1. Introduction

One of the most important unresolved problems of particle physics is the understanding of flavour and the fermion mass spectrum. The observed values of the quark and lepton masses and the quark mixing angles provide our main experimental clues to the underlying flavour dynamics contained in the physics beyond the Standard Model (SM). The most striking qualitative features of the spectroscopy of quarks and charged leptons are:

1. The fermion mass hierarchy: the large mass ratios of order 60 between fermions of a given electric charge, i.e. of the same family.

2. The fermion generation structure: the similarity between the mass spectra of the three families of quarks and charged leptons.

3. The quark mixing hierarchy: the smallness of the off-diagonal elements of the quark weak coupling matrix $V_{CKM}$.

Overall the charged fermion masses range over five orders of magnitude, from 1/2 Mev for the electron to over 100 GeV for the top quark.

A three generation structure is clearly indicated, consisting of $(u,d,e,\nu_e)$, $(c,s,\mu,\nu_\mu)$ and $(t,b,\tau,\nu_\tau)$ respectively. As is well known, each generation forms an anomaly free representation of the SM gauge group (SMG). The LEP measurements of the Z width show that there are just three neutrinos with masses less than $M_Z/2$ or more precisely

$$N_\nu = 2.985 \pm 0.023 \pm 0.004$$

We conclude that there are three generations of quarks and leptons, unless there exists (i) a heavy neutrino at the electroweak scale or (ii) a fourth generation of

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quarks without leptons, but having the SM gauge anomalies cancelled against those of a generation of ‘techniquarks’, associated with an extra non-abelian gauge group extending the SMG.

Neutrino masses, if non-zero, would seem to have a different origin to those of the quarks and charged leptons. In the SM there are no right-handed weak isosinglet neutrino states $\nu_R$ and the Higgs mechanism cannot generate a neutrino mass term. In extensions of the SM it is possible to generate Majorana mass terms connecting the left-handed weak isodoublet neutrinos of the SM with the corresponding set of right-handed weak isodoublet anti-neutrinos. These Majorana mass terms break weak isospin by one unit ($\Delta t = 1$) as well as lepton flavour conservation. Such a $\Delta t = 1$ mass term can be generated by: (i) the exchange of the the usual Higgs tadpole $\langle \phi_{WS} \rangle$ twice, via a superheavy lepton $L^0$ intermediate state having the same gauge quantum numbers as $\nu_R$ (i.e. neutral) under the SM; or (ii) the exchange of a single weak isotriplet Higgs tadpole. Method (i) has become known as the see-saw mechanism, since it generates a neutrino mass scale of $\langle \phi_{WS} \rangle^2/M_{L^0}$, suppressed by a factor of $\langle \phi_{WS} \rangle/M_{L^0}$ relative to the natural charged fermion mass scale of $\langle \phi_{WS} \rangle = 174$ GeV. More details about neutrino masses will be found in other contributions to this meeting.

Here we are really concerned with the charged fermion mass problem and the three main approaches to it:

1. Attempts to derive a fermion mass or mass relation exactly from some dynamical or theoretical principle.
2. Searches for relationships between mass and mixing angle parameters using symmetries and/or ansätze to make detailed fits to the data.
3. Attempts to naturally explain all the qualitative features of the fermion spectrum, fitting all the data within factors of order unity.

We shall illustrate these approaches by reviewing some recent developments in models of the quark and lepton mass matrices. An example of a mass relation following from an a priori theoretical principle is Veltman’s condition:

$$\sum_{\text{leptons}} m_l^2 + \sum_{\text{quarks}} m_q^2 = \frac{3}{2} M_W^2 + \frac{3}{4} M_Z^2 + \frac{3}{4} M_H^2$$

for the cancellation of quadratic divergences to one loop in the SM. In the next section we will consider predictions of the top quark mass based on the strong coupling dynamics of a renormalisation group infrared fixed point. The Fritzsch ansatz and its generalisation to mass matrix ansätze with texture zeros, in the context of supersymmetric grand unified (SUSY-GUT) models, will be considered as examples of the second approach. Finally we will turn to mass protection, by chiral flavour charges beyond the SM, for a natural explanation of the fermion mass hierarchy. We will consider examples of this last approach based on the minimal supersymmetric Standard Model (MSSM), SUSY-GUTS and antigrand unification.
2. Renormalisation Group Fixed Points and the Top Quark Mass

The idea that properties of the observed fermion mass spectrum could be explained in terms of an infrared fixed point of the renormalisation group equations (RGE) for the Yukawa coupling constants was first considered some time ago. It was pointed out that the three generation fermion mass hierarchy does not develop naturally out of the general structure of the RGE. However it was soon realised that the top quark mass might correspond to a fixed point value of the SM RGE, predicting approximately \( m_t \simeq 100 \text{ GeV} \). In practice one finds that such an infrared fixed point behaviour of the running top quark Yukawa coupling constant \( g_t(\mu) \) does not generically set in until \( \mu < 1 \text{ GeV} \), where the QCD coupling constant \( g_3(\mu) \) varies rapidly. The scale relevant for the physical top quark mass prediction is of course \( \mu = m_t \); at this scale \( g_3(\mu) \) is slowly varying and there is an effective infrared stable quasifixed point (which would be an exact fixed point if \( g_3(\mu) \) were constant) behaviour giving a running top quark mass prediction \( m_t(\mu = m_t) \simeq 225 \text{ GeV} \).

