Boundary element method solution for large scale cathodic protection problems

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Abstract. Cathodic protection techniques are widely used for avoiding corrosion sequences in offshore structures. The Boundary Element Method (BEM) is an ideal method for solving such problems because requires only the meshing of the boundary and not the whole domain of the electrolyte as the Finite Element Method does. This advantage becomes more pronounced in cathodic protection systems since electrochemical reactions occur mainly on the surface of the metallic structure. The present work aims to solve numerically a sacrificial cathodic protection problem for a large offshore platform. The solution of that large-scale problem is accomplished by means of “PITHIA Software” a BEM package enhanced by Hierarchical Matrices (HM) and Adaptive Cross Approximation (ACA) techniques that accelerate drastically the computations and reduce memory requirements. The nonlinear polarization curves for steel and aluminium in seawater are employed as boundary condition for the under protection metallic surfaces and aluminium anodes, respectively. The potential as well as the current density at all the surface of the platform are effectively evaluated and presented.

1. Introduction

Damage due to corrosion is the major material degradation factor for underwater and underground metallic structures. The most effective solution to that problem is the combination of painting and the installation of a sacrificial or impressed current cathodic protection (ICCP) system. Painting provides a protective coating that isolates the metal from the surrounding medium, while cathodic protection (CP) system facilitates the metallic structure with an external source of current that protects the areas where the paint is damaged and/or its adhesion with those metallic surfaces has been lost. The sacrificial CP system is a passive protection method where metals with a more negative electrochemical potential than the structure we have to protect are placed on the metallic surface playing the role of anode. On the other hand ICCP system can be considered as an active CP system since an external power supply is used as a current source so that to increase the potential level of the protective surface and thus minimizing corrosion.

In both cases of the aforementioned CP systems, their efficiency is based on the accurate evaluation of the electric potential and protection current density distribution on the surface of the protected metallic structure. This is a potential problem defined in an infinite space surrounding the metallic place where the electrochemical process of corrosion takes place. A robust numerical tool for solving such problems is the Boundary Element Method (BEM) [1-4]. Two remarkable advantages it offers as compared to the Finite Element Method are the reduction of the dimensionality of the problem by one
and its high solution accuracy. Thus, although the electrochemical process occurs at the metallic surface and the surrounding electrolytic medium, the just described CP problems can be solved by discretizing only the under protection metallic surface. However, despite its advantages, the brutal application of BEM to large-scale problems suffers from very time-consuming computations and high demands for computer memory capacity. On the other hand, the design of an efficient CP system demands to model the exact geometry of very large metallic structures, optimize the anodes and impressed current source location, examine the interference with nearby CP systems, predict the life span of the installed CP system, evaluate the efficiency of the CP system in different environments, simulate the transient dynamic response of the CP system under working condition and check the performance of CP system under different damage scenarios [5]. Consequently, the need for an advanced BEM that is able to solve quickly and economically large-scale cathodic protection engineering problems is apparent.

In the present work, a large-scale problem dealing with the protection of an offshore oil steel platform via a sacrificed CP system is treated with the aid of PITHIA [6], an advanced BEM package able to solve large scale potential problems by exploiting hierarchical matrices and adaptive cross approximation technics in conjunction with GMRES iterative solver that accelerates the solution process. The methodology is explained in brief in the section after next and demonstrated in the last section where the abovementioned CP problem is solved and presented.

2. Description of the CP problem
Consider a closed metallic surface \( S \) surrounded by an infinitely extended, homogeneous electrolyte and a large sphere with surface \( S_{\infty} \), where the system structure-electrolyte is embedded (Figure 1). The surface \( S \) consists of the area of anode \( S_c \), the area of cathode \( S_a \) and the painted surface \( S_p \), while the three dimensional (3D) domain of the electrolyte is denoted by \( \Omega \).

![Figure 1. Illustration of a sacrificed CP system.](image)

The electric potential field \( \varphi \) developed in the electrolyte satisfies the Laplace equation [7]:

\[
V^2\varphi = 0
\]  

with \( V \) denoting the gradient operator.
The solution of Equation (1) at a field point \( \mathbf{x} \) admits an integral representation of the form [7]:

\[
c(\mathbf{x})\varphi(\mathbf{x}) + \int_{\partial \Omega} \partial_n G(\mathbf{x}, \mathbf{y}) \varphi(\mathbf{y}) dS_y = \int_{\partial \Omega} G(\mathbf{x}, \mathbf{y}) \partial_n \varphi(\mathbf{y}) dS_y
\]  

