1. INTRODUCTION

Gamma-ray bursts (GRBs) are commonly divided into two classes: short-duration, hard-spectrum bursts, and long-duration, soft-spectrum bursts. Observations have provided growing evidence that short bursts result from the mergers of compact star binaries, while long bursts originate from the collapses of massive stars (for recent reviews see Zhang & Meszérs 2004; Piran 2004; Meszérs 2006; Nakar 2007). It is usually assumed that both compact star mergers and massive star collapse give rise to a central black hole with a debris torus around it. The torus has a mass of about 0.01–1 $M_\odot$, and a large angular momentum, enough to produce a transient accretion disk with a huge accretion rate, up to $\sim 1.0 \times 10^3 M_\odot$ s$^{-1}$. The accretion timescale is short, e.g., a fraction of a second after the merger of two neutron stars, and tens of seconds if a disk forms due to fallback of matter during the collapse process.

The hyperaccretion disk around a black hole is extremely hot and dense. The optical depth of the accreting gas is so enormous that radiation is trapped inside the disk and can only be advected inward. However, in some cases, this hot and dense disk can be cooled via neutrino emission (Narayan et al. 1992). According to the disk structure and different cooling mechanisms, flows in the disk fall into three types: advection-dominated accretion flows (ADAFs), convection-dominated accretion flows (CDAFs), and neutrino-dominated accretion flows (NDAFs). The first two types of flow are radiatively inefficient (Narayan et al. 1998, 2000, 2001), and the final type cools the disk efficiently via neutrino emission (Popham et al. 1999; Di Matteo et al. 2002). Hyperaccretion disks around black holes have been studied both analytically and numerically considering the effects of these three flows (e.g., see Popham et al. 1999; Narayan et al. 2001; Kohri & Mineshige 2002; Di Matteo et al. 2002; Gu et al. 2006; Chen & Beloborodov 2007; Liu et al. 2007; Janiuk et al. 2007).

However, newborn neutron stars have also been suggested as central engines of GRBs in some origin/afterglow models. The discovery of X-ray flares by Swift implies that the central engines of some GRBs have long-living activity (at least hundreds of seconds) after the bursts (Zhang 2007). This provides a challenge for conventional hyperaccretion disk models of black holes. Recently, Dai et al. (2006) argued that the newborn central compact objects in some GRBs could be young, at least transiently existing neutron stars, rather than black holes (for alternative models see Perna et al. 2006 and Proga & Zhang 2006). These neutron stars may have high angular momentum, and their maximum mass may be close to or slightly larger than the upper mass limit of nonrotating Tolman-Oppenheimer-Volkoff neutron stars. Using this argument, Dai et al. (2006) explained X-ray flares of short GRBs as being due to magnetic reconnection-driven events from highly magnetized millisecond pulsars. It is thus reasonable to assume a unified scenario: a prompt burst originates from a highly magnetized, millisecond-period neutron star surrounded by a transient hyperaccretion disk, and subsequent X-ray flares are due to further magnetic activity of the neutron star.

Second, the shallow decay phase of X-ray afterglows observed several hundred seconds after a sizable fraction of GRBs discovered by Swift has been understood as arising from long-lasting energy injection to relativistic forward shocks (Zhang 2007). It was proposed before the Swift observations that pulsars in the unified scenario mentioned above might provide energy injection to a forward shock through magnetic dipole radiation, leading to flattening of an afterglow light curve (Dai & Lu 1998a; Zhang & Meszaros 2001; Dai 2004). Recent model fitting (Fan & Xu 2006; Yu & Dai 2007) and data analysis (Liang et al. 2007) confirm this result. An ultrarelativistic pulsar wind could be dominated by electron/positron pairs, and its interaction with a postburst fireball could give rise to a reverse shock and a forward shock (Dai 2004). The high-energy emission due to inverse-Compton scattering in these shocks is significant enough to be detectable with the upcoming Gamma-ray Large-Area Space Telescope (GLAST, Yu et al. 2007).

We also note that in some origin models of GRBs (e.g., Kluźniak & Ruderman 1998; Dai & Lu 1998b; Wheeler et al.
2000; Wang et al. 2000; Paczyński & Haensel 2005), highly magnetized neutron stars or strange quark stars surrounded by hyperaccretion disks resulting from the fallback of matter could occur during the collapse of massive stars or the merger of two neutron stars. A similar neutron star was recently invoked in numerical simulations of Mazzali et al. (2006) and data analysis of Soderberg et al. (2006) to understand the properties of supernova SN 2006aj, associated with GRB 060218. In addition, hyperaccretion disks could also occur in Type II supernovae if fallback matter has angular momentum. Such disks would be hyperaccretion disks could also occur in Type II supernovae if fallback matter has angular momentum. Such disks would be expected to play an important role in supernova explosions via neutrino emission, similar to some effects reviewed by Bethe (1990).

From these motivations, we here investigate a hyperaccretion disk around a neutron star. To our knowledge, this paper is the first to study hyperaccretion disks around neutron stars possibly related to GRBs. Chevalier (1996) discussed the structure of dense and neutrino-cooled disks around neutron stars. He considered neutron cooling due to electron-positron pair annihilation, which is actually much less important than the cooling due to electron-positron pair capture in the hyperaccreting case of interest here, since the accretion rate assumed in Chevalier (1996) (~M☉ yr⁻¹) is much less than that which concerns us (~0.1 M☉ s⁻¹). In this paper we consider several types of neutrino cooling by using elaborate formulae developed in recent years. In addition, we focus on some differences between black hole and neutron star hyperaccretion. One main difference is that the internal energy in an accretion flow may be advected inward into the event horizon without any energy release if the central object is a black hole, but the internal energy must eventually be released from the disk if the central object is a neutron star, because the stellar surface prevents any heat energy from being advected inward into the star. Since the accretion rate is always very high, the effective cooling mechanism in the disk is still neutrino emission, and as a result, the efficiency of neutrino cooling of the entire disk around a neutron star should be higher than that of a black hole disk.

There have been many studies of accretion onto neutron stars in binary systems (e.g., Shapiro & Salpeter 1975; Kluźniak & Wilson 1991; Medvedev & Narayan 2001; Frank et al. 2002) and supernova explosions (e.g., Chevalier 1989, 1996; Brown & Weingartner 1994; Kohri et al. 2005). For supernovae, spherically symmetric accretion onto neutron stars (the so-called Bondi accretion) has been investigated in detail. In particular, Kohri et al. (2005) tried to use the hyperaccretion disk model with an outflow wind to explain supernova explosions. In binary systems, since the accretion rate is not larger than the Eddington accretion rate (~10⁻⁸ M☉ yr⁻¹), the physical properties of the disk must be very different from those of the hyperaccretion disk discussed here.

This paper is organized as follows. In §2, we describe the plan of our study of the structure of a quasi-steady disk. We propose that the disk around a neutron star can be divided into two regions, the inner and outer disks. Table 1 gives the notation and definition of some quantities in this paper. In order to give clear physical properties, we first adopt a simple model in §3, and give both analytical and numerical results of the disk properties. In §4, we study the disk using a state-of-the-art model with lots of elaborate considerations regarding the thermodynamics and microphysics, and compare the results from this elaborate model with those from the simple model. Section 5 presents our conclusions and a discussion.

### 2. DESCRIPTION OF OUR STUDY

#### 2.1. Two-Region Disk

We study the quasi-steady structure of an accretion disk around a neutron star with a weak outflow, taking the accretion rate to be a parameter. For the accretion rates of interest to us, the disk flow may be an ADAF or NADAF. In this paper, we do not consider a CDAF. The disk around a neutron star differs from the disk around a black hole in that it should eventually release the gravitational binding energy of accreted matter (which is converted to the internal energy of the disk and the rotational kinetic energy) more efficiently.

The energy equation of the disk is (Frank et al. 2002)

\[
\Sigma_s T \frac{d s}{d r} = Q^+ - Q^-, \tag{1}
\]

where \(\Sigma\) is the surface density of the disk, \(r\) is the radial velocity, \(T\) is the temperature, \(s\) is the entropy per unit mass, \(r\) is the radius of a certain position in the disk, and \(Q^+\) and \(Q^-\) are the energy input (heating) and energy loss (cooling) rate in the disk, respectively. From the point of view of evolution, the structure of a hyperaccretion disk around a neutron star should initially be similar to that of the disk around a black hole, because the energy input is mainly due to local viscous dissipation, i.e., \(Q^+ = Q^+_{\text{vis}}\). However, since the stellar surface prevents the matter and heat energy in the disk from further advection inward, the region near the compact object should be extremely dense and hot as accretion proceeds. In addition to the local viscous heating, this inner region can also be heated by the energy \((Q^+_{\text{adv}})\) advected from the

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**TABLE 1**

Notation and Definition of Some Quantities in This Paper

| Notation | Definition | See |
|----------|------------|-----|
| \(m\) | Mass of the central neutron star, \(m = M/1.4 M_\odot\) | 3.1, eq. (20) |
| \(\dot{m}\) | Mass accretion rate, \(\dot{m} = M/0.01 M_\odot \text{ s}^{-1}\) | 3.1, eq. (20) |
| \(Y_e\) | Ratio of the electron to nucleon number density in the disk | 2.2, eq. (9) |
| \(r\) | Radius of the neutron star | 2.3, eq. (13) |
| \(r_{\text{out}}\) | Outer radius of the disk | 2.3, eq. (13) |
| \(\Omega\) | Angular velocity of the stellar surface | 3.3, eq. (49) |
| \(f\) | Efficiency of energy release in the inner disk | 2.3, eq. (17) |
| \(\tilde{r}\) | Radius between the inner and outer disks | 2.3, eq. (12) |
| \(\tilde{r}\) | Parameter to measure the neutrino optically thick region | 3.2, eq. (47) |
| \(f = 1 - \sqrt{\tilde{r}/\tilde{r}}\) | Useful factor as a function of \(\tilde{r}\) | 2.2, eq. (7) |
| \(f = 1 - \sqrt{r/r}\) | Value of \(f\) at radius \(r\) | 3.2, eq. (38) |
| \(\tilde{f}\) | Average efficiency of neutrino cooling in the outer disk | 2.3, eq. (13) |
outer region of the disk. Thus, the heat energy in this region may include both the energy generated by itself and the energy advected from the outer region, that is, \( Q^+ = Q^+_{\text{vis}} + Q^+_{\text{adv}} \). Initially, such a region is so small (i.e., very near the compact star surface) that it cannot be cooled efficiently for a huge accretion rate (~0.01–1 \( M_\odot \) s\(^{-1}\)). As a result, it has to expand its size until an energy balance between heating and cooling is attained in this inner region. Such an energy balance can be expressed by \( Q^+ = Q^- \) in the inner region of the disk.

