Correspondences of matter fluctuations in semiclassical and classical gravity for cosmological spacetime-II

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A correspondence between fluctuations of non-minimally coupled scalar fields and that of an effective fluid with heat flux and anisotropic stresses, is shown. Though the correspondence between respective stress tensors of scalar fields and fluids is known and widely used in literature, the fluctuations in the two cases still await a formal correspondence and are open to investigation in all details. Using results obtained in the newly established theory of semiclassical stochastic gravity which focuses on the fluctuations of the quantum stress tensor, we show new relations in this regard. This development is expected to give insight to the mesoscopic phenomena for gravitating systems, and enable backreaction studies of the fluctuations on the perturbations of astrophysical objects. Such a development is aimed to enhance the perturbative analysis for cosmological spacetimes and astrophysical objects specifically in the decoherence limit. A kinetic theory, which can be based on stochastic fluctuations vs particle picture in curved spacetime may find useful insights from such correspondences in future work.

I. INTRODUCTION

Correspondence between stress tensor for scalar fields and that for general fluids as in a hydrodynamic limit is well known [1, 2]. The area of field fluid correspondence is open for investigations with ongoing attempts in various aspects [3, 4]. In this article, as a sequel of recent work [5], we will show correspondence between the fluctuations of quantum fields and effective general fluid in terms of two point noise kernel which one can obtain from the respective stress tensors. In what follows, we have addressed a more involved case where fluctuations in heat flux and anisotropic stresses along with pressure and energy density contribute to the noise in the matter sector of the gravitating body. This enhanced result is obtained by using exact form of semiclassical noise kernel [4, 7] defining fluctuations of a non-minimally coupled massive quantum field, and relating it to those of effective fluid stress tensor in the classical (decoherence) limit. This also gives us a clue about the yet open directions of research in foundational aspects of decoherence and on the lines of recently established interesting results [5], with possibilities of wider application as discussed in detail towards the concluding sections in the article.

Semiclassical Einstein-Langevin equation which is the base of semiclassical stochastic gravity [7, 9, 10] is aimed at studying the correlations of perturbations of the metric, which in the low energy limit are equivalent to correlations of the quantum perturbations of the metric, as would be obtained in a viable theory of quantum gravity. This is of significance to studying details of the quantum structure of spacetime in the very early universe. The homogeneous solutions of the E-L equation are equivalent to the solutions of perturbed semiclassical equation along with information on the intrinsic fluctuations, however the induced metric fluctuations can only be obtained with the inhomogeneous term proportional to noise kernel. The two point noise kernel plays a central role in this theory, and backreaction of the same is of importance. Similary for the classical E-L equation, the importance of having point separated noise, is that the matter field fluctuations at two separate spacetime points/regions can be probed through the structure of the gravitating body enabling a statistical physics analysis. This makes possible, the study of not just local but also global/extended statistical properties of matter content and geometry. We aim for developments and applications of the fluctuations of matter fields not just for backreaction studies but also for studying dense matter fluids and their mesoscopic properties before moving towards fluid approximations. Such a framework has recently started taking shape with [5, 11, 12], and one can expect a few branched out directions at formulation level ensuing from the basic ideas. In this article, we work on extending the fluid correspondence using a more general fluid model which takes into consideration additional features of a realistic case. One of the applications of such a result is aimed at later stages in the evolution of the universe, where the quantum matter fields that have undergone decoherence and are settling down to classical states. The result that we obtain here also enables one to begin a statistical analysis of extended nonlocal mesoscopic studies to explore the nature of dense fluids (matter) that astrophysical compact objects are composed of. The hydrodynamic approximation of the matter fields would make the solution of the Einstein Langevin equation interesting for applications other than early universe cosmology. The noise kernel that is worked out here, in the hydrodynamic limit then can be used as the central quantity in the corresponding classical Einstein Langevin equation for such a purpose.

The results and connections shown here can also be useful for foundations of field-fluid correspondence, theoretically.

It is known that a stress tensor for a scalar field can
be approximated by a general fluid \[1\], thus
\[
T_{ab}(x) = \phi_a \phi_b - \frac{1}{2} g_{ab} \phi^c \phi_c - \frac{1}{2} g_{ab} m^2 \phi^2
+ \xi (g_{ab} \Box - \nabla_a \nabla_b + G_{ab}) \phi^2
\]
where the Klein Gordon field \(\phi\) satisfies
\[
(\Box + m^2 + \xi R) \phi = 0
\]
has a correspondence with \(u_{\text{fluid}}, \pi\) has a correspondence with \(u\) fluid, \(\pi\) fluid.

When decoherence of the quantum states is effective in the quantum to classical transition of the system \[13\], such as in stochastic inflation the hydrodynamic approximation can be seen to be applicable. Though these correspondences are open to detailed investigations \[4\], they provide a basis for many well established studies.

