Research Article

Frequency Diverse Array Target Localization Based on IPSO-BP

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Abstract

For the traditional target localization algorithms of frequency diverse array (FDA), there are some problems such as angle and distance coupling in single-frequency receiving FDA mode, large amount of calculation, and weak adaptability. This paper introduces a good learning and predictive method of target localization by using BP neural network on FDA, and FDA-IPSO-BP neural network algorithm is formed. The improved particle swarm optimization (IPSO) algorithm with nonlinear weights is developed to optimize the neural network weights and biases to prevent BP neural network from easily falling into local minimum points. In addition, the decoupling of angle and distance with single frequency increment is well solved. The simulation experiments show that the proposed algorithm has better target localization effect and convergence speed, compared with FDA-BP and FDA-MUSIC algorithms.

1. Introduction

Since the frequency diverse array (FDA) was first proposed in 2006 [1], many scholars from all over the world have conducted research on it. Different from the traditional phased array, FDA introduces a small frequency increment on the array elements [2, 3], which induces the beam pattern curved in space and extends the spatial degrees of freedom. In [4], FDA was applied to the detection of ground moving targets by forward-looking radars. Compared with the traditional phased array radar, it can suppress the distance fuzzy clutter to some extent. In [5], FDA was first applied to synthetic aperture radar for high resolution imaging, and it was verified by simulation results that FDA can improve the imaging resolution of azimuth direction and range direction of synthetic aperture radar. In [6], a target localization method was proposed to suppress the interference information in the echo with the nonuniform frequency increment determined by the genetic optimization algorithm, and the position of the targets was estimated by MUSIC algorithm. FDA-MIMO radar technology was studied in [7–9], and it was concluded that the FDA-MIMO radar has a good improvement on side lobe suppression and target detection performance. In [10], the multidimensional information (distance, angle, and amplitude) estimation method of FDA-MIMO radar was proposed based on sparse iteration. By optimizing the objective function of the weighted norm, the distance, angle, and amplitude information of the target were obtained by sparse iterative optimization. In [11], a target localization algorithm with the compressed sensing sparse representation was proposed. The target localization problem was described as a cost function under the sparse representation framework, which was optimized by the convex optimization tool. The non-zero element index in the obtained sparse weight vector mapped the orientation and distance information of the target. In [12], FDA and waveform multiplexing were applied to bistatic radar systems. In [13], a bistatic FDA-MIMO radar was introduced, and nonlinear frequency increment was used to overcome the coupling problem of DOD and distance parameters. The rotation invariance technique and parallel factor algorithm were proposed correspondingly, which can reduce the...
complexity of computational and three-dimensional spectral peak searching.

All the above methods are based on the pure mathematical models to perform a large number of operations to obtain the final result, which causes the adaptability to the environment to be poor; meanwhile, the real-time application is unachievable. In this paper, by introducing the neural network method, the problem of the angle-distance coupling is solved well in the single-frequency receiving FDA mode with single frequency increment, and the trained model has certain environmental adaptability with lower calculation amount [14, 15].

The neural network algorithm has good learning and predictability through the training model, which was applied to target location. In [16], a single linear array combined with appropriately trained linear vector quantization (LVQ) artificial neural networks is used to achieve two-dimensional direction of arrival (2D-DOA) estimation with elevation and azimuth angle estimates. In [17], a fast estimation method based on LVQ neural network for two-dimensional DOA is proposed. By using the symmetry of the UCA array, two different elevation and azimuth data sets were provided, and a LVQ neural network was sequentially trained on each data set to estimate the elevation and the azimuth angles, respectively.

The above documents [16, 17] derived the azimuth and elevation angles of the target from the characteristics of the traditional phased array on the angle information. The distance of the target was obtained by introducing the orientation values into the signal model, which leads to a further increase of computation.

This paper develops a target localization algorithm by combining IPSO (improved particle swarm optimization), BP (backpropagation) neural network, and FDA, named as FDA-IPSO-BP, by which the coupling problem of angle and distance in the FDA can be avoided; in addition, the amount of computation is reduced and the adaptability to the environment is enhanced. Because BP neural network is optimized based on gradient descent, it has the disadvantages of slow convergence and is easy to fall into local minima and sensitive to the initial weight and bias of the network. In this paper, the PSO (particle swarm optimization) algorithm is improved to optimize the BP neural network, which effectively improves the convergence speed of the algorithm and enhances the capabilities of the global and local search. The upper triangle part of the covariance matrix of the received data is selected as the input of the network, and IPSO-BP neural network model is applied to estimate the position of the targets.

