The Hamilton-Jacobi characteristic equations for three dimensional

Ashtekar gravity

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The Hamilton-Jacobi analysis of three dimensional gravity defined in terms of Ashtekar-like variables is performed. We report a detailed analysis where the complete set of Hamilton-Jacobi constraints, the characteristic equations and the gauge transformations of the theory are found. We find from integrability conditions on the Hamilton-Jacobi Hamiltonians that the theory is reduced to a $BF$ field theory defined only in terms of self-dual (or anti-self-dual) variables; we identify the dynamical variables and the counting of physical degrees of freedom is performed. In addition, we compare our results with those reported by using the canonical formalism.

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I. INTRODUCTION

The study of gauge systems is the cornerstone for understand the fundamental interactions of nature. Gauge systems are characterized since there exist equivalent classes among physical states and they are connected via gauge transformations. In fact, the gauge transformations are an important part of the symmetries of the system because they characterize the core of a gauge theory. The identification of the gauge symmetries can be carry out by means different and powerful approaches such as the canonical framework developed by Dirac and Bergmann [1, 2], the symplectic method of Faddeev-Jackiw [3–18] and the Hamilton-Jacobi [HJ] procedure [19–22]. The $HJ$ approach is an economical and elegant scheme for study gauge systems; it is based on the construction of a fundamental differential which has as principal components the $HJ$ constraints called Hamiltonians, which can be involutives and noninvolutives. The former are characterized by considering that their Poisson brackets with all Hamiltonians, including themselves vanish, in otherwise they are noninvolutives. The identification of noninvolutive Hamiltonians allows us to construct the so-
called generalized brackets which are a generalization of the Poisson brackets, at the end of the procedure the fundamental differential will be expressed in terms of involutive Hamiltonians and the generalized brackets. The HJ method has been applied for studying several gauge systems, in particular systems with general covariance just like BF theories\cite{23,24}, topological invariants\cite{25} and field theories\cite{26,27}. In this respect, it has been showed that the development of the HJ scheme is more economical with respect either Dirac or the FJ approaches. Furthermore, in the HJ scheme the identification of the symmetries is made in direct form and we avoid the large procedure of classification of constraints just like in Dirac’s method is done. In this manner, the HJ framework is an interesting alternative for analyzing gauge systems and their symmetries.

With the ideas exposed above, in this paper we will apply the HJ approach to 3D gravity. The model under study is reported in \cite{28} and represents the 3D equivalent version of real gravity theory reported by Holst\cite{29}. In fact, the 3D gravity action is expressed in terms of triads, connexions, a vector field defined on $R^4$ and a Barbero-Immirzi-like parameter ($\gamma$). The analysis reported in \cite{28} was developed by using the canonical scheme and it was showed that the action is reduced to a BF field theory without the presence of the Barbero parameter. In this paper, we will perform a different analysis by introducing a set of Ashtekar-like variables, then the HJ formalism will be developed. We show that the action is reduced to a BF field theory, however the dynamical variables will be identified with the anti-self-dual Ashtekar-like variables such as it is presented in the Holst paper. On the other hand, the $\gamma$ parameter will be present through the Lagrange multipliers; the parameter will not contribute to the constraints because the Lagrange multipliers are not dynamical, this result represent a difference with respect to the Holst action.

The paper is organized as follows. In the Sect. II we will introduce the Ashtekar-like variables, then the HJ is performed. The characteristic equations and the symmetries of the theory will be found. In addition, we will observe that the system is reduced to a BF theory and the Barbero-like parameter will be present at Lagrange multipliers level. Finally in Sect. III some remarks and conclusions are exposed.

II. HAMILTON-JACOBI ANALYSIS

We shall analyze the following action \cite{28}

$$S = \int \tilde{\epsilon}^3 x^\mu \eta_{\mu} \left( \frac{1}{2} \epsilon_{IJKL} x^I e^J_{\mu} F^K_{\nu\rho} + \gamma^{-1} x^I e^\mu x^J F^I_{\mu\nu} \right),$$

where $\epsilon_{IJKL}$ is the volume element of $SO(4)$, $x^I$ is a vector of $R^4$, $F^I_{\mu\nu} = \partial_\mu A^I_{\nu} - \partial_\nu A^I_{\mu} + A^L_{\mu} L^I_{\nu} L^J - A^L_{\nu} L^I_{\mu} L^J$ is the strength curvature of $SO(4)$, $e^\mu$ is the triad and $\gamma$ the Barbero-Immirzi-like parameter. It is straightforward to prove that the action describes 3D Euclidean gravity and we can observe that there is a closed relation with the Holst action in the sense that there is a coupling $\gamma$ parameter.

