Effect of longitudinal steel reinforcement ratio on deflection and ductility in reinforced concrete beams

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Abstract. This search report is an experimental result on load –deflection relationship of various ratios of longitudinal reinforcement for six simply supported reinforced concrete beams. All specimens’ dimensions are (200×300×1750mm) and they were tested under two-point loading. Concrete compressive strength was (37.0-40.7 MPa). The variables studied in this work were tensile steel ratio (0.46-3.0%), bar diameter (12, 16 and 25mm) and number of bars (2 or 3 bars). It is concluded that for low reinforcement values ($\rho_{2025} < 0.013$), an increase in $\rho_{2025}$ is sharply reduced the ductility index $\mu_{d}$. However, this effect decreases with increase in $\rho$ values. All beams exhibit insufficient displacement ductility (less than 3) when reinforced with $\rho_{2025}$ more significant than when using 3 bars in yield and ultimate strengths of specimens as well as the deflection values at yield load, but the deflection values have gradual decrease at the ultimate load as a result of ductility decrease. It was concluded that the increase in longitudinal reinforcement ratio causes an increase in yield and ultimate strength of beams, these increments are compatible with the steel ratio i.e. by increasing the $\rho$ by 50% the strength is increased about this ratio. As well as, the decreasing in ductility compatible with increasing the $\rho$ by 50% the ductility decreased in about 50% on average.

1. Introduction
In designing a flexural member, the structural engineer must not only provide adequate strength, but should ensure that the member exhibits adequate ductility under over loading conditions. However, the ductility of individual structural member, as well as that of an entire structure, becomes a major design consideration in earthquake regions [1], [2]. Ductility in concrete may be related to the fact that bond cracks may not form at the same time as mortar cracks. This time lag affects the deformation capability of concrete before failure. In structural members, the ductility refers to the ability of undergoing large deformations after the yielding of tensile reinforcement. This may save lives by giving warning of failure and preventing total collapse [3], [4].

[5] investigated the strength of bubbled wide reinforced concrete beams using diverse types of shear steel plates with constant longitudinal steel reinforcement ratio. It was concluded that without using bubbles and with aspect ratio of shear steel plates limited between the limits 4.5 - 8 gave improved in ductility by 36%.
[6] studied the variation of strength, stiffness and ductility of normal reinforced concrete beams using moment-curvature diagrams. It was concluded that the moment and curvature are increasing in the concrete strain but it decreases with increase in percentage of tension steel.

[7] investigated the flexural ductility of lap-spliced reinforced concrete of 24 reinforced concrete beams. Results showed that amount of transverse reinforcement over the splice and concrete strength has main effects on ductility. As well as provisions in predicting the bond strength of lap-spliced concrete beams are adequate but may not achieve a satisfactory performance for ductility. Although increasing of longitudinal reinforcement ratio reduce the ductility index but it significant to increase the yield and ultimate load capacities of beams as well as one-way slabs [8], [9], [10] and [11].

2. Research significant
In this work an attempt was made to study the strength, deflection and ductility relations of reinforced concrete beams with compressive strength between (37.0-40.7 MPa) and was conducted using different longitudinal steel reinforcement ratio. Studying the effects of increasing the longitudinal steel reinforcement ratio on strength, deflection and ductility of reinforced concrete beams will give an idea of the differences and similarities between increasing the steel ratio from 0.46% to 3%.

3. Experimental work
Six simply supported beams with dimensions (200×300×1750mm) as shown in Figure 1 were investigated. This figure also shows testing configurations. The materials used for production of concrete were ordinary Portland cement, natural sand with fines modulus of 3.0. The crushed gravel coarse aggregate was used with maximum aggregate size of 14 mm. The mix ingredients of concretes were 200, 400, 728, 1092 kg/m³ for water, cement, sand and gravel. The steel reinforcement diameter and ratio were also varied. Table 1 presents the properties of reinforcement. The tensile steel reinforcement was two bars diameters of 12, 16 and 25mm were used for specimens R1, R2 and R3 but it was three bars diameters of 12, 16 and 25mm were used for specimens R4, R5 and R6, while 10mm steel were used as compressive steel. The tensile steel ratio therefore varied between; all details of specimens are presented in Table 2. Compressive strength was determined using supplementary concrete cylinders (152×305mm) were cast with every concrete beam specimen and tested according with [12].

