THE HUBBLE SPACE TELESCOPE KEY PROJECT ON THE EXTRAGALACTIC DISTANCE SCALE. XV. A CEPHEID DISTANCE TO THE FORNAX CLUSTER AND ITS IMPLICATIONS

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ABSTRACT

Using the Hubble Space Telescope, 37 long-period Cepheids variables have been discovered in the Fornax Cluster spiral galaxy NGC 1365. The resulting V and I period-luminosity relations yield a true distance modulus of μ0 = 31.35 ± 0.07 mag, which corresponds to a distance of 18.6 ± 0.6 Mpc. This measurement provides several routes for estimating the Hubble constant. (1) Assuming this distance for the Fornax Cluster as a whole yields a local Hubble constant of 70 ± 18 (random) ±7 (systematic) km s⁻¹ Mpc⁻¹. (2) Nine Cepheid-based distances to groups of galaxies out to and including the Fornax and Virgo Clusters yield H₀ = 73 ± 16 (random) ±7 (systematic) km s⁻¹ Mpc⁻¹. (3) Recalibrating the I-band Tully-Fisher relation using NGC 1365 and six nearby spiral galaxies, and applying it to 15 galaxy clusters out to 100 Mpc, gives H₀ = 76 ± 3 (random) ±8 (systematic) km s⁻¹ Mpc⁻¹. (4) Using a broad-based set of differential cluster distance moduli ranging from Fornax to Abell 2147 gives H₀ = 72 ± 3 (random) ±6 (systematic) km s⁻¹ Mpc⁻¹. Finally, (5) assuming the NGC 1365 distance for the two additional Type Ia supernovae in Fornax and adding them to the SN Ia calibration (correcting for light-curve shape) gives H₀ = 67 ± 6 (random) ±7 (systematic) km s⁻¹ Mpc⁻¹ out to a distance in excess of 500 Mpc. All five of these H₀ determinations agree to within their statistical errors. The resulting estimate of the Hubble constant, combining all of these determinations, is H₀ = 72 ± 5 (random) ±7 (systematic) km s⁻¹ Mpc⁻¹. An extensive tabulation of identified systematic and statistical errors, and their propagation, is given.

Subject headings: Cepheids — distance scale — galaxies: clusters: individual (Fornax) — galaxies: distances and redshifts — galaxies: individual (NGC 1365)

1. INTRODUCTION

Hubble (1929) announced his discovery of the expansion of the universe nearly 70 years ago. Despite decades of effort and continued improvements in the actual measurement of extragalactic distances, convergence on a consistent value for the absolute expansion rate, the Hubble constant, H₀, has been elusive. However, progress on the absolute calibration of the extragalactic distance scale in the last few years has been rapid and dramatic (see, for instance, Freedman, Madore, & Kennicutt 1997; Mould et al. 1997; Tamman & Federspiel 1997; Madore, Freedman, & Sakai 1997; and also see Jacoby et al. 1992 and Riess, Press, & Kirshner 1996). This accelerated pace has occurred primarily as a result of the improved resolution of the Hubble Space Telescope (HST) and its consequent ability to discover classical Cepheid variables at distances a factor of 10 farther than can routinely be achieved from the ground. As a result, accurate zero points to a number of recently refined methods which can measure precise relative distances beyond the realm of the Cepheids have become available. These combined efforts are providing a more accurate distance scale for local galaxies and indicate a convergence among various secondary distance indicators in establishing an absolute calibration of the far-field Hubble flow.

The discovery of Cepheids with HST has proved to be very efficient out to and even somewhat beyond distances of ~20 Mpc. Soon after the 1993 December HST servicing mission the measurements of Cepheids in the Virgo Cluster (part of the original design specifications for the telescope) became feasible (Freedman et al. 1994a). The subsequent discovery of Cepheids in the Virgo galaxy M100 (Freedman et al. 1994b; Ferrarese et al. 1996) was an important step in resolving outstanding differences in the extragalactic distance scale (Mould et al. 1995). The Virgo Cluster is complex in both its geometric and its kinematic structure, and there still remain large uncertainties in both the velocity and the distance to this cluster. Hence, the Virgo Cluster is not an ideal test site for an unambiguous determination of the cosmological expansion rate or the calibration of secondary distance indicators. In this paper we discuss the implications of a Cepheid distance to the next major cluster of galaxies, Fornax, which is a simpler system than Virgo.

In the companion paper to this one (Silbermann et al.
we present the Cepheid photometry and PL relations for the Cepheids in NGC 1365. In Madore et al. (1998) we briefly discussed the determination of $H_0$, based on the distance of NGC 1365 and the Fornax Cluster, in addition to a calibration of a local Hubble expansion-rate plot. The Fornax Cluster is comparable in distance to the Virgo Cluster (de Vaucouleurs 1975), but it is found almost opposite to Virgo, in the skies of the Southern Hemisphere. The Fornax Cluster is less rich in galaxies than Virgo (Ferguson & Sandage 1988), but it is also substantially more compact than its northern counterpart (Fig. 1). As a result of its lower mass, the influence of Fornax on the local velocity field is less dramatic than that of the Virgo Cluster. Because of its compact nature, questions concerning the membership and location in the cluster of individual galaxies are significantly less problematic; the back-to-front geometry is far simpler and less controversial than that of the Virgo Cluster. Clearly, Fornax provides a much more interesting site for a test of the local expansion rate.

In the context of the Key Project on the Extragalactic Distance Scale (Kennicutt, Freedman, & Mould 1995), there are several important reasons to secure a distance to the Fornax Cluster. The Fornax Cluster serves as both a probe of the local velocity field and a major jumping-off point for several secondary distance indicators, which can be used to probe a volume of space at least 1000 times larger. To obtain a distance to the Fornax Cluster, the $H_0$ Key Project sample includes three member galaxies; the first of these, discussed here, is the Seyfert 1 galaxy NGC 1365, a striking, two-armed, barred-spiral galaxy with an active galactic nucleus. Two additional galaxies, NGC 1425 and NGC 1326A, have also been imaged with HST, and those data are being processed; preliminary reduction shows that the distance to NGC 1326A (Prosser et al. 1999) lies within the uncertainties quoted here for NGC 1365.

