Similarity Decomposition Approach to Leader-Follower Oscillatory Synchronization of Networked Mechanical Systems

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Abstract

This paper addresses the leader-follower oscillatory synchronization problem for multiple uncertain mechanical systems that interact on a digraph containing a spanning tree. Relying on the physically measurable relative information between the neighboring agents, we propose an adaptive controller to achieve the leader-follower synchronization of networked mechanical systems where the virtual leader is governed by the standard mass-spring oscillator dynamics. Using the similarity decomposition approach, we show that the position/velocity of each mechanical follower converges to that of the virtual leader. Simulation results are provided to demonstrate the performance of the proposed adaptive control scheme.

Index Terms

Oscillatory synchronization; Networked mechanical systems; Adaptive Control; Uncertainties.

I. INTRODUCTION

Synchronization problem for multi-agent systems has been intensively studied in recent years due to its universal existence in nature and also many engineering applications. A common practice in the current literature is to adopt the neighboring-information-based control action so that some kind of group behavior is attained (see, e.g., [2], [12]).

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There are roughly two branches of research on synchronization problem (see, e.g., [8]). The first branch focuses on the consensus problem and the second concentrates on the oscillator synchronization problem. The results in [11], [2], [5], [6], [7], [13], [14], [15] can be categorized into the first branch, whose main goal is to synchronize the state of the agents to a common value (in many cases, it is constant). The results in [4], [8], [9], [10] belong to the second branch, and their control objective, different from the first branch, is to achieve certain oscillatory synchronized motion (i.e., the equilibrium, in most cases, is oscillatory). Another interesting result appears in [5] that discusses the oscillatory synchronization of multiple pendulums. Other results (see, e.g., [16], [17], [18], [19]) can also realize certain kind of oscillatory synchronization due to the explicit inclusion/presence of Lipschitz nonlinearity in the closed-loop network dynamics.

Most of the above results under the second branch, however, are confined to agents with exactly known linear dynamics. For example, the results in [8], [10] require that the mass agents are identical or in other words, if the masses of the agents are nonidentical, the exact knowledge of the masses becomes necessary. The pendulum model considered in [5] does not need the accurate knowledge of the masses, yet, it requires the lengths of the pendula to be exactly the same. From a control viewpoint, if we expect to achieve oscillatory motion like the one generated by the networked pendula in [5], the model of the mass agent, again, must be known accurately. The case is similar for the scheme relying on Lipschitz nonlinearity (e.g., [16], [17], [18], [19]). It is emphasized that, here, the Lipschitz nonlinearity and the nonlinear sine function in [5] are not considered to be the model nonlinearity, but simply an approach for realizing oscillatory motion.

However, in many practical applications, it is unrealistic to assume that the model of the agents is linear and exactly known, e.g., robot manipulators, spacecraft, mobile robots, unmanned aerial vehicles (UAVs), etc. The dynamics governing these agents, also known as mechanical dynamics or Euler-Lagrange dynamics, is not only highly nonlinear but often contains parametric uncertainties (see, e.g., [20]). Some attempts along this direction are the leader-follower control schemes in [13], [21], [22]. The leader-follower scheme in [13] is not distributed since it requires the availability of the information of the virtual leader to all followers, and the results in [21], [22] are indeed distributed thanks to the employment of a distributed observer (oscillatory synchronization is certainly achievable by properly designing an oscillatory motion of the virtual leader). This distributed observer, unfortunately, requires the communication of the observed
signal that is not physically measurable (i.e., it is not a physical quantity that describes the position/velocity of the agents, but an artificially generated signal), which is rarely seen in the natural flocks or group behavior of the animals in our physical world. The results in [23], [24] achieve leader-follower oscillatory synchronization for general nonlinear agents relying only on the physically measurable information (i.e., the distributed state observer is not demanded). Nevertheless, the result given in [23] requires the exact knowledge of the dynamic models of the agents. This restrictive assumption is relaxed in [24], which takes into account second-order agents with matched uncertainties on undirected graphs, yet, the extension of [24] to the more general directed graphs is still unclear since the graph Laplacian in this case is usually asymmetrical.

