Volcano deformation monitoring using geodetic method: optimal network design

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Abstract. Until nowadays there is no analytical equation that exactly state number of observing stations that should be installed and how the stations should be spatially distributed in the GNSS geodetic survey for volcano deformation studies. Due to the uniqueness of the deformation pattern of any volcano, which need sensitive monitoring network to detect small changes, the network design depends strongly on the uniqueness underlying physical phenomena. In this research, the geodetic network is designed based on physical equations to find the source including a simple spherical source of Mogi model and 3D-dislocation equation of Okada model for dykes and sills. In principle, those physical equations are used to represent the volcano deformation phenomena from surface horizontal and vertical displacement measurement. Furthermore, those physical equations are used for a reference design and realization of monitoring network. Our calculation applied at Agung and Batur volcanoes show that minimum number of observing stations depends on the total number of parameters, the combination of source shape assumption and field situation and need different type of densification in the next future measurement.

1. Introduction
Since the last decades of satellite geodesy technology implementation, various methods of monitoring position changes or deformations for various objects, in this case the earth surface, have been realized accurately and precisely. Volcano, is the one of the local spatial scale of deformation phenomena on the earth's surface that has a significant impact on the environment if erupted. Various geodetic methods using geodesy satellite have already been applied to study the phenomenon of inflation and deflation of volcanic bodies as an indicator of their volcanic activities (1, 2, 3, and 4). Broadly speaking, the method consists of two major groups, namely point positioning methods (for example: GNSS GPS) and spatial methods (e.g.: InSAR). Each method has its own advantages and disadvantages, and at the moment both are often used together to complement the obtained information. For the point positioning method, the degree of accuracy of the coordinate displacement is very accurate compared to the spatial method, however, in the spatial deformation point of view, the spatial method will provide more comprehensive results even though the accuracy rate is lower than the first method.

For the point positioning method discussed in this study, there is currently no analytical equation that can be used to determine the number of stations and how they are spatially distributed in geodetic surveys for monitoring volcanic deformation. The design of a geodetic deformation monitoring network
is expected to be sensitive in detecting very small changes/deformations that depend on physical phenomena of magma source. The type of observations in this study are episodic and/or periodic/continuous GNSS GPS monitoring with high precision and accuracy results from each point positioning. In the realization of observation, an optimum geodetic deformation monitoring network is needed, that the distribution of GPS GNSS observation stations that is expected to be sensitive to detect position changes or displacement. The change in position is expected to be a quantification of the deformation on the surface of the volcano body as a result of the force response acting on the volcano originating from the source of magma. Thus, to get an accurate picture of the deformation of the volcano body in a spatial-temporal manner, an optimum strategy is needed to put the stations location in observation network and also the minimum number of stations that needed, given the field constraints that usually have difficult terrain to access.

In this study, the deformation geodetic network design was compiled using physical equations used to find the source, namely the Mogi model that uses a simple spherical model and 3D dislocation equation from the Okada model for dyke and sill. Basically, these two equations are used to express the phenomenon of volcanic deformation using measurements of horizontal and vertical displacement on the surface. Furthermore, these two equations are used to design ideal references and for comparison with the realization of the existing monitoring network at Agung and Batur Volcanoes at Bali - Indonesia.

2. Method and data
Simple volcanic deformation sources method used in this study is the point pressure Mogi model for the spherical magma chambers and Elastic dislocation Okada model for Dykes and Sills.

2.1.1 Mogi model
The Mogi model (5) uses the assumption that the earth’s crust is a semi-elastic medium and the deformation that occurs is caused by a source of pressure in the form of a spherical magma shape located at a certain depth. If there is a hydrostatic change in the ball, symmetrical deformation will occur which causes the horizontal surface displacement $U_r$ and vertical surface displacement $U_z$.

$$U_r = \frac{(1-v)pa^3}{\mu} \frac{r}{(r^2+d^2)^{3/2}}$$  \hspace{1cm} (1) \\
$$U_z = \frac{(1-v)pa^3}{\mu} \frac{d}{(r^2+d^2)^{3/2}}$$  \hspace{1cm} (2) \\
$$\Delta V = \frac{\pi pa^3}{\mu}$$  \hspace{1cm} (3) \\
$$C = \frac{(1-v)\Delta V}{\pi}$$  \hspace{1cm} (4)

where $v$: Poisson’s ratio (0.25), $p$: pressurization, $\mu$: shear modulus, $a$: source radius, $r$: radial distance from source, $d$: source depth, $\Delta V$: source volume change, $C$: source strength

