Abstract

Monitoring and managing Earth’s forests in an informed manner is an important requirement for addressing challenges like biodiversity loss and climate change. While traditional in situ or aerial campaigns for forest assessments provide accurate data for analysis at regional level, scaling them to entire countries and beyond with high temporal resolution is hardly possible. In this work, we propose a Bayesian deep learning approach to densely estimate forest structure variables at country-scale with 10-meter resolution, using freely available satellite imagery as input. Our method jointly transforms Sentinel-2 optical images and Sentinel-1 synthetic aperture radar images into maps of five different forest structure variables: 95th height percentile, mean height, density, Gini coefficient, and fractional cover. We train and test our model on reference data from 41 airborne laser scanning missions across Norway and demonstrate that it is able to generalize to unseen test regions, achieving normalized mean absolute errors between 11% and 15%, depending on the variable. Our work is also the first to propose a Bayesian deep learning approach so as to predict forest structure variables with well-calibrated uncertainty estimates. These increase the trustworthiness of the model and its suitability for downstream tasks that require reliable confidence estimates, such as informed decision making. We present an extensive set of experiments to validate the accuracy of the predicted maps as well as the quality of the predicted uncertainties. To demonstrate scalability, we provide Norway-wide maps for the five forest structure variables.

1. Introduction

Forest structure relates to the three-dimensional (3D) spatial arrangement of the plant community within a forest and is both a result and a driver of ecosystem processes and biological diversity (Spies, 1998). Forest structure is strongly correlated with a forest’s ability to store carbon and to provide habitat for a variety of species (Turner et al., 2003; Bergen et al., 2009; Dubayah et al., 2020). Given the large extent and dynamic nature of forest ecosystems, it is important to develop ways of mapping and monitoring their structure consistently through space and time (Valbuena et al., 2020). Such maps can support a more informed and dynamic management of forest resources, required to tackle important global challenges such as the loss of biodiversity; and the mitigation of, as well as adaptation to climate change.

Measuring forest structure has long been an expensive and time-consuming task. Traditional in-situ methods are limited to the recording of a few tree characteristics measurable from the ground (e.g., diameter at breast height, tree species, to some degree tree height), and restricted to small sample plots sparsely scattered across the landscape. Consequently, in-situ measurements are not sufficient for a spatially explicit understanding of forest structures. To collect additional, explicit forest structure measurements, field direct observations can be complemented with terrestrial laser scanning (TLS). Still, TLS in practice only yields local samples, as it has a limited range and is affected by occlusions (Calders et al., 2020). In contrast, airborne laser scanning (ALS) allows one to densely observe forest structure at regional scales. Forest structural variables derived from ALS, such as canopy height, cover, or density, have long been used for the wall-to-wall characterization of single-tree (Hyppa et al., 2001) and forest biophysical properties (Nilsson, 1996; Næsset, 2002). But the high operational cost, which scales more or less linearly with flight time, limits the coverage and revisit time of ALS campaigns and is a bottleneck for country-scale applications.

The natural next step to scale beyond local and single-date ALS campaigns is to take advantage of the wealth of open data from Earth observation satellites. A prominent example is the European Union’s Copernicus program, with multiple satellite missions whose data products are useful to predict forest structure dynamics (Puliti et al., 2021). In particular, the Sentinel-1 (Synthetic-aperture radar, SAR) and Sentinel-2 (multi-spectral optical) missions provide near-global coverage, with both high spatial resolution
In the absence of a tractable radiative transfer model, supervised machine learning offers a statistical approach to retrieve environmental parameters from image data. In recent years, deep neural networks (DNNs) have emerged as the mainstream tool for image analysis, including their applications in remote sensing and, in particular, mapping of forest structure (Lang et al., 2019). DNNs have brought substantial performance gains through their ability to learn the complete functional mapping from raw image values to the desired output variables. While DNNs often achieve high precision, their outputs are typically limited to point estimates and/or are overly confident (Guo et al., 2017; Gast and Roth, 2018). As well-calibrated estimates of uncertainty are important whenever the model outputs feed into critical decisions or probabilistic models, the Bayesian interpretation of model outputs is an active line of research (Blundell et al., 2015; Gal and Ghahramani, 2016; Guo et al., 2017; Kendall and Gal, 2017; Lakshminarayanan et al., 2017).

In this paper, we go a step further and exploit recent advances in Bayesian deep learning (BDL) to densely map forest structure variables and their associated uncertainties from optical and SAR satellite images. Our model predicts an individual per-pixel mean and variance (i.e., a probability distribution) for every variable of interest. BDL can be understood as the deep learning counterpart to traditional Bayesian inference techniques, for instance those for linear regression (Rasmussen, 2004, Ch. 2 to 2.1.1) or Gaussian process regression (Rasmussen, 2004, Ch. 2.1.2 to 2.3). Instead of producing a point estimate of the target variable, one aims to take into consideration the whole range of possible models and (approximately) marginalizes over the corresponding posterior distribution of model parameters. Such a principled approach is particu-
ularly attractive for data-driven models, where the model uncertainty is not a consequence of explicit modelling decisions, but instead is caused by the limited training data. Bayesian modelling may improve the predictive accuracy, but more importantly it can give the user access to well-calibrated estimates of the predictive uncertainty, which take into consideration how well a given model prediction is supported by training data. This ability to self-diagnose the reliability of each individual prediction is indispensable for many downstream tasks that ingest the model output as their input. As an example, when making informed decisions, it is crucial to know how reliable the data are that the decisions are based on. Uncertainty estimates have been successfully used for decision making in many fields of application (Council, 2006; Soroudi and Amraee, 2013; Martin and Johnson, 2019; Sniashko, 2019; Alzate-Mejía et al., 2021), and we argue that they are equally relevant in forest management in order to optimize decisions for cost, carbon stock, biodiversity etc.

Our method (see overview in Fig. 1) is fully supervised and thus consists of a training and a testing phase. During the training phase, a neural network model is fitted to training data, i.e., optical and SAR images with associated, known pixel-wise forest structure reference values computed from ALS data. The fitting employs a loss function that does not just penalize deviations between model outputs and ALS reference data. Instead, the loss minimizes the negative log posterior probability of the model parameters, assuming a zero-mean normally distributed prior that acts as a regularizer. Importantly, for each pixel and variable, the model outputs the parameters of a Gaussian distribution, thereby capturing uncertainty inherent in the input data (aleatoric uncertainty). During the testing phase, multiple trained models are combined into a model ensemble, exploiting the stochastic nature of neural network initialization and training. The predicted distributions of all models are aggregated into an ensemble estimate for the distribution over structure variables. The aggregation of predictions from independently trained models corresponds to a sample-based approximation of Bayesian marginalization. It captures the uncertainty in the model parameters (epistemic uncertainty), which is combined with the aleatoric uncertainty to obtain the overall uncertainty of the predicted forest structure variables.

