Effect of the Reynolds number on the performance and approximate modeling of the small straight-bladed vertical-axis wind turbine

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Abstract
Effect of the Reynolds number on the torque and power characteristics of a small straight-bladed vertical axis wind turbine has been investigated experimentally under various wind velocity. The maximum mean torque coefficient and the maximum mean power coefficient increase with increasing the Reynolds number based on the wind velocity and representative length of the wind turbine, and the dependence of these coefficients on the Reynolds number can be successfully approximated in the logarithmic function. The tip speed ratio for the maximum mean torque coefficient is almost independent of the Reynolds number. Otherwise, the tip speed ratio for the maximum power coefficient increases as increasing Reynolds number, and the dependence of the maximum mean torque coefficient on the Reynolds number can be approximated in the logarithmic function. When the curvature parameter, the aspect ratio, and the solidity represented forms of the wind turbine are same, the wind turbine performance can be successfully explained by an semi-empirical formula including simple analytical functions, namely, the mean torque and the mean power coefficients can be represented well by the logarithmic functions of the Reynolds number and quadratic or cubic function of the tip speed ratio. The proposed approximate equations successfully predict experimental data for the particularly higher tip speed ratio.

Key words : Wind turbine, Vertical axis, Straight blade, Reynolds number, Tip speed ratio, Performance, Torque, Power, Approximate modeling

1. Introduction

The wind turbine is expected to be a candidate that is source of renewable energy and solves the global environmental problems associated with CO₂ emission. The small wind turbines are easily introduced as local electric energy source equipment for residences in suburb. The present experimental study focuses on a small wind turbine which has accessibility of renewable energy.

A straight-bladed vertical-axis wind turbine (This is called commonly Straight Darrieus Turbine) is one of the reliable wind turbines, since it is free from the directional control for variable natural wind, produces low noise for the lower rotational speed, and has simple blade structure (Ushiyama, 2002, Mizuno, 2002). Adequate knowledge on the small wind turbine performance in low Reynolds number range is one of the important issues, because the size of blade is relatively small and it is often used in a low wind velocity condition.

The authors reported that the wing section of the blade has significant influence on the wind turbine performance (Yamada et al., 2011). Aerodynamic characteristics of wing sections at rest in uniform flow are strongly depended Reynolds number in low Reynolds number range (Nishiyama, 1992). Magnitude of the relative wind velocity and the angle of attack to the blade of the straight-bladed vertical-axis wind turbine are supposed to be varied considerably for
a period of rotation (Paraschivoiu, 2007a). The strongly unsteady condition gives us particular difficulties that the wind turbine performance is estimated quantitatively from aerodynamic characteristics of the wing obtained in steady and uniform flow. Effect of the Reynolds number on time mean torque and power characteristics of this wind turbine has been investigated (Paraschivoiu, 2007b, Maeda, 2008), and it is represented that the maximum torque and the maximum power coefficients increase with increasing the Reynolds number (Maeda, 2008). The tip speed ratio for the maximum power determines the power of the wind turbine in actual operation and is the most important value in the issue of Reynolds number dependence. A simple representation with analytic function for the wind turbine performance is required to estimate expected torque and power at any low Reynolds number conditions in practical engineering application. In the present experimental study, the Reynolds number effect on the performance of the small straight-bladed vertical-axis wind turbine is examined by a wind tunnel experiment. A semi-empirical formula to represent the dependence of the mean characteristics of the wind turbine on the Reynolds number will be proposed using adequate modelling with simple analytic functions.

