Strongly Lensed Gravitational Waves and Electromagnetic Signals as Powerful Cosmic Rulers

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ABSTRACT

In this paper, we discuss the possibility of using strongly lensed gravitational waves (GWs) and their electromagnetic (EM) counterparts as powerful cosmic rulers. In the EM domain, it has been suggested that joint observations of the time delay ($\Delta \tau$) between lensed quasar images and the velocity dispersion ($\sigma$) of the lensing galaxy (i.e., the combination $\Delta \tau/\sigma^2$) are able to constrain the cosmological parameters more strongly than $\Delta \tau$ or $\sigma^2$ separately. Here, for the first time, we propose that this $\Delta \tau/\sigma^2$ method can be applied to the strongly lensed systems observed in both GW and EM windows. Combining the redshifts, images and $\sigma$ observed in the EM domain with the very precise $\Delta \tau$ derived from lensed GW signals, we expect that accurate multimesenger cosmology can be achieved in the era of third-generation GW detectors. Comparing with the constraints from the $\Delta \tau$ method, we prove that using $\Delta \tau/\sigma^2$ can improve the discrimination between cosmological models. Furthermore, we demonstrate that with $\sim 50$ strongly lensed GW-EM systems, we can reach a constraint on the dark energy equation of state $w$ comparable to the 580 Union2.1 Type Ia supernovae data. Much more stringent constraints on $w$ can be obtained when combining the $\Delta \tau$ and $\Delta \tau/\sigma^2$ methods.

Key words: gravitational lensing: strong – gravitational waves – cosmological parameters – dark energy – distance scale

1 INTRODUCTION

In the standard $\Lambda$CDM cosmological model, it is currently inferred that $\sim 96\%$ of the total energy density of the Universe consists of dark matter ($\sim 26\%$) and dark energy ($\sim 70\%$). These proportions have been measured precisely via various standard candles or rulers, such as Type Ia supernovae (SNe Ia; Perlmutter et al. 1998; Riess et al. 1998; Schmidt et al. 1998; Suzuki et al. 2012), anisotropy of the cosmic microwave background (CMB) radiation (Hinshaw et al. 2013; Planck Collaboration et al. 2016a), as well as baryon acoustic oscillations (Beutler et al. 2011; Anderson et al. 2012). Though cosmology has entered a new era of precision tests, we should note that all of the cosmological probes are based on electromagnetic (EM) observations alone. In 1986, Schutz (1986) first proposed that the waveform signal of gravitational waves (GWs) from inspiralling and merging compact binaries encodes the luminosity distance $d_L$ information, proving access to the direct measurement of $d_L$. Thus, the GW signals can be considered as standard sirens. The combination of $d_L$ derived from GWs and redshifts $z$ derived from their EM counterparts would make GW events an ideal tool to constrain the cosmological parameters and the equation of state of dark energy.

On 2016 February 11, the Laser Interferometer Gravitational Wave Observatory (LIGO) team reported the first direct detection of the gravitational wave source (GW 150914; Abbott et al. 2016a), opening a brand new window for studying the Universe, which indicates that the era of multimeessenger cosmology is coming. In the past, several studies have investigated the possibility of GWs as standard sirens (e.g. Holz & Hughes 2006; Cai & Yang 2017; Del Pozzo et al. 2017). Particularly, Cai & Yang (2017) found that with about 500–600 simulated GW events they can determine the Hubble constant $H_0$ with an accuracy comparable to the Planck 2015 results; for the dark matter density parameter $\Omega_m$, it should need more than 1000 GW events to match the Planck sensitivity.

Very recently, Fan et al. (2017) presented a new model-
independent method for constraining the speed of GWs, based on future time delay measurements of strongly lensed GWs and their EM counterparts (see also Baker & Trodden 2017; Collett & Bacon 2017). Even more encouragingly, Liao et al. (2017) have shown that such strongly lensed GW-EM systems could also provide strong constraints on cosmological parameters. The GW standard-sirens method in cosmology appeals to the luminosity distance measurement from the GW observation which relies on the fine details of the waveform, but the proposed method of Liao et al. (2017) is waveform independent. Moreover, the GW standard-sirens method would require several hundred GW events to match the Planck sensitivity on $H_0$ (Cai & Yang 2017), while Liao et al. (2017) have shown that the uncertainty of $H_0$ might be better constrained by future time delays of lensed GW-EM events. Note that the time delays between different lensed images ($\sim 10 - 100$ days) obtained from the GW observations would reach an unprecedented accuracy of $\sim 0.1$ s from the detection pipeline. Compared to the uncertainty in lens modelling, the uncertainty of the GW time delay is negligible.

