Lorentz-symmetry Violation and Electrically Charged Vortices in the Planar Regime

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We start from a Lorentz non-invariant Abelian-Higgs model in 1+3 dimensions, and carry out its dimensional reduction to $D = 1 + 2$. The planar model resulting thereof is composed by a Maxwell-Chern-Simons-Proca gauge sector, a massive scalar sector, and a mixing term (involving the fixed background, $v^{\mu}$) that realizes Lorentz violation for the reduced model. Vortex-type solutions of the planar model are investigated, revealing charged vortex configurations that recover the usual Nielsen-Olesen configuration in the asymptotic regime. The Aharonov-Casher Effect in layered superconductors, that shows interference of neutral particles with a magnetic moment moving around a line charge, is also studied. Our charged vortex solutions exhibit a screened electric field that induces the same phase shift as the one caused by the charged wire.

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I. INTRODUCTION

In recent years, some models with Lorentz and CPT breaking have been adopted as a low-energy limit of an extension of the Standard Model, valid at the Plank scale \cite{1,2,3}. An effective action is obtained that incorporates CPT and Lorentz violation and keeps unaffected the $SU(3) \times SU(2) \times U(1)$ gauge structure of the underlying theory. Lorentz covariance is broken only in the particle frame, remaining as a good symmetry at the point of view of the observer frame. A well known way to introduce such a breaking in a QED, due to Carroll, Field and Jackiw \cite{4}, consists in adopting a 4-dimensional version of the Chern-Simons topological term, namely $\epsilon_{\mu\nu\lambda\rho} v^\mu A^\rho F^\nu\lambda$, where $\epsilon_{\mu\nu\lambda\rho}$ is the 4-dimensional Levi-Civita symbol and $v^\mu$ is a fixed four-vector acting as a background. This idea and consequences have been extensively analyzed in refs. \cite{4,5,6}.

Despite the intense activity proposing and analyzing the consequences of a Lorentz-violating electrodynamics, experimental data and theoretical considerations both indicate stringent limits on the parameters responsible by such a breaking \cite{7,8,9} in a factual (1+3)-dimensional electrodynamics. Such evidence raises the question about the feasibility of observation of such effect in a low dimension system and also in an environment distinct from the usual high-energy domain in which this matter has been generally considered so far. It encourages the investigation of the Lorentz-violation phenomenon in Condensed Matter planar systems, for instance. Condensed Matter Systems (CMS), despite belonging to the domain of low-energy physics, are known to exhibit some phenomena well addressed by the mathematical tools of the high-energy physics (field theory) domain, as it occurs in the case of the fractional Hall effect (anyon excitations), kinks, vortices, and other topological defects. Such a correspondence shows that there are some advantages in adopting a Field Theory framework to address...
CMS’s under different perspectives. It is well known that CMS are sometimes endowed with spatial anisotropy which might constitute a nice environment to study Lorentz-violation and to observe correlated effects. Indeed, although Lorentz covariance is not defined in a CMS, Galileo covariance holds as a genuine symmetry in such a system (at least for the case of isotropic low-energy systems). Having in mind that a CMS may be addressed as the low-energy limit of a relativistic model, there follows a straightforward correspondence between the breakdown of Lorentz and Galileo symmetries, in the sense that a CMS with violation of Galileo symmetry may have as counterpart a relativistic system endowed with breaking of Lorentz covariance. Considering the validity of this correspondence, it turns out that anisotropic CMS may be addressed as the low-energy limit of a relativistic model in the presence of a spacelike Lorentz-violating background.

The attainment of an attractive electron-electron \((e^- e^-)\) potential in the context of a model incorporating Lorentz-violation, for instance, is a point that may set up a clear connection between such theoretical models and condensed matter physics. Such issue has been already taken into account both in \((1+3)\) and \((1+2)\) dimensions. In two dimensions, it was adopted as starting point the planar model corresponding to the dimensionally reduced version of the Carroll-Field-Jackiw electrodynamics (11), (12). The electron-electron interaction potential, obtained both for a purely timelike and spacelike Lorentz-violating background, has revealed to be attractive for some radial range, which leads to the real possibility of attaining the formation of electron-electron bound states. Particularly, for the case a purely spacelike background, the resulting interaction potential is endowed with spacial anisotropy, once the Lorentz-violating two-vector \(v\) sets up a fixed direction in space. Therefore, this study may be seen as a first connection with planar superconductivity phenomena, including the possibility of considering the anisotropy of the parameter of order as related with the background.

