Microscopic calculation of the equation of state of nuclear matter and neutron star structure

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\section{ABSTRACT}

We present results for neutron star models constructed with a new equation of state for nuclear matter at zero temperature. The ground state is computed using the Auxiliary Field Diffusion Monte Carlo (AFDMC) technique, with nucleons interacting via a semi-phenomenological Hamiltonian including a realistic two-body interaction. The effect of many-body forces is included by means of additional density-dependent terms in the Hamiltonian. In this letter we compare the properties of the resulting neutron-star models with those obtained using other nuclear Hamiltonians, focusing on the relations between mass and radius, and between the gravitational mass and the baryon number.

\textbf{Key words:} stars: neutron, equation of state

\section{1 INTRODUCTION}

While real neutron stars are very complicated objects, their main global properties can usually be well-approximated by considering simple idealized models consisting of perfect fluid in hydrostatic equilibrium. If rotation can be neglected to a first approximation (as is the case for the spin rates of most currently-known pulsars) then the model can be taken to be spherical and its structure obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations, enabling one to calculate, for example, the stellar mass as a function of radius or of central density. However, this requires specification of an equation of state (EOS) for the neutron-star matter which clearly plays a fundamental role in determining the properties of the resulting models. The EOS needs to take account of the strong spin-isospin correlations induced by realistic interactions, with particular regard to the tensor ones [Raffelt 1996].

The EOS can in principle be computed by means of many-body theories using effective density-dependent interactions, such as those given by Skyrme forces but the phenomenological nature of these can be a disadvantage in making reliable calculations of neutron-star properties [Rikovska Stone et al. 2003]. Making a microscopic calculation based on a Hamiltonian describing the properties of light nuclei and symmetric nuclear matter (SNM), is both challenging and of great relevance. Variational approaches using correlated basis functions (CBF) and Fermi Hyper Netted Chain techniques (FHNC) [Fantoni & Fabrocini 1998] are good candidates for doing this but the strong spin-isospin dependence of the nuclear Hamiltonian requires the introduction of approximations into the FHNC scheme which cannot be fully controlled, such as the single operator chain (SOC) [Pandharipande & Wiringa 1979].

The Auxiliary Field Diffusion Monte Carlo method (AFDMC) [Schmidt & Fantoni 1999] has been employed for computing the ground state energy of nuclei, giving very good agreement with other existing accurate techniques in few-body physics [Gandolfi et al. 2007a] and highlighting important limitations of other many-body theories used for calculations of both nuclear structure [Gandolfi et al. 2007b] and neutron matter [Gandolfi et al. 2009]. In this letter we present results from an AFDMC calculation of the EOS of $\beta$-equilibrium matter (relevant for neutron stars), based on a new class of non-relativistic Hamiltonians using nuclear potentials with a realistic two-body interaction, taken from the Urbana-Argonne scheme.

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(Wiringa et al. [1993] and incorporating many-body interactions via density-dependent terms.

Modern nuclear Hamiltonians are usually constrained so as to reproduce properties of light nuclei as measured in laboratory experiments (Pieper et al. [2003]), but the densities involved are much lower than those found in neutron-star cores for which it becomes necessary to introduce a three-body potential, such as UIX or IL1-IL5 (Pieper et al. [2001]). However, these give very large and very different energy contributions to the EOS of pure neutron matter (PNM) at high density (Sarsa et al. 2003; Gandolfi et al. 2007b). Also, recent AFDMC results by Gandolfi et al. (2007b) indicate that SNM is not well reproduced by the FHNC/SOC techniques used to constrain these Hamiltonians, which are therefore not very suitable for calculating neutron-star models.

Because of this, we have proceeded here in a different way, using density-dependent terms to simulate many-body forces with the forms used coming from explicit integration of these forces over the variables of particle 3 for the three-body force, over those of particles 3 and 4 for the four-body force, and so on. In bulk matter, the neutron and proton densities \( \rho_n \) and \( \rho_p \) can be taken to be constant quantities, whereas in confined systems, like nuclei, they are operators.

As a first step in this direction, we have revisited the old three-parameter density-dependent LP model of Lagaris & Pandharipande (1981) and Friedman & Pandharipande (1983), with the values of the free parameters fixed so as to reproduce the saturation point and compressibility of SNM. The LP calculations were performed using the FHNC/SOC approximation and we then re-fitted the density-dependent term using the AFDMC calculations. By considering the chemical potentials, we have constructed an EOS for a mixture of protons, electrons and muons in \( \beta \)-equilibrium and have then used the resulting EOS for constructing neutron-star models. We stress that (i) the ground states of nuclear matter as a function of \( \rho_n \) and \( \rho_p \) have been calculated with the same AFDMC method which has been shown to be very accurate for calculating the ground states of various nucleonic systems, and (ii) the many-body part of the nuclear interaction has been self-consistently determined by these solutions.

