Synergy of factor analysis based on multiplicative index models and graph theory

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Abstract. This article addresses the issues of solving the problems of factor analysis using the concepts of graph theory.

Factor analysis, a section of multivariate statistical analysis combining mathematical and statistical methods to reduce the dimension of the studied multivariate trait, was important in statistical research.

The index method of factor analysis was used in the study of the development of socio-economic systems and the adoption of appropriate policy decisions. This method provides for the use of indices - in terms of indicators, represented in the form of ratios, i.e. in dimensionless values. Thus, indices are peculiar mathematical models of the investigated processes and phenomena. Indexes and index models are the new categories used in the index method of factor analysis.

Factor analysis is used to identify internal non-measurable patterns in various fields of science. The technique for conducting factor analysis is determined by the initial task.

The central problem of the index method of factor analysis is that the end result of estimating factor increments in the mode depends on the order of the multiplier indices.

In turn, graph theory is one of the most extensive sections of discrete mathematics, widely used in solving economic and managerial problems and other fields of knowledge.

To visually display the order of the factors in the model, you can use the graphoanalytic method, which allows you to present a mathematical factor multiplicative model in the form of gras, which, in turn, allows you to identify the influence of individual factors on the effective indicator.

This article explores the problem of using graph theory in solving the problems of factor analysis, which allows for corresponding generalizations, considering all the many possible solutions, including the influence of the order of the partial indices on the factors under consideration.

1. Introduction

The application of scientific achievements in the system of national economy was a prerequisite for sustained economic growth in promising development. It is this emphasis that makes an enterprise of any type, region, country susceptible to innovation, causes the desire not only to introduce ideas and
developments offered by science, but also creates conditions for independent creativity, turns it into a need, and activates the innovation process.

Constant innovation is the main factor for the success in the market of enterprises of any field of activity. In an effort to maintain and strengthen their market positions in such an environment, enterprises are forced to continuously improve and transform their products, improve production and management processes.

The development of the world economic community was based on intellectual property. The transfer of innovative technologies involved the concentration of financial resources on the priorities of the scientific and technological development of society.

The dominance of command-and-control management methods presupposes the formation of an innovative economy and the effective development of scientific and technological achievements.

According to statistics in the USSR, from 1986 to 1990 more than 70 thousand inventions were recorded annually, but only no more than 30% of them were introduced, in the USA and Japan the indicators of scientific developments amounted to 71-75 thousand inventions of which were introduced 60-80% [3].

Taking into account the fact that Russia has one of the best scientific and technological potential, it is necessary to develop innovative mechanisms aimed at improving the management of socio-economic processes at all levels.

Information is a key element of the management process. The overall objective of the management process is to improve the processing of information, which involves the selection, analysis, processing of initial data, as well as the development of a mechanism for analyzing and evaluating the results.

In the field of managing socio-economic processes, an important link that determines the effectiveness of management is the equipment of managers and analysts with appropriate methods for describing and mathematical modeling of processes and phenomena, for which the methods of factor analysis play an essential role.

Factor analysis, as a branch of applied mathematics, occupies an important place in many subject areas, which include applied statistics in economics, pedagogy, biology, theory of experimental planning; analysis of variance; modeling of processes and systems. In general, in all cases, in various interpretations, the problem of assessing, based on the results of collecting factual and statistical information, the influence of certain factors on the performance indicator is solved, with the aim of identifying the most significant factors in terms of the degree of influence and eliminating insignificant influencing quantities from further analysis, which simplifies the model processes and phenomena.

The urgency of the problem solved in this article is dictated by the following circumstances.

Firstly, there is a list of unresolved scientific tasks. For example, these are tasks related to obtaining requirements for the accuracy of presenting estimates of the impact of random factors on the final result. In this regard, there is a problem with the reliability of the results obtained.

Secondly, at present there is a need to improve the software, which is associated with the constant improvement of the characteristics of computer technology - its ability to quickly process and store large amounts of information.

Thirdly, the methodological aspect of the problem is also important, providing for the improvement of the effectiveness of teaching methods for factor analysis.

