Effects of Fission Fragments on the Angular Distribution of Scission Neutrons

T. Wada\textsuperscript{a*}, T. Asano\textsuperscript{a}, M. Hirokane\textsuperscript{a}, N. Carjan\textsuperscript{b}, M. Rizea\textsuperscript{b}

\textsuperscript{a}Department of Pure and Applied Physics, Kansai University, 3-3-35 Yanate-cho, Saita 564-8680, Japan
\textsuperscript{b}National Institute of Physics and Nuclear Engineering, Reactorului 30, RO-077125, POB-MG6, Magurele-Bucharest, Romania

Abstract

We investigate the effects of the fission fragments on the angular distribution of scission neutrons. The time evolution of the wave function of the scission neutron is obtained by integrating the time-dependent Schrödinger equation. The effects of the re-absorption and scattering by the fission fragments are taken into account by means of the optical potential. The angular distribution is strongly modified by the presence of the fragments. Dependence on the magnitude of the absorption is discussed. Influence of the finiteness of the grid size is also discussed.

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1. Introduction

In low energy fission, such as spontaneous fission and thermal neutron induced fission, there are two main neutron emission processes: scission neutrons and post-scission neutrons. At the moment of scission, the neck that has connected the two fission fragments ruptures, followed by the quick absorption of the neck protrusions by the fragments. On this abrupt change of nuclear shape, it is probable that nucleons are left behind in the neck region and are observed as particle emission. On the other hand, post-scission neutrons are emitted from excited fission fragments; the process is supposed to be a thermal emission. Attempts have been made to separate the yield of scission neutrons in low energy fission (Franklyn, 1978, Kornilov, 2001), they reported that 10-20% of the total neutron yield could be scission neutrons. It is also attempted to estimate the number of scission neutrons theoretically (Carjan, 2007, Carjan, 2010). The results depend on the nuclear shape such as the neck radius before scission. If we extract the reliable number of scission neutrons from experiments, we can get information on the nuclear shape at the time of scission.

* Corresponding author. Tel.: +81-6-6368-1121.
E-mail address: wadataka@kansai-u.ac.jp.
The angular distribution of the scission neutron is a key to separate it from post-scission neutrons. These components can be separated by taking the kinematical condition into account; post-scission neutrons are emitted from the moving source (fully accelerated fragments) while the emission of scission neutrons is supposed to be isotropic in the lowest order approximation. However, since the scission neutrons are emitted in the close vicinity of the fission fragments, the final angular distribution is influenced by the re-absorption and the scattering by the fragments. In the previous work (Wada, 2011), we proposed a formulation based on the time-independent scattering theory. We observed a strongly modified angular distribution due to the scattering and re-absorption by the fission fragments. In this paper, we present an alternative approach based on the time-dependent Schrödinger equation. In the next section, a formulation is given to calculate the angular distribution of the scission neutrons in which the effect of the re-absorption and the scattering is taken into account in terms of the optical potentials. Results are presented for two cases, one is a purely absorptive case and the other is the case that includes the attractive potential. The effects of the fission fragments on the angular distribution of scission neutrons are discussed. Finally, a summary is given.

2. Framework

We start with a time-dependent Schrödinger equation (TDSE),

$$i \frac{\partial \psi}{\partial t} = H \psi, \quad H = -\frac{1}{2m} \nabla^2 + U, \quad \hbar = 1,$$

(1)

where $\psi$ denotes the neutron wave function, $H$ is the Hamiltonian, and $U$ is the potential that represents the effect of the fission fragments. The time development is obtained with the use of the mid-point integration,

$$\psi(t + \Delta t) = \psi(t) - i \Delta t H \psi(t + \Delta t / 2).$$

(2)

By decomposing $\psi$ into the real and the imaginary part, $\psi = R + iI$, the numerical solution is obtained using the following formula (the leapfrog method),

$$\begin{cases}
I(t + \Delta t / 2) = I(t - \Delta t / 2) + \Delta t HR(t) \\
R(t + \Delta t) = R(t) - \Delta t HI(t + \Delta t / 2).
\end{cases}$$

(3)

Some modification is necessary when we introduce an imaginary potential in $H$,

$$\begin{cases}
I(t + \Delta t / 2) = [(1 + \Delta t W / 2)I(t - \Delta t / 2) + \Delta t H_R R(t)]/(1 - \Delta t W / 2) \\
R(t + \Delta t) = [(1 + \Delta t W / 2)R(t) - \Delta t H_R I(t + \Delta t / 2)]/(1 - \Delta t W / 2),
\end{cases}$$

