Aspects of Tachyon theory

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Abstract. Does the Special Theory of Relativity (STR) forbid the existence of particles traveling with speed greater than the speed of light in vacuo (Tachyons)? Prof. Sudarshan and collaborators O.M. Bilaniuk and V.K. Despande examined this question in 1962 and concluded that STR does not rule out such objects. Now, the momentum 4-vector of a Tachyon is necessarily space-like and consequently, the sign of energy no longer Lorentz-invariant. Thus a Tachyon will be found to have negative energy in certain inertial frames. The authors noted that in the latter, the sign of time-intervals will also be reversed. A negative energy Tachyon traveling backward in time could now be reinterpreted as a positive energy particle traveling forward in time. This reinterpretation could be done consistently as was shown by several examples. A quantum field theory of free spinless Tachyons was suggested by Feinberg (1967). M. Arons and Sudarshan reexamined this model and showed that the model did not possess invariance under the Poincaré group. An alternative version was constructed by them that possessed the desired invariance property but no local commutativity. Subsequently, Dhar and Sudarshan constructed a model of a neutral scalar Tachyon with Yukawa coupling to Fermions. The model had unusual features such as the emission of a Tachyon by a Fermion as a real process and the need for an additional non-local interaction. Prof. Sudarshan, in association with J. Narlikar, studied Tachyons in the context of cosmology in 1976. One of their conclusions is that any primordial Tachyons that might have been created at the beginning are unlikely to have survived to the present era.

The question of the possible existence of particles that travel with speed in excess of c, the speed of light in vacuo (hereafter we take c=1), was considered by Sommerfeld,[1] who uncovered many unusual properties of these hypothetical objects. For instance, application of a force will decelerate such a particle even as it gains energy. With the advent of the special theory of relativity (STR) an impression gained ground, supported by Einstein himself,[2] that the existence of such objects was forbidden by the STR. The question was reexamined by Sudarshan and collaborators.[3] A result of this exercise was the conclusion: STR permits particles with speed in excess of light-speed, provided only that they are created that way. The speed of light barrier may not be breached either from below or from above.

For an ordinary particle – a tardyon – the energy-momentum 4-vector is time-like with

$$P_j \ p^j = E^2 - p^2 = m_0^2 , \quad m_0 > 0$$

(1)

and the usual expressions

$$E = \tilde{a} m_0 , \quad \tilde{p} = \tilde{a} m_0 \tilde{v} ; \quad \tilde{a} = \left(1-v^2\right)^{-1/2}$$

(2)
in terms of particle velocity $\tilde{v}$ may be derived via a suitable Lorentz transformation from the rest frame $(E = m_0, \ p = 0)$. For a particle with speed above the speed of light in a vacuum – a tachyon – the momentum 4-vector is space-like

$$P_i p^i = E^2 - p^2 = -m^2$$

and expressions for energy and momentum are

$$E = \Gamma m_\ast, \quad \vec{p} = \Gamma m_\ast \vec{v}, \quad \Gamma = (v^2 - 1)^{-1/2}$$

The above (2a) may be obtained from (2) by formally letting $v > 1$ and simultaneously $m_0 = im_\ast$. This ‘unitary trick’ was devised in Ref. 3, the authors named $m_\ast$ the meta mass. A more direct interpretation is that $m_\ast$ is the minimum value of $p$, i.e. value in a frame in which $E = 0, \ p = m_\ast$. Indeed eq.(2a) may be derived via a Lorentz transformation from the latter frame. For a transformation along the direction of $\vec{p}$, the desired Lorentz transformation has parameter $\tan h^{-1} \beta$ given by $v\beta = 1$. Here $\beta$ is the relative speed between the two frames and $v$ the particle velocity.

