Spontaneous R-parity violation in the minimal gauged $(B - L)$ supersymmetry with a 125 GeV Higgs

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Abstract

We precisely derive the mass squared matrices for charged and neutral (CP-odd and CP-even) Higgs, as well as the mass matrices for neutrino-neutralino and charged lepton-chargino in the minimal R-parity violating supersymmetry with local $U(1)_{B-L}$ symmetry. In the framework the nonzero TeV scale vacuum expectations of right-handed sneutrinos induce the heavy mass of neutral $U(1)_{B-L}$ gauge boson, and result in relatively large mixing between the lightest CP-even Higgs and three generation right-handed sneutrinos when we include the one-loop corrections to the scalar potential. We numerically show that there is parameter space of the considered model to accommodate experimental data on the newly ones of Higgs signal from LHC and experimental observations on the neutrino oscillation simultaneously.

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I. INTRODUCTION

A main destination of the Large Hadron Collider (LHC) is to understand the origin of the electroweak symmetry breaking, and to study the properties of neutral Higgs predicted by the Standard Model (SM) and its various extensions. In the last year, ATLAS and CMS reported significantly excess events in a few channels which are interpreted as the neutral Higgs with mass $m_{h_0} \sim 124 - 126$ GeV\cite{1, 2}, and CP properties and couplings of the particle are also being established\cite{3–6} recently. It implies that the Higgs mechanism to break electroweak symmetry has an experimental cornerstone now. Another important progress of particle physics in the last year is that nonzero experimental observation on the neutrino mixing angle $\theta_{13}$ is obtained with high precision\cite{7}, which opens several prospects for neutrino physics. In this work, we investigate the constraints on parameter space of the minimal R-parity violating supersymmetry with local $U(1)_{B-L}$ symmetry from the updated experimental data mentioned above.

R-parity, as a discrete symmetry, is defined through $R = (-1)^{3(B-L)+2S}$, where $B$, $L$ and $S$ are baryon number, lepton number and spin respectively for a concerned field\cite{8}. When $B - L$ is violated by an even amount, R-parity conservation is guaranteed. However, breaking $B - L$ via nonzero vacuum expectation values (VEVs) of neutral scalar fields with odd $U(1)_{B-L}$ charges will induce the R-parity violation simultaneously. In the minimal supersymmetric extension of SM (MSSM) with local $U(1)_{B-L}$ symmetry, R-parity is spontaneously broken when left- and right-handed sneutrinos acquire nonzero VEVs\cite{9–12}. Actually, both spontaneously violated R-parity and broken local $U(1)_{B_L}$ symmetry replicate the MSSM with conserving baryon number but violating lepton number. The authors of Ref.\cite{13} further propose an extension of the MSSM, which includes right-handed neutrino superfields and two additional superfields $\hat{X}, \hat{X}'$ with even $U(1)_{B-L}$ charges. When sneutrinos and scalar components of $\hat{X}, \hat{X}'$ acquire non zero VEVs simultaneously, local $U(1)_{B-L}$ symmetry and R-parity are broken spontaneously. To account for the neutrino oscillation experiment, tiny neutrino masses are generated through an extended seesaw mechanism in the framework proposed in Ref.\cite{9–13}. Furthermore, the neutral Higgs fields $H_u^0$, $H_d^0$ mix with the scalar components of neutrino superfields and $\hat{X}, \hat{X}'$ superfields after the electroweak symmetry is
broken in those models. Assuming that the scalar components of $\hat{X}$, $\hat{X}'$ and neutral Higgs fields $H^0_u$, $H^0_d$ acquire nonzero VEVs, Ref. [14] studies mass spectrum in the model proposed in Ref. [13].

Here we study the constraints from the observed Higgs signal and neutrino oscillation experimental data on parameter space of the MSSM with local $U(1)_{B-L}$ symmetry in the scenarios where sneutrinos obtain nonzero VEVs [9–12]. Since the tree level mixing between the lightest CP-even Higgs and right-handed sneutrinos is suppressed by the tiny neutrino masses, we include the one-loop corrections to the mixing which are mainly originated from the third generation fermions and their supersymmetric partners. Numerically the MSSM with local $U(1)_{B-L}$ symmetry accommodates naturally the experimental data on the Higgs particle from ATLAS/CMS collaborations and the updated experimental observations on the neutrino oscillation simultaneously. In addition, the model also predicts two sterile neutrinos with sub-eV masses [15, 16], which are favored by the Big-bang nucleosynthesis (BBN) in cosmology [17].

Certainly the deviation from unitarity of the leptonic mixing matrix intervening in charged currents might induce a tree-level enhancement of $R_p = \Gamma(P^+ \to e^+\nu)/\Gamma(P^+ \to \mu^+\nu)$ ($P^+ = K^+$, $\pi^+$) [18] because of additional mixings between the active neutrinos and the sub-eV sterile states. Ignoring the difference between hadronic matrix elements in $P^+ \to e^+\nu$ and that in $P^+ \to \mu^+\nu$, one finds that the experimental observations on $R_p$ also constrain the parameter space of considered model. Furthermore, the experimental data on $Z$ invisible width [19] also constrain the mixings between the active neutrinos and the sub-eV sterile ones. We will address the constraints on the mixings between the active neutrinos and the sub-eV sterile ones from lepton flavor universality (LFU) and $Z$ invisible width elsewhere [20].

Our presentation is organized as follows. In section II we briefly summarize the main ingredients of the MSSM with local $U(1)_{B-L}$ symmetry, then present the mass squared matrices for CP-odd and charged Higgs sectors, respectively. We analyze the loop corrections on the mass squared matrix of CP-even Higgs in section III and present the mass matrices for neutrino-neutralino and charged lepton-chargino in section IV and section V, respectively. Furthermore, we also present the decay widths for $h^0 \to \gamma\gamma$, $VV^*$, ($V = Z, W$) in section.
VI. The numerical analyses are given in section VII and our conclusions are summarized in section VIII.

II. THE MSSM WITH LOCAL $U(1)_{B-L}$ SYMMETRY

When $U(1)_{B-L}$ is a local gauge symmetry, one can enlarge the local gauge group of the SM to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{(B-L)}$. In the model proposed in Ref. [9–12], the exotic superfields are three generation right-handed neutrinos $\tilde{N}^c_i \sim (1, 1, 0, 1)$. Meanwhile, quantum numbers of the matter chiral superfields for quarks and leptons are given by

$$\hat{Q}_i = \begin{pmatrix} \hat{U}_i \\ \hat{D}_i \end{pmatrix} \sim (3, 2, \frac{1}{3}, \frac{1}{3}), \quad \hat{L}_i = \begin{pmatrix} \hat{\nu}_i \\ \hat{E}_i \end{pmatrix} \sim (1, 2, -1, -1),$$

$$\hat{U}^c_i \sim (3, 1, -\frac{4}{3}, -\frac{1}{3}), \quad \hat{D}^c_i \sim (3, 1, \frac{2}{3}, -\frac{1}{3}), \quad \hat{E}^c_i \sim (1, 1, 2, 1),$$

with $I = 1, 2, 3$ denoting the index of generation. In addition, the quantum numbers of two Higgs doublets are assigned as

$$\hat{H}_u = \begin{pmatrix} \hat{H}^+_u \\ \hat{H}^0_u \end{pmatrix} \sim (1, 2, 1, 0), \quad \hat{H}_d = \begin{pmatrix} \hat{H}^0_d \\ \hat{H}^-_d \end{pmatrix} \sim (1, 2, -1, 0).$$

The superpotential of the MSSM with local $U(1)_{B-L}$ symmetry is written as

$$W = W_{MSSM} + W^{(1)}_{(B-L)}.$$  

Here $W_{MSSM}$ is superpotential of the MSSM, and

$$W^{(1)}_{(B-L)} = \left(Y_N\right)_{ij} \hat{H}^+_u i\sigma_2 \hat{L}_i \tilde{N}^c_j.$$  

Correspondingly, the soft breaking terms for the MSSM with local $U(1)_{B-L}$ symmetry are generally given as

$$L_{soft} = L_{soft}^{MSSM} + L_{soft}^{(1)}.$$  

Here $L_{soft}^{MSSM}$ is soft breaking terms of the MSSM, and

$$L_{soft}^{(1)} = -\left(m^2_N\right)_{ij} \tilde{N}^c_i \tilde{N}^c_j - \left(m_{BL} \lambda_{BL} \lambda_{BL} + h.c.\right) + \left\{ \left(A_N\right)_{ij} \hat{H}^+_u i\sigma_2 \hat{L}_i \tilde{N}^c_j + h.c. \right\}.$$
with \( \lambda_{BL} \) denoting the gaugino of \( U(1)_{B-L} \). After the \( SU(2)_L \) doublets \( H_u, H_d, \tilde{L}_I \) and \( SU(2)_L \) singlets \( \tilde{N}_I^c \) acquire the nonzero VEVs,

\[
H_u = \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}} (v_u + H_u^0 + iP_u) \end{pmatrix},
\]
\[
H_d = \begin{pmatrix} H_d^- \\ \frac{1}{\sqrt{2}} (v_d + H_d^0 + iP_d) \end{pmatrix},
\]
\[
\tilde{L}_I = \begin{pmatrix} \tilde{L}_I^- \\ \frac{1}{\sqrt{2}} (v_{L_I} + \tilde{\nu}_{L_I} + iP_{L_I}) \end{pmatrix},
\]
\[
\tilde{N}_I^c = \frac{1}{\sqrt{2}} (v_{N_I} + \tilde{\nu}_{R_I} + iP_{N_I}),
\]

(7)

the R-parity is broken spontaneously, and the local gauge symmetry \( SU(2)_L \otimes U(1)_Y \otimes U(1)_{(B-L)} \) is broken down to the electromagnetic symmetry \( U(1)_e \). Assuming that all parameters are real, we obtain the minimization conditions at one-loop level in the model considered here

\[
T_0^u + \Delta T_u v_u = 0,
\]
\[
T_0^d + \Delta T_d v_d = 0,
\]
\[
T_0^{L_I} + \Delta T_L v_{L_I} = 0,
\]
\[
T_0^{N_I} + \Delta T_{N_I} v_{N_I} = 0,
\]

(8)

where \( T_0^u, T_0^d, T_0^{L_I}, T_0^{N_I} \) denote the tree level tadpole conditions, and \( \Delta T_u, \Delta T_d, \Delta T_L \) as well as \( \Delta T_{N_I} \) are the one-loop radiative corrections to the minimization conditions from top, bottom, tau and their supersymmetric partners respectively, their concrete expressions are given in the appendix. After the local gauge group \( SU(2)_L \otimes U(1)_Y \otimes U(1)_{(B-L)} \) is broken down to the electromagnetic symmetry \( U(1)_e \), the masses of neutral and charged gauge bosons are respectively formulated as

\[
m_Z^2 = \frac{1}{4} (g_1^2 + g_2^2) v_{EW}^2,
\]
\[
m_W^2 = \frac{1}{4} g_2^2 v_{EW}^2,
\]
\[
m_{ZBL}^2 = g_{BL}^2 \left( v_N^2 + v_{EW}^2 - v_{SM}^2 \right),
\]

(9)
Where $v_{SM}^2 = v_u^2 + v_d^2$, $v_{EW}^2 = v_u^2 + v_d^2 + 3 \sum v_{L \alpha}^2$, $v_N^2 = \sum v_{N \alpha}^2$, and $g_2$, $g_1$, $g_{BL}$ denote the gauge couplings of $SU(2)_L$, $U(1)_Y$ and $U(1)_{(B-L)}$, respectively.

To satisfy present electroweak precision observations we assume the mass of neutral $U(1)_{(B-L)}$ gauge boson $m_{z_{BL}} > 1$ TeV which implies $v_N > 1$ TeV when $g_{BL} < 1$, then we derive $\max((Y_N)_{ij}) \leq 10^{-6}$ and $\max(v_{L_1}) \leq 10^{-3}$ GeV [12] to explain experimental data on neutrino oscillation. Ignoring the small terms and assuming that the $3 \times 3$ matrices $m_L^2$, $m_{Nc}^2$ are real, we simplify the minimization conditions in Eq.(8) as

$$
\begin{align*}
&v_u \left\{ \mu^2 + m_{H_u}^2 + \frac{g_1^2 + g_2^2}{8} (2v_u^2 - v_{EW}^2) + \Delta T_u \right\} + B\mu v_u \simeq 0 , \\
&v_d \left\{ \mu^2 + m_{H_d}^2 - \frac{g_1^2 + g_2^2}{8} (2v_u^2 - v_{EW}^2) + \Delta T_d \right\} + B\mu v_d \simeq 0 , \\
&\sum_{\alpha=1}^{3} \left[ (m_{N}^2)_{I\alpha} + \Delta T_{N} \delta_{I\alpha} \right] v_{N\alpha} + \frac{1}{\sqrt{2}} \sum_{\alpha=1}^{3} (A_N)_{I\alpha} v_{N\alpha} + \frac{\mu v_{N}^2}{\sqrt{2}} \simeq 0 , \\
&-v_{L_1} \left\{ \frac{g_1^2 + g_2^2}{8} (2v_u^2 - v_{EW}^2) + \frac{m_{z_{BL}}^2}{2} \right\} \simeq 0 , \\
&\sum_{\alpha=1}^{3} \left[ (m_{Nc}^2)_{I\alpha} + \Delta T_{Nc} \delta_{I\alpha} \right] v_{N\alpha} + \frac{m_{z_{BL}}^2}{2} v_{Nj} \simeq 0 ,
\end{align*}
$$

with $\zeta_I = \sum_{\alpha=1}^{3} (Y_N)_{I\alpha} v_{N\alpha}$. Note here that the first two minimization conditions respectively for $H_u^0$, $H_d^0$ are not greatly modified from that in the MSSM, the third condition keeps the linear terms of $v_{L_1}$ or $Y_N$, and the last equation implies that the vector $(v_{N_1}, v_{N_2}, v_{N_3})$ is an eigenvector of $3 \times 3$ mass squared matrix $m_{Nc}^2$ with eigenvalue $-m_{z_{BL}}^2/2 - \Delta T_{Nc}$. A possible symmetric $3 \times 3$ matrix satisfying the last equation in Eq.(10) is written as

$$
\begin{align*}
m_{Nc}^2 & \approx \begin{pmatrix}
\Lambda_{N_1}^2 - \Lambda_{BL}^2 & 0 & \frac{v_{N_1}}{v_{N_3}} \Lambda_{N_1}^2 \\
0 & \Lambda_{N_2}^2 - \Lambda_{BL}^2 & \frac{v_{N_2}}{v_{N_3}} \Lambda_{N_2}^2 \\
\frac{v_{N_1}}{v_{N_3}} \Lambda_{N_1}^2 & \frac{v_{N_2}}{v_{N_3}} \Lambda_{N_2}^2 & \frac{v_{N_1}}{v_{N_3}} \Lambda_{N_1}^2 + \frac{v_{N_2}}{v_{N_3}} \Lambda_{N_2}^2 - \Lambda_{BL}^2
\end{pmatrix}
\end{align*}
$$

with $\Lambda_{BL}^2 = m_{z_{BL}}^2/2 + \Delta T_{Nc}$. In order to make our final results transparently, we further assume in our following discussion

$$
\begin{align*}
(m_{L}^2)_{IJ} & \simeq m_{L_1}^2 \delta_{IJ} , \quad (I, J = 1, 2, 3) ,
\end{align*}
$$
then we obtain
\[ v_{L_1} \simeq -\frac{4\sqrt{2} \left[ v_u \sum_{\alpha=1}^3 (A_N)_{I\alpha} v_{N\alpha} + \mu v_d \zeta \right]}{8(m_{L_1}^2 + \Delta T_L) - (g_1^2 + g_2^2) \left( v_u^2 - v_d^2 \right) - 4m_{ZBL}^2} . \] (13)

As \( m_{L_1} \sim 1 \) TeV, the condition \( \max(v_{L_1}) \leq 10^{-2} \) GeV requires \( A_N \sim 0.01 \) GeV. This implies that tree level contributions to the mixing between the lightest CP-even Higgs and right-handed sneutrinos can be ignored, leading contributions to the mixing are mainly originated from one-loop radiative corrections.

A. The mass squared matrix for charged Higgs

Using those minimization conditions, we derive the \( 8 \times 8 \) mass squared matrix for charged Higgs

\[
\begin{pmatrix}
\left[ M_{CH}^2 \right]_{2\times2} & \left[ A_{CH} \right]_{2\times6} \\
\left[ A^{T}_{CH} \right]_{6\times2} & \left[ M_{E}^{2} \right]_{6\times6}
\end{pmatrix},
\]

in the interaction eigenstates \( H_{CH}^T = (H_+^-, \ H_d^-, \ \tilde{L}_i^-, \ \tilde{E}_{J}^{e*}) \), \( (I, J = 1, 2, 3) \). Here, elements of the \( 2 \times 2 \) matrix \( M_{CH}^2 \) are given as

\[
\left[ M_{CH}^2 \right]_{11} = (B\mu + \Delta_{odd}) v_d - \frac{g_2^2}{4} (v_{EW}^2 - v_u^2) + \frac{1}{\sqrt{2} v_u} \sum_{\alpha,\beta}^3 (A_N)_{\alpha\beta} v_{N\beta} \\
+ \frac{1}{2} \sum_{\alpha,\beta}^3 v_{La} (Y_{N} Y_{N}^{T})_{\alpha\beta} v_{L\beta} ,
\]

\[
\left[ M_{CH}^2 \right]_{12} = (B\mu + \Delta_{odd}) - \frac{g_2^2}{4} v_u v_d ,
\]

\[
\left[ M_{CH}^2 \right]_{22} = (B\mu + \Delta_{odd}) v_d + \frac{g_2^2}{4} (v_{EW}^2 - v_{SM}^2 - v_u^2) + \frac{\mu_{N}^2}{\sqrt{2} v_d} \\
- \frac{1}{2} \sum_{\alpha,\beta=1}^3 v_{La} (Y_{E} Y_{E}^{T})_{\alpha\beta} v_{L\beta} ,
\]

(15)

where the \( 3 \times 3 \) matrix \( Y_{E} \) is Yukawa couplings in charged lepton sector, and the one-loop radiative correction is written as

\[ \Delta_{odd} = \frac{3g_2^2}{32\pi^2 \sin^2 \beta} \frac{m_r^2 A_r \mu f(m_{r_1}^2) - f(m_{r_2}^2)}{m_w^2 m_{r_1}^2 - m_{r_2}^2} \]
\[
\frac{3g_2^2}{32\pi^2 \cos^2 \beta} \frac{m_b^2 A_b \mu}{m_w^2} f \left( \frac{m_{b_1}^2}{m_{b_2}^2} \right) - f \left( \frac{m_{b_2}^2}{m_{b_1}^2} \right) \\
+ \frac{g_2^2}{32\pi^2 \cos^2 \beta} \frac{m_{\tilde{t}}^2 A_{\tilde{t}} \mu}{m_w^2} f \left( \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} \right) - f \left( \frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \right) .
\] (16)

Here \( m_{\tilde{t}_{1,2}}^2 \), \( m_{\tilde{b}_{1,2}}^2 \) and \( m_{\tilde{\tau}_{1,2}}^2 \) are the eigenvalues of the \( \tilde{t} \), \( \tilde{b} \) and \( \tilde{\tau} \) mass-squared matrices, the form factor \( f \left( m^2 \right) = m^2 \left( \ln \left( m^2 / \Lambda^2 \right) - 1 \right) \) with \( \Lambda \) denoting renormalization scale. Additionally, the concrete expressions for the symmetric matrix \( M^2_E \) and \( A_{CH} \) can be found in appendix B.

