1. INTRODUCTION

It is generally believed that coherent pulsar radio emission is generated by some sort of instability in the relativistic electron–positron plasma ejected from the pulsar along the open field line tube. One of the widely recognized candidates is the curvature instability associated with plasma motion along curved field lines. Goldreich & Keeley (1971) found an electromagnetic instability of a set of charged particles moving on a circular ring. In contrast, Blandford (1975) demonstrated that maser action is impossible in the plasma flow along an infinitely strong magnetic field unless points of inflection and torsion are present in the field geometry. The controversy was resolved by Asseo et al. (1983) and Larroche & Pellat (1987) who showed that the instability develops only if the flow is sharply bounded; the necessary conditions are unlikely to be met in pulsar magnetospheres. Zhelzniakov & Shaposhnikov (1979) noticed that wave amplification is possible even in broad, smooth outflows if the inertial drift of particles is taken into account. This curvature-drift instability was further investigated by Kazbegi et al. (1991), Luo & Melrose (1992), Lyutikov et al. (1999a), Lyutikov et al. (1999b), Shapakidze et al. (2003), and Osmanov et al. (2009).

The curvature-drift instability belongs to the class of resonant instabilities, i.e., it develops when and if particles move in phase with the wave so that the power of the electric force applied to the particles, $E \cdot v$, does not oscillate. The necessary condition for the resonance is that the wave phase velocity is less than the speed of light. The transverse wave in a strongly magnetized plasma does become subluminal; in the electron–positron plasma moving along the magnetic field with the Lorentz factor $\gamma_p$, the phase velocity of the wave propagating at a small angle to the magnetic field is (e.g., Kazbegi et al. 1991)

$$\frac{\omega}{k_c} = 1 - \delta, \quad \delta = \frac{\omega_p^2}{4\omega_c^2 \gamma_p^3}, \quad (1)$$

where

$$\omega_p = \sqrt{\frac{4\pi e^2 n}{m}}, \quad \omega_c = \frac{eB}{mc}. \quad (2)$$

Here, $e$ and $m$ are the electron charge and mass, respectively, and $n$ is the pair number density. The above simple dispersion law is obtained if the wave frequency in the plasma frame is below the cyclotron frequency, $\omega \ll \omega_c/\gamma_p$, and if one neglects the energy spread of the plasma particles.

Conventional wisdom is that the electron–positron plasma is generated near the pulsar polar cap by a highly relativistic primary particle beam accelerated by a rotationally induced electric field. Such a two-component flow ideally suits the curvature-drift instability because a single component could not experience a significant drift, which requires a large Lorentz factor, and at the same time sufficiently affect the wave dispersion properties. In the two-component flow, the dense, relatively low energy secondary plasma makes the wave subluminal according to Equation (1) whereas the energetic primary beam has a drift velocity sufficient to resonantly excite the wave.

Because the magnetic field in the pulsar magnetosphere is very large, the factor $\delta$ is very small; therefore the wave velocity remains very close to the speed of light. Since the beam is highly relativistic, the resonance condition, $\omega = k \cdot v_p$, could be fulfilled only if the wave propagates at a very small angle to the beam. However, the radiation propagates along nearly straight rays (the refraction index is close to unity) whereas the particle beam follows the curved magnetic field lines; therefore for any ray, the resonance condition could be fulfilled only within a small zone where the ray nearly grazes the field line (see Figure 1).

Thus, the wave is resonantly amplified only at a short length scale. This is in spite of the fact that the size of the unstable region could be large, i.e., one can find unstable waves at any point within the region. However, any such wave rapidly leaves the resonant range of the angles when it propagates in the curved magnetic field. It is the resonant growth length that limits the amplification factor of the wave. The curvature instability could be considered as a viable mechanism for pulsar radio emission if the amplification factor is exponentially large; only in this case, small electromagnetic fluctuations in the flow could give rise to the observed high brightness temperature emission. We will show in this paper that this never happens; the amplification factor in any case remains very close to unity.

The paper is organized as follows. In the next section, we outline the basic parameters of the pulsar magnetosphere. In Section 3, we present preliminary, qualitative analysis of the wave amplification by the curvature-drift instability. In Section 4, we derive the wave equations and introduce the coordinate system appropriate for analyzing the wave amplification. In Section 5, we find the response of the medium, i.e., we...
calculate excited currents excited in the main plasma flow and in the high energy beam by the wave. The amplification factor of the wave is limited.

2. PLASMA FLOWS IN PULSAR MAGNETOSPHERE

Let us first briefly outline the basic parameters of the pulsar plasma. Better conditions for the curvature-drift instability are achieved in the external part of the pulsar magnetosphere where the magnetic field is not too high. Assuming a dipole field, one finds the cyclotron frequency at a distance $D$ from the star as

$$\omega_c = 1.8 \times 10^{10} \frac{B_{\text{pol}}}{d^2} \text{ Hz},$$

where $B_*$ is the surface field, $d = D/r_*$, $r_*$ is the star radius, and the usual notation $A = 10^6 A_*$ is used.

The particle density in the primary beam is of the order of the Goldreich–Julian particle density, $n_b = B/(eP)$, where $P$ is the pulsar period. Therefore, the plasma frequency of the beam may be estimated as

$$\omega_p^2 \equiv \frac{4\pi e^2 n_b}{m} = \frac{4\pi \omega_c}{P}.$$  

Note that both the particle density and the magnetic field in the open field line tube vary inversely proportionally to the tube cross section, and therefore expression (4) remains valid at any point.

The Lorentz factor of the primary beam is very high, $\gamma_b \sim 10^6$; therefore, it experiences inertial drift when moving along the curved magnetic field lines. The drift velocity is directed along the binormal to the field line,

$$\mathbf{u}_d = c \frac{\mathbf{F} \times \mathbf{B}}{eB^2}.$$  

where $F = \gamma_B mc^2/R_c$ is the centrifugal force, and $R_c$ is the curvature radius of the field line. Within the open field line tube, the curvature radius could be estimated as

$$R_c = \sqrt{D R_L} = 2 \times 10^9 \sqrt{d_3 P} \text{ cm},$$

where $R_L = P/2\pi$ is the light cylinder radius. Now one gets an estimate

$$|u_d|/c = 8 \times 10^{-4} \gamma_B d_3^{-3} B_{\text{pol}}^{-1} P^{-1/2}.$$  

One sees that the drift velocity is very small. Therefore, the optimal conditions for the curvature-drift instability are achieved in the region of the pulsar light cylinder where the drift velocity is maximal.

The secondary electron–positron plasma is generated with a moderate Lorentz factor $\gamma_p \sim 10–100$. The plasma density is generally assumed to be large as compared with the Goldreich–Julian density. Introducing the multiplicity factor, $\lambda$, defined as the ratio of the plasma to the Goldreich–Julian density, one can write

$$\omega_p^2 = \lambda \omega_B = \frac{4\pi \lambda \omega_c}{P}.$$  

According to the available polar cap models, $\lambda$ varies from a few to a few thousands (Hiibschman & Arons 2001). Observations of pulsar wind nebulae give evidence for much larger multiplicities, $\lambda > \sim 10^5$ (de Jager 2007).

