Research Article

Mathematical Problems in Engineering Joint Structure Stiffness Analysis of an Automotive Body in White with the Finite Element Method

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The finite element model of the seven connection heads of an automotive body in white is established, and the stiffness calculation is carried out by taking the lower joint of the B pillar as an example. Nine stiffness values were obtained for each branch, which can be written as a $3 \times 3$ symmetric matrix. The front and back bending stiffness, torsional stiffness, and internal and external bending stiffness are obtained with the decoupling method. Based on this, the finite element model for unrigid and rigid joints is established, and the three stiffness ratio coefficients of each branch of the joint are calculated and evaluated. Finally, the effects of the thickness and stiffness of the sheet metal parts on the bending condition, torsion condition, first-order bending, and torsion frequency of the body in white are discussed using the regional sensitivity analysis and strain energy calculation.

1. Calculation of Stiffness of Body-in-White Joints

An automotive body in white contains different longitudinal beam structures and the framework of beams and pillars—such as the beam structure, longitudinal beam, and beam and pillar between vertical and horizontal configurations—formed in the area of the junction of joint. As a significant structural component of an automotive body in white, the flexibility of a joint, reflected by its joint deformability, is non-negligible. If these factors were ignored, the stiffness of a BIW (body in white) during rigidity treatment would be 60%–70% higher than the effective stiffness. [1]. Additionally, the stiffness of the joint head is one of the important factors affecting the stiffness of a BIW, so it is necessary to study a joint head.

Shahhosseini et al. [2] proposed a trispring representation of the joint and performed an experimental analysis, Fredricson [3, 4] proposed a simplified model of the 2 joints for the analysis and optimization of joint stiffness, based on which a structure with flexible joints was proposed, and Nguyen and Jang [5, 6] first used higher-order beam elements and shell elements to model a structure consisting of a thin-walled closed beam and shell to the joint, and proposed a new thin-walled beam joint modeling method to deal with the joint flexibility effect of a car body. Gaeta and Monaca [7] performed the contribution volume analysis of joints. Kiani et al. [8] explored the sensitivity of crash and vibration response to joint stiffness. Jia et al. [9] used fatigue analysis of joints and improved joint structure through real vehicle road tests.

This research paper describes the exploration of the mechanical characteristics of a joint structure and the evaluation of the white body joint stiffness using the seven-joint finite element model, with full consideration of joint flexibility. The joint stiffness evaluation method is defined, and the finite element model of rigid and unrigid joints is established. The stiffness of the seven-joint finite element model of the white body is analyzed by calculating and comparing the ratio coefficients of each branch of the joints.
This study reveals the impact of different joints on the performance of an automotive body in white under dynamic and static conditions, based on the discussion of their interactions.

1.1. Joint definition. A joint is defined as the connecting part of the transition between two or more bearing parts on a BIW [9]. The joints mostly include connections between the A pillar, B pillar, and C pillar. As shown in Figure 1, the three joints connected to the A pillar are the upper joint of the A pillar, the middle joint of the A pillar, and the lower joint of the A pillar. The two joints connected to the B pillar are the upper joint of the B pillar and the lower joint of the B pillar. The two joints connected to the C pillar are the upper joint of the C pillar and the lower joint of the C pillar. Figure 1 shows the overall schematic diagram of the body-in-white connector. Figure 2 shows the detailed finite element model of the seven joints.

1.2. Extraction Principle of the Joint Model. To calculate the accuracy of the stiffness, the branch of each joint should contain part of the bearing member. The following principles should be met when the joint is intercepted:

(a) The structural integrity of the severed joint should be ensured. The severed joint should contain internal and external plates, stiffeners, stiffeners, mounting holes, welds, bolts, and so on.

(b) The outward part of the joint should contain part of the bearing member. For example, the upper joint of the B pillar should contain part of the roof structure.

(c) The length of the branch intercepted by the joint should be at its maximum but within the allowable range between the two joint lengths.

