A fit is made to the data for the proton structure function up to $Q^2 = 10\text{ GeV}^2$, including the real $\gamma p$ total cross-section. It is economical and simple, and its form is motivated by physical principles. It is extrapolated down to very small values of $x$. Data for the ratio $\nu W_2^p/\nu W_2^n$ are also fitted.

A FORTRAN program for the fit to $\nu W_2^n$ is available by email on request

May 1993

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The structure function $\nu W_2$ for inelastic electron or muon scattering is constrained by gauge invariance to vanish linearly with $Q^2$ at $Q^2 = 0$. Indeed, the total cross-section for real-photon scattering is

$$\sigma(\gamma p) = \frac{4\pi^2\alpha_{\text{EM}}}{Q^2} \nu W_2$$

(1)
evaluated at $Q^2 = 0$. For this reason, if for no other, the variation of $\nu W_2$ with $Q^2$ at small $Q^2$ cannot be described by perturbative QCD: it is unsafe to use any perturbative evolution equation until $Q^2$ is at least so large that $\nu W_2$ has fully recovered from the need to vanish at $Q^2 = 0$.

A few years ago\cite{1} we parametrised the then-available data for $\sigma(\gamma p)$ and for $\nu W_2$ at small $Q^2$ together in a very simple form. A rather similar analysis has been repeated more recently\cite{2}. Our parametrisation compares quite well with more recent data, but we feel that the time has now come to improve on it. This is for two reasons. The first is concerned with the search for the Lipatov pomeron in the measurements of $\nu W_2$ at small $x$. There is some uncertainty about the effects of kinematic constraints\cite{3}\cite{4} and of shadowing\cite{5} on the solution to the Lipatov equation, and so it may not be immediately obvious from future data whether the Lipatov pomeron is actually present. To help decide this, it will be necessary to know as accurately as possible what is expected from the soft pomeron, which is the pomeron that is seen in all the data that have been collected so far\cite{6}. Our new parametrisation allows what we believe to be a reliable extrapolation to very small $x$ of the effect of soft pomeron exchange. The second reason good fits to small-$Q^2$ data are needed is for calculation of QED radiative corrections. These can be huge at HERA and so they must be calculated as accurately as possible, using more than one parametrisation of existing data as a check.

Our aim is to present a parametrisation of the data that is as simple as possible, with rather few parameters, and whose component parts are motivated by physical considerations.

We have shown recently\cite{7} that all total cross-sections may be parametrised as a simple sum of two Regge powers

$$\sigma^{\text{TOT}} = X s^{0.0808} + Y s^{-0.4525}$$

(2)

We fixed the two powers in this expression from $pp$ and $\bar{p}p$ data, and found that the resulting expression works well also for $\pi p$ and $K p$. In the case of $\gamma p$, the fit (2) is very similar to one we made a few years ago\cite{1}, and it is in agreement with the published data points from HERA\cite{8}. We show this in figure 1a. (Notice that although we have specified the two powers to high accuracy, the data do not determine them to such accuracy; the $pp$ and $\bar{p}p$ data from which we derived them would allow slightly different values, with corresponding changes to the coefficients $X$ and $Y$ – the errors in the parameters are strongly correlated.)
According to the parton model, the same Regge powers appear\cite{9} as powers of $1/x$ in the small-$x$ behaviour of $\nu W_2$. The parton model is drawn in figure 2, where the amplitude $T$ is the amplitude for the emission of a parton of momentum $k$ by the proton. From simple kinematics\cite{9}, the energy variable for this amplitude is

$$(p - k)^2 = -x^{-1}(k^2 + k_T^2) - k^2 + (1 - x)m_p^2$$

and so is large when $x$ is small. Because $T$ is a hadronic amplitude, its high-energy behaviour should involve the same Regge powers of $(p - k)^2$ as appear as powers of $s$ in (2), and these reflect themselves as corresponding powers of $1/x$ in $\nu W_2$. Of course the parton model is only the first term in a perturbative-QCD expansion of $\nu W_2$; the influence of the perturbative corrections on the Regge powers is not understood and is the reason for the great interest in small-$x$ physics at HERA. One possibility is that the Regge powers in (2) survive the perturbative corrections and appear in the small-$x$ behaviour, at least for small, and even perhaps moderately large, values of $Q^2$. We explore this possibility in this paper.

