Prediction trajectory of the fracture crack using the photoelasticity method

L B Shron1, V B Bogutski1, I S Tabolin2 and E E Yagyayev3

1Sevastopol State University, University st., 33, Sevastopol, 299053, Russian Federation
2Closed joint-stock company «Chelyabinsk interactive cable networks», Ordzhonikidze st. 54, Chelyabinsk, 454080, Russian Federation
3Crimean Engineering and Pedagogical University, Academic lane 8, 295015, Simferopol, Russian Federation

* shronlb@mail.ru

Abstract. This paper proposes a method for projecting the propagation of a fillet-weld fracture. The model is based on photoelasticity testing of stress-strain state combined with destructive testing the fracturing resistance of fillet welds. The paper identifies the correlation of weld geometry and stress concentration/gradiente. It shows that reconfiguring the flat body and/or reorienting the resultant external-load vector disrupts the symmetry in general, including the stress tensor field in the vicinity of the fracture apex. To project fracture propagation, this research uses differential geometry, in particular the properties of geodesic lines and gradient descent.

1. Introduction

Visualizing the isochrome field, i.e. the that of the difference of the primary stresses \(\sigma_1 - \sigma_2\), or equivalently, that of maximum tangent stresses \(\tau_{12}\) effective in the model plane, is an undeniable advantage of the photoelasticity method. Despite the dominance of numerical algorithms in the computational mechanics of deformable solids, photoelasticity does have its applications mainly thanks to the increased computing power. Its graphical nature provides qualitative visualization of the non-homogeneous field of stresses \(\sigma_1 - \sigma_2\) for the models of specified configuration and external load. Isochrome field suffices to quantify the factors of stress concentration and intensity without separating \(\sigma_1 - \sigma_2\). Papers [1, 2] well demonstrate the capabilities of this method for calculating the carrying capacity of T or lap joints with fillet weld and released residual stresses. If the problem required finding not only the concentration factor \(\sigma_1\) but also the primary-stress gradient \(\sigma_2\), then separating \(\sigma_1 - \sigma_2\) was done using two fields: isochrome and isopach [2]. Of course, obtaining extra fields (isopach, isocline) in addition to the isochrome field makes the method someone costlier and more difficult to use, as it requires not only special equipment, but also special skills. This justifies research into ways to synthesize strength properties based on isochrome field alone.

This paper seeks to project the propagation of a fracture by analyzing the isochrome field in statically loaded bearing structural elements given the metal is embrittled.
2. Materials and methods

Experimental results [1, 2] as well as the basic theses presented by Ye. M. Morozov [3] show that the trajectory of propagation of brittle and fatigue fractures depends on multiple factors, including the type of weld (T or lap), external-load application direction, and fillet weld configuration.

Figure 1 shows the diagrams of T and lap joints as symmetric halves placed on either side of the vertical axes, as well as the characteristic values, application, and direction of the static-load vector.

The experience of using such metal structures shows that fracturing first occurs at stress concentration spots: at the fillet weld root and/or where the weld transitions to the base metal.

Stress concentration first of all depends on the radii $\rho$ (the rounded lack-of-fusion apex radius and the fillet weld to base metal connection radius), on the relative size of the connected plates as well as on the fillet-weld configuration. Angle $\beta$ is recognized as the descriptor of the fillet-weld legs. At $\beta = \pi/4$ both legs are equal. The angle formed by the intersection of the free (plane) surfaces of the weld and base metal is $\alpha = \pi/2 - \beta$ (Figure 1). At low $\rho \to 0$, these stress concentrators transform into so-called fracture-like defects; at $\rho = 0$, they are classified in the plane problem of the elasticity theory as a canonical singular problem for a two-sided angle (wedge) [4]. In a polar coordinate system, the two-sided angle has the limits: $0 < r < \infty, 0 < \theta \leq \pi$. A particular case is the (infinite) stretching of a plate with an angular cut $\theta = (+/-)\gamma$, both sides of which are unloaded.

Lack of fusion of the T-beam wall and the structural gap between the plates of a lap joint form the two-sided angle $\gamma = (+/-)\pi$ with the apex at the fillet-weld root.

When loading a plate of homogeneous and isotropic material with a two-sided angular cut, singular asymptotic as shown in [4] holds for the elastic components of the stress tensor near the apex of the stationary fracture-like defect ($\rho \to 0$):

$$\sigma_{ij} = r^{-1/(1+n)}.$$

where $n = 1$ for a rectilinear fracture, $n > 1$ for a two-sided angle. The authors hereof believe that for V-shaped concentrators (two-sided angles), including heterogeneous joints, the most detailed research is presented in [5].

