ELECTROMAGNETIC FIELD GENERATED
BY PLASMA FLOWS
AND FEYNMAN AND LIÉNARD-WIECHERT FORMULAS
FOR A MOVING POINT CHARGE

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ABSTRACT. This note represents a stepping stone from the discovery of the
precise mathematical formula for electromagnetic field generated by a moving
point charge, the amended Feynman formula, see Bogdan
http://arxiv.org/abs/0909.5240 and leading to the to the general formula
of gravitational and electromagnetic fields generated by moving matter in a
Lorentzian frame of special theory of relativity, see
http://arxiv.org/abs/0910.0538.

In this note the author introduces the notion of flow of matter in a Lorentzian
frame. This notion is relativistic in the sense of Einstein’s special theory of
relativity.

The author presents explicit formulas suitable for a digital computer per-
mitting one to find time delay for an action from a flow line to any point in
the Lorentzian frame. The time delay field for any flow of plasma in a fixed
Lorentzian frame is unique.

Using this field he introduces the retarded time field and fundamental fields
corresponding to the flow with a free parameter representing the initial posi-
tions of the lines of flow.

By means of these fields one can represent and establish relations between
wave, Lorentz gauge, and Maxwell equations, and Lienard-Wiechert potentials,
and amended Feynman’s formula.

The initial distribution of charges over the initial position of the flow is
given by a signed measure of finite variation defined over Borel sets. It may
include discrete and continuous components.

This note represents a stepping stone from the discovery of the precise mathe-
matical formula for electromagnetic field generated by a moving point charge, the
amended Feynman formula, see Bogdan [5], and leading to the to the general for-
formula of gravitational and electromagnetic fields generated by moving matter in a
Lorentzian frame of special theory of relativity, Bogdan [6].

The the generalized densities of mass and of charge in the above fields may consist
of discrete and continuous components not like in general theory of relativity, for
reference see Dirac [8].

As a byproduct of the paper [6], for the case when the charge field has a Lebesgue
summable density, one obtains precise electromagnetic potentials that correct Feyn-
man’s formulas presented in Feynman-Leighton-Sands [10], vol. 2, chapter 15, page
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The results contained in this note have been presented at the annual meeting of the Society for Applied and Industrial Mathematics held in Denver, Colorado, on 7 July 2009 [6].

Let us get to the essentials. For the sake of simplicity in notation we select units so that the speed of light is \( c = 1 \) and the electrostatic constant satisfies the condition \( 4\pi\varepsilon_0 = 1 \).

We are working here in the environment of Einstein’s special theory of relativity in a fixed Lorentzian frame.

1. **Uniqueness of time delay field for a flow of plasma**

**Definition 1.1** (Plasma flow). Let \( F \subset R^3 \) be a compact set representing the position of plasma at some initial time \( t_0 \).

By a plasma flow we shall understand a continuous function \( r_2(t, r_0) \) from the product \( R \times F \) into \( R^3 \), having third derivative with respect to \( t \) and such that the velocity \( v_2(t, r_0) = \dot{r}_2(t, r_0) \) and the acceleration \( a_2(t, r_0) = \dot{v}_2(t, r_0) \) and the derivative \( \dot{a}_2(t, r_0) \) are continuous on the product \( R \times F \).

Moreover the following two conditions are satisfied:

1. For every time \( t_1 \) there is a velocity \( v_1 < c = 1 \) such that

\[
|v_2(t, r_0)| \leq v_1 \quad \text{for all} \quad t \leq t_1 \text{ and } r_0 \in F.
\]

2. For every time \( t \in R \) the map \( P_t \) given by the formula

\[
P_t(r_0) = r_2(t, r_0) \quad \text{for all} \quad t \in R \text{ and } r_0 \in F
\]

represents a homeomorphism of \( F \) onto \( P_t(F) \).