Strong gravitational lenses are a complementary cosmological probe (see e.g.Refsdal 1964; Zhu 2000; Grillo et al. 2008; Biesiada et al. 2010; Treu 2010; Cao & Zhu 2012; Cao et al. 2012a,b, 2015; Oguri et al. 2012; Collett & Auger 2014). The Einstein ring radius inferred from the deflection angle and the time delay between different lensed images can provide the information of angular-size distance independently, which can be used to measure cosmological parameters (see e.g. Coe & Moustakas 2009; Dobke et al. 2009; Paraficz & Hjorth 2009, 2010; Suyu et al. 2010, 2013; Sereno & Paraficz 2014; Bonvin et al. 2017), to discriminate different cosmological models (see, e.g., Zhu & Sereno 2008; Wei et al. 2014; Melia et al. 2015; Yuan & Wang 2015), and to probe the cosmic distance duality relation (see, e.g., Holanda et al. 2016; Liao et al. 2016; Rana et al. 2017). As of today, the observation of about 70 strong gravitational lensing systems and 12 two-image lensing systems with time delay measurements have provided the data that, in principle, can be used to carry out the study of cosmology. However, this is only the beginning. The upcoming Large Synoptic Survey Telescope (LSST) will find more than $\sim 8000$ lensed quasars, about 3000 of which will have well-measured time delays within 10 yr (Oguri & Marshall 2010). The number of robust time-delay measurements for probing cosmology is estimated to be $\sim 400$, each with precision $< 3\%$ and accuracy of $\sim 1\%$ (Dohler et al. 2015; Liao et al. 2015). In addition, we note that Paraficz & Hjorth (2009) have proposed an interesting cosmic ruler constructed from the joint measurements of the time delay ($\Delta \tau$) between lensed quasar images and the velocity dispersion ($\sigma$) of the lensing galaxy. They have shown that the joint measurement of $\Delta \tau/\sigma^2$ is more effective to constrain cosmological parameters than $\Delta \tau$ or $\sigma^2$ separately.

In the GW window, the fantastic sensitivity of the third-generation GW interferometric detectors, such as the Einstein Telescope (ET), would significantly improve the detection efficiencies of the GW events. With a large number of detectable events, we might expect some of these events to be gravitationally lensed by intervening galaxies. The prospects of observing strongly lensed GWs from merging double compact objects (NS-NS, NS-BH, BH-BH) have been studied in detail (Piorkowska et al. 2013; Biesiada et al. 2014; Ding et al. 2015); these works have predicted that ET would detect about 50–100 strongly lensed GW events per year. This implies that the ET will be able to provide a considerable catalogue of strongly lensed GWs within a few years of successful operation.

As mentioned above, Liao et al. (2017) proposed that future time delay measurements ($\Delta \tau$) of strongly lensed GW signals accompanied by EM counterparts could be used to obtain robust constraints on cosmological parameters. Because $\Delta \tau/\sigma^2$ is more sensitive to the cosmological parameters than $\Delta \tau$ or $\sigma^2$ separately (Paraficz & Hjorth 2009), here, for the first time, we try to explore the cosmological constraint ability by future joint measurements of the precise time delay ($\Delta \tau$) between lensed GW images and the velocity dispersion ($\sigma$) of the lensing galaxy in the era of the third-generation GW detectors.

The paper is organized as follows. In Section 2, we describe the basics of using strong gravitational lensing systems as standard rulers. In Section 3, we demonstrate that the cosmological parameters can be constrained with great accuracy through the combination $\Delta \tau/\sigma^2$ of the lensed GW-EM system, using Monte Carlo simulations. A brief summary and discussion are given in Section 4.

## 2 STRONG LENSES AS COSMIC RULERS

A source lensed by a foreground massive galaxy or galaxy cluster appears in multiple images. For a given image $i$ at angle position $\theta_i$, with the source position at angle $\beta$, the time delay $\Delta \tau_i$ is caused both by the difference in path-length between the straight and deflected rays, and the gravitational time dilation of the light ray traveling through the effective gravitational potential $\Psi(\theta_i)$ of the lens (Blandford & Narayan 1986):

$$\Delta \tau_i = \frac{1 + z_i}{c} \frac{D_{OS}D_{OL}}{D_{LS}} \frac{1}{2} \left( \beta_i^2 - \beta_i^2 - \Psi(\theta_i) \right).$$

