Pioneer Anomaly and Accelerating Universe as Effects of the Minkowski Space Conformal Symmetry

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Abstract

On the basis of the nonisometric transformations subgroup of the SO(4,2) group the nonlinear time inhomogeneity one-parameter conformal transformations are constructed. The connection between the group parameter and the Hubble constant is established. It is shown that the existence of an anomalous blue-shifted frequency drift is the pure kinematic manifestation of the time inhomogeneity induced by the Universe expansion. This conclusion is confirmed via a generalization of the standard Special Relativity clock synchronization procedure to the space expanding case. The obtained formulae are in accordance with the observable Pioneer Anomaly effect. The anomalous blue-shifted drift is universal, does not depend on the presence of graviting centers and can be, in principle, observed on any frequencies under suitable experimental conditions. The explicit analytic expression of speed for the recession — intergalactic distance ratio is received in the form of a function of the redshift $z$, valid in the whole range of its variation. In the small $z$ limit this expression exactly reproduces the Hubble law. The maximum value of this function at $z = 0.475$ quantitatively corresponds to the experimentally found value $z_{exp} = 0.46 \pm 0.13$ of the transition from the decelerated to the accelerated expansion of the Universe.

1. Introduction

A nature of the blue frequency shift with a numerical value $\frac{d\nu}{dt} = (5.99 \pm 0.01) \times 10^{-9}$ Hz/s detected in the signals retransmitted by Pioneer 10 and 11 satellites (Pioneer Anomaly - PA) and interpreted as the Doppler shift, that is due to the uniform anomalous acceleration towards the Sun amounting to $a_p = (8.74 \pm 1.33) \times 10^{-8}$ cm/s$^2$, still remains not clear (see [1-3]).

The fact that the observable $a_p$ is close to the quantity $W_0 = cH_0$ ($c$ – speed of light, $H_0$ – Hubble constant) points to the cosmological origin of the PA effect. The standard GR, however, specifies the contribution of cosmological expansion as a value on the order of $H_0^2$ (rather than $\sim H_0$) [4]. Besides, a sign of this effect is opposite to the observed one [3].

Nevertheless, an interpretation of PA as a local manifestation of the cosmological expansion seems to be acceptable when using the possibilities inherent in the general symmetry group of the Minkowski space.

Most recent experimental data of astrophysics clearly indicate an infinitesimal space curvature of the observable Universe. Because of this, it seems natural to select the Minkowski space as a model for space-time manifold of the modern Metagalaxy. As is well known, its most
wide symmetry group is the group $SO(4,2)$ that, apart from the Poincaré group isometric transformations, includes a subgroup of special conformal transformations and dilatations changing the space and time scales (e.g., see [5]). Naturally, it may be expected that an expansion of the space-time symmetry by the inclusion of this subgroup transformations will result in some generalization of the standard kinematics of the Special Relativity (SR).

In the present paper on the basis of the nonisometric transformations subgroup of the $SO(4,2)$ group the nonlinear time inhomogeneity one-parameter conformal transformations are constructed (Sec. 2). The connection between the group parameter and the Hubble constant $H_0$ is established. It is shown that the existence of an anomalous blue-shifted frequency drift equal to $\nu H_0$ Hz/s in the location-type experiments performed under the condition $\Delta t H_0 \ll 1$ is the pure kinematic manifestation of the time inhomogeneity induced by the Universe expansion. The obtained formulae reproduce the observable Pioneer Anomaly effect. The anomalous blue-shifted drift is universal, does not depend on the presence of graviting centers and can be observed on any frequencies under suitable experimental conditions (Sec. 3). This conclusion is confirmed via a generalization of the standard Special Relativity clock synchronisation procedure to the space expending case (Sec. 4). The explicit analytic expression of speed of recession — intergalactic distance ratio is received in the form of the function of red shift $z$, valid in the whole range of $z$ variation. In the small $z$ limit the expression obtained exactly reproduces the Hubble law. The maximum in this function at $z = 0.475$ quantitatively corresponds to the experimentally founded value ($z_{exp} = 0.46 \pm 0.13$) of the transition of the decelerated to the accelerated expansion of the Universe (Sec. 5).

