Hyperons, $\Delta$ resonances and condensate of charged $\rho$ mesons within relativistic mean-field models with scaled hadron masses and couplings

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Abstract. We study equation of state of cold and dense baryon matter within the relativistic mean-field framework with hadron masses and coupling constants dependent on the mean scalar field. Previously constructed models with included hyperons and $\Delta$ isobars are extended taking into account possibility of the condensation of charged $\rho$ mesons. We demonstrate that the results obtained in the models with the charged $\rho$-meson condensation taken into account exhibit a strong model dependence. In some of our (so-called KVORcut-based) models the charged $\rho$ condensation does not significantly affect the value of the neutron star maximum mass. In other (so called MKVOR-based) models the neutron star maximum mass decreases substantially. All thus constructed models pass the observational constraint on the minimal value of the maximum neutron star mass.

1. Introduction

Knowledge of the equation of state (EoS) of the cold and dense hadronic matter is required for the description of the neutron star (NS) matter. Nowadays there exists a vast number of EoSs and a large set of experimental and observational constraints which an appropriate EoS should satisfy [1]. None of the EoSs satisfies all the existing constraints. Recent measurement of two solar mass ($2M_\odot$) NS [2] rules out many soft EoSs. Possibility of the appearance of additional degrees of freedom in dense NS interiors, such as hyperons and $\Delta$ isobars or meson condensates, leads to a softening of the EoS and a decrease of the maximum NS mass. In our previous works [3, 4, 5] we constructed relativistic mean-field (RMF) models with effective hadron masses and coupling constants dependent on the scalar field with hyperons and $\Delta$ resonances taken into account, which successfully pass the majority of the constraints. However, it has been shown in [6, 7] that with increasing density in the isospin asymmetric matter the condensation of charged $\rho$ mesons becomes possible, provided the model includes the non-Abelian coupling of the $\rho$ fields and the decrease of the $\rho$ meson effective mass with increasing density. In this work we focus on the possibility of the charged $\rho$ meson condensation within our models and its effect on the EoS of the NS matter.
2. RMF models with charged \( \rho \) condensation

We use the framework proposed in [7]. The model is a generalization of the non-linear Walecka model with effective coupling constants \( g_{mb}^\ast = g_{mb} \chi_{mb}(\sigma) \) and hadron masses \( m_i^\ast = m_i \Phi_i(\sigma) \) dependent on the scalar field \( \sigma \). Here \( m = \{\sigma, \omega, \rho, \phi\} \) denotes mesons, \( b = \{N, H, \Delta\} \) lists baryon species, where \( N \) denotes nucleons \( \{p, n\} \), \( H \) stands for hyperons \( \{\Lambda, \Sigma, \Xi\} \) and \( \Delta \) denotes the \( \Delta \)-isobars, and index \( i \) runs through all hadrons \( (m, b) \), \( \chi_{mb}(\sigma) \) and \( \Phi_i(\sigma) \) are some scaling functions. With the charged \( \rho \) meson condensation switched off, the standard solutions for mean meson fields can be obtained, which lead to the following energy density:

\[
E[\{n_b\}, \{n\}, f] = \sum_b E_{\text{kin}}(p_{F,b}, m_b \Phi_b(f), s_b) + \sum_{l=e,\mu} E_{\text{kin}}(p_{F,l}, m_l, s_l) + \frac{m_{\gamma}^2f^2}{2C_\sigma^2} \eta_\sigma(f) \\
+ \frac{1}{2m_N^2} \left[ \frac{C_\rho^2n_i^2}{\eta_\rho(f)} + \frac{C_\omega^2n_i^2}{\eta_\omega(f)} + \frac{C_\phi^2n_i^2}{\eta_\phi(f)} \right], \quad E_{\text{kin}}(p_{F,m}, m, s) = (2s + 1) \int_0^{p_{F,m}} \frac{p^2 dp}{2\pi^2} \sqrt{p^2 + m^2},
\]

