Theory of photon–electron interaction in single-layer graphene sheet

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Abstract
The purpose of this work is to elaborate the quantum theory of photon–electron interaction in a single-layer graphene sheet. Since the light source must be located outside the extremely thin graphene sheet, the problem must be formulated and solved in the three-dimensional physical space, in which the graphene sheet is a thin plane layer. It is convenient to use the orthogonal coordinate system in which the xOy coordinate plane is located in the middle of the plane graphene sheet and therefore the Oz axis is perpendicular to this plane. For the simplicity we assume that the quantum motions of electron in the directions parallel to the coordinate plane xOy and that along the direction of the Oz axis are independent. Then we have a relatively simple formula for the overall Hamiltonian of the electron gas in the graphene sheet. The explicit expressions of the wave functions of the charge carriers are easily derived. The electron–hole formalism is introduced, and the Hamiltonian of the interaction of some external quantum electromagnetic field with the charge carriers in the graphene sheet is established. From the expression of this interaction Hamiltonian it is straightforward to derive the matrix elements of photons with the Dirac fermion–Dirac hole pairs as well as with the electrons in the quantum well along the direction of the Oz axis.

Keywords: graphene, Dirac fermion, quantum well, absorption spectrum, Hamiltonian
Classification numbers: 3.00, 5.04, 5.15

1. Introduction
The discovery of graphene with extraordinary physical properties by Geim and Novoselov [1–4] has opened a new era in the development of condensed matter physics and materials science as well as many fields of high technologies. Right after this discovery the graphene-based optoelectronics has emerged. Xia et al [5] have explored the use of zero-bandgap large-area graphene field effect transistor as ultrafast photodetector. One year later Xia et al [6] have reported again the use of photodetector based on graphene. A broad-band and high-speed waveguide-integrated electroabsorption modulator based on monolayer graphene has been demonstrated by Liu et al [7]. In [8] Wang et al have demonstrated a graphene/silicon-heterostructure waveguide photodetector on silicon-on-insulator material. An ultrawide-band complementary metal-oxide semiconductor-compatible graphene-based photodetector has been fabricated by Muller et al [9]. In [10] Englund et al have demonstrated a waveguide integrated photodetector etc. At the present time the research on graphene photodetectors in still developing [11–13]. In all above-mentioned research works the theoretical reasonings on the light–graphene interaction were limited to the case when the light waves propagate inside very thin graphene layer. However, in the study of the photon–electron interaction in a thin graphene sheet, the light waves always must be sent from the sources located outside the graphene sheet. Therefore the theoretical problem of photon–electron interaction in graphene layers must be formulated and solved...
Figure 1. Graphene single layer is a two-dimensional (2D) lattice of carbon atoms.

Figure 2. The Brillouin zone in the reciprocal lattice of the graphene 2D lattice.

as a problem in the three-dimensional physical space. This is the content of the present work.

In the subsequent section 2 the physical model of the electron gas in a single-layer graphene sheet is formulated and the notations are introduced. In particular, the overall Hamiltonian of the free electron gas in a graphene sheet with some thickness \( d \), which may be extremely small but must be finite, is presented, and the explicit expressions of the wave functions of charge carrier are derived. In section 3 the electron–hole formalism, convenient for the application to the study of the electron–hole pair photo-excitation, is introduced. The theory of the interaction of an external quantum electromagnetic field with charge carriers in a graphene sheet is elaborated in section 4. The explicit expressions of the matrix elements of the photon–electron interaction in the graphene sheet are derived in section 5. The conclusion and discussion are presented in section 6.

2. Physical model of the electron gas in a single-layer graphene sheet

Consider a single graphene sheet as a plane slab of a semi-conducting material with a very small but finite thickness such that the \( xOy \) coordinate plane is parallel to the graphene sheet surface and located in its middle, while the \( Oz \) axis is perpendicular to the graphene surface. It was known \(^{[14]}\) that each graphene single layer is a two-dimensional (2D) lattice of carbon atoms with the hexagonal structure (figure 1), and the first Brillouin zone (BZ) in the reciprocal lattice of the graphene 2D lattice has two corners at two points \( K \) and \( K' \) (figure 2).

