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Adaptive surrounding control for linear multi-agent systems with unknown disturbance

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Abstract. This paper deals with the surrounding control problem of multi-agent systems with linear non-identical dynamics for each agent. In these dynamics, a non-identical time-varying term is considered for each agent. It is assumed that there exist unknown parameter vectors in time-varying terms of the followers. First, an estimator is introduced to estimate the geometric center of the leaders and then, an adaptive law is proposed to compensate the unknown parameter vector of the followers. Using Lyapunov stability theory, the stability of the system with the proposed control law is studied. Finally, a simulation example is given to show the effectiveness of theoretical results.

1. Introduction
In the last decades, cooperative control of network systems or multi-agent systems have been attracted lot of attentions [1]. Multiplicity of agents in these systems has created many applications in different domains such as formation of robotic systems [2], spacecraft formation flying [3], target tracking in sensor networks [4] and etc. Among the applications of cooperative control, the enclosing control has attracted many attentions. In the enclosing problem, a group of agents (called followers) surround other agents or a single agent (called leaders) in a certain geometric configuration. For achieving this purpose, many control methods have been used. For example, the authors in [5] propose cyclic pursuit strategies to achieve cooperative arresting for a single stationary target. In this work, the authors use consensus-based techniques to achieve cooperative enclosing for a moving leader. The authors in [6] utilize sliding mode method with incomplete leader information to build a surrounding control protocol, wherein a single moving target has been considered. Adaptive control method is proposed in [7] and [8] to accomplish enclosing formation for a leader when there exists uncertainty in the system. Also, [9] and [10] implement simultaneously the estimator and control strategy to accomplish enclosing control for targets when the system involves uncertainty or imperfect information.

In the surrounding control problem, all of the followers must encompass dynamic or static leaders. For this goal, the followers need geometric center of the leaders. When the geometric center is not available, an estimator is needed to be built. In the current paper, the surrounding control problem for a non-identical linear multi-agent system with a nonlinear term under undirected graph topology is investigated. Also, it is supposed that the nonlinear term of the followers consists of an unknown parameter vector. For this purpose, a distributed surrounding control protocol with adaptive law is proposed.
The rest of this paper is organized as follows: In section 2 some preliminaries are given. Section 3 presents problem formulation. In Section 4, the main result and the center estimator of the leaders are established. Numerical simulation is provided in Section 5. Finally, the conclusions are drawn in section 6.

2. Preliminaries
In this section, some basic graph concepts are introduced. This paper only considers undirected graphs and does not assume any self-edges. For the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, $\mathcal{V} = \{v_i, i = 1, \ldots, N\}$ means a set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ means a set of edges. An edge denoted as $(i, j) \in \mathcal{E}$ means that agent $i$ has access to the information of agent $j$ and also, agent $i$ is a neighbour of agent $j$. In this paper, the set of neighbours of agent $i$ is represented by $\mathcal{N}_i$. Graph $\mathcal{G}$ is said to be undirected if $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$. An undirected graph is connected if there exists a path, i.e., a sequence of distinct edges such that consecutive edges are joint between any two vertices. The Adjacency matrix of the graph $\mathcal{G}$ is $\mathcal{A} = \{a_{ij}\} \in \mathbb{R}^{N \times N}$ with $a_{ij} = 1$ if the $i$th agent can receive information from $j$th agent, otherwise $a_{ij} = 0$. Throughout this paper, it is assumed that $a_{ii} = 0$ and the topology is fixed. The graph Laplacian matrix is $\mathcal{L} = [l_{ij}] = \mathcal{D} - \mathcal{A}$ where $\mathcal{D} = \text{diag}(d_1, \ldots, d_N)$ is the degree matrix, where $d_i = \sum_{j=1}^{N} a_{ij}$ for $i = 1, \ldots, N$. For an undirected graph, the Adjacency and Laplacian matrices are symmetric.

