What is inside the nucleon?

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Abstract

We briefly review the structure of nucleon in the context of QCD, Constituent Quark Model and Chiral Quark Model.

1 Introduction

The quest to peep into the successive layers of structure of matter has led us from molecules to atoms and from atoms to subatomic particles and so on. This quest, after painstaking efforts by experimentalists and theoreticians together, in the present context has yielded a coherent understanding of matter at the level of $10^{-18} \text{m}$. At the present stage of scrutiny, the fundamental constituents of matter are quarks and leptons interacting through gauge bosons. There are six quarks, set up in well separated three generations, for example,

$$\text{Quarks} : \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}. \quad (1)$$

This pattern is repeated for the leptons, each generation containing a charged lepton and a corresponding neutrino, for example,

$$\text{Leptons} : \begin{pmatrix} e^- \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}. \quad (2)$$

At the present level of our understanding, quarks and leptons are structureless objects having definite quantum numbers and members of both categories carry spin half. All matter, have to be made up of quarks, leptons and the corresponding antiparticles. Unlike leptons, quarks have been “seen”
only inside the hadrons. The theory describing the interaction of these fundamental particles is the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory called the minimal Standard Model (MSM). The MSM has two distinct parts: ‘Quantum Chromodynamics’ (QCD) [1, 2, 3], described by the gauge group $SU(3)_C$ and ‘Electroweak Model’ [4, 5, 6] described by $SU(2)_L \times U(1)_Y$. The electroweak model describes all possible electromagnetic and weak processes, with all the electromagnetic interactions mediated by photons and the weak processes, on the other hand, mediated by three massive vector bosons, two charged ($W^\pm$) and one neutral ($Z^0$). The basic electromagnetic interaction is characterized by the vertex given in Figure 1 where a charged fermion couples to electromagnetic interaction. Interestingly, all other e.m. interactions can be built from this basic interaction. The weak interactions in the Standard Model are characterized by emission and absorption of $W^\pm$ and $Z^0$. In Figure 2(a) we have shown the decay $n \to p + e + \bar{\nu}_e$, mediated by charged vector boson, whereas in Figure 2(b) we have shown the weak interactions mediated by $Z^0$, usually called neutral current interactions.

All hadrons (mesons and baryons) are made up of quarks and antiquarks with $q - q$ and $q - \bar{q}$ interactions mediated through gluons, the theory describing the interactions is called Quantum Chromodynamics (QCD). Naively speaking baryons are made up of three valence quarks and mesons are made up of $q - \bar{q}$ combination. A proton, for example, is made up of two $u$ quarks (each having $+2/3|e|$ charge) and a $d$ quark having charge $-1/3|e|$. Similarly a neutron would consist of two $d$ quarks and a $u$ quark. In Table 1, we have given the valence quark content of some of the well known baryons and mesons. Since the basic purpose of the article is to explore the structure of nucleon, we, therefore, in the sequel detail some of the essentials of QCD.
Figure 2: Weak interaction.

| Mesons | Quark content | Baryons | Quark content |
|--------|---------------|---------|---------------|
| $\pi^\pm$ | $ud, d\bar{u}$ | $p$ | uud |
| $\pi^o$ | $(u\bar{u}, d\bar{d})$ | $n$ | udd |
| $K^\pm$ | $u\bar{s}, s\bar{u}$ | $\Sigma^+$ | uus |
| $K^o, \bar{K}^o$ | $d\bar{s}, s\bar{d}$ | $\Sigma^o$ | dds |
| $\eta$ | $(u\bar{u}, d\bar{d}, s\bar{s})$ | $\Xi^o$ | uss |
+-------+----------------+---------+---------------|
|       |                | $\Xi^-$ | dss       |
|       |                | $\Lambda$ | uds    |

Table 1: Valence quark structure of some of the important mesons and baryons.
2 Quantum Chromodynamics (QCD)

In QCD, quarks are endowed with an additional ‘color’ degree of freedom, for example, ‘red’ (R), ‘green’ (G) and ‘blue’ (B) with the $q - q$ and $q - \bar{q}$ interactions mediated by octet of colored gluons. Gluons are similar to photons in that they have zero rest mass and spin 1. However, photons carry no charge whereas gluons carry color charge. The octet of colored gluons can be characterised as follows:

\[
\begin{align*}
RB & \quad RG & \quad GB & \quad \frac{1}{\sqrt{2}}(BB - GG) \\
BR & \quad GR & \quad BG & \quad \frac{1}{\sqrt{6}}(BB + GG - 2RR)
\end{align*}
\]

Hadrons are colorless implying thereby that they are color-singlet in the color space. Although the purpose of the article is not to go into the technical details of the gluon mediated $q - q$ interactions, however, in order to facilitate the understanding of certain concepts, we would mention the QCD Lagrangian [7], for example,

\[
L_{QCD} = -\frac{1}{4} F^{(a)\mu\nu} F_{(a)\mu\nu} + i \sum_q \bar{\psi}_q \gamma^\mu (D_\mu)_{ij} \psi^j_q - \sum_q m_q \bar{\psi}_q^i \psi^i_q,
\]

\[
(D_\mu)_{ij} = \delta_{ij} \partial_\mu - ig_s \sum_a \frac{\lambda^a_{ij}}{2} A^a_\mu,
\]

where $g_s$ is the QCD coupling constant and $f_{abc}$ are the structure constants of the $SU(3)$ algebra, $\bar{\psi}$ is the quark field and $A$ is the gauge field. The $q - q$ $q - \bar{q}$ interactions are usually discussed in terms of $\alpha_s$ which is related to the QCD gauge coupling, $g_s$ as

\[
\alpha_s = \frac{g^2}{4\pi}.
\]