More precisely the SM quasifixed point prediction for the top quark mass requires the following assumptions:

1. The desert hypothesis of no new interactions beyond those of the SM up to some high energy scale \( \mu = M_X \simeq 10^{15} - 10^{19} \text{ GeV} \), e.g. the grand unification scale or the Planck scale.

2. The SM coupling constants remain positive and finite in the desert, such that perturbation theory and the RGE can be applied up to \( \mu = M_X \).

3. The top quark Yukawa coupling constant is large at \( \mu = M_X \):

\[
1 \leq g_t(M_X) \leq \sqrt{4\pi}
\]

so that it enters the domain of attraction of the infrared quasifixed point.

The nonlinearity of the RGE then strongly focuses \( g_t(\mu) \) at the electroweak scale to its quasifixed point value. The RGE for the Higgs self-coupling \( \lambda(\mu) \) similarly focuses \( \lambda(\mu) \) towards a quasifixed point value, leading to the SM fixed point predictions for the running top quark and Higgs masses:

\[
m_t \simeq 225 \text{ GeV} \quad m_H \simeq 250 \text{ GeV}
\]

Unfortunately the LEP results and the CDF measurement, which require a running top mass \( m_t \simeq 165 \pm 15 \text{ GeV} \) are inconsistent with this fixed point prediction for the top quark mass. Note that the running quark mass \( m_q \) is related to the physical or pole quark mass \( M_q \), defined as the location of the pole in the quark propagator, by

\[
M_q = m_t(M_q)(1 + 4\alpha_3(M_q)/3\pi)
\]

at the one loop QCD level.

There are two interesting modifications to the fixed point top mass prediction in the minimal supersymmetric Standard Model (MSSM) with supersymmetry breaking at the electroweak scale or Tev scale:
The introduction of the supersymmetric partners of the SM particles in the RGE for the Yukawa and gauge coupling constants leads to a 15% reduction in the fixed point value of $g_t(m_t)$.

There are two Higgs doublets in the MSSM and the ratio of Higgs vacuum values, $\tan \beta = v_2/v_1$, is a free parameter; the top quark couples to $v_2$ and so $m_t$ is proportional to $v_2 = (174 \text{ Gev}) \sin \beta$.

The MSSM fixed point prediction for the running top quark mass is

$$m_t(m_t) \simeq (190 \text{ Gev}) \sin \beta$$

which is remarkably close to the LEP and CDF results for $\tan \beta > 1$. This quasifixed point value is of course also the upper bound on the top mass in the MSSM, assuming perturbation theory is valid in the desert up to the SUSY-GUT scale. It then follows that the experimental evidence for a large top mass requires $\tan \beta > 1$. We note that the minimal SU(5) SUSY-GUT symmetry relation between the bottom quark and tau lepton Yukawa coupling constants, $g_b(M_X) = g_\tau(M_X)$, is also only satisfied phenomenologically if the top quark Yukawa coupling is close to its infrared quasifixed point value, so that it contributes significantly to the running of $g_t(\mu)$ and reduces the predicted value of $m_b(m_b)$. In the SM the contribution of the top quark Yukawa coupling has the opposite sign and the SU(5) GUT prediction for $m_b(m_b)$ fails, as it is then phenomenologically too large.

For large $\tan \beta$ it is possible to have a bottom quark Yukawa coupling satisfying $g_b(M_X) \geq 1$ which then approaches an infrared quasifixed point and is no longer negligible in the RGE for $g_t(\mu)$. Indeed with

$$\tan \beta \simeq m_t(m_t)/m_b(m_t) \simeq 60$$

we can trade the mystery of the top to bottom quark mass ratio for that of a hierarchy of vacuum expectation values, $v_2/v_1 \simeq m_t(m_t)/m_b(m_t)$, and have all the third generation Yukawa coupling constants large:

$$g_t(M_X) \geq 1 \quad g_b(M_X) \geq 1 \quad g_\tau(M_X) \geq 1$$

Then $m_t$, $m_b$ and $R = m_b/m_\tau$ all approach infrared quasifixed point values compatible with experiment. This large $\tan \beta$ scenario is consistent with the idea of Yukawa unification as occurs in the SO(10) SUSY-GUT model with the two MSSM Higgs doublets in a single 10 irreducible representation and $g_G \geq 1$ ensures fixed point behaviour. However it should be noted that the equality in Eq. (9) is not necessary. For example in SU(5) finite unified theories the Yukawa couplings are related to the SUSY-GUT coupling constant and satisfy $g_t^2(M_X) = 4g_b^2(M_X)/3 = \mathcal{O}(1)$, giving the same fixed point predictions. In fact one does not need a symmetry assumption at all,
since the weaker assumption of large third generation Yukawa couplings, Eq. (8), is sufficient for the fixed point dynamics to predict the running masses \( m_t \simeq 180 \text{ Gev}, \) \( m_b \simeq 4.1 \text{ Gev} \) and \( m_\tau \simeq 1.8 \text{ Gev} \) in the large \( \tan \beta \) scenario. Also the lightest Higgs particle mass is predicted to be \( m_{h^0} \simeq 120 \text{ Gev} \) (for a top squark mass of order 1 Tev).

