Dynamic stability and vibrations of thin-walled structures considering heredity properties of the material

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Abstract. Dynamic stability and vibrations of thin-walled structures, taking into account the heredity properties of the material are considered in the paper. Mathematical models were constructed in a two-dimensional statement, using the Karman theory of plane plate strains and A.A. Ilyushin aerodynamic theory. When realizing the physico-mechanical properties of the material of the object, the systems of integro-differential equations (IDE) in partial derivatives with the corresponding initial and boundary conditions are taken as mathematical models of the problems under consideration. The obtained nonlinear partial IDEs using the Bubnov-Galerkin method under considered boundary conditions are reduced to solving systems of nonlinear ordinary IDEs with constant or variable coefficients with respect to the time function. The integration of equations obtained using the polynomial approximation of the deflections was carried out by a numerical method based on the use of quadrature formulas. Based on this method, an algorithm for the numerical solution of the problem was developed suitable for all viscoelastic and elastic elements of thin-walled structures of plate type.

1. Introduction

The use of new composite materials in engineering practice, the design and creation of stable, lightweight and reliable structures require the improvement and development of mathematical models of deformable bodies and their calculation, taking into account the real properties of composite materials. As shown by numerous experimental and fundamental studies, most of composite materials have pronounced heredity properties [1-5, 12-20] and must be inhomogeneous [6].

The development and application of composite materials is currently one of the priority areas of scientific and technological progress. The use of composite materials in engineering structures allows not only to significantly improve their operational characteristics, but also in some cases to create structures that are not feasible within the framework of traditional materials. At the same time, the procedure of calculating and designing the structures built of composite materials that require consideration of their real properties is quite complicated. Therefore, the development of effective methods and algorithms for solving nonlinear problems on the stability of thin-walled structures made of composite materials is by far the most urgent problem.

As is well known, an account for heredity effects of deformable materials is of great theoretical and applied value, it guarantees the approximation of the theory of heredity to actual conditions. Therefore, the problems of heredity theory attract a lot of attention of researchers. There is a significant number of publications devoted to solving the problems of calculating thin-walled structures taking into account the heredity properties [7-10, 15].
In [1-3] V.D. Potapov in his study of the stability of integro-differential equations in aerodynamics problems used the Lyapunov exponential method [11].

Nonlinear dynamics problems based on the Kirchhoff-Love hypothesis taking into account the aerodynamic load were studied in [14–16, 21, 22–27].

The present work is devoted to solving the oscillatory processes of viscoelastic thin-walled structures taking into account the aerodynamic load.

2. Statement of the problem

Consider the nonlinear problem of a flutter of viscoelastic plate. Let a plate with sides \( a \) and \( b \) and thickness \( h \) hinge-supported along the entire contour, be flowed around on one side with a supersonic gas flow. Under the assumption adopted in [14, 15], the equation of a viscoelastic plate vibration has the form:

\[
\frac{D}{h} \left( 1 - R^2 \right) \dddot{W} = L(W, \Phi) - \rho \dddot{W} - B \frac{\partial W}{\partial t} + BV \frac{\partial W}{\partial x} - B_1V^2 \left( \frac{\partial W}{\partial x} \right)^2,
\]

\[
\frac{1}{E} \dddot{\Phi} = -\left( 1 - R^2 \right)^{1/2} L(W, W).
\]

The expression approximating the deflection is chosen as follows

\[
W(x, y, t) = \sum_{n=1}^{N} \sum_{m=1}^{L} W_{nm} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}.
\]

Substituting (2) into the right-hand side of the second equation (1), we obtain a linear equation with respect to the force function \( \Phi(x, y, t) \) (time is considered as a parameter in this case). Its general solution, as usual, contains arbitrary functions, using which it is possible to satisfy the boundary conditions for the function \( \Phi(x, y, t) \). However, the operating conditions of the shell together with the reinforcing elements are usually so complex that the exact formulation of the boundary conditions for \( \Phi(x, y, t) \) is devoid of practical meaning. It is enough to formulate these conditions “on average”, characterizing the shell interaction with surrounding structure by some averaged parameters.

