THE IMPACT OF GEOPOLITICAL CONFLICTS ON TRADE, GROWTH, AND INNOVATION

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The Impact of Geopolitical Conflicts on Trade, Growth, and Innovation

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Abstract

Geopolitical conflicts have increasingly been a driver of trade policy. We study the potential effects of global and persistent geopolitical conflicts on trade, technological innovation, and economic growth. In conventional trade models the welfare costs of such conflicts are modest. We build a multi-sector multi-region general equilibrium model with dynamic sector-specific knowledge diffusion, which magnifies welfare losses of trade conflicts. Idea diffusion is mediated by the input-output structure of production, such that both sector cost shares and import trade shares characterize the source distribution of ideas. Using this framework, we explore the potential impact of a “decoupling of the global economy,” a hypothetical scenario under which technology systems would diverge in the global economy. We divide the global economy into two geopolitical blocs—East and West—based on foreign policy similarity and model decoupling through an increase in iceberg trade costs (full decoupling) or tariffs (tariff decoupling). Results yield three main insights. First, the projected welfare losses for the global economy of a decoupling scenario can be drastic, as large as 12% in some regions and are largest in the lower income regions as they would benefit less from technology spillovers from richer areas. Second, the described size and pattern of welfare effects are specific to the model with diffusion of ideas. Without diffusion of ideas the size and variation across regions of the welfare losses would be substantially smaller. Third, a multi-sector framework exacerbates diffusion inefficiencies induced by trade costs relative to a single-sector one.

Keywords: Innovation, International trade, international relations.

JEL codes: F12, F13, O33

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1 Introduction

Open markets and free trade have been a basic tenet of the international order emerging out of World War II. Over that period, a large consensus regarding the need to reduce trade costs and prioritize gains from trade led to a continuous deepening of the international trade regime.

The European Union enlarged eastwards and many countries joined the World Trade Organization (WTO), including Russia, the center-of-gravity of the former Soviet bloc. At the same time China, displayed spectacular growth rates, integrated to world markets, and was recognized as a market economy by also joining the international trade system.

However, the last decade has witnessed the beginning of a backlash against global trade integration. Political scientists conjecture that the emergence of China as a new superpower against an incumbent U.S. might lead to strategic competition between these countries —one in which geopolitical forces and the desire to limit interdependence take primacy over win-win international cooperation\(^1\). Rising support for populist and isolationist parties in many Western countries points towards the same direction\(^2\). Additionally, the 2022 War in Ukraine and the subsequent strong retaliation of the European Union, the United States, and their allies against Russia suggest that the international economic order based on open markets and expanded globalization could be replaced by a more fragmented international economic system.

Using these facts as motivation, this paper aims to determine the potential effects of increased and persistent large-scale geopolitical conflicts on trade, economic growth, and technological innovation. Some of the adverse effects are well-known. Increased trade barriers decrease domestic welfare and gains from trade by shifting production away from the most cost-efficient producers and leaving households with a lower level of total consumption. Canonical trade models capture this static result through the fact that welfare is proportional to the degree of trade openness (Arkolakis, Costinot, and Rodríguez-Clare 2012)\(^3\).

However, some of the main concerns of policymakers and practitioners regarding potentially detrimental effects of limiting trade are abstracted away in standard models. For instance, these models typically assume a fixed technology distribution for domestic firms, thereby limiting gains from trade to static gains. This assumption renders it impossible to address one of the most important long-term consequences of continued geopolitical conflicts or receding globalization —namely, reduced technology and know-how spillovers that happen through trade.

In order to realistically assess the impact of trade conflicts on global innovation, we build a multi-sector multi-region general equilibrium model with dynamic sector-specific knowledge diffusion. As in Buera and Oberfield (2020), we model the arrival of new ideas as a learning process of producers in a certain country-sector. By engaging in international trade, i.e. importing intermediate inputs, domestic innovators have access to new sources of ideas, whose quality depends on the productivity of the countries and sectors from which they source intermediates .

\(^1\) See Wei (2019) and Wyne (2020) for a review of the debate among respectively Chinese and American scholars about the shift in foreign policies towards each other.

\(^2\) For evidence of the impact of trade shocks on the rise of populist parties to power, see Colantone and Stanig (2018).

\(^3\) This is true of all canonical trade models. As Arkolakis, Costinot, and Rodríguez-Clare (2012) show, Armington (1969), Krugman (1980), Eaton and S. Kortum (2002), and Melitz (2003), albeit different in motivation, are isomorphic and summarize gains from trade by some variation of the expression \(G \propto (\pi_{it})^{\frac{1}{\varepsilon}}\), where \(\pi_{it}\) is domestic trade share and \(\varepsilon\) is the elasticity of trade flows with respect to trade costs.
In our model, idea diffusion is mediated by the input-output structure of production, such that both sectoral intermediate input cost shares and import trade shares characterize the source distribution of ideas. Innovation is summarized by describing productivity in different sectors as evolving according to a trade-share weighted-average of trade-partners sectoral productivities. This process is controlled by a parameter which determines the speed of diffusion of ideas, which we calibrate using historical data.

Productivity thus evolves endogenously as a by-product of micro-founded market decisions —i.e., an externality that market agents affect with their behavior but do not take into account when making decisions. In this framework, the outbreak of large-scale trade conflicts will have spillover effects on the future path of sectoral productivities of all countries. Changes in trade costs induce trade diversion and creation, which, in turn, impact productivity dynamics in a way that is not internalized by agents.

After characterizing the model, we use it to perform policy experiments in the context of heightened geopolitical conflicts. We explore the potential impact of a “decoupling of the global economy,” a hypothetical scenario under which technology systems would diverge in the global economy. We divide the global economy in a Western bloc and an Eastern bloc based on differential scores foreign policy similarity.

First, we simulate increased trade costs arising from geopolitical circumstances, which increase frictions prohibitively if one country wants to trade with another one outside its bloc. Alternatively, we simulate a scenario of a global increase in tariffs, in which all countries move from cooperative tariff setting in the context of the WTO to non-cooperative tariff setting. Nicita, Olarreaga, and Silva (2018) estimate to increase global tariffs, on average, by 32 percentage points. For simplicity, we use this average number as a reference and we assume that countries in different blocs raise tariffs against countries in the other bloc by such average amount.

Our analysis leads to five main findings. First, a multi-sector framework exacerbates diffusion inefficiencies induced by trade costs relative to a single-sector one; we explore this issue both through theory and simulations. Second, we show that the projected welfare losses for the global economy of a decoupling scenario can be drastic, as large as 12% in some regions; and are largest in the lower income regions as they would suffer most from reduced technology spillovers from richer areas. Third, the described size and pattern of welfare effects are specific to the model with diffusion of ideas. In a dynamic setting with diffusion of ideas welfare losses are larger and display more variation. Fourth, if one of the middle income regions, Latin America and Caribbean (LAC), would switch from the higher-income Western bloc to the lower income Eastern bloc, its welfare costs of decoupling would be significantly higher. This experiment illustrates that policymakers in low- and middle-income countries would face difficult decisions if decoupling would aggravate. Fifth and finally, the welfare costs of decoupling only in electronic equipment, the sector where decoupling is already taking place, would be much smaller than under full decoupling, albeit sizeable, ranging from 0.4 – 1.9%.

We make four main contributions to the literature. First, we build a multi-sector model of the global economy with Bertrand competition, profits, and technology spillovers which can be solved recursively that permits assessing realistic trade policy experiments. Second, we calibrate the strength of the diffusion of technologies through trade with a tight fit between simulated and historical GDP growth rates. Third, we examine the long-run effects of real-

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4Before conducting simulations with the multi-sector, multi-region model calibrated to real-world data, we explore the discrepancy between actual and optimal levels of idea diffusion This comparison shows that to maximize the total diffusion of ideas, trade shares must be at their optimal point in every sector.
world policy experiments related to the decoupling of the global economy. Last, we draw insights from the Political Science literature in order to incorporate geopolitical conflicts into a workhorse trade model.

Our model builds on the work that evaluates the impact of trade on innovation and shows that trade openness can increase the level of domestic innovation, particularly on the single-sector model of Buera and Oberfield (2020). Compared to previous work, we present a recursive model with intermediate linkages; calibrate the strength of the diffusion of ideas to target historical GDP growth rates across all regions; and explore diffusion inefficiencies in a multi-sector setting.

Related Literature. Our paper is closely related to the literature that adds dynamics to trade models by incorporating knowledge diffusion channels. The earliest explorations of this topic go back to Eaton and S. Kortum (1999), who developed a multi-country dynamic model in which firms innovate by investing in research and development (R&D) and knowledge diffuses, after some lag, to other markets. In this model, diffusion happened somewhat mechanically, was unrelated to trade and eventually reached all countries.

More recently, Alvarez et al. (2013) combined the Eaton and S. Kortum (2002) Ricardian model of trade with an idea diffusion process first presented by S. S. Kortum (1997). Importantly, the authors conjectured that the diffusion process is proportional to the quality of managers of firms whose product reach a given destination market. Ideas flow from one market to another in proportion to the trade linkages between them. Therefore, impediments to trade have not only static, but also dynamic costs—as they decrease knowledge diffusion.

Our model is also closely related to the one described by Santacreu, Li, and Cai (2017), who extended the original model by Eaton and S. Kortum (1999) incorporating lag-diffusion dynamics into a multi-sector framework. Santacreu, Li, and Cai (2017) build a multi-sector model of trade, innovation, and knowledge diffusion, exploring how the welfare gains from trade are affected by knowledge diffusion through their impact on changes in comparative advantage. They show that the welfare gains are larger with endogenous knowledge diffusion because existing specialization patterns tend to get reinforced by knowledge diffusion.

There are three main differences between their work and ours. First, our model emphasizes the nexus between trade and idea diffusion, whereas Santacreu, Li, and Cai (2017) model technology spillovers as being independent of the amount of trade (or other endogenous variables like FDI or migration). Additionally, while they calibrate knowledge spillovers with data on patent citation, we calibrate the strength of the diffusion of ideas based on the fit between actual and simulated historical GDP growth rates. Third, the papers have a different focus: we focus on concrete policy questions and explore how the effect of potential trade policy changes are affected by the inclusion of ideas diffusion in the model; while they highlight how patterns of comparative advantage change with technology spillovers.

Organization. This paper is organized as follows. In Section 2 we present the model, detailing production, demand, and consumption of the global economy. We also describe the dynamic evolution of productivities in different regions and sectors. In Section 3 we describe the discrepancy between the actual and optimal diffusion of ideas in a multi-sector

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5In differentiated varieties trade models, knowledge diffusion shows up in papers like Romer (1990), Rivera-Batiz and Romer (1991), Rivera-Batiz and Romer (1991) and Grossman and Helpman (1989). In the text, we focus on papers that incorporate knowledge diffusion to Ricardian models, which is the class of models that this paper falls in.
framework. In Section 4 we discuss the calibration of the model and underpin the examined policy experiments. In Section 5 we present the results of our main policy experiments and some alternative simulations. Finally, we conclude in Section 6 summarizing the key takeaways.

2 Environment

Time is discrete and indexed by $t \in T$. There are $d \in D$ regions in the global economy, which cover every part of the world economy, either as a stand-alone country, or a regional aggregate of countries. In each region, there are multiple industries $i \in I$.

2.1 Demand

In each region $d$ and each period $t$ a representative agent maximizes Cobb-Douglas preferences over consumption of goods in different sectors $i \in I$, $q_{d,t}^{c,i}$:

$$\max_{(q_{d,t}^{c,i})} \sum_{i \in I} (q_{d,t}^{c,i})^{\kappa_d} \quad s.t. \quad \sum_{i \in I} \kappa_d^i = 1$$

$$\sum_{i \in I} p_{d,t}^{c,i} q_{d,t}^{c,i} \leq (1 - s_{d,t}) Y_{d,t}$$

$$Y_{d,t} = w_{d,t} \ell_{d,t} + r_{d,t} k_{d,t} + T_{d,t} + \sum_{i \in I} \Pi_{d,t}^{i}$$  (1)

$Y_{d,t}$ is gross income determined by $w_{d,t}, \ell_{d,t}$, the wage and measure of workers; $r_{d,t} k_{d,t}$, capital income; $T_{d,t}$, transfers; and $\Pi_{d,t}^{i}$, profits. We set $s_{d,t}$ to be the exogenous savings rate. Hence, we abstract from intertemporal optimization as discussed further below in the subsection on market clearing. Savings are used to finance investment as discussed below. This preference structure implies the following demand function for goods in sector $i \in I$:

$$q_{d,t}^{c,i} = \frac{\kappa_d^i Y_{d,t}}{p_{d,t}^{c,i}}$$  (2)

The aggregate price index satisfies:

$$P_{d,t}^c = K \cdot \Pi_{i \in I} (p_{d,t}^{i})^{\kappa_d^i}$$  (3)

With $K = \Pi_{i \in I} (\kappa_d^i)^{-\kappa_d^i}$, a constant.

2.2 Production

There are many producers of different varieties $\omega$ of each commodity $i$. Firms are endowed with an identical technology and combine factors of production $f_{d,t}^i$ and intermediate inputs $m_{d,t}^i$ to produce variety $q_{d,t}^i(\omega)$:

$$q_{d,t}^i(\omega) = z_{d,t}^i(\omega) \left[ (\Psi_{d,t}^{f_i})^{1 \over \rho_i} (f_{d,t}^i)^{\nu_i - 1 \over \rho_i} + (\Psi_{d,t}^{m_i})^{1 \over \rho_i} (m_{d,t}^i)^{\nu_i - 1 \over \rho_i} \right]^{\rho_i \over \nu_i - 1 \rho_i}$$  (4)
The cost of the unit input bundle, \( c_{i,d,t} \), is a function of the prices of factors of production, \( p_{f_{i,d,t}} \), and prices of commodities used as intermediates, \( p_{i,d,t} \):

\[
c_{i,d,t} = \left[ \Psi_{i,f_{d,t}}(p_{f_{d,t}})^{1-\rho_i} + \Psi_{i,m_{d,t}}(p_{i,d,t})^{1-\rho_i} \right]^{\frac{1}{1-\rho_i}} \tag{5}
\]

Firms combine factors of production (\( f_{i,d,t} \)) and intermediate commodities (\( m_{i,d,t} \)) according to the following sub-production functions:

\[
f_{i,d,t} = \left[ \left( \Psi_{i,k_{d,t}} \right)^{\frac{\nu_i}{\nu_i-1}} k_{d,t} \frac{\nu_i-1}{\nu_i} + \left( \Psi_{i,\ell_{d,t}} \right)^{\frac{\nu_i}{\nu_i-1}} \ell_{d,t} \frac{\nu_i-1}{\nu_i} \right]^{\frac{1}{\nu_i-1}} \tag{6}
\]

\[
m_{i,d,t} = \sum_{j \in I} \left( \Psi_{i,j_{d,t}} \right)^{\frac{\mu_i}{\mu_i-1}} \left( q_{m,j_{d,t}} \right)^{\frac{\mu_i-1}{\mu_i}} \tag{7}
\]

The first aggregator combines capital, \( k_{d,t} \), and labor, \( \ell_{d,t} \), as factors of production, while the second one uses sectoral commodities \( q_{m,j_{d,t}} \) as intermediate inputs.

### 2.3 Supply of Factors of Production

The supply of the three factors of production changes over time. They are perfectly mobile and thus have a uniform price across sectors. For each country, an exogenous path of endowments of labor is imposed based on external projections from the United Nations and the International Monetary Fund as described in the data section below.

