Fermion states on domain wall junctions and the flavor number

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In this paper we address the problem of localizing fermion states on stable domain walls junctions. The study focus on the consequences of intersecting six independent 8d domain walls to form 4d junctions in a ten-dimensional spacetime. This is related to the mechanism of relaxing to three space dimensions through the formation of domain wall junctions. The model is based on six bulk real scalar fields, the \(\phi^4\) model in its broken phase, the prototype of the Higgs field, and is such that the fermion and scalar modes bound to the domain walls are the zero mode and a single massive bound state, which can be regarded as a two level system, at least at sufficiently low energy. Inside the junction, we use the fact that some states are statistically more favored to address the possibility of constraining the flavor number of the elementary fermions.

An interesting issue which has appeared in several investigations concerns the localization of fermion zero modes in order to explain the flavor hierarchy \(\delta\phi\) of the standard model of particle physics, the so-called \(SU(3) \times SU(2) \times U(1)\) model which joins together the strong \(SU(3)\) and electroweak \(SU(2) \times U(1)\) interactions. In this paper, our main purpose is to extend the former work \(\delta\phi\) with the inclusion of fermions, attempting to shed some light in the fact that in our Universe the number of quarks and leptons families seems to be selected to be three. Our results have shown that this number may be related to the number of intersecting domain walls with two bound states, the zero mode and one massive bound state. Interestingly, the presence of the zero mode and a massive bound state is exactly what happens when one investigates a single real scalar field \(\phi\) driven by the \(\phi^4\) model in its broken phase, the prototype of the fundamental scalar, the Higgs field.

In the present study, we will be mainly interested in the localization of both massive and massless fermion states. More specifically, we will consider our system consisting of 8d domain walls embedded in the 9d+1 space-time, where two fermion bound states are living in due to the partner scalar field being driven by \(\phi^4\) model in its broken phase. Six of these domain walls can be joined together to form a stable junction with 2\(^d\) bound states – for sufficiently low energy, there would be no excitation going to the continuum spectrum and so the domain wall can be regarded as a two-level system, the zero mode and a single massive bound state. Thus, if the excited fermion states living on the domain walls have the same energy, say \(m\), which we can construct very naturally, as we will show below, then in the junction made out of six domain walls there would be a distribution of degenerate states that presents a maximum at the energy \(\sqrt{3}m\), supposing normal distribution of states. This value suggests that there exist 20 degenerate massive fermion states with this energy that are statistically favored to live in the junction in addition to one non-degenerate fermion zero mode.

We take advantage of this result to observe that the 21 states might represent six flavors and three colors quark degrees of freedom plus three colorless lepton flavors. One could also think of these 21 fermion degrees of freedom as those to take into account all the matter sector of the Standard Model, given by \(SU(2)\) doublets — the 6 left quarks and 6 left leptons, and \(SU(2)\) singlets — the 6 right quarks and 3 right leptons. According to this scenario anything beyond the Standard Model should be a manifestation of extra-dimensions. This is because our 21 degrees of freedom are based on the requirement that the higher dimensional physics should effectively manifest as a four-dimensional physics on the junction.

We recall that in the model of \(\delta\phi\) one assumes a mechanism of dimensional reduction from the ten-dimensional spacetime down to a four-dimensional spacetime which is generated by the domain wall junction. With this, we can then state that in a 8d domain wall gas in ten
dimensions, the chance of a junction to be formed with the superposition of 20 massive states by combining three 8d domain walls in their fundamental fermion state (the zero mode) and three 8d domain walls in their excited fermion bound state is statistically favored among any other possible combinations of states.

The study starts in the next Sec. II, where we introduce the model in flat ten-dimensional spacetime and discuss how to construct domain wall solutions and how they can be joined together to form stable junctions. In Sec. III we investigate the presence of fermion states in the junction. Next, in Sec. IV we illustrate the investigations with a simple model, described by two real scalar fields in three dimensions and in Sec. V, we extend the analysis to the more involved case of six scalar fields in $D = 9 + 1$ dimensions. In particular, we study how the scalar fields bind to the junction to form bound states which are partners of the fermion bound states. Finally, in Sec. VI we present our ending comments.

