A 3-form Gauge Potential in 5D in connection with a Possible Dark Sector of 4D-Electrodynamics

Abstract

We propose here a 5-dimensional electrodynamic model based on the mixing between the Maxwell gauge potential and a 3-form (Abelian) gauge field by means of a topological mass term. An extended covariant derivative is introduced to minimally couple a Dirac field to the Maxwell potential, while this same covariant derivative couples the 3-form non-minimally to the charged fermion. A number of properties are discussed in 5D; in particular, the appearance of a topological fermionic current. A 4-dimensional reduced version of the model is investigated and there emerges an extra set of electric- and magnetic-like fields which contribute a negative pressure and may be identified as a possible fraction of dark energy. Another consequence we contemplate is the emergence of an extra massive neutral gauge boson which may be taken as a $Z^0$ particle to be detected at accelerator energies.

PACS numbers: 11.15.-q, 11.15.Wx, 03.50.De
I. INTRODUCTION

The possibility of a multidimensional Universe has raised a growing interest over the years. Currently, the reasons for this interest primarily come from models such as Superstring Theory, which is able to incorporate gravity in a natural and consistent way [1].

As a consequence of the superstring landscape, it is nowadays widely accepted that the structure of spacetime must be described as the product of a 5-dimensional Anti-de Sitter space by a hypersphere in 5D as well. Thus, we adopt the viewpoint that the fundamental physics may be derived from five space-time and five compact internal dimensions.

In addition, the possibility of an equivalence between a classical gravity theory defined in a 5-dimensional space-time bulk and a quantum gauge theory (Yang-Mills) on the corresponding 4-dimensional boundary was first proposed by Maldacena in 1997 [2]. Important aspects of the correspondence were elaborated in articles by Gubser, Klebanov, and Polyakov, and by Edward Witten [3–5].

We shall not however adopt the $AdS_5/CFT_4$ correspondence in its full sense. What we borrow from this correspondence is simply the point of view that our fundamental physics takes place in 5D; whether this physics should be specifically analyzed in an $AdS_5$ or a 5D Minkowski scenario will actually depend on the particular phenomenon in question. Here, we shall assume that, so long as electromagnetic interactions are considered, we do not consider the presence of a cosmological constant in the five-dimensional world. For the investigation we aim to pursue, our onset is a 5-dimensional Minkowski space-time.

Actually, in the present study, we explore the consequences of an extra dimension [6], by just considering Minkowski space as the background space-time, for the effect of the curvature of the Anti-de Sitter space (induced by a cosmological constant $10^{-48} \text{GeV}^2$ [7]) yields negligible effects as compared to the scale of masses and lengths of QED [8]. In other words, by neglecting the cosmological constant, the isometry group of $AdS_5$, i.e., $SO(2, 4)$ reduces to the five-dimensional Poincaré group. So, we shall here consider a model for electromagnetic interactions in a five-dimensional Minkowski space and our four-dimensional physics must come out as the result of a specific dimensional reduction scheme rather than by holographic projection.

It is noteworthy that, if we were considering the quantum effects of gravitation, the cosmological constant should not be neglected, for it is known that the latter induces the
production of gravitons with mass of the order of the Planck mass \[9, 10\]. However, in the case we are concerned with, massive gravitons do not couple to the associated fluctuations of the electron due to the fact that they are highly massive, leading us to conclude that the energy regime of Quantum Electrodynamics validity does not provide energy enough to excite those gravitons induced by the cosmological constant.

From this perspective, we present a proposal in favor of an accelerating expansion of the Universe, phenomenon dubbed dark energy \[11\]. Our model may also justify the appearance of an extra neutral massive boson \[12\]. In order to achieve this task, we initially consider whether a rank-2 gauge field in 5D could contain the information of Maxwell’s electrodynamics. We conclude that this field can indeed be more interesting to approach electrodynamics in 5 dimensions \[13\]. After that, we analyze how to introduce a massive photon in a 5-dimensional scenario and we understand that the most natural way would be adding up a topological term built up by the mixing between the Maxwell field and a 3-form gauge potential.

In connection with the study of the 3-form \[14–30\], the mass of the photon is included in order to seek a situation that is as broad as possible, i.e., capable of exploring all the possibilities that a 3-form may offer. According to the work by Koivisto and Nunes \[26\], 3-forms may be used to model dark energy. However, the 3-form was initially studied separately, evaluating only its kinetic term \[27\]. Subsequently, the 3-form was reassessed to include coupling \[29\]. Here, we intend to investigate the 3-form in association with the Maxwell’s field, in a 5D scenario, by means of a topological Chern-Simons-like mass term.

In a recent paper \[31\], the authors show how a vortex gauge field, whenever coupled to charged fermions, induce, by radiative corrections, a gauge invariant mass term for the photon. Rather than as a dynamical effect, like in the paper \[31\], in our work, this mass term arises from a dimensional reduction from 5D model where there is a topological mass term, as it shall be presented further on.

Five-dimensional Chern-Simons theory, in its Abelian version, has recently been used by Qi, Witten and Zhang (QWZ) in the context of modeling topological superconductors \[32\]. As it is known, in a superconductivity process, the massive photon must be present to accommodate the Meissner effect, responsible for the expulsion of the magnetic field from inside the superconductor materials. Thus, due to the physics of topological superconductors being processed in five dimensions, in accordance with the QWZ’s formulation, the photon,
in this case, could also gain mass by a mechanism of topological mass generation according to our proposal.

In summary, we intend to explore an electrodynamic model that uses both a 3-form gauge and a 1-form gauge in a coupled way in order to generate massive fields in a 5-dimensional scenario. By dimensional reduction, we reach a model that presents in its spectrum a massive vector boson, neutral (from the mixing between a one-form and 3-form gauge) and degenerated (i.e., with the same mass) with a neutral scalar associated to the mixing between a genuine scalar field and a vector field of longitudinal nature. Our work follows the outline below.

In Section II, we present the model we adopt to pursue our investigation. We divide it in two subsections in which we obtain the equation fields, the conservation laws and dimensional reduction of the model to 4D. Next, in Section III we add fermions to the action of the model in 5D discussed in the previous section. We obtain the fermionic conserved currents in 5D. This action is reduced to 4D, and we calculate the propagators of the model. Finally, our Concluding Comments are cast in Section IV.

II. DESCRIPTION OF THE MODEL

Taking for granted the importance of understanding physics in our 4-dimensional world from a more fundamental 5-dimensional physics, we focus here on a study of a specific electrodynamic model in 5 dimensions aiming at the possible consequences it yields in a 4-dimensional space-time.

Thus, in this Section, we present the model which consists of a Lagrangian density containing the kinetic terms for each gauge field ($A_{\bar{\mu}}$, and $C_{\bar{\mu}\bar{\nu}\bar{\rho}}$), and a mixing term between them. This mixing term is capable of ensuring that the mass of the associated particle is independent of the metric characteristics of the space. It is known in the literature as a topological term [34–36]. We also exhibit the field equations, the Bianchi identities and the conservation laws.

