Letter

Feasible superadiabatic-based shortcuts for the fast generation of 3D entanglement between two atoms

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Abstract
We propose a scheme to realize the fast generation of three-dimensional entanglement between two atoms via superadiabatic-based shortcuts in an atom–cavity–fiber system. The scheme is feasible because the counterdiabatic Hamiltonian has the same form as the effective Hamiltonian. Numerical simulations are provided to prove that the scheme is robust against variations in various parameters and against decoherence.

Keywords: 3D entanglement, superadiabatic iterations, shortcuts to adiabaticity

(Some figures may appear in colour only in the online journal)

1. Introduction

With the rapid development of quantum information processing, high-dimensional entanglement is increasingly attracting the attention of researchers because its security is superior to that of qubit entanglement in the field of quantum key distribution, and its violation of local realism is greater [1–5]. Thus, the generation of high-dimensional entanglement is of great importance. Up to now, a large number of schemes have been proposed for the generation of high-dimensional entanglement via various techniques [6–16]. Among these techniques, stimulated Raman adiabatic passage (STIRAP) is widely used in the field of time-dependent interaction for many purposes [17, 18] because of its robustness against atomic spontaneous emissions and variations in experimental parameters. However, STIRAP usually requires a relatively long interaction time to restrain non-adiabatic transitions.

A set of techniques called ‘shortcuts to adiabaticity’ (STA) are promising for quantum information processing, and they actually fight against the decoherence, noise and losses that accumulate during long operating times. Hence, many schemes have been proposed for the construction of STA [19–29]. Using STA, many remarkable achievements have been made in quantum information processing [30–35]. Further, numerous schemes have been developed for the fast generation of high-dimensional entanglement [36–40]. For example, Chen et al and He et al prepared a three-atom singlet state [37] and a two-atom three-dimensional (3D) entangled state [39], respectively, using transitionless quantum driving (TQD); Lin et al [36] and Wu et al [38] implemented two-atom 3D entangled states, respectively, based on Lewis–Riesenfeld invariants (LRI); and Wu et al generated three-atom tree-type 3D entangled states with both TQD and LRI [40].

In this paper, we propose a superadiabatic scheme for the fast generation of two-atom 3D entanglement via superadiabatic iterations. Superadiabatic iterations were introduced as an extension of the traditional adiabatic approximation in [41]. The technique was first adopted to speed up the adiabatic process by Ibáñez et al [21, 42]. A short time before that, Song et al extended it to a three-level system [43]. More recently, Huang et al [44] and Kang et al [45] generated a Greenberger–Horne–Zeilinger state and a W state,
Transitions \( AA, 0, 0, 0, 0 \) are resonantly driven by classical laser fields, and the superadiabatic scheme does not need an additional coupling between the initial and final states, while the counteradiabatic Hamiltonian has the same form as the effective Hamiltonian, which guarantees its high experimental feasibility.

This paper is structured as follows. The physical model and effective dynamics are shown in section 2. In section 3, we describe the superadiabatic scheme for the fast generation of 3D entanglement between two atoms. In section 4, numerical simulation results prove that the scheme is fast, valid and robust. The conclusion is given in section 5.

2. Physical model and effective dynamics

A schematic sketch of the physical model and atomic level configurations for the fast generation of two-atom 3D entanglement are shown in figure 1(a). Two atoms are trapped in two spatially separated bimodule cavities connected by a fiber that satisfies the short fiber limit to ensure that only resonant modes of the fiber interact with the cavity modes [46]. Atomic transitions \( |e_0, A \rangle \rightarrow |g_{L(R)} \rangle_A \) and \( |e_{L(R)} \rangle_B \rightarrow |g_0 \rangle_B \) are resonantly coupled to the left- (right)-circularly polarized modes of cavity A and cavity B, respectively, with corresponding coupling constants \( g_{L(R)} \) and \( g_{L(R),R} \). Transitions \( |e_0, A \rangle \rightarrow |g_{L(R)} \rangle_A \) and \( |e_{L(R)} \rangle_B \rightarrow |g_0 \rangle_B \) are resonantly driven by classical laser fields, respectively, with Rabi frequencies \( \Omega (t) \) and \( \Omega (t) \). Quantum information is coded as \( |g_0 \rangle_A \equiv |0 \rangle, |g_{L(R)} \rangle_A \equiv |1 \rangle \) and \( |g_{L(R)} \rangle_A \equiv |2 \rangle \) for atom A, and \( |g_0 \rangle_B \equiv |0 \rangle, |g_{L(R)} \rangle_B \equiv |1 \rangle \) and \( |g_{L(R)} \rangle_B \equiv |2 \rangle \) for atom B. Level structures can be achieved for two atoms by hyperfine-split levels in \( 87 \text{Rb} \) [47, 48], and the corresponding energy levels and transitions are shown in figure 1(b).

