The cosmological solutions of the non-minimal $Y(R)F^2$ theory which are compatible with FRW space-time are investigated. In order to avoid the isotropy violation of a vector field, it can be considered that the presence of a triplet of vector fields satisfying $SO(3)$ symmetry, or the average of randomly oriented electromagnetic fields over the sufficiently large volume. By considering the symmetry arguments, the inflation driven by the non-minimally coupled vector fields is studied. Then the cosmological solutions and corresponding models are obtained with the power law, hyperbolic and hybrid scale functions.
I. INTRODUCTION

Despite the achievements of Einstein’s General Relativity in the solar system, the greatest mysteries of the universe at the large cosmological scale remain unsolved. After the observations which are the accelerated expansion of the universe \cite{1-5} and the flatness of galactic rotation curves \cite{6, 7}, so many theories were proposed to solve the mysteries. Although Einstein’s theory with cosmological constant \cite{8} is consistent with the observational data, it brings out the other problems related with the source of the cosmological constant \cite{9-17}. Thus, unless the dark matter particle is observed directly, one can consider such modifications as $f(R)$ gravity in order to explain these observations, see \cite{18-20} for some reviews.

Recent observations indicate the existence of magnetic fields in the gravitational systems such as galaxies and stars \cite{21-26}. The origin of the magnetic fields is another mystery of the universe. It may be considered that the large scale magnetic fields are seeded from the primordial magnetic fields. There are some mechanism to explain the generation and amplification of the magnetic fields such as plasma vortical motion, inflation, cosmic strings, quark hadron phase transition and electroweak phase transition \cite{26}. In the phase transition mechanism, the motion of dipole charge layers formed on the surface of the phase transition generates a current in the electron-positron plasma, and hence leads to randomly magnetic field fluctuations on a thermal wavelength scale, which mean the stochastic background magnetic fields. Therefore, on the scales much smaller than the Hubble radius, the effect of magnetic fields to the isotropy can be ignored, and the stochastic magnetic fields become compatible with the FRW geometry. Then the energy-momentum tensor of the electromagnetic field turns out to be the energy-momentum tensor of the radiation fluid during inflation phase.

In the presence of electromagnetic fields, firstly, one can consider only the Maxwell Lagrangian minimally coupled to Einstein gravity which known as Einstein-Maxwell theory. However, the minimal Lagrangian has not any solution to explain inflation and the primordial magnetic fields without any couplings. Then it can be considered to modify the minimal theory. Moreover, one can consider to break the conformal invariance of the electromagnetic field Lagrangian by the $RF^2$ type couplings to amplify the magnetic fields at inflation \cite{27}. 


Also the $RF^2$ type couplings and their effects on gravity are studied in \cite{28,30}, and they were derived from reduction of higher dimensional gravity theories \cite{31,33} and the calculation of vacuum polarization in a curved background \cite{34}. Furthermore it is important to note that they also have used to explain the generation of primordial magnetic fields \cite{27,35,41}. However, the amplitude of the produced seed magnetic fields was very small for this case. Then, they were extended to the $R^n F^2$ type modifications in order to produce sufficiently large the seed magnetic fields which lead to the current values. Since the inflationary era has very high gravitational and electromagnetic fields, the non-minimal $Y(R)F^2$ type couplings may arise in such extreme cases. Also they can be reason of the flatness of galactic rotational curves \cite{42,45} and they accept the regular black hole solutions \cite{46}.

Although the energy-momentum tensor is anisotropic in the presence of a vector field, there are interesting methods to obtain isotropic solutions and make the energy-momentum tensor compatible with the FRW geometry. First one is to introduce a triplet of orthogonal vector fields \cite{47,49}, that is, one can build models which lead to isotropic cosmology starting from a non-Abelian gauge theory. Second is to consider a large number of vector fields in random spatial directions, and take the spatial average of the fields which was firstly proposed by Tolman and Ehrenfest \cite{50}. The averaging procedure was applied to the non-linear electromagnetic theory in literature extensively \cite{51,57}. In this procedure, it is assumed that the wavelength of the electromagnetic fields is much smaller than the space-time curvature. Then the spatial average of the fields is considered on the small volumes, that is, the stochastic electromagnetic fields are used. Then the averaged energy-momentum tensor in the stochastic background becomes energy-momentum of a radiation fluid in the minimal case. In this study, we look for the isotropic solutions of the non-minimal $Y(R)F^2$ theory inspired by the symmetry arguments mentioned above. Then we compare the obtained solutions with the previous anisotropic cases.

