Improving the Coherence Time of Superconducting Coplanar Resonators

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(Dated: September 3, 2009)

The quality factor and energy decay time of superconducting resonators have been measured as a function of material, geometry, and magnetic field. Once the dissipation of trapped magnetic vortices is minimized, we identify surface two-level states (TLS) as an important decay mechanism. A wide gap between the center conductor and the ground plane, as well as use of the superconductor Re instead of Al, are shown to decrease loss. We also demonstrate that classical measurements of resonator quality factor at low excitation power are consistent with single-photon decay time measured using qubit-resonator swap experiments.

Superconducting coplanar resonators have many important applications such as photon detection and quantum computation, and recently have been used to host arbitrary photon states generated by coupling to qubits. A key parameter limiting the performance is the energy relaxation time \( T_1 \), while dephasing is relatively unimportant. Resonator performance has typically been determined through classical measurements of the quality factor, and much work has yet to be done to understand the physics of the loss mechanisms and to optimize resonator designs for best performance.

Here we show how several previously untested loss mechanisms can be eliminated or optimized to reach a measured quality factor \( Q_m \) in the 200,000 to 400,000 range at low power, while the intrinsic quality factor \( Q_i \) is even higher after subtraction of the coupling capacitor limited \( Q_c \). We provide detailed evidence that surface loss from two-level state (TLS) defects is an important loss mechanism. Finally, we show how relatively simple quality factor measurements, when taken at low power, can be used to predict the energy decay time of resonators at the single photon level.

For this work, we measured various half-wavelength (\( \lambda/2 \)) and quarter-wavelength (\( \lambda/4 \)) coplanar resonators, as described in Fig. 1 and Table 1. Aluminum (Al) films were sputter deposited and etched with a \( \text{Cl}_2/\text{BCl}_3 \)-based reactive ion etch (RIE), whereas Rhenium (Re) was electron-beam evaporated in a molecular beam epitaxy system using a substrate temperature of 850°C and etched with \( \text{SF}_6/\text{O}_2 \)-based RIE. The films were fabricated as part of a multilayer process to enable testing with qubits. \( Q_m \) of the resonators was determined in an adiabatic demagnetization refrigerator using standard two-port transmission measurements with a vector network analyzer. \( Q_c \)'s estimated from the \( |S_{21}| \) calibration were \( \sim 400,000 \) (\( \sim 1,000,000 \)) for \( \lambda/2 \) (\( \lambda/4 \)) resonators but were not subtracted from \( Q_m \).

For all the resonators we observed an increase in \( Q_m \) as the measurement power increased and temperature \( T \) decreased. The \( T \) dependence is shown in Fig. 2(a) for representative resonators, taken with high excitation power. To avoid complications due to different geometries, we base most of the discussion on \( \lambda/4 \) resonators as they share a similar shape. The decrease in \( Q_m \) with increasing temperature is consistent with quasiparticle dissipation. In Fig. 2(b), the fractional change in the resonance frequency \( \Delta f_0/f_0 \) tends to level off around 100 mK, and its magnitude scales inversely with the center trace width \( w_c \), which is consistent with the kinetic inductance theory. The monotonic variation of resonance frequency (Fig. 2(b), inset) is slightly different than previous studies on Nb resonators, which showed a slight downturn at temperatures below \( T_c/10 \) due to TLS.

In Fig. 3(a) and (b) we plot \( Q_m \) versus excitation...
FIG. 2: (a) Plot of $Q_m$ versus temperature at high excitation power ($V_{\text{rms}} \sim 10^{-2} \text{ V}$) for Re $\lambda/4$ resonators with different center-trace widths $w_c$, as indicated. (b) Fractional variations of the resonance frequency $\Delta f_0/f_0$ versus temperature for resonators shown in (a). The variation scales inversely with $w_c$, characteristic of kinetic inductance. Inset shows the low temperature regime where a monotonic change of $f_0$ is observed down to the lowest temperature. Lines are guides to the eye.

Note that $Q_m$ increases slightly by about a factor of 2 to 3 for an increase in power by a factor $\sim 10^4$. An increase is naturally explained by TLS loss, which scales with the electric field $E$ as $1/\sqrt{1 + E^2/E_s^2}$, where $E_s$ is a saturation field for TLS loss. For a coplanar resonator with a non-uniform field distribution, numerical calculations indicate that TLS loss at the surface of the metal can be well approximated by $(1/Q_{\text{TLS}}) \sqrt{1 + (V_{\text{rms}}/V_\text{rs})^\beta}$, where $V_{\text{rms}}$ is the root-mean-squared voltage on the center conductor, $V_\text{rs} \sim w_g E_s$, and $\beta \approx 1.6$.

