Wideband characteristics of density tapered array antennas

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ABSTRACT

In this paper, wideband characteristics of density tapered arrays are clarified by comparing directly the array factors and radiation patterns of the tapered arrays structures with array factors and radiation patterns of equally spaced arrays. Calculated results for a density tapered distribution array consisting of 30 elements claims that the array can perform within a bandwidth of 2.5:1 with grating lobe levels lower than -7.8 dB. Additionally, this paper shows a method of determining the effectiveness of unequal spacing arrays in the design of actual antennas. This method is based on the calculation and analysis of input impedance of array elements caused by mutual coupling effects among array elements.

Keywords: Grating lobe
Sidelobe level
Unequal spaced array
Wideband

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1. INTRODUCTION

Currently, next generation mobile communication system (5G) is developing [1, 2]. For mobile base station antennas, multi beam and wideband characteristics are requested in addition to low sidelobe characteristics of present base station antennas [3, 4]. In a present mobile base station antenna, an equally spaced linear array configuration is employed as shown in Figure 1(a). Low sidelobe characteristics are achieved by giving adequate excitation coefficients (amplitude and phase) to array elements [5-7], and/or changing the radiation characteristics of elements in the array [8, 9]. In case of providing excitation coefficients to all elements, feeding network includes multiple power dividers and feeder lines that have different values [10-12]. Herein, phase values determined by feeder line lengths, which tend to reduce array antenna’s bandwidth. In addition, the appearance of grating lobes in array factors (AF) [13] when equally spaced arrays (ESA) performing within wideband frequencies makes it more difficult in manufacturing actual antenna models. For the purpose of increasing operation frequency range, the density tapered array (DTA) configuration [14, 15] shown in Figure 1(b) will be promising. The excitation coefficients for all array elements are uniform. Here, the feed line lengths from the input to all array elements are equal. Then the feed network has frequency independent characteristics. The DTA antenna will be expected to achieve low sidelobe characteristics in a wide range of frequency.

For previous researches on unequally spacing or density tapered arrays, some method of designing unequally spaced arrays was proposed in [16-22] which achieved side-lobes level (SLL) lower than -25 dB. In [23, 24], the particle swarm optimization algorithm is applied to determine the allocation of elements to achieve the minimum sidelobe level and null control in the array radiation pattern. The main drawbacks of
these methods are that the spacing between two consecutive elements at the array center was lower than a half of wavelength and/or it required excitation coefficients to array elements with various values. In [25, 26], the evolution algorithm was used to optimize element allocations of 32-element arrays, which resulted in the SLLs around -22.5 dB. Nevertheless, the wideband characteristic of those structures was not clarified. The early studies about the wideband characteristic of unequally spaced arrays were shown in [27-29], and some experiments were reported in [30-34]. Evidence in the method of designing unequal spacing arrays has been stated in [35]. However, the effect of elements radiation, the mutual coupling effect and the effect of changing the density of them to total radiation pattern of the array were not clarified. Thus, more detailed research on tapered distribution arrays is necessary.

In this paper, wideband characteristics of tapered distribution arrays will be shown by considering array factors and radiation pattern of an array consisting of 30 elements that has operating bandwidth of 2.5:1. In section 2, design, array factors calculation method and radiation pattern of 3 DTAs will be discussed. Numerical results based on Method of Moment will be shown in Section 3. Additionally, radiation effects of array elements and mutual effects among elements to the overall radiation pattern of the array will also be clarified.

![Figure 1](image1.png)

Figure 1. Mobile base station antenna configurations, (a) present antenna, (b) density tapered array

2. METHOD TO ACHIEVE TAPERED LINEAR ARRAY

2.1. Allocations of array elements

Figure 2 shows a linear antenna array consisting of 2N elements which are symmetrically allocated along z-axis. The spacing between two consecutive elements is denoted as \( d_i \) \((i = 1 \sim N - 1)\). The total length of the array is denoted as \( L \). \( L \) is fixed and determined by (1).

\[
L = 0.7 \times (2N - 1)\lambda_1
\]

(1)

where, \( \lambda_1 \) is wave-length at lowest frequency.

To achieve tapered distribution, \( d_i \) gaps among array elements is given by

\[
d_i = d_c + i \times \Delta d \quad (i = 1, 2, ..., N - 1).
\]

(2)

where, \( d_c \) is the spacing between two elements at the array center, and \( \Delta d \) expresses the degree of unequally spacing. When \( \Delta d = 0 \), the array is allocated equally. When the number of array elements is fixed at 2N=30, the total length \( L \) of the array determined by (1) and (2) is as followed

\[
L = (29d_c + 210\Delta d)\lambda_1.
\]

(3)
When \( d_c \) obtains the values of 0.7 \( \lambda_1 \), 0.6 \( \lambda_1 \), 0.5 \( \lambda_1 \) and 0.4 \( \lambda_1 \), the value of \( \Delta d \) is given by (3) and shown in Table 1. Herein, the equal spacing array is labeled as ESA and density tapered arrays are labeled as DTA1, DTA2 and DTA3 corresponding with \( d_c=0.6 \lambda_1, 0.5 \lambda_1 \) and 0.4 \( \lambda_1 \).