2. Conformal transformations and time inhomogeneity

It is known that the group $SO(4,2)$ considered as a symmetry group of the Minkowski space, in addition to the ten-parameter Poincaré group, includes also a five-parameter subgroup of the transformations changing the space-time scales. This group involves the four-parameter Abelian subgroup $S$ of the special conformal transformations (SCT).\footnote{Hereinafter the metric used is $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$.}

\[ x'^\mu = \sigma^{-1}(a, x) \{ x^\mu + a^\mu(x^\alpha x_\alpha) \}, \]  

where

\[ \sigma(a, x) = 1 + 2(a^\alpha x_\alpha) + (a^\alpha a_\alpha)(x^\beta x_\beta), \]  

$a^\mu$ — four-vector parameter with a dimension of the reciprocal length varying in the region $-\infty < a^\mu < \infty$, and also the one-parameter group of dilatations $D$

\[ x'^\mu = \lambda x^\mu, \]  

where the positive dimensionless parameter $\lambda$ belongs to the open semi-infinite interval $0 < \lambda < \infty$.

\[ D \] is an external automorphism group of the group $S$. Consequently, the transformations

\[ x'^\mu = \sigma^{-1}(\lambda a, x) \cdot \lambda \{ x^\mu + \lambda a^\mu(x^\alpha x_\alpha) \} \]

form the non-Abelian group $G_{SD}$ isomorphic to the semidirect product of the groups $S$ and $D$ ($G_{SD} \approx S \ltimes D$) with a well-known composition rule for the parameters

\[ (a_2^\mu, \lambda_2)(a_1^\mu, \lambda_1) = (a_2^\mu + \lambda_2(a_1^\mu), \lambda_2 \lambda_1), \]
where $\lambda_2 a_\mu$ – image of the element $g(a_\mu)$ in case of automorphism $\alpha(\lambda_2)$, and $\lambda_2(a_\mu) = \lambda_2^{-1}(a_\mu)$.

The transformations of (4) do not affect invariance of the light cone equation (zero interval).

Subsequently, keeping in mind a well-known kinematic SCT interpretation relating the parameter $a^\mu$ to the uniform four-acceleration [6], the vector parameter $a^\mu$ is assumed to be space-like (i.e. it is set to $(a^\alpha a_\alpha) < 0$). For now, no restrictions are imposed on the vector $x^\mu$.

For simplicity, we select a frame of reference (FR), where the vector $x^\mu$ is of the form $\{x^0, x, 0, 0\}$, whereas the four-vector $a^\mu$, due to its space similitude, may be of the form $\{0, a, 0, 0\}$. In this case the transformations of (4) lead to a two-parameter group that is isomorphic to the group of shifts and dilatations in the two-dimensional subspace $\{x^0, x\}$ of the Minkowski space, with a binary operation meeting the composition rule (5):

$$g(a_2, \lambda_2) \circ q(a_1, \lambda_1) = g(a_2 + \lambda_2^{-1}a_1, \lambda_2\lambda_1).$$

As is known, such a group includes the Abelian invariant subgroup isomorphic to the simple shear group. The elements of this group are of the form $g(b, 1)$, where $-\infty < b < \infty$. The parameter $b$ as a function of the parameters $a$ and $\lambda$ may be selected as follows:

$$b = \mp a\eta,$$

where

$$\begin{cases} 1 - \lambda^{-1} & \text{if } \lambda > 1, \\ 1 - \lambda & \text{if } \lambda < 1, \\ 0 \leq \eta \leq 1. \end{cases}$$

As this takes place, the lower (upper) sign corresponds to expansion (compression) $\lambda > 1$ ($\lambda < 1$). As follows from (4), transformations of the one-parameter group under study are in the explicit form

$$x^0 = \sigma^{-1}(b, x)x^0, \quad x' = \sigma^{-1}(b, x)\{x + b(x_0^2 - x^2)\},$$

where

$$\sigma(b, x) = 1 - 2bx - b^2(x_0^2 - x^2).$$

Most interesting in this aspect is a case of the zero-valued interval (isotropic vector $x^\mu$) when the relations of (8) take the following form:

$$x^0 = \frac{x^0}{1 - 2bx}, \quad x' = \frac{x}{1 - 2bx}$$

on condition $x = \pm ct$, where the signs $(\pm)$ are associated with a signal propagation in counter directions. Let us consider the expansion case ($b = -ax$). With due regard for (7), from the relations of (9) we obtain

$$t' = \frac{t}{1 \pm \frac{t}{t_{max}}},$$

where the parameter $t_{max} = (2a\eta c)^{-1}$ has the dimension of time, the signs $\pm$ being associated with a signal propagation in the forward and backward directions.