To explain the weak power dependence, we postulate an additional loss mechanism $1/Q_0$ that is independent of power. We find the data can be well fit with parameters $Q_0$ and $Q_{\text{TLS}}$ that are plotted in Fig. 3(c) for both the Re and Al films, along with their dependence on the coplanar gap width $w_g$. We note that $E_s$ estimated from fitted $V_\text{rs}$ (not shown) is consistent with previous measurements [14, 15, 16, 19].

TABLE I: Resonator parameters. The thickness of the metal films are 110-130 nm, and widths $w_c$ and $w_g$ were chosen to give a 50 $\Omega$ characteristic impedance, except for the $w_g = 12 \mu$m resonator. $Q_m$ is quoted at low power, and $T_1$ is determined via qubit-resonator swap experiments.

| metal, geometry | $w_c$ ($\mu$m) | $w_g$ ($\mu$m) | $f_0$ (GHz) | $Q_m$ | $Q_m/2\pi f_0$ | $T_1$ ($\mu$s) |
|-----------------|----------------|----------------|-------------|--------|----------------|----------------|
| Re, $\lambda/2$, loop | 5 | 2 | 6.3 | 100 | 2.5 | 2.0 |
| Re, $\lambda/2$, zigzag | 5 | 2 | 6.6 | 40 | 1.0 | 1.0 |
| Re, $\lambda/2$, wrap | 10 | 4 | 6.8 | 200 | 5.1 | 5.1 |
| Al, $\lambda/2$, loop | 5 | 2 | 6.7 | 60 | |
| Al, $\lambda/2$, wrap | 5 | 2 | 7.0 | 60 | |
| Al, $\lambda/2$, zigzag | 10 | 4 | 7.1 | 110 | |
| Re, $\lambda/4$, straight | 5 | 2 | 6.8 | 150 | |
| Re, $\lambda/4$, straight | 8 | 3.2 | 6.9 | 210 | |
| Re, $\lambda/4$, straight | 16 | 6.4 | 7.0 | 330 | |
| Re, $\lambda/4$, straight | 16 | 12 | 7.0 | 230 | 5.8 | 6.4 |
| Al, $\lambda/4$, straight | 5 | 2 | 7.0 | 72 | |
| Al, $\lambda/4$, straight | 8 | 3.2 | 7.0 | 110 | |
| Al, $\lambda/4$, straight | 16 | 6.4 | 7.1 | 170 | |
single photon? In Table I we compare the resonator de-
pendence, suggesting that radiation effects are small with
in Table I. We do not find a significant systematic de-
the effect of the applied field for maximum

Although a wider gap $w_e$ suppresses TLS loss, care
must be taken not to introduce loss from trapped vort-
ces, created when the film is cooled through its super-
conducting transition [14, 15]. The effect of the applied
field on $Q_m$ is shown in Fig. 4(a), which is consistent
with the requirement that the cooling field $B_c \lesssim \Phi_0/w_e^2$
must be reduced as the center trace widens. This condi-
tion indicates a preference for narrow trace widths and
holes in the ground plane. We note that using $\mu$-metal
shielding does not guarantee low magnetic fields at the
sample because components, such as microwave con-
ectors with plated Ni, may introduce stray magnetic fields.
We found that all data had to be taken after optimizing
the applied field for maximum $Q_m$.

The effect of different resonator geometries are listed
in Table I. We do not find a significant systematic de-
pendence, suggesting that radiation effects are small with
these devices.

Does $Q_m$ actually predict the energy decay rate of a
single photon? In Table I we compare the resonator de-

In conclusion, we have identified several loss mech-
nisms in superconducting coplanar resonators. The lay-
out geometry has been determined to be unimportant at
present loss levels, but loss from trapped superconduct-
ing vortices must be minimized by using narrow traces
and cooling through the transition temperature in an op-
timized magnetic field. Surface loss from two-level states
has been found to be an important decay mechanism,
and can be reduced by designing coplanar resonators with
wide gaps and by using superconductors with little sur-
face oxide, such as Re.

