Nonlinear Realization of the General Covariance Group Revisited

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Abstract

A modified algorithm for the construction of nonlinear realizations (sigma models) is applied to the general covariance group. Our method features finite dimensionality of the coset manifold by letting the vacuum stability group be infinite. No decomposition of the symmetry group to its finite-dimensional subgroups is required.

The expected result, i.e. Einstein-Cartan gravity, is finally obtained.

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1 Introduction

Nonlinear realization techniques turned out to be a useful tool in the construction of field theories with spontaneously broken symmetry. As was shown in [1], also Einstein gravitation can be obtained as a nonlinear realization of the general covariance group. In this formulation a gravitational field is the Goldstone boson responsible for breaking the affine group down to the Poincare group. The method was later successfully extended to the $N = 1$ supergravity case [2, 3].

The infinite-parametric (super)group was realized there using the decomposition to its two finite-parametric subgroups, for which then a standard algorithm [4, 5] for nonlinear realizations was used. However, for example Yang-Mills theories, where an analogical decomposition doesn’t exist, were reformulated in a nonlinear realization language without this requirement [6]. Such a decomposition isn’t a necessary condition for the existence of a nonlinear realization of an infinite-parametric group.

Moreover, the method fails in extended supergravity cases. There it leads to fields with too high spins without any hint how to get rid of them. This is maybe a more important reason for another examination of the nonlinear realizations.

We construct a nonlinear realization of the general covariance group by a modified method. It is based on the straightforward realization of the whole infinite-parametric group, in a similar way as was used in the context of $W$-algebras [7] recently. In our approach, however, from the beginning only a finite number of Goldstone fields appear in the theory, because most of the transformations turn out to belong to a vacuum stability group, where they can be handled very easily.

At least two Goldstone fields enter our theory. Apart from the symmetric tensor $h_{\mu\nu}$ there is a third rank tensor $\Gamma^{\mu\nu\rho}$ connected to torsion. Using the inverse Higgs effect [8] this $\Gamma^{\mu\nu\rho}$ can be further eliminated, which is related to the vanishing of the torsion. Finally we arrive to the same invariants as in [1].

In this way we are very close to the results obtained also in [9]. Our approach has the advantage of greater mathematical clarity, because we work with finite-dimensional field manifolds entirely. Furthermore we find a place for matter fields, as well as some other representations, in our theory. We hope our more detailed understanding will be helpful in the extended supergravity cases, where it is difficult to avoid too high spins in the approach like in [1] or [2, 3].
2 Nonlinear Realization of General Covariance Group

Given an infinitesimal analytic coordinate transformation
\[ \delta x^\mu = \lambda^\mu(x), \]  
we may regard it as a transformation encoded by Lie algebra generators
\[ F^\mu_{\nu_1 \ldots \nu_{n+1}} = i x^{\nu_1} \ldots x^{\nu_{n+1}} \frac{\partial}{\partial x^\mu} \]  
with commutation relations
\[ \left[ F^\mu_{\nu_1 \ldots \nu_{n+1}}, F^\nu_{\mu_1 \ldots \mu_{m+1}} \right] = i \sum_{i=1}^{m+1} \delta^\mu_\rho F^m_{\nu_1 \ldots \nu_{n+1} \mu_1 \ldots \mu_{i-1} \mu_{i+1} \ldots \mu_{m+1}} - i \sum_{i=1}^{n+1} \delta^\nu_\kappa F^m_{\mu_1 \ldots \mu_{i-1} \nu_{i+1} \ldots \nu_{n+1} \mu_1 \ldots \mu_{m+1}} \]  
Among these generators we can find translations \( P^\mu = F^{-1}_\mu \), Lorentz transformations \( L_{\mu\nu} = \frac{1}{2}(F^0_{\mu\nu} - F^0_{\nu\mu}) \) and the special affine transformations \( R_{\mu\nu} = \frac{1}{2}(F^0_{\mu\nu} + F^0_{\nu\mu}) \), lowering and raising indices by flat Minkowski
\[ \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1). \]

Furthermore, let us introduce the following decomposition of the generators \( F^1_{\mu\rho} = \eta_{\mu\kappa} \eta_{\rho\kappa} F^1_{\rho \sigma \kappa}: \)
\[ F^1_{\nu\rho\sigma} = \frac{1}{6} \left( F^1_{\nu\rho\sigma} + \text{perm.} \right), \]
\[ T_{\nu\rho\sigma} = F^1_{\nu\rho\sigma} - F^1_{\nu\rho\sigma} = \frac{1}{3} \left( 2F^1_{\nu\rho\sigma} - F^1_{\nu\rho\sigma} - F^1_{\nu\rho\sigma} \right). \]

