High energy particle collisions near the bifurcation surface

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We consider generic nonextremal stationary dirty black holes. It is shown that in the vicinity of any bifurcation surface the energy of collision of two particles in the centre of mass frame can grow unbound. This is a generic property that, in particular, includes collisions near the inner black hole horizon analyzed earlier by different methods. The similar results are also valid for cosmological horizons. The case of the de Sitter metric is discussed.

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I. INTRODUCTION

The effect of acceleration of particles by black holes (called the BSW effect according to the names of its authors) was discovered quite recently [1] and is now under active study. Apart from significant pure theoretical interest, its potential astrophysical applications also attract attention (see, e.g., recent papers [2]). The essence of this effect consists in the unbound growth of the energy $E_{c.m.}$ in the centre of mass frame of two colliding particles. Mainly, it concerns the collision near the event horizon. Meanwhile, this effect was discussed also in a quite different situation - near the inner black hole horizon. It turned out [4] that a crucial role is played in that case by the bifurcation two-dimensional surface where the future and past horizons meet. Namely, for the BSW effect to occur, particles should collide in the vicinity of this surface.

The goal of the present work is to pay attention that any bifurcation surface can lead to the BSW effect. In particular, it includes the case of the inner black hole horizon discussed previously [4] - [10]. But this is also true even if there is no an inner horizon (for example, in the Schwarzschild case). It is worth reminding that the BSW effect was found for extremal
horizons [1] (which do not have a bifurcation sphere at all). Later on, it was shown that it occurs also near the nonextremal event horizons [11, 5] but not on the bifurcation surface. (Hereafter, when mentioning the properties of the BSW effect near the horizon, we imply just one of these two cases, if the bifurcation surface is not mentioned explicitly.) The similar effect was discussed also near singularities [12]. Now, we add a new object to this list of possible particle accelerators in a strong gravitational field.

Some reservations are in order. One of examples of the bifurcation point occurs inside the event horizon where two inner ones intersect. It is known that near such horizon a strong instability develops [13] (see also, e.g. the reviews [14], [15]), so one may ask if it makes sense to study the processes near the object which cannot probably exist in a real world. In our view, there are several points to motivate studies of the BSW effect near the bifurcation surface. (i) It may happen that the BSW effect itself can contribute to further instability of inner horizons but this kind of instability is quite different from that found in [13]. If so, this can be considered as an additional argument against the existence of such horizons. It would be of interest to evaluate the relative role of both effects that seems to be a separate issue. But the first step here is to show that the corresponding property (BSW effect) is indeed inherent to any bifurcation point. The similar conclusions concern also white holes (see Sec. 15.2 of [16]). (ii) The area of potential applications is not restricted by black hole inner horizons. As follows from consideration below, our results apply not only to the inner black hole horizons but also to cosmological ones (the de Sitter spacetime is one example). Such objects not only exist but play an obviously crucial role in modern cosmology. (iii) Moreover, our approach and corresponding results apply to any space-time with nontrivial causal structure where different nonextremal horizons alternate. (iv) Independently of direct applications, the BSW process is a nontrivial gravitational effect. The list of objects to which it applies is gradually growing (see, e.g. the recent work [17]), so it is important problem to understand in which situations and under which conditions it happens. In our view, it looks reasonable to separate two issues - the nature of the BSW effect as such and its potential relevance in realistic astrophysics. The present work concerns the first issue only.

The results of the present work apply to quite generic ”dirty” horizons. In astrophysical circumstance a real black hole is not in vacuum but is surrounded by matter. Therefore, it does not have a simple, say, Kerr or Reissner-Nordström form. Although exact solutions for such systems are absent, universality of black hole physics enabled to derive some general
relationships. Interest to such objects is also connected with such general issues as the black hole entropy, its relation to the horizon symmetries, etc. (see [18], [19], [20], [21]).

