Artificial potential field-based anti-saturation positioning obstacle avoidance control for wheeled robots

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1 Introduction

Robots have played an increasingly important role in daily life in recent years. The research on the application of robots in logistics, education, medical and other fields is also more favored by the majority of scholars [1]. For example, some hospitals use robots to replace manual product delivery and other tasks, and hotels use self-service robots to deliver dishes to guests [2]. Rescuers use robots for search tasks in harsh environments [3].

All of the above applications need to control the position of the robot by designing the speed and angular velocity \((V, \omega)\) of the robot [4,5]. This control action is also called point stability and has been extensively studied in the past few years. However, the robot will inevitably encounter obstacles during the movement. At this time, the obstacle avoidance problem should be added based on the control law for robots’ positioning [6,7]. Caprari et al. [8] conduct a detailed analysis of the positioning problem and propose a solution of considering the final direction of the robot. Still, the robot’s trajectory to the target point may not be optimal. Malu and Majumdar [9] propose a control law based on the robot kinematic model, which provides a reference speed for the PID control of the DC motor. Pourboghrat and Karlsson [10] propose an adaptive control law for mobile robots. Using the method of tracking a reference trajectory, the error between the tracking trajectory and the actual trajectory gradually converges to
zero, reaching a point of stability. De Wit and Sordalen [5] propose a model-based predictive control scheme, which can deal with state or input constraints. Binh et al. [11] designed the controller using the backstepping and Lyapunov direct methods to ensure system stability. Dao and Nguyen [12] separate the robot kinematic model to achieve model predictive trajectory tracking control of the wheeled robotic systems in the presence of external disturbances. But works in [5,10] do not consider the saturation of linear and angular velocities. Fabregas et al. [13] improve the response when the robot is backward-oriented, but obstacle avoidance is not considered. Khatib et al. [14] use the artificial potential field (APF) method to solve obstacle avoidance problems. The process is less computationally intensive, has good real-time performance, and is easy to control [15]. Shi and Zhao [16] add the distance between the robot and the target to the repulsive field by the synthetic potential field method, which improves the local minimum problem of the APF method. Koren and Borenstein [17] propose the robot positioning control algorithm while considering obstacle avoidance by the APF method. Muñoz and Fitz-Coy [18] design a new repulsive field function to solve the obstacle avoidance problem, but it cannot control the influence range of the repulsive field.

Based on the above facts, the main contributions of this paper can be described as follows. First, we improve the DEAF guaranteeing the positive correlation between robot speed and direction error angle. Compared with the literature [13], an $n$-th-order DEAF is designed to enhance the smoothness of the control law. In addition, the hyperbolic tangent function is used to ensure the control law’s boundedness. Second, an $n$-th-order smooth switching function is introduced to design the novel Gaussian BLF. Unlike the inverse proportional BLF in [14], the Gaussian method has a finite gain and fast convergence rate. Compared with the literature [18], due to the introduction of the switching function, the range of the repulsion field can be adjusted by parameters. Finally, combining the novel Gaussian BLF and the DEAF, we obtain the multiple obstacles avoidance control law. Unlike the previous methods [19], the proposed approach achieves multiple obstacles avoidance by adjusting the range of the repulsion field instead of path design. When the distance between obstacles is short, the range of the repulsion field can be reduced. In this situation, the improved DEAF ensures that the robot can slow down when facing and approaching the obstacle to reduce the risk of a collision.

The paper is organized as follows. Section 2 gives the basic model and fundamental equations of positioning and obstacle avoidance control. Section 3 designs the motion algorithm of positioning and obstacle avoidance control and verifies the stability. Section 4 conducts experimental simulations of the positioning and obstacle avoidance algorithms to demonstrate the effectiveness of the proposed algorithms in this paper. Section 5 summarizes the research content of this paper and gives future research directions.

2 Problem formulation

In this section, we describe the kinematic model of the wheeled robot and introduce the basic equations for positioning and obstacle avoidance control.

2.1 Kinematic model

The differential wheeled mobile robot can control the movement direction by changing the relative speed between the driving wheels. Define the left and right driving wheel speeds as $u_L$ and $u_R$. Assume that the wheels roll without slipping, the kinematic model of the robot can be described as follows in the Cartesian coordinate system [13]:

\[ \dot{x} = u \cos \psi, \]
\[ \dot{y} = u \sin \psi, \]
\[ \dot{\psi} = \omega, \]  \hspace{1cm} (1)

where $x, y \in \mathbb{R}$ represent the position states of the robot in the Cartesian coordinate and $\psi$ represents the robot’s heading angle, and its direction is perpendicular to the turning radius. $u$ represents the instantaneous linear velocity of the robot, and $u = \frac{u_L + u_R}{2}$. $\omega$ represents the angular velocity of the robot, and $\omega = \frac{(u_L - u_R)}{l}$, where $l$ is the distance between the two driving wheels.

