Spinorial Structure of $O(3)$ and Application to Dark Sector

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An $O(3)$ spinor, $\Phi$, as a doublet denoted by $2_D$ consists of an $SO(3)$ spinor, $\phi$, and its complex conjugate, $\phi^*$, which form $\Phi = (\phi, \phi^*)^T$ to be identified with a Majorana-type spinor of $O(4)$. The four gamma matrices $\Gamma_\mu (\mu = 1 \sim 4)$ are given by $\Gamma_i = \text{diag}_{\text{2q}}(\tau_i, \tau_i^*)$ $(i = 1, 2, 3)$ and $\Gamma_4 = -\tau_2 \otimes \tau_2$, where $\tau_i$ denote the Pauli matrices. The rotations and axis-reflections of $O(3)$ are, respectively, generated by $\Sigma_{ij}$ and $\Sigma_{i4}$, where $\Sigma_{\mu\nu} = [\Gamma_\mu, \Gamma_\nu]/2i$. While $\Phi$ is regarded as a scalar, a fermionic $O(3)$ spinor is constructed out of an $SO(3)$ doublet Dirac spinor and its charge conjugate. These $O(3)$ spinors are restricted to be neutral and cannot carry the standard model quantum numbers because they contain particles and antiparticles. Our $O(3)$ spinors serve as candidates of dark matter. The $O(3)$ symmetry in particle physics is visible when the invariance of interactions is considered by explicitly including their complex conjugates. It is possible to introduce a gauge symmetry based on $SO(3) \times Z_2$ equivalent to $O(3)$, where the $Z_2$ parity is described by a $U(1)$ charge giving 1 for a particle and $-1$ for an antiparticle. The $SO(3)$ and $U(1)$ gauge bosons turn out to transform as the axial vector of $O(3)$ and the pseudoscalar of $O(3)$, respectively. This property is related to the consistent definition of the nonabelian field strength tensor of $O(3)$ or of the $U(1)$ charge of the $O(3)$-transformed spinor.

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I. INTRODUCTION

The cosmological observation of dark matter [1] has inspired theoretical interest in seeking possible physics of dark matter [2]. However, theoretical description of dark matter to date remains unclear and a candidate of dark matter is not provided by the standard model. A simple candidate exhibits the property that dark matter does not couple to the standard model particles and is present in the so-called dark sector [3]. The dark sector (or hidden sector) may contain new particles that couple only indirectly to ordinary matter. These new particles are expected to have masses well below the weak-scale. The dark sector particles communicate with the standard model particles via “dark matter portals”. The typical dark matter portal includes the coupling of Higgs particle to dark scalars (as a Higgs portal), of flavor neutrinos to sterile neutrinos (as a neutrino portal) and of photons to dark photons (as a vector portal) [4].

The key issue to discuss is how to realize the constraint on the dark sector particles that they do not have direct couplings to ordinary matter. The appropriate invariance of the dark sector based on certain symmetries may forbid the dark sector particles to couple to ordinary matter. Discrete symmetries such as $Z_2$ and a $U(1)$ symmetry are the simpler candidates that also ensure stability of dark matter although the origin of the symmetries is not naturally understood except for the need to constrain the dark sector.

The constraint on the dark sector is naturally satisfied if dark matter transforms as the recently advocated spinorial doublet of the $O(3)$ symmetry [5]. We have presented an irreducible representation $2_D$ as the spinorial doublet of $O(3)$, which is applied to models of quarks and leptons possessing the discrete $S_4$ symmetry that has been considered as a promising flavor symmetry of quarks and leptons [6]. Since $2_D \otimes 2_D = 1 \oplus 3$, the $O(3)$ spinor provides $S_4$-invariant couplings to the standard model particles. Since $S_4$ is known as a subgroup of $O(3)$, the $S_4$ spinor can be based on a four component $O(3)$ spinor, which is composed of the two component $SO(3)$ spinor and its complex conjugate. A remarkable feature is that the $O(3)$ spinor as an elementary particle contains a particle as the $SO(3)$ spinor and an antiparticle as its conjugate in the same multiplet of $2_D$ so that our $O(3)$ spinors cannot have no quantum numbers of the standard model. It is clear that the definition of the $O(3)$ spinor itself forbids dark sector particles to directly couple to ordinary matter if dark matter consists of the $O(3)$ spinor.

In this paper, we would like to enlarge our previous argument [5] on the possible existence of an $O(3)$ spinor on the basis of the four component spinor of the $O(4)$ symmetry and to include the physical aspect of the $O(3)$ spinor. Since particle physics involves fermionic degrees of freedom, our $O(3)$ spinors must include a fermionic $O(3)$ spinor. To introduce the fermionic $O(3)$ spinor needs careful examination because it is simultaneously...
the Dirac spinor. As expected, it is understood that the charge conjugate is employed instead of the complex conjugate to be consistent with the Dirac spinor. We have to confirm that such a fermionic spinor correctly transforms under the $O(3)$ transformation so that the internal $O(3)$ symmetry becomes orthogonal to the Lorentz symmetry.

It is further demonstrated that a gauged $O(3)$ symmetry is based on the mathematical equivalence of $O(3)$ to $SO(3) \times \mathbb{Z}_2$, where $\mathbb{Z}_2$ is generated by the parity inversion. For the $O(3)$ spinor, it is found that $\mathbb{Z}_2$ is described by a spinorial $\mathbb{Z}_2$ parity operator, whose eigenvalues are 1 for the particle as the $SO(3)$ spinor and $-1$ for the antiparticle as its conjugate that does imply the appearance of a $U(1)$ symmetry. The typical feature of the gauged $O(3)$ symmetry is that the gauge bosons transform as the axial vector of $O(3)$ (or the pseudoscalar of $O(3)$). This property is related to the consistent definition of the non-abelian field strength tensor (or of the $U(1)$ charge of the $O(3)$-transformed spinor).

The present article is organized as follows: In Sec. II, we clarify spinorial structure of $O(3)$ based on the $O(4)$ symmetry. In Sec. III, we discuss application of the $O(3)$ spinor to particle physics. For a scalar and a fermion taken as the $O(3)$ spinor, we construct $O(3)$-invariant lagrangians. The consistency with the Dirac spinor is clarified. Also discussed is the possible inclusion of the gauged $O(3)$ symmetry based on $SO(3) \times \mathbb{Z}_2$. Section IV deals with discussions on our candidates of dark matter and dark gauge bosons. A variety of implementation of dark gauge symmetries based on $O(3)$ is discussed. The final section Sec.V is devoted to summary and discussions.

