Method of advanced channel estimation in OFDM systems by regression technique

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Abstract. The article considers the task of increasing the efficiency of estimating the parameters of multipath radio communication channels used in the design of mobile remote access systems with orthogonal frequency division with OFDM multiplexing. In the conditions of an ever-growing number of subscribers, an increase in transmission speed and noise immunity of mobile radio communication systems, the task of increasing the efficiency of channel estimation in multipath signal propagation remains an urgent one. It is also essential that the evaluation function was performed with minimal complexity and with high accuracy of signal recovery. Within the framework of the work, the analysis of existing algorithms, models, and methods for estimating the communication channel parameters of OFDM systems is carried out, the main differences and disadvantages are noted. A technique has been developed for estimating the complex envelope of an OFDM signal in a multipath communication channel based on regression analysis. A mathematical model for estimating a discrete multipath channel by a regression method using pilot symbols is presented. The type of the approximating function is selected, taking into account the requirements for the complexity of implementation and performance indicators. We present an algorithm for estimating the complex envelope of an OFDM signal in a multipath communication channel by the regression method.

1. Introduction
Today, in the era of the development of the digital industry in every area of life, the task of building effective digital radio communication systems that provide resistance to signal fading, a high data transfer rate, and given noise immunity remains an urgent task. Most remote access systems utilize Orthogonal Frequency Division Multiplexing (OFDM) [1].

However, multipath signal propagation in the radio channel is inherent in any system of digital mobile data transmission in modern urban development. A multipath radio channel requires constant monitoring of parameters. For this, in systems with orthogonal frequency multiplexing, along with information data, experimental – pilot symbols with known amplitude and phase are transmitted [2]. At the receiver, the transfer function of the wireless communication channel is estimated by the pilot subcarriers. Then the interpolation of the obtained results on the subcarriers used for transmitting information symbols is used. At the same time, the evaluation function must have high channel tracking accuracy and be implemented with minimal complexity.

2. Fundamentals and Exploring the importance of the issue
A crucial task for developers of mobile systems is to evaluate a discrete multipath communication channel for the subsequent error-free demodulation of the transmitted information.
The multipath channel of an OFDM system can be viewed as a two-dimensional space (2D) of a signal in the time and frequency plane. Then the optimal channel estimation function from the standard error is based on 2D interpolation. But such a structure of the evaluation function is too complicated for practical implementation. The combination of high data rates and reduced bit error in OFDM systems requires the use of evaluation functions, with a compromise solution of low complexity and high accuracy. Therefore, in practice, one-dimensional (1D) channel estimates of two main pilot data allocation models in the OFDM system are used – block and distributed [3, 4]. The block structure is obtained by adding test tones to all subcarrier systems after a certain period. The distributed type is used when a channel changes even from one block to another. In this case, pilots are inserted into specific subcarriers of each symbol [5, 6].

The estimates of the communication channel during the transmission of OFDM signals with a different arrangement of pilot symbols are approximated by linear interpolation. However, this technique has a significant drawback. The methods considered assume that the pilot signal received at the receiver was received correctly. But the pilot symbols themselves, passing through the multipath channel, are subject to noise and distortion [7]. Interpolation based on distorted reference data can lead to significant errors in estimating the value of the function between the nodal points.

The relevance of the task is defined by the need to reduce the bit error rate at a high data transfer rate. This task leads to the following statement of developing and improving the efficiency of methods for estimating the parameters of multipath radio communication channels used in the design of mobile radio communication systems using orthogonal frequency separation technology with OFDM multiplexing. For increasing the efficiency of evaluation functions, it is advisable to develop evaluation algorithms based on regression models.

3. Method
It is proposed to use the regression analysis of the complex envelope of the received signal. This choice is determined by the need to solve the difficulty of degradation of the characteristics of the interpolation algorithm when evaluating the parameters of the OFDM system communication channel under multipath conditions. For obtaining a channel estimate, the use of a regression model allows applying several pilot signals, perform averaging, and, as a result, more accurately restore the received signal.

