Influence of Stark-shift on quantum coherence and non-classical correlations for two two-level atoms interacting with a single-mode cavity field

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An exact analytic solution for two two-level atoms coupled with a multi-photon single-mode electromagnetic cavity field in the presence of the Stark shift is derived. We assume that the field is initially prepared in a coherent state and the two atoms are initially prepared in excited state. Considering the atomic level shifts generated by the Stark shift effect, the dynamical behavior of both quantum coherence (QC) measured using a quantum Jensen-Shannon divergence and of quantum correlations captured by quantum discord (QD) are investigated. It is shown that the intensity-dependent Stark-shift in the cavity and the number of coherent state photons plays a key role in enhancing or destroying both QC and QD during the process of intrinsic decoherence. We remarked that increasing the Stark-shift parameters, the frequencies of the transition for the mode of the cavity field, and photons number destroy both the amount of QC and QD and affected their periodicity. More importantly, QC and QD exhibit similar behavior and both show a revival phenomenon. We believe that the present work shows that the quantum information protocols based on physical resources in optical systems could be controlled by adjusting the Stark-shift parameters.

Keywords: Stark-shift effect, Quantum coherence, Non classical correlations, Decoherence, Two two-level atoms.

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I. INTRODUCTION

Over the past decades, the two-levels atoms constitute valuable tools for both theoretical and experimental investigations in various fields of modern physics such as collision physics [1, 2] and quantum optics [3, 4]. They were considered to be the simplest quantum model used to solve many problems of light-matter interaction and for the fundamental building of modern applications in quantum control [5] and quantum processing [6]. Further, there are many physical systems recognized as a possible candidate for quantum processing such as the cavity QED which studies the interaction between the individual atoms and a single-mode electromagnetic field inside a cavity [7], the artificial two-level atoms based on the superconducting qubits [8, 9], ion traps [10] and quantum dots [11].

On the other hand, the Dicke model [12], describes the interaction between a collection of N two-level atoms and a quantized electromagnetic field, has been extensively used in quantum optics. It gives an opportunities to simulate quantum optical and condensed matter phenomena. This class of models has been employed to investigate quantum phase transition with a coupled optical cavity [13]. Also, a new physical phenomenon of the splitting of the dressed states is explained as a manifestation of an n-photon coupling between them, i.e., it represents an n-photon ac Stark effect parameter [14].

Moreover, a three-level Λ-type atom interacting with a two-mode of electromagnetic cavity field surrounded by a nonlinear Kerr-like medium with decay rates have been considered in [15]. The time-dependent interaction between the time-dependent field and a two-level atom with one mode electromagnetic field have been explored in [16]. In addition, the time-dependent fields were also used for controlling quantum states in several protocols and shown various useful applications, especially in carrying out rapid population transfers by the invariant-based inverse engineering method in a three-level system [17], realizing the perfect adiabatic state transfer protocols via an intermediate lossy system [18] [19] and performing fast population transfer for ground states in multiparticle cavity QED systems via the adiabatic passage [20] [21].

Quantum entanglement (QE), as a special type of quantum correlations, is a valuable resource for several tasks of quantum information processing and has a crucial role in setting the boundary between quantum and classical worlds [22][23]. It has recently attracted significant attention as an essential ingredient for powerful applications including quantum teleportation [24], dense coding [25], quantum cryptography [26] and telecloning [27]. In this regard, various proper entanglement measures have been proposed by employing a rigorous mathematical framework [28] and their time evolution has been studied in several quantum systems of two-level atoms [29][31]. It was believed that QE is tightly related to quantum correlations. However, according to various studies [32][33], it has been proved experimentally that there exist some separable quantum states that are very useful in practical quantum technology and that QE is not the only meaningful type of quantum correlations. For instance, quantum speedup with separable states [34][35], quantum nonlocality without QE

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and the efficiency of deterministic quantum computation with one qubit \cite{38}. In this sense, considerable efforts have been devoted over the last years to characterize and to quantify quantum correlations that may be contained in a multipartite system.

The problem of quantifying non-classical correlations beyond QE in a generic quantum state hasn’t solved yet and their investigation in closed and open multipartite systems is one of the most challenging topics in the literature. Historically, entropic quantum discord (QD) is an alternative approach to investigate quantum correlations for an arbitrary bipartite state even those which are separable. This measure was first proposed independently by Ollivier et al., \cite{39} and Henderson et al., \cite{40}. They have shown that QD is more general and goes beyond QE and it coincides with the entanglement of formation for pure bipartite states. In addition to that, the QD obeys a conservation law with the QE based upon a monogamic principle of the quantum correlation distribution \cite{41} and it is related to the QE irreversibility \cite{42}. Interestingly, Daki et al., \cite{43} showed that a correlated separable state (i.e., the separable state with non-zero QD) can outperform entangled states and it exhibits better performance for remote quantum state preparation. Datta et al., \cite{44} confirmed that QD is a better figure of merit for providing computational speedup compared with classical states in quantum computation models. Moreover, Werlang et al., \cite{45} showed that QD is more robust than QE against decoherence in Markovian environments. It is interesting to stress that other discord-like measures have been introduced such as one-way quantum deficit \cite{46}, local quantum uncertainty \cite{47,49}, local quantum Fisher information \cite{50,51} and quantum interferometric power \cite{52}.

