Determination of $\gamma/Z$ interference in $e^+e^-$ annihilation at LEP

L3 Collaboration

Abstract

An S–Matrix ansatz is used to determine the mass and width of the Z boson, as well as the contributions of $\gamma/Z$ interference and Z boson exchange to fermion–pair production. For this purpose we use hadron and lepton–pair production cross sections and lepton forward–backward asymmetries that have been measured with the L3 detector at centre–of–mass energies between 87 GeV and 189 GeV.

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Introduction

The successful operation of LEP has allowed a precise measurement of fermion–pair production, $e^+e^- \to \bar{f}f$, at centre–of–mass energies near the Z resonance. The mass and the total and partial widths of the Z boson have been determined with excellent accuracy. The Standard Model \cite{1, 2} is confirmed with high precision by the experimental results of L3 \cite{3} and other experiments \cite{4–7}. In these analyses, the contribution of the interference between the photon and the Z boson exchange amplitudes is fixed to the Standard Model expectation.

In this paper, we use an S–Matrix \cite{8} approach to determine the contributions of $\gamma/Z$ interference and Z boson exchange, thus reducing the number of theoretical assumptions. At centre–of–mass energies well above the Z resonance, the reduced importance of Z boson exchange allows the determination, in particular, of $\gamma/Z$ interference with enhanced precision.

The running of LEP in 1997 and 1998 at energies of 182.7 GeV and 188.7 GeV, together with a tenfold increase of integrated luminosity at high energy compared to our previous S–Matrix analysis \cite{9}, improves substantially the sensitivity to the S–Matrix parameters. Similar analyses have been performed by the DELPHI and OPAL collaborations \cite{10, 11}.

Measurements of Fermion–Pair Production

Measurements of cross sections and forward–backward asymmetries for the reactions $e^+e^- \to \bar{f}f$, have been performed with the L3 detector \cite{12} at centre–of–mass energies, $\sqrt{s}$, in the vicinity of the Z resonance \cite{3} and at 130.0, 136.1, 161.3, 172.3, 182.7 and 188.7 GeV \cite{9, 13, 14}.

At energies greater than 130 GeV both leptons of the $e^+e^-$ final state are required to be in the range $44^\circ < \theta < 136^\circ$, where $\theta$ is the angle between the incoming electron and the outgoing lepton. Muon- and tau–pair candidates are selected by the cuts $|\cos \theta| < 0.9$ and $|\cos \theta| < 0.92$, respectively, for both final–state leptons. Hadron events are selected in the full solid angle. In total, 27470 hadron events and 9417 lepton–pair events are selected. These correspond to an integrated luminosity of $258.7 \, \text{pb}^{-1}$.

For centre–of–mass energies in the vicinity of the Z resonance, the sensitivity to photon exchange and $\gamma/Z$ interference is suppressed due to the large Z exchange cross section. Therefore, a minimum effective centre–of–mass energy, $\sqrt{s'}$, or a maximum acollinearity angle in the Bhabha channel, are required to select events without substantial energy loss due to initial state radiation. The remaining event samples at $\sqrt{s} \geq 130$ GeV contain, in total, 7785 hadron and 7704 lepton–pair events. The details of the analyses such as selection procedures, efficiencies, background contributions, measured cross sections and forward–backward asymmetries, together with the statistical and systematic uncertainties, are discussed in References \cite{9, 13} and \cite{14}.

Standard Model expectations are calculated using the ZFITTER \cite{15} and TOPAZ0 \cite{16} programs with the following values \cite{3, 17–20} for the Z boson mass, $m_Z$, the top quark mass, $m_t$, the Higgs boson mass, $m_H$, the electromagnetic coupling constant, $\alpha$, and the strong coupling constant $\alpha_s$:

\[
\begin{align*}
    m_Z &= 91.189.8 \pm 3.1 \, \text{MeV}, \\
    1/\alpha(m_Z^2) &= 128.887 \pm 0.089, \\
    m_t &= 173.8 \pm 5.2 \, \text{GeV}, \\
    \alpha_s(m_Z^2) &= 0.119 \pm 0.002, \\
    95.3 \, \text{GeV} &\leq m_H \leq 1 \, \text{TeV}. 
\end{align*}
\] (1)

The results of our analysis are not sensitive to the uncertainties on these parameters. At energies above the Z resonance, the theoretical uncertainties on the predicted cross sections are estimated to be 0.5% \cite{21} except for the predictions for large–angle Bhabha scattering which
have a larger uncertainty of 2% \cite{22}. Uncertainties on the forward–backward asymmetries are smaller and negligible compared to the statistical uncertainties of the measurements. The theoretical uncertainties are propagated into the systematic uncertainty on our results.