2 THE MODEL

We model the EOS by simulating nuclear matter using a finite number of interacting nucleons in a periodic box. The number of particles is chosen from among the magic numbers giving a rotationally invariant wave function for the corresponding non-interacting system and the volume of the box is fixed by the density. For the two-body interaction, we take the Argonne AV6′ potential (Wiringa & Pieper [2002]), which includes the four central spin-isospin components and the two tensor ones. The six components of the long range OPEP potential are fully included, whereas only the first six of the 18 components of the intermediate and short range parts of the AV18 interaction (Wiringa et al. 1993, \( v_0^b(r_{ij}) \) and \( v_S^b(r_{ij}) \), are kept. The corresponding amplitudes \( I_0 \) and \( S_p \) are re-fitted so as to correctly reproduce the deuteron properties and to give the best fit to NN scattering data (Wiringa & Pieper 2002).

The many-body interactions are represented by density-dependent factors of the structural form given by the LP model. The resulting potential, denoted as DD6′, is given by the following six two-body components

\[
v_{DD6′}(\rho) = v_{OPEP}^b + v_{S}^b e^{-\gamma_1 \rho} + v_S^b + TNA(\rho),
\]

\[
TNA(\rho) = 3 \gamma_2 \rho^2 e^{-\gamma_3 \rho} \left( 1 - \frac{2}{3} \left( \frac{\rho_n - \rho_p}{\rho_n + \rho_p} \right)^2 \right)
\]

with \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) being fixed so as to reproduce the experimental values of the saturation density \( \rho_0 = 0.16 \text{ fm}^{-3} \), the binding energy per particle \( E_0 = -16 \text{ MeV} \) and the compressibility \( K = 9\rho_0^3 (\partial^2 E(\rho)/\partial \rho^2) \rho_0 \approx 240 \text{ MeV} \). (Note that in this paper we follow the nuclear-physics convention of using \( \rho \) to refer to a particle number density rather than a mass density.)

Many nuclear matter calculations of the 1980s were made with the LP model interaction, and gave very good agreement with data for large nuclei, as discussed by Benhar & Pandharipande (1993) and Pandharipande et al. (1997). Afterwards, mainly because of the difficulty of treating the density as an operator when dealing with nuclei, this model was forgotten and attention was switched to the development of increasingly sophisticated three-body potentials. On the basis of the results obtained with AFDMC quantum simulations, we believe that we should instead now proceed in the direction of constructing increasingly sophisticated density-dependent two-body effective interactions, which may also account for N-body interactions with \( N \geq 3 \).

A particular limitation of DD6′ is the exclusion of the non-local components of AV18. The most important of these is the spin-orbit term, particularly when dealing with nuclei. We estimate that the missing non-local components contribute to the EOS of neutron matter by no more than 5% (Gandolfi et al. 2009). Work towards introducing the spin-orbit components is in progress. A second important point concerns the structural form of the density-dependent terms and one may want to increase the number of Feynman diagrams included in the construction of UIX and contributing to the many-body interaction.

3 CALCULATION OF THE EQUATION OF STATE OF NUCLEAR MATTER

We computed the ground state of the system using the AFDMC method (for details, see Gandolfi 2007 and references therein), simulating SNM with \( A = 28 \) nucleons in a periodic box, as described by Gandolfi et al. (2007a). The next magic number of nucleons providing small finite-size corrections is 132, as shown by Gandolfi et al. (2009), which requires an unjustified computational effort given the restriction to local components in the DD6′ model. Table 1 gives the values of the free parameters of DD6′ corresponding to the best fit to experimental data for \( \rho_0^b \), \( E_0 \) and \( K \), and compares these with the values for the original LP model.

The AFDMC density-dependent terms give more attraction than in the original LP model, consistent with the fact that FHNC/SOC overbinds SNM. The TNA term is more than the double at \( 5\rho_0^b \) giving
more attraction, and \( \exp(\gamma_1 \rho) - 1 \) is \( \sim 30\% \) smaller over the whole range \( (\rho_0, 5\rho_0) \), giving less repulsion.