II. TWO VERSIONS OF BSW EFFECT

The crucial role is played in the BSW effect by division of all possible trajectories of particles into two classes - the so-called ”critical” particles (that implies special relationship between particle’s parameters) and so-called ”usual” particles - all other ones [4], [5], [6]. More precise definitions will be given below. As is shown in [4], it is impossible to have configurations that satisfy simultaneously the following properties: (i) $E_{c.m.}$ is infinite (”the strong version of the BSW effect”), (ii) collision does occur in some point. In particular, property (i) requires particle 1 (critical) to pass through the bifurcation point while particle 2 is to be usual and this excludes condition (ii). In this sense, the ”kinematic censorship” forbids actual infinity in any physical event. However, if collision takes place not exactly on the bifurcation surface but somewhere in its vicinity, $E_{c.m.}$ turns out to be finite but can be made as large as one likes. This was called in [4] the ”weak version” of the BSW effect. In what follows, we imply just this version without further reservations. It is the proof that the bifurcation surface can serve as a particle accelerator that we now turn to.

III. BSW EFFECT NEAR BIFURCATION POINT

This proof is surprisingly simple. We follow the general analysis suggested in [6] but apply it not to the event horizon but, instead, to the bifurcation surface. To make presentation self-closed, I remind briefly main ingredients of the approach. Let colliding particles have the masses $m_1, m_2$. Then, by definition,

$$E_{c.m.}^2 = -P_\mu P^\mu, \quad P_\mu = m_1 u_\mu^{(1)} + m_2 u_\mu^{(2)}$$

(1)

is the total momentum. Hereafter, subscript $(i=1,2)$ labels the particles. Then,

$$E_{c.m.}^2 = m_1^2 + m_2^2 + 2m_1 m_2 \gamma$$

(2)

where

$$\gamma = -u_{\mu}^{(1)} u_{\mu(2)}$$

(3)
has the meaning of the Lorentz gamma factor of particles’ relative motion. The BSW effect exists if $\gamma$ can be made arbitrarily large.

Using the basis consisting of null vectors $l^\mu$ and $N^\mu$ and the space-like vectors $a^\mu$ and $b^\mu$ orthogonal to them, one can write

$$u_{(i)}^\mu = \beta_i N^\mu + \frac{l^\mu}{2\alpha_i} + s_{(i)}^\mu$$

where $s^\mu = s_a a^\mu + s_b b^\mu$ and, for definiteness, we use normalization $l^\mu N_\mu = -1$. Then,

$$\beta_i = -u_{(i)}^\mu l_\mu, \quad 2\alpha_i = -[u_{(i)}^\mu N_\mu]^{-1}.$$  

The normalization condition $u_{(i)}^\mu u_{(i)}^\mu = -1$ gives us

$$\frac{\beta_i}{\alpha_i} = 1 + s_{(i)}^\mu s_{\mu(i)}.$$  

It follows from the above formulas that

$$\gamma = \frac{1}{2} \left( \frac{\beta_1}{\alpha_1} + \frac{\beta_2}{\alpha_2} \right) - s_{(1)}^\mu s_{\mu(2)}.$$  

Here, $s_{(i)}^\mu s_{\mu(i)} = s_a^2 + s_b^2$, $s_{(1)}^\mu s_{\mu(2)} = s_{a(1)} s_{a(2)} + s_{b(1)} s_{b(2)}$.

One can rotate the set of basis vectors $(l^\mu, N^\mu, a^\mu, b^\mu)$ to $(\tilde{l}^\mu, \tilde{N}^\mu, \tilde{a}^\mu, \tilde{b}^\mu)$. One can check that $\gamma(\alpha_i, \beta_i, s_{a(i)}) = \gamma(\tilde{\alpha}_i, \tilde{\beta}_i, \tilde{s}_{a(i)})$. There is no need to perform straightforward calculations for this purpose, it follows immediately from the scalar nature of $\gamma$ and (pseudo)orthogonal character of both tetrads. Therefore, the choice of the basis vector is a matter of convenience. In what follows, we assume that $s_a$ and $s_b$ are finite.

