Thermal properties of $\pi$ and $\rho$ meson

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We computed the pole masses and decay constants of $\pi$ and $\rho$ meson at finite temperature in the framework of Dyson-Schwinger equations and Bethe-Salpeter equations approach. Below transition temperature, pion pole mass increases monotonously, while $\rho$ meson seems to be temperature independent. Above transition temperature, pion mass approaches the free field limit, whereas $\rho$ meson is about twice as large as that limit. Pion and the longitudinal projection of $\rho$ meson decay constants have similar behaviour as the order parameter of chiral symmetry, whereas the transverse projection of $\rho$ meson decay constant rises monotonously as temperature increases. The inflection point of decay constant and the chiral susceptibility get the same phase transition temperature. Though there is no access to the thermal width of mesons within this scheme, it is discussed by analyzing the Gell-Mann-Oakes-Renner (GMOR) relation in medium. These thermal properties of hadron observables will help us understand the QCD phases at finite temperature and can be employed to improve the experimental data analysis and heavy ion collision simulations.

I. INTRODUCTION

QCD phase structure at finite temperature is with great interest of investigation both theoretically and experimentally. The investigations will lead to a thorough understanding of the matter formation and the universe evolution. From both theoretical and experimental studies, the existence of the phase transition, crossover specifically, at finite temperature has been confirmed. At low temperature, the QCD matter could be well described by the hadron resonance gas, while it gradually becomes quark gluon plasma at high temperature [1–15]. As the temperature changes, the thermal mass of hadrons will shift, the thermal width will usually get larger and the decay of hadrons will then change. In heavy quark sector, the production and dissociation of quarkonium can be regarded as the signal of the existence of quark gluon plasma, and thus settling down the thermal mass and decay of quarkonium is important for heavy ion collisions simulations [16]. On the other hand, the chiral symmetry breaking or restoration in hot medium can be described by the thermal mass of the light meson and its relevant properties. The appearance of a turning point in the temperature dependence of thermal mass will give explanation to the occurrence of the chiral symmetry transition. Additionally, the deviation of the light scalar resonance thermal mass from the vacuum is relevant to the location of freeze out temperature [17]. Therefore, it is with both desires and difficulties to illustrate the thermal properties directly within hadron observables.

The thermal hadron mass could be separated into the screening mass and pole mass owing to the $O(4)$ symmetry breaking at finite temperature. Screening mass, defined by the large distance behavior of hadron correlation function, is relatively easy to compute. Studies on the light meson screening mass have been carried out with lattice QCD [18–21] (see e.g. [22] for an overview) and the functional approach [23–25]. It has been predicted that the high temperature limit of meson screening mass is $M_{scr} \sim 2\pi T$, as it expected to be the propagation of the thermal quark [26, 27]. Nevertheless, the relation between the screening mass and phenomenologically relevant observables is not clear, and hence, it is difficult but important to study the temperature dependence of the pole mass of hadrons. Though the computation of pole mass in lattice QCD simulation usually encounters the complicated temperature connection between the spectral function and the kernel in temporal correlation functions, there are some pioneering results for pole masses of baryons [28–30].

In the present work we employ the Dyson-Schwinger equations (DSEs) and Bethe-Salpeter equations (BSEs) in imaginary time formalism with Matsubara frequency to study the in medium properties of $\pi$ and $\rho$ meson, which essentially characterize the dynamical chiral symmetry breaking mechanism nonperturbatively. Even though the meson is not on shell on the Matsubara frequency, the eigenvalues of BSEs on the Matsubara frequency could be employed to extract the pole mass and decay constants. In employing the DSEs approach herein, we apply the quark gluon interaction as the one which can reproduce hadronic static properties at zero temperature. An extension of this interaction is assumed to be applicable to studies at finite temperature. Within this scheme, we then obtain the pole masses and decay constants for $\pi$ and $\rho$ meson in a large range of temperature. Moreover, by including the computation of the chiral condensate, we also have the opportunity to verify the GMOR relation in medium. Though the method is not sophisticated enough to extract the thermal width of meson since it is related to the imaginary part of the Bethe-Salpeter amplitudes which can not be directly obtained in the imaginary time formalism, it can be argued that the finite thermal width would cause the deviation

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of GMOR relation at finite temperature, which is then analyzed in this work.

