The Role of Operations Research In Solving The Problem of fuzzy Transport ( An Empirical Study)

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Abstract. Transportation is one of the most important requirements for humans to practice their activities. Also, it considers one of the most essential ingredients for economic development. The applications of operations research are interested in finding the optimal plan for transportation at the lowest costs from their production sources to their requesting bodies to reach the optimal decision as they play the main role in the decision-making process on a scientific basis for planning productive and economic projects. For this purpose, this research has dealt with solving the problem of transportation of oil products in the Midland Oil Company, one of the Iraqi Ministry of Oil formations. Therefore, during this study, the effectiveness of the use of transport models in reducing transportation costs of oil products in production institutions was tested by using fuzzy theory and a modern algorithm (ATM), where the method applied in the Midland Oil Company for distributing its oil products from its four main warehouses to the provinces contracting with the company as the data is fuzzy and compared with using the ready program (Win QSB) in solving the problem of fuzzy transport. As has been found through the results that the methods used to solve the problem are optimal methods for transporting the product at the lowest possible cost. And in light of the results after its analysis, several recommendations and proposals have been presented that help in applying the solution methods as they have contributed to decision-making for decision-makers.

Keywords: Optimized fuzzy solution, Adjacent zero method, Robust fuzzy, Fuzzy zero method (FPZM). Fuzzy Transfer Problem (FTP).

Research problem
The oil sector considered one of the important sectors in the country. Therefore the interest in it has an important impact since it is in direct contact with the life of the citizen and has a major role in achieving the important financial resource in Iraq. So the state is making persuasive efforts to develop this sector. for this, It was necessary to find an optimal method to transport the necessary oil products from its main warehouses to its requesting parties. Because the company had faced such a problem, and for us, we have faced the fuzzy problem of data due to the lack of knowledge of the total cost of transportation, with determining the quantities that will be transferred according to the request to the
company’s approval to transfer its products in basin vehicle by contracting with a private company (Uruk) that facing a high cost of transport as well as traffic accidents, fire and emergency conditions. so it has resorted to the use of fuzzy logic to solve the model to achieve the intended objective of this research.

Research Importance
The importance of research emerges from the importance of decision-making in the various productive and industrial sectors. Especially the large productive enterprises that derive their decisions through the application of operations research systems to reach the right decision in solving the problem of transporting oil products in oil installations and the lowest costs for transport with determining the optimal quantity and focusing its attention on the most important jobs established to transfer goods or products at the lowest costs. Through the methods provided by operations research and application, the best solution is not to find the existence of a budget determined by the company and approved in its plans.

Research objective
The research looks forward to solving a foggy transfer problem to reach an optimal solution by achieving the following goals:

- Reducing the cost of transportation of a fuzzy nature to the Ministry of Oil / Midland Oil Company for the transportation of the product
- Determine the optimum transferred quantities from their main locations to the organizations that required to be supplied.

Research limits
The search limits were as follows:

A- Spatial limits: that relate to the company in question, which is the Midland Refineries Company of the Ministry of Oil

B - Time limits: represents the period in which the data was documented by the company and was processed and analyzed to reach the results for the period from 1/1/2019 to 1/6/2019.

1. Introduction
It’s being considered that the Decision-making for the establishment in planning the resources of industrial and productive institutions on a scientific basis is an important matter. Its pillar represented in analysis and scientific research. so operations research is one of the most important tools for arriving at an optimal solution, and the need for its use in developing countries that need to optimize investment for their resources and limited and available capabilities.

The transportation model is one of the operations research applications that concern with finding an optimal plan for the transfer of the lowest cost between a group of production sources and a group of demand and need centres. It also perceives the presence of a large number of projects in industries, which provides the possibility to take advantage of transport models to develop economic plans to supply their needs and materials with several availabilities from production centres to provide the project with its need for manufactured and primary materials. In light of the technological development, fuzzy programming has become the most effective way to address the scientific reality, so the fuzzy concept (Zadeh and Bellman) was used in the decision-making process, where the programming problem formulated Fuzzy linear programming with fuzzy coefficients, so ensure that search gives a clear picture of fuzzy logic with the use of method of arrangement hippocampus (Robust) for fuzzy data processing method using the (AlRank).

