GENERAL FORMULAE OF INVARIANT 
FUNCTIONS OF THE GENERALIZED REACTION 
$\gamma N \rightarrow \gamma N$ IN THE EFFECTIVE LAGRANGIANS 
METHOD 

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Abstract 

The crossed channels of generalized reaction $\gamma N \rightarrow \gamma N$ have been considered. The transformation coefficients from the independent helicity amplitudes to the invariant functions are calculated. The explicit expressions for invariant functions have been obtained with the subject to the contribution of Born diagrams in $s$-, $u$-, and $t$-channel and six resonances in $s$- and $u$-channel. It has been shown that the obtained invariant functions meet the requirements of crossing-symmetry.

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Interaction of photons with nucleons is one of the basic processes of elementary particle physics with initial energies up to 1 GeV. As an example we can point out the pions photoproduction on nucleons $\gamma N \rightarrow \pi N$, bremsstrahlung off nucleon $NN \rightarrow NN \gamma$, and, finally, Compton scattering of a photon on the nucleon $\gamma N \rightarrow \gamma N$. This paper deals with generalized reaction $\gamma N \rightarrow \gamma N$ which includes three crossed channels: Compton scattering of a photon on the nucleon $\gamma N \rightarrow \gamma N$ (s-channel), Compton scattering of a photon on the antinucleon $\gamma \bar{N} \rightarrow \gamma \bar{N}$ (u-channel), and annihilation of a nucleon-antinucleon pair into two photons $NN \rightarrow \gamma \gamma$ (t-channel).

To describe the processes in the range from the threshold up to $\Delta(1232)$-isobar it is sufficient to take into account the contribution of the Born terms, vector mesons and the $\Delta(1232)$-isobar itself [1, 2, 3]. However to describe the processes in the broader energy area from the threshold up to 1 GeV the calculation of higher nucleon resonances is indispensable. For the reaction of photoproduction of a pion on nucleon $\gamma N \rightarrow \pi N$ such
calculation was carried out in papers \[4, 5\]. In these papers, however, invariant functions of reaction amplitude carrying in themselves the information on dynamics of the process were not calculated. The availability of the explicit form of invariant functions significantly increases the speed of numerical simulation of processes and is indispensable to analytic calculate processes amplitudes on nuclei. In this paper general analytical formulae for invariant functions of generalized reaction $\gamma N \rightarrow \gamma N$ with the subject to the contribution of Born terms in $s$-, $u$- and $t$-channel and six resonances in $s$- and $u$-channel have been calculated.

The article is organized as follows. In Section 2 the general expression for $P$- and $T$-invariant amplitude of reaction $\gamma^- + \frac{1}{2}^+ \rightarrow \gamma^- + \frac{1}{2}^+$ is given. In Section 3 transformation coefficients from helicity amplitudes of reaction $\gamma^- + \frac{1}{2}^+ \rightarrow \gamma^- + \frac{1}{2}^+$ to invariant functions are given. In Section 4 lagrangians used to construct Feynman diagrams, in particular the problems of gauge-invariance and connection with the fermion field of spin $\frac{3}{2}$ are discussed. In Section 5 the properties of crossing-symmetry of invariant functions and crossing transformation of amplitude from $s$-channel into a $u$-, and $t$-channel are discussed. In Section 6 the Feynman diagrams which have been taken into account during amplitude calculation are shown. Methods of calculation of the Born diagrams and six resonances diagrams contribution in $s$-, $u$- and $t$-channel to invariant functions are explained. The Appendix lists the explicit type of lagrangians used and invariant functions obtained with their help.

\section{2}

The general formulae for amplitude and invariant functions are more convenient to obtain considering the reaction in $s$-channel $\gamma N \rightarrow \gamma N$. Transition to amplitudes of other channels can be then accomplished by means of crossing transformation of the amplitude of $s$-channel. To construct the reaction amplitude of $s$-channel it is necessary to define the number of independent invariant functions, which enter the expression for the amplitude. It is equal to the number of independent helicity amplitudes of reaction $\gamma N \rightarrow \gamma N$ with the subject to $P$- and $T$-invariance. The total number of helicity amplitudes of reaction $\gamma^- + \frac{1}{2}^+ \rightarrow \gamma^- + \frac{1}{2}^+$ is equal to $2s_1(2s_2 + 1)2s_3(2s_4 + 1) = 16$. Between these amplitudes there are ratios following from $P$-invariance of electromagnetic interaction:

$$T(\lambda_3, \lambda_4; \lambda_1, \lambda_2) = \eta(-1)^{(\lambda_1 - \lambda_2) - (\lambda_3 - \lambda_4)}T(-\lambda_3, -\lambda_4; -\lambda_1, -\lambda_2),$$

where $\eta = \eta_1\eta_2\eta_3\eta_4(-1)^{s_1+s_4-s_1-s_2}$; $\eta_i$, $s_i$ are internal parities and spins of particles participating in the reaction, $\lambda_3, \lambda_4$ and $\lambda_1, \lambda_2$ are the helicities of the photon and nucleon correspondently both in final and initial state. It is easy to see, that the number of independent helicity amplitudes reduces to 8. The further limitations on the number of
independent helicity amplitudes follow from $T$-invariance of electromagnetic interaction. For elastic processes $T$-invariance leads to the following ratios between helicity amplitudes:

$$T(\lambda_3, \lambda_4; \lambda_1, \lambda_2) = (-1)^{(\lambda_1 - \lambda_2) - (\lambda_3 - \lambda_4)} T(\lambda_1, \lambda_2; \lambda_3, \lambda_4).$$  \hspace{1cm} (2)$$

Thus the number of independent helicity amplitudes of Compton scattering reduces to 6. As independent helicity amplitudes of Compton scattering we can select the following:

$$T(1, \frac{1}{2}; 1, \frac{1}{2}), T(1, -\frac{1}{2}; 1, \frac{1}{2}), T(-1, \frac{1}{2}; 1, \frac{1}{2}), T(-1, -\frac{1}{2}; 1, \frac{1}{2}), T(1, -\frac{1}{2}; 1, -\frac{1}{2}),$$

$$T(-1, \frac{1}{2}; 1, -\frac{1}{2}).$$  \hspace{1cm} (3)$$

The last 10 helicity amplitudes are expressed through independent helicity amplitudes according to (1), (2):

$$T(-1, -\frac{1}{2}; -1, -\frac{1}{2}) = T(1, \frac{1}{2}; 1, \frac{1}{2}), T(-1, -\frac{1}{2}; -1, \frac{1}{2}) = -T(1, -\frac{1}{2}; 1, \frac{1}{2}),$$

$$T(1, \frac{1}{2}; -1, -\frac{1}{2}) = -T(1, -\frac{1}{2}; 1, \frac{1}{2}), T(-1, -\frac{1}{2}; -1, \frac{1}{2}) = T(1, -\frac{1}{2}; 1, \frac{1}{2}),$$

$$T(1, \frac{1}{2}; -1, \frac{1}{2}) = T(-1, \frac{1}{2}; 1, \frac{1}{2}), T(1, \frac{1}{2}; -1, \frac{1}{2}) = -T(-1, -\frac{1}{2}; 1, \frac{1}{2}),$$

$$T(-1, \frac{1}{2}; -1, \frac{1}{2}) = T(1, -\frac{1}{2}; 1, -\frac{1}{2}), T(1, -\frac{1}{2}; -1, \frac{1}{2}) = -T(-1, -\frac{1}{2}; 1, -\frac{1}{2}).$$  \hspace{1cm} (4)$$

Consider the general structure of amplitude of Compton scattering of a photon on nucleon $\gamma^- + \frac{1}{2}^+ \to \gamma^- + \frac{1}{2}^+$. We shall designate four-momenta of initial and final photons $k_1, k_2$ accordingly, and four-momenta of initial and final nucleons $p_1, p_2$ accordingly. We shall designate polarization four-vectors of initial and final photons $\epsilon_1, \epsilon_2$ and bispinors of initial and final nucleons $-u_1, u_2$. To find out the symmetry of the amplitude in relation to spatial reflection and time reversion it is convenient to use symmetric and antisymmetric combination of four-momenta of photons:

$$K = \frac{1}{2}(k_1 + k_2), \quad Q = \frac{1}{2}(k_2 - k_1).$$  \hspace{1cm} (5)$$