Consider the action in 5D whose corresponding Lagrangian density is as follows:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha H_{\mu\nu\rho\kappa\lambda} H^{\mu\nu\rho\kappa\lambda} + \beta \epsilon^{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\kappa}\bar{\lambda}\bar{\rho}} A_{\mu} \partial_{\nu} C_{\kappa\lambda\bar{\rho}},$$  \hspace{1cm} (1)

where $A_{\bar{\mu}}$ is the Maxwell field and $C_{\bar{\mu}\bar{\nu}\bar{\rho}}$ is the 3-form gauge, one of the main elements of this
study. The notation of the indices in 5 dimensions is $\bar{\mu} = \{0, 1, 2, 3, 4\}$. The tensor $F_{\bar{\mu}\bar{\nu}}$ is the usual electromagnetic field strength, and the tensor $H_{\bar{\mu}\bar{\nu}\bar{\lambda}}$ is the completely antisymmetric field strength associated with the 3-form field, $C_{\bar{\mu}\bar{\nu}\bar{\lambda}}$:

$$H_{\bar{\mu}\bar{\nu}\bar{\lambda}} = \partial_{\bar{\mu}} C_{\bar{\nu}\bar{\lambda}} - \partial_{\bar{\nu}} C_{\bar{\lambda}\bar{\mu}} + \partial_{\bar{\lambda}} C_{\bar{\mu}\bar{\nu}} - \partial_{\bar{\lambda}} C_{\bar{\mu}\bar{\nu}}.$$  \hspace{1cm} (2)

The parameters $\alpha$ and $\beta$ are both real. It is not difficult to check that the $\beta$-parameter has mass dimension. The action defined through the Lagrangian (1) is invariant under the following Abelian gauge transformations in 5D:

$$A_{\bar{\mu}} \mapsto A'_{\bar{\mu}} = A_{\bar{\mu}} + \partial_{\bar{\mu}} \Lambda,$$  \hspace{1cm} (3)

$$C_{\bar{\mu}\bar{\nu}\bar{\lambda}} \mapsto C'_{\bar{\mu}\bar{\nu}\bar{\lambda}} = C_{\bar{\mu}\bar{\nu}\bar{\lambda}} + \partial_{\bar{\mu}} \xi_{\bar{\nu}\bar{\lambda}} + \partial_{\bar{\nu}} \xi_{\bar{\lambda}\bar{\mu}} + \partial_{\bar{\lambda}} \xi_{\bar{\mu}\bar{\nu}},$$  \hspace{1cm} (4)

where $\Lambda$ and $\xi_{\bar{\mu}\bar{\nu}}$ are real functions and $\xi_{\bar{\mu}\bar{\nu}}$ is an antisymmetric tensor field. The transformation (3) is the already known one from electrodynamics, $U(1)_{A_{\bar{\mu}}}$, whereas (4) is the antisymmetrized version of the gauge transformation for a rank-3 tensor, $U(1)_{C_{\bar{\mu}\bar{\nu}\bar{\lambda}}}$. Thus, the action is said to be $U(1)_{A_{\bar{\mu}}} \otimes U(1)_{C_{\bar{\mu}\bar{\nu}\bar{\lambda}}}$-invariant. The Lagrangian (1) gives us the field equations

$$\partial_{\bar{\mu}} F_{\bar{\mu}\bar{\nu}} + 6 \beta \tilde{H}^{\bar{\mu}} = 0,$$  \hspace{1cm} (5)

$$8\alpha \partial_{\bar{\mu}} H_{\bar{\mu}\bar{\nu}\bar{\lambda}} - \beta \tilde{F}_{\bar{\mu}\bar{\nu}\bar{\lambda}} = 0.$$  \hspace{1cm} (6)

where the relations between the dual tensors $\tilde{F}_{\bar{\mu}\bar{\nu}\bar{\lambda}}$ and $F_{\bar{\mu}\bar{\nu}}$ are given by the expressions

$$F_{\bar{\mu}\bar{\nu}} = -\frac{1}{3!} \epsilon_{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\lambda}} \tilde{F}_{\bar{\rho}\bar{\lambda}}$$ and $\tilde{F}_{\bar{\mu}\bar{\nu}\bar{\lambda}} = \frac{1}{2!} \epsilon_{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}} F_{\bar{\lambda}\bar{\rho}}$.  \hspace{1cm} (7)

As for the relations between $\tilde{H}^{\bar{\mu}}$ and $H_{\bar{\mu}\bar{\nu}\bar{\lambda}}$, the expressions are given by

$$H_{\bar{\mu}\bar{\nu}\bar{\lambda}} = \epsilon_{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\lambda} \bar{\rho}} \tilde{H}^{\bar{\rho}}$$ and $\tilde{H}^{\bar{\mu}} = \frac{1}{4!} \epsilon_{\bar{\mu}\bar{\nu}\bar{\lambda}\bar{\rho}} H^{\bar{\nu}\bar{\lambda}\bar{\rho}}$.  \hspace{1cm} (8)

The Bianchi identities associated to the fields $F_{\bar{\mu}\bar{\nu}}$ and $H_{\bar{\mu}\bar{\nu}\bar{\lambda}}$ are, respectively:

$$\partial_{\bar{\nu}} F_{\bar{\mu}\bar{\lambda}} + \partial_{\bar{\lambda}} F_{\bar{\nu}\bar{\mu}} + \partial_{\bar{\mu}} F_{\bar{\nu}\bar{\lambda}} = 0,$$  \hspace{1cm} (9)

$$\partial_{\bar{\nu}} H_{\bar{\nu}\bar{\lambda}\bar{\rho}} + \partial_{\bar{\lambda}} H_{\bar{\rho}\bar{\lambda}\bar{\mu}} + \partial_{\bar{\rho}} H_{\bar{\nu}\bar{\lambda}\bar{\mu}} + \partial_{\bar{\lambda}} H_{\bar{\rho}\bar{\mu}\bar{\lambda}} + \partial_{\bar{\mu}} H_{\bar{\rho}\bar{\lambda}\bar{\nu}} = 0.$$  \hspace{1cm} (10)
Expression (10) can also be cast in a more compact form in terms of the dual of $H_{\bar{\mu}\bar{\nu}\bar{\kappa}}$, i.e.:

$$\partial_{\bar{\mu}}\tilde{H}^{\bar{\mu}} = 0 .$$

(11)

The field equations (5) and (6) are coupled and we must necessarily decouple them in order to implement the procedure that will reveal the mass of the particle(s) associated(s) with both fields. The procedure used here to decouple the equations consists in multiplying equation (5) by $\epsilon_{\bar{\nu}\bar{\alpha}\bar{\beta}\bar{\gamma}\bar{\sigma}} \partial_{\bar{\sigma}}$ and proceeding in an analogous way with equation (6). Thus, we obtain

$$\left(\Box - \frac{3}{4\alpha} \beta^2\right) F_{\bar{\mu}\bar{\nu}} = 0 ,$$

(12)

and

$$\left(\Box - \frac{3}{4\alpha} \beta^2\right) H_{\bar{\mu}\bar{\nu}\bar{\kappa}} = 0 .$$

(13)

Therefore, it is noted from (12-13) that both fields obtain the same mass term, which is given by $\xi = -\frac{3\beta^2}{4\alpha}$. Where, it is considered that the parameter $\alpha$ must be restricted to a negative real number. The energy-momentum tensor is obtained multiplying the equation (5) by $F_{\bar{\nu}\bar{\alpha}}$ and using the following relation between the dual fields

$$\tilde{F}^{\bar{\nu}\bar{\kappa}\bar{\lambda}} H_{\bar{\nu}\bar{\kappa}\bar{\lambda}} = -6 F_{\bar{\nu}\bar{\alpha}} \tilde{H}^{\bar{\alpha}} .$$