Suppose that the quantum motion of electrons along any direction parallel to the \( xOy \) coordinate plane and that along the direction of the \( Oz \) axis are independent. Then the electron quantum field \( \psi(\mathbf{r}, z, t) \) is decomposed in terms of the two-component wave functions \( \varphi_{kE, K}(\mathbf{r}) \) and \( \varphi_{kE, K'}(\mathbf{r}) \) of Dirac fermions with momenta \( \mathbf{k} \) close to the corner \( \mathbf{K} \) or \( \mathbf{K}' \) of the first BZ and the \( \mathbf{k} \)-dependent energies \( E \) as well as in terms of the wave functions \( f_{j}(z) \) of electrons with energies \( \varepsilon_{j} \) in the potential well along the \( Oz \) axis. For the simplicity we assume that this potential well has a great depth and therefore wave functions \( f_{j}(z) \) must vanish at the boundary of the potential well.

Since each corner \( \mathbf{K} \) or \( \mathbf{K}' \) is an extreme point of the electron distribution cones and the electron momenta \( \mathbf{k} \) are always close to \( \mathbf{K} \) or \( \mathbf{K}' \), the electron quantum field \( \psi(\mathbf{r}, z, t) \) is composed of two distinct parts

\[
\psi(\mathbf{r}, z, t) = \psi_{\mathbf{K}}(\mathbf{r}, z, t) + \psi_{\mathbf{K}'}(\mathbf{r}, z, t),
\]

where the expression of \( \psi_{\mathbf{K}}(\mathbf{r}, z, t) \) or \( \psi_{\mathbf{K}'}(\mathbf{r}, z, t) \) contains only the wave functions \( \varphi_{kE, K}(\mathbf{r}) \) or \( \varphi_{kE, K'}(\mathbf{r}) \) of corresponding Dirac fermions. Although the graphene sheet may be infinitely large, for the simplicity of the reasoning during the quantization procedure we impose on the wave functions of Dirac fermions the periodic boundary conditions in a square with the large side \( L \) (figure 3).

The overall Hamiltonian of free electron gas (without mutual electron–electron Coulomb interaction) has following expression

\[
H_{G}^{0} = H_{\mathbf{K}}^{0} + H_{\mathbf{K}'}^{0} + H_{\mathbf{L}}^{0},
\]

where

\[
H_{\mathbf{K}}^{0} = \nu_{F} \int d\mathbf{r} \int dz \psi_{\mathbf{K}}^{+}(\mathbf{r}, z, t)(-i\sigma \nabla)\psi_{\mathbf{K}}(\mathbf{r}, z, t),
\]

\[
H_{\mathbf{K}'}^{0} = \nu_{F} \int d\mathbf{r} \int dz \psi_{\mathbf{K}'}^{+}(\mathbf{r}, z, t)(-i\sigma \nabla)\psi_{\mathbf{K}'}(\mathbf{r}, z, t),
\]

Figure 3. Graphene sheet with the width \( d \) and the side \( L \).
In the graphene sheet becomes
and are the destruction operators of Dirac
quantum expression $(E_0)$ is the effective speed of massless Dirac fermion. From
is the effective mass of electron in the potential well, and
where
\[
\sigma \nabla = \sigma_1 \frac{\partial}{\partial x} + \sigma_2 \frac{\partial}{\partial y},
\]
\[
\sigma^* \nabla = \sigma_1 \frac{\partial}{\partial x} - \sigma_2 \frac{\partial}{\partial y},
\]

$m$ is the effective mass of electron in the potential well, and $v_F$ is the effective speed of massless Dirac fermion. From expression (3) and (4) of the Hamiltonians $H_0^k$ and $H_0^{K'}$, we derive the Dirac equations determining two-component wave functions $\varphi_{k,E,K}(r)$ and $\varphi_{k,E,K'}(r)$ of Dirac fermions
\[
\begin{align*}
-\imath (\sigma \nabla) \varphi_{k,E,K}(r) &= E \varphi_{k,E,K}(r), \\
-\imath (\sigma^* \nabla) \varphi_{k,E,K'}(r) &= E \varphi_{k,E,K'}(r).
\end{align*}
\]

