Impact of dimensionless length scale parameter on material dependent thermoelastic attenuation and study of frequency shifts of rectangular microplate resonators

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Abstract. Quality factor is one of the decisive performance parameters of micro/nano plate resonators based sensors and filters. Various energy dissipation mechanisms limit the maximum attainable quality factor, and in micro/nano scales thermoelastic damping is a critical energy loss mechanism. When the devices are scaled down, in order to accurately model micro/nano scale resonators, non-classical elasticity theories like Modified Couple Stress Theory (MCST) are valid. In this paper, by including a length scale parameter ($l$) the size effects were incorporated in the analysis and thermoelastic energy dissipation and related attenuation were found to be diminished by selecting a dimensionless length scale parameter i.e. $l/h$. Thermoeelastic attenuation and frequency shifts related to thermoelastic damping in microplate rectangular resonators with five structural materials (SiC, polySi, diamond, Si and GaAs) were investigated with $l/h = 0$. The attenuation related to thermoelastic energy dissipation and frequency shifts were found to be minimum for polySi material. With the inclusion of size effect, the impact of dimensionless length scale parameter on thermoelastic attenuation of polySi based microplate resonators was also explored. The conventional thermoelastic damping analysis in rectangular plate was modified by applying Modified Couple Stress Theory and the impact of dimensionless length scale ($l/h$) on thermoelastic energy attenuation was simulated numerically using MATLAB 2015.

Keywords: Micro/nanoplate rectangular resonators, thermoelastic attenuation, frequency shifts, size effects, dimensionless length scale parameter.

Introduction

Micro and nanoelectromechanical systems (MEMS and NEMS) based thin rectangular plates are one of the major structures emerged as the essential components for sensors [1] and resonators [2]. Most of these applications exploited the physical and thermal properties of thin film materials for focused applications [2]. Micro/nano structures are universally used due to their unique advantages of high sensitivity, mass and cost, high surface-to-volume ratio etc. [3].

When resonators are used Quality Factor (QF) is one of the essential features to ensure high sensitivity and resolution and gives a representation of dissipated energy due to both energy loss
mechanisms- extrinsic and intrinsic [3]. Thermoelastic damping (TED) is a significant intrinsic energy loss mechanism at micro scales which decides the maximum attainable QF in the resonator denoted by $Q_{TED}$[4]. Thermoelastic damping limited energy attenuation arises due to the coupling between temperature and elasticity of the materials which declines QF. Presence of TED in different structures was identified in various research works [5]. The existence of TED as a prominent energy loss mechanism in homogeneous, isotropic, Euler–Bernoulli micro-beams was first identified by Zener in 1937 [6]. Lifshitz and Roukes studied the impact of different geometrical parameters on TED and derived an exact closed-form expression in slender beams [7]. Nayfeh and Younis expressed $Q_{TED}$ of microplates analytically in terms of their structural mode shapes by utilizing the perturbation method [8].

Classical theories are inadequate to explore the size effects due to the lack of the material length scale parameter; hence, higher order continuum theories must be used to predict the size dependencies. The size dependence effects of materials have been experimentally observed by scaling down the microstructure size [9]. Indeed, the size effect is essential in mechanical analyses of various structures [10]. To include the size effects in the vibration analysis, non-classical continuum theories are used, the most common is the Modified Couple Stress Theory (MCST) developed by Yang et al [11]. Tsiaitas et al. analysed isotropic micro-plates based on the Kirchhoff plate model and suggested using MCST for static analyses that need only one material length scale parameter, which is enough to capture the size effect [9]. Zhong et al. analysed thermoelastic damping and the factors affecting TED in the size-dependent microplate resonators based on modified couple stress theory [12]. Vahid Borjalilou et al. analysed TED in microplates based on MCST and the dual-phase-lag heat conduction model [13]. Resmi R. et al. explored thermoelastic damping dependent quality factor, thermoelastic frequency and figure of merit of rectangular plates applying modified couple stress theory [14].

In this paper, Section 2 gives the analytic equations of thermoelastic attenuation related to energy dissipation and frequencies of a rectangular microplate applying size effect based on MCST. Section 3. presents the results and discussions, where section 3.1. elaborates material dependent thermoelastic attenuation and frequency shifts. Section 3.2. discusses the impact of dimensionless length scale parameter on attenuation. The plots of the attenuation and frequency shifts for different materials are also illustrated in Section 3.1. The conclusions of the work are given in Section 4.

