Experimental Results on Radiative Kaon Decays*

S. Kettell
Brookhaven National Laboratory

Abstract

This paper reviews the current status of experimental results on radiative kaon decays. Several experiments at BNL, CERN and FNAL have recently or will soon complete data collection; as a result, there are several new results.

1 Introduction

Radiative kaon decays provide a testing ground for Chiral Perturbation Theory (ChPT). ChPT provides a framework for calculating the decay rates for several modes, either directly or relative to other measured modes. The radiative modes are important for determining long distance contributions to other decays of interest: the two-photon contribution to $K^0_L \rightarrow \mu^+\mu^-$, and the CP-conserving and indirect CP-violating contributions to $K^0_L \rightarrow \pi^0 e^+e^-$ and $K^0_L \rightarrow \pi^0 \mu^+\mu^-$. They are also important as backgrounds to other modes (e.g. the $K^0_L \rightarrow e^+e^-\gamma\gamma$ background to $K^0_L \rightarrow \pi^0 e^+e^-$).

A number of recent results have been reported in the literature, as well as in several recent conferences[1,2,3,4,5,6].

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2 Radiative $K_{\pi^2}$ Decays

The radiative $K_{\pi^2}$ decays: $K^+ \to \pi^+\pi^0\gamma$, $K^0_L \to \pi^+\pi^-\gamma$ and $K^0_S \to \pi^+\pi^-\gamma$ have two contributions. One is inner bremsstrahlung (IB) radiation from one of the charged particles. The second is direct emission (DE) from the vertex. The branching ratio of the IB contribution scales with the underlying $K_{\pi^2}$ decay rate. Whereas, the rate for direct emission is expected to be roughly comparable for all three modes.

A new result for $K^0_L \to \pi^+\pi^-\gamma$ from KTeV is shown in Fig. 1. The energy of the photon is shown, along with the contributions from IB and DE. The DE component is modified by a “$\rho$-propagator” that serves to soften the DE spectrum. The branching ratio for the direct emission component (see eq.1) is

$$BR(K^0_L \to \pi^+\pi^-\gamma; DE) = (3.70 \pm 0.10) \times 10^{-5}(E_{\gamma} > 20\text{MeV})$$

The ratio of direct emission to DE+IB is (see eq.2)

$$\text{DE}/(\text{DE} + \text{IB}) = 0.685 \pm 0.009 \pm 0.017$$

This result is based on $\sim5\%$ of the total KTeV data for this mode.

There are new results from E787 in the charged decay mode ($K^+ \to \pi^+\pi^0\gamma$) as well. This result is striking, in that the branching ratio is a factor of 4 lower than the previous value. The data is traditionally expressed in
terms of the variable $W$, which is defined as:

$$W^2 \equiv \frac{(p \cdot q)/m_{K^+}}{(p_+ \cdot q)/m_{\pi^+}}$$

$$= E_\gamma^2 \left( E_{\pi^+} - P_{\pi^+} \cos \theta_{\pi^+ \gamma} \right) / (m_{K^+}^2 \times m_{\pi^+}^2)$$

The new result from E787, shown in Figure 2, has about 8 times higher statistics than the old one. The branching ratio for the direct emission component, from a fit to IB and DE (see eq.4) is

$$\text{BR}(K^+ \to \pi^+\pi^0\gamma; \text{DE}) = (4.72 \pm 0.77) \times 10^{-6} \ (55 < T_{\pi^+} < 90 \text{MeV}) \quad (4)$$

This represents half of the E787 data that is currently on tape. The interference term is small, ($-0.4 \pm 1.6\%$) and the direct emission is $(1.85 \pm 0.30\%$).

The decay rate, corrected to full phase space\cite{4}, is now measured to be similar to that for $K_L$: \(\Gamma(K^+ \to \pi^+\pi^0\gamma; DE) = 808 \pm 132 \text{s}^{-1}\) vs. \(\Gamma(K_L^0 \to \pi^+\pi^-\gamma; DE) = 617 \pm 18 \text{s}^{-1}\).

KTeV also has new results on $K_L^0 \to \pi^+\pi^-e^+e^-$, where the photon has internally converted to two electrons. In addition to measuring the branching ratio\cite{5}, a T-odd observable in the angular distribution of the plane of the $\pi$-pair vs. the plane of the electron pair is observed\cite{10}. This data represents one-quarter of the final KTeV sample.

A summary of the current experimental status of radiative $K_{\pi2}$ decays is shown in Table 1.

