Particle and spin transports of spin-orbit coupled Fermi gas through a Quantum Point Contact

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The particle and spin transport through a quantum point contact between two Fermi gases with Raman-induced SO coupling are investigated. We show that the particle and spin conductances both demonstrate the structure of plateau due to the mesoscopic scale of the quantum point contact. Compared with the normal Fermi gases the particle conductance can be significantly enhanced by the spin-orbit coupling effect. Furthermore, the conversion of the particle and spin currents can take place in the spin-orbit coupled system, and we find that it is controlled by the parameter of two-photon detuning. When the parameter of two-photon detuning vanishes the particle and spin currents decouple.

I. INTRODUCTION

Transport measurements are important tools to investigate the fundamental properties of states of matter. Recently, the studies of transport in cold atoms have become one of the frontiers in the area. Many experiments have been conducted, including particle transport \[1\textsuperscript{–}9\], spin transport \[10\textsuperscript{–}14\] and heat transport \[15\textsuperscript{–}17\]. Particularly, the realization of two-terminal set-up by ETH’s group \[14\textsuperscript{–}16\] paved a road to extend this paradigmatic tool from condensed matter physics to various unique states in cold atom physics, for instance, the quantized conductance of neutral matter has been observed \[2]\textsuperscript{,} and the anomalous conductance of unitary Fermi gas has been studied both experimentally \[5]\textsuperscript{,} and theoretically\[18\textsuperscript{–}20\].

The interplay between charge and spin degrees of freedom is ubiquitous in physical systems. By utilizing light-atom interactions the synthetic spin-orbit (SO) coupling in neutral atoms has be realized in both bosonic and fermionic cold atom systems \[21\textsuperscript{,}22\]. These achievements have stimulated intensive studies in this area \[24\textsuperscript{,}25\]. In transport experiments the SO coupling may generate spin and charge currents conversion, which is a key phenomenon in spintronics and can facilitate technological applications, for instance, it’s possible to control spin signals by manipulating the electric signals.

In this work we study the spin and particle transports between two Fermi gases with Raman-induced SO coupling. The two reservoirs are connected by a quantum point contact (QPC). We calculate the particle and spin currents using the Keldysh formulism \[26\]. The chemical potentials of spin-up and spin-down particles are tuned to be different in either of the reservoirs, and then the particle current \(I_p = I_\uparrow + I_\downarrow\) and spin current \(I_s = I_\uparrow - I_\downarrow\) can be generated. In the linear response regime they can be expressed as

\[
\begin{pmatrix}
I_p \\
I_s
\end{pmatrix} = \begin{pmatrix}
\sigma_p & \sigma_o \\
\sigma_o' & \sigma_s
\end{pmatrix} \begin{pmatrix}
\Delta \mu_p \\
\Delta \mu_s
\end{pmatrix}
\]

where \(\Delta \mu_p = \mu_R - \mu_L\) and \(\Delta \mu_s = \delta \mu_R - \delta \mu_L\). \(\mu_j = (\mu_j^\uparrow + \mu_j^\downarrow)/2\) and \(\delta \mu_j = \mu_j^\downarrow - \mu_j^\uparrow\) are the average and difference of the chemical potentials of spin-up and spin-down fermions, where \(j = L, R\) denote the left and right reservoir. \(\sigma_p\) and \(\sigma_s\) are the particle and spin conductances. \(\sigma_o\) and \(\sigma_o'\) describe the conversion of the particle and spin currents. We investigate the variations of the elements \(\sigma_p, \sigma_s, \sigma_o,\) and \(\sigma_o'\) with respect to various parameters in system with Raman-induced SO coupling.

II. MODEL

The whole system, including two reservoirs and the QPC, can be described by the following Hamiltonian (setting \(\hbar = 1\))

\[
\hat{H} = \hat{H}_L + \hat{H}_R + \hat{H}_T,
\]

where \(\hat{H}_L(\hat{H}_R)\) describes the Fermi gas with Raman-induced SO coupling in the left (right) reservoir and is given by

\[
\hat{H}_j = \Psi_j \Gamma^{-1}_j \Psi_j,
\]

where \(\Psi_j = (\hat{\psi}_j^\uparrow, \hat{\psi}_j^\downarrow)\) and

\[
\Gamma_j^{-1} = \left( \begin{array}{cc}
\frac{(k_x + k_0)^2 + k_y^2}{2m} + \frac{\Omega}{2} - \mu_j^\uparrow & \frac{(k_x - k_0)^2 + k_y^2}{2m} - \frac{\Omega}{2} - \mu_j^\downarrow \\
\frac{(k_x + k_0)^2 + k_y^2}{2m} + \frac{\Omega}{2} - \mu_j^\downarrow & \frac{(k_x - k_0)^2 + k_y^2}{2m} - \frac{\Omega}{2} - \mu_j^\uparrow
\end{array} \right).
\]

