Frequency Regularities in $\delta$ Scuti Stars

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Abstract. Space missions have produced an incredibly large database on pulsating stars. The light curves via the frequency content contain a detailed description of each star. The critical point is the identification of modes, especially in the non-asymptotic regime. The best derived parameters from the frequency content of a pulsating star light curve are the frequency differences and ratios. This presentation focuses on the potential of period ratios in mode identification.

1. Introduction

Mode identification is one of the most critical objectives of asteroseismology. In the asymptotic regime (solar type oscillation and white dwarfs) the regularities (patterns) help to identify modes and allow us to reach a level of real asteroseismology. However, over most of the Hertzsprung Russell Diagram the pulsation is outside the asymptotic regime. In particular, $\delta$ Scuti stars exhibit a large discrepancy between the number of observed and theoretically predicted modes, which points to the operation of a mode selection mechanism. Although we recognized that independent identification of modes is impossible in a large number of stars, there always has been a hope that longer and continuous observation would reveal the predicted but missing frequencies, or that we will find some kind of regularity amongst the larger number and larger amplitude modes. With the Wide Field Infrared Explorer (WIRE), Microvariability and Oscillations of Stars (MOST), Convection Rotation and Planetary Transits (CoRoT) and Kepler space missions our dream has become a reality with long and almost continuous data sets now available.

We have started to perform a systematic check as to how we can use the increased number and most precisely derived observables, the frequency differences and period ratios. The primary test case was CoRoT 102749568, a $\delta$ Scuti star where more than one radial period ratio was immediately noticed.

2. Period Spacings of CoRoT 102749568

The period spacings of the independent modes in CoRoT 102749568 have been successfully investigated and presented in [Paparó et al (2013)]. Although this $\delta$ Scuti star is also not in the asymptotic regime, the high level regularity amongst the frequencies allowed us to derive the large separation observationally which assumes a frequent appearance of a characteristic spacing between the consecutive radial orders.
The investigation revealed that not only the radial modes but also nonradial modes with different $\ell$ values contribute to the distinguished peak in the spacing. According to the radial period ratio sequences, modes with three different $\ell$ values were localized in the excited modes including five consecutive radial orders of the radial mode. Not only was the large separation observationally determined and confirmed by modelling but the twelve highest amplitude modes were identified using only frequency regularities. The presence of the large separation between the modes suggest that these modes appear in the region of the near radial period ratio interval. Figure 1 shows a non-homogeneous distribution; the closely spaced period ratios create vertical straight lines in the frequency versus period ratio diagram due to the grouping of frequencies at one value of the frequency pairs. The region of $0.76 – 0.78$ was chosen around the canonical value of the period ratio of the radial fundamental and first overtone. Knowing the identification of modes in CoRoT 102749568, we can state that the vertical lines are around the non-radial modes with $\ell = 1$ and 2.

3. Near Period Ratios for Other $\delta$ Scuti Stars

A large sample of $\delta$ Scuti stars (using both ground-based and space data) were checked for generality in the appearance of radial period ratios or sequences and how we can use them, if we can, for mode identification. The stars chosen according to the type of observation, the evolutionary stage and rotation are given in Table 1.

In Fig. 2 we present four typical cases though we have some general conclusions. The near radial period ratio region is highly populated and the period ratios are not homogeneously or randomly distributed. If frequencies near the frequency pairs separated by the large separation are grouped either at the lower or at the higher frequency value of the pairs, then straight lines are created from the closely spaced values in the near radial period ratio range. The range of grouping is frequency dependent ($\Delta f = 0.3 – 0.6 \text{ d}^{-1}$ for CoRoT 102749568). When we do not know the rich frequency content...
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Table 1. A sample of δ Scuti stars was investigated to see the distribution of period ratios in the near radial period ratio interval.

| Name       | Type of Evol. stage | $T_{\text{eff}}$ (K) | log $g$ / log($L/L_\odot$) | $v \sin i$ (km s$^{-1}$) | Remarks | Ref. $^1$ |
|------------|---------------------|----------------------|-----------------------------|---------------------------|---------|----------|
| 44 Tau     | ground post-MS      | 6900                 | 3.6                         | 3                         | Br08    |          |
| 4 CVn      | ground hi. evolv.   | 6900                 | 3.5                         | Br09                      | H00     |          |
| XX Pyx     | ground early MS     | 8300                 | 4.25                        | 52                        | Br02    |          |
| BI CMi     | ground cool border  | 6925                 | 3.69                        | 76                        | Br05    |          |
| FG Vir     | ground              | –                    | 7500                        | 3.95                      | Br05    |          |
| 101155310  | CoRoT –             | 7300                 | 3.75                        | 23                        | HADS$^2$| P11      |
| 9408694    | Kepler              | 7300                 | 3.5                         | 100                       | HADS$^2$| Ba12b    |
| ε Cep      | WIRE –              | 7340                 | 3.9                         | 90                        | reg. spac. | Br07    |
| α Oph      | MOST –              | 8336                 | 1.496                       | 239                       | binary  | M10      |
| V589 Mon   | MOST pre-MS         | 6800                 | 1.58                        | 60                        | Z11     |          |
| BS Cnc     | MOST cluster        | 7600                 | 4.2                         | 135                       | Br12    |          |
| 10661783   | Kepler evolved      | 8000                 | –                           | –                         | ecl. bin. | S11      |
| 4840675    | Kepler sl. evolv.   | 7400                 | 1.1                         | 220                       | rot. mod. | Ba12a    |
| 9700322    | Kepler –            | 6700                 | 3.7                         | 19                        | rot. mod. | Br11    |
| HD 174936  | CoRoT –             | 8000                 | 4.08                        | 170                       | reg. spac. | G09      |
| HD 50844   | CoRoT evolved       | 7500                 | 3.6                         | 58                        | λ Bootis | P09      |