2. Nomenclature

\[
\begin{align*}
A & \quad \text{Swept area (=2rb) [m²]} \\
A_r & \quad \text{Aspect ratio (=b/(2r)) [-]} \\
b & \quad \text{Span of the blade [m]} \\
c & \quad \text{Chord length [m]} \\
r & \quad \text{Radius of the rotor [m]} \\
C_T & \quad \text{Time mean torque coefficient [-]} \\
C_P & \quad \text{Time mean power coefficient [-]} \\
(C_T)_{\text{max}} & \quad \text{Maximum of } C_T [-] \\
(C_P)_{\text{max}} & \quad \text{Maximum of } C_P [-] \\
N & \quad \text{Number of blades [-]} \\
\text{Re}_r & \quad \text{Reynolds number on the wind turbine size and the wind velocity (=2rU_0/\nu) [-]} \\
\text{Re}_{c_1} & \quad \text{Reynolds number on the chord length and the wind velocity (=cU_0/\nu) [-]} \\
\text{Re}_{c_2} & \quad \text{Reynolds number on the chord length and the circumferential velocity (=cV_\theta/\nu = c(r\bar{\omega})/\nu) [-]} \\
(\text{Re}_u)_{\theta} & \quad \text{Reynolds number on the chord length and the mean relative velocity (=c(W)_{\theta}/\nu) [-]} \\
T & \quad \text{Time mean torque of the rotor [N·m]} \\
U_0 & \quad \text{Wind velocity [m/s]} \\
V_\theta & \quad \text{Circumferential velocity [m/s]} \\
W & \quad \text{Relative velocity (=U_0\sqrt{\lambda^2-2\lambda \sin \theta+1}) [m/s]} \\
\theta & \quad \text{Azimuth angle of the reference blade [deg]} \\
\kappa & \quad \text{Curvature parameter (=c/r) [-]} \\
\lambda & \quad \text{Tip speed ratio (=r\bar{\omega}/U_0) [-]} \\
\lambda_{C_T,\text{max}} & \quad \text{Tip speed ratio at (C_T)_{\text{max}} [-]} \\
\lambda_{C_P,\text{max}} & \quad \text{Tip speed ratio at (C_P)_{\text{max}} [-]} \\
\mu & \quad \text{Viscosity of the air [Pa·s=kg/(m·s)]} \\
\nu & \quad \text{Kinematic viscosity of the air (=\mu/\rho) [m²/s]} \\
\rho & \quad \text{Density of the air [kg/m³]} \\
\sigma & \quad \text{Solidity (=Na/(2\pi\sigma)) [-]} \\
\bar{\omega} & \quad \text{Time mean angular velocity [rad/s]} \\
(\theta)_{\theta} & \quad \text{Mean of * from } \theta = 0 \text{ to } 2\pi
\end{align*}
\]

3. Dimensional analysis and the Reynolds number on the wind turbine performance

3.1 Dimensional analysis on the wind turbine performance

The wind turbine model including various parameters is shown in Fig.1. If the wind turbine is operated in the wind velocity \(U_0\) [m/s], a dimensional group for the time mean torque of the rotor \(T_u\) [N·m] could be defined as

\[
T_u = F(\rho, \mu, U_0, c, b, r, \bar{\omega}, \sigma),
\]

(1)
where \( \rho \) [kg/m\(^3\)] is density of the air, \( \mu \) [Pa·s = kg/(m·s)] is viscosity of the air, \( c \) [m] is chord length, \( b \) [m] is the blade span, \( r \) [m] is the radius of the rotor, \( \varpi \) [rad/s] is the time mean angular velocity, \( \sigma = \frac{Nc}{(2\pi)} \) [-] is the solidity (\( N \) is number of blades). There are six nondimensional parameters \( \Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5 \) and \( \Pi_6 \) defined according to Buckingham’s \( \pi \) number theorem (Nakamura and Osaka, 1986), since there are three independent dimensions [kg], [m], [s] among these nine physical quantity. These six nondimensional parameters are derived by dimensional analysis as follows:

\[
\Pi_1 = \frac{1}{2} \frac{\rho Ar^2 U_T}{C_w T \rho}, \quad \Pi_2 = \frac{U_T}{\rho \mu}, \quad \Pi_3 = \frac{c}{r}, \quad \Pi_4 = \frac{b}{r}, \quad \Pi_5 = \sigma, \quad \Pi_6 = \frac{r c}{4}.
\]

Using these nondimensional parameters, the time mean torque coefficient \( C_T = \frac{\Pi_1}{(1/2)\rho U_0^2 Ar} \) can be represented to be

\[
C_T = f(Re_r, \lambda, \kappa, A_r, \sigma), \quad (2)
\]

The time mean power coefficient \( C_p \) can be defined as

\[
C_p = \frac{\Pi_6}{(1/2)\rho U_0^3 A} = \lambda C_T. \quad (3)
\]

It is expected that \( Re_r \) represents effect of the wind turbine size and the wind velocity, \( \lambda \) represents effect of rotational velocity of the rotor, \( \kappa, A_r \) and \( \sigma \) represent effect of shape of the wind turbine on \( C_T \) and \( C_p \).