In this paper, a multi-agent system with $N$ followers and $M$ leaders is considered. It is supposed that each leader communicates with at least one follower. In this paper, the nodes set of the followers and the leaders are shown by $\mathcal{V}_F$ and $\mathcal{V}_L$, respectively. The interconnection relationship between each follower and corresponding leader is indicated with the $\text{explore}$ matrix and shown by $B = [b_{lk}] \in \mathbb{R}^{N \times M}$. The elements of this matrix is defined as $b_{lk} = 1$ if $l$th follower communicates with $k$th leader and $b_{lk} = 0$, otherwise. Kronecker product is indicated by $\otimes$.

3. Problem formulation
Consider a multi-agent system with $N$ followers and $M$ leaders. Dynamics of the followers and leaders are heterogeneous and described by

$$\dot{x}_i(t) = -A_i x_i(t) + B_i d_i(t) + u_i(t), \quad i \in \mathcal{V}_F$$

where $x_i(t)$ and $u_i(t) \in \mathbb{R}^2$ are the state and control input of the $i$th follower, respectively. $d_i(t)$ is an unknown time-varying disturbance of the $i$th follower and is formulated as $d_i(t) = \varphi_i(t) \theta_i$, $i \in \mathcal{V}_F$, in which $\varphi_i(t) \in \mathbb{R}^{2 \times 2}$ is a known matrix function and $\theta_i \in \mathbb{R}^{2 \times 1}$ is an unknown constant parameter vector. $A_i$ and $B_i$ are constant matrices with compatible dimensions. It is assumed that the leaders move in $\mathbb{R}^2$ with dynamics

$$\dot{x}_k(t) = -A_k x_k(t) + B_k u_k(t), \quad k \in \mathcal{V}_L$$

where $x_k(t) \in \mathbb{R}^2$ is the state of the $k$th leader and $u_k(t) \in \mathbb{R}^2$ is the control input of the leader which is unknown for the followers. $A_k$ and $B_k$ are constant matrices with compatible dimensions. The dynamics equations (1)-(3) have been adopted from [11]. Throughout the paper, we make the following assumption:

Assumption 1: The movement of each leader around the center of the leaders is bounded. That is, there exists a real positive number $\mathcal{M} < \infty$ such that $\mathcal{M} = \sup\{|x_i(t) - \overline{\mathcal{F}}(t)|, \; i = 1, 2, \ldots, N\}$. Here, $\overline{\mathcal{F}}(t)$ denotes the geometric center of the leaders and $\overline{\mathcal{F}}(t) = \frac{1}{\mathcal{M}} \sum_{k=1}^{\mathcal{M}} x_k(t)$. The goal is to find a distributed adaptive control input for the followers which ensures creation of a certain geometrical configuration around the leaders, where the number of followers is allowed to be different from the number of leaders. To achieve this purpose, we should have

$$\lim_{t \to \infty} (x_i(t) - \overline{\mathcal{F}}(t)) = \rho \left[\cos \left(\frac{1.2\pi}{\mathcal{N}}\right) \quad \sin \left(\frac{1.2\pi}{\mathcal{N}}\right) \right]^T$$

where $\rho > \mathcal{M}$.
4. Main result

Suppose that the unknown nonlinear time-varying disturbance of the followers, i.e. \( \mathbf{d}_i(t) \) is parameterized as \( \mathbf{d}_i(t) = \varphi_i(t)\mathbf{\theta}_i \), \( i \in V_F \), where \( \varphi_i(t) \in \mathbb{R}^{2 \times 2} \) is a uniformly bounded function and \( \mathbf{\theta}_i \in \mathbb{R}^{2 \times 1} \) is an unknown constant parameter vector.

Because the unknown parameter vector \( \mathbf{\theta}_i \) is not available to any follower, the \( i \) th follower estimates the unknown vector \( \mathbf{\theta}_i \) by \( \hat{\mathbf{\theta}}_i \). Then, the estimation of \( \mathbf{d}_i(t) \) by the \( i \) th follower is expressed as \( \hat{\mathbf{d}}_i(t) = \varphi_i(t)\hat{\mathbf{\theta}}_i \).

