Large field inflation models from higher-dimensional gauge theories

Kazuyuki Furuuchi\textsuperscript{a} and Yoji Koyama\textsuperscript{b}

\textsuperscript{a}Manipal Centre for Natural Sciences, Manipal University, Manipal, Karnataka 576104, India
\textsuperscript{b}Department of Physics, National Tsing-Hua University, Hsinchu 30013, Taiwan R.O.C.

E-mail: kazuyuki.furuuchi@manipal.edu, ykoyama@phys.nthu.edu.tw

Received July 16, 2014
Revised November 17, 2014
Accepted January 26, 2015
Published February 23, 2015

Abstract. Motivated by the recent detection of B-mode polarization of CMB by BICEP2 which is possibly of primordial origin, we study large field inflation models which can be obtained from higher-dimensional gauge theories. The constraints from CMB observations on the gauge theory parameters are given, and their naturalness are discussed. Among the models analyzed, Dante’s Inferno model turns out to be the most preferred model in this framework.

Keywords: inflation, string theory and cosmology, particle physics - cosmology connection, cosmological applications of theories with extra dimensions

ArXiv ePrint: 1407.1951
1 Introduction

Cosmic inflation [1–6] is a leading paradigm in the study of very early universe. Inflation can explain not only the observed homogeneity and isotropy of the universe over the super-horizon scale but also the tiny deviations from them [7–11]. The agreement between the general theoretical predictions of the standard slow-roll inflation and the recent precise CMB measurements [12] is rather impressive.

Recently, another important clue from CMB observations came in. BICEP2 team reported detection of B-mode polarization at degree angular scales [13]. While the important foreground analysis remains to be worked out in the future, if the detected B-mode polarization turns out to be of primordial origin, it will have tremendous impacts on inflationary cosmology and the understanding of our universe at its very beginning: the tensor-to-scalar ratio fixes the energy scale at the time of inflation; another important consequence of the large tensor-to-scalar ratio is that it requires trans-Planckian inflaton field excursion via the Lyth bound [14]. This poses a challenge for constructing viable inflation models, since it is difficult to protect the flatness of the potential from quantum corrections over trans-Planckian field range in effective field theory framework. Thus the large tensor-to-scalar ratio might require the knowledge of physics near the Planck scale. However, this is not the only theoretical possibility: even if the effective field range of the inflaton is trans-Planckian, field ranges in the defining theory can be sub-Planckian [15–30]. A subclass of this type of models which is specified below will be of our interest.

It has been known that a gauge symmetry in higher dimensions gives rise to an approximate shift symmetry in a four-dimensional scalar potential [31, 32], and this mechanism was

\[1\text{This is a partial list of references on such models, we picked up papers whose interests are relatively close to that of the current paper.}\]
employed in [33] (see also [34]) to construct a version of natural inflation [35] (extra-natural inflation). The original aim of [33] was to construct a large field inflation model (inflation model in which inflaton makes trans-Planckian field excursion) within the framework of ef- fective field theory. But it was already noticed by the authors of [33] that the embedding of extra-natural inflation to string theory was difficult, and this point was further examined in [36]. Then, it was suggested that the underlying reason for the difficulty was the extremely small gauge coupling which was required to explain the CMB data in extra-natural inflation [37]. The authors of [37] proposed that the tiny gauge coupling causes an obstacle for coupling the effective field theory to gravity. It was motivated by the well-known argument against the existence of global symmetry in quantum gravity based on processes involving black holes (see [38] for recent discussions and references for earlier works). When the gauge coupling is turned to zero, the gauge symmetry is physically indistinguishable from a global symmetry. If the limit to the zero gauge coupling is smooth, something must prevent the occurrence of the global symmetry. The answer suggested in [37] was that when the gauge coupling becomes small, the UV cut-off scale where the effective field theory breaks down must be lowered. More precisely, they proposed that there is an upper bound on the UV cut-off scale Λ:

$$\Lambda \lesssim g M_P,$$  \hspace{1cm} (1.1)

where $g$ is the gauge coupling and $M_P$ is the four-dimensional Planck mass. The authors of [37] showed that the bound (1.1) follows from a conjecture that there must be a particle whose mass is smaller than its charge in certain unit (Weak Gravity Conjecture, abbreviated as WGC below). The basis of their arguments which lead to WGC are quite robust, and in this paper we will take WGC seriously. A brief review on WGC is given in appendix B.

In this paper, we examine large field inflation models which can be obtained from higher-dimensional gauge theories. We restrict ourselves to one-form gauge fields in higher dimensions, though these can appear from higher-form gauge fields in even higher dimensions with smaller compactification size. While in this paper we restrict ourselves to the simplest Abelian gauge groups, it is straightforward to extend or embed our models to those with non-Abelian gauge groups. Non-Abelian higher-form fields are known to be theoretically quite involved (see e.g. [39]), and our strategy of first concentrating on one-form gauge fields may have an advantage in bypassing these theoretical complications while still covering large portion of theory space. Such one-form gauge fields are also essential ingredients in the Standard Model of particle physics, and it is natural to expect that one-form gauge fields will continue to be an essential part of the new physics beyond the Standard Model. These constitute our basic motivations to consider one-form gauge theories in higher dimensions.

We are particularly interested in the consequences of WGC, and will assume that it is correct.\footnote{Another possibility would be that WGC does not always hold, but holds in the dominant majority of string vacua. While this is an interesting theoretical possibility, it is not relevant for the discussion of naturalness below as long as it is extremely likely to be in a vacuum in which WGC holds.} Thus the original extra-natural inflation will be excluded from our study.\footnote{There is a possibility that WGC completely excludes natural parameter space for effective field theory. In this case, one may respect the constraints from WGC and accept the unnatural values of the parameters. See [40] for an argument on an example in particle physics model. In this paper we will be interested in natural parameter space allowed by WGC.} This naturally lead us to consider models of the type mentioned above: those in which the field ranges in the defining theory are sub-Planckian but the inflaton effectively travels trans-Planckian field range. As higher-dimensional gauge theories reduce to so-called axion models,
Gauge couplings
Compactification radius
Charges
$-\log_{10}[(L\,Mp)^2] \lesssim \log_{10}[g^2] \lesssim 0$
$\log_{10}[1/(L\,\text{GeV})] \sim 3 - 17$
$n \sim \mathcal{O}(1)$

Table 1. Expected parameter ranges from higher-dimensional gauge theory. $g$ is the gauge coupling in four-dimension. $L$ is the compactification radius of the fifth dimension. $n$ represents charge of a matter measured in unit of the minimal charge in the model.

we examined all the major axion inflation models of the above mentioned type so far known to us, at least in their simplest form. These include: single-field Axion Monodromy model (AM) [16, 17], Dante’s Inferno model (DI) [18], Axion Alignment model (AA) [15, 21–23] and Axion Hierarchy model (AH) [19, 20]. We will examine the constraints from CMB data on gauge theory parameters and discuss their naturalness in the effective field theory framework. However, for the tensor-to-scalar ratio, the above mentioned BICEP2 result does not give conclusive value due to the uncertainty in the foreground [41, 42]. In this paper, we would like to explore the possibility that the large tensor-to-scalar ratio is real considering its impact if it turns out to be the case. We choose $r = 0.16$ at the pivot scale as a reference value [43], but this should be taken as an assumption at this moment.

Table 1 summarizes the expected parameter ranges in our models. While we will not go into full Bayesian model comparison (see e.g. [44, 45]), in principle we can go through it, and in that case our prior can be built based on table 1. In table 1, $g$ stands for four-dimensional gauge coupling which is obtained from higher-dimensional gauge theory as

$$\frac{1}{g^2} = \frac{2\pi L}{g_5^2},$$

(1.2)

where $g_5$ is the five-dimensional gauge coupling and $L$ is the compactification radius of the fifth dimension. $g_5^2$ has dimension of length which can be independent from the compactification radius. A priori, we do not have knowledge of their corresponding energy scales besides the upper bound by the Planck scale and lower bound from high energy experiments like LHC. Therefore, the log-flat prior would be appropriate for $g$ and $L$, if we were to proceed to Bayesian model comparison. The lower bound in $g$ in table 1 is imposed by WGC, while the upper bound comes from applicability of perturbation theory. The expected value of charges is shown in table 1 in unit of the minimal charge in the model. It reflects the theoretical belief of the current authors that extraordinary large charge is unlikely or rare in nature.

