Micropolar ferrofluid flow via natural convective about a radiative isoflux sphere

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Abstract
Current investigation scrutinizes the magnetohydrodynamic (MHD) natural convection flow of micropolar ferrofluid across an isoflux sphere with the impacts of thermal radiation and partial slip. Cobalt-nanoparticles with kerosene as the base fluid are considered. The governing partial differential conservation equations and convenient boundary conditions are rendered into a nondimensional form. The finite difference method (FDM) is then applied to determine the solution of a collection of resultant equations. The outcomes obtained by FDM have also compared with cited investigation. Illustrations describing influences of prominent parameters which provides physical interpretations of velocity, angular velocity, and temperature fields as well as the skin friction coefficient and Nusselt number are examined in detail with the help of graphical representations. This investigation determined that the skin-friction coefficient and heat transport rate reduced along with augmentation in the magnetic force and micropolar parameter, while opposite performance is adhered with elevating in the thermal radiation. Moreover, the boosted nanoparticle volume fraction reduced the skin friction coefficient and improved the Nusselt number.

Keywords
Natural convection, micropolar ferrofluid, isoflux sphere, partial slip, thermal radiation

Date received: 12 November 2020; accepted: 21 January 2021

Handling Editor: James Baldwin

Introduction
In the past few decades, the investigation of nanoparticles attained enormous significance because of its applications in the area of technological industry and biological science like as nano drug delivery, biomedical sciences, electromechanical systems, solar absorption, industrial cooling, and much more. The expression nanofluid was innovated by Choi\textsuperscript{1} which refers to engineering colloids that consists of nanoparticles scattered in a base fluid for enhancing the thermal conductivity. Nanofluid is normally applied in order to improve the heat transmission rate of the base liquid. It is a combined nano-sized particle (1–100 nm) which is suspended inside the base liquid. The nanofluid is normally produced of metals, carbides, oxides, and nano-metals.

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The base fluid is in common, water, blood, ethylene glycol, and so on. Eastman et al. performed an investigation that thermal conductivity is increased as copper nanoparticles are added up into a base fluid (water). In addition, they explored that the thermal conductivity improved by raising the copper nanoparticles into the traditional liquid. Buongiorno debated in his consideration that there are many mechanisms, which are significant to promote the thermal conductivity of the base fluid. Khan et al. studied the mass flux qualifications in the diffusion of nanoparticles with thermal radiation impact. Hayat et al. implemented the suspension of water-based nanoparticles encouraged by a rotating disk with variable thickness. Tlili et al. investigated the heat transport and nanofluid flow across a radiated stretching cylinder in a porous medium. Maleki et al. studied the impact of thermal radiation on nanofluid flow and heat transport along with a permeable plate. Various investigators have worked to augment the characteristics of nanofluids flow.

Magneto-nanofluids (ferrofluids) have a vast use in areas like as magneto-optical fabric floating isolation, wavelength sensors, nonlinear optical devices, optical fibers, hypoxia, pharmacology, optic stimulators, and so on. The magneto-nanofluid has the features of both magnetics and liquids. Each utilized, magnetic strength inspires the reconsists the concentration and dissolved particles within the fluid regime that highly affects the flow emulation of the heat transmission. Magneto-nanofluids are efficient in driving the particles through tissues via magnets up the blood flow, which is due to the verity that the magnetic nanoparticles have been scrutinized to be more viscid to tissue cells than the non-malignant cell kinds. These particles expend more energy than the micro-particulates in reversing present magnetic strengths possible in humans like as in cancer medication. Eid ascertained the MHD mixed convection flow of two-phase chemically reacting nanofluid pattern. Rashad explored the magneto-slip of nanofluid flow on a radiated wedge. Sandeep et al. scrutinized the magneto-nanoparticles by representing fully-accurate numerical exploration. Mishra et al. explored the rheology of nanoliquid to perform the thermo-diffusion features in stretchable surface. Many researchers have examined this particulars study in the view of various elements and, including the magneto-nanofluids flow, for example, see Refs. 18–26