There are certain features of QCD which are quite distinctive compared to Quantum Electrodynamics (QED). As is evident from the definition of $F^{(a)\mu\nu}$, in terms of gluon field $A_\mu^{(a)}$, there is a self interaction represented by $g_s f_{abc} A_\mu^{(b)} A_\nu^{(c)}$ which is absent in the case of photon mediated QED. The self interaction of gluon in fact is a general property of any of the non-Abelian gauge field theory. Another extremely important property of QCD, which is also a general characteristic of the gauge field theories, is the momentum dependence of the coupling constants. The effective QCD coupling, $\alpha_s(Q^2)$, can be shown to have the following momentum dependence at momentum scale $Q$

\[
\alpha_s(Q^2) = \alpha_s(\mu^2) - \alpha_s^2(\mu^2) \beta_0 \ln(Q^2/\mu^2) + \ldots.
\]
where the \( \beta_0 \) is calculated to be

\[
\beta_0 = \frac{11N_c - 2n_f}{12\pi}.
\]  

\( N_c \) is the number of colors (\( =3 \)), \( n_f \) is the number of active flavors, i.e. the number of flavors whose mass threshold is below the momentum scale, \( Q \). The corresponding momentum dependence of the electromagnetic fine structure constant, in the case of QED, is given as

\[
\alpha(Q^2) = \alpha(m^2) - \frac{\alpha^2(m^2)}{3\pi} \log\left(-\frac{Q^2}{m^2}\right) + \ldots.
\]

From the above expression, one can find out that \( \alpha \) has the value 1/137 at energies which are not large compared with the electron mass, however at LEP energies (101 GeV), it takes a value closer to 1/128. In contrast to the electromagnetic interactions, which is an Abelian gauge theory, the coupling constant in the case of non-Abelian gauge theory decreases as the energy increases, as can be checked from Equation (7). Therefore, in the case of QCD, the coupling of one quark to another is weak at very short distances or large four momentum transfer squared (\( Q^2 \)). The property \( \alpha_s \to 0 \) as \( Q \to \infty \) is known as “asymptotic freedom” and helps to explain why quarks deep within hadrons are essentially free particles. Conversely, the effective couplings grow as we go to large distances and tends to infinity at very large distances or small \( Q^2 \) and as a consequence it is impossible to separate quarks. This property of increasing \( \alpha_s \) at large distances and consequently confinement of quarks is called “infrared slavery”. At a deeper level the concept of infrared slavery could be attributed to the self-interaction of gluons in the case of QCD.

Naively speaking, thus we have two different pictures of the inside of the nucleon, one at short distance and the other at large distance. At sufficiently short distances, which can be probed at sufficiently large energies, we can consider quarks and gluons interacting very weakly with each other, hence we can perform calculations of the scattering cross-sections with the perturbative techniques. At large distances, the nonlinearity of gluon-gluon couplings come fully into play, as a consequence it is very difficult to understand.

QCD has registered remarkable success in the limit \( Q^2 \to \infty \), where it is amenable to perturbative calculations. In particular, it has helped in establishing the quark degree of freedom, gluons and the presence of \( q\bar{q} \) pairs inside the nucleon. From the deep inelastic scattering, we learn that the nucleon is composed of spin 1/2 point like particles which can be identified with the quarks. However in the low energy limit, in view of the intractability of the QCD, the progress in this direction is painstakingly slow. However, considerable insight has been achieved through lattice calculations, QCD sum
rules, $1/N_c$ expansions etc. The vast amount of low energy data, however, is usually explained through Constituent Quark Model (CQM). In this context CQM with QCD motivated spin-spin forces [10] has been extremely successful in explaining large amount of spin data. This model, apart from providing considerable insight into the hadronic matrix elements is devoid of complicated technicalities from the calculation point of view. For the benefit of the reader, we include herewith a brief sketch of the CQM, so as to enable one to understand simple calculations.

3 The Constituent Quark Model with spin-spin forces

The constituent quark model or naive quark model is based on certain extremely simplifying assumptions. For example, hadrons are made up of point like valence quarks, baryons consisting of three quark combinations, whereas the mesons consisting of quark-antiquark combinations. The valence quark content of some of the baryons and mesons is given in Table 1. These quarks interact through confining potential, several of these have been used, the most popular being Coulombic + linear and the harmonic oscillator. All hadronic transitions take place through single quark transitions: for example, in a given baryon, two out of the three valence quarks would act as spectators, whereas third quark will participate in the interaction. Apart from the confining potential in the CQM, some extra interactions between the quarks have also been considered.

To understand the essentials of naive quark model, we discuss in somewhat detail a particular model, pioneered by DGG [10], which has been extremely successful. The starting point for CQM with chromodynamic spin-spin forces is the Hamiltonian,

$$H = L(\vec{r}_1, \vec{r}_2, ...) + \sum_i (m_i + \frac{p_i^2}{2m_i} + ...) + \sum_{i>j} k\alpha_s S_{ij}. \quad (10)$$

In the above equation, $L$ describes the universal interaction responsible for quark binding, $\vec{r}_i$, $\vec{p}_i$ and $m_i$ are the position, momentum and mass of the $i^{th}$ quark, and $k$ is -4/3 for mesons and -2/3 for baryons. $S_{ij}$ is the two-body Coulombic interaction and has the form:

$$S_{ij} = \frac{1}{|\vec{r}|} \frac{1}{2m_im_j} (\frac{\vec{p}_i.\vec{p}_j}{|\vec{r}|^2}) + \frac{\vec{r}_i.(\vec{r}_j.\vec{p})}{|\vec{r}|^3} - \frac{\pi}{2} \delta^3(\vec{r})(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16S_i.\vec{S}_j}{3m_im_j}) + ....... \quad (11)$$

where $\vec{r} = \vec{r}_i - \vec{r}_j$ and $S_i$ is the spin of the $i^{th}$ quark. The exact form of the confining potential is not known, however, several kinds of confinement
potential have been used in the literature. To illustrate concrete calculations of hadronic matrix elements we use harmonic oscillator potential \([10]\) because of its exact solvability and simplicity. The corresponding Hamiltonian can be obtained from Equations (10) and (11) by replacing \(H\) by

\[
H = \sum_{i} \frac{p_i^2}{2m_i} + \frac{1}{6}m\omega^2\sum_{i<j}(r_i - r_j)^2. \tag{12}
\]