The interaction Hamiltonian of the atom–cavity–fiber system is \( (b = 1) \):

\[
H(t) = \sum_{i=L,R} \Omega (t) g_{0,i} |e_0, A \rangle |g_{L(R)} \rangle_A + g_{g_{L(R)},R} |e_{L(R)} \rangle_B |g_{0} \rangle_B + v \left( a_{L} + a_{R}^{\dagger} \right) |g_{L(R)} \rangle_A + \text{H.c.},
\]

where \( a_{L(R),R} \) and \( b_{L(R),R} \) are the annihilation operators of the left (right)-circularly polarized mode of cavity A (B) and the fiber, respectively, and \( v \) is the coupling strength between the two cavities and the fiber. For convenience, we assume that \( g_{A,L(R)} = g_{B,L(R)} \)

If the initial state of the whole system is \( |\psi \rangle = |g_0 \rangle_A |g_0 \rangle_B |0 \rangle \), denoting atom A in state \( |g_0 \rangle \), atom B in state \( |g_0 \rangle \) and two cavities and fiber in the vacuum state, Hamiltonian (1) can be rewritten as

\[
H(t) = H_{t} + H_{\text{c.c.}},
\]

for which

\[
|\phi_{L,R} \rangle = |g_0 \rangle_A |g_0 \rangle_B |0 \rangle |A, A, A \rangle |0 \rangle, |B, B, B \rangle,
\]

where \( |A, A, A \rangle \) and \( |B, B, B \rangle \) are the annihilation operators of the left (right)-circularly polarized modes of cavity A (B) and the fiber, respectively, and \( v \) is the coupling strength between the two cavities and the fiber. For convenience, we assume that \( g_{A,L(R)} = g_{B,L(R)} \)

(3)

\[
|\phi_{L,R} \rangle = |g_0 \rangle_A |g_0 \rangle_B |0 \rangle |A, A, A \rangle |0 \rangle, |B, B, B \rangle,
\]

where \( |A, A, A \rangle \) and \( |B, B, B \rangle \) are the annihilation operators of the left (right)-circularly polarized photon state. Now we set a set of orthogonal states

\[
|\psi_{L,R} \rangle = \frac{1}{\sqrt{2}} (|\phi_{2k+1} \rangle + |\phi_{2k+2} \rangle),
\]

where \( k = 1, 2, 3, 4, 5 \). Then Hamiltonian (2) becomes

\[
H(t) = \Omega (t) \left( |\phi_{2k+1} \rangle + |\phi_{2k+2} \rangle \right) + v \left( |\psi_{L,R} \rangle + |\psi_{L,R} \rangle \right) + \text{H.c.}
\]

Because \( |\psi_{L,R} \rangle \) will not be involved during the whole evolution if \( |\phi_{L,R} \rangle \) is the initial state, Hamiltonian (5) becomes

\[
H(t) = \Omega (t) \left( |\phi_{2k+1} \rangle + |\phi_{2k+2} \rangle \right) + v \left( |\psi_{L,R} \rangle + |\psi_{L,R} \rangle \right) + \text{H.c.}
\]

Figure 1. (a) Diagrammatic sketch of the atom–cavity–fiber system, atomic level configurations and related transitions. (b) The energy levels of \( 87 \text{Rb} \). \( \sigma^+ \) and \( \pi \) denote the left, right circular and line polarization light, respectively.
\[ H(t) = \Omega_0 (t)|\psi_1\rangle\langle \psi_1| + \sqrt{2} g A |\psi_2\rangle\langle \psi_2| + n(t)(|\psi_2\rangle\langle \psi_2| + |\psi_1\rangle\langle \psi_1|) + g_B |\psi_3\rangle\langle \psi_3| + \Omega(t)|\psi_3\rangle\langle \psi_3| + \text{H.c.} \]  
\[ (6) \]

