Reflections on Zeilinger-Brukner information interpretation of quantum mechanics

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Abstract
In this short review I present my personal reflections on Zeilinger-Brukner information interpretation of quantum mechanics (QM). In general, this interpretation is very attractive for me. However, its rigid coupling to the notion of irreducible quantum randomness is a very complicated issue which I plan to address in more detail. This note may be useful for general public interested in quantum foundations, especially because I try to analyze essentials of the information interpretation critically (i.e., not just emphasizing its advantages as it is commonly done). This review is written in non-physicist friendly manner. Experts actively exploring this interpretation may be interested in the paper as well, as in the comments of “an external observer” who have been monitoring the development of this approach to QM during the last 18 years. The last part of this review is devoted to the general methodology of science with references to views of de Finetti, Wigner, and Peres.

1 Introduction
Tremendous development of quantum information, both theory and experiment, led to considerable growth of interest to foundational problems of QM. Moreover, nowadays quantum information approaches the stage of technological applications, with startups in quantum cryptography and quantum random generators.\footnote{In particular, recently three leading experimental groups\cite{Ref1,Ref2,Ref3} claimed that they were able to perform the loophole free test of violation of Bell’s inequality. This inequality...}
This recent wave of foundational studies is not surprising, because the aforementioned success in theory and experiment is shadowed by the recognition that the basic foundational problems of QM have not yet been solved, in spite of one hundred years of tremendous efforts of the brightest minds in physics and philosophy. One of these problems is the problem of elaboration of a consistent and commonly acceptable interpretation of QM, an interpretation of mathematical entities used in the quantum formalism. Formal rules of mathematical manipulations are well defined and they function very well in all possible applications. However, there is still no consensus on the physical meaning of (at least) some of them.

The modern situation in QM is characterized by the huge diversity of interpretations. Moreover, as was remarked in my book [3] each of basic interpretations has a variety of individual flavors. For example, suppose that somebody claims to be an adherent of the Copenhagen interpretation of QM. When you ask for details, most probably you will hear a rather specific view on what this interpretation is about. I have the impression that there are so many versions of the Copenhagen interpretation of QM as there are people knowing about it. The same can be said about, e.g., the statistical or Bohmian interpretations of QM. This situation is unacceptable for any rigorous and well established theory. Therefore experts in foundations invest so much effort in the problem of interpretation.

Development of quantum information theory enlightened the fundamental role of information in QM and led to elaboration of a variety of information interpretations of QM. In this short review we would like to present the most widely recognized version - the information interpretation of A. Zeilinger and C. Brukner [5], [6]-[11], [12] (see also Kofler and Zeilinger [13] and Brukner, Zukowski, and Zeilinger [14]). I have been following development of this interpretation from its very beginning observing its evolution and success (quite natural in light of recent development of quantum information). My reflections on the information interpretation can be treated as “external observer” reflections. I hope that this presentation has some degree of objectivity (cf. with the above remark of the role of subjective factor in problem of interpretations of QM).

plays the role of borderline between classical and quantum physics. Experimental verification of its violation is the endpoint in the famous debate between Einstein and Bohr; it also justifies the mentioned quantum technological projects - quantum cryptography and random generators (especially the latter).

2Starting from conferences in Helsinki on quantum foundations in 1990th and conversations with A. Zeilinger at a number of conferences at the beginning of this century to my recent discussions with A. Zeilinger and C. Brukner during the Växjö conferences and my visiting fellowships in Vienna,
The last part of this review is devoted to general scientific methodology with references to contributions of de Finetti, Wigner, and Peres.

2 Zeilinger-Brukner Information Interpretation

In the information interpretation of QM, information is the most fundamental, basic entity. Every quantized system is associated with a definite discrete amount of information (see Zeilinger [5], also [12]). This information content remains constant during evolution of a closed system. Here a quantum state is defined in the spirit of Schrödinger, see [15]: the quantum state is an expectation catalog (of probabilities for all possible outcomes). We remark that Zeilinger elaborated the information interpretation of QM [5] by searching for a fundamental and heuristically clear principle of QM, similar to Einstein's principle of relativity.