Actually, the symmetric matrix in Eq. (14) contains an eigenvector with zero eigenvalue

\[
G^\pm = \frac{v_u}{v_{EW}} H_u^\pm - \frac{v_d}{v_{EW}} H_d^\pm - \sum_{\alpha=1}^{3} \frac{v_{L\alpha}}{v_{EW}} \tilde{f}_\alpha^\pm ,
\] (17)

which corresponds to the charged Goldstone eaten by charged gauge boson as electroweak symmetry broken spontaneously. Applying the \( 8 \times 8 \) orthogonal matrix

\[
Z_{CH}^{(0)} = Z_{CH}^{(0)} \bigoplus 1_{3 \times 3} ,
\] (18)

we separate the charged Goldstone boson from the physical states:

\[
Z_{CH}^{(0)T} \cdot \begin{bmatrix} M_{CH}^2 & A_{CH}^2 \\ A_{CH}^T & M_E^2 \end{bmatrix} \cdot Z_{CH}^{(0)} = \begin{bmatrix} 0 & 0_{1 \times 7} \\ 0_{7 \times 1} & M_{H^\pm}^2 \end{bmatrix} .
\] (19)

Where the \( 5 \times 5 \) orthogonal matrix \( Z_{CH}^{(0)} \) is given as

\[
Z_{CH}^{(0)} = \begin{pmatrix}
\frac{v_u}{v_{EW}}, & \frac{v_d}{v_{SM}}, & \left( \frac{v_u v_{LK}}{v_{SM} v_{EW}} \right)_{1 \times 3}, & -\frac{v_d v_{LK}}{v_{SM} v_{EW}}_{1 \times 3} \\
-\frac{v_d}{v_{EW}}, & \frac{v_u}{v_{SM}}, & \left( \frac{v_u v_{LK}}{v_{SM} v_{EW}} \right)_{1 \times 3}, & -\frac{v_d v_{LK}}{v_{SM} v_{EW}}_{1 \times 3} \\
\left( \frac{-v_u}{v_{EW}} \right)_{3 \times 1}, & 0_{4 \times 1}, & \left( \frac{v_{SM}}{v_{EW}} \delta_{IK} + \sum_{\alpha=1}^{3} \frac{v_{L\alpha}}{v_{EW}} \varepsilon_{IK} \right)_{3 \times 3}
\end{pmatrix} .
\] (20)

Finally, we give the \( 8 \times 8 \) mixing matrix \( Z_{CH} \) in charged Higgs sector as

\[
Z_{CH} = \begin{pmatrix}
1 & 0_{1 \times 7} \\
0_{7 \times 1} & Z_{H^\pm} \end{pmatrix} \cdot Z_{CH}^{(0)}
\] (21)

with \( Z_{H^\pm} \cdot M_{H^\pm} \cdot Z_{H^\pm} = \text{diag}(m_{H^\pm}^2, \ldots, m_{H^\pm}^2) \).
B. The mass squared matrix for CP-odd Higgs

In the interaction basis \( P^{0,T} = (P_{u}^{0}, P_{d}^{0}, P_{L}^{0}, P_{S}^{0}) \), \( (I, J = 1, 2, 3) \), the 8 \( \times \) 8 mass matrix for neutral CP-odd scalars is

\[
\begin{pmatrix}
[M_{CPO}^{2}]_{2 \times 2} & [A^{(0)CPO}]_{2 \times 6} \\
[A^{(0)T}]_{6 \times 2} & [M_{P}^{2}]_{6 \times 6}
\end{pmatrix},
\]

the elements of 2 \( \times \) 2 mass squared matrix are

\[
[M_{CPO}^{2}]_{11} = (B\mu + \Delta_{odd}) \frac{v_{\mu}}{v_{u}} + \frac{1}{\sqrt{2}} u_{\alpha} \sum_{\alpha, \beta} v_{\alpha} \left( A_{N}\right)_{\alpha \beta} v_{N_{\beta}},
\]

\[
[M_{CPO}^{2}]_{12} = B\mu + \Delta_{odd},
\]

\[
[M_{CPO}^{2}]_{22} = (B\mu + \Delta_{odd}) \frac{v_{\mu}}{v_{d}} + \frac{\mu_{N}^{2}}{\sqrt{2}} v_{N_{\mu}}.
\]

As we assign the VEVs of left-handed sneutrinos to zero, the expressions in Eq.(23) recover the elements of mass-squared matrix for CP-odd Higgs in the MSSM. Additionally, the symmetric matrix in Eq.(22) contains two massless eigenstates which correspond to the neutral Goldstones swallowed by neutral gauge bosons \( Z, Z_{BL} \) after the symmetry \( SU(2) \times U_{Y}(1) \times U_{(B-L)} \) is broken down to the electromagnetic symmetry \( U_{e}(1): \)

\[
G^{0} = \frac{v_{u}}{v_{EW}} P_{u}^{0} - \frac{v_{d}}{v_{EW}} P_{d}^{0} - \frac{3}{\alpha} \frac{v_{\alpha}}{v_{u}} P_{\alpha}^{0} L_{\alpha},
\]

\[
G_{(B-L)}^{0} = \eta \frac{v_{u}}{v_{t}} P_{u}^{0} - \frac{v_{d}}{v_{t}} P_{d}^{0} + (1 - \eta) \frac{3}{\alpha} \frac{v_{\alpha}}{v_{t}} P_{\alpha}^{0} L_{\alpha} - \frac{3}{\alpha} \frac{v_{N_{\alpha}}}{v_{N_{\alpha}}} P_{\alpha}^{0} S_{\alpha},
\]

with \( \eta = 1 - \frac{v_{d}^{2}}{v_{EW}^{2}}, \) and \( v_{t}^{2} = v_{S}^{2} + \eta v_{SM}^{2}. \) To separate neutral Goldstones from physical states, we define the 8 \( \times \) 8 orthogonal matrix

\[
Z_{P}^{(0)} = \left\{ Z_{CH}^{(0)} \bigoplus Z_{SC}^{(0)} \right\}
\]

\[
\times \begin{pmatrix}
1_{2 \times 2} \bigoplus \\
1_{2 \times 2}
\end{pmatrix}
\]

\[
\begin{pmatrix}
-\frac{v_{S}^{2}v_{L_{1}}}{v_{t}}, & \frac{v_{N}}{v_{t}}, & \frac{v_{S}^{2}v_{L_{2}}}{v_{t}}, & \frac{v_{N}}{v_{t}}, & \frac{v_{S}^{2}v_{L_{3}}}{v_{t}}, & \frac{v_{N}}{v_{t}}, & \frac{v_{S}^{2}v_{L_{4}}}{v_{t}}, & \frac{v_{N}}{v_{t}} \\
-\frac{v_{S}^{2}v_{L_{2}}}{v_{t}}, & \frac{v_{N}}{v_{t}}, & \frac{v_{S}^{2}v_{L_{3}}}{v_{t}}, & \frac{v_{N}}{v_{t}}, & \frac{v_{S}^{2}v_{L_{4}}}{v_{t}}, & \frac{v_{N}}{v_{t}}, & \frac{v_{S}^{2}v_{L_{1}}}{v_{t}}, & \frac{v_{N}}{v_{t}} \\
-\frac{v_{S}^{2}v_{L_{3}}}{v_{t}}, & \frac{v_{N}}{v_{t}}, & \frac{v_{S}^{2}v_{L_{4}}}{v_{t}}, & \frac{v_{N}}{v_{t}}, & \frac{v_{S}^{2}v_{L_{1}}}{v_{t}}, & \frac{v_{N}}{v_{t}}, & \frac{v_{S}^{2}v_{L_{2}}}{v_{t}}, & \frac{v_{N}}{v_{t}} \\
-\frac{v_{S}^{2}v_{L_{4}}}{v_{t}}, & \frac{v_{N}}{v_{t}}, & \frac{v_{S}^{2}v_{L_{1}}}{v_{t}}, & \frac{v_{N}}{v_{t}}, & \frac{v_{S}^{2}v_{L_{2}}}{v_{t}}, & \frac{v_{N}}{v_{t}}, & \frac{v_{S}^{2}v_{L_{3}}}{v_{t}}, & \frac{v_{N}}{v_{t}} \\
\end{pmatrix}
\]

\[\bigoplus 1_{2 \times 2}\]
\[
\left(\begin{array}{cc}
1, 0, 0 \\
0, 0, 1 \\
0, 1, 0
\end{array}\right) \oplus 1_{5 \times 5}
\]

(25)

then we have
\[
Z_p^{(0)T} \cdot \begin{bmatrix} M^2_{CPO} & A_{CPO} \\ A^T_{CPO} & M^2 \end{bmatrix}_{6 \times 6} \cdot Z_p^{(0)} = \begin{bmatrix} 0_{2 \times 2} & 0_{2 \times 6} \\ 0_{6 \times 2} & [M^2_{p0}]_{6 \times 6} \end{bmatrix}.
\]

Finally, the \(8 \times 8\) mixing matrix \(Z_{A^0}\) in CP-odd Higgs sector is written as
\[
Z_{A^0} = \begin{bmatrix} 1_{2 \times 2} & 0_{2 \times 6} \\ 0_{6 \times 2} & (Z_p)_{6 \times 6} \end{bmatrix} \cdot Z_p^{(0)}
\]

(26)

with \(Z_p^\dagger \cdot M^2_{p0} \cdot Z_p = \text{diag}(m^2_{A^0_1}, \ldots, m^2_{A^0_6})\).

Where
\[
Z_{N^c}^{P,T} m^2_{N^c} Z_{N^c}^P = \text{diag}(0, \frac{\omega_A - \omega_B}{2v^2}, \frac{\omega_A + \omega_B}{2v^2})
\]

(27)

and the concrete expressions for \(\omega_{A,B}\) are
\[
\omega_A = \Lambda_{N^c_1}^2 (v^2 - v_{N_1}^2) + \Lambda_{N^c_2}^2 (v^2 - v_{N_1}^2), \\
\omega_B^2 = \omega_A^2 - 4\Lambda_{N^c_1}^2 \Lambda_{N^c_2}^2 v^2 N_1 v^2 N_3.
\]

(28)

Additionally the orthogonal \(3 \times 3\) rotation is written as
\[
\begin{bmatrix}
\left(\begin{array}{c}
Z_{N^c}^P \end{array}\right)_{11} \\
\left(\begin{array}{c}
Z_{N^c}^P \end{array}\right)_{21} \\
\left(\begin{array}{c}
Z_{N^c}^P \end{array}\right)_{31}
\end{bmatrix} = \frac{1}{v_N} \begin{bmatrix}
\begin{array}{c}
N_1 \\
N_2 \\
N_3
\end{array}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\left(\begin{array}{c}
Z_{N^c}^P \end{array}\right)_{12} \\
\left(\begin{array}{c}
Z_{N^c}^P \end{array}\right)_{22} \\
\left(\begin{array}{c}
Z_{N^c}^P \end{array}\right)_{32}
\end{bmatrix} = \frac{1}{\sqrt{|x_-|^2 + |y|^2 + |z_-|^2}} \begin{bmatrix}
\begin{array}{c}
x_- \\
y \\
z_-
\end{array}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\left(\begin{array}{c}
Z_{N^c}^P \end{array}\right)_{13} \\
\left(\begin{array}{c}
Z_{N^c}^P \end{array}\right)_{23} \\
\left(\begin{array}{c}
Z_{N^c}^P \end{array}\right)_{33}
\end{bmatrix} = \frac{1}{\sqrt{|x_+|^2 + |y|^2 + |z_+|^2}} \begin{bmatrix}
\begin{array}{c}
x_+ \\
y \\
z_+
\end{array}
\end{bmatrix}
\]

(29)
with

\[ x_+ = - \frac{\Lambda^4}{\bar{N}^2} v^2_{N_2} + \left[ \frac{\Lambda^2}{\bar{N}^2} \frac{v_{N_1}}{v_{N_3}} \cdot \frac{v_{N_2}}{v_{N_3}} \cdot \frac{v_{N_3}}{v_{N_3}} - \frac{\omega_A \mp \omega_B}{2 \Lambda^2 \bar{N}^2} \right] \{ v_{N_1} \}, \]

\[ y = \frac{\Lambda^2}{\bar{N}^2} \frac{v_{N_2}}{v_{N_3}} \],

\[ z_+ = \Lambda^2 \frac{\bar{N}^2}{\bar{N}^2} - \frac{\omega_A \mp \omega_B}{2 \Lambda^2 \bar{N}^2}. \]  

(30)

Since one-loop effective potential does not induce corrections to the mixing between \( P_u, P_d \) and \( P_{L_I}, P_{N_I} \), the mixing is dominated by the \( 2 \times 6 \) matrix \( A^{(0)}_{CP} \) originating from tree level contributions. Considering the constraints from neutrino oscillation, we derive the correction to mass of the lightest CP-odd neutral Higgs \( \sim 0.01 \) GeV from the mixing between \( P_u, P_d \) and \( P_{L_I}, P_{N_I} \) as \( m_\tilde{L}_I \simeq \Lambda_{\bar{N}_{1,2}} \simeq m_{Z_{BL}} \sim 1 \) TeV. This fact implies

\[ m^2_{A_1} \simeq \frac{B \mu + \Delta_{odd}}{\sin 2\beta}, \]  

(31)

here we adopt the definition

\[ \tan \beta = \frac{v_u}{\sqrt{v_d^2 + \sum_{\alpha=1}^{3} v_{L_\alpha}^2}}. \]  

(32)

III. THE LIGHTEST CP-EVEN HIGGS MASS MATRIX

It is well known for quite long time that radiative corrections modify the tree level mass squared matrix of neutral Higgs substantially in supersymmetry, and the main effect in those radiative contributions originates from Feynman loops involving the third generation fermions and their supersymmetric partners \[21\]. In order to obtain mass of the lightest neutral CP-even Higgs reasonably, we should also include the one-loop corrections from those fermions and corresponding supersymmetric partner in the MSSM with local \( U(1)_{B-L} \) symmetry. In the interaction basis \( H^{0,T} = (H_u^0, H_d^0, \tilde{\nu}_{L_I}, \tilde{\nu}_{R_J}) (I, J = 1, 2, 3) \), the \( 8 \times 8 \)
symmetric mass squared matrix is written as

\[
\begin{pmatrix}
[M^2_{\mu^0}]_{2\times2} & [A_{CP\,E}]_{2\times6} \\
[A^T_{CP\,E}]_{6\times2} & [M^2_s]_{6\times6}
\end{pmatrix},
\]

(33)

here the 2 × 2 mass squared matrix \(M^2_{\mu^0}\) is

\[
M^2_{\mu^0} = \begin{pmatrix}
[M^2_{CP\,E}]_{11} + \Delta_{11} & [M^2_{CP\,E}]_{12} + \Delta_{12} \\
[M^2_{CP\,E}]_{12} + \Delta_{12} & [M^2_{CP\,E}]_{22} + \Delta_{22}
\end{pmatrix},
\]

(34)

with

\[
[M^2_{CP\,E}]_{11} = (B\mu + \Delta_{odd}) \frac{\nu_d}{\nu_u} + m^2_z \sin^2 \beta,
\]

\[
[M^2_{CP\,E}]_{12} = -(B\mu + \Delta_{odd}) - m^2_z \sin \beta \cos \beta,
\]

\[
[M^2_{CP\,E}]_{22} = (B\mu + \Delta_{odd}) \frac{\nu_u}{\nu_d} + m^2_z \cos^2 \beta.
\]

(35)

Where

\[
\begin{align*}
\Delta_{11} &= \Delta^T_{11} + \Delta^B_{11} + \Delta^L_{11}, \\
\Delta_{12} &= \Delta^T_{12} + \Delta^B_{12} + \Delta^L_{12}, \\
\Delta_{22} &= \Delta^T_{22} + \Delta^B_{22} + \Delta^L_{22},
\end{align*}
\]

(36)

and \(\Delta^T_{11}, \Delta^T_{12}, \Delta^T_{22}\) represent the tree level corrections to CP-even Higgs mass squared matrix from sneutrinos after electroweak symmetry is broken:

\[
\begin{align*}
\Delta^T_{11} &= \frac{1}{\sqrt{2} v_u} \sum_{\alpha,\beta} v_{L\alpha} (A_N)_{\alpha\beta} v_{N\beta}, \\
\Delta^T_{12} &= m^2_z \sin \beta \cos \beta \left\{ 1 - \frac{v_d}{\sqrt{v_d^2 + v_{EW}^2 - v_{SM}^2}} \right\}, \\
\Delta^T_{22} &= \frac{g_1^2 + g_2^2}{4} (v_{EW}^2 - v_{SM}^2) + \frac{\mu^2}{\sqrt{2} v_d}.
\end{align*}
\]

(37)

In fact \(\Delta^T_{11} = \Delta^T_{12} = \Delta^T_{22} = 0\) when the VEVs of left-handed sneutrinos are assigned to zero. The concrete expressions for the radiative corrections from quark sector \(\Delta^B_{ij}\) (i, j =
1, 2) up to two-loop level can be found in literature [22], and the one-loop corrections from lepton sectors can also be found in [23] within framework of the MSSM. Obviously radiative corrections modify the mass spectrum of neutral Higgs drastically, and two-loop corrections decrease that from one-loop in most of the MSSM parameter space. Here it is sufficient to include the dominant one-loop corrections to the mass matrix of CP-even Higgs, and the expressions for $\Delta_{ij}^R, \Delta_{ij}^L$ are given in appendix C.

Furthermore, the $2 \times 6$ matrix $A_{CPE}$ is

$$A_{CPE} = A_{CPE}^{(0)} + \Delta A_{CPE},$$

(38)

where the tree level contribution $A_{CPE}^{(0)}$ is given in appendix B and nontrivial one-loop corrections $\Delta A_{CPE}$ are

$$\left(\Delta A_{CPE}\right)_{1(3+I)} = \frac{G_F m_I^2 g^{g_{BL} v_u v_{N_I}}}{2\sqrt{2} \pi^2} \sin^2 \beta \left(\frac{\ln m_{t_1}^2 - \ln m_{t_2}^2}{m_{t_1}^2 - m_{t_2}^2}\right) \left\{ \frac{\ln m_{t_1}^2 - \ln m_{t_2}^2}{m_{t_1}^2 - m_{t_2}^2} \right\} + \frac{A_i (A_i - \mu \cot \beta)}{(m_{t_1}^2 - m_{t_2}^2)^2} g(m_{t_1}^2, m_{t_2}^2)$$

$$- \frac{G_F m_I^2 g^{g_{BL} v_u v_{N_I}}}{2\sqrt{2} \pi^2} \cos^2 \beta \left(\frac{\mu (A_b - \mu \tan \beta)}{(m_{b_1}^2 - m_{b_2}^2)^2} g(m_{b_1}^2, m_{b_2}^2)\right)$$

$$- \frac{G_F m_I^2 g^{g_{BL} v_u v_{N_I}}}{2\sqrt{2} \pi^2} \left(\frac{\mu (A_{t_i} - \mu \tan \beta)}{(m_{t_i}^2 - m_{t_j}^2)^2} g(m_{t_i}^2, m_{t_j}^2)\right)$$

$$\left(\Delta A_{CPE}\right)_{2(3+I)} = - \frac{G_F m_I^2 g^{g_{BL} v_u v_{N_I}}}{2\sqrt{2} \pi^2} \sin^2 \beta \left(\frac{\ln m_{b_1}^2 - \ln m_{b_2}^2}{m_{b_1}^2 - m_{b_2}^2}\right) \left\{ \frac{\ln m_{b_1}^2 - \ln m_{b_2}^2}{m_{b_1}^2 - m_{b_2}^2} \right\} + \frac{A_b (A_b - \mu \tan \beta)}{(m_{b_1}^2 - m_{b_2}^2)^2} g(m_{b_1}^2, m_{b_2}^2)$$

$$+ \frac{G_F m_I^2 g^{g_{BL} v_u v_{N_I}}}{2\sqrt{2} \pi^2} \cos^2 \beta \left(\frac{\mu (A_{t_i} - \mu \cot \beta)}{(m_{t_i}^2 - m_{t_j}^2)^2} g(m_{t_i}^2, m_{t_j}^2)\right)$$

$$+ \frac{A_i (A_i - \mu \cot \beta)}{(m_{t_i}^2 - m_{t_j}^2)^2} g(m_{t_i}^2, m_{t_j}^2)\right\} \right. \left( I = 1, 2, 3 \right),$$

(39)

with the concrete expression of $g(x, y)$ presented in appendix C. Meanwhile the radiative corrections to $\left(\Delta A_{CPE}\right)_{1I}, \left(\Delta A_{CPE}\right)_{2I}$ are proportional to $v_{\nu_i}$, and can be neglected safely.
here. Note that \((\Delta A_{CPE}^{(1)_{i+j}}, \Delta A_{CPE}^{(2)_{i+j}})\) are independent of the renormalization scale \(\Lambda\), as they should be.