2. Expression for Thermoelastic Energy Attenuation and Frequency Shifts of a Rectangular Microplate

In classical elasticity theory, stress and strain energy is related to strain tensor and independent of the rotation vector. According to MCST, presented by Yang et al. [11] the strain energy density is a function of both strain tensor (conjugated with stress tensor) and curvature tensor (conjugated with couple stress tensor).

In our analysis, a rectangular microplate of length $L$, width $W$, and constant thickness $h$ under plane stress was analysed using five different materials at a temperature of $T_0 = 293K$. In the study, the coupled thermoelastic fields are assumed and the equations to have the simple-harmonic vibrations as [12]

$$W(x,y,t) = w_0(x,y)e^{i\omega t}, \quad \theta(x,y,z,t) = \theta_0(x,y,z)e^{i\omega t}$$ (1)

where $W$ indicates the mid-plane displacement of the microplate, $\theta$ is the temperature distribution in the microplate and $t$ is the time.

The motion equation of the microplate can be given by

$$\left[ P_0 + P_1 + \frac{P_2}{2} \dot{\omega} \right] (1 + f(\omega)) \nabla^4 W - \rho h \dot{W} = 0$$ (2)

where $f(\omega)$ is the complex function; $\dot{\omega} = E\alpha^2 T_0/C_p$ (E-Young’s modulus, $\alpha$-thermal expansion coefficient, $T_0$-equilibrium temperature, $C_p$ -specific heat capacity at constant pressure; $P$ is a parameter depends on material properties such as $E$ (Young’s Modulus), $\nu$ (Poisson’s ratio) and thickness of the microplate and the variables are $P_0 = Eh^3(1 - \nu)/(12(1 + \nu)(1 - 2\nu)$ and
\( P_1 = E \frac{h}{2} (1 + \nu) \).

In Eq. (2), \( P_0 \) \( T^4 W \) and \( \rho h \dot{W} \) are associated with the classical thermoelastic model; \( P_1 \) \( T^4 W \) represents the size effect and \( \frac{P_0}{2} \left( 1 + f(\omega) \right) \) denotes the effect of thermoelastic coupling.

When the microplate is simply supported, the modes of vibration (normal) of the microplate are

\[
\omega_0(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B \sin \frac{m \pi x}{L} \sin \frac{n \pi y}{W} \tag{3}
\]

where B is a constant, m and n are modal numbers.

For a simply supported plate [12],

\[
\pi^4 \left( \frac{m^2}{L^2} + \frac{n^2}{W^2} \right)^2 - \frac{\rho h \omega^2}{P'} = B e^{i \omega t} \sin \frac{m \pi x}{L} \sin \frac{n \pi y}{W} = 0 \tag{4}
\]

where \( P' = P_0 + P_1 + \frac{P_0}{2} \left[ 1 + f(\omega) \right] \).

For a microplate resonator, \( \pi^4 \left( \frac{m^2}{L^2} + \frac{n^2}{W^2} \right)^2 - \frac{\rho h \omega^2}{P'} = 0 \)

The thermoelastic frequency dispersion relation between \( \omega_1 \) and mode numbers (m, n) for the thermoelastic plate can be obtained as

\[
\omega_1 = \pi^2 \sqrt{\frac{P'}{\rho h \left( \frac{m^2}{L^2} + \frac{n^2}{W^2} \right)}} = \omega_0 \sqrt{1 + P_1 \frac{\Delta \rho_0}{P_0} \left[ 1 + f(\omega) \right]} \tag{6}
\]

where \( \omega_0 \) is the isothermal value of the eigenfrequency of the microplate,

\[
\omega_0 = \pi^2 \sqrt{P_0 / \rho h (m^2 / L^2 + n^2 / W^2)}.
\]

From Eq. (6), frequency is complex; the real part \( \text{Re}(\omega) \) indicates the new eigenfrequencies of the rectangular microplate resonators in the presence of thermoelastic coupling, and the imaginary part \( \text{Im}(\omega) \) gives the attenuation of the vibration.

