Majorana versus Dirac neutrino in $\nu - e$ scattering process and its possible applications

J Barranco*, D Delepine†, M Napsuciale‡ and A Yebra§
Departamento de Física, División de Ciencias e Ingenierías, Campus León, Universidad de Guanajuato, León 37150, México

E-mail: *jbarranco@fisica.ugto.mx, †delepine@fisica.ugto.mx, ‡mauro@fisica.ugto.mx
§azarael@fisica.ugto.mx

Abstract. In previous works it was established that if neutrino helicity differs from minus one, then the Dirac and Majorana cross sections in neutrino-electron scattering are different, despite of the smallness of the neutrino mass. Considering this fact, it is possible to employ that difference in order to establish a method to derive an upper bound for neutrino magnetic moment.

1. Introduction
Determining the neutrino nature is an open, important and fundamental question in particle physics. It refers to establish if neutrinos are Dirac or Majorana particles. If neutrinos are Dirac particles, then they will be different from its own antiparticles. On the other hand, if they are Majorana particles, neutrino and anti-neutrino must be the same particle. In that case, certain kind of reactions where the lepton number is no longer conserved are possible. The experimental observation of those reactions would constitute a direct determination of the Majorana nature of the neutrino. One representative example from those reactions is the well known neutrinoless double beta decay [1].

Another way to try to settle this problem is to use the elastic neutrino-electron scattering process. Common wisdom assumes that differences between both cross sections are proportional to terms that depend on the neutrino mass, which is usually very small. As a result, it is very difficult to have a significant difference in the measurement of the scattering cross sections for Dirac or Majorana neutrinos. Nevertheless, if the neutrino polarization is taken into account, or in other words, if we consider that the neutrino longitudinal polarization, $s_{||}$, may differ from minus one ($s_{||} \neq -1$), then, a measurable difference between the Dirac or Majorana cross section is possible [2, 3, 4]. The changes in the neutrino helicity can be induced by the interaction of the neutrino magnetic moment $\mu_\nu$ with an astrophysical external magnetic field. The changes of $s_{||}$ is determined by the Bargmann-Michel-Telegdi equation [5, 6].

Thus, the experimental verification of the neutrino nature will rely on the total number of events detected for astrophysical neutrinos that have changed their longitudinal polarization by the interaction of the neutrinos magnetic moment with an external magnetic field.

Solar neutrinos offer a possible source where such differences should appear. No appreciable differences will imply that the change in $s_{||}$ is very small and thus implies a limit on the neutrino magnetic moment $\mu_\nu$. For example, we will briefly discuss how Borexino [7], which has no seen
any appreciable difference of their expected number of events and the values measured imply that there is no appreciable difference between Dirac and Majorana neutrinos within its relative error. Therefore, we can set a new upper bound for the neutrino magnetic moment [4]. In the next section we will show the differences between the Dirac or Majorana neutrino-electron scattering cross section as a function of the neutrino longitudinal polarization \( s_|| \). Then, we will show how Borexino detection of \(^7\)Be line solar neutrinos imply a determination within a 4\% error in the measured cross section and thus a limit on the neutrino magnetic moment.

2. Dirac and Majorana neutrinos

As we mentioned above, the fact that neutrinos can be its own antiparticles or not is the central question about its nature. Moreover, one direct implication for Majorana neutrinos is that additional contributions from antineutrinos in the scatter on electrons must be included in the calculation of the amplitude, which is not necessary for Dirac neutrinos. Thus, changes on its cross sections are expected.

Let us start with the effective Lagrangian that describes the neutrino-electron scattering process at low energies. It is given by [3]:

\[
\mathcal{L} = \frac{G_F}{\sqrt{2}} \left[ \bar{u}_f^\ell \gamma^\mu (1 - \gamma^5) u_i^\nu \right] \left[ \bar{u}_e^{\ell'} \gamma_\mu \left( g_V^{\ell'} - g_A^{\ell'} \gamma^5 \right) u_e^{i'} \right],
\]

where the effective coupling constants are usually defined as

\[
g_V^{\ell'} = -\frac{1}{2} + 2 \sin^2 \theta_W + \delta_{\ell e}, \quad g_A^{\ell'} = -\frac{1}{2} + \delta_{\ell e},
\]

where \( \theta_W \) is the weak mixing angle and note that \( \delta_{\ell e} \) is added in order to include the charged current contribution.