Table 2–4 show the allowed parameter ranges after taking into account CMB data and assuming $r = 0.16$. Strictly speaking, it is more appropriate to show the allowed parameter range in multi-dimensional parameter space, as the allowed range for one parameter depends on other parameters in general. However, even in the current simplified analysis, one immediately notices that somewhat unusual parameter ranges appear in table 4: AA and AH have at least one charge which is more than $\mathcal{O}(100)$ in unit of minimal charge in the model. Although theories with such a large charge number have been considered, (e.g. see [46] for the so-called milli-charged dark matter, where an issue related to WGC is discussed), such theories look somewhat artificial. This view of the current authors had been reflected in the expected charge number in table 1. On the other hand, the charge of AM is in a natural range, but this model has its own naturalness issue which will be explained in

\footnote{As can be seen from the derivation of table 4 in the main body, this conclusion does not depend on other parameters.}
Model | Gauge coupling(s) \\
--- | --- \\
AM | $-8 \lesssim \log_{10}[g^2] \lesssim 0$ \\
DI | $-1 \lesssim \log_{10}[g_A^2] \lesssim 0$, $-3 \lesssim \log_{10}[g_B^2] \lesssim -2$ \\
AA | $-10 \lesssim \log_{10}[g_A^2], \log_{10}[g_B^2] \lesssim -4$ \\
AH | $-10 \lesssim \log_{10}[g_A^2] \lesssim -4$, $-10 \lesssim \log_{10}[g_B^2] \lesssim 0$

Table 2. Constraints on gauge couplings after taking into account CMB data with the assumption $r = 0.16$.

Model | Compactification radius \\
--- | --- \\
AM | $\log_{10}[1/(L \text{ GeV})] \sim 14 - 16$ \\
DI | $\log_{10}[1/(L \text{ GeV})] \sim 17$ \\
AA | $\log_{10}[1/(L \text{ GeV})] \sim 14 - 17$ \\
AH | $\log_{10}[1/(L \text{ GeV})] \sim 14 - 17$

Table 3. Constraints on compactification radius after taking into account CMB data with the assumption $r = 0.16$.

Model | Charge(s) \\
--- | --- \\
AM | $\mathcal{O}(1)$ \\
DI | $\mathcal{O}(1)$ \\
AA | $\max(|m_1, m_2|) \gtrsim \mathcal{O}(100)$ \\
AH | $m_1 \gtrsim \mathcal{O}(100)$

Table 4. Constraints on charges after taking into account CMB data with the assumption $r = 0.16$.

section 2. Charges in DI are in the expected range given in table 1. From these analysis, one immediately sees that DI is preferred among the models considered.

The organization of the rest of the paper is as follows. We start with single-field Axion Monodromy model in section 2. In section 3 we study Dante’s Inferno model. In section 4 Axion Alignment model and Axion Hierarchy model are studied.\footnote{In\cite{23} aligned natural inflation from higher-dimensional gauge theory similar to ours was studied, but the four-dimensional WGC was not imposed.} For each model we obtain it from higher-dimensional gauge theory, study the constraints from the CMB observations to the parameters of the gauge theory and discuss naturalness of the parameters. We summarize with discussions on future directions in section 5.

2 Single-field axion monodromy

We begin with single-field axion monodromy inflation\cite{16, 17}. The relevant inflaton potential is of the form

$$V(A) = \frac{1}{2} m^2 A^2 + \Lambda^4 \left(1 - \cos \left(\frac{A}{f}\right)\right).$$ (2.1)
The potential (2.1) can be obtained from a five-dimensional gauge theory with an action
\[ S = \int d^5 x \left[ -\frac{1}{4} F_{MN} F^{MN} - \frac{1}{2} m^2 (A_M - g_5 \partial_M \theta)^2 + \text{(matters)} \right]. \] (2.2)

We introduced the Stueckelberg mass term which gives rise to the quadratic potential in (2.1).\(^6\)

We take the gauge group to be compact U(1).\(^7\) Then, the Stueckelberg field \( \theta \) is an angular variable with the identification
\[ \theta \sim \theta + \frac{2\pi}{g_5}. \] (2.3)

This allows \( \theta \) to have a winding mode:
\[ \theta(x, x^5) = \frac{x^5}{g_5 L} w + \sum_n \theta_n(x) e^{i \frac{2\pi n}{L} x^5} \] (2.4)

Here, \( x \) are coordinates in visible large space-time dimensions, and \( x^5 \) is the coordinate of the fifth direction compactified on a circle with radius \( L \). The winding number \( w \) is an integer. If one takes into account all the winding sectors, the spectrum of the model is invariant under the shift of \( A \) by \( 2\pi f \), while starting from a sector with given winding number the shift leads to the monodromy property [16, 17]. At one-loop, the following potential is generated:
\[ V(A) = \frac{1}{2} m^2 (A - 2\pi f w)^2 + \Lambda^4 \left( 1 - \cos \left( \frac{A}{f} \right) \right). \] (2.5)

See appendix A.1 for the outline of the calculation of the one-loop effective potential. For a sector with a given winding number, by redefining \( A \) by a constant shift one obtains (2.1). The inflaton field \( A \) in the potential (2.1) is the zero-mode of the gauge field:
\[ A \equiv A_{5(0)}. \] (2.6)

The parameters of the axion monodromy model (2.1) are related to the parameters of the higher-dimensional gauge theory as follows:
\[ f = \frac{1}{g(2\pi L)}, \quad \Lambda^4 = \frac{c}{\pi^2 (2\pi L)^4}, \quad c \sim \mathcal{O}(1), \] (2.7)

where \( g \) is the four-dimensional gauge coupling which is related to the five-dimensional gauge coupling \( g_5 \) as
\[ g = \frac{g_5}{\sqrt{2\pi L}}. \] (2.8)

The constant \( c \) in (2.7) depends on the matter contents charged under the gauge group. In (2.7) we have assumed that both the number of the matter fields and their charges are of order one, which we think natural.

\(^6\) We chose the massless charged fermion for an illustrative purpose. We can introduce mass term for the fermion or include charged massive scalars in a similar way.

\(^7\) Massive gauge fields can arise via the Higgs mechanism. However, the expectation value of the radial component of the Higgs field, which determines the mass of the gauge field, is affected by the large inflaton expectation value, as the inflaton originates from gauge field in the current model and couples to the Higgs field as such. Then the current analysis does not apply. For a recent review on the use of Stueckelberg fields in axion monodromy inflations in string theory, see [47].

\(^8\) It has been argued that in models which can be consistently coupled to quantum gravity, all the continuous gauge symmetries are compact [38].
If one considers all possible winding numbers of $\theta$, the whole theory is invariant under the shift $A \rightarrow A + 2\pi f$. Thus the field $A$ takes values on a circle with radius $f$. Starting from a given winding number sector, the quadratic potential reveals the phenomenon of monodromy: the potential energy does not return the same under the shift of $A$ by $2\pi f$. Thus one can effectively achieve trans-Planckian field excursion of $A$ even if the original period of $A$ was below the reduced Planck scale $M_P = 2.4 \times 10^{18}$ GeV, by going round the circle several times. This is an important feature of the model, because examples in string theory so far constructed and WGC suggest $2\pi f \lesssim M_P$ for an axion decay constant $f$, which forbids trans-Planckian field excursion of the axion if there were no monodromy (see appendix B for the assertions of WGC we adopt in this paper).

