Anisotropic Fermi surface from holography

Li Qing Fang\textsuperscript{a,b}, Xian-Hui Ge\textsuperscript{a,d,*}, Jian-Pin Wu\textsuperscript{e,d}, Hong-Qiang Leng\textsuperscript{a}

\textsuperscript{a}Department of Physics, Shanghai University, 200444 Shanghai, China
\textsuperscript{b}School of Physics and Electronic Information, Shangrao Normal University, 334001 Shangrao, China
\textsuperscript{c}Department of Physics, School of Mathematics and Physics, Bohai University, 121013 Jinzhou, China
\textsuperscript{d}State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

Abstract

We investigate the probe holographic fermions by using an anisotropic charged black brane solution. We derive the equation of motion of probe bulk fermions with one Fermi momentum along the anisotropic and one along the isotropic directions. We then numerically solve the equation and analysis the properties of Green function with these two momentums. We find in this case the shape of Fermi surface is anisotropic. However, for both Fermi momentums perpendicular to the anisotropic direction, the Fermi surface is isotropic.

Keywords: Anisotropic Fermi surface, AdS/CFT correspondence

1. Introduction

Landau Fermi liquid theory is the standard model for theory of metals and it helps us understand almost all metals, such as semi-conductors, superconductors and so on. Recently, this theory has been challenged by physics by the facts that lots of materials have been found which cannot be described by Landau Fermi liquid. For example, the Landau Fermi theory fails to describe electromagnetic properties of Weyl metal \cite{1,2}. In recent years, using AdS/CFT correspondence \cite{3,4,5} to build a non-Fermi liquid theory has been widely studied in \cite{6,7,8,9,10,11,12,13,14,15}. In ideal systems, the Fermi surface is isotropic, but in real materials, the Fermi surfaces are always anisotropic and inhomogeneous. For instance, the electric structure of superconducting cuprates, is highly anisotropic because of the atomic lattice effects.

In \cite{16,17}, one of us obtained a charged and spatially anisotropic black brane solution, dual to a spatially anisotropic $\mathcal{N} = 4$ super Yang-Mills (SYM) theory at finite chemical potential and finite temperature. The anisotropy is
introduced through deforming the SYM theory by a $\theta$–parameter of the form $\theta \propto z$, which acts as an isotropy-breaking source that forces the system into an anisotropic equilibrium state \cite{20, 21}. The purpose of this paper is to investigate the properties of the probe holographic fermions in this anisotropic but homogeneous background. Note that in our case, the linear axions do not lead to a periodic deformation of the boundary conformal field theory and thus cannot be considered as holographic lattice. Nevertheless, the axions do result in anisotropic Fermi surface.

It is worth noting that there also are some papers \cite{22, 23} working on the anisotropic holographic fermions system. In \cite{22}, the authors studied anisotropic Fermi surface by consider BGP lagrangian. But the gravity background is isotropic. In \cite{23}, the authors numerically constructed an anisotropic holographic lattice background by adding a neutral scalar field with the periodic boundary conditions along the spatial direction, and using this background to study the anisotropic Fermi surface.

This paper is organized as follows: we briefly review the anisotropic and charged black brane solution in section 2. In section 3, we give the equation of motion for the probe fermions with one momentum along the anisotropic direction and one momentum along the isotropic direction. We solve the Dirac equation numerically and study the properties of the Fermi surface in section 4. In particular, we study the holographic fermions with momentums along the isotropic directions and reveal that the resulting Fermi surface is isotropic. The conclusion is presented in the last section.

2. The anisotropic charged black brane solution

The anisotropic charged black brane solution can be derived from the five-dimensional Einstein-Maxwell-Dilaton-Axion truncation of gauge AdS supergravity with compactification of ten-dimensional type IIB supergravity on $S^5$. The non-linear Kuluza-Klein reduction of type IIB supergravity to five dimension, leads to the the presence of an Abelian field in the action. The time component of the Abelian field results in a non-zero chemical potential in the dual gauge theory. Different from the chargless anisotropic black brane solution, the introduction of the U(1) gauge field breaks the SO(6) symmetry and thus leads to the excitations of the Kaluza-Klein modes.