The real and imaginary parts of the frequency can be extracted from Eq. (6)

\[
\text{Re}(\omega) = \omega_0 \left[ \sqrt{1 + P_1 / P_0} + \frac{\dot{\tau}}{4 \sqrt{1 + P_1 / P_0}} \times \left( 1 - \frac{6 \sinh \xi - \sin \xi}{\xi^3 \cosh \xi + \cos \xi} \right) \right] \tag{7}
\]

\[
\text{Im}(\omega) = \omega_0 \frac{\dot{\tau}}{4 \sqrt{1 + P_1 / P_0}} \left( \frac{6 \sinh \xi - \sin \xi}{\xi^3 \cosh \xi + \cos \xi} \right) ^{\frac{6}{\xi^2}} \tag{8}
\]

where \( \xi = b \sqrt{\nu t_0 / 2} \).

The amount of thermoelastic damping, expressed in terms of the inverse of the quality factor, is given by [12]

\[
Q^{-1} = 2 \left| \frac{\text{Im}(\omega)}{\text{Re}(\omega)} \right| \tag{9}
\]

3. Results and Discussions

To validate the analytical study, rectangular plate resonators with simply supported boundary condition, length \( L = 200 \mu m \), width \( W = 200 \mu m \), and thickness \( h = 10 \mu m \) using five different materials were examined at a temperature of \( T_0 = 298 K \). The mechanical and thermodynamic properties of all the materials are taken from [14]. Numerical simulations of the analytical expressions were done with MATLAB 2015.
3.1. Thermoelastic Attenuation and Frequency Analysis

The universal behaviour of the normalized frequency shift \( \frac{\text{Re}(\omega) - \omega_0}{\omega_0} \) and the normalized attenuation \( \frac{\text{Im}(\omega)}{\omega_0} \) as a function of the dimensionless variable \( \xi \) of a polySi based rectangular microplate is shown in Figure 1 (\( \xi = \frac{h}{\sqrt{\omega_0/2\chi}} \) where \( h \) - thickness of the plate; \( \omega_0 \) - isothermal value of the eigenfrequency of the microplate and \( \chi \) - thermal diffusivity of the material used). The normalized attenuation which is a measure of thermoelastic energy dissipation in microplate resonators, increases first, peaks at \( \xi = 2.2246 \), and then decreases again with the dimensionless variable \( \xi \), as depicted in Figure 1. Thermoelastic energy dissipation shows Lorentzian behaviour and is found to be tightly bound between two Lorentzians in the variable \( \xi^2 \) and varies as \( \xi^2 \) for small values of \( \xi \) and as \( \frac{1}{\xi^2} \) for large values of \( \xi \) [7].

Figure 1. Normalized attenuation and frequency shift with \( \xi \) for a polySi rectangular microplate resonator with simply supported boundary condition; \( T_0 = 298 \text{ K} \); mode \( (m, n) \) with \( m = 1 \) and \( n = 1 \); \( L = 200 \mu\text{m} \), width \( W = 200 \mu\text{m} \), and thickness \( h = 10\mu\text{m} \).

To study the effect of material parameters on attenuation and frequency shift, as a measure of the energy dissipation, \( \frac{\text{Im}(\omega)}{\omega_0} \) is plotted with the dimensionless variable \( \xi \), and the variation of TED for different materials is plotted in Figure 2. From the analysis SiC has the largest attenuation, while polySi has the lowest.

Figure 2. (a), (b), (c), (d), and (e) shows the variations of thermoelastic attenuation associated with thermoelastic energy dissipation and the frequency shifts with \( \xi \) for (a) polySi, (b) diamond, (c) Si, (d) GaAs, and (e) SiC. All five selected structural materials show a step for the normalized frequency shift, \( \frac{\text{Re}(\omega) - \omega_0}{\omega_0} \), which is a universal behaviour at \( \xi = 2.2246 \). The rectangular plate model analyzed in this work can be converted to the classical thermoelastic plate model by setting \( l = 0 \), where the size effect is not considered. When rectangular microplate resonators are considered, the energy dissipation peak occurs when the resonance frequency is in the order of its effective relaxation rate. Moreover, the attenuation increases with \( \xi \) for small values and decays for large \( \xi \) values as shown in Figure 2, which is applicable to all five structural materials selected.
Figure 2. Variation of attenuation and frequency shift with $\xi$ for rectangular microplate resonators with simply supported boundary condition; $T_0=298$ K; mode $(m,n)$ with $m=1$ and $n=1$; $L=200$ μm, width $W=200$ μm, and thickness $h=10$μm using five different structural materials (a) polySi, (b) diamond, (c) Si, (d) GaAs, and (e) SiC.