Now we are going to make a coset realization of the general covariance group \( G \) with respect to an infinite-parametric subgroup \( H \) generated by generators \( L_{\mu\nu} \) (the Lorentz ones), \( F^1_{\nu\rho\sigma} \) and all the generators \( F^m; n \geq 2 \). This means what we are going to realize the group of general covariance on the coset, which can be parametrised by coordinates \( x^\mu \) and tensor fields \( h^{\mu\nu}(x) \) and \( \Gamma^{\mu\nu\rho}(x) \), corresponding to the generators \( R_{\mu\nu} \) and \( T_{\mu\nu\rho} \). Here \( h^{\mu\nu} \) is a symmetric tensor and \( \Gamma^{\mu\nu\rho}(x) \) is a nonsymmetric tensor symmetric in the first two indices, i.e.
\[ \begin{align*}
\Gamma^{\mu\nu\rho}(x) &= \Gamma^{\nu\mu\rho}(x), \\
\Gamma^{\mu\nu\rho}(x) + \Gamma^{\nu\mu\rho}(x) + \Gamma^{\rho\mu\nu}(x) &= 0.
\end{align*} \]
Following the algorithm of [4] we parametrise the coset \( G/H \) as
\[
a(x) = e^{ixP}e^{ih(x)R}e^{i\Gamma(x)T}
\] (2.5)
An element \( g \) of the general covariance group determines a transformation \( a(x) \to a'(x') \) according to
\[
a'(x').h'_x = g.a(x),
\] (2.6)
where \( h'_x \) is some element of the subgroup \( H \). This remainder element \( h'_x \) can be further decomposed as
\[
h'_x = l'_x.q'_x,
\] (2.7)
where \( l'_x \) is an element of the Lorentz group and \( q'_x \) is an element of the rest factorgroup.

There is a possibility of accommodating other fields \( \psi(x) \), so called matter fields. They transform according to some representation \( B \) of the subgroup \( H \) as
\[
\psi'(x') = B(h'_x)\psi(x).
\] (2.8)

3 Cartan Forms

Now we are going to find covariant derivatives of both the Goldstone and the matter fields. For this purpose we define Cartan forms, i.e. \( \text{alg}(G) \)–valued one-forms
\[
a^{-1}(x)da(x) = \sum_{n=-1}^{\infty} i\omega_{\nu_1...\nu_n}^n P^{\nu_1...\nu_n} \]
(3.1)
with a nice transformation law
\[
a'^{-1}(x')da'(x') = h'_x a^{-1}(x)(da(x))h'^{-1}_x + h'_x dh'^{-1}_x
\] (3.2)
Using the commutation relations (2.3) we can express Cartan forms in the terms of fields \( h_{\mu\nu} \), \( \Gamma_{\mu\nu\rho} \). The projections onto translations \( P_{\mu} \), Lorentz \( L_{\mu\nu} \) and special affine generators \( R_{\mu\nu} \) read
\[
\omega^\mu_P = dx^\nu (r^{-1})_{\nu}^\mu,
\] (3.3)
\[
\omega^\mu_R = \frac{1}{2}\{r^{-1}(x), dr(x)\}^\mu + \omega_{P\rho} \Gamma_{\mu\nu\rho} \quad \text{and}
\]
\[
\omega^\mu_L = -\frac{1}{2}\left[r^{-1}(x), dr(x)\right]^\mu - \omega_{P\alpha} \Gamma_{\alpha\mu}^\nu + \omega_{P\alpha} \Gamma_{\nu\alpha}^\mu.
\]
with abbreviation \( r_{\mu}^\nu(x) = \left(e^{h(x)}\right)_\mu^\nu \), i.e. a matrix exponential of \( h_{\mu}^\nu(x) \). The \{\}, resp. [ ] denote a matrix anticommutator, resp. commutator.
The forms corresponding to the coset generators, i.e. \( \omega_P, \omega_R \) and \( \omega_T \), do not transform covariantly under the transformations (2.6) by an element of the Lorentz group \( l'x \) obtained by the decomposition (2.7). This is so because generally they obtain a supplement from the \( q'x \) part of \( h'x \). However, apart from the form \( \omega^P \) corresponding to the translations, another exception is \( \omega^{\mu\nu}_R \) which obtains an addition only of the form \( \lambda_{\text{sym}}^{\mu\nu\rho} \omega^{P\rho} \), where \( \lambda_{\text{sym}}^{\mu\nu\rho} \) is some fully symmetric parameter. It means the nonsymmetric part of the projection onto the covariant \( \omega^P \)