(1)

Here, $z_i$ is the lens redshift and $D_{OS}$, $D_{OL}$, and $D_{LS}$ represent the angular-diameter distances between observer and source, observer and lens, and lens and source, respectively. If the lens potential $\Psi$ and the lens geometry $\theta_i - \beta$ are known, the time delay measures the ratio $D_{OS}D_{OL}/D_{LS}$, which depends on the cosmological parameters. Assuming that the time-delay lensing systems have only two images at $\theta_A$ and $\theta_B$, and adopting the single isothermal sphere (SIS) model for the gravitational potential of the lens galaxy, the time delay is therefore given by

$$\Delta \tau = \frac{1 + z_i}{2c} \frac{D_{OS}D_{OL}}{D_{LS}} (\theta_B^2 - \theta_A^2).$$

(2)

The distance ratio that appears in Equation (2) is the time-delay distance, $D_{\Delta \tau} \equiv (1+z_i)D_{OS}D_{OL}/D_{LS}$, which depends primarily on $H_0$ and has a limited sensitivity to other cosmological parameters, such as $\Omega_m$ (more on this below).

Inferring cosmological distances from time-delay lenses also requires accurate models for the mass distribution of the lens galaxy, as well as for any other matter structures along the line-of-sight that might affect the observed time delays between the multiple images (Suyu et al. 2010). A constant external convergence term $\kappa_{ext}$ can be absorbed...
by the lens and source model, leaving the fit to the lensed images unchanged. However, the true time-delay distance $D_{\Delta \tau}$ is altered by a factor of $(1 - \kappa_{\text{ext}})$, i.e.,

$$D_{\Delta \tau} = \frac{D_{\Delta \tau}^{(0)}}{1 - \kappa_{\text{ext}}},$$

where $D_{\Delta \tau}^{(0)}$ is the time-delay distance inferred from a model not accounting for the effects of weak perturbers along the line-of-sight. To break the “mass-sheet degeneracy” (Falco et al. 1985), it is possible to study the lens environment to constrain $\kappa_{\text{ext}}$ within a few percent based on spectroscopy and multifield observations of local galaxy groups and line-of-sight structures (e.g., Fassnacht et al. 2006; Moncheva et al. 2006) in combination with ray-tracing through numerical simulations (e.g., Collett et al. 2013; Greene et al. 2013). According to the recent analysis by Collett & Cunnington (2016), the external convergence over an ensemble of lenses usually does not average to zero. Rusu et al. (2017) presented a robust estimate of the external convergence $\kappa_{\text{ext}}$ for the lensed quasar HE 0435-1223, which has a median of 0.004 and a standard deviation of $\delta_{\text{ext}} = 0.025$. This measured $\delta_{\text{ext}}$ corresponds to a 2.5% uncertainty on $D_{\Delta \tau}$. In sum, the external convergence of each lens is expected to introduce 1 or 2 percent extra uncertainty on $D_{\Delta \tau}$ (Collett et al. 2013; Greene et al. 2013; Rusu et al. 2017). Thus, the uncertainty on $D_{\Delta \tau}$ is given by the quadrature sum of the uncertainties on the time delay, external convergence, and image position measurements (Suyu et al. 2017).

The observed velocity dispersion ($\sigma$) of the lensing galaxy is the result of the superposition of numerous individual stellar spectra, each of which has been Doppler shifted because of the random stellar motions within the galaxy. Hence, it can be measured by analysing the integrated spectrum of the galaxy. According to the virial theorem, the velocity dispersion is related to the mass (i.e. $\sigma^2 \propto M_c R$, where $M_c$ denotes the mass enclosed inside the radius $R$). The mass is determined by the Einstein ring radius $\theta_E$ of the lensing system, and thus the velocity dispersion in the SIS model can be written as

$$\sigma^2 = \theta_E^2 \frac{c^2 D_{OS}}{4\pi D_{LS}}.$$

As shown by Paraficz & Hjorth (2009), two of the angular-diameter distances appearing in Equation (2) could be replaced by the velocity dispersion $\sigma$ and the Einstein radius $\theta_E = (\theta_A + \theta_B)/2$ ($\Delta \tau$ and $\theta_{A,B}$ are defined to be positive here, with $\theta_B > \theta_A$), i.e.

$$D_{OL}(\theta_B - \theta_A) = \frac{c^2}{4\pi \sigma^2(1 + z_L)} \Delta \tau.$$