![Fig. 1 The wind turbine model including various parameters.](image)

### 3.2. Reynolds number on the wind turbine performance

At least, we can define four Reynolds numbers to discuss the dependence of the mean characteristics of the wind turbine.

\[
Re_r = \frac{2ru_o}{v}, \quad (4)
\]

\[
Re_{r1} = \frac{cU_o}{v} = \frac{1}{2} \kappa Re_r, \quad (5)
\]

\[
Re_{r2} = \frac{cV_\theta}{v} = \frac{c(r\varpi)}{v} = \frac{1}{2} \lambda \kappa Re_r, \quad (6)
\]

\[
(Re_w)_{\theta} = \frac{c(W_o)}{v} = \frac{1}{2} \kappa f(\lambda) Re_r, \quad (7)
\]

where \( W = U_o \sqrt{\lambda^2 - 2\lambda \sin \theta + 1} \) is the relative velocity for the blade, when the wind turbine is rotated through the uniform flow \( U_o \) at a circumferential velocity \( V_\theta = r\varpi \), and the influence of the flow field by other blades is not
considered. \( \theta \) [deg] is the azimuth angle of the reference blade, \( \langle \theta \rangle \) represents the mean of * from \( \theta = 0 \) to \( 2 \pi \) (one rotation of the wind turbine). And \( f(\lambda) \) is

\[
f(\lambda) = \frac{1}{2\pi} \int_{0}^{2\pi} \sqrt{\lambda^2 - 2\lambda \sin \theta + 1} d\theta.
\]

As described in the previous section, \( \text{Re}_e \) is considered effects of the wind turbine size and the wind velocity. \( \text{Re}_{c,1} \) is considered effects of the wind turbine size, the wind velocity and the wind turbine shape (curvature parameter). \( \text{Re}_{c,2} \) is considered effects of the wind turbine size, the wind velocity, the wind turbine shape (curvature parameter) and rotational speed. \( \langle \text{Re}_e \rangle \) is obtained by averaging \( \text{Re}_e = cW/\nu \) for the azimuth angle from \( \theta = 0 \) to \( 2\pi \) (one rotation of the rotor). \( \text{Re}_e \) is used when dynamic characteristics of the wind turbine were examined. As well as \( \text{Re}_{c,2} \), \( \langle \text{Re}_e \rangle \) is considered effects of the wind turbine size, the wind velocity, the wind turbine shape (curvature parameter) and rotational speed.

In this study, to investigate effects of the Reynolds number on the performance of the wind turbine focusing on wind turbine size and wind velocity for the same shape, the Reynolds number defined by Eq. (4) is employed. When \( \lambda \) is considered in the approximate model of the wind turbine performance, there are two approaches, namely, use of the Reynolds number in consideration of \( \lambda \) including in Eqs. (6) and (7), or consideration of the Reynolds number and \( \lambda \) separately. The question which approach is better will be discussed in Section 5.2.