Clearly we have

\[
P_{t_0}(r_0) = r_2(t_0, r_0) = r_0 \quad \text{for all} \quad r_0 \in F.
\]

The function \( t \mapsto r_2(t, r_0) \) will be called a line of flow corresponding to the index \( r_0 \).

Let \( T = T(r_1, t, r_0) \) denote the time delay required to reach point \( r_1 \in R^3 \) at time \( t \) from the line of flow corresponding to index \( r_0 \). Its value must satisfy the Lorentz time delay equation

\[
T = |r_1 - r_2(t - T, r_0)|.
\]

**Theorem 1.2** (Time delay is unique and continuous). For every point \( r_1 \in R^3 \), and time \( t \in R \), and index \( r_0 \in F \) there exists one and only one solution \( T \) of equation (1.3). Moreover the function \( T = T(r_1, t, r_0) \) is continuous on its entire domain \( R^3 \times R \times F \).
Notice the following relations
\[ T = 0 \iff \{ r_1 = r_2(t, r_0) \text{ for some } r_0 \in F \} \iff r_1 \in P_t(F). \]

The set \( P_t(F) \) represents the position of the plasma at time \( t \).

Now define sets
\[ G = \{(r_1, t, r_0) \in \mathbb{R}^3 \times \mathbb{R} \times F : T(r_1, t, r_0) > 0 \} \]
and
\[ G_0 = \{(r_1, t) \in \mathbb{R}^3 \times \mathbb{R} : T(r_1, t, r_0) > 0 \text{ for all } r_0 \in F \}. \]

**Theorem 1.3** (Sets \( G \) and \( G_0 \) are open). The set \( G \) is nonempty and open in the product space \( \mathbb{R}^3 \times \mathbb{R} \times F \) and so is the set \( G_0 \) in the product space \( \mathbb{R}^3 \times \mathbb{R} \).

### 2. Fundamental Fields Corresponding to the Flow

**Definition 2.1** (Fundamental fields). Introduce the retarded time function
\[ \tau = \tau(r_1(t, r_0)) = t - T(r_1(t, r_0)) \text{ for all } (r_1, t, r_0) \in \mathbb{R}^3 \times \mathbb{R} \times \mathbb{R}^3, \]
retarded velocity
\[ v = v_2(\tau(r_1(t, r_0), r_0)) \text{ for all } (r_1, t, r_0) \in \mathbb{R}^3 \times \mathbb{R} \times \mathbb{R}^3, \]
and retarded acceleration
\[ a = a_2(\tau(r_1(t, r_0), r_0)) \text{ for all } (r_1, t, r_0) \in \mathbb{R}^3 \times \mathbb{R} \times \mathbb{R}^3, \]
and vector field \( r_{12} \) by
\[ r_{12} = r_1 - r_2(\tau(r_1(t, r_0), r_0)) \text{ for all } (r_1, t, r_0) \in \mathbb{R}^3 \times \mathbb{R} \times \mathbb{R}^3. \]

Introduce the unit vector field \( e \), and the fields \( u \) and \( z \) by the formulas
\[ e = \frac{r_{12}}{T} \text{ and } u = \frac{1}{T} \text{ and } z = \frac{1}{1 - \langle e, v \rangle} \text{ on } G. \]

These functions will be called the **fundamental fields** associated with the flow \( r_2(t, r_0) \), where \( t \in \mathbb{R} \) and \( r_0 \in F \).

Notice that by the definition of flow of plasma the velocities are smaller in magnitude than the speed of light \( c = 1 \). Thus we must have for the dot product \( |\langle e, v \rangle| \leq |v| < 1 \). So the field \( z \) is well defined.

All the above functions consist of compositions of continuous functions, therefore each of them is continuous on its respective domain and thus all of them are continuous on their common domain, the set \( G \).

We would like to stress here that the fundamental fields depend on the Lorentzian frame, in which we consider the trajectory. It is important to find expressions involving fundamental fields that yield fields invariant under Lorentzian transformations.