During the last decade, the requirement to model and shape the liquid that comprises rotating microconstituents have given enchantment to the micropolar liquid theory. The fluids that couple the particle rotary movement and macroscopic velocity distribution are famous as micropolar liquids. Such liquids are synthetic of indeclinable elements that are enfold in a viscous or sticky conduit. Models of such liquids are blood flow, ferrofluids, and bubbly liquid. The industrial applications of these liquids are lubricant fluids, biological structures, and polymer solutions. The concept of micropolar fluid model is primarily coined by Eringen. Later, diverse considerations are performed concentrating on this fundamental non-Newtonian fluid. Ahuja supposed from his experimental research that the improvement in heat transmission may be due to the nanoparticles rotation about their own pivot due to the shear stress impact, and therefore a three-dimensional hydrodynamic boundary layer was also noticed. The rotating micro-constituents’ impacts in nanofluids should be addressed to realize the fluid flow conduct in a preferable way and then the micropolar theory demonstrates the variation between numerical and experimental observations. However, a new type of nanofluids as micropolar fluid has been demonstrate by many investigators. Bourantas and Loukopoulos analyzed the magneto-natural convection flow of micropolar nanofluid driven by inside a square cavity. Bourantas and Loukopoulos have numerically modeled the natural convective flow of micropolar nanoliquids. They explored that the micro-rotations in general decrease overall heat transmission from the heated side and should not hence be ignored. Rashad et al. explained the mixed convective flow of micropolar nanofluid through a cylinder in a porous media. Shah et al. discussed the thermal behavior by magnetic strength on micropolar nanofluid flow between two rotating parallel plates. Rashad et al. investigated the micropolar nanofluid flow by unsteady mixed convective through a stretchable surface. Khan et al. analyzed the magneto-natural convection flow of polar nanoliquid past a truncated radiative cone.

The survey of the above-mentioned literature designates that considerable study is available that reports the notions, about the nanoparticles flow, by applying a diversity of geometrical presumptions. However, to the best of our knowledge, no study far is reported for kerosene carrying Cobalt micropolar nano-particles through isoflux sphere. The main objective of the investigation in hand is to explore numerically the magneto-micropolar ferrofluid flow across the isoflux sphere by natural convective with impacts thermal radiation and partial slip. The finite difference method (FDM) is applied in this investigation to solve the modeled problem. Plots are graphed and exhibited in detail for several causes of embedding parameters by taking into consideration the temperature, velocity, angular velocity, skin friction factor, and Nusselt number.

Problem formulation
Suppose the problem of steady laminar 2D natural convection flow of magneto-micropolar ferrofluid through an isoflux sphere. The ferrofluid is collected
from Cobalt nanoparticle associated with a base fluid (kerosene). Flow model is developed by addressing the influence of thermal radiation and slip boundary conditions. The graphical sight of the investigation and the flow model are revealed in Figure 1. A uniform magnetic field is also utilized in the direction normal to the surface. The sphere surface is kept at a constant heat flux $q_w$ whereas the free stream is kept at a constant temperature by $T_\infty$ far from it. The ferrofluid thermophysical properties are presumed to be constant except the density in the buoyancy force term which is formulated by Boussinesq approximation. According to these presumptions, the governing equations of this investigation can be modeled as Huang and Chen,36 Yih,37 and Chamkha and Al-Mudhaf38:

\[
\frac{\partial(\tilde{u}^2)}{\partial x} + \frac{\partial(\tilde{v}^2)}{\partial y} = 0, \tag{1}
\]

