Impact the parameters of metal-inducer system on its electromagnetic measurands

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Abstract. In this work, we propose the finite-element-based mathematical model of the metal-inducer system. This system consists of a coil with put-on C-shaped ferromagnets. We study how the measurands of the inducer, such as the coil voltage and the uncompensated force acting on metal, depend on the system parameters. We variate the geometry of the system to estimate the boundary effect impact. Also, we study how the coil voltage depends on the ferromagnets' spacing and their number. The inhomogeneity of electric conductivity is modeled via holes in the metal volume. To estimate their impact, we defined the dependences of the measurands on the number and the size of inhomogeneities.

1. Introduction

Magnetohydrodynamic processes [1] are significant for many industries, such as metallurgy and nuclear power. Liquid metals often are very chemically aggressive. Due to that fact, one has to use non-contact methods of impact and characterization [2]. Also, for this reason, it is necessary to use channel geometry as smoothly as possible (often cylindrical).

As the metal flows through the channel, the crystallization process can begin. The electrical conductivity of the liquid phase and the solid phase can differ several times [3]. The appearance of inhomogeneity of electrical conductivity can lead to a change in the flow regime. Thus the estimation of the impact the inhomogeneities becomes an important task.

The inducer is one of the devices used to research processes in media with inhomogeneous conductivity. It consists of a coil with put-on C-shaped ferromagnets. The metal is placed in the center of the coil. Moreover, one can use the inducer for generating different processes in conductive media, such as stirring. It might help obtain metals with homogeneous properties at the mesolevel, which cannot be achieved using additive technologies, such as layer-by-layer deposition [4, 5]. The other application of the inducer is a non-contact characterization of processes in liquid metals by its electromagnetic measurands [6–10].

To pick up the metal-inducer system configuration that will allow us to research interest processes, we use the mathematical simulations. In an experimental study with inducer one can measure the coil voltage or the uncompensated force acting on metal. The coil voltage can depend a lot on the system geometry and the ferromagnets’ existence, whereas the uncompensated electromagnetic forces appear with inhomogeneous conductivity. Thus our research aims to estimate the impact of the metal-inducer system parameters on its experimental measurands.
The mathematical model described in the article can be used when working with much more complex installations of this type [11, 12]. However, this requires careful verification. Therefore, in the present paper, we consider the inhomogeneities that can be realized in an experiment with high accuracy. Inhomogeneities can be modeled as volumes inside the metal with conductivity that differs from the conductivity of the metal. As the simplest case, we model the inhomogeneities through holes in the metal volume.

2. Methods

The geometry of the system consisted of five regions - metal $V_{me}$, coil $V_{coil}$, air $V_{air}$, inhomogeneity $V_{inh}$ and C-shaped ferromagnets $V_{ferro}$.

![Figure 1. Geometry of the problem](image)

The conductivity in the region of metal with inhomogeneity was defined as

$$
\sigma (r) = \begin{cases} 
0, & r \in V_{inh} \\
\sigma_{me}, & r \in V_{me} 
\end{cases}
$$

(1)

Also we defined the permeability of the system as

$$
\mu (r) = \begin{cases} 
\mu_{ferro}, & r \in V_{ferro} \\
1, & r \notin V_{ferro} 
\end{cases}
$$

(2)

The problem statement was formulated in terms of the magnetic vector potential $A$ and the electric scalar potential $\varphi$, defined by the equations:

$$
B = \nabla \times A,
$$

(3)

$$
E = -\nabla \varphi - \frac{\partial A}{\partial t},
$$

(4)

where $B$ is the magnetic induction vector, $E$ is the electric field vector. By substitute these equations into the Ampere’s law $\nabla \times H = j$ and using the constitutive equations $D = \varepsilon \varepsilon_0 E$ and $H = B/\mu \mu_0$ we obtain

$$
\nabla \times \left( \frac{1}{\mu \mu_0} \nabla \times A \right) = j - \varepsilon \varepsilon_0 \frac{\partial^2 A}{\partial t^2} - \varepsilon \varepsilon_0 \nabla \frac{\partial \varphi}{\partial t}.
$$

(5)
Here $j$ is the electric current density, $\mathbf{H}$ is the magnetic field, $\mathbf{D}$ is the electric induction, $\varepsilon$ is the absolute permittivity, $\varepsilon_0$ is the vacuum permittivity and $\mu_0$ is the magnetic constant.

