Evolution of dust grain size distribution and grain porosity in galaxies

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Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

The radiative properties of interstellar dust are affected not only by the grain size distribution but also by the grain porosity. We develop a model for the evolution of size-dependent grain porosity and grain size distribution over the entire history of galaxy evolution. We include stellar dust production, supernova dust destruction, shattering, coagulation, and accretion. Coagulation is assumed to be the source of grain porosity. We use a one-zone model with a constant dense gas fraction ($\eta_{\text{dense}}$, which regulates the balance between shattering and coagulation. We find that porosity develops after small grains are sufficiently created by the interplay between shattering and accretion (at age $t \sim 1$ Gyr for star formation time-scale $\tau_{\text{SF}} = 5$ Gyr) and are coagulated. The filling factor drops down to 0.3 at grain radii $\sim 0.03$ μm for $\eta_{\text{dense}} = 0.5$. The grains are more porous for smaller $\eta_{\text{dense}}$ because small grains, from which porous coagulated grains form, are more abundant. We also calculate the extinction curves based on the above results. The porosity steepens the extinction curve significantly for silicate, but not much for amorphous carbon. The porosity also increases the collisional cross-sections and produces slightly more large grains through the enhanced coagulation; however, the extinction curve does not necessarily become flatter because of the steepening effect by porosity. We also discuss the implication of our results for the Milky Way extinction curve.

Key words: methods: numerical – dust, extinction – galaxies: evolution – ISM: evolution – galaxies: ISM – ultraviolet: ISM.

1 INTRODUCTION

Dust grains play an important role in various processes in the interstellar medium (ISM) of galaxies. As a galaxy is enriched with metals, the dust abundance becomes higher, and the effects of dust on the following processes become more prominent. Dust acts as a catalyst for the formation of various molecular species, especially H$_2$ (e.g. Gould & Salpeter 1963; Cazaux & Tielens 2004). Dust also strongly modifies the spectral energy distributions (SEDs) of galaxies through the extinction of ultraviolet (UV)–optical stellar light and the remissum at infrared (IR) wavelengths (e.g. Désert et al. 1990; Takeuchi et al. 2005). Radiation pressure on dust grains could affect galaxy-scale gas dynamics through gas–dust coupling (e.g. Ferrara et al. 1991; Murray et al. 2011). The above processes are governed by the total surface area (e.g. Yamasawa et al. 2011; Chen et al. 2018) and/or the extinction cross-section (e.g. Draine & Lee 1984), both of which are determined by the grain size distribution. Thus, even with the same total dust abundance, the efficiencies of the above-mentioned processes change with the grain size distribution.

The dust evolution is governed not only by the stellar dust production but also by various processes in the ISM (e.g. Lisenfeld & Ferrara 1998; Draine 2009). Among the interstellar processing mechanisms, dust growth by the accretion of gas–phase metals in the dense ISM is regarded as the dominant source for the dust abundance in evolved (metal-enriched) galaxies (e.g. Draine 1990; Dwek 1998; Hirashita 1999; Zhukovska et al. 2008; Inoue 2011; Mattsson et al. 2014). This process has also been verified by some experiments (Rouillé et al. 2014; Fulvio et al. 2017; Rouillé et al. 2020). Dust growth by accretion is also an important mechanism in explaining the interstellar metal depletion (Weingartner & Draine 1999). The decrease of the dust abundance (dust-to-gas ratio) is, on the other hand, caused by dust destruction in supernova (SN) shocks (e.g. McKee 1989). The grain size distribution is further modified by coagulation in the dense ISM and shattering in the diffuse ISM (e.g. O’Donnell & Mathis 1997; Hirashita & Yan 2009). These two processes determine the relative abundance between large and small grains (Hirashita & Yan 2009). After a pioneering work by O’Donnell & Mathis (1997), a theoretical framework that includes all the above processes to calculate the evolution of grain size distribution has been constructed by Asano et al. (2013b), Nozawa et al. (2015), and Hirashita & Aoyama (2019, hereafter HA19).

In addition to the grain size distribution, inhomogeneity within individual grains in terms of the composition and the shape also affects the observed properties of dust grains such as extinction curves (wavelength dependence of dust extinction). Mathis & Whiffen (1989) assumed interstellar grains to be aggregates of multiple species including vacuum (or voids), and explained the dust properties in the Milky Way (MW). Some studies also suggested fluffy large grains based on the observations of the X-ray scattering halos around point sources, such as X-ray binaries (Woo et al. 1994) and Nova
Cygni 1992 (Mathis et al. 1995), although compact grains are also shown to be consistent with the X-ray halo strengths based on further detailed calculations of grain optical properties (Smith & Dwek 1998; Draine & Tan 2003). In these interpretations, there is an uncertainty arising from the distribution of dust in the line of sight (Draine 2003). On the other hand, from the theoretical point of view, Hirashita et al. (2021, hereafter H21) argued that coagulation develops fluffy or porous grains in the ISM as already shown in the calculations of proto-planetary discs (e.g. Okuzumi et al. 2009, 2012) and of dense molecular cloud cores (Ormel et al. 2009). Fluffy or porous grains produced in these dense environments predict different extinction curves from compact (non-porous) grains (Ormel et al. 2011; Kataoka et al. 2014; Tajaki et al. 2016; Lefèvre et al. 2020).

Studying porous grains in the ISM was motivated by the limited available metals (especially carbon). Mathis (1996) found that, if 25–65 per cent of vacuum is included in the dust grains, the least solid materials (heavy elements) are required to explain the MW extinction curve. However, Dwek et al. (1997) argued the importance of fitting the IR dust emission SED simultaneously, and showed that the above model of Mathis (1996) overpredicts the optical–UV extinction. Li (2005) more robustly showed using the Kramers–Kronig relations that, if we use the Galactic metal abundance derived from B stars, the observed extinction is still underproduced. However, updated extinction-to-column density ratios and protosolar metal abundances have allowed us to construct a dust model consistent with the metal abundance constraints (Draine & Hensley 2021a; Hensley & Draine 2021; Zuo et al. 2021). We still emphasize that the above studies did not necessarily give a definite understanding of the grain porosity; for example, they addressed neither grain-size-dependent porosity nor connection with dust evolution.

The porosity of interstellar dust grains should be determined as a consequence of dust evolution and should be dependent on the grain size. Our previous study mentioned above (H21) solved the evolution of grain size distribution and porosity by taking shattering and coagulation into account. Using H21’s framework, we are able to predict not only the porosity but also its grain size dependence. Thus, in H21’s framework, the porosity is not a free parameter anymore but a quantity predicted along with the grain size distribution. Such a comprehensive understanding is particularly important because the observed optical properties of dust depend on both grain size distribution and grain porosity. H21 also emphasized that the interplay between shattering and coagulation is important in creating porous grains because small grains continuously supplied by shattering become the ingredients for fluffy or porous grains formed by coagulation. This means that including multiple processes for dust evolution is essential in understanding the evolution of grain porosity.

It is yet to be clarified if the grain porosity created by the interplay between coagulation and shattering is maintained in the presence of other processes. H21 focused on shattering and coagulation, but did not include dust enrichment and destruction that occur as a result of galaxy evolution (i.e. star formation and metal enrichment). Specifically, among the processes mentioned above, stellar dust production, dust growth by the accretion of gas-phase metals, and dust destruction in SN shocks are yet to be included in the porosity evolution model. For the evolution of grain size distribution, HA19 provide a comprehensive framework that includes not only coagulation and shattering but also other relevant processes for dust evolution. Since the grain porosity is strongly related to the evolution of grain size distribution, extending HA19’s formulation to include the porosity evolution is a viable way to predict the evolution of grain porosity in the entire history of a galaxy.

Voshchinnikov et al. (2005, 2006) investigated how the extinction curve is affected by porosity. This provides a useful hint for constraining the grain porosity through comparison with observations. Thus, we also predict extinction curves based on the grain size distribution and porosity calculated by our newly developed model in this paper. As a consequence, we will be able to address how the inclusion of porosity affects the predicted extinction curves, and to provide a way to test the porosity evolution against the observed extinction curves such as the MW extinction curve. Previously Voshchinnikov et al. (2006) included porous silicates together with compact graphite to fit the MW extinction curve. They succeeded in finding a fitting solution to the MW extinction curve. However, in previous models (including Mathis 1996 mentioned above), the porosity is a free parameter independent of the grain radius. Thus, it is still necessary to test the grain-size-dependent porosity realized as a result of the dust evolution (especially the evolution of grain size distribution) against the MW extinction curve. Moreover, we aim at treating the porosity and the grain size distribution simultaneously, so that both of them are predicted quantities, not freely adjusted ones.

We note that dust emission SEDs (Dwek et al. 1997) and starlight polarization (Draine & Hensley 2021b) may also be useful to constrain the porosity. Since predictions of these quantities require different extensive frameworks, we leave them for future studies, and concentrate on extinction curves for the first observational test. Since the MW extinction curve is best constrained at UV–near-IR wavelengths (e.g. Fitzpatrick & Massa 2007; Nozawa & Fukugita 2013), we focus on this wavelength range. For the convenience of discussion, we divide the UV wavelength range into far-UV and near-UV separated at 1/λ = 6 μm⁻¹.

The goal of this paper is to clarify the evolution of porous grains in the entire history of a galaxy by extending H21’s framework (i.e. treating coagulation as a source of porosity) to include other processes and a longer evolutionary time comparable to the present galaxy ages. This enables us to predict the grain porosity and its dependence on the grain radius at various epochs in galaxy evolution. In other words, this study predicts for the first time the grain-radius-dependent porosity achieved as a result of galaxy evolution. We also calculate extinction curves to make clear how the evolution of porosity affects the observed dust properties. A particular emphasis is put on the comparison with the MW extinction curve.

This paper is organized as follows. In Section 2, we formulate the evolution of grain size distribution and porosity by developing HA19 and H21’s frameworks. We show the results in Section 3. We make an additional effort of applying our results to the MW extinction curve in Section 4. Based on the results, we provide further discussions in Section 5, and finally give our conclusions in Section 6.

2 MODEL

We construct a model that describes the evolution of grain size distribution and porosity in a galaxy. We follow H21 for the definitions of the terms related to porous grains. The filling factor of a grain is defined as the volume fraction occupied by the grain-composing material. The rest, namely the volume fraction of vacuum, is referred to as the porosity. A grain is compact if the filling factor is unity (i.e. no porosity).

We consider all the processes included by HA19: dust condensation in stellar ejecta (stellar dust production), dust growth by the accretion of gas-phase metals, dust destruction in SN shocks, grain growth by coagulation, and grain disruption by shattering. Accretion and coagulation occur in the dense ISM, while shattering takes place in the diffuse ISM. We only consider the processes treated in HA19,
and we neglect other potentially important processes such as rotational disruption (Hoang 2019), which also depends on quantities (e.g. radiation field intensities) not included in our model. To avoid the complexity arising from compound species, we consider a single dust species in calculating the evolution of grain size distribution and porosity.

We calculate the dust evolution in a galaxy with a one-zone model. Thus, we neglect the spatial variety of grain size distribution within the galaxy (or the grain size distribution can be regarded as being averaged in the galaxy). Nevertheless, we still need to treat the difference between the dense and diffuse ISM since some processes occur in one of these phases as mentioned above. Thus, we introduce the dense gas fraction, $\phi_{d}\text{dense}$, which decides the weight for the processes that only occur in the dense ISM (see Section 2.5 for details). For each process, we assume a homogeneous medium and ignore effects caused by small-scale inhomogeneity such as uniform gas density and dust abundance induced by supersonic turbulence (Hopkins & Lee 2016; Mattsson 2020; Li & Mattsson 2020, 2021).

For simplicity, we neglect the electric charge of grains. As long as the grain motion is driven by turbulence as assumed in this paper, it is not likely that the shattering rate is significantly affected by grain charge because of large grain velocities (H21). For coagulation and accretion, the charge of small grains could be important but the grain charging in the dense ISM is very sensitive to the assumed physical conditions (e.g. Ilev et al. 2015). In coagulation, large ($\geq 0.01 \mu m$) grains still have large velocities so that the Coulomb barrier could be neglected in collisions with large grains. Negative charging for small grains could even enhance dust growth by accretion (Zhukovska et al. 2006), which also depends on quantities $F_{0}$, Ferrarotti & Gail 2006; Zhukovska et al. 2016), (2021) (see also Todini & Ferrara 2001, Li & Mattsson 2020; (2021)).