From this Lagrangian we can obtain the usual amplitudes for neutrino-electron scattering process in Dirac and Majorana case, which are given by:

\[
\mathcal{M}_D = -i \frac{G_F}{\sqrt{2}} \left[ \bar{u}_f^\ell \gamma^\mu (1 - \gamma^5) u_i^\nu \right] \left[ \bar{u}_e^{\ell'} \gamma_\mu \left( g_V^{\ell'} - g_A^{\ell'} \gamma^5 \right) u_e^{i'} \right],
\]

when the neutrino is a Dirac particle, and

\[
\mathcal{M}_M = i \frac{2G_F}{\sqrt{2}} \left[ \bar{u}_f^\ell \gamma_\mu \left( g_V^{\ell'} - g_A^{\ell'} \gamma^5 \right) u_i^\nu \right] \left[ \bar{u}_e^{\ell'} \gamma_\mu \gamma^5 u_e^{i'} \right],
\]

for Majorana neutrinos. It is important to note that one crucial difference from equations (3) and (4) is given by the fact that there is a cancellation of the vector part in second term of Eq. (4) that it is not present in the Dirac case. Last two amplitudes look very different. At this point the computation of the cross sections will lead to different results. Nevertheless, the left-handedness of the neutrino implies an important role: since neutrinos are born with \( s_|| \approx -1 \) then an extra term \( (1 - \gamma^5)/2 \) is usually added in both eqs. (3) and (4) making them identical and thus, no appreciable difference arises. Addition of this factor \( (1 - \gamma^5)/2 \) in fact leads to the usual cross section, which in the massless limit is:

\[
\frac{d^2\sigma}{dTdE} = \frac{m_eG_F^2}{2\pi} \left[ (g_A + g_V)^2 + (g_V - g_A)^2 \left( 1 - \frac{T}{E_\nu} \right)^2 - (g_V^2 - g_A^2) \frac{m_e^2T}{E_\nu} \right].
\]
That is the reason why it is very difficult to distinguish between Dirac and Majorana neutrinos in neutrino-electron scattering processes. In Eq. (5) the electron recoil energy is $T$ and the energy of the incident neutrino is $E_\nu$.

Nevertheless, let us consider that $s_\parallel \neq -1$ and then compute the corresponding cross sections with Eqs. (3) and (4). In previous works [2, 3] authors have demonstrated that when $s_\parallel \neq -1$ there is an additional contribution to the usual cross sections, proportional to $s_\parallel$, that can improve the difference between Dirac and Majorana neutrinos. In other words, the longitudinal neutrino polarization would be able to enhance the difference between Dirac and Majorana cross sections for $\nu-e$ elastic scattering process, but only if it differs from minus one. When the incident neutrino polarization if defined in its rest frame as $s_\nu = (0, s_\perp, 0, s_\parallel)$, with a straightforward calculation we can obtain [2, 3, 4] for Dirac case:

$$
\frac{d^2\sigma^D}{dEdT} = \frac{m_e G_F^2}{4\pi F^2} \left\{ (g_A - g_V)^2 (E_\nu - T)^2 + m_e T (g_A^2 - g_V^2) \right\} \left( 1 - \frac{E_\nu}{P} s_\parallel \right) + m_e^2 \left( (g_A^2 - g_V^2) \left( 1 - \frac{T}{P} s_\parallel \right) + (g_A - g_V)^2 \left( E_\nu - T \left( 1 + \frac{T}{m_e} \right) s_\parallel \right) \right) + (g_A + g_V)^2 E_\nu^2 \left( 1 - \frac{P}{E_\nu} s_\parallel \right),
$$