\[
\tilde{u} \frac{\partial \tilde{u}}{\partial x} + \tilde{v} \frac{\partial \tilde{u}}{\partial y} = \frac{\mu_{nf}}{\rho_{ff}} \frac{\partial^2 \tilde{u}}{\partial y^2} + \frac{(\rho \beta_{ff})}{\rho_{ff}} g (T - T_\infty) \sin \left( \frac{\tilde{x}}{a} \right)
+ \frac{K}{\rho_{ff}} \frac{\partial \tilde{H}}{\partial y} - \frac{\sigma_{ff} B_0^2}{\rho_{ff}} \tilde{u}, \tag{2}
\]

\[
\tilde{u} \frac{\partial \tilde{T}}{\partial x} + \tilde{v} \frac{\partial \tilde{T}}{\partial y} = \frac{k_{ff}}{(\rho C_p)_{ff}} \frac{\partial^2 \tilde{T}}{\partial y^2} - \frac{1}{(\rho C_p)_{ff}} \frac{\partial q_w}{k_{ff}} \tilde{H} = 0, \tag{4}
\]

\[
\tilde{T} \to 0 : \tilde{u} = 0, \tilde{v} = 0, \tilde{T} \to T, \tilde{H} \to 0, \tag{5}
\]

Subjected to the corresponding boundary conditions (see Rashad15 and Huang and Chen36):

\[
\tilde{y} = 0 : \tilde{u} = A \mu_{ff} \frac{\partial \tilde{u}}{\partial y} \tilde{v} = 0 \quad \frac{\partial \tilde{T}}{\partial y} = - \frac{q_w}{k_{ff}} \tilde{H} = 0
\]

\[
\tilde{y} \to \infty : \tilde{u} \to 0, T \to T_\infty, \tilde{H} \to 0,
\]

Where $\tilde{x}, \tilde{y}$ are the stream-wise or circumferential and the transverse distances, respectively. $\tilde{r}(\tilde{x}) = a \sin(\tilde{x}/a)$ shows the radial distance from the symmetric axis to the sphere surface, $a$ indicates the sphere radius, $\tilde{u}$ and $\tilde{v}$ stands for the velocity components along $\tilde{x}, \tilde{y}$ axes. $T$ is the temperature. $g$ is the gravitational acceleration. $\rho_{ff}$ symbolizes the density, $\mu_{ff}$ signifies the dynamic viscosity, $\beta_{ff}$ signifies the thermal expansion coefficient. $\sigma_{ff}$ electrical conductivity, $(\rho C_p)_{ff}$ stands for the specific heat at a uniform pressure. $\alpha_{ff}$ is the thermal diffusivity of the ferrofluid. $A$ is the slip coefficient and $\tilde{H}$ stands for the angular velocity. $q_w$ signifies the surface heat flux. $k_{ff}$ stands for the thermal conductivity of ferrofluid. The radiative heat flux $q_r$ is approached according to the Rosseland approximations (see Raptis37):

\[
\frac{\partial q_r}{\partial y} = \frac{4 \sigma_1 \tilde{T}^4}{3 \beta_R \frac{\partial \tilde{T}}{\partial y}}, \quad \tag{6}
\]

where $\beta_R$ and $\sigma_1$ stand for the mean absorption coefficient and Stefan-Boltzmann constant. As carried out by Raptis,39 the fluid-phase temperature variations within the flow are approached to be adequately small so that $T^4$ may be obvious as a linear function of temperature. This is created by extending $T^4$ in a Taylor series on the free-stream temperature $T_\infty$ and removing higher-order terms to yield:

\[
T^4 = 4 T_\infty^4 - 3 T_\infty^2, \quad \tag{7}
\]

Utilizing equations (6) and (7) in the last term of equation (4), we obtain

\[
\frac{\partial q_r}{\partial y} = - \frac{16 \sigma_1 T_\infty^3 \frac{\partial \tilde{T}}{\partial y}}{3 \beta_R} \tag{8}
\]