In contrast to $D_{\Delta \tau}$, Jee et al. (2015) found that the mass external to the lens along the line-of-sight (external convergence) has no effect on the inferred $D_{OL}$. The reason is as follows. Assume that there is a lens system which has a time delay of $\Delta \tau$ and a velocity dispersion of $\sigma^2$. We then try to model this system by a lens plus an external convergence, $\kappa_{\text{ext}}$. The modelled $\Delta \tau$ and $\sigma^2$ would be different from the original ones by a factor of $(1 - \kappa_{\text{ext}})$, but the ratio of the two is invariant. Because $D_{OL}$ is proportional to the ratio $\Delta \tau/\sigma^2$, we can determine the same $D_{OL}$ as before, regardless of the existence of the external convergence. Thus, the uncertainty on $D_{OL}$ is given by the quadrature sum of the uncertainties on the time delay, velocity dispersion, and image position measurements (Jee et al. 2015, 2016).

From Equations (2), (4), and (5), we can see that the time-delay $\Delta \tau$ is proportional to $D_{OS} D_{OL}/D_{LS}$, the square of the velocity dispersion $\sigma^2$ is proportional to $D_{OS}^2/D_{LS}$, and the ratio $\Delta \tau/\sigma^2$ is dependent only on $D_{OL}$. That is to say,

$$\Delta \tau \propto \frac{D_{OS} D_{OL}}{D_{LS}}, \quad \sigma^2 \propto \frac{D_{OS}^2}{D_{LS}}, \quad \frac{\Delta \tau}{\sigma^2} \propto D_{OL}.$$

In Fig. 1, we show the three quantities ($\Delta \tau$, $\sigma^2$, and $\Delta \tau/\sigma^2$) as a function of the lens redshift $z_L$ in the flat $\Lambda$CDM model with a fixed source redshift $z_S = 3$ (see also Paraficz & Hjorth 2009). To illustrate the sensitivity of the three functions to $\Omega_m$, we plot them for several cases of a flat Universe with $\Omega_m = 0.1, 0.3, 0.5, 0.7$ and 0.9, relative to the Einstein-de Sitter Universe ($\Omega_m = 1, \Omega_{\Lambda} = 0$). We can see from this plot that the $\Delta \tau/\sigma^2$ curves have a wider separation than the $\Delta \tau$ or $\sigma^2$ curves to allow an easier discrimination among different cosmological models. This is especially so at high redshifts, and thus it is of special significance for the $\Delta \tau/\sigma^2$ method to study high-redshift lenses. Moreover, this method has the advantage of being independent of the source redshift. However, before we put the $\Delta \tau/\sigma^2$ method into practical cosmological use, we must consider its
3 TESTING THE CAPABILITY OF LENSED GW-EM EVENTS TO CONDUCT COSMOGRAPHY

3.1 Monte Carlo simulations

We perform Monte Carlo simulations to test how well the two quantities (\( \Delta \tau \) and \( \Delta \tau / \sigma^2 \)) from strongly lensed GW-EM systems can be used to constrain cosmological parameters. To do so, we have to choose a fiducial cosmological model and then simulate a sample of lensed GW-EM systems. Here we adopt the following cosmological parameters of the flat ΛCDM model derived from the Planck Collaboration Planck Collaboration Planck Collaboration et al. (2016a) in our simulations: \( H_0 = 67.8 \text{ km s}^{-1} \text{ Mpc}^{-1}, \Omega_m = 0.308, \) and \( \Omega_\Lambda = 1 - \Omega_m. \)

Our detailed simulation steps are described as follows:

1. The redshifts of source \( z_s \) and lens \( z_l \) are randomly generated from the expected redshift probability distribution functions (PDFs) of lensed GW events (Biesiada et al. 2014; Ding et al. 2015). These redshift PDFs were calculated using the following procedure. First, considering the intrinsic merger rates of the whole class of double compact objects located at different redshifts as calculated by Dominik et al. (2013) and the designed sensitivity of the ET, the yearly detection rate of GW events was estimated. Secondly, the probability that individual GW signals from inspiralling double compact objects could be lensed by an early-type galaxy was then calculated. Finally, adding all the double-compact-objects merging systems together, the yearly detection rate of lensed GW events detected by the ET was predicted. This prediction is accompanied by the redshift PDF (see Fig. 2 of Ding et al. (2015)), which enables us to randomly generate the samples of \( z_s \) and \( z_l \). Note that short gamma-ray bursts (short GRBs), on-beam GRB afterglow emission, and kilonovae/mergerenovae are considered as promising EM counterparts of GW signals. Because \( z < 3 \) for the current short GRBs, the range of the source redshift \( z_s \) for our analysis is from 0 to 3.

