The ability to Understanding of the Concept of Derivative Functions for Inter-Level Students During Ethnomathematics Learning

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Abstract. The mathematical abilities can be measured through students' mental and physical activities. One of them is using genetic decomposition analysis of APOS theory. The purpose of this study was to describe the ability to understand the concept of derivative functions for inter-level students during ethnomathematics learning. The research is part of our development research. The subjects were high school students majoring in mathematics and science in Bengkulu. The research was carried out in a participatory manner in the regular learning process by applying the ethnomathematics approach. Subjects were interviewed based on the assignments given. Data were analyzed qualitatively by applying genetic decomposition. To obtain inter-level characteristics, the description of genetic decomposition continued with fixed-comparison methods. The result was able students to encapsulate the process of function properties or intervals in the domain so that the object is formed about the sketch of the function graph.

1. Introduction

The mathematical learning based on ethnomathematics can change people's perceptions of the application of mathematics in everyday life [1]. Also, knowledge structures consist of fact development schemes, and mathematical activities are based on situations arising from experience [2]. Therefore, understanding the mathematical concept will be easier if the objects that are near the environment are started.

During learning ethnomathematics, students are able to abstract mathematical concepts correctly [3][2][4]. Results of research by Widada, et. al. [5], through ethnomathematics students reach abstract levels. The characters of abstract level students are able to use all statements given to solve problems, can explain the relationship of statements given to arguments in problem solving, able to explain the usefulness of each statement used to solve problems, as a result of a proven statement, can explain statements that are composed as a result of existing statements using good arguments and drawing conclusions that have been made on paper and pencil, but have not been able to make the proof [6], and he tries to make a new statement more than the original statement refers to the statement, but fails prove the truth.
Students was understanding the concept carried out through the process of abstraction, generalization and idealization [7]. Understanding the concept was done through understanding abstract ideas using concrete objects to be classified into examples/non-examples of concepts [8]. Students can define a concept by connecting the "concept name", "proximum genus" and "special differentiator" [7]. This makes it easier to understand a mathematical concept. But students often experience errors understanding the concept of derivative functions. The students had barriers to applying the concepts and the properties of derivative [9][6][10].

Derivation is one of the fundamental concepts in the learning of university mathematics. Students have difficulties in the learning of this concept which mostly come back to lack of conceptual understanding [11]. The students do not understands of derivatives conceptually, the properties of functions logically, and the difficulty of understanding the function domain \( h = \mathbb{R} \) [9] [10].

The students have serious difficulties in understanding conceptually of derivation. They were difficulties in conceptual understanding of derivation come back to focusing on symbolic aspect more than embodied aspect, lack of making logical connection between these aspects, and weakness of dealing with generalized question [11]. Function concept is an important part of cumulative blocks of concepts in advanced level mathematics and engineering courses. The function concept also requires knowledge of limits, derivatives, and asymptotes [12].

Teachers should be more serious in handling students who have limit learning difficulties and derivative functions. Teachers should be more serious in handling students who have limit learning difficulties and derivative functions. Students need to learn in an atmosphere that is close to their culture. Students solved mathematical problems through mathematization process based on ethnomathematics. Students were aware that Rejang Lebong's ethnomathematics was the starting point of horizontal mathematical activity [2]. This allows students to make a realistic mathematical process [13][14][15]. The results showed that the students studied using the realistic mathematics learning approach, the mathematics representation ability of students given the ethnomathematics-oriented materials was higher than the students learning with the non-ethnomathematics materials [16]. Therefore, mathematical pedagogy based on ethnomathematics makes it easy for teachers to manage mathematics learning [17]. What is the cognitive process during the learning? This is answered through genetic decomposition of students during and after learning. Genetic decomposition (or cognition model) is a structured collection of mental activities that build blocks (categories) to describe how concepts/principles can be developed in the mind of an individual [18][10][19]. It can be analyzed using APOS theory (action-process-object and schema) [20].

According to DeVries & Arnon [21], a genetic decomposition (GD) of a particular concept consists of detailed descriptions of possible actions, and typical mathematical behaviors and reactions of a student who has developed the same concept, starting with that action, across different levels (Action, Process, Object and Schema). Therefore, GD can be used as a diagnostic tool, giving teachers and researchers insight into the situation of learners in developing the concept. In addition, this helps teachers and material developers to provide students with activities that will improve their progress in developing their understanding of concepts through various levels of Action-Process-Object-Schema. As applied in research that shows that successful calculus students, who seem to operate at various stages of development of their calculus graph schemes, illustrate the complexities involved in achieving schematic thematization, thus indicating that thematization is possible [22].

