Superalgebra and Conservative Quantities in $N = 1$ Complex Supergravity

Sze-Shiang Feng$^{1,2,3,\#}$, Bin Wang$^4$

1. *High Energy Section, ICTP, Trieste, 34100, Italy*
   e-mail: fengss@ictp.trieste.it

2. *CCAST (World Lab.), P.O. Box 8730, Beijing 100080*

3. *Modern Physics Department, University of Science and Technology of China, 230026, Hefei, China*
   e-mail: zhdp@USTC.EDU.CN

4. *Physics Department, Shanghai Normal University, 100234, Shanghai, China*

Abstract

The $N = 1$ self-dual supergravity has $SL(2, \mathbb{C})$ symmetry and the left-handed and right-handed local supersymmetries. These symmetries result in $SU(2)$ charges as the angular-momentum and the supercharges. The model possesses also the invariance under the general translation transforms and this invariance leads to the energy-momentum. All the definitions are generally covariant. As the $SU(2)$ charges and the energy-momentum we obtained previously constituting the 3-Poincare algebra in the Ashtekar’ complex gravity, the $SU(2)$ charges, the supercharges and the energy-momentum in simple supergravity also restore the super-Poincare algebra, and this serves to support the reasonableness of their interpretations.

PACS number(s): 11.30.Cp, 11.30.Pb, 04.20.Me, 04.65.+e.

Key words: superalgebra, conservative quantities, supergravity

---

# Corresponding adress

* On leave of absence from Physics Department, Shanghai University, 201800, Shanghai, China
1 Introduction

The study of self-dual gravities has drawn much attention in the past decade since the discovery
of Ashtekar's new variables, in terms of which the constraints can be greatly simplified [1]-[2]. The
new phase variables consist of densitized $SU(2)$ soldering forms $\tilde{e}^{i} A^{B}$ from which a metric density
is obtained according to the definition $q_{ij} = -\text{Tr} \tilde{e}^{i} \tilde{e}^{j}$, and a complexified connection $A_{iA}^{B}$ which carries
the momentum dependence in its imaginary part. The original Ashtekar's self-dual canonical gravity permits also a Lagrangian formulation [3]-[4]. The supersymmetric extension of this Lagrangian formulation, which is equivalent to the simple real supergravity, was proposed by Jacobson [5], and the corresponding Ashtekar complex canonical transform was given by Gorobey et al [6].

In our previous works, we have obtained the $SU(2)$ charges and the energy-momentum in the
Ashtekar’s formulation of Einstein gravity [7]-[8] and they are closely related to the angular-momentum
[9]-[11] and the energy-momentum [12] in the vierbein formalism of Einstein gravity. The fact that the algebra formed by their Poisson brackets do constitute the 3-Poincare algebra on the Cauchy surface supports from another aspect that their definitions are reasonable.

Out of the same reason, the definitions of $SU(2)$ charges, which are to be interpreted as the angular-momentum, the supercharges and the energy-momentum are also interesting and important aspects in the simple self-dual supergravity. In this paper, we will exploit the $SL(2,\mathbb{C})$ invariance, the left-handed and right-handed supersymmetry and the invariance under the general translation transform [12] to obtain the conservative charges under consideration. This paper is arranged as follows. In section 2, we will give a brief review of the $N = 1$ self-dual supergravity. In section 3, we will derive the $SU(2)$ charges from the original Lagrangian of Jacobson. In section 4, we will derive the energy-momentum from a slightly different Lagrangian and the general translation. In section 5, we derive the supercharges from the invariance under left-handed and right-handed local supersymmetric transforms. The last section is devoted to summary and discussions.

2 A Brief Review of the Model

The Lagrangian density is [3]

$$\mathcal{L}_{J} = \frac{1}{\sqrt{2}} (e^{AA'} \wedge e_{BA'} \wedge F_{A}^{B} + i e^{AA'} \wedge \bar{\psi}_{A'} \wedge \mathcal{D} \psi_{A}) \quad (1)$$

The dynamical variables are the real tetrad $e^{AA'}$ (the ”real” means $\bar{e}^{A'A} = e^{AA'}$), the traceless left-handed $SL(2,\mathbb{C})$ connection $A_{\mu MN}$ and the complex anticommuting spin-$\frac{3}{2}$ gravitino field $\psi_{\mu A}$. The
superalgebra complex supergravity

\( SL(2, \mathbb{C}) \) covariant exterior derivative is defined by

\[ D\psi_M := d\psi_M + A_M^N \wedge \psi_N \]

and the curvature 2-form is

\[ F_M^N := dA_M^N + A_M^P \wedge A_P^N \]

The indices are lowered and raised with the antisymmetric \( SL(2, \mathbb{C}) \) spinor \( \epsilon^{AB} \) and its inverse \( \epsilon_{AB} \) according to the convention \( \lambda_B = \lambda^A \epsilon_{AB}, \lambda^A = \epsilon^{AB} \lambda_B \), and the implied summations are always in north-westerly fashion: from the left-upper to the right-lower. The Lagrangian eq.(1) is a holomorphic functions of the connection and the equation for \( A_{\mu A B} \) is equivalent to

\[ D\epsilon^{AA'} = \frac{i}{2} \psi^A \wedge \bar{\psi}^{A'} \]

provided \( \epsilon^{AA'} \) is real. The Lagrangian \( \frac{1}{2}(\mathcal{L}_J + \bar{\mathcal{L}}_J) \) for real supergravity is a non-holomorphic function, but leads to no surfeit of field equations. Under the left-handed local supersymmetric transform generated by anticommuting parametres \( \epsilon_A \)

\[ \delta\psi_A = 2D\epsilon_A, \quad \delta\bar{\psi}_A = 0, \quad \delta\epsilon_{AA'} = -i\bar{\psi}_A \epsilon_A \]

the Lagrangian \( \mathcal{L}_J \) is invariant without using any one of the Euler-Lagrangian equations while under the right-handed transform

\[ \delta\psi_A = 0, \quad \delta\bar{\psi}_A = 2D\bar{\epsilon}_{A'}, \quad \delta\epsilon_{AA'} = -i\psi_A \bar{\epsilon}_{A'} \]

\( \mathcal{L}_J \) is invariant modulo the field equations.

