Measuring Human-perceived Similarity in Heterogeneous Collections

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ABSTRACT

We present a technique for estimating the similarity between objects such as movies or foods whose proper representation depends on human perception. Our technique combines a modest number of human similarity assessments to infer a pairwise similarity function between the objects. This similarity function captures some human notion of similarity which may be difficult or impossible to automatically extract, such as which movie from a collection would be a better substitute when the desired one is unavailable. In contrast to prior techniques, our method does not assume that all similarity questions on the collection can be answered or that all users perceive similarity in the same way. When combined with a user model, we find how each assessor’s tastes vary, affecting their perception of similarity.

Keywords

Recommender systems, Feature selection, Sentiment and opinion mining, User modeling

1. INTRODUCTION

Many common tasks in Data Mining, Machine Learning, Information Retrieval and related fields depend on an embedding of data objects into some feature space, either explicitly through numeric features or implicitly through a pairwise similarity function (kernel). A significant amount of work has gone into developing the correct feature space for a given task, often attempting to model user intentions, or model a notion of similarity between objects in the collection. For instance, density based methods or SVMs depend explicitly on similarity (or kernel) mapping; in collaborative filtering, a product/item recommendation often depends on how similar a product is to other products the user has purchased, or products that other users similar to the potential customer have purchased. Although similarity is central to many of these tasks, and great strides have been made in producing automatic measures of similarity for many tasks \[10\, 15, 12\], we find that the ability to directly measure a human perception of the similarity between objects in a collection is currently lacking. This is unfortunate because the success of many learning tasks with realistically difficult data depends on the ability to accurately model user satisfaction or preference, particularly in cases where the users’ notion of similarity inherently depends on subjective information.

We propose a framework allowing researchers to construct pairwise object similarity functions for domains that require subjective or user-specific information to achieve high accuracy. These similarity functions, or similarity kernels, can be used to (1) run learning algorithms on raw data such as movies; (2) analyze an existing set of features, pointing out deficiencies in matching human judgements and providing guidance toward developing more natural features; (3) model user differences in perception; or (4) evaluate a variety of tasks such as Information Retrieval rankings, query diversity, and product recommendation. Rather than imposing a set of features or a similarity function explicitly upon a data set, we propose to instead learn either a pairwise object similarity function or, equivalently, an embedding of those objects into a metric space which best fits the observed human-generated similarity information. Different collections of objects, users, and tasks pose different challenges:

- to define what is meant by similarity in a given task or context,
- to collect human perception as feedback from appropriate questions/answers,
- to design backend likelihood models for human response actuation on the learning model for similarity, and
- to use optimization/training procedures to infer optimal model parameters.

Existing methods currently fall short in one or more of these areas, preventing their use in more complex data collections. The first difficulty is that humans are very inconsistent in assessing similarity, so quantitative questions like "how similar is X to Y?" usually are not good user feedback \[3\]. For most tasks, it has been shown that preference questions like "is X more like Y, or more like Z?" make for better, consistent, feedback \[16\]; but even then, similarity is not at all straightforward. For example, if we know a user rates the dish eggplant parmesan very highly, which of the following is more similar to it: baingan bharta (an Indian eggplant dish), or lasagna (an Italian dish)? While the first
is closer in terms of ingredients but quite different meal experience, the second is closer in terms of cuisine but different in ingredients and nutrition. Thus a correct similarity notion has to account for who is making the preference, and possibly for the contexts in which these food dishes are discussed. A movie example: If Joe wants to see the movie Star Wars, which is unfortunately unavailable, would he prefer Lord of the Rings or The Matrix as an alternative? Not only is it hard to answer such a question solely from superficial features like actors, genre, director etc; different users will legitimately answer differently, so even with good movie features, the models have to account for user personality or taste on movies. In short, high accuracy depends on measuring subjective perception.

1.1 Omer Tamuz et. al 2011 Kernel paper

Omer Tamuz et al presented in their 2011 ICML paper “Adaptively Learning the Crowd Kernel” [16] an original framework for doing just that: learning similarity from human input in the form of preferences “is X more like Y or more like Z”? This allows characteristics of an image and relationships between the images to be inferred from feedback, and not imposed artificially. They demonstrated good results on pictures of ties and of human faces. Our study is a follow-up on their excellent idea.