Once this energy balance is achieved, the disk is in a steady state as long as the accretion rate is not significantly changed. For such a steady disk, the structure of the outer region is still similar to that of the disk around a black hole, but the inner region has to be hotter and denser than the disk around a black hole, and could have a structure different from both its initial structure and the outer region, which is not affected by the neutron star surface.

Based on the above consideration, the hyperaccretion disk around a neutron star could have two different regions. In order to discuss their structure clearly using a mathematical method, as a reasonable approximation we here divide the steady accretion disk into an inner region and an outer region, called the inner and outer disks, respectively. The outer disk is similar to that of a black hole. The inner disk, depending on its heating and cooling mechanisms, as discussed above, should satisfy the entropy conservation condition \( \frac{dt}{dr} \propto r^{\gamma} = 0 \), and thus we obtain \( P \propto \rho^\gamma \), where \( P \) and \( \rho \) are the pressure and the density of the disk, and \( \gamma \) is the adiabatic index of the disk gas. The radial momentum equation is

\[
(\Omega^2 - \Omega_K^2)r - \frac{1}{\rho} \frac{d}{dr}(\rho c_s^2) = 0, \tag{2}
\]

where \( \Omega \) and \( \Omega_K \) are the angular velocity and Keplerian angular velocity of the inner disk, and \( c_s = (P/\rho)^{1/2} \) is the isothermal sound speed. We here neglect the radial velocity term \( v_r d v_r / dr \), since we consider the situation \( v_r \ll r \omega_K \) with \( \Omega \ll \Omega_K \) but \( |\Omega - \Omega_K| \geq v_r / r \). Equation (2) gives \( \Omega \propto r^{-3/2} \) and \( c_s \propto r^{-1/2} \). Moreover, from the continuity equation, we have \( v_T \propto (\rho H)^{-1} \) with the disk’s half-thickness \( H = c_s / \Omega \propto r \). Thus, we can derive a self-similar structure in the inner region of the hyperaccretion disk of a neutron star,

\[
\rho \propto r^{-1/(\gamma-1)}, \quad P \propto r^{-\gamma/(\gamma-1)}, \quad v_T \propto r^{(3-2\gamma)/(\gamma-1)}. \tag{3}
\]

This self-similar structure has been given by Chevalier (1989) and Brown & Weingartner (1994) for Bondi accretion under the adiabatic condition, and by Medvedev & Narayan (2001) and Medvedev (2004) for disk accretion under the entropy conservation condition. In addition, if the gas pressure is dominant in the disk, we have \( \gamma = 5/3 \), so that equation (3) becomes \( \rho \propto r^{-3/2}, \quad P \propto r^{-5/2}, \) and \( v_T \propto r^{-1/2} \), which have been discussed by Spruit et al. (1987) and Narayan & Yi (1994).

An important problem we next solve is to determine the size of the inner disk. Since the total luminosity of neutrinos emitted from the whole disk varies significantly with the inner region size, we can estimate the inner region size by solving an energy balance between neutrino cooling and heating in the entire disk. The details will be discussed in § 2.3.

Finally, we focus on two problems here. First, we discuss physical definition of the boundary layer between the outer and inner disks. There are two possible boundary conditions. One approach is to assume that a stalled shock exists at the boundary layer. Under this assumption, the inner disk is a postshock region, in which the pressure, temperature, and density just behind the shock are much higher than in the outer disk in front of the shock. The other approach is to assume that no shock exists in the disk, and that all the physical variables of two sides of this boundary change continuously. Which condition is reasonable?

Let us define the mass density, pressure, radial velocity, and internal energy density of the outer disk along the boundary layer as \( \rho_1, P_1, v_1, \) and \( u_1 \), and the corresponding physical variables of the inner disk as \( \rho_2, P_2, v_2, \) and \( u_2 \) at the same radius. Thus the conservation equations are

\[
\begin{align*}
\rho_1 v_1 &= \rho_2 v_2 \\
P_1 + \rho_1 v_1^2 &= P_2 + \rho_2 v_2^2 \\
(u_1 + P_1 + \rho_1 v_1^2/2)/\rho_1 &= (u_2 + P_2 + \rho_2 v_2^2/2)/\rho_2. \tag{4}
\end{align*}
\]

From the Rankine-Hugoniot relations, we know that if \( P_2 \gg P_1 \), the densities of two sides of the boundary layer are discontinuous, which means that a strong shock exists between the inner and outer disks. On the other hand, if \( P_1 \sim P_2 \), we can obtain \( \rho_1 \sim \rho_2 \), which means that only a very weak shock forms at this boundary layer, or we can say that no shock exists. Therefore, we compare \( P_1 \) and \( \rho_1 v_1^2 \) of the outer disk. If \( P_1 \ll \rho_1 v_1^2 \) or \( c_s < v_1 \), we can assume that a stalled shock exists at the boundary layer, i.e., the first boundary condition is correct. If \( P_1 \gg \rho_1 v_1^2 \) or \( c_s > v_1 \), then \( P_1 \sim P_2 \), and thus no shock exists. In § 3 and § 4, we will use this method to discuss which boundary condition is reasonable.

We also note that the effect of the magnetic field of the central neutron star is not considered in this paper. We estimate the order of magnitude of the Alfvén radius using the expression (Frank et al. 2002) \( r_{A} \approx \frac{2}{3} \times 10^{4} \frac{m_{2}^{-2/3}}{\mu_{42}^{1/3}} \left(\frac{\dot{m}}{10^{-8}}\right)^{3/4} \left(\frac{M_{5}}{0.01}\right)^{-1/2} \left(\frac{r_{c}}{10^{6}}\right)^{-3/4} \), where \( m_{2} = M/0.01 \) \( M_{5} \) \( s^{-1} \) is the accretion rate, \( M = M/1.4 M_{\odot} \) is the mass of the neutron star, and \( \mu_{42} \) is the magnetic moment of the neutron star in units of \( 10^{26} \) G cm\(^3\). Let \( r_{c} \) be the neutron star radius. If the stellar surface magnetic field \( B_{s} \leq B_{c, \alpha} = 2.8 \times 10^{20} \frac{m_{2}^{1/4}}{\mu_{42}^{1/4}} \left(\frac{\dot{m}}{10^{-8}}\right)^{1/4} \frac{\dot{m}_{2}^{-1/4}}{10^{-10}} \) G for typical accretion rates. This implies that the stellar surface magnetic field affects the structure of the disk significantly if \( B_{s} \geq B_{c, \alpha} \approx 10^{-5} \)–\( 10^{-6} \) G for typical accretion rates. Therefore, in this paper we assume a neutron star with surface magnetic field weaker than \( B_{c, \alpha} \). This assumption is consistent with some GRB-origin models, such as those of Kluźniak & Ruderman (1998), Dai & Lu (1998b), Wang et al. (2000), Pacyński & Haensel (2005), and Dai et al. (2006), because these models require a neutron star or strange quark star with surface magnetic field much weaker than \( B_{c, \alpha} \).

### 2.2. Structure of the Outer Disk

Here we discuss the structure of the outer disk based on Newtonian dynamics and a standard \( \alpha \)-viscosity disk model, for simplicity. The structure of the hyperaccretion disk around a stellar-mass black hole has been discussed in many previous works, and here we use similar methods and equations, since the outer disk is very similar to the disk around a black hole.

We approximately consider the angular velocity of the outer disk to be Keplerian, \( \Omega_K = (GM/r^3)^{1/2} \). The velocity \( \Omega_K \) should be modified in a relativistic model of accretion disks (Popham et al. 1999; Chen & Beloborodov 2007), but we do not consider it in this paper. We can write four equations to describe the outer
disk, i.e., the continuity equation, the energy equation, the angular momentum equation, and the equation of state. The continuity equation is

$$\dot{M} = 4\pi r \rho v_c H \equiv 2\pi r \Sigma \dot{r},$$

(5)

where the notations of the physical quantities have been introduced in § 2.1.

In the outer disk, the heat energy could be advected inward, and we take $Q_{\text{adv}} = \Sigma T \nu \dot{r}$ as the energy advection rate, where the factor 1/2 is added because we only study the vertically integrated disk over a half-thickness $H$. Thus, the energy-conservation equation (1) is rewritten as

$$\dot{Q}^+ = Q_{\text{rad}}^+ + Q_{\text{adv}}^+ + Q_{\text{v}}^-,$$

(6)

The quantity $Q^+$ in equation (6) is the viscous heat energy generation rate per unit surface area. According to the standard viscosity disk model, we have

$$Q^+ = \frac{3GM\dot{M}}{8\pi r^3} \frac{f}{\rho},$$

(7)

where $f = 1 - (r/r_c)^{1/2}$ (Frank et al. 2002).

The quantity $Q_{\text{rad}}$ in equation (6) is the photon cooling rate per unit surface area of the disk. Since the disk is extremely hot and hot, the optical depth of photons is always very large, and thus we can take $Q_{\text{rad}} = 0$ as a good approximation.

The entropy per unit mass of the disk is (similar to Kohri & Mineshige 2002)

$$s = s_{\text{gas}} + s_{\text{rad}} = \frac{S_{\text{gas}}}{\rho} + \frac{S_{\text{rad}}}{\rho} = \sum_i n_i \left[ \frac{k_B}{2} + \frac{k_B}{\rho} \ln \left( \frac{(2\pi k_B T)^{3/2}}{\hbar^3 n_i} \right) \right] + S_0 + \frac{2}{3} g_e \frac{a T^3}{\rho},$$

(8)

where the summation runs over nucleons and electrons, $k_B$ is the Boltzmann constant, $h$ is the Planck constant, $a$ is the radiation constant, $S_0$ is the integration constant of the gas entropy, and the term $2g_e a T^3/3 \rho$ is the entropy density of the radiation, with $g_e = 2$ for photons and $g_e = 11/2$ for a plasma of electrons and relativistic $\gamma^+\gamma^-$ pairs. We assume that electrons and nucleons have the same temperature. Then we use equation (8) to calculate $ds/dr$ and approximately take $dT/dr \approx T/r$ and $d\rho/dr \approx \rho/r$ to obtain the energy advection rate,

$$Q_{\text{adv}} = v_i T \left[ \sum_i n_i \left( 1 + Y_i \right) + \frac{4}{3} g_e \frac{a T^3}{\rho} \right],$$

(9)

where the gas constant is $R = 8.315 \times 10^7$ ergs mole$^{-1}$ K$^{-1}$, and $Y_i$ is the ratio of electron to nucleon number density. The first term in the right-hand bracket of equation (9) comes from the contribution of the gas entropy, and the second term comes from the contribution of radiation.