4. Experimental Techniques

The experimental apparatus and the coordinate system are shown in Fig.2. \( U_0 [\text{m/s}] \) is the wind velocity and \( \theta \) [deg] is the azimuth angle of the reference blade. The wind turbine is located in open space in front of the exit of wind tunnel test section with the 1000mm\( \times \)1000mmH rectangular cross section, so that it is not necessary to consider the blockage effect of the wind turbine on the wind tunnel test section. While effect of non-uniformity of the velocity would be some. The distance of between the wind tunnel nozzle exit and the center of rotation of the wind turbine is 450mm. The wind turbine was located on the center of the wind tunnel and within the potential core of the flow produced by the wind tunnel, so that it is not necessary to consider the effect of non-uniformity of the velocity. The origin of coordinate is located the center of the wind turbine height on the rotation axis of the rotor. The Cartesian coordinate is employed; the \( x \)-axis is parallel to the main flow direction and the \( z \)-axis is vertical direction. The position of \( 0[\text{deg}] \) for the azimuth angle \( \theta \) is located at the most upstream against the main stream, and positive direction is clockwise to the \( z \)-axis direction. The velocity of the flow approaching to the wind turbine is measured by a pitot tube located at just exit of the wind tunnel. The wind turbine is connected to the motor via the torque converter and controlled in a constant rotational speed by the inverter. The torque of the rotor, the angle of rotation, and the phase detection signal of the reference blade are measured simultaneously, and the time series data converted from the analog signal to the digital signal at 10 kHz for 30 seconds are stored in a computer for the various statistical calculations. The time mean torque and the time mean angular velocity are calculated from the time series data analysis. Conventional time mean of arbitrary function \( F(t) \) are defined as

![Fig. 2 The experimental apparatus and the coordinate system.](image-url)
\[ F(t) = \lim_{\Delta t \to \infty} \frac{1}{\Delta T} \int_{t_0}^{t_0+\Delta T} F(t) \, dt \quad (8) \]

The test wind turbine has a rotor radius of \( r = 0.3\text{[m]} \), a blade span of \( b = 0.6\text{[m]} \), a chord length of \( c = 150\text{[mm]} \), number of blades of \( N = 2 \), and a solidity \( \sigma = Ne/(2\pi r) = 0.16 \). \( r \) is defined as the distance between 0.3 \( c \) wing chord position on the mean line and the center of rotation of the wind turbine. The blades are attached to arms at those ends in the angle for the power coefficient to be maximum. The wing section of the blade is NACA6520 designed according to NACA 4 digit wing section. The plane shape of the blade is rectangle. Experiments were made at the wind velocity \( U_0 = 2, 3, 4, 6, 8, 10\text{[m/s]} \) with \( \kappa, A, \) and \( \sigma \) kept to be constant in order to be examined effects of the Reynolds number on the performance of the same shape wind turbine. Then the Reynolds number \( \text{Re}_r = 2rU_0/\nu \) is from \( 8.3 \times 10^4 \) to \( 4.2 \times 10^5 \).

5. Experimental Results and Discussion
5.1 Effect of the Reynolds number on mean characteristics of the wind turbine

Figure 3 shows time mean torque and power characteristics for \( U_0 = 2 - 10\text{[m/s]} \) (\( \text{Re}_r = 8.3 \times 10^4 \) to \( 4.2 \times 10^5 \)). The horizontal axis is the time mean tip speed ratio \( \lambda = r\omega / U_0 \), and the vertical axis is the time mean torque coefficient \( C_T \) (upper side) and the time mean power coefficient \( C_p \) (lower side). Plots at \( U_0 = 10\text{m/s} \) in \( C_p \) are omitted, because we were not able to increase rotational speed to the tip speed ratio indicating the max power coefficient in a condition of \( U_0 = 10\text{m/s} \) in the limitation of the experimental device. \( C_T \) increases with the higher wind velocity without depending on \( \lambda \) in most area and the maximum \( (C_T)_{\text{max}} \) also increases with the higher wind velocity. This wind velocity dependence of \( C_T \) is discussed as follows. \( C_T \) is strongly affected by the maximum of the phase torque coefficient at the phase where the blade is located near the upstream (Yamada et al., 2011). The contribution of lift force acting on the blade to the rotational torque is the most significant near the upstream position. The angle of attack of the wing becomes larger near the upstream position. The lift coefficient increases with a larger Reynolds number at a high angle of attack. The Reynolds number increases with the higher wind velocity. Therefore it suggests that the maximum of the phase torque coefficient increases with the higher wind velocity, consequently \( C_T \) increases with the higher wind velocity. The tip speed ratio \( \lambda_{CT_{\text{max}}} \) at \( (C_T)_{\text{max}} \) is independent of the wind velocity, and \( \lambda_{CT_{\text{max}}} \) is almost constant from 1.41 to 1.45. The torque characteristic reflect to the mean power coefficient \( C_p \). \( C_p \) increases with the higher wind velocity.