Aggregate capital, \( k_{d,t} \), is a function of capital in the previous period, \( k_{d,t-1} \), depreciation, \( \delta \), and investment, \( in_{d,t} \), evolving according to the following law of motion:

\[
k_{d,t} = (1 - \delta_{d})k_{d,t-1} + in_{d,t} \tag{8}
\]

There is an investment sector that combines different sectoral commodities \( q_{d,t}^{j_{d,t}} \) with Leontief technology under perfect competition:

\[
in_{d,t} = \min \left\{ \frac{q_{d,t}^{in,1}}{\chi_{d,t}}, \ldots, \frac{q_{d,t}^{in,|I|}}{\chi_{d,t}} \right\} \tag{9}
\]

Hence, the demand for any particular input \( i \) and the price of the aggregate investment good are, respectively:

\[
q_{d,t}^{in,i} = \frac{\chi_{d,t} \cdot in_{d,t}}{\chi_{d,t}^{in,i}} \tag{10}
\]

\[
p_{d,t}^{in} = \sum_{i \in \mathcal{I}} \frac{\chi_{d,t}^{i} \cdot in_{d,t}}{\chi_{d,t}^{in,i}} \tag{11}
\]

We assume that the ratio of a region’s trade balance to its total income is fixed. Abstracting from other components of the current account, the capital account is equal to the trade balance. Assuming a fixed trade balance ratio (relative to income) thus implies that the investment rate is equal to the savings rate. Hence, in equilibrium we have:

\[
\sum_{i \in \mathcal{I}} p_{d,t}^{in,i} in_{d,t} = s_{d,t} Y_{d,t} \tag{12}
\]
2.4 International trade

Consumers, investors and firms demand sectoral commodities \( q_{d,t}^j \) by amounts \( q_{d,t}^{c,j} \), \( q_{d,t}^{in,j} \) and \( q_{d,t}^{m,j} \), respectively. A local producer source the cheapest landed varieties \( \{ q_{d,t}^j (\omega) : \omega \in [0, 1] \} \) from all countries \( s \in D \) and produces the sectoral commodity according to the following technology:

\[
q_{d,t}^j = \left[ \int_{[0,1]} q_{d,t}^j (\omega) \frac{\sigma_{j-1}^{\sigma_j-1}}{\sigma_{j-1}^{\sigma_j-1}} \omega \right]^{\frac{1}{1-\sigma_j}}
\]

The price of commodity \( j \in I \) thus satisfies:

\[
p_{d,t}^j = \left[ \int_{[0,1]} p_{d,t}^j (\omega)^{1-\sigma_j} \omega \right]^{\frac{1}{1-\sigma_j}}
\]

Trade happens through demand for varieties used as inputs in the production of sectoral goods \( q_{s,t}^i \). These goods, in turn, are used in two different ways: as intermediate inputs in the production of varieties and investment goods; and in final consumption. Let \( x_{s,t}^i (\omega) \) be the landed unit cost of supplying variety \( \omega \) of commodity \( i \in I \) produced in source region \( s \in D \) and delivered to region \( d \in D \):

\[
x_{s,t}^i (\omega) \equiv \frac{tm_{s,t}^i \cdot \tau_{s,t}^i \cdot c_{s,t}^i}{z_{s,t}^i (\omega)} = \frac{\tilde{x}_{s,t}^i}{z_{s,t}^i (\omega)}
\]

where \( tm_{s,t}^i \) are gross import taxes, which can be source and destination specific; \( \tau_{s,t}^i \geq 1 \) are bilateral iceberg trade costs; and \( z_{s,t}^i (\omega) \) is the firm’s productivity. The last equality follows from defining \( \tilde{x}_{s,t}^i = tm_{s,t}^i \cdot \tau_{s,t}^i \cdot c_{s,t}^i \) as the landed input bundle costs.

Since varieties can be sourced from every region \( s \in D \) consumers in destination region \( d \in D \) will only buy variety \( \omega \) from the source with the lowest landed price. Following Bernard et al. (2003), producers engage in Bertrand competition. For each country, order firms \( k = [1, 2, \cdots] \) such that \( z_{1,s,t}^i (\omega) > z_{2,s,t}^i (\omega), \cdots \). If the lowest-cost provider of the variety \( \omega \) to country \( d \in D \) is a producer from country \( s \in D \), the price in \( d \) satisfies:

\[
p_{d,t}^i (\omega) = \min \left\{ \frac{\sigma_i}{\sigma_i - 1} \frac{\tilde{x}_{s,t}^i}{z_{s,t}^i (\omega)} , \min_{n \neq s} \frac{\tilde{x}_{n,t}^i}{z_{n,t}^i (\omega)} \right\}
\]

Assumption 1 (Productivity draws). We follow the canonical Eaton and S. Kortum (2002) assumption that and take \( z_{s,t}^i (\omega) : A \times D \times I \times \Omega \rightarrow \mathbb{R}_+ \) to be the realization of an i.i.d. random variable, where \( A \) is the set of states of the world. Productivity is distributed according to a Type II Extreme Value Distribution (Fréchet).

\[
F_{s,t}^i (z) = \exp \{ -\lambda_{s,t}^i z^{-\theta_i} \}
\]
variability in productivity and lower potential for diversification according to comparative advantage).

We show in the Appendix that prices in the destination region $d \in D$ will be, respectively:

$$p_{d,t}^i = \Gamma_1(\Phi_{d,t}^i)^{-\theta_i}$$

(18)

where $\Gamma_1$ is a constant $^6$; $\Phi_{d,t}^i = \sum_{s \in D} \lambda_{s,t}^i(\bar{x}_{s,t}^i)^{-\theta_i}$.

As there are infinitely many varieties in the unit interval, by the law of large numbers, the expenditure share of destination region $d \in D$ on goods coming from source country $s \in D$ converges to their expected values. $\pi_{s,t}^i$ denotes expenditures of consumers in region $d \in D$ on goods coming from region $s \in D$ as a share of their total expenditure on commodity $i \in I$ and is given by:

$$\pi_{s,t}^i = \frac{\lambda_{s,t}^i(\bar{x}_{s,t}^i)^{-\theta_i}}{\Phi_{d,t}^i}$$

(19)

Sectoral commodities can be used both for final consumption and as intermediate inputs. Given the assumptions above, $\pi_{s,t}^i$, which is the expenditure on goods coming from $s$ to be used as intermediate inputs in sector $i$ of country $d$ as a share of total expenditures of that sector on goods from sector $j$ is equal to the trade shares for final consumption, i.e: $(\forall i,j) \pi_{s,t}^{i,j} = \pi_{s,t}^j$.

In the presence of Bertrand Competition, we show in the Appendix that source firms realize a profit which is proportional to the total expenditure of destination countries. In particular, profits are:

$$\Pi_{s,t}^i = \frac{1}{1 + \theta_i} \sum_{d \in D} \pi_{s,t}^i e_{d,t}^i; \quad e_{d,t}^i = \sum_{ag \in \{c,in,m\}} e_{ag,i}^{d,t}$$

(20)

with $i^{ag,i} = p_{d,t}^{ag,i} q_{d,t}^{ag,i}$.

2.5 Market Clearing

In every region, factor markets must clear, such that the use of each factor of production summed over all sectors $i \in I$ in region $s \in D$ by producers of varieties must equal its supply:

$$\sum_{i \in I} k_{s,t}^i = k_s, \quad \sum_{i \in I} l_{s,t}^i = l_{s,t}$$

(21)

The main equilibrium conditions are $|D| \cdot |I|$ expenditure equations that satisfy:

$$e_{s,t}^j = \sum_{d \in D} \pi_{s,t}^j \sum_{i \in I} \eta_{d,t}^i \eta_{d,t}^{m.i} \cdot \frac{\theta_i}{1 + \theta_i} e_{d,t}^m \cdot \sum_{d \in D} \pi_{s,t}^j \cdot (\kappa_{d,t}^j \cdot (1 - s_{d,t}) Y_{d,t} + e_{d,t}^{i,n,i})$$

(22)

The first line of equation (22) denotes the purchasing by firms of intermediate inputs. $\pi_{s,t}^j$ is the trade share of $s$ in varieties demanded by the producer of sectoral good $j$ in

$^6$Specifically, $\Gamma_1 = [1 - \frac{\sigma_i - 1}{\theta_i} + \frac{\sigma_i - 1}{\theta_i} (\frac{\alpha_i}{\sigma_i - 1})^{-\theta_i}] \Gamma\left(\frac{1 - \sigma_i + \theta_i}{\theta_i}\right)$, where $\Gamma(\cdot)$ is the Gamma function
region \( d \); \( \eta_{d,t}^{i,j} \) is the cost share of sector \( j \) in the total intermediate expenditure use in sector \( i \); \( \kappa_{d,t}^{i,m} \) is the cost share of intermediates in total cost in sector \( i \); and \( \frac{\theta_i}{1 + \theta_i} \epsilon_{d,t}^j \) is the total cost payments\(^7\).

The second line represents expenditure related to the use of varieties in the production of sectoral goods for final consumption and for investment goods. \( \kappa_{d,t}^{i,f} \) is the Cobb-Douglas parameter that denotes expenditure share in sector \( i \) as a fraction of total final goods expenditures.

Prices of factors of production are proportional to their use and total cost. Since factors are used in every sector, we aggregate over sectors to calculate aggregate payments to each factor of production:

\[
w_{s,t} \ell_{s,t} = \sum_{d \in D} \sum_{i \in I} \eta_{d,t}^{i,f} \cdot \frac{\theta_i}{1 + \theta_i} \cdot \pi_{s,d,t}^i \cdot e_{d,t}^j
\]

\[
r_{s,t} k_s = \sum_{d \in D} \sum_{i \in I} \eta_{d,t}^{i,k} \cdot \frac{\theta_i}{1 + \theta_i} \cdot \pi_{s,d,t}^i \cdot e_{d,t}^j
\]

where \( \eta_{d,t}^{i,f} \) is the cost share of value added in total cost; and, for each factor of production \( n \in \{t, k\}, \eta_{d,t}^{i,n} \) is the cost share of factor \( n \) in total expenditure on factors of production\(^8\).

The government collects tariffs and other taxes and directs them to the representative household as lump-sum transfers:

\[
T_{d,t} = \sum_{s \in D} \sum_{i \in I} \frac{tm_{s,d,t} - 1}{tm_{s,d,t}} \cdot \pi_{s,d,t}^i \cdot e_{d,t}^j
\]

Recalling that profits are \( \Pi_{s,t}^j = \frac{1}{1 + \theta_i} \sum_{d \in D} \pi_{s,d,t}^i \cdot e_{d,t}^j \) completes all elements necessary to characterize domestic income:

\[
Y_{s,t} = w_{s,t} \ell_{s,t} + r_{s,t} k_s + T_{s,t} + \sum_{j \in I} \Pi_{s,t}^j
\]

Our model is characterized by a sequence of static equilibria satisfying the equilibrium equations in each of the periods \( t \). Solutions in the different periods are related in two ways. First, the capital stock in period \( t \) is determined by investment in period \( t \) and the capital stock in period \( t - 1 \) (plus depreciation) as specified in equation (8). Second, the country-sector-specific technology parameters \( \lambda_{d,t} \) change over time as specified in equation (24) below.

We abstract from intertemporal optimization of consumption, imposing instead a fixed savings rate. This makes the model computationally more tractable and leads to a more straightforward interpretation of the simulation results. We focus on the long-run effects of decoupling of the global economy from 2020 to 2040 and not on the effects of decoupling

\(^7\)The cost shares are, respectively: \( \eta_{d,t}^{i,f} = \psi_{d,t}^{i,j}(p_{d,t}^{j})^{1-\rho_j}/(\sum_{k \in I} \psi_{d,t}^{i,k}(p_{d,t}^{k})^{1-\rho_k}) \) and \( \eta_{d,t}^{i,m} = \psi_{d,t}^{i,m}(pm_{d,t}^{j})^{1-\rho_j}/(\sum_{k \in I} \psi_{d,t}^{i,m}(pm_{d,t}^{k})^{1-\rho_k}) \).

\(^8\)Once again, the mathematical expressions for the cost shares are: \( \eta_{d,t}^{i,f} = \psi_{d,t}^{i,f}(pf_{d,t}^{j})^{1-\rho_j}/(\psi_{d,t}^{i,f}(pf_{d,t}^{j})^{1-\rho_j} + \psi_{d,t}^{i,m}(pm_{d,t}^{j})^{1-\rho_j}) \); and, for each factor of production \( n \in \{t, k\}, \eta_{d,t}^{i,n} = \psi_{d,t}^{i,n}(pm_{d,t}^{j})^{1-\nu_j}/\sum_{q \in \{t, k\}} \psi_{d,t}^{i,n}(pq_{d,t}^{j})^{1-\nu_j}, \) with \( pm_{d,t}^{j} \) standing for the price of factor \( n \).
on the trajectory followed by the economy to 2040. However, abstracting from intertemporal optimization implies that potential effects through changes in savings rates on capital accumulation are also abstracted from.

### 2.6 Dynamic innovation

Unlike in the standard Eaton and S. Kortum (2002) model or in the Bertrand-competition version developed in Bernard et al. (2003), we assume that each region’s location parameter evolves over time. Each commodity \( i \in I \) and each country \( d \in D \) has a different period-specific productivity distribution \( F_{d,t}(z) \).

Our model follows a strand of the literature which models ideas diffusion through random matches between domestic and foreign managers\(^9\). Seminal examples of this work include Jovanovic and Rob (1989) and S. S. Kortum (1997). More recently, Alvarez et al. (2013) explored how idea diffusion is intertwined with trade linkages. Like Buera and Oberfield (2020), we assume that a manager draws new insights as a by-product of sourcing a basket of inputs.

We extend this framework to a model with diffusion of ideas in a multi-sector context and solve it in a recursive fashion that permits forward-looking assessment of policy experiments. The idea diffusion mechanism is mediated by the input-output structure of production, such that both sector cost shares and import trade shares characterize the source distribution of ideas\(^10\).

**Assumption 2** (Idea formation). New ideas are the transformation of two random variables, namely: (i) original insights \( o \), which arrive according to a power law: \( O_t(o) = \Pr(O < o) = 1 - \alpha_t o^{-\beta} \); (ii) derived insights \( z' \), drawn from a source distribution \( G_{d,t}(z) \).

After the realization of those two random variables, the new idea has productivity \( z = o(z')^\beta \), where \( o \) is the original component of the new idea, \( z' \) is the derived insight, and \( \beta \in [0,1] \) captures the contribution of the derived insights to new ideas. Local producers only adopt new ideas if their quality dominates the quality of local varieties. Therefore, for any period, domestic technological frontier evolves according to\(^11\):

\[
F_{d,t+\Delta}(z) = \frac{F_{d,t}(z)}{\Pr(\text{productivity} < z \text{ at } t)} \left( 1 - \int_t^{t+\Delta} \int \alpha_t z'^{-\theta} (z')^{\beta \theta} dG_{d,t} (z') d\tau \right) / \Pr(\text{no better draws in } (t,t+\Delta])
\]

**Lemma** (Generic Law of Motion, Buera and Oberfield 2020). Given Assumption 2, if, for any \( t \), \( F_{d,t}(z) \) is Fréchet with location parameter \( \lambda_{d,t} = \int_0^{\infty} \alpha_t z'^{\beta} dG_{d,t}(z') d\tau \) and

\(^9\)The assumption of a fixed trade balance implies that the capital stock is also not affected by potential changes in the capital balance in response to shocks. However, the international finance literature suggests that standard open economy models with intertemporal optimization have generated counterfactual predictions on the direction of capital flows between developed and emerging countries in the 1990s and 2000s. Capital was flowing on net from emerging to developed economies instead of capital flowing to the emerging economies with higher growth rates as predicted by the standard models.

\(^10\)For a detailed review of this literature, see the comprehensive review chapter published by Buera and Lucas (2018).

\(^11\)As mentioned earlier, our work is closely related to Santacreu, Li, and Cai (2017), who extend S. S. Kortum (1997) to a multi-sector framework. We differ in that they model diffusion as happening separately from trade, rather than as a trade-externality.

\(^12\)Here we simply use the fact that \( o = z(z')^{-\beta} \) and note that, given an insight \( z' \), at any moment \( t \) the arrival rate of ideas of quality better than \( z \) is \( \Pr (O > o) = \Pr (O > z(z')^{-\beta}) = \alpha_t z^{-\theta} (z')^{\beta \theta} \). We then integrate over all possible values of \( z' \).
scale parameter \( \theta_i \), the former evolving according to the following law of motion:

\[
\Delta \lambda_{i,d,t} = \alpha_t \int (z')^{\beta \theta_i} dG_{i,d,t}(z')
\]  

(23)

where \( \alpha_t \) is a parameter that controls the arrival rate of ideas and \( \beta \) is the sensitivity of current productivity to derived insights. The integral on the right hand side of the equation denotes the average productivity of ideas drawn from source distribution \( G_{i,d,t}^i(z') \)\(^{13}\).

Proof. Appendix. □

To fully characterize (23), we need to define the source distribution. We assume that managers learn from their suppliers, such that \( G_{i,d,t}^i(z') \) is proportional to the sourcing decisions in production of commodity \( i \) in country \( d \). Productivity thus evolves endogenously as a by-product of sourcing decisions. Additionally, we assume that insights take time to come to fruition. Rather than drawing insights from interactions with suppliers in the current period, we assume that insights take one period to materialize. Intuitively, we are assuming that entrepreneurs have to study their purchases for one period and only then draw insights. This assumption will be convenient because it will allow us to compute the law of motion for technology without relying on present period trade shares. Therefore, we will be able to solve the model recursively and use it for forward-looking counterfactual analysis.

**Assumption 3** (Source Distribution from Intermediates). The source distribution \( G_{i,d,t}^i(z') = \sum_{j \in I} \eta_{i,j,d,t}^i \sum_{s \in D} H_{i,j,s,d,t-1}^i(z') \), where \( \eta_{i,j,d,t}^i \) is the intermediate cost share of sector \( j \) when producing good \( i \) in region \( d \); and \( H_{i,j,s,d,t-1}^i(z') \) is the fraction of commodities for which the lowest cost supplier in period \( t-1 \) is a firm located in \( s \) in country \( d \) with productivity weakly less than \( z' \).

**Proposition 1** (Law of Motion in a Multi-Sector Framework). Given Assumptions 1-3, in the multi-sector multi-region economy described in the previous section, the country-sector-specific technology parameter evolves according to the following process:

\[
\Delta \lambda_{i,d,t} = \alpha_t \sum_{j \in I} \Gamma(1 - \beta) \eta_{i,j,d,t-1}^i \sum_{s \in D} \left( \pi_{i,j,s,d,t-1}^{i,j} \right)^{1-\beta} \left( \lambda_{j,s,t-1}^{i,j} \right)^{\beta}
\]  

(24)

where \( \Gamma(\cdot) \) is the gamma function, \( \eta_{i,j,d,t-1}^i \) are cost shares, and \( \pi_{i,j,s,d,t-1}^{i,j} \) are intermediate input trade shares.

