Electroweak symmetry breaking in supersymmetric models with heavy scalar superpartners

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Abstract

We propose a novel mechanism of electroweak symmetry breaking in supersymmetric models, as the one recently discussed by Birkedal, Chacko and Gaillard, in which the Standard Model Higgs doublet is a pseudo-Goldstone boson of some global symmetry. The Higgs mass parameter is generated at one loop level by two different, moderately fine-tuned sources of the global symmetry breaking. The mechanism works for scalar superpartner masses of order 10 TeV, but gauginos can be light. The scale at which supersymmetry breaking is mediated to the visible sector has to be low, of order 100 TeV. Fine-tuning in the scalar potential is at least two orders of magnitude smaller than in the MSSM with similar soft scalar masses. The physical Higgs boson mass is (for tan β \gg 1) in the range 120 – 135 GeV.

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1 Introduction

Explanation of the origin of the Fermi scale is a challenge for physics beyond the Standard Model (SM). An interesting possibility is that the electroweak symmetry breaking is generated by quantum corrections to the Higgs doublet potential. For such a mechanism to be under theoretical control, the tree level Higgs mass squared parameter $m_H^2$ has to be calculable (at least in principle) at some scale $\Lambda_S$ in terms of some more fundamental parameters. Secondly, the dependence of quantum corrections to the Higgs doublet potential on dimensionful parameters of new physics should be moderate. Otherwise large cancellations between the tree level parameters and quantum corrections would be necessary, rendering such a mechanism doubtful. Although fine-tuning is difficult to quantify in a precise way, it is usually easy to give a rough estimate of its order of magnitude. As a reference number it is worth remembering that in the SM cut-off by the neutrino see-saw scale of order $10^{13}$ GeV, the Higgs potential has to be fine-tuned in 1 part to $10^{20}$.

Radiative electroweak symmetry breaking, triggered by the large top quark Yukawa coupling, has been a successful prediction of the Minimal Supersymmetric Standard Model (MSSM). However, in the MSSM the necessary degree of fine-tuning in the Higgs potential grows exponentially with the value of the lightest CP-even Higgs boson mass. The present LEP limits on the Higgs boson mass push the stop masses into the range $500$ GeV–$1$ TeV and, in consequence, the necessary fine-tuning in the MSSM Higgs potential is estimated to be of order of $1\%$. This fact may be taken as somewhat disappointing for a supersymmetric model and it stimulated several authors to look for alternatives to the MSSM and to supersymmetry itself, which could explain the origin of the Fermi scale. However, no convincing idea has emerged yet that would lead to fine-tuning significantly lower than $O(1\%)$ needed in the MSSM with the present limits on the Higgs boson mass.

We believe, therefore, it may be worthwhile to ask a different question. After all, the questions about an acceptable degree of fine-tuning and even about its definition do not have any sensible quantitative answer at the level of effective theories, for instance, when the theory of soft supersymmetry breaking terms is not known. More bothersome is the quadratic dependence of the fine-tuning in the MSSM on the mass of the scalar supersymmetry breaking terms.

\footnote{A different point of view has recently been expressed in refs. [1, 2].}
perparners of the top quark. Although the FCNC effects in the MSSM are controlled by the squark masses of the first two families, a light stop is not easy (although not impossible) to reconcile with the observed suppression of the FCNC effects [12]. Indeed, unless the first two sfermion families are degenerate in mass, they have to be heavy, with masses at least $\mathcal{O}(10 \text{ TeV})$, and the large splitting with the third one needs some explanation by the mechanism of supersymmetry breaking [13]. Thus it is of some interest to ask if in models in which radiative electroweak symmetry breaking occurs one can avoid quadratic dependence of the Fermi scale on the stop masses, i.e. if one can significantly rise the scalar superpartner masses without jeopardizing naturalness.

An interesting and very simple combination of the idea of the Higgs doublet as a pseudo-Goldstone boson (revived in the non-supersymmetric Little Higgs models [8], inspired by the so-called deconstruction [14]) and of supersymmetry has been proposed recently by Birkedal, Chacko and Gaillard [15]. In their model the Higgs doublet is a Goldstone boson of a spontaneously broken global $SU(3)$ symmetry. Global $SU(3)$ is also explicitly broken by a supersymmetric fermion mass term and by the SM $SU(2)_{L} \times U(1)_{Y}$ electroweak interactions. In the present paper we show that in this model, in certain range of its parameters, an interesting mechanism of radiative electroweak symmetry breaking can be realized. The Higgs doublet mass parameter is then generated at one-loop level by the two moderately fine-tuned sources of the global $SU(3)$ symmetry breaking. The mechanism works for scalar superpartner masses of order $\mathcal{O}(10 \text{ TeV})$. Fine-tuning in the scalar potential is at least two orders of magnitude smaller than in the MSSM with similar soft scalar masses, i.e. stays at the level of (1%). The physical Higgs boson mass is predicted to be (for $\tan \beta \gg 1$) in the range $120 - 135 \text{ GeV}$, where the main source of uncertainty are unknown two-loop effects.