Having added to $K$ and $Q$ a symmetric four-vector $P' = P - \frac{P \cdot K}{K^2} K$, where $P = \frac{1}{2}(p_1 + p_2)$ and pseudovector $N^{\mu} = i\epsilon^{\mu\nu\lambda\sigma} P'_{\nu} K_{\lambda} Q_{\sigma}$ we shall obtain four mutually orthogonal vectors with the help of which it is convenient to describe the gauge-invariant structure of the amplitude of reaction $\gamma^- + \frac{1}{2}^+ \to \gamma^- + \frac{1}{2}^+$. Invariant variables $s, t$ and $u$ are expressed through four-vectors $K, P, Q$ in the following way:

$$s = (P + K)^2, \quad t = 4Q^2, \quad u = (P - K)^2, \quad s + t + u = 2M^2,$$  \hspace{1cm} (6)$$
where $M$ is nucleon mass. The general expression for the amplitude of reaction with arbitrary spins of particles, participating in it, looks like:

$$T(p_2, p_1; P) = \sum_i f_i(s, t) R_i^i,$$  \hspace{0.5cm} (7)

where $f_i(s, t)$ are the invariant functions depending on four-momenta of initial and final particles through invariant variables $s$ and $t$ only, $R_i^i$ invariant combinations, which should be linearly composed of wave functions of all particles, participating in the reaction. In case of reaction $\gamma^- + \frac{1}{2}^+ \rightarrow \gamma^- + \frac{1}{2}^+$ the number of independent invariant functions $f_i(s, t)$ with the subject to $P$- and $T$-invariance are equal to 6. We can select the following independent invariant combinations as:

$$R_1^i = \bar{u}(p_2)(P' \cdot \epsilon^* (k_2))(P' \cdot \epsilon(k_1))P'^{-2}u(p_1), \quad R_2^i = \bar{u}(p_2)(P' \cdot \epsilon^* (k_2))(P' \cdot \epsilon(k_1))\hat{K}P'^{-2}u(p_1),$$
$$R_3^i = \bar{u}(p_2)(N' \cdot \epsilon^* (k_2))(N' \cdot \epsilon(k_1))N^{-2}u(p_1), \quad R_4^i = \bar{u}(p_2)(N' \cdot \epsilon^* (k_2))(N' \cdot \epsilon(k_1))\hat{K}N^{-2}u(p_1),$$
$$R_5^i = \bar{u}(p_2)((P' \cdot \epsilon^* (k_2))(N' \cdot \epsilon(k_1)) - (P' \cdot \epsilon(k_1))(N' \cdot \epsilon^* (k_2)))\gamma_5 P'^{-2}K^{-2}u(p_1),$$
$$R_6^i = \bar{u}(p_2)((P' \cdot \epsilon^* (k_2))(N' \cdot \epsilon(k_1)) + (P' \cdot \epsilon(k_1))(N' \cdot \epsilon^* (k_2)))\gamma_5 \hat{K}P'^{-2}K^{-2}u(p_1),$$  \hspace{0.5cm} (8)

where $\hat{K} = K_{\mu} \gamma^\mu$.

It can be easily seen that $R_i^i$ are $P$-invariant. Let’s show that $R_i^i$ are also $C$-invariant. With the charge conjugation of the amplitude of reaction $\gamma^- + \frac{1}{2}^+ \rightarrow \gamma^- + \frac{1}{2}^+$ the following permutations of wave functions and four-momenta of the particles are fulfilled [9]:

$$R^i_{k'l} (\epsilon^* (k_2), k_2, p_2; \epsilon(k_1), k_1, p_1) \rightarrow C_{km} R_{nm}^i (-\epsilon(k_1), -k_1, -p_1; -\epsilon^* (k_2), -k_2, -p_2) C_{nt}.$$  \hspace{0.5cm} (9)

Moreover there is a need to transpose $R^i_{k'l}$ along the Dirac indexes $k$ and $l$ and to multiply from the left and from the right on the matrix of charge conjugation $C = \gamma^2\gamma^0$. The resulting transformation of $R^i_{k'l}$ may be presented in the following way:

$$R^i_{k'l} (\epsilon^* (k_2), k_2, p_2; \epsilon(k_1), k_1, p_1) \rightarrow C_{km} R_{nm}^i (-\epsilon(k_1), -k_1, -p_1; -\epsilon^* (k_2), -k_2, -p_2) C_{nt}.$$  \hspace{0.5cm} (10)

Using the properties of the charge conjugation matrix [7, 8] we can easily show that all the combinations [8] during the transformation [10] transfer into themselves, i.e. they are $C$-invariant. Invariant variables $s$, $t$, $u$, and therefore invariant amplitudes $f_i(s, t)$ do not change during the transformation [9]. As amplitude [7] is $P$- and $C$-invariant, it is also $T$-invariant according to the CPT theorem.

Invariant combinations $R_i^i$ are at the same time gauge-invariant, as all $R_i^i$ during the substitution $\epsilon(k_1) \rightarrow k_1$ or $\epsilon(k_2) \rightarrow k_2$ turn to the zero. As $k_1 = K - Q$, $k_2 = K + Q$ we have:

$$P' \cdot k_1 = P' \cdot (K - Q) = P' \cdot K - P' \cdot Q = 0,$$
$$P' \cdot k_2 = P' \cdot (K + Q) = P' \cdot K + P' \cdot Q = 0,$$  \hspace{0.5cm} (11)
due to the orthogonality of four-vectors $P', K, Q$. Scalar products of four-vectors $N$ and $k_1, k_2$ also equal to zero:

\[
N \cdot k_1 = i \epsilon^{\mu \nu \lambda} \sigma P'_\mu K_\lambda Q_\sigma k_{1\mu} = i \epsilon^{\mu \nu \lambda} \sigma P'_\mu K_\lambda Q_\sigma (K_\mu - Q_\mu) = 0,
\]
\[
N \cdot k_2 = i \epsilon^{\mu \nu \lambda} \sigma P'_\mu K_\lambda Q_\sigma k_{2\mu} = i \epsilon^{\mu \nu \lambda} \sigma P'_\mu K_\lambda Q_\sigma (K_\mu + Q_\mu) = 0,
\]

as the products of tensors, one of which are absolutely antisymmetric and another symmetric in two indexes. It results to the fact that $R_i$ are gauge-invariant.

Finally let's write down the expression for $P^-, C^-, T^-$ and gauge-invariant helicity amplitude of the reaction $\gamma^- + \frac{1}{2} \rightarrow \gamma^- + \frac{1}{2}^+$:

\[
T(\lambda_3, \lambda_4, \lambda_1, \lambda_2) = f_1(s, t) \tilde{u}(p_2, \lambda_4)(P' \cdot \epsilon^*(k_2, \lambda_3))(P' \cdot \epsilon(k_1, \lambda_1))P'^{-2}u(p_1, \lambda_2) +
\]
\[
+ f_2(s, t) \tilde{u}(p_2, \lambda_4)(P' \cdot \epsilon^*(k_2, \lambda_3))(P' \cdot \epsilon(k_1, \lambda_1))\tilde{K}P'^{-2}u(p_1, \lambda_2) +
\]
\[
+ f_3(s, t) \tilde{u}(p_2, \lambda_4)(N \cdot \epsilon^*(k_2, \lambda_3))(N \cdot \epsilon(k_1, \lambda_1))N^{-2}u(p_1, \lambda_2) +
\]
\[
+ f_4(s, t) \tilde{u}(p_2, \lambda_4)(N \cdot \epsilon^*(k_2, \lambda_3))(N \cdot \epsilon(k_1, \lambda_1))\tilde{K}N^{-2}u(p_1, \lambda_2) +
\]
\[
+ f_5(s, t) \tilde{u}(p_2, \lambda_4)((P' \cdot \epsilon^*(k_2, \lambda_3))(N \cdot \epsilon(k_1, \lambda_1)) -
\]
\[
- (P' \cdot \epsilon(k_1, \lambda_1))(N \cdot \epsilon^*(k_2, \lambda_3))\gamma_5\tilde{K}P'^{-2}u(p_1, \lambda_2) +
\]
\[
+ f_6(s, t) \tilde{u}(p_2, \lambda_4)((P' \cdot \epsilon^*(k_2, \lambda_3))(N \cdot \epsilon(k_1, \lambda_1)) +
\]
\[
+ (P' \cdot \epsilon(k_1, \lambda_1))(N \cdot \epsilon^*(k_2, \lambda_3))\gamma_5\tilde{K}P'^{-2}K^{-2}u(p_1, \lambda_2).
\]

In definition of helicity bispinors of the nucleons the convention [9] about phase of second particle is taken into account.

3

Let’s get the matrix of the transformation between the six independent helicity amplitudes [3] and six invariant function $f_i(s, t)$. Having numbered the independent helicity amplitudes [3] from 1 to 6 we may write them down in the explicit form. In order to do this there is a need to put into the general expression of the helicity amplitude [13] the explicit expressions for four-vectors $K, Q, P'$ and $N$ and also for the helicity bispinors of the nucleons $u(p_1, \lambda_2), u(p_4, \lambda_4)$, taking into consideration the convention [9], and four-vectors of photon polarization $\epsilon(k_1, \lambda_1), \epsilon(k_2, \lambda_3)$. As a result we will have a linear heterogeneous equation system of the following type:

\[
T_i(s, t) = \sum_{j=1}^{6} A_{ij} f_j(s, t),
\]

where $T_i(s, t)$ are independent helicity amplitudes [3].

