1. NEUTRON STAR ELECTRODYNAMICS

We explore black hole electrodynamics through an analogy with an axisymmetric pulsar (Goldreich & Julian 1969; Contopoulos et al. 1999). We begin with three simple statements:

1. A neutron star may be charged. One might naively argue that once you embed a charged star in an ionized medium, it will attract carriers of the opposite charge and very quickly lose its charge. A relativistic astrophysicist, however, will argue that a neutron star with a dipole magnetic field spinning along its magnetic axis inside an ionized medium (not vacuum), induces a distribution of radial electric field

   \[ E_r = B_0 \frac{\Omega r_s \sin \theta}{c} = B \frac{\Omega r_s}{c} \sin^2 \theta \]  

   (Goldreich & Julian 1969), and therefore an electric charge

   \[ Q = \int_0^\theta 2\pi r_s^2 \sin \theta E_r d\theta = \frac{8\pi}{3} r_s^2 B \frac{\Omega r_s}{c}. \]  

   Here, \( B \) is the equatorial value of the dipole magnetic field as measured by a non-rotating observer, \( \Omega \) and \( r_s \) are the angular velocity and radius of the star, and \( \theta \) is the polar angle. This charge is distributed in the neutron star interior in such a way as to satisfy the infinite conductivity condition \( E \cdot B = 0 \) everywhere. It may be sitting inside an ionized magnetosphere, but it is not “sitting idle” waiting to be discharged. The spinning neutron star is an astrophysical engine that electrically polarizes its magnetosphere, generates large scale electric currents, and emits electromagnetic (Poynting) radiation. As long as this engine operates, the neutron star is not discharged. We will see in the next section that a similar result may also apply to black holes.

2. A neutron star supports its own magnetic field. What is of interest here is that an observer co-rotating with the neutron star measures an intrinsic dipole magnetic field \( B_s \) generated by toroidal electric currents in the neutron star interior. However, the magnetic field \( B \) measured by a stationary observer is different. \( B \) is the Lorentz transformation of \( B_s \), namely

   \[ B = \frac{B_s}{\sqrt{1 - (\Omega r_s \sin \theta/c)^2}} \approx B_s + \frac{1}{2} \left( \frac{\Omega r_s \sin \theta}{c} \right)^2 B_s, \]  

   with \( \Omega r_s/c \) typically less than about 0.1. An equivalent way to view this result is that the intrinsic stellar magnetic field induces a certain distribution of charge in the stellar interior and in the rotating magnetosphere, and thus forms a distribution of toroidal currents that generates an extra poloidal magnetic field component

   \[ \delta B \sim \frac{Q}{r_s^2} \left( \frac{\Omega r_s}{c} \right). \]  

   As we will see below, a similar result may also apply to black holes.

3. Isolated neutron stars spin down electrodynamically. We remind the reader that the magnetosphere of the axisymmetric pulsar consists of closed and open field lines, and only the open field lines (those that cross the light cylinder) contribute to the neutron star spindown as

   \[ \dot{E} = \frac{2}{5} M a r_s^2 \Omega \dot{\Omega} = -\frac{\Psi_{\text{open}}^2}{6\pi^2 c^2} \frac{\Omega^2}{r_s^2} = -B^2 r_s^2 c \left( \frac{\Omega r_s}{c} \right)^4 \]  

   (Contopoulos 2005). Here, \( M_a \) is the mass of the neutron star. One can solve Equation (5) to obtain \( \Omega = \Omega(t) \) and \( \dot{E} = \dot{E}(t) \), and thus easily show that at late times, \( \Omega \propto t^{-1/2} \), and

   \[ \dot{E} \propto t^{-2}. \]  

   We will now see that, under certain astrophysical circumstances, rotating black holes may also function as axisymmetric pulsars.

2. BLACK HOLE ELECTRODYNAMICS

We will consider black holes that form in the core collapse of supermassive stars. If the star is magnetized, magnetic flux
will be advected with the collapse. The material that is going to collapse into a black hole will be strongly magnetized, and therefore its core will pass through a spinning magnetized neutron star stage. A certain amount of magnetic flux and electric charge is then going to cross the horizon. What happens next is most interesting.

