Hysteretic Models Considering Axial-Shear-Flexure Interaction

Paola Ceresa 1, Giorgio Negrisoli 2

1 Institute for Advanced Study of Pavia (IUSS), Piazza della Vittoria 15, 27100, Pavia, Italy
2 UME School, Institute for Advanced Study of Pavia (IUSS), Piazza della Vittoria 15, 27100, Pavia, Italy

paola.ceresa@iusspavia.it

Abstract. Most of the existing numerical models implemented in finite element (FE) software, at the current state of the art, are not capable to describe, with enough reliability, the interaction between axial, shear and flexural actions under cyclic loading (e.g. seismic actions), neglecting crucial effects for predicting the nature of the collapse of reinforced concrete (RC) structural elements. Just a few existing 3D volume models or fibre beam models can lead to a quite accurate response, but they are still computationally inefficient for typical applications in earthquake engineering and also characterized by very complex formulation. Thus, discrete models with lumped plasticity hinges may be the preferred choice for modelling the hysteretic behaviour due to cyclic loading conditions, in particular with reference to its implementation in a commercial software package. These considerations lead to this research work focused on the development of a model for RC beam-column elements able to consider degradation effects and interaction between the actions under cyclic loading conditions. In order to develop a model for a general 3D discrete hinge element able to take into account the axial-shear-flexural interaction, it is necessary to provide an implementation which involves a corrector-predictor iterative scheme. Furthermore, a reliable constitutive model based on damage plasticity theory is formulated and implemented for its numerical validation. Aim of this research work is to provide the formulation of a numerical model, which will allow implementation within a FE software package for nonlinear cyclic analysis of RC structural members. The developed model accounts for stiffness degradation effect and stiffness recovery for loading reversal.

1. Introduction
With the increasing of densely populated and economically developed areas in high-risk seismic zones, the consequence of a strong earthquake becomes every day potentially more catastrophic, having a significant impact on the structural design of buildings and infrastructures. Elastic structural response corresponding to high-seismicity loading level can be often extremely high. However, the use of conventional structural design solutions usually leads to not economically justifiable design choices for making the structure able to resist to so high demand. A remedy to this problem can be the reduction of the level of seismic loading by allowing the inelastic response of the structure: the evaluation of the performance of the structure becomes therefore even more a necessity.
The modern seismic approach is to design the structures so they can ensure large post-elastic deformation and an adequate energy dissipation due to earthquake ground shaking through stable hysteretic behaviour of their structural components. The assessment of the dynamic response of the structure depends to great extent on the possibility to describe the cyclic behaviour of preselected inelastic regions (plastic hinges) through reliable hysteretic models able to capture degradation effects due to cyclic deterioration of the material mechanical proprieties, accounting for the interaction between axial, shear and flexural deformations.

An increasing interest for using a model that includes degradation effects is no longer shown only by research world, but also by construction codes. In recent instructions about seismic assessment of existing building by Italian Code Development Commission (CNR) it is pointed out that the choice of a model without degradation is not enough accurate, mainly for advanced limit states (life safety and collapse) for which the degradation of the material mechanical properties plays a fundamental role [1].

2. Motivations and objectives
The main motivations of this research are related to the multi-directional nature of an earthquake and the catastrophic collapses occurred during past seismic events. Furthermore, several experimental tests to simulate the hysteretic effects due to earthquake excitations show a tendency for strength and stiffness degradation and for other types of deterioration such as pinching or slip of the reinforcement. The axial load also plays an important role in the overall response of a structure because the member flexural strength depends on the axial load level, which in general can vary in magnitude and direction during an earthquake excitation. These phenomena are very important to be captured from a modelling point of view. This research aims to formulate a mathematical model that can reflect the sources of deterioration effect due to cyclic loading. Then the formulation has to be extended to account for complex 3D interaction between actions.

3. Literature review
In this section a brief overview of the main sources of cyclic degradation of reinforced concrete (RC) structural elements is given. Then the mathematical models able to describe the sources of degradation and the modelling strategies for axial-shear-flexure interaction are presented.