2.2 Control objectives

2.2.1 Positioning control

The wheeled robot owns a limited velocity and a minimum turning radius. Assume that its maximum linear
velocity is \( u_{\text{max}} \) and its minimum turning radius is \( R_{\text{min}} \). In this case, the maximum angular velocity of the robot is: \( \omega_{\text{max}} = \frac{u_{\text{max}}}{R_{\text{min}}} \). As shown in Fig. 1, for the purpose of positioning control, the distance \( d \) and angle \( \alpha \) between the points \( T_c(x_c, y_c) \) and \( T_p(x_p, y_p) \) are as follows,

\[
d = \sqrt{(y_p - y_c)^2 + (x_p - x_c)^2}, \tag{2}
\]

\[
\alpha = \arctan2(y_p - y_c, x_p - x_c), \tag{3}
\]

Equations (2)–(3) mean that \( y_p - y_c = d \sin(\alpha) \) and \( x_p - x_c = d \cos(\alpha) \). Then define the directional error variable as follows,

\[
e(\psi) = \alpha - \psi. \tag{4}
\]

It can be seen that \( e(\psi) \in [-\pi, \pi] \). Taking the time derivative of Eq. (2) and combining it with Eq. (1), we get:

\[
d = -\frac{\dot{y}_c(y_p - y_c) + \dot{x}(x_p - x_c)}{d} = -u(\sin(\alpha) \sin(\psi) + \cos(\alpha) \cos(\psi)) = -u \cos(\alpha - \psi).
\]

From Eq. (4), it has:

\[
\dot{e}(\psi) = \dot{\alpha} - \dot{\psi} = \frac{(y_p - y_c)\dot{x}_c - (x_p - x_c)\dot{y}_c}{d^2} = \frac{u \sin(\alpha - \psi)}{d} - \omega = \frac{u \sin(e(\psi))}{d} - \omega.
\]

Then we can combine the above two equations to obtain the following system,

\[
\dot{d} = -u \cos(e(\psi)),
\]

\[
\dot{e}(\psi) = \frac{u}{d} \sin(e(\psi)) - \omega. \tag{5}
\]

Therefore, the robot positioning control problem is transformed into the stabilization control problem of state \( d \) in Eq. (5). That is to say, by designing the linear velocity \( u \) and angular velocity \( \omega \) to realize \( d \to 0 \), the robot can reach the target point and remain stable.

### 2.2.2 Obstacle avoidance control

The artificial potential field method mainly includes the gravitational and repulsive fields. As shown in Fig. 2, the target point generates the gravitational field \( \phi_f(q) \), which produces the gravitational force, and its direction points from the robot to the target point. The obstacle makes the repulsive field \( \phi_r(q) \), which generates the repulsive force whose direction points from the obstacle to the robot. The resultant force of gravitational and repulsive forces determines the direction of motion of the robot [18].

The gravitational field \( \phi_f(q) \) and the gravitational force \( F_f(q) \) can be expressed as:

\[
\phi_f(q) = \frac{1}{2} \mu \|q\|^2, \quad F_f(q) = -\phi_f(q) = -\mu \|q\|, \tag{6}
\]

where \( \mu \) is the gravitational field gain coefficient, \( q \) is the current position of the robot, and the original point is the position of the target point.
The repulsive field \( \phi_t(q) \) and the repulsive force \( F_t(q) \) can be expressed as:

\[
\phi_t(q) = \begin{cases} 
\frac{1}{2} \beta \left( \frac{1}{\chi} - \frac{1}{\chi_0} \right)^2, & \chi \leq \chi_0, \\
0, & \chi > \chi_0,
\end{cases}
\]

\[
F_t(q) = -\dot{\phi}_t(q) = \begin{cases} 
\frac{1}{2} \beta \frac{\partial x}{\partial q}, & \chi \leq \chi_0, \\
0, & \chi > \chi_0.
\end{cases}
\]

where \( \beta \) is the gain coefficient of the repulsive field, \( \chi \) is the distance between the robot and the obstacle, and \( \chi_0 \) is the influence range of the obstacle repulsive field, that is, the safety distance between the robot and the obstacle.

Therefore, the resultant force field \( U(q) \) and the resultant force \( F(q) \) received by the robot \( q \) are:

\[
U(q) = \phi_t(q) + \phi_r(q),
\]

\[
F(q) = F_t(q) + F_r(q).
\]

Then based on Eq. (8), the next objective is to achieve the positioning obstacle avoidance control for the wheeled robot.

3 Main results

In this section, the positioning control law and obstacle avoidance control law are proposed and their stability is analyzed.

3.1 Anti-saturation positioning controller design

In [20], the robot speed controller is designed based on the distance between the robot \( T_c(x_c, y_c) \) and the target point \( T_p(x_p, y_p) \). The velocity designed in [20] drives the robot forward at the maximum speed, while the robot heading angle may be opposite to the direction of the target point, which causes the robot cannot reach the target of positioning control. Therefore, the robot is required to adjust the heading angle while moving. The literature [13] designs a controller based on the direction error angle \( e_\psi \), aiming to propose a velocity control guided by the correct heading angle. However, the problem of speed saturation and smoothness is not solved.

Inspired by the literature [13,20], we propose the following continuous controller,

\[
u = \tanh[K_1 dp^n(e_\psi)] u_{max},
\]

\[
\omega(t) = K_p \sin(e_\psi(t)) + K_i \int_0^t e_\psi(s) ds,
\]

where

\[
p^n(x) = \frac{x^n - \pi^n}{\pi^n},
\]

where \( n \) is a positive even number larger than one.

The parameters \( K_1, K_p \) and \( K_i > 0 \) meet the following constraints,

\[
0 < u_{max} K_1 < K_p,
K_p + \sqrt{K_i} \pi < \omega_{max}.
\]

It can be seen from (10) that \( 0 \leq p^n(x) \leq 1 \) when \( x \in [-\pi, \pi] \).

