THE MASS FUNCTION AND DISTRIBUTION OF VELOCITY DISPERSIONS FOR UZC GROUPS OF GALAXIES

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ABSTRACT

We measure the distribution of velocity dispersions of groups of galaxies identified in the Updated Zwicky Catalog, and we use the distribution to derive the group mass function. We introduce a new method that makes efficient use of the entire magnitude-limited catalog. Our determination of \( n(\geq \sigma_T) \) includes a significant contribution from low-luminosity systems that would be missing in a volume-limited sample. We start from a model for the probability density function of the number of group members and reproduce the observed distribution. We take several effects, such as the local fluctuations in volume density, limited sampling, and group selection, into account. We estimate the relation between total number of members, total luminosity, and true velocity dispersion. We can then reproduce not only the observed distribution of \( \sigma_v \), but also the distributions of the number of group members, the total group luminosity, and virial mass. The best fit to the data in the true velocity dispersion range 100 km s\(^{-1}\) ≤ \( \sigma_T \) ≤ 750 km s\(^{-1}\) is a power-law model with a slope \( d \log n(\geq \sigma_T)/d \log \sigma_T = -3.4 \) (−2.1, −5) and a normalization \( n(\sigma_T \geq 750 \text{ km s}^{-1}) = (1.27 \pm 0.21) \times 10^{-5} h^3 \text{Mpc}^{-3} \) where \( \sigma_T \) is the true velocity dispersion of a group. The predictions of cold dark matter models are consistent with the mass function we derive from this distribution of velocity dispersions.

Key words: cosmology: observations — cosmology: theory — galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

Groups of galaxies contain approximately half of the galaxies within a magnitude-limited redshift survey (see, e.g., Ramella, Pisani, & Geller 1997; Trasarti-Battistoni 1998; Girardi et al. 2000; Tucker et al. 2000; Adami & Mazure 2002; Merchán & Zandivarez 2002; Ramella et al. 2002). In hierarchical structure formation theories, these abundant systems are the natural progenitors of the galaxy clusters. Thus, measurement of the distribution of the physical parameters of groups provides part of the fundamental basis for understanding the large-scale structure of the universe (Zabludoff & Geller 1994; Nolthenius, Klypin, & Primack 1994; Diaferio et al. 1999; Carlberg et al. 2000; Girardi & Giruricin 2000).

The velocity dispersion of a system of galaxies is an indicator of the depth of the potential associated with the system. Hence the distribution of velocity dispersions is the basis for the determination of the group mass function. Although the relation linking the velocity dispersion with the gravitational mass depends on the relative distribution of visible and dark mass and on the dynamical state of the galaxy system (Diaferio et al. 1993; but see also Xu, Fang, & Wu 2000), the velocity dispersion itself is a physically interesting parameter. The data readily provide an estimate of the group velocity dispersion that is robust against the inclusion of faint members (Ramella et al. 1995; Ramella, Focardi, & Geller 1996; Mahdavi et al. 1999). The small median number of group members introduces a large scatter in the observational estimate, \( \sigma_v \), of the dispersion of a system relative to the true value, \( \sigma_T \). This finite-sampling effect also introduces distortion in the distribution of \( \sigma_v \). Several investigators have determined \( n(\geq \sigma_T) \) for optical galaxy clusters (e.g., Biviano et al. 1993; Bahcall & Cen 1993; Mazure et al. 1996; Fadda et al. 1996; Girardi et al. 1998; Borgani et al. 1999; Reiprich & Böhringer 2002; Bahcall et al. 2003). In addition, there are measurements of the cluster X-ray temperature function (Edge et al. 1990; Henry & Arnaud 1991; Markevitch 1998; Henry 2000; Blanchard et al. 2000; Pierpaoli, Scott, & White 2001; Ikebe et al. 2002). On the theoretical side, large numerical simulations provide estimates of the predicted group/cluster mass function (e.g., Lacey & Cole 1994; Thomas et al. 1998; Diaferio et al. 1999; Governato et al. 1999; Jenkins et al. 2001) for a variety of cosmological models.

For galaxy groups with low luminosity and hence low velocity dispersion, some determinations of \( n(\geq \sigma_T) \) rest on small samples of systems in volume-limited catalogs (e.g., Zabludoff et al. 1993). Others contain residual biases resulting from the magnitude limit of the samples (e.g., Moore, Frenk, & White 1993; Girardi & Giruricin 2000; Heinämäki et al. 2003). In principle, restriction to a volume-limited catalog mitigates the problem of selection effects introduced by the apparent magnitude limit of the redshift survey (Zabludoff et al. 1993). However, low-luminosity systems are virtually absent from volume-limited catalogs.
The determination of \( n(\geq \sigma_T) \) from magnitude-limited samples requires correction for the selection function. The standard approach is to weight each group by the maximum volume in which a group could still be recognized as a triplet. The weighting scheme assumes that throughout the mass range the catalog covers a fair sample of the universe. These determinations also assume that the mass-to-light ratio, \( M/L \), is independent of velocity dispersion.

A reliable determination of the mass function of groups is particularly interesting because there are still significant discrepancies between theory and observations at the low-mass end of the distribution (Martínez et al. 2002; Heinämäki et al. 2003). Standard observational approaches to the determination of \( n(\geq \sigma_T) \) are most uncertain at the low-mass end. For example, Zabludoff et al. (1993) lack low-mass systems because of the absolute magnitude cut of their volume-limited sample. For larger samples, the assumption of a constant number of groups per unit volume is a potential problem. Even in these larger surveys, low-mass systems probe a small volume where the distribution of systems is not homogeneous.

Here we introduce a method for constructing \( n(\geq \sigma_T) \) from all the groups identified in the magnitude-limited Updated Zwicky Catalog (UZC) redshift survey. We improve the statistics of the \( n(\geq \sigma_T) \) determination in the range 100 km s\(^{-1}\) \( \leq \sigma_T \leq 750 \) km s\(^{-1}\). We determine \( n(\geq \sigma_T) \) in a homogeneous way throughout the entire range in \( \sigma_T \). We account for the effects of limited sampling on the estimated velocity dispersion of each group and for local fluctuations in the number density of groups.

We demonstrate that in the UZC, the intrinsic relations among the luminosity, velocity dispersion, and richness of groups are well represented by power laws. By using these model relations we can satisfactorily account for the observed distributions of all the group parameters. These scaling relations for groups may be useful for evaluating the evolution of group properties in catalogs extracted from deeper surveys.

Section 2 discusses the group catalog. Section 3 outlines the main problems in estimating \( n(\geq \sigma_T) \) and the basic method we apply. Section 4 discusses a power-law model for the relation between group richness and velocity dispersion. Section 5 describes the entire analysis method in detail and shows how we use the relation in § 4. Section 6 contains the distribution of velocity dispersions and the group mass function. In § 7, we discuss the scaling relations and demonstrate the broad consistency of the method we use. We summarize in § 8.

2. THE GROUP CATALOG

We base our analysis on the UZC (Falco et al. 1999), in the region defined by \(-2.5 \leq \delta \leq 50^\circ\) and \(8^h \leq \alpha \leq 17^h\) (B1950) in the north Galactic cap and by \(20^h \leq \alpha \leq 4^h\) in the south Galactic cap. In this region the completeness in redshift is 98%. We discard the region \(-13^\circ \leq b \leq 13^\circ\) because of the greater Galactic absorption there (Padmanabhan, Tegmark, & Hamilton 2001).