As shown in Figure 1b, the external-load vector is orthogonal to the plane of the fracture-like lack of fusion in the T-beam wall. For such positioning of the lack-of-fusion plane and the resultant external-load vector, the stress tensor $\sigma_{ij}$ near the apex of the acute ($\rho \to 0$) lack-of-fusion must correspond to a normal fracture. This implies the fracture will propagate in the lack-of-fusion plane as the stress intensity factor (SIF) $K_I$ reaches its critical value $K_{IC}$, meaning that for better fracture resistance, the horizontal leg of the fillet weld must be longer. In-situ experimentation has disproved these
assumptions. Figure 2 shows photographs of welded T-beams with fracture-like lack of fusion destroyed due to the embrittlement of the fillet-weld metal.

![Figure 2](image)

**Figure 2.** Fragments of brittle fracture of T joints: A – angle $\beta = 30^\circ$; B – $\beta = 45^\circ$; C – $\beta = 60^\circ$.

All the specimens had the same fillet-weld cross-section area and lack-of-fusion length $2\iota$, only differing in the angle $\beta$ (vertical-to-horizontal leg ratio). Apparently:
1) fracturing begins at the lack-of-fusion apex (A) at a specific angle $\theta_c$, which is not $0^\circ$;
2) the fracture trajectory is an arc that goes from the lack-of-fusion apex (A) and proceeds to the free surfaces of the fillet weld (B);
3) the arc curvature is reduced at smaller $\beta$;
4) increasing the angle $\beta$ raises the limit of fracturing load.

In case of fillet-weld metal embrittlement, the fracturing of welded T-specimens matches the criteria of generalized normal fracture [4]. From the qualitative point of view, the emergence of the fracture initiation angle $\theta_c$ (starting from the fracture apex in the system of polar coordinates, see Figure 1,A) is due to the emergences of SIF $K_{II}$. The specific value of $\theta_c$ depends on the $K_{II}/K_I$ ratio.

Note that exclusively normal fracturing is rare. Such defects may only occur in elements that feature double symmetry with respect to the coordinate axes. The load vector must be directed along the vertical axis while being orthogonal to the rectilinear defect, which in its turn must coincide with the horizontal coordinate axis. Reconfiguring the flat body and/or reorienting the resultant external-load vector disrupts the symmetry in general, including the stress tensor field in the vicinity of the fracture apex. This is shown in Figure 3,A. Apparently, the isochrome field is not symmetric with respect to the fracture plane, and the fracturing trajectory is not within the initial fracture plane. The line atop the isochrome field is the expected fracture propagation trajectory. The actual fracture propagation trajectory in shown in Figure 3,B and is limited by the arc $AB$. See the fracture initiation angle $\theta_c$. Apparently, both the expected trajectory and its actual counterpart are nearly coincident.

![Figure 3](image)

**Figure 3.** Fragments of T joint ($\beta = 45^\circ$) with internal lack of fusion (symmetric with respect to the vertical axis and stretching-loaded): (A) – isochrome field; (B) – brittle fracture of the fillet weld, $\theta_c$ is the fracture initiation angle.

Thus, the isochrome field suffices for finding the brittle fracture propagation trajectory to an engineering-appropriate accuracy.
In the loaded-model plane, the isochrome field comprises level lines, along each of which the primary-stress difference \( \sigma_1 - \sigma_2 \) has a constant value proportional to the isochrome number \( m \), i.e.:

\[
\sigma_1 - \sigma_2 = m
\]  

The parameter \( m \) starts at \( m = 0 \) and can only be an integer (1, 2, 3, etc.), corresponding to the complete blackout of bands. As a rule, the bands of black intensity are used to analyze the stress state of the model. Figure 3A shows the location of the isochrome \( m = 0 \). When loaded, a model of a light-sensitive material generates its (unique) plane scalar field (the black level line with an integer-valued isochrome order), which is a complex geometric object. To study that object, refer to the basic concepts of differential geometry [6, 7]. Note the following:

1. The colour intensity changes between two isochromes of integer-valued order monotonously and continuously in any direction, with the isochrome order being of a real value from the interval \([m, m + 1]\);

2. The field of stress difference \( \sigma_1 - \sigma_2 \) is continuous in simply-connected regions of photoelastic models (the so-called simple surface fragments).

