The Bose-Einstein effect in Monte Carlo generators: weight methods

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Abstract

We present a method which incorporates the Bose - Einstein effect into Monte Carlo generators for multiple production by weighting the events. Various aspects of weight calculations are discussed in detail. We show that our method allows to describe reasonably well a sample of data and we outline the future tests and applications.

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1 Introduction

Recently there was much activity on the subject of the second order interference effects in multiple production due to the Bose - Einstein statistics for pions \cite{1}. In particular, many authors have stressed the importance of incorporating these effects into Monte Carlo generators which well describe other features of multiple production processes. It seems most natural to implement the Bose-Einstein (BE) effect into Monte Carlo generators at the level of constructing the matrix element for the generation of events. At present, however, this has been done only for a single Lund string \cite{2} and for many models such an implementation seems to be difficult. Therefore the standard procedure is to generate first the events according to the existing Monte Carlo programs (without the BE effect), and then modify them. The most popular method \cite{3} applies the momentum shifts for final state particles to reproduce the experimental two-body "Bose-Einstein ratio"

\[ R_2(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)}. \] (1)

This method should be regarded as an imitation rather than an implementation of the effect, as it has no theoretical justification. The method has many other deficiencies, although some of them are recently claimed to be removed \cite{4}.

Another way of implementing the effect is to attach to the generated events different weights, which depend on the momentum configuration, and thus to reconstruct the necessary enhancement in Bose-Einstein ratios. This method has a better theoretical justification. Following Pratt \cite{5}, Bialas and Krzywicki \cite{6} have presented recently a derivation of the formula for weights based on the approximation for Wigner functions\footnote{A different method of implementing the BE effect using Wigner functions (see \cite{7} and references quoted therein) requires a Monte Carlo generator which defines for final state particles both momenta and coordinates of the generation (or the last interaction) points in space. Therefore it seems to be reliable only for heavy ion collisions. We will not discuss it in this paper.}.

This formula reads:

\[ W(n) = \sum_{\{P_n\}} \prod_{i=1}^{n} w_{i,k_i}. \] (2)

Here \( n \) is the number of identical particles, \( k_i = P_n(i), w_{i,k_i} \) is a two particle weight factor calculated for the pair of momenta (of the \( i-th \) particle and the particle which occupies the \( i-th \) place in the permutation \( P_n \)). The sum extends over all the permutations of \( n \) elements. In fact, such prescription should be applied separately for each sign of identical pions, and the full weight of the event is a product of three weight factors (one for each sign). The two-particle weight factor \( w_{i,k} \) should be parametrized in a way reflecting the assumed space-time structure of the source. For pions coming from the decay of long-living resonances the effective source size is very large, and the effects appear for unmeasurably small differences of momenta. Thus only "direct" pions and the decay products of wide resonances for each event should be counted.

The main difficulty with formula (2) is the factorial increase of the number of terms in the sum with increasing multiplicity of identical pions \( n \) (other problems will be discussed later on). For high energies, when \( n \) often exceeds 20, a straightforward application of
formula (2) is impractical, and some authors [8,9] replaced it with simpler expressions, motivated by some models. It is, however, rather difficult to estimate their reliability.

We have recently proposed two ways of dealing with this problem. One method consists of a truncation of the sum up to terms, for which the permutation \( P(i) \) moves no more than 5 particles from their places [10]. This has a simple justification: for \( P(i) = i \) a two-particle weight factor \( w_{i,P(i)} \) is equal 1, and for non-equal indices it should decrease fast (usually Gaussian shape is assumed) with increasing difference between the momenta of the \( i \)-th and the \( P(i) \)-th particle. Thus the magnitude of terms, for which more than \( k \) factors in the product are different from 1, should decrease quickly with \( k \). We have checked for the \( p\bar{p} \) minimum bias events at 630 GeV [10] and for the \( e^+e^- \rightarrow W^+W^- \) events at LEP II [11] that changing the maximal value of \( k \) from 4 to 5 hardly influences the distributions, and in particular the two-particle ratio (1). However, it is difficult to claim a priori that such a truncation does not change the results which would be obtained using the full series (2).

