On chirality of the vorticity of the Universe

Davor Palle

Zavod za teorijsku fiziku, Institut Rugjer Bošković

Bijenička cesta 54, 10001 Zagreb, Croatia

(Dated: September 16, 2008)

Abstract

We study chirality of the vorticity of the Universe within the Einstein-Cartan cosmology. It is shown that spin density of the light Majorana neutrinos acts as a seed for vorticity at early stages of the evolution of the Universe. Its chirality can be evaluated at spacelike infinity. It turns out that vorticity of the Universe has right-handed chirality.

PACS numbers: 98.80.Es; 12.10.Dm; 04.60.-m
To comprehend all the phenomenology in particle physics and cosmology one has to construct mathematically consistent and complete theories based on few basic physical principles. The theory presented in Ref. [1] is an attempt of solving two main obstacles in the Standard Model (SM) and relativistic quantum field theory: zero-distance singularity and causality-violating $SU(2)$ global anomaly.

The resulting theory (called BY in Ref. [1]) is UV finite (not only renormalizable) with heavy and light Majorana neutrinos as cold and hot dark matter [2, 3]. There is a perfect balance between bosonic (electroweak gauge bosons) and fermionic (leptons and quarks) particles, owing to exact cancellation of anomalous effective actions and the constraint relation between boson and fermion mixing angles $\theta_W = 2(\theta_{12} + \theta_{23} + \theta_{31})$. The left-handed chirally-asymmetric weak interactions appear as an inevitable consequence of the assumed dimensionality and noncontractibility of the physical spacetime.

The absence of Higgs particles is crucial for the cosmological stability of heavy Majorana neutrinos $\tau_{N_i} \gg \tau_U$ [2]. The lepton-number violation, the conservation of $B-L$, as well as lepton and baryon CP violation, lead to leptogenesis and baryogenesis.

Formulating the theory of the local structure of spacetime as local $SU(3) \times SU(2) \times U(1)$ gauge theories, we choose the Einstein-Cartan theory as formulated by Sciama and Kibble to be the theory of the global structure of spacetime. Trautman was the first who realized the possibility of the nonsingular Einstein-Cartan (EC) cosmology [4]. In addition, there is more freedom to avoid noncausal Goedel cosmological solutions [5, 6]. The Einstein-Cartan gravity is also a quantum theory of gravity. Namely, the EC gravity relates rotational degrees of freedom of matter and spacetime, i.e. total angular momentum as a conserved quantity in the Special Theory of Relativity consisting of the orbital angular momentum and spin of matter vs. torsion of spacetime. Spin, as an internal angular momentum of particles, is a quantum mechanical quantity. One can introduce the angular momentum in General Relativity only as a nonconserved quantity and it does not obey tensorial transformations [7].

The torsion and curvature are in balance at the beginning and at the end of the evolution of the Universe [9] at spacelike infinity. At the weak interaction scale $R_{\text{min}} = \mathcal{O}(10^{-16} \text{cm})$ torsion is dominated by fermion spin densities, while at the largest scale $R = \infty$ torsion is dominated by the angular momentum of the whole Universe (galaxies, groups, clusters, ...). However, the contribution of the torsion is even at present ($R_0 = \mathcal{O}(10^{28} \text{cm})$) much smaller
than the mass density, thus allowing the usage of Einstein, not Einstein-Cartan equations. The mass density and the cosmological constant are uniquely defined at spacelike infinity: \( \Omega_m = 2, \Omega_\Lambda = -1 \). The primordial mass density contrast is evolved from quantum fluctuations of the spin to the value at the photon decoupling defined by parameters of metric beyond that of Robertson-Walker.

The question is whether it is possible to find the chirality of the vorticity of the Universe? The answer is positive, provided the local and global theories of spacetime are BY and EC theories.