(14)

Then, we insert the equation (6) and after the application of the Leibniz’s rule, we obtain

$$- \partial_{\bar{\mu}} (8\alpha H^{\bar{\mu}\bar{\nu}\bar{\kappa}} H_{\bar{\nu}\bar{\kappa}\bar{\lambda}}) + 8\alpha H^{\bar{\mu}\bar{\nu}\bar{\kappa}} \partial_{\bar{\mu}} H_{\bar{\nu}\bar{\kappa}\bar{\lambda}} +$$

$$+ \partial_{\bar{\mu}} (F^{\bar{\mu}\bar{\nu}} F_{\bar{\nu}\bar{\alpha}}) - F^{\bar{\mu}\bar{\nu}} \partial_{\bar{\mu}} F_{\bar{\nu}\bar{\alpha}} = 0 .$$

(15)

Thus, replacing the Bianchi identities (9-10) and using the following relation

$$2 F_{\bar{\nu}\bar{\alpha}} \partial_{\bar{\mu}} F^{\bar{\mu}\bar{\nu}} = \partial_{\bar{\mu}} (16\alpha H^{\bar{\mu}\bar{\nu}\bar{\kappa}} H_{\bar{\nu}\bar{\kappa}\bar{\lambda}}) + \partial_{\bar{\alpha}} (2\alpha H^{2}_{\bar{\mu}\bar{\nu}\bar{\kappa}}) ,$$

we obtain the continuity equation

$$\partial_{\bar{\mu}} \Theta^{\bar{\mu}}_{\bar{\alpha}} = 0 .$$

(17)

where $\Theta^{\bar{\mu}}_{\bar{\alpha}}$, the energy-momentum tensor associated with the Lagrangian (11), is given by

$$\Theta^{\bar{\mu}}_{\bar{\alpha}} = -8\alpha \left( H^{\bar{\mu}\bar{\nu}\bar{\kappa}} H_{\bar{\nu}\bar{\kappa}\bar{\lambda}} + \delta^{\bar{\mu}}_{\bar{\alpha}} \frac{1}{8} H^{2}_{\bar{\mu}\bar{\nu}\bar{\kappa}} \right) +$$

$$+ F^{\bar{\mu}\bar{\nu}} F_{\bar{\nu}\bar{\alpha}} + \delta^{\bar{\mu}}_{\bar{\alpha}} \frac{1}{4} F^{2}_{\bar{\mu}\bar{\nu}} .$$

(18)
Comparing the second term of $\Theta^{\bar{\alpha}}_{\bar{\alpha}}$ with the kinetic term of the rank-3 tensor field in (1), we can set the value of the parameter as $\alpha = -1/8$. Thus, we rewrite the mass as $m^2 := \frac{-3\beta^2}{4\alpha}$ and thus the value of $\beta$ is fixed. Therefore, the topological mass term, $\Delta$, is given by

$$
\Delta = \frac{m}{\sqrt{6}} \epsilon^{\bar{\mu}\bar{\nu}\bar{\kappa}\bar{\lambda}} A_{\bar{\mu}} \partial_{\bar{\nu}} C_{\bar{\kappa}\bar{\lambda}} .
$$

Because the energy-momentum tensor is written in terms of the field strength tensors, it is invariant under the gauge transformations (3) and (4). It is also symmetrical for exchanging indexes. The expression (18) can be rewritten in terms of the dual field of $H_{\mu\bar{\nu}\bar{\kappa}}$ as stated by

$$
\Theta^{\bar{\mu}}_{\bar{\alpha}} = 6 \tilde{H}^{\bar{\mu}} \tilde{H}_{\bar{\alpha}} - \delta^{\bar{\mu}}_{\bar{\alpha}} 3 \tilde{H}^2_{\bar{\mu}} + F^{\mu\bar{\nu}} F_{\nu\bar{\alpha}} + \frac{1}{4} F_{\mu\bar{\nu}}^2 .
$$

A. Decomposition into irreducible components of $SO(3)$.

To carry out the decomposition of the energy-momentum tensor (20), the field equations (5-6) and the Bianchi identities (9) and (11) in terms of irreducible components of $SO(3)$, we initially make the identification of each sector of $F_{\mu\nu}$ and $\tilde{H}_{\mu}$ with the corresponding irreducible components of $SO(3)$ as listed on Table I:

From (17), we extract the components of the conserved energy-momentum tensor $\Theta^0_{\bar{\alpha}}$, so that the energy, the Poynting vector, and a new density pressure associated with the extra dimension are expressed respectively by:

$$
\Theta^0 = \frac{1}{2}(E^2 + B^2 + b^2 + e^2) - 3(\chi^2 + Y^2 + S^2) \quad (21)
$$

$$
\Theta^0_i = -(\vec{E} \times \vec{B})_i + b \vec{e}_i + 6\chi \vec{Y}_i \quad (22)
$$

$$
\Theta^0_4 = -\vec{E} \cdot \vec{e} + 6\chi S \quad (23)
$$

Going on with the procedure for extracting the components of the energy-momentum tensor, we have that the stress tensors are presented as follows:

$$
\Theta^i_j = \vec{E}_i \vec{E}_j + \vec{B}_i \vec{B}_j - \vec{e}_i \vec{e}_j + 6\vec{Y}_i \vec{Y}_j \quad (24)
$$

$$
\Theta^i_4 = -(b \vec{E} + \vec{e} \times \vec{B})_i + 6\vec{Y}_i S \quad (25)
$$

$$
\Theta^4_4 = -\frac{1}{2}(E^2 - B^2 + e^2 - b^2) + 3(S^2 + \chi^2 - Y^2) \quad (26)
$$
Table I: Components of tensor field $F^\mu\nu$ and dual tensor of $H^\mu\nu\bar{\rho}\bar{\lambda}$. The indices $i$, $j$, $k = 1, 2, 3$ refer to the space components in 4 dimensional space-time.

In Table II, $\vec{E}$, $\vec{B}$, $\chi$ and $\vec{Y}$ are the field strengths associated to the Maxwell and the 2-form potential. On the other hand, $\vec{e}$, $b$ and $S$ constitute what we call the dark sector of our extended 4-dimensional electrodynamics. At this point, we would like to point out the work of Reference [37] were the author introduces a second photon, which he refers to as the shadow photon or paraphoton, and unobserved photon. In our case, what we could call the dark photon is the particle associated to the propagation of $\vec{e}$ and $b$. In our model, there remains a scalar, $S$, which is also part of what we call the dark sector. It would be interesting, but we are not doing this here, if we later work out astrophysical constraints on this dark sector as it is done in the series of papers quoted in References [38–40].

The right-hand side of Einstein’s equation is essentially described by the energy-momentum tensor. This constitutes a unified relation (arising from the space-time symmetry) between the energy density and the pressure in the system. In a five-dimensional model, one identifies in the energy-momentum tensor the presence of a sector able to submit the system, through a particular configuration of the fields [26], to a negative pressure which, in its turn, characterizes the effect of accelerated inflation of the Universe, the effect of the so-called dark energy. As a result of the observations, the inflationary profile of the Universe
changes over time \[41,42\]. Currently, it presents itself as accelerated \[43,44\]. This changing behavior in the inflationary profile may be the result of changes in the configuration of the present fields in each phase of the history of the Universe.