Lorentz [11] and Einstein [9], Part II, section 6, established that fields satisfying Maxwell equations are invariant under Lorentzian transformations.

Our main goal is to prove that fields constructed for flows of plasma will satisfy Maxwell equations. We shall do this by showing that these fields are representable
by means of fundamental fields and using the formulas for partial derivatives of the fundamental fields prove that such fields generate fields satisfying Maxwell equations.

Introduce operators $D = \frac{\partial}{\partial t}$ and $D_i = \frac{\partial}{\partial x_i}$ for $i = 1, 2, 3$ and $\nabla = (D_1, D_2, D_3)$. Observe that $\delta_i$ in the following formulas denotes the $i$-th unit vector of the standard base in $\mathbb{R}^3$ that is $\delta_1 = (1, 0, 0)$, $\delta_2 = (0, 1, 0)$, $\delta_3 = (0, 0, 1)$.

The symbols $e_i$, $v_i$, $a_i$, denote the corresponding component of the vector fields $e$, $v$, $a$, respectively.

**Theorem 2.2** (Partial derivatives of fundamental fields). Assume that in some Lorentzian frame we are given a plasma flow $(t, r_0) \mapsto r_2(t, r_0)$. For partial derivatives with respect to coordinates of the vector $r_1$ we have the following identities on the set $G$

\begin{align*}
(2.2) & \quad D_i T = ze_i, \\
(2.3) & \quad D_i u = -zu^2 e_i, \\
(2.4) & \quad D_i v = -e_i a, \\
(2.5) & \quad D_i \tau = -ze_i, \\
(2.6) & \quad D_i e = -uze_i e + u\delta_i + uze_i v \quad \text{where} \quad \delta_i = (\delta_{ij}), \\
(2.7) & \quad D_i z = -z^3 e_i (e, a) - uz^3 e_i + uz^2 e_i + uz^2 v + uz^3 e_i (v, v) \\
(2.8) & \quad \nabla T = ze, \\
(2.9) & \quad \nabla u = -zu^2 e, \\
(2.10) & \quad \nabla z = -z^3 (e, a)e - uz^3 e + uz^2 e + uz^2 v + uz^3 (v, v)e.
\end{align*}

and for the partial derivative with respect to time we have

\begin{align*}
(2.11) & \quad DT = 1 - z, \\
(2.12) & \quad Du = zu^2 - u^2, \\
(2.13) & \quad D\tau = z, \\
(2.14) & \quad Dv = za, \\
(2.15) & \quad De = -ue + uze - uzv, \\
(2.16) & \quad Dz = uz - 2uz^2 + z^3 (e, a) + uz^3 - uz^3 (v, v).
\end{align*}

Since the expression on the right side of each formula represents a continuous function, the fundamental fields are at least of class $C^\infty$ on the set $G$.

The proof of the above theorem is similar to the proof of analogous theorem in Bogdan [5].

3. Integration with respect to a signed measure

Let $V$ be a prering of subsets of $F$ consisting of sets of the form $Q \cap B$ where $Q$ is compact and $B$ is open. See Bogdanowicz [1] page 498 available on the web.

Assume that the set functions $q^+(A)$ and $q^-(A)$ represent, respectively, the total positive and total negative charge contained in the body covered by the set $A \in V$. We shall assume that these functions are countably additive.

Remark
A heuristic argument relying on assumption that charge of an electron is indivisible can be presented as follows: Take a decomposition of a set $A \in V$ into a countable union of disjoint sets

$$A = A_1 \cup A_2 \cup \ldots A_n \cup \ldots.$$ 

Since every charge comes in the form of finite number of indivisible unit charges, that are all equal to the charge of a single electron, only a finite number of the sets may contain a charge. Thus starting from a sufficiently large index $n_0$ all sets $A_n$ will have charge zero. Thus

$$q^+(A) = \sum_{n \leq n_0} q^+(A_n) + \sum_{n > n_0} 0 = \sum_{n=1}^{\infty} q^+(A_n).$$

Similarly we can get countable additivity of $q^-$. 