(4)

where $H$ is given as $H = H_R + iW$. The potential $U$ is parameterized in Woods-Saxon form centered at the position of the fragments,

$$U(\rho, z) = \frac{V_0 + iW_0}{1 + \exp\left(\frac{\sqrt{\rho^2 + (z + B)^2} - r_F}{a}\right)} + \frac{V_0 + iW_0}{1 + \exp\left(\frac{\sqrt{\rho^2 + (z - B)^2} - r_F}{a}\right)},$$

(5)

where $B$ is the half-distance between the fragments, $a$ is the diffuseness, and $r_F$ is the radius of the potentials. For simplicity, we assume symmetric fission.
Assuming the axial symmetry, we solve the TDSE in two-dimensional grid space \((\rho, z)\). The original emission of the scission neutrons is assumed to be isotropic, and we adopt a Gaussian wave packet for the initial wave function, \(\psi(t=0) = C \exp\left(-\alpha(\rho^2 + z^2)\right)\), where \(C\) is a normalization constant and \(\alpha\) is the width of the wave packet. We now set a sphere of radius \(R\) and calculate the neutron flux on this spherical surface,

\[
j(r, t) = \frac{1}{2im} (\psi \nabla \psi^* - \psi^* \nabla \psi).
\]  

(6)

We then calculate the number of outgoing neutrons per unit solid angle per unit time and integrate it with time to obtain the neutron angular distribution, \(i.e.,\) the number of neutrons per unit solid angle that passed the surface up to time \(t\),

\[
\frac{d\nu(\theta, t)}{d\Omega} = \int_0^t d\tau \int_0^{2\pi} d\phi \int_0^\pi d\phi' \frac{\partial^2 \nu(\theta', t')}{\partial \theta \partial \phi'} = \int_0^\pi j(R, \theta) R^2 \sin \theta d\theta'\ , \quad n = e_z.
\]  

(7)

In order to avoid artificial reflections of the wave function at the border of the grid, we put a week absorbing potential outside of the sphere of radius \(R\). We take a quadratic form for the absorbing potential. The strength is determined to minimize the effect of the reflection.

3. Results

We investigate the fission of \(^{236}\text{U}\) that corresponds to the neutron induced fission of \(^{235}\text{U}\) as an example. An important parameter in the calculation is the initial separation between the fragments. From a systematic study of the average total kinetic energy (TKE) of the fragments, Zhao et al. (Zhao, 2000) deduced the elongation parameter \(\beta\) which is the ratio between the average distance between the fragments to the contact distance \(r_0(A_1^{1/3} + A_2^{1/3})\), where \(A_1\) and \(A_2\) are the mass numbers of the fragments and \(r_0 = 1.17\) fm. The average distance between the fragments was determined so that the corresponding point charge Coulomb energy is equal to the average TKE. They obtained \(\beta = 1.53\) for the asymmetric fission in U region. In the calculation, we set the parameter \(B\) as \(B = \beta r_0(A_1^{1/3} + A_2^{1/3})/2\). The distance between the grid points is typically 0.1 fm in both \(z\)- and \(\rho\)-directions. The time step for the integration is typically \(\Delta t = 0.02\) fm/c. The radius \(R\) is set to 50 fm and the integration is performed up to \(t = 4 \times 10^{-21}\) s.

Figure 1 shows the calculated angular distribution of scission neutrons for the case of purely absorptive potentials \((V_0 = 0)\). The width of the initial wave packet is determined to give the average energy of neutron \(<\varepsilon> = 1.5\) MeV. In calculating the time development, we take account of the motion of the fragments due to the Coulomb repulsion between the fragments. The pre-scission kinetic energy of 10 MeV is assumed. The fragments lie along \(z\)-axis, \(i.e.,\) we have the absorptive potentials around 0 and 180 degrees. As we increase the magnitude of the absorption, the yields around 0 and 180 degrees decrease significantly, while the yield at 90 degrees does not change much. The resulting angular distribution has a peak at 90 degrees and can be easily distinguished from that of the post-scission component.