II. The prototype model. For our purposes in the present study, we restrict ourselves to the fermionic and bosonic scalar sectors of a larger supersymmetric theory in ten dimensions [8] to find junctions of co-dimension one objects. The number of dimensions is suggested by superstring theory, and so the D8-branes are co-dimension one objects in the $(9,1)$ spacetime dimensions. In this set up, the branes are classical 8d domain wall solutions (8-branes) embedded in 10d spacetime. Let us refer to our theory as a softly broken supersymmetric theory, in the sense of Ref. [8], where a supersymmetric theory is perturbed under the action of a small parameter $\varepsilon$ — here, this parameter is responsible for the stability of the junctions.

Thus the softly broken supersymmetric Lagrangian is written in the following form

$$\mathcal{L} = \frac{1}{2} \partial_m \phi^i \partial^m \phi^i + \bar{\psi} \Gamma^m \partial_m \psi^i + W_{\phi^i \phi^j} \bar{\psi} \psi^j - V(\phi^i) - \frac{1}{2} \varepsilon F(\phi^i),$$

(1)

where $m = 0, 1, 2, ..., D - 1$, and $i, j = 1, 2, ..., N$. The scalar potential is given in terms of the superpotential $W$ by

$$V(\phi^1, \phi^2, ..., \phi^N) = \frac{1}{2} \frac{\partial W}{\partial \phi^1} \frac{\partial W}{\partial \phi^1} + \frac{1}{2} \frac{\partial W}{\partial \phi^2} \frac{\partial W}{\partial \phi^2} + ... + \frac{1}{2} \frac{\partial W}{\partial \phi^N} \frac{\partial W}{\partial \phi^N}.$$  

(2)

The scalar fields in the superpotential are such that

$$W_{\phi^i \phi^j} = \delta_{ij} W_{\phi^i \phi^i},$$

(3)

and

$$V(\phi^1, \phi^2, ..., \phi^N) = V(\phi^1) + V(\phi^2) + ... + V(\phi^N),$$

(4)

where $W_{\phi^i \phi^j}$ stands for the second derivative of the superpotential.

The domain wall junctions [8] and networks [9] of domain walls have been addressed in the literature in several contexts. In spite of the difficulty of finding analytical junction solutions, there are known cases in the literature [8, 10]. Furthermore, other investigations have been used to address the study of the vacua and energy balance among the intersecting domain walls [8], where individual domain walls are assumed to exist in a way such that they could be joined together to form a stable junction. These domain walls may form stable junctions, but in this case they should have their tensions satisfying the inequality

$$|T_{i1} + T_{i2} + ... + T_{iN}| < |T_{i1}| + |T_{i2}| + ... + |T_{iN}|,$$

where $i_1, i_2, ..., i_N = 1, 2, ..., N$ and $T_i = \Delta W_i$ behaves like a vector since it measures $\Delta W$ along different directions in the scalar field space $(\phi_1, \phi_2, ..., \phi_N)$. In this paper we shall follow the lines of Ref. [8], where the individual domain wall tensions were found to satisfy

$$|T_{i1} + T_{i2} + ... + T_{iN}| = |T_{i1}| + |T_{i2}| + ... + |T_{iN}|$$

$$+ \lambda \varepsilon < |T_{i1}| + |T_{i2}| + ... + |T_{iN}|,$$

(5)

for $\varepsilon < 0$, with $\lambda$ being a positive number which depends on the choice of $F(\phi^i) = \sum_{i=1}^{N} F(\phi^i, \phi^j)$. To our present purpose it is enough to use [8]

$$F(\phi^i, \phi^j) = \frac{1}{2} (\phi^{i^4} + \phi^{j^4}) - 3 \phi^{i^2 \phi^{j^2}} + \frac{9}{2}.$$

