TESTING COLD DARK MATTER WITH THE LOW-MASS TULLY-FISHER RELATION

MICHAEL R. BLANTON,1 MARLA GEHA,2 AND ANDREW A. WEST3

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ABSTRACT

In most cosmological theories, the galaxy mass function at small masses is related to the small-scale matter power spectrum. The circular velocity function (closely related to the mass function) is well studied for Local Group dwarfs. However, predictions and measurements are difficult for satellite galaxies because of ram pressure and tidal stripping. By contrast, isolated dwarf galaxies are less affected by these processes and almost always have 21 cm emission to trace their dynamics. Here we use isolated low-mass dwarf galaxies from the SDSS, with measured 21 cm widths, to test CDM cosmology. We find consistency between the predicted and observed number density of isolated galaxies down to \( V_{\text{max}} \sim 50 \text{ km s}^{-1} \). Our technique yields a direct test of small-scale cosmology that is independent of but weaker than the Ly\( \alpha \) forest: warm dark matter particles heavier than 0.5 keV cannot be ruled out (for the case of thermal gravitinos). New, blind H\( \text{i} \) surveys are expected to uncover a much larger number of isolated low-mass galaxies and will increase the power of our constraints at small scales. Using our sample, we also find that the Tully-Fisher relation for dwarfs is a strong function of environment, and that the baryon fraction is only a weak function of galaxy mass. Together with the strong dependence of gas fraction on environment, these results indicate that for dwarf galaxies, gas loss and the end of star formation are dominated by external, not internal, processes.

Subject headings: cosmology: observations — galaxies: dwarf — galaxies: kinematics and dynamics

Online material: color figures

1. INTRODUCTION: THE MASS FUNCTION AND COSMOLOGY

A critical test for any theory of cosmology and structure formation is whether it correctly predicts the galaxy mass function. The current cold dark matter model with a cosmological constant (\( \Lambda \)CDM; Spergel et al. 2007) makes robust predictions for the number of dark matter halos as a function of mass, finding roughly that \( dN/dM \propto M^{-1.8} \) (Sheth et al. 2001; Jenkins et al. 2001; Reed et al. 2003; Yahagi et al. 2004). A similar mass function describes the distribution of surviving subclumps within the dark matter halos (“subhalos”) that most investigators associate with the sites of galaxy formation (Colin et al. 1999; Gao et al. 2004; Kravtsov et al. 2004a; Reed et al. 2005; Zentner et al. 2005; Conroy et al. 2006). Of course, extending this model to predict the number density of galaxies in the universe requires including physical effects such as gas cooling, star formation, supernova feedback, and possibly the formation of supermassive black holes and their feedback, which are too complex to follow exactly in numerical predictions. Nevertheless, in the last three decades numerous approximate approaches to the problem have lent us some understanding of what the correct predictions might be and what physical processes a successful model may involve (e.g., Rees & Ostriker 1977; White & Rees 1978; White & Frenk 1991; Blanton et al. 1999; Somerville et al. 2001; Benson et al. 2003; Robertson et al. 2005; Croton et al. 2006).

Recently, various investigators have tested the halo mass function at high masses using galaxy clusters and found consistency describes the distribution of surviving subclumps within the dark matter halos as a function of mass, finding roughly that \( dN/dM \propto M^{-1.8} \) (Sheth et al. 2001; Jenkins et al. 2001; Reed et al. 2003; Yahagi et al. 2004). A similar mass function describes the distribution of surviving subclumps within the dark matter halos (“subhalos”) that most investigators associate with the sites of galaxy formation (Colin et al. 1999; Gao et al. 2004; Kravtsov et al. 2004a; Reed et al. 2005; Zentner et al. 2005; Conroy et al. 2006). Of course, extending this model to predict the number density of galaxies in the universe requires including physical effects such as gas cooling, star formation, supernova feedback, and possibly the formation of supermassive black holes and their feedback, which are too complex to follow exactly in numerical predictions. Nevertheless, in the last three decades numerous approximate approaches to the problem have lent us some understanding of what the correct predictions might be and what physical processes a successful model may involve (e.g., Rees & Ostriker 1977; White & Rees 1978; White & Frenk 1991; Blanton et al. 1999; Somerville et al. 2001; Benson et al. 2003; Robertson et al. 2005; Croton et al. 2006).

In addition, for individual, high-luminosity galaxies one can perform a similar test. If galaxy luminosity is related monotonically (with some scatter) to halo mass, then \( \Lambda \)CDM predicts the weak-lensing signal as a function of galaxy abundance. Tasitsiomi et al. (2004) have verified this prediction for galaxies with \( L_r > L_{r,*} \), and Seljak et al. (2005) have performed a similar test extending down to about \( 0.1L_{r,*} \).

Below \( 0.1L_{r,*} \), the luminosity function is closer to a power law, with a slope that varies somewhat over luminosity but is never steeper than about \( N \propto L^{-1.5} \) at most, significantly shallower than the prediction for the number as a function of halo mass (Trentham et al. 2005; Blanton et al. 2005b). If galaxies and subhalos are associated one to one, then the mass-to-light ratios of galaxies must increase substantially with decreasing luminosity. However, data on galaxies at the lowest luminosities have in the past been rather scarce. At circular velocities below \( L_r \sim 0.1L_{r,*} \) nobody has demonstrated consistency between the \( \Lambda \)CDM prediction and the galaxy mass function. Some valiant attempts exist, but rely on extrapolating the Tully & Fisher (1977) and Faber & Jackson (1976) relations into the low-luminosity regime (Desai et al. 2004; Goldberg et al. 2005).

The most extreme example of this issue is the “substructure problem” in the Local Group (Klypin et al. 1999; Kravtsov et al. 2004b). \( \Lambda \)CDM predicts hundreds of low-mass satellites of the Milky Way, but only approximately 20 are known (although the number is growing month by month; Ibata et al. 1995; Mateo 1998; Willman et al. 2005a, 2005b; Belokurov et al. 2006; Zucker et al. 2006a, 2006b). Simon & Geha (2007) show that the newly discovered galaxies have eased this discrepancy: a factor of 2–4 difference remains, but by taking into account baryonic physics, reasonable agreement is achieved. This test of cosmology probes the lowest mass galaxies possible and so is extremely sensitive to differences in the mass function slope. However, both predictions and observations are quite difficult in the vicinity of a luminous galaxy. From the point of view of theory, a number of processes, such as tidal stripping, ram
pressure stripping, dynamical friction, and merging onto the large galaxy, can occur near luminous galaxies (Kravtsov et al. 2004b; Bullock & Johnston 2005; Zentner et al. 2005; Mayer et al. 2006). When a dwarf galaxy enters the environment of a large galaxy, tidal stripping can reduce its mass. In addition, the larger dwarf galaxies are preferentially dragged to the center and merge with the large galaxy. All of these processes alter the predicted mass function, but to follow them all requires difficult-to-execute and physically uncertain simulations. From the point of view of observations, the ram pressure stripping removes any existing neutral gas disks, the component of dwarf galaxies known to extend furthest out into the dark matter halo (Stoehr et al. 2002). With only the stars, it is difficult to probe any dark halo that might still surround a Local Group dwarf.

