Cooperative effects on Optical forces- Dicke’s bullet

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Abstract
We investigate the cooperative effects on optical forces in a system of $N$ two level atoms confined to a volume of dimension less than $\lambda^3$, where $\lambda$ is radiation wavelength and driven by a coherent radiation field with a spatial profile like Laguerre-Gaussian or ideal Bessel beam. We show a dramatic enhancement on optical forces as well as the angular momentum imparted to the atom by a factor of $N^2$.

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Recently, there is a considerable interest in cooling and trapping of alkali atoms at low temperatures and subsequent observation of Bose-Einstein (BEC) condensation\cite{1,2} which has a property of a generating laser like source of matter waves. As demonstrated by S. Inoyu et al., these neutral matter waves confined in a cigar shaped BEC\cite{3} show a highly directional super radiant Rayleigh scattering when pumped by an off resonant laser beam\cite{4}. This is attributed to a cooperative effect due to $N$ atoms spaced to within a wavelength of the laser and is similar (though not identical) to Dicke’s superradiance\cite{5} phenomenon. It required nearly twenty years to observe the superradiant intensity which is proportional to $N^2$ accompanied by a subsequent reduction in life time\cite{6}. There has since been a resurgence of Dicke’s idea after its observation in atomic system of two ions by DeVoe and Brewer\cite{7}. They observed a transition rate of two ions ($\Gamma(R)$) as a function of separation($R$) and found that $\Gamma(R)/\Gamma_0 \approx 2$ when $R \to 0$; where $\Gamma_0$ is transition rate of a single isolated ion\cite{7}. Very recently, a clear experimental demonstration of optical superradiance is demonstrated in a thin material sample by Greiner et al.\cite{8}.

Dicke’s superradiance is observed in many electronic systems of lower dimension too. The systems studied are quantum dots by Brandes et al.\cite{9}, and theoretical possibility of the Dicke superradiance is predicted by the same group in semiconductor heterostructure\cite{10} where electrons and holes in lowest Landau levels act as two level atom--like system. Dicke’s superradiance has been proposed as an effective means to reduce spontaneous decay time of an excited atom to cool and manipulate atoms with narrow width by Djotyan et al.\cite{11}.

In many applications of condensate matter waves, one needs to handle the motion of a single atom \cite{12} or of an atomic built--up in a trap\cite{21} or amplification of input matter wave\cite{13}. Individual atomic motion can also be guided either in free space or in a metallic or dielectric interface\cite{14} or in the wave guide\cite{15}. Most of these studies are confined to manipulate single atom in external coherent field. Of late, Gangl and Ritsch have theoretically shown that an ensemble of two level atom, coupled to high Q cavity mode could be cooled by considering the interaction between the cavity field and the motion of atoms in the ensemble\cite{16} indicating an interest in modification of properties due to collective effect. In the light of the resurgence of interest in studying collective effects like superradiance, it would be interesting to investigate the nature of optical forces in a collective system of N atoms in Dicke state interacting with a common radiation field. The aim of this letter is to show that cooperative effects modify optical forces radically.

As a model of cooperative effects we consider a collection of $N$ identical
two level atoms of transition frequency \( \omega_o \) confined to a volume less than \( \lambda^3 \) where \( \lambda \) is a wavelength of transition and driven by a coherent c.w. laser field of frequency \( \omega_L \). Such a model was first considered by Dicke\[5\] to explain superradiance. In a typical superradiance phenomenon, the intensity of emitted radiation by the \( N \) atom system scales as \( N^2 \). This implies that a collective decay coefficient of \( N\gamma \) where \( 2\gamma \) is Einstein’s \( A \) coefficient. We show in this letter that the optical forces (dipolar and dissipative) on the above \( N \) atom system also scale as \( N^2 \) thereby justifying the title of this letter. We also show that the maximum angular momentum imparted to the system by the beam is \( N^2 l \hbar \) where \( l \) is quantum number associated with the coherent field.