Next, for further simplification, we set \( \sqrt{2} g A = v = g_B = g \) and rewrite Hamiltonian (6) as

\[ H(t) = H_0 + V(t), \]
\[ H_0 = g(|\psi_1\rangle\langle \psi_1| - |\psi_2\rangle\langle \psi_2|) + \sqrt{3} g(|\psi_2\rangle\langle \psi_2| - |\psi_3\rangle\langle \psi_3|), \]
\[ V(t) = \frac{\Omega_0(t)}{\sqrt{3}} |\psi_2\rangle\langle \psi_2| - \frac{\Omega_0(t)}{\sqrt{2}} (|\psi_1\rangle\langle \psi_1| + |\psi_3\rangle\langle \psi_3|) + \frac{\Omega_0(t)}{\sqrt{3}} |\psi_1\rangle\langle \psi_1| + \frac{\Omega_0(t)}{\sqrt{2}} (|\psi_3\rangle\langle \psi_3| + |\psi_2\rangle\langle \psi_2|)), \]
\[ + \text{H.c.}, \]  
\[ (7) \]

Then, after performing the unitary transformation \( U = \exp(-i H_0 t) \) and neglecting high oscillating terms under the limit condition \( \Omega_{AB}(t) \ll 2g \), we simplify Hamiltonian (7) to an effective Hamiltonian:

\[ H_{\text{eff}}(t) = \Omega_1(t)|\psi_1\rangle\langle \psi_1| + \Omega_2(t)|\psi_2\rangle\langle \psi_2| + \text{H.c.}, \]
\[ (9) \]

with \( \Omega_1(t) = \Omega_0(t)\sqrt{3} \) and \( \Omega_2(t) = \Omega_0(t)/\sqrt{3} \).

3. Superadiabatic scheme for the fast generation of two-atom 3D entanglement

The instantaneous eigenstates of Hamiltonian (9) with eigenvalues \( n_{\pm} = \pm \Omega(t) \) and \( n_0 = 0 \), respectively, are

\[ |n_\pm(t)\rangle = \frac{1}{\sqrt{2}} [\sin \theta(t)|\psi_1\rangle \pm |\psi_2\rangle + |\psi_3\rangle, \]
\[ |n_0(t)\rangle = \cos \theta(t)|\psi_1\rangle - \sin \theta(t)|\psi_2\rangle, \]
\[ (10) \]

where \( \Omega(t) = \sqrt{\Omega_0(t)^2 + \Omega_2(t)^2} \) and \( \tan \theta(t) = \Omega_0(t)/\Omega_2(t) \). We transform \( H_{\text{eff}}(t) \) to the adiabatic frame by performing the unitary transformation \( U_0(t) = \sum_{n_{\pm}} n_{\pm}(t) \). At each instant in time, \( U_0(t) \) maps the instantaneous eigenstate \( n_\pm(t) \) onto the time-independent state \( |n_\pm\rangle \). In the adiabatic frame, the Hamiltonian (9) becomes

\[ H(t) = U_0(t) H_{\text{eff}}(t) U_0(t) + i U_0(t) \dot{U_0}(t), \]
\[ = \Omega(t)[|n_+\rangle\langle n_+| - |n_-\rangle\langle n_-|] \]
\[ + \frac{\dot{\theta}(t)}{\sqrt{3}} [i |n_+\rangle\langle n_0| + i |n_-\rangle\langle n_0| + \text{H.c.}], \]
\[ (11) \]

The effective system evolution will adiabatically follow one of the states \( \{|n_{\pm0}\rangle\} \) with the adiabatic approximation \( \theta(t) = \sqrt{2} \Omega(t) \), which needs a very long running time. To shorten the running time, Demirplack and Rice [49] and Berry [50] proposed that adding a suitable counterdiabatic (CD) Hamiltonian \( H_{\text{CD}}(t) \) to the original Hamiltonian can suppress the transitions between different eigenstates. In the adiabatic frame, the CD Hamiltonian may be \(-iU_0(t) U_0(t) \), which is written in the \( \{|\phi_1\rangle, |\psi_2\rangle, |\psi_3\rangle\} \) frame by

\[ H_{\text{CD}}^{(1)}(t) = -iU_0(t)^\dagger U_0(t) \dot{U_0}(t) \]
\[ = \delta(t) (|\phi_1\rangle\langle \psi_3| - |\psi_3\rangle\langle \phi_1|), \]
\[ (12) \]

CD Hamiltonian (12) needs a direct coupling between \( |\phi_1\rangle \) and \( |\psi_3\rangle \), which is too hard to implement in practice for such a complex system.