Acknowledgements. Devices were made at the
UCSB Nanofabrication Facility, a part of the NSF-funded
National Nanotechnology Infrastructure Network. This
work was supported by IARPA under grant W911NF-04-
1-0204 and by the NSF under grant CCF-0507227.

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SUPPLEMENTARY MATERIAL

We provide detailed calculations for Ref. [S1], mainly showing how a non-uniform electric field distribution can be accounted for in TLS dielectric loss.

We consider a coplanar resonator with a non-uniform surface charge distribution, with a cross-section as illustrated in Fig. S1. For the case of a uniform dielectric, e.g. \( \epsilon_1 = \epsilon_2 \), we use matrix inversion of the inverse capacitance matrix, defined through the equation

\[
V_i = \frac{1}{2\pi \epsilon_1 \epsilon_0} \sum_j q_j \ln r_{ij},
\]

to find the surface charge distribution \( q_j \) (and thus the field distribution) at position \( j \). Here, \( V_i \) is the potential at position \( i \), set to \( V \) at the center trace and 0 on ground pads. The separation between elements \( i \) and \( j \) is \( r_{ij} \), and \( \epsilon_0 \) is the vacuum permittivity.

For \( \epsilon_1 \neq \epsilon_2 \), the \( \epsilon_1 \) and \( \epsilon_2 \) regions can be, respectively, conformally mapped into two rectangles where field distributions are easily calculable [S2]. We find that both approaches yield similar electric field distributions. Note that \( E \) fields have tangential components that are continuous at the interface between the two dielectrics. For simplicity, we discuss the results based on uniform dielectrics. Note that these calculations were also checked with commercial software (COMSOL), which were roughly in agreement with our simplified model.

Most of the energy of the \( E \) field is concentrated around the gap of the coplanar line. For the \( w_c = 5 \) \( \mu \)m resonator, for example, \( \approx 90\% \) of the metal-oxide surface energy is stored within 1 \( \mu \)m around the gap region. Finite element analysis using COMSOL shows that \( \approx 2000 \) ppm of the total resonator energy is stored in the substrate surface and \( \approx 600 \) ppm is in the metal surface, assuming a thickness of 3 nm and a dielectric constant of 10 for surface layers.

We now calculate the power dependence of the resonator quality factor \( Q \) coming from dielectric loss of two-level states (TLS) at the metal surface. We first consider the approximation of a uniform electric field \( E \) coming from the surface region around the middle of the center-trace side wall, as indicated in Fig. S1. From the loss theory of TLS, we find

\[
\frac{1}{Q_{\text{uniform}}} \propto \frac{1}{\sqrt{1 + E_s^2/E_s^2}}
\]

\[
= \frac{1}{\sqrt{1 + (\gamma V/v_g)^2/V_s^2}}
\]

\[
= \frac{1}{\sqrt{1 + V^2/V_s^2}}
\]

(S2)

where \( E_s \) is the saturation field for the TLS, and \( \gamma = E/(V/w_g) \) is a factor obtained from numerical simulations (Eq. S1) and tabulated for three common parameter sets in Table S1. An exact relation is obtained by incorporating the computed field distribution and using a weighted sum of the TLS loss over all exposed metal surfaces

\[
\frac{1}{Q_{\text{exact}}} \propto \sum_i \frac{1}{\sqrt{1 + E_i^2/E_s^2}} \cdot \frac{E_i^2}{\sum_i E_i^2},
\]

(S3)

where the surface fields \( E_i \) are proportional to the resonator voltage \( V \).

In Fig. S2 we plot \( Q_{\text{exact}} \) (dots) versus \( V/V_s \) for a resonator with \( w_c = 5 \) \( \mu \)m, using the exact field distribution from Eqs. S1 and S3. For reference, \( Q_{\text{uniform}} \) from Eq. S2 is also plotted as the blue line. To more simply describe the results of the numerical calculations, we fit a line to the dots at the low voltage region according to

\[
\frac{1}{Q_{\text{exact}}} \propto \frac{1}{\sqrt{1 + (V/\alpha V_s)^3}},
\]

(S4)

FIG. S2: \( Q \) versus \( V/V_s \) for both uniform (blue line, Eq. S2) and exact (red dots, Eq. S3) field distributions. Red line passing through dots is a fit using Eq. S4.

