FIELD-THEORETICAL TREATMENT OF NEUTRINO OSCILLATIONS

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Abstract

We discuss the field-theoretical approach to neutrino oscillations. This approach includes the neutrino source and detector processes and allows to obtain the neutrino transition or survival probabilities as cross sections derived from the Feynman diagram of the combined source – detection process. In this context, the neutrinos which are supposed to oscillate appear as propagators of the neutrino mass eigenfields, connecting the source and detection processes.

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I. INTRODUCTION

Neutrino oscillations [1] play a central role in neutrino physics. The most important condition for this phenomenon is given by neutrino mixing which is described by

$$\nu_{L\alpha} = \sum_j U_{\alpha j} \nu_{Lj}$$

with $\alpha = e, \mu, \tau, \ldots$ and $j = 1, 2, 3, \ldots$ labelling neutrino flavours (types) and mass eigenfields, respectively. All neutrino oscillation experiments are evaluated with the formula [1]

$$P_{\nu_\alpha \rightarrow \nu_\beta} (L/E_\nu) = \left| \sum_j U_{\beta j} U_{\alpha j}^* \exp \left( -i \frac{m_j^2 L}{2E_\nu} \right) \right|^2,$$  \hspace{1cm} (1)

where $U$ denotes the unitary mixing matrix, $L$ the distance between source and detector and $E_\nu$ the neutrino energy. The neutrino masses $m_j$ are associated with the mass eigenfields $\nu_j$. It has been indicated in several publications that the standard derivation of Eq. (1) raises a number of conceptual questions (see, e.g., Ref. [4] for a clear exposition). Some of these questions are solved by the wave packet approach [2] (see also the review [3] where a list of references can be found), however, the size and form of the wave packet is not determined in this approach and remains a subject to reasonable estimates.

The idea has been put forward to include the neutrino production and detection processes into the consideration of neutrino oscillations. [4] Such an approach can be realized with quantum mechanics – in which case the neutrinos with definite mass are unobserved intermediate states between the source and detection processes [4] – or with quantum field theory where the massive neutrinos are represented by inner lines in a big Feynman diagram depicting the combined source – detection process. [5,6] In the following we will discuss the field-theoretical treatment. The aims and hopes of such an approach are the following:

1. The elimination of the arbitrariness associated with the wave packet approach, 2. the description of neutrino oscillations by means of the particles in neutrino production and detection which are really manipulated in an experiment, 3. a more complete and realistic description in order to find possible limitations of formula (1) in specific experimental situations.

Considering laboratory experiments, there are two typical situations for neutrino oscillation experiments. The first one is decay at rest (DAR) of the neutrino source. Its corresponding Feynman diagram is depicted in Fig. 1. The wave functions of the source and detector particles are localized (peaked) at $\vec{x}_S$ and $\vec{x}_D$, respectively. The other situation is decay in flight (DIF) of the neutrino source as represented in Fig. 2 where it is assumed that a proton hits a target localized at $\vec{x}_T$. The detector particle sits again at $\vec{x}_D$ but the source is not localized. In both situations the distance between source and detection is given by $L = |\vec{x}_D - \vec{x}_S|$. Note that in the Feynman diagrams of Figs. 1 and 2 the neutrinos with definite mass occur as inner lines. In the spirit of our approach, neutrino oscillation probabilities are proportional to the cross sections derived from the amplitudes represented by these diagrams.

II. ASSUMPTIONS AND THE RESULTING AMPLITUDE

The further discussion is based on the following assumptions:
I. The wave function \( \phi_D \) of the detector particle does not spread with time which amounts to

\[
\phi(\vec{x}, t) = \psi_D(\vec{x} - \vec{x}_D) e^{-iE_D pt},
\]

where \( E_{DP} \) is the sharp energy of the detector particle and \( \psi_D(\vec{y}) \) is peaked at \( \vec{y} = \vec{0} \).

II. The detector is sensitive to momenta (energies) and possibly to observables commuting with momenta (charges, spin).

III. The usual prescription for the calculation of the cross section is valid.

With the amplitudes symbolized by Figs. 1 and 2 the oscillation probabilities are obtained by

\[
\left\langle P_{\nu_\alpha \rightarrow \nu_\beta} \right\rangle \propto \int dP_S \int d^3p'_D \ldots \int d^3p'_{DnD} \left| A_{\nu_\alpha \rightarrow \nu_\beta} \right|^2.
\]

In this equation we have indicated the average over some region \( P \) in the phase space of the final particle of the detection process. If no final particle of the neutrino production process is measured then one has to integrate over the total phase space of these final states. By definition, at the source (detector) a neutrino \( \nu_\alpha \) \( \nu_\beta \) is produced (detected) if there is a charged lepton \( \alpha \) \( \beta \) among the final states.