The methodological foundations of this direction are the work on the theory of economic growth and innovative development of the economy of such scientists like LI Abalkin [1], A.G. Aganbegyan [2], K.A. Bagrinovsky [4, 5], M.A. Bendikov [5], A.R. Belousov [6], A.E. Varshavsky [8], S.Yu. Glazyev [10,11], PN Zavlin [12], K. Marx [13], E. Mansfield [14], D. Sakhal [15], J. Stiglitz [16], J. Tinbergen [17], N.L. Frolova [18], E.Yu. Khrustalev [19] and others.

Currently, various methods of factor analysis are generally recognized and are widely used. At the same time, it must be recognized that all known methods have their own specific areas of application and do not always allow to effectively solve special problems due to the presence of limitations of a very diverse nature, incompleteness and errors in the collection of initial data.
The problem of solving problems of factor analysis is so diverse that at present there are 8-10 original approaches to its solution. This variety is due to the peculiarities of the presentation of the initial data - the volume of samples and the accuracy of their receipt. Currently, in science, especially in economics, the subjective factor plays an important role, when indices are divided into quantitative and qualitative indicators. At the same time, the so-called qualitative indicators are presented in numerical form, being purely formally quantitative.

Currently, there is a problem with the non-transitivity of the index method of factor analysis. The lack of knowledge, relevance, theoretical and practical relevance of factor analysis, as one of the areas of innovation in socio-economic systems, was a prerequisite for the study and conditioned its purpose and objectives.

The purpose of this study is to improve the methods of factor analysis based on multiplicative index models of factor analysis, which involves solving the following problems:

• research of classification issues of factor analysis methods and systematization of factors and calculation formulas in order to provide an integrated and systematic approach to assessing their impact on the results of economic activity and on the indicators of socio-economic processes;
• carrying out the classification of factor models and identifying the most effective ones for describing socio-economic processes;
• development of a mathematical description of factor models and obtaining, using graph theory, algorithms for deriving formulas for calculating the values and increments of the effective indicator as a function of the increment of factors.

In this article, the main subject of research is the method of factor analysis based on index multiplicative models, which allow studying phenomena in a dimensionless form. As well as elements of the factor analysis methodology in socio-economic systems: methods for mapping formulas and algorithms for calculating indicators in the form of graphs; mathematical and topological formulas for calculating the influence of random factors on performance indicators; evaluation of the performance indicator, taking into account increments by factors.

It can be noted that applied graph theory is the field of discretely mathematics, the purpose of which is to study applied graph models. A graph is a collection of vertices and edges connecting vertices. In the form of graphs, information and material values, a map of a city and a country, computer programs, models of the human and animal brain, chemical compounds, etc. can be represented. Application of the results of studying applied graph theory has wide application in science.

In turn, synergy is a field of scientific research that studies the principles that underlie the processes of self-organization in systems of different nature: physical, chemical, biological, technical, social and others.

The German theoretical physicist Hermann Haken is considered the founder of synergetics as a science. (H. Haken) [19, 20]. In 1977, in his book "Synergetics", he introduced a definition of the term close to its modern understanding. ... "Synergy is a science that studies the processes of emergence, self-organization, maintaining the stability and decay of systems of the most diverse nature..." [18, 19]. The field of research on synergy is quite broad, since its interests affect all branches of science. Being a "young" science, synergy currently does not have a single terminology, including a single name for the whole theory, since it is still far from its conclusion.

At the moment, one of the leading schools in which the synergistic approach is considered is the Brussels School, which rightfully holds the primacy in the formulation and development of the first theorems and the corresponding mathematical apparatus.

Central to synergy research was the analysis of the dynamics of complex systems in a state of imbalance, in which, under certain conditions, qualitative changes with new properties arose.

Consider a variant of factor analysis based on multiplicative index models, in which the general index is calculated as the product of partial indices by the factors considered. A distinctive feature of this method is the lack of commutativity, as a result of which the amount of increments by factors depends on the order of the somnobody indices in the model [3].
2. Formulation of the problem
The noted property represents certain difficulties in determining the exact order of the multiplier indices. A method of "techno-economic analysis" was used, in which indices of factors were divided into qualitative and quantitative ones. Qualitative characteristics include attributes of an attributive nature, and quantitative ones, as a rule, include factors indices for which a specific analysis is carried out. This division can be considered very arbitrary. It does not have any kind of logical rationale based on the strict rules of mathematics.