2.1.2. Okada model
The 3D dislocation that occurs at point i is stated by [6] as follows, for the Green function $u_{ij}(x_1, x_2, x_3)$ from a rectangular field $\Sigma$ half-space isotropic, the deformation due to dislocation is $\Delta u_j(\xi_1, \xi_2, \xi_3)$ centered on a point $(\xi_1, \xi_2, \xi_3)$ with $j$ direction.
\[ u_i = \int_{\Sigma} \Delta u_j \left[ \lambda \delta_{jk} \frac{\partial u_i^n}{\partial \xi^k} + \mu \left( \frac{\partial u_i^j}{\partial \xi^k} + \frac{\partial u_i^k}{\partial \xi^j} \right) \right] v_k d\Sigma \]  

where \( \lambda \) and \( \mu \) are Lame constants, \( \delta_{jk} \) is delta Kronecker and \( v_k \) is normal cosines w.r.t \( d\Sigma \).

By applying geodetic adjustment theory, the rule is that the minimum number of observation stations that must be realized will depend on the parameters total number that must be solved in a physical equation. Furthermore, more measures are applied to prevent singularity problems or nearly singular in technical calculations. Whereas the optimum number of observation stations and laying their position depend on a priori information, geological conditions and monitoring criteria for deformation sources of objects by applying general equations for non-linear optimization objective functions

\[ \Phi = \sum C_i P_i \rightarrow \text{minimum} \]  

where \( P \) is the observation weight and \( C \) is the parameter variance-covariance.

The equations above are then used to design the ideal monitoring network in the area of Agung Volcano and Batur Volcano located on the island of Bali, both are active volcanoes which have erupted several times. Both of these volcanoes were chosen referring to [7] which has modeled the magmatic plumbing system in Agung and Batur volcano to facilitate the application of the Mogi model and the Okada model in deformation monitoring network design.

3. Result and discussion

The application of the simple volcanic information sources model, namely Mogi and Okada at Agung volcano and Batur volcano is represented in figure 1. The source position and suitability of the model with the source form are determined using the magmatic plumbing system (7), i.e. at a depth of 5 km from the ground for the Mogi model of Agung volcano and a depth of 3.5 km from the ground for the Okada opening model at Mount volcano. The ideal deformation model based on the equation (1), (2) and (5), is realized using a tightly arranged grid model every 250 m to anticipate various obstacles, including real terrain conditions that may be difficult or inaccessible to carry out monitoring stations or conditions geological rocks on the surface that are not sensitive to deformation due to pressure from source. The assumptions used in the deformation model in this study are homogeneous and sensitive to source pressure. In figure 1 it can be seen that the application of the Mogi model for Agung volcano and the application of the Okada opening model for Batur volcano showed slightly different results for horizontal deformation and vertical deformation at the ground/surface level. The Mogi model shows horizontal deformation that is larger than the Okada model, and smaller vertical deformation deformations in almost the same radius.

Based on the equation (6), the minimum number of station criteria depends on the number of deformation parameters sought and the station location will depend on the physical model used. For the Mogi model, where the parameters that must be solved consist of 4 source parameters, a minimum of 3 or 4 observation stations are needed. As for the Okada model, at least 5 observation stations are needed to solve 8 source parameters. For Mogi model, the station distribution pattern is spread evenly with almost the same radius from the source of deformation to the source model approaching the spherical source. As for the Okada model, the distribution pattern is slightly different from prioritizing a more tight distribution above the opening zone. This is done to apply geometric and physical optimization aimed at obtaining accuracy in determining deformation parameters. However, at the stage of implementation in the field, some theoretical designs will be modified because they must be adapted to the real morphological and geological conditions, this is mainly related to the urgency of shifting the location of the observation and densification stations. Although adjustments can be made to the observation station location, the minimum number of stations must be realized so that the deformation parameters sought can be estimated. The deformation monitoring network design for volcanoes depends on the initial prediction of the source position. Modifications from the initial design will depend on at least two minimal epoch observations, which can be studied about the phenomenon of strain tendencies.
that occur. Thus densification and frequency of observations when using episodic methods can be carried out more optimally.