\[
D^\rho h^{\mu\nu} = \frac{1}{3} \left( 2 \frac{\omega^{\mu\nu}_R}{\omega^P} - \frac{\omega^{\nu\rho}_R}{\omega^P} - \frac{\omega^{\rho\mu}_R}{\omega^P} \right) \tag{3.4}
\]
does transform covariantly by a Lorentz group element.

Inserting for the Cartan forms and exploiting the symmetry properties of \( \Gamma^{\mu\nu\rho} \) (2.4) we get

\[
D^\rho h^{\mu\nu} = \frac{1}{3} \left( 2 \nabla^\rho h^{\mu\nu} - \nabla^\mu h^{\nu\rho} - \nabla^\nu h^{\rho\mu} \right) + \Gamma^{\mu\nu\rho}, \tag{3.5}
\]
where we have abbreviated

\[
\nabla^\rho h^{\mu\nu} \equiv \frac{1}{2} r^{\rho\alpha} \{ r^{-1} \partial_\alpha r \}^{\mu\nu}. \tag{3.6}
\]

4 Covariant Derivative of the Matter Fields

The linearly realized subgroup \( H \) consists of two parts - Lorentz group \( L \) and the remaining transformations \( Q \). The remainder \( Q \) itself is also a subgroup. In fact it is even a normal subgroup of \( H \), though not of the whole group of general covariance.

This normality of \( Q \) inside \( H \) gives us a possibility of representing it trivially \( B(q'_x) = 1 \). Then the matter fields \( \psi(x) \) are transformed only in some representation \( B \) of the Lorentz group

\[
\psi'(x') = B(l'_x) \psi(x). \tag{4.1}
\]
A covariant derivative then contains only a Lorentz algebra part, because the remaining generators are effectively zero.

This means that the covariant derivative of matter fields \( \psi(x) \)

\[
D^\rho \psi = \frac{d\psi(x) + i\omega^{\mu\nu}_L B(L_{\mu\nu}) \psi(x)}{\omega_{P\rho}} \tag{4.2}
\]
is, exploiting the shape of \( \omega^{\mu\nu}_L \) (3.3),

\[
D^\rho \psi = r^{\rho\alpha} \partial_\alpha \psi - \left( \frac{1}{2} r^{\rho\alpha} \{ r^{-1}(x), \partial_\alpha r(x) \}^{\mu\nu} + (\Gamma^{\rho\mu\nu} - \Gamma^{\rho\nu\mu}) \right) iB(L_{\mu\nu}) \psi. \tag{4.3}
\]
This covariant derivative is of course defined by its transformation properties only up to a homogeneously transforming addition.

Equivalently, with the use of covariant derivative $D^\rho h^{\mu\nu}$ (3.4) instead of $\Gamma^{\mu\nu\rho}$, we can rewrite

$$D^\rho \psi = r^\rho_\alpha \partial_\alpha \psi - \left( \frac{1}{2} r^\rho_\alpha \left[ r^{-1}(x), \partial_\alpha r(x) \right]^{\mu\nu} - \frac{1}{2} r^\rho_\alpha \left\{ r^{-1}(x), \partial_\alpha r(x) \right\}^{\mu\rho} + \right. \left. \frac{1}{2} r^\rho_\alpha \left\{ r^{-1}(x), \partial_\alpha r(x) \right\}^{\nu\rho} + D^\nu h^{\rho\mu} - D^\mu h^{\rho\nu} \right) i B(L_{\mu\nu} \psi). \quad (4.4)$$

We leave here open the question of that representations of the vacuum stability group $H$, where the $Q$-part doesn’t act trivially. However, as we will show in the next section, known matter fields fit to the above considered case.