2.1. Background solution

The effective action for the Einstein-Maxwell-Dilaton-Axion theory can be written as\cite{18, 19}

$$S = \frac{1}{2\kappa^2} \int_M \sqrt{-g} \left( R + 12 - \frac{1}{2} e^{2\phi} (\partial \chi)^2 - \frac{1}{4} F_{MN} F^{MN} \right) + \frac{1}{2\kappa^2} \int_{\partial M} \sqrt{-g} 2K,$$

(1)
where \( \kappa^2 = 8\pi G_5 = 4\pi^2/N_5^2 \), \( L = 1 \), \( F_{MN} = \partial_M A_N - \partial_N A_M \), and \( \chi \) is an axion field. The black brane solution takes the following form

\[
\begin{align*}
\frac{ds^2}{r^2} &= e^{-\frac{4}{3}\phi(r)} \left(-F \mathcal{B} dt^2 + \mathcal{H} dx^2 + dy^2 + dz^2\right) + \frac{e^{-\frac{2}{3}\phi(r)}}{r^2} \mathcal{F} \\
\chi &= a x, \quad A = A_0 dt, \quad \phi = \phi(r)
\end{align*}
\]

The metric was already solved numerically and analytically in [18, 19]. While \( a^2 > 0 \) corresponds to the prolate anisotropy, the analytic continuation of the anisotropy parameter \( a^2 < 0 \), gives rise to an oblate anisotropy. Note that the form of the metric given here is slightly different from that of [18, 19] and we choose \( \chi = a x \) only for the convenience of computation.

In the following, we will mainly utilize the analytic black brane solution. The metric functions given in the \( r \)-coordinate can be solved by perturbing around the isotropic Resseiner – Nordström-AdS black brane [18, 19]

\[
\begin{align*}
\mathcal{F} &= 1 - \left(\frac{r_H}{r}\right)^4 + \left(\frac{r_H}{r}\right)^6 - \left(\frac{r_H}{r}\right)^4 q^2 + a^2 \mathcal{F}_2(r) + \mathcal{O}(a^4), \\
\mathcal{B} &= 1 + a^2 \mathcal{B}_2(r) + \mathcal{O}(a^4), \\
\mathcal{H} &= e^{-\phi(r)}, \quad \text{with} \quad \phi(r) = a^2 \phi_2(r) + \mathcal{O}(a^4),
\end{align*}
\]

where

\[
\begin{align*}
\mathcal{F}_2(r) &= \frac{r_H^4}{24\sqrt{1 + 4q^2}} \left\{3(-4q^2 + \frac{1}{r_H^6}) \ln \left(\frac{1 + \sqrt{1 + 4q^2} r_H^2}{1 - \sqrt{1 + 4q^2} r_H^2} + 2r^2\right) + \frac{1}{r_H^4} \left[8\sqrt{1 + 4q^2}(-\frac{1}{r^2} + \frac{1}{r_H^4}) + \frac{1}{r^2} \left(3\ln \left(-2 - 2q^2 + 2\sqrt{1 + 4q^2}\right) + 5(-2 + q^2) \ln \left(-1 + 2q^2 + \sqrt{1 + 4q^2}\right) - 12q^2 \ln \left(-2 - 2q^2 + 2\sqrt{1 + 4q^2}\right) + 7(1 + q^2) \ln \left(\frac{-1 + 2q^2}{r_H^2} + \sqrt{1 + 4q^2}\right) + \ln \left(\frac{2q^2}{r^2} - \frac{1 + \sqrt{1 + 4q^2}}{r_H^2}\right)\right]\right]\}, \\
\mathcal{B}_2(r) &= \frac{1}{24}\left(\frac{10r^2}{q^2 r_H^4 - r^2 r_H^2 - r^4} + \frac{1}{r_H^4 \sqrt{1 + 4q^2}} \ln \left(\frac{1 + \sqrt{1 + 4q^2} r_H^2}{1 - \sqrt{1 + 4q^2} r_H^2} + 2r^2\right)\right), \\
\phi_2(r) &= -\frac{1}{4r_H^2 \sqrt{1 + 4q^2}} \ln \left(\frac{1 + \sqrt{1 + 4q^2} r_H^2}{1 - \sqrt{1 + 4q^2} r_H^2} + 2r^2\right) + \mathcal{O}(a^4)
\end{align*}
\]

and dimensionless charge \( q = \frac{Q}{2\sqrt{3} r_H} \). The constant \( Q \) is a dimensional charge.
which corresponds to the $U(1)$ gauge field. The gauge field $A_t$ is given by

$$A_t = \sqrt{3} qr\left(\frac{1}{r^2} - \frac{5r_H}{24(\sqrt{4q^2+1})}\right) \ln \left[\frac{\sqrt{4q^2+1} + 1}{\sqrt{4q^2+1} - 1}\right] - \frac{\sqrt{3} q}{r^2} \ln \left[\frac{\sqrt{4q^2+1} + 1}{\sqrt{4q^2+1} - 1}\right] - \frac{\sqrt{3} q}{r_H^2} \ln \left[\frac{\sqrt{4q^2+1} + 1}{\sqrt{4q^2+1} - 1}\right] \right] a^2.$$  \hspace{1cm} (6)