The (3+1) decomposition is effected as

\[ \mathcal{L}_J = \epsilon^{kAB} \dot{A}_{kAB} + \bar{\pi}^{kA} \dot{\psi}_{kA} - \mathcal{H} \]

\[ \mathcal{H} := \epsilon_{0AA'} \mathcal{H}^{AA'} + \psi_{0A} \mathcal{S}^A + \bar{S}^{A'} \bar{\bar{\psi}}_{0A'} + A_{0AB} \mathcal{J}^{AB} + \text{(total divergence)} \]

The canonical momenta are

\[ \bar{\pi}^{kA} := \frac{i}{\sqrt{2}} \bar{\epsilon}^{ijk} \epsilon_i^{AA'} \bar{\psi}_{jA'} \]

\[ \bar{\pi}^{kA} := \frac{i}{\sqrt{2}} \epsilon^{ijk} \epsilon_i^{AA'} \bar{\psi}_{jA'} \]

and the constraints are

\[ \mathcal{H}^{AA'} := \frac{1}{\sqrt{2}} \epsilon^{ijk} (\epsilon_i \mathcal{J}_{jk} + \mathcal{D}_j \psi_{kA}) \]
The 0-components $e_{0AA'}$, $\psi_{0A}$, $\bar{\psi}_{0A'}$ and $A_{0AB}$ are just the Lagrange multipliers and the dynamical conjugate pairs are $(\tilde{e}^{kAB}, A_{jAB})$, $(\tilde{\pi}^kA, \psi_kA)$. The constraints $H^{AA'} = 0$ and $\hat{S}^{A'} = 0$ generate the following two
\begin{equation}
\hat{H}^{AB} := (\tilde{e}^j e^k F_{jk})^{AB} + 2\tilde{\pi}^j \tilde{e}^k D_j \psi_k \epsilon^{AB} + 2(\tilde{\pi}^j D_j \psi_k) \tilde{e}^{kAB} = 0
\end{equation}
\begin{equation}
S^{\dagger A} := \frac{1}{\sqrt{2}} e^{ijk} \tilde{e}_i A^B D_j \psi_k B = 0
\end{equation}
The equations of motion will be properly expressed in Hamiltonian form $\dot{f} = \{H, f\}$ if we assign the Poisson brackets
\begin{equation}
\{e^{kAB}(x), A_{jAB}(y)\} = \delta_j^k \delta(M^A \delta_N^B) \delta^3(x, y)
\end{equation}
\begin{equation}
\{\tilde{\pi}^kA(x), \psi_jA(y)\} = -\delta_j^k \delta^3(x, y)
\end{equation}
all other brackets among these quantities being zero.

This is the outline of the theory.

## 3 The SU(2) Charge

Under any $SL(2, \mathbb{C})$ transform
\begin{equation}
e_{\mu AA'} \rightarrow L_A^B R_{A'}^{B'} e_{\mu BB'}, \quad \psi_A \rightarrow L_A^B \psi_B, \quad \bar{\psi}_{A'} \rightarrow R_{A'}^{B'} \bar{\psi}_{B'}
\end{equation}
\begin{equation}
A_{\mu MN} \rightarrow L_M^A A_{\mu A} B(L^{-1})_{BN} + L_M^A \partial_\mu (L^{-1})_{AN}
\end{equation}
$L_J$ is invariant. $L$ and $\bar{R}$ may not necessarily related by complex conjugation. Note that $L_{AB} = -(L^{-1})_{BA}$, the transform of $A$ may also be written as
\begin{equation}
A_{\mu MN} \rightarrow L_M^A L_N^B A_{\mu AB} - L_M^A \partial_\mu L_{NA}
\end{equation}
For infinitesimal transform, $L_A^B = \delta_A^B + \xi_A^B$ where $\xi_{AB} = -\xi_{BA}$ are infinitesimal parameters. Thus we have
\begin{equation}
\delta \xi A = [\xi, A] - d\xi, \quad \delta \psi = \xi \psi
\end{equation}
When calculating the variation of the Lagrangian, one must take into consideration of the anticommuting feature of the gravitino field. We write the variation in the way that

$$\delta\mathcal{L}_J = \delta\phi^A (\frac{\partial}{\partial\phi^A} - \partial_\mu \frac{\partial}{\partial\phi^A})\mathcal{L}_J + \partial_\mu (\delta\phi^A \frac{\partial}{\partial\phi^A}\mathcal{L}_J)$$  \hspace{1cm} (22)

where $\phi^A$ denotes any field involved in the first order Lagrangian. Now both $\frac{\partial}{\partial\phi^A}$ and $\frac{\partial}{\partial\phi^A}$ are (anti-)commuting if $\phi^A$ is (anti-)commuting, and so there is no ordering problem.

The invariance of $\mathcal{L}_J$ under the infinitesimal $SL(2,\mathbb{C})$ transform is equivalent to the following modulo the field equations

$$\partial_\rho (\delta A_{\sigma A}^B \frac{\partial}{\partial\rho} \mathcal{L}_J + \delta \psi_\sigma A \frac{\partial}{\partial\psi_\sigma A}) = 0$$  \hspace{1cm} (23)

For constant $\xi$, we have

$$\partial_\rho \left( \frac{1}{\sqrt{2}} \epsilon^{\mu\nu\rho\sigma} e_{\mu A} A_{\nu BA}^*[\xi, A_\sigma]^B + \frac{i}{\sqrt{2}} \epsilon^{\mu\nu\rho\sigma} e_{\mu A} A_{\nu BA}^* \bar{\psi}_\sigma A (\xi_\sigma)_A \right) = 0$$  \hspace{1cm} (24)

we have therefore the conservation of $SU(2)$ charges

$$\partial_\mu \tilde{j}_{AB}^\mu = 0$$  \hspace{1cm} (25)

where

$$\tilde{j}_{AB}^\rho = \frac{1}{\sqrt{2}} \epsilon^{\mu\nu\rho\sigma} (e_{\mu A} A_{\nu BA}^* A_{\sigma MA}^M - e_{\mu A} A_{\nu BA}^* A_{\sigma MA}) + \frac{i}{2} e_{\mu A} A_{\nu BA}^* \bar{\psi}_\sigma A B + \frac{i}{2} e_{\mu B} A_{\nu BA}^* \bar{\psi}_\sigma A A$$  \hspace{1cm} (26)

Thus

$$J_{AB} = \int_\Sigma \tilde{j}_{AB}^0 d^3 x$$  \hspace{1cm} (27)

where

$$\tilde{j}_{AB}^0 = \frac{1}{\sqrt{2}} \epsilon^{ijk} (e_{iA} A_{jMA}^* A_{kMB}^M - e_{iA} e_{jMA} A_{kMA} + \frac{i}{2} e_{iA} A_{jMA} \bar{\psi}_k A + \frac{i}{2} e_{iB} A_{jMA} \bar{\psi}_k A)$$  \hspace{1cm} (28)

Using eq(9) and eq(10), $\tilde{j}_{AB}^0$ can be written as

$$\tilde{j}_{AB}^0 = [\tilde{e}_k^A, A_{AB}] + \bar{\pi}_k(A_{AB})$$  \hspace{1cm} (29)

