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STUDY OF THE THREE-NUCLEON (e,e’) LONGITUDINAL RESPONSE FUNCTION WITH A NEW APPROACH

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A new method for studying the many-body response functions is elaborated and first applied to the $^3$He longitudinal response. An integral transform of the response function is calculated from the bound-state-type equations for several versions of the N-N force. The equations are solved with the help of the hyperspherical expansion. The final-state interaction is completely taken into account. The results are compared with the integral transform of the experimental response function for $250 \text{ MeV/c} \leq q \leq 500 \text{ MeV/c}$. The difference amounts to 20-25% with the experimental response substantially exceeding the theoretical one in the low-energy region.

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Many calculations testify to the fact that the conventional form of the nuclear charge density is inapplicable to the description of the elastic form factors of three- and four-nucleon nuclei at \( q > 2.5 \, \text{fm}^{-1} \) values. However the elastic scattering occurs with a quite low probability at such \( q \) values and some non-typical nucleon configurations including those where all the nucleons are close together may make a substantial contribution. In this connection it seems important to test a form of the nuclear 4-current in the inelastic processes and to study the \((e,e')\) response functions. This requires a proper account of the nuclear final-state interaction.

In this paper we present a microscopical analysis of the \( ^3\text{He} \) longitudinal response function \( R_l \) with a full account of the final-state interaction. The conventional approach calculates the response functions directly from the definition

\[
R(q,\omega) = \sum_{M_0} \int df | \langle \psi_f | 0 \rangle |^2 \delta(E_f - E_0 - \epsilon)
\]

and it requires obtaining the whole set of the complicated final-state continuum wave functions \( \psi_f \). (Here \( \epsilon \approx \omega - q^2/(2AM) \) is the nuclear excitation energy.) Therefore some model approximations always were used in such a calculation. We apply a new approach [1-3] which enables us to avoid calculating \( \psi_f \) and thereby to obtain accurate results once the underlying nuclear dynamics is specified.

Define the reduced transition operator and the response function

\[
\tilde{O} = [\tilde{G}^E_p(Q)]^{-1} \tilde{O}, \quad \tilde{R}(q,\omega) = [\tilde{G}^E_p(Q)]^{-2} R(q,\omega).
\]

Here \( Q^2 = q^2 - \omega^2 \) and \[4\] \( \tilde{G}^E_p(Q) = [1 + Q^2/(4M^2)]^{-1/2} G^E_p(Q) \) where \( G^E_p \) is the proton Sachs form factor. We calculate the integral transform of the response

\[
\Phi(q,\sigma) = \int_{\epsilon_{\text{min}}}^{\infty} (\sigma + \epsilon)^{-1} \tilde{R}(q,\omega) d\epsilon
\]

instead of the response itself. We use the conventional single-nucleon expression for the charge density \( \tilde{O} \). Then to a very good approximation one can disregard the \( \omega \)-dependence of the \( \tilde{O} \) operator and use the expression

\[
\tilde{O}(q) = \sum_{n=1}^{A} \left[ \frac{1 - \tau_{zn}}{2} + \frac{G^E_n(q)}{G^E_p(q)} \frac{1 + \tau_{zn}}{2} \right] e^{iq \rho_n}
\]

1
where \( \rho_n = r_n - R_{c.m.} \). It has been shown [1,5] that \( \Phi(q, \sigma) \) can be calculated by first solving for the localized solution to the following inhomogeneous equation

\[
(H - E_0 + \sigma) \tilde{\Psi} = \tilde{O}\psi_0
\]  

(4)

where as in Eq. (1) \( \psi_0 \) is the ground-state wave function and \( E_0 \) is the ground-state energy. In terms of \( \tilde{\Psi} \) we have [1,5]

\[
\Phi(q, \sigma) = \langle \tilde{\Psi} | \tilde{O}\psi_0 > - \sigma^{-1} \tilde{R}_{el}
\]  

(5)

where \( \tilde{R}_0 \) is the elastic contribution to the response.

