Multi-Task Offloading over Vehicular Clouds under Graph-based Representation

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Abstract—Vehicular cloud computing has emerged as a promising paradigm for realizing user requirements in computation-intensive tasks in modern driving environments. In this paper, a novel framework of multi-task offloading over vehicular clouds (VCs) is introduced where tasks and VCs are modeled as undirected weighted graphs. Aiming to achieve a trade-off between minimizing task completion time and data exchange costs, task components are efficiently mapped to available virtual machines in the related VCs. The problem is formulated as a non-linear integer programming problem, mainly under constraints of limited contact between vehicles as well as available resources, and addressed in low-traffic and rush-hour scenarios. In low-traffic cases, we determine optimal solutions; in rush-hour cases, a connection-restricted random-matching-based subgraph isomorphism algorithm is proposed that presents low computational complexity. Evaluations of the proposed algorithms against greedy-based baseline methods are conducted via extensive simulations.

Index Terms—Computation-intensive task, multi-task offloading, vehicular cloud computing, subgraph isomorphism

I. INTRODUCTION

The Internet of vehicles (IoV) is an emerging paradigm that enables innovations related to immersive vehicular applications such as autonomous driving and advanced driver assistants, offering safety, convenience and entertainment experiences for drivers and passengers [1]. Moreover, technological advances in computing processors and sensing devices facilitate tasks with innovative and computation-intensive features (e.g., self-driving and simultaneous localization and mapping), which require massive computational resources. Graph-based representation can be used to characterize some of these tasks: each task is modeled as a graph, where the vertices (components) represent either data sources or data processing units while edges describe the dependency (data flows) between the vertices [2]–[4]. However, the computational resources and capability limitations of on-board equipments [4], [5] pose major challenges to IoV, such that the inherent limitation of a single smart vehicle may hinder fulfillment of task execution requirements. Vehicular cloud computing (VCC) has thus been proposed, wherein vehicles act as computing servers to form vehicular clouds (VCs) by sharing surplus resources with users facing heavy on-board workloads via opportunistic vehicle-to-vehicle (V2V) communications [5].

The literature on computation-intensive task offloading can be roughly divided into two categories: 1) tasks that are directly mapped as bit streams and considered a collection of sub-tasks without considering inherent dependencies [6]–[9]; and 2) tasks that are modeled as directed/undirected graphs by considering the inner dependencies of sub-tasks, such as [2]–[4], [10]–[13]. Furthermore, graph-based task offloading can be classified into three types: static, semi-static, and dynamic, according to the features of network topology. Assuming static topologies of servers and users in the cloud computing context, a randomized scheduling algorithm is proposed in [4], which stabilizes a system with graph job arrivals/departures and facilitates a smooth trade-off between minimizing the average execution cost and queue length.

For the semi-static environment where the locations of either servers or users are fixed, in [2], applications were modeled as directed tasks, and sequential and concurrent task offloading mechanisms were presented to minimize application completion time. The authors in [3] put forth a novel framework for energy-efficient graph job allocation in geodistributed cloud networks, where solutions were provided for data center networks of varying scales. A Lyapunov-optimization-based dynamic offloading approach for directed-graph-based jobs was described in [10]. The proposed approach satisfies the constraints on both energy conservation and application execution time. In [11], scheduling of parallel jobs composed of a set of independent tasks was studied by considering energy consumption and job completion time. A fast hybrid multi-site computation offloading mechanism was proposed in [12], where offloading solutions considering application sizes were obtained in a timely manner. Different from the abovementioned scenarios, a VC-based computation offloading mechanism was studied in [13] where computing missions were modeled as tasks with interdependency and executed in different vehicles to minimize overall response time while enhancing the capacity of users (edge clouds).