Proof. Combining the result of the Lemma stated above and Assumption 2, we can express the law of motion as:

\[
\Delta \lambda_{i,d,t} = \alpha_t \int z^{\beta \theta_i} dG_{i,d,t}^i(z)
\]

\[
\quad = \alpha_t \sum_{j \in I} \eta_{i,d,t-1}^{i,j} \sum_{s \in D} \int z^{\beta \theta_i} dH_{i,j,s,d,t-1}^i(z)
\]

The strategy of the proof relies on Bertrand competition and Assumption 1, regarding productivity draws. The joint distribution of the two most productive firms in a given industry \( i \) in country \( s \) is given by \( F_{i,s,t}(z_1, z_2) = (1 + \lambda_{i,s,t}^i) \exp\{-\lambda_{i,s,t}^i(z_2 - z_1)\} \). Incorporating landed input bundle costs \( \tilde{x}_{i,s,d,t} \), we calculate the probability that the lowest

\(^{13}\)Equation (23) is a discrete-time approximation of the continuous-time law of motion derived in the Appendix.
cost producer at destination $d$ is from $s$ and has productivity lower than $z_2$ or in the range $[z_2, z_1)$. In the Appendix, we use the work the each integral $\int z^{\beta d} \cdot dH_{sd,t-1}^{i,j}(z)$ and derive the result stated in the Proposition.

This result extends the main proposition of Buera and Oberfield (2020) to a multi-sector framework. We will use equation (24) and a calibrated path of $\alpha_t$ to solve for an endogenous path for $\lambda^i_{d,t}$.

3 Discussion and Intuition of Ideas Diffusion in a Multi-sector Framework

In this section, we provide some intuition regarding how the idea diffusion mechanism operates in the multi-sector framework. We will use a simplified two-country, two-region economy to show how the actual market equilibrium outcome exhibits both within- and between sector deviations from an outcome maximizing ideas diffusion.

Below we denote industries as $i, \neg i$ and label regions as home ($h$) and foreign ($f$). $\eta^i$ denotes own-cost share of industry $i$, assumed to be identical in both countries; $\lambda^i_h$ is the productivity in sector $i$ at home; and $\pi^{i,-i}_h$ stands for the domestic trade share of $h$ in the total intermediate cost of inputs from industry $i$ in the production of industry $\neg i$. We drop time subscripts for simplicity. In a two-by-two symmetric economy, equation (24) for industry $i$ at home is proportional to a weighted average of the two sector input shares:

$$\Delta \lambda^i \propto \eta^i \left[ \pi^{i,i}_h \right]^{1-\beta} \lambda^i + (1 - \pi^{i,i}_h) \left( \lambda^i \right)^\beta \right] + (1 - \eta^i) \left[ \left( \pi^{i,-i}_h \right)^{1-\beta} \lambda^i + (1 - \pi^{i,-i}_h) \left( \lambda^i \right)^\beta \right]$$

What would the optimal total domestic trade shares in sectors $i, \neg i$ be? If a planner were to choose $\pi^{i,i}_h, \pi^{i,-i}_h$ to maximize idea diffusion\footnote{Specifically, a planner is maximizing $\max_{\pi^{i,i}_h, \pi^{i,-i}_h} \eta^i \left[ \left( \pi^{i,i}_h \right)^{1-\beta} \lambda^i + (1 - \pi^{i,i}_h) \left( \lambda^i \right)^\beta \right] + (1 - \eta^i) \left[ \left( \pi^{i,-i}_h \right)^{1-\beta} \lambda^i + (1 - \pi^{i,-i}_h) \left( \lambda^i \right)^\beta \right]$, which is a separable and strictly concave programming problem in $\pi^{i,i}_h, \pi^{i,-i}_h$.}, the ratio of optimal total expenditure shares within a given sector would satisfy:

$$\left( \frac{\eta^i \pi^{i,i}_h}{\eta^i \left( 1 - \pi^{i,i}_h \right)} \right)_{\text{Diffusion Optimum}} = \frac{\lambda^i_h}{\lambda^i_f}$$

How does this compare with the total domestic trade shares that results from market optimization by private agents? Market allocations incorporate unit costs $x^i_h$ and trade costs $\tau \geq 1$ (assumed to be symmetric):

$$\left( \frac{\eta^i \pi^{i,i}_h}{\eta^i \left( 1 - \pi^{i,i}_h \right)} \right)_{\text{Actual Trade}} = \frac{\lambda^i_h (x^i_h)^{-\theta}}{\lambda^i_f \left( \tau \cdot x^i_f \right)^{-\theta}}$$

In general, the market allocation will be different from the diffusion optimal one, except if differences in trade and unit costs exactly cancel out, i.e.: $\tau = x^i_f / x^i_f$. This within sector distortion mimics the single-sector results of Buera and Oberfield (2020). Below, we show that in a multi-sector framework not only there are deviations within each sector,
but also that in general they are not proportional across sectors: i.e., there are cross-sector distortions. Consider first the ratio of domestic shares in total trade expenditures in sectors $i,-i$ that induce optimal idea diffusion:

$$
\left( \frac{\eta^i \pi^i_h}{(1-\eta^i)\pi_{h,i}} \right) \frac{\text{Diffusion Optimum}}{\text{Actual Trade}} = \frac{\eta^i}{1-\eta^i} \times \frac{\lambda_h^i}{\lambda_{h,i}} \times \frac{(\lambda_h^i + \lambda_{f,i})^{-1}}{(\lambda_{h,i} + \lambda_{f,i})^{-1}} \tag{25}
$$

The ratio can be decomposed into a cost share component, a relative own-productivity component; and a industry-wise relative productivity component. Intuitively, optimal domestic trade allocation in industry $i$ will increase relative to industry $-i$ if intermediate cost share of industry $i$ increases and if the relative domestic productivity of industry $i$ goes up. The ratio is decreasing in industry-wise relative productivity: if the productivity gap between foreign and home is larger in industry $i$ relative to industry $-i$, optimal domestic trade share of industry $i$ will decrease relative to industry $-i$.

How does this compare with the total domestic trade shares that results from market optimization? The industry-wise productivity ratio is adjusted by unit and trade costs:

$$
\left( \frac{\eta^i \pi^i_h}{(1-\eta^i)\pi_{h,i}} \right) \frac{\text{Actual Trade}}{\text{Diffusion Optimum}} = \frac{\eta^i}{1-\eta^i} \times \frac{\lambda_h^i(x_h^i)^{-\theta}}{\lambda_{h,i}^{1}(x_{h,i}^{1})^{-\theta}} \times \frac{(\lambda_{h,i}^{1}(x_{h,i}^{1})^{-\theta} + \lambda_{f,i}^{1}(\tau \cdot x_{f,i}^{1})^{-\theta})^{-1}}{(\lambda_h^i(x_h^i)^{-\theta} + \lambda_{f,i}^i(\tau \cdot x_{f,i}^i)^{-\theta})^{-1}} \tag{26}
$$

These differences induce a gap between the diffusion optimal allocation and the market allocation. Define $\kappa$ as the ratio of equations (26) for (25):

$$
\kappa = \frac{\left( \frac{x_h^i}{x_{h,i}^{1}} \right)^{-\theta} \times \frac{(\lambda_{h,i}^{1}(x_{h,i}^{1})^{-\theta} + \lambda_{f,i}^{1}(\tau \cdot x_{f,i}^{1})^{-\theta})^{-1}}{(\lambda_h^i(x_h^i)^{-\theta} + \lambda_{f,i}^i(\tau \cdot x_{f,i}^i)^{-\theta})^{-1}}}{\left( \frac{x_h^i}{x_{h,i}^{1}} \right)^{-\theta} \times \frac{\lambda_h^i(x_h^i)^{-\theta} + \lambda_{f,i}^i(\tau \cdot x_{f,i}^i)^{-\theta}}{\lambda_h^i(x_h^i)^{-\theta} + \lambda_{f,i}^i(\tau \cdot x_{f,i}^i)^{-\theta}}} \tag{27}
$$

Whenever $\kappa \neq 1$, there is a sectoral distortion in domestic trade expenditure shares. If $\kappa > 1$ ($\kappa < 1$), domestic trade expenditure share on sector $i$ relative to sector $-i$ is above (below) the diffusion-optimal ratio. Even if $\kappa = 1$, that does not guarantee absence of deviations from the optimal diffusion point. Rather, it means that deviations (or absence thereof) are proportional in both sectors, such that domestic trade share in one sector is not disproportionately higher (lower) in sector $i$ relative to sector $-i$.

In general, deviations need not be proportional. In fact, only in knife edge cases $\kappa = 1$\textsuperscript{15}. This underscores that, in a multi-sector framework, there will not only be within sector distortions, but also between sector distortions. Domestic sourcing will be biased towards the industry with lowest relative cost, even if that industry is not very productive. For instance, if costs are disproportionately low in one industry $i$ relative to industry $-i$,

\textsuperscript{15}For instance, if countries have identical input costs across industries $x_h^i = x_{h,i}^{1}, x_f^i = x_{f,i}^{1}$; and either industries in each country have identical productivity ($\lambda_h^i = \lambda_h^i, \lambda_f^i = \lambda_f^i$); or sector-specific productivities at home are a linear transformation of the sector-specific productivities at foreign ($\lambda_h^i = \kappa \cdot \lambda_f^i, \lambda_{f,i}^{1} = \kappa \cdot \lambda_f^{1}$, $\kappa \in \mathbb{R}_{++}$)
either domestically or industry-wise, domestic trade share will be disproportionately high in industry \(i\) under market relative to the optimal allocation and \(N > 1\).

There are additional complexities that arise in a multi-sector framework, which we illustrate geometrically. First, consider what happens within sector \(i\) in a fully symmetric two-by-two economy. Even with identical countries, the strict concavity of diffusion equation implies that idea diffusion is not uniform as \(\pi_{h,i}^{i,i}\) varies. The optimal diffusion point is \((\pi_{h,i}^{i,i})^{\text{Diffusion Optimum}} = \lambda_{h,i}^{i} / (\lambda_{h,i}^{i} + \lambda_{f,i}^{i}) = 1/2\). Under the market allocation, trade costs induce home bias such that domestic share is \((\pi_{h,i}^{i,i})^{\text{Actual Trade}} = 1/(1 + \tau - \theta) > 1/2\) and ideas diffusion is below the optimal point. If trade costs increase and \(\tau \to \infty\), the home country moves to autarky and deviations from the optimal idea diffusion reach a maximum. We plot the optimal, actual trade, and autarky points along the ideas diffusion function for sector \(i\) on the left hand side panel of Figure 1.

![Figure 1: Within sector idea diffusion functions in a two-by-two economy.](image)

Both panels plot the idea diffusion functions for the home country in a two-by-two model within sector \(i\): \(\eta^{i}((\pi_{h,i}^{i,i})^{1-\beta}(\lambda_{h,i}^{i})^{\beta} + (1 - \pi_{h,i}^{i,i})^{1-\beta}(\lambda_{f,i}^{i})^{\beta})\). The left hand side panel shows the optimal, actual trade, and autarky points along the ideas diffusion function when countries are fully symmetric \((\lambda_{h,i}^{i} = \lambda_{f,i}^{i})\). The right hand side panel plots the functions and diffusion optimal points for the cases when countries have identical productivities \(\lambda_{h,i}^{i} = \lambda_{f,i}^{i}\) and the home country is less productive \(\lambda_{h,i}^{i} < \lambda_{f,i}^{i}\).

The right hand side panel illustrates what happens when the home country has a lower productivity in sector \(i\). The curve shifts down at the autarky point and the optimal solution moves to the left (smaller domestic trade share)\(^{16}\). When \(\lambda_{h,i}^{i} < \lambda_{f,i}^{i}\), diffusion losses from high trade costs are higher. This highlights a key characteristic of this class of models: countries that are less productive in a given sector have higher dynamic gains from trade\(^{17}\).

When considering a multi-sector framework, within sector inefficiencies accumulate. For

\(^{16}\)Formally, once countries are no longer symmetric, we need to make the following regularity condition to guarantee convergence to the autarky equilibrium: \(\lim_{\tau \to \infty} (\tau x_{f,i}^{i}) / x_{h,i}^{i} = +\infty\). Most models make this assumption either explicitly or implicitly.

\(^{17}\)In fact, for any \(\pi_{h,i}^{i} \in (0,1]\), the marginal change in diffusion as \(\pi_{h,i}^{i}\) increases will be increasing in a
instance, suppose that in sector $i$ domestic trade share $\pi_i = 0$ while, in sector $-i$, $\pi_{-i} = 1$. If $\eta = 1/2$, deviations from optimal idea diffusion will be at a maximum even though total trade share will be at the optimal point ($1/2$). In a multi-sector framework, the fact that total domestic trade share is at its optimal point is necessary but no longer sufficient for optimal diffusion. To maximize total diffusion, trade shares must be at their optimal point in every sector.

Figure 2 underlines this fact. It shows that there is a unique point in the $[0,1]^2 \times [0,\infty)$ space that maximizes idea diffusion in a two-by-two symmetric model as $\pi_{-i} = 1 - \pi_i$ vary. With $\eta = 1/2$, every point in the diagonal $\pi_{-i} = 1 - \pi_i$ will have total trade share at its optimal point $1/2$. Additionally, every point in the counterdiagonal $\pi_{-i} = \pi_i$ has absence of between sector distortion ($8 = 1$). However, neither fact is sufficient to guarantee optimal diffusion. Only if trade shares are optimal in both sectors (i.e., $\pi_{-i} = \pi_i = 1/2$) diffusion is maximized in this simplified economy. Any other point will have some degree of inefficiency.

Figure 2: Idea diffusion function in a two-by-two economy. The graph shows $\Delta \lambda \propto \eta \cdot (1 - \beta) \cdot (\pi_{-i} - \beta (\lambda_{-i}))(1 - (1 - \beta) \cdot (\pi_{-i} - \beta (\lambda_{-i})))$ as a function of domestic trade share in sectors $i, -i$: $\pi_{-i}, \pi_i$. If countries and sectors are identical and $\eta = 1/2$, Optimal, Actual Trade, and Autarky allocations are as represented in this figure. The marginal contribution of each sector to total diffusion are as shown in the left panel of Figure 1.

It is easy to see how the problem increases in complexity and inefficiencies accumulate as the number of countries and sectors increase. With $N$ countries and $J$ sectors, there are $(N-1)J$ free parameters that would need to be at their optimal points if diffusion were to country’s productivity. To see that, take $\frac{\partial \Delta \lambda_i}{\partial \pi_i} = \alpha \cdot (1 - \beta) \cdot \eta \cdot (1 - \beta) \cdot [(\pi_{-i})^\beta (\lambda_{-i})^\beta - (1 - \pi_{-i})^\beta (\lambda_{-i})^\beta]$, which is increasing in $\lambda_{-i}$. 


15
be maximized. In a market equilibrium, those parameters will not be at their optimal points and each one of them will contribute to some deviation from optimal diffusion. Therefore, a multi-sector framework is important to have a more realistic assessment of diffusion in policy experiments. We derive multi-sector, multi-region versions of equations 25, 26, and 27 in Appendix D, but most of the intuition can be represented with the simplified version presented in this section.

4 Calibration and Setup of Policy Experiments

In this section we first outline the employed baseline data and behavioral parameters. The parameter determining the strength of diffusion of ideas is calibrated by minimizing the difference between the historical and simulated GDP growth rate across many countries in the world economy. We then motivate and describe the detail set-up of the experiments.

4.1 Data and Behavioral Parameters

4.1.1 Baseline Data

The model is calibrated to trade and production data from the 2014 version of the GTAP Data Base, Version GTAP10A. This means that all spending and cost shares are set equal to the shares in the 2014 database, following the same calibration procedure as in models employing exact hat algebra (Dekle, Eaton, and S. Kortum 2007). The data are aggregated to 10 regions and 6 sectors as specified in Table 1. The model is solved until 2040 in a sequence of recursive dynamic simulations, thus solving the model period per period, using the model solution in the previous period as the starting point for the next period. Population grows based on UN population projections and labor supply grows based on International Monetary Fund projections for employment (until 2025) and United Nations projections regarding working age population (from 2026 until 2040).

| Region Code | Description           | Sector Code | Description                  |
|-------------|-----------------------|-------------|------------------------------|
| chn         | China                 | pri         | Primary (agri & natres)     |
| e27         | European Union 27     | lmn         | Light manufacturing          |
| jpn         | Japan                 | hmn         | Heavy manufacturing          |
| ind         | India                 | elm         | Electronic Equipment         |
| lac         | Latin America         | tas         | Business services            |
| ode         | Other developed       | ots         | Other Services               |
| rwc         | Rest of the World - Eastern bloc |
| rwu         | Rest of the World - Western bloc |
| rus         | Russia                |
| usa         | United States         |

The data in the GTAP Data Base do not include profit income as in our model with Bertrand competition. Therefore, we have to modify the baseline data employed, considering that profit income $\Pi_i^s$ is a share $\frac{1}{1+\theta_i}$ of the total value of sales in sector $i$ in region $s$. We have done this as follows, proceeding in two steps. First, we reduced the value of payments to the
production factor capital (capital income) by 50% and reallocated it to profit income. With this step the share of profit income in the value of sales is not yet equal to \( \frac{1}{1+\theta_i} \). Therefore, in a second step we employ our model to modify the base data to target the share of profit income in the value of sales for each country and sector. The reason to proceed in two steps is that capital income in some cases is smaller than profit income required by the model. This is especially the case in sectors with large intermediate linkages and a small trade elasticity, because profit income is a share of gross output in the Bertrand model, whereas capital income is part of net output.\(^{18}\)

### 4.1.2 Behavioral Parameters

The dispersion parameter of the Fréchet distribution, \( \theta_i \), equal to the trade elasticity, is based on the estimates of trade elasticities in Hertel et al. (2007). The substitution elasticity between value added and intermediates, \( \rho \), between intermediates from different sectors, \( \mu \), are equal to zero, implying a Leontief structure. As such we follow the approach employed in most CGE models, which finds empirical support in recent estimates with US data (Atalay 2017). The substitution elasticities between production factors, \( \nu_i \), are based on the values in the GTAP Data Base.