Quite unlike the case with a time-like 4-momentum; for a space-like momentum 4-vector, the sign of energy is not Lorentz-invariant. Thus for a tachyon there will exist inertial frames in which it will be found to have negative energy. Thus, let a tachyon with positive energy $E$ have velocity $\vec{v}$ in a frame $\Sigma$. In the frame $\Sigma'$, moving with velocity $\vec{\beta}$ with respect to $\Sigma$, the energy $E'$ will be

$$E' = \gamma E (1 - \vec{v} \cdot \vec{\beta}) \quad \text{where} \quad \gamma = (1 - \beta^2)^{-1/2}.$$  

Thus $E'$ will be negative if $\vec{v} \cdot \vec{\beta} > 1$. What is the meaning of a negative energy particle? The authors of ref. 3 noticed that whenever energy changes sign so will a time interval. That is, let $\Delta t = t_2 - t_1$, be positive, $t_2 > t_1$. Then, in frame $\Sigma'$, $\Delta t' < 0$, $t'_2 < t'_1$ when $\vec{v} \cdot \vec{\beta} > 1$. Thus a negative energy particle will necessarily move backward in time. An observer in $\Sigma'$ could simply then interpret the tachyon as having a positive energy and moving forward in time (according to his sense of time-ordering). Specifically, consider the sequence of events as seen from $\Sigma$: a tachyon with $E > 0$ is emitted at source $S_1$ at time $t_1$ and subsequently absorbed at sink $S_2$ at time $t_2$, with $t_2 > t_1$. In $\Sigma'$ a negative energy tachyon will be found to be absorbed first at the instant $t'_2$ at $S_2$ and emitted later at $t'_1$ at $S_1$. Since the absorption of a negative energy is the same as the emission of a positive energy, in so far as the effect on the absorber is concerned, it is possible for an observer to simply describe the sequence as the emission of a positive energy tachyon at $S_2$ followed by the absorption of the same at $S_1$. This is the reinterpretation principle, illustrated in ref. 3 by a number of examples. This principle is the magic wand that makes possible a consistent physical explanation of phenomena associated with a tachyon. Without it, one could not seriously entertain the hypothesis of the existence of tachyons. It should be emphasized that the principle operates at a purely classical level. Furthermore, it is called upon to explain phenomena only in certain (but not all!) inertial frames. These features distinguish it from, say, the Feynman picture of a positron. In subsequent publications, Bilaniuk and Sudarshan[4] showed that the reinterpretation principle also resolved the apparent paradoxes and a few persistent misconceptions regarding the alleged violation of the causality principle by tachyons.

A deeper connection with STR will require that the states of a tachyon furnish a projective unitary irreducible representation of the Poincaré group. According to the general theory,[5] these are to be ‘induced’ from those of the corresponding stability group (a subgroup that leaves fixed a chosen 4-momentum). For a space-like 4-momentum, the stability group is $SO(2,1)$, whose unitary representations are all infinite dimensional, except for the one-dimensional singlet representation.[6] Thus we have the following: a tachyon will come either as a single component object or as an infinite supermultiplet whose members are distinguished by a quantum number, which can be interpreted as the helicity. The supermultiplets themselves (there are four classes of these) are distinguished by the value of the Casimir invariant, built out of operators from the enveloping algebra of the Lie algebra of
the stability group \( \text{SO}(2,1) \). These remarks should be kept in mind while thinking of a possible quantum field theory of a tachyon.

Field theories have been proposed for the singlet tachyon, represented by a scalar field. The first to attempt this was Tanaka.[7] As Tanaka noted, his model had difficulties with Lorentz-invariance: a Lorentz-invariant unique vacuum state did not exist, nor did an unitary operator implementing a Lorentz transformation. Next attempt is due to Feinberg.[8] The most unusual feature of this model is that the free, scalar field had to be quantized in conformity with Fermi (rather than Bose!) statistics. This comes about as follows: the free scalar field satisfied a Klein-Gordon-like equation (with \( m_0^2 \) replaced by \(-m_0^2\) ) that follows from eq. (1a). A set of plane wave solutions of this is indexed by the pair \((\omega_k, \vec{k})\), where \( \omega_k = \sqrt{k^2 - m_0^2} \) and the condition \( k = |\vec{k}| \geq m_0 \) is imposed to ensure that \( \omega_k \) stays real. The hermitian field \( \phi(x,t) \) is then expanded in terms of these solutions, in the usual way, and the coefficients of positive-(negative-) frequency waves are identified with annihilation (creation) operators. However, the separation of the field into positive- and negative-frequency parts is not Lorentz-invariant, for much the same reason as the sign of energy is not. Concomitantly, the distinction between creation (a’\(^{-}\)) and destruction (a) operators also is not Lorentz-invariant. The latter can mix them up; in particular, there exists Lorentz-transformations under which \( a \rightarrow a^- \) and \( a^- \rightarrow a \). This situation makes it impossible to impose the canonical commutation relations (CCR)