The remainder of this paper is organized as follows. In Sec. II we reiterate briefly the DSEs and BSEs approach at finite temperature. We highlight in this section how the pole mass is extracted from the BSEs in imaginary time formalism with Matsubara frequency. Sec. III contains our results of the temperature dependence of pole masses and decay constants of $\pi$ and $\rho$ meson, as well as the discussion on the GMOR relation at finite temperature. Finally, we summarize in Sec.IV.

II. DSE-BSE SCHEME AT FINITE TEMPERATURE

A. Dyson-Schwinger Equations at finite temperature

At finite temperature the quark propagator can be written as [8]

$$S^{-1}(p) = i\gamma \cdot \vec{p}A(p) + i\gamma_0 p_0 C(p) + B(p),$$

where $p = (\vec{p}, \omega_n)$ with $\omega_n = (2n + 1)\pi T$ the fermion Matsubara frequency. The quark propagator satisfies the Dyson-Schwinger equation as

$$S^{-1}(p) = Z_2^{-1}i\gamma \cdot \vec{p} + Z_2^0 i\gamma_0 p_0 + Z_4 m_0 - Z_1 \Sigma(p),$$

$$\Sigma(p) = T \sum_n \int \frac{d^3q}{(2\pi)^3} g^2 D_{\mu\nu}(p - q; T)\times \lambda^a_\mu \gamma^a_\nu S(q) \frac{\lambda^a_\nu}{2} \Gamma_{\nu},$$

where $m_0$ is the current quark mass; $q = (\vec{q}, \omega_n); D_{\mu\nu}$ the gluon propagator; $\Gamma_{\nu}$, the quark-gluon vertex; $Z_1(\zeta)$ and $Z_4(\zeta)$ respectively, the vertex and mass renormalisation constants; $\zeta$ the renormalisation point; $Z_2(\zeta)$, respectively, the spatial and temporal quark wave function renormalisation constants. With this, the quark condensate can be defined as

$$\Delta_q \simeq -m_0^2 T \sum_{n \in \mathbb{Z}} \int \frac{d^3q}{(2\pi)^3} \text{tr} \ S_{q,q}(q),$$

with $q_i = u, d, s$, three light quark fields. The renormalized light quark condensate comprises the thermal part of the chiral condensate. In particular, the renormalized light quark condensate is given by [9, 31–33]

$$\Delta_{q_i} R = \frac{1}{2 N} \sum_{q_i = u,d} \left[ \Delta_{q_i}(T) - \Delta_{q_i}(0) \right].$$

The quark DSE can be solved under a certain truncation of quark-gluon vertex. The rainbow-ladder truncation is the first systematic, symmetry-preserving DSE truncation scheme which is accurate for ground-state vector- and isospin-nonzero-pseudoscalar-mesons owing to the corrections of these channels cancel via the Ward-Takahashi identities [34–36]. Therefore, here we focus on computing the properties of $\pi$ and $\rho$ meson with the rainbow-ladder truncation. The truncation scheme employs the tree level quark-gluon vertex with modeling the interaction kernel introduced in Ref. [37], $D_{\mu\nu}(s) = P_{\mu\nu}G(s)$:

$$G(s) = \frac{8 \pi^2}{\omega^4} \frac{D e^{-s/\omega^2}}{\ln[\tau + (1 + s/\Lambda_{QCD}^2)]},$$

where: $P_{\mu\nu} = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}; \gamma_m = 12/(33 - 2N_f), N_f = 4, \Lambda_{QCD} = 0.234 \text{GeV}; \tau = e^2 - 1$; and $\mathcal{F}(s) = [1 - \exp(-s/[4m_t^2])]/s, m_t = 0.5 \text{GeV}$. The interaction kernel involves a massive gluon propagator on the domain at $s = 0$, which is consistent with that determined in recent studies of QCD’s gauge sector [38–50]. At finite temperature, the gluon propagator separates into longitudinal and transverse modes due to the breaking of $O(4)$ symmetry, i.e., the dimension corresponding to temperature will be isolated in order to allow the introduction of $O(4)$ symmetry breaking [8]. This requirement can be considered as an improvement in further studies, and given the difficulties of computational complexity, it will nevertheless allow us to apply the gluon propagator in Eq.(5) as the first step to exploit hadron properties at finite temperature.