A. Zeilinger who presented the basic principles of the information interpretation in 1999 [5] always emphasized its close connection with the Copenhagen interpretation; in particular, he often cited N. Bohr to emphasize connection with Bohr’s ideas. The same line of presentation continues in joint publications of Zeilinger and Brukner [6]-[11], Kofler and Zeilinger [13] and Brukner, Zukowski, and Zeilinger [14]. Indeed, the information interpretation of QM can be considered as a modern information-theoretic version of the orthodox Copenhagen interpretation. It has some commonality with von Neumann’s version of this interpretation. In particular, Zeilinger and Brukner explore heavily the concept of irreducible quantum randomness which was invented by von Neumann [16]. They also consider a quantum state as a state of an individual physical system.

As all interpretations in the spirit of Copenhagen [17]-[19], the information interpretation is non-realistic (see also [20]). By this interpretation QM is not about features of quantum systems, but about information about these systems gained with the aid of (classical) measurement devices. In [19] it was emphasized that one has to distinguish realism and reality. For example, Bohr used the non-realist interpretation of QM [21], [22], but he definitely did not deny reality of atoms, electrons (and later even photons). It is not clear whether the information interpretation needs reality - beyond

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3 Surprisingly they did not refer to von Neumann at all (of course, it might be that I missed some of their other papers with a corresponding reference to von Neumann as the inventor of irreducible quantum randomness). They use the term “objective randomness”, but the meaning of this term coincides with the meaning of von Neumann’s irreducible randomness.
outputs of measurement devices (e.g., here photon is treated as a click of a detector).  

A. Zeilinger has put forward an idea which connects the concept of information with the notion of elementary systems. Here we follow the presentation from his joint paper with J. Kofler [13]:

The description of the physical world is based on propositions. Any physical object can be described by a set of true propositions. Then Zeilinger pointed out that “we have knowledge or information about an object only through observations.” Everybody would agree with this. But further line of reasoning is not unquestionable.

“Any complex object which is represented by numerous propositions can be decomposed into constituent systems which need fewer propositions to be specified. The process of subdividing reaches its limit when the individual subsystems only represent a single proposition, and such a system is denoted as an elementary system. The truth value of a single proposition about an elementary system can be represented by one bit of information with “true” being identified with the bit value “1” and “false” with “0”. ” It is then suggested to assume

**Principle of quantization of information:** An elementary system carries one bit of information.

For me, the main questionable point of this reasoning is validity of such subdivision. Why it is always possible? Cannot it be the case that two bits of information belong to a system and cannot be separated? Anyway, the possibility of decomposition as described above looks more like another postulate. From the principle of quantization of information, Zeilinger derives objectivity (irreducibility) of quantum randomness. (In contrast to von Neumann, he needs no mathematically complicated no-go theorem.) Further we again follow the paper of Kofler and Zeilinger [13]:

4In general, Viennese do not like realism and even reality. The following story represents perfectly Viennese’s viewpoint on reality. Once my friend Johan Summhammer (Atom Institute, Vienna) spent two weeks in Växjö. This town is surrounded by beautiful lakes ("sjö" is a lake in Swedish). Once I met him looking at a lake, really excited. I asked him about reason of the excitement, expecting a typical comment about beauty of Swedish nature. But, Johan answered that he is excited by this great picture created by clicks of detectors composing his brain.” From long conversations with Johan I learned that there is no objective reality, and bio-systems developed an ability to select some patterns from noisy and unstructured environment and cognition was developed in this way. Thus this world is composed of clicks of detectors. May be not all Viennese share this view of “clicks-made reality”, but the above story definitey reflects the general Venennese attitude. There is something in the spirit of the town...

5Here one of the important problems of this interpretations lies in assigning the meaning to “we”. “Whose knowledge?” as was asked by D. Mermin [23]. We shall come back to this problem later.
“Disregarding the mass, charge, position and momentum of the electron, its
spin is such an elementary system. If it is prepared “up along $z$”, we have used
up our single bit and a measurement along any other direction must necessarily
contain an element of randomness. This randomness must be objective and ir-
reducible. It cannot be reduced to some unknown hidden properties as then the
system would carry more than a single bit of information. Since there are more
possible experimental questions than the system can answer definitely, it has to
“guess”. Objective randomness is a consequence of the principle lack of informa-
tion.”

I appreciate the elegance of this derivation of irreducibility of quantum
randomness. In contrast to von Neumann, Zeilinger has no need for condi-
tions of coupling between mathematical formalisms of a possible subquantum
model and QM. His fundamental principle is formulated in heuristically clear
terms, i.e., without any direct relation to, e.g., microworld. However, I stress
again that Zeilinger’s basic principle of quantization of information is ques-
tionable. Other Copenhagenists (Bohr, Heisenberg, Pauli, von Neumann)
proceeded without it (as well as recently Plotnitsky [19] with his “statistical
Copenhagen interpretation” ).