Now one can estimate the parameter $\delta$, which defines, according to Equation (1), the dispersion properties of the electromagnetic waves in pulsar magnetospheres:

$$\delta = 1.7 \times 10^{-6} \frac{\lambda d_3^3}{\gamma_p^3 B_{\text{pol}} P}.$$  

One sees that the phase velocity of the waves is very close to the speed of light at any reasonable pulsar condition.

3. PRELIMINARY ANALYSIS OF THE PROBLEM

We are going to study the resonant amplification of radio waves when they propagate across the pulsar plasma. As already discussed in the introduction, the wave could be resonantly amplified only if it propagates close to the direction of the particle beam. The particles move along the curved magnetic field lines and could also experience a weak inertial drift along the binormal to the field line. Therefore, the amplifying wave should be directed at a small angle to the magnetic field and polarized predominantly along the direction of the drift velocity. Since the radiation propagates along nearly straight rays whereas the beam particles follow the curved magnetic field lines, the amplification could occur only in a small zone (Figure 1). The magnetic field lines within this zone could be considered as arches of concentric circles along which both the plasma and the beam circulate. This configuration locally approximates real plasma motion along curved magnetic field lines in the pulsar magnetosphere. Therefore, one can conveniently use the cylindrical coordinates $(r, \phi, z)$ with the $\phi$ direction along the background magnetic field.

In this configuration, one can expand the wave in cylindrical modes,

$$\mathbf{E}(r, t) = \sum_{k} \mathbf{E}_{\omega, r, k}(r) \exp(-i\omega t + ik_z z + i\delta \phi),$$  

thus reducing the problem to an ordinary differential equation for $\mathbf{E}_{\omega, r, k}(r)$. The wavelength is very small compared with the characteristic inhomogeneity scale,

$$\omega r/c = 4 \times 10^8 f_{\text{GHz}} \sqrt{d_3 P}.$$  

where $f_{\text{crit}}$ is the wave frequency in GHz; we used estimate (6) for the curvature radius. Therefore, one can try to use the WKB approximation.

In this case, the solution should take the form of locally plane waves satisfying the dispersion equation (1), i.e., there should be

$$E(r, t) \propto A_{\text{out}} e^{i k_\theta dr} + A_{\text{in}} e^{-i k_\theta dr},$$  \hspace{0.5cm} (12)

where the radial wave vector, $k_r$, should be found from the dispersion equation (1) as

$$k_r^2 = \frac{\omega^2}{c^2(1 - \delta^2)} - \frac{s^2}{r^2} - k_\theta^2.$$  \hspace{0.5cm} (13)

The first term in Equation (12) represents rays propagating away from the axis. At any point, the direction of such a ray is along the wave vector $(k_r, k_\theta = s/r, k_z)$. Taking into account Equation (13), one finds the impact parameter of the ray,

$$r_0 = \frac{cs}{\sqrt{\left(\frac{\omega}{c} \right)^2 - c^2 k_z^2}}.$$  \hspace{0.5cm} (14)

We are interested in rays propagating together with the beam; therefore, $k_z/\omega \sim u_d \ll 1$. Then one can expand

$$r_0 = \frac{cs}{\omega} \left(1 - \delta + \frac{1}{2} \frac{c^2 k_z^2}{\omega^2}\right).$$  \hspace{0.5cm} (15)

The second term in Equation (12) represents rays propagating toward the axis with the impact parameter (Equation (15)). Therefore, the cylindrical wave could be considered as a set of rays with the same impact parameter (Figure 2(a)). These rays converge to the axis until the minimal distance $r_0$ and then go to infinity. Two rays, incoming and outgoing, pass through any point at $r > r_0$. At $r \approx r_0$, a caustic is formed where the WKB approximation is violated. The structure of the solution should resemble that of the Airy function (Figure 2(b)): the maximum near $r_0$, oscillations at $r > r_0$, and the exponential decay (the evanescent wave) at $r < r_0$. The width of the caustic is very small, $\Delta r = r_0^{2/3}(c/\omega)^{2/3} \ll r_0$. The oscillating part approaches the WKB solution (12) at $r - r_0 \gg \Delta r$.

In the $r$-$\phi$ plane, the lines of constant phase rotate with a constant angular velocity $\omega/s$. At $r > r_0$, they form unwinding spirals (see Figure 2 in Lyutikov et al. 1999b) that eventually approach the lines $\int k_z dr + s \phi/r = \text{const}$. Normals to these lines form rays plotted in Figure 2(a). The azimuthal component of the wave phase velocity $v_{\phi, \theta} \equiv \omega / k_\theta = r_0 / s$ varies with the radius. Therefore, the resonant interaction with particles could occur only in a narrower layer where the particle azimuthal velocity is close to $r_0 / cs$.

More exactly, the resonant condition is $\omega = \mathbf{k} \cdot \mathbf{v}$. The particle azimuthal velocity is $v_\phi = \sqrt{c^2 - u_d^2 - 1/(2\gamma_\phi^2)} \approx c(1 - u_d^2/(2c^2))$, therefore the resonance condition is written as

$$\omega = k_z u_d + (cs/r)[1 - u_d^2/(2c^2)].$$  \hspace{0.5cm} (16)

Resolving this equation with respect to $r$, one finds the radius of the resonance layer

$$r_{\text{res}} = \frac{cs}{\omega} \left[1 + \frac{u_d}{c} \left(\frac{ck_z}{\omega} - \frac{u_d}{2c}\right)\right].$$  \hspace{0.5cm} (17)

The condition that the wave reaches the resonance radius, $r_0 < r_{\text{res}}$, can now be written as (Lyutikov et al. 1999a)

$$\delta > \frac{1}{2} \left(\frac{k_z c}{\omega} - \frac{u_d}{c}\right)^2.$$  \hspace{0.5cm} (18)

The right-hand side of this expression goes to zero if the ray is inclined to the magnetic field line plane at the same angle as the particle beam (more accurate calculation shows that the right-hand side does not go to zero but to $\gamma_\phi^{-2}$, which could be neglected); therefore at first glance, this condition does not impose any restrictions on $\delta$. However, we will see that the amplification also ceases in this case, and therefore some discrepancy between the directions of the ray and the beam is necessary. The reason is that the resonance interaction occurs via the component of the wave electric field parallel to the particle velocity, $\mathbf{E} \cdot \mathbf{v}$. The transverse wave should be inclined to the beam in order for this component be non-zero. Then, Equation (18) implies that the curvature-drift mechanism could operate only if the magnetospheric plasma is dense enough.