Furthermore, in this paper, we will work with the temporal gauge, by fixing $x^I = (1, 0, 0, 0)$. Although there are other gauge fixing options, we are interested to report the closer relation with the
Holst action, and the temporal gauge provides us that aim.

In this manner, by performing the 2+1 decomposition the action takes the following form

\[ S = \int d^3x \frac{1}{2} \epsilon^{abc} e_{0i} \left( e^i_{jk} F_{ab}^{jk} + \frac{2}{\gamma} F_{ab}^{0i} \right) - \int d^3x \epsilon^{abc} e_{ai} \left( e^i_{jk} F_{0b}^{jk} + \frac{2}{\gamma} F_{0b}^{0i} \right), \]  

(2)

where the strength curvature components are given by

\[ F_{ab}^{jk} = \partial_a A_b^{jk} - \partial_b A_a^{jk} + A_a^j A_b^k - A_b^j A_a^k - A_a^j A_b^k + A_b^j A_a^k, \]

\[ F_{ab}^{0i} = \partial_a A_b^{0i} - \partial_b A_a^{0i} + A_a^0 A_b^i - A_b^0 A_a^i + A_a^0 A_b^i - A_b^0 A_a^i, \]

\[ F_{0b}^{jk} = \partial_0 A_b^{jk} - \partial_b A_0^{jk} + A_0^j A_b^k - A_b^j A_0^k + A_0^j A_b^k - A_b^j A_0^k, \]

\[ F_{0b}^{0i} = \partial_0 A_b^{0i} - \partial_b A_0^{0i} + A_0^0 A_b^i - A_b^0 A_0^i + A_0^0 A_b^i - A_b^0 A_0^i. \]

(3)

(4)

Now we introduce the following Ashtekar-like variables

\[ \pm \mathcal{A}_b^{ij} = e^i_{jk} A_b^{jk} \pm \frac{2}{\gamma} A_b^{0j}. \]

(5)

and thus we obtain

\[ A_b^{0j} = \frac{\gamma}{4} \left[ \pm \mathcal{A}_b^{ij} - \mathcal{A}_b^{ij} \right], \]

\[ A_b^{jk} = \frac{1}{4} e^{ijk} \left[ \pm \mathcal{A}_b^{ij} + \mathcal{A}_b^{ij} \right]. \]

(6)

(7)

Then, by using these variables, the action reads

\[ S = \int d^3x \epsilon^{abc} e_{0i} \left[ \partial_a - \partial_b A_a^{0\dot{i}} + e^i_{jk} \left( \frac{\gamma^2 - 1}{16} + A_a^j A_b^k + \frac{\gamma^2 + 3}{16} - A_a^j - A_b^k - \frac{\gamma^2 - 1}{8} A_a^j - A_b^k \right) \right] \]

\[ + \int d^3x \epsilon^{abc} e_{ai} \left[ \partial_b \left( e^i_{jk} A_b^{jk} + \frac{2}{\gamma} A_b^{0i} \right) - \partial_a - \partial_b A_a^{0\dot{i}} + \left( \frac{\gamma^2 - 1}{2\gamma} \right) e^i_{jk} A_b^{0\dot{j}} + A_b^{0\dot{i}} - \left( \frac{\gamma^2 + 1}{2\gamma} \right) e^i_{jk} A_b^{0\dot{j}} + A_b^{0\dot{i}} - A_b^{0\dot{i}} \right], \]

(8)

we can observe that if \( \gamma = 1 \), then the self-dual-connexion disappears. Because of the action is under a variational principle, we will not fix the value of \( \gamma \) until the end of the calculations. In this manner, according the \( HJ \) method, from the definition of the momenta \((p^0_i, p^a_i, \pi_{ij}, +\pi^a_i, -\pi^a_i)\) canonically conjugated to \((\epsilon^i_0, \epsilon^a_i, A_0^{0\dot{i}}, A_0^{ij}, +A_a^{ij}, -A_a^{ij})\), and from the action \( (8) \) we identify the following Hamiltonians

\[ H' \equiv \pi + H_0 = 0, \]

\[ \phi_i = p_i^0 = 0, \]

\[ \phi_i^a = p_i^a = 0, \]

\[ \phi_{ij} = \pi_{ij} = 0, \]

\[ +\phi_i^a = +\pi_i^a = 0, \]

\[ -\phi_i^a = -\pi_i^a - \epsilon^{ab} e_{bi} = 0, \]