We note that Monte Carlo models are commonly in error by taking both $k^2$ and $k_T$ to be zero. While this may be a good approximation for many purposes, it is not good when $x$ is small, because, according to (3), it fails to make $(p - k)^2$ large and so does not correctly take account of the nonperturbative Regge behaviour.

Our first fit, then, is to the small-$x$ data from NMC\cite{10}. We use two simple powers of $x$, each multiplied by a simple function of $Q^2$ that vanishes linearly with $Q^2$ as $Q^2 \to 0$ and goes to 1 for large $Q^2$:

$$\nu W_2 \sim A x^{-0.0808} \left( \frac{Q^2}{Q^2 + a} \right)^{1.0808} + B x^{0.4525} \left( \frac{Q^2}{Q^2 + b} \right)^{0.5475}$$

with the constraints that

$$Aa^{-1.0808} = 0.604 \quad \quad Bb^{-0.5475} = 1.15$$

so as to retrieve the fit of figure 1a to the real-photon data when $Q^2 \to 0$. This two-parameter fit is compared with the NMC data in figure 3, for the choices $A = 0.324$ and $B = 0.098$. Our fit has used only the data up to $Q^2 = 10$.

Strictly speaking, because $x = Q^2/2\nu$, the $Q^2 \to 0$ limit of (4) is not a sum of powers of $s$ as in (2), but rather a sum of powers of $2\nu = s - m_N^2$. At the lowest $s$-value of the data in figure 1a the difference is negligible, though if we continue to smaller $s$ it becomes important. We show the curve obtained for $\sigma(\gamma p)$ from the $Q^2 \to 0$ limit of (4) in figure 1b. Even though
we have not used the data below $\sqrt{s} = 6$ to determine the fit, it works satisfactorily down into the region where the resonances begin to be important.

If we extrapolate (4) to very small $x$, we obtain a predicted value just less than 0.8 for $\nu W^2$ at $x = 10^{-5}, Q^2 = 10$. This prediction remains rather stable under the refinements to our fit that we report below. We make refinements to the fit partly in order to extend it to larger values of $x$, as is necessary if it is to be useful for making radiative corrections. But also we want to incorporate the contribution from heavy flavours into the analysis, since part of the rapid rise with $Q^2$ seen in the data in figure 3 is to be attributed to their very rapid switching-on.

The charmed-quark contribution to $\nu W^2$ in muon scattering has been measured\[11\] and found to vary extremely rapidly with $Q^2$. It seems that this is a threshold effect, though we must emphasise that there is no good theoretical understanding of how one should take account of threshold effects\[12\]. Such an understanding would require more knowledge than we have of the effects of confinement. Nevertheless, a successful phenomenology of the data is available\[11\][13]: the threshold effect is well described by supposing that the contribution $F^{cc\bar{c}}_2$ to $\nu W^2$ from charmed quarks is a function not of the Bjorken variable $x$ but of

$$\xi_c = x \left(1 + \frac{\mu_c^2}{Q^2}\right)$$
(5a)

Then

$$(1 - \xi_c) = \frac{W^2 - W_0^2}{2\nu}$$
(5b)

with $W_0^2 = \mu_c^2 + m_p^2$, and the best description of the data corresponds to choosing the parameter $\mu_c$ such that $W_0 = m_D + m_{\Lambda_c}$, the threshold value of $W$. (Strictly speaking, the threshold is different according to whether it is a quark or an antiquark that has absorbed the virtual photon, but the data are not sufficiently accurate for this to matter, and we simply take the lowest physical threshold.) Hence, by using the variable $\xi_c$ instead of $x$, we ensure that $F^{cc\bar{c}}_2$ goes to zero at threshold for each $Q^2$. According to the spectator-counting rule\[14\] $F^{cc\bar{c}}_2$ should behave as $(1 - \xi_c)^7$ as $\xi_c \rightarrow 1$, while Regge theory requires it to have power behaviour close to $\xi_c^{-0.08}$ at small $\xi_c$. In addition, it must vanish linearly with $Q^2$ when $Q^2 \rightarrow 0$, which we achieve as in (4a). This led us in reference 13 to the fit