### 5 Interpretation in Terms of Differential Geometry

We have obtained a theory invariant with respect to general coordinate transformations. The field content is apart from matter fields a symmetric tensor $h_{\mu\nu}$ and a tensor of the third rank $\Gamma_{\mu\nu\rho}$ with the symmetry properties (2.4). This reminds us slightly of the formulation of gravity in terms of a metric $g_{ab}$ and an affine connection $\kappa^a_{bc}$.

The relation of our formulation to the classical one can be established as follows. First of all let us mention that the group of stability of the origin $G_o$, which includes all the transformations of the general covariance group up to translations, has a covariant vector representation $R_v$. By this we mean the representation defined as the ordinary matrix representation for $GL(4)$ part of $G_o$, i.e.

$$\left( R_v (L^\mu_\nu) \right)_\beta^\alpha = -\frac{i}{2} \left( \delta^\mu_\alpha \delta^\beta_\nu - \eta^{\mu\beta} \eta_{\nu\alpha} \right), \quad (5.1)$$

$$\left( R_v (R^\alpha_\nu) \right)_\beta^\alpha = -\frac{i}{2} \left( \delta^\alpha_\alpha \delta^\beta_\nu + \eta^{\alpha\beta} \eta_{\nu\alpha} \right),$$

and all the other generators represent trivially

$$F^n_\mu \eta^{\nu_1...\nu_{n+1}} = 0 \quad \text{for } n \geq 1. \quad (5.2)$$

In an analogical way other tensor representations can be defined.

Now let us return to the transformation law (2.3). We can rewrite $g.a(x) \equiv g.e^{ixP}.e^{ih(x)R}.e^{i\Gamma(x)T}$ as

$$g.e^{ixP}.e^{ih(x)R}.e^{i\Gamma(x)T} = e^{ix'P}.g_o(x,g).e^{ih(x)R}.e^{i\Gamma(x)T}, \quad (5.3)$$
where \( g_o(x, g) \) is just an element of the stability group of origin \( G_o \) determined by \( g \) and \( x \).

This element \( g_o(x) \), taken in the covariant vector representation, is exactly
\[
\frac{\partial x'_{\mu}}{\partial x^\nu} = \frac{\partial x'_{\mu}}{\partial x^\nu}.
\]
This is a key relation in the relationship between the two formulations.

The proof is very straightforward. First we are going to calculate \( g_o \) for an infinitesimal transformation and then insert the vector representation. The calculation of \( g_o \) can best be done in the original representation (2.3), using however a variable \( y \) instead of \( x \) to prevent mixing it up with the coset coordinate. Then we obtain
\[
g_o e^{ixP} = e^{-\lambda^\mu(y)} \frac{\partial}{\partial y^\nu} e^{-x^\nu \frac{\partial}{\partial y^\nu}} = e^{-x^\nu \frac{\partial}{\partial y^\nu}} e^{-\lambda^\mu(y+x) \frac{\partial}{\partial y^\nu}} = (5.5)
\]
From this it follows that in the covariant vector representation (5.1)
\[
(R_v (g_o(x)))^\mu_{\nu} = \delta^\mu_{\nu} + \frac{\partial \lambda^\mu(x)}{\partial x^\nu},
\]
which is exactly the key relation (5.4).

Now we are able to identify tetrades and other quantities according to their transformation properties. We observe that coset elements in the covariant vector representation \( r^\mu_{\nu}(x) = R^\mu_{\nu}(e^{ih(x)R}) \) transform as
\[
r'^\mu_{\nu}(x') = \frac{\partial x'^\mu}{\partial x^\rho} r^\rho_{\lambda}(x) \left( R_v (l_x^{-1}) \right)^\lambda_{\nu}.
\]
This means that although \( r^\mu_{\nu} \) is a symmetrical matrix, each of its indices transforms differently. Thus we arrive at two types of indices - holonomic, resp. anholonomic ones, i.e. those transforming with \( \frac{\partial x'^\mu}{\partial x^\rho} \) and those transforming by the Lorentz transformation \( (R_v (l_x^{-1}) \right)^\lambda_{\nu} \), respectively. From now on we will carefully distinguish these two types by denoting the holonomic indices by latin letters and the anholonomic ones by greek letters. Thus we have \( r^a_{\mu} \) and \( r^{-1}_a \mu \).