In this work, we propose a Bayesian deep learning approach to predict structural forest variables 95th height percentile, mean height, density, Gini coefficient, and fractional cover, using optical and SAR satellite images as input. Our method makes it possible to densely retrieve forest structure with 10-meter ground sampling distance (GSD) at country-scale and with a high update frequency (in the extreme case down to five days), at a low cost. Moreover, the estimated forest structure maps come with an individual, calibrated uncertainty estimate for every structure variable at every single pixel. We conduct extensive experiments on a test region in Norway, for which reference data from a full-waveform ALS campaign is available. We then apply our method to compute a country-wide forest structure map for all of Norway, that we make available online. Source code for training and testing the model is provided, too.

2. Related Work

2.1. Remote sensing of structural forest variables

Since its early days, ALS research has seen a tremendous growth, and forest structural variables derived from ALS are broadly used as a source of auxiliary information for the large scale characterization of forest ecosystems. Many different types of modelling techniques have been tested over the years, most of which rely on linking field measured forest biophysical properties (e.g. biomass) with forest structural predictor variables derived from ALS, like height percentiles, vegetation density, forest cover, or foliage height diversity (Næsset et al., 2004; Næsset, 2007). More recently, there has been a trend to directly use ALS-derived structural variables to map forest structural (Coops et al., 2016; Valbuena et al., 2017; Adnan et al., 2021) and functional diversity (Schneider et al., 2017; Zheng et al., 2021). While ALS data are the most detailed source of information to characterize forest structure, their geographic and temporal availability remains limited.

If the goal is frequent, or even continuous, monitoring of large forest areas, a more promising data source are freely available satellite images, such as those provided by the Landsat and Copernicus programs. Landsat-based approaches have mostly been relying on time series features to map forest structure (Tyukavina et al., 2015; Hansen et al., 2016; Potapov et al., 2019, 2021). Also Copernicus’ Sentinel-1 and Sentinel-2 missions, with 10 m GSD and less than five days revisit time, offer dense time series of observations that are suitable for forest monitoring. Previous studies have demonstrated the usefulness of Sentinel data to map and estimate key forest biophysical variables (e.g., above ground biomass) (Laurin et al., 2018; Puliti et al., 2020; Breidenbach et al., 2021) and their dynamics through time (Puliti et al., 2021). In the past two years, a growing body of literature has shown the possibility to map forest canopy height with Sentinel-2 (Lang et al., 2019; Shimizu et al., 2020; Astola et al., 2021; Lang et al., 2021). Since canopy height is only one of many ecosystem characteristics that can be obtained from ALS, our work aims to understand whether Sentinel data can provide maps for a more comprehensive spectrum of structural variables, like vegetation density, cover, and complexity. Like Lang et al. (2019) our model learns to extract predictive texture features from single input images.

2.2. Deep learning in remote sensing

In the last decade, deep learning has revolutionized the way information is extracted from images. In particular, convolutional neural networks (CNNs) have achieved
unprecedented results in areas like image classification [Krizhevsky et al., 2012] Simonoyan and Zisserman [2014] He et al. [2015], semantic segmentation [Long et al., 2015] Chen et al. [2016], object detection [Szegedy et al., 2013] Girshick et al. [2014] Redmon et al. [2015] and further perception tasks.

In addition, deep learning is increasingly being applied to vision tasks in remote sensing such as super resolution [Lanaras et al., 2015] or change detection [Caye Daudt et al., 2018]. Traditional applications include land cover classification from aerial [Kaiser et al., 2017] Marmans et al. et al. 2018] or satellite images [Helber [2018]. Agricultural crop type classification, a specific type of land cover, is a well-studied task, where most authors exploit features from time-series data [Rußwurm and Körner, 2018] or satellite images [He et al., 2015]. More related to our work are methods that seek to predict biophysical indicators such as crop yield from climate data and Enhanced Vegetation Index maps [Kuwata and Shibasaki, 2015], the prediction of sea ice concentration from RADARSAT synthetic aperture radar (SAR) satellite images [Wang et al., 2016] or tree density estimation from Sentinel-2 optical images [Rodriguez and Wegner, 2018] Rodriguez et al. [2021]. The prior work most relevant for our paper is Lang et al. [2019], where a CNN was developed to predict country-wide vegetation height maps from Sentinel-2 optical images. Their model was trained and evaluated for Gabon and Switzerland, where training data was derived from LiDAR measurements and photogrammetric surface reconstruction, respectively. Here, we predict five forest structural variables jointly and estimate their predictive uncertainties with a Bayesian deep learning approach. Moreover, we explore the potential of data fusion using Sentinel-1 SAR images as an additional input signal on top of the Sentinel-2 optical bands.

2.3. Uncertainty in deep learning

Although deep learning has become the most-widely used paradigm in computer vision, deep learning models have the disadvantage of being overconfident about their predictions [Guo et al., 2017] Lakshminarayanan et al. [2017]. To mitigate this effect and to develop more trustworthy models, reliably quantifying the predictive uncertainty is important. However, uncertainty estimation in deep learning is challenging [Gustafsson et al., 2020]. Usually, uncertainty in machine learning is decomposed into aleatoric and epistemic uncertainty [Gal and Ghahramani, 2016]. The former is assumed to be inherent in the observations – e.g. resulting from sensor noise or lack of signal – and can thus not be “explained away” with more training data. On the contrary, epistemic uncertainty captures uncertainty in the model itself, i.e., the lack of knowledge due to not having seen enough training data for the respective region of the input space. Unlike aleatoric uncertainty, it cannot simply be learned from data – instead, it generally requires marginalization over an (approximate) posterior distribution over model parameters. These methods are colloquially referred to as Bayesian deep learning [Kendall and Gal, 2017]: common approaches include variational Bayesian inference [Ranganath et al., 2013] Blundell et al. [2015] and Markov chain Monte Carlo (MCMC) sampling [Welling and Teh, 2011] Chen et al. [2014]. In this work, we use a deep ensemble [Lakshminarayanan et al., 2017], an approximate Bayesian inference method specific to deep neural networks. The general idea is to train an ensemble of M independent models on the same data, each initialized with a different set of random weights. The randomness inherent in the weight initialization, as well as random sampling of training batches, causes each model to converge to a different local minimum in the solution space, and the resulting weights can be interpreted as samples from an approximate posterior distribution [Gustafsson et al., 2020] Wilson and Izmailov [2020]. In practice, among all approximate Bayesian methods for deep learning, ensembles are generally reported to perform best in terms of predictive performance and reliability of the produced uncertainty estimates [Ovadia et al., 2019] Gustafsson et al. [2020] Ashukha et al. [2020].