II. $O(3)$ SPINOR

A. Manifestation of Spinorial Structure

We have advocated the use of the $O(3)$ spinor doublet as an $S_4$ spinor representation, which is composed of an $SO(3)$ spinor $\phi$:

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$$

(1)

from which $\Phi$ as the $O(3)$ spinor is defined to be:

$$\Phi = \begin{pmatrix} \phi \\ \phi^* \end{pmatrix},$$

(2)

as a four component spinor, transforming as $2D[5]$. Since $O(3)$ is contained in the $O(4)$ symmetry, which enables us to discuss the parity inversion, we first introduce the standard form of the gamma matrices $\gamma_\mu (\mu = 1 \sim 4)$:

$$\gamma_i = \begin{pmatrix} \tau_i & 0 \\ 0 & -\tau_i \end{pmatrix} = \tau_i \otimes \tau_3, \quad \gamma_4 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} = I \otimes \tau_1,$$

(3)

where $\tau_i$ ($i = 1, 2, 3$) stand for the three Pauli matrices. The action of generators $\gamma_{ij} = [\gamma_i, \gamma_j]/2i$ on $\Phi$ must be compatible with the definition of $\Phi = (\phi, \phi^*)^T$. It is known that this kind of $\Phi$ takes the form of $(\phi, i\tau_2 \phi^*)^T$ as the proper spinor of $O(4)$, which can be regarded as a Majorana representation of the $O(4)$ spinor. For $(\phi, \phi^*)$, it is straightforward to find $\Gamma_\mu$ from $\gamma_\mu$ by $W = \text{diag.}(I, i\tau_2)$:

$$\Gamma_i = W^\dagger \gamma_i W = \begin{pmatrix} \tau_i & 0 \\ 0 & \tau_i^* \end{pmatrix},$$

$$\Gamma_4 = W^\dagger \gamma_4 W = \begin{pmatrix} 0 & i\tau_2 \\ -i\tau_2 & 0 \end{pmatrix} = -\tau_2 \otimes \tau_2,$$

(4)

leading to the generators $\Sigma_{\mu \nu} = [\Gamma_\mu, \Gamma_\nu]/2i$ of $O(4)$ acting on $\Phi$.

The $i$-th axis reflection of $O(3)$ on $\Phi^\dagger \Gamma_i \Phi$ as the vector of $O(3)$ is induced by $\Sigma_{ij}$, which are explicitly written as

$$\Sigma_{14} = \begin{pmatrix} 0 & i\tau_3 \\ -i\tau_3 & 0 \end{pmatrix}, \quad \Sigma_{24} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix},$$

$$\Sigma_{34} = \begin{pmatrix} 0 & -i\tau_1 \\ i\tau_1 & 0 \end{pmatrix}.$$

(5)

Altogether, we obtain the following rotation matrices of $O(3)$ denoted by $D_{ij}(\sigma, \theta)$ for $\sigma = 1$ and $\sigma = -1$, respectively, taking care of the $SO(3)$ rotations and those including the parity inversion:

$$D_{ij} (1, \theta) = \exp \left( -i \frac{\Sigma_{ij} \theta}{2} \right),$$

$$D_{ij} (-1, \theta) = \exp \left( -i \frac{\Sigma_{ij} \theta}{2} \right) \Sigma_{4i4}.$$

(6)

In terms of the $SO(3)$ rotations on $\Phi$ denoted by $S_{ij}(1, \theta) = \exp(-i\tau_{ij}\theta/2)$, where $\tau_{ij} = [\tau_i, \tau_j]/2i$, $D_{ij}(\sigma, \theta)$ are expressed as follows [5]:

$$D_{ij} (1, \theta) = \begin{pmatrix} S_{ij}(1, \theta) & 0 \\ 0 & S_{ij}^*(1, \theta) \end{pmatrix},$$

(7)

and

$$D_{ij} (-1, \theta) = \begin{pmatrix} 0 & S_{ij}^*(-1, \theta) \\ S_{ij}(-1, \theta) & 0 \end{pmatrix},$$

(8)

where

$$S_{12}(-1, \theta) = iS_{12}(1, \theta)\tau_3, \quad S_{23}(-1, \theta) = S_{23}(1, \theta),$$

$$S_{31}(-1, \theta) = -iS_{31}(1, \theta)\tau_1.$$

(9)

The definition of $\Phi$ given by Eq. (2) can be generalized to include a parameter $\eta$, which commutes with $S_{ij}(1, \theta)$. The generalized form of the $O(3)$ spinor takes the form of

$$\Phi = \begin{pmatrix} \phi \\ \eta \phi^* \end{pmatrix},$$

(10)

for $\eta^* \eta = I$, or equivalently,

$$\Phi_{a+2} = \eta \Phi_{a},$$

(11)
for $a = 1, 2$, as the consistency condition on $\Phi$, which will be used to find the fermionic $O(3)$ spinor. The action of $D_{ij}(-1, \theta)$ on $\Phi$ yields
\begin{align}
\Phi' &= S_{ij}(-1, \theta)_{ab} \Phi_{b+2}, \\
\Phi'_{a+2} &= S_{ij}(-1, \theta)_{ab} \Phi_{b},
\end{align}

where $P(3)$ denotes the parity operator given by $P(3) = \text{diag}(-1, -1, -1)$ and
\begin{align}
T_{12}(\sigma, \theta) &= \begin{pmatrix} \sigma \cos \theta & -\sin \theta & 0 \\ \sigma \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
T_{23}(\sigma, \theta) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sigma \cos \theta & -\sin \theta \\ 0 & \sigma \sin \theta & \cos \theta \end{pmatrix}, \\
T_{31}(\sigma, \theta) &= \begin{pmatrix} \cos \theta & 0 & \sigma \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \sigma \cos \theta \end{pmatrix}.
\end{align}

The operator $\tilde{P}$ corresponding to $P(3)$ plays a rôle of the spinorial parity operator, whose eigenvalues are $(1, 1, -1, -1)$. As a result, $\tilde{P}$ is equivalent to the following operator $P$:
\begin{align}
P = -i \Gamma_1 \Gamma_2 \Gamma_3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},
\end{align}

which is responsible for the $Z_2$ parity in the spinorial space. As a result, $\phi$ and $\phi^*$ are distinguished by the parity operator, implying the appearance of a $U(1)$ symmetry associated with $P$. Note that there is a relation of
\begin{align}
\Sigma_i = PT_i = \Gamma_i P.
\end{align}

C. G-Parity

We introduce a G-conjugate state, $\Phi^G$, which corresponds to a complex conjugate transforming as $2D$. The G-conjugate state can be defined by
\begin{align}
\Phi^G = G\Phi^*,
\end{align}

where
\begin{align}
G = i \Sigma_2.
\end{align}

Instead of Eq. (22), we also find that
\begin{align}
\Phi^G = i \tilde{P} \Phi^*.
\end{align}

This spinor doublet is transformed by $D^G_{ij}(\sigma, \theta)$:
\begin{align}
D^G_{ij}(\sigma, \theta) = GD^*_{ij}(\sigma, \theta)G^{-1},
\end{align}

which should be equal to $D_{ij}(\sigma, \theta)$ up to phases so that $\Phi^G$ transforms as $2D$. For Eq. (17), $D^G_{ij}(\sigma, \theta) = D_{ij}(\sigma, \theta)$ is satisfied.