The rest of the paper is organized as follows. In Section 2, the basic FDA model is described and the basic criterion is derived for the FDA-BP through mathematical analysis. In Section 3, the FDA-BP model is optimized by using IPSO. In Section 4, the simulation analysis in MATLAB data and FEKO data is performed.

2. Signal Model

2.1. Model of FDA. The FDA model is shown in Figure 1. Unlike the conventional phased arrays, FDA has a small frequency increment between two adjacent array elements [18]. The transmission frequency $f_n$ of the $n$-th array element is shown as follows:

$$f_n = f_0 + (n - 1)\Delta f, \quad n = 0, 1, \ldots, N - 1,$$

where $\Delta f$ is the frequency increment with reference to the carrier frequency $f_0$ and $N$ is the number of array elements. The transmitted signal of the $n$-th element is represented as

$$s_n(t) = a(t)e^{j2\pi f_n t},$$

where $a(t)$ is the complex envelope function of the transmitter signal and $f_n$ is the carrier frequency of the $n$-th element. In this paper, the carrier frequency increment between two adjacent array elements is $\Delta f$.

The received signal can be written as

$$y_n(t) = a(t)\left(t - \frac{2R_k}{c} + \frac{2nd\sin\theta_k}{c}\right) + n_n(t),$$

where $n_n(t)$ denotes the additive noise and is uncorrelated with signal, $\theta_k$ and $R_k$ are the angle and the distance of the $k$-th target, respectively, $d$ is the spacing of the array elements, and $c$ is the speed of light. Due to the narrow bandwidth signal and small amplitude change, the difference of the envelope of each element can be ignored [20]. Therefore, the equation

$$a\left(t - (2R_k/c) + (2n\sin\theta_k/c)\right) = x_k(t)$$

is obtained. Through matching filtering and down frequency conversion processing [21], the signal model can be expressed as

$$Y(t) = A(R, \theta)X(t) + N(t),$$

where the received signal vector is $Y(t) = [y_0(t), y_1(t), \ldots, y_{N-1}(t)]^T$, the source vector of targets is $X(t) = [x_1(t), x_2(t), \ldots, x_K(t)]^T$, the noise vector is $N(t) = [n_0(t), n_1(t), \ldots, n_{N-1}(t)]^T$, and $K$ is the number of targets. The steering vector is described as

$$A(R, \theta) = [a(R_1, \theta_1), a(R_2, \theta_2), \ldots, a(R_K, \theta_K)],$$

where

$$a(R_k, \theta_k) = [a_0(R_k, \theta_k), a_1(R_k, \theta_k), \ldots, a_{N-1}(R_k, \theta_k)]^T, a_n(R_k, \theta_k) = \exp\{j2\pi (f_0 + n\Delta f][- (2R_k/c) + 2n\sin\theta_k/c]\}.$$
The main task of this paper is to determine the angle and distance according to equation (3) by using the BP network. The space-frequency covariance matrix $R$ is estimated with the definition as follows:

$$R = E[Y(t)Y^H(t)].$$

The upper triangular matrix of $R$ consisting of $N(N - 1)/2$ elements is chosen to construct the input vector of the model as follows [22]:

$$R_1 = [R_{12}, R_{13}, \ldots, R_{1N}, R_{23}, R_{24}, \ldots, R_{(N-1)N}]^T.$$  

Meanwhile, the real and imaginary parts of each element in the vector $R_1$ are separated to form a $N(N - 1) \times 1$-dimensional vector $R_2$, and the input set of the BP network is expressed as

$$R_{in} = [R^1_2, R^2_2, R^3_2, \ldots, R^2_N, R^3_N, \ldots, R^K_N].$$

where $R_{in}$ is a $N(N - 1) \times K$-dimensional matrix.

