Waterflooding simulation of reservoir containing horizontal well stimulated by multistage hydraulic fracturing

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Abstract. The article presents a three-dimensional mathematical model for two-phase fluid flow near a multistage hydraulically fractured horizontal well (MSHFHW). The flow in the reservoir and in the fractures is simulated separately, and the flow rate is governed by Darcy's law. Finite volume method is used for spatial approximation. The obtained systems of linear equations for pressure in the reservoir and in the fractures are solved simultaneously, which allows us to avoid using iterative process for solution adjustment both in the fractures and the reservoir. Saturation is calculated by the implicit adaptive scheme AIM.

Keywords. horizontal well, multistage hydraulic fracturing, two-phase flow in porous media

1. Introduction
Multistage hydraulically fractured horizontal wells (MSHFHW) are often used for development of oil fields characterized by weak reservoir properties, high heterogeneity, or low permeability. With the growing popularity of this technique, there is a need to develop mathematical models describing complex filtration processes in the “reservoir–hydraulic fractures–well” system.

Paper [1] proposes a mathematical model of two-phase flow in porous media with fractures. Sampling of equations is performed on an unstructured hybrid grid using implicit schemes for pressure and saturation. Work [2] is devoted to solving the problem of two-phase flow near a horizontal well with MSHF in a three-dimensional formulation in the Cartesian coordinate system.

A model of two-phase flow near HW with MSHF, which takes into account the variation of fractures conductivity in time, is presented in [3]. Spatial approximation of the equations was performed using finite difference method. The IMPES scheme was used for temporal sampling [4]. The pressure in this case was calculated by the implicit scheme, and saturation – by the explicit one. The results showed that consideration of saturation changes in the fracture has a significant effect on the pressure distribution near the fractures. Paper [5] proposes an iterative method for simultaneous solution of the problems for saturation and pressure, which accelerates calculations by 30% compared to the full implicit method (FIM) [6] and, unlike IMPES, is convergent at a bigger Courant number:

$$C = \frac{\tau \max |u|}{h} \ll 1,$$

where $h$, $\tau$ are the grid steps in space and time, $u$ is the flow rate.

This paper presents a three-dimensional mathematical model describing a two-phase flow near multistage hydraulically fractured horizontal wells.
2. Constitutive equations

We consider the problem of two-phase flow near HW stimulated by transverse hydraulic fractures in a three-dimensional formulation. The solution region $D$ is a part of the reservoir, having the form of a rectangular parallelepiped with the height $2H$ and rounded edges, at the center of which there is a cut of a cylindrical shape – a well with the length $L$ and the radius $r_w$ (figure 1). The region $D$ is vertically bounded by the planes $z = \pm H$, modeling the top and bottom of the reservoir. The lateral surface $\Gamma$ located at the distance $l$ from the well $\gamma$ models the external boundary. The $Oy$ axis is directed along the borehole, and the hydraulic fractures are located orthogonally to it. The walls of the fracture are two rectangular planes $F_i^-$ and $F_i^+$ with the normals in the directions $n^\pm = \pm y_i$ and having the sizes $2H \times 2l_i$, the distance between them being $2\delta$ (fracture opening). The fractures have permeability far exceeding the absolute permeability of the reservoir. The reservoir is considered to be uniform. Capillary and gravitational forces are neglected.

In dimensionless variables, the equations for pressure and saturation in the reservoir $D$ have the form:

\[
\beta \frac{\partial p}{\partial t} + \text{div} \ u = 0, \quad u = -\sigma \text{ grad } p , \quad (1)
\]
\[
\frac{\partial s}{\partial t} + \text{div} \ (f(s)u) = 0; \quad (2)
\]
\[
f(s) = \frac{k_w(s)}{\sigma}, \quad \sigma = k_w(s) + K_\mu k_o(s),
\]
\[
k_w(s) = s^A, \quad k_o(s) = (1-s)^B.
\]

Here $\beta \sim 10^{-3}$ is the compressibility; $f(s)$ is the Buckley-Leverett function; $k_w(s), k_o(s)$ are the relative permeabilities (RP) of water and oil respectively; $A, B$ are the RP coefficients; $\sigma(S)$ is the transmissibility; $K_\mu < 1$ is the water and oil viscosity ratio.