The constraint $J_{AB} = 0$ guarantees that

$$J_{AB} \approx \int_\Sigma \partial_k \tilde{e}_{AB}^k = \int_{\partial\Sigma} \tilde{e}_{AB}^k ds_i$$  \hspace{1cm} (30)
where \( ds_i = \frac{1}{2} \epsilon_{ijk} dx^j \wedge dx^k \). It can also be obtained in the following way. Using the field equation:

\[
e^{A'}(A \wedge (D e^B)_{A'} - \frac{i}{2} \psi^B) \wedge \bar{\psi}_{A'}) = 0,
\]

we have

\[
e^{\mu
u\sigma}[e_{\mu A}'(\partial_{\sigma}e_{\nu BA'} + A_{\sigma B}^{M}e_{\nu MA'} + \frac{i}{2} \bar{\psi}_{\nu A'}\psi_{\sigma B}) + e_{\mu B}'(\partial_{\sigma}e_{\nu AA'} + A_{\sigma A}^{M}e_{\nu MA'} + \frac{i}{2} \bar{\psi}_{\nu A'\psi_{\sigma A}})] = 0 \tag{31}
\]

so

\[
\bar{\tilde{j}}_{AB}^{\rho} = -\frac{1}{\sqrt{2}} e^{\rho\mu\nu\sigma} \partial_{\sigma}(e_{\mu A}'e_{\nu BA'}) \tag{32}
\]

Using

\[
e_{[\mu A}'e_{\nu BA']} = e_{[\mu AC}e_{\nu B]C} - i\sqrt{2}n_{[\mu e_{\nu]}AB} \tag{33}
\]

we have

\[
\bar{\tilde{j}}_{AB}^{0} = -\frac{1}{\sqrt{2}} \epsilon^{ijk} \partial_{k}(e_{[i A}'e_{j BA']} = -\frac{1}{\sqrt{2}} \epsilon^{ijk} \partial_{k}(e_{[i AC}e_{j B]C} - i\sqrt{2}n_{[i e_{j}]}AB) \tag{34}
\]

which is exactly the same as eq.(30) We can thus have the Poisson brackets

\[
\{J_{AB}, J_{MN}\} = \left\{ \int_{\partial \Sigma} \tilde{e}_{AB}^{k} ds_{k}, \int_{\Sigma} (\tilde{e}_{i}^{i M P} A_{i PN} + \tilde{e}_{i}^{i N P} A_{i PN}) d^{3}x \right\}
\]

\[
= \frac{1}{2}(J_{MA}^{B} + J_{MB}^{A} + J_{NA}^{B} + J_{NB}^{A}) \tag{35}
\]

Now the flat dreibein on \( \Sigma \) is needed in order to find the angular-momentum \( J_{i} \). To clarify the notions, we use the following conventions: \( \mu, \nu, ... \) denote the 4-dim curved indices and \( i, j, k, ... \) denote the 3-dim curved indices on \( \Sigma \); \( a, b, c, ... \) denote the flat 4-dim indices and \( l, m, n, ... \) denote the flat 3-dim indices on \( \Sigma \). The rigid flat vierbein is denoted as \( E_{a A}^{A} \) and the rigid flat dreibein is denoted by \( E_{m AB}^{m} \). Then define

\[
J_{m} := \frac{1}{\sqrt{2}} E_{m AB}^{AB} J_{AB} \tag{36}
\]

and using the relation \( \epsilon^{mn l} E_{m} E_{n} = \sqrt{2} E^{l} \) we have

\[
\{J_{m}, J_{n}\} = \epsilon^{mn l} J^{l} \tag{37}
\]

Therefore the \( su(2) \) algebra is restored. One may doubt the finiteness of \( J_{MN} \) for isolated system.

They are indeed finite because, in the non-supersymmetric case, \( J_{MN} \) is related to \( J_{ab} \) by a linear transform\[8\], where \( J_{ab} \) is the angular-momentum obtained in the vierbein formalism and are proved finite for general isolated systems, further, it can give the correct formula of radiation of angular-momentum. \[9\]-\[10\]. As in the non-supersymmetric case\[8\], we can also obtain only the \( SU(2) \).
superalgebra complex supergravity

charges instead of the whole $SL(2,\mathbb{C})$ charges. Yet, the angular-momentum $J_{ab}$ obtained in [9]-[10] is completely contained in $J_{MN}$ (see section 5).

4 The Energy-momentum

In order to obtain the energy-momentum, we do not exploit the Lagrangian $L_J$. Instead, we use the following $L$

$$L = \frac{1}{\sqrt{2}} \epsilon^{\mu\nu\rho\sigma} \left[ -\partial_\rho (e_\mu^{AA'} e_{\nu BA'}) A_{\sigma A} B + e_\mu^{AA'} e_{\nu BA'} A_{\rho A}^M A_{\sigma M} B + \frac{i}{2} e_\mu^{AA'} \bar{\psi}_{BA'} \mathcal{D}_\rho \psi_{\sigma A} ight]$$

i.e.

$$L = L_J - \frac{1}{\sqrt{2}} \epsilon^{\mu\nu\rho\sigma} \partial_\rho (e_\mu^{AA'} e_{\nu BA'} A_{\sigma A} B + i e_\mu^{AA'} \bar{\psi}_{BA'} \psi_{\rho A})$$

(39)

So as far as the Euler-Lagrange equations are concerned, $L$ and $L_J$ are equivalent. But why do we use $L$ rather than $L_J$? As discussed in [13], conservative quantities in general relativity are often quasi-local, i.e. can be expressed as a surface integral at $\partial \Sigma$. Whence total divergences in the Lagrangian may do non-trivial contribution to the conservative quantities though they do not affect the motion equations. In the non-supersymmetric case, we also used a different Lagrangian in order to obtain the energy-momentum thereof [7]. In order to agree with the definition of energy-momentum in the non-supersymmetric case, in which the energy-momentum agree exactly with the ADM definition, we use the Lagrangian (38) here.

