A new strong bound on sub-GeV dark matter from Migdal effect

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Migdal effect provides a new way to search for sub-GeV dark matter. However, due to the limitation of the energy of ionization and/or de-excitation of the intrinsic atomic electron state, the current limits on light DM exist in the mass range of $O(10)\text{ MeV} < m_\chi < O(1)\text{ GeV}$. On the other hand, since the ionization form factor in Migdal effects can be enhanced by large momentum transfer, the resulting event rate of S2 signal may be increased for fast-moving DM, and then improves the bounds on light DM. As a proof-of-concept, we consider a fast-moving sub-GeV DM, namely atmospheric DM (ADM), which is produced by the interaction between high energy cosmic rays (CRs) and the atmosphere. With Xenon1T data, we find that there will be nearly three orders of magnitude improvement on the DM-nucleus scattering cross section by including Migdal effect.

INTRODUCTION

The existence of dark matter (DM) has been supported by cosmological and astrophysical observations, however its properties, including its mass and interactions, are still elusive. As an attractive dark matter candidate, the Weakly Interacting Massive Particle (WIMP) [1] has been extensively studied in direct detection, indirect detection and collider experiments. With unprecedented sensitivity, the current direct detections that measure the nuclear recoils have produced very stringent limits on the WIMP dark matter. This motivates a broad investigation of light dark matter particles [2, 3].

However, the sub-GeV DM particles usually produce small energy deposited in an elastic nuclear recoil, $E_R \sim m_\chi^2 v_\chi^2/m_N \sim 0.1\text{ eV}$, which is much below the threshold of traditional noble liquid detectors. Therefore, new technologies and detectors have been developed to search for sub-GeV scale DM particles, see e.g. [4–12].

Migdal effect [13] that can take place in a very low energy nuclear recoil has been used to extend the coverage of the low mass DM in noble liquid targets and semiconductors [8, 9, 14–20]. Due to non-instantaneous movement of electron cloud during a nuclear recoil event, the recoil atom is either excited or ionized, and then immediately de-excites through the X-ray transition, the Auger transition, or the Coster-Kronig transition, which deposits additional energy inside the detector [21]. It should be mentioned that the rate of Migdal scattering depends on the momentum transfer to individual electron $q_e \sim q_N m_e/m_N$, which is usually suppressed in the case of light DM. For example, when $m_\chi \sim \text{MeV}$, $|q_e|$ is typically of order of $10^{-6}\text{ keV}$. While if the momentum transfer is sufficiently large, then the Migdal rate may become sizable.

On the other hand, there are some astrophysical sources that can naturally boost the non-relativistic light dark matter by imparting the energy from the Standard Model particles to the dark matter. For examples, a small component of light dark matter in the halo can be elastically scattered by the high energy primary cosmic rays, and then obtains the sufficient kinetic energy to reach the sensitivity of traditional detectors [22–30]. Besides, light dark matter can also come from light meson decays produced in the inelastic collision of cosmic rays with the atmosphere[31, 32]. By inheriting the parent velocity, such a light dark matter can move fast as well, and generate the detectable scintillation or ionization signal in direct detection. Also, light dark matter interacting with fast moving nuclei or electrons inside the Sun can be accelerated above threshold [33].

Therefore, such a inevitable component of the dark matter flux may enhance the Migdal scattering. In this work, we will for the first time investigate the Migdal rate of fast-moving light DM. As an example, we consider a fast-moving DM produced in collision of CRs and atmosphere and demonstrate that the DM-nucleus scat-
tering cross section can be greatly improved by including Migdal effect.