After discussion with the authors and reimplementation of the procedures, we found several shortcomings: (1) the image data is not the most telling for assessing performance: the applicability of such framework is rather useful on data objects for which human perception is subjective and far less computable than it is on images (an automatic method for face detection or tie pattern recognition can do quite well); (2) their crowd answers are limiting the human input by not allowing certain natural answers “I dont know,” “neither,” “both,” etc. — we found that these answers are returned a significant amount of the time, and that not explicitly using them as feedback slows down the learning process; (3) the original framework does not allow for any contradiction or associative disagreement between answers, hoping that the kernel eventually averages out the feedback — but subjective feedback on objects like movies or foods or even text does contain large disagreement, thus better handled explicitly by accounting for various users; (4) the optimization method given, based on gradient descent, is slow and quite inefficient at finding a global optimum for any number of data dimensions more than two.

Section 2 presents an adaptation of the kernel framework to embed user models. It greatly improves the kernel fit onto observations by allowing disagreement across users, something not possible in the original framework. It also makes better predictions given a user model that matches the test observations, for example when the same user is known to have contributed to the training observation set, or when the testing set user profile is known as a combination of trained users.

Figure 1 illustrates the results of the data collection process after several grad students tested the feedback interface with preference questions about movies, before running the Mechanical Turks study presented below. The actual
object positions are revealing beyond mere cluster membership. For instance, “The Princess Bride” (PB in the figure) falls in the “action comedy” cluster along with spy/cop movies, but is positioned very near the “romantic comedy” cluster. “Forrest Gump” (FG) shares a cluster with both “Pay it Forward” (PiF) and “Philadelphia,” (Ph) but is much nearer the upbeat former than the more dramatic latter film. “Crouching Tiger, Hidden Dragon” (CT) is very near “Kill Bill: Vol. 1” (KB) and more distant from the comic “Kung Fu Hustle,” (KFH) though all three martial arts films are fairly near each other.

1.2 Other Related Work

Human Input and Image Similarity Considering the space of image similarity, we see that a large amount of work has been done to perform human relevance feedback for content-based image retrieval [6][9][14]. This is mainly because a machine has a harder time deducing the content of an image than of text, and as such, human judgments are far more valuable than machine features. Human assessments for images are far easier than for text, however, because humans have a harder time processing an entire document than a single image.

There are, however, many parallels that can be drawn between image and text feedback. For both image and text retrieval, the user model can be thought of in two ways: a user can search for a category of information or images [8], or for a target piece of information or image [8]. In the former case, the feedback is binary—whether or not the document or image fits in the category; the latter is a continuous measure of how close the document or image is to the target. It is the latter user model which we use in the similarity kernel, as relative judgments have been shown to have less bias and therefore less noise than absolute judgments [8][13][17]. Additionally, relative judgments allow for arbitrarily fine-grained notions of graded relevance [2], or in our case, notions of relevance across multiple dimensions.

Preferences vs. Nominal Grades on IR document assessment. Some work is being done using preference pair judgments for retrieval [9] or preference triples [5], but these preferences either require many judgments or focus only on ordering documents for relevance and diversity; they do not examine the similarity between objects.

The current approach to evaluating search engine diversity is subtractive relevance [4][7]. However, these subtopics are created in advance by posting query facets or manually inspecting query logs for query variants. This is artificially imposing the dimensions of relevance, which as such do not necessarily correlate to the idea of relevance held by a group of users [15]. Additionally, the documents for a query are crawled and pooled with no knowledge of the subtopics, and as such do not necessarily match them.
find that the results are good even for the local optima we converge on. We have employed a quasi-Newtonian/Hessian method which performs gradient descent on the loss function using learning rates computed from the second derivative of $L$, with multiple random initialization points to find a reasonably good local optimum.

### 2.1 Experiments and Results

To test our method, we crowdsourced assessments for two data sets, described below. We compare our three-answer model to the two-answer model described by Tamuz et al.