II. DEFINING GENERALIZED RANDOMNESS AND STOCHASTICITY FOR A SPACETIME STRUCTURE

The concept of stochastic processes assumes a physical quantity of interest to vary randomly w.r.t time. However for an underlying general spacetime structure the notion of time is more involved and specific cases call for associating specific concepts of time for a given problem, like the 3+1 split.

Here our attempt is rather to generalize the concept of stochasticity for classical macroscopic variables defined on an underlying spacetime structure, in order to include randomness with respect to the space-time coordinates, rather than just the temporal (or time) coordinate. Hence we introduce a new terminology of "generalized stochasticity", which enables probabilistic approach in its simple form to be extended for physical variables w.r.t the space-time coordinates. Enabling one to address spatial randomness or roughness on a par with stochastic fluctuations for the physical parameters, enhances the framework of probabilistic and statistical analysis for such systems. An example of this is a random variation of pressure or energy density with respect to spatial coordinates in an astrophysical system composed of dense matter. It is understood that such macroscopic quantities in any system are a result of smoothened ideal approximations above a certain scale (e.g a hydrostatic/hydrodynamic scale ), hence to probe scales which are below this and lie inbetween micro and macro scales in curved spacetime, the generalized stochasticity concept can be useful.

In principle the transition from macro to micro or vice versa is expected to have a regime inbetween which is not very well understood or formulated, for various astrophysical systems and compact bodies, with underlying spacetime geometries. Extending the concept of stochasticity in the way we propose here is useful, not just for the conventional processes w.r.t time, but also because it allows one to probe the details of the physical quantities at an intermediate scale, which may shed light on interesting physical effects in curved spacetime. An application of interest could be that of dense matter in strong gravity regions, which may also affect transport properties and non-equilibrium phenomena in the interiors of the gravitating system.

We define a classical random field \(X_gab(x), x\) as \(X : \{x^i\} \rightarrow \mathbb{R}\) (thus a scalar field, similarly vector and tensor random fields can be defined ) where \(\{x^i\}\) are coordinates on a pseudo-Reimannian manifold with metric \(g_{ab}\) and a 3 + 1 spacetime split. Note that \(g_{ab}\) metric itself is not to be considered as a random tensor, nor the coordinates are random variables. Its probability distribution is denoted by \(P[X] = \int P[X] DX, \ P[X]\) being its probability density and \(DX\) is a functional integral. Also \(X\) would reduce to a regular stochastic field if it were a random function of \(t\) only. It is a generalized stochastic variable or field having randomness w.r.t the temporal as well as spatial components of \(x^i\) . This extension also calls for the possibility that \(X\) may only depend on the spatial components, or show randomness only w.r.t space coordinates. Such cases may relate to roughness effects, accounting for randomness in physical parameters of matter content over the spatial structure of the system. Such concepts have been used in DDFT theories \[15\] and give us useful ideas for basic new constructs in our formulation. To probe this roughness would be of interest then, for looking into structures of matter and its consequences which have not been possible with the regular smooth approximations leading to classical matter.

III. THE GENERALIZED NOISE IN THE SYSTEM

Models of noise with the above construct have been used recently in \[2,11,12\] and toy models of perturbed spherically symmetric relativistic stars with such stochastic effects have been considered. However, it is here in the above section, that we have introduced the terminology of generalized stochasticity, formally. This may not seem much different from the way quantum scalar field \(\hat{\phi}(x)\) and its expectation \(< \hat{\phi}(x) > = \int \hat{\phi}(x) P[\hat{\phi}] D\hat{\phi}\) are usually considered, where \(D\phi\) is the functional integral with \(\phi\) being a quantum operator. Regarding fluctuations of quantum scalar field as given in equation \[6,7,8\] (which have been obtained in \[6,7\] ), these are defined on the underlying spacetime structure, following from the first principles of quantum field theory, which is not the case with classical fields. The importance of introducing generalized stochasticity as given in the above section is for extending a similar treatment for random variations of a classical field w.r.t the space-time coordinates rather
than just the time coordinate. We show the relevance of such a framework in the following sections.

In the present article, we work out a correspondence of the generalized classical noise of a fluid with the semiclassical noise in matter fields. The results is valid at mesoscopic scales, which lie a little below the hydrodynamic smooth scales, and above the microscopic scales, where one needs to trace out each fluid particle separately. We discuss details of our results and scales of validity in the concluding section.

It is known that fluctuations are inherent to quantum phenomena, while in the decoherence limit the fields become classical. It is expected then that the quantum fluctuations of fields would vanish once they decohere, however, structure formation is known to take place due to the initial quantum fluctuations in the universe. Though these correspondences are expected to be seen in the quantum level description of matter fields, stochastic effects. In what follows, we address the classical noise kernel expressed in terms which can be arranged in the following way.