2 The model

In this section we introduce our model which is a slight modification of the one proposed in [15]. The Higgs and the 3rd generation weak doublet superfields are extended to fit the fundamental ($\hat{\mathcal{H}}_u$) and anti-fundamental ($\hat{\mathcal{H}}_d$ and $\hat{Q}$) representations of an approximate global $SU(3)$ symmetry:

$$
\hat{\mathcal{H}}_u = \begin{pmatrix} \hat{H}_u \\ \hat{S}_u \end{pmatrix}, \quad \hat{\mathcal{H}}_d^T = \begin{pmatrix} \hat{H}_d \\ \hat{S}_d \end{pmatrix}, \quad \hat{Q}^T = \begin{pmatrix} \hat{Q}_3 \\ \hat{T} \end{pmatrix}.
$$

(1)
In addition there is a new $SU(3)$ singlet quark supermultiplet $\hat{T}^c$. At some high scale $\Lambda_S$ the $SU(3)$ symmetry is respected by the top Yukawa couplings in the superpotential, but is explicitly broken by the $\mu_T$ term:

$$w_t = Y_T \hat{Q}_3 \hat{H}_u \hat{T}^c + \mu_T \hat{T}^c \hat{T}^c$$  \hspace{1cm} (2)

In order to preserve the symmetry between the up- and down-type sectors one can introduce another global $SU(3)'$ symmetry, that controls the bottom sector. To this end we extend the quark doublet $\hat{Q}_3$ by the $SU(2)_L$ singlet quark superfield $\hat{B}$, so that

$$\hat{Q}' = \left( \begin{array}{c} \hat{Q}_3 \\ \hat{B} \end{array} \right)$$  \hspace{1cm} (3)

is in the fundamental representation of $SU(3)'$ (with respect to which $\hat{H}_u$ and $\hat{H}_d$ form the fundamental and anti-fundamental representations, respectively). We also introduce a corresponding $SU(3)'$ singlet quark superfield $B^c$. The bottom sector superpotential then reads

$$w_b = Y_B \hat{H}_d \hat{Q}' \hat{B} + \mu_B \hat{B}^c \hat{B}$$  \hspace{1cm} (4)

Here $SU(3)'$ is explicitly broken by $\mu_B$. Of course, $w_b$ breaks $SU(3)$ while $w_t$ breaks $SU(3)'$. We also assume that at the high scale $\Lambda_S$ the $SU(3)$ and $SU(3)'$ symmetries are respected by the soft mass terms and trilinear couplings in the top and bottom sectors, respectively. Thus, the most general form of these mass terms (at $\Lambda_S$) is:

$$L_{tb} = -m_3^2 (|\hat{Q}_3|^2 + |\hat{T}|^2 + |\hat{B}|^2) - m_{3T}^2 |\hat{t}^c|^2 - m_{3c}^2 |\hat{T}^c|^2 - m_{3B}^2 |\hat{B}^c|^2 - m_{3c}^2 |\hat{B}^c|^2$$  \hspace{1cm} (5)

$$+ A_T (\hat{Q}_3 \hat{H}_d \hat{t}^c + \hat{T} \hat{S}_d \hat{t}^c) + A_B (\hat{H}_d \hat{Q}_3 \hat{b}^c + \hat{S}_d \hat{B} \hat{b}^c) .$$

Finally, the Higgs doublet $\mu$ term is assumed to respect the $SU(3)$ (and in fact, also the $SU(3)'$) symmetry

$$w_h = \mu \hat{H}_d \hat{H}_u .$$  \hspace{1cm} (6)

As in ref. [15] the gauge symmetry of the MSSM is extended by the $U(1)_E$ group, which commutes with the global $SU(3)$ and $SU(3)'$ symmetries. The requirement of the $SU(3)$ symmetry of the soft terms eliminates the otherwise allowed $m_{3T}^2 \hat{T}T^c$ and $m_{3B}^2 \hat{B}B^c$ terms.

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$U(1)_E$ gauge coupling $g_E$ is normalized in such a way that $\hat{H}_u$ and $\hat{H}_d$ have $U(1)_E$ charges +1 and −1 respectively. The quantum numbers of the relevant fields of the model are summarized in Table 1. Since the electroweak symmetry should not be broken by the vevs of the $S_u$ and $S_d$ fields one must assume that both these superfields are SM singlets. The SM quantum numbers of all the new superfields are uniquely determined by the form of the superpotential. Furthermore, the anomaly cancellation requirement constrains the $U(1)_E$ charges of the SM fields to be proportional to the hypercharge (the other possibility that the additional $U(1)$ is proportional to $B - L$ cannot be realized here as the Higgs fields must be charged under $U(1)_E$) [18]. With our normalization of the Higgs $U(1)_E$ charges this determines all the remaining $U(1)_E$ charges uniquely.

The $SU(3)$ preserving part of the scalar potential of the Higgs fields reads

$$V_{\text{symm}} = m_1^2 |H_d|^2 + m_2^2 |H_u|^2 - (m_3^2 H_d H_u + \text{H.c.}) + \frac{1}{2} g_E^2 (|H_u|^2 - |H_d|^2)^2 , \quad (7)$$

where $m_1^2 = \mu^2 + m_{H_d}^2$, $m_2^2 = \mu^2 + m_{H_u}^2$ and the parameters $m_{H_d}^2$, $m_{H_u}^2$, $m_3^2$ are the soft breaking mass terms. As we discuss in more detail in Sec. 3, spontaneous breaking of the global $SU(3)$ symmetry, such that the Higgs triplets $H_u$ and $H_d$ acquire vacuum expectation values in the $S_u$ and $S_d$ directions

$$f_u \equiv \langle S_u \rangle = f \sin \beta , \quad f_d \equiv \langle S_d \rangle = f \cos \beta , \quad (8)$$

can be achieved in a similar way as the electroweak symmetry breaking in the MSSM, i.e. by quantum corrections induced by the large Yukawa coupling $Y_T$ in the superpotential [2] [15]. Similarly as in the MSSM, the mass of the additional neutral $Z'$ boson is then given by:

$$\frac{1}{2} M_{Z'}^2 = g_E^2 f^2 = \frac{m_1^2 - m_2^2 \tan^2 \beta}{\tan^2 \beta - 1} = \frac{m_1^2 + m_2^2}{2} \left[ \frac{m_1^2 - m_2^2}{\sqrt{(m_1^2 + m_2^2)^2 - 4 m_3^4}} - 1 \right]$$