In this paper, we aim to design a distributed adaptive controller to realize leader-follower oscillatory synchronization of networked uncertain mechanical systems on digraphs using only the physically measurable information (mimicking the flocks in our natural world in contrast to [21], [22]), relying on the similarity decomposition approach [14]. Using Lyapunov-like analysis and input-output analysis, we demonstrate that the positions and velocities of the mechanical agents converge to those of the virtual leader with oscillatory motion. Unlike the results in [8], [5] that require the exact knowledge of the mass properties of the linear agents if the masses of the agents are nonidentical (very common in practice), our control, via adaptive techniques, no longer relies on this relatively restrictive assumption and additionally considers the mechanical agents that are governed by nonlinear dynamics rather than linear dynamics. In this sense, our result extends [8], [5] to the case of nonidentical mechanical systems with high nonlinearities and parametric uncertainties. Although many results have already made some extensions to agents with Lipschitz nonlinearity (e.g., [16], [17]), this kind of nonlinearity is rather weak compared with the nonlinear terms contained in mechanical systems which are the squares and the mutual multiplications of the generalized coordinates. In addition, our result allows the interaction topology to be directed and thus is more general than the undirected topology case considered in [24]. Other results that are possibly related to the present work are consensus/flocking control schemes for multiple mechanical systems without leaders in [13], [25], [21], [14], [15], [26]. The consensus equilibrium in [13], [25], [21] is unknown (possibly unbounded), and the consensus equilibrium in [14], [15], [26] is constant. The result presented here, however, guarantees oscillatory coordination of multiple uncertain mechanical systems with
a virtual leader, i.e., the consensus equilibrium is oscillatory.

II. PRELIMINARIES

A. Graph Theory

Let us first present a brief introduction to digraph theory [2], [3], [12]. We take into consideration $n$ mechanical followers with a dynamic leader. The $i$-th mechanical follower is denoted by vertex $i$, $i = 1, 2, \ldots, n$, and all the vertices that represent the $n$ mechanical followers constitute a set $\mathcal{V} = \{1, 2, \ldots, n\}$. The unique virtual leader is denoted by vertex 0 (also called agent 0). All the vertices constitute the vertex set $\mathcal{V} = \{0, 1, 2, \ldots, n\}$, where it is noted that vertex 0 is included, and all the directed edges constitute the edge set $\mathcal{E}^* \subseteq \mathcal{V}^* \times \mathcal{V}^*$ which describes the interaction among all agents. For two agents $i$ and $j$, if agent $i$ is capable of receiving information from agent $j$, there will appear an edge $(i, j)$ in the graph. The set of neighbors of agent $i$ is defined as $\mathcal{N}_i^* = \{j|j \in \mathcal{V}^*, (i, j) \in \mathcal{E}^*\}$. A graph is said to have a directed spanning tree if there is a vertex $k^*$ such that any other vertex of the graph has a directed path to vertex $k^*$. The weighted adjacency matrix associated with the graph $\mathcal{W}^* = [w_{ij}]$ is defined according to the rule that $w_{ij} > 0$ if $j \in \mathcal{N}_i^*$ and $w_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L}_w^* = [\ell_{w,ij}]$ is defined as $\ell_{w,ij} = \sum_{k=1}^n w_{ik}$ if $i = j$ and $\ell_{w,ij} = -w_{ij}$ otherwise. Some fundamental properties of the Laplacian matrix $\mathcal{L}_w^*$ can be described by the following lemma.