B. Bethe-Salpeter Equation in imaginary time formula

The practical way of computing BSEs for mesons at finite temperature is through the imaginary time formula which is simply to change the fourth component of all the momentum in Euclidean space to Matsubara frequency [23–25]. Applying the rainbow-ladder truncation, the homogeneous BSE at finite temperature can be described as:

$$\lambda(\Omega_m^2, \vec{P}^2) \Gamma_{\pi,\rho}(k; P) = T \sum_n \int \frac{d^3q}{(2\pi)^3} g^2 D_{\mu\nu}(k - q; T)\times \gamma_\mu \chi^{ab}_{\pi,\rho}(q; P) \gamma_\nu,$$

where

$$\chi^{ab}_{\pi,\rho}(q; P) := S^a(\omega_n + \Omega_m, \vec{q} + \vec{P}) \Gamma^{ab}_{\pi,\rho}(q, P) S^b(\omega_n, \vec{q})$$

and $P = (\Omega_m, \vec{P})$ with $\Omega_m = 2m_\pi T$. $\lambda(\Omega_m^2, \vec{P}^2)$ is the eigenvalue of the meson BSE. The eigenvalue of the homogeneous BSE becomes 1 when the meson propagator is on shell, i.e.,

$$\Omega_m^2 + \vec{P}^2 + M(\Omega_m^2, \vec{P}^2) = 0,$$

where $M(\Omega_m^2, \vec{P}^2)$ is the meson mass. On one hand, people could define the so called screening mass $M_{scr}$ via putting $\Omega_m = 0$, extending $\vec{P}$ into complex plane and
then locating the screening mass at $\lambda(0, -M_{\pi}^2) = 1$ [18–21, 23–25]. On the other hand, the pole mass is in principle difficult to define since an analytic continuation of the Matsubara frequency in the form of spectral representation is required, which is [51]:

$$\frac{1}{\Omega_m^2 + \vec{P}^2 + M(\Omega_m^2, \vec{P}^2)} = \int_{-\infty}^{\infty} d\omega \frac{\rho(\omega, \vec{P})}{\omega - i\Omega_m}. \quad (7)$$

The pole mass is located at $\lambda(P^2 = -M_{\text{pole}}^2, \vec{P} = 0) = 1$ through replacing $i\Omega_m$ with $M_{\text{pole}}$ in the above spectral representation. Therefore, if people try to obtain the pole mass directly, the BSE in real time formula with the spectral representation is required. However, no matter how the formula is changed, the eigenvalue $\lambda(P^2)$ keeps to be an analytic function at least in a broad range of $P^2 \in [-M_{\text{pole}}^2, \infty)$ [52]. Therefore, one could proceed the way of constructing the meson pole mass as follows: We compute the eigenvalues $\lambda(P^2 = \Omega_m^2)$ at each $\Omega_m$ with $m = 1, 2, ..., m_{\text{Max}}$, and extrapolate them to obtain the pole mass of the meson $M_{\pi, \rho}$ at $\lambda(P^2 = -M_{\pi, \rho}^2) = 1$. The larger number of $m$ will certainly leads to a more precise extrapolation, and here we employ $m_{\text{Max}} = 30$ practically.

1. $\pi$ meson

The essential case of interest is the temperature dependent behaviour of pion, which is the simplest two-body system as well as the Goldstone mode of QCD [53]. The Bethe-Salpeter amplitude of pion outlined in Eq.(6) is of the general form

$$\Gamma_\pi(q; P) = i\gamma_5 \tau_1^\pi(q; P) + \gamma_5 P \tau_2^\pi(q; P). \quad (8)$$

We here drop the other two terms with higher order Dirac structures, which is the most practical choice of theoretical studies on hadron phenomenological observables at finite temperature. We limit ourselves to this case, and further investigations with the complete set of Dirac basis can be the supplement of this work.