At the tree level, i.e. \(\Delta B_{ij} = \Delta L_{ij} = 0\) (\(i, j = 1, 2\)) and \(\Delta A_{CPE} = 0\), there are relations between the CP-even and CP-odd Higgs masses\[24\]

\[
\begin{align*}
\sum_{i=1}^{8} m_{H^0_i}^2 &= m_{Z}^2 + m_{Z_{BL}}^2 + \sum_{i=1}^{6} m_{A_{(2+i)}}^2 , \\
\prod_{i=1}^{8} m_{\tilde{H}_i}^2 &= \left( \frac{\nu_{2}^2 - 2 \nu_{u}^2}{\nu_{u}^2} \right) \left( \frac{\nu_{3}^2 - \nu_{2}^2 + 2 \nu_{3} \nu_{SM}}{\nu_{u}^2 + \nu_{2}^2} \right) m_{Z}^2 m_{Z_{BL}}^2 \prod_{i=1}^{6} m_{A_{(2+i)}}^2 .
\end{align*}
\] (40)

Certainly, radiative corrections to the neutral Higgs mass squared matrices destroy the relations in Eq.\[40\] strongly.

Considering the constraints from neutrino oscillation on parameter space of the model considered here, we find that the radiative correction from right-handed neutrinos/sneutrinos on the lightest CP-even Higgs mass is negligible. This conclusion coincides with that presented in Ref\[25\].

Applying above equations, one finds that the mass squared matrices for real part of sneutrinos is approximately approached as

\[
M_{S}^2 \cong \begin{pmatrix}
(m_{L_i}^2 + \Delta T_L) \delta_{IJ} \\
+ \frac{1}{4} m_{Z}^2 \cos 2\beta - \frac{m_{Z_{BL}}^2}{2} \delta_{IJ} \\
- \frac{g_{BL}^2 v_{L_i} v_{N_j}}{\sqrt{2}} v_{Y} A_{N_{IJ}} \\
- \frac{\mu v_{Y}}{\sqrt{2}} Y_{N_{IJ}}
\end{pmatrix}_{3 \times 3} , \quad \begin{pmatrix}
g_{BL} v_{L_i} v_{N_j} + \frac{v_{Y}}{\sqrt{2}} A_{N_{IJ}} \\
- \frac{\mu v_{Y}}{\sqrt{2}} Y_{N_{IJ}}
\end{pmatrix}_{3 \times 3}
\] (41)

where the 3×3 mass squared matrix \(M_{S}^2\) is

\[
M_{S}^2 \cong \begin{pmatrix}
\Lambda_{S_1}^2 + \frac{g_{BL}^2}{2} v_{N_1}^2 v_{N_2}^2 , \\
\frac{g_{BL}^2}{2} v_{N_1}^2 v_{N_2}^2 , \\
\Lambda_{S_2}^2 + \frac{g_{BL}^2}{2} v_{N_2}^2 v_{N_3}^2 , \\
\frac{g_{BL}^2}{2} v_{N_2}^2 v_{N_3}^2 + \frac{v_{N_1}}{v_{N_3}} \Lambda_{S_1}^2 , \\
\frac{g_{BL}^2}{2} v_{N_1}^2 v_{N_3}^2 - \frac{v_{N_1}}{v_{N_3}} \Lambda_{S_1}^2 , \\
\frac{g_{BL}^2}{2} v_{N_2}^2 v_{N_3}^2 - \frac{v_{N_2}}{v_{N_3}} \Lambda_{S_2}^2 , \\
\frac{g_{BL}^2}{2} v_{N_3}^2 + \frac{v_{N_3}}{v_{N_1}^2} \Lambda_{S_2}^2 + \frac{v_{N_2}}{v_{N_3}} \Lambda_{S_3}^2
\end{pmatrix}
\] (42)

Defining the orthogonal 3×3 rotation \(Z_{S}^T\), we get

\[
Z_{S}^T M_{S}^2 Z_{S} = \text{diag}(M_{S_1}^2, M_{S_2}^2, M_{S_3}^2) ,
\] (43)
where

\[
M_{\nu_k}^2 = \frac{1}{2} m_{z_{BL}}^2,
M_{\nu_k}^2 = \frac{\omega_A - \omega_B}{2 v_{N_3}},
M_{\nu_k}^2 = \frac{\omega_A + \omega_B}{2 v_{N_3}}.
\] (44)

Additionally the orthogonal 3 × 3 rotation is written as

\[
\begin{pmatrix}
Z_{\nu e}^{11} \\
Z_{\nu e}^{21} \\
Z_{\nu e}^{31}
\end{pmatrix} = \frac{1}{v_N} \begin{pmatrix} u_{N_1} \\ u_{N_2} \\ u_{N_3}
\end{pmatrix},
\begin{pmatrix}
Z_{\nu e}^{12} \\
Z_{\nu e}^{22} \\
Z_{\nu e}^{32}
\end{pmatrix} = \frac{1}{\sqrt{|X_-|^2 + |Y_-|^2 + |Z_-|^2}} \begin{pmatrix} X_- \\ Y_- \\ Z_-
\end{pmatrix},
\begin{pmatrix}
Z_{\nu e}^{13} \\
Z_{\nu e}^{23} \\
Z_{\nu e}^{33}
\end{pmatrix} = \frac{1}{\sqrt{|X_+|^2 + |Y_+|^2 + |Z_+|^2}} \begin{pmatrix} X_+ \\ Y_+ \\ Z_+
\end{pmatrix},
\] (45)

with

\[
X_+ = \frac{v_{N_3}}{v_{N_1}} \left[ 2 \Lambda_{\nu e}^2 \left( v_N^2 - v_{N_1}^2 \right) \right] \left[ \Lambda_{\nu e}^2 \left( v_N^2 - v_{N_1}^2 \right) \pm \omega_B \right] \\
-2 \Lambda_{\nu e}^2 \left( v_N^2 - v_{N_2}^2 \right) - g_{BL}^2 \Lambda_{\nu e}^2 \left( v_{N_1} v_{N_2} + g_{BL}^2 \Lambda_{\nu e}^2 v_{N_2}^2 v_{N_3} \right) \\
Z_+ = 2 \Lambda_{\nu e}^4 \left( v_N^2 - v_{N_2}^2 \right) + 2 \Lambda_{\nu e}^2 \left( v_{N_2} - v_{N_3}^2 \right) - g_{BL}^2 \Lambda_{\nu e}^2 \left( v_N^2 + v_{N_2}^2 \right) v_{N_3}^2 \\
+ g_{BL}^2 \Lambda_{\nu e}^2 \left( v_N^2 - v_{N_1}^2 \right) v_{N_3}^2 \mp \left( 2 \Lambda_{\nu e}^2 - g_{BL}^2 v_{N_3}^2 \right) \omega_B.
\] (46)

To continue with our analysis on the mass spectrum and mixing in the neutral scalar sector, we assume \( \max(|v_a(A_N)_{IJ}|, |\mu v_d(Y_N)_{IJ}|, g_{BL}^2 v_{L_j} v_{N_j}) \ll \min(|m_{L_j}^2 + \Delta T_L + \left( \frac{1}{2} m_{z}^2 \cos 2\beta - m_{z_{BL}}^2 \right)|, m_{\nu_k}^2), \) then obtain

\[
m_{\nu_k}^2 \simeq m_{L_i}^2 + \Delta T_L + \frac{1}{2} \left( m_{z}^2 \cos 2\beta - m_{z_{BL}}^2 \right)
\]
Meanwhile, the mixing matrix $Z_S$ is

$$Z_S \simeq \begin{pmatrix}
\sum_{\alpha=1}^{3} & (E^2)_{ij}^\alpha, \\
\left[\begin{array}{c}
\frac{2M^2 H_0}{v_R^2} - 2m^2 L_i - 2\Delta T_L - m^2 Z_{BL} \cos 2\beta + m^2 R_{BL} \\
\frac{2(\sum_{\alpha=1}^{3} (Z_{S}^2)^\alpha (Z_{S}^2)^\alpha)_{ij}}{m^2 - \lambda_i}
\end{array}\right]^{13\times3},
\end{pmatrix},$$

and the $8 \times 8$ mixing matrix is written as

$$Z_H = \left[\begin{array}{c}
\left[(Z_R)_{ij}\right]_{2\times2}, \\
\left[\sum_{\alpha=1}^{2} (Z_R)_{ij} (Z_{S}^T A_{CP E} Z_S)_{ij}\right]_{6\times6},
\end{array}\right].$$

Then we formulate the mass squared for those CP-even neutral scalars as

$$m^2_{h^0_1} \simeq \lambda_1 + \sum_{\alpha=1}^{6} \frac{(Z_R^T A_{CP E} Z_S)_{i\alpha}}{(m^2_{S\alpha} - \lambda_1)^2} m^2_{S\alpha},$$

$$m^2_{h^0_2} \simeq \lambda_2 + \sum_{\alpha=1}^{6} \frac{(Z_R^T A_{CP E} Z_S)_{i\alpha}}{(m^2_{S\alpha} - \lambda_2)^2} m^2_{S\alpha},$$

$$m^2_{h^0_{3+}} = m^2_{S_3} + \frac{(Z_R^T A_{CP E} Z_S)_{1\alpha}^2}{(m^2_{S\alpha} - \lambda_1)^2} \lambda_1 + \frac{(Z_R^T A_{CP E} Z_S)_{2\alpha}^2}{(m^2_{S\alpha} - \lambda_2)^2} \lambda_2,$$

with

$$Z_R = \begin{pmatrix}
\cos \alpha_H & -\sin \alpha_H \\
\sin \alpha_H & \cos \alpha_H
\end{pmatrix}.$$
\[
\tan 2\alpha_H = \frac{2(M^2_{\tilde{H}^0})_{12}}{(M^2_{\tilde{H}^0})_{11} - (M^2_{\tilde{H}^0})_{22}},
\]

\[
\lambda_{1,2} = \frac{1}{2} \left( (M^2_{\tilde{H}^0})_{11} + (M^2_{\tilde{H}^0})_{22} \right) \mp \sqrt{\frac{1}{4} \left( (M^2_{\tilde{H}^0})_{11} - (M^2_{\tilde{H}^0})_{22} \right)^2 + (M^2_{\tilde{H}^0})_{12}^2}.
\]

One most stringent constraint on parameter space of the model is that the mass squared matrix in Eq. (33) should produce an eigenvalue around \((125 \text{ GeV})^2\) as mass squared of the lightest neutral CP-even Higgs. The current combination of the ATLAS and CMS data gives:

\[
m_{h^0} = 125.9 \pm 2.1 \text{ GeV},
\]

this fact constrains parameter space of the MSSM with local \(U(1)_{B-L}\) symmetry strongly. Considering the constraints from neutrino oscillation experimental data, the mixing between \(H^0_u, H^0_d\) and real part of right-handed sneutrinos is only about \(10^{-6}\) at tree level. Nevertheless one-loop radiative corrections can enhance the mixing drastically when \(A_t, A_b, A_\tau, \nu_N \geq 1 \text{ TeV}\) and \(\tan \beta > 1\), and the corrections to mass of the lightest CP-even Higgs from this mixture increase the corresponding MSSM radiative contributions.

IV. THE MASS MATRIX FOR NEUTRALINOS AND NEUTRINOS

After the local gauge symmetry \(SU(2)_L \otimes U(1)_Y \otimes U(1)_{(B-L)}\) is broken down, the nonzero VEVs of left- and right-handed sneutrinos induce the mixing between neutralinos (charginos) and neutrinos (charged leptons). As mentioned above, the MSSM with local \(U(1)_{B-L}\) symmetry naturally predicates two sterile neutrinos with sub-eV masses. In the basis \((\nu^{c}_{Lj}, N^c_j, i\lambda^B_{BL}, i\lambda^B_{BL}, i\lambda^3_A, \psi^{1}_{H_d}, \psi^{2}_{H_d})\), the mass matrix for neutralino-neutrino is formulated as

\[
M_N = \begin{pmatrix}
0_{3 \times 3} & (A^{(1)}_N)_{3 \times 4} & (A^{(2)}_N)_{3 \times 4} \\
(A^{(1)T}_N)_{4 \times 3} & (M^{(0)}_N)_{4 \times 4} & (A^{(3)}_N)_{4 \times 4} \\
(A^{(2)T}_N)_{4 \times 3} & (A^{(3)T}_N)_{4 \times 4} & (M_N)_{4 \times 4}
\end{pmatrix},
\]

17
where $\mathcal{M}_N$ denotes the $4 \times 4$ mass matrix for neutralinos in the MSSM, the concrete expressions for $\mathcal{M}^{(0)}_N$, $\mathcal{A}^{(1)}_N$, $\mathcal{A}^{(2)}_N$ and $\mathcal{A}^{(3)}_N$ are

$$
\mathcal{M}^{(0)}_N = \begin{pmatrix}
0_{3 \times 3} & \begin{pmatrix} g_{BL} v_{N_{1}^I J} \end{pmatrix}_{3 \times 1} \\
\begin{pmatrix} g_{BL} v_{N_{1}^I} \end{pmatrix}_{1 \times 3} & 2m_{BL}
\end{pmatrix},
$$

$$
\mathcal{A}^{(1)}_N = \begin{pmatrix}
\begin{pmatrix} \sqrt{\frac{3}{2}} (Y_N)_{I J} \end{pmatrix}_{3 \times 3} & \begin{pmatrix} -g_{BL} v_{L_{1}^I} \end{pmatrix}_{3 \times 1}
\end{pmatrix},
$$

$$
\mathcal{A}^{(2)}_N = \begin{pmatrix}
\begin{pmatrix} -g_{BL} v_{L_{1}^I} \end{pmatrix}_{3 \times 1} & \begin{pmatrix} g_{BL} v_{L_{1}^I} \end{pmatrix}_{3 \times 1} & 0_{3 \times 1} & \begin{pmatrix} \nu^I \end{pmatrix}_{3 \times 1}
\end{pmatrix},
$$

$$
\mathcal{A}^{(3)}_N = \begin{pmatrix}
0_{3 \times 1} & 0_{3 \times 1} & 0_{3 \times 1} & \begin{pmatrix} \frac{1}{\sqrt{2}} \sum_{\alpha=1}^{3} v_{L_{\alpha}} (Y_N)_{\alpha J} \end{pmatrix}_{3 \times 1}
\end{pmatrix},
$$

with the row indices of matrix $I'$, $J' = 1$, 2, 3, and the column indices of matrix $I$, $J = 1$, 2, 3, respectively. Defining the $4 \times 4$ orthogonal matrix

$$
\mathcal{Z}_N^{(0)} = \begin{pmatrix}
-v_{N_{1}} \sqrt{\frac{1}{2} \Delta_{BL} \eta_{BL}} & v_{N_{2}} \sqrt{\frac{1}{2} \Delta_{BL} \eta_{BL}} & -ig_{BL} v_{N_{1}} \sqrt{\frac{1}{2} \Delta_{BL} \eta_{BL}} & g_{BL} v_{N_{1}} \sqrt{\frac{1}{2} \Delta_{BL} \eta_{BL}} \\
0 & v_{N_{1}} \sqrt{\frac{1}{2} \Delta_{BL} \eta_{BL}} & -ig_{BL} v_{N_{2}} \sqrt{\frac{1}{2} \Delta_{BL} \eta_{BL}} & g_{BL} v_{N_{2}} \sqrt{\frac{1}{2} \Delta_{BL} \eta_{BL}} \\
v_{N_{1}} \sqrt{\frac{1}{2} \Delta_{BL} \eta_{BL}} & 0 & -ig_{BL} v_{N_{3}} \sqrt{\frac{1}{2} \Delta_{BL} \eta_{BL}} & g_{BL} v_{N_{3}} \sqrt{\frac{1}{2} \Delta_{BL} \eta_{BL}} \\
0 & 0 & \sqrt{\frac{1}{2} \Delta_{BL} \eta_{BL}} & \frac{1}{\sqrt{2}} \eta_{BL}
\end{pmatrix},
$$

one obtains

$$
\begin{pmatrix}
1_{3 \times 3} & 0_{3 \times 4} & 0_{3 \times 4} \\
0_{4 \times 3} & \mathcal{Z}_N^{(0) T} & 0_{4 \times 4} \\
0_{4 \times 3} & 0_{4 \times 4} & 1_{4 \times 4}
\end{pmatrix} \mathcal{M}_N \begin{pmatrix}
1_{3 \times 3} & 0_{3 \times 4} & 0_{3 \times 4} \\
0_{4 \times 3} & \mathcal{Z}_N^{(0) T} & 0_{4 \times 4} \\
0_{4 \times 3} & 0_{4 \times 4} & 1_{4 \times 4}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
\mathcal{A}_N^{(1) T} \mathcal{Z}_N^{(0) T} & \mathcal{Z}_N^{(0) T} \mathcal{M}_N^{(0)} & \mathcal{Z}_N^{(0) T} \mathcal{A}_N^{(2) T} \\
\mathcal{A}_N^{(2) T} \mathcal{Z}_N^{(0) T} & \mathcal{Z}_N^{(0) T} \mathcal{M}_N^{(0)} & \mathcal{Z}_N^{(0) T} \mathcal{A}_N^{(3) T} \\
\mathcal{A}_N^{(3) T} \mathcal{Z}_N^{(0) T} & \mathcal{Z}_N^{(0) T} \mathcal{M}_N^{(0)} & \mathcal{M}_N^{(4) T}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
\mathcal{M}_N^{(5)} & \mathcal{M}_N^{(6)} \\
\mathcal{M}_N^{(7)} & \mathcal{M}_N^{(8)}
\end{pmatrix}.
$$
Where $\Delta_{BL} = \sqrt{m_{BL}^2 + g_{BL}^2 v_{N}^2}$, $\eta_{BL}^\pm = \sqrt{1 \pm \frac{m_{BL}}{\Delta_{BL}}}$, and $Z_N^{(0)^T} M_N^{(0)} Z_N^{(0)} = diag(0, 0, \Delta_{BL} - m_{BL}, \Delta_{BL} + m_{BL})$, respectively.