The current density consists of two parts – induced eddy currents density $j_e$ and source currents density $j_s$. In considered system the source current was an alternating current with a frequency of $f$ in the coil volume $V_{coil}$:

\begin{align}
 j_s &= \{0, j_\varphi, 0\}, \\
 j_\varphi &= j_0 e^{i2\pi ft}.
\end{align}

Here $\varphi$ denotes the azimuthal direction of a cylindrical coordinate system associated with the coil. For the coil consisted of $N_c$ turns with coil cross-section area $S_c$

\begin{equation}
 j_0 = \frac{N_c}{S_c} I,
\end{equation}

where $I$ is the amperage. The magnetic flux is defined as

\begin{equation}
 \Phi = \frac{N_c}{S_c} \int_{V_{coil}} \mathbf{A} \cdot \mathbf{e}_\varphi \, dV.
\end{equation}

Here $\mathbf{e}_\varphi$ is the unit vector in the azimuthal direction of the coil. The coil voltage is given by the equation

\begin{equation}
 U = RI - \frac{\partial \Phi}{\partial t},
\end{equation}

where $R$ is the total DC resistance of the coil winding.

The induced eddy currents density are defined by Faraday’s law of induction:

\begin{equation}
 j_e = -\sigma \frac{\partial \mathbf{A}}{\partial t}.
\end{equation}

The electromagnetic force acting on conductive medium is given by relation

\begin{equation}
 \mathbf{F} = j_e \times \mathbf{B}.
\end{equation}

The numerical simulations were carried out using the finite element method via the ANSYS Emag. In all calculations we set the amperage $I = 0.5 A$, $N_c = 133$, $\mu = 1000 \, \text{H/m}$. The size of mesh elements was 10 mm in $V_{air}$ and 4 mm in $V_{me}$, $V_{ferro}$, $V_{inh}$ and $V_{coil}$.

To estimate the influence of the geometry of the system and the boundary effects, we considered configurations with different heights of the metal $L_m$ and the coil $L_c$, as well as the aspect ratio $\Gamma_m = \frac{L_m}{2R_m}$, where $R_m$ is the radius of the cylindrical volume of metal (Figure 2). The size of the air region was chosen according to the geometrical parameters of the system, such as the height of the metal or the coil and the radius of the coil).

In the others series of simulations, we studied the effects of C-shaped ferromagnets and inhomogeneities with fixed geometric parameters of the metal and the coil. The geometric parameters were $L_m = 19 \, \text{mm}$, $R_m = 36 \, \text{mm}$, $L_c = 12 \, \text{mm}$. We variated the number of ferromagnets as well as the number and the radius of the inhomogeneities.

3. Results and discussion

3.1. The influence of the geometry

The figure 3 shows the plots of the relative coil voltage as a function of electrical conductivity of metal for various values of $\Gamma_m$ and frequencies of alternating current. We set $L_m = L_c$, and the radius of the coil was chosen according to the radius of metal. For large values of $\Gamma_m$ (so-called
Figure 2. Drawing of the metal-inducer system without C-shaped ferromagnets and inhomogeneities

![Diagram of the metal-inducer system without C-shaped ferromagnets and inhomogeneities](image)

Figure 3. The relative coil voltage as a function of electrical conductivity of metal: a) $f = 50$ Hz, b) $f = 100$ Hz

![Graphs showing relative coil voltage as a function of conductivity](image)

rod configurations), edge effects play an insignificant role, and a linear dependence is observed. Nonlinear behavior is observed at small values of the of $\Gamma_m$ (so-called tablet configurations). It can be associated with the fact that the induced magnetic field in the metal turns out to be substantially inhomogeneous. For high values of the frequency, the linear section becomes small, and the system demonstrates nonlinear behavior even at small values of $\sigma$.