Therefore a second way of an approximate calculation of the sum (2) was proposed [12]. Since this sum, called a permanent of a matrix built from weight factors \( w_{i,k} \), is quite familiar in field theory, one may use a known integral representation and approximate the integral by the saddle point method. This approximation improves in fact with increasing number of particles \( n \). However, this method is reliable only if in each row (and column) of the matrix there is at least one non-diagonal element significantly different from zero (as already noted, all diagonal elements are equal to 1). Thus the prescription should not be applied to the full events, but to the clusters, in which each momentum is not far from at least one other momentum. The full weight is then a product of weights calculated for clusters, in which the full event is divided.

These considerations suggest the necessity of combining two methods. After dividing the final state momenta of identical particles into clusters, one should use for small clusters exact formulae presented in [10,11]. For large clusters (with more than five particles) one should compare two approximations (truncated series and the integral representation) to estimate their reliability and the sensitivity of the final results to the method. Obviously, the results will depend also on the clustering algorithm used: if we restrict each cluster to particles very close in momentum space, the neglected contributions to the sum (2) from permutations exchanging pions from different clusters may be non-negligible, and if the cluster definition is very loose, the saddle point approximation may be unreliable. This should be then also checked to optimize the algorithm used.

In this paper we investigate and compare various approximations to formula (2) for the events generated by default PYTHIA/JETSET generator [13] for the proton-antiproton collisions at 630 GeV (simulating conditions of the UA1 experiment [14], for which many data on the BE effect were collected). In the next section we define used procedures and compare the weight distributions obtained with different assumptions. In Section 3 the problem of weight rescaling (necessary to reproduce the experimental multiplicity distributions) is discussed and some comparison with data is presented. We conclude with Section 4.
2 Event generation and the procedures calculating weights

As already noted, we have chosen for our analysis the proton-antiproton collisions at CM energy of 630GeV. We generate minimum bias events according to the default PYTHIA-JETSET generator [13]. The imitation of the Bose-Einstein effect by momenta shifting is switched off. For the trial runs, samples of 10 000 events were generated.

For each event the weight was calculated according to different procedures and parameter choices. In all cases we counted only the direct pions and the decay products of wide resonances (dominated by $\rho$). This is easily done if the procedures which calculate weights are called not after the generation of final state, but in the same place, in which the original LUBOEI procedure was called [3].

The first procedure, referred further to as "no-clustering with restricted permutations" is the same as applied before to the rough description of UA1 data [10] and to the discussion of W mass shifts [11]. As already noted, it consists of separating in the sum (2) for each sign of pions into the classes of permutations which change places of exactly k momenta,

$$W(n) = \sum_k W^{(k)}(n),$$  

and neglecting all terms with $k > k_{\text{max}}$. As previously, we used $k_{\text{max}} = 5$. The detailed formulae used to calculate this approximation were given before [10,11]. The full weight of the event is the product of three weight factors calculated for pions of each sign.

The shape of the two-particle weight factor $w_{i,j}$ in (2) should be chosen to fit the "BE ratio". For practical reasons we do not use the function $R_2(p_1, p_2)$ but the ratio of integrals

$$c_2(Q) = \frac{\int d^3p_1d^3p_2\rho_2(p_1, p_2)\delta[Q - \sqrt{-(p_1 - p_2)^2}] < n >}{\int d^3p_1d^3p_2\rho_1(p_1)p_1(p_2)\delta[Q - \sqrt{-(p_1 - p_2)^2}] < n(n-1) >},$$  

which is a function of a single variable $Q = \sqrt{-(p_1 - p_2)^2}$. Motivated by many experimental fits we parametrize $w_{i,j}$ as

$$w_{i,j} = e^{(p_i - p_j)^2/2\sigma^2}$$  

Of course, different components of momentum difference squared may be multiplied by different coefficients, and the shape may be modified. In this note we do not discuss these possibilities. Therefore the only parameter is a Gaussian half-width of the distribution $\sigma$. As noted in [10] the values of $\sigma$ of 0.1-0.2 GeV reproduce quite well the experimental width of the Bose-Einstein enhancement, and we will use the values from this range.