In the paper of Reference \[26\], the author argues that the tiny value of the cosmological constant can be phenomenologically explained by the use of a 3-form. We also adopt the 3-form, but we consider that, for the sake of electromagnetic effects, the cosmological constant is tiny enough, so that we neglect the curvature of the (Anti-de Sitter) space. In view of that, we adopt Minkowski space as the spacetime background. Then, we attribute to the presence of a specific sector of the energy-momentum tensor in 5D, the effect that mimics dark energy, by virtue of the use of a 3-form in our model.

From what we have discussed above, our work sets out as a possible theoretical support to the paper \[26\] in order to provide a justification to the fact that 3-form comprises a negative cosmological constant as suggested by the presence of the sector \(\Theta^4\), which can become negative depending on the particular configuration of the fields \((\vec{E}, \vec{B}, \vec{e}, \chi, S, \text{and } \vec{Y})\).

The topological mass term \((19)\) used in our action does not affect - by construction - the energy-momentum tensor \((20)\). Hence, if the sector \(\Theta^4\) presents itself as negative, it happens regardless of the mass term we adopt. This \(\Theta^4\), which is negative in 5D, may play the role of the cosmological constant in 4D.

The presence of a negative cosmological constant in 4D can be attributed to the presence of a 3-form gauge potential in 5D. If the rank-3 field can provide a negative contribution to \(\Theta^4\), so if we put a topological mass term, this will not affect the energy-momentum tensor, so we do not lose the property of a negative \(\Theta^4\) while we explore other aspects of the rank-3 field in 5D.

B. Radiation fields in 4D.

Next we exhibit the field equations in 5D extracted from the Lagrangian \((1)\) where it is considered fixed constants \(\alpha\) and \(\beta\) as has been detailed in the previous section. The equations are expressed in terms of the components \(\vec{E}, \vec{B}, \vec{e}\) and \(b\) of \(F^{\mu\nu}\) and of the components \(\chi, S\) and \(\vec{Y}\) of \(H_{\mu\nu\kappa\lambda}\) including the mass terms.

We will adopt a dimensional reduction scheme known as Scherk-Schwarz reduction \[33\]
where it is considered that all potentials and fields do not depend on the extra dimension, i.e., it is considered that the derivatives of any field to the fifth coordinate is null, i.e., \( \partial_4 \) (any field) = 0. The equation (5) in the presence of an external source \( J^\mu \), when it is decomposed reveals the following equations:

\[
\nabla \cdot \vec{E} + m \sqrt{6} \chi = \rho 
\]

(27)

\[
\nabla \times \vec{B} + m \sqrt{6} \vec{Y} = \vec{j} + \frac{\partial \vec{E}}{\partial t} 
\]

(28)

\[
\nabla \cdot \vec{\epsilon} + m \sqrt{6} S = j_s + \frac{\partial b}{\partial t} 
\]

(29)

When the equation (6) is decomposed it reveals the following equations:

\[
\partial_\mu H^{\rho \kappa \lambda} + \frac{m}{\sqrt{6}} \tilde{F}^{\rho \kappa \lambda} = J^{\rho \kappa \lambda} 
\]

\[
\nabla S + \frac{m}{\sqrt{6}} (\vec{\epsilon}) = \chi 
\]

\[
\lambda_k = \frac{1}{2} \varepsilon_{ijk} J^{0ij} 
\]

(30)

\[
\nabla \times \vec{Y} + \frac{m}{\sqrt{6}} \vec{B} = \zeta 
\]

\[
\zeta^i = J^{0i4} 
\]

(31)

\[
\frac{\partial \vec{Y}}{\partial t} + \nabla \chi + \frac{m}{\sqrt{6}} \vec{E} = \vec{\sigma} 
\]

\[
\sigma_i = \frac{1}{2} \varepsilon_{ijk} J^{jk4} 
\]

(32)

\[
\frac{\partial S}{\partial t} + \frac{m}{\sqrt{6}} b = \tau 
\]

\[
\tau = -\varepsilon_{ijk} J^{ijk} 
\]

(33)

As for the Bianchi identity (9), when it is decomposed reveals:

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, 
\]

(30)

\[
\nabla \cdot \vec{B} = 0, 
\]

(31)

\[
\nabla \times \vec{\epsilon} = 0, 
\]

(32)
\[ \vec{\nabla} b = \frac{\partial \vec{e}}{\partial t} . \] (33)

And finally, the second Bianchi identity gives us just one expression:

\[ \frac{\partial \chi}{\partial t} + \vec{\nabla} \cdot \vec{Y} = 0 . \] (34)

This is a continuity equation involving the components \((\chi, \vec{Y})\). It appoints that

\[ \Xi := \int_{\mathbb{R}} d^5x \, \chi(x, t) \] (35)

is a conserved quantity of model.

It is important to make clear that, although we write down and study Maxwell’s equations in the 5 dimensions, we shall actually carry out a dimensional reduction to \((1+3)D\) and, whenever we consider our electromagnetic fields confined to the 4-dimensional space, there appear extra fields which are inherited from 5 dimensions upon dimensional reduction. So, we are truly considering our electromagnetic interaction in \((1+3)D\), but we take into account new fields that show up as a by-product of the 5-dimensional space-time where we have set up our physical scenario.

III. THE FERMIONIC SECTOR IN 5D AND ITS DIMENSIONAL REDUCTION TO 4D.

In this section, we add to the action corresponding to (1) a fermionic sector in 5 dimensions

\[ S_{5D} = \int d^5x \left[ \bar{\psi} (i\gamma^\mu D_\mu - m_f) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8} H_{\mu\nu\kappa\lambda} H^{\mu\nu\kappa\lambda} + \frac{m}{\sqrt{6}} \epsilon^{\alpha\beta\kappa\lambda\rho} A_\mu \partial_\nu C^\alpha_\kappa C^\lambda_\rho \right] , \] (36)

where we insert the covariant derivative in order to study the interaction of the Dirac field with the gauge fields

\[ D_\mu := \partial_\mu + ieA_\mu + ig\tilde{H}_\mu , \] (37)

and the spinor \(\psi\) is a Dirac fermionic field in 5D. The \(\gamma\)-matrices are defined as \(\gamma^\mu = (\gamma^\mu, \gamma^4)\), with \(\gamma^4 = i\gamma_5\) and \(\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3\) such that they satisfy the anti-commutation relations

\[ \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} , \quad \{\gamma^\mu, \gamma_5\} = 0 \] (38)

and the conditions \((\gamma_5)^\dagger = \gamma_5\), and \((\gamma_5)^2 = 1\).
The field equations derived from for the gauge fields in the presence of fermions are given by
\[ \partial_{\mu} F^{\bar{\mu} \mu} + \sqrt{6} m \bar{H}^\rho = e \bar{\psi} \gamma^\rho \psi , \]
(39)
and
\[ \partial_{\mu} H^{\bar{\mu} \nu \rho \lambda} + \frac{m}{\sqrt{6}} \bar{F}^{\rho \kappa \lambda} = 4 g e^{\bar{\rho} \rho \kappa \lambda \mu} \partial_{\mu} ( \bar{\psi} \gamma^\rho \psi ) . \]
(40)
from which we identify the source terms for each equation:
\[ J^\mu_F = e \bar{\psi} \gamma^\mu \psi \]
(41)
and
\[ J^{\bar{\rho} \rho \kappa \lambda}_H = 4 g e^{\bar{\rho} \rho \kappa \lambda \mu} \partial_{\mu} ( \bar{\psi} \gamma^\rho \psi ) . \]
(42)
We may notice that these currents arise due to the presence of the mixing term between the
gauge fields in the Lagrangian. \( J^{\bar{\rho} \rho \kappa \lambda}_H \) is a topological current, which means that we have a
current that is conserved without any reference to the equations of motion and no continuous
symmetry of the Lagrangian or the action is associated to this conservation equation. In
other words, we have an identically conserved current.