The spectrum of hadrons made up of \(u, d\) and \(s\) quarks, in the CQM follows the SU(6)×O(3) symmetry. According to SU(6)×O(3) symmetry, mesons fall into multiplets \((35 + 1)\). The 35 containing the nonet of scalar and vector mesons. The baryons fall into the \((56 + 70' + 70'' + 20)\) representations of \(6 \times 6 \times 6\). The extension to hadrons involving heavier quarks \(c, b\) and \(t\), can be carried out in the same manner. The O(3) symmetry controls the spatial part of the wavefunction which could be state of definite angular momentum and radial excitations while constructing explicit wavefunctions of CQM. The baryon wavefunction in CQM can be written as

\[
\psi_{\text{Baryon}} = \phi_{\text{unitary spin}} \chi_{\text{spin}} \eta_{\text{color}}. \tag{13}
\]

As the quarks are fermion spin 1/2 objects then the total wave function has to be antisymmetric. The wave functions are constructed in such a manner that the antisymmetricity resides in the color space. Baryons and mesons are colorless objects which one ensures by considering them to be color singlets. In the case of mesons this is ensured by the \(q - \bar{q}\) combinations, whereas in the case of baryons this is ensured by taking the wave function completely antisymmetric in color space. We discard the color part of the wavefunction for rest of our discussion as it does not play any dynamical role in the low energy hadronic matrix elements. As an explicit example of the symmetrized wave function in unitary and spin space, we consider the ground state octet of baryons, for which the wave function is expressed as

\[
\psi_o(8, \frac{1}{2}^+ ) = \frac{1}{\sqrt{2}}(\phi' \chi' + \phi'' \chi'')\psi_o^s; \tag{14}
\]

where \(\phi, \chi\) and \(\psi\) denote respectively the SU(3), spin and space wave functions, with the various types of symmetry under quark exchange. For the proton and neutron, we mention the explicit form of \(\phi', \chi', \phi'', \chi''\), for example,

\[
\chi' = \frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\uparrow), \quad \chi'' = \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \tag{15}
\]

\[
\phi'_p = \frac{1}{\sqrt{2}}(udd - duu), \quad \phi''_p = \frac{1}{\sqrt{6}}(2uud - udu - duu). \tag{16}
\]
\[
\phi'_n = \frac{1}{\sqrt{2}} (udd - dud), \quad \phi''_n = \frac{1}{\sqrt{6}} (udd + dud - 2duu).
\] (17)

The spatial wavefunctions, denoted by \( \psi \), are solutions of the Hamiltonian. Each level will have definite symmetry and will be associated with the SU(6) to build a symmetric function. The levels and their symmetries are dependent on the potential, generally, the ground-state level is symmetric with \( L^P = 0^+ \), and the next level is of mixed symmetry with \( L^P = 1^- \). The wavefunctions of the first few spatial states are listed as under. The ground state is given by

\[
\psi^s(56, 0^+) = \psi_0(\rho, \lambda),
\] (18)

the N=1 states are as

\[
\psi'(70, 1^-) = \left( \frac{8}{3} \pi \right)^{1/2} R^{-1} Y^M_1(\rho) \psi_0(\rho, \lambda),
\] (19)

\[
\psi''(70, 1^-) = \left( \frac{8}{3} \pi \right)^{1/2} R^{-1} Y^M_1(\lambda) \psi_0(\rho, \lambda),
\] (20)

where as the N=2 states are

\[
\psi^s(56, 0^+) = \left( \frac{1}{3} \right)^{1/2} R^{-2}[3R^2 - (\rho^2 + \lambda^2)] \psi_0(\rho, \lambda),
\] (21)

\[
\psi'(70, 0^+) = \left( \frac{1}{3} \right)^{1/2} R^{-2}[2\lambda, \rho] \psi_0(\rho, \lambda),
\] (22)

\[
\psi''(70, 0^+) = \left( \frac{1}{3} \right)^{1/2} R^{-2}[2\rho^2 - \lambda^2] \psi_0(\rho, \lambda),
\] (23)

\[
\psi''(70, 2^+) = \left( \frac{8}{3} \pi \right)^{1/2} R^{-2}[Y^M_2(\rho) - Y^M_2(\lambda)] \psi_0(\rho, \lambda).
\] (24)

For rest of spatial wavefunctions we refer the reader to reference [11]. The variables \( \rho \) and \( \lambda \) are defined as

\[
\rho = \frac{1}{\sqrt{2}}(r_1 - r_2), \quad \lambda = \frac{1}{\sqrt{2}}(r_1 + r_2 - 2r_3), \quad R = \frac{1}{3}(r_1 + r_2 + r_3).
\] (25)

\( \rho \) being antisymmetric in 1 and 2 and \( \lambda \) being symmetric in 1 and 2.

Spin-spin forces lead to interband mixing, for example, the octet wavefunction \( \psi(8, \frac{1}{2}^+) \) not only gets contribution from the octet \( |56, 0^+ >_{N=0} \) but also from \( |56, 0^+ >_{N=2}, |70, 0^+ >_{N=2} \) and \( |70, 2^+ >_{N=2} \). Therefore, the nucleon wavefunction is expressed as,
ψ(8, 1/2) = |56, 0+ >_{N=0} + \alpha |56, 0+ >_{N=2} \\
+ \beta |70, 0^+ >_{N=2} + \epsilon |70, 2^+ >_{N=2} . \quad (26)

Using the above Hamiltonian of DGG, Isgur and Collaborators have carried an extremely detailed analysis of hadronic spectra and hadronic matrix elements. For detailed exposure in this regard we refer the reader to references [10, 11, 12, 13]. For the special case of nucleon they arrive at the following wave function

\[ 8, \frac{1}{2}^+ \rangle_n = 0.90 |56, 0^+ >_{N=0} - 0.34 |56, 0^+ >_{N=2} \\
- 0.27 |70, 0^+ >_{N=2} - 0.06 |70, 2^+ >_{N=2} . \quad (27) \]

In Equation (26) it should be noted that \((56, 0^+)_N=2\) does not affect the spin-isospin structure of \((56, 0^+)_N=0\), whereas \((70, 2^+)_N=2\) does not affect the spin-isospin structure of \((70, 2^+)_N=2\). Therefore, Equation (26) can be simplified to

\[ 8, \frac{1}{2}^+ \rangle = \cos \phi |56, 0^+ >_{N=0} + \sin \phi |70, 0^+ >_{N=2}, \quad (28) \]

with \(\phi = 20^\circ\). This has been referred to as non-trivial mixing in the literature [12].