Lemma 1 ([3], [12]): If each mechanical follower has a directed path to the virtual leader, then:

1) $\mathcal{L}_w^*$ has a simple zero eigenvalue and all the other eigenvalues of $\mathcal{L}_w^*$ are in the open right half plane (RHP);
2) the vectors $\gamma^* = [1, 0, \ldots, 0]^T \in R^{n+1}$ and $1_{n+1} = [1, 1, \ldots, 1]^T$ are the left and right eigenvectors of $\mathcal{L}_w^*$ associated with its zero eigenvalue, i.e., $\gamma^T \mathcal{L}_w^* = 0$ and $\mathcal{L}_w^* 1_{n+1} = 0$;
3) $\text{rank}(\mathcal{L}_w^*) = n$.

B. Equations of Motion of Mechanical Systems

The equations of motion of the $i$-th mechanical system can be written as [27], [20]

$$M_i(q_i)\ddot{q}_i + C_i(g_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i$$

(1)
where \( q_i \in \mathbb{R}^m \) is the configuration variable, \( M_i(q_i) \in \mathbb{R}^{m \times m} \) is the inertia matrix, \( C_i(q_i, \dot{q}_i) \in \mathbb{R}^{m \times m} \) is the Coriolis and centrifugal matrix, \( g_i(q_i) \in \mathbb{R}^m \) is the gravitational torque, and \( \tau_i \in \mathbb{R}^m \) is the control torque exerted on the system.

Three basic properties of the dynamic model (1) are given as follows [27], [20].

**Property 1:** The inertia matrix \( M_i(q_i) \) is symmetric and uniformly positive definite.

**Property 2:** The Coriolis and centrifugal matrix \( C_i(q_i, \dot{q}_i) \) can be appropriately chosen such that \( \dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i) \) be skew-symmetric.

**Property 3:** The dynamic model (1) depends linearly on a constant parameter vector \( a_i \), thus yielding

\[
M_i(q_i)\dot{\zeta} + C_i(q_i, \dot{q}_i)\zeta + g_i(q_i) = Y_i(q_i, \dot{q}_i, \zeta, \dot{\zeta})a_i
\]

where \( Y_i(q_i, \dot{q}_i, \zeta, \dot{\zeta}) \) is the regressor matrix, \( \zeta \in \mathbb{R}^m \) is a differentiable vector and \( \dot{\zeta} \) is the time derivative of \( \zeta \).

### III. LEADER-FOLLOWER ADAPTIVE OSCILLATORY SYNCHRONIZATION

In this section, we seek an adaptive control algorithm to realize the leader-follower oscillatory synchronization of networked uncertain mechanical systems, and the major step for achieving this goal is the definition of a reference velocity for each mechanical system.

Following [8], we consider a virtual leader (denoted by vertex 0) that is governed by the standard oscillatory dynamics

\[
\ddot{q}_0 = -\alpha q_0
\]

where \( \alpha > 0 \) is a design constant, and \( q_0 \in \mathbb{R}^m \) denotes the configuration variable of the virtual leader. Then, the control objective is to drive the network of the mechanical followers to converge to the oscillatory trajectory generated by the virtual leader.

For the \( i \)-th mechanical follower, we define a new reference velocity of the following form

\[
\dot{q}_{r,i} = -\sum_{j \in N^*_i} w_{ij} (q_i - q_j) - \alpha \int_0^t q_i(r) dr
\]

where the integral action will be demonstrated to be essential for realizing the oscillatory synchronization of the mechanical agents.

Differentiating equation (4) yields the reference acceleration

\[
\ddot{q}_{r,i} = -\sum_{j \in N^*_i} w_{ij} (\dot{q}_i - \dot{q}_j) - \alpha q_i
\]
Then, let us define a sliding vector $s_i$ as

$$s_i = \dot{q}_i - \dot{q}_{r,i}$$

(6)

whose derivative with respect to time can be written as

$$\dot{s}_i = \ddot{q}_i + \sum_{j \in N_i^*} w_{ij} (\dot{q}_i - \dot{q}_j) + \alpha q_i$$

(7)