Then, the estimate of the complex amplitude of the OFDM signal is reduced to finding the function of the frequency of the subcarrier oscillation and unknown parameters $a_i$:

$$Q(\Omega) = f(\Omega, a_1, a_2, ..., a_n)$$  

Hence the mathematical model of regression analysis for famous pilots (least squares method):

$$\min_{a_1, a_2, ..., a_n} \sum_{j=1}^{M} \left| Q(\Omega_j) - f(\Omega_j, a_1, a_2, ..., a_n) \right|^2$$  

where $Q(\Omega_j)$ – channel estimation at pilot frequencies, $M$ – number of pilot subcarriers, $a_1, ..., a_n$ – unknown parameters.

The practical implementation of the regression is reduced to solving two problems:

1) choose the type of approximating function $f(\Omega, a_1, a_2, ..., a_n)$;

2) find unknown parameters $a_1, a_2, ..., a_n$

An OFDM signal is transmitted at subcarrier frequencies, of which a certain amount is a pilot (known); for the rest, it is necessary to evaluate the complex envelope. We consider reports on time for block formation of pilot data, and on frequency for distributed pilot construction, respectively. The second method is more relevant for OFDM systems because it allows each OFDM symbol to be received independently.

The received OFDM signal in the time domain corresponds to the vector in the spectral:

$$x(t) \rightarrow X(\Omega),$$  

where $\Omega$ – subcarrier frequency.
A signal delay for a specific time (t-τ) leads to a phase shift by the same value, of the properties of the Fourier transform, then the spectrum of the delayed signal is written in the form:

\[ X(\Omega)e^{-j\Omega \tau}, \]  

(4)

where \( \tau \) – signal delay. Then, having passed through a fading communication channel, the sum of two rays arrive at the receiver:

\[ a_1 x(t) + a_2 x(t - \tau), \]  

(5)

where \( a_1 \) and \( a_2 \) – channel amplification (attenuation) of the signal amplitude. Then the spectral record of the sum of two signals is:

\[ a_1 X(\Omega) + a_2 X(\Omega)e^{-j\Omega \tau} = (a_1 + a_2 e^{-j\Omega \tau}) X(\Omega), \]  

(6)

Equation (6) is the basic OFDM equation, showing that the presence of delayed beams does not lead to intersymbol interference, i.e., spectra of delayed rays do not mix. However, each subcarrier \( \Omega_i \) received an unknown amplitude-phase factor \( (a_1 + a_2 e^{-j\Omega \tau}) \). Hence the task of evaluating the communication channel and subsequent signal demodulation is reduced to finding this factor.

In real conditions of signal transmission via a multipath communication channel, not only the sum of the delayed signals but also the noise component comes to the receiver input \( n(t) \):

\[ \sum_{i=1}^{N} a_i x(t - \tau_i) + n(t), \]  

(7)

At a sufficiently high level of noise, distortions of nodal points are observed, according to which an interpolation estimation of the transmitted information takes place. Hence, interpolation based on noisy data introduce additional errors in signal recovery.

For a multipath communication channel, the unknown amplitude-phase factor from (6) can be written as:

\[ \sum_{i=1}^{N} a_i x(t - \tau_i) + n(t) = \sum_{i=1}^{N} c_i e^{-j\Omega \tau_i}, \]  

(8)

Function (8) is nonlinear and has 3 unknown parameters: \( c_i \) – amplitude factor, \( \tau_i \) – signal delay time, \( N \) – the number of rays received on the receiver (reflectors in the system).

In real conditions of signal transmission, the exact number of reflectors \( N \) and, accordingly, the delays are unknown \( \tau_i \), hence, it is challenging to solve approximation (8) by a system of simple linear equations. An unknown number of incoming signals is one of the main tasks in the synthesis of an optimal algorithm for multipath signal reception.

To find unknown parameters in (8), we add an empirical restriction on the class of the approximating function. Let us examine how the complex amplitude of the freezing OFDM signal for a discrete multipath radio communication channel with Rayleigh fading changes.

The changes are such that a polynomial can describe them in a small interval. In modern high-speed wireless communication systems, data transmission is carried out on frames (forms) with a duration of 1-20 ms. We consider the interval of 5 ms – WiMax Remote Access Frame Size.

The complex envelope of the OFDM signal in the multipath communication channel was estimated by the regression method.

