Antisymmetric neutrino Yukawa matrices and neutrino mixing

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Abstract

We study leptonic CKM mixing matrices when the neutrino Yukawa matrices are antisymmetric which gives rise to mass patterns suitable to explain solar, atmospheric and LSND neutrino oscillation experiments. Taking diagonal leptonic matrices which can give rise to hierarchical lepton masses, we compute the leptonic CKM matrix.
1 Introduction

The structure of the Yukawa coupling matrices in the generation space are left unconstrained by gauge symmetries which can experimentally be tracked by measuring the masses and the mixing angles of the fermions [1]. Although the masses (eigenvalues) of the quarks and the charged leptons are hierarchical in pattern, the solar [2] and atmospheric [3] neutrino oscillation results, if caused by $\nu_e \leftrightarrow \nu_\mu$ and $\nu_\mu \leftrightarrow \nu_\tau$ oscillations, hint towards an approximate degeneracy of squared neutrino masses in the three generation scenario whether in the case of four neutrino species including a sterile neutrino $\nu_s$, the approximate degeneracy is at least partial [4]; In this case the LSND result on $\nu_e \leftrightarrow \nu_\mu$ indicating $\Delta m^2_{e\mu} \sim 1 \text{eV}^2$ can also be accommodated. It is worth noting that whereas the quark mixing angle $V_{cb}$ is approximately 0.04 the above explanation of the atmospheric neutrino anomaly requires that the corresponding mixing angle in the leptonic sector is maximal, requiring $\sin^2 2\theta_{\mu\tau} \sim O(1)$. Mass matrices of the quark and leptonic sectors can indeed be very different. Because the Yukawa coupling matrices are independent of the gauge symmetries, and also even though the underlying GUT symmetries [5, 6] may relate various Yukawa matrices they do not describe the form of the Yukawa matrices [7, 8] themselves vis-a-vis the experimental informations on fermion masses, in particular the neutrinos.

2 Completely antisymmetric masses

Let us consider that an $n$ dimensional antisymmetric matrix $M$ has an eigenvalue $\lambda_0$. Then we have,

$$\text{Det}[\lambda_0 - M] = (-)^n \text{Det}[\lambda_0 - M] = 0$$

implying that $-\lambda_0$ is also an eigenvalue. Thus a $3 \times 3$ antisymmetric matrix must have a zero eigenvalue. However, as noted above if the apparent solar neutrino deficit has to be explained in terms of matter induced resonant Mikhayev-Smirnov-Wolfenstein (MSW) or vacuum two-flavor oscillations and if the atmospheric neutrino anomaly is due to $\nu_\mu \leftrightarrow \nu_\tau$ oscillations the three neutrino species have to be almost degenerate in mass. Thus, within an antisymmetric scenario all three neutrinos must have almost vanishing mass.
ruling out neutrino masses in the eV range. Thus it is difficult to consider neutrino as a hot dark matter candidate. Therefore we consider the following mass matrix of the four neutrino species including a sterile neutrino,

\[
M = \begin{pmatrix}
0 & m_{12} & m_{13} & m_{14} \\
-m_{12} & 0 & m_{23} & m_{24} \\
-m_{13} & -m_{23} & 0 & m_{34} \\
-m_{14} & -m_{24} & -m_{34} & 0
\end{pmatrix}.
\] (2)

The eigenvalues of \( M \) satisfies the equation,

\[
\lambda^4 + (m_{12}^2 + m_{13}^2 + m_{14}^2 + m_{23}^2 + m_{24}^2 + m_{34}^2) \lambda^2 + (m_{14}m_{23} - m_{13}m_{24} + m_{12}m_{34})^2 = 0.
\] (3)

Firstly, the equation is invariant under \( \lambda \leftrightarrow -\lambda \) in accordance with Eqn (1). However, in this case we have two non-zero solutions of \( \lambda^2 \). Hence, four solutions are grouped into two sets, \( \{\nu_e, \nu_\tau\} \) and \( \{\nu_\mu, \nu_s\} \). Each set having a pair of eigenvalues with equal magnitude and opposite sign as a result of the antisymmetry independent of the entries of the matrix! This guarantees a mass squared degeneracy among each set, hence solar neutrino problem can in principle be described by \( \nu_e \leftrightarrow \nu_\tau \) oscillations and atmospheric neutrino problem by the \( \nu_\mu \leftrightarrow \nu_s \) oscillations. Moreover \( \Delta m^2_{\nu\mu} \) is of the order of \( m_{ij}^2 \) where \( m_{ij} \) is a typical entry of the matrix \( M^D_\nu \). Choosing \( m_{ij} \) 1 eV we can get the mass squared difference required by the LSND result, and as a bonus \( \sum_{i=1}^{4} |\lambda_i| \sim O(1) \) eV, making neutrino favorable as a Dark Matter candidate.