The research on understanding mathematics can be analyzed qualitatively and quantitatively using the triad classification in the theory of Action-Process-Object-Schema (APOS) [12]. Evolution of a schema, as described by APOS theory, is a possible form of construction for complex and abstract concepts [22]. Piaget and Garcia hypothesized that these levels can be found when one analyzes any developing schema, and the nature of the triad stages is functional, not structural, and we describe the triad's general psycho-dynamical aspects [9]. According to Tokgoz & Gualpa [12], considering the APOS theory-based data classification, post-interview data collection indicated a uniform triad classification of the participants about the graphing problem indicated students’ misconception of derivative. The results,
the main misconception of the participants appeared to be the first and second derivative related information, due to the post-interview responses to the question. Baker, et.al. [9] used APOS theory in terms of the triad and schema development as the initial framework for categorizing student understanding from the interviews. The two dimensions of the calculus graphing schema emerged. Using the model allowed researcher to understand certain student behaviors, caused by the interaction of schemas, that would have been overlooked without the consideration of their mutual effects. Based on the previous description, then this paper discusses the ability to understanding of the concept of derivative functions for inter-level students during ethnomathematics learning.

2. Methods
The research is part of our development research. We were described the cognitive characteristics of a student who is at inter-level. The student was selected from this research subject. The subjects were high school students majoring in mathematics and science in Bengkulu. A total of 10 subjects were selected from 215 students based on their cognitive abilities. The research was carried out in a participatory manner in the regular learning process by applying the ethnomathematics approach. Subjects were interviewed based on the assignments given. Students are asked to sketch the graph of the function h that meets the following conditions (adoption of [9]). Let h continuous function; h(0) = 2, h'(-2)=h'(3)=0, and lim_{x→0} h'(x) = ∞; h'(x)>0, if -4<x<-2 and -2<x<3; h'(x)<0, if x<-4 and x>3; h''(x)<0, if x<-4, -4<x<-2, and 0<x<5; h''(x)>0, if -2<x<0, and x>5; lim_{x→-∞} h(x) = ∞; and lim_{x→∞} h(x) = -2. [9].

Subjects are explored about their cognitive processes about interconnectivity between property schema, and interval domain schema based on assignments. Data were analyzed qualitatively by applying genetic decomposition. To obtain inter-level characteristics, the description of genetic decomposition continued with fixed-comparison methods [23].

3. Results and Discussions
Based on the genetic decomposition of the research subject (call it F) collected through in-depth interviews during ethnomathematics learning, obtained as follows. Subject F can sketch the graph of the function h correctly for x≥0; can describe the condition of the graph around x = 0 using two properties for the limit of the first derivative, and h (0) = 2. Also, it is able to describe at intervals of 0 <x <3 using two existing properties, namely the first derivative, and the second derivative. He coordinated the conditions at x = 3, and 3 <x <5 using the first derivative, and the second derivative. Whereas for x> 5 it is deviated by using 3 functions, namely the first derivative, the second derivative, and the limit of h (x) for x to infinity. However, F sketches the graph incorrectly for x <0, it misrepresents the first derivative. As can be seen in Figure 1.

![Figure 1. The Graph Sketch of h by F](image)

The genetic decomposition analysis of mental and physical activity from F is presented with interview footage. This is classified in the action-process-object-scheme activity. First, the results of our interview (R = researcher) with F were:
R: Okay! Please read and understand the problems I gave, then you solve the problem.
F.01: Yes .......... [...] [F reads, understands and tries to solve problem 1 for 8 minutes.]
R: Well ... you have solved the problem. Try to reveal your thoughts?
F.02: From here it is known that \( h(0) = 2 \) means curve \( h \) through \((0.2)\). The first derivative of \( h \) at \( x = -2 \) and 3 is equal to 0 meaning the tangent at that point is parallel to the x-axis. [F shows the chart h chart through point \((0.2)\), also the tangent line is called F.02.]
F.03: Then \( h'(x) > 0 \) at the interval \(-4 < x < 2\) means the graph goes up [to the right].
R: Okay!
F.04: then when \(-2 < x < 3\) the chart also rises up [to the right] [because at that interval \( h'(x) > 0 \)]. Then \( h'(-2) = h'(3) = 0 \) so that at these points it becomes [a] critical point. [F shows the critical point in the sketch of the h chart as referred to in F.04.]
R: Why is it a critical point?
F.05: Maximum point of both! ...
R: Hmm ...
F.06: Then for \( x < -4 \), \( h'(x) < 0 \) means the graph will go down [to the right] ... [F points to the curve that means F.06.]
R: Okay! Good, then what about the limit of \( h(x) \) for \( x \) towards \(-\infty \) equal to \( \infty \)?
F.13: ... After \( x < -4 \), the graph will go up [to the left]. [F shows a graphic sketch of function h, as intended F.13]
R: ...... Now try to look around \( x = -4 \), you make the point \(-4\) as a critical point, why?
F.09: Actually it is not a requirement [that states it], but from the existing conditions, I see [graph h] after going down to the right, right up to the right, like this. [F points to the terms \( x < -4, h'(x) < 0 \), and \( h'(x) > 0 \) if \(-4 < x < -2\).]
R: Next!
F.07: Then for \( x > 3 \) the chart also goes down to the Right ... Because \( x < -4 \), \( h'(x) < 0 \), the graph is facing down [concave down], also for \(-4 < x < -2\) the curve is facing down. Then for \( 0 < x < 5 \) it also faces down [concave down]. [F shows a graphic sketch as disclosed F.07]
R: Yes ... yes ...
F.08: Then for intervals of \(-2 < x < 0\), the graph faces up [concave up], and for \( x > 5 \) it also faces up [concave up]. [F shows graphic sketches as disclosed F.08]
R: Okay! ...... Now, there is a given condition, that the limit of \( h'(x) \) for \( x \) approaches \( 0 \) is infinite, what does that mean?
F.10: ...... the tangent at that point [around \( x = 0 \)] approaches the y axis. [F shows a graph sketch of function h, as meant by F.10]
R: How do you apply it to the graph?
F.11: Like this [see graph sketch made by U]. [U draws the tangent line.] R: What is the graph like?
F.12: Here [around 0 left] goes right up. [F shows a graphic sketch of the function h, as intended F.12]
R: What is the limit of \( h(x) \) for \( x \) towards \( \infty \) equal to -2? 
F.14: At the interval \( x > 5 \), the graph will approach -2 but never cut [line] \( y = -2 \), as an asymptot line. [F shows a graphic sketch of function h, as intended F.14]