Table 1: Quantum numbers of the multiplets in the model

|          | $\hat{H}_u$ | $\hat{S}_u$ | $\hat{H}_d$ | $\hat{S}_d$ | $\hat{Q}$ | $\hat{T}$ | $\hat{t}^c$ | $\hat{T}^c$ | $\hat{B}$ | $\hat{b}^c$ | $\hat{B}^c$ |
|----------|-------------|-------------|-------------|-------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $SU(3)$  | 1           | 1           | 1           | 1           | 3         | 3         | 3         | 3         | 3         | 3         | 3         |
| $SU(2)$  | 2           | 2           | 2           | 2           | 1         | 1         | 1         | 1         | 1         | 1         | 1         |
| $U(1)_Y$ | 1/2         | 0           | −1/2        | 0           | 1/6       | 2/3       | −2/3      | −2/3      | −1/3      | 1/3       | 1/3       |
| $U(1)_E$ | 1           | 1           | −1          | −1          | 1/3       | 1/3       | −4/3      | −1/3      | 1/3       | 2/3       | −1/3      |
\[
\sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2}
\]  

(9)

There is also the tree-level \( SU(3) \) breaking potential originating from the electroweak \( D \)-terms:

\[
V_{EW} = \frac{g_2^2 + g_y^2}{8} (|H_u|^2 - |H_d|^2)^2 - \frac{g_2^2}{2} (|H_d H_u|^2 - |H_u|^2 |H_d|^2) .
\]

(10)

Let us now identify the SM Higgs doublet. The \( SU(3) \) global symmetry is spontaneously broken down to \( SU(2) \) by the vevs (8). Hence five pseudo-Goldstone bosons emerge, of which one becomes the longitudinal component of the massive \( U(1)_E \) gauge boson. The four remaining physical degrees of freedom are identified with the SM Higgs doublet. Below the scale \( f \) of the \( SU(3) \) breaking we can work with the non-linear realization of the \( SU(3) \) symmetry and ignore all heavy scalars from the Higgs sector. In this approach the Higgs triplets are parametrized as

\[
\mathcal{H}_u = e^{i\Pi/f} \begin{pmatrix} 0 \\ f_u \end{pmatrix}, \quad \mathcal{H}_d = (0, f_d) e^{-i\Pi/f}
\]

(11)

where\(^7\)

\[
i\Pi = \begin{pmatrix} 0_{2\times 2} & H \\ -H^\dagger & 0 \end{pmatrix}
\]

(12)

which leads to

\[
\begin{pmatrix} H_u \\ S_u \end{pmatrix} = \sin \beta \begin{pmatrix} H^{\sin(|H|/f)} \\ f \cos(|H|/f) \end{pmatrix}, \quad \begin{pmatrix} H_d \\ S_d \end{pmatrix} = \cos \beta \begin{pmatrix} H^{* \sin(|H|/f)} \\ f \cos(|H|/f) \end{pmatrix}
\]

(13)

where \(|H| \equiv \sqrt{|H^\dagger H|} \). In the following we will keep track only of the real neutral component of the Higgs doublet \( H \), i.e. we will substitute \((0, h)^T\) for \( H \) and \( h \) for \(|H|\). With this parametrization it is explicit that the \( SU(3) \) preserving Higgs potential (7) does not contribute to the potential of the SM Higgs doublet \( H \). At the tree level the SM Higgs potential has only the quartic part which arises from the electroweak \( D \)-terms (10) explicitly breaking global \( SU(3) \):

\[
V_{\text{tree}} = \frac{1}{8} (g_2^2 + g_y^2) \cos^2(2\beta) f^4 \sin^4(h/f) .
\]

(14)

\(^7\)The scalar field associated with the broken generator \( T^8 \) of \( SU(3) \) becomes the longitudinal component of the massive \( Z' \) boson and needs not be included in \( \Pi \).
3 One loop SM Higgs potential

Since the $SU(3)$ symmetry is only approximate, corrections to the SM Higgs potential appear at loop level. We therefore calculate the one-loop effective potential $V = V_{\text{tree}} + \Delta V_{1\text{-loop}}$ in terms of the Lagrangian parameters renormalized at the scale $\Lambda_S$. In a supersymmetric model such a calculation may be equivalently viewed as the calculation in terms of the bare parameters and with the momentum cut-off $\Lambda_S$. In a consistent one-loop calculation of the effective potential the $SU(3)$ symmetric parameters defined by eq. (5), (7), (4), (2) and (1), must be used. An $SU(3)$ splitting of these parameters is generated at one loop level, too, by the two sources of explicit $SU(3)$ breaking: the nonzero electroweak gauge couplings $g_2$, $g_y$ and the $\mu_T$ and $\mu_B$ terms. It enters the effective potential only in the two-loop approximation. For the mechanism of the electroweak symmetry breaking we propose in this paper such higher order effects must be negligible and this constrains the scale $\Lambda_S$ at which the soft mass terms are generated. As we shall see, for squarks masses $\mathcal{O}(10 \text{ TeV})$ our mechanism works anyway only for $\Lambda_S \sim \mathcal{O}(100 \text{ TeV})$, consistently with the above requirement. Moreover, we assume that the $SU(3)$ breaking corrections to the Yukawa couplings also vanish above $\Lambda_S$, i.e. that above $\Lambda_S$ the model described by eqs. (2), (4), (5), (6) and (7) is embedded in some $SU(3)$ invariant theory as in [15].