**CALCULATION FRAMEWORK**

In order to derive the sensitivity of the Migdal effect of fast-moving DM, we have to calculate the differential event rate of ionization electron with units per tonne per year per keV,

\[
\frac{dR_{\text{ion}}}{dE_e} = N_T \Phi_{\text{halo}} \sum_{nl} \frac{d\langle \sigma_{\text{ion}} nl v \rangle}{dE_e}.
\]

(1)

where \( N_T = 4.2 \times 10^{27} \) is number density of Xenon per tonne and \( \Phi_{\text{halo}} \) is the flux of halo DM. The velocity-averaged ionization cross section \( \langle \sigma_{\text{ion}} nl v \rangle \) can be written as [8]

\[
\frac{d\langle \sigma_{\text{ion}} nl v \rangle}{dE_e} = \int \int \int dE_R dE_N \frac{d\langle \sigma_{\text{ion}} nl v \rangle}{dE_R dE_N} \times \frac{1}{2\pi} \frac{dP_{nl \rightarrow E_e}(q)}{dE_e}
\]

\[
= \frac{\sigma_n}{8\mu_n^2 E_e} A^2 \int_{q_{\text{min}}} dq q |F_N(q)|^2 |F_{\text{DM}}(q)|^2 
\]

\[
\times |f_{nl}^{\text{ion}}(p_e, q_e)|^2 \eta(E_{\text{min}}(q, E_{EM})).
\]

(2)

with the momentum independent DM-nucleon scattering cross section \( \sigma_n \),

\[
\sigma_n \equiv \mu_n^2 \frac{\langle M_n(q = q_0) \rangle^2}{(16\pi m_e^2 m_N^2)}.
\]

(3)

Here \( A \) is the mass number of atom and \( \mu_n = m_e m_n / (m_e + m_n) \) is the DM and nucleon reduced mass. The minimal value of momentum transfer \( q \) in Eq. 2 should be able to ionize the electrons in the first shell, while the maximal value of \( |q| \) is determined by \( T_{\chi}^{\text{max}} \) in relativistic limit.

For xenon, we use the dipole form factor for the nucleus form factor \( F_N(q) \), which is given by [34],

\[
|F_N|^2 = \left( \frac{3j_1(q R_1)}{q R_1} \right)^2 e^{-q^2 s^2}
\]

(4)

with

\[
j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}
\]

(5)

being a spherical Bessel function of the first kind. \( R_1 \) and \( s \) are the effective nuclear radius and skin thickness, respectively. Their values for the Helm form factor that can reproduce the numerical fourier transform of a Two-Parameter Fermi distribution are taken as

\[
R_1(x) = \sqrt{c^2 + \frac{7}{3} \pi^2 a^2 - 5s^2}, \quad s = 0.9 \text{ fm}
\]

(6)

where \( a = 0.52 \text{ fm} \) and \( c = (1.23A^{1/3} - 0.60) \text{ fm} \). In Fig. 2, we present the numerical result of \( |F_N|^2 \) as a function of the momentum transfer \( q \). It can be seen that \( |F_N|^2 \) equals one in the range of small \( q \), and decreases rapidly when \( q \) becomes large.

Besides, we also define the DM form factor \( F_{\text{DM}}(q) \) as

\[
|F_{\text{DM}}(q)|^2 = \frac{\langle |M_n(q)|^2 \rangle}{\langle |M_n(q = q_0)|^2 \rangle}
\]

(7)

where the reference momentum \( q_0 = am_e \). It should be mentioned that the form of \( F_{\text{DM}(q)} \) is model-dependent. For the contact interaction, \( F_{\text{DM}}(q) = 1 \). While including the light mediator effect, \( F_{\text{DM}}(q) \) will rely on the momentum transfer and has to be calculated in specific models, which will be given in next section.

Since we are considering the DM accelerated by some astrophysical sources, the DM velocity distribution \( \eta \) has to be generalized to include such a contribution [35],

\[
\eta(E_{\text{min}}) = \int_{E_{\text{min}}} dE_e \frac{d\Phi_{\text{halo}}}{dE_e} \frac{m_e^2}{p_x \Phi_X} \frac{d\Phi_X}{dE_e}
\]