In Taylor series expansion of $a$, the Hawking temperature and entropy density can be expressed as

$$T = \frac{(2 - q^2)r_H}{2\pi} + \frac{-4\sqrt{1+4q^2} + 5(2 + 5q^2)\ln \left[\frac{3 + \sqrt{1+4q^2}}{3 - \sqrt{1+4q^2}}\right]}{96\pi r_H \sqrt{1+4q^2}} a^2 + O(a^4).$$  \hspace{1cm} (7)

and

$$s = \frac{N^2 r_H^3}{2\pi} + \frac{5r_H N_c^2 \ln \left[\frac{3 + \sqrt{1+4q^2}}{3 - \sqrt{1+4q^2}}\right]}{32\pi \sqrt{1+4q^2}} a^2 + O(a^4).$$  \hspace{1cm} (8)

### 2.2. Thermodynamic properties

The anisotropic black brane yields very interesting thermodynamic properties, as discussed in [18, 19]. For the prolate anisotropy $a^2 > 0$, the black brane suffers thermodynamic instabilities. For a fixed temperature there are two branches of allowed black brane solutions, a branch with larger horizon radii and one with smaller. The smaller branch of solution is unstable with negative specific heat. This situation is very similar to the case of Schwarzschild-AdS black holes with a spherically horizon. As to the oblate anisotropy, the black brane is qualitatively the same as the planar black brane. That is to say, there is only one stable branch of black brane solution and the thermodynamics is dominated by this phase for all temperature.

Before we discuss the property of holographic fermions, we should clarify the parameters used in the numerical computation. In order to work near the zero temperature, we will fixed the dimensionless charge as $q = 1.4$. After that, we vary the anisotropy parameter $a^2$ from the “prolate anisotropy” to the “oblate anisotropy”. Figure 1 shows how the entropy density varies as the temperature and $a^2$ changes. The parameter range ($q = 1.4$ and $a^2$) is shown by the red line of the figure 1. For the prolate anisotropy $a^2 > 0$, there exists two branches as shown in the left plot of figure 1. We will work with the stable branch. As to the oblate anisotropy $a^2 < 0$, it was demonstrated in the right plot of figure 1. Note that the background thermodynamic is exactly the same as RN-AdS when $a^2 = 0$. We will study the properties of holographic fermions for the cases $a^2 = 0$, $a^2 > 0$ and $a^2 < 0$ in the following section, respectively.
3. Dirac Equation

In order to study the properties of the probe fermions on the dual boundary theory, we consider the bulk action for a probe Dirac fermion with the mass $m$, charge $q_f$. The action of bulk fermion is

$$S_{\text{bulk}} = \int d^5x \sqrt{-g} \bar{\psi} \left( \Gamma^a D_a - m \right) \psi,$$

where $\Gamma^a = (e_\mu)^a_{\Gamma^\mu}$, the covariant derivative $D_a = \partial_a + \frac{1}{4}(\omega_{\mu\nu})_a \Gamma^\mu_{\nu} - iq_f A_a$. The spin connection 1-forms $(\omega_{\mu\nu})_a = (e_\mu)^b_{\Gamma^\nu} \nabla_a (e_\nu)_b$. Here $(e_\mu)^a$ form a set of orthogonal normal vector bases. From above action, the Dirac equation can be written as

$$\Gamma^a D_a \psi - m \psi = 0.$$  \hspace{1cm} (10)

Then, we make a Fourier transformation $\psi = (-g g^{rr})^{-\frac{1}{2}} e^{-i(\omega t + k_x x + k_y y)} \tilde{\phi}$, and use the following gamma matrices basis

$$\Gamma^r = \begin{pmatrix} -\sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \quad \Gamma^t = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix},$$

$$\Gamma^x = \begin{pmatrix} -\sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}, \quad \Gamma^y = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}. \hspace{1cm} (11)$$

Note that we choose $k_x$ along the anisotropic direction and $k_y$ along the isotropic direction. We set $\tilde{\phi} = \begin{pmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2 \end{pmatrix}$. The Dirac equation becomes to two coupled equations

$$\sqrt{g^{rr}} \partial_r \tilde{\phi}_1 + m\sigma_3 \tilde{\phi}_1 = \sqrt{g^{tt}} (\omega + q_f A_t) i\sigma_2 \tilde{\phi}_1 - \sqrt{g^{xx}} k_x \sigma_1 \tilde{\phi}_1 + \sqrt{g^{yy}} k_y \sigma_1 \tilde{\phi}_2,$$