3.1. Sources of cyclic degradation of RC structural elements
During a cyclic loading many effects of degradation are taking place in a RC structural element, ranging from strength or stiffness degradation to problems related to local loss of anchorage of the bars (bond-slip). Some of these sources of deterioration cannot be easily described by a material model because of their complex nature, and also because from experimental results there is no unique response in the hysteretic loop even between samples that should describe the same curve (i.e. the same behaviour). Too many uncertainties are present during a cyclic test on a RC member. For this reason, it is difficult to find experimental tests that can be considered good references for the calibration of a model. This is in particular more critical in case of a squat element for which the orientation of the cracks with respect to the principal directions of the applied stresses can strongly influence the pinched shape of the post-yielding part of the hysteretic loop. Another effect of degradation that is difficult to capture and to calibrate is the bond-slip effect. Strength and stiffness degradation can be related to the bar buckling and bar rupture. Moreover, the bar pull-out, as well as the crack-closure can be factors affecting the pinched shape of the hysteretic loop in the re-loading branch. Additionally, many other microscopic phenomena can cause deterioration in a RC member such as tensile cracking due to rebar buckling (low cycle fatigue), crushing or spalling, prying dowel action, reduction in aggregate interlock and sliding at cracked interfaces and construction joints [2]. Bar slip due to strain penetration greatly affects the local response measures that are indicative of damage in the plastic hinge region [3].
3.2. Mathematical models able to describe the sources of degradation

Focusing on models that can be used as work law for a rotational spring or a translational spring element, representing respectively plastic hinge region and shear force-deformation, there are mathematical models that incorporate rules that are more appropriate to describe the shear deformation effect like the picking rule in Figure 1 [4] or the peak-oriented rule in Figure 2 [5]. Generally models that include deterioration effects can be implemented, especially for unidirectional problems, using a backbone curve that represents the reference force-deformation relation for the work law and that defines the boundary of the hysteretic loop response (Figure 2). The Italian Code Development Commission [1] considers the model of Ibarra, Medina and Krawinkler [5] as the reference constitutive model for describing the cyclic response of beam-column RC elements.

![Figure 1. Basic options for hysteretic characteristics: bilinear (a), peak oriented (b), pinching (c) [4]](image1)

Even though for earthquake engineering applications, basic models like those in Figure 1. have been used, it is increasing the motivation to model the behaviour of the structure near to the collapse.

![Figure 2. Individual deterioration models for a peak-oriented model: basic strength deterioration (a), post-capping strength deterioration (b), unloading stiffness deterioration (c), acceleration reloading stiffness deterioration (d) [5]](image2)

In 1970 a tri-linear backbone curve model – the Takeda model - able to distinguish between the stiffness of the uncracked and cracked section was developed [6]. The model was built mainly for the RC members and accounted for stiffness degradation with a non-linear law depending on the ductility.
This model was modified later by several authors. Other sources of deterioration, such as a strength degradation and a pinching effect as a result of a combination of not-reversible behaviour of the crack opening-closing and bar bond-slip in the cracked region, were added in the model of Takayanagi and Schnobrich [7]. Otani [8] modified the original Takeda model moving from a tri-linear to a bi-linear backbone curve. Another important model obtained from the original Takeda was proposed by Allahabadi and Powell [9]. Their modifications were related to the modelling of pinching effect and a peculiar behavior for the peak-orientation as a function of the small or large amplitude cycles. Saatcioglu and Ozcebe [10] can be considered the pioneers of a shear hysteretic model. They derived the experimental significance of the axial load and transverse reinforcement influence on the strength and stiffness degradation. Song and Pincheira [11] developed a shear spring hysteric loop able to describe the stiffness and strength deterioration with a peak-oriented model that considers pinching and a backbone curve including a post-capping negative tangent stiffness and residual strength branch.

3.3. Modelling strategies for axial-shear-flexural interaction

Capturing in a proper way the axial-shear-flexural interaction is very challenging, because the introduction of a relationship between different actions generate a coupling between equations that can make the solution more difficult to achieve, introducing more complexity in the model, increasing the possibility of encountering numerical problems for convergence failure [12]. There are two main different approaches for studying the nonlinear behaviour of a RC structural member under cyclic loading – with a lumped plastic hinge or with a fibre beam-column Finite Element (or a 3D volume FE).