This CQM with one gluon mediated \(q - q\) and \(q - \bar{q}\) forces have been applied very successfully to large variety of low energy hadronic matrix elements [14]. It has not only given a remarkably accurate description of hadron spectroscopy data [13] but has also been able to describe some very subtle features of the data, such as neutron charge radius [11, 15], \(N - \Delta\) mass difference, photohelicty amplitudes [16], baryon magnetic moments, etc.

To illustrate the success of CQM, in the sequel we discuss two cases, for example, nucleon magnetic moments and neutron charge radius. The magnetic moment for a nucleon is defined as

\[ \mu(B) = \Delta u^B \mu_u + \Delta d^B \mu_d + \Delta s^B \mu_s, \quad (29) \]

where \(\Delta u^B, \Delta d^B\) and \(\Delta s^B\) for the given nucleon are the spin polarizations defined as:

\[ \Delta q = q^\uparrow - q^\downarrow, \quad (30) \]

\(q^\uparrow\) and \(q^\downarrow\) being the number of quarks with spin up and spin down. The total spin

\[ \Delta \Sigma = \Delta u + \Delta d + \Delta s, \quad (31) \]
is twice the value of \( S \) (spin of proton).

Assuming \( u \) and \( d \) to be point particles, the magnetic moments associated with these \((\mu_u, \mu_d)\) can be defined as

\[
\mu_u = \frac{q_u}{2m_u} = \frac{2}{2m_N} = 2\mu_N, \quad (32)
\]

\[
\mu_d = \frac{q_d}{2m_d} = \frac{-1}{2m_N} = -\mu_N. \quad (33)
\]

Here the masses of \( u \) and \( d \) quarks are considered to be 1/3 rd of the nucleon mass and are taken to be equal. To find the number of quarks with spin up and spin down, \( q^\uparrow \) and \( q^\downarrow \), say in proton, one has to consider the symmetrized wavefunction for the proton given in Equation (14). Let us calculate the number of \( u \) quarks with spin up. This can be calculated by considering the expectation value of the operator \( n_u(i)P^\uparrow(i) \), where \( i \) stands for the \( i^{th} \) quark and \( P^\uparrow(i) \) is the projection operator for spin up and is 1 if the \( i^{th} \) quark has spin up and 0 otherwise. \( n_u(i) \) is 1 if the \( i^{th} \) quark is \( u \) and 0 otherwise. Then,

\[
u^\uparrow = \langle 56, 0^+ | \sum_{i=1}^{3} n_u(i)P^\uparrow(i) | 56, 0^+ \rangle. \quad (34)
\]

Since the wavefunction in Equation (14) is symmetrized, as well as the operator \( n_u(i)P^\uparrow(i) \) does not affect the spatial part of the wavefunction, therefore one can write

\[
\nu^\uparrow = \frac{3}{2} \langle \chi'^\prime \phi'^\prime + \chi'^\prime \phi'^\prime | n_u(3)P^\uparrow(3) | \chi'^\prime \phi'^\prime + \chi'^\prime \phi'^\prime \rangle.
\]

By carrying out simple calculation one finds \( \nu^\uparrow = \frac{5}{3} \). Similarly we can find \( \nu^\downarrow, d^\uparrow \) and \( d^\downarrow \), for example,

\[
\nu^\downarrow = \frac{1}{3}, \quad d^\uparrow = \frac{1}{3}, \quad d^\downarrow = \frac{2}{3}. \quad (35)
\]

This can be repeated for other baryons, for example, in neutron we interchange \( u \) and \( d \), in \( \Sigma^+ \) we replace \( d \) by \( s \) and so on.

Thus, the contribution by each of the quark flavors to the proton spin can be written as:

\[
\Delta u = \frac{4}{3}, \quad \Delta d = -\frac{1}{3}, \quad \Delta s = 0. \quad (36)
\]

From Equations (29) and (36) we get the magnetic moment of the proton and the neutron as,

\[
\mu(p) = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d, \quad (37)
\]

\[
\mu(n) = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u. \quad (38)
\]
If we substitute $\mu_u = -2\mu_d$, then we obtain $\mu_n = -\frac{3}{2}$ which is very well in agreement with the experiment. The same thing can be repeated when we use the wavefunction with non-trivial mixing. The magnetic moments are

$$\mu(p) = \cos^2\phi\left(\frac{4}{3}\mu_u - \frac{1}{3}\mu_d\right) + \sin^2\phi\left(\frac{2}{3}\mu_u + \frac{1}{3}\mu_d\right),$$

(39)

and

$$\mu(n) = \cos^2\phi\left(\frac{4}{3}\mu_d - \frac{1}{3}\mu_u\right) + \sin^2\phi\left(\frac{2}{3}\mu_d + \frac{1}{3}\mu_u\right).$$

(40)

One can calculate magnetic moment of other baryons, and in Table 2, we have presented the results of a particular calculation [12].