2.1. The Blandford–Znajek Phase

The rotational collapse will naturally form a thick equatorial disk of ionized material around the central black hole. That material will hold the magnetic flux \( \Phi \) advected initially through the horizon and will prevent it from escaping to infinity. In that phase of the system’s evolution, the black hole will spin down very dramatically according to the Blandford–Znajek prescription

\[
\dot{E} \sim -\frac{1}{6\pi^2 c} \psi_o^2 \Omega^2 \quad (8)
\]

(Blandford & Znajek 1977; Tchekhovskoy et al. 2010; Contopoulos et al. 2013). Here,

\[
\Omega = \Omega_o \frac{\alpha}{1 + \sqrt{1 - \alpha^2}} \quad (9)
\]

is the angular velocity of a maximally rotating black hole, and \( \psi_o \) is the gravitational constant. As in pulsars, the radiated energy is extracted from the available (reducible) black hole “rotational” energy \( \alpha \mathcal{G} M^2 \Omega/c \) (Christodoulou & Ruffini 1971). The black hole will therefore spin down as

\[
\dot{E} = \frac{\mathcal{G} M^2 d(\alpha \Omega)}{c} \quad (11)
\]

We can reverse Equation (9) to obtain \( \alpha \) as a function of \( \Omega/\Omega_o \), and rewrite the equation that describes the black hole spindown as

\[
\tau_{\text{BZ}} \frac{d}{dt} \left( \frac{2(\Omega/\Omega_o)^2}{1 + (\Omega/\Omega_o)^2} \right) = - \left( \frac{\Omega}{\Omega_o} \right)^2 \quad (12)
\]

where

\[
\tau_{\text{BZ}} = \frac{12 c^5}{\mathcal{G}^2 \psi_o^2 B_o^2 M} = 33 B_o^{-2} M_{10}^{-1} \text{ s} \quad (13)
\]

As we will see in the next section, the decay time \( \tau_{\text{BZ}} \) is a very important physical parameter. We have defined here a typical value for the initial black hole magnetic field

\[
B_o = \frac{\psi_o}{\pi \tau_{\text{BZ}}} = \frac{\psi_0 c^4}{\pi \mathcal{G}^2 M^2} \quad (14)
\]

\( B_{16} \) is \( B_o \) in units of \( 10^{16} \text{ G} \), and \( M_{10} \) is the black hole mass in units of \( 10 M_{\odot} \).

It is reasonable to assume that, when the black hole forms, its core is maximally rotating. This allows us to integrate Equation (12) as

\[
\frac{1}{1 + (\Omega/\Omega_o)^2} + \ln \left( \frac{2(\Omega/\Omega_o)^2}{1 + (\Omega/\Omega_o)^2} \right) = \frac{1 - (t/\tau_{\text{BZ}})}{2} \quad (15)
\]

We can solve Equation (15) numerically to obtain \( \Omega = \Omega(t) \), and thus

\[
\dot{E} = \frac{\dot{E}_o}{1 + \left[ W\left(-\frac{1}{2} e^{-t/\tau_{\text{BZ}}}ight) \right]^{-1}} \quad (16)
\]

where

\[
\dot{E}_o \equiv -\frac{\psi_o^2 \Omega_o^2}{6\pi^2 c} = -3 \times 10^{33} B_{16}^2 M_{10}^2 \text{ erg s}^{-1} \quad (17)
\]

\( W(x) \) is the Lambert W function which solves the equation \( x = W(x)e^{W(x)} \). Note that, for a fixed black hole mass, \( \dot{E}_o \) is inversely proportional to \( \tau_{\text{BZ}} \). An approximation to Equation (16) is

\[
\dot{E} \approx \dot{E}_o \frac{e^{-t/2\tau_{\text{BZ}}}}{2} \quad (18)
\]

We would like to emphasize that during this phase, the equatorial disk surrounding the black hole keeps the advected flux in place, and the black hole magnetic field does not diminish.

2.2. The Pulsar-like Phase

During the Blandford–Znajek phase, the accumulated black hole electric charge may be estimated by the Wald value

\[
Q \sim B_o r_h \frac{\Omega r_h}{c} \quad (19)
\]

(Wald 1974). Obviously, this phase will not last for too long. After a transition period that may last anywhere between a few minutes to a few weeks, the surrounding material will either be dispersed or will be engulfed by the black hole. The black hole will still be spinning, but it is not clear how much charge it will be left with, so we can only estimate it through Equation (19).

Let us assume for the moment that all external charges and currents are removed. An isolated charged and spinning black hole corresponds to a vacuum solution of the Einstein equations, but a solution that describes a so-called electro-vacuum will still be spinning, but it is not clear how much charge it will be left with, so we can only estimate it through Equation (19).