The current \( J^{\bar{\rho} \rho \kappa \lambda}_H \) above, when dimensionally reduced to 4D, gives rise precisely to the
pseudo-tensor current to which the vortex gauge field of \([31]\) couples. In our case, the current
stems from the non-minimal coupling present in the covariant derivative \([37]\) as an imprint
of the five-dimensional world. So, this topological current in 5D plays the crucial role of
inducing the gauge invariant mass term of reference \([31]\) upon its coupling to the vortex
gauge field.

**A. Dimensional reduction.**

Next, one redefines the complete action, but now having undergone a procedure of di-
mensional reduction from five to four dimensions. The Greek indices follow the notation
\( \bar{\mu} = (\mu, 4) \) where \( \mu \) indicates the usual four dimensions and \( \bar{\mu} \) indicate five dimensions, i.e.,
the four usual dimensions plus an extra spatial dimension.

Here, the 1-form \( A^{\bar{\mu}} \) can be divided into a vector sector and a scalar sector: \( A^{\bar{\mu}} = (A^\mu, A^4) \).
As for the 3-form, it can be split into two tensor sectors \( C^{\bar{\mu} \rho \kappa} = (C^{\mu \rho \kappa}, C^{\nu \mu \rho}) \).
One redefines the scalar component as \( A^4 = \phi \) and one then identifies the sector \( C^{\mu \nu \lambda} = \frac{1}{\sqrt{3}} B^{\mu \nu} \) as the one
known in the literature as Kalb-Ramond field \([45]\).
Thus, the 5D action is reduced to 4D and can be expressed as follows:

$$S_{4D} = \int d^4x \left[ \bar{\psi} \left( i\gamma^\mu D_\mu - m_f \right) \psi - \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{6} G_{\mu\nu\kappa}^2 \right. $$

$$+ \frac{1}{2} \left( \partial_\phi \phi \right)^2 + 3(\partial_\mu X^\mu)^2 - \frac{2\sqrt{2}}{3} m \epsilon^{\mu\nu\kappa\lambda} A_\mu \partial_\nu B_{\kappa\lambda} $$

$$- \sqrt{6} m \phi \partial_\mu X^\mu - ie \bar{\psi} \gamma_5 \psi \phi $$

$$+ ig \bar{\psi} \gamma_5 \psi (\partial_\mu X^\mu) \right] $$

(43)

where

$$G_{\mu\nu\kappa} = \partial_\mu B_{\nu\kappa} + \partial_\nu B_{\kappa\mu} + \partial_\kappa B_{\mu\nu} ,$$

(44)

is the field strength associated with the Kalb-Ramond field. The vector field $X_\mu$ is the dual of $C_{\mu\nu\kappa}$

$$X^\mu := \frac{1}{6} \epsilon^{\mu\nu\kappa\lambda} C_{\nu\kappa\lambda} ,$$

(45)

and by using the gauge transformation (44) of $C_{\mu\nu\kappa}$ we obtain

$$X'^\mu = X^\mu + \frac{1}{2} \epsilon^{\mu\nu\kappa\lambda} \partial_\nu \xi_{\kappa\lambda} ,$$

(46)

and hence

$$\partial_\mu X'^\mu = \partial_\mu X^\mu ,$$

(47)

e.i., the vector field $X^\mu$ is purely longitudinal. By using the field equations (12-13), this dimensional reduction shows that the bosonic fields in the reduced action (43) acquire a mass $m^2$. The field $\tilde{H}^\mu = (\tilde{G}^\mu, \tilde{H}^4)$ may be split in $\tilde{H}^4 = \partial_\mu X^\mu$ and $\tilde{G}^\mu$, i.e., the dual of $G_{\mu\nu\kappa}$: $\tilde{G}^\mu = (\chi, \vec{Y})$. The dual of $G_{\mu\nu\kappa}$ is given by

$$\tilde{G}_\mu = \frac{1}{6} \epsilon_{\mu\nu\kappa\lambda} G^{\nu\kappa\lambda} .$$

(48)

Therefore, the covariant derivative of (43) in four dimensions is

$$D_\mu = \partial_\mu + ie A_\mu + ig \tilde{G}_\mu .$$

(49)

Here the 3-form gauge field in 4D is nothing but a longitudinal vector, because it propagates its longitudinal part and suppresses its transverse component, as equation (47) suggests. The light-shining-through-a-wall experiments (LSW) [46, 47] are capable of detecting longitudinal radiation [48].
From the works by Antoniadis et al. [49, 50] and Ringwald et al. [48], what they consider in 4D as a scalar (in the Axionic Electrodynamics), turns out to originate from the 3-form \((X^\mu)\). So, the Antoniadis’ axion is for us a remnant of the 3-form gauge potential in 5D.

The articles by Antoniadis [49, 50] show that the 3-form which appears in 4D may be in fact a scalar. Our vector field \(X^\mu\) just propagates the longitudinal part because this is the gauge invariant component, i.e., this vector field carries the spin-\(0\) and the spin-\(1\) components, but gauge symmetry acts to gauge away the spin-\(1\) piece.

These two new bosons (vector and scalar) that appear simultaneously in our model can be interpreted, in fact, as "two sides of the same coin". A "coin" that is conceived in a 5-dimensional scenario, but, from the point of view of our 4-dimensional world, leads us to see it as if there were two separate entities. However, from the point of view of the five-dimensional bulk, it is only one entity, since the 5 dimensions provide a unified view of these two fields. In 4D, we see two entities, the vector and scalar bosons, as a result of dimensional reduction. Under this unified interpretation, the masses of the "two particles" being the same would also suggest an that there is a common entity the propagates in the bulk between the branes.