The operator \(\hat{\psi}_j^\sigma(\hat{\psi}_j^\sigma)\) describes the creation (annihilation) of a fermion atom with spin \(\sigma = \uparrow, \downarrow\) in the \(j\)-th reservoir. \(m\) is the mass of fermions and \(k_0^2 = k_x^2 + k_y^2\). \(\Omega/2\) is the strength of the Raman coupling and \(k_0\) is the

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wave vector of the laser. $\delta$ is the two-photon detuning \[2\delta\]. With the Pauli matrices the Eq. (11) can be cast as
\[
G_j^{-1} = \frac{k^2}{2m} + B_k \cdot \sigma + E_r - \tilde{\mu}_j,
\]
where $B_k = (\Omega/2, 0, k_x k_0/m + \delta/2 - \delta \mu_j /2)$, and $E_r = k_0^2/2m$ is the recoil energy. With SO coupling the spin is not a good quantum number anymore. However, the single particle Hamiltonian can be diagonalized to two eigenstates with another good quantum number, the helicity $\pm \delta$, which denote being spin parallel or anti-parallel to the Zeeman field $B_k$. With a unitary transformation $U_j \hat{H}_j U_j^\dagger$ the eigenenergies can be obtained as
\[
E_{k, \pm} = \frac{k^2}{2m} \pm \xi(k_x) + E_r - (\tilde{\mu}_j \pm S(k_x) \delta \mu_j),
\]
where
\[
\xi(k_x) = \sqrt{\left(k_x k_0/m + \delta/2\right)^2 + \Omega^2/4},
S(k_x) = \frac{2\sqrt{\left(k_x k_0/m + \delta/2\right)^2 + \Omega^2/4}}{2\sqrt{\left(k_x k_0/m + \delta/2\right)^2 + \Omega^2/4}}.
\]
In this work we study the cases where $\delta \mu_j$ is small compared with the chemical potentials $\mu_{j, r}$. Hence, in Eq. (6) and the following work we only keep the terms up to the first order of $\delta \mu_j$. The fields $\hat{\psi}_{j, \pm}$ corresponding to these two branches can be found by a unitary transformation as the following
\[
\begin{pmatrix}
\hat{\psi}_{j, +}(k) \\
\hat{\psi}_{j, -}(k)
\end{pmatrix} =
\begin{pmatrix}
D_j(k_x) & -O_j(k_x) \\
O_j(k_x) & D_j(k_x)
\end{pmatrix}
\begin{pmatrix}
\hat{\psi}_{j, \uparrow}(k) \\
\hat{\psi}_{j, \downarrow}(k)
\end{pmatrix},
\]
and the elements in above matrix are presented as
\[
D_j(k_x) = \frac{\Lambda(k_x)}{\sqrt{\Lambda(k_x)^2 + \Omega^2/4}} + \frac{\Lambda(k_x)/2 \Lambda(k_x)^2/4}{2\xi(k_x) \Lambda(k_x)^2 + \Omega^2/4} \delta \mu_j,
\]
\[
O_j(k_x) = \frac{\Omega/2}{\sqrt{\Lambda(k_x)^2 + \Omega^2/4}} - \frac{\Lambda(k_x)^2/2 \Lambda(k_x)^2/4}{2\xi(k_x) \Lambda(k_x)^2 + \Omega^2/4} \delta \mu_j,
\]
where
\[
\Lambda(k_x) = k_x k_0/m + \delta/2 + \xi(k_x).
\]
The tunneling between two reservoirs through the QPC is described by $\hat{H}_T$. In real space it can be written as
\[
\hat{H}_T = \sum_{n=0}^{\infty} [T_n^{(+)} \hat{\psi}_{L, +}^\dagger(0) \hat{\psi}_{R, +}(0) + T_n^{(-)} \hat{\psi}_{L, -}^\dagger(0) \hat{\psi}_{R, -}(0)] + h.c.,
\]
Here we assume the eigenstates $\psi_{R, +}$ and $\psi_{R, -}$ are transported through point $x = 0$ between the two reservoirs. In the experimental setup the QPC is formed by the confinement in $\hat{y}$ and $\hat{z}$ directions, which lead to the transport channels with energies of $\epsilon_\pm(n_y, n_z) \equiv (\frac{1}{2} + n_y) \omega_y + (\frac{1}{2} + n_z) \omega_z + V_g [\ref{2} \ref{3}]$. For simplicity an effective gate potential $V_g = \frac{1}{2} \omega_y + \frac{1}{2} \omega_z$ can be defined and we assume $\omega_z \gg \omega_y$, then the several lowest transport channels would be $n_y \omega_y + V_g$, and they are non-degenerate. Then the tunneling amplitude $T_n^{(\pm)}$ can be written as
\[
T_n^{(\pm)}(k_L, k_R) = T \prod_{j=L,R} \Theta(\epsilon_{\pm}(k_j) - n \omega_y - V_g),
\]
where $\epsilon_{\pm}(k) = \frac{k^2}{2m} \pm \xi(k_x) + E_r$ is the single particle energy of field $\hat{\psi}_{j, \pm}$, and $\Theta(\epsilon_{\pm}(k_j) - n \omega_y - V_g)$ is the heaviside step function. Above tunneling amplitude indicates that only the particle with energy $\epsilon_{k, \pm} > n \omega + V_g$ can enter the $n$-th channel and will come out from the same channel. That is, there is no inter-channel scattering within the QPC region.