The dimensionless initial conditions

\[ t = 0, \ \ (x, y, z) \in D: \quad p = 1, \quad s = 0.. \quad (3)\]

show that the reservoir is saturated by oil under hydrostatic pressure. The top and bottom of the reservoir (spatial coordinates are normalized on $H$) are impermeable,

\[ z = \pm 1: \quad u_n = \sigma \frac{\partial p}{\partial n} = 0. \quad (4)\]

The process of waterflooding is modeled at $t > 0$, when the constant pressure $p = 1$ and water saturation $s = 1$ are maintained at the external boundary $\Gamma$. In this case the pressure $p = p_\gamma = 0$ is set
on the well \( \gamma \). The hydrodynamic interaction between the reservoir, the surface \( \gamma \) of the well and the surfaces \( F_i^+ \) of the fractures is expressed in the continuity of the pressure and the normal flow rate.

The equation for the dimensionless pressure \( p_f \), averaged by the fracture opening \( 2\delta \), is written as follows:

\[
\Delta_{xz} p_f^+ + \frac{1}{2M} \sigma \frac{\partial p}{\partial y}{}^F_{F^-} = 0 \quad -h_i < x < h_i, \quad -1 < z < 1,
\]

\( x = \pm h_i, \quad z = \pm 1: \quad \frac{\partial p}{\partial n} = 0, \quad (x, y, z) \in \gamma: \quad p_f^+ = p_f^- = 0. \) (5)

The equation for the water saturation \( s_f \) in the dimensionless variables has the form:

\[
m_f \frac{\partial s_f}{\partial t} + \gamma \left( f \left( S_f \right) u_f \right) + \frac{H}{2\delta} \left( f \left( S \right) u_y \right)_{F_i^+} = 0. \) (6)

Let us note that the Buckley-Leverett function in the last term of this equation, modeling the water inflow from the reservoir to the well, is calculated at the fracture faces \( F_i^+ \) and \( F_i^- \) from the reservoir’s side, just as the total flow rates \( u_y^\pm = -\alpha \partial p / \partial y \). The boundary conditions for this equation are not necessary.

Thus, the mathematical model of two-phase flow towards the production horizontal well with MSHF in the dimensionless form consists of Equations (1)–(4) in the reservoir \( D \) and Equations (5), (6) in the fractures.

The method of covering the area by the condensing grid, the approximation of Equations (1) and (5), and the algorithms for solving the problem for pressure using the implicit scheme are presented in [7]. To calculate saturation, we used the adaptive implicit method AIM [8].

3. Numerical scheme

To solve the problem, we were seeking the average pressure and saturation in the finite volumes (FV) \( V \)

\[
P = \frac{1}{|V|} \int_V p \, dV, \quad S = \frac{1}{|V|} \int_V s \, dV.
\]

(7)

For each FV, Equation (2) integrated through the volume \( V_j \) can be written (taking into account the divergence theorem) in the form:

\[
m_f \frac{S_f^j - S_f^i}{\tau} + \sum_s f^{i,j} \bar{u}_n^{i,j} \left| \Gamma_{V_j} \right| = 0,
\]

in the fractures

(8)

\[
m_f \frac{S_f^j - S_f^i}{\tau} + \sum_s (\bar{u}_f)^{i,j} \left| \Gamma_{V_j} \right| + \frac{1}{M} \left| V_j \right| P_f^j P_i^j = 0,
\]

where \( \Gamma_{V_j} = \bigcup_j \Gamma_{V_j} \) is the surface of the finite volume \( V_j \) which consists of the plane faces \( \Gamma_{V_j} \); \( \left| \Gamma_{V_j} \right| \) is the area of the face \( \Gamma_{V_j} \); \( \bar{u}_n^{i,j} \) is the average projection of the rate on the external normal of this face.