Since both $\delta$ and $u_d/c$ are very small at any reasonable pulsar conditions, the ray should be inclined to the plane of the magnetic field line by a small angle. In this case, the resonance and the caustic radii are close to each other. Therefore, in order to find the amplification factor of the wave, we could solve the wave equations in a relatively narrow range of radii containing...
both $r_0$ and $r_{\text{res}}$. At the external boundary of this region, the solution should be matched with the WKB solution (12). At the internal boundary, the solution should be matched with the evanescent wave. Then, the amplification factor is found as the ratio of amplitudes of the outgoing and incoming waves, $a = |A_{\text{in}}|/|A_{\text{out}}|$. With such a setup, we have in fact to solve the reflection problem for the radial wave $E_{\omega,s,k}(r)$. Therefore, the curvature instability may be thought of as a partial case of the general effect, namely, the overreflection of waves from a rotating dielectric body (Zel'dovich 1972).

4. WAVE EQUATIONS

The propagation of the wave is described by Maxwell’s equations

$$\nabla \cdot E = 4\pi \rho, \quad \nabla \times B = 4\pi j + \frac{\partial E}{\partial t},$$

(19)

$$\nabla \cdot E = 0, \quad \nabla \times E = -\frac{\partial B}{\partial t},$$

(20)

where currents and charges are excited in the plasma by the wave. From here on we take the speed of light to be unity. One can conveniently study the curvature instability by expanding the wave in cylindrical modes (Equation (10)). Eliminating $B$, one writes the equations for the amplitude of a cylindrical wave as

$$\frac{d^2 E_\phi}{dr^2} + \frac{1}{r} \frac{dE_\phi}{dr} + \left( \omega^2 - k_z^2 - \frac{1}{r^2} \right) E_\phi - i \frac{s}{r} \frac{d}{dr} \frac{E_r}{r} + k_z \frac{s}{r} E_z = -4\pi \omega i j_\phi,$$

(21)

$$\frac{d^2 E_z}{dr^2} + \frac{1}{r} \frac{dE_z}{dr} + \left( \omega^2 - \frac{s^2}{r^2} \right) E_z - i \frac{k_z}{r} \frac{d}{dr} \frac{E_r}{r} + k_z \frac{s}{r} E_\phi = -4\pi \omega i j_z,$$

(22)

$$\left( \omega^2 - k_z^2 - \frac{s^2}{r^2} \right) E_r - i s \frac{d}{dr} E_\phi - ik_z \frac{d}{dr} E_z = -4\pi \omega i j_r,$$

(23)

We are going to solve these equations close to the caustic zone where the wave propagates nearly along the $\phi$ direction (which coincides with the direction of the background magnetic field). We will see that the wave remains nearly transverse, and therefore one can conveniently introduce the variables

$$E_+ = \frac{s}{\omega r} E_\phi + \frac{k_z}{\omega} E_z, \quad E_- = \frac{s}{\omega r} E_z - \frac{k_z}{\omega} E_\phi$$

(24)

such that in the region of interest, $E_+$ becomes the longitudinal component of the field while $E_-$ and $E_r$ are the transverse components. Combining Equations (21) and (22), one gets the equation for $E_+$:

$$\frac{d^2 E_+}{dr^2} + \frac{1}{r} \left( 1 + \frac{s^2}{\omega^2 r^2} \right) \frac{dE_+}{dr} + \left[ \omega^2 - k_z^2 - \frac{s^2}{r^2} \left( 1 + \frac{1}{s^2} + \frac{1}{\omega^2 r^2} \right) \right] E_+ - \frac{k_z s}{\omega^2 r^2} \frac{dE_+}{dr} - 2s k_z \frac{dE_+}{dr} E_r = -4\pi i \omega j_-, \quad (25)$$

where

$$j_+ = \frac{s}{\omega r} j_\phi - \frac{k_z}{\omega} j_\phi. \quad (26)$$

We complement this equation by Equation (23) for $E_-$, and by the Gauss law

$$\frac{1}{r} \frac{d}{dr} E_r + i \omega E_+ = 4\pi \rho. \quad (27)$$

Since $\omega r \approx s \gg 1$, we can solve these equations in the short-wave approximation (e.g., Nayfeh 1973). The resonance occurs close to the caustic zone, so one can conveniently use the caustic coordinates. The plasma refraction index is very close to unity, therefore the characteristic width of the caustic zone is the same as in the vacuum case, $\Delta r \sim r/s^{2/3}$, except the reflection point is shifted toward a smaller radius. Therefore, let us define the new radial dimensionless coordinate, $x$, as

$$r = r_0 \left( 1 + \frac{x}{s^{2/3}} \right), \quad (28)$$

where $r_0$ is the WKB impact parameter (Equation (15)), which is in fact the caustic radius. Transforming to the new variable and retaining only the leading order in the $s^{-1/3} \approx (\omega r_0)^{-1/3}$ terms, we can reduce Equations (25), (23), and (27) to the forms

$$E''_+ + 2(x - s^{2/3})E_+ = -\frac{4\pi s^{2/3}}{\omega} i j_-, \quad (29)$$

$$i s^{-1/3} E'_* - 2(x - s^{2/3})E_r = \frac{4\pi s^{2/3}}{\omega} i j_r, \quad (30)$$

$$E'_r + i s^{-1/3} E_+ = \frac{4\pi s^{2/3}}{\omega} \rho, \quad (31)$$

where the prime denotes differentiation with respect to $x$.

In order to write the equations in the closed form, we have to find the charge and current densities excited by the wave in the plasma and in the beam. Here, we consider the simplest case when both the plasma and the beam are cold so that they are described only by their densities and velocities. Then, the current and the charge densities are presented as

$$\rho = \sum_a q_a n_a, \quad j = \sum_a q_a n_a v_a, \quad (32)$$

where the summation is over the particle species (electrons and positrons in the plasma and, for example, positrons in the beam). Now we have to find the perturbation of the particle motion in the electromagnetic field of the wave.

5. THE RESPONSE OF THE MEDIUM

In this section, we find the current and charge density induced in the medium by the wave. For this, we have to solve the equations of particle motion in the field of the wave:

$$\frac{\partial y}{\partial t} + (v \cdot \nabla) y = \frac{q}{m} (E + v \times B_0 + v \times B). \quad (33)$$

Here, $q = \pm e$ is the particle charge; the background field, $B_0$, is assumed to have only a $\phi$ component. The oscillatory motion of the particles in the electromagnetic field of the wave implies perturbation of the particle density according to the continuity equation

$$\frac{\partial n}{\partial t} + (\nabla \cdot n v) = 0. \quad (34)$$
Without the wave, the particles move in the $\phi$ direction along the background magnetic field and also experience the inertial drift in the $z$-direction. In order to find the response of the medium to the wave, one has to linearize Equations (33) and (34) with respect to small perturbations of the velocity $v = v^{(0)} + v^{(1)}$ and of the density $n = n^{(0)} + n^{(1)}$. The response is expressed via the solution to the linearized equations as

$$\rho = \sum_a q_a n_a^{(1)}, \quad j = \sum_a q_a (n_a^{(1)} v_a^{(0)} + n_a^{(0)} v_a^{(1)}). \quad (35)$$

Even though the procedure is straightforward, it leads to rather cumbersome expressions, so we consider the responses of the plasma and of the beam separately, exploiting from the beginning appropriate approximations.

