Light-Quark Decays in Heavy Hadrons

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Abstract

We consider weak decays of heavy hadrons (bottom and charmed) where the heavy quark acts as a spectator. These decays are heavily phase-space suppressed but may become experimentally accessible in the near future. These decays are interesting as a QCD laboratory to study the behaviour of the light quarks in the colour-background field of the heavy spectator.

Keywords: heavy-flavour conserving hadron decays

1. Introduction

Weak decays of heavy hadrons play an important role in shaping our understanding of heavy quark physics, see [1] and references therein. Aside from the decays where the heavy quark undergoes a weak transition, there is also a class of decays in which the heavy quark acts as a spectator and the light quark decays in a weak transition. Depending on phase space, these can be either $s \rightarrow u$ or in one case also $d \rightarrow u$ transitions.

Due to the very small phase space available in this class of decays, for charmed (strange) and bottom (strange) mesons only semi-electronic decays are possible. While the small phase space substantially suppresses these decay modes, making them difficult to be observed, the special kinematics allows for solid theoretical predictions, since all form factors need to be known only at the non-recoil point.

For some of the heavy baryons we can have - aside form the semi-electronic and semi-muonic decays - also nonleptonic (pionic) modes. However, due to the small phase space the pion is quite soft in the rest frame of the decaying baryon, which will make the observation of these modes difficult.

Since the $s \rightarrow u$ and $d \rightarrow u$ transitions have been investigated in all detail in ordinary beta decays as well as in kaon and hyperon decays, there are no expectations to become sensitive to any physics effects beyond the standard model in these heavy-flavour conserving weak processes. On the other hand, these decays could serve as an interesting cross check of our understanding of light quark physics, since the heavy quark in all cases acts as a spectator. Thus the physics case for an investigation of such processes is to test the behaviour of light-quark systems moving in the (static) colour-background of a heavy quark.

In the next section we first gather all the decays which are possible from the viewpoint of phase space and discuss the hadronic matrix elements for a weak transition of the light quarks. It turns out that the fact that we are basically at zero recoil (i.e. the velocity of the heavy quark does not change) allows to have on the one hand normalization statements for the form factors derived from the flavor symmetry of the light quarks, on the other hand the heavy quark spin symmetry allows us to obtain relations between various decays. We then first discuss the semi-electronic and semi-muonic decays for which we can get accurate predictions; in a second step we look at the pionic decays, which cannot be predicted that reliably; however, we obtain a few benchmark numbers from applying naive factorization.

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Table 1: List of heavy charm and ground-state baryons [3]. Mass for the $\Sigma_c^0$ baryon taken from Ref. [3] and masses for $\Xi_c^+$ and $\Xi_c^-$ baryons are taken from the latest LHCb measurement [4]. In the second column we list the total angular momentum $J$ and parity $P$ of the hadron and in the third column we give the total spin $s_f$ of the light degrees of freedom.

| Baryon   | Mass [MeV] | $J^P$ | $s_f$ | Quark Content | $I, I_J$ |
|----------|------------|-------|-------|---------------|----------|
| $\Lambda_c^+$ | 2286.46    | $1/2^+$ | 0     | $c(ud)_0$    | (0, 0)   |
| $\Sigma_c^+$ | 2453.98    | $1/2^+$ | 1     | $c(ud)_1$    | (1, 1)   |
| $\Sigma_c^0$ | 2452.9     | $1/2^+$ | 1     | $c(ud)_1$    | (1, 0)   |
| $\Sigma_c^0$ | 2453.74    | $1/2^+$ | 1     | $c(ud)_1$    | (1, 0)   |
| $\Sigma_c^{++}$ | 2517.9     | $3/2^+$ | 1     | $c(ud)_1$    | (1, 1)   |
| $\Sigma_c^+$ | 2517.5     | $3/2^+$ | 1     | $c(ud)_1$    | (1, 0)   |
| $\Sigma_c^{*0}$ | 2518.8     | $3/2^*$ | 1     | $c(ud)_1$    | (1, −1)  |
| $\Xi_c^+$ | 2467.8     | $1/2^+$ | 0     | $c(su)_0$    | (1/2, 1/2) |
| $\Xi_c^0$ | 2470.88    | $1/2^*$ | 0     | $c(su)_1$    | (1/2, −1/2) |
| $\Xi_c^0$ | 2575.6     | $1/2^+$ | 1     | $c(su)_1$    | (1/2, 1/2) |
| $\Xi_c^0$ | 2577.9     | $1/2^+$ | 1     | $c(sd)_1$    | (1/2, −1/2) |
| $\Xi_c^{*0}$ | 2645.9     | $3/2^*$ | 1     | $c(su)_1$    | (1/2, 1/2) |
| $\Xi_c^{*0}$ | 2645.9     | $3/2^*$ | 1     | $c(sd)_1$    | (1/2, −1/2) |
| $\Omega_c^+$ | 2695.2     | $1/2^+$ | 1     | $c(sx)_1$    | (0, 0)   |
| $\Lambda_b^0$ | 5619.5     | $1/2^+$ | 0     | $b(ud)_0$    | (0, 0)   |
| $\Sigma_b^+$ | 5810.3     | $1/2^+$ | 1     | $b(ud)_1$    | (1, 0)   |
| $\Sigma_b^0$ | 5811.3     | $1/2^+$ | 1     | $b(ud)_1$    | (1, 1)   |
| $\Sigma_b^{*0}$ | 5815.5     | $1/2^*$ | 1     | $b(ud)_1$    | (1, −1)  |
| $\Xi_b^+$ | 5949.3     | $3/2^+$ | 1     | $b(ud)_1$    | (1, 0)   |
| $\Xi_b^0$ | 5832.1     | $3/2^+$ | 1     | $b(uu)_1$    | (1, 1)   |
| $\Xi_b^{*0}$ | 5835.1     | $3/2^*$ | 1     | $b(ud)_1$    | (1, −1)  |
| $\Xi_b^+$ | 5793.1     | $1/2^+$ | 0     | $b(su)_0$    | (1/2, 1/2) |
| $\Xi_b^0$ | 5794.9     | $1/2^*$ | 0     | $b(su)_1$    | (1/2, −1/2) |
| $\Xi_b^{*0}$ | 5935.02    | $1/2^*$ | 1     | $b(su)_1$    | (1/2, −1/2) |
| $\Xi_b^-$ | 5949.3     | $3/2^*$ | 1     | $b(su)_1$    | (1/2, −1/2) |
| $\Xi_b^{*0}$ | 5955.33    | $3/2^*$ | 1     | $b(su)_1$    | (1/2, −1/2) |
| $\Omega_b^+$ | 6048.8     | $1/2^+$ | 1     | $b(sx)_1$    | (0, 0)   |