There are large number of hadronic matrix elements which can be calculated and these agree very well with the data. For the sake of readability of the article we include another calculation of CQM which involves calculations of spatial wavefunctions. The neutron charge radius is usually expressed in terms of the slope of the electric form factor $G_E^n(q^2)$, the experimental value [17] of which is given as

$$\left(\frac{dG_E^n(q^2)}{dq^2}\right)_{q^2=0} = 0.47 \pm 0.01 \text{ GeV}^{-2}.$$

(41)

If we assign the nucleon to a pure $56$ (with the spin expressed in terms of Pauli spinors), the neutron electric form factor vanishes for all $q^2$. Considering our complete wavefunction with the $56-70$ mixing and performing the calculations, keeping the lowest order in $\tan\phi$, we obtain

$$\mu_n G_E^n(q^2) = -\tan\phi \sqrt{2} < \psi_{N=1}''|\psi_{N=1}'\rangle, \quad (42)$$

Table 2: Magnetic moments of baryons in CQM.
and from Equations (21) and (23) we have
\[
<\psi''_N|e^{i\vec{q}.\vec{r}_3}\psi^s> = \frac{|q^2|R^2}{\sqrt{3}}.
\] (43)

Neutron charge radius can be expressed as
\[
<r^2_n> = 6(\frac{dG_n(q^2)}{dq^2})|q^2=0 = \sqrt{\frac{2}{3}}R^2(-\tan\phi),
\] (44)

where \(\phi\) is the mixing angle and is negative and \(R^2\) is the shape factor [11, 12] for the harmonic oscillator wave function. The calculated values of neutron charge radius \(<r^2_n>(=6b)\) as function of \(\phi\) and \(R^2\) are presented in Table 3. From the table one can immediately find out that CQM with spin spin forces is able to give an excellent fit to neutron charge radius.

### 4 Difficulties with CQM

Despite amazing success in explaining large and diverse amount of hadronic data, the basic tenents of CQM raise many questions about their justifications. Neither one can deduce it from basic QCD considerations nor can one provide justification for its basic assumptions. So, from aesthetic considerations it is very unsatisfactory situation.

Besides the philosophical inadequacy of CQM, there are a few parameters which have defied explanation within CQM. For example, \(G_A/G_V\), defined in
terms of the spin distribution functions, is given as
\[ \frac{G_A}{G_V} = \Delta u - \Delta d. \] (45)

In comparison to the experimental value of 1.26, Equation (45) predicts it to be $\frac{2}{3}$ in the case of CQM. Introduction of configuration mixing does not help much in this case, for example, with a mixing characterised by $\phi = 20^\circ$, $G_A/G_V$ doesn’t change much. Similarly, there are several other parameters which require one to go beyond CQM.

The most important challenge to CQM was, however, posed by the observations in the deep inelastic scattering of the polarised leptons off polarised nucleons made by the European Muon Collaboration (EMC) [18]. The deep inelastic polarized muon-proton scattering measurements made by the EMC indicated that the entire spin of the proton is not carried by the valence quarks but only 30% of the spin is carried by the valence quarks. It also indicated that the $qq$ sea is not unpolarised and there is a significant contribution to the proton spin by the strange quarks in the sea. This is contrasted with the CQM assumptions that entire spin of the proton is carried by the valence quarks. This was called “spin crisis”.

Further, in CQM, the similarity of the $u$ and $d$ quark masses and the flavor independent nature of the gluon couplings led to expect $\bar{d} = \bar{u}$, thus to the validity of the Gottfried sum rule in CQM. The NMC measurements of the muon scatterings off proton and neutron targets [19], however, show that the Gottfried sum rule [20] is violated. It has been interpreted as showing $\bar{d} > \bar{u}$ in the proton. This conclusion has been confirmed by NA51 [21] in the Drell-Yan process with proton and neutron targets.

From the above discussion it seems that many of the successes of CQM are primarily due to cancellation which are taking place due to various degrees of freedom inside the nucleon. It therefore becomes interesting to introduce components into the wavefunction having angular momentum, a polarized sea of quark-antiquark pairs, gluons and Goldstone bosons etc. In this context we would like to discuss a particular successful model, Chiral Quark Model, which not only incorporates the basic features of CQM but also some other degrees of freedom.

5 Chiral Quark Model ($\chi$QM)

Chiral quark model was developed [14, 22] essentially to understand the successes of CQM. The idea of the $\chi$QM is based on the picture that a quark inside a nucleon emits quark-antiquark pairs via Goldstone bosons (GB), for example,
\[ q_\pm \rightarrow GB^0 + q'_\mp \rightarrow (q\bar{q}') + q'_\mp. \] (46)
Figure 3: Production of a $q - \bar{q}$ pair via a Goldstone Boson emission.

The basic interaction causes a modification of the spin content because a quark changes its helicity by emitting a spin zero meson. It causes a modification of the flavor content because the GB fluctuation, unlike gluon emission, is flavor dependent. The spin flip process makes it possible to understand the spin content of the nucleon, which was not possible in the conventional constituent quark model. With the admixture of mesons to the nucleon wavefunction, one finds that only $\frac{1}{3}$rd of the nucleon is carried by the quarks. Moreover, for the other spin-flavor observables, such as magnetic moments, sea quark distributions and the Gottfried sum rule, the agreement with experimental data is also improved using this model. Thus, inside the nucleon, but not deep inside where perturbative QCD is applicable, the effective degrees of freedom are constituent quarks, gluons, the $\chi_{SB}$ Goldstone bosons and the $q - \bar{q}$ pairs. In order to make the article self contained we reproduce in the sequel some of the essential details of $\chi$QM.

The basic idea of $\chi$QM is that the chiral symmetry breaking takes place at a distance significantly smaller than the confinement scale. For example, the QCD confinement scale is characterised by $\Lambda_{QCD}=0.1-0.3$ GeV, whereas the chiral symmetry breaking scale, $\Lambda_{\chi_{SB}}$ is characterised by 1 GeV. The Lagrangian based on the chiral quark model is

$$L = g_8 \bar{q} \phi q,$$

where $g_8$ is the coupling constant,

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}.$$
and
\[
\phi = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\
\pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \alpha K^0 \\
\alpha K^- & \alpha K^o & -\beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}}
\end{pmatrix}.
\] (48)

SU(3) symmetry breaking is introduced by considering different quark masses \(m_s > m_{u,d}\) as well as by considering the masses of Goldstone Bosons to be non-degenerate \((M_{K,\eta} > M_\pi)\) \([23, 24, 25]\), whereas the axial U(1) breaking is introduced by \(M_{\eta'} > M_{K,\eta}\) \([23, 24, 25, 26]\). The parameter \(a(=|g_8|^2)\) denotes the transition probability of chiral fluctuation of the splittings \(u(d) \rightarrow d(u) + \pi^+(-)\), whereas \(\alpha^2 a\) denotes the probability of transition of \(u(d) \rightarrow s + K^{-(0)}\). Similarly \(\beta^2 a\) and \(\zeta^2 a\) denote the probability of \(u(d,s) \rightarrow u(d,s) + \eta\) and \(u(d,s) \rightarrow u(d,s) + \eta'\), respectively.