With the help of the package of symbolic calculations Mathematica it is possible to show that the determinant of the system matrix [14] is not equal to zero. This may serve
as a proof for the linear independence of invariant combinations (8). Using the package Mathematica we may solve the linear heterogeneous system (14) with regard to invariant functions $f_i(s, t)$:

$$f_i(s, t) = \sum_{j=1}^{6} u_{ij} T_j(s, t).$$

(15)

In the matrix $\|u_{ij}\|$ 8 elements out of 36 are equal to zero and the remaining 28 nonzero elements may be divided into 4 groups:

$$u_{11} = -\frac{1}{2}u_{13} = u_{15} = -u_{31} = -\frac{1}{2}u_{33} = -u_{35} = Mu_{61} = -Mu_{65} = -\frac{M^2 \sec(\theta)}{s - M^2},$$

$$u_{12} = -2u_{14} = 2u_{16} = -u_{32} = -2u_{34} = 2u_{36} = -2u_{54} = -2u_{56} = \frac{2M}{s} \frac{\sqrt{s}}{\csc(\theta)},$$

$$u_{21} = -\frac{1}{2}u_{23} = u_{25} = -u_{41} = -\frac{1}{2}u_{43} = -u_{45} = \frac{M}{(M^2 + s) \sec(\theta)},$$

$$u_{22} = -2u_{24} = 2u_{26} = -u_{42} = -2u_{44} = 2u_{46} = \frac{4M}{s} \frac{\sqrt{s} \csc(\theta)}{(M^2 - s)^2},$$

$$u_{51} = u_{52} = u_{53} = u_{55} = u_{62} = u_{63} = u_{64} = u_{66} = 0.$$  

(16)

In the expression (16) $\theta$ is the scattering angle of photon in the center of mass system of the s-channel:

$$\cos(\theta) = 1 + \frac{2st}{(M^2 - s)^2}.$$  

(17)

In the present work we took into consideration the contribution into the amplitude of Born terms in s-, u- and t-channel and the contribution of six resonances $P_{33}(1232)$, $P_{11}(1430)$, $S_{11}(1500)$, $D_{13}(1505)$, $S_{31}(1620)$, $D_{33}(1700)$ in s- and u-channel. At the same time we face the question of choice of corresponding lagrangians of interaction, that correspond to the vertices of Feynman diagrams. In this work while choosing the lagrangians of interaction no simplifying assumptions were made, i.e. lagrangians of interaction was written in the most general form, compatible with the requirements of hermiticity, $P$-, $T$- and $C$-invariance. As for the gauge-invariance we may say that the lagrangians of interaction with $\gamma$-quantum, which the electromagnetic field enters by means of gauge-invariant tensor $F_{\mu\nu} = -i(k_{\mu}\epsilon_{\nu} - k_{\nu}\epsilon_{\mu})$ are also gradient invariant, regardless of whether they are on mass shell or not, because during the substitution $\epsilon \to k$ tensor $F_{\mu\nu}$ identically turns to zero. Almost all the lagrangians, used in this work, refer to this type, except the Dirac’s
part of the interaction lagrangian of photon with nucleon (30). This last lagrangian is
gauge-invariant only when two nucleons are on mass shell, which is the consequence of
the Dirac equation for free nucleon. If one or both nucleons are not on the mass shell, the
vertex corresponding to the Dirac’s part of interaction lagrangian of photon with nucleon
(30) formally won’t be gauge-invariant. However it can be easily shown that the sum of
two nucleon Born diagrams in s- and u-channels will be gauge-invariant the same as for
Compton scattering on the electron, thus all the observables of reactions will be gauge-

invariant. As the lagrangian πN interaction in the present work we have used pseudoscalar

variant. We have to mention that in case of Compton scattering in the Born diagram of
t-channel, which corresponds to the interaction of pion with nucleon, both nucleons are
on mass shell, that’s why pseudovector variant πN interaction in this case is equivalent
to pseudoscalar one. The difference between the pseudoscalar and pseudovector coupling
occurs in case of Compton scattering only when observing diagrams with loops, in which
one or both nucleons aren’t on the mass shell. At the same time the usage of pseudovector
coupling leads to the extra degrees of momentum in the numerator of integrand, which
worsens the degree of integral convergence. That’s why from the mathematicial point of
view the usage of pseudoscalar connection in loop diagrams is more preferable. In contrast
to this, in Born diagrams of photoproduction of pion on the nucleon, one of the nucleons
of πNN vertex always is beyond the mass shell, which leads to the nonequivalence of
pseudoscalar and pseudovector coupling already on the level of Born diagrams.

Let’s consider lagrangian, which corresponds to the possible contact four-particle in-
teraction of two photons and two nucleons. This lagrangian come from Pauli’s part
of interaction lagrangian of photon with nucleon (30) with aid of minimal substitution
∂µ → ∂µ − ieAµ and should have the following form:

\[ \mathcal{L}_{\text{cont}} = \frac{e^2}{2M} A^\mu \nabla_\mu \sigma_{\mu\nu} N A^\nu. \]  

(18)

We can show that the vertex, corresponding to the lagrangian (18) during the gauge
replacement \( e^\mu \rightarrow k^\mu \), in case when nucleons are on mass shell, doesn’t turn identically
to zero but is the value, proportional to the multiplier \( (s - M^2) \). Thus, in relativistic

theory of perturbation the lagrangian (18) is not gauge-invariant. It is important to note,
however, that there are also contact terms in the recording of three-momentum Compton

scattering amplitude in the center of mass system, proportional to \( \epsilon_1 \cdot \epsilon_2 \), or \( \epsilon_1 \times \epsilon_2 \). But

they occur as three-dimension reduction of non-contact relativistic diagrams.

Three of six resonances, which contribution is taken into account in the s- and u-
channe, have a spin equal to \( \frac{3}{2} \). The Rarita-Schwinger propagator, corresponding to
these resonances, describes the transfer between the \( \frac{3}{2} \) spins states only on the mass shell
\( P^2 = M^*^2 \) and does not possess this property when the particle with spin \( \frac{3}{2} \) is not located
on the mass shell. If such propagator is used when the particle with \( \frac{3}{2} \) spin does not lie on
the mass shell it is necessarily to provide interaction lagrangian with additional condition,
ensuring the connection only with the \( \frac{3}{2} \) spin field. Let us consider the interaction of the
\( \frac{3}{2} \) spin particle with the tensor of electromagnetic field \( F^{\mu\nu} \). In this case the interaction lagrangian has the following expression:

\[
\mathcal{L}_{\text{int}} = \overline{N}^{*\mu} O_{\mu\nu\sigma} N F^{\nu\sigma} + h.c.,
\]

(19)

where \( N^{*\mu} \) is the field of \( \frac{3}{2} \) spin, \( N \) – a nucleon field. In order to ensure the connection with the \( \frac{3}{2} \) spin only, vertex matrix \( O_{\mu\nu\sigma} \) must meet the following requirements \([4, 10]\):

\[
\gamma^{\mu} O_{\mu\nu\sigma} = 0.
\]

(20)

Acquiring this condition is possible under the following circumstances. Let the vertex matrix \( \Gamma_{\mu\nu\sigma} \) possess all required properties of relativistic invariance and \( C-, P-, T- \)-invariance on the mass shell. Now let us write \([10]\) the new vertex matrix:

\[
O_{\mu\nu\sigma} = \Gamma_{\mu\nu\sigma} - \frac{1}{4} \gamma^{\rho} \Gamma_{\rho\nu\sigma}.
\]

(21)

Matrix \( O_{\mu\nu\sigma} \) will obviously meet the requirement \((20)\), and when located on the mass shell it becomes equal to matrix \( \Gamma_{\mu\nu\sigma} \), because the Rarita-Schwinger spinors \( N^{*\mu} \) meet the requirement of \( \gamma^{\mu} N^{*\mu} = 0 \). All interaction lagrangians of the \( \frac{3}{2} \) spins particles used in this paper meet requirement \((20)\). Lagrangians, used for the diagrams construction, and the explicit form of the propagators of particles with spin \( \frac{1}{2} \) and \( \frac{3}{2} \) is shown in the Appendix.

5

If \( \gamma N \rightarrow \gamma N \) is regarded as the generalized reaction, which takes place in three crossed channels \( s-, u- \) and \( t- \), it is possible to established crossing-symmetry properties of the invariant amplitudes \( f_i \). The crossing-symmetry of the arbitrary generalized reaction is possible if four particles include two identical ones (the particles, which relate to one isomultiplet, are regarded as identical, for example, nucleons and pions). In our case there are two pairs of identical particles – two photons and two nucleons. The crossing of any of these particle pairs results in transformation of \( s \)-channel into \( u \)-channel. In this case, due to generalized Pauli’s principle, the received amplitude must be identical to the initial one during the crossing of two photons and differ only in sign during the crossing of two nucleons. Examining of two photons crossing is less complicated. To carry out the crossing of two photons in initial amplitude \((13)\) the following changes would be necessary \([6]\):

\[
\begin{align*}
k_1 &\rightarrow -k_2, \quad k_2 \rightarrow -k_1, \quad K \rightarrow -K, \quad P \rightarrow P, \quad P' \rightarrow P', \quad Q \rightarrow Q, \quad N \rightarrow -N, \\
s &\rightarrow u, \quad u \rightarrow s, \quad t \rightarrow t, \quad \epsilon(k_1) \rightarrow -\epsilon^*(k_2), \quad \epsilon(k_2) \rightarrow -\epsilon^*(k_1).
\end{align*}
\]

(22)
Here the invariant combinations \( R^i \) are converted as follows:

\[
R^1 \to R^1, \quad R^2 \to -R^2, \quad R^3 \to R^3, \quad R^4 \to -R^4, \quad R^5 \to R^5, \quad R^6 \to R^6. \tag{23}
\]

In view of generalized Pauli’s principle during the permutation of two identical photons, the amplitude \((13)\) must not change its sign, the invariant functions \( f_1 - f_6 \) are converted as follows:

\[
f_1(s,t) \to f_1(u,t), \quad f_2(s,t) \to -f_2(u,t), \quad f_3(s,t) \to f_3(u,t), \quad f_4(s,t) \to -f_4(u,t), \quad f_5(s,t) \to f_5(u,t), \quad f_6(s,t) \to f_6(u,t). \tag{24}
\]

The definition of the invariant functions symmetry properties \( f_1 - f_6 \) with transposition \( s \leftrightarrow u \) allows accomplishing the crossing of identical nucleons instead of the crossing of identical photons on the same base. In is case instead of \((22)\) we will have:

\[
p_1 \to -p_2, \quad p_2 \to -p_1, \quad K \to K, \quad P \to -P, \quad P' \to -P', \quad Q \to Q, \quad N \to -N, \quad s \to u, \quad u \to s, \quad t \to t, \quad u(p_1) \to u(-p_2) \equiv v(p_2), \quad u(p_2) \to u(-p_1) \equiv v(p_1), \tag{25}
\]

where the last line has the equalities of \( v(p) = C\bar{u}(p), \bar{v}(p) = -\bar{u}(p)C \) and the matrix of charge conjugation properties. It is clear that under this condition the invariant spin combinations \( R^i \) are converted similarly as in \((23)\), and we again may state that the invariant functions \( f_i \) have the crossing-symmetry properties as in \((24)\).