TABLE S1: Fitting and scaling parameters obtained from numerical calculations based on Eqs. S1 S2 and S3 as explained in text.

| \( w_c \) (\( \mu \)m) | \( w_g \) (\( \mu \)m) | \( \alpha \) | \( \beta \) | \( \gamma \) |
|----------------|----------------|----------|----------|--------|
| 5              | 2              | 1.31     | 1.64     | 2.40   |
| 8              | 3.2            | 1.40     | 1.59     | 2.93   |
| 16             | 6.4            | 1.48     | 1.56     | 3.85   |

FIG. S1: Cross-section of a coplanar resonator showing the center trace width \( w_c \), the gap separation \( w_g \) between the center trace and the ground plane, and the metal film thickness \( t \). The middle point of the center trace side wall is indicated.
TABLE S2: Parameters from fits to the $Q_m$ versus $V$ (or $V_{\text{rms}}$) data (Fig. 3 in Ref. [S1]), according to Eqs. S5 and S6.

| Material | $w_c$ (μm) | $w_g$ (μm) | $Q_0$ ($10^5$) | $Q_{\text{TLS}}$ ($10^5$) | $V_s'$ (10$^{-5}$V) | $E_s$ (V/m) |
|----------|-------------|-------------|-----------------|---------------------------|---------------------|-------------|
| Al       | 5           | 2           | 3.16            | 1.23                      | 5.0                 | 46          |
|          | 8           | 3.2         | 3.85            | 1.41                      | 5.4                 | 35          |
|          | 16          | 6.4         | 4.39            | 2.92                      | 11.8                | 48          |
| Re       | 5           | 2           | 2.79            | 2.82                      | 6.0                 | 55          |
|          | 8           | 3.2         | 3.33            | 5.43                      | 7.8                 | 51          |
|          | 16          | 6.4         | 5.84            | 8.41                      | 10.7                | 43          |

where $\alpha$ and $\beta$ are rescaling factors, obtained from the fits, that are also listed in Table S1. Accordingly, we use $\beta = 1.6$ to fit the experimental data of $Q_m$ versus $V$ (or $V_{\text{rms}}$ as in Ref. [S1]) such that

$$\frac{1}{Q_m} = \frac{1}{Q_0} + \frac{1}{Q_{\text{TLS}}} \cdot \frac{1}{\sqrt{1 + (V/V_s')^{1.6}}}$$  \quad (S5)

with $Q_0$, $Q_{\text{TLS}}$, and $V_s'$ as fitting parameters. These fit parameters are listed in Table S2. We find that varying $\beta$ slightly does not affect the systematic trend of $Q_0$ and $Q_{\text{TLS}}$, as shown in Fig. 3(c) in Ref. [S1].

From the fitted $V_s'$ we obtain the saturation field for the metal surface layer as

$$E_s = \frac{\gamma V_s'}{\alpha w_g}.$$  \quad (S6)

In Table S2 we list the fitted saturation fields $E_s$’s from different resonators. They are reasonably close to each other, and have a magnitude close to that of a 250 nm-thick LC pancake resonator from Fig. 1 in Ref. [S3], and, taking $V_{\text{rms}} \sim 10^{-5}$ V, especially considering possible systematic errors in the absolute calibration of the transmission $|S_{21}|$ (also see Refs. [S4, S5]). $Q_0$ is power independent and comes from coupling capacitor related loss, vortex loss, and other loss mechanisms. It should also be noted that $Q_0$ may also be partly from the surface layer of the sapphire substrate, for which the saturation field might be significantly higher than $E_s$’s listed in Table S2. Support of this hypothesis is that $Q_0$ also increases with increasing $w_g$. We tried fitting the data with two $E_s$’s of different magnitudes to account for this effect, but it was unsuccessful as it introduced too many degrees of freedom for the limited number of data points. Singling out the substrate surface TLS contribution to $Q_0$ will require further measurements.

Finally we comment on the connection between results from the power measurement (Fig. 3 in Ref. [S1]) and the temperature measurement (Fig. 2 in Ref. [S1]). It has been shown in previous studies (see references in Ref. [S1]) that a downturn in resonance frequency at temperatures below $T_c/10$ indicates the existence of surface TLS. This feature is missing in our data presumably because we use lower $T_c$ materials (1 K versus 10 K). At the lowest temperatures, the TLS loss mechanism is not dominant (though important) as $Q_0$ and $Q_{\text{TLS}}$ are comparable (see Table S2). Measurements of temperature dependence are mostly consistent with quasiparticle dissipation.

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