Then, if, say, $\alpha_1 \to 0$ but $\alpha_2 \neq 0$, $\gamma \to \infty$. Particle 1 is called critical and particle 2 is called usual. As for spacelike vectors $s_{(i)}^\mu s_{\mu(i)} \geq 0$, it is seen from (3) that the condition $\beta_i = 0$ entails $\alpha_i = 0$ that will be used below. (It is worth noting that the reverse is true with the additional condition that $s_{(i)}^2 = s_{(i)}^\mu s_{\mu(i)}$ is finite. Then, the aforementioned conditions on $\alpha$ and $\beta$ become equivalent.)

In what follows, we assume that when the point of collision approaches the future horizon, the light-like vector $l_\mu$ approaches its generator. Let both particles move from infinity (or any location outside) towards the future event horizon. Then, the condition $\beta_1 = 0$ (hence, also $\alpha_1 = 0$) gives rise to a special relationship between the particle’s parameters and in this sense selects a special type of trajectories among all possible ones. Thus for the Reissner-Nordström black hole it gives $E_1 = q_1 \varphi_H$ where $q$ is the electric charge of the
particle, \( \varphi_H \) is the electric potential, subscript ”H” refers to the horizon. For rotating black holes \( E_1 = \omega_H L_1 \) where \( L_1 \) is the particle’s angular momentum, \( \omega \) is the metric coefficient responsible for rotation of the space-time (see [6] for details). Geometrically, the condition \( \beta \to 0 \) means that the component of the velocity across the horizon vanishes and the critical particle cannot cross it all. It only approaches it asymptotically, and an infinite proper time is required to reach the extremal horizon [23], [11]. (If the horizon is nonextremal, the critical particle cannot reach it and the explanation of the BSW effect somewhat changes [11], [5].)

Let us now consider the bifurcation surface instead of the future horizon. First, we assume that a black hole is axially-symmetric and stationary. In general, it is surrounded by matter (so it is ”dirty”) and does not necessarily coincides with the Kerr metric or any other exact solution of the field equations. Then, one can take advantage of the known properties (see, for example, Sec. 5.1.10, 5.3.11 and 5.4.2 of textbook [24]). On the horizon, \( l^\mu = \xi^\mu + \omega_H \eta^\mu \) where \( \xi^\mu \) is the Killing vector \( \xi^\mu \) generates time translation and the Killing vector \( \eta^\mu \) generates rotations. For a static metric, \( \omega = 0 \). On the bifurcation surface, \( l^\mu \to 0 \). According to (5), it means that \( \beta_1 \to 0 \) and, from (6), \( \alpha_1 \to 0 \) as well. Meanwhile, particle 2 is by assumption usual, it foes not pass through the bifurcation surface, so \( \alpha_2 \neq 0 \). As a result, (7) grows unbound. This completes the proof. We stress that in contrast to previous works [4] - [10], we did not analyze the motion of test particles in the vicinity of the bifurcation surface at all.

We can also do without the referring to the Killing vector. If, say, the horizon lies at \( r = r_+ \), the normal vector \( l_\mu \sim -\frac{\partial r}{\partial x^\mu} \) satisfies the condition \( l_\mu l^\mu = 0 \). In the coordinate system like

\[
ds^2 = -F dU dV + r^2 d\omega^2,
\]

it reduces to \( l_U l_V = 0 \). The bifurcation point, by its very meaning is singled out by the condition that both factors tends to zero, so the vector \( l_\mu \) itself vanishes (see below the explicit example for the Reisnner-Nordström metric). Generalization to spacetimes without spherical symmetry is straightforward.