Taking the lower joint of the B pillar as an example, the branch length of \( L = 180 \text{ mm} \) [10] is intercepted from the end of the chamfer radius of the joint. Figure 3 shows the joint structure and the finite element model of the branch length. The distance between two yellow temporary joints, illustrated below, is the length of the extraction.

1.3. Detailed Stiffness Calculation of the Finite Element Model of the Joint Head

1.3.1. Determination of Centroid of the Joint. The centroid of the section is the geometric center of the section model. To calculate the joint stiffness, the positions of the load and the constraint point act on the centroid, which is determined by HyperBeam. Additionally, the shape of the branch section of each joint in the HyperBeam module is shown by the one-dimensional beam section. The beam section needs to be treated as a closed body to accurately show the position of the centroid. Figure 4 shows the three branch sections of the lower B pillar joint and the location of the form center point. The green circle with a cross mark in the figure represents the form center point.

1.3.2. Simulation of Joint Boundary Conditions. To transfer the loading moment to the edge of the section, the rigid joint of the RBE2 element is adopted. The centroid of RBE2 is the main node, i.e., the independent node. The rigid connection of a branch of the lower joint of the B pillar is shown in Figure 5.

1.3.3. Joint Local Coordinate System. The joint local coordinate system is identified by referring to the global coordinate system of the white body. The global coordinate system of the white body is shown in the bottom left in Figure 6. The \( X \)-axis in the global coordinate system of the white body is along the longitudinal direction of the white body, the \( Y \)-axis is along the direction of the horizontal direction of the white body, and the \( Z \)-axis direction is determined according to the right-hand spiral law to be upward perpendicular to the XY plane in the joint in the global coordinate system. In the joint local coordinate system, the center, \( O \), is located at the centroid. The \( X \)-axis is perpendicular to the section direction of the joint, the \( Y \)-axis is perpendicular to the section direction of the joint and parallel to the XY plane in the global coordinate system, and the \( Z \)-axis is perpendicular to the XY plane according to the right-hand screw rule.

1.3.4. Head Branch Constraints and Loading Conditions. For a joint unit with three branches, each branch endpoint has three linear displacements and three angular displacements. Thus, the joint has six degrees of freedom. For the transition region of the body beam, the linear displacement requirement is very small, so only three angular displacement variations are left. In the stiffness calculation of the joint model, only the angular displacement variation of the joint unit is considered for the convenience of solving the simulation.

The stiffness characteristics of the joint can be explained by using the theory of a supercell, and the stiffness matrix of the joint substructure model is as follows:

\[
[K]u = F,
\]

where \([K]\) is the stiffness matrix, and \([u]\) and \([F]\) are the displacement pillar vector and the load pillar vector, respectively.

Equation (1) is written in block form and is expressed as follows:
Figure 2: The detailed finite element models of the seven joints. (a) A pillar upper joint. (b) Middle sub of pillar A. (c) Lower sub of pillar A. (d) B pillar upper joint. (e) B pillar lower joint. (f) C pillar upper joint. (g) C pillar lower joint.

Figure 3: B pillar branch extraction.

Figure 4: Three branch sections of the lower joint of pillar B and the location of the center of the form. (a) Branch 1 cross section. (b) Branch 2 cross section. (c) Branch 3 cross section.
where \( \{u_m\} \) and \( \{u_s\} \) are called the master and slave degrees of freedom, respectively. The angular degrees of freedom are the master degrees of freedom, and the linear degrees of freedom are the slave degrees of freedom.

