Elastic theory of pinned flux lattices

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Abstract

The pinning of flux lattices by weak impurity disorder is studied in the absence of free dislocations using both the gaussian variational method and, to $O(\epsilon = 4 - d)$, the functional renormalization group. We find universal logarithmic growth of displacements for $2 < d < 4$: $\langle (u(x) - u(0))^2 \rangle \sim A_d \log |x|$ and persistence of algebraic quasi-long range translational order. When the two methods can be compared they agree within 10% on the value of $A_d$. We compute the function describing the crossover between the “random manifold” regime and the logarithmic regime. This crossover should be observable in present decoration experiments.

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It has been argued on the basis of various elastic models for vortex lattices, such as Larkin’s model of independent random forces acting on each vortex, that arbitrarily weak disorder destroys translational order below four dimensions [3]. There is considerable disagreement, however, on the exact behaviour of the density-density correlation function $C_K(r) = \langle \rho_K(0) \rho_K(r) \rangle$ even in the simpler case where dislocations are excluded. While direct extensions of Larkin’s model predict exponential decay in $d = 3$ [4], it has been pointed out that beyond the Larkin-Ovchinikov pinning length $L_c$ the lattice behaves collectively as an elastic manifold in a random potential with many metastable states, leading to a different power-law roughening of the lattice and to stretched exponential decay of $C_K(r)$ [5–7]. On the other hand, Flory-type arguments were proposed making explicit use of the periodicity of the lattice leading to logarithmic roughening [8]. The role of dislocations at weak disorder above two dimensions is presently unsettled, but Bitter decoration experiments [9] show remarkably large regions free of dislocations and provide a strong motivation for a better understanding of the elastic model. Other related pinned elastic systems such as charge density waves, magnetic bubbles, Wigner crystal, are under current active experimental study [10,11].

In this Letter we take into account both the existence of many metastable states and the periodicity of the lattice. We are primarily interested in the triangular Abrikosov lattice $(d = 2 + 1)$. We also mention the case of $d = 2 + 0$ (point vortices in thin films) or $d = 1 + 1$ (lines in a plane). We show that in the absence of dislocations, the translational correlation function has a slow algebraic decay in dimension larger than two, and thus quasi-long range order persists. Two important length scales control the crossover towards this asymptotic decay. i) When the mean square of the relative displacement $\tilde{B}(x) = \frac{1}{2} \langle |u(x) - u(0)|^2 \rangle$ of two lines as a function of their separation $x$ is shorter than the square of the Lindemann length $l_T^2 = \langle u^2 \rangle$, the thermal wandering of the lines averages enough over the random potential and the model becomes equivalent to the random force Larkin model for which $\tilde{B}(x) \sim |x|^{4-d}$. At low enough temperature, $l_T$ is replaced by the superconducting coherence length $\xi_0$ (i.e. the correlation length of the random potential [3–4]). ii) For $l_T^2 \ll \tilde{B}(x) \leq a^2$, $\tilde{B}(x) \sim x^{2\nu}\ldots$
this is the random manifold regime where each line sees effectively an independent random potential. iii) For $x > \xi$, where $\xi$ corresponds to a relative displacement of the order of the lattice spacing $a$, $\tilde{B}(x = \xi) \sim a^2$, the periodicity of the lattice becomes important. We find $\tilde{B}(x) \sim A_d \log |x|$ where $A_d$ is a universal amplitude depending on dimension only and isotropy is recovered at large distances. This leads to quasi long range order $C_{K_0}(r) \sim (1/r)^{A_d}$. We have computed the full crossover function in $d = 3$ (Fig. 1). It suggests that all the above regimes could be observed by analysis of the dislocation-free decoration samples.