The quantity $Q_{\text{v}}^-$ in equation (6) is the neutrino cooling rate per unit area. The expression of $Q_{\text{v}}^-$ will be discussed in detail in § 3 and § 4.

In this paper we ignore the cooling term of photodisintegration, $Q_{\text{photodis}}$, and approximately take the free nucleon fraction $X_{\text{nuc}} \approx 1$. For the disks formed by the collapses of massive stars, the photodisintegration process that breaks down $\alpha$-particles into neutrons and protons is important in a disk region at very large radius $r$. However, the effect of photodisintegration becomes less significant for a region at small radius, which contains fewer $\alpha$-particles.\footnote{Kohri et al. (2005), Chen & Beloborodov (2007), and Liu et al. (2007) discussed the value of the nucleon fraction $X_{\text{nuc}}$ and the effect of photodisintegration as a function of radius for particular parameters such as the accretion rate and the viscosity parameter $\alpha$. The first two papers show that $X_{\text{nuc}} \approx 1$ and $Q_{\text{photodis}} \approx 0$ for $r \leq 10^3 r_c$. Although the value of $X_{\text{nuc}}$ from Liu et al. (2007) is somewhat different from the previous works, the ratio of $Q_{\text{photodis}}/Q^+$ in their work also drops dramatically for $r \leq 10^3 r_c$. Therefore, it is convenient for us to neglect the photodisintegration process for $r \leq 10^2 r_c$ or $r \leq 400$ km for the central star mass $M = 1.4 M_\odot$.}

On the other hand, for disks formed by the mergers of double compact stars, there will be rare $\alpha$-particles over the entire disk. As a result, we reasonably take all the nucleons to be free ($X_{\text{nuc}} \approx 1$) and neglect the photodisintegration process, since we mainly focus on small disks or small regions of the disks (as we discuss in § 3.4, with outer boundary $r_{\text{out}} = 150$ km.)

The angular momentum conservation and the equation of state can be written as

$$\nu \Sigma = \frac{M}{3\pi f},$$

(10)

$$P = P_e + P_{\text{nuc}} + P_{\text{rad}} + P_{\nu},$$

(11)

where $\nu = \alpha C H$ in equation (10) is the kinematic viscosity, $\alpha$ is the classical viscosity parameter, and $P_e$, $P_{\text{nuc}}$, $P_{\text{rad}}$, and $P_{\nu}$ in equation (11) are the electron, nucleon, radiation, and neutrino pressure, respectively. In § 3 we consider the electron pressure in extreme cases, and in § 4 we calculate the $e^+ e^-$ pressure using the exact Fermi-Dirac distribution function and the condition of $\beta$-equilibrium.

Equations (5), (6), (10), and (11) are the basic equations for solving the structure of the outer disk, which is important for our study of the inner disk.

2.3. Self-Similar Structure of the Inner Disk

In § 2.1 we introduced a self-similar structure of the inner disk, and described the method used to determine the size of the inner disk. Now we establish the energy conservation equation in the inner disk. We define $\tilde{r}$ as the radius of the boundary layer between the inner and outer disks, and $\tilde{\rho}$, $\tilde{P}$, and $\tilde{v}_r$ as the density, pressure, and radial velocity of the inner disk just at the boundary layer, respectively. From equation (3), the variables in the inner disk at any given radius $r$ can be written as

$$\rho = \tilde{\rho} \left( \tilde{r}/r \right)^{1/(\gamma - 1)}, \quad P = \tilde{P} \left( \tilde{r}/r \right)^{2/(\gamma - 1)}, \quad v_r = \tilde{v}_r \left( \tilde{r}/r \right)^{(\gamma - 3)/(\gamma - 1)}.$$

(12)

We take the outer radius of the accretion disk to be $r_{\text{out}}$. The total energy per unit time that the outer disk advects into the inner disk is (Frank et al. 2002)

$$\tilde{E}_{\text{adv}} = \left( 1 - \tilde{f}_v \right) \frac{3GM\dot{M}}{4} \times \left[ \frac{1}{\tilde{r}} - \frac{2}{3} \left( \frac{r_c}{\tilde{r}} \right)^{1/2} \right] - \frac{1}{r_{\text{out}}} \left[ 1 - \frac{2}{3} \left( \frac{r_c}{r_{\text{out}}} \right)^{1/2} \right],$$

(13)

where $\tilde{f}_v$ is the average neutrino cooling efficiency of the outer disk,

$$\tilde{f}_v = \frac{\int_{r_{\text{out}}}^{r_{\text{out}}} Q_{\nu}^- 2\pi r dr}{\int_{r_{\text{out}}}^{r_{\text{out}}} \dot{Q}^+ 2\pi r dr}.$$
If the outer disk flow is mainly an ADAF, \( Q_{\text{out}}^{-} \ll Q_{\text{in}}^{+} \), then \( f_{\nu} \sim 0 \); if the outer disk flow is mainly an NDAF, \( Q_{\text{out}}^{-} \gg Q_{\text{rad}} \) and \( Q_{\text{out}}^{-} \ll Q_{\text{adv}} \), we have \( f_{\nu} \sim 1 \). While the heat energy in the inner disk should be released more efficiently than in the outer disk, we can still approximately take \( \Omega \approx \Omega_{K} \) in the inner disk, and the maximum power that the inner disk can release is

\[
L_{\nu, \text{max}} \approx \frac{3 G M M}{4} \left( \frac{1}{3 r_{s}} - \frac{\tilde{r}_{\nu}}{\tilde{r}} \left[ 1 - \frac{2}{3} \left( \frac{r_{s}}{\tilde{r}} \right)^{1/2} \right] \right),
\]

(15)

where we have integrated vertically over the half-thickness. The first term in the right-hand of this equation is the total heat energy per unit time of the entire disk, and the second term is the power lost through neutrino cooling in the outer disk. Considering \( r_{\text{out}} \gg r_{s} \), we have

\[
L_{\nu, \text{max}} \approx \frac{3 G M M}{4} \left( \frac{1}{3 r_{s}} - \frac{\tilde{r}_{\nu}}{\tilde{r}} \left[ 1 - \frac{2}{3} \left( \frac{r_{s}}{\tilde{r}} \right)^{1/2} \right] \right),
\]

(16)

The maximum energy release rate of the inner disk is \( G M M/4 r_{s} \), if the outer disk flow is mainly an ADAF and \( f_{\nu} \sim 0 \). This value is just one-half of the gravitational binding energy, and satisfies the virial theorem. If the outer disk flow is an NDAF, then the energy release of the inner disk mainly results from the heat energy generated by itself.

Following the above consideration, most of the energy generated in the disk around a neutron star is released from the disk, so we have an energy-conservation equation,

\[
\int_{r_{s}}^{\tilde{r}} Q_{\nu} 2 \pi r \, dr = \varepsilon \frac{3 G M M}{4} \left( \frac{1}{3 r_{s}} - \frac{\tilde{r}_{\nu}}{\tilde{r}} \left[ 1 - \frac{2}{3} \left( \frac{r_{s}}{\tilde{r}} \right)^{1/2} \right] \right),
\]

(17)

where \( \varepsilon \) is a parameter that measures the efficiency of the energy release. If the central compact object is a black hole, we have \( \varepsilon \approx 0 \), and the inner disk cannot exist. If the central compact object is a neutron star, \( \varepsilon \approx 1 \), and thus we are able to use equation (17) to determine the size of the inner disk.

3. A SIMPLE MODEL OF THE DISK

In § 2 we gave the equations to describe the structure of a hyperaccretion disk. However, additional equations for microphysics in the disk are also needed. In order to see the physical properties of the entire disk clearly, we first adopt a simple model, for analytical purposes. Comparing with § 4, we here adopt a relatively simple treatment for the disk microphysics, similar to that of Popham et al. (1999) and Narayan et al. (2001), and discuss some important physical properties; we then compare analytical results with numerical ones, which are also based on the simple model.

If the disk is optically thin to its own neutrino emission, the neutrino cooling rate can be written as a summation of four terms, the electron-positron pair capture rate, the electron-positron pair annihilation rate, the nucleon bremsstrahlung rate, and the plasmon decay rate, that is, \( Q_{\nu}^{-} = \dot{q}_{\nu} + \dot{q}_{e^{-}e^{+}} + \dot{q}_{\text{brems}} + \dot{q}_{\text{plas}} \) H (Kohri & Mineshige 2002). We take two major contributions of these four terms and use the approximate formulae \( \dot{q}_{e^{-}e^{+}} = 5 \times 10^{33} f_{11}^{9} \) ergs cm\(^{-3}\) s\(^{-1}\) and \( \dot{q}_{\nu} = 9 \times 10^{23} \rho T_{11}^{6} \) ergs cm\(^{-3}\) s\(^{-1}\). Thus, equation (6) can be rewritten as

\[
\frac{3 G M M}{8 \pi r_{s}^{3} f_{\nu}} = \frac{M T}{4 \pi r_{s}^{2}} \left[ \frac{R}{2} (1 + \gamma_{e}) + \frac{22}{3} \frac{a T^{3}}{3} \right] + (5 \times 10^{33}) f^{9} + 9 \times 10^{23} \rho T_{11}^{6} \frac{C_{2}}{\Omega_{K}},
\]

(18)

If neutrinos are trapped in the disk, we use the blackbody limit for the neutrino luminosity: \( Q_{\nu}^{-} \sim (\frac{2}{3} \sigma T^{4})/\tau \), where \( \tau \) is the neutrino optical depth. We approximately estimate the neutrino optical depth as \( (\dot{q}_{e^{-}e^{+}} + \dot{q}_{\nu})/(4 \times 3 \sigma T^{4}) \).

Moreover, we take the total pressure in the disk to be a summation of the three terms \( P_{e}, P_{\text{muc}}, \) and \( P_{\text{rad}} \), and neglect the pressure of neutrinos; thus \( P_{e} = n_{e} k T + K_{1}(\rho Y_{e})^{4/3}, P_{\text{muc}} = n_{\text{muc}} k T, \) and \( P_{\text{rad}} = 11 a T^{4}/12, \) where \( K_{1}(\rho Y_{e})^{4/3} \) is the relativistic degeneracy pressure of electrons, and \( n_{e} \) and \( n_{\text{muc}} \) are the number densities of electrons and nucleons, respectively. Here we also neglect the nonrelativistic degeneracy pressure of nucleons.