Table 2 displays the values of the dispersion parameter of the Fréchet distribution, \( \theta_i \) and the substitution elasticity between production factors \( \nu_i \).

| Sector                      | \( \theta_i \) | \( \nu_i \) |
|-----------------------------|----------------|------------|
| Primary (agriculture & natres) | 10.09          | 0.27       |
| Light manufacturing         | 4.60           | 1.20       |
| Heavy manufacturing         | 5.99           | 1.26       |
| Electronic Equipment        | 7.80           | 1.26       |
| Business services           | 2.80           | 1.26       |
| Other Services              | 2.90           | 1.42       |

Source: Hertel et al. 2007

Even though the location-parameter of the sector-country specific Fréchet distribution \( \lambda_{s,i}^i \) evolves endogenously in this model, their starting values need to be calibrated. We calibrate the starting values \( \{\lambda_{s,0}^i\}_{s \in D, i \in I} \) using the assumption that this parameter is proportional to PPP-adjusted labor productivity in each sector-country in our baseline year, 2014.

We constructed a database of sectoral productivity combining two sources: the World Input-Output Database and the World Bank’s Global Productivity Database. We provide details of how we aggregated sectors and country groups in the Appendix B. Figure 3 shows the distribution of the calibrated \( \lambda_{s,0}^i \) parameters across industries \( i \) of each region \( s \) in our model.

Buera and Oberfield 2020 set the growth rate of \( \alpha_t \) equal to the population growth rate of the US. We adopt the same heuristics and set the growth rate of the autonomous arrival

\(^{18}\) As an alternative approach, we can reduce the value of payments to factors of production by an identical share for all production factors and reallocate this to profit income. The reallocation is set such that profit income \( \Pi_{s,t}^i \) becomes a share \( \frac{1}{1+\theta_i} \) of the value of sales. However, this approach is not followed because there is a risk that factor income in the data is smaller than profit income implied by the model. As discussed, this is especially a risk in sectors with large intermediate linkages and a small trade elasticity.
Figure 3: Distribution of the calibrated $\lambda_{s,0}$ parameters across industries $i$ of each region $s$ in our model. We assume that this parameter is proportional to PPP-adjusted labor productivity in each sector-country. After the initial period, the location-parameter of the sector-country specific Fréchet distribution $\lambda_{s,t}$ evolves endogenously according to the model.

rate of ideas $\alpha_t$ at 1.18% per year, equal to the projected global population growth rate from 2021 to 2040.

The ideas diffusion parameter $\beta$ is uniform across sectors and determined based on model validation, using simulated methods of moments, as described below. The variance of the growth rates of GDP rise as $\beta$ increases, because at higher levels of $\beta$ there is more diffusion of ideas leading to convergence of income levels thus implying larger differences between growth rates of poor and rich countries. As discussed in Section 3, countries starting with a lower productivity parameters have larger dynamic gains from trade. That effect increases in the value of $\beta$\textsuperscript{19}.

We simulate the model from 2004 to 2019 imposing historical growth rates of population and the labor force (based on IMF data) without further policy changes varying the level of $\beta$ and evaluate for which values of $\beta$ the mean and variance of the growth rates of GDP are closest to the mean and variance in historical data.\textsuperscript{20} Formally, we are minimizing the following loss function with $m$ the historical moment and $m(\beta)$ the simulated moment for either real GDP per capita growth or real GDP growth in the 2004-19 period:

$$\min_\beta \sum_{m \in \{\mu, \sigma\}} w^{GDPPc}(m(\beta)^{GDPPc,model} - m^{GDPPc,hist})^2 + (1 - w^{GDPPc})(m(\beta)^{GDPPc,model} - m^{GDPPc,hist})^2$$

where $w^{GDPPc}$ is the exogenous weight put on real GDP per capita growth, rather than aggregate real GDP growth.

\textsuperscript{19}In the limiting point $\beta \to 1$, ideas diffuse instantly and every country that is not in autarky experiences equal productivity gains in absolute terms. With equal gains in absolute terms, those countries with lower productivity experience larger growth in relative terms as $\beta$ increases.

\textsuperscript{20}In this exercise the growth rate of $\alpha_t$ is set at 1.18% per year, the global population growth rate between 2004 and 2019.
As a first step, we raise $\beta$ in steps of 0.05 from 0 to 0.6.\textsuperscript{21} This exercise indicates that the simulated growth rates are closest to historical growth rates for $\beta$ between 0.4 and 0.5 (See Appendix Table A3). Therefore, as a second step we simulate the model raising $\beta$ in steps of 0.01 from 0.4 to 0.5.

Figure (4) plots the loss function (28) for values of $\beta \in [0.4, 0.5]$. Table A1 in the Appendix displays additional summary statistics (mean, standard deviation, maximum and minimum) of average growth rates of GDP and GDP per capita between 2004 and 2019. Regardless of the weight, the loss function takes a minimum when $\beta \in [0.44, 0.45]$, a rather short interval. Taking an agnostic stance and setting the weight $w_{GDPpc} = 1/2$, we find that the loss function (28) is minimized for $\beta = 0.44$, but results are virtually unchanged by setting it to any value in the aforementioned interval.

Figure 4: Plots of loss function (28) for values of $\beta \in [0.4, 0.5]$. The solid grey line shows the loss function with the parametrization of $w_{GDPpc} = 0$, which is minimized at $\beta = 0.44$. The dotted grey line shows the loss function with the parametrization of $w_{GDPpc} = 1$, which is minimized at $\beta = 0.45$. The thicker black line shows the loss function with the parametrization of $w_{GDPpc} = 1/2$, which is minimized at $\beta = 0.44$.

Figure 5 compares projected GDP growth rates for $\beta = 0.44$ in individual regions with historical GDP growth rates. This figure shows that the simulated GDP growth rates are relatively close to the historical growth rates, suggesting that also for individual regions the model does a good job at replicating historical growth rates. Furthermore, these results can also be interpreted as an analysis of the under- and overperformance of different regions compared to the projections of the model with diffusion of ideas through trade.

In Appendix Table A3 we display the same summary statistics for $\beta$ ranging from 0 to 0.6, in steps of 0.05. This table makes clear that calibration to historical data is important. An excessively high value of $\beta$ will lead to higher than historically observed growth rates in low-income regions. Conversely, a too low value $\beta$ will fail to show the convergence patterns exhibited in the historical data.

\textsuperscript{21}For values larger than 0.6 the variance in the simulations becomes unrealistically high, so these are disregarded.
4.2 Motivation and Set-up of Policy Experiments

4.2.1 Motivation of Policy Experiments

Our main motivation for simulating large-scale trade conflicts is the possibility of receding globalization due to a political backlash. Challenges to the international trade regime (and to globalization at large) might seem like some circumstantial discontinuity in a long-run trend towards increasing openness. However, as we show below, political scientists argue that there is reason to believe that strategic geopolitical rivalries could trump economic gains—at least partially—in their relationships between the U.S. and its allies with China and Russia their allies.

There is ample evidence of substantial gains from trade openness, which can be as large as 50% of national income (Ossa 2015) even in a static setting. At the same time, recent empirical evidence about frictions in local labor markets (Autor, Dorn, and Hanson 2013; Dix-Carneiro and Kovak 2017) highlights the distributional aspects of trade liberalization.

These concerns can translate into political grievances and may have led to an increase in the number of populist and isolationist parties in Western countries calling for less open trade policies (Colantone and Stanig 2018). An increasing number of political parties use anti-globalization rhetoric to rally support of constituents that have grievances against the distributional consequences of automation, structural change, offshoring, and trade opening, as shown in the review of the political science literature by Walter (2021).

A clear example of the shift in consensus during the last decade is the trade conflict between the U.S. and China, which started under the Trump Administration. The economic discourse shifted away from emphasizing the gains from trade to a framing of trade as a zero-sum game and to the use of national-security provisions of the international trade regime to engage in protectionist policy-making.\(^2\)

These geopolitical disputes are exemplified not only in the trade conflict between the U.S. and China but also in industry-specific policy changes, such as the U.S. government

\(^2\)For a contemporaneous review of the policies implemented, see Bown and Irwin (2019).
pressuring allies against allowing participation of Chinese telecommunications companies in new infrastructure developments or limited cooperation in science and education between the two countries (Tang et al. 2021).

Wei (2019) provides a review of debates among Chinese scholars. Some Chinese analysts see an escalating and continuous conflict between China and the U.S. as a natural and “structural” development of a shifting international system that is moving from a unipolar (the U.S. being the only superpower) to a bipolar (China becoming a superpower on an equal footing to the U.S.) balance of power23. They tie a scenario of a continuous confrontation between the U.S. and China either to strategic geopolitical forces or to domestic political forces in America (Zhao 2019).

In the West, political scientist Joseph S. Nye Jr. (2020) highlights that, while an abrupt decoupling between the U.S. and China is unlikely, both parties will try to decrease their (inter-)dependence with respect to each other’s actions, except where the costs of disengagement are too high to bear24. American policymakers and academics also motivate the conflict between China and the U.S. on geopolitical grounds. Although the tone of confrontation is stronger when coming from right-of-center policymakers and scholars25, both sides of the political spectrum in the U.S. discuss readjustment of the economic relationship with China due to geopolitical concerns (Wyne 2020).

The 2022 War in Ukraine and the global scale retaliation against the Russian Federation that followed is another example of how geopolitical interests can take precedence over gains from economic integration. Escalation began in 2014, after Russia’s annexation of Crimea. The U.S. and its allies approved several rounds of sanctions against Russia, culminating in its expulsion from the G8. Confrontation reached another level in the aftermath of Russia’s invasion of Ukraine. Western countries imposed sanctions on banking transactions, froze foreign reserves, and closed the airspace for Russian planes. In March 2022, the G7 and the European Union revoked their recognition of Russia’s most-favored-nation status, opening the door for large tariff increases, and limited the operation of multinational corporations Russian subsidiaries26.

Like in the case of China, the relationship between Russia and the West can also be interpreted through the lens of a strategic game amongst great powers. Scholars argue about the geopolitical nature of the conflict (Mearsheimer 2014) and the “reawakening” of geopolitics (Weber and Scheffer 2022). Therefore, in either case, sanctions and trade conflicts fall within the larger backdrop of a strategic confrontation. These simultaneous conflicts can potentially be interpreted as a larger clash between the U.S. and its allies —

23In the context above, the balance of power between functionally equivalent states (the “international structure”) provides incentives for strategic behavior by governments that try to maximize their power. We can interpret the unipolar or bipolar configurations as equilibria and disruptions between such equilibria as transition paths. This is known as the “structural realism” theory of international politics, developed by Kenneth Waltz (2010).

24Nye Jr. is mostly known for his joint work with Robert Keohane on “complex interdependence” during the post-World War II era (Keohane and Nye Jr 2011). The authors focus their analysis on the creation of international rules and practices in a world in which the use of military force is very costly due to interdependence between multiple agents that engage both internationally and domestically. For instance, a great degree of trade integration increases the costs (and decreases the probability) of outright military conflict.

25See, for instance, the remarks of former White House Trade Council Director to Congress (Navarro, Peter 2018) or a policy blueprint for decoupling by Scissors (2020), who is a scholar at the America Enterprise Institute, a right-of-center think tank.

26For a timeline of sanctions against Russia in the context of the War in Ukraine, see this website maintained by the Peterson Institute for International Economics: https://www.piie.com/blogs/realtimeneconomic-issues-watch/russias-war-ukraine-sanctions-timeline
a Western bloc; and Russia, China, and their allies — an Eastern bloc. As scholars have argued, we could be observing the emergence of a “China-Russia entente” (Lukin 2021), which could lead to a “New Cold War” (Abrams 2022).

We use these facts as motivation to apply our model to conduct hypothetical policy experiments of trade decoupling between East and West: namely, to simulate the effects of large scale geopolitical conflicts between these blocs, in which players try to limit the level of interdependence between each bloc due to political drivers, even if that leads to economic costs.

4.2.2 Set-up of Policy Experiments

We are agnostic about the degree of future decoupling between East and West. Nonetheless, the fact that international relations scholars envisage disengagement as a real possibility underscores that estimating the potential economic consequences of decoupling is an important exercise. As our model highlights, changes in trade patterns and sourcing decisions have not only static effects, but also dynamic effects with respect to potential growth and innovation. Our policy experiments try to disentangle the static and dynamic costs of decoupling.

In order to develop the decoupling scenarios, we classify different regions as belonging to either a Western or an Eastern bloc. We do so resorting to the Foreign Policy Similarity Database, which uses UN General Assembly voting records for a large set of countries to calculate foreign policy similarity indices for each country pair (Häge 2011). Intuitively, the index takes countries who vote similarly in the United Nations (compared to the expected level of similarity of a random voting pattern) as being similar in their foreign policy.

We ordered country groups in terms of their foreign policy similarity with China and the United States in order to place the ten regions of our model either in a Western (U.S.) or Eastern (China) bloc. The results shown in Figure 6 shows that Europe, Canada, Australia, Japan, South Korea fall in the Western bloc. Latin America and Sub-Saharan Africa fall somewhere in between, with the former being closer to the U.S. than the latter. India, Russia, and most of North Africa and Southeast Asia fall closer to China.

As a robustness exercise, we repeated the same procedure but replaced China for Russia as the center of gravity of the Eastern bloc. We plot the two Differential Foreign Policy Similarity Indices for each country in our sample in Figure (7). Most countries fall very close to the 45-degree line (i.e., the regression coefficient is close to 1) and the correlation between the two series is very high. This suggests that, qualitatively, results will be very similar using either Russia or China as the main country in the Eastern bloc.

After classifying the countries into Eastern and Western geopolitical blocs, we design two different policy experiments. We first increase iceberg trade costs $\tau_{sd,t}$ to a point where virtually all of the trade happens exclusively within each bloc. In total, we increase bilateral trade costs by $\sim 160$ percentage points. We label this scenario full decouple. This provides an important limiting case that can be useful for putting bounds on potential effects.

The second scenario relies on work by Nicita, Olarreaga, and Silva (2018), who estimate that a move from cooperative to non-cooperative tariff setting would increase average tariffs by 32 percentage points globally. We simulate what would happen if countries kept cooperative tariff setting within their trade blocs but moved to non-cooperative tariff setting across trade blocs. For simplicity, we assume that regions in different blocs increase bilateral tariffs $tm_{sd,t}$ by the globally average increase in tariffs when moving from cooperative to non-cooperative tariffs: 32 percentage point increases in tariff rates against regions outside the bloc. These tariff increases seem high. However, the average tariff increase in the
Figure 6: Differential Foreign Policy Similarity Index. Values are normalized such that 1 represents maximum relative similarity with the U.S. and \(-1\) represents maximum relative similarity with China. The map shows the difference between pairwise similarity indices \(\kappa_{i,US} - \kappa_{i,China}\). The parameter \(\kappa_{i,j}\) represents the foreign policy similarity of countries \(i, j\), based on vote similarity in the United Nations General Assembly. Given vote possibilities \(n, m \in \{1, \cdots, k\}\), one can calculate a matrix \(P = [p_{nm}]\), where entry \(p_{nm}\) represents the share of votes in which country \(i\) took position \(n\) and country \(j\) took position \(m\). Given matrix \(P\), \(\kappa_{i,j} = 1 - \sum_{m \neq n} p_{mn} / \sum_{m \neq n} p_{mn} p_n\), where \(p_m, p_n\) are expected marginal propensity of any country to take position \(m, n\) at a random vote. For more details, see Häge (2011).

China-U.S. trade war was higher than 21pp. We call this scenario **tariff decouple**.

Besides the full and tariff decouple scenarios, we explore two additional policy experiments. First, we evaluate what the impact would be of a switch of the region Latin America and Caribbean (LAC) from the Western bloc to the Eastern bloc. This sheds some light on the question many countries would face if the decoupling would aggravate: which bloc to stay closest to?

Second, we explore a less hypothetical scenario: trade decoupling only in the electronic equipment sector. We perform a full decoupling between the original blocs (with LAC in the Western bloc) but restricting the increase in iceberg trade costs \(\tau_{sd,t}\) only to the electronic equipment (\textit{elm}) sector. This scenario is motivated by U.S. and Chinese authorities being increasingly at loggerheads with each other in the technological arena.