1 – References: Br08: Breger & Lenz (2008), Br99: Breger et al. (1999), H00: Handler et al. (2000), Br02: Breger et al. (2002), Br05: Breger et al. (2005), P11: Poretti et al. (2011), Ba12b: Balona et al. (2012b), Br07: Bruntt et al. (2007), M10: Monnier et al. (2010), Z11: Zwintz et al. (2011), B12: Breger et al. (2012), S11: Southworth et al. (2011), Ba12a: Balona et al. (2012a), Br11: Breger et al. (2011), G09: Garcia Hernández et al. (2009), P09: Poretti et al. (2009).

2 – High Amplitude Delta Scuti (HADS)

Regimes (most of the ground-based samples) then we do not have a structure. The opposite case appears when the frequency content is very rich (some of the space data) with many low amplitude modes, then the distribution is very dense and any kind of regularity is hardly noticed. In this case we investigated only the frequency content with the highest amplitude (HD 50844, Poretti et al. 2009). Our sample shows that in fast rotating stars there are no regular structures in the near radial period ratio range (KIC 4840675, Balona et al. 2012a). In ε Cep (Bruntt et al. 2007) straight lines appear around both the radial ($\ell = 0$) and nonradial ($\ell = 1, 2$) modes. A grouping around radial modes may appear if non-radial trapped modes are close to the radial modes. KIC 9700322 (Breger et al. 2011) shows a simple case of a rotationally modulated star. The rotational splittings also create a straight line in the near radial period ratio range.

4. Test for Period Ratios

The previously investigated near radial period region is important to get the grouping of frequencies around modes separated by the large separation. However, a wider range of period ratios (which has never been used before) can be derived from the frequency content as we show in Fig 3. These values contain information not only for the radial
The period ratios (equivalent to frequency ratios) were calculated for the 52 independent modes of CoRoT 102749568 such as \( f_1/f_2, f_1/f_3, f_1/f_4, \ldots, f_2/f_3, f_2/f_4, \ldots, f_{51}/f_{52} \). Here indices indicate the position of the frequency in the order: \( f_1 \) is the lowest frequency and \( f_{52} \) is the highest one. When we plot the period ratio versus the frequency (in the counters) we get Fig. 2. Some structures of the figure can simply be explained from its construction. The “vertical structures” show the period ratios connected to the frequencies in the counters, the slope of the “inclined lines” indicates the frequencies in the denominators. Finally, the distribution of the period ratios in both lines reflects the frequency distribution itself. Closely spaced period ratios appear everywhere but their actual place depends on the grouping of the frequencies.

The important question is whether the distribution of ratios really reflects the structure of the pulsation or not. To check this we carried out a similar process for a randomly generated frequency content with the same averages and standard deviations as the observed set. We ordered the total number of ratios by their values and plotted them successively; namely the index 1 belongs to the lowest ratio while the highest ratio has the index 1326 (see Fig. 4).
We did not detect any serious agglomerations in the ratios since we did not see a kind of staircase function. The distribution was rather smooth in both cases and seemed to be similar on the face of it. To verify the similarity in a mathematical way we performed a Monte Carlo simulation: 1 000 random frequency sets were prepared and processed in the same manner as the observed one. Then a two-sample Kolmogorov-Smirnov (KS) test [Press et al. 2001] was applied for each pair formed from a random sample and the original data set. The test rejected the null hypothesis with a probability of 0.92 ± 0.17. That is, the period ratio distribution of CoRoT 102749568 definitely has a different structure than the random one.

5. Mode Identification According to Period Ratios in CoRoT 102749568

All information about the star, the frequency content, the grouping of frequencies, the spacing of the consecutive radial orders of a given ℓ and the spacings of the modes with different ℓ values are given in the whole range of the period ratios (Fig. 3). The key to mode identification is knowing which horizontal segments have to be used. Know-
Figure 4. Comparison of the random distribution to CoRoT 102749568. The thick (red) curve gives the observed distribution of ratios. The thin (green) curves represent some distributions of ratios from random samples.