2. NGC 1365 AND THE FORNAX CLUSTER

Three lines of evidence independently suggest that NGC 1365 is a representative, physical member of the Fornax Cluster. First, NGC 1365 is almost directly along our line of sight to Fornax: the galaxy is projected only $\sim 70'$ (380 kpc) from the geometric center of the cluster, whereas the radius of the cluster is $\sim 100'$ (540 kpc; Ferguson 1989, and see also Fig. 2). In addition, NGC 1365 is also coincident with the Fornax Cluster in velocity space. The observed velocity of NGC 1365 (1636 km s$^{-1}$) is only 195 km s$^{-1}$ larger than the cluster mean and is well inside the cluster velocity dis-

![Fig. 1. Comparison of the distribution of galaxies as projected on the sky for the Virgo Cluster (right) and the Fornax Cluster (left). M100 and NGC 1365 are each individually marked by arrows showing their relative disposition with respect to the main body and cores of their respective clusters. Units are arcminutes.](image1)

![Fig. 2. Fornax galaxies with published radial velocities within 6° of the cluster center and having apparent velocities less than 2500 km s$^{-1}$. All 117 galaxies used to define the mean velocity (and velocity dispersion) for the Fornax Cluster are shown plotted as they appear on the sky. The 78 early-type galaxies are depicted by filled circles; the 39 late-type galaxies are shown as open circles. NGC 1365, near the center of the cluster, is individually marked.](image2)
shown, fitting well within the distribution. At the base of each of the plots the velocity of NGC 1365 is a direct reflection of the highly Gaussian nature of the Fornax velocity distribution plotted. Horizontal lines at CPD \(0.50, 0.64, 0.84\) cross the distribution curve at the mean velocity and at \(\pm 1\) \(\sigma\), respectively. These are to be compared to the simple average and standard deviation shown by the centrally plotted error bar. The close coincidence of the two estimates is a direct reflection of the highly Gaussian nature of the Fornax velocity distribution. At the base of each of the plots the velocity of NGC 1365 is shown, fitting well within the \(1\) \(\sigma\) velocity dispersion.

The solid curve is the sum of the individual Gaussian distributions. Another representation of the velocity density distribution is given in Figure 4. The sample distribution in position-velocity space are shown in Figure 2, and two "pie diagrams" illustrating the sample distribution of these 117 objects projected on the sky is shown in Figure 3. In all three representations, elliptical galaxies are shown as filled circles and spiral galaxies as open circles. While the core of the cluster is demonstrably dominated by E/S0 galaxies, there is no other obvious segregation of the two populations, spiral and elliptical galaxies being coincident and largely cospatial. After subdividing the sample by morphological type, 39 spiral/irregular galaxies give \(V = 1399\) km s\(^{-1}\) and \(\sigma = \pm 334\) km s\(^{-1}\), and 78 E/S0 galaxies give \(V = 1463\) km s\(^{-1}\) with \(\sigma = \pm 347\) km s\(^{-1}\). The mean velocity of the spirals agrees with the mean for the ellipticals to within \(0.2\) \(\sigma\) of the velocity dispersion of the system. The combined sample of 117 galaxies has an unweighted mean of \(V = 1441\) km s\(^{-1}\) and \(\sigma = \pm 342\) km s\(^{-1}\) which we adopt hereafter (see also Schröder 1995; Han & Mould 1990). The velocity offset of 195 km s\(^{-1}\) for NGC 1365 with respect to this mean is less than two-thirds of the cluster velocity dispersion.\(^{14}\)

The systemic (heliocentric) velocity and velocity dispersion of the main population of galaxies in Fornax are well defined. A search of the NASA/IPAC Extragalactic Database\(^{13}\) for galaxies within 6° of the Fornax Cluster center and having published redshifts of \(\leq 2500\) km s\(^{-1}\) produced a sample of 106 galaxies; this was was then supplemented with four additional redshifts from ZCAT (Huchra et al. 1992; the 1998 edition of ZCAT is available via anonymous ftp from fang.harvard.edu) and seven recently published dwarf galaxy redshifts from Drinkwater & Gregg (1998), giving a total of 117 redshifts. The distribution of these 117 objects projected on the sky is shown in Figure 2, and two "pie diagrams" illustrating the sample distribution in position-velocity space are shown in Figure 3. In all three representations, elliptical galaxies are shown as filled circles and spiral galaxies as open circles. While the core of the cluster is demonstrably dominated by E/S0 galaxies, there is no other obvious segregation of the two populations, spiral and elliptical galaxies being coincident and largely cospatial. After subdividing the sample by morphological type, 39 spiral/irregular galaxies give \(V = 1399\) km s\(^{-1}\) and \(\sigma = \pm 334\) km s\(^{-1}\), and 78 E/S0 galaxies give \(V = 1463\) km s\(^{-1}\) with \(\sigma = \pm 347\) km s\(^{-1}\). The mean velocity of the spirals agrees with the mean for the ellipticals to within \(0.2\) \(\sigma\) of the velocity dispersion of the system. The combined sample of 117 galaxies has an unweighted mean of \(V = 1441\) km s\(^{-1}\) and \(\sigma = \pm 342\) km s\(^{-1}\) which we adopt hereafter (see also Schröder 1995; Han & Mould 1990). The velocity offset of 195 km s\(^{-1}\) for NGC 1365 with respect to this mean is less than two-thirds of the cluster velocity dispersion.\(^{14}\)