In fact, the mathematical law of commutativity is not fulfilled, according to which the result should not change from a change in the places of the factors. To overcome this drawback and obtain a formally strict result, the present work studies the properties of factor analysis when presenting a generalizing effective feature in a multiplicative form. In this case, the task is to obtain a correct result that would satisfy the commutativity requirement.

An algorithm for solving using multivariate index models is known from the literature [7]. Let's consider it in general terms to calculate the absolute growth and the growth rate of the effective indicator due to individual factors.

The economic and mathematical model of dependence in general form in multiplicative form is presented as follows:

\[ Y = a \cdot b \cdot c, \ldots, \cdot k, \]

where \( Y \) - effective feature; \( a, b, c, \ldots, k \) - factors.

The calculation of the rate of growth of the effective indicator due to individual factors is carried out by subtracting the corresponding conditional growth rates of the effective indicator. The conditional growth rate means the growth rate of the effective indicator taking into account changes in only the factor "\( a \)" , only the factors "\( a \)" и "\( b \)" , etc. Conditional growth rates of the effective indicator will be represented by indices \( I_{a}, I_{a}I_{b}, \ldots, I_{k} \), respectively.

The overall growth rate of the effective indicator

\[ \Delta I_{y} = I_{y} - 1 \text{ или } \Delta I_{y} = (Y_{y}/Y_{o}) - 1 \] (1)

due to the factor "\( a \)"

\[ \Delta I_{ya} = I_{a} - 1 \] (2)

due to factor "\( b \)"

\[ \Delta I_{yb} = I_{a}I_{b} - I_{a} \] (3)

due to factor "\( k \)"

\[ \Delta I_{yk} = I_{a}I_{b}, \ldots, I_{k} - I_{a}I_{b}I_{k-1} \] (4)

where the influence of factors is taken into account by individual indices. The given solution algorithm is applicable for models in which the quantitative indicator is in the first place. In the case when a qualitative indicator is in the first place of the index model, when they are decided, the weighing of factor values located on the left side of the studied indicator is carried out according to the basic data, and which stand on the right - according to the reporting data.

2.1 Research methods
In this case, the graphical analytical method can be used for calculations. For example, to calculate the growth rates by factors, it is required to build the following digraph, which is shown in Figure 1.
Let us explain the principle of constructing a graph.

First. Signs "+" and "-" mean weights of arcs +1 and -1.

The second. This graph is a system of equations.

The graph shown in Figure 2a illustrates equation (2). In turn, the graph in Figure 2b represents equation (3). Since the graphs in Figures 2a and 2b have only one common arc, and are weakly bound, combining equations (2) and (3) into a system, the graph shown in Figure 2b is radiant. This becomes possible due to the fact that the graphs in Figures 2, a and 2, b are weakly connected graphs - they have only one common arc - $I_a$. The calculations are recursive.

To synthesize increment calculation formulas, you must compute path weights from the initial vertex to the nodes of interest. So, for verse, we get the expression. For example, for the top $\Delta I_{yc}$ we get the expression

$$\Delta I_{yc} = P\{I_a, I_b, I_c, 1\} + P\{I_a, I_b, -1\} = I_a I_b I_c - I_a I_b,$$

where $P\{\}$ - weights indicated along path arcs.
Using this algorithm, you can construct a graph for any number of factors. In general, expression (5) can be represented as a formula for the aggregate growth rate:

$$\Delta I_y = \sum_i P_i,$$

where $P_i$ - the weights of all possible paths from the top of the source "0" to the top $\Delta I_y$.

It can be noted that the graph presented in Figure 1 allows you to see the consequence of the order of the indices of the multipliers in the products (2)... (4). This is achieved by rearrangements of arcs $I_a, I_b, \ldots, I_k$ in the considered graph. It is necessary to emphasize the fact that it is precisely in the possibility of displaying the consequences of the sequence of the indices of the multipliers in formulae (2)... (4) that the synergy of graph theory and factor analysis lies.