Thus, we neglect the spatial variety of grain size distribution within the galaxy. We define the distribution function of $\phi_{d}\text{dense}$, which also depends on quantities $F_{0}$, Ferrarotti & Gail 2006; Zhukovska et al. 2016), (2021) (see also Todini & Ferrara 2001, Li & Mattsson 2020; (2021)).

We formulate the evolution of grain size distribution and grain porosity using the distribution functions of grain mass ($m$) and grain volume ($V$) at time $t$. To treat porous grains, we introduce the following two types of grain radius: characteristic radius ($a_{ch}$) and mass-equivalent radius ($a_{m}$). The characteristic radius, as defined by Okuzumi et al. (2009), is related to the grain volume (including vacuum) as $V = (4/3)\pi a_{ch}^{3}$ and the mass-equivalent radius is linked to the grain mass as $m = (4\pi/3)a_{ch}^{3}s$, where $s$ represents the bulk material density. The filling factor, denoted as $\phi_{m}$, is expressed as $\phi_{m} \equiv (a_{m}/a_{ch})^{3}(\leq 1)$ (note that $a_{ch} > a_{m}$ for porous grains), while the porosity is $1 - \phi_{m}$ (see also Ormel et al. 2009). For compact grains, $a_{ch} = a_{m}$ or equivalently $m = sV$. Since the variation of $s$ in a reasonable range does not affect our conclusion significantly, we adopt the value of astronomical silicate (hereafter silicate; $s = 3.5$ g cm$^{-3}$; Weingartner & Draine 2001) for the calculations of grain size distribution and grain porosity.

Here we introduce some quantities used to construct the basic equations. Because there is ambiguity in the grain size (i.e. $a_{dm}$ or $a_{ch}$), we use the grain mass distribution instead of the grain size distribution. We define the distribution function of $m$ and $V$ at time $t$ as the number density of dust grains in the $R^{3} \times (m, V)$ space ($R^{3}$ is a real 3-dimensional volume in the galaxy, for which we assume uniformity because of the one-zone treatment), and denote it as $f(m, V, t)$. We do not directly solve for $f(m, V, t)$ but use moment equations to save the computational cost. The grain mass distribution at time $t$, $\bar{n}(m, t)$ is introduced by the zeroth moment of the above distribution function for $V$:

$$\bar{n}(m, t) \equiv \int_{0}^{\infty} f(m, V, t) \, dV.$$  (1)

We note that $\bar{n}(m, t) \, dm$ is the number density of grains whose mass is between $m$ and $m + dm$. We also use the first moment of $f$ for $V$:

$$\bar{V}(m, t) \equiv \frac{1}{\bar{n}(m, t)} \int_{0}^{\infty} V \, f(m, V, t) \, dV,$$  (2)

which is the mean volume of grains with mass $m$. We also define

$$\varrho(m, t) \equiv m \bar{n}(m, t),$$  (3)

$$\psi(m, t) \equiv \bar{V}(m, t) \bar{n}(m, t),$$  (4)

The set of $\varrho$ and $\psi$ contains the information on the distribution function of grain mass and volume.

We construct the basic equations for $\varrho$ and $\psi$, which are equivalent to the moments, $\bar{n}$ and $\bar{V}$. In other words, we solve the moment equations. To save the computational cost, we further adopt the volume-averaging approximation (Okuzumi et al. 2009) (see also Appendix A and H21), in which we represent the volume by $\bar{V}$; that is, all grains with the same mass are approximated to have the same volume. Note that

$$\bar{V}(m, t) = m \varrho(m, t)/\varrho(m, t)$$  (5)

holds from equations (3) and (4), so that the two differential equations for $\varrho$ and $\psi$ are closed. Because of the volume-averaging approximation, the filling factor reduces to a function of $m$ (at each $t$) as

$$\phi_{m} = \frac{m}{sV}.$$  (6)

The dust mass density (per volume) is the integration of $\varrho(m, t)$ for $m$, so that the dust-to-gas mass ratio at time $t$, $D(t)$, is written as

$$\mu_{H}m_{H}\varrho_{H}(t)D(t) = \int_{0}^{\infty} \varrho(m, t) \, dm,$$  (7)

where $\mu_{H} = 1.4$ is the gas mass per hydrogen, $m_{H}$ is the hydrogen atom mass, and $\varrho_{H}$ is the hydrogen number density. Note that the gas density, $\rho_{gas}$, is estimated as $\rho_{gas} = \mu_{H}m_{H}\varrho_{H}$.

In the following subsections, we present the basic equations for $\varrho(m, t)$ and $\psi(m, t)$. For convenience, we formulate individual processes separately; in reality, we solve all processes together at each time-step. The continuous differential equations given below are solved by discretizing the entire grain radius range ($a_{m} = 3 \times 10^{-4} - 10 \mu m$) into 128 grid points with logarithmically equal spacing. The discretization algorithm is described in appendix B of HA19. We set $\varrho_{d}(m, t) = 0$ and $\varrho_{d}(m, t) = 0$ at the maximum and minimum grain radii for the boundary conditions. The integration in the range $[0, \infty]$ is practically performed between the minimum and maximum grain masses ($m_{\text{min}}$ and $m_{\text{max}}$, respectively) corresponding to the above minimum and maximum $a_{m}$, respectively (or equivalently we regard $\varrho$ and $\psi$ out of the above grain mass range as zero).

### 2.2 Stellar dust production

A certain fraction of the metals ejected from SNe and AGB stars are condensed into dust (e.g. Kozasa et al. 1989; Todini & Ferrara 2001; Nozawa et al. 2003; Ferrarotti & Gail 2006; Ventura et al. 2014). Although the dust condensation efficiency in the stellar ejecta could vary depending on the progenitor star mass, the dependence is uncertain (Inoue 2011; Kuo et al. 2013, and references therein). Therefore, following HA19, we adopt a constant parameter ($f_{d}$) that describes the condensation efficiency of metals in stellar ejecta.
We write the change of the grain size distribution by stellar dust production as
\[
\frac{\partial \phi(m, t)}{\partial t} = f_{\text{in}} \phi_{\text{gas}} Z m \dot{\phi}(m),
\]
(8)
where \( Z \) is the metallicity (including both gas and dust phases) with the dot meaning the increasing rate, and \( \dot{\phi}(m) \) is the mass distribution function of the dust grains produced by stars. The normalization of \( \dot{\phi}(m) \) is determined so that the integration for the whole grain mass range is unity. We adopt \( f_{\text{in}} = 0.1 \) (HA19). It is often convenient to define the grain size distribution corresponding to \( \phi(m) \) as \( \varphi(a) \equiv \phi(m) \, dm \). We assume the following lognormal form for \( \varphi(a) \):
\[
\varphi(a) = \frac{C_\varphi}{a} \exp \left\{ -\frac{[\ln(a/a_0)]^2}{2\sigma^2} \right\},
\]
(9)
where \( C_\varphi \) is the normalization factor, \( \sigma \) is the standard deviation, and \( a_0 \) is the central grain radius (\( \sigma = 0.47 \) and \( a_0 = 0.1 \mu m \) following Asano et al. 2013b; HA19). This functional form assumes that stars produce large (~ 0.1 \( \mu m \)) grains (see the arguments in e.g. Nozawa et al. 2007 and Yasuda & Kozasa 2012 for the typical grain radii condensed in SNe and in AGB star winds, respectively). For the chemical evolution (evolution of \( Z \)), we adopt a simple functional form later in Section 2.5.

We assume that dust condensation in stellar ejecta occurs predominantly through the accretion of atoms or monomers (i.e. growth at an atomic level, not through the attachment of macroscopic grains). In atomic-level condensation, we do not expect that porosity develops. However, as shown by Sarangi & Cherchneff (2015), coagulation can take place in SN ejecta (see also Studer et al. 2018) although its efficiency depends on the clumpiness of the ejecta. Coagulation is also included in a dust condensation calculation for AGB stars (Gobrecht et al. 2016); however, whether coagulation could play a significant role compared with dust condensation (nucleation and accretion) is not clear. Since the general importance of coagulation in stellar ejecta is still uncertain, we simply assume that stardust grains are compact. This treatment gives a conservative estimate of the porosity evolution, and also serves to focus on the creation of porosity through interstellar processing. We note that pre-solar grains originating from SNe and AGB stars found in meteorites are compact (e.g. Anders & Zimmer 1993; Amari et al. 1994).

The compactness of the grains formed in stellar ejecta leads to the following expression for the change of the volume-weighted distribution function by stellar dust production:
\[
\frac{\partial \psi(m, t)}{\partial t} = \frac{1}{V} \frac{\partial \phi(m, t)}{\partial t} \phi_{\text{coag}},
\]
(10)
which is based on the relation \( \psi_m = \psi/(s \psi) \) with \( \phi_m = 1 \) (equations 5 and 6).

2.3 Coagulation and shattering

The evolution of grain size distribution and porosity in shattering and coagulation was already formulated by H21. For compact grains, the equations for the evolution of grain size distribution had already been developed (Jones et al. 1994, 1996; Hirashita & Yan 2009). We use the moment equations of 2-dimensional \((m, V)\) Smoluchowski equation for coagulation with the volume-averaging approximation based on Okuzumi et al. (2009, 2012), who investigated how coagulation develops grain porosity. H21 extended their equation to describe shattering. Below we only describe the final equations we use in this paper, and refer the interested reader to H21 for the derivation.

The time evolution of the two distribution functions, \( \varphi \) and \( \psi \), is described by the following equations with the subscripts ‘coag’ and ‘shat’ indicating coagulation and shattering, respectively:
\[
\frac{\partial \psi(m, t)}{\partial t} = -m \varphi(m, t) \int_0^\infty \frac{K_{m,m_1}}{m_{m_1}} \varphi(m_1, t) dm_1
\]
\[
+ \int_0^\infty \int_0^\infty \frac{K_{m,m_2}}{m_{m_1} m_{m_2}} \varphi(m_1, t) \varphi(m_2, t) m \psi(m_1, m_2) dm_1 dm_2.
\]
(11)
\[
\frac{\partial \psi(m, t)}{\partial t} = -V(m, t) \psi(m, t) \int_0^\infty \frac{K_{m,m_1}}{V(m, t) V(m_1, t)} \psi(m_1, t) dm_1
\]
\[
+ \int_0^\infty \int_0^\infty \frac{K_{m,m_2}}{V(m_1) V(m_2)} \psi(m_1, t) \psi(m_2, t) (V_1 + V_2)_{m_1, m_2}^m m dm_1 dm_2.
\]
(12)
where \( K_{m,m_2} \) is the Kernel function in the collision between grains with mass \( m_1 \) and \( m_2 \) (referred to as grains 1 and 2, respectively), \( \theta(m_1, m_2) \) is the distribution function of mass produced from grain 1 in the collision between grains 1 and 2, and \( (V_1 + V_2)_{m_1, m_2}^m \) is the volume of the newly produced grain with mass \( m \).

The collision kernel for the collision between grains 1 and 2 is estimated as
\[
K_{m_1, m_2} = \sigma_{1,2} v_{1,2},
\]
(13)
where \( \sigma_{1,2} = \pi (a_{ch1} + a_{ch2})^2 \) (\( a_{ch1} \) and \( a_{ch2} \) are characteristic radii of grain 1 and 2, respectively) is the collisional cross-section, and \( v_{1,2} \) is the relative velocity between grains 1 and 2. We assume that the grain velocity is induced by interstellar turbulence, and the resulting typical velocity as a function of grain radius is described as (Ormel et al. 2009; H21)
\[
\nu_{\text{gr}}(m) = 1.1 M^{3/2} \phi_m^{1/3} \left( \frac{a_m}{0.1 \mu m} \right)^{1/2} \left( \frac{T_{\text{gas}}}{10^4 \text{ K}} \right)^{1/4} \left( \frac{m_H}{1 \text{ cm}^{-3}} \right)^{-1/4}
\times \left( \frac{s}{3.5 \text{ g cm}^{-3}} \right)^{1/2} \text{ km s}^{-1},
\]
(14)
where \( M \) is the Mach number of the largest-eddy velocity (practically used to normalize the grain velocity), and \( T_{\text{gas}} \) is the gas temperature. We apply \( M = 3 \) for shattering and \( M = 1 \) for coagulation (Hirashita & Murga 2020) to obtain similar grain velocities to those in Yan et al. (2004). We also fix \( s = 3.5 \text{ g cm}^{-3} \) as mentioned above. The gas density and temperature are set in Section 2.5. We estimate the relative velocity \( v_{1,2} \) between grains 1 and 2 by assuming a random direction in every calculation of the collision kernel (Hirashita & Li 2013). We neglect grain–grain collisions in other environments such as supernova shocks (Jones et al. 1996; Kirchschlager et al. 2019) for simplicity and note that additional shattering and coagulation cannot be distinguished from these processes associated with interstellar turbulence in our model.

