Generation of tripartite squeezed state by cascaded four-wave mixing in single hot rubidium atomic system

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Abstract
We report the quantum correlated triple beams via cascaded four-wave mixing (CFWM) amplified in single hot atomic vapor. Experimentally, we show that strong quantum correlation of three light beams, among them any two’s quantum correlation is characterized by the maximum value of intensity-difference squeezing (IDS) about $-7.8 \pm 0.3$ dB. We found there is IDS between two idler beams, because two pairs of Einstein–Podolsky–Rosen injections potentially exist in our system. Besides, CFWM can emit three-mode beams at three different frequencies, in which these beams can be well separated in the spatial domain. Moreover, much difference with other methods, the injecting probe field can manipulate the gain and IDS of output three-mode light beams, which is resulting from competition relationship between cascaded two four-wave mixing processes. More interestingly, Autler–Townes splitting of gain peaks of output signals due to dressing effect of pumping fields, will lead to the evolution of measured two- and three-mode IDS from single-mode to multi-mode at frequency domain. This result will provide a multimode quantum resource which can potentially realize multimode entanglement and quantum networks.

1. Introduction
The non-classical states of light are the benchmark tools in the field of quantum optics for exploring the fundamental problems of quantum physics [1–6] and emerging quantum technologies [5–14]. Since the generation of squeezed light using four-wave mixing (FWM) in sodium vapor by Slusher et al [15], many different proposals to generate squeezed states have been studied and experimentally implemented. The fundamental and frequent technique being used to generate squeezed states is optical parametric oscillator (OPO) in nonlinear crystals [2, 3, 16–18]. Although very large amounts of quantum noise reduction have been achieved in OPO [19, 20], OPO has disadvantage on spatially coincident generated beams, and broad bandwidth limit it for certain application [21–23]. Following that, matching to an atomic transition naturally, the narrowband correlated and entangled light beams created in FWM process gradually become popular method for generating squeezing states [4, 21–26]. FWM process has several strengths compared with spontaneous parameter down transformation in practical implementations, for example, it does not need a cavity to the system and has strong nonlinearity along spatial separation of correlated beams and so on.

Researcher would like to increase the degree of the intensity-difference squeezing (IDS) and the modes of correlated beams for wide applications. In this regard, the degree of IDS can be enhanced by modulating the internal energy levels in FWM process with high-gain atomic media [23]. By cascading FWM setups, the quantum correlated triple or quadruple beams was theoretically proposed and experimentally realized [27–30], where the number of quantum modes is increased that so does degree of quantum correlation [27]. Also using cascading, cluster-state or entanglement of multi-mode beams have been realized by two or more OPOs [31, 32]. However, in this cascaded repeating process case, the systems entail further alignment
and appended loss [33]. Besides, other methods have been proposed to achieve direct generation of multipartite correlations and entanglement without going through the cascading processes, such as using third-order nonlinear materials or optical frequency combs [34–36]. Moreover, using two pump beams of the same frequency in cascaded multimode FWM process with single vapor cell, the four-mode even ten-mode correlated light beams are generated with two different frequency [33, 37–39]. These multi-modes squeezed light have widely promising applications in quantum technologies such as quantum images [5, 6], quantum communication [7, 8], quantum networks [9, 10] and quantum metrology [11–14], in which all of them have been experimentally demonstrated.

In this paper, we implement a novel scheme to obtain the three-body squeezed beams in cascaded FWM (CFWM) amplifier, which shows the simpler experimental setup and stronger quantum correlations. The mature FWM process is constructed with one pump beam and one seed beam, in which the seed beam is amplified and one idler beam is generated [4, 21–26]. Based on this FWM process, we add another pump beam with different frequency. In this case, the new pump beam not only can suppress the gain of FWM process, but also assist in the phase-matching of another FWM nonlinear process that can generate a new idler beam, suggesting all incident beams could manipulate the gain and relationship between these two cascaded FWM. We therefore measure the IDS of these beams, and examine the transition between two different third-order nonlinear processes with different power of pump beams and seed beam. Therefore, CFWM can emit three-body light beams at three different frequencies, and strong quantum correlations are generated between three light beams or any two of them due to strong nonlinear process, which have a great potential for offering wide applications.