(8)

where \( E_X = m_\chi + T_X \) and \( d\Phi_X/dE_e \) is a model-dependent flux of fast-moving DM (see details in following section). The interval of integration of the DM incoming energy \( E_X \) in Eq. 8 can be obtained from DM-atom scattering process \( \chi(p) + A(k) \rightarrow \chi(p') + A(k') \). With energy conservation, we can have the minimal value of DM momentum,

\[
p_{\text{min}} = \frac{q}{2} - \frac{q^2 + 2m_N E_{EM}}{4m_N}
\]

\[
\times \sqrt{1 + \frac{2m_N E_{EM} q^2 - 16m_e^2 m_N^2}{q^2 + 2m_N E_{EM} q^2 + 4m_N^2 (E_{EM} - q^2)}}
\]

(9)

where the energy transferred to the scattered electron is defined as \( E_{EM} = E_nl + \Delta E_e \). Then, we can obtain \( E_{\text{min}} = \sqrt{p_{\text{min}}^2 + m_N^2} \).

Next, we discuss ionization form factor \( |f_{nl}^{\text{ion}}(p_e, q_e)| \) in Eq. 2. It relates with Migdal effect that primarily refers to inelastic collisions between dark matter and atom where electrons in an atom are ionized during a nuclear recoil (see Fig. 1). The secondary contributions from de-excitation electron and X-rays will not be taken into account in this work. We assume that \( |i\rangle \) and \( |i'\rangle \) are the states of electron cloud of atom before and after a nuclear recoil, respectively. The transition matrix of the state \( |i'\rangle \) to the ionization/excitation state \( |f\rangle \) of atom can be given by [8],

\[
\langle f| i' \rangle M = \langle f| e^{-i\frac{m_e}{m_N} \vec{q} \cdot \vec{x}'} |i\rangle,
\]

(10)

where \( m_e \) and \( m_N \) are the masses of electron and nucleus, respectively. \( \vec{q} \) is three moment transfer in the nucleus scattering with DM. \( \vec{x}' \) denotes the position of \( s^{th} \) electron in electron cloud. With the dipole approximation, \( (m_e / m_N) \vec{q} \cdot \vec{x}' \ll 1 \), we can obtain

\[
\langle f| i' \rangle M \simeq \langle f| 1 - i\frac{m_e}{m_N} \vec{q} \sum_s \vec{d}_{f_s} |i\rangle = -i \frac{m_e}{m_N} \vec{q} \cdot \vec{d}_{f_1}.
\]
where $\tilde{d}_{fi} = \langle f | e \sum_x \tilde{x}^x | i \rangle$ is the atom transition dipole moment. Migdal effect can be related with the direct ionization rate of single electron in DM-electron scattering [14], in which the transfer momentum $q$ is directly impeded to the electron. However, in the Migdal transition, there is a suppression factor $m_e/m_N \sim 10^{-6}$ for Xenon. For a very low $q$, Migdal scattering is weak, however, which can be enhanced as $q$ is sufficiently large.

In isolated atom model, the probability of an electron ionized from the $(n, l)$ energy level to continuum final state $(p_e, l')$ can be written as,

$$\frac{dP_{nl \rightarrow E_e}(q)}{dE_e} = 2\pi \left| \langle p_e, l' | e^{-q \sum_x \hat{x}^x} | n, l \rangle \right|^2. \quad (11)$$

where $E_e$ is electron recoil and $p_e = \sqrt{2m_eE_e}$ and $l'$ are electron momentum and orbital quantum number, respectively. In Ref. [8], the bound and continuum state wave functions have been numerically calculated with the FAC atomic code. On the other hand, this transition probability is proportional to the ionization form factor $f_{nl}^{\text{ion}}(p_e, q)$ of DM-electron scattering \(^1\) [14]

$$\frac{dP_{nl \rightarrow E_e}(q_e)}{dE_e} = \frac{\pi}{2E_e} \left| f_{nl}^{\text{ion}}(p_e, q_e) \right|^2. \quad (12)$$

Here the momentum transfer $q_e \simeq \frac{m_e}{m_N}q$. The ionization form factor is given by

$$|f_{nl}^{\text{ion}}(p_e, q_e)|^2 = (2l + 1) \frac{P_e^2}{4\pi q_e} \int_{|p_e-q_e|}^{p_e+q_e} k dk |\chi_{nl}(k)|^2. \quad (13)$$

where the radial wave function $\chi_{nl}(k)$ in momentum space can be calculated by Hartree-Fock method [38, 39], see Appendix.