Both situations (the BSW effect near the event horizon and near the bifurcation surface) are complimentary to each other in the following sense. The key property of a critical particle \( \alpha_1 = \beta_1 = 0 \) can be realized in two ways. If one uses the Kruskal-like coordinates
and $V$ in which the metric is nondegenerate, the property under discussion reduces to
the condition of the type $u^U l_U \to 0$. Then, either $u^U \to 0$ or $l_U \to 0$. The first case is
typical of the BSW effect near the event horizon [6], while the second one is the property
of the bifurcation surface. From another hand, for a usual particle, $\alpha_2 \neq 0$ and $\beta_2 \neq 0$
notwithstanding the fact that $l_U \to 0$. This is because $u^U \to \infty$ in such a way that the
product $u^U l_U$ remains finite nonzero (actually, this is just the definition of a usual particle).
The fact that $u^U \to \infty$ is consistent with the normalization $u^\mu u_\mu = -1$, it simply implies
that $u_V \to 0$. Geometrically, this means that a particle moves along a leg of the horizon [6].
In other words, we require that for a usual particle $\beta \neq 0$ and, as a consequences, obtain
the aforementioned properties of its velocity.

It is also worth stressing that the fact that our tetrad becomes singular near the surface of
interest causes no difficulties. Indeed, we already pointed out that the results of calculations
do not depend on the choice of the tetrad which can be subjected to rotation. Apart from
this, the metric itself remains regular near the bifurcation surface. It is also worth reminding
that for the "standard" BSW effect near the event horizon far from the bifurcation surface
the components of the four-velocity also singular in the sense that $u^U \to 0$, $u^V \to \infty$ (or
vice versa) in the regular Kruskal-like coordinates but this does not lead to any difficulties
- instead, this is one of manifestation of the BSW effect (see the end of Sec. 3 in Ref. [6]
for details).

IV. EXACTLY SOLVABLE EXAMPLE: REISSNER-NORDSTRÖM METRIC

To illustrate general features discussed above, it is instructive to consider a concrete
example. To test the approach, I choose the metric for which the results were already found
by quite different methods. I choose the part of the Reissner-Nordström metric near the
inner horizon and will show that the result agrees with that obtained in [4]. I express the
vectors $l^\mu$, $N^\mu$ in terms of the coordinates and, using the equations of motion, trace how the
general results of the previous section are reproduced.

The detailed analysis of trajectories passing through in the immediate vicinity of the
bifurcation point was carried out in Sections III B and III C of Ref. [4], so I will not repeat
here the corresponding results. Instead, I will concentrate on the properties of the null
tetrad in this vicinity and those of the particle’s four-velocity which were not discussed in
The metric can be written in the form
\[ ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 (d\theta^2 + d\phi^2 \sin^2 \theta) \] (9)

where
\[ f = \left( 1 - \frac{r_+}{r} \right) \left( 1 - \frac{r_-}{r} \right), \] (10)

\( r_+ \) has the meaning of the event horizon radius, \( r_- \) corresponds to the inner horizon. We are interested in the inner region \( r_- < r < r_+ \). It is convenient to rewrite the metric there as
\[ ds^2 = -\frac{dT^2}{g(T)} + g(T) dy^2 + T^2 (d\theta^2 + d\phi^2 \sin^2 \theta), \] (11)

\( r = -T \) is a timelike coordinate and \( t \equiv y \) is spacelike, \( g = -f > 0 \). Now, we can choose the lightlike basis vectors as follows:
\[ l^\mu = (g, -1, 0, 0), \quad l_\mu = (-1, -g, 0, 0), \] (12)
\[ N^\mu = \frac{1}{2g} (g, 1, 0, 0), \quad N_\mu = \frac{1}{2g} (-1, g, 0, 0), \] (13)

where we use the coordinates \( (T, y, \theta, \phi) \). It follows from the equations of motion that
\[ u^y = \dot{y} = \frac{X}{mg}, \] (14)
\[ u^T = \dot{T} = \sqrt{g + \frac{X^2}{m^2}}, \] (15)

where dot denotes differentiation with respect to the proper time,
\[ X = P - \frac{qQ}{T}, \] (16)

\( q \) is the particle’s charge, \( Q \) is the charge of a black hole, \( P \) has the meaning of the conserved momentum in \( y \)-direction.

