degrees of freedom.

Performing the transformation $\chi = \chi'' - \psi$ in (11), we obtain

$$Z'_I = \int D\chi'' D\chi'' D\xi' D\psi D\psi \exp \left\{ i \int d^2 x \left[ -i\overline{\chi}'' \partial \chi'' + i\overline{\psi} \partial \psi - m\overline{\psi} \psi + m\overline{\xi} \partial \partial \xi' \right] \right\}. \hspace{1cm} (14)$$

In this way, we have reexpressed the interpolating partition function $Z_I$ in terms of a product of decoupled partition functions for the massless field $\chi''$, quantized with negative metric, the canonical massive fermion field $\psi$ and a second order massless field $\xi'$. We must remark that although the decoupling procedure to lowering the third-order term just replaces the original second-order mass term $m\overline{\xi} \partial \partial \xi'$ by $m\overline{\xi} \partial \partial \xi'$, its presence in the original theory is crucial to perform the transition to the first-order fields $\chi''$ and $\psi$.

The last step is the reduction of the second-order theory $m\overline{\xi} \partial \partial \xi'$ to first order. This can be done by adding a term $\delta \overline{\xi} \partial \partial \xi'$ to the second order theory and, at the final stage of the derivative-order lowering procedure, taking the limit $\delta \to 0$. This is done in detail in Appendix B.

The counting of the resulting number of dynamical degrees of freedom involved in the interpolating theories gives two massless fields $\chi''$, $\xi''$, quantized with negative metric, one massless field $\psi'$, quantized with positive metric and one massive Dirac field $\psi$. Since the enlarged interpolating theories possess one more negative metric dynamical degree of freedom than the original theory, we can infer that the third-order theory described by $Z_a$ contains two massless fields quantized with opposite metrics and a massive free fermion field.
As a matter of fact, the usefulness of the decoupling procedure depends, however, on generalizing it from the partition functions to the generating functionals. To this end we consider the generating functional

$$Z_0[\eta_i, \eta_j] = \int D\xi D\xi \exp \left\{ i \int d^2x \left[ -i\overline{\xi}(x)\phi_0 \phi_0 \xi(x) + m\overline{\xi}(x)\phi_0 \phi_0 \xi(x) + \right. \right.$$  
$$+ \eta_1 \xi + \eta_2 \phi^0 \phi_0 + \eta_3 \phi^0 \phi_0 + h.c. \right\},$$  

(15)

This is the functional that generates the whole set of correlation functions of the phase space variables $\xi, \phi^0 \phi_0$ and $\phi^0 \phi_0$. For the interpolating theory, we consider the generating functional

$$Z_I[\eta_i, \eta_j, \eta_j, j, j] = \int D\xi D\xi D\chi D\chi \exp \left\{ i \int d^2x \left[ -i\overline{\chi}(x)\chi + i\chi \phi^0 \phi_0 \xi(x) - i\overline{\xi}(x)\phi^0 \phi_0 \chi(x) + \right. \right.$$  
$$+ m\overline{\xi}(x)\phi_0 \phi_0 \xi(x) + \eta_1 \xi + \eta_2 \phi^0 \phi_0 + \eta_3 \phi^0 \phi_0 + h.c. + \overline{\chi} + \chi j \right\}. $$  

(16)

Performing the transformation (7) on the interpolating generating functional (16), although the source $j$ couples with $\chi'$ and $\phi^0 \phi_0$ we see that, up to a normalization factor corresponding to the partition function of the field $\chi'$, the two generating functionals (15) and (16) are isomorphic in the space generated by the functional derivatives of the sources $\eta_i$ and $\eta_j$, leading to the same correlation functions. Thus, for any polynomial $\mathcal{O}$ of the functional derivatives of $\eta_i$, we obtain

$$\frac{1}{Z_0[0]} \left. \mathcal{O} \left\{ \frac{\delta}{\delta \eta_i}, \frac{\delta}{\delta \eta_j} \right\} Z_0[\eta_i, \eta_j] \bigg|_{\eta_i=\eta_j=0} \right. = \frac{1}{Z_I[0]} \left. \mathcal{O} \left\{ \frac{\delta}{\delta \eta_i}, \frac{\delta}{\delta \eta_j} \right\} Z_I[\eta_i, \eta_j, j, j] \bigg|_{\eta_i=\eta_j=j=0} \right. $$  

(17)

Following the same steps from (5) to (14) we can perform the decoupling mechanism for the generating functional. For the source term associated with the field $\xi$ we get

$$\eta_i \xi = \eta_i \left[ \xi' + \frac{i}{m} \overline{\phi^0 \phi_0} (\chi'' - \psi) \right].$$  

(18)

---

3See section III for definition of the phase space variables.
The correlation function for $\xi$ can be obtained from Eq.(18). Indeed, making use of $\chi = \chi'' - \psi$ we find

$$
\langle \xi \overline{\xi} \rangle_0 = \langle \xi \overline{\xi} \rangle_0 + \frac{1}{m^2} \left( \frac{\partial \phi^0}{\partial \phi^0} - \chi \frac{\partial \phi^0}{\partial \phi^0} \right)_0
= \frac{1}{m^2} + \frac{1}{m^2} \left( -\frac{1}{i\phi^0} + \frac{1}{m^2} \right) \frac{\partial \phi^0}{\partial \phi^0} \\
\equiv \frac{1}{-i\phi^0\phi^0 + m^2}
$$

which coincides with the inverse of the operator that appears in the original formulation of the third order theory.