(9)
with \( \pi = \partial_0 S \) where \( S \) is the action and the canonical Hamiltonian \( H_0 \) reads

\[
H_0 = -\epsilon^{ab}_{\alpha\beta} \left[ \partial_a - A_b \right] - \epsilon_{jk} \left[ \left( \frac{\gamma^2 - 1}{16} \right) + \frac{\gamma^2}{16} - \frac{\gamma^2 - 1}{8} \right] - B_a \left[ \left( \frac{\gamma^2 + 3}{16} \right) - \frac{\gamma^2 + 3}{8} \right] - \frac{\gamma^2 + 1}{2} \right] \epsilon_{jk} A_b - \left( \frac{\gamma^2}{2} \right) \epsilon_{jk} A_b^0 + A_b^j A_b^k \right] - A_b^k. \tag{10}
\]

Now with the Hamiltonians we construct the following fundamental \( HJ \) differential

\[
df(x) = \int d^3y \left\{ f(x), H' \right\} dt + \left\{ f(x), \phi_i \right\} d\xi^i + \left\{ f(x), \phi^a_i \right\} d\xi_a^i + \left\{ f(x), \phi_i \right\} d\xi^i + \left\{ f(x), \phi_i \right\} d\xi^i + \left\{ f(x), \phi^a_i \right\} d\xi_a^i\right\}
\]

where \((\xi^i, \xi_a^i, \xi^i, + \xi_a^i, - \xi_a^i)\) are parameters related to the Hamiltonians. It is worth to mention that these parameters play a fundamental roll; for involutives Hamiltonians they correspond to parameters related with the gauge transformations, this fact will be discussed bellow.

On the other hand, the fundamental Poisson brackets between the canonical variables are given by

\[
\{\epsilon^i(x), p^j(y)\} = \delta^i_j \delta^2(x - y), \quad \{A_0^a(x), \pi_j(y)\} = \delta^i_j \delta^2(x - y), \quad \{A_0^a(x), \pi_{kl}(y)\} = \frac{1}{2} \delta_{kl} \delta^2(x - y), \quad \{+ A_a^i(x), + \pi_b^j(y)\} = \delta^a_b \delta^2(x - y), \quad \{- A_a^i(x), - \pi_b^j(y)\} = \delta^a_b \delta^2(x - y). \tag{12}
\]

Once defined the Poisson brackets, all Hamiltonians having vanishing Poisson brackets to each other are called involutives, otherwise, they are non-involutive Hamiltonians. Thus, by using the fundamental brackets we observe that the Hamiltonians \((\phi_i, \tilde{\phi}_i, \phi_{ij}, + \phi^a_i)\) are involutives and \((\phi^a_i, - \phi^a_i)\) are noninvolutives. Furthermore, due to there are noninvolutives Hamiltonians, we introduce the generalized brackets by constructing the matrix whose entries are the Poisson brackets between all noninvolutives Hamiltonians, this is

\[
C_{\alpha\beta} = \begin{pmatrix}
0 & -\epsilon^{ab}_{\alpha\beta} \\
\epsilon^{ba}_{\alpha\beta} & 0
\end{pmatrix} \delta^2(x - y),
\]

and its inverse reads

\[
(C_{\alpha\beta})^{-1} = \begin{pmatrix}
0 & \epsilon_{ad} \eta^{ij} \\
-\epsilon_{bd} \eta^{ij} & 0
\end{pmatrix} \delta^2(x - y),
\]

thus, by using \((C_{\alpha\beta})^{-1}\) we can introduce the generalized brackets given by

\[
\{A, B\}^* = \{A, B\} - \{A, H_a^i\}(C_{\alpha\beta})^{-1}\{H_b^i, B\}, \tag{13}
\]

where \(H_a^i\) are the non-involutive Hamiltonians. In this manner, by using the generalized brackets
are given by
\[
\{c^i_0(x), p^0_j(y)\}^* = \delta^i_0 \delta^2(x - y),
\{c^i_a(x), p^0_j(y)\}^* = 0,
\{A^i_0(x), \pi_j(y)\}^* = \delta^i_j \delta^2(x - y),
\{A^i_0(x), \pi_{kl}(y)\}^* = \frac{1}{2} \delta^i_{kj} \delta^2(x - y),
\{+A^i_a(x), +\pi^0_j(y)\}^* = \delta^a_0 \delta^j \delta^2(x - y),
\{-A^i_a(x), -\pi^0_j(y)\}^* = \delta^a_0 \delta^j \delta^2(x - y).
\]

The introduction of the generalized brackets redefine the dynamics. In fact, the non-involutive constraints are removed from the fundamental differential and it can be expressed in terms of the generalized brackets and involutive Hamiltonians.