3. Data

3.1. ALS structural forest variables

In this study, the target variables are forest structural variables normally extracted from ALS data. In Norway, the corresponding ALS data is available as part of a national program aimed at the production of a high resolution digital terrain model (DTM) of the entire country. Amongst the many available ALS projects, we selected a sample of 41 projects covering a range of latitude (58°N – 69°N), longitude (5°E–18°E), and the corresponding diversity in landscapes and forest types in Norway. Our selected area is covered by the following main forest types:

- East: productive boreal forest in the south eastern part, characterized by mild slopes and continental climate.
- North: predominantly deciduous areas with a large portion of low-productivity mountain birch (Betula nana) in the north. These forests represent the transition from subpolar oceanic vegetation towards the tundra and sub-artic climates.
- West: deciduous forest alternating with patches of coniferous forests and un-productive forests in the coastal areas. These areas are characterized by a milder oceanic climate that, due to the steep slopes (i.e., fjords), can transition to tundra and alpine vegetation within short distances.

The selected samples of ALS data were collected between 2015 and 2018, with ≈80% being from the 2016 - 2017 period. For ≈35% of the area, the ALS data were collected under leaf-off conditions (Oct.-Dec.). The point
density in the selected ALS projects was 2 pts./m² for 51% of the area, 5 pts./m² for a further 48%, and 10 pts./m² for the remaining 1%. Overall, the sample covers a broad range of climatic, vegetation, terrain, and ALS data characteristics, thus providing a suitable training set for deep learning models that shall generalize across a range of conditions.

The height values $z$ (in meters above sea level) from the raw ALS point clouds were normalized to $D_z$ (in meters above ground) by subtracting the ground elevation from each point’s $z$ value. Furthermore, all points with $D_z > 1.3$ m were classified as vegetation points. We extracted a selection of commonly used ALS variables to describe forest structure and ultimately ecosystem characteristics (Valbuena et al., 2020): height, density/cover, complexity, and habitat area. These variables were selected with the aim to minimize the number of variables while maintaining complementary information on the vertical and horizontal distribution of the forest canopy. The following variables – illustrated in Fig. 4 – were computed for the entire area where ALS was available:

- **P95** (m above ground): the 95th percentile of the $D_z$ values of all vegetation points. This variable is close to the maximum height but it removes potential noise from spurious high ALS returns (e.g., from birds or power lines). The use of the height percentiles dates back to the some of the first area-based forest inventories (Næsset, 2002). Among them, the 95th percentile is useful as a measure of canopy top height, which is known to be correlated with the developmental stage of the forest and hence its biomass stock.

- **MeanH** (m above ground): mean of the $D_z$ values of vegetation points. This measure is of interest because it not only captures the height of a forest but also the vertical distribution of the plant material within the canopy. We note that, thanks to its ability to encompass these two levels of information, MeanH is at present the only predictor variable used to produce country-wide maps of forest biomass in Norway (Astrup et al., 2019).

- **Dens** (%): proportion of vegetation points in the entire set of LiDAR returns. This variable describes the density of the forest canopy and, along with the height percentiles, has long been used in area-based ALS forest inventories. Its is complementary to the canopy height, as it provides information about the vertical distribution of the plant material through the canopy (Næsset, 2002).

- **Gini** (index): The Gini coefficient is equal to half of the relative mean difference in $D_z$ values among all the height values. This measure is of interest because it not only captures the height of a forest but also the vertical distribution of the plant material within the canopy.
the vegetation returns, and was calculated using the function implemented in the leafR package (Alves de Almeida et al., 2020). The Gini coefficient is a measure of the inequality among the members of a data distribution, and it has been used as a proxy for tree size variation (Knox et al., 1989) and to map differences in forest structures and management regimes (Valbuena et al., 2017; Adnan et al., 2019). While typically the Gini coefficient has been calculated using single-tree data, a recent work by (Adnan et al., 2021) demonstrated the usefulness of the Gini coefficient calculated from the ALS Dz values, showing that it could reliably describe the structural heterogeneity of the forest.

• Cover (%): Forest cover in terms of the proportion of projected canopy area relative to the entire area of a pixel (100 m²). Cover was computed by projecting the vegetation points onto a (x, y) plane and converting them to a binary occupancy grid with 1 m resolution. The forest cover was then derived as the percentage of pixels occupied by forest. While somewhat correlated to Dens, cover remains a fully 2-dimensional variable, describing the horizontal vegetation cover, rather than the density of points in the vertical canopy profile. Time series of forest cover maps have been widely used in remote sensing as a measure of the canopy openness, and to assess land use changes with particular interest on forest cover losses (Hansen et al., 2013).

We divide up the dataset geographically into horizontal stripes of 1.8 km width and assign the stripes to the training, validation and test set, such that the different regions and modalities in each ALS area are evenly distributed between the three sets. Overall, the data set consists of 105,022,419 pixels (10,502 km²) of ALS reference data, divided into 64,487,551 training pixels (6,449 km²), 20,784,407 validation pixels (2,078 km²) and 19,750,461 test pixels (1,975 km²), following the “60-20-20” ratio often used in machine learning studies.

3.2. Sentinel satellite imagery

Sentinel-1 and Sentinel-2 are satellite missions that belong to ESA’s Copernicus Earth observation programme (European Space Agency, 2021), and provide high-resolution synthetic aperture radar (SAR) and optical imagery of land and coastal water areas between 56° South and 84° North. Each mission consists of a ground segment and a constellation of two satellites in sun-synchronous low earth orbits, phased 180° from each other. Jointly, both satellites in each mission achieve high revisit frequencies of >1 visit every five days, depending on latitude. The Sentinel-2 satellites are equipped with multi-spectral instruments that detect light in 13 spectral bands ranging from visible blue to short-wave infrared, with band-dependent spatial resolutions between 10 and 60 m. This spectral profile gives rise to a multitude of applications, including forest and vegetation monitoring as well as the inference of vegetation parameters such as leaf area index or carbon stock. For this work, we have collected Sentinel-2 Bottom-of-Atmosphere (BOA) reflectance images (level 2A, Main-Knorn et al., 2017) that have already been atmospherically corrected. For each ALS acquisition area, we select largely cloud-free images captured between May and October of the respective year, such that the area of interest is fully covered. Because BOA reflectance images still contain small amounts of atmospheric variation, we collect between two and seven images for each ground point (depending on cloud conditions), such that our model learns to cover a range of conditions.