Note that Eq.(18) suggests that the representation of $\xi$ in terms of infrafermions is nonlocal. This result will be confirmed by the operator approach to be taken up next. We remark however that Eq.(18) is not an infrafermion expansion for $\xi$. It provides the correct dimensional and Lorentz properties of $\xi$ but its dynamical aspects are somewhat obscure within this context. The physical aspects of the model shall become more clear within the canonical procedure.

**IV. CANONICAL QUANTIZATION**

In order to perform the canonical quantization of the theory, we must obtain the basic Poisson brackets. In accordance with the third-order character of the Lagrangian density (4), we take $\xi(1)$, $\partial_+ \xi(1)$, $\partial^2 \xi(1) + im\partial_+ \xi(2)$, $\xi(2)$, $\partial_+ \xi(2)$, $\partial^2 \xi(2) + im\partial_+ \xi(1)$ as the basic space phase variables. The associated canonical momenta 4, obtained by variation of the action around the equations of motion, are: $-i\partial^2 \xi^* - m\partial_+ \xi^*$, $i\partial_+ \xi^*$, $-i\xi^*_{(1)}$, $-i\partial^2 \xi^*$, $m\partial_+ \xi^*_{(1)}$, $i\partial_+ \xi^*_{(2)}$, $-i\xi^*_{(2)}$, respectively.

Using these variables, a systematic quantization (carried out using either a Dirac bracket formalism, if $\xi$ is treated as an independent variable, or Poisson brackets, if $\xi$ is taken as a function of $\xi$) furnishes the following nonvanishing equal-time anticommutators

4 Our choice for basic variables, different from the one in Ref. [4], has the advantage of providing momenta with homogenous Lorentz properties for the massive case.
\[ \{ \xi_{(1)}(x), \partial_+^2 \xi_{(1)}^*(y) \} = -\{ \partial_- \xi_{(1)}(x), \partial_- \xi_{(1)}^*(y) \} = -\delta(x^1 - y^1), \]
\[ \{ \partial_- \xi_{(1)}(x), \partial_+ \xi_{(1)}^*(y) \} = \frac{i}{m} \{ \partial_- \xi_{(1)}(x), \partial_- \xi_{(1)}^*(y) \} = im\delta(x^1 - y^1). \tag{20} \]

The other nonvanishing equal-time anticommutators are obtained from \[\text{(20)}\] by switching the spinor indices and \(x^1\) to \(-x^1\).

Introducing the Fourier decomposition

\[ \xi_{(\alpha)}(x) = \int d^2k e^{-ikx} \tilde{\xi}_{(\alpha)}(k), \tag{21} \]

we obtain the general solution of the equations of motion

\[ \tilde{\xi}_{(1)}(k) = a(k)\delta(k^2 - m^2) + b_{(2)}(k_-)\delta(k_-) + c_{(1)}(k_+)\delta(k_+) - \frac{k^2}{m} b_{(1)}(k_+) \frac{d}{dk_+} \delta(k_+), \]
\[ \tilde{\xi}_{(2)}(k) = \frac{k^4}{m^2} a(k)\delta(k^2 - m^2) + b_{(1)}(k_-)\delta(k_-) + c_{(2)}(k_+)\delta(k_+) - \frac{k^2}{m} b_{(2)}(k_-) \frac{d}{dk_-} \delta(k_-). \tag{22} \]

With the help of fields \(\chi^i\), with dispersion relations described by

\[ \chi_{(1,2)}(x) = -i \int d^2k a(k) k_\mp \delta(k^2 - m^2) e^{-ikx}, \]
\[ \partial_\pm^{-1} \chi_{(1,2)}^2(x) = \int dk_\pm c_{(1,2)}(k_\pm) e^{-ik_\pm x_\pm}, \]
\[ \partial_\pm^{-1} \chi_{(1,2)}^3(x) = \int dk_\pm b_{(1,2)}(k_\pm) e^{-ik_\pm x_\pm}, \tag{23} \]

and consistently defining the operators \[\text{5}\]

\[ \partial_\mp^{-1} \chi_{(1,2)}^1(x) = -\frac{1}{m^2} \partial_\pm^2 \chi_{(1,2)}^1(x), \tag{24} \]

we come back to the configuration space arriving at

\[ \xi_{(1)}(x) = \partial_\pm^{-1} \chi_{(1)}^1(x) + \partial_\mp^{-1} \chi_{(1)}^2(x^+) + \partial_\pm^{-1} \chi_{(1)}^3(x^-) + i \frac{2}{m} \partial_\mp \chi_{(1)}^3(x^+), \]
\[ \xi_{(2)}(x) = \partial_\mp^{-1} \chi_{(2)}^1(x) + \partial_\pm^{-1} \chi_{(2)}^2(x^-) + \partial_\mp^{-1} \chi_{(2)}^3(x^+) + i \frac{2}{m} \partial_\mp \chi_{(2)}^3(x^-). \tag{25} \]

\[\text{5}\] A convenient definition for the non-local operator \(\partial^{-1}\) acting on the massless modes is \(\partial^{-1}_\pm \chi_{(1,2)} = 2^{-1} \int_{-\infty}^{x_\pm} dz \chi_{(1,2)}.\) However, for calculating physical quantities, other definitions can be applied, so long as the identity \(\partial_\pm \left\{ \partial_\pm^{-1} \chi_{(1,2)} \right\} = \chi_{(1,2)}\) is ensured to hold.
The mode $\chi^1$ is massive, whereas $\chi^2$ and $\chi^3$ are massless. In the general case (1) this decomposition would generate one massive mode and $2N$ other massless modes. Tachyon excitations do not appear.