On the other hand, isolated dwarf galaxies are possibly much simpler systems. Without a large galaxy nearby, the physical processes described in the previous paragraph cannot occur, simplifying the prediction of their mass function. In addition, as it happens, isolated dwarf galaxies essentially always (95% of the time) have intact H i disks (Geha et al. 2006a), allowing us to probe their masses out to large radii with relative ease (Swaters et al. 2002). The disadvantage of isolated dwarf galaxies is that they are far away, making them difficult to find at the lowest luminosities.

This paper represents a first step at examining the low-mass end of the mass function for isolated galaxies. We use a sample of dwarf galaxies selected from the Sloan Digital Sky Survey (SDSS; York et al. 2000). From the SDSS, we evaluate their number densities, and we estimate their maximum circular velocity using single-beam H i profiles taken using the Arecibo Observatory and the Green Bank Telescope. We compare these observations to theoretical predictions for the number density of isolated halos of the same circular velocity. By comparing circular velocities, we avoid the considerable theoretical and observational uncertainties in determining the total mass of halos and galaxies. While this approach represents a beginning, we discuss in the conclusions how we can improve our observational estimates of the masses and number densities of these objects and also find lower mass galaxies. With a much larger sample of isolated low-mass galaxies, our technique can provide a stringent and unique constraint on the power spectrum at small scales.

In § 2 we describe the optical and radio observations our results are based on. In § 3 we describe the Tully-Fisher relationship for isolated galaxies. In § 4 we describe our theoretical models. In § 5 we compare the observations to the theory. In § 6 we explore the robustness of our results to our definition of “isolation” in this context. In § 7 we examine the relationships among the baryonic, stellar, and total mass estimates in our sample. In § 8 we compare our cosmological constraints to those of other techniques. In § 9 we describe how these results might be improved in the future. Finally, in § 10 we summarize our results.

For determining luminosity distances and other derived parameters from observations, we have assumed cosmological parameters $\Omega_0 = 0.3$, $\Omega_m = 0.7$, and $H_0 = 100\ h\ km\ s^{-1}\ Mpc^{-1}$. Where necessary, we have used $h = 0.7$; otherwise, we have left the dependence on $h$ explicit. All magnitudes in this paper are $K$-corrected to rest-frame bandpasses using the method of Blanton et al. (2003) and $k\correct v4.1_4$, unless otherwise specified. Because of the small range of look-back times in our sample (a maximum of around 700 Myr), we do not evolution-correct any of our magnitudes.

2. OBSERVATIONS

2.1. Sloan Digital Sky Survey

To evaluate the luminosity function of dwarf galaxies, we use a modified version of the SDSS spectroscopic catalog. Blanton et al. (2005b) describe our sample, which is a subsample of the New York University Value-Added Galaxy Catalog (NYU-VAGC; Blanton et al. 2005a). We have updated that catalog from SDSS Data Release 2 to Data Release 4 (DR4; Adelman-McCarthy et al. 2006). The Blanton et al. (2005b) catalog represents a significant improvement over naively selecting galaxies from the SDSS catalog, which is not optimized for nearby, low surface brightness galaxies. Our catalog extends down to the spectroscopic flux limit of $m_r \sim 17.8$ used by the SDSS. For each galaxy, the catalog provides the SDSS redshift, emission-line measurements, multiband photometry, structural measurements, and environment estimates (for more catalog details see Blanton et al. 2005b). Distances are estimated based on a model of the local velocity field (Willick et al. 1997). Distance errors have been folded into error estimates of all distance-dependent quantities such as absolute magnitude and $H_i$ mass.

For our purposes, this catalog suffers from one major selection effect, due to the difficulty of detecting low surface brightness galaxies in the optical. As shown in Blanton et al. (2005b), the completeness as a function of half-light surface brightness drops below 50% at $\mu_{50, r} \sim 23.5\ mag\ arcsec^{-2}$. Blanton et al. (2005b) present a simple model for the effect that surface brightness has on the completeness, which assumes a lognormal surface brightness distribution with a mean that decreases as luminosity decreases. Figure 1 shows what this model implies about the missing fraction of galaxies as a function of luminosity in our sample, in terms of the correction factor $c$ we must apply to recover the “correct” number density at each luminosity. For comparison, we also show the equivalent factor for dwarf galaxies in the Local Group (Mateo 1998), given the SDSS surface brightness cutoff, and in the local catalog of Karachentsev et al. (2004). Note that the Karachentsev et al. (2004) curve is complicated by the fact that the quantities in the catalog (Holmberg radius and flux) are not enough to infer a half-light surface brightness (the terms in which we have calculated the completeness) uniquely in general, even if we assume an exponential profile. The Mateo (1998) and Karachentsev et al. (2004) catalogs bracket our correction factor of $1.7$ at $M_r = 5\ log_{10}h \sim 15$, indicating that it is roughly correct to within 20%. Therefore, we use the model of Blanton et al. (2005b) to correct the luminosity functions we present here, without any a posteriori corrections. Nevertheless, the uncertainty of how many galaxies we are missing due to this effect is the most worrying one affecting our results.

2.2. Environments of Galaxies

As argued above, there are advantages to finding isolated galaxies with which to test the circular velocity and mass functions. We cannot rely on the SDSS alone to determine whether a galaxy is isolated, for several reasons. First, the angular distances between nearest neighbor galaxies can be large for this nearby sample: for example, searching a 1 Mpc region around a galaxy 30 Mpc away corresponds to $2^\circ$ on the sky. Many of our dwarf galaxies are on the SDSS southern stripes, which are only 2.5° wide. In addition, because the SDSS reduction software is not optimized for large, extended objects and fails to process them correctly, the SDSS catalog does not contain many of the bright galaxies within 30 Mpc. Thus, to calculate the environments of our dwarf galaxy sample, we need a supplemental
for galaxies on the Tully-Fisher relationship (see context, we define galaxies as luminous when the distance of each to its nearest "luminous" neighbor. In this mine environments for our dwarf galaxies, we must determine which is a nearly complete catalog of nearby galaxies. To determine Catalog of Galaxies (RC3; de Vaucouleurs et al. 1991), brightest galaxies.

For galaxies that have the relevant entries listed, we call galaxies luminous if $\frac{M_r}{h} > 19$, corresponding to circular velocities of $V_c > 140 \, \text{km s}^{-1}$ for galaxies on the Tully-Fisher relationship (see §3). From the $B$ and $V$ photometry listed in RC3, we infer $M_r$ for each galaxy. For galaxies that have the relevant entries listed, we call galaxies luminous if $M_r < -19$. For galaxies that do not have the relevant entries but do have H I data listed, we call them luminous if $W_{20} > 300 \, \text{km s}^{-1}$ (as described in §2.3, $W_{20}$ is twice the maximum circular velocity of the H I gas). Finally, there are some galaxies with neither H I data nor optical photometry listed in RC3. For this small set, we extract the "magnitude" from the NASA Extragalactic Database (NED) (which empirically is very similar to the $B$-band RC3 magnitude for galaxies that have both) and apply an offset $M_r = M_{\text{NED}} - 1$. We call these galaxies luminous if $M_r < -19$. In addition, we update the coordinates in RC3 using NED coordinates for each of the catalog objects. This set of bright galaxies is not perfectly uniform but is suitable for our purposes.