The solution to Eq. (4) is much easier to obtain than the functions \( \psi_f \) entering Eq. (1). Indeed, in contrast to the latter functions there is no need to impose the complicated large-distance boundary conditions in order to get the solution. Only the condition that the solution vanishes at large distances is needed in solving Eq. (4). Therefore methods that are used in solving bound-state problems can be utilized here. In particular for many-body systems Monte-Carlo Green functions technique can be applied.

We have two possible ways to connect our theoretical calculations with experimental measurements. One way [5] is to compare \( \Phi(q, \sigma) \) with the same quantity obtained from the experimental \( \tilde{R}(q, \epsilon) \) using Eq. (2). Another way [1] is to consider Eq. (2) as the integral transform and invert to obtain theoretical \( \tilde{R}(q, \omega) \) and then compare the responses themselves. In the present work we use the first approach. It is worth mentioning that there exists a generalization [1] of this method to exclusive reactions including \( 2 \rightarrow N \) reactions induced by strong interaction.

In this first calculation we use effective central \( N - N \) forces [6-8] which are supposed to act in the s-wave. We supplement them with a realistic singlet \( p \)-wave \( N - N \) force [9]. Its contribution is 4% at the lowest \( q \) values and it is negligible at the highest \( q \) values. The contribution from the triplet \( p \)-wave force is believed to be of the same size. Only the components of the proton-proton Coulomb interaction which are diagonal in the isospin \( T = 1/2, 3/2 \) quantum numbers are retained in the calculation. Even these components change the results at most by 3% at the lowest \( q \) values considered.

Under these assumptions on the nuclear dynamics, Eq. (4) is split into independent sets of equations with a given orbital momentum \( L \), and isospin \( T \) of the system. It is convenient to calculate the right-hand sides of these equations in the following way. Since \( \psi_0 \) has \( L = 0 \) then only the components
$\sim Y_{LM}(\hat{\rho}_n)$ from the expansion of $exp(iq\rho_n)$ from Eq. (3) contribute to the problem for a given $L$ value. Let $q$ be directed along the $z$ axis. Then only the components with $M_L = 0$ give non-zero contributions and hence only the components of $\tilde{\Psi}$ with $M_L = 0$ are different from zero. We have

$$\tilde{O}_{LT} = T \sum_{n=1}^{3} \left[ \frac{1 - \tau_{xn}}{2} + \frac{G_n^E(q)}{G_p^E(q)} \frac{1 + \tau_{xn}}{2} \right] j_L(q\rho_n) Y_{L0}(\hat{\rho}_n),$$

$$\Phi(q, \sigma) = 4\pi \sum_{L=0}^{\infty} (2L + 1) \sum_{T=1/2, 3/2} \Phi_{LT}(q, \sigma).$$

The functions $\tilde{\Psi}_{LT}$ have the same spin $S = 1/2$ as $\psi_0$.

We solve Eqs. (6) by an expansion in the hyperspherical harmonics. Denote $K$ the hyperspherical momentum and $[f]$ the type of symmetry of the spatial components of the basis functions. We have developed a computer code to construct complete sets of the basis functions with arbitrary $K, L, S, T$ and $[f]$ values using the Raynal-Revai transformation [10]. The coefficients of this transformation are evaluated using the recurrent formula of the $K \to K + 2$ type [11]. Although the net number of the basis functions with the same $K, L, S, T$ and $[f]$ values grows linearly with $K$ there exists a possibility indicated in Ref. 12 to specify the basis states in such a way that only two of them contribute to the problem in our case if one disregards the Coulomb interaction. It is because the nuclear forces that we use here only act in two N-N orbital states. Only such states are retained in our calculation.

We make a comment concerning calculation of $\Phi$ at small $\sigma$ values. It is necessary to avoid large cancellations in the right-hand side of Eq. (3). This is achieved if one uses in Eq. (3) for $L = 0, T = 1/2$ the same $K_{\text{max}}$ value as that at calculating $\psi_0$. Then the pole terms cancel exactly.