It should be noted that the value of $q$ affects the ionization form factor significantly. For a sub-GeV DM, the value of $q$ typically varies from $\text{keV}$ to $\text{MeV}$, which indicates $q_e \ll 1 \text{keV}$. So we can use the dipole approximation to simplify ionization form factor as,

$$|f_{nl}^{\text{ion}}(p_e, q_e)|^2 = \frac{q_e}{q_0} \times |f_{nl}^{\text{ion}}(p_e, q_0)|^2. \quad (14)$$

where $q_0$ is a reference momentum and $q_0 \simeq O(\text{keV})$ for Xenon. In Fig. 2, we show the dependence of ionization factor $|f_{nl}^{\text{ion}}(p_e, q_e)|^2$ on nucleus momentum transfer $q$ at $E_e = 1 \text{keV}$. We include the contributions from electron configurations of $4s$, $4p$, $4d$, $5s$ and $5p$. We can see that the $|f_{nl}^{\text{ion}}(p_e, q_e)|^2$ is enhanced by about eight orders when $q$ is varied from $10 \text{keV}$ to $0.1 \text{GeV}$.

Finally, we can use the differential ionization rate $dR/dE_e$ in Eq. 1 and S2 data of the Xenon1T to derive the exclusion limit by

$$\frac{dR_{nl}}{dS^2} = \int dE_e c(S^2) P(S^2 | E_{EM}) dR_{nl}/dE_e \quad (15)$$

where $c(S^2)$ is the detector efficiency. The probability function $P$ that converts energy transfer into the photo-electron (PE) in S2 is given by

$$P(S^2 | E_{EM}) = \sum_{n_e, n_e} P(n_e^s | n_e) \cdot P(n_e^s | n_e) \cdot P(n_e | \langle n_e \rangle) \quad (16)$$

Here $P(n_e | \langle n_e \rangle)$ is the number of electrons escaping the interaction point, which follows a binomial distribution

$$P(n_e | \langle n_e \rangle) = \text{binom}(n_e | N_Q, f_e) = C_{N_Q}^{n_e} f_e^{n_e}(1 - f_e)^{N_Q - n_e} \quad (17)$$

where $N_Q = E_{EM}/(13.8 \text{eV})$ and $f_e = \langle n_e \rangle / N_Q \sim 0.8$. In our calculations, we assume that the possibility of electrons surviving the drift in Xenon1T is $P(n_e | n_e) = 80\%$. Finally the survival electrons have to be converted into PE in S2 with the possibility,

$$P(S^2 | n_e^s) = \text{gauss}(S^2 | g_2 n_e^s, \sigma_{S2}) \quad (18)$$

where $\sigma_{S2} = 7\sqrt{n_e^s}$ and $g_2 = 33$ for Xenon1T.

\(^1\) Since in Migdal scattering, the momentum transfer to electron is smaller than $O(\text{keV})$, we will not include the relativistic correction to ionization form factor $f_{nl}^{\text{ion}}$ [38]. Instead, we use the plane wave approximation to calculate $f_{nl}^{\text{ion}}$, which will give a similar result as non-relativistic form factor by solving Schrodinger equation [57].
**BENCHMARK MODEL: ATMOSPHERIC DM**

We will consider a fast-moving DM, namely atmospheric DM (ADM), which is produced in the inelastic collision of high energy CRs with the atmosphere on Earth. In our study, we only include the contribution of proton in the CRs for simplicity and parameterize it as Ref. [40]. We focus on the production process of ADM, where

\[ p + N \to M \to \chi \bar{\chi} + X \]  

(19)

where \( p \) is proton and \( N \) is nitrogen. The light meson \( M \) produced in collision is taken as \( \eta \) for example, which will decay to a pair of DM \( \chi \bar{\chi} \) via an on-shell scalar mediator. \(^2\) \( X \) denotes other SM particle.