3. THE MEAN VELOCITY AND VELOCITY DISPERSION OF FORNAX

The systemic (heliocentric) velocity and velocity dispersion of the main population of galaxies in Fornax are well defined. A search of the NASA/IPAC Extragalactic Database\(^{13}\) for galaxies within 6° of the Fornax Cluster center and having published redshifts of \(\leq 2500\) km s\(^{-1}\) produced a sample of 106 galaxies; this was was then supplemented with four additional redshifts from ZCAT (Huchra et al. 1992; the 1998 edition of ZCAT is available via anonymous ftp from fang.harvard.edu) and seven recently published dwarf galaxy redshifts from Drinkwater & Gregg (1998), giving a total of 117 redshifts. The distribution of these 117 objects projected on the sky is shown in Figure 2, and two "pie diagrams" illustrating the sample distribution in position-velocity space are shown in Figure 3. In all three representations, elliptical galaxies are shown as filled circles and spiral galaxies as open circles. While the core of the cluster is demonstrably dominated by E/S0 galaxies, there is no other obvious segregation of the two populations, spiral and elliptical galaxies being coincident and largely cospatial. After subdividing the sample by morphological type, 39 spiral/irregular galaxies give \(V = 1399\) km s\(^{-1}\) and \(\sigma = \pm 334\) km s\(^{-1}\), and 78 E/S0 galaxies give \(V = 1463\) km s\(^{-1}\) with \(\sigma = \pm 347\) km s\(^{-1}\). The mean velocity of the spirals agrees with the mean for the ellipticals to within \(0.2\) \(\sigma\) of the velocity dispersion of the system. The combined sample of 117 galaxies has an unweighted mean of \(V = 1441\) km s\(^{-1}\) and \(\sigma = \pm 342\) km s\(^{-1}\) which we adopt hereafter (see also Schröder 1995; Han & Mould 1990). The velocity offset of 195 km s\(^{-1}\) for NGC 1365 with respect to this mean is less than two-thirds of the cluster velocity dispersion.\(^{14}\)

4. HST OBSERVATIONS AND THE CEPHEIDS IN NGC 1365

Using the Wide Field and Planetary Camera 2 on HST, we have obtained a set of 12 epoch observations of NGC 1365. The observing window of 44 days, beginning August 6 and continuing until 1995 September 24, was selected to...
maximize target visibility, without necessitating any roll of the targeted field of view. Sampling within the window was prescribed by a power-law distribution, tailored to optimally cover the light and color curves of Cepheids with anticipated periods in the range 10 to 60 days (see § 3 for additional details). Contiguous with four of the 12 $V$-band epochs (5100 s each through the F555W filter), $I$-band exposures (5400 s each through the F814W filter) were also obtained to allow a determination of reddening corrections for the Cepheids.

All frames were pipeline preprocessed at the Space Telescope Science Institute in Baltimore, MD, and were subsequently analyzed using two stellar photometry packages, ALLFRAME (Stetson 1994) and DoPhot (Schechter, Mateo, & Saha 1993), in order to quantify potential systematic differences in the two reduction programs. Zero-point calibrations for the photometry were adopted from Holtzman et al. (1995) and Hill et al. (1998), which agree to 0.05 mag on average. Details on the DoPhot and ALLFRAME reduction and analysis of this data set are presented elsewhere (Silbermann et al. 1999). We are also currently undertaking artificial star tests on these frames to quantify the uncertainty due to crowding (Ferrarese et al. 1999).

Detailed information on the 52 Cepheid candidates discovered in NGC 1365 can be found in Silbermann et al. (1999). The phase coverage in all cases is sufficiently dense and uniform that the form of the light curves is clearly delineated. We have adopted a sample of 37 of these variables as being unambiguously classified as high-quality Cepheids on the basis of their distinctively rapid brightening, followed by a long, linear decline phase (for both the DoPhot and ALLFRAME variable-star candidates). Periods, obtained using a modified Laffer-Kinman algorithm (Laffer & Kinman 1965), are statistically good to a few percent, although in some cases ambiguities larger than this do exist as a consequence of the narrow observing window and the restricted number of cycles (between 1 and 5) covered within the 44 day window. A variety of other samples, selected and reduced in a number of different ways, are discussed in Silbermann et al. To the degree that the distances all agree to within their quoted errors the broad conclusions resulting from this paper are not affected by choice of sample.

The resulting $V$ and $I$ period-luminosity relations for the select set of 37 Cepheids (using intensity-averaged magnitudes) are shown in the upper and lower panels of Figure 5, respectively. This sample differs slightly from that adopted in Silbermann et al. (1999) only in the fact that the three 50 day Cepheids are retained in this analysis. The derived apparent moduli are $\mu_V = 31.68 \pm 0.05$ (random) mag and $\mu_I = 31.55 \pm 0.05$ (random) mag. Correcting for a derived total line-of-sight reddening of $E(V-I)_{N1365} = 0.14$ mag (derived from the Cepheids themselves) gives a true distance modulus of $\mu_0 = 31.35 \pm 0.07$ (random) mag. This corresponds to a distance to NGC 1365 of 18.6 $\pm$ 0.6 (random) Mpc, which is within 2% of the value derived from phase-weighted magnitudes of the slightly smaller Cepheid data set as given in Silbermann et al. The quoted error at this step in the discussion quantifies only the statistical (random) uncertainty generated by photometric errors in the ALLFRAME data combined with the intrinsic magnitude and color width of the Cepheid instability strip.

Extensive reviews of the distance to the Fornax Cluster (especially in the context of a differential comparison with the distance to the Virgo Cluster) can be found in two recent publications (Bureau et al. 1996, Table 3; Schröder 1996, Table 6.1). Bureau et al. quote a distance of 16.6 $\pm$ 3.4 Mpc, which in turn was consistent with a value of 16.9 $\pm$ 1.1 Mpc reported in the same year by McMillan, Ciardullo, & Jacoby (1996).

5. THE HUBBLE CONSTANT

We now discuss the impact of a Cepheid distance to the Fornax Cluster in estimating the Hubble constant. Before doing so we must make clear the limited context and focused nature of this paper. We are interested in exploring the consequences of Cepheid-based distances in general and the impact of a Cepheid distance to Fornax in the specific case, on the determinations of the extragalactic distance scale (and the Hubble constant) directly dependent upon the Cepheids. This is not intended to be a review of all measures of the Hubble constant, nor do we revisit methods that do not penetrate the flow any further than the Cepheids themselves now do. At the time of writing, this latter exclusion applied to the planetary nebula luminosity function (PLNF) method and to the surface brightness fluctuation (SBF) method, neither of which (see Jacoby et al. 1992 for an extensive discussion and review) extended further than Fornax, the subject of this Cepheid paper. In the meantime, $HST$ observations by Lauer et al. (1998) and by Jensen, Tonry, & Luppino (1998) have extended the SBF method out to the far field, and they determine values of $H_0 = 89$ and $87 \pm 10$ km s$^{-1}$ Mpc$^{-1}$. A comprehensive reanalysis of SBF and other methods will be presented in a later series of Key Project team papers. This is an interim report.