On the other side, one of the main challenges in the practical implementation of quantum computing and in the physical realization of quantum technologies is how to protect quantum coherence (QC) and overcome decoherence \cite{53}. This is due to the realistic quantum systems are naturally open and very brittle, which leads to loss of information and the intrinsic physical quantum properties from the system to the environment. So, it is important to study both QD and QC dynamics in open quantum systems when loses their coherence. QC also provides essential power for quantum information processing and a theorectical framework for their quantification in quantum states has been developed \cite{54,56}. The new fundamental and challenging task in this field, from both theoretical and experimental perspectives, consists of how to characterize and quantify the interplay between these different kinds of quantum correlation. In this sense, the role of QC to characterize QE and quantum thermodynamics was discussed in \cite{57} and in \cite{58,59} respectively.

Inspired by these works, we would study the QD and QC dynamics in an open quantum system consisting of two two-level atoms that are connected by a single-mode electromagnetic cavity field in the presence of the Stark-shift effect. We are especially interested in how intensity-dependent Stark-shift in the cavity and the number of coherent state photons would affect both QD and QC behaviors in this model. This paper is structured as follows: In sec.II, we firstly introduce the considered physical model and then we find an exact solution of the density matrix in the Schrödinger picture when these two atoms are initially prepared in their excited state. In sec.III, we give the explicit expression of QC by employing the concepts of the quantum Jensen-Shannon divergence. The time evolution behavior of the QC as a function of the parameters of the considered system and its environment is also discussed. Sec.IV is devoted to the effect of Stark-shift in the cavity on the temporal evolution of correlations captured by QD. A brief summary of the main obtained results closes this paper.

![FIG. 1: (a) Energy level diagram for two two-level atoms interacting to a single-mode cavity field (b) graphical representation of the dynamic Stark splitting of the effective two two-level atoms.](image-url)
II. THE MODEL: HAMILTONIAN AND SOLUTION

In this work, the system under consideration consists of two 2-level atoms (considered as two qubits $A$ and $B$) having ground states $|g_i\rangle$ and excited states $|e_i\rangle$ ($i = 1, 2$) which are coupled with a single-mode electromagnetic cavity field. In our consideration, we assume that the interaction occurs in the presence of the Stark-shift effect as shown schematically in Fig. 1. This Stark effect induces the splitting and shifting of spectral energy levels due to the presence of an electromagnetic cavity field [60][62]. In this picture, the total Hamiltonian of the system is given by \( h = 1 \)

\[ \hat{H} = \hat{H}_A + \hat{H}_F + \hat{H}_{A-F} + \hat{H}_{\text{Stark}}, \tag{1} \]

where \( \hat{H}_A \) and \( \hat{H}_F \) describes respectively the two atoms Hamiltonian and the quantized electromagnetic cavity field Hamiltonian, which can be written as

\[ \hat{H}_A + \hat{H}_F = \frac{1}{2} \sum_{j=1}^{2} w_z \hat{S}_j^z + w_c \hat{a} \hat{a}^\dagger, \tag{2} \]

where \( \hat{S}_j^z = |e_j\rangle \langle e_j| - |g_j\rangle \langle g_j| \) is the energy operator of the \( j \)th atom, \( w_z \) and \( w_c \) denotes the transition frequency and the frequency of the mode of the cavity field, respectively and \( \hat{a} \) (\( \hat{a}^\dagger \)) is the creation (annihilation) operators of the cavity field which they satisfy the usual bosonic commutation relations. The interaction Hamiltonian \( \hat{H}_{A-F} \) between the atoms and the electromagnetic cavity field is given by

\[ \hat{H}_{A-F} = \sum_{j=1}^{2} \lambda_j \left( \hat{a} \hat{S}_j^+ + \hat{a}^\dagger \hat{S}_j^- \right), \tag{3} \]

where \( \lambda_j \) ($j = 1, 2$) is the coupling strength between the atom \( j \) and the mode of the cavity field and \( \hat{S}_j^+ = |e_j\rangle \langle g_j| \) and \( \hat{S}_j^- = |g_j\rangle \langle e_j| \) represents the dipole raising and lowering operators, respectively. The Hamiltonian describing the effect of Stark shift in the cavity can be written as

\[ \hat{H}_{\text{Stark}} = (\gamma_1 + \gamma_2) \sum_{j=1}^{2} \hat{S}_j^z + \frac{1}{2} (\gamma_1 + \gamma_2) \hat{a} \hat{a} \sum_{j=1}^{2} \hat{S}_j^z, \tag{4} \]

where \( \gamma_1 \) and \( \gamma_2 \) describe the intensity-dependent Stark-shift in the cavity. The dynamic Stark shift parameters and effect has been included in two-photon absorption studies in the past. The Stark effect to first order in a perturbative calculation in two-photon equal frequency absorption has been studied [63]. Furthermore, the importance of the Stark-shift terms in an exact calculation of two-photon equal-frequency absorption has been demonstrated [64]. Moreover, Ashraf and Zubairy [65] included this power-dependent effect in their study of the equal-frequency two-photon micromaser. Also, Gou [66] discussed how to eliminate the Stark shifts through the use of correlated two-mode field states in unequal-frequency absorption. All of these studies investigated the case of exact two-photon resonance. The general expression of the time-dependent wave function of an atom \( j \) ($j = 1, 2$) in the system under consideration can be written as