2. Experimental setup and basic theory

Our scheme uses a CFWM amplifier to generate three-mode light beams. We consider a three-level atomic system at 3 cm long thermal rubidium atomic vapor at 130 °C, and the relevant energy level diagram is shown in figure 1(a). A strong pump beam $E_1$ (frequency $\omega_1$, wave vector $k_1$, Rabi frequency $G_1$, wavelength 795 nm, vertical polarization) is applied to the atomic transition $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |3\rangle$, with detuning $\Delta_1$ and $\Delta_1'$, respectively. The detuning $\Delta_1 = \Omega_1 - \omega_1$ is defined as the difference between the resonant transition frequency $\Omega_1$ and laser frequency $\omega_1$ of field $E_1$. A weak probe beam $E_2$ ($\omega_2$, $k_2$, $G_2$, horizontal polarization) via two double passed acousto-optic modulators (AOM1) of 1.52 GHz to have a blue frequency difference of 0.8 GHz and is send into the center of rubidium atomic vapor cell and propagates at $E_1$ direction with an angle of 0.5° in horizontal plane. The phase matched FWM parametric amplifier with $k_3 + k_s = 2k_1$ is formed.

More, in presence of a coupling beam $E_3$ ($\omega_3$, $k_3$, $G_3$, vertical polarization) via passed AOM2 of 0.8 GHz to have a red frequency difference of 0.8 GHz and is applied to the atomic transition $|2\rangle \rightarrow |3\rangle$ with a detuning $\Delta_3$, which propagate at $E_1$ direction. The angle between fields $E_1$ and $E_3$ is either 0.2° or 0.4°. In the first case, when the angle between two fields $E_1$ and $E_3$ is 0.2° in vertical plane, the geometric distribution of two beams and three signals is shown in CS1 in figure 1(c). Therefore, CFWM1 will be generated form the energy level diagram in figure 1(a), and satisfies the phase-matching condition $k_1 + k_3 = k_s + k_3$. In the second case when angle is 0.4° in vertical plane, the geometric distribution of beams and signals is shown in CS2 in figure 1(c). The generated CFWM2 satisfies the phase-matching condition $k_1 + k_3 = k_s + k_3$, and its energy level diagram is shown in figure 1(b). Therefore, we can experimentally realize the conversion of the two CFWM processes by changing the angle between two fields $E_1$ and $E_3$. In two cases, the three output lights $E_{31}$, $E_{32}$ and $E_{33}$ (or $E_{34}$) are detected respectively by balanced homodyne detectors with a transimpedance gain of $10^5$ V A⁻¹ and 96% quantum efficiency, and then are analyzed through the frequency spectrum analyzer (SA).

The interaction Hamiltonians of the CFWM1 and CFWM2 amplifier can be expressed as

$$H_1 = i\hbar \kappa_1 \hat{a}_{1s} \hat{a}_{1s}^\dagger + i\hbar \kappa_2 \hat{a}_{2s} \hat{a}_{3s}^\dagger + H.c.,$$  

$$H_{II} = i\hbar \kappa_3 \hat{a}_{2s} \hat{a}_{3s}^\dagger + i\hbar \kappa_4 \hat{a}_{3s} \hat{a}_{4s}^\dagger + H.c.,$$

where $\hat{a}_{1s}^\dagger$, $\hat{a}_{2s}^\dagger$, $\hat{a}_{3s}^\dagger$, $\hat{a}_{4s}^\dagger$ are the boson creation operators of the three generated fields. The pumping parameter for FWM and CFWM1 processes are $\kappa_1 = |\chi^{(3)}| E_1 E_1^\dagger = |N \mu_\Omega (3/2;1/2) / \hbar_0 G_{32}/\hbar_0 G_{23}/\hbar_0 G_{13}|$ and $\kappa_2 = |\chi^{(3)}| E_1 E_3 = |N \mu_\Omega (3/2;1/2) / \hbar_0 G_{32}/\hbar_0 G_{23}/\hbar_0 G_{13}|$, $\chi^{(3)}$ can be described by the following perturbation chains [40]: $\rho^{(0)}_{12} \rightarrow \rho^{(1)}_{12} \rightarrow \rho^{(2)}_{12} \rightarrow \rho^{(3)}_{12} (E_{31})$, $\rho^{(0)}_{13} \rightarrow \rho^{(1)}_{13} \rightarrow \rho^{(2)}_{13} \rightarrow \rho^{(3)}_{13} (E_{32})$, $\rho^{(0)}_{23} \rightarrow \rho^{(1)}_{23} \rightarrow \rho^{(2)}_{23} \rightarrow \rho^{(3)}_{23} (E_{33})$ & $\rho^{(0)}_{11} \rightarrow \rho^{(1)}_{11} \rightarrow \rho^{(2)}_{11} \rightarrow \rho^{(3)}_{11} (E_{34})$. 