The testing of the beams was carried out on effective simple span of 1850 mm. The loads were applied at third points of the beam. The beams were tested up to failure. Deflections at the center of the beams were measured using a mechanical strain gauges with an accuracy of 0.01mm.

![Figure 1. Loading details.](image-url)
Table 1. Reinforcement properties.

| Nominal Diameter (mm) | Measured Diameter (mm) | Area (mm²) | Modulus of Elasticity (GPa) | $f_y$ (MPa) | $f_u$ (MPa) |
|-----------------------|------------------------|------------|-----------------------------|------------|------------|
| 25                    | 25.18                  | 498.0      | 201                         | 440        | 720        |
| 16                    | 15.70                  | 193.6      | 201                         | 459        | 698        |
| 12                    | 12.35                  | 119.8      | 207                         | 460        | 648        |
| 10                    | 10.03                  | 79.01      | 208                         | 452        | 652        |

Table 2. Properties of beam specimens.

| Beam | $f_c$ (MPa) | Ten. Rein. | ρ x10⁻² | Comp. Rein |
|------|-------------|------------|---------|------------|
| R1   | 38.2        | 2 Ø 12     | 0.46    | 2 Ø 10     |
| R2   | 37.5        | 2 Ø 16     | 0.75    | 2 Ø 10     |
| R3   | 37.3        | 2 Ø 25     | 2.05    | 2 Ø 10     |
| R4   | 37.0        | 3 Ø 12     | 0.69    | 2 Ø 10     |
| R5   | 39.1        | 3 Ø 16     | 1.30    | 2 Ø 10     |
| R6   | 40.7        | 3 Ø 25     | 3.00    | 2 Ø 10     |

4. Experimental results and discussion

Table 3 presents all experimental results of six specimens, compressive strength of concrete, yield and ultimate loads with deflections in two stages, as well as the ductility index.

Table 3. Experimental results of beam specimens.

| Beam | Yield | Ultimate | | |
|------|-------|----------|---|
| Load (kN) | % | Δy (mm) | % | Load (kN) | % | Δu (mm) | % | μd |
| R1   | 111   | -       | 4.71 | - | 152 | - | 33 | - | 7.01 | - |
| R2   | 185   | 67      | 4.95 | 5 | 239 | 58 | 24 | -27 | 4.85 | -30 |
| R3   | 340   | 207     | 5.65 | 20 | 371 | 144 | 15 | -54 | 2.66 | -62 |
| R4   | 189   | 71      | 4.97 | 6 | 237 | 56 | 21 | -36 | 4.23 | -39 |
| R5   | 308   | 178     | 5.72 | 22 | 341 | 125 | 14 | -57 | 2.45 | -65 |
| R6   | 495   | 346     | 6.12 | 30 | 516 | 240 | 7.0 | -78 | 1.15 | -83 |
4.1. Load-deflection behavior

All beams were tested under two-point loads with 1850mm length as illustrated in Figure 2. The load increments were 10kN by a hydraulic jack machine with 2000kN capacity, and the deflection for every beam recorded by three dial gauge (one in center of beam and two under the point loads) till failure stage. The load-deflection behaviours of all beams are shown in Figure 3. In the pre-cracking stage, the deflection is linearly increased with the load applying increase. This is expected subsequently the strains in the concrete and steel are comparatively small and both the materials concrete and steel are in the elastic portion of their corresponding responses. Initial cracking was observed at loads ranging from 35.3% (for R1) to 12.5 percent (for R3) of the ultimate load.