These results are obtained using the Mezard-Parisi variational method [12] first applied in this context by Bouchaud, Mezard and Yedidia (BMY) [6,7]. Our results are at variance with BMY, for reasons detailed below. In addition, we perform an $\epsilon = 4 - d$ expansion using the functional renormalization group. The amplitudes $A_d$ obtained by these two rather different methods agree at order $\epsilon$ within 10%. In $d = 2$, thermal fluctuations are important ($l_T = \infty$) and the random manifold regime is much reduced. We find a modified Larkin regime with T-dependent exponents $\tilde{B}(x) \sim |x|^{2\nu(T)}$ and a long distance logarithmic regime. Details can be found in [13].

We denote by $R_i$ the equilibrium position of the lines labeled by an integer $i$, in the $x - y$ plane, and by $u(R_i, z)$ their in-plane displacements. $z$ denotes the coordinate perpendicular to the planes. For weak disorder $a/\xi \ll 1$ it is legitimate to assume that $u(R_i, z)$ is slowly varying on the scale of the lattice and to use a continuum elastic energy, as a function of the continuous variable $u(x, z)$. Impurity disorder is modeled by a gaussian random potential $V(x, z)$ with correlations: $\overline{V(x, z)V(x', z')} = \Delta(x - x')\delta(z - z')$ where $\Delta(x)$ is a short range function of range $\xi_0$ and Fourier transform $\Delta_q$. The total energy is:

$$H_{el} = \frac{1}{2} \int d^2xdz[(c_{11} - c_{66})(\partial_\alpha u_\alpha)^2 + c_{66}(\partial_\alpha u_\beta)^2 + c_{44}(\partial_z u_\alpha)^2] + \int d^2xdzV(x, z)\rho(x, z) \quad (1)$$

where $\alpha, \beta$ denote in-plane coordinates and the density is $\rho(x, z) = \sum_i \delta(x - R_i - u(R_i, z))$. Although we have also performed the calculations directly on the Hamiltonian [13, 14] it is more enlightening to use the following decomposition of the density that keeps track of the discreteness of the lines. In the absence of dislocations, generalizing [14], one intro-
duces the slowly varying field $\phi(x, z) = x - u(\phi(x, z), z)$. The density can be rewritten as $\rho(x, z) = \rho_0 \det[\partial_\alpha \phi] \sum_K e^{iK \cdot \phi(x, z)} \simeq \rho_0 (1 - \partial_\alpha u_\alpha(\phi(x, z), z) + \sum_{K \neq 0} e^{iK \cdot x} \rho_K(x))$, where $\rho_K(x) = e^{-iK \cdot u(\phi(x, z), z)}$ is the usual translational order parameter defined in terms of the reciprocal lattice vectors $K$, and $\rho_0$ is the average density.

Using the replica trick on (1) the disorder term gives $-1/(2T) \sum_{a, b} \int d^2xdx'dz \Delta(x - x') \rho^a(x, z)\rho^b(x', z)$. The above decomposition for the density leads to our starting model:

$$H_{\text{eff}} = \int d^2qdq_z \sum_a G^{-1}_{0, \alpha\beta} u^a_\alpha(q, q_z) u^b_\beta(q, q_z)$$

$$- \int d^2xdz \left[ \sum_{a, b} \frac{\rho^2 \Delta_0}{2T} \partial_\alpha u^a_\alpha \partial_\beta u^b_\beta + \sum_{K \neq 0} \frac{\rho^2_0 \Delta_K}{2T} \cos(K \cdot (u^a(\phi(x, z), z) - u^b(x, z))) \right]$$

with $G^{-1}_0 = (c_{44}q_z^2 + c_{60}q^2)P^T_{\alpha\beta} + (c_{44}q_z^2 + c_{11}q^2)P^L_{\alpha\beta}$ in the case of (1), where $P^T_{\alpha\beta} = \delta_{\alpha\beta} - q_\alpha q_\beta/q^2$ and $P^L_{\alpha\beta} = q_\alpha q_\beta/q^2$. To be rigorous (2) should be written in term of $u(\phi(x, z), z)$. This however has no effect on our results. It leads only to corrections of higher order in $\nabla u$ which we neglect since we work here in the elastic limit $a/\xi \ll 1$ [13]. For clarity we present the calculation for the isotropic version of (2) in $d$ dimensions with $G^{-1}_0 = c q^2 \delta_{\alpha\beta}$ where $c$ is an elastic constant and $\rho^2_0 \Delta_K = \Delta$ for all $K (x \equiv (x, z))$. The results for (1) is presented at the end. A related single cosine model was studied by Villain and Fernandez [15] using a real space RG. Our results confirm and extend their analysis.