![Figure 1](image.png)

**Figure 1.** Magmatic plumbing system model [3], GPS station position at Agung and Batur Volcanoes, horizontal and vertical deformation from Mogi source modelling at Agung Volcano and Okada source modelling at Batur Volcano.

The distribution of the station for continuous GPS GNSS observation in the Agung volcano along with horizontal-vertical deformation using the Mogi model is shown in table 1 and its sensitivity representation is shown in Figure 2.

| station | latitude  | longitude  | distance (km) | Ur (mm)  | Uz (mm)  |
|---------|-----------|------------|---------------|----------|----------|
| CEGI    | -8.30232  | 115.47160  | 6.11610       | 1.24059  | 1.01420  |
| DKUH    | -8.29601  | 115.53434  | 6.00072       | 1.25926  | 1.04926  |

**Table 1.** Agung volcano periodic GPS station, distance from the source and horizontal $U_r$ – vertical $U_z$ deformation using Mogi Model
Figure 2. Horizontal and vertical surface deformation from the source of Mogi modelling for Agung Volcano.

The realization of GPS observation stations distribution at Agung volcano, lies in the closest radius of 4.132 km from the projection of vertical sources on the earth's surface, namely YHKR and farthest on the RNDG with a distance of 12.249 km. The vertical source projection at ground level is assumed to be in the coordinates (-8.344 S; 115.508 E). Theoretically, the minimum number of GPS observation stations to solve the source parameters on Agung volcano using the Mogi modelling has been fulfilled, which has 9 observation stations (more than the minimum requirements of 3 or 4 observation stations). Based on terrain conditions, 5 stations have a distribution that is still within the scope of the volcano body, while the rest is estimated to be located some distance from the volcano body. This also fulfils the minimum criteria mentioned above, namely the minimum number of stations expected to be sensitive to deformation. The sensitivity of YHKR station located at a radius of 4.132 km looks very high, i.e. vertical deformation looks higher reaching 1.83 mm compared to the 1.51 mm horizontal deformation. For stations in the radius interval of 5.434 km up to 6.116 km, namely TTK1, PGBN, TTK2, PCNG, DKUH and CEGI, there is a similar sensitivity pattern, which is greater horizontal deformation than vertical deformation. This shows that the station located within the radius is sensitive to deformation that occurs. For the last 2 stations with an interval of 11.498 to 12.249 km, namely SUTR and RNDG, although it shows a similar pattern to the previous station, it has a low sensitivity to deformation or can indicate that the area is not deformed or stable because of its horizontal and vertical deformation values is below the maximum deformation detection capability of GNSS GPS (i.e. 1 mm). Thus, these two last points can be used as tie points of deformation monitoring network adjustment at Agung volcano with the assumption that they are in a stable region. For densification, if possible, the deformation of Agung volcano is expected to be more detectable if stations are placed within a radius of 1 km to 4 km from the estimated source. The densification distribution is at least 2 stations with the pattern expected to be spread evenly over the same radius from the source.

|     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|
| PCNG | -8.29900 | 115.47900 | 5.91280 | 1.27347 | 1.07687 |
| PGBN | -8.36372 | 115.46102 | 5.64520 | 1.31636 | 1.16591 |
| RNDG | -8.42472 | 115.43168 | 12.24991 | 0.52887 | 0.21587 |
| SUTR | -8.32300 | 115.40600 | 11.49843 | 0.58331 | 0.25365 |
| TTK1 | -8.38900 | 115.48800 | 5.43489 | 1.34940 | 1.24143 |
| TTK2 | -8.34400 | 115.56100 | 5.78600 | 1.29387 | 1.11811 |
| YHKR | -8.38157 | 115.50838 | 4.13270 | 1.51402 | 1.83176 |

Figure 2. Horizontal and vertical surface deformation from the source of Mogi modelling for Agung Volcano.
Furthermore, the distribution of the episodic GPS GNSS observation station in Batur volcano along with horizontal-vertical deformation using the Okada model is shown in Table 2 and the representation of sensitivity is shown in Figure 3.