### 2.2. Model of FDA-BP

A typical BP neural network model of FDA radar (FDA-BP) is shown in Figure 2 with three-layer neuron operating [23], the input of the FDA-BP model is $R_{in}$, and the output is the estimates of the azimuth angle and the distance of targets. Every neuron in each layer is connected to every neuron in the adjacent forward layer, and no connections are permitted between the neurons belonging to the same layer. Each neuron is characterized by a transfer function and bias, and each connection between two neurons is by a weight vector. Weight coefficients are typically adjustable and calculated by means of an adaptive algorithm combined with training samples (input-output signals) presented during the training (or learning) phase.

Signals propagate gradually through the network from the input layer and the hidden layers up to the output layer. The outputs of the hidden layer and output layer [23], $H_h$ and $o_l = (\tilde{\theta}_l, \tilde{R}_l)$, can be, respectively, written as

$$H_h = f\left(\sum_{i=1}^M w_{ih} - a_{ih}\right), \quad h = 1, 2, \ldots, Q,$$

$$o_l = \sum_{i=1}^Q H_i w_{ih} - b_{ih}, \quad l = 1, 2, \ldots, L,$$

where $M$, $Q$, and $L$ are the numbers of neurons in the input layer, the hidden layer, and the output layer, respectively, $w_{ih}$ and $a_{ih}$ are the weight coefficient and bias value between input layer and hidden layer, respectively, and $w_{ih}$ and $b_{ih}$ are the weight matrix and bias value between hidden layer and output layer. Function $f(\cdot)$ is the activation function of each neuron and linear for input and output layers but is sigmoid (tan-sigmoid in the particular case) for hidden layer as follows:

$$f(u) = \frac{1}{1 + e^{-u}}.$$
considered to be massless and un-volumetric, and the process of optimization is carried out with the basis of particles’ position and speed. Assuming that there exists a particle swarm in Q-dimensional space, which consists of m particles, the position and speed of the i-th particle are given by

\[ X^t_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,Q}), \]
\[ V^t_i = (v_{i,1}, v_{i,2}, \ldots, v_{i,Q}). \tag{13} \]

The particle swarm will get its own optimal solution and acquire the global optimal solution during each iteration, which is denoted as \( P^t = (p_{1,1}, p_{1,2}, \ldots, p_{1,Q}) \). The position and speed are updated, respectively, according to the following equations:

\[ v_{i,j}(t + 1) = w \times v_{i,j}(t) + c_1 \times r_1 \times \left( p_{i,j} - x_{i,j}(t) \right) + c_2 \times r_2 \times \left( p_{g,j} - x_{i,j}(t) \right), \tag{14} \]
\[ x_{i,j}(t + 1) = x_{i,j}(t) + v_{i,j}(t + 1), \quad j = 1, 2, \ldots, Q, \]
\[ i = 1, 2, \ldots, m, \tag{15} \]

where \( w \) denotes the inertia weight, \( c_1 \) and \( c_2 \) are learning factors, and \( r_1 \) and \( r_2 \) are random numbers in the range of [0, 1], and \( p_{g,j} \) is the global optimal value. If \( v_{i,j} > V_{\text{max}} \), we get \( v_{i,j} = V_{\text{max}} \); while \( v_{i,j} < -V_{\text{max}} \), \( v_{i,j} = -V_{\text{max}} \), where \( V_{\text{max}} \) is the maximum speed limit value.

The above analysis shows the inertia weight is an important parameter to control the deviation of particles movement from global solution. In general, it is linearly updated with the expression \( w = w_{\text{max}} - t \times (w_{\text{max}} - w_{\text{min}})/t_{\text{max}} \); therefore, the convergence speed of the inertia weight PSO algorithm is slow [25]. On the other hand, with the decrease of inertia weight value in the later stage, the global search ability becomes weak. Once the optimal position of particle swarm is not the optimal value of the objective function, the algorithm will fall into the precocity state, resulting in the lack of diversity and local optimization of the particle swarm. Therefore, in this paper, the inertia weight is nonlinearly modified to ensure the global optimization of the particle swarm, meanwhile improving the local search ability of the PSO algorithm, and avoiding the premature convergence.