\[ |\psi^j(t)\rangle = A_j(t) |e_j, n\rangle + B_j(t) |g_j, n+1\rangle, \tag{5} \]

with \( A_j(t) \) and \( B_j(t) \) are the probability amplitudes. Consequently, after interacting with a single-mode electromagnetic cavity field, the state vector of the whole system \( |\psi(t)\rangle \) takes the following form

\[ |\psi(t)\rangle = \sum_{n=0}^{\infty} [A_n(t) |e_1, e_2, n\rangle + C_{n+1}(t) |e_1, g_2, n+1\rangle + D_{n+1}(t) |g_1, e_2, n+1\rangle + B_{n+2}(t) |g_1, g_2, n+2\rangle] \tag{6} \]

After some simplifications and by using the following creation and annihilation operators actions

\[ a |n\rangle = \sqrt{n} |n-1\rangle, \quad a^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle, \tag{8} \]

we obtain the following coupled differential equations

\[ i \frac{\partial A_n(t)}{\partial t} = \xi_1 A_n(t) + \lambda_2 \sqrt{n+1} C_{n+1}(t) + \lambda_1 \sqrt{n+1} D_{n+1}(t), \tag{9} \]

\[ i \frac{\partial C_{n+1}(t)}{\partial t} = \lambda_2 \sqrt{n+1} A_n(t) + \xi_2 C_{n+1}(t) + \lambda_1 \sqrt{n+2} B_{n+2}(t), \tag{10} \]

\[ i \frac{\partial D_{n+1}(t)}{\partial t} = \lambda_1 \sqrt{n+1} A_n(t) + \xi_3 D_{n+1}(t) + \lambda_2 \sqrt{n+2} B_{n+2}(t), \tag{11} \]

and

\[ i \frac{\partial B_{n+2}(t)}{\partial t} = \lambda_1 \sqrt{n+2} C_{n+1}(t) + \lambda_2 \sqrt{n+2} D_{n+1}(t) + \xi_4 B_{n+2}(t), \tag{12} \]

The parameters appearing in these differential equations are

\[ \xi_1 = w_z + w_c (\gamma_1 + \gamma_2) (2 + n), \tag{13} \]

\[ \xi_2 = \xi_3 = w_c (n + 1), \tag{14} \]

and

\[ \xi_4 = -w_z + w_c (n + 2) - (\gamma_1 + \gamma_2)(n + 4). \tag{15} \]
We assume that two atoms are initially prepared in their excited states $|e_1\rangle$ and $|e_2\rangle$. We also assume that the cavity electromagnetic field is prepared in the Glauber coherent states

$$|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

(16)

where $\alpha \in \mathbb{C}$. In this scheme, the initial state is given by

$$|\psi(0)\rangle = |e_1, e_2, \alpha\rangle.$$  

(17)

Comparing with the equation (6), the initial probability amplitudes are

$$A_n(0) = \exp\left(-\frac{|\alpha|^2}{2}\right) \frac{\alpha^n}{\sqrt{n!}},$$

(18)

and

$$C_{n+1}(0) = D_{n+1}(0) = B_{n+2}(0) = 0.$$  

(19)

To solve the differential equations (9), (10), (11) and (12), we set

$$A_n(t) = a_n(t) \frac{\alpha^n}{\sqrt{n!}},$$

(20)

$$C_{n+1}(t) = c_{n+1}(t) \frac{\alpha^n}{\sqrt{n!}},$$

(21)

where

$$D_{n+1}(t) = d_{n+1}(t) \frac{\alpha^n}{\sqrt{n!}},$$

(22)

$$B_{n+2}(t) = b_{n+2}(t) \frac{\alpha^n}{\sqrt{n!}}.$$  

(23)

The exact solution for the coupled differential equations is given by

$$a_n(t) = \sum_{l=1}^{4} \tilde{a}_l \exp (iM_l t),$$

(24)

$$c_{n+1}(t) = \sum_{l=1}^{4} \tilde{c}_l \exp (iM_l t),$$

(25)

$$d_{n+1}(t) = \sum_{l=1}^{4} \tilde{d}_l \exp (iM_l t),$$

(26)

$$b_{n+2}(t) = \sum_{l=1}^{4} \tilde{b}_l \exp (iM_l t).$$  

(27)

$$\tilde{a}_l = \frac{R_1 + \xi_1 (M_k + M_m + M_n) + R_2 (M_k M_m + M_k M_n + M_m M_n) + M_k M_m M_n}{M_{lk} M_{lm} M_{tn}},$$

(28)

$$\tilde{c}_l = \frac{R_3 + f_1 (M_k + M_m + M_n) + R_4 (M_k M_m + M_k M_n + M_m M_n)}{M_{lk} M_{lm} M_{tn}},$$

(29)

$$\tilde{d}_l = \frac{R_5 + f_1 (M_k + M_m + M_n) + R_6 (M_k M_m + M_k M_n + M_m M_n)}{M_{lk} M_{lm} M_{tn}},$$