**Determination of $\gamma/Z$ interference**

The measurements of cross sections and forward–backward asymmetries are analysed in the framework of the S–Matrix ansatz. The programs SMATASY \cite{23}, together with ZFITTER and TOPAZ0, are used for the calculation of the theoretical predictions and QED radiative corrections to the cross sections and forward–backward asymmetries.

The lowest–order total cross section, $\sigma_{a}^0$, and forward–backward asymmetry, $A_{fb}^0$, for $e^+e^- \rightarrow f\bar{f}$ \cite{8} are:

$$
\sigma_a^0(s) = \frac{4}{3} \pi \alpha^2 \left[ \frac{g_{f}^0}{s} + \frac{j_{f}^0(s - m_{Z}^2) + r_{f}^0 s}{(s - m_{Z}^2)^2 + m_{Z}^2 \Gamma_{Z}^2} \right], \quad \text{where } a = \text{tot, fb},
$$

$$
A_{fb}^0(s) = \frac{3}{4} \sigma_{fb}^0(s), \quad \text{with } \sigma_{fb}^0 = \frac{4}{3} (\sigma_{f} - \sigma_{b}). \quad (2)
$$

The S–Matrix ansatz defines the Z resonance using a Breit–Wigner denominator with an $s$–independent width. In other approaches, a Breit–Wigner denominator with an $s$–dependent width is used, which implies the following transformation of the values of the Z boson mass and width \cite{8}: $m_{Z} = \bar{m}_{Z} + 34.1$ MeV and $\Gamma_{Z} = \bar{\Gamma}_{Z} + 0.9$ MeV. In the following, the fit results are quoted after applying these transformations. The S–Matrix parameters $r_{f}$, $j_{f}$ and $g_{f}$ give the Z exchange, $\gamma/Z$ interference and photon exchange contributions for fermions of type $f$, respectively. For hadronic final states the parameters $r_{\text{tot had}}$, $j_{\text{tot had}}$ and $g_{\text{tot had}}$ are sums over all produced quark flavours.

In Figure 1 the measured hadronic cross sections are compared to the theory predictions for different values of $j_{\text{tot had}}$. The S–Matrix parameters are determined in a $\chi^2$ fit to our published measurements \cite{14} at centre–of–mass energies of 130 to 189 GeV, using our results of S–Matrix fits to the Z–peak data \cite{3} as constraints. As a cross–check, a fit to all cross–section and asymmetry measurements is performed, which gives identical results. Correlations between measurements taken close to the Z resonance and measurements at high centre–of–mass energies are estimated and their influence on the fit results is found to be negligible. Correlations between different measurements at high centre–of–mass energy are taken into account in the fits. The photon exchange contributions, $g_{f}$, are fixed in the fits to their QED predictions. The fit results do not depend on the uncertainty on $\alpha(m_{Z}^2)$.

The fitted S–Matrix parameters for electrons, muons, taus and hadrons, and their correlations, are listed in Tables 1, 3 and 2. The fits are performed with and without the assumption of lepton universality. The parameters obtained for the individual lepton types are compatible with each other and support this assumption.