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2.1. Theoretical simulation of three-mode IDS

Where equations (4) and (5).

\[ \Omega = \text{frequency detuning of the} \]

The intensities of \( E_{S1}, E_{S2} \) and \( E_{S3} \) signals can be given by

\[
\begin{align*}
\rho_{32(31)}^{(3)} &= -iG_1^2G_{S2}/d_{32}d_{311}, \\
\rho_{31(32)}^{(3)} &= -iG_1^2G_{S1}/d_{31}d_{321} - iG_3G_{S3}/d_{312}d_{321}d_{312}, \\
\rho_{23(32)}^{(3)} &= -iG_1^2G_{S2}/d_{32}d_{312}d_{323},
\end{align*}
\]

where \( d_{32} = \Gamma_{32} + i\Delta_{s1}, d_{31} = \Gamma_{31} + i\Delta_{s2}, d_{311} = \Gamma_{311} + i\Delta_{s31} + \Delta_{s31}, d_{321} = \Gamma_{321} + i\Delta_{s31} + \Delta_{s321}, d_{312} = \Gamma_{312} + i\Delta_{s32} + \Delta_{s312}, d_{321} = \Gamma_{321} + i\Delta_{s32} + \Delta_{s312}, d_{323} = \Gamma_{323} + i\Delta_{s32} + \Delta_{s323} \) with \( \Delta_{s1}, \Delta_{s2} \) and \( \Delta_{s3} \) representing the frequency detuning of the \( E_{S1}, E_{S2} \) and \( E_{S3} \) signals. \( \Gamma = (\Gamma_1 + \Gamma_2)/2 \) is the de-coherence rate between \( |j\rangle \) and \( |j\rangle \); \( G_i = \mu_iE_i/\hbar \) is the Rabi frequency of \( E_i \).

2.2. Frequency multi-mode analysis

By ignoring the dressing effect of field \( E_i \), one can see that the spectra of \( E_{S1}, E_{S2}, E_{S3} \) signals show only one peak (figures 2(a)–(c)), as scanning \( \Delta_{s2}/\Delta_{s1} \) [scanning \( \Delta_{s2} \) and \( \Delta_{s1} \) are equivalent because they satisfy energy conservation condition \( 2\omega_1 = \omega_2 + \omega_{S1} \)]. These correspond to two-photon peaks of \( E_{S1}, E_{S2} \) and \( E_{S3} \) signals in equation (2), respectively. Then, the spectra of \( E_{S1}, E_{S2}, E_{S3} \) signals becomes two peaks when the dressing effect of field \( E_i \) is considered, as shown in figures 2(e) and (f), respectively. This is mainly due

Figure 1. The relevant \(^{85}\text{Rb} \) atomic energy-level diagram under two different FWM conditions. (a) CFWM1 and (b) CFWM2. (c) Spatial beam (square–box) geometry used in the experiment. CS1 and CS2 are the geometric distribution of wave vectors of CFWM1 and CFWM2, respectively. (d) Coherent channels energy-level diagram. (e) Spatial images corresponding to (c).
to the dressing effect of field $E_1$, which leads to the splitting of energy level $|3\rangle$ [41]. So, the intensities of $E_{S1}, E_{S2}, E_{S3}$ can be described by the density matrix elements $\rho^{(3)}_{32(31)}, \rho^{(3)}_{31(32)}$ and $\rho^{(3)}_{32(33)}$ by using a perturbation chain method [40], which are given by

\[
\rho^{(3)}_{32(31)} = -iG^3_1 G_{SS}/d_{12D}d_{321}, \tag{6a}
\]

\[
\rho^{(3)}_{31(32)} = -iG^3_2 G_{SS}/d_{312}d_{321} - iG_3 G_3 G_{SS}/d_{312}d_{212D}d_{312}, \tag{6b}
\]