Graph-based task offloading in dynamic network environments have rarely been investigated up to now, where the mobility of servers and users as well as the interdependency of components pose challenges to the design of offloading mechanisms. Consequently, approaches in static network systems are difficult to implement in dynamic environments. In our previous study [19], a randomized graph job allocation
mechanism over VCs based on hierarchical tree decomposition was proposed. This paper is regarded as follow-up work of [19], in which multi-task offloading and various attributes of components and service providers (SPs) are considered; such factors present additional challenges related to the problem size and algorithm efficiency. Moreover, potential competition for resources between components must be considered due to task concurrency. To the best of our knowledge, we are among the first to examine the multi-task offloading problem over VCs while considering their inherent characteristics.

In this paper, we introduce a novel VC-assisted computation offloading framework that captures a multi-server and multi-user environment, where each user (task owner (TO)) has a task modeled as an undirected weighted graph with specific requirements for execution time. Virtual machine (VM)-based [14] representation is utilized to quantify available resources of vehicles (SPs) in the VC. The VC is abstracted as an undirected weighted graph of SPs with VMs that can provide heterogeneous computational capabilities. Through the proposed mechanism, each component is efficiently mapped (offloaded) to an appropriate VM under constraints of opportunistic communications and available resources. The main contributions of this paper are as follows:

- A novel VC-assisted computation offloading framework is presented that accounts for concurrent multi-tasks under graph-based representation. Our framework alleviates heavy TO workloads caused by limitations in on-board resources and computational capabilities.
- To achieve the trade-off between minimizing the task completion time and data exchange costs, the graph-based concurrent multi-task offloading problem is formulated as a nonlinear integer programming (NIP) problem under limited opportunistic contact between SPs and available VMs.
- To tackle the aforementioned NIP problem, an optimal algorithm is first developed to find the best offloading solution in low-traffic scenarios. This approach relies on addressing the subgraph isomorphism problem [3, 18] which is known to be NP-complete [3, 18]. The optimal algorithm is thus ineffective when encountering a large number of tasks and SPs or when facing complicated task and network topologies. To address this issue, a connection-restricted random matching algorithm is proposed, which exhibits low computational complexity and works well for larger and fast-changing IoV networks.
- Two greedy-based methods are proposed to serve as baselines for performance evaluation. We demonstrate that the proposed connection-restricted random matching algorithm achieves a similar performance to the optimal algorithm with a significantly lower running time while outperforming baseline methods under various scenarios.

The rest of this paper is organized as follows. The system models and problem formulation are presented in Section II. The optimal and the connection-restricted random matching algorithms are presented in Section III and IV, respectively. In Section V, the performance evaluation through comprehensive simulations is introduced before conclusion in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this paper, a novel computation offloading framework is proposed where multi-tasks can be mapped to SPs in the relevant VC via one-hop V2V interactions. SPs own different quantities of available VMs, which can process task components by providing various computational capabilities measured by execution time. Note that structural characteristics exist in computation-intensive tasks and IoV topology, due to which the tasks of TOs and the VC network are both modeled as weighted undirected graphs. Under constraints of limited opportunistic vehicular contact duration and available resources, each VC aims to effectively allocate all of the task components to SPs while achieving a trade-off between task completion time and data exchange costs. Assuming that a VC contains SP set \( S \) and TO set \( O \), the related models are introduced below.

A. Vehicular contact model

The interaction between vehicles \( x \) and \( y \) during \( t \in (\tau_1, \tau_2) \) occurs when the following conditions are satisfied: \( \| L_x(\tau_1) - L_y(\tau_1) \| > R, \| L_x(t) - L_y(t) \| \leq R, \) and \( \| L_x(\tau_2) - L_y(\tau_2) \| > R, \) where \( L_x(\tau) \) and \( L_y(\tau) \) denote vehicles’ locations at time \( \tau \), and \( \| \cdot \| \) and \( R \) represent the Euclidean distance and the vehicular communication radius, respectively. Generally, the contact duration between vehicle \( x \) and \( y \) obeys an exponential distribution \([15, 16]\) with parameter \( \lambda_{xy} \); therefore, the probability that the contact duration \( \Delta t_{xy} \) between vehicles \( x \) and \( y \) is larger than \( T \) is given by \( P(\Delta t_{xy} > T, \lambda_{xy}) = e^{-T \lambda_{xy}} \). The larger the value of \( e^{-T \lambda_{xy}} \), the more assurance can be achieved to protect the required data interaction between moving vehicles.

