Statistical properties of currents flowing through tunnel junctions

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This paper presents an overview of the statistical properties arising from the broadness of the distribution of tunnel currents in metal-insulator-metal junctions. Experimental current inhomogeneities can be modelled by a lognormal distribution and the size dependence of the tunnel current is modified at small sizes by the effect of broad distributions.

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I. INTRODUCTION: MIM JUNCTIONS AND BROAD DISTRIBUTIONS

Metal-Insulator-Metal (MIM) tunnel junctions have been introduced into the physics toolbox four decades ago [1]. They have given rise to several landmarks in condensed matter physics such as the Josephson effect and the Coulomb blockade. Since the discovery of large room temperature Tunnel Magneto-Resistance (TMR) [2, 3], MIM junctions have been under intense scrutiny again. This paper will summarize recent studies of the disorder effects in MIM junctions, which is one of the aspects of MIM junctions’ physics. This topic is not new [4, 5] but it can be revisited thanks to recent experimental and theoretical developments.

Practitioners know how difficult it is to achieve reproducibility of the conductances of MIM junctions, even when the junctions are prepared on the same wafer. This is becoming a crucial problem with the prospect of applications of TMR to Magnetic Random Access Memories and magnetic read heads, which require the conductance dispersion to be typically less than 10%. This raises the question of whether the conductance irreproducibility is a purely technical problem or whether there is something more fundamental behind it.

The observed large dispersion of conductances from one junction to another one is statistically unusual and this provides a clue on the nature of the problem. Consider, for instance, a 10 \times 10 \mu m^2 junction with a typical interatomic distance of 0.3 nm, so that the cross-section contains about 10^9 atoms. According to the central limit theorem, relative fluctuations of an ensemble of n components scale as 1/\sqrt{n}. Thus, fluctuations of 10% at the junction scale would correspond to fluctuations of 3000 at the atomic scale. This suggests that either the distribution of tunnel currents is extremely broad, or that fluctuations do not average out as in the central limit theorem, or both.

During the last fifteen years, the importance of such broad distributions has emerged in several areas of statistical physics related mostly to anomalous diffusion [6, 7, 8]. The paradigm of broad distributions is the Lévy flight, i.e., random walks in which the length l of the free flight has a power law distribution P(l) = \alpha l_0^\alpha /l^{1+\alpha} (l > l_0) with a diverging second moment (0 < \alpha < 2). With such distributions, the variance is infinite, the usual central limit theorem does not apply and the relative fluctuations of a sum of n terms do not decrease with the number of terms. This reminds of the large fluctuations observed even in large junctions. Moreover, the sum of n terms displacements tends to be dominated by a few of them which reminds of the infamous 'hot spots', i.e., of filamentary like structures carrying most of the current. At last, with Lévy flights, even the law of large numbers can fail to apply, i.e., the typical sum of n terms might not be proportional to n even at large n (case \alpha \leq 1). This suggests that the tunnel current might not always be proportional to the size of the junction.

There seems to be a connection between broad distributions and MIM tunnel junctions. To clarify the matter, one needs to know experimentally the distribution of tunnel conductances (Section II). Then one can study the consequences of the current distribution (Section IV), in particular the scale effects (Section V).

II. EXPERIMENTAL DISTRIBUTION OF TUNNEL CURRENTS

Conducting Atomic Force Microscopy (C-AFM) can map the tunnel current flowing through an oxide barrier [9]. In this technique, the conducting tip of an atomic force microscope is scanned in contact with the surface of the barrier, while a bias voltage between the tip and the bottom metallic electrode creates a current flowing through the barrier. In this way, one records simultaneously the topography and the tunnel current. The actual resolution of the C-AFM is difficult to estimate. Correlations studies show that current structures smaller than 1 nm^2 are resolved. There must, however, be some sort of convolution by the finite tip size, which produces some...
The topography and current images for a typical oxide (AlO₂) barrier are shown in fig. 1. The topography (1a) is very smooth (roughness ≃ 0.2 nm), whereas the logarithm of the current image (1b) exhibits a continuum of current values going from ∼ 40 pA to ∼ 1 nA. The local $I-V$ characteristics obtained with C-AFM are consistent with tunnelling with transmission much less than 1, even at the highest current points which are, thus, not pinholes. This type of experiments has been reproduced by several groups with similar results [10, 11, 12, 13, 14, 15]. They are especially useful for barrier optimization, for instance with respect to oxidation [16, 17] or annealing [18].