2. Heavy flavour conserving weak decays

Looking at the spectroscopy of the ground state mesons of bottom and charm we infer that only semi-electronic decays are allowed, if we assume that the heavy flavour remains conserved. The mass difference between the charged and neutral $D$ meson allows for a semi electronic decay through a $d \rightarrow u$ transition, all other decays we consider will be induced by an $s \rightarrow d$ transition.

Strange mesons with a heavy flavour can decay semi-electronically through an $s \rightarrow u$ transition; for the $B_s$-meson decay, the final state can be a $B$- or a $B^*$-meson, while for the $D_s$-meson the only possible final state is a $D$-meson, since the $D^*$ is too heavy. In all mesonic cases no hadronic decay is possible since the phase space is too narrow.

Table 2 shows the spectroscopy of heavy favoured baryonic ground states. From the point of view of the heavy mass limit, the spin of the heavy quark decouples, making the baryonic ground states particularly simple [5, 6]. They consist of a heavy quark, acting as a source of a static colour field, and a system of light degrees of freedom having either spin $s_f = 0$ or 1.

Out of these many baryons, only the $\Xi_c$, the $\Omega_c$ states as well as the $\Xi_b$, the $\Omega_b$ states can undergo a heavy flavour conserving weak transition. Unlike for the mesons, the phase space of the baryonic weak decays allows for a semi-muonic as well as for a hadronic decay with a pion in the final state.

Table 2 lists all possible heavy flavour weak decays for bottom and charm hadrons. The second column in the
Table 2: List of heavy flavour conserving weak decays as discussed in the text. The mass difference is \( \Delta m = \sqrt{(M - m)^2 - m^2} \) for the semi-muonic decays and \( \Delta m = M - m \) for all the other decays.

| Decay                                      | \( \Delta m \) [MeV] | \( J^{P} \rightarrow J'^{P} \) | \( s \rightarrow s' \) | Quark Transition |
|--------------------------------------------|----------------------|---------------------------------|-------------------------|------------------|
| Semi-electronic decays                     |                      |                                 |                         |                  |
| \( D_c^+ \rightarrow D^0 e^+ \nu \)      | 4.8                  | 0 \( \rightarrow \) 0          | 1/2 \( \rightarrow \) 1/2 | \( d \rightarrow u \) |
| \( D_s^+ \rightarrow D^0 e^+ \nu \)      | 103.5                | 0 \( \rightarrow \) 0          | 1/2 \( \rightarrow \) 1/2 | \( s \rightarrow u \) |
| \( B^0 \rightarrow B^- e^+ \nu \)        | 87.5                 | 0 \( \rightarrow \) 0          | 1/2 \( \rightarrow \) 1/2 | \( s \rightarrow u \) |
| \( B^0 \rightarrow B^- e^+ \nu \)        | 41.6                 | 0 \( \rightarrow \) 1          | 1/2 \( \rightarrow \) 1/2 | \( s \rightarrow u \) |
| \( \Xi_c^0 \rightarrow \Lambda^0 e^- \nu \) | 184.4                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 0 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Xi_c^+ \rightarrow \Sigma^+_c e^- \nu \) | 18.0                 | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 0 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Xi_c^+ \rightarrow \Sigma^{++}_c e^- \nu \) | 13.8                | 1/2\( ^+ \rightarrow \) 3/2\( ^+ \) | 0 \( \rightarrow \) 1 | \( s \rightarrow u \) |
| \( \Omega_c^0 \rightarrow \Xi^0 e^- \nu \) | 227.4                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 1 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Omega_c^0 \rightarrow \Xi^{+} e^- \nu \) | 119.7               | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 1 \( \rightarrow \) 1 | \( s \rightarrow u \) |
| \( \Omega_c^0 \rightarrow \Xi^0 e^- \nu \) | 49.3                 | 1/2\( ^+ \rightarrow \) 3/2\( ^+ \) | 1 \( \rightarrow \) 1 | \( s \rightarrow u \) |
| \( \Xi_b^0 \rightarrow \Lambda_b^0 e^- \nu \) | 175.4                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 0 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Omega_b^0 \rightarrow \Xi_b^0 e^- \nu \) | 255.7                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 1 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Xi_b^0 \rightarrow \Xi_b^0 e^- \nu \) | 99.5                 | 1/2\( ^+ \rightarrow \) 3/2\( ^+ \) | 1 \( \rightarrow \) 1 | \( s \rightarrow u \) |
| Semi-muonic decays                         |                      |                                 |                         |                  |
| \( \Xi_c^0 \rightarrow \Lambda^0 \mu^- \nu \) | 151.2                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 0 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Omega_c^0 \rightarrow \Xi^0 \mu^- \nu \) | 201.4                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 1 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Omega_c^0 \rightarrow \Xi^{+} \mu^- \nu \) | 56.1                 | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 1 \( \rightarrow \) 1 | \( s \rightarrow u \) |
| \( \Xi_b^0 \rightarrow \Lambda_b^0 \mu^- \nu \) | 140.0                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 0 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Omega_b^0 \rightarrow \Xi_b^0 \mu^- \nu \) | 232.8                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 1 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| Pionic decays                              |                      |                                 |                         |                  |
| \( \Xi_c^0 \rightarrow \Lambda^0 \pi^- \) | 184.4                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 0 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Xi_c^+ \rightarrow \Xi^+_c \pi^0 \) | 181.3                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 0 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Omega_c^0 \rightarrow \Xi^0 \pi^- \) | 227.4                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 1 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Omega_c^0 \rightarrow \Xi^0 \pi^0 \) | 224.3                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 1 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Xi_b^0 \rightarrow \Lambda_b^0 \pi^- \) | 175.4                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 0 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Xi_b^+ \rightarrow \Xi_b^0 \pi^0 \) | 173.6                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 0 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Omega_b^0 \rightarrow \Xi_b^0 \pi^- \) | 255.7                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 1 \( \rightarrow \) 0 | \( s \rightarrow u \) |
| \( \Omega_b^0 \rightarrow \Xi_b^0 \pi^0 \) | 253.9                | 1/2\( ^+ \rightarrow \) 1/2\( ^+ \) | 1 \( \rightarrow \) 0 | \( s \rightarrow u \) |