One important example of this process has been the conflict involving Chinese telecom giant Huawei Technologies. Since 2019, American corporations have been banned from doing business with Huawei. In a similar move, the New York Exchange delisted China Unicom, China Mobile, and China Telecom. Despite legal challenges and a new administration, as of April 2021 neither decision has been reversed.

Additionally, the U.S. has been using its foreign policy arsenal to pressure allies to join them in limiting Chinese telecom companies reach. In particular, there is a desire to limit Chinese participation in 5G technology auctions, citing national security and privacy concerns\(^{27}\). So far, Australia, the United Kingdom and some European allies have chosen

\(^{27}\)North American Treaty Organization (NATO) researchers Kaska, Beckvard, and Minárik (2019) review the arguments put forth from a Western national security perspective. This topic is extremely contentious and some Chinese commentators argue that the U.S. is using national security concerns as an excuse to
to ban or limit Chinese participation in technological auctions.

This conflict suggests that a large increase in trade costs between the U.S. allies and Chinese allies regarding technological equipment is a positive probability scenario in the future. In this case, decoupling would mean a near total separation of electronic equipment sectors of the two blocs.

Huawei and Google breaking their business connections after the U.S. government sanctions against the Chinese corporation is a good illustration of what this separation could look like in real life. Huawei used Google’s Android ecosystem in their smartphones, which gave their users access to Google-approved updates and apps. After the ban issued by the Trump administration, however, Google announced it would comply with the U.S. government directives and Huawei was forced to shift away from Google software and design their own operating system HarmonyOS.

Since this separation is driven primarily by regulation rather than tariffs, it is appropriate to think of it as an increase in iceberg trade costs \( \tau_{sd,t} \) between blocs in the electronic equipment sector and so this is the scenario we will explore.

## 5 Main Results

We have four main scenarios. We simulate full decouple and tariff decouple, defined as explained above. We simulate either scenario both with and without diffusion of ideas, in order to assess the additional impact of the diffusion mechanism. After a discussion of the results of the full and tariff decouple scenarios, we compare the impact of decoupling on productivity in a multi-sector and single-sector setting. We finish this section with a description of the results of the two additional policy experiments that restrict decoupling implementation.
to the electronics and equipment sector or change LAC from the Western bloc to the Eastern bloc.

In the results below we report the results of a comparison of the simulation results with and without policy experiment. We first simulate the dynamic world economy with no policy change, then do the same with the policy change, and report the long run cumulative percentage difference between the two: \( \hat{x} = \frac{\sum_{t=p}^{T} (x'_t - x_t)}{\sum_{t=p}^{T} x_t} \), where \( p \) is the date of the first policy change, \( x'_t, x_t \) are the values of variable \( x \) with and without the policy change, respectively.

5.1 Full and Tariff Decouple

As expected, all scenarios show large negative impacts on cross-bloc trade after the introduction of the policy intervention. In the full decouple scenario, trade between the countries in the Western bloc and the Eastern bloc is virtually shut down, with imports and exports dropping by 98%. Those countries also shift a substantial part of their trade to the U.S., with trade flows increasing anywhere between 10 – 42% depending on the region and scenario. The domestic spending share in the U.S. increases by about 7%. The converse happens in the Eastern bloc but with larger dispersion across regions. Trade with the U.S. drops by 65 – 90% while trade with China increases by 9 – 60%. The domestic trade share in China increases by 3%. The tariff decouple scenario yields qualitatively similar results but with smaller magnitudes. We show the results by region and scenario in Figure 8.

One of the reasons behind the asymmetry in trade decreases between blocs is the assumption of a fixed trade-balance-to-income ratio in all regions but one. This implies that regions with large trade surpluses will shift proportionally less of their trade flows away from regions in other trade blocs in order to satisfy the fixed trade balance assumption.

Figure (9) shows that both the increases in iceberg trade costs (full decouple) and retaliatory tariff hikes (tariff decouple) induce substantial welfare decreases for all countries. The effects, however, are asymmetric. While welfare losses in the Western bloc range anywhere between \(-1\%\) and \(-8\%\) (median: \(-4\%\)) in the Eastern bloc it falls in the \(-8\%\) to \(-12\%\) range (median: \(-10.5\%\)).

The underlying factor driving the divergence in results between the two blocs is a difference in the evolution of productivity, represented by the scale parameter of the Fréchet distribution of different sectors. Sourcing goods from high productive countries puts domestic managers in contact with better quality designs that inspire better ideas through innovation or imitation.

Importantly, the dynamics governed by equation (24) incorporate the input-output structure of production, such that domestic managers in each sector innovate in proportion to the quality and share of their inputs. Losing access to high quality designs does not only lead to static losses, but also to a lower level of future innovation, which implies larger dynamic losses. Additionally, the input-output structure of the model implies that cutting ties to innovative regions is particularly costly if the destination country sources many intermediate inputs from such regions prior to the policy change.

For those reasons, in our policy experiments, countries in the Eastern bloc that currently have a lower level of productivity and have larger ties with innovative countries have larger losses. By looking at results in Figure 10, one can see the stark contrast between the differential evolution of \( \lambda_{i,t} \) for those countries in the Western bloc and those in the Eastern bloc. By cutting ties with richer and innovative markets such as OECD countries, destination countries such as China, India, and parts of Asia and Africa shift their supply chains...
Figure 8: Cumulative Percentage Change in Trade Flows with China and the United States, respectively, after policy change, by 2040. Full Decouple increases iceberg trade costs $\tau_{i,s,t,d}^{*}$ by 160 percentage points. Tariff decouple increases bilateral tariffs $tm_{i,s,t,d}^{*}$ across groups, by 32 percentage points. $\beta$ is a parameter that controls the diffusion of ideas according to equation 24, assumed to be homogeneous across sectors. Country codes: chn, China; ind, India; rus, Russia; rwc, Rest of Eastern bloc; rwu, Rest of Western bloc; lac, Latin America; e27, European Union; ode, Other Developed; usa, United States. Tables with the values for these charts can be found in the Appendix.
Figure 9: Cumulative Percentage Change in Real Income, after policy change, by 2040. *Full Decouple* increases iceberg trade costs \( \tau^*_i \) by 160 percentage points. *Tariff decouple* increases bilateral tariffs \( t_{m,i} \) across groups, by 32 percentage points. \( \beta \) is a parameter that controls the diffusion of ideas according to equation 24, assumed to be homogeneous across sectors. Country codes: chn, China; ind, India; rus, Russia; rwc, Rest of Eastern bloc; rwu, Rest of Western bloc; lac, Latin America; e27, European Union; ode, Other Developed; usa, United States. Tables with the values for these charts can be found in the Appendix.

towards lower quality inputs, which, in turn, induce less innovation. By contrast, while countries in the Western bloc do suffer welfare losses, their innovation paths are virtually unchanged after decoupling, suggesting that nearly all of their losses are static, rather than dynamic.

There is large dispersion across both sectors and countries in differential productivity losses. The two most affected regions are India and the “rest of the Eastern bloc” region. Starting from a lower income level than China and Russia, those regions have a much slower productivity catch-up after severing trade ties with the West. Sectors with larger supply chain linkages to the West prior to the policy change, such as manufacturing in India, experience larger losses.

Among those regions in the Eastern bloc, differential productivity losses are larger in the manufacturing sectors (−1.5% and −3% with full decoupling and tariff decoupling, respectively; this includes elm, lmn, and hmn) than in the services (−0.8% and −1.6%, respectively; ots tas) or primary (−0.5% and −1%, respectively; pri) sectors.

Finally, we address the contrast between the static effect (when the diffusion of ideas mechanism is shut down) and the dynamic effect. For the two poorer regions of the Eastern bloc, dynamic losses far outsize static losses, which can be explained through the loss of access to higher quality inputs. In the right panel of Figure (9), we show the dynamic losses for each region.

In India, static welfare losses amount to 1 – 2% while dynamic losses range from 7 – 10%, depending on the decoupling scenario. Static losses to real income are small because India is a relatively large country and its domestic trade share in the market equilibrium is large, which limits the range of goods affects by changes in terms of trade. However, because it is relatively poor, its losses in the diffusion of ideas version of the model are very large. By severing ties with the Western bloc, it limits the role of trade-induced innovation, which is
a by-product of having access to high quality suppliers.

By contrast, in Russia including dynamics leads only to small additional effects: welfare losses are very similar with or without the ideas diffusion mechanism. As explained above, this stems both from a higher income starting point and relatively limited input-output linkages with the West.

5.2 Diffusion Inefficiencies a Multi-sector vs. a Single-sector Framework

In Section 3, we stressed that, except in knife-edge cases, within- and between-sector inefficiencies accumulate as the number of countries and sectors increase. The concavity of the diffusion process implies that total trade shares being at their optimal points is no longer sufficient for optimal diffusion. Optimal diffusion requires trade shares to be at their optimal points at every sector. This suggests that, in most cases, diffusion inefficiencies should increase with the number of sectors.

Our empirical results confirm that theoretical intuition. Figure 11 contrasts the results of either the full decouple or the tariff decouple scenarios under the baseline specification presented in the previous section and an alternative simulation in which we collapse the model to a single-sector framework.

In both scenarios, every country whose firms lose access to the most productive suppliers face higher cumulative diffusion inefficiencies (as measured by the reduction in the Fréchet parameters $\lambda_{d,t}^i$) in a multi-sector framework after trade costs go up. In fact, the single
sector dynamic productivity losses is outside the min-max range of the sectoral productivity changes for all countries in the Eastern bloc. These results underscore one important takeaway of this paper: modeling trade diffusion in a simplified single-sector framework can underestimate the level of dynamic losses induced by an increase in trade costs.

5.3 Consequences of Bloc Membership

In this section, we consider the consequences of moving one of the regions — Latin America and the Caribbean (LAC) — from the Western to the Eastern bloc. Intuitively, we expect that, by losing access to the highest productivity suppliers, LAC will experience less productivity growth. Nonetheless, the quantitative exercise allows us to have a sense of the magnitude induced by the change in group membership.

Figure 12 compares the results of identical decoupling scenarios, simulating either full decouple or tariff decouple. The only difference is LAC bloc membership. As expected, most of the changes are concentrated in the LAC region. The left panel of Figure 12 shows that welfare losses in LAC are about 100 – 150% larger when it is included in the Eastern bloc, for both scenarios. The domestic trade share in LAC is virtually identical under both settings (with LAC in the Western or in the Eastern bloc), implying similar static welfare losses. This suggests that the increased losses from switching bloc stem almost entirely from dynamic losses.

Moving LAC to the Eastern bloc reduces the welfare losses of decoupling in India and China by about 2p.p. (16%) and 1p.p. (15%), respectively (results not reported). The reason is twofold. First, LAC has higher income than India and the Rest of the Eastern bloc. All else equal, on average, its inclusion in the bloc raises average productivity and decreases...
dynamic losses. Second, lower tariff or iceberg trade costs between the Eastern bloc and LAC induce lower static losses for those countries.

The right hand side panel of Figure 12 shows the differential productivity changes in the LAC region for different sectors. When LAC is included in the Western bloc, there are essentially no dynamic productivity losses in any sector: the evolution of the Fréchet Distribution scale parameter $\lambda_{d,t}^i$ in the LAC Region behaves very similarly to a scenario with no policy changes.

In contrast, all sectors have dynamic productivity losses weakly greater than 1% when we simulate decoupling with LAC as part of the Eastern bloc. There is large sectoral heterogeneity. Under full decoupling, productivity losses range from 1% in Electronic Equipment ($\text{elm}$) to 0.4% in Business Services ($\text{tas}$). These differences are induced by input-output linkages.

This experiment underscores that the costs of decoupling might be unbearably high for low and middle income countries who are excluded from the Western bloc. Many countries in Latin America and Africa benefit from increasingly large trade ties to China through both having larger market access and access to lower input costs. However, as the dynamic costs of severing ties with the West would be very high, political leaders in those countries might have an incentive to keep an equidistant relationship between East and West, by preserving both mid-term gains from the relationship with China and longer term dynamic gains from having access to Western supply chains.
5.4 Electronic Equipment Decoupling

Finally, we compare our baseline scenario of full decouple in all sectors with a full decouple restricted to the electronics equipment sector. In both scenarios, we assume that the ideas diffusion mechanism works as described by equation (24) and we set $\beta = 0.44$, according to the calibration described before.

Note that, due to the multi-sector structure of the model, an increase in iceberg trade costs in one particular sector potentially has an indirect effect in all sectors of the economy. The magnitude of such impact in a given sector can be split between a direct effect (proportional to input use from the elm sector as intermediates) and an indirect effect (proportional to the use of the elm sector in the production of intermediates inputs).

Results in Figure 13 show the productivity losses induced by policy changes represented by the evolution of the Fréchet Distribution scale parameter $\lambda_{d,t}^i$ for those regions in the Eastern bloc. Contrasting the full decoupling in all sectors and one restricted to electronic equipment shows that, across all regions, productivity losses are substantially reduced and
mostly restricted to the elm sector.

While there is some negative spillover effect to other sectors due to input-output linkages, particularly to business services (tas), these are very small for most regions. Regions such as Russia, which already had limited exposure to Western intermediate sourcing in the main scenario, see productivity losses go down to nearly zero across all sectors under the scenario that limits decoupling to the elm sector. China’s losses in the elm sector are roughly similar to losses when decoupling happens in all products, but other sectors are not substantially affected.

All other regions have non-negligible losses in the elm sector. The largest changes happen for India and the Rest of the Eastern bloc. Those regions have a lower productivity starting point and benefit proportionately more from exposure to higher quality intermediate inputs. For that reason, full decoupling in all products leads to large differential losses in productivity in those regions. The more restricted full decoupling in elm scenario limits losses, since those are proportional to the use of Western electronic equipment as inputs in the production of other sectors.

![Figure 14: Cumulative Percentage Change in Welfare (Real Income), by scenario.](image)

Table 4.4. Country codes: chn, China; ind, India; rus, Russia; rwc, Rest of Eastern bloc; rwu, Rest of Western bloc; lac, Latin America; e27, European Union; ode, Other Developed; usa, United States. Tables with the values for these charts can be found in the Appendix.

Changes in productivity map into changes in welfare, pictured in Figure 14. While welfare losses are substantial, ranging from 0.4 – 1.9%, they are very different in magnitude to the devastating results of a full decoupling in all products, in which losses range between 8 – 12%.

These results underline two important observations. First, the costs of sector-specific decoupling might be limited enough for this scenario to be feasible. Second, input-output structures play an important role in magnifying dynamic losses. Limiting decoupling to one specific sector tapers down indirect magnification effects that happen through the input-output network.
6 Conclusion

We build a multi-sector multi-region general equilibrium model with dynamic sector-specific knowledge diffusion in order to realistically investigate the impact of large and persistent geopolitical conflicts on global trade patterns, economic growth, and innovation. Canonical trade models typically start from a fixed technology assumption, which thus misses a crucial source of gains from trade through the diffusion of ideas.

In our theoretical contribution, we show that large trade costs can lead to dynamic inefficiencies in sectoral knowledge diffusion. Furthermore, we show that in a multi-sector framework deviations from optimal knowledge diffusion happen both within- and between-sectors. Additionally, sectoral deviations accumulate, such that total trade shares being close to their aggregate optimal diffusion points is no longer sufficient to guarantee optimal diffusion. A takeaway of our theoretical discussion is that, as the number of sectors increases, so do the number of deviations from optimality and diffusion losses tend to be higher with multiple sectors.

We then use this toolkit to simulate the trade, innovation, and welfare effects of potential receding globalization characterized by economic decouple between the East and West, yielding three main insights. First, the projected welfare losses for the global economy of a decoupling scenario can be drastic, being as large as 12% in some regions, and are largest in the lower income regions as they would benefit less from technology spillovers from richer areas. Second, the described size and pattern of welfare effects are specific to the model with diffusion of ideas. Without diffusion of ideas the size and variation across regions of the welfare losses would be substantially smaller. Third, a multi-sector framework exacerbates diffusion inefficiencies induced by trade costs relative to a single-sector one.

This has important implications for the role of the multilateral trading system. First, the current system with global trade rules guaranteeing open and free trade between all major players is especially important for the lowest income regions. Second, if geopolitical considerations would lead to a split of the big players into two blocs, it would be important that an institutional framework remains in place for smaller countries to keep open trade relations with both blocs, in particular for the lowest income regions.

The toolkit we have built is versatile and can be employed for many other research questions, in particular focused on the analysis of policy. Future research could be extended in various directions. We mention two. First, there is ample empirical evidence for spillover effects from FDI, so the model could be extended with FDI and sales by multinational affiliates. Second, the framework with both technology spillovers and profits can be employed to analyze the economic effects of subsidy policies in different regions, an important policy topic in the multilateral trading system.