The origin of the large value of \( \tan \beta \) is of course a puzzle, which must be solved before the large \( \tan \beta \) scenario can be said to explain the large \( m_t/m_b \) ratio. It is possible to introduce approximate symmetries \(^{22,23}\) of the Higgs potential which ensure a hierarchy of vacuum expectation values - a Peccei-Quinn symmetry and a continuous \( \mathcal{R} \) symmetry have been used. However these symmetries then result in a light chargino \(^{24}\), in conflict with the LEP lower bound of order 45 Gev on the chargino mass, unless the SUSY breaking scale \( M_{SUSY}^2 \) is fine-tuned to be much larger than the electroweak scale: \( M_{SUSY}^2 \geq \tan \beta M_Z^2 \). The Peccei-Quinn and \( \mathcal{R} \) symmetries require a hierarchical SUSY spectrum with the squark and slepton masses much larger than the gauginos, Higgsinos and \( Z \) masses. In particular they are inconsistent with the popular scenario of universal soft SUSY breaking mass parameters at the unification scale and radiative electroweak symmetry breaking \(^{25}\).

Also, in the large \( \tan \beta \) scenario, SUSY radiative corrections to \( m_b \) are generically large: the bottom quark mass gets a contribution proportional to \( v_2 \) from some one-loop diagrams with internal superpartners, such as top squark-charged Higgsino exchange, whereas its tree level mass is proportional to \( v_1 = v_2/\tan \beta \). Consequently these loop diagrams give a fractional correction \( \delta m_b/m_b \) to the bottom quark mass proportional to \( \tan \beta \) and generically of order unity \(^{23,25}\). The presence of the above-mentioned Peccei-Quinn and \( \mathcal{R} \) symmetries and the associated hierarchical SUSY spectrum (with the squarks much heavier than the gauginos and Higgsinos) would protect \( m_b \) from large radiative corrections, by providing a suppression factor in the loop diagrams and giving \( \delta m_b/m_b \ll 1 \). The hierarchical superpartner mass spectrum would also suppress a similar \( \mathcal{O}(\tan \beta) \) enhancement of the rare \( b \rightarrow s\gamma \) decay amplitude, which would otherwise be in conflict with the CLEO data \(^{26}\). However, in the absence of experimental information on the superpartner spectrum, the predictions of the third generation quark-lepton masses in the large \( \tan \beta \) scenario must, unfortunately, be considered unreliable.

3. Mass Matrix Ansätze and Texture Zeros

The motivation for considering mass matrix ansätze is to obtain testable relationships between fermion masses and mixing angles, thereby reducing the number of free parameters in the SM and providing a hint to the physics beyond the SM. The best known ansatz for the quark mass matrices is due to Fritzsch \(^{27}\):

\[
M_U = \begin{pmatrix}
0 & C & 0 \\
C & 0 & B \\
0 & B & A
\end{pmatrix} \quad M_D = \begin{pmatrix}
0 & C' & 0 \\
C' & 0 & B' \\
0 & B' & A'
\end{pmatrix}
\]

(10)

It contains 6 complex parameters \( A,B,C,A',B',C' \). Four of the phases can be rotated away by redefining the phases of the quark fields, leaving just 8 real parameters.
(the magnitudes of $A,B,C,A',B'$ and $C'$ and two phases $\phi_1$ and $\phi_2$) to reproduce 6 quark masses and 4 angles parameterising $V_{CKM}$. There are thus two relationships predicted by the Fritzsch ansatz. It is necessary to assume:

$$|A| \gg |B| \gg |C|, \quad |A'| \gg |B'| \gg |C'|$$

(11)

in order to obtain a good fermion mass hierarchy.

The first prediction is a generalised version of the relation $\theta_c \simeq \sqrt{\frac{m_d}{m_s}}$ for the Cabibbo angle, which originally motivated the ansatz:

$$|V_{us}| \simeq \sqrt{\frac{m_d}{m_s}} - e^{-i\phi_1} \sqrt{\frac{m_u}{m_c}}$$

(12)

and is well satisfied experimentally. However the second relationship:

$$|V_{cb}| \simeq \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}}$$

(13)

cannot be satisfied with a heavy top quark. Using $\sqrt{\frac{m_t}{m_b}} \simeq 0.18$ and $|V_{cb}| \leq 0.055$, an upper limit of $m_t < 100$ GeV is obtained. The limit is valid in the SM whether the ansatz is applied at the electroweak scale or at the GUT scale. This is also true in the MSSM. So, using the standard quark masses, the Fritzsch ansatz is excluded by the data.