D. Axial vector and pseudoscalar of $O(3)$

Since $\Gamma_m$ is the vector of $O(4)$, $\Gamma_i$ is the $O(3)$ vector. By the action of $P$ as the spinorial version of $P(3)$, we
have the following results,
\[
\begin{align*}
\mathcal{P}^\dagger \Gamma_i \mathcal{P} &= -\Gamma_i, \quad \mathcal{P}^\dagger \tilde{\Gamma}_i \mathcal{P} = -\tilde{\Gamma}_i, \\
\mathcal{P}^\dagger \Sigma_i \mathcal{P} &= \Sigma_i, \quad \mathcal{P}^\dagger \tilde{\Sigma}_i \mathcal{P} = \tilde{\Sigma}_i, \\
\mathcal{P}^\dagger \tilde{P} \mathcal{P} &= -\tilde{P},
\end{align*}
\]
where \( \Sigma_i = \Sigma_{4i} \) and \( \tilde{\Gamma}_i = P \tilde{\Sigma}_i \). These relations correctly describe the property of the parity inversion that the vector changes its sign but the axial vector does not change its sign and that the pseudoscalar changes its sign. Therefore, we obtain that

- \( \tilde{\Gamma} \) and \( \mathcal{P} \tilde{\Gamma} \) are the vectors of \( O(3) \);
- \( \tilde{\Sigma} \) and \( \mathcal{P} \tilde{\Sigma} \) are the axial vectors of \( O(3) \);
- \( P \) is the pseudoscalars of \( O(3) \),

where
\[
\tilde{\Gamma} = \Gamma_1 \mathcal{P} \tilde{\tau}_1 + \Gamma_2 \mathcal{P} \tilde{\tau}_2 + \Gamma_3 \mathcal{P} \tilde{\tau}_3, \tag{27}
\]
and similarly for \( \tilde{\Sigma} \) and \( \mathcal{P} \tilde{\Sigma} \). The rotations generated by \( D_{ij}(\sigma, \theta) \) provide the correct description of the pseudoscalar-rotation because \( D_{ij}(\sigma, \theta) \) turn out to give
\[
\begin{align*}
D_{ij}^\dagger (\sigma, \theta) \Gamma_m D_{ij} (\sigma, \theta) &= T_{ij} (\sigma, \theta)_{mn} \Gamma_n, \\
D_{ij}^\dagger (\sigma, \theta) \Sigma_m D_{ij} (\sigma, \theta) &= \sigma T_{ij} (\sigma, \theta)_{mn} \Sigma_n, \\
D_{ij}^\dagger (\sigma, \theta) \tilde{P} D_{ij} (\sigma, \theta) &= \mathcal{P} P.
\end{align*}
\]
These transformation properties are consistent with those indicated by Eq. (26). For the remaining \( SO(3) \) vectors, \( \tilde{\Gamma} \) and \( \tilde{\Sigma} \), since there are relations of \( \tilde{\Gamma} \Phi = -i \mathcal{P} \tilde{\Sigma} \Phi \) and \( \tilde{\Sigma} \Phi = -i \mathcal{P} \tilde{\Sigma} \Phi \), the action on \( \tilde{\Gamma} \) and \( \tilde{\Sigma} \) can be understood from that on \( \Gamma \) and \( \Sigma \).

We note that the property of \( P \) being the pseudoscalar operator provides us an important observation on the transformed state of \( \Phi \) given by \( \Phi' = D_{ij}(-1, \theta) \Phi \). The eigenvalues of \( P \) for \( \Phi' \) are calculated by
\[
P \Phi' = D_{ij}(-1, \theta)(-P \Phi), \tag{29}
\]
from which we find that \(-P \Phi \) is transformed into \( P \Phi' \). The eigenvalues of \( P \) for \( \Phi' \) are opposite in signs to \( \Phi \). In terms of \( \phi, \phi' = S_{ij}(-1, \theta) \phi^* \) given by \( D_{ij}(-1, \theta) \) indicates that \( P \) for \( \phi' \) is equal to \( P \) for \( \phi^* \). In this pseudoscalar case, \( \phi' = S_{ij}(-1, \theta) \phi^* \) and its complex conjugate are consistent with \( P \Phi' = D_{ij}(-1, \theta)(-P \Phi) \). The parity operator \( P \) being the pseudoscalar is a key ingredient to later introduce a \( U(1) \) symmetry into the \( O(3) \) spinor.

III. PARTICLE PHYSICS

To see the feasibility of the \( O(3) \) spinors in particle physics, we construct \( O(3) \)-invariant lagrangians for a bosonic and fermionic \( O(3) \) spinor. It is found that the appearance of the \( O(3) \) symmetry in particle physics can be rephrased as “the \( O(3) \) symmetry gets visible when we consider the invariance of interactions by including their complex conjugates”.

A. Scalar

Let us start by utilizing \( \Phi \) as a scalar. As already demonstrated, \( \Phi^\dagger \Phi \) and \( \Phi^\dagger \Gamma_i \Phi \) behave as \( 1 \) and \( 3 \), respectively. Because of \( \tilde{X} \Phi = -i X \Phi \), where \( \tilde{X} \) represents \( \Gamma_i \) and \( \Sigma_i \), bilinears containing \( \tilde{\Gamma}_i \) or \( \tilde{\Sigma}_i \) are equivalent to those containing \( \Gamma_i \) or \( \Sigma_i \). The relevant bilinears are \( \tilde{\Phi} \tilde{\Phi} \), \( \tilde{\Phi} \Sigma_i \Phi \), \( \Phi^\dagger \Phi \), \( \Phi^\dagger \Sigma_i \Phi \), \( \Phi^\dagger \Phi \Gamma_i \Phi \), and \( \Phi^\dagger \Sigma_i \Phi \). The computations are straightforward to obtain that

1. \( \Phi^\dagger \Phi(\Phi^\dagger \Phi \Phi^\dagger \Phi) = \phi^\dagger \phi + \phi^T \phi^* = 2 \phi^\dagger \phi; \)
2. \( \Phi^\dagger \Phi \Phi(\Phi^\dagger \Phi \Phi^\dagger \Phi) = \phi^\dagger \phi - \phi^T \phi^* = 0; \)
3. \( \Phi^\dagger \Sigma_i \Phi(\Phi^\dagger \Phi \Phi^\dagger \Phi) = \phi^\dagger \tau_i \phi + \phi^T \tau_i^* \phi^* = 2 \phi^\dagger \tau_i \phi; \)
4. \( \Phi^\dagger \Sigma_i \Phi(\Phi^\dagger \Phi \Phi^\dagger \Phi) = \phi^\dagger \tau_i \phi - \phi^T \tau_i^* \phi^* = 0; \)
5. \( \Phi^\dagger \Sigma_i \Phi = -i (\phi^T \tau_2 \phi - \phi^\dagger \tau_2^* \phi^*) = 0, \quad \tau_2 = -\tau_2^T; \)
6. \( \Phi^\dagger \Sigma_i \Phi = -i (\phi^T \tau_2 \phi + \phi^\dagger \tau_2^* \phi^*) = 0; \)
7. \( \Phi^\dagger \Sigma_i \Phi = -i (\phi^T \tau_2 \phi + \phi^\dagger \tau_2^* \phi^*), \quad \tau_2 \tau_i = (\tau_2 \tau_i)^T; \)
8. \( \Phi^\dagger \Sigma_i \Phi = -i (\phi^T \tau_2 \phi + \phi^\dagger \tau_2^* \phi^*), \)