A. The Semiclassical Noise Kernel in decoherence limit

The correspondence of the fluctuations of the two stress tensors for scalar fields and the classical fluid approximation can be obtained by using the exact form of the semiclassical noise kernel as is given in [2, 6] for quantum scalar fields. This is defined as

\[ 8\langle \hat{T}_{ab}(x, x') \rangle = -2 \langle \hat{T}_{ab}(x) \rangle < \hat{T}_{v'(x')} > \]

where \( \hat{T}_{ab} \) denotes the quantum stress tensor, obtained by raising \( \phi \) in (1) to an operator. The expectation of such a quantum stress tensor \( \langle \hat{T}_{ab}(x) \rangle \) after regularization is used as the matter content in the semiclassical Einstein’s equations. These fluctuations via a noise kernel (4) form the central quantity of importance in the theory of semiclassical stochastic gravity as mentioned earlier. An elaborate procedure using the point splitting formalism to deal with ill defined operators like \( \phi^2 \) in the quantum stress tensor, in (4) have been used in a straightforward but elaborate way [17] to obtain the noise kernel bitensor in semiclassical stochastic gravity in terms of two point (positive) Wightman functions. These functions denoted and defined as \( G \equiv G(x, x') = \langle \phi(x)\phi(x') \rangle \), follow easily in the expressions thus obtained. This has been as worked out in [2, 6], with the final form of the semiclassical noise kernel expressed in terms which can be arranged in the following way.

\[ N_{abc'd'} = \tilde{N}_{abc'd'} + g_{ab} \tilde{N}_{c'd'} + g_{c'd'} \tilde{N}_{ab} + g_{ab} g_{c'd'} \tilde{N} \] (5)

with

\[
8\tilde{N}_{abcd}(x, x') = (1 - 2\xi)^2(G_{c'd'b}G_{d'a} + G_{c'a}G_{d'b}) + 4\xi^2(G_{c'd'}G_{ab} + GG_{abc'd'}) - 2\xi(1 - 2\xi)(G_{ab}G_{c'd'} + G_{a}G_{c'd'})
\]

\[
+ G_{d'b}G_{abc'} + G_{c'a}G_{abd'}) + 2\xi(1 - 2\xi)(G_{ab}G_{c'd'} + G_{a}G_{d'}R_{ab}) - 4\xi^2(G_{a}G_{b}R_{cd} + G_{c'd'}R_{ab})G
\]

\[ + 2\xi^2R_{d'b}G_{ab}G^2 \] (6)

\[
8\tilde{N}_{ab}(x, x') = 2(1 - 2\xi)[(2\xi - \frac{1}{2})G_{d'b}G_{a} + \xi(G_{gb}G_{p'a} + G_{gb}G_{bp'}) - 4\xi[(2\xi - \frac{1}{2})G_{d'p'}G_{ab}p' + \xi(G_{gp'}G_{ab} + GG_{abp'}) - (m^2 + 1\xi R'][(1 - 2\xi)G_{ab} + 2\xi G_{ab}G_{ab} + 2\xi[(2\xi - \frac{1}{2})G_{d'p'}G_{a} + 2\xi G_{d'p'}G_{ab}p' + G_{d'b}G_{abc'} + G_{c'a}G_{abd'})]
\]

\[ + (m^2 + \xi R')G_{ab}G^2 \] (7)

\[
8\tilde{N} = 2(2\xi - \frac{1}{2})G_{gp'q}G_{i'} + 4\xi^2(G_{g'p}G_{i} + GG_{g'p'}) + 4\xi(2\xi - \frac{1}{2})G_{g'p}G_{i} + (m^2 + \xi R')G_{i}G_{p'} - 2\xi[(m^2 + \xi R')G_{i}G_{p'} + (m^2 + \xi R')G_{i}G_{p'}G]
\]

\[ + \frac{1}{2}(m^2 + 1\xi R')(m^2 + \xi R')G^2 \] (8)
The quantity $\tilde{N}_{c'd'}$ in equation (5) can be easily obtained from equation (6) by taking $g^{ab} \tilde{N}_{a'b'} = \tilde{N}_{c'd'}$, using the properties satisfied by the noise kernel.

$$8N_{abcd}(x,x') = 2(<T_{ab}(x)T_{c'd'}(x')>) - <T_{ab}(x)><T_{c'd'}(x')> = 2Cov[T_{ab}(x)T_{cd}(x')]$$ (9)