(6)

The equations of motion for boson and fermion obtained from the Lagrangian (1) are

$$\Box \phi^i + \frac{\partial V}{\partial \phi^i} - W_{\phi^i \phi^j} \bar{\psi} \psi^j + \varepsilon \frac{\partial F}{2 \partial \phi^i} = 0,$$

(7)

$$\Gamma^m \partial_m \psi^i + W_{\phi^i \phi^j} \psi^j = 0.$$    

(8)

By a suitable choice of the superpotential, one finds individual domain wall solutions in the bosonic sector whose dynamics is governed by the equation of motion

$$\Box \phi^i + \frac{\partial V}{\partial \phi^i} = 0.$$    

(9)

For domain wall junctions we are interested in domain wall solutions that can be joined orthogonally together to form stable junctions. Thus, we shall consider that each scalar field depends on a single spatial coordinate $x^k$, i.e., $\phi(x^1, x^2, ..., x^N) \rightarrow \phi^k(x^k) \in \{\phi^1(x^1), \phi^2(x^2), ..., \phi^N(x^N)\}$, where $x^k$ is a spatial coordinate transverse to the domain wall described by $\phi^k$. Since each domain wall has co-dimension one, this is very well defined operation. Under such considerations, static domain walls are governed by the following equations

$$\frac{d^2 \phi^k}{dx^k_k} = \frac{\partial V}{\partial \phi^k}, \quad k = 1, 2, ..., N.$$    

(10)
A first integral of (11) enables us to work with the first-order equations

\[ \frac{d\phi^k}{dx^k} = \frac{\partial W}{\partial \phi^k}, \quad k = 1, 2, ..., N. \]  

(11)

These first-order equations naturally appear in supersymmetric theories and domain walls are BPS solutions preserving half of the supersymmetries.

Before going to the next section, some important comments are in order. To show that a domain wall junction is a mechanism of compactification from ten down to four-dimensional spacetime, one should also show that gravity is localized on it [4]. This is achieved by considering that the bulk is an AdS spacetime. Thus, it is important to show that the bulk cosmological constant \( \Lambda \) in our setup is indeed negative. Using the fact that the scalar potential in (2) is perturbed by the \( \varepsilon \)-term of (1), we find that the perturbed vacua are generically able to localize massless fields on them.

In a bulk flat space (where \( \Lambda \) should vanish) are not present, this is achieved by considering the junctions. This result is crucial for the present investigation, because it circumvents the former result of Ref. [14], which stresses that domain wall junctions in a bulk flat space (where \( \Lambda \) should vanish) are not generally able to localize massless fields on them.

### III. The fermion modes.

Let us now investigate the fermion states in the presence of the domain walls background [1]. The fermion equation of motion (8) can be written in terms of positive and negative energy components \( \psi^\pm_\chi \). We look for fermion solutions in the form

\[ \psi^k = e^{ip^k(x^\mu)\chi(x^k)}, \]  

(14)

where \( \mu = 0, 1, 2, ..., d-2 \) are indices labeling coordinates along the domain walls, thus excluding \( x^k \) itself from the sum \( p^k(x^\mu) \). Now substituting (14) into (8), we find

\[ i\Gamma^\mu p^k(\chi^k) - \Gamma^k \partial_k \chi^k + W_{\phi^k\phi^k} \chi^k = 0. \]

(15)

Without loosening generality, we take the reference frame where \( p^k = (E^k, 0, ..., 0) \). In this frame the equation (15) takes the simpler form

\[ iE^k \Gamma^0 \chi^k - \Gamma^k \partial_k \chi^k + W_{\phi^k\phi^k} \chi^k = 0. \]

(16)