Using Eq. (54) and Eq. (56), we formulate the submatrices in Eq. (57) respectively as

\[
\mathcal{M} = \begin{pmatrix}
\Delta_{BL} - m_{BL} & 0 & 0 & 0 & 0 & -i\varepsilon_-
0 & \Delta_{BL} + m_{BL} & 0 & 0 & 0 & \varepsilon_+
0 & 0 & 2m_1 & 0 & -\frac{g_1 v_L}{2} & \frac{g_1 v_L}{2}
0 & 0 & 0 & 2m_2 & \frac{g_2 v_L}{2} & -\frac{g_2 v_L}{2}
0 & 0 & -\frac{g_1 v_L}{2} & \frac{g_2 v_L}{2} & 0 & \mu
-i\varepsilon_- & \varepsilon_+ & \frac{g_1 v_L}{2} & -\frac{g_2 v_L}{2} & \mu & 0
\end{pmatrix} ,
\]

\[
m_D = \begin{pmatrix}
-i\delta_1 & \delta_1^+ & -\frac{g_1}{2} v_{L_1} & -\frac{g_2}{2} v_{L_1} & 0 & \frac{1}{\sqrt{2}} \zeta_1
-i\delta_2 & \delta_2^+ & -\frac{g_1}{2} v_{L_2} & -\frac{g_2}{2} v_{L_2} & 0 & \frac{1}{\sqrt{2}} \zeta_2
-i\delta_3 & \delta_3^+ & -\frac{g_1}{2} v_{L_3} & -\frac{g_2}{2} v_{L_3} & 0 & \frac{1}{\sqrt{2}} \zeta_3
0 & 0 & 0 & 0 & 0 & \varepsilon_{13}
0 & 0 & 0 & 0 & 0 & \varepsilon_{12}
\end{pmatrix} ,
\]

\[
m_\nu = \begin{pmatrix}
0 & 0 & 0 & \delta_{13} & \delta_{12}
0 & 0 & 0 & \delta_{23} & \delta_{22}
0 & 0 & 0 & \delta_{33} & \delta_{32}
\delta_{13} & \delta_{23} & \delta_{33} & 0 & 0
\delta_{12} & \delta_{22} & \delta_{32} & 0 & 0
\end{pmatrix} ,
\]

(58)

where the abbreviations are

\[
\varepsilon_\pm = \frac{g_{BL} \xi_N^2}{2 \Delta_{BL} \eta_{BL}^\pm} ,
\]

\[
\delta_{12} = \frac{v_u}{\sqrt{2} (v_{N_1}^2 + v_{N_2}^2)} \left[ - (Y_N)_{i1} v_{N_2} + (Y_N)_{i2} v_{N_1} \right] ,
\]

\[
\delta_{13} = \frac{v_u}{\sqrt{2} (v_{N_1}^2 + v_{N_3}^2)} \left[ - (Y_N)_{i1} v_{N_3} + (Y_N)_{i3} v_{N_1} \right] ,
\]

\[
\varepsilon_{12} = \frac{1}{v_u} \sum_{\alpha=1}^3 v_{L_{\alpha}} \delta_{\alpha 2} ,
\]

\[
\varepsilon_{13} = \frac{1}{v_u} \sum_{\alpha=1}^3 v_{L_{\alpha}} \delta_{\alpha 3} ,
\]

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\[
\delta_i^\pm = \frac{g_{BL} v_u}{\sqrt{2} \Delta_{BL} \eta_{BL}^\pm} \zeta_i \mp \frac{g_{BL} \eta_{BL}^\pm v_L}{\sqrt{2}}.
\]

Defining a \(11 \times 11\) approximated orthogonal transformation matrix \(Z_N\)

\[
Z_N = \begin{pmatrix} 
1 - \frac{1}{2} m_D \cdot \mathcal{M}^{-2} \cdot m_D^T & [m_D \cdot \mathcal{M}^{-1} + m_D \cdot \mathcal{M}^{-2}] \cdot m_D \cdot \mathcal{M}^{-1} \end{pmatrix}_{5 \times 5} 
- [m_D \cdot \mathcal{M}^{-2} \cdot m_D^T \cdot m_D]_{6 \times 5} \end{pmatrix}
\]

we finally write the effective mass matrix for five light neutrinos (three active and two sterile) as

\[
m_{\nu}^{\text{eff}} \simeq m_{\nu} - m_D \cdot \mathcal{M}^{-1} \cdot m_D^T - \frac{1}{2} m_{\nu} \cdot m_D \cdot \mathcal{M}^{-2} \cdot m_D^T - \frac{1}{2} m_D \cdot \mathcal{M}^{-2} \cdot m_D^T \cdot m_D \cdot m_{\nu} \\
\downarrow \\
\mathcal{M}_{\nu}^{LL} \begin{pmatrix} \mathcal{M}_{\nu}^{LR} \\ \mathcal{M}_{\nu}^{LR,T} \end{pmatrix}_{3 \times 3} \begin{pmatrix} \mathcal{M}_{\nu}^{RR} \end{pmatrix}_{2 \times 2}
\]

Where

\[
(M_{\nu}^{LL})_{ij} = \frac{v_{L_i} v_{L_j}}{\Lambda_{\nu}} + \frac{\zeta_i \zeta_j}{\Lambda_{\zeta}} + \frac{v_{L_i} \zeta_j + v_{L_j} \zeta_i}{\Lambda_{v \zeta}} \\
- \frac{1}{2} \frac{\Lambda_1^2}{\Lambda_1^2} \left[ \left( \delta_{i2} \zeta_j + \delta_{j2} \zeta_i \right) \varepsilon_{12} + \left( \delta_{i3} \zeta_j + \delta_{j3} \zeta_i \right) \varepsilon_{13} \right] \\
- \frac{1}{2} \frac{\Lambda_2^2}{\Lambda_2^2} \left[ \left( \delta_{i2} v_{L_j} + \delta_{j2} v_{L_i} \right) \varepsilon_{12} + \left( \delta_{i3} v_{L_j} + \delta_{j3} v_{L_i} \right) \varepsilon_{13} \right],
\]

\[
(M_{\nu}^{LR})_{i1} = \delta_{i3} + \frac{\delta_i^+ \sum_{a=1}^3 \delta_{a} \delta_{a3}}{2 (\Delta_{BL} - m_{BL})} - \frac{\delta_i^- \sum_{a=1}^3 \delta_{a} \delta_{a3}}{2 (\Delta_{BL} + m_{BL})} \\
- \frac{\mu^2 \bar{m}^2 v_{SM}^2}{8 m^8} + \frac{\alpha (g_1^2 m_2^2 + g_2^2 m_1^2)}{2 \mu^8} v_{L_i} \sum_{a=1}^3 v_{L_a} \delta_{a3} \\
- \frac{1}{\sqrt{2} \Lambda_1^2} \zeta_i \sum_{a=1}^3 \zeta_a \delta_{a3} - \frac{\bar{m} v_{SM}^2}{2 \sqrt{2} \mu^4} \varepsilon_{13} \zeta_i + \frac{\bar{m} \mu v_{SM}}{2 \mu^4} \varepsilon_{13} \zeta_{v_i} \\
- \frac{1}{\sqrt{2} \Lambda_2^2} \left[ v_{L_i} \sum_{a=1}^3 \zeta_a \delta_{a3} + \zeta_i \sum_{a=1}^3 v_{L_a} \delta_{a3} \right],
\]

\[
(M_{\nu}^{LR})_{i2} = \left( M_{\nu}^{LR} \right)_{i1} \left( \delta_{i3} \rightarrow \delta_{i2}, \varepsilon_{13} \rightarrow \varepsilon_{12} \right),
\]

\[
(M_{\nu}^{RR})_{11} = - \frac{2}{\Lambda_2^2} \varepsilon_{13} \sum_{a=1}^3 v_{L_a} \delta_{a3} - \frac{2}{\Lambda_2^2} \varepsilon_{13} \sum_{a=1}^3 \zeta_a \delta_{a3} - \frac{\bar{m} v_{SM}^2}{2 \mu^4} \varepsilon_{13}^2,
\]

\[
(M_{\nu}^{RR})_{12} = - \frac{1}{\Lambda_2^2} \left[ \varepsilon_{12} \sum_{a=1}^3 v_{L_a} \delta_{a3} + \varepsilon_{13} \sum_{a=1}^3 v_{L_a} \delta_{a2} \right].
\]
Three active neutrinos with sub-eV masses require max$(\nu_\ell, \zeta) \leq 10^{-2}\text{GeV}$, where $i, j = 1, 2, 3$ are the generation indices.
Only including the leading contributions to the neutrino mass matrix in Eq. (61), one finds that only two left-handed neutrinos acquire nonzero masses. Two sterile neutrinos and another active left-handed neutrino acquire their physical masses after we consider next-to-leading corrections to the neutrino mass matrix in Eq. (61). To guarantee decoupling two light sterile neutrinos from the active ones, we choose the Yukawa couplings for right-handed neutrinos as

$$Y_N \simeq \frac{1}{v_N} \begin{pmatrix} v_{N_1} Y_1, & v_{N_2} Y_1, & v_{N_3} Y_1 \\ v_{N_1} Y_2, & v_{N_2} Y_2, & v_{N_3} Y_2 \\ v_{N_1} Y_3, & v_{N_2} Y_3, & v_{N_3} Y_3 \end{pmatrix} = \frac{1}{v_N} \begin{pmatrix} Y_1, & 0, & 0 \\ 0, & Y_2, & 0 \\ 0, & 0, & Y_3 \end{pmatrix} \begin{pmatrix} v_{N_1}, & v_{N_2}, & v_{N_3} \\ v_{N_1}, & v_{N_2}, & v_{N_3} \\ v_{N_1}, & v_{N_2}, & v_{N_3} \end{pmatrix},$$

then get

$$\zeta_i \simeq Y_i v_N \quad (i = 1, 2, 3).$$

Theoretical evaluations on the mass squared differences of neutrinos are approximately formulated as

$$\Delta m^2_A \simeq \sqrt{b^2 - 4c^2},$$

$$\Delta m^2_{\odot} \simeq \frac{b - \sqrt{b^2 - 4c^2}}{2}$$

with

$$b = \frac{(v_{\text{EW}}^2 - v_{\text{SM}}^2)^2}{\Lambda^2 v_L^2} + \frac{\zeta^4}{\Lambda^2 v_L^2} + \frac{2}{\Lambda^2 v_L^2} \left\{ (v_{\text{EW}}^2 - v_{\text{SM}}^2) \zeta^2 + (\zeta \cdot v_L)^2 \right\}$$

$$+ \frac{2(\zeta \cdot v_L)^2}{\Lambda^2 v_L^2} + \frac{4(v_{\text{EW}}^2 - v_{\text{SM}}^2)^2}{\Lambda^2 v_L^2} (\zeta \cdot v_L) + \frac{4\zeta^2}{\Lambda^2 v_L^2} (\zeta \cdot v_L),$$

$$c = \left( \frac{1}{\Lambda^2 v_L^2} - \frac{1}{\Lambda^2 v_L^2} \right) \left[ (v_{\text{EW}}^2 - v_{\text{SM}}^2) \zeta^2 - (\zeta \cdot v_L)^2 \right].$$

Here, we adopt the abbreviations $\zeta = (\zeta_1, \zeta_2, \zeta_3)$, $v_L = (v_{L_1}, v_{L_2}, v_{L_3})$, $\zeta \cdot v_L = \sum_{\alpha=1}^3 v_{L_\alpha} \zeta_\alpha$, and $\zeta^2 = \sum_{\alpha=1}^3 \zeta_\alpha^2$, respectively.

In the case of three active neutrino mixing, so far the available measurements on the neutrino oscillations can determine the neutrino mass spectrum up to two possible solutions:

- the normal ordering (NO) spectrum:

$$m_{\nu_1} < m_{\nu_2} < m_{\nu_3}, \quad \Delta m^2_A = \Delta m^2_{A_1}, \quad \Delta m^2_{\odot} = \Delta m^2_{A_2};$$

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• the inverted ordering (IO) spectrum:

\[ m_{\nu_3} < m_{\nu_1} < m_{\nu_2}, \quad \Delta m^2_A = \Delta m^2_{23}, \quad \Delta m^2_\odot = \Delta m^2_{13}. \quad (70) \]

The flavor neutrinos are mixed into massive neutrinos \( \nu_{1,2,3} \) during their flight, and the mixings are described by the Pontecorvo-Maki-Nakagawa-Sakata matrix \( U_{PMNS} \) \[26, 27\]:

\[
\begin{align*}
\sin \theta_{13} &= \left| (U_{PMNS})_{13} \right|, \\
\cos \theta_{13} &= \sqrt{1 - \left| (U_{PMNS})_{13} \right|^2}, \\
\sin \theta_{23} &= \frac{\left| (U_{PMNS})_{23} \right|}{\sqrt{1 - \left| (U_{PMNS})_{13} \right|^2}}, \\
\cos \theta_{23} &= \frac{\left| (U_{PMNS})_{33} \right|}{\sqrt{1 - \left| (U_{PMNS})_{13} \right|^2}}, \\
\sin \theta_{12} &= \frac{\left| (U_{PMNS})_{12} \right|}{\sqrt{1 - \left| (U_{PMNS})_{13} \right|^2}}, \\
\cos \theta_{12} &= \frac{\left| (U_{PMNS})_{11} \right|}{\sqrt{1 - \left| (U_{PMNS})_{13} \right|^2}}. 
\end{align*}
\quad (71)\]

Through several recent reactor oscillation experiments \[7, 28–31\], the mixing angle \( \theta_{13} \) is now precisely known. The global fit of \( \theta_{13} \) gives \[32\]

\[ \sin^2 \theta_{13} = 0.023 \pm 0.0023, \quad (72) \]

and other experimental observations relating solar and atmospheric neutrino oscillations are shown as \[19\]:

\[
\begin{align*}
\Delta m^2_\odot &= 7.58^{+0.22}_{-0.26} \times 10^{-5} \text{eV}^2, \\
\Delta m^2_A &= 2.35^{+0.12}_{-0.09} \times 10^{-3} \text{eV}^2, \\
\sin^2 \theta_\odot &= 0.306^{+0.018}_{-0.015}, \\
\sin^2 \theta_A &= 0.42^{+0.08}_{-0.03}. 
\end{align*}
\quad (73)\]

V. THE MASS MATRIX FOR CHARGINOS AND CHARGED LEPTONS

The mass terms of charginos are written as

\[
-L_{\text{charginos}} = \left( e^-_{L_I}, \ i\lambda^-_A, \ \psi^2_{H_d} \right) \mathcal{M}_c \left( e^+_{R_J}, \ i\lambda^+_A, \ \psi^1_{H_u} \right) + h.c. \quad (74)
\]

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with

\[
M_c = \left( \begin{array}{ccc}
-\frac{1}{\sqrt{2}} (Y_e)_{I J} v_d, & \left( \frac{g_{2 L}}{\sqrt{2}} (Y_\ell)_{I J} \right)_{3 \times 1}, & \left( -\frac{1}{\sqrt{2}} \xi_{L I J} \right)_{3 \times 1} \\
0_{1 \times 3}, & 2m_2, & \frac{\xi_{e I J}}{\sqrt{2} s_w} \\
\frac{1}{\sqrt{2}} \sum_{a=1}^3 v_{L a} (Y_\ell)_{a I}, & \frac{\xi_{e I J}}{\sqrt{2} s_w}, & -\mu \\
\end{array} \right). \tag{75}
\]

Defining

\[
Z_- = \left( \begin{array}{ccc}
[1 - \frac{1}{2} \xi_T \xi_L]_{3 \times 3}, & [- \xi_T]_{3 \times 2}, & \left( V_L 0 \right) \\
[\xi_L]_{2 \times 3}, & \left( 1 - \frac{1}{2} \xi_{T L} \xi_{L I} \right)_{2 \times 2}, & \left( 0 U_- \right) \\
\end{array} \right), \tag{76}
\]

and

\[
Z_+ = \left( \begin{array}{ccc}
[1 - \frac{1}{2} \xi_T \xi_R]_{3 \times 3}, & [- \xi_T]_{3 \times 2}, & \left( V_R 0 \right) \\
[\xi_R]_{2 \times 3}, & \left( 1 - \frac{1}{2} \xi_{T R} \xi_{R I} \right)_{2 \times 2}, & \left( 0 U_+ \right) \\
\end{array} \right), \tag{77}
\]

we obtain

\[
(\xi_L)_{1 I} = -\frac{1}{\Delta_+} \left\{ \frac{e \mu v_d}{\sqrt{2} s_w} - \frac{e v_d}{2 s_w} \xi_L + \frac{e v_d (2m_2 v_u - \mu v_d)}{2 \sqrt{2} s_w \Delta_+} \sum_{a=1}^3 v_{L a} (Y_e Y_e^T)_{a I} \right\},
\]

\[
(\xi_L)_{2 I} = -\frac{1}{\Delta_+} \left\{ \frac{e^2 v_u v_d}{2 s_w^2} + \sqrt{2} m_2 \xi_L - \frac{v_d (8s_w^2 m_2 + e^2 v_d^2)}{4 s_w^2 \Delta_+} \sum_{a=1}^3 v_{L a} (Y_e Y_e^T)_{a I} \right\}, \tag{78}
\]

and

\[
(\xi_R)_{1 I} = -\frac{1}{\Delta_\pm} \left\{ \left[ \frac{e v_u}{2 s_w} - \frac{e v_d (2s_w^2 \mu^2 + e^2 v_d^2)}{4 s_w^2 \Delta_\pm} \right] \sum_{a=1}^3 v_{L a} (Y_e^T Y_e)_{a I} - \frac{e v_d (2s_w^2 \mu^2 + e^2 v_d^2)}{4 s_w^2 \Delta_\pm} \sum_{a=1}^3 v_{N a} (Y_N^T Y_N)_{a I} \right\},
\]

\[
(\xi_R)_{2 I} = \frac{1}{\Delta_\pm} \left\{ \left[ \sqrt{2} m_2 - \frac{e^2 v_u v_d (2m_2 v_u - \mu v_d)}{2 \sqrt{2} s_w \Delta_\pm} \right] \sum_{a=1}^3 v_{L a} (Y_e)_{a I} - \frac{v_d (8s_w^2 m_2^2 + e^2 v_d^2)}{4 s_w^2 \Delta_\pm} \sum_{a=1}^3 v_{N a} (Y_N^T Y_N)_{a I} \right\}. \tag{79}
\]

Here, \( I = 1, 2, 3 \) and \( \Delta_\pm = 2m_2 \mu + e^2 v_u v_d / (2s_w^2) \). Considering corrections from the mixing between charged leptons and charginos, we approximate the elements of \( 3 \times 3 \) mass matrix
for charged leptons as

\[
\left( m_E \right)_{IJ} = -\frac{1}{\sqrt{2}} \left( Y_E \right)_{IJ} v_d + \frac{e^2 v_u}{2 \sqrt{2} s_w^2} \Delta_\pm v_{L_I} \sum_{\alpha=1}^{3} \nu_{L_{\alpha}} \left( Y_E \right)_{\alpha J} \\
+ \frac{m_2}{\Delta_\pm} \sum_{\alpha=1}^{3} \nu_{L_{\alpha}} \left( Y_E \right)_{\alpha J} . \tag{80}
\]

Similarly, the elements of 2 × 2 mass matrix for charginos are approached as

\[
\left( M_{\pm} \right)_{11} = 2 m_2 + \frac{e^2 \mu}{4 s_w^2} \left( \nu_{E_W}^2 - \nu_{SM}^2 \right) - \frac{e^2 v_d \Delta_\pm}{4 \sqrt{2} s_w^2}, \\
\left( M_{\pm} \right)_{12} = \frac{e \nu_u}{\sqrt{2} s_w} - \frac{e \mu \Delta_\pm}{4 s_w^2} - \frac{e v_d \zeta^2}{4 \sqrt{2} s_w^2}, \\
\left( M_{\pm} \right)_{21} = \frac{e \nu_d}{\sqrt{2} s_w} + \frac{e^2 \nu_u}{4 \sqrt{2} s_w^2} \left( \nu_{E_W}^2 - \nu_{SM}^2 \right) + \frac{m_2 \Delta_\pm}{4 s_w^2} + \frac{\nu_{T} \Delta_\pm}{4 \sqrt{2} s_w^2} u_{L_{\alpha}} \left( Y_E \right)_{\alpha J} u_{L_{\beta}}, \\
\left( M_{\pm} \right)_{22} = \frac{e \nu_u}{\sqrt{2} s_w} + \frac{m_2 \Delta_\pm}{2 \Delta_\pm} \sum_{\alpha, \beta} u_{L_{\alpha}} \left( Y_E \right)_{\alpha J} u_{L_{\beta}} . \tag{81}
\]

Furthermore, the submatrices \( V_{L,R} \) and \( U_{\pm} \) respectively diagonalize \( m_E \) and \( M_{\pm} \) in the following manner

\[
V^T \cdot m_E \cdot V = \text{diag}(m_e, m_\mu, m_\tau), \\
U^T \cdot M_{\pm} \cdot U = \text{diag}(m_{\chi^\pm_1}, m_{\chi^\pm_2}). \tag{82}
\]

Finally the interactions between CP-even Higgs and charginos/charged leptons are formulated as

\[
\mathcal{L}_{\mathcal{M}_{\chi^\pm_1 \alpha}} = \sum_{i=1}^{10} \sum_{\alpha, \beta} \xi_{i,\alpha, \beta} H_i^{0} \tilde{\chi}^{-}_{\alpha} \omega_{-}^{-} + \sum_{i=1}^{10} \sum_{\alpha=1}^{3} \xi_{i,\alpha} H_i^{0} \tilde{e}_{\alpha} \omega_{-}^{-} \\
+ \sum_{i=1}^{10} \sum_{\alpha=1}^{3} \sum_{I,J} \xi_{i,\alpha}^{2} H_i^{0} \tilde{\chi}^{-}_{\alpha} \omega_{-}^{-} e_{I,J} + \sum_{i=1}^{10} \xi_{i,J}^{2} H_i^{0} \tilde{e}_{I,J} \omega_{-}^{-} e_{I,J} + \text{h.c.} \tag{83}
\]

with

\[
\xi_{i,\alpha, \beta} = \frac{e}{\sqrt{2} s_w} \left[ \left( Z_{H^0} \right)_{2i} \left( \nu_{U_+} \right)_{1\alpha} \left( \nu_{U_-} \right)_{2\beta} + \left( Z_{H^0} \right)_{1i} \left( \nu_{U_+} \right)_{2\alpha} \left( \nu_{U_-} \right)_{1\beta} \\
- \left( \nu_{U_+} \right)_{1\alpha} \sum_{I=1}^{3} \left( Z_{H^0} \right)_{(2+I)i} \left( \xi_{L} U_{-} \right)_{I\beta} \right] 
\]
VI. $gg \rightarrow h^0$ AND $h^0 \rightarrow \gamma\gamma$, $ZZ^*$, $WW^*$