In two other procedures we separate first the momenta of pions of each sign into clusters. The pion is assigned to a given cluster, if the (minus) square of the difference of its four-momentum with at least one pion from this cluster is smaller than the assumed value of a parameter $\epsilon$. Then, for each cluster, a cluster weight factor is calculated by the formula (2) using one of two approximations to be described below, and the final weight
factor for each sign is a product of all cluster weight factors. Obviously, if there is only one pion in the cluster, its weight factor equals one. Note that the results may depend strongly on the ratio $\varepsilon/\sigma^2$. If this ratio is too small, the neglected terms in (2) (corresponding to the permutations, which exchange pions from different clusters) may be quite big. If it is large, the clusters will contain on the average many particles and the computing time will increase. Moreover, the matrix built out of two-particle weight factors for a cluster may have zero modes, and the saddle point approximation [12] becomes unreliable. Thus the results for different values of $\varepsilon$ should be compared.

If the number of pions in a cluster is smaller than six, the sum (2) is calculated exactly with the formulae from refs. [10,11]. For (rather few) larger clusters two alternative approaches are used. In the first version, referred to as ”clusters with restricted permutations”, the same approximation to the sum (2) as described above is applied. Note that now, for the events containing many clusters with more than one pion, this takes into account permutations, which may change places of many more than five momenta. However, the global number of used permutations is usually lower than for this approximation applied to the full final state. In the second version, referred to as ”clusters with the saddle point method” the method from ref. [12] is used.

We have started with the parameter values of $\varepsilon = 0.1 \, GeV^2$ and $0.15 \, GeV^2$, and $\sigma = 0.17 \, GeV$. The first observation is that the original ”no clustering” algorithm needs much more computing time (here by an order of magnitude) than the two others. Moreover both algorithms with restricted permutations approximate the full weight factor for each sign from below, since in both cases positive terms in the full sum (2) are neglected. Thus it is evident that the approximation which gives bigger weights is better. We find the average value (taken only for weight factors below 100) of 1.84 without clustering, and 1.88 or 1.90 for ”clusters with restricted permutations” with the two values of $\varepsilon$ quoted above. Even though these differences seem small, we feel that with our statistics they are significant. Moreover, without clustering there are only five cases (out of 30 000) of weight factors above 100, and with clustering there are 13 or 14 such cases. We see that the procedure with clustering gives slightly larger values of weights, which means that it approximates the full sum better than the other one, even though it neglects more terms and uses much less computer time. In the following we will thus discuss only procedures with clustering.

The comparison of weight factors calculated with these two procedures gives less unambiguous results. The average values of weight factors are bigger for ”restricted permutations”, but the number of cases of values above 100 is bigger for ”saddle point method”. More precisely, the average weight factor for the saddle point method is 1.82 or 1.83, and the number of values above 100 is 19 or 21 (to be compared with numbers quoted above for ”restricted permutations”). However, analyzing in more detail the events with anomalously high weight values, and adding the results for smaller (and more realistic) value of $\sigma = 0.14 \, GeV$ we find that the saddle point method gives larger values of weights only in these cases, when they are anyway very high. For smaller $\sigma$ for both procedures only in 3 cases (out of 30 000) the weight factor exceeds 100, and the averages are 1.35 and 1.33 for the two procedures. Thus the saddle point algorithm gives slightly better approximation of weight factor only for anomalous cases, which are not very relevant for the analysis, as will be discussed later.
We conclude that the main improvement consists of introducing clustering into the analysis. With this modification both algorithms, i.e. "restricted permutations" and "saddle point approximation", have similar performance. Restricted permutations gives slightly better average while the saddle point method is advantageous at reproducing large weights. The situation may change for the processes with different density of particles in phase-space, or after improving the approximations used in the saddle point method. In the following we use the restricted permutation algorithm.