Recently ansätze incorporating relationships between the fermion mass parameters at the grand unified or the Planck scale have been studied. We have already mentioned the best known result: the simple SU(5) relation $m_b(M_X) = m_\tau(M_X)$ which is satisfied in SUSY-GUTs provided the top quark mass is near to its quasifixed point value. However the corresponding relations for the first two generations are not satisfied, as they predict for example

$$m_d/m_s = m_e/m_\mu$$

(14)

which fails phenomenologically by an order of magnitude. This led Georgi and Jarlskog to postulate the mass relations $m_b(M_X) = m_\tau(M_X)$, $m_s(M_X) = m_\mu(M_X)/3$ and $m_d(M_X) = 3m_e(M_X)$ at the GUT scale. Dimopoulos, Hall and Raby revived these relations in the context of an SO(10) SUSY-GUT, combining the Fritzsch form for the up quark mass matrix $M_U = Y_u v_2$ with the Georgi-Jarlskog form for the down quark and charged lepton mass matrices $M_D = Y_d v_1$ and $M_L = Y_l v_1$:

$$Y_u = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix}, \quad Y_d = \begin{pmatrix} 0 & F e^{i\phi} & 0 \\ F^{-i\phi} & E & 0 \\ 0 & 0 & D \end{pmatrix}, \quad Y_l = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix}$$

(15)

The phase freedom in the definition of the fermion fields has been used to make the parameters $A$, $B$, $C$, $D$, $E$ and $F$ real and we have again to assume:

$$|A| \gg |B| \gg |C|, \quad |D| \gg |E| \gg |F|$$

(16)
Thus there are 7 free parameters in the Yukawa coupling ansatz and \( \tan \beta \) available to fit 13 observables. Using the RGE from the SUSY-GUT scale to the electroweak scale, this ansatz gives 5 predictions which are, within errors, in agreement with data for \( 1 < \tan \beta < 60 \). The failed simple SU(5) prediction Eq. (14) is replaced by the successful mass ratio prediction

\[
(m_d/m_s)(1 - m_d/m_s)^{-2} = 9(m_e/m_\mu)(1 - m_e/m_\mu)^{-2}
\]

Since the down quark matrix \( Y_d \) is diagonal in the two heaviest generations, one of the SUSY-GUT scale predictions is \( V_{cb} \approx \sqrt{\frac{m_c}{m_t}} \). Fits give \( m_t \) close to its fixed point and the large top Yukawa coupling causes \( V_{cb} \) to run between the GUT and electroweak scales to a somewhat lower value. Nonetheless the fits still tend to make \( V_{cb} \) too large. A fit satisfying Yukawa unification is obtained by setting \( A = D \) and \( \tan \beta \approx 60 \). It is of course subject to uncertainties due to the possibly large SUSY radiative corrections to \( m_b \) mentioned in the previous section.

Table 1. Approximate forms for the symmetric textures. The parameter \( \lambda \approx 0.2 \) is the CKM matrix element \( V_{us} \) used in the Wolfenstein parameterisation of \( V_{CKM} \).

|   | U                                      | D                                      |
|---|----------------------------------------|----------------------------------------|
| 1 | \( \begin{pmatrix} 0 & \sqrt{2}\lambda^6 & 0 \\ \frac{\sqrt{2}\lambda^6}{\lambda^4} & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 4\lambda^3 \\ 0 & 4\lambda^3 & 1 \end{pmatrix} \) |
| 2 | \( \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & 0 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 2\lambda^3 \\ 0 & 2\lambda^3 & 1 \end{pmatrix} \) |
| 3 | \( \begin{pmatrix} 0 & 0 & \sqrt{2}\lambda^4 \\ 0 & \lambda^4 & 0 \\ \frac{\sqrt{2}\lambda^4}{\sqrt{3}\lambda^4} & \lambda^2 & 0 \\ 0 & \lambda^2 & 1 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 4\lambda^3 \\ 0 & 4\lambda^3 & 1 \end{pmatrix} \) |
| 4 | \( \begin{pmatrix} 0 & \sqrt{2}\lambda^6 & 0 \\ \sqrt{2}\lambda^6 & \frac{\sqrt{3}\lambda^4}{\lambda^2} & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \) |
| 5 | \( \begin{pmatrix} 0 & 0 & \lambda^4 \\ 0 & \sqrt{2}\lambda^4 & \frac{\lambda^2}{\sqrt{2}} \\ \lambda^4 & \frac{\lambda^2}{\sqrt{2}} & 1 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \) |

The predictions arise due to the reduction in the number of free parameters, obtained by requiring the presence of zeros and symmetries between mass matrix elements. A systematic analysis of symmetric quark mass matrices with 5 or 6 “texture” zeros at the SUSY-GUT scale has recently been made. There are just 6 possible forms of symmetric mass matrix with an hierarchy of three non-zero eigen-
values and three texture zeros. These are:

\[
\begin{pmatrix}
a_1 & 0 & 0 \\
0 & b_1 & 0 \\
0 & 0 & c_1
\end{pmatrix}
\begin{pmatrix}
a_2 & 0 & 0 \\
0 & b_2 & 0 \\
0 & 0 & c_2
\end{pmatrix}
\begin{pmatrix}
a_3 & 0 & 0 \\
0 & b_3 & c_3
\end{pmatrix}
\tag{18}
\]

and

\[
\begin{pmatrix}
0 & 0 & a_4 \\
0 & b_4 & 0 \\
a_4 & 0 & c_4
\end{pmatrix}
\begin{pmatrix}
0 & a_5 & 0 \\
0 & b_5 & b_5 \\
a_5 & 0 & c_5
\end{pmatrix}
\begin{pmatrix}
0 & a_6 & b_6 \\
a_6 & 0 & 0 \\
b_6 & 0 & c_6
\end{pmatrix}
\tag{19}
\]

Comparison with the measured values of quark masses and mixing angles yields another 5 quark mass matrix ansätze consistent with experiment. The hierarchical structure of the parameters in the ansätze (cf. Eq. (16)) suggests a parameterisation of the form shown in Table 1, analogous to that of Wolfenstein for the quark mixing matrix. It is natural to interpret \( \lambda \) as a symmetry breaking parameter for some approximate symmetry beyond those of the Standard Model Group (SMG). The nature of this symmetry is discussed in the next section.