The force function is represented approximately in the form:

\[
\Phi(x, y, t) = E \sum_{i,j=1}^{N} \sum_{k=1}^{L} (1 - R^2) W_{ik} W_{jk} \ast
\]

\[
\ast \left[ C_{ij} \ast \cos \frac{(i + j) \pi x}{a} \cos \frac{(r + s) \pi y}{b} + A_{ij} \cos \frac{(i + j) \pi x}{a} \cos \frac{(r - s) \pi y}{b} + D_{ij} \cos \frac{(i - j) \pi x}{a} \cos \frac{(r + s) \pi y}{b} + B_{ij} \cos \frac{(i - j) \pi x}{a} \cos \frac{(r - s) \pi y}{b} \right],
\]

where

\[
C_{ij} = \frac{\lambda^2 \pi r (ir - js)}{4((i + j)^2 + \lambda^2 (r - s)^2)} , \quad B_{ij} = \frac{\lambda^2 \pi r (ir - js)}{4((i - j)^2 + \lambda^2 (r - s)^2)},
\]

\[
A_{ij} = \frac{\lambda^2 \pi r (ir + js)}{4((i + j)^2 + \lambda^2 (r - s)^2)} , \quad D_{ij} = \frac{\lambda^2 \pi r (ir + js)}{4((i - j)^2 + \lambda^2 (r - s)^2)}, \quad \lambda = \frac{a}{b},
\]

Satisfying the conditions:

\[
\left. \frac{\partial^2 \Phi}{\partial y^2} \right|_{y=0} dy = \left. \frac{\partial^2 \Phi}{\partial y^2} \right|_{y=L} dy = 0.
\]
Expressions (2) and (3) are substituted into the first expression of equations (1) and, performing the Bubnov-Galerkin procedure for determining \( W_{kl} \), the following system of nonlinear IDEs is obtained:

\[
\frac{Eh^2\pi^2}{b^4}\left\{\frac{\pi^2}{12(1-\mu^2)}\left[k^2/\lambda^2+f^2\right]^{\frac{2}{3}}\right\}\left(1-R^2\right)W_{kl} = -\frac{Eh^2\pi^2}{b^4}\sum_{a,i,j,l}^N \sum_{m,r,s=1}^1 a_{kmnirj} W_{mn} \left(1-R^2\right)W_{ul}W_{pl} - \rho \frac{W_{kl}}{h} \frac{BV}{ha} - \frac{2BV}{ha} \sum_{n=1}^N \gamma_{kn} W_{el} - \frac{B_i V^2}{ha} \sum_{n=1}^N \sum_{m,r,s=1}^1 \Gamma_{kmnirj} W_{mn} W_{pl},
\]

where

\[
\gamma_{kn} = n\left(\gamma_{n+k} - \gamma_{n-k}\right) ; \quad \Gamma_{knm} = n_i \left(\gamma_{n+i+k} - \gamma_{n+i-k} + \gamma_{n-i+k} - \gamma_{n-i-k}\right).
\]

\[
\alpha_s = \begin{cases} 1, & \text{if } k\text{-odd;} \\ 0, & \text{if } k = 0 \text{ if } k\text{-even;} \end{cases} \quad \gamma_s = \begin{cases} \alpha_k, & \text{if } k\text{-odd;} \\ 0, & \text{if } k = 0 \text{ if } k\text{-even;} \end{cases}
\]

\[
a_{kmnirj} = -\frac{\pi^2}{4\lambda^2}\left[n^2\left[(r+s)^2 C_{irjs} \delta_{1nkj} \delta_{1mlrs} + (r-s)^2 A_{irjs} \delta_{1nkj} \delta_{2mlrs} + (r+s)^2 \delta_{1nkj} \delta_{1mlrs} + (r-s)^2 \delta_{2nkj} \delta_{1mlrs} + \right] + m^2\left[(i+j)^2 C_{irjs} \delta_{1nkj} \delta_{1mlrs} + (i+j)^2 A_{irjs} \delta_{1nkj} \delta_{2mlrs} + (i-j)^2 D_{irjs} \delta_{2nkj} \delta_{1mlrs} + (i-j)^2 B_{irjs} \delta_{2nkj} \delta_{2mlrs} + \right] - 2nm\left[(i+j)(r+s) C_{irjs} \delta_{3nkj} \delta_{2mlrs} + (i+j)(r-s) A_{irjs} \delta_{3nkj} \delta_{4mlrs} + \right] + (i-j)(r+s) D_{irjs} \delta_{3nkj} \delta_{3mlrs} + \right]\right];
\]

