Matter Wave Interference Pattern in the collision of bright solitons (Bose Einstein condensates) in a time dependent trap

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We show that it is possible to observe matter wave interference patterns in the collision of bright solitons (Bose-Einstein condensates) without free ballistic expansion for suitable choices of scattering length and time dependent trap.

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I. INTRODUCTION

When a gas of massive bosons are cooled to a temperature very close to absolute zero in an external potential, a large fraction of the atoms collapse into the lowest quantum state of the external potential forming a condensate known as a Bose-Einstein condensate (BEC) [1-4]. Bose-Einstein condensation is an exotic quantum phenomenon observed in dilute atomic gases and has made a huge turnaround in the fields of atom optics and condensed matter physics. The recent experimental realization of BECs in rubidium [5] has really kickstarted the upsurge in this area of research leading to flurry of activities in matter physics. The recent experimental realization of BECs in rubidium [5] has really kickstarted the upsurge in this area of research leading to flurry of activities in matter physics. The recent experimental realization of BECs in rubidium [5] has really kickstarted the upsurge in this area of research leading to flurry of activities in matter physics. 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of eq. (2) for both regular and expulsive potentials, we introduce the following modified lens transformation

$$\psi(z,t) = \sqrt{A(t)}Q(z,t)\exp(i\Phi(z,t))$$

(3)

where the phase has the following simple quadratic form

$$\Phi(z,t) = -\frac{1}{2}c(t)z^2.$$  

(4)

Substituting the modified lens transformation given by eq. (3) in eq. (2), we obtain the modified NLS equation

$$iQ_t + \frac{1}{2}Q_{zz} - ic(t)zQ_z - ic(t)Q + \gamma(t)A(t)Q^2Q = 0,$$  

(5)

with

$$M(t) = c'(t) - c(t)^2,$$  

(6)

and

$$c(t) = -\frac{d}{dt}\ln A(t).$$  

(7)

Equation (5) admits the following linear eigen value problem

$$\phi_z = U\phi, \quad U = \begin{pmatrix} i\zeta(t) & Q \\ -Q^* & -i\zeta(t) \end{pmatrix}$$

(8)

$$\phi_t = V\phi,$$

$$V = \begin{pmatrix} -i\zeta(t)^2 + ic(t)\zeta(t) & (c(t)z - \zeta(t))Q \\ +\frac{i}{2}\gamma(t)A(t)|Q|^2 & +\frac{i}{2}Q_z \\ -(c(t)z - \zeta(t))Q^* & i\zeta(t)^2 - ic(t)z\zeta(t) \\ +\frac{i}{2}Q_z^* & -\frac{i}{2}\gamma(t)A(t)|Q|^2 \end{pmatrix}$$  

(9)

In the above linear eigenvalue problem, the spectral parameter which is complex is nonisospectral obeying the following equation

$$\zeta'(t) = c(t)\zeta(t)$$

(10)

with $\gamma(t) = 1/A(t)$. It is obvious that the compatibility condition $(\phi_z)_t = (\phi_t)_z$ generates eq. (5).

Substituting eq. (7) with $\gamma(t) = 1/A(t)$ in eq. (6), we get

$$\gamma''(t)\gamma(t) - 2\gamma'(t)^2 - M(t)\gamma(t) = 0$$  

(11)

Thus, the solvability of the GP eqn. (2) depends on the suitable choices of scattering length $\gamma(t)$ and the trap frequency $M(t)$ consistent with eq. (11). Recently eq. (2) has been investigated and bright solitons have been generated employing gauge transformation approach [11].

III. BRIGHT SOLITON INTERACTION AND MATTER WAVE INTERFERENCE

To investigate the collisional dynamics of condensates in the presence of a time dependent trap, we now consider the two bright soliton of the following form,

$$\psi^2(z,t) = \sqrt{\frac{1}{\gamma(t)}}\frac{A_1 + A_2 + A_3 + A_4}{B_1 + B_2} e^{-\frac{1}{2}c(t)z^2}$$  

(12)

FIG. 1: The variation of scattering length $\gamma(t)=0.2sec\sqrt{2t}$ corresponding to case (i).