Ramella et al. (2002) describes the catalog of UZC groups and lists the membership of all the groups analyzed here. Ramella et al. (2002) identifies the UZC groups with a friends-of-friends algorithm (FOFA; Huchra & Geller 1982; Ramella et al. 1997). This algorithm is currently in wide use (Trasarti-Battistoni 1998; Tucker et al. 2000; Girardi & Giuricin 2000; Merchán & Zandivarez 2002; Adami & Mazure 2002) along with some applications of the dendrogram analysis applied by Materne (1979) and by Tully (1987). The groups identified in Ramella et al. (2002) are number density enhancements with \( \delta \rho_N/\rho_N \geq 80 \) in redshift space (\( \alpha, \delta, V \)).

The FOFA associates galaxies with a projected separation on the sky less than \( D_{\text{proj}} = 0.25 \text{ h}^{-1} \text{ Mpc} \) and a line-of-sight velocity difference less than \( V_{\text{LOS}} = 350 \text{ km s}^{-1} \) at a reference fiducial velocity \( v_F = 1000 \text{ km s}^{-1} \). We scale the linking parameters \( D_L = R(V)D_0 \) and \( V_L = R(V)V_0 \) with a function \( R(V) \) according to the prescription of Huchra & Geller (1982). We assume that the UZC galaxy luminosity function is in the Schechter (1976) form and adopt \( M_r = -19.1 \), \( \alpha = -1.1 \), and \( \phi_0 = 0.04 \text{ h}^3 \text{ Mpc}^{-3} \). We obtain these parameters by convolving the luminosity function determined by Marzke et al. (1994) for a very similar sample with a Gaussian 0.3 mag error. These parameters improve those used in Ramella et al. (1997) to identify groups within the northern region of the CfA2 redshift survey (CfA2N). The differences between the Ramella et al. (1997) groups and the groups we identify now within the same region are negligible.

Within the UZC, we identify 301 groups with at least five members and with mean velocities 500 km s\(^{-1}\) \( < V < 12,000 \text{ km s}^{-1} \). At the minimum group radial velocity \( V_{\text{min}} = 500 \text{ km s}^{-1} \), the apparent magnitude limit of the UZC, \( m_{\text{lim}} = 15.5 \), corresponds to an absolute magnitude

\[
M_{\text{lim}} = m_{\text{lim}} - 25 - 5 \log (V_{\text{min}}/H_0) = -13.00 + 5 \log h
\]

and, with \( h = 1 \), to a luminosity log \( (L_{\text{lim}}/L_\odot) = 7.39 \).

To eliminate 39 groups with very low intrinsic luminosity, we limit our analysis to groups with mean velocity \( V \geq 3000 \text{ km s}^{-1} \). We refer to the remaining \( N_C = 262 \) groups as the UZCGG sample. These groups have true velocity dispersions larger than \( \sim 2.5 \) times the errors in the individual redshift determinations. The lower limit in the true velocity dispersion of our 262 groups is comparable to the lower limit of the velocity dispersion of the X-ray–emitting groups detected within the UZCGG sample (Mahdavi et al. 1999).

In a group catalog selected from a redshift survey with an FOFA, some fraction of the groups are accidental superpositions. We have two measures of the fraction of true physical systems in the UZCGG sample. Ramella et al. (1997) used geometric simulations of the large-scale structure in the northern UZC region to demonstrate that 80% of the groups are probably physical systems. Diaferio et al. (1999) compared \( \Lambda \)CDM simulations with the northern portion of the UZC and concluded that the linking parameters that we have chosen for group selection are in the optimal range for reproducing the catalog derived from N-body simulations. In that range, the fraction of real groups is 70%-80%. Mahdavi et al. (2000) cross-correlate a large portion of the UZCGG sample with the ROSAT All-Sky Survey (RASS). Sixty-one groups in the Mahdavi et al. (1999) sample have associated extended X-ray emission. The presence of hot X-ray–emitting gas bound in the group potential well is a confirmation of the physical reality of the system. Mahdavi et al. (2000) use the groups detected in X-rays to show that a minimum fraction of 40% of the groups in the UZCGG subsample are similar X-ray–emitting systems undetectable with the RASS; thus they set a lower limit of 40% on the fraction of real physical systems in the group catalog. At least 40% and probably 70%-80% of the UZCGG systems
are real, and it is thus reasonable to use their properties to derive physical constraints on the nature of groups of galaxies.

3. THE VELOCITY DISPERSION DISTRIBUTION: THE METHOD

The group velocity dispersion is a fundamental quantity both for studying the internal dynamics of the group and for understanding the processes that produce these galaxy systems (Frenk et al. 1990; Weinberg & Cole 1992; Moore et al. 1993; Zabludoff et al. 1993; Crone & Geller 1995). It is therefore important to have a reliable estimate of the distribution of velocity dispersions for groups, the most common bound systems in the universe.

Our goal is measurement of both the probability density of groups with a given velocity dispersion, \( f(\sigma_T) \), and the space density of groups as a function of velocity dispersion, \( n(\geq \sigma_T) \). One direct approach to the problem is analysis of a volume-limited catalog of groups. This procedure significantly reduces the number of groups in the sample, and more importantly, it discards a large number of low-luminosity systems, largely with low velocity dispersions. For example, in the UZCGG sample there are 34 groups within the volume-limited sample with velocity limit \( V_* = 8300 \text{ km s}^{-1} \), and only 20 groups within \( V_M = 12,000 \text{ km s}^{-1} \).

Undersampling the internal velocity field of each system also affects the determination of \( n(\geq \sigma_T) \) for groups. The UZCGG groups have a median of seven members; the estimate of their velocity dispersion \( \sigma_v \) (corrected for radial velocity uncertainty according to Danese, De Zotti, & di Tullio 1980) has a large scatter around the true underlying velocity dispersion \( \sigma_T \). We use the unbiased estimator of Ledermann (1984) to compute \( \sigma_v \). To illustrate the scatter, we use a Monte Carlo simulation to extract 1000 groups at random from a specified true distribution of \( \sigma_T \). We simulate the “observed” group velocity dispersion \( \sigma_v \) under the assumption that peculiar velocities within the group are distributed according to a Gaussian with true dispersion \( \sigma_T \). The distribution of mean radial velocity and richness of the simulated groups is the same as for the groups observed in the UZCGG sample. Figure 1 shows the relation between the true \( \sigma_T \) and the Monte Carlo–sampled \( \sigma_v \) (the straight line represents equality of the two measures). The spread is quite large; it decreases at larger \( \sigma_T \) because the number of members observed is typically larger. For poorly sampled systems, the observed value of \( \sigma_v \) is a poor estimator of the true \( \sigma_T \). In conclusion, the estimate of the distribution of the velocity dispersion is severely affected by several observational and sampling effects.

To avoid these difficulties, we assume that the peculiar velocities of group member galaxies are Gaussian-distributed. This assumption is warranted by our deeper
sampling of groups with associated X-ray emission (Mahdavi et al. 1999). With an average of 35 galaxies per group, the velocity distributions are consistent with a Gaussian.

We simulate the observed value of $\sigma_r$ by Monte Carlo sampling of the Gaussian velocity field with dispersion $\sigma_T$. We use the data to demonstrate that there is a relation between $\sigma_T$ and the absolute group richness $N$. We confirm the relation by examining a set of 43 more deeply sampled groups from Mahdavi et al. (1999) and Mahdavi & Geller (2001). We account for the selection effects introduced by (1) the apparent magnitude limit of the galaxy catalog, (2) the distance-dependent criterion of inclusion of a group in our analysis, $N_{mem} \geq N_{min} = 5$, and finally (3) fluctuation in the space density of groups, $\rho(V)$.