In the decoherence limit thus, the stress tensor can be said to contain the classical scalar field $\phi$, with a mesoscopic scale randomness giving it a probability distribution. The averages above in the classical limit are then, the statistical averages, where the classical scalar field expectation denoted by $<\phi(x)> = \int \phi(x) P[\phi] D\phi$, while the Wightman functions reduce to the two point form $<\phi(x)\phi(x')> = \int \phi(x)\phi(x') P[\phi] D\phi$. We connect with this the fluid approximation in the section below. Further in this article for the sake of clarity in notation, we denote by $T_{ab}^{(fluid)}$ and $N_{a'b'c'd'}^{(fluid)}$ the stress tensor and noise kernel in the fluid approximation, while for the scalar fields we use $T_{ab}^{(field)}$ and $N_{a'b'c'd'}^{(field)}$.

### B. Scalar field and fluid approximation with statistical averages

We follow on the lines of [1] to show the relation between the stress tensor for scalar fields and fluids. The

$$u_a = [\partial_c(\phi)\partial^c(\phi)]^{-1/2} \partial_a \phi \quad \epsilon = (1 - \xi \phi^2)^{-1} \frac{1}{2} \partial_c \phi \partial^c \phi + V(\phi) + \xi \{ (\square(\phi^2) - (\partial^c \phi \partial_c \phi^{-1} \partial^b \phi \partial_a \phi \nabla_a \nabla_b(\phi^2)) \}$$ (12)

$$q_a = \xi (1 - \xi \phi^2)^{-1} (\partial^c \phi \partial_c \phi^{-1})^{-3/2} \partial_b \phi \nabla_c \nabla_a (\phi^2) \partial_a \phi - \nabla_a \nabla_b (\phi^2) \partial_d \phi$$ (13)

$$p = (1 - \xi \phi^2)^{-1} \frac{1}{2} \partial_c \phi \partial^c \phi - V(\phi) - \xi \{ \frac{2}{3} \partial(\phi^2) + \frac{1}{3} (\partial^c \phi \partial_c \phi^{-1} \nabla_a \nabla_b (\phi^2) \partial_a \phi \partial_b \phi) \}$$ (14)

$$\pi_{ab} = \xi (1 - \xi \phi^2)^{-1} (\partial^c \phi \partial_c \phi^{-1})^{-1/2} \{ \partial_a \phi \partial_b \phi - g_{ab} \partial^c \phi \partial_c \phi \} \{ \partial_c(\phi^2) - (\partial^c \phi \partial_c \phi^{-1} \nabla_c \nabla_a (\phi^2) \partial_a \phi \partial_b \phi) \} + \partial^c \phi \{ \nabla_a \nabla_b (\phi^2) \partial_a \phi - \nabla_a \nabla_p (\phi^2) \partial_b \phi - \nabla_p \nabla_b (\phi^2) \partial_a \phi + (\partial_c \phi \partial^c \phi^{-1} \partial^d \phi \nabla_d \nabla_p (\phi^2) \partial_a \phi \partial_b \phi) \}$$ (15)

For our work, we assign averaged values to the scalar field terms in the above expression as mentioned earlier. The random fluctuations that we intend to focus on further can then be defined as $\delta_R \phi(x) = \phi(x) - <\phi(x)>$, where $\delta_R$ stands for the random fluctuations such that $<\delta_R \phi(x)> = 0$. It is important to realise that, here the randomness to the stress tensor is not imparted by any classical particles performing random motion in the fluid, and is different from the standard picture of thermal effects giving rise to stochasticity, though the thermal effects can be inclusive. For the case discussed in this article, it is the scalar field distribution, which accounts for the stochastic behaviour of the stress tensor. The picture of the fluid in our work is essentially that of an effective fluid, which has subtle differences with the regular fluids. This is a topic of active interest in research, with many details still open to investigations cite. We address here stochasticity which is a step ahead of the basic correspondence as established in the above equations taking into account the well understood aspects, and building over them.

In general the fluid velocity needs to be considered as a stochastic vector, since though its modulus remains unchanged, its direction which is normal to the hyper-
surfaces of constant $\phi$, varies for different realizations of the stochastic field $\phi(x)$. This argument would hold for a quantum scalar field surely where this is possible due to coherence of states. But once the states decohere and the scalar field is considered a classical scalar field, it is only the magnitude which changes in different realizations of the stochastic field, while the underlying geometry is deterministic. We are interested in taking the decoherence limit of the quantum scalar field for our work, thus we assume that in this limit the operator valued quantum field collapses into a classical field state. The randomness in the the scalar field then arises due the magnitude mainly. It is known that the decoherence phenomena does not lead to complete vanishing of the phases, but one can consider a classical picture of the fields, with the assumption that the randomness w.r.t the direction of the four-velocity is negligible. One way is to move from the quantum scalar fields to the classical picture first and then define the velocity in the hydrodynamic limit. The second approximation which one may be able to consider also for the quantum scalar fields is that, even without losing the quantum nature, these fluctuations in the directions of the unit velocity vector are negligible compared to the "bulk properties" fluctuating and their dependence on the magnitude of the fluctuating fields. However the correspondence that we work out here, is specifically for the decoherence limit of fields which is closer to the former case.