We refer the reader to H21 (see their section 2.3) for the treatment of collisional products. We only give summaries in what follows.

2.3.1 Collisional products in coagulation

The porosity of a coagulated grain in a grain–grain collision is broadly determined by the ratio between the impact energy and the rolling energy (Okuzumi et al. 2012). H21 kept this physical essence, but somewhat simplified the formulae that describe the porosity of
collisional products. We here summarize H21’s treatment of porosity, focusing on the physical essence without repeating the full equations.

The rolling energy, denoted as $E_{\text{roll}}$, is the energy necessary for a monomer to roll over 90 degrees on the surface of another monomer (Dominik & Tielens 1997; Wada et al. 2007). The rolling energy is written as $E_{\text{roll}} = 12\pi^2 \gamma R_1 \xi_{\text{crit}}$, where $\gamma$ is the surface energy per unit contact area, $R_{1/2}$ is the reduced grain radius (assumed to be equal to the reduced radius of the two grains: $a_{m1}/a_{m2}/(a_{m1} + a_{m2})$; H21), and $\xi_{\text{crit}}$ is the critical displacement of rolling (beyond which particle motions enter the inelastic regime). We estimate the volume of the coagulated grain $(V_{1+2})$ from grains 1 and 2 (with volumes $V_1$ and $V_2$, respectively). The volume of the coagulated grain depends on the impact (kinetic) energy, $E_{\text{imp}}$, relative to $E_{\text{roll}}$ (see H21 for the actual equations we adopted): (i) $V_{1+2} \propto V_1 + V_2$ for $E_{\text{imp}} \ll E_{\text{roll}}$ (reflecting the hit-and-stick regime). (ii) The newly created volume $(V_{\text{void}})$ is compressed if $E_{\text{imp}} \geq E_{\text{roll}}$. (iii) If $E_{\text{imp}}$ becomes comparable to or higher than $n_c E_{\text{roll}}$, the grain is compressed further, approaching $V_{1+2} \propto (1 + \epsilon_V)(V_{1,\text{comp}} + V_{2,\text{comp}})$ at $E_{\text{imp}} \gg n_c E_{\text{roll}}$, where $V_{i,\text{comp}}$ is $m_i/s$ for $i = 1$ and 2, $n_c$ is the number of contact points (treated as a fixed parameter), and $\epsilon_V$ is the parameter that regulates the maximum compression. The coagulated grain has a mass of $m_1 + m_2$, and it is put in the appropriate grain mass bin.

Since H21 already investigated detailed parameter dependence for coagulation, we basically fix the parameter values necessary for the calculation of coagulation. See H21 for the discussions on the meaning and choice of each parameter. We adopt $\gamma = 25$ erg cm$^{-2}$ (value for silicate, originally for quartz; Chokshi et al. 1993; note that, as mentioned above, we adopt the silicate properties for the calculation of grain size distribution and porosity), $\xi_{\text{crit}} = 10$ Å, $\epsilon_V = 0.5$ and $n_c = 30$ unless otherwise stated. We should also keep in mind that the surface energy of dry silica may be larger than the above values (Kimura et al. 2015; Steinpiz et al. 2019). This implies that $E_{\text{roll}}$ is underestimated. Graphite and water ice may have larger $E_{\text{roll}}$. To examine a possibility of larger $E_{\text{roll}}$, we also examine a higher $E_{\text{roll}}$ with $\gamma = 100$ erg cm$^{-2}$ (appropriate for water ice; Israelachvili 1992; Wada et al. 2007). The abovementioned $E_{\text{roll}}$ is based on intermediate values (75 erg cm$^{-2}$; Dominik & Tielens 1997) and $\xi_{\text{crit}} = 30$ Å (these are the largest values examined in H21 based on Dominik & Tielens 1997; Heim et al. 1999; Wada et al. 2007) in Section 5.3.

### 2.3.2 Collisional products in shattering

The total ejected mass ($m_{\text{ej}}$) of shattered fragments from the original grain $m_1$ is determined by the ratio between the impact energy per grain mass (specific impact energy) and the specific energy necessary for the catastrophic disruption ($Q_{\text{c}}^*$/n$^*$) (Kobayashi & Tanaka 2010). The catastrophic disruption is defined as half of the grain mass being shattered; that is, $m_{\text{ej}} = m_1/2$. If the specific impact energy is much smaller (larger) than $Q_{\text{c}}^*/n^*$, $m_{\text{ej}}$ is negligible (comparable to $m_1$). We adopt $Q_{\text{c}}^* = 4.3 \times 10^{10}$ erg g$^{-1}$, which is valid for silicate (HA19). If we adopt the value of graphite, which is 5 times smaller, shattering produces larger numbers of fragments, leading to an increase of porosity (Section 5.3). The total ejected mass $m_{\text{ej}}$ is distributed into fragments, of which the size distribution is assumed to be a power-law with an index of $-3.5$ (Jones et al. 1996). The remnant $(m_1 - m_{\text{ej}})$ is put in the appropriate grain radius bin. The maximum and minimum masses of the fragments are assumed to be $m_{\text{f,max}} = 0.02 m_{\text{ej}}$ and $m_{\text{f,min}} = 10^{-6} m_{\text{f,max}}$, respectively (Guillet et al. 2011); if $m_{\text{f,min}}$ is smaller than the minimum grain mass (corresponding $a_m = 3$ Å), we apply $m_{\text{f,min}} = m_{\text{min}}$. We remove grains if the mass-equivalent grain radius becomes smaller than 3 Å (this happens when the maximum fragment mass is less than $m_{\text{min}}$).

The remnant can be compressed after the collision. H21 considered two cases: one is that we neglect this compression, and the other is that we assume compaction of a volume equivalent to $m_{\text{ej}}$. We adopt the latter case in this paper, since it gives a more conservative estimates for the porosity (however, this compaction only affects the grains at $a_m \gtrsim 0.1$ μm, and its effect is minor compared with the change of $\gamma$ and $\xi_{\text{crit}}$). Thus, we assume that the volume fraction equal to $m_{\text{ej}}/m_1$ becomes compact after the collision. We also set a limit of $\phi_m \lesssim 1$ to avoid the compression proceeding beyond that point.

### 2.4 Dust destruction by SN shocks and dust growth by accretion

Here we consider the evolution of grain size distribution and porosity through dust destruction by SN shocks and dust growth by the accretion of gas-phase metals. The evolution of grain size distribution by these two processes can be described by a conservation law of the number of grains (HA19). As a consequence, the evolution is described by an advection equation in the grain radius (or grain mass) space. In this paper, we extend the equations to include the total grain volume. As derived in Appendix A, the evolution of $\bar{\rho}$ and $\bar{\psi}$ through the above two processes is described by

$$\frac{\partial \rho(m,t)}{\partial t}_{\text{dest/acc}} = - \frac{\bar{\rho}}{\bar{m}} \left[ \bar{m} \phi(m,t) \right]_{\text{dest/acc}} + \frac{\bar{m}}{\bar{\rho}} \rho(m,t),$$

and

$$\frac{\partial \psi(m,t)}{\partial t}_{\text{dest/acc}} = - \frac{\bar{\psi}}{\bar{m}} \left[ \bar{m} \psi(m,t) \right]_{\text{dest/acc}} + \frac{\bar{\psi}}{\bar{V}} \psi(m,t).$$

where $\bar{m} \equiv m/\bar{V}$, $\bar{\rho}$ and $\bar{\psi}$ are functions of $m$ and $t$ (Appendix A). The subscripts ‘dest’ and ‘acc’ indicate dust destruction and accretion, respectively. The above equations indicate that $\bar{\rho}$ and $\bar{\psi}$ follow the same ‘advection’ equations except for the factor $\bar{m}/\bar{V}$ or $\bar{V}/\bar{V}$ in the source term. This means that the difference between the growth rates of mass and volume changes the porosity. In what follows, we evaluate $\bar{m}$ and $\bar{V}$ for each process.

#### 2.4.1 Dust destruction by SN shocks

Since our model is not capable of treating the inhomogeneity within a grain, we simply assume that the mass and volume are destroyed at the same rate in SN destruction. This is correct if the material and vacuum are mixed homogeneously within a grain. Based on this assumption, we write $\bar{m}$ and $\bar{V}$ as

$$\bar{m} = - m/\tau_{\text{dest}}(m, \bar{V}),$$

and

$$\bar{V} = - \bar{V}/\tau_{\text{dest}}(m, \bar{V}),$$

where $\tau_{\text{dest}}(m, \bar{V})$ is the destruction time-scale as a function of grain mass and volume, which is given below. Since $\bar{m}/\bar{V} = \bar{V}/\bar{V}$, the filling factor (porosity) does not change in dust destruction in our model.

For the destruction time-scale, we basically adopt the functional form from HA19 (originally from McKee 1989) with a modification for porosity. The time-scale on which the ISM is swept once by an average SN energy is estimated by $\tau_{\text{sw}} \equiv M_{\text{gas}}/(M_* \gamma)$, where $M_{\text{gas}}$ is the total gas mass of the ISM, $M_*$ is the gas mass swept by a single SN blast, and $\gamma$ is the SN rate. Using the sweeping time-scale, we evaluate the destruction time-scale as

$$\tau_{\text{dest}}(m) = \frac{\tau_{\text{sw}} \phi_m^{2/3}}{\epsilon_{\text{dest}}(m)},$$

where $\epsilon_{\text{dest}}(m)$ is the fraction of destroyed dust in a single passage of
SN shock as a function of grain mass, and $\phi_m^{2/3}$ represents the effective increase of the grain surface area (explained below). We adopt the following functional form for the efficiency (HA19): $\epsilon_{\text{degr}}(m) = 1 - \exp[-0.1(\alpha m/0.1\,\mu m)^{-1}]$. The SN rate $\gamma$ is tightly coupled with the chemical enrichment and is given later in Section 2.5 together with $M_{\odot}$.

The destruction occurs by sputtering, which is a surface process. Thus, the destruction rate is proportional to the surface-to-volume ratio. To be precise, the surface-to-volume ratio of fluffy grain depends on the fractal dimension (e.g. Okuzumi et al. 2009); in this paper, however, we simply assume that the porosity increases the surface by a factor of $(\alpha_c/\alpha_m)^2 = \phi_m^{-2/3}$, which is regarded as the effective increase of the projected area of a grain. Thus, the dust destruction time-scale is assumed to be proportional to $\phi_m^{-2/3}$.

As mentioned in Section 2.3, we neglect shattering associated with SN shocks, although shattering enhances the dust destruction by sputtering owing to the resulting smaller grain sizes (Kirchschlager et al. 2021). We also ignore the metallicity dependence of SN destruction efficiency, which could have a significant imprint on the evolution of dust abundance (Yamasawa et al. 2011; Priestley et al. 2021). The metallicity dependence of SN destruction could delay/enhance the increase of grain abundance at low/high metallicity. However, SN dust destruction has a minor influence on the porosity compared with coagulation and shattering, so that the detailed treatment of SN destruction does not influence the conclusion (Section 5.1).