One of the most important reasons for choosing this topic is an attempt to study the problem of transporting oil products, as it is one of the problems that have a direct impact on the life of the citizen.
Further and in-depth study of the active role in reducing the costs of transporting the product by modern methods. In addition to the current stage of the country in which the productive institutions of the country live similarly the lack of studies. Besides the lack of field and applied studies that deal with the problem of transportation, since the research has to do with the researcher's specialization as well as linking the theoretical study to what is on the ground. In addition to trying to create a competitive advantage for the corporation by reducing costs and thereby raising its competitiveness of the product and thus obtaining an optimal plan to transfer the symmetric product from several sources to several centres through which to highlight the importance of transportation models in reducing costs (the Algerian message is taken from recommendations).

The research structure is to:
First: The theoretical aspect includes the theoretical concept of the fuzzy group
Secondly: The applied aspect: which includes the problem of fuzzy transport of the company by using methods to process data with fuzzy coefficient and solve them to extract the results and analyze them to reach the optimal solution.
Third: It included the most important conclusions and recommendations in light of the findings of the research to provide support to researchers specialized in this field in the future. The importance of research appears through reaching the right decision to solve the problem of transporting health materials and the lowest costs for transporting them, while determining the optimal quantity, through the methods provided by operations research and their application. The optimal solution is not found in the presence of a budget determined by the company and approved in its plans.

2. The theoretical side

2.1. Fuzzy Theory.
Fuzzy theory is one of the forms of logic that has used in systems and applications of artificial intelligence. It is a highly effective technique for finding a solution to applied problems. Since it has a great influence on decision-making to solve the various problems of decision-makers and researchers more accurately. Through the use of fuzzy logic, specific results have obtained from false data unlike the classic logic that includes determining numerical values of values, so it was adopted in the methods of accurate inference, this logic was used by the Iranian scientist Lotfi Zadeh, he developed it as the best way to process fuzzy data and he intended to provide a mathematical system, where he used mathematical laws followed by statistical methods. Then he combined mathematical laws and statistical methods with modern development. Fuzzy logic has become one of the methods for building modern systems, where it has been applied in neural networks and manufacturing processes. As it has an important role in making the optimal decision through the use of mathematical models of a fuzzy nature to address problems [6].

2.2. Fuzzy Numbers.
The classical group (crisp set) is a well-defined group, that is, a specific and countable group, which takes one of the two values (0,1). When the element belongs to the group, it takes the value (1), and when not affiliated it takes the value (0) and when the degree of affiliation of the element is equal to (0.9), (0.8) This means that the degree of affiliation of the element is high, but when the degree of affiliation is equal to (0.5) it means that the degree of affiliation is of equal value for non-affiliation. Whereas if the degree of affiliation of the element is less than (0.5), this indicates that the affiliation of the element to the group is weak. As shown in the figure below shown in the simple group:
Figure 1. Shows the fuzzy group

The scientist (Zimmerman) defined in (1988) the fuzzy group: Let $X$ have a group of elements and be denoted by the symbol $x$ and let a group $A$ be a fuzzy group in $X$ be a group of arranged pairs, and let the fuzzy group be known as $(A^-)$ where\[5]:

\[
\{ (x_i), (\mu_{A^-}(x_i)) \} \quad \text{where} \quad i=1,2,\ldots, n
\]

So that $\mu_{A^-}(x_i) \in [0, 1]$

2.3. Types of function membership liner

A. The Triangular membership function is a linear membership function with three parameters that are of a straight line shape, so when they are:

\[ A^- = (a, b, c) \]

When it has been said that $A^-$ represent a fuzzy triangular number and the triangular membership function can be expressed in the following formula:

\[
(x) = \begin{cases} 
\frac{x-a}{b-a} & \text{if } a \leq x < b \\
1 & \text{if } x = b \\
\frac{c-x}{c-b} & \text{if } b < x \leq c \\
0 & \text{if } \text{otherwise}
\end{cases}
\]

whereas:

a. represents the lower bound of the function
b. represents the value of the centre
c. represents the upper bound of the function

$\mu_A(x)$ Degree of affiliation $x$ whose value ranges between $(0, 1)$

The function can be illustrated as follows:

Figure 2. shows the trigonometric affiliation function
B. Trapezoidal Function
Linear membership function with four parameters, where $A^-$ has four parameters for fuzzy number, where it represents [4]:

$$A^- = (a, b, c, d)$$

$$\mu_A(x) = \begin{cases} 
\frac{x - a}{b - a} & \text{if } a \leq x < b \\
1 & \text{if } b \leq x \leq c \\
\frac{d - x}{d - c} & \text{if } b \leq x < c \\
0 & \text{if } otherwise 
\end{cases} \quad (3)$$

It can be represented as follows [11]