The problem of finding the pressure \( P, P_f \) by Equations (1), (5) and the saturation on a new temporal layer \( \hat{S}, \hat{S}_f \) by Equations (2), (6) is solved in several stages.
1) We calculate the pressures \( P_i, P_{fi} \) on the new temporal layer using the saturation from the previous layer \( S_i, S_{fi} \).

2) We calculate the total flow rates through all the faces of the finite volumes:

\[
\tilde{u}_{n}^{i,j} = -\sigma^{i,j} \frac{P_{j} - P_{i}}{h_{d}^{i,j}},
\]

where \( h_{d}^{i,j} \) is the distance from the center of the \( i^{th} \) FV to the center of the \( j^{th} \) one. If the centers of the FV are not located on the line, through which the vector of the external normal passes to the face separating the two FV, then we use the four-point linear approximation [9] through the average values of pressure at collocation points located at the centers of the corresponding finite volumes.

3) We calculate the saturations \( S_i, S_{fi} \) using

a) explicit scheme

\[
S^i = S^i - \frac{\tau}{m_{j}} \sum_{j} f^{i,j} (S) \tilde{u}_{n}^{i,j} (S) \mid_{\Gamma_{V_{i}}} = 0;
\]

b) implicit scheme

\[
S^{i+\tau} + \frac{\tau}{m_{j}} \sum_{j} f^{i,j} (S) \tilde{u}_{n}^{i,j} (S) \mid_{\Gamma_{V_{i}}} = S^{i}.
\]

4. Results

Figure 2 shows the pressure distribution map and the isobar field in a uniform reservoir containing a HW with a single hydraulic fracture for \( h = 5, M = 100 \). It can be seen that the fracture substantially changes the pressure distribution, and the concentric surfaces of the isobars stretch with the increase of antigradient due to the presence of the fracture.

![Figure 2](image)

**Figure 2.** Pressure distribution and isobar map in computational domain D.

Figure 3 shows the saturation maps for the case of the reservoir waterflooding at different moments of time. We can see that the presence of the fracture accelerates the movement of the isosats in that part of the reservoir where there are low-pressure zones (figure 2). It is worth mentioning that the isosats move towards the well ends much more slowly than towards the main part of the borehole. This is due to the fact that the total withdrawal of the fluid in the case of spherical symmetry is carried out over a longer period of time than in the case of radial symmetry [10].
Figure 3. Saturation map for waterflooding of the reservoir containing HW with MSHF.

The pressure distribution and streamlines near the wellbore area and the high-conductivity hydraulic fracture are shown in figure 4. It is clearly seen how the fluid inflow separates from the external boundary: one part enters the fractures, the other gets directly to the well.

Figure 4. Pressure distribution and streamlines in cross-section areas of computational domain D

Figure 5 demonstrates the dynamics of water cut and cumulative oil production of the well for the case of: a) the absence of hydraulic fractures, b) the presence of a single fracture at \( h = 5 \), \( M = 10^2 \), intersecting the borehole in the middle, and c) the presence of two fractures with similar parameters equidistant from each other and from the ends of the well. It is evident that water content grows earlier in HW with MSHF, but at the same time the increase in the well production rate results in the growth of the cumulative oil production at a particular moment of time as compared to HW without MSHF.
Figure 5. Dynamics of water cut and cumulative oil production of the well: 1 – HW without MSHF, 2 – HW with a single fracture, 3 – HW with two hydraulic fractures.

5. Conclusion

We presented a mathematical model of two-phase flow near a horizontal well with multistage hydraulic fracturing. Calculation of saturation was accelerated by using the adaptive implicit scheme. We analyzed the effect of hydraulic fractures on the reservoir waterflooding. It was shown that water cut grows earlier in HW with MSHF compared to the traditional HW, but the total cumulative oil production increases significantly.

Acknowledgments

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