C. The generalized noise in the system

Considering the generalized random effects and fluctuations in the classical scalar field, with a spacetime structure underneath, leads to directions of investigation for capturing "partial microscopic" or mesoscopic scale effects in a classical description of the system. An important consequence of such a formulation could be the possibility of studying and addressing roughness in physical parameters of dense matter in the system at intermediate length scales, or its dynamical effects. This has not been explored yet for astrophysical systems with dense matter. Any externally induced mechanical effects (not necessarily thermal) which are random in nature can also cause such fluctuations/roughness in physical parameters at different scales in the system.

Our aim in this article is to find a correspondence between fluctuations of a scalar field coupled to a curved spacetime with those of the generalized fluctuations of an ideal fluid. We do this by comparing two point noise kernels, which are obtained in terms of the respective stress tensors as discussed above. The reason for taking two point correlations of the fluctuations for our work instead of that at a single spacetime point, is partly due to the nature of stochastic fluctuations in both the quantum and classical approximations, that at a particular spacetime the statistical averages of fluctuations necessarily vanish by definition. It is also due to this reason that the semiclassical noise which is the first closed form expression (intended for the applications in early universe cosmology) with the two point noise acts as the central quantity of interest, later taking point coincidence when needed for analysis at a single point. The emphasis in that case for keeping the points separated is the reliance towards extended spacetime structure in the very early Universe. We see that the analytical form of the semiclassical noise kernel which was an elaborate and extensive work, makes it possible for us to find the correspondence here in the decoherence limit of the scalar fields with further extension to the fluid approximation. Hence we obtain our results in terms of two point fluctuations of fluid variables which we show below. We begin with the form of two point noise kernel in the effective fluid approximation. Thus the effective noise kernel, in terms of the non-ideal fluid variables can be obtained by using (11) in (9) as follows

$$4N_{abcd}(x, x') = \text{Cov}[T(x)T'(x')] = u_{a}(x)u_{b}(x)u_{c}(x')u_{d}(x')\{\text{Cov}[\epsilon(x)\epsilon(x')] + \text{Cov}[\epsilon(x)p(x')] + \text{Cov}[p(x)\epsilon(x')] + \text{Cov}[\epsilon(x)p(x')]\} - u_{a}(x)u_{b}(x)g_{cd}(x')\{\text{Cov}[\epsilon(x)\epsilon(x')] + \text{Cov}[p(x)p(x')]\} - g_{ab}(x)u_{c}(x')u_{d}(x')\{\text{Cov}[\epsilon(x)p(x')] + \text{Cov}[p(x)p(x')]\} + g_{ab}(x)g_{cd}(x')$$

$$\text{Cov}[\pi_{a}(x)\pi_{c}(x')] + u_{a}(x)u_{c}(x')\text{Cov}[\rho_{a}(x), \rho_{d}(x')] + u_{a}(x)u_{d}(x')\text{Cov}[\rho_{a}(x), \rho_{c}(x')] + \text{Cov}[\pi_{a}(x)\pi_{c}(x')]$$

(17)

Similar to the expectation value for any macroscopic physical variables as defined earlier all the fluid variables showing generalized stochastic nature, are defined through the common form in the curved spacetime, e.g pressure given by $<p(x)> \equiv \int p(x)P[p]Dp$, while $p(x)p(x') \equiv \int p(x)p(x')P[p]Dp$. To be more specific $p \equiv p[g_{ab}, x]$ for all our considerations, and similarly rest of the fluid variables. We can compare the above expression with the form of equation (5) of the semiclassical case, which is expressed in terms of coefficients with the spacetime metric $g_{ab}$, being common to semiclassical and classical cases.
IV. CORRESPONDENCE BETWEEN THE
SCALAR FIELD AND FLUID FLUCTUATIONS