Figure 3. shows the trapezoidal function

2.4. Operations on triangular intuitionistic fuzzy number. Suppose we have $A^-$, $B^-$ two sets of fuzzy numbers, which are known as [12]:

$$a_2, a_3, a_1 = (A^-)$$

$$B^- = (b_1, b_2, b_3)$$

Therefore, basic mathematical operations can be performed on fuzzy numbers (addition, subtraction and multiplication) as follows:

1. Addition of the fuzzy number to $\tilde{A}$, $\tilde{B}$ is defined by

$$\tilde{A} + \tilde{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

2. Subtraction the fuzzy numbers

Let the groups $\tilde{A}$ and $\tilde{B}$ be the subtraction process for the two groups:

$$= (a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3) \tilde{A} - \tilde{B}$$

3. Multiplication to the fuzzy numbers for the two sets

$$A^- \ast B^- = (a_1, a_2, a_3) \ast (b_1, b_2, b_3) = (a_1 b_1, a_1 b_2, a_3 b_1, a_3 b_2)$$

4. Scalar multiplication the fuzzy numbers

Stable $k$,
\((a_1, a_2, a_3 ) = (ka_1, ka_2, ka_3 )\)

5. Numbers are positive if \(A^+ \sim 0\)
   And zero if \(A^± \sim 0\)
   And equivalent if \(A^± = B^±\)

Where:
- \(a_1\) represents the first number in the first set
- \(a_2\) represents the second number in the first set
- \(a_3\) represents the third number in the first set
- \(b_1\) represents the first number in the second set
- \(b_2\) represents the second number in the second set
- \(b_3\) represents the third number in the second set

### 2.5. Fuzzy Transport Problem (FTP)

It is a linear programming problem that is similar to the usual transportation problem in its structure, but its variables have a fuzzy shape (its transactions are represented by the target function, the cost of transportation, the amount of demand, the amount of supply), the importance of transportation problems in distributing the product from production sources such as factories to their requesting destinations such as markets and stores with a specific absorptive capacity with a limited requesting parties. The transportation issues are looking at finding a plan for the transfer and distribution of materials at the lowest possible cost to transfer the product between points of production and consumption, where we symbolize production centers by \((m)\) and the number of points of demand in \((n)\) and the level of producing the product in each center with \((a_i)\) and for the need of each requesting point in \((b_j)\), as well \((X_{ij})\) denotes the quantity of the product that will be transferred from the production source \((i)\) to the consumption point \((j)\) where [10]:

The fuzzy transport model can be expressed in the following table:

|        | \(c_{11}\) | \(\ldots\) | \(c_{in}\) | \(\tilde{a}_1\) |Supply
|--------|------------|------------|------------|----------------|
| Demand | \(b_1\)    | \(\ldots\) | \(b_n\)    |                |

As for mathematically, the mathematical formula for the transfer model is written as follows:

\[
\text{minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \tilde{x}_{ij}
\]

According to the restrictions below

\[
\sum_{j=1}^{n} x_{ij} = \tilde{a}_i, \text{ for } i = 1,2,\ldots,m
\]

\[
\sum_{i=1}^{m} x_{ij} = \tilde{b}_j, \text{ for } j = 1,2,\ldots,n
\]

whereas:
- \(M\): the total number of sources
- \(N\): The total number of applicants
- \(\tilde{a}_i\): Quantities displayed that are fuzzy for sources
- \(\tilde{b}_j\): Fuzzy quantities required for demand directions
The cost of the fuzzy unit transferred from the source (i) to the demanding entity (j) $c_{ij}$. The optimum b amount that will transfer the source to the requesting side.

With the necessity that the transportation problem should be balanced that is:

\[ \sum_{j=1}^{n} b_{j} = \sum_{i=1}^{m} a_{i} \]

A New Technique to Obtain Initial Basic Feasible Solution for the Transportation Problem

As for the opposite, the goal that the model seeks to achieve is a distribution plan that has minimal transportation costs and can be expressed in the following form:

\[ \text{Min} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \]

2.5.1 Methods to solve fuzzy transport problems.

To solve the problem of fuzzy transport, several methods have proven their efficiency in solving the problem. Most research attempts to develop a solution method for fuzzy transport problems, as the (Rank) function method is considered one of the most important modern techniques in arriving at solutions to complex transport problems, as it was distinguished by being a function with specific criteria. It aims to find the optimal solution by converting the problem from fuzzy to non-fuzzy. Since most problems in the real world have fuzzy conditions, which makes the decision-making task often difficult, especially when defining the main goal in light of lack of clarity and accuracy, for all of this contributed in being an important tool to deal with those conditions to solve the problem of optimization.

