Challenges for Inverted Hybrid Inflation

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ABSTRACT

Inverted hybrid inflation (in which the inflaton field slowly rolls away from the origin, giving a spectral index $n \lesssim 1$) is an appealing variant of the more usually studied hybrid model. Analysing the model alongside the ordinary hybrid case, we show that, in order to provide the correct density perturbations consistent with the COBE measurements, the dimensionless coupling constants of the inverted hybrid potential must be very small indeed. For example, the quartic coupling in a typical such potential is found to be $\lesssim 10^{-12}$. A supersymmetric model of inverted hybrid inflation, which does not involve the troublesome quartic coupling is found to lead to a potential which is unbounded from below.
1 Introduction

The hybrid inflation scenario proposed by Linde [1, 2] is probably the most attractive model of inflation, at present. Not only does it remove any necessity for fine-tuning, but, after many years, it reconnects inflation with particle physics, as in the original models of inflation developed in the early 1980s [3]. In particular, it is potentially well suited to realizations in supersymmetry (SUSY).

The relevance of hybrid inflation to SUSY models has been widely discussed [4, 5]. Inflation generally requires a flat potential in order to satisfy the slow roll conditions. More exactly, one requires flat directions in field space. If one wishes to avoid fine tuning at each order in perturbation theory, there are two strategies to achieve flat directions: Goldstone bosons (which roll around the flat bottom of the wine bottle potential), or flat directions in SUSY theories. The importance of SUSY is that flat directions are not spoiled by radiative corrections due to the non-renormalisation theorem. In the context of SUSY the renormalisable scalar potential is a sum of squares of F-terms and D-terms, which may both vanish identically along certain directions in field space called the flat directions. The massless chiral superfields which parametrise the flat directions are known as the moduli fields. In fact in realistic theories, the flat directions are not exactly flat since they are lifted by two effects: SUSY-breaking effects (such as soft masses for the moduli fields), and effects due to non-renormalisable terms in the superpotential. For example, in a particular class of “supernatural” theories of hybrid inflation [5], the renormalisable superpotential is postulated to be zero, and the moduli fields are assumed to have zero D-terms, with the potential for the moduli fields being provided by SUSY-breaking effects. This occurs via the Kahler potential in the context of supergravity theories based on hidden sector SUSY-breaking at an intermediate scale, $M_I \sim 10^{11}$ GeV. However this is just one class of model, and more generally it is not necessary to have a vanishing superpotential, nor is it necessary to identify all the fields in the superpotential as moduli fields of flat directions. It is sufficient in hybrid inflation for the potential to have a flat direction along the “inflaton” direction in the case when the other field takes a zero value. To understand this statement it is necessary to explain a little more about hybrid inflation in general terms.

In hybrid inflation more than one scalar field is relevant to inflation. A scalar field $\psi$ whose eventual vacuum expectation value (VEV) is, say, of order $10^{11}$ GeV is initially prevented from attaining this VEV by a second scalar field $\phi$ which takes rather large values initially (but well below the Planck scale $M_P \approx 1.2 \times 10^{19}$ GeV) as it rolls towards its eventual VEV at the origin. While the “inflaton” field $\phi$ takes values larger than some critical value $\phi_c$, the field $\psi$ is pinned at the false vacuum $\psi = 0$ and inflation takes place. When the inflaton reaches its critical value $\phi = \phi_c$, a cascade takes place.

\footnote{There are well known difficulties with such models based on supergravity due to the flatness parameter $(M_P^2 V'')/(8\pi V)$ being of order unity as a result of the non-renormalisable supergravity contributions, but we shall not address this problem here.}
of the fields to their eventual VEVs $\langle \psi \rangle \sim 10^{11}$ GeV and $\langle \phi \rangle = 0$, and inflation is abruptly ended. Consequently this scenario is able to avoid the standard naturalness problems of single-field models. Not surprisingly then, hybrid inflation has received a good deal of attention recently and many different versions have resulted. Generally, hybrid models give a blue tilted spectrum of adiabatic density perturbations, corresponding to a spectral index $n > 1$. In the context of SUSY theories, it is not necessary to identify both $\psi$ and $\phi$ as the scalar components of moduli fields. All that is required is that when $\psi = 0$ the potential is flat in the $\phi$ direction, with the flatness lifted by a soft SUSY breaking mass $m^2|\phi|^2$, where $m \sim 1$ TeV. In other words only $\psi = 0, \phi \neq 0$ must be a SUSY flat direction, with $\phi$ being the scalar component of a moduli field.

Given the success of hybrid inflation, it was subsequently suggested that the hybrid mechanism could be adapted to create an “inverted” model in which the inflaton field $\phi$ has a negative mass squared and rolls away from the origin, predicting a spectral index which can be significantly below 1 in contrast to virtually all other hybrid models. In this “inverted hybrid inflation” the field $\phi$ is supposed to obtain a non-zero VEV eventually, and this is typically achieved by adding to the potential a $\phi^4$ term. The purpose of the present paper is to point out that this model is not viable unless all its dimensionless couplings are extremely small. Already at a qualitative level, the inverted hybrid scenario looks mildly uncomfortable since the quartic inflaton coupling does not fit in with the notion of flat directions outlined above. Even given this assumption, it is almost obvious without doing any calculation that the inverted hybrid scenario cannot be viable if all the dimensionless couplings are of order unity. The argument runs as follows. Let us assume that all dimensionless couplings are of order unity. Then if the mass of the $\phi$ field is of order 1 TeV, the natural scale for its VEV (in the limit that the $\psi$ field is set to zero) must also be about 1 TeV. However we know that in inverted hybrid inflation as in hybrid inflation, for a $\phi$ mass of order 1 TeV, the $\psi$ mass and VEV must be around $10^{11}$ GeV (still assuming all dimensionless couplings of order unity), and so the critical value of the $\phi$ field must also be around $10^{11}$ GeV. Now we can see the problem, - the $\phi$ field which is rolling from the origin towards its critical value must necessarily pass through its natural minimum at 1 TeV, and so it will get hung up there and never reach its critical value. The only way round this problem is by fine tuning the dimensionless couplings so that the critical value of $\phi$ is below the $\phi$ VEV (in the limit that $\psi = 0$) and this leads to the stated conclusion that in inverted hybrid inflation the dimensionless couplings are necessarily extremely small.