Below we present and discuss several independent estimates of the local expansion rate, where the analysis is based both on the new Fornax distance and the distances to
other Key Project galaxies, consistently scaled to a true distance modulus of 18.50 mag (50 kpc) for the Large Magellanic Cloud. At the end we intercompare the results for convergence and consistency. The first estimate is based solely on the Fornax Cluster, its velocity, and the Cepheid-based distance to one of its members. It samples the flow in one particular direction at a distance of ~20 Mpc. We then examine the inner volume of space, leading up to and including both the Virgo and Fornax Clusters. This has the added advantage of averaging over different samples and a variety of directions, but it is still limited in volume (to an average distance of ~10 Mpc), and it is subject to the usual caveats concerning bulk flows and the adopted Virgocentric flow model (Table 1). The third estimate comes from using the Cepheid distance to Fornax to lock into secondary distance indicators, thereby allowing us to step out to cosmo-logically significant velocities (10,000 km s⁻¹ and beyond) corresponding to distances greater than 100 Mpc. Averaging over the sky and working at large redshifts alleviate the flow problems. Examining consistency between the independent secondary distance estimates and then averaging over their far-field estimates should provide a more systematically secure value of and, more importantly, a measure of its external error. Comparison of the three “regional” estimates (Fornax, local, and far-field) then can be used to provide a check on the systematics resulting from the various assumptions made independently at each step.

6. UNCERTAINTIES IN THE FORNAX CLUSTER DISTANCE AND VELOCITY

Figure 1 shows a comparison of the Virgo and Fornax Clusters of galaxies drawn to scale, as seen projected on the sky. The comparison of apparent sizes is appropriate given that the two clusters are at approximately the same distance from us. In the extensive Virgo Cluster (right), the galaxy M100 can be seen marked ~4° to the northwest of the elliptical-galaxy-rich core; this corresponds to an impact parameter of 1.3 Mpc, or 8% of the distance from the Local Group to the Virgo Cluster. The Fornax Cluster (left) is more centrally concentrated than Virgo, so that the back-to-front uncertainty associated with its three-dimensional spatial extent is reduced for any randomly selected member. Roughly speaking, converting the total angular extent of the cluster on the sky (~3° in diameter; Ferguson & Sandage 1988) into a back-to-front extent, the error associated with any randomly chosen galaxy in the Fornax Cluster, translates into a few percent uncertainty in distance; this uncertainty in distance can be reduced when more distances to spirals in Fornax have been measured.

Here we note that the infall-velocity correction for the Local Group motion with respect to the Virgo Cluster (and its associated uncertainty) becomes a minor issue for the Fornax Cluster. This is the result of a fortuitous combination of geometry and kinematics. We now have Cepheid distances from the Local Group to both the Fornax and Virgo Clusters. Combined with their angular separation on the sky, this immediately leads to the physical separation between the two clusters. Under the assumption that the Virgo Cluster dominates the local velocity perturbation field at the Local Group and at Fornax, we can calculate the velocity perturbation at Fornax (assuming that the flow-field amplitude scales with 1/R_{virgo} and characterized by a R⁻² density distribution; Sandage 1980).

From this we then derive the flow contribution to the measured line-of-sight radial velocity, as seen from the Local Group. Figure 6 shows the distance scale structure (left) and the velocity-field geometry (right) of the Local Group—Virgo—Fornax system. An infall velocity of the Local Group

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**TABLE 1**

| Source of Uncertainty on the Mean | Description of Uncertainty | Error (%)
|----------------------------------|-----------------------------|----------
| LMC: Cepheid PL Calibration      |                            |          |
| [A] LMC true modulus .............. | Independent estimates = 18.50 ± 0.10 mag | 5        |
| [B] V PL zero point .............. | LMC PL σ_v = (0.27)/√31 = ±0.05 mag | 3        |
| [C] I PL zero point .............. | LMC PL σ_i = (0.18)/√31 = ±0.03 mag | 2        |
| [SC] Systematic uncertainty ...... | [A] + [B] + [C] combined in quadrature | 6        |
| NGC 1365: Cepheid True Distance Modulus |                           |          |
| (D) HST V-band zero point ....... | On-orbit calibration: ±0.05 mag | 3        |
| (E) HST I-band zero point ....... | On-orbit calibration: ±0.05 mag | 3        |
| (M1) Cepheid true modulus ....... | (D)(E) are uncorrelated, but coupled by reddening law: σ_v = ±0.15 mag | 7        |
| (F) Cepheid V modulus .......... | NGC 1365 PL σ_v = (0.32)/√37 = ±0.05 mag | 3        |
| (G) Cepheid I modulus ........... | NGC 1365 PL σ_i = (0.31)/√37 = ±0.05 mag | 3        |
| (M2) Cepheid true modulus ....... | (F)(G) are partially correlated, giving σ_v = ±0.06 mag | 3        |
| (P1) V-band aperture correction | Silbermann et al. (1999) give ±0.067 mag | 3        |
| (P2) I-band aperture correction | Silbermann et al. (1999) give ±0.061 mag | 3        |
| (M3) Cepheid true modulus ....... | (P1)(P2) are uncorrelated, but coupled by reddening law: σ_v = ±0.15 mag | 7        |
| [Z] Metallicity ................. | M31 metallicity gradient test gives σ_vz = ±0.08 mag | 4        |
| [H] Reduction package ........... | M101 calibration gives +0.14 mag of Δμ_0/μ[Fe/H] = -0.25 mag dex⁻¹ | 7        |
| [J] Random errors ............... | (M1) + (M2) + (M3) combined in quadrature | 10       |
| [K] Systematic errors ........... | [SC] + [Z] + [H] combined in quadrature | 10       |

**Note:** There are 32 Cepheids in the LMC with published V/I photometry (Madore & Freedman 1991). The measured dispersions in the period-luminosity relations at V and I are ±0.27 and ±0.18 mag, respectively.
Fig. 6.—Relative geometry (left) and the corresponding velocity vectors (right) for the disposition and flow of Fornax and the Local Group with respect to the Virgo Cluster. The circles plotted at the positions of the Virgo and Fornax Clusters have the same angular size as the circles enclosing M100 and NGC 1365 in the two panels of Fig. 1.

toward Virgo of 200 km s\(^{-1}\) is obtained by minimizing the velocity residuals for the galaxies with Cepheid-based distances. This value is in good agreement with that estimated by Han & Mould (1990). We adopt 200 ± 100 km s\(^{-1}\), which results in a projected Virgocentric flow correction for Fornax of \(- 45 \pm 23\) km s\(^{-1}\).  