\footnote{This correction assumes that the form factor has no energy dependence.}
| Decay Mode                              | Branching Ratio          | Citation     |
|----------------------------------------|--------------------------|--------------|
| $K_L^0 \rightarrow \pi^+ \pi^- \gamma$(DE) | $(3.70 \pm 0.10) \times 10^{-5}$ | KTeV-99[7]   |
| $K^+ \rightarrow \pi^+ \pi^0 \gamma$(DE)  | $(4.72 \pm 0.77) \times 10^{-6}$ | E787-99[8]   |
| $K_L^0 \rightarrow \pi^+ \pi^- e^+ e^-$   | $(3.63 \pm .11 \pm .14) \times 10^{-7}$ | KTeV-99[9]   |
| $K_L^0 \rightarrow \pi^0 \pi^0 \gamma$  | $< 5.6 \times 10^{-6}$   | NA31-94[11]  |
| $K_S^0 \rightarrow \pi^+ \pi^- \gamma$  | $(1.78 \pm 0.05) \times 10^{-3}$ | E731-93[12]  |
| $K_S^0 \rightarrow \pi^+ \pi^- \gamma$(DE) | $< 0.06 \times 10^{-3}$ | CERN-76[13]  |

3 $K \rightarrow \pi \gamma \gamma$ Decays

The decay $K_L^0 \rightarrow \pi^0 \gamma \gamma$ is predicted in $O(p^4)$ of ChPT, without any free parameters[14]. This is in good agreement with the experimental value[25] (see Table 3), although the experimental

4 $K^0$ to Two Real or Off-shell Photons

The decay $K_S^0 \rightarrow \gamma \gamma$ is predicted in $O(p^4)$ of ChPT, without any free parameters, to occur with $\text{BR}(K_S^0 \rightarrow \gamma \gamma) = 2.0 \times 10^{-6}$[14].
Table 2: Summary of $K \rightarrow \pi\gamma\gamma$ results.

| Decay Mode | Branching Ratio                  | Citation    |
|------------|----------------------------------|-------------|
| $K^+_L \rightarrow \pi^+\gamma\gamma$ | $(6.0 \pm 1.5 \pm 0.7) \times 10^{-7}$ | E787-97[17] |
| $K^{0}_S \rightarrow \pi^0\gamma\gamma$ | no limit | (NA48-02?) |
| $K^+_L \rightarrow \pi^+e^+e^-$ | $< 5.6 \times 10^{-10}$ | KTeV-99[18] |
| $K^{0}_S \rightarrow \pi^0\mu^+\mu^-$ | $< 3.4 \times 10^{-10}$ | KTeV-99[19] |
| $K^+_L \rightarrow \pi^+e^+e^-$ | $(2.94 \pm 0.05 \pm 0.13) \times 10^{-7}$ | E865-99[20] |
| $K^+_S \rightarrow \pi^+\mu^+\mu^-$ | $(9.22 \pm 0.60 \pm 0.49) \times 10^{-8}$ | E865-99[21] |
| $K^+_L \rightarrow \pi^0\gamma\gamma$ | $< 1.1 \times 10^{-6}$ | NA31-93[22] |
| $K^+_L \rightarrow \pi^0e^+e^-\gamma$ | $(2.20 \pm 0.48 \pm 0.11) \times 10^{-8}$ | KTeV-99[23] |
| $K^+_L \rightarrow \pi^+e^+e^-\gamma$ | $\sim 30$ events | E865-99[24] |

errors need to be reduced.

The decay $K^+_L \rightarrow \gamma\gamma$ is of interest for its importance in interpreting the measurement of $K^0_L \rightarrow \mu^+\mu^-$. The decay $K^+_L \rightarrow \mu^+\mu^-$ is sensitive to internal top quark loops, that would allow a determination of the fundamental SM parameter $\rho$. The decay is, however, dominated by the decay $K^+_L \rightarrow \gamma\gamma$ with the photons converting to a $\mu^\pm$ pair. For this reason a precise measure of $K^+_L \rightarrow \gamma\gamma$ is needed. With the improved precision on $K^0_L \rightarrow \mu^+\mu^-$ from E871, the uncertainties on $K^+_L \rightarrow \gamma\gamma$ and $\frac{B(K^+_L \rightarrow \gamma\gamma)}{B(K^0_S \rightarrow \pi^+\pi^-)}$ are now contributing significantly[26,27,28] to the uncertainty on the ratio