where we use the dimensionless scalar field variable \( f = g_{\sigma N} \chi_{\sigma N}(\sigma)/m_N \), the coupling constant ratios \( x_{mb} = g_{mb}/g_{N m} \). The \( \tilde{\Phi}_b \) is the isospin projection of baryon \( b \), and the Fermi momentum of a fermion \( j \) is \( p_{F,j} = (6\pi^2 n_j/2s_j + 1)^{1/3} \), where \( s_j \) is the fermion spin, \( j = (b, l) \). In case of infinite matter meson coupling constants, masses and scaling functions enter the energy density only in combinations \( C_M = g_{MN} m_N/m_M \), \( M = \sigma, \omega, \rho \), \( C_\phi = g_{\omega N} m_N/m_\phi \), \( \eta_\omega(f) = \Phi_m^2(f)/x_\omega N(f), \eta_\rho(f) = \Phi_m^2(f)/x_\rho N(f), \eta_\phi(f) = \Phi_m^2(f)/x_\phi N(f) \) (cf. Brown-Rho scaling), \( \chi_{\omega N}(f) = 1 \), \( \chi_{\rho N}(f) = \chi_{\phi N}(f) = 0 \). The scaling function of the baryon mass is \( \Phi_b(f) = 1 - x_{mb} \xi_{mb} m_N f/m_b \), where \( \xi_{mb} = \chi_{mb}/\chi_{\omega N} \), and we suppose that \( \chi_{\omega N}(f), \chi_{\phi N}(f) = \chi_{\rho N}(f) \). Explicit expressions for the scaling functions \( \eta_M(f) \) are given in [3, 4, 5].

The vector-meson coupling constants to hyperons are chosen following the quark SU(6) symmetry:

\[
x_{\omega \Lambda} = x_{\omega \Sigma} = 2x_{\omega \Xi} = \frac{2}{3}, \quad x_{\rho \Lambda} = x_{\rho \Sigma} = 2x_{\rho \Xi} = \frac{4}{3}, \quad x_{\phi \Lambda} = x_{\phi \Sigma} = x_{\phi \Xi} = -\frac{1}{3}, \quad x_{\omega \Lambda} = x_{\phi \Lambda} = 0.
\]

The hyperon coupling constants with the scalar field are deduced from the hyperon binding energies \( \varepsilon_{\text{bind}}^H \) in isospin-symmetric matter (ISM) at the saturation density \( n = n_0 \) given by \( \varepsilon_{\text{bind}}^H(n_0) = C_\omega^2 n_0^2 x_{\omega H} n_0 - m_N + m_N^\ast(n_0) \), and the empirical values \( \varepsilon_{\text{bind}}^\Lambda(n_0) = -28 \text{ MeV}, \varepsilon_{\text{bind}}^\Sigma(n_0) = 30 \text{ MeV} \) and \( \varepsilon_{\text{bind}}^\Xi(n_0) = -15 \text{ MeV} \). The value of the \( \Delta \) potential \( U_\Delta(n_0) \) is poorly constrained by the data. Here we use \( U_\Delta(n_0) = -50 \text{ MeV} \) as the most realistic estimate. We incorporate the \( \phi \) meson field assuming the universal mass scaling with other hadrons, \( \Phi_\phi = 1 - f \), and taking the vacuum coupling constant \( \chi_{\phi N} = 1 \), that leads to \( \eta_\phi(f) = (1 - f)^2 \). We denote models with hyperons included with this choice of \( \eta_\phi \) by "H\(\phi\)" suffix.

The free parameters of the model are fitted to reproduce properties of nuclear matter near the saturation point. These properties are defined as the coefficients of the Taylor expansion of the energy per particle in the ISM in terms of \( \epsilon = (n - n_0)/3n_0 \) and \( \beta = (n - n_\rho)/n \), \( \varepsilon = \varepsilon_0 + \frac{K_\rho}{2} \epsilon^2 - \frac{K_6}{6} \epsilon^3 + \beta^2 J(n) + \ldots \) and \( J(n) = \tilde{J} + L \epsilon + \frac{K_{\rho 2}}{4} \epsilon^2 + \ldots \).