In the current investigation, the following thermophysical relations are utilized; see Tiwari and Das40 formulation:

\[
\rho_{ff} = (1 - \phi) \rho_f + \phi \rho_s, \mu_{ff} = \frac{\mu_f}{(1 - \phi)^2},
\]

\[
\alpha_{ff} = \frac{k_{ff}}{(\rho C_p)_{ff}},
\]

\[
(\rho C_p)_{ff} = (1 - \phi) (\rho C_p) + \phi (\rho C_p),
\]

\[
(\rho \beta)_{ff} = (1 - \phi) (\rho \beta) + \phi (\rho \beta),
\]

\[
\frac{k_{ff}}{k_f} = \frac{(k_s + 2k_f) - 2 \phi (k_f - k_s)}{(k_s + 2k_f) + \phi (k_f - k_s)},
\]

\[
\frac{\sigma_{ff}}{\sigma_f} = 1 + \frac{3(\gamma - 1) \phi}{(\gamma + 2) - (\gamma - 1) \phi} \quad \text{where} \quad \gamma = \frac{\sigma_p}{\sigma_f}
\]

Here subscripts “s,” “f,” and “ff” stand for the magnetic nanoparticle (Cobalt), base fluid (kerosene) and
ferrofluid, respectively. \( \phi \) stands for nanoparticles volume fraction. The efficient thermal and physical properties of ferrofluid have been presented in Table 1. Also, the spin-gradient nanofluid viseid \( \gamma_{ff} \) is defined as:

\[
\gamma_{ff} = \left( \frac{\mu_{ff} + K}{2} \right) j = \mu_{f} \left( \frac{\mu_{ff} + \kappa}{2} \right) j, \tag{10}
\]

Where, \( \mu_{f} \) is the dynamic viscosity of the regular fluid, \( \kappa = \frac{K}{\mu_{f}} \) where \( \kappa \) is the micropolar parameter. Proceeding with analysis, let us consider the following dimensionless quantities:

\[
x = \frac{\bar{x}}{a}, \quad y = \frac{\bar{y}}{a} Gr^{1/5}, \quad u = \frac{a}{\nu Gr^{2/5}} \bar{u}, \quad v = \frac{a}{\nu Gr^{1/5}} \bar{v}, \quad Gr = \frac{g \beta_f q_w a^4}{k_f \nu_f^2},
\]

\[
\theta = \frac{k_f Gr^{1/5}}{q_w a^3} (T - T_c), \quad H = \frac{a^2}{\nu Gr^{2/5}} \bar{H}, \quad j = \frac{a^2}{Gr^{2/5}}.
\tag{11}
\]

Using equation (11) in equations (1)–(8), we have following equations:

\[
\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial y} = 0, \tag{12}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\rho_f}{\rho_{ff}} \left( \frac{\mu_{ff} + \kappa}{\mu_f} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\rho_f}{\rho_{ff}} \left( \frac{(\rho \beta_f) c_p x}{x} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\rho_f}{\rho_{ff}} \frac{\partial H}{\partial y} \kappa \left( 2 + \frac{\partial w}{\partial y} \right),
\tag{13}
\]

\[
u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} = \frac{\rho_f}{\rho_{ff}} \left( \frac{\mu_{ff} + \kappa}{\mu_f} \right) \frac{\partial^2 H}{\partial y^2} - \frac{\rho_f}{\rho_{ff}} \kappa \left( 2H + \frac{\partial w}{\partial y} \right), \tag{14}
\]

\[
u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr (\rho C_p) f} \left( \frac{k_f}{k_f} + \frac{4}{3 N_r} \right) \frac{\partial^2 \theta}{\partial y^2}, \tag{15}
\]

and

\[
y = 0 : u = \delta \frac{\mu_{ff}}{\mu_f} \frac{\partial u}{\partial y}, \quad v = 0, \quad \frac{\partial \theta}{\partial y} = -\frac{k_f}{k_{ff}} H = 0
\]

\[
y \to \infty : u = 0, \quad \theta = 0, \quad H = 0,
\tag{16}
\]

where \( Ha \) stands for Hartmann number, \( Nr \) stands for radiation parameter, \( Pr \) signifies the Prandtl number and \( \delta \) signifies the slip parameter which are given respectively as:

\[
Ha^2 = \frac{\sigma_f B_0^2 a^2}{\mu_f Gr^{2/5}}, \quad Nr = k_f \beta_f / 4 \sigma_1 \left( \frac{q_w a}{k_f Gr^{1/5}} \right)^3,
\tag{17}
\]

\[
Pr = \frac{\nu_f (\rho C_p) f}{k_f}, \quad \delta = A Gr^{1/5} \mu_f / a.
\]