By using the properties of gamma matrices we have \( \Gamma^k \chi^k = \pm \chi^\pm \) and \( i\Gamma^0 \chi^k = \chi^\pm \), which lead to system of equations

\[ (\partial_k - W_{\phi^k\phi^k}) \chi^+_k = E^k \chi^+_k, \]

(17a)

\[ (\partial_k + W_{\phi^k\phi^k}) \chi^-_k = -E^k \chi^-_k. \]

(17b)

These equations allow us to write the Schrödinger-like equations

\[ [-\partial^2_k + U^k_\chi(x^k)] \chi^k_\pm = E^k \chi^k_\pm, \]

(18a)

\[ U^k_\chi(x^k) = W_{\phi^k\phi^k}(\chi^k_\pm) \pm W_{\phi^k\phi^k}(x^k). \]

(18b)

These equations govern the dynamics of the fermion bound states which are linked to the several independent domain walls. To describe domains walls joined together to form a junction we consider the following Schrödinger-like equation

\[ [-\nabla^2 + U_{\text{junc}}] \Psi_\pm(n_1 ... n_N) = E^2 \Psi_\pm(n_1 ... n_N), \]

(19)

where

\[ U_{\text{junc}} = U_{\chi_1}^1(x^1) + U_{\chi_2}^2(x^2) + ... + U_{\chi_N}^N(x^N), \]

(20a)

\[ E^2(n_1 ... n_N) = E^2_1(n_1)x_1 + E^2_2(n_2)x_2^2 + ... + E^2_3(n_3)x_3^3, \]

(20b)

\[ \Psi_\pm(n_1 ... n_N) = \chi_1^{n_1}(x^1) \times ... \chi_3^{n_3}(x^3), \]

(20c)

where the components \( \chi_1^{n_1}(x^1) \) and \( \chi_3^{n_3}(x^3) \) are normalizable functions. We are considering the numbers \( n_1 = 1 \), i.e., only two bound states, because they can be localized on the individual domain walls – recall that the sector forms a collection of fields in their broken phase. The infinite tower of continuum states are non-normalizable states that cannot be localized neither on individual domain walls nor on the junction.

They would fill the bulk, but they should be seen at higher energy. For the zero modes, \( E = 0 \), only one of them is normalizable, i.e., the one associated with the chiral fermion on domain walls and junction, as we have learned long ago from Ref. [11, 12].

### IV. The two-field example.

Let us consider the example with \( N = 2 \) bulk scalar fields to form \( N = 2 \) independent domain walls to be joined together to form a junction in \( D = 3 + 1 \) dimensions. The extension of the results to the case of \( N \) arbitrary can be done straightforwardly. Consider the following superpotential

\[ W(\phi_1, \phi_2) = \lambda_1 \left( \frac{\phi_1^2}{3} - a^2 \phi_1^3 \right) + \lambda_2 \left( \frac{\phi_2^2}{3} - a^2 \phi_2 \right). \]

(21)

For this case the first-order equations (11) reduce to

\[ \frac{d\phi^1}{dx^1} = \frac{\partial W}{\partial \phi^1}, \quad \frac{d\phi^2}{dx^2} = \frac{\partial W}{\partial \phi^2}. \]

(22)

There are solutions satisfying these differential equations such as

\[ \phi^1(x^1) = -a \tanh(\lambda_1 ax^1), \quad \phi^2(x^2) = -a \tanh(\lambda_2 ax^2). \]