After identifying twelve modes in CoRoT 102749568, we selected some critical horizontal segments which reflect the large separation and the characteristic spacings between the modes with different $\ell$ values. The $\ell$ and $n$ for a given period ratio and the actual ranges (marked on Fig. 3) are given in Table 2. Each segment determines two subsets of frequencies that are connected to each other by the given period ratio. Due to the grouping, some frequencies are missing. For mode identification we used the fact

| $\ell, n$ | $\ell, n$ | range | $\ell, n$ | $\ell, n$ | range | $\ell, n$ | $\ell, n$ | range |
|----------|----------|-------|----------|----------|-------|----------|----------|-------|
| 0, 0     | 0, 1     | 0.76 – 0.78 | 0, 2 | 2, 2 | 0.72 – 0.73 | 0, 0 | 2, 1 | 0.525 – 0.535 |
| 0, 0     | 0, 2     | 0.60 – 0.62 | 1, 0 | 2, 0 | 0.71 – 0.72 | 0, 1 | 2, 2 | 0.57 – 0.58 |
| 0, 0     | 0, 3     | 0.50 – 0.515 | 1, 1 | 2, 1 | 0.75 – 0.76 | 1, 1 | 2, 0 | 0.925 – 0.935 |
| 0, 0     | 0, 4     | 0.42 – 0.43 | 0, 0 | 1, 1 | 0.69 – 0.71 | 1, 2 | 2, 1 | 0.96 – 0.97 |
| 2, 0     | 2, 1     | 0.81 – 0.82 | 2, 0 | 1, 2 | 0.84 – 0.85 | 1, 3 | 2, 2 | 0.97 – 0.98 |
| 2, 0     | 2, 2     | 0.69 – 0.70 | 0, 2 | 1, 3 | 0.74 – 0.75 | 2, 0 | 0, 2 | 0.95 – 0.96 |
| 0, 0     | 2, 0     | 0.64 – 0.65 | 1, 0 | 0, 1 | 0.85 – 0.86 | 2, 1 | 1, 3 | 0.865 – 0.875 |
| 0, 1     | 2, 1     | 0.68 – 0.69 | 1, 1 | 0, 2 | 0.88 – 0.89 |
that the missing frequencies may not have the same quantum numbers as we used for the segment. As a consequence of the exclusions, we got a mode identification of the twelve modes with rather high probability. For three modes we got a unique identification ($\ell = 0$, $n = 1$; $\ell = 2$, $n = 1$ and $\ell = 1$, $n = 0$). For 3 modes we got 83% probability and 75% for another 3 modes. Only three modes had a weak identification (60%). The real identification of only one mode ($\ell = 0$, $n = 4$) was excluded in our process which means that we did not select segments sensitive enough for this mode.

A known identification of modes was used in our test case to select the segments. In real cases we do not have any a priori knowledge. However, if the critical range of segments are calibrated by the quantum numbers of the modes excited in models in general, we could use the process to get a tentative identification of modes in the large number of stars observed by space instruments.

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References

Balona, L. A., Breger, M., Catanzaro, G., Cunha, M. S., Handler, G., Kołaczkowski, Z., Kurtz, D. W., Murphy, S., Niemczura, E., Paparó, M., Smalley, B., Szabó, R., Uytterhoeven, K., Christiansen, J. L., Uddin, K., & Stumpe, M. C. 2012a, MNRAS, 424, 1187

Balona, L. A., Lenz, P., Antoci, V., Bernabei, S., Catanzaro, G., Daszyńska-Daszkiewicz, J., di Criscienzo, M., Grigahcène, A., Handler, G., Kurtz, D. W., Marconi, M., Molenda-Zakowicz, J., Moya, A., Nemec, J. M., Pigulski, A., Pricopi, D., Ripepi, V., Smalley, B., Suárez, J. C., Suran, M., Hall, J. R., Kinemuchi, K., & Klaus, T. C. 2012b, MNRAS, 419, 3028

Breger, M., Balona, L., Lenz, P., Hollek, J. K., Kurtz, D. W., Catanzaro, G., Marconi, M., Pamyatnykh, A. A., Smalley, B., Suárez, J. C., Szabó, R., Uytterhoeven, K., Ripepi, V., Christiansen-Dalsgaard, J., Kjeldsen, H., Fanelli, M. N., Ibrahim, K. A., & Uddin, K. 2011, MNRAS, 414, 1721

Breger, M., Garrido, R., Handler, G., Wood, M. A., Shobbrook, R. R., Bischof, K. M., Rodler, F., Gray, R. O., Stankov, A., Martinez, P., O’Donoghue, D., Szabó, R., Zima, W., Kaye, A. B., Barban, C., & Heiter, U. 2002, MNRAS, 329, 531