Inverting the relations (26) and using the anticommutation laws (20) we obtain the following nonvanishing anticommutation relations

\[
\{\chi^1_{(\alpha)}(x), \chi^1_{(\alpha)}(y)\} = \delta(x^1 - y^1),
\]
\[
\{\chi^2_{(\alpha)}(x), \chi^2_{(\alpha)}(y)\} = -\frac{16}{m^4} \partial^4 \delta(x^1 - y^1),
\]
\[
\{\chi^3_{(\alpha)}(x), \chi^3_{(\alpha)}(y)\} = -\frac{2i \gamma_5}{m} \partial^4 \delta(x^1 - y^1). \tag{26}
\]

The mode $\chi^1$ is a free massive Dirac field quantized with positive metric and the other two modes are noncanonical, $\chi^3$ being quantized with null metric. Nevertheless, the anticommutation structure (29) can be cast into a diagonal form by introducing a free massive field $\psi^1$ and two other free massless fields $\psi^2$ and $\psi^3$ quantized with opposite metrics

\[
\{\psi^1(x), \psi^1(y)\} = \{\psi^2(x), \psi^2(y)\} = -\{\psi^3(x), \psi^3(y)\} = \delta(x^1 - y^1). \tag{27}
\]

In terms of these fields we have (see Appendix A),

\[
\chi^1_{(\alpha)}(x) = \psi^1_{(\alpha)}(x),
\]
\[
\chi^2_{(1,2)}(x) = \left[ \frac{1}{2M^{p+1}} \partial_\pm^p + (-1)^p \frac{M^{p+1}}{2m^4} \partial_\mp^{3-p} \right] \psi^2_{(1,2)}(x)
\]
\[
+ \left[ \frac{1}{2M^{p+1}} \partial_\mp^p - (-1)^p \frac{M^{p+1}}{2m^4} \partial_\pm^{3-p} \right] \psi^3_{(1,2)}(x),
\]
\[
\chi^3_{(1,2)}(x) = i(-1)^p \frac{M^{p+1}}{m} \partial_\mp^p \psi^2_{(1,2)}(x) - i(-1)^p \frac{M^{p+1}}{m} \partial_\pm^p \psi^3_{(1,2)}(x), \tag{28}
\]

where $M$ is an arbitrary parameter of same dimension of $m$, and $p$ an arbitrary integer.

Under Lorentz transformations (7) we require that $M \to \lambda^{(1-p)/(p+1)} M$. Using this mapping, the original general solution turns out to be

\[
\xi_{(1,2)}(x) = -\frac{1}{m^2} \partial_\pm \psi^1_{(1,2)}(x) + i(-1)^p \frac{M^{p+1}}{m} \partial_\mp^{3-p} \left[ \psi^2_{(2,1)}(x) - \psi^3_{(2,1)}(x) \right] + 
\]
\[
+ \left[ \frac{1}{2M^{p+1}} \partial_\pm^p + (-1)^p \frac{M^{p+1}}{2m^4} \partial_\mp^{3-p} - (-1)^p \frac{M^{p+1}}{m^2} x^\mp \partial_\pm^{1-p} \right] \psi^2_{(1,2)}(x)
\]
\[
+ \left[ \frac{1}{2M^{p+1}} \partial_\mp^p - (-1)^p \frac{M^{p+1}}{2m^4} \partial_\pm^{2-p} + (-1)^p \frac{M^{p+1}}{m^2} x^\pm \partial_\mp^{1-p} \right] \psi^3_{(1,2)}(x), \tag{29}
\]
and is not a genuine operator-valued field. Note that it is impossible to adjust $p$ to describe the original fields locally in terms of usual fermions, while the corresponding relationship is local in the massless case [6,7]. However, the choice $p = 1$ appears to be particularly convenient since it makes the parameter $M$ a scalar under Lorentz transformations.

The correlation functions do not depend on $p$ or $M$:

$$
\langle 0| \xi_{(1)}^I (x) \xi_{(1)}^I (y) |0 \rangle = \frac{1}{m^4} \frac{\partial_x^2 \partial_y^2}{2 \pi m^2} S^{(+)}(x-y)_{11} - \frac{i}{\pi m^4} \frac{1}{(x^+ - y^+ - i\epsilon)^3} \frac{1}{(x^- - y^-)} ; \tag{30}
$$

$$
\langle 0| \xi_{(1)}^I (x) \xi_{(2)}^I (y) |0 \rangle = \frac{1}{m^4} \frac{\partial_x^2 \partial_y^2}{4 \pi m} \int_{-\infty}^{x^+} dz^+ \frac{1}{z^+ - y^+ - i\epsilon} . \tag{31}
$$