B. Vehicular cloud modeling via graphs

Each SP \( s_y \in S \) has a collection of idle VMs \( m_y \) that are fully connected, with various computational capabilities related to the execution time for processing one component. Moreover, each available VM can run only one component at a time. A VC covers a region containing several TOs and SPs, where at least one SP exists in each TO’s communication range. It is assumed that TOs are under consistent pressure due to insufficient local resources, such that TOs prefer to integrate resources from SPs in the related VC; thus, we do not consider VMs on TOs as available resources. Consequently, a VC is represented as a graph \( G^s = (V^s, E^s, W^s) \) containing a set of SPs \( V^s = \{ s_y | s_y \in S \} \), in which each SP \( s_y \) has a collection of available VMs \( m_y = \{ s_{yj} | j \in \{1, 2, \ldots, |m_y|\} \} \), where \( s_{yj} \) denotes the \( j \)-th VM of \( s_y \); and the related computational capability set \( t_{yj} = \{ t_{yj} | j \in \{1, 2, \ldots, |m_y|\} \} \), where \( t_{yj} \) represents the execution time that VM \( s_{yj} \) can provide to process one component in a graph job. The edge set \( E^s = \{ e_{yy'} | s_y \in S, s_{y'} \in S, y \neq y' \} \), where \( e_{yy'} \) indicates that \( s_y \) can communicate with \( s_{y'} \) via a one-hop V2V channel; and the weight of the edge set \( W^s = \{ \lambda_{yy'} | s_y \in S, s_{y'} \in S, y \neq y' \} \)
describes the mean of the corresponding parameters of the
exponential distribution of contact duration between vehicles.

C. Task modeling via graphs

The task of a TO $o_x \in O$ is depicted as a graph $G^{o_x} = (V^{o_x}, E^{o_x}, W^{o_x})$, which contains a set of components $V^{o_x} = \{v^{o_x}\}_{i \in n_x}$ where $n_x = \{1, \ldots, |V^{o_x}|\}$; here, each component $v^{o_x}$ has the maximum tolerant execution time $t^{o_x}_i$ (seconds) and a set of edges $E^{o_x} = \{e^{o_x}_{i'j'}|i \in n_x, i' \neq i\}$ with associated weights $W^{o_x} = \{\omega^{o_x}_{i'j'}|i \in n_x, i' \neq i\}$. Graph $G^{o_x}$ represents how the computation should be split among components in $V^{o_x}$. Specifically, edges represent required data flows between components, and the weight $\omega^{o_x}_{i'j'}$ indicates the requested connecting duration between components $v^{o_x}_{i'}$ and $v^{o_x}_{j'}$, which is considered to be the lowest execution time between these components. Also, the contact duration of SPs that handle these components should be equal to or greater than $\omega^{o_x}_{i'j'}$. For example, if two connected components $v^{o_x}_{i_1}$ and $v^{o_x}_{i_2}$ are offloaded to different VMs $s^1_y$ and $s^2_y$ that can provide execution time 1.5 and 3 seconds, respectively. Then, $\omega^{o_x}_{i_1i_2} = min(1.5, 3) = 1.5$ seconds are required for the intermediate data collection and transmission between these two VMs.

D. Data exchange cost

We assume the data exchange cost $c^{exch}_{yx}$ to be strictly larger than zero when two connected components are assigned to VMs on different SPs $s_y$ and $s_{y'}$ \[^1\]; otherwise, $c^{exch}_{yx} = 0$. This element captures the cost incurred from traffic exchange among different SPs in a VC.