where relations of
\[
(\Phi^\dagger \Gamma_i \Phi)^\dagger = \Phi^\dagger \Gamma_i \Phi, \quad (\Phi^\dagger \Sigma_i \Phi)^\dagger = -\Phi^\dagger \Sigma_i \Phi, \tag{30}
\]
are satisfied. Therefore, interactions of scalars consist of \( \Phi^\dagger \Phi \) as the scalar, \( \Phi^\dagger \Gamma_i \Phi \) and \( \Phi^\dagger \Sigma_i \Phi \) as the vector and \( \Phi^\dagger \Sigma_i \Phi \) as the axial vector. The mass and kinetic terms are described by the form of \( \Phi^\dagger \Phi \). Quartic couplings consist of \( (\Phi^\dagger \Phi)^2 \) and of the appropriate products out of \( \Phi^\dagger \Gamma_i \Phi \), \( \Phi^\dagger \Sigma_i \Phi \) and \( \Phi^\dagger \Sigma_i \Phi \). A Fierz identity relates these quartic terms. In fact,
\[
(\Phi^\dagger \Gamma_i \Phi)(\Phi^\dagger \Gamma_i \Phi) = (\Phi^\dagger \Phi)^2, \tag{31}
\]
is satisfied. Similarly, we find that
\[
(\Phi^\dagger \Gamma_i \Phi)(\Phi^\dagger \Sigma_i \Phi) = (\Phi^\dagger \Sigma_i \Phi)(\Phi^\dagger \Gamma_i \Phi) = (\Phi^\dagger \Phi)^2, \tag{32}
\]
\[
(\Phi^\dagger \Gamma_i \Phi)(\Phi^\dagger \Sigma_i \Phi) = (\Phi^\dagger \Sigma_i \Phi)(\Phi^\dagger \Gamma_i \Phi) = 0. \tag{33}
\]
As quartic couplings, it is sufficient to use \((\Phi^4)^2\). Since \(\Phi^4 = 2\phi^4\), the lagrangian of \(\Phi\), \(\mathcal{L}_\Phi\), is determined to be

\[
\mathcal{L}_\Phi = \frac{1}{2} \partial^\mu \Phi^\dagger \partial_\mu \Phi - V_\Phi,
\]

\[
V_\Phi = \frac{\mu_\phi^2}{2} \Phi^4 + \frac{\lambda_\phi}{4} (\Phi^\dagger \Phi)^2,
\]

where \(\mu_\phi\) and \(\lambda_\phi\) stand for a mass and a quartic coupling, respectively. \(\mathcal{L}_\Phi\) is also invariant under a phase transformation induced by \(\phi' = e^{-iq\phi}\) as a global \(U(1)\) symmetry, where \(q\) is real. In terms of \(\Phi\), the invariance of \(\mathcal{L}_\Phi\) is associated with the transformation of

\[
\Phi' = D_{ij} (\sigma, \theta) e^{-iq^\mu \Phi} \frac{e^{-iq\Phi}}{2} D_{ij} (\sigma, \theta) \Phi,
\]

where \(P\) changes its sign for \(\sigma = -1\) because of Eq. (28), reflecting the fact that \(P\) for \(\phi^*\) is equal to \(P\) for \(\phi\). This charge given by \(P\) can be interpreted as a \(\Phi\)-number, which is referred to the “dark number”. It should be noted that the transformation of \(\Phi' = e^{-iq\Phi}\) such as in \(U(1)\) of the standard model is incompatible with the definition of \(\Phi\) because the transformed state \(\Phi'\) cannot satisfy the consistency condition of Eq. (14).

Although \(\mathcal{L}_\Phi\) expressed in terms of \(\Phi\) is nothing but a lagrangian expressed in terms of \(\phi_{1,2}\), the \(O(3)\) structure can be recognized if its complex conjugate is treated on the same footing. For example, the identity of \(2\phi^4 = \sum_{a=1}^2 (\phi^a_\dagger \phi_a + \phi^a_\dagger \phi_a)\) gives \(\Phi^4\). In this simple case, it is invariant under the interchange of \(\phi \leftrightarrow \phi^*\), which is caused by the parity operator \(iP\) of Eq. (15).

### B. Vectorlike fermion

#### 1. Consistency with the Dirac fermion

For the Dirac fermion denoted by \(\psi\), the fermionic \(O(3)\) spinor, which is consistent with the Dirac spinor, should be composed of \(\psi^C\) instead of \(\psi^*\) to form \((\psi, \psi^C)^T\), where \(\psi^C\) is a charge conjugation of \(\psi\) defined by \(\psi = C\bar{\psi}^T\). The charge conjugation operator, \(C\), satisfies that

\[
C^{-1} \gamma^\mu C = -\gamma^\mu T, \quad C^\dagger = C^{-1}, \quad CT = -C,
\]

where \(\gamma^\mu\) (\(\mu = 0, 1, 2, 3\)) represents the Dirac gamma matrices. The definition of the \(O(3)\) spinor demands that \(\psi^C \neq \psi\). Since the \(O(3)\) spinor is required to be neutral, the possible candidate is a Majorana fermion, which is, however, excluded because of \(\psi^C = \psi\). Our neutral \(O(3)\) spinor carries the different \(Z_2\) charge; therefore, \(\psi\) and \(\psi^C\) are distinguished.

Let us start with a pair of Dirac fields, \(\psi_1\) and \(\psi_2\), which form

\[
\xi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix},
\]

as the \(SO(3)\) doublet spinor and \(\xi^C = (\psi_1^C, \psi_2^C)^T\) as its charge conjugate. In terms of \(\xi\), the \(O(3)\) doublet spinor is expressed to be \(\Psi\):

\[
\Psi = \begin{pmatrix} \xi \\ \xi^C \end{pmatrix}.
\]

The fermionic spinor behaves as the \(O(3)\) spinor if \(\Psi\) obeys the \(O(3)\) transformation of \(\Psi' = D_{ij} (\sigma, \theta) \Psi\). The \(O(3)\) spinor compatible with the transformation must satisfy Eq. (11). For the fermionic \(O(3)\) spinor, since \(\Psi_{a+2} = \xi^a = C\gamma^0 \xi^a\) holds, the parameter \(\eta\) in Eq. (11) is given by

\[
\eta = C\gamma^0 T,
\]

which turns out satisfy the necessary condition of \(\eta^* \eta = 1\). As a result, we confirm that \(\Psi\) is the \(O(3)\) spinor, which is subject to the correct \(O(3)\) transformation:

\[
\Psi' = D_{ij} (\sigma, \theta) \Psi.
\]

The use of \(\psi^C\) instead of \(\psi^*\) to form \(\Psi\) assures the orthogonality of the \(O(3)\) symmetry to the Lorentz symmetry. It is obvious that the fermionic \(O(3)\) spinor is vectorlike. If it is not vectorlike, \(\xi\) and \(\xi^C\) have different chiralities so that \(\xi'^C = S^C_{0,1}(-1, \theta) \xi\) given by Eq. (40) cannot be satisfied.