We propose the following control law for the $i$-th mechanical follower

$$\tau_i = Y_i(q_i, \dot{q}_i, \dot{q}_{r,i}, \ddot{q}_{r,i}) \hat{a}_i - K_i s_i$$

(8)

where $K_i$ is a symmetric positive definite matrix, and $\hat{a}_i$ is the estimate of the parameter $a_i$ which is updated by the adaptation law

$$\dot{\hat{a}}_i = -\Gamma_i Y_i^T(q_i, \dot{q}_i, \dot{q}_{r,i}, \ddot{q}_{r,i}) s_i$$

(9)

where $\Gamma_i$ is a symmetric positive definite matrix.

**Remark 1:** The adaptive controller (8), (9) is basically the same as the well-known Slotine and Li adaptive control [28], and the difference lies in the definition of new reference velocities and accelerations which take into consideration the neighboring information of each mechanical agent.

Substituting the control law (8) into the dynamics of the $i$-th mechanical system yields the closed-loop dynamics

$$M_i(q_i) \dot{s}_i + C_i(q_i, \dot{q}_i) s_i = -K_i s_i + Y_i(q_i, \dot{q}_i, \dot{q}_{r,i}, \ddot{q}_{r,i}) \Delta a_i$$

(10)

where $\Delta a_i = \hat{a}_i - a_i$ is the parameter estimation error.

The network dynamics can be adequately described by the following cascade system [i.e., the composition of equations (3), (6) and (10)]

$$\begin{cases}
\dot{q}_0 = -\alpha \int_0^t q_0(r) dr + \dot{q}_0(0) \\
\dot{q}_i = -\sum_{j \in N_i^*} w_{ij} (q_i - q_j) - \alpha \int_0^t q_i(r) dr + s_i, \\
M_i(q_i) \dot{s}_i + C_i(q_i, \dot{q}_i) s_i \\
= -K_i s_i + Y_i(q_i, \dot{q}_i, \dot{q}_{r,i}, \ddot{q}_{r,i}) \Delta a_i, \forall \ i \in V,
\end{cases}$$

(11)

where the integration of equation (3) is incorporated for the convenience of the subsequent analysis.
The first and second subsystems in (11) can be rewritten as (using the standard Kronecker product [29])

\[
\dot{\Psi} = -(L^* \otimes I_m) q^* - \alpha \int_0^t q^*(r) dr + s^* + d^*_0
\]

where \( q^* = [q^T_0, q^T_1, q^T_2, \ldots, q^T_n]^T \), \( s^* = [0^T_m, s^T_1, s^T_2, \ldots, s^T_n]^T \) and the initial-state-dependent constant vector \( d^*_0 = [\dot{q}^T_0(0), 0^T_{mn}]^T \).

Although the work reported in [8] has already presented the stability and convergence properties of the differentiated form of the system \( \dot{\Psi} \), i.e.,

\[
\dot{\Psi} = -(L^* \otimes I_m) \dot{q}^* - \alpha q^*, \\
\frac{d\Psi}{dt}
\]

it is not so clear about the input-output property of \( \Psi \), namely, the property of the output \( q^* \) under an external input signal \( s^* \).

Let us adopt the similarity decomposition in [14] to analyze the system (12) which relies on the following coordinate transformation [7], [30]

\[
\xi = (T \otimes I_m) q^*
\]

where the transformation matrix \( T \in \mathbb{R}^{(n+1)\times(n+1)} \) is

\[
T = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
1 & -1 & 0 & \cdots & 0 \\
0 & 1 & -1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 1 & -1 \\
\end{bmatrix}
\]

and the vector \( \xi = [\xi_1^T, \xi_E]^T \), in which \( \xi_1 = q_0 \) and \( \xi_E = [q^T_0 - q^T_1, q^T_1 - q^T_2, \ldots, q^T_{n-1} - q^T_n]^T \).

