Physical Meaning of the Current Vertex Corrections: DC and AC Transport Phenomena in High-$T_c$ Superconductors

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Abstract. Famous non-Fermi liquid-like behaviors of the transport phenomena in high-$T_c$ cuprates ($R_H$, $\Delta \rho / \rho$, $S$, $\nu$, etc) are caused by the current vertex corrections in nearly antiferromagnetic (AF) Fermi liquid, which was called the backflow by Landau. We present a simple explanation why the backflow is prominent in strongly correlated systems. In nearly AF Fermi liquid, $R_H$ is enhanced by the backflow because it changes the effective curvature of the Fermi surfaces. Therefore, the relaxation time approximation is not applicable to a system nearby a magnetic quantum critical point (QCP).

Keywords: transport phenomena, high-$T_c$ superconductors, current vertex correction, backflow

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In the present article, we explain that the backflow always plays important roles in Fermi liquids. In a Fermi liquid, excited quasiparticles (QP) interact with each other. Landau showed that the QP energy at $p$ in the presence of QP excitations is expressed as

$$\tilde{\epsilon}_p = \epsilon_p + \sum_k f_{p,k} \delta n_k,$$  

(1)

where $\epsilon_p = p^2/2m^*$ and $\delta n_k$ represents the deviation of the QP distribution function from the ground state. $f_{p,k}$ is the QP interaction (i.e., Landau interaction function) which arises from the electron-electron correlations. Hereafter, we consider the paramagnetic state.

First, we discuss an isotropic Fermi liquid. Here, we consider to add a QP just above the Fermi surface (FS) at $k$ as shown in Fig.1 whose lifetime $\tau$ is infinitesimally long. Then, the QP energy at $p$ shifts by $f_{p,k}$, which induces the change of QP velocity $\tilde{\epsilon}_p$ due to the backflow by Landau. In a spherical system, the QP velocity at $k$ and the backflow are given by $v_k = k/m^*$ and $\frac{1}{2}F_1 k/m^*$ ($F_1$ being a Landau parameter), respectively. Because $1 + \frac{1}{2}F_1 = m^*/m$ due to Galilei invariance, the total current $J_k$ is $k/m^*$. Thus, the backflow dominates the QP velocity in a strongly correlated Fermi liquid where $m^* \gg m$.

The backflow plays an important role in DC transport phenomena. To show this fact, we study the current vertex correction in the hydrodynamic limit ($\tau \ll 1$) based on the microscopic Fermi liquid theory. The resistivity of interacting electrons without umklapp scatterings should be completely zero, as consequence of the momentum conservation law. However, the relaxation time approximation (RTA) gives a finite resistivity because $\rho_{\text{RTA}} \propto \tau > 0$. Yamada and Yosida solved this discrepancy by taking the backflow (i.e., the current vertex correction) into account; they succeeded in reproducing $\rho = 0$ in the absence of the umklapp scatterings, even when $\tau$ is finite.

In 1999, we found that the backflow in nearly antiferromagnetic (AF) metals plays highly nontrivial roles; the total current $J_k$ is no more parallel to the QP velocity $v_k$ due to the backflow. The Bethe-Salpeter equation in the microscopic Fermi liquid theory is given by

$$J_k = \tilde{v}_k + \oint_{FS} dk || \Gamma(k,p) J_p,$$  

(2)

where $\Gamma(k,p)$ corresponds to the Landau interaction function in the hydrodynamic limit, and $k_0$ is the momentum along the FS. In the FLEX approximation, $\Gamma(k,p) = \int d\epsilon [\epsilon \theta(\epsilon/2T) - \epsilon \theta(\epsilon/2T)] \Im \nu_{k-p}^{\text{FLEX}}(\epsilon) \tau_\rho$ and $\nu_{k-p}^{\text{FLEX}}(\epsilon) \propto \chi_\epsilon'(\epsilon) \propto \xi^{-2}(1 + \xi^2(k-Q)^2 - i\epsilon/\omega_0)^{-1}$, where $\chi_\epsilon'$ is the AF correlation length and $Q = (\pm\pi, \pm\pi)$. In the SCR or FLEX approximation, $\xi^2 \propto T^{-1}$ and $\xi \gg 1$ at lower temperatures. Then, $\Gamma(k,p)$ takes a large value only when $|k-p-Q| \lesssim \xi^{-2}$. As shown in Fig.2 a QP excitation at $k$ causes the backflow through the QP interaction.
In summary, the highly anisotropic Landau interaction $J_{k}$ around $v_{k}$ only on the small portion of the FS in eq. (2), only on the small portion of the FS for the Hall conductivity is given by \[2\] Fig. 2 leads to the enhancement of $-\varepsilon_{\Gamma}$ in eq.(4). In fact, $\varepsilon_{\Gamma}$ above $k_{\parallel}$ of QP. Because $\varepsilon_{\Gamma}$ is determined by the FSC

$$C_{k}^{0}$$.