When the slope of the sinusoidal potential is much smaller than that of the mass term in (2.1), the model effectively reduces to chaotic inflation.\(^9\) This condition is written as

$$\frac{\Lambda^4}{f} \ll m^2 A_*, \quad (2.9)$$

where $A_*$ is the value of $A$ when the pivot scale exited the horizon. Using (2.7), this condition becomes

$$\frac{3g}{\pi^2} \frac{1}{(2\pi L)^3} \ll m^2 A_*, \quad (2.10)$$

or

$$\frac{1}{L} < 2\pi \left( \frac{\pi^2}{3g} m^2 A_* \right)^{1/3}. \quad (2.11)$$

We review the constraints from CMB observations on chaotic inflation in appendix C.1. Putting the values of $m^2$ and $A_*$ given in (C.15) and (C.14) for $r = 0.16$ and $N_* \approx 50$, we obtain

$$\frac{1}{L} < g^{-1/3} \times 3.2 \times 10^{16}$GeV. \quad (2.12)$$

Note that the energy scale of the compactification should not be smaller than the Hubble scale during inflation, otherwise the use of the four-dimensional Einstein equation is not justified. From (C.11), this gives

$$\frac{1}{L} > 1.0 \times 10^{14}$GeV. \quad (2.13)$$

If there were no sinusoidal potential, when one takes $m^2$ to zero the shift symmetry $A \rightarrow A + c$ ($c$: constant) recovers. Thus small $m^2$ is natural in the sense of ’t Hooft [50]. In order for the inflaton to achieve trans-Planckian field excursion, this shift symmetry must be a good symmetry at the Planck scale. Whether this is the case or not is a problem beyond the scope of the higher-dimensional gauge theory, which is an effective field theory. One needs to work in a theory of quantum gravity to study this issue. In other words, while the whole theory is invariant under the shift of the field $A$ by $2\pi f$, starting from a given winding number the potential of $A$ is not periodic. And the large $A$ behavior of the non-periodic part of the potential has the usual UV issue of effective field theory.

\(^9\)See [48, 49] for the case in which the sinusoidal potential is not totally negligible. From appendix A of [49] one can show that the effect of the sinusoidal potential is proportional to $L^{-3}$ and thus quickly suppressed as one moves away from the bound in (2.11).
3 Dante’s Inferno

Next we study Dante’s Inferno model [18], which is a two-axion model with the following potential:

$$V(A, B) = \frac{1}{2} m_A^2 A^2 + \Lambda^4 \left( 1 - \cos \left( \frac{A}{f_A} - \frac{B}{f_B} \right) \right). \quad (3.1)$$

The potential (3.1) can be obtained from a gauge theory in higher dimensions with the action

$$S = \int d^5x \left[ -\frac{1}{4} F_{MN}^A F^{AMN} - \frac{1}{2} m_A^2 (A_M - g_{A5} \partial_M \theta)^2 - \frac{1}{4} F_{MN}^B F^{BMN} 
- i \bar{\psi} \Gamma^M (\partial_M + ig_{A5} A_M - ig_{B5} B_M) \psi \right]. \quad (3.2)$$

We consider the case where both of the gauge groups are compact $U(1)$, which we refer to as $U_A(1)$ and $U_B(1)$. Here, as an illustration, we consider fermionic matter, but the case with bosonic matters can be studied in essentially the same way. The one-loop effective potential of this model produces the second term in (3.1) with

$$f_A = \frac{1}{g_A(2\pi L)}, \quad f_B = \frac{1}{g_B(2\pi L)}, \quad (3.3)$$

and

$$\Lambda^4 \simeq \frac{3}{\pi^2} \left( \frac{1}{(2\pi L)^4} \right). \quad (3.4)$$

Here, $g_A$ and $g_B$ are four-dimensional gauge couplings which are related to the five-dimensional gauge couplings $g_{A5}$ and $g_{B5}$ as

$$g_A = \frac{g_{A5}}{\sqrt{2\pi L}}, \quad g_B = \frac{g_{B5}}{\sqrt{2\pi L}}. \quad (3.5)$$

It is convenient to rotate the fields as

$$\begin{pmatrix} \tilde{B} \\ \tilde{A} \end{pmatrix} = \begin{pmatrix} \cos \gamma & \sin \gamma \\ -\sin \gamma & \cos \gamma \end{pmatrix} \begin{pmatrix} B \\ A \end{pmatrix}, \quad (3.6)$$

where

$$\sin \gamma = \frac{f_A}{\sqrt{f_A^2 + f_B^2}}, \quad \cos \gamma = \frac{f_B}{\sqrt{f_A^2 + f_B^2}}. \quad (3.7)$$

Then the potential (3.1) takes the form

$$V(\tilde{A}, \tilde{B}) = \frac{m_A^2}{2} \left( \tilde{A} \cos \gamma + \tilde{B} \sin \gamma \right)^2 + \Lambda^4 \left( 1 - \cos \frac{\tilde{A}}{f} \right), \quad (3.8)$$

where

$$f = \frac{f_A f_B}{\sqrt{f_A^2 + f_B^2}}. \quad (3.9)$$
In this model, the regime of interest is\(^{10}\)

\[
2\pi f_A \ll 2\pi f_B \lesssim M_P, \tag{3.10}
\]

\[
\Lambda^4 f \gg m_A^2 A_{\text{in}}, \tag{3.11}
\]

where \(A_{\text{in}}\) is the initial condition set at the beginning of the observable inflation and we require it to be in the range \(f \ll A_{\text{in}} < M_P\). Notice that the condition (3.10) implies in the leading order in \(f_A/f_B\)

\[
\cos \gamma \simeq 1, \quad \sin \gamma \simeq \frac{f_A}{f_B}, \quad f \simeq f_A. \tag{3.12}
\]

We require that the excitation in \(\tilde{A}\) direction is much heavier than the Hubble scale during inflation so that they can be safely integrated out:

\[
\frac{\partial^2}{\partial A^2} V(\tilde{A}, \tilde{B}) > H^2. \tag{3.13}
\]

From (3.11) and \(f \ll A_{\text{in}}\) this reads

\[
\frac{3g_A^2}{\pi^2 L^2} > H^2. \tag{3.14}
\]

After integrating out \(\tilde{A}\), we obtain the following effective potential for \(\tilde{B}\) which we rewrite as \(\phi \equiv \tilde{B}\) \([18]\):

\[
V_{\text{eff}}(\phi) = \frac{m^2}{2} \phi^2, \quad m \equiv \frac{f_A}{f_B} m_A, \tag{3.15}
\]

to leading order in \(f_A/f_B\). Thus Dante’s Inferno model effectively reduces to chaotic inflation, with \(\phi\) being the inflaton. The constraints from CMB observations on chaotic inflation are summarized in appendix C.1. Using these inputs, now we examine the CMB constraints on the parameters of the higher-dimensional gauge theory. We will take the number of e-fold \(N_e \simeq 50\) and the tensor-to-scalar ratio \(r = 0.16\) (see appendix C for the detail and the notations used below). From (3.3), the condition (3.10) reads in terms of gauge theory parameters as

\[
g_A \gg g_B, \tag{3.16}
\]

and

\[
\frac{1}{g_B(2\pi L)} \lesssim M_P. \tag{3.17}
\]

Chaotic inflation is a large field inflation model in which the inflaton travels trans-Planckian field distance \(\Delta \phi \equiv \phi_e - \phi_e \simeq 14 M_P\), see (C.14). However, the original fields in the current model, \(A\) and \(B\) (which were the zero-modes of the higher-dimensional gauge theory), do not need to make trans-Planckian field excursion.

Regarding the field \(A\), its initial value \(A_{\text{in}}\) is restricted as

\[
A_{\text{in}} \simeq \frac{f_A}{\sqrt{f_A^2 + f_B^2}} \phi_e \simeq \frac{f_A}{f_B} \times 14 M_P. \tag{3.18}
\]

\(^{10}\)Be aware of the difference between (2.9) and (3.11).
Thus $A_{in}$ is sub-Planckian if

$$f_A < \frac{1}{14} f_B.$$  (3.19)

From (3.3), in terms of gauge couplings (3.19) amounts to

$$g_A > 14 g_B.$$  (3.20)

This condition should be compared with (3.16). On the other hand, field $B$ is periodic and its field range $2\pi f_B$ is bounded from above by $M_P$, as noted in (3.10).