$$xc(x, Q^2) = \frac{9}{8} F^{cc\bar{c}}_2 = C_c \frac{Q^2}{Q^2 + 6.25} \xi_c^{-\epsilon}(1 - \xi_c)^7$$
(5c)

where we took $\epsilon$ to be 0.086 and $C_c = 0.045$. For the reasons we have explained, we now adopt the value 0.0808 for $\epsilon$, but the difference is negligible. Since we made this fit, the data
have been scaled downwards, because of a revised experimental value of the branching ratio of the charmed quark decaying to a muon; we therefore shall use the value $C_c = 0.032$. The fit, with the renormalised data, is shown in figure 4.

Some discussion of the use of $\xi_c$ is called for. As we have indicated, it is suggested by the data. More naive theoretical considerations might lead to the “slow rescaling” choice $\mu_c = m_c$, the charmed quark mass, instead of the 2 GeV that is needed. To derive such slow rescaling one would need two assumptions: (i) that the mass scale associated with the fragmentation of the charmed quark after it has absorbed the virtual photon is $m_c$ rather than that of the lightest hadron to which it can fragment; and (ii) one can ignore the fact that the momentum $k$ of the quark before it absorbs the photon is not on shell. The fact that $\mu_c$ needs to be so large indicates that neither of these assumptions is tenable: not only is the minimum mass of the hadron to which the quark fragments important, so also is the minimum mass of the residual fragments of the proton left behind when the quark has been pulled out of it. The only way that such a mass can be supported is for the quark momentum $k$ to go off shell and become negative: see (3).

Given that the phenomenology of $F_c^{\bar{c}c}$ points to the use of the variable $\xi_c$ defined in (5a), it is natural to assume that the strange antiquark distribution $\bar{s}(x, Q^2)$ should be handled similarly. The corresponding variable $\xi_s$ should be defined similarly, but with a scale $\mu_s$ such that the threshold $W_0 = m_K + m_\Lambda$. From (5b), this requires $\mu_s^2 = 1.7$ GeV$^2$. We shall work with

$$x\bar{s}(x, Q^2) = C \frac{Q^2}{Q^2 + a_s \xi_s^{-0.0808}(1 - \xi_s)^7}$$  \hspace{1cm} (6)$$

The threshold for the strange quark distribution $xs(x, Q^2)$ is somewhat higher, but we shall not take account of this refinement, because we also have to ignore the possibility that the $s$ and $\bar{s}$ quarks recombine and do not appear in the final state, with then a much smaller value for $W_0$ and hence for $\mu_s$. We can only guess what to take for the parameter $a_s$; we choose the value 1 GeV$^2$, which is in between the corresponding value for light quarks (see (4)) and charmed quarks. Fortunately, its precise value will not be too important for us. Our first guess for the value of the constant $C$ is that it is the same as for light quarks; (4) then gives $C \approx 0.22$. We shall confirm from our later fit that this is the appropriate value in the case of the light quarks, but of course it is far from obvious that we should use it also for strange quarks. There is some confusion about the magnitude of the strange-quark content of the proton. A direct measurement from two-muon events in neutrino scattering finds that the strange quark distribution is half as large as the light antiquark distributions, but
the less direct method\cite{17} of measuring

\[ xs(x, Q^2) \approx \frac{5}{3} F_2^{\nu N} - 3 F_2^{\mu D} \] (7)
gives a result that is significantly larger\cite{18}, even allowing for that fact that it is risky to measure a small quantity as the difference between two large ones, as in (7). As a contribution to this debate, we point out that the threshold effects, being sensitive to the minimum mass that can be produced in the final state, will vary according to how one measures \( xs(x, Q^2) \). In \( F_2^{\nu N} \) there are two contributions. In the first, which is Cabbibo suppressed, the strange quark absorbs the weak current and becomes a light quark, so that the threshold is \( W_0 = m_\pi + m_\Lambda \) corresponding to \( \mu_s = 0.8 \) GeV. In the second, the strange quark absorbs the weak current and becomes a charm quark, so that the threshold is \( W_0 = m_D + m_\Lambda \) corresponding to \( \mu_s = 2.9 \) GeV. The latter process is also the source of the dimuon events in neutrino interactions, with the same value of \( \mu_s \) (a much larger value than was assumed by the experimentalists\cite{14} in the analysis of their data!). In \( F_2^{\mu D} \) the strange quark survives and \( W_0 = m_K + m_\Lambda \), with then \( \mu_s = 1.3 \) GeV. In the latter case there is, however, the possibility that the \( s \) and \( \bar{s} \) quarks recombine with consequently a smaller value for \( \mu_s \). As an additional complication, the requirement that \( \nu W_2 \) vanishes as \( Q^2 \to 0 \) applies only to photon-exchange processes and not to processes involving the non-conserved weak current. To some extent one can take this into account\cite{13} through PCAC, but only for light-quark distributions.