The local surrounding error for \( i \)th follower is defined as

\[
e_i(t) = x_i(t) - \bar{r}(t) - \rho \left( \cos \left( \frac{i \pi}{N} \right) \sin \left( \frac{i \pi}{N} \right) \right)^T
\]

and global error vector is expressed as

\[
e(t) = x(t) - 1_N \otimes \bar{r}(t) - \rho \left( 1_N \otimes \left[ \cos \left( \frac{i \pi}{N} \right) \sin \left( \frac{i \pi}{N} \right) \right]^T \right)
\]

Here, \( 1_N \) represents a column vector and it’s all elements are 1. It is obvious that the surrounding control is achieved when \( \lim_{t \to \infty} e(t) \to 0 \).

As it can be seen, to achieve certain geometrical configuration, the geometric center of leaders should be available for each follower. In general, each follower has access to only a subset of the leaders. Therefore, a decentralized estimator is constructed for each follower to estimate the geometric center of the leaders. Then, the estimated center is used to design the controller.

4.1. Estimator for the center of Leaders

To perform a certain geometrical configuration, we introduce the following estimator for the center of leaders.

\[
\hat{x}_i(t) = \phi_i(t) + \hat{l}_i(t), \quad i \in V_F
\]

Here, \( \hat{x}_i(t) \) is the estimation of the center of leaders for the \( i \)th follower and \( \phi_i(t) \) is a dynamic variable with the following dynamics.

\[
\dot{\phi}_i(t) = \alpha \sum_{k=1}^{N} d_{ik} \left[ \hat{x}_k(t) - \hat{x}_i(t) \right] \left\| \hat{x}_k(t) - \hat{x}_i(t) \right\|
\]

where \( \phi_i(0) = 0, \alpha > 0 \) is a constant, and \( \hat{l}_i(t) \) is defined as

\[
\begin{bmatrix}
\hat{l}_1(t) \\
\hat{l}_2(t) \\
\vdots \\
\hat{l}_N(t)
\end{bmatrix} = \frac{1}{M} \otimes \begin{bmatrix}
\sum_{k=1}^{N} b_{11} \delta_{k1} b_{k1} \\
\sum_{k=1}^{N} b_{12} \delta_{k2} b_{k2} \\
\vdots \\
\sum_{k=1}^{N} b_{MN} \delta_{MN} b_{kM}
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t) \\
\vdots \\
x_M(t)
\end{bmatrix}
\]

in which \( x_j(t) \) denotes the position of the leaders.

In [12], it has been shown that the estimator defined by (6)-(8) converges to the leaders’ center in a finite time \( T \). Thus

\[
\hat{x}_i(t) = \frac{1}{M} \sum_{k=1}^{M} x_k(t), \quad t > T = \frac{1}{\delta} \sqrt{V(0)}
\]

where \( \delta = \sqrt{\frac{2N(c_1 - (N-1) c_2)}{2(N-1)}} \) with \( c_1 > c_2 \), wherein \( c_1 \) and \( c_2 \) are real positive constants. For more details, one can refer to [13].

From (9), it can be seen that \( \hat{x}_i(t) = \bar{r}(t) \) when \( t > T \). Then, the following modification is made for the local surrounding error of \( i \)th follower.

\[
e_i(t) = x_i(t) - \hat{x}_i(t) - \rho \left( \cos \left( \frac{i \pi}{N} \right) \sin \left( \frac{i \pi}{N} \right) \right)^T
\]

Also, the global error vector is changed to:

\[
x(t) = x(t) - 1_N \otimes \bar{r}(t) - \rho \left( 1_N \otimes \left[ \cos \left( \frac{i \pi}{N} \right) \sin \left( \frac{i \pi}{N} \right) \right]^T \right)
\]
where $\hat{x}(t) = [\hat{x}_1(t) \ \hat{x}_2(t) \ \ldots \ \hat{x}_N(t)]^T$.