In this manner, the fundamental differential written in terms of the generalized brackets and involutive Hamiltonians takes the form
\[
df(x) = \int d^3y \left( \{f(x), H'\}^* dt + \{f(x), \phi_i\}^* d\xi^i + \{f(x), \tilde{\phi}_i\}^* d\tilde{\xi}^i + \{f(x), \varphi_{ij}\}^* d\varphi^{ij} + \{f(x), \phi^a_i\}^* d^+\varphi^a_i \right),
\]
thus, the Frobenius integrability conditions for the Hamiltonians \[19, 20\], say \((\phi_i, \tilde{\phi}_i, \varphi_{ij}, +\phi^a_i)\), introduce new Hamiltonians
\[
d\phi_i, = \int d^3y \left( \{\phi_i, H'\} dt + \{\phi_i, \varphi_{ij}\} d\xi^j + \{\phi_i, \tilde{\phi}_j\} d\tilde{\xi}^j + \{\phi_i, \phi_{kl}\} d\xi^{kl} + \{\phi_i, +\phi^a_j\} d^+\varphi^a_j \right) = \epsilon^{ab} \left[ \partial_d A^b d - \epsilon_d j k \left[ \frac{\gamma^2 - 1}{16} A^j A^k + \frac{\gamma^2 + 3}{16} A^j A^k - \gamma \right] \right] = 0,
\]
\[
d\tilde{\phi}_i, = \int d^3y \left( \{\tilde{\phi}_i, H'\} dt + \{\tilde{\phi}_i, \varphi_{ij}\} d\xi^j + \{\tilde{\phi}_i, \tilde{\phi}_j\} d\tilde{\xi}^j + \{\tilde{\phi}_i, \phi_{kl}\} d\xi^{kl} + \{\tilde{\phi}_i, +\phi^a_j\} d^+\varphi^a_j \right) = \frac{2}{\gamma} \partial_d \pi^a_i d + \epsilon_d j k \left[ \frac{\gamma^2 - 1}{2\gamma} A^j A^k - \gamma \right] = 0,
\]
\[
d\varphi_{ij}, = \int d^3y \left( \{\varphi_{ij}, H'\} dt + \{\varphi_{ij}, \varphi_{kl}\} d\xi^k + \{\varphi_{ij}, \tilde{\varphi}_k\} d\tilde{\xi}^k + \{\varphi_{ij}, \phi_{kl}\} d\xi^{kl} + \{\varphi_{ij}, +\phi^a_k\} d^+\varphi^a_k \right) = \epsilon^{ij} \partial_d \pi^a_i d + \epsilon_d j k \left[ \frac{\gamma^2 - 1}{2\gamma} A^j A^k - \gamma \right] = 0,
\]
\[
d^+\phi^a_i, = \int d^3y \left( \{+\phi^a_i, H'\} dt + \{+\phi^a_i, \varphi_{ij}\} d\xi^j + \{+\phi^a_i, \tilde{\varphi}_j\} d\tilde{\xi}^j + \{+\phi^a_i, \phi_{kl}\} d\xi^{kl} + \{+\phi^a_i, +\phi^a_j\} d^+\varphi^a_j \right) = \frac{1}{4} \epsilon^{ab} \epsilon^{ij} \left[ A^b A^k - \gamma A^k \right] + \frac{1}{4} \epsilon^{ab} \epsilon^{ij} \left[ A^b A^k - \gamma A^k \right] = 0,
\]
where we identify the following Hamiltonians,
\[
\chi^i = \epsilon^{ab} \left[ \partial_d A^b d - \epsilon_d j k \left[ \frac{\gamma^2 - 1}{16} A^j A^k + \frac{\gamma^2 + 3}{16} A^j A^k - \gamma \right] \right] = 0,
\]
\[
\tilde{\chi}_i = \frac{2}{\gamma} \partial_d \pi^a_i + \epsilon_d j k \left[ \frac{\gamma^2 - 1}{2\gamma} A^j A^k - \gamma \right] = 0,
\]
\[
\psi_{ij} = \epsilon_d j k D_a \pi^a_i = 0,
\]
\[
\tilde{\chi}^a = \frac{1}{4} \epsilon^{ab} \epsilon^{ij} \left[ A^b A^k - \gamma A^k \right] + \frac{1}{4} \epsilon^{ab} \epsilon^{ij} \left[ A^b A^k - \gamma A^k \right] = 0.
\]
here $D_a^{-1} \pi^a_1 = \partial_a \pi^a_1 - \frac{1}{2} \epsilon_{ij}^k A_a^j \dot{\pi}^a_1$. On the other hand, from the Hamiltonian $\dot{\chi}^a_1$ we observe that $\dot{\chi}^a_i - \dot{\pi}^a_i = 0$, and thus $\dot{A}_b^k = -\dot{A}_b^k = 0$; this result implies that in three dimensional Ashtekar gravity the dynamical variables are given by the adjoint representation of $SO(3)$. In order to follow the Holst work we choose $A_{b}^{jk} = \frac{1}{4} \epsilon^{ijk} A_{a}^i$. In this manner, with these results at hand the Hamiltonians take the form