3.1.1 Stability analysis

In order to facilitate subsequent controller design and analysis, the following related lemmas are introduced.

**Lemma 1** [22] If \( x(t) \) is a continuous function and \( \lim_{t \to \infty} |x(\tau)| d\tau < 0 \) (bounded), then \( \lim_{t \to \infty} x(t) = 0 \).

**Lemma 2** [13] Consider the system \( \dot{x}(t) = f(t, x) \), assume that \( f(t, x) \) is piecewise continuous in \( t \) and locally Lipschitz in \( x \). Furthermore, assume that \( f(t, 0) \) is uniformly bounded for all \( t \geq 0 \). Let \( L : \mathbb{R}^n \to \mathbb{R}^n \) as a continuous positive definite differential function such that \( \dot{L}(x(t)) \leq -W(x), \forall t \geq 0, \forall x \in \mathbb{R}^n \), where \( W(x) \) is a continuous positive semidefinite function, then \( W(x(t)) \to 0 \) as \( t \to \infty \).

Introducing the auxiliary state \( z = \int_0^t e_\psi(s) ds \), dynamic system (5) can be rewritten as:

\[
\dot{d} = -u \cos(e_\psi),
\]

\[
\dot{e}_\psi = e_\psi - \omega,
\]

\[
\dot{z} = e_\psi.
\]

**Theorem 1** Considering wheeled robot dynamic system (1) and designing control law (9) based on constraints (11), dynamic system (12) is globally asymptotically stable, that is to say, \( \lim_{t \to \infty} y_p - y = 0 \) and \( \lim_{t \to \infty} x_p - x = 0 \).

**Proof** Substituting controller (9) into system (12), we get:

\[
\dot{d} = -u_{max} \tanh[K_1 dp^n(e_\psi)] \cos(e_\psi),
\]

\[
\dot{e}_\psi = \frac{u_{max} \tanh[K_1 dp^n(e_\psi)]}{d} \sin(e_\psi) - K_p \sin(e_\psi(t)) - K_i z,
\]

\[
\dot{z} = e_\psi.
\]
To analyze the stability, we first consider subsystems (14)–(15) whose states are \( e_\psi \) and \( z \), and control input is \( d(t) \). Design the following Lyapunov function,

\[
L_1 = \frac{e_\psi^2}{2} + K_1 z^2.
\]

and its time derivative is

\[
\dot{L}_1 = e_\psi \left( u_{\text{max}} \left[ \frac{\tanh(K_1 p^n(e_\psi))}{d} \right] - K_p \right) \sin(e_\psi),
\]

with \( d \geq 0 \), \( K_1 > 0 \). Because \(-\pi < e_\psi \leq \pi\), \( 0 \leq p^n(e_\psi) \leq 1 \), then \( K_1 p^n(e_\psi) \geq 0 \). Therefore,

\[
\dot{L}_1 = e_\psi \left( u_{\text{max}} K_1 p^n(e_\psi) - K_p \right) \sin(e_\psi)
\leq e_\psi [u_{\text{max}} K_1 p^n(e_\psi) - K_p] \sin(e_\psi)
= -W(e_\psi).
\]

Then consider subsystem (13) with state \( d(t) \) and input \( e_\psi \), the following Lyapunov function is proposed,

\[
L_2 = \frac{d^2}{2}.
\]

\( u_{\text{max}} \) is consistent with literature [13], and the proof of saturation problem for state \( \omega \) is consistent with literature [13], but the constraint condition of the stability proof of this paper is \( u_{\text{max}} K_1 p^n(e_\psi) - K_p \leq 0 \). Therefore, even though \( u_{\text{max}} > 1 \), by selecting appropriate \( K_1 \) and \( K_p \), the constraint conditions can also be established.

Remark 1 When \( 0 \leq u_{\text{max}} \leq 1 \), the stability proof of (16) is similar with literature [13], and the proof of saturation problem for state \( \omega \) is consistent with literature [13], but the constraint condition of the stability proof of this paper is \( u_{\text{max}} K_1 p^n(e_\psi) - K_p \leq 0 \). Therefore, even though \( u_{\text{max}} > 1 \), by selecting appropriate \( K_1 \) and \( K_p \), the constraint conditions can also be established.

Remark 2 This paper introduces the hyperbolic tangent function into the controller to ensure anti-saturation performance. Meanwhile, the DAEF in Eq. (10) is of \( n \)-th-order smoothness, and the function \( p^n(e_\psi) \) makes the robot adjust to the correct heading angle before reaching the maximum speed.

It is worth noting that the effect of \( K_1 \) on the settling time needs further analysis. According to the above statement, one has

\[
\frac{\partial \dot{L}_2}{\partial K_1} = -L_2 u_{\text{max}} [1 - \tanh^2(K_1 p^n(e_\psi))] p^n(e_\psi) \cos e_\psi.
\]

When \( |e_\psi| > \pi/2 \), we can have \( \cos e_\psi < 0 \); it implies the fact that \( \frac{\partial \dot{L}_2}{\partial K_1} > 0 \). Under this situation, \( K_1 \) is inversely related to the convergence rate, but the proposed DAEF can hinder this trend and keep the robot running at a low speed. Although a large \( K_1 \) makes the robot move in the opposite direction of the target point, it is known from the above statement that \( d(t) \) is bounded by \( d(t) < d(0) + V_{\text{max}} * T = d(0) + u_{\text{max}} \tanh(K_1 p^n(e_\psi)) * T \); hence, the negative impact of excessive on the settling time is eliminated. When \( |e_\psi| < \pi/2 \), \( \cos e_\psi > 0 \), thus we can have \( \frac{\partial \dot{L}_2}{\partial K_1} < 0 \). Under this condition, \( K_1 \) is positively correlated with the convergence rate. Combined with formula (17), it can be obtained that, under condition (11), increasing \( K_1 \) can speed up the convergence rate of \( d \), and this result is illustrated in subsequent simulations.