By defining $e(t)$ as (11), if $\lim_{t \to \infty} e(t) \to 0$ the surrounding control problem is solved asymptotically.

### 4.2. Surrounding Control Protocol

In this subsection, we propose a surrounding control strategy which achieves control objective (3) for any initial position of the followers and leaders. With considering the decentralized center estimator (6)-(8), the adaptive control protocol and decentralized adaptive laws for $i$th follower are proposed as follows.

\begin{equation}
\begin{align*}
\dot{u}_i(t) &= \rho A_i(1_N \otimes \cos \left(\frac{1.2 \pi}{N}\right) - \sin \left(\frac{1.2 \pi}{N}\right)) - cK_i e_i(t) + \hat{x}_i(t) - B_i \varphi_i(t) \hat{\theta}_i + A_i \hat{x}_i(t) \\
\hat{\theta}_i(t) &= e_i(t)P_i^{-1}B_i \varphi_i(t)
\end{align*}
\end{equation}

In (12), $c > 0$ is a constant number, $\hat{x}_i(t)$ denotes the velocity of estimation of the center of leaders for the $i$th follower, $K_i = \frac{1}{2}P_i^{-1}$, and $P_i$ is a positive definite solution to the following linear matrix inequality (LMI).

\begin{equation}
A_iP_i + P_iA_i^T - cI > 0
\end{equation}

Equations (12) and (13) can be written in vector form as

\begin{equation}
\begin{align*}
\mathbf{u}(t) &= -c\text{diag}(K_1 \ K_2 \ \ldots \ \ K_N)\mathbf{e}(t) - \text{diag}(A_1 \ A_2 \ \ldots \ \ A_N)\hat{x}(t) - \text{diag}(B_1 \ B_2 \ \ldots \ \ B_N)\varphi(t)\hat{\theta}(t) + \rho\text{diag}(A_1 \ A_2 \ \ldots \ \ A_N)(1_N \otimes \cos \left(\frac{1.2 \pi}{N}\right) - \sin \left(\frac{1.2 \pi}{N}\right))^T
\end{align*}
\end{equation}

and

\begin{equation}
\hat{\theta}(t) = e^T(t)\text{diag}(P_1^{-1} \ P_2^{-1} \ \ldots \ P_N^{-1})\text{diag}(B_1 \ B_2 \ \ldots \ B_N)\varphi(t)
\end{equation}

where $\mathbf{e}(t) = [e_1(t) \ e_2(t) \ \ldots \ e_N(t)]^T$, $\mathbf{u}(t) = [u_1(t) \ u_2(t) \ \ldots \ u_N(t)]^T$, $\hat{\theta}(t) = [\hat{\theta}_1 \ \hat{\theta}_2 \ \ldots \ \hat{\theta}_N]$ and $\varphi(t) = [\varphi_1(t) \ \varphi_2(t) \ \ldots \ \varphi_N(t)]$.

**Theorem 1.** Suppose that assumption 1 holds and $d_i(t)$ and $u_k(t)$ are uniformly bounded for all agents. Under control law (12) and the parameter adaptive law (13) with $K_i = \frac{1}{2}P_i^{-1}$, where $P_i > 0$ is a solution of the LMI (14), the followers described by (1) create a certain geometrical configuration around the leaders.

**Proof.** Before presenting the proof, we define $\tilde{\theta}(t) = \hat{\theta}(t) - \Theta$. The derivative of the global error vector (11) is $\dot{e}(t) = \dot{x}(t) - \tilde{x}(t)$.