\[
[a(\kappa), a^+(\kappa')] = \delta^3 (\kappa - \kappa') \tag{3}
\]

Feinberg, accordingly, replaced the above commutator by an anti-commutator. Although, one difficulty was thus resolved, the others remained. The fact that a(\(\kappa\)) could be transformed into a’(\(\kappa\)) means that the vacuum state of one observer was non-vacuum for certain other observers. A careful reexamination of the Feinberg model was next undertaken by Arons and Sudarshan.\(^9\) These authors chose to quantize in an infinite (spatial) volume, to keep the translational invariance intact and also used normalisable wave packet states instead of plane-waves. What these authors then found is that the vacuum state may be transformed by a Lorentz-transformation into a state with an infinite number of particles. The corresponding Fock spaces must be inequivalent representations of the canonical anti-commutation relations and hence, the Lorentz-transformation was not unitarily implementable. Arons and Sudarshan also gave a more direct proof that an unitary operator \( U(\Lambda, a) \) representing the element \( (\Lambda, a) \) of the Poincaré group and acting as an automorphism

\[
U(\Lambda, a)\phi(x)U^{-1}(\Lambda, a) = \phi(\Lambda x + a) \tag{4}
\]

does not exist in the Feinberg model. An alternative quantization procedure was next proposed by these authors.\(^9\) Since the root of all troubles was in the separation of the field into positive- and negative-frequency parts, and interpreting the expansion coefficients as annihilation and creation operators, it was proposed not to do it at all. The complex scalar field \( \phi(x,t) \) was now expanded into annihilation operators alone, for both positive and negative energies, and its hermitian adjoint \( \phi^+(x,t) \) in terms of creation operators of positive and negative energies. This procedure, while leading to a Lorentz-invariant formulation, was at the expense of the explicit appearance of states of negative energy. To interpret these the following \textit{physical postulate} was proposed: In a transition amplitude any negative energy tachyon in the initial state is to be interpreted as an outgoing positive energy anti-tachyon with the sign of all additive quantum numbers reversed and similarly for negative energy particles in the final state. This \textit{physical postulate} was the quantum mechanical analogue of the classical \textit{reinterpretation principle}. The scalar field was quantized via commutators, in conformity with the spin-statistics theorem. The resulting field theory did not have the property of local commutativity. This last feature is not unexpected, it is, indeed, necessary for the inner consistency of theory. In a field theory with local commutativity, the field could not annihilate the vacuum state without itself vanishing identically.\(^10\) The field theory of tachyons, proposed in ref. 9, was fully developed in a subsequent publication by Dhar and Sudarshan.\(^11\) Quantization procedure was carried out for a neutral as well as for a complex scalar field. An Yukawa interaction connecting
fermions to the scalar tachyon was introduced. The assumed coupling permits emission of a tachyon by a fermion, of course. This could occur as a real process (permitted by energy-momentum conservation) provided that the energy of the initial fermion is suitably high. This has the implication that what appears to be a single fermion in its rest frame will be seen as a fermion plus an emitted tachyon in suitable other Lorentz frames. This is another instance of a general feature involving tachyons; the frame dependence of phenomena observed. As another example, we may consider tachyon-fermion elastic scattering as seen in one frame. In a different Lorentz-frame, where the initial tachyon energy might be negative, the process will be observed as the decay of the initial fermion into the final fermion plus a tachyon-antitachyon pair. To date no field theory based on infinite component fields that provide projective unitary representations of SO(2,1) has been constructed.