There is also a lower bound on the inverse compactification radius. Using (3.15) and (3.18), the condition (3.11) can be rewritten as

$$\frac{3}{\pi^2 (2\pi L)^4} \gg f_A \left( m f_B f_A / f_B \right)^2 f_A f_B \times 14 M_P.$$  (3.21)

Using (3.3) and putting the value of $m$ in (C.15), we obtain

$$g_B^{1/3} > 3.2 \times 10^{16} \text{GeV}.$$  (3.22)

Together with (3.17) we have

$$g_B^{-1/3} \times 3.2 \times 10^{16} \text{GeV} < \frac{1}{L} \lesssim g_B \times 2.4 \times 10^{18} \text{GeV}.$$  (3.23)

(3.23) immediately implies $g_B \gtrsim 0.04$. On the other hand, in order for our one-loop effective potential to be valid, the gauge coupling should not be large, $g_A \lesssim \mathcal{O}(1)$. Together with (3.20), we have

$$0.04 \lesssim g_B \lesssim \mathcal{O}(0.1).$$  (3.24)

For $g_B = 0.04$ we have

$$9.2 \times 10^{16} \text{GeV} < \frac{1}{L} \lesssim 9.6 \times 10^{16} \text{GeV},$$  (3.25)

while for $g_B = 0.1$ we have

$$6.8 \times 10^{16} \text{GeV} < \frac{1}{L} \lesssim 2.4 \times 10^{17} \text{GeV}.$$  (3.26)

See figure 1 for the values of $g_B$ in between. We observe that the allowed values of the gauge couplings and the compactification radius of the gauge theory are rather restricted, which will be advantageous for the model to be predictive. Note that the above compactification scales are high enough so that the use of the four-dimensional Einstein gravity is justified, $1/L \gg H \sim 10^{14} \text{GeV}$ (see (C.11)).

For completeness, we check that (3.13) is satisfied. It gives

$$\frac{g_A}{2\pi L} \gtrsim \frac{\pi}{\sqrt{3}} H.$$  (3.27)

Putting the value from appendix C (C.11) we obtain

$$\frac{g_A}{2\pi L} \gtrsim 2 \times 10^{14} \text{GeV}.$$  (3.28)

This is readily satisfied for the above values of $g_A$ and $L$. 
Now we turn to another feature of the model which could be potentially constrained by CMB data. The shift symmetry allows the following axionic coupling to gauge fields:

\[ S_{AC} = \int d^4x \frac{\alpha_i \sigma_i}{4f_i} F_{\mu\nu} \tilde{F}^{\mu\nu}, \]  

(3.29)

where \( \sigma_i \) is an axion, \( f_i \) is its decay constant and \( \alpha_i \) is a constant parameter. \( i \) labels axions when there are more than one, in the current case \( i \) labels the field \( A \) and the field \( B \) (we just label them as \( i = A \) and \( i = B \), respectively). How the coupling (3.29) arises from higher-dimensional gauge theory is explained in appendix A.2. Contributions to CMB power spectrum, non-Gaussianity and primordial gravitational waves through this coupling have been studied in [51–55]. These effects are mainly controlled by the following parameter:

\[ \xi_i \equiv \frac{\alpha_i \dot{\sigma}_i}{2f_i H}. \]  

(3.30)

The current observational bound is given as [53, 55]

\[ \xi_i \lesssim 3. \]  

(3.31)

To obtain \( \xi_i \ (i = A, B) \) in (3.30), we first need to know the time derivatives of fields \( A \) and \( B \). In \( \tilde{A} \) direction, we had

\[ \dot{\tilde{A}} = 0. \]  

(3.32)

On the other hand, \( \tilde{B} \) is the inflaton which slowly rolls down the potential. From (C.9) we estimate

\[ \frac{\dot{\tilde{B}}^2}{H^2M_P^2} \sim \frac{M_P^2}{2} \left( \frac{V'(\tilde{B})}{V(\tilde{B})} \right)^2 \sim 0.01. \]  

(3.33)

From (3.32) and (3.33) we can estimate \( \dot{A} \) and \( \dot{B} \) through

\[ \dot{A} = \sin \gamma \dot{\tilde{B}} + \cos \gamma \dot{A} \sim \frac{f_A}{f_B} \dot{\tilde{B}}, \]  

(3.34)

\[ \dot{\tilde{B}} = \cos \gamma \dot{\tilde{B}} - \sin \gamma \dot{A} \sim \dot{\tilde{B}}. \]  

(3.35)
On the other hand, $\alpha_i (i = A, B)$ can be obtained as in (A.17):

$$
\alpha_A = \frac{g_A^2 k_A}{4\pi^2}, \quad \alpha_B = \frac{g_B^2 k_B}{4\pi^2}
$$  \quad (3.36)

Putting (3.33), (3.34), (3.35) and (3.36) into the definition (3.30), we arrive at

$$
\xi_A \lesssim \frac{g_A^3 k_A}{4\pi} L M_P \times \frac{0.1}{14},
$$  \quad (3.37)

$$
\xi_B \sim \frac{g_B^3 k_B}{4\pi} L M_P \times 0.1,
$$  \quad (3.38)

In deriving (3.37) we have used (3.19). As we have assumed $g_A \lesssim \mathcal{O}(1)$, by putting $L \sim \mathcal{O}(10^{17})$ GeV, we obtain

$$
\xi_A \lesssim k_A \times \mathcal{O}(10^{-2}).
$$  \quad (3.39)

On the other hand, from $g_B \lesssim \mathcal{O}(0.1)$ in (3.24), we obtain

$$
\xi_B \lesssim k_B \times \mathcal{O}(10^{-3}).
$$  \quad (3.40)

As argued in appendix A.2, we expect $k_A, k_B \sim \mathcal{O}(1 - 10)$. In this case the observational bound $\xi_i \lesssim 3$ is satisfied for $i = A, B$.

4 Axion alignment and axion hierarchy

In this section we study aligned axion inflation [15, 21, 22] and hierarchical axion inflation [19, 20] from higher-dimensional gauge theory perspective. Both models can be described by the potential of the form

$$
V(A, B) = \Lambda_1^4 \left(1 - \cos \left(\frac{m_1}{f_A} A + \frac{n_1}{f_B} B\right)\right) + \Lambda_2^4 \left(1 - \cos \left(\frac{m_2}{f_A} A + \frac{n_2}{f_B} B\right)\right).
$$  \quad (4.1)

Upon field rotation

$$
\begin{pmatrix}
\phi_s \\
\phi_l
\end{pmatrix} = \begin{pmatrix}
\cos \zeta & \sin \zeta \\
-\sin \zeta & \cos \zeta
\end{pmatrix} \begin{pmatrix}
A \\
B
\end{pmatrix},
$$  \quad (4.2)

with

$$
\cos \zeta = \frac{f_s}{f_A} m_1, \quad \sin \zeta = \frac{f_s}{f_B} n_1,
$$  \quad (4.3)

$$
f_s = \frac{1}{\sqrt{\frac{m_1^2}{f_A^2} + \frac{n_1^2}{f_B^2}}},
$$  \quad (4.4)

the potential (4.1) takes the form

$$
V(\phi_s, \phi_l) = \Lambda_1^4 \left(1 - \cos \left(\frac{\phi_s}{f_s} \right)\right) + \Lambda_2^4 \left(1 - \cos \left(\frac{\phi_s}{f_s} + \frac{\phi_l}{f_l}\right)\right),
$$  \quad (4.5)

where

$$
f_l = \frac{\sqrt{m_1^2 f_B^2 + n_1^2 f_A^2}}{m_1 n_2 - m_2 n_1}, \quad f_s' = \frac{1}{f_s} \left(\frac{m_1 n_2}{f_A^2} + \frac{n_1 m_2}{f_B^2}\right).
$$  \quad (4.6)
The potential (4.1) can be obtained from a higher-dimensional gauge theory with following action:

\[
S = \int d^5x \left[ -\frac{1}{4} F_{MN}^A F^{AMN} - \frac{1}{4} F_{MN}^B F^{BMN} 
    - i\bar{\psi} \Gamma^M (\partial_M + ig_{A5} m_1 A_M + ig_{B5} n_1 B_M) \psi
    - i\bar{\chi} \Gamma^M (\partial_M + ig_{A5} m_2 A_M - ig_{B5} n_2 B_M) \chi \right].
\]

The parameters in the potential (4.1) and the higher-dimensional gauge theory are related as

\[
f_A = \frac{1}{g_A (2\pi L)}, \quad f_B = \frac{1}{g_B (2\pi L)},
\]

where \(g_A\) and \(g_B\) are four-dimensional gauge couplings

\[
g_A \equiv g_{A5} / \sqrt{2\pi L}, \quad g_B \equiv g_{B5} / \sqrt{2\pi L}.
\]

Anticipating UV completions such as string theory, it is natural that charges are quantized with respect to the unit charge. Thus we assume \(m_1, m_2, n_1, n_2\) are all integers.