We thus obtain

\[
P = P_{e} + P_{\text{muc}} + P_{\text{rad}} = \rho(1 + Y_{e}) R T + K_{1}(\rho Y_{e})^{4/3} + \frac{11}{12} a T^{4},
\]

(19)

where \( K_{1} = (2 \pi h C/3)[3/(8 \pi m_{p})]^{4/3} = 1.24 \times 10^{15} \) cgs.

Equations (10), (18), and (19) can be solved for three unknowns (density, temperature, and pressure) of the steady outer disk as functions of radius \( r \) for four given parameters, \( \alpha, Y_{e}, M, \) and \( M_{*} \) in the simple model. Once the density, temperature, and pressure profiles are determined, we can present the structure of the outer disk and further establish the size and structure of the inner disk.

3.1. The Outer Disk

Here we analytically solve equations (10), (18), and (19). Our method is similar to that of Narayan et al. (2001). However, we differ from their work in using the same equations to obtain both ADAF and NDF solutions in different conditions. We also find that the factor \( f = 1 - (r/r_{*})^{1/2} \) cannot be omitted, because it plays an important role in determining the disk structure. For convenience, we here expand the solution range to the entire disk (i.e., \( r_{s} < r < r_{\text{out}} \)) rather than just considering it in the outer region. The size of the inner disk, which depends on the structure of the outer disk, will be solved in § 3.2.

First we give a general picture. If the accretion rate is not very high, most of the energy generated in the disk is advected inward, and we call the disk as an advection-dominated disk. As the accretion rate increases, the density and temperature of the disk also increase, and neutrino cooling becomes the dominant cooling mechanism in some regions of the disk. Thus, we say that this region becomes neutrino-dominated. When the accretion rate is sufficiently large, the disk may become entirely neutrino-dominated. In addition to the accretion rate, there are some other factors, such as the mass of the central neutron star, \( M \), and the electron-nucleon ratio, \( Y_{e} \), that can influence the disk structure.

We take the mass of neutron star as \( M = 1.4 M_{\odot} \), the accretion rate as \( \dot{M} = M_{\odot} 0.01 M_{\odot} \) s\(^{-1}\), \( \alpha = 0.1 \), \( r = 10^{7/3} \) cm, \( \rho = 10^{11} \) g cm\(^{-3}\), \( T = 10^{11} R_{11} \) K, and \( P = 10^{29} P_{29} \) ergs cm\(^{-3}\). For an ADAF disk flow with radiation
pressure dominant, we find that the density and temperature in the disk are

\[ \rho_{11} = 0.0953 \mu_d m^{1/2} f^{1/2} \alpha^{-1} r_6^{-3/2} \]
\[ T_{11} = 0.832 m^{1/8} \mu_d^{1/4} f^{1/8} \alpha^{-1} r_6^{-5/8}. \]  

Also, the pressure of the disk from equation (19) becomes

\[ P_{29} = 3.32 m^{1/2} \mu_d f^{1/2} \alpha^{-1} r_6^{-5/2}. \]  

We have to check the validity of the assumption made in deriving the above solution, i.e., we need the relations \( Q_{\text{adv}} > Q_e \) and \( P_{\text{rad}} > P_{\text{gas}} \), \( P_{\text{rad}} > P_{\text{deg}} \) to be satisfied, where \( P_{\text{rad}} \) and \( P_{\text{deg}} \) are the gas and degeneracy pressure in the disk. Then we get the relations

\[ r_6 f^{1/5} > 2.28 m^{-3/5} \mu_d^{6/5} \alpha^{-2}; \]  
\[ r_6 f^{-7/3} < 74.6 (1 + \gamma_e)^{-8/3} m^{-7/3} \mu_d^{-2/3} \alpha^{-2/3}; \]  
\[ r_6 f^{-7/3} < 174 m^{-7/3} \mu_d^{-2/3} \alpha^{-2/3} \gamma_e^{-1/3}. \]  

In particular, for a fixed radius \( r \), we can rewrite equation (22) as

\[ \dot{m}_{\text{d}} < 0.504 m^{1/2} \mu_d^{-5/3} \alpha^{-1/2} r_6^{-1/6}, \]  

which means that a larger radius allows a larger upper limit of the accretion rate for the radiation-pressure-dominated region. On the other hand, if the parameters \( m, \alpha, \) and \( \mu_d \) in some region of the disk do not satisfy the conditions (22) or (25), other types of pressure can exceed the radiation pressure, but the region can still be advection-dominated. For analytical purposes, we examine the range of different types of pressure in the two extreme cases where \( Y_e \sim 1 \) or \( Y_e \ll 1 \). From equation (23), we can see that, since the minimum value of \( r_6 f^{-7/3} \) is 19.9 when \( \mu_d > 0.453 m^{7/2} \alpha^{-1} \), for \( Y_e \sim 1 \), or \( \mu_d > 7.25 m^{2/2} \alpha^{-1} \), for \( Y_e \ll 1 \), the gas pressure takes over the radiation pressure in the disk and the entire disk becomes gas-pressure-dominated. On the other hand, the degeneracy pressure is larger than the radiation pressure at a very large radius if the electron fraction \( Y_e \) is not very small. However, we do not consider this situation for an ADAF region of the disk, because the degeneracy pressure, even if larger than the radiation pressure, cannot exceed the gas pressure.

When the gas pressure is dominant and the outer disk flow is still an ADAF, we obtain the density, temperature, and pressure of the disk as

\[ \rho_{11} = 0.556 (1 + \gamma_e)^{-12/11} m^{5/11} \mu_d^{8/11} f^{1/11} \alpha^{-8/11} r_6^{-21/11} \]
\[ T_{11} = 1.29 (1 + \gamma_e)^{-3/11} m^{4/11} \mu_d^{2/11} f^{4/11} \alpha^{-2/11} r_6^{-8/11} \]
\[ P_{29} = 5.98 (1 + \gamma_e)^{-4/11} m^{9/11} \mu_d^{10/11} f^{9/11} \alpha^{-10/11} r_6^{-29/11}. \]  

The assumption of a gas-pressure-dominated ADAF disk \( Q_{\text{adv}} > Q_e \) requires

\[ r_6 f^{47/22} f^{-20/11} > 128 (1 + \gamma_e)^{-26/11} m^{29/22} \mu_d^{10/11} \alpha^{-21/11}. \]  

\( P_{\text{gas}} > P_{\text{rad}} \) can also be expressed as

\[ r_6 f^{-7/3} > 74.6 (1 + \gamma_e)^{-8/3} m^{7/3} \mu_d^{-2/3} \alpha^{-2/3}. \]  

\( P_{\text{gas}} > P_{\text{deg}} \) leads to

\[ r_6 f^{-7/3} < 8.07 \times 10^3 (1 + \gamma_e)^{12} m^{-4/3} \mu_d^{-2/3} \alpha^{-2/3}. \]  

which is satisfied for a large parameter space.

If \( Y_e \sim 1 \) and the parameters \( m \) and \( \alpha \) satisfy 0.453 \( m^{7/2} \alpha^{-1} \), \( \mu_d \ll 1.73 m^{2/2} \alpha^{-1} \), the entire disk becomes an advection-dominated disk with gas pressure dominant. However, if \( Y_e \ll 1 \), such a disk cannot exist, since equation (27) cannot always be satisfied in the entire disk, and some region of the disk would become neutrino-dominated. In addition, when the mass accretion rate \( \dot{m}_d \) becomes higher, most of the disk becomes neutrino-dominated.

Furthermore, in the region where neutrino cooling is efficient and the gas pressure dominates over the degeneracy pressure, we have another particular solution,

\[ \rho_{11} = 2.38 (1 + \gamma_e)^{-9/5} m^{17/20} \mu_d f^{13/10} \gamma_e^{-1/5} r_6^{-51/20} \]
\[ T_{11} = 0.490 (1 + \gamma_e)^{3/5} m^{1/10} \alpha^{-1} r_6^{-3/10} \]
\[ P_{29} = 9.71 (1 + \gamma_e)^{-3/5} m^{19/20} \mu_d f^{11/10} \gamma_e^{-1/10} r_6^{-57/20}. \]  

In this case, the temperature is independent of the accretion rate \( \dot{m}_d \). Finally, we check the gas-pressure-dominated assumption. Using equation (30) and assuming \( P_{\text{gas}} > P_{\text{deg}} \), we obtain

\[ r_6 f^{-20/33} > 3.18 \gamma_e^{80/33} (1 + \gamma_e)^{-36/11} m^{1/3} \mu_d^{20/33} \alpha^{-38/33}. \]  

The condition (31) is always satisfied if \( \gamma_e \ll 1 \), and thus we can say that the gas-pressure-dominated assumption is valid. However, if \( \gamma_e \sim 1 \), a part of the disk becomes degeneracy-pressure-dominated if \( \mu_d > 64.5 \gamma_e^{19/10} m^{-11/20} \). In particular, in the case of \( \gamma_e \sim 1 \) and large accretion rate \( \dot{m}_d \), the solutions for part of the disk are

\[ \rho_{11} = 1.26 (1 + \gamma_e)^{-4/3} m^{2/3} \mu_d^{2/3} \alpha^{-2/3} \gamma_e^{-2} \]
\[ T_{11} = 0.526 \gamma_e^{12/7} m^{13/10} \mu_d^{1/27} f^{1/27} \gamma_e^{-13/36} \]
\[ P_{29} = 7.85 \gamma_e^{4/9} m^{8/9} \mu_d^{8/9} \gamma_e^{-8/9} r_6^{-8/3}, \]  

which describe an NDAF with degeneracy pressure dominant. However, we should remember that in deriving the above solutions, we have not considered the constraint of \( r > \tilde{r} \).

Here we have used an analytical method to solve the density, pressure, and temperature of the outer disk based on the simple model discussed at the beginning of § 3. The accretion flow may be ADAF or NDAF, with the radiation, gas, or degeneracy pressure dominant. We fix the radius \( r \) and show several possible cases in the disk with different accretion rate \( \dot{m}_d \) in Table 2. We also calculate the upper limit of \( \dot{m}_d \) for different electron fractions \( Y_e \) and fixing \( m = 1 \) and \( \alpha = 1 \). If \( Y_e \sim 1 \), the advection-dominated region in the disk can be radiation- or gas-pressure-dominated, and the neutrino-cooled region can be gas- or degeneracy-pressure-dominated. However, if \( Y_e \ll 1 \), the gas-pressure-dominated region in the ADAF case is very small, and the degeneracy-pressure-dominated region cannot exist. In § 3.4 we obtain similar results using a numerical method.