With the transformation equation (13) and applying the similarity transformation (following [14]) to (12) yields

\[
\dot{\xi} = - \left( (TL^*_{t}T^{-1}) \otimes I_m \right) \xi - \alpha \int_0^t \xi(r) dr \\
+ (T \otimes I_m) (s^* + d^*_0)
\]

where the matrix \( TL^*_{t}T^{-1} \) can be decomposed as [14]

\[
TL^*_{t}T^{-1} = \text{diag} \left[ 0, \hat{L}^* \right]
\]
where the matrix $\bar{L}_w \in R^{n \times n}$ satisfies the property that all its eigenvalues are in the open RHP if the interaction graph among the $n$ mechanical agents and the virtual leader contains a spanning tree.

**Remark 2:** The Jordan form of a Laplacian matrix dates back to the result in [2] (concerning strongly connected digraphs), and the extension to digraphs containing a spanning tree appears in, e.g., [31]. Let $\mathcal{L}_w \in R^{m \times m}$ be the Laplacian matrix associated with a digraph containing a spanning tree. The relationship between $\mathcal{L}_w$ and its Jordan form can be written as [2], [31]

$$\mathcal{L}_w = DJD^{-1}$$  \hspace{1cm} (17)

where the Jordan form $J = \text{diag} [0, \bar{J}]$ with $\bar{J} \in R^{(m-1) \times (m-1)}$ having the property that all its eigenvalues are in the open RHP, the first column of $D$ is $1_m$, and the first row of $D^{-1}$ is $\gamma$ satisfying the property that $\gamma^T \mathcal{L}_w = 0$ and $\sum_{k=1}^{m} \gamma_k = 1$. The transformation (17) and the property of $\bar{J}$ are also exploited in [32] to handle the consensus problem under input and communication delays. A more intuitive formulation of equation (17) can be obtained by letting $T_0 = D^{-1}$ as

$$T_0 \mathcal{L}_w T_0^{-1} = \text{diag} [0, \bar{J}].$$  \hspace{1cm} (18)

Due to [2], [31], $T_0$ can be written as

$$T_0 = \begin{bmatrix} \gamma^T \\ \bar{T}_0 \end{bmatrix}$$  \hspace{1cm} (19)

where $\bar{T}_0 \in R^{(m-1) \times m}$ and $\text{rank}(\bar{T}_0) = m-1$, and in addition, $\bar{T}_0$ obviously satisfies the following property (since $T_0T_0^{-1} = I_m$)

$$\bar{T}_0 1_m = 0.$$  \hspace{1cm} (20)

The transformation $T$ in [7], [30] (other forms of $T$ can be found in, e.g., [33]) is

$$T = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \cdots & \gamma_n \\ 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & -1 \end{bmatrix}$$  \hspace{1cm} (21)

which obviously satisfies property (20). However, in general, $T$ is different from $T_0$ and the similarity decomposition [e.g., equation (16)] in [14] is also unlike (17) in that, usually, it does not give rise to the Jordan form of $\mathcal{L}_w$.  

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Using the similarity decomposition (16) and exploiting the standard constant disturbance compensation capability of the integral action, we can rewrite equation (15) as

\[
\begin{align*}
\dot{q}_0 &= -\alpha \int_0^t q_0(r)dr + \dot{q}_0(0) \\
\dot{\xi}_E &= - (\dot{L}_w \otimes I_m) \xi_E - \alpha \left[ \int_0^t \xi_E(r)dr - \alpha^{-1} d_0 \right]
\end{align*}
\]

where \( s_E^* = [q_m^T - s_1^T, s_1^T - s_2^T, s_2^T - s_3^T, \ldots, s_{n-1}^T - s_n^T]^T \), \( d_0 = [q_0^T(0), 0_{m(n-1)}]^T \), and, similar to [7], [34], the first and second subsystems in (22) can be interpreted as the group dynamics and formation dynamics, respectively. It is quite natural and also well understood that in the leader-follower control problem, the group dynamics is determined only by the leader dynamics, as can be explicitly seen in the above equation. For convenience, let \( \sigma_E = \int_0^t \xi_E(r)dr - \alpha^{-1} d_0 \), and then, equation (22) can be rewritten as