We show that the nontrivial behavior of $J_{k}$ shown in Fig. 2 leads to the enhancement of $R_{H}$. The expression for the Hall conductivity is given by

$$\sigma_{xy}^{\text{RTA}}/B_{c} = \left|\varepsilon_{\Gamma}^{0}\right|^{3} \int_{\text{FS}} k_{\parallel} \tau_{k} |C_{k}^{0}|, \quad \sigma_{xy}/B_{c} = \left|\varepsilon_{\Gamma}^{0}\right|^{3} \int_{\text{FS}} k_{\parallel} \tau_{k} |C_{k}^{0}|, \quad (3)$$

where $k_{\parallel}$ is the momentum parallel to the FS, $C_{k}^{0} = -\partial \theta_{k}/\partial k_{\parallel}, C_{k}^{0} = \theta_{k}^{0}, C_{k}^{0} = \theta_{k}^{0}, C_{k}^{0} = \tau_{k} = 1/2 \Im \Sigma_{k}(-\delta)$ is the lifetime of QP. Because $C_{k}^{0}$ represents the FS curvature (FSC) at $k$, we recognize the well-known result of the RTA: $\sigma_{xy}$ is determined by the FSC. However, this statement does not hold in nearly AF systems any more, because the effective FSC in eq.(4), $C_{k}^{0}$, strongly deviates from the true FSC. In Fig. 2, $C_{k}^{0} > 0$ around A whereas it is negative around B. In contrast, $C_{k}^{0} > 0$ everywhere. Moreover, the "cold spot" where $\tau_{k}$ takes the maximum value on the FS is around A (B) in the hole-doped (electron-doped) systems. As a result, $\sigma_{xy}$ is positive (negative) in hole-doped (electron-doped) systems, as recognized in eq.(4). In fact, $R_{H}$ in electron-doped systems is negative although its FSC observed by ARPES is positive. This fact cannot be explained by the idea, "deformation of the FS due to SDW transition", because $R_{H} < 0$ even above $T_{N}$. In Ref. [3], we solved this discrepancy by taking the fact that the effective FSC around the cold-spot becomes negative due to the backflow. We have shown that $C_{k}^{0} \propto \xi_{z}^{2}$ in both hole-doped and electron-doped systems in terms of the conserving approximation, so $R_{H}$ in under-doped system is strongly enhanced at lower temperatures in proportion to $T^{-1}$. Moreover, characteristic frequency and temperature dependences of the AC Hall coefficient are also reproduced very well by taking the backflow into account, as shown in Fig. 3.

In summary, the backflow is a natural consequence of the basic Fermi liquid equation, eq.\(\text{(4)}\), and it cannot be ignored in strongly correlated systems. The total current in nearly AF metals is no more perpendicular to the FS due to the backflow, because the Landau interaction function caused by the AF fluctuations has a prominent momentum dependence. This fact means that the RTA is totally broken down. The Hall coefficient shows a strong temperature dependence because the backflow changes the "effective FSC" around the cold spot. By taking the backflow into account, we succeeded in reproducing characteristic non-Fermi liquid-like behaviors in various DC and AC transport phenomena in high-$T_{c}$ cuprates, even in the pseudo-gap region [3-4].

REFERENCES

1. D. Pines and P. Nozieres, \textit{The Theory of Quantum Liquids}, Benjamin, New York, 1966.
2. H. Kontani and K. Yamada, J. Phys. Soc. Jpn. 74, 155 (2005), and references are therein.
3. H. Kontani et al., Phys. Rev. B 59, 14723 (1999).
4. H. Kontani, Phys. Rev. B 67, 014408 (2003).
5. H. Kontani, cond-mat/0507664, cond-mat/0511015.