Before presenting our results for the one-loop effective potential we recall the structure of the effective potential in other models. In the MSSM quadratically divergent corrections to the Higgs mass parameter are absent at any order of perturbation theory due to supersymmetry. Logarithmically divergent contribution is determined by $\text{STr} \mathcal{M}^4$ and depends quadratically on the supersymmetry breaking mass parameters. It consists of two parts: one proportional to the top Yukawa coupling quadratically dependent on the sfermion and Higgs fields soft mass parameters and one proportional to the gauge couplings and quadratically dependent on gaugino masses. In the non-supersymmetric Little Higgs Models [8] global symmetries forbid quadratically divergent corrections to the Higgs mass parameter at one-loop (but such divergences are present already at two-loops). In the language of the effective potential, the mass matrix squared $\mathcal{M}^2$ in these models does not depend on the SM Higgs field $h$ and, what is important, the cancellations occur independently in fermionic and bosonic sectors. Logarithmically divergent corrections proportional to the top Yukawa coupling are generically present already at the one-loop level and they, as well as the two-loop quadrati-
cally divergent ones, require some ultraviolet completion of the models at the scales of order 10 TeV. Finally, there are models with “hard” supersymmetry breaking but such, that the leading contribution to the effective potential proportional to the top quark Yukawa coupling is finite at one-loop \([7]\). However quadratically divergent contributions appear from \(D\)-terms \([19]\) and at higher orders and require a low cut-off.

In the model discussed in this paper the situation is still different and more elegant, as long as all \(SU(3)\) breaking quantum effects in the parameters entering \(\Delta V_{1\text{-loop}}\) can be neglected (i.e. for a low scale \(\Lambda_S\)). Because the model is supersymmetric, all quadratic divergences are absent to all orders in perturbation theory. Moreover, the SM Higgs potential must be proportional to some parameter that breaks \(SU(3)\) symmetry and also leads to supersymmetry breaking for \(h \neq 0\) (in the sense of forcing some \(F\)- or \(D\)-terms to be nonvanishing for \(h \neq 0\)). The most important consequence is that the contribution proportional to the top (and also bottom) Yukawa coupling is finite at one-loop and only logarithmically sensitive to the sfermion mass scale \(m_\tilde{q}^2\). Indeed, since the \(Y_T\) Yukawa coupling and stop soft mass terms are \(SU(3)\) symmetric the Higgs potential should be proportional to \(Y_T^2 \mu_T^2\). But \(\mu_T\) does not lead to supersymmetry breaking for \(h \neq 0\), therefore such a contribution cannot occur in \(\text{STr}\mathcal{M}^4\). It can only enter the finite part of the Higgs potential such as \(\Delta V_{1\text{-loop}} \sim Y_T^2 \mu_T^2 h^2 \log(m_\tilde{q}/f)\). The mild logarithmic sensitivity to \(m_\tilde{q}\) allows us to raise the squark masses far above 1 TeV without introducing too much fine-tuning. Logarithmically divergent contributions that arise at one loop are suppressed by electroweak gauge couplings. In our model with a low cutoff scale \(\Lambda_S\), these will be of the same order of magnitude as the finite part dependent on the top Yukawa couplings.

Let us first discuss the top/stop contribution to the one-loop correction \(\Delta V_{1\text{-loop}}\) to the SM Higgs potential in our model. As usually, it is given by:

\[
\Delta V_{1\text{-loop}} = \frac{1}{64\pi^2} \text{STr} \left\{ \mathcal{M}^4 \left( \log \frac{\mathcal{M}^2}{\Lambda_S} - \frac{3}{2} \right) \right\} . \tag{15}
\]

where \(\mathcal{M}^2\) is the field dependent mass squared matrix of the theory. For a nonzero background value \(h\) of the SM Higgs field the contribution of the top sector to the fermionic mass matrix reads

\[
\mathcal{L}_{\text{mass}} = -(t^c, T^c) \mathcal{M}_{\text{tops}} (t, T)^T + \text{H.c.} \quad \mathcal{M}_{\text{tops}} = \begin{pmatrix} Y_T f_u s_h & Y_T f_u c_h \\ 0 & \mu_T \end{pmatrix} \tag{16}
\]
where we have used the abbreviations $s_h = \sin(h/f)$, $c_h = \cos(h/f)$. Hence,

$$
\mathcal{M}_{\text{tops}}^2 \equiv \mathcal{M}_{\text{tops}}^{\dagger} \mathcal{M}_{\text{tops}} = \begin{pmatrix}
Y_T^2 f_u^2 s_h^2 & Y_T^2 f_u^2 s_h c_h \\
Y_T^2 f_u^2 s_h c_h & \mu_T^2 + Y_T^2 f_u^2 c_h^2
\end{pmatrix} .
$$

(17)

In the stop sector

$$
\mathcal{L}_{\text{mass}} = -(\tilde{t}^*, \tilde{T}^*, \tilde{c}^*, \tilde{T}^c) \mathcal{M}_{\text{stops}}^2 (\tilde{t}, \tilde{T}, \tilde{c}, \tilde{T}^c)^T
$$

(18)

where for vanishing gauge couplings $\mathcal{M}_{\text{stops}}^2$ takes the form

$$
\begin{pmatrix}
m_Q^2 + Y_T^2 f_u^2 s_h^2 & Y_T^2 f_u^2 s_h c_h & (Y_T f_d \mu - A_T f_u) s_h & 0 \\
Y_T^2 f_u^2 s_h c_h & m_Q^2 + \mu_T^2 + Y_T^2 f_u^2 c_h^2 & (Y_T f_d \mu - A_T f_u) c_h & 0 \\
(Y_T f_d \mu - A_T f_u) s_h & (Y_T f_d \mu - A_T f_u) c_h & m_{U_3}^2 + Y_T^2 f_u^2 & Y_T f_u \mu_T c_h \\
0 & 0 & Y_T f_u \mu_T c_h & m_{T^c}^2 + \mu_T^2
\end{pmatrix}
$$

(19)