#### 2. Lagrangian of the fermionic \(O(3)\) spinor

Treating charge conjugates on the same footing, we use the familiar relations given by \(\bar{\psi}_a i\partial \psi_a = \bar{\psi}^C_a i\partial \psi^C_a\) up to the total derivative and \(\bar{\psi}_a \psi_a = \bar{\psi}^C_a \psi^C_a\). The lagrangian of \(\psi_a\) can be defined by

\[
\mathcal{L}_\Psi = \frac{1}{2} \sum_{a=1}^2 \bar{\psi}_a \left( i\partial - m_\psi \right) \psi_a
\]

\[
= \frac{1}{2} \sum_{a=1}^2 \left[ \bar{\psi}_a \left( i\partial - m_\psi \right) \psi_a + \bar{\psi}^C_a \left( i\partial - m_\psi \right) \psi^C_a \right]
\]

\[
= \frac{1}{2} \bar{\Psi} \left( i\partial - m_\psi \right) \Psi,
\]

where \(m_\psi\) is a mass of \(\psi_a\). The invariance under the \(O(3)\) transformation of \(D_{ij} (\sigma, \theta)\) acting on \(\Psi\) is satisfied by \(\mathcal{L}_\Psi\) because of \(\bar{\Psi} = \Psi^T \gamma^0\), where \(\gamma^\mu\) commutes with \(D_{ij} (\sigma, \theta)\). The possible bilinears including \(\bar{\Psi} \Psi\) are expressed in terms of \(\xi\) as follows:

\[
\bar{\Psi}\Psi = 2\xi_5 \xi, \quad \bar{\Psi} P \Psi = 0,
\]

\[
\bar{\Psi} \Gamma_i \Psi = 2\xi_{5,1} \xi, \quad \bar{\Psi} \Sigma_i \Psi = 0,
\]

\[
\bar{\Psi} \gamma^\mu \Psi = 0, \quad \bar{\Psi} \gamma^\mu P \Psi = 2\xi_{5,1} \xi,
\]

\[
\bar{\Psi} \gamma^\mu \Gamma_i \Psi = 0, \quad \bar{\Psi} \gamma^\mu \Sigma_i \Psi = 2\xi_{5,1} \gamma^\mu \xi, \quad \bar{\Psi} \gamma^\mu \Sigma_i \Psi = 0.
\]
As for Yukawa coupling of $\Psi$, $\Phi$ accompanied by an $O(3)$-singlet Majorana fermion $N$ satisfying that $N^C = N$ can yield

$$\mathcal{L}_Y = -fN \sum_{a=1}^{2} \bar{\phi}_a \phi_a + \text{h.c.}, \quad (44)$$

where $f$ is a coupling constant. Considering relations of $N^C \phi_a = \bar{\phi}_a^C \phi_a N$ and $\bar{\phi}_a \phi_a N = N\phi_a \bar{\phi}_a^C$ to form $\Phi$ and $\Psi$, $\mathcal{L}_Y$ turns out to be

$$\mathcal{L}_Y = -\frac{1}{2} f \bar{\Phi} \Phi + \text{h.c.}. \quad (45)$$

The invariance under the $O(3)$ symmetry is satisfied because $\Phi^T \Psi$ is a singlet of $O(3)$. In $\mathcal{L}_Y$, $\Phi$ can be replaced by its G-conjugate $\Phi^G$. If both couplings of $\Phi$ and $\Phi^G$ are present, the $U(1)$ symmetry is not respected.

The lagrangian for $N$ can be expressed in terms of $N_R$ and $\Psi_L$. Since

$$N^C \phi_a = N^C \bar{\phi}_a \phi_a N = \bar{\phi}_a \phi_a N,$$

the resulting lagrangian is

$$\mathcal{L}_Y = -\frac{1}{2} f \bar{\Phi} \Phi + \text{h.c.}. \quad (47)$$

In a practical sense, $N_R$ can be taken as a right-handed neutrino that couples to flavor neutrinos (to be denoted by $\nu$). The Majorana mass of $N_R$ generates very tiny masses of $\nu$ due to the seesaw mechanism.

C. Gauge boson

Let us start with a lagrangian for an $SO(3)$-triplet vector, $V_{i\mu}$, and an $SO(3)$-singlet vector $U_\mu$ accompanied by $\phi$ and $\xi$:

$$\mathcal{L} = -\frac{1}{4} V_{i\mu}^\nu V_{i\mu}^\nu - \frac{1}{4} U_{\mu}^\nu U_{\mu}^\nu + \mathcal{L}_\phi + \mathcal{L}_\xi,$$

$$\mathcal{L}_\phi = \left| \left( \partial_\mu + igV_{i\mu} + ig' \frac{Y_U}{2} U_\mu \right) \phi \right|^2,$$

$$\mathcal{L}_\xi = \bar{\xi} \gamma^\mu \left( i\partial_\mu - gV_\mu - g' \frac{Y_U}{2} U_\mu \right) \xi,$$

for $g$ and $g'$ as coupling constants and $Y_U$ stands for an $U(1)$ charge, where mass terms are omitted for simplicity and

$$V_{i\mu} = \partial_\mu V_{i\mu} - \partial_\nu V_{i\nu} + g \varepsilon_{ijk} V_{j\mu} V_{k\nu},$$

$$U_\mu = \partial_\mu U_\mu - \partial_\nu U_\nu,$$

$$V_\mu = \frac{\gamma_5}{2} V_{i\mu}.$$

To introduce $\Phi$ and $\Psi$, the relations to the complex conjugate of $\mathcal{L}_{\phi, \xi}$:

$$\left| \left( \partial_\mu - igV_{i\mu} - ig' \frac{Y_U}{2} U_\mu \right) \phi \right|^2,$$

$$\overline{\xi} \gamma^\mu \left( i\partial_\mu + gV_\mu + g' \frac{Y_U}{2} U_\mu \right) \xi,$$

are explicitly used.

Now, after $V_\mu^T = V_{i\mu}^*$ is used for $\xi^C$, $\tau_i$ for $\phi$ and $\xi$ can be replaced by diag.$(\tau_i, -\tau_i^*)$ for $\Phi$ and $\Psi$, which is nothing but $\Sigma_i$. As a result, we find that

$$\mathcal{L}_\phi = \frac{1}{2} \left( \partial_\mu + igV_\mu + ig' \frac{Y_U}{2} U_\mu \right) \phi^2,$$

$$\mathcal{L}_\xi = \frac{1}{2} \bar{\Psi} \gamma^\mu \left( i\partial_\mu - gV_\mu - g' \frac{Y_U}{2} U_\mu \right) \Psi,$$

where

$$V_\mu = \frac{\Sigma_i^2}{2} V_{i\mu}, \quad U_\mu = PU_\mu. \quad (52)$$

Hereafter, $\Psi$ is assumed to possess the same $Y_U$ as $\Phi$ so as to yield the vanishing $U(1)$ change of $\Phi^T \Psi$ in $\mathcal{L}_Y$. In $\mathcal{L}_{\phi, \xi}$, $\Phi \Sigma_i \partial_\mu \Phi, \Psi \gamma^\mu \Sigma_i \Psi$ and $\Phi^T P \partial_\mu \Phi, \Psi \gamma^\mu P \Psi$, respectively, transform as the axial vector and the pseudoscalar as indicated by Eqs. (26) and (28). It is concluded that

- $V_{i\mu}$ is the axial vector transformed by $V_{k\mu}' = \sigma T_{ij}(\sigma, \theta)_{ik} V_{i\mu}$;
- $U_\mu$ is the pseudoscalar transformed by $U_{\mu}' = \sigma U_\mu$,

to get the $O(3)$-invariant lagrangian. This feature may represent a remnant of the finite $O(3)$ transformation. It is understood that the gauged $SU(2) \times U(1)$ symmetry is generated by $\Sigma_i$ and $P$ linked to $SO(3) \times Z_2$.