Several types of tiny barrier inhomogeneities can be responsible for the large current inhomogeneities which are observed in both C-AFM and numerical simulations [1]. First, due to the amorphous nature of most insulator barriers, the metal-oxide interfaces can not be perfectly smooth and fluctuations of the barrier thickness are unavoidable. What is of interest here is the ‘roughness’ of the barrier thickness which is usually much smaller than the topography roughness [1, 14]. Thickness roughness of less than 0.1 nm can generate the fluctuations visible in fig. 1b. There must also exist inhomogeneities of the barrier height. Indeed changing a single atom at a metal-oxide interface can induce local barrier changes larger than 1 eV [20]. Statistics on the barrier parameters obtained from many local C-AFM $I-V$ curves seem to indicate that barrier height inhomogeneities play a more important role than thickness inhomogeneities [1]. At last, metal oxides are known for containing electron traps which are clearly evidenced in noise studies [21] (see also Section [IV]) and impurity states [12, 22]. Further studies (C-AFM, physico-chemistry of oxides, electronic and structural simulation, ballistic electron emission spectroscopy [23, ...]) are needed to clarify the origin of current inhomogeneities.

Even without knowing the origin of the current inhomogeneities, important consequences can be drawn from the knowledge of the statistical distribution $P(i)$ of currents. The distribution $P(i)$ presents in many cases [9, 24] a lognormal shape (fig. 2):

$$P(i) = \frac{1}{\sqrt{2\pi\sigma i}} \exp \left[ \frac{-(\ln i - \mu)^2}{2\sigma^2} \right] = \text{LN}(\mu, \sigma^2)(i),$$

where $\mu$ is a scale parameter and $\sigma$ is a shape parameter, hereafter called the 'disorder strength'. Note that standard $\chi^2$ fitting procedures are not adequate to fit lognormal distributions because most of the current is usually carried by the tail of the distribution while $\chi^2$ fitting weights heavily the peak of the distribution, which does not carry here a significant current and is strongly affected by spurious noise. To take into account the large value tail more properly, one can, e.g., fit a parabola to the large ln $i$ branch of ln $P(I)$ vs ln $i$ (a lognormal distribution is a parabola in log-log scale).

The occurrence of lognormal distribution of tunnel currents is not a surprise [27]. Suppose, indeed, that the distribution $P(d)$ of barrier thickness $d$ is Gaussian with mean $\mu_d$ and standard deviation $\sigma_d$. The current $i$ varies typically exponentially with $d$:

$$i = i_0 e^{-d/\lambda},$$

where $i_0$ is a current scale and $\lambda$ is the attenuation length of the electronic wave functions in the barrier. By definition, the exponential of a Gaussian random variable is a lognormal random variable. Thus, the current $i$ has a lognormal distribution LN($\mu, \sigma^2$) with $\mu = \ln i_0 - \mu_d/\lambda$ and $\sigma = \sigma_d/\lambda$ [25]. Importantly, the disorder strength $\sigma$ is $\sigma_d/\lambda$ and not $\sigma_d/\mu_d$ as one might expect naively. Thus, a barrier which appears geometrically smooth ($\sigma_d/\mu_d \ll 1$) might be ‘rough’ ($\sigma = \sigma_d/\lambda \gtrsim 1$) from the point of view of current statistics, since typically $\lambda \ll \mu_d$.
(λ ≈ 0.05 – 0.1 nm, μd ≈ 1 – 2 nm), and generate large current inhomogeneities.