\subsection{Dust growth by accretion}

For accretion, the growth rate $\dot{m}$ is estimated as

$$\dot{m} = \xi(t)m_\text{\small{acc}}(m, V),$$

$$V = \dot{m}/s,$$

where $\xi(t) \equiv 1 - D(t)/Z(t)$ is the fraction of metals in the gas phase and $\tau_{\text{acc}}(m, V)$ is the accretion (dust growth) time-scale given later. We assume that the newly increased volume by accretion is compact (equation 21); that is, new condensation of material creates compact solid (like dust condensation in stellar ejecta). We adopt the following accretion time-scale (HA19):\footnote{The sticking efficiency $S$ is fixed to 0.3.}

$$\tau_{\text{acc}} = \tau_0\phi_m^{2/3} \left( \frac{\alpha_m}{0.1 \mu m} \right) \left( \frac{Z}{Z_\odot} \right)^{-1} \left( \frac{n_H}{10^2 \text{ cm}^{-3}} \right)^{-1} \left( \frac{T_{\text{gas}}}{10 \text{ K}} \right)^{-1/2},$$

where $\tau_0$ is a constant given below, $Z_\odot$ is the solar metallicity (we adopt the same solar metallicity $Z_\odot = 0.02$ as in HA19). Because we fix $n_H = 10^3 \text{ cm}^{-3}$ and $T_{\text{gas}} = 10 \text{ K}$ (Section 2.5) in the dense ISM where accretion occurs, $\tau_{\text{acc}}$ is a function of $a$ and $Z$. We adopt $\tau_0 = 5.37 \times 10^7 \text{ yr}$ (HA19). Since accretion is a surface process, the factor $\phi_m^{2/3}$ is multiplied to take into account the effect of porosity on the grain surface area as also done in equation (19).

For accretion, $V/V = \dot{m}/(sV) = (\dot{m}/m)(m/sV) = \phi_m(m/\dot{m}) \leq \dot{m}/m$ using equations (6), (20), and (21). This means that the volume grows less rapidly than the mass. Thus, the porosity decreases (or the filling factor increases) as a result of accretion. This is due to our assumption that newly condensed portion of a grain is compact.

\subsection{All processes together}

As mentioned in the beginning of this section, some processes occur only in a limited ISM phase. Coagulation and accretion take place in the dense ISM, while shattering occurs in the diffuse ISM. For simplicity, we divide the ISM into the two phases: the dense and diffuse ISM. We adopt $n_H = 10^3 \text{ cm}^{-3}$ and $T_{\text{gas}} = 10 \text{ K}$ (typical of molecular clouds) for the dense ISM and $n_H = 0.3 \text{ cm}^{-3}$ and $T_{\text{gas}} = 10^4 \text{ K}$ for the diffuse ISM, following H21.

Because of the one-zone nature of the model, we adopt the common grain size distribution for the two ISM phases. The contributions from each of the two ISM phases to the time variation of the grain size distribution are added with an appropriate weight, for which we use the dense gas fraction, $\eta_{\text{dense}}$. Although spatially resolved simulations showed that the grain size distributions are different between the dense and diffuse ISM (Aoyama et al. 2020), our approach of a single (averaged) grain size distribution is expected to give a reasonable approximation (i) for a volume large enough to contain a statistically significant mass of both ISM phases and/or (ii) for a time much longer than the mixing time-scale of the ISM phases ($\sim 10^7 \text{ yr}$; e.g. McKee 1989). Given that detailed modelling of the ISM is beyond the scope of this paper, we simply fix $\eta_{\text{dense}}$ as done by Hirashita & Mura (2020) (or we regard the constant value of $\eta_{\text{dense}}$ as the average over the entire history of the galaxy). Since the two ISM phases have different gas densities, it is convenient to define the distribution functions per hydrogen, $\bar{\phi} = \phi/n_H$ and $\bar{\psi} = \psi/n_H$, which are independent of the density scaling. We calculate the evolution of $\bar{\phi}$ and $\bar{\psi}$ by

$$\frac{d\bar{\phi}(m, t)}{dt} = \sum_i \frac{1}{n_{H,i}} \left[ \frac{d\bar{\phi}(m, t)}{dt} \right]_i \eta_i,$$

$$\frac{d\bar{\psi}(m, t)}{dt} = \sum_i \frac{1}{n_{H,i}} \left[ \frac{d\bar{\psi}(m, t)}{dt} \right]_i \eta_i,$$

where the subscript $i$ indicates the process (‘star’, ‘coag’, ‘shat’, ‘dest’, or ‘acc’), $\eta_i$ is the weight for the ISM phase ($\eta_i = \eta_{\text{dense}}$ for coagulation and accretion, $\eta_i = 1 - \eta_{\text{dense}}$ for shattering, and $\eta_i = 1$ for stellar dust production and dust destruction). $n_{H,i}$ is the hydrogen number density in the appropriate ISM phase ($10^3 \text{ cm}^{-3}$ for coagulation and accretion, 0.3 cm$^{-3}$ for shattering, and arbitrary for stellar dust production and dust destruction; note that the equations for the last two processes have a linear dependence on the density). As is clear from the above equations, $\eta_{\text{dense}}$ is practically a parameter that determines the balance between dust growth (coagulation and accretion) and dust fragmentation in our model.

We also need the evolution of metallicity $Z$ and SN rate $\gamma$. Because the construction of a detailed chemical evolution model is not the purpose of this paper, we adopt a simple functional form used by HA19: $Z = 0.6\gamma(t/T_{\text{SFR}}) Z_\odot$, where $T_{\text{SFR}}$ is the star formation timescale (i.e. the star formation rate $\psi$ is given by $\psi = M_{\text{gas}}/T_{\text{SFR}}$). Since there are no metals at $t = 0$, we adopt $\gamma(m, t = 0) = 0$ and $\psi(m, t = 0) = 0$ for the initial condition. The above metallicity evolution indeed fits the one in Asano et al. (2013b, see their fig. 6).

For the SN rate, we only consider core-collapse SNe with progenitor mass $> 8 M_\odot$. The lifetimes of such massive stars are short enough to be regarded as instantaneous, so that $\gamma$ is proportional to $\psi$. Thus, the sweeping time-scale $\tau_{\text{sw}}$ defined in Section 2.4.1 is evaluated as $\tau_{\text{sw}} = M_{\text{gas}}/(\langle M \gamma \rangle) = M_{\text{gas}}/(\nu_{\text{SN}} \psi M_\odot) = T_{\text{SFR}}/(\nu_{\text{SN}} M_\odot)$, where $\nu_{\text{SN}}$ is the proportionality constant between $\psi$ and $\gamma$. Adopting the Chabrier (2003) initial mass function with a stellar mass range of 0.1–100 $M_\odot$, we estimate that $\nu_{\text{SN}} = 1.0 \times 10^{-2} M_\odot^{-1}$. We adopt
For the setup of galaxy parameters, we choose $\eta_{\text{dense}} = 0.5$ and $\tau_{\text{SF}} = 5$ Gyr as fiducial values. We vary these two parameters to examine the effects of the balance among various processes and of the metal enrichment time-scale.

### 2.6 Calculation of extinction curves

Following H21, we calculate extinction curves to predict an observable property. We focus on two representative grain materials: silicate and carbonaceous species. Note that we used the silicate properties for the calculations of grain size distribution and porosity. Since the uncertainty in the parameters has as large an influence as the material properties, we simply use the above grain size distribution and porosity for both materials. Moreover, the comparison under a common grain size distribution is useful to observe the difference purely caused by the grain material.

For the carbonaceous material, as mentioned in H21, graphite has a problem that the central wavelength of the 2175 Å bump shifts as porosity increases, which is not observed in the MW. Other than the 2175 Å bump, the effects of porosity are similar between amorphous carbon (amC) and graphite. Thus, for the carbonaceous species, we focus on amC. Voshchinnikov et al. (2006), in their fitting to the MW extinction curve, treated graphite as a compact component, while they treated silicate grains as porous. Thus, we also discuss the 2175 Å carrier as a separate component from our porosity modeling. We only summarize the calculation method of extinction curves and refer the interested reader to H21 for further details. For silicate and carbonaceous dust, we adopt the optical constants of astronomical silicate from Weingartner & Draine (2001) and those of amC from ‘ACAR’ in Zubko et al. (1996), respectively.

The effective dielectric permittivity ($\bar{\varepsilon}$) is calculated using the effective medium theory with the Bruggeman mixing rule. We separately calculate the extinction curves for silicate and amC using the grain size distribution calculated above. We assume each dust grain to be spherical with radius $a$, and refractive index $\bar{\varepsilon} = \sqrt{\bar{\varepsilon}_s \bar{\varepsilon}_m}$. The cross-section $C_{\text{ext},m}$ is calculated by the Mie theory (Bohren & Huffman 1983). To concentrate on the effect of porosity, we do not consider more complicated structures, such as compounds and grain mantles. Such structures may be important if we aim at more detailed modeling of the MW extinction curve (Mathis 1996; Jones et al. 2017).

The extinction at wavelength $\lambda$, $A_\lambda$ (mag) is calculated as

$$ A_\lambda = (2.5 \log_{10} \bar{\varepsilon}) L \int_0^\infty \bar{n}(m) C_{\text{ext},m} dm, $$

where $L$ is the path length. We present the extinction per hydrogen $A_\lambda/N_H$ ($N_H = n_H L$ is the column density of hydrogen nuclei), which scales with the dust abundance. We also show the normalized extinction $A_\lambda/A_V$ (the $V$ band wavelength corresponds to $\lambda^{-1} = 1.8 \mu m^{-1}$) to focus on the wavelength dependence. The path length $L$ is cancelled out in both expressions.

To clarify the effect of porosity, we calculate the extinction curve without porosity by forcing the filling factor of all grains to be unity (i.e. $\phi_m = 1$). The extinction calculated in this way is denoted as $A_{\lambda,1}$. The effect of porosity is quantified by $A_\lambda/A_{\lambda,1}$.

### 3 RESULTS

We present the evolution of grain size distribution, filling factor and extinction curve in this section. As shown by HA19, different processes are dominant at different ages; thus, we expect that the effect of each process on the porosity will be clear if we examine the time evolution. We also change $\eta_{\text{dense}}$ and $\tau_{\text{SF}}$, which affect the balance among the processes.

For the presentations, the grain size distribution is shown in the form of $a_m n(a_m)/n_H$, where $n(a_m)$ is defined as $n(a_m) da_m = \bar{n}(m) dm$. The quantity $a_m n(a_m)/n_H$ is interpreted as the mass-weighted ($\times a_m^3$) grain size distribution per log $a_m$. The grain size distribution is divided by $n_H$ to cancel out the density difference between the ISM phases. We refer to $a_m n(a_m)/n_H$ as the grain size distribution as long as there is no risk of confusion.

### 3.1 Evolution of grain size distribution and filling factor

We present the evolution of grain size distribution and filling factor. We show the results in Fig. 1 for various $\eta_{\text{dense}}$ with a fixed $\tau_{\text{SF}} = 5$ Gyr.

First, we discuss the fiducial case ($\eta_{\text{dense}} = 0.5$; Fig. 1a). The evolution of grain size distribution is overall similar to that presented in HA19. In the early epoch ($t \lesssim 0.3$ Gyr), the grain abundance is dominated by large ($a_m \sim 0.1$ μm) grains from stellar sources. At $t \sim 0.3$ Gyr, shattering gradually becomes prominent as we observe in the tail of the grain size distribution at small $a_m$. Between $t \sim 0.3$ and 1 Gyr, the small grain abundance drastically increases because of dust growth by accretion. Note that accretion is more efficient for smaller grains because of their larger surface-to-volume ratios. After that, the abundance of large grains continuously grows because of coagulation. As a consequence of continuous coagulation, the overall grain size distribution approaches a power-law-like shape whose slope is similar to that derived by Mathis et al. (1977, hereafter MRN) for the MW extinction curve ($n \propto a_m^{-3.5}$). Some analytic studies suggested that the slope of the grain size distribution reaches an equilibrium and becomes similar to the MRN value if the fragmentation and/or coagulation reach an equilibrium state (e.g. Dohnanyi 1969; Tanaka et al. 1996; Kobayashi & Tanaka 2010).

The filling factor in the fiducial case decreases around $a_m \sim 0.03$ μm after $t \sim 1$ Gyr. Since coagulation is the unique process that creates porosity in our model, the decrease of the filling factor coincident with the increase of large grains is attributed to the onset of efficient coagulation. The filling factor drops down to $\phi_m \sim 0.3$ around $a_m \sim 0.03$ μm. At $a_m \gtrsim 0.1$ μm, compaction occurs (recall that the grain velocity increases with $a_m$; equation 14). The filling factor stays almost unchanged after $t \sim 1$ Gyr; in this ‘equilibrium’ state, the porosity creation by coagulation is balanced by the compact grain formation by shattering and accretion. This balance also realizes the convergence of the slope in the grain size distribution.