The proper time
\[ \tau = m \int_r^{r_1} \frac{dr}{\sqrt{m^2 g + X^2}}. \] (17)

We will consider two cases - \( X_- = X(r_-) \neq 0 \) and \( X_- = 0 \). We will see below that this is completely equivalent to the choice of usual \( (\beta \neq 0) \) and critical \( (\beta \to 0) \) particles, respectively. For what follows, we need explicit asymptotic behavior of the four-velocity near the horizon.
For a usual particle, one can easily obtain from (14), (15) that in the coordinates $(T, y, \theta, \phi)$

$$u^T \approx \frac{X(r_-)}{m}, \quad u^y = \frac{X(r_-)}{2m \kappa_-(r - r_-)}.$$  \hspace{1cm} (18)

Here, $\kappa_- = \frac{1}{2} \left. \left( \frac{dg}{dr} \right) \right|_{r = r_-}$ has a meaning of the surface gravity of the inner horizon, it is chosen for definiteness that $X_- > 0$.

For the critical particle,

$$u^T \approx \sqrt{\frac{\kappa_-}{2}(r - r_-)},$$  \hspace{1cm} (19)

$$u^y = \frac{X}{mg} \approx \frac{X_1}{2m \kappa_-} = (u^y)_-.$$  \hspace{1cm} (20)

where $X \approx X_1(r - r_-)$ near the inner horizon, $X_1$ is some constant.

The coordinates (11) become degenerate near the horizon since $g \to 0$ there. To remedy this shortcoming, one can introduce the Kruskal-like coordinates in the standard manner. This can be done as follows:

$$U = \exp[-\kappa_-(t - r^*)],$$  \hspace{1cm} (21)

$$V = \exp[\kappa_-(t + r^*)],$$  \hspace{1cm} (22)

the tortoise coordinate

$$r^* = \int \frac{dr}{g}.$$  \hspace{1cm} (23)

When $r \to r_-$, the tortoise coordinate diverges,

$$r^* \approx \frac{1}{2\kappa_-} \ln(r - r_-) + r_0^*$$  \hspace{1cm} (24)

where $r_0^*$ is a constant. It follows from (21), (22) that

$$\frac{U}{V} = \exp(-2\kappa_-t), \quad UV = \exp(2\kappa_-r^*) = (r - r_-)\chi(r),$$  \hspace{1cm} (25)

Near the horizon the function $\chi(r)$ is finite (its exact form is irrelevant for our purposes), one can choose $\chi(r_-) = 1$.

Then, the metric acquires the form (8) with

$$F = g\kappa_-^2 \exp(-2\kappa_-r^*) = \frac{g}{\kappa_-^2 UV}.$$  \hspace{1cm} (26)
Now, in coordinates \((U, V, \theta, \phi)\) our basis null vectors read
\[
l_\mu = \kappa_- F(V, 0, 0, 0), \tag{27}
\]
\[
N_\mu = \frac{1}{2\kappa_-}(0, \frac{1}{V}, 0, 0). \tag{28}
\]

The vector \(l_\mu\) is tangent to the horizon \(U = 0\). It does vanish in the bifurcation point where both \(U\) and \(V\) vanish. It is instructive to note that the vector \(l_\mu\) can be also presented in another form. It follows from \((21), (22)\) that \(\frac{\partial r}{\partial U} = \frac{\kappa_- V}{2} F, \ \frac{\partial r}{\partial V} = \frac{\kappa_- U}{2} F\). On the horizon \(U = 0\) we have that \(\frac{\partial r}{\partial V} = 0\). As a result, we can write \((l_\mu)_{U=0} = 2 \left(\frac{\partial r}{\partial x_\mu}\right)_{U=0}\) Near the bifurcation point, also \(\frac{\partial r}{\partial U} \to 0\) in agreement with discussion in the previous Section, so \(l_\nu \to 0\) as well.