II. FIELD EQUATIONS FOR THE SYMMETRY ARGUMENTS

We start with the following Lagrangian of the non-minimal $Y(R)F^2$ theory \cite{42,45}

\[ L(e^a, \omega^a_b, A) = \frac{1}{2\kappa^2} R * 1 - Y(R)F \wedge \ast F + \lambda_a \wedge T^a, \]  

(1)
where $\kappa$ is the gravitational coupling constant, $R$ is the Ricci scalar obtained from the curvature 2-form $R_{ab} = d\omega_{ab} + \omega_{ac} \wedge \omega^c_b$ by the interior product $\iota_a$ such as $\iota_b \iota_a R^{ab}$ and $\omega_{ab}$ is the connection 1-form. In this Lagrangian, the Hodge star operator $\ast$ maps $p$ form to $4-p$ form, and $\ast 1 = \epsilon^{0123}$ corresponds to the oriented volume element given by the abbreviated notation $e^{abc} = e^a \wedge e^b \wedge \cdots$ for the orthonormal basis 1-forms $e^a, e^b, \cdots$. Then the curved space-time metric is determined by $g = \eta_{ab} e^a \otimes e^b$, where the flat Minkowski metric $\eta_{ab}$ has the diagonal elements $(-1, 1, 1, 1)$. Here, the non-minimal modification is constituted by multiplying an arbitrary function of Ricci scalar $Y(R)$ with the Maxwell Lagrangian $F \wedge \ast F$. Here $F = dA$ is the Maxwell tensor and $A$ is the potential one form. Then the torsion two form $T^a = De^a = de^a + \omega_{ab} \wedge e^b$ is constrained to zero by the Lagrange multiplier $\lambda_a$ and it leads to the Levi-Civita connection. By taking the variation of the Lagrangian, we obtain the following gravitational field equation,

$$
-\frac{1}{2\kappa^2} R^{bc} \wedge \ast e_{abc} = Y(\iota_a F \wedge \ast F - F \wedge \iota_a \ast F) + Y_R F_{mn} F^{mn} \ast R_a \\
+ D[\iota^b d(Y_R F_{mn} F^{mn})] \wedge \ast e_{ab},
$$

up to a closed form, where $Y_R = dY/dR$. The trace of the field equation gives the following constraint

$$
Y_R F_{mn} F^{mn} = -\frac{1}{\kappa^2}.
$$

The constraint equation eliminate the last term in the field equation (2) and the possible instabilities and complexities of the higher order derivatives. Thus we have the following field equations for the theory which have no more than second order derivatives,

$$
-\frac{1}{2\kappa^2} R^{bc} \wedge \ast e_{abc} = Y(\iota_a F \wedge \ast F - F \wedge \iota_a \ast F) - \frac{1}{\kappa^2} \ast R_a.
$$

We can also start with a Lagrangian which has the constraint (3) by a Lagrange multiplier. By taking the variation of the Lagrangian we can obtain again the field equation (4). It is interesting to note that the constraint (3) is not linearly independent from the field equation (4). When we take the exterior covariant derivative of the field equation (4), we obtain the constraint (3), see [58] for details. Thus the constraint corresponds to conservation of the energy-momentum of this model. We consider the following flat isotropic FRW metric,

$$
g = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)
$$
where \( a(t) \) is the scale factor of the universe. The presence of electromagnetic fields leads to a preferential direction and anisotropy in the space-time geometry. The following averaging procedure which was firstly proposed by Tolman and Ehrenfest \[50\] can be applied over the spatial volume to obtain isotropic energy momentum tensor

\[
< E_i > = 0, \quad < B_i > = 0,
\]

\[
< E_i B_j > = 0, \quad < E_i E_j > = \frac{1}{3} E^2 \eta_{ij}, \quad < B_i B_j > = \frac{1}{3} B^2 \eta_{ij}.
\]

(6)

Here \( E_i \) and \( B_i \) electric and magnetic components of the electromagnetic field and the average of the electric component \( E_i \) is defined by \[55\]

\[
< E_i > = \lim_{V \to V_0} \frac{1}{V} \int E_i a^3 d^3 x
\]

(8)

where \( V = \int a^3 d^3 x \) and \( V_0 \) is the sufficiently large spatial volume satisfying these relations.