Wightman functions satisfying positive-definiteness and the cluster decomposition property can be obtained by considering correlations between appropriate derivatives of $\xi$, which represent genuine operator-valued fields. For example:

$$
\langle 0| \partial_- \xi_{(1)}^I (x) \partial_- \xi_{(1)}^I (y) |0 \rangle = S^{(+)}(x-y)_{11} ,
$$

$$
\langle 0| \partial_- \xi_{(1)}^I (x) \partial_+ \xi_{(2)}^I (y) |0 \rangle = S^{(+)}(x-y)_{12} . \tag{32}
$$

If one recalls the remarks concerning Eq.(3), it is not surprising that these derivatives of $\xi$ have the physical properties of the first order massive fermion field components. Indeed taking the corresponding derivatives of Eqs.(29) the massless infrafermions appear only in the combination $(\psi^2 - \psi^3)$ giving thereof no contribution to the correlation functions. On the other hand, although $\xi$ does not represent a genuine spinor operator-valued field the corresponding spinorial field nature is carried by the infrafermion fields $\psi^i$, which ensure for instance the correct microcausality requirements. In this sense, the general quantum field features of the general solution $\xi$ are implemented by the infrafermion operators. The (infrafermion) massless operators are nevertheless nonlocal functions of the original fields.

According to the general principles of QFT the Hilbert space of any theory should be constructed from the basic field operators of the theory [6,7]. They make up the basic building blocks, and in terms of local functions of them the algebra of fields is defined. In a theory
with fields obeying higher-derivative equations the algebra of fields should be enlarged by including all independent derivatives of the fields. Computing their correlation functions and then building the Hilbert space is the route to discussing the physical content of the theory. Under this perspective the infrafermion fields are in principle artifacts to obtain the solution and it is not granted that they represent “physical” properties of the model. In our case the basic field is $\xi$. But as we have seen $\xi$ is an unacceptable field from which to build a physical Hilbert space since general rules of QFT such as positivity and cluster decomposition property would be violated. To cure these pathologies let us construct the physical sub-algebra only through the derivatives $\partial -\xi_1$ and $\partial +\xi_2$ and local functions of them. Since the massless fields do not contribute to correlation functions of the derivatives of the field $\xi$ the algebra of fields becomes isomorphic to the algebra of the massive infrafermion field. The higher-derivative theory is reduced in the free case to the usual massive fermion field.

The phase space variables and their associated momenta generate an indefinite metric Hilbert space $\mathcal{H}$ violating the cluster decomposition property. The *physical* Hilbert subspace $\tilde{\mathcal{H}}$ is selected by requiring that

$$\tilde{\mathcal{H}} = \left\{ |\Phi\rangle \in \mathcal{H} \mid (\psi_2^\alpha - \psi_3^\alpha)|\Phi\rangle = 0 \right\},$$

where the field combination $(\psi^2 - \psi^3)$ generates from the vacuum $\Psi_o$ zero norm states

$$(\Psi_o(\psi_2^\alpha(x) - \psi_3^\alpha(x)), (\psi_2^\alpha(y) - \psi_3^\alpha(y))\Psi_o) = 0.$$

This condition ensures that the states generated from the derivatives of the field $\xi$

$$(\partial^n \xi) \Psi_o \in \tilde{\mathcal{H}}, \ n \geq 1,$$

and the quocient space $\hat{\mathcal{H}} = \tilde{\mathcal{H}}/\mathcal{H}^o$ is isomorphic to the positive definite metric Hilbert space of the free massive Fermi theory.
1. The Generator of Global Gauge Symmetry

The conserved current associated with the global gauge symmetry is given by the products of fields and conjugate momenta. The light-cone components are

\begin{align*}
    j^- &= i \xi^{(1)} (i \partial_+^2 \xi^{*}_{(1)} + m \partial_- \xi^{*}_{(1)}) + (\partial_- \xi^{(1)})(\partial_+ \xi^{*}_{(1)}) - i(-i \partial_+^2 \xi^{(1)} + m \partial_- \xi^{*}_{(1)}) \xi^{*}_{(1)}, \\
    j^+ &= i \xi^{(2)} (i \partial_-^2 \xi^{*}_{(2)} + m \partial_+ \xi^{*}_{(2)}) + (\partial_+ \xi^{(2)})(\partial_- \xi^{*}_{(2)}) - i(-i \partial_-^2 \xi^{(2)} + m \partial_+ \xi^{*}_{(2)}) \xi^{*}_{(2)}.
\end{align*}

Using these expressions and the diagonal expansions of \( \xi \) and dropping out total derivatives we obtain

\begin{equation}
    j^\mu(x) = \bar{\psi}^1(x) \gamma^\mu \psi^1(x) + \bar{\psi}^2(x) \gamma^\mu \psi^2(x) - \bar{\psi}^3(x) \gamma^\mu \psi^3(x).
\end{equation}