As seen, the parameter $t_{max}$ at each fixed value of the parameters $a, \eta$ sets the upper limit on possible values of $t'(t)$ when a signal is propagating in the forward (backward) direction. If it turns out that possible values of the parameter $|b| = a\eta$ are bounded below by $b_0 = a_0\eta_0$, the permissible time interval is bounded above by $t_{max} = (2a_0\eta_0 c)^{-1}$. A nonlinear character of the transformations (10) suggests nonuniformity of time. Geometrically, these transformations represent a deformation (respectively compression or extension) of the light cone generating
lines. And identifying \((t,x)\) with the coordinates of an event in the ordinary IFR, we find that \((t',x')\) will be representing the coordinates of the same event in the deformed FR. As will be shown later, it may be interpreted as a special-form noninertial FR. Clearly the transformations of (10) cause a change in the length of any finite segment of the light-cone generating lines. We write the required transformations for the following sequence of events: emission of a signal at the world point \(A\) at the instant of time \(t^0_A\); its transfer to the world point \(B\), instantaneous reemission, and subsequent return to the world point \(A\) at the instant of time \(t_A\). That is to say, we have

\[
t'_B - t^0_A = \frac{t_B - t^0_A}{1 + \frac{t_B-t'_A}{t_{max}}}, \tag{11}
\]

\[
t'_A - t'_B = \frac{t_A - t_B}{1 - \frac{t_A-t_B}{t_{max}}}. \tag{12}
\]

Here the primes denote the coordinates of the associated events in the deformed FR. As seen, for the coincident time intervals \(t_A - t_B\) and \(t_B - t^0_A\) the time intervals \(t'_A - t'_B\) and \(t'_B - t^0_A\) are not equal to each other, and \(t'_A - t'_B > t'_B - t^0_A\). This disagreement should result in the experimentally observed results; primarily in the experiments on location of the space-separated objects by electromagnetic signals. In experiments of this type the measured time of the whole process is determined as a time interval between the transmission of a signal \((t^0_A)\) and its return \((t_A)\) because \(\Delta t_{obs} = t_A - t^0_A\). Since time coincidence is assumed both in the forward and backward direction, the time interval required for a signal to cover a distance to the object equals

\[
\Delta t = \frac{1}{2}(t_A - t^0_A), \tag{13}
\]

And the distance itself, due to the universal constancy of a signal rate (first of all its independence of the source motion rate), is determined by the following relation:

\[
\Delta r = c\Delta t. \tag{14}
\]

### 3. Time inhomogeneity and Pioneer Anomaly

For the comparison with experiment, we consider a small-time approximation when the condition \(t/t_{max} \ll 1\) is valid. In this case, restricting ourselves to consideration of the first-order infinitesimal terms, from (12) we obtain

\[
\Delta t' = \Delta t, \tag{15}
\]

\[
t'_B - t^0_A = \Delta t - \frac{(\Delta t)^2}{t_{max}}, \quad t'_A - t'_B = \Delta t + \frac{(\Delta t)^2}{t_{max}}. \tag{16}
\]

As follows from formulae (15) and (16), the time interval \(\Delta t'\) determined by the relation of (15) is smaller than the actual length of time \(t'_A - t'_B\) required for a signal to cover the distance from the source to the observation point. It is easily seen that this results in the observable radiation-frequency shift, in the direction of increasing values, in accordance with the following relation:

\[
\nu_{obs} = \nu(1 + \frac{2\Delta t}{t_{max}}). \tag{17}
\]

Here \(\nu\)– frequency of the signal from a stationary, in the ordinary sense, source, \(\nu_{obs}\) – really observed frequency.
Whence the expression for a relative frequency shift in unit time is as follows:

$$\frac{\nu_{\text{obs}} - \nu}{\Delta t} = 2t_{\text{max}}^{-1} \nu = 4a_0\eta_0 c \nu.$$  \hspace{1cm} (18)

Within the scope of the approach under study, the blue frequency shift is universal, and in conditions of the expanding space-time manifold it should be observed, in principal, at all frequencies. Proceeding from this point of view, the Pioneer Anomaly is the first experimentally recorded effect of this type that may form a basis for experimental estimation of the numerical value of $t_{\text{max}}$.