5.1. Response of the Beam

The Lorentz factor of the beam is very large, $\gamma_b \sim 10^6$; therefore, when moving along the curved magnetic field line, it experiences an inertial drift in the $z$-direction,

$$v_z^{(0)} \text{beam} = u_d = -\frac{\gamma_b V^2}{r \omega_r}. \quad (36)$$

The unperturbed velocity in the $\phi$ direction may be presented as

$$v_\phi^{(0)} \text{beam} = V = \sqrt{1 - u_d^2 - \gamma_b^{-2}}. \quad (37)$$

In this subsection, only velocities and density of the beam appear, therefore we drop the index “beam” in $v^{(1)}$ and $n^{(1)}$.

Linearizing continuity Equation (34), one gets

$$n^{(1)} = \frac{n_b}{\Omega} \left( k_z v^{(1)}_z + \frac{2V}{r} v^{(1)}_\phi - i \frac{\partial v^{(1)}_\phi}{\partial r} \right), \quad (38)$$

where

$$\Omega = \omega - \frac{V}{r} - k_z u_d + i 0, \quad (39)$$

and we take into account that according to the Landau prescription, a small imaginary part should be added to the frequency.

The linearized equations of motion are written as

$$i \Omega v^{(1)}_\phi + \frac{\omega}{\gamma_b} v^{(1)}_z - \frac{2V}{r} v^{(1)}_\phi = \frac{e}{m\gamma_b} \left( VB_z - E_r - u_d B_\phi \right), \quad (40)$$

$$i \Omega v^{(1)}_\phi - \frac{V}{r} v^{(1)}_\phi = \frac{e}{m\gamma_b} \left[ V u_d E_z - \left( u_d^2 + \frac{1}{\gamma_b^2} \right) E_\phi + u_d B_r \right], \quad (41)$$

$$i \Omega v^{(1)}_z - \frac{i}{\gamma_b} v^{(1)}_\phi = -\frac{e}{m\gamma_b} \left( (1 - u_d^2) E_z + V B_r - u_d V E_\phi \right). \quad (42)$$

The terms with $V/r$ on the left-hand side of Equations (40) and (41) arise from the centrifugal and Coriolis forces, respectively. These terms were neglected in all previous works on the curvature instability. We will see that they are important near the resonance where $\Omega$ goes to zero.

The density of the beam is extremely small, and therefore the contribution of the beam to the current and charge densities of the medium is significant only near the resonance layer where the wave moves together with the beam so that $\Omega$ goes to zero. The resonant radius is defined from the condition $\Omega = 0$, which is nothing more than the condition $\omega = k \cdot v$ already discussed in Section 3. Therefore, we can use Equation (17) for $r_{res}$. Expressing $r_{res}$ via the radial coordinate $x$ defined by Equation (28) and making use of Equation (15) for $r_0$, one gets the position of the resonance layer in the dimensionless variable

$$x_{res} = \delta s^{2/3} - \frac{1}{2} \delta s^{2/3} (1 - \kappa)^2 u_d^2. \quad (43)$$

Taking into account that the ray is directed nearly along the particle beam, one can expect $k_z/\omega \sim u_d$; therefore, we introduced the parameter

$$\kappa = \frac{k_z}{u_d \omega}. \quad (44)$$

The function $\Omega$ can now be written via the dimensionless variable as

$$\Omega = \frac{\omega}{s^{2/3}} (x - x_{res} + i 0). \quad (45)$$

When solving Equations (40)–(42) for $v^{(1)}_\phi$ and $v^{(1)}_z$, we retain only resonant terms (those proportional either to $\Omega^{-1}$ or $\partial/\partial r$). Expressing $B$ via $E$ with the aid of Maxwell’s equations, one finds

$$v^{(1)}_\phi = -i \frac{e}{m\gamma_b V} \left[ \frac{1}{\Omega} \left( \frac{1}{\gamma_b^2} - \kappa (1 - \kappa) u_d^4 \right) - \frac{u_d^2}{\omega V^2} \frac{d}{dr} \right] \times (VE_\phi + u_d E_z), \quad (46)$$

$$v^{(1)}_z = -i \frac{e u_d}{m\gamma_b V^2} \left[ \frac{2}{\Omega} \left( \frac{1}{\gamma_b^2} - \kappa (1 - \kappa) u_d^4 \right) + \frac{u_d r}{\omega} \frac{d}{dr} \right] \times (VE_\phi + u_d E_z). \quad (47)$$

The component $v^{(1)}_\phi$ has no resonant terms; however, it should be taken into account in the expression (38) for $n^{(1)}$ because being differentiated, it could contribute to the resonance interaction provided that the field amplitudes are singular. Dropping the terms proportional to $\Omega$ one finds

$$v^{(1)}_z = -\frac{e u_d}{m\omega \kappa} (1 - \kappa) (VE_\phi + u_d E_z). \quad (48)$$

One sees that the resonance interaction is determined by the projection of the wave electric field onto the direction of the beam. This projection is expressed via the “longitudinal” and “transverse” components of the electric field (Equation (24)) as

$$VE_\phi + u_d E_z \approx E_\phi + (1 - \kappa) u_d E_z - \frac{s V}{c \omega} \frac{u_d k_z}{\omega} E_\phi + \frac{u_d s}{c \omega} \frac{V k_z}{\omega} E_z \approx E_\phi + (1 - \kappa) u_d E_z. \quad (49)$$

Substitution of Equations (46)–(48) into Equation (38) yields the beam contribution to the charge density in the form

$$4\pi i \rho^{\text{beam}} = 4\pi i e n^{(1)} = -\frac{e^2 \delta s^{2/3} u_d^4 (1 - \kappa)}{\omega \gamma_b (x - x_{res} + i 0)^2} \times \left[ 1 + (x - x_{res}) \frac{d}{dx} \right] \left[ E_\phi + (1 - \kappa) u_d E_z \right]. \quad (50)$$
Here, we made a transformation to the dimensionless coordinate $x$ and also dropped the term $1/\gamma_p^2$ as compared with $u_d^2$ in the square brackets in Equations (46) and (47). One sees that the density perturbation is of the second order in $\Omega^{-1} \propto (x-x_{\mathrm{res}})^{-1}$ whereas the velocities are only of the first order. Therefore, the beam contribution to the current could be written as

$$j_{\mathrm{beam}} = \rho_{\mathrm{beam}} v_{(0)}^{(0)},$$

which yields

$$j_{\perp} = \rho_{\mathrm{beam}} \left( k^2 \omega V - s \frac{u_d}{\omega r_0} \right) \approx (1 - \kappa) u_d \rho_{\mathrm{beam}}.$$  \hspace{1cm} (52)

Note that if we neglected the $V/r$ terms (arising from the Coriolis and centrifugal forces) in the linearized equations of motion (40) and (41), we would come to an expression similar to (50) but with the coefficient proportional to $u_d^2$ instead of $u_d$. Thus, taking these terms into account is crucially important.