On the other hand, the Sentinel-1 satellites are equipped with a C-band SAR sensor that actively monitors the surface with radiation of about 6 cm wavelength (European Space Agency, 2021). Sentinel-1 is invariant against meteorological factors like clouds, and also against illumination conditions. Although C-Band SAR does not enter deep into the canopy, it may superficially penetrate it. Thus, we explore this as a complementary signal to the optical image. For every collected optical image, we query a Sentinel-1 image that was acquired within ten days of the optical image and that covers the same geographic area. As preprocessing, we carry out orbit correction and terrain correction, where for the latter we rely on the Copernicus
digital elevation model with a resolution of one arc second. We intentionally keep the preprocessing simple because we expect our model to automatically infer the optimal transformations given the task at hand. For all preprocessing steps, we use SNAP (European Space Agency 2020a) with the Sentinel-1 Toolbox (European Space Agency 2020b) as provided by ESA.

4. Method

4.1. Forest structure model architecture

We utilize a multi-branch convolutional network (CNN) to transform co-registered optical and SAR images into maps of forest structure variables, and of their associated uncertainties. A CNN gradually transforms its inputs into outputs with a series of simple transformations (layers). One can think of the early layers as a pre-processing of the input modalities, the middle layers as a feature extractor and the last layers as a regression, but there is no clear distinction between those parts. At the heart of all of them is the (discrete) convolution operator, i.e., a linear filtering of a c-channel input image with a $k \times k \times c$ kernel of weights. In each convolutional layer, multiple convolutions with different filter weights are applied, so the output is again an “image” with as many channels as there were filters. The filter weights are the trainable parameters of the model. Convolutional layers are interleaved with some element-wise, non-linear activation functions. The series of transformations gradually abstracts the bare image pixels into a sequence of feature maps until, at the output level, they have become the desired forest structure maps.

Our model architecture is inspired by the design principles of ResNet (He et al., 2015). Our model is a multi-branch convolutional network (CNN) with $N$ stages. As commonly done in deep learning, we iteratively learn the model parameters with stochastic gradient descend, such that the block learns an additive residual update to its input, making it possible to training much deeper networks than the classical ResNet design (He et al., 2015). Due to the grouped convolution, each block effectively implements a multi-branch computational graph, where each branch can be interpreted as a lower-dimensional feature embedding, and where all branches are eventually combined by summation. This layout has been shown to offer superior predictive performance (Xie et al., 2017), while being less complex than the classical ResNet design (He et al., 2015). Regression Heads. Finally, from the resulting feature map, two parallel heads regress the means and log-variances of the five structural variables at every pixel. Each head consists of two convolution layers with kernel size one and an intermediate ReLU operation, gradually reducing the dimensional feature embedding, and where all branches are eventually combined by summation. This layout has been shown to offer superior predictive performance (Xie et al., 2017), while being less complex than the classical ResNet design (He et al., 2015).

4.2. Model training

As commonly done in deep learning, we iteratively learn the model parameters with stochastic gradient descend, starting from a random initialization. In each iteration, we randomly sample a batch of $B = 64$ reference data patches of size $15 \times 15$ pixels, where a patch is only considered for training if the center pixel is forested. We consider a pixel forested if and only if it contains vegetation points (points with $Dz > 1.3$ m, see Sec. 3.1) and also is considered forested based on NIBIO’s Norway-wide timber volume map (Astrup et al., 2019). We use the latter as an additional precautionary measure to avoid unnecessary noise from non-forested areas, as we are interested in learning forest characteristics only. For every reference data
patch, we randomly pick an optical image from the correct year and two SAR images (one ascending and one descending orbit) with acquisition dates near the one of the optical image. Using SAR with both ascending and descending orbits is expected to add robustness against terrain-induced geometric distortions such as shadowing, foreshortening and layover \cite{Smalletal1995,Carrascoetal1997}. In total the model input is composed of 12 optical bands forming a tensor of size $B \times 12 \times 15 \times 15$, and four SAR bands (two polarizations $\times$ two orbital directions), forming a corresponding $B \times 4 \times 15 \times 15$ tensor. The model output are two tensors of shape $B \times 15$ each, and one for their variances. An appropriate loss function measures the deviations between the model output and the ALS reference data, see below. Note that we only calculate $D$ with the training data for $n, c$ in zip($N_{\text{blocks}}, N_{\text{channels}}$):

\[ D(x; \theta) = \left( \frac{1}{N} \sum_{n=1}^{N} \sum_{c=1}^{C} \| y_n - \hat{y}_n \|_2 \right)^{1/2} \]

During training, we periodically evaluate the prediction against the statistics of previous updates to speed up convergence. Typically scales the magnitude of the weight updates based on the steepest descent. We use the Adam \cite{Kingmaetal2015} variant of stochastic gradient descent (SGD), which adaptively scales the magnitude of the weight updates based on the statistics of previous updates to speed up convergence. During training, we periodically evaluate the prediction error of the model (i.e., the current set of weights) on a held-out validation set and keep the configuration $\theta^*$ with the lowest error as the final model.

4.3. Loss function

The loss function, which is minimized during training, measures how well a given set of model parameters $\theta$ agree with the training data $D = \{ (x_n, y_n) \}_{n=1}^{N}$. We use a loss function $\mathcal{L}(D; \theta)$ whose minimization corresponds to maximizing the posterior probability of the parameters given the data, assuming a zero-mean Gaussian prior for the weights and a Gaussian likelihood function with mean $\mu_i := \hat{\mu}_i(x_i; \theta) \in \mathbb{R}^5$ and diagonal covariance matrix with logarithmic elements $\delta_i := \hat{s}(x_i; \theta) \in \mathbb{R}^5$:

\[ \mathcal{L}(D; \theta) = \lambda \| \theta \|_2^2 + \sum_{i,j} \left[ \delta_{ij} + \exp(-\delta_{ij})(\hat{\mu}_{ij} - y_{ij})^2 \right] \]  

Here, $i \in \{1 \ldots N\}$ indexes the data point while $j \in \{1 \ldots 5\}$ indexes one of the five structure variables we are aiming to predict. The hyperparameter $\lambda$ is inversely proportional to the variance of the weight prior. Note that by explicitly predicting aleatoric uncertainty, in the form of log variances $\hat{s}(x_i; \theta)$ of the output variables, the model learns to reduce the influence of data points on the loss that it deems particularly noisy, which in turn improves model performance \cite{KendallGal2017}.