The Higgs is produced chiefly through the gluon fusion at the LHC. In the SM, the leading order (LO) contributions originate from the one-loop diagram which involves virtual top quarks. The cross section for this process is known to the next-to-next-to-leading order (NNLO)\cite{33} which can enhance the LO result by 80-100%. Furthermore, any new particle which strongly couples with the Higgs can significantly modify this cross section. In extension of the SM considered here, the LO decay width for the process $h^0 \rightarrow gg$ is given by (see \textit{...}):(84)
Ref.\textsuperscript{[34]} and references therein)
\begin{equation}
\Gamma_{NP}(h^0 \rightarrow gg) = \frac{G_F \alpha^2 m^3 h^0}{64 \sqrt{2} \pi^3} \left| \sum_q g_{h^0 q q} A_{1/2}(x_q) + \sum_{\tilde{q}} g_{h^0 \tilde{q} \tilde{q}} \frac{m^2_{\tilde{q}}}{m^2_{\tilde{q}}} A_0(x_{\tilde{q}}) \right|^2, \tag{85}
\end{equation}
with \(x_a = \frac{m^2_{h^0}}{4m_a^2}\). In the sum above, \(q = t, b\) and \(\tilde{q} = \tilde{U}, \tilde{D},\) \((i = 1, \ldots, 6)\). The concrete expressions for \(g_{h^0 tt}, g_{h^0 bb}, g_{h^0 \tilde{U}, \tilde{U}}, g_{h^0 \tilde{D}, \tilde{D}},\) \((i = 1, 6)\) are
\begin{align*}
g_{h^0 tt} &= \frac{1}{\sin \beta} (Z_{h^0})_{11}, \\
g_{h^0 bb} &= -\frac{1}{\cos \beta} (Z_{h^0})_{21} \sqrt{1 + \sum_{a=1}^3 \frac{v^2_a}{v^2_d}}, \\
g_{h^0 \tilde{U}, \tilde{U}} &= -\frac{s_w}{e m_w} \xi_{1ii}, \quad (i = 1, \ldots, 6), \\
g_{h^0 \tilde{D}, \tilde{D}} &= -\frac{s_w}{e m_w} \xi_{1ii}, \quad (i = 1, \ldots, 6). \tag{86}
\end{align*}
Here, we adopt the abbreviation \(s_w = \sin \theta_w\) with \(\theta_w\) denoting the Weinberg angle. Furthermore, \(e\) is the electromagnetic coupling constant, and the concrete expressions of \(\xi_{1ii}, \xi_{1ii}\) can be found in appendix E. The form factors \(A_{1/2}, A_0\) in Eq.(85) are defined as
\begin{align*}
A_{1/2}(x) &= 2 \left[ x + (x - 1)g(x) \right]/x^2, \\
A_0(x) &= -(x - g(x))/x^2, \tag{87}
\end{align*}
with
\begin{equation}
g(x) = \begin{cases} 
\arcsin^2 \sqrt{x}, & x \leq 1 \\
-\frac{1}{4} \left[ \ln \frac{1 + \sqrt{1 - 1/x}}{1 - \sqrt{1 - 1/x}} - i \pi \right]^2, & x > 1 \end{cases} \tag{88}
\end{equation}

The Higgs to diphoton decay is also obtained from loop diagrams, the LO contributions are derived from the one-loop diagrams containing virtual charged gauge boson \(W^\pm\) or virtual top quarks in the SM. In this model, the charged Higgs together with corresponding supersymmetric partner, and the supersymmetric partners of charged standard particles also contribute the corrections to the decay width of Higgs to diphoton at LO, the corresponding correction is written as
\begin{equation}
\Gamma_{NP}(h^0 \rightarrow \gamma \gamma) = \frac{G_F \alpha^2 m^3 h^0}{128 \sqrt{2} \pi^3} \left| \sum_f N_c Q^2_f g_{h^0 f f} A_{1/2}(x_f) + g_{h^0 WW} A_1(x_w) \right|^2, 
\end{equation}
with
\begin{equation}
N_c = 2, \quad Q^2_f = +1, \quad g_{h^0 WW} = \frac{1}{\sqrt{2}}, \quad A_1(x_w) = \frac{1}{4} \ln \frac{1 + \sqrt{1 - 1/x}}{1 - \sqrt{1 - 1/x}} + i \pi. \tag{89}
\end{equation}
\[ \sum_{i=2}^{8} g_{h^0 H^+ H^-} \frac{m^2_w}{m^2_{H_i}} A_0(x_{H_i^\pm}) + \sum_{i=1}^{2} g_{h^0 x_i^+ x_i^-} \frac{m^2_w}{m^2_{x_i}} A_1/2(x_{x_i}) \]

\[ + \sum_{\bar{q}} N_c Q^2 \frac{m^2_w}{m^2_{\bar{q}}} A_0(x_{\bar{q}}) \]

(89)

where the concrete expression for the loop functions \( A_1(x) \) is

\[ A_1(x) = -\left[ 2x^2 + 3x + 3(2x - 1)g(x) \right]/x^2. \]

(90)

In addition, the couplings \( g_{h^0 \tau \tau}, g_{h^0 WW}, \) and \( g_{h^0 H^+ H^-} \) are expressed as

\[ g_{h^0 \tau \tau} \simeq -\frac{1}{\cos \beta} \left( Z_{h^0} \right)_{21} \left[ 1 + \sum_{\alpha=1}^{3} \frac{v_{l_{\alpha}}^2}{v_d^2} \right], \]

\[ g_{h^0 WW} = \sin \beta \left( Z_{h^0} \right)_{11} + \frac{\cos \beta}{\sqrt{v_u^2 + \sum_{\alpha=1}^{3} v_{l_{\alpha}}^2}} \left\{ v_d \left( Z_{h^0} \right)_{21} + \sum_{i=1}^{3} v_{l_{i1}} \left( Z_{h^0} \right)_{(2+i)1} \right\}, \]

\[ g_{h^0 H^+ H^-} = -\frac{s_w}{e m_w} \xi_{1i1}^{H^\pm}, \ (i = 2, \ldots, 8), \]

(91)

and the couplings between the lightest neutral CP-even Higgs and charginos \( g_{h^0 x_i^+ x_i^-} \) are

\[ g_{h^0 x_i^+ x_i^-} = -\frac{2}{e} \Re \left[ \xi_{1i1}^{H^\pm} \right], \ (i = 1, 2). \]

(92)

The lightest neutral CP-even Higgs with 125 GeV mass can also decay through the modes \( h^0 \to ZZ^*, \ h^0 \to WW^* \), where \( Z^*/W^* \) denote the off-shell neutral/charged electroweak gauge bosons. Summing over all channels available to \( W^* \) or \( Z^* \), one writes the widths as

\[ \Gamma(h^0 \to WW^*) = \frac{3e^4 m^6_{h^0}}{512 \pi^3 s^3_w c^4_w} |g_{h^0 WW}|^2 F\left( \frac{m_w}{m_{h^0}} \right), \]

\[ \Gamma(h^0 \to ZZ^*) = \frac{e^4 m^6_{h^0}}{2048 \pi^3 s^3_w c^4_w} |g_{h^0 ZZ}|^2 \left( 7 - \frac{40}{3} s^2_w + \frac{160}{9} s^4_w \right) F\left( \frac{m_Z}{m_{h^0}} \right), \]

(93)

with \( g_{h^0 ZZ} = g_{h^0 WW} \) and the abbreviation \( c_w = \cos \theta_w \). The form factor \( F(x) \) is given as

\[ F(x) = -\left( 1 - x^2 \right) \left( \frac{47}{2} x^2 - \frac{13}{2} + \frac{1}{x^2} \right) - 3(1 - 6x^2 + 4x^4) \ln x \]

\[ + \frac{3(1 - 8x^2 + 20x^4)}{\sqrt{4x^2 - 1}} \cos^{-1} \left( \frac{3x^2 - 1}{2x^2} \right). \]

(94)
Besides the Higgs discovery the ATLAS and CMS experiments have both observed an excess in Higgs production and decay into diphoton channel which slightly differs from the SM expectations. The observed signals for the Higgs decaying channels are quantified by the ratio
\[
R_{\gamma\gamma} = \frac{\sigma_{NP}(h_0 \to gg) \text{BR}_{NP}(h_0 \to \gamma\gamma)}{\sigma_{SM}(h_0 \to gg) \text{BR}_{SM}(h_0 \to \gamma\gamma)} \approx \frac{\Gamma_{NP}(h_0 \to gg) \text{BR}_{NP}(h_0 \to \gamma\gamma)}{\Gamma_{SM}(h_0 \to gg) \text{BR}_{SM}(h_0 \to \gamma\gamma)},
\]
\[
R_{VV^*} = \frac{\sigma_{NP}(h_0 \to gg) \text{BR}_{NP}(h_0 \to VV^*)}{\sigma_{SM}(h_0 \to gg) \text{BR}_{SM}(h_0 \to VV^*)} \approx \frac{\Gamma_{NP}(h_0 \to gg) \text{BR}_{NP}(h_0 \to VV^*)}{\Gamma_{SM}(h_0 \to gg) \text{BR}_{SM}(h_0 \to VV^*)}, \quad (V = Z, W).
\]

To obtain the Higgs production cross sections normalised to the SM values, we adopt
\[
\frac{\sigma_{NP}(h_0 \to gg)}{\sigma_{SM}(h_0 \to gg)} \approx \frac{\Gamma_{NP}^{h_0} \text{BR}_{NP}(h_0 \to gg)}{\Gamma_{SM}^{h_0} \text{BR}_{SM}(h_0 \to gg)}
\]
where \(\Gamma_{SM}^{h_0}\) denotes the total decay width of Higgs in the SM, and \(\Gamma_{NP}^{h_0}\) denotes the total decay width of the lightest Higgs \(h_0\) in the supersymmetry with local \(U(1)_{B-L}\) symmetry, respectively. To accommodate the observed Higgs signals at CMS and ATLAS, we require the theoretical predictions on \(R_{\gamma\gamma}, R_{VV^*}\) satisfying
\[
0.9 < R_{\gamma\gamma} < 2.2, \\
0.2 < R_{VV^*} < 1.4, \quad (V = Z, W).
\]
The lower bounds of the ranges originate from the lower limits of 95% C.L. range for observed Higgs strength\cite{3,4}, and the upper bounds of the ranges originate from 95% C.L. exclusion from Higgs searching\cite{5,6}.

VII. NUMERICAL ANALYSES

As mentioned above, the most stringent constraint on the parameter space is that the \(8 \times 8\) mass squared matrix in Eq.(34) should produce the lightest eigenstate with a mass
FIG. 1: Assuming neutrino mass spectrum with NO and taking $m_{BL} = 1$ TeV, we plot the mass squared differences and mixing angles of neutrinos versus $\tan \beta$. Where (a) the solid line stands $\Delta m^2_{31}$ versus $\tan \beta$, the dashed line stands $\Delta m^2_{21}$ versus $\tan \beta$, together with the gray band A represents the points which deviate the experimental central value on $\Delta m^2_{A} = \Delta m^2_{31}$ within 1 standard deviation, the gray band B represents the points which deviate the experimental central value on $\Delta m^2_{B} = \Delta m^2_{21}$ within 1 standard deviation; and (b) solid line stands $\sin^2 \theta_{23}$ versus $\tan \beta$, the dashed line stands $\sin^2 \theta_{12}$ versus $\tan \beta$, as well as the dotted line stands $\sin^2 \theta_{13}$ versus $\tan \beta$, as well as the gray band A represents the points which deviate the experimental central value on $\sin^2 \theta_{23}$ within 1 standard deviation, the gray band B represents the points which deviate the experimental central value on $\sin^2 \theta_{12}$ within 1 standard deviation, the gray band C represents the points which deviate the experimental central value on $\sin^2 \theta_{13}$ within 1 standard deviation, respectively.

$m_{h_0} \approx 125.9$ GeV. In numerical analysis, we choose the mass of the lightest CP-odd Higgs $m_{A^0_1}$ as an input. In addition, we also adopt the ansatz on the parameter space

\[
\begin{align*}
    m_{\tilde{U}_3} &= 1 \text{ TeV}, \quad m_{\tilde{D}_3} = 2 \text{ TeV}, \\
    m_{\tilde{E}_3} &= m_{\tilde{L}_3} = \Lambda_{\tilde{N}_1^c} = \Lambda_{\tilde{N}_2^c} = 3 \text{ TeV}, \\
    m_1 &= 200 \text{ GeV}, \quad m_2 = 400 \text{ GeV}, \quad \mu = 1 \text{ TeV}, \\
    \nu_{N_1} &= \nu_{N_2} = \nu_{N_3} = 2 \text{ TeV}, \\
    m_{A^0_1} &= A_r = A_b = 1 \text{ TeV}, \\
\end{align*}
\]
FIG. 2: Assuming neutrino mass spectrum with NO and taking $\tan \beta = 3$, we plot the mass squared differences and mixing angles of neutrinos versus $U(1)_{(B-L)}$ gauging mass $m_{BL}$. Where (a) the solid line denotes $\Delta m^2_{31}$ versus $m_{BL}$, the dashed line denotes $\Delta m^2_{21}$ versus $m_{BL}$, together with the gray band A represents the points which deviate the experimental central value on $\Delta m^2_A = \Delta m^2_{31}$ within 1 standard deviation, the gray band B represents the points which deviate the experimental central value on $\Delta m^2_B = \Delta m^2_{21}$ within 1 standard deviation; and (b) solid line denotes $\sin^2 \theta_{23}$ versus $m_{BL}$, the dashed line denotes $\sin^2 \theta_{13}$ versus $m_{BL}$, as well as the dotted line denotes $\sin^2 \theta_{13}$ versus $m_{BL}$, as well as the gray band A represents the points which deviate the experimental central value on $\sin^2 \theta_{23}$ within 1 standard deviation, the gray band B represents the points which deviate the experimental central value on $\sin^2 \theta_{12}$ within 1 standard deviation, the gray band C represents the points which deviate the experimental central value on $\sin^2 \theta_{13}$ within 1 standard deviation, respectively.

$$m_{z_{BL}} = 2.4 \text{ TeV}$$ \hspace{1cm} (98)

to reduce the number of free parameters in the model considered here. For relevant parameters in the SM, we choose [19]

$$\alpha_s(m_Z) = 0.118 , \quad \alpha(m_Z) = 1/128 , \quad s_w^2(m_Z) = 0.23 ,$$
$$m_t = 174.2 \text{ GeV} , \quad m_b = 4.18 \text{ GeV} , \quad m_w = 80.4 \text{ GeV} .$$ \hspace{1cm} (99)

In order to fit the experimental data on neutrino oscillations with two solutions of the neutrino mass spectrum, we choose the VEVs of left-handed sneutrinos and the Yukawa
FIG. 3: Assuming neutrino mass spectrum with IO and taking $m_{\beta L} = 1 \text{ TeV}$, we plot the mass squared differences and mixing angles of neutrinos versus $\tan \beta$. Where (a) the solid line stands $\Delta m^2_{31}$ versus $\tan \beta$, the dashed line stands $\Delta m^2_{23}$ versus $\tan \beta$, together with the gray band A represents the points which deviate the experimental central value on $\Delta m^2_A = \Delta m^2_{13}$ within 1 standard deviation, the gray band B represents the points which deviate the experimental central value on $\Delta m^2_\odot = \Delta m^2_{21}$ within 1 standard deviation; and (b) solid line stands $\sin^2 \theta_{23}$ versus $\tan \beta$, the dashed line stands $\sin^2 \theta_{12}$ versus $\tan \beta$, as well as the dotted line stands $\sin^2 \theta_{13}$ versus $\tan \beta$, as well as the gray band A represents the points which deviate the experimental central value on $\sin^2 \theta_{23}$ within 1 standard deviation, the gray band B represents the points which deviate the experimental central value on $\sin^2 \theta_{12}$ within 1 standard deviation, the gray band C represents the points which deviate the experimental central value on $\sin^2 \theta_{13}$ within 1 standard deviation, respectively.

couplings of right-handed neutrinos respectively as

- for the NO spectrum:

$$v_{e_1} = 3.26 \times 10^{-4} \text{GeV}, \quad v_{e_2} = 7.45 \times 10^{-4} \text{GeV},$$

$$v_{e_3} = 4.06 \times 10^{-4} \text{GeV},$$

$$(Y_1, Y_2, Y_3) = (0, 1.53 \times 10^{-7}, 2.93 \times 10^{-7})$$

and issue the theoretical predictions on neutrino masses and mixing angles as

$$\Delta m^2_\odot = m^2_{\nu_2} - m^2_{\nu_1} \simeq 7.58 \times 10^{-23} \text{ GeV}^2.$$
\[
\Delta m_A^2 = m_{\nu_3}^2 - m_{\nu_1}^2 \approx 2.35 \times 10^{-21} \text{ GeV}^2,
\]
\[
\sin^2 \theta_{12} \approx 0.306, \quad \sin^2 \theta_{23} \approx 0.420, \quad \sin^2 \theta_{13} \approx 0.023 \quad (101)
\]
when \( \tan \beta = 3 \), \( m_{BL} = 1 \text{ TeV} \), \( m_{ZBL} = 2.4 \text{ TeV} \);

- for the IO spectrum:

\[
v_{l_1} = 3.11 \times 10^{-4} \text{ GeV}, \quad v_{l_2} = 1.22 \times 10^{-3} \text{ GeV}, \quad v_{l_3} = 1.10 \times 10^{-3} \text{ GeV},
\]

\[(Y_1, Y_2, Y_3) = (-1.10 \times 10^{-7}, \quad 2.61 \times 10^{-8}, \quad 0), \quad (102)\]

and issue the theoretical predictions on neutrino masses and mixing angles as

\[
\Delta m_{\odot}^2 = m_{\nu_1}^2 - m_{\nu_3}^2 \approx 7.58 \times 10^{-23} \text{ GeV}^2,
\]
\[
\Delta m_A^2 = m_{\nu_2}^2 - m_{\nu_3}^2 \approx 2.35 \times 10^{-21} \text{ GeV}^2,
\]
\[
\sin^2 \theta_{12} \approx 0.306, \quad \sin^2 \theta_{23} \approx 0.420, \quad \sin^2 \theta_{13} \approx 0.023 \quad (103)
\]
when \( \tan \beta = 3 \), \( m_{BL} = 1 \text{ TeV} \), \( m_{ZBL} = 2.4 \text{ TeV} \).

Additionally we can safely neglect the last two terms in Eq.(80) which are originate from the mixing between charginos and charged leptons, when we adopt the choices in Eq.(100) and Eq.(102). Further assuming the \( 3 \times 3 \) Yukawa couplings \( Y_E \) diagonally we obtain the solution as \( Y_E = \text{diag} (\sqrt{2} m_e / v_d, \sqrt{2} m_\mu / v_d, \sqrt{2} m_\tau / v_d) \).

Assuming neutrino mass spectrum with NO and taking \( m_{BL} = 1 \text{ TeV} \), we depict the mass squared differences of neutrinos versus \( \tan \beta \) in Fig(1)(a). Where the solid line denotes \( \Delta m_{31}^2 = \Delta m_A^2 \) varying with \( \tan \beta \), the dashed line denotes \( \Delta m_{21}^2 = \Delta m_{\odot}^2 \) varying with \( \tan \beta \), respectively. With increasing of \( \tan \beta \), the theoretical evaluations on \( \Delta m_{31}^2 \), \( \Delta m_{21}^2 \) decrease steeply as \( \tan \beta \leq 10 \), and diminish mildly as \( \tan \beta > 15 \). Using the same choice on parameter space, we also draw the mixing angles of neutrinos versus \( \tan \beta \) in Fig(1)(b). Where the solid line denotes \( \sin^2 \theta_{23} \) versus \( \tan \beta \), the dashed line denotes \( \sin^2 \theta_{12} \) versus \( \tan \beta \), as well as the dotted line denotes \( \sin^2 \theta_{13} \) versus \( \tan \beta \), respectively. Actually, those mixing angles \( \theta_{12}, \theta_{23}, \theta_{13} \) vary with \( \tan \beta \) gently.