The equation of motion for the reduced atomic density operator \( \rho \) for \( N \) identical two level atoms confined to a single site and driven by a single mode c-w laser of frequency \( \omega_L \) is,

\[
i\hbar \frac{\partial \rho}{\partial t} = [H, \rho] + i\hbar \gamma (2\pi_{12}\rho\pi_{21} - \rho\pi_{21}\pi_{12} - \pi_{21}\pi_{12}\rho)
\]

where,

\[
\pi_{ij}^{(\alpha)} = |i^{(\alpha)}\rangle \langle j^{(\alpha)}|
\]

for \( \alpha^{th} \) atom with \( i \) and \( j \) taking values 1 and 2 for level \( |1> \) (ground state) and \( |2> \) (excited state). \( 2\gamma \) is Einstein’s \( A \) coefficient. The total \( \pi \) operator is \( \pi_{ij} = \sum_{\alpha=1}^{N} \pi_{ij}^{(\alpha)} \) and has an algebra

\[
[\pi_{21}, \pi_{12}] = \pi_{22} - \pi_{11} \equiv 2\pi_3
\]

\[
[\pi_3, \pi_{21}^n] = n\pi_{21}^n
\]

\[
[\pi_3, \pi_{12}^n] = -n\pi_{12}^n
\]

\[
[\pi_{21}, \pi_{12}^n] = -m(m-1)\pi_{12}^{m-1} + 2m\pi_{12}^{m-1}\pi_3
\]

\[
[\pi_{12}, \pi_{21}^n] = -n(n-1)\pi_{21}^{n-1} - 2n\pi_{21}^{n-1}\pi_3
\]

Hamiltonian of atoms and of field is

\[
H = \hbar \omega_o \pi_{22} - i\hbar (\pi_{21} \alpha f(r)e^{i\theta - i\omega_L t} - h.c.)
\]

It is convenient to make a transformation to the frame of reference of the applied coherent field by defining,
\[
\tilde{\rho} = \exp(i\omega_L t\pi_{22}) \rho \exp(-i\omega_L t\pi_{22}).
\]

The equation (1) is then changed to,
\[
\frac{\partial \tilde{\rho}}{\partial t} = -i\Delta [\pi_{22}, \tilde{\rho}] - [\pi_{21}\alpha f(r)e^{i\theta} - \pi_{12}\alpha^* f^*(r)e^{-i\theta}, \tilde{\rho}] \\
+ \gamma(2\pi_{12}\tilde{\rho}\pi_{21} - \pi_{21}\pi_{12}\tilde{\rho} - \tilde{\rho}\pi_{21}\pi_{12})
\]

The steady state solution \(\tilde{\rho}_{ss}\) of equation (3) is defined by \(\frac{\partial \tilde{\rho}}{\partial t} = 0\) and has a form [17]
\[
\tilde{\rho}_{ss} = \sum_{m,n=0}^{N} a_{mn} (-g)^{-m}(-g^*)^{-n} \pi_{12}^m \pi_{21}^n
\]

where,
\[
a_{mn} = \frac{\Gamma(m + \delta + 1)\Gamma(n - \delta + 1)}{\Gamma(m + 1)\Gamma(n + 1)\Gamma(1 + \delta)\Gamma(1 - \delta)},
\]
\[
D = \sum_{m=0}^{N} a_{mm} \frac{(N + m + 1)!(m)!}{(N - m)!(2m + 1)! |g|^2m}
\]

and
\[
\Delta = \omega_0 - \omega_L, \quad \delta = i\Delta \gamma, \quad g = \alpha f e^{i\theta} \gamma.
\]

Calculations of average values of dipolar as well as dissipative forces involve averages \(\langle \pi_{21} \rangle\) and \(\langle \pi_{12} \rangle\) where,
\[
\langle \pi_{ij} \rangle = Tr(\tilde{\rho}\pi_{ij}).
\]

The trace is taken over Dicke states
\[
| \frac{N}{2}, k >
\]

where k lies between -N/2 and +N/2 and k is appropriately summed. The expressions for dipolar force \(\langle \mathbf{F}_{\text{dipole}} \rangle\) and the dissipative force \(\langle \mathbf{F}_{\text{diss}} \rangle\) are,
\[
\langle \mathbf{F}_{\text{dipole}} \rangle = i\hbar \nabla | \alpha f(r) | (e^{-i\theta} \langle \pi_{21} \rangle - e^{i\theta} \langle \pi_{12} \rangle)
\]

and
\[
< F_{\text{diss}} > = h| \alpha f(\mathbf{r}) | \nabla \theta (e^{-i\theta} < \pi_{21} > + e^{i\theta} < \pi_{12} >) \tag{9}
\]