Table lists the mass differences of the initial and final state heavy hadrons. We note that all mass differences are large compared to the electron mass, so we can neglect the electron mass in the following, while we have to keep the pion and the muon mass.

2.1. Form factors for light-quark currents

To describe the decays shown in Table 2 we need matrix elements of light-quark currents with heavy-hadron states. The heavy quark is in these decays only a spectator and acts in the infinite-mass limit as a static source of colour. In other words, we need to look at the transition in light-quark system in the colour background created by the (static) heavy quark. This picture allows us to obtain information on the form factors.

The four-momenta of the initial \( H_i \) and final \( H_f \) heavy hadrons are \( p^i = Mv^i \) and \( p^f = m v^f \), respectively, and \( q^2 = (p - p')^2 \) is the momentum transfer squared from the hadronic to the leptonic systems. Instead of the momentum transfer squared we use the variable \( w = v \cdot v' \),

\[
w = \frac{M^2 + m^2 - q^2}{2Mm},
\]

(1)
where the kinematic boundaries are given by
\[ 1 \leq w \leq w_{\text{max}} = \frac{M^2 + m^2}{2Mm} = 1 + \frac{(M - m)^2}{2Mm} \sim 1, \]  
(2)
showing that the range of \( w \) is tiny for all decays listed in Table 2 since in all cases \((M - m) \ll M\). Assuming that the form factors are slowly varying functions of the kinematic variables, we may replace all form factors by their values where
\[ u \]

the usual symmetry, which is generated by the currents in (3) and (4). However, this symmetry is spontaneously broken to
while the light-quark flavour symmetry does not tell us anything about
\[ f \]
transition of a light state with the quantum number of the light quark in the meson polarization vector of the excited final state meson
state in
\[ f \]
showing that the range of \( f \) is tiny for all decays listed in Table 2, since in all cases
Thus in the following we only need to obtain some insight into the form factor in the region \( v \sim v' \).

For the mesonic decays we define the relevant form factors as \((q, q' = u, d, s)\)
\[
\frac{\langle H_f(p')|\bar{q}'\gamma_\mu q|H_f(p)\rangle}{\sqrt{Mm}} = (v + v')_\mu \Phi_s(w) + \ldots, \]  
(3)
\[
\frac{\langle H_A(p', e)|\bar{q}'\gamma_\mu \gamma_5 q|H_f(p)\rangle}{\sqrt{Mm}} = i(w + 1)\epsilon_\mu^s \Phi_A(w) + \ldots, \]  
(4)
where we only show the form factors relevant for the leading contribution in the limit \( v \to v' \). In Eq. (4), \( \epsilon_\mu \) is the polarization vector of the excited final state meson \( H'(p', e) \). Taking the heavy quark as static, we need to look at the transition of a light state with the quantum number of the light quark in the meson \( H_f \) into the corresponding final light state in \( H_f \) via the vector and axial-vector (light quark) current.

Furthermore, despite of the heavy quark's colour field, the light quark system has an \( SU(3) \times SU(3) \) chiral symmetry, which is generated by the currents in (3) and (4). However, this symmetry is spontaneously broken to the usual \( SU(3)_{L,R} \) flavour symmetry of the light quarks. Assuming that this symmetry is exact, we derive from the conservation of the vector light-quark current the normalization statement
\[ \Phi_s(1) = 1, \]  
(5)
while the light-quark flavour symmetry does not tell us anything about \( \Phi_A(1) \).

The case of the baryonic decays is more interesting, since the light-quark systems are composed of two valence quarks. For the case of a transition between two “A-like” heavy baryons (i.e. baryons in a \( h(qq')_0 \) configuration) the light quark current mediates a transition between two spinless states. Furthermore, in the heavy mass limit the spin of the baryons is the spin of the heavy quark, which in the infinite mass limit remains unchanged; consequently, the relevant matrix elements in the region \( v \sim v' \) can be written in terms of a form factor \( B(w) \) as
\[
\langle \Xi_H(v, s) | \gamma_\mu | \Lambda_H(v', s') \rangle = \bar{u}_v(v, s)u_{A}(v', s')B(w)(v + v')_\mu + \ldots, \]  
(6)
\[
\langle \Xi_H(v, s) | \gamma_\mu \gamma_5 u | \Lambda_H(v', s') \rangle = 0 + \ldots, \]  
(7)
where the ellipses denote subleading contributions in the limit \( v \to v' \).