2.6. Rank Function Method. It is a method for dealing with fuzzy numbers for specific numbers by applying the formula. The rank function was used to get the result. as follow [8]:

So the set of fuzzy number $F(R)$

\[ F(R) = \{ (a, b, c, d) \} \]

Where $F(R)$ is the rank function: the set of all the fuzzy numbers on the real number line

\[ (R^-) = (a + b + c + d)/4 \]

So to be the $A^-$, $B^-$ two set of a fuzzy number

\[ a, \quad a = (A^-) \]

\[ B^- = (b, b, b, b) \]

(i) $A^- \geq B^- \quad R(A^-) \geq R(B^-)$

(ii) $A^- \leq B^- \quad R(A^-) \leq R(B^-)$

(iii) $A^- = B^- \quad R(A^-) = R(B^-)$

2.7. Robust Ranking Method (RRM)

is one of the most important modern techniques for solving fuzzy transport problems, which provide results compatible with human intuition. Where the method is used in the decision-making process when the costs and quantities offered and required are fuzzy. Where this method was used in research published in the year (2010) by the two scientists (R. Nagajajan, A. Solairaju), where the function takes the following form [2]:

Let $\bar{a}$ be: A set of fuzzy numbers where the Robust Ranking function can be defined as follows:

\[ R(\bar{a}) = \int_{0}^{1} (0.5)(a_{1}^{\alpha} a_{2}^{\beta}) \, d\alpha \]
Where \( \tilde{\alpha} = \begin{pmatrix} a_1, a_2, a_3, a_4 \\ a_1^l, a_1^u \end{pmatrix} \)

\[
\begin{pmatrix} a_1^l & a_1^u \end{pmatrix} = \begin{pmatrix} (a_2 - a_1)\alpha + a_1, a_4 - (a_4 - a_3)\alpha \end{pmatrix}
\]

where \((a_1^l, a_1^u)\) represents the level of intersect \(\alpha\) to the fuzzy number \(\tilde{\alpha}\).

It can be extracted in the following mathematical formula Source 1:

\[
\begin{pmatrix} a_1^l & a_1^u \end{pmatrix} = \begin{pmatrix} (b - a)\alpha + a + c - (c - b)\alpha \end{pmatrix}
\]

The robust function considers among the important functions to solve the problem of transport and allocation of a fuzzy nature [1].

2.8. Algorithm of zero suffix method (M.Z.S.M)

One of the modern and efficient methods to reach the optimal solution to the problem of transportation, the idea of this method is that it does not need many iterative processes, as researchers (Thiagarajan, & Saravanan, & et al) presented in 2013 a research under the title (Finding the best solution to the problem of transportation using The adjacent zero method)[13] It was called by that name because the allocation in it is in cells that have been assigned zeros and that have the largest non-zero rate of the zero site, and it is one of the suggested methods that give good results. The solution steps for this method are summarized in the following [14].

- Create a schedule for the transportation problem, then ensure that the schedule is balanced.
- Subtracting the lowest element found in each of the row or column elements in the transfer matrix.
- After the previous step, we obtain the cost reduction transfer schedule, then we make sure that there is at least one zero in each row or column in the reduced schedule.
- We extract the non-zero adjacent average at the location of every zero in the table. The non-zero rate can be defined as dividing the non-zero cells that are adjacent to the zero location and calculate the average for them by dividing their sum by the number of those values.

We perform the allocation process at the location of every zero which carries the largest non-zero rate by allocating the specified offer and demand and the cell is allocated at that time.

Repeating the previous steps from (1) to (5) and verifying every process of having each row or column of zero and at least one zero.

After allocating all quantities shown in the transfer schedule, we start calculating the final cost of transportation.

2.9. Fuzzy Zero Point Method (IFZPM)

It is a modern and effective way to solve the problem of fuzzy transport, proposed by the researcher (Edward Samuel) in (2012) to solve transport problems and problems of fuzzy transport, and it was distinguished by being simple and considered an important tool for decision-makers in taking and analysing activities and administrative and economic decisions that face them to find a solution to the transport problem of a Fuzzy Nature [9] the algorithm method of the modified fuzzy zero point.

To apply the above method, we follow the following steps

The first step - to ensure the balance of the transfer schedule for the problem of fuzzy transport, and in the event of the opposite, the schedule is balanced.

The second step - determining the highest fuzzy cost of all transportation costs in the schedule.

In the third step - we compensate for the cost of the column or fictitious ghost row with the highest transport cost in the table instead of zero costs.

Step 4 - We determine from each row of the transfer table in every column in the table columns the lowest transport cost and subtract this cost from all cells of this transport column and row.
Step Five - After the previous steps, we get a table called the Table of Fuzzy Costs.