(2)

where \( S = S_a \cup S_p \cup S_c \cup S_0 \), \( \partial_n = \hat{n} \cdot \nabla \) is the directional derivative with respect to the normal unit vector \( \hat{n} \) defined at all surfaces and externally to domain \( \Omega \), \( \mathbf{y} \) represents source points lying across the boundaries of the problem, \( c(\mathbf{x}) \) is a coefficient obtaining the values 1 and 0.5 when point \( \mathbf{x} \) lies in space \( \Omega \) and its boundaries, respectively and \( G(\mathbf{x}, \mathbf{y}) \) stands for the fundamental solution of 3D Laplace Equation (1) written as

\[
G(\mathbf{x}, \mathbf{y}) = \frac{1}{4\pi r}
\]

\[
\partial_n G(\mathbf{x}, \mathbf{y}) = \frac{1}{4\pi r^3} \hat{n} \cdot \mathbf{r}
\]  

(3)

At the boundaries of domain \( \Omega \), three types of boundary conditions can be assigned. At the perfectly painted surfaces the current density is vanished, while for the exposed metal surfaces, which play the role of a cathode in order to be protected, a nonlinear Robin boundary condition is imposed, usually derived experimentally. At the anodic surface \( S_a \) the potential can be considered either constant and known for standard material properties and geometry or can be treated as an anodic material obeying to a Robin nonlinear relation between potential and current density, as in the case of cathodic surfaces. For the case of sacrificial CP system that utilises aluminium anodes to protect steel in seawater, the nonlinear polarization curves are depicted in Figure 2 [8].

![Figure 2. Polarization curves for steel and aluminum anodes in sea-water [8].](image)

For the problem of Figure 1, the just described boundary conditions read:

\[
\dot{i}(\mathbf{x}) = -\sigma \partial_n \varphi(\mathbf{x}) = 0, \quad \mathbf{x} \in S_p
\]  

(4)
where \( \sigma \) is the conductivity of seawater, \( i \) is the current density and \( g, f \) represent the upper branch of the blue polarization curve and the lower branch of steel polarization curve shown in Figure 2, respectively.

The boundary condition (7) implies that the constraint equation

\[
\int_{S_\ell \cup S_\ell \cup S_o} \partial_n \varphi(x) dS_i = 0
\]

which expresses the conservation of charge on the considered structure, is automatically fulfilled.

3. BEM solution

In the present section, an advanced BEM used for the numerical solution of the above-described cathodic protection problem is described in brief. To this end the boundaries of the problem of Figure 1 are discretized into 3-noded triangular or 4-noded quadrilateral linear elements. Then the integral Equation (2) written for the surface node \( i \), obtains the discrete form:

\[
\frac{1}{2} \varphi_i + \sum_{e} \sum_{a} H_{ea} \varphi_a - \sum_{a} G_{ea} q_a
\]

where \( e, a \) denote the number of elements and nodes per element, respectively, \( \varphi \) is the potential flux, while the coefficients \( H_{ea}, G_{ea} \) have the form:

\[
H_{ea} = \int_{-1}^{1} \int_{-1}^{1} \partial_n G \left( x', y', \xi_1, \xi_2 \right) \mathcal{N} \left( \xi_1, \xi_2 \right) \mathcal{J} \left( d\xi_1 d\xi_2 \right)
\]

\[
G_{ea} = \int_{-1}^{1} \int_{-1}^{1} G \left( x', y', \xi_1, \xi_2 \right) \mathcal{N} \left( \xi_1, \xi_2 \right) \mathcal{J} \left( d\xi_1 d\xi_2 \right)
\]

where \( \mathcal{N} \left( \xi_1, \xi_2 \right) \) represents the functions used for the interpolation of potentials and potential fluxes in a local coordinate system \( \xi_1, \xi_2 \), while \( \mathcal{J} \) stands for the Jacobian of the transformation from the global Cartesian coordinate system to the local one.

Collocating Equation (9) at all nodes one obtains the following system of algebraic equations:

\[
[H] \cdot [\varphi] = [G] \cdot [q]
\]

where the elements of matrices \([H]\) and \([G]\) are integrals of the form of Equations (10) and (11), respectively, evaluated numerically via highly accurate, direct integration techniques illustrated in [9], while the vectors \([\varphi]\) and \([q]\) contain all the known and unknown nodal values of potential and potential fluxes, respectively.
Finally, applying a Newton-Raphson iterative procedure for the nonlinear boundary condition (6) and satisfying all the boundary conditions of the problem, Equation (11) can be written as

\[ |A|x = b \]  \hspace{1cm} (13)

with the vectors \( x \) and \( b \) containing all the unknown and known nodal values of the CP problem, respectively.