3.2 Anti-saturation obstacle avoidance control based on artificial potential field

In this subsection, the obstacle avoidance control algorithm is analyzed. The BLF combined with the artificial potential field method is designed, and the DAEF is improved in the control law to achieve obstacle avoidance control.

3.2.1 Improved repulsive field function

As shown in Eq. (7), the conventional repulsive field function needs to switch when \( \chi = \delta_0 \), and there are singularities in the repulsive function, which will cause the robot to chatter when entering or getting rid of the obstacles in the repulsive field. In this paper a Gaussian function in the literature [18] is introduced to design the repulsive field function,

\[
\gamma_r(q) = \beta e^{-\frac{q^2}{\delta_0^2}}.
\]

In order to solve the problem of unsmooth phenomenon caused by switching control, this paper introduces the \( n \)-th-order smooth switching function \( h(\chi) \)
obstacle repulsion field by choosing parameters \( \chi \) and the target point. The interval \([0, \chi_b]\) can be expressed as

\[
\begin{align*}
\phi'(q) &= \sum_{i=1}^{N} \beta_i e^{\frac{\chi^2}{\pi h(\chi_i)}}, \\
\end{align*}
\]

and the repulsion field function of multiple obstacles can be expressed as

\[
\begin{align*}
\phi'(q) &= \sum_{i=1}^{N} \beta_i e^{\frac{\chi^2}{\pi h(\chi_i)}}, \\
\end{align*}
\]

where \( \chi_a \) is the maximum range of the repulsion field, \( \chi_b \) is the minimum safety range of the obstacle under the influence of the repulsion field, and \( \chi_b \) is greater than the radius of the circular obstacle.

With (18) and (19), this paper constructs a new repulsion field function below,

\[
\phi(q) = \beta e^{-\frac{\chi^2}{\pi h(\chi)}},
\]

and the repulsion field function of multiple obstacles can be selected as

\[
\phi'_i(q) = \sum_{i=1}^{N} \beta_i e^{\frac{\chi^2}{\pi h(\chi_i)}},
\]

with

\[
\begin{align*}
\phi'_i(q) &= \sum_{i=1}^{N} \beta_i e^{\frac{\chi^2}{\pi h(\chi_i)}}, \\
\end{align*}
\]

where \( \chi_a \) and \( \chi_b \) can be selected according to the physical limitations of obstacles, which is stated as follows.

**Remark 3** The switching function \( h(\chi_i) \), \( i = 1, \ldots, N \) in Eq. (22) can adjust the influence range of the repulsion field by choosing parameters \( \chi_{b_i} \) and \( \chi_{a_i} \). Compared with the literature [21], parameters \( \chi_{b_i} \) and \( \chi_{a_i} \) need to be designed. Specifically, the interval \([0, \chi_{a_i}]\) represents the traditional repulsion field range. When considering multiple obstacles, the sum of parameters \( \chi_{a_i} \) between any two obstacles needs to be less than the distance between obstacles. Meanwhile, \( \chi_{a_i} \) should be greater than the distance between the obstacle and the target point. The interval \([\chi_{b_i}, \chi_{a_i}]\) represents the buffer range. Hence, the selection of \( \chi_{b_i} \) needs to consider the size of the obstacle, satisfying the value is greater than the radius of the obstacle. Moreover, the distance between two obstacles should be larger than the sum of the repulsion field radius.

3.2.2 Switching function analysis

The switching function \( h(\chi) \) as shown in Fig. 3 gradually starts working as \( |\chi| \) decreases. As a result, the strength of obstacle repulsive field gradually increases from \( \chi = \chi_a \) until the repulsive field reaches its maximum at \( \chi = \chi_b \).

This represents the obstacle enters repulsive field range \([0, \chi_a]\), and the robot has a safe distance to avoid the obstacle. At the same time, to enhance the smoothness, the change of the repulsion field is gradually improved from \( \chi_a \) to \( \chi_b \).

To illustrate the effect of the smoothness of the \( n \)-th-order switching function, the first and second derivatives of \( h(\chi) \) are given below,

\[
\frac{d h(\chi)}{d \chi} = \begin{cases} 
\lambda, & |\chi| \leq \chi_a, \\
0, & \text{otherwise}, 
\end{cases}
\]

\[
\frac{d^2 h(\chi)}{d \chi^2} = \begin{cases} 
-(\Theta_1 + \Theta_2 + \Theta_3), & |\chi| \leq \chi_a, \\
0, & \text{otherwise}, 
\end{cases}
\]

where \( \rho = \frac{\pi \chi^2}{\chi_a^2}, \sigma = \frac{\chi^2 - \chi_b^2}{\chi_a^2}, \zeta = \frac{\pi}{2} \sin^2(\sigma), \lambda = -\frac{\pi}{4} n^2 \cos^{n-1}(\zeta) \sin^2(\zeta) \sin^{n-1}(\sigma) \cos(\sigma), \)

\[
\Theta_1 = (n - 1) \rho \cos^2(\sigma) \sin(\zeta) \sin^2(\zeta) \sin^{n-2}(\sigma), \Theta_2 = \sin(\sigma) \cos(\sigma) \sin(\zeta) \sin^{n-1}(\sigma) \cos(\sigma), \Theta_3 = \frac{n \pi}{2} \cos^2(\zeta) \sin^{n-2}(\sigma) \cos^2(\sigma) \rho.
\]