The current for spin-$\sigma$ is defined as
\[
I_\sigma = \frac{1}{2} \left\langle \frac{\partial}{\partial t} (N_L \sigma - N_R \sigma) \right\rangle,
\]
where $N_{j, \sigma} = \sum_{k} \hat{\psi}_{j, \sigma}^\dagger(k) \hat{\psi}_{j, \sigma}(k)$. In above expression the averages $\langle \cdot \cdot \cdot \rangle$ is taken over a time-evolving many-body state, which can be calculated using the Keldysh formalism. Please see the appendix A for details. In the linear response regime the particle and spin currents are expressed as the Eq. (14). The elements $\sigma_p$, $\sigma_x$, $\sigma_y$, and $\sigma_z$ in Eq. (14) can be calculated as the following
\[
\sigma_p = \frac{\alpha}{\hbar} \sum_{n=0}^{\infty} \int_{\epsilon_+(k_x) < \epsilon_F} dk_{1x} \int_{-\infty}^{\infty} dk_{2x} \Theta(\epsilon_+(k_x) - n \omega_y - V_g) F_1(k_{1x}, k_{2x}) n_+(k_{2x})
+ \int_{\epsilon_-(k_x) < \epsilon_F} dk_{1x} \int_{-\infty}^{\infty} dk_{2x} \Theta(\epsilon_-(k_x) - n \omega_y - V_g) F_1(k_{1x}, k_{2x}) n_-(k_{2x})
\]
\[
\sigma_x = \frac{\alpha}{\hbar} \sum_{n=0}^{\infty} \int_{\epsilon_+(k_x) < \epsilon_F} dk_{1x} \int_{-\infty}^{\infty} dk_{2x} \Theta(\epsilon_+(k_x) - n \omega_y - V_g) F_1(k_{1x}, k_{2x}) S(k_{2x}) n_+(k_{2x})
+ \int_{\epsilon_-(k_x) < \epsilon_F} dk_{1x} \int_{-\infty}^{\infty} dk_{2x} \Theta(\epsilon_-(k_x) - n \omega_y - V_g) F_1(k_{1x}, k_{2x}) S(k_{2x}) n_-(k_{2x})
\]
\[
\sigma_y = \frac{\alpha}{\hbar} \sum_{n=0}^{\infty} \int_{\epsilon_+(k_x) < \epsilon_F} dk_{1x} \int_{-\infty}^{\infty} dk_{2x} \Theta(\epsilon_+(k_x) - n \omega_y - V_g) F_1(k_{1x}, k_{2x}) D(k_{2x}) n_+(k_{2x})
+ \int_{\epsilon_-(k_x) < \epsilon_F} dk_{1x} \int_{-\infty}^{\infty} dk_{2x} \Theta(\epsilon_-(k_x) - n \omega_y - V_g) F_1(k_{1x}, k_{2x}) D(k_{2x}) n_-(k_{2x})
\]
\[
\sigma_z = \frac{\alpha}{\hbar} \sum_{n=0}^{\infty} \int_{\epsilon_+(k_x) < \epsilon_F} dk_{1x} \int_{-\infty}^{\infty} dk_{2x} \Theta(\epsilon_+(k_x) - n \omega_y - V_g) F_1(k_{1x}, k_{2x}) O(k_{2x}) n_+(k_{2x})
+ \int_{\epsilon_-(k_x) < \epsilon_F} dk_{1x} \int_{-\infty}^{\infty} dk_{2x} \Theta(\epsilon_-(k_x) - n \omega_y - V_g) F_1(k_{1x}, k_{2x}) O(k_{2x}) n_-(k_{2x})
\]
\[ \sigma'_{\alpha} = \frac{\alpha}{\hbar} \sum_{n=0}^{\infty} \left\{ \int_{\epsilon_{+}(k_{x})<\epsilon_F} \frac{dk_{1x}}{\epsilon_{+}(k_{x})} \int_{-\infty}^{\infty} \frac{dk_{2x}}{\epsilon_{+}(k_{x})} \Theta(\epsilon_{+}(k_{x}) - n\omega_y - \bar{V}_g) F_1(k_{1x}, k_{2x}) n_+(k_{2x}) + \int_{\epsilon-(k_{x})<\epsilon_F} \frac{dk_{1x}}{\epsilon_{-}(k_{x})} \int_{-\infty}^{\infty} \frac{dk_{2x}}{\epsilon_{-}(k_{x})} \Theta(\epsilon_{-}(k_{x}) - n\omega_y - \bar{V}_g) F_2(k_{1x}, k_{2x}) n_-(k_{2x}) \right\}, \]