![Fig.3](image)

Fig.3 The effect of the wind velocity on the time mean torque and the time mean power for \( U_0 = 2 - 10\text{[m/s]} \).

The time mean torque coefficient \( C_T \) (upper side). The time mean power coefficient \( C_p \) (lower side).
without dependence on $\lambda$ for the most range of $\lambda$ and the maximum power coefficient $(C_p)_{Max}$ increases with the higher wind velocity. The tip speed ratio $\lambda_{CPmax}$ at $(C_p)_{Max}$ increases with the higher wind velocity.

From the above the mean torque and the mean power coefficients increase with increasing the wind velocity, namely with increasing $Re_r$, in the most range of $\lambda$ except for the range of the low tip speed ratio.

Next effects of the Reynolds number on $(C_T)_{Max}$, $(C_p)_{Max}$, $\lambda_{CTmax}$ and $\lambda_{CPmax}$ are examined in detail. The Reynolds number $Re_r$ defined by Eq. (4) is used in order to treat the effect of the wind velocity on the performance of the wind turbine. Figure 4 shows variations of the maximum of the time mean torque coefficient $(C_T)_{Max}$ and the maximum of the time mean power coefficient $(C_p)_{Max}$ as a function of the Reynolds number $Re_r$. $(C_T)_{Max}$ and $(C_p)_{Max}$ increase with increasing the Reynolds number $Re_r$. These relations can be approximated by logarithmic functions to be shown following equations,

$$
(C_T)_{Max} = a_{Tmax} \ln Re_r + b_{Tmax},
$$

where $a_{Tmax} = 0.0525240$, $b_{Tmax} = -0.492475$, and

$$
(C_p)_{Max} = a_{pmax} \ln Re_r + b_{pmax},
$$

where $a_{pmax} = 0.0928484$, $b_{pmax} = -0.904097$.

Next the Reynolds number dependence of $\lambda_{CTmax}$ and $\lambda_{CPmax}$ is examined. Figure 5 shows variation of $\lambda_{CTmax}$ and $\lambda_{CPmax}$ as a function of the Reynolds number $Re_r$. $\lambda_{CTmax}$ takes an approximately constant value from 1.41 to 1.45 without depending on $Re_r$. While $\lambda_{CPmax}$ increases with increasing $Re_r$, and its relation can be approximated by the logarithmic function given as following equation,

$$
\lambda_{CPmax} = a_{Pmax} \ln Re_r + b_{Pmax},
$$

where $a_{Pmax} = 0.209664$, $b_{Pmax} = -0.875214$.

![Fig.4 Dependence of maximum torque and power coefficients on the Reynolds number $Re_r$.](image)

![Fig.5 Dependence of the tip speed ratio at maximum torque and power coefficients on the Reynolds number $Re_r$.](image)
From the above, \((C_T)_{\text{max}}, (C_p)_{\text{max}}\) and \(\lambda_{CP_{\text{max}}}\) can be approximated by logarithmic functions. These behaviors indicate that the performance of the wind turbine and the tip speed ratio at the maximum power can be predicted for any wind velocity and the wind turbine size about the given shape of the wind turbine.