B. Propagators

The propagators associated with the Lagrangian \([43]\) are obtained after insertion of the corresponding gauge fixing terms

\[
\mathcal{L}_{gf} = -\frac{1}{2\alpha} (\partial_\mu A^\mu)^2 - \frac{1}{2\beta} (\partial_\mu B^{\mu\nu})^2 ,
\]

and adding it to the free part of \([43]\), we have \(\mathcal{L}_0 = \mathcal{L}_{04D} + \mathcal{L}_{gf}\), thus

\[
\mathcal{L}_0 = \bar{\psi} (i\gamma^\mu D_\mu - m_f) \psi - \frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 + \frac{1}{6} G_{\mu\nu\kappa}^2

- \frac{1}{2\alpha} (\partial_\mu B^{\mu\nu})^2 - \frac{2\sqrt{2}}{3} m\epsilon^{\mu\nu\kappa\lambda} A_\mu \partial_\nu B_{\kappa\lambda}
+ 3 (\partial_\mu X^\mu)^2 + \frac{1}{2} (\partial_\mu \phi)^2 - \sqrt{6} m\phi \partial_\mu X^\mu .
\]
In the sector of the gauge fields, we cast the Lagrangian into the form below:

\[
\mathcal{L}_{0g} = \frac{1}{2} A^\mu \left( \Box \theta_{\mu\nu} - \frac{1}{\alpha} \Box \omega_{\mu\nu} \right) A^\nu \\
- \frac{1}{2} B^\mu{}_{\nu\alpha} \left[ \Box \left( P_b^1 \right)_{\mu\nu,\kappa\lambda} + \frac{1}{2\beta} \Box \left( P_e^1 \right)_{\mu\nu,\kappa\lambda} \right] B^{\kappa\lambda} \\
- \frac{1}{2} B^\mu{}_{\nu\alpha} - X^\mu 3 \Box \omega_{\mu\nu} X^\nu - \frac{\sqrt{2}}{3} mA^\mu S_{\mu\kappa\lambda} B^{\kappa\lambda} \\
+ \frac{\sqrt{2}}{3} mB^{\kappa\lambda} S_{\kappa\lambda\mu} A^\mu - \frac{\sqrt{6}}{2} m\phi \partial_\mu X^\mu \\
+ \frac{\sqrt{6}}{2} mX^\mu \partial_\mu \phi,
\]

where it is written in terms of the projection operators

\[
\theta_{\mu\nu} = g_{\mu\nu} - \partial_\mu \partial_\nu \Box ,
\]

\[
\theta_{\mu\nu} = g_{\mu\nu} - \partial_\mu \partial_\nu \Box ,
\]

\[
(P_b^1)_{\mu\nu,\kappa\lambda} = \frac{1}{2} \left( \theta_{\mu\nu} \theta_{\kappa\lambda} - \theta_{\mu\kappa} \theta_{\nu\lambda} \right) ,
\]

\[
(P_e^1)_{\mu\nu,\kappa\lambda} = \frac{1}{2} \left( \theta_{\mu\nu} \omega_{\kappa\lambda} - \theta_{\mu\kappa} \omega_{\nu\lambda} - \theta_{\nu\kappa} \omega_{\mu\lambda} + \theta_{\nu\lambda} \omega_{\mu\kappa} \right) ,
\]

\[
S_{\mu\nu\kappa} = -m\epsilon_{\mu\nu\kappa\lambda} \partial_\lambda .
\]

which satisfy the relations

\[
(P_b^1)_{\mu\nu,\kappa\lambda} \left( P_b^1 \right)_{\nu\sigma,\kappa\lambda} = \left( P_b^1 \right)_{\mu\nu,\rho\sigma} ,
\]

\[
(P_e^1)_{\mu\nu,\kappa\lambda} \left( P_e^1 \right)_{\nu\sigma,\kappa\lambda} = \left( P_e^1 \right)_{\mu\nu,\rho\sigma} ,
\]

\[
(P_b^1)_{\mu\nu,\kappa\lambda} \left( P_e^1 \right)_{\nu\sigma,\kappa\lambda} = 0 ,
\]

\[
(P_e^1)_{\mu\nu,\kappa\lambda} \left( P_b^1 \right)_{\nu\sigma,\kappa\lambda} = 0 ,
\]

\[
S_{\mu\nu\alpha\sigma} S^{\sigma\alpha\kappa} = -2 \Box \left( P_b^1 \right)_{\mu\nu,\kappa\lambda} ,
\]

\[
\left( P_b^1 \right)_{\mu\nu,\alpha\beta} S^{\alpha\beta\kappa} = S_{\mu\nu}^\kappa ,
\]

\[
S^{\kappa\alpha\beta} \left( P_b^1 \right)_{\alpha\beta,\mu\nu} = S_{\kappa\mu\nu} ,
\]

\[
\left( P_b^1 \right)_{\mu\nu,\alpha\beta} S^{\alpha\beta\kappa} = 0 ,
\]

\[
\left( P_e^1 \right)_{\mu\nu,\alpha\beta} S^{\alpha\beta\kappa} = 0 ,
\]

\[
\left( P_e^1 \right)_{\mu\nu,\alpha\beta} S^{\alpha\beta\kappa} = 0 ,
\]
\[
S^\kappa_{\alpha\beta}(P_\nu^1)^{\alpha\beta,\mu\nu} = 0. \tag{67}
\]

It is convenient to write the Lagrangian in the matrix form. For this task, we set some of the matrix elements as

\[
P_{\mu\nu} = \Box \left( \theta_{\mu\nu} - \frac{1}{\alpha} \omega_{\mu\nu} \right), \tag{68}
\]

\[
Q_{\mu\rho\sigma} = -R_{\mu\rho\sigma} = \frac{2\sqrt{2}}{3} m S_{\mu\rho\sigma}, \tag{69}
\]

\[
S_{\kappa\lambda,\rho\sigma} = -\Box (P_\nu^1)^{\kappa\lambda,\rho\sigma} - \frac{2\beta}{\alpha} (P_\nu^1)^{\kappa\lambda,\rho\sigma}. \tag{70}
\]

Thus, \( L_{0g} = \frac{1}{2} N^t M N \). Where \( N^t = \left( A^\mu \ B_\kappa X^\alpha \ \phi \right) \) and

\[
M = \begin{pmatrix}
P_{\mu\nu} & R_{\mu\rho\sigma} & 0 & 0 \\
Q_{\kappa\lambda\nu} & S_{\kappa\lambda,\rho\sigma} & 0 & 0 \\
0 & 0 & -6\Box \omega_{\alpha\beta} \sqrt{6} m \partial_\alpha & \Box \\
0 & 0 & -\sqrt{6} m \partial_\beta & -\Box
\end{pmatrix}. \tag{71}
\]

After that, we inverse the \( M \)-matrix in order to find the following propagators:

\[
\langle \Phi \Phi \rangle = \frac{i}{k^2 - m^2}, \tag{72}
\]

\[
\langle X^\mu X^\nu \rangle = \frac{1}{3} \frac{i}{k^2 - m^2} \frac{k_\mu k_\nu}{k^2}, \tag{73}
\]

\[
\langle \Phi X^\mu \rangle = \langle X^\mu \Phi \rangle = \sqrt{6} \frac{im}{3} \frac{k_\mu}{k^2 - m^2} \tag{74}
\]

\[
\langle A_\mu B_\nu \kappa \rangle = \frac{im}{k^2 - m^2} \epsilon_{\mu\nu\kappa\alpha} \partial^\alpha, \tag{75}
\]

\[
\langle B_\mu A_\nu \rangle = -\frac{im}{k^2 - m^2} \epsilon_{\mu\nu\kappa\alpha} \partial^\alpha, \tag{76}
\]

\[
\langle A_\mu A_\nu \rangle = -\frac{i}{k^2 - m^2} \left[ g_{\mu\nu} + (\alpha - 1) \frac{k_\mu k_\nu}{k^2} \right] + \alpha \frac{im^2}{k^2 - m^2} \frac{k_\mu k_\nu}{(k^2)^2}, \tag{77}
\]

\[
\langle B_\mu B_\nu \kappa \rangle = \frac{i}{k^2 - m^2} \left[ 1_{\mu\nu,\kappa\lambda} + \left( \beta - \frac{1}{2} \right) \mathbb{K} \right] - \frac{\beta m^2}{k^2 - m^2} \frac{1}{k^2 \mathbb{K}}, \tag{78}
\]