The solutions given here can also be used to discuss the properties of the disk around a black hole. Our analytical solutions of
the outer disk are similar to those of Narayan et al. (2001) and Di Matteo et al. (2002), who took $Y_e$ to be a parameter. Narayan et al. (2001) found that advection-dominated disks can be radiation-or gas-pressure-dominated, and neutrino-dominated disks can be gas- or degeneracy-pressure-dominated. This is consistent with our conclusion for $Y_e \sim 1$. However, these authors did not consider the factor $f = 1 - (r_s/r)^{1/2}$, which is important because a small $r_s$ as we have mentioned above, can dramatically change the parameter space of the outer disk. Di Matteo et al. (2002) discussed different pressure components (see their Fig. 2), also consistent with our conclusions. Chen & Beloborodov (2007) calculated the value of $Y_e$ and showed that $Y_e \ll 1$ when $r$ is small. According to the above discussion, therefore, the degeneracy-pressure-dominated region cannot exist in NDAF disks. This is consistent with the results of Chen & Beloborodov (2007), who found that pressure in a neutrino-cooled disk is dominated by baryons (gas).

However, our analytical results differ in part from those of Kohri et al. (2005) and Chen & Beloborodov (2007), who showed that the electron pressure is dominant in some advection-dominated regions of the disk. This difference is mainly due to the fact that we take $P_{\text{rad}} = 11aT^4/12$ in our analytical model, which includes the contribution of relativistic electron-positron pairs, while Kohri et al. (2005) and Chen & Beloborodov (2007) took $P_{\text{rad}} = at^3/3$ and calculated the pressure of $e^+e^-$ pairs in the electron pressure term $P_e$. As a result, the radiation pressure we consider here is actually the pressure of a photon and $e^+e^-$ pair plasma.

In the following subsection, we establish the structure of the inner disk, depending on the outer disk solutions here, and use the value of $\tilde{r}$ to further constrain the solutions that we have obtained.

### 3.2. The Inner Disk

#### 3.2.1. Boundary Layer between the Inner and Outer Disk

We use the method discussed in § 2.1 and compare the radial velocity with the local speed of sound of the outer disk using the results given in § 3.1.

The radial velocity of the outer disk at radius $r$ is

$$v_1 = \frac{\dot{M}}{2\pi \rho \Sigma} = \frac{\dot{M}}{\rho \pi r_1^2 H} = \frac{\dot{M} \sqrt{GM}}{4\pi r^{5/2} \sqrt{P_1 \rho_1}}.$$  \hspace{1cm} (33)

Hence, we have

$$\frac{v_1^2}{c_s^2} \sim \frac{\rho_1 v_1^2}{P_1} = \frac{\dot{M}^2 GM}{16\pi r^5 P_r^2} = 0.0465 \frac{m_r^2 m}{r_6^2 P_1^{29}}.$$  \hspace{1cm} (34)

In the case where the outer disk is an ADAF and radiation pressure is dominant, using solutions (20) and (21), we have

$$\frac{v_1^2}{c_s^2} \sim 4.22 \times 10^{-3} \alpha^{-2} f^{-1}. \hspace{1cm} (35)$$

Therefore, we see that $v_1 \ll c_s$ for typical values of the parameters.

In the case of an ADAF with gas pressure dominant, using solution (26), we have

$$\frac{v_1^2}{c_s^2} \sim 1.302 \times 10^{-3} (1 + Y_e)^{8/11} \times m^{-7/11} \tilde{m}_d^{2/11} \alpha^{-1/2} f^{-18/11} r_6^{3/11}. \hspace{1cm} (36)$$

For a NDAF, using expression (30) to compare the radial velocity with the speed of sound, we have

$$\frac{v_1^2}{c_s^2} \sim 4.94 \times 10^{-4} (1 + Y_e)^{6/5} m^{-9/10} \alpha^{-1/5} f^{-2} r_6^{7/10}. \hspace{1cm} (37)$$

Note that this ratio is independent of the accretion rate. From equations (36) and (37), we still find that $v_1 \ll c_s$ is always satisfied except for a region very near the stellar surface, where the factor $f$ is very small. This region, however, is so small that it belongs to the inner disk, where we use self-similar structure, as discussed in detail in later sections.

For the hyperaccretion disk discussed in this paper, because the disk is extremely hot and dense, the radial velocity is always subsonic. Thus, there is no stalled shock between the inner and outer disks, and all physical variables change continuously on both sides of the boundary layer between the two regions of the disk. In addition, the rotational velocity is assumed to have the Keplerian value, with no jump at the boundary layer.

#### 3.2.2. Solution of the Inner Disk

We now study the inner disk analytically, based on the results given in § 3.1. The main problem that we should solve in this subsection is to determine the size of the inner disk for a range of parameters and to describe the structure of the inner disk. In the case where the radiation pressure is dominant, by using the self-similar structure given by equation (12), we obtain the temperature of the inner disk,

$$T_{11} = \left( P_{29}/6.931 \right)^{1/4},$$

$$= 1.01 m^{1/6} r_1^{1/6} \alpha^{-1/6} r_6^{(2-\gamma)/(4-\gamma)} f^{-1} r_6^{-\gamma/(4-\gamma)}, \hspace{1cm} (38)$$
From the energy-conservation equation (17), the position of the disk, the actual adiabatic index 

gives solutions of equation (45) with different sets of parameters. We can see that \( \tilde{r} \) also decreases with increasing \( \gamma \) or \( m \), or decreasing \( Y_e \).

Here we have examined the case of an advection-dominated outer disk, and find that the size of the inner disk always increases with the accretion rate. For an outer disk that is mainly neutrino-dominated, using equations (30) and (17), we obtain an energy-conservation equation in the inner disk of

\[
T_{11} = (1 + Y_e)^{-1} \frac{1}{8.315} \frac{\rho_{29}}{\rho_{11}}
- 0.874m^2/3 \tilde{r}_6^2/3 f^2/2 \tilde{r}_6^{-2/3} \tilde{r}_6^{-1} - 1.
\]

or

\[
\tilde{r}_6 = \left( \frac{1 - \sqrt{r_s/\tilde{r}_6}}{\rho_{29}/\rho_{11}} \right)^{-1} \tilde{r}_6^{(2/3)}/(\gamma - 1) - \tilde{r}_6^{(2/3)}/(\gamma - 1).
\]

From equation (45), we see that the size of the inner disk \((\tilde{r})\) also decreases with increasing \( \tilde{m}_d \). Table 4 gives solutions of equation (45) with different sets of parameters. We can see that \( \tilde{r} \) also decreases with increasing \( \gamma \) or \( m \), or decreasing \( Y_e \).

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or

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- 0.874m^2/3 \tilde{r}_6^2/3 f^2/2 \tilde{r}_6^{-2/3} \tilde{r}_6^{-1} - 1.
\]

or

\[
\tilde{r}_6 = \left( \frac{1 - \sqrt{r_s/\tilde{r}_6}}{\rho_{29}/\rho_{11}} \right)^{-1} \tilde{r}_6^{(2/3)}/(\gamma - 1) - \tilde{r}_6^{(2/3)}/(\gamma - 1).
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- 0.874m^2/3 \tilde{r}_6^2/3 f^2/2 \tilde{r}_6^{-2/3} \tilde{r}_6^{-1} - 1.
\]

or

\[
\tilde{r}_6 = \left( \frac{1 - \sqrt{r_s/\tilde{r}_6}}{\rho_{29}/\rho_{11}} \right)^{-1} \tilde{r}_6^{(2/3)}/(\gamma - 1) - \tilde{r}_6^{(2/3)}/(\gamma - 1).
\]
This equation shows us that $\tilde{r}_6$ in NDAF is independent of $Y_e$, $m$, $\dot{m}_d$, and $\alpha$, and only depends on $\gamma$ and $\tilde{f}_r$. The size of the inner disk is constant no matter how many components and what size the accretion rate of the disk and the mass of the central neutron star are. If the outer disk is mainly an NDAF, we have $f_r \sim 1$. We choose several different sets of parameters to obtain the solution of equation (46) (see Table 5). As $f_r$ increases, the value of $\tilde{r}$ decreases slightly. Here we also consider an intermediate case of $\gamma$ between 5/3 and 4/3; the decline in $\gamma$ makes the size of the inner disk decrease.

However, in our discussion of NDAFs above, we have not considered the effect of neutrino opacity, but simply assumed that neutrinos escape freely. In fact, if the accretion rate is sufficiently large and the disk flow is mainly an NDAF, the disk’s region near the neutron star surface can be optically thick to neutrino emission. When the accretion rate is increased, the area of this optically thick region also increases. We now estimate the effect of the neutrino opacity on the structure of the inner disk. Let the region $r_t < r < \tilde{r}$ be optically thick to neutrino emission, the region of $\tilde{r} < r < r_t$ be optically thin, and the electron/positron pair capture reactions be the dominant cooling mechanism. Thus, equation (17) becomes

$$
\int_{r_t}^{\tilde{r}} \frac{d\dot{M}_d}{\tau} + \int_{r_t}^{r_t} 9 \times 10^{34} \rho_1 T_1^4 H \pi r dr
$$

$$= \frac{3GM^2 M}{4} \left\{ \frac{1}{3r_t^3} \tilde{r} \left[ 1 - \frac{2}{3} \left( \frac{r_t}{\tilde{r}} \right)^{1/2} \right] \right\},
$$

where we take $\epsilon \approx 1$. Using the self-similar relations and performing some derivations, we find

$$18.7(1 + Y_e)^{8/3} \frac{\gamma - 1}{\gamma - 2} m^{-1/5} \dot{m}_d^{-1/5} \alpha_x^{-1/5}$$

$$\times \tilde{r}_6^{18(8/5 - 1)/\gamma - 1)} \left[ \tilde{r}_6^{11/\gamma - 1)} - \tilde{r}_6^{11/\gamma - 1)} \right]
$$

$$+ 2.77 \left[ \frac{\gamma - 1}{3\gamma - 2} \tilde{r}_6^{12(1/2)/\gamma - 1)} \left( \frac{\gamma - 3}{\gamma - 4} \right) \tilde{r}_6^{(2 - 3)/\gamma - 1)} \right]
$$

$$= 0.924 \dot{m}_d \left[ \frac{2\tilde{r}_6}{r_t} \tilde{r}_6 + \frac{2\tilde{r}_6}{r_t} \tilde{r}_6^{1/2} \right].
$$

The solution of equation (48) gives $\tilde{r}$. We take $f_r \approx 1$ and $\alpha_x = 0.1$. We also define a new parameter $k = \tilde{r}_6/\tilde{r}_6$, and assume several sets of parameters to give the solution of equation (48).