$$\sqrt{g^{rr}} \partial_r \tilde{\phi}_2 + m\sigma_3 \tilde{\phi}_2 = \sqrt{g^{tt}} (\omega + q_f A_t) i\sigma_2 \tilde{\phi}_2 + \sqrt{g^{xx}} k_x \sigma_1 \tilde{\phi}_2 + \sqrt{g^{yy}} k_y \sigma_1 \tilde{\phi}_1. \hspace{1cm} (12)$$
In order to decouple the equation of motion, we assume \( \hat{\phi}_I = \begin{pmatrix} y_I \\ z_I \end{pmatrix} \), with \( I = 1, 2 \). The equation of motion (12) yields

\[
\sqrt{g^{rr}} \partial_r y_I + y_I (\omega + q_I A_I) z_I - \sqrt{g^{zz}} k_z y_I = \sqrt{g^{\eta\eta}} k_\eta y_I z_I + \sqrt{g^{yy}} k_y y_I z_I \\
\sqrt{g^{rr}} \partial_r y_I - y_I (\omega + q_I A_I) z_I - \sqrt{g^{zz}} k_z y_I = \sqrt{g^{\eta\eta}} k_\eta y_I z_I + \sqrt{g^{yy}} k_y y_I z_I \\
\sqrt{g^{rr}} \partial_r y_I + m_I y_I = \sqrt{g^{zz}} (\omega + q_I A_I) z_I + \sqrt{g^{\eta\eta}} k_\eta y_I z_I + \sqrt{g^{yy}} k_y y_I z_I \\
\sqrt{g^{rr}} \partial_r z_I - m_I z_I = - \sqrt{g^{zz}} (\omega + q_I A_I) y_I + \sqrt{g^{\eta\eta}} k_\eta y_I z_I + \sqrt{g^{yy}} k_y y_I z_I \\
\sqrt{g^{rr}} \partial_r y_I + m_I y_I = \sqrt{g^{zz}} (\omega + q_I A_I) z_I + \sqrt{g^{\eta\eta}} k_\eta y_I z_I + \sqrt{g^{yy}} k_y y_I z_I \\
\sqrt{g^{rr}} \partial_r z_I - m_I z_I = - \sqrt{g^{zz}} (\omega + q_I A_I) y_I + \sqrt{g^{\eta\eta}} k_\eta y_I z_I + \sqrt{g^{yy}} k_y y_I z_I.
\]

The ingoing boundary condition for \( \hat{\phi}_I \) at the event horizon can be imposed as

\[
\hat{\phi}_I \propto \begin{pmatrix} i \\ 1 \end{pmatrix} e^{-i \omega r},
\]

with \( r_+ = \int \frac{dr}{r^2 \sqrt{g^{rr}}} \). Near the AdS boundary, the solution of the Dirac equation (12) can be written as

\[
\begin{align*}
\hat{\phi}_1^I & \overset{r \to \infty}{\approx} \begin{pmatrix} c_1^I r^{-m} \\ d_1^I r^m \end{pmatrix}, & \hat{\phi}_{II}^1 & \overset{r \to \infty}{\approx} \begin{pmatrix} c_1^{II} r^{-m} \\ d_1^{II} r^m \end{pmatrix}, \\
\hat{\phi}_2^I & \overset{r \to \infty}{\approx} \begin{pmatrix} c_2^I r^{-m} \\ d_2^I r^m \end{pmatrix}, & \hat{\phi}_{II}^2 & \overset{r \to \infty}{\approx} \begin{pmatrix} c_2^{II} r^{-m} \\ d_2^{II} r^m \end{pmatrix},
\end{align*}
\]

where I, II correspond to two independent ingoing boundary conditions. From the holographic dictionary, the retarded Green function is

\[
G = \mathbb{C} \mathbb{D}^{-1},
\]

where we have defined

\[
G \equiv \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}, \quad \mathbb{C} \equiv \begin{pmatrix} c_1^I & c_1^{II} \\ c_2^I & c_2^{II} \end{pmatrix}, \quad \mathbb{D} \equiv \begin{pmatrix} d_1^I & d_1^{II} \\ d_2^I & d_2^{II} \end{pmatrix}.
\]

For numerical convenience, we can define the following matrices

\[
Y \equiv \begin{pmatrix} y_1^I & y_1^{II} \\ y_2^I & y_2^{II} \end{pmatrix}, \quad Z \equiv \begin{pmatrix} z_1^I & z_1^{II} \\ z_2^I & z_2^{II} \end{pmatrix}, \quad \tilde{G} \equiv Y Z^{-1}.
\]