\[ \sigma'_{\alpha} = \frac{\alpha}{\hbar} \sum_{n=0}^{\infty} \left\{ \int_{\epsilon_{+}(k_{x})<\epsilon_F} \frac{dk_{1x}}{\epsilon_{+}(k_{x})} \int_{-\infty}^{\infty} \frac{dk_{2x}}{\epsilon_{+}(k_{x})} \Theta(\epsilon_{+}(k_{x}) - n\omega_y - \bar{V}_g) F_1(k_{1x}, k_{2x}) n_+(k_{2x}) + \int_{\epsilon-(k_{x})<\epsilon_F} \frac{dk_{1x}}{\epsilon_{-}(k_{x})} \int_{-\infty}^{\infty} \frac{dk_{2x}}{\epsilon_{-}(k_{x})} \Theta(\epsilon_{-}(k_{x}) - n\omega_y - \bar{V}_g) F_2(k_{1x}, k_{2x}) n_-(k_{2x}) \right\}, \]

where

\[ n_{\pm}(k_{x}) = \frac{1}{\exp\{(\epsilon_{\pm}(k_{x}) - \epsilon_F)/k_B T\} + 1} \]

\[ F_1(k_{1x}, k_{2x}) = \frac{2(\Lambda(k_{1x})^2 + \Omega^2/4)(\Lambda(k_{2x})^2 + \Omega^2/4)}{\Lambda(k_{1x})^2\Lambda(k_{2x})^2 - \Omega^2/16}, \]

\[ F_2(k_{1x}, k_{2x}) = \frac{2(\Lambda(k_{1x})^2 + \Omega^2/4)(\Lambda(k_{2x})^2 + \Omega^2/4)}{\Lambda(k_{1x})^2\Lambda(k_{2x})^2 - \Omega^2/16}. \]

In this work we use \( \bar{\mu}_L \) as the energy scale and define the Fermi energy as \( \epsilon_F = \bar{\mu}_L \). The so-called transparency is defined as \( \alpha = \frac{\sigma_{\alpha}}{\sigma_{\alpha}^0} \). Here we assume perfectly transparent junction and set \( \alpha = 1 \).

