We present a detailed study of the effect of different three-nucleon interaction models in $p - {^3}\text{He}$ elastic scattering at low energies. In particular, two models have been considered: one derived from effective field theory at next-to-next-to-leading order and one derived from a more phenomenological point of view – the so-called Illinois model. The four-nucleon scattering observables are calculated using the Kohn variational principle and the hyperspherical harmonics technique and the results are compared with available experimental data. We have found that the inclusion of either one of the other force model improves the agreement with the experimental data, in particular for the proton vector analyzing power.

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The complete knowledge of the three-nucleon (3N) interaction is one of the open questions in Nuclear Physics nowadays. As is well known, there exist a number of different realistic nuclear-nucleon (NN) interaction models capable to reproduce almost perfectly the experimental NN scattering data up to energies of 350 MeV. However, with only this component of the nuclear interaction, one encounters several problems in the description of $A \geq 3$ nuclear systems (see, e.g., Refs. [1,2]). To improve that situation, different 3N forces have been introduced.

The recent development of 3N forces has followed mainly two lines. First, there are 3N force models derived within a chiral effective field theory (EFT) approach [4,5]. Models derived at next-to-next-to-leading order (N2LO) of the so-called chiral expansion have been used so far. At this particular order, the 3N force contains two unknown constants [5] usually determined either by fitting the 3N and four-nucleon (4N) binding energies [6] or, alternatively, the 3N binding energy and the Gamow-Teller matrix element (GTME) in the tritium $\beta$-decay [7,8]. The 3N force depends also on a cutoff function, which in general includes a cutoff parameter $\Lambda$. With a particular choice of the cutoff function, a local version of the N2LO 3N interaction has been derived [6]. The parameter $\Lambda$ is chosen to be for physical reason of the order of 500 MeV (for a discussion about the size of the $\Lambda$, see Ref. [5]). The derivation of chiral 3N force at successive orders is now in rapid progress [10,12].

Alternatively, within a more phenomenological approach, the so-called Illinois model for the 3N force model has been derived [13]. This model has been constructed to include specific two- and three-pion exchange mechanisms between the three nucleons. The model contains a few unknown parameters, which have been determined by fitting the spectra of $A = 4-12$ nuclei.

Clearly, it is very important to test these models to understand how they describe nuclear dynamics. The $A = 3$ and 4 scattering observables are between the best testing grounds to this aim. However, most of the $A = 3$ scattering observables are not very sensitive to the effect of the 3N force [1,3]. It is therefore of relevance to study their effect in 4N systems.

In recent years, there has been a rapid advance in solving the 4N scattering problem with realistic Hamiltonians. Accurate calculations of four-body scattering observables have been achieved in the framework of the Alt-Grassberger-Sandhas (AGS) equations [14,15], solved in momentum space, where the long-range Coulomb interaction is treated using the screening-renormalization method [16,17]. Solutions of the Faddeev-Yakubovsky (FY) equations in configuration space [18,19] and several calculations using the resonating group model [20,21] were also reported. In this contribution, the four-body scattering problem is solved using the Kohn variational method and expanding the internal part of the wave function in terms of the hyperspherical harmonic (HH) functions (for a review, see Ref. [22]). Very recently, the efforts of the various groups have culminated in a benchmark paper [23], where it was shown that $p - {^3}\text{He}$ and $n - {^3}\text{H}$ phase-shifts calculated using the AGS, FY, and HH techniques and using several types of NN potentials are in very close agreement with each other (at the level of or less than 1%).

Since 4N scattering observables can be calculated with high accuracy, it is timely to investigate the effect of the 3N force in these systems. It is important to note that the 4N studies performed so far have revealed the presence of several discrepancies between theoretical predictions and experimental data. In $p - {^3}\text{He}$ elastic scattering several accurate measurements exist for the unpolarized cross section [24,25], the proton analyzing power $A_y$ [26,28], and other polarization observables [29]. The calculations performed with a variety of NN interactions have shown a glaring discrepancy between theory and experiment for $A_y$ [14,20,26,28,30]. This discrepancy is very similar to the well known “$A_y$ Puzzle” in $N - d$ scattering. This is a fairly old problem, already reported about 20 years ago [31,32] in the case of $n - d$ and later confirmed also...
TABLE I: NN+3N interaction models used in this work. In columns 2–4 the values of the cutoff parameter $\Lambda$ and the coefficients $c_D$ and $c_E$ entering the EFT force models are reported (the coefficients are adimensional). In the last column we have reported the corresponding $^3$He binding energy.