The effective Lagrangian in the region between \(\Lambda_{\chi_{SB}}\) and \(\Lambda_{QCD}\) has fundamental quark and gluon fields, because these particles are not bound into color-singlet hadrons at such short distances. Since the \(SU(3)_L \times SU(3)_R\) global chiral symmetry is spontaneously broken, there is also an octet of pseudoscalar Goldstone boson, which are put in as fundamental fields. The Goldstone boson fields are essential if the Lagrangian is to consistently reproduce the effects of a spontaneously broken global symmetry. In this energy range the quarks and GBs propagate in the QCD vacuum which is filled with the \(q-\bar{q}\) condensate. The interaction of a quark with the condensate will cause it to gain an extra mass of \(\simeq 350\) MeV. This is the \(\chi\text{QM}\) explanation of the large constituent quark mass. The QCD Lagrangian is also invariant under the axial U(1) symmetry, which would imply the ninth GB \(m_{\eta'} \simeq m_\eta\). But the existence of axial anomaly breaks the symmetry and in this way the \(\eta'\) picks up an extra mass.

The interaction of the GBs is weak enough to be treated by perturbation theory. This means that on long enough time scales for the low energy parameters to develop we have

\[
u = (d^\dagger + \pi^+) + (s^\dagger + K^+) + (u^\dagger + \pi^0, \eta, \eta'),
\]
\[
d^\dagger \leftarrow (u^\dagger + \pi^-) + (s^\dagger + K^0) + (d^\dagger + \pi^0, \eta, \eta'),
\]
\[
s^\dagger \leftarrow (u^\dagger + K^-) + (d^\dagger + \bar{K}^0) + (s^\dagger + \eta, \eta').
\]

In the absence of interactions, the proton is made up of two \(u\) quarks and one \(d\) quark. Proton’s flavor content can be calculated after any one of these
quarks turns into part of the quark sea by ‘disintegrating’, via GB emissions, into a quark plus a quark-antiquark pair.

χQM with SU(3) symmetry (α = β = 1) is also able to provide fairly satisfactory explanation for various quark flavor contributions to the proton spin [27], baryon magnetic moments [26, 27] as well as the absence of polarizations of the antiquark sea in the nucleon [23, 24]. However, in the case of hyperon decay parameters the predictions of the χQM are not in tune with the data [29], for example, in comparison to the experimental numbers 0.21 and 2.17 the χQM with SU(3) symmetry predicts $f_3/f_8$ and $\Delta_3/\Delta_8$ to be $\frac{1}{3}$ and $\frac{5}{3}$ respectively. It has been shown [24, 26] that when SU(3) breaking effects are taken into consideration within χQM, the predictions of the χQM regarding the above mentioned ratios have much better overlap with the data.

However, as mentioned earlier that the constituent quark model with one gluon mediated configuration mixing (CQMgcm) gives a fairly satisfactory explanation of host of low energy hadronic matrix elements [10, 13, 15]. In view of the fact that constituent quarks constitute one of the important degrees of freedom, therefore it becomes interesting to examine, within the χQM, the implications of one gluon mediated configuration mixing. This is particularly interesting as some of the low energy data are responsive only to configuration mixing.

6 Chiral Quark Model with configuration mixing

In order to make the article self contained we discuss here some of the essential details of the chiral quark model with one gluon generated configuration mixing (χQMgcm), for the details we refer the reader to reference [30]. To begin with, we consider the spin distribution functions for nucleon wave function affected by spin-spin forces. However, for the sake of simplicity we consider nucleon wave function described by Equation (28). The spin distribution functions for proton are defined as

$$\left\langle 8, \frac{1}{2} | N | 8, \frac{1}{2} \right\rangle = \cos^2 \phi < 56, 0^+ | N | 56, 0^+ >$$

$$+ \sin^2 \phi < 70, 0^+ | N | 70, 0^+ > .$$

For the $| 56 >$ part we have

$$< 56, 0^+ | N | 56, 0^+ > = \frac{5}{3} u^\uparrow + \frac{1}{3} u^\downarrow + \frac{1}{3} d^\uparrow + \frac{2}{3} d^\downarrow ,$$

(50)
as derived earlier. In the case of \(|70^+\rangle\), one can find the spin in the similar manner and are defined as

\[
< 70, 0^+ | N | 70, 0^+ > = \frac{4}{3} u^\uparrow + \frac{2}{3} u^\downarrow + \frac{2}{3} d^\uparrow + \frac{1}{3} d^\downarrow.
\] (51)

In the \(\chi\)QM, the basic process is the emission of a Goldstone Boson which further splits into \(q\bar{q}\) pair as mentioned in Equation (46) of the text. Following reference [25], the spin structure after one interaction can be obtained by substituting for every quark, for example,

\[
q^\uparrow \to P_q q^\uparrow + |\psi(q^\uparrow)|^2, \tag{52}
\]

where \(P_q\) is the probability of no emission of GB from a \(q\) quark and the probabilities of transforming a \(q^\uparrow\downarrow\) quark are \(|\psi(q^\uparrow)|^2\), given as

\[
|\psi(u^\uparrow)|^2 = \frac{a}{6}(3 + \beta^2 + 2\zeta^2)u^\downarrow + a\alpha^2 s^\downarrow, \tag{53}
\]
\[
|\psi(d^\uparrow)|^2 = au^\uparrow + \frac{a}{6}(3 + \beta^2 + 2\zeta^2)d^\uparrow + a\alpha^2 s^\uparrow, \tag{54}
\]
\[
|\psi(s^\uparrow)|^2 = a\alpha^2 u^\downarrow + a\alpha^2 d^\downarrow + \frac{a}{3}(2\beta^2 + \zeta^2)s^\downarrow. \tag{55}
\]