2. We simulate the velocity dispersion \( \sigma \) and time delay \( \Delta \tau \) separately from the probability distributions of \( \sigma \) and \( \Delta \tau \) from the OM10 catalogue (Oguri & Marshall 2010). The OM10 catalogue provides mock observations of lensed quasars expected for the baseline survey planned with the LSST, based on realistic distributions of quasars and elliptical galaxies as well as the observational condition of this telescope.

3. For the \( \Delta \tau \) method, the mock external convergence \( \kappa_{\text{ext}} \) is obtained by sampling the PDF of \( \kappa_{\text{ext}} \) given by Rusu et al. (2017). We then infer the fiducial value of \( \Theta \equiv (\theta_3^2 - \theta_5^2) \) from Equations (2) and (3) using the mock \( z_s, z_l, \) \( \kappa_{\text{ext}}, \) and \( \Delta \tau \). For the \( \Delta \tau / \sigma^2 \) method, the fiducial value of \( \Delta \theta \equiv (\theta_B - \theta_A) \) is inferred from Equation (5) with the mock \( z_l, \sigma, \) and \( \Delta \tau. \)

4. Large numbers of new strong gravitational lenses will be discovered by dedicated surveys, including the LSST project (Marshall et al. 2011; Chang et al. 2013), the Dark Energy Survey (DES; Banerji et al. 2008; Buckley-Geer & Dark Energy Survey Collaboration 2014; Schneider 2014), and the VST ATLAS survey (Koposov et al. 2014). Also, time delays will be accurately constrained for a subsample of these with subsequent monitoring observations. The precision of time-delay measurements is estimated to be < 3% (Dobler et al. 2015; Liao et al. 2015). While the dedicated observations of lensed quasar systems in the EM domain give ~ 3% uncertainty of the time-delay measurement (Liao et al. 2015), \( \Delta \tau \) obtained from the GW signals are supposed to be very accurate with negligible uncertainty. Therefore, we assign an uncertainty \( \delta_{\Delta \tau} = 3\% \Delta \tau \) to the lensed quasar system, and \( \delta_{\Delta \tau} \approx 0 \) to the lensed GW-EM system.

5. The current techniques concerning lensed quasar systems in the EM window give a few percent uncertainty of the determination of lens modelling, that is, ~ 4% uncertainty on image position measurement \( \Theta \) (or \( \Delta \theta \)) and ~ 10% uncertainty for the observed velocity dispersion \( \sigma^2 \) (Jee et al. 2015). Note that the bright point spread functions (PSFs) of active galaxy nuclei (AGNs) could induce large systematic errors. In order to extract bright AGN images during the lens modelling procedure, instead one has to use a nearby star’s PSF or to adopt an iterative PSF modelling process, which can accurately recover the PSF for real observations (Chen et al. 2016; Ding et al. 2017; Wong et al. 2017). However, the systematic errors cannot be completely eliminated by these operations. Unlike AGNs, the EM counterparts of GW signals, such as short GRBs and kilonovae, are not always so bright. Therefore, the lensed images might not be affected much by the bright PSFs, which are difficult to extract, and the exposure time could be longer, making lens modelling so much easier. Based on the current lensing project H0LiCOW, Liao et al. (2017) simulated two sets of realistic lensed images with and without the AGN (see Ding et al. 2017 for more details on the simulations). The corresponding exposure time and noise level were set as close as possible the deep Hubble Space Telescope observations. They found that the effect of bright PSFs can significantly influence the uncertainties of the parameters in the lens model. Therefore, Liao et al. (2017) suggested that the accuracy of lens modelling would be improved to some extent with gravitationally lensed GWs and EM signals. For each lensed GW-EM event, we assign the uncertainties \( \delta_{\theta_3} = 2\% \Theta \) (or \( \delta_{\Delta \theta} = 2\% \Delta \theta \)) and \( \delta_{\sigma^2} = 5\% \sigma^2 \) to the mock \( \Theta \) (or \( \Delta \theta \)) and velocity dispersion \( \sigma^2 \) separately. These are supposed to be the best-case scenarios in the GW era, and they are two times smaller than those of the lensed quasar system in the EM domain. With large-aperture telescopes (e.g. the Thirty Meter Telescope and the European Extremely Large Telescope) or the James Webb Space Telescope, the required precision of velocity dispersions could be achieved. This should be coupled with high-resolution imaging, which can effectively constrain the density structures of the lenses and mass structures affecting the lensing. Density structures and extrinsic mass can also be constrained by modelling the lensing configuration, including positions and flux ratios of the images (Paraficz & Hjorth 2009).
6. It should be underlined that lensed GWs might provide some help with improving lens modelling uncertainty, but they do not help with the uncertainty of external convergence $\kappa_{\text{ext}}$. This is because in order to accurately quantify the mass distribution along the line-of-sight, wide-field imaging and spectroscopy are required (see Treu & Marshall 2016 for a recent review), either in lensed quasars or in lensed GWs. Rusu et al. (2017) have shown that the uncertainty of $\kappa_{\text{ext}}$ would contribute a root-mean-square error of 1 or 2 percent to the value of $D_{\Delta\tau}$ (see also Collett et al. 2013; Greene et al. 2013). Thus, we assign an uncertainty $\delta_{\kappa_{\text{ext}}} = 1\% (1 - \kappa_{\text{ext}})$ to the mock $\kappa_{\text{ext}}$ for both the lensed quasar system and the lensed GW-EM system.

