Design of QAM-OFDM Radar-Communication Integrated Signal Based on Golay Complementary sequences

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Abstract. The integration of radar and communication is conducive to improving the utilization rate of hardware resources and spectrum. To address the issue that the communication information in orthogonal frequency division multiplexing (OFDM) integrated signal for radar and communication affects detection performance, an OFDM integrated signal for radar and communication based on precoding was proposed. The communication information of the integrated signal was pre-coded by using the Gray Complementary Sequences (GCS) with better correlation performance to improve the performance of the ambiguity function of the integrated signal and its detection ability. Simulation results show that GCS have better correlation performance compared to M and Gold sequences, and the OFDM integrated signal pre-coded by GCS has lower Peak Sidelobes Ratio (PSLR) and comparable radar performance to LFM.

1. Introduction

As the number of users and wireless devices increases, so does the demand for spectrum resources. Therefore, the integration and allocation of spectrum resources have become important hotspots in recent years. The combination of the radar system and communication system has gained widely attention for its equipment miniaturizing and high efficiency of the spectrum [1]. In addition, the demand is generated by 5G high-speed communication and high-performance radar, which promotes the rapid development of Dual-Functional Radar-Communication system (DFRC) [2]. Vehicles need to locate and transmit information to each other in the intelligent transportation system. The combat platform of the radar needs to detect targets and transmit information to the intelligence center [3]. Unmanned Aerial vehicles (UAV) need to perceive the environment and transmit environmental data to the target user [4].

There are two main ways to realize the integrated waveform design of DFRC. One is that the radar signal is modulated to carry communication information, and the other is that the communication signal is converted to pulse for detecting radar. OFDM integrated waveform is realized by using the second method, which is important for communication and radar detection [5]. Communication symbols are embedded into OFDM waveforms which are used in waveforms for delayed Doppler radar [6]. In [7], the Direct Sequence Spread Spectrum (DSSS) coding is used to form an integrated waveform. A design scheme for modulating Multicarrier Complementary Phase-Coded (MCPC) using OFDM was proposed in [8]. But, OFDM waveforms have problems of high autocorrelation sidelobe and Peak to Average Power Ratio (PAPR) which will cause serious distortion and poor performance of radar detection and tracking when passing the non-linear region of the high-power amplifier.
A method of pre-coding OFDM integrated signal of DFRC is proposed in this paper. GCS are introduced to modify communication symbols to improve the auto-correlation and cross-correlation performance of the original sequence and reduce the PSLR characteristics of ambiguity functions.

2. QAM-OFDM integrated signal

2.1. Signal model

Assuming that an integrated signal pulse consists of $N$ OFDM symbols, it can be represented as

$$x(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} d_{n,m} \exp \left\{ j2\pi m\Delta f (t-nT) \right\} \text{rect} \left( \frac{t-nT}{T} \right)$$

(1)

Given the number of subcarriers $M$, the duration of an OFDM symbol $T$, the subcarrier frequency interval $\Delta f = 1/T$, the rectangle window $\text{rect}(\cdot)$, and the QAM symbol $d_{n,m}$ of the $m$th subcarrier in the $n$th OFDM symbol, the signal with delay $\tau$ is represented as

$$x(t-\tau) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} d_{n,m} \exp \left\{ j2\pi m\Delta f (t-nT-\tau) \right\} \text{rect} \left( \frac{t-nT-\tau}{T} \right)$$

(2)

2.2. Ambiguity function

The output of matched filter represents the radar ambiguity function, which is widely used to analysis the target resolution of waveforms [9]. The integrated signal ambiguity function is defined as

$$\chi(\tau, f_d) = \int_{-\infty}^{\infty} x(t)x^*(t-\tau) \exp \left\{ j2\pi f_d t \right\} dt$$

(3)

where $\tau$ is the delay time, $f_d$ is the Doppler shift, $s^*(t)$ is the conjugate of $s(t)$. It can be obtained from equation (1)-(3)

$$\chi(\tau, f_d) = \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \sum_{p=0}^{M-1} \sum_{q=0}^{M-1} b(m,n)b^*(q,p) \exp \left\{ j2\pi m\Delta f \right\} \text{rect} \left( \frac{t-nT}{T} \right) \exp \left\{ j2\pi f_d t \right\} dt$$

(4)

![Figure 1. Integral range of ambiguity function](image)

The ambiguity function has different cases according to the $\tau$, $k$ is fix($\tau/T$) and fix($\cdot$) round down the numbers in figure 1. when $\tau = 0$, $f_d \neq 0$, the doppler resolution is

$$\chi(0, f_d) = \sum_{n=1}^{N} \sum_{m=1}^{M} \sum_{p=1}^{M} \sum_{q=1}^{M} \left( b(m,n)b^*(q,p) \right) \exp \left\{ j2\pi f_d T_{avg} \right\} T_{diff}$$

(5)

$$\text{sinc} \left[ (f_q - f_m - f_d) T_{diff} \right] \exp \left\{ -j2\pi (f_q - f_m) T_{avg} \right\}$$

when $\tau \neq 0$, $f_d = 0$, the delay resolution is

$$\chi(\tau, 0) = \sum_{n=1}^{N} \sum_{m=1}^{M} \sum_{p=1}^{M} \sum_{q=1}^{M} \left( b(m,n)b^*(q,p) \right) \exp \left\{ j2\pi f_d T_{avg} \right\} T_{diff}$$

