SHOCKED SELF-SIMILAR COLLAPSES AND FLOWS IN STAR FORMATION PROCESSES

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ABSTRACT

We propose self-similar shocked flow models for certain dynamical evolution phases of young stellar objects, “champagne flows” of H II regions surrounding OB stars, and shaping processes of planetary nebulae. We analyze an isothermal fluid of spherical symmetry and construct families of self-similar shocked flow solutions that feature (1) either a core expansion with a finite central density or a core accretion at a constant rate with a density scaling proportional to $r^{-3/2}$ (a free-fall state); (2) a shock moving outward at a constant speed (a wind, a breeze, or an accretion flow); and (3) a preshock gas approaching a constant speed at large $r$ with a density scaling proportional to $r^{-2}$. In addition to testing numerical codes, our models can accommodate diverse shocked flows with or without a core collapse or outflow and with or without an envelope expansion or contraction. As an application, we introduce our model analysis to observations of Bok globule B335.

Subject headings: H II regions — hydrodynamics — ISM: clouds — shock waves — stars: formation — stars: winds, outflows

1. INTRODUCTION

Far away from the initial and boundary conditions, the dynamical evolution of a fluid may lead to self-similar phases for physical variables with shape-invariant profiles and properly scaled magnitudes (Sedov 1959; Landau & Lifshitz 1959). For star formation, self-similar solutions were found to describe collapses of isothermal gas clouds (Larson 1969; Penston 1969; Shu 1977, hereafter S77). From an initial time $t = 0$ when the core becomes singular to the final stage $t \rightarrow \infty$, Shu (1977) derived the “expansion-wave collapse solution” (EWCS) that has a free-fall core with a density $\rho \propto r^{-3/2}$, a radial infall speed $u \propto r^{-1/2}$, a constant mass accretion rate at $r \rightarrow 0$, and a static envelope of a singular isothermal sphere (SIS) with the stagnation point moving outward at the sound speed $a$. This EWCS scenario of “inside-out” collapses for star formation has been advocated by Shu et al. (1987) and compared with observations (e.g., Zhou et al. 1993, hereafter Z93; Choi et al. 1995; Saito et al. 1999, hereafter Sa99; Harvey et al. 2001).

Lou & Shen (2004, hereafter LS04) reexamined this problem and derived new solutions in the “semiconcrete space” ($0 < t < +\infty$), in contrast to the “complete space” ($-\infty < t < +\infty$) of Hunter (1977) and Whitworth & Summers (1985, hereafter WS). The gas flows at large $r$, together with the two distinct asymptotic behaviors near the origin, give rise to an infinite number of self-similar solutions. Those solutions of “envelope expansion with core collapse” (EECC) are especially interesting.

Besides these shock-free solutions, Tsai & Hsu (1995, hereafter TH95) constructed a shocked self-collapse solution for a situation in which a central energy release initiates an outgoing shock during a protostellar collapse of a low-mass star. Such a shocked solution, matched with a static SIS envelope, has a free-fall core, but with a lower mass accretion rate than the EWCS. TH95 also constructed a shocked expansion solution of a finite core density matched with a static SIS envelope, which was recently generalized and subsumed into a family of shocked “champagne flow” solutions by Shu et al. (2002, hereafter S02) for expansions of H II regions around OB stars (Strömgren 1939; Tenorio-Tagle 1979, 1982; Newman & Axford 1968; Mathews & O’Dell 1969; Franco et al. 1990, hereafter FTB).

The main thrust of this Letter is to show that in the solution framework of LS04, a variety of shocked self-similar flow solutions can be constructed and applied to astrophysical systems.