$$\frac{\Gamma(K^+_L \rightarrow \mu^+\mu^-)}{\Gamma(K^+_L \rightarrow \gamma\gamma)} = \left[ \frac{B(K^+_L \rightarrow \mu^+\mu^-)}{B(K^+_L \rightarrow \gamma\gamma)} \times \left[ \frac{\eta_{+-}}{\eta_{oo}} \right] \frac{B(K^0_S \rightarrow \pi^+\pi^-)}{B(K^0_S \rightarrow \pi^0\pi^0)} \times \left[ \frac{B(K^0_L \rightarrow \pi^0\pi^0)}{B(K^0_L \rightarrow \gamma\gamma)} \right] \right] \times [1.55\%][0.23\%] (1.28\%) [1.42\%] = (1.213 \pm 0.030) \times 10^{-5}$$

KLOE should be able to contribute to improving both of these measurements. Finally there is a long distance dispersive contribution, from two off-shell photons, for which additional input from ChPT and measurements of the decays $K^0_L \rightarrow e^+e^-\gamma$, $K^0_L \rightarrow \mu^+\mu^-e^+e^-$ and $K^0_L \rightarrow e^+e^-e^+e^-$ are needed[29].
Results of kaon decays to two real or off-shell photons are summarized in Table 3. The KTeV measurements of $K^0_s \to e^+e^-\gamma\gamma$ and $K^0_L \to \mu^+\mu^-\gamma\gamma$ should improve by $\times 4$ and the modes $K^0_L \to e^+e^-\gamma\gamma$ and $K^0_L \to \mu^+\mu^-\gamma\gamma$ should improve by $\times 3$ with the final KTeV data set. The $K^0_L \to e^+e^-\gamma$ should improve by $\times 20$ and $K^0_L \to \mu^+\mu^-\gamma$ should improve by $\times 30$. The $K^0_S$ modes may be improved by NA48 in a special run, after $\epsilon'$. The $K^0_S \to \gamma\gamma$ and $K^0_S \to \gamma\gamma$ as well as several other modes will be improved by KLOE. There is no improvement in the foreseeable future for $K^0_L \to e^+e^-$ or $K^0_L \to \mu^+\mu^-$. 

| Decay Mode            | Branching Ratio            | Citation         |
|-----------------------|----------------------------|------------------|
| $K^0_S \to \gamma\gamma$ | $(2.4 \pm 0.9) \times 10^{-6}$ | NA31-95 [25]   |
| $K^0_L \to \gamma\gamma$ | $(5.92 \pm 0.15) \times 10^{-4}$ | NA31-87 [26]   |
| $K^0_L \to \mu^+\mu^-$  | $(7.24 \pm 0.17) \times 10^{-9}$ | E871-99 [27]   |
| $K^0_L \to e^+e^-$     | $8.7_{-4.1}^{+6.4} \times 10^{-12}$ | E871-98 [28]   |
| $K^0_L \to e^+e^-\gamma$ | $(1.06 \pm 0.02 \pm 0.04) \times 10^{-9}$ | NA48-99 [30]   |
| $K^0_L \to \mu^+\mu^-\gamma$ | $(3.23 \pm 0.23 \pm 0.19) \times 10^{-7}$ | E799-95 [32]   |
| $K^0_L \to e^+e^-\mu^+\mu^-$ | $(4.14 \pm 0.27 \pm 0.31) \times 10^{-8}$ | KTeV-98 [33]   |
| $K^0_L \to \mu^+\mu^-\mu^+\mu^-$ | $\sim 40$ events | KTeV-99 [34]   |
| $K^0_S \to \mu^+\mu^-$  | $< 3.2 \times 10^{-4}$ | CERN-73 [35]    |
| $K^0_S \to e^+e^-$     | $< 1.4 \times 10^{-4}$ | CPLEAR [36]     |
| $K^0_L \to e^+e^-\gamma\gamma$ | $(6.31 \pm 0.14 \pm 0.42) \times 10^{-7}$ | KTeV-99 [37]   |
| $K^0_L \to \mu^+\mu^-\gamma\gamma$ | $(1.42_{-0.81}^{+1.02} \pm 0.14) \times 10^{-9}$ | KTeV-99 [38]   |

5 Radiative $K_{\ell 2}$ Decays

The form factors in the decays $K^+ \to \ell^+\nu\ell\gamma$, A and V, and, R, in the decays $K^+ \to \ell^+\nu\ell^+\ell^-$, are predicted by ChPT. Recent measurements should allow precise experimental determinations of all three parameters. The most recent determination of $|F_V + F_A| = 0.165 \pm 0.007 \pm 0.011$ from the E787 measurement of the direct emission component of $K^+ \to \mu^+\nu\gamma$, usually called
Structure Dependent (SD+) radiation, is consistent with the previous determination of \(|F_V + F_A| = 0.148 \pm 0.010\) from \(K^+ \to e^+\nu\gamma\). A limit of \(-0.25 < F_V - F_A < 0.07\) is derived from the \(K^+ \to \mu^+\nu\gamma(SD+)\). An improved measure of \(F_V - F_A\) along with a measure of \(R\) should be available soon from E865.