The quantity of interest here is \(\hat{B}\), defined using the above Equations

\[
\hat{B} = \cos^2 \phi \left[ \frac{5}{3}(P_u u^\uparrow + |\psi(u^\uparrow)|^2) + \frac{1}{3}(P_u u^\downarrow + |\psi(u^\downarrow)|^2) + \frac{1}{3}(P_d d^\uparrow + |\psi(d^\uparrow)|^2) + \frac{2}{3}(P_d d^\downarrow + |\psi(d^\downarrow)|^2) \right] \\
+ \sin^2 \phi \left[ \frac{4}{3}(P_u u^\uparrow + |\psi(u^\uparrow)|^2) + \frac{2}{3}(P_u u^\downarrow + |\psi(u^\downarrow)|^2) + \frac{1}{3}(P_d d^\uparrow + |\psi(d^\uparrow)|^2) \right]. \tag{56}
\]

Using the spin structure from the above Equation we can calculate the spin polarizations, which come out to be

\[
\Delta u = \cos^2 \phi \left[ \frac{4}{3} - \frac{a}{3}(7 + 4\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2) \right] \\
+ \sin^2 \phi \left[ \frac{2}{3} - \frac{a}{3}(5 + 2\alpha^2 + \frac{2}{3}\beta^2 + \frac{4}{3}\zeta^2) \right], \tag{57}
\]
| Parameter | Expt. value | Without configuration mixing | With configuration mixing |
|-----------|-------------|-----------------------------|---------------------------|
|           |             | CQM with SU(3) symmetry     | CQM<sub>gcm</sub> with SU(3) symmetry |
|           |             | NMC | E866 | NMC | E866 | NMC | E866 |
| Δ u       | 0.85 ± 0.05 | 1.33 | .79 | .81 | .96 | .99 | 20° | 1.26 | .74 | .76 | .90 | .92 | 18° | 1.27 | .75 | .77 | .91 | .93 | 16° | 1.28 | .76 | .78 | .92 | .94 |
| Δ d       | -0.41 ± 0.05 | -0.33 | -0.35 | -0.37 | -0.40 | -0.41 | 20° | -0.26 | -0.30 | -0.31 | -0.32 | -0.34 | 18° | -0.27 | -0.31 | -0.32 | -0.33 | -0.35 | 16° | -0.28 | -0.32 | -0.33 | -0.34 | -0.36 |
| Δ s       | -0.07 ± 0.05 | 0 | -0.1 | -0.12 | -0.02 | -0.02 | 20° | 0 | -0.1 | -0.12 | -0.02 | -0.02 | 18° | 0 | -0.1 | -0.12 | -0.02 | -0.02 | 16° | 0 | -0.1 | -0.12 | -0.02 | -0.02 |
| G<sub>A</sub>/G<sub>V</sub> | 1.267 ± 0.0035 | 1.66 | 1.14 | 1.18 | 1.35 | 1.40 | 20° | 1.52 | 1.04 | 1.07 | 1.22 | 1.26 | 18° | 1.54 | 1.06 | 1.09 | 1.24 | 1.28 | 16° | 1.56 | 1.08 | 1.11 | 1.26 | 1.30 |
| Δ<sub>8</sub> | 0.58 ± 0.025 | 1 | 0.64 | 0.68 | 0.60 | 0.62 | 20° | 1 | 0.64 | 0.69 | 0.62 | 0.62 | 18° | 1 | 0.64 | 0.69 | 0.62 | 0.62 | 16° | 1 | 0.64 | 0.69 | 0.62 | 0.62 |
| F-D       | -0.34       | -0.33 | -0.25 | -0.25 | -0.38 | -0.39 | 20° | -0.26 | -0.20 | -0.19 | -0.30 | -0.32 | 18° | -0.27 | -0.21 | -0.20 | -0.31 | -0.33 | 16° | -0.28 | -0.22 | -0.21 | -0.32 | -0.34 |
| F/D       | 0.575       | 0.67 | 0.64 | 0.65 | 0.56 | 0.56 | 18° | 0.70 | 0.67 | 0.69 | 0.60 | 0.58 | 16° | 0.69 | 0.66 | 0.68 | 0.59 | 0.57 |
| F         | 0.462       | 0.665 | 0.445 | 0.465 | 0.49 | 0.505 | 20° | 0.63 | 0.42 | 0.44 | 0.46 | 0.47 | 18° | 0.635 | 0.425 | 0.445 | 0.465 | 0.475 | 16° | 0.64 | 0.43 | 0.45 | 0.47 | 0.48 |
| D         | 0.794       | 1 | 0.695 | 0.715 | 0.87 | 0.895 | 20° | 0.89 | 0.62 | 0.63 | 0.76 | 0.79 | 18° | 0.905 | 0.635 | 0.645 | 0.775 | 0.805 | 16° | 0.920 | 0.65 | 0.66 | 0.79 | 0.82 |

Table 4: The calculated values of spin polarization functions Δ<sub>u</sub>, Δ<sub>d</sub>, Δ<sub>s</sub>, and quantities dependent on these: G<sub>A</sub>/G<sub>V</sub> and Δ<sub>8</sub>, both for NMC and E866 data with the symmetry breaking parameters obtained by χ<sup>2</sup> minimization in the χQM with one gluon generated configuration mixing (χQM<sub>gcm</sub>) and SU(3) symmetry breaking.
\[
\Delta d = \cos^2 \phi \left[ -\frac{1}{3} - \frac{a}{3}(2 - \alpha^2 - \frac{1}{3} \beta^2 - \frac{2}{3} \zeta^2) \right] \\
+ \sin^2 \phi \left[ \frac{1}{3} - \frac{a}{3}(4 + \alpha^2 + \frac{1}{3} \beta^2 + \frac{2}{3} \zeta^2) \right], \quad (58)
\]

\[
\Delta s = \cos^2 \phi [-a \alpha^2] + \sin^2 \phi [-a \alpha^2] = -a \alpha^2. \quad (59)
\]

Before we present our results it is perhaps desirable to discuss certain aspects of the symmetry breaking parameters employed here. As has been considered by Cheng and Li [23], the singlet octet symmetry breaking parameter \( \zeta \) is related to \( \bar{u} - \bar{d} \) asymmetry [19, 20, 31]. We have also taken \( \zeta \) to be responsible for the \( \bar{u} - \bar{d} \) asymmetry in the \( \chi \)QM with SU(3) symmetry breaking and configuration mixing. Further the parameter \( \zeta \) is constrained [19, 25, 31] by the expressions \( \zeta = -0.7 - \beta/2 \) and \( \zeta = -\beta/2 \) for the NMC and E866 experiments respectively, which essentially represent the fitting of deviation from Gottfried sum rule [20].