\[
\delta_{1nkj} = \delta_{n-k+i-j} + \delta_{n-k+i+j} - \delta_{n+k-i-j} - \delta_{n+k+i+j}; \quad \delta_{2nkj} = \delta_{n-k+i-j} + \delta_{n-k+i+j} - \delta_{n+k-i-j} - \delta_{n+k+i+j}; \quad \delta_{3nkj} = \delta_{n-k+i-j} + \delta_{n+k-i-j} - \delta_{n-k+i+j} - \delta_{n+k+i+j}; \quad \delta_{4nkj} = \delta_{n-k+i-j} - \delta_{n+k-i-j} + \delta_{n-k+i+j} - \delta_{n+k+i+j}.
\]

After applying the Bubnov-Galerkin method, the problem under consideration is reduced to solving a system of nonlinear IDEs (1.33). Introducing into IDE (5) the following dimensionless quantities

\[
\frac{W}{h}, \quad \frac{V_{a,t}}{a}, \quad \frac{a}{V_{\infty}} R(t),
\]

while maintaining the previous notation, we have
\[ \ddot{W}_{kl} + \lambda^2 \Omega^2 \left[ \left( \frac{k}{\lambda} \right)^2 + I^2 \right] (1 - R^*) W_{kl} + \frac{12 \lambda^4 (1 - \mu^2) \Omega^2}{\pi^2} \times \]
\[ \times \sum_{n,l,j=1}^{\infty} \sum_{m,r,s=1}^{\infty} a_{klmnirs} W_{nm} (1 - R^*) W_{ir} W_{js} + M \dot{W}_{kl} - 
- 2M M^* \sum_{n=1}^{\infty} \gamma_{kl} W_{nl} + M_1 M^2 \sum_{m,i}^{\infty} \sum_{m,r}^{\infty} \Gamma_{klmnir} W_{nm} W_{ir} = 0, \]

where \( \Omega^2 = \frac{\pi^4}{12(1 - \mu^2)} M_2^2 \left( \frac{h}{a} \right)^2 \), \( M = \infty M_p^2 \left( \frac{a}{h} \right) \), and \( M_1 = \infty \left( \alpha + 1 \right) \frac{M_2^2}{4} \); 
\( M^* = \frac{V}{V_\infty} \) is the Mach number; \( M_E = \sqrt{\frac{E}{\rho V_\infty^2}} \); \( M_p = \sqrt{\frac{P_\infty}{\rho V_\infty^2}} \); \( \lambda = \frac{a}{b} \); \( \gamma_k \) and \( \gamma_{kl} \) are the dimensionless coefficients [25-27].

3. Solution method
Integration of system (6) was carried out by the numerical method proposed in [14 - 17]. To do this, it is written in integral form, then the formula for numerical integration with the Koltunov - Rzhanitsyn kernel \( R(t) = A \cdot \exp(-\beta t) \cdot t^{\alpha - 1} \) takes the form:

\[ W_{kl}(0) = W_{0kl}, \quad W_{kl}(0) = W_{0kl}, \quad k = \overline{1,N}; \quad I = \overline{1,L}, \]

where
\[ W_{kl}(0) = \frac{1}{1 + \alpha M} \left( W_{0kl} + \left( W_{0kl} + MW_{0ld} \right) I_i - \sum_{j=0}^{L-1} A_j \left( M W_{jkl} - \left( t_j - t_i \right) \right) \right) \]
\[ - \left( t_i - t_j \right) \sum_{n=1}^{N} \gamma_{kn} W_{nml} - \lambda^4 \Omega^2 \left[ \left( \frac{k}{\lambda} \right)^2 + I^2 \right] \left( W_{jkl} - \frac{A}{\alpha} \right) \times \]
\[ \times \sum_{s=0}^{L} B_s e^{-\beta s} W_{jkl} - \frac{12 \lambda^4 (1 - \mu^2) \Omega^2}{\pi^2} \sum_{n,l,j=1}^{\infty} \sum_{m,r}^{\infty} a_{klmnirs} W_{nm} \times \]
\[ \frac{1}{\alpha} \sum_{x=0}^{L} B_x e^{-\beta x} W_{jkl} - \frac{A}{\alpha} \sum_{x=0}^{L} B_x e^{-\beta x} W_{jkl} \right) \] 
\[ - M_1 M^2 \sum_{n,k,l=1}^{\infty} \sum_{i,j=1}^{\infty} \Gamma_{klmnir} W_{jkl} W_{jir} \]

\( i = 1, 2, \ldots; \quad n = \overline{1,N}; \quad m = \overline{1,L} \); where \( A_j, B_s \) are the numerical coefficients that are independent of the choice of integrands and take different values depending on the quadrature formulas used.