The charge operator becomes the sum of the charges of the infrafermions. Indeed defining

\begin{equation}
    Q = \int dz \, j^0(z),
\end{equation}

it is straightforward to show from (35), (29) and (34) that

\begin{equation}
    \{Q, \xi(x)\} = -\xi(x).
\end{equation}

V. BOSONIZATION

As emphasized in [7], the Hamiltonian \( \mathcal{H}_0 \) obtained from the Legendre transformation of the Lagrangian (4) evolves the \( \xi \) field. The Hamiltonian \( \mathcal{H}'_0 \) evolving the infrafermions is obtained from it by recognizing the time-dependent relationship between the basic variables and the infrafermions as a point transformation. The generating function may be constructed as in [7] and the Hamiltonian \( \mathcal{H}'_0 \) computed. The result is the Hamiltonian for the three independent and canonical (except for metrics) first-derivative infrafermions:

\begin{equation}
    \mathcal{H}'_0 = -i \bar{\psi}^1 \gamma^1 \partial_\psi^1 - i \bar{\psi}^2 \gamma^1 \partial_\psi^2 + i \bar{\psi}^3 \gamma^1 \partial_\psi^3 + m \bar{\psi}^1 \psi^1.
\end{equation}

By means of a Legendre transformation one finds
\[ L'(x) = \bar{\psi}^1(x)(i\partial - m)\psi^1(x) + \bar{\psi}^2(x)(i\partial)\psi^2(x) - \bar{\psi}^3(x)(i\partial)\psi^3(x). \] (38)

It is the Hamiltonian (37) and the infrafermions that we are going to bosonize. The bosonization scheme we employ is the standard one [10,11]. Using the Mandelstam representation [10] for the Fermi field operators, we obtain

\[
\psi^1_{(\alpha)}(x) = \left( \frac{\mu}{2\pi} \right)^{1/2} e^{-i\frac{\pi}{4} \gamma^a_{\alpha}} \mathcal{K}(\phi_2) \mathcal{K}(\phi_3) : e^{-i\sqrt{\pi} \int_{-\infty}^{x} dz_1 \pi_1(z) + \gamma^a_{\alpha} \phi_1(x)} : ,
\]

\[
\psi^2_{(\alpha)}(x) = \left( \frac{\mu}{2\pi} \right)^{1/2} \mathcal{K}(\phi_1) \mathcal{K}(\phi_3) : e^{-i\sqrt{\pi} \int_{-\infty}^{x} dz_1 \pi_2(z) + \gamma^a_{\alpha} \phi_2(x)} : ,
\]

\[
\psi^3_{(\alpha)}(x) = \left( \frac{\mu}{2\pi} \right)^{1/2} \mathcal{K}(\phi_1) \mathcal{K}(\phi_2) \hat{\mathcal{K}}(\phi_3) : e^{-i\sqrt{\pi} \int_{-\infty}^{x} dz_1 \pi_j(z) + \gamma^a_{\alpha} \phi_j(x)} : ,
\]

where \( \mu \) is an arbitrary finite mass scale, \( \phi_2 \) and \( \phi_3 \) are free and massless scalar fields, \( \phi_1 \) is a sine-Gordon field and \( \pi_j = \dot{\phi}_j \). The Klein factors that ensure the correct anticommutation relations between the independent fields \( \psi^j(x) \) are given by

\[
\mathcal{K}(\phi_j) = e^{i\frac{\pi}{2} \int_{-\infty}^{x} dz_1 \pi_j(z)}.
\] (40)

Opposite metrics are also ensured by Klein factors [6]. The Klein factor \( \hat{\mathcal{K}}(\phi_3) \) is introduced to ensure the negative metric for the field \( \psi^3(x) \) and is defined by

\[
\hat{\mathcal{K}}(\phi_3) = e^{i\pi \int_{-\infty}^{x} dz_1 \pi_3(z)}.
\] (41)

The equivalent boson field theory Hamiltonian is

\[
\mathcal{H}_0^B = \frac{1}{2} \left[ \pi_1^2 + (\partial_1 \phi_1)^2 \right] - \frac{m}{\pi} \mu \cos(2\sqrt{\pi} \phi_1) + \frac{1}{2} \left[ \pi_2^2 + (\partial_1 \phi_2)^2 \right] + \frac{1}{2} \left[ \pi_3^2 + (\partial_1 \phi_3)^2 \right].
\] (42)

For the conserved current (34) we find

\[
J^\mu(x) = -\frac{1}{\sqrt{\pi}} \varepsilon^{\mu \nu \rho} \partial_\nu \{ \phi_1(x) + \phi_2(x) + \phi_3(x) \}.
\] (43)

The bosonization of the higher-derivative fermion fields \( \xi(x) \) is obtained by using (39) in (29).
VI. CURRENT-CURRENT INTERACTION

The discussion in the final of section III suggests that the higher derivative theory, when implemented in such a way to avoid unphysical properties, reduces to the first derivative theory. It is desirable to address this issue when there is interaction. Under the functional integral framework all decoupling procedure can be reproduced if the fermion couples minimally with a gauge field. One should essentially replace usual by covariant derivatives. The introduction of extra degrees of freedom in the decoupling partition function becomes now more critical. We obtain the original partition function from the decoupling one discarding the partition function of the Schwinger model with inverse fermion metric instead of the free fermion with inverted metric. It is more suitable to proceed to a study in the canonical formalism.