Since all factors in the sum (2) are positive and \( w_{i,i} = 1 \), the resulting weight is not smaller than one (a contribution from identity permutation). One may rescale the weights to keep, e.g., the average number of particles fixed; we return to this point later.

As already noted, we are using the final weight of the event given by a product of weight factors calculated separately for positive, negative and neutral pions. In fact, the BE interference for neutral particles is not observable (apart from the possible effects for direct photons [15]): neutral pions decay before detection, and for the resulting photons the effective source size is so big that the BE effects must be negligible for momentum differences above a few eV. However, the procedure should not change the observable correlations between the numbers of charged and neutral pions. Therefore weights for all signs of pions must be really taken into account.

3 Results and comparison with data

Before using our prescription for weights to calculate the BE ratio (4) let us note that weights do change not only this ratio (which for the default JETSET/PYTHIA generator is always close to one in clear disagreement with data) but also many other distributions. Thus with the default values of free model parameters (fitted to the data without weights) we find inevitably some discrepancies with data after introducing weights.

We want to make clear that this cannot be taken as a flaw of the weight method. There is no measurable world "without the BE effect", and it makes no sense to ask, if this effect changes e.g. the multiplicity distributions. If any model is compared to the data without taking the BE effect into account, the fitted values of its free parameters are simply not correct. They should be refitted with weights, and then the weights recalculated in an iterative procedure. This, however, may be a rather tedious task.

Therefore we use, as in the previous papers [10,11] a simple rescaling method proposed by Jadach and Zalewski [8]. Instead of refitting the free parameters of the MC generator, we rescale the BE weights (calculated according to the procedure outlined above) with a simple factor \( cV^n \), where \( n \) is the global multiplicity of "direct" pions, and \( c \) and \( V \) are fit parameters. Their values are fitted to minimize the differences between the multiplicity distribution obtained from MC without weights, and the one obtained with the rescaled weights. As noted before [10], one may even avoid fitting if the distributions without- and with weights (non-rescaled) are well parametrized with the negative binominal distributions (NBD) with parameters \( < n >, k \) and \( < n' >, k' \), respectively. In our case, however, the distribution with weights becomes rather jagged for large multiplicities due to the few events with anomalously high weight values. Therefore the values of NBD parameters would be unreliable and direct fitting of \( c \) and \( V \) is necessary.
The results, however, are fortunately rather insensitive to the "anomalous" cases. Even if we remove them from the fitting procedure and then include again for rescaling, the rescaled weight distribution exhibits an "anomalous" tail greatly reduced. Out of 300 000 values of weight factors only 5 exceed 100. Moreover, the resulting multiplicity distribution with rescaled weights is quite smooth and approximates well the original distribution. Specifically, with ε and σ values as quoted above, the average multiplicity and dispersion do not change more than by a few percent. This is illustrated in Fig.1 for still smaller (but realistic) value σ = 0.1 GeV and ε = 0.1 GeV². It should be contrasted with the 20% increase of the average multiplicity which occurs when weights are introduced without rescaling (not shown).

![Figure 1](image_url)

**Fig.1.** The multiplicity distribution (in number of events) without weights (diamonds) and with rescaled weights (crosses).

Moreover, the rescaling in n gives also satisfactory corrections to the inclusive distributions in transverse- and longitudinal momenta (or Feynman x). Whereas the unrescaled weights shift the average value of ln(1/x) by about 0.7, rescaling restores the original value with the accuracy of a few percent. The corresponding distributions of ln(1/x) are shown in Fig.2.
Fig. 2. The distribution of $\ln(1/x)$ (in thousands of pions) without weights (diamonds) and with rescaled weights (crosses).