4.4. Acquiring uncertainty estimates

Let $\hat{\mu}(x; \theta)$ and $\hat{\sigma}^2(x; \theta)$ be the mean and variance of the predicted distribution that the model with parameters $\theta$ outputs when shown the test image $x$. Furthermore, let $p(\theta \mid D)$ denote the posterior distribution over the model parameters, given the prior and the likelihood function defined in Sec. 4.3.

The exact predictive distribution of our model is intractable to compute, since the posterior $p(\theta \mid D)$ is intractable and we would further require integration over the extremely high-dimensional parameter space. However, it is possible to sample from an approximate $p(\theta \mid D)$ by training an ensemble of multiple neural networks from the same data (but with different random initializations and random batches for SGD). With $M$ different networks this
We chose \( \approx 0 \) where the validation loss has not improved for 15 consecutive epochs. Further, a correlation between predicted uncertainty and the reference data can be observed, although the latter is more "grainy", an attribute that we believe is transferred from the reference data that can vary strongly between adjacent pixels (this is not as visible as in the error maps, because the overall value range is larger). The correlation between those quantities is not as visually obvious as the previous one, which calls for a more extensive investigation into the predicted uncertainties.

In the following subsections, among other experiments, we will quantitatively validate the observations made above (good reconstruction of the true structural variables as well as uncertainty estimates being representative of the error) for the entire test set.

### 5.1. Evaluation of forest variables prediction

For each variable, indexed by \( j \in \{1, \ldots, 5\} \), we report the Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and the Mean Bias Error (MBE) defined as follows:

\[
\text{MAE} = \frac{1}{N_{\text{test}}} \sum_{i=1}^{N_{\text{test}}} |\bar{y}_{ij} - y_{ij}| \tag{5}
\]

\[
\text{RMSE} = \sqrt{\frac{1}{N_{\text{test}}} \sum_{i=1}^{N_{\text{test}}} (\bar{y}_{ij} - y_{ij})^2} \tag{6}
\]

\[
\text{MBE} = \frac{1}{N_{\text{test}}} \sum_{i=1}^{N_{\text{test}}} (\bar{y}_{ij} - y_{ij}) \tag{7}
\]

To represent the predictive distribution for each pixel \( i \) and variable \( j \) as a point estimate, we use its approximate mean \( \bar{y}_{ij} \) in accordance with Eq. 5. The MAE and RMSE metrics measure the average deviation between model prediction and reference data (with RMSE giving higher penalty to large deviations), while the MBE serves to identify systematic biases in the predictions. In addition to the above metrics, we calculate their normalized counterparts MAE%, RMSE% and MBE% by dividing by the corresponding mean values over the training data.

Tab. 1 shows the results of the test set evaluation of the final model. Overall (subtable a), we achieve low mean absolute errors for all predicted variables, e.g., 1.65 m for the 95th height percentile (\( P_{95} \)). Assessing the relative errors, the experiments show that the developed model produces consistently accurate predictions with an MAE% always smaller than 15%. The variables ranking in order of increasing relative error (MAE%) are forest cover (\( \text{Cover}, 11.2\% \)), vegetation density (\( \text{Dens}, 11.8\% \)), Gini index (\( \text{Gini}, 12.4\% \)), the 95th height percentile (\( P_{95}, 12.9\% \)) and lastly the mean height (\( \text{MeanH}, 14.4\% \)). When evaluating the individual geographic regions shown in Fig. 2, we observe comparable values for the eastern region (subtable b), which can be explained by this region making up for the majority of reference data points. Interestingly, the model performs even better in the northern test region in terms of mean absolute error of \( P_{95} \) and \( \text{MeanH} \), however the error for the remaining structural variables is slightly
Figure 6: Result for a sample region within the test set. **Left tile:** The model input is composed of a Sentinel-2 optical image (12 bands), and two preprocessed Sentinel-1 SAR images taken from an ascending and descending orbit direction, respectively. For visualization, the amplitudes of the VH and VV polarizations have been assigned to the red and green channels of the RGB image. **Right tile:** ALS reference data, the predicted mean and standard variation as well as the absolute prediction error $|\mu_{ij} - y_{ij}|$ for each of the five structural variables. The figure shows high correspondence between reference data and the predicted mean, indicating that our model is able to accurately regress the structural variables. There is also a clear (albeit not as crisp) correlation between predicted standard variation and absolute error, indicating that erroneous predictions are assigned higher uncertainty. We analyze the latter in more detail in Sec. 5.2.

bigger. For the western test region, evaluation errors are consistently higher, yielding e.g. an MAE in P95 of 1.85 meters and an MAE% of 14.4%. Possible reasons for the slightly lower performance in the western region are the scarcity of cloud-free data due to oceanic climate, and the increased topographic complexity. The fjord landscape in these western areas is characterized by very steep and narrow valleys causing both topographic shading and large variations in sun-target-sensor geometry. All of these factors have a negative impact on the quality of the satellite observations and thus the predictions obtainable in such areas. In addition, the forests in these areas are more diverse in terms of species compositions and structurally complex compared to the other areas. It is noteworthy that, for all of the tested regions, the MBE% is <0.7%, indicating the absence of any systematic model bias, regardless of the predicted variable and geographic region. Our model can therefore be considered an unbiased estimator for forest structure, a significant benefit for downstream tasks such as decision making based on the resulting maps.