Along with the invariant functions calculation we considered the reaction in \( s \)-channel \( \gamma N \to \gamma N \). Accomplishing the crossing transformation provided transition of the reaction amplitude in \( s \)-channel \((13)\) to the reaction amplitude in \( u \)- and \( t \)-channel. The transition of the amplitude \((13)\) in \( s \)-channel \( \gamma(k_1)N(p_1) \to \gamma(k_2)N(p_2) \) into the amplitude of reaction in \( u \)-channel \( \gamma(k_1)\bar{N}(\bar{p}_1) \to \gamma(k_2)\bar{N}(\bar{p}_2) \), would require the following replacements:

\[
p_1 \to -\bar{p}_2, \quad p_2 \to -\bar{p}_1, \quad u(p_1) \to u(-\bar{p}_2) \equiv v(\bar{p}_2), \quad \bar{u}(p_2) \to \bar{u}(-\bar{p}_1) \equiv \bar{v}(\bar{p}_1). \tag{26}
\]

In this case four-vectors \( K, Q \), remain constant, and four-vectors \( P, P', N \) as well as the invariant variables \( s, t, u \) are determined as follows:

\[
P = -\frac{1}{2}(p_1 + p_2), \quad P' = P - \frac{P \cdot K}{K^2} K, \quad N^\mu = i\epsilon^{\mu\nu\lambda\sigma} P'_\nu K_\lambda Q_\sigma, \quad s = (k_1 - \bar{p}_2)^2, \quad t = (k_1 - k_2)^2, \quad u = (\bar{p}_1 + k_1)^2. \tag{27}
\]

The transition of the \( s \)-channel reaction amplitude into the \( t \)-channel reaction amplitude \( N(p)\bar{N}(\bar{p}) \to \gamma(k)\gamma(k') \), requires the following replacements in \((13)\):

\[
p_1 \to p, \quad p_2 \to -\bar{p}, \quad k_1 \to -k, \quad k_2 \to k', \quad u(p_1) \to u(p), \quad \bar{u}(p_2) \to \bar{u}(-\bar{p}) \equiv \bar{v}(\bar{p}), \quad \epsilon(k_1) \to \epsilon(-k) = -\epsilon^*(k), \quad \epsilon^*(k_2) \to \epsilon^*(k'). \tag{28}
\]
Diagrams are considered on calculation of the invariant functions $f_1 - f_6$. Diagrams $a$, $b$, $c$ are corresponding to Born terms in $s$-, $u$- and $t$-channel, diagrams $d$, $e$ are corresponding to resonances terms in $s$- and $u$-channel.

Four-vectors $K, Q, P, P', N$ and the invariant variables $s, t, u$ in this case are determined as follows:

$$
K = \frac{1}{2}(k' - k), \quad Q = \frac{1}{2}(k + k'), \quad P = \frac{1}{2}(p - \bar{p}), \quad P' = P - \frac{P \cdot K}{K^2} K,
$$

$$
N^\mu = i\epsilon^{\mu\nu\lambda\sigma} P'_\nu K_\lambda Q_\sigma, \quad s = (p - k)^2, \quad t = (k + k')^2, \quad u = (\bar{p} - k)^2. \quad (29)
$$

When calculating invariant functions $f_1 - f_6$ the amplitude of Compton scattering reaction was recorded in accordance with Feynman rules for diagrams in figure 1.

Vertices of diagrams correspond to lagrangians are cited in the Appendix. Propagators of $\frac{1}{2}$ and $\frac{3}{2}$ spin resonances, used when recording the amplitude, are cited in [11], [12].

Using direct expressions for helicity bispinors and four-vectors of photon polarization [11] it is possible to get expressions for six independent helicity amplitudes [3] of Compton scattering with the help of symbolic calculations package Mathematica. Then using
the calculated transformation matrix (16) of independent helicity amplitudes (3) to invariant functions $f_1 - f_6$ it is possible to get explicit expressions for the latter. Calculated contributions of Born diagrams in $s$-, $u$- and $t$-channel and contributions of resonances diagrams $P_{33}(1232)$, $P_{11}(1430)$, $S_{11}(1500)$, $S_{31}(1620)$, $D_{33}(1700)$, $D_{13}(1505)$ in $s$- and $u$- channel into invariant functions of Compton scattering are sited in the Appendix. Cases concerning Compton scattering on proton and neutron are considered. Invariant variables $s$, $t$ are used as independent arguments of invariant functions $f_1 - f_6$. All the given expressions of contributions in invariant functions meet the requirements for crossing symmetry (24). Invariant functions $f_1 - f_6$ of Compton scattering after crossing transformation (26), (28) may be used for the construction of reaction amplitude in $u$-channel and $t$-channel correspondingly.

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**Appendix**

Below there is a explicit structure of interaction lagrangians, used in the work. The following indications are accepted: $N^*{^\mu}$ is $\frac{3}{2}$ spin field, which corresponds to the given resonance, $N^*$ is $\frac{1}{2}$ spin field, which corresponds to the given resonance, $N$ is nucleon field, $\pi_i$ is pionic field with isotopic index $i$, $\eta$ is $\eta$-meson field. $A^\mu$ is four-potential of electromagnetic field, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is gauge-invariant tensor of electromagnetic field. Mass indications: $M$ is nucleon mass, $\mu$ is pion mass, $\mu_{\eta}$ is $\eta$-meson mass. Dirac matrixes $\gamma_\mu$, $\gamma_5$ and antisymmetric matrix tensor $\sigma_{\mu\nu}$ are determined as in [12]. The following indications were used in isotopic part of lagrangians: $\tau_i$ is isotopic Pauli matrix, $S_i$ is isotopic transition matrix $\frac{3}{2} \rightarrow \frac{1}{2}$ [13].

Lagrangian of photon interaction with nucleon:

\[ \mathcal{L}_{NN\gamma} = \frac{N}{2} \left( F_i^S + F_i^V \tau_3 \right) \hat{A} N - \frac{F^{\alpha\beta}}{4M} \bar{N} \sigma_{\alpha\beta} \frac{1}{2} \left( F_2^S + F_2^V \tau_3 \right) N, \]  

(30)

where $F_i^S = F_i^p + F_i^n$, $F_i^V = F_i^p - F_i^n$ are isoscalar and isovector form factors of nucleon, $\hat{A} = A_\alpha \gamma^\alpha$.

Lagrangians of interaction, used in the calculations of Born diagrams in $t$-channel of
Compton scattering:

\[ \mathcal{L}_{\pi\gamma\gamma} = \frac{1}{4} F_{\pi}^{\mu}\epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha\beta} \gamma_5 \, \pi_0, \]  
\[ \mathcal{L}_{\pi NN} = 2i M f_{\pi} N_{\gamma_5} \gamma_5 N_{\pi_1}, \]  
\[ \mathcal{L}_{\eta\gamma\gamma} = \frac{1}{4} F_{\eta}^{\mu}\epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha\beta} \eta, \]  
\[ \mathcal{L}_{\eta NN} = 2i M f_{\eta} N_{\gamma_5} \gamma_5 N_{\eta}, \]  

where \( F_{\pi}, F_{\eta}, f_{\pi}, f_{\eta} \) are corresponding coupling constants. Lagrangians (32), (33) correspond to pseudoscalar meson-nucleon coupling, lagrangians (31), (35) are obviously gauge-invariant.

Lagrangian of \( NS_{11}\gamma \) interaction:

\[ \mathcal{L}_{NS_{11}\gamma} = -\frac{F_{\pi\gamma}}{4M} \left[ \nabla \sigma_{\alpha\beta} \gamma_5 \left( G_{NS_{11}\gamma}^S + \tau_3 G_{NS_{11}\gamma}^V \right) N^* - \nabla^* \sigma_{\alpha\beta} \gamma_5 \left( G_{NS_{11}\gamma}^S + \tau_3 G_{NS_{11}\gamma}^V \right) N \right], \]  

where \( G_{NS_{11}\gamma}^S = G_{NS_{11}\gamma}^p + G_{NS_{11}\gamma}^n, G_{NS_{11}\gamma}^V = G_{NS_{11}\gamma}^p - G_{NS_{11}\gamma}^n \) are isoscalar and isovector form factors of \( NS_{11}\gamma \) interaction.

Lagrangian of \( NS_{31}\gamma \) interaction:

\[ \mathcal{L}_{NS_{31}\gamma} = -\frac{G_{NS_{31}\gamma}}{2M} F_{\alpha\beta} \left[ \nabla \sigma_{\alpha\beta} \gamma_5 S_3 N^* - \nabla^* \sigma_{\alpha\beta} \gamma_5 S_3^\dagger N \right], \]  

where \( G_{NS_{31}\gamma} \) is isovector form factor of \( NS_{31}\gamma \) interaction. In this case isoscalar form factor is absent, because absorption or emission of only isovector \( \gamma \)-quantum is possible in \( NS_{31}\gamma \) vertex. The same may be said about other resonances with \( \frac{3}{2} \) isospin.