We ran 20 rounds of active learning independently for each model and data set. In each round, we asked crowd workers one question with each object in the collection serving as $a$ in an $(a, b, c)$ triple. We sent the questions to Amazon Mechanical Turk (AMT), asking 12 questions per HIT. These questions were divided between active learning questions selected by the model being trained, randomly sampled test questions used for evaluation, and trap questions used for accepting or rejecting a batch of crowdsourced answers. The trap questions were selected using a manual clustering of the objects, with either $b$ or $c$ drawn from the same cluster as $a$ and the other object drawn from a different cluster. A HIT was accepted if the worker selected the object from the same cluster as $a$ for at least one of the two trap questions. This serves as a mild quality filter, since even totally random workers can be expected to pass the trap questions fairly often. In practice, we observed about a 90% acceptance rate of submitted work.

Each HIT for the baseline model included 10 active learning questions and 2 trap questions. For our three-answer model, we asked 2 trap questions, 5 active learning questions, and 5 test questions per HIT. We mixed training and test questions in the same HIT so we can later evaluate the predictive power of our user model, described in the next section. Further, the test questions were only asked for the three-answer model because we interpret an answer of "neither" on a test question as meaning that the comparison is uninteresting, or at least unreliable; we excluded these test triples from our evaluation.

### Movie data

Our first data set is a collection of 100 popular movies. We decided to measure the extent to which a user who wants to see one movie would be satisfied with some other movie.

| Answer | Kernel two answers | Kernel three answers |
|--------|-------------------|---------------------|
| $\hat{p}_{bc}$ | $\frac{\lambda + \delta_{ac}}{2\lambda + \delta_{ab} + \delta_{ac}}$ | $(1 - \hat{p}_{neither}) \cdot \frac{\lambda + \delta_{ac}}{2\lambda + \delta_{ab} + \delta_{ac}}$ |
| $\hat{p}_{bb}$ | $\frac{\lambda + \delta_{ab}}{2\lambda + \delta_{ac} + \delta_{ab}}$ | $(1 - \hat{p}_{neither}) \cdot \frac{\lambda + \delta_{ab}}{2\lambda + \delta_{ac} + \delta_{ab}}$ |
| $\hat{p}_{neither}$ | 0 (N/A) | $\frac{\mu + \delta_{ab}}{\mu + d^{2} + \delta_{ab}}$, $\frac{\mu + \delta_{ac}}{\mu + d^{2} + \delta_{ac}}$ |

Table 1: Probabilistic model for Kernel likelihood. $\hat{p}_{bc}$ here means the estimated probability of the user claiming $b$ is more similar than $c$ to $a$. The original two-answer Kernel is a particular case of the three-answer Kernel with $\hat{p}_{neither} = 0$ (equivalently $d = \infty$).

For instance, perhaps they have a movie in mind but have to select from a limited menu which doesn't include their movie of interest.

We chose movies because many people are familiar with the task of choosing a movie, but also because there is enough variety among them that many movies are sufficiently dissimilar that they seem to have nothing to do with each other. For instance, if you want to see a light martial arts comedy you are presumably very unlikely to choose an intense relationship drama as a replacement. This collection is culminated as the Rebel's, including Skywalker and flying with X-Wing Amble make an impact on the Empire's most powerful and ominous weapon, the Death Star.

![Image of a movie with a sword and a lightsaber, titled "Mystic River" (2003).](http://www.imdb.com/title/tt0375706)

Figure 2: Preference Feedback interface shown for movies.

![Image of a movie with a sword and a lightsaber, titled "The Perfect Storm" (2000).](http://www.imdb.com/title/tt0117731)

| Director | Wolfgang Petersen |
| Cast | George Clooney, Mark Wahlberg, Diane Lane, John C. Reilly |
| Genre | Adventure, Drama, Thriller |
| Plot | In October 1991, a confluence of weather conditions combined to form a killer storm in the North Atlantic. Caught in the storm was the sword-fishing boat Andrea Gail. Magnificent tornadowing and anticipation fill this true-life drama while minute details of the fishing boat, their gear and the weather are unpurposed with the sea adventure. |
| IMDB | http://www.imdb.com/title/tt0117731 |

The movies in our collection were selected to fit into a few large scale genres (e.g., comedy, drama, or science fiction), each with several sub-genres (such as war drama, relationship drama, or a personal quest) and some genre-crossing features (relationship drama vs. romantic comedy, or serious science fiction).