### 3.2. Improved PSO Algorithm for the FDA-BP Model

A nonlinear weight \( w' \) is applied in equation (14), and the improved PSO (IPSO) algorithm in this paper is formed, which can improve the slow convergence and local optimization. The nonlinear weight \( w' \) is written as

\[ w' = w_{\text{max}} - \left( w_{\text{max}} - w_{\text{min}} \right) \times \tan \left( \frac{t}{t_{\text{max}}} \times \frac{\pi}{4} \right), \tag{16} \]

where \( w_{\text{max}} = 0.9 \) and \( w_{\text{min}} = 0.4 \) and \( t \) and \( t_{\text{max}} \) are the numbers of the current iteration and maximum iteration. From equation (16), \( w' \) is close to \( w_{\text{max}} \) at the beginning of the iteration processing, which leads to the large step for global optimization. With the increase of the iteration number \( t \), the weight \( w' \) decreases, which ensures the smaller step for local optimization.

In order to evaluate the performance of the proposed algorithm, the Ackley fitness function is used to compare the convergence of standard PSO, inertia weight PSO, and IPSO algorithms. The expression of the Ackley fitness function is written as [26]

\[
\min f(x) = -20 \exp \left( -0.2 \sqrt{\frac{1}{n} \sum_{j=1}^{n} x_j^2} \right) + \exp \left( \frac{1}{n} \sum_{j=1}^{n} \cos(2\pi x_j) \right) + 22.71282, \quad x_j \in [-5, 5], \quad j = 1, 2, \ldots, n, \tag{17}
\]

where \( x \) is the value of the function argument.

In this experiment, the weight of the PSO algorithm without the inertia weight (named as standard PSO, abbreviated as PSO) is 1, and that of the inertia weights PSO and IPSO is \( w \) and \( w' \), respectively. The Ackley function has many local optimal traps and is an unconstrained optimization problem. When \( n = 2 \), the global optimal solution is 0. For the parameters of the three algorithms, the particle number is \( m = 20 \), the learning factors are \( c_1 = c_2 = 1.49445 \), \( V_{\text{max}} = 1 \), and \( t_{\text{max}} = 100 \). The fitness curves of the test function are shown in Figure 3; the X-axis is the number of evolution and the Y-axis is the fitness value. Compared with the first two algorithms, the IPSO algorithm has better global and local search ability to obtain the global optimal solution faster and more accurately. The IPSO method is combined with FAD-BP into FAD-IPSO-BP algorithm in this paper.

FAD-IPSO-BP algorithm improves the connection weights and thresholds in the standard BP neural network, which can reach the vicinity of the global optimal solution. In this vicinity region, the BP neural network with strong local search ability is used to perform the optimal value search. Therefore, the complete network model is established.

The implementation process of FAD-IPSO-BP algorithm can be summarized as follows. Firstly, the received data are generated by FDA radar, weights and offsets of the network are initialized, and a three-layer BP neural network is pretrained. Secondly, IPSO algorithm is introduced to optimize and train the weight coefficients of FDA-BP neural network. Finally, the azimuth and distance of the targets are estimated by the trained BP neural network.

In order to test the performance of the proposed algorithm, the simulation experiments in MATLAB and FEKO simulation software are performed in the next section.

### 4. Simulation

#### 4.1. Estimation of Target Position Using MATLAB Simulation Data

In this section, two experiments shown in Table 1 are performed in the MATLAB simulation software. In
In experiment 1, different targets are localized by using the FDA-IPSO-BP, FDA-BP, and FDA-MUSIC algorithms. In experiment 2, multiple estimates are made using the FDA-IPSO-BP and FDA-BP algorithms for targets (5°, 8 km), (15°, 8 km), and (20°, 12 km), respectively, under the different SNRs and snapshots.

In experiment 1, an FDA uniform linear array consists of 11 array elements, and the signal sources are selected as narrow-band signals with a reference carrier frequency of \( f_0 = 10 \) GHz. The spacing of the array elements is \( d = \lambda/2 \), the noise is independent zero-mean Gaussian white noise, the SNR is 10 dB, and the number of snapshots is 100. 24 different targets are utilized for positioning, where 8 angle parameters include 5°, 10°, . . . , 40° with the interval 5°, and 3 distance parameters are 4 km, 8 km, 12 km, respectively. The different angles and distances are constituted into 24 target positions.