In the context of a \((1+3)\)-dimensional CMS, a fixed four-vector \(-v^\mu = (v_0, v)\), acting as a Lorentz-violating background, may be used to describe anisotropy of a particular material, once it can select a preferential plane in which phenomena such as planar superconductivity and Quantum Hall Effect take place. In this way, some informations of the matter environment may be associated with the geometric orientation of the fixed four-vector. It is well known that superconducting ceramics are strong anisotropic materials, in which the charge transport is essentially two-dimensional. A suitable choice of the background spacial orientation can be used to simulate anisotropy of such materials. Indeed, if one considers the z-component of the four-vector orthogonal to the conducting plane, the x and y spacial components will lay parallel to this plane. One has thus a scenario in which the component \(v_z\) defines the conduction plane, whereas \(v_x\) and \(v_y\) are useful to simulate anisotropy on such a plane. This is a general prescription for addressing four dimensional systems in the presence of a Lorentz-violating background.

It should be still remarked that in Lorentz-violating theories endowed with a fixed background, it is understood that such a background must be kept invariant under Lorentz transformations in the particle frame, fact to which the breakdown of the covariance is intrinsically related to. Therefore, we can effectively propose a gauge theory suitable to such a requirement, that is, for which the fixed background does not undergo Lorentz transformations (basically, rotations). Generally, in CMS investigations one does not move the bulk, which may be associated with an occasional background; only the fields are allowed to change. Such a general description establishes a good compatibility between CMS and our theoretical general conditions which define the Lorentz violation.

Presently, the investigation of vortex configurations is of interest of both Field Theory and Condensed Matter physics. This interplay was born since the pioneering work of Nielsen & Olesen (14) that argued the existence of (chargeless) vortices in the Abelian Higgs model (the relativistic version of the Landau-Ginzburg theory, suitable for addressing superconducting systems in a Field Theory approach). The presence of stable (chargeless) vortex solutions in a non-Abelian Higgs context was soon demonstrated as well (15). Some time later, the introduction of the Chern-Simons term provided the attainment of charged vortex solutions in the context of both Abelian (16) and non-Abelian Higgs model (17), which are characterized by a linear relation involving the charge and magnetic flux of the vortex. In the sequel, it was shown that these vortex configurations satisfy a set of self-duality equations in the pure anyonic limit (18), in which the dynamics of the gauge field is ruled only by Chern-Simons term.

In the present work, we are particularly interested in studying vortex solutions in the context of a Lorentz-violating Higgs Abelian model in \((1+2)\) dimensions. One possibility to simulate this kind of system is taking a
spatial component of $\nu^\mu$ perpendicular to the plane in which the supercurrent runs, according to the description given above. Another way involves the definition of a planar theoretical model for which the third spacial component is gotten rid of, although it provides a remanent in the two-dimensional world. This is accomplished by means of the dimensional reduction prescription in which the third spacial component is frozen. Such a procedure will be suitably elucidated in Sec. II. The dimensional reduction procedure is then implemented on the recently developed Abelian-Higgs version of Carroll-Field-Jackiw electrodynamics. The resulting $(1+2)$-dimensional model consists of a planar electrodynamics composed of a Maxwell-Chern-Simons-Proca gauge field, two scalar fields, and a mixing term (responsible for the violation of Lorentz symmetry). The consistency of this model was also classically analyzed, revealing that the causality, unitarity and stability of the modes are assured for both purely timelike and spacelike backgrounds. This planar model constitutes our theoretical framework for studying vortex-like configurations in a Lorentz-violating Higgs-Abelian framework. It corresponds to the relativistic analogue of the Landau-Ginzburg free energy, which describes superconductivity and vortex solutions in superconductors, here supplemented by the Lorentz-violating mixing term. This model differ from the usual Abelian-Higgs model analyzed by Paul & Khare by the presence of two terms containing the $\varphi$-scalar field (the remanent of the $A^{(3)}$ component). Working with the resulting differential equations, it is possible to show that the $\varphi$ field exhibits an exponential decaying asymptotic behavior, limit in which our model recovers the usual Nielsen-Olesen vortex configurations.