3 See-saw mechanism

In the discussion above we did not make an attempt to explain the smallness of the entries in \( M^D_\nu \), and indeed the absence of Majorana mass terms forbid see-saw mechanism forbidding a natural way to explain the smallness of neutrino mass. Now we present a modified model to include Majorana terms and thereby see-saw mechanism. The skeleton key to the following discussion is that the eigenvalues of a matrix \( MM^\dagger \) are the squares of those of \( M \). The

\footnote{At this stage we could have also chosen the pairs as \( \{\nu_e, \nu_s\} \) and \( \{\nu_\tau, \nu_\mu\} \), because they are indistinguishable from the point of view of the mass matrix as long as antisymmetry is unbroken. At later stage this choice will be justified by the mixing angles favoured by experiments.}
see-saw mechanism suppresses the Dirac mass term and we get back a light left handed Majorana neutrino mass matrix,

\[ M_{\nu}^{ij} = \left( \frac{V_F^2}{2V_R} \right) \left( \frac{\tan^2 \beta}{1 + \tan^2 \beta} \right) \epsilon_{ik} [\chi]^{-1}_{kl} \epsilon_{lj} \]  \hspace{1cm} (4)

Where \( \tan \beta \equiv \frac{\langle H_u \rangle}{\langle H_d \rangle} \) and \( \chi \) is the right handed Majorana type Yukawa texture. We note that if \( \chi \) is an approximately diagonal matrix, the light neutrino mass eigenvalues keep the underlying pattern dictated by those of the Dirac mass textures \( \epsilon \).

### 4 Four neutrino textures

Here we postulate a \( 4 \times 4 \) Dirac mass textures as,

\[ M_D^{\nu} = \left( \frac{V_F^2}{\sqrt{2}} \right) \left( \frac{\tan^2 \beta}{1 + \tan^2 \beta} \right) \begin{pmatrix}
0 & 1 & 1 & 1 \\
-1 & 0 & 1 & 1 \\
-1 & -1 & 0 & 1 \\
-1 & -1 & -1 & 0
\end{pmatrix} \] \hspace{1cm} ←− DIRAC TEXTURE

(5)

The Majorana Yukawa texture is defined as,

\[ M_R = V_R \begin{pmatrix}
\nu_R^e & \nu_R^\tau & \nu_R^\mu & \nu_R^s \\
\nu_R^e & 0 & 0 & 0 \\
\nu_R^\tau & 0 & 0 & 0 \\
\nu_R^\mu & \nu_R^s & 0 & 1 + \eta
\end{pmatrix} \] \hspace{1cm} ←− MAJORANA TEXTURE

(6)

Where \( \eta V_R \) is the mass of the sterile neutrino. We expect it to be a little different from the other right handed masses. Using the expression for the Dirac and Majorana masses given in Eqn(5) and Eqn.(6) and inserting them to Eqn(4) we can describe the \( 4 \times 4 \) light neutrino Majorana mass matrix in terms of three parameters \( V_R, \tan \beta \) and \( \eta \). The dependence of the results on \( \tan \beta \) is milder than that on \( \eta \), and \( V_R \) is restricted from unification requirements. Our results are summarized in Table 1.

\(^2\)For renormalization group analysis of the scale \( V_R \) in the presence of right handed triplet scalars see Ref[3].
5 Solar, atmospheric and LSND neutrino oscillations