In terms of lumped plasticity hinge models, Filippou et al. [13], Pincheira et al. [14], Lee and Elnashai [15] and Xu [12] considered models accounting for inelastic shear combined with non-linear flexural behaviour. Xu presented a modelling approach for axial-shear-flexural interaction first considering a model with shear-flexure interaction for a constant axial load and a second more complex model able to consider more properly the behaviour with variation of axial load level, that becomes a controlling factor for the interaction between shear and flexural failure mode and strength level in RC members. Another recent and still in progress contribution is the work of Reshotkina [16], based on the development of a damage-based lumped plasticity formulation accounting for different degradation effects and considering a 3D interaction between shear, flexure and axial loads.

Petrangeli [17], Martinelli [18], Mullapudi et al. [19], Ceresa et al. [20, 21] and Kagermanov and Ceresa [22, 23, 24] proposed interesting fibre beam-column modelling strategies. The last two formulations are able to capture different sources of degradation even for shear-critical RC structures.

Interaction between different actions is very rare to be found implemented within software frameworks for structural engineering and also extended to consider bi-axial behaviour and shear-flexural interaction. If the lumped plastic hinge models are concerned, there are many analysis software packages where the response of the element is independent on the aspect ratio and the axial load level. Furthermore, at the current state of the art, there are no commercial software that include shear deformations at fibre level. An investigation of the state-of-the-art software packages for the analysis of shear-critical reinforced concrete elements is presented in [25].

After a comprehensive study of the literature, discrete models with lumped hinges seem to remain the preferred choice for modelling the interaction between different actions and several degradation effects in RC members subjected to cyclic loading. The adopted modelling strategy is justified by the final aim of developing a model to be implemented and managed in a more computationally efficient way within a commercial software such as SOFiSTiK [26].

4. Hysteretic Takeda model implementation for 1D spring element

The implementation of a model that describes different types of cyclic deterioration has been initially carried out considering the Takeda-modified hysteretic rules: the model can be used for a 1D spring element to account for stiffness degradation effect due to cyclic excitation in uniaxial conditions. The logic of the implementation is to build a series of rules in a routine able to robustly determine the
response of the hysteretic behaviour of the spring due to cyclic load, independently from the applied load history. The developed routine is built with reference to the hysteric rules of [9]: the assumed force-deformation relationship can be derived from the moment-curvature relationship described in the user guide of the code Drain-2DX [9]. A second reference was the model of Lestuzzi [27], which was modified in order to obtain a more routine more robust and independent of the loading history. Details of the implemented rules are presented elsewhere [28]. Numerical simulations with the developed Takeda inelastic model were performed considering a single degree of freedom (SDOF) system (1D spring element) under quasi-static and dynamic load-histories, such as the “El Centro NS” ground motion. Very good results were obtained, also when compared with the model developed by Lestuzzi.

5. Damage-based formulation for 3D discrete hysteretic model

The adopted geometrical-based formulation becomes inappropriate and too complex to be extended for considering the presence of multiple actions through 3D interaction. A new model based on the damage mechanics concepts and coupled with plasticity is developed. Many models can be found in literature in the last 10 years considering the coupling between plasticity and damage theories providing different approaches for a novel constitutive model for concrete, merging the advantages of damage mechanics and plasticity in one formulation. Recent studies [29-31] are showing enhanced versions of the earlier models of reference [32, 33]. The plastic-damage model implemented in the software Abaqus [34] is based on the models [32, 35]. Among the existing models, a good reference for the implementation of the present formulation was the work in [36]. The latter proposes a numerical method for elastic-plastic-damage for a 1D system, where, through the introduction of the concept of effective stress and the proposed implementation, the plastic and damage internal mechanisms can be considered decoupled. The core of the numerical procedure is an incremental modified Newton-Raphson scheme that amounts to an elastic-predictor, plastic corrector and damage-corrector strategy.

5.1. Model formulation and implementation aspects

The damage-plasticity formulation is presented following the steps of its implementation.