We find that the AFDMC results for the binding energy per nucleon of SNM at densities larger than \( \sim 0.08 \) fm\(^{-3} \) can be very well described by:

\[
E_{SNM}(\rho) = E_0 + a(\rho - \rho_0)^2 + b(\rho - \rho_0)^3 e^{\gamma (\rho - \rho_0)},
\]

where \( E_0 = -16.0 \) MeV, \( \rho_0 = 0.16 \) fm\(^{-3} \), \( a = 520.0 \) MeV fm\(^6 \), \( b = -1297.4 \) MeV fm\(^9 \) and \( \gamma = -2.213 \) fm\(^3 \). This parametrization was chosen to represent the EOS of nuclear matter, reproducing properties constrained by terrestrial experiments on nuclei \cite{danielewicz2002}

The DD6' Hamiltonian was then used to compute the EOS of PNM, by making a simulation with 66 neutrons in a periodic box. The EOS for nuclear matter as a function of the total nucleon density, computed considering the presence of just electrons as negatively-charged particles (dotted line) and with both electrons and muons (full line).

\[
\begin{array}{c|c|c}
\text{parameter} & \text{FHNC/SOC} & \text{AFDMC} \\
\hline
\gamma_1 & 0.15 \text{ fm}^3 & 0.10 \text{ fm}^3 \\
\gamma_2 & -700 \text{ fm}^6 & -750 \text{ fm}^6 \\
\gamma_3 & 13.6 \text{ fm}^3 & 13.9 \text{ fm}^3 \\
\end{array}
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\begin{center}
\textbf{Table 1.} The AFDMC results for the free parameters of the DD6' interaction as compared with the original values of the LP model calculated within the FHNC/SOC approximation.
\end{center}

\[
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\end{array}
\]

their contribution must then also be considered. For our EOS, muons begin to appear at \( \rho \approx 0.133 \) fm\(^{-3} \).

The EOSs calculated for SNM, PNM and \( \beta \)-equilibrium matter are shown in Fig. 1 with the energy per nucleon being plotted against the nucleon number density \( \rho \); in the inset, we show the proton fraction \( x_p \) plotted as a function of the total nucleon density, computed considering the presence of just electrons as negatively-charged particles (dotted line) and with both electrons and muons (full line).

At very high densities, the chemical potential of the nucleonic matter becomes larger than the threshold for creation of heavier particles. Such states are due to the up and down quarks transforming to strange quarks, so that particles with strangeness (hyperons) start to appear. A realistic EOS should include these when they appear and this can seriously modify the structure of the star. We do not include this in our present calculations (although we make some comment about its likely effects in the next section); we leave inclusion of this until a subsequent paper.

4 RESULTING NEUTRON STAR MODELS

When the EOS of the neutron-star matter has been specified, the structure of an idealized spherically-symmetric neutron star model can be calculated by integrating the Tolman-Oppenheimer-Volkoff (TOV) equations:

![Figure 1. The equation of state for symmetric nuclear matter (dashed line), pure neutron matter (dot-dashed line) and \( \beta \)-equilibrium nuclear matter with both electrons and muons (full line) and with electrons only (dotted line). The points show the AFDMC results for symmetric nuclear matter (SNM, squares) and pure neutron matter (PNM, circles) which have been used to fit the EOS. In the inset, we show the proton fraction \( x_p \) plotted as a function of the total nucleon density, computed considering the presence of just electrons as negatively-charged particles (dotted line) and with both electrons and muons (full line).](image-url)
\[
\frac{dP}{dr} = -\frac{G(m(r) + 4\pi r^3 P/c^2)[\epsilon + P/c^2]}{r[r - 2Gm(r)/c^2]}, \tag{6}
\]
\[
\frac{dm(r)}{dr} = 4\pi cr^2, \tag{7}
\]
where \(P = \rho^2(\partial E/\partial \rho)\) and \(\epsilon = \rho(E + m_N)\) are the pressure and the energy density, \(m_N\) is the average nucleon mass, \(m(r)\) is the gravitational mass enclosed within a radius \(r\), and \(G\) is the gravitational constant. The solution of the TOV equations for a given central density gives the profiles of \(\rho\), \(\epsilon\) and \(P\) as functions of radius \(r\), and also the total radius \(R\) and mass \(M = m(R)\). A sequence of models can be generated by specifying a succession of values for the central density. In Fig. 2 we plot the mass \(M\) (measured in solar masses \(M_\odot\)) as a function of the radius \(R\) (measured in km), as obtained from calculations with four different prescriptions for the EOS: the \(\beta\)-equilibrium and PNM EOSs discussed in Sections 2 and 3, the equivalent one for PNM with just two-body interactions (using AV6'), and a previous one from Gandolfi et al. (2009), for PNM with three-body interactions (using AVS' + UIX). Models to the right of the maximum of each curve are stable to radial perturbations and these are the ones of interest for astrophysical neutron stars. The maximum mass obtained with the two-body interaction AVS' is very similar to that for AV6'.