(23)
The potentials with upper signs in (13) are given by
\begin{align}
U_1^2(x^1) &= 4\lambda_1^2 a^2 - 6\lambda_1^2 a^2 \sech^2 \lambda_1 a x^1, \\
U_2^2(x^2) &= 4\lambda_2^2 a^2 - 6\lambda_2^2 a^2 \sech^2 \lambda_2 a x^2.
\end{align}
These are modified Pöschl-Teller potentials \(^1\) of the general form \(U(x^k) = A - B \sech^2(x^k)\) for \(k = 1, 2\), with \(A\) and \(B\) being real constants. The normalizable bound states have the following energies
\[E_n = A - \left[\sqrt{B + \frac{1}{4} - (n + \frac{1}{2})}\right]^2,\]
where
\[n = 0, 1, \ldots < \sqrt{B + \frac{1}{4} - \frac{1}{2}}.\]
The discrete spectrum is composed by two bound states, the zero mode and the excited state given by
\begin{align}
E^2_{(0)1,2} &= 0, \\
\chi^{1.2} &= C_0 \sech^2(\lambda_{1,2} a x^{1,2}), \\
E^2_{(1)1,2} &= 3\lambda_{1,2} a^2, \\
\chi^{1.2} &= C_1 \tanh(\lambda_{1,2} a x^{1,2}) \sech(\lambda_{1,2} a x^{1,2}).
\end{align}
They are the spectrum of the fermions bound to the domain wall. The spectrum bound to a domain wall junction can be found by using (19). There are four combinations using zero mode and the excited state that are described by
\begin{align}
E^2_{(0)\text{junc}} &= 0, \\
\Psi^{(0)} &= C_1 \sech^2(\lambda_1 a x^1) \times \sech^2(\lambda_2 a x^2), \\
E^2_{(0)\text{junc}} &= 3\lambda_1^2 a^2, \\
\Psi^{(1)} &= C_2 \sech^2(\lambda_1 a x^1) \times \tanh(\lambda_2 a x^2) \times \sech(\lambda_2 a x^2), \\
E^2_{(10)\text{junc}} &= 3\lambda_2^2 a^2, \\
\Psi^{(10)} &= C_3 \tanh(\lambda_1 a x^1) \times \sech(\lambda_1 a x^1) \times \sech^2(\lambda_2 a x^2), \\
E^2_{(11)\text{junc}} &= 3(\lambda_1^2 + \lambda_2^2) a^2, \\
\Psi^{(11)} &= C_4 \tanh(\lambda_1 a x^1) \times \sech(\lambda_1 a x^1) \times \tanh(\lambda_2 a x^2) \times \sech(\lambda_2 a x^2).
\end{align}

In the two-field example, the first-order equations give the independent BPS domain walls with the kink profiles in (22).

Let us now consider the perturbation theory by writing the kink solutions \(\phi_k^e\) as a sum of all vibrational normal modes for \(k = 1, 2\), i.e.
\[\phi_k^e(x^k, y^\mu) = \phi_k^0(x^k) + \sum_n n_n(x^k) \xi_n(y^\mu),\]
Here \(\mu = 0, 1, 2, 3\) stand for the junction world-volume index. Substituting the perturbation \(\Psi^{(n)}\) into the equations of motion (19), we obtain two Schrödinger-like equations for the fluctuations \(n^{(1)}(x^1)\) and \(n^{(2)}(x^2)\) that can be written as
\[\frac{d^2 n^{(k)}}{dx^k} + V_{k j}^{(n)} = E_{(n_k)}^{2} n^{(k)}, \quad k = 1, 2\]
where we have used \(\square_{n}^{(k)}(y^\mu) = E_{(n_k)}^{2} \xi^{(k)}(y^\mu)\). Here \(V_{k j}\) are components of the matrix
\[V = \begin{pmatrix} V_{\phi_1 \phi_1} & V_{\phi_1 \phi_2} \\ V_{\phi_2 \phi_1} & V_{\phi_2 \phi_2} \end{pmatrix}.
\]