Having decided, albeit somewhat tentatively, how to handle the contributions to \( \nu W_2 \) from the heavy quarks (fortunately they are small), we now turn to the \( u \) and \( d \) quarks and their antiquarks. With the threshold \( W_0 = m_\pi + m_p \), \( \mu =0.53 \) GeV, and we use this to define a variable \( \xi \) similar to (5a). When \( Q^2 \to 0 \), \( \xi \sim \mu^2/2\nu \), so that in order to have \( \nu W_2 \) vanishing linearly with \( Q^2 \) we replace the small-\( x \) behaviour (4) with

\[ \nu W_2 \sim A \xi^{-0.0808} \phi(Q^2) + B \xi^{0.4525} \psi(Q^2) \] (8a)

where

\[ \phi(Q^2) = \frac{Q^2}{Q^2 + a} \quad \psi(Q^2) = \frac{Q^2}{Q^2 + b} \] (8b)

and

\[ A a^{-1}(\mu^2)^{-0.0808} = 0.604 \quad B b^{-1}(\mu^2)^{0.4525} = 1.15 \] (8c)

This is the behaviour we want when \( x \) or \( \xi \) is small; when \( \xi \) is close to 1 we impose instead the spectator-counting rules\cite{14} to determine the powers of \( (1 - \xi) \). However, we find that if we simply multiply the terms in (4a) by such powers, they spoil the good fit of figure 3 to the small-\( x \) data. This is because, while at the smallest \( x \)-value 0.008 of the data in the figure,
$(1-x)^7$ is close to 1, at the largest $x$-value of 0.07 it is already as small as 0.6, and the effect of using instead $(1-\xi)^7$ is even more marked. In order to overcome this problem, we shall use the simple sum of powers for values of $\xi$ less than some fixed value $\xi_0$, which we leave as a free parameter. For $\xi > \xi_0$ we match to each term in (8a) expressions of the form

$$\text{const } \xi^\lambda (1-\xi)^m$$

where the power $\lambda$ is fixed by requiring such a form to fit smoothly on to the simple power of $\xi$ at $\xi = \xi_0$, that is the two forms and their first derivatives are equal there. We shall use the same value of $\xi_0$ throughout, and find that the best fit requires it to be quite small, about 0.07.

Consider first the valence distribution $xu_V(x,Q^2)$. At small $\xi$ it should have the power behaviour 0.4525, corresponding to $\omega/\rho$ exchange. As $\xi \to 1$ its behaviour should be $(1-\xi)^3$, according to the spectator-counting rule\cite{14}. Hence we take

$$xu_V(x,Q^2) = U(\xi)\psi(Q^2)$$

$$U(\xi) = \begin{cases} B_u \xi^{0.4525} & \xi < \xi_0 \\ \beta_u \xi^{\lambda_u} (1-\xi)^3 & \xi > \xi_0 \end{cases} \quad (9a)$$