There exists a test which enables us to check the calculation as a whole. Namely, the leading term of $\Phi(q, \sigma)$ at high $\sigma$ values behaves as $\sigma^{-1}$. This term can be calculated independently from the sum rule,

$$\lim_{\sigma \to \infty} \sigma \Phi(q, \sigma) = \int_{\epsilon_{\text{min}}}^{\infty} \tilde{R}(q, \omega) d\epsilon = \sum_{M_0} <\psi_0 | \tilde{O}^\dagger \tilde{O} | \psi_0 > - \tilde{R}_{el}. \quad (10)$$
This allows one to check the right-hand side of Eq. (9). Besides the correctness of the calculation the test allows one to verify whether at high $\sigma$ values the results are stable against increasing $K_{max}$ and $L_{max}$.

The calculations are performed at $q = 250$, $400$ and $500$ MeV/c. The required accuracy of the calculated $\Phi$ is determined by the accuracy of $\Phi$ extracted from experimental data. The latter is predominantly determined by systematic errors of the data and it is typically [13] about several per cent. We use $L_{max} = 10$ while $K_{max} = 20$ for $L = 0$ and $K_{max} = 10$ for other $L$ values. We have verified that these values are sufficient for our results to converge within the experimental accuracy.

Let us first discuss the relative contributions from various $L$ and $T$ values to the right-hand side of Eq. (9). The relative contributions of lower $L$ values increase as $q$ decreases in accordance with Eq. (7). For example, at $\sigma \to \infty$ the $L = 0$ contribution dominates at $q = 250$ MeV/c contributing about 60% to the net sum while at $q = 500$ MeV/c the $L = 2$ contribution dominates with the $L = 0$ component contributing only about 12%. Besides, the relative $L = 0$ contribution proves to increase as $\sigma$ decreases. At $\sigma = 1$ MeV and $q = 500$ MeV/c the $L = 0$ component contributes more than 70%. At $\sigma = 1$ MeV and $q = 250$ MeV/c it contributes more than 98%. The reason probably is as follows. At small $\sigma$ values the values of $\tilde{R}$ with low excitation energies enter the integral from Eq. (2) with the highest weights. At low excitation energies the hyperspherical centrifugal barrier hinders particles from being inside the reaction zone. Such a barrier is absent when $K = 0$, which value exists only for the $L = 0$ component. The effect is more pronounced at lower $q$ values since the spectrum is shifted to lower energies.

The $T = 3/2$ contribution proves to be suppressed by one or two orders relative to the $T = 1/2$ contribution at $L = 0$. The reason is that $K = 0$ is forbidden for $T = 3/2$. Indeed, the $K = 0$ basis state is symmetrical under particle interchange. But the symmetrical spatial components of the final-state wave functions appear at $T = 1/2$ only. For higher $L$ values the $T = 3/2$ contributions are typically several times smaller than the $T = 1/2$ contributions as well. This may be due to the fact that the spatially symmetrical final-state components provide more inter-particle attraction that increases the amplitudes of the final-state wave functions inside the reaction zone.

We compare our results with $\Phi(q, \sigma)$ obtained from $^3$He experimental
responses [13] using Eq. (2). We note that at $q \geq 550$ MeV/c the longitudinal data of Ref. 13 become less accurate and, besides, the sums over the data greatly exceed the theoretical sum rule values (see Eq. (10)). We therefore only consider the lower $q$ values. In order to perform the integration in Eq. (2) with a sufficient accuracy and in particular to estimate the contribution from the unavailable high-$\epsilon$ tails of the spectra we approximated the spectra by the following analytical expressions:

\[ a(\omega - \omega_{\text{thresh}})^{1/2} \] in the low $\omega$ region $\omega_{\text{thresh}} \leq \omega \leq \omega_1$,

\[ \sum_{n=0}^{N_b} b_n \omega^n \] in the region of the peak $\omega_1 \leq \omega \leq \omega_2$ and