E. Problem formulation

To better analyze the proposed multi-task offloading framework, we mainly study the problem in one VC because the proposed mechanism is universal across all VCs. Furthermore, we focus on the snapshot of one VC in the network, that contains $|O|$ TOs, $o_x \in O$, and $|S|$ SPs, $s_y \in S$, where each $o_x$ has a graph-based task $G^{o_x}$ waiting to be offloaded to available VMs in $G^s$. Notably, the time of data transmission and the resulting feedback regarding execution between TOs and SPs are ignored \[^{19, 17}\]. The notation $s_y \in R_{o_x}$, where $o_x \in O$ and $s_y \in S$ indicates that $s_y$ is in $o_x$‘s communication range. We only consider one-hop V2V computing offloading in this paper, meaning that $o_x$ cannot offload any data to $s_y$ when $s_y \notin R_{o_x}$.

Let the binary indicator $\kappa^{xy}_{ij} = 1$ denote the assignment of component $v^{o_x}_{i}$ of TO $o_x$ to VM $s^1_y$ of SP $s_y$; and $\kappa^{xy}_{ij} = 0$ otherwise. For the data exchange cost model of task partitioning among different SPs, let the binary indicator $\nu^{yy'}_{jj'} = 1$ denote data exchange between $s_y$ and $s_{y'}$ upon allocation of components; otherwise, $\nu^{yy'}_{jj'} = 0$.

Based on the above notations, $\nu^{yy'}_{jj'}$ is defined as a piecewise function of $\kappa^{xy}_{ij}$ and $\kappa^{xy}_{ij'}$, given in (1), describing the case where two connected components of a task assigned to VMs on different SPs will incur data exchange cost.

$$
\nu^{yy'}_{jj'} = \begin{cases} 
1, & \forall e^{o_x}_{ij} \in E^{o_x}, \\
0, & \text{if } y \neq y' \text{ and } \kappa^{xy}_{ij} \times \kappa^{xy}_{ij'} = 1 \\
0, & \text{otherwise}. 
\end{cases} \quad (1)
$$

The task completion time is given by the processing time of the slowest processed component of the task as:

$$
U^{t}_{o_x}(\mathbb{K}) = \max \{\kappa^{xy}_{ij} \times t^{o_x}_{ij} \}_{1 \leq i \leq |n_x|, 1 \leq j \leq |S|, 1 \leq y \leq |m_y|} \quad (2)
$$

where $\mathbb{K} = [\kappa^{xy}_{ij}]_{1 \leq i \leq |O|, 1 \leq j \leq |n_x|, 1 \leq y \leq |S|, 1 \leq y \leq |m_y|}$ defines the matrix of indicator $\kappa^{xy}_{ij}$, for notational simplicity. Then, \[^4\] Here, it is assumed that the transmission time of component data and the resulting feedback between TOs to SPs are ignored for two main reasons \[^{19}\]. First, some computation-intensive applications have small input/output data sizes that can be ignored, such as vehicular navigation. Second, advanced communication standards and technology can achieve low end-to-end latency (e.g., 5G) \[^{17}\], which leads to cases where the data transmission time of each component can be ignored.
the total cost of multi-task offloading is formulated as the following function:

\[ U^c(\mathcal{V}(\mathcal{K})) = \frac{1}{2} \sum_{s=1}^{O} \sum_{m_y} |S| m_y \sum_{s} |m_y| \sum_{j,j' \neq j} \nu_{j,j'}^{y} \times \epsilon_{y}^{exch} \]  

(3)

where \( \mathcal{V}(\mathcal{K}) = [\nu_{j,j'}^{y}]_{1 \leq y \leq |S|, 1 \leq j \leq m_y, 1 \leq y' \leq |S|, 1 \leq j' \leq m_{y'}} \) denotes the matrix of indicator \( \nu_{j,j'}^{y} \), the elements of which are defined in (1). The normalization factor \( \frac{1}{2} \) is considered since data exchange cost will be calculated twice due to the undirected graph-based task model.