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## Additional tables calibration exercise

Table A1: Growth Rate of Real GDP and Real GDP per Capita, respectively, Historically and in Simulations, using different values of $\beta$

| $\beta$ | Mean | St.Dev. | max | min |
|---------|------|---------|-----|-----|
| GDP     |      |         |     |     |
| Historical | 3.60 | 2.66    | 8.90| 0.67|
| 0.40    | 3.09 | 2.02    | 6.26| 0.41|
| 0.41    | 3.21 | 2.13    | 6.58| 0.43|
| 0.42    | 3.34 | 2.26    | 6.94| 0.44|
| 0.43    | 3.48 | 2.40    | 7.34| 0.46|
| 0.44    | 3.64 | 2.56    | 7.78| 0.48|
| 0.45    | 3.82 | 2.73    | 8.26| 0.50|
| 0.46    | 4.02 | 2.92    | 8.79| 0.53|
| 0.47    | 4.24 | 3.13    | 9.36| 0.56|
| 0.48    | 4.48 | 3.36    | 9.97| 0.59|
| 0.49    | 4.75 | 3.60    | 10.62| 0.63|
| 0.50    | 5.04 | 3.87    | 11.32| 0.68|
| GDP per capita |      |         |     |     |
| Historical | 2.70 | 2.51    | 8.36| 0.75|
| 0.40    | 2.20 | 1.65    | 4.91| 0.47|
| 0.41    | 2.32 | 1.76    | 5.23| 0.48|
| 0.42    | 2.44 | 1.89    | 5.59| 0.50|
| 0.43    | 2.59 | 2.03    | 5.98| 0.51|
| 0.44    | 2.75 | 2.19    | 6.41| 0.53|
| 0.45    | 2.92 | 2.36    | 6.89| 0.54|
| 0.46    | 3.12 | 2.55    | 7.41| 0.57|
| 0.47    | 3.34 | 2.76    | 7.97| 0.59|
| 0.48    | 3.58 | 2.98    | 8.57| 0.62|
| 0.49    | 3.84 | 3.22    | 9.22| 0.65|
| 0.50    | 4.13 | 3.48    | 9.91| 0.69|
Table A2: The squared difference between the sum of the historical and simulated mean and standard deviation of GDP, GDP per capita and their sum

| $\beta$ | GDP | GDP pc | Sum  |
|---------|-----|--------|------|
| 0.40    | 0.67| 1.00   | 1.67 |
| 0.41    | 0.43| 0.72   | 1.15 |
| 0.42    | 0.23| 0.46   | 0.69 |
| 0.43    | 0.08| 0.25   | 0.33 |
| 0.44    | 0.01| 0.11   | 0.12 |
| 0.45    | 0.06| 0.07   | 0.13 |
| 0.46    | 0.25| 0.17   | 0.42 |
| 0.47    | 0.64| 0.46   | 1.10 |
| 0.48    | 1.28| 0.98   | 2.26 |
| 0.49    | 2.22| 1.79   | 4.01 |
| 0.50    | 3.54| 2.96   | 6.50 |
Table A3: Growth Rate of Real GDP and Real GDP per Capita, respectively, Historically and in Simulations, using different values of $\beta$ between 0 and 0.6

| $\beta$ | Mean | St.Dev. | max | min |
|--------|------|---------|-----|-----|
| GDP    |      |         |     |     |
| Historical | 3.60 | 2.66    | 8.90| 0.67|
| 0      | 2.17 | 1.10    | 3.79| 0.32|
| 0.5    | 2.19 | 1.12    | 3.84| 0.32|
| 0.10   | 2.20 | 1.13    | 3.87| 0.32|
| 0.15   | 2.23 | 1.16    | 3.93| 0.32|
| 0.20   | 2.27 | 1.20    | 4.02| 0.33|
| 0.25   | 2.34 | 1.26    | 4.19| 0.33|
| 0.30   | 2.46 | 1.39    | 4.49| 0.35|
| 0.35   | 2.68 | 1.61    | 5.08| 0.37|
| 0.40   | 3.09 | 2.02    | 6.26| 0.41|
| 0.45   | 3.82 | 2.73    | 8.26| 0.50|
| 0.50   | 5.04 | 3.87    | 11.32| 0.68|
| 0.55   | 6.89 | 5.42    | 15.39| 1.01|
| 0.60   | 9.50 | 7.32    | 20.53| 1.66|

| GDP per capita |      |         |     |     |
|----------------|------|---------|-----|-----|
| Historical     | 2.70 | 2.51    | 8.36| 0.75|
| 0              | 1.29 | 0.72    | 2.27| 0.40|
| 0.5            | 1.31 | 0.75    | 2.33| 0.40|
| 0.10           | 1.32 | 0.76    | 2.36| 0.40|
| 0.15           | 1.35 | 0.78    | 2.41| 0.40|
| 0.20           | 1.39 | 0.82    | 2.53| 0.40|
| 0.25           | 1.46 | 0.89    | 2.74| 0.41|
| 0.30           | 1.58 | 1.01    | 3.10| 0.42|
| 0.35           | 1.80 | 1.24    | 3.75| 0.44|
| 0.40           | 2.20 | 1.65    | 4.91| 0.47|
| 0.45           | 2.92 | 2.36    | 6.89| 0.54|
| 0.50           | 4.13 | 3.48    | 9.91| 0.69|
| 0.55           | 5.96 | 5.02    | 13.92| 0.98|
| 0.60           | 8.54 | 6.88    | 18.97| 1.55|
Table A4: Growth Rate of Real GDP and Real GDP per Capita, respectively, Historically and in Simulations, using different values of $\beta$ between 0 and 0.6 with an Autonomous Technology Growth Rate of $\alpha = 2.36$.

| $\beta$ | Mean | St.Dev. | max | min |
|---------|------|---------|-----|-----|
| GDP     |      |         |     |     |
| Historical | 3.60 | 2.66    | 8.90 | 0.67 |
| 0       | 2.17 | 1.10    | 3.79 | 0.32 |
| 0.5     | 2.19 | 1.12    | 3.84 | 0.32 |
| 0.10    | 2.21 | 1.14    | 3.88 | 0.32 |
| 0.15    | 2.23 | 1.16    | 3.94 | 0.32 |
| 0.20    | 2.28 | 1.21    | 4.04 | 0.33 |
| 0.25    | 2.35 | 1.28    | 4.23 | 0.33 |
| 0.30    | 2.49 | 1.41    | 4.56 | 0.35 |
| 0.35    | 2.73 | 1.65    | 5.22 | 0.37 |
| 0.40    | 3.17 | 2.09    | 6.48 | 0.42 |
| 0.45    | 3.95 | 2.85    | 8.60 | 0.52 |
| 0.50    | 5.23 | 4.03    | 11.76| 0.71 |
| 0.55    | 7.16 | 5.62    | 15.92| 1.08 |
| 0.60    | 9.84 | 7.52    | 21.17| 1.77 |
| GDP per capita |      |         |     |     |
| Historical | 2.70 | 2.51    | 8.36 | 0.75 |
| 0       | 1.29 | 0.72    | 2.27 | 0.40 |
| 0.5     | 1.31 | 0.75    | 2.33 | 0.40 |
| 0.10    | 1.33 | 0.77    | 2.36 | 0.40 |
| 0.15    | 1.35 | 0.79    | 2.43 | 0.40 |
| 0.20    | 1.40 | 0.83    | 2.56 | 0.40 |
| 0.25    | 1.47 | 0.91    | 2.79 | 0.41 |
| 0.30    | 1.61 | 1.04    | 3.18 | 0.42 |
| 0.35    | 1.85 | 1.28    | 3.89 | 0.44 |
| 0.40    | 2.28 | 1.72    | 5.13 | 0.48 |
| 0.45    | 3.05 | 2.48    | 7.22 | 0.56 |
| 0.50    | 4.32 | 3.64    | 10.35| 0.72 |
| 0.55    | 6.22 | 5.21    | 14.45| 1.04 |
| 0.60    | 8.87 | 7.08    | 19.56| 1.66 |
B  Construction of Initial Sectoral Productivity Parameters $\lambda_{d,0}^i$

In our model, the location parameter of Fréchet distribution of a given industry-country $\lambda_{d,t}^i$ evolves endogenously according to a law of motion, as described by equation (24). To calibrate the model, we need initial values $(\lambda_{d,0}^i)_{d \in D, i \in I}$. We proxy for the initial values using labor productivity in different sectors for each aggregate region and industry in our sample in the base-year 2014.

We do so by combining two different databases: the World Input Output Database’s Social Economic Accounts (WIOD-SEA — http://wiod.org/database/seas16) and the World Bank’s Global Productivity Database (WB-GPD — https://www.worldbank.org/en/research/publication/global-productivity). WIOD-SEA reports value added in local currency and employed population for 42 countries and 56 industries. WB-GPD reports value added in local currency and employed population for 103 countries and 9 industries. For countries whose data are available in both databases, we use the data from WIOD-SEA, which is more granular.

The first step is to create a cross-walk between WIOD-SEA industries and the more aggregate sectors in our model, namely: $elm$, Electronic Equipment; $hmn$, Heavy manufacturing; $lmn$, Light manufacturing; $ots$, Other Services; $pri$, Primary Sector; $tas$, Business services. We then convert value added in local currency to PPP-USD and market-rate USD using a panel of PPP and market exchange rates from the World Bank’s World Development Indicators.

Afterwards, we did a similar cross-walk for the country-sector pairs in the WB-GPD database. The detailed cross-walk can be found at the end of this Appendix. However, WB-GPD only reports one aggregate manufacturing sector, while our model disaggregates manufacturing into three subsectors. In order to make them compatible, we take the following steps: (a) we classify countries as advanced and emerging markets in both the WIOD-SEA and the WB-GPD databases; (b) we calculate the average share of value added and employed workers in total manufacturing for each of the manufacturing subsectors ($elm$, $hmn$, and $lmn$) for emerging markets and advanced economies, respectively, in the WIOD-SEA database; and (c) we use those shares and reported value added and employed workers from the WB-GPD database in order to input, for each country, a disaggregation of total manufacturing into $elm$, $hmn$, and $lmn$. We then convert value added in local currency to PPP-USD and market-rate USD using a panel of PPP and market exchange rates from the World Bank’s World Development Indicators.

Finally, we collapse PPP-USD value added, market-rate USD value added, and number of workers for the regions of our model ($chn$, China; $ind$, India; $rus$, Russia; $rwc$, Rest of Eastern bloc; $rwu$, Rest of Western bloc; $lac$, Latin America; $e27$, European Union; $ode$, Other Developed; $usa$, United States), and calculate, for each region-industry pair, labor productivity as:

$$\lambda_{d,0}^i = \frac{PPPSVA_{d,0}^i}{L_{d,0}}$$

using PPP-USD value added per worker.
### Table A5: Cross Walk Between WIOD-SEA and Model

| WIOD-SEA Sector | Model Sector |
|-----------------|-------------|
| A01-03, B       | pri         |
| C10-19, C31-32  | lmn         |
| C20-25, C28-30  | lmn         |
| C26-27          | elm         |
| C33, D35, E36, F, G45-47, H50-53, I, L68, N, O84, P85, Q, R, S, T, U J58, J61, K64-66, M71-73 | ots, tas |

### Table A6: Cross Walk Between WB-GPD and Model

| WB-GPD Sector                  | Model Sector |
|--------------------------------|--------------|
| 1. Agriculture                 | pri          |
| 2. Mining                      | pri          |
| 3. Manufacturing               | (see methodology) |
| 4. Utilities                   | ots          |
| 5. Construction                | ots          |
| 6. Trade services              | tas          |
| 7. Transport services          | ots          |
| 8. Finance and business services | tas         |
| 9. Other services              | ots          |

- **Countries in WIOD-SEA**: Austria, Belgium, Brazil, Bulgaria, Canada, China, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Korea, Rep., Latvia, Lithuania, Luxembourg, Malta, Mexico, Netherlands, Norway, Poland, Portugal, Romania, Russian Federation, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States.

- **Countries in WB-GPD**: Angola, Argentina, Australia, Austria, Azerbaijan, Belgium, Burkina Faso, Bangladesh, Bulgaria, Belize, Bolivia, Brazil, Botswana, Canada, Switzerland, Chile, China, Cameroon, Colombia, Costa Rica, Cyprus, Czech Republic, Germany, Denmark, Dominican Republic, Algeria, Ecuador, Egypt, Spain, Estonia, Ethiopia, Finland, Fiji, France, United Kingdom, Georgia, Ghana, Greece, Guatemala, China, Hong Kong SAR, Honduras, Croatia, Hungary, Indonesia, India, Ireland, Iran, Iceland, Italy, Jamaica, Jordan, Japan, Kenya, Republic of Korea, Lao People’s Dem Rep, Saint Lucia, Sri Lanka, Lesotho, Lithuania, Luxembourg, Latvia, Morocco, Mexico, Montenegro, Mongolia, Mozambique, Mauritius, Malawi, Malaysia, Namibia, Nigeria, Netherlands, Norway, Nepal, New Zealand, Pakistan, Philippines, Poland, Portugal, Paraguay, Qatar, Romania, Russian Federation, Rwanda, Senegal, Singapore, Sierra Leone, Serbia, Slovakia, Slovenia, Sweden, Eswatini, Thailand, Turkey, Chinese Taipei, United Republic of Tanzania, Uganda, Uruguay, United States, St. Vincent and the Grenadines, Viet Nam, South Africa, Zambia.
C Mathematical Derivation of Dynamic Innovation

C.1 Evolution of the Productivity Frontier

In this section, we largely follow the steps of the mathematical appendix to Buera and Oberfield (2020) to the particularities of our model. In the proofs, for simplicity, we initially abstract away from sectoral specific elasticities, using $\theta$ rather than $\theta_i$, but at the end generalize the results to accommodate them. For any period, domestic technological frontier evolves according to:

$$F_{i,d,t+\Delta}(z) = F_{i,d,t}(z) \times \left(1 - \int_t^{t+\Delta} \alpha_t z^{-\theta}(z')^\beta dG_{d,t}(z')d\tau\right)$$

Rearranging and using the definition of the derivative:

$$\frac{d}{dt} \ln F_{i,s,t}(z) = \lim_{\Delta \to 0} \frac{F_{i,s,t+\Delta}(z) - F_{i,s,t}(z)}{F_{i,s,t}(z)} = -\int \alpha_t z^{-\theta}(z')^\beta dG_{d,t}(z')$$

Define $\lambda_{i,s,t} = \int_{-\infty}^t \alpha_t \int (z')^\beta dG_{s,t}(z')d\tau$ and integrate both sides wrt to time:

$$\int_0^t \frac{d}{d\tau} \ln F_{i,s,\tau}(z)d\tau = -z^{-\theta} \int_0^t \alpha_t (z')^\beta dG_{d,\tau}(z')d\tau$$

$\ln \left(\frac{F_{i,s,\tau}(z)}{F_{i,s,0}(z)}\right) = -z^{-\theta}(\lambda_{i,s,t} - \lambda_{i,s,0})$

$$F_{i,s,t}(z) = F_{i,s,0}(z) \exp\left(-z^{-\theta}(\lambda_{i,s,t} - \lambda_{i,s,0})\right)$$

Assuming that the initial distribution is Fréchet $F_{i,s,0}(z) = \exp\{-\lambda_{i,s,0}z^{-\theta}\}$ guarantees that the distribution will be Fréchet at any point in time:

$$F_{i,s,t}(z) = \exp\{-\lambda_{i,s,t}z^{-\theta}\} \quad (A-1)$$

C.2 Law of Motion of Productivity

As seen above, we have defined:

$$\lambda_{i,s,t} = \int_{-\infty}^t \alpha_t \int (z')^\beta dG_{s,\tau}(z')d\tau$$

Differentiating this definition with respect to time and applying Leibnitz’s Lemma yields:

$$\dot{\lambda}_{i,s,t} = \alpha_t \int (z')^\beta dG_{s,t}(z')$$

We use these results and work with a discrete approximation of the law of motion for productivity:

$$\Delta \lambda_{i,s,t} = \alpha_t \int (z')^\beta dG_{s,t}(z') \quad (A-2)$$
The source distribution \( G_{d,t}^i(z') = \sum_{j \in \mathcal{J}} \eta_{d,t}^{i,j} \sum_{s \in \mathcal{D}} H_{sd,t-1}^{i,j}(z') \), where \( \eta_{d,t}^{i,j} \) is the expenditure share of sector \( j \) in the cost of intermediates when producing good \( i \) in region \( d \); and \( H_{sd,t-1}^{i,j}(z') \) is the fraction of commodities for which the lowest cost supplier in period \( t-1 \) is a firm located in \( s \in \mathcal{D} \) with productivity weakly less than \( z' \).

We focus our attention on the integral \( \int z^\theta dH_{sd,t}^{i,j}(z) \). Let \( F_{s,t}^{i,j}(z_1, z_2) = \exp\{-\lambda_{s,t}^{i,j} z_2^{-\theta} \} \) and \( F_{s,t}^{i,j}(z_1, z_2) = (1 + \lambda_{s,t}^{i,j} [z_2^{\theta} - z_1^{\theta}]) \exp\{-\lambda_{s,t}^{i,j} z_2^{\theta} \} \) are, respectively, the probability that a productivity draw is below \( z_2 \), and that the maximum productivity is \( z_1 \) and the second highest productivity is \( z_2 \).