It is known (see [1-3]) that the directly observable effect of PA consists in revealing of the uniform rate of a positive shift $\dot{\nu}_{\text{obs}}$ at the operating frequency $\nu = 2,29 \cdot 10^9$ Hz for satellite repeaters by the amount calculated as

$$\dot{\nu}_{\text{obs}} = (5,99 \pm 0,01) \cdot 10^{-9} \text{Hz/s}. \hspace{1cm} (19)$$

Selecting for a minimum value of the parameter $a$ the quantity $H_0/c$ ($H_0 = 2,4 \cdot 10^{-18}$ s$^{-1}$ – Hubble constant), assuming in (18) $\nu = 2,29 \cdot 10^9$ Hz and $d\nu/dt = \dot{\nu}_{\text{obs}}$, by comparison with the numerical value $\dot{\nu}_{\text{obs}}$ from (19), we can find that for $\eta_0 \approx 0,272$ equation (17) reproduces the observable PA effect, both quantitatively and qualitatively. And as this takes place, the value of the parameter $t_{\text{max}}$ is equal to $t_{\text{max}} = (2\eta_0 H_0)^{-1} \approx 1,84 \cdot H_0^{-1}$.

It should be noted that, within the scope of the proposed approach, the effect is purely kinematic (in the same sense as the effects of time "retardation" and Lorentzian compression are kinematic in a standard Special Relativity). Because of this, the approach requires no specific dynamic substantiation. In fact, the question is about the exhibited time inhomogeneity induced by the cosmological expansion and described in the general case by equation (10). In the approximation of $H_0 t \ll 1$, valid in a very wide time interval, this inhomogeneity is exhibited as a term quadratic in $t$, that appears in the expression determining the nonlinear dependence of $t^\prime$ on $t$.

For the distances $\Delta r^\prime = ct^\prime$ covered by a signal along the light-cone generating lines in the forward and backward directions we obtain the following expression:

$$\Delta r^\prime = c\Delta t \mp \frac{c(\Delta t)^2}{t_{\text{max}}} = c\Delta t \mp W_0(\Delta t)^2/2. \hspace{1cm} (20)$$

As is demonstrated, in conditions of the expanding space-time manifold (in the small-time approximation $t \ll H_0^{-1}$) the distances covered by a signal along the light-cone generating lines in the forward and backward directions are distinguished from $r_0 = c\Delta t$ by the quantity $\delta r = \frac{W_0 t_{\text{max}}^2}{2}$, where

$$W_0 = 2c t_{\text{max}}^{-1} = 4a_0\eta_0 c^2 \hspace{1cm} (21)$$

is a constant with the dimension of acceleration.

From the conventional point of view, the situation seems to be caused by a radiation source experiencing at all space points the uniform acceleration $W_0$ directed to the origin of coordinates (observation point).

Substituting into (21) the numerical values $c = 3 \cdot 10^{10}$ cm, $H_0 = 2.4 \cdot 10^{-18}$ s$^{-1}$, $\eta_0 = 0.272$ for the acceleration $W_0$ we obtain the numerical value $W_0 \approx 7.83 \cdot 10^{-8}$ cm/s$^2$ that is in a good agreement with the experimentally recorded value $a_p = (8,74 \pm 1.33) \cdot 10^{-8}$ cm/s$^2$ cited in [1-3].

Note that the uniform acceleration $W_0$ is actually background in character. Consequently, one may consider that it is characteristic for "expanding" Frame of Reference (FR) regarded as a specific case of the noninertial one. As this takes place, the corresponding four-acceleration...
$W_\mu$ is given by the four-vector having in the co-moving FR the following form: $W_\mu = \{0, W_0, \bar{z}, \bar{r}\}$. And the $a^\mu$ is related to $W^\mu$ by the relation $a^\mu = \frac{1}{c^2} W^\mu$ that is adopted in the concept associated with a well-known kinematic interpretation of SCT (see [6]).

According the proposed approach the appearance of the quadratic in time term in the formula (16) and (20) under condition $t/t_{max} \ll 1$ mimics the effect of the constant acceleration (21) directed in the accordance with (20) towards the point of observation. It is easy to see that the condition $t/t_{max} \sim tH_0 \ll 1$ is sure satisfied in any real location-type experiment in our solar system.

4. The clock synchronization procedure in expanding space

Let us demonstrate that the same result may be acquired when we consider a standard location procedure for the clock synchronization in conditions of the expanding space-time manifold.

It is common knowledge that the kinematic physical basis for the SR concept is formed using the location procedure for synchronization of the space-separated clocks at rest in each fixed IFR (see [7]). In so doing it stands to reason that the condition used should set an invariable distance between each clock pair in the process of synchronization. As shown in [8], with a change in the space scales the synchronization condition is modified due to inclusion of the cosmological expansion.