Table 2. Batur episodic GPS station, distance from the source and horizontal $U_r$ – vertical $U_z$ deformation using Okada Model

| station | latitude | longitude | distance (km) | $U_r$ (mm) | $U_z$ (mm) |
|---------|----------|-----------|---------------|------------|------------|
| DB01    | -8.28065 | 115.36150 | 7.69266       | 1.27433    | 0.57980    |
| DB13    | -8.28725 | 115.37850 | 8.77407       | 1.04089    | 0.41521    |
| PJ01    | -8.26472 | 115.38642 | 6.84191       | 1.50736    | 0.77110    |
| DB17    | -8.27130 | 115.38026 | 7.18820       | 1.40657    | 0.68487    |
| DB06    | -8.25344 | 115.39736 | 6.59178       | 1.58563    | 0.84191    |
| DB25    | -8.23002 | 115.38849 | 4.23230       | 2.55493    | 2.11286    |
| TABU    | -8.22035 | 115.39420 | 4.42865       | 2.46231    | 1.94598    |
| DB05    | -8.23534 | 115.39983 | 5.60681       | 1.94177    | 1.21213    |
| KW01    | -8.24299 | 115.37817 | 4.34179       | 2.50325    | 2.01792    |
| KW02    | -8.24483 | 115.37423 | 4.27687       | 2.53389    | 2.07362    |
| MUSE    | -8.28359 | 115.36473 | 8.05464       | 1.18914    | 0.51672    |
| DB07    | -8.24748 | 115.34825 | 4.08020       | 2.62651    | 2.25302    |
| DB19    | -8.23789 | 115.35287 | 2.96589       | 3.07174    | 3.62492    |
| DB02    | -8.21813 | 115.35582 | 0.78765       | 1.70586    | 7.58020    |
| DB03    | -8.21101 | 115.37584 | 2.28782       | 3.12937    | 4.78742    |
| DB21    | -8.25986 | 115.35281 | 5.37882       | 2.03531    | 1.32437    |
| DB10    | -8.24843 | 115.35504 | 4.11567       | 2.60986    | 2.21945    |

Figure 3. Horizontal and vertical surface deformation from the source of Okada modelling for Batur Volcano.

The realization of episodic GPS observation stations distribution in Batur lies in the closest radius of 0.787 km of vertical source projections on the earth's surface, namely in DB02 and farthest in the DB13 with a distance of 8.774 km. The vertical source projection at ground level is assumed to be in the coordinates (-8.211 S; 115.355 E). Based on the radius of the source, the distribution of the observation station at Batur volcano is closer to the ideal condition than the distribution on Agung volcano. This is
due to terrain conditions and ease of location accessibility. Theoretically, the minimum number of GPS observation stations to solve the source parameters at Batur volcano using Okada modelling has been fulfilled, which has 17 observation stations (more than the minimum requirement of 5 observation stations). Based on terrain conditions, there are few obstacles to achieving radially dispersed conditions, namely the presence of Lake Batur on the south-east side of the source which does not allow installation of GPS stations. 16 stations have a distribution that is still within the scope of the volcano body, while 1 station, namely DB13, is estimated to be located some distance from the volcano body. This also fulfill the minimum criteria mentioned above, namely the minimum number of stations expected to be sensitive to deformation.

The sensitivity of the 3 closest stations namely DB02, DB03 and DB19 which are located at a radius of 0.787 km to 2.956 km looks very high, i.e. vertical deformation looks higher reaching at intervals of 3.63 - 7.58 mm compared to horizontal deformation at 1.71 - 3.07 mm. This shows a high sensitivity to deformation for the station which is located not far from the source. For stations in the interval of 4.080 km to 4.429 km, namely DB07, DB10, DB25, KW02, KW01 and TABU show a similar sensitivity pattern, namely horizontal deformation in the range of 2.46 - 2.46 mm which has a greater value than vertical deformation which ranges from 1.95 - 2.25 mm. This shows that in the region the influence of source pressure provides a different deformation pattern compared to the previous 3 station dg located near the source. For the next radius, which is between 5.378 km and 8.774 km, DB21, DB05, DB06, PJ01, DB01, MUSE and DB13 stations show a similar pattern to the 6 previous stations, i.e. horizontal component deformation is greater than the vertical component. The first two stations, namely DB21 and DB05, still showed significant deformation patterns, namely in the interval 1.94 - 2.03 mm for horizontal components and 1.21-1.32 mm for vertical components. Starting from a radius of 6.591 km, namely DB06 station to the farthest station, DB13, visible sensitivity to vertical component deformation is below the minimum range of GPS technology capability, which is 0.84 - 0.41 mm even though the horizontal component is still above the minimum GPS capability above 1.04 mm. This could indicate that the area is estimated to be a stable zone and a deformed zone. Thus, it is recommended that it will be better to use another tie point with radius larger than the DB13 station.