By the symmetry arguments and the process, the Maxwell energy-momentum tensor becomes

\[
< F_{a} \wedge * F - F \wedge \tau_{a} * F > = (\rho_F + p_F) u_a * u + p_F * e_a ,
\]

(9)

which is the energy-momentum tensor of the isotropic radiation fluid. Here \( u = u_a e^a \) and \( u_a \) time-like, unit vector field. The energy density and pressure of the electromagnetic field are given as

\[
\rho_F = E^2 + B^2 ,
\]

\[
p_F = \frac{1}{3} (E^2 + B^2) = \frac{\rho_F}{3} .
\]

(10)

(11)

By considering the definitions \(< E_1^2 + E_2^2 + E_3^2 > = E^2 \) and \(< B_1^2 + B_2^2 + B_3^2 > = B^2 \), we calculate

\[
< F_{mn} F^{mn} > = 2(B^2 - E^2) .
\]

(12)

Alternatively, it is interesting to note that the symmetry arguments given in \[47-49\] can be used to satisfy homogeneity and isotropy of the early universe. In these studies, a vector
potential 1-form which has a triplet of identical and orthogonal vector fields is considered
as \( A = A^I T_I = \phi(t) \delta^I_\mu dx^\mu T_I \). Here \( I = 1, 2, 3 \) and \( T_I \) is the generator of the internal non-
Abelian SU(2) gauge group space. Then this gauge potential leads to \( E_1^2 = E_2^2 = E_3^2 = \frac{E^2}{3} \),
and \( B_1^2 = B_2^2 = B_3^2 = \frac{B^2}{3} \) as in the averaging procedure.

Thus the gravitational field equation (4) turns out to be

\[
-\frac{1}{2\kappa^2} R_{bc} \wedge \ast e^{abc} = \tau^a = Y(\rho_F + p_F)u^a * u + Y p_F * e^a - \frac{1}{\kappa^2} * R^a .
\]  

(13)

We use the definition of effective energy-momentum tensor \( \tau_a = \tau_{ab} * e^b \), and obtain the total
effective energy density and pressure from \( \rho = \tau_{00}, \ p = \tau_{11} = \tau_{22} = \tau_{33} \).

\[
\rho = Y \rho_F + \frac{3}{\kappa^2} \dot{a}, \quad p = Y p_F - \frac{1}{\kappa^2} (\dot{a} + 2 \ddot{a}^2)
\]

(14)

Here \( \dot{a} = \frac{da}{dt} \). Also we can write them in terms of the Hubble parameter \( H = \frac{\dot{a}}{a} \) as

\[
\rho = Y \rho_F + \frac{3}{\kappa^2} (\dot{H} + H^2), \quad p = \frac{\rho}{3} - \frac{2}{\kappa^2} (\dot{H} + 2H^2).
\]  

(15)

Thus the modified gravitational field equation (13) gives us the following two linearly de-
pendent differential equations for the flat FRW metric (5)

\[
\frac{3}{\kappa^2} \ddot{a}^2 = \frac{3}{\kappa^2} \frac{\ddot{a}}{a} + Y(E^2 + B^2)
\]

(16)

\[
\frac{1}{\kappa^2} (2\dot{a} + \frac{\ddot{a}^2}{a^2}) = \frac{1}{\kappa^2} (\frac{\ddot{a}}{a} + 2 \dddot{a}^2) - \frac{1}{3} Y(E^2 + B^2)
\]

(17)

which lead to only the following differential equation

\[
\frac{3}{\kappa^2} (\frac{\ddot{a}}{a} - \frac{\dddot{a}^2}{a^2}) + Y(E^2 + B^2) = 0 .
\]