Let us denote the two eigenvalues of the $(h)$ dependent top mass matrix squared $\mathcal{M}_{\text{tops}}^2$ by $t_1^2$ and $t_2^2$ and the four eigenvalues of the stop mass squared matrix $\mathcal{M}_{\text{stops}}^2$ by $m_{\tilde{q}}^2 + s_i^2$, where $m_{\tilde{q}}$ is the overall scale of the soft supersymmetry breaking in the stop sector. In the following we assume\(^8\) that $m_{\tilde{q}}^2 \gg s_i^2$ and expand the one-loop effective potential in powers of $1/m_{\tilde{q}}^2$. Up to terms of order $1/m_{\tilde{q}}^2$ we can rewrite the top/stop sector contribution as:

$$
\Delta V_{1\text{-loop}} = \frac{N_c}{32 \pi^2} \left\{ \left[ \text{Tr} \mathcal{M}_{\text{tops}}^4 - 2 \text{Tr} \mathcal{M}_{\text{tops}}^4 \right] \log \frac{m_{\tilde{q}}^2}{\Lambda_S^2} - 2 \sum_{i=1}^2 t_i^4 \left[ \log \frac{t_i^2}{m_{\tilde{q}}^2} - \frac{3}{2} \right] \right\} + \text{const.} + O(1/m_{\tilde{q}}^2)
$$

(20)

where the color factor $N_c = 3$. We also used the fact that both $\text{Tr} \mathcal{M}_{\text{tops}}^4$ and $\text{Tr} \mathcal{M}_{\text{stops}}^4$ do not depend on $h$ as a consequence of $SU(3)$ symmetry. As explained before, $\text{Tr} \mathcal{M}_{\text{tops}}^4 - 2 \text{Tr} \mathcal{M}_{\text{tops}}^4$ does not depend on $Y_T$. Therefore, in order to calculate the part of $\Delta V_{1\text{-loop}}$ proportional to the Yukawa couplings in the limit $m_{\tilde{q}}^2 \gg s_i^2$ it is sufficient to find the eigenvalues of the top mass matrix squared $\mathcal{M}_{\text{tops}}^2$.\(^8\)

\(^8\)The reasoning used here implicitly assumes that the soft mass terms in the stop sector are almost degenerate, but as long as the entries mixing left and and right stops in the matrix $\mathcal{M}_{\text{stops}}^2$ are small, the results are independent of this assumption. This can be checked by directly computing the eigenvalues of $\mathcal{M}_{\text{stops}}^2$.\(^9\)

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\(^9\)
The necessary eigenvalues of the top mass matrix are given by:

\[ t_{1,2}^2 = \frac{1}{2} \left[ Y_T^2 f_u^2 + \mu_T^2 \pm \sqrt{(Y_T^2 f_u^2 + \mu_T^2)^2 - 4\mu_T^2 Y_T^2 f_u^2 s_h^2} \right] \]  \hspace{1cm} (21)

The lower sign corresponds to the ordinary top quark mass, which approximately equals \( m_t \approx y_t f \sin(h/f) \) where

\[ y_t = \frac{\mu_T Y_T \sin \beta}{\sqrt{Y_T^2 f_u^2 + \mu_T^2}} \]  \hspace{1cm} (22)

is the physical top quark Yukawa coupling. The higher eigenvalue

\[ t_2^2 \approx m_T^2 \equiv Y_T^2 f_u^2 + \mu_T^2 \]  \hspace{1cm} (23)

corresponds to the mass squared of the \( SU(3) \) fermionic partner of the top quark. Inserting (22) in the last term in the curly brackets in (20) we can determine the contribution to the effective potential proportional to the top Yukawa coupling \( Y_T \). Expanding in powers of \( \sin(h/f) \) to the quartic order we find:

\[ \Delta V_{1-\text{loop}}^{(1)} = -\frac{3}{8\pi^2} Y_T^2 \mu_T^2 f^2 \sin^2 \beta \left( \log \frac{m_q^2}{m_T^2} + 1 \right) \sin^2(h/f) - \frac{3}{8\pi^2} \frac{Y_T^2 \mu_T^4 \sin^4 \beta}{m_T^4} f^4 \left[ \log \left( \frac{Y_T \mu_T f \sin \beta}{m_T^2} \sin(h/f) \right) + \frac{1}{4} \right] \sin^4(h/f) \]  \hspace{1cm} (24)

The first term of (24) gives the \( Y_T \) dependent one-loop contribution to the SM Higgs mass squared \( m_H^2 \). It is negative, which enables the electroweak symmetry breaking. Moreover, as advertised, it is not proportional to \( m_q^2 \) but rather to the square of the \( SU(3) \) breaking supersymmetric parameter \( \mu_T \), which may be much smaller than the soft supersymmetry breaking scale \( m_q \). The analogous contribution from the bottom-sbottom sector, can be easily obtained by using the substitutions: \( Y_T \rightarrow Y_B, \mu_T \rightarrow \mu_B, f_u \rightarrow f_d, \sin \beta \rightarrow \cos \beta. \) It is however small compared to the top-stop sector because \( Y_B \cos \beta \ll Y_T \sin \beta \).

Terms in \( \Delta V_{1-\text{loop}} \) logarithmically depending on the scale \( \Lambda_S \), i.e. proportional to \( \text{STr} \mathcal{M}^4 \) arise from two different sources which depend on the \( SU(3) \) breaking SM gauge interactions. One is the gauge coupling dependent contribution to the sfermion mass matrices, that enters via \( \text{Tr} \mathcal{M}^4_{\text{stops}} - 2\text{Tr} \mathcal{M}^4_{\text{tops}} \).
in (20). In this way we get the $h$ dependent contribution

$$\Delta V_{1-\text{loop}}^{(2)} = \frac{1}{32\pi^2} g_y^2 \cos(2\beta) f^2 \text{Tr}[Y m^2] \log \frac{\Lambda^2_S}{m_{\tilde{q}}^2} \sin^2(h/f)$$

$$- \frac{3}{64\pi^2} (g_2^2 + g_y^2) \cos(2\beta) \sin^2(\beta f^4 Y^2_T \log \frac{\Lambda^2_S}{m_{\tilde{q}}^2} \sin^4(h/f))$$