The neutrino Majorana mass matrices generated by the see-saw mechanism in many extensions of the SM naturally have the above type of symmetric texture. Due to the hierarchical structure of their elements, there are two qualitatively different types of eigenstate that can arise. In the first case, a neutrino can dominantly combine with its own antineutrino to form a Majorana particle. The second case occurs when a neutrino combines dominantly with an antineutrino, which is not the CP conjugate state, to form a 2-component massive neutrino. For example the electron neutrino might combine with the muon antineutrino. Such states naturally occur in pairs with order of magnitude-wise degenerate masses. In the example given, the other member of the pair of Majorana states would be formed by combining the electron neutrino with the muon antineutrino. The hierarchical structure which gives rise to this second case is of course ruled out phenomenologically for the quark and charged lepton mass matrices, as none have a pair of states with order of magnitude-wise degenerate masses. However, considering two generations for simplicity, a neutrino mass matrix of the form

\[
M_\nu = \nu_1 \nu_2
\begin{pmatrix}
\nu_1 & \nu_2 \\
0 & B \\
B & A
\end{pmatrix}
\]

with the assumed hierarchy

\[|B| \gg |A|\]

could be phenomenologically relevant. The mass eigenvalues are \( m_1 = B + A/2 \) and \( m_2 = B - A/2 \), giving a neutrino mass squared difference \( \Delta m^2 = 2AB \), and the neutrino mixing angle is \( \theta \simeq \pi/4 \) giving maximal mixing. Maximal neutrino mixing, \( \sin^2 2\theta \simeq 1 \), provides a candidate explanation for (i) the atmospheric muon neutrino deficit with \( \Delta m^2 = 10^{-2}eV^2 \) and \( \nu_\mu-\nu_\tau \) oscillations, or (ii) the solar neutrino problem with \( \Delta m^2 = 10^{-10}eV^2 \) and \( \nu_e-\nu_\mu \) vacuum oscillations.
4. Chiral Flavour Symmetries and Mass Protection

It is natural to try to explain the occurrence of large mass ratios in terms of selection rules due to approximate conservation laws. A Dirac mass term:

\[-m \bar{\psi}_R \psi_L + h.c.\]  \hspace{1cm} (22)

connects a left-handed fermion component \(\psi_L\) to its right-handed partner \(\psi_R\). If \(\psi_L\) and \(\psi_R\) have different quantum numbers, i.e. belong to inequivalent irreducible representations (IRs) of a symmetry group \(G\) (\(G\) is then called a chiral symmetry), then the mass term is forbidden in the limit of exact \(G\) symmetry and they represent two massless Weyl particles. \(G\) thus “protects” the fermion from gaining a mass. Note that this is exactly the situation for all the SM fermions, which are mass-protected by \(SU(2)_L \times U(1)_Y\) (but not by \(SU(3)_c\)). The \(SU(2)_L \times U(1)_Y\) symmetry is spontaneously broken and the SM fermions gain masses suppressed relative to the presumed fundamental (GUT or Planck) mass scale \(M\) by the symmetry breaking parameter:

\[\epsilon = \langle \phi_{WS} \rangle / M\]  \hspace{1cm} (23)

The extreme smallness of this parameter \(\epsilon\) constitutes, of course, the gauge hierarchy problem.

Here we are interested in the further suppression of the quark and lepton mass matrix elements relative to \(\langle \phi_{WS} \rangle\). We take the view \[\Box\] that this hierarchy is due to the existence of further approximately conserved chiral quantum numbers beyond those of the SMG. The SMG is then a low energy remnant of some larger group \(G\) and the fermion mass and mixing hierarchies are consequences of the spontaneous breaking of \(G\) to the SMG. The mass matrix element suppression factors depend on how the fermions behave w.r.t. \(G\) and on the symmetry breaking mechanism itself.