7. \(H_0\) AT FORNAX AND ITS UNCERTAINITIES  
Correcting to the barycenter of the Local Group (\(-90\) km s\(^{-1}\) and for the \(-45\) km s\(^{-1}\) component of the Virgocentric flow derived above, we calculate that the cosmological expansion velocity of Fornax is 1306 km s\(^{-1}\). Using our Cepheid distance of 18.6 Mpc for Fornax gives \(H_0 = 70 \pm 18\) (random) \(\pm 7\) (systematic) km s\(^{-1}\) Mpc\(^{-1}\). The first uncertainty includes random errors in the distance derived from the PL fit to the Cepheid data (see Table 1), as well as random velocity errors in the adopted Virgocentric flow, combined with the distance uncertainties to Virgo propagated through the flow model. The second uncertainty quantifies the currently identifiable systematic errors associated with the adopted mean velocity of Fornax, and the adopted zero point of the PL relation (combining in quadrature the LMC distance error, a measure of the metallicity uncertainty, and a conservative estimate of the stellar photometry errors). Finally, we note that according to the Han-Mould model (Han & Mould 1990), the so-called “Local Anomaly,” gives the Local Group an extra velocity component of approximately 73 km s\(^{-1}\) toward Fornax. If we were to add that correction to our local estimate, the Hubble constant would increase to \(H_0 = 74\) km s\(^{-1}\) Mpc\(^{-1}\).

Given the highly clumped nature of the local universe and the existence of large-scale streaming velocities, there is still a lingering uncertainty about the total peculiar motion of the Fornax Cluster with respect to the cosmic microwave background rest frame. Observations of flows and the determination of the absolute motion of the Milky Way with respect to the background radiation suggest that line-of-sight velocities \(\sim 300\) km s\(^{-1}\) are not uncommon (e.g., Coles & Lucchin 1995, p. 399 and references therein). The uncertainty in absolute motion of Fornax with respect to the Local Group then becomes the largest outstanding uncertainty at this point in our error analysis: a 300 km s\(^{-1}\) flow velocity for Fornax would result in a systematic error in the Hubble constant of \(\sim 20\%\). We can revisit this issue, however, following an analysis of more distant galaxies made later in this section.

8. THE NEARBY FLOW FIELD  
We now step back somewhat and investigate the Hubble flow between us and Fornax, derived from galaxies and groups of galaxies inside 20 Mpc, each having Cepheid-based distances and expansion velocities individually corrected for a Virgocentric flow model (see Kraan-Korteweg 1986, for example). These data are presented in Figure 7. At 3 Mpc the M81–NGC 2403 Group (for which both galaxies of this pair have Cepheid distance determinations) gives \(H_0 = 75\) km s\(^{-1}\) Mpc\(^{-1}\) after averaging their two veloc-
ties. Working further out to M101, the NGC 1023 Group, and the Leo Group, the calculated values of $H_0$ range from 62 to 99 km s$^{-1}$ Mpc$^{-1}$. An average of these independent determinations, including Virgo and Fornax, gives $H_0 = 73 \pm 16$ (random) km s$^{-1}$ Mpc$^{-1}$, where flow uncertainties are added to the random error estimate for later intercomparisons of Hubble constants derived from independent methods and volumes of space. This determination, as before, uses a Virgocentric flow model with a $1/R_{Virgo}$ infall-velocity falloff, scaled to a Local Group infall velocity of 200 km s$^{-1}$.

The foregoing determination of $H_0$ is again predicated on the assumption that the infall flow-corrected velocities of both Fornax and Virgo are not further perturbed by other mass concentrations or large-scale flows, and that the 25,000 Mpc$^3$ volume of space delineated by them is at rest with respect to the distant galaxy frame. To avoid these local uncertainties we now step out from Fornax to the distant flow field. There we explore three applications: (1) Using the Tully-Fisher relation calibrated by published Cepheid distances locally, and now including NGC 1365 and about two dozen additional galaxies in the Fornax cluster; (2) using the distance to Fornax to tie into averages over previously published differential moduli for independently selected distant-field clusters; (3) recalibrating the Type Ia supernova luminosities at maximum light and applying that calibration to events as distant as 30,000 km s$^{-1}$.

Ultimately the calibrators in application (1) are tied into the distant flow field at 10,000 km s$^{-1}$ defined by the the Tully-Fisher sample of galaxies in clusters (Aaronson et al. 1980; Han 1992).

9. BEYOND FORNAX: THE TULLY-FISHER RELATION

Quite independent of its association with the Fornax Cluster as a whole, NGC 1365 provides an important calibration point for the Tully-Fisher relation which links the (distance-independent) peak rotation rate of a galaxy to its intrinsic luminosity. In Figure 8 (left) we show NGC 1365 (in addition to NGC 925) (Silbermann et al. 1996), NGC 4536 (Saha et al. 1996), and NGC 4639 (Sandage et al. 1996) added to the ensemble of calibrators having published Cepheid distances from ground-based data (Freedman 1990), and $I$-band magnitudes and line widths, measured at 20% of the peak height (Pierce 1994; Pierce & Tully 1992, and references therein). As mentioned earlier, NGC 1365 provides the brightest data point in the relation; additional galaxies recently having Cepheid distances measured include NGC 3621 (Rawson 1997), NGC 3351 (Graham et al. 1997), and NGC 2090 (Phelps et al. 1998) and will be included once $I$-band magnitudes become available.