### III. PARTICLE AND SPIN CONDUCTANCES FOR \( \delta = 0 \)

In this section we investigate the variation of the conductance matrix in Eq. 1 for the case of symmetric dispersion, where the two-photo detuning \( \delta = 0 \). Several properties of the conductances can be found. First, the particle conductance \( \sigma_p \) demonstrates the structure of quantized plateaus analogous to the system without SO coupling. In Fig. 2 (a), one observes that \( \sigma_p \) can reach 4/\( h \) when \( E_r \) approaches 0, which is double of the height of the plateau in the system without SO coupling. That is, in the region of \( E_r \ll \Omega \) the coupling of spin-up and spin-down fields can enhance the particle transport ability. Then, when \( E_r \) increases to the region \( \Omega \ll E_r \ll \epsilon_F \) the particle conductance \( \sigma_p \) drops rapidly and can reach the value of 2/\( h \). In this region the spin-up and spin-down fields are roughly decoupled. It’s reduced to a system without SO coupling. Hence, \( \sigma_p \) reaches the value of 2/\( h \), which is the conductance of normal gas. When \( E_r \) increases further and becomes comparable to \( \epsilon_F \) one observes that \( \sigma_p \) increases up to 4/\( h \) again. This can be explained by the graphs of Fig. 3 (a) and (b), in which we plot the transport of particles around the Fermi surface in one-dimensional case. The single particle dispersion of region \( E_r \ll \epsilon_F \) is sketched in Fig. 3 (a). One observes that a particle on the lower branch in the left reservoir can be transported to two states in the right reservoir, since the momentum is not conserved in the tunneling process as show in Eq. 1A in the appendix A. For the region \( E_r \gg \epsilon_F \) in Fig. 3 (b), we demonstrate that a particle on

![Fig. 1: (Color online) The particle conductance \( \sigma_p \) and spin conductance \( \sigma_s \) as functions of gate potential \( \bar{V}_g \) for fixed \( \Omega \) and \( E_r \).](image1)

![Fig. 2: (Color online) The particle conductance \( \sigma_p \) and spin conductance \( \sigma_s \) as functions of energy \( E_r \) for \( \Omega/\epsilon_F = 0.01, 0.1, 0.5 \) and 1. In both (a) and (b), we set \( \bar{V}_g/\epsilon_F = 0.5 \) and \( \delta = 0 \).](image2)
the lower branch in the left reservoir can be transported to four state in the right reservoir. This explains the increasing of the conductance as \( E_r \) increases. Fig. 3 (a) and (b) are examples in one-dimensional case. Our system is three dimensional. The Fermi surface is not a set of points but a sphere. The situation is more complicated but the logic beneath is the same. Comparing the four curves of \( \Omega/\epsilon_F = 0.01, 0.1, 0.5, \) and 1 in Fig. 4 (a) one observe that for any fixed \( E_r \) the particle conductance \( \sigma_p \) increases as \( \Omega \) increases. There are two effects that help to increase the particle conductance. One is the mixing of spin-up and spin-down fields. The other effect is the density of state. The low energy density of state in Raman-induced SO coupled system is larger than the case of \( k_x^2 \) dispersion. The curves of \( \Omega/\epsilon_F = 0.5 \) and 1 demonstrated clear peaks, which correspond to the cases when the Fermi surfaces reach the middle peak of the dispersion of the lower branch, where the density of state is large.

Third, in Fig. 2 (b) one observes that the spin conductance \( \sigma_s \) increases as \( E_r \) increases and saturates at the value of \( 2/h \). The variation of \( \Omega \) doesn’t affect \( \sigma_s \) much.

For the region (a) a particle around the Fermi surface on the lower branch can be transported to two states in the right reservoir. For the region (b) a particle on the lower branch can be transported to four states in the right reservoir. In both (a) and (b) we set \( \delta\mu_j = 0 \). (c), the schematic plot of the tilted Fermi surface due to the non-zero \( \delta\mu_j \). The red straight line represents the Fermi surface of the lower branch, and the blue one is for the upper branch.