To get the solutions to equations (13)–(15) utilizing equation (17), the following functions are introduced

\[
\psi = \psi(x,y), \quad \theta = \theta(x,y), \quad H = H(x,y), \quad r = a \sin x
\]

\[
\tag{18}
\]

Where \( \psi \) stands for the stream function which is given by \( u = (1/r) \partial \psi / \partial y \) and \( v = -(1/r) \partial \psi / \partial x \), that satisfies equation (12), and \( \theta \) stands for the non-dimensional temperature of ferrofluid. Substituting equation (18) into (13)–(15), the following converted PDEs are obtained:

\[
\frac{\rho_f}{\rho_{ff}} \left( \frac{\mu_{ff} + \kappa}{\mu_f} \right) \frac{\partial^2 \psi}{\partial y^2} + \left( 1 + \frac{x \cos x}{\sin x} \right) \frac{\partial^2 \psi}{\partial y^2} - \left( \frac{\partial \psi}{\partial y} \right)^2 + \frac{\rho_f}{\rho_{ff}} \left( \frac{(\rho \beta_f) c_p x}{x} \right) \frac{\partial^2 \psi}{\partial y^2} \tag{19}
\]

\[
+ \frac{\rho_f}{\rho_{ff}} \frac{\partial h}{\partial y} - \frac{\rho_f}{\rho_{ff}} \frac{\partial \theta}{\partial y} - \frac{\rho_f}{\rho_{ff}} \frac{\partial \theta}{\partial y} - \frac{\rho_f}{\rho_{ff}} \kappa \frac{\partial H}{\partial y} - \frac{\rho_f}{\rho_{ff}} \frac{\partial H}{\partial y} - \frac{\rho_f}{\rho_{ff}} \kappa \left( 2 + \frac{\partial w}{\partial y} \right),
\tag{20}
\]

\[
\frac{\rho_f}{\rho_{ff}} \left( \frac{\mu_{ff} + \kappa}{\mu_f} \right) \frac{\partial^2 \theta}{\partial y^2} + \left( 1 + \frac{x \cos x}{\sin x} \right) \frac{\partial \theta}{\partial y} - \frac{\partial \theta}{\partial y} - \frac{\partial \theta}{\partial y} - \frac{\partial \theta}{\partial y} - \frac{\partial \theta}{\partial y} \tag{21}
\]

and
\( y = 0 : \frac{\partial f}{\partial y} = \delta \frac{\mu_f f'}{\mu_f} f \left( 1 + \frac{x \cos x}{\sin x} \right) + \frac{\partial f}{\partial x} = 0, \)

\[ \frac{\partial \theta}{\partial y} = \frac{k_f}{k_{ff}}, h = 0 \]

\[ y \rightarrow \infty : \frac{\partial f}{\partial y} = 0, \theta = 0, h = 0 \] (22)