A large correlation is observed between the mass of the Z boson and the hadronic $\gamma/Z$ interference term, $j_{\text{tot had}}$. This correlation causes an increase in the uncertainty on $m_{Z}$ with respect to fits where the hadronic $\gamma/Z$ interference term is fixed to its Standard Model prediction. The value of this correlation is reduced from -0.95 when using Z peak data alone \cite{3} to -0.57. Under the assumption of lepton universality the fitted hadronic $\gamma/Z$ interference term is:

$$
j_{\text{tot had}} = 0.31 \pm 0.13 \text{(exp)} \pm 0.04 \text{(th)}.
$$
This value agrees with the Standard Model prediction of 0.21 and improves significantly the precision of our previous S–Matrix analyses \[3, 9\]. The theoretical uncertainty is due to the uncertainty of 0.5% on the calculation of cross sections for \( \sqrt{s} > 130 \) GeV of the ZFITTER program. For the leptonic \( \gamma/Z \) interference terms we obtain similar improvements. The fitted value for \( m_Z \) is:

\[
m_Z = 91.187.5 \pm 3.1(\text{exp}) \pm 2.3(\Delta j_{\text{had}}^{\text{tot}}) \pm 0.6(\text{th}) \text{ MeV}.
\]

The contribution to the uncertainty due to the \( \gamma/Z \) interference is separated from the rest of the experimental uncertainty. It is reduced significantly with respect to our previous results \[3, 9\]. Again, the theoretical uncertainty is due to the limited precision of the cross section calculations with the ZFITTER program. Figure 2 shows the 68% confidence level contours in the \((m_Z, j_{\text{had}}^{\text{tot}})} \) plane for the data taken at the Z–pole and after including the 130–189 GeV measurements. The improvement resulting from the inclusion of the high energy measurements is clearly visible.

Our measurement of the hadronic interference term agrees with results from data taken at energies below the Z resonance \[24\]. Adding these low energy measurements to the fits, we obtain:

\[
j_{\text{had}}^{\text{tot}} = 0.159 \pm 0.082(\text{exp}) \pm 0.015(\text{th}),
m_Z = 91.190.4 \pm 3.1(\text{exp}) \pm 1.5(\Delta j_{\text{had}}^{\text{tot}}) \pm 0.3(\text{th}) \text{ MeV},
\]

with a correlation coefficient of -0.40 between these quantities.

In summary, we use the S–Matrix framework to analyse data taken at the Z resonance and at higher centre–of–mass energies. Our measurements provide an improved determination of the \( \gamma/Z \) interference terms and the Z boson mass. All parameters show good agreement with the Standard Model predictions.

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| Parameter          | Treatment of Charged Leptons | Theory uncertainty | Standard Model |
|-------------------|------------------------------|--------------------|----------------|
|                   | Non–Universality             | Universality       |                |
| $m_Z$ [MeV]       | 91188.3 ± 3.9                | 91187.5 ± 3.9      | 0.6            | —              |
| $\Gamma_Z$ [MeV]  | 2502.8 ± 4.1                 | 2502.5 ± 4.1       | 0.1            | 2492.7 ± 3.8   |
| $r_{\text{had}}^t$ | 2.9856 ± 0.0092              | 2.9848 ± 0.0092    | 0.0003         | 2.9584 ± 0.0088|
| $r_e^t$           | 0.14317 ± 0.00075            | —                  | 0.00002        |                |
| $r_\mu^t$         | 0.14287 ± 0.00079            | —                  | 0.00002        |                |
| $r_\tau^t$        | 0.14375 ± 0.00102            | —                  | 0.00002        |                |
| $r_\ell^t$        | —                            | 0.14318 ± 0.00059  | 0.00002        | 0.14242 ± 0.00035|
| $j_{\text{had}}^t$| 0.30 ± 0.13                  | 0.31 ± 0.13        | 0.04           | 0.21 ± 0.01    |
| $j_e^t$           | −0.030 ± 0.045               | —                  | 0.002          |                |
| $j_\mu^t$         | −0.001 ± 0.027               | —                  | 0.002          |                |
| $j_\tau^t$        | 0.061 ± 0.031                | —                  | 0.002          |                |
| $j_\ell^t$        | —                            | 0.017 ± 0.019      | 0.002          | 0.0041 ± 0.0003|
| $r_e^f$           | 0.00177 ± 0.00111            | —                  | 0.000002       |                |
| $r_\mu^f$         | 0.00333 ± 0.00064            | —                  | 0.000002       |                |
| $r_\tau^f$        | 0.00448 ± 0.00092            | —                  | 0.000002       |                |
| $r_\ell^f$        | —                            | 0.00332 ± 0.00047  | 0.000002       | 0.00255 ± 0.00023|
| $j_e^f$           | 0.700 ± 0.075                | —                  | 0.001          |                |
| $j_\mu^f$         | 0.807 ± 0.034                | —                  | 0.001          |                |
| $j_\tau^f$        | 0.732 ± 0.044                | —                  | 0.001          |                |
| $j_\ell^f$        | —                            | 0.770 ± 0.026      | 0.001          | 0.799 ± 0.001  |