(6)

$$\text{sinc} \left[ (f_q - f_m) T_{diff} \right] \exp \left\{ -j2\pi (f_q - f_m) T_{avg} \right\}$$

and
\[ T_{avg} = \max \{ (m-1)T, (p-1)T \} - \min \{ mT, pT \} \]  
\[ T_{avg} = \left( \max \{ (m-1)T, (p-1)T \} + \min \{ mT, pT \} \right) / 2 \]  

It can be seen from equations (5) and (6), the correlation of communication symbols will affect the radar resolution of the integrated signal. The aperiodic auto-correlation and cross-correlation function of the modulation information is reduced in the OFDM signals, which decrease the maximum sidelobe of the ambiguity function between different symbols. GCS with excellent autocorrelation and cross-correlation performances are introduced to pre-code communication data, it can improve the correlation of communication information and reduce the interference to radar resolution. Thus, an integrated signal with better radar performance can be obtained.

2.3. Golay complementary sequences
Two sequences \( a = (a_0, a_1, \ldots, a_{n-1}) \) and \( b = (b_0, b_1, \ldots, b_{n-1}) \) are GCS if the sum of their auto-correlations satisfies
\[ \sum_{k=0}^{n-1} a_k a_{k+i} + b_k b_{k+i} = \begin{cases} 2n, & i = 0 \\ 0, & i \neq 0 \end{cases} \]  

Equation (9) after the Fourier transform is
\[ |A(f)|^2 + |B(f)|^2 = 2n \]  
where \( A(f) \) is the Fourier transform of sequence \( a \) and \( B(f) \) is the Fourier transform of sequence \( b \). From equation (10) we can obtain
\[ |A(f)|^2, 2n \]  

If the power of sequence \( a \) is 1, the average power is \( n \), and the Peak-to-Average Power Ratio (PAPR) of GCS satisfies
\[ \text{PAPR,} \frac{2n}{n} = 2(\text{i.e. 3dB}) \]  

Therefore, the integrated signal with excellent radar detection performance can be obtained by pre-coding to generate OFDM symbols with good correlation performances and low PAPR. The DFRC of QAM-OFDM is shown in figure 2.

![Diagram](image)

Figure 2. The DFRC of QAM-OFDM

3. Simulations and discussion

3.1. Communication symbol sequence characteristics
The aperiodic auto-correlation and cross-correlation functions of M, Gold, and GCS are in figure 3. In this simulation, a 31-bit pair is used to generate the GCS. The M sequences with series 5 and feedback
coefficients \((0, 0, 1, 0, 1)\) and \((1, 1, 0, 1, 0)\) are used to generate the Gold sequences. The results show that GCS has excellent cross-correlation and as good auto-correlation as M sequences, which agree well with theoretical analysis.

![Gold Sequences](image)

Figure 3. (a) Auto-correlation property (b) Cross-correlation property

## 3.2. Performance analysis of ambiguity function

In this simulation, the OFDM signal is modulated by 16QAM, the protection interval is one-quarter of the length of OFDM data, and the parameters are shown in table 1. Compared with LFM, the central peak of the modulated signal is narrower, indicating that the modulated signal has good radar performance in figure 4. Due to the excellent correlation characteristics of GCS, the OFDM signal modulated by GCS has narrower main lobes and lower sidelobes by analysis the zero-doppler cut and zero-delay cut in figure 5 and figure 6.

Table 1. The parameters of OFDM signal in simulation

| Parameter                  | Value |
|----------------------------|-------|
| The number of subcarriers  | \(M = 52\) |
| The number of OFDM symbol  | \(N = 4\) |
| The duration of OFDM symbol| \(T = 1\mu s\) |
| The sampling frequency     | \(f_s = 5\text{MHz}\) |

![Ambiguity Functions](image)

Figure 4. Ambiguity functions of (a) LFM (b) OFDM modulated by direct information (c) OFDM modulated by GCS
The Peak Sidelobes Ratio (PSLR) was used to quantitatively compare the performance of the ambiguity function. The ratio PSLR of the highest side-lobe peak $P_s$ to the main-lobe peak $P_m$ is defined as

$$\text{PSLR} = 10 \log\left(\frac{P_s}{P_m}\right)\text{(dB)}$$

Table 2 shows that the PSLR of the OFDM signal modulated by GCS is $-8.43$ dB in the range dimension and $-6.98$ dB in the velocity dimension, which is significantly lower than the OFDM signal modulated by other sequences. This indicates that the pre-coded OFDM integrated signal by using GCS with good correlation characteristics can improve the radar performance.

Table 2. PSLR of different integrated waveforms

| Integrated signal                  | Delay resolution PSLR/dB | Doppler resolution PSLR/dB |
|------------------------------------|--------------------------|----------------------------|
| OFDM modulated by direct information | -5.52                    | -6.38                      |
| OFDM modulated by M sequence       | -7.21                    | -6.45                      |
| OFDM modulated by Gold sequence    | -6.19                    | -6.41                      |
| OFDM modulated by GCS             | -8.43                    | -6.98                      |

4. Conclusion

To improve the integration waveform performance of radar and communication, a method of pre-coding OFDM radar communication integrated signal by GCS is proposed in this paper. Firstly, the signal generation model is constructed by QAM symbol mapping, secondly, the ambiguity function of the signal is analyzed, and finally, the GCS is used to optimize the PSLR of the ambiguity function. The simulation shows that the GCS has excellent correlation performance, the PSLR of the improved
signal is lower and the radar detection performance is better when compared to M and Gold sequences.

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