The light degrees of freedom \( s_L \) in the “A-like” heavy baryons form a colour anti-triplet as well as an anti-triplet with respect to the flavour symmetry \( SU(3)_{L,R} \) of the light quarks. By the same argument as for the mesonic case, one obtains a normalization statement for the form factor \( B(w) \),
\[ B(1) = 1. \]  
(8)

With the same reasoning we can obtain some insight into the form factors for the transition from a “A-like” heavy baryon to a “Σ-like” heavy baryon, i.e. baryons in a \( h(qq')_1 \) configuration. In the heavy mass limit, the heavy quark spin remains unchanged, and the amplitude is determined by the transition of the 0* state of the light degrees of freedom into a 1* state. In this way we get for \( v \sim v' \),
\[
\langle \Xi_H(v, s) | \gamma_\mu \gamma_5 u | \Sigma_H(v', s') \rangle = \bar{u}_v(v, s)u_f(v', s')\epsilon_\mu A(w) + \ldots, \]  
(9)
\[
\langle \Xi_H(v, s) | \bar{\gamma}_\mu u | \Sigma_H(v', s') \rangle = 0 + \ldots, \]  
(10)
where \( u_v (u_f) \) is the spinor of the heavy quark in the initial (final) state, \( A(w) \) is an unknown form factor, and the ellipses again denote subleading terms.
We have not yet specified the spin of the “Σ-like” heavy baryon which can be either 1/2 or 3/2. Projecting out the relevant components by combining the polarization vector of the light degrees of freedom $\epsilon'_c$ with the heavy quark spin $[S \otimes S]$,

$$\psi^{(3/2)}_\mu = \epsilon'_c \left[ \delta_\mu - \frac{1}{3} (\gamma_\mu + v_\mu') y_\nu^\nu \right] u_\nu (v', s') = R^{3/2}_\mu (v', s'),$$

$$\psi^{(1/2)}_\mu = \epsilon'_c \left[ \frac{1}{3} (\gamma_\mu + v_\mu') y_\nu^\nu \right] u_\nu (v', s') = \frac{1}{\sqrt{3}} (\gamma_\mu + v_\mu') y_5 u_5 \sqrt{1/2} (v', s'),$$

we get for the relevant matrix elements from Eq. (9),

$$\langle \Xi_H (v, s) | 3 y_\mu y_5 u | \Xi^{(3/2)}_H (v', s') \rangle = \bar{u}_{\Xi}(v, s) R^{3/2}_\mu (v', s') A(w) + \ldots,$$

$$\langle \Xi_H (v, s) | 3 y_\mu y_5 u | \Xi^{(1/2)}_H (v', s') \rangle = \frac{1}{\sqrt{3}} \bar{u}_{\Xi}(v, s) (\gamma_\mu + v_\mu') y_5 u_5 \sqrt{1/2} (v', s') A(w) + \ldots,$$

where $R^{3/2}_\mu$ is the Rarita-Schwinger field for the spin 3/2 baryon and $u^{1/2}_\Xi$ is the spinor for the spin 1/2 baryon. Note also that we have replaced the heavy quark spin of the initial state with the one of the “Σ-like” heavy baryon of the initial state.

Close to $w = 1$ we can replace $A(w)$ by $A(1)$; however, in this case we do not have a normalization statement, since the axial current generates a broken symmetry. Still due to the heavy quark’s spin symmetry we get the same factor $A(1)$ for both the spin 1/2 and the spin 3/2 case.

Finally, the heavy Ω-baryons also decay weakly, so we also have the case of a colour-antitriplet $1^+$ state decaying into a heavy Ξ or Ξ′ baryon. For the case of a $1^+ \rightarrow 0^+$ transition we get the same structure as for the $0^+ \rightarrow 1^+$ (up to complex conjugation), while the case $1^+ \rightarrow 1^+$ needs a new discussion.

For this we start again form the heavy mass limit and note that the heavy quark spin remind unchanged. The underlying $1^+ \rightarrow 1^+$ transition via a vector current is usually described in terms of six form factors out of which five vanish as $v \rightarrow v'$. The transition amplitudes via the axial vector has to have a Levi-Civita-tensor and hence will vanish for $v \rightarrow v'$. To this end we get in terms of a form factor $C(w)$,

$$\langle \Omega_H (v, s) | 3 y_\mu y_5 u | \Xi^{(3/2)}_H (v', s') \rangle = \bar{u}_\Omega(v, s) u_\mu (v', s') (\epsilon^* \cdot \epsilon') (v_\mu + v'_\mu) C(w) + \ldots, \quad (15)$$

$$\langle \Omega_H (v, s) | 3 y_\mu y_5 u | \Xi^{(1/2)}_H (v', s') \rangle = 0 + \ldots, \quad (16)$$

where the ellipses denote terms which vanish as $v \rightarrow v$.

Again we have not yet specified the total spin of the Ω baryons. While the initial $\Omega_H$ will have total spin 1/2, the final states can either be spin 1/2 or 3/2. Using Eqs. (11) and (12), we can project out the relevant components and obtain

$$\langle \Omega_H (v, s) | 3 y_\mu y_5 u | \Xi^{(3/2)}_H (v', s') \rangle = - \frac{1}{\sqrt{3}} \bar{u}_\Omega(v, s) y_5 (\gamma^\rho + v^\rho) R^{3/2}_\mu (v', s') (v + v'_\mu) C(w) + \ldots, \quad (17)$$

$$\langle \Omega_H (v, s) | 3 y_\mu y_5 u | \Xi^{(1/2)}_H (v', s') \rangle = - \frac{1}{3} \bar{u}_\Omega(v, s) y_5 (\gamma^\rho + v^\rho) (\gamma_\mu + v'_\mu) y_5 u_5 (v', s') (v' + v'_\mu) C(w) + \ldots. \quad (18)$$

With the same arguments as above, we can replace $C(w)$ by $C(1)$ in the limit $w \rightarrow 1$. Since the transition proceeds through the vector current, and the light quark states in the initial and final state belong to the same $SU(3)_{f \otimes R}$ multiplet, we infer

$$C(1) = 1. \quad (19)$$

2.2. Semi-electronic decays with conserved heavy flavor

In this section we will calculate the decay rates of heavy-flavour conserving semi-leptonic decays. Table 2 lists all the possible semi-electronic heavy-flavour conserving decays of bottom and charm hadrons. The differential decay rates for exclusive semileptonic decays are in general given by

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M^5}{192\pi^3} |V_{CKM}|^2 \sqrt{w^2 - 1} P(w), \quad (20)$$

where $M$ is the mass of the heavy quark and $P(w)$ is the quark wave function at the mass shell, $w = (p_f \cdot k)/(m_H + m_f)$.