Lagrangian of \( NP_{11}\gamma \) interaction:

\[ \mathcal{L}_{NP_{11}\gamma} = -\frac{F_{\pi\gamma}}{4M} \left[ \nabla \sigma_{\alpha\beta} \left( G_{NP_{11}\gamma}^S + \tau_3 G_{NP_{11}\gamma}^V \right) N^* + \nabla^* \sigma_{\alpha\beta} \left( G_{NP_{11}\gamma}^S + \tau_3 G_{NP_{11}\gamma}^V \right) N \right], \]  

where \( G_{NP_{11}\gamma}^S = G_{NP_{11}\gamma}^p + G_{NP_{11}\gamma}^n, G_{NP_{11}\gamma}^V = G_{NP_{11}\gamma}^p - G_{NP_{11}\gamma}^n \) are isoscalar and isovector form factors of \( NP_{11}\gamma \) interaction.

Lagrangian of \( NP_{33}\gamma \) interaction:

\[ \mathcal{L}_{NP_{33}\gamma} = \frac{G_{1NP_{33}\gamma}}{4M} \left\{ \nabla \sigma_{\alpha\beta} \left( g_{\lambda\gamma} \gamma_5 \gamma_5 - g_{\lambda\beta} \gamma_5 \gamma_5 \right) + \frac{i}{2} \gamma_5 \sigma_{\alpha\beta} \right\} \gamma_5 S_3 N + h.c. \]  
\[ - \frac{G_{2NP_{33}\gamma}}{8M^2} \left\{ \nabla \sigma_{\alpha\beta} \left( i \left( g_{\lambda\alpha} \gamma_5 \gamma_5 - g_{\lambda\alpha} \gamma_5 \gamma_5 \right) + \frac{i}{4} \gamma_5 \gamma_5 \sigma_{\alpha\beta} \left( \gamma_5 S_3 \gamma_5 S_3 \right) + h.c. \right) \right\} \]  
\[ + \frac{G_{3NP_{33}\gamma}}{4M^2} \left\{ \nabla \sigma_{\alpha\beta} \left( -g_{\lambda\beta} + \frac{1}{4} \gamma_5 \gamma_5 \gamma_5 \right) i \gamma_5 S_3 N + h.c. \right\} \]  
\[ + \left\{ \nabla \sigma_{\alpha\beta} \left( g_{\lambda\alpha} - \frac{1}{4} \gamma_5 \gamma_5 \right) i \gamma_5 S_3 N + h.c. \right\} \]  
\[ + \partial_{\alpha} F_{\alpha\beta} + \partial_{\beta} F_{\alpha\beta}, \]  

(38)
where $G_{1NP_{33}}$, $G_{2NP_{33}}$, $G_{3NP_{33}}$ are isovector form factors of $NP_{33\gamma}$ interaction. Symbol $\frac{\partial}{\partial t}$ in formulas (38), (39), (40) operates on the function in such a way: $u \frac{\partial}{\partial t} v = u(\partial v) - (\partial u)v$.

Lagrangian of $ND_{33\gamma}$ interaction:

$$
\mathcal{L}_{ND_{33\gamma}} = \frac{G_{1ND_{33\gamma}}}{4M} \left\{ \nabla^\lambda \left[ (g_{\lambda\alpha} \gamma_\beta - g_{\lambda\beta} \gamma_\alpha) + \frac{1}{2} \gamma_\lambda \sigma_{\alpha\beta} \right] S_3^\dagger N + h.c. \right\} F^{\alpha\beta} - \frac{G_{2ND_{33\gamma}}}{8M^2} \left\{ \nabla^\lambda \left[ i \left( g_{\lambda\beta} \frac{\partial_{\alpha}}{\partial x} - g_{\lambda\alpha} \frac{\partial_{\beta}}{\partial x} \right) - \frac{i}{4} \gamma_\lambda \left( \gamma_\beta \frac{\partial_{\alpha}}{\partial x} - \gamma_\alpha \frac{\partial_{\beta}}{\partial x} \right) \right] S_3^\dagger N + h.c. \right\} F^{\alpha\beta} + \frac{G_{3ND_{33\gamma}}}{4M^2} \left\{ \nabla^\lambda \left[ -g_{\lambda\beta} + \frac{1}{4} \gamma_\lambda \gamma_\beta \right] S_3^\dagger N + h.c. \right\} \partial_\alpha F^{\alpha\beta} + \left[ \nabla^\lambda \left( g_{\lambda\alpha} - \frac{1}{4} \gamma_\lambda \gamma_\alpha \right) S_3^\dagger N + h.c. \right] \partial_\beta F^{\alpha\beta},
$$

(39)

where $G_{1ND_{33\gamma}}$, $G_{2ND_{33\gamma}}$, $G_{3ND_{33\gamma}}$ are isovector form factors of $ND_{33\gamma}$ interaction.

Lagrangian of $ND_{13\gamma}$ interaction:

$$
\mathcal{L}_{ND_{13\gamma}} = \frac{1}{4M} \left\{ \nabla^\lambda \left[ (g_{\lambda\alpha} \gamma_\beta - g_{\lambda\beta} \gamma_\alpha) + \frac{1}{2} \gamma_\lambda \sigma_{\alpha\beta} \right] \frac{1}{2} \left[ G_{1ND_{13\gamma}}^S + G_{1ND_{13\gamma}}^V \right] N + h.c. \right\} F^{\alpha\beta} - \frac{1}{8M^2} \left\{ \nabla^\lambda \left[ i \left( g_{\lambda\beta} \frac{\partial_{\alpha}}{\partial x} - g_{\lambda\alpha} \frac{\partial_{\beta}}{\partial x} \right) - \frac{i}{4} \gamma_\lambda \left( \gamma_\beta \frac{\partial_{\alpha}}{\partial x} - \gamma_\alpha \frac{\partial_{\beta}}{\partial x} \right) \right] \times \frac{1}{2} \left[ G_{2ND_{13\gamma}}^S + G_{2ND_{13\gamma}}^V \right] N + h.c. \right\} F^{\alpha\beta} + \frac{1}{4M^2} \left\{ \nabla^\lambda \left[ -g_{\lambda\beta} + \frac{1}{4} \gamma_\lambda \gamma_\beta \right] \frac{1}{2} \left[ G_{3ND_{13\gamma}}^S + G_{3ND_{13\gamma}}^V \right] N + h.c. \right\} \partial_\alpha F^{\alpha\beta} + \frac{1}{4M^2} \left\{ \nabla^\lambda \left( g_{\lambda\alpha} - \frac{1}{4} \gamma_\lambda \gamma_\alpha \right) \frac{1}{2} \left[ G_{3ND_{13\gamma}}^S + G_{3ND_{13\gamma}}^V \right] N + h.c. \right\} \partial_\beta F^{\alpha\beta},
$$

(40)

where $G_{i,ND_{13\gamma}}^S = G_{i,ND_{13\gamma}}^p + G_{i,ND_{13\gamma}}^m$, $G_{i,ND_{13\gamma}}^V = G_{i,ND_{13\gamma}}^p - G_{i,ND_{13\gamma}}^m$ are isoscalar and isovector form factors of $NP_{11\gamma}$ interaction.

Lagrangians (38) - (40) are gauge-invariant, because four-potential $A^\mu$ is also involved due to gauge-invariant tensor $F^{\mu\nu}$. It is necessary to note that the contribution of components, proportional $\partial_\mu F^{\mu\nu}$, in lagrangians (38) - (40) turns into zero for real photons ($k^2 = 0, \epsilon \cdot k = 0$).

Propagator was used in this work for $\frac{1}{2}$ spin resonances ($S_{11}$, $S_{31}$, $P_{11}$):

$$
G(P^2; P) = \frac{\hat{P} + M^*}{P^2 - M^*^2 + iM^*\Gamma(P^2)}.
$$

(41)
Rarita-Schwinger propagator was used for $\frac{3}{2}$ spin resonances ($P_{33}$, $D_{33}$, $D_{13}$):

$$G^{\mu\nu}(P^2; P) = \frac{\hat{P} + M^*}{P^2 - M^{*2} + iM^*\Gamma(P^2)} \times \left\{ -g^{\mu\nu} + \frac{\gamma^\mu\gamma^\nu}{3} + \frac{2P^\mu P^\nu}{3M^*^2} - \frac{P^\mu\gamma^\nu - \gamma^\mu P^\nu}{3M^*} \right\}. \quad (42)$$

In expressions (41), (42) $M^* \Gamma(P^2)$ are mass and width of the corresponding resonance with four-momentum $P$.

Below there are explicit expressions for Born diagrams contribution $s$ and diagrams of six resonances $P_{33}(1232)$, $P_{11}(1430)$, $S_{11}(1500)$, $S_{31}(1620)$, $D_{33}(1700)$, $D_{13}(1505)$ into invariant functions of $f_1 - f_6$ amplitude.