5.2. Response of the Plasma

The Lorentz factor of the plasma particles, $\gamma_p = (1 - v_p^2)^{-1/2}$, is not very large, so that one can neglect the inertial forces. Then, the unperturbed velocity is in the $\phi$ direction, $v_{(0)}^{(0)} = 0, v_{\phi}^{(0)} \equiv v_p$. In this subsection, we drop the index “plasma” in $v_{(1)}$ and $n_{(1)}$.

The linearized equations of motion are now written as

$$i \left( \omega - \frac{sv_p}{r} \right) v_p^{(1)} + i \frac{\omega_c}{\gamma_p} v_z^{(1)} = \frac{q}{m\gamma_p} (v_p B_z - E_r),$$

$$i \left( \omega - \frac{sv_p}{r} \right) v_\phi^{(1)} = - \frac{q}{m\gamma_p^2} E_\phi,$$

$$i \left( \omega - \frac{sv_p}{r} \right) v_z^{(1)} - i \frac{\omega_c}{\gamma_p} v_\phi^{(1)} = - \frac{q}{m\gamma_p} (E_z + v_p B_r).$$

Since the plasma is electrically neutral in the sense that the densities and unperturbed velocities of electrons and positrons are equal, the plasma response is determined only by the terms with odd powers of $q$ in the perturbed velocity. These terms are easily found as

$$v_p^{(1)} = - i \frac{e\gamma_p (\omega - v_p s/r)}{m\gamma_p^2} \left[ (\omega - v_p s/r) E_r - i v_p \frac{dE_\phi}{dr} \right] + \{ \text{terms} \} \propto e^2,$$

$$v_\phi^{(1)} = i \frac{e}{m\gamma_p^2 (\omega - v_p s/r)} E_\phi,$$

$$v_z^{(1)} = - i \frac{e\gamma_p (\omega - v_p s/r)}{m\gamma_p^2} \left[ (\omega - v_p s/r) E_z + v_p k_z E_\phi \right] + \{ \text{terms} \} \propto e^2.$$  \hspace{1cm} (56)

Here, we used the strong field approximation

$$\omega_c \gg \gamma_p (\omega - sv_p/r) \approx \omega/2\gamma_p.$$  \hspace{1cm} (59)

The charge and current densities are written as

$$\rho_{\mathrm{plasma}} = v_p \rho + 2en_p v_\phi^{(1)},$$

$$j_z^{\mathrm{plasma}} = 2en_p v_z^{(1)},$$

$$j_\perp^{\mathrm{plasma}} = 2en_p v_\phi^{(1)}.$$  \hspace{1cm} (61)

Here, $n_p$ is the pair number density (the number density of electrons or positrons); the factor 2 takes into account that the contributions of electrons and positrons are equal. Note that the plasma current and charge densities are small as $\delta \sim (\omega_p/\omega_c)^2$ (see Equation (1)). Then, the “longitudinal” component of the electric field, $E_\perp$, is also small as $\delta$ (or as $s^{-1/3}$; see Equation (31)), therefore one can neglect the contribution of this component to the plasma current and charge densities. Then, one gets

$$4\pi ij_\perp^{\mathrm{plasma}} = 2 \left\{ \frac{\omega_p^2 \gamma_p}{\omega^3} \left[ \frac{\omega}{r} - v_p \left( k_z^2 + \frac{s^2}{r^2} \right) \right] \right\} \times \left\{ \frac{\omega - v_p s^2/r^2 - k_z^2}{\gamma_p^2 (\omega - v_p s/r)} \right\} E_\perp - 2 \frac{\omega_p^2 k_z^2}{\omega^3} \frac{d^2 E_\perp}{dr^2} + \frac{\omega_p^2 k_z}{\gamma_p \omega^2} \frac{dE_\perp}{dr}. $$  \hspace{1cm} (62)

Here, we took into account that $dE_\perp/dr \gg E_\perp/r$.

These expressions could be significantly simplified by taking into account that we are interested in the region close to the resonance layer defined by Equation (17) so that we can substitute $r_{\mathrm{res}}$ instead of $r$. Then, one can write, e.g.,

$$\omega - sv_p/r = \omega \left( 1 - v_p - \frac{\kappa u_d}{\sqrt{V}} \right) \approx \frac{\omega}{2\gamma_p^2}.$$  \hspace{1cm} (63)

In the last equality, we take into account that $\gamma_p |u_d| \ll 1$. Making use of this approximation and also taking into account condition (59), one finds

$$4\pi ij_\perp^{\mathrm{plasma}} = \omega_p^2 \left( \frac{\omega}{2\omega_c^2 \gamma_p} - \frac{8\gamma_p k_z^2}{\omega^3} \right) E_\perp + \frac{\omega_p^2 k_z}{\omega^2} \left( i \frac{\omega}{\gamma_p^2 s^{1/3}} E_r' - \frac{2k_z}{s^{2/3}} E_\perp' \right). $$  \hspace{1cm} (64)

The term with $E_r'$ can be neglected because one can show with the aid of Equation (31) that this term is small compared with terms with $E_\perp$ (small as $E_r/E_\perp$ or $E_r/r \sim \delta$). The term with $E_\perp'$ could also be neglected because when substituting the expression for $j_\perp^{\mathrm{plasma}}$ into the right-hand side of Equation (29), one sees that this term is small compared with the corresponding term on the left-hand side. Assuming also that $k_z/\omega \sim u_d \ll \omega/(4\omega_c \gamma_p^2)$, one finally finds

$$4\pi ij_\perp^{\mathrm{plasma}} = \frac{\omega_p^2 \omega}{2\omega_c^2 \gamma_p} E_\perp = 2\delta \omega E_\perp.$$  \hspace{1cm} (65)

As a consistency check, let us show that this expression for the plasma response leads to dispersion law (1).

With this purpose, let us substitute this expression into wave equation (29) and return to the original variable $r$. This yields

$$\frac{r_0^2}{s^{4/3}} \frac{d^2 E_\perp}{dr^2} + 2s^{2/3} r - r_0 E_\perp = 0.$$  \hspace{1cm} (66)
In the WKB approximation, \( E_- = \exp(i \int k_r dr) \), and one gets
\[
k_r^2 = 2 \frac{s^2}{r_0^2} (r - r_0).
\]
(67)

On the other hand, the dispersion equation (1) is written in form (13), which reduces to Equation (67) at \( r - r_0 \ll r_0 \).