The organization of our work is as follows: Section II begins with a brief description of fundamental mechanisms in low-temperature superconductivity in connection with the scenario in which the planar charged vortices appear. Next, we perform the dimensional reduction of the Maxwell-Chern-Simons-Higgs Electrodynamics. In Section III, we propose the investigation of vortex-like solutions, discuss its electric charge and the possibility of the appearance of a phase difference as a consequence of their non-neutrality. In Section IV, we propose the discussion of the interaction of particles in a superconducting medium with vortices and the natural appearance of the Aharonov-Casher effect. Finally, in Sec. V, we present our Concluding Comments and Perspectives, where we also present a possible connection of our theoretical model, vortices and Aharonov-Casher effect in the so-called pseudo-gap regime of planar superconductors.

II. THE DIMENSIONALLY-REDUCED MODEL

One starts from the Maxwell Lagrangian in $1+3$ dimensions supplemented with the Carroll-Field-Jackiw term and a scalar sector endowed with spontaneous symmetry breaking, as it appears in ref. 8:

$$L_{1+3} = \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \epsilon^{\mu\nu\lambda\chi} D_\mu A_\nu F_{\lambda\chi} + (D^\mu \phi)^* D_\mu \phi - V(\phi^* \phi) + A_\mu J^\phi \right\},$$

(1)

where the $\hat{\mu}$ runs from 0 to 3, $D_\mu \phi = (\partial_\mu + ic A_\mu) \phi$ is the covariant derivative, and $V(\phi^* \phi) = m^2 \phi^* \phi + \lambda (\phi^* \phi)^2$ represents the scalar potential responsible for the spontaneous symmetry breaking ($m^2 < 0$ and $\lambda > 0$). This model is gauge invariant but does not preserve the Lorentz and CTP symmetries, once the fixed 4-vector, $\nu^\mu$, does not undergo Lorentz transformation (it can be viewed as a set of four scalars). It should be remarked that the spontaneous symmetry breaking yields a Proca mass to the photon and induces the transition to a non-trivial vacuum. Therefore, the light propagation in such a material medium constitutes a short range interaction. This background fluid, provided by the Higgs mechanism in a coherent way, does not suffer scattering or phase decoherence by components of the crystalline lattice. This scenario describes, actually, BSC superconductivity.

To study the planar counterpart of this model, one performs its dimensional reduction to $1+2$ dimensions, which consists effectively in adopting the following ansatz over any 4-vector: (i) one keeps unaffected the temporal and also the first two spatial components; (ii) one freezes the third spatial dimension by splitting it from the body of the new 3-vector and requiring that the new quantities ($\chi$), defined in $1+2$ dimensions, do not depend on the third spatial dimension: $\partial_3 \chi \rightarrow 0$. It is also advisable to remark here that the prescription adopted for the dimensional reduction ensures automatically gauge invariance in $(1+2)$ dimensions. Actually, the scalar, $s$, associated to $A^{(3)}$ is a gauge-invariant scalar ($\partial_3 \alpha = 0$, $\alpha$ being the gauge parameter), where as $A^{(1)}$ ($\mu = 0, 1, 2$) is subject to the usual gauge $U(1)$ transformation with parameter $\alpha$. So, gauge invariance does match with our dimensional reduction procedure. Other possible dimensional reduction schemes could be eventually
adopted, such as spontaneous compactification of the third spatial component. However, this particular reduction procedure introduces non-zero massive modes which do not fit the phenomenology of excitations appearing in CMS. It brings about the so-called towers of massive modes and this is not pertinent with the physics of CMS. Actually, it seems that, if one wishes to build field-theoretic models to describe low-dimensional CMS, the procedure we choose to carry out here appears as the most suitable one. A Legendre-type reduction could also be an interesting proposal to produce models in lower dimensions. It avoids the non-zero massive models and it may give a new insight to the problem of the dependence on the third space direction.

Now, applying our adopted prescription to the gauge 4-vector, $A^\mu$, and the fixed external 4-vector, $v^\mu$, one has:

$$A^\mu \rightarrow (A^\nu; \varphi)$$

$$v^\mu \rightarrow (v^\mu; s)$$

where: $A^{(3)} = \varphi, v^{(3)} = s$ and $\nu = 0, 1, 2$, (note that all indices without hat have this range). According to this process, there appear two scalars: the scalar field, $\varphi$, that exhibits dynamics, and $s$, a constant scalar (without dynamics). Carrying out this prescription for eq. (1), one then obtains:

$$\mathcal{L}_{1+2} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{s}{2}\varepsilon_{\mu\nu\kappa}A^\mu \partial^\nu A^\kappa + \varphi \varepsilon_{\mu\nu\kappa}v^\mu \partial^\nu A^\kappa + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi + (D^\mu \varphi)^* D_\mu \varphi - \varepsilon^2 \varphi^2 \phi^* \phi$$

$$- V(\varphi^2) + A_\nu J^\nu - \varphi J,$$

The scalar field, $\varphi$, exhibits a typical Klein-Gordon massless dynamics behavior and it also appears as the parameter that couples $v^\mu$ to the gauge sector of the model by means of the Lorentz-violating mixing term: $\varphi \varepsilon_{\mu\nu\kappa}v^\mu \partial^\nu A^\kappa$. The presence of the Chern-Simons term in Lagrangian (1), here with mass $s$, will amount also to the breakdown of the parity and time-reversal symmetries, a phenomenon already observed in planar superconducting systems which behave as a two-dimensional Fermi liquid.

The mass dimensions of the fields and parameters contained in Lagrangian (1) are the following: $[\varphi] = [A^\mu] = [\phi] = 1/2, [s] = [v^\mu] = 1, [\varepsilon] = 1/2$. One thus notes that the dimensional reduction procedure changes the dimension of the fields and coupling constants defined in the (1+2) space-time in relation to their (1+3) counterparts. Indeed, while the four-dimensional U(1) coupling constant, $e_4$, is dimensionless, the tree-dimensional one exhibits a $[\text{mass}]^{1/2}$ dimension, a usual result for planar systems, ascribed to a possible effect remanent of the third spacial dimension. In this sense, such constant is taken as an effective coupling ($e_3 = e_4/\sqrt{l_\perp}$) by some authors, with $l_\perp$ standing for a characteristic length orthogonal to the plane. It is also instructive to discuss the possible role of the fields contained in Lagrangian (1) in the context of a CMS. The first two terms define the well-known Maxwell-Chern-Simons electrodynamics, which finds many applications in connection with topological defects, anyonic excitations, etc... The scalar field $\phi$ represents a Higgs sector responsible for the appearance of stable vortex configurations, as already analyzed in the context of both Abelian and non-Abelian scalar electrodynamics. In the absence of currents and with a vanishing $\varphi$-field ($\varphi = 0$), one obtains exactly the same planar Lagrangian which provides the existence of charged vortices and self-dual configurations in the absence of the Maxwell term. In this context, we should point out that the Lorentz-violating mixing term constitutes the key difference of our planar model and models already analyzed in literature. It is also of importance to discuss the physical role played by the fixed background, $v^\mu$, in this theoretical framework. It acts inducing both Lorentz violation and spatial anisotropy in the plane (in the case it is spacelike). Therefore, the fixed background may be seen as a physical element able to promote not only a

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1 In high-Tc superconductors, this effect may be associated with the coupling between parallel conduction planes, taken into account in some theoretical models.

2 Such anisotropy reflects the directional dependence of the solutions on the angle between this vector and the point where the solutions are being considered. The electron-electron interaction potential evaluated for the case of a purely spacelike background exhibits direct dependence on such angle, confirming this role of the background.
shift at the energy spectrum of the system, but also anisotropy and even changes in the physical behavior of the systems\(^3\).

The equations of motions obtained from (4) are:

\[
\partial_\mu F^{\mu \nu} = -s \varepsilon^{\mu \nu \rho} \partial_\nu A_\rho + \varepsilon^{\mu \nu \rho \sigma} v_\rho \partial_\sigma \phi + i e (\phi \partial^\mu \phi^* - \phi^* \partial^\mu \phi) + 2 e^2 \phi^* \phi A^\mu + J^\mu,
\]

(5)

\[
(\Box + M_A^2) \varphi = \varepsilon_{\mu \nu k} \partial^\nu A^k + J,
\]

(6)

\[
(D^\mu)^s D_\mu \phi = -e^2 \varphi^2 \phi - m^2 \phi - 2 \lambda |\phi|^2 \phi,
\]

(7)

where we have \(M_A^2 = 2 e^2 \phi^* \phi\). Note that, by the eq. (7) we have a increasing in the minimum of \(V(\phi^* \phi)\). This implies that we obtain a planar Landau-Ginzburg theory with critical field stronger that if we would start with a planar model.