The superadiabatic states (instantaneous eigenstates of \( H_1(t) \)) with the eigenvalues \( n_{\pm} = \pm \Omega(t) \) and \( n_0 = 0 \), respectively, are

\[ |n_\pm(t)\rangle = \frac{1}{\sqrt{2}} [i |1 \pm \cos \theta(t)|n_\pm\rangle \pm \sqrt{2} \sin \theta(t)|n_\mp\rangle, \]
\[ i |1 \mp \cos \theta(t)|n_\mp\rangle, \]
\[ (13) \]

for which \( \Omega(t) = \sqrt{\delta(t)^2 + \Omega(t)^2} \) and \( \tan \theta(t) = \delta(t)/\Omega(t) \). Then we transform \( H_1(t) \) to the superadiabatic frame by the unitary transformation \( U_1(t) = \sum_{n_{\pm}} n_{\pm}(t) \). Analogously to the adiabatic CD Hamiltonian (12), the superadiabatic CD Hamiltonian \(-iU_1(t)^\dagger U_1(t) \) is written in the \( \{|\phi_1\rangle, |\psi_2\rangle, |\psi_3\rangle\} \) frame by

\[ H_{\text{CD}}^{(2)}(t) = -iU_1(t)^\dagger U_1(t) \dot{U_1}(t) \]
\[ = \delta(t) [-\cos \theta(t)|\phi_1\rangle\langle \psi_3| + \sin \theta(t)|\psi_2\rangle\langle \psi_3| + \text{H.c.}], \]
\[ (14) \]

\[ H_{\text{CD}}^{(2)}(t) \] is satisfactory because it has the same form as the effective Hamiltonian (9).

We regard \( \Omega_1(t) = -\dot{\theta}(t) \cos \theta(t) \) and \( \Omega_2(t) = \dot{\theta}(t) \sin \theta(t) \) as two auxiliary pulses added to the pulses \( \Omega_1(t) \) and \( \Omega_2(t) \), respectively. Then the modified pulses \( \Omega_1'(t) = \Omega_1(t) + \Omega_1(t) \) and \( \Omega_2'(t) = \Omega_2(t) + \Omega_2(t) \) can drive the effective system to evolve along one of the superadiabatic states in equation (13). Therefore, if the related parameters meet the time boundary conditions \( \theta(0) = \theta(t_f) = 0, \dot{\theta}(0) = 0, \dot{\theta}(t_f) = -\arctan \sqrt{2} \) (\( t_f \) is the final time), \( |n_{\pm0}(t_f)| \) will act as a medium state to achieve the expected transformation \( |\phi_1\rangle \to (|\phi_1\rangle + \sqrt{2} |\psi_3\rangle)/\sqrt{3} \). In this way we obtain the 3D entanglement between two atoms in the superadiabatic scheme

\[ |\Psi_{3D}\rangle = \frac{1}{\sqrt{3}} (|\phi_1\rangle + |\phi_1\rangle + |\phi_1\rangle), \]
\[ = \frac{1}{\sqrt{3}} (|00\rangle + |11\rangle + |22\rangle)_{AB} \otimes |0\rangle_A |0\rangle_B \text{.} \]
\[ (15) \]
4. Numerical simulations

To meet \( \theta(0) = \theta(t_f) = 0 \), we choose \( \Omega_2(t) \) and \( \Omega_1(t) \) as [51]

\[
\begin{align*}
\Omega_2(t) &= \frac{1}{\sqrt{3}} \Omega_0 \exp[-(t-t_f/2-t_0)^2/t_c^2] \\
&\quad + \Omega_0 \exp[-(t-t_f/2+2)^2/t_c^2], \\
\Omega_1(t) &= -\frac{\sqrt{2}}{\sqrt{3}} \Omega_0 \exp[-(t-t_f/2-t_0)^2/t_c^2],
\end{align*}
\]

(16)

with two related Gaussian parameters \( t_0 = 0.18t_f \) and \( t_c = 0.24t_f \). In figure 2, we plot the time dependence of \( \theta_0(t) \) and \( \theta_1(t) \) with different values of \( \Omega_0 \). By converting the relation \( \sim g \) into \( \sim A_{BM}(t) \), we easily find that, for the same high final fidelity (e.g. \( F(t_f) = 0.999 \)), the operation time of the superadiabatic scheme is reduced to about 1/4 of that of the STIRAP scheme, which proves that the scheme we propose is indeed fast. Further, from both figures 3(a) and (b), we also see that the limit condition \( \Omega_{BM}(t) \ll 2g \) is in action. Therefore, we adopt a pair of parameters \( \Omega_0 = 8t_f^{-1} \) and \( g = 70t_f^{-1} \) for the superadiabatic scheme in the following discussions.