Rewriting equation (2),

\[
\begin{bmatrix}
K_{mm} & K_{ms} \\
K_{sm} & K_{ss}
\end{bmatrix}
\begin{bmatrix}
\theta \\
u_s
\end{bmatrix} = \begin{bmatrix}
M \\
0
\end{bmatrix},
\]

(3)

According to the supercell theory, the following supercell stiffness equation can be obtained by using the method of static coalescence for the degrees of freedom.

\[
[K^*]_{9 \times 9}\{\theta\} = [M].
\]

(4)

The effect of the joint on the overall structural stiffness is transmitted through the nine degrees of freedom, and the stiffness properties of the joint are expressed as shown in equation (5).

\[
[K^*] = K_{mm} - K_{ms}K_{ss}^{-1}K_{sm}.
\]

(5)

When calculating the stiffness of each branch of the joint, for the B pillar joints with constraints for three branches, we constrain the translational and rotational degrees of freedom of two branches. The independent node for one of the branches in the centroid is applied along the X, Y, and Z directions of the unit torque, and the torque size is 1 Nmm. The torque direction is along the direction of the joint local coordinate system.

Each branch of the joint has three working conditions in the X, Y, and Z directions, for a total of nine loading conditions. The constraints and the loading conditions of the three branches of the lower joint of the B pillar are shown in Figure 7.

1.3.5. Stiffness Analysis of Joint Model. The angular displacements at the center points of each branch are calculated using the finite element method. The angular displacements of the three branches of the lower joint of the B pillar are shown in Tables 1–3.

The table shows that the angular displacements of the joint branches are equivalent to an angular symmetric matrix with a size of \(3 \times 3\). When the unit torque load is applied in a certain direction, the angular displacements can be obtained in both the loaded and unloaded directions.

When the unit torque along the X-axis is applied, the angular displacements in the X, Y, and Z directions are obtained. For the applied load in the X direction and Y direction of the angular displacement, the same joint mutual influence exists between each branch. The angular displacement coupling is calculated for the B pillar under sub-branches. It is necessary to use the decoupling method to eliminate the coupling model of the coupling phenomenon and then calculate the stiffness of the joint branch.

1.4. Study of the Decoupling Method of the Finite Element Model of the Joint. It is assumed that there are two points in the spatial coordinate system, and there is the following relationship between these points and the angular symmetric matrix, as shown in equation (6) [11].

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
A_{XX} & A_{XY} & A_{XZ} \\
A_{YX} & A_{YY} & A_{YZ} \\
A_{ZX} & A_{ZY} & A_{ZZ}
\end{bmatrix}\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix}.
\]

(6)

By diagonalizing the angular symmetric matrix, the main diagonal matrix and its corresponding eigenvectors can be decomposed, as shown in equation (7).

\[
\begin{bmatrix}
A_{XX} & A_{XY} & A_{XZ} \\
A_{YX} & A_{YY} & A_{YZ} \\
A_{ZX} & A_{ZY} & A_{ZZ}
\end{bmatrix} = \begin{bmatrix}
C_{XX} & C_{XY} & C_{XZ} \\
C_{YX} & C_{YY} & C_{YZ} \\
C_{ZX} & C_{ZY} & C_{ZZ}
\end{bmatrix}\begin{bmatrix}
\beta_{XX} & 0 & 0 \\
0 & \beta_{YY} & 0 \\
0 & 0 & \beta_{ZZ}
\end{bmatrix}\begin{bmatrix}
C_{XX}^T & C_{XY}^T & C_{XZ}^T \\
C_{YX}^T & C_{YY}^T & C_{YZ}^T \\
C_{ZX}^T & C_{ZY}^T & C_{ZZ}^T
\end{bmatrix}.
\]

(7)

After substituting equation (7) into equation (6), equation (8) can be simplified to obtain equation (9).