The decay constant of pion can also be extrapolated from $P^2 = \Omega_m^2$ to $P^2 = -M_{\pi}^2$ after the mass is located. The definition of pion decay constant is given as:

$$f_\pi(P^2) = \frac{Z_2}{P^2} T \sum_n \int \frac{d^3q}{(2\pi)^3} \text{tr} \left[ i\gamma_5 P \chi_\pi(q; \Omega_m) \right], \quad (9)$$

which is the residue at the pion pole in the axial-vector vertex [34, 53].

By projecting pion Bethe-Salpeter wave function onto $\gamma_5$ channel, we could also define a quantity related to quark condensate, which is

$$ir_\pi(P) = Z_4 T \sum_n \int \frac{d^3q}{(2\pi)^3} \text{tr} \left[ i\gamma_5 \chi_\pi(q; \Omega_m) \right]. \quad (10)$$

In particular, the preservation of the axial-vector Ward-Green-Takahashi identity at zero temperature yields the mass relation [53]

$$f_\pi M_{\pi}^2 = 2m(\zeta)r_\pi(\zeta). \quad (11)$$

The quantity $r_\pi$ is related to quark condensate in chiral limit with

$$\lim_{m \rightarrow 0} r_\pi(\zeta) = \frac{\langle \bar{q}q \rangle^0}{f_\pi^2}, \quad (12)$$

where $\langle \bar{q}q \rangle^0$ is the chiral condensate; $f_\pi^2$ the pion decay constant in chiral limit. It indicates that the mass relation is equivalent to the GMOR relation:

$$f_\pi^2 M_{\pi}^2 = 2m(\zeta)\langle \bar{q}q \rangle^0. \quad (13)$$

The mass relation in Eq.(11) and/or the related GMOR relation are essentially on-shell properties of pion, therefore, as the temperature becomes nonzero, the thermal width of pion could drive the deviation of such relations. It is then interesting to check the behaviour of GMOR relation at finite temperature.

2. $\rho$ meson

The other case of great interest is the $\rho$ meson, with its Bethe-Salpeter amplitude outlined in Eq.(6) takes the general form [54]

$$\Gamma_{\mu, \rho}(q; P) = i\gamma_5 T \tau_1^\rho(q; P) + q_\mu^T \tau_2^\rho(q; P), \quad (14)$$

with $F_{\mu}^T = P_{\mu\nu}F_{\nu}$. Here we practically consider two Dirac structures for $\rho$ meson, which are the dominant two terms while in principle there are eight Dirac structures in the complete set of the vector Bethe-Salpeter amplitude. Besides that, if trying to reflect the impact of $O(4)$ symmetry breaking, people need to split $\gamma_5^T$ and $q_\mu^T$ into their longitudinal and transverse modes [23]. Consequently, one shall see distinguishing temperature dependence of the longitudinal $\rho$ meson from the transverse one. Instead of doing that, we keep their original form in the Bethe-Salpeter amplitude, leaving that possibility for further investigation.

It is also straightforward to consider the decay constants of $\rho$ meson, and they are

$$f_{\rho}(P^2) = \frac{Z_2}{3\Omega_m} T \sum_n \int \frac{d^3q}{(2\pi)^3} \text{tr} \left[ i\gamma_\lambda \chi_\lambda(q; \Omega_m) \right], \quad (15)$$

$$f_{\rho}^T(P^2) = \frac{Z_T}{3P^2} T \sum_n \int \frac{d^3q}{(2\pi)^3} \text{tr} \left[ i\sigma_{\mu\lambda} P_\mu \chi_\lambda(q; \Omega_m) \right],$$