In order to simplify matters let us consider here the current-current interaction. Consider the theory described by

$$\mathcal{L}_1(x) = \mathcal{L}_0(x) + g j^+(x) j^-(x), \quad (44)$$

where $\mathcal{L}_0$ is the Lagrangian density (4), $j^\pm$ are given by (33), $g$ is a constant and all the fields are in the Heisenberg picture. This is an extension of the Thirring model as a third-order Lagrangian theory. A natural candidate to be the infrafermion Lagrangian density for this theory is built by adding the current-current interaction $j^\mu j^\mu$ with $j^\mu$ given by (20) to the Lagrangian density (38) in the Heisenberg picture. This identification is correct in the interaction picture, since the solution (29) has led us to relate the third-order Lagrangian density (4) to the first-order fermion theory (38) and the current (33) to (34). However, it is not clear that this direct identification remains true in the Heisenberg picture. It depends on generalizing the solution (29), an issue we do not address here. In order to gain insight into this new theory we shall consider the first-order fermion theory

$$\mathcal{L} = \bar{\Psi}^1 (i\partial - m)\Psi^1 + \bar{\Psi}^2 (i\partial)\Psi^2 - \bar{\Psi}^3 (i\partial)\Psi^3 - g(\bar{\Psi}^1 \gamma^\mu \Psi^1 + \bar{\Psi}^2 \gamma^\mu \Psi^2 - \bar{\Psi}^3 \gamma^\mu \Psi^3)^2. \quad (45)$$

From now on, we shall use lowercase letters to denote fields in the interaction picture and
the uppercase ones to those in the Heisenberg picture. We have thus been led to a Thirring model with global $SU(2, 1)$ symmetry explicitly broken by the mass term.

Following \[11\], in the interaction picture the current-current term (34) should be written as

$$H_{I}^{B} = \frac{g}{2} \left[ (j_{1F}^{0})^{2} - \lambda (j_{1F}^{1})^{2} \right],$$  \hspace{1cm} (46)

where $\lambda$ is a parameter that has to be introduced in the interaction picture and is fixed by requiring Lorentz invariance. The subscript $F$ was inserted in order to emphasize that $j_{\mu}^{\nu}$ is a functional of free quantities, since we are in the interaction picture. After bosonization we find that

$$H_{I} = \frac{g}{2} \left\{ (\partial_{1} \phi_{1} + \partial_{1} \phi_{2} - \partial_{1} \phi_{3})^{2} - \lambda (\pi_{1} + \pi_{2} - \pi_{3})^{2} \right\}.$$ \hspace{1cm} (47)

The full Heisenberg picture bosonized Hamiltonian density is immediately found:

$$H = H_{0}^{\prime}[\Phi, \Pi] + H_{I}[\Phi, \Pi] = \frac{1}{2} \left[ \Pi_{1}^{2} + (\partial_{1} \Phi_{1})^{2} \right] - \frac{m}{\pi} \mu \cos(2\sqrt{\pi} \Phi_{1}) + \frac{1}{2} \left[ \Pi_{2}^{2} + (\partial_{2} \Phi_{2})^{2} \right]
+ \frac{1}{2} \left[ \Pi_{3}^{2} + (\partial_{1} \Phi_{3})^{2} \right] + \frac{g}{2} (\partial_{1} \Phi_{1} + \partial_{1} \Phi_{2} - \partial_{1} \Phi_{3})^{2} - \frac{g\lambda}{2} (\Pi_{1} + \Pi_{2} - \Pi_{3})^{2}.$$ \hspace{1cm} (48)

The Heisenberg picture momenta $\Pi_{i}$ are

$$\Pi_{1} = \frac{1 - 2g\lambda}{1 - 3g\lambda} \dot{\phi}_{1} + \frac{g\lambda}{1 - 3g\lambda} \dot{\phi}_{2} - \frac{g\lambda}{1 - 3g\lambda} \dot{\phi}_{3},$$
$$\Pi_{2} = \frac{g\lambda}{1 - 3g\lambda} \dot{\phi}_{1} + \frac{1 - 2g\lambda}{1 - 3g\lambda} \dot{\phi}_{2} - \frac{g\lambda}{1 - 3g\lambda} \dot{\phi}_{3},$$
$$\Pi_{3} = -\frac{g\lambda}{1 - 3g\lambda} \dot{\phi}_{1} - \frac{g\lambda}{1 - 3g\lambda} \dot{\phi}_{2} + \frac{1 - 2g\lambda}{1 - 3g\lambda} \dot{\phi}_{3},$$ \hspace{1cm} (49)

and the fields in the two pictures are related by $A_{H} = U^{\dagger} A_{I} U$, $\dot{U} = -iH_{I} U$.

A Legendre transformation yields the full Heisenberg picture Lagrangian $\mathcal{L}$. Requiring Lorentz invariance of $\mathcal{L}$ we obtain $\lambda = \frac{1}{1 + 3g}$. This result could also be achieved by imposing Schwinger’s condition \[11\]. Thus,