One can get an idea of the effect of various terms in (2), by using arguments similar to [16]. Assuming that $u$ varies of $\sim a$ over a length $\xi$, the density of kinetic energy is $\sim c(a/\xi)^2$.

The long wavelength part of the disorder is $H_{q \rightarrow 0}^{\text{dis}} \sim \int d^d x V(x) \partial_\alpha u_\alpha(x) \sim \Delta^{1/2}/\xi^{1+d/2}$ and $H_{q \sim K_0}^{\text{dis}} \sim \int d^d x V(x) \cos(K_0 \cdot (x - u(x))) \sim \Delta^{1/2}/\xi^{d/2}$ where $K_0$ is the first reciprocal lattice vector. There are thus two length scales $\xi_{q \rightarrow 0} \sim a \left(c^2 a^d/\Delta\right)^{1/(2-d)}$ and $\xi \sim a \left(c^2 a^d/\Delta\right)^{1/(4-d)}$. The $q \sim 0$ component of the disorder is therefore relevant only for $d \leq 2$ and the second term in (2) can be dropped for $d > 2$. Higher Fourier components $V_{q \sim K}$ disorder the lattice below $d = 4$.

We now look for the best trial Gaussian Hamiltonian $H_0$ in replica space, of the form [12]:
\[ H_0 = \frac{1}{2} \int \frac{d^d q}{(2\pi)^d} G^{-1}_{ab}(q) u_a(q) \cdot u_b(-q) \]  

Defining the self-energy \( G^{-1}_{ab} = cq^2 \delta_{ab} - \sigma_{ab} \), and \( G^{-1}_c(q) = \sum_b G^{-1}_{ab}(q) \) we obtain by minimization of the variational free energy \( F_{\text{var}} = F_0 + \langle H - H_0 \rangle H_0 \) the saddle point equations:

\[ G_c(q) = \frac{1}{cq^2}, \quad \sigma_{a \neq b} = \frac{\Delta}{nt} \sum_K K^2 e^{-\frac{K^2}{2} B_{ab}(x=0)} \]  

\[ B_{ab}(x) = \frac{1}{n} \langle [u_a(x) - u_b(0)]^2 \rangle \]  

\[ = T \int \frac{d^d q}{(2\pi)^d} (G_{aa}(q) + G_{bb}(q) - 2 \cos(qx)G_{ab}(q)) \]

where \( n \) is the number of components of \( u \).

The replica symmetric solution \( B_{a \neq b}(x) = B(x) \), which mimics the distribution of displacements by a single Gaussian, is always unstable for \( 2 < d < 4 \). This is seen from the eigenvalue \( \lambda \) of the replicon mode \[12]:

\[ \lambda = 1 - \frac{\Delta}{n} \sum_K K^4 e^{-\frac{K^2}{2} T} \int \frac{d^d p}{(2\pi)^d} G_c(p) \int \frac{d^d q}{(2\pi)^d} G_c^2(q) \]

Introducing a small regularizing mass in \( G_c \): \( G_c^{-1}(q) = cq^2 + \mu^2 \) we find, when \( \mu \to 0 \), that for \( d < 2 \) the replica symmetric solution is always stable and disorder is irrelevant. For \( d = 2 \) the condition becomes \( \mu^{-2(1 - \frac{d}{4})} < 1 \) for small \( \mu \). Thus there is a transition at \( T = T_c = \frac{4\pi c}{K_0^2} \) between a stable high-T phase where disorder is irrelevant and a low-T glassy phase where the symmetric saddle point is unstable. For \( 2 < d < 4 \) it is always unstable and disorder is always relevant.