We select groups according to the number of members. We therefore must estimate the probability density function, $p(N)$, of the group absolute richness $N$, that is, the total number of group members brighter than $L_{lim}$ (or $M_{lim} = -13.00$). We use a power-law model for $p(N)$ and constrain both its slope and normalization. From $p(N)$ we predict the number density of groups with more than $N$ members.

By using the relation between group richness and velocity dispersion along with the estimate of $p(N)$, we can compute both $f(\sigma_T)$ and $n(\geq \sigma_T)$. From these two distributions, we can use Monte Carlo simulations to predict the observed distribution of $\sigma_r$. We can also predict the distribution of observed group luminosities, $L_{mem}$, provided that the luminosity function is universal.

The procedure includes the following steps:

1. We estimate the relation between the number of group members (richness) $N$ and the true velocity dispersion $\sigma_T$ (§ 4).
2. We choose a model for the probability density function of the group richness $N$: $p(N)$ (§ 5).
3. We estimate the group selection function by using $p(N)$ (step 2), and then, by comparison with the data, we obtain the radial distribution of group density $\rho(V)$ (§ 5).
4. By using the $N$ versus $\sigma_T$ relation (step 1), the model for $p(N)$ (step 2), and the estimate of $\rho(V)$ (step 3), we compute the cumulative distribution of the velocity dispersions $n(\geq \sigma_r)$ and compare it with the real data in order to assess the agreement with $p(N)$ from step 2 (§ 6).
5. By using the relations among richness, total luminosity, $\sigma_T$, and the group mass, we compare the model distributions of all these quantities with the observations (§ 7).

The main advantages of this procedure are that (a) we determine $n(\geq \sigma_T)$ for groups with $L_T$ an order of magnitude fainter than for a volume-limited catalog of groups identified within the same region, (b) we need not assume a constant space density of groups, (c) we use a sample of groups and an order of magnitude larger than the volume-limited sample, and (d) the mass function estimate does not require the assumption of constant $M/L$.

4. GROUP RICHNESS AND VELOCITY DISPERSION

Several investigators have explored the relationship between various measures of “richness” and the velocity dispersion of rich clusters of galaxies (Bahcall & Cen 1993): richer clusters tend to have larger velocity dispersions. Bahcall (1988), for example, fitted a power-law relation between the galaxy count within 0.25 $h^{-1}$ Mpc and the system velocity dispersion for 23 Abell clusters and a sample of groups (Turner & Gott 1976). More recently, Yee & Ellington (2003) have shown that the richness of clusters of galaxies in the CNOC1 survey is well correlated with their main physical parameters as derived from both optical and X-ray observations. Here we use the UZCGG data along with a set of well-sampled X-ray–emitting groups (Mahdavi et al. 1999; Mahdavi & Geller 2001) to examine the relationship between group richness and velocity dispersion.

We expect some relation between the total number of group members $N$ and the true velocity dispersion $\sigma_T$ because $N$ and $L_T$ are related through the luminosity function, $L_T$ and mass are related for bound systems, and mass and velocity dispersion are also related (if light traces mass). We do not require the same $M/L$ for groups at every velocity dispersion. As a simple approximation, we choose a power-law model for the relation between $n$ and $\sigma_T$:

$$N = N_0(\sigma_T/\sigma_0)^a,$$

where $N_0$ and $\sigma_0$ are scale factors and $a$ is the exponent.

To estimate the parameters $\sigma_0$ and $a$, we fix an arbitrary normalization in number, $N_0 = 100$, and proceed with the following steps:

1. We select the $i$th group from the sample at random. This group has $N_{mem}(i)$ members at a mean radial velocity $V(i)$ and a total number of galaxies brighter than $L_{lim} = -13.00$ of $N(i) = N_{mem}(i)/\nu(V(i))$. The quantity $\nu(V(i))$ is the fraction of galaxies detected in a group at mean radial velocity $V(i)$:

$$\nu(V(i)) = \frac{\int_{L_{lim}}^{\infty} \phi(x)dx}{\int_{L_{lim}}^{\infty} \phi(x)dx}.$$

The luminosity $L_{cut}(V(i))$ corresponds to the UZC apparent magnitude limit at radial velocity $V(i)$, and $\phi(L)$ is the luminosity function. The true velocity dispersion of the group is, according to our model,

$$\sigma_T(i) = \sigma_0[N(i)/N_0]^{1/a}.$$

2. We sample $N_{mem}(i)$ velocities from a Gaussian with standard deviation $\sigma_T(i)$ and compute the simulated velocity dispersion $\sigma_{sim,i}(i)$.

3. We repeat these steps until we obtain the desired number of simulated groups.

4. We perform a Kolmogorov-Smirnov (K-S) test between the simulated distribution of $\sigma_{sim,i}$ and the distribution of observed $\sigma_r$.

5. We vary $\sigma_0$ and $a$ in the interval $a \in [1, 3]$ and $\sigma_0 \in [400, 600]$ km s$^{-1}$ in a $21 \times 21$ grid and maximize the K-S significance level.

We iterate these five steps 10 times with 5000 groups in each iteration and compute the median of $S_{KS}$. The K-S test significance of the comparison between the predicted and the observed distributions.

With this procedure applied to the UZCGG sample, we obtain $\sigma_0 = 510$ km s$^{-1}$ and $a = 1.8$ with median $S_{KS}(\sigma_r) = 0.44$. The 99% confidence interval is $(1.4, 2.1)$ for $a$ (and $(460, 580)$ km s$^{-1}$ for $\sigma_0$). The value of $\sigma_0$ is sensitive to the definition and scale of “richness.”

Figure 2a shows the contours of constant $S_{KS}(\sigma_r)$ in the parameter space. The outermost isopleth is the 99% confidence level, the middle contour traces the 90% confidence
level, and the inner contour traces the 50% confidence level. Note that the values of $c_0$ are scaled for group richness estimated at $V \geq 3000$ km s$^{-1}$ and hence for members brighter than $-16.89$. The corresponding value of the best-fit $c_0$ scaled to $V = 500$ km s$^{-1}$ and $M_{\text{lim}} = -13.00$ is $230$ km s$^{-1}$.

To test the model in equation (1) further, we obtain $c_0$ and $a$ for a set of X-ray–emitting groups kindly provided to us by A. Mahdavi (1999, private communication). Mahdavi et al. (2000) identified a set of groups with extended X-ray emission by cross-correlating a portion of the UZC with the RASS. The extended X-ray emission essentially guarantees the reality of the system. Mahdavi (1999, 2003, private communications) obtained deeper redshift surveys of the groups and increased the typical membership to 30 galaxies. We use these enhanced data here to test the scaling relations derived for the UZC groups.

There are 43 groups in the Mahdavi sample with mean radial velocities $V \geq 5000$ km s$^{-1}$ and galaxies brighter than $m = 16.5$. The average number of group members is 20, and hence the value of the velocity dispersion is estimated more reliably than for the UZCGG sample. Moreover, the diffuse X-ray emission strongly reduces the false-group identification problem. Figure 2b shows the K-S test confidence level contours for the X-ray sample. The best-fit value for the slope $a$ is 1.7. The 99% confidence intervals are (1.0, 2.7) for $a$ and (410, 590) km s$^{-1}$ for $c_0$. Because of the different completeness limits and different cuts in mean radial velocity, the values of the $c_0$ normalization in the two plots for X-ray and UZCGG groups differ by a factor of 1.06. This offset improves the overlap of the contours in the $c_0$ direction. Although the area covered by the contours is quite large, it is quite remarkable that X-ray groups yield values for $a$ and $c_0$ nearly coincident with the UZCGG sample.