Generally speaking, $V_{i\mu}$ should transform as the axial vector of $O(3)$. If $V_{i\mu}$ transforms as the vector of $O(3)$, the nonabelian term of $\varepsilon_{ijk} V_{j\mu} V_{k\nu}$ transforms as the axial vector of $O(3)$ because this term exhibits an antisymmetric configuration as the cross product of two vectors. The nonabelian term, which is the axial vector, is inconsistent with the abelian term of $\partial_\mu V_{i\nu} - \partial_\nu V_{i\mu}$, which is the vector. The consistent assignment to $V_{i\mu}$ is clearly to use the axial vector of $O(3)$. There is no such a constraint on matter field in the triplet representation because the similar nonabelian term involved in the covariant derivative is always consistent with the abelian term. For $U_\mu$ as the pseudoscalar of $O(3)$, the consistency is related to the fact that the eigenvalues of the $U(1)$ charge of the $O(3)$-transformed spinor are opposite in signs to the original spinor as stressed in Eq. (29).
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FIG. 1: The decay of $h \to \tilde{\gamma}\tilde{\gamma}$ in (A) unbroken $U(1)$ symmetry, (B) spontaneously broken $O(3)/U(1)$ and $O(3)$ symmetries, respectively, for the $V$-loop and the $W^\pm$-loop, and (C) spontaneously broken $O(3)/U(1)$ symmetry.

IV. DARK GAUGE SYMMETRY

In this section, we argue how our $O(3)$ spinors together with the gauge bosons are suitable to describe dark sector of particle physics. Our $O(3)$ spinors are restricted to be neutral and this restriction may be good news for dark matter, which is considered to be neutral. We do not intend to show detailed numerical analyses of effects of our $O(3)$ spinors on dark matter but how the spinorial $O(3)$ structure affects the dark sector physics, especially, based on a new realization of the $O(3)$ gauge symmetry. The $O(3)$ symmetry can be elevated to the $SU(2) \times U(1)$ gauge symmetry with $V_\mu$ in the axial vector representation and $U_\mu$ in the pseudoscalar representation, which couple to neutral $O(3)$ spinors. The $U(1)$ symmetry is related to the $Z_2$ parity of $O(3)$. Since our dark fermions are vectorlike, no chiral anomalies are present. Our gauge symmetry may serve as a dark gauge symmetry, which includes a dark photon as a dark gauge boson [17, 18]. Our gauged $O(3)$ model provides a vector portal dark matter [19].

The dark photon sometimes remains massless [17]. As the worst case, $\Phi$ and $\Psi$, provided that they are dark charged, are annihilated into the massless dark photon if the dominance of dark matter over dark antimatter is not effective. In the rest of discussions, we assume that this dominance is based on the universal mechanism over the universe [20, 21] so that dark matter remains in the dark sector after the annihilation is completed. For instance, since our interactions may contain $N_R$ as a right-handed neutrino in $L_Y$, the dominance of matter over antimatter can be based on the leptogenesis utilizing the lepton number violation due to $N_R$ [22], which in turn produces a dark matter particle-antiparticle asymmetry in the dark sector [23, 24]. Reviews on the dark matter and baryon abundances are presented in Ref. [25, 26], where further references can be found.

Dark matter scenarios based on the gauged $O(3)$ symmetry as a maximal gauge symmetry provide the similar scenarios to the existing ones [2]. However, the essential difference from the existing scenarios lies in the fact that our dark matter and dark gauge bosons are theoretically forced to be all neutral once the spinorial structure of $O(3)$ is built in the dark sector. From this feature, the kinetic mixing with the photon [27] is not a primary mixing with the standard model gauge bosons. Instead, the mixing with the $Z$ boson becomes important and arises as an induced loop effect that allows $Z$ to couple to intermediate flavor neutrinos. Another difference is that our gauge bosons do not transform as the triplet vector for $V_\mu$ and the singlet scalar for $U_\mu$ but as the axial vector and the pseudoscalar, respectively.

A. Unbroken $U(1)$ symmetry

The simplest realization of the dark photon denoted by $\tilde{\gamma}$ can be based on the $U(1)$ gauge model. When the $U(1)$ symmetry is not spontaneously broken, the dark photon stays massless. The dark photon interacts with the standard model particles via $H$. The relevant part of the lagrangian of $\Phi$ including $H$ is

$$L = \frac{1}{2} \bar{\Psi} \left[ \gamma^\mu \left( i\partial_\mu - g \frac{g_\Phi}{2} U_\mu \right) - m_\Phi \right] \Psi + L_Y + \frac{1}{2} \left( \partial_\mu + ig \frac{Y_\Phi}{2} U_\mu \right)^2 - V_S,$$

$$V_S = \mu^2_H H^\dagger H + \lambda_H (H^\dagger H)^2 + \frac{\mu^2_\Phi}{2} \Phi^\dagger \Phi + \frac{\lambda_\Phi}{4} (\Phi^\dagger \Phi)^2 + \frac{\lambda_{H,\Phi}}{2} H^\dagger H \Phi^\dagger \Phi,$$  

(53)

where $\mu_{H,\Phi}$ and $\lambda_{H,\Phi}$, respectively, stand for mass parameters and quartic couplings. The physical Higgs scalar $h$ is parameterized in $H = (0, (v + h)/\sqrt{2})$, where $v$ is a VEV of $H$.

Since $\Phi$ couples to the $U(1)$ gauge boson but does not couple to the standard model gauge bosons, the decay of $h$ proceeds via

$$\bullet \ h \to \tilde{\gamma}\tilde{\gamma} \text{ triggered by the one loop of } \Phi,$$

which is generated by $\Phi^\dagger \Phi H^\dagger H$ [28]. The decay of $h \to \tilde{\gamma}\tilde{\gamma}$ is depicted in FIG. including other two cases to be
discussed. The dark matter consists of the lightest $O(3)$ spinors. Because of $\Phi \to \Psi \overline{\nu}$ or $\overline{\Psi} \to \Phi \nu$ induced by $\mathcal{L}_Y$, depending upon the magnitude relation of masses of $\Phi$ and $\Psi$, the lightest $O(3)$ spinor is either $\varphi$ or $\Psi$, which has two degenerated components.