with $Z_2$ is the quark wave function renormalisation constant and $Z_T$ is the renormalisation constant for the tensor vertex. These two decay constants are both gauge- and Poincaré-invariant, but $f_{\rho}^T$ is renormalisation scale dependent [55].
the interaction strength \( D_\omega \) increases, the mass of pion increases monotonously, as shown in Fig. 1. As the temperature increases, the decay constant \( \pi \) gets close to the free field limit above the critical temperature \( T_c \), while the \( \rho \) meson pole mass is twice as large as this limit. This gap indicates that \( \rho \) meson would likely to be a \( \pi - \pi \) resonance state. Moreover, the discrepancy of the pole masses here between pseudoscalar and vector meson is qualitatively different from that of the screening masses. The screening masses approach the free field limit \( M_{scr} \sim 2\pi T \) for both pseudoscalar and vector meson at \( T \approx 3T_c \). However, it has also been found in lattice QCD simulation [21] that up to \( T \sim 1 \) GeV, pion screening mass overshoots the free field limit and is not only smaller than \( \rho \) meson screening mass, which is qualitatively consistent with our finding of a large discrepancy between \( \pi \) and \( \rho \) meson pole mass at large \( T \). The overshooting behaviour of screening mass towards the free field limit could be well explained by the positive correction from the leading order perturbative computation predicted by both dimensional reduction [59] and the hard thermal loop methods [60], meanwhile, the notable difference of \( \pi \) and \( \rho \) meson reveals that it still remains considerable non-perturbative effect of QCD on the thermal properties associated with bound states.

### B. Decay constants of \( \pi \) and \( \rho \) meson

Hitherto we have canvassed \( \pi \) and \( \rho \) meson thermal pole masses, it is also important in understanding their corresponding decay properties. The temperature dependence of \( \pi \) and \( \rho \) meson decay constants is illustrated in Fig. 2. As the temperature increases, the decay constant of pion goes up very slightly till around \( T = 0.12 \) GeV and then declines rapidly to zero. Noticing that the light quark dynamical mass function in the quark propagator, is also almost \( T \)-independent below a critical temperature, and then goes to zero. It should not be surprising of this resembling behaviour since the Bethe-Salpeter amplitude of pseudoscalar meson could be directly related to the quark mass function via the Ward identity [53]. It is evident that both pion decay constant and the light quark dynamical mass function are equivalent order pa-
rameters for chiral symmetry restoration. Below transition temperature, chiral symmetry is broken, and its order parameters become nonzero. Above $T_c$, chiral symmetry get restored, and order parameters vanish quickly. Compared to other studies, temperature dependence of pion decay constant here is qualitatively consistent with DSE results [24, 25] and lattice QCD simulation [61].

The longitudinal decay constant of pion meson has similar behaviour as pion’s. It slightly depends on temperature within the hadronic phase, and then chiral symmetry is rapidly restored above the transition temperature, apart from the explicit symmetry breaking by the current quark mass. The transverse decay constant behaves completely different however, which rises monotonously as temperature increases. The ratio of $f_\rho/f_\rho^T$ can be related to the proportion of $S-$ and $D-$ wave content of the $\rho$ meson [55]. Therefore, considering the behaviour of two decay constants, as the temperature increases, one would find that the $D-$ wave contribution becomes larger. Additionally, the higher order Dirac structures in the $\rho$ meson Bethe-Salpeter amplitude could play an important role in computing an accurate value for $f_\rho^T$ at finite temperature, because they contain the detailed contributions of angular momentum.

Noticing that the decay constants $f_\pi$ and $f_\rho$ own similar behaviour as the order parameter, quark condensate, we then compare the temperature derivative of the decay constants with the chiral susceptibility, defined by the temperature derivative of quark condensate, i.e., $\chi = \partial \langle \bar q q \rangle / \partial T$. In Fig. 3 we can see the inflection point of pion decay constant, i.e., $\partial^2 f_\pi/\partial T^2 = 0$ almost coincides with that of $\rho$ meson decay constant. In detail, the transition temperature associated with the inflection point of pion decay constant is $T_{c}^{f_\pi} = 146$ MeV, and that of $\rho$ meson is $T_{c}^{f_\rho} = 149$ MeV compared to that determined by the inflection point of quark condensate as $T_{c} = 150$ MeV [62]. On average, our estimate is

$$T_c = (148 \pm 2) \text{ MeV}.$$  

It is consistent with the chiral phase transition temperature from functional methods [9, 11, 63] and also lattice QCD which is in a range from 147 to 165 MeV in Ref. [64, 65] and $T_c = (154 \pm 9)$ MeV in Ref. [66].