7. For every synthetic lens, we add a deviation to the fiducial value of $\Theta_{\text{fid}}$ (or $\Delta \Theta_{\text{fid}}$). That is, we sample the $\Theta_{\text{mea}}$ (or $\Delta \Theta_{\text{mea}}$) measurement according to the Gaussian distribution $\Theta_{\text{mea}} = N(\Theta_{\text{fid}}, \sigma_{\Theta})$ or $\Delta \Theta_{\text{mea}} = N(\Delta \Theta_{\text{fid}}, \sigma_{\Delta \Theta})$.

8. Repeat the above steps to obtain a sample of 50 strong lenses.

3.2 Estimation of cosmological parameters

For a set of 50 simulated lenses, the likelihood for the cosmological parameters can be determined from the minimum $\chi^2$ statistic:

$$\chi^2(p) = \sum_i \frac{[D_{\text{obs}}^i - D_{\text{th}}^i(p)]^2}{\delta D_i^2}. \quad (7)$$

Here, $D_{\text{th}}^i$ is the theoretical distance calculated from the set of cosmological parameters $p$, $D_{\text{obs}}^i = D_{\text{OS}}D_{\text{OL}}/D_{\text{LS}}$ and $D_{\text{th}}^i = D_{\text{OL}}$ correspond to the $\Delta \tau$ method and the $\Delta \tau/\sigma^2$ method, respectively. $D_{\text{obs}}$ is the distance of the simulated observational data sets and $\delta D$ is the error of $D_{\text{obs}}$. With the measured distance $D_{\text{obs}}$ (see Equations 2 and 3), the propagated error $\delta D_{\Delta\tau}$ in $D_{\text{obs}}$ using the $\Delta \tau$ method is

$$\delta D_{\Delta\tau} = D_{\text{obs}} \left[ \left( \frac{\delta \Delta \tau}{\Delta \tau} \right)^2 + \left( \frac{\delta D_{\text{LS}}}{D_{\text{LS}}} \right)^2 + \left( \frac{\delta_{\kappa_{\text{ext}}}}{1 - \kappa_{\text{ext}}} \right)^2 \right]^{1/2}. \quad (8)$$

From Equation (5), the propagated error $\delta D_{\Delta\tau/\sigma^2}$ in $D_{\text{obs}}$ using the $\Delta \tau/\sigma^2$ method can be written as

$$\delta D_{\Delta\tau/\sigma^2} = D_{\text{obs}} \left[ \left( \frac{\delta \Delta \tau}{\Delta \tau} \right)^2 + \left( \frac{\delta \Delta \theta}{\Delta \theta} \right)^2 + \left( \frac{\delta \sigma}{\sigma} \right)^2 \right]^{1/2}, \quad (9)$$

which is dominated by the uncertainty of the velocity dispersion $\sigma^2$ (Jee et al. 2015). To ensure the final constraint results are unbiased, we repeat this process 1000 times for each data set by using different noise seeds.