For instance, they might want to see "The Perfect Storm," but the tickets were sold out. They're trying to decide between two other movies playing at the theater, and are asking you for advice. Knowing only that they wanted to see "The Perfect Storm," do you think they would enjoy "Star Wars: Episode IV - A New Hope" or "Mystic River" movie?
action versus comedic action). We constructed a web-based interface which presents the movies’ titles, posters, director, genres, main actors, and summaries. Users are shown three movies at a time along with the following instructions:

Your friend calls and says that they wanted to see the movie “Movie A,” but the tickets were sold out. They’re trying to decide between two other movies playing at the theater, and are asking you for advice. Knowing only that they wanted to see “Movie A,” do you think they would enjoy “Movie B” or “Movie C” more? Note that this is not a question about which movie you like more, but instead a question about which movie is most similar to “Movie A.”

**Food data.**

Our second data set repeats the same experiment using a different collection of objects. In this case, we used 100 different types of food. We chose a similar similarity question to produce comparable results: if someone wants food a but must instead choose between foods b and c, which would be a better replacement? We expected foods to be easy for crowd workers to compare, but also to have triples where neither option is similar at all to a and to have subjective answers. For instance, a vegetarian can be expected to have different views from a non-vegetarian when trying to replace a vegetable-based entree. Our interface presented the food name, a photo, and a description drawn from the food type’s Wikipedia article – usually its first paragraph. We selected mainly dinner entrees from various cultures, with many themes linking different kinds of foods: main ingredient, ethnicity, style (soups, dumplings, sandwiches, etc.) and so on. As with movies, we ran a separate active learning process for each of the two models we compare.

This data set proved more challenging than the movies data. The most important contributing factor may be that foods don’t fall as neatly into pre-defined clusters. When comparing movies, users seem inclined to compare based on genre first, and then based on similarity within genre. Foods don’t have such clear-cut categories. There are many ambiguous foods which sit somewhere between the natural groups. (For instance, is a hamburger more like a ham sandwich or like a steak?) We believe this led to less consistent answers, both between assessors and for observations from the same assessor.

### 2.2 Results for Updated Model

**Evaluation.**

We compare the kernels output from each of the two active learning processes by evaluating them against the test data. For any given test observation in which the user expressed a preference, stating that either b or c is more similar to a than the other, we consider the kernel to be correct when the preferred object is more similar to a and incorrect otherwise. This allows us to build the learning curve in Figure 3 and Figure 4 showing accuracy as a function of number of observations collected.

#### 3. USER-SPECIFIC SIMILARITY

In the prior section, we showed how to relax the baseline model’s assumption that all objects in the collection can be compared by the assessors. Here we relax a further assumption: that all users answer similarity questions consistently with each other.

The proper representation for objects in many learning tasks has a subjective component. That is, users will disagree with each other when answering similarity questions, but will answer consistently with themselves. This can pose interesting challenges in a machine learning setting, where assessor disagreement can look like label or feature noise. In this work, we approach the problem by first assuming that there is an objective representation of the objects that all users can observe. This representation can be expressed as a global kernel: an embedding M of the objects into a d-dimensional metric space shared by all users, exactly as in the prior models. However, each user has his or her own scaling of the dimensions of this metric space. This scaling, which we represent as a diagonal matrix $U_k^k$ for each user $k$, produces a subjective personalized kernel $K_k = MU_k^kM^T$. 
for each user. The global kernel, used in our prior three-answers model, is equivalent to a personalized kernel with an identity matrix for $U^k$.

To draw an example from the movies data set, consider that movie watchers enjoy different movies for a wide variety of reasons. They see the same movies, and likely observe more or less the same features of those movies, but with different sensitivities to those features. One person will pay particular attention to the mood of the film, while another is interested in the special effects. When comparing different movies, these viewers will each emphasize the features they care more about, applying a larger scale to those features as compared to the features which are less important to them.

We update our three-answer model to draw distances from personalized kernels instead of the previously-used global kernel. Where the three-answer model uses distances $\delta_{a,b} = \|M_a \cdot M_b\|^2_2$, we instead use $\delta_{a,b}^k = \|M_a \cdot \text{diag}(U^k) \cdot M_b\|^2_2$. We also train the distance $d_{\text{other}}$ per-user instead of training a single global value to account for different users’ abilities to perceive similarity between objects.