An then one can obtain the evolution equation as follows

\[
\begin{align*}
\sqrt{g^{rr}} \partial_r \tilde{G}_{11} & + 2 m \tilde{G}_{11} - \sqrt{g^{tt}} (\omega + q_I A_I) \tilde{G}_{11}^2 + \tilde{G}_{11} \tilde{G}_{21} + \tilde{G}_{12} \tilde{G}_{21} + 1 \\
& - \sqrt{g^{zz}} k_z (\tilde{G}_{11}^2 - \tilde{G}_{12} \tilde{G}_{21} - 1) + \sqrt{g^{\eta\eta}} k_\eta \tilde{G}_{11} (\tilde{G}_{12} + \tilde{G}_{21}) = 0, \\
\sqrt{g^{rr}} \partial_r \tilde{G}_{22} & + 2 m \tilde{G}_{22} - \sqrt{g^{tt}} (\omega + q_I A_I) \tilde{G}_{22}^2 + \tilde{G}_{12} \tilde{G}_{21} + \tilde{G}_{12} \tilde{G}_{22} + 1 \\
& + \sqrt{g^{zz}} k_z (\tilde{G}_{22}^2 - \tilde{G}_{12} \tilde{G}_{21} - 1) + \sqrt{g^{\eta\eta}} k_\eta \tilde{G}_{22} (\tilde{G}_{12} + \tilde{G}_{21}) = 0, \\
\sqrt{g^{rr}} \partial_r \tilde{G}_{12} & + 2 m \tilde{G}_{12} - \sqrt{g^{tt}} (\omega + q_I A_I) \tilde{G}_{12} (\tilde{G}_{11} + \tilde{G}_{22}) \\
& + \sqrt{g^{zz}} k_z (\tilde{G}_{12} - \tilde{G}_{12} \tilde{G}_{22} - \tilde{G}_{11}) - \sqrt{g^{\eta\eta}} k_\eta (1 - \tilde{G}_{12} - \tilde{G}_{11} \tilde{G}_{22}) = 0, \\
\sqrt{g^{rr}} \partial_r \tilde{G}_{21} & + 2 m \tilde{G}_{21} - \sqrt{g^{tt}} (\omega + q_I A_I) \tilde{G}_{21} (\tilde{G}_{11} + \tilde{G}_{22}) + \sqrt{g^{zz}} k_z (\tilde{G}_{22} - \tilde{G}_{22} - \tilde{G}_{11} \tilde{G}_{22}) \\
& - \sqrt{g^{\eta\eta}} k_\eta (1 - \tilde{G}_{21} - \tilde{G}_{11} \tilde{G}_{22}) = 0.
\end{align*}
\]
In term of Eq. (14), we can easily find the boundary condition for the above evolution equation at the horizon

\[ \tilde{G} \rightarrow r_H \approx \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}, \]  

(20)

After solving the evolution equations (19), we can read off the boundary Green’s function as

\[ G = \lim_{r \to \infty} r^{2m} \tilde{G}. \]  

(21)

4. The properties of anisotropic Fermi surface

In this section, we mainly focus on the property of the spectral function \( A(\omega, k_x, 0) = \text{Im}[G_{11} + G_{22}] \). In the discussion below, we will study the case of \( m = 0, q_f = 1 \) and \( r_H = 1 \) for simplicity but without loss of generality. Before we start to study the property of holographic fermions, the parameters \( a \) which determines the level of anisotropy and the dimensionless charge \( q \) should be determined. For fixed \( q \), the temperature of the system will be low whatever \( a \) changes. Thus, we choose \( q = 1.4 \) in order to make sure the temperature of system is finite and close to zero. From (19), we can discover that \( G_{11}(\omega, -k_x, k_y) = G_{22}(\omega, k_x, k_y) \) and \( G_{12}(\omega, k_x, k_y) = G_{21}(\omega, k_x, k_y) \).

4.1. The shape of Fermi surface

Now, we investigate the Fermi momentum along \( x- \) and \( y- \) axes. With this in mind, we solve the equations of motion (19) numerically.

Firstly, we will find the Fermi momentum along the \( x- \) direction (i.e. \( k_y = 0 \)) for \( a^2 = 0 \). For the imaginary part of retard Green function, a peak appear in the region \( \omega > 0 \) which has a broad maximum (see the left plot of figure 2). When \( k_x = 2 \), this peak is sharper than \( k_x = 2.2 \). As the value of momentum approach 1.84318392, a sharp quasi particle like peak is generate near \( \omega = 0 \), that height go to infinity and width close to zero. By studying the spectrum function \( A(\omega, k_x, 0) \) for a given \( \omega = 10^{-9} \), we can determine Fermi momentum.
\[ k_F = 1.84318392 \] along the \( x^- \) direction for \( a^2 = 0 \) (see the right plot of figure [2]). That is to say, when \( a = 0 \), the background is the same as five-dimensional RN-AdS black hole. And above result agrees with that of [7, 8], although the form of Green function is different to (21). This was also testified that our equation of motion (19) is correct.