Correspondingly, attempting to minimize the completion time of each task and data exchange costs under opportunistic contact and resource constraints, we formulate the concurrent multi-task offloading problem under graph-based representation as shown in (4). For notational simplicity, \( U^t = [U^t_{\alpha}(\mathcal{K})]_{1 \leq x \leq |O|} \) denotes the vector of TOs’ task completion time.

\[ \arg\min_{\mathcal{K}} \xi_{i} \|U^t\|_2 + \xi_{c} U^c(\mathcal{V}(\mathcal{K})) \]  

(4)

s.t.

\[ \sum_{x=1}^{O} |n_{x}| \nu_{j,j'}^{y} \leq m_{y'}, \forall s_{y} \in S \]  

(4a)

\[ e^{-\lambda \nu_{j,j'}^{y} x_{j,j'}^{y}} \geq \xi, \forall e_{i,j}^{x_{j,j'}^{y}} \in E_{x_{j,j'}^{y}}, y \neq y', \nu_{j,j'}^{y} \times \nu_{j,j'}^{y'} = 1 \]  

(4b)

\[ \nu_{j,j'}^{y} \geq 0, \text{if } t_{x_{j,j'}^{y}} < t_{x_{j,j'}^{y'}} \text{ or } s_{y} \notin \mathcal{R}_{x_{j,j'}^{y'}} \]  

(4c)

where \( \| \cdot \|_2 \) represents the vector’s 2-norm defined as (5), with the significance of minimizing the completion time of each task rather than the overall completion time of all the tasks, in order to better achieve the fairness among TOs:

\[ \|U^t\|_2 = \left( \sum_{x=1}^{O} (U^t_{\alpha}(\mathcal{K}))^2 \right)^{\frac{1}{2}} \]  

(5)

Here, \( \xi_{i} \) and \( \xi_{c} \) are non-negative weight coefficients that indicate the preference between task completion time and data exchange costs, respectively. The system prefers to reduce the completion time of tasks with a larger value of \( \xi_{i} \); comparatively, the data exchange cost between vehicles is more likely to be reduced as the value of \( \xi_{c} \) becomes higher.

In (4), constraint (4a) prevents overloading of the available resources, where \( |m_{y}| \) denotes the number of idle VMs on \( s_{y} \). Constraint (4b) is a probabilistic constraint handling the cases in which components \( v_{i,x}^{y} \) and \( v_{i,y}^{y'} \) are assigned to VMs on different SPs \( s_{y} \) and \( s_{y'} \). In such cases, the probability of the contact duration between \( s_{y} \) and \( s_{y'} \) exceeding \( \omega_{i,x}^{y} \) must be greater than the tunable threshold \( \xi \in [0, 1] \). Constraint (4c) ensures component \( v_{i,x}^{y} \) is allocated to a VM that meets \( v_{i,x}^{y} \)’s requirement \( t_{x}^{e_x} \) to guarantee successful task execution, and one-hop V2V computation offloading from TOs to SPs.

The objective function in (4) represents a nonlinear integer programming problem that is NP-hard. Moreover, the constraints related to (4) impose solving the sub-graph isomorphism problem, which is NP-complete [3], [18]. Consequently, the system can rarely identify solutions to reconfigure the IoV extemporaneously, as the running time required to solve large and real-life network cases increases sharply with increasing vehicular density (and with the complexity of the VC topology and task structures). To solve the multi-task offloading problem under graph-based representation, an optimal algorithm for low-traffic (e.g., fewer than three TOs and six SPs in a VC) scenarios is first presented. Then, a novel multi-task offloading algorithm based on connection-restricted random matching is proposed for rush-hour scenarios, through which near-optimal solutions can be obtained by achieving much lower computational complexity.