As we have noted above, if the apparent solar neutrino deficit has to be explained in terms of matter induced resonant Mikhayev-Smirnov-Wolfenstein (MSW) or vacuum two-flavor oscillations, the most recent 'best fit' mass squared differences for $\delta m^2_{e\mu}$ or $\Delta m^2_{e\tau}$ is in the range $10^{-5}$ and also, if the atmospheric neutrino anomaly is due to $\nu_\mu \leftrightarrow \nu_\tau$ or $\nu_s$ oscillations, we need $\delta m^2_{\mu\tau/s} \sim 10^{-2} - 10^{-4} eV^2$ with large mixing. Moreover LSND oscillation requires $\Delta m^2_{e\mu} > 0.3 eV^2$. Our textures can predict $\Delta m^2_{e\tau} = 10^{-5} eV^2$ and $\Delta m^2_{\mu s} = 10^{-2} eV^2$ and $\Delta m^2_{e\mu} = 1 eV^2$ for ranges of parameters as displayed in table 1. We note that the corresponding mixing angles are not solely the properties of the neutrino mass matrices, as neutrino mixing has to be folded in with the mixing of the charged leptons, whereas the required mass squared differences depend exclusively on the mass matrices. However, to convince ourselves that there exists compatible mixing angles we assume that the leptonic mass matrix is diagonal which makes the leptonic mixing matrix identical to the neutrino mixing matrix. Thereafter, we parametrize the $4 \times 4$ light neutrino mixing matrix according to the convention of Barger et al\cite{17}, which gives $\sin^2 2\Theta_{e\tau} \approx 0.75$, $\sin^2 2\Theta_{e\mu} \approx 0.88$ and $\sin^2 2\Theta_{\mu s} \approx 0.93$: large angles for all the transitions under consideration. These mixing angles can be altered by choices of the Yukawa coupling matrix of the charged leptons which is beyond the scope of the present discussion.

6 Constraints from double beta decay

Neutrinoless double beta decay is unobserved in nature. The Heidelberg-Moscow Experiments quote\cite{11} the lower limits on the half life as $\Gamma_{1/2} \geq 1.1 \times 10^{25}$ y; which already restricts the $< \nu_e \nu_e >$ Majorana mass term to be less than 0.60 eV at 90% confidence level\cite{5}. In future the limits may go

\cite{3} A $4 \times 4$ unitary matrix can be parametrized by six angles and two phases. In our case the rotation matrix is real. Out of the three remaining angles two are large and one vanish.

\cite{4} Zenith angle distribution of atmospheric sub GeV data favours large angle solution of the solar neutrino anomaly\cite{10}, at least in the three flavour case.

\cite{5} The limit depends on nuclear matrix elements. See Table (4) of Ref.\cite{11}.
down to 0.1 eV with the present experimental setup. Thus in combination with LSND results, which require $\Delta m^2_{\text{e\mu}} \approx 0.3$ eV$^2$ or higher (a lower limit on mass$^2$ implies an upper limit on $V_{eR}$), neutrino-less double beta decay constraints (vice-versa) may establish or rule out these textures.

7 Hot Dark Matter

$\Omega = 1$ with $h \approx 0.5$ Cold Dark Matter cosmological models provide a good fit to the observational data in the presence of massive neutrinos, when $m_\nu \approx 5$eV is equally shared between two relatively heavy neutrinos, contributing a tiny Hot component (CHDM) to the dark matter[14]. Our textures do not achieve this as that will give too much Majorana mass to the electron neutrino. At best these textures can give a pair of neutrinos approximately at 1 eV. On the contrary, neutrinos in this mass range might modify the power spectrum to agree better with the data on galaxy distribution in the $\Omega \approx 0.4$, $\Omega_\Lambda \approx 0.6$ cosmology indicated by the high-red-shift supernova data[15], which may resolve the problems in r-process nucleosynthesis[16] in Type II supernovae.

8 Neucleosynthesis bounds

There exist bounds on the product $\Delta m^2_{i8} \sin^4 2 \Theta_{i8}$ from Big bang nucleosynthesis. These bounds are derived by demanding that oscillations do not bring the sterile neutrino in equilibrium with the known neutrino species. However, these bounds depends on the value of the primordial lepton asymmetry. If the primordial asymmetry is of the order of $10^{-9}$ and the mixing is maximal, the bound upper bound from nucleosynthesis is nearly $\delta m^2 \approx 10^{-8}$[18]. However, if the initial lepton asymmetry is large enough $L_e \approx 10^{-5}$ the bound on the mixing between an ordinary and sterile neutrino is weakened and large-angle-mixing solution of the atmospheric neutrino anomaly becomes feasible[19].
9 Theoretical scenario