| $\chi^2$/d.o.f.  | 30.4/28                      | 33.0/36            |                |

Table 1: Results of the fits in the S–Matrix framework without and with the assumption of lepton universality. The theory uncertainties on the S–Matrix parameters are determined from the 0.5% uncertainty on the ZFITTER predictions for cross sections. The Standard Model expectations are calculated using the parameters listed in Equation 1. [1]
|      | $m_Z$ | $\Gamma_Z$ | $r_{\text{had}}^\text{tot}$ | $r_e^\text{tot}$ | $r_\mu^\text{tot}$ | $r_\tau^\text{tot}$ | $j_{\text{had}}^\text{tot}$ | $j_e^\text{tot}$ | $j_\mu^\text{tot}$ | $j_\tau^\text{tot}$ | $r_{\text{fb}}^\text{tot}$ | $j_{\text{fb}}^\text{tot}$ | $j_{\text{fb}}^\text{tot}$ | $j_{\text{fb}}^\text{tot}$ |
|------|-------|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $m_Z$ | 1.00  | 0.38      | 0.07           | -0.08         | 0.01          | 0.01          | -0.58         | -0.14         | -0.15         | -0.14         | -0.05         | 0.08          | 0.03          | -0.01         |
| $\Gamma_Z$ | 1.00  | 0.91      | 0.54           | 0.52          | 0.40          | 0.01          | -0.02         | 0.02          | 0.02          | 0.02          | 0.02          | 0.02          | -0.01         | 0.04          |
| $r_{\text{had}}^\text{tot}$ | 1.00  | 0.56      | 0.53           | 0.41          | 0.01          | -0.03         | 0.01          | 0.01          | 0.00          | 0.02          | 0.02          | -0.01         | 0.04          | 0.04          |
| $r_e^\text{tot}$ | 1.00  | 0.33      | 0.25           | 0.08          | 0.05          | 0.04          | 0.13          | -0.01         | 0.00          | -0.02         | 0.04          | 0.02          | 0.02          | 0.02          |
| $r_\mu^\text{tot}$ | 1.00  | 0.23      | 0.00           | -0.02         | 0.10          | 0.02          | 0.00          | 0.03          | 0.01          | -0.01         | 0.11          | 0.02          | 0.02          | 0.02          |
| $r_\tau^\text{tot}$ | 1.00  | 0.00      | -0.01         | 0.02          | 0.09          | 0.00          | 0.01          | 0.03          | -0.01         | 0.02          | 0.10          | 0.05          | 0.02          |
| $j_{\text{had}}^\text{tot}$ | 1.00  | 0.13      | 0.13           | 0.13          | 0.04          | -0.04         | -0.03         | 0.01          | 0.05          | 0.02          | 0.02          | 0.02          | 0.02          |
| $j_e^\text{tot}$ | 1.00  | 0.03      | 0.03           | 0.00          | -0.01         | -0.01         | 0.30          | 0.01          | 0.01          | 0.01          | 0.01          | 0.01          | 0.01          |
| $j_\mu^\text{tot}$ | 1.00  | 0.03      | 0.01           | 0.07          | -0.01         | 0.00          | 0.31          | 0.01          | 0.01          | 0.01          | 0.01          | 0.01          | 0.01          |
| $j_\tau^\text{tot}$ | 1.00  | 0.01      | -0.01         | 0.04          | 0.00          | 0.00          | 0.15          | 0.01          | 0.01          | 0.01          | 0.01          | 0.01          | 0.01          |
| $r_{\text{fb}}^\text{tot}$ | 1.00  | -0.01     | 0.00          | 0.03          | 0.00          | 0.00          | 0.00          | 0.00          | 0.00          | 0.00          | 0.00          | 0.00          | 0.00          |
| $r_\mu^\text{fb}$ | 1.00  | 0.01      | 0.00           | 0.13          | 0.00          | 0.00          | 0.11          | 0.00          | 0.00          | 0.11          | 0.00          | 0.00          | 0.11          |
| $r_\tau^\text{fb}$ | 1.00  | 0.00      | 0.00           | 0.00          | 0.00          | 0.00          | 1.00          | 0.00          | 1.00          | 1.00          | 1.00          | 1.00          | 1.00          |
| $j_e^\text{fb}$ | 1.00  | 0.00      | 0.00           | 0.00          | 0.00          | 0.00          | 1.00          | 0.00          | 1.00          | 1.00          | 1.00          | 1.00          | 1.00          |
| $j_\mu^\text{fb}$ | 1.00  | 0.00      | 0.00           | 0.00          | 0.00          | 0.00          | 1.00          | 0.00          | 1.00          | 1.00          | 1.00          | 1.00          | 1.00          |
| $j_\tau^\text{fb}$ | 1.00  | 0.00      | 0.00           | 0.00          | 0.00          | 0.00          | 1.00          | 0.00          | 1.00          | 1.00          | 1.00          | 1.00          | 1.00          |

