A unitary and relativistic model for $\pi\eta$ and $\pi\pi$ photoproduction

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Abstract. We describe a model of $\pi\eta N$ photoproduction based on a phenomenological Lagrangian approach that satisfies two-body unitary and is relativistic. Unitarity is ensured by using the Lippmann-Schwinger equation to iterate the vertices and dress the propagators to all orders, and by including all possible two-body and quasi-two-body intermediate channels. This model has been tested by investigating $\pi\pi N$ photoproduction. A preliminary comparison of our calculation to an existing $\pi\pi N$ photoproduction study is made, and is shown to produce consistent results. We have also calculated the sum of the nonresonant Born diagrams in $\pi\eta N$ photoproduction and compared the resulting observables to invariant mass distributions from the CB-ELSA collaboration.

1. Introduction
The scattering region of roughly 1.7 GeV center-of-mass energy and beyond is generally full of many uncertainties with regard to baryon resonance properties (masses, width, etc.). There are also expectations of finding unidentified baryon resonances, since most of our knowledge of baryon resonances comes from $\pi N$ scattering, and it is conceivable that some baryon resonances do not couple strongly to this channel. This picture is shown to be valid by model calculations. For example, Capstick and Roberts show that many baryon resonances in the 1.7 GeV region tend to couple more strongly to $\eta N$, $\omega N$, and various quasi-two-body $N\pi\pi$ channels than to $N\pi[1]$.

In view of this situation, it is not surprising that both theorists and experimentalists have been looking at a number of reactions in which they expect to find new baryon resonances. Using the $\pi\eta$ photoproduction reaction, $\gamma N \rightarrow N\pi\eta$, known baryon resonances are also more easily studied and understood. This is because the threshold is quite high, around 1.6 GeV center-of-mass energy, which reduces the influence of nonresonant interactions in the region of interest compared to using a low-threshold reaction.

The Crystal-Barrel collaboration at ELSA is now performing an experiment on this reaction, while Jefferson Laboratory in principle has $N\pi\eta$ data which awaits analysis. The CB-ELSA collaboration expects that they will be able to produce an analysis of the data by one or two years from now. As with any experimental effort, theoretical guidance from a model of this reaction would be welcome.

When the data has been analyzed, it should be compared to predictions of quark model
calculations. This comparison cannot be made directly since quark models produce baryon properties like masses and widths, while the experiments produce scattering amplitudes. A model that links these quantities is required.

The model that we mention above needs to contain a set of parameters that can be obtained from experiments on other reactions and quark models and whose output is the scattering observables of the reaction. This is important if we would like to guide the experimental effort using our knowledge of baryon resonances. When the experimental data has been analyzed, it is very important also that this model is able to extract the masses and widths of the baryon resonances involved in the reaction from a fit to the data.

This reaction bears a close resemblance to $\pi\pi$ photoproduction, which can be calculated using the same model with minor changes. The latter reaction has been studied extensively, and much data and many models are available. We will test our model by comparing its $\pi\pi$ photoproduction results to data or to other models.

2. A proposed model

We propose a model which is unitary and relativistic based on a dynamical coupled-channel approach. Unitarity is related to conservation of probability, which is an important physical requirement. The dynamical coupled-channel approach treats the intermediate rescattering effects using the correct dynamics, and introduces important effects as shown in our previous work [2] as well as the work of others. This is also important if we would like to compare masses and widths extracted from data using our model to those calculated using quark models. Quark model results are bare quantities, as opposed to dressed quantities where the rescattering effects have been taken into account. Finally, using a relativistic approach is important if we want to study a reaction in which the kinetic energy of the interacting particles is the same order of magnitude as the rest energy.

In our model, unitarity is imposed by including rescattering to all orders. We begin the construction of our model by writing down the tree-level diagrams of the reaction, given in Fig. 1. Here, $B$, $B'$, and $M$ label all possible baryon and meson intermediate states. We can start by including low spin and energy resonances and work toward higher spin and energy, and then drop those that do not significantly contribute to the scattering amplitudes.