Since the term \( \frac{d^2 h}{d \chi^2} \) has a significant value when the parameter \( n \) is large, it is difficult to show trajectories of the term \( \frac{d^2 h}{d \chi^2} \). Hence this paper uses \( \ln(a) \) to represent the relationship between \( \frac{d^2 h}{d \chi^2} \) and \( \chi \), where

\[
a = \frac{d^2 h}{d \chi^2} + 1.
\]

It can be seen that \( h(\chi) \) is twice differentiable, which can meet the design requirements of the robot controller. From Fig. 4a, b, it can be found that

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**Fig. 3** \( n \)-th-order smooth switching function \( h(\chi) \) with \( \chi_a = 2 \) and \( \chi_b = 1 \)
Artificial potential field-based anti-saturation positioning obstacle avoidance control

![Graph](image)

Fig. 4 Performance analysis of switching function

as the order increases, the change rate of the switching function \( \frac{dh}{d\chi} \) at (1, 0) becomes smaller, so the smoothness at this node becomes stronger.

### 3.2.3 Barrier Lyapunov function design

By combining Eqs. (6) and (21), we design the following Gaussian BLF for the existence of multiple obstacles,

\[
V = \mu \frac{x^2 + y^2}{2} + \sum_{i=1}^{N} \beta_i e^{-\frac{x^2 + y^2}{2} h(\chi_i)}. \tag{23}
\]

This paper designs novel Gaussian BLF (23) based on Gaussian function (18) and smooth switching function (19). The most important significance of using Gaussian BLF (23) instead of inverse proportion BLF is that the Gaussian BLF does not produce infinite gain. In addition, the exponential term of Eq. (18) converges to origin faster than the conventional inverse distance term [18].

Define \( V_{\text{obj}} = \mu \frac{x^2 + y^2}{2} + \beta_j \) as the Gaussian BLF when the robot collides with the \( j \)-th obstacle. Since \( \mu \frac{x^2 + y^2}{2} \geq 0 \), it is easy to get that \( V_{\text{obj}} \geq \beta_j \). Hence, if \( V \) is guaranteed to be less than \( \beta_j \), then one has \( V < V_{\text{obj}} \) and collision avoidance for \( j \)-th obstacle can be achieved. In this paper, according to the adjustment of the effective obstacle repulsion field, only one obstacle needs to be considered at the same time. Define the initial value of \( V \) as \( V(0) \). Hence, under the condition \( V < 0 \), if the parameters \( \beta_i \) \( (i = 1, \cdots, N) \) satisfy \( V(0) < \min \beta_i \), then \( V \) can be ensured to be \( V < \min \beta_i \), which means collision avoidance for all obstacles can be achieved [18].

**Remark 4** For the convenience of reading, define the target point in this section as \((0, 0)\), the robot position coordinates as \((x, y)\), and \( \mu = 1 \), but the effect of parameter \( \mu \) on the path will be further discussed in the subsequent simulations.

### 3.2.4 Anti-saturation obstacle avoidance controller

In order to achieve the purpose of obstacle avoidance, the following control law is designed,

\[
\begin{align*}
    u &= -\tanh[K_1 \sqrt{x^2_s + y^2_s} p\theta(e_{\psi_s})] u_{\text{max}}, \\
    \omega &= \frac{\dot{y}_s x_s - y_s \dot{x}_s}{x^2_s + y^2_s} + K_s \tanh(e_{\psi_s}), \tag{24}
\end{align*}
\]

where

\[
\begin{align*}
    x_s &= x + \sum_{i=1}^{N} \beta_i e^{-\sigma_i \frac{2h(\chi_i)}{\delta_i} - \Delta_s} (x - x_i), \\
    y_s &= y + \sum_{i=1}^{N} \beta_i e^{-\sigma_i \frac{2h(\chi_i)}{\delta_i} - \Delta_s} (y - y_i). \tag{25}
\end{align*}
\]

and define the direction error angle \( e_{\psi_s} \) as

\[
e_{\psi_s} = \psi_s - \psi, \tag{26}
\]

where

\[
\psi_s = \text{atan2}(y_s, x_s). \tag{27}
\]

When designing obstacle avoidance control law (24), we use \( \psi_s \) to improve the DEAF. \( \psi_s \) also considers the
angle between the robot, the obstacle, and the target point. When the robot is near the obstacle, it pushes the robot away from the obstacle. At this time, the distance between the robot and the obstacle dominates. So the DEAF can make the robot slow down when heading toward the obstacle and accelerate when it is away from the obstacle. The parameter $\Delta_s$ will be defined later, and the parameters $K_1$ and $K_s$ meet the following constraints,

$$K_1u_{\text{max}} \leq \omega_{\text{max}}, \quad K_s > 0. \quad (28)$$

In this paper, the robot’s path depends on the coefficients in BLF (23). Therefore, we first explore the influence of the characteristic parameters $\beta$ and $\delta$ on the repulsion field in BLF. It can be seen from Fig. 5 that $\beta$ is related to the maximum strength of the repulsive field. The larger the $\beta$, the stronger the center of the repulsive field. $\delta$ is related to the width of the repulsive field. When the value of $\delta$ is smaller, the range of the repulsion field is narrower. When the value of $\beta$ is more significant and the value of $\delta$ is smaller, the curve of the repulsive field is steeper, so the robot may be bounced off due to the sudden increase of repulsive force, resulting a faster trajectory change of the robot.