\[
\rho^{(3)}_{32(33)} = -iG_1 G_2 G_{SS}/d_{21D}d_{323} - iG_2 G_3 G_{SS}/d_{312}d_{21D}d_{321}, \tag{6c}
\]

where $d_{12D} = \Gamma_1 + i(\Delta_1 - \Delta_{S2}) + G^2_1/|\Gamma_3 - i\Delta_3 - \Delta_{S3}|$, $d_{21D} = \Gamma_1 + i(\Delta_1 - \Delta_{S1}) + G^2_1/|\Gamma_3 - i\Delta_3 - \Delta_{S3}|$, $d_{312} = \Gamma_1 + i(\Delta_1 - \Delta_{S1}) + G^2_2/|\Gamma_3 - i\Delta_3 - \Delta_{S3}|$.

From equations (6a)–(6c), one can see that two-photon peak split from single peak into two peaks, and finally form three peaks as shown in figures 2(g)–(i). The detuning width of Autler–Townes (AT)-spitting peaks is $\Delta_S/2$, where $\hbar\Omega_{e1} = \sqrt{\Delta^2_1 + 4\Gamma^2_1} + 4\Gamma_1\Gamma_3\Gamma_1$, and the linewidth of resonance peak is $\Gamma_{e1} + \Gamma_{e2} + \Gamma_{e3}$. For example, the left peak in figure 2(e) is split into the first and second peaks in figure 2(h) correspond to the dressed states $|G_1+\rangle$ and $|G_1-\rangle$, respectively (figure 1(d)). At the same time, the $E_{S1}$ signal of one of the two coherent channels split by the $E_1$ field satisfies the two-photon resonance condition [meeting suppression condition $(\Delta_1 - \hbar\Omega_{e1})/2 - \Delta_3 = 0$, thus we see only one peak of $E_{S1}$ signal in figure 2(d). Further, $E_3$ drives the transition from $|1\rangle$ to $|3\rangle$ and creates the dressed states $|G_{S3}\rangle$ from $|3\rangle$ when the dressing effect of field $E_3$ is considered. Therefore, one can see that the peaks marked $d1$ in figure 2(d), peak e1 in figure 2(e) and peak f2 in figure 2(f) are split into two peaks, as shown figures 2(g)–(i) [41].

### 3. Experimental results

Now, we discuss the degree of IDS of FWM and CFWM1 in cascade FWM process system when the dressing effect of fields $E_1$ and $E_3$ is not considered. First, the strong pumping beam $E_1$ is coupled into the rubidium cell and FWM will occur in this system, which can generate $E_3$ and $E_{S3}$ fields. Then, a weak probe beam $E_2$ is injected into $E_{S3}$ field. The probe beam waist at the crossing point is 360 $\mu$m, while the pumping beam $E_1$ waist is 600 $\mu$m. In this case, the FWM serves as amplifier, and the signal beam $E_{S3}$ and idler beam $E_{S3}$ are generated. This scheme also is one of the most popular candidates for the generation of quantum correlated twin beams [21, 22]. Then, another pump beam $E_3$ (beam waist of 400 $\mu$m) is sending into two FWM processes. In this case, we adjust the angles of three input beams, and the second idler beam is generated.

\[\Delta_1 = \sqrt{\Delta^2_1 + 4\Gamma^2_1 + 4\Gamma_1\Gamma_3\Gamma_1}\]

\[\Delta_2 = \sqrt{\Delta^2_1 + 4\Gamma^2_1 + 4\Gamma_1\Gamma_3\Gamma_1}\]

\[\Delta_3 = \sqrt{\Delta^2_1 + 4\Gamma^2_1 + 4\Gamma_1\Gamma_3\Gamma_1}\]

\[\Delta_4 = \sqrt{\Delta^2_1 + 4\Gamma^2_1 + 4\Gamma_1\Gamma_3\Gamma_1}\]

\[\Delta_5 = \sqrt{\Delta^2_1 + 4\Gamma^2_1 + 4\Gamma_1\Gamma_3\Gamma_1}\]

\[\Delta_6 = \sqrt{\Delta^2_1 + 4\Gamma^2_1 + 4\Gamma_1\Gamma_3\Gamma_1}\]
Similarly, we observe that the IDS of $E_1$ and $E_2$ changes on the spot. The corresponding light beam's geometric distribution is shown in CS2 in figure 1. We then record the noise power of the difference of the photocurrents $D_{1,2}$ and obtain an accurate measure of the SNL. Subsequently, the noise spectra of the relative intensities between the three-mode light beams $I_{1,2}$ and $I_3$ are measured. The results of their subtractions $I_2 - I_1$, $I_2 - I_3$, $I_1 - I_2$, $I_1 - I_3$ are shown in figure 3(a). It should be noted that each noise power curve is normalized here, emphasizing the quantum correlated noise subtraction characteristics of each combination of different beams. While the normalized traces of noise power are below the normalized SNL suggesting quantum squeezing.