(30)

$$\tilde{b}_l = \frac{R_7 + R_8 (M_k M_m + M_k M_n + M_m M_n)}{M_{lk} M_{lm} M_{tn}},$$

(31)

where

$$R_1 = \xi_1^3 + (f_1^2 + f_2^2) (2\xi_1 + \xi_2),$$

(32)

$$R_3 = f_2 \left(3f_1^2 + f_2^2 + \xi_1^2 + \xi_1\xi_2 + \xi_2^2\right),$$

(35)

$$R_4 = 2f_1 f_2 (\xi_1 + \xi_2 + \xi_3),$$

(36)

$$R_5 = f_1 (f_2^2 + 3f_1^2 + 5\xi_1^2 + \xi_1\xi_2 + \xi_2^2),$$

(37)

$$R_6 = f_1 (\xi_1 + \xi_2),$$

(38)

$$R_7 = f_1 \xi_1 + f_2 \xi_2,$$

(39)

with

$$f_1 = \lambda_2 \sqrt{n + 1}, \quad f_2 = \lambda_1 \sqrt{n + 1},$$

(33)

$$R_2 = f_1^2 + f_2^2 + \xi_1^2,$$

(34)

and

$$R_3 = f_2 \left(3f_1^2 + f_2^2 + \xi_1^2 + \xi_1\xi_2 + \xi_2^2\right),$$

(35)

$$R_4 = 2f_1 f_2 (\xi_1 + \xi_2 + \xi_3),$$

(36)

$$R_5 = f_1 (f_2^2 + 3f_1^2 + 5\xi_1^2 + \xi_1\xi_2 + \xi_2^2),$$

(37)

$$R_6 = f_1 (\xi_1 + \xi_2),$$

(38)

$$R_7 = f_1 \xi_1 + f_2 \xi_2,$$

(39)
The solutions of the equation (41) write as

\[ e_{n+1} (t) = - \frac{1}{(f_1 + f_2)} \sum_{j=1}^{3} k_j (\chi_j + \xi_4) \exp [i\chi_j t], \]  

and

\[ a_n (t) = \frac{1}{f_1 (f_1 + f_2)} \sum_{j=1}^{3} k_j [(\chi_j + \xi_4) (\chi_j + \xi_2) - f_2 (f_1 + f_2)] \exp [i\chi_j t]. \]

where \( k_j \) (\( i = 1, 2, 3 \)) are time-independent functions and \( \chi_j \) are the eigenvalues of the matrix

\[ W(t) = \begin{bmatrix} & \xi_1 & f_1 & f_2 & 0 \\ & f_1 & \xi_2 & f_2 & 0 \\ & 0 & f_1 & f_2 & \xi_4 \end{bmatrix}. \]

It is simple to verify that the eigenvalues of the matrix \( W(t) \) are analytically obtained as

\[ \chi_1 = \frac{1}{2} \left[ \xi_1 + \xi_2 - \sqrt{(\xi_1 - \xi_2)^2 + 4f_1 (f_1 + f_2)} \right], \]

\[ \chi_2 = \frac{1}{2} \left[ \xi_1 + \xi_2 + \sqrt{(\xi_1 - \xi_2)^2 + 4f_1 (f_1 + f_2)} \right], \]

and

\[ \chi_3 = \xi_4. \]

From Eqs. (9) and (11), we obtain the following conditions

\[ k_1 \chi_1^2 + k_2 \chi_2^2 + k_3 \chi_3^2 = f_1 (f_1 + f_2), \]

\[ k_1 + k_2 + k_3 = 0, \]

\[ k_1 \chi_1 + k_2 \chi_2 + k_3 \chi_3 = 0. \]  

In this picture, the eigenvalues of the matrix \( W(t) \) as a function of the transition frequency \( w_z \), the frequency of the mode of the cavity field \( w_c \) and the photon number \( n \) are

\[ \chi_1 = \frac{1}{2} \left[ w_z + w_c (2n + 1) + (\gamma_1 + \gamma_2) (n + 2) - \delta \right], \]

\[ \chi_2 = \frac{1}{2} \left[ w_z + w_c (2n + 1) + (\gamma_1 + \gamma_2) (n + 2) + \delta \right], \]

and

\[ \chi_3 = -w_z + w_c (n + 2) - (\gamma_1 + \gamma_2) (n + 4), \]

where the function \( \delta \) is defined by

\[ \delta = (w_z - w_c + (\gamma_1 + \gamma_2) (n + 2))^2 + 4\lambda_2 (\lambda_1 + \lambda_2) (n + 1). \]