Since the action $I = \int L d^4x$ is invariant under the infinitesimal transform $x'^\mu = x^\mu + \delta x^\mu, \phi^\ell(x')$ $\phi^\ell(x) + \delta \phi^\ell, \text{here } \phi^\ell = e_\mu^{AA'}, A_{\mu MN}, \bar{\psi}_{\mu A'}, \psi_{\mu}$ we have the Noether theorem

$$\partial_\mu (L dx^\mu + \delta_0 \phi^\ell \frac{\partial L}{\partial \phi^\ell}) + \delta_0 \phi^\ell [L]_{\phi^\ell} = 0$$

(40)

where $[L]_{\phi^\ell} = (\frac{\partial L}{\partial \phi^\ell} - \partial_\mu \frac{\partial L}{\partial \mu \phi^\ell}) L$ and $\delta_0 \phi^\ell = \delta \phi^\ell - \partial_\mu \phi^\ell d^\mu x$. Using the field equations, we have

$$\partial_\mu (L dx^\mu + \delta_0 \phi^\ell \frac{\partial L}{\partial \mu \phi^\ell}) = 0$$

(41)

Since all the fields $\phi^\ell_\mu$ have a lower curved index, we have $\delta \phi^\ell_\nu = -\delta x^\mu_\nu \phi^\ell_\mu$. (The $\,$,$\,$ denotes "partial derivative"). Therefore, $\delta \phi^\ell_\nu = -\delta x^\lambda_\nu \phi^\ell_\lambda - \partial_\lambda \phi^\ell_\nu \delta x^\lambda$. Hence we have

$$\partial_\mu [(L \delta^\lambda_\nu - \partial_\lambda \phi^\ell_\nu \frac{\partial L}{\partial \mu \phi^\ell_\nu}) \delta x^\lambda - (\phi^\ell_\lambda \frac{\partial L}{\partial \mu \phi^\ell_\nu}) \delta x^\lambda_\nu] = 0$$

(42)
superalgebra complex supergravity

which can be expressed as

$$\partial_\mu [\tilde{I}_\lambda^\mu \delta x^\lambda + \tilde{V}_{\lambda}^{\mu\nu} \delta x^\nu] = 0$$  \hspace{1cm} (43)

Therefore the independence of $\delta x^\mu$, $\delta x_{\nu}$, and $\delta x_{\nu\lambda}$ implies

$$\partial_\mu \tilde{I}_\lambda^\mu = 0, \quad \tilde{I}_\lambda^\nu = -\partial_\mu \tilde{V}_{\lambda}^{\mu\nu}, \quad \tilde{V}_{\lambda}^{\mu\nu} = -\tilde{V}_{\lambda}^{\nu\mu}$$  \hspace{1cm} (44)

Finally, we use the general translation: $\delta x^\mu = e_{\mu A} b^{AA'}$, $b^{AA'}$ is an arbitrary infinitesimal four-vector.

This step is crucial. Note that as discussed in [7], the general coordinate transform $\delta x^\mu = \xi^\mu$ does not in fact effect a translation because $x^\mu$ can be any curvilinear coordinate. Now we have

$$\partial_\mu (\tilde{I}_\lambda^\mu e_{AA'}^\lambda + \tilde{V}_{\lambda}^{\mu\nu} \partial_\nu e_{AA'}^\lambda) = 0$$  \hspace{1cm} (45)

The energy-momentum tensor is defined to be

$$\tilde{t}_{AA'}^{\mu} := e_{AA'}^\mu := \tilde{I}_\lambda^\mu e_{AA'}^\lambda + \tilde{V}_{\lambda}^{\mu\nu} \partial_\nu e_{AA'}^\lambda$$  \hspace{1cm} (46)

So

$$\partial_\mu (e_{AA'}^\mu) = 0$$  \hspace{1cm} (47)

Using $\tilde{I}_\lambda^\mu = \partial_\nu \tilde{V}_{\lambda}^{\mu\nu}$, we have

$$\tilde{t}_{AA'}^{\mu} = \partial_\nu \tilde{V}_{AA'}^{\mu\nu}, \quad \tilde{V}_{AA'}^{\mu\nu} := \tilde{V}_{\lambda}^{\mu\nu} e_{AA'}^\lambda$$  \hspace{1cm} (48)

Since

$$\frac{\partial \mathcal{L}}{\partial \partial_\mu e_{AA'}^\nu} = -\sqrt{2} e_{\mu\nu\alpha\beta} e_{\alpha BA} A_{\beta A}^B - \frac{i}{2\sqrt{2}} e_{\mu\nu\alpha\beta} \bar{\psi}_{\alpha A'} \psi_{\beta A}$$  \hspace{1cm} (49)

$$\frac{\partial \mathcal{L}}{\partial \partial_\mu A_{\nu MN}} = 0, \quad \frac{\partial \mathcal{L}}{\partial \partial_\mu \bar{\psi}_{AA'}} = \frac{i}{2\sqrt{2}} e_{\mu\nu\alpha\beta} e_{\alpha A} A_{\alpha}^A \bar{\psi}_{\beta A}$$  \hspace{1cm} (50)

$$\frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_{AA'}} = -\frac{i}{2\sqrt{2}} e_{\mu\nu\alpha\beta} e_{\alpha A} A_{\alpha}^A \bar{\psi}_{\beta A}$$  \hspace{1cm} (51)

we have

$$\tilde{V}_{\lambda}^{\mu\nu} = e_{\mu\nu\alpha\beta} [\sqrt{2} e_{\lambda}^{AA'} e_{\alpha BA'} A_{\beta A}^B + \frac{i}{2\sqrt{2}} (e_{\alpha}^{AA'} \bar{\psi}_{\beta A} - e_{\alpha} A_{\lambda A'} \bar{\psi}_{\beta A} - e_{\alpha} A_{\lambda A} \bar{\psi}_{\beta A} \psi_{AA'})]$$  \hspace{1cm} (52)

and

$$\tilde{V}_{NN'}^{\mu\nu} = e_{\mu\nu\alpha\beta} [\sqrt{2} e_{\lambda}^{AA'} e_{\alpha BA'} e_{\nu}^{\lambda N} A_{\beta A}^B + \frac{i}{2\sqrt{2}} (e_{\alpha}^{AA'} \bar{\psi}_{\nu A'} e_{\lambda}^{\lambda N} \bar{\psi}_{\beta A} - e_{\alpha} A_{\lambda A'} \bar{\psi}_{\nu A} \psi_{AA'} e_{\lambda}^{\lambda N} \bar{\psi}_{\beta A} \psi_{AA'})]$$  \hspace{1cm} (53)
For a closed system, the conservative energy-momentum is

\[ P_{NN'} = \int_{\Sigma} e \rho^0_{NN'} d^3 x = \int_{\Sigma} \partial_i \tilde{V}^{0i}_{NN'} d^3 x = \int_{\partial \Sigma} \tilde{V}^{0i}_{NN'} ds_i \] (54)

where

\[ \tilde{V}^{0i}_{NN'} = \epsilon^{ijk} [\sqrt{2} \epsilon^{AA'} e_j e^{\lambda}_{NN'} e^{\lambda}_{NN'} A_k A^B + \frac{i}{2 \sqrt{2}} (\epsilon^{AA'} e^{\lambda}_{NN'} \bar{\psi}_{jA'} \psi_{kA} - e_j e_{AA'} \bar{\psi}_{kA} \psi_{jA} e^{\lambda}_{NN'})] \] (55)