(6)

and in the case where the neutrino is a Majorana particle:

$$
\frac{d^2\sigma^M}{dEdT} = \frac{m_e G_F^2}{2\pi F^2} \left\{ (g_A + g_V - 2 \frac{E_\nu}{P} g_A g_V s_\parallel) E_\nu^2 + (g_A^2 - g_V^2) m_e T \right\} + m_e^2 \frac{T}{m_e} \left( 2 \frac{m_e}{T} (2g_A^2 - g_V^2) - 4g_A g_V s_\parallel \frac{E_\nu}{P} \left( 1 - \frac{T}{2E_\nu} \right) + g_A^2 + g_V^2 \right) + \left( g_A^2 + g_V^2 + 2 \frac{E_\nu}{P} g_A g_V s_\parallel \right) (E_\nu - T)^2 \right\},
$$

(7)

Equations (7) and (6) are the differential cross sections in the laboratory frame. Here, $P = |\vec{P}_\nu|$ is the magnitude of the momentum of the incident neutrino.

Note that in the case $s_\parallel = -1$, those two cross sections differ only by terms proportional to the neutrino mass, as common wisdom said. Nevertheless, even if $m_\nu \to 0$, IF $s_\parallel \neq -1$ those two cross sections are different regardless the neutrino energy and thus, measurable differences should be observable at current neutrino detectors.

3. Applications

According to last equations, the differential cross sections depend on the longitudinal polarization $s_\parallel$. The total cross section is the obtained by integrations over recoil and neutrino incident energies and it will depend on $s_\parallel$, i.e.

$$
\sigma^{M,D}(s_\parallel) \equiv \int_{T_{\min}}^{T_{\max}} dT \int_0^\infty dE_\nu \lambda(E_\nu) \frac{d^2\sigma^{M,D}}{dE_\nu dT}(E_\nu, T, s_\parallel).
$$

(8)

The ingredients that we need to know in order to obtain the total cross section given by Eq. (8) are i) the expression for differential cross section in Dirac or Majorana case, ii) a spectrum $\lambda(E_\nu)$ of energy for neutrinos, and finally iii) an interval of integration over recoil energy $T$. For
example, in a detector with the features of Borexino, the electron recoil energy can be integrated 
over the interval $T \in [250, 750]$ keV while the neutrino spectrum is the well know $^7$Be line of 
solar neutrino spectrum given by $\lambda(E_\nu) = \delta(E_\nu - 0.862$ MeV).

Once we have calculated the total cross section the next step is to construct a function so as to 
to be able to quantify the percentage of difference between Dirac and Majorana cross sections:

$$D(s_\parallel) \equiv \frac{\left|\sigma^M(s_\parallel) - \sigma^D(s_\parallel)\right|}{\sigma^D(s_\parallel)}. \quad (9)$$

This function depends only on $s_\parallel$ and allows to evaluated how difference between Dirac and 
Majorana changes for different values of the polarization. The next question related to last 
equation is how can we change the neutrino helicity. We actually know that although neutrino 
magnetic moment is very small [10], it is not equal to zero. This fact recovers importance 
because it has been showed that particles with magnetic moment different from zero that are 
embedded in an external magnetic field can change its helicity according to the equation [5, 6, 9]

$$\frac{ds_\parallel}{dr} = -2\mu_\nu B_\perp s_\parallel, \quad (10)$$

Here, $B_\perp$ is the perpendicular component of that external magnetic field, with respect to the 
direction in which the particle is propagating and $\mu_\nu$ is the neutrino magnetic moment. In 
the case of solar neutrinos, a magnetic profile of the Sun is needed. For definitiveness we 
will use the profile proposed in [11]. In that work, authors have obtained a magnetic profile 
through self-consistent and analytical solutions of the magnetohydrodynamic equations inside 
the Sun (see reference [11] for more details). Solution of eq. 10 can be obtained at the surface 
of the Sun, $r = R_\odot$, for a given value of $\mu_\nu$ and a maximum value of $B_\perp$ at the convective 
zone. Current limits fixes $B_{\perp}^{\max} = 10 - 20$ KG. Thus we can obtain the final neutrino polarization 
$s_\parallel(r = R_\odot, \mu_\nu, B_{\perp}^{\max} = 10$KG)