In general, the decay constants could be regarded as a criterion of chiral transition. The fact that chiral phase transition temperature defined with the temperature dependence of $\pi$ and $\rho$ meson decay constants and from the chiral condensate coincide can be viewed as a direct evidence of the chiral phase transition from the physical observables.

C. GMOR relation at finite temperature

GMOR relation as in Eq.(13) can be derived by putting the axial vector Ward identity on shell. It has been argued that the GMOR relation still holds at finite temperature [24, 67, 68], and hence the deviation of GMOR relation indicates a finite thermal width of pion, which leads pion spectral function away from a pole structure. The deviation of the GMOR relation is shown in Fig. 4. At zero temperature, the GMOR relation is exactly preserved, while a clearly remarkable increase of the deviation has emerged when $T$ is approaching the critical temperature, and above $T_c$, it vanishes drastically. The experiments indicate that the matter near the phase transition is in a strongly-coupled state, and the ratio of shear viscosity and entropy density is nearly the lower bound at phase transition point [69–71]. The deviation of GMOR relation can be then regarded as a signal for this strongly-coupled property of the matter in the phase transition region, where the thermal width of pion is generated via Landau damping mechanism [72]. Moreover,
the non monotonous behaviour exhibits the change of the pion structure during the phase transition, and the rapid decrease after phase transition also indicates the dissociation of pion.

### IV. SUMMARY

In this work, the hadronic observables at finite temperature have been studied in the framework of DSEs and BSEs approach. As the temperature increases, the pole mass of pion becomes larger monotonously and after the chiral phase transition, the pole mass gradually reaches the same limit as screening mass, $M_{scr} \sim 2\pi T$ at high temperature. The pole mass of $\rho$ meson is quite stable till $T \sim 0.8T_c$, and then rapidly grows at high temperature. The mass of $\rho$ meson approaches twice as large as pion’s at high temperature, which could be a signal of the $\rho$ meson as the resonance of two pions.

After obtaining the location of the masses, we compute the decay constants of $\pi$ and $\rho$ mesons. The decay constant of pion and the longitudinal decay constant of $\rho$ meson show similar behaviour as a function of temperature. In the hadronic phase, these quantities are barely dependent on temperature, and then drop rapidly in the phase transition region. Thus, the decay constant is strongly related to the phase transition and can be employed as the criterion of chiral phase transition. They give the consistent chiral phase transition temperature as the quark condensate. The transversal decay constant of $\rho$ meson here shows a monotonously increasing behaviour as temperature increases.

Even though this method cannot directly give the information of the thermal width of mesons, the strong deviation of GMOR relation indicates the strongly-coupled property of QCD matter in the phase transition region. The non monotonous behaviour also exhibits the change of the inside structure of mesons during the phase transition.

A straightforward and worthwhile extension of this work is the consideration in the pole masses of the $\sigma$ and $\omega_1$ meson. The proper calculation of the scalar and the axial-vector channel is complicated even at zero temperature, since one must include other Lorentz structures in quark-gluon vertex beyond Rainbow-Ladder approximation in order to give a correct description of the angular momentum. Despite of this, the research on the temperature dependence of scalar and the axial-vector channel with nevertheless provide us insights into the difference of parity partners. The other possible extension is to consider the mesons with strange quark and also quarkonium. It has been brought out by lattice QCD simulation that the screening masses of meson including strange quark will give a higher $T_c$ [19, 21]. For the quarkonium, the $J/\Psi$ production is especially important for helping understand experimental data of heavy ion collisions [73, 74]. Therefore, it will be of high value to extend the computation to these observables.

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