2. SHOCKED SELF-SIMILAR FLOW MODELS

To study the basic properties of shocked self-similar flows of spherical symmetry, we assume isothermality with a constant sound speed $a$. The standard self-similar nonlinear ordinary differential equations (S77; TH95; S02; LS04) are

$$[(x - v)^2 - 1] \frac{dv}{dx} = \left[\alpha (x - v) - \frac{2}{x}\right] (x - v),$$

$$[(x - v)^2 - 1] \frac{d\alpha}{dx} = \left[\alpha - \frac{2}{x} (x - v)\right] (x - v),$$

$$m = x^3 \alpha (x - v),$$

where the independent self-similar variable $x \equiv rl(\alpha t)$ and $\alpha (x), v(x)$, and $m(x)$ are the reduced density, radial speed, and enclosed mass, respectively. The physical density $\rho$, radial speed $u$, and enclosed mass $M$ are obtained by the self-similar transformation

$$\rho(r,t) = \frac{\alpha(x)}{4\pi G r^3}, \quad u(r,t) = av(x), \quad M(r,t) = \frac{\alpha^3 t}{G} m(x).$$

The analytical asymptotic solutions are that for $x \rightarrow +\infty$,

$$v \rightarrow V, \quad \alpha \rightarrow A/x^2, \quad m \rightarrow Ax,$$

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special downstream solution, referred to as the CP1 EECC solution in LS04, can

to cross the sonic critical line \[ x = v = 1 \] [the dash-dotted line for \( t(x) \) at \( x = 0.23 \) with \( m_0 = 0.406 \). An infinite number of solutions can be constructed for various shock locations \( x \), to match the asymptotic behavior of solution (5) as \( x \to +\infty \) with various \( V \) and \( A \) (either a wind, a breeze, or a contraction). This special downstream solution, referred to as the CP1 EECC solution in LS04, can analytically cross the sonic critical line again at \( x = 0.23 \) (dashed line). For Class II solutions in parallel, we choose the downstream solution of \( B = 0.1 \) and various \( x \), to match solution (5) with various \( V \) and \( A \). The parameters of these examples are summarized in Table 1. The dotted lines stand for the EWCS (S02).

where \( V \) and \( A \) are two constant parameters; and that for \( x \to 0 \), either

\[ v \to -(2m_0/\alpha)^{1/2}, \quad \alpha \to [m_0/(2\alpha^3)]^{1/2}, \quad m \to m_0 \]  

or

\[ v \to 2\alpha/3, \quad \alpha \to B, \quad m \to Bx^2/3, \]  

where \( m_0 \) and \( B \) are the two constant parameters (e.g., LS04).

There exist two exact solutions, viz., the SIS solution (Bonner 1956; Chandrasekhar 1957) and the homogeneous “Hubble flow” solution (WS; S02). The ordinary differential equations (1)–(3) can be solved numerically to connect the proper asymptotic solutions (5)–(7).

The isothermal shock conditions are the mass conservation

\[ \alpha_d(v_d - x_s, d)\alpha_d = \alpha_s(v_u - x_s, u)\alpha_u, \]

and the radial momentum conservation

\[ \alpha_d[1 + v_d(v_d - x_s, d)]\alpha_d = \alpha_s[1 + v_u(v_u - x_s, u)]\alpha_u, \]

with the energy conservation involving radiative losses (e.g., Courant & Friedrichs 1976; Spitzer 1978), where subscript \( d \) (\( u \)) denotes the downstream (upstream) of a shock, \( a_d x_{s, d} = a_s x_{s, u} \) is the shock speed, \( a_d \) and \( a_u \) are the downstream and upstream isothermal sound speeds, and \( x_d = r/(a_d t) \) and \( x_u = r/(a_u t) \) are the downstream and upstream independent self-similar variables. For a sound speed ratio \( \tau = a_d/a_u \) with \( \tau x_{s, d} = x_{s, u} \), we have

\[ v_d - x_{s, d} - \tau(v_u - x_{s, u}) = (\tau v_d - v_u)(v_u - x_{s, u})(v_d - x_{s, d}), \]

\[ \alpha_d/\alpha_u = (v_u - x_{s, u})/[(\tau v_d - v_u)]. \]  

The special isothermal shock conditions of \( \tau = 1 \) with \( x_{s, d} = x_{s, u} \) (TH95; S02) are adopted in our numerical examples.