Comparing equations (13), (14). (17), (18) with equation (17) it is straightforward to relate terms which are coefficients of $g_{ab}(x), g_{cd}(x'), g_{ab}(x)g_{cd}(x')$ as corresponding terms in the two cases to get,

\[
8\tilde{N}_{abc'd'}^{(field)} = (1 - 2\xi^2)(G_{c'b}G_{d'a} + G_{c'a}G_{d'b}) + 4\xi(2G_{c'd'}G_{ab} + GG_{abc'd'}) - 2\xi(1 - 2\xi)(G_{b}G_{c'ad'} + G_{a}G_{c'bd'}) + 8G_{abc'd'}(2\xi(1 - 2\xi)G_{a}G_{b}R_{c'd'} + G_{a}G_{d'}R_{ab}) - 4\xi^2(G_{ab}R_{c'd'} + G_{c'd'}R_{ab})G + 2\xi^2R_{c'd'}R_{ab}G^2 \rightarrow
\]

\[
8\tilde{N}_{abc'}^{(fluid)} = 2\{u_a(x)u_b(x)u_c(x)u_d(x')[\text{Cov}[e(x)e(x')] + \text{Cov}[e(x)p(x')] + \text{Cov}[p(x)e(x')] + \text{Cov}[p(x)p(x')]}\} + \text{u}_a(x)u_b(x')\text{Cov}[q_a(x)q_b(x')] + u_a(x)u_c(x')\text{Cov}[q_a(x)q_c(x')] + \text{u}_a(x)u_c(x')\text{Cov}[q_a(x)q_d(x')] + \text{u}_a(x)u_d(x')\text{Cov}[q_a(x)q_e(x')] + \text{Cov}[\pi_{ab}(x)\pi_{cd}(x')]}\} \rightarrow (18)
\]

\[
8\tilde{N}_{ab}^{(fluid)} = -2u_a(x)u_b(x)\{\text{Cov}[e(x)p(x')] + \text{Cov}[p(x)p(x')]\} \rightarrow (19)
\]

\[
8\tilde{N}_{c'd'}^{(field)} = 2(2\xi - \frac{1}{2})G_{c'b}G_{d'a} + \xi(G_{b}G_{c'ad'} + G_{a}G_{c'bd'}) - 4\xi(2\xi - \frac{1}{2})G_{b}G_{c'ad'} + G_{a}G_{c'bd'} \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow 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\[
\text{Cov}[\epsilon(x)\epsilon(x')] + u_a u_c \text{Cov}[q_b q_{d'}] + \text{Cov}[\pi_{ab} \pi_{c'd'}] = \frac{4\tilde{N}_{abc'd'}^{(field)}}{\partial_a \phi \partial_b \phi \partial_{c'} \partial_{d'} \phi} (\partial_{c'} \phi \partial_{d'} \phi)^2 + \frac{4\tilde{N}_{ab}^{(field)}}{\partial_a \phi \partial_b \phi} (\partial_{c'} \phi \partial_{d'} \phi) + \frac{4\tilde{N}_{c'd'}^{(field)}}{\partial_{c'} \phi \partial_{d'} \phi} (\partial_{c'} \phi \partial_{d'} \phi) + 4\tilde{N}^{(field)}
\]

where \(\tilde{N}_{abc'd'}^{(field)}, \tilde{N}_{ab}^{(field)}, \tilde{N}_{c'd'}^{(field)}\) and \(\tilde{N}^{(field)}\) are given by LHS of equation (18), (19), (20), (21) in terms of the covariant derivatives of Wightman functions. On the RHS of the above equation we have used the relation \(u_a = (\partial_c \phi \partial_d \phi)^{-1/2}\partial_a \phi\), to get our expressions in consistent form in terms of scalar field only.

A. Relations for the effective ideal fluid approximation

The case of ideal or perfect fluid approximation and its fluctuations has been discussed in [5], where we have obtained the correspondences equivalent to expressions (18), (19) and (20). We present here the result in reverse terms giving the fluctuations in fluid variables in terms of those for scalar field, on the lines of equations (22), (23), (24), (25). Revising the semiclassical noise kernel when a perfect fluid approximation can be assumed to hold, the stress tensor is that of a perfect fluid so that \(\xi = 0\) in all the equations starting from (1), thus giving

\[
8\tilde{N}_{abc'd'}^{(field)} = G_{c'b} G_{d'a} + G_{c'a} G_{d'b} \quad \rightarrow \\
8\tilde{N}_{ab}^{(fluid)} = 2u_a(x)u_b(x)\{\text{Cov}[\epsilon(x)\epsilon(x')] + \text{Cov}[\epsilon(x)p(x')] + \text{Cov}[p(x)\epsilon(x')] + \text{Cov}[p(x)p(x')]\} \quad (26)
\]

\[
8\tilde{N}_{ab}^{(field)} = -G_{p'a} G_{p'b} - m^2 G_{p'a} G_{p'b} \quad \rightarrow \\
8\tilde{N}_{ab}^{(fluid)} = -2u_a(x)u_b(x)\{\text{Cov}[\epsilon(x)p(x')] + \text{Cov}[p(x)\epsilon(x')]\} \quad (27)
\]

\[
8\tilde{N}_{ab}^{(field)} = \frac{1}{2} G_{p'a} G_{p'b} + \frac{1}{2} m^2 [G_{p'a} G_{p'b} + G_{p'a} G_{p'b} + m^2 G^2] \quad \rightarrow \\
8\tilde{N}_{ab}^{(fluid)} = 2\text{Cov}[p(x)p(x')] \quad (28)
\]

\[
8\tilde{N}_{ab}^{(field)} = 2G_{p'a} G_{p'b} \quad \rightarrow \\
8\tilde{N}_{ab}^{(fluid)} = -2u_a u_b \{\text{Cov}[p(x)\epsilon(x')] + \text{Cov}[\epsilon(x)p(x')]\} \quad (29)
\]