Next, we will study the shape of Fermi surface. When \( a^2 = 0 \), the background is isotropic which became to RN-AdS black hole. We find that the shape of Fermi surface is also isotropic for \( a^2 = 0 \). In other word, the Fermi momentums are equal on each direction of the \( k_x - k_y \) plane. We can fit our numerical result by a function as follows [23]:

\[ \frac{k_x^2}{c_x^2} + \frac{k_y^2}{c_y^2} = 1. \] (22)

For convenience, we can also introduce two quantities that are the difference between the \( k_x \) axis and \( k_y \) axis \( d = c_x - c_y \) and the flattening factor \( f = \frac{c_x - c_y}{c_x} \).

From the above result, we see that the shape of Fermi surface for \( a^2 = 0 \) case is a sphere. It means \( c_x = c_y = 1.84318392 \) and \( d = f = 0 \).

Turning to the prolate anisotropy \( a^2 > 0 \), we expect that the shape of the Fermi surface would be deformed by the anisotropy parameter. Under this circumstances, we work with the stable branch and study the influence of \( a^2 \) to the Fermi surface. As the result showed in Table 1 the Fermi momentum on \( k_x \) direction is larger than that on \( k_y \), and the flattening of the shape of Fermi surface decreases as \( a^2 \) decreases. In other word, the shape of Fermi surface is a prolate sphere when \( a^2 > 0 \).

As to the oblate anisotropy, the spectral function can still be solved numerically although the anisotropy parameter becomes imaginary. We find \( c_x \) is smaller than \( c_y \), and the difference between the \( c_x \) and \( c_y \) increases as \( a^2 \) decreases. The flattening factor of Fermi surface is minus, so the Fermi surface is an “oblate”-like one.

To have a clear physical picture about the effect of anisotropy parameter \( a^2 \) on the shape of the Fermi surface, we plot the Fermi surface with larger

| \( a^2 \) | \( 0.002 \) | \( 0.004 \) | \( 0.006 \) | \( 0.008 \) | \( 0.01 \) |
|---|---|---|---|---|---|
| \( c_x \) | 1.83646078 | 1.82973202 | 1.82303540 | 1.81637499 | 1.80975576 |
| \( c_y \) | 1.82694040 | 1.82033522 | 1.81376704 | 1.80723888 | 1.80075680 |
| \( d \) | 0.00046673 | 0.00037680 | 0.00028569 | 0.00019211 | 0.00009708 |
| \( f \) | 0.00025554 | 0.00020593 | 0.00015564 | 0.00010461 | 0.00005277 |

| \( a^2 \) | \( -0.002 \) | \( -0.004 \) | \( -0.006 \) | \( -0.008 \) | \( -0.01 \) |
|---|---|---|---|---|---|
| \( c_x \) | 1.84666749 | 1.85021698 | 1.85384682 | 1.85757781 | 1.86144205 |
| \( c_y \) | 1.84766681 | 1.85041818 | 1.85415283 | 1.85799214 | 1.86189696 |
| \( d \) | -0.00009932 | -0.00020120 | -0.00030601 | -0.00041433 | -0.00052701 |
| \( f \) | -0.00005378 | -0.00010874 | -0.00016507 | -0.00022305 | -0.00028312 |
The shape of Fermi surface for $a^2 = 1$ (left), $a^2 = 0$ (middle) and $a^2 = -1$ (right). The red dashed line and blue line represent standard circle and numerical result with different $a^2$, respectively.

Figure 3: The shape of Fermi surface for $a^2 = 1$ (left), $a^2 = 0$ (middle) and $a^2 = -1$ (right). The red dashed line and blue line represent standard circle and numerical result with different $a^2$, respectively.

Figure 4: Fermi momentum $k_F(k_y = 0)$ (Red) and $k_F(k_x = 0)$ (Blue) varies with $a^2$.

From above relations, we find that the influence of the anisotropy of the background to Fermi momentum on $k_x$ is bigger than $k_y$ direction. The “prolate” solution correspond to “prolate” Fermi surface and the “oblate” solution correspond to “oblate” Fermi surface. In a word, the anisotropy of the background effect the shape of Fermi surface in case the momentum is chosen along the anisotropic direction.
4.2. The scaling behavior

In the following, we try to ascertain type of the dual system. Thus, we must acquire two scaling behaviors at \( k_\perp = k - k_F \to 0 \). For convenience, we analyze the scaling behaviors with two cases \( (k_x = 0 \text{ or } k_y = 0) \). The dispersion relations is given by

\[
\omega_s(k_\perp) \sim k_\perp^\alpha. \tag{24}
\]

and the scaling relation of the height of \( \text{Im}G_{22} \) scales as \( k_\perp \)

\[
\text{Im}G_{22}(\omega_s(k_\perp), k_\perp) \sim k_\perp^{-\beta}. \tag{25}
\]

As to the case \( a^2 = 0.01 \), we find the dispersion relation for this anisotropic background is almost linear (i.e. \( \alpha \approx 1 \))(see the left plot of figure 6). To make sure whether the dual liquid is that of the Landau Fermi liquid type, we must check another scaling relation. To be more clearly, we take the logarithm of the both sides of the equation (25). As demonstrated in the left plot of figure 6, we find \( \beta \neq 1 \). From figure 5 and figure 6, we can see that \( a \) cannot change basic type of two scaling behaviors both for \( k_x \) and \( k_y \).