(25)

where the trace runs over all sfermions charged under $U(1)_Y$. We have dropped the terms with fourth powers of the gauge couplings as they are small (potentially large terms $\propto g_2^2$ cancel out between the up and down type squark contributions). For non-universal soft breaking scalar masses, when $\text{Tr}[Y m^2] \neq 0$, this term depends quadratically on the scale $m_{\tilde{q}}$. However it is suppressed by the small coupling $g_y$. It is also interesting to notice that in contrast to the Little Higgs models, in which a complicated gauge structure is used to cancel the $g_2^2 \Lambda^2$ contribution to $m^2_{H}$, here it is the supersymmetric structure alone which ensures the absence of the $g_2^2 m_{\tilde{q}}^2$ piece.

The complete $h$ dependent contribution of the gaugino/gauge boson and higgsino/Higgs boson sectors to $\Delta V_{1-\text{loop}}$ is rather lengthy. It is most relevant parts can be approximated by

$$\Delta V_{1-\text{loop}}^{(3)} = \frac{1}{64\pi^2} \left[ 3(3g^2 + g_y^2) \mu^2 + 12g_2^2 M_2^2 + 4g_y^2 M_y^2 \
- 4 \sin 2\beta (3g_2^2 M_2 + g_y^2 M_y) \mu \
- g_2^2 (3g^2 + g_y^2) \cos^2(2\beta f^2) \right] f^2 \log \frac{\Lambda^2_S}{\mu^2} \sin^2(h/f)$$

$$- \frac{1}{64\pi^2} g_E^2 (g_2^2 + g_y^2) f^4 \log \frac{\Lambda^2_S}{\mu^2} \sin^4(h/f)$$

(26)

We have neglected the contributions proportional to the soft mass terms of the Higgs fields and those with electroweak gauge couplings $g_2$ and $g_y$ in the fourth power. We have also approximated $\mathcal{M}^2$ under the logarithm in (15) by $\mu^2$.

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In order to compute the Higgs sector contribution to $\Delta V_{1-\text{loop}}$, taking into account also heavy degrees of freedom necessary for vanishing of the $h$ dependent part of $\text{STr}\mathcal{M}^2$, we split the neutral CP-even components of the fields in (7) and (10) into $H_{u,d}^0 + \text{quantum fluctuations}$, $S_{u,d} + \text{quantum fluctuations}$, compute the mass squared matrices of the quantum neutral CP-even and CP-odd and charged Higgs fields and only at the end substitute $H_{u,d}^0 \rightarrow f_{u,d}s_h$, $S_{u,d} \rightarrow f_{u,d}c_h$. 

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\text{10}
4 $SU(3)$ and electroweak breaking

We turn to analyzing in a more quantitative way radiatively induced $SU(3)$ and electroweak breaking. Our basic assumption is that the sfermion soft masses set the largest mass scale in the model. Breaking of the gauge $U(1)_E$ and the global $SU(3)$ symmetries occurs if $m_1^2 m_2^2 < m_3^4$ in eq. (7). This condition is obviously satisfied for $m_1^2 > 0$ and $m_2^2 < 0$. Similarly as in the MSSM, the quantum corrections generated primarily by the top/stop loops can make the parameter $m_{H_u}^2$ negative. In the present model this effect will be enhanced by the largeness of soft squark masses. The leading one-loop effect of the renormalization of $m_{H_u}^2$ between the scales $\Lambda_S$ and $m_{\tilde{q}}$ is given by

$$m_{H_u}^2(m_{\tilde{q}}) \approx m_{H_u}^2(\Lambda_S) - \frac{1}{(4\pi)^2} \left[ g_E^2 \text{Tr}(E m^2) + 6 Y_T^2 (m_Q^2 + m_U^2) \right] \log \frac{\Lambda_S}{m_{\tilde{q}}} \tag{27}$$

We have not displayed here the much smaller $g_y^2 \text{Tr}(Y m^2)$ and the gaugino contribution. Moreover, since we will assume that $m_{H_u}^2(\Lambda_S) \ll m_{\tilde{q}}^2(\Lambda_S)$, we have dropped the Higgs soft masses in the second term of eq. (27) as well. With this assumption, as long as $\text{Tr}(E m^2)$ is not too large and negative, the soft mass $m_{H_u}^2$ is driven towards a negative value. For very large $\tan \beta \sim 50$ one should check whether $m_{H_d}^2$ is not driven to negative values too but even in that case the situation can be improved for positive $\text{Tr}(E m^2)$.

For $\tan \beta \geq 5$ and $Y_T^2 \sim 1.3$ (see below) the scale $f$ of the $SU(3)$ breaking can be estimated as:

$$g_E^2 f^2 \approx -m_2^2 \approx -\mu^2 + 0.1 m_{\tilde{q}}^2 \log \frac{\Lambda_S}{m_{\tilde{q}}} \tag{28}$$

Since the $U(1)_E$ gauge coupling $g_E$ does not contribute to the SM Higgs mass it does not have to be small and in the following we assume $g_E \approx 1$. This choice minimizes the fine-tuning necessary for $SU(3)$ breaking. Moreover, we shall soon see that increasing $f$ increases fine-tuning in the electroweak symmetry breaking. Therefore adopt the minimal phenomenologically allowed value $f \sim 2.5 \text{ TeV}$ [16]. Then eq. (28) sets the lower bound on the squark masses $m_{\tilde{q}} > 8/\log^{1/2}(\Lambda_S/m_{\tilde{q}}) \text{ TeV}$. It also correlates the values of $\mu$, $m_{\tilde{q}}$ and $\Lambda_S$ so that $\mu^2 \sim 0.1 m_{\tilde{q}}^2 \log(\Lambda_S/m_{\tilde{q}})$. Essentially no fine-tuning is required for $f \approx 2.5 \text{ TeV}$ as long as $m_{\tilde{q}} \lesssim 10 \text{ TeV}$ and $\Lambda_S \sim 100 \text{ TeV}$. A fine-tuning of order 1% is required for $m_{\tilde{q}} \sim 50 \text{ TeV}$.