Aligned axion inflation is obtained in the regime

\[
|f_l| \gg f_A, f_B \text{ from (4.6). Notice that } |f_l| \text{ is at largest the order of max}(|m_1| f_B, |n_1| f_A).\]

On the other hand, as explained in appendix C.2, \(r \approx 0.16\) requires \(|f_l| \gtrsim 20 M_P\). Since from WGC we have \(2\pi f_A, 2\pi f_B \lesssim M_P\), this requires \(\max(|m_1|, |n_1|) \gtrsim 20 \times 2\pi\). A matter with such a large charge seems to us quite unnatural, considering that the energy scale under consideration is rather high (\(H \sim 10^{14}\) GeV).

Next we turn to the hierarchical axion inflation in higher-dimensional gauge theory. This model corresponds to taking \(n_2 = 0\) in (4.1). Then (4.6) reduces to

\[
|f_l| = \sqrt{m_2^2 f_B^2 + n_1^2 f_A^2} / |n_1 m_2|.
\]

One further requires a hierarchy

\[
\left| \frac{f_A}{m_1} \right| \ll \frac{f_A}{|m_2|}, \frac{f_B}{|n_1|}.
\]

Then (4.11) can be approximated as

\[
|f_l| \approx \left| \frac{m_1}{n_1 m_2} \right| f_B.
\]

From WGC we have \(2\pi f_B \lesssim M_P\), thus \(|f_l| \gtrsim 20 M_P\) requires \(|m_1| \gtrsim 20 |n_1 m_2| \times 2\pi\). Such a large hierarchy between the charges in the same gauge group seems quite unnatural.\(^{12}\)

---

\(^{11}\)As we have assumed that the gauge groups are compact U(1), charges are quantized. Here we made a stronger assumption that charges are all integer multiples of the minimal charge in the theory. This can be regarded as for simplicity, the result does not change qualitatively unless one assumes highly exotic charge spectrum.

\(^{12}\)The upper bound of the gauge coupling in table 2 for AA and AH were obtained by requiring applicability of perturbation theory with these large charge number: in order for the perturbation theory to be appropriate, we need \(gn \sim O(1)\), where \(g\) is the gauge coupling and \(n\) is the maximal charge in the model.
5 Summary and discussions

In this paper we studied large field inflation models which can be obtained from higher-dimensional gauge theories. We accept WGC as our working hypothesis, and studied the constraints from CMB data on the gauge theory parameters. We consider the case with large tensor-to-scalar ratio, and used $r = 0.16$ as a reference value. We found that the allowed range of gauge theory parameters are quite constrained. Among the models studied in this paper, Dante’s Inferno model appears as the most preferred model. The allowed values of the gauge couplings and the compactification radius turned out to be quite restricted but fell within a natural range, making the model attractive for being predictive. Single-field axion monodromy model leaves the problem that whether the shift symmetry is a good symmetry or not to its UV completion. Axion alignment model and axion hierarchy model require large hierarchy among charges in the same gauge group, which makes the models rather unnatural.

The allowed values of gauge couplings in Dante’s Inferno model are in the range $0.04 - \mathcal{O}(1)$. This is in contrast to the extremely small gauge coupling $\lesssim \mathcal{O}(10^{-3})$ required for extra-natural inflation [33, 56]. The above values of gauge couplings for Dante’s Inferno model would be large enough to have interesting consequences in cosmological history or particle physics experiments in model dependent ways, which will be interesting to investigate. In particular, since gauge symmetry is a basic ingredient of the Standard Model of particle physics, it is natural to expect that the higher-dimensional gauge theories responsible for inflation are also relevant for the new physics beyond the Standard Model. If this is the case, particle physics experiments would provide complimentary data for such models. See [56, 57] for earlier investigations along this line in the case of extra-natural inflation.

Acknowledgments

KF benefited from the discussions on the BICEP2 results, naturalness and WGC at the Physical Research Laboratory (PRL) and Indian Institute of Astrophysics (IIA) during his visits including the workshop “Aspects of Cosmology” at IIA held in April 9-11, 2014. In particular, he would like to express special thanks to Namit Mahajan at PRL and Pravabati Chingangbam at IIA for the hospitality. YK’s work is supported in part by the National Science Council of Taiwan under Grant No. NSC-101-2112-M-007-021 and Taiwan String Theory Focus Group of NCTS under Grant No. NSC-103-2119-M-002-001. The authors would also like to express their gratitude to the anonymous referee for various suggestions for improvements, in particular for pointing out possible relevance of the effects of the axionic couplings in the current analysis.

A Four-dimensional effective action from higher-dimensional gauge theory

A.1 One-loop effective potential

In this appendix we outline the calculation of the one-loop effective potential in higher-dimensional gauge theories compactified on a circle. We start with the five-dimensional action

$$S = \int d^5 x \left[ -\frac{1}{4} F_{MN}^A F^{AMN} - \frac{1}{4} F_{MN}^B F^{B,MN} + \frac{1}{2} m_A^2 (A_M - g A_M \partial_M \theta)^2 + \bar{\psi} i \Gamma^M D_M \psi \right] + S_{g.f.} ,$$

(A.1)
where space-time indices $M$ and $N$ run $0, \cdots, 3$ and 5,

\[ F^A_{MN} = \partial_M A_N - \partial_N A_M, \quad F^B_{MN} = \partial_M B_N - \partial_N B_M, \]  

(A.2)

and

\[ D_M \psi = \partial_M \psi - ig_{A5} p A_M \psi - ig_{B5} q B_M \psi. \]  

(A.3)

We choose the gauge fixing term as

\[ S_{g.f.} = \int d^5x \left[ -\frac{1}{2} (\partial_M A^M)^2 - \frac{1}{2} (\partial_M B^M)^2 \right]. \]  

(A.4)

Then the total action becomes

\[ S = \int d^5x \left[ \frac{1}{2} A_N \partial_M \partial^M A_N + \frac{1}{2} B_N \partial_M \partial^M B_N + \frac{m^2}{2} (A_M - g_A \partial_M \theta)^2 + \bar{\psi} i \Gamma^M D_M \psi \right]. \]  

(A.5)

We compactify the fifth dimension on a circle with radius $L$. The Fourier expansions of the fields in the fifth dimension are

\[ A_M(x, x^5) = \frac{1}{\sqrt{2\pi L}} \sum_{n=-\infty}^{\infty} A_M(n) e^{i n x^5}, \text{ similar for } B_M, \psi, \]  

(A.6)

\[ \theta(x, x^5) = \frac{x^5}{g_5 L} + \frac{1}{\sqrt{2\pi L}} \sum_{n=-\infty}^{\infty} \theta(n) e^{i n x^5}. \]  

(A.7)

We will be interested in the effective potential for the zero-modes of the gauge fields, $A_{5(0)} \equiv A$ and $B_{5(0)} \equiv B$. At one-loop level, only the quadratic part of the matter action is relevant:

\[ S^{(2)}_{\psi} = \int d^4x \sum_{n=-\infty}^{\infty} \bar{\psi}(n) \left( i \Gamma^\mu \partial_\mu + g_A p A 5 + g_B q B 5 + \Gamma^5 \frac{n}{L} \right) \psi(n). \]  

(A.8)