Now we use the reality of \( P_{AA'} \) to simplify the expression. Since both sides of eq.(4) are real, the first term is real. It is also because of the reality of \( i \psi_{A} \wedge \bar{\psi}_{A'} \), the last two terms contribute nothing to the real \( P_{AA'} \). So we have

\[ P_{NN'} = \int_{\partial \Sigma} \epsilon^{ijk} [\sqrt{2} \epsilon^{AA'} e_j e^{\lambda}_{NN'} e^{\lambda}_{NN'} A_k A^B + \frac{i}{2 \sqrt{2}} e_{AA'} \bar{\psi}_{kA} \psi_{jA} e^{\lambda}_{NN'}] ds_i \] (56)

From eq.(4) we have \( \bar{\psi}_{[jA'} \psi_{k]A} = 2i (\mathcal{D}_{[j} e_{k]})_{AA'} \). therefore

\[ P_{NN'} = \frac{1}{\sqrt{2}} \int_{\partial \Sigma} \epsilon^{ijk} \epsilon^{AA'} e_j e^{\lambda}_{NN'} e^{\lambda}_{NN'} A_k A^B ds_i \] (57)

To make the 3+1 decomposition of \( P_{AA'} \), we may use two ways. The first one is to use the relationship between the flat SL(2, C) soldering form \( E_{aAA'} \) and the sigma matrices in ref.[14]

\[ E_{aAA'} = \frac{1}{\sqrt{2}} \sigma_{AA'}^a \] (58)

and

\[ \sigma_{aAA'}^a \sigma_{B'B'}^b = 2 \delta_{AB} \delta_{B'B'}^a \] (59)

we have

\[ e^{AA'} e^{\lambda}_{NN'} = e^a e^b e_{AA'} E_{aAA'} E_{bBB'} = \delta_{AN'} \delta_{AN'} \] (60)

so

\[ P_{AA'} = \frac{1}{\sqrt{2}} \int_{\partial \Sigma} \epsilon^{ijk} e_j e_{AA'} A_k A^B ds_i \] (61)

Use \( P_a = E_{aAA'} P_{AA'} \) and

\[ E_{aAA'} = -i \sqrt{2} E_{aAB} n_B A' + n_a n_{AA'} \] (62)

we have

\[ P_0 = P_{NN'} n_{NN'} = \frac{1}{\sqrt{2}} \int_{\partial \Sigma} \epsilon^{ijk} e_j e_{AA'} A_k A^B n_{AA'} ds_i = \frac{-i}{2} \int_{\partial \Sigma} \epsilon^{ijk} e_j e_{AA'} e_{kA} B^B ds_i \] (63)
superalgebra complex supergravity

\[ P_0 = -\frac{i}{2} \int_{\partial \Sigma} \epsilon^{ijk} \text{tr} e_j A_k ds_i \]  
(64)

\[ P_m = P_{NN'}(-i\sqrt{2})E_m^{NC}n_{C'}^{N'} = \frac{1}{\sqrt{2}} \int_{\partial \Sigma} \epsilon^{ijk}(A_j e_k)_{MN}E_m^{MN} ds_i = \frac{1}{2} E_m^{MN} P_{MN} \]  
(65)

The second way is to use

\[ \epsilon_\lambda^{AA'} e_\lambda^{AB} = 2 \epsilon_\lambda^{AB} e_\lambda^{AB} n_B n_{A'} n_{MN'} + n_{A A'} n_{N N'} \]  
(66)

and

\[ \epsilon^0_{MN} = 0, \quad \epsilon_{AB}^i = \delta^i_{AB} \]  
(67)

(Note that \( e_{\mu AA'} := g_{\mu\nu} e^{\nu AA'} \) while \( e^\mu_{AB} := -g^{\mu
u} e_{\nu AB}, n_\mu = (n_0, 0, 0, 0) \)). Substituting the first term which is spatial of the r.h.s of eq.(66) into eq(57) gives the momentum \( P_m \) and the second term gives the energy \( P_0 \). The result is the same.

We now rescale the energy-momentum by a constant factor \( 2\sqrt{2} \). i.e.

\[ P_{AA'} = 2 \int_{\partial \Sigma} \epsilon^{ijk} e_j B A' A_k B ds_i \]  
(68)

\[ P_0 = -i\sqrt{2} \int_{\partial \Sigma} \epsilon^{ijk} \text{tr} e_j A_k ds_i \]  
(69)

\[ P_m = 2 \int_{\partial \Sigma} \epsilon^{ijk}(A_j e_k)_{MN}E_m^{MN} ds_i = \frac{1}{\sqrt{2}} E_m^{MN} P_{MN} \]  
(70)

so that it agrees exactly with that of the non-syuppiesymmetric case[7].

The Poisson bracket \( \{ J_{MN}, P_{AB} \} \) can be calculated using \( (e^i e_i)_{(MN)} = 0, \sqrt{2} e^i e^k = q^{-1/2} \epsilon^{ijk} \) and

\[ \{ e^i_{AB}(x), A_{jMN}(y) \} = 2q^{-1/2} \epsilon^i_{[AB} e^j_{MN]} \delta^3(x,y) - \frac{1}{2\sqrt{2}} q^{-1} \epsilon_{lmj}[e^l e^m]_{(MN)} e^i_{AB} \delta^3(x,y) \]  
(71)

The result is

\[ \{ J_{MN}, P_{AB} \} = \frac{1}{2} (\epsilon_{AM} P_{NB} + \epsilon_{BM} P_{AN} + \epsilon_{AN} P_{MB} + \epsilon_{BN} P_{MA}) \]  
(72)

therefore

\[ \{ J_m, P_n \} = \{ \frac{1}{\sqrt{2}} E_m^{AB} J_{AB}, \frac{1}{\sqrt{2}} E_n^{MN} P_{MN} \} = \epsilon_{mnl} P_l \]  
(73)

To calculate \( \{ P_{MN}, P_{AB} \} \), one of the two \( P \)'s must be expressed as a 3-dim integral.

\[ P_{AB} = \int_{\partial \Sigma} 4q^{-1/2}(A_j e^i e^j)_{(AB)} ds_i = \int_{\Sigma} 4\partial_i(q^{-1/2}(A_j e^i e^j)_{(AB)}) d^3 x \]  
(74)