According to the regression, the equation of the envelope of the OFDM signal on the jth subcarrier is written as a polynomial of the kth degree:

\[ f(\Omega_j) = a_0 + a_1 x(\Omega_j) + a_2 x(\Omega_j)^2 + \ldots + a_k x(\Omega_j)^k + \tilde{c}_j, \]  

(9)

where \( \tilde{c}_j \) – random model error (remainder), k – polynomial order, \( x(\Omega_j) \) – report received on the modem of the receiver.

\[ x(\Omega_j) = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_z \end{pmatrix}, \]

\( z \) – number of reports in a frame.
According to the least-squares method, the desired parameter vector \( \vec{a} = (a_0, a_1, \ldots, a_k)^T \) is a solution to the standard equation

\[
\vec{a} = (B^T B)^{-1} B^T \vec{x}_i
\]  

(10)

where \( \vec{x}_i \) – pilot report, \( i = 1, \ldots, p \), \( p \) – the number of pilots in the \( l \)-th frame.

The matrix of solutions \( B \) of polynomial regression (Vandermond matrix) takes the form

\[
B = \begin{bmatrix}
1 & x_1 & x_1^2 & \cdots & x_1^k \\
1 & x_2 & x_2^2 & \cdots & x_2^k \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & x_p & x_p^2 & \cdots & x_p^k 
\end{bmatrix}
\]  

(11)

Estimated Complex Envelope Function \( f_i^* (\Omega_j) \) recovered from the calculated weights and values received by the receiver from the pilot data \( \vec{x}_i \).

\[
f_i^* = B \vec{a}
\]  

(12)

To assess the quality of the obtained methodology, we use the criterion for the minimum of the sum of squares of regression residues of SCRO:

\[
H = \sum_{i=1}^{p} (y_i - f_i^*)^2 = (y - f^*)^T (y - f^*),
\]  

(13)

where \( p \) – number of pilots.

Subcarrier rating \( j \):

\[
Q(\Omega_j) = \sum_{l=1}^{m} H_l
\]  

(14)

\( l \) – the number of frames in the \( j \)-th envelope of the signal, \( y_i \) – submitted reports.

To restore the signal at unknown frequencies, we use weights at \( Q < 1\% \).

### 4. Results

In modern radio communication systems, there is a tendency to reduce the coverage of base stations, due to the need to increase the transmission speed and the ever-growing number of subscribers. This tendency leads to the growth of base stations; from here, the mobile subscriber can be in the direct line of sight of the station. Also, it receives delayed signal beams reflected from complex geometric shapes of modern urban development and uneven terrain. According to the technical specification, 4-5 beams are enough for modeling a discrete multipath radio channel 3GPP TR 25.996 [8]. The simulation data are shown in Table 1.

| Data | Mobile subscriber |
|------|--------------------|
| **Data** | **Number of rays** | **Delay level, dB** | **Delay time, ns** |
| 1) 4+1 (there are rays in line of sight, K = 6dB) | Ray +1 | 1) 0.0 | 0 |
| 2) 4 (no rays in line of sight) | Ray1 | 1) -6.51 | 0 |
| 1) 0.0 | 2) -\( \infty \) | 0 |
| Ray 2 | 1) -16.21 | 110 |
| 2) -9.7 | Ray 3 | 1) -25.71 | 190 |
| 1) -29.31 | Ray 4 | 1) -22.8 | 410 |
In the experiment, the signal is shifted in the time domain by the number of samples of the complex envelope of the OFDM symbol and varies from 1 to 400 samples in the frequency range 10–20 MHz. The independent tests were performed for calculating the probability of bit error (BER) from the signal-to-noise ratio (SNR). In each experiment, one OFDM symbol is transmitted.

5. Conclusion
According to the presented modeling, we can conclude that the application of the developed methodology improves the efficiency of the radio access system. With a bit error probability of 10^{-5}, the use of a regression algorithm for estimating the complex envelope of an OFDM signal in a multipath communication channel gives again in the Signal / Noise ratio of about 1 – 1.2 dB concerning linear interpolation methods. Thus, the proposed algorithm for estimating the complex envelope of an OFDM signal in a discrete multipath communication channel increases the data transfer rate by 20%.

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