We combine the SDSS galaxies with $M_r < -19$ with the RC3 luminous galaxy catalog (removing repeats between the two). Then, we determine the nearest neighbor distance by asking whether there is a luminous RC3 or SDSS neighbor within $2 \, \text{h}^{-1} \, \text{Mpc}$ and $400 \, \text{km s}^{-1}$ in redshift for each galaxy in our sample. In this paper we generally refer to isolated galaxies as those with no such neighbor within a projected separation $r_p = r_{\text{lim}} = 1.0 \, \text{h}^{-1} \, \text{Mpc}$. However, in order to test the robustness of our results, we vary our procedure below by using alternate limits of $M_r < -19$ (a "fainter tracer sample") and $M_r < -20$ (a "brighter tracer sample"). We also vary $r_{\text{lim}}$, as we describe explicitly below.

We can calculate the number density of galaxies by weighting with the $1/V_{\text{max}}$ values that Blanton et al. (2005a) describe. In this context $V_{\text{max}}$ is the maximum volume over which a given galaxy is observable; thus, $1/V_{\text{max}}$ is its contribution in terms of number density. We calculate this maximum volume by integrating over angle and comoving distance and multiplying by the angular completeness and the odds of observing a galaxy given its luminosity and the flux limit in each direction. Of course, some of the volume actually is not available (e.g., in a cluster a galaxy will never appear isolated), and in principle we might need to account for this fact. To do so, we can measure the "isolated fraction," the fraction of the volume of our sample that is isolated from luminous galaxies, by randomly placing points within the volume and testing their environment. This fraction is 0.95 and varies by less than 0.05 when we vary our definition of "isolated" as noted above, minor compared to our other uncertainties. Thus, we simply ignore this correction and calculate the number density of isolated galaxies over the entire volume.

The thick line in Figure 2 shows the cumulative number density of such isolated galaxies as a function of absolute magnitude, corrected for surface brightness incompleteness. This statistic depends on the definition of "isolated" that we use; the two thin lines use the fainter and brighter tracer samples described above. As this comparison shows, a change in magnitude of the tracer by about 1 mag is equivalent to a 20% change in the cumulative number density at $M_r < -14.7$ is $(6.83 \pm 2.3) \times 10^{-2} \, \text{h}^3 \, \text{Mpc}^{-3}$, with the uncertainty dominated by the definition
of “isolated” and the fraction of galaxies missing due to surface brightness effects.

2.3. Radio Observations at 21 cm

Geha et al. (2006a) present the radio observations of dwarf galaxies selected from the SDSS sample described above. The data were obtained on the Green Bank 100 m Telescope (GBT) and at Arecibo Observatory. Each observation had a velocity resolution better than about 3 km s$^{-1}$. The optical half-light radii of the dwarf galaxies in our sample are typically $\sim$8$''$ and should be completely contained with the radio beam size of 3$'$ and 9$'$ for Arecibo and GBT, respectively. We restrict ourselves to galaxies with detected H i. Geha et al. (2006a) showed that such galaxies dominated the isolated galaxy population, comprising about 95% of the galaxy population under our definition of “isolated” here (see their Fig. 4). In order to minimize inclination effects, we also restrict to $b/a < 0.5$.

We compute the 20% H i line width (W20) by finding the peak H i flux within 150 km s$^{-1}$ of the optical radial velocity of each galaxy and computing the difference between the nearest points having 20% of the peak flux. The integrated H i flux is calculated by expanding the W20 values by 20 km s$^{-1}$ on each side and integrating the flux in this region. Errors bars on the line widths and integrated fluxes were computed using a Monte Carlo bootstrap method: noise was added to the stacked one-dimensional radio spectra (based on the observed variance in the baseline) and the observed quantities remeasured. We calculated error bars on the line width and integrated flux from the scatter in the mean quantities recovered from Monte Carlo simulations.

As Geha et al. (2006a) showed, the dwarf galaxies in our sample have a significant rotation component. We derive the maximum rotation speeds as follows. We correct the observed H i line widths for line broadening due to turbulent velocity dispersion and inclination using the formula first proposed by Bottinelli et al. (1983):

$$V_{\text{max}} = \frac{W20 - W20_i}{\sin i},$$

where W20 is the observed H i line width, W20$_i$ is the turbulent velocity correction term, and $i$ is the inclination angle inferred from the axis ratios of the optical images.

We determine the axis ratios from a two-dimensional, single-component Sérsic fit and use the method of Haynes et al. (1984) to infer $i$, assuming that the intrinsic axis ratio is 0.2. For reference, assuming an intrinsic axis ratio of 0.5 would result in about a 10% decrease in our inferred $V_{\text{max}}$ values, and our general conclusions would remain the same.

We confirm the validity of a linear turbulence correction by modeling the integrated velocity profiles of simulated galaxies constructed from Hernquist (1993) model disk galaxies. For nearby dwarf galaxies with rotation velocities similar to our sample, Begum et al. (2006) have measured a velocity dispersion in the gas component of $\sigma_{\text{gas}} = 8$ km s$^{-1}$ from two-dimensional velocity maps. Using the Begum et al. (2006) value in our model disk galaxies results in a turbulence correction of W20$_i$ = 16 km s$^{-1}$, which we use here. Altering this turbulence correction (say, to 25 km s$^{-1}$) does not change our results below.

A valid concern about our method is that the H i does not extend far enough out in the galaxy halo to probe the full $V_{\text{max}}$ of the halo. Although the majority (8 out of 12) of our isolated, edge-on dwarf galaxies are flat-topped or double-horned profiles, indicating that the rotation curve is likely turning over, it remains possible that we are underestimating the $V_{\text{max}}$ values. As we show below, underestimating $V_{\text{max}}$ is conservative in the sense that warm dark matter models will predict smaller values than CDM.

Figure 3 shows our best estimate of $V_{\text{max}}$ for our dwarf galaxies (with $b/a < 0.5$) as a function of the distance to the nearest luminous galaxy. For the purposes of demonstrating the importance of environment, we have here included even galaxies that are isolated. Clearly there is a strong relationship between $V_{\text{max}}$ and projected separation. Within a projected separation $r_p = 1$ h$^{-1}$ Mpc of luminous galaxies there is a population of objects with $V_{\text{max}} < 40$ km s$^{-1}$, which is much rarer at large separations. Our galaxies are distributed in absolute magnitude roughly evenly in the range $-13.5 > M_r > -15.5$, independent of environment. Thus, at these luminosities the “forward” Tully-Fisher relationship—the circular velocity at a fixed luminosity—appears to be a strong function of nearest neighbor distance. This result implies that if low circular velocity galaxies exist in the field, they are too low luminosity to make it into our sample.

As Geha et al. (2006a) and others have found, dwarf galaxies near a luminous neighbor also tend to be red and gas-poor. Taken all together, these results suggest that some important physical effects are shaping the gas content, star formation histories, and inferred dynamics of dwarf satellite galaxies relative to isolated dwarfs. For this reason, we choose to concentrate our attention here on the isolated galaxies, whose $V_{\text{max}}$ values have a smaller dispersion and whose properties in general we expect to be less altered since formation, relative to those dwarfs perturbed by a massive neighbor.