In the second stage we study breaking of the electroweak symmetry. The leading contributions to the SM Higgs potential are those of eq. (24), eq. (25)
and eq. (26). The resulting SM Higgs doublet potential has the approximate form:

\[ V = m_{H}^{2}h^{2} + \lambda h^{4} + \kappa h^{4} \log \frac{h^{2}}{m_{T}^{2}} \]  

At tree-level \( m_{H}^{2} = 0 \), \( \lambda = \frac{1}{8}(g_{2}^{2} + g_{y}^{2}) \cos^{2}(2\beta) \) and \( \kappa = 0 \). At one loop \( \Delta V^{(1)}_{1-\text{loop}}, \Delta V^{(2)}_{1-\text{loop}} \) and \( \Delta V^{(3)}_{1-\text{loop}} \) contribute to \( m_{H}^{2} \):

\[ m_{H}^{2} \approx -\frac{3}{4\pi^{2}}g_{y}^{2}m_{T}^{2} \log \frac{m_{T}^{2}}{m_{T}^{2}} + \frac{g_{y}^{2}}{16\pi^{2}} \cos(2\beta) \text{Tr}[Y m^{2}] \log \frac{\Lambda_{S}}{m_{T}} \] 

In eq. (30) the Higgs mass parameter at the electroweak scale is expressed in terms of the other parameters renormalized at the scale \( \Lambda_{S} \). In the first term we have used the tree-level relations (22) and (23) to express \( Y_{T}(\Lambda_{S}) \) in terms of the top quark Yukawa coupling \( y_{t} \), consistently with the systematic one-loop calculation of \( \Delta V^{(1)}_{1-\text{loop}} \). Then for \( m_{t} = (178 \pm 4.3) \text{ GeV} \) and the corresponding \( \overline{\text{MS}} \) mass 169 GeV we have

\[ \frac{\mu_{T}Y_{T} \sin \beta}{\sqrt{Y_{T}^{2}f_{u}^{2} + \mu_{T}^{2}}} = 0.97 \]  

which relates the values of \( Y_{T}(\Lambda_{S}) \) and \( \mu_{T}(\Lambda_{S}) \). We can now analyze the contribution of the first term on the rhs of eq. (30). It is quadratically dependent on the mass \( m_{T} \) of the heavy top quark partner. For \( f = 2.5 \text{ TeV} \), taking into account the relation (31), \( m_{T} \) has a minimum for \( \mu_{T} \approx 3 \text{ TeV} \) and \( Y_{T} \approx 1.35 \) at which the contribution to the Higgs mass parameter is \( m_{H}^{2} \approx -1.6 \text{ TeV}^{2} \). We choose this value of \( \mu_{T} \) as it minimizes the fine-tuning in electroweak breaking. Note that these arguments do not depend on the value of stop masses \( m_{\tilde{q}} \). Also perturbativity of \( Y_{T} \), i.e. \( Y_{T}^{2}/4\pi < 1 \) up to the scale \( \Lambda_{S} \), does not impose any new bound.

Since for the electroweak breaking we need \( m_{H}^{2} \sim 0.01 \text{ TeV}^{2} \) the finite Yukawa contribution is by itself too large by a factor of hundred. For our mechanism to work, it must be partly cancelled by the other contributions in eq. (30). Thus, our first conclusion is that fine-tuning at 1% level is required.
here. However the squark masses can be large. Note that in the MSSM for squark masses $m_{\tilde{q}} \sim 10 \text{ TeV}$ the fine-tuning is of order 0.01%.

Before discussing the remaining terms in eq. (30) it is important to note that the scale dependence of the top quark Yukawa coupling is relatively strong. Although this is formally a two-loop effect, it may introduce important corrections to our systematic one-loop calculation. To estimate this uncertainty we may use eq. (22) for $y_t(\Lambda_S)$ rather than $y_t(m_t)$, where $y_t(\Lambda_S)$ is obtained by evolving the top Yukawa coupling from the scale $m_t$ to $\Lambda_S$ using the appropriate RG equations. Then the corresponding numbers are $\mu_T \approx 3 \text{ TeV}$, $Y_T \approx 1.15$ and the contribution (24) to the mass parameter is $m_H^2 \approx -1.2 \text{ TeV}^2$. The fine-tuning is then slightly smaller but still of order 1%. Of course, using the present procedure for extracting $y_t(\Lambda_S)$ from the physical top quark mass we should, for consistency, calculate the effective potential at two loops. However, since the loop corrections to the top Yukawa coupling are likely the most important, the comparison of the two methods is at least a good estimate of the uncertainty of our results.