We fix $\beta_u$ and $\lambda_u$ in terms of $B_u$ by requiring these two forms to join smoothly at $\xi = \xi_0$, and then determine $B_u$ by imposing the number sum rule

$$\int_0^1 \frac{d\xi}{\xi} U(\xi) = 2 \quad (10a)$$

The valence distribution $xd_V(x,Q^2)$ should have the same power behaviour 0.4525. The spectator-counting rule would make it also behave as $(1-\xi)^3$ as $\xi \to 1$; however, it is well known that this conflicts with the measured ratio\cite{19} $\nu W_2^n/\nu W_2^p$, which indicates that in $xd_V$ the coefficient of $(1-\xi)^3$ is very small. So we take

$$xd_V(x,Q^2) = D(\xi)\psi(Q^2)$$

$$D(\xi) = \begin{cases} B_d \xi^{0.4525} & \xi < \xi_0 \\ \beta_d \xi^{\lambda_d} (1-\xi)^4 & \xi > \xi_0 \end{cases} \quad (9b)$$

with $\beta_d$ and $\lambda_d$ again being fixed in terms of $B_d$ by joining the two forms smoothly and then $B_d$ determined from

$$\int_0^1 \frac{d\xi}{\xi} D(\xi) = 1 \quad (10b)$$
For the nonvalence distributions, for each light quark and antiquark we include a term that behaves as \( \xi^{-0.0808} \) for small \( \xi \):

\[
\begin{cases}
C\xi^{-0.0808}\phi(Q^2) & \xi < \xi_0 \\
\gamma\xi^{\lambda_s}(1 - \xi)^{7}\phi(Q^2) & \xi > \xi_0
\end{cases}
\]  

(11a)

The constants \( \gamma \) and \( \lambda_s \) are fixed in terms of \( C \) by joining these two forms smoothly at \( \xi = \xi_0 \), with the same \( \xi_0 \) as for the valence terms. There is no number sum rule for the nonvalence distributions, so \( C \) is a free parameter. In \( \nu W_2 \), (11a) is multiplied by \( 10/9 \), that is the constant \( A \) in (8a) is \( 10C/9 \).

When the two-component parton model was first formulated\(^{[20]} \), the component that later came to be called nonvalence\(^{[21]} \) had no contribution from \( \rho, \omega, f, a_2 \) exchange. This is because, at that time, the idea of exchange degeneracy was taken very seriously, not just for the Regge trajectories but also for their couplings. Since then, it has become clear that the degeneracy of the trajectories is satisfied very well but there is no basis for supposing that it extends also to the couplings\(^{[7]} \). So we include in each nonvalence distribution also a term behaving as \( \xi^{0.4525} \) at small \( \xi \). We write the total such contribution to \( \nu W_2^p \) as

\[
\begin{cases}
(B - B_u - B_d)\xi^{0.4525}\psi(Q^2) & \xi < \xi_0 \\
\beta\xi^{\lambda}(1 - \xi)^{9}\psi(Q^2) & \xi > \xi_0
\end{cases}
\]  

(11b)

where again there is only one free parameter, \( B \), after we have joined the two forms smoothly. Notice that we have not specified how this term divides among the quark flavours: while by definition (because the term is nonvalence) \( u \) and \( \bar{u} \) receive equal contributions, as do \( d \) and \( \bar{d} \), it is likely\(^{[22]} \) that \( \bar{u} \neq \bar{d} \). Notice also that we have, somewhat arbitrarily, chosen the power \( (1 - \xi)^9 \); the spectator-counting rule gives no guidance here, but we need a power greater than 7 to prevent the nonvalence distribution from becoming negative for large values of \( \xi \), since while \( B_u \) is close to \( \frac{3}{2} \) and \( B_d \) is about half that, the best fit for \( B \) is at quite a small value, as we already saw in figure 3. Again somewhat arbitrarily, we have used the same factor \( \psi(Q^2) \) to make all the \( \xi^{0.4525} \) terms vanish linearly with \( Q^2 \) at small \( Q^2 \).

The terms we have included so far have the the property that, at each \( x \), \( \nu W_2 \) increases as \( Q^2 \) increases, while the data show the opposite when \( x \) is not small. At large \( Q^2 \) this is a consequence of the perturbative evolution, but we are constructing a fit at \( Q^2 \)-values where perturbation theory is not applicable. We therefore include an additional “higher-twist” term

\[
ht(x, Q^2) = D\frac{x^2(1 - \xi)^2}{1 + Q^2/Q_0^2}
\]  

(12)