\[ \sum_{n=0}^{N_c} c_n \omega^{-(\alpha+n)} \] in the region beyond the peak. The parameters $a, b_n, c_n, \alpha$ and $\omega_2$ are chosen from the least-square procedure. Additional requirements of continuity of the fitting spectra and their first derivatives at $\omega_1$ and $\omega_2$ points are imposed. It turns out that a good description at a quite wide ranges $\omega_2 \leq \omega \leq \omega_{\text{max}}$ of the spectra beyond the peaks are provided with a single term $\sim \omega^{-\alpha}$. (In case, say, exponentially decreasing tail-terms the description is worse.) The best $\alpha$ values range between about 4 and 5 for all the $q$ values considered. Similar $\alpha$ values were found [14] in the $^4\text{He}$ case. We extrapolate the fits obtained beyond the $\omega_{\text{max}}$ values in order to take into account the contributions from the unavailable tails of the spectra. These contributions prove to be quite small in our case. They reach the maxima at high $\sigma$ values where they are between 1 and 2%.

Four versions of central $N - N$ forces acting in the $s$-wave are used in our calculation. These include the S2 and S3 potentials from Ref. 6, the MT(I+III) potential from Ref. 7 and the EH potential from Ref. 8. The S2 potential and the MT(I+III) potential reproduce the $N - N$ low-energy properties and $s$-wave $N - N$ phases up to high energies. The S3 potential fits the low-energy data and yields nearly correct values for the binding energies and rms radii of $^3\text{He}$ and $^4\text{He}$. The EH potential fits the $s$-wave $N - N$ phases up to high energies as well but it does not reproduce properly the low-energy $N - N$ data (see Ref. 6).

The results of our calculation for $q=250$ MeV/c and 500 MeV/c are shown in Fig. 1 and 2 respectively. For $q=400$ MeV/c the results are similar. We plot the quantity $[\Phi_{\text{theor}}(q,\sigma) - \Phi_{\text{exp}}(q,\sigma)]/\Phi_{\text{exp}}(q,\sigma)$ against $\sigma$. At the lowest $\sigma$ values this quantity amounts to -25% for the S2 potential and -21% for the MT(I+III) potential in case of $q = 250$ MeV/c and -(22-23)% for both potentials in case of $q = 500$ MeV/c. For the more phenomenological S3 potential it equals -17% at $q = 250$ MeV/c and -24% at $q = 500$ MeV/c for the same $\sigma$ values. For the EH potential the corresponding numbers are -8%
and -20\%. (The drawback of the EH force in reproducing the $N - N$ data is pointed out above.) At higher $\sigma$ values the differences between $\Phi_{\text{exp}}$ and $\Phi_{\text{theor}}$ monotonically decrease and they do not exceed 6.5\% in their absolute values for all the potentials as $\sigma$ goes to infinity. (At $q = 250\text{ MeV}/c$ they are about 2\% in cases of S2 and MT potentials. The differences in 6\% in case of $q = 500\text{ MeV}/c$ are perhaps connected in part with some systematic errors of the data at high $q$ values, cf. above.) The results obtained indicate that the theoretical response functions are lower than the experimental ones at the low-energy wings. The deviation from experiment obtained may be caused by some deficiencies of the adopted conventional description of the nuclear states or, in principle, by some deficiencies of the conventional nuclear charge density operator. Further investigations are planned in this connection. In any case the results presented here show that a microscopic study of the nuclear $(e, e')$ response functions with the final-state interaction fully taken into account is accessible and fruitful.

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FIG. 1. The relative deviation of the calculated transformant $\Phi_{\text{theor}}(q, \sigma)$ of the longitudinal $^3$He response function (see Eq. (2)) from that obtained from the experimental data [13]. 1-S2 potential, 2-MT(I+III) potential, 3-S3 potential, 4-EH potential (see the text). The $q$ value is 250 MeV/c.

FIG. 2. Same as Fig. 1 but for $q = 500$ MeV/c.