where

$$A_1 = \{ -2\beta_2[(\alpha_2 - \alpha_1)^2 - (\beta_1^2 - \beta_2^2)] \}
- 4i\beta_1\beta_2(\alpha_2 - \alpha_1)e^{(\theta_1 + \xi_2)}$$

$$A_2 = -2\beta_2[(\alpha_2 - \alpha_1)^2 + (\beta_1^2 + \beta_2^2)]e^{(-\theta_1 + \xi_2)}$$

$$A_3 = \{ -2\beta_1[(\alpha_2 - \alpha_1)^2 + (\beta_1^2 - \beta_2^2)] \}
+ 4i\beta_1\beta_2(\alpha_2 - \alpha_1)e^{(\xi_1 + \theta_2)}$$

$$A_4 = -4i\beta_1\beta_2[(\alpha_2 - \alpha_1) - i(\beta_1 - \beta_2)]e^{(n_1 - \theta_2)}$$

$$B_1 = -4\beta_1\beta_2[\sinh(\theta_1)\sinh(\theta_2) + \cos(\xi_1 - \xi_2)]$$

$$B_2 = 2\cosh(\theta_1)\cosh(\theta_2) [(\alpha_2 - \alpha_1)^2 + (\beta_1^2 + \beta_2^2)]$$

and

$$\theta_1 = 2\beta_1z - 4\int_0^t(\alpha_1\beta_1)dt' + 2\delta_1$$

$$\xi_1 = 2\alpha_1z - 2\int_0^t(\alpha_1^2 - \beta_1^2)dt' - 2\phi_1$$

$$\theta_2 = 2\beta_2z - 4\int_0^t(\alpha_2\beta_2)dt' + 2\delta_2$$

$$\xi_2 = 2\alpha_2z - 2\int_0^t(\alpha_2^2 - \beta_2^2)dt' - 2\phi_2$$

$$\alpha_1 = \alpha_{10}e^{\int_0^t\beta_1c(t')dt'}, \quad \beta_1 = \beta_{10}e^{\int_0^t\beta_1c(t')dt'}$$

$$\alpha_2 = \alpha_{20}e^{\int_0^t\beta_2c(t')dt'}, \quad \beta_2 = \beta_{20}e^{\int_0^t\beta_2c(t')dt'}$$

$$\gamma(t) = \gamma_0e^{\int_0^t\gamma(t')dt'}$$

To generate the interference pattern, we now allow the bright solitons to collide with each other in the presence of trap for suitable choices of scattering length $\gamma(t)$ and trap frequency $M(t)$ (or $c(t)$) consistent with eq. (11).

Case (i): When $c(t)=\sqrt{2tan\sqrt{2t}}$, the variation of scattering length $\gamma(t)=0.2sec\sqrt{2t}$ is shown in fig.1. Accordingly, the trap frequency $M(t)$ becomes a constant ($M(t)=2$). Under this condition, the collisional dynamics of bright solitons in the harmonic trap is shown in
Case(ii): When \(c(t)=\sqrt{2}\tan[\sqrt{2}t]\), the scattering length \(\gamma(t)=0.2\sin[\sqrt{2}t]\) corresponds to case (ii).

From the interference pattern, one observes that the matter wave density is maximum at the origin and it decreases gradually on either sides. Again, the phase change between the condensates oscillates with time as shown in fig.6.

Thus, our results reinforce the fact that the matter waves originating from the condensates (bright solitons) vary sinusoidally with time as in the case of Feshbach resonance (shown in fig.4) while the trap becomes \(M(t)=-2c^2t\sqrt{2t} - 2\csc^2\sqrt{2t}\). The evolution of the condensates in the above time dependent trap and the corresponding density evolution are now shown in figs 5a and 5b. From the interference pattern, one observes that the matter wave density is maximum at the origin and it decreases gradually on either sides. Again, the phase change between the condensates oscillates with time as shown in fig.6.
do interfere and produce a fringe pattern analogous to the coherent laser beams and the interference pattern is a clear signature of the long range spatial coherence of the condensates. It should be mentioned that the interference patterns were obtained earlier by Andrews et al. [13] under the condition of free ballistic expansion (atoms are released from the trap) while we selectively tune the frequency of the trap M(t) in accordance with the scattering length $\gamma(t)$ (consistent with eq.(11)) to observe matter wave interference in the collisional dynamics of bright solitons. It should be mentioned that the optical traps have opened up the possibility of realizing different types of temporal variation of the trap frequency M(t) while the scattering length $\gamma(t)$ can be controlled both by Feshbach resonance as well as through the trap frequency M(t). The phase change evolution which gives a measure of the coherence of the condensates shown in figs.3 and 6 shows that it oscillates with time and this is reminiscent of Josephsen effect wherein the phase difference between the superconducting wave functions oscillates with time.

IV. CONCLUSION

In conclusion, we have shown that it is possible to observe matter wave interference pattern by studying the collisional dynamics of condensates (bright solitons) for suitable choices of scattering length $\gamma(t)$ and trap frequency M(t) and this could happen without free ballistic expansion. We also observe that the phase difference between the condensates oscillates with time clearly showing the manifestation of Josephson effect. It would be interesting to interpret the matter wave destructive interference and whether this would mean that atoms plus atoms add up to vacuum.

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