Born diagrams contribution to invariant functions $f_1 - f_6$ of Compton scattering on proton:

$$\left\{ \begin{align*}
-2M^2 F_1^{p2} & \frac{(2M^2 - 2s - t) F_1^{p2}}{(M^2 - s)(M^2 - s - t)}, \\
(2F_1^p - F_2^p) F_2^p & \frac{( - 2M^2 + 2s + t)(F_1^p - F_2^p)^2}{M(M^2 - s)(M^2 - s - t)}, \\
2M^2 t F_1^{p2} & \frac{2(M^4 - 2M^2(s + t) + s(s + t)) F_1^p F_2^p - (M^2 - s)(M^2 - s - t) F_2^{p2}}{2M(M^2 - s)(M^2 - s - t)} + \\
+ \frac{2f M^2 F_1^\pi}{\mu(t - \mu^2)} & + \frac{2F_1^\eta M^2 F_1^\eta}{\mu_\eta(t - \mu_\eta^2)}, \\
-2M^2 t F_1^{p2} & + 2M^2 t F_1^p F_2^p + (M^2 - s)(M^2 - s - t) F_2^{p2} \\
2M^2(M^2 - s)(M^2 - s - t) & \end{align*} \right\}. \quad (43)$$

It is necessary to make a change in case of Compton scattering on neutron in formulae (43):

$$F_1^p \rightarrow F_1^n, \quad F_2^p \rightarrow F_2^n, \quad f \rightarrow -f. \quad (44)$$

Contribution of $P_{33}$ into invariant functions $f_1 - f_6$ of Compton scattering (is the
same for Compton scattering on proton and neutron):

\[
\left\{-\frac{1}{576 M^4 D_{\Delta} M_{\Delta}^2} \left( 16 M^2 (M - M_{\Delta}) \left( M^4 - M^2 s + 3 (M^3 - M s) M_{\Delta} + 3 t M_{\Delta}^2 \right) G_{\Delta N \gamma}^1 e^{-2} - \\
-16 M (M^2 (M^2 - s) s + (M^5 - M s^2) M_{\Delta} - 3 s (-M^2 + s + 2 t) M_{\Delta}^2 + \\
+6 M (-M^2 + s + t) M_{\Delta}^3) G_{\Delta N \gamma}^1 G_{\Delta N \gamma}^2 + (M s (M^4 + 2 M^2 s - 3 s^2) - \\
-2 s (-3 M^4 + 2 M^2 s + s^2) M_{\Delta} + 3 M (5 M^4 - 22 M^2 s + s (17 s + 16 t)) M_{\Delta}^2 - \\
-6 (M^4 - 6 M^2 s + s (5 s + 8 t)) M_{\Delta}^3) G_{\Delta N \gamma}^2 e^{-2} \right) + \\
+ \frac{1}{576 M^4 D_{\Delta} M_{\Delta}^2} \left( 16 M^2 (M - M_{\Delta}) \left( M^2 (M^2 - s - t) + 3 M (M^2 - s - t) M_{\Delta} - 3 t M_{\Delta}^2 \right) \times \\
\times G_{\Delta N \gamma}^1 e^{-2} - 16 M (M^2 (2 M^4 - 3 M^2 (s + t) + (s + t)^2) + M (3 M^4 - 4 M^2 (s + t) + \\
+(s + t)^2) M_{\Delta} + 3 \left( 2 M^4 + s^2 - t^2 + M^2 (t - 3 s) \right) M_{\Delta}^2 - 6 (M^3 - M s) M_{\Delta}^3) G_{\Delta N \gamma}^1 G_{\Delta N \gamma}^2 + \\
+ (M (14 M^6 - 27 M^4 (s + t) + 16 M^2 (s + t)^2 - 3 (s + t)^3) + (20 M^6 - 34 M^4 (s + t) + \\
+ 16 M^2 (s + t)^2 - 2 (s + t)^3) M_{\Delta} - 3 M (29 M^4 + 17 s^2 + 18 s t + t^2 - 2 M^2 (23 s + 7 t)) M_{\Delta}^2 + \\
+ 6 (9 M^4 + 5 s^2 + 2 s t - 3 t^2 + 2 M^2 (-7 s + t)) M_{\Delta}^3) G_{\Delta N \gamma}^2 e^{-2} \right), \\
- \frac{1}{576 M^4 D_{\Delta} M_{\Delta}^2} \left( 16 M^2 (M^2 + s) + 4 M^3 M_{\Delta} + 3 (-3 M^2 + s + t) M_{\Delta}^2) G_{\Delta N \gamma}^1 e^{-2} - \\
-8 M (M s (3 M^2 + s) + 2 \left( M^4 + 3 M^2 s \right) M_{\Delta} + 3 M (3 M^2 + s) M_{\Delta}^2 + \\
+ 6 \left( 5 M^2 + s + 2 t \right) M_{\Delta}^3) G_{\Delta N \gamma}^1 G_{\Delta N \gamma}^2 + (s (M^4 + 6 M^2 s + s^2) + 8 M s (M^2 + s) M_{\Delta} + \\
+ 3 \left( 5 M^4 - 34 M^2 s + s (5 s + 16 t) \right) M_{\Delta}^2 + 24 M (M^2 + s) M_{\Delta}^3) G_{\Delta N \gamma}^2 e^{-2} \right) + \\
+ \frac{1}{576 M^4 D_{\Delta} M_{\Delta}^2} \left( 16 (M^4 (3 M^2 - s - t) + 4 M^5 M_{\Delta} - 3 M^2 (M^2 + s) M_{\Delta}^2) G_{\Delta N \gamma}^1 e^{-2} - \\
-8 M (M (10 M^4 - 7 M^2 (s + t) + (s + t)^2) + 2 M^2 (7 M^2 - 3 (s + t)) M_{\Delta} + \\
+ 3 M (5 M^2 - s - t) M_{\Delta}^2 - 6 \left( 3 M^2 + s - t \right) M_{\Delta}^3) G_{\Delta N \gamma}^1 G_{\Delta N \gamma}^2 e^{-2} + \\
+ (34 M^6 - 37 M^4 (s + t) + 12 M^2 (s + t)^2 - (s + t)^3) + \\
+ 8 M (6 M^4 - 5 M^2 (s + t) + (s + t)^2) M_{\Delta} - 3 \left( 43 M^4 - 5 s^2 + 6 s t + \\
+ 11 t^2 - 2 M^2 (7 s + 23 t) \right) M_{\Delta}^2 + 24 M (3 M^2 - s - t) M_{\Delta}^3) G_{\Delta N \gamma}^2 e^{-2} \right), \\
\right\}
\]
\[
\frac{1}{576 M^4 D_{s\Delta} M_{\Delta}^2} \left( 16 M^2 (M + M_{\Delta}) (M^4 - M^2 s - 3 (M^3 - M s) M_{\Delta} + 3 t M_{\Delta}^2) G_{\Delta N\gamma}^1 - 16 M (M^2 - s)^2 (M - 3 M_{\Delta}) M_{\Delta} G_{\Delta N\gamma}^1 G_{\Delta N\gamma}^2 + (M^2 - s)^2 (M s + 2 s M_{\Delta} + +15 M M_{\Delta}^2 - 18 M_{\Delta}^3) G_{\Delta N\gamma}^2 \right) - \\
\frac{1}{576 M^4 D_{u\Delta} M_{\Delta}^2} \left( 16 M^2 (M + M_{\Delta}) (M^2 (M^2 - s - t) + 3 M (-M^2 + s + t) M_{\Delta} - -3 t M_{\Delta}^2) G_{\Delta N\gamma}^1 + 16 M (-M^2 + s + t)^2 (M - 3 M_{\Delta}) M_{\Delta} G_{\Delta N\gamma}^1 G_{\Delta N\gamma}^2 - (-M^2 + s + t)^2 \times \times (M (2 M^2 - s - t) (4 M^2 - 2 (s + t)) M_{\Delta} + 15 M M_{\Delta}^2 - 18 M_{\Delta}^3) G_{\Delta N\gamma}^2 \right), \\
\frac{1}{288 M^4 D_{s\Delta} M_{\Delta}^2} \left( 8 M^2 M_{\Delta} (2 M^4 - 2 M^2 s + 3 t M_{\Delta}^2) G_{\Delta N\gamma}^1 - 4 M (M^2 (M^2 - s) s+ + M_{\Delta} (2 M (M^2 - s) s + 3 M_{\Delta} (M^4 - M^2 s + 2 s t + 2 M (-M^2 + s) M_{\Delta})) \times \times G_{\Delta N\gamma}^1 G_{\Delta N\gamma}^2 + (M^2 - s) (M s^2 + s (M^2 + s) M_{\Delta} - 9 M s M_{\Delta}^2+ +3 (M^2 + s) M_{\Delta}^3) G_{\Delta N\gamma}^2 \right) - \\
\frac{1}{288 M^4 D_{u\Delta} M_{\Delta}^2} \left( -8 M^2 M_{\Delta} (2 M^2 (M^2 - s - t) - 3 t M_{\Delta}^2) G_{\Delta N\gamma}^1 + +4 M (M^2 (M^2 - s - t) (2 M^2 - s - t) + M_{\Delta} (2 M (M^2 - s - t) (2 M^2 - s - t) + +3 M (M^4 + 2 t (s + t) + M_{\Delta} (M - 2 M^2 + s + t) M_{\Delta}))) G_{\Delta N\gamma}^1 G_{\Delta N\gamma}^2 - - (M^2 - s - t) (M (2 M^2 + s + t)^2 + (2 M^2 - s - t) (3 M^2 - s - t) M_{\Delta}+ + 9 M (2 M^2 + s + t) M_{\Delta}^2 + (9 M^2 - 3 (s + t)) M_{\Delta}^3) G_{\Delta N\gamma}^2 \right), 
\]
\[-\frac{1}{576 M^4 D_{s}\Delta M_{\Delta}^2} \left( 16 M^2 (M^4 - M^2 s + 3 (-M^2 + s + t) M_{\Delta}^2) G_{\Delta N \gamma}^1 \right) \]
\[-8 M (M (M^2 - s) s + 2 (M^4 - M^2 s) M_{\Delta} + 3 (M^3 - M s) M_{\Delta}^2 + \]
\[+ 6 (-M^2 + s + t) M_{\Delta}^3) G_{\Delta N \gamma}^1 G_{\Delta N \gamma}^2 + (s (M^4 - s^2) + 4 M (M^2 - s) s M_{\Delta} - \]
\[-9 (M^4 - s^2) M_{\Delta}^2 + 12 (M^3 - M s) M_{\Delta}^3) G_{\Delta N \gamma}^2 \right) + \]
\[+ \frac{1}{576 M^4 D_{u}\Delta M_{\Delta}^2} \left( 16 M^2 (M^2 - s - t) - 3 (M^4 - M^2 s) M_{\Delta}^2) G_{\Delta N \gamma}^1 \right) \]
\[-8 M (M (2M^4 - 3 M^2 (s + t) + (s + t)^2) + 2 M^2 (M^2 - s - t) M_{\Delta} + \]
\[+ 3 M (M^2 - s - t) M_{\Delta}^2 - 6 (M^2 - s) M_{\Delta}^3) G_{\Delta N \gamma}^1 G_{\Delta N \gamma}^2 + \]
\[+ (6 M^6 - 11 M^4 (s + t) + 6 M^2 (s + t)^2 - (s + t)^3 + 4 M (2 M^4 - \]
\[-3 M^2 (s + t) + (s + t)^2) M_{\Delta} - 9 (3 M^4 - 4 M^2 (s + t) + (s + t)^2) M_{\Delta}^2 + \]
\[+ 12 M (M^2 - s - t) M_{\Delta}^3) G_{\Delta N \gamma}^2 \right) \). \tag{45} \]

