Small-x Growth of Nucleon Structure Functions and Its Manifestations in Ultrahigh-Energy Neutrino Astrophysics

A. Z. Gazizov and S. I. Yanush

B. I. Stepanov Institute of Physics of the National Academy of Sciences of Belarus
F. Skarny Ave. 68, 220072 Minsk, Belarus
E-mail: gazizov@dragon.bas-net.by
E-mail: yanush@dragon.bas-net.by

Abstract. Rapid growth of neutrino-nucleon cross-sections at high energies due to hypothetical hard pomeron enhancement of $\nu N$-structure functions is discussed. Differential and integral hadron moments, which, together with cross-sections, define rates of hadron-electromagnetic cascades in a neutrino detector are calculated for different power-law decreasing neutrino spectra. For comparison two small-x extrapolation schemes are discussed. First includes Regge theory inspired hard pomeron enhancement of $\nu N$-structure functions. The second is obtained with the help of trivial extrapolation of perturbative QCD structure functions from the large $x$ region to the small $x$ one. Implications of hard pomeron effects for cross-sections and hadron moments are demonstrated. The most pronounced manifestations are found in integral hadron moments for the case of charged current electron (anti)neutrinos scattering off nucleons.

1. Introduction

This paper continues the discussion of some specific features of $\nu N$ Deep Inelastic Scattering (DIS) at extremely high, up to $E_\nu \sim 1 \times 10^{12}$ GeV, energies that was started at the previous Seminar ‘NPCS-2000’[1]. The main idea of our approach is to extend the successful small-$x$ description of $F_2^{ep}(x, Q^2)$ Structure Function (SF) by A. Donnachie and P. V. Landshoff (DL) [2] to $\nu N$-SFs, namely, to $F_2^{\nu N}(x, Q^2)$, $F_3^{\nu N}(x, Q^2)$. DL claim that record small-$x$ ep-scattering data by HERA [3] may be successfully explained with the help of a simple combination of several Regge theory inspired non-perturbative pomerons. The most important are ‘soft’ pomeron (with intercept $\sim 1.08$) and ‘hard’ pomeron (with intercept $\approx 1.4$). First prevails at small $Q^2$, while the latter dominates at large $Q^2$. Moreover, DL argue that perturbative QCD ($pQCD$) fails at small $x < 10^{-5}$ and that its validity at $x \sim 10^{-4} \div 10^{-5}$ is a pure fluke. However, one should keep in mind that DL’s approach neglects the other, non-leading, poles and cuts in the complex angular momentum $l$-plane. This common feature of pomeron physics causes this model to violate the unitarity at $E_\nu \rightarrow \infty$.

Using a nontrivial generalization of DL’s $F_2^{ep}(x, Q^2)$ SF description to the $\nu N$-scattering case, we have constructed $F_2^{\nu N}(x, Q^2)$ and $F_3^{\nu N}(x, Q^2)$ SFs, presumably valid in the whole range of kinematic variables $0 \leq x \leq 1$ and $0 \leq Q^2 \leq \infty$[1, 4]. At $x \gtrsim 10^{-5}$ they are chosen to coincide with $pQCD$ parameterization by CTEQ5 collaboration [5], while in the small-$x$ region these SFs are driven by the analogous Regge theory inspired description. A special interpolation procedure, developed in Ref. [4], allows to meet smoothly these different, both over $x$ and $Q^2$, descriptions of SFs at low and high $x$. In Ref. [1] these SFs were denoted...
by DL + CTEQ5, indicating that they have their origin in the interpolation between DL and pQCD descriptions.

In parallel there were considered SFs obtained via simple extrapolation of pQCD SFs from \( x \geq 1 \times 10^{-5} \) to the small-\( x \) region:

\[
F_{i}^{\nu N, \text{Log+CTEQ5}}(x < x_{\text{min}}, Q^{2}) = F_{i}^{\nu N, \text{CTEQ5}}(x_{\text{min}}, Q^{2}) \left( \frac{x}{x_{\text{min}}} \right)^{\beta_{i}(Q^{2})},
\]

\[
\beta_{i}(Q^{2}) = \frac{\partial \ln F_{i}^{\nu N, \text{CTEQ5}}(x, Q^{2})}{\partial \ln x} \bigg|_{x=x_{\text{min}}} ; \ x_{\text{min}} = 1 \times 10^{-5}.
\]

These SFs smoothly shoot to the low-\( x \) region from the CTEQ5 defined high-\( x \) one. Starting values of functions and of their logarithm derivatives over \( \ln x \) are taken here at the \( x = x_{\text{min}} \) boundary of CTEQ5. This parameterization was designated as Log+CTEQ5.

Below we shall evaluate several observables involved in High Energy Neutrino Astrophysics (HENA) using both parameterization. We shall compare them so that to reveal the manifestations of hard pomeron enhancement in these values.

