An interpolation type anisotropic yield function and its application in sheet metal forming simulation

R Q Xiao, F Peng and X H Dong*
Shanghai Jiao Tong University, 1954 Huashan Road, Shanghai, 200030, China

E-mail: dongxh@sjtu.edu.cn

Abstract. Affected by texture, Bauschinger effect and different deformation mechanisms, plastic deformation behavior of the sheet metals are complicated, especially for HCP metals, such as magnesium and titanium. With more and more enhanced demand to describe the materials’ yield behavior precisely and efficiently in numerical simulation, application of traditional continuous type yield functions encounters great challenge. So an interpolation type anisotropic yield function for plane stress is proposed. This yield function is represented by a yield surface in the polar coordinate system. The radius vector to the yield surface represents the yield stress at yielding, while the outer normal is related with the R values. The physical meaning of the parameters is directly defined. Accuracy, efficiency and flexibility can be achieved by application of such yield functions.

1. Introduction

The accuracy of yield function has essential effect in the simulation results of sheet metal forming process. During the last decades, a number of yield functions have been proposed to describe the anisotropic yield behavior of materials, most of which take the form of continuous functions. A review of these yield functions has been presented in [1]. Generally, in order to further improve the prediction accuracy and depict more strong anisotropy, the form of the yield function is bound to be more complex. In order to overcome the drawbacks of continuous yield function, Vegter and van den Boogaard [2] proposed a yield function based on the second order Bézier curve interpolation, which is adaptive in describing highly anisotropic materials without increasing the complexity of yield function. Then Peng et.al [3] proposed a new interpolation-type yield function based on the Hermite interpolation, which takes simple function form and the parameters possessing clear physical meaning, but the convexity of the yield locus may not be guaranteed. In this work, an adjusted Hermite interpolation-type yield function is proposed which consists of a base function of Hermite polynomial and an adjusting function to insure the convexity and fit additional experimental data. Yield locus calculated by the proposed yield function, some continuous yield function and experimental data are compared to verify the effectiveness of the proposed yield function.

2. The interpolation-type anisotropic yield function

2.1. The yield locus

For orthotropic sheet metals, the stress and plastic strain increment are usually expressed in the material coordinate system, with x-axis, y-axis and z-axis corresponding to the rolling, transverse and
normal directions respectively. In this paper, only biaxial tension and compression loading conditions are considered, thus the yield locus in $\sigma_x-\sigma_y$ plane is investigated. For convenience, the interpolation type yield function is expressed by polar coordinate, in which a planar stress condition is represented by a stress vector $r(\theta)$, where $r$ is the modulus, and $\theta$ is the angle between the stress vector and $\sigma_x$-axis. The direction of the plastic strain increment is described by $\beta$, the angle between the in-plane plastic strain increment and $x$-axis, which is related with the R value, i.e. the strain ratio.

As the associated flow rule being adopted, the tangent of yield locus will be normal to the direction of plastic strain increment, thus the first derivative of yield locus can be expressed by plastic strain increment direction $\beta$ as follows

$$
\frac{dr}{d\theta} = \frac{r(\sin \theta - \cos \theta \tan \beta)}{\sin \theta \tan \beta + \cos \theta}
$$

### 2.2. Adjusted Hermite interpolation-type yield function

To define a segment of the yield function between the polar angle $[\theta_a, \theta_b]$, the radius $r$ and first order derivative $(dr/d\theta)$ at its two nodes are calculated from experimentally obtained corresponding yield stress and the R value, and then substituted into the Hermite interpolation function $r(\theta)=r_H(\theta)$ [3]. However, the convexity of the Hermite interpolation-type yield function may not be guaranteed, and the yield locus between the nodes may deviate from additional experimental data. In order to solve these problems, an adjusting function $\xi(\theta)$ is added to the Hermite interpolation function (base function) as follows

$$
r(\theta) = r_H(\theta) + \xi(\theta)
$$

The adjusting function $\xi(\theta)$ is the linear combination of $\xi_i(\theta)$

$$
\xi(\theta) = \sum_{i=1}^{n} \lambda_i \xi_i(\theta)
$$

with

$$
\xi_i(\theta) = \frac{\bar{\theta} - \theta}{\theta_i - \theta_a}
$$

where $\bar{\theta} = \theta - \theta_a$, $\theta_a \leq \theta \leq \theta_i$, $i \geq 1$; $\lambda_i$ is the undetermined parameters.

After introducing the adjusting function $\xi(\theta)$, the $C_1$ continuity of the yield function at the interpolation nodes is still hold for the fact that

$$
\xi_i(\theta_a) = \xi_i(\theta_b) = \xi_i'(\theta_a) = \xi_i'(\theta_b) = 0
$$

It can be proved that $\xi(\theta)$ is convex. Thus, if convexity of the yield locus isn’t fulfilled by the base function, e.g. Hermite interpolation function, it can be insured by introducing the adjusting function in corresponding segment. Moreover, the number of adjusting function’s term can be selected, which provides great flexibility to fit the experimental data between the interpolation nodes.