where \( \mathbb{K} = \left( g_{\mu\alpha} \frac{k_\nu k_\beta}{k^2} - g_{\mu\beta} \frac{k_\alpha k_\nu}{k^2} - g_{\nu\alpha} \frac{k_\beta k_\alpha}{k^2} + g_{\nu\beta} \frac{k_\alpha k_\nu}{k^2} \right). \)
In 5D, a 1-form gauge potential contains 3 on-shell degrees of freedom (d.f.); a 3-form gauge propagates just 1 on-shell d.f.. Therefore, we have 4 physical degrees of freedom in the sector of gauge bosons. In 4D, we consequently have these 4 d.f. distributed among the fields we end up with by dimensional reduction. Considering the propagators from the $\Phi$ and $X^\mu$ sector, remembering that $\Phi$ comes from the Maxwell field in 5D and behaves as a genuine scalar Klein-Gordon in 4D, $X^\mu$ comes from the 3-form in 5D and is the dual of a 3-form in 4D and does not propagate degrees of freedom in 4D. Thus, it becomes clear that this sector describes a single scalar degree of freedom.

IV. CONCLUDING COMMENTS.

One proposes here to investigate a 5D electromagnetic model with a topological mass term for the photon built up in terms of a 1-form and a 3-form gauge potential. Such a description may offer some hint for modeling the so-called dark energy, due to the presence of the $\Theta^{4}_4$-component of our energy-momentum tensor that may correspond to a negative pressure and may then be describing an expanding system.

In addition, one identifies the emergence of a sector which we refer to as the extra dark sector. This extends Maxwell’s Electrodynamics by including an excitation which acts as a scalar photon, to which a scalar magnetic-like field is associated. In this scenario, one obtains, in 4D, a massive neutral vector boson (mass $m$) along with a scalar (with the same mass). The 3-form used to describe the negative contribution to the pressure contributes (with its coupling to the Maxwell field) to the presence of two massive bosons (one vector and one scalar) in 4D. This massive neutral vector boson may be identified as a sort of $Z^{01}$-boson [51] that may be detected at present high-energy accelerators. On the other hand, the massive scalar may be interpreted as the axion remnant of the Electrodynamics in 5D considering that the Chern-Simons term (Abelian) in 5D is defined as $\epsilon^{\mu\nu\lambda\rho\bar{\mu}\bar{\rho}}A_{\nu}F_{\lambda\rho}F_{\bar{\mu}\bar{\rho}}$ and its dimensional reduction to 4D leads to the axionic term type: $\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$ where $A^4 = \theta [46]$.

By setting $g = 0$, i.e., by eliminating the non-minimal coupling described by $\tilde{G}^\mu$ in the covariant derivative, the field $X^\mu$ decouples from the fermions; however, the axionic-like particle remains coupled, for its coupling is electromagnetic. We then point out that it is possible to decouple the field $X^\mu$, and, at the same time, to keep the axion coupled with the charged fermionic matter. We believe that it would be interesting to consider, from the
onset, a Chern-Simons term in five dimensions which would naturally induce the axionic coupling in 4D: $\theta F_{\mu\nu} \tilde{F}^{\mu\nu}$. The 5D Abelian Chern-Simons term is cubic in the gauge field and may provide a very natural scenario to discuss photon self-interactions and non-linear effects, with potentially interesting consequences for the electromagnetic interaction in 4 dimensions. We shall concentrate some efforts on this particular issue and we intend to report on that in a forthcoming paper.

As a final open question, one highlights the study of magnetic monopoles in a 5-dimensional scenario, where they are extended one-dimensional objects (i.e., strings) which appear as the dual of point-like charges. So, in 5D, magnetic monopoles have their interaction mediated by the 2-form Kalb-Ramond field. As a follow-up of the present work, we shall be making efforts to pursue an investigation of 5D Electrodynamics in the presence of (extended) magnetic monopoles, so that a 1-, a 2- and a 3-form should be all be present and their effect on the phenomenon of dark energy in 4 dimensions should be reassessed.

Acknowledgments

The authors express their gratitude to the agencies "Conselho Nacional de Desenvolvimento Científico e Tecnológico" (CNPq-Brazil) and "Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro" (FAPERJ) for the financial support.

[1] M.B. Green, Classical and Quantum Gravity 16(12), A77 (1999). URL http://stacks.iop.org/0264-9381/16/i=12A/a=304
[2] J. Maldacena, International Journal of Theoretical Physics 38(4), 1113 (1999). DOI 10.1023/A:1026654312961
[3] S. Gubser, I. Klebanov, A. Polyakov, Physics Letters B 428(1-2), 105 (1998). DOI http://dx.doi.org/10.1016/S0370-2693(98)00377-3
[4] E. Witten, ArXiv e-prints (1998). URL http://arxiv.org/abs/9802150
[5] O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri, Y. Oz, Physics Reports 323(3-4), 183 (2000). DOI http://dx.doi.org/10.1016/S0370-1573(99)00083-6
[6] Nima Arkani-Hamed and Savas Dimopoulos and Gia Dvali, Physics Letters B 429(3-4), 263
[7] P. Ade, et al., ArXiv e-prints (2013). URL http://arxiv.org/abs/1303.5062

[8] P. MÃ¶lsÃ¤ros, "High-Energy Radiation from Magnetized Neutron Stars" (University of Chicago Press, 1992)

[9] K.S. Stelle, Phys. Rev. D 16, 953 (1977). DOI 10.1103/PhysRevD.16.953. URL http://link.aps.org/doi/10.1103/PhysRevD.16.953

[10] E. Sezgin, P. van Nieuwenhuizen, Phys. Rev. D 21, 3269 (1980). DOI 10.1103/PhysRevD.21.3269. URL http://link.aps.org/doi/10.1103/PhysRevD.21.3269

[11] P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. 75, 559 (2003). DOI 10.1103/RevModPhys.75.559

[12] J. Blas, J. Lizana, M. PÂ¶rez-Victoria, Journal of High Energy Physics 2013(1), 1 (2013). DOI 10.1007/JHEP01(2013)166

[13] D. Cocuroci, "On the presence of a dark sector of radiation in the Classical Electrodynamics". Master’s thesis, Presented at the Graduate Program of the Brazilian Centre for Research in Physics, CBPF, Rio de Janeiro, Brazil (2009). URL http://cbpfindex.cbpf.br/publication_pdfs/denis%20cocuroci_%20tese%20de%20mestrado.2009_10_0

[14] B.A. Ovrut, D. Waldram, Nuclear Physics B 506(1-2), 236 (1997). DOI http://dx.doi.org/10.1016/S0550-3213(97)00510-5

[15] N. Hitchin, ArXiv Mathematics e-prints (2000). URL http://arxiv.org/abs/math/0010054

[16] D. Youm, Phys. Rev. D 63, 045004 (2001). DOI 10.1103/PhysRevD.63.045004

[17] M. Graña, J. Polchinski, Phys. Rev. D 65, 126005 (2002). DOI 10.1103/PhysRevD.65.126005

[18] A.A. Gerasimov, S.L. Shatashvili, Journal of High Energy Physics 2004(11), 074 (2004). URL http://stacks.iop.org/1126-6708/2004/i=11/a=074