It is interesting to make a comparison between these curves so as to see the changes caused by introduction of the various different features. The solid curve (\(\beta\)-equilibrium) is our best proposal for the neutron-star EOS but it can be seen that it differs only very little from the pure neutron matter EOS (where the radii for a given mass are just slightly larger within the main range of interest). There is a considerable difference, however, with respect to the previous AVS' + UIX curve for pure neutron matter, with the maximum mass being reduced from \(\sim 2.5 M_\odot\) to \(\sim 2.2 M_\odot\) and the radii in the main region of interest also being substantially reduced. This reflects the effective softening of the EOS caused by the different treatment of many-body effects.

An objective of this type of work is to attempt to constrain microphysical models for neutron-star matter by making comparison with astronomical observations (see Lattimer & Prakash 2003). This is just starting to be possible now and further progress is anticipated within the next few years. At present, the only neutron stars for which masses are accurately known are the components of the best-observed binary pulsars, for which timing measurements give results correct to many significant figures. The maximum mass for any of these is the 1.441 M_\odot for the Hulse/Taylor binary pulsar PSR 1913+16 (see Weisberg & Taylor 2003). However, there is accumulating evidence for higher masses, particularly for neutron stars in binary systems together with white dwarfs (see Ransom et al. 2003) and there is now a widespread belief that the maximum should probably be in the range 1.8 M_\odot to 2.1 M_\odot (at least when the rotation is sufficiently slow, as is the case for almost all pulsars so far observed). At high enough densities, it is expected that the composition of the matter would change because of the appearance of either hyperons or deconfined quarks, both of which are likely to decrease the maximum mass (see, for example, Schulze et al. 2006 for the case of hyperons, and Akmal et al. 1998 for an example of the inclusion of quark matter). The central density corresponding to our maximum mass for \(\beta\)-equilibrium matter is \(\rho_c \approx 1.2 \text{ fm}^{-3}\) which is well within the range where these changes are likely to have occurred, and so we expect that the maximum mass would be slightly lower than that shown in Fig. 2. This brings us well within the expected range.

Observational constraints for the radius are more problematic, but one of the best of these seems to be the indirect constraint suggested by Podsiadlowski et al. (2003) in the case of the less massive component of the double pulsar PSR J0737-3059. If, as seems likely, this neutron star was the product of an electron-capture supernova, then the total pre-collapse baryon number of the stellar core is rather precisely known from model calculations and, since only a very small loss of material is expected to have occurred in the subsequent collapse, the baryon number of the neutron star is itself also well-known. Together with the very accurate value for the gravitational mass (calculated from pulsar timing), this can be used to place a quite stringent constraint on the EOS. The baryon number \(A\) of a neutron-star model can be readily calculated from

\[
\frac{dA(r)}{dr} = 4\pi r^2 \left(1 - \frac{2Gm}{rc^2}\right)^{-2}, \tag{8}
\]
which needs to be solved together with equations 6 and 7 (we recall that \(\rho\) is here the baryon number density).

In practice, it is convenient to talk in terms of the baryonic mass, defined as \(M_0 = m_N A(R)\), rather than \(A(R)\) itself: the difference between the baryonic mass and the gravitational mass depends on the compactness of the neutron star, and hence indirectly on the radius. If \(M\) is plotted against \(M_0\) for a given EOS, then the curve needs to pass through a certain error box in order to be consistent with the observations for PSR J0737-3059, subject to the assumptions being made in the analysis. This plot is shown in Fig. 3 for...
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Figure 3. The gravitational mass of stellar models plotted as a function of the baryonic mass. See the caption of Fig. 2 for details. The shaded rectangle is the error box inferred from observations of the lower-mass component of the double pulsar PSR J0737-3059 and associated modelling (following Podsiadlowski et al. 2005, with the modification mentioned in the text). Kitaura et al. (2009) have proposed a smaller error box just to the left of this, but we prefer the one shown here.

the same EOSs shown in Fig. 2 (We use here the error box as suggested by Podsiadlowski et al. 2005, extended to the left by $3 \times 10^{-3} M_\odot$ to include a plausible upper limit for the matter lost during the core collapse.) The curve for our $\beta$-equilibrium EOS just touches the top left-hand corner of the error box. However, the central density of our model corresponding to the mass of the pulsar concerned ($M = 1.249 M_\odot$) is more than three times nuclear matter density, by which point hyperons have probably already appeared, giving some softening of the EOS which would move the curve slightly downwards.

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