Our calculation shows that the off-diagonal terms \( \sigma_o \) and \( \sigma'_o \) both vanish in the case of \( \delta = 0 \). The spin current is driven by the bias of the spin difference \( \Delta\mu_s = \delta\mu_R - \delta\mu_L \). In Eq. (6) we can see that the spin difference \( \delta\mu_j \) deforms the Fermi surface. Roughly speaking, it tilts the Fermi surface as sketched in Fig. 3 (c). During the spin transport, the Fermi surface of the left reservoir will become more horizontal. The particles on the lower branch with momenta \( k_x < 0 \) in the left reservoir will be transported to the right, while the particles with momenta \( k_x > 0 \) in the right reservoir will be transported to the left. Since the dispersion is symmetric about the axis \( k_x = 0 \), the number of particles transported from the left to the right reservoir equals to the one transported from the right to the left reservoir. Hence, the bias of the spin difference \( \Delta\mu_s \) will not drive a net particle current. Analogously, \( \Delta\mu_p \) will not drive a spin current. Hence, \( \sigma_o \) and \( \sigma'_o \) are both zero for \( \delta = 0 \) case.

### IV. PARTICLE AND SPIN CONDUCTANCES FOR \( \delta \neq 0 \)

As we discussed in the last section the particle and spin currents decouple when \( \delta \) vanishes. In this section we investigate how a non-zero \( \delta \) affect the transport. In Fig. 4 we plot \( \sigma_p, \sigma_o, \sigma'_o \), and \( \sigma_s \) as functions of \( E_r \) and \( \delta \) for a typical value of \( \Omega/\epsilon_F = 0.1 \). One observe that as \( \delta \) increases the \( \sigma_p \) and \( \sigma_s \) are both enhanced in Fig. 4 (a) and (d). Of particular interests are the graph Fig. 4 (b) and (d).

![Figure 3](image-url)  
**FIG. 3:** (Color online) the schematic plot of dispersions and Fermi surfaces for the region of (a) \( \Omega \ll E_r \ll \epsilon_F \) and (b) \( \Omega \ll \epsilon_F \ll E_r \), the red part of the dispersion represents the lower branch and the blue one represents the upper branch. For the region (a) a particle around the Fermi surface on the lower branch can be transported to two states in the right reservoir. For the region (b) a particle on the lower branch can be transported to four states in the right reservoir. In both (a) and (b) we set \( \delta\mu_j = 0 \). (c), the schematic plot of the tilted Fermi surface due to the non-zero \( \delta\mu_j \). The red straight line represents the Fermi surface of the lower branch, and the blue one is for the upper branch.

![Figure 4](image-url)  
**FIG. 4:** (Color online) \( \sigma_p, \sigma_o, \sigma'_o \), and \( \sigma_s \) as functions of \( E_r \) for different values of \( \delta \). Here we set \( \Omega/\epsilon_F = 0.1 \). When \( \delta \) changes sign, \( \sigma_p \) and \( \sigma_s \) remain the same, while \( \sigma_o \) and \( \sigma'_o \) changes sign.
(c), in which we see that $\sigma_{o}$ and $\sigma'_{o}$ become non-zero for non-zero $\delta$, that is, the strength of the coupling between particle and spin currents can be controlled by the two photon detuning $\delta$. Furthermore, the signs of $\sigma_{o}$ and $\sigma'_{o}$ are also tunable. When $\delta$ changes sign, $\sigma_{o}$ and $\sigma'_{o}$ change signs either, which gives more flexibility to manipulate the conversion of the particle and spin currents.

V. CONCLUSIONS

In summary, the particle and spin transports through a QPC have been studied in a fermionic system with Raman-induced SO coupling. Due to the mesoscopic scale of the QPC both particle and spin conductance exhibit plateau structure. Compared with the system without SO couplings the height of the plateau of the particle conductance is enlarged by the SO coupling effects. The magnitude of the spin conductance $\sigma_{s}$ majorly depends on the laser wave vector $k_{0}$. As $k_{0}$ increases $\sigma_{s}$ increases and saturates at a fixed value. Large two-photon detuning $\delta$ can help to enhance $\sigma_{s}$ when $k_{0}$ is small. Furthermore, the conversion of the particle and spin currents are exclusively controlled by the two-photon detuning $\delta$. When $\delta$ is non-zero, the particle and spin currents couple together.