The just described conventional BEM formulation produces full populated and non-symmetric collocation matrices \( A \) increasing thus the computational cost and confining the method to the solution of relatively small problems with no more than 100,000 degrees of freedom (DOFs). A very efficient method that circumvents that problem and accelerates drastically the solution process of a BEM code is that of the Fast Multipole BEM (FM/BEM). The FM/BEM accelerates the computation of matrix \( A \) of the final system of algebraic equations \( A|x = b \) to \( O(N) \) operations and reduces the memory requirements to \( O(N) \) as well. This is accomplished, first, by analytically expanding the fundamental solutions of the differential operator of the problem around the centres of a hierarchical cell structure through fast multipole expansions, second, by performing analytical integrations utilizing constant elements and third, by using an iterative solver (e.g. GMRES) instead of a direct one. The use of FM/BEM for the solution of elastic and acoustic problems is explained in the book of Liu [7].

An alternative to the FM/BEM is the use of a BEM enhanced by Hierarchical Matrices (HM) and Adaptive Cross Approximation (ACA) techniques that accelerate drastically the computation of matrix \( A \) and also reduce the memory requirements. That acceleration is possible due to the nature of the fundamental solutions, which are functions of the distance between the source and field points and thus only a small number of elements of the collocation matrix \( A \) are calculated, while the rest of them are approximated via the already evaluated elements. According to ACA/BEM, the matrix \( A \) is organized into a hierarchical structure of blocks depended on the geometry of the problem. Applying a geometrical criterion the blocks are characterized either as non-admissible, where the ACA algorithm is inefficient and thus the conventional BEM is employed or admissible where ACA is effective and is used to calculate only a small number of their rows and columns. Each admissible block is represented in a low rank matrix format via the product of two matrices formed by the previously calculated rows and columns, respectively. This low rank format in conjunction with an iterative solver, like GMRES, leads to significant reductions in memory requirements and CPU time due to the acceleration of the matrix vector multiplication.

Table 1 presents the gain coming from the application of ACA/BEM to large-scale elastic problems solved by Gortsas et al. [10]. Although the FMM/BEM seems to be faster than ACA/BEM, it appears some disadvantages as it is compared to the ACA/BEM, namely, it requires the knowledge of the multipole expansions of the fundamental solution of the problem and significant and complex modifications in a conventional BEM code in order to be implemented. On the other hand, ACA/BEM is a black box algorithm applied upon the collocation matrix \( A \) and thus its implementation is the same regardless of the differential operator of the problem. More details one can find in the work of Gortsas et al. [10].

Table 1. CPU time and memory requirements for conventional BEM and ACA/BEM appearing in [9]

| Degrees of Freedom x 10^6 | Memory requirements (GB) | CPU time (sec) |
|--------------------------|--------------------------|----------------|
|                          | Conventional BEM | ACA/BEM | Conventional BEM | ACA/BEM |
| 0.1                      | 78                 | 2       | 30000            | 2000   |
| 0.5                      | -                  | 23      | -                | 52000  |
| 1.0                      | -                  | 49      | -                | 11700  |
| 1.2                      | -                  | 68      | -                | 14500  |
Depending on the problem, ACA/BEM can be applied either to the final matrix $[A]$ or to the matrices $[H]$ and $[G]$ of Equation (12) and before the satisfaction of the boundary conditions of the problem. The later solution is adopted in the present work because of the nonlinear nature of the boundary conditions (5) and (6), where a Newton-Raphson iterative procedure is applied according to the scheme:

$$\varphi_a(x) = \varphi_0$$

$$\partial_\alpha \varphi_a^k(x) = \frac{1}{\alpha} \tilde{g}(\varphi_a^{k-1}(x)) + \frac{1}{\alpha} \frac{\partial \tilde{g}(\varphi_a^{k-1}(x))}{\partial \varphi} \Delta \varphi_a^k(x) \quad x \in S_a \quad (14)$$

$$\partial_\alpha \varphi_c^k(x) = \frac{1}{\alpha} \tilde{f}(\varphi_c^{k-1}(x)) + \frac{1}{\alpha} \frac{\partial \tilde{f}(\varphi_c^{k-1}(x))}{\partial \varphi} \Delta \varphi_c^k(x) \quad x \in S_c \quad (15)$$

where $k$ is the step where the boundary element method machinery is applied, while $\tilde{g}$, $\tilde{f}$ represent the best fitting functions of the polarization curves $g$ and $f$, respectively.