Finally, we close with a discussion on the magnitude of the semi-electronic decay rates. While the semi-electronic decay rates for charm hadron are typically smaller than the corresponding rates for bottom hadrons, the suppression is less pronounced for the semi-electronic heavy-flavour conserving decays.
where

\[ P(w) = H_{\mu\nu}(v, v')L^{\mu\nu}(v, v'), \]  

(21)

with \( H_{\mu\nu} \) and \( L_{\mu\nu} \) are the hadronic and leptonic tensors, respectively.

The integration over \( w \) can be performed when setting \( w = 1 \) in the hadronic form factors. To this end, it is useful to expand in the small velocity difference

\[ v' = v - \Delta, \quad \Delta = v - v'. \]

The leptonic tensor becomes for \( q = Mv - mv' = (M - m)v + m\Delta \), neglecting the electron mass

\[ L_{\mu\nu} = g_{\mu\nu}q^2 - q_{\mu}q_{\nu} = (M - m)^2(g_{\mu\nu} - v_{\mu}v_{\nu}) - 2Mm g_{\mu\nu}(w - 1) - m(M - m)(\Delta_{\mu}v_{\nu} + v_{\mu}\Delta_{\nu}) - m^2\Delta_{\mu}\Delta_{\nu}. \]  

(22)

We shall compute the total rate, including only the leading term in the mass difference \( (M - m) \). The integration over \( w \) yields the expressions

\[
\int_{w_{\text{max}}} dw \left( w^2 - 1 \right) = \frac{(M - m)^3}{3M^4} + O\left(\frac{(M - m)^4}{M^5}\right),
\]

(23)

\[
\int_{w_{\text{max}}} dw (w - 1) \left( w^2 - 1 \right) = \frac{(M - m)^5}{10M^5} + O\left(\frac{(M - m)^6}{M^6}\right),
\]

(24)

\[
\int_{w_{\text{max}}} dw (w - 1)^2 \left( w^2 - 1 \right) = \frac{(M - m)^7}{28M^6} + O\left(\frac{(M - m)^8}{M^7}\right),
\]

(25)

\[
\int_{w_{\text{max}}} dw (w - 1)^n \left( w^2 - 1 \right) = O\left(\frac{(M - m)^{2n+3}}{M^{2n+3}}\right),
\]

(26)

where \( w_{\text{max}} \) is given by Eq. (24). This shows that a one power of \((w - 1)\) in the differential rate counts as two powers of \((M - m)\) in the total rate. Hence, looking at the expansion (22) of the leptonic tensor we note that the leading terms of \( L_{\mu\nu} \) are already of order \((M - m)^2\). Note that, depending on the hadronic tensor, even the last term involving \( \Delta_{\mu}\Delta_{\nu} \) needs to be kept, since \( \Delta^2 = 2v_\cdot\Delta = 2(1 - w) \sim (M - m)^2 \).

For the hadronic tensor we need to include only the leading term with \( \Delta = 0 \), which is in all cases of order \((M - m)^3\). The simplest process is the decay \( 0^- \rightarrow 0^- \) between ground states, where we have a light quark transition in the background field of the heavy quark. Using the discussion from the previous section, we insert for the hadronic tensor Eq. (6)

\[ H_{\mu\nu} = 4M^2v_{\mu}v_{\nu}. \]  

(27)

Inserting the integral (24), we get

\[ \Gamma^{0^- \rightarrow 0^-} = \frac{G_F^2}{60\pi^3}\left| V_{\text{CKM}} \right|^2(M - m)^5. \]  

(28)

For the transition \( 0^- \rightarrow 1^- \) mesons we obtain for the hadronic tensor from (4)

\[ H_{\mu\nu} = 4M^2|\Phi_A(1)|^2 \sum_{\text{Pol}} \epsilon_\mu\epsilon_\nu = 4M^2|\Phi_A(1)|^2(g_{\mu\nu} - v_{\mu}v_{\nu}). \]  

(29)

Using the integrals (23) and (24), and keeping only the leading order, we get using (4) for the total decay rate

\[ \Gamma^{0^- \rightarrow 1^-} = \frac{G_F^2}{20\pi^3}\left| V_{\text{CKM}} \right|^2(M - m)^5|\Phi_A(1)|^2. \]  

(30)
Table 3: Branching ratios for semileptonic meson decays as discussed in the text.

| Mode          | Decay Rate [GeV] | Branching Ratio |
|---------------|------------------|-----------------|
| $D^+ \rightarrow D^0 e^+ \nu$ | $1.72 \times 10^{-20}$ | $2.71 \times 10^{-14}$ |
| $D_s^+ \rightarrow D^0 e^+ \nu$ | $4.40 \times 10^{-20}$ | $3.34 \times 10^{-8}$ |
| $B_s^+ \rightarrow B^- e^+ \nu$ | $1.90 \times 10^{-20}$ | $4.37 \times 10^{-8}$ |
| $B_s^0 \rightarrow B^- e^+ \nu$ | $1.38 \times 10^{-21}$ | $3.17 \times 10^{-9}$ |

For our numerical estimates of the decay rates and branching ratios for the mesonic semileptonic decays shown in Table 3 we shall set $\Phi_a(1) = 1$. Note that the result for $\Phi_a(1) = 1$ just reflects spin counting, furthermore, the sum of the two rates is just the total, spin-summed decay rate for the spin 1/2 light system decaying in the colour background of the heavy quark. Note that the $D^+ \rightarrow D^0$ decay is a $d \rightarrow u$ transitions, while all other decays are $s \rightarrow u$.