Although we have only the Fornax Cluster for comparison at the present time, it is interesting to note that there is no obvious discrepancy in the Tully-Fisher relation between galaxies in the (low-density) field and galaxies in this (high-density) cluster environment. The NGC 1365 data point is consistent with the data for other Cepheid calibrators. Adding in all of the other Fornax galaxies for which there are published $I$-band magnitudes and inclination-corrected $H_1$ line widths provides us with another comparison of field and cluster spirals. In Figure 8 (right) we see that the 21 Fornax galaxies (shifted by the true modulus of NGC 1365) agree extremely well with the nine
brightest Cepheid-based calibrators. The slope of the relation is virtually unchanged by this augmentation. The scatter in the individually Cepheid-calibrated data (left) is ±0.35 mag. This increases to ±0.48 mag if the entire Fornax Cluster sample is included (right). In following applications we adopt $M_I = -8.80[\log (\Delta V) - 2.445] + 20.47$ as the best-fitting least-squares solution (derived from equal weighting of all galaxies and minimizing magnitude residuals) for the calibrating galaxies.

Han (1992) has presented $I$-band photometry and neutral-hydrogen line widths for the determination of Tully-Fisher distances to individual galaxies in 16 clusters out to redshifts exceeding 10,000 km s$^{-1}$. We have rederived distances and uncertainties to each of these clusters using the above-calibrated expression for the Tully-Fisher relation. The results are contained in Figure 9. A linear fit to the data in Figure 9 gives a Hubble constant of $H_0 = 76$ km s$^{-1}$ Mpc$^{-1}$ with a total observed scatter giving a formal (random) uncertainty on the mean of only ±2 km s$^{-1}$ Mpc$^{-1}$, increasing to ±3 (Table 2) when flow uncertainties are added in quadrature. It is significant that neither Fornax nor Virgo deviate to any significant degree from an inward extrapolation of this far-field solution. At face value, these results provide evidence for both of these clusters having only small motions with respect to their local Hubble flow. This value compares favorably with other recent calibrations of the Tully-Fisher relation by Giovanelli et al. (1997), who obtain $H_0 = 69 \pm 5$ km s$^{-1}$ Mpc$^{-1}$ ($1 \sigma$), and then by Tully (1998), who finds $H_0 = 82 \pm 16$ km s$^{-1}$ Mpc$^{-1}$ (95% confidence).

10. OTHER RELATIVE DISTANCE DETERMINATIONS

In addition to the relative distances using the Tully-Fisher relation discussed above, a set of relative distance moduli based on a number of independent secondary distance indicators, including brightest cluster galaxies, Tully-Fisher, and supernovae, is also available (Jerjen & Tammann 1993). We adopt, without modification, their differential distance scale and tie into the Cepheid distance to the Fornax Cluster, which was part of their cluster sample. The results are shown in Figure 10, which extends the velocity-distance relation out to more than 160 Mpc. No error bars are given in the published compilation. For a discussion of uncertainties in this sample, see Huchra (1995). This sample yields a value of $H_0 = 72 \pm 3$ (random) km s$^{-1}$ Mpc$^{-1}$ (random), with a systematic error of 9% being associated with the distance (but not the velocity) of the Fornax Cluster.

11. BEYOND FORNAX: TYPE Ia SUPERNOVAE

The Fornax Cluster elliptical galaxies NGC 1316 and NGC 1380 are host to the well-observed Type Ia supernovae 1980N and 1992A, respectively. Although the distances to these galaxies are not measured directly, the new Cepheid distance to NGC 1365 and associated estimate of the distance to the Fornax Cluster allow two additional very high quality objects to be added to the calibration of Type Ia supernovae, allowing for the uncertainty in their distances. A preliminary discussion of these objects was given by Freedman, Madore, & Kennicutt (1997). A more extensive discussion of the Type Ia supernova distance scale (including a reanalysis of all of the Cepheid data for Type Ia supernova-host galaxies) will be presented in Gibson et al. (1999).

The galaxies hosting Type Ia supernovae for which Cepheid distances have been measured to date include IC 4182 (1937C), NGC 5253 (1895B, 1972E), NGC 4536 (19811B), NGC 4496 (1960F), NGC 4639 (1990N) (see Sandage et al. 1996), and NGC 4414 (1974G) (Turner et al. 1998). NGC 3627 was host to 1989B, a galaxy which in the Leo Triplet is assumed to be at the same distance as the Leo I Group (given by Cepheids) by Sandage et al. The quality of the supernova observations for this sample is quite mixed; with the exception of 1972E, the (mainly photographic) photometry for the earlier, historical supernovae is of significantly lower quality than the more recent supernovae.

We have undertaken a preliminary recalibration of the Type Ia supernovae, including SN 1980N and SN 1992A in the analysis. We assume for this purpose that the Cepheid distance to NGC 1365 is representative of the Fornax Cluster and give these two supernovae only half-weight
For 1980N, we adopt peak supernova magnitudes of et al. (1991) and Phillips (1998, private communication). For SN 1980N and SN 1992A were obtained from Hamuy et al. (1995, 1996). This procedure differs markedly from that of Sandage et al. (1996), but is consistent with that of Hamuy et al. (1996). Along with SN 1895B, SN 1937C, SN 1960F, SN 1973R, but photographic observations only are available for this object. The Cepheid-calibrated supernova sample including the best-observed supernovae SN 1972E, SN 1981B, SN 1990N, and SN 1992G, are among the fastest decliners in the Cepheid-calibrating sample. A decline-rate absolute-magnitude relation for the Cepheid calibrators has been presented by Freedman (1997). It is consistent with that observed for distant supernovae (e.g., Hamuy et al. 1995, 1996).