A summary of the recent radiative \(K_{\ell 2}\) results is presented in Table 4.

### Table 4: Summary of Radiative \(K_{\ell 2}\) results.

| Decay Mode | Branching Ratio | Citation |
|------------|-----------------|----------|
| \(K^+ \to \mu^+\nu\gamma\) | \((5.50 \pm 0.28) \times 10^{-3}\) | KEK-85 [39] |
| \(K^+ \to \mu^+\nu\gamma(DE)\) | \((1.33 \pm 0.12 \pm 0.18) \times 10^{-5}\) | E787-97 [40] |
| \(K^+ \to e^+\nu\gamma(DE)\) | \((1.52 \pm 0.23) \times 10^{-5}\) | CERN-79 [41] |
| \(K^+ \to \mu^+\nu\mu^-\mu^-\) | \(< 4.1 \times 10^{-7}\) | E787-89 [42] |
| \(K^+ \to e^+\nu\mu^+\mu^-\) | \(< 5.0 \times 10^{-7}\) | E787-98 [43] |
| \(K^+ \to \mu^+\nu e^+e^-\) | \(\sim 1500\) events | E865-99 [44] |
| \(K^+ \to e^+\nu e^+e^-\) | \(\sim 400\) events | E865-99 [44] |

6 Other Radiative Kaon Decays

The experimental sensitivity for the other radiative kaon decays \(K_{\pi 3\gamma}\), \(K_{\ell 3\gamma}\) and \(K_{\pi 4\gamma}\) are such as to only be sensitive to IB contributions. All of these measurements are consistent with theoretical predictions. A summary of the results is given in Table 4.

A couple of modes should be seen for the first time in existing data, \(K^+ \to \pi^0\mu^+\nu\gamma\) (E787) and \(K^+ \to \pi^+\pi^-e^+\nu\gamma\) (E865). Improvements in other modes may be possible, particularly at IHEP.

7 Conclusions

Several new results are expected from KTeV and NA48, as well as a few more from E787,E865 and E871. With the turn on of DAΦNE and KLOE, which is well equipped for the radiative modes, we can expect another round
Table 5: Summary of Radiative 3- and 4-body decays.

| Decay Mode       | Branching Ratio                        | Citation  |
|------------------|----------------------------------------|-----------|
| $K^+ \rightarrow \pi^+\pi^+\pi^-\gamma$ | $(1.04 \pm 0.31) \times 10^{-4}$        | ITEP-89[45] |
| $K^- \rightarrow \pi^-\pi^0\pi^0\gamma$ | $(7.5^{+5.0}_{-3.0}) \times 10^{-6}$    | IHEP-95[46] |
| $K^+ \rightarrow \pi^+\mu^+\nu_\mu\gamma$ | $<6.1 \times 10^{-5}$                     | ZGS-73[47] |
| $K^0_L \rightarrow \pi\mu\nu_\mu\gamma$ | $(5.7^{+0.5}_{-0.7}) \times 10^{-4}$    | NA48-98[48] |
| $K^+ \rightarrow \pi^+e^+\nu_e\gamma$   | $(2.62 \pm 0.20) \times 10^{-4}$        | ITEP-91[49] |
| $K^0_L \rightarrow e\nu_e\gamma$        | $(3.62^{+0.20}_{-0.21}) \times 10^{-4}$ | NA31-96[50] |
| $K^+ \rightarrow \pi^0\pi^0e^+\nu_e\gamma$ | $<5 \times 10^{-6}$                     | ITEP-92[51] |
| $K^+ \rightarrow \pi^+\pi^-e^+\nu_e\gamma$ | no limit                                 | (E865-97?) |

of new measurements. Finally, the next generation of rare kaon experiments, designed to fully constrain the CKM unitarity triangle, by measuring the ‘Golden modes’ $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K^0_L \rightarrow \pi^0\nu\bar{\nu}$, are under construction (E391a, E949) or being designed (KOPIO, CKM, KAMI). These experiments will provide even more precise measurements of several radiative modes.

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