Finally, we remark that in the derivation of irreducibility of quantum
randomness there was encrypted the application of the principle of comple-
mentarity in the form: ”Since there are more possible experimental questions
than the system can answer definitely...”. Thus Zeilinger’s information in-
terpretation is based on two fundamental principles:

1. Principle of quantization of information.

2. Principle of complementarity.

We note that if two observables (experimental questions) are not comple-
mentary, that is, can be asked simultaneously or in any order, it means,
mathematically, that their operators commute. There is a theorem, liked by
von Neumann [16], that for commuting operators $A$ and $B$ a new observable
$C$ can be constructed such that values of $A$ and $B$ can be unambiguously
extracted from the result of $C$ measurement, i.e. there are two functions
$f = f(x)$ and $g = g(x), x \in \mathbb{R}$, such that $A = f(C)$ and $B = g(C)$. Hence,
no complementarity means no inherent randomness.

As was pointed out by one of the reviewers, the principle of complemen-
tarity could be replaced by an assumption that there are independent propo-
sitions – those which do not have joint true values. This would already imply
that there must exist complementarity, as a consequence of the principle of
quantization of information and the independence of propositions.
Now it is really the time to turn to the questions: “Whose knowledge? Whose information?” I think that these are the “hard questions” of the information approach to QM. Surprisingly here the positions of Zeilinger and Brukner do not coincide.

In June 2014 at the conference in Vienna devoted to 50th anniversary of Bell’s inequality, Zeilinger gave the talk presenting the personal agent viewpoint on information encoded in a quantum state. The wave function is in the head and not in nature (von Neumann would definitely disagree, Bohr would probably agree, Wigner would applaud). And the main point is that it is in the concrete head, his head or my head. This viewpoint is close to the views of Fuchs, Schack, Caves, Mermin, \cite{24}-\cite{28} (see also my recent critical review on QBism \cite{29} and even more critical paper of Marchildon \cite{30}). This viewpoint on a quantum state as an information entity used in decision making regarding experimentally observed probabilities well matches interpretations of QM as the machinery for update of probabilities - QBism and the Växjö interpretation. I cannot confidently say how Zeilinger interprets probability: statistically or subjectively? During my lectures on foundations of probability at his seminar in May-June 2014 the subjective interpretation of probability was not mentioned at all. I used consistently the statistical interpretation and it seemed that all participants of the seminar were fine with this.

At the same time recently Brukner published a paper on what I interpret as the universal (i.e., not private as in QBism) agent perspective on the information interpretation, we cite Brukner \cite{9}:

“The quantum state is a representation of knowledge necessary for a hypothetical observer respecting her experimental capabilities to compute probabilities of outcomes of all possible future experiments.”

Here an explicit reference to the observer’s experimental capabilities is crucial, cf. with my analysis of QBism in \cite{29}, section : Agents constrained by Born’s rule.

It has to be noted that in this paper the author emphasized the closeness

\footnote{The talk of Zeilinger generated very polarized reactions. For example, Aspect strongly commented that in the two slit experiment the photon “knows” from the very beginning that it is forbidden to go to some regions on the registration screen, independently of what happens in Zeilinger’s head. Then another provocative question was asked to Zeilinger: Is it important to be a human being in order to have a wave function in the head? The reply was in the spirit that this is not important and dog’s head is also a good machine to present a wave function! Of course, it might be that this was just a provocative joke response to this provocative question. If not, then additionally to the trouble with Schrödinger cat we got a new trouble - with a Zeilinger’s dog. I remark that all these strange creatures were born in Vienna. (But in general this discussion after Zeilinger’s lecture reminds the koan about whether a six-trunk white elephant can have a Buddha nature.)}
to QBism. With this I strongly disagree. From the QBism perspective, the wave function is in the head of a concrete private agent, e.g., in Fuchs’ head, not in the head of a hypothetical observer. Similarly, Zeilinger spoke (at least at the aforementioned occasion) about his private wave function (or even his dog’s wave function, again the latter was may be just a joke, but later we shall discuss this point seriously), not the wave function used by a hypothetical observer.