(18)

Here we note that we have also the constraint equation from (3)

\[
\frac{dY}{dR} = \frac{1}{2\kappa^2(E^2 - B^2)} .
\]

(19)

It is easy to check that the constraint equation (19) can also be obtained by taking the
derivative of the differential equation (18). Also, the conservation of energy momentum
tensor can be calculated as

\[
D \tau^a = D(Y(\rho_F + p_F)u^a u_b) \wedge \ast e^b + d(Y p_F) \wedge \ast e^a - \frac{1}{\kappa^2} D \ast R^a = 0
\]

(20)
and the following differential equation is obtained from (20) by using the condition (19) and the metric (5),
\[
\frac{\dot{R}}{2\kappa^2} \left( 1 + \frac{E^2 + B^2}{E^2 - B^2} \right) + Y(E^2 + B^2) + 4Y(E^2 + B^2)H = 0
\]
(21)
where \( R = 6\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a} \right) \). We also note that the conservation equation (21) is equivalent to the constraint equation (19) and it satisfies the solution
\[
B = \frac{B_0}{a^2}, \quad E = \frac{E_0}{Ya^2}
\]
(22)
where \( E_0 \) and \( B_0 \) are integration constants. The non-zero electric fields can also be important at the beginning. Since it is considered that the universe has the plasma phase with the free electric charges, there exist non-zero electric fields in the plasma. Therefore we can consider both cases with non-zero electric fields or magnetic fields.

For all these models which have the electric and magnetic fields, we have one differential equation (18) and two unknown functions \( Y(R(t)) \) and \( a(t) \). That is, each non-minimal function \( Y(R) \) gives a scale factor \( a(t) \), and vice versa. Therefore, instead of starting with a specific \( Y(R)F^2 \) model, we start with some known scale factors and determine the corresponding models.

III. THE HYPERBOLIC EXPANSION

We firstly consider the following hyperbolic scale function
\[
a(t) = \sinh^k(\alpha t)
\]
(23)
where \( k \) and \( \alpha \) are positive constants. Then the differential equation (18) gives the following solution for \( E^2 = 0 \)
\[
Y(t) = \frac{3k\alpha^2 \sinh^{4k-2}(\alpha t)}{k^2 B_0^2},
\]
(24)
\[
B(t) = B_0 \sinh^{-2k}(\alpha t).
\]
(25)
On the other hand, we can also find the solution with $B^2 = 0$

$$
Y(t) = \frac{\kappa^2 E_0^2}{3k\alpha^2 \sinh^{4k-2}(\alpha t)}, \quad (26)
$$

$$
E(t) = \frac{3\alpha^2 \sinh^{2k-2}(\alpha t)}{E_0\kappa^2}. \quad (27)
$$

We note that the equations (16)-(22) and the solutions (24)-(27) satisfy the duality symmetry given by $B \rightarrow -Y E$, $Y \rightarrow \frac{1}{Y}$ and $B_0 \rightarrow -E_0$.

For these solutions, the Ricci curvature scalar is calculated as

$$
R(t) = \frac{6\alpha^2 k(2k - 1)}{\sinh^2(\alpha t)} + 12\alpha^2 k^2. \quad (28)
$$

By solving $\sinh(\alpha t)$ in terms of $R$ from (28) and substituting it in (26) and (24), the non-minimal function $Y(R)$ can be obtained as follows for the existence of magnetic fields and electric fields, respectively

$$
Y(R) = \frac{Y_0}{B_0^2} (R - 12\alpha^2 k^2)^{1-2k}, \quad Y(R) = \frac{E_0^2}{Y_0} (R - 12\alpha^2 k^2)^{2k-1}. \quad (29)
$$

where $Y_0 = \frac{3\alpha^2}{\kappa^2} (6\alpha^2 (2k - 1))^{2k-1}$. We calculate the Hubble parameter and deceleration parameter as

$$
H = \frac{\dot{a}}{a} = \alpha k \coth(\alpha t) = \alpha k \left(1 + (1 + z)^{2/k}\right)^{1/2}, \quad (30)
$$