\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
\beta_{XX} & 0 & 0 \\
0 & \beta_{YY} & 0 \\
0 & 0 & \beta_{ZZ}
\end{bmatrix}\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix}.
\]

(8)

\[
\begin{bmatrix}
x_1' \\
x_2' \\
x_3'
\end{bmatrix} = \begin{bmatrix}
\beta_{XX} & 0 & 0 \\
0 & \beta_{YY} & 0 \\
0 & 0 & \beta_{ZZ}
\end{bmatrix}\begin{bmatrix}
y_1' \\
y_2' \\
y_3'
\end{bmatrix}.
\]

(9)
integrated for the condition of the type of the corresponding angular displacement matrix and the stiffness matrix are conditions, and the working conditions are similar. For three pillar joints under each branch have three kinds of load placement matrix and the stiffness matrix type 11. To the B direction after applying the unit moment is used to obtain the relationship between the loading moment and the angular displacement of the B pillar.

**Table 1:** Data table for the angular displacement of the sub-branch of the B pillar.

| Unit load | $\theta_x$(rad) | $\theta_y$(rad) | $\theta_z$(rad) |
|-----------|-----------------|-----------------|-----------------|
| Mx        | $3.790E-09$     | $3.819E-11$     | $-5.970E-11$    |
| My        | $3.819E-11$     | $9.260E-10$     | $4.006E-11$     |
| Mz        | $-5.970E-11$    | $4.006E-11$     | $1.242E-09$     |

**Table 2:** Data table for the angular displacement of the sub-branch of the B pillar.

| Unit load | $\theta_x$(rad) | $\theta_y$(rad) | $\theta_z$(rad) |
|-----------|-----------------|-----------------|-----------------|
| Mx        | $3.609E-09$     | $-1.473E-11$    | $2.564E-10$     |
| My        | $-1.473E-11$    | $9.886E-10$     | $9.654E-11$     |
| Mz        | $2.564E-10$     | $9.654E-11$     | $1.205E-09$     |

**Table 3:** Trigonal displacement data table for the B pillar lower joint branch.

| Unit load | $\theta_x$(rad) | $\theta_y$(rad) | $\theta_z$(rad) |
|-----------|-----------------|-----------------|-----------------|
| Mx        | $2.145E-08$     | $-1.294E-10$    | $7.646E-10$     |
| My        | $-1.294E-10$    | $2.265E-09$     | $2.718E-09$     |
| Mz        | $7.646E-10$     | $2.718E-09$     | $1.274E-08$     |

To further solve the stiffness of the joint branch, the relationship between the loading moment and the angular displacement is established, as shown in equation (10), where $j$ ($j = \alpha, \beta, \gamma$) is the branch number, and $x, y, z$ is the coordinate axis.

$$
\begin{bmatrix}
M_{jx} \\
M_{jy} \\
M_{jz}
\end{bmatrix} =
\begin{bmatrix}
K_{jxx} & K_{jxy} & K_{jxz} \\
K_{jyx} & K_{jyy} & K_{jyz} \\
K_{jzx} & K_{jzy} & K_{jzz}
\end{bmatrix}
\begin{bmatrix}
\theta_x \\
\theta_y \\
\theta_z
\end{bmatrix}.
$$

For a branch, the joint at the centroid coordinate $X$-axis direction after applying the unit moment is used to obtain the corresponding relationship between the angular displacement matrix and the stiffness matrix type 11. The B pillar joints under each branch have three kinds of load conditions, and the working conditions are similar. Three angular displacement matrixes and the stiffness matrix are integrated for the condition of the type of the corresponding relationship between 12.

After the angular displacement matrix is decoupled, the principal diagonal matrix and its corresponding eigenvectors can be obtained. The reciprocal relationship between the angular displacement matrix and the stiffness matrix in equation (12) can be used to solve the front and rear bending stiffness, torsional stiffness, and internal and external bending stiffness of the joint branches [12].