Important entities namely drag friction \( C_f(x) \) and local Nusselt number \( Nu(x) \) are determined for physical interest as follows;

\[ \tilde{C}_f(x, 0) = \frac{\tau_w}{\rho_f U^2} = \left. \left[ \frac{(\mu_f + \kappa) \partial u / \partial y} {\rho_f U^2} \right] \right|_{y=0} \]

\[ = Gr^{-1/5} \left( \frac{\mu_f}{\mu_f} + \kappa \right) x^m \tilde{f}(x, 0) \]

\[ C_f(x, 0) = \tilde{C}_f(x) Gr^{1/5} = \left( \frac{\mu_f}{\mu_f} + \kappa \right) x^m \tilde{f}(x, 0), \] (23)

\[ \tilde{Nu}(x, 0) = \frac{q_w a}{k_f(T - T_c)} = - \frac{\partial (\partial T / \partial y)}{k_f(T - T_c)} \]

\[ = Gr^{1/5} \frac{k_f}{k_{ff}} \left( \frac{1}{\theta(x, 0)} \right) \] (24)

\[ Nu(x, 0) = Gr^{-1/5} \tilde{Nu}(x) = \frac{k_{ff}}{k_f} \left( \frac{1}{\theta(x, 0)} \right). \]

**Numerical method**

The non-linear, non-similar partial differential equations (19)–(21) are solved numerically with the boundary conditions (22) using a finite difference method provided by Gorla et al.\(^4\) The main steps used in this method are summarized below:

- At the boundary layer edge, the boundary conditions are replaced by \( f'(x,y_{max}) = 0; h(x,y_{max}) = 0; \theta(x,y_{max}) = 0 \) where \( y_{max} \) is selected such that the boundary conditions (22) are satisfied. In this case, we assumed \( y_{max} = 15 \) for each computation.

- The domain of interest \((x,y)\) is discretized with an equispaced mesh in the \(x\)- and \(y\)-direction, respectively.

- The partial derivatives with respect to \(x\) and \(y\) are determined using central difference approximations.

- Based on successive substitution, two iteration loops are used since the equations are non-linear.

- The value of \( x \) is fixed in each inner iteration loop, and the governing equations (19)–(21) are solved in the \(y\) domain. After convergence, the solution is halted.

- The value of \( x \) is progressed in small intervals from the lower stagnation point to the upper stagnation point, and the derivatives of the functions are revised after every outer iteration step.

The accuracy of this numerical method was validated by comparing the present results with the results reported by Huang and Chen,\(^3\) Yih,\(^3\) and Chamkha and Al-Mudhaf,\(^3\) in the absence of magnetic field, and thermal radiation and partial slip effects. Tables 2 and 3 present the results of these various comparisons. The present results are found in an excellent agreement with the existing results.

**Results and discussions**

The numerical investigation are reported to explore the magneto-natural convective flow of micropolar Cobalt-kerosene ferrofluid adjacent an isoflux sphere in presence of partial slip and radiation impacts. To visualize the physical consequences of pertinent parameters such as micropolar parameter \( \kappa \), radiation parameter \( N_r \), dimensionless coordinate \( x \), velocity slip parameter \( \delta \),
Hartmann number $\text{Ha}$, and solid volume fraction of ferrofluid $\phi$, several plots are prepared for the distribution of velocity, angular velocity, temperature, local skin friction coefficient, and Nusselt number.

Figures 2 to 4 are contrived to exhibit the impact of Hartmann number $\text{Ha}$ on dimensionless velocity $f(x,y)$, temperature $\theta(x,y)$, and angular velocity $h(x,y)$, for pure fluid and ferrofluid ($\phi = 0.0, 0.05$) at different positions along the sphere. From Figure 2(a) through 3(b), it is manifested that an evolution in $\text{Ha}$ has a tendency to decline the maximum velocity, and to enlarge the ferrofluid temperature. Physically, the motivation of Lorentz strength through the growing in the magnetic constraint caused a deceleration to flow and increased the temperature for pure fluid and ferrofluid at two positions. Moreover, it is important to note that, for a pure fluid, the maximum velocity is higher and decreases with an increase in the solid volume fraction of ferroparticles $\phi$. As the fluid moves upwards, the maximum velocity increases due to a decrease in viscous forces. The comparison shows the dimensionless velocity at the lower stagnation point is lower and rises along the sphere surface. Furthermore, it is noticed from Figure 4(a) and (b) that the magnetic field tends to suppress the dimensionless angular velocity at the surface, but close to the boundary layer, the behavior of the angular velocity is just opposite. In the absence of ferroparticles, the angular velocity is smaller at the surface and increases with increasing solid volume fraction of ferroparticles in both cases. It is due to higher density of ferroparticles. About the boundary layer, the behavior is just opposite to the surface.