In the second series of simulations, we estimated the influence of $\Gamma_m$ for a rod configurations with $L_m \gg L_c$. We set $L_c = 16.5$ cm and $R_m = 18$ mm. As it shown on the figure 4, starting with $\Gamma_m = 8$ the influence of boundary effects becomes negligible.
The coil voltage as a function of electrical conductivity of metal for rod configurations with $L_m \gg L_c$

3.2. The influence of the C-shaped ferromagnets

C-shaped ferromagnets changed the current density in the coil and in the metal. The figure 5 shows the plots of the relative coil voltage as a function of electrical conductivity of metal for various number of C-shaped ferromagnets.

We considered the dependence of the coil voltage on the spacing between C-shaped ferromagnets and the metal. As expected, a decrease in the spacing leads to an increase in the coil voltage. With sufficient spacing, the effect of C-shaped ferromagnets disappears. On the figure 6 vertical dashed lines indicate coil boundaries.

One way to obtain an uncompensated force is to change the location of the metal relative to the coil. We moved the metal along the radial axis with angular coordinate $\theta = 0$ in polar coordinate system $\{\rho, \theta\}$, associated with the coil. Also, we considered configurations with different numbers of C-shaped ferromagnets. The simulation results are shown at the figure 7. Here $d$ means the displacement of the metal volume center relative to the center of the coil. The coil voltage was normalized to the value at the undisplaced position.
Figure 6. The coil voltage as a function of the spacing between C-shaped ferromagnets and the metal

![Figure 6](image_url)

Figure 7. a) The relative coil voltage as a function of the displacement of the metal relative to the coil; b) The value of the modulus of the force acting on the metal as a function of the displacement of the metal relative to the coil

![Figure 7](image_url)

The dissimilarity of the results at \( N = 1 \) is explained by the fact that this configuration is essentially asymmetric. It should be noted that the force about \( 10^{-5} \) N can be measured experimentally.

3.3. The influence of the inhomogeneities in metal

As shown earlier, the most significant response should be expected from the system with the highest symmetry breaking. For this reason, we considered configurations with one inhomogeneity in the metal. The orientation of the metal was specified by the position of the inhomogeneity center in the polar coordinate system, associated with the coil. As expected, the impact of the inhomogeneity rotation angle relative to the coil on coil voltage is negligible. Figure 8 represents the dependence of the force module acting on metal on the orientation of the inhomogeneity.

The value of the force at \( N = 1 \) is due to the influence of the ferromagnet. In general, we can
Figure 8. The value of the modulus of the force acting on the metal as a function of the angle of orientation of the inhomogeneity

say that the orientation of the inhomogeneity gives the least influence to the system response compared to others.

Figure 9. a) The relative coil voltage as a function of the diameter of the inhomogeneity; b) The value of the modulus of the force acting on the metal as a function of diameter of the inhomogeneity

The figure 9 shows the plots of the coil voltage and the force acting on metal as a function of the diameter of the inhomogeneity. Note that the dependence of the relative coil voltage increases monotonically, while the dependence of the force has a local maximum.

4. Conclusion
In our work, we have studied the impact of the parameters of the metal-inducer system on its electromagnetic measurands via a finite-element-based mathematical model. The simulations were carried out with ANSYS Emag. We have shown that so-called rod configurations give an
almost linear dependence of the coil voltage on the electric conductivity of metal at moderate
alternating current frequencies. Also, it should be noted that sufficient electromagnetic force
appears only in the most asymmetric case. The defined dependencies will be used for creating
the experimental setup for verifying the numerical approach.

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