Since the analytic functions of \( \Omega_1(t) \) and \( \Omega_2(t) \) are too complicated, for experimental feasibility we seek two superpositions of the Gaussian functions by curve fitting to replace them, respectively

\[
\begin{align*}
\tilde{\Omega}_1(t) &= -\sum_{m=0}^{4} \Omega_{1m} \exp[-(t-t_{1m})^2/\chi_{1m}^2], \\
\tilde{\Omega}_2(t) &= \sum_{m=0}^{4} \Omega_{2m} \exp[-(t-t_{2m})^2/\chi_{2m}^2],
\end{align*}
\]

(17)
with related parameters \( \Omega_1 = 1.4695 t_f, \Omega_2 = 2.4114 t_f, \Omega_3 = 1.9854 t_f, \Omega_4 = 4.4491 t_f, \tau_1 = 0.7373 t_f, \tau_2 = 0.4424 t_f, \tau_3 = 0.6547 t_f, \tau_4 = 0.7568 t_f, \chi_1 = 0.1494 t_f, \chi_2 = 0.0939 t_f, \chi_3 = 0.1358 t_f, \chi_4 = 0.2044 t_f \) for \( \tilde{\Omega}_1 (t) \) and \( \{ \tilde{\Omega}_1 (t), \tilde{\Omega}_2 (t) \} \). Through plotting figure 4, we see that the curve for \( \Omega_1 (t) \) is very close to that for \( \tilde{\Omega}_1 (t) \) (\( \Omega_2 (t) \)). In the following, to show the effectiveness of two alternative Rabi frequencies, in figure 5(a) we plot the time dependence of the fidelity when adopting \( \Omega_1 (t) \) and \( \Omega_2 (t) \) or \( \tilde{\Omega}_1 (t) \) and \( \tilde{\Omega}_2 (t) \). The very approximate coincidence of two curves indicates that the alternative Rabi frequencies are quite valid. For a further illustration, with \( \tilde{\Omega}_1 (t) \) and \( \tilde{\Omega}_2 (t) \), in figure 5(b) we plot the time evolution of the populations for states in equation (3), and the results show that the desired two-atom 3D entanglement \( |\Psi_{3D} \rangle \) can be obtained almost perfectly at \( t = t_f \). What is more, we also see that the states not involved in \( |\Psi_{3D} \rangle \) are almost unpopulated.

Since it is impossible to control most parameters perfectly in experiments, we should investigate the robustness of the scheme against variations in control parameters. Here we define \( \Delta x = x' - x \) as the deviation of \( x \), in which \( x \) denotes the ideal value and \( x' \) denotes the actual value. In figure 6, we consider the effects of variations in the parameters involved in the superadiabatic scheme on the final fidelity for the fast generation of 3D entanglement between two atoms. As we can see from figure 6, the final fidelity is always over 0.993,
even when the variations in two of the parameters are up to \(|\delta x| = 0.1x\), which indicates that the superadiabatic scheme for the fast generation of two-atom 3D entanglement is extremely robust against variations in control parameters. We also note that figure 6(d) shows that the condition \(v = 1\) is slightly critical for high-fidelity generation of the target state.

Next we take the decoherence caused by spontaneous atomic emissions and photon leakages from the cavity–fiber system into account. There are six possible paths for the atomic emissions and photon leakages from the cavity–fiber state. In figures 7(a)–7(c), there will also be seven possible spontaneous emission paths.

For the fast generation of two-atom 3D entanglement is extremely robust against variations in control parameters. We also note that figure 6(d) shows that the condition \(v = 1\) is slightly critical for high-fidelity generation of the target state.