The 240 sets of data as input data \( \mathbf{R}_m \) for the network are obtained from each of the above 24 targets calculated 10 times in equation (7), of which 216 sets obtained from the first 9 calculations of the 24 targets are selected as input data for network training period and 24 sets of data from the last calculation are selected as predictive input data to estimate the azimuth and range of the targets as stated above. According to the dimension of vector \( \mathbf{R}_m \), the number of parameters in the hidden layer is \( N(N - 1) = 11 \times 10 = 110 \). In order to reduce the complexity of neural network, PCA (principal component analysis) algorithm is used for dimensionality reduction of \( \mathbf{R}_m \), and the number of neurons in the input layer is selected as 14 through comprehensive consideration of the calculation ability of MATLAB software and complexity of neural network. The output is two parameters (distance and angle), and the output layer is composed of two neurons. The number of neurons in the hidden layer is calculated according to the empirical formula, \( Q = \sqrt{M + L + \alpha} \), where \( Q \), \( M \), and \( L \) are the numbers of the neurons in hidden layer, input layer, and output layer, respectively, and \( \alpha \) is a constant of \([0, 10]\). In this paper, \( M = 14 \), \( L = 2 \), and \( \alpha = 7 \); then, \( Q = 11 \). The same treatment is used in Section 4.2.

Under the condition of the single-frequency receiving FDA mode, the FDA-MUSIC algorithm requires two frequency increments, \( \Delta f_1 = 5 \) kHz and \( \Delta f_2 = -5 \) kHz, to decouple the azimuth and range of the targets, and the target position is obtained by using two-dimensional spectral peak search. However, the FDA-IPSO-BP and FDA-BP algorithms only need a single frequency increment, \( \Delta f_1 = 5 \) kHz, to determine the localization of target, which is an obvious advantage of the two algorithms. The experimental results are shown in Figure 4, where X-axis and Y-axis represent the angle and the range, respectively.

The average errors are shown in Table 2, where the range of two-dimensional spectrum peak search angle in FDA-MUSIC algorithm is from 0° to 180° with the interval 6° and the distance from 3 km to 15 km with the interval 50 m. The network training errors of FDA-IPSO-BP algorithm and FDA-BP algorithm are set to 1e - 3. From Figure 4 and Table 1, it can be seen that the FDA-IPSO-BP algorithm has high target localization accuracy.

In experiment 2, the performance of the FDA-IPSO-BP algorithm and the FDA-BP algorithm will be numerically compared through the root mean square error (RMSE) with the expression as follows:

\[
\text{RMSE}_{\theta} = \sqrt{\frac{1}{H \times K} \sum_{k=1}^{K} \sum_{h=1}^{H} (\theta_k - \bar{\theta}_{kh})^2}, \tag{18}
\]

\[
\text{RMSE}_{R} = \sqrt{\frac{1}{H \times K} \sum_{k=1}^{K} \sum_{h=1}^{H} (R_k - \bar{R}_{kh})^2}, \tag{19}
\]

where \( H \) is the number of Monte Carlos, \( \bar{\theta}_{kh} \) and \( \bar{R}_{kh} \) are the estimates of the \( h \)-th experiment of the angle and distance, respectively, and \( \theta_k \) and \( R_k \) are the actual values of the \( k \)-th source parameters of the angle and the distance.

The network structure and the training sets of FDA-IPSO-BP algorithm and FDA-BP algorithm are consistent with experiment 1. The target positions are selected as (5°, 8 km), (15°, 8 km), and (20°, 12 km), and the accuracy of target position estimation is compared by changing the signal-to-noise ratio and the number of snapshots of the algorithm, respectively.

The number of snapshots is fixed at 100, SNR value varies from -5 dB to 15 dB with the step 5 dB, and the Monte Carlo number is 700. The total 2100 test sets are computed for the three targets with 700 times for each one in equation (7). The root mean square error (RMSE) of these test sets is calculated using equations (18) and (19) between the estimates and the true location of the targets. Finally, the averages of the RMSE of the three targets are calculated and shown in Figures 5 and 6, where the X-axis
Table 1: The simulation experiments of target location.