Although it is immaterial to our analysis below, it is interesting to ask what physical effects are causing the trend in Figure 3. We can think of three explanations. First, for dwarfs in the vicinity of luminous galaxies, ram pressure stripping could remove gas at the largest galactocentric radii first, reducing the maximum circular velocity traced by H i emission. Second, tidal stripping could reduce the total mass and thus the circular velocities. Third, interaction-triggered star formation could either raise the
luminosities of satellite galaxies relative to isolated galaxies, bringing lower mass systems into our sample if they are near bright galaxies, or speed up star formation and use up the gas in the outer disk of the galaxy. To help investigate this question, in Figure 3 we have distinguished between single-peaked H I profiles (open circles) and double-peaked or flat-topped profiles (filled circles), as classified by Geha et al. (2006a). Because the dwarfs with low $V_{\text{max}}$ are predominantly single-peaked profiles (and we have enough resolution to see double-peaked profiles if they existed), we favor the explanation that gas has been stripped from the outsides. However, a definitive conclusion awaits a comprehensive analysis, including more detailed dynamics of these galaxies.

3. TULLY-FISHER RELATION FOR ISOLATED GALAXIES

Here we give an estimate of the Tully-Fisher relationship for isolated galaxies. We base our estimate on the sample described in the previous section for low luminosities, plus the recently published samples of Pizagno et al. (2007) and Springob et al. (2005) for higher luminosities.

Pizagno et al. (2007) presented a Tully-Fisher survey of galaxies found in the SDSS, using follow-up Hα rotation curves. They have performed model fits to the disk components of these galaxies to inclination-correct their maximum circular velocities. Unlike previous Tully-Fisher samples (e.g., Courteau 1997), which were selected to be very homogeneous sets of galaxies, the Pizagno et al. (2007) sample spans a large range of galaxy types and colors, resulting in a somewhat larger scatter in the Tully-Fisher relationship than found in other studies. For each of their galaxies we have determined its environment in the same manner as for our low-luminosity sample, and we only consider isolated galaxies here. However, as Pizagno et al. (2007) show, and as we have confirmed using our own measurements of environment, there is very little dependence of the Tully-Fisher relation on environment at high luminosities. As with the low-luminosity sample, we restrict ourselves to galaxies with $b/a < 0.5$, leaving 35 galaxies from Pizagno et al. (2007).

Springob et al. (2005) have compiled H I spectra from archival data sets for around 9000 galaxies in the local universe, observed originally with Arecibo, the 91 and 42 m Green Bank telescopes, the Nançay telescope, and the Effelsberg 100 m telescope. They have homogeneously analyzed these spectra, measuring their widths and H I fluxes. All their galaxies have optical data associated with them, including a measure of their axis ratio ($b/a$). We use the turbulence and inclination corrections to account for this fact. We used the outputs from TF Used in Fig. 4 (second column), this table yields the comparable $V_{\text{max}}$ resulting from matching to the Tully-Fisher relation in Fig. 4 (second column). It also lists the $V_{\text{max}}$ values we actually tried in Fig. 10 (third column).

| $M_r - 5 \log h$ | $V_{\text{max}}$ (km s$^{-1}$) |
|-----------------|-------------------------------|
| $-14.7$         | $56 \pm 3$                    |
| $-18.0$         | $108 \pm 5$                   |
| $-18.5$         | $116 \pm 6$                   |
| $-19.0$         | $143 \pm 7$                   |
| $-19.5$         | $156 \pm 7$                   |
| $-20.0$         | $180 \pm 4$                   |
| $-20.5$         | $214 \pm 7$                   |

Notes.—For each choice of $M_r$ (first column), this table yields the comparable $V_{\text{max}}$ resulting from matching to the Tully-Fisher relation in Fig. 4 (second column). It also lists the $V_{\text{max}}$ values we actually tried in Fig. 10 (third column).
To define “isolated,” we use tracer halos with $V_{\text{max}}$ values corresponding to the Tully-Fisher results listed in Table 1. We project the distribution of halos in a random direction and perturb the “redshift-space” positions of the halos to account for their peculiar velocities. Then we define “isolated” in the simulation using the same geometrical considerations used in the observations (a velocity difference of $|\Delta V| < 400$ km s$^{-1}$ and a projected separation $r_p < 1$ h$^{-1}$ Mpc) but relative to the tracer halo population. The velocity function of these isolated halos is shown in Figure 5 as the histograms. Down to 70 km s$^{-1}$ each histogram comes from the simulation, but below that we extrapolate the velocity function as a power law [roughly $\Phi(>V_{\text{max}}) \propto V^{-2.7}$]. The three histograms, as labeled, correspond to tracers with $V_{\text{max}} > 110$ km s$^{-1}$ (corresponding to galaxies with $M_r - 5 \log_{10} h < -18$), $V_{\text{max}} > 140$ km s$^{-1}$ (corresponding to galaxies with $M_r - 5 \log_{10} h < -19$), and $V_{\text{max}} > 180$ km s$^{-1}$ (corresponding to galaxies with $M_r - 5 \log_{10} h < -20$). For the bulk of this paper, we are concerned with the central class, shown as the thick histogram, but we use the other results to quantify how much our results depend on the choice of tracer population.

Because of the complex geometrical definition of “isolated,” it is useful to have this N-body estimate of the mass function. However, the cosmology used for the simulation does not correspond precisely to the current best-fit cosmology (e.g., Tegmark et al. 2006); in particular, it does not include the effects of baryons on the initial power spectrum. We adjust for this difference by using the excursion set formalism and the transfer functions of Eisenstein & Hu (1998) along with the mass function approximation from Warren et al. (2006). These methods are able to predict the mass function for any hierarchical cosmology, as a function of redshift and of large-scale environment. We use a particular implementation provided by A. Berlind (2006, private communication). These methods yield the mass function of galaxies, which we convert into a circular velocity function using the methods of Bullock et al. (2001b) using $M_\odot = 1.5 \times 10^{12} h^{-1} M_\odot$ and $\Omega_m = 0.24$.

First, we need to evaluate what large-scale environment our definition of “isolated” corresponds to. The smooth line in Figure 5 is the prediction for the mass function of halos in large-scale underdensities of $\delta = -0.4$, for the cosmology used in Kravtsov et al. (2004a). From this agreement, we conclude that in the excursion set mass functions, $\delta = -0.4$ is the underdensity that is most comparable to our isolation criterion.

Second, in order to adjust the results of Kravtsov et al. (2004a) for cosmology, we evaluate the ratio $f_\theta$ between the velocity function for the cosmology of Kravtsov et al. (2004a) (listed above) and the best fit of Tegmark et al. (2006) (the only differences are that in the latter $\sigma_8 = 0.76$ and $\Omega_m h^2 = 0.022$). Figure 6 shows this ratio as a function of maximum circular velocity. Over the range we use here, this correction is never more than about 10%. In order to compare our results from the Kravtsov et al. (2004a) simulations to observations, we first apply this correction factor to the predictions from the simulations.