1.4.1. Lower Joint Stiffness of the B Pillar after Decoupling.

The main diagonal eigenvalue matrix and the eigenvector matrix of the B pillar lower joint branch 1 after decoupling are shown in equations (13) and (14), respectively.

$$
\begin{bmatrix}
\theta_{x1} \\
\theta_{y1} \\
\theta_{z1}
\end{bmatrix} =
\begin{bmatrix}
K_{\alpha \alpha} & K_{\alpha \beta} & K_{\alpha \gamma} \\
K_{\beta \alpha} & K_{\beta \beta} & K_{\beta \gamma} \\
K_{\gamma \alpha} & K_{\gamma \beta} & K_{\gamma \gamma}
\end{bmatrix}^{-1}
\begin{bmatrix}
\theta_{x1} \\
\theta_{y1} \\
\theta_{z1}
\end{bmatrix}.
$$

In equation (14), the pencil vector of the feature vector is the vector direction of the corresponding joint stiffness after decoupling. In these new coordinates, the angular displacement no longer has a coupling phenomenon. The values of the front and rear bending stiffness, internal and external bending stiffness, and torsional stiffness of the three branches of the B pillar lower joint are shown in Table 4, and the stiffness units are N-mm/rad. The directions of the bending stiffness and the torsional stiffness before and after and inside and outside the B pillar lower joint are shown in Figure 8.
Table 4: Bending stiffness and torsional stiffness values of branch 3 of the B pillar lower joint.

| Connector branch | Front and back bending stiffness (N·mm/rad) | Internal and external bending stiffness (N·mm/rad) | Torsional rigidity (N·mm/rad) |
|------------------|-------------------------------------------|--------------------------------------------------|-------------------------------|
| Branch 1         | 8.026E + 08                              | 2.637E + 08                                      | 1.078E + 09                   |
| Branch 2         | 8.203E + 08                              | 2.750E + 08                                      | 1.056E + 09                   |
| Branch 3         | 7.496E + 07                              | 6.250E + 08                                      | 4.647E + 07                   |

The histograms of the ratios of front and back stiffness, inner and outer bending stiffness, and torsional stiffness of each branch of the seven joints are shown in Figures 11–13. As shown in Figure 11, the higher ratios of the front and back flexural stiffness are those for branch 2 (0.8) of the B pillar lower joint, branch 1 (0.785) and branch 2 (0.781) of the C pillar lower joint, and the lower ratios are those for branch 3 (0.22); and branch 1 (0.332) and branch 2 (0.332) of the A pillar lower joint. As shown in Figure 12, the higher ratios of the internal and external bending stiffness are those for the lower joint branch 3 of B pillar (0.745), the upper joint branch 2 of B pillar (0.689), and the middle joint branch 1 of A pillar (0.667). The lower ratios are those for the lower joint branch 2 of B pillar (0.349), the lower joint branch 1 of B pillar (0.247), and the middle joint branch 2 of A pillar (0.216). As shown in Figure 13, the higher ratios of the torsional stiffness are those for the second branch of the upper joint of B pillar (0.753), the second branch of the upper joint of A pillar (0.703), and the first branch of the middle joint of A pillar (0.854). The lower ratios are those for the third branch of the upper joint of B pillar (0.343), the first branch of the lower joint of A pillar (0.216), and the second branch of A pillar (0.247).

2. Stiffness Evaluation of the Finite Element Model of the Joint Head

2.1. Calculation and Evaluation of Joint Stiffness under B Pillar. The stiffnesses of the unrigid joint and rigid joint are calculated with the finite element model. The ratio of the two is used to evaluate the stiffness of the joint. The greater the ratio is, the stronger the stiffness in the relevant direction of the joints is [13]. The B pillar lower joint is taken as an example for calculation.

The length between the B pillar lower joint and the adjacent joint is the branch length; for calculating the stiffness contrast, the rigid joint models and unrigid joint models are required. When the B pillar lower joint is a rigid finite element model (as shown in Figure 9(a)), the elastic modulus is set to 1E + 8 MPa. As shown in Figure 9(b), the purple area indicates that the B pillar lower joint is rigid. The load and boundary conditions in Figures 9(a) and 9(b) are consistent [14].

The model consists of 34150 nodes and about 32782 shells. The material of sheet metal parts is mild steel, and the material of welded joints is common cold-rolled steel (ST12). The modulus of elasticity of mild steel and cold-rolled steel is 210,000 MPa, and Poisson’s ratio is 0.3.