To verify the predicted strong quantum correlations in our system, we measure the IDS of signal–idler pair, idler–idler pair and triple beams. Figure 3(a) show the degree of IDS of $I_2 - I_1$ (A curve), $I_2 - I_3$ (B curve), $I_1 - I_3$ (C curve), $I_2 - I_1 - I_3$ (D curve), respectively. According to figure 3(a), we observe that the degree of IDS from FWM and CFWM are $-5.8$ dB and $-5.3$ dB, respectively. And the IDS value of triple beams reaches $-7.8$ dB below the SNL. More interestingly, the IDS of idler–idler pair also exist, and the value is $-2.9$ dB, this is because the outputs of FWM and CFWM ($E_{S1}$ and $E_{S2}$) turn into two pairs of EPR entangled injections, and therefore there is quantum correlation between two idler beams. The large peaks shown below 1 MHz are classical noise from our lasers. The presence of IDS not only exists among the three light beams, but also exists between signal–idler pair. Moreover, the degree of IDS from FWM and CFWM1 can be approximately equal by adjusting the power of two pumping fields ($E_i$ and $E_3$). These results have potential applications in the construction of practical quantum networks.

In addition, figure 3(b) shows the degree of IDS from FWM (A curve), CFWM2 (B curve), two idler light beams (C curve) and all three light beams (D curve), when the direction of injection of field $E_3$ changes on the spot. The corresponding light beams geometric distribution is shown in CS2 in figure 1. Similarly, we observe that the IDS of $I_1 - I_2$, $I_1 - I_4$, $I_2 - I_4$ (C curve), $I_1 - I_2 - I_4$ are $-5.8$ dB, $-3.9$ dB, $-2.7$ dB and $-7.2$ dB respectively. However, the degree of IDS of FWM and CFWM2 is different from that in figure 3(a). This is mainly attributed to the larger angle between two pumping field $E_1$ and $E_3$ in CFWM2 results in the lower conversion efficiency, the gain of CFWM2 becomes lower and the degree of IDS of CFWM2 and triple beams become smaller. In addition, in CFWM1, the seeded beam $E_3$ interacts with $E_1$ and $E_3$ in the Hamiltonian (equation (1a)), two pairs of EPR beams ($E_{S1}$ and $E_{S2}$) potentially exist and the total three input signals may be EPR entangled beams. However, $E_2$ only interacts with $E_1$ in CFWM2 (equation (1b)). There is a single pair of EPR beams $E_{S1}$ and $E_{S2}$, which the correlations of the input signals are less than that compared with CFWM1. So, the degree of IDS of two idle beams in CFWM1 is larger than that in CFWM2.

To comprehensively demonstrate the squeezing enhancement as predicted by the theory, we study the intensity-difference noise power for the triple beams (trace B) and twin beams of two idler, FWM and CFWM1 (traces C, D and E, respectively) at 1 MHz as a function of total optical power, as shown in

![Figure 3. Relative IDS versus SA frequency. (a) A–E curves $S_{1–5}$, $S_{1–3}$, $S_{1–2–3}$, $S_{1–2–4}$, $S_{1–3}$, SNL of CFWM1, respectively. (b) A–E curves $S_{1–2}$, $S_{1–4}$, $S_{1–4}$, $S_{1–2–4}$, SNL of CFWM2, respectively. The background noise is subtracted from all of the traces.](image-url)
Figure 4. Relative intensity noise power at different total optical power. (a) Triple beams (circle curve), (b) twin beams from FWM (circle curve), and CFWM1 (triangle curve), SNL (square curve). The electronic noise floor and background noise are subtracted from all the traces and data points.