[19] A. Aurilia, E. Spallucci, Phys. Rev. D 69, 105005 (2004). DOI 10.1103/PhysRevD.69.105005

[20] G. Dvali, ArXiv e-prints (2005). URL http://arxiv.org/abs/hep-th/0507215

[21] C. Bizdadea, E.M. Cioroianu, S.C. Săraru, International Journal of Modern Physics A 21(31), 6477 (2006). DOI 10.1142/S0217751X06034331

[22] P.M. Ho, Y. Imamura, Y. Matsuo, S. Shiba, Journal of High Energy Physics 2008(08), 014 (2008). URL http://stacks.iop.org/1126-6708/2008/i=08/a=014

[23] Cioroianu, E.M. and Diaconu, E. and Săraru, S.-C., Fortschritte der Physik 57(5-7), 535 (2009). DOI 10.1002/prop.200900056

[24] J.B. JimÃ¡nez, T.S. Koivisto, A.L. Maroto, , D.F. Mota, Journal
of Cosmology and Astroparticle Physics 2009(10), 029 (2009). URL http://stacks.iop.org/1475-7516/2009/i=10/a=029

[25] T.S. Koivisto, D.F. Mota, C. Pitrou, Journal of High Energy Physics 2009(09), 092 (2009). URL http://stacks.iop.org/1126-6708/2009/i=09/a=092

[26] T.S. Koivisto, N.J. Nunes, Phys. Rev. D 80, 103509 (2009). DOI 10.1103/PhysRevD.80.103509

[27] T.S. Koivisto, N.J. Nunes, Physics Letters B 685(2-3), 105 (2010). DOI http://dx.doi.org/10.1016/j.physletb.2010.01.051

[28] T. Ngampitipan, P. Wongjun, Journal of Cosmology and Astroparticle Physics 2011(11), 036 (2011). URL http://stacks.iop.org/1475-7516/2011/i=11/a=036

[29] T.S. Koivisto, N.J. Nunes, ArXiv e-prints (2012). URL http://arxiv.org/abs/1212.2541

[30] J. Schmude, ArXiv e-prints (2012). URL http://arxiv.org/abs/1201.1621

[31] M.C. Diamantini, G. Guarnaccia, C.A. Trugenberger, ArXiv e-prints (2013). URL http://arxiv.org/abs/1310.2103

[32] X.L. Qi, E. Witten, S.C. Zhang, Phys. Rev. B 87, 134519 (2013). DOI 10.1103/PhysRevB.87.134519

[33] J. Scherk, J.H. Schwarz, Physics Letters B 82(1), 60 (1979). DOI http://dx.doi.org/10.1016/0370-2693(79)90425-8

[34] L.F. Abbott, Acta Phys. Polon. B 13(33), 5 (1982). URL #http://isites.harvard.edu/fs/docs/icb.topic538993.files/Background_field_abbott.pdf#

[35] H.R. Christiansen, M.S. Cunha, J.A. Helayël-Neto, L.R.U. Manssur, A.L.M.A. Nogueira, International Journal of Modern Physics A 14(01), 147 (1999). DOI 10.1142/S0217751X99000075

[36] C.N. Ferreira and J.A. Helayël-Neto and M.B.D.S.M. Porto, Nuclear Physics B 620(1-2), 181 (2002). DOI http://dx.doi.org/10.1016/S0550-3213(01)00571-5

[37] B. Holdom, Physics Letters B 166(2), 196 (1986). DOI http://dx.doi.org/10.1016/0370-2693(86)91377-8

[38] B. Holdom, Phys.Lett. B259, 329 (1991). DOI 10.1016/0370-2693(91)90836-F

[39] S. Davidson, B. Campbell, D. Bailey, Phys. Rev. D 43, 2314 (1991). DOI 10.1103/PhysRevD.43.2314

[40] S. Davidson, M. Peskin, Phys. Rev. D 49, 2114 (1994). DOI 10.1103/PhysRevD.49.2114

[41] A.G. Riess, P.E. Nugent, R.L. Gilliland, B.P. Schmidt, J. Tonry, M. Dickinson, R.I. Thompson, T. Budavári, S. Casertano, A.S. Evans, A.V. Filippenko, M. Livio, D.B. Sanders, A.E. Shapley,
H. Spinrad, C.C. Steidel, D. Stern, J. Surace, S. Veilleux, The Astrophysical Journal 560(1), 49 (2001). URL http://stacks.iop.org/0004-637X/560/i=1/a=49

[42] M.S. Turner, A.G. Riess, The Astrophysical Journal 569(1), 18 (2002). URL http://stacks.iop.org/0004-637X/569/i=1/a=18

[43] S. Perlmutter, G. Aldering, G. Goldhaber, R.A. Knop, P. Nugent, P.G. Castro, S. Deustua, S. Fabbro, A. Goobar, D.E. Groom, I.M. Hook, A.G. Kim, M.Y. Kim, J.C. Lee, N.J. Nunes, R. Pain, C.R. Pennypacker, R. Quimby, C. Lidman, R.S. Ellis, M. Irwin, R.G. McMahon, P. Ruiz-Lapuente, N. Walton, B. Schaefer, B.J. Boyle, A.V. Filippenko, T. Matheson, A.S. Fruchter, N. Panagia, H.J.M. Newberg, W.J. Couch, T.S.C. Project, The Astrophysical Journal 517(2), 565 (1999). URL http://stacks.iop.org/0004-637X/517/i=2/a=565

[44] A.G. Riess, A.V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P.M. Garnavich, R.L. Gilliland, C.J. Hogan, S. Jha, R.P. Kirshner, B. Leibundgut, M.M. Phillips, D. Reiss, B.P. Schmidt, R.A. Schommer, R.C. Smith, J. Spyromilio, C. Stubbs, N.B. Suntzeff, J. Tonry, The Astronomical Journal 116(3), 1009 (1998). URL http://stacks.iop.org/1538-3881/116/i=3/a=1009

[45] M. Kalb, P. Ramond, Phys. Rev. D 9, 2273 (1974). DOI 10.1103/PhysRevD.9.2273

[46] J. Redondo, A. Ringwald, Contemporary Physics 52(3), 211 (2011). DOI 10.1080/00107514.2011.563516. URL http://www.tandfonline.com/doi/abs/10.1080/00107514.2011.563516

[47] M. Betz, F. Caspers, M. Gasior, M. Thumm, ArXiv e-prints (2013)

[48] P. Arias, J. Jaeckel, J. Redondo, A. Ringwald, Phys. Rev. D 82, 115018 (2010). DOI 10.1103/PhysRevD.82.115018

[49] I. Antoniadis, A. Boyarsky, O. Ruchayskiy, ArXiv High Energy Physics - Phenomenology e-prints (2006). URL http://arxiv.org/abs/hep-ph/0606306

[50] I. Antoniadis, A. Boyarsky, O. Ruchayskiy, Nuclear Physics B 793(1-2), 246 (2008). DOI http://dx.doi.org/10.1016/j.nuclphysb.2007.10.006

[51] A. Leike, Physics Reports 317(3Á«áéÇñáÁÌJ4), 143 (1999). DOI http://dx.doi.org/10.1016/S0370-1573(98)00133-1. URL http://www.sciencedirect.com/science/article/pii/S0370157398001331