VI. ACKNOWLEDGEMENTS

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Appendix A: The calculation of current in Keldysh formulism

To calculate the particle and spin currents we employ the Keldysh formulism in which the whole system is on a closed time contour. The partition function can be written as

$$Z = \frac{1}{Z_0} \int D[\bar{\psi}_{j+}, \psi_{j+}, \bar{\psi}_{j-}, \psi_{j-}] \exp(iS),$$  \hfill (A1)

where $S = S_0 + S_T + S_s$ and $S_0$, $S_T$ and $S_s$ correspond to the free, tunneling and source terms, respectively. $S_0$ can be expressed in the momentum space as the following

$$S_0 = \int dkd\omega \sum_{j=L,R} \left\{ \bar{\Psi}_{j+} [G_{j+}]^{-1} \Psi_{j+} + \bar{\Psi}_{j-} [G_{j-}]^{-1} \Psi_{j-} \right\},$$  \hfill (A2)

where the fields are given by

$$\bar{\Psi}_{j\pm} = \left( \bar{\psi}_{j1\pm}, \bar{\psi}_{j2\pm} \right), \Psi_{j\pm} = \left( \psi_{j1\pm}, \psi_{j2\pm} \right).$$

The field $\bar{\psi}_{j\pm}$ and $\psi_{j\pm}$ are the Keldysh rotation of the fields $\bar{\psi}_{j\pm}^+$ and $\psi_{j\pm}^+$, which are the fields of $\bar{\psi}_{j\pm}$ on forward and backward time directions. The Keldysh rotation are given by

$$\bar{\psi}_{j\pm} = \frac{1}{\sqrt{2}} (\psi_{j\pm}^+ + \psi_{j\pm}^-), \psi_{j\pm} = \frac{1}{\sqrt{2}} (\psi_{j\pm}^+ - \psi_{j\pm}^-).$$  \hfill (A4)

The Green’s functions in the Keldysh space are expressed as

$$G_{j\pm} = \left( \begin{array}{cc} G_{R}^{K} & G_{A}^{K} \\ 0 & G_{A}^{K} \end{array} \right),$$  \hfill (A5)

The retard(advanced) Green’s function is given by

$$G_{R}^{K} = \frac{1}{\omega - k^2/2m + \xi(x) - E_r + (\bar{\mu}_j + S(k_x))\Delta \mu_j) + i0^+},$$

$$G_{A}^{K} = \frac{1}{\omega - k^2/2m + \xi(x) - E_r + (\bar{\mu}_j + S(k_x))\Delta \mu_j) - i0^+},$$  \hfill (A6)

and the Keldysh Green function is

$$G_{K}^{K} = (1 - 2n_T(\omega))(G_{R}^{K} - G_{A}^{K}).$$  \hfill (A7)

To write down the action $S_T$ for the tunneling Hamiltonian $H_T$ we use the single particle Hamiltonian $H + \sum_{j\uparrow}(\hat{n}_{j\uparrow}^+ \hat{n}_{j\uparrow} + \bar{\mu}_j + \hat{n}_{j\uparrow} \hat{n}_{j\downarrow})$ to construct the time evolution operator

$$U(t) = e^{i[H + \sum_{j}(\bar{n}_{j\uparrow} \hat{n}_{j\uparrow} + \bar{\mu}_j \hat{n}_{j\downarrow})]t}$$  \hfill (A8)

Then the time evolution of the tunneling part is given by $H_T(t) = U(t)H_TU^\dagger(t)$. In momentum space it’s written as

$$H_T = \sum_{n=0}^{\infty} \int \frac{d\omega}{2\pi} \frac{d^3k_L}{(2\pi)^3} \frac{d^3k_R}{(2\pi)^3} \left[ \tilde{T}_n^{(+)}(\omega, \bar{\kappa}_L)\bar{\psi}_{R+}^{+}(\omega - \Delta \mu_p + S(k_{Rx})\Delta \mu_s, k_R) \right. \right.$$  \hfill (A9)

$$+ \tilde{T}_n^{(-)}(\omega, \bar{\kappa}_L)\bar{\psi}_{R-}(\omega - \Delta \mu_p - S(k_{Rx})\Delta \mu_s, k_R) \right] + h.c.$$  