Consider, for example, an \(SMG \times U(1)_f\) model, whose fundamental mass scale is \(M\), broken to the SMG by the VEV of a scalar field \(\phi_S\) where \(\langle \phi_S \rangle < M\) and \(\phi_S\) carries \(U(1)_f\) charge \(Q_f(\phi_S) = 1\). Suppose further that \(Q_f(\phi_{WS}) = 0\), \(Q_f(b_L) = 0\) and \(Q_f(b_R) = 2\). Then it is natural to expect the generation of a \(b\) mass of order:

\[\left(\frac{\langle \phi_S \rangle}{M}\right)^2 \langle \phi_{WS} \rangle\]  \hspace{1cm} (24)

via (see Fig. \[\Box\]) the exchange of two \(\langle \phi_S \rangle\) tadpoles, in addition to the usual \(\langle \phi_{WS} \rangle\) tadpole, through two appropriately charged vector-like superheavy (i.e. of mass \(M\)) fermion intermediate states \[\Box\]. We identify

\[\epsilon_f = \frac{\langle \phi_S \rangle}{M}\]  \hspace{1cm} (25)

as the \(U(1)_f\) flavour symmetry breaking parameter. In general we expect mass matrix elements of order

\[M(i, j) \simeq \epsilon_f^{n_f} \langle \phi_{WS} \rangle\]  \hspace{1cm} (26)
\[ Q_f = 0 \quad Q_f = 1 \quad Q_f = 1 \quad Q_f = 2 \]
\[ b_L \quad M \quad M \quad b_R \]

\[ \langle \phi_S \rangle \quad \langle \phi_{WS} \rangle \quad \langle \phi_S \rangle \]

Fig. 1. Feynman diagram which generates the b quark mass via superheavy intermediate states.

where

\[ n_{ij} = | Q_f(\psi_{Li}) - Q_f(\psi_{Rj}) | \]  

is the degree of forbiddenness due to the \( U(1)_f \) quantum number difference between the left- and right-handed fermion components. So the effective SM Yukawa couplings of the quarks and leptons to the Weinberg-Salam Higgs

\[ y_{ij} \simeq \epsilon_f^{n_{ij}} \]

can consequently be small even though all fundamental Yukawa couplings of the “true” underlying theory are of \( \mathcal{O}(1) \). We are implicitly assuming here that there exists a superheavy spectrum of states which can mediate all of the symmetry breaking transitions; in particular we do not postulate the absence of appropriate superheavy states in order to obtain exact texture zeroes in the mass matrices \( \mathbb{F} \). We now consider models based on this idea.

Recently a systematic analysis of fermion masses in SO(10) SUSY-GUT models has been made \( \mathbb{E} \) in terms of effective operators obtained by integrating out the superheavy states, which are presumed to belong to vector-like SO(10) \( 16 + \overline{16} \) representations, in tree diagrams like Fig. \( \mathbb{I} \). The minimal number of effective operators contributing to mass matrices consistent with the low energy data is four, which leads to the consideration of GUT scale Yukawa coupling matrices satisfying Yukawa unification, Eq. \( \mathbb{I} \), and having the following texture

\[ Y_i = \begin{pmatrix} 0 & z'C & 0 \\ z_iC & y_iEe^{i\phi} & x'_iB \\ 0 & x_iB & A \end{pmatrix} \]

where \( i = u, d, l \). Here the \( x_i, x'_i, y_i, z_i \) and \( z'_i \) are SO(10) Clebsch Gordon coefficients. These Clebschs can take on a very large number of discrete values, which are determined once the set of 4 effective operators (tree diagrams) is specified. A scan of millions of operators leads to just 9 solutions consistent with experiment, having Yukawa coupling matrices with a partial Georgi-Jarlskog structure of the form:
\[
Y_u = \begin{pmatrix}
0 & \frac{-1}{2} C & 0 \\
-\frac{1}{2} C & 0 & x_u' B \\
0 & x_u B & A
\end{pmatrix}
\]
\[
Y_d = \begin{pmatrix}
0 & C & 0 \\
C & E e^{i \phi} & x_d' B \\
0 & x_d B & A
\end{pmatrix}
\]
\[
Y_l = \begin{pmatrix}
0 & C & 0 \\
C & 3 E e^{i \phi} & x_l' B \\
0 & x_l B & A
\end{pmatrix}
\]

For each of the 9 models the Clebschs \(x_i\) and \(x_i'\) have fixed values and the Yukawa matrices depend on 6 free parameters: \(A, B, C, E, \phi\) and \(\tan \beta\). Each solution gives 8 predictions consistent with the data, as illustrated in Table 2 for one of the models.

Table 2. Predictions for Model 6 with \(\alpha_s(M_Z) = 0.115\). The so-called Bag constant \(\hat{B}_K\) has been determined by lattice calculations to be in the range \(\hat{B}_K = 0.7 \pm 0.2\).

| Input Quantity | Input Value   | Predicted Quantity | Predicted Value |
|----------------|--------------|--------------------|-----------------|
| \(m_b(m_b)\)  | 4.35 GeV    | \(M_t\)           | 176 GeV         |
| \(m_c(m_c)\)  | 1.777 GeV   | \(\tan \beta\)    | 55              |
| \(m_c(m_c)\)  | 1.22 GeV    | \(V_{cb}\)        | .048            |
| \(m_\mu\)     | 105.6 MeV   | \(V_{ub}/V_{cb}\) | .059            |
| \(m_\tau\)    | 5.11 MeV    | \(m_s(1\text{GeV})\) | 172 MeV         |
| \(V_{us}\)    | 0.221       | \(\hat{B}_K\)     | 0.64            |
|                |             | \(m_u/m_d\)       | 0.64            |
|                |             | \(m_s/m_d\)       | 24.             |

The parameter hierarchy \(A \gg B, E \gg C\) and the texture zeros are interpreted as due to an approximately conserved global \(U(1)_f\) symmetry and the chosen superheavy fermion spectrum. The global \(U(1)_f\) charges are assigned in such a way that only the 4 selected tree diagrams are allowed. In particular the texture zeros reflect the assumed absence of superheavy fermion states which could mediate the transition between the corresponding Weyl states. A more detailed analysis of this \(U(1)_f\) flavour symmetry is promised.