Based on the interview exposure, Subject F can coordinate cognitively the adjacent intervals and overlaps, but not all intervals in the function domain h. Uncoordinated intervals are intervals of \(-2 < x < 3\) (see F.04), intervals \(-2 < x < 0\), and \( 0 < x < 5 \) (see F.07, and F.08) and see argument F on the worksheet. So that the coordination has not arrived at a mature scheme, and only as a process encapsulation on the object. Also, F can do encapsulation of the process properties of the function given at intervals in the domain h. As in the interval \( x < -4 \), F coordinates the description \( h'(x) \) and the limit h \((x)\) for \( x \) to infinity, at intervals of \(-4 < x < -2\), F coordinates the description \( h''(x) \) and \( h'''(x) \), at intervals of \(-2 < x < 0\), F coordinates the description \( h''(x) \) and \( h'(x) \), but F is wrong in describing \( h'(x) \), see interesting things, around \( x = 0 \), F coordinate the description description of the limit of \( h'(x) \) for \( x \) approaches 0, \( h'(x) \) and
\( h'(x), \) for intervals \( 0 < x < 5, \) \( F \) coordinates the description \( h'(x), h''(x), \) or \( h'(3) = 0. \) For \( x > 5, \) \( F \) coordinates the complete description of the three properties of the functions in the interval, \( h'(x), h''(x), \) and the limit of \( h(x) \) for \( x \) to infinity.

Analysis of genetic decomposition can be concluded that \( F \) can coordinate at least two adjacent and overlapping intervals, but not for all intervals in the domain, and the coordination forms an object about the interval. He can encapsulate the process of the function properties that are given at intervals in domain \( h \) so that an object is formed about the sketch of the function graph \( h. \) This characteristic indicates that \( F \) is at an inter level.

The results of this study support several other studies, such as Baker, et. al. [9], at the trans-property, inter-interval (trans-inter) level, the student could coordinate all properties across parts, but not all, of the domain. The student linked some contiguous segments of the domain but still expressed difficulty at some points. Among the four students at the trans-inter level, who experienced problems while trying to join the intervals, such as at \( x = 0. \) He was able to coordinate properties across some intervals of the domain, but he hesitated when he had to subdivide intervals to coordinate properties. The APOS theory is not only for calculus, but also for others. Meagher, et. al. [24] state that the learning of Linear Algebra course is a constructivist-based education course which employs the Action-Process-Object-Schema (APOS) theory of learning math. APOS theory represents an extension of Piagetian theories on children’s reflective learning to the realm of higher level abstract mathematics. Analysing mathematics from the APOS standpoint allows for the development of ways of thinking about how abstract mathematics can be assimilated and learned.

Cooley, et. al.[22] state that future research needs to explore more fully the notion of thematization of various schemata. Given that this is the type of mathematical understanding that most professors would state they strive to instill in their students, what it means to have thematized a schema needs to be clarified in terms of the mathematical topic in question and its genetic decomposition. These genetic decompositions would provide a framework by which one could analyze student understanding and which could also inform the focus of mathematics instruction. Thus, genetic decomposition analysis becomes important to be applied for teachers and researchers in describing students' ability to understand mathematics.

4. Conclusion
The conclusion of this study is that the ability of inter-level students to understand derivatives during ethnomathematics learning as follows. The student can apply the properties of derivatives, he coordinates at least two adjacent and overlapping intervals, but not for all intervals in the domain, and coordination forms objects about intervals. Students are able to encapsulate the process of function properties at an interval in domain \( h \) so that an object is formed about the sketch of the function graph \( h. \)

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