Put $q(A) = q^+(A) + q^-(A)$ and $\eta(A) = q^+(A) - q^-(A)$. The value $q(A)$ represents the total charge in the body covered by the set $A$ and $\eta(A)$ represents a non-negative countably additive set function on $V$ such that $|q(A)| \leq \eta(A)$.

Such a function satisfies the requirements of a volume function as defined in [1, page 492]. Observe that the function $q$ belongs to the space $M$, defined on page 492, and its norm $\|q\| \leq 1$. Therefore we can use the trilinear integral $\int u(f, dq)$ developed there. In our case for the bilinear operator $u(y, r) = ry = yr$ defined for $y \in Y$ and $r \in R$, where $Y$ stands for either the vector space $R^3$ or the space of $R$ of reals.

Thus we can use the theory developed in the papers Bogdanowicz [1] and [2]. The latter one is also available on the web. These two papers provide all the tools of Lebesgue and Bochner theory on measure and integration, based on measure on sigma rings of sets, available if needed in applications.

Concerning notation: We are using the symbol $\int u(f, dq)$ to denote the integral over the entire space $F$ of integration. When it is desirable to indicate the variable of integration we shall write $\int u(f(r_0), q(dr_0))$.

If we have a set $A \subset F$ and a function $f : F \mapsto Y$ such that the product $\chi_A f$ yields an $\eta$-summable function, where $\chi_A$ denotes the characteristic function of the set $A$, then we shall say that the function $f$ is summable on the set $A$ and by its integral over the set we shall understand the following

$$\int_A u(f, dq) = \int u(\chi_A f, dq).$$

Since $\chi_f f = f$ for all functions defined on $F$, the two notions for the set $F$ coincide, that is

$$\int u(f, dq) = \int_F u(f, dq).$$

In the case when the bilinear form $u(r, \lambda) = r\lambda = \lambda r$ we shall write the integral with respect to $u$ just as $\int f dq$.

From [1], Theorem 8, page 498, and Theorem 5, page 497, we can get the following theorem.

**Theorem 3.1 (Commutativity of differential and integral operators).** Assume that $h$ is either a scalar or a vector function on the open set $G$. If $h$ is continuous on $G$ and for every fixed $r_0 \in F$ the function $(r_1, t) \mapsto h(r_1, t, r_0)$ has partial
derivatives with respect to the coordinates of the point \((r_1, t)\) and these derivatives are continuous on the set \(G\), then the function
\[ H(r_1, t) = \int_F h(r_1, t, r_0) q(dr_0) \quad \text{for all} \quad (r_1, t) \in G_0 \]
is well defined and has continuous partial derivatives on \(G_0\).
Moreover we have the following formulas
\[ D \int_F h \, dq = \int_F Dh \, dq \quad \text{and} \quad D_i \int_F h \, dq = \int_F D_i h \, dq. \]

4. Fields with free parameter representing index of the line of flow

In the following \(r_0\), index of the line of flow, represents a free parameter from the compact set \(F\). All the partial derivatives are with respect to coordinates of the point \((r_1, t)\) \(\in R^3 \times R\).

**Theorem 4.1.** On the set \(G = \{(r_1, t, r_0) : T(r_1, t, r_0) > 0\}\) we have the following identities involving wave equations and Lorentz gauge equation
\[ (\nabla^2 - D^2)[uz] = 0, \quad (\nabla^2 - D^2)[uzv] = 0, \quad \nabla \cdot [uzv] + D[uz] = 0 \]

**Theorem 4.2** (Wave equation with gauge imply Maxwell equations). Assume that on the set
\[ G = \{(r_1, t, r_0) : T(r_1, t, r_0) > 0\}\]there are given two scalar fields \(\phi\) and \(S\) and two vector fields \(A\) and \(J\).
Assume that the fields \(\phi\) and \(A\) have second partial derivatives with respect to the coordinates of the point \((r_1, t)\) and these derivatives are continuous on the set \(G\).