Step Six - Ensure that each element of the demand is less or equal to the sum of the fuzzy supply and each component of the bid is less or equal to the sum of the fuzzy demand.

Step Seven - When there is in each row or column one cell that costs at least zero, we move to step (the tenth), but if the opposite is the case, we move to step (eight).

Step Eight - draw the lowest possible number of vertical and horizontal lines to cover all zeros from the reduced fuzzy transfer schedule as a result of step (the fifth).

Step 9 - We start developing the solution schedule as
A. We determine the lowest cost and let (c) be not covered by any line among all cells.
B. We subtract the cost (c) from all cells not covered by any line.
C - We add cost (c) to each cell located below the intersection of two lines.
D - Cell elements below one line remain the same.
E. - We move to step (seventh).

Step 10 - We start by selecting the cells to allocate them within the specified supply and demand limits as follows:
- a. Choose the largest transport cost in the reduced fuzzy transfer schedule, and when the largest equal costs are equal we choose any cell and this cell is called (i, j).
- B. After selecting the cell (i, j) we choose a zero cell from the row (i) and column (j) then we assign them by investigation between demand and supply and cross out as usual.
- C. If there are not in a row (i) and column (j) any of zero cells, we choose the second largest cost in the reduced transport schedule. Thus, we continue to repeat the previous steps until all the requests and supply elements are met in the transfer schedule.

2.10. Allocation table method (ATM)
It is one of the modern ways to achieve the optimal solution to the problems of transportation because of the instability of the oil economy, so it is distinguished by its simple application and sports work. As the cost of transportation is one of the most important concerns for success in managing productive projects, the research aims to find the lowest costs for transport under conditions of fluctuating oil prices and through the results, it shows that the method was efficient and accurate in the application in predicting the future of transportation costs, whereas the procedures used in the method are [3].

Formulating a schedule for the transportation problem within the limits of specific supply and demand
- 2- After checking the balance of the transfer schedule and if it is unbalanced, we should balance the schedule.
- 3- We define the lowest individual costs within the costs of the schedule, whether they are an integer, decimal, or individual fractional costs.
- 4- We subtract all table costs from that cost while maintaining this individual cost as it is then we get the reduced schedule by its values.
- 5- We start by allocating the lowest individual cost in the table within the specified supply and demand of the cell, while deleting the allocated row or column.
- 6- We continue with the previous steps until all quantities are allocated to the requesting parties in the transportation matrix and we start calculating the final cost of transportation. It can also be clarified what will be applied from the algorithms to solve the problem of fuzzy transport in the following figure [7].
3. The applied side

In order to apply the theoretical aspect of the research, we have applied the modern algorithm to demonstrate that it is a modern and important way to reach the optimal solution, which has a major impact "in reducing transportation costs to national institutions of a productive nature. The choice was made on the Central Refineries Company, one of the formations of the Ministry of Oil Which contains four refineries (Al-Dora Refinery, Al-Najaf Refinery, Al-Samawa Refinery, Al-Diwaniya Refinery) for the period (1/1/2019 - 1/6/2019) that supply the governorates of the country with black oil (fuel oil) as it is an important resource for Bricks factories and electrical generators, as well as the cost of transporting them, has an important the economy of the country. thus obtaining the lowest total cost for transporting black oil from the main warehouses to the governorates requesting this material, and using the formula below (1) approved by calculating the cost of transporting black oil in the Midland Refineries Company.

The transportation costs are calculated according to Table (3-1) after documenting the fuzzy data. Transport cost = load (tons) * distance (km) * transport price (dinar per ton) .... (1)

| Governor | Baghdad | Basra | Karbala | Babylon | Al-Kut | Al-Najaf | Al-Diwaniya | Al-Samawa | Offer |
|----------|---------|-------|---------|---------|--------|----------|-------------|-----------|-------|
| Al Dora refinery warehouse | 10,15, 20 | 50,60, 80 | 10,30, 45 | 10,15, 35 | 30, 40, 45 | 32,50, 56 | 20, 35, 40 | 35, 50, 53 | 4000, 4600, 5000 |
| Najaf refinery warehouse | 34,45, 63 | 50,55, 35 | 10,16, 25 | 10,15, 25 | 30, 45, 55 | 15,20, 35 | 30,40, 45 | 2100, 2500, 2555 |
After a table has been figured for the problem fuzzy transport with a fuzzy of cost, offer, and demand, then the robust ranking function is used to convert the fuzzy numbers to specific numbers according to the robust ranking function below:

\[ R(\tilde{a}) = \int_{0}^{1} (0.5)(a^L_i, a^U_i) \, d\alpha \]