Next, we discuss the effect of $\eta_{\text{dense}}$. We compare the results for $\eta_{\text{dense}} = 0.2$ and 0.8 with the fiducial case in Fig. 1. In the early epochs ($t \lesssim 0.3$ Gyr), the grain size distribution and the porosity are insensitive to $\eta_{\text{dense}}$ because their evolution is dominated by stellar dust production, not by interstellar processing. The evolution after $t \sim 1$ Gyr is, however, very different.

Here we discuss the case of $\eta_{\text{dense}} = 0.2$ (Fig. 1b). In this case, coagulation is inefficient (compared with the fiducial case), which makes the following differences in the grain size distribution at $t \gtrsim 1$ Gyr. At $t = 1$ Gyr, the small grain abundance is less for $\eta_{\text{dense}} = 0.2$ than for $\eta_{\text{dense}} = 0.5$ because accretion is less efficient. After $t \sim 3$ Gyr, the difference in the grain size distribution is explained by the inefficient coagulation, which has the following two influences: (i) The small-grain-dominated phase is more prominent because small grains do not coagulate efficiently; and (ii) the grain size distributions at later times ($t \sim 3$–10 Gyr) are
The solid, dotted, dashed, dot–dashed, and triple-dot–dashed lines show the mass-weighted grain size distribution per log $a_m$ relative to the gas mass. The thin straight solid line shows the MRN slope ($n \propto a_m^{-3.5}$).

More dominated by small grains as seen in the slope (compared with the MRN distribution) and the maximum grain radius. The filling factor is also affected by $\eta_{\text{dense}}$ at later epochs. Compared with the above fiducial case, the filling factor drops more (down to $\phi_m \sim 0.2$ at $a_m \sim 0.03 \mu m$) or the porosity develops more. This is because more efficient shattering for smaller $\eta_{\text{dense}}$ provides more small grains from which coagulation creates porosity (H21). The grain radius ($a_m$) at which $\phi_m$ becomes minimum is, however, insensitive to $\eta_{\text{dense}}$. This radius corresponds to the value above which compaction occurs. Indeed, using equation (14), the impact energy is roughly estimated as $E_{\text{imp}} \propto \frac{1}{m} m_v^2 \sim 3.2 \times 10^{-10} (a_m/0.03 \mu m)^3 (\gamma/3.5 \text{ g cm}^{-3})^2 \text{ erg}$ in the dense ISM. The rolling energy is, on the other hand, approximately given by $E_{\text{roll}} \sim 8.9 \times 10^{-10} (a_m/0.03 \mu m) (\gamma/25 \text{ erg cm}^{-2}) (\xi_{\text{crit}}/10 \text{ Å}) \text{ erg}$. Thus, $E_{\text{imp}} \gtrsim E_{\text{roll}}$ is satisfied if $a_m \gtrsim 0.04 \mu m$, which explains the grain radius above which compaction (increase of $\phi_m$) occurs. The filling factor also increases towards small $a_m$ because shattering continuously supplies compact small grains. Note that the above condition of compaction does not depend on $\eta_{\text{dense}}$.

In the case of $\eta_{\text{dense}} = 0.8$, the maximum grain radius is larger than in the fiducial case at later epochs because of more efficient coagulation. The filling factor is overall larger because the production of small grains, from which porosity is created through coagulation, is less efficient. The filling factor continues to increase up to $t \sim 10$ Gyr. For large ($a \gtrsim 0.3 \mu m$) grains, the porosity is determined by the maximum compaction regulated by $E_{\text{f}}$ in Section 2.3.1; that is, the grains are not completely compressed. This treatment has already been discussed by H21 and it does not affect our discussions below significantly.

3.2 Effect of star-formation time-scale

As shown by Asano et al. (2013a), metal enrichment plays an important role in determining the dust abundance, especially because dust growth by accretion is governed by the metallicity. Indeed, as argued by Hirashita & Murga (2020), a similar grain size distribution is realized at the same $t/\sqrt{\tau_{\text{SF}}}$ (asano et al. 2013a). The stellar dust yield has a minor influence on the dust enrichment after dust growth starts to dominate the total dust abundance. Since the metal-enrichment time-scale is determined by $\tau_{\text{SF}}$, it is useful to examine the evolution of grain size distribution for various $\tau_{\text{SF}}$.

We present the evolution of grain size distribution and filling factor for various $\tau_{\text{SF}}$ (0.5 and 50 Gyr) with fixed $\eta_{\text{dense}} = 0.5$ in Fig. 2. Note that the fiducial case with $\tau_{\text{SF}} = 5$ Gyr is presented in Fig. 1a. For $\tau_{\text{SF}} = 0.5$ Gyr, since the metal enrichment occurs faster, we also show $t = 0.03$ Gyr and stop at $t = 3$ Gyr. In this case, except for the faster evolution, the overall evolutionary behaviour of grain size distribution is similar to the results in the fiducial case. The filling factor and its $a_m$ dependence are also similar between the two cases. As mentioned above, the evolutionary time-scale of grain size distribution is scaled with $\sqrt{\tau_{\text{SF}}}$; for example, a grain size distribution smoothed by coagulation appears at $t \sim 1$ Gyr for $\tau_{\text{SF}} = 0.5$ Gyr while it appears at $t \sim 3$ Gyr for $\tau_{\text{SF}} = 5$ Gyr. The porosity also develops when coagulation starts to be efficient; thus, considering the above scaling, the time when the effect of coagulation prominently appears is roughly evaluated as $t \sim 3(\tau_{\text{SF}}/5 \text{ Gyr})^{1/2}$ Gyr.

For $\tau_{\text{SF}} = 50$ Gyr, the dust enrichment proceeds more slowly and stops at the lower dust abundance at $t = 10$ Gyr than in the other cases with shorter $\tau_{\text{SF}}$. This is due to slow chemical enrichment. However, interstellar processing of dust still makes the grain size distribution

Figure 1. Evolution of grain size distribution (upper window) and filling factor (lower window) in each panel. Panels (a), (b), and (c) show the results for $\eta_{\text{dense}} = 0.5$ (fiducial), 0.2, and 0.8, respectively. The grain size distribution is multiplied by $a_m^n$ and divided by $n_H$; the resulting quantity is proportional to the mass-weighted grain size distribution per log $a_m$ relative to the gas mass. The solid, dotted, dashed, dot–dashed, and triple-dot–dashed lines show the results at $t = 0.1$, 0.3, 1, 3, and 10 Gyr, respectively. For $\phi_m$, we do not present the result at $t = 0.1$ Gyr, which is similar to that at $t = 0.3$ Gyr (i.e. unity for all $a_m$). The thin straight solid line shows the MRN slope ($n \propto a_m^{-3.5}$).
Dust porosity in galaxies

Figure 2. Same as Fig. 1 but for various star formation time-scales $\tau_{\text{SF}}$: (a) $\tau_{\text{SF}} = 0.5$ Gyr and (b) 50 Gyr (with $n_{\text{dense}} = 0.5$). The correspondence between the line species and the age is shown in the legend. Note that the case with $\tau_{\text{SF}} = 5$ Gyr (fiducial case) is shown in Fig. 1a.

approach the MRN slope. The filling factor achieved is similar to the above. Thus, the porosity, once it is created by coagulation, is insensitive to $\tau_{\text{SF}}$. Also, the grain radius at which the filling factor becomes minimum does not depend on $\tau_{\text{SF}}$ from the above argument in Section 3.1. We can also confirm that the time-scale on which the grain size distribution and the porosity are modified by interstellar processing follows the above scaling $\propto \tau_{\text{SF}}^{1/2}$.

3.3 Extinction curves

We calculate the evolution of extinction curve (shown in two forms: $A_\lambda/N_H$ and $A_\lambda/A_V$) for silicate and amC separately based on the grain size distributions and the filling factors presented above, using the method in Section 2.6. We also show the extinction curve without porosity, $A_{\lambda,1}$, defined in Section 2.6 so that the effect of porosity can be quantified by $A_\lambda/A_{\lambda,1}$. We display the results in Fig. 3.

First, we examine the case of silicate. From Fig. 3 (upper panel), we observe that $A_\lambda/N_H$ monotonically rises at all wavelength because of dust enrichment. Compared with the extinction without porosity ($A_{\lambda,1}$), $A_\lambda/N_H$ is higher in the far-UV and lower in the

Figure 3. Extinction curves for silicate and amC in the upper and lower panels, respectively. The thick solid, dotted, dashed, dot-dashed, and triple-dot-dashed lines show the extinction curves at $t = 0.1, 0.3, 1, 3,$ and $10$ Gyr, respectively. The thin lines show $A_{\lambda,1}$ (extinction without porosity) with the same line species. In each panel, the upper, middle, and lower windows present the extinction per hydrogen, the extinction normalized to the $V$-band value, and the ratio of $A_\lambda$ to $A_{\lambda,1}$ (an indicator of the porosity effect), respectively.
near-UV at \( t \geq 3 \) Gyr, when the porosity has developed. This is also clear in the ratio \( A_\lambda / A_{\lambda,1} \). Thus, the porosity can both increase and decrease the extinction depending on the wavelength as noted by Voshchinnikov et al. (2006) and Shen et al. (2008) (and discussed in H21). The effect of dust enrichment is cancelled out if we show \( A_\lambda / A_V \), from which we observe that the extinction curve steepens up to \( t \approx 1 \) Gyr and flattens after that. A similar evolutionary behaviour was also found by our previous calculation without porosity (HA19), and is interpreted by the evolution of grain size distribution shown in Section 3.1. The extinction curve is flat in the beginning because the grain abundance is dominated by large grains (stellar dust production). After that, the extinction curve becomes steep because of small grain production by shattering and accretion. The steepening at \( t = 1 \) Gyr corresponds to the drastic increase of grains with \( a_m \leq 0.02 \) \( \mu \)m, which contribute to the extinction at \( \lambda \leq 2\pi r_{\text{ch}} \approx 0.19 \) \( \mu \)m (1/\( \lambda \geq 5.3 \) \( \mu \)m\(^{-1} \)) if we adopt \( \phi_m = 0.3 \) from Fig. 1 (recall that \( a_{\text{ch}} = a_m \phi_m^{1/3} \)). This explains the steep rise for silicate around 1/\( \lambda \sim 5 \) \( \mu \)m\(^{-1} \) at \( t = 1 \) Gyr. After \( t = 1 \) Gyr, since coagulation efficiently converts small grains to large grains, the extinction curve becomes flatter, but not as flat as the one in the early evolutionary stage. At such later ages, the porosity makes the extinction curve shape (\( A_\lambda / A_V \)) steeper, especially in the far-UV. This is because of the above-mentioned behaviours of \( A_\lambda / A_{\lambda,1} \); that is, the enhancement of far-UV extinction by porosity.

Next, we discuss the extinction curves of amC. Fig. 3 shows monotonic rise of \( A_{\text{SF}} / N_{\text{H}} \) as it increases because of dust enrichment as also seen for silicate. The porosity created at \( t \geq 3 \) Gyr enhances \( A_{\text{SF}} / N_{\text{H}} \) at almost all wavelengths, which is clear also in \( A_\lambda / A_{\lambda,1} \). Recall that the porosity does not necessarily enhance the extinction for silicate. This difference between silicate and amC was already noted by H21. The evolution of \( A_\lambda / A_V \) for amC is qualitatively similar to that for silicate: steepening up to \( t \sim 1 \) Gyr and subsequent flattening. However, the overall steepness is less than in the case of silicate. At \( t > 1 \) Gyr, if we compare \( A_\lambda \) with \( A_{\lambda,1} \), we find that the extinction curve shape is flattened by the porosity. This is because the porosity enhances the extinction more around the \( V \) band wavelength, where the extinction curve is normalized, than in the UV. Except at \( t \sim 1 \) Gyr, when the rapid enhancement of the small grain abundance occurs, porosity has only a moderate effect on the resulting extinction curve shape of amC.

The porosity also enhances the opacity at IR wavelengths (1/\( \lambda \ll 1 \) \( \mu \)m\(^{-1} \)) for both dust materials as also shown by previous studies (e.g. Voshchinnikov et al. 2006). Naturally, the enhancement stays finite, so that \( A_\lambda \) always converges to zero as 1/\( \lambda \) approaches to zero. As mentioned in the Introduction, we concentrate on the wavelengths shorter than near-IR and do not discuss the behaviour at such long wavelengths in this paper.