With the same method we can discuss the semi-electronic decays of heavy baryons. As discussed above the light degrees of freedom are more complicated in this case. For this reason we introduce the notation, where the superscript denotes the spin-parity of the baryon transitions, while the subscripts denote the spin-parity of the corresponding degrees of freedom.

For the final states with a “$\Sigma$-like” baryon we obtain from Eqs. (14) and (15).

\[
\Gamma_{1/2^+ \rightarrow 3/2^+}^{0^- \rightarrow 1^+} = \frac{G_F^2 |V_{CKM}|^2}{30 \pi^3} (M - m)^5, \quad (32)
\]

where we again note that the sum of the two rates is just the result we obtained for the mesonic $0^- \rightarrow 0^-$ transition, which is not surprising, since this is just a spinless system of light degrees of freedom decaying in the colour-background of the heavy quark.

For the final states with a “$\Xi$-like” baryon we obtain from Eqs. (14) and (15),

\[
\Gamma_{1/2^- \rightarrow 3/2^-}^{0^- \rightarrow 1^+} = \frac{G_F^2 |V_{CKM}|^2}{30 \pi^3} (M - m)^5 |A(1)|^2, \quad (33)
\]

where we again note that the sum of the two rates is just the result we obtained for the mesonic $0^- \rightarrow 1^-$ transition. Again this is due to spin counting, since in both decays we observe a transition of a light $0^-$ state into a light $1^-$ state, however, with different spin combinations with the heavy quark.

Finally, utilizing (17) and (18) we find for final states with a “$\Xi$-like” baryon,

\[
\Gamma_{1/2^+ \rightarrow 1/2^-}^{0^- \rightarrow 1^+} = \frac{G_F^2 |V_{CKM}|^2}{15 \pi^3} (M - m)^5, \quad (34)
\]

\[
\Gamma_{1/2^- \rightarrow 3/2^-}^{0^- \rightarrow 1^+} = O((M - m)^5). \quad (35)
\]

where the last line means that this transition has an additional suppression factor $(M - m)^2/M^2$ compared to the other decays, the rates of which are all of the order $G_F^2 (M - m)^5$. Since we only considered the leading terms of the form factors for $v \sim v'$, we cannot obtain a result for these decay on the basis of the discussion in section 2.1.

For our numerical estimates we shall set $|A(1)|^2 = 1$; in Table 4 we list the branching ratios for possible semi-electronic baryon decays with conserved heavy flavour.

2.3. Semi-Muonic Decays

For a few of the baryonic decays phase space is large enough to allow for semi-muonic decay. In this case we have to take into account the mass $m_\mu$ of the muon in the leptonic tensor

\[
L_{\mu\nu} = \frac{(q^2 - m_\mu^2)^2(2q^2 + m_\mu^2)}{2q^2} g_{\mu\nu} = \frac{(q^2)^3 - 3m_\mu^2 q^2 + 2m_\mu^6}{(q^2)^3} q_\mu q_\nu, \quad (36)
\]
The decay rate for the decays is given by

\[ 2.4. Non-leptonic (pionic) decays \]

The pionic decays listed in Table 2 are more difficult to predict, since the pion is soft and methods based on some kind of factorization are doomed to fail. The decay rate for the decays is given by

\[ \Gamma = \frac{1}{16\pi M^2} \frac{1}{2} \sum_{\text{spins}} |\mathcal{A}|^2 \sqrt{\lambda(M^2, m^2, m_\pi^2)} \],

(37)

where

\[ \lambda(M^2, m^2, m_\pi^2) = \left[ M^2 - (m + m_\pi)^2 \right] \left[ M^2 - (m - m_\pi)^2 \right] \]

(38)

is the Källén function and \( \mathcal{A} \) is the amplitude for the transition under consideration:

\[ \mathcal{A} = \langle B_i | H_{\text{eff}} | B_f \rangle \],

(39)

and \( M \) (\( m \)) is the mass of the initial (final) state baryon \( B_i \) (\( B_f \)).

The effective Hamiltonian is the usual \( \Delta S = \pm 1 \) weak-transition Hamiltonian, which consists of a \( \Delta I = 1/2 \) and a \( \Delta I = 3/2 \) piece

\[ H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{us} V_{ud}^{*} \left[ C_1 (\bar{\delta}_L \gamma_\mu u_L)(\bar{u}_L \gamma_\mu d_L) + C_2 (\bar{\delta}_L \gamma_\mu d_L)(\bar{u}_L \gamma_\mu u_L) \right] + \text{QCD penguins}. \]

(40)

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(40)

Table 4: Decay rates and branching ratios for semi-electronic baryon decays as explained in the text.

| Mode          | Decay Rate [GeV] | Branching Ratio |
|---------------|------------------|-----------------|
| \( \Xi^- \rightarrow \Lambda^0 \mu^- \bar{\nu} \) | \( 7.91 \times 10^{-19} \) | \( 1.35 \times 10^{-3} \) |
| \( \Omega^- \rightarrow \Xi^0 \mu^- \bar{\nu} \) | \( 3.74 \times 10^{-24} \) | \( 2.36 \times 10^{-7} \) |
| \( \Omega^- \rightarrow \Xi^0 e^- \bar{\nu} \) | \( 3.63 \times 10^{-19} \) | \( 3.81 \times 10^{-8} \) |
| \( \Xi^- \rightarrow \Lambda^0 e^- \bar{\nu} \) | \( 1.49 \times 10^{-20} \) | \( 1.57 \times 10^{-18} \) |
| \( \Xi^- \rightarrow \Lambda^+ e^- \bar{\nu} \) | \( 6.16 \times 10^{-19} \) | \( 1.46 \times 10^{-9} \) |
| \( \Omega^- \rightarrow \Xi^0 e^- \bar{\nu} \) | \( 4.05 \times 10^{-18} \) | \( 6.78 \times 10^{-6} \) |
| \( \Omega^- \rightarrow \Xi^0 e^- \bar{\nu} \) | \( 3.27 \times 10^{-28} \) | \( 5.47 \times 10^{-16} \) |

with \( q = Mv - m' v' \).