That is \( \tilde{A} = (a_1, a_2, a_3, a_4) \)

Where \( (a^L_i, a^U_i) \) represent the intersection \( \alpha \) to the fuzzy numbers \( \tilde{a} \)

It is extracted according to the mathematical equation where:

\[ (a^L_i, a^U_i) = ((a_2 - a_1)\alpha + a_1, a_4 - (a_4 - a_2)\alpha) \]

Thus, after applying the fuzzy ranking function to the parameters of the fuzzy transport
\( R(10,15,20) = 15, R(50,60,80) = 62.5, R(10,30,45) = 30, R(10,15,35) = 18.75, R(30,40,45) \)
\( = 40, R(32,50,60) = 48, R(20,35,40) = 32.5, R(35,50,53) = 47, R(30,34,45) \)
\( = 35.75, R(50,55,63) = 55.75, R(10,16,35) = 19.25, R(10,15,25) \)
\( = 16.25, R(25,30,45) = 32.5, R(10,15,33) = 18.25, R(15,20,35) \)
\( = 22.5, R(30,40,45) = 38.75, R(35,40,55) = 42.5, R(48,50,65) \)
\( = 53.25, R(20,28,40) = 29, R(25,33,40) = 32.75, R(18,25,30) \)
\( = 24.5, R(15,20,35) = 22.5, R(15,20,25) = 20, R(33,39,42) = 38.25, R(40,50,60) \)
\( = 50, R(40,42,53) = 44.25, R(30,36,40) = 35.5, R(36,40,45) \)
\( = 40.45, R(30,36,50) = 38, R(32,35,46) = 37, R(20,21,30) = 23, R(9,15,30) \)
\( = 17.25 \)

port schedule, the results appear as follows:

As for the fuzzy offer amounts after the Robust function is applied, they are as follows:

\( R(4000,4600,5000) = 4550, R(2100,2500,2555) = 2413.75, R(1250,1550,1600) \)
\( = 1487.5, R(2000,2500,2550) = 2387.5 \)

As from the demand directions

\( R(3100,3500,4000) = 3525, R(3800,3999,4398) = 4049, R(1500,2500,2900) \)
\( = 2350, R(800,850,955) = 863.75, R(600,750,950) = 762.5, R(500,650,700) \)
\( = 625, R(445,800,910) = 738.75, R(400,550,600) = 525 \)

3.1. Application of modern transportation methods to solve fuzzy transportation problems

The three solution methods will be applied to the transport matrix, the methods are (zero fuzzy point, zero suffix method, method (ATM)) on the black oil product model (fuel oil) in the Midland refineries company to transfer it from the main warehouses to the provinces requesting the product and compare the results of these methods with a method Linear programming to find any of the above methods closest to an optimal solution. After documenting the data, the transport schedule will be according to the features (supply, demand, cost) for the problem of fuzzy transportation of the product as in table (3-2) below.
After balancing the transportation schedule by adding an imaginary row that cost its cells equal to zero, the supply amount is (2600.25) tons of black oil, as in Table (3-3):

| Province               | Baghdad | Basra | Karbala | Babylon | Al-Kut | Al-Najaf | Al-Diwaniya | Al-Sama wa | Offer   |
|------------------------|---------|-------|---------|---------|--------|----------|-------------|------------|---------|
| Al Dora refinery warehouse | 15      | 62.5  | 30      | 18.75   | 40     | 48       | 32.5        | 47         | 4550    |
| Najaf refinery warehouse | 35.75   | 55.75 | 19.25   | 16.25   | 32.5   | 18.25    | 22.5        | 38.75      | 2413.75 |
| Diwanyiah refinery warehouse | 42.5    | 53.25 | 29      | 32.75   | 24.5   | 22.5     | 20          | 38.25      | 1487.5  |
| Samawa refinery warehouse | 50      | 44.25 | 35.5    | 40.25   | 38     | 37       | 23          | 17.25      | 2387.5  |
| Imaginary row           | 0       | 0     | 0       | 0       | 0      | 0        | 0           | 0          | 2600.25 |
| Demand                 | 3525    | 4049  | 2350    | 863.75  | 762.5  | 625      | 738.75      | 525        |