As in the non-supersymmetric case[7], it is not differentiable with respect to \( A_{iMN} \). To circumvent this difficulty, we use the same trick as in [7], which stems from the construction of the Hamiltonian.
generating time translations. Suppose $N$ is a scalar density of weight $-1$ and equals $q^{-1}$ outside a compact set of $\Sigma$. So

$$P_{AB} = \int_\Sigma 4[\partial_i(N e^{[i} \tilde{\epsilon}^{j]})]_{(AB)} d^3 x$$

$$= \int_\Sigma 4[\partial_i(N e^{[i} \tilde{\epsilon}^{j]})A_j + N e^{[i} \tilde{\epsilon}^{j]}(\frac{1}{2} F_{ij} - A_i A_j)]_{(AB)} d^3 x$$

Use the constraint eq(15), we have

$$P_{AB} \approx \int_\Sigma 4[\partial_i(N e^{[i} \tilde{\epsilon}^{j]})A_j + N e^{[i} \tilde{\epsilon}^{j]}A_j - N \pi^j \epsilon^k D_{[j} \psi_{k]} \epsilon - N \pi^j D_{[j} \psi_{k]} \epsilon^k]_{(AB)} d^3 x$$

When taking into consideration the falloff $d\tilde{e} \sim r^{-2}, A \sim r^{-2}, \psi, \bar{\psi}, \bar{\pi} \sim r^{-1}$, we have

$$\{P_{AB}, P_{MN}\} \approx 0$$

i.e.

$$\{P_i, P_j\} \approx 0$$

## 5 The Supercharges

Since the Lagrangian varies as

$$\delta \mathcal{L} = \delta \phi^\ell_\mu [\mathcal{L}]_{\phi^\ell_\mu} + \partial_\mu (\delta \phi^\ell_\nu \frac{\partial \mathcal{L}}{\partial \phi^\ell_\nu})$$

we have on-shell that

$$\delta \mathcal{L} = \partial_\mu (\delta \phi^\ell_\nu \frac{\partial \mathcal{L}}{\partial \phi^\ell_\nu})$$

On the other hand, one can calculate $\delta \mathcal{L}$ directly. Using eq(39) and the invariance of $\mathcal{L}_J$ under the left-handed transform eq.(5), we have the variation of $\mathcal{L}$ under the left-handed transform eq.(5).

$$\delta \mathcal{L} = \sqrt{2} i e^{\mu \rho \sigma} \partial_\rho (e^{AA'} \bar{\psi}_{\nu A'} A_{\sigma A} B_{\epsilon B} - \bar{\psi}_{\nu A'} e^{AA'} D_{\sigma A} \epsilon)$$

Whence

$$\partial_\mu (\delta \phi^\ell_\nu \frac{\partial \mathcal{L}}{\partial \phi^\ell_\nu}) = -\sqrt{2} i e^{\mu \rho \sigma} \partial_\rho (e^{AA'} \bar{\psi}_{\nu A'} \partial_\sigma A)$$

Since

$$\partial_\mu (\delta \phi^\ell_\nu \frac{\partial \mathcal{L}}{\partial \phi^\ell_\nu}) = e^{\alpha \beta \mu} \partial_\mu [(\sqrt{2} e^{\alpha BA} A_{\beta A} B_{\epsilon B} - \frac{i}{\sqrt{2}} \bar{\psi}_{\alpha A'} \psi_{\beta A' })(-i \bar{\psi}_{\nu A'} \epsilon)]$$

$$+ i \frac{1}{\sqrt{2} e^{AA'} \bar{\psi}_{\nu A'} D_{\nu} \epsilon A}$$

$$= e^{\alpha \beta \mu} \partial_\mu [i \sqrt{2} e_{\alpha A B A} A_{\beta A} B_{\epsilon B} \bar{\psi}_{\nu A'} \psi_{\beta A' } \epsilon A + i \frac{1}{\sqrt{2} e^{AA'} \bar{\psi}_{\nu A'} D_{\nu} \epsilon A}]$$
we have
\[
\epsilon^{\alpha\beta\mu\nu} \partial_\mu [i\sqrt{2}e_{A\beta}A_{\beta A} B^A \bar{\psi}_\nu' e^A + i \frac{1}{\sqrt{2}} e_{\alpha A} A^{\alpha A'} \bar{\psi}_{A'} D_\nu e_A + \sqrt{2}i e_{\alpha A} A^{\alpha A'} \bar{\psi}_{A'} \partial_\nu e_A] = 0
\] (84)

Therefore, we have
\[
\partial_\mu (\tilde{Q}^\mu_B e^B + \tilde{Q}^{\mu\nu} B \partial_\nu e^B) = 0
\] (85)

where
\[
\tilde{Q}^\mu_B := 2\epsilon^{\mu\nu\rho\sigma} e_{A\sigma A'} A_{\beta A} B^A \bar{\psi}_{A'} e^A, \quad \tilde{Q}^{\mu\nu} B := -2\epsilon^{\mu\nu\rho\sigma} e_{\alpha B} B' \bar{\psi}_{B'} e^A
\] (86)

The independence of \(\epsilon_A, \partial_\mu \epsilon_A\) and \(\partial_{\mu\nu} \epsilon_A\) implies that
\[
\partial_\mu \tilde{Q}^\mu_A = 0, \quad \tilde{Q}^\nu_B = \partial_\mu \tilde{Q}^{\mu\nu}_B, \quad \tilde{Q}^{\mu\nu} = -\tilde{Q}^{\nu\mu}_B
\] (87)

So we have the left-handed supercharge
\[
Q_A = 2 \int_\Sigma \tilde{Q}^0_A d^3x = 2 \int_\Sigma \tilde{Q}^0_B ds_i = 2 \int_\Sigma e^{ijk} e_{jA\sigma A'} \bar{\psi}_k e^A ds_i = i2\sqrt{2} \int_\Sigma \pi^i_A ds_i
\] (88)

To obtain the right-handed supercharge, we use the right-handed transform eq.(6) under which \(\mathcal{L}_J\) transforms as
\[
\mathcal{L}_J = -\mathcal{D}(\sqrt{2}i e^{AA'} \wedge \bar{\epsilon}_{A'} D\psi_A) + (\sqrt{2}i D e^{AA'} + \psi_A \wedge \bar{\psi}_A') \wedge \bar{\epsilon}_{A'} D\psi_A
\] (89)

Using the field equation \(e^{AA'} \wedge D\psi_A = 0\) and eq.(4), we have \(\delta \mathcal{L}_J = 0\). Thus
\[
\delta \mathcal{L} = -\epsilon^{\mu\nu\rho\sigma} \partial_\mu [\sqrt{2}i \bar{\psi}_\mu A_{\beta A} e_{\nu B A} A_{\sigma A} B^A + \epsilon^{\alpha A} A^{\alpha A'} \bar{\psi}_{A'} D_\nu \psi_A
\]
\[
+\sqrt{2}i e_{\alpha A} A^{\alpha A'} \bar{\psi}_{A'} \partial_\nu e_A] = 0
\] (90)