If there is a matter field \( \psi^\mu(x) \) transforming as a covariant vector with respect to the Lorentz group elements \( l_x^{-1} \), the quantity \( \psi^a = r^a_{\mu}(x) \psi^\mu(x) \) is a covariant holonomic vector. Similarly, we can construct a holonomic
contravariant vector as \( \psi_a = r_a^{-1}\lambda \psi_\lambda \). This suggest that \( r_\mu^a \) plays the role of a tetrade.

Using the fact that any tensorial (i.e., non–spinorial) representation of the Lorentz group can be extended to the representation of the whole \( GL(4) \), we can construct a holonomic tensor \( t^{a_1 \ldots a_m}_{b_1 \ldots b_n} \) from a given matter field \( \phi^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} \) and the coset element \( e^{\hbar(x)R} \) in a proper representation analogously. One arrives at

\[
t^{a_1 \ldots a_m}_{b_1 \ldots b_n} = g^{a_1 \mu_1}_{\mu_1^1} \ldots g^{a_m \mu_m}_{\mu_m^1} r_{b_1}^{-1} \ldots r_{b_n}^{-1} \phi^{\mu_1 \ldots \mu_m}_{\nu_1 \ldots \nu_n}.
\]

Furthermore, the quantities \( g_{ab} = r_a^{-1} r_b^{-1} \eta_{\mu \nu} \), resp. their inverse \( g^{ab} = r_\mu^a r_\nu^b = (g_{ab})^{-1} \) determine an invariant scalar product of covariant, resp. contravariant vectors and thus play the role of a metric field.

We can conclude that quantity \( r_\mu \), the coset element in a vector representation, plays the role of a tetrade which is preserved symmetric by a proper the Lorentz transformation of the anholonomic index when subject to a general covariance transformation.

It remains to answer what are the counterparts of the covariant derivatives \( \nabla^\rho \phi^{\mu \nu \rho} \) and, related to this, what is the role of the field \( \Gamma^{\mu \nu \rho} \).

The covariant derivative \( D^\rho \phi^{\mu \nu} \) transforms as a second rank anholonomic tensor. The corresponding holonomic tensor \( \Delta^a \phi^b = r^a_\rho r^b_\mu D^\rho \phi^{\mu \nu} \) can be rewritten, using the formula \( (4.3) \) for the covariant derivative and the abbreviation \( (3.6) \), as

\[
\Delta^a \phi^b = r_\rho^{-1} r_\mu^b D^\rho \phi^{\mu \nu} = \partial_\rho \phi^b + \kappa^b_{ac} \phi^c
\]

where, with help of the identity \( \nabla^\rho h^{\mu \nu} = -\partial_\rho g_{bc} r^a_\rho r^b_\mu r^c_\nu \),

\[
\kappa^b_{ac} = - (\nabla^\rho h^{\mu \nu} - \Gamma^{\rho \mu \nu} + \Gamma^{\rho \nu \mu}) r_\rho^{-1} r_\mu^{-1} r_\nu^{-1}
\approx \frac{1}{2} g^{bd} (\partial_d g_{ac} + \partial_a g_{cd} + \partial_c g_{ad}) + (D^\rho h^{\mu \nu} - D^\mu h^{\rho \nu}) r_\rho^{-1} r_\mu^{-1} r_\nu^{-1}.
\]

Similarly, for each tensorial representation

\[
\Delta^a t^{a_1 \ldots a_m}_{b_1 \ldots b_n} \equiv r_{a_1}^{-1} \ldots r_{a_m}^{-1} r_{b_1}^{-1} \ldots r_{b_n}^{-1} D^\rho t^{\mu_1 \ldots \mu_m}_{\nu_1 \ldots \nu_n} =
\]

\[
= \partial_\mu t^{a_1 \ldots a_m}_{b_1 \ldots b_n} + \sum_{i=1}^m \kappa_{cd} t^{a_1 \ldots a_i-1 b_{i+1} \ldots a_m}_{b_1 \ldots b_n} - \sum_{j=1}^n \kappa_{cd} t^{a_1 \ldots a_m}_{b_1 \ldots b_{j-1} b_{j+1} \ldots b_n}.
\]

Let us note that it implies together with \( D^\rho \eta^{\mu \nu} = D^\rho \eta_{\mu \nu} = 0 \) that

\[
\Delta^a g^{bc} = \Delta_a g_{bc} = 0.
\]

