Automatization of calculation of dynamic characteristics of the car suspension system

M Lahtyukhov\textsuperscript{1,2} and L Zheglov\textsuperscript{1}

\textsuperscript{1}Bauman Moscow State Technical University

\textsuperscript{2}E-mail: motor@bmstu.ru

Abstract. The article deals with the questions of finding the parameters of generalized coordinates for plane linear dynamic systems of any structure with degrees of freedom, used in the study of suspension systems. The appropriateness of separation of a number of subsystems for each of which potential energy and dissipative function are calculated by the same type equations is shown. As an example, the differential equations describing in the general case the linear movements of the masses of the dynamic system are given. It is shown that in order to find the parameters of generalized coordinates of a flat linear dynamic system with degrees of freedom at the kinematic harmonic disturbance determined by the structure of the road surface, a solution of the system of linear algebraic equations is required.

Introduction

The solution of problems of vibration safety and loading of vehicle components (systems) is carried out on condition of choosing a rational scheme of dynamic system of the analyzed object. The design of the main suspension system can be carried out using multi-mass dynamic systems [1-6]. In case of research of automatic suspension systems, the structure of the dynamic system is determined taking into account the control algorithm being applied [7-12]. The linear [13-16] and nonlinear [17-20] definition of the problem for plane and three-dimensional schemes of dynamic systems can be used. Execution of the algorithm of structural synthesis of a dynamic system should be implemented using the modular approach of its construction [21-25].

Plane linear dynamic systems consisting of a finite number of masses connected to each other and in contact with the road surface through inertial elastic and dissipative bonds are considered. Two types of masses with one or two degrees of freedom, respectively, are usually used to construct a dynamic system scheme. Afterwards, for the convenience of presentation, depending on the number of degrees of freedom, the corresponding masses will be called masses of the first or second type. The masses of both types can be moved vertically, and the masses of the second type can also be rotated around the transverse axis of the vehicle passing through the center of mass. The arbitrary $i$-th mass of the first type is only the mass $m_i$, and $i$-th mass of the second type is the mass $m_i$ and the moment of inertia $J_i$.

If the dynamic system has $n_1$ masses of the first type and $n_2$ masses of the second type, the total number of degrees of freedom is $n = n_1 + 2n_2$. 

\textsuperscript{Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Published under licence by IOP Publishing Ltd.
Calculation of amplitude-frequency response of plane linear dynamic systems

Differential equations of the holonomic dynamic system with lumped constants and stationary constraints can be obtained using the Lagrange second order differential equation

\[ \frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial \dot{q}_i} - \frac{\partial U}{\partial q_i} = Q_i, \quad i = 1, n, \]  

(1)

where \( T, U \) – the kinetic and potential energies of the system, respectively; \( V \) – dissipative function of the system; \( Q_i \) – \( i \)-th generalized disturbing force; \( q_i, \dot{q}_i \) – \( i \)-th generalized coordinate and speed; \( t \) – time.

For the generalized coordinates for all masses of the system, we will select the vertical displacements, calculated upwards from the position of static equilibrium of the centers of masses, and for masses of the second type, in addition, the angular displacements, calculated from the position of equilibrium of these masses counterclockwise.

At small oscillations near the static equilibrium position, the kinetic and potential energies of the system, as well as its dissipative function, can be expressed by quadratic forms from generalized coordinates:

\[ T = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} q_i q_j, \quad U = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} q_i q_j, \quad V = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} q_i q_j, \]  

(2)

where \( a_{ij} \) – inertial coefficients, \( c_{ij}, b_{ij} \) – stiffness and dissipation coefficients, respectively.

In general, the numbering of masses can be done in random order. We will assign a number from 1 to \( n_1 \) to the masses of the first type for simplicity. To the masses of the second type from \( 1 \) to \( n_2 \). Then the first \( n_1 + n_2 \) vector \( q \) components are vertical displacements of the first and second type masses:

\[ q_i = x_i, \quad i = 1, n_1 + n_2, \]

and the last ones \( n_2 \) vector \( q \) components is the angular displacement of type two masses

\[ q_{n_2+i} = \alpha_i, \quad i = n_1 + 1, n_1 + n_2. \]

The kinetic energy of the system is determined by the equation:

\[ T = \frac{1}{2} \left[ \sum_{i=1}^{n_1+n_2} m_i \dot{q}_i^2 + \sum_{j=n_1+n_2+1}^{n} J_{j-n_2} \dot{q}_j^2 \right]. \]  