### 2.3. Identification of the adjusting function parameters

For an adjusting function in the interval $\theta \in [\theta_a, \theta_b]$, parameters can be determined by solving the following optimization problem

$$
\min f(\lambda) = \sum_{i=1}^{n} (r(\theta_i) - r_i)^2
$$

s.t. \( g(\theta, \lambda) \geq 0, \theta \in [\theta_a, \theta_b] \)
The objective function $f(\lambda)$ is the error function of yield stress vector’s modulus; $r_i$ is the yield stress vector’s modulus obtained by experiment; $r(\theta)$ is the yield stress vector’s modulus predicted by the yield function; $n$ is the number of experimental yield points; $\lambda=(\lambda_1, \lambda_2, \ldots, \lambda_m)$ is the vector of the adjusting function parameters; $m$ is the number of the parameters. The constraint function $g(\lambda)$ is imposed to insure the convexity of the yield locus, which takes the following form

$$g(\theta, \lambda) = r^2(\theta, \lambda) + 2\left(\frac{dr}{d\theta}(\theta, \lambda)\right)^2 - r(\theta, \lambda)\left(\frac{d^2r}{d\theta^2}(\theta, \lambda)\right)$$  \hspace{1cm}(8)

The constraint can be treated as

$$g_{\text{min}}(\lambda) \geq 0, \theta \in [\theta_a, \theta_b]$$  \hspace{1cm}(9)

Then the minimum $g_{\text{min}}(\lambda)$ can be obtained by another optimization process

$$\min g(\theta, \lambda)$$  \hspace{1cm}(10)

s.t. $\theta \in [\theta_a, \theta_b]$  \hspace{1cm}(11)

Therefore, a double optimization process is adopted to solve the problem. The main optimization process is to minimize the error function of yield stress vector’s modulus, while the subordinate optimization process is to find the minimum $g_{\text{min}}(\lambda)$. The two optimization process are implemented simultaneously in MATLAB. The undetermined parameter $\lambda_i (i=1\sim m)$ are obtained from the double optimization process.

3. Application to an HCP magnesium alloy

The experiment results of an AZ31 magnesium alloy sheet [4], which show a strong tendency of tension/compression asymmetry, are adopted to compare the effectiveness of the proposed yield function with some widely-used continuous yield functions, including Hill48 [5], BBC2005 [6] and CPB2006 [7].

The proposed interpolation yield locus consists of five interpolation segments separated by 5 yield stress points, i.e., $P_1(\sigma_0,0)$, $P_2(\sigma_0,\sigma_0)$, $P_3(0,\sigma_0)$, $P_4(\sigma_0,0)$, $P_5(0,\sigma_0)$, where the superscript $t$ and $c$ denote the tension and compression tests respectively. Besides the $R$ values corresponding to $P_1 \sim P_5$ are used for calculating $(dr/d\theta) _i (i=1\sim5)$.

The yield loci predicted by Hermite interpolation (denoted as Hermite) and the adjusted Hermite interpolation (denoted as Hermite($\lambda$)) are compared in Fig.1(a). It can be seen that the yield locus obtained by Hermite interpolation base function alone shows concave near points $P_1(\sigma_0,0)$ and $P_2(0,\sigma_0)$. However, the convexity is fulfilled after introducing adjusting functions with one parameter $\lambda$ in the two segments of the first quadrant. The example shows that the convexity problem of Hermite interpolation may occur and the adjusting function can fix it. Moreover, the parameter $\lambda$ is obtained by the aforementioned optimization process, in which the additional yield points have been used. Thus, the accuracy of yield locus in the two segments has been improved as well. In other segments, it is unnecessary to introduce the adjusting function, because the yield locus by Hermite interpolation is convex and no additional experimental data is provided, so only the Hermite interpolation base functions are adopted.

The comparison of the yield loci of normalized stress on the $\sigma_r$-$\sigma_t$ plane by different yield functions and experimental data is shown in Fig.1(b). Comparing the yield loci in Fig.1(b), it can be seen that the accuracy of a yield function is determined by its function form and the number of its undetermined parameters. The Hill48 yield function can be determined by only 3 experimental data, i.e., $\sigma_0$, $\sigma_0$, $R_c$, thus the yield locus by Hill48 yield function is obviously deviated from the experimental data. In the identification of BBC2005 yield function, the yield stress and $R$ values of 3 points $P_1$-$P_3$ are used, so the yield locus matches the experimental data in the first quadrant well, but the tension/compression
asymmetry can’t be predicted, because of its tension/compression symmetric function form. To
determine the parameters of CPB2006 yield function, the yield stress and R values of 5 points P1~P5
are used, as the proposed yield function. Thus, in the first quadrant the yield locus is predicted by
CPB2006 yield function with a certain degree of accuracy; and the tension/compression asymmetry is
described accurately, since the compression experimental data are used in parameter determination.
For the proposed yield function, as mentioned above, the adjusting functions are adopted in the two
segments of first quadrant to keep convex and fit the additional experimental data. Though only one
adjusting parameter λ is adopted in each of the two effective segments, the accuracy is improved
obviously, and further improvement can be achieved by taking more parameters in the adjusting
function.

Fig 1. Comparison between experimental data and yield loci predicted by various yield
functions(ε̅p=0.004). Data after Andar M O, Kuwabara T and Steglich D [4].

4. Conclusions
In this paper, an adjusted Hermite interpolation-type yield function is proposed. The yield function is
constructed in the base-adjusting function framework, the base function, such as Hermite interpolation,
describes the shape of the yield locus, and the expandable adjusting function is introduced to insure
convexity and fit additional experimental data. Compared with continuous type yield function, more
parameters can be easily introduced into the proposed interpolation type yield function, thus more
accurate yield locus can be predicted, as shown in the comparison of different predicted yield loci with
the experimental data of an AZ31 magnesium alloy sheet.

Acknowledgements
The authors are grateful to the financial support of the National Natural Science Foundation of China
by Grant no. 51275297.

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