5.1.1. Damage-plasticity model formulation. In order to build a general formulation of the plasticity problem coupled with damage, the case of isotropic hardening is considered but the laws for hardening and damage evolution are general and they are functions of two internal variables \( \kappa^d \) and \( \kappa^t - \sigma_Y = \sigma_o + h(\kappa^t) \) and \( d = g(\kappa^d) \), where \( h \) is the hardening function and \( g \) is the damage evolution law. The approach is based on the plasticity theory, formulated in the effective undamaged stress space, for which the stress-strain (\( \sigma - \varepsilon \)) law is in the form of Equation (1). In the space of the nominal stresses, the hardening function is depending on the inelastic deformation \( \varepsilon^t \), that corresponds to the plastic deformation \( \varepsilon^P \) in case of a pure plastic theory without the damage. The geometrical representation of the implemented nominal stress-deformation and effective stress-deformation is plotted on the left of Figure 3, with the damage evolution law and the hardening evolution law (top), and the definition of unloading stiffness, plastic and inelastic deformations (bottom). While the damage evolution law is function of \( \varepsilon^P \), the hardening function depends instead from the variable \( \varepsilon^t \), where \( \varepsilon^t = \varepsilon^P + \varepsilon^d \). This justifies the introduction of the second internal variable \( \kappa^t \), where its rate \( \dot{k}^t \) can be defined as the norm of the inelastic deformation rate \( \dot{\varepsilon}^t \) as well as the damage internal variable \( \kappa^d \) can be defined as the norm of the plastic deformation rate \( \dot{\varepsilon}^P \) (fourth and fifth equations of Equation (5). The first focus of the formulation is the derivation of a set of algebraic equations, then linearized through backward Euler integration scheme in order to achieve a system of discrete equations that are representing the plastic correction step in a form of the return mapping algorithm followed by the damage correction. The formulation is derived in terms of stresses and strains because of the potential application to a more general framework of continuous constitutive models. The general governing equations which are valid for isotropic hardening plasticity can be reformulated in the effective stress space. The effective stress \( \bar{\sigma} \) and the yielding surface \( \bar{\psi} \) are defined in Equations (1) and (2). The Kuhn-Tucker
loading/unloading complementary conditions that capture the irreversible nature of plastic flow [36] are given in Equation (3). The potential relationship in Equation (4) is chosen with an associative flow-rule for which
\[
\sigma = k_0 \cdot (\varepsilon - \varepsilon^p) ; \quad \sigma = (1 - d) \cdot \bar{\sigma}
\]
\[
\phi(\bar{\sigma}, \kappa^i, \kappa^d) = \bar{\sigma}(\bar{\sigma}) - \bar{\sigma}_t(\kappa^i, \kappa^d)
\]
\[
\bar{\sigma} \leq 0 \quad \lambda \geq 0 \quad \lambda \cdot \bar{\phi} = 0
\]
\[
\varepsilon^p = \dot{\lambda} \cdot (\partial \sigma / \partial \bar{\sigma}) = \dot{\lambda} \cdot (\partial \bar{\phi} / \partial \bar{\sigma})
\]

where \( G \) is a general function that for associative flow-rule is equal to the effective yielding function \( \bar{\phi} \). The set of equations that are governing the plastic-damage correction step can be summarized in Equation (5).

\[
\begin{align*}
\bar{\sigma} &= k_0 \cdot (\varepsilon - \varepsilon^p) \\
\bar{\phi}(\bar{\sigma}, \kappa^i, \kappa^d) &= 0 \\
\dot{\varepsilon}^p &= \dot{\lambda} \cdot (\partial \bar{\phi} / \partial \bar{\sigma}) \\
\kappa^d &= ||\varepsilon^d|| \\
\kappa^i &= ||\varepsilon^i||
\end{align*}
\]