![Figure 2. Geometry of a linear symmetrical array](image)

### Table 1. Array element spacing (normalize by \( \lambda_1 \))

|       | Equal spacing (ES) | DTA1 | DTA2 | DTA3 |
|-------|--------------------|------|------|------|
| \( d_c \) | 0.7                | 0.6  | 0.5  | 0.4  |
| \( \Delta d \) | 0.01381           | 0.02762 | 0.04143 |

#### 2.2. Radiation pattern

Radiation pattern of array in Figure 1 given by the following equation [13]

\[
\vec{E}(\theta) = \vec{E}_{ele}(\theta) \times AF(\theta),
\]

where

\[
AF(\theta) = \sum_{n=-N, n \neq 0}^{N} I_n e^{j(k_s n \sin \theta + \beta_n)}.
\]

is the array factor, and \( E_{ele}(\theta) \) is the elemental radiation. \( I_n \) and \( \beta_n \) denote excitation current and excitation phasor of \( n \)th element. \( S_n \) is the location of \( n \)th element. To simplify the feeding network and ensure that its operation is independent of frequencies, excitation phasor and excitation amplitude to all of the elements will be maintained unchanged. Hence (5) is rewritten as

\[
AF(\theta) = \sum_{n=-N, n \neq 0}^{N} I_0 e^{j k_s n \sin \theta}.
\]

where, \( I_0 \) is excitation current of each elements in an isolated system.

### 3. Numerical Results and Analysis

In this section, to evaluate wideband characteristics of the array structure shown in subsection 2.1, array factors and radiation pattern of DTAs will be directly compared with array factors and radiation pattern of the ESA. Four different frequencies \( f_1, f_2=1.5f_1, f_3=2f_1 \) and \( f_4=2.5f_1 \) corresponding with bandwidth \( f_2: f_1 \) of 2.5:1 is chosen for calculation. Note that when calculating frequencies change, the locations of elements remain unchanged. In other words, the total length of the array is fixed according to the wavelength \( \lambda_1 \) when the operating frequencies of the array change. Herein, \( \lambda_1 \) is the corresponding wavelength at the lowest frequency \( f_1 \).
3.1. Array factor

3.1.1. Equally spaced array \((d_i=0.7\lambda_i)\)

Frequency responses of equally spaced arrays are shown in Figure 3. Obviously, grating lobes are produced at high frequencies. The angle \((\theta_i)\) of grating lobe position is given by the expression

\[
\sin \theta_i = \frac{\lambda_i}{0.7 \lambda_i} \quad (i = 2, 3, 4).
\]

(7)

at \(f_2\), \(f_3\) and \(f_4\), \(\theta_0\) becomes 72.24 degrees, 45.58 degrees and 34.85 degrees, respectively.

![Figure 3. Array factor of equally spaced array, (a) \(f_1\), (b) \(f_2\), (c) \(f_3\), (d) \(f_4\)](image)

3.1.2. Density tapered array

Frequency responses of density tapered array configurations with \(d_i=0.6 \lambda_i\) (DTA1) are shown in Figure 4. Typically, the first sidelobe level (FSLL) of DTA1 reduces 2.4 dB compared to that of ESA. Interestingly, FSLL remains unchanged at -15.6 dB at all frequencies. The next effect is that the grating lobe levels are stretched and those peaks decrease 5 dB compared to those in ESA. Moreover, the grating lobe peaks remain unchanged at all frequencies.

For the DTA of \(d_i=0.5 \lambda_i\), the array factors are shown in Figure 5. The FSLL and the maximum grating lobe level keep reducing compared to those in Figure 4. Herein, the FSLL and maximum level of grating lobe are -19.1 dB and -7.8 dB, respectively. It can be seen that grating lobe variation is becoming larger.

For the DTA of \(d_i=0.4 \lambda_i\), the array factors are shown in Figure 6. The FSLL keeps decreasing, reaching -25.4 dB. In contrast, the second sidelobe level increases compared to that in Figure 5, reaching -17.2 dB. The reason for this increment is that the spacing among elements at the array edge become too large.
Figure 4. Array factor of density tapered array (DTA1), (a) $f_1$, (b) $f_2$, (c) $f_3$, (d) $f_4$

Figure 5. Array factor of density tapered array (DTA2), (a) $f_1$, (b) $f_2$, (c) $f_3$, (d) $f_4$
3.1.3. Summary

To summary the features of array factors, the FSLL and grating lobe levels are chosen and shown in Figure 7. The FSLL characteristic is shown in Figure 7(a), in which the FSLL decreases gradually when the density of elements at the array center increases. Moreover, those values remain unchanged at all frequencies. For grating lobe levels, the maximum levels of grating lobe are likely to decrease when the density of elements at the array center increases. However, at \( d_r = 0.4 \lambda_1 \) and \( f_c = 2.5 f_1 \) the grating lobe peaks tend to increase because the spacings between elements at the edge of the array become too large.