When the space scales are varying in accordance with the postulates of a homogeneous and isotropic model, it can be assumed that the radial distances $R$ have the exponential time-dependence

$$R(t) = R(t_0) \exp\left\{ \pm \int_{t_0}^{t} H(\tau) d\tau \right\}. \tag{22}$$

Here $H$ – Hubble parameter that is generally dependent on time, and the signs of $(\pm)$ are associated with expansion and compression. Hereinafter our consideration is confined to the expansion case only.

Let the identical clocks located at the points $A$ and $B$ be separated by the space interval $R_{AB}$. The synchronization procedure remains the same as previously: a synchronizing signal is transmitted from the point $A$ at the instant of time $t_0^A$, reflected at the point $B$ at the instant of time $t_B$, and returned back to the point $A$ at time $t_A$. Now the distances $R_{AB}$ and $R_{BA}$ covered by the signal in both directions are different ($R_{BA} > R_{AB}$).

If $c_1$ and $c_2$ – average rates of the synchronizing signal in the "forward" and "backward" directions, with the use of the postulate that the synchronizing-signal rate is independent of the source motion rate, the following relations may be written:

$$R_{AB} = c_1(t_B - t_0^A), \quad R_{BA} = c_2(t_A - t_B). \tag{23}$$

Taking them into account, we have

$$\frac{R_{BA}}{R_{AB}} = \frac{c_2(t_A - t_B)}{c_1(t_B - t_0^A)} = \exp\left\{ \bar{H}(t_A - t_B) \right\}, \tag{24}$$

where $\bar{H} = (t_A - t_B)^{-1} \int_{t_0^A}^{t_B} H dt$.

As the right-hand side of (24) is above 1, the Einstein conditions $t_A - t_B = t_B - t_0^A$ and $c_1 = c_2$ may be simultaneously fulfilled only in the absence of expansion. And in the case under study there is an alternative:

(I) $t_A - t_B > t_B - t_0^A$, $c_1 = c_2$; (II) $t_A - t_B = t_B - t_0^A$, $c_2 > c_1$. 
With due regard for a key role played in the SR concept and modern metrology by the representation for the universal stability of the synchronizing signal rate (i.e. constant \( c = 3 \cdot 10^{10} \) m/s), we choose the first variant (as variant II has been considered in [8]).

On condition \( c_1 = c_2 = c \), from the relation of (24) we derive the following expression:

\[
\frac{t_A - t_B}{t_B - t_A^0} = \exp \{ \bar{H}(t_A - t_B) \},
\]

that comprises an transcendental equation for the definition of the unknown \( t_B \) in terms of the given values \( t_A^0 \) and \( t_A \). Under the condition \( \bar{H}(t_A - t_B) \ll 1 \) (25) leads to the relation

\[
t_A - t_B = (t_B - t_A^0)\{1 + \bar{H}(t_A - t_B)\}
\]

representing the following quadratic equation for \( t_B \):

\[
t_B^2 - \frac{2}{\bar{H}} \left\{ 1 + \frac{\bar{H}}{2}(t_A + t_A^0) \right\} t_B + \frac{1}{\bar{H}}(t_A + t_A^0) + t_A t_A^0 = 0.
\]

Its solutions, in the approximation linear with respect to \( \bar{H}(t_A - t_A^0) \), are of the form

\[
t_B(\pm) = \frac{1}{\bar{H}} + \frac{t_A^0 + t_A}{2} \pm \frac{1}{\bar{H}} \left\{ 1 + \frac{\bar{H}^2(t_A - t_A^0)^2}{8} \right\}.
\]

Proceeding from the correspondence principle, in accordance with which for \( \bar{H} = 0 \) the result of the standard SR should be reproduced by \( t_B = \frac{1}{2}(t_A^0 + t_A) \), we have to choose the solution \( t_B(-) \). Then we get

\[
t_B = \frac{1}{2}(t_A^0 + t_A) - \frac{\bar{H}(t_A - t_A^0)^2}{8}.
\]

As seen, \( t_B \) has a linear dependence on the difference in indications \( (t_A - t_A^0) \) of the "reference" clocks and, within the approximation used, includes an additional quadratic term.