Figure 4, A gives a schematic of a smooth surface fragment built within the isochromes \( m = 3 \) between the points A (lack-of-fusion apex) and the point C (weld to base metal transition, see Figure 3,A). The surface is a hyperbolic paraboloid (a «saddle»). The surface is configured parametrically. The parameters are the two coordinates \( u \) and \( v \) within the limits 1*2*A*C*3*4* of the simply-connected region \( U \). \( U \) belongs to the real plane \( \mathbb{R}^2 \), i.e. \( U \in \mathbb{R}^2 \). In Cartesian coordinates, each point of the surface \( D \) (see Figure 3,A) corresponds to a vector radius

\[
\vec{r} = \vec{r}(u, v) = \begin{bmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{bmatrix}.
\]  

Thus, a fragment of the surface \( D \) is an image of the region \( U \) as mapped per (3). The plane \( U \) is a map, and \( D \) is the mapped surface fragment, i.e. the mapping (3) projects the map \( U \) onto \( D \).

Figure 4,C shows a fragment of the parameterized surface \( D \) that «covers» the brittle fracture propagation trajectory (dotted line). The fracture apex is a special point that does not belong to \( D \). Note that \( D \) is both convex (Figure 4,A) and concave (Figure 4,C). The research team has analyzed the isochrome field and the in-situ test results, assuming that the brittle fracture trajectory is an orthogonal projection of the curve \( \chi \in D \) onto the phase plane, see Figure 4,C. The isochrome level lines formed by the intersection of \( D \) and also belong to the surface \( D \). The
curves \( \chi \) intersect the isochromes at right angles at the maximum curvature points, see Figures 4.A and 4.C. Thus, the curve \( \chi \) is a trajectory of gradient descent that illustrates the travel to a minimum in the direction of the fastest descent of the specified objective function. The function descent direction is defined by its anti-gradient. At the same time, the curve \( \chi \) is part of the internal geometry of the surface D and is its geodesic line. The release of the fracturing energy along the geodesic lines is the basis of the theory that stipulates the real fracture will propagate along the geodetics [8].

One property that makes the curve \( \chi \in D \) a geodetic is the zero-ness of the geodesic curvature vector \( \overrightarrow{k_g} \) at all of its points [7].

Consider a trough-shaped surface fragment, see Figure 4.C. Consider the concept of a ball-shaped material point forced to move freely along the surface D. The trajectory coincides with a smooth curve \( \chi \). At \( t = 0 \), the ball is at the point a (the fracture apex); it ends up at the point b (the free surface of the fillet weld). The movement of the material point is described by the vector function \( \overrightarrow{r} = \overrightarrow{r}(t) \), where the parameter is the time \( t \). The ball is affected by neither external forces nor by the friction, as the surface D is perfectly smooth. Then, the ball movement will meet Newton’s second law:

\[
m \cdot \overrightarrow{r''}(t) = -\overrightarrow{N},
\]

where \( \overrightarrow{r''}(t) \) is the second derivative by \( t \); \( \overrightarrow{N} \) is the normal response of the support at the point \( M(t) \). In Figure 5, the vector \( \overrightarrow{N} \) is collinear to the normal vector \( \overrightarrow{n} \) to the surface D at the point \( M(t) \). The vector normal \( \overrightarrow{n} \) is the product of orthogonal unit vectors \( [\overrightarrow{\tilde{t}}_1(t), \overrightarrow{\tilde{t}}_2(t)] \), tangent to the plane level line \( m_{i+1} \) and to the curve \( \chi \) at their intersection point \( M(t) \), respectively. The unit vector \( \overrightarrow{\tilde{r}}(t) = \overrightarrow{\tilde{r}}(t) / |\overrightarrow{\tilde{r}}(t)| \); the complanar vectors \( \overrightarrow{\tilde{t}}_1(t) \) and \( \overrightarrow{\tilde{t}}_2(t) \) are located in a plane tangent to D at \( M(t) \).

In general, the geodesic curvature vector \( \overrightarrow{k_g} \) is an orthogonal projection of the primary-curvature vector \( \overrightarrow{k\partial_1} \) onto the tangent plane to the surface D at the point \( M(t) \) [7]. Figure 4c shows the projection of the primary-curvature vector \( \overrightarrow{k\partial_1} \) of the level line \( m_{i+1} \) at the point \( M(t) \) of intersection with the curve \( \chi \). The intersection of curves at \( M(t) \) means the vectors \( \overrightarrow{\tilde{k}}_g \) and \( \overrightarrow{\tilde{t}}_2(t) \) are collinear. Level line of order \( m_{i+1} \) belongs to D but is not a geodetic as \( \overrightarrow{k_g} \neq 0 \). The vector \( \overrightarrow{k_g} = 0 \) if the primary-curvature vector of the curve \( \chi \) is collinear to the normal vector \( \overrightarrow{n} \) to D at \( M(t) \), i.e. \( k\partial_1 \parallel \overrightarrow{n} \).