\[
\begin{align*}
\dot{q}_0 &= -\alpha \int_0^t q_0(r)dr + \dot{q}_0(0) \\
\dot{\sigma}_E &= - (\dot{L}_w \otimes I_m) \dot{\sigma}_E - \alpha \sigma_E + s_E^*
\end{align*}
\]

where the property of system \( \Psi_r \) can be characterized by the following lemma.

**Lemma 2:** If the interaction graph among the \( n \) mechanical agents and the virtual leader contains a spanning tree, then all the poles of \( \Psi_r \) are located in the open left half plane (LHP).

**Proof:** The system (23) reduces to the one considered in [8] if the external input \( s_E^* \) is ruled out, or more precisely, the system \( \Psi \) in (12). It is demonstrated in [8] that \( \Psi \) contains two simple poles on the imaginary axis (the locations of which are determined by the parameter \( \alpha \)), and all the other poles of \( \Psi \) are in the open LHP. Here, by the similarity decomposition, the two simple poles are both contained in the first subsystem of (23). In fact, according to the standard linear system theory, the first subsystem in (23) indeed include two poles on the imaginary axis, i.e., \( p_1 = j^* \sqrt{\alpha} \) and \( p_2 = -j^* \sqrt{\alpha} \), where \( j^* = \sqrt{-1} \) denotes the imaginary unit. Therefore, all the poles of the linear system \( \Psi_r \) are those of \( \Psi \) that are located in the open LHP.

We are presently ready to give the following theorem.

**Theorem 1:** The control law (8) and the adaptation law (9) for the mechanical followers and the virtual leader governed by dynamics (3) ensure that all the mechanical followers converge
to the oscillatory trajectory of the virtual leader provided that the graph among the mechanical followers and the virtual leader contains a spanning tree, i.e., \( q_i(t) \to q_0(t) \) and \( \dot{q}_i(t) \to \dot{q}_0(t) \) as \( t \to \infty \), \( \forall i \in \mathcal{V} \).

**Proof:** Following [28], [35], we consider the Lyapunov-like function candidate for the third subsystem in (11) \( V_i = \frac{1}{2} s_i^T M_i(q_i) s_i + \frac{1}{2} \Delta a_i^T T_i^{-1} \Delta a_i \) and exploiting Property 2, we have \( \dot{V}_i = -s_i^T K_i s_i \leq 0 \), which then yields the result that \( s_i \in L_2 \cap L_\infty \) and \( \dot{a}_i \in L_\infty \), \( \forall i \in \mathcal{V} \).

The result \( s_i \in L_2 \cap L_\infty \), \( \forall i \in \mathcal{V} \) yields \( s_E^* \in L_2 \cap L_\infty \). From Lemma 2, we know that all the poles of \( \Psi_r \) are in the open LHP in the case that the graph contains a spanning tree. In addition, it is obvious that the relative degree of the second subsystem in (23) is two if \( \sigma_E \) is taken as the output and \( s_E^* \) taken as the input. Therefore, the input-output mapping described by the second subsystem in (23) is exponentially stable and strictly proper. From the input-output property of linear systems [36], we obtain \( \sigma_E \in L_2 \cap L_\infty \), \( \dot{\sigma}_E \cap L_2 \), and \( \sigma_E \to 0 \) as \( t \to \infty \). Rewrite the second subsystem in (22) as

\[
\dot{\xi}_E = - (\overline{L}_{w} \otimes I_m) \xi_E + \underbrace{\left[ \begin{array}{c} \alpha \sigma_E + s_E^* \end{array} \right]}_{\text{system input}}
\]