It is well known that a straight-forward generalization of three generation scenario of the Dirac sector to four generations is problematic from the point of view of the invisible $Z$ width which implies that only three light neutrino species couple to the $Z$ bosons. This experimental restriction can be circumvented by adding a pair of singlet sterile neutrinos $\nu^s_L$ and $\nu^s_R$ to the three generation of quarks and leptons. Let us write down the neutrino Dirac type mass terms in a matrix form,

$$\mathcal{L}^D = \begin{pmatrix}
\nu^e_R & \nu^e_R & \nu^e_R & \nu^s_R \\
0 & H_2 & H_2 & H_2 \\
-H_2 & 0 & H_2 & H_2 \\
-H_2 & -H_2 & 0 & H_2 \\
-H_s & -H_s & -H_s & 0 
\end{pmatrix}$$

Where the Yukawa couplings could be either zero or one, and the mass spectrum is generated due to the combination of variation in VEVs and the structure of the matrices. The inclusion of the singlet Higgs field, which enters in the fourth row, is not ad-hoc. Actually we need it also to stabilize the supersymmetric $\mu H_1 H_2$ term $[20]$ from the VEV of the singlet from the trilinear interaction $H_s H_2 H_1$. However, the zero in the $(4,4)$ element is needs to be justified by some discrete symmetry. In the case $<H_s> = <H_2>$ we get back two sets of degenerate eigenvalues of the Dirac mass matrix. The Majorana mass term of the right handed sterile neutrino breaks the $B-L$ symmetry by two units. We expect that its mass is tied to the $B-L$ symmetry breaking scale $M_R$.

10 Conclusions

In conclusion, we have postulated a set of textures for the neutrino Dirac and Majorana mass matrices including a sterile neutrino $\nu_s$ with one parameter. The Dirac texture is antisymmetric in the generation space whereas the Majorana texture is diagonal at the leading order displayed in Eqns (5) and (6). We have calculated the light neutrino masses upon see-saw mechanism. Solar and atmospheric neutrino anomalies can be described by $\nu_e \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_s$ oscillations, whereas the LSND results can be described by the
\[\nu_e \leftrightarrow \nu_\mu\] oscillations. A pair of neutrinos emerge at 0.5-1 eV range whereas another pair remain at around \(10^{-2}\) eV. It is possible that the heavier pair is in suitable mass range for dark matter for the \(\Omega \approx 0.4, \Omega_\Lambda \approx 0.6\)\(^{[4]}\). The structure of the textures provides large mixing angles for \(\{e\tau\}, \{e\mu\}\) and \(\{\mu s\}\) sectors. The right handed symmetry breaking scale \(V_R\) is bounded from below from the upper bound on the left handed Majorana mass from neutrinoless double beta decay, whereas it is also bounded from above from the requirement of mass difference squared (\(> 0.3\) eV\(^2\)) by the LSND results. Hence the scenario can be experimentally tested or ruled out in near future.

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Table 1: For ranges of the parameters in the first three columns, the predictions are given in the last five columns. A crucial prediction in the entry $(\nu_e^c)^c \nu^c_L$ of the mass matrix, which is accessible by present day experiments apart from neutrino oscillations.

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| $V_R$  | $\tan \beta$ | $\log_{10}[\eta]$ | $\log_{10}[\Delta m^2_{ee}]$ | $\log_{10}[\Delta m^2_{\mu\mu}]$ | $\Delta m^2_{\nu\mu}$ | $(\nu_e^c)^c \nu_L^c$ | $m_{\nu_e}$ (eV) | $m_{\nu_\mu}$ (eV) |
|-------|--------------|-------------------|--------------------------|--------------------------|----------------|----------------|----------------|----------------|
| $10^{14.2}$ | 2            | -2.0             | -5.08                    | -2.03                    | 0.93           | 0.49           | 0.03           | 0.97           |
| $10^{14.2}$ | 2            | -2.5             | -5.59                    | -2.53                    | 0.93           | 0.49           | 0.03           | 0.96           |
| $10^{14.2}$ | 3            | -2.5             | -5.49                    | -2.42                    | 1.17           | 0.56           | 0.03           | 1.08           |
| $10^{14.5}$ | 2            | -1.5             | -5.17                    | -2.12                    | 0.24           | 0.25           | 0.01           | 0.50           |
| $10^{14.5}$ | 2            | -2.0             | -5.68                    | -2.62                    | 0.23           | 0.25           | 0.01           | 0.48           |
| $10^{14.5}$ | 3            | -2.0             | -5.58                    | -2.52                    | 0.29           | 0.28           | 0.02           | 0.55           |