As pointed out in [24, 25], the exponent of scaling behavior obeys \( \alpha = \beta = 1 \) for Landau Fermi liquid. We checked \( a^2 > 0, a^2 = 0 \) and \( a^2 < 0 \) cases, which the scaling behavior \( \alpha \approx 1 \) and \( \beta \neq 1 \) is still valid. So in this regard, the dual liquid does not behave as a Landau Fermi liquid one, which means the type of dual liquid is non-Fermi liquid for this anisotropy background geometry.

4.3. Influence of axion field with isotropic spatial direction

In this subsection, we will discuss the holographic fermions with momentums perpendicular to the anisotropic direction. In order to do this, the background metric can be written as

\[
ds^2 = e^{-\frac{1}{2}\phi}r^2 \left(-F dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{e^{-\frac{1}{2}\phi}dr^2}{r^2F}, \tag{26}
\]

\[
\chi = az, \quad A = A_t(r)dt, \quad \phi = \phi(r). \tag{27}
\]
The result shows that for any cases, the Fermi surface is isotropic and the equation can still be solved numerically as given in the previous subsections.

We may take a transformation $\psi = (-g g^T)^{-\frac{1}{4}} e^{-i \omega t + ik_x x + i k_y y} \tilde{\phi}$. The Dirac equation can still be solved numerically as given in the previous subsections. The result shows that for any cases, the Fermi surface is isotropic and $c_x = c_y$. This implies that although the background geometry is anisotropic, the Fermi surface is isotropic if the momentums are chosen along the isotropic directions. We also find smaller $a^2$ can make Fermi momentum bigger.

5. Discussion and Conclusion

We have investigated the anisotropic fermions system at low temperature by using a five dimensional charged and anisotropic geometry. The properties of Fermi surface can be summarized as follows:

- The Fermi momentum decreases as $a^2$ increases for all cases.
- For the case the momentum along the anisotropic direction, the “prolate anisotropy” results in “prolate” Fermi surfaces and the “oblate” solution leads to “oblate” Fermi surfaces. On the other hand, the Fermi surface still remains isotropic if the momentums of the Dirac wave function is chosen perpendicular to the anisotropic direction.

| $a^2$  | 0.01 | 0   | -0.01 |
|-------|------|-----|-------|
| $c_x$ | 1.82599405 | 1.84318392 | 1.86196906 |
| $c_y$ | 1.82599405 | 1.84318392 | 1.86196906 |
| $d$   | 0    | 0   | 0     |
| $f$   | 0    | 0   | 0     |

Table 3: Fermi momentums with different $a^2$ with isotropic spatial direction.
• The scaling behavior has been studied in this background and the dual system is a non-Fermi type liquid.

In general, we obtain an anisotropic non-Fermi liquid from holographic system. For real materials, it always has the lattice structure that corresponds to Brillouin zone, and the first Brillouin zone is important. In this paper, the axion field does not result in a periodic deformation of the boundary conformal field theory. So the Fermi surface observed here is different from the anisotropic Fermi surface obtained by using a neutral scalar field with periodic boundary conditions \[23\]. The Fermi surface which we discussed in this work should locate in the first Brillouin zone.

Acknowledgements

The authors would like to thank Long Cheng and Chao Niu for their helpful discussions. This work was partly supported by NSFC, China (No. 11375110, 11305018, 11275208). XGH would like to thank APCTP for hospitality during the focus program “Aspects of Holography”. JPW was also Supported by Program for Liaoning Excellent Talents in University (No. LJQ2014123).

References

[1] F. D. M. Haldane, “Berry Curvature on the Fermi Surface: Anomalous Hall Effect as a Topological Fermi-Liquid Property”, Phys. Rev. Lett. 93, 206602 (2004).

[2] Ki-Seok Kim, Heon-Jung Kim, M. Sasaki, “Anomalous transport phenomena in Weyl metal beyond the Drude model for Landau’s Fermi liquids”, arXiv:1407.3056.

[3] J. M. Maldacena, “The Large N Limit of Superconformal Field Theories and Supergravity”, Adv. Theor. Math. Phys. 2 (1998) 231, arXiv:hep-th/9711200.