Next, we investigate the case with attractive real potentials together with the absorption. Figure 2 shows the results with \(V_0 = -40\) MeV. The difference from the purely absorptive case is clearly seen in particular at 0 and 180 degrees. The yield at 0 degrees is strongly enhanced by the attraction of the real potential and it decreases drastically as the absorption becomes stronger. In particular, in the case where we take the attractive potential alone, \(i.e.,\) \(W_0 = 0\), the distribution is strongly peaked around 0 and 180 degrees. For the case with \(W_0 = -5\) MeV, the yield around 90 degrees also decreases significantly, resulting in a rather flat angular distribution. Thus, the angular distribution of the scission neutrons depends strongly on the strength of the absorption.
In Fig. 3, we display the time development of the angular distribution of scission neutrons. It is seen that the yield around 0 (180) degrees grows rapidly first. We may say that this is because of the higher velocity of neutrons in the fragments caused by the attractive potentials. At $t = 2 \times 10^{-21}$s, we do not yet observe a peak around 90 degrees. Then at $t = 3 \times 10^{-21}$s, the peak around 90 degrees starts to grow and it keeps growing up to $t = 4 \times 10^{-21}$s. It should be noted here that the time scale mentioned above depends crucially on the radius $R$ of the sphere on which we observe the outgoing flux of neutrons.

![Fig. 1. Angular distribution with purely absorptive potentials: $W_0 = -1$ MeV (dot-dashed line), $W_0 = -2$ MeV (dashed line), and $W_0 = -5$ MeV (solid line).](image1)

![Fig. 2. Angular distribution with attractive potentials $V_0 = -40$ MeV and absorptive potentials: $W_0 = 0$ (dot-dot-dashed line), $W_0 = -1$ MeV (dot-dashed line), $W_0 = -2$ MeV (dashed line), and $W_0 = -5$ MeV (solid line).](image2)
Fig. 3. Time development of the angular distribution of scission neutrons with attractive ($V_0 = 40\text{MeV}$) and absorptive ($W_0 = 5\text{MeV}$) potentials: $t = 1 \times 10^{-21}\text{s}$ (dot-dot-dashed), $t = 2 \times 10^{-21}\text{s}$ (dot-dashed), $t = 3 \times 10^{-21}\text{s}$ (dashed), and $t = 4 \times 10^{-21}\text{s}$ (solid).

Fig. 4. (a) Illustration of the relationship between the radius $R$ and the emission angle $\theta$; (b) Comparison of the angular distributions with different $R$; $R = 30\text{ fm}$ (dashed) and $R = 50\text{ fm}$ (solid).

Because we are working in a finite grid size, the radius $R$ of the sphere on which we count the outgoing neutron flux must be finite. When a neutron is scattered by the fragment whose position is shifted from the origin, the emission angle is modified because of the finiteness of the radius $R$. Therefore, the angular distribution calculated in this way depends on the size of $R$. The situation is illustrated in Fig. 4(a). A particle
is emitted to an angle $\theta_1$ from a point that is displaced from the origin. When we detect the particle on the sphere with radius $R_1$, we count that the particle is emitted to the angle $\theta_1$, while if we detect it at a larger radius $R_2$, we count that it is emitted to the angle $\theta_2$. As can be seen in Fig. 4(a), we obtain $\theta_1 < \theta_2 < \theta_0$. When $R$ approaches infinity, we have no ambiguities in the definition of the angle. In Fig. 4(b), we display the comparison of the angular distribution with two values of the radius, $R = 50$ fm and $R = 30$ fm, for the case with $V_0 = -40$ MeV and $W_0 = -5$ MeV. In these calculations, the motion of the fragments was not taken into account. Compared with the case with $R = 50$ fm, the case with $R = 30$ fm shows larger yields around 0 and 180 degrees, and the peak around 40 degrees is shifted to a smaller angle. Though, some differences are seen between the two cases, the essential features of the angular distribution have not changed.

We investigated more cases starting from different initial distribution, e.g., changing the distance between the fragments or changing the width of the initial Gaussian wave packet. It is found that the final angular distribution depends considerably on the initial distribution of the neutrons. Since the information on the angular distribution of the scission neutrons is very important to separate them from other neutron sources, further investigation with more realistic initial wave function is necessary and is under progress. Because of the simplicity of the framework, it is rather easy to extend the calculation to mass-asymmetric fission. It is of interest to investigate the right-left asymmetry of the angular distribution since it is supposed to depend sensitively on the distribution of the emission points.

4. Summary

The effects of the re-absorption and scattering by the fission fragments on the angular distribution of scission neutrons have been investigated in the framework of the time-dependent Schrödinger equation. A formulation has been given to calculate the angular distribution from the neutron flux on a spherical surface with finite radius $R$. It has been demonstrated that the absorptive potential diminishes the yields around 0 and 180 degrees, resulting in the angular distribution that has a peak around 90 degrees. On the other hand, the attractive potential enhances the yields around 0 and 180 degrees. The final angular distribution depends strongly on the magnitude of the absorption and on the initial distribution of the scission neutrons.

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