We take the so-called type-2 solutions (Hunter 1977, 1986; LS04) that cross the sonic critical line \( x = v = 1 \) once at \( x < 1 \) and approach the asymptotic diverging behavior of solution (6) for \( x \to 0 \) as downstream (postshock) flows. By integrating a type-2 solution toward the sonic critical line, imposing shock condition (8) at each integration step and continuing further toward \( x \to +\infty \) (upstream), we construct a family of shocked self-similar flows, which is displayed in Figure 1 with \( \tau = 1 \). The resulting upstream solutions the approach asymptotic behavior of solution (5) as \( x \to +\infty \). Such a solution is uniquely determined by two parameters: the central mass accretion rate \( m_0 \) (or, equivalently, the crossing point \( x = 1 \)) and the shock location \( x \). The velocity and density parameters \( V \) and \( A \) as \( x \to +\infty \) in solution (5) are simultaneously determined. For each such solution (denoted as Class I), a core collapses with a constant mass accretion rate \( m_0 = \alpha^2/G \), and a shock travels at speed \( ax \) outward to match with either an expanding or a contracting envelope; at very large \( r \), the flow approaches a constant speed as either wind (including a breeze and an SIS) or inflow—all with density scalings proportional to \( r^{-3/2} \) (LS04).

For downstream solutions with the asymptotic behavior of solution (7), we can readily construct Class II solutions of shocked flows in parallel with the Class I solutions. Such a solution is uniquely determined by the finite core parameter \( B \) and the shock location \( x \). For each Class II solution, a core expands, and a shock travels outward at a constant speed \( ax \), to match with envelope expansion or contraction of a constant speed \( V \neq 0 \) or a breeze \( V = 0 \) (S02) as \( x \to +\infty \), shown by the heavy solid curves in Figure 1.

A few important features are noted here (see Fig. 1 and Table 1). (1) Shocked self-similar solutions of Class I qualitatively share properties of the EWCS (i.e., inside-out collapse) at small \( r \) with density \( \rho \propto r^{-3/2} \), inflow speed \( u \propto r^{-1/2} \), and constant core mass accretion rate \( m_0 \). (2) The main distinction is that the upstream solutions for both classes can be an inflow,
a wind (LS04), a breeze (S02), or an SIS (TH95). (3) For very large \( B \) or small \( m_0 \) (or small \( x \)), the downstream solution \( v(x) \) can oscillate (see Hunter 1977 for the former, and LS04 for the latter). The core mass accretion rate can be extremely low for the latter.

3. STAR FORMATION PROCESSES

3.1. Protostellar Collapses and Outflows (Class I Solutions)

Regarding the EWCS scenario for star formation (Shu et al. 1987), TH95 argued that a central energy output during a protostellar collapse can initiate a shock into an SIS envelope with a lower core mass accretion rate. Based on the solutions of LS04, we envision a much more general scenario for a central inside-out collapse with an adjustable central mass accretion rate and with an outgoing shock into various possible envelope flows such as winds, breezes, inflows, or SISs at large \( r \). The core mass accretion rate is \( M(0, t) = m_0 a^1/5 \). While the EWCS has \( m_0 = 0.975 \) and the example of TH95 has \( m_0 = 0.105 \), our Class I solutions for shocked flows give rise to a continuous range of \( 0 < m_0 < 0.975 \). These solutions can describe certain self-similar evolution phases during protostellar collapses and outflows. With these physical concepts, we now turn to the well-observed protostellar cloud B335 that possesses a collapsing core.

Besides the observed bipolar CO outflows on large scales, an SIS environment is presumed to surround the central collapse region and has been modeled using the EWCS (Zhou, Z93; Sa99). From a temperature profile estimated from dust emissions (Zhou et al. 1990), the B335 globule was approximated as an isothermal cloud with an effective sound speed \( c_{\text{eff}} \sim 0.23 \text{ km s}^{-1} \). In essence, a gross spherical symmetry is assumed for the B335 cloud by regarding the bipolar outflows as additional features produced by a circumstellar disk within \( \approx 100 \) AU (Harvey et al. 2003). In the EWCS modeling (Z93), a core collapses as a free fall while an envelope remains static with an estimated infall radius \( r_{\text{inf}} \sim 0.03 \text{ pc} \).