We can extract from (5) the followings modified Maxwell equations without current:

\[
\partial_t \vec{E} + \vec{\nabla} B = -s \vec{E} - \left( \vec{\nabla} \varphi - v_0 \vec{\nabla} \phi \right) - i e (\phi \vec{\nabla} \phi^* - \phi^* \vec{\nabla} \phi) - 2 e^2 |\phi|^2 \vec{A},
\]

(8)

\[
\vec{\nabla} \cdot \vec{E} + \frac{s}{2} B = - \vec{\nabla} \times \vec{\nabla} \varphi - i e (\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi) - 2 e^2 |\phi|^2 A_0,
\]

(9)

where a tilde on the operators and fields stands for the dual in 2 space-dimensions, given as follows: \(\vec{\nabla}_i = \varepsilon_{ij} \nabla_j\), and \(\vec{E}_i = \varepsilon_{ij} E_j\).

With these equations, we can establish a discussion of vortex-like configuration in \((1 + 2)\) dimensions. In the next section, we shall obtain a charged vortex solution with \(A_0\) generating a screened electric field. We discuss the stability of the vortex and recover the traditional Nielsen-Olesen solution in \((1 + 2)\) dimensions.

### III. A DISCUSSION ON VORTEX-LIKE CONFIGURATIONS

Abrikosov\(^{20}\) showed that the Ginzburg-Landau theory admits vortex solutions. On the other hand, Nielsen and Olesen\(^{14}\) discovered that the relativistic Abelian Higgs model also presents vortex solutions while the Higgs field represents the superconducting order parameter. These vortex solutions are important in the Landau-Ginsburg model of superconductivity since the static energy functional for the relativistic Abelian-Higgs model coincides with the non-relativistic Landau-Ginzburg free energy in the description of type II superconductors. It is well-known that cuprate superconductors exhibit an enhanced effective mass anisotropy \((M_z \approx M_y \ll M_x = M_A)\). In other words, the charges are confined to planes, in which the vortices can build up a lattice and amount to the destruction of the coherence of the Cooper pairs and the planar superconducting current. Therefore, vortex configurations in high-Tc superconductors should be addressed by a planar model. Recently, it was also discovered the presence of charged fluxons in high-Tc superconductors\(^{25}\), which enforces the quest for an anisotropic planar theory able for yielding charged vortices.

To analyze the vortex-type solutions, we consider the charged scalar field in 2-dimensional space. Its asymptotic solution is proposed to be a circle \((S^1)\)

\[
\phi = a e^{i n \theta}; \quad (r \to \infty),
\]

(10)

where \(r\) and \(\theta\) are polar coordinates in the plane, \(a\) is a constant and \(n\) is an integer. In this limit we have \(M_A^2 = 2 e^2 a^2\).

We intend to study the asymptotic limit, and in this region the magnetic field is vanishing. Then, the eq. (10), considering that no current term is present, becomes:

\(^3\) In ref.\(^{12}\), it was verified that the Lorentz-violating background was able to change the long distance behavior of the electric field interaction in a Maxwell-Chern-Simons electrodynamics. Indeed, the screened scalar potential \((A_0)\) has been replaced by a logarithmic interaction in the presence of the background.
and we have a Klein-Gordon equation for the $A^{(3)}$ component. In the static regime, we have:

$$\left(\nabla^2 - M_A^2\right)\phi = 0$$

The solution is $\phi \simeq \frac{1}{r} \exp(-M_A r)$, and asymptotically, we have $\phi = 0$.

On the other hand, the gauge field takes the following asymptotic behavior:

$$A = e^{-i\theta}n \quad (r \to \infty),$$

or, in terms of its components:

$$A_r \to 0, \quad A_\theta \to -\frac{n}{er} \quad (r \to \infty).$$

The breaking of Lorentz covariance prevents us from setting $A_\mu$ as a pure gauge at infinity, as usually done for the Nielsen-Olesen vortices. This means that $A_0 = A_0(r)$, as $r \to \infty$. In this limit, we consider that $\phi \to 0$.

To avoid singularity for $r \to 0$, and to keep an asymptotic solution, we adopt the limits

$$\lim_{r \to 0} \chi(r) = 0 \quad \text{(14)}$$

and

$$\lim_{r \to \infty} \chi(r) = a. \quad \text{(15)}$$

Notice that, when the dimensional reduction is carried out, we obtain a new potential:

$$V(\phi^* \phi) = e^2 \phi^2 \phi^* \phi + V(\phi^* \phi),$$

but the analysis of stability sets the point $(\phi = 0, \chi = \sqrt{-\frac{m^2}{2\lambda}} = a)$ as a new minimum.