After including the tree-level diagrams, the next thing to do is to include rescattering effects by
dressing the bare vertices and propagators. For hadronic dressed vertices

$$\Gamma_{MB \rightarrow N^*_i}(E) = \Gamma_{MB \rightarrow N^*_i}^0(E) + \sum_{M'B'} t_{MB,M'B'}(E) G_{M'B'}(E) \Gamma_{M'B' \rightarrow N^*_i}^0(E),$$

(1)

and for electromagnetic dressed vertices

$$\Gamma_{\gamma N \rightarrow N^*_i}(E) = \Gamma_{\gamma N \rightarrow N^*_i}^0(E) + \sum_{M'B'} t_{\gamma N,M'B'}(E) G_{M'B'}(E) \Gamma_{M'B' \rightarrow N^*_i}^0(E).$$

(2)

The quantity $G_{MB}$ in Eqs. (1) and (2) is the two-body propagator of the propagating baryon and meson intermediate states. The dressed baryon propagator is given by

$$G_{ij}(E) = G_{ij}^0(E) + \sum_{kl} G_{ik}^0(E) \Sigma_{kl}(E) G_{lj}(E),$$

(3)

which can be solved to give

$$G(E)_{ij} = \frac{1}{(E - M_{N^*_i}^0)\delta_{ij} - \Sigma_{ij}(E)},$$

(4)

where the indices $i$ and $j$ in $G_{ij}(E)$ refer to baryon states under consideration. The self-energy term $\Sigma_{ij}(E)$ is

$$\Sigma_{ij}(E) = \sum_{M'B'} \Gamma_{N^*_i \rightarrow M'B'}^0(E) G_{M'B'}(E) \Gamma_{M'B' \rightarrow N^*_i}(E).$$

(5)

The corrections presented here are enough to implement quasi-two-body unitarity. We further restrict the set of intermediate channels to $N \pi$, $N \eta$, $\Delta(1232) \eta$, $N(1535) \pi$, and $N p(770)$ in our initial calculation. Since $N \pi \eta$ is a three-body final state, a rigorous treatment of unitarity would involve solving the Faddeev equation, which we do not do here.

3. Present results

3.1. Preliminary study of $\pi \pi$ photoproduction

Besides studying $\pi \eta$ photoproduction, we also study $\pi \pi$ photoproduction as a check of $\pi \eta$ photoproduction. In addition, few models of $\pi \pi$ photoproduction include correction from rescattering and unitarity, and we are interested in seeing their effects.

At this point, we have only included two baryon resonances $\Delta(1232)$ and $N(1520)$ in our model. Our choice not to include $N(1440)$, which lies between the two, is simply because its contribution has been shown to be small by Tejedor and Oset [3], and more recently by Hirata, Katagiri, and Takaki [4]. The latter do not include it and still obtain a very good fit to the data.

We use coupling constants at the vertices which are fit to the partial widths of the baryon resonances involved. As a first step, we do not include the unitarity corrections described above. Instead, we include a rudimentary form of rescattering effect in the propagators of the excited baryon states.

The results for total cross-sections are given in Fig. 2. Our results are similar to those of Tejedor and Oset, including missing strength in the $\gamma p \rightarrow n \pi^+ \pi^0$ and $\gamma n \rightarrow p \pi^- \pi^0$ reactions. This gives us confidence that our results are correct.

3.2. Preliminary study of $\pi \eta$ photoproduction

The invariant mass distributions of this reaction show that a substantial fraction of $N \pi \eta$ events go through $\Delta(1232) \eta$ and $N(1535) \pi \eta$ quasi-two-body channels [5]. Little experimental evidence exists for decays to those quasi-two body final states, so no such decays are listed for known
baryons by the PDG [6]. A strong signal in the invariant mass distribution of $\pi\eta$ is also observed around the mass of $a_0(980)$. Our model for $\pi\eta$ photoproduction is less developed than that of $\pi\pi$, so we show only the results for total cross-section of the nonresonant diagrams to get some sense of its size. Fig. 3 shows the total cross-section of the nonresonant contributions to $\pi\eta$ photoproduction compared to $\pi\pi$ photoproduction, calculated without form-factors in all vertices. The total cross-section of the nonresonant contribution of the $\pi\eta$ photoproduction is much smaller than that of $\pi\pi$ photoproduction. This may also be true of a preliminary cross-section measurement by CB-ELSA [5].

Figure 2. Predicted total cross-section for $\pi\pi$ photoproduction in our preliminary model. Full lines uses a pseudovector $\pi N N$ vertex, dashed lines uses a pseudoscalar $\pi N N$ vertex. Data for $\gamma n \rightarrow p\pi^-\pi^0$ is taken from ref. [7], while the rest are from ref. [8].

4. Conclusion
The scattering region of about 1.7 GeV and beyond is full of many uncertainties with regard to baryon resonance properties, which can be better understood by studying $\pi\eta$ photoproduction. A model is needed for this reaction and we propose one which is unitary, relativistic, and based on a dynamical coupled-channel approach. Preliminary results for $\pi\pi$ photoproduction, calculated with an approximate treatment of unitarity agree with those of other groups using the same approach. The nonresonant tree-level diagrams for $\pi\eta$ photoproduction have also been studied and the resulting cross-section has been shown to be of a reasonable size.

5. Bibliography
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