The reverse equations can be obtained easily from the above in the following form

\[
\text{Cov}[p(x)p(x')] = \frac{1}{4} G_{p'a} G_{p'b} + \frac{1}{4} m^2 [G_{p'a} G_{p'b} + G_{p'a} G_{p'b} + m^2 G^2] \quad (30)
\]

\[
\text{Cov}[\epsilon(x)p(x')] = \frac{1}{2} (G_{p'a} G_{p'b} + m^2 G_{p'a} G_{p'b}) \frac{\partial \phi}{\partial a} \frac{\partial \phi}{\partial b} - \frac{1}{4} G_{p'a} G_{p'b} + \frac{m^2}{4} G_{p'a} G_{p'b} + m^2 G^2 \quad (31)
\]

\[
\text{Cov}[p(x)\epsilon(x')] = -\left(\frac{\partial \phi}{\partial a} \frac{\partial \phi}{\partial b} G_{p'a} G_{p'b} - \frac{1}{4} G_{p'a} G_{p'b} + m^2 (G_{p'a} G_{p'b} + G_{p'a} G_{p'b} + m^2 G^2) \right) \quad (32)
\]

\[
\text{Cov}[\epsilon(x)\epsilon(x')] = \frac{1}{2} \left(\frac{\partial \phi}{\partial a} \frac{\partial \phi}{\partial b} \frac{\partial \phi}{\partial c'} \frac{\partial \phi}{\partial d'} \right)^2 (G_{c'd'a} G_{d'b} + G_{c'd'a} G_{d'b}) - \frac{1}{2} \frac{\partial (\partial_c \phi \partial_d \phi)}{\partial a \partial b \phi} (G_{p'a} G_{p'b} + m^2 G_{p'a} G_{p'b}) \quad (33)
\]

Thus we see that, there is one to one correspondence in the ideal fluid approximation of scalar field fluctuations. For the non-ideal case, as seen from equation (25) the two point covariances of energy density, heat flux and anisotropic stresses appear together in one expression and it is not possible to get separate relations for these using the the present approach. This is an operational level difficulty at present given the complexity
of equations and the number of equations are less compared to the number of fluid variables in the non-ideal case. However this glitch shows up while trying to find out reverse relations only, and not for the forward expressions that we get. In the next section, we discuss the usefulness of our work.

V. DISCUSSIONS OF THE RESULTS

The results in the above section are obtained by assuming negligible randomness and fluctuations for the four-velocity of the effective fluid approximation for scalar fields. Thus the noise in the fluid approximation which is the decoherence limit of the semiclassical noise kernel, can be encapsulated in terms of the bulk fluid variables, like pressure, energy density etc.

We explain below in more detail, the possible cause for the physically validity of such non-thermal fluctuations in the system.