Assume the lengths of the arcs \( s \) of the curves \( \chi \) and \( m_{i+1} \) as the natural parameter. Then for \( \chi \), the first derivative:

\[
\overrightarrow{\tilde{r}}_2(t) = \overrightarrow{\tilde{r}}_2(s) \cdot \frac{ds}{dt},
\]

the second derivative:

\[
\overrightarrow{\tilde{r}}_2''(t) = \overrightarrow{\tilde{r}}_2''(s) \cdot \left( \frac{ds}{dt} \right)^2 + \overrightarrow{\tilde{r}}_2'(s) \cdot \frac{d^2 s}{dt^2}.
\]

According [6], a tangent unit vector to the curve \( \chi \) equals \( \overrightarrow{\tilde{r}}_2(s) = \overrightarrow{\tilde{r}}_1(s) \), i.e. the primary-curvature vector \( k\partial_2(s) = \overrightarrow{\tilde{r}}_2(s) = \overrightarrow{\tilde{r}}_1(s) \). Now find the dot product of both sides of the equation (4) and the tangent unit vector \( \overrightarrow{\tilde{t}}_1(s) \). Since \( \overrightarrow{N} \perp \overrightarrow{\tilde{t}}_1(t) \), the right side of (4) is zero. Rewrite (6) equivalently:

\[
\overrightarrow{\tilde{r}}_2''(t) = k\partial_2(s) \cdot \left( \frac{ds}{dt} \right)^2 + \overrightarrow{\tilde{r}}_2'(s) \cdot \frac{d^2 s}{dt^2}.
\]

Since \( \overrightarrow{\tilde{t}}_1(t) \perp \overrightarrow{\tilde{t}}_2(t) \), then \( \langle \overrightarrow{\tilde{t}}_1(s), \overrightarrow{\tilde{t}}_2(s) \rangle = 0 \), thus: \( \langle k\partial_2(s), \overrightarrow{\tilde{t}}_1(s) \rangle = 0 \). This condition can be met if the vectors

\[ \tilde{g}_2(s) \perp \tilde{r}_1(s). \] In turn, \( \tilde{r}_1(s) \) lies in the plane tangent to the surface \( D \) at the point \( M(t) \), which means the vector \( \tilde{g}_2(s) \) is orthogonal to this tangent plane. Thus, \( \tilde{k} \tilde{g}_2(s) \| \tilde{n} \) and the curve \( \chi \) is geodetic.

The saddle-shaped fragment of the parameterised surface \( D \) shown in Figure 4 shows that the ball first moves from \( A \) along the gradient descent trajectory to the point \( O \), then follows the gradient ascent trajectory to the point \( C \); however, in an in-situ experiment the ball will inevitably leave the AOC “saddle” trajectory unless exerted external influence upon. The trough movement is however stable in a real setting, too. Therefore, we shall use another method to prove the curve \( \chi \) geodetic.

Consider that the points \( A \) and \( C \) inherently have a fixed weightless non-stretching thread of an arbitrary length. The thread lies freely on the surface \( D \) and is tangent to it for the entire length. The surface \( D \) is perfectly smooth. The thread is held in place by the initial tension, the absolute value of which is \( T_0 \). Assume that over a finite time \( t \), the absolute tension will reach the ultimate value \( T \). This will cause the thread to tangentially move along the surface \( D \) and reach equilibrium in such a position, at which any increase in the tension \( \Delta T \) will detach the thread from the surface \( D \) in the final segment of the arc \( \Delta s \).