From Table 6, we can see that the size of the inner disk increases with the accretion rate. In addition, an increase of $m$ or $k$, or an decrease of $Y_e$, also makes the inner disk size larger. We compare the analytical results from Tables 3–6 with numerical results in §3.4 in more detail.

### Table 5

| $\tilde{f}_r$ | $\gamma$ = 5/3 | $\gamma$ = 3/2 | $\gamma$ = 4/3 |
|-----|-----|-----|-----|
| 0.9 | 1.31 | 1.28 | 1.26 |
| 0.7 | 1.46 | 1.43 | 1.39 |
| 0.5 | 1.56 | 1.53 | 1.47 |

**Note:** Eq. (46) gives $\tilde{r}_6$.

### Table 6

| $\gamma$ | $\tilde{r}_6$ | $\tilde{r}_6$ | $\tilde{r}_6$ |
|-----|-----|-----|-----|
| Case 1 | 4.28 | 5.13 | 5.94 | 7.10 |
| Case 2 | 6.26 | 7.48 | 8.65 | 10.33 |
| Case 3 | 5.09 | 6.09 | 7.05 | 8.43 |
| Case 4 | 4.10 | 4.99 | 5.86 | 7.11 |
| Case 5 | 4.87 | 5.90 | 6.89 | 8.32 |

**Note:** Eq. (48) gives $\tilde{r}_6$ for several different cases. Case 1: $Y_e = 1, m = 1, k = 1, \gamma = 5/3$; case 2: $Y_e = 1/9, m = 1, k = 1, \gamma = 5/3$; case 3: $Y_e = 1, m = 2.0/1.4, k = 1, \gamma = 5/3$. Case 4: $Y_e = 1, m = 1, k = 0.7, \gamma = 5/3$; case 5: $Y_e = 1, m = 1, k = 1, \gamma = 3/2$.

### 3.3. The Stellar Surface Boundary

Here we turn to a discussion of the physical conditions near the neutron star surface. We know that if the rotational velocity of the neutron star surface is different from that of the inner boundary of the disk, then the disk can exert a torque on the star at the stellar radius. Here we take the rotational velocity of the inner disk to be $\Omega \simeq \Omega_0$, as mentioned in §2.1 and §2.3. Then the stellar surface angular velocity $\Omega_s$ is slower than that of the inner disk $\Omega_s$ (i.e., $\Omega_s < \Omega_0$). As a result, the kinetic energy of the accreted matter is released when the angular velocity of the matter decreases to the angular velocity of the neutron star surface. From Newtonian dynamics, we obtain the differential equation

$$G_\ast r_s = \frac{dI}{dt},$$

where $G_\ast r_s = \dot{M}r_s^2(\Omega_K - \Omega_s)$ is the torque acting on the star surface from the disk, and $I = \dot{m}r_s^2$ is the moment of inertia of the star, where $\xi$ is a coefficient. This equation can be further written as

$$[\Omega_K - \Omega_s(1 + \xi)]dM = \xi \dot{M} d\Omega_s.$$

From equation (50), we can see that if $\Omega_s < \Omega_s(1 + \xi)$ initially, we always have $d\Omega_s/dM > 0$, i.e., the neutron star is spun up as accretion proceeds. On the other hand, if $\Omega_s > \Omega_s(1 + \xi)$ initially, then the star is always spun down by the disk. The limit value of $\Omega_s$ is $\Omega_s(1 + \xi)$ in both cases. The solution of equation (50) is then

$$\left[ \Omega_s - \Omega_s(1 + \xi) \right] = \begin{pmatrix} M_0 + \Delta M \\ M_0 \end{pmatrix} \frac{(\xi + 1)}{(\xi + 1)} \left[ \Omega_s - \Omega_s(1 + \xi) \right] \approx \frac{M_0}{M_0 + \Delta M} \frac{(\xi + 1)}{(\xi + 1)} \left[ \Omega_s - \Omega_s \right],
$$

where $M_0$ is the initial mass of the neutron star, and $\Delta M$ is the mass of the matter accreted from the disk by the star. The change of $\Omega_s$ depends on the ratio $\Delta M/M_0$. For example, if we assume $M_0 = 1.4 M_\odot$ and $\Delta M = 0.01 M_\odot$, and set $\xi = 2/5$, then we have $[\Omega_0(1 + \xi) - \Omega_s] = 0.98[\Omega_0/1 + \xi) - \Omega_s].$ Or if we take $\Delta M = 0.1 M_\odot, \xi = 2/5$, and leave the central stellar mass unchanged, then we find $[\Omega_0(1 + \xi) - \Omega_s] = 0.79[\Omega_0/1 + \xi) - \Omega_s].$

Here we consider that the energy released from the surface boundary is also carried away by neutrino emission, and assume...
that neutrinos emitted around the stellar surface are opaque. We can estimate the temperature of the neutron star surface through

$$\frac{7}{8} \sigma T^4 4\pi r, H \sim \frac{G M}{2r_*}(2 - \varepsilon),$$

where $H$ is the half-thickness of the inner boundary of the disk, and the parameter $\varepsilon$ has the same meaning as in § 2.3. Then we can estimate the surface temperature as

$$T_{11} \sim 0.415(2 - \varepsilon)^{1/4} \frac{m^{1/4} m_d^{1/4}}{H_0^{1/4} r_*^{1/2}}.$$

This estimation of the temperature is only valid if the inner disk is optically thin to neutrinos, with only the surface boundary optically thick. If the inner disk becomes optically thick to neutrino emission (due to a large accretion rate), the surface temperature should be higher.

### 3.4. Comparison with Numerical Results

In order to give an analytical solution of the accretion disk around a neutron star in the simple model, we can choose the dominant terms in equations (18) and (19). We now consider one type of pressure to be dominant, and assume extreme cases from ADAF to NDAF. For example, we consider the neutrino-dominated region with $f_\nu \sim 1$ and the advection-dominated region with $f_\nu = 0$. Here we solve the hyperaccretion disk structure numerically, based on the simple model. In order to compare the numerical results with what we have obtained analytically, we keep the same equations and definitions of all the parameters as introduced above. However, we also need to point out several approximations and some differences between numerical and analytical methods based on the simple model.

First, we choose the range of $r_0$ to be from 1 to 15 in our numerical calculations. In other words, we take the range of the accretion disk to be from the surface of the neutron star to the radius of 150 km as the outer boundary. During a compact star merger or massive star collapse, only part of the torus around a neutron star has an angular momentum large enough to form a debris disk, so the mass of the disk may be smaller than the total mass of the torus. In simulations (Lee & Ramirez-Ruiz 2007), if the debris disk forms through the merger of two neutron stars, its outer radius may be slightly smaller than the value we give above. On the other hand, the disk size may be slightly larger if the disk forms during the collapse of a massive star. The changes in physical variables of the disk due to changes in the outer radius may be insignificant, and we do not discuss the effect of the outer radius in this paper.

Second, we fix the viscosity parameter $\alpha$ to be 0.1. If $\alpha$ decreases (or increases), the variables of the disk increase (or decrease). More information can be seen in the analytical solutions in § 3.1 and § 3.2. In numerical calculations, we take $\alpha$ to be a constant.

Third, we still set $Y_e$ as a parameter in numerical calculations in § 3.4. In order to show the results clearly, we consider two conditions: one is an extreme condition with $Y_e = 1$, which means that the disk is made mainly of electrons and protons but no neutrons; the other is $Y_e = 1/9$, which means that the numerical ratio of electrons, protons, and neutrons is $1:1:8$. A complete analysis should consider the effect of $\beta$-equilibrium, and we discuss it in detail in § 4.

For numerical calculations, we consider all the pressure terms and an intermediate case between ADAF and NDAF. In analytical calculations we take the adiabatic index $\gamma$ of the inner disk to be $5/3$ if the disk is gas-pressure-dominated, or $\gamma = 4/3$ if the disk is radiation- or degeneracy-pressure-dominated. In numerical calculations, however, it is convenient to introduce an “equivalent” adiabatic index $\gamma$ based on the original definition of $\gamma$ from the first law of thermodynamics, $\gamma = 1 + P/u$, where $P$ is the pressure of the disk at a given $r$, and $u$ is the internal energy density at the same radius. Therefore, $\gamma$ is a variable function of radius. We obtain the self-similar structure,

$$\frac{\rho(r)}{\rho(r + dr)} = \left( \frac{r}{r + dr} \right)^{-\gamma(r)/\gamma(r)} \left( \frac{r}{r + dr} \right)^{-1/(\gamma(r) - 1)}.$$

If $\gamma$ does not vary significantly in the inner disk, the difference between the approximate solution where $\gamma$ is a constant and the numerical solution where we introduce an “equivalent” $\gamma$ is not obvious.

In addition, some portion of the accretion disk is optically thick to neutrino emission when $\dot{m}_d$ is sufficiently large. We use the same expressions of neutrino optical depth and emission as in § 3. We also require that the neutrino emission luminosity per unit area is continuous when the optical depth crosses $\tau = 1$.

We first calculate the value of $\tilde{r}$, which is the radius of the boundary layer between the inner and outer disks. Figure 1 shows $\tilde{r}$ as a function of $\dot{m}_d$ for different values of $m$. We can see that when the disk flow is an ADAF at a low accretion rate, $\tilde{r}$ decreases monotonically as the accretion rate increases until the value of $\tilde{r}$ reaches a minimum. At this minimum, the outer disk is covered, no part of the disk is similar to an accretion disk around a black hole, as discussed in § 3.1, and thus
we say that the entire disk becomes a self-similar structure, and the physical variables of the entire disk are adjusted to build an energy balance between heating and neutrino cooling. Here we choose to focus on the accretion rate that allows two steady parts of disks to exist, so we do not discuss the self-similar case in any greater detail.