Figure 3 shows $N$ as a function of $c_v$ for the X-ray sample. The thick solid line is the result we obtained with the Monte Carlo procedure. The thin line is the bisector of the two fits obtained from a straightforward $\chi^2$ fit to $N$ versus $c_v$ and to $c_v$ versus $N$, respectively. We find

$$\log N = 1.27^{+0.83}_{-0.42} \log c_v - 1.47^{+1.14}_{-2.04},$$

consistent with the Monte Carlo approach. The Monte Carlo yields a steeper slope because it assigns less weight to the rare groups with very low velocity dispersion, which also depart from the $L_X$-$\sigma$ scaling relations (Mahdavi & Geller 2001).

Fig. 2.—(a) Contour levels of the $N$ vs. $s_T$ model (eq. [1]) for UZCGG groups with five or more members. The outer contour traces the 99% confidence region, the middle contour traces the 90% confidence level, and the inner contour traces the 50% confidence level. (b) Same as (a), but for X-ray–emitting groups.

Fig. 3.—Relation between the group richness $N$ and the velocity dispersion $c_v$ for the X-ray–emitting groups. The thin line results from the bisector of the two least-squares straight lines (one by fitting $N$ vs. $c_v$ and the other by fitting $c_v$ vs. $N$); the thick line shows that our Monte Carlo approach has less sensitivity to low-$c_v$ groups.
Finally, we note that our result agrees with the power-law relation that Yee & Ellingson (2003) find between richness parameter, $B_{\text{gc}}$, and radial velocity dispersion, $\sigma_r$, for CNOC1 clusters. Yee & Ellingson find $B_{\text{gc}} \propto \sigma_r^{1.83 \pm 0.2}$ within the range $600 \text{ km s}^{-1} \leq \sigma_r \leq 1300 \text{ km s}^{-1}$. The agreement is interesting also, because there is only a marginal overlap between our high-$\sigma$ range and the low-$\sigma$ range of CNOC1 clusters.

5. THE PROBABILITY DENSITY FUNCTION OF GROUPS

Here we investigate the probability density function of groups, $p(N)$, for the UZCGG sample. Most (92%) of the groups have $N \leq N_r = 840$ (or $\sigma_T \leq \sigma_r = 750 \text{ km s}^{-1}$). Sparse sampling of the richest systems (clusters) limits the range over which we can reliably estimate $p(N)$ to $20 < N < 840$. In our determination of $p(N)$, we include all groups. Groups with $N > N_r$ are visible (i.e., $N_{\text{obs}} > 5$) throughout the whole sample volume. These groups have marginal weight in the determination of the slope of $p(N)$, but they are important for the normalization of $n(\geq \sigma_T)$.

We assume a power-law model for $p(N)$:

$$p(N) = A(N/N_{\text{norm}})^{-\gamma},$$

(4)

where $N_{\text{norm}}$ is a scale factor and $A$ is a normalization constant. Here we set $N_{\text{norm}} = N_{\text{min}}$ with $N_{\text{min}} = 5$ (see the previous section). We note that the choice of $N_{\text{norm}}$ is arbitrary because the power law is scale-free.

The Press & Schechter (1974) formalism, P-S hereafter, provides a basis for this assumption. Valageas & Schaeffer (1997) argue that in the low-mass range occupied by poor galaxy systems, the probability density function $p(N)$ is well described by a power law. Recently, Jenkins et al. (2001) used numerical simulations to obtain similar results. We make the argument appropriate for our data here.

The P-S model describes the mass function as

$$n_{\text{PS}}(M) dM = K \left( \frac{M}{M_*} \right)^{\alpha} \exp \left[ - \left( \frac{M}{M_*} \right)^\beta \right] d \left( \frac{M}{M_*} \right),$$

(5)

where $K$ is a normalization factor, $M_*$ is a scale factor, and the exponents $\alpha$ and $\beta$ are linked to the spectral index, $n_{\text{eff}}$, of primordial mass fluctuations: $\alpha = n_{\text{eff}}/6 - 3/2$ and $\beta = 1 + n_{\text{eff}}/3$. If we suppose that the group mass $M$ is a power-law function of the group number of galaxies $N$,

$$M \propto N^k,$$

(6)

we can rewrite the mass function in terms of $N$:

$$n_{\text{PS}}(N) dN = K' \left( \frac{N}{N_*} \right)^{\alpha k + k - 1} \exp \left[ - \left( \frac{N}{N_*} \right)^\beta \right] d \left( \frac{N}{N_*} \right).$$

(7)

The local slope of the P-S model is

$$\gamma = \frac{d}{d \ln x} \ln n_{\text{PS}}(x),$$

(8)

where $x = N/N_*$. In other words,

$$\gamma = \beta k x^\beta - \alpha k + 1.$$

(9)

The slope $\gamma$ depends on $x$ and hence on $N$, but for the range of interest here ($20 \leq N \leq 840$), $\gamma$ goes from 2.2 to 3.3. This range is in good agreement with the 90% confidence interval in our fit, namely, $\langle 2.7, 3.2 \rangle$ (see § 6). By taking $N$ as the average value $\langle N \rangle$ of the $N$ estimated for the observed groups, we can conclude that equation (9) links the two parameters $n_{\text{eff}}$ and $N_*$ that characterize the P-S model. Given a value of $n_{\text{eff}}$, we should obtain a "best-fit" $N_*$,

$$N_* = \langle N \rangle \left( \frac{\beta k}{\gamma + \alpha k + k - 1} \right)^{1/(\beta k)},$$

(10)

shown in Figure 4. Over the range covered by our data, the P-S model is well approximated by a simple power-law model for the mass function and $N$-function.

We next evaluate $\gamma$ in equation (4) from the observed group catalog. We start by estimating the probability of finding a group at mean velocity $V$ with at least five observed members, $N_{\text{mem}} \geq N_{\text{min}} = 5$. We call this function the group selection function, $\Sigma(V)$.

The probability that a group has at least $N$ total members is

$$S(N) = \int_N^\infty p(x) dx$$

(11)

with $S(N)$ normalized so that $S(N_{\text{norm}} = N_{\text{min}} = 5) = 1$. Consequently, the group selection function $\Sigma(V)$ is

$$\Sigma(V) = S(N_{\text{min}}/\mu(V)).$$

(12)

Clearly, $\Sigma(V)$ depends on both $\phi(L)$ and $p(N)$.