In addition, we have incorporated a “warm dark matter” version of the Tegmark et al. (2006) cosmology that is identical in its large-scale structure but includes a light dark matter particle, with $m_{\text{DM}} = 0.5$ keV. The lightness of this particle increases its free-streaming length, which smooths fluctuations on scales approaching $1 h^{-1}$ Mpc. Here we apply the adjustment required to the transfer function as outlined by Bardeen et al. (1986) for the case of a thermal gravitino–like particle (their Appendix G, case 3). We simply input this new transfer function into the excursion set calculation of the mass function. Of course, on the scales comparable to the free-streaming length, the collapse of structure will cease to be hierarchical, as described by Bode et al. (2001). Thus, almost by definition this prediction will be incorrect; however, we use it as a rough approximation to a more correct prediction that might be available in the future.
shows the “correction” as a function of $V_{\text{max}}$ as the dashed line. In order to predict the warm dark matter case, we apply this correction factor to the simulations.

Figure 7 shows the resulting $V_{\text{max}}$ functions in the CDM and warm dark matter cases. In the next section we describe how we compare these theoretical predictions to the observations.

5. COMPARING THEORY TO OBSERVATIONS

The simplest possible comparison we can make between simulations and the observations is to try to put some observed points on the velocity function. After all, we have measured the number density of galaxies as a function of luminosity, and from the Tully-Fisher measurements we know the relationship between luminosity and the circular velocities. Given the luminosity function of Figure 2 (using the tracers with $M_r - 5 \log_{10} h < -19$), plus the relationship between luminosity and $V_{\text{max}}$ from Figure 4, we also plot the observed number density as a function of $V_{\text{max}}$ as the four points in Figure 7. In this comparison, there appear to be slight discrepancies between the CDM model and the observations, particularly at the bright end. The warm dark matter model is nearly as good a fit to the data but somewhat underpredicts (at a bit more than 1 $\sigma$) the number of isolated low circular velocity galaxies.

However, this method of comparison is sensitive to bias related to scatter in the relationship between luminosity and circular velocity. A more robust comparison can be achieved as follows. For a given predicted circular velocity function and a given observed luminosity function, we can find the relationship between circular velocity and luminosity that makes them consistent with one another. We do so here by parameterizing the conditional distribution $P(M_r | V_{\text{max}})$ of absolute magnitude $M_r$ for each $V_{\text{max}}$ as a piecewise linear mean with Gaussian scatter about that mean. We vary the parameters of the piecewise linear relationship to fit the luminosity function (using the Levenberg-Marquardt method implemented in the IDL routine mpfit distributed by Craig B. Markwardt). However, we fix the Gaussian scatter to have $\sigma_M = 2.5$ for $V_{\text{max}} \leq 10 \text{ km s}^{-1}$, to have $\sigma_M = 0.4$ for $V_{\text{max}} \geq 100 \text{ km s}^{-1}$, and to vary linearly with circular velocity in between.

Figure 8 shows the conditional distribution $P(V_{\text{max}} | M_r)$, now of $V_{\text{max}}$ as a function of $M_r$ (note that this is the converse relationship to that which we model). Here we use our best-fit relationship (using tracers in the simulation with $V_{\text{max}} > 140 \text{ km s}^{-1}$ and corresponding tracers in the observations with $M_r - 5 \log_{10} h < -19$). The lines are the quartiles of the distribution. The overplotted points are the data from Figure 4. Clearly the median values (shown as the boxes) agree rather well with the predictions. This constitutes a confirmation that the Tully-Fisher relationship and the luminosity function together are consistent with CDM predictions.

How well can these data exclude alternate scenarios? We explore this question by considering the warm dark matter model described above, with a light thermal gravitino–like dark matter particle with a mass of 0.5 keV. We perform the same procedure as described above and obtain the predictions shown in Figure 9. Here the observed circular velocities of low-mass galaxies tend to be higher than predicted, but not by significant amounts. The median is about 4 $\sigma$ away from the predicted median, which, given the systematic uncertainties here, we regard as a marginal exclusion of this model and therefore a lower limit on the mass of such a particle.

As noted above, there is some uncertainty in our inference of $V_{\text{max}}$ because we cannot be certain that we are probing out to the real maximum circular velocity of the halo. If so, that means that the points in Figure 7 should actually sit slightly to the right of their current position, making warm dark matter models less likely (although if the effect is big enough, also creating a problem with the comparison with CDM).

6. ROBUSTNESS RELATIVE TO OUR DEFINITION OF “ISOLATED”

Here we examine the robustness of our results to our definition of “isolation,” and how we relate isolation in the observations to
isolation in the simulations. There are a number of arbitrary decisions we have made here to define “isolated” in the observations. In particular, we chose a certain projected distance \( r_{\text{lim}} \) from galaxies of a certain absolute magnitude \( M_{r,\text{bright}} \) (or brighter). We must examine the sensitivity of our results to these arbitrary choices. In addition, we have also had to define “isolated” in the theoretical predictions. To do so, we have had to relate the absolute magnitude \( M_r \) used above to a \( V_{\text{max}} \) of the halos in the simulation. Of course, we do not know the exact correspondence, and so we need to examine how our results depend on errors in our estimate of it.

Figure 10 examines the sensitivity of our results to both sets of decisions. The top panels show the ratio of the predicted isolated galaxy circular velocity function \( f_{\text{iso}}(>V_{\text{max}}) \) at \( V_{\text{max}} = 56 \text{ km s}^{-1} \) to the observed \( f_{\text{iso}}(<M_r - 5 \log_{10} h = -14.7) \). The bottom panels show another comparison of the observations to the theory: the ratio of the observed \( V_{\text{max}} = 56 \text{ km s}^{-1} \) at \( M_r - 5 \log_{10} h = -14.7 \) to that predicted by equating the number density of halos larger than a given \( V_{\text{max}} \) to the number density of galaxies brighter than \( M_r - 5 \log_{10} h = -14.7 \). The left panels correspond to our CDM model, and the right panels correspond to our warm dark matter model with \( m_{\text{DM}} = 0.5 \text{ keV} \).

There are 27 points shown on each plot, each corresponding to a different definition of “isolated” in the observations and the theory. First, we check three different choices of tracer sample \( (M_{r,\text{bright}} - 5 \log_{10} h = -18, -19, \text{ and } -20) \), as shown by the rough horizontal position. We cannot observe fainter tracer samples over a large volume, and using brighter tracer samples would mean that we called some dwarfs “isolated” when they had a neighbor large enough to affect their observed \( V_{\text{max}} \) (see Fig. 3). Thus, these choices span the possible space of parameters.

Second, for each of these three choices of galaxy sample, we choose three different \( V_{\text{max}} \) values with which to define our halo sample for comparison. Our choices of predicted circular velocities for each observed sample are listed in Table 1 and are motivated by the observed uncertainty in the Tully-Fisher relation (corresponding to about 2–3 \( \sigma \)). In Figure 10, the larger symbols correspond to higher circular velocities.

Third, for each of those nine choices, we choose three different values of \( r_{\text{lim}} \) (0.7, 1.0, and 1.3 \( h^{-1} \text{ Mpc} \)), using the same radius for theory and observation. Smaller choices would include nonisolated dwarf galaxies in the sample; larger choices would make our resulting sample too small to be statistically useful. These three cases are offset from each other slightly in Figure 10 (left to right, respectively) for clarity.