Here, $\mu$ and $\nu$ run four-dimensional space-time indices $0, \cdots, 3$. Then, the one-loop effective potential is expressed as

\[ V(A, B)_{1\text{-loop}} = \text{Tr} \ln \left( -i \Gamma^5 \partial_\mu - g_A p A 5 + g_B q B 5 + \Gamma^5 \frac{n}{L} \right) \]

\[ = \frac{1}{2} \text{Tr} \ln \left( -\partial_\mu^2 + \left( \frac{n}{L} - (g_A p A + g_B q B) \right)^2 \right), \]  

(A.9)

where we have made Wick rotation and the subscript $E$ indicates the Euclidean space. The four-dimensional gauge couplings are related to the five-dimensional ones as

\[ g_A = \frac{g_{A5}}{\sqrt{2\pi L}}, \quad g_B = \frac{g_{B5}}{\sqrt{2\pi L}}. \]  

(A.10)

Employing the $\zeta$ function regularization, the effective potential becomes

\[ V(A, B)_{1\text{-loop}} = \frac{3}{\pi^2 (2\pi L)^4} \sum_{n=1}^{\infty} \frac{1}{n^5} \cos \left[ n \left( \frac{p A}{f_A} + \frac{q B}{f_B} \right) \right], \]  

(A.11)

where

\[ f_A = \frac{1}{g_A (2\pi L)} \quad f_B = \frac{1}{g_A (2\pi L)}. \]  

(A.12)
In (A.11) we have dropped the constant part, the fine tuning of which is the cosmological constant problem which we will not address in this paper. Taking the leading term $n = 1$ in (A.11) together with the tree-level potential coming from the Stueckelberg mass term, we arrive at the potential

$$V(A, B) \simeq \frac{m_2^2}{2} (A - 2\pi f w)^2 + \frac{3}{\pi^2(2\pi L)^4} \left[ 1 - \cos \left( \frac{pA}{f_A} + \frac{qB}{f_B} \right) \right],$$  \hspace{1cm} (A.13)

where we have redefined the field $B$ by an appropriate constant shift.

### A.2 Axionic couplings

The shift symmetry allows the following axionic coupling

$$S_{AC} = \int d^4x \frac{\alpha \sigma}{4f} F_{\mu\nu} \tilde{F}^{\mu\nu},$$  \hspace{1cm} (A.14)

where $\sigma$ is an axion and $\alpha$ is some constant.

In higher-dimensional gauge theory, the axionic coupling (A.14) follows from the Chern-Simons term in five-dimensional gauge theory [56]:

$$S_{CS} = \frac{k}{24\pi^2} \int \mathcal{A} F^2,$$  \hspace{1cm} (A.15)

where $\mathcal{A} = A_M dx^M$, $\mathcal{F} = d\mathcal{A} = \frac{1}{2} F_{MN} dx^M dx^N$ and $k$ is an integer. Quantum corrections to $k$ due to parity-violating charged matters are one-loop exact and proportional to the cubic powers of charges [58]. As we assume charges to be $O(1)$, we may expect $k \sim O(1 - 10)$. The 1-form $A_M dx^M$ is related to the canonically normalized gauge field $A_\mu$ in five dimensions as

$$A_M = \frac{1}{g_5} A_M,$$  \hspace{1cm} (A.16)

where $g_5$ is the five-dimensional gauge coupling. After integrating KK modes of the fifth direction we obtain the axionic coupling (A.14) with

$$\alpha = \frac{g_5^2 k}{4\pi^2},$$  \hspace{1cm} (A.17)

and

$$\sigma = \frac{A_5(0)}{g_4}.$$  \hspace{1cm} (A.18)

### B Weak Gravity Conjecture

Weak Gravity Conjecture (WGC) [37] asserts the existence of a state with charge and mass $(q, m)$ which satisfy

$$\frac{gq}{\sqrt{4\pi}} \geq \sqrt{G_N m} = \frac{m}{\sqrt{8\pi} M_P},$$  \hspace{1cm} (B.1)

(B.1) is estimated from requiring that the Coulomb repulsive force is greater than the Newtonian attractive force so that extremal black holes can loose their charge by emitting such particles. In this paper we assume the existence of a particle with the smallest unit charge, with respect to which all charges are integers. Generalization is straightforward and does not
change the result qualitatively, unless one assumes highly exotic charge spectrum. Then, the Dirac monopole with unit magnetic charge has charge and mass

\[ q_m = \frac{4\pi}{g}, \quad m_m \simeq \frac{4\pi \Lambda_{UV}}{g^2}, \]  

(B.2)

where \( \Lambda_{UV} \) is a UV scale which regularizes the mass of the Dirac monopole. Here, we used non-Abelian gauge-Higgs system as the UV completion to estimate the mass of the Dirac monopole. An important constraint for our study is obtained by applying WGC the Dirac monopole:

\[ \frac{4\pi}{g} \gtrsim \frac{4\pi \Lambda_{UV}}{g^2} \frac{1}{\sqrt{2}M_P}. \]  

(B.3)

It follows that

\[ \Lambda_{UV} \lesssim \sqrt{2}gM_P. \]  

(B.4)

This condition also follows by requiring that the Dirac monopole with unit magnetic charge is not a black hole [37]. Strictly speaking, one should take into account the running of the couplings. We assume that those runnings are not significant so that they do not alter our order of magnitude estimate. In order for the higher-dimensional gauge theory to be applicable, the compactification scale should be sufficiently below the UV cut-off scale:

\[ \frac{1}{L} \ll \sqrt{2}gM_P. \]  

(B.5)

In terms of the axion decay constant \( f = 1/(g2\pi L) \),

\[ 2\pi f \ll \sqrt{2}M_P. \]  

(B.6)

Since the above argument is an order estimate, in the main body we adopted slightly milder bound \( 2\pi f \lesssim M_P \).

C Relevant inflation models in light of BICEP2

In this appendix we review the constraints from CMB observations, in particular the possible detection of primordial tensor perturbation by BICEP2 [13], on inflation models which are relevant in this paper. The detection of the B-mode polarization by BICEP2 indicates large tensor-to-scalar ratio \( r \). In this paper we adopt a conservative value \( r = 0.16 \) at the pivot scale \( k = 0.05 \) Mpc\(^{-1} \) as a reference value, considering the uncertainty in the foreground [41] and the constraint from Planck 2013 [12, 43].

C.1 Chaotic inflation with quadratic potential

Consider quadratic potential for the inflaton

\[ V(\phi) = \frac{m^2}{2}\phi^2. \]  

(C.1)

More precisely we consider WGC in five dimensions [37]. In this case electro-magnetic dual to the one-form gauge potential is two-form gauge potential which couples to magnetic strings. Then the analysis of the forces in three spacial dimensions transverse to the string is the same.
We assume canonical kinetic term for the inflaton $\phi$. The slow-roll parameters are given by

$$\epsilon(\phi) = \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2 = \frac{2M_P^2}{\phi^2}, \quad (C.2)$$

$$\eta(\phi) = \frac{M_P^2}{2} \frac{V''}{V} = \frac{2M_P^2}{\phi^2}. \quad (C.3)$$

We will use suffix $\ast$ to indicate that it is the value when the pivot scale exited the horizon. The scalar spectral index is given by

$$n_s = 1 - 6\epsilon_\ast + 2\eta_\ast. \quad (C.4)$$

Using (C.2) and (C.3) we obtain

$$n_s = 1 - 0.04 \times \frac{r_{0.16}}{16}, \quad (C.5)$$

The scalar power spectrum and the tensor power spectrum are given as

$$P_s = \frac{V(\phi_\ast)}{24\pi^2 M_P^4 \epsilon_\ast} = 2.2 \times 10^{-9}, \quad (C.6)$$

$$P_t = \frac{2V(\phi_\ast)}{3\pi^2 M_P^4}, \quad (C.7)$$

where the last value in (C.6) is the COBE normalization. The tensor-to-scalar ratio $r$ is given by

$$r \equiv \frac{P_t}{P_s} = 16\epsilon_\ast, \quad (C.8)$$

or equivalently

$$\epsilon_\ast = 0.01 \times \left( \frac{r_{0.16}}{16} \right). \quad (C.9)$$

From (C.6) this requires

$$V(\phi_\ast) \simeq (2.0 \times 10^{16} \text{ GeV})^4 \times \left( \frac{r_{0.16}}{16} \right). \quad (C.10)$$