$$
q = -1 + \frac{d}{dt} \left(\frac{1}{H}\right) = \frac{1}{k \sinh^2(\alpha t)} - 1 = \frac{1}{k + k(1 + z)^{-2/k} - 1} \quad (31)
$$

which are equivalent for the above non-minimal functions or models. In the last step, the parameters are rewritten in terms of the cosmic redshift $z$ by using the relation $a = \frac{1}{1+z}$. We see that the Hubble parameter is infinity at $t = 0$ and it decreases to the constant value $H = \alpha k$ as $t \rightarrow \infty$. The deceleration parameter decreases from $q(0) = \frac{1-k}{k}$ to $-1$, and the phase transition occurs at $t = \frac{1}{\alpha} \text{arccosh}(\frac{1}{\sqrt{k}})$. It must be $0 < k < 1$ to obtain the phase transition from deceleration to acceleration. For $k > 1$, $q$ is negative at the beginning of the universe. The effective energy density $\rho$ and pressure $p$ are calculated as

$$
\rho = \frac{3\alpha^2 k^2 \cosh^2(\alpha t)}{\kappa^2 \sinh^2(\alpha t)} = \frac{3\alpha^2 k^2}{\kappa^2} (1 + (1 + z)^{2/k}), \quad (32)
$$

$$
p = -\frac{\alpha^2 k (3k \sinh^2(\alpha t) - 2)}{\kappa^2 \sinh^2(\alpha t)} = -\frac{\alpha^2 k}{\kappa^2} (3k + (3k - 2)(1 + z)^{2/k}). \quad (33)
$$
and then we have the ratio

\[ w = \frac{p}{\rho} = \frac{2}{3k \cosh^2(\alpha t)} - 1 = \frac{2(1 + z)^{2/k}}{3k(1 + (1 + z)^{2/k})} - 1 \tag{34} \]

It is interesting to note that the Hubble parameter, energy density and pressure are singular at \( t = 0 \) or \( a = 0 \) and they decrease to constant values later times. This shows that the existence of the Big Bang singularity at the starting point of the universe.

We can write the pressure in terms of \( \rho \) by using (32) and (33) as the equation of state

\[ p = -\rho + f(\rho), \quad f(\rho) = \frac{2}{3k}(\rho - \frac{3(3\alpha^2k^2)}{\kappa^2}) \tag{35} \]

Under the condition \( \left| \frac{f(\rho)}{\rho} \right| \ll 1 \), the inflationary parameters such as the spectral index \( n_s \), the tensor to scalar ratio \( r \) and the running spectral index \( \alpha_s \) can be expressed as follows

\[ n_s \approx 1 - 6 \frac{f(\rho)}{\rho}, \quad r \approx 24 \frac{f(\rho)}{\rho}, \quad \alpha_s \approx -9 \frac{f^2(\rho)}{\rho^2} \tag{36} \]

which was given in [60]. Then we can write

\[ n_s = 1 - \frac{r}{4} = 1 - 2\sqrt{-\alpha_s} = 1 - \frac{4}{k\rho}(\rho - \frac{3(3\alpha^2k^2)}{\kappa^2}) \tag{37} \]

for the model. Recent Planck data analysis [9, 10] gives the constraints on the parameters as \( n_s = 0.968 \pm 0.006(68\% CL), r < 0.11(95\% CL) \) and \( \alpha_s = -0.003 \pm 0.007(68\% CL) \). When we take \( n_s = 0.968 \) in (37), we obtain \( r = 0.128 \) and \( \alpha_s = 2.56 \times 10^{-4} \) which leads to

\[ \frac{f(\rho)}{\rho} = \frac{2}{3k\rho}(\rho - \frac{3(3\alpha^2k^2)}{\kappa^2}) = 5.3 \times 10^{-3} \tag{38} \]

From (38) the parameters \( \alpha \) and \( k \) can be chosen properly to obtain a certain energy density or magnetic field during the inflation.