After some straightforward calculations, we can obtain the functions \( k_1, k_2 \) and \( k_3 \) as

\[ k_1 = \frac{2\lambda_2 (\lambda_1 + \lambda_2) (n + 1)}{\delta - F\sqrt{\delta}}, \]
\[ k_2 = \frac{2\lambda_2 (\lambda_1 + \lambda_2) (n + 1)}{\delta + F\sqrt{\delta}}, \]  
\[ k_3 = \frac{4\lambda_2 (\lambda_1 + \lambda_2) (n + 1)}{F^2 - \delta}, \]
and
\[ F = 3(w_z - w_c) + (\gamma_1 + \gamma_2) (3n + 10). \]
Using these results, the atom-atom density operator \( \rho_{AB} \) can be obtained by tracing out the electromagnetic cavity field degrees of freedom. It is given by
\[ \rho_{AB} = \sum_{n=0}^{\infty} \langle n' | \psi(t) \rangle \langle \psi(t) | n' \rangle. \] In the computational basis \( \{ |e_1, e_2 \rangle, |e_1, g_2 \rangle, |g_1, e_2 \rangle, |g_1, g_2 \rangle \} \), the density matrix \( \rho_{AB} \) takes the form
\[ \rho_{AB} = \begin{pmatrix} |a_n(t)|^2 & 0 & 0 & 0 \\ 0 & |e_{n+1}(t)|^2 & |e_{n+1}(t)|^2 & 0 \\ 0 & |e_{n+1}(t)|^2 & |e_{n+1}(t)|^2 & 0 \\ 0 & 0 & 0 & |b_{n+2}(t)|^2 \end{pmatrix}, \]
which is a two-qubit state of \( X \) type and their entries are given by
\[ |e_{n+1}(t)|^2 = \lambda_2^2 (n + 1) \left[ \sum_{\pm} -w_z + w_c (4n + 5) - (\gamma_1 + \gamma_2) (n + 6) + \sqrt{\delta} \left( \frac{\delta + F\sqrt{\delta}}{\delta^2 - F^2} \right)^2 + \frac{64(-w_z + w_c (n + 2) - (\gamma_1 + \gamma_2) (n + 4))^2}{(F^2 - \delta)^2} \right] \]
\[ + \frac{2\lambda_2^2 (n + 1) \cos \left( \sqrt{\delta} t \right)}{\delta^2 - F^2} \left[ (-w_z + w_c (n + 2) - (\gamma_1 + \gamma_2) (n + 4)) \times \left( \frac{\delta^2 + F^2}{\delta^2 - F^2} \right) \right] \]
\[ + \sum_{\pm} 16\lambda_2^2 (n + 1) \cos \left( \frac{F\pm\sqrt{\delta}}{2} t \right) \left[ (-w_z + w_c (n + 2) - (\gamma_1 + \gamma_2) (n + 4)) \times \left( \frac{\delta^2 + F^2}{\delta^2 - F^2} \right) \right], \]
\[ |b_{n+2}(t)|^2 = \frac{8\lambda_2^2 (\lambda_1 + \lambda_2)^2 (n + 1)^2}{\delta^2 - F^2} \left[ \frac{3\delta + F^2}{\delta (\delta - F^2)} + \frac{\cos \left( \sqrt{\delta} t \right)}{\sqrt{\delta}} \right] \]
\[ + \sum_{\pm} \cos \left( \frac{F\pm\sqrt{\delta}}{2} t \right) \left( \frac{\delta^2 + F^2}{\delta^2 - F^2} \right) \]
and
\[ |a_n(t)|^2 = 1 - 2|e_{n+1}(t)|^2 - |b_{n+2}(t)|^2. \]

Having determined the bipartite atomic density matrix operator, we shall investigate the quantum coherence and quantum discord. In particular, we shall investigate the influence of effect of the stark-shift parameter on these quantum measures.

### III. QUANTUM COHERENCE

Arising from the superposition principle, quantum coherence (QC) is a key concept to understand the weirdness of the fundamental aspects of quantum mechanics. QC is an important resource in different quantum information processing tasks. The characterization of QC in composite quantum systems and in particular how it can be quantified is one of the challenges facing both quantum information theory and quantum technologies. For this reason, a rigorous framework to quantify the degree of superposition in quantum states has been established by Berg and recently updated by Baumgratz et al. By analogy with the resource theory of quantum entanglement which establishes the sets of separable states and local operations and classical communication (LOCC), the quantification of QC is based on the concepts of incoherent states and incoherent operations. For a quantum state associated with a \( d \)-dimensional Hilbert space \( \mathcal{H} \), we fix the orthogonal basis \( \{|j\rangle \}_{j=1}^d \) as the reference basis. If a quantum state is diagonal when expressed on this local reference basis, it is called incoherent state. On the other hand, an incoherent operation is a completely positive and trace-preserving linear map (CPTP) that maps an incoherent state to an incoherent state and no creation of coherence could be observed.

The Baumgratz et al. analysis has attracted deep attention of many researchers and various measures of QC, which satisfy the physical requirements of noncontractivity and monotonicity, have been proposed since then. We quote, for instance, the \( l_1 \)-norm coherence, the coherence of formation, the relative entropy coherence, the
geometric measure of coherence \cite{69}, the distillable coherence \cite{68, 70}, the coherence measures based on entanglement \cite{69, 71}, the coherence measures based on trace norm \cite{67}, the coherence measure via quantum skew information \cite{72}, via Tsallis relative entropy \cite{73, 74}, via fidelity \cite{75} and via relative entropy in Gaussian states \cite{76}.