The muon mass is of the same order as the mass difference \( (M - m) \) between the initial and the final state baryon, and thus an expansion in \( (M - m) \) as in the massless case is spoiled by the presence of the ratio \( m_\mu/(M - m) = O(1) \). Hence we perform the integration over the phase space after contracting the leptonic tensor (36) with the hadronic tensors taken at \( v = v' \) without the expansions (23) and (24) performed in the massless case. The results for the rates and branching fractions are shown in Table 5. It is interesting to note that the branching ratios for the semi-muonic channels are not that much smaller as it is suggested by phase space; this effect is due to the presence of the muon mass in the leptonic tensor.

### Table 4: Decay rates and branching ratios for semi-electronic baryon decays as explained in the text.

| Mode          | Decay Rate [GeV] | Branching Ratio |
|---------------|------------------|-----------------|
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| \( \Omega^- \rightarrow \Xi^0 \mu^- \bar{\nu} \) | \( 7.1 \times 10^{-19} \) | \( 7.4 \times 10^{-8} \) |
| \( \Omega^- \rightarrow \Xi^0 \mu^- \bar{\nu} \) | \( 1.0 \times 10^{-21} \) | \( 1.1 \times 10^{-10} \) |
| \( \Xi^- \rightarrow \Lambda^0 \mu^- \bar{\nu} \) | \( 9.1 \times 10^{-20} \) | \( 2.2 \times 10^{-7} \) |
| \( \Omega^- \rightarrow \Xi^0 \mu^- \bar{\nu} \) | \( 1.7 \times 10^{-18} \) | \( 2.8 \times 10^{-6} \) |
Naive factorization is not expected to work, but may serve as a benchmark; ignoring the possible effects from the QCD penguins we get for \( B_i \to B_f \pi \) in naive factorization,

\[
\mathcal{A} = \frac{G_f}{\sqrt{2}} f_\pi q_\mu \left( B_i |\bar{q} q^\mu (1 - \gamma_5) u| B_f \right) a_{if},
\]

where \( f_\pi = (130.41 \pm 0.20) \text{ MeV} \) is the pion decay constant [3] and \( a_{if} \) is a combination of Wilson coefficients calculated from the usual naive factorization prescription: \( a_{if} = C_1 + C_2/3 \) for the colour allowed decays, and \( a_{if} = C_2 + C_1/3 \) for the colour-suppressed ones.

Making use of the form factors discussed in Sec. 2.1 we obtain in naive factorization for a pionic transition between "\( \Lambda \)-like" baryons Eq. (3),

\[
\Gamma(\Xi^0 \to \Lambda^0 \pi^-) = \frac{G_F^2 f_\pi^2 |V_{\text{CKM}}|^2}{2\pi M} (M - m)^2 \sqrt{\lambda(M^2, m^2, m_\pi^2)} \left( C_1 + \frac{1}{3} C_2 \right)^2,
\]

\[
\Gamma(\Xi^+ \to \Lambda^+ \pi^0) = \frac{G_F^2 f_\pi^2 |V_{\text{CKM}}|^2}{4\pi M} (M - m)^2 \sqrt{\lambda(M^2, m^2, m_\pi^2)} \left( C_2 + \frac{1}{3} C_1 \right)^2,
\]

and similar relations for the isodoublet of \( \Xi_b \) baryons.

Likewise, we find for a pionic transition from a \( \Omega_b \)-baryon to a "\( \Sigma \)-like" baryon the relations

\[
\Gamma(\Omega^0_b \to \Sigma^0 \pi^0) = \frac{G_F^2 f_\pi^2 |V_{\text{CKM}}|^2}{6\pi M} \left( M^2 - m^2 \right)^2 \sqrt{\lambda(M^2, m^2, m_\pi^2)} \left( C_1 + \frac{1}{3} C_2 \right)^2,
\]

\[
\Gamma(\Omega^0_b \to \Xi^0 \pi^0) = \frac{G_F^2 f_\pi^2 |V_{\text{CKM}}|^2}{12\pi M} \left( M^2 - m^2 \right)^2 \sqrt{\lambda(M^2, m^2, m_\pi^2)} \left( C_2 + \frac{1}{3} C_1 \right)^2,
\]

and the analogous relations for the \( \Omega_c \) decays. Table 6 lists the benchmark values obtained in naive factorization, where we have set the combinations of Wilson coefficients \((C_1 + C_2/3)\) and \((C_2 + C_1/3)\) equal to unity.

However, on the basis of what we known from kaon decays we do not expect the values shown in Table 6 to be realistic. More appropriate is, also with respect to a comparison with the kaon system, an isospin decomposition of the amplitudes. To this end, the relevant effective Hamiltonian consist of an isospin 1/2 and an isospin 3/2 contribution:

\[
\mathcal{H}_{\text{eff}} = \mathcal{H}_{1/2} + \mathcal{H}_{3/2}.
\]

The heavy \( \Xi \) states form an isodoublet, while the heavy \( \Lambda \) baryons are isosinglets, thus the final state isospin in the heavy \( \Xi \) decays into heavy \( \Lambda \) baryons is determined by the pion. Thus we get for the \( \Xi_b \) system

\[
\left\langle \Xi^+_b |\mathcal{H}_{\text{eff}}| \Lambda^+_\pi^0 \right\rangle = -\frac{1}{\sqrt{6}} \left( \mathcal{A}_{1/2}^{(c)} + \mathcal{A}_{3/2}^{(c)} \right),
\]

\[
\left\langle \Xi^0_b |\mathcal{H}_{\text{eff}}| \Lambda^+_\pi^0 \right\rangle = -\frac{1}{\sqrt{3}} \mathcal{A}_{1/2}^{(c)} + \frac{1}{\sqrt{12}} \mathcal{A}_{3/2}^{(c)},
\]