The Cepheid-calibrated supernova sample including the best-observed supernovae SN 1972E, SN 1981B, SN 1990N, giving half-weight to SN 1989B, SN 1980N, and SN 1992A and applied to the distant Type Ia supernovae of Hamuy (1995), gives $H_0 = 67 \text{ km s}^{-1} \text{ Mpc}^{-1}$. We have also experimented with various weighting schemes (e.g., including the photographic data; while allowing for its larger uncertainty; excluding the Fornax data completely; or analyzing the $B$ and $V$ data alone). Resulting values of $H_0$ lie in the range of 63–67 km s$^{-1}$ Mpc$^{-1}$. Half of the difference between the results of Sandage et al. (1996) (giving $H_0 = 57$...
km s$^{-1}$ Mpc$^{-1}$) is the lower weight placed in the current analysis on the (poorer quality) photographic data. The remaining difference is due to our adoption of the Hamuy et al. (1995) decline-rate correction, and the inclusion of the Fornax supernovae. Sandage et al. adopt no decline-rate correction. The relation that we have adopted is consistent with that of Phillips (1993), Hamuy et al. (1996), and Riess et al. (1996). Finally, we note that eliminating the new Fornax calibrators from the analysis changes the Hubble constant by $-1$ to 3 km s$^{-1}$ Mpc$^{-1}$, for a variety of calibrating samples and weighting schemes.

12. COMPARING AND COMBINING THE RESULTS

The results of the previous five sections are presented in Table 3. What is the summary conclusion? In the first instance we can simply state that, based on a number of different methods calibrated here, $H_0$ falls within the range of $67 \pm 6$ (random) and $76 \pm 3$ km s$^{-1}$ Mpc$^{-1}$, with no obvious dependence on the indicative volume of space being probed. Hence, a variety of independent distance determination methods are yielding agreement at the 10% level. With the exception of the common Cepheid PL relation zero point, these various determinations are largely independent; thus, their differences are indicative of the true systematic errors affecting each of the methods and their individual underlying assumptions. No single determination stands out either as markedly anomalous or as undeniably superior.

How then do we combine these individual results in a summary number with its own uncertainty? We have undertaken two types of approach: a Frequentist approach and a Bayesian one. In the end they only differ in their resulting confidence intervals. We begin by first considering the random errors.

In our application of the Frequentist approach (e.g., Wall 1997 and references therein) we simply represent each determination as a probability distribution having its mean at $H(i)$, a dispersion of $\sigma(i)_{\text{random}}$ and unit integral (i.e., equal total weight in the sum). These are shown as the connected dotted lines in Figure 11 (left). The solid enveloping line is the resulting sum of the five probability density distributions. The composite probability distribution is somewhat non-Gaussian, but it is still centrally peaked, with both the mode and the median coinciding at 72–73 km s$^{-1}$ Mpc$^{-1}$. An estimate of the traditional ($\pm 1\sigma$) errors can be easily obtained from this distribution by identifying where the cumulative probabilities hit 0.16 and 0.84, respectively. This procedure gives the cited error on the mean for five estimates of $\pm 5$ km s$^{-1}$ Mpc$^{-1}$. [At the suggestion of the referee, this exercise was repeated using identical errors of 10% on each of the five estimates. The result is $71 \pm 4$ km s$^{-1}$ Mpc$^{-1}$.] The Bayesian estimate is equally straightforward (see Press 1997). Again, taking the individual Hubble constant estimates to be represented by Gaussians, we combine them by multiplication, assuming a minimum-bias (flat) prior (Sivia 1996). We note that the Bayesian approach assumes statistical independence: strictly speaking, the considered samples are not completely independent given that they explicitly share a common (Cepheid) zero point, and the Jerjen & Tamman hybrid sample overlaps in part with the pure Tully-Fisher application. Nevertheless, we have done the exercise of computing the posterior probability distribution with these caveats clearly stated. Because of the strong overlap in the various estimates the combined solution is both very strongly peaked and symmetric, giving a value of $H_0 = 74 \pm 3$ (random) km s$^{-1}$ Mpc$^{-1}$ as depicted in Figure 11 (right). The $1\sigma$ error on the mean was again determined from the 0.16 and 0.84 cumulative probability distribution points. As already anticipated above, the results of the two

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### Table 3

| Method               | Hubble Constant (km s$^{-1}$ Mpc$^{-1}$) | Random | Systematic |
|---------------------|----------------------------------------|--------|------------|
| Fornax Cluster      | 70                                     | $\pm 18$ | $\pm 7$   |
| Local flow          | 73                                     | $\pm 16$ | $\pm 7$   |
| Tully-Fisher        | 76                                     | $\pm 3$  | $\pm 8$   |
| Hybrid methods      | 72                                     | $\pm 3$  | $\pm 6$   |
| Type Ia SNe         | 67                                     | $\pm 6$  | $\pm 7$   |
| Modal average       | 72                                     | $\pm 5$  | $\pm 7$   |
| Major systematics   | $\pm 2\%$ [FLOWS]                     | $\pm 5\%$ [LMC] | $\pm 4\%$ [Fe/H] |

Note: (1) The measured scatter of the $N = 5$ tabulated values of the Hubble constant about the derived mean of 72 km s$^{-1}$ Mpc$^{-1}$ is $\pm 3.4$ km s$^{-1}$ Mpc$^{-1}$; the formal error on the unweighted mean (due to random errors between methods) is then $3.4(N - 1)^{-1/2} = 1.7$ km s$^{-1}$ Mpc$^{-1}$. (2) The concordance between the local and far-field values of the the Hubble constant argue that there is no large flow of the local supercluster with respect to the 20,000 km s$^{-1}$ volume probed by the SNe. At face value the differences in Hubble constants admit a local flow of $\sim 100$ km s$^{-1}$. If so, the (averaged) systematic error due to large-scale flow perturbations is less than 2%–3%, leaving the LMC distance as the leading source of systematic error on $H_0$, it being at the $\pm 5\%$ level. (3) The systematic uncertainty due to metallicity is derived from the work on M31 (Freedman & Madore 1990), which reports a marginal detection of a dependence of Cepheid luminosities on metallicity, at the level of $\sim 0.2$ mag dex$^{-1}$. A similar test using Cepheids at various radii in M101 has been undertaken by the Key Project Team (Kennicutt et al. 1998) and gives a slightly larger dependence of $\sim 0.25$ mag dex$^{-1}$. In any case, this uncertainty is expected to mainly introduce scatter rather than any large residual systematic effect given that the averaged metallicity of the galaxies so far observed by the HST Key Project is closely approximated by that of the LMC metallicity.
analyses are indistinguishable except for the high confidence attributed to the number by the Bayesian analysis. These results are summarized graphically in Figure 11 and numerically in Table 3.