The differential flux of ADM is given by,

\[
\frac{d\phi_{\chi}}{dT_X} = G \int_{T_{p_{\text{min}}}}^{T_p} \frac{d\phi_p}{dT_p} \frac{d\phi_p(h_{\text{max}})}{dT_p} \frac{d\sigma_{pN \to \chi \bar{\chi}X}}{dT_X} 
\]  

(20)

where \( T_p \) is the proton energy and \( h \) is the height from the ground level. The geometrical factor \( G \) of ADM reaching the detector can be calculated by

\[
G = \int_0^{h_{\text{max}}} dh \left( R_E + h \right)^2 2\pi d\phi \int_{-1}^{+1} d\cos \theta \frac{y_d}{\ell_d^2} y_N(h),
\]  

(21)

where \( n_N \) is the number density of nitrogen. \( l_d \) is the line of sight distance between the production point of DM and the detector,

\[
l_d^2 = (R_E + h)^2 + (R_E - h_d)^2 - 2(R_E + h)(R_E - h_d) \cos \theta
\]  

(22)

where \( h_d = 1.4 \text{ km} \) is the depth of the detector, and \( \theta \) is the polar angle of the DM production point. The dilution factor of incoming CRs in atmosphere \( y_p \) and attenuation factor of DM in the Earth \( y_d \) are given by,

\[
y_p = \exp \left( -\sigma_{pN} \int_h^{h_{\text{max}}} dh n_N(h) \right),
\]  

(23)

\[
y_d = \exp \left( -\sigma_{\chi N} \int_0^{l_d} dz n(r(z) - R_E) \right)
\]  

(24)

where \( h_{\text{max}} = 180 \text{ km} \) and \( R_E = 6378.1 \text{ km} \). \( n \) is the number density of nucleus at the depth of \( r(z) - R_E \). \( \sigma_{pN} \) and \( \sigma_{\chi N} \) are the proton-nitrogen and DM-nucleus scattering cross section, respectively. We use the package CRMC [41–43] to simulate the above inelastic collision of incoming CRs with the nitrogen.

The CRs differential flux \( d\phi_p/dT_p \), and the inelastic differential cross section \( d\sigma_{pN \to \chi \bar{\chi}X}/dT_X \) can be obtained by

\[
\frac{d\phi_p(T_p, h)}{dT_p} = y_p(h) \frac{d\phi_p(T_p, h_{\text{max}})}{dT_p}
\]

\[
\frac{d\sigma_{pN \to \chi \bar{\chi}X}}{dT_X} = \frac{d\sigma_{pN \to \chi \bar{\chi}X}}{dT_X} T_{\text{tot}} \approx \frac{\sigma_{pN}}{T_{\text{max}}} \text{BR}(M \to \chi \bar{\chi} X) \tag{26}
\]

where we assume an isotropic scattering and take a uniform distribution of the ADM kinetic energy.

Since our ADM only needs to interact with the SM quarks, we embed it in a hadrophilic DM model, where a flavor-specific singlet scalar mediator \( S \) couples to up-type quarks and DM \[44\]. The effective interactions can be written as follows,

\[
\mathcal{L} = i\bar{\chi} (\not{\! p} - m_\chi) \chi + \frac{1}{2} \partial_\mu S \partial^\mu S - \frac{1}{2} m_S^2 S^2 - (g_\chi S \chi \eta \chi + g_{u} S u \bar{u} + \text{h.c.}) \tag{27}
\]

where \( m_S \) and \( m_\chi \) are the masses of mediator and DM, respectively. \( g_\chi \) and \( g_u \) are the couplings of mediator \( S \) with the DM and up-quarks, respectively. With Eq. 27, we can obtain the momentum independent DM-nucleon scattering cross section \( \sigma_n \) in Eq. 3,

\[
\sigma_n = \mu_n^2 \frac{[M_n(q = q_0)]^2}{16\pi m_n^2 m_N^2} = \frac{g_n^2 g_n^2 \mu_n^2}{\pi (\alpha^2 m_n^2 + m_N^2)^2} \tag{28}
\]

where \( g_n = 0.014 g_{u}(m_p/m_u) \), \( q_0 = \alpha m_e \) and the DM form factor \( F_{DM} \) in Eq. 7,

\[
|F_{DM}(q)|^2 = \frac{(4m_n^2 + q^2)}{16m_n^2 m_N^2} \frac{(m_n^2 + q^2)^2 (4m_n^2 + q^2)}{16m_n^2 m_N^2 (m_n^2 + q^2)^2} \tag{29}
\]