It is worth noticing that all of the QC measures of a quantum state can be classified into two categories; the first concerns the entropic measures which are based on the entropic function and the second situation concerns the geometric class of the measures which has a metric character and is quantified as its distance to the closest incoherent state. Recently, a new measure of CQ based on the quantum version of the Jensen-Shannon divergence has been introduced by Radhakrishnan et al \cite{77}. This measure has both entropic and geometric characters and is easy to compute analytically for a generic quantum state. In addition, it satisfies the full physical requirements that every good QC quantifier should satisfy \cite{56}. The quantum Jensen-Shannon divergence, as a measure of the distance between two quantum states, is defined as \cite{78–81}

\[ \mathcal{J}(\rho, \sigma) = S\left(\frac{\rho + \sigma}{2}\right) - \frac{1}{2}(S(\rho) + S(\sigma)), \]  

where \( S(\rho) = -\text{Tr}(\rho \log_2 \rho) \) is the von Neumann entropy and \( \rho_d \) is the diagonal part of quantum state \( \rho \) in the computational basis.

Hence, the expression of QC for two 2-level atoms interacting with a single-mode electromagnetic cavity field in the presence of the Stark-shift effect can be easily computed. Indeed, using the density matrix \eqref{69}, one can check that the quantum coherence defined by \eqref{74} is given by

\[ C(\rho) = \sqrt{\mathcal{J}(\rho, \rho_d)}, \]  

and the QC is defined as the square root of the quantum Jensen-Shannon divergence, i.e.,

\[ C(\rho) = \sqrt{\mathcal{J}(\rho, \rho_d)}, \]  

In order to analyze in detail the influence of the Stark shift on the QC dynamics of two two-level atoms interacting with a

\[ C(\rho) = \sqrt{\mathcal{J}(\rho, \rho_d)}, \]  

FIG. 2: Time evolution of quantum coherence in terms of the time \( t \) for different values of the Stark shift; \( \gamma_1 = 1 \) and \( \gamma_2 = 2 \) for blue curve, \( \gamma_1 = 2 \) and \( \gamma_2 = 3 \) for brown curve, \( \gamma_1 = 3 \) and \( \gamma_2 = 4 \) for red curve, \( \gamma_1 = 4 \) and \( \gamma_2 = 5 \) for green curve and for different values of the number of coherent state photons; \( n = 0 \) for panel (A), \( n = 1 \) for panel (B), \( n = 3 \) for panel (C), when \( w_z = w_c = 0 \).

FIG. 3: The same as in figure 2 but for different parameters of \( w_z = 0.5 \) and \( w_c = 1 \).
single-mode electromagnetic cavity field, we display the evolution of QC based on the quantum Jensen-Shannon divergence versus the time for various values of the Stark shift parameters in Fig.(4). To simplify the numerical analysis, we consider the situation where $w_z$ (transition frequency) and $w_c$ (mode field cavity frequency) vanishes. From the plot of Fig.(4), it is observed that the QC behavior of this model is a periodic function of time $t$ and their periods decrease with the increase of the Stark shift parameters $\gamma_{1,2}$. Also, the increase of the Stark shift parameters in the cavity leads to a rapid decrease of QC due to the intrinsic decoherence and the two atoms can remain in a stationary incoherent state for large values of the Stark shift parameters. As depicted in Fig.(2), the amount of QC decreases over a certain interval of time showing the degradation of the superposition of the initial state. After, it increases and survives which shows the revival phenomenon of QC in the system. This revival phenomenon can be explained by the transfer of QC between the total system including the cavity field and the two atoms. Furthermore, we noticed that the decrease in the amount of QC becomes more pronounced when passing from $n = 0$ (Fig.2(A)) to $n = 3$ (Fig.2(C)). This means that QC decreases abruptly with the increasing number of photons. In Fig.(3), we depict the behavior of QC versus the time $t$ for different values of the Stark shift parameters with $w_z = 0.5$ and $w_c = 1$. These results are in agreement with the results obtained in Fig.(2) which shows that for fixed Stark shift parameters, the QC decreases as the number of coherent state photons increases. Also, for a fixed number of coherent state photons, the QC decreases as the Stark shift parameters increases. On the other hand, the results reported in Fig.(3) show that increasing both the transition frequency and frequency of the mode of the cavity field destroys the amount of QC in the system and modify their periodicity.

IV. QUANTUM DISCORD

In order to investigate the dynamic evolutions of the quantum correlations in our system, one can use the quantum discord (QD) measure which characterizes the nonclassicality of correlations in multipartite quantum system \[39\, 40\]. It is noteworthy that QD is captured by strong measurements that lead to the loss of its coherence, but it reveals more quantum correlations than entanglement in the sense that it may not disappear even for mixed separable quantum states. It is defined as the difference between two classically-equivalent expressions of the mutual information; the original quantum mutual information and the local measurement-induced quantum mutual information. According to ref.\[39\], the QD $Q(\rho_{AB})$ in a two-qubit state $\rho_{AB}$ is defined by:

$$Q(\rho_{AB}) := I(\rho_{AB}) - J(\rho_{AB}),$$

with the total correlation is quantified by the quantum mutual information $I(\rho_{AB}) := S(\rho_A) + S(\rho_B) - S(\rho_{AB})$, and the quantity $J(\rho_{AB})$ is defined as a measure of classical correlation

$$J(\rho_{AB}) = \max_{\pi_{B,j}} \left( S(\rho_B) - \sum_j p_{B,j} S(\rho_{B,j}) \right),$$