Systematic errors must be dealt with separately and independently from the random error discussion. While some of the identified systematic errors affect all of the above determinations equally and in the same sense (the LMC distance modulus for instance), others are more "randomly" distributed among the methods and their contributing galaxies. For instance, the (as yet unknown) effects of flows are estimated and scaled for each of the methods here, but they may be large for the Fornax Cluster but smaller and perhaps have a different sign for the ensemble of Type Ia supernovae. It seems prudent therefore to simply average the systematic errors while listing the main components individually. The main systematic error on the finally adopted value of the Hubble constant is threefold:

1. Large-scale flow fields.—These contribute large fractional uncertainties to the nearest estimates of $H_0$, but they progressively drop to only a few percent at large distances and/or for samples averaged over many directions (see Table 2 and further discussion below).

2. The zero point of the adopted PL relation.—In our case, this is tied directly to the adopted true distance modulus of the LMC. A variety of independent estimates are reviewed by Westerlund (1996) and more recently by Walker (1999); they each conclude that the uncertainty is at the 5% level in distance. Westerlund prefers a true distance modulus of $18.45 \pm 0.10$ mag; Walker adopts $18.55 \pm 0.10$ mag. We have consistently used $18.50 \pm 0.10$ mag throughout this series of papers. Therefore, if the distance to the LMC systematically changes downward/upward (by 10% or 0.2 mag, say) from that adopted here, then our entire distance scale shifts by the same amount, and the derived value of $H_0$ would increase/decrease by the same (10%) factor.

3. The metallicity dependence of the Cepheid PL relation.—This is a complex and much debated topic, and the interested reader is referred to Sasselov et al. (1997), Kochanek (1997), Kennicutt et al. (1998), and earlier reviews by Freedman & Madore (1990) for an introduction to the literature. The metallicity of the galaxies for which Cepheid searches have been undertaken span a range in [O/H] abundance of almost an order of magnitude, with a median value of $-0.3$ dex. The calibrating sample of Cepheids in the LMC have a very similar abundance of $[O/H] = -0.4$ dex. These results suggest that even if in individual cases the metallicity effect amounted to 10%–20%, the overall effect on the calibration of secondary distance indicators will be less than 5%. Recently concluded observations with NICMOS on HST should further help to constrain the magnitude of this effect.

The importance of bulk flow motions in the determination of $H_0$ varies significantly depending on how far a
particular distance indicator can be extended (see Table 2). For local distance indicators the uncertainties due to unknown bulk motions are the largest contributing source of systematic error to the determination of $H_0$, amounting to an uncertainty of 20%–25% in the local value of the Hubble constant. The Tully-Fisher relation extends to a velocity distance of about 10,000 km s$^{-1}$, although most of the observed clusters are not this remote. At 6000 km s$^{-1}$, peculiar motions of $\sim 300$ km s$^{-1}$ would individually contribute 5% perturbations; however, with many clusters distributed over the sky, peculiar motions of this magnitude will give only a few percent uncertainty. For Type la supernovae which extend to beyond 30,000 km s$^{-1}$, the problem is even less severe. These estimates are consistent with recent studies by Shi & Turner (1998) and Zehavi et al. (1998), which place limits on the variation of $H_0$ with distance, based on theoretical and empirical considerations, respectively.

13. COSMOLOGICAL IMPLICATIONS

A value of the Hubble constant, in combination with an independent estimate of the average density of the universe, can be used to estimate a dynamical age for the universe (e.g., see Fig. 12). For a value of $H_0 = 72 \pm 5$ (random) km s$^{-1}$ Mpc$^{-1}$, the age ranges from a high of $\sim 12$ Gyr for a low-density ($\Omega = 0.15$) universe, to a young age of $\sim 9$ Gyr for a critical-density ($\Omega = 1.0$) universe. These ages change to 15 and 7.5 Gyr, respectively, allowing for an error of $\pm 10$ km s$^{-1}$ Mpc$^{-1}$.

The ages of Galactic globular clusters have until recently tended to fall in the range of $14 \pm 2$ Gyr (Chaboyer et al. 1996); however, the subdwarf parallaxes obtained by the Hipparcos satellite (e.g., Reid 1997) may reduce these ages considerably. For $\tau = 14$ Gyr and $\Omega = 1.0$, $H_0$ would have to be $\sim 45$ km s$^{-1}$ Mpc$^{-1}$; interpreted within the context of the standard Einstein–de Sitter model, our value of $H_0 = 72$ km s$^{-1}$ Mpc$^{-1}$, is incompatible with a high-density ($\Omega = 1.0$) model universe without a cosmological constant (at the 2 $\sigma$ level defined by the identified systematic errors). If, however, $\tau = 11$ Gyr, then the globular cluster and the expansion ages would be consistent to within their mutually quoted uncertainties.

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Fig. 12.—Lines of fixed time representing the theoretical ages of the oldest globular cluster stars are shown for 12, 14, and 16 Gyr, plotted as a function of the expansion rate $H_0$ and density parameter $\Omega_0$, for an Einstein–de Sitter universe with the cosmological constant $\Lambda = 0$. The thick dashed horizontal line at $H = 72 \pm 5$ (random) km s$^{-1}$ Mpc$^{-1}$ is the average value of the Hubble constant given in Table 3. The parallel (solid) lines on either side of that solution represent the 1 $\sigma$ random errors on that solution. Systematic errors on the solution for $H_0$ are represented by thin dashed lines at 65 and 79 km s$^{-1}$ Mpc$^{-1}$. The only region of (marginal) overlap between these two constraints is in the low density ($\Omega < 0.2$) regime, unless $\Lambda \neq 0$. If the globular cluster ages are assumed to place a lower bound on the age of the universe, the region of plausible overlap between the two solutions is more severely restricted to even lower density models.

$H_0 = 72 \pm (5)_{\tau} \pm (7)_{\Lambda}$ km/sec/Mpc

$\Lambda = 0$

$\Omega_0$

\begin{align*}
\tau = 12 & \\
\tau = 14 & \\
\tau = 16 & \\
\Lambda = 0 & \\
\Omega_0 & 
\end{align*}
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