More generally, the tunnel current i depends on the barrier parameters pb (thickness, height, voltage, ...) typically as:

\[ i = g_b (p_b) \exp [f_b (p_b)] \]

(3)

where \( g_b (p_b) \) and \( f_b (p_b) \) vary less strongly than an exponential. If \( p_b \) presents small Gaussian fluctuations of standard deviation \( \sigma_{p_b} \) (\( \sigma_{p_b} \ll \mu_{p_b} \)) around its average value \( \mu_{p_b} \), then one has \( p_b = \mu_{p_b} + \epsilon \sigma_{p_b} \) where \( \epsilon \) is a Gaussian random variable of order 1 (mean 0, standard deviation = 1). As \( \sigma_{p_b} \ll \mu_{p_b} \), one can write \( f_b(p_b) = f_b(\mu_{p_b}) + \epsilon \sigma_{p_b} f'_b(\mu_{p_b}) \) and thus

\[ i = g_b (p_b) \exp [f_b (\mu_{p_b})] \exp [\epsilon \sigma_{p_b} f'_b (\mu_{p_b})]. \]

(4)

As \( g_b(p_b) \) varies slowly, one can neglect its fluctuations. Thus, \( i \) appears as the product of a fixed term, \( g_b (p_b) \exp [f_b (\mu_{p_b})] \), by the exponential of a Gaussian random variable, \( \epsilon \sigma_{p_b} f'_b (\mu_{p_b}) \), which is, by definition, a lognormal random variable.

Thus lognormal distributions of tunnel currents emerge as the consequence of small Gaussian fluctuations of the tunnelling parameters and are a good starting point to investigate the tunnel current statistics.

### III. SIMPLE CONSEQUENCES OF THE LOGNORMAL MODEL

The lognormal distribution of currents (eq. (1)) gives rise to peculiar statistical properties. If the disorder strength is small (\( \sigma \ll 1 \)), the lognormal is close to a Gaussian and the usual statistical behaviours, related to narrow distributions, appear. On the contrary, if \( \sigma \) is on the order of 1 or larger, the lognormal distribution is broad and it presents a long tail, just as a Lévy flight, even if, unlike a Lévy flight, it has finite average and standard deviation. The broadness of the lognormal also appears in the fact that the typical current \( i^\ast \) (i.e., most probable value),

\[ i^\ast = e^{\mu - \sigma^2}, \]

(5)

can be much smaller than the average current \( \mu_i \),

\[ \mu_i = e^{\mu + \sigma^2/2}, \]

(6)

indicating a large dispersion.

Fig. 3 illustrates the differences between narrow and broad distributions. Fig. 3(a) represents random values of a narrow distribution, a Gaussian of arbitrary mean \( \mu \) and standard deviation \( \sigma = 2 \). The X-coordinate may represent the position \( k \) in a 1D tunnel barrier while the Y-coordinate may represent the thickness or height of the barrier. All values are of the same order of magnitude, \( \mu \), to within about \( \sigma \). For the Gaussian, the typical value and the mean are equal. Fig. 3(b) represents random values of a broad distribution, which is the lognormal arising from the exponential of the Gaussian values of fig. 3(a). Fig. 3(b) may represent tunnel currents. The appearance of the fluctuations is now completely different. The possible values cover several orders of magnitude. Neither the typical value nor the mean, which differ by a factor of 400, characterize well the range of possible values. Thus, practically, the tunnel current through a disordered barrier is not well characterized by a single value like its mean but rather by the full distribution (two parameters for a lognormal).

The large values observed in fig. 3(b) correspond to the **hotspots** in tunnelling. These are not pinholes: the large currents arise from tiny fluctuations of barrier parameters (fig. 3(b)) because the exponential dependence acts as a 'fluctuation amplifier'. There is a continuous range of large current values corresponding to sites that have nothing qualitatively special, but rather present small quantitative fluctuations of the barrier parameters. With the lognormal model, one can estimate the current in-
homogeneity [26] by calculating the proportion $p_A$ of the surface ($p_A = \int P(i)di$ where $i_a$ is a parameter) carrying the proportion $p_i$ of the average current ($i$) ($p_i = \int P(i)di/i$)). For small $\sigma$ ($\sigma < 0.25$), half of the total current is carried by roughly half of the sites (no hot spots). For large $\sigma$, the current proportion carried by hot spots becomes more and more important. For instance, for $\sigma = 1$ (respectively 2), $p_i = 50\%$ of the total current is carried on average by $p_A = 16\%$ (respectively $3\%$) of the sites with highest current.