where the system input \( -\alpha \sigma_E + s_E^* \in L_2 \cap L_\infty \), and the system \( \Psi_{rr} : \dot{\xi}_E = - (\overline{L}_{w} \otimes I_m) \xi_E \) is obviously exponentially stable and strictly proper if \( \xi_E \) is taken as the output signal since, from the similarity decomposition (16), all the eigenvalues of \( \overline{L}_{w} \) are in the open RHP (implying that \( -\overline{L}_{w} \) is Hurwitz) if the graph contains a spanning tree. Therefore, again from the input-output property of linear systems [36], we have \( \xi_E \in L_2 \cap L_\infty \), \( \dot{\xi}_E \in L_2 \) and \( \xi_E \to 0 \) as \( t \to \infty \). It is also obvious that \( \dot{\xi}_E \in L_\infty \) since the right side of equation (24) is bounded.

From the standard linear system theory, the explicit solution of the first subsystem in (23) can be given as

\[
\int_0^t q_0(r)dr = \left[ \begin{array}{c} \frac{1}{\sqrt{\alpha}} \sin(\sqrt{\alpha} t) \\ \cos(\sqrt{\alpha} t) \end{array} \right] \left[ \begin{array}{c} 1 - \cos(\sqrt{\alpha} t) \\ \frac{1}{\sqrt{\alpha}} \sin(\sqrt{\alpha} t) \end{array} \right] \left[ \begin{array}{c} q_0(0) \\ \dot{q}_0(0) \end{array} \right]
\]

where it is obvious that \( \int_0^t q_0(r)dr \in L_\infty \), \( q_0(t) \in L_\infty \) and \( \dot{q}_0(t) = -\sqrt{\alpha} \sin(\sqrt{\alpha} t)q_0(0) + \cos(\sqrt{\alpha} t)\dot{q}_0(0) \in L_\infty \), \( \forall t \geq 0 \). From \( \int_0^t \xi_E(r)dr = \sigma_E + \alpha^{-1} d_0 \in L_\infty \), \( \xi_E \in L_\infty \) and \( \dot{\xi}_E \in L_\infty \), we obtain \( \int_0^t q_i(r)dr \in L_\infty \), \( q_i \in L_\infty \) and \( \dot{q}_i \in L_\infty \), \( \forall i \in \mathcal{V} \). From equations (4) and (5), we get the boundedness of \( \dot{q}_{r,i} \) and \( \ddot{q}_{r,i} \). The boundedness of \( \dot{s}_i \) can then be derived from the closed-loop dynamics (10) since \( M_i(q_i) \) is uniformly positive definite (i.e., Property 1), \( \forall i \in \mathcal{V} \). From the
definition of \( \dot{s}_i \), i.e., equation (7), we obtain \( \ddot{q}_i \in L_\infty, \forall i \in \mathcal{V} \). It can also be easily observed that
\[
\ddot{q}_0(t) = -\alpha \cos(\sqrt{\alpha t})q_0(0) - \sqrt{\alpha} \sin(\sqrt{\alpha t})\dot{q}_0(0) \in L_\infty, \forall t \geq 0.
\]
Therefore, \( \dddot{\xi}_E \) must be bounded, implying the uniform continuity of \( \dddot{\xi}_E \). Then, from Barbalat’s Lemma [20], we have \( \dddot{\xi}_E \to 0 \) as \( t \to \infty \). The result that \( \xi_E \to 0 \) and \( \dddot{\xi}_E \to 0 \) as \( t \to \infty \) directly gives the conclusion that \( q_i(t) \to q_0(t) \) and \( \dddot{q}_i(t) \to \dddot{q}_0(t) \) as \( t \to \infty \), \( \forall i \in \mathcal{V} \).