Now, in the Kruskal coordinates, we obtain from \((18)\) that for a usual particle,
\[
u^U \approx -\frac{A}{V}, \tag{29}\]
where
\[
A = \frac{2}{F(r_-)\kappa_-} \frac{X(r_-)}{m}, \tag{30}\]
and
\[
u^V \approx \frac{2V}{F(r_-)A}. \tag{31}\]

It means for that for small \(V\) a particle moves with almost vanishing \(\nu^V\), i.e. almost along the leg \(V = 0\). Using \((27)\) and \((29)\), one can calculate the horizon value
\[
\beta = -\nu^\mu l_\mu \approx 2 \frac{X(r_-)}{m} \neq 0 \tag{32}\]
which corresponds exactly to its counterpart derived near the event horizon (see discussion between eq.s 23 and 24 in Ref. [6]).

For the critical particle, one finds from \((14), (15)\) that \(y\) remains finite, so \((25)\) entails that \(U \sim V \sim r - r_-\). Then, one can find easily from \((19), (20)\) that \(u^U\) and \(u^V\) remain finite. As near the bifurcation point
\[
U = V = 0. \tag{33}\]
the vector \(l_\mu \to 0\) according to \((27)\), we obtain that \(\beta \to 0\). For the radial motion in the Reissner-Nordström metric \(\beta = \alpha\) according to \((6)\) with \(s_\mu = 0\), so \(\alpha \to 0\) as well.
As a result, we see that the ratio \( \frac{\beta}{\alpha} \) in (7) diverges, so the BSW effect takes place. It is worth stressing that if we rescale (27), (28) (say, due to another reparametrization), it does not affect the final result for the gamma factor (7) corresponding to the energy in the centre of mass frame. Indeed, if \( l_\mu \rightarrow \lambda l_\mu \), we must change \( N_\mu \rightarrow \lambda^{-1} N_\mu \) to preserve normalization \( l^\mu N_\mu = -1 \). Then, \( \beta_i \rightarrow \lambda \beta_i \), \( \alpha_i \rightarrow \lambda \alpha_i \) according to (5), so the ratio \( \frac{\beta}{\alpha} \) remains unaffected.

It is convenient to summarize the situation with the help of Table 1. Here, for definiteness, it is implied that a usual particle moves towards the horizon \( U = 0 \). We compare the situation near the bifurcation surface and near the event horizon far from it (see Sec. 3.2 of [6]).

| Behavior of \( \beta = -u^\mu l_\mu \) | usual particle | critical particle | \( l^\mu \) |
|-----------------------------------------|----------------|-------------------|---------|
| Event horizon                           | \( u^U \) finite | \( u^U \rightarrow 0 \) | \( \neq 0 \) |
| Vicinity of bifurcation surface         | \( u^U \rightarrow \infty \) | \( u^U \) finite | \( = 0 \) |

Table 1. Behavior of different factors in the expression for the coefficient \( \beta \).

Thus in case 1 the critical particle moves almost along the horizon \( U = 0 \) while a usual one crosses it arbitrarily. In case 2 the critical particle passes through the bifurcation point (the near-critical passes through very closely to it) while a usual one moves almost along the horizon \( V = 0 \).

We see that quite different mechanisms act to ensure in each case that one of colliding particle have \( \beta \rightarrow 0 \) (critical) and the other one have \( \beta \neq 0 \) (usual).