The porosity enhances the far-UV extinction by \( \sim 20 \) per cent for silicate and \( \sim 10 \) per cent for amC at \( t \geq 3 \) Gyr. This means that we could ‘save’ \( 10–20 \) per cent of metals to realize a certain amount of far-UV extinction. However, the porosity has an optically opposite effect for the near-UV extinction of silicate, which is diminished by \( \sim 10 \) per cent. Thus, it is not obvious if porosity really saves the metals. We will further investigate this point in Section 4.

We also discuss the results with different \( \eta_{\text{dense}} = 0.2 \) and 0.8 based on the grain size distributions shown in Fig. 1. Since the results are quite obvious, we show the figures in Appendix B.

In Fig. B1, we present the evolution of extinction curve for \( \eta_{\text{dense}} = 0.2 \). The following discussions hold for both silicate and amC (unless the species is specified). The difference from the fiducial case appears after \( t \geq 1 \) Gyr, when interstellar processing starts to dominate the grain size distribution. At \( t = 1 \) Gyr, the extinction curves in the case of \( \eta_{\text{dense}} = 0.2 \) are rather flatter than those in the fiducial case because the increase of the small grain abundance by accretion is slower (recall that accretion occurs in the dense ISM). At later epochs (\( t \geq 3 \) Gyr), the extinction curves are steeper than in the fiducial case because of the dominance of small grains in the grain size distribution (Fig. 1b). For silicate, the steepness is further enhanced by the effect of porosity, which is clear from \( A_\lambda / A_{\lambda,1} \). The extinction at 1/\( \lambda \sim 1.5–6 \) \( \mu \)m\(^{-1} \) is diminished, while that in the far-UV is enhanced. This behaviour for silicate is already noted by Voshchinnikov et al. (2006), and the large porosity achieved in the case of \( \eta_{\text{dense}} = 0.2 \) makes the effect of porosity prominent. Because of inefficient coagulation in this case, the extinction curve shape (\( A_\lambda / A_V \)) stays steep after \( t = 3 \) Gyr for the case of silicate, contrary to the significant flattening in the case of \( \eta_{\text{dense}} = 0.5 \).

We also show the extinction curves for the dense-gas-dominated case with \( \eta_{\text{dense}} = 0.8 \) in Fig. B2. The extinction curves are overall flatter compared with the fiducial case for both silicate and amC. This is because of the overall large grain radii (Fig. 1c). The flatness of the extinction curve compared with the other cases is confirmed in the plots of \( A_\lambda / A_V \) except for the steep phase at \( t = 1 \) Gyr. For the case of silicate, the flatness is not only due to efficient coagulation but also because of less porosity (as discussed above, porosity makes the silicate extinction curve slope at optical–UV wavelengths steeper). However, the extinction curves of silicate with porosity are still slightly steeper than those without porosity at \( t \geq 3 \) Gyr. The porosity effect is less prominent in amC.

Finally, we discuss the effect of \( \tau_{\text{SF}} \). We calculate the extinction curves for \( \tau_{\text{SF}} = 0.5 \) and 50 Gyr based on the grain size distributions presented in Fig. 2. We show the results for \( \tau_{\text{SF}} = 0.5 \) and 50 Gyr in Appendix B (Figs. B3 and B4, respectively). As mentioned above, \( \tau_{\text{SF}} \) regulates the overall time-scale of dust enrichment. Thus, the evolution of extinction curve occurs on a different time-scale depending on \( \tau_{\text{SF}} \). In particular, the level of \( A_\lambda / N_{\text{H}} \) reflects the dust abundance, so that it rises more quickly for shorter \( \tau_{\text{SF}} \). As mentioned above, the time-scale of interstellar processing is scaled as \( \tau_{\text{SF}}^{1/2} \). For example, in the case of \( \tau_{\text{SF}} = 0.5 \) Gyr (50 Gyr), the steepening of extinction curve starts around \( t = 0.3 \) Gyr (3 Gyr), while a similar steepening appears around \( t = 1 \) Gyr in the fiducial case with \( \tau_{\text{SF}} = 5 \) Gyr. Thus, the steepening of the extinction curve happens around \( t \sim (\tau_{\text{SF}} / 5 \text{ Gyr})^{1/2} \) Gyr, considering the above scaling, although the extinction curves are not exactly the same because of the different scaling of the chemical enrichment time-scale (\( \propto \tau_{\text{SF}} \)).

4 THE MW EXTINCTION CURVE

Although the main purpose of this paper is to give basic predictions on the porosity formation in galaxy evolution, it may be useful to discuss how the above results fit the actually observed extinction curves. Here, we focus on the most investigated target – the MW extinction curve. The purpose of this section is to examine if the predicted porosity is still accepted by the observed MW extinction curve. We do not aim at detailed fitting to the MW extinction curve. This section will provide a key for further making a detailed model of the MW dust based on our dust evolution calculations.

There is still clearly a missing component in our model to explain the MW extinction curve – the 2175 Å bump carriers. As mentioned in Section 2.6, if graphite has porosity, the central wavelength of the bump changes, which is not observed in the MW. Our interpretation is that the bump is attributed to a component separated from the general porosity evolution. This implies that the modelling of the 2175 Å bump carrier needs a special treatment. We propose a possibility
Table 1. MW extinction curve fitting.

| Model          | $D_{\text{sil}}$ | $D_{\text{amC}}$ | $D_{\text{PAH}}$ | $D_{\text{tot}}$ |
|---------------|-----------------|-----------------|-----------------|-----------------|
| Fiducial      | 2.2             | 2.0             | 0.65            | 4.8             |
| $\eta_{\text{dense}} = 0.2$ | 0.34            | 2.2             | 0.43            | 3.0             |
| $\eta_{\text{dense}} = 0.8$ | 9.8             | 0.67            | 0.85            | 11              |
| w/o porosity  | 3.2             | 2.1             | 0.55            | 5.8             |
| Larger porosity | 2.1             | 2.1             | 0.65            | 4.9             |

*Using the fiducial result but forcing the porosity to be unity.

*Case with enhanced porosity examined in Section 5.2.*

that polycyclic aromatic hydrocarbons (PAHs) are responsible for the 2175 Å bump (Li & Draine 2001), and may not be suitable for being modelled using the bulk properties as done in this paper. In this section, we use the PAH component, of which the abundance is adjusted to fit the 2175 Å bump, but we adopt the above calculation results for other grain materials (silicate and amC).

We use $A_I/N_H$ for the extinction curve, since it includes the information on the dust abundance. We use the extinction curves in the fiducial case ($\eta_{\text{dense}} = 0.5$ and $t_{\text{SF}} = 5$ Gyr) at $t = 10$ Gyr (comparable to the age of the MW). The above extinction curves are normalized by the dust-to-gas ratio (equation 7) and obtain $A_I/N_H/D$ for silicate and amC. We also calculate $A_I/N_H/D$ for the PAH component in the following way. We take the absorption cross-section of PAHs per carbon atom, which is denoted as $C_{\text{abs,C}}$, from Li & Draine (2001). For the PAH component, the dust-to-gas ratio, which is defined as the PAH abundance per mass gas is evaluated as $D = 12N_C/(\mu N_H)$, where $N_C$ is the column density of carbon atoms (we neglect the contribution of hydrogen to the PAH mass because it only changes the mass by 4 per cent even if we assume $H/C = 0.5$). Since the extinction is evaluated as $A_I = (2.5 \log e)C_{\text{abs,C}}N_C$, we obtain $A_I/N_H/D = (2.5 \log e)\mu C_{\text{abs,C}}/12$ (recall that $\mu = 1.4$) for PAHs.

In the fitting, we use $A_I/N_H/D$ of each component from the model, and we derive the abundance of each dust component from the fitting. (In other words, we do not use the dust abundance calculated in the model.) The total extinction per hydrogen is now written as

$$
\frac{A_I}{N_H} = D_{\text{sil}}(A_I/N_H/D)_{\text{sil}} + D_{\text{amC}}(A_I/N_H/D)_{\text{amC}} + D_{\text{PAH}}(A_I/N_H/D)_{\text{PAH}},
$$

(26)

where $(A_I/N_H/D)$ for each component is distinguished by the subscript (‘sil’, ‘amC’, and ‘PAH’ for the silicate, amC, and PAH components, respectively), and $D$ is the dust-to-gas ratio (the mass abundance relative to the gas mass) of each component (the component is indicated by the subscript).

We adopt $A_I/N_H$ for the Milky Way extinction curve from Cardelli et al. (1989) [for $R_V = 3.1$, where $R_V \equiv A_V/(A_B - A_V)$] with the normalization determined by $A_V/N_H = 5.3 \times 10^{-22}$ mag cm$^2$ (Weingartner & Draine 2001). We derive the values of $D$ for the three components by performing least square fitting. We list the best-fitting values of $D$ in Table 1, and show the fitting results in Fig. 4.

From Fig. 4a, we observe that the MW extinction curve is broadly reproduced by using the fiducial model: At optical–near-IR wavelengths, the extinction level is determined by amC, since the other two components are relatively transparent. As expected, the PAH component dominates the extinction around the 2175 Å bump. The steep far-UV rise is further supplemented by silicate and PAHs. In this

Figure 4. Fitting to the MW extinction curve. The extinction curves for the silicate and amC components are based on the models of $\eta_{\text{dense}} = 0.5$ (fiducial), 0.2, and 0.8 ($t_{\text{SF}} = 5$ Gyr and $t = 10$ Gyr for all the models) in Panels (a), (b), and (c), respectively. We also add the PAH component. The abundances of the three dust components are adjusted to fit the observational data points. The solid line shows the best-fit total extinction, and the dotted, dashed, and dot-dashed lines present the contribution from the silicate, amC, and PAH components, respectively. The diamonds show the MW extinction curve adopted for the fitting.
fitting, the abundances of silicate and amC are comparable as shown in Table 1. The total dust-to-gas ratio is $4.8 \times 10^{-3}$, slightly lower than the often used value ($\sim 6 \times 10^{-3}$; e.g. Weingartner & Draine 2001). However, it is not fair to compare these values directly since the goodness of fitting is different. It is natural that our fitting to the MW extinction curve is worse than previous papers (because we do not change the grain size distribution freely); nevertheless, our results still fit the MW extinction curve quite successfully. The slight overprediction in the near-IR and underproduction in the optical are worth improving in the future.

To clarify the effect of porosity, we also perform another fitting using the extinction curves without porosity ($\phi_m = 1$; i.e. using $A_{1.1}$ for the extinction curves of silicate and amC). We show the fitting result in Fig. 5, and the resulting abundance of each component in Table 1. We observe that the best fit solution in this case has a 1.5 times higher silicate abundance and shows a slightly better fit to the near-IR extinction than that in the above fit. This is because the less steep extinction curve of compact silicate grains fits the overall steepness of the MW extinction curve. Nevertheless, we also need slightly more amC in this case because compact amC has less extinction per dust mass (see Fig. 3). However, less PAHs are required because of higher contribution of silicate to the overall UV extinction. Thus, in total, the carbon abundance required for the MW fitting is almost the same between the porous (fiducial) and non-porous cases. The fact that the resulting extinction curves for the porous and non-porous cases are similar means that the porosity is not the main driver to regulate the goodness of fitting. Note, though, that the resulting best-fit silicate abundance is affected by the porosity.

We also examine the fitting using the results with different $n_{\text{dense}}$. If we use the grain size distribution for $n_{\text{dense}} = 0.2$ (with the other parameters same as the fiducial case), we obtain the fitting result shown in Fig. 4b. As shown in Table 1, the resulting abundance of silicate is particularly small. This is mainly because of the difference in the grain size distribution. The extinction curve of amC is steep enough to reproduce the overall slope of the MW extinction curve. This leads to an extinction curve dominated by amC. Silicate has too steep an extinction curve in this case, so that it is disfavoured. The large porosity in this case also contributes to steepening the silicate extinction curve. As a result, the silicate abundance in the best fit solution is only $\sim 3 \times 10^{-4}$, which is too small to explain the interstellar depletion (Weingartner & Draine 2001). The fitting with $n_{\text{dense}} = 0.8$ provides an opposite extreme as shown in Fig. 4c. In this case, the fitting solution is dominated by silicate, since the amC extinction curve is too flat. The required silicate abundance is $\sim 0.01$ (Table 1), which exceeds the elemental abundance constraint (Weingartner & Draine 2001). Moreover, the fitting in the near-IR—optical is bad; the extinction curve at these wavelengths is too flat because of the dominance of large grains (Fig. 1c).