For further analysis of our model, we provide confusion plots for all variables in Fig. 7. For the plots, all test samples are binned with regard to both their true, ALS-derived reference value ($x$ axis) and their predicted mean
Table 1: Test set evaluation results of the proposed model for all ALS areas (subtable a) as well as for the individual geographic regions shown in Fig. 2 (subtables b - d). P95 and MeanH metrics are given in meters, while Dens, Gini and Cover are reported as fractions. All normalized metrics (postfixed with %) are given as a fraction over the respective training set mean.

|          | P95 | MeanH | Dens | Gini | Cover |
|----------|-----|-------|------|------|-------|
| **a) All regions** |     |       |      |      |       |
| MAE      | 1.648 | 1.127 | 0.061 | 0.028 | 0.078 |
| MAE%     | 0.129 | 0.144 | 0.118 | 0.123 | 0.112 |
| RMSE     | 2.298 | 1.595 | 0.082 | 0.039 | 0.107 |
| RMSE%    | 0.179 | 0.204 | 0.158 | 0.170 | 0.154 |
| MBE      | -0.086 | -0.040 | -0.003 | -0.001 | 0.001 |
| MBE%     | -0.007 | -0.005 | -0.006 | -0.004 | 0.002 |
| **b) East** |     |       |      |      |       |
| MAE      | 1.631 | 1.127 | 0.060 | 0.027 | 0.075 |
| MAE%     | 0.127 | 0.144 | 0.114 | 0.121 | 0.107 |
| RMSE     | 2.250 | 1.589 | 0.080 | 0.038 | 0.101 |
| RMSE%    | 0.176 | 0.203 | 0.153 | 0.166 | 0.145 |
| MBE      | -0.090 | -0.040 | -0.004 | -0.001 | 0.000 |
| MBE%     | -0.007 | -0.005 | -0.007 | -0.005 | 0.000 |
| **c) North** |     |       |      |      |       |
| MAE      | 1.451 | 0.904 | 0.071 | 0.031 | 0.106 |
| MAE%     | 0.113 | 0.115 | 0.136 | 0.136 | 0.152 |
| RMSE     | 2.238 | 1.329 | 0.095 | 0.042 | 0.140 |
| RMSE%    | 0.175 | 0.170 | 0.182 | 0.186 | 0.200 |
| MBE      | -0.068 | -0.038 | -0.003 | -0.001 | 0.009 |
| MBE%     | -0.005 | -0.005 | -0.005 | -0.005 | 0.013 |
| **d) West** |     |       |      |      |       |
| MAE      | 1.845 | 1.224 | 0.069 | 0.031 | 0.091 |
| MAE%     | 0.144 | 0.156 | 0.132 | 0.136 | 0.131 |
| RMSE     | 2.609 | 1.740 | 0.093 | 0.042 | 0.127 |
| RMSE%    | 0.204 | 0.222 | 0.178 | 0.184 | 0.182 |
| MBE      | -0.069 | -0.041 | -0.001 | -0.000 | 0.003 |
| MBE%     | -0.005 | -0.005 | -0.002 | -0.001 | 0.005 |

(y axis), the color of a bin indicating how many samples fall into the respective bin. Higher densities along the identity line imply higher accuracy of the model, and indeed we can observe very high agreement between prediction and reference data, for all variables.

5.2. Uncertainty evaluation

We evaluate the quality of the model’s uncertainty estimates (Eq. 4), i.e., how well the predicted uncertainty correlates with the expected error of a given prediction. If a model is perfectly calibrated then, for any value of the predicted variance, the expected squared error between the true value and the predicted mean should be equal to the predicted variance (by definition of the variance). To obtain an estimate of the expected squared error, we group predictions into bins based on the predicted uncertainty and compare, for each bin, the mean squared error with the mean predicted variance of all samples in the bin. For a more intuitive interpretation, in the units of the original data, we take square roots and compare the empirical RMSE and the root mean variance.

In Fig. 8, the result of this comparison is shown for 20 bins for the P95 variable. It can be observed that the plot corresponding to the ensemble follows the identity line closely, implying highly reliable uncertainty estimates. Only for higher-variance predictions (Var(y, | x, D) ≥ 12) we can observe a slight under-estimation of uncertainty. Each of the individual models, on the other hand, suffers from much more severe under-estimation of uncertainty (i.e., they are over-confident) throughout all uncertainty levels. This is consistent with the literature, where it has repeatedly been observed that neural network models tend to be overconfident about their predictions (Guo et al. [2017]).

Another way to validate the usefulness of the predicted uncertainty values is to observe how error metrics change when retaining a decreasing fraction p ∈ (0, 1] of least uncertain predictions. The intuition is that if one discards predictions with high uncertainty and the latter is indeed a good predictor of the expected error, then the average error of the remaining predictions should decrease. Fig. 9 displays this relationship between the fraction of retained pixels and the MAE%, for all structural variables. Indeed, all curves increase monotonically, implying that the predicted uncertainty correlates (in expectation) with the actual error at a given test sample.

5.3. Sensor ablation study

We investigate how much the reported accuracy depends on the individual optical and SAR image inputs. To this end, Tab. 2 reports the achieved MAE when training a network with different input data configurations. Unsurprisingly, the default configuration that uses one Sentinel-2 optical and two Sentinel-1 SAR images (“S2+S1”) performs best w.r.t. all predicted structure variables. What stands out is that removing the optical information (“S1 only”) is significantly more detrimental than removing the SAR input (“S2 only”), indicating that the former is more predictive of forest structure. The ablation study further shows that including SAR data from both ascending and descending orbits is beneficial in terms of regression performance, compared to using only one (randomly chosen) orbit direction during training and testing. As an explanation, we suspect a combination of two effects: (1) The model has a second observation for any given pixel, thus
Figure 7: Confusion plots for the predicted structure variables. In each plot, the $x$ axis denotes the ALS reference data value and the $y$ axis denotes the mean of the predicted likelihood distribution. The number of data points that fall into each 2D bin is indicated by the color. The plots demonstrate that most predictions are located very close to the identity line, where prediction and ALS reference data agree.

Figure 8: Calibration plot for the P95 variable, using 20 uncertainty bins. For the ensemble, the predicted uncertainty agrees very well with the actually observed error, whereas we observe systematic over-confidence in the individual models’ predictions. Note that for this plot, we have removed the first and last percentile of predictions (according to predicted uncertainty) to avoid bins with a very small number of samples at the tails of the distribution, because with very few samples the MSE is a noisy estimate of the expected squared error.

Figure 9: MAE% for all variables measured on a fraction $p \in (0, 1]$ of test pixels that have the lowest associated uncertainty (as defined by Eq. $\text{[4]}$). The plot shows a strong correlation between estimated uncertainty and expected normalized absolute error. We show the normalized MAE% (instead of the absolute MAE) so we can compare the results for the five structural variables in a single plot.
benefiting from redundancy to suppress noise; and (2) the scene is illuminated from two different directions, which helps to mitigate geometric distortions specific to SAR, such as shadowing, foreshortening and layover (Small et al. 1995; Carrasco et al. 1997).