Given some successes of the EWCS model in interpreting the density and velocity profiles (via simulations of spectral lines; Z93; Choi et al. 1995; Velusamy et al. 1995), departures from the EWCS model do exist. In Sa99, the estimated infall speed at \( r \sim 2200 \) AU is \( -0.14 \text{ km s}^{-1} \) for \( r_{\text{inf}} \sim 7200 \) AU, which is smaller than the EWCS prediction of \( 0.34 \text{ km s}^{-1} \) by a factor of \( \sim 2.4 \), and the estimated mass accretion rate is \( (1.2-2.4) \times 10^{-6} M_\odot \text{ yr}^{-1} \), corresponding to \( m_0 \sim 0.418-0.836 \), which tends to be less than the EWCS prediction of \( m_0 = 0.975 \) (Z93). Meanwhile, the column density of the outer envelope at \( r = 10,000 \) AU is \( N \sim 6.3 \times 10^{21} \text{ cm}^{-2} \), which is slightly larger than the EWCS prediction of \( N \sim 5.4 \times 10^{21} \text{ cm}^{-2} \) (Sa99). This last difference is aggravated by the near-IR extinction study of Harvey et al. (2001) that gives an envelope density \( 3-5 \) times higher than EWCS prediction while the density \( p \) scales as \( r^{-2} \). HCN spectral analysis of B335 seems to indicate a central core collapse with an envelope expansion distinctly different from bipolar CO outflows.

The major advantage of our shocked self-similar flow model is its capability of accommodating observations of the B335 globule while making testable predictions for envelope expansion in contrast to an SIS of the EWCS. As an example, we introduce the unshocked CP1 EEC solution (dashed curves in Fig. 1) that analytically passes the sonic critical line twice (LS04). While the inner core freely falls with \( \rho \sim r^{-3/2} \), the envelope expands with \( \rho \sim r^{-2} \). In contrast to the EWCS, the transition across \( r_{\text{inf}} \) is smooth (without a “kink”). Still taking \( x = 1 \) for \( r_{\text{inf}} \sim 7200 \) AU (or the age of the self-similar process is \( \sim 1.5 \times 10^5 \) yr in the EWCS model; Z93), the infall speed at \( r \sim 2200 \) AU is \( \sim 0.12 \text{ km s}^{-1} \), and the mass accretion rate given by the CP1 EEC solution is \( m_0 \sim 0.406 \), less than a half of 0.975 in the EWCS. At large \( r \), the density approaches \( \rho \sim A_1 a_{\text{eff}}^3/(4\pi Gr^2) \), with \( A = 5.158 \) (Table 3 of LS04). That is, our CP1 EEC model gives an envelope density \( 2.5 \) times higher than that of an SIS in the EWCS (i.e., \( A = 2 \)). This tends to agree better with the inference of \( 3-5 \) times by Harvey et al. (2001). One distinctive consequence of our isothermal model is that the envelope, instead of an SIS, expands with a speed of \( \sim 0.25 \text{ km s}^{-1} \) at \( r = 25,000 \) AU. This is qualitatively consistent with the preliminary HCN spectral analysis (A. Lapinov, private communications). By inserting a shock at the proper location, it has a range to adjust the data fit for density and speed profiles, and there is preliminary evidence of shock in the B335 system (e.g., Nisini et al. 1999). Sharing the qualitative features of the EWCS in the central region, our Class I solutions (including the CP1 EEC solution) of shocked self-similar flows are versatile enough to model other protostellar clouds or Bok globule systems.