By substituting (1) into $\dot{e}(t)$, the derivative of the global error vector is obtained as

\begin{equation}
\dot{e}(t) = -\text{diag}(A_1 \ A_2 \ \ldots \ A_N)\mathbf{x}(t) + \text{diag}(B_1 \ B_2 \ \ldots \ B_N)d(t) + \mathbf{u}(t) - \dot{\mathbf{x}}(t)
\end{equation}

where $d(t) = [d_1(t) \ d_2(t) \ \ldots \ d_N(t)]^T$ and $\mathbf{x}(t) = [x_1(t) \ x_2(t) \ \ldots \ x_N(t)]^T$. Now, consider a Lyapunov function candidate as follows.

\begin{equation}
V(t) = \frac{1}{2}e^T(t)\text{diag}(P_1^{-1} \ P_2^{-1} \ \ldots \ P_N^{-1})e(t) + \frac{1}{2}\tilde{\theta}^T(t)\tilde{\theta}(t)
\end{equation}

The derivative of $V(t)$ gives

\begin{equation}
\dot{V}(t) = e^T(t)\text{diag}(P_1^{-1} \ P_2^{-1} \ \ldots \ P_N^{-1})e(t) + \tilde{\theta}^T(t)\tilde{\theta}(t)
\end{equation}

By substituting equation (17), we have

\begin{equation}
\dot{V}(t) = e^T(t)\text{diag}(P_1^{-1} \ P_2^{-1} \ \ldots \ P_N^{-1})[\text{diag}(A_1 \ A_2 \ \ldots \ A_N)\mathbf{x}(t) + \mathbf{u}(t) - \dot{\mathbf{x}}(t)]
\end{equation}

From (15), one has
\[
\dot{V}(t) = e^T(t) \text{diag}(P_1^{-1} \quad P_2^{-1} \quad \ldots \quad P_N^{-1}) \{ -\text{diag}(A_1 \quad A_2 \quad \ldots \quad A_N) x(t) - \dot{x}(t) \\
+ \text{diag}(A_1 \quad A_2 \quad \ldots \quad A_N) \ddot{x}(t) - c\text{diag}(K_1 \quad K_2 \quad \ldots \quad K_N) e(t) + \dot{x}(t) \\
+ \rho \text{diag}(A_1 \quad A_2 \quad \ldots \quad A_N) (1_N \hat{\Theta} \begin{bmatrix} \cos \left( \frac{i.2\pi}{N} \right) \\
\sin \left( \frac{i.2\pi}{N} \right) \end{bmatrix}^T) \\
+ \text{diag}(B_1 \quad B_2 \quad \ldots \quad B_N) d(t)) + \hat{\Theta}^T(t) \hat{\Theta}(t) - \text{diag}(B_1 \quad B_2 \quad \ldots \quad B_N) \Phi(t) \hat{\Theta}(t) \}
\]

Using (11), it follows that
\[
\dot{V}(t) = e^T(t) \text{diag}(P_1^{-1} \quad P_2^{-1} \quad \ldots \quad P_N^{-1}) \{ -\text{diag}(A_1 \quad A_2 \quad \ldots \quad A_N) e(t) \\
- c\text{diag}(K_1 \quad K_2 \quad \ldots \quad K_N) e(t) - \text{diag}(B_1 \quad B_2 \quad \ldots \quad B_N) \Phi(t) \hat{\Theta}(t) \} + \hat{\Theta}^T(t) \hat{\Theta}(t)
\]

By using adaptive laws (13), one can write
\[
\dot{V}(t) = -e^T(t) \text{diag}(P_1^{-1} \quad P_2^{-1} \quad \ldots \quad P_N^{-1}) \text{diag}(A_1 \quad A_2 \quad \ldots \quad A_N) - c\text{diag}(K_1 \quad K_2 \quad \ldots \quad K_N) e(t)
\]

If we define \( P_E = \text{diag}(P_1^{-1} \quad P_2^{-1} \quad \ldots \quad P_N^{-1}) \), \( A_E = \text{diag}(A_1 \quad A_2 \quad \ldots \quad A_N) \) and \( K_E = \text{diag}(K_1 \quad K_2 \quad \ldots \quad K_N) \), we can rewrite (23) as
\[
\dot{V}(t) = -\frac{1}{2} e^T(t) (P_E A_E + A_E^T P_E - 2 c P_E K_E) e(t)
\]

Defining \( K_E = \frac{1}{2} P_E \), multiplying both side with \( P_E^{-1} \) and using LMI (14), \( \dot{V}(t) \) gets negative. \( e(t) \) and \( \hat{\Theta}(t) \) are uniformly bounded. The assumption on \( \phi(t) \) results that \( \dot{e}(t) \) in equation (22) is uniformly bounded. Because \( e(t) \in L_2 \) and \( \dot{e}(t) \in L_\infty \), using Barbalat’s lemma, we have \( \lim_{t \to \infty} e(t) \to 0 \).