In Figure 9, for the models in Figures 9(a) and 9(b), unit torque is applied along the X, Y, and Z axes of the joint local coordinate system at each branch, and the magnitude of the torque is set to 1 N·mm to obtain the ratio coefficient of stiffness between the nonrigid joint and the rigid joint of the three branches of the B pillar lower joint, as shown in Tables 5–8.

Table 8 shows the strength of front and rear bending stiffness, torsional stiffness, and internal and external bending stiffness of each branch of the B pillar lower joint. The internal and external bending stiffness of branches 1 and 2 is the lowest. Additionally, the front and rear bending stiffness of branch 3 is the lowest, so this stiffness needs to be further improved. The comparison of the stiffness in the same direction shows that the bending stiffnesses for the front and back of branch 3, inside and outside of branch 2, and the torsional rigidity of branch 2 are lower.

In this section, the inner and outer bending stiffness, front and rear bending stiffness, and torsional bending stiffness of rigid and unrigid B pillar lower joints are evaluated by stiffness ratio coefficients. For branch one, the ratio coefficient of its internal and external bending stiffness is 0.428. For branch 2, its internal and external bending stiffness ratio coefficients and torsional stiffness ratio coefficients are 0.349 and 0.44, respectively. For branch 3, the ratio coefficient of its front and rear bending stiffness is 0.445. It can be seen that the stiffness of the three branches of the lower joint of the B pillar has room for improvement.

2.2. Evaluation of the Overall Stiffness of Seven Joints. According to the calculation and evaluation method of joint stiffness under the B pillar, the stiffness of the remaining six joints is calculated, including joints three and two. The branching order of the six joints is shown in Figure 10.

The stiffnesses of the unrigid joint and rigid joint are calculated with the finite element model. The ratio of the two is used to evaluate the stiffness of the joint. The greater the ratio is, the stronger the stiffness in the relevant direction of the joints is [13]. The B pillar lower joint is taken as an example for calculation.

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3. Regional Sensitivity Analysis and Strain Energy Calculation

Taking the body in white as the research object, the regional sensitivity analysis and strain energy calculation are used to study the influence of the seven joints on the bending condition, torsion condition, first-order bending frequency, and first-order torsion frequency of the body in white.
Figure 9: Finite element model of nonrigid and rigid B pillar lower joint. (a) Model of unrigid B pillar lower joint. (b) Rigid B pillar lower joint model.

Table 5: Stiffness ratio of unrigid and rigid joints of sub-branch 1 of pillar B.

| B pillar lower joint | Front and back bending stiffness (N-mm/rad) | Internal and external bending stiffness (N-mm/rad) | Torsional rigidity (N-mm/rad) |
|----------------------|--------------------------------------------|---------------------------------------------------|-----------------------------|
| Unrigid joint        | $4.651E+08$                                | $1.609E+08$                                       | $5.734E+08$                 |
| Rigid joint          | $7.782E+08$                                | $3.757E+08$                                       | $9.116E+08$                 |

Table 6: Ratio of stiffness between unrigid and rigid joints of sub-branch 2 of pillar B.

| B pillar lower joint | Front and back bending stiffness (N-mm/rad) | Internal and external bending stiffness (N-mm/rad) | Torsional rigidity (N-mm/rad) |
|----------------------|--------------------------------------------|---------------------------------------------------|-----------------------------|
| Unrigid joint        | $4.808E+08$                                | $1.731E+08$                                       | $8.143E+08$                 |
| Rigid joint          | $9.311E+08$                                | $4.963E+08$                                       | $1.852E+08$                 |

Table 7: Ratio of stiffness between unrigid and rigid joints of sub-branch 3 of pillar B.