The material order in which attenuation and frequency shift diminishes with $\xi$ in our work is found to be in the order of SiC > GaAs > Si > diamond > polySi as shown in Table 1. As the frequency shift decreases, attenuation also declines and the quality factor gets enhanced.
Table 1. Attenuation and frequency shift of rectangular microplate resonators with simply supported boundary condition; mode (m,n) with m=1 and n=1; L= 200 μm, width W= 200 μm, and thickness h=10μm using five different structural materials

| Material | Attenuation | Frequency Shift |
|----------|-------------|----------------|
| PolySi   | 6.71E-05    | 1.31E-04       |
| Diamond  | 7.23E-05    | 1.43E-04       |
| Si       | 8.29E-05    | 1.68E-04       |
| GaAs     | 1.43E-04    | 2.89E-04       |
| SiC      | 1.84E-04    | 3.74E-04       |

3.2. Impact of Dimensionless Length Scale Parameter (l/h) on Thermoelastic Attenuation

As size effects are included in the study, the size-dependence of the material properties enhances the bending rigidity and hence the values of thermoelastic frequencies. By considering the material length scale parameter $l$, the normalized frequency shift $\frac{\text{Re}(\omega_0 - \omega_0')}{\omega_0}$ is elevated, and the normalized attenuation $\frac{\text{Im}(\omega_0 - \omega_0')}{\omega_0}$ is diminished more than that in the classical model. This demonstrates that the energy dissipation in the microplate resonators may have a large reduction due to the size effect. Figure 3 depicts the variation of attenuation without size effect (l/h=0) for different structural materials. The material order in which attenuation reduces with $\xi$ is in the order of SiC > GaAs > Si > diamond > polySi. When size effects are included, the peak damping diminishes with the material length scale parameter. It is thus confirmed that energy dissipation decreases with the dimensionless material length scale $l/h$, and, as a result, the size-dependent quality factor increases. Figure 4 illustrates the size effects i.e. $l/h=0,0.2,0.5$, and 1 when polySi is applied as the structural material. The thermoelastic damping limited quality factor increases with the dimensionless material length scale parameter ($l/h$) for microplate resonators with all boundary conditions and in all vibrating modes.

![Figure 3. Variation of attenuation and frequency shift with $\xi$ for rectangular microplate resonators with simply supported boundary condition; T0=298 K; mode (m,n) with m=1 and n=1; L= 200 μm; width W= 200 μm; and thickness h=10μm; l/h=0 using five different structural materials.](image-url)
Figure 4. Variation of attenuation and frequency shift with $\xi$ for a polySi based rectangular microplate resonator with simply supported boundary condition; $T_0=298$ K; mode $(m,n)$ with $m=1$ and $n=1$; $L=200$ μm, width $W=200$ μm, and thickness $h=10$ μm; $l/h=0, 0.2, 0.5, 0.8, 1$.

4. Conclusion

Various energy dissipation mechanisms limit the maximum attainable quality factor and in micro/nano scales, thermoelastic energy dissipation is a critical energy loss mechanism. When the devices are scaled down, in order to accurately model micro/nano scale resonators, non-classical elasticity theories like Modified Couple Stress Theory (MCST) are valid. By including a dimensionless length scale parameter ($l/h$), the size effects are incorporated in the analysis and thermoelastic energy dissipation and related attenuation were found to be diminished. In this paper, thermoelastic attenuation and frequency shift related to thermoelastic damping in microplate resonators were investigated without applying size effects ($l/h=0$). The material order in which thermoelastic attenuation diminishes is SiC > GaAs > Si diamond > polySi. The impact of dimensionless length scale parameter on thermoelastic attenuation was also explored with polySi as the structural material. Thin beams with properly selected structural material with length scale parameters vibrating in higher modes provide large $Q_{TED}$ and helps engineers to design microplate resonators with less energy dissipation and high quality factors.

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