![Fig. 4.-The P-S model. The contours show the K-S test significance level $S_{\text{KS}}(N_{\text{mem}})$ for the comparison between the observed distribution of the groups and the P-S model described by $N_*$ and the effective spectral index $n_{\text{eff}}$. The outer contours indicate the 99% confidence level, the middle contours indicate the 50% confidence level, and the inner contours show the 10% confidence level. The dashed line shows the relation for data distributed according to a power law (eq. [10]) and with $\langle N \rangle = 300$, $\gamma = 2.9$, and $k = 1.43$. The power-law line lies well within the 50% confidence level. The relation between the group richness $N$ and mass is $M \propto N^{\gamma}$ (see Table 1).](image-url)
From the group selection function, we can compute the distributions of $V$ and $N_{\text{mem}}$ from the model for $p(N)$. The distribution of group radial velocities is

$$C(V) = \int_{V_m}^{V} \Delta \Omega v^2 \rho(v) \Sigma(v) dv,$$

where $V_m = 3000$ km s$^{-1}$, $\Delta \Omega = 3.16$ sr is the solid angle of the UZCGG sample, and $\rho(V)$ is the density of groups as a function of their mean radial velocity. The distribution of observed members is

$$Q(N_{\text{mem}}) = \int_{V_m}^{V} \Delta \Omega v^2 \rho(v) S(N_{\text{mem}}/\nu(v)) dv$$

with $V_M = 12,000$ km s$^{-1}$, the redshift limit of the group catalog.

We compare the simulated $C(V)$ and $Q(N_{\text{mem}})$ with the observed distributions of the same quantities. To make the comparison, we must solve a system of two equations in the two functions $p(N)$ and $\rho(V)$. We have an assumed form for $p(N)$. To estimate $\rho(V)$ we invert the equation for $C(V)$ (eq. [13]). We approximate $\rho(V)$ with a sequence of $N_{\text{shell}}$ radial velocity shells, each with constant density; that is,

$$\rho_i = \frac{\Delta N_i}{\Delta \Omega \int_{V_{i-1}}^{V_i} v^2 \Sigma(v) dv}.$$

We choose the limits of the shells, $V_i$, so that all shells contain the same number of groups. We start with $N_{\text{shell}} = 1$ and perform a K-S test to estimate the significance $S_{\text{KS}}(V)$ of the agreement between the model prediction $C(V)$ and the observed distribution of $V$. We increase $N_{\text{shell}}$ until $S_{\text{KS}}(V) \geq 0.5$.

We have thus derived a reasonable approximation to $\rho(V)$, and we next use it in equation (14) to compute $Q(N_{\text{mem}})$. We use a K-S test to evaluate the agreement between $Q(N_{\text{mem}})$ and the observed distribution of $N_{\text{mem}}$. If the agreement is satisfactory, we conclude that we have a satisfactory slope for $p(N)$. If, on the contrary, the distributions are inconsistent, we repeat the whole procedure with a different slope for $p(N)$. Once we determine $\rho(V)$ and $p(N)$, we can compute the total abundance of seen and unseen groups richer than a fixed threshold within the UZCGG volume, that is, the group multiplicity function $\mu(\geq N)$.

The total number $N_G$ of groups within the UZCGG sample is

$$N_G = \Delta \Omega \int_{V_{\text{min}}}^{V_{\text{max}}} v^2 \rho(v) dv,$$

and the group average density is

$$\bar{\rho} = N_G/\varphi,$$

where $\varphi = 1.79 \times 10^6$ h$^{-3}$ Mpc$^3$ is the UZCGG volume. Under the assumption that $p(N)$ does not depend on position within the UZCGG volume, the multiplicity function is

$$\mu(\geq N) = \bar{\rho} \int_N^{+\infty} p(x) dx.$$

For groups with $N$ large enough to be observable throughout the UZCGG volume, we can compare our model prediction $\mu(N)$ directly with the observations. We call these groups robust and use them to normalize the function $\mu(\geq N)$.

To go from $p(N)$ and $\mu(\geq N)$ to $n(\geq \sigma_T)$, we use the relation between $N$ and $\sigma_T$ (eq. [1]). The data constrain the power-law slope $a$ and the normalization $\sigma_0$. The relations we need are

$$f(\sigma_T) = p(N) \frac{dn}{d\sigma_T},$$

$$n(\geq \sigma_T) = \rho \int_{\sigma_T}^{+\infty} f(x) dx.$$

We can compute $n(\geq \sigma_T)$ by inserting equation (4) into equation (20).

Because $\sigma_T$ is related to $N$ as in equation (1), we can write

$$f(\sigma_T) = (\sigma_T/\sigma_{\text{norm}})^{-\gamma},$$

where $\gamma = \gamma_0 - a + 1$ and $\sigma_{\text{norm}}$ is a normalization scale.

To estimate the value of the exponent $\gamma$, we compare the observations with the predictions of the $p(N)$ model obtained by inserting equation (4) for $p(N)$ into equations (13) and (14). The best-fit value is $\gamma = 2.9$ with a 90% confidence level interval (2.7, 3.2). Figure 5 shows the K-S significance level $S_{\text{KS}}$ versus $\gamma$. The curve is very well behaved. This fit requires $N_{\text{shell}} = 7$ homogeneous density shells (see eq. [15]). We show the density function $\rho(V)$ for the seven shells in Figure 6; the density rises steeply at large $V$ (the last two shells). If we remove these shells from the analysis, the results do not change significantly. With $\gamma = 2.9$, our model is consistent with both the observed cumulative distribution of $N_{\text{mem}}$ and the radial velocity distribution, with significance levels of $S_{\text{KS}}(N_{\text{mem}}) = 0.73$ and $S_{\text{KS}}(V) = 0.62$, respectively.

![Fig. 5.—The power-law model: the K-S test significance level $S_{\text{KS}}(N_{\text{mem}})$ for the comparison between the model and observed distributions of the group members.](image-url)
6. THE DISTRIBUTION OF VELOCITY DISPERSIONS AND THE GROUP MASS FUNCTION

To compare with models, we derive the velocity dispersion distribution and the mass function. Diaferio et al. (1999) have made an exhaustive comparison of group catalogs derived from the $\Lambda$CDM and $\sigma$CDM simulations by Kauffmann et al. (1999) with catalogs derived from CfA2N, a subset of the UZC. They use the same FOFA we employ and vary the linking parameters over a wide range. They also test the sensitivity of their results to variations in the luminosity function and to the presence of structures such as the Great Wall. They demonstrate that for the linking parameters we use here, the catalog derived from the data agree reasonably well with the catalogs derived from $\Lambda$CDM provided that the luminosity density is the same in both cases. They show that the normalization of the distribution of velocity dispersions (and of the mass function) is much more sensitive to the luminosity density than to variation in the linking parameters in the group-finding algorithm over a reasonable range. They emphasize that the Great Wall plays an important role in biasing the distribution of velocity dispersions and the mass function toward higher values relative to the simulations. We see a similar effect and comment further below.

In the previous section, we obtained the best-fit value $\gamma = 2.9$ [90% c.l. (2.7, 3.2)] for the exponent of the power-law probability density function of groups, $p(N)$. We conclude that, at high confidence level, the density of galaxy systems with true velocity dispersion larger than $\sigma_T$ is

$$n(\geq \sigma_T) = (1.27 \pm 0.21) \times 10^{-5} \ h^3 \ Mpc^{-3}$$

$$\times \left( \frac{\sigma_T}{750 \ \text{km s}^{-1}} \right)^{-3.4^{+13}_{-16}}. \quad (23)$$

This distribution applies over the range 100 km s$^{-1}$ $\leq \sigma \leq 750$ km s$^{-1}$. The volume covered by the group cata-

log is too small to include many rich clusters. Within the UZCGG sample, there are 20 (robust) groups visible throughout the entire UZCGG volume. These groups have $N \geq N_r = 840$, corresponding to a true velocity dispersion $\sigma_t = 750$ km s$^{-1}$ (see eq. [1]), the actual upper limit of the range of $N$ (and $\sigma_T$) over which we determine $p(N)$. For the high-$\sigma_T$ systems, our analysis does not provide an estimate of the slope of $p(N)$, but it does provide an estimate of the abundance of these systems: $p(N_r) = 1.27 \times 10^{-5} \ h^3 \ Mpc^{-3}$. We compute the Poisson uncertainty in $p(N_r)$ in the number of observed groups $N_G$. $\delta p(N_r) = 0.25 \times 10^{-5} \ h^3 \ Mpc^{-3}$, and conclude that our prediction agrees with the observed value $p(N_r)$ to within 0.68$\delta p(N_r)$.