5.2 The approximate model of mean characteristics of the wind turbine

As described in the above section, mean characteristics of the small wind turbine are strongly affected by the Reynolds number. Now considering the tip speed ratio \(\lambda\), we attempt to propose an approximate model to represent mean characteristics of the wind turbine for the strong Reynolds number dependence range of the wind turbine performance. To take account of the parameter \(\lambda\), we have two choices, namely, consideration of the Reynolds number including \(\lambda\) or consideration of \(\lambda\) itself and the Reynolds number, respectively. We have already definitions of the Reynolds number including \(\lambda\) in Eqs. (6) and (7). The consideration has been made based on dependence of the mean torque coefficient \(C_T\) and the mean power coefficient \(C_p\) on the Reynolds number \(Re_\lambda\), represented by the Eq. (6) with an additional parameter \(\lambda\). (These behaviors are not given here.) The Reynolds number dependence of those coefficients is strongly influenced by the tip speed ratio \(\lambda\). Consequently \(C_T\) and \(C_p\) cannot be expressed in single-valued function of the Reynolds number based on the size, the wind velocity and rotational speed. This indicates that the effect of the size, the wind velocity and the effect of the rotational speed must be considered individually. Then functional relation of coefficient \(C_T\) to the Reynolds number \(Re_\lambda\) without \(\lambda\) are examined for various \(\lambda\). Figure 6 shows the relation of \(C_T\) to \(Re_\lambda\) for \(\lambda\) of 0.63 to 2.10. For all \(\lambda\), \(C_T\) increases with increasing \(Re_\lambda\), and those relations can be represented by the following logarithmic function

\[
C_T = a_T \log_{10} Re_\lambda + b_T,
\]

where \(a_T\) and \(b_T\) varies according to \(\lambda\).

Next, dependences for \(\lambda\) of \(a_T\) and \(b_T\) are examined in detail. Figures 7(a) and (b) show variation of \(a_T\) and \(b_T\) for various tip speed ratios \(\lambda\). The relation of \(a_T\) and \(b_T\) for various \(\lambda\) is significantly different between \(\lambda < 1.1\) and \(\lambda \geq 1.1\). \(a_T\) decreases with the smaller \(\lambda\) in \(\lambda < 1.1\), and \(a_T\) is much smaller in \(\lambda < 0.9\). This suggests that the Reynolds number dependence could be ignored for the smaller \(\lambda\). Thus, it seems to be associated with differences of physical phenomena in the flow field around the wing between \(\lambda < 1.1\) and \(\lambda \geq 1.1\). The unsteady effect as represented by the dynamic stall is larger at lower tip speed ratios, while the viscous effect in the boundary layer on the wing is larger at higher tip speed ratios (Amet et al., 2009). There is the way that two different equations are derived approximate expressions respectively for \(\lambda < 1.1\) and \(\lambda \geq 1.1\) in consideration of these experimental facts. Now for simplicity \(a_T\) and \(b_T\) can be approximated by quadratic functions of \(\lambda\) considering the variation of \(C_T\).
Fig.7 Variations of $a_t$ and $b_t$ for $\lambda$.

(a) $a_t$  
(b) $b_t$

The horizontal axis is $\lambda$ in Fig.3. Then they are given as following equations

$$a_t = a_{t1}\lambda^2 + a_{t2}\lambda + a_{t3}$$

(13)

and

$$b_t = b_{t1}\lambda^2 + b_{t2}\lambda + b_{t3}$$

(14)

where $a_{t1} = -0.102637$, $a_{t2} = 0.334523$, $a_{t3} = -0.125606$, $b_{t1} = 0.397272$, $b_{t2} = -1.33957$, and $b_{t3} = 0.487711$.

Next the mean power coefficient can be expressed by the following equation according to Eq. (3).

$$C_p = \lambda C_r = \lambda(a_t \log_{10}Re_t + b_t)$$

(15)

Consequently $C_p$ becomes a cubic function of $\lambda$. Now approximate quantities of the mean torque coefficient and the mean power coefficient given by Eqs. (12) and (15) are compared with experimental data obtained by the wind tunnel experiment. Figure 8 shows that approximate quantities are compared with experimental facts for $C_r$. The horizontal axis is the mean torque coefficient calculated by the proposed Eq. (12), and the vertical axis is the mean torque coefficient obtained by the experiment. They are well correlated for various tip speed ratios. Figure 9 shows that approximate quantities are compared with experimental facts for $C_p$. The horizontal axis is the mean power coefficient calculated by the proposed Eq. (15), and the vertical axis is the mean power coefficient obtained by the experiment. They are also well correlated for various tip speed ratios. These comparisons certainly show that proposed equations for the
Fig. 9 Comparison of calculated value by Eq. (15) to the experimental data for $C_p$.

mean torque and the power coefficients successfully represent dependence on the Reynolds number and the tip speed ratio.