$$H^i := \frac{1}{2} \epsilon_{ab} F_{ab}^i = \frac{1}{2} \epsilon_{ab} \left[ \partial_a - \partial_b - A_a^i - \frac{1}{2} \epsilon_{ijk} \dot{A}_b^j - \dot{A}_b^k \right] = 0,$$

$$G_j := \frac{2}{\gamma} \partial_j - \dot{\pi}^b_j - \frac{1}{\gamma} \epsilon_{jik} \dot{\pi}^b_i - \dot{A}_b^k = 0,$$

$$\tilde{G}_{ij} := \epsilon_{ij}^k D_a - \dot{\pi}^a_i = 0,$$

and the Hamiltonian $H_0$ is reduced to

$$H_0 = -\frac{\epsilon_{ab}}{2} \epsilon_0 F_{ab}^k - \Lambda^i D_b^b \dot{\pi}_i^a,$$

where $\Lambda^i = \epsilon_{ijk} A_0^{jk} - \frac{2}{\gamma} A_0^{i0}$. We can observe that the contribution of the $\gamma$ parameter is only present in $\Lambda^i$ that will be identified as Lagrange multipliers; this result is a difference respect to that reported in [28], where the Barbero-like parameter is eliminated completely. Furthermore, from Eq. (18) we observe that the variables $\pi_{ij}$ and $\pi_i$ generate the same involutive Hamiltonian $G_i$, however we will not remove that Hamiltonian until the end of the analysis. The Hamiltonians are involutives; their generalized algebra is closed

$$\{\mathcal{H}^i(x), \mathcal{H}^j(y)\} = 0,$$

$$\{\mathcal{H}^i(x), G_j(y)\} = -\frac{1}{2} \epsilon_{ijk} \mathcal{H}^k,$$

$$\{G_i(x), G_j(y)\} = -\frac{1}{2} \epsilon_{ijk} \tilde{G}_k,$$

because the algebra is closed, then there are not more Hamiltonians.

Thus, by using all involutive Hamiltonians we construct a new fundamental differential

$$df(x) = \int d^3 y \left( \{f(x), H_0(y)\} dt + \{f(x), \phi_i(y)\} \dot{\xi}^i + \{f(x), \tilde{\phi}_i(y)\} \dot{\xi}^i + \{f(x), \phi_{ij}(y)\} \dot{\xi}^{ij} + \{f(x), \phi_k(y)\} \dot{\omega}^k + \{f(x), \tilde{G}_k(y)\} \dot{\omega}^k \right),$$

where all noninvolutive Hamiltonians have been removed. Now, from the fundamental differential we can identify the characteristics equations, then the symmetries. The characteristic equations are given by

$$d^{-1} A_a^i = D_a \Lambda^i dt + D_a d\tilde{\omega}^i,$$

$$d^{-1} \pi_a^i = \left[ \epsilon^{ab} D_b \epsilon_{oi} - \frac{A^k}{2} \epsilon_{kij} - \dot{\pi}^a_j \right] dt + \frac{\epsilon_{klj}}{2} \dot{\pi}^a_j d\tilde{\omega}^k,$$