Within this framework the evolution scenario is the following:

(1) Assuming CP violation in lepton sector, a dynamics of heavy Majorana neutrinos produces imbalance between leptons and antileptons before their decoupling from primordial plasma. This happens around \( T > \mathcal{O}(10^3 TeV) \).

(2) Before light Majorana neutrino decoupling (\( T_{\text{dec}} = \text{few MeV} \)) the imbalance in the number of baryons and antibaryons appears as a consequence of the surplus of leptons against antileptons and conserved B-L.

(3) Now follows the crucial observation: Part of the survived leptons, like electron, are produced together with neutrinos through charged current: \( W^- \to e^- \nu_M \) and the helicity \( \lambda(\nu_M) \) is predominantly positive (\( \lambda(\nu_M) = +1 \)). This is the consequence of the two facts: (a) helicity of Dirac antineutrinos produced in \( W^- \to e^- \bar{\nu}_D \) is positive for left-handed weak interactions and (b) production of the negative helicity relativistic Majorana neutrinos is suppressed in the same process by the kinematical factor \( \frac{m_{\nu}}{E} \ll 1 \).

(4) It is easy to estimate number of neutrinos and other particles at the epoch of neutrino decoupling:

\[
n_\nu(T_0 = 2.73K) = \mathcal{O}(10^2 cm^{-3}), T_{\text{dec}}(\nu) = \text{few MeV},
\]

\[
n_{e^-}(T_0) = n_p(T_0) \simeq n_B(T_0) = \mathcal{O}(10^{-7} cm^{-3}),
\]

\[
m_{N_i} = \mathcal{O}(10 TeV) - \mathcal{O}(100 TeV), m_{\nu_i} = \mathcal{O}(10^{-3} eV) - \mathcal{O}(1 eV)
\]

\[
\Rightarrow \frac{n_{\nu}}{n_{e^-}}, \frac{n_{\nu}}{n_N}, \frac{n_{\nu}}{n_B} \gg 1, \text{ at } T = \text{few MeV}.
\]
We conclude that the spin density of the matter is dominated by the spin of the light neutrinos with an excess of the positive helicity states. The neutral weak currents obviously cannot contribute to helicity asymmetry. In addition, the sum of the orbital angular momenta of particles vanishes because of the isotropy.

The spin alignment hypothesis together with the positive helicity abundance guarantee the nonvanishing torsion. This is a seed for the small vorticity with the well defined chirality.

(5) The particles that can compete with neutrinos in abundances are the background photons. However, photons, as a massless gauge boson particles, do not generate the torsion in a gauge invariant way.

(6) Electromagnetic and strong forces, as well as the curvature of spacetime, as chirally-symmetric interactions, cannot alter the chirality of vorticity in later stages of evolution, after neutrino decoupling. Torsion is growing relatively to curvature as a consequence of the growth of the angular momentum of large scale structures, reaching finally the strength of the curvature at spacelike infinity \( R = \infty \).

(7) A term of EC equations linear in torsion (spin) allows us to uniquely determine chirality of the vorticity:

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}^{\text{eff}},
\]

\[
T_{\mu\nu}^{\text{eff}} = -p_{\text{eff}} g_{\mu\nu} + u_\mu u_\nu (p_{\text{eff}} + \rho_{\text{eff}}) - 2(g^{\alpha\beta} + u^\alpha u^\beta) \nabla_\alpha [u_{(\mu} S_{\nu)}^{\alpha\beta}],
\]

\[
\kappa = 8 \pi G N c^{-4}, \quad \rho_{\text{eff}} = \rho - \kappa S^2 + \Lambda, \quad p_{\text{eff}} = p - \kappa S^2 - \Lambda,
\]

\[
S^2 = \frac{1}{2} S_{\alpha\beta} S^{\alpha\beta}, \quad S_{\mu, \alpha\beta} = u^\mu S_{\alpha\beta}, \quad (\alpha\beta) = \frac{1}{2} (\alpha\beta + \beta\alpha),
\]