If there will be a future densification plan at Batur volcano network, it is recommended to be placed on the east side of the wall of the lake Batur caldera with a radius not too far away to determine the sensitivity of the possibility of deformation that maybe occurs. And if it is planned to create an integrated monitoring network with Agung volcano which is located side by side, then the observation station can be installed along a straight line connecting source of Agung and Batur volcanoes with distance intervals adjusted to terrain conditions and minimal sensitivity observations from at least two episodic epochs of monitoring.

4. Conclusion
The design of the geodetic deformation monitoring network for volcanoes depends on the initial prediction of the source position along with the minimum number of parameters to be solved using the source model. Source-related a priori information can be estimated from the distribution of volcanic earthquake patterns if there is no source location modeling information from previous research studies. The thing that most influences the realization of the model in the field is terrain condition and accessibility. The modification of the initial design will depend on at least the results of observing surface deformation from at least two epochs of monitoring, where information on the sensitivity of the position of the observation station due to surface deformation can be obtained as a result of source pressure.

From the results of the Mogi model application for geodetic deformation network design at Agung volcano, it shows that the current monitoring network is sufficiently sensitive to horizontal and vertical deformation patterns, with the suggestion of densification for the radius approaching to the source. Whereas, the application of the Okada model to Batur volcano shows that the net currently available is overall sensitive to surface deformation due to source pressure. Suggestion densification for the Batur
region is the installation of a tie point station in a stable area and for the eastern zone after the Batur Lake Caldera extends towards Agung volcano to find out the interaction pattern between both volcanoes.

5. References

[1] Burgmann R, Rosen P and Fielding E 2000 Synthetic Aperture Radar Interferometry to Measure Earth’s Surface Topography and Its Deformation. *Annual Reviews of Earth and Planetary Sciences* **28** 169-209

[2] Dzurisin D 2006 *Volcano deformation: new geodetic monitoring techniques* (Springer Science & Business Media)

[3] Abidin H Z, Andreas H, Gamal M, Suganda O K, Meilano I, Hendrasto M, Kusuma M A, Darmawan D, Purbawinata M A, Wirakusumah A D and Kimata F 2006 Ground deformation of Papandayan volcano before, during, and after the 2002 eruption as detected by GPS surveys, *GPS Solutions* **10** 75-84

[4] Janssen Vand Rizos C 2003 Processing mixed-mode GPS networks for deformation monitoring applications, *zfv – Zeitschrift für Geodäsie, Geoinformation und Landmanagement* **128** 87-96.

[5] Mogi K 1958 Relation between the Eruption of Various Volcanoes and the Deformation of the Ground Surface Around Them. *Bulletin Earthquake Research Institute*, Tokyo Univ **36** 99-134.

[6] Okada Y 1985 Surface deformation due to shear and tensile faults in a half-space. *Bull. Seism. Soc. Am* **75**, 1135–1154

[7] Syahbana D K, K Kasbani, G Suantika, O Prambada, A S Andreas, U B Saing, S L Kunrat, S Andreastuti, M Martanto, E Kriswati, Y Suparman , H Humaida, S Ogburn , P J Kelly , J Wellik, H M N Wright, J D Pesicek, R Wessels, C Kern, M Lisowski, A Diefenbach, M Poland, F Beauducel, J Pallister, R G Vaughan and J B Lowenstern 2019 The 2017–19 activity at Mount Agung in Bali (Indonesia): Intense unrest, monitoring, crisis response, evacuation, and eruption. *Nature Scientific Reports* **vol 9**, Article number: 8848