Similar effect is seen for transverse momenta, as shown in Fig. 3.

Fig. 3. The transverse momentum distribution (in thousands of pions) without weights (diamonds) and with rescaled weights (crosses).
Thus it seems to be sufficient to rescale weights with respect to a single quantity \( n \); the single-particle distributions recover then automatically their original shape. Let us note that, as we have already checked before [11], the shape of BE ratio (4) is practically the same with- and without rescaling. This suggests that the rescaling was indeed well chosen and corresponds to the refitting of such parameters, which do not influence the BE ratio.

Obviously, for the more detailed analysis of the final states, single rescaling may be not enough. E.g., since different parameters govern the average number of jets, and the average multiplicity of a single jet, both should be rescaled separately to avoid discrepancy with the data. Let us stress once again, however, that such problems arise only due to the use of generators with improperly fitted free parameters, and do not suggest any flaw of the weight method.

Now we may check if the new version of our procedure yields the same Bose - Enstein ratio (4) as the old one. In Fig.4 we show that within the statistical fluctuations this is indeed the case.

![Graph](image)

**Fig.4.** The BE ratio (4) as a function of \( \log_2(Q^2/1\text{GeV}^2) \). Crosses, squares and diamonds correspond to the results with weights from the new procedure, from the old one, and without weights, respectively.

Since the data of UA1 shown in Ref. [10] are bracketed by the results of the old procedure with the values of \( \sigma \) parameter equal 0.1 GeV and 0.14 GeV, we conclude that our new procedure can describe the data as well. Obviously, the results are meaningful only for \( \log_2(Q^2/1\text{GeV}^2) > -8 \) \( (Q^2 > 0.004\text{GeV}^2) \). For smaller \( Q \) the statistical fluctuations (visualized by the spread of points) are too large to draw any conclusions. As already mentioned, the default version of PYTHIA/JETSET Monte Carlo (without weights) yields a rather flat distribution at the level close to one.
4 Summary and conclusions

We have presented a new version of the weight method implementing the BE effect into Monte Carlo generators of multiple production, which uses a clustering procedure for the final state prompt pions. It provides a better approximation to the (rather impractical) full formula (2) than the previous method, although it uses considerably less computer time. The shape of the BE ratio is practically the same for various variants of the new method and for the old one. This suggests the stability of the results, and in particular their insensitivity to the internal parameter $\epsilon$ of the clustering algorithm. We discuss also how the weights influence other distributions. We show that a simple rescaling of the weights, introduced to restore the shape of the multiplicity distribution, reproduces at the same time the original inclusive momentum distributions.

The satisfactory description of two-particle BE ratio in proton-proton collisions at 630 GeV should be obviously just a first step in testing the applicability of our method. In particular, one should check if the data for other energies and colliding particles may be described as well using the simple form of two-particle weight factor with only one free parameter. One should also investigate the three-particle effect and semi-inclusive data (e.g. in bins of restricted multiplicity and transverse energy). The BE effect in more than one variable (distinguishing the transverse-, longitudinal- and time-like dimensions of the source) may be analyzed by introducing two-particle weight factor which depends on more variables. Finally, less simplistic distinction of direct pions and decay products of various resonances would be welcome.

It is particularly interesting if the new algorithm may be applied as well to the heavy ion collisions (with very high multiplicities $n$) and how the results will compare with the description obtained with other methods [7]. Since the computer time needed for clustering grows only as $n^2$, and the number of clusters is proportional to $n$, whereas our previous method [10] required a calculation of $n^5$ terms, we may now expect to deal with nuclear data in a reasonable computer time. All the applications listed above are under investigation, and for some of them encouraging preliminary results are obtained.

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