Let for each \( n, A_{n,t} = \bar{x}_{n,d,t}/\bar{x}_{s,t} \), such that \( s \) will have a lower cost than \( d \) iff \( A_{n,t}^{d} z_{n,t}^{i} (\omega) < z_{s,t}^{i} (\omega) \). Region \( s \) with highest productivity producers \( z_1, z_2 \) will supply the commodity \( i \in \mathcal{I} \) in region \( d \) with the following probability:

\[
F_{sd,t-1}^{i,j}(z_1, z_2) = \int_0^{z_2} \Pi_{n \neq s} F_{n,t-1}^{i,j}(A_{n,t}^{i} y, A_{n,t}^{i-1} y) dF_{s,t-1}^{j}(y, y) + \int_{z_2}^{z_1} \Pi_{n \neq s} F_{n,t-1}^{i,j}(A_{n,t}^{i} z_2, A_{n,t}^{i} z_2) \frac{d}{dz_1} F_{s,t-1}^{i,j}(z_1, z_2)
\]

The first term in the right hand side denotes the probability that the lowest cost producer at destination \( d \) is from \( s \) and has productivity lower than \( z_2 \), while the second term accounts for the probability that the lowest cost producer at destination \( d \) is from \( s \) and has productivity in the range \([z_2, z_1] \). We will evaluate each integral separately. First, take the first term:

\[
\int_0^{z_2} \Pi_{n \neq s} F_{n,t-1}^{i,j}(A_{n,t-1}^{i} y, A_{n,t-1}^{i-1} y) dF_{s,t-1}^{j}(y, y)
\]

\[
= \int_0^{z_2} \exp \left\{ - \sum_{n \neq s} \lambda_{n,t-1}^{i,j} (A_{n,t-1}^{i} y)^{-\theta} \right\} \theta \lambda_{s,t-1}^{i,j} y^{-\theta-1} \exp\{-\lambda_{s,t-1}^{i,j} y^{-\theta}\} dy
\]

\[
= \lambda_{s,t-1}^{i,j} \int_0^{z_2} \theta y^{-\theta-1} \exp \left\{ - \sum_{n} \lambda_{n,t-1}^{i,j} (A_{n,t-1}^{i})^{-\theta} y^{-\theta} \right\} dy
\]

\[
= \lambda_{s,t-1}^{i,j} \left[ \frac{1}{\sum_{n} \lambda_{n,t-1}^{i,j} (A_{n,t-1}^{i})^{-\theta}} \exp \left\{ - \sum_{n} \lambda_{n,t-1}^{i,j} (A_{n,t-1}^{i})^{-\theta} y^{-\theta} \right\} \right]_{y=0}^{y=z_2}
\]

\[
= \pi_{s,t-1}^{i,j} \exp \left\{ - \frac{\lambda_{s,t-1}^{i,j} z_2^{-\theta}}{\pi_{s,t-1}^{i,j}} \right\}
\]

Now consider the second term.

To see the latter, note that:

\[
Prob(z_1 \leq Z_1, z_2 \leq Z_2) = F_{s,t}^{i,j}(Z_2) + \int_0^{Z_1} \int_{Z_2}^{Z_2} f_{s,t}^{i}(y) f_{s,t}^{i}(y') dy' dy
\]

\[
= F_{s,t}^{i,j}(Z_2) + F_{s,t}^{i,j}(Z_2) (F_{s,t}^{i,j}(Z_1) - F_{s,t}^{i,j}(Z_2))
\]

\[
= (1 + \lambda_{s,t}^{i,j} [Z_2^{\theta} - Z_1^{\theta}]) \exp\{-\lambda_{s,t}^{i,j} Z_2^{\theta}\}
\]
\[
\int_{z_2}^{z_1} \Pi_{n \neq s} F_{n,t-1}^{-j} \left( A_{n,t}^i z_2, A_{n,t}^i z_2 \right) \frac{d}{dz_1} F_{s,t-1}^{j}(z_1, z_2) \\
= \int_{z_2}^{z_1} \exp \left\{- \sum_{n \neq s} \lambda_{n,t-1}^i \left( A_{n,t}^i z_2 \right)^{-\theta} \right\} \theta \lambda_{s,t}^{m,j} z_1^{-\theta-1} \exp \left\{ -\lambda_{s,t-1}^{i,j} z_2^{-\theta} \right\} dz_1 \\
= \exp \left\{- \sum_n \lambda_{n,t-1}^j \left( A_{n,t}^i z_2 \right)^{-\theta} \right\} \lambda_{s,t-1}^j \int_{z_2}^{z_1} \theta z_1^{-\theta-1} dz_1 \\
= \exp \left\{- \frac{\lambda_{s,t-1}^j}{\pi_{s,t-1}^{i,j}} z_2^{-\theta} \right\} \lambda_{s,t-1}^j (z_2^{-\theta} - z_1^{-\theta})
\]

Therefore:

\[
F_{s,t-1}^{i,j}(z_1, z_2) = \exp \left\{- \frac{\lambda_{s,t-1}^j}{\pi_{s,t-1}^{i,j}} z_2^{-\theta} \right\} \left( \frac{\pi_{s,t-1}^{i,j}}{\pi_{s,t-1}^{i,j}} + \lambda_{s,t-1}^j (z_2^{-\theta} - z_1^{-\theta}) \right) \tag{A-3}
\]

Note that:

\[
\int z^{\theta \theta} dH_{s,t}^{i,j}(z) = \int_{z_2}^{z_1} \int_{z_2}^{z_1} z_1^{\theta} \frac{\partial^2 F_{s,t-1}^{i,j}(z_1, z_2)}{\partial z_1 \partial z_2} d\gamma d\gamma \tag{A-4}
\]

and that we can calculate the joint density explicitly:

\[
\frac{\partial^2 F_{s,t-1}^{i,j}(z_1, z_2)}{\partial z_1 \partial z_2} = \frac{\partial}{\partial z_2} \exp \left\{- \frac{\lambda_{s,t-1}^j}{\pi_{s,t-1}^{i,j}} z_2^{-\theta} \right\} \theta \lambda_{s,t-1}^j z_1^{-\theta-1} \\
= \frac{1}{\pi_{s,t-1}^{i,j}} \exp \left\{- \frac{\lambda_{s,t-1}^j}{\pi_{s,t-1}^{i,j}} z_2^{-\theta} \right\} (\theta \lambda_{s,t-1}^j z_1^{-\theta-1})(\theta \lambda_{n,t-1}^j z_2^{-\theta-1})
\]

Plugging this into (A-4):

\[
\int_0^\infty \int_0^\infty \frac{1}{\pi_{s,t-1}^{i,j}} \exp \left\{- \frac{\lambda_{s,t-1}^j}{\pi_{s,t-1}^{i,j}} z_2^{-\theta} \right\} (\theta \lambda_{s,t-1}^j z_2^{-\theta-1})(\theta \lambda_{n,t-1}^j z_2^{-\theta-1}) d\gamma d\gamma \\
= \int_0^\infty \exp \left\{- \frac{\lambda_{s,t-1}^j}{\pi_{s,t-1}^{i,j}} z_2^{-\theta} \right\} (\theta \lambda_{s,t-1}^j z_2^{-\theta-1}) \lambda_{s,t-1}^j \int_0^\infty (\theta \lambda_{s,t-1}^j z_2^{-\theta-1}) d\gamma d\gamma \\
= \int_0^\infty \exp \left\{- \frac{\lambda_{s,t-1}^j}{\pi_{s,t-1}^{i,j}} z_2^{-\theta} \right\} (\theta \lambda_{s,t-1}^j z_2^{-\theta-1}) \lambda_{s,t-1}^j \frac{1}{1 - \beta} z_2^{-\theta(1-\beta)} d\gamma d\gamma
\]

Using a change of variables, let \( \gamma = \frac{\lambda_{s,t-1}^j}{\pi_{s,t-1}^{i,j}} z_2^{-\theta} \), which implies that \( d\gamma = -\theta \frac{\lambda_{s,t-1}^j}{\pi_{s,t-1}^{i,j}} z_2^{-\theta-1} d\gamma \)

Replacing above:
\[
(\lambda_{s,t-1}^j)^{\beta}(\pi_{sd,t-1}^{i,j})^{1-\beta} \left[ \frac{1}{1-\beta} \int_0^\infty \exp \{-\gamma\} \eta^{(1-\beta)}d\gamma \right] \\
= (\lambda_{s,t-1}^j)^{\beta}(\pi_{sd,t-1}^{i,j})^{1-\beta} \frac{1}{1-\beta} \Gamma(2-\beta) \\
= (\lambda_{s,t-1}^j)^{\beta}(\pi_{sd,t-1}^{i,j})^{1-\beta} \Gamma(1-\beta) \\
\quad \text{\small (since } \Gamma(y + 1) = y\Gamma(y)\text{)}
\]

Therefore, replacing into the law of motion for the location parameter of the Fréchet distribution:

\[
\Delta \lambda_{d,t}^i = \alpha_t \int z^{\theta} dG_{d,t}^i(z) \\
= \alpha_t \sum_{j \in I} \eta_{d,t-1}^{i,j} \sum_{s \in D} z^{\theta} dH_{sd,t-1}^{i,j}(z) \\
= \alpha_t \Gamma(1-\beta) \sum_{j \in I} \eta_{d,t-1}^{i,j} \sum_{s \in D} (\lambda_{s,t-1}^j)^{\beta}(\pi_{sd,t-1}^{i,j})^{1-\beta}
\]

which is the same expression as in equation (24).
D Optimal Diffusion Levels

D.1 Two-by-Two Economy

If a Benevolent Planner were to chose domestic trade shares to maximize idea diffusion to a given sector at the home economy, she would solve the following concave programming problem:

\[
\max_{\pi_h^{i,1}, \pi_h^{i,-1}} \eta^i [\pi_h^{i,1} (\lambda_h^i)^\beta + (1 - \pi_h^{i,1}) (\lambda_f^i)^\beta] + (1 - \eta^i_h) [\pi_h^{i,-1} (\lambda_h^{-1})^\beta + (1 - \pi_h^{i,-1}) (\lambda_f^{-1})^\beta]
\]  

(A-5)

For \(\pi_h^{i,i}\), the first order condition satisfies:

\[
\eta^i (1 - \beta) [\pi_h^{i,1} (\lambda_h^i)^\beta - (1 - \pi_h^{i,1}) (\lambda_f^i)^\beta] = 0
\]

\[
(\pi_h^{i,1})^{\beta} (\lambda_h^i)^\beta = (1 - \pi_h^{i,1})^{-\beta} (\lambda_f^i)^\beta
\]

\[
(\pi_h^{i,i}) \text{ Diffusion Optimum } = \frac{\lambda_h^i}{\lambda_f^i + \lambda_h^i}
\]

This result is the building block of the ratios that we express in Section 3. If we want to calculate the within sector ratio of total domestic trade expenditure, we can write:

\[
\left(\frac{\eta^i \pi_h^{i,i}}{\eta^i (1 - \pi_h^{i,1})}\right) \text{ Diffusion Optimum } = \frac{\lambda_h^i}{\lambda_f^i + \lambda_h^i} \times \left(\frac{\lambda_f^i}{\lambda_f^i + \lambda_h^i}\right)^{-1} = \frac{\lambda_h^i}{\lambda_f^i + \lambda_h^i}
\]

Similarly, if we want to write a cross-sector ratio of total domestic trade expenditure shares, we can write:

\[
\left(\frac{\eta^i \pi_h^{i,i}}{(1 - \eta^i) \pi_h^{i,-1}}\right) \text{ Diffusion Optimum } = \frac{\eta^i}{1 - \eta^i} \times \frac{\lambda_h^i}{\lambda_h^i} \times \left(\frac{\lambda_f^i}{\lambda_f^i + \lambda_h^i}\right)^{-1}
\]

which is the same as equation (25).

D.2 Multi-Sector, Multi-Region Economy

For each commodity \(i\) in macrosector \(j\), the Benevolent Planner maximizes:

\[
\max_{\{\pi_{sd,t-1}^{i,j}\}_{s \in D, t \in T}} \sum_{j \in J} \eta_{d,j-1}^{i,j} \sum_{s \in D} (\pi_{sd,t-1}^{i,j})^{1-\beta} (\lambda_{s,t-1}^j)^\beta
\]

(A-6)

s.t. \(\forall (i, j) \in I \times I\) \(\sum_{s \in D} \pi_{sd,t-1}^{i,j} = 1\)

Let \(\varphi\) be the Lagrange multiplier. Then, for each \((s, i, j)\) first order conditions satisfy:

\[
(1 - \beta) \eta_{d,t-1}^{i,j} (\pi_{sd,t-1}^{i,j})^{\beta} (\lambda_{s,t-1}^j)^\beta = \varphi
\]

\[
(\pi_{sd,t-1}^{i,j}) \text{ Diffusion Optimum } = \varphi^{-\frac{1}{\beta}} [(1 - \beta) \eta_{d,t-1}^{i,j}]^{\frac{1}{\beta}} \lambda_{s,t-1}^j
\]
using the constraint:

$$\sum_{s \in D} (\varphi^{-\frac{1}{2}}[(1 - \beta)\eta_{d,t-1}^{i,j}]^\frac{1}{2} \lambda_{s,t-1}^j) = 1 \iff \varphi^{-\frac{1}{2}} = [(1 - \beta)\eta_{d,t-1}^{i,j}]^{-\frac{1}{2}} (\sum_{s \in D} \lambda_{s,t-1}^j)^{-1}$$

Therefore:

$$\left(\frac{\pi_{sd,t-1}^i}{\pi_{pd,t-1}^i}\right)^{\text{Diffusion Optimum}} = \frac{\lambda_{s,t-1}^j}{\sum_{k \in D} \lambda_{k,t-1}^{m,j}}$$ (A-7)

If we want to calculate the within sector ratio of total domestic trade expenditure, we can write:

$$\left(\frac{\eta_{d,t-1}^{i,j}}{\eta_{d,t-1}^{i,p}}\right)^{\text{Diffusion Optimum}} = \frac{\lambda_{s,t-1}^j}{\sum_{k \in D} \lambda_{k,t-1}^{m,j}} \times \left(\frac{\lambda_{n,t-1}^j}{\sum_{k \in D} \lambda_{k,t-1}^{m,j}}\right)^{-1} = \frac{\lambda_{s,t-1}^j}{\lambda_{n,t-1}^j}$$

Similarly, if we want to write a cross-sector ratio of total domestic trade expenditure shares, we can write:

$$\left(\frac{\eta_{d,t-1}^{i,j}}{\eta_{d,t-1}^{i,p}}\right)^{\text{Diffusion Optimum}} = \frac{\lambda_{s,t-1}^j}{\sum_{k \in D} \lambda_{k,t-1}^{m,j}} \times \left(\sum_{k \in D} \lambda_{k,t-1}^{j,k} \lambda_{s,t-1}^j (\tilde{x}_{j,k,t}^{d,t-1}) - \theta \sum_{k \in D} \lambda_{k,t-1}^{j,k} (\tilde{x}_{k,t}^{d,t-1}) - \theta\right)^{-1}$$

which is analogous to equation 25. The actual trade allocation under the multi-country, multi-sector framework satisfies:

$$\left(\frac{\eta_{d,t-1}^{i,j}}{\eta_{d,t-1}^{i,p}}\right)^{\text{Actual Trade}} = \frac{\lambda_{s,t-1}^j}{\sum_{k \in D} \lambda_{k,t-1}^{m,j}} \times \left(\sum_{k \in D} \lambda_{k,t-1}^{j,k} (\tilde{x}_{j,k,t}^{d,t-1}) - \theta \sum_{k \in D} \lambda_{k,t-1}^{j,k} (\tilde{x}_{k,t}^{d,t-1}) - \theta\right)^{-1}$$

which is analogous to equation 26. The ratio the actual trade allocation for the planner’s allocation satisfies:

$$N = \frac{(\tilde{x}_{sd,t-1}^{j})^{-\theta}}{(\tilde{x}_{nd,t-1}^{p})^{-\theta}} \times \left(\sum_{k \in D} \lambda_{k,t-1}^{j,k} (\tilde{x}_{k,t}^{d,t-1})^{-\theta}\right)^{-1} \times \left(\sum_{k \in D} \lambda_{k,t-1}^{j,k} (\tilde{x}_{k,t}^{d,t-1})^{-\theta}\right)^{-1}$$
E Other Mathematical Derivations

E.1 Trade shares

In this model, since there are infinitely many varieties in the unit interval, the expenditure share of destination region $d \in D$ on goods coming from source country $s \in D$ converge to their expected values. Let $\pi_{sd,t}^i$ denote the share of expenditures of consumers in region $d \in D$ on commodity $i \in I_m$ coming from region $s \in D$ and, let for each $n$, $A_{n,t}^i - 1 \equiv \tilde{x}_{sd,t}^i / \tilde{x}_{nd,t}^i$. This share will satisfy:

$$\pi_{sd,t}^i = \Pr \left( \frac{\tilde{x}_{sd,t}^i}{\tilde{x}_{s,t}^i(\omega)} < \min_{(n \neq s)} \left\{ \frac{\tilde{x}_{nd,t}^i}{\tilde{x}_{n,t}^i(\omega)} \right\} \right)$$

$$= \int_0^\infty \Pr (z_{s,t}^i(\omega) = z) \Pr (z_{n,t}^i(\omega) < zA_n^i) dz$$

$$= \int_0^\infty f_{s,t}(z) \Pi_{(n \neq s)} F_{n,t}(zA_n^i) dz$$

$$= \int_0^\infty \theta \lambda_{s,t}^i z^{-(1+\theta)} e^{-\left(\sum_{n \in D} \lambda_{n,t}^i A_n^i\right)-\theta} dz$$

$$= \frac{\lambda_{s,t}^i (\tilde{x}_{sd,t}^i)^{-\theta}}{\sum_{n \in D} \lambda_{n,t}^i (\tilde{x}_{nd,t}^i)^{-\theta}}$$

$$= \frac{\lambda_{s,t}^i (\tilde{x}_{sd,t}^i)^{-\theta}}{\Phi_{d,t}^i}$$

\hspace{1cm} (A-8)

Similarly, since countries use the same aggregate final goods as intermediate inputs, cost shares in intermediates for each supplying sector $j$ and region $s$ used in the production of good $i$ in region $d$ satisfies:

$$\pi_{sd,t}^{i,j} = \frac{\lambda_{s,j}^i (\tilde{x}_{sd,t}^j)^{-\theta}}{\Phi_{d,t}^j}$$

\hspace{1cm} (A-9)

which are the same as expressed in (19).