This approach has a few potential problems. We would like the global kernel, parameterized with $U^k$ set to an identity matrix, to be a reasonable prior for some user about whom we have no information. However, the model as stated has no such constraint. To see this problem, observe that the distance between two objects in a particular user’s personalized kernel is a function both of the global object positions in $M$ and the user’s scaling in $U^k$. If every value along some dimension $d$ of $M$ were reduced by half and the value of $U^k_{a,d}$ was doubled for every user, the distances between objects would not be affected. This has no impact on the predictive power of personalized kernels, but it means that the information in the global kernel is somewhat deficient. Second, we have a varying amount of information per user, ranging from five to hundreds of assessments from particular users in our training set. This is relatively common in practice, both in a commercial setting (where some customers rate only a few items, while others rate many) and in crowdsourced data (where a worker may or may not choose to accept additional tasks). Our ability to accurately fit a personalized kernel is dependent on the number of assessments available, and with few assessments it’s easy to overfit the user parameters.

We resolve these problems by imposing a prior on our user kernels. We want this prior to have a mode of 1, to keep the identity user for the global kernel meaningful, and to assign zero probability mass to negative values, so mirroring dimensions relative to other users isn’t possible. We accomplish this by putting a Gaussian prior with mean zero and per-dimension standard deviation $\sigma_d$ on the logarithm of the values of each $U^k_{a,d}$. This makes our loss function

$$L \left( \{M_a\}, \{\Omega^k\} \right) = -\sum_{a, d} N(\log U^k_{a,d}, \sigma_d)$$

$$- \sum_{i=1}^n \log \rho(r_i | M_{a_i}, M_{b_i}, M_{c_i}, \Omega)$$

with $\Omega^k = \{\mu^k, d^k\}$ parameterized per user.

Although you could imagine learning $\sigma_d$ per dimension, justified by the intuition that variance of user feature sensitivity is likely to differ from feature to feature, for our results here we simply fix $\sigma_d = 0.18$ for all dimensions.

## 3.1 Results for User Model

We evaluate our user model on the same data described in the prior section. We compare three different methods of obtaining similarity predictions:

- the three-answer kernel used in the prior section,
- the general kernel trained with user-specific parameters but making predictions for an identity user ($U^k = I$), and
- the personalized kernels trained with user-specific parameters, and making predictions for test data observations using users’ own personalized kernels.

While the three-answer kernel outperforms the original two-answer kernel, both the general and personalized kernels outperform the three-answer kernel. This helps validate our hypothesis that variation between users can be effectively modeled as a scaling of features.

## 4. FUTURE WORK

We believe this framework has a great deal of potential, particularly in the domain of constructing evaluation collec-
Figure 5: Learning curves for user models on movies data. Note that these plots are not directly comparable to the two answer baseline, because all the models here were allowed to predict “neither.” A random answer lies somewhere around 1/3 probability, depending on the prior value of \( d \).

Figure 6: Learning curves for user models on foods data. The personalized kernel has a higher log-likelihood for the test data, but as in our prior plot this does not translate to a much higher test accuracy.

tions for machine learning and information retrieval tasks. We see several opportunities for its further development.

- Use on text collections: the current technique is best suited to notions of similarity that can be easily captured by simple questions that people can answer without training. This makes it somewhat ill-suited to use on several useful notions of text similarity, such as entailment or query relevance. We seek to develop the framework further to target these.

- Active learning: We have not modified the active learning process from the original, but we see an opportunity for improvement here. The process considers only a few discrete future object positions given some question it’s considering, and we find empirically that it often chooses sub-optimal questions.

- Scalability: Our current model is often still learning after asking 20 questions per head. This is many fewer questions than the total \( n \cdot \binom{n-1}{2} \), but still difficult to scale to very large object collections. We would like to address this, perhaps by quickly finding a large-scale positioning of all objects and then considering only questions which position objects locally. Another possible approach is to find ways to incorporate existing data collected from slightly different questions, such as pair preferences or clickthrough data.

- Better optimization: the optimization method we’re using is somewhat prone to getting trapped in local minima. We believe that this can be avoided by either using a better optimization technique, or perhaps by adjusting the model to be somewhat more convex.