### B. Spontaneously broken $O(3)/U(1)$ symmetry

The next example is based on the $O(3)$ gauge symmetry without the $U(1)$ symmetry, which we refer as an $^{\prime}O(3)/U(1)^{\prime}$ symmetry. The relevant part of the lagrangian of $\Phi$ and $\Psi$ is

$$\mathcal{L} = \frac{1}{2} \bar{\Psi} \left[ i \partial_\mu - g V_\mu \right] \Psi + \mathcal{L}_Y + \frac{1}{2} \left[ (\partial_\mu + ig V_\mu) \Phi \right]^2 - V_S. \quad (54)$$

In addition to $\mathcal{L}_Y$, another $\mathcal{L}_Y$ as $\mathcal{L}'_Y$ given by

$$\mathcal{L}'_Y = -f \bar{N}_R \Phi \Phi^C \bar{N}_L + \text{h.c.}, \quad (55)$$

can be included because of the absence of $U(1)$. To generate spontaneous breakdown, we assume that $\phi_2$ in $\phi$ acquires a VEV. For $\mu_3^2 < 0$ in Eq. (54), $\phi_2$ develops a VEV that spontaneously breaks $O(3)/U(1)$. The physical scalar denoted by $\varphi$ is parameterized in $\phi = (0, (v_\phi + \varphi)/\sqrt{2})$ for $v_\phi$ as a VEV of $\Phi$. Three of the four real components in $\Phi$ are absorbed into three dark gauge bosons, which become massive. The remaining scalar is a massive $\varphi$. Interactions of scalars provide the following results:

- $h$ and $\varphi$ are mixed via the mass term of $M_{mix}$ calculated to be:
  $$M_{mix} = \begin{pmatrix} m_h^2 & 2 \lambda_\Phi H v_\Phi v \\ 2 \lambda_\Phi H v_\Phi v & m_\varphi^2 \end{pmatrix}, \quad (56)$$
  where $m_h^2 = 2 \lambda_H v^2$ and $m_\varphi^2 = 2 \lambda_\Phi v_\Phi^2$;
- The massive dark gauge bosons have a degenerated mass of $g v_\Phi/2$ and play a role of $\tilde{\gamma}$;
- $h \to \tilde{\gamma} \tilde{\gamma}$ via the one-loop by $V$ (FIG. 4(B) and by $\Psi$ and $N_R$ (FIG. 4(C)) if the decay is kinematically allowed.

The mixing of $h$ and $\varphi$ should be carefully examined to be consistent with the currently established Higgs phenomenology [30] [33]. For a reference value of the Higgs mixing angle $\theta_H$ associated with Eq. (56), we quote that $|\tan \theta_H| \lesssim 2.2 \times 10^{-3} (v_\Phi/10 \text{ GeV})$ derived from the

1. Needless to say, the condition of $\psi \neq \psi^C$, which is ascribed to $U(1)$, is also fulfilled by the $SO(3)$ spinor property that ensures $\psi \neq \psi^C$
2. For $\langle \Phi \rangle \propto \Gamma_3$ leading to $U(1)$ as a residual symmetry, see Ref. [29].

### C. Spontaneously broken $O(3)$ symmetry

The relevant part of the lagrangian of $\Phi$ and $\Psi$ is

$$\mathcal{L} = \frac{1}{2} \bar{\Psi} \left[ i \partial_\mu - g V_\mu - \frac{Y_{\Phi}}{2} U_\mu \right] \Psi + \mathcal{L}_Y + \frac{1}{2} \left[ (\partial_\mu + ig V_\mu - \frac{Y_{\Phi}}{2} U_\mu) \Phi \right]^2 - V_S. \quad (57)$$

In the case that the $O(3)$ symmetry is spontaneously broken, if the appropriately defined $U(1)$ charge is not carried by $\phi_2$, there remains a massless gauge boson chosen to be $\tilde{\gamma}$ as a mixed state of $V_{3\mu}$ and $U_\mu$ as in the standard model [54]. Since the $U(1)$ invariance is assumed in the $O(3)$ symmetry, $\mathcal{L}'_Y$ cannot be included. To be specific, without the loss of the generality, $Y_\Phi = 1$ for $\Phi$ can be chosen. Since everything is the same as in the standard model, we use the tilde to denote dark gauge bosons with the obvious notations.

The massless dark photon identified with $\tilde{A}$ can couple to dark charged particles. For $\Psi$, $\psi_1$ is a dark charged particle while $\psi_2$ is a dark neutral particle because $Y_{\Psi} = Y_{\phi}$ is taken in Eq. (57). The remaining dark charged particles are $W^\pm$. The decay of $h \to \tilde{\gamma} \tilde{\gamma}$ needs both $\varphi$ and $\tilde{\gamma}$ to couple to intermediate particles. The possible intermediate particles are $W^\pm$ since $\varphi$ does not couple to the dark charged $\psi_1$ while $\tilde{\gamma}$ does not couple to the dark neutral $\phi_2$ and $\varphi$. The diagram as in FIG. 4(C) is absent. The decay of $h$ proceeds via $W^\pm$:

- $h \to \tilde{\gamma} \tilde{\gamma}$ via the one loop by $W^\pm$,
  because $W^\pm$ interact with both $\varphi$ and $\tilde{\gamma}$ as shown in FIG. 4(B).

Our dark matter consists of $\varphi$ and $\psi_1$. For other candidates of $W^\pm$ and $Z$, $W^\pm$ disappear because $W^+ W^-$ are annihilated into $\tilde{\gamma} \tilde{\gamma}$ and $Z$ decays into $\nu \bar{\nu}$ owing to the $\tilde{Z}$ mixing with the $Z$ boson (see Sec. IV D). There arises an interaction between $\varphi$ and $\psi_2$, which is $\bar{\nu}_L \nu_R$ induced by the collaboration of $\bar{N}_R \varphi \nu_2$ and $\bar{N}_L \nu_2$. Due to this interaction, the heavier $O(3)$ spinor will decay according to the decay mode of either $\varphi \to \nu_2 \overline{\nu}_2$ or $\psi_2 \to \nu \nu / \phi_2 \bar{\nu}$. For $\psi_1$, these decay modes yielding $\nu$ are absent because $\mathcal{L}'_Y$ is absent. Since $\psi_1, 2$ are degenerated, $\psi_1$ is stable. The dark charged $W^\pm$ are annihilated into dark photons while $Z$ may decay into $\ell^+ \ell^-$, $\nu \bar{\nu}$ and so on. We conclude that

- our dark matter consists of $\psi_1$ and the lighter spinor, which is either $\varphi$ or $\psi_2$.
FIG. 2: The $\tilde{\gamma}$-$Z$ mixing in (A) unbroken $U(1)$ symmetry, where $\Phi$ and $\Psi$ can be interchanged, (B) spontaneously broken $O(3)/U(1)$ symmetry and (C) spontaneously broken $O(3)$ symmetry, where $\tilde{W}^+$ and $\psi_1$ can be interchanged.

TABLE I: Particle content of the dark sector.

| model | Sec IV A | Sec IV B | Sec IV C |
|-------|----------|----------|----------|
| gauge symmetry | $U(1)$ | $O(3)/U(1)$ | $O(3)$ |
| dark matter candidates | $\psi_{1,2}$ or $\phi_{1,2}$ | $\psi_{1,2}$ or $\varphi$ | $\psi_1/\psi_2$ or $\varphi$ |
| dark charge | charged $\psi_{1,2}$, $\phi_{1,2}$ | --- | charged $\psi_1$, neutral $\psi_2, \varphi$ |
| dark photon $\tilde{\gamma}$ | massless $U$ | massive $V_{1,2,3}$ (degenerated) | massless $A$ |

- $\tilde{\gamma}$ has a direct coupling to the dark charged $\psi_1$ while $\varphi$ has a direct coupling to the dark neutral $\psi_2$.

The recent discussions on the phenomenology of the massless dark photon of this type can be found in Ref. [35, 36].