We also recorded the noise powers of a coherent beam at different optical powers using SNL measurement method as described above (black square curve). After fitting all these three noise power curves to straight lines, the ratio between two slopes of trace A and trace B, C, D, E is 0.16, 0.51, 0.29, 0.26, which correspond to $-7.8$ dB, $-2.9$ dB, $-5.3$ dB, $-5.8$ dB of IDS. As well known to all, the relative noise reduction will decrease rapidly with the increase of loss. In our optical configuration, the optical loss is about 3%, resulting in a total detection efficiency of 0.93; the uncertainty is estimated at 1 standard.

Finally, we emphasize on measuring the change of degree of IDS of FWM, CFWM1 and triple beams with the power of two pumping field ($E_1$ and $E_3$) as shown in figure 5. According to figure 5(a), the degree of IDS of twin beams and triple beams increases first with the increase of power of field $E_1$, then the degree of IDS of twin beams being to flatten or even decrease. Specially, the degree of IDS of triple beams rapidly decreases with increases of power of field $E_1$. The main reason is that when power of field $E_1$ increases, $\kappa_1$ and $\kappa_2$ in equation (1) increases, leading to the increase of gain of FWM and CFWM1, resulting in the increase of degree of IDS of twin beams and triple beams, as shown in figure 5(a). However, when power of field $E_1$ increases further, one should consider the dressing effect of field $E_1$ effect on intensities and lineshape of FWM and CFWM1 signals. The small windows in figure 5(a) is the DC voltages of the three output signals in the experiment. The DC voltage of $I_2$ is determined by two FWM processes together, therefore the DC voltages of $I_1$ and $I_3$ can characterize the gain of two independent FWM processes. With increasing $P_1$, the gain of the first FWM process becomes larger, and the second becomes smaller. The IDS shown in figure 5(a) first increase with increasing gain, and the maximum value appears at $P_1 = 170$ mW, and then it decreases, which is due to the dressing effect becoming stronger. In addition, when power of field $E_3$ gradually increases, the degree of IDS of CFWM1 and triple beams is increasing, as shown in figure 5(b). The reason is that the gain of CFWM1 increases due to the increase of the power of field $E_3$, resulting in the increase of degree of IDS of CFWM1 and triple beams. But the degree of FWM decreases with the power of field $E_3$. This is because competitive relationship between the two FWM processes, the increased gain of CFWM1 causes FWM to be correspondingly suppressed, resulting in a decrease in the degree of IDS of FWM. However, when the power of $E_3$ field continues to increase, the degree of IDS of FWM, CFWM1 and triple beams is all decreasing, which is mainly caused by the dressing effect of the $E_3$ field. Furthermore, when the power of field $E_3$ is set to a very large value of 66 mW and gradually increase the $E_1$ power, we can see that due to the common dressing effect of the two pump fields $E_1$ and $E_3$, the degree of IDS of FWM, CFWM1 and triple beams is decreasing sharply as showing in figure 5(c).

In order to discuss the dressing effect of field $E_1$ effect on the degree of IDS of FWM and CFWM1, we experimentally observe spectra of $E_{S1}$, $E_{S2}$ and $E_{S3}$ channels when the power of field $E_1$ is large. Figure 6 shows signal amplitude of $E_{S1}$, $E_{S2}$ and $E_{S3}$ channels as scanning the frequency of probe field $E_2$ (come from a separate laser). It can be seen that when the power of $E_1$ field and $E_3$ field is low, their dressing effect is so low that can be ignored, the gain peak does not split at this time (figures 6(a)–(c)), so the degree of IDS of FWM, CFWM1 and triple beams increases with the power of $E_1$. One can see that the gain spectra of origin FWM and CFWM1 signals are split into two peaks at $P_1 = 210$ mW; correspondingly, the amplitude of FWM and CFWM1 signals is decreased at the position of the origin single gain peak, as shown in figures 6(d)–(f). Since, the generated signal beams are loaded in electromagnetic induced windows, and left peak in figure 2(d) and left peak in figure 6(d) is absorbed by the sample, only twoAT splitting peaks are
Figure 5. Relative IDS versus power of pumping field. (a) Power of \(E_1, P_1 = 41\) mW, (b) theoretical curves of squeezing in (a), \(S_{2-1}\) (the middle curve), \(S_{2-3}\) (the upper curve), \(S_{2-1-3}\) (the bottom curve). (c) The power of \(E_1, P_1 = 170\) mW and (d) the power of \(E_3, P_1 = 66\) mW. \(S_{2-1}\) (square curve), \(S_{2-3}\) (circle curve), \(S_{2-1-3}\) (triangle curve). The background noise is subtracted from all of the traces.