We also compare our determination of the abundance of massive systems with previous surveys. Direct comparison is possible with Mazure et al. (1996), Fadda et al. (1996), and Zabludoff et al. (1993). These authors give $p(\geq \sigma_T)$ for clusters. Their system abundances at $\sigma = 750$ km s$^{-1}$ are 0.2, 0.6, and 0.6 $\times 10^{-5} \ h^3 \ Mpc^{-3}$, respectively. Our system abundance is $n(\geq 750$ km s$^{-1}) = 1.3 \times 10^{-5} \ h^3 \ Mpc^{-3}$. Internal errors are on the order of a few tenths in units of $10^{-5}$. These errors reflect the size of the samples and may be less important than the systematic errors introduced by various selection effects. To examine the discrepancy between our estimate of $n(\geq 750$ km s$^{-1}$) and previous ones, we summarize the characteristics of the different samples.

The ENACS clusters (Mazure et al. 1996) lead to the lowest abundance of systems. ENACS is a sample of Abell clusters selected according to a richness criterion. Abell clusters are an incomplete set of systems (see, e.g., Gal et al. 2003, DPOSS2). More importantly, selection according to Abell richness produces a biased sample of velocity dispersions. Mazure et al. (1996) were aware of this problem and discuss it in detail. The velocity dispersions of clusters selected according to richness are biased high because $\sigma$ increases with richness and because there is a broad scatter around the mean relation. Mazure et al. used reasonable arguments to identify a threshold, $\sigma = 800$ km s$^{-1}$, above which their cluster catalog is unbiased. In the end, however, unbiasedness remains an assumption. The abundance derived by Fadda et al. (1996) is a factor of 3 larger than that of Mazure et al. (1996). Fadda et al. added poorer systems to the ENACS sample. In order to have a $\sigma$-distribution that takes into account poor systems, Fadda et al. scaled their richness distribution to mimic that of the Edinburgh-Durham Cluster Catalog (EDCC; Lumsden et al. 1992). EDCC is probably a more complete catalog than Abell/ACO, especially at the low-richness end. Fadda et al. performed 10,000 random samplings of the velocity dispersions of their systems, constraining the richness distribution of the bootstrapped sample to be the same as that of EDCC systems. This analysis leads to the same cluster abundance as found by Zabludoff et al. (1993), who used a combined sample of Abell clusters and the 30 densest groups selected within the first two slices of the CfA2N redshift survey (Ramella, Geller, & Huchra 1989).

We select systems in redshift space from a complete magnitude-limited galaxy catalog similar to but more extensive than that used by Zabludoff et al. (1993). In contrast with Zabludoff et al., we include relatively poor systems that would enter neither a volume-limited sample nor a two-dimensionally selected sample. In fact, because of the large scatter around the mean relation between $\sigma$ and richness, several of these relatively poor systems have velocity

![Figure 6. Density of groups as a function of the radial distance $p(R)$. The function is the best-fit power law model for $p(N)$ with $N_{\text{null}} = 7$ intervals. The error bars in $p$ are the width of the bins; for $p$ they are the Poisson uncertainty $\delta p$ in the number of groups per bin.](image-url)
dispersions $\sigma \geq 750$ km s$^{-1}$. It is therefore not surprising that we find a higher abundance of systems above this threshold.

Other optical and X-ray studies have returned lower system abundances. Bahcall & Cen (1993), for example, argue that the abundance of clusters with velocity dispersion larger than 750 km s$^{-1}$ is $0.2 \times 10^{-5} h^3$ Mpc$^{-3}$, much lower than our estimate. They use a richness-limited sample of Abell clusters and obtain a scaling relation $M \propto \sigma^2$, significantly different from ours (see Table 1).

The abundance of clusters can also be estimated from X-ray–selected samples. The comparison of our abundance of systems with that of X-ray clusters requires a relation between X-ray temperature and velocity dispersion, mass, or both. The scatter around this relation is the source of considerable uncertainty. Reiprich & Böhringer (2002) have carefully analyzed the abundance of clusters derived from X-ray data. In the range $2.55 \times 10^{14} M_\odot \leq M \leq 4.4 \times 10^{14} M_\odot$, the cluster abundance is $1.4 \times 10^{-6} h^3$ Mpc$^{-3} \lesssim n(M) \lesssim 7.0 \times 10^{-6} h^3$ Mpc$^{-3}$. They conclude that the abundance of X-ray clusters is generally lower by a factor of 1.2–6.2 than that of optically selected clusters (Girardi et al. 1998). The origin of the discrepancy either may be intrinsic—optical and X-ray clusters may belong to different populations—or may result from observational biases affecting the mass estimates.

We may overestimate $n(\geq 750$ km s$^{-1})$ somewhat because of (1) projection effects in redshift space, (2) large-scale inhomogeneities in the galaxy distribution, and (3) systematic errors in catalog magnitude limits. N-body and geometric simulations indicate that 20% of the five-member groups we study are accidental superpositions. X-ray observations (Mahdavi et al. 2000) indicate that at least 40% of the groups we study are true physical systems. It is difficult to correct our $n(\geq \sigma_T)$ for the presence of false groups, since we know neither their abundance nor their velocity dispersion distribution. Whatever the correction, our $n(\geq 750$ km s$^{-1})$ would decrease to come into closer agreement with the estimates of Zabludoff et al. (1993) and Fadda et al. (1996).

Another possible effect driving our abundance toward a high value is the possible overluminosity of the UZC, from which we derive our group catalog (Ramella et al. 2002). The Zwicky magnitude system may be deeper than implied by its formal magnitude limit. In this case, we would detect more systems than we should within the surveyed volume of the universe. The density of systems could be too high by as much as a factor of 2. Correction for this effect would bring our estimate of massive systems into close agreement with Fadda et al. (1996) and with Zabludoff et al. (1993).

Finally, it is possible that the region of the UZC is overdense. The abundance of groups is proportional to the galaxy density across many different surveys (see, e.g., Ramella et al. 1999). Variations in density on the scale of the UZC may be as large as a factor of 2, but not more. Further, Diaferio et al. (1999) emphasize that the presence of the Great Wall in the northern portion of the UZC biases the group sample toward higher luminosity and velocity dispersion relative to typical regions extracted from a $\Lambda$CDM simulation. The groups in the Great Wall are at a higher velocity ($\sim 8000$ km s$^{-1}$) than the median group redshift ($\sim 6000$ km s$^{-1}$) in a simulated region. These effects may contribute substantially to the high normalization of the mass function that we derive.

We next review estimates of the mass function of groups. Girardi & Giuricin (2000) have measured the mass function of a sample of groups. A direct comparison between their masses and ours is not straightforward, because their masses are obtained in a model-dependent way. Taken at face value (see Fig. 9 below), their mass function is not significantly different from ours. Our estimate, however, extends toward lower masses by almost an order of magnitude.