It can be shown [14] that for each momentum $k$ there exist two eigenvalues of each of two equations (7) and (8)
\[ E_{\pm}(k) = \pm v_F k, \]
and the corresponding eigenfunctions are
\[
\varphi_{k,E,K}(r) = e^{ikr} \frac{1}{\sqrt{2}} \left( e^{-\imath (k/2)} \right)
\]
\[
\varphi_{k,E,K'}(r) = e^{ikr} \frac{1}{\sqrt{2}} \left( e^{\imath (k/2)} \right),
\]
\[
\tan \theta(k) = \frac{k_2}{k_1}.
\]

The quantum fields $\psi_{K}(r, z, t)$ and $\psi_{K'}(r, z, t)$ have following expansions in terms of wave functions $\varphi_{k,E,K}(r)$ and $f_i(z)$
\[
\psi_{K}(r, z, t) = \sum_{k} \sum_{l=\pm 1} a_{k,l,K} \varphi_{k,E,K}(r) \alpha_i f_l(z) e^{-i(E_{\pm} + \imath t)}
\]
and similar formula with $K \rightarrow K'$. where $a_{k,l,K}$ and $a_{k,l,K'}$ are the destruction operators of Dirac fermion with wave functions (10) and (11) respectively, $\alpha_i$ are the electron destruction operators with wave functions $f_i(z)$ in the quantum well along the direction of the $Oz$ axis. Therefore $a_{k,l,K}$, $a_{k,l,K'}$, and $a_{k,l,K'}$ are the corresponding creation operators. The quantum fields $\psi_{K}(r, z, t)$ and $\psi_{K'}(r, z, t)$ satisfy the Heisenberg equation of motion
\[
\frac{\partial \psi_{K}(r, z, t)}{\partial t} = \left[ \psi_{K}(r, z, t), H_0^k \right]
\]
and similar equation with $K \rightarrow K'$.

3. Electron–hole formalism in graphene

As usual, for the study of graphene as a semiconductor we work in the electron–hole formalism. We shall use following notations

\[
\begin{align*}
\{ k, E_i(k) \} &\rightarrow I \ or \ J, \\
E_i(k) &\rightarrow E_i \ or \ E_j, \\
a_{k,v,K} &\rightarrow a_{K} \ or \ a_{K'}, \\
a_{k,v,K'} &\rightarrow a_{K'} \ or \ a_{K''}.
\end{align*}
\]

Denote $E_F$ the Fermi level of the system of Dirac fermions. In order to distinguish the wave functions of electrons from those of holes it is convenient to substitute
\[
\varphi_{k,E,K}(r) = \begin{cases} u_{K}(r) & \text{if } E_i > E_F, \\
u_{K}(r) & \text{if } E_i \leq E_F, \end{cases}
\]
and similarly with $K \rightarrow K'$. Then the expansion (13) of the quantum field $\psi_{K}(r, z, t)$ in the graphene sheet becomes
\[
\psi_{K}(r, z, t) = \sum_{E_i > E_F} \sum_{l=\pm 1} a_{Kl} \alpha_i u_{K}(r)f_l(z) e^{-i(E_{\pm} + \imath t)} + \sum_{E_i < E_F} \sum_{l=\pm 1} a_{Kl} \alpha_i u_{K}(r)f_l(z) e^{-i(E_{\pm} + \imath t)},
\]
and similar formula with $K \rightarrow K'$. Note that all operators $a_{K}$, $a_{K'}$, and $a_{K''}$ are the destruction operators of Dirac fermions, while $a_i$ are those of electron in the quantum well along the direction of the $Oz$ axis. Denote $|0\rangle$ the vacuum state vector. We have following formula, by definition,
\[ a_{K}|0\rangle = a_{K'}|0\rangle = a_{K''}|0\rangle = a_i|0\rangle = 0 \]
for all indices $I$, $J$, $K$, $K'$ and $i$.

Consider now the ground state $|\Phi_0\rangle$ of the system at $T = 0$. In this state all energy levels of Dirac fermions below the Fermi level are occupied and all those higher than the Fermi level are empty. Suppose that all states at the Fermi level are also occupied. Then we have
\[ |\Phi_0\rangle = \prod_{E_i > E_F} \prod_{l=\pm 1} a_{Kl}\alpha_i u_{K}(r)|0\rangle. \]

The destruction and creation operators of holes are defined as follows:
\[ b_{K} = a_{K}, \quad b_{K'} = a_{K'}, \quad b_{K''} = a_{K''}. \]