4. Numerical solution

In order to demonstrate the applicability of the above described ACA/BEM to CP engineering problems, a typical offshore oil steel platform, galvanically protected by a number of sacrificial anodes, is analysed. The structure of the platform is a frame consisted by hollow circular columns and beams, shown in Figure 3. The dimensions of the platform are (55m) x (36m) x (41m), with 47m of its height being immersed in the seawater. Thus, the total surface of the platform lying below the waterline is 3694 m² and 75 identical sacrificial anodes are utilized for its corrosion protection. Sacrificial anodes are rectangular parallelepiped in shape with dimensions (4m)x(0.3m)x(0.3m) and total surface area of 4.98m², mounded at columns and beams as it is shown in Figure 4. The conductivity of the seawater was taken equal to $4 \ \Omega^{-1}m^{-1}$, while on the surface of the anodes first the constant potential of -1.1V has been assigned and next the polarization curve of aluminium anodes has been considered. The immersed metallic surface of the platform is treated as a cathode obeying to the polarization curve of bare steel in sea-water depicted in Figure 2.

According to the BEM described in the previous section, only the surface of the electrolyte touching the outer surface of the frame, is needed to be discretized. Additionally, in order to ensure the conservation of charge on the structure, the semi-infinite medium of the problem is bounded by a large semi-sphere, with a radius of 100 m. The surface of the semi-sphere is also discretized and a zero current density boundary condition is imposed. The surface of the electrolyte being in contact with the platform is discretized by 20594 eight-noded quadratic quadrilateral and six-noded quadratic triangular elements, shown in Figure 4, while for the interpolation of both potential and current density fields, linear four-noded quadrilateral and three-noded triangular elements has been employed, thus resulting to a problem with 24716 DOFs.
Figure 3. Geometry and dimensions of the offshore oil steel platform analysed in the present work.

Figure 4. Placement of rectangular parallelepiped sacrificial anodes for the protection of the platform of Figure 3.
Figure 5. Boundary element discretization of the electrolyte surface being in contact with the outer surfaces of the platform, used for the ACA/BEM solution of the exterior potential CP problem.

For comparison reasons, the problem has been solved via both the conventional BEM and the ACA/BEM version of PITHIA package. The solution of the problem via the ACA/BEM code was accomplished in 35 minutes, almost 50 times faster than in the case of conventional BEM code. The difference here between conventional BEM and ACA/BEM is more pronounced than that of Table I due to the iterations required for the treatment of the nonlinear boundary condition valid at the cathodic surface of the platform.

The calculated potential and current density fields on the sunken platform surface are depicted in Figures 6 and 7, respectively. The minimum potential value of -0.55V and the maximum value of -0.2V are observed at platform points being very close and away from the considered anodes, respectively. On the other hand, the current density varies in the range of 2 mA/m² to 50 mA/m². Although the above CP problem has been solved for demonstration purposes, from engineering point of view the examined platform is not protected adequately since the steel in seawater is protected when it is polarized below to -0.58V. Thus, the jacket of seventy five sacrificial anodes is not enough and a larger number of distributed anodes, perhaps two anodes per beam/column, should be tested.

Figure 6. Potential distribution (V) at the submerged surface of the metallic platform
5. Conclusions

PITHIA, an advanced Boundary Element Method (BEM) enhanced by Adaptive Cross Approximation (ACA) and Hierarchical Matrices (HM) techniques, has been employed for the solution of a potential problem dealing with the sacrificial Cathodic Protection (CP) of a large metallic oil-platform. The nonlinear polarization curve for steel in sea-water was employed as boundary condition for the metallic surfaces we need to protect. Due to the nonlinear form of polarization curve the problem has been solved iteratively via a Newton-Raphson scheme. The solution process of the final BEM system of linear algebraic equations was accelerated via an iterative GMRES solver and the potential as well as the current density at all the surface of the platform were effectively evaluated and presented. Due to the efficiency of the utilized ACA/BEM, the problem was solved 50 times faster than in the case of using conventional BEM, while the gain in storage memory was significant so that the problem to be solved in a desktop PC with 64 GB RAM. The convergence, the efficiency and the accuracy of the obtained results indicate that PITHIA is a robust numerical tool for simulating large-scale CP problems.

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