This means that nonmetricity is essentially zero in this formulation.
Furthermore we observe that the connection $\kappa_{ac}^b$ isn’t symmetric. The torsion $S_{ac}^b$, i.e. the part of $\kappa_{ac}^b$ antisymmetric in $a$, $c$, is then

$$S_{ac}^b = \kappa_{ac}^b - \kappa_{ca}^b = (D^\nu h^{\mu\nu} - D^\rho h^{\nu\mu}) (r^{-1})_{\nu\rho} r_{\mu\alpha} r^{-1}.$$

(5.13)

Let us mention that there is a possibility of another choice of the subgroup $H$. We could leave some more generators in the "broken" (coset) part of the general covariance group $G$. For example leaving there the generator $F_{\mu\nu\rho}^{\text{sym}}$, $\kappa_{ac}^b$ would also yield the fully symmetric part, thus obtaining all the degrees of freedom of the connection. However, considering the transformation properties only, this additional nonlinearly transforming field would be superfluous, because it can be expressed in terms of the metric and a linearly transforming field.

6 Elimination of $\Gamma^{\mu\nu\rho}$

The covariant derivative of the field $h^{\mu\nu}$ (3.5) gives us the possibility of expressing $\Gamma^{\mu\nu\rho}$ in terms of $h^{\mu\nu}$ and its derivatives via so called inverse Higgs effect, i.e. fixing $D^\rho h^{\mu\nu}$ to be zero

$$D^\rho h^{\mu\nu} = 0. \quad (6.1)$$

Then we have

$$\Gamma^{\mu\nu\rho} = -\frac{1}{3} (2\nabla^\rho h^{\mu\nu} - \nabla^\mu h^{\nu\rho} - \nabla^\nu h^{\rho\mu}). \quad (6.2)$$

This elimination can be inserted into the Cartan form $\tilde{\omega}^{\mu\nu}_L$ corresponding to the Lorentz generators $L_{\mu\nu}$, which affects the covariant derivative (4.2) of matter fields $\psi(x)$.

$$D^\rho \psi = r^{\alpha\rho} \partial_\alpha \psi - \left( r^{\alpha\rho} \left[ r^{-1} (x), \partial_\alpha r (x) \right] \right)^\mu + \left[ \nabla^\mu h^{\rho\nu} - \nabla^\nu h^{\rho\mu} \right] iB(L_{\mu\nu}) \psi. \quad (6.3)$$

The condition (6.4) is equivalent to vanishing of the torsion. Really, from the shape of the torsion (5.13) and the symmetries of $D^\rho h^{\mu\nu}$ it follows that

$$D^\rho h^{\mu\nu} = (r^{-1})^\mu_b r^{\alpha\nu}_{\rho} S_{ac}^b + (r^{-1})^\nu_c r^{\alpha\mu}_{\rho} S_{ac}^b. \quad (6.4)$$

which together with (5.13) gives

$$D^\rho h^{\mu\nu} = 0 \iff S_{bc}^a = 0 \quad (6.5)$$
7 Invariant Action

In order to construct a suitable action we need a list of invariants of the theory. First of all we construct an invariant volume, which can be made of the Cartan forms $\omega_P$ corresponding to the translations

$$\omega_V = \omega_P^0 \wedge \omega_P^1 \wedge \omega_P^2 \wedge \omega_P^3 = \det(r_{\mu}^a)^{-1}d^4x.$$ (7.1)

The invariant Lagrangian for the matter fields can be constructed by replacing ordinary derivatives by the covariant ones (6.3) in their Lorentz invariant Lagrangian. It remains to find a Lagrangian for the gravity field itself.

There is a curvature $R_{abcd}$ which can be defined as a commutator of the covariant derivatives of a vector. This completes the list of invariants in terms of the tetrade field $r_{\mu}^a$ without more than two derivatives.

One may wonder if it would not be possible to construct a well-transforming object by adding to the Cartan form $\omega_T^{\mu\nu}$ some suitable combination of the other forms. This is indeed possible, the result however is again the curvature because of the Cartan equations.

Anyway, we have all the needed components of the gravitational Lagrangian.

8 Conclusions

We have rederived Einstein–Cartan theory using a modified non-linear realization algorithm, in which from the beginning only a finite number of generators of the general covariance group are broken. The symmetric tensor field arising as a Goldstone boson responsible for breaking the special affine generators is finally the only essential nonlinearly-transforming field in the theory.

The method used doesn’t require an infinitely-parametric symmetry to be decomposable onto finite-parametric subgroups, so it is more general than the previously used algorithm.

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