For the propagation time of a synchronizing signal in the forward \( (t_{AB} = t_B - t_A^0) \) and backward \( (t_{BA} = t_A - t_B) \) directions we find that

\[
t_{AB} = t - \frac{\bar{H}t^2}{2}, \quad t_{BA} = t + \frac{\bar{H}t^2}{2},
\]

where \( t = \frac{1}{2}(t_A - t_A^0) \) is the propagation time of a synchronizing signal as used in the standard SR.

From the relation of (29) we derive the following expressions to determine the distances covered by a signal in the forward and backward directions:

\[
R_{AB} = r_0 - \Delta r = ct - \frac{c\bar{H}t^2}{2}, \quad R_{BA} = r_0 + \Delta r = ct + \frac{c\bar{H}t^2}{2}.
\]

From the customary point of view, the situation looks as though in the process of synchronization the clock \( B \) were moving with respect to the "reference" clock \( A \) with the acceleration, equal to

\[
W = c\bar{H}
\]

and directed towards the clock \( A \) (i.e. to the observation point).

It should be noted that the presence of a term quadratic with respect to \( t \) in the relation (29) makes it possible to suggest the "clock acceleration" effect as \( \frac{d^2 t_{AB}}{dt^2} = \bar{H} \neq 0 \) (see[9] and reference [9] ibidem).
In essence, the expressions (29) and (30) thus obtained are coincident with the relations (16) and (20) providing an identity of $\ddot{H}$ with the Hubble constant $H_0$ and assuming $\eta_0 = \frac{1}{4}$.

It is clear that a choice of the sign (-) in the initial relation (22) - compression case - leads to the reciprocal substitution of signs in the resultant relations (29) and (30), that is an additional argument in support of the sign choice in definition (7) of the parameter $b$. The frequency shift in case of the compressed space-time manifold would turn to be red, and the associated "minimum acceleration" were directed from the observation point.

It is interesting that the real measuring procedure performed in the process of tracking the satellite motion completely replicates the principal features of clock synchronization in SR: signal transmission, its repeating by a satellite transmitter, and return to the observation point.

5. The dependence of distance on red shift. The Hubble law

Let us consider the experimentally verifiable cosmological consequences of the conformal time transformations (10) which are not restricted to the case of small values of $t/t_{\text{max}}$. First of all we show that the explicit expression for the distance between the signal emitter and the point of observation can be derived directly from conformal time transformations (10) in the form of a simple function of the red shift $z$. For this propose we consider the case of a signal traveling in the direction of negative generatrix of light cone (lower sign in the formula (10)):  

$$ t' = \frac{t}{1 - \frac{t}{t_{\text{max}}}}. $$ (32)

For small time increments $\Delta t'$ and $\Delta t$ we have from (32)

$$ \Delta t' = \Delta t(1 - \frac{t}{t_{\text{max}}})^{-2}. $$ (33)

If $\Delta t$ and $\Delta t'$ are the periods of oscillations of emitted ($\Delta t = T_{\text{emitted}}$) and received ($\Delta t' = T_{\text{observable}}$) signals correspondingly, then using the standard definition of red shift

$$ \frac{\lambda_{\text{observable}}}{\lambda_{\text{emitted}}} = z + 1, $$ (34)

where $\lambda_{\text{observable}} = cT_{\text{observable}}$ and $\lambda_{\text{emitted}} = cT_{\text{emitted}}$, we find from (33) the expression

$$ \frac{\lambda_{\text{observable}}}{\lambda_{\text{emitted}}} = (1 - \frac{t}{t_{\text{max}}})^{-2} = z + 1, $$ (35)

which gives

$$ t(z) = t_{\text{max}} \frac{(z + 1)^{1/2} - 1}{(z + 1)^{1/2}}. $$ (36)

Here $t(z)$ represents by definition the time interval between the moments of emitting and receiving the light (electromagnetic) signal. So, supposing that the speed of light is constant and does not depend on the velocity of the emitter, the quantity $R = ct$ can be regarded as the distance covered by the signal.

From the formula (36) we obtain an expression which determines an explicit form of dependence of $R$ on the red shift $z$:

$$ R(z) = R_u \frac{(z + 1)^{1/2} - 1}{(z + 1)^{1/2}} = R_u \left(1 - \frac{1}{(z + 1)^{1/2}}\right). $$ (37)

\footnote{Relation of the type $R(z) = \text{const} \cdot z$, that connect cosmological distance $R$ with the red shift, seems to be firstly obtained from the conformal symmetry arguments by Ingraham [10] in 1954.}
Here $R_u = c t_{\text{max}}$ is a parameter, which, within in the model suggested, has the sense of the limit (maximal) distance. We will assume its value to be equal to

$$R_u = 2cH_0^{-1}. \quad (38)$$

Quantity $R(z)$ defined by (37) corresponds to the distance, which in cosmology is referred to as a location distance. In principle the relation (37) allows direct experimental verification in the whole range of $z$ variation and can be confirmed or refused by observations.