The \( \rho \) meson part of the Lagrangian is as follows [6, 7],

\[
\mathcal{L}_\rho = -\frac{1}{4} \bar{\rho}_{\mu\nu} \tilde{\rho}^{\mu\nu} + \frac{1}{2} g_{\rho} \chi_{\rho} \bar{\rho}^{\mu} \rho^{\mu} - g_{\rho} \chi_{\rho} \bar{\rho}^{\mu} \rho^{\mu}, \quad \tilde{J}_\rho = \delta^{\alpha\beta} \bar{\rho}^{\mu} \psi_{\alpha}^{\mu} \tau_{\beta} \psi,
\]

\[
\bar{R}_{\mu\nu} = \partial_{\mu} \bar{\rho}_{\nu} - \partial_{\nu} \bar{\rho}_{\mu} + g_{\rho} \chi_{\rho} \bar{\rho}^{\nu} \bar{\rho}_{\mu} + \mu_{\chi_{\rho}} \delta_{\alpha\beta} \bar{\rho}_{\mu} \bar{\rho}_{\nu} \psi_{\alpha}^{\mu} \tau_{\beta} \psi.
\]
where \((\vec{n}_3)^a = \delta^{a3}\) is the unit vector in the isospin space and \(\mu_{\text{ch},\rho}\) is the chemical potential for charged mesons. From the hidden local symmetry arguments it follows that \(g'_\rho = g_\rho\), which we adopt here. The new ansatz for \(\rho\) meson field includes non-zero \(\rho_0^{(3)}\) and \(\rho_i^\pm = (\rho_i^{(1)} + \rho_i^{(2)})/\sqrt{2}\), \(i = 1, 2, 3\) components, with the condition \(\rho_i^{(+)}\rho_j^{(-)} - \rho_i^{(-)}\rho_j^{(+)} = 0\), which implies that the ratio \(\rho_i^{(+)}/\rho_i^{(-)}\) is constant, independent of the spatial index \(i\). Thus we assume \(\rho_i^{(-)} = a_i\rho_c\) and \(\rho_i^{(+)} = a_i\rho_c^\dagger\), where \(a = \{a_i\}\) is the spatial unit vector, and \(\rho_c\) is a complex amplitude of the charged \(\rho\) meson field. With this ansatz the \(\rho\) field contribution to the thermodynamic potential \(\Omega[\mu]\) is as follows

\[
\Omega_{\rho}[\{n_b\}, \mu_{\text{ch},\rho}, f, \rho_0^{(3)}, \rho_c] = g_\rho \chi_\rho n_I \rho_0^{(3)} - \frac{1}{2} (\rho_0^{(3)})^2 m^2_\rho \Phi^2_\rho \\
- \left\{(g_\rho \chi_\rho \rho_0^{(3)} - \mu_{\text{ch},\rho})^2 - m^2_\rho \Phi^2_\rho\right\} |\rho_c|^2 .
\] (4)

Equations of motion for the \(\rho_0^{(3)}\) and \(\rho_c\) fields given by minimization of the thermodynamic potential have two solutions. One of them is the traditional one

\[
\rho_0^{(3)} = \frac{g_\rho \chi_\rho}{m^2_\rho \Phi^2_\rho} n_I, \quad \Omega^{(1)}_{\rho} = \frac{C_\rho^2 n_I^2}{2m^2_\rho \eta_\rho(f)}.
\] (5)

The second solution is

\[
\rho_0^{(3)} = \frac{\mu_{\text{ch},\rho} - m^2_\rho \Phi_\rho}{g_\rho \chi_\rho}, \quad |\rho_c|^2 = \frac{(-n_I - n_{\rho})\theta(-n_I - n_{\rho})}{2m^2_\rho \eta_\rho}.
\] (6)

where

\[
n_{\rho} = a (m^2_\rho \Phi_\rho - \mu_{\text{ch},\rho}) , \quad a = \frac{m^2_\rho \eta_\rho^{1/2} \Phi_\rho}{C_\rho^2 \chi_\rho}.
\] (7)

resulting in

\[
\Omega^{(2)}_{\rho} = \Omega^{(1)}_{\rho} - \frac{C_\rho^2}{2m^2_\rho \eta_\rho} (n_I + n_{\rho})^2 \theta(-n_I - n_{\rho}) .
\] (8)

The \(\rho\) charge density is given by \(n_{\text{ch},\rho} = \partial \Omega / \partial \mu_{\text{ch},\rho} = -2m^2_\rho \Phi_\rho(f)|\rho_c|^2 < 0\). The contribution to the energy density from the charged \(\rho\) meson condensate (1) is then given by

\[
\Delta E_{\text{ch},\rho}[\{n_b\}; f] = - \frac{C_\rho^2}{2m^2_\rho \eta_\rho} (n_I + n_{\rho})^2 \theta(-n_I - n_{\rho}) - \mu_{\text{ch},\rho} n_{\text{ch},\rho} ,
\] (9)

\(\theta(-n_I - n_{\rho}) = 1\) for \(n_I + n_{\rho} < 0\) and zero otherwise.