If these fields satisfy the following wave equations with Lorentz gauge formula
\[ (4.1) \quad \nabla^2 \phi - \frac{\partial^2}{\partial t^2} \phi = -S, \quad \nabla^2 A - \frac{\partial^2}{\partial t^2} A = -J, \quad \nabla \cdot A + \frac{\partial}{\partial t} \phi = 0 \quad \text{on the set} \quad G, \]
then the fields \(E\) and \(B\) defined by the formulas
\[ E = -\nabla \phi - \frac{\partial}{\partial t} A \quad \text{and} \quad B = \nabla \times A \quad \text{on the set} \quad G, \]
will satisfy the following Maxwell equations
\[ (4.2) \quad (a) \quad \nabla \cdot E = S, \quad (b) \quad \nabla \times E = -\frac{\partial}{\partial t} B, \quad (c) \quad \nabla \cdot B = 0, \quad (d) \quad \nabla \times B = \frac{\partial}{\partial t} E + J \]
on the set \(G\).

**Theorem 4.3.** On the set \(G = \{(r_1, t, r_0) : T(r_1, t, r_0) > 0\}\) introduce fields \(\phi\) and \(A\) by Liénard-Wiechert formulas
\[ \phi = uz \quad \text{and} \quad A = uzv \]
and define fields \(E\), and \(B\), by the formulas
\[ E = -\nabla \phi - DA \quad \text{and} \quad B = \nabla \times A. \]
and the fields $E_f$ and $B_f$ by the formulas

$$E_f = u^2e + u^{-1}D(u^2e) + D^2e \quad \text{and} \quad B_f = e \times E.$$  

Then the following Maxwell equations are satisfied on the entire set $G$

$$\nabla \cdot E = 0, \quad \nabla \times E = -DB, \quad \nabla \cdot B = 0, \quad \nabla \times B = DE$$

and for the fields $E$ and $B$, we have the following representations

$$E_f = E = u^2e + u^{-1}D(u^2e) + D^2e$$

$$B_f = B = -uz^2e \times a - uz^3\langle e, a \rangle e \times v - u^2z^3e \times v + u^2z^3\langle v, v \rangle e \times v$$

at every point $(r_1, t, r_0)$ the set $G$.

We shall remind the reader the meaning of the notation. The set $P_t(F)$ represents the position of the plasma at time $t$.

We have defined sets

$$G = \{(r_1, t, r_0) \in R^3 \times R \times F : T(r_1, t, r_0) > 0\}$$

and

$$G_0 = \{(r_1, t) \in R^3 \times R : T(r_1, t, r_0) > 0 \quad \text{for all} \quad r_0 \in F\}.$$  

Introduce here the set $B$ by the formula

$$B = \{(r_1, t) \in R^3 \times R : r_1 = r_2(t, r_0) \text{ for some } r_0 \in F\}.$$  

The set $B$ will be called the trace of the flow. It follows from continuity of the function $r_2$ that the trace $B$ represents a closed set.

Notice the relation

$$G_0 = R^3 \setminus B.$$  

This means that $G_0$ as a complement of the closed set $B$ is open.

5. Fields over the complement of the trace

Definition 5.1 (Scalar and vector potentials). For any flow of plasma and any measure $q(Q)$ over $F$ define the scalar potential $\phi$, and the vector potential $A$, are well defined on the open set $G_0$ by the formulas

$$\phi(r_1, t) = \int_F [(uz)(r_1, t, r_0)] q(dr_0),$$

$$A(r_1, t) = \int_F [(uzv)(r_1, t, r_0)] q(dr_0).$$

Since for any point $(r_1, t)$ not in the trace of the flow the functions under the integral sign are continuous with respect to $r_0$, we can conclude that the following is true.