As a ‘brand new’ parameter, the \( U(1)_{B-L} \) gaugino mass \( m_{BL} \) also affects the finally numerical results on neutrino sector in the MSSM with local \( U(1)_{B-L} \) symmetry. Supposing
FIG. 4: Assuming neutrino mass spectrum with IO and taking $\tan \beta = 3$, we plot the mass squared differences and mixing angles of neutrinos versus $U(1)_{(B-L)}$ gauging mass $m_{BL}$. Where (a) the solid line denotes $\Delta m_{23}^2$ versus $m_{BL}$, the dashed line denotes $\Delta m_{13}^2$ versus $m_{BL}$, together with the gray band A represents the points which deviate the experimental central value on $\Delta m_{23}^2 = \Delta m_{31}^2$ within 1 standard deviation, the gray band B represents the points which deviate the experimental central value on $\Delta m_{13}^2 = \Delta m_{21}^2$ within 1 standard deviation; and (b) solid line denotes $\sin^2 \theta_{23}$ versus $m_{BL}$, the dashed line denotes $\sin^2 \theta_{12}$ versus $m_{BL}$, as well as the dotted line denotes $\sin^2 \theta_{13}$ versus $m_{BL}$, as well as the gray band A represents the points which deviate the experimental central value on $\sin^2 \theta_{23}$ within 1 standard deviation, the gray band B represents the points which deviate the experimental central value on $\sin^2 \theta_{12}$ within 1 standard deviation, the gray band C represents the points which deviate the experimental central value on $\sin^2 \theta_{13}$ within 1 standard deviation, respectively.

neutrino mass spectrum with NO and taking $\tan \beta = 3$, we plot the mass squared differences of neutrinos versus $m_{BL}$ in Fig[2](a). Where the solid line represents $\Delta m_{31}^2 = \Delta m_{A}^2$ varying with $m_{BL}$, the dashed line represents $\Delta m_{21}^2 = \Delta m_{B}^2$ varying with $m_{BL}$, respectively. With increasing of $m_{BL}$, the theoretical prediction on $\Delta m_{31}^2$ diminishes steeply, and that on $\Delta m_{21}^2$ raises quickly. Using the same assumption on parameter space, we also present the mixing angles of neutrinos versus $m_{BL}$ in Fig[2](b). Where the solid line denotes $\sin^2 \theta_{23}$ versus $m_{BL}$, the dashed line denotes $\sin^2 \theta_{12}$ versus $m_{BL}$, as well as the dotted line denotes $\sin^2 \theta_{13}$ versus $m_{BL}$, respectively. Actually, the mixing angle $\theta_{12}$ depends on $m_{BL}$ mildly, and other mixing
angles $\theta_{23}$, $\theta_{13}$ vary with $m_{BL}$ acutely.

FIG. 5: Assuming neutrino mass spectrum with IO and taking $m_{A_b} = 9$ TeV, $m_{BL} = A_t = 1$ TeV, we plot $m_{h,0}^2$, $\Delta m_{h,0}^2$, $\left|\left(Z_{h0}\right)_{12}\right|$, $Z_{h0,Ne}$, $R_{\gamma\gamma}$ and $R_{VV^*}$ ($V = Z, W$) versus tan $\beta$. Where (a) the solid line (I) denotes $\Delta m_{h,0}^2$/GeV versus tan $\beta$, the dash-dot line (II) denotes $Z_{h0,Ne} \times 10^2$ versus tan $\beta$, the dash line (III) denotes $m_{h,0}^2$/125.9GeV versus tan $\beta$, as well as the dot line (IV) denotes $\left|\left(Z_{h0}\right)_{12}\right|$ versus tan $\beta$; and (b) the solid line denotes $R_{\gamma\gamma}$ versus tan $\beta$, the dashed line denotes $R_{VV^*}$ versus tan $\beta$, respectively.

When the neutrino mass spectrum is IO, the manners of parameters tan $\beta$, $m_{BL}$ affecting the numerical results on neutrino sector differ from that of the neutrino mass spectrum with NO. Assuming neutrino mass spectrum with IO and taking $m_{BL} = 1$ TeV, we plot the mass squared differences of neutrinos versus tan $\beta$ in Fig.3(a). Where the solid line denotes $\Delta m_{23}^2 = \Delta m_A^2$ varying with tan $\beta$, the dashed line denotes $\Delta m_{13}^2 = \Delta m_{\odot}^2$ varying with tan $\beta$, respectively. Obviously the theoretical predictions on $\Delta m_{23}^2$, $\Delta m_{13}^2$ vary with tan $\beta$ steeply as tan $\beta < 10$, and the theoretical predictions on $\Delta m_{23}^2$, $\Delta m_{13}^2$ depend on tan $\beta$ relatively mildly when tan $\beta > 15$. Adopting the same choice on parameter space, we also show the mixing angles of neutrinos versus tan $\beta$ in Fig.3(b). Where the solid line denotes $\sin^2 \theta_{23}$ versus tan $\beta$, the dashed line denotes $\sin^2 \theta_{12}$ versus tan $\beta$, as well as the dotted line denotes $\sin^2 \theta_{13}$ versus tan $\beta$, respectively. Obviously the mixing angle $\theta_{12}$ decreases quickly with increasing of tan $\beta$, and the mixing angles $\theta_{23}$, $\theta_{13}$ vary with tan $\beta$ slowly.

As mentioned above, the $U(1)_{(B-L)}$ gaugino mass $m_{BL}$ also affects the numerical eval-
uations in neutrino sector when neutrino mass spectrum is IO. Assuming neutrino mass spectrum with IO and taking $\tan \beta = 3$, we depict the mass squared differences of neutrinos versus the $U(1)_{B-L}$ gaugino mass $m_{BL}$ in Fig.4(a). Where the solid line represents $\Delta m_{23}^2 = \Delta m_A^2$ varying with $m_{BL}$, the dashed line represents $\Delta m_{13}^2 = \Delta m_\odot^2$ varying with $m_{BL}$, respectively. With increasing of $m_{BL}$, the theoretical prediction on $\Delta m_{23}^2$ raises steeply, and that on $\Delta m_{13}^2$ decreases quickly. Adopting the same assumption on parameter space, we also draw the mixing angles of neutrinos versus $m_{BL}$ in Fig.4(b). Where the solid line stands $\sin^2 \theta_{23}$ versus $m_{BL}$, the dashed line stands $\sin^2 \theta_{12}$ versus $m_{BL}$, as well as the dotted line stands $\sin^2 \theta_{13}$ versus $m_{BL}$, respectively. Obviously the mixing angle $\theta_{12}$ decreases quickly with increasing of $m_{BL}$, and the mixing angles $\theta_{23}$, $\theta_{13}$ vary with $m_{BL}$ mildly.

FIG. 4: Assuming neutrino mass spectrum with IO and taking $\tan \beta = 3$, $m_{\tilde{Q}_3} = 9$ TeV, $m_{BL} = 1$ TeV, we plot $m_{h^0}$, $\Delta m_{h^0}$, $\left| \left( Z_{h^0} \right)_{12} \right|$, $Z_{h^0 N_c}$, $R_{\gamma\gamma}$ and $R_{VV^\ast}$ ($V = Z$, $W$) versus $A_t$. Where (a) the solid line (I) denotes $\Delta m_{h^0}/\text{GeV}$ versus $A_t$, the dash-dot line (II) denotes $Z_{h^0 N_c} \times 10^2$ versus $A_t$, the dash line (III) denotes $m_{h^0}/125.9\text{GeV}$ versus $A_t$, as well as the dot line (IV) denotes $\left| \left( Z_{h^0} \right)_{12} \right|$ versus $A_t$; and (b) the solid line denotes $R_{\gamma\gamma}$ versus $A_t$, the dashed line denotes $R_{VV^\ast}$ versus $A_t$, respectively.

Choosing the assumptions presented in Eq.(100) and Eq.(102), one finds that the radiative corrections from right-handed neutrino sector to the CP-even Higgs mass squared matrix can be neglected safely[25]. This fact implies that theoretical predictions on Higgs sector almost do not depend on our hypothesis of neutrino mass spectrum. Assuming neutrino
mass spectrum with IO and taking $\tan \beta = 3$, $m_{\tilde{Q}_3} = 9$ TeV, $m_{BL} = A_t = 1$ TeV, we obtain the following numerical results in Higgs sector as

$$m_{h^0} \simeq 125.8 \text{ GeV}, \quad \Delta m_{h^0} = m_{h^0} - m_{h^0}^{MSSM} \simeq 2.24 \text{ GeV},$$

$$m_{h_2^0} \simeq 1.40 \text{ TeV}, \quad \left| (Z_{h^0})_{12} \right| \simeq 0.32,$$

$$Z_{h^0,\tilde{N}_c} = \sqrt{\sum_{i=1}^{3} \left| (Z_{h^0})_{1(3+i)} \right|^2} \simeq 1.38 \times 10^{-2},$$

$$R_{\gamma \gamma} = 1.21, \quad R_{VV^*} = 0.89 . \quad (104)$$

Here $m_{h^0}^{MSSM}$ is theoretical evaluation on the lightest CP-even mass including one-loop corrections, and $m_{h^0}$ is corresponding evaluation in the MSSM with local $U(1)_{B-L}$ symmetry. Furthermore $m_{h_2^0}$ represents the heavier CP-even Higgs in the MSSM, $(Z_{h^0})_{12}$ represents the mixing between $H_2^0$ and $h^0$, $Z_{h^0,\tilde{N}_c}$ represents the mixing between the lightest CP-even Higgs and real parts of right-handed sneutrinos, respectively.

Here the parameters $\tan \beta$, $m_{\tilde{Q}_3}$ and $A_t$ all affect theoretical predictions for Higgs sector in the MSSM with local $U(1)_{B-L}$ symmetry. Assuming neutrino mass spectrum with IO and

FIG. 7: Assuming neutrino mass spectrum with IO and taking $\tan \beta = 3$, $m_{BL} = A_t = 1$ TeV, we plot $m_{h^0}$, $\Delta m_{h^0}$, $\left| (Z_{h^0})_{12} \right|$, $Z_{h^0,\tilde{N}_c}$, $R_{\gamma \gamma}$ and $R_{VV^*}$ ($V = Z$, $W$) versus $m_{\tilde{Q}_3}$. Where (a) the solid line (I) denotes $\Delta m_{h^0}/\text{GeV}$ versus $m_{\tilde{Q}_3}$, the dash-dot line (II) denotes $Z_{h^0,\tilde{N}_c} \times 10^2$ versus $m_{\tilde{Q}_3}$, the dash line (III) denotes $m_{h^0}/125.9 \text{GeV}$ versus $m_{\tilde{Q}_3}$, as well as the dot line (IV) denotes $\left| (Z_{h^0})_{12} \right|$ versus $m_{\tilde{Q}_3}$; and (b) the solid line denotes $R_{\gamma \gamma}$ versus $A_t$, the dashed line denotes $R_{VV^*}$ versus $m_{\tilde{Q}_3}$, respectively.
taking $m_{\tilde{Q}_3} = 9$ TeV, $m_{BL} = A_t = 1$ TeV, we plot $m_{h^0}$, $\Delta m_{h^0}$, $\left| \left( Z_{h_0} \right)_{12} \right|$, and $Z_{h^0, \tilde{N}^c}$ versus $\tan \beta$ in Fig.5(a). Where the solid line (I) denotes $\Delta m_{h^0}/\text{GeV}$ versus $\tan \beta$, the dash-dot line (II) denotes $Z_{h^0, \tilde{N}^c} \times 10^2$ versus $\tan \beta$, the dash line (III) denotes $m_{h^0}/125.9 \text{GeV}$ versus $\tan \beta$, as well as the dot line (IV) denotes $\left| \left( Z_{h_0} \right)_{12} \right|$ versus $\tan \beta$, respectively. The evaluation on the lightest CP-even Higgs mass $m_{h^0}$ coincides with the ATLAS/CMS data in one standard deviation: $123.8 \text{ GeV} \leq m_{h^0} \leq 128.0 \text{ GeV}$ as $2 \leq \tan \beta \leq 5$. With increasing of $\tan \beta$, the evaluation on $m_{h^0}$ raises gently. Meanwhile the correction to $m_{h^0}$ from the mixing between the lightest CP-even Higgs and right-handed sneutrinos exceeds 2 GeV. The mixing $Z_{h^0, \tilde{N}^c}$ between the lightest CP-even Higgs and right-handed sneutinos is dominated by the loop corrections, and is well above $10^{-2}$ with our assumptions on parameter space. The mixing between $h^0$ and $H_2^0$ decreases with increasing of $\tan \beta$. Similarly, we also depict $R_{\gamma \gamma}$ and $R_{VV^*}$ versus $\tan \beta$ in Fig.5(b). With increasing of $\tan \beta$, $R_{\gamma \gamma}$ raises relatively steeply as $\tan \beta < 10$, and varies gently as $\tan \beta > 15$. $R_{VV^*}$ ($V = Z, W$) decreases smoothly with increasing of $\tan \beta$. Although the loop induced mixing between $h^0$ and right-handed sneutrinos exceeds 0.01, this mixing cannot modify theoretical evaluation on $R_{\gamma \gamma}$ drastically. The deviation between theoretical evaluation of the model and that of the SM originates from the mixing between $h^0$ and $H_2^0$, as well as radiative corrections from virtual stops, charged Higgs and charginos when $\tan \beta \leq 5$. Since the mixing between $h^0$ and $H_2^0$ is small as $\tan \beta \geq 10$, the deviation mainly originates from virtual corrections of charged Higgs, stops, sbottoms, staus, and charginos.

Assuming neutrino mass spectrum with IO and taking $\tan \beta = 3$, $m_{\tilde{Q}_3} = 9$ TeV, $m_{BL} = 1$ TeV, we plot $m_{h^0}$, $\Delta m_{h^0}$, $\left| \left( Z_{h_0} \right)_{12} \right|$, and $Z_{h^0, \tilde{N}^c}$ versus $A_t$ in Fig.5(a). The evaluation on the lightest CP-even Higgs mass $m_{h^0}$ coincides with the ATLAS/CMS data in one standard deviation as $|A_t| \simeq 1$ TeV. With increasing of $|A_t|$, the evaluation on $m_{h^0}$ raises mildly. Meanwhile the correction to $m_{h^0}$ from the mixing between the lightest CP-even Higgs and right-handed sneutrinos exceeds 2 GeV around $|A_t| \simeq 1$ TeV, and decreases with increasing of $|A_t|$. The mixing between the lightest CP-even Higgs and right-handed sneutrinos $Z_{h^0, \tilde{N}^c}$ is dominated by the radiative corrections, is well above $10^{-2}$ around $|A_t| \simeq 1$ TeV, and diminishes quickly with increasing of $|A_t|$. The mixing between $h^0$ and $H_2^0$ changes mildly with $A_t$. Similarly, we also depict $R_{\gamma \gamma}$ and $R_{VV^*}$ versus $A_t$ in Fig.5(b). $R_{\gamma \gamma}$ and $R_{VV^*}$ ($V = Z, W$)
vary gently with $A_t$. As mentioned above, the deviation between theoretical evaluation on $R_{\gamma\gamma}$ of the model considered here and that of the SM originates from the mixing between $h^0$ and $H^0_2$, as well as one-loop corrections from virtual stops, charged Higgs and charginos here.

Assuming neutrino mass spectrum with IO and taking $\tan\beta = 3$, $m_{B_L} = A_t = 1$ TeV, we plot $m_{h^0}$, $\Delta m_{h^0}$, $|\langle Z_{h^0}\rangle|_{12}$, and $Z_{h^0}\bar{N}_c$ versus $m_{\tilde{Q}_3}$ in Fig.7(a). The evaluation on the lightest CP-even Higgs mass $m_{h^0}$ coincides with the ATLAS/CMS data in one standard deviation around $m_{\tilde{Q}_3} \simeq 9$ TeV. Because the one-loop corrections to the mixing between the lightest CP-even Higgs and right-handed sneutrinos are proportional to $m^2_{\tilde{t}_L} - m^2_{\tilde{t}_R}$, the correction to $m_{h^0}$ from the mixing between the lightest CP-even Higgs and right-handed sneutrinos exceeds 1 GeV when $m_{\tilde{Q}_3} \geq 2$ TeV, and raises with increasing of $m_{\tilde{Q}_3}$. The mixing between the lightest CP-even Higgs and right-handed sneutrinos $Z_{h^0}\bar{N}_c$ is well above $10^{-2}$ as $m_{\tilde{Q}_3} \geq 2$ TeV. The mixing between $h^0$ and $H^0_2$ increases mildly with increasing of $m_{\tilde{Q}_3}$. Similarly, we also depict $R_{\gamma\gamma}$ and $R_{VV^*}$ versus $A_t$ in Fig.7(b). Actually $R_{\gamma\gamma}$ and $R_{VV^*}$ ($V = Z, W$) vary mildly with $m_{\tilde{Q}_3}$.

VIII. SUMMARY

In the scenarios where sneutrinos all obtain nonzero VEVs, we study the constraints from the observed Higgs signal and neutrino oscillation experimental data on parameter space of the MSSM with local $U(1)_{B-L}$ symmetry\cite{9,12}. Considering the constraints from neutrino oscillation, the mixing between real parts of right-handed sneutrinos and the lightest CP-even Higgs is below $10^{-12}$ at tree level, and the mixing between real parts of left-handed sneutrinos and the lightest CP-even Higgs is about $10^{-6}$ at tree level. Including one-loop virtual corrections, we find that the mixing between real parts of right-handed sneutrinos and the lightest CP-even Higgs can reach $1.5 \times 10^{-2}$, and this mixing increases the MSSM theoretical evaluation on the mass of the lightest CP-even Higgs exceeding 2.0 GeV. Meanwhile, we can safely neglect the one-loop corrections to the mixing between real parts of left-handed sneutrinos and the lightest CP-even Higgs which are proportional to the tiny nonzero VEVs of left-handed sneutrinos. Numerically the MSSM with local $U(1)_{B-L}$ sym-
metry accommodates naturally the observed Higgs signal from ATLAS/CMS collaborations and the updated experimental data on neutrino oscillation simultaneously. In addition, the model also predicts two sterile neutrinos with sub-eV masses [15, 16] which are favored by the BBN in cosmology [17].

Acknowledgments

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Appendix A: The mass squared matrices for squarks

With the minimal flavor violation assumption, the $2 \times 2$ mass squared matrix for scalar tops is given as

\[
\mathbf{Z}_t^\dagger \begin{pmatrix} m^2_{\tilde{t}_L} & m^2_{\tilde{t}_X} \\ m^2_{\tilde{t}_X} & m^2_{\tilde{t}_R} \end{pmatrix} \mathbf{Z}_t = \text{diag}(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}), \tag{A1}
\]

with

\[
m^2_{\tilde{t}_L} = \frac{(g_1^2 + g_2^2) v^2_{\text{EW}}}{24} \left( 1 - 2 \cos^2 \beta \right) \left( 1 - 4 c^2_W \right) \\
+ \frac{g_2^2}{6} \left( v^2_N - v^2_{\text{EW}} + v^2_{\text{SM}} \right) + m_t^2 + m_{\tilde{q}_3}^2,
\]

\[
m^2_{\tilde{t}_R} = -\frac{g_2^2 v^2_{\text{EW}}}{6} \left( 1 - 2 \cos^2 \beta \right) \\
- \frac{g_2^2}{6} \left( v^2_N - v^2_{\text{EW}} + v^2_{\text{SM}} \right) + m_t^2 + m_{\tilde{q}_3}^2,
\]

\[
m^2_{\tilde{t}_X} = -\frac{v_t}{\sqrt{2}} A_t Y_t + \frac{\mu}{\sqrt{2}} Y_t.
\]

\[\]
Here $Y_t$, $A_t$ denote Yukawa coupling and trilinear soft-breaking parameters in top quark sector, respectively. In a similar way, the mass-squared matrix for scalar bottoms is

$$Z^t_b = \begin{pmatrix} m^2_{b_L} & m^2_{b_X} \\ m^2_{b_X} & m^2_{b_R} \end{pmatrix}$$

with

$$m^2_{b_L} = \frac{(g_1^2 + g_2^2)\nu_{EW}^2}{24} \left[ 1 - 2 \cos^2 \beta \right] \left( 1 + 2 \cos^2 \beta \right)$$

$$+ \frac{g_2^2}{6} \left( v_N^2 - v_{EW}^2 + v_{SM}^2 \right) + m^2_b + m^2_{Q_3},$$

$$m^2_{b_R} = \frac{g_1^2 v^2_{EW}}{12} \left( 1 - 2 \cos^2 \beta \right)$$

$$- \frac{g_2^2}{6} \left( v_N^2 - v_{EW}^2 + v_{SM}^2 \right) + m^2_b + m^2_{D_3},$$

$$m^2_{b_X} = \frac{v_d}{\sqrt{2}} A_b Y_b - \frac{\mu \nu_d}{\sqrt{2}},$$

here $Y_b$, $A_b$ denote Yukawas couplings and trilinear soft-breaking parameters in b quark sector, respectively.