\[
\text{torsion} = Q^\mu_{\alpha\beta}, \quad Q^\mu_{\alpha\beta} + 2 h^\mu_{[a} Q^\alpha_{b]} = \kappa S^\mu_{\alpha\beta},
\]

\[
Q_a = h^\mu_a Q_\mu, \quad Q_\mu = Q^\nu_{\mu \nu}, \quad [\mu \nu] = \frac{1}{2} (\mu \nu - \nu \mu),
\]

\[
a, b = \text{local Lorentzian indices}, \quad h^\mu_a = \text{tetrad basis},
\]

\[
\eta_{ab} = \text{diag}(+1, -1, -1, -1).
\]

Cosmic mass-density \((\Omega_m = 2)\), the cosmological constant \((\Omega_\Lambda = -1)\) and torsion are fixed by three algebraic equations at spacelike infinity:

\[
\text{by definition} : \quad S^{\alpha\beta} = -\frac{1}{2} \rho h^\alpha_i h^\beta_j \mu_{ij},
\]
metric: \[ ds^2 = dt^2 - dx^2 - (1 - \Sigma)e^{2mx}dy^2 - dz^2 - 2\sqrt{\Sigma}e^{mx}dydt, \]
\[ \Sigma, \ m = \text{constant parameters}, \]
\[ R = \infty: \ Q = Q^0_{i\hat{2}} = -\frac{m(2 - \Sigma)}{2\sqrt{\Sigma}}, \ \Sigma = \mathcal{O}(10^{-3}) \ll 1, \]
\[ G_N\rho_\infty H_{\infty}^2 = \frac{3}{4\pi}, \ H_\infty = \text{Hubble constant at } \infty, \]
by definition: \[ Q = \frac{1}{2}\kappa n\mu_\hat{1}\hat{2}B_\infty, \ \mu_\hat{1}\hat{2} = -\mu_\hat{2}\hat{1} = +\frac{1}{2}h, \]
\[ n = n_\nu(\lambda = +1) - n_\nu(\lambda = -1) > 0. \]

\( B_\infty \) is some positive amplification factor as a result of the evolution. Its magnitude can be estimated \( B_\infty \geq Q(\text{Universe})_0/\kappa \text{Spin(neutrinos)}_0 = \mathcal{O}(10^{34}) \).

We incorporate the abundant positive-helicity states of neutrinos to the spin density. One can conclude then:

\[ Q < 0 \Rightarrow m > 0. \]

Definition and chirality of vorticity are naturally defined as [17, 18, 19]:

\[ \omega_{\nu\mu} = \frac{1}{2}(\nabla_\alpha u_\beta - \nabla_\beta u_\alpha)P^\alpha_\mu P^\beta_\nu, \]
\[ P_{\alpha\beta} = g_{\alpha\beta} - u_\alpha u_\beta, \ \omega_{ij} = h^\mu_i h^\nu_j \omega_{\mu\nu}, \]
\[ \Sigma H_\infty \approx \frac{2}{\sqrt{3}}\omega_\infty, \ \omega_\infty^2 = \frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu}, \]
\[ \omega_{\mu\nu}(m = +|m|, xyz) = -\omega_{\mu\nu}(m = -|m|, xyz) = -\omega_{\mu\nu}(m = +|m|, yxz), \]
\[ m > 0 \text{ and } \omega_{\hat{1}\hat{2}} = \omega_{\hat{2}\hat{1}} = +m\frac{\sqrt{\Sigma}}{2} > 0 \Rightarrow \text{right-handed vorticity}. \]

Hence, if the positive definite helicity light neutrinos define the torsion, the chirality of the vorticity of the Universe is right-handed.

The magnitude and chirality of the vorticity and angular momentum of the Universe, as well as the relic-neutrino helicities, are well defined observables.