For individual domain walls solutions we simply have
\begin{align}
V_{\phi_1 \phi_1} &= 4\lambda_1^2 a^2 - 6\lambda_1^2 a^2 \sech^2 \lambda_1 a x^1, \\
V_{\phi_2 \phi_2} &= 4\lambda_2^2 a^2 - 6\lambda_2^2 a^2 \sech^2 \lambda_2 a x^2, \\
V_{\phi_1 \phi_2} &= V_{\phi_2 \phi_1} = 0.
\end{align}
These potentials were also found for the fermionic case. Just as in the case of fermions, for each scalar field component, the discrete spectrum is composed of two bound states, the zero mode and the excited state given by
\begin{align}
E^2_{(0)1,2} &= 0, \\
\eta^{1.2} &= C_0 \sech^2(\lambda_{1,2} a x^{1.2}), \\
E^2_{(1)1,2} &= 3\lambda_{1,2} a^2, \\
\eta^{1.2} &= C_1 \tanh(\lambda_{1,2} a x^{1.2}) \sech(\lambda_{1,2} a x^{1.2}).
\end{align}

They are the spectrum of the scalar modes bound to the domain wall. The spectrum bound to a domain wall junction can be found with the use of (19). There are four combinations using the zero mode and the excited state that are described by the same set (28), (29).

Before closing this section, we should mention that these results in \(D = 3 + 1\) can also be useful in other scenarios, for instance, in the cosmological investigations presented in Refs. \(14\), where domain wall networks are supposed to fill the spacetime, acting as a possible candidate to describe the dark energy.

**V. The four-dimensional model.** We now focus on the extension of the previous results to the case of six scalars in ten-dimensions. We want to find an effective four-dimensional theory for the fields localized on the junction of six orthogonal eight-dimensional domain walls (8-brane) in ten-dimensions. This is required by the framework under consideration, since we are considering \((9,1)\) spacetime dimensions, and the Universe is described by \((3,1)\) dimensions. In this case, each one of the six extra (spatial) dimensions requires a scalar field, leading to the six scalar fields which we will use below. Moreover, since we are considering the Universe as a 3-brane which evolves in time to make its worldvolume a four dimensional spacetime with \((3,1)\)
spacetime dimensions, with the 3-brane being a junction of
domain walls, we have to impose that all the six
scalar fields are immersed in full ten dimensional space,
and then they will form 8d domain walls. They are all
codimension one global defect structures or domain
walls in the usual sense.

The Lagrangian for localized fermions states on the 4d
junction is given by integrating out the 10d Lagrangian.
We start with
\[ \mathcal{L}^F_{4d} = \int \mathcal{L}^F_{10d} dx_1 dx_2 dx_3 dx_4 dx_5, \]
where the fermion dynamics and Yukawa couplings are
governed by the Lagrangian
\[ \mathcal{L}_{10d} = \bar{\Psi} \Gamma^M \partial_M \Psi + (W_{\phi_1 \phi_4} + \ldots + W_{\phi_6 \phi_8}) \bar{\Psi} \Psi. \]
The scalar and fermion fields are given by the spectral
decomposition
\[ \Phi - \Phi_s = \eta(y^\mu; x_1, \ldots, x_6) = \sum_{n_1 \ldots n_6} ^\text{junc} \eta(y^\mu) \psi^{n_1 \ldots n_6}, \]
\[ \Psi (y^\mu; x_1, \ldots, x_6) = \sum_{n_1 \ldots n_6} ^\text{junc} \eta(y^\mu) \psi^{n_1 \ldots n_6}, \]
where \( n_i \) is 0, 1 and \( \psi^{n_1 \ldots n_6} = \chi^{n_1}(x_1) \ldots \chi^{n_6}(x_6) \),
and the \( \chi(x) \) are functions that satisfy the equation \[ 19 \],
valid for both fermions and bosons. Since the system
has two bound states, there are \( 2^N \) superpartners, i.e.,
for \( N = 6 \) there are \( 2^6 = 64 \) four-dimensional scalars
\( \epsilon^{n_1 \ldots n_6}(y^\mu) \) and \( 2^6 = 64 \) four-dimensional Dirac fermions
\( \tau^{n_1 \ldots n_6}(y^\mu) \) living on the junction. Thus we have the
four-dimensional Lagrangian
\[ \mathcal{L}^F_{4d} = \sum_{0, \ldots, 0} ^\text{junc} \Gamma^M \partial_M \eta^{n_1 \ldots n_6} + \sum_{n_1 \ldots n_6} \tau^{n_1 \ldots n_6}(\Gamma^M \partial_M - E^{n_1 \ldots n_6}) \eta^{n_1 \ldots n_6} + \sum_{l_1 \ldots l_6} \sum_{m_1 \ldots m_6} \sum_{n_1 \ldots n_6} g \epsilon^{l_1 \ldots l_6} \epsilon^{m_1 \ldots m_6} \tau^{n_1 \ldots n_6} \sqrt{s} \]
Note that the first term describes massless four-
dimensional fermions, whereas the second one leads to
massive four-dimensional fermions. The Yukawa
couplings are controlled by \( g \), a constant that is computed
by integrating the Yukawa couplings in the six extra
dimensions.