It is interesting to point out that the fiducial case provides the best fit solution for the MW extinction curve with a balanced grain abundance. We also find that the grain size distribution predominantly determines the goodness of the fit, and the porosity, as far as it is around our predicted value, does not affect the shape of the best-fit extinction curve significantly. Thus, extinction curves do not provide a strong constraint on the porosity, but rather constrain the grain size distribution. In other words, the moderate grain porosities predicted in our paper do not alter the overall understanding of extinction curves obtained from fitting efforts in previous studies (e.g. MRN; Kim et al. 1994; Weingartner & Draine 2001; Zubko et al. 2004).

5 DISCUSSION

5.1 Uncertainties

The grain-size-dependent porosity is the prediction made possible in our new framework that treats the porosity and the grain size distribution simultaneously. As shown above (and in H21), the interplay between coagulation and shattering is important for the porosity. However, the grain radius at which the porosity peaks as well as the peak porosity value still depends on some parameters in coagulation and shattering. We discuss which parameter could change the porosity evolution.

For coagulation, the results depends on $\gamma$, $\xi_{\text{crit}}$ and $\epsilon_V$ ($n_c$ has a smaller influence than $\xi_{\text{crit}}$, H21). Note that $\gamma$ is degenerate with $\xi_{\text{crit}}$. A small value of $\gamma$ or $\xi_{\text{crit}}$ (equivalent to small $E_{\text{roll}}$) leads to small porosity, especially at large ($a_m \geq 0.1 \mu m$) radii, because compaction easily occurs. The parameter $\epsilon_V$, which determines the maximum compaction, affects the porosity at large grain radii, where strong compaction occurs ($a_m \geq 0.3 \mu m$). Since such large grains are also processed by shattering, $\epsilon_V$ is also degenerate with the treatment of compaction in shattered remnants. Also, shattering effectively disrupt large grains at $a_m \geq 0.3 \mu m$. Thus, $\epsilon_V$ is less important than $\gamma$ and $\xi_{\text{crit}}$. In summary, among the parameters that regulate coagulation, $\xi_{\text{crit}}$ and $\gamma$, which directly affect $E_{\text{roll}}$, most efficiently influence the porosity in our model.

The treatment of shattering also has some freedom. The degree of compaction of shattered remnants may affect the porosity evolution. However, this only has a minor influence on the porosity unless the porosity is very large (such as $\phi_m \leq 0.1$; H21). As shown above, shattering has a positive effect on the porosity by supplying fragments from which coagulation builds up porous grains. Thus, the efficiency of fragment formation (i.e. the shattered fraction in a grain–grain collision) is important for the porosity. The shattered fraction is determined by the ratio between the specific impact energy and $Q_D^*$; thus, both grain velocities and $Q_D^*$ affect the resulting grain porosity. If we adopt a smaller value of $Q_D^*$ (or larger grain velocities) than adopted above, shattering becomes more efficient, leading to a result similar to lower $n_{\text{dense}}$; that is, the grain size distribution is more biased towards smaller radii and the porosity becomes larger. In this sense, $n_{\text{dense}}$ and $Q_D^*$ are degenerate, and a case with smaller $Q_D^*$ (or
larger grain velocities) can be effectively investigated by adopting smaller \( \eta_{\text{dense}} \).

The parameters related to the other processes (stellar dust production, SN destruction and accretion) have minor effects on the porosity compared with those of shattering and coagulation. This, however, does not mean that these processes are unimportant: They play important roles in the evolution of grains size distribution and dust abundance (H19) and indirectly influence the efficiency of coagulation and shattering. In particular, accretion drastically increases the abundance of small grains, which are later coagulated. Moreover, it is also interesting to point out that the 'bombardments' associated with sputtering and shattering could imprint some holes in dust grains, which can be taken as a creation of porosity. Thus, in the future, it is still worth including some other ways of creating grain inhomogeneity that could lead to the formation of porosity.

To summarize, in our model, the parameters concerning shattering and coagulation potentially affect the results significantly. Since the effects of these processes on the porosity have already been investigated in H21, we do not repeat the parameter surveys. Nevertheless, since we newly included all processes, it is worth reexamining some parameter dependence in the context of our new framework. We discuss the effect of porosity creation further and implications for the MW extinction curve in the following subsection.

### 5.2 Effects of porosity on the evolution of grain size distribution

As discussed in the previous subsection, the parameters concerning shattering and coagulation are important in regulating the porosity; in particular, \( \gamma \) and \( \xi_{\text{crit}} \) directly regulate the rolling energy \( E_{\text{roll}} \) (important for compaction), and the ratio between the specific impact energy and \( Q_{\text{D}}^* \) governs the efficiency of producing shattered fragments (from which porous grains form through coagulation). In order to examine how much the effects of porosity could be enhanced, we focus on the cases where porosity increases compared with the above results.

First, we consider a case with decreased \( Q_{\text{D}}^* \), so that shattering becomes more efficient than in the fiducial case. As mentioned in the previous subsection, the result with smaller \( Q_{\text{D}}^* \) becomes similar to that with smaller \( \eta_{\text{dense}} \). Thus, the fitting to the MW extinction curve becomes similar to the case shown in Fig. 4b (Section 4). In this fitting solution, silicate is not favoured, leading to too small a silicate abundance. This is not supported by the interstellar depletion as discussed above.

Next, we change \( E_{\text{roll}} \), which affects the porosity directly. The rolling energy is regulated by the product of \( \xi_{\text{crit}} \) and \( \gamma \). As mentioned in Section 2.3.1, \( \gamma \) depends on the grain material and its value is also uncertain. There is a possibility that \( \gamma \) is larger than assumed in this paper. Also, a larger value of \( E_{\text{roll}} \propto \gamma \xi_{\text{crit}} \) is interesting to examine how much the effect of porosity is enhanced within the uncertainties in the parameters. As discussed in Section 2.3.1, \( \xi_{\text{crit}} \) can be 3 times larger (\( \sim 30 \) Å) and \( \gamma \) can also be 4 times larger (\( \sim 100 \) erg cm\(^{-2} \)). Thus, we examine the case where \( E_{\text{roll}} \) is 12 times larger than in the above calculations (this case with larger \( E_{\text{roll}} \) is referred to as the case with enhanced porosity). The fiducial parameters are used (\( \eta_{\text{dense}} = 0.5 \) and \( \tau_{\text{SF}} = 5 \) Gyr). The difference in the porosity appears after \( t \sim 1 \) Gyr, when coagulation starts to affect the grain size distribution significantly. The difference is the largest at the latest epoch; thus, we compare the results at \( t = 10 \) Gyr in Fig. 6.

From Fig. 6 (top), we observe that the porosity is indeed enhanced at \( a_{m} \sim 0.1 \) \( \mu \)m in the case of enhanced porosity. This is because compaction does not occur efficiently. Compaction is still significant in the submicron regime because the grain velocities are sufficiently

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![Figure 6](image_url)
large. The enhanced porosity increases the grain cross-sections, so that the coagulation becomes more efficient. As a consequence, the upper cutoff of the grain radius is slightly larger. In this case, the porosity indeed increases (the filling factor drops down to ~ 0.2 at $\alpha_m \sim 0.1 \mu$m).

In Fig. 6 (middle and bottom), we compare the extinction curves between the case with enhanced porosity and the fiducial case. Because the grain size distributions are different between the two cases, the extinction curves without porosity (i.e. forcing $\phi_m$ to be unity, shown by the thin lines) are different. However, the extinction curves which take the porosity into account (shown by the thick lines) are similar between the two cases. This is because the larger porosity tends to enhance the extinction curve slopes (as shown in the plot of $A_{14}/A_{4,1}$). This steepening effect almost cancels out the flattening by the increase of large grains. Thus, different efficiencies of porosity formation lead to different grain size distributions and porosities, but can predict very similar extinction curves. Because of the similar extinction curves, if we use the extinction curves predicted by the case of enhanced porosity for the fitting to the MW extinction curve, the resulting best fit solution and the mass fraction of each component are almost the same as those shown in Section 4 (Table 1; indicated by ‘Large porosity’).

In summary, the enhancement of grain radius by porosity affects the grain size distribution through the increase of grain cross-sections, which leads to more large grains through more efficient coagulation. However, the effect on the extinction curve is not obvious: Although the increase of large grains by porosity tends to flatten the extinction curve, the porosity can steepen it. These two effects can cancel out each other. To distinguish the two effects, calculating dust emission SEDs would be useful, particularly because the porosity also affects the far-IR opacity (e.g. Voshchinnikov et al. 2006; Ysard et al. 2012, 2018) while the mid-IR emission is sensitive to the grain size distribution (e.g. Désert et al. 1990; Dwek et al. 1997; Draine & Li 2001; Li & Draine 2001; Hirashita et al. 2020). Starlight polarization may also be useful to constrain the porosity (Draine & Hensley 2021b), although non-sphericity is also essential in modelling polarization. Draine & Hensley (2021b) obtained low porosity ($\phi_m \lesssim 0.5$) when a moderate axial ratio is assumed for spheroidal aligned grains. Considering that their porosity constraint is applicable for $\alpha_m \gtrsim 0.05 \mu$m, where grain alignment is efficient, our model predicts $\phi_m \lesssim 0.5$ in the fiducial model. Compaction plays an important role in keeping the porosity small at large grain radii (Section 3.1). For further quantitative comparisons with observational data, the dust emission SED and the starlight polarization are worth modelling by extending our framework.

5.3 Prospect for the fitting to the Milky Way extinction curve

In Section 4, we showed that our fiducial model with the additional inclusion of PAHs is broadly successful in fitting the MW extinction curve. The shape of the best-fit extinction curve does not significantly change even if we impose $\phi_m = 1$ (no porosity). This implies that our porosity model does not significantly alter the previous fitting to the MW extinction curve. As shown in the previous subsection, the uncertainties in the parameters that affect the porosity do not substantially alter the output extinction curves.

We also note that the 2175 Å carriers are modelled by an ad hoc PAH component, which is not treated by the evolution model in this paper. Hirashita & Mura (2020) modelled PAHs as small aromatic carbon grains in their evolution model of grain size distribution (see also Seok et al. 2014; Rau et al. 2019). However, PAHs cannot be included in our model because their porosity is not well defined. Moreover, as mentioned above, graphite, which is another candidate of the 2175 Å carriers, has a problem since its central wavelength shifts as porosity increases. The difficulty in fitting the 2175 Å carriers to our framework implies that we need some special treatment for them; for example, the bulk material properties assumed in this paper may not be applicable to them. Hirashita et al. (2020) also showed that enhancement of the diffuse gas fraction is necessary to explain the PAH emission but is not needed for small grain emission. If the major formation sites of PAHs are segregated from those of other dust components, our one-zone model is not capable of treating the PAH formation appropriately. Moreover, hydrodynamic effects such as enhanced densities caused by supersonic turbulence could further promote interstellar processing as mentioned in Section 2 (e.g. Hopkins & Lee 2016). This effect could not only enhance the spatial inhomogeneity in the grain size distribution but also accelerate shattering, coagulation, and accretion, which may lead to faster evolution of grain size distribution than predicted in this paper. In parallel, as done by Hirashita et al. (2020), it is also useful to predict mid-IR emission SED, where PAHs have prominent features (e.g. Tielens 2008), to further constrain the PAH abundance.

6 CONCLUSIONS

We formulate and compute the evolution of grain size distribution and filling factor (porosity) using a one-zone galaxy evolution model. We treat all the following processes considered previously to calculate the evolution of grain size distribution: stellar dust production, dust destruction by SN shocks sweeping the ISM, dust disruption by shattering in the diffuse ISM, grain growth by coagulation in the dense ISM, and dust growth by the accretion of gas-phase metals in the dense ISM. We extend the framework to include the evolution of grain porosity. We assume that the dust grains formed in the stellar ejecta are compact. For coagulation and shattering, we solve moment equations derived from the 2-dimensional Smoluchowski equation by adopting the volume-averaging approximation; that is, the grain volume is represented by an averaged value for each grain mass (Okuzumi et al. 2009, 2012). In particular, coagulation is assumed to be the source of porosity. H21 already provided detailed results for shattering and coagulation, and we newly include the porosity evolution with accretion and SN destruction by developing the moment equations based on the conservation of the grain number. We assume that dust destruction does not change the porosity while the newly condensed portion in accretion is compact. As a consequence, our model is able to treat the grain size distribution and the porosity simultaneously, and to predict the grain-size-dependent porosity in the entire evolutionary history of a galaxy.