We now find a replica symmetry breaking solution for \( 2 < d < 4 \), the \( d = 2 \) case being discussed later. Following \[12\] we denote \( \tilde{G}(q) = G_{aa}(q) \) and parametrize \( G_{ab}(q) \) by \( G(q, v) \) where \( 0 < v < 1 \), and similarly for \( B_{ab}(x) \). The saddle point equations become:

\[ \sigma(v) = \frac{\Delta}{nt} \sum_K K^2 e^{-\frac{K^2}{2} B(0,v)} \]

Expression (4) and the algebraic rules for inversion of hierarchical matrices \[12\] give:

\[ B(0, v) = B(0, v_c) + \int_v^{v_c} dw \int \frac{d^d q}{(2\pi)^d} \frac{2T \sigma'(w)}{(G_c(q)^{-1} + [\sigma](w))^2} \]
where \( [\sigma](v) = u\sigma(v) - \int_0^v dw \sigma(w) \) and \( v_c \) is the breakpoint such that \( \sigma(v) \) is constant for \( v > v_c \). \( B(0, v_c) \) is a nonuniversal quantity \( B(0, v_c) \simeq \xi_0^2 + l_T^2 \).

To discuss the large distance behaviour \( x \gg \xi \) it is enough to keep \( K^2 = K_0^2 \) in (7) since \( B(0, v_c) \gg a^2 \). In that case, taking the derivative of (7) with respect to \( v \), using \( [\sigma]'(v) = v\sigma'(v) \) and (8) one finds the effective self energy:

\[
[\sigma](v) = (v/v_0)^{2/\theta}
\]

and \( v_0 = 2K_0^2Tc_0c^{-d/2}/(4 - d) \). The energy fluctuation exponent is \( \theta = d - 2 \). Energy fluctuations are of order \( T/v \) and the large scale behaviour is controlled by small \( v \). One can now compute the correlation functions:

\[
\langle (u(x) - u(0))^2 \rangle = 2nT \int \frac{d^d q}{(2\pi)^d} (1 - \cos(qx)) \tilde{G}(q)
\]

\[
\tilde{G}(q) = \frac{1}{cq_2} (1 + \int_0^1 \frac{dv}{v^2} \frac{[\sigma](v)}{cq_2 + [\sigma](v)}) \sim \frac{Z_d}{q^d}
\]

with \( Z_d = (4 - d)/(TK_0^2S_d) \) and \( 1/S_d = 2^{d-1}\pi^{d/2}\Gamma[d/2] \). Thus for \( 2 < d < 4 \) we find logarithmic growth:

\[
\langle (u(x) - u(0))^2 \rangle = \frac{2n}{K_0^2} A_d \log |x|
\]

with \( A_d = 4 - d \).

We now give the full crossover function for \( d = 3 \) for model (2) with \( \rho_0^2\Delta_{K_0} = \Delta \). The crossover length is \( \xi = a^4c^2/(2\pi^3\Delta) \). Defining \( h(z) = \sum_{P}(\Delta_K/\Delta_{K_0})P^4e^{-zP^2} \), where \( K = 2\pi P/a \), the solution is, in parametric form:

\[
v = v_\xi h'(z) \quad [\sigma]^{1/2} = c^{1/2}\xi^{-1}h(z)
\]

with \( B(0, v) = a^2z/(2\pi^2) \) and \( v_\xi = 2\pi^4T\Delta/(a^6\xi^3) \sim l_T/\xi \). The mean square displacement \( \tilde{B} \) is:

\[
\tilde{B}(x) = \frac{a^2}{2\pi^2} \tilde{b}(\frac{x}{\xi}) \quad \tilde{b}(x) = \int_0^\infty dt \frac{h^n(0)h(t)}{h'(t)^2} f(xh(t))
\]

\[
f(x) = 1 - \frac{1}{x}(1 - e^{-x})
\]
For $x \ll \xi$, $B(0, v)$ is very small and $h(z) \sim 1/z^{n/2+2}$ and therefore $\tilde{B}(x) \sim x^{2\nu}$ with $\nu = 1/6$ for $n = 2$. This corresponds to the random manifold regime \cite{13,14}. The crossover function \cite{13,14} was derived assuming $v_\xi \ll v_c$, equivalent to $l_T \ll \xi$. At scales such that $\tilde{B}(x)$ is smaller than $l_T^2$ or $\xi_0^2$ one is in the regime $v > v_c$ and one recovers the replica symmetric propagator $\tilde{G}(q) \sim 1/q^4$ for $q^2 \gg [\sigma](v_c)$, and Larkin’s model behavior.