The second one is obtained starting from the definition of the yielding surface combined with the Kuhn-Tucker condition, satisfying the yielding surface activation condition, equivalent to assume \( \bar{\phi} = 0 \). Each of these 5 equations can be integrated using backward Euler integration scheme in order to obtain a nonlinear system of discrete equations in the pseudo-time \( t_{n+1} \). The first and the second equations of the set can be directly evaluated at \( t_{n+1} \). The third, fourth and the fifth equations, because expressed in terms of internal variables rate, have to be integrated. At the end of the calculations, those details are given elsewhere [28], the set of discrete non-linear equations in (6) is derived. The solution of the system, whose unknowns are \( \bar{\sigma}_{n+1}, \kappa^i_{n+1}, \kappa^d_{n+1}, \varepsilon^p_{n+1} \) and \( \Delta \lambda \), completes the plastic-damage correction step of the algorithm shown in BOX 1 (Figure 3). The derived return-mapping algorithm has been validated through simple benchmark examples [28].

\[
\begin{align*}
\bar{\sigma}_{n+1} &= k_0 \cdot (\varepsilon_{n+1} - \varepsilon^p_{n+1}) \\
\bar{\phi}(\bar{\sigma}_{n+1}, \kappa^i_{n+1}, \kappa^d_{n+1}) &= 0 \\
\varepsilon^p_{n+1} &= \varepsilon^p_n + \Delta \lambda_{n+1} \cdot (\partial \bar{\phi} / \partial \bar{\sigma}) \bigg|_{n+1} \\
\kappa^d_{n+1} &= \kappa^d_n + \Delta \lambda_{n+1} \cdot (\partial \bar{\phi} / \partial \bar{\sigma}) \bigg|_{n+1} \\
\kappa^i_{n+1} &= \kappa^i_n + I(\kappa^d_{n+1}, \varepsilon^p_{n+1}) \text{ with } I(...) = \int_{t_n}^{t_{n+1}} \kappa^i \cdot dt
\end{align*}
\]

5.1.2. Bi-dissipative damage and load reversal peak-orientation effect. The formulation in the subsection 5.1.1 has been then enhanced, providing a more general capability for the analysis of concrete structures and dynamic loading. The model is modified, distinguishing the behaviour in compression and in tension, offering to the user more flexibility in the definition of different mechanical parameters. Furthermore, the hardening law and the damage evolution law are described differently depending on the state of compression (\( c \)) or tension (\( t \)). The stress-strain law is then expressed as in Equation (7), where \( d_c \) and \( d_t \) are two independent damage variables, which are functions of different internal damage variables. Moreover, the model accounts for stiffness deterioration in the reversal loading branch depending on the damage level of the previous state and on the history of the damage for reloading of previous cycles. This stiffness recovery effect, also known as the “unilateral effect” [34], is considered due to the introduction of two parameters \( s_c \) and \( s_t \), functions of the stress state. This enhanced formulation can find application for describing a special hinge work law for RC as well as a constitutive model for concrete, reaching a wider range of applicability for cyclic response evaluation. The mathematical formulation of the recovery of the
elastic stiffness behaviour, as the load changes sign, can be expressed starting from the definition of stiffness of the reversal load branch, $K_{rev}$ (Figure 4). Also the evolution of the plastic strain can be generalized using the definition of $r^*(\sigma)$, as function of the total plastic strain deformation rate, $\dot{\varepsilon}^P$. Furthermore, the proposed formulation for stiffness recovery can be used to describe the peak orientation effect, assuming the weights $w_c$ and $w_t$ to be functions depending on damage, rather than fixed parameters defined at the beginning of the analysis. At each loop the value of weight factors is varying with the value of the damage variable in compression and in tension of previous cycles (Figure 4). The use of the weight function to describe the peak-orientation represents a new contribution of the proposed formulation when compared to other models that use the weight function in a range between 0 and 1 only [34].

$$\begin{cases} \bar{\sigma} = k_0 \cdot (\varepsilon - \varepsilon_c^P), & \sigma = (1 - d_c) \cdot \bar{\sigma}, \quad \text{if } \sigma > 0 \\ \bar{\sigma} = k_0 \cdot (\varepsilon - \varepsilon_t^P), & \sigma = (1 - d_t) \cdot \bar{\sigma}, \quad \text{if } \sigma < 0 \end{cases}$$