For \( N \) intersecting domain walls with two bound states,
there are \( 2^N \) bound states on the junction. There exists
a number of degenerate states as \( \lambda_s = \lambda_s \) for any \( k \).
For \( N = 6 \) intersecting domain walls there is a single
state, the zero mode, with vanishing energy. Also, for
\( m = \sqrt{3} \lambda \) the other states are given by: 6 states with
energy \( m \), 15 states with energy \( \sqrt{2m} \), 20 states with
energy \( \sqrt{3m} \), 15 states with energy \( \sqrt{4m} \), 6 states with
energy \( \sqrt{5m} \), and a single state with energy \( \sqrt{6m} \). So we
have the following distribution:
\[ (N_f, m) = \{(1, 0), (6, m), (15, \sqrt{2m}), (20, \sqrt{3m}),
(15, \sqrt{4m}), (6, \sqrt{5m}), (1, \sqrt{6m})\}. \]

Thus the fermions in the Lagrangian have a mass ‘hierarchy’ which goes as follows
\[ \mathcal{L}^F_{4d} = \bar{\psi} (0)^\text{H} \Gamma^M \partial_M \psi (0) + \sum_{n=1}^{N_s} \tau^{(n)}(\Gamma^M \partial_M - \sqrt{s} m) \tau^{(n)} + \sum_{l', m', n, n'} g \Gamma^{l'm'n} \langle l'/m' \rangle \langle n' \rangle \]
where \( N_1 = 6, N_2 = 15, N_3 = 30, N_4 = 15, N_5 = 6, N_6 = 1 \) and \( l', m', n' = 0, 1, \ldots, 6 \).

The partition function for a gas of junctions of six 8d
domain walls can be found by considering the energy of
all fermion states on 6M 8d domain wall gas in (9+1)
dimensions
\[ \bar{E} = \sum_{n=1}^{6M} n_i \epsilon_i, \quad \epsilon_i = 0, \epsilon, \quad i = 1, 2, \ldots, 6M, \]
where \( \epsilon = 3 \) and \( \bar{E} \) is normalized in the sense of \[ 20 \], i.e.,
in the form \( E^2/\lambda^2 a^2 \). Thus the partition function gets the form
\[ Z = \sum_{n_1 \ldots n_{6M}} \exp \left(-\beta \sum_{i=1}^{6M} n_i \epsilon_i \right) \]
The mean energy per domain wall on the junction is given by
\[ \bar{e}_{\text{junc}} = -\frac{\partial}{\partial \beta} \ln Z \quad M = 6 \epsilon e^{-\beta \epsilon} \]
Thus, \( \bar{e}_{\text{junc}} \rightarrow 3 \epsilon \) at sufficiently high temperature, for \( \epsilon/T \ll 1 \). In this regime the junction energy per domain wall is precisely the same as the energy of three
excited domain walls intersecting three domain walls in
their fundamental state (\( \epsilon = 0 \)). As we have mentioned
before there are 20 massive states that contribute to the
junction energy in this case.