[4] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge Theory Correlators from Non-Critical String Theory”, Phys. Lett. B 428 (1998) 105, arXiv:hep-th/9802109.

[5] E. Witten, “Anti De Sitter Space And Holography”, Adv. Theor. Math. Phys. (1998) 253, arXiv:hep-th/9802150.

[6] S. S. Lee, “A Non-Fermi Liquid from a Charged Black Hole; A Critical Fermi Ball,” Phys. Rev. D 79 (2009) 086006, arXiv:0809.3402 [hep-th].

[7] H. Liu, J. McGreevy and D. Vegh, “Non-Fermi liquids from holography,” Phys. Rev. D 83 (2011) 065029, arXiv:0903.2477 [hep-th].
[8] T. Faulkner, H. Liu, J. McGreevy and D. Vegh, “Emergent quantum criticality, Fermi surfaces, and AdS2,” Phys. Rev. D 83 (2011) 125002, arXiv:0907.2694 [hep-th].

[9] M. Cubrovic, J. Zaanen and K. Schalm, “String Theory, Quantum Phase Transitions and the Emergent Fermi-Liquid,” Science 325 (2009) 439, arXiv:0904.1993 [hep-th].

[10] N. Iqbal and H. Liu, “Real-time response in AdS/CFT with application to spinors,” Fortsch. Phys. 57, 367 (2009), arXiv:0903.2596 [hep-th].

[11] J. P. Wu, “Holographic fermions in charged Gauss-Bonnet black hole,” JHEP 07 (2011) 106, arXiv:1103.3982 [hep-th].

[12] J. P. Wu, “Some properties of the holographic fermions in an extremal charged dilatonic black hole,” Phys. Rev. D 84, 064008 (2011), arXiv:1108.6134 [hep-th].

[13] L. Q. Fang, X. H. Ge, X. M. Kuang, “Holographic fermions in charged Lifshitz theory”, Phys. Rev. D 86, 105037 (2012).

[14] L. Q. Fang, X. H. Ge, X. M. Kuang, “Holographic fermions with running chemical potential and dipole coupling”, Nucl. Phys. B 877, 807 (2013).

[15] X. M. Kuang, E. Papantonopoulos, B. Wang, J. P. Wu, “Formation of Fermi surfaces and the appearance of liquid phases in holographic theories with hyperscaling violation”, arXiv:1409.2945.

[16] K. Sugawara, T. Sato, S. Souma, T. Takahashi, M. Arai, and T. Sasaki, “Fermi Surface and Anisotropic Spin-Orbit Coupling of Sb(111) Studied by Angle-Resolved Photoemission Spectroscopy”, Phys. Rev. Lett. 96 046411 (2006).

[17] S. B. Dugdale, M. A. Alam, I. Wilkinson, R. J. Hughes, I. R. Fisher, P. C. Canfield, T. Jarlborg, G. Santi, “Nesting Properties and Anisotropy of the Fermi Surface of LuNi$_2$B$_2$C”, Phys. Rev. Lett. 83 (2003) 4824.

[18] L. Cheng, X. H. Ge and S. J. Sin, “Anisotropic plasma with a chemical potential and scheme-independent instabilities”, Phys. Lett. B 734 (2014) 116 arXiv:1404.1994[hep-th].

[19] L. Cheng, X. H. Ge and S. J. Sin, “Anisotropic plasma at finite $U(1)$ chemical potential”, JHEP 07 (2014) 083 arXiv:1404.5027[hep-th].

[20] D. Mateos and D. Trancanelli, “The anisotropic N = 4 super Yang-Mills plasma and its instabilities”, Phys. Rev. Lett. 107 (2011) 101601

[21] D. Mateos, D. Trancanelli, “Thermodynamics and instabilities of a strongly coupled anisotropic plasma”, JHEP 1107 (2011) 054.
[22] M. Edalati, K. W. Lo, P. W. Phillips, “Pomeranchuk instability in a non-Fermi liquid from holography”, Phys. Rev. D 86 (2012) 086003.

[23] Y. Ling, C. Niu, J. P. Wu, Z. Y. Xian, H. Zhang, “Holographic Fermionic Liquid with Lattices”, JHEP 07 (2013) 045, [arXiv:1304.2128]

[24] T. Senthil, Critical fermi surfaces and non-fermi liquid metals, Phys. Rev. B 78, 035103 (2008), [arXiv:0803.4009 [cond-mat.str-el]].

[25] T. Senthil, Theory of a continuous Mott transition in two dimensions, Phys. Rev. B 78, 045109 (2008), [arXiv:0804.1555 [cond-mat.str-el]].