Then we have following condition
\[ a_{K}|\Phi_0\rangle = a_{K'}|\Phi_0\rangle = b_{K}|\Phi_0\rangle = b_{K'}|\Phi_0\rangle = 0 \]
for all indices $I$ and $J$, meaning that in the ground state there does not exist any Dirac fermion above the Fermi level as well as any hole of Dirac fermion on or below the Fermi level. In the sequel the hole of Dirac fermion on or below the Fermi level will be shortly called Dirac hole. The energies of Dirac
fermion and Dirac hole relative to the Fermi level are
\[ \tilde{E}_i = E_i - E_F > 0, \]
\[ \tilde{E}_g = E_F - E_i \leq 0. \]  
(22)

Instead of the quantum field operators \( \psi_K(r, z, t) \) and \( \psi_{K'}(r, z, t) \), in the electron–hole formalism it is more convenient to use new quantum field operators
\[ \Psi_K(r, z, t) = e^{iE_FT} \psi_K(r, z, t), \]
\[ \Psi_{K'}(r, z, t) = e^{iE_FT} \psi_{K'}(r, z, t). \]  
(23)

They have following expansions in terms of Dirac fermion destruction operators \( a_{0K} \) and Dirac hole creation operators \( b^+_{0K} \):
\[ \Psi_K(r, z, t) = \sum_{\ell_i < \ell_p} \sum_{\ell_p} a_{\ell_p K} a_{\ell_i K}(r) f_{\ell_i}(z) e^{-i(\tilde{E}_i + \tilde{r}_i)t}, \]
\[ + \sum_{\ell_i < \ell_p} \sum_{\ell_p} b^+_{\ell_p K} a_{\ell_i K}(r) f_{\ell_i}(z) e^{-i(\tilde{E}_i - \tilde{r}_i)t}, \]  
(24)

and similar formula with \( K \to K' \).

Denote \( N_K \) and \( N_{K'} \) the electron number operators corresponding to the fields \( \psi_K(r, z, t) \) and \( \psi_{K'}(r, z, t) \)
\[ N_K = \int \! dr \int \! dz \langle \psi_K(r, z, t)^+ \psi_K(r, z, t), \]
\[ N_{K'} = \int \! dr \int \! dz \langle \psi_{K'}(r, z, t)^+ \psi_{K'}(r, z, t), \]  
(25)

and introduce the new definition of Hamiltonians
\[ \tilde{H}_0^0 = H_0^0 - E_F N_K, \]
\[ \tilde{H}_0^{K'} = H_0^{K'} - E_F N_{K'}. \]  
(26)

From the Heisenberg equations of motion (14) for the fields \( \psi_K(r, z, t) \) and \( \psi_{K'}(r, z, t) \) it follows the Heisenberg equation of motion for the field \( \Psi_K(r, z, t) \)
\[ \frac{i}{\hbar} \frac{\partial \Psi_K(r, z, t)}{\partial t} = \left[ \tilde{H}_0^0, \Psi_K(r, z, t) \right], \]  
(27)

and similar equation with \( K \to K' \).

4. Photon–electron interaction in graphene

The overall Hamiltonian \( H_G \) of the single-layer graphene sheet interacting with the transverse electromagnetic field \( A(r, z, t) \)
\[ \nabla A(r, z, t) = 0, \]  
(28)
can be obtained from the expressions (2)–(5) of the overall Hamiltonian \( H_G^0 \) of the free electron gas in this sheet by substituting
\[-i \nabla \rightarrow - i \nabla + eA(t, r, z, t), \]
\[-i \frac{\partial}{\partial t} \rightarrow - i \frac{\partial}{\partial t} + eA(t, r, z, t), \]  
(29)