These results indicate that our systematic uncertainties are (fractionally) about 0.3 in the comparison of number densities and 0.1 in the comparison of circular velocities. The difference is at least partly due to the approximate dependence \( \Phi(V_{\text{max}}) \propto V_{\text{max}}^{-2.7} \). In addition, we expect the comparison of circular velocities to be more robust, since it depends less on the scatter in the relationship between luminosity and circular velocity. While for our sample these systematics are about equivalent to the systematics associated with our surface brightness completeness selection, the systematics shown in Figure 10 will ultimately be the most difficult uncertainties to overcome, even when much more complete galaxy samples are available.

7. BARYONIC AND STELLAR CONTENT OF LOW-MASS GALAXIES

In the previous section we showed that the CDM model reasonably explains the number densities and circular velocities of low-luminosity galaxies. Assuming that the relationship between circular velocity and total mass that CDM theory predicts is correct, we can now investigate the baryonic and stellar mass content (relative to the total mass) of these low-mass galaxies. This census of the matter in dwarf galaxies may help us understand their creation and development over time.

To infer the total mass from \( V_{\text{max}} \), we use the methods of Bullock et al. (2001b) using \( M_* = 1.5 \times 10^{12} h^{-1} M_{\odot} \) and \( \Omega_m = 0.24 \). These methods have been calibrated down to masses of \( 10^{11} h^{-1} M_{\odot} \), or about 85 km s\(^{-1}\), so our use of them for the lowest luminosity galaxies represents an extrapolation of the current theoretical understanding. At our typical \( V_{\text{max}} \approx 56 \text{ km s}^{-1} \), the virial mass of the halo determined by this method is \( 2.5 \times 10^{10} h^{-1} M_{\odot} \).

To infer the stellar mass, we use the optical broadband SDSS data and the methods of Blanton & Roweis (2007). Any method for inferring the total stellar mass is sensitive to the initial mass function (IMF) of stars, since the lowest mass stars (most of those below 0.5 \( M_{\odot} \)) contribute almost no optical light but are a significant fraction of the mass. The differences between different reasonable choices of IMF can be up to 50%. Blanton & Roweis (2007) have chosen the Chabrier (2003) IMF and find stellar masses within about 30% of those found for the same galaxies by Kauffmann et al. (2003) using spectroscopic techniques. The median stellar mass for our isolated dwarf galaxy sample is \( 2.2 \times 10^7 h^{-2} M_{\odot} \).

By “baryonic mass” we mean here the sum of the stellar mass and neutral gas content, which we take to be \( M_\text{HI} = M_* + 1.4 M_{\text{H}_1} \), where \( M_{\text{H}_1} \) is the neutral hydrogen mass inferred from 21 cm observations and the factor 1.4 accounts for helium, molecular clouds, and metals. The median baryonic mass so defined is \( 2.5 \times 10^6 h^{-2} M_{\odot} \), much larger than the stellar mass contribution. Naturally, there may also be ionized hydrogen in the galaxy, which we do not try to account for here. Note that for the data from Geha et al. (2006a) we do not try to correct for self-absorption of \( \text{H}_1 \) because little evidence for any inclination dependence of the \( \text{H}_1 \)-to—stellar mass ratio is found in our sample. However,
Springob et al. (2005) did make such corrections, which can be up to 20%.

For the samples of isolated galaxies used in this paper, Figure 11 shows the ratio of baryonic mass to total mass as a function of \( r \)-band absolute magnitude (for \( h = 0.7 \)). The dashed line is the cosmic mean based on the results of Tegmark et al. (2006) \((\Omega_b/\Omega_m = 0.17)\), and the dotted line is the mean for the sample of Springob et al. (2005) taken alone. The mean of the low-luminosity galaxies from Geha et al. (2006a) is somewhat less than that of higher luminosity galaxies, about 8% of the cosmic value rather than 14%. The difference in the treatment of self-absorption may account for about half of this difference.

The baryonic fraction may continue to decrease at lower masses. However, at \( V_{\text{max}} \approx 50 \text{ km s}^{-1} \) isolated low-luminosity galaxies do not show much evidence that they have expelled or ionized very much more of their cold gas than have their more massive counterparts. This measurement supports detailed models of the physics of gas blowout and blow-away due to supernovae, which predict a loss of only a few percent for galaxies in this mass range (Mac Low & Ferrara 1999; Ferrara & Tolstoy 2000; Stinson et al. 2007). However, it disfavors models that prevent star formation in low-mass galaxies through significant baryonic mass loss (Dekel & Silk 1986; Cole et al. 2000; Mori & Burkert 2000; Bullock et al. 2000; Benson et al. 2003; Dekel & Woo 2003; Tremonti et al. 2004; Croton et al. 2006). Of course, reasonable modifications to those models in which feedback prevents the gas from forming stars but keeps it mostly in neutral form and within the galaxy disk are probably tenable. Furthermore, although some investigators have invoked outflows to explain the mass-metallicity relationship, Dalcanton (2007) has shown that alternate models without significant outflow can explain the observations.

We can also look at the relationship between the stellar mass of the galaxies and their total mass. Figure 12 shows the

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Fig. 10.—Dependence of our results on our definition of “isolated.” The left panels refer to our CDM model. The right panels refer to the warm dark matter model with \( m_{DM} = 0.5 \text{ keV} \). The top panels show the ratio of the observed number density of isolated galaxies with \( M_r - 5 \log_{10} h < -14.7 \) to the predicted number of isolated halos with \( V_{\text{max}} > 56 \text{ km s}^{-1} \) (that is, the vertical offset in Fig. 7). The bottom panels show the ratio of the median observed circular velocity at \( M_r - 5 \log_{10} h < -14.7 \) to that predicted in Figs. 8 and 9. All of these results are shown as a function of how we define “isolated,” which we do in 27 different ways here. We choose (1) three different absolute magnitude limits for the tracers in the observed sample, \( M_r - 5 \log_{10} h < -18, -19, \) and \(-20\),  as shown by the rough horizontal position; (2) for each sort of observed tracer, three choices of minimum \( V_{\text{max}} \) for tracers in the predicted sample, as listed in Table 1 and as shown by the size of the symbols (larger corresponds to higher circular velocity of tracer); and (3) three different minimum projected radii from the tracers, \( r_p = 0.7, 1.0, \) and \( 1.3 \text{ h}^{-1} \text{ Mpc} \), as shown by small offsets in the horizontal positions (left to right). [See the electronic edition of the Journal for a color version of this figure.]
The mean of the Springob et al. (2005) measurements. The dotted line at 0.025 is the cosmic mean based on cosmological measurements (Tegmark et al. 2006). The dashed line at 0.17 is the cosmic mean based on baryonic mass, as described in the text. The dashed line at 0.17 is the mean of the Springob et al. (2005) measurements.

The dynamical-to-stellar mass ratio as a function of stellar mass for the three samples used here. This relationship shows a strong trend, illustrating the strong dependence of star formation efficiency on mass, at least for disk galaxies. It is, of course, exactly this dependence that causes the luminosity and stellar mass functions to be shallow while the total mass function of galaxies is so steep, as described in § 1.