**Appendix B: The minimization conditions and mass squared matrices for Higgs**

The tree level minimization conditions are formulated as

$$T^0_u = v_u \left\{ \mu^2 + m^2_{h_u} + \frac{g_1^2 + g_2^2}{8} \left[ 2v_u^2 - v_{EW}^2 \right] \right\} + \frac{1}{2} \sum_{\alpha,\beta = 1}^{3} \left[ v_{N\alpha} \left( Y_N^T Y_N \right)_{\alpha\beta} v_{N\beta} \right]$$

$$+ v_{L\alpha} \left( Y_N^T Y_N \right)_{\alpha\beta} v_{L\beta} \right\} + \frac{1}{\sqrt{2}} \sum_{\alpha,\beta = 1}^{3} v_{L\alpha} \left( A_N \right)_{\alpha\beta} v_{N\beta} + B \mu v_d,$$

$$T^0_d = v_d \left\{ \mu^2 + m^2_{h_d} - \frac{g_1^2 + g_2^2}{8} \left[ 2v_u^2 - v_{EW}^2 \right] \right\} + \frac{\mu v_d^2}{\sqrt{2}} + B \mu v_u,$$

$$T^0_{L_l} = \frac{1}{2} \sum_{\alpha = 1}^{3} \left[ (m^2_{L_l})_{\alpha\alpha} + (m^2_{L_l})_{\alpha\beta} \right] v_{L\alpha} + \frac{v_L}{\sqrt{2}} \sum_{\alpha = 1}^{3} \left( A_N \right)_{\alpha\alpha} v_{N\alpha} + \frac{\mu v_d}{\sqrt{2}} \zeta_l$$

$$+ \frac{\epsilon_N}{2} \sum_{\alpha = 1}^{3} \left( Y_N^T Y_N \right)_{\alpha\alpha} v_{L\alpha} - v_{L\alpha} \left\{ \frac{g_1^2 + g_2^2}{8} \left( 2v_u^2 - v_{EW}^2 \right) \right\}$$

$$+ \frac{g_2^2}{2} \left( v_N^2 - v_{EW}^2 + v_{SM}^2 \right) \right\}.$$

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The radiative corrections from top, bottom and tau sectors to the minimization conditions are

\[
\Delta T_u = \frac{3}{(4\pi)^2} \left\{ \left( Y_t^2 - \frac{g_t^2}{8} \right) \left( f(m_{t_1}^2) + f(m_{t_2}^2) \right) - 2Y_t^2 f(m_{t_1}^2) \right\} \\
+ \left( Y_t^2 A_t (A_t - \mu - \frac{\nu_d}{\nu_u}) - \frac{3g_t^2 - 5g_t^2}{24} (m_{t_L}^2 - m_{t_R}^2) \right) \frac{f(m_{t_1}^2) - f(m_{t_2}^2)}{m_{t_1}^2 - m_{t_2}^2} \\
+ \frac{g_t^2 + g_b^2}{8} \left( f(m_{b_1}^2) + f(m_{b_2}^2) \right) \\
- \left( Y_b^2 \mu (A_b \frac{\nu_d}{\nu_u} - \mu) - \frac{3g_b^2 - 5g_t^2}{24} (m_{b_L}^2 - m_{b_R}^2) \right) \frac{f(m_{b_1}^2) - f(m_{b_2}^2)}{m_{b_1}^2 - m_{b_2}^2} \\
+ \frac{1}{(4\pi)^2} \left\{ \left( \frac{g_t^2 + g_b^2}{8} \right) \left( f(m_{r_1}^2) + f(m_{r_2}^2) \right) - \left( Y_r^2 \mu (A_r \frac{\nu_d}{\nu_u} - \mu) \right) \right\} \\
- \frac{g_r^2 - 3g_t^2}{8} (m_{r_L}^2 - m_{r_R}^2) \frac{f(m_{r_1}^2) - f(m_{r_2}^2)}{m_{r_1}^2 - m_{r_2}^2} \\
\Delta T_d = \frac{3}{(4\pi)^2} \left\{ \left( Y_t^2 - \frac{g_t^2}{8} \right) \left( f(m_{t_1}^2) + f(m_{t_2}^2) \right) \right\} \\
+ \left( Y_b^2 A_b \frac{\nu_d}{\nu_u} - \mu - \frac{\nu_d}{\nu_u} \right) - \frac{3g_t^2 - 5g_t^2}{24} (m_{t_L}^2 - m_{t_R}^2) \frac{f(m_{t_1}^2) - f(m_{t_2}^2)}{m_{t_1}^2 - m_{t_2}^2} \\
\Delta T_L = \frac{3}{(4\pi)^2} \left\{ \left( \frac{g_t^2 + g_b^2}{8} \right) \left( f(m_{t_1}^2) + f(m_{t_2}^2) \right) \right\} \\
\]
\[
\begin{align*}
&+ \left( \frac{3g_2^2 - 5g_1^2}{24} - \frac{g_{LL}^2}{3} \right) (m_{tL}^2 - m_{tR}^2) \frac{f(m_{tL}^2) - f(m_{tR}^2)}{m_{tL}^2 - m_{tR}^2} \\
&- \left[ \frac{g_1^2 + g_2^2}{8} \left( f(m_{tL}^2) + f(m_{tR}^2) \right) \right. \\
&\left. + \left( \frac{3g_2^2 - g_1^2}{24} + \frac{g_{LL}^2}{3} \right) (m_{bL}^2 - m_{bR}^2) \frac{f(m_{bL}^2) - f(m_{bR}^2)}{m_{bL}^2 - m_{bR}^2} \right] \\
&- \frac{1}{(4\pi)^2} \left( \frac{g_1^2 + g_2^2}{8} \left( f(m_{tL}^2) + f(m_{tR}^2) \right) \right. \\
&\left. + \left( \frac{g_2^2 - 3g_1^2}{8} + \frac{g_{LL}^2}{2} \right) (m_{bL}^2 - m_{bR}^2) \frac{f(m_{bL}^2) - f(m_{bR}^2)}{m_{bL}^2 - m_{bR}^2} \right), \\
\Delta T_N &= \frac{g_{BL}^2}{16\pi^2} \left\{ (m_{tL}^2 - m_{tR}^2) \frac{f(m_{tL}^2) - f(m_{tR}^2)}{m_{tL}^2 - m_{tR}^2} + (m_{bL}^2 - m_{bR}^2) \frac{f(m_{bL}^2) - f(m_{bR}^2)}{m_{bL}^2 - m_{bR}^2} \right. \\
&\left. + (m_{tL}^2 - m_{tR}^2) \frac{f(m_{tL}^2) - f(m_{tR}^2)}{m_{tL}^2 - m_{tR}^2} \right\}
\end{align*}
\]

with \( \varepsilon_N^2 = \sum_{\alpha,\beta=1}^3 v_L \alpha \left( Y_N \right)_{\alpha \beta} v_{N'\beta} \).

In the interaction basis \( H_{CH}^T = (H_I^-, H_I^-, \bar{L}_I^-, \bar{E}_J^\alpha), (I, J = 1, 2, 3) \), elements of \( A_{CH} \) and the symmetric matrix \( M_{\tilde{E}}^2 \) are written as

\[
A_{CH}^{1p} = \frac{1}{\sqrt{2}} \sum_{\alpha=1}^3 \left( A_{N} \right)_\alpha v_{N\alpha} + \frac{v_u}{2} \sum_{\alpha=1}^3 \left( Y_N Y_N^T \right)_{\alpha \alpha} v_{N\alpha} - \frac{g_2^2}{4} v_u v_{Lp}, \\
A_{CH}^{1(3+J')} = \frac{\mu}{\sqrt{2}} \sum_{\alpha=1}^3 \left( Y_E \right)_{J\alpha} v_{L\alpha} + \frac{1}{2} v_d \sum_{\alpha=1}^3 \left( Y_E Y_N^T \right)_{J\alpha} v_{N\alpha}, \\
A_{CH}^{2p} = -\frac{g_2^2}{4} v_d v_{Lp} - \frac{\mu \xi_{J'}}{\sqrt{2}} - \frac{1}{2} v_d \sum_{\alpha=1}^3 \left( Y_E Y_E^T \right)_{J\alpha} v_{L\alpha}, \\
A_{CH}^{2(3+J')} = \frac{1}{\sqrt{2}} \sum_{\alpha=1}^3 v_{L\alpha} \left( A_{E} Y_E \right)_{\alpha J'}^* + \frac{v_d}{2} \sum_{\alpha=1}^3 v_{N\alpha} \left( Y_E Y_E^T \right)_{\alpha J'}, \\
M_{\tilde{E}}^2_{IJ'} = \left( m_{L}^2 \right)_{IJ'} - \frac{g_2^2}{8} \left( 2v_u^2 - v_{EW}^2 \right) \delta_{IJ'} - \frac{g_2^2}{4} v_{L1} v_{Lp} + \frac{v_2^2}{2} \left( Y_E Y_E^T \right)_{IJ'}, \\
- \frac{1}{2} \xi_{I} \xi_{J'} + \frac{g_{BL}^2}{2} \left( v_N^2 - v_{EW}^2 + v_{SM}^2 \right) \delta_{IJ'} + \Delta T_L \delta_{IJ'}, \\
M_{\tilde{E}}^2_{IJ(3+J')} = \frac{v_d}{\sqrt{2}} \left( A_{E} Y_E \right)_{IJ'} - \frac{\mu \xi_{J'}}{\sqrt{2}} \left( Y_E \right)_{IJ'}, \\
M_{\tilde{E}}^2_{IJ(3+J')(3+J')} = \left( m_{E} \right)_{IJ'} + \frac{g_2^2}{4} \left( 2v_u^2 - v_{EW}^2 \right) \delta_{IJ'} - \frac{g_{BL}^2}{2} \left( v_N^2 - v_{EW}^2 + v_{SM}^2 \right) \delta_{IJ'},
\]
\[ + \frac{1}{2} v_d^2 (Y_R^T Y_E)_{J\alpha} + \frac{1}{2} \sum_{\alpha, \beta = 1}^3 (Y_E^T)_{J\alpha} v_{L\alpha} v_{L\beta} (Y_E)_{\beta J}. \]  

(B3)

In the interaction basis \( P^{0,T} = (P^0_u, P^0_d, P^0_{L_i}, P^0_{S_j}) \), \((I, J = 1, 2, 3)\), elements of \( A_{CPO} \) are written as

\[
[A^{(0)}_{CPO}]_{1J'} = \frac{1}{\sqrt{2}} \sum_{\alpha = 1}^3 (A_N)_{I\alpha} v_{N\alpha},
\]

\[
[A^{(0)}_{CPO}]_{1(3+J')} = \frac{1}{\sqrt{2}} \sum_{\alpha = 1}^3 (A_N)_{\alpha J'} v_{La},
\]

\[
[A^{(0)}_{CPO}]_{2J'} = -\frac{\mu}{\sqrt{2}} \zeta_j',
\]

\[
[A^{(0)}_{CPO}]_{2(3+J')} = -\frac{\mu}{\sqrt{2}} \sum_{\alpha = 1}^3 v_{La} (Y_N)_{\alpha J'}, \quad (B4)
\]

and elements of the symmetric matrix \( M_p^2 \) are similarly given as

\[
[M_p^2]_{1J'} = (m_{L_e}^2)_{1J'} - g_1^2 + g_2^2 (2v_u^2 - v_{EW}^2) \delta_{1J'} - \frac{g_{BL}^2}{2} (v_N^2 + v_{EW}^2 + v_{SM}^2) \delta_{1J'} - \frac{1}{2} \zeta_{J'} - \frac{1}{2} (Y_N Y_N^T)_{1J'} v_u^2 + \Delta T_L \delta_{1J'},
\]

\[
[M_p^2]_{1(3+J')} = \frac{v_u}{\sqrt{2}} (A_N)_{1J'} + \frac{\mu u_d}{2} (Y_N)_{1J'} + \frac{\varepsilon^2}{2} (Y_N)_{1J'} - \frac{1}{2} \sum_{\alpha = 1}^3 v_{La} (Y_N^T)_{J\alpha},
\]

\[
[M_p^2]_{(3+J')(3+J')} = \left( m_{N_e}^2 \right)_{J\alpha} + \frac{g_{BL}^2}{2} (v_N^2 - v_{EW}^2 + v_{SM}^2) \delta_{JJ'} - \frac{1}{2} v_u^2 (Y_N Y_N^T)_{J\alpha} - \frac{1}{2} \sum_{\alpha, \beta = 1}^3 (Y_N^T)_{J\alpha} v_{La} v_{La} (Y_N)_{\beta J'} + \Delta T_N \delta_{JJ'}. \quad (B5)
\]

In the interaction basis \( H^{0,T} = (H^0_u, H^0_d, \tilde{v}_{L_i}, \tilde{v}_{R_j}) \), \((I, J = 1, 2, 3)\), elements of the matrix \( A^{(0)}_{CPE} \) are respectively written as

\[
(A^{(0)}_{CPE})_{1J'} = -\frac{g_1^2 + g_5^2}{4} v_d v_{LJ'} - \frac{1}{\sqrt{2}} \sum_{\alpha = 1}^3 (A_N)_{I\alpha} v_{N\alpha} - v_u \sum_{\alpha = 1}^3 v_{La} (Y_N Y_N^T)_{\alpha I'},
\]

\[
(A^{(0)}_{CPE})_{1(3+J')} = -\frac{1}{\sqrt{2}} \sum_{\alpha = 1}^3 (A_N)_{\alpha J'} v_{La} - v_u \sum_{\alpha = 1}^3 v_{Na} (Y_N Y_N^T)_{\alpha J'},
\]

\[
(A^{(0)}_{CPE})_{2J'} = \frac{g_1^2 + g_5^2}{4} v_d v_{LJ'} - \frac{\mu \zeta_j'}{\sqrt{2}},
\]

\[
(A^{(0)}_{CPE})_{2(3+J')} = -\frac{\mu}{\sqrt{2}} \sum_{\alpha = 1}^3 v_{La} (Y_N)_{\alpha J'}, \quad (B6)
\]
and elements of the symmetric matrix $M_S^2$ are similarly given as

$$
\begin{align*}
(M_S^2)_{II} &= (m_L^2)_{II} - \frac{g_1^2 + g_2^2}{8} \left[ (2v_u^2 - v_{ew}^2)\delta_{II} - 2v_{l_i} v_{l_i'} \right] - \frac{1}{2} \zeta_i \zeta_{i'} \\
&\quad - \frac{g_{BL}^2}{2} \left[ (v_N^2 - v_{ew}^2 + v_{SM}^2)\delta_{II} - 2v_{l_i} v_{l_i'} \right] - \frac{1}{2} \left( Y_N Y_T \right)_{II} \left( Y_N \right)_{II} + \Delta T_L \delta_{II} , \\
(M_S^2)_{I(3+)j'} &= -g_{BL}^2 v_{l_j} v_{N,j'} - \frac{v_u}{\sqrt{2}} \left( A_N \right)_{I,j'} + \frac{\mu v}{\sqrt{2}} \left( Y_N \right)_{I,j'} \\
&\quad - \frac{1}{2} \varepsilon N \left( Y_N \right)_{I,j'} - \frac{1}{2} \zeta_i \sum_{a=1}^3 v_{la} \left( Y_N \right)_{a,j'} , \\
(M_S^2)_{(3+)(3+j')} &= (m_N^2)_{J,j'} + \frac{g_{BL}^2}{2} \left[ (v_N^2 - v_{ew}^2 + v_{SM}^2)\delta_{JJ'} + v_{N,j} v_{N,j'} \right] \\
&\quad - \frac{1}{2} \sum_{a,b=1}^3 \left( Y_N^T \right)_{J,a} v_{la} v_{l_b} \left( Y_N \right)_{J,b} - \frac{1}{2} v_u^2 \left( Y_N Y_T \right)_{J,j'} + \Delta T_N \delta_{JJ'} .
\end{align*}
$$

(B7)

Appendix C: Radiative corrections to the CP-even Higgs mass squared matrix

The radiative corrections from quark sector are formulated as

$$
\begin{align*}
\Delta_{B_{11}} &= \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2 \beta} \left\{ \ln \frac{m^2_{t_1} m^2_{t_2}}{m^4_t} + \frac{2A_i(A_i - \mu \cot \beta)}{m^2_{t_1} - m^2_{t_2}} \ln \frac{m^2_{t_1}}{m^2_{t_2}} \right\} \\
&\quad + \frac{A_i^2(A_i - \mu \cot \beta)^2}{(m^2_{t_1} - m^2_{t_2})^2} g(m^2_{t_1}, m^2_{t_2}) \\
&\quad + \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \cos^2 \beta} \cdot \frac{\mu^2(A_b - \mu \tan \beta)^2}{(m_{b_1}^2 - m_{b_2}^2)^2} g(m^2_{b_1}, m^2_{b_2}) , \\
\Delta_{B_{12}} &= -\frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \sin^2 \beta} \cdot \frac{\mu(A_i - \mu \cot \beta)}{m^2_{t_1} - m^2_{t_2}} \left\{ \ln \frac{m^2_{t_1}}{m^2_{t_2}} \right\} \\
&\quad + \frac{A_i(A_i - \mu \cot \beta)}{m^2_{t_1} - m^2_{t_2}} g(m^2_{t_1}, m^2_{t_2}) \\
&\quad - \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 \cos^2 \beta} \cdot \frac{\mu(A_b - \mu \tan \beta)}{m^2_{b_1} - m^2_{b_2}} \left\{ \ln \frac{m^2_{b_1}}{m^2_{b_2}} \right\} \\
&\quad + \frac{A_i(A_b - \mu \tan \beta)}{m^2_{b_1} - m^2_{b_2}} g(m^2_{b_1}, m^2_{b_2}) , \\
\Delta_{B_{22}} &= \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \sin^2 \beta} \cdot \frac{\mu^2(A_i - \mu \cot \beta)^2}{(m^2_{b_1} - m^2_{b_2})^2} g(m^2_{b_1}, m^2_{b_2}) \\
&\quad + \frac{3G_F m_b^4}{2\sqrt{2}\pi^2 \cos^2 \beta} \left\{ \ln \frac{m^2_{b_1} m^2_{b_2}}{m^4_b} + \frac{2A_b(A_b - \mu \tan \beta)}{m^2_{b_1} - m^2_{b_2}} \ln \frac{m^2_{b_1}}{m^2_{b_2}} \right\} .
\end{align*}
$$
Similarly the contributions from lepton sector to the mass-squared matrix of CP-even Higgs are

\[
\begin{align*}
\Delta^r_{11} &= \frac{G_F m^4 \mu^2 (A_\tau - \mu \tan \beta)^2}{2 \sqrt{2} \pi^2 \cos^2 \beta} \left\{ \ln \frac{m^2_{\tau_1}}{m^2_{\tau_2}} + \frac{A_\tau (A_\tau - \mu \tan \beta)}{m^2_{\tau_1} - m^2_{\tau_2}} g(m^2_{\tau_1}, m^2_{\tau_2}) \right\}, \\
\Delta^r_{12} &= -\frac{G_F m^4 \mu^2 (A_\tau - \mu \tan \beta)^2}{2 \sqrt{2} \pi^2 \cos^2 \beta} \left\{ \ln \frac{m^2_{\tau_1}}{m^2_{\tau_2}} + \frac{A_\tau (A_\tau - \mu \tan \beta)}{m^2_{\tau_1} - m^2_{\tau_2}} g(m^2_{\tau_1}, m^2_{\tau_2}) \right\}, \\
\Delta^r_{22} &= \frac{G_F m^4 \mu^2 (A_\tau - \mu \tan \beta)^2}{2 \sqrt{2} \pi^2 \cos^2 \beta} \left\{ \ln \frac{m^2_{\tau_1}}{m^2_{\tau_2}} + \frac{2 A_\tau (A_\tau - \mu \tan \beta)}{m^2_{\tau_1} - m^2_{\tau_2}} \ln \frac{m^2_{\tau_1}}{m^2_{\tau_2}} \right\}, \\
&\quad + \frac{A^2_\tau (A_\tau - \mu \tan \beta)^2}{(m^2_{\tau_1} - m^2_{\tau_2})^2} g(m^2_{\tau_1}, m^2_{\tau_2}),
\end{align*}
\]

where

\[
g(x, y) = 2 - \frac{x + y}{x - y} \ln \frac{x}{y}.
\]