(7)

given $K_0 = \omega_n^2$, $H = \text{isotropic coef.}$, $d = \text{damage}$

$$\begin{align*}
\sigma_{n+1}^{\text{trial}} &= g(k_n^d) \\
\bar{\sigma}_{n+1}^{\text{trial}} &= k_0 \cdot (\varepsilon_{n+1} - \varepsilon_n^P) \\
\bar{\varphi}_{n+1}^{\text{trial}} &= \bar{\sigma}_{n+1}^{\text{trial}} - \left( \sigma_n^d + h(k_n^d) \right) \\
&= 1 - g(k_n^d)
\end{align*}$$

IF $\bar{\sigma}_{n+1}^{\text{trial}} > 0$ THEN

plastic – damage corrector step

Solution of system (Eq. 6):

$$\begin{align*}
\bar{\sigma}_{n+1}, \kappa_{n+1}, k_{n+1}^d, \varepsilon_{n+1}^P, \Delta \lambda \\
\text{ELSEIF } \bar{\varphi}_{n+1}^{\text{trial}} \leq 0 \text{ THEN}
\text{Elastic step: set } \varepsilon_{n+1} = \varepsilon_{n+1}^{\text{trial}} \\
\bar{\sigma}_{n+1} = \bar{\sigma}_{n+1}^{\text{trial}} \\
\text{ENDIF}
\end{align*}$$

FIGURE 3. Geometrical representation of the implemented nominal and effective stress-deformation (left). Return-mapping algorithm for 1D damage-isotropic hardening plasticity (BOX 1)

5.2. Validation of damage-plasticity formulation

Different numerical analyses to test the applicability of the proposed formulation and the possibility to describe stress-strain curves with different damage evolution and hardening laws have been performed. Among the others, two verifications are presented. Figure 5, on the left, describes a cyclic loading-unloading analysis with a hyperbolic damage evolution law and with a quadratic hardening law, comprised of an initial hardening followed by the softening region. The plot on the right of Figure 5 refers to a cyclic reversal loading-unloading test from compression to tension, showing the degradation effects accounted for in the model – stiffness degradation, stiffness recovery for loading reversal – with a linear hardening for compression and a non-linear quadratic hardening in tension with a softening branch. The chosen damage evolution law is exponential for compression and hyperbolic for tension.
Figure 4. Stiffness recovery formulation and weight functions for peak-orientation effect

\[
k_{\text{rev}} = k_0 \cdot (1 - s_t \cdot d_t) \cdot (1 - s_c \cdot d_c)
\]
\[
s_t = 1 - w_t \cdot [1 - r'(\sigma)]
\]
\[
s_c = 1 - w_c \cdot r'(\sigma)
\]
\[
r'(\sigma) = H(\sigma) = 1 \text{ if } \sigma > 0
\]
\[
r'(\sigma) = H(\sigma) = 0 \text{ if } \sigma < 0
\]

\[
\varepsilon^e = r^e \cdot \dot{\varepsilon}^e \text{ and } \varepsilon^f = -(1-r^e) \cdot \dot{\varepsilon}^f
\]

Figure 5. Cyclic loading/unloading test (left). Cyclic reversal loading/unloading test (right)

6. Conclusions
The damage-plasticity formulation presented in Section 5 starts from the definition of the work laws with respect to nominal stresses, thus enabling the user to define the material input parameters by using the material properties evaluated directly through a calibration with the results of the experimental tests. The approach followed in other formulations [29-31] starts from the effective stresses, forcing the user to calibrate the material properties on the effective stress-strain curves, applying a transformation to the experimental curves. The choice of formulating the model in terms of nominal stresses represents a good improvement and a clear advantage of this research with respect to existing models. However, the implemented model needs further improvements, mainly in terms of: i) Axial dependency, which is a basic issue for connecting the model with the interaction 3D surface between biaxial flexural and axial load. ii) Implementation in an existing FE framework, such as SOFiSTiK, in order to obtain more robust and stable solutions. iii) Shear model and shear-flexural interaction, modelling the pinching effect which is typical of shear-critical members; iv) Modelling other degradation effects, such as strength degradation or bond-slip, in order to further improve the obtained results and extend the range of its applicability.
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