Turning now to the other contributions in eq. (30), the second one in importance is the $\mu^2$ dependent part of the last term. Indeed, because eq. (28) fixes $\mu^2$ as a function of $m_{\tilde{q}}$ and $\Lambda_S$, this contribution reads

\[
10^{-3} m_{\tilde{q}}^2 \log^2(\Lambda_S/m_{\tilde{q}}) \text{ and for heavy sfermions it is similar in magnitude (but with opposite sign) to the Yukawa contribution. In fact, the requirement that it does not make } m_H^2 \text{ positive sets a stringent bound on the cutoff scale } \Lambda_S, \Lambda_S \lesssim m_{\tilde{q}} e^3 \text{ TeV} / m_{\tilde{q}}. \text{ For } m_{\tilde{q}} = 10 \text{ TeV, we get } \Lambda_S \lesssim 250 \text{ TeV}. \text{ This is a post-factum justification of our assumption that the cutoff scale is not high. As for the remaining contributions in eq. (30), the term proportional to } \text{Tr}[Y m^2] \text{ is small even for non-universal squark masses. In that case we would expect } \text{Tr}[Y m^2] \sim m_{\tilde{q}}^2, \text{ but it is strongly suppressed by the loop factor and the small hypercharge gauge coupling. The positive contribution of gaugino masses to } m_H^2 \text{ can be larger and can complement the } \mu^2 \text{ contribution in canceling the too large negative contribution of (24). For } \Lambda_S \approx 100 \text{ TeV the requirement that } m_H^2 < 0 \text{ puts the upper bound } M_2 \lesssim 4.5 \text{ TeV.}

Collecting all the relevant contributions to $m_H^2$ we see that with a 1% fine-tuning the necessary value for this parameter $m_H^2 \sim (10 \text{ GeV})^2$ is easy to obtain for $m_{\tilde{q}} \sim \mathcal{O}(10 \text{ TeV})$ and $\Lambda_S \sim \mathcal{O}(100 \text{ TeV})$. 

13
For the one-loop contributions to the quartic couplings $\lambda$ and $\kappa$ we get\footnote{The corrections to the quartic coupling $\lambda$ contained in the $\sin^2(h/f)$ terms in eq. (24), eq. (25) and eq. (26), once the large contribution to $m_H^2$ proportional to $Y_T^2\mu^2$ is canceled against other contributions, becomes of order $m_H^2/6f^2 \sim -(100 \text{ GeV})^2/6(2.5 \text{ TeV})^2$ and is negligible.}

\[
\lambda = \frac{1}{8}(g_y^2 + g_2^2)\cos^22\beta - \frac{3}{8\pi^2}y_t^4 \left[ \log(y_t) + \frac{1}{4} \right] - \frac{3}{64\pi^2}Y_T^2 \cos(2\beta) \sin^2\beta \log \frac{\Lambda}{m_{\tilde{q}}} - \frac{1}{16\pi^2}g^2 g_y^2 \log \frac{\Lambda^2}{\mu^2}
\]

\[
\kappa = -\frac{3}{16\pi^2}y_t^4
\]

From this we can estimate the SM Higgs mass:

\[
M^2_h = 2v^2 \left( \lambda + \frac{3}{2} \kappa + \kappa \log(v^2/2m_T^2) \right)
\]

(33)

The term proportional to $\kappa \log(v^2/2m_T^2)$ turns out to be the dominant correction to the SM Higgs mass. Its effect is to change the tree-level prediction for the Higgs mass, and to raise it above $M_Z$. Note that $\kappa$ in eq. (32) is set by the top quark Yukawa coupling $y_t$ and so this contribution depends only logarithmically on $m_T$ (and thus on $\mu_T$ and $f$). Inserting the numbers we get the Higgs mass (for $\tan\beta \geq 5$):

\[
M_h \approx 120 - 135 \text{ GeV}.
\]

(34)

This is the prediction of present model. The lower value is obtained when the higher order effects associated with the RG running of the top quark Yukawa coupling from $m_t$ to $\Lambda_S$ is taken into account. For this reason we believe it approximates slightly better the true Higgs mass in our model. There is of course also the dependence of $M_h$ on $\tan\beta$, which becomes significant for $\tan\beta \leq 5$.

5 Conclusions

In this paper we have discussed electroweak symmetry breaking in a supersymmetric model in which the SM Higgs doublet is a pseudo-Goldstone boson of $SU(3)$ global symmetry. The Higgs mass parameter is generated at
one loop level by two different, moderately fine-tuned sources of the global symmetry breaking. The mechanism works well for heavy sfermion masses $m_{\tilde{q}} \sim 10 \text{ TeV}$, but the fine-tuning is, nevertheless, of order 1%, two orders of magnitude less than in the MSSM with similar sfermion masses. The scale $\Lambda_S$ at which supersymmetry breaking is mediated to the visible sector has to be low, of order $100 \text{ TeV}$.

Several of the phenomenological consequences of our scenario are similar to those of the so-called split supersymmetry model of Arkani-Hamed and Dimopoulos [2]. The sfermions as well as additional scalars in the Higgs sector should be beyond reach of the LHC. Also, the heavy top quark partner has mass $m_T \gtrsim 4 \text{ TeV}$, too large to be seen in the LHC [20]. Chargino and neutralino masses are more model dependent but they should be smaller than 4 TeV. The important difference with the split supersymmetry scenario is that although the gluino mass is not bounded by the mechanism of the electroweak symmetry breaking, gluino is not expected to be long lived because squarks are not so heavy. The Higgs boson mass is predicted (for $\tan \beta \gg 1$) in the range $120 - 135 \text{ GeV}$, whereas it is in the range $130 - 170 \text{ GeV}$ in split supersymmetry [21]. Another potential signature of our mechanism is the presence of the $Z'$ gauge boson with $m_{Z'} \sim 3 \text{ TeV}$, which should easily be discovered in the LHC [20].

**Acknowledgments**

S.P. would like to thank the Physics Departments at the University of Bonn and at the University of Hamburg and the APC Institute at the University Paris VII for their hospitality. His visits to the University of Bonn and at the University of Hamburg were possible thanks to the research award of the Humboldt Foundation. P.H.Ch. was partially supported by the RTN European Program HPRN-CT-2000-00152 and by the Polish KBN grant 2 P03B 040 24 for years 2003–2005. A.F. and S.P. were partially supported by the RTN European Program HPRN-CT-2000-00148 and by the Polish KBN grant 2 P03B 129 24 for years 2003–2005.
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