Lastly, the ablation study demonstrates that the impact of removing one of the SAR images is far less pronounced when additional optical information is available ("S2+S1" vs. "S2+S1Rand"), compared to a configuration that solely relies on SAR ("S1 only" vs. "S1Rand only"). This outcome further supports the finding that optical images are the more predictive data source for forest structure.

5.4. Country-wide forest structure map

To demonstrate the applicability of our method at country-scale, we compute a Norway-wide map of forest structure variables for the year 2020 and make it publicly available. To be more robust against clouds and to avoid gaps, we use $T=10$ optical images per location that were acquired throughout the leaf-on-period June to October. We also add one SAR image to every location and pair it with the optical data. We feed all eleven inputs per location when additional optical information is available ("S2+S1" vs. "S2+S1Rand"), compared to a configuration that solely relies on SAR ("S1 only" vs. "S1Rand only"). This outcome further supports the finding that optical images are the more predictive data source for forest structure.

| Input          | P95  | MeanH | Dens | Gini  | Cover |
|----------------|------|-------|------|-------|-------|
| S2+S1          | 1.697| 1.162 | 0.063| 0.029 | 0.080 |
| S2+S1Rand      | 1.778| 1.219 | 0.066| 0.029 | 0.083 |
| S2 only        | 1.813| 1.243 | 0.067| 0.029 | 0.084 |
| S1 only        | 3.052| 2.123 | 0.120| 0.038 | 0.148 |
| S1Rand only    | 3.480| 2.421 | 0.144| 0.040 | 0.174 |

The main finding of our research is that forest structure indicators can be mapped at country-scale from publicly available Sentinel data, in combination with modern deep machine learning and open airborne laser scanning (ALS) data as reference data for training. The availability of these maps at almost any point in time (starting from the beginning of the Copernicus program in 2015) opens up new possibilities for large scale forest resource mapping and is thus a promising tool for understanding and tackling challenges such as climate change or biodiversity loss. A straightforward application of the predicted structural variables could be updating existing nationwide forest resource maps hitherto produced on the basis of the same variables, but derived from ALS data (Nord-Larsen and Schumacher 2012; Nilsson et al. 2017; Astrup et al. 2019). Given the large costs for updating these maps, our Sentinel-derived forest structure variables could provide inexpensive auxiliary data for producing dense time series of forest resource maps. The availability of such dynamic maps can help to better understand the effect of forest management practices on forest biomass dynamics and on the variations in functional diversity. In addition to their use for mapping traditional variables (e.g., above-ground biomass), dense time series of canopy height (e.g., P95) from Sentinel data allow one to derive the canopy height vertical growth, which in turn can be used to measure site productivity (Lennart Noordermeer and Næsset 2018; Svein Solberg and Puliti 2019).

An innovative aspect of our method is that it additionally predicts variables describing the vegetation density (Dens) and its variation in the vertical profile of the canopy (Gini), different to previous studies that focussed on canopy height or cover only. These additional variables are important as they provide complementary information to the height and the cover and thus allow for a more comprehensive understanding of forest structures on a large scale. The combined use all of these variables further opens up new possibilities to address complex issues such as the quantification of biomass losses and gains from subtle land-use changes, e.g., forest degradation and forest restoration.

6. Conclusion

Fig. 10 shows overview images of the generated map for each of the five structural forest variables. We mask out areas that are not considered forested according to the NIBIO timber volume map (see Sec. 4.2) as already done for training our model. We also add one SAR image to every location and pair it with the optical data. We feed all eleven inputs per location when additional optical information is available ("S2+S1" vs. "S2+S1Rand"), compared to a configuration that solely relies on SAR ("S1 only" vs. "S1Rand only"). This outcome further supports the finding that optical images are the more predictive data source for forest structure.
A further novelty of the presented method, compared to previous applications of deep learning to remotely sensed data, is that it outputs pixel-wise, well-calibrated output distributions instead of mere point estimates. The quantification of the predictive uncertainty makes the system more reliable and more trustworthy. In particular, estimates of the expected model error can be propagated to downstream tasks, such as higher-level mapping systems that rely on our forest structure estimates as input. Even further downstream, maps ultimately serve as a basis for forest management, where well-calibrated uncertainty estimates ("knowing what you don’t know") translate to better decision making.

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Appendix A. Derivations

In the following, we give detailed mathematical justifications of the formulas employed in the paper.

Appendix A.1. Loss function

We start with the maximizer of the posterior parameter probability (i.e., its mode) and deduce the equivalence to the loss function presented in Eq. 4.

\[
\text{arg max}_\theta p(\theta \mid \mathcal{D}) = \text{arg max}_\theta \log p(\theta) + \log p(\mathcal{D} \mid \theta) \tag{A.1a}
\]

\[
= \text{arg min}_\theta - \log p(\theta) - \sum_{i=1}^N \log p(y_i \mid x_i, \theta) \tag{A.1b}
\]

\[
= \text{arg min}_\theta - \log \mathcal{N}(\theta; 0, \Sigma_p) - \sum_{i=1}^N \log \mathcal{N}(y_i; \bar{\mu}_i, \bar{\Sigma}_i) \tag{A.1c}
\]

\[
= \text{arg min}_\theta \frac{\lambda}{2} \|\theta\|^2_2 + \frac{1}{2} \sum_{i=1}^N (y_i - \bar{\mu}_i)^T \bar{\Sigma}_i^{-1} (y_i - \bar{\mu}_i) \tag{A.1d}
\]

\[
= \text{arg min}_\theta \frac{\lambda}{2} \|\theta\|^2_2 + \sum_{i,j} \left[ \bar{s}_{ij} + \exp(-\bar{s}_{ij})(\bar{\mu}_{ij} - y_{ij})^2 \right]. \tag{A.1e}
\]

In Eq. (A.1b) we make use of the logarithm being a monotonic function. In Eq. (A.1d) \( \mathcal{N}(\cdot \mid \mu, \Sigma) \) denotes the probability density function of the multivariate Gaussian normal distribution with mean \( \mu \) and covariance \( \Sigma \). In the same line, \( \bar{\Sigma}_i = \text{diag}(\bar{\sigma}_i^2) \) is a diagonal matrix containing the predicted variances for all structural variables. In Eq. (A.1f) \( j \in \{1 \ldots 5\} \) indexes the structure variables and we again use \( \bar{s}_{ij} = \log \bar{\sigma}_{ij}^2 \).