III. THE OPTIMAL MULTI-TASK OFFLOADING UNDER GRAPH-BASED REPRESENTATION

Our optimal algorithm aims to solve the graph-based multi-task offloading problem by enumerating all possible solutions. The pseudocode is given in Algorithm I, where \( V^t_{\alpha} \) and \( V^c_{\alpha} \) are sequence of permutation of the components and VMs. Notation \( \mathcal{M} \) and \( \mathcal{K}^* \) indicate the set of possible solutions and the optimal mapping from components to VMs, respectively. The algorithm contains two primary stages: In Stage 1, i.e., lines 2–10, we look for every possible solution \( \mathcal{K}_{\alpha} \), which is seen as a mapping from component set to possible VM set, based on merging the adjacency matrices of tasks into a sparse matrix to handle irrelevant features between these concurrent tasks; in Stage 2, i.e., lines 12–13, the best solution with the minimum value of (4) is selected. In this algorithm, going through all possible solutions ensures identification of the optimal solution for the offloading problem via graph-based representation.

However, obtaining the optimal solution requires high computational complexity of \( \mathcal{O}((\sum_{o_{\alpha} \in O} |n_{x}|)! \times C(\sum_{s_{y} \in S} |m_{y}|, \sum_{o_{\alpha} \in O} |n_{x}|)) \) where \( \sum_{o_{\alpha} \in O} |n_{x}| \) represents the total number of components of multi-tasks and “!” is the factorial notation. \( \sum_{s_{y} \in S} |m_{y}| \) indicates the number of available VMs in the related VC. Notably, \( C(m, n) \) stands for the m – choose – n operation, as tasks are modeled as undirected graphs where components do not have a particular execution sequence. This computational complexity becomes prohibitive as the number of TOs and SPs and the available VMs grow larger.

IV. MULTI-TASK OFFLOADING BASED ON CONNECTION-RESTRICTED RANDOM MATCHING

To better handle the multi-task offloading problem under graph-based representation in real-life IoV scenarios with large numbers of SPs and TOs as well as complicated structures, we develop an efficient low-complexity multi-task offloading algorithm called connection-restricted random matching (CRRM). Through the proposed CRRM algorithm, we have only one visit to each component of tasks, for which an available VM can be randomly chosen for execution while satisfying the inner structures of tasks and the VC, modeled as graphs. Consequently, the computational complexity of \( \mathcal{O}((\sum_{o_{\alpha} \in O} |n_{x}|)) \) in each iteration can be achieved to substantially improve the running time performance. Details appear in Algorithm II, where \( \mathcal{K}_{\alpha} \) and \( \mathcal{K}^* \) indicates the mapping between the components and VMs in iteration r and the near-optimal solution, respectively.
Algorithm 1: The Optimal multi-task offloading algorithm under graph-based representation