8. COMPARISON TO OTHER CONSTRAINTS

While methods like the cosmic microwave background and the galaxy power spectrum constrain the cosmological model on large scales, our method essentially tests the predictions for the amount of power at around $1\ h^{-1}\ Mpc$ or a bit less. Other investigations also inform us about constraints on these scales. Some are more powerful statistically (such as the small-scale Lyc forest), and others probe smaller scales (such as using dwarf galaxies in the Local Group). As with large-scale cosmology, the complex nature of all such constraints means that we want to test the CDM model using numerous techniques.

As we discuss these techniques, we will talk about them in the context of the thermal gravitino mass. It is worth remembering that other models for the dark matter particle affect the power spectrum in somewhat different ways at a chosen mass (Abazajian & Koushiappas 2006), and certain models have other observable consequences (e.g., the sterile neutrino; Seljak et al. 2006).

The most statistically powerful current method is the small-scale Lyc forest power spectrum from quasars. Narayanan et al. (2000), for example, have published lower limits on the mass of the CDM particle based on the power spectrum of these fluctuations, compared to those predicted by an approximation to gas physics applied to a pure $N$-body dark matter simulation. More recent work by Seljak et al. (2006) and Viel et al. (2008) has found much more stringent limits, corresponding to about a 4 keV thermal gravitino at 2 $\sigma$ (and in combination with other constraints ruling out sterile neutrinos entirely). Although the physics of these fluctuations is simpler than that of galaxy formation, it is not trivial, and these limits could be significantly changed with better modeling (e.g., Abazajian & Koushiappas 2006).

Both the method used in this paper and the Lyc forest power spectrum probe the nature of the power spectrum on close to megaparsec scales. By using much less massive dwarf galaxies, one might probe much smaller scales in the initial power spectrum and significantly raise the lower limit on the dark matter particle mass. Dwarf galaxies at sufficiently low masses are currently only observable in the Local Group. Predicting the circular velocity function of dwarf satellites near a large galaxy is difficult: dwarf satellites tend to be gas-poor and have probably experienced ram pressure and/or tidal stripping, as well as tidally induced star formation. These complications introduce large systematic uncertainties into the predictions for the circular velocities of dwarfs near bright galaxies. A number of authors have shown that the observed counts of Milky Way dwarfs can be explained in the context of CDM using certain models for the formation of these dwarfs (e.g., Kravtsov et al. 2004b; Simon & Geha 2007). However, because such analyses invoke largely uncertain physical effects, their quantitative constraints on alternative models are weak (and, in fact, not usually discussed!).

Another common way of studying small scales is to use the inner profiles of dark matter–dominated, low surface brightness galaxies (McGaugh et al. 2003; van den Bosch et al. 2003; Swaters et al. 2003; Rhee et al. 2004; de Blok 2005; Kuzio de Naray et al. 2008). These inner profiles are usually found to be not as steep as those predicted for typical pure dark matter halos. While these tests probe dark matter physics on the smallest scales, they suffer theoretical and observational difficulties much worse than those facing mass function analyses. The inner profile involves physics occurring in the inner 1 kpc of galaxies, where cooling and gas physics is important. The theoretical difficulty is that the actual profile predicted by CDM, when baryons are included, cannot be reliably determined by current simulations because the physics of the baryons is so complex, involving shocks,
cooling, star formation, and perhaps massive black hole formation. The observational difficulty is that interpreting the observed velocity fields in the inner parts of galaxies in terms of a mass profile is very difficult, and details of the geometry can wash out a steep profile. Thus, while in principle these techniques can probe the dark matter small-scale initial conditions on extremely small scales, in practice it is too difficult to do so yet in a way that is competitive with the mass function constraints.

A less direct method of probing small-scale CDM fluctuations uses gravitational lensing anomalies. Substructure in lens galaxy halos alters the fluxes, but not positions, of lens images and can result in so-called flux anomalies (Dalal & Kochanek 2002; Schechter & Wambsganss 2002). However, it is hard to extract from observed anomalies a precise constraint on the small-scale power spectrum, since it is a highly degenerate situation and large-scale structure along the line of sight may be relevant (Metcalf 2005). There are no published constraints on dark matter properties using this technique.

9. FUTURE DIRECTIONS

The analysis of this paper, while consistent with the CDM model, puts only mild constraints on alternative models (for example, a warm dark matter particle as light as 0.5 keV is barely ruled out). How can this analysis be improved in the future? Two paths are possible: first, increasing the depth and completeness of our optical plus HI sample; second, using upcoming blind HI surveys to push to considerably lower masses. Both paths require improving our understanding of the theoretical predictions at the low-mass end.

Our analysis of the current SDSS sample would be substantially improved with more HI follow-up observations. Because Geha et al. (2006a) were studying the general HI properties of dwarfs, we only targeted about 12 systems edge-on enough and isolated enough to include in the analysis of this paper. Our results could be put on a much firmer footing with an increase in our follow-up sample. In DR4 there are 64 galaxies with $b/a < 0.5$, $M_r - 5 \log_{10} h > -15$, and $r_p > 1 h^{-1}$ Mpc, and obviously there are still more in later releases. Additional follow-up or deeper HI sky surveys should fill this gap in the future.

The optical SDSS sample on which our analysis here is based will increase by DR8, the final SDSS release, perhaps by a factor of 2. However, this will not substantially reduce the uncertainties in the luminosity function, which are already dominated by the surface brightness completeness correction (Fig. 1). Blanton et al. (2005b) concluded, based on introducing simulated galaxies into raw SDSS imaging data, that the SDSS photometric pipeline (optimized for reasonably high surface brightness galaxies around $z \sim 0.1$) was probably failing at a brighter surface brightness limit than the data required, and that with a differently optimized pipeline it might be possible to push the surface brightness limits to 25 mag or more. Doing so would allow us to probe magnitudes as faint as $M_r = 5 \log_{10} h \sim -12$ over cosmological volumes. Another possibility would be to wait for upcoming, deeper surveys such as the Dark Energy Survey (DES; Wester et al. 2005), which has first light in late 2010, Pan-STARRS 4 (Hodapp et al. 2004), whose prototype telescope PS1 will probably see first light in 2007, or the Large Scale Synoptic Telescope, which has first light in 2014. Searches for low surface brightness galaxies (through their diffuse light) tend to be dominated by the scattered light background, so it is difficult to anticipate how well any of these surveys can do. Any of these possibilities would require spectroscopic follow-up, probably searching for 21 cm emission in the radio or targeting HI regions in the galaxies.