As stated in an earlier section, four-velocity of the effective fluid is a normalized quantity, hence the only possibility that it carries a random nature could occur, would be due to the corresponding unit vector fluctuating in various directions for different realizations of the field. This would happen surely if the scalar field retains all its quantum mechanical properties and coherence. Such that, for different realizations of the scalar field, the unit vector which is orthogonal to the hypersurface with constant $\phi(x)$ at a given $t$, becomes randomly oriented. This picture can approximately be associated with the fluid particles performing random motion due to thermal effects as in Brownian motion. As we have stated above, our results are obtained in the decoherence limit, the scalar field attains a classical decohered state, such that the phase information is lost or negligible. The issue of decoherence of states is yet an open area of research, however it is known that we do not attain absolute classical results in the sense that the uncertainty due to Heisenberg’s principle does not go exactly to zero in the decoherence limit. However there are other more relevant possibilities which can explain things giving connections between fundamental aspects in physics and experimental results. There are very recent advances in this regard which have been of observational consequences about foundations in quantum mechanics [9], which clearly state that its the magnitude of the scalar field responsible for such "kicks" which can be observed on macroscopics objects, particularly in terms of radiation pressure. Though the origin of these kicks originates in the quantum noise, one can see the effects it has using macroscopic variables like pressure. Similarly the role of a scalar field in our work, aimed towards fluid approximation, which is physically a different system than a laser as is the case in the above reference is common in this regard of fluctuations in the decoherence limit towards fluid approximation at hydro scales. Thus in the decoherence limit one can have the fluid approximation such that thermal effects in the fluid are vanishing with no randomness in the four-velocity of the fluid particles, but with fluctuations in pressure and other bulk variables due to the magnitude of the scalar field fluctuating. For such a case one can raise questions about the other bulk quantities that show randomness. This is definitely an interesting point, while we see that it follows very simple in a theoretical way from our results. Such fluctuations can in fact be associated with a superfluid state of gravitating matter in relativistic stars. Thus the generalized fluctuations and randomness in the bulk quantities can show up as partial remanants of the fully quantum nature of the fields retained at mesoscopic scales that we intend to address, moving towards the hydrodynamic macroscopic approximation with smooth variables at larger scales. How are these quantum fluctuations captured in bulk macroscopic quantites or are filtered through, in the way we have show here, is a deeper question, which needs further detailed research. The indication that we give here is towards the quantum potential and its possible fluctuations being responsible for the fluctuations of bulk quantities in the hydrodynamic limit. The quantum potential, is not a kinetic quantity for scalar fields, and is essentially due to the inherent nature of the fields. For gravitating matter in bulk, under the influence of strong gravity against which it supports the matter from collapsing this has several roles to play, like for scalar field models of dark matter [19] (and references therein) , it is the quantum potential that gives rise to the pressure in the fluid approximation. This is one of the directions which we would further like to explore in all details in upcoming work.

VI. CONCLUSIONS AND FURTHER DIRECTIONS

Our results indicate that, fluctuations of quantum fields can induce mesoscopic effects in the fluid description of matter, given by covariances (or variances) of pressure, energy density, heat flux etc in the background spacetime. These are the first theoretical results in closed analytical form regarding fluctuation of matter fields as correspondences in the fluid approximation that we have obtained. In addition to being used in the perturbative theory in general relativity as the noise source, these fluctuations characterize grossly, partial microscopic effects in the matter fields coupled to a spacetime of interest. The significance of these fluctuations also lies in realising their importance for compact astrophysical objects which are coupled to say thermal fields as discussed in [13, 16] and are of interest to collapsing clouds, towards critical phases and end states of collapse.

The importance of what we have shown here, lies in realising that the quantum sourced noise can be captured partially via the generalized fluctuations of classical parameters of an effective fluid model of matter in a mesoscopic range. This can be used to analyse properties of
dense matter in strong gravity regions at intermediate length scales which are not yet explored, where nonlocal or extended statistical structure may influence the dynamical as well as thermal properties. Such an extended structure given in terms of two point correlations of matter fields being filtered out from microscale to effective classical variables in the mesoscopic description, is the key feature of what we present in this article. In the currently developing area of semiclassical stochastic gravity, solutions of the semiclassical Einstein-Langevin equations pose a challenge due to the intricate nature and presence of quantum stress tensor and its fluctuations and few results have been worked out \[20\] \[22\] with all the rigour. With the version of matter fields in the hydrodynamic limit, it makes possible finding solutions applicable to epoch after decoherence of states sets in during the evolution of the universe or for compact astrophysical systems. Characterizing such a generalized noise in the system is a first step towards this.

The usefulness of this correspondence, can also be seen to give a direction for studying microscopic structure and its connections with kinetic theory in curved spacetime \[13\]. One can begin such an endeavour by trying to consider generalized fluctuations of matter fields instead of trying to define particles in the microscopic picture in a curved spacetime as being fundamental and construct the theory accordingly. We know that a global definition to particles and to vacuum in a curved spacetime background is not unique, one may then attempt to formulate a kinetic theory using the field fluctuations and its generalization as the basic entity. It is our future endeavour to investigate and explore on these lines of thought, with the framework of two point or higher correlations of generalized fluctuations of matter fields as a tool to study non-local and extended structure of matter in the curved spacetime. A kinetic theory of matter in such a curved spacetime then can be based on these fluctuations rather than the ambiguous localized particles. For such an aim, we may need to consider the full set of fluctuations including the kinetic term which gives the velocity vector and consider its fluctuations too as non-vanishing giving rise to more terms in equations \(17\) and the set \(18\) \(19\) \(20\) \(21\). We plan to carry this out in upcoming work and address the difference between what we have done here, and how it would add to the research program to follow up on the other case. Regarding that in a curved spacetime, the matter fields can be defined more appropriately than particles, in principle, the next step is to show a similar benefit of the field fluctuations in a curved spacetime. This calls for more elaborate work and confirmation, further attempts of investigation and study in this regard are on the way.

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