Acceleration time scale at ultrarelativistic shock waves

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Abstract
The first-order cosmic ray acceleration at ultrarelativistic shocks is investigated using the Monte Carlo method. We apply a method of discrete particle momentum scattering as a model of particle pitch angle diffusion to reproduce highly anisotropic conditions at the shock wave. SHocks with Lorentz factors \(\gamma\) up to 320 and varying magnetic field inclinations \(\psi\) are considered. Values of diffusion coefficients upstream in the point where energy spectral indices stabilize to the limit 2.2 were calculated. The obtained acceleration time does not depend on shock conditions.

Introduction.
Till quite recently the discussion of first order Fermi acceleration at relativistic shocks was restricted to small Lorentz \(\gamma\)-factors (for review see Ostrowski[1] and Kirk[2]). Bednarz and Ostrowski[3] found a simple mechanism that allows particles to be accelerated at shock waves with large Lorentz factors. It implies that an inclination of formed spectrum does not depend on downstream but only on upstream conditions and a shock velocity. The mechanism is a natural process accelerating particles in astronomical objects where such shocks occur.

Among these phenomena where ultrarelativistic shocks are anticipated are gamma-ray bursts. They are non-thermal bursts of low energy \(\gamma\)-rays. Typical GRB lasts for about 10 sec. Their broad spectra usually peak between a few 100 keV and a few MeV (for review see Piran [4]). The Lorentz factor of an expanding fireball larger then 300 is required in order to the pair production optical depth to be sufficiently small (Baring [5]). Recently the host galaxy with redshift 3.42 was indentified for GRB971214 (Kulkarni et al. [6]) which confirms the suggestion that they originate at cosmological sources (Meegan et al. [7]).

Numerical simulations of the acceleration process.
In simulations we follow the procedure used in Bednarz and Ostrowski [8]. Seed particles are injected downstream the shock with the same initial energy. Theirs trajectories are derived in homogenous magnetic field \(B_0\) perturbed by inhomogeneities which are simulated by particle momentum scattering within a cone with small angular opening \(\Delta\Omega\) less than the particle anisotropy \(\sim 1/\gamma\) (see Ostrowski [9]). A particle is excluded from the simulation either if it escapes through the free-escape boundary placed far downstream the shock or if it reaches a time or energy larger than the assumed upper limit. These particles are replaced with ones arising from splitting the remaining high-weight particles with preserving their physical parameters. They are not replaced if they reach upper limit of time.
Figure 1: The simulated spectral indices $\sigma$ for particles accelerated at shocks with different upstream diffusion represented by $\log_{10}(\kappa_{\perp}/\kappa_{\parallel})$. Downstream we assume $\kappa_{\perp} = 0$. Perpendicular dashed line points to ‘boundary point’.

Figure 2: The simulated acceleration times $t_{acc}$ for particles accelerated at shocks with different Lorentz factors $\gamma$. The considered upstream magnetic field inclination are represent by different lines with indicated simulation errors. The $t_{acc}$ value near 1.5 is for $\psi = 30^\circ$ and results from large simulation error at $\gamma = 20$. 
Computations are performed in the respective - upstream or downstream - plasma rest frame. If particles cross the shock their parameters are transformed to the shock rest frame. The respective contribution divided by their velocity component normal to the shock added to momentum bin forms particle spectra with shifting cut-off point toward higher energies. These spectra allow us to derive their inclination and acceleration time (see Bednarz and Ostrowski [8]). Time measured in the shock rest frame in downstream $r_g/c$ units was transformed to the downstream plasma rest frame ($r_g$ is a particle gyroradius). That can be done because we know the ratio of mean time that particle spends upstream to mean time it spends downstream. This value is small and due to this the time measured in the shock rest frame is of 1.05 factor shorter than downstream for large $\gamma$.

In this paper we simulate different conditions upstream and downstream the shock, represented by cross-field diffusion coefficient $\kappa_\perp$ and the parallel diffusion coefficient $\kappa_\parallel$.

**Results.**

We take into consideration shocks with the Lorentz factors equal $\gamma = 20, 40, 80, 160, 320$ and magnetic field inclinations upstream the shock $\psi = 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ$.

At first we consider downstream conditions without magnetic fluctuations. We search by simple data inspection as in Fig. 1 for a value $\kappa_\perp/\kappa_\parallel$ where the spectral index reaches limiting value of 2.2. It yields a fit:

$$\frac{\kappa_\perp}{\kappa_\parallel} = 0.25 \cdot \gamma^{-1.22} \psi$$

The fit is good in full considered parameter space.

The acceleration time $t_{acc}$ for the boundary points is presented in Fig. 2. where one can seen the lack of any change with $\psi$. The figure exhibits also a slow $t_{acc}$ decrease from 1.2 to approach the value 1.1 at $\gamma = 80$. We observe small tendency of $t_{acc}$ to grow if $\sigma$ increases to 2.3-2.4 and no further change if $\kappa_\perp/\kappa_\parallel$ grows. In the next step we introduce magnetic field fluctuations downstream to be $\kappa_\perp/\kappa_\parallel = 7.1 \cdot 10^{-4}$ or $1.1 \cdot 10^{-1}$. $t_{acc}$ behaves nearly as in the case without fluctuations, with only a small reduction (circa 0.1) for the larger fluctuation. Thus the resulting acceleration times are always approximately $t_{acc} = 1$ (in units of $r_g/c$ downstream the shock).

**Discussion.**

We have obtained the result that is not surprising if we recall the way particles are accelerated (Bednarz and Ostrowski [3]) and that they do not spend much time upstream.

We expect our results can be applied in modelling relativistic shock waves expected to occur in gamma-ray burst sources and at pulsar wind terminal shocks. The model of Kenel and Coroniti [10] assumes for Crab Nebula initial flow with Lorentz factor equal to $10^6$. One should note that the acceleration time is short, close to the value of $r_g/c$ downstream the shock. However, it is not very short with energy gains $\sim \gamma^2$ at single particle-shock interaction. It results from the fact that it is not possible to reflect a particle from the ultrarelativistic shock (cf., Begelman and Kirk [11], Ostrowski [1]).
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