This can yield that
\[
\partial_\mu (\tilde{Q}^{\mu}_A e^A + \tilde{Q}^{\mu\nu}_A \bar{\psi}_\nu e^A) = 0
\] (91)

where
\[
\tilde{Q}^\rho_A := -2\epsilon^{\mu\rho\nu} e_{\mu B A} A_{\sigma A} B^A, \quad \tilde{Q}^{\mu\nu}_A := -2\epsilon^{\mu\nu\rho\sigma} e_{\mu A A'} \bar{\psi}_\sigma
\] (92)

Similar to eq.(87), we have
\[
\partial_\mu \tilde{Q}^{\mu}_A = 0, \quad \tilde{Q}^\nu_B = \partial_\mu \tilde{Q}^{\mu\nu}_B, \quad \tilde{Q}^{\mu\nu}_B = -\tilde{Q}^{\nu\mu}_B
\] (93)
and the right-handed supercharge is

\[ \tilde{Q}_A = \int_\Sigma \tilde{Q}_A^0 d^3x = \int_{\partial \Sigma} \tilde{Q}_A^0 \gamma_5 d\Sigma = -2 \int_\Sigma \epsilon^{ijk} \psi_i^A e_j B A' A k A B d^3x = 2 \int_{\partial \Sigma} \epsilon^{ijk} e_j A A' \psi_k^A d\Sigma \]  

(94)

We can easily see that \( Q_A \) and \( \tilde{Q}_A \) are complex conjugate to each other by comparing eq.(88) and eq.(94). Their Poisson bracket gives that

\[ \{Q_A, \tilde{Q}_A\} = \{\bar{Q}_A, \bar{Q}_A'\} = \{i \sqrt{2} \tilde{\pi}_A, -2 \int_{\Sigma} \epsilon^{ijk} \psi_i^C e_j B A' A k C B d^3x\} = i \sqrt{2} P_{AA'} = i \sigma_{AA'}^a P_a \]  

(95)

Using the volume integral of the supercharges and the surface integral of the energy-momentum and taking into the fall-off of the fields, one may easily obtain that

\[ \{Q_A, P_{BB'}\} = \{\bar{Q}_A, P_{BB'}\} = 0 \]  

(96)

Finally, we calculate the Poisson bracket of the supercharges and the \( SU(2) \) charges. Note that the quantal commutator of them is [13]

\[ [Q_A, J_{ab}] = \frac{1}{2} (\sigma_{ab})_A^B Q_B \]  

(97)

where \( (\sigma_{ab})_A^B = -\frac{1}{2} (\sigma_{aA} B \sigma_{b} B B' - \sigma_{bA} B \sigma_{a} B B') \) (here the \( \sigma \)-matrices are those in [14] not in [15] which the \( \sigma \)-matrices with one lower primed index differ by a sign from those in [14]). So

\[ [Q_A, J_{ab}] = -E_{[aA} B' E_{b]}^B Q_B \]  

(98)

Using

\[ E_{[aA} B' E_{b]}^B = E_{[aA} E_{b]} B C - i \sqrt{2} n_{[a} E_{b]} A B \]  

(99)

we have

\[ [Q_A, J_{ij}] = -E_{[iAC} E_{j]} B C Q_B \]  

(100)

and

\[ [Q_A, J_{0i}] = i \sqrt{2} n_{[i0} E_{j]} A B Q_B = \frac{i}{\sqrt{2}} E_{iA} B Q_B \]  

(101)

On the other hand, we have from eq(32) that

\[ \tilde{J}_{AB}^\rho = -\frac{1}{2} \tilde{j}_{ab}^\rho E_a A' E_{b BA'} \]  

(102)

where \( \tilde{j}_{ab}^\rho \) is the angular-momentum current obtained in [3-10],

\[ \tilde{j}_{ab}^\rho = \sqrt{2} e^{\rho \mu \nu} \partial_\nu (e_{\mu a} e_{\nu b}) \]  

(103)
superalgebra complex supergravity

and the angular-momentum is

$$J_{ab} = \int_\Sigma \tilde{J}_0^a d^3x$$  \hspace{1cm} (104)

Hence

$$J_{MN} = -\frac{1}{2} J^{ab} E_{[aM} A^b_{N]} A' + i \sqrt{2} J^{0i} n_0 E_{iA}' B$$  \hspace{1cm} (105)

where $L_i = \frac{1}{2} \epsilon_{ijk} J^{jk}$ are the spatial rotations and $K_i = J_{0i} = -J^{0i}$ are the Lorentz boosts. Therefore

$$J_i = \frac{1}{2} (L_i - iK_i)$$  \hspace{1cm} (106)

(Bearing in mind that both $\frac{1}{2}(L_i - iK_i)$ and $\frac{1}{2}(L_i + iK_i)$ obey the $su(2)$ algebra [16]-[17]) From eq.(100) and eq. (101) we have

$$[Q_A, J_k] = \frac{1}{\sqrt{2}} E_{kA} B Q_B$$  \hspace{1cm} (107)

This can really be realized by the Poisson bracket because

$$\{Q_A, J_i\} = \{Q_A, \frac{1}{\sqrt{2}} E_{iA}^M N \int_\partial \Sigma \tilde{e}_i^k d s_k \} = \frac{1}{\sqrt{2}} E_{iA} B Q_B$$  \hspace{1cm} (108)

Actually, the boost charges are vanishing here as can be seen from eq(30).

6 Summary and Discussions

In this paper, we have obtained the angular-momentum, supercharges and the energy-momentum in the self-dual simple supergravity. The conservation laws possess the common feature of the conservation laws obtained previously, i.e., the currents are identically conservative because they can be expressed as divergences of antisymmetric tensor densities which are often referred to as potential.