## 2. Observables of HENA

An incident cosmic high-energy \( \nu \)-flux can be detected only by registration of secondary particles, the products of \( \nu N \)- and/or \( \nu e \)-collisions with matter (basic ideas of HENA are expounded in Ref. [7]). Hence, the observables are mostly rates of high-energy muons and/or of nuclear-electromagnetic cascades. In this paper we shall discuss only \( \nu N \)-interactions, though the most remarkable process in HENA is resonance cascade production via

\[
\bar{\nu}_{e} + e^{-} \rightarrow W^{-} \rightarrow \text{hadrons}
\]

at \( E_{\bar{\nu}_{e}} \simeq 6.4 \times 10^{15} \text{ eV} \) [8, 9]. This resonance should show itself as a narrow high spike in the differential energy spectrum of cascades; at resonance energy it exceeds essentially the ordinary \( \nu N \)-interaction background.

Muons and cascades are produced via CC-

\[
\nu_{\mu}(\bar{\nu}_{\mu}) + N \rightarrow \mu^{\mp} + X
\]

and NC-

\[
\nu_{\mu}(\bar{\nu}_{\mu}) + N \rightarrow \nu_{\mu}(\bar{\nu}_{\mu}) + X
\]

DIS. And in special case of electron (anti)neutrino CC-interaction,

\[
\nu_{e}(\bar{\nu}_{e}) + N \rightarrow e^{\mp} + X,
\]

both final states contribute to the same cascade, so that the whole energy of an incident neutrino is transferred in it. But the NC-scattering case of \( \nu_{e}(\bar{\nu}_{e}) N \) does not differ from (3).

Differential and integral rates of cascade production in a detector for a model neutrino flux with a power-law decreasing energy spectrum,

\[
F_{\nu}(E_{\nu}) = A \times E_{\nu}^{-(\gamma+1)},
\]

where \( \gamma \) is the index of an integral neutrino spectrum \( F_{\nu}(> E) \) (it is commonly assumed that \( 1.1 \leq \gamma \leq 2.1 \)), may be calculated with the help of so-called differential,

\[
Z_{h}(E_{h}, \gamma) = \int_{0}^{1} dyy^{\gamma} \frac{d\sigma_{\nu N}(E_{h}/y, y)}{\sigma_{0}dy},
\]
and integral,

\[ Y_h(E_h, \gamma) = \gamma \int_0^1 du u^{\gamma - 1} Z_h \left( \frac{E_h}{u}, \gamma \right), \]  

hadron moments [10, 11]. Here \( E_h \) is the energy of a hadron-electromagnetic cascade, \( y = E_h/E_\nu \) and \( \sigma_0 \) is the normalization cross-section; for \( m_W = 81 \text{ GeV} \) \( \sigma_0 = 1.09 \times 10^{-34} \text{ cm}^2 \).

These rates in CC-scattering case (2) are

\[ \frac{dN_h(E_h)}{dt} = Z_h^{CC}(E_h, \gamma) N_N \sigma_0 \Omega F_\nu(E_h), \]  

\[ \frac{dN_h(> E_h)}{dt} = Y_h^{CC}(E_h, \gamma) N_N \sigma_0 \Omega F_\nu(> E_h). \]  

Here \( N_N \) is the number of nucleons in a detector and \( \Omega \) is an effective solid angle the neutrino flux comes from. The cases of \( \bar{\nu}N \)- and \( NC \)-interactions may be accounted for in a similar way: one is to substitute in (6) the appropriate differential cross-section for the CC one.

Note that in Eq.s (8,9) both differential and integral neutrino fluxes are taken at the cascade energy \( E_h \) and that for power-law decreasing spectra (5) the following useful relation is valid:

\[ \gamma \times F_\nu(> E) = E \times F_\nu(E). \]  

In the case of (anti)neutrino CC-scattering (4) a role of differential hadron moment belongs to the normalized CC-cross-section, \( \sigma_{\nu N}^{CC}(E_\nu)/\sigma_0 \); since now \( E_h = E_\nu \), \( \nu_\nu \)-flux in (8,9) is to be taken at the energy of incident neutrino.

3. Cross-sections of \( \nu N \)-scattering

In parton picture \( \nu N \)-cross-section increases with the energy due to multiplication in number of the nucleon small-\( x \) ‘sea’-quark contents. In the framework of non-perturbative pomeron approach such growth occurs due to specific poles in the complex \( l \)-plane. Calculated within DL+CTEQ5 and Log+CTEQ5 parameterizations, CC and NC \( \nu(\bar{\nu})N \)-cross-sections are shown in Fig. [1]. The most part of high-energy cross-section is accumulated at small \( x \) and high \( Q^2 \), where hard pomeron term dominates SFs. Since hard pomeron enhanced...


Small-x growth of nucleon structure functions and its manifestations in ultrahigh-energy neutrino astrophysics

DL+CTEQ5 small-x SFs are higher than corresponding Log+CTEQ5 ones, their cross-sections prevail over ‘perturbative’ at high energies. Discrepancies become especially clear in Fig. 2 where ratios of corresponding DL+CTEQ5 and Log+CTEQ5 cross-sections, \( r_\sigma(E_\nu) \), are plotted versus neutrino energy. The curves for \( \nu N \) and \( \bar{\nu} N \) cases practically coincide in this graph; ratios for NC-interactions are very close to corresponding CC ones.