Let us now consider the gravitational interaction of a tachyon. We may extend the Einstein principle of equivalence to a tachyon, whose motion will then obey the geodesic equation (for a free particle). Raychaudhuri[12] obtained the following two results: 1) In a weak gravitational field a tachyon will experience a repulsive inverse square force and 2) for a tachyon falling radially into a Schwarzschild black-hole there exists a minimum value of the Schwarzschild radial coordinate at which the tachyon will be forced to turn around and move in the opposite direction. This happens at a point inside the event horizon. The first result is immediately understandable. As a tachyon falls in a gravitational field, its energy will increase and consequently, its speed must decrease (according to eq. (2a) ). The effective repulsive force must, moreover, be inverse square in the weak field limit, since the gravitational potential in that limit is just the Newtonian potential. Consider now the motion of a tachyon in the Schwarzschild geometry. For a purely radial motion, the expression for the proper velocity is

$$\left(\frac{dl}{d\tau}\right)^2 = 1 - \frac{\eta}{c^2} B(r)$$

$$B(r) = g_{00}(r) = 1 - \frac{2M}{r}$$

(5)

where \(d\tau = B^{1/2}dt\), \(dl = B^{-1/2}dr\) are elements of proper time and length and \(r\) and \(t\) are the Schwarzschild radial and time coordinates. In the above we use units \(G = 1, c = 1\). In eq. (5), \(\kappa\) is a constant of motion (the energy-at-infinity per unit mass/meta mass) and \(\eta = +1\) for time-like, \(\eta = -1\) for space-like and \(\eta = 0\) for light-like geodesics. The above eq. (5) shows that there exists minimum value of \(r\) given by

$$r_{min} = \frac{2M}{1 + \kappa^2}$$

(6)

at which a tachyon (\(\eta = -1\)) will be forced to turn back. The bounce occurs inside the event horizon. Additional results are obtained by Dhurandhar and Narlikar[13] including the one that for a tachyon the event horizon \((r = 2M)\) is no longer a one-way membrane. These authors also studied tachyon geodesics in the Kerr metric.[14] One of their conclusions is that the capture of a tachyon by a Kerr black hole may lead to a violation of the second law[15] of black hole physics, i.e. the black hole surface area may decrease.

In the context of standard cosmology, the problem of tachyons was considered by Narlikar and Sudarshan[16] and by others.[12,17] An integration of the equations of motion yielded the result that a tachyon obeys the same relaxation rule as ordinary tardyons or massless particles; namely

$$P R(t) = \text{const}$$

(7)

where \(P\) is the magnitude of particle momentum, in the co-moving frame, and \(R(t)\) the cosmic scale factor. The result is not altogether unexpected since it arises, essentially, from the very high degree of symmetry of standard cosmology. However, for tachyons there is a problem with eqn. (7). Here \(P\) falls as the universe expands. But the momentum of a tachyon cannot fall below \(m_\ast\), the value of its metamass. What happens when \(P = m_\ast\) is reached (say at time \(t = t_\ast\))? At that instant of ‘Nirvana’ \((E = 0, v \rightarrow \infty)\) a tachyon will simply disappear from view. Alternatively,[16] one may notice from the equation of motion, that \(t_\ast\) is actually a maximum on the trajectory and hence the tachyon will be
forced to travel backward in time starting \( t = t_m \). This could then be interpreted as tachyon-antitachyon annihilation with zero energy release. Narlikar and Sudarshan go on to show that these conclusions remain valid when the tachyon is treated via a quantum field; a scalar field coupled conformally to the background metric. These authors further considered the question if any tachyons that might have been produced in the early universe (say, at the era of helium synthesis or earlier) could have survived to the present cosmic era. The answer depends on the assumed (meta)-mass\(^{[16]}\)(M) of the tachyon. For survival till present it was estimated \( M/m_e \sim 10^{-11} \), where \( m_e \) is the electron mass. Thus it appears highly unlikely that any possible primordial tachyon survives today. An immediate consequence of this is that tachyons have no role in the resolution of the ‘missing-mass’ problem.