6. AMPLIFICATION FACTOR

6.1. The Wave Equation in the Closed Form

With the results of the previous section, we can write the wave equations in closed form. Substituting Equations (50), (52), and (65) into the equation for the “transverse” component of the electric field (Equation (29)), one gets
\[
E'' + 2\chi E_\perp = \frac{\alpha}{(x - x_{res} + i\theta)^2} \left( 1 + (x - x_{res}) \frac{d}{dx} \right) \times \left( E_\perp + \frac{E_\parallel}{(1 - \kappa)u_d} \right),
\]
where
\[
\alpha = \frac{\omega_0^3 s^3 \delta k (1 - \kappa)^3}{\omega^2 y_0}.
\]
(68)

Taking into account that \( s \approx \omega r \), one can present \( \alpha \), with the aid of Equations (4), (6), and (36), as
\[
\alpha = 2(D/R_L)^{1/2} u_d^5 \kappa (1 - \kappa)^3.
\]
(70)

Taking into account that \( D \ll R_L, \kappa \sim 1 \), and the drift velocity is always small (recall that here \( u_d \) is measured in units of \( c \)), one sees that \( \alpha \) is a very small quantity. This already ensures that the amplification factor is small, but let us calculate it explicitly.

Equation (68) should generally be complemented by Equations (30) and (31). However, Equation (68) is reduced to a closed equation for \( E_\perp \) provided
\[
E_\perp \ll (1 - \kappa)u_d E_\perp.
\]
(71)

The conditions for such an approximation could be found as follows.

One can estimate \( E_\perp \) from Equation (31). Outside the resonance region, one can write \( E_\perp \sim s^{-1/3} E_r \) because the right-hand side of this equation is of the order of \( \delta E_\perp \ll s^{-1/3} E_\perp \). We are interested in the wave polarized predominantly in the \( z \)-direction; therefore, \( E_r \ll E_\perp \). Then, condition (71) is fulfilled if
\[
(1 - \kappa) s^{1/3} u_d \geq 1,
\]
(72)
i.e., if the drift velocity is not too small. Taking into account that the effect under consideration assumes non-negligible \( u_d \) so that the wave amplification is suppressed when \( u_d \) goes to zero, we assume that condition (72) is fulfilled, which justifies inequality (71) outside the resonance region. It is shown in the Appendix that if inequality (71) is fulfilled outside the resonance region, it also remains valid within the resonance region. Therefore, condition (72) justifies neglecting \( E_r \) in Equation (68). Now we come to the closed equation for \( E_\perp \),
\[
E'' + 2\chi E_\perp = \frac{\alpha}{(x - x_{res} + i\theta)^2} \left( E_\perp + (x - x_{res}) \frac{dE_\perp}{dx} \right).
\]
(73)

Inasmuch as \( \alpha \ll 1 \), this equation could be solved by the method of matching asymptotic expansions (e.g., Nayfeh 1973).

6.2. Solutions Outside and Inside the Resonance Zone

At \( |x - x_{res}| \gg \sqrt{\alpha} \), one can neglect the term on the right-hand side of Equation (73); then the solutions are expressed via the Airy functions. Specifically, to the left of the resonance, at \( x_{res} - x \gg \sqrt{\alpha} \), we have to choose the evanescent wave solution
\[
E_\perp = \text{Ai}(-2^{1/3} \xi).
\]
(74)

To the right of the resonance, at \( x - x_{res} \gg \sqrt{\alpha} \), the external solution should be chosen in the general form
\[
E_\perp = A_1 \text{Ai}(-2^{1/3} \xi) + A_2 \text{Bi}(-2^{1/3} \xi),
\]
(75)

and the amplitudes \( A_1 \) and \( A_2 \) should be found by matching solution (75) to solution (74) via the resonance layer. In the wave zone, \( x \gg 1 \), function (75) is reduced to a superposition of incoming and outgoing waves
\[
E_\perp = \frac{1}{2^{7/6} \chi^{1/2}} \left[ (A_1 + iA_2)e^{i[2^{4/3}z^{2/3} - \xi]} + (A_1 - iA_2)e^{-i[2^{4/3}z^{2/3} - \xi]} \right].
\]
(76)

The amplification factor is defined as the ratio of the amplitudes of the outgoing and incoming waves
\[
a = \left| \frac{A_1 + iA_2}{A_1 - iA_2} \right|.
\]
(77)

We will see that \( |A_2| \ll |A_1| \) (at \( \alpha = 0 \), the full solution is described by Equation (74), which yields \( A_1 = 1, A_2 = 0 \); therefore at \( \alpha \ll 1 \), we obtain \( A_2 \ll A_1 \)). Then one can write
\[
a = 1 + 2\Re A_2,
\]
(78)
so that the amplification factor is determined only by the imaginary part of the amplitude \( A_2 \). Now let us proceed to matching the solutions \( E_\perp \) via the resonance zone.

First we have to find the “inner” solution, i.e., that in the resonant zone \( |x - x_{res}| \ll 1 \). Introducing the “inner” variable \( z = (x - x_{res})/\sqrt{\alpha} \), one writes Equation (73) as
\[
\frac{d^2 E_\perp}{dz^2} + 2\alpha(\sqrt{\alpha} z + x_{res})E_\perp = \frac{1}{(z + i\theta)^2} \left( E_\perp + z \frac{dE_\perp}{dz} \right).
\]
(79)

One sees that one can neglect the second term on the left-hand side, then the solution is easily found in the power-law form. Returning to the original variable, \( x \), one can write the solution in the resonant zone as
\[
E_\perp = Q(x - x_{res} + i\theta)^{-\alpha} + P(x - x_{res} + i\theta)^{1+2\alpha},
\]
(80)
where \( Q \) and \( P \) are constants. Now we have to match the solutions (74), (75), and (80). This could be done because the “outer” solutions (74) and (75) are valid at \( x_{res} - x \gg \sqrt{\alpha} \) and \( x - x_{res} \gg \sqrt{\alpha} \), respectively, therefore at small \( \alpha \), the domains of validity of these solutions are overlapped with the domain of validity of the “inner” solution, \( |x - x_{res}| \ll 1 \).

6.3. Matching the Solutions

We first find the constants \( Q \) and \( P \) by matching the “inner” solution (80) with solution (74). In the region
\[
\sqrt{\alpha} \ll x_{res} - x \ll 1,
\]
(81)
solution (80) is still valid. Having found Q and P one can find the amplitudes A1 and A2 by matching solution (80) with (74) in the region
\[ \sqrt{\alpha} \ll x - x_{res} \ll 1. \] (82)
We will see that it is not necessary to perform the full matching procedure if we are interested only in the reflection coefficient, which is determined only by the imaginary part of the amplitudes.