D. Dark gauge bosons

The dark gauge bosons imported into these three subsections consist of

- the massless $U$ boson identified with $\tilde{\gamma}$ in Sec IV A
- three degenerated massive $V$ bosons identified with $\tilde{\gamma}$ in Sec IV B
- the massless $\tilde{A}$ identified with $\tilde{\gamma}$ and three massive bosons of $\tilde{W}^{\pm}$ and $\tilde{Z}$ in Sec IV C

It is a general feature that $\tilde{\gamma}$ cannot possess the hypercharge associated with the $U(1)_{\gamma}$ symmetry of the standard model, which enables $\tilde{\gamma}$ to kinematically mix with the photon. Instead, there appears the mixing between $\tilde{\gamma}$ and $Z$.

The mixing of $\tilde{\gamma}$ with $Z$ arises as the following loop effects due to

- two loops by $\Psi$, $N_R$ and $\nu$ in the external loop involving the internal exchange of $\Phi$ for Sec IV A or one loop with the same external loop as in Sec IV A but involving the nonvanishing $\langle \Phi \rangle$ for Sec IV B

These induced couplings are depicted in FIG. 2. The similar mixing with $Z$ is applied to $\tilde{Z}$. Due to the same one-loop coupling as the $\tilde{\gamma}$-$Z$ mixing but one of $\langle H^0 \rangle$'s is replaced with $h$, the $h$ decay proceeds via

- $h \rightarrow Z\tilde{\gamma}$, which is necessarily induced for the massless $\tilde{\gamma}$.

The particle content of each realization of the dark gauge symmetry is summarized in TABLE I.

For the massless $\tilde{\gamma}$, the mixing between $\tilde{\gamma}$ and $Z$ is restricted to the kinetic mixing. For the massive $\tilde{\gamma}$, the induced decay of $\tilde{\gamma}$: $\tilde{\gamma} \rightarrow \nu \bar{\nu}$ is possible to occur via the intermediate $Z$ boson decaying into $\nu \bar{\nu}$. Considering higher loop effects involving the induced $\tilde{\gamma}$-$Z$ mixing, we have the kinetic $\tilde{\gamma}$-$\gamma$ mixing, which is generated by the $Z$-boson exchanged between the loop(s) giving the $\tilde{\gamma}$-$Z$ mixing and the one loop by quarks and leptons, which finally couples to the photon (FIG. 3). Our kinetic $\tilde{\gamma}$-$\gamma$ mixing turns out to be further suppressed by the $Z$ boson mass.

3 The Furry theorem is applied to quantum corrections containing the dark photons, whose contributions can be evaluated according to the ordinary rules since both particle and antiparticle are involved in the $O(3)$ spinors.
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are, respectively, indicated by

\[ \bar{P}^\dagger \tilde{\Gamma} \bar{P} = \tilde{\Gamma}, \quad \bar{P}^\dagger \Sigma \bar{P} = \Sigma. \]  (59)

Altogether, the rotation by \( D_{ij}(-1, \theta) = D_{ij}(1, \theta)\Sigma_{i4} \) in the spinor space describes the corresponding \( O(3) \) rotation in the vector space. Furthermore, \( \bar{P} \) takes care of the interchange of \( \phi \leftrightarrow \phi^* \) as \( \Psi = i\bar{P}\Phi \), which is related to the \( \mathbb{Z}_2 \) parity realized on the \( O(3) \) spinor. In fact, \( \bar{P} \) is equivalent to the spinorial \( \mathbb{Z}_2 \) parity operator \( P \):

\[ P = \text{diag.}(I, -I), \]  (60)

for \( (\phi, \phi^*) \), which describes the \( U(1) \) charge of \( \Phi \). We have also noted that \( P \) as the pseudoscalar of \( O(3) \) ensures that the transformed \( O(3) \) spinor with the reflections included has the opposite \( U(1) \) charge to the original \( O(3) \) spinor.

In particle physics, the \( O(3) \) spinor consisting of \( (\phi, \phi^*) \) serve as the bosonic \( O(3) \) spinor. The fermionic spinor is composed of the Dirac spinor and its charge conjugate instead of the complex conjugate so that the internal \( O(3) \) symmetry gets orthogonal to the Lorentz symmetry. The \( O(3) \) symmetry gets visible when the invariance of interactions is extended to explicitly include contributions from their complex conjugates. Owing to the inherent property that both the particle and antiparticle are contained in the \( O(3) \) spinor, which is constrained to be neutral, the application to particle physics is limited. One of the viable possibilities is to regard the \( O(3) \) spinors as dark matter. To see the interactions of the dark sector particles, we have constructed the lagrangians of the bosonic spinor, the fermionic spinor and the gauge bosons. The results indicate that the fermionic spinor is vectorlike and that the gauge bosons, \( V_{\mu} \) and \( U_{\mu} \), associated with \( SO(3) \times \mathbb{Z}_2 \) behave as the axial vector and pseudoscalar, respectively.

We have described the following scenarios of dark gauge bosons:

1. The massless dark photon is associated with the unbroken \( U(1) \) symmetry generated by the spinorial \( \mathbb{Z}_2 \) parity, and \( h \rightarrow \tilde{\gamma}\tilde{\gamma} \) proceeds via the one loop by \( \Phi \);
2. Three degenerated massive dark photons are associated with the spontaneously broken \( O(3)/U(1) \) symmetry and \( h \rightarrow \tilde{\gamma}\tilde{\gamma} \) proceeds via the one loop by \( V \) with the gauge coupling to \( \tilde{\gamma} \) as well as by \( \Psi \) and \( N_R \) with the \( \Psi \) coupling to \( \tilde{\gamma} \) if the decay is kinematically allowed;
3. The massless dark photon is associated with the spontaneously broken \( O(3) \) symmetry and \( h \rightarrow \tilde{\gamma}\tilde{\gamma} \) proceeds via the one loop by \( \tilde{W}^\pm \) with the gauge coupling to \( \tilde{\gamma} \).

We have further obtained that

1. the \( \tilde{\gamma}\tilde{\gamma} \) mixing is generated as loop corrections to give the decay of \( h \) as \( h \rightarrow Z\tilde{\gamma} \) and the kinetic \( \tilde{\gamma}\tilde{\gamma} \) mixing further suppressed by the \( Z \) boson mass;
2. massive dark gauge bosons decay into \( \nu\bar{\nu} \).

In case of the nonvanishing \( (\Phi) \), the \( O(3) \)-doublet vectorlike fermion supplies four species of Majorana fermions of \( \psi_{1,2} \) and \( \psi^C_{1,2} \), which all couple to \( N_R \) as in Eq. (46), and will mix with flavor neutrinos via the seesaw mechanism. These Majorana fermions can be sterile neutrinos as dark matter [37]. To clarify detailed dark matter physics influenced by the \( O(3) \)-doublet vectorlike fermion as sterile neutrinos as well as to estimate their effects on neutrino oscillations is left for our future study. Finally, since the \( CP \) transformation exchanges particles and antiparticles, which are also exchanged by \( D_{23}(-1, 0) \), there might be a chance to implement the \( CP \) violation in the dark sector associated with the breaking of \( O(3) \).

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