The “knowledge” here refers to Wigner’s definition of the quantum state: “... the state vector is only a shorthand expression of that part of our information concerning the past of the system which is relevant for predicting (as far as possible) the future behavior thereof.”

Unfortunately, it seems that de Finetti and Wigner were not familiar with the works of each other. And the reader can see [31], [32] (see also [29]) that de Finetti’s viewpoint on probability - its subjective (personal knowledge) interpretation - is very close to Wigner’s viewpoint on quantum state.

As Brukner pointed out, “Peres correctly notes that considering hypothetical observers is not a prerogative of quantum theory [...]. They are also used in thermodynamics, when we say that a perpetual-motion machine of the second kind cannot be built, or in the theory of special relativity, when we say that no signal can be transferred faster than the speed of light.”

This is a natural conclusion; it would be surprising if hypothetical observers and Wigner’s interpretation of a state were applicable only in quantum physics. We can refer to de Finetti [31], [32] who starting with the subjective interpretation of probability developed a subjective experience methodology of science, see [29] for detailed presentation. De Finetti, of course, did not reduce his methodology to quantum physics at all. (However, in contrast to Brukner, de Finetti advertised the private and not universal agent perspective; so Fuchs et al. (and, it seems, Zeilinger as well) are closer to de Finetti than Brukner).

In the light of this statement of Peres (completed by views of de Finetti, Wigner, Zeilinger, Brukner) the following question naturally arise: What are distinguishing features of “information concerning the past of the system which is relevant for predicting” related to quantum systems? Why does information about them is represented mathematically in such a special way, by vectors of complex Hilbert space? De Finetti’s framework [31], [32] (see also [29]) covered homogeneously all areas of science (natural and humanities); in the same way Wigner’s viewpoint of a state as a collection of information is applicable to both classical and quantum states of both biological and physical systems. How can the general framework of de Finetti be reduced to the special quantum representation?

One can refer to the principle of quantization of information as the key
principle restricting de Finetti’s general viewpoint on scientific method; so to say,

“classical systems” are those where we are not able to approach the level of a single proposition description, the single bit level.

This viewpoint presumes the possibility to derive the quantum (complex Hilbert space) representation from this principle. And we point that there are already several complete reconstructions of QM from information-theoretical principles [33]–[40], [8]; we point especially to the work of Dakic and Brukner [34] as their axiom 1 is based on Zeilinger’s principle of quantization.

I think that to explain peculiarity of the quantum representation of information one has to discuss the logical structure of information processing by humans. Besides classical Boolean logic, there exist various nonclassical logics, nonclassical rules for information processing. In spite of his revolutionary treatment of the concept of probability and the scientific method in general, de Finetti was still rigidly devoted to Boolean logic (and hence to the use of measure-theoretic probability concept). In spite of wide applications of Boolean logic, e.g., in artificial intelligence and computer science, we do not forget that it is just one special model of information processing which was created by a concrete person. Meanwhile, the brain may use more complex logical systems. In particular, it may use quantum logic. Thus the quantum representation of information is a mathematical signature of the use of quantum logic in reasoning. Of course, from this viewpoint the context of quantum mechanical reasoning is only a special context in which the brain uses nonclassical logic. Such nonclassical reasoning may be profitable for the brain in other situations, see [47] on applications of the quantum formalism in cognitive science, psychology, economics.

Now we come back to Zeilinger’s dog issue. Applying the quantum formalism outside of physics we discover [47], [48], [49] that nonclassical processing of information is a feature not only of humans, but of all bio-systems, from cells and proteins to, e.g., dogs. From this viewpoint, Zeilinger’s dog also processes information (in some situations) by using nonclassical logic of reasoning and hence (roughly speaking) has wave functions in its head. Of

\[\text{This problem also was understood well by QBists who tried to reconstruct the quantum formalism from their fundamental principle: QM is a special machinery for probability (information) update based on the special nonclassical version of the formula of total probability } [29]. \text{ But QBists succeeded only partially in approaching this great aim. The same problem was actively studied in development the Växjö interpretation of QM which is based on the same principle as QBism, but explores another version of this formula, see [29], [41]–[46] for details. Nor this interpretation managed to solve the problem of quantum reconstruction.}\]
course, the above attempt to couple the information interpretation to general theory of reasoning and decision making is my own speculation; it has nothing to do with the views of adherents of this interpretation (see [49] for further discussions).

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