The particle and spin currents are defined in Eq. (1). In momentum space they can be expressed in terms of field $\bar{\psi}_{j\pm}$ as the following

$$\bar{\psi}_{j\pm} = \left( \bar{\psi}_{j1\pm}, \bar{\psi}_{j2\pm} \right),$$

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In above equations the $D_j$ and $O_j$ are the elements of the unitary matrix in Eq. (8). Then in the Keldysh formulism the action parts of the tunneling term and source term can be cast as

\[ S_T = \sum_{n=0}^{\infty} \int \frac{d\omega}{2\pi} \frac{d^3k_L}{(2\pi)^3} \frac{d^3k_R}{(2\pi)^3} \left( J^+_n(0, \omega, k_L, k_R) + J^-_n(0, \omega, k_L, k_R) \right) + h.c., \]

\[ S_s = -i \sum_{n=0}^{\infty} \int d\Omega \frac{d\omega}{2\pi} \frac{d^3k_L}{(2\pi)^3} \frac{d^3k_R}{(2\pi)^3} \left\{ \begin{array}{l}
A^{cl}_1(\Omega)[(D_D D_R + O_L O_R) J^+_n(0, \omega, k_L, k_R) + O_L O_R J^-_n(\Omega, \omega, k_L, k_R)] \\
+ A^{cl}_2(\Omega)[(D_D D_R + O_L O_R) J^+_n(\Omega, \omega, k_L, k_R) + O_L O_R J^-_n(\Omega, \omega, k_L, k_R)] \\
+ A^{cl}_3(\Omega)[(D_D D_R + O_L O_R) J^+_n(\Omega, \omega, k_L, k_R) + D_D D_R J^+_n(\Omega, \omega, k_L, k_R)] \\
+ A^{cl}_4(\Omega)[(D_D D_R + O_L O_R) J^+_n(\Omega, \omega, k_L, k_R) + D_D D_R J^+_n(\Omega, \omega, k_L, k_R)] \\
\end{array} \right\} + h.c., \]  

where

\[ J^+_n(\Omega, \omega, k_L, k_R) = \sum_{n=0}^{\infty} T_n^{(+)} \]

\[ \left( \tilde{\psi}_{L1}(\omega, k_L) \psi_{R2-}(\Omega + \omega - \Delta \mu_p + S(k_{Rz})\Delta \mu_s, k_R) + \tilde{\psi}_{L2-}(\omega, k_L) \psi_{R1+}(\Omega + \omega - \Delta \mu_p + S(k_{Rz})\Delta \mu_s, k_R) \right), \]

\[ J^-_n(\Omega, \omega, k_L, k_R) = \sum_{n=0}^{\infty} T_n^{(-)} \]

\[ \left( \tilde{\psi}_{L1-}(\omega, k_L) \psi_{R2+}(\Omega + \omega - \Delta \mu_p - S(k_{Rz})\Delta \mu_s, k_R) + \tilde{\psi}_{L2+}(\omega, k_L) \psi_{R1-}(\Omega + \omega - \Delta \mu_p - S(k_{Rz})\Delta \mu_s, k_R) \right), \]

\[ J^n_+(\Omega, \omega, k_L, k_R) = \sum_{n=0}^{\infty} T_n^{(+)} \]

\[ \left( \tilde{\psi}_{L1}(\omega, k_L) \psi_{R1+}(\Omega + \omega - \Delta \mu_p + S(k_{Rz})\Delta \mu_s, k_R) + \tilde{\psi}_{L2+}(\omega, k_L) \psi_{R2+}(\Omega + \omega - \Delta \mu_p + S(k_{Rz})\Delta \mu_s, k_R) \right), \]

\[ J^n_-(\Omega, \omega, k_L, k_R) = \sum_{n=0}^{\infty} T_n^{(-)} \]

\[ \left( \tilde{\psi}_{L1-}(\omega, k_L) \psi_{R1-}(\Omega + \omega - \Delta \mu_p - S(k_{Rz})\Delta \mu_s, k_R) + \tilde{\psi}_{L2-}(\omega, k_L) \psi_{R2-}(\Omega + \omega - \Delta \mu_p - S(k_{Rz})\Delta \mu_s, k_R) \right). \]

The currents of spin-up and spin-down can be calculated by

\[ I_\sigma(t) = \frac{\Omega}{2\pi} e^{i\Omega t} I_\sigma(\Omega), \]
where $I_\sigma(\Omega)$ is given by

$$I_\sigma(\Omega) = \frac{i}{2} \frac{\partial Z}{\partial A^\dagger} \bigg|_{A^\dagger_{\sigma}, A^\dagger_{\sigma} = 0}.$$  \hfill (A14)