Breger, M., Handler, G., Garrido, R., Audard, N., Zima, W., Paparó, M., Beichbuchner, F., Li, Z.-P., Jiang, S.-Y., Liu, Z.-L., Zhou, A.-Y., Pikall, H., Stankov, A., Guzik, J. A., Sperl, M., Krzesinski, J., Ogloza, W., Pajdosz, G., Zola, S., Thomassen, T., Solheim, J.-E., Serkowitsch, E., Reegen, P., Rumpf, T., Schmalwieser, A., & Montgomery, M. H. 1999, A&A, 349, 225

Breger, M., Hareter, M., Endl, M., Kuschnig, R., Weiss, W. W., Matthews, J. M., Guenther, D. B., Moffat, A. F. J., Rowe, J. F., Rucinski, S. M., & Sasselov, D. 2012, Astronomische Nachrichten, 333, 131

Breger, M., & Lenz, P. 2008, A&A, 488, 643

Breger, M., Lenz, P., Antoci, V., Guggenberger, E., Shobbrook, R. R., Handler, G., Ngwato, B., Rodler, F., Rodriguez, E., López de Coca, P., Rolland, A., & Costa, V. 2005, A&A, 435, 955

Bruntt, H., Suárez, J. C., Bedding, T. R., Buzasi, D. L., Moya, A., Amado, P. J., Martín-Ruiz, S., Garrido, R., López de Coca, P., Rolland, A., Costa, V., Olivares, I., & García-Pelayo, J. M. 2007, A&A, 461, 619

García Hernández, A., Moya, A., Michel, E., Garrido, R., Suárez, J. C., Rodríguez, E., Amado, P. J., Martín-Ruiz, S., Rolland, A., Poretti, E., Samadi, R., Baglin, A., Auvergne, M., Catala, C., Lefevre, L., & Baudin, F. 2009, A&A, 506, 79
Handler, G., Arentoft, T., Shobbrook, R. R., Wood, M. A., Crause, L. A., Crake, P., Podmore, F., Habanyama, A., Oswalt, T., Birch, P. V., Lowe, G., Sterken, C., Meintjes, P., Brink, J., Claver, C. F., Medupe, R., Guzik, J. A., Beach, T. E., Martinez, P., Leibowitz, E. M., Ibbetson, P. A., Smith, T., Ashoka, B. N., Raj, N. E., Kurtz, D. W., Balona, L. A., O’Donoghue, D., Costa, J. E. S., & Breger, M. 2000, MNRAS, 318, 511

Monnier, J. D., Townsend, R. H. D., Che, X., Zhao, M., Kallinger, T., Matthews, J., & Moffat, A. F. J. 2010, ApJ, 725, 1192

Paparó, M., Bognár, Z., Benkő, J. M., Gandolfi, D., Moya, A., Suárez, J. C., Sógor, A., Hareter, M., Poretti, E., Auvergne, M., Baglin, A., Weiss, W. W., & Guenther, E. W. 2013, CoRoT 102749568: mode identification of a δ Scuti star based on regular spacings. Accepted by A&A, arXiv:astro-ph/1307.2553

Poretti, E., Michel, E., Garrido, R., Lefèvre, L., Mantegazza, L., Rainer, M., Rodríguez, E., Uytterhoeven, K., Amado, P. J., Martín-Ruiz, S., Moya, A., Niemczura, E., Suárez, J. C., Zima, W., Baglin, A., Auvergne, M., Baudin, F., Catala, C., Samadi, R., Alvarez, M., Mathias, P., Paparó, M., Pápics, P., & Plachy, E. 2009, A&A, 506, 85

Poretti, E., Rainer, M., Weiss, W. W., Bognár, Z., Moya, A., Niemczura, E., Suárez, J. C., Auvergne, M., Baglin, A., Baudin, F., Benkő, J. M., Debosscher, J., Garrido, R., Mantegazza, L., & Paparó, M. 2011, A&A, 528, A147

Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 2001, Numerical recipes. The art of scientific computing (Cambridge University Press)

Southworth, J., Zima, W., Aerts, C., Bruntt, H., Lehmann, H., Kim, S.-L., Kurtz, D. W., Pavlovski, K., Prša, A., Smalley, B., Gilliland, R. L., Christensen-Dalsgaard, J., Kawaler, S. D., Kjeldsen, H., Cote, M. T., Tenenbaum, P., & Twicken, J. D. 2011, MNRAS, 414, 2413

Zwintz, K., Kallinger, T., Guenther, D. B., Gruberbauer, M., Kuschnig, R., Weiss, W. W., Auvergne, M., Jorda, L., Favata, F., Matthews, J., & Fischer, M. 2011, ApJ, 729, 20