In $\Lambda$CDM, the dark-energy equation-of-state parameter, $w$, is exactly $-1$. Assuming a flat Universe, $\Omega_m = 1 - \Omega_m$, there are only two free parameters: $\Omega_m$ and $H_0$. We first fix the flat $\Lambda$CDM model with $\Omega_m = 0.308$, but keep $H_0$ as a free parameter. Fig. 2 shows the constraints on $H_0$ using two different quantities ($\Delta \tau$ and $\Delta \tau/\sigma^2$) from 50 strongly lensed GW-EM systems (solid lines). For comparison, we also plot those constraints obtained from 50 lensed quasars (dashed lines) in the EM domain. We can see that lensed systems observed jointly in GW and EM windows place much more stringent constraints on $H_0$ than pure EM lensed systems, independent of what kind of observed quantity ($\Delta \tau$ or $\Delta \tau/\sigma^2$) is adopted. This is mainly because the uncertainties of both the time delay and lens modelling in the lensed GW-EM systems are smaller than those of the lensed quasar systems in the EM domain. Using $\Delta \tau$, we find that the uncertainty of $H_0$ from 50 lensed GW-EM systems is $\sim 0.3\%$, compared to $\sim 0.7\%$ from pure EM lensed systems. Similarly, $H_0$ is better constrained by 50 lensed GW-EM systems than by pure EM lensed systems with uncertainties of $\sim 0.8\%$ versus $\sim 1.6\%$ using $\Delta \tau/\sigma^2$. Our results are in good agreement with Liao et al. (2017). Not surprisingly, a comparison of Figs. (a) and (b) shows that the $\Delta \tau/\sigma^2$ method gives weaker constraints on $H_0$ than the other method, because the joint observations of time delay and velocity dispersion bring the extra uncertainty from the velocity dispersion (see the comparison between Equations (8) and (9)).

If we relax the priors, and allow both $H_0$ and $\Omega_m$ to be free parameters, we obtain the constraints set in the $\Omega_m - H_0$ plane, as illustrated in Fig. 3. In the traditional approach using lensed quasar systems observed in the EM domain, we need a larger sample to increase the significance of the constraints. In contrary, future observations of lensed GWs and their EM counterparts will enable us to achieve precise cosmography from around 50 such systems. The constraints on the parameter space from the $\Delta \tau$ method (Fig. 3(a)) give a good constraint on $H_0$, but a weak constraint on $\Omega_m$. As expected, the $\Delta \tau/\sigma^2$ method (Fig. 3(b)) gives tighter constraints on $\Omega_m$ than the other method (i.e. the $\Delta \tau/\sigma^2$ method can improve the discrimination between cosmological models).

For the $w$CDM model, $w$ is constant but possibly different from $-1$. For a flat Universe ($\Omega_m = 0$), there are three free parameters: $\Omega_m$, $w$, and $H_0$. Here, we marginalize $H_0$ in the $w$CDM model to find the confidence levels in the $\Omega_m - w$ plane. We demonstrate that lensed GW-EM systems can be a viable way to constrain the dark-energy equation of state. To gauge the impact of these constraints more clearly, we show in Fig. 4 the confidence regions (red solid lines) for $\Omega_m$. 

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Constraints on the Hubble constant, $H_0$, using 50 lensed GW-EM systems (red solid lines) and 50 lensed quasars (blue dashed lines): (a) simulations for the $\Delta \tau$ method; (b) simulations for the $\Delta \tau/\sigma^2$ method.}
\end{figure}
and $w$ using 50 simulated strongly lensed GW-EM systems, and we compare these to the constraint contours for the 580 Union2.1 SNe Ia data (Suzuki et al. 2012) (represented by the cyan contours in Fig. 4). It is straightforward to see how effectively the lensed GW-EM systems could be used as a cosmological probe. With a sample size of $\sim 50$, the contour size of lensed GW-EM systems is already comparable to that of 580 SNe Ia data. Furthermore, we note that the constraints obtained from the $\Delta \tau$ method (Fig. 4(a)) and the $\Delta \tau/\sigma^2$ method (Fig. 4(b)) are intersecting, much better constraints can be achieved when combining these two methods (see the black contours in Fig. 5). That is, if we are lucky enough to have the joint measurements of $\Delta \tau$ and $\sigma^2$ for each lens, the better cosmological constraints would be obtained by combining these two methods.

To illustrate the degeneracy breaking power of the proposed methods, we also plot the constraint contours of CMB from Planck 2015 measurements (Planck Collaboration et al. 2016b, see Wen & Wang 2017 for more details on the calculations of CMB) (represented by the green contours in Figs. 3 and 4). We can see that when the observations of lensed GW-EM systems are verified in the future, CMB constraints would benefit from having the constraints of lensed GW-EM systems overlaid.