### 3.2.5 Stability analysis

**Theorem 2** Considering wheeled robot system (1), $n$-th order smooth switching function (19) and Gaussian BLF (23), under constraints (28), control law (24) can globally asymptotically stabilize the wheeled robot without obstacle collision.

**Proof** The time derivative of Eq. (23) is

$$\dot{V} = xu \cos \psi + yu \sin \psi + \sum_{i=1}^{N} \beta_i e^{-\sigma_i} \times$$

$$\left\{ - \frac{2h}{\delta_i} [(x-x_i)u \cos \psi + (y-y_i)u \sin \psi] + \dot{h} \right\},$$

(29)

where $\sigma_i = \frac{(x-x_i)^2+(y-y_i)^2}{\delta_i}$, to facilitate calculation, let

$$\dot{h} = \frac{d\Delta_s}{d\chi} \dot{\chi},$$

$$\dot{\chi} = -\Delta_s[(x-x_i)u \cos \psi + (y-y_i)u \sin \psi],$$

where $\Delta_s = \frac{d\chi}{dK_i}$ and $\dot{h}$ is the derivative of $h(\chi_i)$ for simplification. Therefore, we can get

$$\dot{V} = u \cos \psi \left[ \sum_{i=1}^{N} \beta_i e^{-\sigma_i} \left( - \frac{2h}{\delta_i} - \Delta_s \right) (x-x_i) \right] +$$

$$u \sin \psi \left[ \sum_{i=1}^{N} \beta_i e^{-\sigma_i} \left( - \frac{2h}{\delta_i} - \Delta_s \right) (y-y_i) + y \right].$$

From Eq. (25), the above equation can be simplified as $\dot{V} = u \sqrt{x_s^2 + y_s^2} \cos(e_{\psi_s})$. According to the definition of $e_{\psi_s}$, $x_s$, $y_s$, $\psi_s$, we get

$$\dot{e}_{\psi_s} = \frac{\dot{y}_s x_s - y_s \dot{x}_s}{x_s^2 + y_s^2} - \omega.$$ Here

$$\dot{y}_s x_s - y_s \dot{x}_s = \sqrt{x_s^2 + y_s^2} \left( \frac{\dot{y}_s}{\sqrt{x_s^2 + y_s^2}} - \dot{x}_s \frac{y_s}{\sqrt{x_s^2 + y_s^2}} \right) = u \sqrt{x_s^2 + y_s^2} (y_s \cos \psi_s - x_s \sin \psi_s).$$

With the above two equations, we can get

$$\dot{e}_{\psi_s} = \frac{u \sqrt{x_s^2 + y_s^2} (y_s \cos \psi_s - x_s \sin \psi_s) - \omega}{x_s^2 + y_s^2} \frac{d_s}{d_s}$$

(31)

Among them, $\dot{x}_s = u x_{s1}$, $\dot{y}_s = u y_{s1}$, $d_s = \sqrt{x_s^2 + y_s^2}$. Because of the boundedness of $e_{\psi_s}$, then the states $x_{s1}$ and $y_{s1}$ can be expressed as: $x_{s1} = \cos \psi + \sum_{i=1}^{N} \beta_i e^{-\sigma_i} \left[ \frac{\Delta_s}{\delta_i} (\frac{\Delta_s}{\delta_i} + \Delta_s) - \frac{\Delta_s}{\delta_i} \right] (x-x_i) - \frac{\Delta_s}{\delta_i}$

$$\sum_{i=1}^{N} \beta_i e^{-\sigma_i} \left[ \frac{\Delta_s}{\delta_i} (\frac{\Delta_s}{\delta_i} + \Delta_s) - \frac{\Delta_s}{\delta_i} \right] (y-y_i) + y.$$
Artificial potential field-based anti-saturation positioning obstacle avoidance control

\( s \) cos \( \psi_i \), \( y_1 = \sin \psi + \sum_{i=1}^{N} B_i e^{-\sigma_i} \left[ \frac{4}{\tau_i} \left( \frac{b}{s_i} + \Delta_i \right) - \frac{2}{\tau_i} \right] (y - y_i) - \frac{2}{\tau_i} + s \) sin \( \psi \). Here \( \dot{z} = -\frac{\pi^2}{2} n^2 \left[ A \frac{\pi^2}{2} n (n - 1) \right. \cos^{-1}(\xi) \sin^{-1}(\sigma) \gamma + B \frac{\pi^2}{2} n \cos(\xi) \sin^{-1}(\sigma) \gamma + C \pi(n - 1) \gamma \times \sin^{-1}(\sigma) - D \pi \sin(\sigma) \gamma \right] \dot{x}_i - \frac{\dot{h}}{\dot{x}_i}, \dot{q} = (x - x_i) \cos \psi + (y - y_i) \sin \psi. \) And \( y = \frac{1}{x^2 - x_i^2}, \ A = \sin(\xi) \sin^{-1}(\sigma) \times \cos(\sigma) \gamma, \ B = \cos^{-1}(\xi) \sin^{-1}(\sigma) \cos(\sigma) \gamma, \ C = \sin(\xi) \cos^{-1}(\xi) \cos(\sigma) \gamma, \ D = \cos^{-1}(\xi) \sin(\xi) \sin^{-1}(\sigma) \gamma, \) and \( \dot{\chi}_i = u \frac{\dot{\psi}_s}{\dot{x}_i}. \)

In the obstacle avoidance control process, it is necessary to control not only the distance between the robot and target but also the distance between the robot and the obstacle. The convergence of the state \( d_i \) can meet the needs of approaching the target and moving away from obstacles.