In perturbation theory with respect to weak interactions, according to the Feynman diagrams Figs. 1 and 2 one has to perform integrations \( \int d^4x_1, \int d^4x_2 \) and \( \int d^4q \) corresponding to the Hamiltonian densities for neutrino production and detection and the propagators of the mass eigenfields, respectively. These integrations are non-trivial because \( \psi_D \) and the source (target) wave functions are not plane waves, but are localized at \( \vec{x}_D \) and \( \vec{x}_S (\vec{x}_T) \), respectively. After having performed the integrations over \( x_{1,2} \) and \( q^0 \), in the asymptotic limit \( L \rightarrow \infty \) only the neutrinos on mass shell contribute to the amplitude [5] which can be written as [5,6]

\[
A_{\nu_\alpha \rightarrow \nu_\beta}^\infty = \sum_j A_j^S A_j^D U_{\beta j} U^*_{\alpha j} e^{iq_j L}
\]

with

\[
E_D = \sum_{b=1}^{n_D} E'_{Db} - E_{DP}, \quad q_j = \sqrt{E_D^2 - m_j^2}.
\]

\( A_j^S \) and \( A_j^D \) denote the amplitudes for production and detection, respectively, of a neutrino with mass \( m_j \). Note that \( E_D \) is the energy on the neutrino line in Figs. 1 and 2 and it is independent of \( m_j \). This is an immediate consequence of the assumptions in this section. Furthermore, due to the above-mentioned integrations and the asymptotic limit we obtain

\[
A_j^D \propto \tilde{\psi}_D(-q_j \vec{\ell} + \vec{p}'_D) \quad \text{with} \quad \vec{\ell} = (\vec{x}_D - \vec{x}_S)/L \quad \text{and} \quad \vec{p}'_D = \sum_{b=1}^{n_D} \vec{p}'_{Db},
\]

where \( \tilde{\psi}_D \) is the Fourier transform of \( \psi_D \).
III. RESULTS

The preceding discussion leads us to the conclusion that with the assumptions stated in Section 2 the neutrino mass eigenstates are characterized by the energy $E_\nu \equiv E_D$ and momenta $q_j$ \(\text{(3)}\). Thus they have all the same energy determined by the detection process, but the momenta are different. The summation over $E_D$ is incoherent, i.e., it occurs in the cross section (see Eq. \(\text{(3)}\)), not in the amplitude \(\text{(4)}\). In this sense there are no neutrino wave packets in experiments conforming with our assumptions. Note that it has been pointed out in \(\text{[8]}\) that a coherent or incoherent neutrino energy spread cannot be distinguished in neutrino oscillation experiments. Since the neutrino energy can in principle be determined with arbitrary precision the coherence length can theoretically be increased solely by detector manipulations. \(\text{[8]}\) From Eq. \(\text{(3)}\) it follows that with $\Delta m^2 \equiv |m_j^2 - m_k^2|$ the condition

$$|q_j - q_k| \lesssim \frac{\Delta m^2}{2E_D} \lesssim \sigma_D \quad \text{or} \quad \sigma_{xD} \lesssim \frac{1}{4\pi}L_{\text{osc}}$$

\(\text{(7)}\)

is necessary for neutrino oscillations, where $\sigma_D$ and $\sigma_{xD}$ are the widths of the wave function of the detector particle in momentum and coordinate space, respectively, and $L_{\text{osc}}$ is the oscillation length. \(\text{[2,4,5]}\) In realistic experiments condition \(\text{(6)}\) holds because $\sigma_{xD}$ is a microscopic whereas $L_{\text{osc}}$ a macroscopic quantity. For DAR an analogous condition exists for the width of the neutrino source wave function.

For more details of the field-theoretical approach to neutrino oscillations, for a consideration of the finite lifetime of the neutrino source in the case of DAR and for an application of the results to the LSND and KARMEN experiments we refer the reader to \(\text{[5,6]}\). We have shown that in these experiments effects of the finite lifetime can be neglected. For a discussion of DIF see \(\text{[7]}\). Thus in the framework discussed here all corrections to Eq. \(\text{(4)}\) are negligible. Note that we have not taken into account or discussed the interaction of the final state particles in the source with the environment (“interruption of neutrino emission”), a possible intermediate range of the asymptotic limit $L \to \infty$ as found in \(\text{[10]}\) and the possibility that in some cases (e.g., the KARMEN experiment) it is not realistic to use the conventional procedure to calculate the cross section \(\text{(3)}\) by taking the asymptotic limit of the final time to infinity.
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FIG. 1. Feynman diagram for decay at rest (DAR) of the neutrino source particle. The source (S) and detector (D) processes are symbolized by the circles. The labels $\vec{x}_S$ and $\vec{x}_D$ represent the coordinates where the wave functions of the source and detector particles are peaked, respectively. We have also indicated the $n_S$ ($n_D$) momenta of the final particles originating from the source (detector) process and the neutrino propagator of the neutrino field with mass $m_j$.

FIG. 2. Feynman diagram for decay in flight (DIF) of the neutrino source particle which is produced by a proton with momentum $\vec{P}$ hitting a target (T) particle localized at $\vec{x}_T$. In addition to the final momenta of DAR there are $n_T$ final momenta originating from the target process.