To clarify the significance of the influence of permutations of the succession of indices in a product, the question is of interest: "How to formally synthesize permutations?" This is important when programming and especially when verifying programs. Studies have shown that a convenient tool is the representation of the permutation field in the form of digraphs.

When solving the problem of choosing means of displaying multipliers on a graph, empirically, it was revealed that it is necessary to depict in-dexes in the form of arcs, with corresponding weights. Consider the problem in a number of examples presented in Figure 3.

![Figure 3. Graph of possible permutations of multiplier indices for a two-factor model.](image)

The two-factor model can be illustrated as a graph in Figure 3. By analogy with the above, the order of the indices in the model is determined by the order of the arcs in the paths, as a result of which two products can be obtained $I_aI_b \text{ and } I_bI_a$.

Following logic, for three factors, the graph representing the possible products of the indices is shown in Figure 4.

![Figure 4. Graph of possible permutations of multiplier indices for a three-factor model.](image)

We can conclude that there are 6 variants of permutations of index co-factors in the model, since there are 6 paths from source to drain.
It is essential to note that the topology of the graph depicts the edges of a cube, i.e. displays a 3D function. This explains the complexity of building graphs for four-dimensional (factor) models. This requires the introduction of pseudo-arcs with weights +1. Such a graph is shown in Figure 5. This graph describes 24 paths and outwardly is perceived as very cumbersome. This can be commented as follows - a convenient way to represent multidimensional functions on a plane has not yet been invented. At the same time, the examples considered can serve as a sufficient principle for mapping the properties of multiplicative n-dimensional functions - their domains of definition - in the form of graphs.

![Figure 5. A graph of permutation options for a four-factor model.](image)

It can be noted that the graphs allow you to obtain simple analytical estimates of the average growth rate of the effective indicator. To simplify the indicators, we introduce the concept of incremental indices for factors, which for the m-th factor are defined as:

\[ \Delta I_m = I_m - 1, \]  

by indicating the increment of the index compared to one, i.e. with the case when the baseline and reporting indicators have equal values.

Using formulas (7) ... (9) for two factors we get:

\[
\begin{align*}
\begin{bmatrix}
\Delta I_{ya} \\
\Delta I_{yb}
\end{bmatrix}
&= \frac{1}{2} \begin{bmatrix} 1 & I_a \\ I_b & 1 \end{bmatrix}^T \begin{bmatrix} \Delta I_a \\ \Delta I_b \end{bmatrix}.
\end{align*}
\]  

This expression is built on the basis of the graph in Figure 3 and in matrix form we display the calculation equations:
\[ \Delta I_{ya} = \frac{1}{2} (1 + I_b) \Delta I_a; \]  
\[ \Delta I_{yb} = \frac{1}{2} (1 + I_a) \Delta I_b. \]  

Using the graph in Figure 4, we repeat the considered procedure for three factors, where, as a result of averaging, we obtain:

\[ \Delta I_{ya} = \frac{1}{6} (2 + 2I_b I_c + I_b + I_c) \Delta I_a; \]  
\[ \Delta I_{yb} = \frac{1}{6} (2 + 2I_a I_c + I_a + I_c) \Delta I_b; \]  
\[ \Delta I_{yc} = \frac{1}{6} (2 + 2I_a I_b + I_a + I_b) \Delta I_c. \]

Similarly, using the graph in Figure 5, get expressions for the increments by four factors:

\[ \Delta I_{ya} = \frac{1}{24} [6(1 + I_b I_c I_d) + 2(I_b I_c + I_c I_d + \ldots + I_b I_d + I_c I_d)] \Delta I_a \]  
\[ \Delta I_{yb} = \frac{1}{24} [6(1 + I_a I_c I_d) + 2(I_a I_c + I_c I_d + \ldots + I_a I_d + I_c I_d)] \Delta I_b \]  
\[ \Delta I_{yc} = \frac{1}{24} [6(1 + I_a I_b I_d) + 2(I_a I_b + I_b I_d + \ldots + I_a I_d + I_b I_d)] \Delta I_c \]  
\[ \Delta I_{yd} = \frac{1}{24} [6(1 + I_a I_b I_c) + 2(I_a I_b + I_b I_c + \ldots + I_a I_c + I_b I_c)] \Delta I_d \]  

Similar to formulae (13) to (14), you can sequentially write you to estimate the growth rate for any number of factors.