where the minimum is taken over the set of positive operator valued measurements (POVM) $\{\pi_{B,j}\}$ on subsystem $B$ which satisfy $\sum_j \pi_{B,j} = I$. $S(\rho)$ is the von Neumann entropy, $p_{B,j} = \text{Tr}_{AB} \left[ (I \otimes \pi_{B,j}) \rho_{AB} (I \otimes \pi_{B,j}) \right]$ and $\rho_{B,j} = \text{Tr}_{A} \left( (I \otimes \pi_{B,j}) \rho_{AB} (I \otimes \pi_{B,j}) \right)/p_{B,j}$ are the probability and the conditional state of subsystem $B$ associated with outcome $j$. It is worth pointing out that the main idea of calculating the QD is to quantify the amount of information that is not accessible by a local measurement by extracting some information about subsystem $B$ by reading the state of subsystem $B$ without disturbing state $A$. The difficult step in evaluating QD is to find an analytical expression of classical correlations because it requires a minimization process in optimizing the conditional entropy over all possible local measurements. Due to this fact, the analytical derivation of QD may be performed only for some specific two-qubit \[82\, 83\]. For the bipartite system under consideration in this paper, the analytic expressions of QD can be computed by using the method reported by C.Z Wang et al in \[84\]. First, for a two qubit state of $X$-type, the density matrix has taken the following form

$$\rho_{AB} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32}^* & \rho_{33} & 0 \\ \rho_{41}^* & 0 & 0 & \rho_{44} \end{pmatrix},$$

and the corresponding eigenvalues are given by

$$\eta_{1,2} = \frac{1}{2} \left[ \rho_{11} + \rho_{44} \pm \sqrt{(\rho_{11} - \rho_{44})^2 + 4|\rho_{14}|^2} \right],$$

and

$$\eta_{3,4} = \frac{1}{2} \left[ \rho_{22} + \rho_{33} \pm \sqrt{(\rho_{22} - \rho_{33})^2 + 4|\rho_{23}|^2} \right].$$

The entropy of the reduced matrix of $\rho_A$ and $\rho_B$ is given by

$$S(\rho_A) = - (\rho_{11} + \rho_{22}) \log_2 (\rho_{11} + \rho_{22}) - (\rho_{33} + \rho_{44}) \log_2 (\rho_{33} + \rho_{44}),$$

and

$$S(\rho_B) = - (\rho_{11} + \rho_{33}) \log_2 (\rho_{11} + \rho_{33}) - (\rho_{22} + \rho_{44}) \log_2 (\rho_{22} + \rho_{44}).$$

To minimize the classical correlations \[77\], we take a complete set of projective measures on the subsystem $B$ which are represented by the operators $\{\pi_{B,j} = |B_j\rangle \langle B_j|, j = 1, 2\}$ with

$$|B_1\rangle = \cos \left( \frac{\theta}{2} \right) |1\rangle + e^{i\varphi} \sin \left( \frac{\theta}{2} \right) |0\rangle,$$

$$|B_2\rangle = \sin \left( \frac{\theta}{2} \right) |1\rangle - e^{i\varphi} \cos \left( \frac{\theta}{2} \right) |0\rangle.$$
with $0 \leq \theta \leq \frac{\pi}{2}$ and $0 \leq \varphi < 2\pi$. The probability $p_{B,j}$ corresponding to the result $j$ and the two eigenvalues of the corresponding $\rho_{B,j}$ after the measurement are given by

$$p_{B,j} = \frac{1}{2} \left[ 1 + (-1)^j \cos \theta (1 - 2\rho_{11} - 2\rho_{33}) \right] \quad (82)$$

and

$$\eta_{\pm}(\rho_{B,j}) = \frac{1}{2} \left( 1 \pm \sqrt{\frac{v_j}{p_{B,j}}} \right), \quad (83)$$

with

$$v_j = \frac{1}{4} \left[ 1 - 2(\rho_{33} + \rho_{44}) + (-1)^j \cos \theta (1 - 2\rho_{11} - 2\rho_{44}) \right]^2 +$$

$$\sin^2 \theta \left[ |\rho_{14}|^2 + |\rho_{23}|^2 - 2|\rho_{14}||\rho_{23}| \sin (2\varphi + \phi) \right]. \quad (84)$$

The entropy of $\rho_{B,j}$ can be written in terms of its eigenvalues as follows

$$S(\rho_{B,j}) = H(\eta_{+}(\rho_{B,j})) \quad (85)$$

where

$$H(x) = -x \log_2 x - (1 - x) \log_2 (1 - x)$$

is the binary Shannon entropy. Thus, the conditional entropy is

$$S(\rho_{A|B}) = \sum_{j=1}^{2} p_{B,j} S(\rho_{B,j}) = p_{B,1} S(\rho_{B,1}) + p_{B,2} S(\rho_{B,2}). \quad (86)$$

By setting partial derivatives of this conditional entropy with respect to $\theta$ and $\varphi$ equal to zero, it is easy to show that the conditional entropy is minimal when $\theta = \frac{\pi}{2}$ (in this case $p_{B,1} = p_{B,2}$ and $S(\rho_{B,1}) = S(\rho_{B,2})$). Thus, the minimum value of the conditional entropy is

$$\zeta_1 = H \left( 1 + \sqrt{(1 - 2(\rho_{33} + \rho_{44}))^2 + 4(|\rho_{14}| + |\rho_{23}|)^2} \right) \quad (87) \frac{2}{2}$$