If only closed transitions (the final states of the spontaneous emission processes are involved in \(|\Psi_{3D}\rangle\)) are considered in spontaneous atomic emissions, there will be seven possible spontaneous emission paths \(|\psi_2\rangle \rightarrow \{|\phi_1\rangle, |\phi_1\rangle, |\phi_1\rangle, |\phi_2\rangle, |\phi_2\rangle, |\phi_2\rangle, |\phi_2\rangle\}\) and \(|\phi_1\rangle \rightarrow \{|\phi_1\rangle, |\phi_1\rangle, |\phi_1\rangle, |\phi_2\rangle, |\phi_2\rangle, |\phi_2\rangle, |\phi_2\rangle\\}. In addition, unclosed transitions (the final states of the spontaneous emission processes are not involved in \(|\Psi_{3D}\rangle\)) may also occur, and there will also be seven possible spontaneous emission paths \(|\psi_2\rangle \rightarrow \{|\phi_1\rangle, |\phi_1\rangle, |\phi_1\rangle, |\phi_2\rangle, |\phi_2\rangle, |\phi_2\rangle, |\phi_2\rangle\}\) and \(|\phi_1\rangle \rightarrow \{|\phi_1\rangle, |\phi_1\rangle, |\phi_1\rangle, |\phi_2\rangle, |\phi_2\rangle, |\phi_2\rangle, |\phi_2\rangle\\}. For which

\[
|\phi_1\rangle = |g_0\rangle|g_0\rangle|0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0,
|\phi_2\rangle = |g_0\rangle|g_0\rangle|0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0,
|\phi_3\rangle = |g_0\rangle|g_0\rangle|0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0,
|\phi_4\rangle = |g_0\rangle|g_0\rangle|0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0,
|\phi_5\rangle = |g_0\rangle|g_0\rangle|0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0,
|\phi_6\rangle = |g_0\rangle|g_0\rangle|0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0, |0\rangle_0.
\]

Then the evolution of the whole system can be dominated by the master equation

\[
\dot{\rho}(t) = -i[H(t), \rho(t)] - \sum_{j=1,2,3} \frac{\gamma^A_{j}}{2}[\sigma^A_{j}, \rho(t)](1, 2, 3) + \rho(t)[1, 2, 3],
\]

where \(H(t)\) is Hamiltonian (1), \(\gamma^A_{j}\) \((n = 1, 2, 3)\) is the spontaneous emission rate of atom \(l\) from the excited states to the lower state \(|j_1\rangle\), \(\kappa^B_{j}\) is the \(k\)-circular polarized photon leakage rate of cavity \(l\), \(\kappa^B_{j}\) is the \(k\)-circular polarized photon leakage rate of the fiber and \(\sigma^A_{j} = |j\rangle\langle j|n\rangle\). For simplicity, we assume photon leakage rate \(\kappa^B_{j} = \kappa^B_{j} = \kappa\), closed-transition spontaneous emission rates \(\gamma^A_{l_j, l_{j+1}} = \gamma^B_{l_j, l_{j+1}} = \gamma_{l_j, l_{j+1}} = \gamma_{l, l} / 2\) and unclosed-transition spontaneous emission rates \(\gamma^A_{l_j, l_{j+1}} = \gamma^B_{l_j, l_{j+1}} = \gamma_{l_j, l_{j+1}} = \gamma_{l, l} / 2\).

By solving the master equation numerically, in figure 7 we plot the effects of the decoherence on the final fidelity \(F(\tau) = |\langle \Psi_{3D}\rangle \rangle \rho(t) \langle \Psi_{3D}\rangle \rangle|\). In figures 7(a)–(c), by considering...
the effects of any two decoherence factors of the closed-transition and unclosed-transition spontaneous emission and the photon leakage on the final fidelity, we can see that the final fidelity will remain over 0.955, even when the rates of any two decoherence factors are both set by 0.02. In figure 7(d), all three decoherence factors are taken into account, for which $\gamma_1 = \gamma_2 = \gamma$ has been set. The final fidelity is still beyond 0.94, even though $\kappa = \gamma = 0.02$ is chosen, which implies that the scheme for the fast generation of two-atom 3D entanglement is robust against decoherence.