In region (81), the “inner” solution (80) is reduced to
\[ E_+ = Q \left[ 1 - \alpha \ln |x - x_{res}| - i\pi \alpha \right] + P(x - x_{res}) \left[ 1 + 2\alpha \ln |x - x_{res}| + 2i\pi \alpha \right]. \] (83)
The “external” solution is found from Equation (73) as a slightly perturbed evanescent wave. Namely, the right-hand side of this equation is small in region (81), therefore one can substitute there the evanescent wave (74) thus arriving at an inhomogeneous Airy equation. The solution to this equation which goes to Equation (74) at \( x \to -\infty \) is written as
\[ E_- = E_-^{left} + \alpha \int_{-\infty}^{x} G(x, x_1) \left[ 1 + (x_1 - x_{res}) \frac{d}{dx_1} \right] E_-^{left}(x_1) dx_1, \] (84)
\[ G(x, x_1) = 21/3 \pi [Ai(-21/3 x_1)Bi(-21/3 x) - Bi(-21/3 x_1)Ai(-21/3 x)]. \] (85)
At \( x \to x_{res} \), the first term in this solution is reduced to the linear function
\[ E_-^{left}(x \to x_{res}) = Ai(-21/3 x_{res}) - 21/3 Ai'(-21/3 x_{res})(x - x_{res}), \] (86)
so that matching with the “inner” solution (83) to within the zeroth order in \( \alpha \) yields
\[ Q = Ai(-21/3 x_{res}) + O(\alpha), \quad P = -21/3 Ai'(-21/3 x_{res}) + O(\alpha). \] (87)
As \( x \to x_{res} \), the second term in Equation (84) gives the terms of the order of \( \alpha \ln |x - x_{res}| \) and \( \alpha \), therefore the solution (84) could be smoothly matched with the “inner” solution (83).

We are interested only in the imaginary parts of the coefficients \( Q \) and \( P \), which are sufficient in order to find the imaginary part of the amplitude \( A_2 \). Since solution (84) is real, it is matched with solution (83) if \( \Im Q = \pi \alpha \Im Q \) and \( \Im P = -2\pi \alpha \Im P \) to within the first order in \( \alpha \). Making use of Equation (87), one gets
\[ \Im Q = -\pi \alpha Ai(-21/3 x_{res}), \quad \Im P = -21/3 \pi \alpha Ai'(-21/3 x_{res}). \] (88)

In region (82), the resonance solution (80) is written as
\[ E_- = Q \left[ 1 - \alpha \ln (x - x_{res}) \right] + P(x - x_{res}) \left[ 1 + 2\alpha \ln (x - x_{res}) \right], \] (89)
whereas the “external” solution is presented as
\[ E_- = E_-^{right} - \alpha \int_{x}^{\infty} \frac{G(x, x_1)}{(x_1 - x_{res})^2} \left[ 1 + (x_1 - x_{res}) \frac{d}{dx_1} \right] E_-^{right}(x_1) dx_1. \] (90)
At \( x \to x_{res} \), one can substitute \( E_-^{right} \) by a linear function
\[ E_-^{right}(x) = A_1 Ai(-21/3 x_{res}) + A_2 Bi(-21/3 x_{res}) - 21/3 [A_1 Ai'(-21/3 x_{res}) + A_2 Bi'(-21/3 x_{res})](x - x_{res}). \] (91)
The second term in Equation (90) has the order of \( \alpha \ln(1/\alpha) \) in region (82) because the integral diverges only logarithmically. Therefore, matching solution (90) with (89) in the zeroth in \( \alpha \) approximation yields, taking into account Equation (87),
\[ A_1 = 1 + O(\alpha), \quad A_2 = O(\alpha). \] (92)
In this case, the general expression for the amplification factor (77) is reduced to Equation (78). Therefore, it suffices to find only the imaginary part of \( A_2 \).

With this purpose, we can only match the imaginary parts of Equations (89) and (90) at \( x \to x_{res} \). Taking into account Equation (88), one gets from Equation (89)
\[ \Im E_- = -\pi \alpha [Ai(-21/3 x_{res}) + 21/3 Ai'(-21/3 x_{res})(x - x_{res})] + O(\alpha^2). \] (93)
Equation (90) yields at \( x \to x_{res} \), taking into account Equations (91) and (92),
\[ \Im E_- = \Im A_1 Ai(-21/3 x_{res}) + \Im A_2 Bi(-21/3 x_{res}) - 21/3 [\Im A_1 Ai'(-21/3 x_{res}) + \Im A_2 Bi'(-21/3 x_{res})] \times (x - x_{res}) + O(\alpha^2). \] (94)
Comparing the coefficients at \( (x - x_{res})^0 \) and \( (x - x_{res}) \) in the last two equations, one gets
\[ \Im A_1 = -\pi \alpha^2 [Ai(-21/3 x_{res})Bi'(-21/3 x_{res}) + 21/3 Ai'(-21/3 x_{res})Bi(-21/3 x_{res})], \] (95)
\[ \Im A_2 = (1 + 21/3 \pi \alpha^2 Ai(-21/3 x_{res})Ai'(-21/3 x_{res}). \] (96)
Here, we used the formula for Wronskian, \( Ai'Bi - AiBi' = 1/\pi \).

Now Equation (78) finally yields
\[ a = 1 + 2(1 + 21/3)\pi \alpha^2 Ai(-21/3 x_{res})Ai'(-21/3 x_{res}). \] (97)
One sees that the wave is amplified, \( a > 1 \), provided \( Ai(-21/3 x_{res})Ai'(-21/3 x_{res}) > 0 \), which happens if \( x_{res} \) falls within the intervals
\[ 1 < 21/3 x_{res} < 2.4, \quad 3.2 < 21/3 x_{res} < 4, \quad \ldots \]
\[ \left[ \frac{3}{8}(1 + 4k)\pi \right]^{2/3} < 21/3 x_{res} < \left[ \frac{3}{8}(3 + 4k)\pi \right]^{2/3}, \] (98)
where \( k \) is a large integer number.

It follows immediately from Equation (43) that the necessary condition for the amplification, \( x_{res} > 2^{1/3} \), is achieved only if \( \delta \), i.e., the deviation of the wave velocity from \( c \), is not too small:
\[ \delta > \frac{1}{2} \left[ 1 - \kappa \right]^2 u_d^2 + \frac{1}{21/3 \kappa^{2/3}}. \] (99)
Taking into account condition (72), one sees that condition (99) roughly coincides with condition (18) found in the WKB approximation. This condition implies that the curvature-drift instability is possible only if the pulsar plasma density is high enough.

In any case, one sees from Equation (97) that even if the instability condition is fulfilled, the amplification factor (97) is very close to unity because it follows from Equation (70) that \( \alpha \) remains small at any reasonable conditions. Therefore, the curvature-drift instability could not amplify small fluctuations.
up to high brightness temperatures characteristic of pulsar radio emission.