IV. THE POWER LAW EXPANSION

Secondly, we take into account solutions with the well-known power law scale factor

\[ a(t) = a_0 t^n \tag{39} \]
where $n$ positive real constant. When we substitute the power law function (39) and the magnetic field $B$ (22) in equation (18), we obtain

$$Y(t) = \frac{3nt^{4n-2}}{\kappa^2 B_0^2}, \quad B(t) = \frac{B_0}{t^{2n}}$$

(40)

for $E^2 = 0$, we also obtain solutions with $B^2 = 0$ as result of the duality transformation (45)

$$Y(t) = \frac{E_0^2 \kappa^2 t^{2-4n}}{3n}, \quad E(t) = \frac{3nt^{2n-2}}{E_0 \kappa^2}.$$  

(41)

For the scale function (39), the Ricci scalar becomes

$$R(t) = \frac{12n^2 - 6n}{t^2}.$$  

(42)

By taking the inverse function of $R(t)$, we obtain the non-minimal functions depending on $R$ in the models without electric fields or magnetic fields, respectively

$$Y(R) = \frac{3n(12n^2 - 6n)^{2n-1}}{\kappa^2 B_0^2} R^{1-2n}, \quad Y(R) = \frac{\kappa^2 E_0^2}{3n(12n^2 - 6n)^{2n-1}} R^{2n-1}.$$  

(43)

Then the cosmological parameters become

$$H = \frac{n}{t} = na_0^{1/n} (1 + z)^{1/n}, \quad q = \frac{1}{n} - 1.$$  

(44)

We note that we have the constant deceleration parameter for a specific $n$ value in these models. The effective energy density and pressure turn out to be

$$\rho = \frac{3n^2}{\kappa^2 t^2} = \frac{3n^2 a_0^{2/n}}{\kappa^2} (1 + z)^{2/n}, \quad p = -\frac{n(3n - 2)}{\kappa^2 t^2} = -\frac{n(3n - 2)a_0^{2/n}}{\kappa^2} (1 + z)^{2/n},$$

(45)

which gives the equation of state parameter

$$w = \frac{2}{3n} - 1.$$  

(46)

We also see that the models shows the Big Bang singularity at $t = 0$, since $H$, $p$ and $\rho$ goes to infinity as $a \to 0$. From (45) we see that

$$p = -\rho + \frac{2\rho}{3n}$$  

(47)
and by considering the Planck data and perfect fluid approach $\frac{f(\rho)}{\rho} = \frac{2}{3n} = 5.3 \times 10^{-3}$ gives $n \approx 126$ at the inflation. We note that the anisotropic solutions which obtained in [40] with the mean scale factor $v = v_1 t^{n-1/3}$ have the same cosmological parameters (the mean Hubble and deceleration parameters) with the isotropic scale factor $a = a_0 t^n$, where $n = \alpha - 1/3$. Therefore the analysis of the mean quantities in [40] gives the same results with this isotropic case. The both approach give approximately the same energy density. The total energy density can be written in terms of e-folds $N$. This analysis shows that the modified theory gives reasonable inflationary parameters during the inflation with the power law expansion.

It is worth to note that for $n = 2/3$ the model reproduce the matter dominated universe with $Y(R) = \frac{2}{\kappa^2 B_0^2} (\frac{4}{3})^{1/3} R^{-1/3}$ or $Y(R) = \frac{\kappa^2 E_0^2}{2} (\frac{4}{3})^{-1/3} R^{1/3}$ and $R(t) = \frac{4}{3n^2}$.

V. HYBRID EXPANSION LAW

Let generalize the previous power law model considering by the hybrid expansion law

$$a(t) = a_0 t^n e^{\alpha t}$$

(48)

where $n$ and $\alpha$ positive constants. Then we obtain

$$B(t) = \frac{B_0}{a_0^2 t^{2n} e^{2\alpha t}}, \quad Y(t) = \frac{3n a_0^4 t^{4n-2} e^{4\alpha t}}{B_0^2 \kappa^2},$$

(49)

$$R(t) = \frac{12\alpha^2 t^2 + 24\alpha nt + 12n^2 - 6n}{t^2}.$$  

(50)

By solving $t$ from (50), we find the non-minimal function in term of $R$ for the model as

$$Y(R) = \frac{3n a_0^4}{B_0^2 \kappa^2} \left( \frac{12n\alpha + X}{R - 12\alpha^2} \right)^{4n-2} e^{4\alpha (12n\alpha + X) / R - 12\alpha^2}$$