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The four-potential of the Kerr–Newman electromagnetic field along the equator is given by \( A_\phi = Q/r \) (e.g., Misner et al. 1973; Poisson 2004). It is then straightforward to calculate the magnetic flux \( \Psi_{\text{KN}} \) that threads the horizon as

\[
\Psi_{\text{KN}} \equiv \int_0^{2\pi} A_\phi(r_h) \frac{2\mathcal{G} M}{c^2} d\phi = 2\pi Q \frac{2\mathcal{G} M}{r_h c^2} \quad (20)
\]

For a slowly rotating black hole,

\[
\Psi_{\text{KN}} \approx 4\pi Q \left( \frac{\Omega r_h}{c} \right) \quad (21)
\]

(Equation (9)).

The reader can check that a “maximally charged” slowly rotating Kerr–Newman black hole corresponds to \( Q_{\text{max}} \lesssim G^{1/2} M \), i.e., to \( B_{\text{max}} \sim 10^{18} M_{10}^{-2} G \) (Equation (2)),

\[
\text{if we estimate } Q \text{ through Equation (19), our result differs from those of Lyutikov & McKinney (2011) and Lyutikov (2011) by one extra factor for } (\Omega r_h/c) \).
hence values of $B_r \lesssim 10^{16}$ G justify the use of the Kerr metric as an excellent approximation to the Kerr–Newman one. The point we would like to emphasize here is that a stellar mass black hole is very naturally charged during its formation in the collapse of its progenitor star, and therefore it can naturally generate its own dipole magnetic field, even after the external currents are removed.

The astrophysical problem is more complicated. Obviously, the electromagnetic field cannot remain that of the electro-vacuum Kerr–Newman solution. Microphysical processes will generate a distribution of electron–positron pair plasma charges and currents that will short out the electric field component parallel to the magnetic field. The black hole will absorb opposite charges and reduce its charge. This effect will be balanced by an equivalent increase of the rotating magnetospheric charge which is naturally expected to support an amount of dipolar magnetic flux, given approximately in Equation (21). In this picture, the source of the exterior magnetic field has moved from inside the event horizon (the Kerr–Newman solution) to just outside (Petterson 1975; Takahashi & Koyama 2009). We must acknowledge that we do not know anything about the stability of such a configuration apart from the fact that if the black hole engulfs the above magnetospheric charge, it will revert to the Kerr–Newman solution, so the whole process will start all over again. Moreover, matching the above exterior solution to an interior black hole solution is a problem of considerable astrophysical importance (Ghosh 2000).

During that later pulsar-like phase of the core collapse, the spindown of the isolated magnetized black hole will proceed in analogy to the spindown of the axisymmetric pulsar (e.g., Punsly 1998). Notice that this is an electrodynamic (not static) system that holds a rotating magnetospheric electric charge which we can only assume to decrease as

$$Q \sim B_n r_n^2 \left( \frac{\Omega r_n}{c} \right)^n,$$

with $n \geq 0$.

The black hole is no longer maximally rotating. The magnetosphere will consist of closed and open field lines, and only the open field lines (those that cross the light cylinder) will contribute to the black hole spindown (see Figure 1). Notice that now $\alpha \ll 1$, therefore, $\Omega = \alpha \Omega_0 / 2$, $r_h = 2GM/c^2$, and the reducible black hole "rotational" energy is equal to $M(r_h \Omega)^2$ to an excellent approximation. The axisymmetric pulsar theory now yields

$$\dot{E} = 2Mr_h^2 \Omega \dot{\Omega} = -\frac{V_{KN}^{2 \text{open}}}{6\pi^2 c} \Omega^2$$

$$= - B^2 r_n^2 c \left( \frac{\Omega r_n}{c} \right)^4 \sim -Q^2 r_n^{-2} c \left( \frac{\Omega r_n}{c} \right)^6,$$

which is proportional to $\Omega^{6+2n}$. As before, one can solve Equation (23) to obtain $\dot{\Omega} = \Omega(t)$ and $\dot{E} = \dot{E}(t)$ during this later phase of the black hole electrodynamic evolution, and show that

$$\Omega \propto t^{-1/(4+2n)}, \quad \text{and}$$

$$\dot{E} \propto t^{-(3+2n)/(2+3n)}.$$  

Figure 1. Poloidal magnetic field lines near the black hole horizon. Top: initial Blandford–Znajek phase when the black hole is maximally rotating and the magnetic flux that threads the horizon is held in place by the surrounding equatorial material (Contopoulos et al. 2013). Bottom: pulsar-like phase when the black hole has slowed down by a factor of about four (A. Nathanail et al. 2013, in preparation). Thicker lines: ergosphere. Dashed lines: light cylinder and inner light surface.

(A color version of this figure is available in the online journal.)