The charge neutrality condition is now \(\sum_b Q_b n_b - n_e - n_\mu + n_{\text{ch},\rho} = 0\) and the relations between chemical potentials in beta-equilibrium matter (BEM) \(\mu_e = \mu_\mu = \mu_{\text{ch},\rho}, \mu_b = \mu_n - Q_b \mu_\mu\).

All equations are solved self-consistently with the equation of motion for the scalar field \(\partial E / \partial f = 0\). The pressure is given by \(P = \sum_j \mu_j n_j - E, \quad j = b \cup l \cup \{\text{ch}, \rho\}\). To be specific in our numerical calculations we use \(\chi' = 1\).
3. Numerical results

First, we study two models constructed in [4, 5], which use the ”cut-mechanism” suggested in [8] to make the EoS stiffer at high densities without altering its properties near the saturation density. The KVORcut03 model is a modification of the KVOR model (labeled as MW(nu) model in [7]). The models labeled as KVORcut exploit a sharp decrease of the $\eta_{\omega}(f)$ for $f > f_{c}^{\text{KVORcut}}$, thus making the EoS stiffer in both ISM and BEM at high densities. This allows to pass the maximum NS mass constraint even provided hyperons and $\Delta$s are included. The saturation properties of the model are the same as in the original KVOR model. Below we choose the KVORcut03-based models, cf. [7]. Then we consider extensions of the MKVOR* model, cf. [5]. In the MKVOR* model a sharp decrease in the $\eta_{\rho}(f)$ with increasing $f$ for $f > f_{c}^{\text{MKVOR*}}$ is used to make the EoS stiffer in the BEM keeping it unchanged in the ISM. The input parameters for the MKVOR* model are: $n_{0} = 0.16\text{ fm}^{-3}$, the binding energy $E_{0} = -16\text{ MeV}$, the incompressibility $K = 250\text{ MeV}$, the symmetry energy $J = 32\text{ MeV}$ and the nucleon effective mass $m_{N}^{*}(n_{0}) = 0.73m_{N}$. The models satisfy many constraints from NS observations, HICs analyses and nuclear phenomenology.

3.1. KVORcut03-based models

On the left panel in Fig. 1 we show particle fractions together with the scalar field for the KVORcut03 model (KVORcut03 model with included possibility of the charged $\rho$ condensation) in the BEM. The charged $\rho$ condensate appears with a second order phase transition with the critical density $n_{c,\rho} \approx 4.62n_{0}$, which is larger than that found in [7] for the original KVOR model. The reason of this shift of the charged $\rho$ condensation threshold to a higher density is the limiting of the hadron effective mass decrease, which is a general feature of models with the ”cut-mechanism” included in the scaling functions $\eta_{\sigma}$ or $\eta_{\omega}$. On the right panel in Fig. 1 we show the pressure in the BEM as a function of the density for the KVORcut03 model. The effect of the charged $\rho$ condensation on the EoS is minor, and the maximum NS mass decreases only by $0.01M_{\odot}$. In the KVORcut03 model with the hyperons and/or $\Delta$s included into calculation the charged $\rho$ condensation does not occur at densities relevant for NSs.
3.2. MKVOR*-based models