There are a number of issues that are important in solving problems by factor analysis methods:

1. What is the effect of the non-transitivity property of factor analysis based on multiplicative index models on the accuracy of the final result in calculations?

2. What effect do random errors in the source data have on factor analysis performance?

To answer the questions posed, two methods are used: 1) the method of analytical research and 2) the method of imitative statistical modeling.

The analytical research and simulation statistical modus operandi should be used to answer the questions raised.

We estimate the effect of the initial data errors on the example of the study of conditional growth rates, where the total growth rate is given by formula (2), and the growth rate by individual factors by expressions (3). (4). To be sure, we would limit ourselves to two factors. Taking into account the errors in the initial data, the following formulas will be correct:

\[ \Delta \tilde{I}_{ya} = I_a + \xi_a(t) - 1; \]  
\[ \Delta \tilde{I}_{yb} = [I_a + \xi_a(t)] \cdot [I_a + \xi_b(t) - 1], \]  

where it is assumed that the errors in the representation of the growth rates by factors \( a \) and \( b \), etc. \( \Delta \tilde{I}_{ya} \) and \( \Delta \tilde{I}_{yb} \), are some random functions of time \( \xi_a(t) \) and \( \xi_b(t) \) with the known laws of probability density distribution and zero mathematical expectation. We get
$$M\{\Delta I_{ya}\} = \frac{1}{T} \int_0^T [I_a - 1] dt + \frac{1}{T} \int_0^T \xi_a(t) dt = I_a - 1 + M\{\xi_a(t)\} = I_a - 1. \quad (19)$$

Since the expectation of the function $\xi_a(t)$ is equal to zero i.e.
$$M\{\xi_a(t)\} = \frac{1}{T} \int_0^T \xi_a(t) dt = 0,$$
then the results will not be biased.

In the case of assessing the growth rate by factor $b$ we have
$$M\{\Delta I_{yb}\} = \frac{1}{T} \int_0^T (I_a I_b - I_a) dt + \frac{(I_b - 1)}{T} \int_0^T \xi_a(t) dt + \frac{I_a}{T} \int_0^T \xi_b(t) dt +$$
$$+ \frac{1}{T} \int_0^T \xi_a(t) \cdot \xi_b(t) dt. \quad (21)$$

The second and third terms are obviously equal to zero. With this in mind, we get:
$$M\{\Delta I_{yb}\} = I_a I_b - I_a + \frac{1}{T} \int_0^T \xi_a(t) \cdot \xi_b(t) dt,$$
then, to the ideal result (3.10) some random bias is added, represented by the correlation between the factors $a$ и $b$. When the functions $\xi_a(t)$ и $\xi_b(t)$ are correlated, an error appears. In most cases, the size of this error can be neglected, since $\xi_a(t) << 1$ и $\xi_b(t) << 1$.

Let us further define the corresponding variances for which:
$$D\{\Delta I_{ya}\} = \frac{1}{T} \int_0^T \xi_a^2(t) dt = D\{\xi_a\};$$
$$D\{\Delta I_{yb}\} = \frac{1}{T} \int_0^T [I_a \xi_b(t) + I_b \xi_a(t)]^2 dt; \quad (23)$$
$$D\{\Delta I_{yb}\} = \frac{1}{T} \int_0^T [I_a \xi_a(t) + I_b \xi_b(t)]^2 dt; \quad (24)$$

Taking into account (24), we define the variance:
$$D\{\Delta I_{yb}\} = I_a^2 D\{\xi_a\} + 2I_a (I_b - 1) K_{ab} +$$
$$+ \frac{2I_a}{T} \int_0^T \xi_a(t) \cdot \xi_b^2(t) dt + (I_b - 1)^2 D\{\xi_a\} +$$
$$+ \frac{2(I_b - 1)}{T} \int_0^T \xi_b(t) dt + \frac{1}{T} \int_0^T \xi_a^2(t) \xi_b^2(t) dt,$$
where $K_{ab}$ - factor correlation coefficient $a$ и $b$.