The thread configuration is described by the vector function \( \tilde{r} = \tilde{r}(s) \) its position is shown in Figure 4a and is designated by the curve \( \chi \). Assume the tension vector \( \tilde{T} = T \cdot \tilde{r}(s) \) and write the full differential:

\[
dd T = dT \cdot \dd (\tilde{r}(s)) + T \cdot d \dd (\tilde{r}(s)).
\] (7)

In (7), the first term \( dT \cdot \dd (\tilde{r}(s)) = 0 \), as \( T = \text{const} \). The second term in (7) is structured as the expression (6.1), i.e.:

\[
dd T = T \cdot \tilde{k} \dd (\tilde{g}(s)) \cdot \left[ \frac{ds}{dt} \right]^2 dt + T \cdot \dd (\tilde{r}(s)) \cdot \frac{d^2 s}{dt^2} dt.
\] (8)

(8) means the vector \( d\tilde{T} \) is the resultant of the total of two orthogonal vectors plus the primary-normal unit axis \( \tilde{g}(s) \) and the tangent unit axis \( \tilde{r}(s) \). The first term of (8) is a vector codirectional with the primary-curvature vector \( \tilde{g}(s) \) of the curve \( \chi \), the effect of which is balanced by the response of the surface \( D \) that is collinear to the normal vector \( \tilde{n} \). Thus, \( k \tilde{g}(s) \perp \tilde{n} \) and the curve \( \chi \) is also a geodetic. Note that the thread will be “detached” at the point where the curvature \( k \) peaks (Figure 4a, point \( O \)).

Figure 5a shows the fracture trajectory of viscous weld metal.

**Figure 5.** Fragments of T joint (\( \beta = 30^\circ \)) with internal lack of fusion (symmetric with respect to the vertical axis and stretching-loaded): (a) viscous weld fracture; (b) isochrome field

Note the sloping line in the macrophotograph of the fracture (Figure 5a) that connects the points 1 and 2. This line delimits the plastic shear strain of the welded metal concentrated inside an irregularly shaped quadrangle with apices 1, 2, a, c. A considerable region in the weld metal within the triangle 1-2-
3 contains a zero-order isochrome ($m=0$, Figure 5b) in the photoelastic model and has virtually no resistance to viscous fracturing ($\tau_{12} = 0$).

**Conclusions**

Thus, the fracture trajectory is an arc that goes from the lack-of-fusion apex (a) and proceeds to the free surface (c) near the weld to base metal transition.

It has been found that the isochrome-field gradient descent trajectory is a projection of the geodetic $\chi$ onto the phase plane (the weld plane).

*In the loving memory of Sergey Yurievich Googe, PhD of Engineering, a brilliant researcher and practitioner; this dedication does not have to be explained to anyone who knew Mr. Googe, was his friend, coworker, or postgraduate coursemate.*

**References**

[1] S.Yu. Googe, I.S. Tabolin, Ye.I. Shiryaev, L.B. Shron, Application of the plane section method for defining of stress intensity factor, Vestnik of Kuzbass State Technical University. 2012. No. 1 (89). PP. 137-140.

[2] L.B. Shron, V.B. Bogutsky, E.E. Yagyayev, Peculiarities of stress concentrators at fracture origin points in fillet-weld joints (Analiz osobennostey kontsentratorov napryazheny v zonakh zarozhdeniya ustalostnykh treshchin v soedineniyah s uglovymi shvami), Notes of the Crimean Engineering and Pedagogical University. 2015. No. 2 (50). PP. 109-116.

[3] Morozov Ye. M. Is It Possible to Project the Propagation of Fracture Once and For All? (Vozmozhno li otskaniye trayektorii treshchiny srazu v sleme?) // World Community: problems and solutions (Mirovoye soobshchestvo: problemy i puti resheniya): collection of papers, Ufa: USPTU Publ., 2001, No. 11. PP. 27-38.

[4] Cherepanov G. P. Mechanics of Brittle Fracturing (Mekhanika khrupkogo razrushenia). — Moscow: Nauka, 1974. — 640 p.

[5] Gumerov A.K. Better Safety Assessment Methods for Trunklines with V-Shaped Stress Concentrators (Sovershenstvovaniye metodov otsenki bezopasnosti magistralnykh truboprovodov s V-obraznymi kontsentratorami napryazheny): author’s abstract of a PhD of Engineering thesis. — Ufa: Institute of Energy Transport Problems, 2009. — 24 p.

[6] Laptev G. F. Elements of Vector Calculus (Elementy vektornogo ischisleniya). — Moscow: Nauka, 1975. — 336 p.

[7] Norden A. P. A Short Course of Differential Geometry (Kratky kurs differentzialnoy geometrii). 2nd ed. — Moscow: Fizmatgiz, 1958. 244 p.

[8] Morozov Ye. M., Friedman Ya. B. Trajectories of Brittle Fractures as Geodesic Lines on a Body’s Surface (Trajektorii treshchini khrupkogo razrusheniya kak geodezicheskiye linii na poverkhnosti tela) // Report of of the USSR Academy of Sciences. 1961. Vol. 139, No. 1. PP. 87–90.