where
\[ A_H(r, z, t) = i A_{0}(r, z, t) + j A_{0}(r, z, t), \]  
(30)
i and \( j \) being unit vectors along the directions of the \( Ox \) and \( Oy \) coordinate axes. Then we have
\[ H_G = H_0^0 + H_G^{int} \]  
(31)
and
\[ H_G^{int} = e v_F \int \! dz \int \! dz \left[ \tilde{\Psi}_K(r, z, t)^+ \sigma \tilde{\Psi}_K(r, z, t) \right] A_H(r, z, t) \]
\[ - i \frac{e}{2m} \int \! dz \int \! dz \left[ \tilde{\Psi}_K(r, z, t)^+ \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial z} \right) \tilde{\Psi}_K(r, z, t) \right] A(z, z, t) \]
\[ + \frac{e^2}{2m} \int \! dz \int \! dz \left[ \tilde{\Psi}_K(r, z, t)^+ \tilde{\Psi}_K(r, z, t) \right] A(z, z, t)^2, \]  
(32)
where \( e \) is the absolute value of the electron charge. The transverse vector electromagnetic field \( A(r, z, t) \) is expanded in terms of the photon destruction and creation operators \( c_{0k} \) and \( c_{0k}^+ \), respectively, of the photon with momentum \( k, l \) \( xOy \), \( l \) along \( Oz \) and in the polarization state labeled by the index \( \sigma \). We have
\[ A(r, z, t) = \sum_k \sum_l \sum_\sigma \frac{1}{\sqrt{2 \omega_k l}} \left[ c_{0k}^\sigma \xi_{0kl} e^{-i(\omega_k t - kr - \ell z)}, \right. \]
\[ \left. + c_{0k}^\sigma \xi_{0kl}^* e^{i(\omega_k t - kr - \ell z)} \right], \]  
(33)
where \( \omega_kl = \sqrt{k^2 + \ell^2} \), \( \xi_{0kl} \) is the complex unit vector characterizing the polarization state of photon and satisfying following condition
\[ k_l \xi_{0kl}^* + l_k \xi_{0kl} = 0, \]  
(34)
\( \xi_{0kl} \) and \( \xi_{0kl}^* \) being the components parallel and perpendicular to the coordinate plane \( xOy \) of the vector \( \xi_{0kl} \).

In the first order of the perturbation theory with respect to the electron–photon interaction the scattering matrix (5-matrix) is
\[ S = -i \int \! dt \int \! dz \int \! dz \left[ e v_F \left[ \tilde{\Psi}_K(r, z, t)^+ \sigma \tilde{\Psi}_K(r, z, t) \right] A_H(r, z, t) \right. \]
\[ \left. + \tilde{\Psi}_K'(r, z, t)^+ \sigma \tilde{\Psi}_K'(r, z, t) \right] A_H(r, z, t) \]
\[ - i \frac{e}{2m} \left[ \tilde{\Psi}_K(r, z, t)^+ \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial z} \right) \tilde{\Psi}_K(r, z, t) \right] A(z, z, t) \]
\[ \left. + \tilde{\Psi}_K'(r, z, t)^+ \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial z} \right) \tilde{\Psi}_K'(r, z, t) \right] \times A(z, z, t), \]  
(35)

Consider the photoexcitation of a Dirac fermion–Dirac hole pair simultaneously taking place together with the
transition of an electron from the initial state \( f_i(z) \) to the final one \( f_f(z) \) in the quantum well along the direction of the \( Oz \) axis. The incoming state of the whole system has the state vector

\[
|\text{in}\rangle = a_i^+ c_{ek} |\psi_0\rangle,
\]

while for the outgoing state vector there may be two different cases: either

\[
|\text{out}\rangle = a_f^+ a_{bK} b_{jk}^+ |\psi_0\rangle
\]

or

\[
|\text{out}\rangle = a_f^+ a_{K} b_{jk}^+ |\psi_0\rangle.
\]

By means of lengthy but standard calculations it can be shown that the matrix elements of the scattering matrix between the incoming state (36) and one of two outgoing states (37) or (38) have following general form

\[
S^{K,K'}_{\text{fi}} = \langle \text{out} | \mathcal{S}^{K,K'}_{\text{fi}} | \text{in}\rangle
\]

where

\[
M_{\text{fi}}^{K} = evF\xi_{ek}\int dz e^{ikz} f_i^*(z)\phi_f(z)
\]

\[
-\frac{e}{2m}\int dz e^{ikz} f_i^*(z)\phi_f(z)
\]

\[
\times \int dz e^{ikz} \left[ f_j(z) \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial z} \right) f_i(z) \right]
\]

in the case of outgoing state vector (37), and

\[
M_{\text{fi}}^{K'} = evF\xi_{ek}\int dz e^{ikz} f_i^*(z)\phi_f(z)
\]

\[
-\frac{e}{2m}\int dz e^{ikz} f_i^*(z)\phi_f(z)
\]