Then Eq. (31) can be expressed as

\[
\dot{e}_{\psi_s} = \frac{u}{s} \sqrt{x_s^2 + y_s^2} (y_1 \cos \psi_s - x_1 \sin \psi_s) - \omega \tag{32}
\]

From Eqs. (26) and (32), the signs of \( e_{\psi_s} \) and \( \dot{e}_{\psi_s} \) are always opposite, so \( e_{\psi_s} \rightarrow 0. \)

In summary, substituting controller (24) into (30), then we will get

\[
\dot{V} = u \sqrt{x_s^2 + y_s^2} \cos(e_{\psi_s}) = u d_s \cos(e_{\psi_s})
\]

\[
= - \tanh[K_1 d_s p^n(e_{\psi_s})] d_s u_{\max} \cos(e_{\psi_s}) \leq 0.
\]

According to Lemma 1, we can get \( x_s \rightarrow 0, y_s \rightarrow 0 \) as \( t \rightarrow \infty. \) Due to the advantage of the Gaussian BLF, the range of the repulsion field can be adjusted by parameter selection for satisfying \( l_{TO_i} > x_0 (i = 1, \ldots, N), \) where \( l_{TO_i} \) represents the distance between target point and the \( i \)-th obstacle. Then in the case of \( \chi_i > \chi_a, \) one has \( h(\chi_i) = 0 \) and further \( x \rightarrow 0, y \rightarrow 0 \) as \( t \rightarrow \infty \) based on Eq. (25), which means the robot is stable at the target point. When the robot stops, then \( x \rightarrow 0, y \rightarrow 0 \) as \( t \rightarrow \infty, \) that is, the robot is stable at the target. Therefore, this paper sets the constraint \( u_t > \chi_a \) where \( u_t \) represents the distance between the obstacle and the robot. The proof is completed. □

Remark 5 The proposed method in Theorem 2 is developed from the works in [13, 18, 20]. Fabregas et al. [13] and Villela et al. [20] propose a sort of DEAF, but its smoothness cannot be guaranteed, and the speed anti-saturation problem is not considered. In contrast, Theorem 2 suggests an improved DEAF \( p^n(e_{\psi_s}) \) in Eq. (10), which can guarantee the smoothness of the control law by introducing the parameter \( n. \) Meanwhile, the hyperbolic tangent function is introduced to deal with the anti-saturation problem of robot speed. Furthermore, although a Gaussian repulsive field function is designed in the literature [18], the smoothness of the repulsive field cannot be guaranteed. This paper introduces the smooth switching function (20) to design a novel Gaussian BLF and establishes a buffer range. Subsequently, we continue to improve the DEAF by adding a distance factor between the obstacle and the robot. Different from the previous method in [19], the avoidance of multiple obstacles is achieved with the novel Gaussian BLF by adjusting the range of the repulsion field. In addition, the improved DEAF \( p^n(e_{\psi_s}) \) ensures that the robot reduces its speed when heading toward and approaching obstacles.

4 Simulation results

In this section, the detailed simulations of the positioning control law and the obstacle avoidance control law proposed in this paper are carried out, respectively, and the influence of each parameter on the control law is analyzed.

4.1 Performance analysis of positioning control law

Figure 6 represents the simulation results of control method (9). Here, control parameters are selected as \( K_1 = 4, K_p = 0.75, K_i = 0.00007, u_{\max} = 0.05. \) To verify that the robot can reach the target point from different starting points under control law (9), we choose the following five starting points \((-0.4, 0), (0.4, 0), (-0.8, -0.8), (0.1, -0.4), (-0.4, 0.5). \) We select the target point \((x_p, y_p) = (0, 0). \) The simulation results show that with any initial conditions, the robot with control law designed in Eq. (9) can reach the target point.

In order to verify the advantages of control law (9) designed in this paper, it is simulated and compared with the control law of the literature [13], as shown in Figs. 7 and 8. Because restriction condition (11) of the control law in this paper is different from that of the literature [13], the selection of \( K_1 \) parameter is also different. The parameters of the control law in this paper are selected as follows, \( K_1 = 4, K_p = 0.75, \)
$K_i = 0.00007$, $u_{\text{max}} = 0.05$. In the literature [13], the parameters selected for the control law simulation experiment are as follows, $K_1 = 0.1$, $K_p = 0.75$, $K_i = 0.00007$, $u_{\text{max}} = 0.05$. Select the starting point as $T_p(-0.4, 0)$ and the target point as $T_d(0.8, 0)$.

Figure 7 shows the velocities of the robot under our control laws and literature [13]. It can be seen that control law (9) reaches the target point about 20 s earlier than that of the literature [13], and the convergence rate is greatly improved. The improved DEAF $p^k(e_\psi)$ with the introduced hyperbolic tangent function makes the robot velocity variation curve smoother while ensuring the anti-saturation of the velocity, and the robot velocity does not reach the maximum value at the beginning, which makes the robot move more smoothly.