5. Simulation result

In this section, we present an example to evaluate our theoretical results. Consider a group of agents with four followers, three leaders and \( n=2 \). The network connections are given in Fig. 1. In this figure, to show the communication between the followers and leaders, we use solid line and dashed line, respectively. The constant matrices of the followers are selected as

\[
A_1 = \begin{bmatrix} 1.5 & -1 \\
-1 & 2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 \\
0 & 2 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & -2 \\
0 & 3 \end{bmatrix}, \quad A_4 = 2I_2
\]

\[
B_1 = \begin{bmatrix} 1.5 & -0.1 \\
-0.2 & 2 \end{bmatrix}, \quad B_2 = 2I_2, \quad B_3 = \begin{bmatrix} -2 & -0.1 \\
-0.1 & -3 \end{bmatrix}, \quad B_4 = \begin{bmatrix} -2 & 0 \\
-2 & -3 \end{bmatrix}
\]

The unknown nonlinear disturbance dynamics of the followers are chosen as:
\[
\phi_1(t) = \text{diag}(0.5 \cos(4t), e^{-2t} - \sin(2t)) \theta_1, \quad \phi_2(t) = \text{diag}(0.9e^{-t}, -2) \theta_2, \quad \phi_3(t) = \text{diag}(1, -t) \theta_3, \quad \phi_4(t) = \text{diag}(0.5t, 0.9 \cos(3t)) \theta_4.
\]

The unknown parameters \( \theta_1 \) are chosen as:
\[
\theta_1 = [1 \quad 0.5]^T, \quad \theta_2 = [-0.9 \quad 0.6]^T, \quad \theta_3 = [0.07 \quad 0.9]^T, \quad \theta_4 = [0.98 \quad -1.5]^T.
\]

The time-varying control input of the leaders are selected as follows:
\[
A_1 = 2.5I_2, \quad A_2 = \begin{bmatrix} 2 & 1 \\
2 & 3 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 4 & 1 \\
1 & 3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -2 & -1 \\
-2 & 3 \end{bmatrix}
\]

Figure 1. Communication topology
The leader’s constant matrices are selected as $B_2 = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$, $B_3 = \begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix}$.

From (19) and (20) with $c = 2$, $\rho = 5\mathcal{M}$. Also, $\alpha = 9$ in the leaders center estimator (10)-(12).

The simulation results are presented in Fig. 2 and Fig. 3. In Fig. 2 the trajectories of the followers and leaders are depicted. In this figure, the surrounding of the followers around the leaders are shown with dashed line for better illustrations. It is clear that four followers enclose three leaders during the control process. Also, by enlarging the moving radius of the leaders, the moving radius of the followers increase to surround the leaders. Fig. 3 shows the surrounding error of the followers. It is clear that all of the trajectory errors tend to zeros as time increases.

**Remark.** If the goal is faster convergence of the global surrounding error vector reaching zero at shorter time, then it is necessary to change the control input to achieve finite-time stability or fixed-time stability. The authors suggest this as a future work.

**Figure 2.** Trajectory of all agents

**Figure 3.** The global surrounding error vector

### 6. Conclusion

This paper addressed the adaptive surrounding control problem for non-identical linear multi-agent systems with nonlinear non-identical time-varying terms. It was demonstrated that the proposed method made the followers efficiently enclose the moving leaders in the presence of nonlinear unknown terms in the followers’ dynamics. Numerical simulation revealed the efficacy of the results.

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