This resembles a term we introduced some years ago\(^{[23]} \), when we indentified it as a contribution from the virtual photon being absorbed by a diquark within the proton, though we
do not necessarily adhere to this interpretation now. By making it vanish quadratically as \( x \to 0 \), we have ensured that it does not contribute to the real-photon cross-section, while the power \((1 - \xi)^2\) gives a better fit than would \((1 - \xi)\). This term has two free parameters, \(D\) and \(Q_0\), in addition to the \(\xi_0\), \(C\) and \(B\) we already have. Our best fit to the data for \(Q^2 < 10\) GeV\(^2\) corresponds to

\[
C = 0.220 \quad B = 0.279 \quad \xi_0 = 0.071 \quad D = 15.88 \quad Q_0 = 550 \text{ MeV} \quad (13a)
\]

and is shown in figure 5. The \(\chi^2\) per data point is just less than 0.5. Again we have quoted the values of the parameters to high accuracy because the errors, particularly in \(D\) and \(Q_0\), are strongly correlated. For example,

\[
C = 0.220 \quad B = 0.274 \quad \xi_0 = 0.075 \quad D = 36.74 \quad Q_0 = 338 \text{ MeV}
\]

gives almost exactly the same \(\chi^2\). The values we obtain for \(C\) and \(B\) are rather stable, particularly that for \(C\). The values of the subsidiary parameters corresponding to the choices (13a) are

\[
B_u = 1.456 \quad B_d = 0.772 \quad \lambda_u = 0.683 \quad \lambda_d = 0.760 \quad \lambda = 1.144 \quad \lambda_s = 0.457 \quad (14)
\]

We emphasise that our nonperturbative fit is supposed only to apply for \(Q^2 < 10\). For larger values of \(Q^2\) it lies above the data at moderate and large \(x\), allowing room for the perturbative evolution to take over at \(Q^2 = 10\), or at some smaller value. See figure 6. We remark also that our choice to fit the data up to \(Q^2 = 10\) is somewhat arbitrary. If we replace this with \(Q^2 = 5\) the values of the parameters hardly change:

\[
C = 0.213 \quad B = 0.312 \quad \xi_0 = 0.069 \quad D = 15.88 \quad Q_0 = 554 \text{ MeV}
\]

and the \(\chi^2\) per data point is reduced to 0.3.

We have extended our fit to \(\nu W_2^n\), though the data here are rather more uncertain. Not only is there a lack of basic knowledge on exactly how to make deuterium corrections\(^{[24]}\), but also the NMC data are in the course of being changed slightly\(^{[25]}\). We use the NMC data, as published\(^{[19]}\), for \(\nu W_2^n/\nu W_2^p\). Our fit to \(\nu W_2^n\) uses the same values for \(\xi_0\) and \(C\) as for \(\nu W_2^p\), but allows for a different \(B\), corresponding to \(\bar{u} \neq \bar{d}\), and different “higher twist”. We choose to work with the same mass scale \(Q_0\) in the latter, but allow its normalisation \(D\) to be chosen by the fit to the data. So we have 2 new free parameters. For \(\nu W_2^n\) we end up with

\[
C = 0.220 \quad B = 0.169 \quad \xi_0 = 0.071 \quad D = 4.94 \quad Q_0 = 550 \text{ MeV} \quad (13b)
\]
which gives the fit shown in figure 7. Once again, the data do not determine the “higher twist” term at all precisely: we could have used a very different value of $D$ with $Q_0$ changed correspondingly. But it is clear that we need different terms for $\nu W_2^p$ and $\nu W_2^n$, not only for the “higher twist” but also for the nonvalence $\xi^{-0.0808}$ term: for $\nu W_2^p$ its coefficient is -0.454, while for $\nu W_2^n$ it is -0.337, so that indeed $\bar{u} \neq \bar{d}$.

We have succeeded in obtaining an excellent description of $\nu W_2^p$ and $\nu W_2^n/\nu W_2^p$ in the range $0 < Q^2 < 10$ with very few parameters. The form of our fit is simple, and motivated by theoretical principles. Our analysis should be regarded as complementary to the standard ones[17][22], which are mainly driven by considerations of perturbative QCD. We do not use any neutrino-scattering data to determine the free parameters. As we have explained, until $Q^2$ is rather larger than the range with which we are concerned, the neutrino data are not expected to be related at all precisely to the photon-exchange data, because they approach the limit $Q^2 \to 0$ differently. On the other hand, we fit the photon-exchange data down to small $Q^2$, even $Q^2 = 0$. The urgent question still remains of how properly to combine the perturbative and nonperturbative analyses, such as has been attempted for example by Badelek and Kwiecinski[26].