At this point we can put all ingredients together. As we discussed above, function (9) gives 
a measure of the difference between Dirac and Majorana cross sections. On the other hand, 
if we take into account the number of events reported by Borexino collaboration, which is 
$N = 49 \pm 1.5_{-1.6}^{+1.5}$ stat syst counts/day/100 ton [7, 8], and assuming that no error arises from the 
neutrino probability, this measurement gives a determination of the cross section within a error 
given by the error in the number of events. That is, the relative error in the determination of the 
number of events reported by Borexino is 4.4%.

In other words, if there is a difference between the Majorana and the Dirac cross section this 
should be less than the relative error reported by Borexino, thus $D(s_\parallel) < 0.044$. Using Eqs. 6 
and 7 we can solve $D(s_\parallel) < 0.044$ for $s_\parallel$ and this gives $s_\parallel < -0.68$.

Finally, solving $s_\parallel(r = R_\odot, \mu_\nu, B_{\perp}^{\max} = 10$KG) $< -0.68$ for the neutrino magnetic moment, 
we obtain the new upper bound $\mu_\nu < 1.4 \times 10^{-12} \mu_B$. See [4] for more details.

4. Conclusion
To determine the neutrino nature through neutrino-electron scattering process is difficult because any 
possible difference will be suppressed by terms that are proportional to neutrino mass unless 
$s_\parallel \neq -1$, [2, 3, 4]. In that case, measurable differences are possible. Solar neutrinos offer 
a possible source where such differences should appear since its magnetic field may induce a 
change in $s_\parallel$ due to the interaction of the magnetic moment of the neutrino with the external 
magnetic field of the Sun. No appreciable differences will imply that the change in $s_\parallel$ is very 
small and thus implies a limit on the neutrino magnetic moment $\mu_\nu$.

The number of events reported by Borexino collaboration of Solar neutrinos of the $^7$Be line implies a determination of the cross section within a error given by the error in the number of
events. which is of the order of the 4%. Then, if there is a difference between the Majorana and the Dirac cross section this should be less than the relative error reported by Borexino, which implies a limit on the neutrino magnetic moment given by $\mu_\nu < 1.4 \times 10^{-12}\mu_B$.

**Acknowledgments**
This work was partially supported by Conacyt project CB-259228, Conacyt SNI and DAIP-UG.

**References**
1. Schechter J and Valle J W F 1982 *Phys. Rev.* D **25** 2951
2. Kayser B 1982 *Phys. Lett.* B **112** 137-42
3. Barranco J, Delepine D, Gonzalez-Macias V, Lujan-Peschard C and Napsuciale M 2014 *Phys. Lett.* B **739** 343-7
4. Barranco J, Delepine D, Napsuciale M and Yebra A, arXiv:1704.01549 [hep-ph].
5. V. Bargmann, L. Michel and V. L. Telegdi, *Phys. Rev. Lett.* **2**, 435 (1959). doi:10.1103/PhysRevLett.2.435
6. V. Semikoz, *Phys. Rev.* D **48** (1993) 5264 [Erratum-ibid. D **49** (1994) 6246].
7. Arpesella C et al. [Borexino Collaboration] 2008 *Phys. Rev. Lett.* **101** 091302
8. Bellini G et al. 2011 *Phys. Rev. Lett.* **107** 141302
9. Semikoz V 1993 *Phys. Rev.* D **49** 5264 [Erratum-ibid. D **49** (1994) 6246]
10. K. Fujikawa and R. Shrock, *Phys. Rev. Lett.* **45** (1980) 963.
11. Miranda O G, Pena-Garay C, Rashbe T I, Semikoz V and Valle J W F 2001 *Nucl. Phys.* B **595** 360