input : All graph-based tasks $G^o$, $x \in \{1, \ldots, |\mathcal{O}|\}$, VC graph $G^*$
output: Optimal solution $\mathcal{K}^*$ for distributing all $G^o$, $x \in \{1, \ldots, |\mathcal{O}|\}$ over $G^*$
1 // Stage 1 Solution search procedure
2 Initialization: $\mathcal{V}^* \leftarrow \{\mathcal{V}^m \cup \mathcal{V}^a \cup \ldots \cup \mathcal{V}^{o(0)}\}$; $\mathcal{V}^{**} \leftarrow \{m_1 \cup m_2 \cup \ldots \cup m_{|\mathcal{O}|}\}$; $\mathcal{M} \leftarrow \emptyset$; $\mathcal{K}^* \leftarrow \emptyset$; $\%
3$ Deriving the set of all the permutations of the elements in $\mathcal{V}^*$, i.e., $\mathcal{V}^{comp}$, and the set of all the permutations of any $\mathcal{V}^*$ elements in $\mathcal{V}^{**}$, i.e., $\mathcal{V}^{VM}$.
4 $\mathcal{V}^{VM} \leftarrow \{\mathcal{V}^{j}_{vm} \mid j \in \{1, 2, \ldots, (\sum_{o \in \mathcal{O}} |n_x|)\}, |\mathcal{V}^{j}_{vm}| = |\mathcal{V}^*|$, where each sequence $\mathcal{V}^{j}_{vm}$ is a permutation of the elements in $\mathcal{V}^*$;
5 for $i = 1$ to $(\sum_{o \in \mathcal{O}} |n_x|)!$ do
6   for $j = 1$ to $\mathcal{C}(\sum_{y \in \mathcal{S}} |m_y|, \sum_{o \in \mathcal{O}} |n_x|)$ do
7     if components in $\mathcal{V}^{j}_{vm}$ can be offloaded to VMs in $\mathcal{V}^{j}_{vm}$ sequentially under constraint (4b), (4c) in (4) then
8       $\mathcal{K}_{ij} \leftarrow \{\mathcal{V}^{j}_{vm}, \mathcal{V}^{j}_{vm}\}$; % match the components in $\mathcal{V}^{j}_{vm}$ to the VMs in $\mathcal{V}^{j}_{vm}$ one by one, as a mapping $\mathcal{K}_{ij}$;
9     else
10       $\mathcal{K}_{ij} \leftarrow \emptyset$;
11 // Stage 2 The optimal solution selection procedure
12 $\mathcal{K}^* \leftarrow$ the solution with minimum value of (4) in $\mathcal{M}$;
13 end algorithm

In CRRM, Steps 2–3 describe the initialization procedure, and Steps 5–6 handle the unsuccessful offloading cases due to insufficient available VMs. Steps 7–22 describe how each component is matched to an available VM. Specifically, Steps 14–15 and 18–19 indicate that a component is randomly mapped to an available VM while satisfying V2V communication coverage and required execution time limitations as well as opportunistic connection restrictions between SPs. Notably, potential competition on VMs between components exists during matching; thus, Steps 16–17 and 20–21 deal with unsuccessful offloading cases in each iteration, where no available VMs can be used to handle the randomly selected component. A better solution is considered after every iteration by comparing the value of the objective function given in (4), among which the best solution will be reserved as the final solution $\mathcal{K}^*$ through Steps 23-27. This process ensures convergence of the proposed algorithm.

V. NUMERICAL RESULTS AND PERFORMANCE EVALUATION

This section presents numerical results illustrating the performance of the proposed algorithms. Moreover, two greedy-based algorithms serve as baseline methods are shown below to better evaluate the advantages of the optimal and proposed CRRM algorithms.

Degree preferred mechanism (DPM): Components are sorted by degree\(^5\) from largest to smallest as a list. Each component in the list is matched one by one to the available VM with the largest degree at present, while satisfying all the constraints in (4), until all components have been allocated successfully. Otherwise, tasks must be executed locally.

Execution time preferred mechanism (ETPM): Randomly select one task component at a time and match it to the available VM that can provide the lowest execution time at present, while satisfying all the constraints in (4), until all components have been allocated successfully. Otherwise, tasks must be executed locally.

The completion time of a locally executed task depends on the number and execution time of available VMs the TO can provide, which is considered a serial mode rather than parallel, and thus excluded from the solution space of the proposed optimal and CRRM algorithms. Thus, local execution cases are not considered in our performance comparisons and evaluations. In this paper, task types are randomly chosen from those depicted in Fig. 1(b) and the related parameters are randomly set in the following intervals: $e \in [0.9, 1]$, $\omega_{tt}^{l} \in [0.1, 0.3]$, $t_i^{ov} \in [0.1, 0.2]$, $t_j^{ov} \in [0.05, 0.25]$, $c_{exch}^{ov} \in [0.05, 0.15]$, $t_i = \xi_c = 0.5$, $\lambda_{yy} \in [0.04, 0.05]$, $\lambda_{yy} \in [0.01, 0.02]$ for rush-hour cases.