| B pillar lower joint | Front and back bending stiffness (N-mm/rad) | Internal and external bending stiffness (N-mm/rad) | Torsional rigidity (N-mm/rad) |
|----------------------|--------------------------------------------|---------------------------------------------------|-----------------------------|
| Unrigid joint        | $2.206E+07$                                | $1.258E+08$                                       | $1.855E+07$                 |
| Rigid joint          | $4.850E+07$                                | $1.689E+08$                                       | $2.825E+07$                 |

Table 8: Ratio of stiffness between unrigid and rigid joints of each branch of the B pillar lower joint.

|                | Front and back bending stiffness ratio | Internal and external bending stiffness ratio | Torsional rigidity ratio |
|----------------|---------------------------------------|---------------------------------------------|-------------------------|
| Sub-branch 1 of pillar B | 0.598                                 | 0.428                                       | 0.629                   |
| Sub-branch 2 of pillar B | 0.516                                 | 0.349                                       | 0.440                   |
| Sub-branch 3 of pillar B | 0.455                                 | 0.745                                       | 0.657                   |

Figure 10: Schematic diagram of the branch sequence of the six joints. (a) B pillar upper joint. (b) A pillar upper joint. (c) Middle joint of pillar A. (d) Lower sub of pillar A. (e) C pillar upper joint. (f) C pillar lower joint.
Figure 11: Ratio of the bending stiffness between the front and rear of each branch of the seven joints.

Figure 12: The ratio of the inner and outer bending stiffness of each branch of the seven joints.

Figure 13: Ratio of the torsional stiffness of each branch of the seven joints.
3.1. Sensitivity Analysis of Bending and Torsion Regions.
The joint is welded by different sheet metal parts such as the inner plate, reinforcing plate, and outer plate. The thickness of the different sheet metal parts of a joint is set to \( t_1, t_2, t_3 \ldots \). The surface is recreated with a sheet metal part with a thickness of \( \Delta t \). The new sheet metal part with a thickness of \( \Delta t \) shares the same node as the original sheet metal part. To avoid affecting the mechanical properties of the body in white, the thickness \( \Delta t \) of the new sheet metal part is set to be approximately 0. The thickness of the new sheet metal is set as a regional variable. When the regional variable changes, the thickness of each sheet metal of the joint is expressed as \( t_1 + \Delta t, t_2 + \Delta t, t_3 + \Delta t \ldots \), thereby establishing the regional variable and the original sheet metal. The relationship between the thicknesses can indirectly optimize the thickness of the original sheet metal parts by optimizing the thickness of the regional variables. Assuming that the body-in-white performance response is represented by \( F(\Delta t) \) and \( \Delta t \) is the regional design variable, the regional sensitivity of the joint is interpreted as the first-order differential of the body-in-white performance response to the regional variable from the perspective of mathematical theory.

\[
S(\Delta t_i) = \frac{\partial F(\Delta t_i)}{\partial \Delta t_i} \quad (15)
\]

The design variable \( \Delta t_i \) in equation (15) is the regional variable of a certain joint, and the joint’s sensitivity to the dynamic and static performance of the white body can be determined through the regional sensitivity analysis of the seven joints. To study the sensitivity of the seven joints to the static performance of the body in white with bending and torsion conditions, each connector area variable is used as a design parameter, the mass of the body in white is used as a constraint condition, and the bending stiffness or torsional stiffness is maximized as the goal. The displacement constraint setting of the finite element model is referred to in reference [15]. Performing the calculation with the finite element method, the sensitivity of the seven joint areas for the bending condition is as shown in Figure 14(a), and the sensitivity of the seven connector areas for the torsion condition is as shown in Figure 13.