The notion of spacelike infinity (frequently used in Minkowski spacetime [7, 20]) could be useful in numerical simulation to evaluate angular momentum of the Universe in the evolution up to \( R = \infty \Leftrightarrow T_\gamma = 0 \) (one can use also the concept of negative redshift: \[ T = \frac{1}{

5
\[ R/R_0 = 1/(1 + z), \ R \geq R_0, \ z \in [-1,0] \]. N-body simulations are indispensable in the study of nonlinear evolution with a feedback of the angular momentum to the background geometry of the Universe.

Let us add some final comments. Extremely small power of density contrast at large scales, observed by COBE and WMAP, leads to the negative cosmological constant because of the negative contribution of the integrated Sachs-Wolfe effect. All-over fit of cosmological data requires then small Hubble constant [10], but fixing the absolute distance scale in cosmology by standard candles is a difficult task [21].

Recent results of MiniBooNE with anomalous behaviour of the spectrum and no oscillations, contrary to LSND results ("neutrinos" vs. "antineutrinos") could be the first signal for the lepton CP violation. Note that the CP violating phase need not to be accompanied with \( \theta_{31} \) mixing angle, but can be attached to \( \theta_{12} \) mixing angle. The inclusion of large CP violating phase can substantially change the present estimate of the mixing angles.

The assumption of noncontractible space will be tested soon by background events at LHC, differing essentially from SM expectations [22]. Heavy Majorana neutrinos are probably already observed by H.E.S.S. and Magic at the center of our galaxy.

[1] D. Palle, Nuovo Cim. A 109, 1535 (1996).
[2] D. Palle, Nuovo Cim. B 115, 445 (2000).
[3] D. Palle, Nuovo Cim. B 118, 747 (2003).
[4] A. Trautman, Nature 242, 7 (1973).
[5] J. R. Ray and L. L. Smalley, Phys. Rev. Lett. 49, 1059 (1982).
[6] Yu. N. Obukhov and V. A. Korotky, Class. Quantum Grav. 4, 1633 (1987).
[7] J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields (McGraw-Hill Book Company, New York, 1965); N. N. Bogoljubov and D. V. Širkov, Vvedenie v teoriju kvantovannyh poljei (Izdateljstvo Nauka, Moskva, 1973).
[8] S. Weinberg, Gravitation and Cosmology (J. Wiley and Sons, New York, 1972).
[9] D. Palle, Nuovo Cim. B 111, 671 (1996).
[10] D. Palle, Nuovo Cim. B 114, 853 (1999); ibidem 121, 129 (2006); ibidem 122, 67 (2007).
[11] G. ’t Hooft, Phys. Rev. D 14, 3432 (1976); V. A. Kuzmin, V. A. Rubakov, and M. E. Sha-
[12] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, 1990).

[13] B. Kayser, F. Gibrat-Debu, and F. Perrier, *The Physics of Massive Neutrinos* (World Scientific, Singapore, 1989).

[14] K. Hayashi and A. Bregman, *Ann. Phys. (N.Y.)* **75**, 562 (1973).

[15] J. Binney and S. Tremaine, *Galactic Dynamics* (Princeton University Press, Princeton, 1994).

[16] R. Penrose, *Proc. Roy. Soc.* **284 A**, 159 (1965).

[17] M. L. Bedran, E. P. Vasconcellos-Vaidya, and M. M. Som, *Nuovo Cim. B* **87**, 101 (1985).

[18] J. Ehlers, in *Abh. Mainz Akad. Wiss. u. Lit. (Math. Nat. Kl.)*, pp. 791 (1961).

[19] E. Kreyszig, *Differentialgeometrie* (A. V. Geest & Portig K.-G., Leipzig, 1957).

[20] D. Palle, *arXiv:hep-ph/0703203* (2007).

[21] F. Mannucci, M. Della Valle, and N. Panagia, *M.N.R.A.S.* **370**, 773 (2006).

[22] D. Palle, *Hadronic J.* **24**, 87 (2001); ibidem **24**, 469 (2001).