We now turn to models in which the chiral flavour charges are part of the extended gauge group. The values of the chiral charges are then strongly constrained by the anomaly conditions for the gauge theory. It will also be assumed that any superheavy state needed to mediate a symmetry breaking transition exists, so that the results are insensitive to the details of the superheavy spectrum. Consequently there will be no exact texture zeros but just highly suppressed elements given by expressions like Eq. (26). The aim in these models is to reproduce all quark-lepton masses and mixing angles within a factor of 2 or 3.

The \(SMG \times U(1)_f\) model obtained by extending the SM with a gauged abelian flavour group appears unable to explain the fermion masses and mixings using an anomaly-free set of flavour charges. Models extending the SM (or the MSSM) with discrete gauge symmetries and having new interactions at energies as low as 1 Tev have also been investigated.

In a recent paper, Ibanez and Ross consider the extension of the MSSM by an abelian flavour group \(U(1)_f\). They then consider the construction of an anomaly free
The non-abelian contributions are given by:

\[
\begin{pmatrix}
\begin{bmatrix} d_L & u_R & d_R & e_L & e_R \\ s_L & c_R & s_R & \mu_L & \mu_R \\ b_L & t_R & b_R & \tau_L & \tau_R \end{bmatrix} & \begin{bmatrix} -4 & 4 & 4 & -7/2 & 7/2 \\ 1 & -1 & -1 & 1/2 & -1/2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\end{pmatrix}
\]

Since the charge assignments are axial, the quark and charged lepton mass matrices are symmetric up to factors of order unity. In addition to the two Higgs doublets of MSSM, which are taken to be neutral under \( U(1)_f \), two Higgs singlets, \( \theta \) and \( \bar{\theta} \), are introduced with \( U(1)_f \) charges +1 and −1 respectively and equal vacuum expectation values. The \( U(1)^2_2U(1)_Y \) gauge anomaly vanishes. The \( U(1)^3_f \) anomaly and the mixed \( U(1)_f \) gravitational anomaly could be cancelled against spectator particles neutral under the SMG. However cancellation of the mixed \( SU(3)^2U(1)_f, SU(2)^2U(1)_f \) and \( U(1)^3_2U(1)_f \) anomalies is only possible in the context of superstring theories via the Green Schwarz mechanism with \( \sin^2\theta_W = 3/8 \). Consequently the \( U(1)_f \) symmetry is spontaneously broken slightly below the string scale.

The \( U(1)_f \) charge assignments of Eq. 31 generate Yukawa matrices, via Eq. 28, of the following form:

\[
Y_u \approx \begin{pmatrix} \epsilon^8 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon & 1 \end{pmatrix} \quad Y_d \approx \begin{pmatrix} \epsilon^8 & \bar{\epsilon}^3 & \bar{\epsilon}^4 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon} \\ \bar{\epsilon}^4 & \bar{\epsilon} & 1 \end{pmatrix} \quad Y_l \approx \begin{pmatrix} \epsilon^5 & \epsilon^3 & 0 \\ \epsilon^3 & \bar{\epsilon} & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

The correct order of magnitude for all the masses and mixing angles are obtained by fitting \( \epsilon \), \( \bar{\epsilon} \) and \( \tan \beta \). This is a large \( \tan \beta \approx m_t/m_b \) model, but not necessarily having exact Yukawa unification.

In the antigrand unified model the fundamental non-simple gauge group \( SMG^3 \equiv SMG_1 \times SMG_2 \times SMG_3 \) (where each factor \( SMG_a \) acts non-trivially only on the \( a \)th generation) breaks down near the Planck scale to the usual SMG. This model has several broken chiral flavour charges, corresponding to the gauge subgroups \( SU_a(3), SU_a(2) \) and \( U_a(1) \), which can suppress fermion mass matrix elements. Any matrix element affected by a particular approximately conserved non-abelian subgroup will be suppressed by the same factor, because all suppressed transitions are identical (triplet \( \leftrightarrow \) singlet for \( SU_a(3) \) or doublet \( \leftrightarrow \) singlet for \( SU_a(2) \)). However the matrix elements affected by an abelian subgroup \( U_a(1) \) are not suppressed identically, since the differences in the \( a \)th generation weak hypercharge between the corresponding left- and right-handed Weyl components vary. The overall suppression of the mass matrix elements can be written in the form:

\[
M(i, j) = y_{ij}^{non-ab} y_{ij}' \phi_{WS}
\]