Heinämäki et al. (2003) estimate the mass function of the Las Campanas Redshift Survey groups (Tucker et al. 2000). The amplitude of their mass function is very low (less than $0.1 \times 10^{-5} h^3$ Mpc$^{-3}$ for masses roughly corresponding to $\sigma = 750$ km s$^{-1}$). Heinämäki et al. compare their mass function with the results of Girardi & Giuricin and with N-body simulations (see Fig. 7 of Heinämäki et al. 2003). The comparison demonstrates the generally low abundance of systems and the marked flattening of the LCRS group mass function with respect to the simulated mass function at the low-mass end. We obtain a similar flattening when we do not take the large-scale variation of group number density into account. Low-mass groups are undersampled, and a $V/V_{\text{max}}$ weighting scheme does not fully recover them.

Martínez et al. (2002) analyze a sample of groups from the 2dF Galaxy Redshift Survey (Merchán & Zandivarez 2002). In this case, as the authors recognize, the flattening of the mass function at low mass results from the low-number cutoff imposed on their group identification procedure. Martínez et al. compare their mass function with N-body simulations (Jenkins et al. 2001) and claim good agreement. Because their best fit agrees with the $\Lambda$CDM model, their result is marginally consistent ($\sim 95\%$) with ours.

In Figure 7, we plot $n(\geq \sigma_T)$ and its 95% confidence level corridor. There we also plot the velocity dispersion distribution obtained by Zabludoff et al. (1993, long-dashed line). As expected, the Zabludoff et al. volume-limited sample does not include low velocity dispersion systems.

By using the relations between richness and mass (see eq. [6] and Table 1) and between richness and velocity dispersion (eq. [1]), we can convert $n(\geq \sigma_T)$ into an estimate of the mass function. We compare this determination with the simulations of Jenkins et al. (2001; the Virgo Consortium), who derive predictions for the mass function of systems of galaxies over our observed mass range. Because the theoretical mass functions predicted by Jenkins et al. apply to masses of critical overdensities, $(\delta_{p}/\rho_N) = 80$, we perform the scaling as in Borgani et al. (1999) for both of the cosmological models in Jenkins et al. (2001), $\Lambda$CDM ($\Omega_m = 0.3$, $\Omega_L = 0.7$, $\sigma_8 = 0.9$) and $\tau$CDM ($\Omega_m = 1$, $\Omega_L = 0$, $\sigma_8 = 0.6$).

**TABLE 1**

| Relation | Slope | Intercept |
|----------|-------|-----------|
| $\log M_T$ vs. $\log N$ | $1.43 \pm 0.04$ | $10.70^{+0.08}_{-0.09}$ |
| $\log M_T$ vs. $\log L_T$ | $1.44 \pm 0.06$ | $-2.21^{+0.04}_{-0.07}$ |
| $\log M_T$ vs. $\log \sigma_T$ | $2.58 \pm 0.07$ | $7.48^{+0.16}_{-0.13}$ |
| $\log L_T$ vs. $\log \sigma_T$ | $1.80^{+0.05}_{-0.04}$ | $6.71 \pm 0.11$ |
| $\log N$ vs. $\log R_{\text{ng}}$ | $2.00^{+1.54}_{-1.38}$ | $2.11^{+0.06}_{-0.02}$ |
| $\log \sigma_T$ vs. $\log R_{\text{ng}}$ | $1.18^{+0.08}_{-0.45}$ | $2.42^{+0.03}_{-0.08}$ |
Figure 8 compares our power-law determination of the mass function with the theoretical results of Jenkins et al. (2001). We plot the cumulative mass function \( n(\geq M) \) versus the mass \( M \). The solid line represents our power-law estimate, and the dotted lines are 95% confidence levels. The \( \Lambda \)CDM mass function lies within the 95% confidence level over the mass range. The mass function predicted by the \( \Lambda \)CDM cosmology is only marginally consistent with our observations. If we correct the mass function normalization for the presence of unphysical groups, the agreement with the \( \Lambda \)CDM model improves.

The agreement of our observations with the theoretical mass function is remarkable. The use of our weighting procedure is critical in obtaining this result. In fact, if we use the standard approach and weight each group by its maximum accessible volume, \( V/V_{\text{max}} \), we obtain the dashed curve in Figure 9. This curve shows the characteristic bending to a shallower slope at low masses. As discussed above, this shallower slope appears in other observed mass functions (e.g., Girardi & Giuricin 2000; Martínez et al. 2002). Our procedure extends the determination of the mass function to masses almost 1 order of magnitude lower than previous estimates.

7. CONSISTENCY TESTS AND SCALING RELATIONS

The mass function we derive depends on the estimate of the two functions \( p(V) \) and \( p(N) \). We now show that, starting from these two functions, we can reproduce the observed distributions of all the main physical parameters of groups, that is, \( V, N_{\text{mem}}, L_{\text{mem}}, \sigma_T \), and virial mass \( M_{\text{vir}} \).

We proceed in the following way: We randomly sample \( p(V) \) and generate a group with \( N \) members brighter than \( L_{\text{lim}} \). We estimate the number of visible members, \( N_{\text{mem}} \), as \( N_{\text{mem}} = N p(V) \), according to equation (2). We discard the group if \( N_{\text{mem}} \) is less than 5. If we have at least five members, we estimate \( L_{\text{mem}} \) by randomly sampling the Schechter (1976) luminosity function \( N_{\text{mem}} \) times. Finally, we estimate \( \sigma_T \) by randomly sampling a Gaussian with dispersion \( \sigma_T \), where we compute \( \sigma_T \) from the best-fit model (eq. [1]). We
could estimate the virial mass by using the true velocity dispersion $\sigma_T$: $M_T = 3\sigma_T^2R_{\text{vir}}/G$. However, in the usual analysis of an observed group catalog, the value of $\sigma_T$ is not available, and the standard estimate of the virial mass is $M_{\text{vir}} = 3\sigma^2R_{\text{vir}}/G$. We simulate the observationally derived value of $M_{\text{vir}}$ in the following way: We compute the scaling relations between, for example, $N$ and $M_T$ by using the bisector of the two straight lines we obtain first by least squares of log $M_T$ versus log $N$ and second by least squares of log $N$ versus log $M_T$. We compute the simulated value of $M_{\text{vir}}$ as $M_{\text{vir}} = M_T(\sigma_T/\sigma_X)^2$.

We stop the previous Monte Carlo simulations when we reach a total of 1000 observable groups. We compare the simulated distribution with the observed distribution and compute the significance of the agreement between the two with a K-S test. We repeat the whole procedure 10 times and take the median K-S significance level as a measure of the agreement between the distributions.

We obtain the following median values of $S_{\text{KS}}$: 0.43, 0.76, 0.29, 0.15, and 0.34 for $V$, $N_{\text{mem}}$, $L_{\text{mem}}$, $\sigma_X$, and $M_{\text{vir}}$ respectively. Figure 10 shows the distributions of the physical parameters of 1000 simulated groups (thick lines) together with the corresponding distributions for the observed groups (thin lines). The agreement between the observations and the predictions of the simulations indicates that our model is a satisfactory representation of the data.

To test the sensitivity of the exponent $\gamma$ in the $p(N)$ model to the parameters of the $N$ versus $\sigma_T$ law, we use the most extreme values of the parameters $a$ and $N_0$ in equation (1) within the 99% confidence level contour (see Fig. 2a). With these parameter values, it is impossible to recover the observed distributions of both the group members $N_{\text{mem}}$ and velocity dispersion $\sigma_X$.