The second extremal value is obtained when $\theta = 0$ for any arbitrary value of $\varphi$ and the second minimal value of the conditional entropy is

$$\zeta_2 = -\sum_{i=1}^{4} \rho_{ii} \log_2 \rho_{ii} - H(\rho_{11} + \rho_{33}). \quad (88)$$

Consequently, the explicit expression of classical correlations for the $X$-states (78) takes the form

$$\mathcal{J}(\rho_{AB}) = \max(\mathcal{J}_1, \mathcal{J}_2), \quad (89)$$

with

$$\mathcal{J}_i = H(\rho_{11} + \rho_{22}) - \zeta_i. \quad (90)$$

Finally, the analytical expression of QD can be determined from Eq. (76) as

$$Q(\rho_{AB}) = \min(Q_1, Q_2), \quad (91)$$

where

$$Q_i = H(\rho_{11} + \rho_{33}) + \sum_{i=1}^{4} \eta_i \log_2 \eta_i + \zeta_i. \quad (92)$$

For the two-qubit state (69), the quantum discord is given by the expression (91) with

$$Q_1 = H \left( 1 - |e_{n+1}|^2 - |b_{n+2}|^2 \right) + |a_n|^2 \log_2 \left( |a_n|^2 \right) +$$

$$+ |b_{n+2}|^2 \log_2 \left( |b_{n+2}|^2 \right) - 2|e_{n+1}|^2 \log_2 \left( \frac{2|e_{n+1}|^2}{4|e_{n+1}|^2} \right) +$$

$$H \left( \frac{1}{2} \left( 1 + \sqrt{(1 - 2|e_{n+1}|^2 - 2|b_{n+2}|^2)^2 + 4|e_{n+1}|^4} \right) \right), \quad (93)$$

and

$$Q_2 = 2|e_{n+1}|^2. \quad (94)$$

The time evolution of the quantum discord for different values of the Stark shift parameters, when the two atoms are initially prepared in their excited state, is depicted in Fig. 4 for $w_z = w_c = 0$ and in Fig. 5 when $w_z = 0.5$ and $w_c = 1$. We observe from these figures that QD and QC have similar behaviors. As mentioned earlier for QC, the QD evolves periodically with respect to time and their periods depend heavily on the Stark-shift parameters. Also, increasing the Stark shift parameters leads to reduce the amount of quantum correla-

...ions contained in the system. Moreover, QD has confirmed the revival phenomenon like QC and we can find more quantum correlation oscillations for the large values of the Stark shift parameters. We also remark that QD decreases with increasing of photon number as the frequency $w_z$ and $w_c$ increase. More importantly, by comparing the Figs. (A)-(C) with Figs. (A)-(C) and Figs. (A)-(C) with Figs. (A)-(C), we remark that the amount of QC is greater and goes beyond QD. This result is completely in concordance with the physical in...
terpretation given in [85] where the authors show that QC (via the quantum relative entropy) is more fundamental than other manifestations of quantum correlations (QE and QD) since it can also appear in single-partite systems.

V. CONCLUSIONS

To summarize, we have investigated the quantum coherence dynamics and non-classical correlation dynamics in two two-level atoms interacting with a single-mode electromagnetic cavity field. A special attention is dedicated to the presence of the Stark shifts. Considering two atoms initially in their excited states and the field in coherent state, we have obtained the exact analytic solution of the time-dependent Schrödinger equation and extracted the time evolution of the relevant density matrix elements. In analyzing the dynamics of QC and QD, we observed that the time evolution of quantum coherence quantified by quantum Jensen-Shannon divergence is almost similar to the time evolution of quantum correlations measured by entropic quantum discord. But we have noticed the amount of quantum coherence is always greater and goes beyond quantum discord due to the fact that the total quantum coherence in multipartite systems has contributions from local coherence on subsystems and collective coherence between them. Another remarkable feature for both QC and QD behaviors is the revival phenomenon which reflects the transfer of quantum features from the cavity to the two-qubit system. Furthermore, these two quantifiers decrease with increasing values of the Stark shift parameters as photons number increases. Because of the degeneracy of the levels in the two atoms produced by the electromagnetic cavity field, we remarked that the Stark-shift parameters can dramatically influence both the quantity of the initial quantum correlations and the quantum superposition of the system and can also change the other properties of the physical resources needed for implementing quantum information processing tasks.

As prolongation of this work, we believe that it will be interesting to study the effects of the reflecting boundary on the coherence dynamics in this model. Very recently, Huang [86] has shown that the presence of the boundary in two identical two-level atoms weakly interacting with a bath of fluctuating electromagnetic fields in a vacuum has a significant impact on the generation, revival and degradation of quantum correlations. Furthermore, Cheng et al.[87] showed that the boundary executes important influences on the entanglement dynamics behaviors in the same model. On the other hand, this research raised many questions that need further investigation, the most important of which is related to the influence of Stark-shift when the field of photons coupled with a modulated coupling parameter which depends explicitly on time. In other words, our results are encouraging and should be validated in the case of time-dependent fields. We hope to report on this subject in another work.
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