$$\mathcal{L} = \frac{1 + g}{2} (\partial_{\mu} \Phi_{1})^{2} + \frac{1 + g}{2} (\partial_{\mu} \Phi_{2})^{2} + \frac{1 + g}{2} (\partial_{\mu} \Phi_{3})^{2} + (\partial_{\mu} \Phi_{1})(\partial^{\mu} \Phi_{2})
- g(\partial_{\mu} \Phi_{1})(\partial^{\mu} \Phi_{3}) - g(\partial_{\mu} \Phi_{2})(\partial^{\mu} \Phi_{3}) + \frac{m}{\pi} \mu \cos(2\sqrt{\pi} \Phi_{1}).$$ \hspace{1cm} (50)
The transformations
\[
\begin{align*}
\Phi_1 &= \sqrt{1 + 2g} \sqrt{1 + 3g} \Phi'_1, \\
\Phi_2 &= -\frac{g}{\sqrt{(1 + 2g)(1 + 3g)}} \Phi'_1 + \frac{\sqrt{1 + g}}{\sqrt{1 + 2g}} \Phi'_2, \\
\Phi_3 &= \frac{g}{\sqrt{(1 + 2g)(1 + 3g)}} \Phi'_1 + \frac{g}{\sqrt{(1 + g)(1 + 2g)}} \Phi'_2 + \frac{1}{\sqrt{1 + g}} \Phi'_3
\end{align*}
\] (51)
applied to \( \mathcal{L} \) cast it in the diagonal form
\[
\mathcal{L} = \frac{1}{2} (\partial_\mu \Phi'_1)^2 + \frac{1}{2} (\partial_\mu \Phi'_2)^2 + \frac{1}{2} (\partial_\mu \Phi'_3)^2 + \frac{m}{\pi} \mu \cos(2a\sqrt{\pi}\Phi'_1),
\] (52)
where \( a = [(1 + 2g)/(1 + 3g)]^{1/2} \). The dynamics of the interacting infrafermions is described in terms of two free massless scalar fields plus a scalar field that satisfies a sine-Gordon equation modified by the appearance of the parameter \( a \). Note that this factor in the argument of the cosine term differs from the corresponding factor in the usual Thirring model. This suggests that after introduction of interaction the physical Hilbert space is not isomorphic to that of the first-order theory, in contrast to the free case.

Having obtained the canonical scalar fields, let us derive the bosonized expression of the infrafermions in the Heisenberg picture, which amounts to writing all operators in (29) as Heisenberg field operators. This means to apply the transformations (49) and (51) on
\[
\Psi_{(a)}(x) = (\frac{\mu}{2\pi})^{1/2} : e^{-i\sqrt{\pi}(\int_x^{-\infty} ds^1 \Pi_j(x)+\gamma_{\alpha\beta} \Phi_j(x))} :.
\] (53)
The dynamics of the fields \( \Phi'_i \) is found from (52). From (53) and (52) all expectation values of infrafermion fields can be computed. It is worthwhile to remark that in the general case (1) one would be led to a Thirring model with \( SU(N+1, N) \) explicitly broken global symmetry.

VII. CONCLUSION

We have discussed the generalization of the massive fermion theory by introducing higher derivatives. The requirements of Lorentz symmetry, hermiticity of the Hamiltonian, and absence of tachyon excitations suffice to fix the mass term. The mode expansion of the
fermion fields has been explicitly made and it has been seen that one needs two massless first-order (infra) fermion fields and one massive free fermion field to express the solution in usual form. In contrast to the massless case the relation between the higher-derivative field and the infrafermions is non-local. A family of (equivalent) solutions has been constructed but all of them are non-local in some degree. This nonlocal expression of the infrafermions in terms of the original field precludes the interpretation of their associated particles as belonging to the spectrum of the theory, irrespective of the negative metric problem. The spectrum reduces to the free massive fermion when local acceptable derivatives of the field are chosen for the definition of the physical sub-algebra of fields of the model. An interesting point is that, in spite of the non-local relationship among the fields, the charge operator is obtained from the sum of the currents associated with each infrafermion including the negative sign expected for the negative metric infrafermion.

As an example of application we have bosonized the model resulting from the current-current interaction expressed in terms of the infrafermions. The new infrafermion fields have been obtained, allowing the computation of any number of correlation functions. The bosonized model was written in terms of one massive and two massless scalar fields. The effect of the interaction appeared through a change in the value of the mass and in the dependence of the infrafermions in all scalar fields introduced.

The generalization of the model by considering coupling with a gauge field, as in Ref. [6], is presently under investigation. Due to the presence of derivatives in the mass term this generalization is not a trivial rewriting of the treatment of the massive Schwinger model.

Acknowledgments

The authors express their thanks to Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and to Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), Brazil, for partial financial support.
APPENDIX: A

For simplicity, we shall concentrate on the first component alone and shall derive the results for the case $p = 1$. The generalizations can be obtained by following the same procedure. Our first step consists in finding combinations $\alpha$ and $\beta$ of $\chi^2_{(1)}$ and $\chi^3_{(1)}$ such that

\[
\{\alpha(x^+), \alpha^*(y^+)\} = 0, \quad (A1)
\]
\[
\{\beta(x^+), \beta^*(y^+)\} = 0, \quad (A2)
\]
\[
\{\alpha(x^+), \beta^*(y^+)\} = \delta(x^1 - y^1). \quad (A3)
\]

From the anticommutation relations (26) we get

\[
\{\partial^{-1}_+ \chi^2_{(1)}(x^+), \partial^{-1}_+ \chi^2_{(1)}^*(y^+)\} = \frac{4}{m^4} \partial^2_1 \delta(x^1 - y^1), \quad (A4)
\]
\[
\{\partial_+ \chi^3_{(1)}(x^+), \partial^{-1}_+ \chi^2_{(1)}^*(y^+)\} = -\frac{i}{m} \partial^2_1 \delta(x^1 - y^1). \quad (A5)
\]