In Table 4, we have presented the results of our calculations pertaining to spin polarization functions \( \Delta u, \Delta d, \Delta s \) and related parameters including the hyperon \( \beta \)-decay parameters dependent on spin polarizations functions. The value of the mixing angle is taken to be \( \phi \simeq 20^\circ \), a value dictated by consideration of neutron charge radius, as discussed earlier. In the table, however, we have considered a few more values of the mixing parameter \( \phi \) in order to study the variation of spin distribution functions with \( \phi \). The parameter \( a \) is taken to be 0.1, as considered by other authors [24, 23, 26, 27]. Further, while presenting the results of SU(3) symmetry breaking case without configuration mixing (\( \phi = 0^\circ \)), we have used the same values of parameters \( \alpha \) and \( \beta \), primarily to understand the role of configuration mixing for this case. The SU(3) symmetry calculations are obtained by taking \( \alpha = \beta = 1, \phi = 20^\circ \) and \( \alpha = \beta = 1, \phi = 0^\circ \) respectively for with and without configuration mixing. For the sake of completion, we have also presented the results of CQM with and without configuration mixing.

In order to appreciate the role of configuration mixing in affecting the fit, we first compare the results of CQM with those of CQM\textsuperscript{gcm} [30]. One observes that configuration mixing corrects the result of the quantities in the right direction but this is not to the desirable level. Further, in order to understand the role of configuration mixing and SU(3) symmetry with and without breaking in \( \chi \)QM, we can compare the results of \( \chi \)QM with SU(3) symmetry to those of \( \chi \)QM\textsuperscript{gcm} with SU(3) symmetry. Curiously \( \chi \)QM\textsuperscript{gcm} compares unfavourably with \( \chi \)QM in case of most of the calculated quantities. This indicates that configuration mixing alone is not enough to generate an appropriate fit in
Table 5: The calculated values of quark distribution functions and other dependent quantities as calculated in the $\chi$QM with and without SU(3) symmetry breaking both for NMC and E866 data, with the same values of symmetry breaking parameters as used in spin distribution functions and hyperon $\beta$ decay parameters.

$\chi$QM. However when $\chi$QM$_{gcm}$ is used with SU(3) and axial U(1) symmetry breakings then the results show uniform improvement over the corresponding results of $\chi$QM with SU(3) and axial U(1) symmetry breakings. To summarize the discussion of these results, one finds that both configuration mixing and symmetry breaking are very much needed to fit the data within $\chi$QM.

In order to have a unified fit to spin polarization functions as well as quark distribution functions, we have presented in Table 5 the various quark distribution functions with the symmetry breaking parameters used in the case of $\chi$QM with symmetry breaking and configuration mixing both for NMC and E866 data. The general survey of Table 5 immediately makes it clear that the success achieved in the case of spin polarization functions is very well maintained in this case also. The calculated values hardly leave anything to be desired both for the NMC and E866 data.

We find that $\chi$QM$_{gcm}$ with SU(3) symmetry breaking is able to give a satisfactory unified fit for spin and quark distribution functions, with the symmetry breaking parameters $\alpha = .4$, $\beta = .7$ and the mixing angle $\phi = 20^\circ$, both for NMC as well as the most recent E866 data. In particular, the agreement in the case of $G_A/G_V$, $\Delta_8$, F, D, $f_s$ and $f_3/f_8$, is quite striking. It is found
that configuration mixing improves the CQM results, however in the case of \(\chi\)QM with SU(3) symmetry the results become worse. The situation changes completely when SU(3) symmetry breaking and configuration mixing are included simultaneously. Thus, it seems that both configuration mixing as well as symmetry breaking are very much needed to fit the data within \(\chi\)QM.

7 Summary and conclusion

In the last 5 decades there has been phenomenal growth in the understanding of the question: What is inside the nucleon? The early 60’s saw the emergence of unitary symmetry and consequently the Quark Model. That quark are point like constituents of nucleon was formally established by deep inelastic scattering experiments. The emergence of the Standard Model as an extension of electroweak unification laid the foundation of the basic tenants of Quantum Chromodynamics — the theory describing the \(q-q\) and \(q-\bar{q}\) interactions inside the hadrons. QCD being non-Abelian in nature with non-linear interactions between gluons cannot be solved exactly in all limits, this gives rise to various effective models describing the inside of the nucleon for different energy regions.

As has been emphasized earlier, for the low energy hadronic matrix elements or phenomena involving the surface of the nucleon, the CQM with QCD inspired spin-spin forces provides a simplistic but satisfactory description of the data.

Below the surface of the nucleon, in the energy scale \(\Lambda_{QCD} < Q < \Lambda_{\chi QM}\), the effective degrees of freedom change from constituent quarks to quarks, \(q-\bar{q}\) pairs, gluons and Goldstone bosons. Mathematically, in this region the wave function of the nucleon is described by Equation (47). The wavefunction of nucleon in this region becomes more complicated as it has to incorporate more degrees of freedom. The Deep Inelastic region, with the dynamics of the quarks described by the QCD Lagrangian is given by Equation (3). The deep inside of the nucleon is characterized by quarks, gluons, \(q-\bar{q}\) pairs, with hardly any interactions among themselves.

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