To search for vortices, we need to analyze the modified Maxwell equations in the static regime to obtain the corresponding differential equations, and then consider the asymptotic limit. Therefore, we can eliminate the terms that have time dependence in our differential equations [19]. Taking as starting point eq. (7),

$$(\partial_i - ieA_i) (\partial_i - ieA_i) \phi - m^2 \phi - 2\lambda^2 |\phi|^2 \phi = 0,$$

adopting the field parametrization [19], and summing over the components, it becomes

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{d\chi}{dr} \right) - \left( \frac{n}{r} - eA \right)^2 + (e^2 \phi^2 + m^2) + 2\lambda^2 + e^2 A_0^2 \chi = 0,$$

while the modified Maxwell equations [19] take on the form:

$$\nabla^2 A_0 - \nabla \times \nabla \phi - \frac{s}{2} B - 2e^2 \chi^2 A_0 = 0.$$
Also, in the asymptotic region, this equation takes is read as:

\[ \nabla^2 A_0 - M_A^2 A_0 = 0, \]

(19)
whose simple solution is \( A_0 \simeq \frac{1}{r} \exp(-M_A r) \). Considering now the \( \theta \)-component of eq. (5), it is written as follows:

\[ \frac{d}{dr} \left( \frac{1}{r} \frac{d}{dr} (r A) \right) - 2e\chi^2 \left( \frac{n}{r} + eA \right) - s \frac{dA_0}{dr} - \chi^0 \frac{d}{dr} \varphi = 0, \]

(20)
which asymptotically is reduced to the form below:

\[ \frac{d}{dr} \left( \frac{1}{r} \frac{d}{dr} (r A) \right) - 2ea^2 \left( \frac{n}{r} + eA \right) = 0. \]

(21)
We then recover the Nielsen-Olensen solution:

\[ A(r) = -\frac{n}{er} - \frac{c}{e} K_1 \left( \sqrt{2a} |e| r \right), \]

(22)
where \( c \) is a constant, and \( K_1 \) is the modified Bessel function.

We notice that, asymptotically, the complex scalar field \( \phi = \chi(r) e^{in\theta} \) goes to a non-trivial vacuum and becomes \( \phi = ae^{in\theta} \). Then, the topology of the vacuum manifold is \( S^1 \). We also remark that the asymptotic behavior of the fields \( \varphi \) and \( A_0 \) is dominated by a Yukawa-like solution, whose corresponding mass is given by \( M_A^2 = 2e^2a^2 \). We can trace the screening of these fields back to the spontaneous symmetry breaking mechanism.

Following the results of ref \[13\], which states the stability, causality and unitarity of the reduced model here adopted, we conclude that the stability of the vortex configuration is already assured. Furthermore, we can note that, whenever \( sB \neq 0 \), \( A_0 \) must necessarily be non-trivial, and an electric field appears along with the magnetic flux. If this is the situation, in the asymptotic region, \( A_0 \) falls off exponentially.

The appearance of an electrostatic field attached to the magnetic vortex, whenever \( sB \neq 0 \), is not surprising. Its origin may be traced back to the Lorentz-breaking term: indeed, being a Chern-Simons-like term, the electrostatic problem induces a magnetic field and the magnetostatic regime demands an electric field too. So, a non-vanishing \( A_0 \), and therefore a non-trivial \( E \) is a response to the Chern-Simons Lorentz-breaking term. The presence of a non-vanishing electrostatic potential points to a charged vortex. In the next section, we shall continue exploiting this feature of vortex configurations in a Lorentz breaking theory.

IV. ON THE AHARONOV-CASHER EFFECT

A question which becomes relevant in Quantum Mechanics is to understand how to attribute to a phase factor in the wave function effects that classically are trivial due to the vanishing of the force. The Aharonov-Bohm (AB) effect \[26\] is a remarkable example of how we can provide with a real meaning a quantity that could, apparently, appear as simple a mathematical artifact. In this effect, the electromagnetic vector potential induces a change in the wave function associated to an electron moving in a region where there is no magnetic field. In the work by Aharonov and Casher \[27\], we have a dual aspect of this effect. They predicted a phase shift in the wave function of a neutral particle with magnetic dipole moment \( \mu \), due to the action of an external electrostatic field:

\[ \Delta \Phi_{AC} = \frac{1}{\hbar c} \oint \hat{\mu} \times \vec{E} \cdot \vec{r}, \]