(3)

In order to simplify the following calculations in the initial system depending on the type of mass to be connected, it is useful to identify a number of subsystems, for each of which both the potential energy and the dissipative function are calculated by the same type of equations. There are usually five subsystems:

- two masses of the first type with elastic linkage between them (Fig. 1, a);
- mass of the first type and mass of the second type with elastic linkage between them (Fig. 1, b);
- mass of the first type, supported on the road surface by means of elastic linkage (Fig. 1, c);
- two masses of the second type with several elastic linkages between them (Fig. 1, d);
- mass of the second type, supported on the road surface by several elastic linkages (Fig. 1, e), if necessary, other subsystems can be added.

Then both potential energy and dissipative function of the dynamic system can be represented in the sum of potential energies and dissipative functions of separate subsystems (Table 1)
Fig. 1. Typical dynamic subsystems

![Diagram of dynamic subsystems](image)

\[ U = \sum_{i=1}^{nl} U_i^{(1-1)} + \sum_{m=1}^{nm} U_m^{(1-2)} + \sum_{p=1}^{np} U_p^{(1-0)} + \sum_{s=1}^{ns} U_s^{(2-2)} + \sum_{t=1}^{nt} U_t^{(2-0)} \quad (4) \]

\[ V = \sum_{i=1}^{nl} V_i^{(1-1)} + \sum_{m=1}^{nm} V_m^{(1-2)} + \sum_{p=1}^{np} V_p^{(1-0)} + \sum_{s=1}^{ns} V_s^{(2-2)} + \sum_{t=1}^{nt} V_t^{(2-0)} \quad (5) \]

where \( nl \) – the number of mass pairs of the first type connected to each other; \( nm \) – the number of pairs of masses of the first and second type connected to each other; \( np \) and \( nt \) – the number of masses of the first and second type supported by the road surface, respectively; \( ns \) – the number of pairs of masses of the second type connected to each other.

As the dynamic system is exposed only to kinematic disturbance from the support surface when calculating the amplitude-frequency response, all generalized disturbing forces in (1) are equal to zero:

\[ Q_i = 0, \quad i = 1, n. \quad (6) \]

Using equations (1), (3), (4), (5) and (6), it is not difficult to obtain an equation of motion for any of the masses of the dynamic system. For example, if we assume that the random mass of the first type has elastic linkages with all masses and with the road surface, then the differential equation of its motion after the rearrangement of the terms will take the form:
The matrix equation describing the motion of the linear dynamic model of the suspension system with degrees of freedom looks like:

$$\begin{align*}
\begin{bmatrix} A & B \\ S & 0 \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} f \end{bmatrix} &= 0,
\end{align*}$$

where $A$ - matrix of inertial coefficients (dimension $n \times n$), $S,B$ - Matrix of stiffness and dissipation (damping) coefficients $(n \times n)$, $q$ - $n$- dimensional vector of generalized coordinates, $f$ - $n$- dimensional vector of external forces, caused by the microprofile of the road surface.

It is easy to make sure that the right side $i$-th of the differential equation $f_i$ can be represented by

$$f_i = f_{1,i} \cos \omega t + f_{2,i} \sin \omega t.$$
### Table 1: Typical dynamic subsystems, their potential energy and dissipative function