Let us finally consider the situation where a tachyon field may exist without an attendant tachyon. In fact, Tanaka’s original publication was an attempt in that direction; to arrange to have tachyons only as virtual particles around more conventional particles (tardyons). A tachyon-cloud around a slower-than-light particle is also treated in ref. 18. Another interesting phenomenon involves the concept of tachyon-condensation. The point is this: a Lagrangian may contain a term with the wrong sign of the mass-square. Instead of accepting it to signify a tachyon, one interprets it to herald the onset of an instability, caused by the use of a ‘wrong’ vacuum state which is a maximum of the given potential. Any sort of disturbance will make the field roll over to the ‘true vacuum’ state, which is a minimum of the potential. When the field theory is reformulated around the true vacuum, tachyon states disappear from the spectrum. An example of such behavior is the usual Higg’s mechanism involving a quartic potential of a scalar field. Actually, excitations with wrong sign of mass-square term appear copiously in the string theories. These, according to Sen \([19]\) are to be interpreted via tachyon condensation. This suggestion has interesting cosmological consequences, particularly relating to the question of dark energy. For a discussion of these speculations, see refs. 20 and 21.

My initiation into the community of practitioners of the art of doing physics was at the hands of Prof. E.C.G. Sudarshan. It gives me genuine pleasure to dedicate this modest contribution to him.

References

[1] Sommerfeld A 1904 *K. Akad. Wet. Amsterdam, Proc.* 8 346
[2] Einstein A 1905 *Ann. Physik* 17
[3] Bilaniuk O M, Despande V K and Sudarshan E C G 1962 *Am. J. Physics* 30 718
[4] Bilaniuk O M and Sudarshan E C G 1969 *Nature* 223 386; 1969 *Physics Today* 22 43
[5] Mackey G 1968 *Induced Representation of Groups and Quantum Mechanics* (New York: W.A. Benjamin)
[6] Bergmann V 1947 *Ann. Math.* 48 568
[7] Tanaka S 1960 *Prog. Theo. Physics (Kyoto)* 24 171
[8] Feinberg G 1967 *Phys. Rev.* 159 1089
[9] Arons M E and Sudarshan E C G 1968 *Phys. Rev.* 173 1622
[10] Streater R F and Wightman A S 1964 *TCP Spin, Statistics and All That*, (New York: W.A. Benjamin, Inc.)
[11] Dhar J and Sudarshan E C G 1968 *Phys. Rev.* 174 1808
[12] Raychaudhuri A 1974 *J. Math. Phys.* 15 856
[13] Dhurandhar S V and Narlikar J V 1976 *Pramana* 6 388
[14] Dhurandhar S V and Narlikar J V 1978 *Gen. Rel. and Gravit.* 9 1089; Dhurandhar S V 1979 *Phys. Rev. D* 19 2310
[15] Hawking S W 1972 *Comm. Math. Phys.* 25 152
[16] Narlikar J V and Sudarshan E C G 1976 *Mon. Not. R. Astr. Soc.* 175 105
[17] Davies P C W 1975 *Nuovo Cimento* 258 571; 2004 *Int. J. Theo. Phys.* 43 141
[18] Boulware D S 1970 *Phys. Rev. D* 1 2426; Sudarshan E C G 1970 *Phys. Rev. D* 1 2428
[19] Sen A 2002 *J. High Energy Phys.* 04 048; 07 065; 2002 *Mod. Phys. Lett. A* 17 1797
[20] Gibbons G W 2003 *Class. Quan. Gravity* 20 5321
[21] Bagla J S, Jassal H K and Padmanabhan T 2003 *Phys. Rev. D* 67 063504