$$d\pi^0_i = \frac{\epsilon^{ab}}{2} F_{iab} dt = 0,$$

$$d\pi_i = G_i dt = 0,$$

$$d\pi_{ij} = \epsilon_{ij}^k \tilde{G}_k dt = 0,$$

(22)
\[ \begin{align*}
de c_0^i &= d\xi^i, \\
d A_0^{ij} &= d\xi^{ij}, \\
d A_0^{i0} &= d\tilde{\xi}^i,
\end{align*} \]

where we defined \( d\tilde{\omega}^i \equiv d\omega^i + \epsilon^i_{jk} d\omega^{jk} \). From (22) we can identify the equations of motion

\[ \begin{align*}
\partial_0 - A_a^i &= D_a \Lambda_i, \\
\partial_0 - \pi_a^i &= [\epsilon^{ab} D_b c_{0i} - \Lambda_k^i \frac{1}{2} \epsilon_k^{ij} \pi_{ij}],
\end{align*} \]

and we observe that the evolution of \( p_0^i, \pi_i, \pi_{ij} \) due to the noninvolutive Hamiltonians vanishes. Furthermore, Eq. (23), implies that \( c_0^i, A_0^{ij}, A_0^{i0} \) are identified as Lagrange multipliers, thus, \( \Lambda^i \) is also identified as Lagrange multiplier. Moreover, by taking \( dt = 0 \) in (22) we can also identify the gauge transformations of the theory

\[ \begin{align*}
\delta - A_a^i &= D_a \epsilon^i, \\
\delta - \pi_a^i &= \frac{1}{2} \epsilon_k^{ij} \pi_{ij} \epsilon^k,
\end{align*} \]

where \( \epsilon^i \equiv d\tilde{\omega}^i \). On the other hand, we commented above that the variables \( A_0^{jk} \) and \( A_0^{i0} \) generate the same Hamiltonian \( G_i \) and this fact will be taken into account to perform the counting of physical degrees of freedom. In fact, the counting of physical degrees of freedom is performed as follows: there are 12 dynamical variables \( (-A_a^i, -\pi_a^i) \), and 18 involutive Hamiltonians \( (H^i, G_i, \tilde{G}_{ij}, p_0^i, \pi_i, \pi_{ij}) \), however, \( \pi_i \) and \( \pi_{ij} \) generate the same hamiltonian \( \tilde{G}_i \); therefore there are 18-6=12 independent involutive Hamiltonians, hence the system is lacking of physical degrees of freedom as expected; in the counting of degrees of freedom only independent involutive Hamiltonians must be involved. Moreover, if we fix \( A_0^{jk} = 0 \), we still have the presence of the \( \gamma \) parameter in the Lagrange multiplier \( \Lambda^i \); the theory will take a \( BF \) form with \( \gamma \) present only at the level of Lagrange multipliers and there will be a contribution in the equation of motion (24). On the other hand, if we remove \( \pi_i \) by taking \( A_0^{i0} = 0 \), then there is not any contribution from \( \gamma \) because \( \Lambda^i = \epsilon^i_{jk} A_0^{jk} \); the theory will be a \( BF \) theory just like that reported in [28], and hence our results extend the results reported in the literature.

### III. CONCLUSIONS

In this paper, the HJ analysis of 3D gravity written in terms of Ashtekar-Like variables was performed. We obtained an action with a close relation to the Holst Lagrangian where self-dual and anti-self-dual connexions are present. We identified all \( HJ \) Hamiltonians of the theory, then a fundamental differential was constructed. From the fundamental differential the characteristic equations were found; we reported the gauge transformations and we identified the dynamical variables corresponding to the adjoint representation of the \( SO(3) \) connexion. We observed that the theory is reduced to a \( BF \) theory where the presence of the Barbero-like parameter is present at level of the
Lagrange multipliers, in particular, under a fixing gauge on the multipliers the results reported in the literature were reproduced. It is worth to mention that the coupling of 3D gravity with degrees of freedom as for instance matter degrees of freedom, will be an interesting scenario to analyze. In fact, the coupling with matter degrees of freedom could provide us an understanding of the role of the $\gamma$ parameter in 3D gravity just like it is present in the 4D case \[30\]. In this sense, it is well-known that in 4D gravity with the coupling of fermions, there is a contribution of the $\gamma$ parameter; it does not vanish and it determines the coupling constant of a four-fermion interaction. In this manner, the $HJ$ framework will be a good alternative for analyzing these problems and we expect to find in the future advantages in relation with other approaches.

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