In Figure 2, we choose several special conditions to plot the density, temperature, and pressure of the whole disk as functions of radius \( r \). In particular, we take \( \rho, P, \) and \( T \) to be continuous at the boundary between the inner and outer disks. If the parameter \( \dot{m}_d \) is larger, or if the disk contains more neutrons (i.e., \( Y_e \) becomes smaller), then the density, temperature, and pressure of the disk all increase, although the change in the density and pressure will be more dramatic than the change in temperature, since the change in temperature cannot be too large without greatly affecting the neutrino cooling rate of the disk.

If the mass of the central neutron star \( m \) is larger or the electron fraction \( Y_e \) is smaller, then the value of \( \tilde{r} \) in the monotonically decreasing segment of \( \tilde{m}_d - \tilde{r} \) becomes smaller, and the value of \( \tilde{r} \) in the monotonically increasing segment of \( \tilde{m}_d - \tilde{r} \) is larger. Furthermore, the minimum value of \( \tilde{r} \) is almost independent of \( Y_e \) and \( m \). All of these conclusions are consistent with the analytical solutions, except in the case of an advection-dominated outer disk with radiation pressure dominant. In § 3.2.2, we found that the size of the inner disk increases with increasing \( m \) for our analytical solutions. However, by calculating the “equivalent” adiabatic index \( \gamma \), we find that \( \gamma \) decreases slightly with increasing \( m \). This makes the value of \( \tilde{r}_0 \) decrease, which is also consistent with the analytical results (see Table 3).

Figure 3 shows the equivalent adiabatic index \( \gamma \) as a function of radius of the entire disk for several different sets of parameters. In the case where the accretion rate is low, the radiation pressure is important. As the accretion rate increases, the gas pressure becomes more dominant, and the value of \( \gamma \) is larger. On the other hand, the ratio of the degeneracy pressure to the total pressure is larger in the case of a higher accretion rate and \( Y_e \sim 1 \). However, the gas pressure is always dominant for the accretion rate chosen here. We see from Figure 3 that the change of \( \gamma \) in the inner disk is insignificant.

Figure 4 shows the ratio of the radial velocity \( v_r \) and local speed of sound \( c_s \) as a function of \( r \). The ratio is always much smaller than unity, which means that the accretion flow is always subsonic, and no stalled shock exists in the disk. In many cases, the peak of this ratio is just at the boundary between the inner and outer disks. The reason for this can be found in § 3.2.1, where we give the analytical expression of the ratio; \( f = 1 - (r_i/r)^{1/2} \) is the major factor that affects the value of \( v_r/c_s \) of the outer disk. However, in the inner disk \( v_r/c_s \) always decreases, since the isothermal sound speed can be greater at smaller radii (i.e., \( c_s \propto r^{-1/2} \)), and the radial velocity of the accreting gas, which satisfies the self-similar equation (12), cannot change dramatically for the disk matter to strike the neutron star surface.

Figure 5 shows the total neutrino emission luminosity of the entire disk around a neutron star as a function of accretion rate for the parameters \( M \) and \( Y_e \) (where we do not consider neutrino emission from the stellar surface, discussed in § 3.3), and we compare it with the neutrino luminosity from a black hole disk. We also calculate the total neutrino luminosity from the inner
and outer disks. Here we roughly take the mass of the black hole to be the same as that of the neutron star, and the innermost stable circular orbit of the disk has a radius that is equal to the radius of the neutron star. We use approximately Newtonian dynamics for simplicity. In Figure 5, we find that the difference in neutrino luminosity between the neutron star and black hole cases is a strong function of the accretion rate. When the accretion rate is low ($\dot{m}_d \lesssim 10$), the total neutrino luminosity of the black hole disk $L_{\nu,\mathrm{BH}}$ is much smaller than that of the neutron star disk $L_{\nu,\mathrm{NS}}$, but $L_{\nu,\mathrm{BH}}$ and $L_{\nu,\mathrm{NS}}$ are similar for a moderate accretion rate ($\dot{m}_d = 10$–100). In fact, this result is consistent with the general scenario introduced in § 2.1 and the basic result shown in Figure 1: for a low accretion rate, the black hole disk is mainly advection-dominated, with most of the viscous dissipation-driven energy advected into the event horizon of the black hole, and we have $L_{\nu,\mathrm{BH}} \ll G M (4 \pi r)$. On the other hand, a large inner disk for a neutron star disk with a low accretion rate makes the neutrino emission efficiency much higher than its black hole counterpart. However, for a moderate accretion rate, the black hole disk is similar to the neutron star disk, which in this case has quite a small inner disk, and we have $L_{\nu,\mathrm{BH}} \sim L_{\nu,\mathrm{NS}}$. Moreover, neutrino opacity causes the value of $L_{\nu,\mathrm{BH}}$ to be less again compared to $L_{\nu,\mathrm{NS}}$ for a high accretion rate, as this opacity decreases the neutrino emission efficiency in the black hole disk, but increases the size of the neutron star disk again to balance the heat energy release.

4. AN ELABORATE MODEL OF THE DISK

In the last section we first studied the disk structure analytically. To do this, we used several approximations. First, we took the pressure as a summation of several extreme contributions, such as the gas pressure of nucleons and electrons, and the radiation pressure of a plasma of photons and $e^+e^-$ pairs. However, electrons may actually be degenerate or partially degenerate, and the neutrino pressure should also be added to the total pressure. Following Kohri et al. (2005), many works on hyperaccretion disks have used the Fermi-Dirac distribution to calculate the pressure of electrons and even the pressure of nucleons. Second, the neutrino cooling we used in § 3 is simplified, following Popham et al. (1999) and Narayan et al. (2001), and neglecting the effects of electron degeneracy and different types of neutrinos with different optical depths. In fact, these effects may be significant in some cases. Third, we took the electron-nucleon radio $Y_e$ as a constant parameter in our analytical model in § 3. Realistically, $Y_e$ should be calculated based on $\beta$-equilibrium and neutronization in hyperaccretion disks. In this section, we retain the assumption of outer and inner disks, as discussed in § 2, but consider a state-of-the-art model with lots of elaborate (more physical) considerations regarding the thermodynamics and microphysics in the disk, which was recently developed in studying the neutrino-cooled disk of a black hole. We then compare results from this elaborate model with those of the simple model discussed in § 3.

4.1. Thermodynamics and Microphysics

The total pressure in the disk can be written as $P = P_{\text{me}} + P_{\text{rad}} + P_e + P_{\nu}$. We still consider all the nucleons to be free ($X_{\text{me}} \approx 1$), as mentioned in § 2.2, and ignore the photodisintegration process. We replace the radiation pressure term $11aT^4/12$ in the simple model with $aT^4/3$ in this section, because the pressure of $e^+e^-$ pairs can be calculated in the electron pressure $P_e$ using the Fermi-Dirac distribution,

$$ P_e = \frac{1}{3} \frac{m_e^3 c^5}{\pi^2 h^3} \int_0^\infty \frac{x^4}{\sqrt{x^2 + 1}} e^x e^x e^{\left(c^2/(2m_e^2\sqrt{x^2 + 1})\right)/k_B T} + 1, \quad (53) $$

where $x = p/m_e c$ is the dimensionless momentum of an electron, $\mu_e$ is the chemical potential of the electron gas, and $P_e$ is the summation of $P_e$ and $P_{\nu}$. In addition, we take the neutrino pressure to be

$$ P_{\nu} = u_{\nu}/3, \quad (54) $$

where $u_{\nu}$ is the energy density of neutrinos.
The “equivalent” adiabatic index can be expressed by

$$\gamma = 1 + \frac{P_{\text{rad}} + P_e + P_i}{u_{\text{rad}} + u_e + u_i},$$

with the inner energy density

$$u_{\text{gas}} = \frac{3}{2} P_{\text{gas}},$$

$$u_{\text{rad}} = 3 P_{\text{rad}},$$

$$u_e = \frac{m_e^4 c^2}{\pi^2 \hbar^2} \int_0^\infty \frac{x^2 \sqrt{x^2 + 1}}{e^{(x^2 + 1 + \rho_0)/(kT)} + 1} dx.$$
accretion rate is high enough. In addition, $\gamma$ decreases as the radius decreases in the inner disk. These are consistent with the results of the simple model (see Fig. 3). From the right panel of Figure 7, we can see that $Y_e \sim 1$ when the accretion rate is low, and $Y_e \ll 1$ when the accretion rate becomes sufficiently high. This result is consistent with Kohri et al. (2005, their Fig. 6b). Chen & Beloborodov (2007) and Liu et al. (2007) showed the electron fraction $Y_e \leq 0.5$ in the disk, since they supposed that initial neutrons and protons come from photodisintegration of $\alpha$-particles at some large radius far from a central black hole. However, since the hyperaccretion disk around a neutron star is smaller than a black hole disk, we consider the mass fraction of free nucleons $X_{\text{nuc}} = 1$ in the inner disk. Therefore, the fraction of protons may be slightly higher, and it is possible that the protons are richer than neutrons in the disk, or $Y_e \gtrsim 0.5$, if the accretion rate is low enough.

Figures 8 and the right panel of 9 show the density, temperature, pressure, and neutrino luminosity per unit area in the entire disk for three different accretion rates. We fix $M = 1.4 M_\odot$, and also plot two other curves of solutions in the simple model with $Y_e = 1$ and $1/9$. The density $\rho$ and pressure $P$ in the elaborate model are smaller than those in the simple model when the accretion rate is low ($\dot{m}_d = 1.0$), but are similar to the solution of the simple model with $Y_e = 1/9$ for a high accretion rate ($\dot{m}_d = 100$). In addition, $\rho$ and $P$ in the elaborate model change from one solution ($Y_e = 1$) to the next ($Y_e = 1/9$) in the simple model for an intermediate accretion rate (e.g., $\dot{m}_d \sim 10$), since $Y_e \sim 0.5$ in the outer edge of the disk and $Y_e \ll 1$ in the inner disk. The distribution of the neutrino cooling rate $Q^\nu_r$ (luminosity per unit area) in the elaborate model is almost the same as that in the simple model with $Y_e = 1$ for a low accretion rate or $Y_e = 1/9$ for a high accretion rate. However, the value of $Q^\nu_r$ is still different in these two models for the region that is optically thick to neutrino emission in the disk.

We also plot the total neutrino emission luminosity of the entire disk and the outer and inner disks as a function of accretion rate for a central neutron star mass of $1.4 M_\odot$, and compare the total neutrino luminosity with that of a black hole disk (Fig. 9, right). The results are similar to what we have found in the simple model (Fig. 5).