Figures 5 to 7 indicate the behaviors of the velocity slip parameter $\delta$ and micropolar parameter $\kappa$ on dimensionless velocity $f(x,y)$, temperature $\theta(x,y)$, and angular velocity $h(x,y)$, at two positions along the sphere. It is apparent that the encouragement in slip factor $\delta$ produces a notable growth in both angular velocity and the maximum velocity close to the surface, while the opposite behavior is happened for the ferrofluid temperature. This is because the magnify in the $\delta$ has a tendency to accelerate the motion, and then the
temperature and thermal boundary layer thickness decreases slightly. Moreover, both the ferrofluid velocity and angular velocity are higher in the vicinity of the surface due to accelerated estimates of $\kappa$, as exhibited in Figure 5(a) through 6(b). In contrast, an enhancement temperature profiles has been occurred slightly due to greater values of $\kappa$ at two positions along the sphere. The physical consequences of such accelerated behavior involved the fact that $\kappa$ conquers opposite relation with viscosity. A lower viscosity corresponds to greater $\kappa$, which promotes the ferrofluid temperature. However, Figure 7(a) and (b) show almost negligible effects of velocity slip and micropolar parameters on the temperature at different positions along sphere. At the lower stagnation point, the temperature is lesser and increases with the position along the sphere.

Figures 8 and 9 examine the variation of local skin friction $C_f(x, 0)$ and local Nusselt number $Nu(x, 0)$ with Hartmann number $Ha$ and solid volume fraction of ferroparticles $\phi$ with several values of velocity slip parameter $\delta$ at different positions along the sphere. In the absence of a magnetic field ($Ha = 0.0$), the skin friction is higher and reduces with the increasing magnetic field, as uncovered in Figure 8(a) and (b). This is primarily because the rise in $Ha$ affects the augmentation in Lorentz force, and thus, decreases momentum in the boundary layer as noted above. This decelerates the ferrofluid flow on the surface and decreases surface shear stress, which dwindles both the skin friction coefficient and the Nusselt number. Similarly, in the absence of a solid volume fraction of ferroparticles ($\phi = 0.0$), the skin friction is higher and decreases with increasing $\phi$ in both cases. This is due to higher density of ferroparticles (Table 1). Physically, the intension in the nanoparticles volume fraction seemed to improve the performance of fluid viscosity. Therefore, this notice suggests that boosted nanoparticles volume fraction can reduce the surface shear stress, thus decreasing skin friction, and hence produces a considerable enhancement in the heat transfer, as manifested in Figure 9(a) and (b). Moreover, the skin friction also decreases, while the heat transfer boosts with the increment in the velocity slip parameter $\delta$ at all positions. This because the
momentum boundary layer obstructs caused by the augmentation in the velocity slip parameter. This appoints that coefficient of a weak skin friction can be carried out for the largest value of $\delta$ on boosting nanoparticles volume fraction and enhancement in the Nusselt number.

Figures 10 and 11 explain the consequences of the skin friction local skin friction $C_f(x, 0)$ and local Nusselt number $Nu(x, 0)$ against the radiation parameter $Nr$ and micropolar parameter $\kappa$ at different positions for water and kerosene oil. It is illustrated that upsurge in values of micropolar parameter $\kappa$ encourage the vortex viscosity of the nanofluid flow which drops opposition to rotate the ferrofluid that yields a sufficient reduction in the skin friction coefficient and heat transfer rate. On other side, according to the definition in equation (17), the influence of conductive radiation becomes more massive as $Nr = 0$ and can be ignored when $Nr \to \infty$. Hence, the diminishing of the thermal radiation parameter $Nr$ yields a great elevation in the thermal state of the ferrofluid causing its temperature to boost. Hence, a reduction in $Nr$ causes an improvement in both the skin friction coefficient and Nusselt number. This corresponds with the physical pattern that, as mentioned from equation (17), the heat transfer is very greater with the existence of the radiation influence, and hence the shear stress boosts.