E.2 Price levels

Recall from equations (14) that the prices of commodities and intermediate goods can be expressed, respectively, as:

$$p_{d,t}^i = \left[ \int_{[0,1]} p_{d,t}^i(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

Let $\Omega_{sd,t}^i$ and $\Omega_{sd,t}^{i,j}$ denote the subsets of $\Omega = [0,1]$ for which the region $s \in D$ is a supplier in destination region $d \in D$. We can then rewrite price levels above as:

$$p_{d,t}^i = \left[ \sum_{s \in D} \int_{\Omega_{sd,t}^i} p_{d,t}^i(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$
Similarly, we restate \( F_{sd,t}(z_1, z_2) \) and the analogous measure \( F_{sd,t}^{i,j}(z_1, z_2) \):

\[
F_{sd,t}(z_1, z_2) = \exp \left\{ -\frac{\lambda_{s,t}^i}{\pi_{sd,t}^i} z_1^{-\theta} \right\} \left( \pi_{sd,t}^i + \lambda_{s,t}^i (z_2^{-\theta} - z_1^{-\theta}) \right) \tag{A-10}
\]

which denote the fraction of varieties that \( d \) purchases from \( s \) with productivity up to \( z_1 \) and whose second best producer is not more efficient than \( z_2 \). Recall that, from the Bertrand competition assumption, we can write, for each variety \( \omega \):

\[
p_{d,t}^i(\omega) = \min \left\{ \frac{\sigma}{\sigma - 1} \frac{\bar{x}_{sd,t}^i(z_1(\omega))}{\pi_{sd,t}^i}, \frac{\bar{x}_{sd,t}^i(z_2(\omega))}{\pi_{sd,t}^i} \right\}
\]

So we can rewrite the equation \( \int_{\Omega_{sd,t}} p_{d,t}^i(\omega)^{1-\sigma} d\omega \) in the following fashion:

\[
\int_{\Omega_{sd,t}} p_{d,t}^i(\omega)^{1-\sigma} d\omega = \int_0^\infty \int_{z_2}^\infty (p_{d,t}^i)^{1-\sigma} \frac{\partial^2 F_{sd,t}(z_1, z_2)}{\partial z_1 \partial z_2} dz_1 dz_2
\]

\[
= \int_0^\infty \int_{z_2}^\infty \min \left\{ \frac{\sigma}{\sigma - 1} \frac{\bar{x}_{sd,t}^i(z_1(\omega))}{z_1}, \frac{\bar{x}_{sd,t}^i(z_2(\omega))}{z_2} \right\} \frac{1}{\pi_{sd,t}^i} \exp \left\{ -\frac{\lambda_{s,t}^i}{\pi_{sd,t}^i} z_2^{-\theta} \right\} (\theta \lambda_{s,t}^i z_1^{-\theta-1})(\theta \lambda_{s,t}^i z_2^{-\theta-1}) dz_1 dz_2
\]

With a change of variables, denote \( \eta_1 \equiv \frac{\lambda_{s,t}^i}{\pi_{sd,t}^i} z_1^{-\theta} \) and \( \eta_2 \equiv \frac{\lambda_{s,t}^i}{\pi_{sd,t}^i} z_2^{-\theta} \) and \( d\eta_1 = -\frac{\theta \lambda_{s,t}^i z_1^{-\theta-1}}{\pi_{sd,t}^i} dz_1 \),

\[
d\eta_2 = -\frac{\theta \lambda_{s,t}^i z_2^{-\theta-1}}{\pi_{sd,t}^i} dz_2,
\]

which allows us to rewrite the equation above as:
\[
\int_{\Omega_{s,t}} p_{d,t}^i(\omega)^{1-\sigma} d\omega \\
= \pi_{s,t}^{i} \int_{0}^{\infty} \int_{0}^{\sigma^2} \min \left\{ \frac{\sigma}{\sigma - 1} \frac{x^{i}_{s,t}}{z_1} \frac{x^{i}_{s,t}}{z_2} \right\}^{1-\sigma} \exp \left\{- \eta_2 \right\} d\eta_1 d\eta_2 \\
= \pi_{s,t}^{i} \frac{\lambda_{s,t}^i}{\pi_{s,t}^{i}} \frac{1}{\pi^i} (x^{i}_{s,t})^{1-\sigma} \int_{0}^{\infty} \int_{0}^{\sigma^2} \min \left\{ \frac{\sigma}{\sigma - 1} \eta_1, \eta_2 \right\}^{1-\sigma} \exp \left\{- \eta_2 \right\} d\eta_1 d\eta_2 \\
= \pi_{s,t}^{i} \frac{\lambda_{s,t}^i}{\pi_{s,t}^{i}} \frac{1}{\pi^i} (x^{i}_{s,t})^{1-\sigma} \left[ \int_{0}^{\infty} \int_{\frac{\sigma^2}{\eta_1 + \theta}}^{\infty} \eta_2^{1-\sigma} \exp \left\{- \eta_2 \right\} d\eta_1 d\eta_2 \right] \\
+ \int_{0}^{\infty} \int_{0}^{\sigma^2} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \eta_1^{1-\sigma} \exp \left\{- \eta_2 \right\} d\eta_1 d\eta_2 \\
= \pi_{s,t}^{i} \frac{\lambda_{s,t}^i}{\pi_{s,t}^{i}} \frac{1}{\pi^i} (x^{i}_{s,t})^{1-\sigma} \left[ 1 - \left( \frac{\sigma - 1}{\sigma} \right)^{-\theta} + \frac{\theta}{1 - \sigma + \theta \left( \frac{\sigma}{\sigma - 1} \right)^{-\theta}} \cdot \int_{0}^{\infty} \eta_2^{1-\sigma} d\eta_2 \right] \\
= \pi_{s,t}^{i} \frac{\lambda_{s,t}^i}{\pi_{s,t}^{i}} \frac{1}{\pi^i} (x^{i}_{s,t})^{1-\sigma} \left[ 1 - \left( \frac{\sigma - 1}{\sigma} \right)^{-\theta} + \frac{\theta}{1 - \sigma + \theta \left( \frac{\sigma}{\sigma - 1} \right)^{-\theta}} \right] \pi_{s,t}^{i} \frac{1}{\pi^i} (x^{i}_{s,t})^{1-\sigma} \\
= \pi_{s,t}^{i} \frac{\lambda_{s,t}^i}{\pi_{s,t}^{i}} \frac{1}{\pi^i} (x^{i}_{s,t})^{1-\sigma} \left[ 1 - \left( \frac{\sigma - 1}{\sigma} \right)^{-\theta} + \frac{\theta}{1 - \sigma + \theta \left( \frac{\sigma}{\sigma - 1} \right)^{-\theta}} \right] \pi_{s,t}^{i} \frac{1}{\pi^i} (x^{i}_{s,t})^{1-\sigma} \\
= \frac{1}{\theta} \left[ 1 - \frac{\sigma - 1}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{-\theta} \right] \cdot \Gamma \left( \frac{1 - \sigma + \theta}{\theta} \right) \cdot \pi_{s,t}^{i} \left( \sum_{n \in D} \lambda_{n,t}^{i}(x^{i}_{n,t})^{-\theta} \right) \\
\]

Therefore:

\[
\begin{align*}
p_{d,t}^i &= \left[ \sum_{s \in D} \int_{\Omega_{s,t}} p_{d,t}^i(\omega)^{1-\sigma} d\omega \right]^{1-\sigma} \\
\tilde{p}_{d,t}^i &= \left[ 1 - \frac{\sigma - 1}{\theta} \frac{\sigma - 1}{\theta} \right]^{-\theta} \cdot \Gamma \left( \frac{1 - \sigma + \theta}{\theta} \right) \cdot \frac{1}{\theta} \left( \sum_{s \in D} \lambda_{n,t}^{i}(x^{i}_{n,t})^{-\theta} \right)^{-\theta} \cdot \left[ \sum_{s \in D} \pi_{s,t}^{i} \right]^{1-\sigma} \\
\tilde{p}_{d,t}^i &= \left[ 1 - \frac{\sigma - 1}{\theta} \frac{\sigma - 1}{\theta} \right]^{-\theta} \cdot \Gamma \left( \frac{1 - \sigma + \theta}{\theta} \right) \cdot \frac{1}{\theta} \left( \sum_{s \in D} \lambda_{n,t}^{i}(x^{i}_{n,t})^{-\theta} \right)^{-\theta} \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \}}
\[ q_{d,t}^i(\omega) = \left( \frac{p_{d,t}^i(\omega)}{p_{d,t}^i} \right) p_{d,t}^i(\omega)^{-\sigma} \epsilon_{d,t}^i = \left( \frac{p_{m,j}^m(\omega)}{p_{m,j}^d} \right) p_{m,j}^d(\omega)^{-\sigma} \epsilon_{d,t}^{i,j} \]  
\[ c_{d,t}^{i,j}(\omega) = \left( \frac{p_{d,t}^i(\omega)}{p_{d,t}^i} \right) p_{d,t}^i(\omega)^{-\sigma} \epsilon_{d,t}^{i,j} \]

where \( p_{d,t}^i \) satisfies equations (18); \( \epsilon_{d,t}^i \) denotes expenditure on commodity \( i \) of macro-sector \( m \) in country \( d \); and \( \epsilon_{d,t}^{i,j} \) denotes expenditure on intermediate input \( j \) used in the production of commodity \( i \) of macro-sector \( m \) in country \( d \).

As in previous subsections of the Appendix, we will derive the expression for the marginal cost and mark-up for the production of variety \( q_{d,t}^i(\omega) \) and state a corresponding expression for \( c_{d,t}^{i,j}(\omega) \). The marginal cost of producing variety \( \omega \) sourced in country \( s \) and consumed in country \( s \) is:

\[ \frac{\tilde{x}_{d,t}^i}{z_1(\omega)}q_{d,t}^i(\omega) \]

and total cost of varieties sourced in country \( s \) and consumed in country \( s \) can be expressed as:

\[ \int_{\Omega_{sd,t}} \frac{\tilde{x}_{d,t}^i}{z_1(\omega)}q_{d,t}^i(\omega) d\omega = \int_{\Omega_{sd,t}} \frac{\tilde{x}_{d,t}^i}{z_1(\omega)} \left( \frac{p_{d,t}^i(\omega)}{p_{d,t}^i} \right) p_{d,t}^i(\omega)^{-\sigma} \epsilon_{d,t}^i d\omega \]

As in the previous section of the Appendix, we let \( \Omega_{sd,t}^i \) and \( \Omega_{sd,t}^{i,j} \) denote the subsets of \( \Omega = [0, 1] \) for which the region \( s \in D \) is a supplier in destination region \( d \in D \). We can then rewrite the integral above as:

\[ \int_{\Omega_{sd,t}^i} \frac{\tilde{x}_{d,t}^i}{z_1(\omega)} p_{d,t}^i(\omega)^{-\sigma} \epsilon_{d,t}^i d\omega \]

\[ = \frac{\tilde{x}_{d,t}^i}{p_{d,t}^i} \epsilon_{d,t}^i \int_0^\infty \int_0^\infty (z_1)^{-\sigma} \left( \frac{\partial^2 F_{sd,t}(z_1, z_2)}{\partial z_1 \partial z_2} \right) dz_1 dz_2 \]

\[ = \frac{\tilde{x}_{d,t}^i}{p_{d,t}^i} \epsilon_{d,t}^i \int_0^\infty \int_0^\infty \min \left\{ \frac{\sigma}{\sigma - 1}, \frac{\tilde{x}_{sd,t}^i}{z_1}, \frac{\tilde{x}_{sd,t}^i}{z_2} \right\} - \frac{1}{\pi_{sd,t}^i} \exp \left\{ - \frac{\lambda_{s,t}^i z_1^{-\theta} \theta}{\pi_{sd,t}^i z_1} \right\} \left( \theta \lambda_{s,t}^i z_2^{-\theta} \right) dz_1 dz_2 \]

Once again, use a change of variables, denote \( \eta_1 = \frac{\lambda_{s,t}^i z_1^{-\theta} \theta}{\pi_{sd,t}^i z_1} \) and \( \eta_2 = \frac{\lambda_{s,t}^i z_2^{-\theta} \theta}{\pi_{sd,t}^i z_2} \) and

\[ d\eta_1 = -\frac{\theta \lambda_{s,t}^i z_1^{-\theta} \theta - 1}{\pi_{sd,t}^i z_1} dz_1, d\eta_2 = -\frac{\theta \lambda_{s,t}^i z_2^{-\theta} \theta - 1}{\pi_{sd,t}^i z_2} dz_2 \]

which allows U.S. to rewrite the equation above as:
\[
\int_{\Omega_{d,t}} \frac{\tilde{x}_{d,t}^i}{z_1(\omega)} \left( \frac{p_{d,t}(\omega)}{p_{d,t}^i} \right)^{-\sigma} e_{d,t}^i d\omega
\]
\[
= \pi_{d,t} e_{d,t}^i \left( \frac{\lambda_{d,t}^i}{\pi_{d,t}} \right)^{-\frac{1}{\sigma}} \int_{0}^{\infty} \int_{0}^{\infty} \eta_1 \eta_2 \min \left\{ \left( \frac{\sigma}{\sigma - 1} \right)^{\theta} \eta_1, \eta_2 \right\}^{-\frac{\theta}{\sigma}} \exp \left\{ - \eta_2 \right\} d\eta_1 d\eta_2
\]
\[
+ \int_{0}^{\infty} \int_{0}^{\infty} \frac{\left( \frac{\sigma}{\sigma - 1} \right)^{-\theta} \eta_1^{-\frac{1}{\sigma}} \exp \left\{ - \eta_2 \right\} d\eta_1 d\eta_2
\]
\[
= \pi_{d,t} e_{d,t}^i \left( \frac{\lambda_{d,t}^i}{\pi_{d,t}} \right)^{-\frac{1}{\sigma}} \int_{0}^{\infty} \int_{0}^{\infty} \frac{\theta}{1 + \theta} \left( 1 - \left( \frac{\sigma}{\sigma - 1} \right)^{1-\theta} \right) \eta_2^{-\frac{1-\theta}{\sigma}} \exp \left\{ - \eta_2 \right\} d\eta_1 d\eta_2
\]
\[
+ \int_{0}^{\infty} \frac{\theta}{1 + \theta} \left( \frac{\sigma}{\sigma - 1} \right)^{-\theta} \eta_1^{-\frac{1-\theta}{\sigma}} \exp \left\{ - \eta_2 \right\} d\eta_1 d\eta_2
\]
\[
= \pi_{d,t} e_{d,t}^i \left( \frac{\lambda_{d,t}^i}{\pi_{d,t}} \right)^{-\frac{1}{\sigma}} \left( \frac{\lambda_{d,t}^i}{\pi_{d,t}} \right)^{-\frac{1}{\sigma}} \int_{0}^{\infty} \int_{0}^{\infty} \frac{\theta}{1 + \theta} \left( 1 - \left( \frac{\sigma}{\sigma - 1} \right)^{1-\theta} \right) \eta_2^{-\frac{1-\theta}{\sigma}} \exp \left\{ - \eta_2 \right\} d\eta_1 d\eta_2
\]
\[
+ \int_{0}^{\infty} \frac{\theta}{1 + \theta} \left( \frac{\sigma}{\sigma - 1} \right)^{-\theta} \eta_1^{-\frac{1-\theta}{\sigma}} \exp \left\{ - \eta_2 \right\} d\eta_1 d\eta_2
\]

Using the expression for \( p_{d,t}^{-\sigma} \):
\[
\Pi^i_{s,t} = \sum_{d \in D} \pi^i_{sd,t} e^i_{d,t} - \frac{\theta}{1 + \theta} \sum_{d \in D} \pi^i_{sd,t} e^i_{d,t} = \frac{1}{1 + \theta} \sum_{d \in D} \pi^i_{sd,t} e^i_{d,t} \quad (A-15)
\]

Analogously, total costs and profits of intermediary producers are, respectively:

\[
c^i_{s,t} = \frac{\theta}{1 + \theta} \sum_{d \in D} \pi^{i,j}_{sd,t} e^{i,j}_{d,t}, \quad \Pi^i_{s,t} = \frac{1}{1 + \theta} \sum_{d \in D} \pi^{i,j}_{sd,t} e^{i,j}_{d,t} \quad (A-16)
\]

These are allow analogous to the expression in the paper after allowing the elasticities to be sector-specific.