As can be seen from formula (25), a bias appears in the variance estimates. When evaluating, assuming $D\{\xi_a\} = D\{\xi_b\} = D\{\xi\}$ and, considering that
The formula (25) is written as follows:

$$\frac{2(I_b - 1)}{T} \int_0^T \xi(t) dt = 0.$$  \hspace{1cm} (26)

The formula (25) is written as follows:

$$D\{\Delta I_{yb}\} = I_a^2 D\{\xi\} + 2I_a (I_b - 1) K_{ab} +$$

$$+ \frac{2I_a}{T} \int_0^T \xi_a(t) \xi_b^2(t) dt + (I_b - 1)^2 D\{\xi\} +$$

$$+ \frac{1}{T} \int_0^T \xi_a^2(t) \xi_b^2(t) dt.$$  \hspace{1cm} (27)

Using formulas (23) and (27), let us compare the formulas for calculating variances in the first approximation. According to (23), we have:

$$D\{\Delta I_{ya}\} = D\{\xi\};$$  \hspace{1cm} (28)

$$D\{\Delta I_{yb}\} = I_a^2 D\{\xi\} + 2I_a (I_b - 1) + (I_b - 1)^2 D\{\xi\}. \hspace{1cm} (29)$$

On the basis of formula (29), it could be argued that with increasing performance and relatively small random variations in partial factor indices, inequality would be met:

$$D\{\Delta I_{yb}\} > D\{\Delta I_{ya}\}. \hspace{1cm} (30)$$

Example, \( I_a = 1.2; \ I_b = 1.3; \ D\{\xi\} = 0.02 \). Result is \( D\{\Delta I_{ya}\} = 0.02 \), but \( D\{\Delta I_{yb}\} = 0.755 \), those. the difference is more than obvious.

If \( I_a < 1 \) и \( I_b < 1 \), for initial data \( I_a = 0.8 \) и \( I_b = 0.7 \) with the same variance we get \( D\{\Delta I_{ya}\} = 0.02 \) и \( D\{\Delta I_{yb}\} = 0.005 \), variance decreases \( D\{\Delta I_{yb}\} \).

3. Results

The results are the following:

1. Factor analysis algorithms for the index models were investigated for the levels of the effective indicator, its absolute increments and growth rates, for which it was proposed to represent the systems of the corresponding equations in the form of directed graphs.

2. The study of the nontransitivity property of factor analysis algorithms based on multiplicative index models was carried out.

3. A method is proposed for displaying options for permutations of indices in multiplicative models in the form of directed graphs.

4. A method for the synthesis of formulas for factor analysis is proposed, which uses the representation of the area of determining the effective indicator in the form of a directed graph. The proposed technique can serve as an effective mnemonic scheme for teaching factor analysis.

5. Issues of graphical representation of the resulting indicator in the form of a combination of an oriented graph have been investigated.

6. A method is proposed for displaying algorithms for calculating the increments of the effective and growth rates of indicators in the form of an oriented graph and the corresponding topological formulas for calculating the indicated parameters of socio-economic processes are obtained.

7. A method of analytical analysis has been developed and appropriate formulas have been obtained to assess the influence of random variations in the initial data on the results of factor analysis.
8. When solving problems of factor analysis based on multiplicative index models, in order to take into account the nontransitivity of the applied algorithms of calculations, it is proposed to represent variants of permutations in the form of directed graphs, which is an effective tool for programming and verification of analysis programs.

4. Conclusion

Thus, it becomes clear that, by analogy, it is possible to display the properties of the procedure for calculating the influence of factors for any number of influencing quantities in the form of graphs.

When solving the problems of factor analysis based on multiplicative index models, taking into account the lack of commutativity in the known calculation algorithms, the authors of the article proposed to present permutation options in the form of oriented graphs, which is an effective tool when programming and verifying factorial analysis programs.

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