To obtain explicit expression for the Hubble law, we make use of the well-known formulae describing the Doppler effect in Special Relativity. However in the context of our approach, to find the explicit expression for the longitudinal Doppler effect, it is convenient to apply the formulae which immediately follow from the Lorentz boosts written in the light-cone variables:

$$u = \frac{1}{2}(x_0 + x), \quad v = \frac{1}{2}(x_0 - x).$$

These expressions are (see, for example, [11]):

$$u' = k(\beta)u, \quad v = k^{-1}(\beta)v, \quad (39)$$

where $k(\beta) = \left(\frac{1 - \beta}{1 + \beta}\right)^{1/2}$, $\beta = V/c$, $V$ is the velocity which can be identified as the radial component of relative velocity of emitter and receiver motion.

Clearly, in terms of $u$ and $v$, the Lorentz boosts have the form of dilatations. For the description of the Doppler effect we need use the equation of light cone $4uv = 0$. Then for the case of the signal traveling in positive ($v = 0$) and negative ($u = 0$) directions we have in coordinates $(x, t)$:

$$t' = \left(\frac{1 \pm \beta}{1 \pm \beta}\right)^{1/2}t, \quad (40)$$

In the case of emitter and receiver moving away from each other, we find from (40) for small time increments $\Delta t'$ and $\Delta t$:

$$\Delta t' = \Delta t \left(\frac{1 + \beta}{1 - \beta}\right)^{1/2},$$

whence, taking into account (40), the known expression for function $V(z)$ follows:

$$\frac{V(z)}{c} = \frac{(z + 1)^2 - 1}{(z + 1)^2 + 1}. \quad (41)$$

Equation (A10) is valid in the whole range of velocity $V$ variation.

In the approximation $z \ll 1$ we obtain from (37) and (40) to accuracy of the terms $\sim z^2$ $V(z)/c \approx z$ and $R(z)/R_u \approx z/2$, whence, taking into account (38), the expression for the Hubble law in its standard form follows:

$$V = H_0 R. \quad (42)$$

In the general case we find by (37) and (41) the following expression for the ratio $V/R$:

$$\frac{V(z)}{H_0 R(z)} = f(z) = \frac{1}{2} \frac{(z + 1)^{1/2}}{(z + 1)^2 + 1} \cdot \frac{(z + 1)^2 - 1}{(z + 1)^{1/2} - 1}. \quad (43)$$

Clearly, $\lim_{z \to 0} f(z) = 1$ and $\lim_{z \to \infty} f(z) = 1/2$. Asymptotic behavior of the function $f(z)$ is determined as follows:

$$f(z)|_{z \ll 1} = 1 + z, \quad f(z)|_{z \gg 1} = \frac{1}{2} \left(1 + \frac{1}{\sqrt{z}}\right).$$
The function $f(z)$ and its derivative $f'(z) = \frac{df(z)}{dz}$ are shown in Fig. 1 and Fig. 2. Obviously, $f(z)$ has a maximum at $z_0 \approx 0.475$ (see Fig. 2 and Fig. 3). Overall variation of $f(z)$ demonstrates that in the interval $0 \leq z < z_0$ the distance $R(z)$ increases with $z$ more slowly, and in the interval $z_0 < z < \infty$ approaches its limit value $R_u$ more rapidly, than the velocity $V(z)$ approaches its limit $c$. Horizontal line in Fig. 1 represents strict Hubble law (42).

As regards a possible treatment of the behavior of the function (43) in terms of the standard
approach using the deceleration parameter, we are to notice the following. According to the
pure kinematic approach proposed in our paper, the source of the effects induced by the cos-
alogic expansion is the time conformal inhomogeneity. The "acceleration" attributed to the
emitting source arises because of treating the actual nonlinear time dependence $t'(t)$ in terms
of the traditional theoretical paradigm based on the time homogeneity concept. The uniform
"acceleration" $W_0 = cH_0$ which appears in the formula (21) in the $t/t_{\text{max}} \ll 1$ approxima-
tion is not a "true" but the "effective" acceleration in the reality. The "acceleration" in the general
case must be treated as time-dependent one. Pure formally it can be determined as the second
derivative of the function

$$R(t) = ct' = ct \left(1 - \frac{t}{t_{\text{max}}}\right)^{-1} \quad (44)$$

and it possess the following form

$$W(t) = \frac{2c}{t_{\text{max}}} \left(1 - \frac{t}{t_{\text{max}}}\right)^{-3}. \quad (45)$$