In the vortex lattice \cite{11}, shear deformations dominate ($c_{66} \ll c_{11}$) and the crossover length becomes $\xi = c_{66}^{3/2} c_{14}^{1/2} a^4/(2\pi^3 \Delta)$. $\tilde{B}_{\alpha\beta}(x) = \tilde{B}_{T,P}^T(x) + \tilde{B}_{L,P}^L(x)$ depends on the direction of $x$ and we find:

$$
\tilde{B}_{L,T}(x) = \frac{a^2}{2\pi^2} \bar{b}_{T,L}(\frac{x}{\xi}) + \frac{c_{66}}{c_{11}} \bar{b}_{L,T}((\frac{x}{\xi}) \sqrt{\frac{c_{66}}{c_{11}}})
$$

(16)

where $b_{L,T}$ have an expression similar to (14) with $f$ replaced by

$$
f_L(x) = \frac{1}{2} - \frac{1}{x^2} + \left(\frac{1}{x} + \frac{1}{x^2}\right)e^{-x} \quad f_T(x) = f(x) - f_L(x)
$$

(17)

The crossover function is shown in Fig. 1. We find that if $\xi/a \gg 1$ all curves should scale when plotted in units of $x/\xi$. Such a crossover should be observable in decoration experiments. The ratio $R = \tilde{B}_T(x)/\tilde{B}_L(x)$ crosses over at $x = \xi$ from $R = 4/3$ towards $R = 1$ as isotropy is restored at large scale.

As $d \to 2^+$ the function $[\sigma](v)$ in \cite{9} vanishes for $v < v_0 = TK_0^2/(4\pi c)$. Thus in $d = 2$ for $T < T_c$ there is a one-step replica symmetry broken solution with $[\sigma](v) = 0$ for $v < v_c$ and $[\sigma](v) = [\sigma](v_c)$ for $v_c < v < 1$ \cite{13}. In the glassy phase $T < T_c$ the variational method predicts the following two regimes. For $T$ moderate, the short distance Larkin regime $\tilde{B}(x) \sim x^2$ crosses over directly towards the asymptotic logarithmic growth \cite{12} at a length $\xi = a(c^2 a^2/\Delta)^{1/(2-TK_0^2/2\pi c)}$. At low $T$, $T \log(\xi/a)/(ca^2) \ll 1$, there is, in addition, an intermediate random manifold regime. In $d = 1 + 1$ the starting model \cite{2} becomes exact due to the absence of dislocations and a RG calculation for $T \approx T_c$ \cite{13,14,15} finds $\tilde{B}(x) \sim \log^2(x)$ at large $x$. If one believes in the validity of this RG in a glassy phase, the simple gaussian ansatz does not give the exact long range behaviour, due to the importance of fluctuations in $d = 2$. However it gives $T_c$ exactly and captures correctly the crossover
towards a slower logarithmic regime. Using RG we have shown \cite{13} that in $d = 2$ the Larkin regime is in fact anomalous with a continuously variable exponent:

$$\tilde{B}(x) \sim a^2 (x/\xi)^{2-\frac{TK_0^2}{2\pi}}$$

for $x < \xi$ (in the low-T regime $\xi$ is replaced by another length). Model (2) will probably apply in $d = 2 + 0$ at scales shorter than the distance between dislocations. It could also describe a polymerized membrane on a disordered substrate, with very high dislocation core energy.