In the post-cracking stage, the slope in the load-deflection curve is changed as a result of cracking in concrete, which causes a reduction in the effective moment of inertia of the beam cross section. After cracking, the deflections increased again almost linearly with the load increase up to the point at which the tensile steel starts to yield. Tensile steel yielding load varied between 73% (for beam R1) to 96 percent (for beam R6) of the ultimate load. In the post-yielding stage, due to yielding of the tensile longitudinal steel reinforcement, all load-deflection curves exhibit changes in slope. The deflections immediately increased after yielding due to the reduction in neutral axis depth. Each beam showed a different post-yield load-deflection response.
Figure 3. Load-central deflection of all tested specimens.
4.2. Ductility
In this study the ductility of beams is taken as in Eq.(1) below:

\[ \mu_d = \frac{\Delta_u}{\Delta_y} \]  

(1)

Where \( \Delta_u \) is member deflection at ultimate load and \( \Delta_y \) is member deflection at yielding of the tension in steel reinforcement. Ultimate is definite as the stage beyond which the specimen would not be able to sustain further deformations at the same load strength. A summary of the test results, including the load and deflection at yield and ultimate stages are presented in Table 3. The table also includes the ductility index of the six investigated beams.

4.3. Effect of tensile reinforcement for strength
The displacement-ductility index \( \mu_d \) decreases with an increase in tensile steel content \( \rho \) as shown in Table 3 and Figure 3. The effects of increasing the tensile steel ratio decrease for the higher amount of tensile steel beams. However, for the beams with 0.0046<\( \rho \) <0.0075, a sharp reduction in \( \mu_d \) occurs with an increase in \( \rho \), while for beams with 0.0113<\( \rho \) <0.03 the values of \( \mu_d \) appears to be less sensitive to change in tensile reinforcement ratio \( \rho \).

It can be seen from load-deflection of all specimens illustrated in Figure 4, that the longitudinal steel reinforcement ratio is increased using 2 bars with steel ratio from ratio 0.46%, to 0.75%, 2.05% increased the yield and ultimate strength by 67%, 207% and 58%, 144% respectively. And by increasing the longitudinal steel reinforcement ratio by using 3 steel bars with steel from 0.69% ratio to 1.3% and 3% increased the yield and ultimate loads by 63%, 162% and 44%, 117% respectively. The increasing of longitudinal steel ratio by using 2 bars is more significant than when using 3 bars in strength of specimens. In general, increasing the longitudinal reinforcement caused increasing the yield and ultimate strength of beams, these increments were (71%, 67%, 178%, 207%, 346%) and (56%, 58%, 125%, 144%, 240%) respectively when the ratio of longitudinal reinforcement increased from 0.46% to 0.69%, 0.75%, 1.3% 2.05% and 3%).

4.4. Effect of tensile reinforcement for deflection
From the other hand, it can be seen from load-deflection of all specimens shown in Figure 4 and Table 3, when the longitudinal steel reinforcement ratio is increased by using two bars with steel ratio from ration 0.46%, to 0.75%, 2.05%, the deflection at yield strength is increased by 5%, 20% respectively as increasing of yield stress but this ratio was decreased in ultimate load by 27%, 54% respectively as a result of the ductility decrease of these specimens. As well as, when the longitudinal steel reinforcement ratio is increased by using 3 steel bars with steel from 0.69% ratio to 1.3% and 3%, the deflection at yield load is increased by 15%, 23% respectively but this ratio was decreased in ultimate load by 33%, 67% respectively for the same above reason. The longitudinal steel ratio increase using 2 bars is more significant than that when using 3 bars as a deflection decreasing.

In general, the longitudinal reinforcement increase caused the deflection increase at yield strength of beams, these increments were (5%, 6%, 20%, 22% and 30%) when the ratio of longitudinal reinforcement increased from 0.46% to 0.69%, 0.75%, 1.3% 2.05% and 3%) respectively, but this increase caused decrease in the deflection at ultimate load by (36%, 27%, 57%, 54%, 78%) as a result of the ductility decrease.
4.5. Comparison with the minimum limits of ductility

Furlong recommends [1], that the ductility index $\mu_d$ should satisfy the criterion in Eq. (2):

$$\mu_d \geq 1 + 0.25 \frac{L'}{d}$$

where: $L'$ is the effective length ($L=1850$ mm for present study), and $d$ is the effective depth which has 261, 259, and 254.5 mm values for the bars diameters 12, 16, and 25 mm, respectively.