| Subsystem type | Subsystem number | Number and type of connected mass | Potential energy | Dissipative function |
|----------------|------------------|-----------------------------------|------------------|---------------------|
| $U_{p}^{(1-1)}$ | $p$              | $i$-th mass of the first type and $j$-th mass of the first type | $\frac{1}{2} c_{p} \left( x_{i} - x_{j} \right)^2$ | $\frac{1}{2} k_{p} \left( \dot{x}_{i} - \dot{x}_{j} \right)^2$ |
| $U_{s}^{(1-2)}$ | $s$              | $i$-th mass of the first type and $k$-th mass of the second type | $\frac{1}{2} c_{s} \left[ x_{i} - \left( x_{k} + \left( L_{i} - L_{k} \right) \alpha_{k} \right) \right]^2$ | $\frac{1}{2} k_{s} \left[ \dot{x}_{i} - \left( \dot{x}_{k} + \left( L_{i} - L_{k} \right) \alpha_{k} \right) \right]^2$ |
| $U_{t}^{(1-0)}$ | $t$              | $i$-th mass of the first type and road surface | $\frac{1}{2} c_{t} \left( x_{i} - g_{i} \right)^2$ | $\frac{1}{2} k_{t} \left( \dot{x}_{i} - \dot{g}_{i} \right)^2$ |
| $U_{u}^{(2-2)}$ | $u$              | $i$-th mass of the second type and $m$-th mass of the second type | $\frac{1}{2} \sum_{j=1}^{nf} c_{ua} \left[ \left( x_{i} + i_{r_{-im}} \alpha_{j} \right) - \left( x_{m} + i_{r_{-im}} \alpha_{m} \right) \right]^2$ | $\frac{1}{2} \sum_{j=1}^{nf} k_{ua} \left[ \left( \dot{x}_{i} + i_{r_{-im}} \alpha_{j} \right) - \left( \dot{x}_{m} + i_{r_{-im}} \alpha_{m} \right) \right]^2$ |
| $U_{v}^{(2-0)}$ | $v$              | $i$-th mass of the second type and road surface | $\frac{1}{2} \sum_{j=1}^{nf} c_{va} \left[ \left( x_{i} + i_{f_{-ia}} \alpha_{j} \right) - g_{f_{-ia}} \right]^2$ | $\frac{1}{2} \sum_{j=1}^{nf} k_{va} \left[ \left( \dot{x}_{i} + i_{f_{-ia}} \alpha_{j} \right) - g_{f_{-ia}} \right]^2$ |

Note: Numeric and alphabetic indexes in upper brackets for $U$ and $V$, separated by a hyphen, represent the type of mass to be connected or the road surface; $c_{ij}$ and $k_{ij}$ - stiffness and damping coefficients of the elastic linkage between the $i$-th and $j$-th masses; $c_{s}$ and $k_{s} -$ stiffness and damping coefficients of the elastic linkage of the $i$-th mass of the first type with the road surface; $c_{u}$ and $k_{u} -$ stiffness and damping coefficients $r$-th elastic linkage between $i$-th and $m$-th masses of the second type; $nr_{u} -$ the number of elastic linkages between $i$-th and $m$-th masses of the second type in $u$-th subsystem; $c_{f_{-ia}}$ and $k_{f_{-ia}} -$ stiffness and damping coefficients $f$-th elastic linkage between $i$-th mass of the second type with the road surface; $nf_{r} - nr_{u} -$ the number of elastic linkages between $i-1$-th mass of the second type with the road surface in $v$-th subsystem; $L_{i} -$ distance from the first axle of the vehicle to the centre of mass (CM); $i$-th mass (distance is taken with sign "-", if the CM is located to the left of the front axle of the vehicle, and with a sign "+", if the CM is located to the right of the front axle of the vehicle); $L_{r_{-im}}$ and $L_{r_{-im}} -$ the distance between $r$-th elastic linkage and CM $i$-th and $m$-th mass of the second type respectively (distance is taken with sign "-", if the $r$-th elastic linkage is located to the left of the CM, and with a sign "+", if the $r$-th elastic linkage is located to the right of the CM), and $L_{r_{-im}} = L_{i} + i_{r_{-im}} - L_{m}$; $i_{f_{-ia}} -$ distance between $f$-th elastic linkage and CM $i$-th mass of the second type (distance is taken with sign "-", if the $f$-th elastic linkage is located to the left of the CM $i$-th mass, and with a sign "+", if the $f$-th elastic linkage is located to the right of the CM $i$-th mass); $g_{i} -$ road surface shape under $i$-th mass of the first type; $g_{f_{-ia}} -$ road surface shape under $f$-th elastic linkage, connecting the $i$-th mass of the second type with the road surface.
Due to the energy dissipation, free vibrations with frequencies different from the disturbance frequency \( \omega \) are out over time and stationary vibrations with the frequency \( \omega \), independent of the initial conditions, are installed. The solution of equation (9) corresponding to such stationary oscillations is presented in the form

\[
q_i = q_{Mi} \sin(\omega t + \alpha_{Mi}) = q_{1i} \cos \omega t + q_{2i} \sin \omega t,
\]

(11)

where \( q_{Mi} \) and \( \alpha_{Mi} \) – amplitude and phase angle \( i \)-th generalized coordinate, \( q_{1i} = q_{Mi} \sin \alpha_{Mi} \), \( q_{2i} = q_{Mi} \cos \alpha_{Mi} \).