The domination of the current by the few largest transmission sites is indirectly confirmed by the time fluctuations of the total current. In certain conditions (small enough conditions, low temperatures), the current exhibits strong telegraph noise indicative of electron trapping and detrapping on a single trap in the barrier [21, 27, 28]. The large effect of a single electron trap on the total current flowing through a junction is a strong clue for the dominant role of a few hot spots.

At last, we comment eq. (1) for the average current. One should first be cautious that, contrary to what is frequently assumed [4, 5], the average $\mu_i$ is not in general what is measured on a single tunnel junctions (see Section IV). For a perfect barrier ($\sigma = 0$), we recover the current $e^\mu$ without inhomogeneities, as expected. The inhomogeneities generate a correcting ‘disorder term’, $e^{\sigma^2/2}$. This term is always larger than 1: the barrier inhomogeneities always increase the average current, due to the non-linear dependence of the current on the fluctuating parameters. Thus, the average current $\mu_i$ flowing through an inhomogeneous barrier of given average thickness corresponds to the current $i(d_{\text{eff}})$ flowing through a thinner effective homogeneous barrier ($d_{\text{eff}} < \mu_d$), as already noticed in [4]. Yet, in the model based on thickness fluctuations, as $\mu = \ln i_0 - \mu_d/\lambda$, the average current $\mu_i$ still depends exponentially on the average thickness $\mu_d$, just as the current for a homogeneous barrier. This is contrast with the modification of the $I - V$ characteristics shape by the presence of disorder [5].

IV. SCALE EFFECTS

The broad distribution of currents affects the size dependence of tunnel junctions giving rise to anomalous scaling laws. To understand this intuitively, we have plotted in figs. 3a and 3b (solid lines) the quantities

$$D_n/n = \sum_{k=1}^n d_k/n$$

and

$$I_n/n = \sum_{k=1}^n i_k/n$$

which represent, physically, the measured quantities at scale $n$. For instance $I_n/n$ is proportional to the current per unit area flowing through a junction of size $n$. For the Gaussian variable $d$, $D_n/n$ is statistically distributed around the mean $\mu$ and statistically converges to $\mu$ as $1/\sqrt{n}$ when $n$ increases (central limit theorem). At any scale, we have $D_n \propto n$. For the lognormal variable $i$, the sum $I_n$ behaves completely differently. At small scales, $I_n$ takes small values very different from the mean and, as $n$ increases, there is a slow upward trend of $I_n$ towards the mean $\mu_i$: this is the anomalous scaling we investigate here. This upward trend is created by the higher probability of larger samples (larger $n$’s) to have a very large current peak $i_k$ that will significantly draw the sum $I_n$ towards larger values. This effect does not occur with narrow distributions like Gaussians for which the largest terms in a statistical sample are not large enough to modify the sums significantly.

These scale effects can be studied theoretically [23, 29]. The problem reduces to finding the distribution of the sum $I_n$ of $n$ independent currents $i_k$ with the same lognormal distribution $P(i) = \ln(\mu, \sigma^2)$($i$). For moderately broad lognormal distributions, we find that $I_n$ is also approximately lognormally distributed, as $\ln(\mu_n, \sigma^2/n)$, and has a typical value

$$I_n \approx n\mu_i \left(1 + C^2/n\right)^{-3/2}$$

where $\mu_i$ is the average current (eq. (1)) and $C^2 = \sigma^2 - 1$ is the coefficient of variation. The typical current is thus the product of the usual term $n\mu_i$ by a correction term $(1 + C^2/n)^{-3/2}$. For small junctions ($n \ll C^2$), the correction term is important. Above a characteristic size $n_c = C^2$ related to the disorder strength, the correction term tends slowly to 1 and the usual behaviour $n\mu_i$, related to the law of large numbers is recovered.