7. CONCLUSIONS

In this paper, we rederived the curvature-drift instability, which has long been considered a viable mechanism for pulsar radio emission. Our approach differs from that adopted in the previous works on the topic. Namely, we did not look for the growth rate of the instability, but instead explicitly calculated the propagation of the electromagnetic wave through the plasma moving along the curved magnetic field lines. With such an approach, we found the amplification factor of the waves. In the standard approach, the amplification factor depends on the growth rate and also on the size of the resonance zone. In the case under consideration, both the growth rate and the resonance zone are determined by the curvature radius, so that it makes no sense to calculate these quantities separately. Moreover, in the previous works, the growth rate was obtained using the dielectric tensor method, which implies that the waves are considered locally plane. Then, one could define the dielectric tensor of the medium such that the growth rate is an imaginary part of an appropriate eigenvalue. Even in the most advanced work by Lyutikov et al. (1999a, 1999b), where it was explicitly demonstrated that the resonance occurs in the vicinity of the caustic zone so that the wave should be described by the Airy type functions, the amplification was calculated only at caustic zone so that the wave should be described by the demonstrated that the resonance occurs in the vicinity of the caustic zone such that the growth rate is an imaginary part of an appropriate eigenvalue. Even in the most advanced work by Lyutikov et al. (1999a, 1999b), where it was explicitly demonstrated that the resonance occurs in the vicinity of the caustic zone so that the wave should be described by the Airy type functions, the amplification was calculated only at the WKB condition $x_{res} \gg 1$. Our approach is free from these limitations and moreover provides the amplification factor straightforwardly without intermediate steps.

The advantage of explicit solutions over the dielectric tensor approach was clearly demonstrated by the long-lasting discussion initiated by the works of Beskin et al. (1987, 1988a). These authors derived the dielectric tensor of a weakly inhomogeneous plasma in the infinitely strong magnetic field and found unstable modes. The correctness of the obtained dielectric tensor has been extensively debated (see Nambu 1989, 1996; Machabeli 1991, 1995; Istomine 1994; Bornatici & Kravtsov 2000). However, the claim of Beskin et al. was disproved by Larroche & Pellat (1987), who presented an explicit solution to the wave equations and demonstrated that the wave amplitude does not grow unless the flow is sharply bounded (see also discussions in Beskin et al. 1988b and Larroche & Pellat 1988). The specific property of the obtained solution is that it becomes singular in a narrow resonance layer. The dielectric tensor method assumes implicitly that the global solution could be described by a locally plane wave with smoothly varying parameters. In shear flows, there are typically no such solutions satisfying reasonable boundary conditions; the wave becomes singular in the critical layer where the phase velocity becomes equal to the flow velocity (e.g., Stepanyants & Fabrikant 1989). Therefore, the dielectric tensor method does not provide unambiguous conclusions on the stability of shear flows; one has to find explicit solutions as was done in the present paper.

Another novel aspect of the present research is taking into account the inertial forces (Coriolis and centrifugal) when considering interaction of the waves with resonant particles. These terms were ignored in all previous works on the curvature-drift instability. We have shown that these forces significantly suppress the growth rate.

The net result of our study is that there is no condition at which the amplification factor could become large; therefore the curvature-drift instability should be excluded from the list of potential mechanisms for pulsar radio emission.

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APPENDIX

GENERAL SOLUTION TO THE WAVE EQUATIONS IN THE RESONANCE REGION

Within the resonance region, $|x - x_{res}| \ll 1$, one has to retain only the resonant terms so that Equations (29)–(31) could be written, taking into account Equations (50) and (52), in the form

$$E_x'' = \frac{\alpha}{(x - x_{res} + i0)^2} \left(1 + (x - x_{res}) \frac{d}{dx}\right) \left(E_- + \frac{E_+}{(1 - \kappa)\mu_d}\right),$$  \hspace{1cm} (A1)

$$is^{1/3}E_x' = 2x_{res}E_r,$$  \hspace{1cm} (A2)

$$(1 - \kappa)\mu_d s^{1/3} E_y' = \frac{i\alpha}{(x - x_{res} + i0)^2} \left(1 + (x - x_{res}) \frac{d}{dx}\right)$$

$$\times \left(E_- + \frac{E_+}{(1 - \kappa)\mu_d}\right).$$  \hspace{1cm} (A3)

Eliminating $E_r$ from the last two equations and making use of Equation (43), one gets

$$E_x'' = -\frac{(1 - \kappa)\mu_d\alpha}{(x - x_{res} + i0)^2} \left(1 + (x - x_{res}) \frac{d}{dx}\right)$$

$$\times \left(E_- + \frac{E_+}{(1 - \kappa)\mu_d}\right),$$  \hspace{1cm} (A4)

thus reducing the system to two equations for $E_-$ and $E_+$. It follows immediately from Equations (A1) and (A4) that $E_x'' + (1 - \kappa)\mu_d E_x'' = 0$, which implies

$$E_+ + (1 - \kappa)\mu_d E_- = C_1 + C_2(x - x_{res}),$$  \hspace{1cm} (A5)

where $C_1$ and $C_2$ are constants. Now one can find the fields in the resonance region in the form

$$E_+ = Q + P(x - x_{res}) + \frac{\alpha[S(x - x_{res}) + T]}{(1 - \kappa)\mu_d} \ln(x - x_{res} + i0),$$  \hspace{1cm} (A6)

$$E_x = -T - (1 - \kappa)\mu_d Q + \frac{1}{2} [S - 2(1 - \kappa)\mu_d P] (x - x_{res})$$

$$- \alpha[S(x - x_{res}) + T] \ln(x - x_{res} + i0),$$  \hspace{1cm} (A7)

where $P$, $Q$, $S$, and $T$ are constants. These constants could be found by matching them at $|x - x_{res}| \sim 1$, with the solution outside of the resonance region.

It has been shown in Section 4 that at condition (72), the “longitudinal” component, $E_x$, satisfies the inequality (71) outside the resonance region. Inspecting Equations (A6) and (A7), one sees that this inequality is fulfilled at $|x - x_{res}| \sim 1$ provided

$$|T + (1 - \kappa)\mu_d Q| \ll (1 - \kappa)\mu_d Q,$$  \hspace{1cm} (A8)

$$|S - 2(1 - \kappa)\mu_d P| \ll (1 - \kappa)\mu_d P,$$  \hspace{1cm} (A9)
which yields

\[ T = -(1 - \kappa)u_d Q, \quad S = 2(1 - \kappa)u_d P. \quad (A10) \]

One immediately sees that in this case the inequality (71) is also satisfied within the resonance region with the exception of the negligibly small region \( |x - x_{\text{res}}| \lesssim \exp(-1/\alpha) \). Therefore, one can neglect \( E_+ \) in the resonance term (that on the right-hand side of Equation (68)) thus coming to the closed equation (73). This justifies the use of Equation (73) in order to find the reflection coefficient. Note that upon substituting Equation (A10) into the general resonance solution (A6), one comes to Equation (89) obtained as the solution to Equation (73).

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