Appendix A.2. Posterior Predictive Distribution

The posterior predictive distribution is obtained by approximately marginalizing over the posterior parameter distribution \( p(\theta \mid \mathcal{D}) \):

\[
p(y_* \mid x_*, \mathcal{D}) = \int p(y_* \mid x_*, \theta) p(\theta \mid \mathcal{D}) \, d\theta \tag{A.2a}
\]

\[
= \mathbb{E}_{\theta \sim p(\theta \mid \mathcal{D})} \left[ p(y_* \mid x_*, \theta) \right] \tag{A.2b}
\]

\[
\approx \frac{1}{M} \sum_{k=1}^M \mathcal{N}(y_* \mid \bar{\mu}_{*,k}, \text{diag}(\bar{\sigma}_{*,k}^2)) \tag{A.2c}
\]

where \( \bar{\mu}_{*,k} := \bar{\mu}(x_*; \theta_k) \) and \( \bar{\sigma}_{*,k}^2 := \bar{\sigma}^2(x_*; \theta_k) \) and \( \theta_k \sim p(\theta \mid \mathcal{D}) \). In practise, we sample multiple \( \theta_k \) by training an ensemble and treat each parameter set as a sample from an approximate \( p(\theta \mid \mathcal{D}) \). Given the resulting approximate predictive distribution (Eq. (A.2d)), which is a mixture of Gaussians, the mean is obtained by the tower rule,

\[
\mathbb{E}[y_* \mid x_*, \mathcal{D}] = \mathbb{E}_{\theta \mid \mathcal{D}} \left[ \mathbb{E}[y_* \mid \theta, x_*] \right] \approx \frac{1}{M} \sum_{k=1}^M \bar{\mu}_{*,k}, \tag{A.3}
\]

and the variance by the law of total variance:

\[
\text{Var}[y_* \mid x_*, \mathcal{D}] = \mathbb{E}_{\theta \mid \mathcal{D}} \left[ \text{Var}[y_* \mid \theta, x_*] \right] + \text{Var} \left[ \mathbb{E}[y_* \mid \theta, x_*] \right] \tag{A.4a}
\]

\[
\approx \frac{1}{M} \sum_{k=1}^M \bar{\sigma}_{*,k}^2 + \frac{1}{M} \sum_{k=1}^M (\bar{\mu}_{*,k} - \bar{\mu}_*)^2 \tag{A.4b}
\]

\[
= \frac{1}{M} \sum_{k=1}^M \bar{\sigma}_{*,k}^2 + (\bar{\mu}_{*,k} - \bar{\mu}_*)^2, \tag{A.4c}
\]

where we again abbreviate \( \bar{\mu}_* := \sum_{k=1}^M \bar{\mu}_{*,k} \).

Appendix B. Additional Experimental Results

Supplementary to Tab. 2 where we reported the achieved MAE of our method for various input configurations, Tab. B.3 reports the results of these experiments in terms of all metrics.

Fig. B.11 shows the calibration plots similar to Fig. 8 for the remaining structure variables. As for P95, we observe a systematic under-estimation of the variance for the individual networks, and a clearly better uncertainty calibration for the ensemble. The effect is more or less pronounced, depending on the variable. For all plots we have again used 20 uncertainty bins and the same noise removal technique as explained for Fig. 8.
Table B.3: Detailed experimental results of the input ablation study conducted in Sec. 5.3, reporting the performance of our method on different input configurations.

|     | P95 | MeanH | Dens | Gini | Cover |
|-----|-----|-------|------|------|-------|
| a) S2+S1 |     |       |      |      |       |
| MAE  | 1.697 | 1.162 | 0.063 | 0.029 | 0.080 |
| MAE% | 0.132 | 0.148 | 0.121 | 0.125 | 0.115 |
| RMSE | 2.366 | 1.645 | 0.085 | 0.039 | 0.110 |
| RMSE%| 0.185 | 0.210 | 0.163 | 0.172 | 0.158 |
| MBE  | -0.039 | -0.019 | -0.002 | -0.000 | -0.001 |
| MBE% | -0.003 | -0.002 | -0.004 | -0.002 | -0.001 |
| b) S2+SI Rand |     |       |      |      |       |
| MAE  | 1.778 | 1.219 | 0.066 | 0.029 | 0.085 |
| MAE% | 0.139 | 0.156 | 0.126 | 0.128 | 0.119 |
| RMSE | 2.470 | 1.723 | 0.089 | 0.040 | 0.114 |
| RMSE%| 0.193 | 0.220 | 0.170 | 0.175 | 0.164 |
| MBE  | -0.075 | -0.041 | -0.004 | -0.001 | 0.002 |
| MBE% | -0.006 | -0.005 | -0.007 | -0.003 | 0.003 |
| c) S2 only |     |       |      |      |       |
| MAE  | 1.813 | 1.243 | 0.067 | 0.029 | 0.084 |
| MAE% | 0.141 | 0.159 | 0.129 | 0.129 | 0.121 |
| RMSE | 2.511 | 1.753 | 0.091 | 0.040 | 0.116 |
| RMSE%| 0.196 | 0.224 | 0.174 | 0.177 | 0.167 |
| MBE  | -0.037 | -0.006 | -0.000 | -0.001 | 0.005 |
| MBE% | -0.003 | -0.001 | -0.001 | -0.003 | 0.007 |
| d) S1 only |     |       |      |      |       |
| MAE  | 3.052 | 2.123 | 0.120 | 0.038 | 0.148 |
| MAE% | 0.238 | 0.271 | 0.231 | 0.167 | 0.212 |
| RMSE | 3.932 | 2.791 | 0.154 | 0.050 | 0.190 |
| RMSE%| 0.307 | 0.356 | 0.295 | 0.219 | 0.272 |
| MBE  | -0.018 | -0.004 | 0.002 | 0.000 | 0.007 |
| MBE% | -0.001 | -0.001 | 0.004 | 0.000 | 0.010 |
| e) S1 Rand only |     |       |      |      |       |
| MAE  | 3.480 | 2.421 | 0.144 | 0.040 | 0.174 |
| MAE% | 0.272 | 0.309 | 0.276 | 0.176 | 0.248 |
| RMSE | 4.411 | 3.124 | 0.180 | 0.052 | 0.218 |
| RMSE%| 0.344 | 0.399 | 0.346 | 0.229 | 0.312 |
| MBE  | -0.066 | -0.054 | 0.001 | 0.000 | 0.006 |
| MBE% | -0.005 | -0.007 | 0.002 | 0.000 | 0.009 |

Figure B.11: Calibration plots for the MeanH, Gini, Dens and Cover variables, similar to those in Fig. 8.