| The number of targets | Algorithm | Performance index | Experimental scene |
|-----------------------|-----------|-------------------|--------------------|
| Experiment 1 24       | FDA-IPSO-BP, FDA-BP, FDA-MUSIC | Average error | $f_0 = 10$ GHz, $SNR = 10$ dB, The number of snapshots = 100 |
| Experiment 2 3        | FDA-IPSO-BP, FDA-BP | The root mean square error | $f_0 = 10$ GHz, $SNR = (-5:5:15)$ dB, The number of snapshots = 100:100:600 |

Table 2: The average error of the three algorithms.

| Algorithm   | Average error of 24 target angles (°) | Average error of 24 target distances (m) |
|-------------|---------------------------------------|------------------------------------------|
| FDA-IPSO-BP | 0.269                                  | 75.000                                   |
| FDA-BP      | 0.334                                  | 185.250                                  |
| FDA-MUSIC   | 4.833                                  | 1.837e3                                  |

Figure 4: Target localization result of the three algorithms.

Figure 5: RMSE of azimuth estimation versus SNR.

Figure 6: RMSE of range estimation versus SNR.

Figure 7: RMSE of azimuth estimation versus snapshot.
indicates SNR and the Y-axis represents the RMSE of the azimuth and range, respectively.

Similarly, the average of the RMSE for the three targets is calculated by varying the number of snapshots shown in Figures 7 and 8, where the SNR is fixed at 10 dB, the number of snapshots is constantly changing from 100 to 600 with the step 100, and the Monte Carlo number is 700.

From the above experiment results from Figures 5–8, it can be seen that the FDA-IPSO-BP algorithm has higher estimation accuracy in azimuth and distance than the FDA-BP algorithm, where the IPSO method is utilized to prevent BP network from falling into local optimum as much as possible.

4.2. Estimation of Target Position Using Data from FEKO Simulation Software. In experiment 3, FEKO simulation data is used to further verify the application of FDA-IPSO-BP close to the actual environment. FEKO software is a powerful three-dimensional full-wave electromagnetic simulation software. As shown in Figure 9, a 7-array FDA radar is constructed in FEKO. The target is a rectangular sheet of 1 square meter. The initial carrier frequency of the array is $f_0 = 1 \text{GHz}$, and the frequency deviation is $\Delta f = 1050 \text{kHz}$.

The received data obtained by moving the location of the target are used as samples for this experiment at angles from 25.0° to 35.0° with interval 0.1°, and distances from 95.0 m to 105.0 m with interval 0.1 m. Considering the operation time of the FEKO software to generate simulation data, the training data are obtained from the 101 target positions composed of the one-to-one point matching between the angles and distances. The 101 target position set is $\{(25.0^\circ, 95.0\text{ m}), (25.1^\circ, 95.1\text{ m}), (25.2^\circ, 95.2\text{ m}), \ldots, (35.0^\circ, 105.0\text{ m})\}$. The received data from the target position $(30^\circ, 100\text{ m})$ are used as an input sample for the testing period.

The numbers of neurons in the input layer, the hidden layer, and the output layer are 14, 11, and 2, respectively. Because there is no signal-to-noiseratio parameter setting option in FEKO software, we add the white noise to the received data from FEKO software with SNR = 15 dB.

The azimuth and distance information estimates are shown in Table 3. As can be seen from Table 3, the target position obtained by the FDA-IPSO-BP algorithm matches the actual target. The proposed algorithm avoids the coupling problem of angle and distance and saves time and computing resources to a large extent compared with MUSIC-type algorithms.

5. Conclusion

In order to improve the precision and robustness of target localization in frequency diverse array (FDA) radar, the BP neural network is merged with FDA radar, and the improved particle swarm optimization (IPSO) algorithm nonlinearly updates the weights to keep away from local optimum and sensitivity to initialization of weights in this paper. Furthermore, the angle and distance are decoupled by single frequency increment. It can be seen from the experiment results under MATLAB and FEKO simulation environments that the proposed method has an effective improvement in the accuracy of target localization, compared with FDA-BP and FDA-MUSIC algorithms. The future work is to build experimental platform and use the proposed algorithm to achieve radar target positioning with the actual measurement data in the near-field and far-field environment.

**Notations**

$\mathbf{A}^T$: Matrix transpose

$\mathbf{A}^H$: Matrix conjugate transpose.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.
Conflicts of Interest

The authors declare that they have no conflicts of interest.

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