(23)
with \( E \) being the electric field applied to magnetic dipole due to a charged wire (for the sake of calculating the phase, we have restored the constants \( c \) and \( \hbar \), up to now taken both equal to 1 ). In the last section of ref. \[27\], it is shown the term \( \vec{V} \cdot \vec{E} \times \hat{\mu} \) that generates no force and is responsible for generating an A-B phase. In such
a work, taking a Maxwell-Higgs model with spontaneous symmetry breaking in \((1 + 2)\) dimensions, the vortex solution is presented, but the charged wire is put in by hand. Our model \(\Phi\), by virtue of the Lorentz-breaking Chern-Simons term, has a richer structure for the vortices, and an electrostatic potential \(A^0 \neq 0\) is a natural solution from the equations of motion. Then, the effect of the charged wire in the work of the ref. \([27]\) is, in our proposal, replaced by the non-neutral vortex that is naturally formed as a response to the vector background that breaks Lorentz symmetry by means of a Chern-Simons type action term.

The vortex charge is viewed as a source and the electric field generated seems to be screened by the environment \(\left( A_0 \simeq \frac{1}{r} \exp(-M_A r) \right)\). Charged vortices in high-\(T_c\) superconductors are proposed in \([28]\) as consequence of the variation of the chemical potential in the phase transition. The superconductor fluid screens all the moment of the electric field, but does not screens the topological effect \((23)\). Our purpose is to establish a phase shift from the interaction between two vortices. Lets suppose a planar circular superconductor with two vortices: one pinned at the center and another circulating around. To obtain the corresponding effect in this case, we can identify \(\tilde{\mu} \simeq \Phi_0 \hat{z}\); in so doing, the analogous expression for the phase of the circulating vortex is given as below:

\[
\vec{V} \cdot \vec{E} \times \Phi_0 \hat{z}, \tag{24}
\]

where \(\vec{V}\) is the vortex velocity, \(\vec{E}\) is the electric field, \(\hat{z}\) is the unit vector perpendicular to the plane, and \(\Phi_0 = \frac{h}{q}\) is called a flux quantum. As it is well known, a high-\(T_c\) superconductor may support quantized localized magnetic fluxons with a flux \(\Phi_0\) in a fluid environment with unit charge \(q = 2e\). The idea of a circular geometry for superconductors was first explored in ref. \([29]\). The proposal of introducing a charge potential vector that yields a phase shift (in the same sense that a normal potential vector creates the A-B phase) was suggested by van Wess in the work of ref. \([30]\). This charge potential vector can also produce a persistent voltage in a ring made up of arrays of Josephson junctions. The dynamics of a single vortex present in a ring-shaped (Corbino geometry) two-dimensional array of low-capacity Josephson junction is analyzed. The vortex is seen as a macroscopic quantum particle, whose energy levels \(E_n(Q_0)\) are periodic functions of the externally induced gauge charge \(Q_0\) which is enclosed by the vortex, with a period \(2e\). The quantization of the voltage generated by ballistic vortices with a mass \(m_v\) in a two-dimensional superconductor ring is assessed in \([31]\). The experimental observation of this quantum interference is verified in \([32]\) and it enforces the relevance of the study of charged-vortex interaction. In our case, the configuration of the gauge field which describes the vortex is steady. A possible connection amongst charged vortex, A-C effect and planar superconductor will be pointed out in our next section.

V. CONCLUDING COMMENTS AND PERSPECTIVES

In this work, we have studied a planar Abelian gauge models attained as a by-product of a 4-dimensional theory under a dimensional reduction process. This procedure has provided reasonable results under both the theoretical and phenomenological points of view, with the advantage that some specific 3-dimensional aspects may be understood as a manifestation of mechanisms or characteristics of the 4-dimensional system. In this sense, for instance, the topological mass parameter \((s)\) of the planar system is nothing but the axial component of the 4-dimensional vector that breaks Lorentz and CPT symmetries. In other words: topologically massive planar gauge theories may be justified as an inheritance from the Carroll-Field-Jackiw Lorentz-violating gauge theory in \((1+3)\)-D.