(51)

where $X = \sqrt{(12n^2 - 6n)R + 72n\alpha^2}$. Furthermore, we have also a dual model with an electric field, which obtained by the duality transformation $Y \rightarrow \frac{1}{Y}$. We calculate the related parameters for these models

$$H = \alpha + \frac{n}{t}, \quad q = \frac{n}{(\alpha t + n)^2} - 1$$

(52)
then the energy density, pressure and $\omega$ can be expressed as

$$
\rho = \frac{3(\alpha t + n)^2}{\kappa^2 t^2}, \quad p = \frac{2n - 3(\alpha t + n)^2}{\kappa^2 t^2}, \quad \omega = \frac{2n}{3(\alpha t + n)^2} - 1. \quad (53)
$$

We note that $\alpha = 0$ gives us the previous model with the well-known power law expansion and $n = 0$ is the exponential expansion. Also, in the hybrid model the constants can be fixed so that the power law is more effective in the early universe. Thus the non-minimal $Y(R)F^2$ model with the non-minimal function $Y$ (or dual of it with $1/Y$) has the solution with the hybrid expansion. From (53) we obtain

$$
p = -\rho + \frac{2(3\alpha^2 - \kappa^2 \rho)^2}{n\kappa^2(\sqrt{3\kappa^2 \rho} - 3\alpha)^2}. \quad (54)
$$

By using the perfect fluid description of the inflationary parameters, we can write

$$
\frac{f(\rho)}{\rho} = \frac{2(3\alpha^2 - \kappa^2 \rho)^2}{n\kappa^2\rho(\sqrt{3\kappa^2 \rho} - 3\alpha)^2}. \quad (55)
$$

By taking the spectral index value $n_s$ from the Planck observations we get

$$
\frac{2(3\alpha^2 - \kappa^2 \rho)^2}{n\kappa^2\rho(\sqrt{3\kappa^2 \rho} - 3\alpha)^2} = 5.3 \times 10^{-3} \quad (56)
$$

We see that it is possible to choose the parameters of the model $\alpha$ and $n$ as appropriately to satisfy the cosmological indices. Then the model is also give consistent solutions with observations.

VI. CONCLUSIONS

In this paper, we have shown that there are a wide class of isotropic inflationary solutions of the non-minimal $Y(R)F^2$ gravity by the symmetry arguments of the electromagnetic fields. Firstly, we have derived the modified field equations under the averaging procedure which leads to isotropic energy momentum tensor. Here we note that the isotropy can also be obtained by considering a triplet of vector fields satisfying $SO(3)$ symmetry [47,49]. Then we obtained isotropic solutions of the model with the power law, hyperbolic and hybrid expansions which have electric and magnetic fields.
We note that the studied anisotropic case in [40] have the same features with the power-law model for the averaged field approach, and it approaches to the isotropic case. Furthermore, the obtained isotropic solutions with power law, hyperbolic and hybrid expansions have the Big Bang singularity at \( t = 0 \). The deceleration parameter monotonically decreases from \( q = \frac{1-k}{k} \) to the value \(-1\) for the hyperbolic and hybrid expansion, while it is constant for the power law expansion. The constants in the model can be chosen for satisfying the latest Planck observations as consistent with the inflationary parameters. We pointed out that the non-minimal gravitational coupling of the electromagnetic field can be used to describe the inflationary phase for certain coupling values of the model with the magnetic monopole field.

The magnetic monopoles can be considered as topological defects and they can be source of inflation which known as topological inflation proposed by Linde [63] and Vilenkin [62]. When the size of the defects is much larger than the Hubble radius, the topological inflation can occur in the center of the magnetic monopole for the model with the scalar field \( \phi \) which vanishes in the center of the topological defect and corresponds to a local maximum of the effective potential. Then even if inflation phase of the universe ends in the surrounding space, inflation of the monopole continues without end. On the other hand, in this study we consider the effects of the non-minimally coupled electromagnetic fields to gravity without using any scalar field or other exotic fields. However, in the both models once inflation begins, it can be driven by the primordial magnetic monopole fields.

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