In formulae (45) $M_{\Delta}$ is $P_{33}$ resonance mass, $D_{s}\Delta = s - M_{\Delta}^2 + i M_{\Delta} \Gamma_{\Delta}(s)$, $D_{u}\Delta = u - M_{\Delta}^2 + i M_{\Delta} \Gamma_{\Delta}(u)$ are denominators of $P_{33}$ resonance propagator in $s$- and $u$-channel correspondingly.

$P_{11}$ contribution to invariant functions $f_1 - f_6$ of Compton scattering on proton:
\[
\begin{align*}
\left\{ - \frac{(M^2 - s)}{4M^2 D_{s}\rho_{11}} G_{P11\rho_{11}}^2 (M - M_{P_{11}}) + & \frac{(M^2 - s - t)}{4M^2 D_{u}\rho_{11}} G_{P11\rho_{11}}^2 (M - M_{P_{11}}), \\
- \frac{G_{P11\rho_{11}}^2 (M + s - 2 M M_{P_{11}})}{4M^2 D_{s}\rho_{11}} + & \frac{G_{P11\rho_{11}}^2 (3 M^2 - s - t - 2 M M_{P_{11}})}{4M^2 D_{u}\rho_{11}}, \\
- \frac{(M^2 - s)}{4M^2 D_{s}\rho_{11}} G_{P11\rho_{11}}^2 (M + M_{P_{11}}) - & \frac{G_{P11\rho_{11}}^2 (3 M^2 - s - t + 2 M M_{P_{11}})}{4M^2 D_{u}\rho_{11}}, \\
\frac{G_{P11\rho_{11}}^2 (M^2 + s + 2 M M_{P_{11}})}{4M^2 D_{s}\rho_{11}} - & \frac{G_{P11\rho_{11}}^2 (3 M^2 - s - t + 2 M M_{P_{11}})}{4M^2 D_{u}\rho_{11}}, \\
- \frac{(M^2 - s)}{4M^2 D_{s}\rho_{11}} G_{P11\rho_{11}}^2 M_{P_{11}} - & \frac{(M^2 - s - t)}{4M^2 D_{u}\rho_{11}} G_{P11\rho_{11}}^2 M_{P_{11}}, \\
- \frac{G_{P11\rho_{11}}^2}{4M^2 D_{s}\rho_{11}} + & \frac{(M^2 - s - t)}{4M^2 D_{u}\rho_{11}} \right\}. \tag{46} \]

In formulae (46) $M_{P_{11}}$ is $P_{11}$ resonance mass, $D_{s}\rho_{11} = s - M_{P_{11}}^2 + i M_{P_{11}} \Gamma_{P_{11}}(s)$, $D_{u}\rho_{11} = u - M_{P_{11}}^2 + i M_{P_{11}} \Gamma_{P_{11}}(u)$ are denominators of resonance $P_{11}$ propagator in $s$- and $u$-channel correspondingly. It is necessary to make a change in case of Compton scattering on neutron in formulae (46): $G_{P11\rho_{11}} \rightarrow G_{P11\rho_{11}}^n$. 

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correspondingly. It is necessary to make a change in case of Compton scattering on neutron in formulae (47): $G_{S11\gamma} \rightarrow G_{S11n\gamma}$.

$S_{31}$ contribution in invariant functions $f_1 - f_6$ of Compton scattering (is the same for Compton scattering on proton and neutron):

\[
\left\{ \frac{2 \left( M^2 - s \right) G_{S31N\gamma}^2 \left( M + M_{S31} \right)}{3M^2 D_{sS31}} + \frac{2 \left( M^2 - s - t \right) G_{S31N\gamma}^2 \left( M + M_{S31} \right)}{3M^2 D_{uS31}},
\frac{2 G_{S31N\gamma}^2 \left( M^2 + s + 2 M_{S31} \right)}{3M^2 D_{sS31}} + \frac{2 G_{S31N\gamma}^2 \left( 3 M^2 - s - t + 2 M_{S31} \right)}{3M^2 D_{uS31}},
\frac{2 \left( M^2 - s \right) G_{S31N\gamma}^2 \left( M - M_{S31} \right)}{3M^2 D_{sS31}} - \frac{2 \left( M^2 - s - t \right) G_{S31N\gamma}^2 \left( M - M_{S31} \right)}{3M^2 D_{uS31}},
\frac{2 G_{S31N\gamma}^2 \left( M^2 + s - 2 M_{S31} \right)}{3M^2 D_{sS31}} - \frac{2 G_{S31N\gamma}^2 \left( 3 M^2 - s - t - 2 M_{S31} \right)}{3M^2 D_{uS31}} \right\}. \tag{48}
\]

In formulae (48) $M_{S31}$ is $S_{31}$ resonance mass, $D_{sS31} = s - M_{S31}^2 + i M_{S31} \Gamma_{S31}(s)$, $D_{uS31} = u - M_{S31}^2 + i M_{S31} \Gamma_{S31}(u)$ are denominators of $S_{31}$ resonance propagator in $s$- and $u$-channel correspondingly. $D_{s33}$ contribution to invariant functions $f_1 - f_6$ of Compton scattering (is the same for
Compton scattering on proton and neutron):

\[
\left\{ \begin{array}{c}
\frac{1}{576 M^4 D_{sD33} M_{D33}^2} (16 M^2 (M + M_{D33}) (M^4 + 3 M^2 s - 4 s^2 - 3 (M^3 - M s) M_{D33} + \\
+ (-4 M^2 + 4 s + 3 t) M_{D33}^2) G_{D33N}^1, \gamma^2 - 8 M (s (-3 M^4 + 2 M^2 s + s^2) + \\
+ 2 (M^5 - M s^2) M_{D33} + (M^4 - 6 M^2 s + s (5 s + 12 t)) M_{D33}^2 - 12 M (M^2 - s - t) M_{D33}^3) \times \\
\times G_{D33N}^1 G_{D33N}^2 G_{D33N}^3 + (M s (M^4 + 2 M^2 s - 3 s^2) + 2 s (2 M^2 s + s^2 - 3 M^4) M_{D33} + 3 M (5 M^4 - \\
- 22 M^2 s + s (17 s + 16 t)) M_{D33}^2 + 6 (M^4 - 6 M^2 s + s (5 s + 8 t)) M_{D33}^3) G_{D33N}^2 \right) + \\
+ \frac{1}{576 M^4 D_{sD33} M_{D33}^2} (16 M^2 (M + M_{D33}) (9 M^4 - 13 M^2 (s + t) + 4 (s + t)^2 + 3 M (s - M^2 + \\
t) M_{D33} + (4 s + t - 4 M^2) M_{D33}^2) G_{D33N}^1, \gamma^2 + 8 M (10 M^6 - 17 M^4 (s + t) + 8 M^2 (s + t)^2 - \\
- (s + t)^3 - 2 M (3 M^4 - 4 M^2 (s + t) + (s + t)^2) M_{D33} + (9 M^4 + 5 s^2 - 2 M^2 (7 s - 5 t) - \\
- 2 s t - 7 t^2) M_{D33}^2 + 12 (M^3 - M s) M_{D33}^3) G_{D33N}^1 G_{D33N}^2 G_{D33N}^3 + (M (14 M^6 - 27 M^4 (s + t) + \\
+ 16 M^2 (s + t)^2 - 3 (s + t)^3) + 2 (-10 M^6 + 17 M^4 (s + t) - 8 M^2 (s + t)^2 + (s + t)^3) M_{D33} - \\
- 3 M (29 M^4 + 17 s^2 + 18 s t + t^2 - 2 M^2 (23 s + 7 t)) M_{D33}^2 - 6 (9 M^4 + 5 s^2 + 2 s t - \\
- 3 t^2 + 2 M^2 (-7 s + t)) M_{D33}^3) G_{D33N}^2 \right),
\end{array} \right.
\]