Theorem 5.2 (Potentials are well defined). For any flow of plasma and any measure $q(Q)$ over $F$ the scalar and vector potentials are well defined on the set $G_0$ and represent continuous functions.
Theorem 5.3 (Potentials and Maxwell’s equations). For any flow of plasma and any measure \( q(Q) \) of finite variation over \( F \) define fields \( S \) and \( J \) by the following formulas

\[
\nabla^2 \phi - \frac{\partial^2}{\partial t^2} \phi = -S, \quad \nabla^2 A - \frac{\partial^2}{\partial t^2} A = -J.
\]

Then the fields defined by \( E = -\nabla \phi - \frac{\partial}{\partial t} A \) and \( B = \nabla \times A \) will satisfy the Maxwell equations

\[
\nabla \cdot E = S, \quad \nabla \times E = -\frac{\partial}{\partial t} B, \quad \nabla \cdot B = 0, \quad \nabla \times B = \frac{\partial}{\partial t} E + J
\]

and can be represented by means of the integral formulas

\[
E = \int_F \left( u^2 e + u^{-1} \frac{\partial}{\partial t}(u^2 e) + \frac{\partial^2}{\partial t^2} e \right) dq,
\]

\[
B = \int_F e \times \left( u^2 e + u^{-1} \frac{\partial}{\partial t}(u^2 e) + \frac{\partial^2}{\partial t^2} e \right) dq.
\]

Moreover the field \( S \) represents the general density of charges and the field \( J \) represents the generalized density of currents. They satisfy the equation of continuity

\[
\nabla \cdot J + \frac{\partial}{\partial t} S = 0
\]

of flow of charge. Here \( F \) represents the initial position of the plasma in \( \mathbb{R}^3 \), and the scalar field \( u \) and the vector field \( e \) are defined in formula (2.1).

6. The Independence of the Fields from Initial Measure

Assume that \( q(A) \) represents as before the total charge contained in the body covered by a set \( A \subset F \) at time \( t_0 \). Assume that at some later time \( \tilde{t}_0 \) the position of the plasma is in the set \( \tilde{F} \) and

\[
P(r) = r_2(\tilde{t}_0, r) \quad \text{for all} \quad r \in F
\]

represents transformation of points in \( F \) at time \( t_0 \) to points in \( \tilde{F} \) at time \( \tilde{t}_0 \). Since by definition of a plasma flow the transformation \( P \) is homeomorphism the set \( \tilde{F} \) is compact since \( F \) is.

Let \( \tilde{V} \) be the prering consisting of intersections of compact sets with open sets of the space \( \tilde{F} \). Sets of this prering can be represented as set differences of two compact sets. Define set function

\[
\tilde{q}(\tilde{A}) = q(P^{-1}(\tilde{A})) \quad \text{for all} \quad \tilde{A} \in \tilde{F}.
\]

Since the transformation \( P^{-1} \) preserves compact sets and set differences, the set function \( \tilde{q} \) is well defined. We shall prove that it represents distribution of charges at time \( t_0 \).

The following theorem shows that the formulas for potentials in integral form do not depend on transition from one initial time \( t_0 \) to another \( \tilde{t}_0 \).
Theorem 6.1. Let $h(r)$ be a continuous function on the set $F$ with values in either the vector space $\mathbb{R}^3$ or the space $\mathbb{R}$ of reals. Let

$$\tilde{h}(r) = h(P^{-1}(r)) \quad \text{for all} \quad r \in \tilde{F}.$$ 

Then we have the equality

$$\int_{\tilde{F}} \tilde{h} \, d\tilde{q} = \int_{F} h \, dq.$$ 

(6.1)

Corollary 6.2. The scalar potential $\phi$ and the vector potential $A$ are independent of the initial time $t_0$ when the distribution $q$ of charges was observed, and as a consequence the electric field $E$ and the magnetic field $B$ also do not depend on the initial time when the distribution was observed.

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