Due to that the running time can be too large as the task structure becomes more complex, we mainly consider different numbers of Task type 1 in various scenarios, for which a running time comparison between the optimal and CRRM algorithms appears in Fig. 2 under 10-based logarithm presentation. Compared with the CRRM algorithm under 3,000 iterations, Fig. 2 indicates that the running time of the optimal algorithm may rise sharply as the vehicular density grows (e.g., approximately 2,261 seconds (37.68 minutes) are needed to obtain the optimal offloading solution in a VC containing seven SPs and three tasks), which makes it unsuitable for fast-changing and large IoV networks. Notably, VC configurations

\(^5\)The degree of a vertex in a graph is defined as the number of edges associated with this vertex.
with different topological complexity (e.g., the presence of more connections between vehicles) will also lead to dramatic changes in running time performance. However, Fig. 2 and Table I reveal that the proposed CRRM algorithm can always maintain a low running time under different numbers of iterations, enabling which to be implemented efficiently in rapidly changing and large-scale IoV networks particularly during rush hour, rather than the lowerbound on running time of the

![Algorithm 2: Connection-restricted random matching for multi-task offloading under graph-based representation](image)

Table I

| Tasks, SPs, and VMs in a VC | Optimal | Iterations (CRRM) |
|-----------------------------|---------|-------------------|
|                            | 1,000   | 2,000             | 3,000             |
| 2 × type 2, 4 × SPs, 16 × VMs | 5 × 10^3 s | 0.1413s | 0.2816s | 0.4194s |
| 5 × type 2, 8 × SPs, 40 × VMs | 1 × 10^3 s | 0.1439s | 0.2745s | 0.4275s |
| 2 × type 3, 4 × SPs, 25 × VMs | 1 × 10^3 s | 0.1613s | 0.3120s | 0.4806s |
| 5 × type 3, 8 × SPs, 48 × VMs | 2 × 10^3 s | 0.1702s | 0.3442s | 0.5103s |
| 2 × type 4, 5 × SPs, 30 × VMs | 3 × 10^3 s | 0.1688s | 0.3234s | 0.4956s |
| 5 × type 4, 10 × SPs, 50 × VMs | 3 × 10^3 s | 0.1801s | 0.3628s | 0.5279s |

Fig. 2. Running time comparison of Task type 1 between the optimal and CRRM algorithms.
optimal algorithm. Considering in Fig. 2, comparing with the optimal algorithm, utilizing the proposed CRRM algorithm leads to 90.19% and 99.84% performance improvement on the average running time in scenarios with two tasks and three tasks, respectively.

Performance comparisons on the value of (4) between baseline methods and the proposed algorithms in low-traffic and rush-hour scenarios with different numbers of tasks, SPs, and available VMs are presented in Fig. 3. As the number of iterations increases, the proposed CRRM outperforms the DPM and ETPM methods. Although these methods may perform similarly initially, when the number of iterations is small (e.g., Fig. 3(c)), the gap becomes larger as iterations increase. Due to that a very long time is needed to obtain optimal solutions in rush-hour scenarios, the performance of the optimal algorithm is ignored in Fig. 3(c) and Fig. 3(d). Solutions of the proposed CRRM approach those of the optimal algorithm shown in Fig. 3 while enjoying much lower computational complexity for different task types and vehicular densities.

VI. CONCLUSION

This paper studies a novel multi-task offloading mechanism over VCs using graph-based representation, which is modeled as a nonlinear integer programming problem under constraints. For low-traffic scenarios, an optimal algorithm is introduced; for rush-hour scenarios, a CRRM algorithm is proposed with low computational complexity. The effectiveness of the proposed algorithms is revealed through comprehensive simulations. One potential future direction for research is considering tasks modeled by directed graphs.

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