\[
\times \int dz e^{ikz} \left[ f_j(z) \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial z} \right) f_i(z) \right]
\]

in the case of outgoing state vector (38), \( \bar{E}_f \) and \( \bar{E}_j \) are energies of Dirac fermion and Dirac hole relative to the Fermi level, \( \epsilon_f \) and \( \epsilon_j \) are energies of electrons in the final and initial states in the quantum well along the \( Oz \) axis, \( \omega_{ki} \) is the angular frequency (energy) of photon. Calculations of matrix elements \( M_{\text{fi}}^{K} \) and \( M_{\text{fi}}^{K'} \) will be done in the next section.

5. Matrix elements of photoexcitation processes

In this section we derive explicit expressions of the matrix elements determined by formulae (40) and (41). Functions \( u_{K}(r) \), \( u_{K'}(r) \) and \( v_{K}(r) \), \( v_{K'}(r) \) in these formulae are the Bloch wave functions

\[
u_{K}(r) = e^{i\Phi} \phi_{K}(p),
\]

\[
u_{K'}(r) = e^{-i\Phi} \phi_{K'}(-q).\]

and similar relations with \( K \rightarrow K' \), \( p \) and \( q \) being the momenta of the Dirac fermion and the Dirac hole, respectively. The concrete forms of the functions \( \phi_{K}(p) \), \( \phi_{K'}(p) \) and \( \phi_{K}(-q) \), \( \phi_{K'}(-q) \) depend on the position of the Fermi level \( E_F \). Using expressions (42) and similar expressions with \( K \rightarrow K' \), we obtain

\[
\int d\mathbf{r} e^{ikr} u_{K}(r)^+ \phi_{K}(r) = \epsilon_{K} \phi_{K}(r)
\]

\[
\int d\mathbf{r} e^{ikr} u_{K'}(r)^+ \phi_{K'}(r) = \epsilon_{K'} \phi_{K'}(r)
\]

where

\[
\Gamma_{K}(p, q) = \phi_{K}(p)^+ \phi_{K}(q)
\]

\[
\Gamma_{K'}(p, q) = \phi_{K'}(p)^+ \phi_{K'}(q).
\]

Both vector functions (45) certainly depend also on the position of the Fermi level \( E_F \). There are three different cases. Using expressions (10) and (11) of the Dirac spinors for calculating vector functions (45) in each case, we obtain following results:

Case 1. \( E_F = 0 \) (figure 4(a))

In this case we have

\[
\Gamma_{K}(p, q) = -\Gamma_{K'}(p, q)
\]

\[
= -i \left[ \sin \left( \frac{\theta(p) + \theta(q)}{2} \right) - j \cos \left( \frac{\theta(p) + \theta(q)}{2} \right) \right].
\]

Case 2. \( E_F > 0 \) (figure 4(b))

In the upper part (U) with

\[
E_{f}(q) = v_F |q|
\]

of the valence band we have

\[
\Gamma_{K}(p, q) = \Gamma_{K'}(p, q)
\]

\[
= i \cos \left( \frac{\theta(p) + \theta(q)}{2} \right) + j \sin \left( \frac{\theta(p) + \theta(q)}{2} \right).
\]

while in the lower part (L) with

\[
E_{f}(q) = -v_F |q|
\]

of the valence band formula (46) holds.

Case 3. \( E_F < 0 \) (figure 4(c))

In the upper part (U) with

\[
E_{f}(p) = v_F |p|
\]

of the conduction band we still have formula (46), while in the lower part (L) with

\[
E_{f}(p) = -v_F |p|
\]
of the conduction band we obtain
\[
\Gamma_k(p, q) = \Gamma_k'(p, q) = -\left\{ i \cos \left[ \frac{\theta(p) + \theta(q)}{2} \right] + j \sin \left[ \frac{\theta(p) + \theta(q)}{2} \right] \right\},
\]
(48)
Together with the integrals (43) and (44), the matrix elements (40) and (41) contain also following similar integrals
\[
\int dr \ e^{ikr_{ij}k}(r) \lambda_k(r) = \delta_{k,p+q} \lambda_k(p, q),
\]
(49)
\[
\int dr \ e^{ikr_{ij}k'}(r) \lambda_k'(r) = \delta_{k,p+q} \lambda_k'(p, q),
\]
(50)
where
\[
\lambda_k(p, q) = \varphi_k(p) \varphi_k(-q),
\]
\[
\lambda_k'(p, q) = \varphi_k'(p) \varphi_k'(-q).
\]
(51)
These functions also depend on the position of the Fermi level \( E_F \). Calculations in each case give following results:

**Case 1.** \( E_F = 0 \)
In this case we have
\[
\lambda_k(p, q) = -\lambda_k'(p, q) = i \sin \left[ \frac{\theta(p) - \theta(q)}{2} \right],
\]
(52)
**Case 2.** \( E_F > 0 \)
In the upper part (U) with
\[
E_f(q) = v_F |q|
\]
of the valence band we have
\[
\lambda_k(p, q) = \lambda_k'(p, q) = \cos \left[ \frac{\theta(p) - \theta(q)}{2} \right],
\]
(53)
while in the lower part (L) with
\[
E_f(q) = -v_F |q|
\]
of the valence band formula (52) holds.

**Case 3.** \( E_F < 0 \)
In the upper part (U) with
\[
E_f(p) = v_F |p|
\]
of the conduction band we still have formula (52), while in the lower part (L)
\[
E_f(p) = -v_F |p|
\]
of the conduction band we have again formula (53).

In order to complete the determination of matrix elements (40) and (41) it still remains to find the possible expressions of the wave functions \( f_i(z) \) and \( f_f(z) \) of the initial and final states, respectively, of electrons in the quantum well along the direction of the Oz axis, and to calculate the integrals containing \( f_i'(z) \) and \( f_f(z) \) with respect to the variable \( z \). These integrals can be considered as the functionals of the wave functions \( f_i(z) \), \( f_f(z) \) and they are denoted as follows:
\[
B[f_f, f_i] = \int_{-d/2}^{d/2} dz \ e^{ikf(z)y_i(z)} f_f(z)
\]
and
\[
C[f_f, f_i] = \int_{-d/2}^{d/2} dz \ e^{ikf(z)} \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial z} f_f(z) \right) f_i(z),
\]
(54)
(55)
It can be shown that electrons in the quantum well along the direction of the $Oz$ axis have following wave functions

$$f_n^{(+)}(z) = \sqrt{\frac{2}{d}} \cos \left( n + \frac{1}{2} \right) \frac{2\pi}{d} z, $$

$$f_n^{(-)}(z) = \sqrt{\frac{2}{d}} \sin n \frac{2\pi}{d} z. $$

(56)

Corresponding eigenvalues of energy are

$$\varepsilon_n^{(+)} = \frac{1}{2m} \left( n + \frac{1}{2} \right)^2 \left( \frac{2\pi}{d} \right)^2, $$

$$\varepsilon_n^{(-)} = \frac{1}{2m} n^2 \left( \frac{2\pi}{d} \right)^2. $$

(57)

6. Conclusion and discussions

In this work the quantum field theory of the photon–electron interaction in a thin graphene single layer was elaborated. With the simplifying assumption on the independence of the quantum motion of electrons in the directions parallel to the plane of the graphene sheet and that in the direction perpendicular to this plane, a simple expression of the overall Hamiltonian of free electron in the graphene sheet was established. After introducing the electron–hole formalism, the expression of the overall Hamiltonian of the interaction between the charge carriers in the graphene sheet and the external quantum electromagnetic field was derived. From this interaction Hamiltonian it follows immediately the matrix elements of the photon absorption processes in different cases with different positions of the Fermi level of the Dirac fermion gas. The obtained results can be used to numerically calculate the corresponding photon absorption rates.

The determination of the photon absorption spectra is the simplest problem related to the photon–electron interaction in the electron gas of the graphene sheet. The method elaborated in the present work can be generalized for the application to the study of any photon–electron interaction process in the single-layer graphene sheet, for example the electronic Raman scattering, the multiphoton absorption processes and, in general, the non-linear optical processes and phenomena. Note that there always exists the interaction of the charge carriers in the single-layer graphene sheet with the phonons, so that all above presented results should be extended to include the electron–phonon interaction. Moreover, the electronic structures of graphene multilayers are more complicated than that of the graphene single layer, and the study of optical processes and phenomena in graphene multilayers certainly requires our strong effort.

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