Appendix D: The couplings between CP-even Higgs and charged scalars

The interaction between the CP-even Higgs and charged Higgs is written as

\[
\mathcal{L}^H_{H^+ H^+} = \sum_{i=1}^{10} \sum_{\alpha, \beta=1}^{8} \xi_{i,\alpha\beta}^{H^+} H^0_i H^-_\alpha H^+_\beta
\]  

(D1)

with

\[
x_{i,\alpha\beta}^{H^+} = \frac{g_1^2 + g_2^2}{4} v^2_{EW} R_{1i} A_{\alpha\beta}^{H^+} + \frac{g_3^2 v^2_{EW}}{4} \left\{ \left[ (Z_{H_0}^i)_{1i} (Z_{CH})_{2i} + (Z_{H_0}^i)_{2i} (Z_{CH})_{1i} \right] \nu^\alpha_1 + \left[ (Z_{H_0}^i)_{1i} (Z_{CH})_{2i} + (Z_{H_0}^i)_{2i} (Z_{CH})_{1i} \right] \nu^\alpha_1 \right\} \\
+ \left[ (Z_{H_0}^i)_{1i} (Z_{CH})_{1i} + (Z_{H_0}^i)_{2i} (Z_{CH})_{2i} \right] \nu^\alpha_1 + 2 R_{1i} \sum_{I=1}^{3} (Z_{CH})_{(2+I)\alpha} (Z_{CH})_{(2+I)\beta} \\
+ 2 \sum_{I=1}^{3} \frac{v_{Li}}{v^2_{EW}} (Z_{H_0}^i)_{(2+I)\alpha} \left[ (Z_{CH})_{1i} (Z_{CH})_{1i} + (Z_{CH})_{2i} (Z_{CH})_{2i} \right] \\
+ \sum_{I=1}^{3} \frac{3}{v^2_{EW}} (Z_{H_0}^i)_{(2+I)\alpha} \left[ (Z_{CH})_{(2+I)\alpha} (Z_{CH})_{(2+I)\beta} + (Z_{CH})_{(2+I)\beta} (Z_{CH})_{(2+I)\alpha} \right]
\]

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\[

+ \left[ \frac{v_{\text{EW}}}{v_{\text{EW}}} (Z_{H_0})_{1i} + \frac{v_{\text{EW}}}{v_{\text{EW}}} (Z_{H_0})_{(2+I)i} \right] \left[ (Z_{CH})_{(2+I)\alpha} (Z_{CH}^\dagger)_{\beta 1} + (Z_{CH})_{1\alpha} (Z_{CH}^\dagger)_{\beta (2+I)} \right] \\

+ \left[ \frac{v_{\text{EW}}}{v_{\text{EW}}} (Z_{H_0})_{2i} + \frac{v_{\text{EW}}}{v_{\text{EW}}} (Z_{H_0})_{(2+I)i} \right] \left[ (Z_{CH})_{(2+I)\alpha} (Z_{CH}^\dagger)_{\beta 2} + (Z_{CH})_{2\alpha} (Z_{CH}^\dagger)_{\beta (2+I)} \right] \\

+ \frac{g_{\text{BL}}^2}{2} v_{\text{EW}} R_{1i} \sum_{i=1}^3 (Z_{CH})_{(5+I)\alpha} (Z_{CH}^\dagger)_{\beta (5+I)} \\

+ g_{\beta L} v_t R_{2i} \sum_{i=1}^3 \left[ (Z_{CH})_{(2+I)\alpha} (Z_{CH}^\dagger)_{\beta (2+I)} - (Z_{CH})_{(5+I)\alpha} (Z_{CH}^\dagger)_{\beta (5+I)} \right] \\

+ \left\{ \frac{\mu^*}{\sqrt{2}} \sum_{I,J}^3 (Z_{CH})_{(2+I)\alpha} (Y_N^\dagger)_{IJ} (Z_{H_0})_{(5+J)i} (Z_{CH}^\dagger)_{\beta 2} \right\} \\

+ \frac{\mu}{\sqrt{2}} \sum_{I,J}^3 (Z_{H_0}^T)_{i(5+I)} (Y_N^\dagger)_{IJ} (Z_{CH}^\dagger)_{(2+J)\beta} (Z_{CH})_{2\alpha} \\

+ \frac{1}{2} \sum_{I,J,J'}^3 [\xi_{\beta} (Z_{CH})_{(2+I)\alpha} (Z_{CH}^\dagger)_{(2+J)\beta} (Y_N^\dagger)_{IJ} (Z_{H_0})_{(5+I)i} \\

+ (Z_{H_0}^T)_{i(5+I)} (Y_N^\dagger)_{IJ} (Z_{CH}^\dagger)_{(2+J)\alpha} (Z_{CH}^\dagger)_{\beta (2+I)\beta} ] \\

+ \left\{ - \frac{\mu^*}{\sqrt{2}} \sum_{I,J}^3 (Z_{H_0}^T)_{i(2+I)} (Y_E^\dagger)_{IJ} (Z_{CH})_{(5+J)\alpha} (Z_{CH}^\dagger)_{\beta 1} \\

- \frac{\mu}{\sqrt{2}} \sum_{I,J}^3 (Z_{CH}^\dagger)_{\beta (5+I)} (Y_E^\dagger)_{IJ} (Z_{H_0})_{(2+J)i} (Z_{CH})_{1\alpha} \right\} \\

+ \frac{1}{2} \sum_{I,J,J'}^3 \sum_{I,J}^3 [(Z_{CH}^\dagger)_{\beta (5+I)} (Y_E^\dagger)_{IJ} v_{L_I} (Z_{H_0}^T)_{i(2+J)'} (Y_E^\dagger)_{J'I'} (Z_{CH})_{(5+I\alpha)} \\

+ (Z_{CH}^\dagger)_{\beta (5+I)} (Y_E^\dagger)_{IJ} (Z_{H_0})_{(2+J)'} v_{L_I'} (Y_E^\dagger)_{J'I'} (Z_{CH})_{(5+I\alpha)} ] \\

+ \sum_{I,J}^3 \left\{ v_u (Z_{CH}^\dagger)_{\beta (5+I)} (Y_E^\dagger Y_E^\dagger)_{IJ} (Z_{CH})_{(5+J)\alpha} (Z_{H_0})_{2i} \right\} \\

+ \frac{1}{2} \left[ v_{Y_{NI}} (Y_{NI}^\dagger Y_{NI}^\dagger)_{IJ} (Z_{H_0})_{(5+J)i} + (Z_{H_0}^T)_{i(5+I)} (Y_{NI}^\dagger Y_{NI})_{IJ} v_{Y_{NI}} (Z_{CH}^\dagger)_{\beta 1} (Z_{CH})_{1\alpha} \right] \\

+ \frac{1}{2} (Z_{CH}^\dagger)_{\beta (5+I)} (Y_E^\dagger Y_{NI}^\dagger)_{IJ} v_{Y_{NI}} (Z_{H_0})_{2i} [(Z_{CH})_{1\alpha} + (Z_{CH})_{2\alpha}] \\

+ \frac{1}{2} v_{\text{EW}} (Z_{H_0}^T)_{i(5+I)} (Y_{NI}^\dagger Y_{NI})_{IJ} (Z_{CH})_{(5+J)\beta} Y_{\alpha} \\

+ \frac{1}{2} [(Z_{CH})_{\beta 1} + (Z_{CH}^\dagger)_{\beta 2}] v_{Y_{NI}} (Y_{NI}^\dagger Y_E^\dagger)_{IJ} (Z_{CH}^\dagger)_{(5+J)\alpha} (Z_{H_0})_{2i} \\

+ \frac{1}{2} v_{\text{EW}} (Z_{H_0}^\dagger)_{(5+I)} (Y_{NI}^\dagger Y_E^\dagger)_{IJ} (Z_{CH})_{(5+J)\alpha} \right\} \\

+ \sum_{I,J}^3 \left\{ v_d (Z_{H_0})_{2i} (Z_{CH}^\dagger)_{\beta (2+I)} (Y_E^\dagger Y_{NI}^\dagger)_{IJ} (Z_{CH})_{(2+J)\alpha} \right\}
\]
\[
+ \frac{1}{2} \left[ v_L I (Y_E Y_E^\dagger)_{IJ} (Z_{H_0})_{(2+J)i} + (Z_{H_0}^T)_{i(2+I)} (Y_E Y_E^\dagger)_{IJ} v_L J \right] (Z_{CH})_{\beta 2} (Z_{CH})_{2\alpha} \\
- \frac{1}{2} v_d (Z_{CH})_{\beta(2+I)} (Y_E Y_E^\dagger)_{IJ} (Z_{H_0})_{(2+J)i} (Z_{CH})_{2\alpha} \\
- \frac{1}{2} v_d (Z_{CH})_{\beta 2} (Z_{H_0}^T)_{i(2+I)} (Y_E Y_E^T)_{IJ} (Z_{CH})_{(2+J)\alpha} \\
- \frac{1}{2} (Z_{H_0})_{2i} (Z_{CH})_{\beta(2+I)} (Y_E Y_E^\dagger)_{IJ} v_L J (Z_{CH})_{2\alpha} \\
- \frac{1}{2} (Z_{H_0})_{2i} (Z_{CH})_{\beta 2} v_L I (Y_E Y_E^T)_{IJ} (Z_{CH})_{(2+J)\alpha} \\
+ \sum_{I,J} \left\{ v_u (Z_{H_0})_{1i} (Z_{CH})_{\beta(2+I)} (Y_N Y_N^T)_{IJ} (Z_{CH})_{(2+J)\alpha} \\
+ \frac{1}{2} \left[ v_L I (Y_N Y_N^\dagger)_{IJ} (Z_{H_0})_{(2+J)i} + (Z_{H_0}^T)_{i(2+I)} (Y_N Y_N^\dagger)_{IJ} v_L J \right] (Z_{CH})_{\beta 1} (Z_{CH})_{1\alpha} \\
- \frac{1}{2} v_u (Z_{CH})_{\beta(2+I)} (Y_N Y_N^\dagger)_{IJ} (Z_{H_0})_{(2+J)i} (Z_{CH})_{1\alpha} \\
- \frac{1}{2} v_u (Z_{CH})_{\beta 1} (Z_{H_0}^T)_{i(2+I)} (Y_N Y_N^T)_{IJ} (Z_{CH})_{(2+J)\alpha} \\
- \frac{1}{2} (Z_{H_0})_{1i} (Z_{CH})_{\beta(2+I)} (Y_N Y_N^\dagger)_{IJ} v_L J (Z_{CH})_{1\alpha} \\
- \frac{1}{2} (Z_{H_0})_{1i} (Z_{CH})_{\beta 1} v_L I (Y_N Y_N^T)_{IJ} (Z_{CH})_{(2+J)\alpha} \\
+ \frac{3}{\sqrt{2}} \sum_{I,J} \left\{ (Z_{CH})_{\beta(2+I)} (A_E Y_E^\dagger)_{IJ} (Z_{CH})_{(5+J)\alpha} (Z_{H_0})_{2i} \\
+ (Z_{CH})_{\beta(5+I)} (A_E Y_E^\dagger)_{IJ} (Z_{CH})_{(2+J)\alpha} (Z_{H_0})_{2i} \\
+ (Z_{CH})_{\beta(2+I)} (A_N Y_N^\dagger)_{IJ} (Z_{H_0})_{(5+J)i} (Z_{CH})_{1\alpha} \\
+ (Z_{CH})_{\beta 1} (Z_{H_0}^T)_{i(5+I)} (A_N Y_N^\dagger)_{IJ} (Z_{CH})_{(2+J)\alpha} \right\} \right).
\]

\[
\text{(D2)}
\]

Where

\[
\mathcal{R}_{1i} = \frac{v_u}{v_{EW}} (Z_{H_0})_{1i} - \frac{v_d}{v_{EW}} (Z_{H_0})_{2i} - \sum_{j=1}^{3} \frac{v_{LJ}}{v_{EW}} (Z_{H_0})_{(2+J)i},
\]

\[
\mathcal{R}_{2i} = \sum_{j=1}^{3} \left[ \frac{v_{LJ}}{v_{t}} (Z_{H_0})_{(2+J)i} - \frac{v_{NJ}}{v_{t}} (Z_{H_0})_{(5+J)i} \right],
\]

\[
V_\alpha = \frac{v_u}{v_{EW}} (Z_{CH})_{2\alpha} + \frac{v_d}{v_{EW}} (Z_{CH})_{1\alpha},
\]

\[
A_{\alpha\beta}^{H\pm} = (Z_{CH})_{1\alpha} (Z_{CH})_{1\beta} - (Z_{CH})_{2\alpha} (Z_{CH})_{2\beta} - \sum_{i=1}^{3} (Z_{CH})_{(2+I)\alpha} (Z_{CH})_{(2+I)\beta}
\]

(D3)

and \( Y_E = \text{diag}(Y_e, Y_\mu, Y_\tau) \).
The couplings between CP-even Higgs and stops are formulated as

\[ \mathcal{L}_{\mu_i^0 \bar{t}_\beta} = \sum_{i=1}^{8} \sum_{\alpha, \beta} \xi_{i, \alpha}^{\hat{\beta}} H_{i}^{0} \bar{t}_{\beta} \bar{t}_{\alpha}, \]  

(D4)

with

\[ \xi_{i, \alpha}^{\hat{\beta}} = -\frac{\epsilon^2}{4s_w^2 c_w^2} v_{ew} R_{11} \left\{ \left[ 1 - \left( 1 + Y_q \right) s_w^2 \right] \left( Z_{\hat{\beta} \alpha}^{\top} \right) \beta_1 \left( Z_{\hat{\beta} \alpha} \right) \right\} \]

\[ - Y_u s_w^2 \left( Z_{\hat{\beta} \alpha} \right) \beta_2 \left( Z_{\hat{\beta} \alpha} \right) \]

\[ - \frac{g_{BL}^2 v_{t}}{3} R_{2i} \left\{ \left( Z_{\hat{\beta} \alpha}^{\top} \right) \beta_1 \left( Z_{\hat{\beta} \alpha} \right) \right\} \]

\[ + \frac{\mu Y_b}{\sqrt{2}} \left( Z_{h_0} \right)_{i1} \left( Z_{\hat{\beta} \alpha}^{\top} \right) \beta_2 \left( Z_{\hat{\beta} \alpha} \right) \]

\[ + \frac{Y_b^*}{3} \sum_{i, j \neq i} \left[ v_{i1} Y_i \left( Z_{h_0} \right)_{ij} \right] \]

\[ \times \left( Z_{\hat{\beta} \alpha} \right)_{\beta_1} \left( Z_{\hat{\beta} \alpha} \right) \]

\[ + \frac{Y_b^*}{\sqrt{2}} \left( Z_{h_0} \right)_{1i} \left( Z_{\hat{\beta} \alpha}^{\top} \right) \beta_2 \left( Z_{\hat{\beta} \alpha} \right) \]

\[ - \frac{A_Y^*}{\sqrt{2}} \left( Z_{h_0} \right)_{i1} \left( Z_{\hat{\beta} \alpha}^{\top} \right) \beta_1 \left( Z_{\hat{\beta} \alpha} \right) \]  

(D5)

where \( Y_q = 1/3 \), \( Y_u = -4/3 \).

The couplings between CP-even Higgs and sbottoms are formulated as

\[ \mathcal{L}_{\mu_i^0 \tilde{s}_\beta} = \sum_{i=1}^{8} \sum_{\alpha, \beta} \xi_{i, \alpha}^{\hat{\beta}} H_{i}^{0} \tilde{s}_{\beta} \tilde{s}_{\alpha}, \]  

(D6)

with

\[ \xi_{i, \alpha}^{\hat{\beta}} = -\frac{\epsilon^2}{4s_w^2 c_w^2} v_{ew} R_{11} \left\{ \left[ 1 - \left( 1 + Y_q \right) s_w^2 \right] \left( Z_{\hat{\beta} \alpha}^{\top} \right) \beta_1 \left( Z_{\hat{\beta} \alpha} \right) \right\} \]

\[ + Y_u s_w^2 \left( Z_{\hat{\beta} \alpha} \right) \beta_2 \left( Z_{\hat{\beta} \alpha} \right) \]

\[ - \frac{g_{BL}^2 v_{t}}{3} R_{2i} \left\{ \left( Z_{\hat{\beta} \alpha}^{\top} \right) \beta_1 \left( Z_{\hat{\beta} \alpha} \right) \right\} \]

\[ - \frac{\mu Y_b}{\sqrt{2}} \left( Z_{h_0} \right)_{i1} \left( Z_{\hat{\beta} \alpha}^{\top} \right) \beta_2 \left( Z_{\hat{\beta} \alpha} \right) \]

\[ + \frac{Y_b^*}{\sqrt{2}} \left( Z_{h_0} \right)_{i1} \left( Z_{\hat{\beta} \alpha}^{\top} \right) \beta_1 \left( Z_{\hat{\beta} \alpha} \right) \]  

(D7)
The inverse mass matrix of six heavy majorana fermions is

\[
M^{-1} = \begin{pmatrix}
[M_N^{(1)}]^{-1}_{2\times 2} & [M_N^{(2)}]^{-1}_{2\times 4} \\
[M_N^{(2)}]^{-1}_{4\times 2} & [M_N]^{-1}_{4\times 4}
\end{pmatrix}
\]

with

\[
[M_N]^{-1} = \begin{pmatrix}
\frac{4m_2^2+2\mu_1\mu_\nu v_d}{2\mu^4} & \frac{g_1g_2\mu_1\nu_1\nu_2}{2\mu^4} & \frac{g_1m_2\mu_\nu}{\mu^4} & \frac{g_1m_2\mu_\nu^2}{\mu^4} \\
\frac{g_1g_2\mu_1\nu_1\nu_2}{2\mu^4} & \frac{4m_1^2+2\mu_1\mu_\nu v_d}{2\mu^4} & \frac{g_2m_1\mu_\nu}{\mu^4} & \frac{g_2m_1\mu_\nu^2}{\mu^4} \\
\frac{g_1m_2\mu_\nu}{\mu^4} & \frac{g_2m_1\mu_\nu}{\mu^4} & \frac{8\mu_1m_2+\mu_\nu v_d}{2\mu^4} & \frac{\mu_\nu^2}{2\mu^4} \\
\frac{g_1m_2\mu_\nu}{\mu^4} & \frac{g_2m_1\mu_\nu}{\mu^4} & \frac{\mu_\nu^2}{2\mu^4} & \frac{8\mu_1m_2+\mu_\nu v_d}{2\mu^4}
\end{pmatrix}
\]

\[
[M_N^{(1)}]^{-1} = \begin{pmatrix}
\frac{1}{\Delta_{BL-m_{BL}}} & 0 \\
0 & \frac{1}{\Delta_{BL+m_{BL}}}
\end{pmatrix}
\]

\[
[M_N^{(2)}]^{-1} = \begin{pmatrix}
\frac{i g_1 m_2 \mu_\nu \nu \epsilon_+}{\mu^4 (\Delta_{BL} - m_{BL})} & -\frac{g_1 m_2 \mu_\nu \nu \epsilon_+}{\mu^4 (\Delta_{BL} + m_{BL})} \\
\frac{-i g_2 m_1 \mu_\nu \nu \epsilon_-}{\mu^4 (\Delta_{BL} - m_{BL})} & \frac{g_2 m_1 \mu_\nu \nu \epsilon_-}{\mu^4 (\Delta_{BL} + m_{BL})} \\
\frac{-i (\Delta_{BL} - m_{BL})}{8 \mu_1 m_2 + \mu_\nu v_d \nu_1 \nu_2 \epsilon_-}{2 \mu^4 (\Delta_{BL} - m_{BL})} & \frac{8 \mu_1 m_2 + \mu_\nu v_d \nu_1 \nu_2 \epsilon_+}{2 \mu^4 (\Delta_{BL} + m_{BL})} \\
\frac{-(\Delta_{BL} + m_{BL})}{8 \mu_1 m_2 + \mu_\nu v_d \nu_1 \nu_2 \epsilon_-}{2 \mu^4 (\Delta_{BL} - m_{BL})} & \frac{-8 \mu_1 m_2 + \mu_\nu v_d \nu_1 \nu_2 \epsilon_+}{2 \mu^4 (\Delta_{BL} + m_{BL})}
\end{pmatrix}
\]
Where the abbreviations are

\[
\tilde{m} = g_1^2 m_2 + g_2^2 m_1,
\]

\[
\tilde{\mu}^4 = 4 m_1 m_2 \mu^2 + \tilde{m}_\mu u_d.
\]

(E3)

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