4 SUMMARY AND DISCUSSION

Although the constraints on cosmological parameters have reached a high precision, all of the constraints so far have relied on EM observations alone. New multimessenger signals exploiting different emission channels are worth exploring in cosmology. Recently, Liao et al. (2017) proposed that future time-delay measurements of strongly lensed GW signals and their EM counterparts have great potential to infer cosmological parameters. Compared with the traditional approach of using strongly lensed quasar systems observed in the EM domain, the approach with lensed systems observed in both GW and EM windows has two advantages in constraining cosmological parameters. First, the time delays ($\Delta \tau$) between lensed images inferred from the GW signals would reach an extremely high accuracy ($\sim 0.1$ s) from the detection pipeline, and such accurate measurements of $\Delta \tau$ $^2$

For the $w$-CDM model, because CMB observables explicitly depend on $H_0$, we use a Gaussian prior on its value, $H_0 = 67.8 \pm 0.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$, to guide the minimization procedure over $H_0$. 

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Figure 3. The $\sigma - 3\sigma$ constraint contours of $(\Omega_m, H_0)$ in the flat $\Lambda$CDM model from 50 lensed GW-EM systems (red solid lines), 50 lensed quasars (blue dashed lines), and CMB data (green contours): (a) simulations for the $\Delta \tau$ method; (b) simulations for the $\Delta \tau/\sigma^2$ method.

Figure 4. Constraint results for the $w$-CDM model using 50 lensed GW-EM systems (red solid lines), compared with those associated with the 580 Union2.1 SNe Ia data (cyan contours) and CMB data (green contours): (a) simulations for the $\Delta \tau$ method; (b) simulations for the $\Delta \tau/\sigma^2$ method.

Figure 5. Cosmological constraints on the $w$-CDM model from 50 lensed GW-EM systems for three different cases: $\Delta \tau$ (red dashed lines), $\Delta \tau/\sigma^2$ (magenta dot lines) and $\Delta \tau + \Delta \tau/\sigma^2$ (black contours).
would play an important role in boosting the development of precision cosmology. Secondly, with gravitationally lensed GWs and EM signals, the accuracy of lens modelling could be improved to some extent, leading to better constraints on cosmological parameters.

In the EM window, Paraficz & Hjorth (2009) suggested that the joint observations of the time delay (\(\Delta \tau\)) between lensed quasar images and the velocity dispersion (\(\sigma\)) of the lensing galaxy are more effective to constrain cosmological parameters than \(\Delta \tau\) or \(\sigma^2\) separately. In this work, we apply the \(\Delta \tau/\sigma^2\) method, for the first time, to the strongly lensed systems observed in both GW and EM windows. We prove that both \(\Delta \tau\) and \(\Delta \tau/\sigma^2\) from strongly lensed GW-EM systems can serve as powerful cosmic rulers. From the comparison of the two different methods, we confirm that the \(\Delta \tau/\sigma^2\) method can provide tighter constraints on \(\Omega_m\) than the \(\Delta \tau\) method, (i.e. using \(\Delta \tau/\sigma^2\) can make it easier to differentiate different cosmological models). Furthermore, we show that with a moderate sample size of ~ 50, a constraint on the dark energy equation of state \(w\) can be reached that is comparable to the 580 Union2.1 SNe Ia sample. Combining the \(\Delta \tau\) and \(\Delta \tau/\sigma^2\) methods, it is possible to achieve higher accuracy in constraining \(\omega\).

The recent Advanced LIGO observations of binary black hole mergers GW150914 (Abbott et al. 2016a), GW151226 (Abbott et al. 2016b), and GW170104 (Abbott et al. 2017) have initiated the era of GW astronomy. Because of the high sensitivity, the planned third-generation GW detectors, such as the ET, could observe strongly lensed GWs. Recent works (Piorkowska et al. 2013; Biesiada et al. 2014; Ding et al. 2015) have carefully studied the prospects of observing strongly lensed GWs from merging double compact objects, which predicted that the ET would detect about 50–100 strongly lensed GW events per year. Although a considerable catalogue of lensed GWs would be obtained, the measurements of strongly lensed GW-EM systems suggested by our method will still be extremely hard in practice. The measurements must meet three requirements: (i) we need an EM counterpart to give the exact location of the lensed images; (ii) we need to get a source redshift from that EM counterpart; (iii) we need the GW source to have a detectable host galaxy so we can carry out detailed lens modelling. It is not clear what fraction of lensed GW events will actually satisfy these three requirements. If, in the future, gravitationally lensed GWs and their EM counterparts are detected simultaneously, the prospects for the study of cosmology with such lensing systems, as discussed in this work, will be very promising.

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