shown figures 6(e) and (f). The main reason is that the dressing effect of field \(E_1\) must be considered when the power of filed \(E_1\) increases to a certain level. At this time, field \(E_1\) split energy level \(|3\rangle\) and creating two new energy level \(|G_1\rangle\) and \(|G_{-1}\rangle|41, 42\rangle\) subjected to suppression conditions \(\Delta_{S1} - \Delta_1 = 0\) and \(\Delta_{S2} - \Delta_{S1} = 0\) are satisfied. Therefore, the coefficient \(\kappa_1\) and \(\kappa_2\) decreases, resulting in degree of IDS of FWM, CFWM1 and triple beams deceases, as shown in figure 5(a). In addition, \(E_{S1}, E_{S2}\) and \(E_{S3}\) have two dressed states (figure 1(d)), and their corresponding spectra are shown in figures 6(d)–(f). This results indicates that triple beams quantum correlations have two coherent channels (satisfying energy conservation conditions \(\Delta_{S1} + \Delta_{S2} + \Delta_{S3} = \Delta_1 + \Delta_1' + \Delta_3, \Delta_{S1} + \Delta_{S2} + \Delta_{S3} = \Delta_1 + \Delta_1' + \Delta_3,\) where \(\Delta_{S1}\) and \(\Delta_{S2}\) are location of AT-splitting peaks), and show the property of the high-dimensional time–energy entangled state in cascade FWM process system. This result will provide multi-mode quantum resources which can potentially realize multimode entanglement and quantum networks.

In addition, when power of field \(E_3\) gradually increases, the degree of IDS of CFWM1 and triple beams is increasing, as shown in figure 5(b). The reason is that the gain of CFWM1 increases due to the increase of the power of field \(E_1\), resulting in the increase of degree of IDS of CFWM1 and triple beams. Although field \(E_3\) is independent of FWM process, field \(E_3\) can suppress the gain of FWM signal though the dressing effect. So, the dressing effect of field \(E_3\) increases as the power of field \(E_3\), leading to a gradual decrease in degree of IDS of FWM, as shown in the square curve in figure 5(b). Specially, when the power of field \(E_1\) is large enough large, such as \(P_3 = 50\) mW, field \(E_1\) is the same as field \(E_3\), which will also suppress the gain of CFWM1 and triple beams, resulting in the reduction of degree of IDS of CFWM1 and triple beams, as shown in circle and triangle curves in figure 5(b). More interesting, when the power of field \(E_3\) is changed from 200 mW to 260 mW and \(P_1 = 66\) mW, the degree of IDS of FWM, CFWM1 and triple beams is all decreased, even become anti-squeezing as shown in figure 5(c). Considered the dressing effect of two fields \(E_1\) and \(E_3\), the intensities of \(E_{S1}, E_{S2}\) and \(E_{S3}\) in figures 6(g)–(i) compared with that in figures 6(d)–(f),
which means that the gain of FWM and CFWM1 decrease. When the influence of system loss and noise is considered, the degree of IDS of FWM, CFWM1 and triple beams becomes positive, which means anti-squeezing, as shown in figure 5(c). Figure 5(d) shows corresponding to theoretical curves of figure 5(a).

4. Conclusion

In summary, we have discussed cascaded FWM amplifier in single hot rubidium vapor. We have observed the enhanced IDS for the three-mode process in the same hot atomic system while input probe field assisting the nonlinear gain of cascaded FWM process. When the tripartite squeezed states are generated by cascading two rubidium atomic cells, the quantum correlation does not occur between the two idler beams [27]. However, due to the potential existence of two pairs of EPR injections in our system, there can be experimentally observed degree of IDS between two idler beams. This system with IDS between any two modes and all three modes may help construct large-scale practical quantum networks. Compared to the simple FWM amplifier, the degree of IDS for three-mode process can be greatly improved in theory. We can also observe the transformation of two- and three-mode light beams from single-mode to multi-mode at frequencies domain by dressing effect of two pumping fields. In addition, the proposed method is also immune to phase instabilities due to its phase insensitive nature, can directly obtain three-mode in one process as it avoids the phase locking required by linear beam splitter method.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).
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