By substitution (10) and (11) in (9) and separating the variables containing sinuses and cosines, the initial system from \( n \) differential equations is transformed to a system of linear algebraic equations:

\[
DQ = h,
\]

(12)

where \( D = \begin{bmatrix} S - \omega^2 A & \omega B \\ -\omega B & S - \omega^2 A \end{bmatrix} \) – matrix of \( 2n \times 2n \) dimension; \( Q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \) – vector \( 2n \) length;

\( h = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \) – vector \( 2n \) length; \( q_1, q_2 \) – vectors \( n \) length containing \( q_{11}, q_{21} \), respectively; \( i = 1, n \),

\( f_1, f_2 \) – vectors \( n \) length containing \( f_{11}, f_{21} \), respectively.

Obtained by solving the system of linear equations (12) elements of vector \( Q \) can be used to find the amplitudes and phases of all generalized coordinates. Thus, to find the amplitudes and phases of the generalized coordinates of the dynamic system with \( n \) degrees of freedom at the kinematic harmonic perturbation the solution of the system of \( 2n \) linear algebraic equations is required.

At a given speed of \( v_a \) vehicle movement, the formation of \( A, B, S \) matrices and \( h \) vector (12) can be automated. The system of linear algebraic equations (12) is solved using numerically stable matrix methods of linear algebra.

**Conclusions**

1. The method of automated calculation of amplitude-frequency response of a plane linear dynamic system with degrees of freedom and random connections is developed.
2. The expediency of separation of a number of subsystems is presented, for each of which both the potential energy and the dissipative function are calculated by the same type of equations.
3. Examples of differential equations are given that generally describe linear mass displacements.
4. It is shown that the solution of the system of linear algebraic equations is required to find the amplitudes and phases of the generalized coordinates of the plane linear dynamic model with degrees of freedom at the kinematic harmonic perturbation from the road surface.

**References**

[1] Belousov B.N. and Popov S.D., Heavy-duty wheeled vehicles: Design, Theory, Calculations SAE International ISBN 978-0-7680-7723-0, 2014, 800 p.

[2] Chumakov, D.A., Chernyshov, K.V., Novikov, V.V., Diakov, A.S., Suchenina, A.S. Mathematical model of motor vehicle air suspension with a combined damping system – Journal of Physics: Conference Series, Volume 1177, Issue 1, 10 April 2019, article No012049, 3rd Science and Technology Seminar on Mobility of Transport and Technological Machines, MTTM 2018; Nizhny Novgorod; Russian Federation; 28 August 2018; 147340

[3] Vdovin D., Chichekin I. Loads and Stress Analysis Cycle Automation in the Automotive Suspension Development Process– Procedia Engineering, Volume 150, 2016, Pages 1276-1279, 2nd International Conference on Industrial Engineering, ICIE 2016; Chelyabinsk; Russian Federation; 19 May 2016 to 20 May 2016; 123270
[4] Gorelov, V.A., Komissarov, A.I., Miroshnichenko, A.V. 8×8 wheeled vehicle modeling in a multibody dynamics simulation software (2015) Procedia Engineering, 129, pp. 300-307.

[5] Keller, A.V., Gorelov, V.A., Vdovin, D.S., Taranenko, P.A., Anchukov, V.V. Mathematical model of all-terrain truck (2015) Proceedings of the ECCOMAS Thematic Conference on Multibody Dynamics 2015, Multibody Dynamics 2015, pp. 1285-1296.

[6] Evseev, K.B., Kartashov, A.B., Dashtiev, I.Z., Pozdeev, A.V. Analysis viscoelastic properties of fiber-reinforced composite spring for the all-terrain vehicle, MATEC Web of Conferences 224, 02039 (2018).

[7] Velupillai S., Guvenc L., Oncu S., Ozcan D. Vehicle chassis control using adaptive semi-active suspension– IFAC Proceedings Volumes (IFAC-PapersOnline), Volume 17, Issue 1, PART 1, 2008, 17th World Congress, International Federation of Automatic Control, IFAC; Seoul; South Korea; 6 July 2008 to 11 July 2008; 79403

[8] FüsunAlışverişçi G., Bayiroğlu H., Balthazar J.M., Felix J.L.P., da Fonseca Brasil R.M.L.R. On dynamic behavior of a nonideal torsional machine suspension structure(Conference Paper) – Springer Proceedings in Mathematics and Statistics, Volume 181, 2016, Pages 1-10, 13th International Conference on Dynamical Systems, 2015; Łódź; Poland; 7 December 2015 to 10 December 2015; 181529