In Fig. 2, we present the dependence of DM form factor \( |F_{DM}(q)|^2 \) on the momentum transfer \( q \). It can be seen that \( |F_{DM}(q)|^2 \) for \( m_\chi = 5 \text{ MeV} \) can be much greatly enhanced as \( q \) increases in the range of our interest, while \( |F_{DM}(q)|^2 = 1 \) for \( m_\chi = 100 \text{ MeV} \).

Since our ADMS are produced via the decay of on-shell mediator \( S \), we will investigate the mass range of \( m_S < m_\eta - m_\pi \). When \( m_S < m_\eta \), the coupling \( g_u \) is tightly constrained by the null result of \( K \to \pi \nu \bar{\nu} \) in E787/949 experiment \[45–48\], which requires \( g_u \leq 4 \times 10^{-6} \). If \( m_S < 2m_\pi \), the decay of \( S \) to diphotons will increase the deuterium abundance \( D/H = (2.53 \pm 0.04) \times 10^{-5} \) and further affect the baryon-to-photon ratio. In terms of existing bounds, we find the bound \( m_S > 20 \text{ MeV} \) and \( g_u > (2 \times 10^{-8}) (m_S/\text{MeV})^{-3/2} \). On the other hand, when \( m_\pi < m_S < m_K - m_\pi \), MINIBoone experiment \[49\] gives a upper bound \( g_u \leq 4 \times 10^{-5} \). This is stronger than

\(^2\) In principle, neutral \( \pi \) can also be the source of ADM. However, the current limit on \( Br(\pi \to \gamma + \text{invisible}) < 10^{-9} \) make its contribution to the ADM flux negligible. On the other hand, the decays of other charged mesons are more model-dependent. We will not include them in our calculations.
the bound derived from the Xenon1T data. Therefore, in order to escape these constraints, we take $g_a = 10^{-5}$ and $m_S = 300$ MeV as our benchmark point, which corresponds to $Br(\eta \rightarrow \pi \bar{\chi} \chi) \simeq 1 \times 10^{-5}$.

Figure 3. Differential event rates of DM-nucleus scattering process in the cases of ADM, Migdal effect (non-relativistic DM) and ADM+Migdal effect, where $E_R$ is the nucleus recoil and $E_e$ is the energy of ionization electron. The solid and dashed lines correspond to DM mass $m_s = 5$ MeV and 100 MeV, respectively. We assume the mediator mass $m_S = 300$ MeV, the branching ratio $Br(\eta \rightarrow \pi \bar{\chi} \chi) = 1 \times 10^{-5}$ and momentum-independent DM-nucleus scattering cross section $\sigma_n = 1 \times 10^{-34} \text{cm}^2$.

In Fig. 3, we compare the differential event rates of DM-nucleus scattering process in the cases of ADM, Migdal effect (non-relativistic DM) and ADM+Migdal effect. We can see that the nucleus recoil $E_R$ of ADM can be as large as tens of keV because of the boosted effect of the meson decay. This can make the light ADM reach the threshold of nucleus recoil signal ($E_R > 1 \text{ keV}$) in Xenon detectors. On the other hand, we can find that the Migdal scattering has sizable events in the low ionization electron energy $E_e$. In contrast with the nucleus recoil, these electrons with $E_e > 0.2 \text{ keV}$ can directly contribute to the S2 signal, and thus enhance the sensitivity of light DM. For a given event rate, the heavy DM can produce a larger $E_R$ than the light DM. Including the Migdal effect in ADM, we can see that its event rate is much smaller than the Migdal scattering, which is still larger than the ADM. This is because that the ADM has a high velocity and can produce a larger momentum transfer, which will lead to the suppression of the nucleus form factor $F_N$ (see Fig. 2). However, due to inheriting the energy of ADM, the $E_e$ of ADM+Migdal can be much larger than that of Migdal scattering.