Since our model neglects the spatial variation within the galaxy, we simply adopt a constant mass ratio between the diffuse and dense ISM by specifying the dense gas fraction $\eta_{\text{dense}}$ (the diffuse ISM occupies $1 - \eta_{\text{dense}}$). We also vary the star formation time-scale $\tau_{\text{SF}}$ to regulate the chemical enrichment time-scale. We adopt $\eta_{\text{dense}} = 0.5$ and $\tau_{\text{SF}} = 5$ Gyr for the fiducial values.

In the fiducial case, the result is described as follows. The evolution of grain size distribution is similar to the case without porosity evolution as previously calculated by HA19. In the early epoch, the grain size distribution is dominated by large grains produced by stars and shattering gradually produces small grains at $t \sim 0.3$ Gyr. In this phase, the grains are compact because these dominant processes (stellar dust destruction and shattering) do not produce any porosity. Porosity appreciably increases around $t \sim 1$ Gyr when coagulation starts to become efficient owing to the increased abundance of small
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grains by accretion. The filling factor decreases down to $\sim 0.3$ at $a_m \sim 0.03 \, \mu m$, and reaches an equilibrium. Compaction increases the filling factor at $a_m > 0.03 \, \mu m$. The grain size distribution converges to an MRN-like shape at $t \geq 3 \, Gyr$.

The filling factor at $t \geq 1 \, Gyr$ is strongly affected by $\eta_{\text{dense}}$. The filling factor drops to 0.2 around $a_m \sim 0.03 \, \mu m$ when $\eta_{\text{dense}}$ is low (0.2). In this case, shattering is more enhanced and coagulation is less efficient than in the fiducial case. Although coagulation is the source of porosity, shattering produces small grains, from which porous grains are built up by coagulation. This confirms the conclusion in H21 that strong shattering with weak coagulation is favourable for creating grain porosity. In the case of $\eta_{\text{dense}} = 0.8$, large sub-micron grains are efficiently formed by coagulation, and the filling factor is relatively large ($\sim 0.5$ at $a_m \sim 0.03 \, \mu m$).

We also examine the dependence on $\tau_{\text{SF}}$. The filling factor achieved after the evolution (e.g. at $t \geq 3 \, Gyr$) is insensitive to $\tau_{\text{SF}}$. We confirm that the time-scale of interstellar processing scales as $\tau_{\text{SF}}^{1/2}$ (Hirashita & Murga 2020). That is, a similar grain size distribution and porosity are obtained at the same $t/\sqrt{\tau_{\text{SF}}}$. This scaling is not exact (especially for the total dust abundance) since the overall dust enrichment time-scale is scaled as $\sim \tau_{\text{SF}}$.

To investigate the effect of porosity evolution on observed dust properties, we calculate optical–UV extinction curves using the effective medium theory for silicate and amC separately. Porosity enhances the far-UV extinction by $\sim 20$ per cent and decreases the near-UV extinction by $\sim 10$ per cent for silicate in the fiducial case. As a consequence, porosity makes the silicate extinction curve steeper. The extinction of amC is enhanced by $\sim 10$ per cent in the UV and $\sim 20$ per cent in the optical, and the overall slope is not sensitive to the porosity. The decrease/increase of silicate extinction in the near/far-UV becomes larger for $\eta_{\text{dense}} = 0.2$ and smaller for $\eta_{\text{dense}} = 0.8$, reflecting the difference in the porosity achieved. The wavelength range of diminished silicate extinction becomes wider for smaller $\eta_{\text{dense}}$.

The above predictions will serve to examine or constrain the porosity evolution against the observed extinction curve in the MW. The 2175 Å bump is difficult to model in our framework, since porous graphite grains show a shift in the peak wavelength of the bump, which is not observed in the MW. Thus, we model the bump separately from the two components (silicate and amC), using PAHs, and leave the origin of the bump carriers for future work. We use the calculation results for the fiducial case at $t = 10 \, Gyr$, and fit the MW extinction curve. We find that the porosity does not affect the goodness of fitting. This means that the grain size distribution, rather than the porosity, is more important in fitting the MW extinction curve (as long as the porosity values calculated in our model are applicable). Our fitting also requires similar amounts of silicate and amC, and the total dust-to-gas ratio of $\sim 5 \times 10^{-3}$. We also used the results for non-fiducial values for $\eta_{\text{dense}}$ ($= 0.2$ and $0.8$) at $t = 10 \, Gyr$ to fit the MW extinction curve. For $\eta_{\text{eta}} = 0.2$, because the extinction curve of amC is steep enough, the overall slope of the MW extinction curve is mostly reproduced by amC; thus, the fitting result predicts negligible amount of silicate, which is not consistent with the interstellar depletion. For $\eta_{\text{dense}} = 0.8$, the large-grain-dominated size distribution predicts too flat an extinction curve in the optical and near-IR. Overall, the grain size distribution plays a more dominant role than the porosity in determining the fitting solution for the MW extinction curve.

Grain porosity also affects the evolution of grain size distribution because it effectively increases the grain cross-sections. To examine this effect, we increase the values of the parameters ($\gamma$ and $\xi_{\text{crit}}$) that regulate the rolling energy. If the rolling energy is $\sim$ ten times higher (allowing for the uncertainties in the parameters), the abundance of large grains slightly increases. This is because less compaction leads to higher porosity at $a_m \geq 0.1 \, \mu m$, which increases the coagulation rate. The resulting extinction curves are not necessarily flatter even with the larger abundance of large grains. This is because larger porosity tends to make the extinction curve steeper (especially for silicate), which counteracts the flattening of extinction curve by the enhanced large-grain abundance. Thus, although the enhancement of coagulation by porosity affects the grain size distribution, it does not necessarily change the extinction curve significantly.

The calculations in this paper serve to give a basis for further extension of the model to, for example, dust emission SEDs. Furthermore, a modelling effort of including the 2175 Å bump carriers is necessary. If PAHs contribute not only to the 2175 Å bump but also to mid-IR emission, predictions of dust emission SEDs are a crucial step towards a comprehensive understanding of dust evolution in galaxies. Polarization could also be important to predict, since it may also constrain the porosity (Draine & Hensley 2021b). We could also extend our predictions to high redshift galaxies to investigate the evolution of dust porosity as well as grain size distribution (see Liu & Hirashita 2019 for a model at high redshift) in the history of the Universe.

ACKNOWLEDGEMENTS

We are grateful to the anonymous referee, L. Pagani, and B. T. Draine for useful comments. HH thanks the Ministry of Science and Technology (MOST) for support through grant MOST 107-2923-M-001-003-MY3 and MOST 108-2112-M-001-007-MY3, and the Academia Sinica for Investigator Award AS-IA-109-M02. VBI acknowledges the support from the RFFR grant 18-52-52006 and the SUAI grant FSRF-2020-0004.

DATA AVAILABILITY

Data related to this publication and its figures are available on request from the corresponding author.

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APPENDIX A: DERIVATION OF THE EQUATIONS FOR DESTRUCTION AND ACCRETION
We derive equations (15) and (16) for dust destruction by SN shocks and dust growth by accretion. The most important characteristic of these processes is the conservation of the grain number in the (m, V) space (except at the boundary) (see Hirashita & Kuo 2011; Mattsson 2016).
\[
\frac{\partial f(m, V, t)}{\partial t} + \frac{\partial}{\partial m} [n f(m, V, t)] + \frac{\partial}{\partial V} [V f(m, V, t)] = 0. \tag{A1}
\]

Here the ranges of \( m \) and \( V \) are assumed to be \( 0 \leq m < \infty \) and \( 0 \leq V < \infty \). We take the zeroth moment of this equation (but multiplied by \( m \)) as

\[
\frac{\partial}{\partial t} \int m f dV + \frac{\partial}{\partial m} \int m f dV - \int \dot{m} f dV + [\int V f dV]_0^\infty = 0, \tag{A2}
\]

where \( [F(V)]_0^\infty \equiv F(\infty) - F(0) \), and we omit the variables in \( f \) and the integration range \([0, \infty]\). The last term on the left-hand side in equation (A2) vanishes since \( m = 0 \) at \( V = 0 \) and \( f \) is expected to approach to zero quickly enough as \( V \to \infty \) (precisely speaking, we assume that \( V \) is finite for \( V \in [0, \infty] \) and that \( m \to 0 \) as \( V \to \infty \)).

Next, we take the first moment of equation (A1) and obtain

\[
\frac{\partial}{\partial t} \int f V dV + \frac{\partial}{\partial m} \int m V dV - \int \dot{V} f dV + [\int V^2 f dV]_0^\infty = 0. \tag{A3}
\]

The last term on the left-hand side of this equation vanishes for the same reasons as above.

We define the following mean quantity of \( Q(m, V, t) \) (\( Q \) is an arbitrary function of \( m, V, \) and \( t \)):

\[
\bar{Q}(m, t) \equiv \frac{\int Q(m, V, t) f((m, V, t) dV}{\int f(m, V, t) dV}. \tag{A4}
\]

Using \( \bar{n}(m, t) \) defined in equation (1) together with some mean quantities (following equation A4), we obtain from equations (A2) and (A3)

\[
\frac{\partial}{\partial t} [m \bar{n}(m, t)] + \frac{\partial}{\partial m} [\bar{n}(m, t) m \bar{n}(m, t)] - \bar{n}(m, t) \bar{\dot{n}}(m, t) = 0, \tag{A5}
\]

\[
\frac{\partial}{\partial t} [\bar{V}(m, t) \bar{n}(m, t)] + \frac{\partial}{\partial m} [m \bar{V}(m, t) \bar{n}(m, t)] - \bar{V}(m, t) \bar{\dot{n}}(m, t) = 0. \tag{A6}
\]

To close this hierarchy of moment equations, we adopt the volume-averaging assumption (Okuzumi et al. 2009); that is, the volume is replaced with the mean value at each \( m \). This approximation is mathematically expressed as \( f(m, V) = \bar{n}(m) \delta[V - \bar{V}(m)] \), where \( \delta \) is Dirac’s delta function. Using this expression, we obtain

\[
\dot{n}(m, t) = \dot{\bar{n}}(m, \bar{V}(m, t), t), \tag{A7}
\]

\[
\bar{m} \bar{V}(m, t) = \bar{n}(m, \bar{V}(m, t), t) \bar{V}(m, t), \tag{A8}
\]

\[
\bar{V}(m, t) = \bar{V}(m, \bar{V}(m, t), t). \tag{A9}
\]

Using equations (A7)–(A9) together with equations (3) and (4) for the definitions of \( \varphi \) and \( \psi \), equations (A5) and (A6) are reduced to equations (15) and (16). Note that in the main text, we simplify the notations as \( \dot{m} \equiv \dot{\bar{n}}(m, \bar{V}(m, t), t) \) and \( \bar{V} \equiv \bar{V}(m, \bar{V}(m, t), t) \).

**APPENDIX B: EXTINCTION CURVES FOR VARIOUS PARAMETER VALUES**

We show the extinction curves for silicate and amC together with the increment of extinction by porosity \((A_{d}/A_{A_{d}})\) for the cases other than the fiducial case \((\eta_{dense} = 0.2\) and \(\tau_{SF} = 5\) Gyr), which is shown in the text (Section 3.3; Fig. 3). We present the cases with \(\eta_{dense} = 0.2\) and 0.8 (with \(\tau_{SF} = 5\) Gyr) in Figs. B1 and B2, respectively. We also show the dependence on \(\tau_{SF}\) with a fixed \(\eta_{dense} = 0.5\) (\(\tau_{SF} = 0.5\) and 50 Gyr in Figs. B3 and B4, respectively). The discussions on these figures are provided in Section 3.3.
Figure B1. Same as Fig. 3 but for \( \eta_{\text{dense}} = 0.2 \).

Figure B2. Same as Fig. 3 but for \( \eta_{\text{dense}} = 0.8 \).
Figure B3. Same as Fig. 3 but for \( \tau_{\text{SF}} = 0.5 \) Gyr.

Figure B4. Same as Fig. 3 but for \( \tau_{\text{SF}} = 50 \) Gyr.