Figure 7. Comparison of FSLL and grating lobe level in terms of spacing at the array center
3.2. Dipole elements

To consider effects of array elements to its overall radiation pattern, dipole antennas with the length of 0.48 $\lambda_i$ and the diameter of $10^{-3} \lambda_i$ are selected, where $\lambda_i$ ($i=1,2,3,4$) are the wavelengths of corresponding frequencies $f_1$, $f_2$, $f_3$ and $f_4$. Herein, typical examples at $f_1$ and $f_3$ are selected.

3.2.1. Equally spaced array

Radiation pattern of ESA is shown in Figure 8. Obviously, the FSLL in Figure 8 is kept unchanged and the FSLL values in this case are equivalent to the results in Figure 3. Sidelobes which are far from the main lobe tend to decrease because of radiation characteristics of array elements. As a result, the maximum level of grating lobe is likely to decrease.

![Figure 8](image)

Figure 8. Radiation pattern of equally spaced array, (a) $f_1$, (b) $f_2$

The input impedances of elements in ESA are shown in Figure 9. Clearly, the input resistance (Rm) and reactance (Xm) components are approximately uniform among array elements. When the input impedance is uniform, the amplitude and phasor of each element are similar to the others, which makes it convenient to determine excitation in terms of amplitude and phasor to array elements.

![Figure 9](image)

Figure 9. The input impedances of half-wavelength elements in equally spaced array, (a) $f_1$, (b) $f_2$

3.2.2. Density tapered array

Radiation pattern of DTA when $d_i=0.5 \lambda_1$ is shown in Figure 10. The FSLLs are much more different compared to those in Figure 5. In particular, at the frequency of $f_1$, the FSLL in Figure 10(a) increases 5.1 dB compared to that of Figure 5(a). The reason for this discrepancy can be explained through results shown in Figure 11(a). Accordingly, the intense mutual coupling effects among elements at the array
center cause the increase of input impedance of elements at the array center. Since the input impedance of array center elements increases, the excitation current at the array center elements decreases. This results in the increase in the FSLL. For sidelobes far from the main lobe, the maximum levels of those lobes tend to decrease due to the radiation characteristic of array elements. At the frequency of \( f_1 \) shown in Figure 10(b), the FSLL is 2.2 dB lower than that in Figure 5(c), which can be explained by results in Figure 11(b). Herein, the input impedance of elements at array center is lower than those at array edge. This result makes the current distribution at the array center higher than that at the array edge. This feature makes the FSLL in Figure 10(b) lower than that in Figure 5(c). For grating lobe levels, the peak levels decrease significantly due to the radiation characteristic of array elements.

![Figure 10. Radiation pattern of density tapered array (DTA2). (a) \( f_1 \), (b) \( f_2 \)](image)

![Figure 11. The input impedances of half-wavelength elements in density tapered array (DTA2). (a) \( f_1 \), (b) \( f_2 \)](image)

3.2.3. Summary

The typical characteristics of DTA including FSLL and grating lobe levels are shown in Figure 12. As can be seen from Figure 12(a), the FSLL of DTA structure is equivalent to that in ESA at the frequency of \( f_1 \). This is because of the intense mutual coupling effects of elements at the array center. At high frequencies, the mutual coupling effect decreases which results in the reduction of FSLL. Those results show that, the mutual effects among array elements have to be taken into account in calculation and design of tapered arrays. Hence, the techniques of reducing mutual effects among array elements should be considered in unequally spaced array antenna designs. The variation trend of grating lobe peaks in Figure 12(b) is the same to that in Figure 7(b). The different results in Figure 12(b) and Figure 7(b) are caused by radiation characteristics of array elements which make the grating lobe levels in Figure 12(b) lower than those in Figure 7(b). Again, those results claim the ability of compressing grating lobe in a wide frequency range of DTA structure.

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Figure 12. Comparison FSLL and grating lobe level in terms of spacing at the array center. (a) $f_1$, (b) $f_2$

4. CONCLUSION

In this paper, the authors have proposed a method to examine the effectiveness of density tapered array, which is based on determining input impedance of array elements. In calculations, arrays consisting of 30 elements with various distributions were chosen. The results show that when there is no mutual coupling effect, DTA structures achieved sidelobe levels lower than ESA as well as were able to compress grating lobes within frequency range of 2.5:1. When mutual coupling effect between elements is considered, the wideband characteristic of DTA structure is ensured. Furthermore, the mutual coupling effect is likely to enhance or lower the sidelobe levels which are near the main lobe. This feature shows that mutual coupling effect has to be considered while designing density tapered array antenna.

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