Averaging on the nonzero fermion masses under the
distribution for \( N_f \) we find
\[ < m > = \frac{\sum_{m=1}^{6} N_s \sqrt{s} m}{\sum_{m=1}^{6} N_s} = 1.709 m \simeq \sqrt{3} m. \]
This shows that the class of \( N_3 = 20 \) distinct fermions
with masses \( \sqrt{3} m \) is favored. This means that in a
domain wall gas in ten dimensions, the probability of a
junction to be formed with the superposition of 20 mas-
sive domain walls is less than the probability of a
junction to be formed with a single excited junction state.

other combination. Thus, the observed fermions in our 4d world are governed by the ‘averaged’ Lagrangian
\[
\mathcal{L}_{4d}^\xi \simeq \frac{1}{\sqrt{2}} \sum_{n=1}^{20} \sum_{l,l'} \sum_{m,m',n,n'} g_{l'm'n'n} \xi_l^{(l')} \xi_m^{(m')} \tau_n^{(n')}. \tag{51}
\]
Assuming that these fermions states can be collected in a vector column which transforms under the local SU(3) group
\[
q_n = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix},
\]
we can give \( N_c = 3 \) colors to six quarks \((n = 1, 2, \ldots, 6)\), i.e., the quark flavor number is \( N_F = 6 \). This comprises \( N_c N_F = 18 \) fermions degree of freedom. There are still two colorless fermions left, that can be put together with the first term (the zero mode) to give rise to three leptons. Thus, the simple model \((51)\) seems to appear as a good approximation to describe the six quarks and three lepton generations. All the masses are corrected by the Yukawa approximation to describe the six quarks and three lepton combinations. We would like to thank CAPES, CNPq, PROCAD-CAPES, and PRONEX-CNPq-FAPESQ for partial financial support.

VI. Ending comments. In this paper we have studied domain wall solutions associated with six scalar fields \( \phi^k(x_k), k = 1, 2, \ldots, 6 \). They are responsible for forming a four-dimensional junction with \( 2^6 \) possibly localized modes. However, from the statistical point of view, only 20 degenerate massive states and the zero mode are favored to live on the junction. These 21 localized fermion degrees of freedom are considered to take into account the \( N_c = 3 \) colors and the \( N_F = 6 \) quark flavors quantum numbers, plus three colorless leptons. As we have mentioned before, one could also think of these 21 fermion degrees of freedom as those to take into account all the matter sector of the Standard Model, given by SU(2) doublets — the 6 left quarks and 6 left leptons, and SU(2) singlets — the 6 right quarks and 3 right leptons. According to this scenario anything beyond the Standard Model should be a manifestation of extra-dimensions. This is because our 21 degrees of freedom are based on the requirement that the higher dimensional physics should effectively manifest as a four-dimensional physics on the junction.

In the charicatured setup we suggest that our Universe has selected such quantum numbers because this is statistically favored, for the Universe being a 3-brane formed as a 3-dimensional junction which evolves in time, with its worldvolume being the standard (3,1) spacetime dimensions. In our opinion, it seems interesting to see how a simple model, suggested in \([3]\) to drive the (9,1) spacetime to form our (3,1) Universe, can capture some important features of the elementary particles, as we see them today.

Although the proposed model is very simple, it seems to unveil some properties which are of current interest. Evidently, at sufficiently higher energies other excitations should appear, in particular the tower of continuum states should be taken into account. However, since they are not bound to the junction, they must live in the bulk and then one should consider them to probe the extra dimensions. The model includes several boson states, which appear in consequence of our starting model, which is based on superstring theory, and up to now we know nothing about their existence. To comply with more realistic models, we should extend the model to add other degrees of freedom. A direct possibility is the inclusion of gauge fields, and we hope to report on this issue in the near future.

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