3.2. Applying the zero fuzzy point method on the transport schedule with its procedures. The assigned transport schedule is as follows according to Table (3-4) below:

| Province               | Baghdad | Basra | Karbala | Babylon | Al-Kut | Al-Najaf | Al-Diwaniya | Al-Sama wa | Offer   |
|------------------------|---------|-------|---------|---------|--------|----------|-------------|------------|---------|
| Al Dora refinery warehouse | 15      | 62.5  | 30      | 18.75   | 40     | 48       | 32.5        | 47         | 4550    |
| Najaf refinery warehouse | 35.75   | 55.75 | 19.25   | 16.25   | 32.5   | 18.25    | 22.5        | 38.75      | 2413.75 |
| Diwanyiah refinery warehouse | 42.5    | 53.25 | 29      | 32.75   | 24.5   | 22.5     | 20          | 38.25      | 1487.5  |
| Samawa refinery warehouse | 50      | 44.25 | 35.5    | 40.25   | 38     | 37       | 23          | 17.25      | 2387.5  |
| Imaginary row           | 0       | 0     | 0       | 0       | 0      | 0        | 0           | 0          | 2600.25 |

| Province               | Baghdad | Basra | Karbala | Babylon | Al-Kut | Al-Najaf | Al-Diwaniya | Al-Sama wa | Offer   |
|------------------------|---------|-------|---------|---------|--------|----------|-------------|------------|---------|
| Al Dora refinery warehouse | 15      | 62.5  | 30      | 18.75   | 40     | 48       | 32.5        | 47         | 4550    |
| Najaf refinery warehouse | 35.75   | 55.75 | 19.25   | 16.25   | 32.5   | 18.25    | 22.5        | 38.75      | 2413.75 |
| Diwanyiah refinery warehouse | 42.5    | 53.25 | 29      | 32.75   | 24.5   | 22.5     | 20          | 38.25      | 1487.5  |
| Samawa refinery warehouse | 50      | 44.25 | 35.5    | 40.25   | 38     | 37       | 23          | 17.25      | 2387.5  |
| Imaginary row           | 0       | 0     | 0       | 0       | 0      | 0        | 0           | 0          | 2600.25 |
After that the final cost of transporting the product in this way is calculated after allocating all the offered quantities and required as follows:

$$MIN(Z) = 247878.75$$

3.3. Applying the zero suffix method to the transport schedule after balancing the schedule, so the assigned schedule will be in Table (3-5):

| Province     | Baghdad | Basra | Karbala | Babylon | Al-Kut | Al-Najaf | Al-Diwaniya | Al-Samawa | Offer |
|--------------|---------|-------|---------|---------|--------|----------|-------------|-----------|-------|
| Al Dora refinery warehouse | 15      | 62.5  | 30      | 18.75   | 40     | 48        | 32.5        | 47        | 4550  |
| Najaf refinery warehouse | 3       | 3525  | 5.75    | 1025    | 16.25  | 32.5      | 18.25       | 22.5      | 2413.75 |
| Diwanyiah refinery warehouse | 42.5    | 53.25 | 925     | 863.75  | 34.5   | 625       | 20          | 38.25     | 1487.5 |
| Samawa refinery warehouse | 50      | 44.25 | 35.5    | 40.25   | 38     | 37        | 23          | 17.25     | 2387.5 |
| Imaginary row | 0       | 0     | 400     | 13.75   | 0      | 0         | 0           | 0         | 2600.25 |

| Demand | 3525 | 4049 | 2350 | 863.75 | 762.5 | 625 | 738.75 | 525 |

Then we calculate the final cost of transportation in a zero suffix method with its previously mentioned procedures as follows:

$$MIN(Z) = 247878.75$$

3.4. Applying the ATM method to the transport schedule after balancing the schedule, so the assigned schedule will be as in Table (3-6):

| Province     | Baghdad | Basra | Karbala | Babylon | Al-Kut | Al-Najaf | Al-Diwaniya | Al-Samawa | Offer |
|--------------|---------|-------|---------|---------|--------|----------|-------------|-----------|-------|
| Al Dora refinery warehouse | 15      | 62.5  | 30      | 18.75   | 40     | 48        | 32.5        | 47        | 4550  |
| Najaf refinery warehouse | 35.75   | 55.75 | 19.25   | 16.25   | 32.5   | 18.25     | 22.5        | 38.75     | 2413.75 |
| Diwanyiah refinery warehouse | 42.5    | 53.25 | 29      | 32.75   | 24.5   | 22.5      | 20          | 38.25     | 1487.5 |
| Samawa refinery warehouse | 50      | 44.25 | 35.5    | 40.25   | 38     | 37        | 23          | 17.25     | 2387.5 |
| Imaginary row | 0       | 0     | 400     | 13.75   | 0      | 0         | 0           | 0         | 2600.25 |

| Demand | 3525 | 4049 | 2350 | 863.75 | 762.5 | 625 | 738.75 | 525 |

13
Determining the lowest individual costs from the costs of the transportation schedule, which is (15) from the costs of the schedule, then this cost is subtracted from the individual costs of the schedule, which are found in Table (3-7) as shown below.