| Model                        | $\Lambda$ [MeV] | $c_D$       | $c_E$       | $B(^3\text{He})$ [MeV] |
|------------------------------|------------------|-------------|-------------|------------------------|
| N3LO500/N2LO500*             | 500              | 1.0         | -0.192      | 28.36                  |
| N3LO500/N2LO500              | 500              | -0.12       | -0.196      | 28.49                  |
| N3LO600/N2LO600              | 600              | -0.26       | -0.846      | 28.64                  |
| AV18/IL7                     |                  |             |             | 28.44                  |

in the $p-d$ case. For other $p-^3\text{He}$ observables, as the $^3\text{He}$ analyzing power $A_{0y}$ and some spin correlation observables, discrepancies have been also observed. Recently at the Triangle University National Laboratory (TUNL) there has been a new set of accurate measurements (at $E_p = 1.60, 2.25, 4$ and $5.54$ MeV) of various spin correlation coefficients, which has allowed for a phase-shift analysis (PSA).

In this letter we report a study of the effect of 3N force models in $p-^3\text{He}$ elastic scattering in order to see whether their inclusion allows to reduce the above mentioned discrepancies. Clearly, it is important to specify which NN potential is used together with a particular model of 3N interaction. The N2LO 3N force derived from EFT has been used together with the NN potential models constructed within the same approach, in particular the next-to-next-to-next-to-leading order (N3LO) interaction derived by Entem and Machleidt. We have considered two cutoff values, $\Lambda = 500$ MeV and $\Lambda = 600$ MeV, labeled respectively N3LO500 and N3LO600. Correspondingly, we have to fix the two parameters $c_D$ and $c_E$ present in the N2LO 3N force. Together with the N3LO500 interaction model we have considered two versions of the N2LO 3N force; in the first one, label-led N2LO500*, $c_D$ and $c_E$ have been chosen so as to reproduce the $\Lambda = 3, 4$ binding energies as in Ref. [6]. In the second one, labeled N2LO500, the two parameters have been fixed reproducing the 3N binding energy and the tritium GTME. These two models have been used to explore the dependence of the results on $c_D$ and $c_E$.

With the N3LO600 NN interaction model, we have considered the 3N N2LO force label-led N2LO600 with $c_D$ and $c_E$ fixed to reproduce the 3N binding energy and the tritium GTME. In this way we can explore the dependence on $\Lambda$ of the 4N observables. The specific values of the parameters $c_D$ and $c_E$ are summarized in Table I.

The Illinois 3N model has been used in conjunction with the Argonne $v_{18}$ (AV18) NN potential. Between the different Illinois models, we have considered the most recent one, the so called Illinois-7 model (IL7). In Table II we have also reported the corresponding $^3\text{He}$ binding energy, which results rather close to the experimental value of 28.30 MeV. Therefore, eventual 4N forces should be rather tiny and their effect in $p-^3\text{He}$ scattering at low energy can be safely neglected.

For this study we have focused our attention to the effect of the 3N interaction. For this reason we have restricted the electromagnetic interaction between the nucleons to just the point Coulomb interaction between the protons. To be noticed that with the AV18 potential one should include the full electromagnetic interaction, including two-photon exchange, Darwin-Foldy term, vacuum polarization, and magnetic moment interactions as discussed in Ref. [23]. The effect of these additional terms for $N-d$ scattering was studied in Refs. [27, 28] and found to have a sizeable effect for some polarization observables. Regarding the N3LO500 and N3LO600 NN interactions, one should include only the effect of the two-photon exchange, Darwin-Foldy term, and vacuum polarization interactions in the $^3S_0$ partial wave [29]. Again, we have disregarded them in this work. The effect of these additional electromagnetic interactions will be the subject of a forthcoming paper.

In the energy range considered here ($E_p \leq 6$ MeV), the various $p-^3\text{He}$ observables are dominated by $S$-wave and $P$-wave phase shifts ($D$-wave phase shifts give only a marginal contribution, and more peripheral phase shifts are negligible). A comparison of a selected set of calculated phase-shifts and mixing parameters with those obtained by the recent PSA reveals that, using the interaction models with only a NN potential, both $S$- and $P$-wave phase-shifts result to be at variance with the PSA. Including the 3N force, we observe a general improvement of the description of the $S$- and $P$-wave phase shifts and mixing parameters. A detailed comparison between the calculated phase-shifts and those obtained from the PSA has been reported in Ref. [41].