[9] Song X., Cao D. Design of semiactive vehicle suspension controls in frequency domain– Proceedings of the ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference 2009, DETC2009, Volume 6, 2010, Pages 1047-1052, 2009 ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, DETC2009; San Diego, CA; United States; 30 August 2009 to 2 September 2009; 80638

[10] Connolly T.J., Longoria R.G. An approach for actuation specification and synthesis of dynamic systems– Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME, Volume 131, Issue 3, May 2009, Pages 1-15

[11] Margolis, D.L. Bond graphs for automated simulation and control of nonlinear vehicle systems– SAE Technical Pa pers1987Future Transportation Technology Conference and Exposition; Seattle, WA; United States; 10 August 1987 to 13 August 1987; 90059

[12] Zhileynik, M.M., Kotiev, G.O., Nagatsev, M.V. Synthesis of the adaptive continuous system for the multi-axle wheeled vehicle body oscillation damping(2018) IOP Conference Series: Materials Science and Engineering, 315 (1), article № 012031.

[13] Chernikov, S.A. Expansion of the suppression band of a vibroprotective system by a feedback dynamic damper (2015) Journal of Machinery Manufacture and Reliability44(5), c. 439-444

[14] Jabeen, S.D. Vehicle vibration and passengers comfort– Advances in Intelligent Systems and Computing, Volume 509, 2017, Pages 357-372, 1st International Conference on Computational Intelligence, ICCI 2015; Ranchi; India; 10 December 2015 to 11 December 2015; 186569

[15] Chen K.-C., Shih H.-Y. Research and Simulation of the Electrical Vehicle Based Dynamical System – Ma thematical Problems in Engineering, Volume 2015, 2015, article № 929701

[16] Tlibekov, A.K., Yakhotlov, M.M., Bathyrov, U.D., Dosko, S.I. (2017) Identification of mechanical systems in the frequency area. Proceedings of the 2017 International Conference "Quality Management, Transport and Information Security, Information Technologies", IT and QM and IS, 2017 8085812, c. 284-286

[17] Le V.Q., Zhang J., Liu X., Wang Y. Nonlinear dynamic analysis of interaction between vehicle and road surfaces for 5-axle heavy truck– Journal of Southeast University (English Edition), Volume 27, Issue 4, December 2011, Pages 405-409

[18] Liang Y.-J., Li N., Gao D.-X., Wang Z.-S. Optimal vibration control for nonlinear systems of tracked vehicle half-car suspensions– International Journal of Control, Automation and Systems, Volume 15, Issue 4, 1 August 2017, Pages 1675-1683
[19] Milanese M., Novara C., Pivano L. Structured SM identification of vehicle vertical dynamics–
Mathematical and Computer Modelling of Dynamical Systems, Volume 11, Issue 2, June 2005, Pages 195-207

[20] Zhileynkin, M.M., Kotiev, G.O., Nagatsev, M.V.Comparative analysis of the operation efficiency of the continuous and relay control systems of a multi-axle wheeled vehicle suspension(2018) IOP Conference Series: Materials Science and Engineering, 315 (1), article № 012030.

[21] Belousov B., Ksenevich T.I., NaumovS. Automated system to control steering and wheel springing parameters in vehicle locomotion module– SAE Technical Papers, 14 January 2015. 14th Symposium on International Automotive Technology, SIAT 2015; ARAI CampusPune; India; 21 January 2015 to 23 January 2015; 110846

[22] Metallidis P., Stavrakis I., Natsiavas S. Parametric identification and health monitoring of large scale strongly nonlinear vehicle models– 2007 Proceedings of the ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, DETC2007, Volume 1, PART B, 2008, Pages 1397-1406, 21st Biennial Conference on Mechanical Vibration and Noise, presented at - 2007 ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, IDETC/CIE2007; Las Vegas, NV; United States; 4 September 2007 to 7 September 2007; 72165

[23] Novikov, V.V., Pozdeev, A.V., Diakov, A.S. Research and testing complex for analysis of vehicle suspension units(2015) Procedia Engineering, 129, pp. 465-470.

[24] Ovchinnikov, I.N. Its own spectrum band - A property of mechanical systems (2018) IOP Conference Series: Materials Science and Engineering 468(1), 012045

[25] Tlibekov, A.K., Yakhutlov, M.M., Batyrov, U.D., Dosko, S.I. Identification of mechanical systems in the frequency area (2017) Proceedings of the 2017 International Conference "Quality Management, Transport and Information Security, Information Technologies", IT and QM and IS 20178085812, c. 284-286