3.2. “Champagne Flows” in H ii Regions (Class II Solutions)

As massive OB stars form in molecular clouds, intense ultraviolet radiation ionizes the surrounding H I clouds and carves out H ii regions (Strömgren 1939). As an ionization front passes by, a pressure gradient develops between the H ii and H i regions that drives the flows. Shocks can naturally occur in such an impulsive expansion phase. An expansion of an H ii region embedded in a uniform ambient medium has been studied (e.g., Newman & Axford 1968; Mathews & O’Dell 1969). Other possible environments may involve power-law cloud density profiles proportional to \( r^{-n} \) with \( 1 \leq n \leq 3 \) and a mean of \( n = 2 \) (e.g., Arquilla & Goldsmith 1985). As ionization or shock fronts encounter a strong negative density gradient, the ensuing supersonic expansion of the ionized gas is referred to as the “champagne phase” (FTB). When a fast-moving ionization front cannot be confined under certain conditions (FTB) and it ionizes the entire cloud in a relatively short time, the inevitable expansion of H ii regions might evolve into a self-similar phase (S02).

In the scenario above, a cloud is presumed to be static initially, and the H ii region forms instantaneously at \( t = 0 \) when the stellar nuclear burning turns on. Our more general scenario would allow a molecular cloud with asymptotic flows (either winds and breezes or accretions and contractions) at large \( r \). To model expansions of H ii regions as the ionization front travels, we focus on various possible large-scale systematic flows at large \( r \) and put aside the messy central star-forming processes that occur on much smaller scales. Within the ionized

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4 There are an infinite number of discrete downstream solutions with extremely large \( B \) or small \( m_0 \) and oscillating \( v(x) \) around zero to drive shocks into an SIS envelope.

5 In reality, a polytropic model should be more pertinent as discussed in LS04. For a polytropic gas, the asymptotic wind speed should tend to vanish at large \( r \).

6 Practically, it might be far easier to detect the presence of large-scale shocks than to estimate the velocity and density profiles since shocks will leave nonkinematic signatures amenable to observations, i.e., through emission diagnostics from highly compressed postshock gas or from chemical or radiative cooling processes.
cloud sphere, asymptotic winds and accretions at large $r$ can be connected to inner champagne flows via transonic shocks. As transients peter out, an expanding H II region may become self-similar with an outgoing shock to match with an asymptotic wind, breeze, or accretion.

The shocked breeze solutions have been used to model self-similar expansions of H II regions (S02). For each “champagne breeze” solution of S02, we can use the same downstream to construct Class II solutions with shocked upstream flows that approach breezes, winds, and accretions at large $r$. As $B \to 0$, the downstream solution becomes invariant (S02), and the resulting family of upstream solutions is displayed in Figure 2.

Observations do suggest the presence of systematic flows in H II regions, and models involving stellar winds were proposed (e.g., Dyson 1977; Hippelein & Münch 1981; Comerón 1997). Such outflows may be supersonic at a few kilometers per second (e.g., Relaño et al. 2003). In reference to our Class II models, shocked self-similar “champagne” flows in H II regions distinctly differ from “champagne breezes” or the SIS by having both constant wind speeds at large $r$ and outgoing shocks at constant speed. Compared with breezes, champagne winds would dynamically impact the surrounding interstellar medium more efficiently.

3.3. Implication for Planetary Nebulae

In the hydrodynamic models for shaping planetary nebulae (PNs), the interacting stellar wind (ISW) scenario (Kwok et al. 1978) is often invoked. In a few spherical cases, a much faster stellar wind from the core catches up with a slower dense wind—the remnant of the asymptotic giant branch phase—and together they form shocks. Our shocked self-similar flow solutions of Class I clearly show that the shaping of a PN involving an ISW can be concurrently accompanied by a central accretion or infall toward a proto–white dwarf in a self-similar manner. In order to model bipolar PNs other than round or mildly elliptical morphologies, aspherical aspects are included in the so-called generalized ISW model (e.g., Balick & Frank 2002). By flow instabilities, a shocked envelope can expand to create various nebula morphologies while the central core continuously accretes material that forms a white dwarf. As a proto–white dwarf continues to accrete material in order to approach the Chandrasekhar limit of $\sim 1.39 M_\odot$, there might even be the possibility of igniting a Type Ia supernova explosion.

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