This observation leads us to associate the pulsar-like phase (Equation (25)) with the GRB afterglow. Eventually, the black hole will stop spinning down electrodynamically when its magnetosphere stops producing the electron–positrons pairs required to satisfy the force-free condition everywhere, in analogy to pulsar “death.”

power law decay exponent observed during the gamma-ray burst (GRB) afterglow phase has a value between $-1$ and $-1.5$ (Nousek et al. 2006), in agreement with Equation (25). This observation leads us to associate the pulsar-like phase (Equation (25)) with the GRB afterglow.
3. GRB OBSERVATIONS

Our GRB model, of a 10 $M_\odot$ newly formed maximally rotating black hole spinning down electro-dynamically, explores an analogy with an axisymmetric pulsar. It is interesting that neither system forms relativistic jets on its own, except of course if there is a surrounding medium that collimates the black hole/pulsar wind which is nearly isotropic beyond the light cylinder (Figure 1). As in pulsars, high energy radiation is generated through reconnection and particle acceleration processes in the equatorial magnetospheric current sheet (Lyubarsky & Kirk 2001; Li et al. 2012; Kalapotharakos et al. 2012). In that respect, our model is “orthogonal” to the standard GRB model where all the action takes place along a relativistic jet emitted along the rotation and magnetic axis.

We can compare our model with observations. In order to do that, we need to take into account the source’s cosmological redshift $z$. The observed decay time $\tau_{BZ\, \text{obs}}$ is related to $\tau_{BZ}$ as

$$\tau_{BZ\, \text{obs}} = \tau_{BZ}(1+z).$$

Straightforward fits of typical GRB light curves (Evans et al. 2007, 2009) with initial exponential decay and known redshifts yield

$$\tau_{BZ} \sim 10-100 \text{ s}$$

(Table 1, Figure 2). Our model predicts that

$$\dot{E}_\odot \tau_{BZ} = 10^{55} M_{10} \text{ erg},$$

and therefore, Equation (27) yields $\dot{E}_\odot \sim 10^{53}-10^{54} \text{ erg s}^{-1}$, and $B_{\odot 6} \sim 1$ (Equation (17)). Notice that the black hole spindown time is much longer than the initial rotational period of about 1 ms.

The black hole spindown luminosity $\dot{E}_\odot$ is not directly observable. In analogy to pulsars, though, some fraction of it $f$ will be emitted in the form of high energy radiation (X-rays, $\gamma$-rays) generated by electrons/positrons accelerated electrostatically in the equatorial magnetospheric current sheet. For a given luminosity distance $d_L$ and observed high energy radiation flux $F$,

$$\dot{E}_{\text{rad}} \approx f \dot{E}_\odot = 4\pi d_L^2 F$$

under the assumption of isotropic emission. Therefore, in order to test Equation (28), one needs to know $f$. If a correlation between $\tau_{BZ}$ and $\dot{E}_{\text{rad}}$ is confirmed in the few GRB cases with known redshift and a clear exponential luminosity decay during the initial phase of the burst, this will allow us to use GRBs as standard candles in cosmology.

We conclude that the light curves of long duration GRBs may yield important information about the electrodynamic processes that take place on the horizon of a spinning black hole.

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Table 1

| Name      | $z$  | $d_L$ (Gpc) | $F$ (erg s$^{-1}$ cm$^{-2}$) | $\tau_{BZ\, \text{obs}}$ (s) | $\dot{E}_{\text{rad}} \cdot \tau_{BZ}$ ($10^{53}$ erg) |
|-----------|------|-------------|----------------------------|-----------------------------|--------------------------------------------------|
| 050502B   | 5.2  | 50.2        | $10^{-7}$                  | 74                          | 4                                                 |
| 060614A   | 0.125| 0.6         | $10^{-6}$                  | 48                          | 0.02                                              |
| 080307    | 10   | 10^{-7}     | 200                        |                             |                                                   |
| 090814A   | 2.2  | 17.8        | $10^{-7}$                  | 85                          | 1                                                 |
| 120401A   | 10   | 10^{-8}     | 150                        |                             |                                                   |
| 130701A   | 1.155| 8           | $10^{-6}$                  | 180                        | 6                                                 |

Notes.

a Estimates of $F$ from peak 15–150 keV photon flux (Swift data archive).
b Redshift estimate from Afonso et al. (2011).

Figure 2. X-ray light curve data for GRB 060614A (dots) and fits of early Blandford-Znajek phase (red line) and late pulsar-like afterglow (blue line). (A color version of this figure is available in the online journal.)