IV. SIMULATION RESULTS

In this section, we provide simulation results to demonstrate the performance of the proposed adaptive control, using a network of 9 standard mass agents (the same as the case in [11], [21]) and a virtual leader interacting on a digraph containing a spanning tree (Fig. 1). The mass agents are assumed to move in the X-Y plane and be governed by the following dynamics [11]

\[
m_i\ddot{q}_i + c_i\dot{q}_i = \tau_i
\]

where \( m_i, c_i \) denote the mass and the damping coefficient of the \( i \)-th agent, respectively, \( \tau_i \) is the control input, and \( \dot{q}_i = [x_i, y_i]^T \) denotes the position of the \( i \)-th agent, \( i = 1, 2, \ldots, 9 \). The mass parameters of the agents are \( m_1 = 1.0, m_2 = 1.5, m_3 = 1.6, m_4 = 1.2, m_5 = 0.5, m_6 = 2.5, m_7 = 2.2, m_8 = 1.8, \) and \( m_9 = 2.1 \), respectively. The damping coefficients of the agents are \( c_1 = 0.3, c_2 = 0.5, c_3 = 0.7, c_4 = 0.35, c_5 = 0.6, c_6 = 0.8, c_7 = 0.9, c_8 = 0.75, \) and \( c_9 = 0.85 \), respectively. The parameter \( \alpha \) that determines the behavior of the virtual leader is given as \( \alpha = 1.0 \). The sampling period in the following simulation is chosen to be 5 ms.

The entries of the weighted adjacency matrix \( \mathcal{W}^* \) are chosen according to the rule that \( w_{ij} = 1.0 \) if \( j \in N_i^* \) and \( w_{ij} = 0 \) otherwise. The controller parameters \( K_i, \Gamma_i \) are chosen as \( K_i = 20.0I_2 \) and \( \Gamma_i = 2.0I_2, i = 1, 2, \ldots, 9 \). The parameters of the agents \( a_i = [m_i, c_i]^T, i = 1, 2, \ldots, 9 \) are assumed to be unknown. The initial estimates of \( a_i \) are chosen as \( \hat{a}_i(0) = 0, i = 1, 2, \ldots, 9 \). The initial state of the virtual leader (i.e., agent 0) is determined as \( \dot{q}_0(0) = [2, 0]^T, \ddot{q}_0(0) = [0, 1]^T \), which means that the path of the virtual leader is an ellipse in the X-Y plane centered at \( (0, 0) \) (i.e., \( q_0(t) = [2 \cos(t), \sin(t)]^T \)). The initial positions of the mechanical followers are set to be \( q_1(0) = [3, 2]^T, q_2(0) = [-3, 2]^T, q_3(0) = [-3, -2]^T, q_4(0) = [3, -2]^T, q_5(0) = [3, 0]^T, q_6(0) = [-3, 0]^T, q_7(0) = [3, 3]^T, q_8(0) = [-3, 3]^T, \) and \( q_9(0) = [-3, -3]^T \), respectively, and their initial velocities are determined to be \( \dot{q}_i(0) = 0, i = 1, 2, \ldots, 9 \). The simulation results are plotted in Fig. 2, Fig. 3 and Fig. 4, which demonstrate that the proposed adaptive controller ensures the leader-follower oscillatory synchronization of multiple uncertain mass agents.
Fig. 1. The interconnection graph among the mechanical agents and the virtual leader

Fig. 2. Positions of the agents (X-axis)

V. CONCLUSION

In this paper, we have studied the synchronization problem for multiple nonidentical mechanical followers and a virtual leader with mass-spring oscillator dynamics. We propose an adaptive control scheme to ensure the leader-follower oscillatory synchronization, which only requires the physically measurable information of the agents (e.g., the relative position/velocity between the agents). Using similarity decomposition approach, we show that the positions/velocities of the mechanical followers converge to the position/velocity of the virtual leader provided that the interaction graph among the mechanical followers and the virtual leader contains a spanning tree. Simulation results are presented to illustrate the performance of the proposed control scheme.
Fig. 3. Positions of the agents (Y-axis)

Fig. 4. The paths of the agents in the X-Y plane

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