The non-abelian contributions are given by:

\[
Y^{non-ab}_{u} \approx Y^{non-ab}_{d} \approx \begin{pmatrix} \epsilon_1 & \epsilon_2 & \bar{\epsilon}_1 & \bar{\epsilon}_2 & \bar{\epsilon}_3 \\ \epsilon_1 & \epsilon_2 & \bar{\epsilon}_1 & \bar{\epsilon}_2 & \bar{\epsilon}_3 \\ \epsilon_1 & \epsilon_2 & \bar{\epsilon}_1 & \bar{\epsilon}_2 & \bar{\epsilon}_3 \\ \epsilon_1 & \epsilon_2 & \bar{\epsilon}_1 & \bar{\epsilon}_2 & \bar{\epsilon}_3 \\ \epsilon_1 & \epsilon_2 & \bar{\epsilon}_1 & \bar{\epsilon}_2 & \bar{\epsilon}_3 \end{pmatrix} \quad Y^{non-ab}_{l} \approx \begin{pmatrix} \epsilon_1 & \epsilon_2 & \epsilon_3 \\ \epsilon_1 & \epsilon_2 & \epsilon_3 \\ \epsilon_1 & \epsilon_2 & \epsilon_3 \\ \epsilon_1 & \epsilon_2 & \epsilon_3 \\ \epsilon_1 & \epsilon_2 & \epsilon_3 \end{pmatrix}
\]
where \( \epsilon_a \) is the symmetry breaking parameter for \( SU_a(2) \) and \( \delta_a \) is the symmetry breaking parameter for \( SU_a(3) \). A natural measure of the degree of suppression by the abelian \( U_a(1) \) components is given by the distance in abelian charge space parameterised by a general metric \( g_{ab} \):

\[
y^{ab}_{ij} = \exp[-\sqrt{(Q_{ai} - Q_{aj})g_{ab}(Q_{bi} - Q_{bj})}]
\]

(35)

where \( Q_{ai} \) is the value of the \( a \)'th generation weak hypercharge carried by the \( i \)'th Weyl state.

The above ansatz Eq. (33) can readily explain the generation mass gaps but not the mass splittings within each generation, as it inevitably predicts:

\[
m_u m_c m_t \simeq m_e m_\mu m_\tau \leq m_d m_s m_b
\]

(36)

So we are led to extend the gauge group further and \( SMG^3 \times U(1)_f \) is the only non-trivial anomaly-free extension with no new fermions and the \( U(1)_f \) charges are essentially unique:

\[
\left( \begin{array}{ccccccc}
    d_L & u_R & d_R & e_L & e_R \\
    s_L & c_R & s_R & \mu_L & \mu_R \\
    b_L & t_R & b_R & \tau_L & \tau_R \\
\end{array} \right) = \left( \begin{array}{ccccccc}
    0 & 0 & 0 & 0 & 0 \\
    0 & 1 & -1 & 0 & -1 \\
    0 & -1 & 1 & 0 & 1 \\
\end{array} \right)
\]

(37)

Table 3. Results of an \( SMG^3 \times U(1)_f \) model fit to fermion masses and mixing angles. All masses are running masses evaluated at 1 GeV unless otherwise stated. The third column shows a fit biased in favour of obtaining \( m_c > m_s \).

| Fit Results | \( m_t^{phys} = 100 \text{ GeV} \) | \( m_t^{phys} = 200 \text{ GeV} \) |
|-------------|----------------|----------------|
| \( \chi^2 \) | 3.7 | 5.6 | 6.9 |
| \( m_e \) (MeV) | 1.0 | 1.0 | 1.0 |
| \( m_\mu \) (MeV) | 120 | 160 | 110 |
| \( m_\tau \) (GeV) | 1.4 | 1.5 | 1.5 |
| \( m_d \) (MeV) | 4.9 | 4.9 | 4.9 |
| \( m_s \) (MeV) | 600 | 790 | 530 |
| \( m_b^{phys} \) (GeV) | 5.4 | 5.5 | 5.3 |
| \( m_u \) (MeV) | 4.9 | 4.9 | 4.9 |
| \( m_c \) (GeV) | 0.73 | 0.53 | 0.84 |
| \( V_{us} \) | 0.19 | 0.22 | 0.22 |
| \( V_{cb} \) | 0.016 | 0.012 | 0.0048 |
| \( V_{ub} \) | 0.0030 | 0.0027 | 0.0027 |

A good order of magnitude fit to the data can now be obtained\(^{12}\) using 5 degrees of freedom and results are shown in the first column of Table 3. All the data are fitted within a factor of 2, except for \( m_s \) and \( V_{cb} \) which are fitted within a factor of 3.
5. Conclusion

All the fermions except the top quark are light compared to the electroweak scale $\langle \phi_{WS} \rangle$. So we might obtain a dynamical understanding of $m_t$ - the SUSY fixed point value is particularly promising - before understanding the electron mass and the rest of the spectrum. The large top to bottom quark mass ratio is a mystery, which can be exchanged for the mystery of a hierarchy of Higgs vacuum values; all the third generation masses are then consistent with quasifixed point values and/or Yukawa unification. However, in this large $\tan \beta$ scenario, SUSY radiative corrections to $m_b$ are generically large. There exist several mass matrix ansätze with texture zeros giving typically 5 successful relations between mass and mixing parameters (including the 3 Georgi Jarlskog GUT scale relations: $m_b = m_\tau$, $m_s = m_\mu/3$ and $m_d = 3m_e$). However these ansätze merely incorporate the mass hierarchy. Their hierarchical structure strongly suggests the existence of approximately conserved chiral gauge quantum numbers beyond those of the SMG responsible for mass protection.

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