After evaluating the requisite averages in equations (8) and (9), we obtain expressions for forces as,

\[
\langle F_{\text{dipole}} \rangle = \frac{2h\Delta (\nabla | \alpha f(\mathbf{r}) |)}{D} \sum_{m=1}^{N} \frac{m\Gamma(m+\delta)\Gamma(m-\delta)(N+m+1)!}{(2m+1)!(N-m)!\Gamma(1-\delta)\Gamma(1+\delta)} \left\{ \frac{| \alpha f(\mathbf{r}) |}{\gamma} \right\}^{-(2m-1)} \tag{10}
\]

and

\[
< F_{\text{diss}} > = \frac{2h | \alpha f(\mathbf{r}) | \nabla \theta}{D} \sum_{m=1}^{N} \frac{m^2\Gamma(m+\delta)\Gamma(m-\delta)(N+m+1)!}{\Gamma(1-\delta)\Gamma(1+\delta)(2m+1)!(N-m)!} \left\{ \frac{| \alpha f(\mathbf{r}) |}{\gamma} \right\}^{-(2m-1)} \tag{11}
\]

It is easy to see that the expressions (10) and (11) reduce to the forces on a single atom in an e.m. field when we take \(N=1\) as

\[
< F_{\text{dipole}} >= 2h | \alpha f(\mathbf{r}) | (\nabla | \alpha f(\mathbf{r}) |) \frac{\Delta}{\Delta^2 + 2 | \alpha f(\mathbf{r}) |^2 + \gamma^2} \tag{12}
\]

and

\[
< F_{\text{diss}} >= 2h\gamma | \alpha f(\mathbf{r}) |^2 \frac{\nabla \theta}{\gamma^2 + 2 | \alpha f(\mathbf{r}) |^2 + \Delta^2} \tag{13}
\]

We have plotted in Fig.(1)

\[
f_{\text{dip}} = \frac{\langle F_{\text{dipole}} \rangle}{[2h\gamma \Delta (\nabla g)/\gamma^2]}
\]

and

\[
f_{\text{diss}} = \frac{\langle F_{\text{diss}} \rangle}{[2h\gamma g^2 \nabla \theta]}
\]

as a function of \(N\) for parameters \(| \delta | = 1, g = 1.5\) and observe that \(f_{\text{diss}}\) and \(f_{\text{dip}}\) tend to a constant value for large \(N\). It means that the sums in equations (10) and (11) and the denominator \(D\) behave similarly for large \(N\).

Also \(f_{\text{dip}}\) being a difference of two terms(see eq.8) shows a peak whose position is almost insensitive to \(N\) for given parameters but its value decreases with increasing detuning \(\delta\). Note that \(f_{\text{diss}}\) is a dimensionless quantity which
tends to a value 1, an asymptotic limit. We have plotted similar graphs for different values of the parameters $|\delta|$ and $g$ and observe that the smaller the value of the Rabi frequency, smaller is a value of $N$ around which $f_{\text{diss}}$ tends to one. It is natural to scale the quantities like $f_{\text{dip}}$ and $f_{\text{diss}}$ by $\frac{1}{N\gamma}$ rather than $\frac{1}{\gamma}$ since the width of super radiant pulse is $N\gamma$ and the peak intensity behaves as $N^2\gamma$ [18]. We then see that $<F_{\text{dipole}}>$ and $<F_{\text{diss}}>$ behave as $N^2$ in the reduced variables $g/N$ and $\frac{\Delta}{N\gamma}$.

For large $N$, asymptotic analysis of equations (6), (10) and (11) can be made following the methods developed by Lawande et al [17]. These authors discuss asymptotic expansion of general operator averages of the type $<\pi_{21}p(\pi_{22} - \pi_{11})^r\pi_{12}q>$. Operator averages involved in our case are much simpler, namely, $<\pi_{21}>$ and $<\pi_{12}>$. The method, even though exact is complicated. The results obtained thus can be obtained in much more simple fashion by mean field method. We write $\pi_{12} = \pi_-, \pi_{21} = \pi_+$ and $\pi_z = (\pi_{22} - \pi_{11})/2$ and write equation of motion for $<\pi_{\pm}>$ and $\pi_z$. The equations involve averages like $<\pi_{\pm}\pi_z>$ which we approximate in mean field as,

$$<\pi_{\pm}\pi_z> \approx <\pi_{\pm}><\pi_z> \quad (14)$$

We then go over to reduced variables $m_\pm = \frac{<\pi_{\pm}>e^{\mp i\theta}}{N\gamma}$, $m_o = \frac{<\pi_z>}{N\gamma}$, $\tau = N\gamma t$, $\mu = \frac{2\alpha f(r)}{N\gamma}$ and $\nu = \frac{2\delta}{N\gamma}$. The moment equations in the mean field approximation are,