Conclusions

This investigation reflects the influence of thermal radiation on magneto-natural convection flow of micropolar ferrofluid past an isoflux sphere with the impact of velocity slip. Cobalt-nanoparticles with kerosene based-ferrofluid are considered. Non-dimensional factors were exploited to transmute the governing PDEs into non-similar form. The transmuted model subject to analogous BCs was then solved numerically with the help of finite difference method. The influences of prominent parameters on velocity, angular velocity and temperature fields as well as the skin friction coefficient and Nusselt number are visualized and analyzed through graphs. The main achieved results are as follows:

- Both skin friction coefficient and Nusselt number reduce with upsurging in the micropolar parameter.
- Magnetic force contributes to the dwindling skin friction coefficient and heat transport rate ever-growing. Similar performance is adhered with elevating in the micropolar parameter.
- Boosted nanoparticles volume fraction reduces the surface shear stress, and enhances the heat transport rate.
- Both the skin-friction coefficient and heat transport rate show a considerable improvement in the presence of thermal radiation.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by the Deanship of Scientific Research at Prince Sattam bin Abdulaziz University under the research project No. 2020/01/16413.

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Figure 10. Variation of skin friction with radiation parameter $Nr$ and micropolar parameter $\kappa$ at different positions along sphere: (a) at $x=0$ and (b) $x=0.85$.

Figure 11. Variation of Nusselt number with radiation parameter $Nr$ and micropolar parameter $\kappa$ for different micropolar ferrofluids at different positions along sphere: (a) at $x=0$ and (b) $x=0.85$. 
Availability of data
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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**Appendix**

**Notation**

- $a$: sphere radius
- $A$: velocity slip coefficient
- $C_p$: specific heat at constant pressure
- $C_f$: skin-friction coefficient
- $f$: dimensionless stream function
- $f^*$: dimensionless velocity
- $g$: acceleration due to gravity
- $Gr$: Grashof number
- $H$: dimensionless of angular velocity
- $H$: Hartmann number
- $k$: thermal conductivity
- $K$: dimensional material parameter
- $N_R$: thermal radiation parameter
- $Nu$: local Nusselt number
- $Pr$: Prandtl number
- $q_r$: radiative heat flux
- $q_w$: surface heat flux
- $r(\tilde{x})$: radial distance from the symmetric axis to the sphere surface
- $r$: dimensionless radial distance
- $T$: temperature of the fluid in the boundary layer
- $T_a$: temperature of the ambient fluid
- $T_f$: uniform temperature of the cylinder surface
- $u$, $v$: dimensionless fluid velocities in the $x$, $y$ directions
- $\tilde{u}$, $\tilde{v}$: velocity components along $\tilde{x}$, $\tilde{y}$ axes.
- $\tilde{x}$, $\tilde{y}$: dimensional axes in the direction along and normal to the surface
- $\beta$: volumetric coefficient of thermal expansion
- $\beta_R$: mean absorption coefficient
- $\psi$: stream function
- $\rho$: fluid density
- $\mu$: dynamic viscosity of the fluid
- $\nu$: kinematic viscosity of the fluid
- $\theta$: dimensionless temperature function
- $\sigma$: nanoparticles volume fraction
- $\sigma_1$: Stefan-Boltzmann constant
- $\sigma$: electrical conductivity
- $\kappa$: dimensionless micropolar parameter
- $\delta$: velocity slip parameter

**Subscripts**

- $f$: pure fluid
- $ff$: ferrofluid
- $s$: solid nanoparticle