It should be noted, that hard pomeron enhanced growth of \( \nu N \)-cross-sections is the most rapid among all presently known, which have been obtained under different ‘ordinary’ (no extra dimensions and so on) assumptions (see Ref. \[4\] and references therein).

4. Differential and integral hadron moments

According to Eq.s (6,7), hadron moments depend on high-energy part of \( \nu \)-spectrum

\[
\begin{align*}
\text{Fig. 3. } & \nu N \text{ and } \bar{\nu} N \text{ DL+CTEQ5 CC and NC differential hadron moments for integral } \nu \text{-spectrum index } \gamma = 1.1. \\
\text{Fig. 4. The same curves as in Fig. 3, but for integral hadron moments. } \nu_e N \text{- and } \bar{\nu}_e N \text{-scattering cases are shown separately.}
\end{align*}
\]

cross-sections are higher. As a consequence, hard pomeron effects look even more pronounced in these observables. To illustrate a common trend, the differential and integral \( \nu N \) and \( \bar{\nu} N \) hadron moments are plotted in Fig. 3 and Fig. 4, respectively, for DL+CTEQ5 parameterization and \( \gamma = 1.1 \), both for CC- and NC-interactions.

Dependencies of CC and NC hadron moments on \( \gamma \) are shown in Fig.s 3,4,5,6 with the help of ratios between corresponding moments with indicated \( \gamma \) and those with \( \gamma = 1.1 \).

And, finally, ratios between corresponding CC hadron moments of DL+CTEQ5 and Log+CTEQ5 parameterizations are demonstrated in Fig.s 7,8,9,10 for \( \gamma = 1.1 \). Due to sensitivity to higher energies, these ratios are higher than ratios of cross-sections. The most salient difference is seen in \( Y_h^{CC}(E_h, \gamma) \) for \( \nu_e(\bar{\nu}_e) N \) scattering, where the whole energy goes to the cascade.

5. Conclusions

We have demonstrated that small-x hard pomeron enhancement of \( \nu N \) structure functions evinces itself via essential growth of some HENA observables at high energies. For example, cross-sections and hadron moments, defining rates of cascades, grow more rapidly with the energy than in the case of trivial pQCD extrapolation. The calculated hadron moments may be used for estimation of cascade rates in future giant high-energy neutrino detectors.
Fig. 5. Ratios for $DL + CTEQ5$ $CC$ differential hadron moments.

Fig. 6. Ratios for $DL + CTEQ5$ $CC$ integral hadron moments.

Fig. 7. Ratios for $DL + CTEQ5$ $NC$ differential hadron moments.

Fig. 8. Ratios for $DL + CTEQ5$ $NC$ integral hadron moments.

Fig. 9. Ratios between $CC$ differential hadron moments of $DL + CTEQ5$ and $Log + CTEQ5$ parameterizations for $\gamma = 1.1$.

Fig. 10. The same as in Fig. 9, but for integral hadron moments.

Acknowledgments

The authors are grateful to Prof. V. S. Berezinsky for his partial participation, useful comments and encouragements. This work was supported by the INTAS grant No: 99-1065.
References

[1] A. Z. Gazizov and S. I. Yanush, in Proc. of 9th Annual Seminar NPCS’2000, edited by L. Babichev and V. Kuvshinov (Institute of Physics, Minsk, 2000), Vol. 9, p. 313.

[2] A. Donnachie and P. V. Landshoff, Phys. Lett. 437B, 408 (1998).
A. Donnachie and P. V. Landshoff, Phys. Lett. B, (in press); hep-ph/0105088.

[3] H1: C. Adloff et al., Nucl. Phys. B497, 3 (1997).
ZEUS: J. Breitweg et al., Phys. Lett. 407B, 432 (1997).
H1: C. Adloff et al., Eur. Phys. J. C21, 33 (2001).

[4] A. Z. Gazizov and S. I. Yanush, Phys. Rev. D (2002), (in press); astro-ph/0105368.

[5] CTEQ collaboration: H. L. La et al., hep-ph/9903282; see also http://www.phys.psu.edu/~cteq/.

[6] V. S. Berezinsky, A. Z. Gazizov, G. T. Zatsepin and I. L. Rozental, Sov. J. Nucl. Phys. 43, 637 (1986).

[7] V. S. Berezinsky, S. V. Bulanov, V. A. Dogiel, V. L. Ginzburg and V. S. Ptuskin, Astrophysics of Cosmic Rays (North-Holland, Amsterdam, 1990).

[8] V. S. Berezinsky and A. Z. Gazizov, Sov. Phys. JETP Lett. 25, 276 (1977).

[9] V. S. Berezinsky and A. Z. Gazizov, Sov. J. Nucl. Phys. 33, 230 (1981).

[10] V. S. Berezinsky and A. Z. Gazizov, Sov. J. Nucl. Phys. 29, 1589 (1979).

[11] V. S. Berezinsky and A. Z. Gazizov, in Proc. of 1979 DUMAND Summer Workshop at Khabarovsk and Lake Baikal (Hawaii, Honolulu, 1980), p. 202.