On the left panel in Fig. 2 we show the particle concentrations and the scalar field $f$ in the BEM for the MKVOR*$\Delta \rho$ (MKVOR* model with included $\Delta$ and charged $\rho$ condensation) and MKVOR*$H\Delta \phi \rho$ models. In the presence of the charged $\rho$ condensation the hyperon concentrations are suppressed, so the lines for MKVOR*$\Delta \rho$, MKVOR*$H\Delta \phi \rho$ visually coincide. There exists a region with three solutions for equilibrium particle concentrations for a given total baryon density, which is a feature of the first order phase transition. One of these three solutions corresponds to the maximum of the energy density and the configuration is unstable (shown by the dashed line regions), while two other solutions are the local minima of the energy density. The stable solution with greater $n_{ch,\rho}$ becomes energetically favorable at $n^* = 2.81 n_0$, at which the second stable branch becomes energetically favorable for both models. The pressure for the MKVOR*$\Delta$, MKVOR*$H\Delta \phi$, MKVOR*$\Delta \rho$, MKVOR*$H\Delta \phi \rho$ models in the BEM is shown on the right panel of Fig. 2. The result of calculation with self-consistent particle concentrations is shown by thin lines, and horizontal lines indicate the Maxwell construction, which spans from $2.36 n_0$ to $3.37 n_0$.

In Fig. 3 we show the resulting NS mass as a function of the central density (left panel), mass-radius relation (central panel), and NS radius as a function of the central density (right panel) for the MKVOR*$[\Delta, H\Delta \phi, \Delta \rho, H\Delta \phi \rho]$ models. Due to the description of the phase transition with the Maxwell construction, the region of central densities corresponding to the Maxwell construction (horizontal dotted lines between bars in left and right panels) cannot be realized in stable NS configurations. Contrary to the KVORcut03 model, in the MKVOR*-based models the appearance of the charged $\rho$ condensate leads to a substantial decrease of the maximum NS mass from $2.3 M_\odot$ for MKVOR*$\Delta$ model ($2.21 M_\odot$ for MKVOR*$H\Delta \phi$) to $2.03 M_\odot$ for MKVOR*$\Delta \rho$ and MKVOR*$H\Delta \phi \rho$ models (for latter two models the curves visually
Figure 3. NS masses versus the central density (left panel) and NS radii (central panel), and NS radius as a function of the central density (right panel) for MKVOR*Δ, MKVOR*HΔφ, MKVOR*Δρ, MKVOR*HΔφρ models. Dotted lines between bars indicate the central density interval corresponding to the Maxwell construction line in Fig. 2 (right), which is not realized in NSs.

coincide). However, the resulting models still pass the maximum NS mass constraint with \( M_{\text{max}} = 2.03 M_\odot \). On the mass-radius relationship the phase transition leads to a connected stable branch. The radius, corresponding to the NS with the maximum mass, decreases from 12 km to 9.75 km.

4. Conclusion
In this contribution we studied the possibility of the charged \( \rho \) meson condensation in the KVORecut03 and MKVOR* relativistic mean-field models and their extensions with scaled hadron masses and coupling constants including hyperons and \( \Delta \) isobars, have been constructed in our previous works. Results are model dependent. In the KVORecut03-based models the charged \( \rho \) meson condensate appears by a second order phase transition, and the effect on the EoS and on the maximum neutron star mass is minor. In the MKVOR*-based models the charged \( \rho \) meson condensate appears by the first order phase transition, that results in a substantial decrease of the maximum neutron star mass.

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References
[1] Klähn T et al. 2006 Phys. Rev. C 74 035802.
[2] Antoniadis J et al. 2013 Science 340 6131.
[3] Maslov K A, Kolomeitsev E E and Voskresensky D N 2015 Phys. Lett. B 748 369.
[4] Maslov K A, Kolomeitsev E E and Voskresensky D N 2016 Nucl. Phys. A 950 64.
[5] Kolomeitsev E E, Maslov K A and Voskresensky D N 2017 Nucl. Phys. A 961 106.
[6] Voskresensky D N 1997 Phys. Lett. B 392 262.
[7] Kolomeitsev E E and Voskresensky D N 2005 Nucl. Phys. A 759 373.
[8] Maslov K A, Kolomeitsev E E and Voskresensky D N 2015 Phys. Rev. C 92 052801.
[9] Danielewicz P, Lacey R and Lynch W G 2002 Science 298 1592.
[10] Vidana I 2016 J. Phys. Conf. Ser. 668 012031.
[11] van Dalen E N E, Colucci G and Sedrakian A 2014 Phys. Lett. B 734 383.