Figures 10(a) – 10(d) show that calculated variations are compared with experimental data for relation of $C_T$ or $C_P$ to $\lambda$ at $U_0 = 2, 4, 6, 8$[m/s]. For all $U_0$, calculated variations and experimental facts for $C_T$ and $C_P$ show well agreement, except for the range of the lower tip speed ratio $\lambda < 0.6$. For the lower tip speed ratio, more sophisticated relation would be required to understand complicated flow phenomena. Agreement of approximately calculated quantities and experimental data for $C_T$ and $C_P$ is well in the higher tip speed ratio range $\lambda \geq 1$ in comparison with the lower tip speed ratio range $\lambda < 1$.

Fig. 10 Approximate quantities are compared with experimental data for relation of $C_T$ or $C_P$ to $\lambda$ at $U_0 = 2, 4, 6, 8$[m/s]. (a) $U_0 = 2$[m/s], (b) $U_0 = 4$[m/s], (c) $U_0 = 6$[m/s], (d) $U_0 = 8$[m/s].
Fig. 10 Approximate quantities are compared with experimental data for relation of $C_T$ or $C_p$ to $\lambda$ at $U_0 = 2, 4, 6, 8$[m/s]. (a) $U_0 = 2$[m/s], (b) $U_0 = 4$[m/s], (c) $U_0 = 6$[m/s], (d) $U_0 = 8$[m/s].

Now the difference of calculated quantities and experimental data is discussed in detail. In the variation of $C_T$ for $\lambda$ in Fig.3, the curve of $C_T$ for $\lambda$ is convex in $\lambda \geq 1$, while it is concave in $\lambda < 1$. However the coefficient of $\lambda^2$ in the Eq. (12) has a positive value, so that the curve of $C_T$ for $\lambda$ must be convex. Probably, these trends explain that difference of calculated variations and experimental data in $\lambda < 1$ is larger than in $\lambda \geq 1$. In this way, because we attempt to model using single approximate expression in all tip speed ratio range in this study, tendency of matching calculated variations and experimental facts is different between the low tip speed ratio range and the high tip speed ratio range. However, the proposed approximate Eqs. (12) and (15) are able to give reasonable prediction of the effect of the Reynolds number and the tip speed ratio on $C_T$ and $C_p$ as shown in Fig.8 to 9.

In this experiment, for any given shape parameters $\kappa$, $A$, and $\sigma$ on specifics of the wind turbine, it is shown that the mean torque and power coefficients can be well represented by the logarithmic functions of $Re_{\kappa}$ and quadratic or cubic function of the tip speed ratio $\lambda$. While it is required that appropriate functional forms of $a_\tau$ and $b_\tau$ for any different wing section and shape parameter are derived. Proposing the single-valued functional forms considered these parameters is a future issue.

6. Conclusions

(1) The mean torque and the mean power coefficients increase with increasing the Reynolds number $Re_{\kappa}$ in the most region of $\lambda$ except for the region of the low tip speed ratio.

(2) The maximum torque and the maximum power coefficients increase with increasing the Reynolds number $Re_{\kappa}$ and the dependence of these coefficients on $Re_{\kappa}$ can be successfully approximated in the logarithmic function.

(3) $\lambda_{CT\text{\scriptsize{max}}}$ takes an approximately constant value with respect to $Re_{\kappa}$, while $\lambda_{CP\text{\scriptsize{max}}}$ increases with increasing $Re_{\kappa}$.

(4) The dependence of $\lambda_{CP\text{\scriptsize{max}}}$ on $Re_{\kappa}$ can be approximated in the logarithmic function.

(5) For any given shape parameters $\kappa$, $A$, and $\sigma$ on specifics of the wind turbine, the mean torque and power coefficients can be well represented by the logarithmic functions of $Re_{\kappa}$ and quadratic or cubic function of the tip speed ratio $\lambda$. 

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