Via the Friedmann equation $V \simeq 3H^2M_P^2$, (C.10) corresponds to the Hubble scale

$$H_\ast \simeq 1.0 \times 10^{14} \times \left( \frac{r_{0.16}}{16} \right)^{1/2} \text{ GeV}. \quad (C.11)$$

The slow-roll inflation ends when $\epsilon(\phi_e) \sim 1$. This gives

$$\phi_e \sim \sqrt{2}M_P. \quad (C.12)$$

The number of e-folds is given as

$$N_\ast = \left| \frac{1}{M_P^2} \int_{\phi_e}^{\phi_\ast} d\phi \frac{V}{V'} \right| = \frac{1}{4M_P^2} \left[ \phi_\ast^2 - \phi_e^2 \right]. \quad (C.13)$$

Thus

$$\phi_\ast = 2M_P \sqrt{N_\ast - \frac{1}{2}} \simeq 14M_P \times \left( \frac{N_\ast - \frac{1}{2}}{50} \right)^{1/2}. \quad (C.14)$$

Putting this value to (C.1) and comparing it with (C.10), we obtain

$$m = \sqrt{\frac{2V_s}{\phi_\ast^2}} = 3.4 \times 10^{13} \text{ GeV} \times \left( \frac{50}{N_\ast - \frac{1}{2}} \right)^{1/2} \times \left( \frac{r_{0.16}}{16} \right)^{1/2}. \quad (C.15)$$
C.2 Natural inflation

The typical form of the potential for natural inflation is given by

\[ V(\phi) = \frac{V_0}{2} \left[ 1 - \cos \left( \frac{\phi}{f} \right) \right]. \]  

(C.16)

From (C.16) the slow-roll parameters are given as

\[ \epsilon(\phi) \equiv \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2 = \frac{M_P^2}{2f^2} \frac{1 + \cos \left( \frac{\phi}{f} \right)}{1 - \cos \left( \frac{\phi}{f} \right)}, \]  

(C.17)

\[ \eta(\phi) \equiv \frac{M_P^2}{2} \frac{V''}{V} = \frac{M_P^2}{f^2} \cos \left( \frac{\phi}{f} \right) \frac{1}{1 - \cos \left( \frac{\phi}{f} \right)}. \]  

(C.18)

The number of e-folds as a function of \( \phi \) is given by

\[ N(\phi) \simeq \left| \int_{\phi_e}^{\phi} d\phi \frac{1}{M_P^2 V'} V \right| = \left| \int_{\phi_e}^{\phi} d\phi \frac{f}{M_P^2} \frac{1 - \cos \left( \frac{\phi}{f} \right)}{\sin \left( \frac{\phi}{f} \right)} \right| \]

\[ = \left( \frac{f}{M_P} \right)^2 \left| \log \left[ \frac{1}{2} \left( 1 + \cos \left( \frac{\phi}{f} \right) \right) \right] \right|_{\phi_e}. \]  

(C.19)

The slow-roll inflation ends when \( \epsilon(\phi_e) \simeq 1 \), which gives

\[ \cos \left( \frac{\phi_e}{f} \right) = \frac{1 - \frac{M_P^2}{2f^2}}{1 + \frac{M_P^2}{2f^2}}. \]  

(C.20)

Plugging (C.20) into (C.19) we obtain

\[ \cos \left( \frac{\phi}{f} \right) = \frac{2e^{-\frac{M_P^2}{2f^2} N}}{1 + \frac{M_P^2}{2f^2} - 1}. \]  

(C.21)

From (C.17) and (C.21), for a given \( N_\ast \), \( r \) is determined as a function of \( f \). This is plotted in figure 2. Notice that to obtain the tensor-to-scalar ratio as large as \( r \simeq 0.16 \), we need \( f \gtrsim 20M_P \) and \( N_\ast \simeq 50 \). These values were adopted in the main body.
References

[1] A.H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, Phys. Rev. D 23 (1981) 347 [SPIRE].

[2] A.D. Linde, *A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems*, Phys. Lett. B 108 (1982) 389 [SPIRE].

[3] A.A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, Phys. Lett. B 91 (1980) 99 [SPIRE].

[4] K. Sato, *First Order Phase Transition of a Vacuum and Expansion of the Universe*, Mon. Not. Roy. Astron. Soc. 195 (1981) 467 [SPIRE].

[5] D. Kazanas, *Dynamics of the Universe and Spontaneous Symmetry Breaking*, Astrophys. J. 241 (1980) L59 [SPIRE].

[6] A. Albrecht and P.J. Steinhardt, *Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking*, Phys. Rev. Lett. 48 (1982) 1220 [SPIRE].

[7] V.F. Mukhanov and G.V. Chibisov, *Quantum Fluctuation and Nonsingular Universe. (In Russian)*, JETP Lett. 33 (1981) 532 [SPIRE].

[8] G.V. Chibisov and V.F. Mukhanov, *Galaxy formation and phonons*, Mon. Not. Roy. Astron. Soc. 200 (1982) 535 [SPIRE].

[9] A.H. Guth and S.Y. Pi, *Fluctuations in the New Inflationary Universe*, Phys. Rev. Lett. 49 (1982) 1110 [SPIRE].

[10] S.W. Hawking, *The Development of Irregularities in a Single Bubble Inflationary Universe*, Phys. Lett. B 115 (1982) 295 [SPIRE].

[11] A.A. Starobinsky, *Dynamics of Phase Transition in the New Inflationary Universe Scenario and Generation of Perturbations*, Phys. Lett. B 117 (1982) 175 [SPIRE].

[12] PLANCK collaboration, P.A.R. Ade et al., *Planck 2013 results. XXII. Constraints on inflation*, Astron. Astrophys. 571 (2014) A22 [arXiv:1303.5082] [SPIRE].

[13] BICEP2 collaboration, P.A.R. Ade et al., *Detection of B-Mode Polarization at Degree Angular Scales by BICEP2*, Phys. Rev. Lett. 112 (2014) 241101 [arXiv:1403.3985] [SPIRE].

[14] D.H. Lyth, *What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy?*, Phys. Rev. Lett. 78 (1997) 1861 [hep-ph/9606387] [SPIRE].

[15] J.E. Kim, H.P. Nilles and M. Peloso, *Completing natural inflation*, JCAP 01 (2005) 005 [hep-ph/0409138] [SPIRE].

[16] E. Silverstein and A. Westphal, *Monodromy in the CMB: Gravity Waves and String Inflation*, Phys. Rev. D 78 (2008) 106003 [arXiv:0803.3085] [SPIRE].

[17] L. McAllister, E. Silverstein and A. Westphal, *Gravity Waves and Linear Inflation from Axion Monodromy*, Phys. Rev. D 82 (2010) 046003 [arXiv:0808.0706] [SPIRE].

[18] M. Berg, E. Pajer and S. Sjors, *Dante’s Inferno*, Phys. Rev. D 81 (2010) 103535 [arXiv:0912.1341] [SPIRE].

[19] S.H.H. Tye and S.S.C. Wong, *Helical Inflation and Cosmic Strings*, arXiv:1404.6988 [SPIRE].

[20] I. Ben-Dayan, F.G. Pedro and A. Westphal, *Hierarchical Axion Inflation*, Phys. Rev. Lett. 113 (2014) 261301 [arXiv:1404.7773] [SPIRE].
[21] R. Kappl, S. Krippendorf and H.P. Nilles, \textit{Aligned Natural Inflation: Monodromies of two Axions}, \textit{Phys. Lett. B} \textbf{737} (2014) 124 [arXiv:1404.7127] [inSPIRE].

[22] C. Long, L. McAllister and P. McGuirk, \textit{Aligned Natural Inflation in String Theory}, \textit{Phys. Rev. D} \textbf{90} (2014) 023501 [arXiv:1404.7852] [inSPIRE].

[23] Y. Bai and B.A. Stefanean, \textit{Natural Milli-Charged Inflation}, arXiv:1405.6720 [inSPIRE].