Table 1 lists the scaling relations we obtain. We determine the scaling relations as the angular coefficient and intercept of the bisector of the two straight-line fits $X$ versus $Y$ and $X$ versus $X$, with $X$ and $Y$ any two related quantities in Table 1. We use the results of the two fits to characterize the uncertainty in the scaling relations.

We actually use the relation between mass $M$ and richness $N$ (eq. [6]), or, equivalently, between $M$ and the velocity dispersion $\sigma$, to translate $n(\geq \sigma T)$ into a mass function, $n(\geq M)$. We also use this relation to recover the observed distribution of the virial mass $M_{\text{vir}}$.

Once we establish the relation between $M$ and $N$, we also have a relation between $M$ and luminosity, $L$. As expected, the slope of log $M$ versus log $N$ and the slope of log $M$ vs. log $L$ are indistinguishable. Girardi et al. (2002) analyze the relation between $M$ and $L$ for a different sample of groups and clusters. They obtain $M \propto L^{1.34\pm0.03}$, within about 1 $\sigma$ of our relation.

$N$-body simulations and semianalytic modeling of galaxy formation performed by Kauffmann et al. (1999) and Benson et al. (2000) gave a reliable prediction of the mass-to-light ratio for galaxy systems and also provided an estimate of the dependence of $M/L$ on $L$ (or $M$). It is generally found that $M/L$ increases from poor to rich systems and eventually flattens on large scales.

Yee & Ellingson (2003) measure the relation between mass and richness ($B_{200}$) within the CNOC1 sample of clusters of galaxies. These authors find $M \propto B_{200}^{0.64\pm0.28}$ in very good agreement with our result. Because the intercept of this relation depends on the magnitude completeness limit, we can compare only slopes and not intercepts.

As a consistency check, we also compute the relation between log $L_T$ and log $\sigma_T$. Reassuringly, the slope of this scaling law is the same as our best fit of the power-law model in equation (1), $N$ versus $\sigma_T$.

Finally, for the sake of completeness, in Table 1 we list the relations between log $M$ and log $\sigma_T$, log $N$ and log $R_{\text{vir}}$, and log $\sigma_T$ and log $R_{\text{vir}}$. Yee & Ellingson (2002) fitted the scaling between the virial radius and richness and find $R_{\text{vir}} \propto R^{2.4\pm0.7}$, in very good agreement with our result. Our scaling relations significantly extend the data range spanned by cluster analyses. These scaling relations are remarkably consistent with those derived for clusters.

We also explore different forms for $p(N)$. For example, we consider a $p(N)$ that gives an exponential law for $f(\sigma_T)$, that is, the law used by Zabludoff et al. (1993). This form for $p(N)$ fails to reproduce the space density of robust groups, $\mu_{\text{obs}}(\geq N_0)$, and leads to a discordant distribution of $L_{\text{mem}}$. Another model we test is the P-S model. The problem with this model (eq. [7]) is that it has two free parameters, $n_{\text{eff}}$ and $N_0$; our procedure can constrain only one parameter (see § 5). However, we can test the relation between $n_{\text{eff}}$ and $N_0$ if $p(N)$ can be approximated by a power law (see eq. [10]). We compare the P-S model for $p(N)$ with our data using a K-S test. In Figure 4, we plot the 10%, 50%, and 99% confidence level contours in the $n_{\text{eff}}$-$N_0$ plane. In the same plane we also plot $N_0$ as a function of $n_{\text{eff}}$ according to equation (1), where we assume $(N) = 300$, close to the average value for our sample. For $\gamma$ we use our best-fit value, $\gamma = 2.9$. Figure 4 clearly shows good agreement between the power-law approximation and the P-S model (note that the rejection region is external to the outermost contour [99% confidence level]).

Sato et al. (2000) determine $n_{\text{eff}} = -1.2 \pm 0.3$ for masses in the range $10^{12}$-$10^{15} M_\odot$ based on analysis of an ASCA X-ray cluster sample. Zandivarez, Abadi, & Lambas (2001) find similar results for these scales. If we use the value $n_{\text{eff}} = -1.2$ in equation (7), we obtain a P-S model with $N_0 \approx 280$, corresponding to $M_\odot = 1.6 \times 10^{14} h^{-1} M_\odot$ (95% confidence interval goes from 6.5 $\times 10^{13}$ to 3.6 $\times 10^{14} h^{-1} M_\odot$). This value of $M_\odot$ is in reasonable agreement with the determinations of Bahcall & Cen (1993), namely, $M_\odot = (1.8 \pm 0.3) \times 10^{14} h^{-1} M_\odot$, and Girardi et al. (1998), $2.6^{+0.8}_{-0.6} \times 10^{14} h^{-1} M_\odot$. This result indicates that our analysis of low-mass systems of galaxies leads to an estimate of the mass function parameters that is consistent with the results determined in the higher mass range of galaxy clusters.

8. SUMMARY

We measure the probability density function for the velocity dispersion of groups in the UZC galaxy catalog. Our method (1) does not require that the group spatial density be constant, (2) predicts the observed distributions of all the fundamental quantities of groups, that is, radial velocity, number of members, luminosity, velocity dispersion, and virial mass, and (3) includes low-luminosity systems that would be neglected with the usual analysis of a volume-limited sample. The best fit for the slope of $n(\geq \sigma T)$ is $-3.4 (-2.6, -4.7)$ over the range 100 km $s^{-1}$ $\leq \sigma \leq 750$ km $s^{-1}$. Over this range, the Press-Schechter model is well approximated by a simple power-law model. Our model predictions agree quite well with the predictions of Jenkins et al. (2001) for the cold dark matter models.
Fig. 10—Power-law model for the group probability density function: observed and model cumulative distributions of (a) radial velocity $V$, (b) $N_{\text{mem}}$, (c) $\log L_{\text{mem}}$, (d) $\sigma_v$, and (e) virial mass $M_{\text{vir}}$ of UZCGG groups (thick lines) and a 1000-group Monte Carlo simulation of our best-fit power-law model (thin lines).
Our weighting procedure is critical in the mass function determination. If we use the standard approach and weight each group by its maximum accessible volume, $V/V_{\text{max}}$, we obtain the characteristic bending of the mass function to a shallower slope at low masses. This shallower slope appears in other observed mass functions (e.g., Girardi & Giuricin 2000; Martínez et al. 2002). Our procedure extends the determination of the mass function to masses almost 1 order of magnitude lower than previous estimates.

For massive systems, the density of groups with true velocity dispersion larger than 750 km s$^{-1}$ is $(1.27 \pm 0.21) \times 10^{-5} h^3$ Mpc$^{-3}$. Our determination is larger by a factor of 2 than previous determinations. We examine this discrepancy and identify several possible sources: (1) incompleteness of cluster catalogs used in previous works, (2) the presence of false groups in our catalog produced by projection effects in redshift space, (3) systematic errors of the Zwicky magnitude system (which may be deeper than its nominal magnitude limit would imply), and (4) real overdensity of the region of the UZC. Corrections for any of these effects would bring the abundance determinations into closer agreement.

The mass function we derive depends on estimates of the two functions $\rho(V)$ and $\rho(N)$. Starting from these two functions, we reproduce the observed distributions of all the main physical parameters of groups, that is, $V$, $N_{\text{mem}}$, $L_{\text{mem}}$, $\sigma_{\text{vir}}$, and virial mass $M_{\text{vir}}$. We also obtain scaling relations between these physical parameters. Our scaling relations significantly extend the data range spanned by cluster analyses and are remarkably consistent with those derived for clusters.

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