Defining

\[
\alpha(x^+) = M \partial^{-1}_+ \chi^2_{(1)}(x^+) + b \partial_+ \chi^3_{(1)}(x^+), \quad (A6)
\]
\[
\beta(x^+) = c \partial_+ \chi^3_{(1)}(x^+), \quad (A7)
\]

the relations (A1)-(A3) are satisfied if one takes

\[
b = i \frac{M}{2m^3}, \quad c = i \frac{m}{M}. \quad (A8)
\]

Now all one needs is to adjust $M$ in order to obtain $\psi^2_{(1)}$ and $\psi^3_{(1)}$ from the combinations $\alpha + \beta$ and $\alpha - \beta$, respectively. Inverting these relations we obtain (28) with $p = 1$. 
In order to decouple the second-order theory for $\xi'$, we consider the following partition function

$$Z = \int D\xi' D\xi' \exp \left\{ i \int d^2 z \left[ -i\delta \xi' \partial \xi' + m\overline{\xi'} \partial \overline{\xi'} \right] \right\}, \quad (A1)$$

and the corresponding interpolating theory

$$Z_I = \int D\xi' D\xi' D\psi' D\psi' \exp \left\{ i \int d^2 z \left[ -i\delta \xi' \partial \xi' + \frac{\delta^2}{m} \overline{\psi'} \psi' - i\delta \psi' \partial \overline{\psi'} - i\delta \overline{\psi'} \partial \psi' \right] \right\}. \quad (A2)$$

The connection between $Z_I$ and $Z$ can be seen by performing in the interpolating partition function the change of variables

$$\psi' = \psi'' + i\frac{m}{\delta} \partial \xi', \quad (A3)$$

which leads to

$$Z_I = Z \times \left( \int D\psi'' D\psi'' e^{i \int d^2 z \left[ \frac{\delta^2}{m} \overline{\psi'} \psi'' \right]} \right). \quad (A4)$$

and $\psi''$ is not a dynamical field.

Introducing in the partition function $Z_I$, given by (B2), the decoupling transformation

$$\xi' = \xi'' - \psi', \quad (A5)$$

we obtain

$$Z_I = \int D\xi'' D\xi'' D\psi' D\psi' \exp \left\{ i \int d^2 z \left[ -i\delta \xi'' \partial \xi'' + i\delta \overline{\psi'} \partial \psi' + \frac{\delta^2}{m} \overline{\psi'} \psi' \right] \right\}. \quad (A6)$$

The regulating parameter $\delta$ makes $\psi'$ a massive field with mass $\delta/m$.

The limit $\delta \to 0$ is singular for the correlation functions of the fields $\xi''$ and $\psi'$ individually. Nevertheless, this limit is well-defined for the correlation functions of the second-order field $\xi' = \xi'' - \psi'$, which is the field that contributes for the correlation functions of the original
third order field $\xi$. From Eq. (A.6) we can obtain the two point function of the field $\xi'$, which yields

$$\langle \Psi_0 \xi', \xi' \Psi_0 \rangle = \langle \Psi_0 (\xi'' - \psi'), (\xi'' - \psi') \Psi_0 \rangle = \frac{1}{m} \left[ \partial(\partial - i\delta m) \right]^{-1}. \quad (A7)$$

The limit $\delta \to 0$ is well defined for general $n$-point functions of the field $\xi'$. 
REFERENCES

[1] C.Batlle, J.Gomis, J.M.Pons and N.Roman-Roy, J. Phys. A21 (1988) 2963; V.V. Nesterenko, ibid. A22 (1989) 1673; C.A.P.Galvão and N.A.Lemos, J. Math. Phys. 29 (1988) 1588; J.Barcelos-Neto and N.R.F.Braga, Acta Phys. Polonica 20 (1989) 205; J.Barcelos-Neto and C.P.Natividade, Z. Phys. C49 (1991) 511; ibid C51 (1991) 313; J.J.Giambiagi, Nuovo Cimento 104 A (1991) 1841; C.G.Bolini and J.J.Giambiagi, J. Math. Phys. 34 (1993) 610.

[2] D.G.Boulware, in Quantum Theory of Gravity, edited by S.M.Christensen (Hilger, Bristol, 1984).

[3] P.Fayet and S.Ferrara, Phys.Rep. C32 (1977) 249; P.van Nieuwenhuizen, ibid. 68 (1981) 264; J.Barcelos-Neto and N.R.F.Braga, Phys. Rev. D39 (1989) 494.

[4] A.M.Chervyakov and V.V.Nesterenko, Phys. Rev.D48 (1993) 5811.

[5] D.G.Boulware and S.Deser, Phys. Rev. Lett. 55 (1984) 2656.

[6] R.L.P.G.Amaral, L.V.Belvedere, N.A. Lemos and C.P.Natividade, Phys. Rev. D47 (1993) 3443.

[7] L.V.Belvedere, R.L.P.G. Amaral and N.A.Lemos, Z. Phys. C66 (1995) 613.

[8] G. Morchio, D. Pierotti and F. Strocchi, Annals of Physics 188,(1988),217;

[9] C. G. Carvalhaes, L. V. Belvedere, C. Natividade and H. Boschi-Filho, Annals of Physics (N.Y.),258 (1997) 210.

[10] S.Mandelstam, Phys. Rev. D11 (1975) 3026.

[11] M.B.Halpern, Phys. Rev. D12 (1975) 1684.

[12] E.Fradkin and F.A.Schaposnik, Phys. Lett. B338 (1994) 253.