We have already remarked that the extrapolation of our fit down to very small $x$ is finally not very different from that obtained from (4). In figure 8 we show our curves plotted against $x$. If the HERA experiments find results for $\nu W_2$ significantly larger at small $x$ than our extrapolations, we claim that this will be a clear signal that they have discovered new physics. Of course, the hope is that they will discover the Lipatov pomeron. In relation to this, the question arises whether the effect of the soft pomeron, which is included in our curves, should be subtracted off from the data[3], with the Lipatov pomeron being identified with anything that may remain, or whether instead the Lipatov pomeron simply replaces the soft pomeron in the small-$x$ behaviour of $\nu W_2$. This is all part of the general study of the interface between perturbative and nonperturbative QCD, which will be so important at HERA.

One of us (PVL) is pleased to acknowledge vigorous discussions with Jean-René Cudell, Steve Ellis and Dieter Haidt.
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Figure captions

1 Data for the real-photon cross-section, with the fit from reference 7.

2 The parton model

3 NMC data at small $x$, with simple-power fit. Reading from top to bottom, the values of $x$ are

$$0.008, 0.0125, 0.0175, 0.025, 0.035, 0.05, 0.07$$

For clarity of presentation, the curves and data have been scaled by a different factor at each value of $x$; the scale factors are

$$4, 3.2, 2.5, 2, 1.5, 1.2, 1$$

4 EMC data for $x c(x, Q^2)$ with fit from reference 13. The data have been renormalised to take account of the revised branching ratio for a charm quark decaying to a muon. Reading from top to bottom, the values of $x$ are 0.00422, 0.0075, 0.0133, 0.0237, 0.0422, 0.075, 0.133, 0.237. For clarity of presentation, the curves and data have been scaled by a different factor at each value of $x$; the scale factors are 128, 64, 32, 16, 8, 4, 2, 1 respectively.

5 Data from NMC, SLAC and BCDMS with fit described in the text. For clarity of presentation, the curves and data have been scaled by a different factor at each value of $x$. Reading from top to bottom, the values of $x$, with the scale factors in brackets, are

(a) $0.008 (5), 0.0125 (4), 0.0175 (3.2), 0.025 (2.5), 0.035 (2), 0.05 (1.5), 0.07 (1)$

(b) $0.09 (7), 0.1 (5), 0.11 (3.5), 0.14 (2.5), 0.18 (2), 0.225 (1.5), 0.275 (1)$

(c) $0.35 (32), 0.45 (16), 0.5 (8), 0.55 (4), 0.65 (2), 0.75 (1)$

6 The curves of figure 6b extrapolated to larger $Q^2$. Reading from top to bottom, the values of $x$ are 0.09, 0.18, 0.275, 0.45 and 0.65, with scale factors 8, 4, 2, 1, and 1.

7 NMC data for $\nu W^2_n / \nu W^2_p$, with fit. The data and curve correspond to values of $Q^2$ that vary with $x$, from $\langle Q^2 \rangle = 0.4 \text{ GeV}^2$ at the smallest $x$ to 10.8 at the largest.

8 Fits to $\nu W^2_p$ extrapolated to very small $x$ for three values of $Q^2$. The curves are the total, together with its valence, nonvalence and “higher-twist” components
\[ \gamma p \quad 0.0677s^{0.0808} + 0.129s^{-0.4525} \]
Figure 1b

\( \gamma p \)

\[ 0.0677 \ (2\nu)^{0.0808} + 0.129 \ (2\nu)^{-0.4525} \]
Figure 2
Figure 4
Figure 5c
Figure 6
Figure 7
Figure 8
\[ \gamma p = 0.0677s^{0.0808} + 0.129s^{-0.4525} \]
Figure 1b

\[ \gamma p = 0.0677 \ (2\nu)^{0.0808} + 0.129 \ (2\nu)^{-0.4525} \]
Figure 2
Figure 5c
Figure 7
Figure 8