A previous application of the variational method by BMY \cite{6,7} led to the conclusion which we believe is erroneous, that the fluctuations are enhanced at large distances. They applied the same method to a model in which each line sees a different disorder. This amounts to introduce an extra and unphysical disorder in the original model (11), with correlations decaying as $1/|R_i - R_j|^\lambda$. The long wavelength part of such a disorder dominates since large global translations of the lattice can improve the bulk energy, whereas in the physical model the gain in energy from the long wavelengths components can only come from surface terms and is thus irrelevant for $d > 2$, as shown in the discussion following (2). Indeed for $\lambda \to 0$ the amplitude they obtain vanishes. In fact one can simplify the the saddle point equations of \cite{6,7} by noting that the $x$ dependence of $B(x, u)$ in these equations is unimportant, up to higher order terms in $\nabla u$. The resulting local model (2) is simple enough to allow for the exact solution \cite{16}.

To complement the variational method we perform a functional renormalization group calculation on the isotropic model. To simplify we take $u$ to be a scalar field ($n = 1$) and set $c = 1$. One defines the replicated Hamiltonian as:

$$H_{\text{imp}} = -\frac{1}{2T} \sum_{a,b} \int d^d x \Delta(u_a(x) - u_b(x))$$

(19)

The function $\Delta(z)$ is periodic of period 1 ($2\pi K_0 = 1$). The RG equations to order $\epsilon = 4 - d$ have been derived by D.S. Fisher \cite{19} for the random manifold problem:

$$\frac{d\Delta}{dl} = (\epsilon - 4\zeta)\Delta + \zeta z\Delta' + \frac{1}{2}(\Delta'')^2 - \Delta''\Delta''(0)$$

(20)
A factor $S_d$ has been absorbed into $\Delta$. The periodicity of the function implies that the roughening exponent is $\zeta = 0$. This allows to obtain the fixed point function $\Delta^*(z)$ in the interval $[0, 1]$:

$$\Delta^*(z) = \frac{\epsilon}{72} \left( \frac{1}{36} - z^2 (1 - z)^2 \right)$$ \hspace{1cm} (21)

Values for other $z$ are obtained by periodicity. The fixed point is stable except for a constant shift. The linearized spectrum is discrete and the eigenvectors are Jacobi polynomials. This function has a nonanalyticity \cite{19} $\Delta^{*(4)}(0) = \infty$. To compute the correlation function $\tilde{\Gamma} = T \tilde{G}$ one uses the RG flow equation:

$$\tilde{\Gamma}(q, T, \Delta) = e^{dl} \tilde{\Gamma}(qe^l, Te^{(2-d)l}, \Delta(l))$$ \hspace{1cm} (22)

choosing $e^l q = 1/a$ allows to obtain perturbatively $\tilde{\Gamma}(q) = -\Delta^{''*}(0)/q^4 = a^{(4-d)}\epsilon / (36 S_d q^d)$. Thus we find, at order $\epsilon$, a logarithmic growth \cite{12} of line displacement with $A_{d, RG} = \epsilon (2\pi)^2 / 36 = 1.10 \epsilon$. This compares within 10% with $A_{d, VAR} = \epsilon$ which slightly underestimate fluctuations. For $2 \leq d < 4$ the real space RG method of \cite{13} also predicts a log but does not allow to compute the universal prefactor $A_d$ or the crossover function. The agreement between these methods, none being rigorous, lends credibility to the additional results in $d = 3$ obtained using the variational method.

To conclude, we have shown that due to the periodicity of the lattice, the pinning by impurities becomes less effective at large length scales. This has consequences on the transport properties. In the flux creep regime energy barrier arguments \cite{3}, and the exponents found here, lead to a non linear voltage-current relation $V \sim \exp[-1/(T j^n)]$ where $\mu$ crosses over from $\mu \approx 0.7 - 0.8$ to $\mu = 1/2$ as $j$ decreases.

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FIGURES

FIG. 1. Plot of $\tilde{b}_L$ versus $\log |x/\xi|$ (solid line), as defined in the text, for the Abrikosov triangular lattice. The dashed line is the random manifold result.