The positive value of the regional sensitivity of the joint to the body-in-white stiffness indicates that the stiffness increases with the increase in the regional variable, and the negative value means that the increase in the regional variable reduces the stiffness [1]. As shown in Figure 14, the bending stiffness of the joint for the B pillar in Figure 14(a) is the most sensitive and the sensitivity is positive. The sensitivity of the joint for the A pillar and C pillar to the bending stiffness is negative. Increasing the thickness of the joint decreases the bending stiffness. The sensitivity values of the seven joints in Figure 14(b) for the torsional stiffness are all positive values, and the joints on the A pillar are the most sensitive to the torsional stiffness, followed by the joints on the C pillar.

3.2. Sensitivity Analysis of First-Order Torsion and First-Order Bending Modes. The regional sensitivity of the first-order torsion mode is shown in Figure 15(a), and the regional sensitivity of the first-order bending mode is shown in Figure 15(b).

In Figure 15(a), the first-order torsional mode sensitivity is negative. There are six joints. The most negative impact on the first-order torsional mode is the B pillar upper joint, followed by the C pillar lower joint and the C pillar upper joint. The first-order torsion mode has a great positive effect, and the sensitivity value is positive. The sensitivity value of the first-order bending mode with six joints in Figure 15(b) is positive, and the joint on the C pillar has the greatest positive effect on the first-order bending mode. The joint whose first-order bending mode sensitivity is negative is the A pillar upper joint.

3.3. Strain Energy Calculation for Bending and Torsion Conditions. The stiffness for bending conditions and torsion conditions reflects the ability to resist bending deformation.
and torsion deformation. A greater deformation means a greater stored strain energy and weaker stiffness. Therefore, a greater local strain energy can be determined with the strain energy calculations, which can effectively improve the rigidity of the local area. The strain energy ratio diagrams of the seven joints for bending conditions and torsion conditions are shown in Figures 16(a) and 16(b).

As shown in Figure 16(a), for the bending condition, the joint under the C pillar has the largest deformation, and the ratio of the strain energy to the total energy reaches 5.40%. The ratio of the energy is 0.38%, which indicates that the stiffness of the upper joint of the A pillar is strong. As shown in Figure 16(b), the stiffness of the joint of the A pillar is the weakest for torsion conditions, followed by the upper joint of the C pillar, and the upper joint of the B pillar and the lower joint of the B pillar have strong regional rigidity.

3.4. Calculation of Strain Energy in First-Order Torsion and First-Order Bending Modes. The strain energy ratio graphs of the seven joints in the first-order torsion mode and the first-order bending mode are shown in Figures 17(a) and 17(b).

Figure 17(a) illustrates the first-order torsional mode strain energy ratio, and it shows that the stiffness of the joint area on the C pillar is the weakest, and the joint area stiffness in the A pillar is the strongest. Figure 17(b) displays the first-order bending mode strain energy ratio. The stiffness of the
joint in the A pillar and the lower joint of the A pillar is weak, and the stiffness of the upper joint of the A pillar is the strongest.

4. Conclusions

(1) In this study, the seven joints’ finite element models of the body in white are examined; the mechanical characteristics of the joint structure are explored; the coupling phenomenon in the angular displacement matrix of the joint is calculated and analyzed; and the decoupling method of the joint is further explored, thereby obtaining the joint’s decoupling method, front and rear bending rigidity, torsion rigidity, and inner and outer bending rigidity.

(2) The method for evaluating the stiffness of the joint is explored; the finite element model of the unrigid and rigid joint is established; the stiffness ratio coefficient of each branch of the different joint is calculated; and the front and back bending stiffness and torsional stiffness of each branch of the seven joints models are evaluated. Additionally, for the strength of the internal and external bending stiffness, and the overall comparison of the strength of all the connector branches for the same stiffness condition, it is concluded that the stiffness ratio coefficient is high and low for the front and back bending stiffness, the inner and outer bending stiffness, and the torsion stiffness of the joint branch.

(3) Taking the body in white as the research object, through regional sensitivity analysis and strain energy calculation, the influence of the thickness and stiffness of the sheet metal parts of each joint on the bending conditions, torsion conditions, first-order bending, and torsion frequency is studied.

Data Availability

The simulation models used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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