An interesting feature considered here is the appearance of electrically charged magnetic vortices in the set-up of a planar gauge theory for which the mass parameter \(s\) is non-vanishing. Indeed, in the framework of this model there appear charges vortex configurations that recover the Nielsen-Olesen structure in the asymptotic limit. Once vortex configurations are associated with a intrinsic electric field, it settles down the full scenario for the study of Aharonov-Casher effect. It was then shown that an A-Casher phase is induced whenever a spin-\(1/2\) neutral particle move around the vortex. The point is that our vortex configurations need not be supplemented by a thin charged wire in order to incorporate the effect of a charge. The screening of the field
configurations and the net charge of the vortex come naturally out as an inheritance of Maxwell-Chern-Simons sector generated from the Carroll-Field-Jackiw term. As an example of possible properties of charged vortices in high-Tc superconductors, we refer to the $H_{g0.82Re0.18Ba2Ca2Cu3O6+\delta}$. We are interested in investigating the properties of the planar charged vortices before the phase transition to superconductivity in ceramic superconductors: the so-called pseudo gap regime. The phase transition diagram in high Tc-superconductors is quite complex. First of all, we can say that there occurs the formation of puddles’s superfluid below the $T^*$ (temperature of pseudo-gap transition). When the temperature decreases, the frontiers of the puddles grow and percolate. At this point, it settles down the formation of the superconducting planes, and we can treat the system in the regime of a macroscopic planar physics. The 2D character of the fluctuation spectrum in this temperature region, which extends down to (T-Tc)/Tc of the order of 0.01, was justified by the strong planar anisotropy that characterizes the Hg,Re-1223 high-Tc superconductors.

In this environment, below $T^*$ and down to $T_c$, the gap’s screening is not effective because we do not have a stabilized percolative superfluid. The expression for the electric field generated by the charged vortex is:

$$\vec{E} = \left( \frac{1}{r^2} + \frac{MA}{r} \right) \exp(-MAr)\hat{r},$$

and we have the Aharonov-Casher term promoting interaction among vortices. If we take consider electrons moving in the region close to the core of the vortex, which amounts to taking into account the approximation $\exp(-MAr) \simeq 1 - MAr$, we can compute the stable phase factor:

$$\Delta\Phi_{AC} = \frac{\Phi_0/\hbar c^2}{2\pi} \frac{2\pi}{d},$$

where $d$ stands for the distance of the electron to the center of the vortex. Another interesting scenario for the application appears when the superconducting regime is established. The high-Tc superconductors are built by planes containing Cu and O, which are sandwiched between two planes: one containing Ba and O and another containing Hg and O. The outer planes (Ba-O and Hg-O) present isolant properties and the Cu-O planes are conducting ones. Linear combinations of Cu and O atomic orbitals, called "hybrid orbitals", are used to form covalent bindings that hold those atoms together. For these high-Tc superconductors, the conducting planes are formed by $3d(x^2-y^2)-2p$ hybrid orbitals of Cu and O. The c-axis presents a different behavior, which can be compared with semiconducting materials (the resistance is not linear as a function of the temperature). According to Allen et al., the fundamental physics of the oxide superconductors is contained in the Hubbard Hamiltonian on a two-dimensional square lattice for small numbers of holes. In this kind of model, the superconducting planes interact by charge tunneling through the z axis, which can be interpreted as a strong anisotropy or low z-axis charge mobility. In this situation, the z-axis charge can be modeled as a static charge, and the plane charge as a moving charge. If we consider the z-direction formed by hybridized $3d(x^2)\cdot 2p$ bound as a charged wire with linear densities, $\lambda$, the vortices can present a stable phase upon the circulation around this wire given by:

$$\Delta\Phi_{AC} = \frac{\Phi_0/\hbar c^2}{4\pi\lambda}.$$ With these results, we have presented possible effects that charged vortices could suffer in planar superconductivity, when it the phase transition region $T^*$ down to $T_c$ was considered.

In a recent work, it has been shown that a non-minimal coupling of fermions to a Lorentz-violating background induces an Aharonov-Casher phase. It is known that spinless neutral particles may develop a magnetic dipole moment whenever non-minimally coupled to an electromagnetic field. If this is the case, it would be worthwhile to explicitly derive the Aharonov-Casher phase of a spinless neutral particle placed in the electric field of a charged wire. We argue that the Lorentz breaking in 4 D may induce the magnetic moment for spinless particles and then, the reduced 3-dimensional system may indicate how the scalar particle may pick up the Aharonov-Casher phase by means of its induced magnetic moment. This issue is under investigation.

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