In Fig. 4, we use the S2 data of Xenon1T [50] to derive the exclusion limits on the momentum-independent DM-nucleus scattering cross section $\sigma_n$ for above cases. As a comparison with the result in Ref. [31], we present their limit by scaling the branching ratio $Br(\eta \rightarrow \pi \bar{\chi} \chi)$ from $10^{-3}$ to $10^{-5}$ as well. It can be seen that the Migdal scattering can exclude the DM mass down to about 300 MeV for $\sigma_n = 10^{-37} \text{ cm}^2$, while the ADM and ADM+Migdal can further exclude lower DM mass. Besides, when $m_\chi < \mathcal{O}(10) \text{ MeV}$, it should be noted that the enhancement effect of DM form factor in Eq. 29 will be sizable and has to be taken into account, which leads to a much stronger limit than that in Ref. [31]. Furthermore, including the Migdal effect, our bound on $\sigma_n$ can be further improved by nearly three orders of magnitude. Since these enhancement effects are almost independent on the acceleration mechanism, we expect that other fast-moving DMs, such as cosmic ray up-scattering DM [23], will have similar conclusions.

Finally, we discuss the relic density of ADM in our interest of mass range, i.e. $m_\chi < m_S$, in which $\chi$ will annihilate directly to the SM quarks via a s-channel mediator. The cross section of annihilation is given by

$$\sigma v_{\text{rel}}(\chi \bar{\chi} \rightarrow q\bar{q}) = \frac{g_a^2 m_\chi v_{\text{rel}}^2 |\Gamma_S|_{m_S=2m_\chi}^2}{2 \left( (m_S^2 - 4m_\chi^2)^2 + m_S^2 v_{\text{rel}}^2 \right)} , \quad (30)$$

which is suppressed by velocity. $|\Gamma_S|_{m_S=2m_\chi}$ is the decay width of the scalar at $m_S = 2m_\chi$. The freeze-out scenario is guaranteed that $\Gamma_S > |H[m_S]|$, which in turn favors relatively larger coupling $g_a$. Thus DM is in chemical equilibrium with the early universe bath as long as it contains quarks. After the QCD phase transition takes place, DM pairs could still annihilate into two pions. Therefore achieving the correct relic density via thermal
freeze-out would require \( m_\chi > m_\pi \). For lighter DM, we can allow for a small coupling of the mediator to neutrinos to achieve the correct relic abundance via thermal freeze-out [51].

**CONCLUSIONS**

In this work, we investigate the sensitivity of light fast-moving DM by including Migdal effects. Such a DM can be produced by the interaction between DM and high energy cosmic rays. Due to the double enhancement of boost effect and Migdal effect, it is expected the current limit on sub-GeV DM will be greatly extended. As a proof of concept, we consider the atmospheric DM with a light scalar mediator that only couples to the SM quarks. Using the Xenon1T data, we demonstrate that the DM-nucleus scattering cross section can be improved by nearly three orders of magnitude.

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**APPENDIX: HARTREE-FOCK METHOD FOR IONIZATION FACTOR**

In the process \( \chi(p_1) + e(k_1) \rightarrow \chi(p_2) + e(k_2) \), the ionization form factors \( |f_{nl}^{\text{ion}}(k_2, q)|^2 \) can be written as:

\[
|f_{nl}^{\text{ion}}(k_2, q)|^2 = \frac{2k_2^3}{(2\pi)^3} \sum_{\text{deg}} |f_{nl}(q)|^2
\]

where \( k_2 = \sqrt{2m_\pi E_R} \) is energy of the electron in final states. The ionization form factor \( f_{nl}(q) \) is defined as:

\[
\sum_{\text{deg}} |f_{nl}(q)|^2 = \sum_{\text{deg}} \sum_{l} \left| \langle k_2 | e^{iq \cdot r} | nlm \rangle \right|^2
\]

where \( k = k_2 - q \). \( \psi_{nlm}(k) \) can be determined by solving Schrödinger equation,

\[
|f_{nl}^{\text{ion}}(k_2, q)|^2 = \frac{2k_2^3}{(2\pi)^3} \sum_{\text{deg}, m} |\psi_{nlm}(k)|^2
\]

\[
= \frac{2k_2^3}{(2\pi)^3} \sum_{\text{deg}, m} |\chi_{nl}(k) Y_{lm}(\theta, \phi)|^2
\]

\[
= \frac{2k_2^3}{(2\pi)^3} \int \frac{2l + 1}{2\pi} \sum_{\text{deg}} |\chi_{nl}(k)|^2
\]

\[
= \frac{2k_2^3}{(2\pi)^3} \int \frac{2l + 1}{2\pi} \chi_{nl}(\sqrt{k_2^2 + q^2 - 2k_2q \cos \theta})^2
\]

\[
= (2l + 1) \frac{k_2^3}{4\pi q} \int_{|k_2 - q|} kdk |\chi_{nl}(k)|^2. \quad (33)
\]

In the third line, we use the orthogonality of the spherical harmonics \( \sum_{m=-l}^{l} |Y_{lm}(\theta, \phi)|^2 = \frac{2l+1}{4\pi} \), and make coordinate transformation \( \theta \rightarrow k \) in the last line.

Then, the radial wave function \( \chi_{nl}(k) \) in the momentum space is given by

\[
\chi_{nl}(k) = \frac{4\pi}{2l + 1} \sum_{m} \psi_{nlm}(k) Y_{lm}(\theta, \phi)
\]

\[
= 2\pi \int r^2 \sin \theta d\theta d\phi R_{nl}(r) P_l(\cos \theta) e^{i k r \cos \theta}
\]

\[
= 2\pi \int r^2 dr R_{nl}(r) \sum_{l'} (2l' + 1) i^{l'} j_{l'}(kr)
\]

\[
\cdot \int d(\cos \theta) P_l(\cos \theta) P_{l'}(\cos \theta)
\]

\[
= 4\pi i^l \int r^2 dr R_{nl}(r) j_{l}(kr), \quad (34)
\]

where \( P_l(\cos \theta) \) and \( j_{l}(k) \) are the Legendre polynomial and the spherical Bessel function, respectively. \( R_{nl}(r) \) can be approximately given by a linear combination of Slater type orbitals,

\[
R_{nl}(r) = \sum_{k} C_{nlk} \left( \frac{Z_{lk}}{a_0} \right)^{n_k+1/2} \left( \frac{r}{a_0} \right)^{n_k-1}
\]

\[
\cdot \exp \left( -\frac{Z_{lk}}{a_0} r \right) \quad (35)
\]

In which, \( a_0 = 1/(\alpha e) \) is Bohr radius. The \( C_{nlk}, n_k \) and \( Z_{lk} \) are some parametric coefficient, can be find in [39].

Therefore, the radial wave function in the momentum
space can be written as:

\[
\chi_{nl}(p) = \sum_k C_{nlk} (\frac{2Z_{lk}}{a_0})^{n_{lk}+1/2} \left( \frac{1}{a_0} \right)^{n_{lk}+1/2} \cdot \int_0^\infty \frac{d\rho r^{n_{lk}+1}}{\sqrt{2(2n_{lk})!}} \exp \left( -\frac{Z_{lk}}{a_0} r \right) j_l(\rho l) \cdot 2F_1 \left[ \frac{2 + l + n_{lk}}{2}, \frac{3 + l + n_{lk}}{2}, \frac{l + 3}{2}, \left( \frac{\rho a_0}{Z_{lk}} \right)^2 \right] 
\]

where \(2F_1(a, b, c, x)\) is a hypergeometric function.