| province                  | Baghda d | Basra | Karbala | Babylon | Al-Kut | Al-Najaf | Al-Diwan Iyya | Al-Sama wa | Offer |
|---------------------------|----------|-------|---------|---------|--------|-----------|---------------|------------|-------|
| Al-Dora refinery warehouse| 7.5      | 31.25 | 15      | 9.375   | 20     | 24        | 16.25         | 23.5       | 4550  |
| Najaf refinery warehouse  | 17.875   | 27.875| 9.625   | 8.125   | 16.25  | 9.125     | 11.25         | 19.37      | 2413.75|
| Diwanyiah refinery warehouse| 21.25  | 26.625| 14.5    | 16.375  | 12.25  | 11.25     | 10            | 19.12      | 1487.5|
| Samawa refinery warehouse | 25       | 22.125| 17.75   | 20.125  | 19     | 18.5      | 11.5          | 8.625      | 2387.5|
| Imaginary row             | 0        | 0     | 0       | 0       | 0      | 0         | 0             | 0          | 2600.25|

Demand | 3525 | 4049 | 2350 | 863.75 | 762.5 | 625 | 738.75 | 525 |

The next step is to obtain the reduced schedule. The lowest individual cost is allocated in the transport matrix within the specified demand and supply limits, and then we start allocating the lowest cost of the schedule in succession until all the quantities required for this product are exhausted, as shown in Table No. (3-8).

| Governorate | Baghda d | Basra | Karbala | Babylon | Al-Kut | Al-Najaf | Al-Diwan Iyya | Al-Sama wa | Offer |
|-------------|----------|-------|---------|---------|--------|-----------|---------------|------------|-------|
| Al-Dora refinery warehouse | 7.5      | 31.25 | 15      | 9.375   | 20     | 24        | 16.25         | 23.5       | 4550  |
| Najaf refinery warehouse  | 17.875   | 27.875| 9.625   | 8.125   | 16.25  | 9.125     | 11.25         | 19.37      | 2413.75|
| Diwanyiah refinery warehouse| 21.25  | 26.625| 14.5    | 16.375  | 12.25  | 11.25     | 10            | 19.12      | 1487.5|
| Samawa refinery warehouse | 25       | 22.125| 17.75   | 20.125  | 19     | 18.5      | 11.5          | 8.625      | 2387.5|
| Imaginary row             | 0        | 0     | 0       | 0       | 0      | 0         | 0             | 0          | 2600.25|

Demand | 3525 | 4049 | 2350 | 863.75 | 762.5 | 625 | 738.75 | 525 |

After measuring the total cost of transporting the product from the main warehouses to the provinces requesting the black oil product, it was as follow:

\[
MIN(Z) = 240908.4375
\]
3.5. Apply linear programming method

(Win Q.S.B) program is one of the ready-made applications for computer systems, the importance comes from that it collects operations research applications and administrative applications to solve mathematical models with accuracy and efficiency, in this research we used the program to find the optimal solution to the problem to compare and analyse the results. The final results by applying the methods above on the fuzzy transportation problem with the linear programming method of the (WINQSP) program. The methods above are the results of the solution as follows:

| Method                             | Result solution |
|------------------------------------|-----------------|
| 1. Zero point method               | 247878.75       |
| 2. Zero suffix method              | 247878.75       |
| 3. ATM method                      | 240908.4375     |
| 4. Linear programming method (optimal solution) | 237008.4000 |

4. The researcher reached to

1. The ATM method came first, and it is a modern method that is mathematically easy to apply and gives a solution close to optimization, compared to the linear programming method that gives an “optimal” solution to solve fuzzy transport problems.
2. The researchers’ reliance in the future on modern methods to solve fuzzy transportation problems, as they give accurate solutions efficiently.

5. Recommendations

1. We recommend researching the application of the ATM method to find the optimal solution for fuzzy transport problems, so that the optimal solution becomes simpler.
2. Paying attention to oil products and increasing their warehouses stores, as it is the important economic resource for the country’s economy and has a fundamental impact on the growth of its economy.
3. It is preferable to use modern methods and modern software to solve any transport problem that is lost in the economy of productive, service and economic establishments.
4. Establishing a database for each productive facility, as it has an important impact on researchers’ future reliance on it in their research.

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