\[
\frac{1}{576 M^4 D_{sD33} M_{D33}^2} (16 M^2 (M^4 + 7 M^2 s + 2 s^2 - 4 (M^3 - 2 M s) M_{D33} + (-15 M^2 + s + \\
+ 3 t) M_{D33}^2 - 8 M M_{D33}^3) G_{D33N}^1, \gamma^2 - 16 M (-2 M s (M^2 + s) + (M^4 + 3 M^2 s) M_{D33} - \\
- 4 M^3 M_{D33}^2 + 3 (-5 M^2 + s + 2 t) M_{D33}^3) G_{D33N}^1 G_{D33N}^2 G_{D33N}^3 + (s (M^4 + 6 M^2 s + s^2) - \\
- 8 M s (M^2 + s)) M_{D33} + 3 (5 M^4 - 34 M^2 s + s (5 s + 16 t)) M_{D33}^2 - 24 M (M^2 + s) \times \\
\times M_{D33}^3) G_{D33N}^2 \right) + \\
+ \frac{1}{576 M^4 D_{uD33} M_{D33}^2} (16 M^2 (23 M^4 - 15 M^2 (s + t) + 2 (s + t)^2 + 4 M (3 M^2 - 2 (s + t)) \times \\
\times M_{D33} - (13 M^2 + s - 2 t) M_{D33}^2 - 8 M M_{D33}^3) G_{D33N}^1, \gamma^2 + 16 M (2 M (6 M^4 - 5 M^2 (s + \\
t) + (s + t)^2) + M^2 (-7 M^2 + 3 (s + t))) M_{D33} + 4 M^3 M_{D33}^2 + 3 (3 M^2 + s - t) M_{D33}^3 \times \\
\times G_{D33N}^1, \gamma^2 G_{D33N}^2 + (34 M^6 - 37 M^4 (s + t) + 12 M^2 (s + t)^2 - (s + t)^3 - 8 M (6 M^4 - \\
- 5 M^2 (s + t) + (s + t)^2) M_{D33} - 3 (43 M^4 - 5 s^2 + 6 s t + 11 t^2 - 2 M^2 (7 s + 23 t)) M_{D33}^2 - \\
- 24 M (3 M^2 - s - t) M_{D33}^3) G_{D33N}^2 \right),
\]

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\[
\frac{1}{576 M^4 D_{sD33} M_{D33}^2} (16 M^2 (M^5 - M^3 s + 2 (M^4 - 3 M^2 s + 2 s^2) M_{D33} + 3 M (-M^2 + s + t) M_{D33} + (4 M^2 - 4 s - 3 t) M_{D33}^3) G_{D33N\gamma}^1 - 8 M (M^2 - s)^2 (-s + 2 M M_{D33} + 7 M_{D33}^2) G_{D33N\gamma}^2 + (M^2 - s)^2 (M s - 2 s M_{D33} + 15 M M_{D33}^2 + 18 M_{D33}^3) \times G_{D33N\gamma}^2) - \\
\frac{1}{576 M^4 D_{aD33} M_{D33}^2} (16 M^2 (M^3 (M^2 - s - t) - 2 (3 M^4 - 5 M^2 (s + t) + 2 (s + t)^2) M_{D33} - 3 (M^3 - M s) M_{D33}^2 + (4 M^2 - 4 s - t) M_{D33}^3) G_{D33N\gamma}^1 - 8 M (-M^2 + s + t)^2 (2 M^2 - s - t - 2 M M_{D33} - 7 M_{D33}^2) G_{D33N\gamma}^2 - (M^2 + s + t)^2 (M (2 M^2 - s - t) + 2 (-2 M^2 + s + t) M_{D33} + 15 M M_{D33}^2 + 18 M_{D33}^3) G_{D33N\gamma}^2), \\
\frac{1}{576 M^4 D_{sD33} M_{D33}^2} (16 M^2 (M^4 - M^2 s + 2 s^2 + 4 (M^3 - 2 M s) M_{D33} + (-7 M^2 + s + 3 t) M_{D33} + 8 M M_{D33}^2) G_{D33N\gamma}^1 - 16 M (M^2 - s) M_{D33} (M^2 + 2 M M_{D33} - 3 M_{D33}^2) \times G_{D33N\gamma}^2 + (M^2 - s)^2 (s + 15 M_{D33}^2) G_{D33N\gamma}^2) - \\
\frac{1}{576 M^4 D_{aD33} M_{D33}^2} (16 M^2 (7 M^4 - 7 M^2 (s + t) + 2 (s + t)^2 - 4 M (3 M^2 - 2 (s + t)) M_{D33} - (5 M^2 + s - 2 t) M_{D33}^2 + 8 M M_{D33}^3) G_{D33N\gamma}^1 + 16 M (M^2 - s - t) M_{D33} (M^2 + 2 M M_{D33} - 3 M_{D33}^2) G_{D33N\gamma}^2 + (s + t - M^2)^2 (2 M^2 - s - t + 15 M_{D33}^2) G_{D33N\gamma}^2), \\
\frac{1}{288 M^4 D_{sD33} M_{D33}^2} (8 M^2 (2 M s (s - M^2) + 2 (M^4 - 3 M^2 s + 2 s^2) M_{D33} + 2 (M^3 - M s) M_{D33}^2 + (4 M^2 - 4 s + 3 t) M_{D33}^3) G_{D33N\gamma}^1 - 4 M (s (M^4 - s^2) + 2 M s (-M^2 + s) M_{D33} + (3 M^4 - 4 M^2 s + s (s + 6 t)) M_{D33}^2 + 6 (M^3 - M s) M_{D33}^3) G_{D33N\gamma}^2 + (M s^2 (s - s^2) M_{D33} + 9 M (M^2 - s) s M_{D33}^2 + 3 (M^4 - s^2) M_{D33}^3) G_{D33N\gamma}^2 + \\
+ \frac{1}{288 M^4 D_{aD33} M_{D33}^2} (8 M^2 (2 M (2 M^4 - 3 M^2 (s + t) + (s + t)^2) + 2 (3 M^4 - 5 M^2 (s + t) + 2 M (s + t)^2)^2 M_{D33} + 2 M (s + t - M^2) M_{D33}^2 + (4 M^2 - 4 s + 7 t) M_{D33}^3) G_{D33N\gamma}^1 + 4 M \times (6 M^6 - 11 M^4 (s + t) + 6 M^2 (s + t)^2 - (s + t)^3) - 2 M (2 M^4 - 3 M^2 (s + t) + (s + t)^2) M_{D33}^2 + 6 M (M^2 - s - t) M_{D33}^3) G_{D33N\gamma}^1 \times G_{D33N\gamma}^2 + \left(M (M^2 - s - t) (s + t - 2 M^2)^2 + (-6 M^6 + 11 M^4 (s + t) - 6 M^2 (s + t)^2) + (s + t)^2 M_{D33}^2 - 9 M (2 M^4 - 3 M^2 (s + t) + (s + t)^2) M_{D33}^2 - 3 (3 M^4 - 4 M^2 (s + t) + (s + t)^2) M_{D33}^3 \right) G_{D33N\gamma}^2),
\]
\[
\frac{1}{576 \, M^4 \, D_{sD33} \, M_{D33}^2} \left( 16 \, M^2 \, (M^4 + M^2 \, s - 2 \, s^2 + (-5 \, M^2 + 5 \, s + 3 \, t) \, M_{D33}^2) \, G_{D33N\gamma}^1 - 16 \, M \, (M \, s \, (-M^2 + s) + (M^4 - M^2 \, s) \, M_{D33} + (-M^3 + M \, s) \, M_{D33}^2 + 3 \, (-M^2 + s + t) \times M_{D33}^3 \right) \, G_{D33N\gamma}^1 \, G_{D33N\gamma}^2 + (s \, (M^4 - s^2) + 4 \, M \, s \, (-M^2 + s) \, M_{D33} - 9 \, (M^4 - s^2) \, M_{D33}^2 - 12 \, (M^3 - M \, s) \, M_{D33}^3 \right) \, G_{D33N\gamma}^2 \). \tag{49}
\]

In formulae (49) \( M_{D33} \) is \( D_{33} \) resonance mass, \( D_{sD33} = s - M_{D33}^2 + iM_{D33} \Gamma_{D33}(s) \), \( D_{uD33} = u - M_{D33}^2 + iM_{D33} \Gamma_{D33}(u) \) are denominators of \( M_{D33} \) resonance propagator in \( s \)- and \( u \)-channel correspondingly. Since resonances \( D_{33} \) and \( D_{13} \) have the same spin and parity \( \frac{3}{2}^- \) and differ only by isospins and masses, \( D_{13} \) contribution to invariant functions \( f_1 \) - \( f_6 \) in case of Compton scattering on proton may be gained from the corresponding formulae (49) for \( D_{33} \) by replacement:

\[
D_{sD33} \rightarrow D_{sD13}, \quad D_{uD33} \rightarrow D_{uD13}, \quad M_{D33} \rightarrow M_{D13}, \quad G_{D33N\gamma}^1 \rightarrow \sqrt{\frac{3}{2}} \, G_{D13\gamma\gamma}^1, \quad G_{D33N\gamma}^2 \rightarrow \sqrt{\frac{3}{2}} \, G_{D13\gamma\gamma}^2. \tag{50}
\]

The second line in (50) must be replaced by the following one for Compton scattering on neutron:

\[
G_{D33N\gamma}^1 \rightarrow \sqrt{\frac{3}{2}} \, G_{D13\gamma\gamma}^1, \quad G_{D33N\gamma}^2 \rightarrow \sqrt{\frac{3}{2}} \, G_{D13\gamma\gamma}^2. \tag{51}
\]

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