Figure 8 shows the effect of $K_1$ on the convergence rate in control law (9). The initial value of the orientation $\psi$ is 178°. Under condition (11), clearly $K_1$ is positively correlated with the convergence rate. In addition, with the increase of $K_1$, the robot will run more distance in the reverse direction before approaching the target point.

4.2 Performance analysis of obstacle avoidance control law

The simulation results of obstacle avoidance for control law (24)–(28) are shown in Figs. 9, 10, 11, 12, 13, 14, 15 and 16. Let the robot starting point as $T_p(10, 10)$ and target point as $T_d(0, 0)$. The black circle is the maximum influence range $\chi_a$ of the obstacle repulsion field, and the purple circle is the minimum safety distance $\chi_b$ of the obstacle.

For the convenience of observation, the simulation of avoiding single obstacle adopts the same algorithm parameters, as shown in Table 1. When avoiding multiple obstacles, we assume $\chi_b = 1$, and $\chi_a$ can refer to specific situations.

Figure 9 illustrates the path of the robot under different obstacle avoidance strategies. The dashed blue line shows the robot’s positioning path without obstacles, and the orange line is the robot’s path under our method. Compared with other methods, the paths of our approach are relatively smoother. The reason is the smooth switching function reduces the conservativeness of the controller, which ensures that the robot
Table 1 Algorithm parameters

| Simulation parameters | Value |
|-----------------------|-------|
| $\beta_i$             | 100   |
| $\delta_i$            | 1     |
| $n$                   | 1     |
| $K_1$                 | 0.1   |
| $u_{\text{max}}$      | 0.2   |
| $K_s$                 | 1     |
| $\chi_a$              | 2     |
| $\chi_b$              | 1     |

Fig. 9 Comparison of obstacle avoidance strategies

subjects to gradually increasing repulsive forces in the repulsive field. The dark purple line is the path under Gaussian function (18) in literature [18] without the switching function of this paper. It can be seen that the equilibrium point will be offset from the target point. The green line is the path under the inverse proportional BLF in the literature [14]. It can be found that it is not smooth enough and will enter into the minimum safe distance designed by the obstacle. Simulation comparisons have shown that the obstacle avoidance control approach based on Gaussian BLF has significant advantages.

4.3 Parameter selection for path optimization

Figure 10 shows the effect of the repulsive field coefficients $\beta$ and $\delta$ on the robot path. With a small $\delta$, the distribution of the repulsion field is concentrated near obstacles. This case is suitable for slow robot velocity and has less impact on the path. Conversely, a significant $\delta$ is ideal for faster robots. In addition, when the repulsive field coefficient $\beta$ becomes more extensive, as shown by the yellow line, the repulsive force of the robot will increase. On the contrary, a small coefficient $\beta$ reduces the repulsive force in the blue line. Therefore, choosing the appropriate coefficients optimizes the paths under the obstacle avoidance control law.

Figure 11 shows the effect of the parameter $\mu$ in Eq. (23) on the path of the robot. Clearly, the value $\mu$ affects the robot’s forward velocity $u$ and the directional error angle $e_{\psi_s}$, and a larger $\mu$ leads to a closer turning point to the obstacle.

Figure 12 shows the effect of the value $\chi_a$ in the switching function $h(\chi)$ on the robot path. Clearly, only
the distance between the robot and the obstacle belongs to the range \([0, \chi_a]\); the path begins to change so that the robot is far away from obstacles. Hence, the value of \(\chi_a\) determines the maximum influence range of the repulsive field.

Figure 13 shows the robot’s path in the case of a large distance between obstacles. The coordinates of obstacles are \((3, 3)\) and \((8, 8)\), respectively. The result illustrates the effectiveness of our avoidance method for multiple obstacles.

4.4 Avoidance for multiple obstacles

Figure 14 shows the robot’s path in the case of a small distance between the obstacles. The range of obstacles is adjusted by choosing \(\chi_a\) to avoid mutual interference between the repulsion fields. To better illustrate how the robot avoids obstacles in this situation, we will next show the relationship of states \(u, e_{\psi_s}\) and robot position.

In Figs. 15 and 16, it can be found that when the robot approaches the obstacle near \((8.8, 9.0)\), \(e_{\psi_s}\) decreases, and the robot starts to decelerate. When the robot approaches \((8.7, 8.4)\), the robot’s forward direction is away from the obstacle, and \(e_{\psi_s}\) increases, prompting the robot to start accelerating away from the obstacle. It shows the DEAF and speed decrease when the robot approaches and faces the obstacle. Otherwise, the DEAF and speed increase when the robot faces away from the obstacle. This conclusion illustrates the effectiveness of the avoidance control algorithm for multiple obstacles in this paper.
5 Conclusion

This paper studies the anti-saturation positioning and obstacle avoidance control of wheeled robots. First, an improved $n$-th-order DEAF is proposed to ensure the smoothness of the control method. The hyperbolic tangent function is also introduced to ensure the boundedness and continuity of the control law. Then the novel Gaussian BLF based on the smooth switching function is introduced while avoiding the problem of infinite gain. Finally, multiple obstacles avoidance is achieved by combining the DEAF and the novel Gaussian BLF. In the future, we will further combine the particle swarm optimization algorithm to analyze the robot path, and slipping angle and the environment disturbances are other issues to be considered.

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