5. CONCLUSIONS AND DISCUSSION

In this paper we have studied the structure, energy advection and conversion, and neutrino emission of a hyperaccretion disk around a neutron star. We considered a quasi-steady disk model without any outflow. Similar to the disk around a black hole, a neutron star disk with a huge mass accretion rate is extremely hot and dense, opaque to photons, and thus can only be cooled via neutrino emission, or even may be optically thick to neutrino emission in some region of the disk if the accretion rate is sufficiently high. However, a significant difference between black hole and neutron star disks is that the heat energy of the disk can
be advected into the event horizon if the central object is a black hole, but if the central object is a neutron star, the heat energy should eventually be released from a region of the disk near the stellar surface. As a result, the neutrino luminosity of a neutron star disk should be much larger than that in black hole accretion. We took the disk to be approximately Keplerian. According to the virial theorem, one-half of the gravitational energy in such a disk is used to heat the disk, and the other half to increase the rotational kinetic energy of the disk. We assumed that most of the heat energy generated from the disk is still cooled from the disk via neutrino emission, and that the rotational energy is used to spin up the neutron star or is released at the stellar surface via neutrino emission.

Fig. 8.—Density (in units of $10^{11}$ g cm$^{-3}$), pressure (in units of $10^{11}$ ergs cm$^{-3}$), and temperature (in units of $10^{11}$ K) of the entire disk in two models for $M = 1.4 M_\odot$. The profiles include three groups of lines, and each group also includes three lines, which are shown for three values of the accretion rate: $\dot{M} = 0.01, 0.1,$ and $1.0 M_\odot$ s$^{-1}$ (bottom to top). The solution in the simple model with $Y_e = 1$ is shown by the thin dashed line, and $Y_e = 1/9$ by the thin dotted line. The solution in the elaborate model is shown by the thick solid line.

For a certain range of hypercritical accretion rates, depending on the mechanisms of energy heating and cooling in the disk, we considered a two-region, steady-state disk model. The outer disk is similar to the outer region of a black hole disk. We used the standard viscosity assumption, Newtonian dynamics, and a vertically integrated method to study the structure of the outer disk. Since the radial velocity of the disk flow is always subsonic, no stalled shock exists in the disk, and thus we postulate that physical variables in the disk change continuously when

Fig. 9.—Left: Neutrino luminosity per unit area (in units of $10^{39}$ ergs cm$^{-2}$ s$^{-1}$) in both the simple model and the elaborate model. Lines are as in Fig. 8. Right: Neutrino luminosity from the disk in the elaborate model. Lines are as in Fig. 5.
crossing the boundary layer between the inner and outer disks. The inner disk, which expands until a heating-cooling balance is achieved, could satisfy a self-similar structure, as shown by equation (12).

In this paper we first studied the disk structure analytically. To do this, we adopted a simple disk model based on the analytical method. We took the pressure as a summation of several extra contributions and simple formulae for neutrino cooling. We also took the electron fraction $Y_e$ as a parameter in the simple model. We used an analytical method to find the dominant pressure distribution (Table 2) and the radial distributions of the density, temperature, and pressure (eqs. [20], [21], [26], [30], and [32]) in the outer disk. Then we used the equation of energy balance between heating and neutrino cooling to calculate the size of the inner disk in four different cases, with the advection-dominated outer disk either radiation- or gas-pressure-dominated, and the neutrino-cooled outer disk either optically thin or thick to neutrino emission (Tables 3–6). Subsequently, we numerically calculated the size of the inner disk, the structure, and energy conversion and emission of the entire disk in the simple model (Figs. 2–5) and compared the numerical results with the analytical results. The numerical results are consistent with the analytical ones from the simple model.

When the accretion rate is sufficiently low, most of the disk is advection-dominated, and the energy is advected inward to heat the inner disk and eventually released via neutrino emission in the inner disk. In this case, the inner disk is very large, and quite different from a black hole disk, which advects most of the energy inward into the event horizon. If the accretion rate is higher, then physical variables such as the density, temperature, and pressure become larger, the disk flow becomes NDAF, the advected energy becomes smaller, and heating of the inner disk becomes less significant. As a result, the size of the inner disk is much smaller, and the difference between the entire disk and the black hole disk becomes less significant. Furthermore, if the accretion rate is large enough to make neutrino emission optically thick, then the effect of neutrino opacity becomes important, decreasing the efficiency of neutrino emission from most of the disk, and the size of the inner disk again increases until the entire disk becomes self-similar. A different mass of the central star or a different electron-nucleon ratio also makes the physical variables and properties of the disk different. However, the accretion rate plays a more significant role in the disk structure and energy conversion, as it varies much more widely than the other parameters.

The simple model is based on early works of, e.g., Popham et al. (1999) and Narayan et al. (2001). We found that the simple model in fact gives us a clear physical picture of the hyper-accretion disk around a neutron star, even if we use some simplified formulae for thermodynamics and microphysics in the disk. In § 4 we considered an elaborate model, in which we calculate the pressure of electrons and positrons using the Fermi-Dirac distribution, and replace the factor $11/12$ by $1/3$ in the radiation pressure equation. In this model we also adopted more advanced expressions of the neutrino cooling rates, including the effect of all three types of neutrinos and the electron degeneracy, and considered $\beta$-equilibrium in the disk to calculate the electron fraction $Y_e$. Then we calculated the size of the inner disk (Fig. 6), the radial distributions of the “equivalent” adiabatic index $\gamma$, the electron fraction (Fig. 7), the density, temperature, and pressure in the disk (Fig. 8), the neutrino cooling rate distribution of the disk (Fig. 9, left), the neutrino emission luminosity from the inner and outer disks, and the total neutrino luminosity of a neutron star disk compared with that of a black hole disk (Fig. 9, right).

The electron fraction $Y_e$ was also determined in the elaborate model. We found that $Y_e$ drops with increasing accretion rate in the outer disk. The value of $Y_e$ can be greater than 0.5 at large radii if the accretion rate is sufficiently low, and $Y_e \ll 1$ in the disk when the accretion rate is sufficiently high. If we incorporate these results for $Y_e$ from the elaborate model into the simple model correctly (i.e., $Y_e \sim 1$ for low accretion rate and $Y_e \ll 1$ for high accretion rate), we find that they are basically consistent with each other (see Figs. 6–9), and also consistent with most earlier work (see the discussion in § 3.1).

One main difference in structure between the simple and elaborate models is caused by the different expressions for pressure adopted in the two models. In order to see this clearly, we introduce a third model here. We still keep $P_{\text{rad}} = 11aT^4/12$ as in the simple model, but change the relativistic degeneracy pressure term of electrons (the second term in eq. [19]) to the Fermi-Dirac distribution (eq. [53] for $P_e$), and use all the other formulae as given in § 4. In other words, the third model is generated by changing only one pressure term in the elaborate model. We find that the results from the third model are even more consistent with those of the simple model than the elaborate model in § 4. Take Figure 10 as an example. We compare the solution of the inner disk of the third model with that of the simple model. From Figure 10, we can see that if the accretion rate is low and $Y_e \sim 1$, the thick solid line is much closer to the thin dashed line, which shows the simple model with $Y_e = 1$; if the accretion rate is high enough and $Y_e \ll 1$, the solid line is much closer to the thick dotted line, which is the result from the simple model with $Y_e = 1/9$. Compared with the left panel of Figure 8, this result is even more consistent with the simple model.

The values of the density $\rho$ and pressure $P$ in Figure 10, for a low accretion rate, are smaller than those of the simple model, which is also due to the different expressions for the radiation pressure used in these two models in §§ 3 and 4. Therefore, we conclude that the main difference of the results between the simple model in § 3 and the elaborate model in § 4 come from different expressions of the pressure in disks. However, we believe that the pressure formulae given in the elaborate model are more realistic, since $11aT^4/12$ is only an approximate formula for the pressure of a plasma of photons and $e^+e^-$ pairs.

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**Fig. 10.** Comparison of $r_6$ in three models with $M = 1.4 M_\odot$: the simple model with $Y_e = 1$ (dashed line), the simple model with $Y_e = 1/9$ (dotted line), and the third model, discussed in § 5 (thick solid line). The solution of $r_6$ in the third model is even more consistent with the simple model than that of the elaborate model discussed in § 4.
On the other hand, as has been pointed out by Lee et al. (2005) and Liu et al. (2007), the formulae $P_{\text{rad}} = \alpha T^4/3$ and equation (53) in § 4.1 are better and can automatically take relativistic $e^+e^-$ pairs into account in the expression of $P_{\nu}$.

The different expression of the neutrino cooling rate $Q_{\nu}$ makes the neutrino luminosity distribution different in the region that is optically thick to neutrino emission. A more advanced expression of the neutrino cooling rate $Q_{\nu}$ gives better results for the neutrino luminosity per unit area than that given by the rough expression $\frac{4}{3}\sigma T^4/\tau$ in § 3.

In this paper we studied a disk without any outflow, which may exist if the disk flow is an ADAF. However, it is still unclear whether an outflow or neutrino cooling might play a more important role, since the size of the disk is quite small. The other case that we ignored is if the radius of the central neutron star is smaller than that of the innermost stable circular orbit of the accretion disk, and the accreting gas eventually falls freely onto the neutron star. In this case, a shock could form in the region between the innermost stable circular orbit and neutron star surface (Medvedev 2004). This effect could be studied if other effects, such as the equation of state of a differentially rotating neutron star and its mass-radius relation, are taken into account.

Neutrinos from a hyperaccretion disk around a neutron star may possibly be annihilated to electron/positron pairs, which could further produce a jet. Such a jet would be expected to be more energetic than that from the neutrino-cooled disk of a black hole with same mass and accretion rate as the disk of a neutron star (D. Zhang & Z. G. Dai 2008, in preparation). This could further support the conclusion that some GRBs originate from neutrino annihilation rather than magnetic effects such as the Blandford-Znajek effect.

In this paper we considered a central neutron star with surface magnetic field weaker than $B_{\text{star}} \sim 10^{15} - 10^{16}$ G for typical hyperaccretion rates. For magnetars (i.e., neutron stars with ultrastrong magnetic fields of $\sim B_{\text{star}}$), however, the magnetic fields could play a significant role in the global structure of hyperaccretion disks as well as underlying microphysical processes, e.g., quantum effect (Landau levels) on the electron distribution and magnetic pressure in the disks could become important. Thus, the effects of an ultrastrong magnetic field on hyperaccretion disks around neutron stars are an interesting topic, which deserve more detailed study.

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