Table 2: Correlation coefficients of the S–Matrix parameters listed in Table 1, not assuming lepton universality.
|       | $m_Z$ | $\Gamma_Z$ | $r^{\text{tot}}_{\text{had}}$ | $r^{\text{tot}}_{\ell}$ | $i^{\text{tot}}_{\text{had}}$ | $i^{\text{tot}}_{\ell}$ | $r^{\text{fb}}_{\ell}$ | $j^{\text{fb}}_{\ell}$ |
|-------|-------|------------|------------------|------------------|------------------|------------------|------------------|------------------|
| $m_Z$ | 1.00  | 0.05       | 0.06             | -0.02            | -0.57            | -0.24            | 0.05             | -0.06            |
| $\Gamma_Z$ | 1.00  | 0.92       | 0.69             | 0.01             | 0.01             | 0.01             | 0.02             | 0.05             |
| $r^{\text{tot}}_{\text{had}}$ | 1.00  | 0.71       | 0.04             | 0.08             | 0.03             | 0.05             | 0.08             | 0.05             |
| $r^{\text{tot}}_{\ell}$ | 1.00  | 0.04       | 0.21             | -0.03            | 0.06             | 0.25             | 0.04             | 0.25             |
| $i^{\text{tot}}_{\text{had}}$ | 1.00  | 0.21       | 1.00             | 0.04             | 1.00             | 0.11             | 1.00             | 1.00             |
| $i^{\text{tot}}_{\ell}$ |       |            |                   |                  |                  |                  |                  |                  |
| $r^{\text{fb}}_{\ell}$ |       |            |                   |                  |                  |                  |                  |                  |
| $j^{\text{fb}}_{\ell}$ |       |            |                   |                  |                  |                  |                  |                  |

Table 3: Correlation coefficients of the S–Matrix parameters listed in Table 1 assuming lepton universality.
Figure 1: Cross sections for the process $e^+e^- \rightarrow \text{hadrons}(\gamma)$, for $\sqrt{s'}/s > 0.85$. The lines represent the theory predictions for different values of $j^\text{tot}_\text{had}$. The lower plot normalises the measurements and predictions to that of $j^\text{tot}_\text{had} = 0.31$. 
Figure 2: Contours in the \((m_Z, j_{\text{had}}^\text{tot})\) plane at 68\% confidence level under the assumption of lepton universality. The dashed line is obtained from Z data only; the inclusion of 130 GeV to 189 GeV data gives the solid line. The circle (Z data) and the cross (all data) indicate the central values of the fits. The Standard Model prediction for \(j_{\text{had}}^\text{tot}\) is shown as the horizontal band. The vertical band corresponds to the 68\% confidence level interval on \(m_Z\) in a fit assuming the Standard Model value for \(\gamma/Z\) interference.