Let us compare the theoretical results directly with a selected set of available experimental data. To see the effect of the 3N interaction, we have reported in Fig. I two bands, one collecting the results obtained using only NN interaction models and one obtained including also a 3N interaction. We have reported the results for the $p-^3\text{He}$ unpolarized differential cross section, two analyzing power observables, and some spin correlation observables. We note that the differential cross section, the $^3\text{He}$ analyzing power $A_{0y}$, and the spin correlation coefficients are not particularly sensitive to the adopted interaction models, and in general we observe a good agreement with the experimental values in all considered cases.

On the contrary, for the proton analyzing power $A_y$ shown in the upper right panel, we note a large sensitivity to the inclusion of the 3N interaction. The calculations performed using N3LO500 and AV18, in fact, largely underpredict the experimental points, a fact already observed before [23, 24, 28]. A sizable improvement is found by including the 3N interaction. The underprediction of the experimental data is now around 8-10%.
FIG. 1: (color online) $p - ^3$He differential cross section, analyzing powers and various spin correlation coefficients at $E_p = 5.54$ MeV calculated with only the NN potential (light cyan band) or including also the 3N interaction (darker blue band). The experimental data are from Refs. [26–28].

To better point out the sensitivity to the particular interaction model, in Fig. 2 an enlargement of $A_y$ and $A_{0y}$ in the peak region is shown. From the inspection of the figure, we can see that the results obtained using the N3LO500/N2LO500* and N3LO500/N2LO500 interaction models are very similar, showing that there is not much sensitivity to the parameters $c_D$ and $c_E$. The observables are more sensitive to the choice of the cutoff $\Lambda$, in particular $A_y$ calculated with the $\Lambda = 600$ MeV interaction model is slightly closer to the experimental data. Finally, the $A_y$ calculated with AV18/IL7 is very similar to those obtained with the chiral models, while $A_{0y}$ is in better agreement with the data (however, for this observable the experimental uncertainties are rather large).

The previously observed large underprediction of the $p - ^3$He $A_y$ observable was considered to be due to some deficiencies of the interaction in $P$-waves [28, 30], as, for example, due to the appearance of an unconventional “spin-orbit” interaction in $A > 2$ systems [42]. The IL7 model has been fitted to reproduce the $P$-shell nuclei spectra and, in particular, the two low-lying states in $^7$Li. This may explain the improvement in the description of the $p - ^3$He $A_y$ obtained with this interaction model. Regarding the N2LO 3N force models, its two parameters have been fitted either to the $A = 3$ and 4 binding energies, or to reproduce the 3N binding energy and the tritium GTME, quantities which are more sensitive to $S$-waves. Therefore, its capability to improve the description of the $p - ^3$He $A_y$ observable is not imposed but it is somewhat built-in.

It is interesting to examine the effect of the same interaction models in $p - d$ scattering. To this aim, we report in Fig. 3 two vector polarization observables at $E_p = 3$ MeV. In this figure, the light (cyan) band has been obtained using the NN chiral interaction only (in this case, the N3LO500 and N3LO600 models). The dark (blue) band has been obtained adding the corresponding N2LO 3N interaction. In this figure, the results obtained with AV18/IL7 are shown by the dashed (orange) lines (in this case, we have included the effect of the magnetic moment asymmetries, though rather tiny, show a large sensitivity to the $P$-waves phase-shift splitting [1, 33]). Accordingly, they can be used to fine tune the strength of subleading 3N spin-orbit appearing at next-to-next-to-next-to-next-to-leading order (N4LO) [12].

In conclusion, we have presented for the first time an analysis of $p - ^3$He elastic scattering observables including the effect of different 3N force models. The results obtained have been compared with the available experimental data. We have found that the phase shifts obtained with both the chiral and AV18/IL7 models are very close [41] with those derived from the recent PSA performed at TUNL [29]. The direct comparison of the
FIG. 3: (color online) $p - d$ vector polarization observables at $E_p = 3$ MeV calculated with only the NN potentials (light cyan band) or including also the 3N interactions (dark blue band) obtained within EFT. The results obtained with the AV18/IL7 interaction models are reported as the dashed (orange) lines. The experimental data are from Refs. [43].

due to the discrepancies, but the $A_0$ problem is noticeably reduced. In fact, we observe that the discrepancy is reduced to be of the order of $10\%$ at the peak, much less than before. We have also found that the results obtained with the N3LO/N2LO models and AV18/IL7 model are always rather close with each other (except for $A_0$). Since the frameworks used to derive these 3N force models are rather different, this outcome is somewhat surprising. Finally, it will be certainly very interesting to test the effect of the inclusion of the N3LO and N4LO 3N forces derived from EFT. Work in this direction is in progress.

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