Using a set of predicted cavity QED parameters $(g, \kappa, \gamma) = (750, 3.5, 2.62) \times 2\pi$ MHz [52, 53], we can easily deduce the final time of the scheme $t_f = 70/g \simeq 15$ ns, which is quite short. Finally, with the parameters $g = 750 \times 2\pi$ MHz and $\Omega_0 = 8g/70 \simeq 85.7 \times 2\pi$ MHz, in figure 8 we investigate the effects of the final time on the final fidelity in the presence of different values of $\gamma$ and $\kappa$. Apparently, with non-zero $\gamma$ and $\kappa$, the final fidelities are increasingly spoiled as $t_f$ becomes large enough. However, there is no need to worry because, as we can see from the red triangle line in figure 8, there is still a wide range of values for $t_f$, guaranteeing a very high final fidelity.

5. Conclusion

In conclusion, we have implemented the fast generation of 3D entanglement between two atoms in a feasible superadiabatic scheme. Under certain limiting conditions, the complicated system is simplified into a three-state system, which
makes the superadiabatic scheme more convenient to apply to the generation of two-atom 3D entanglement. Because of the compensation for non-adiabatic couplings, the adiabatic approximation is not needed. Compared with TQD, the superadiabatic scheme is more complicated in terms of mathematical calculations, but it does not require direct coupling between the initial and final states, which greatly enhances the experimental feasibility. In addition, the results of our numerical simulations show that the superadiabatic scheme is very robust against variations in various parameters and the decoherence caused by atomic spontaneous emissions and cavity–fiber photon leakages.