[24] K. Choi, H. Kim and S. Yun, \textit{Natural inflation with multiple sub-Planckian axions}, \textit{Phys. Rev. D} \textbf{90} (2014) 023545 [arXiv:1404.6209] [inSPIRE].

[25] N. Kaloper and L. Sorbo, \textit{A Natural Framework for Chaotic Inflation}, \textit{Phys. Rev. Lett.} \textbf{102} (2009) 121301 [arXiv:0811.1989] [INSPIRE].

[26] N. Kaloper, A. Lawrence and L. Sorbo, \textit{An Ignoble Approach to Large Field Inflation}, \textit{JCAP} \textbf{03} (2011) 023 [arXiv:1101.0026] [INSPIRE].

[27] F. Marchesano, G. Shiu and A.M. Uranga, \textit{F-term Axion Monodromy Inflation}, \textit{JHEP} \textbf{09} (2014) 184 [arXiv:1404.3040] [INSPIRE].

[28] A. Ashoorioon, H. Firouzjahi and M.M. Sheikh-Jabbari, \textit{M-flation: Inflation From Matrix Valued Scalar Fields}, \textit{JCAP} \textbf{06} (2009) 018 [arXiv:0903.1481] [INSPIRE].

[29] K. Harigaya and M. Ibe, \textit{Simple realization of inflaton potential on a Riemann surface}, \textit{Phys. Lett. B} \textbf{738} (2014) 301 [arXiv:1404.3511] [INSPIRE].

[30] J. McDonald, \textit{Sub-Planckian Two-Field Inflation Consistent with the Lyth Bound}, \textit{JCAP} \textbf{09} (2014) 027 [arXiv:1404.4620] [INSPIRE].

[31] Y. Hosotani, \textit{Dynamical Mass Generation by Compact Extra Dimensions}, \textit{Phys. Lett. B} \textbf{126} (1983) 309 [INSPIRE].

[32] H. Hatanaka, T. Inami and C.S. Lim, \textit{The Gauge hierarchy problem and higher dimensional gauge theories}, \textit{Mod. Phys. Lett. A} \textbf{13} (1998) 2601 [hep-th/9805067] [INSPIRE].

[33] N. Arkani-Hamed, H.-C. Cheng, P. Creminelli and L. Randall, \textit{Extra natural inflation}, \textit{Phys. Rev. Lett.} \textbf{90} (2003) 221302 [hep-th/0301218] [INSPER].

[34] D.E. Kaplan and N.J. Weiner, \textit{Little inflatons and gauge inflation}, \textit{JCAP} \textbf{02} (2004) 005 [hep-ph/0302014] [INSPIRE].

[35] K. Freese, J.A. Frieman and A.V. Olinto, \textit{Natural inflation with pseudo-Nambu-Goldstone bosons}, \textit{Phys. Rev. Lett.} \textbf{65} (1990) 3233 [INSPIRE].

[36] T. Banks, M. Dine, P.J. Fox and E. Gorbatov, \textit{On the possibility of large axion decay constants}, \textit{JCAP} \textbf{06} (2003) 001 [hep-th/0303252] [INSPIRE].

[37] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, \textit{The String landscape, black holes and gravity as the weakest force}, \textit{JHEP} \textbf{06} (2007) 060 [hep-th/0601001] [INSPIRE].

[38] T. Banks and N. Seiberg, \textit{Symmetries and Strings in Field Theory and Gravity}, \textit{Phys. Rev. D} \textbf{83} (2011) 084019 [arXiv:1011.5120] [INSPIRE].

[39] M. Henneaux and B. Knaepen, \textit{All consistent interactions for exterior form gauge fields}, \textit{Phys. Rev. D} \textbf{56} (1997) 6076 [hep-th/9706119] [INSPIRE].

[40] C. Cheung and G.N. Remmen, \textit{Naturalness and the Weak Gravity Conjecture}, \textit{Phys. Rev. Lett.} \textbf{113} (2014) 051601 [arXiv:1402.2287] [INSPIRE].

[41] R. Flauger, J.C. Hill and D.N. Spergel, \textit{Toward an Understanding of Foreground Emission in the BICEP2 Region}, \textit{JCAP} \textbf{08} (2014) 039 [arXiv:1405.7351] [INSPIRE].

[42] PLANCK collaboration, R. Adam et al., \textit{Planck intermediate results. XXX. The angular power spectrum of polarized dust emission at intermediate and high Galactic latitudes}, arXiv:1409.5738 [INSPIRE].
B. Audren, D.G. Figueroa and T. Tram, *A note of clarification: BICEP2 and Planck are not in tension*, arXiv:1405.1390 [inSPIRE].

R. Trotta, *Bayes in the sky: Bayesian inference and model selection in cosmology*, *Contemp. Phys.* **49** (2008) 71 [arXiv:0803.4089] [inSPIRE].

M. P. Hobson, A.H. Jaffe, A.R. Liddle and P. Mukherjee eds., *Bayesian Methods in Cosmology*, Cambridge University Press, Cambridge U.K. (2009).

G. Shiu, P. Soler and F. Ye, *Millicharged Dark Matter in Quantum Gravity and String Theory*, *Phys. Rev. Lett.* **110** (2013) 241304 [arXiv:1302.5471] [inSPIRE].

R. Trotta, *Bayes in the sky: Bayesian inference and model selection in cosmology*, *Contemp. Phys.* **49** (2008) 71 [arXiv:0803.4089] [inSPIRE].

M. P. Hobson, A.H. Jaffe, A.R. Liddle and P. Mukherjee eds., *Bayesian Methods in Cosmology*, Cambridge University Press, Cambridge U.K. (2009).

G. Shiu, P. Soler and F. Ye, *Millicharged Dark Matter in Quantum Gravity and String Theory*, *Phys. Rev. Lett.* **110** (2013) 241304 [arXiv:1302.5471] [inSPIRE].

R. Flauger, L. McAllister, E. Pajer, A. Westphal and G. Xu, *Oscillations in the CMB from Axion Monodromy Inflation*, *JCAP* **09** (2010) 009 [arXiv:0907.2916] [inSPIRE].

P.D. Meerburg, *Alleviating the tension at low ℓ through axion monodromy*, *Phys. Rev.* **D** **90** (2014) 063529 [arXiv:1406.3243] [inSPIRE].

G. ’t Hooft, *Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking*, NATO Sci. Ser. **B** **59** (1980) 135.

M.M. Anber and L. Sorbo, *N-flationary magnetic fields*, *JCAP* **10** (2006) 018 [astro-ph/0606534] [inSPIRE].

N. Barnaby, E. Pajer and M. Peloso, *Gauge Field Production in Axion Inflation: Consequences for Monodromy, non-Gaussianity in the CMB and Gravitational Waves at Interferometers*, *Phys. Rev.* **D** **85** (2012) 023525 [arXiv:1110.3327] [inSPIRE].

P.D. Meerburg and E. Pajer, *Observational Constraints on Gauge Field Production in Axion Inflation*, *JCAP* **02** (2013) 017 [arXiv:1203.6076] [inSPIRE].

A. Linde, S. Mooij and E. Pajer, *Gauge field production in supergravity inflation: Local non-Gaussianity and primordial black holes*, *Phys. Rev.* **D** **87** (2013) 103506 [arXiv:1212.1693] [inSPIRE].

R.Z. Ferreira and M.S. Sloth, *Universal Constraints on Axions from Inflation*, *JHEP* **12** (2014) 139 [arXiv:1409.5799] [inSPIRE].

K. Furuuchi and J.M.S. Wu, *U(1)_{B−L} extra-natural inflation with Standard Model on a brane*, *Phys. Lett.* **B** **729** (2014) 56 [arXiv:1310.4646] [inSPIRE].

T. Inami, Y. Koyama, C.S. Lim and S. Minakami, *Higgs-Inflaton Potential in 5D Super Yang-Mills Theory*, *Prog. Theor. Phys.* **122** (2009) 543 [arXiv:0903.3637] [inSPIRE].

F. Bonetti, T.W. Grimm and S. Hohenegger, *One-loop Chern-Simons terms in five dimensions*, *JHEP* **07** (2013) 043 [arXiv:1302.2918] [inSPIRE].