The "time-dependent effective acceleration" is directed to the point of observation and coincides
in the first approximation in $t/t_{\text{max}}$ with $W_0 = cH_0$. This "acceleration" can be presented
according to (35) in the form

$$W(z) = W_0 (1 + z)^{3/2}.$$

One can say that the numerical value of such an "acceleration" decreases during the process of the
Universe expansion starting from the very large magnitude. Evidently, the interpretation of $f(z)$
behavior from the position of common treatment seems as follows. In the interval $\infty > z > z_0$
there is the deceleration of cosmological expansion, and at the point of $z_0 \approx 0.475$ it changes to
the acceleration. Numerical value of $z_0$ agrees quite well with experimentally founded "point of
change" $z_{\text{exp}} = 0.46 \pm 0.13$ from the deceleration of cosmological expansion to the acceleration.

At last, we again emphasize the essentially kinematic nature of the relation (37). It is the
manifestation of the nonlinear time deformation (10) which follows from Special Conformal
Transformations exactly in the same manner, as the Doppler effect follows from the linear
time deformation (40), which arise from the Lorentz boosts leaving the equation of light cone
unaltered.

It should be emphasized that the basic formula (10) for the conformal transformations of
the time, as well as all its consequences are valid on the assumption that the Hubble parameter
$H_0$ is constant. Hopefully this assumption is reasonable as applied to at least later stages of
the Universe evolution. In this case the proposed formulae (37) and (43) can be valid for the
experimentally obtained values of the red shift having the order of several units.

6. Conclusion

Provided the proposed interpretation of PA is correct, the frequency shift measured may be
considered as a new independent high-precision measurement of the Hubble constant.

Moreover, the concept as a whole may be directly tested by the experiment. The use of
the appropriate sources and monochromatic detectors at required experimental conditions en-
ables observation of the described anomalous blue shift in its "pure form", when the source
and detector are mutually motionless, practically at all frequencies. As according to (18) the
effect is linearly growing with the frequency, it is expedient to use high-frequency radiation. In
principle, this allows for a considerable decrease in the observation time. To study this effect,
it is required to provide (i) maximum immobility of the source and detector, (ii) elimination of
the gravitational field effect of massive bodies, (iii) minimization of thermal fluctuations and mechanical deformations. One should not rule out a possibility of providing all these conditions in zero gravity at the satellites orbiting along the circumterrestrial orbits. This proposal has been made in [12].

By the proposed approach, the anomalous blue-shifted drift is a pure kinematic effect arising from homogeneous and isotropic expansion of the flat four-dimensional Minkowski manifold. In a sense, this approach is phenomenological with respect to the General Relativity, in itself being a certain natural (and minimal) extension of the SR concept, with evident adherence to the correspondence principle. And most important is the problem concerning modification of the standard relativistic kinematics necessitating further investigation.

It should be emphasized that, according to the suggested interpretation, the effect should be independent of any gravitating centers present. In other words, the acceleration $W_0 \sim cH_0$ is background in character and can be treated as the manifestation of the noninertial character of local frame of reference associated with any point of expanding Universe. It would be noted that such a "noninertiality" can mimic in accordance with the equivalence principle the dark energy effect. It is highly probable that a similar origin may be also attributed to a minimum acceleration $\sim 10^{-8}$ cm/s$^2$ (Milgrom parameter) used as a fundamental dimensional constant in the MOND concept (see [13-15]). Also, note that the parameter $W_0$ arise as the so-called Mass Dependent Maximal Acceleration (MDMA) (see [16] and references ibidem), if we substitute the value $M = M_u = \frac{4\pi}{3}R_u^3\rho_c$ that is equal to the Metagalaxy mass ($R_u = cH_0^{-1}$ - "radius" of the Universe, $\rho_c = \frac{3H_0^2}{8\pi G}$ - critical density) into the expression $W(M) = F_0/M$ for MDMA, where $F_0 = c^4/G$ (G - gravitational Newton constant) is the Maximum Force (MF) introduced in [16] and independently in [17,18]. All these problems call for special consideration in further publications of the author.

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