4. Discussion of results
The calculation results are presented in the table. Based on formula (7), the critical flutter velocity of viscoelastic plates was determined.
The table shows the critical values of flutter velocity depending on the physicomechanical and geometrical characteristics of a plate.

As a criterion that determines the critical velocity of flutter, a condition was accepted that at these velocities the vibration amplitude changes according to a harmonic law. At velocities higher than the critical ones, there occurs an oscillatory motion with rapidly increasing amplitudes, which can lead to structure destruction. In the case \( V < V_{cr} \), the oscillation amplitude damps.

As seen from the analysis of results given in the table, the values of the coefficient \( V_{cr} \) turn out to be 854.15 m/s in elastic case (A = 0) and 753 m/s in viscoelastic case (A = 0.1). Thus, the viscoelastic properties of the material lead to a decrease in flutter velocity.

**Table 1. Dependence of critical flow rate on physico-mechanical and geometrical parameters of the plate**

| A     | \( \alpha \) | \( \beta \) | \( a/h \) | \( M_E \) | \( M_p \) | \( M_{cr} \) | \( V_{cr} \) (m/s) |
|-------|-------------|-------------|----------|---------|---------|--------|-----------------|
| 0.001 | 0.25        | 0.05        | 400      | 14.893  | 0.0106  | 2.512  | 854.15          |
| 0.04  | 0.25        | 0.01        | 400      | 14.893  | 0.0106  | 2.241  | 762             |
| 0.1   | 0.5         | 0.05        | 400      | 14.893  | 0.0106  | 2.157  | 753             |
| 0.1   | 0.25        | 0.1         | 400      | 14.893  | 0.0106  | 2.157  | 746.2           |
| 0.1   | 0.25        | 0.05        | 400      | 14.893  | 0.0106  | 2.157  | 740             |
| 0.05  | 0.25        | 0.05        | 350      | 14.893  | 0.0106  | 1.544  | 525             |
|       |             |             | 450      |         |         | 1.279  |                 |
|       |             |             | 480      |         |         | 1.279  |                 |
|       |             |             | 753      |         |         | 3.224  | 1130            |

Note a significant increase in \( V_{cr} \) for the values of parameter \( \alpha \). For \( \alpha = 0.1; 0.75 \) the critical flutter velocity determined by formula (7) is 624 m/s and 863.5 m/s, respectively, and differs from each other by 38.4%. In calculations, the following constant values are accepted: \( \mu = 0.32; \ P_\infty =1.014 \text{ kg/cm}^2 \).

The influence of parameter \( a/h \) on plate behavior was investigated. At \( a/h = 350 \), \( a/h = 450 \), and \( a/h = 480 \) the critical flutter velocity is 1130 m/s; 525 m/s and 435 m/s, respectively. A slight decrease in the plate thickness \( h = a/450 \) leads to a sharp change in oscillation pattern: we have a flutter motion with rapidly increasing amplitudes at a velocity of \( V = 550 \text{ m/s} \).

Changes in dimensionless rigidity parameter and pressure parameter play an important role in behavior of a thin-walled structure with heredity material properties. Studies show that an increase in these parameters leads to an intensive increase in critical velocity.

5. Conclusion

In conclusion, note that the influence of the viscosity parameters \( A \) and the singularity parameter \( \alpha \) plays a dominant role not only on viscoelastic systems vibrations, but also on the values of critical flutter velocity, when compared to rheological parameters \( \beta \) of the heredity kernel. When studying nonlinear problems, a number of new effects were obtained: an account for heredity properties of the material of thin-walled structures leads to a decrease in critical flow rate; an account for geometrical nonlinearity leads to an increase in critical velocity.
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