The total charges take the same integral forms as those in the non-supersymmetric case. Though we can obtain the $SU(2)$ sector of the $SL(2, \mathbb{C})$ charges, the information of the angular-momentum is completely contained in the $SU(2)$ charges. It can be seen from the surface integrals that the angular-momentum is governed by the $r^{-2}$ part of $\tilde{e}^i$, the energy-momentum is determined by the $r^0$ part of $\tilde{e}^i$ and the $r^{-2}$ part of $A_i$, and the supercharges are governed by the $r^{-2}$ part of $\tilde{\pi}^i$. As in [1]-[2], we always assume that the phase space variables are subject to the boundary conditions.

$$e_{\mu AB}^{\sigma \Sigma} = (1 + \frac{M(\theta, \phi)}{r})^2 e_{\mu AB}^{\sigma \Sigma} + O(1/r^2), \quad A_{\mu MN} |_{\partial \Sigma} = O(1/r^2)$$  \hspace{1cm} (109)

$$\tilde{\pi}^i_A = O(1/r), \quad \psi_{\mu A} = O(1/r)$$  \hspace{1cm} (110)
superalgebra complex supergravity

where $e^\mu_{AB}$ denote the flat $SU(2)$ soldering forms. As a consequence, under the $SL(2, \mathbb{C})$ transforms, behaving as

$$L_A^B = \Lambda_A^B + O(1/r^{1+\epsilon}), \quad (\epsilon > 0)$$

where $\Lambda$ are rigid transforms. The charges transform as

$$J_{MN} \rightarrow \Lambda_M^A J_{AN}, \quad P_{AA'} \rightarrow \Lambda_A^B \bar{\Lambda}_{A'}^{B'} P_{BB'}$$

$$Q_A \rightarrow \Lambda_A^B Q_B, \quad \bar{Q}_{A'} \rightarrow \bar{\Lambda}_{A'}^{B'} \bar{Q}_{B'}$$

i.e., they gauge covariant. Their conservation is generally covariant. Upon quantization, the Poisson brackets correspond to the quantal commutators or anti-commutators and their algebra realizes indeed the super-Poincare algebra. This shows that their interpretations convincing, especially that the approaches used previously to obtain generally covariant conservation laws are reasonable.

It is novel that the relationship among the conservative quantities and the first class constraints is the same as the gauge charges and the constraints in the usual Yang-Mills gauge field models. To see this, consider the example of interacting Yang-Mills and spinor fields. The Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu \nu}^a F_{\mu \nu}^a + i \bar{\psi} \gamma^\mu D_\mu \psi - m \bar{\psi} \psi$$

is invariant under gauge transforms and this leads to the conservative Noether currents

$$J_a^\mu = t^{abc} F_{b \mu}^c A_c^\mu + i \bar{\psi} \gamma^\mu T^c \psi$$

where $[T^a, T^b] = t^{abc} T^c$. Among the equations of motion, there are the constraints

$$C^a(x) = \partial_k F_{k \mu}^a - t^{abc} A_b^k F_{k \mu}^c + i \bar{\psi} \gamma_0 T^a \psi \approx 0$$

generating time-independent gauge transforms. The zero component of $J_a^\mu$ is just the last two terms of the constraints. So we have the gauge charges

$$Q_a = \int_\Sigma J_a^0 d^3 x = \int_{\partial \Sigma} F_{a \mu}^0 ds_k$$

i.e., it is also a surface integral modulo the constraints. The surface integral expressions of $J_{MN}, Q_A$ et al in this paper are also obtained in this way.

The supergravity considered here is an extension of the non-supersymmetric case. This can be seen by setting the anticommuting fields to be zero. Then not only the field equations but also the
constraints reduces to the constraints in [1]-[2]. The Hamiltonian constraint and the diffeomorphism constraint in [1]-[2] are implied in the constraint $\mathcal{H}^{AA'}$. Since eq(15) which stems from eq(11) reduces to the Hamiltonian constraint

$$\text{tr}(\bar{e}^i \bar{e}^j F_{ij}) = 0$$

and eq(11) reduces to

$$\epsilon^{ijk} e_i^{BD} F_{jkD}^A = 0$$

so

$$\epsilon^{ijk} e_i^{BD} F_{jkD}^A E_{LAB} = 0$$

i.e.

$$\epsilon^{ijk} e_i^m (E_m E_l)_A^D F_{jkD}^A = 0$$

Using $(E_m E_l)_A^D = \frac{1}{2}(\delta ml \delta A^D + \epsilon_{mln} E_{nA}^D), \text{tr} F_{ij} = 0$ and $\epsilon^{ijk} \epsilon_{mln} e_i^m \sim e_i^j e_n^k$, we can obtain the diffeomorphism constraint

$$\text{tr}(\bar{e}^i F_{ij}) = 0$$

Thus we can say that the Hamiltonian constraint and the diffeomorphism constraint can be combined together.

Acknowledgement S.S. Feng is indebted to Prof. S. Randjbar-Daemi for his invitation for three months at ICTP. This work is supported by the National Science Foundation of China under Grant No. 19805004 and in part by the Funds for Young Teachers of Shanghai Education Council.

References

[1] A. Ashtekar *Phys. Rev. Lett.* 57 (1986):2244; *Phys. Rev. D* 36 (1987):1587.

[2] A. Ashtekar *New Perspectives in Canonical Gravity* (Lecture Notes, 1988, Naples:Biblipolis).

[3] T. Jacobson & L. Smolin *Phys. Lett. B* 196 (1987):39.

[4] J. Samuel *Pramana J Phys.* 28 (1987):L429.

[5] T. Jacobson *Class. Quan. Grav.* 5 (1988):923.

[6] N.N. Gorobey & A.S. Lukyanenko *Class. Quan. Grav.* 7 (1990):67.
superalgebra complex supergravity

[7] S.S. Feng & Y.S. Duan Gen. Rel. Grav. 27(8) (1995):887.

[8] S.S. Feng & Y.S. Duan Comm. Theor. Phys. 25(1996):485.

[9] Y.S. Duan & S.S. Feng Comm. Theor. Phys. 25 (1996):99.

[10] S.S. Feng & H.S. Zong Inter. J. Theor.Phys. 35 (1996):267. S.S. Feng & Y.S. Duan Grav. & Cos. 1 (1995):319.

[11] S.S. Feng Nucl. Phys. B 468(1996):163.

[12] Y.S. Duan & J.Y. Zhang Acta. Phys. Sini 19 (1963):589.

[13] S.S. Feng & X.J. Qiu Phys. Lett. B 411 (1997):256.

[14] J.Wess & J. Bagger Supersymmetry and Supergravity second edition. (Princeton University Press, 1992).

[15] Peter West Introduction to Supersymmetry and Supergravity (Extended Second Edition) 1990, World Scientific Publishing Co.Pte.Ltd.

[16] L.H. Ryder Quantum Field Theory (Cambridge University Press,1985)

[17] S. Weinberg The Quantum Theory of Fields Vol.I. (Cambridge University Press,1995).

[18] R.Casalbuoni Nuovo Cim, 33A (1976):115; Nuovo Com. 33A (1976):389.

[19] Peter. G. . Freund Introduction to Supersymmetry ( Cambridge University Press, 1986).

[20] L.D. Faddeev & A.A.Slavnov Gauge Fields Introduction to Quantum Theory (Addison-Wesley Publishing Company ,1991).