Blind searches for galaxies in 21 cm may show even more promise, particularly since isolated dwarf galaxies virtually always exhibit HI (Geha et al. 2006a) and since the dynamics of each galaxy will be measured simultaneously with its detection. Unfortunately, HIPASS appears to not be deep enough to provide a competitive sample in this respect (many of the SDSS galaxies in our sample are undetectable in HIPASS; Geha et al. 2006a). ALFALFA (Giovanelli et al. 2005) can detect galaxies with HI masses of $10^7 h^{-2} M_\odot$ at distances of $20 h^{-1}$ Mpc. If the full 7000 deg$^2$ planned survey is completed, the overall volume mapped will be about $6 \times 10^4 h^3$ Mpc$^{-3}$. That large a volume has an expected cosmic variance of around 30%, although given our restriction to isolated regions, the actual cosmic variance uncertainties will be lower. Assuming a baryonic-to-total mass fraction of 0.02 (see Fig. 11) and using the methods of Bullock et al. (2001a), this mass corresponds to $V_{\text{max}} \sim 20$ km s$^{-1}$. Galaxies of this mass in the preliminary ALFALFA catalog of Giovanelli et al. (2007) have a median $W_{50}$ measurement of $\sim 40$ km s$^{-1}$, consistent with this estimate. Assuming (conservatively) that 30% cosmic variance errors dominate the uncertainties, using our techniques here one could marginally exclude a warm dark matter particle as high as 2 keV in mass. Future surveys such as AMI/ALFALFA will contribute a similar volume (of a distinct chunk of the universe) and increase this precision somewhat. In any case, these new HI surveys will push this technique into a low circular velocity regime that is currently only tested with observations of Local Group satellites.

Even to make use of the current data, however, we probably require a better understanding of the theoretical predictions. For example, as we noted above, the excursion set predictions we are making for “warm dark matter” are not entirely self-consistent, since we expect in this regime that the hierarchical picture will start to break down. Under these conditions, Bode et al. (2001) found that the number of forming halos was much smaller than the excursion set prediction. Thus, it may be that our observations would put stronger constraints on a correctly calculated prediction. However, doing so is difficult, since the effective mass resolution for warm dark matter simulations appears to scale much less favorably than for CDM (see Wang & White [2007], who indeed argue that even the number of halos predicted by Bode et al. [2001] is an overestimate).

Finally, it is worth noting two possible fundamental limits to the technique we describe here. First, we rely on the 21 cm emission to probe the dynamics in the flat part of the rotation curve. While this appears to be the case for most of our galaxies here (based on their double-peaked morphology), it will not necessarily be true of the typical very low mass galaxy. If so, one would need to resort to comparing to CDM predictions in the very inner parts of halos. Second, at lower masses, it may happen that reionization evaporates the lowest mass halos, preventing the formation of stars and the existence of any neutral gas at all. If so, our counting technique will fail. For the galaxies in the mass range we study here, this appears to be a weak effect, but it may occur at smaller masses.

10. SUMMARY

We demonstrate that the predictions of CDM are consistent with the number density of isolated low circular velocity galaxies ($V_{\text{max}} \sim 50$ km s$^{-1}$), at a precision of about 30% (Figs. 7 and 8). Our major systematic uncertainties (which dominate our error budget) are related to our definition of “isolated” and our model of the surface brightness completeness of the SDSS at low luminosities. These results represent a valuable independent check of the small-scale predictions of the CDM model. Our
technique avoids many of the systematic uncertainties associated with observations of satellite galaxies in the Local Group, related to the complex physics of ram pressure and tidal stripping. From a statistical point of view, our results are less powerful than the current constraints from the Ly$\alpha$ forest power spectrum (Narayanan et al. 2000; Abazajian 2006; Seljak et al. 2006; Viel et al. 2008) and only marginally exclude a thermal gravitino-like dark matter particle with $m_{DM} \sim 0.5$ keV. With an improved sample, this technique will offer a powerful and unique constraint at small scales.

We also find several secondary results that are relevant to the formation of these dwarf galaxies:

1. At low luminosities, the Tully-Fisher relationship appears to be a function of environment, with dwarf satellite galaxies having lower circular velocities than isolated dwarf galaxies, by up to a factor of 2 or more. There is circumstantial evidence from the nature of the H$\alpha$ profiles that this effect is due to stripping of the outer gas in satellite galaxies, although other processes may be at work.

2. The baryonic mass fraction of galaxies (counting neutral gas plus stars) appears to be a weak function of luminosity down to $M_r = -5 \log h \sim -14$, decreasing by 40% at most (from about 14% to a minimum of about 8% the cosmic mean; Fig. 12). This result disfavors models that call for a preferentially large amount of baryonic outflow in dwarf galaxies (due to internal processes such as feedback).

3. The ratio of total to stellar mass is a very strong function of stellar mass, ranging from 50 or so at the highest luminosities to over 1000 at the lowest luminosities.

Taken together with the deficit of neutral gas in dwarf galaxies near luminous neighbors (Geha et al. 2006a), these results suggest that the primary effects that remove gas and end star formation in dwarf galaxies (with circular velocities of 50 km s$^{-1}$ or so) are external interactions with bright neighbors, rather than internal processes such as feedback and outflows.

Increasing our sample of isolated dwarf galaxies with H$\alpha$ follow-up (it is straightforward to increase the current sample by more than a factor of 5) would improve the precision of all of these results, which are based on a relatively small sample of isolated galaxies. Upcoming blind H$\alpha$ surveys such as the ALFALFA survey on Arecibo are going to impose better constraints on cosmological models at small scales, as well as better explore the issues of dwarf galaxy formation. They will propel the study of field dwarf galaxies into a low circular velocity and low-mass regime previously studied only in the Local Group.

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REFERENCES

Abazajian, K. 2006, Phys. Rev. D, 73, 063513
Abazajian, K., & Koushiappas, S. M. 2006, Phys. Rev. D, 74, 023527
Adelman-McCarthy, J. K., et al. 2006, ApJS, 162, 38
Bahcall, N. A., et al. 2003, ApJ, 585, 182
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Begum, A., Chengalur, J. N., Karachentsev, I. D., Kaisin, S. S., & Sharina, M. E. 2006, MNRAS, 365, 1220
Belokurov, V., et al. 2006, ApJ, 647, L111
Benson, A. J., Bower, R. G., Frenk, C. S., Lacey, C. G., Baugh, C. M., & Cole, S. 2003, ApJ, 599, 38
Blanton, M. R., et al. 2005, ApJ, 635, 208
Bode, P., Ostriker, J. P., & Turok, N. 2001, ApJ, 556, 93
Bottinelli, L., Guigouneim, P., Patoulet, G., & de Vaucouleurs, G. 1983, A&A, 118, 4
Bullock, J. S., Dekel, A., Kolatt, T. S., Kravtsov, A. V., Klypin, A. A., Porciani, C., & Primack, J. R. 2001a, ApJ, 555, 240
Bullock, J. S., & Johnston, K. V. 2005, ApJ, 635, 931
Bullock, J. S., Kolatt, T. S., Sigad, Y., Somerville, R. S., Kravtsov, A. V., Klypin, A. A., Primack, J. R., & Dekel, A. 2001b, MNRAS, 321, 559
Bullock, J. S., Kravtsov, A. V., & Weinberg, D. H. 2000, ApJ, 539, 517
Chabrier, G. 2003, PASP, 115, 763
Cole, S., Lacey, C. G., Baugh, C. M., & Frenk, C. S. 2000, MNRAS, 319, 168
Colin, P., Klypin, A. A., Kravtsov, A. V., & Khokhlov, A. M. 1999, ApJ, 523, 32...