V. DE SITTER METRIC

Another venue where the results of the present paper can be used is the BSW effect near the cosmological horizon. It is clear from our approach that the proof concerning the role of the bifurcation point applies to such horizons as well. Let us consider the de Sitter spacetime as one of simplest examples. Let the metric have the form (28) with 

\[
f = 1 - \frac{\Lambda}{3} r^2, \quad \Lambda > 0. \tag{34}
\]

In the T-region the metric takes the form (11) with 

\[
g = \frac{\Lambda}{3} T^2 - 1 = g = \frac{\Lambda}{3} (r^2 - r_h^2), \tag{35}
\]

where \( r_h = \frac{1}{\sqrt{\Lambda}} \) corresponds to the horizon. Equations of motion in the T-region are given by (14), (15).
Now, although the concrete formulas for the metric change, division to the critical \((X = 0)\) and usual \((X \neq 0)\) particles is completely similar to that described in the previous Section. As now \(X\) is the integral of motion, \(X = X(r_h) = \text{const}\), so a particle if critical if \(X = 0\) and is usual if \(X \neq 0\). The quantity \(X\) has the meaning of the momentum.

A. Critical particle, \(X = 0\)

Then, it follows from (14) that \(y = y_0 = \text{const}\). It is seen from (21), (22) that this trajectory passes through the bifurcation point \(U = V = 0\). The integral in (15) takes a very simple form:

\[
\tau = \int_{r_1}^{r} \frac{dr}{\sqrt{g}} = -\sqrt{\frac{3}{\Lambda}} \ln(r + \sqrt{r^2 - r_h^2}) + \text{const},
\]

whence it is clear that \(\tau - \tau_h \sim \sqrt{r - r_h}\) near the horizon similarly to eq. (15) of [4], where \(\tau_h\) is the value of \(\tau\) when the horizon is crossed.

B. Usual particle, \(X \neq 0\)

The integrals can be calculated exactly in terms of elementary functions but the corresponding expressions are rather cumbersome and are not listed here. It follows from (14) that for such a particle \(y \sim \ln(r - r_h)\) near the horizon. Therefore, in the point of collision where coordinates of both particles should coincide, the absolute value of \(y_0\) should also have the order \(|\ln(r - r_h)|\) (cf. [4]).

Let us consider collision of two particles, where particle 1 is critical \((X_1 = 0)\) but particle 2 is usual \((X_2 \neq 0)\). Then, it follows from eqs. (2) and (3) and equators of motion (14), (15) that

\[
E_{c.m.}^2 = m_1^2 + m_2^2 + \frac{m_1 Z_2}{\sqrt{g}}, \quad Z_2 = \sqrt{X_2^2 + m_2 g}
\]

Thus in the horizon limit \(g \to 0\) the energy in the centre of mass frame does diverge. We would like to stress one more time that for getting growing energies it is necessary that one of particles be critical. In turn, it means that it passes through the bifurcation point as is explained above. The closer the point of collision to the bifurcation point for a fixed \(X_2\), the larger \(E_{c.m.}^2\).
It is worth noting that collision of particles near the inner horizon of the de Sitter-Reissner-Nordström metric (with the electric charge $Q \neq 0$) was considered in [22]. In doing so, it was essential that a critical particle was also electrically charged. However, the role of the bifurcation surface was not revealed there. If this is done, it follows from our consideration that the BSW effect occurs even if $Q = 0$ and particles are neutral.

VI. CONCLUSION

Thus we have managed to generalize the BSW effect for the vicinity of an arbitrary bifurcation surface not using the details of the metric. Using the Reissner-Nordström metric as an example, we checked that this approach and that based on equations of motion agree with each other. Meanwhile, now the knowledge of these equations is not required and was used for comparison of both approaches only. The corresponding conclusions apply to the inner region of nonextremal black holes having an inner horizon. What is especially important, they are also valid for cosmological horizons. Here, we also compared the results obtained in a general approach with that based on equations of motion. This confirmed that the BSW effect near the bifurcation point does exist.

At present, it is far from being clear whether and how this effect can influence cosmological evolution but, in any case, the present results can serve as motivation to pose such a question.

To summarize, we showed that a generic bifurcation surface is an accelerator of particles in a strong gravitational field.

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