5. CONCLUSION

We have presented an extension of the Crowd Kernel model by Tamuz et al which corrects for two major sources of noise: users’ inability to compare dissimilar objects, and inter-assessor disagreement between users. This allows this useful model to be applied more effectively on heterogeneous data sets. Our results showed the most dramatic improvements for data that falls more or less into distinct classes, with relatively few objects defying categorization. However, the model improves somewhat over the original even for our more ambiguous data set.
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APPENDIX

We perform Newton-Raphson approach to optimize for the object embeddings \( M \) and parameters \( \Omega \):

\[
\{ M, \Omega \} := \{ M, \Omega \} - H^{-1} \nabla_{(M, \Omega)} \sum_{i=1}^{n} \ln \hat{p}(r_i | M_{a_i}, M_{b_i}, M_{c_i}, \Omega)
\]

To speed up computation, we consider Hessian matrix with zeroes outside the main diagonal.

Assuming

\[
\lambda = 1
\]

\[
c_1 = (\mu_k + \delta_{ab}^k)^{-1}
\]

\[
c_2 = (\mu_k + \delta_{ac}^k)^{-1}
\]

\[
c_3 = (\mu_k + \delta_{ac}^k)^{-1}
\]

\[
c_4 = (\mu_k + \delta_{ac}^k)^{-1}
\]

\[
c_5 = (\mu_k + \delta_{ac}^k)^{-1}
\]

\[
c_6 = (2 + \delta_{ab}^k + \delta_{ac}^k)^{-1}
\]

\[
c_7 = (d_k^2)^{-1}
\]

\[
c_8 = (2\mu_k + d_k^2 + \delta_{ab}^k + \delta_{ac}^k)^{-1}
\]

\[
\frac{\partial \ln \hat{p}_a}{\partial \delta_{ab}^k} = c_8 - c_3 - c_6
\]

\[
\frac{\partial \ln \hat{p}_a}{\partial \delta_{ac}^k} = c_8 - c_4 + c_5 - c_6
\]

\[
\frac{\partial \ln \hat{p}_a}{\partial d_k^2} = c_7 + c_8 - c_3 - c_4
\]

\[
\frac{\partial \ln \hat{p}_a}{\partial \mu_k} = 2c_8 - c_3 - c_4
\]

\[
\frac{\partial \ln \hat{p}_{neither}}{\partial \delta_{ab}^k} = c_1 - c_3
\]

\[
\frac{\partial \ln \hat{p}_{neither}}{\partial \delta_{ac}^k} = c_2 - c_4
\]

\[
\frac{\partial \ln \hat{p}_{neither}}{\partial d_k^2} = -c_3 - c_4
\]

\[
\frac{\partial \ln \hat{p}_{neither}}{\partial \mu_k} = c_1 + c_2 - c_3 - c_4
\]

\[
\frac{\partial^2 \ln \hat{p}_a}{\partial \delta_{ab}^k \partial \delta_{ab}^k} = -(c_1)^2 + (c_2)^2 + (c_3)^2
\]

\[
\frac{\partial^2 \ln \hat{p}_a}{\partial \delta_{ac}^k \partial \delta_{ac}^k} = -(c_2)^2 + (c_3)^2
\]

\[
\frac{\partial^2 \ln \hat{p}_a}{\partial d_k^2 \partial d_k^2} = -4(c_8)^2 + (c_3)^2
\]

\[
\frac{\partial^2 \ln \hat{p}_{neither}}{\partial \delta_{ab}^k \partial \delta_{ab}^k} = -(c_1)^2 + (c_3)^2 + (c_6)^2
\]

\[
\frac{\partial^2 \ln \hat{p}_{neither}}{\partial \delta_{ac}^k \partial \delta_{ac}^k} = -(c_2)^2 + (c_3)^2
\]

\[
\frac{\partial^2 \ln \hat{p}_{neither}}{\partial d_k^2 \partial d_k^2} = -(c_3)^2 + (c_4)^2
\]

\[
\frac{\partial^2 \ln \hat{p}_{neither}}{\partial \mu_k^2} = -4(c_8)^2 + (c_3)^2
\]