For the present study, the minimum limits of $\mu_d$ according to Furlong equation are 2.67, 2.69, and 2.72 for the bar diameters 12, 16, and 25, respectively. The minimum limit according to Furlong criterion is satisfied for beams with $\rho - \rho' \leq 0.75 \rho_b$. [13] code provides a maximum tensile reinforcement limitation (Eqs. (3) and (4)):

$$\rho - \rho' \leq 0.75 \rho_b$$

Where: $\rho_b = 0.85 \beta_1' f'c \sqrt[0.003E_s]}{f_y \cdot \sqrt[0.003E_s+F_y]}$

It can be notified from Table 1 that the ductility diminished when reinforcing the beams with $\rho - \rho' > 0.75 \rho_b$. Fig. 5 shows that the ductility is decreased when increasing the longitudinal steel reinforcement ratio by 50% to 40%, 50% and 57% for 12, 16 and 25 mm respectively.
5. Comparison with Other studies
A comparison of displacement ductility between the results of the present study and those of other studies for the normal and lightweight high-strength reinforced concrete beams is detected. It can be concluded that the values of $\mu_d$ obtained in this study confirm with those of a similar study by [14], [15] and [16], in normal weight high-strength reinforced concrete beams, while they are higher than those reported by [1], [17] for lightweight high-strength reinforced concrete beams.

6. Conclusions
The displacement ductility $\mu_d$ (deflection at ultimate to the deflection at yield of tensile steel) was investigated for the six reinforced concrete beams, and the following conclusions were drawn:
1. For low steel reinforcement values ($\rho < 0.013$), an increase in $\rho$ sharply reduces the ductility index $\mu_d$. However, this effect decreases with increasing values of $\rho$. 
2. All specimen’s exhibition insufficient displacement ductility (less than 3) when reinforced with $\rho/\rho_b > 0.4$.

3. The ratio $\rho/\rho_b$ was found to be the most dominant factor influencing the ductility. This fact also was concluded by Shin, et al (1989). The ductility decreases with increasing the ratio $\rho/\rho_b$. However, for the same value of $\rho/\rho_b$, the displacement ductility was not substantially affected by the concrete compressive strength.

4. Furlong minimum limit is satisfied for values of $\rho_{3096}/\rho_{2879}/\rho_{4594} < 0.35$. The ductility was diminished when value of $\rho_{3096}/\rho_{3277}$ exceeded 0.75. However, this result confirms the code requirement to prevent the brittle failure.

5. The longitudinal steel ratio increase using 2 bars is more significant than that when using 3 bars in yield and ultimate strengths of specimens as well as the deflection at yield load, but the deflection slightly decrease at ultimate load as a result of the ductility decrease.

6. It was concluded that increase of the longitudinal reinforcement ratio causes an increase in yield and ultimate strength of beams, this increments compatible with the steel ratio i.e. by increasing the $\rho$ by 50% the strength is increased about this ratio. As well as, the ductility decrease is compatible with increasing the $\rho$ by 50% the ductility is decreased by about 50% an average.

7. The yield and ultimate strengths of beams increased by (71%, 67%, 178%, 207% and 346%) and (56%, 58%, 125%, 144% and 240%) respectively when the ratio of longitudinal reinforcement increased from 0.46% to 0.69%, 0.75%, 1.3% 2.05% and 3%.

8. The deflection at yield strength of beams increased by (5%, 6%, 20%, 22% and 30%) when the ratio of longitudinal reinforcement increased from 0.46% to 0.69%, 0.75%, 1.3% 2.05% and 3% respectively, but it caused a decrease in the ultimate load by (36%, 27%, 57%, 54% and 78%) as a result of the ductility decrease.

5. References

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