Table 6: Branching ratios for pionic baryon decays as explained in the text.

| Mode       | Decay Rate [GeV] | Branching Ratio [10^-3] |
|------------|-----------------|-------------------------|
| \( \Xi^+_b \to \Lambda^+_\pi^- \) | 1.30 \times 10^{-16} | 0.022 |
| \( \Xi^+_b \to \Lambda^+_\pi^0 \) | 6.31 \times 10^{-17} | 0.042 |
| \( \Omega^0_b \to \Xi^+_\pi^- \) | 1.16 \times 10^{-15} | 0.121 |
| \( \Omega^0_b \to \Xi^+_\pi^0 \) | 5.62 \times 10^{-16} | 0.059 |
| \( \Xi^+_b \to \Lambda'^+_\pi^- \) | 1.11 \times 10^{-16} | 0.262 |
| \( \Xi^+_b \to \Lambda'^+_\pi^0 \) | 5.58 \times 10^{-17} | 0.126 |
| \( \Omega^0_b \to \Xi'^+_\pi^- \) | 1.87 \times 10^{-15} | 3.116 |
| \( \Omega^0_b \to \Xi'^+_\pi^0 \) | 9.23 \times 10^{-16} | 1.543 |
and likewise for the $\Xi_b$ system:

$$\langle \Xi_b^0 | H_{\text{eff}} | \Lambda_b^0 \pi^0 \rangle = -\frac{1}{\sqrt{6}} (\mathcal{A}_{1/2}^{(b)} + \mathcal{A}_{3/2}^{(b)}) ,$$  \hspace{1cm} (49) \\
$$\langle \Xi_b^- | H_{\text{eff}} | \Lambda_b^0 \pi^- \rangle = -\frac{1}{\sqrt{3}} \mathcal{A}_{1/2}^{(b)} + \frac{1}{\sqrt{12}} \mathcal{A}_{3/2}^{(b)} .$$  \hspace{1cm} (50)

The heavy $\Omega$ baryons are isosinglets, while the finals states consisting of a heavy $\Xi$ baryon and a pion need to be decomposed into its $I = 1/2$ and $I = 3/2$ components: for the $\Omega^+$ system we get

$$\langle \Omega^+_c | H_{\text{eff}} | \Xi_c^0 \pi^0 \rangle = -\frac{1}{\sqrt{6}} (\mathcal{A}_{1/2}^{(c)} + \mathcal{A}_{3/2}^{(c)}) ,$$  \hspace{1cm} (51) \\
$$\langle \Omega^+_c | H_{\text{eff}} | \Xi_c^+ \pi^- \rangle = \frac{1}{\sqrt{3}} \mathcal{A}_{1/2}^{(c)} + \frac{1}{\sqrt{12}} \mathcal{A}_{3/2}^{(c)} ,$$  \hspace{1cm} (52)

and for the $\Omega_b$ system likewise

$$\langle \Omega_b^- | H_{\text{eff}} | \Xi_b^0 \pi^0 \rangle = -\frac{1}{\sqrt{6}} (\mathcal{A}_{1/2}^{(b)} + \mathcal{A}_{3/2}^{(b)}) ,$$  \hspace{1cm} (53) \\
$$\langle \Omega_b^- | H_{\text{eff}} | \Xi_b^- \pi^- \rangle = \frac{1}{\sqrt{3}} \mathcal{A}_{1/2}^{(b)} - \frac{1}{\sqrt{12}} \mathcal{A}_{3/2}^{(b)} .$$  \hspace{1cm} (54)

This isospin decomposition has a priori no predictive power; nevertheless one may speculate, if a similar mechanism as the $\Delta I = 1/2$ rule known from kaons and hyperons may be at work here as well, since the kinematics are quite similar. This would mean that the $\mathcal{A}_{1/2}$ amplitudes would be dominant compared to the $\mathcal{A}_{3/2}$ contributions.

The physics origin of the $\Delta I = 1/2$ rule is still not entirely clarified although an enhancement of the corresponding amplitudes would be dominant compared to the $\mathcal{A}_{3/2}$ contributions.

The semi-electronic modes are under reasonable theoretical control and thus may serve as a benchmark test for the pionic modes. Like in non-leptonic kaon processes, naive factorization will probably not work, but the numbers obtained in this way may give a hint of the size of the branching fractions. Here it will be interesting to see, if some patterns observed in the kaon system also appear, if the light-quark systems decay in a colour background field.

3. Summary

Heavy flavour conserving weak decays will very likely not advance our insight into weak interactions; however, they may be an interesting QCD laboratory for the study of light-quark systems in the colour-background field of a heavy quark. While for heavy mesons this mainly is the decay of a light quark in such a background field, the situation for a heavy baryon may be more interesting in this respect, since the light degrees of freedom form a more complicated system.

The semi-electronic modes are under reasonable theoretical control and thus may serve as a benchmark test for the pionic modes. Like in non-leptonic kaon processes, naive factorization will probably not work, but the numbers obtained in this way may give a hint of the size of the branching fractions. Here it will be interesting to see, if some patterns observed in the kaon system also appear, if the light-quark systems decay in a colour background field.

One obvious disadvantage of these decays is their suppression through the small phase space. Relative to the major decay modes, these decays suffer from a suppression factor $(M - m)^3/M^5$ for the semi-electronic modes, and the phase space suppression for the pionic modes is numerically about the same. This leaves branching fractions of the order of $10^{-6}$ in the best cases, typically $10^{-7}$ to $10^{-8}$, making the investigation of these decays a challenge for the $B$ physics experiments.

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