$$\frac{dm_o}{d\tau} = -\frac{\mu}{2}(m_+ + m_-) - 2m_+m_-$$

$$\frac{dm_+}{d\tau} = \mu m_o - \frac{i\nu m_+}{2} + 2m_+m_o$$

$$\frac{dm_-}{d\tau} = \mu m_o + \frac{i\nu m_-}{2} + 2m_-m_o \quad (15)$$

Using steady state solutions of equations (15) and equations (8) and (9), we get the dipole and dissipative force per particle to be

$$\frac{<F_{\text{dipole}}>}{N} = \frac{\hbar(N\gamma)\nu\mu(\nabla\mu)}{4\eta(1 + \xi^2)} \quad (16)$$

and

$$\frac{<F_{\text{diss}}>}{N} = \frac{\hbar(N\gamma)\xi\mu^2\nabla\theta}{2(1 + \xi^2)} \quad (17)$$

where, $\xi^2 = \frac{1}{2}[(\sqrt{\beta^2 + \nu^2}) + \beta]$ and $\eta^2 = \frac{1}{2}[(\sqrt{\beta^2 + \nu^2}) - \beta]$ and $4\beta = 4\mu^2 + \nu^2 - 4$. 6
We now consider an angular momentum imparted to the system of atoms by the beams that carry angular momentum such as Laguerre-Gaussian (LG) beam or an ideal Bessel beam. The torque imparted will be due to the dissipative force. With $\theta$ as a Guoy phase for the beams, we see[19],

$$\nabla \theta = \left[ \frac{krz}{z^2 + z_R^2} \right] \hat{e}_r + \frac{l}{r} \hat{e}_\phi$$

$$+ \left[ \frac{k_r^2}{2(z^2 + z_R^2)} (1 - \frac{2z^2}{z^2 + z_R^2}) + k + \frac{(2p + l + 1)z_R}{z^2 + z_R^2} \right] \hat{e}_z$$

(18)

for LG beam and,

$$\nabla \theta = \left( \frac{l}{r} \right) \hat{e}_\phi + (k - \frac{k^2_{\perp}}{2k}) \hat{e}_z$$

(19)

for ideal Bessel beam. The torque imparted is,

$$|\langle \tau \rangle| = |\langle r \times F_{\text{diss}} \rangle|$$

(20)

The torque imparted by either beams is,

$$|\langle \tau \rangle| = \frac{\hbar N^2 \gamma \xi \mu^2 l}{2(1 + \xi^2)}$$

(21)

Equations (16),(17) and (21) are principal results of this letter. It is clear that there is a cooperative effect on forces as well as on torque of an external beam if the number of atoms N are within some coherence distance like $\lambda$. Then such a system of atoms experiences a large force and a very large torque and very large angular momentum-justifying the part title Dicke’s bullet.

Possibility of observing Dicke’s bullet is best in gaseous systems. Even though, the systems of transition metal ions in solid state matrix as used by DeVoe and Brewer [7] and by Greiner et al [8] show superradiance in most clear terms it probably is difficult to observe Dicke’s bullet in these systems. In practice, it is impossible to arrange N atoms at a point. However, the correlations between atoms within a distance $\frac{\xi}{2N\gamma}$ can also give rise to superradiance. Gaseous samples of sufficiently low density ensure negligible interaction between the dipoles. Among many gaseous systems, superradiance in Tl-Hg [20] and Cs atomic beam was studied sometime ago. If a Laguerre-Gaussian (LG) beam of superradiant frequency falls on such a system in a direction opposite to the existing pulse immediately
after extinction of the exciting pulse, then, the superradiant atom will experience a force as given by Eq. (17) and subsequent acceleration \( a \). The c.m. velocity imparted to the radiating atoms is \( \frac{a}{N\gamma} \). This will Doppler shift the peak frequency of \( \omega \) of superradiant transition to \( \omega - \frac{2\pi a}{\lambda N\gamma} \) which may be observable.

In conclusion, we have shown that the dissipative force and the torque for the Dicke system scale as \( N^2 \) if \( N \) atoms, radiating cooperatively, are exposed to a coherent radiation field with spatial profile such as LG beam or ideal Bessel beam.

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Figure Caption

Figure 1 shows $f_{dip}$ and $f_{diss}$ as a function of number of atoms $N$. $f_{dip}$ and $f_{diss}$ are dimensionless as defined in the text.

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Fig. 1  PVP and SVL