The scaling relation eq. (1) can be tested experimentally with the current image of fig. 1. For each $n$, we first sum the currents of groups of $n$ pixels to obtain a statistical ensemble of values $I_n$ and then construct the histograms presented in fig. 2. The histograms’ peaks give $I_n$ (one must not compute the mean but the typical value of $I_n$’s: the mean presents no special scaling behaviour). Before grouping them, the pixels have been spatially randomized to satisfy the condition of statistical independence of the $i_k$’s. The result of this procedure for $I_n/n$ is plotted in fig. 4. As predicted, $I_n/n$ deviates strongly at small scales from the constant $\mu_i$ that one would expect naively and the deviation is well described by eq. (1). The maximum deviation is a factor 2.8 $\approx e^{3\sigma^2/2}$ for this good quality junction ($\sigma = 0.83$). For poorer quality junctions, deviations larger than $10^2$ have been observed [24].

If one takes into account the spatial correlations existing in the barrier by not randomizing the pixels (black squares in fig. 4), there is still an anomalous scaling of $I_n/n$ and the convergence towards $\mu_i$ is much slower than without correlations. Thus the correlations play a crucial role in the statistical properties of tunnel junctions as they combine with the broadness of the current distri-
ally checks that dependences have been reported recently [30]. For 1 nm the normal size dependence. However, anomalous size found, from 60 $\Omega$ barriers embedded in GaAs [29]. Again, the typical cur-
tions of different sizes to obtain the typical current at these sizes. This has been done for semi-conducting AlAs
ter reproducibility, apart from the obvious reduction of strength and to the spatial correlations. To achieve bet-
scales but the transition from small scale behaviour to large scale behaviour is slow. This transition is further slowed down by spatial correlations. Correspondingly, the large current inhomogeneities that exist at small scale average out slowly when increasing the size of the system. Thus, even relatively large junctions exhibit large dispers-
sions of conductance which are the relics of the poorly averaged small scale inhomogeneities.

For applications, it is worth mentioning that the effects of disorder increase rapidly when decreasing the junction size below a characteristic size related to the disorder strength and to the spatial correlations. To achieve better reproducibility, apart from the obvious reduction of the barrier disorder, one could also aim at reducing the spatial correlations.

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V. CONCLUSIONS AND OVERVIEW

Several types of experiments (conductive AFM, noise studies, scaling studies) provide a body of evidence that the distribution of tunnel currents flowing through MIM junctions is broad, even in good junctions. This is a natural consequence of the exponential dependence of quantum tunnelling with the parameters. With such broad distributions, the typical value is much smaller than the average value so that tunnel currents should not be characterized by a single number but instead by the full statistical distribution. The lognormal distribution is found to fit the experimental data in many cases. The shape parameter $\sigma$ of lognormal distributions is a convenient figure of merit to compare the quality of different junctions.

The broad character of the current distribution has several implications, some obvious, some less obvious. First, the current flows heterogeneously through the junction, in a way that the lognormal model can quantify. Second, large spatial variations of the current imply large time variations, i.e., large noise. Third, the average current varies strongly, typically as $e^{\sigma^2}$ with the strength $\sigma$ of the disorder. Fourth, the size dependence of the typical properties of tunnel junctions (resistance or conductance) is affected by the disorder, especially at small scales. One recovers the usual size dependences at large scales but the transition from small scale behaviour to large scale behaviour is slow. This transition is further slowed down by spatial correlations. Correspondingly, the large current inhomogeneities that exist at small scale average out slowly when increasing the size of the system. Thus, even relatively large junctions exhibit large dispers-
sions of conductances which are the relics of the poorly averaged small scale inhomogeneities.

\begin{equation}
(R \times A)^{1/3} \propto (1 + C^2/n)^{-1/2},
\end{equation}

which shows how counter-intuitive broad distributions can be.

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