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References

[1] Bourennane M, Karlsisson A and Björk G 2001 Phys. Rev. A 64 012306
[2] Bruß D and Macchiavello C 2002 Phys. Rev. Lett. 88 127901
[3] Cerf N J, Bourennane M, Karlsson A and Gisin N 2002 Phys. Rev. Lett. 88 127902
[4] Jo Y and Son W 2016 Phys. Rev. A 94 052316
[5] Kaszlikowski D, Gnacinski P, Żukowski M, Miklaszewski W and Zeilinger A 2000 Phys. Rev. Lett. 85 4418–21
[6] Li X Y, Liu J B, Ding C L and Li J H 2008 Phys. Rev. A 78 032305
[7] Shao X Q, Wang H F, Chen L, Zhang S, Zhao Y F and Yeon K H 2010 New J. Phys. 12 023040
[8] Chen L B, Shi P, Zheng C H and Gu Y J 2012 Opt. Express 20 14547–55
[9] Liang Y, Su S L, Wu Q C, Ji X and Zhang S 2015 Opt. Express 23 5064–77
[10] Song C, Su S L, Wu J L, Wang D Y, Ji X and Zhang S 2016 Phys. Rev. A 93 062321
[11] Li W A and Huang G Y 2011 Phys. Rev. A 83 022322
[12] Liu S, Li J, Yu R and Wu Y 2013 Phys. Rev. A 87 062316
[13] Wu Q C and Ji X 2013 Quantum Inf. Process. 12 3167–78
[14] Shao X Q, Zheng T Y, Öh C H and Zhang S 2014 Phys. Rev. A 89 012319
[15] Su S L, Shao X Q, Wang H F and Zhang S 2014 Sci. Rep. 4 07566
[16] Wang D Y, Wen J J, Bai C H, Hu S, Cui W X, Wang H F, Zhu A D and Zhang S 2015 Ann. Phys., NY 360 228–36
[17] Bergmann K, Theuer H and Shore B W 1998 Rev. Mod. Phys. 70 1003–25
[18] Kraš P, Thanopulos I and Shapiro M 2007 Rev. Mod. Phys. 79 53–77
[19] Chen X, Rusciahaut A, Schmidt S, del Campo A, Guéry-Odelin D and Muga J G 2010 Phys. Rev. Lett. 104 063002
[20] Chen X, Lizuain I, Rusciahaut A, Guéry-Odelin D and Muga J G 2010 Phys. Rev. Lett. 105 123003
[21] Ibáñez S, Chen X, Torrontegui E, Muga J G and Rusciahaut A 2012 Phys. Rev. Lett. 109 100403
[22] del Campo A, Rams M M and Zurek W H 2012 Phys. Rev. Lett. 109 115703
[23] Rusciahaut A, Chen X, Alonso D and Muga J G 2012 New J. Phys. 14 093040
[24] Martínez-Garaot S, Torrontegui E, Chen X, Modugno M, Guéry-Odelin D, Tseng S Y and Muga J G 2013 Phys. Rev. Lett. 111 213001
[25] del Campo A 2013 Phys. Rev. Lett. 111 100502
[26] Torrontegui E, Ibáñez S, Martínez-Garaot S, Modugno M, del Campo A, Guéry-Odelin D, Rusciahaut A, Chen X and Muga J G 2013 Adv. At. Mol. Opt. Phys. 62 117
[27] Guéry-Odelin D, Muga J G, Ruiz-Montero M J and Trizac E 2014 Phys. Rev. Lett. 112 180602
[28] Chen Y H, Xia Y, Wu Q C, Huang B H and Song J 2016 Phys. Rev. A 93 052109
[29] Baksic A, Ribeiro H and Clerk A A 2016 Phys. Rev. Lett. 116 230503
[30] Lu M, Xia Y, Shen L T, Song J and An N B 2014 Phys. Rev. A 89 012326
[31] Lu M, Xia Y, Shen L T and Song J 2014 Laser Phys. 24 105201
[32] Chen Y H, Xia Y, Chen Q Q and Song J 2014 Phys. Rev. A 89 033856
[33] Chen Y H, Xia Y, Song J and Chen Q Q 2015 Sci. Rep. 5 15616
[34] Chen Y H, Xia Y, Chen Q Q and Song J 2015 Phys. Rev. A 91 012325
[35] Chen Y H, Xia Y, Chen Q Q and Song J 2014 Laser Phys. Lett. 11 115201
[36] Shi X and Wei L F 2015 Laser Phys. Lett. 12 015204
[37] Zhang J, Kyaw T H, Tong D M, Sjöqvist E and Kwek L C 2015 Sci. Rep. 5 18414
[38] Liang Y, Wu Q C, Su S L, Ji X and Zhang S 2015 Phys. Rev. A 91 032304
[39] Liang Y, Song C, Ji X and Zhang S 2015 Opt. Express 23 23798–810
[40] Liang Y, Ji X, Wang H F and Zhang S 2015 Laser Phys. Lett. 12 115201
[41] Song X K, Zhang H, Ai Q, Qiu J and Deng F G 2016 New J. Phys. 18 023001
[42] Lin J B, Liang Y, Song C, Ji X and Zhang S 2016 J. Opt. Soc. Am. B 33 519–24
[43] Chen Z, Chen Y H, Xia Y, Song J and Huang B H 2016 Sci. Rep. 6 22202
[44] Wu J L, Song C, Ji X and Zhang S 2016 J. Opt. Soc. Am. B 33 2026–32
[45] He S, Su S L, Wang D Y, Sun W M, Bai C H, Zhu A D, Wang H F and Zhang S 2016 Sci. Rep. 6 30929
[46] Wu J L, Ji X and Zhang S 2016 Sci. Rep. 6 33669
[47] Berry M V 1987 Proc. R. Soc. Lond. A 414 31
[48] Ibáñez S, Chen X and Muga J G 2013 Phys. Rev. A 87 043402
[49] Song X K, Ai Q, Qiu J and Deng F G 2016 Phys. Rev. A 93 052324
[50] Huang H H, Chen Y H, Wu Q C, Song J and Xia Y 2016 Laser Phys. Lett. 13 105202
[51] Kang Y H, Chen Y H, Wu Q C, Huang B H, Song J and Xia Y 2016 Sci. Rep. 6 36737
[52] Serafini A, Mancini S and Bose S 2006 Phys. Rev. Lett. 96 010503
[53] Wilk T, Simon C, Kuhn A and Remp G 2007 Science 317 488
[54] Weber B, Specht H P, Müller T, Bochmann J, Mücke M, Moehring D L and Remp G 2009 Phys. Rev. Lett. 102 030501
[55] Demirplak M and Rice S A 2003 J. Phys. Chem. A 107 9937
[56] Demirplak M and Rice S A 2008 J. Chem. Phys. 129 154111
[57] Berry M V 2009 J. Phys. A: Math. Theor. 42 365303
[58] Vitanov N V, Halfmann T, Shore B W and Bergmann K 2001 Annu. Rev. Chem. Phys. 52 763
[59] Spillane S M, Kippenberg T J, Painter O J and Vahala K J 2003 Phys. Rev. Lett. 91 043902
[60] Spillane S M, Kippenberg T J, Vahala K J, Goh K W, Wilcut E and Kimble H J 2005 Phys. Rev. A 71 013817