The remainder of the paper consists of fleshing out the above argument. In sections 2 and 3 we shall focus on a definite form of the potential which enables easy comparison between the hybrid and inverted hybrid cases. The discussion will be at the level of the scalar potential, but we shall have SUSY in the back of our mind and so occasionally refer to the superpotential from which the potential may be derived. In addition we shall assume that the $\phi$ mass arises from a soft SUSY breaking effect and so has a value of around 1 TeV, which effectively fixes the scale of the $\psi$ VEV to be around $10^{11}$ GeV, as we shall discuss. In section 3 we shall determine exactly how small
the dimensionless couplings of inverted hybrid inflation must be. In section 4 we shall discuss the problems with obtaining an inverted hybrid potential from a superpotential.

2 Hybrid and Inverted Hybrid Inflation

We shall take the potential of hybrid inflation to be of the form [7]:

\[ V_H(\phi, \psi) = \frac{1}{4}\lambda_\psi(\psi^2 - M^2)^2 + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}\lambda\phi^2\psi^2 \]  
\[ = V_0 - \frac{1}{2}m_\phi^2\psi^2 + \frac{1}{4}\lambda_\psi^4 + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}\lambda\phi^2\psi^2, \]

where \( \phi \) and \( \psi \) are real scalar fields, \( V_0 = \frac{1}{4}\lambda_\psi M^4 \), \( m_\phi^2 = \lambda_\psi M^2 \) and the subscript H stands for “Hybrid”. This scalar potential can easily be realized in SUSY. It may be derived from the superpotential \( W = \sqrt{\lambda_\psi/2} (M^2 - 2\Psi^2)\Phi \), where \( \Phi \) and \( \Psi \) are complex chiral superfields, which is the simplest superpotential that breaks a U(1) symmetry, [4]. Adding a soft SUSY-breaking \( \phi \) mass term whose mass, \( m_\phi \sim 1 \text{ TeV} \), we obtain the potential \( (1) \) in terms of the real scalar fields \( \phi \) and \( \psi \).

Perhaps the simplest way of achieving inverted hybrid inflation is, as suggested [6, 7], to just reverse certain signs in this potential, giving

\[ V_{IH}(\phi, \psi) = \frac{1}{4}\lambda_\psi(\psi^2 + M^2)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}\lambda\phi^2\psi^2 + \frac{1}{4}\lambda_\phi^4 \]  
\[ = V_0 + \frac{1}{2}m_\phi^2\psi^2 + \frac{1}{4}\lambda_\psi\psi^4 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}\lambda\phi^2\psi^2 + \frac{1}{4}\lambda_\phi^4, \]

where we have also added a \( \phi^4 \) term, enabling \( \phi \) to possess a vacuum expectation value (VEV), and the subscript IH stands for Inverted Hybrid. The necessity of having this term is the essential difference between the two models. This is a general renormalizable potential leading to inverted hybrid inflation.

The scalar field \( \phi \) in these models acts as the inflaton which slowly rolls during inflation while the potential is dominated by the vacuum energy of the \( \psi \) field. The field \( \psi \) is held in a false vacuum \( \psi = 0 \) by the presence of the \( \phi \) field, until \( \phi \) reaches some critical value, \( \phi_c \), when the effective \( \psi \) mass squared becomes negative, allowing \( \psi \) to roll out to its VEV through a second order phase transition, and inflation consequently ends promptly.
In this case, it is easy to see that for both models,

$$\phi_c = \frac{m_\psi}{\sqrt{\lambda}}. \quad (5)$$

In the original hybrid model, $\phi$ starts out greater than this value and for $\phi > \phi_c$, we shall set $\psi = 0$. In this regime, inflation occurs with the quadratic potential

$$V_H = V_0 + \frac{1}{2}m^2_\phi \phi^2. \quad (6)$$

Then, if $V_0$ dominates, inflation ends when $\phi$ becomes less than $\phi_c$, and the fields reach their true vacuum values; $\langle \psi \rangle = M, \langle \phi \rangle = 0$, at which the potential vanishes, and the previous $\psi \leftrightarrow -\psi$ symmetry of the potential is broken.

In the inverted case, $\phi$ is originally less than $\phi_c$, so that inflation occurs (and the false vacuum exists) when $\phi < \phi_c$. However now, whilst $\psi = 0$ during inflation, the potential is given by

$$V_{IH} = V_0 - \frac{1}{2}m^2_\phi \phi^2 + \frac{1}{4}\lambda_\phi \phi^4, \quad (7)$$

which is minimised at

$$\phi_m = \frac{m_\phi}{\sqrt{\lambda_\phi}}. \quad (8)$$

Clearly for the hybrid mechanism to work we require

$$\phi_c < \phi_m, \quad (9)$$

otherwise $\phi$ would reach its minimum, $\phi_m$ with $\psi$ still trapped in its false vacuum state, $\psi$ would never reach its VEV and the flatness conditions (see below) would have been violated long ago, ending inflation in the usual manner and there would be no difference to ordinary single-field slow-rollover inflation. If we impose the stronger condition:

$$\phi_c \ll \phi_m, \quad (10)$$

then we may neglect the $\phi^4$ term in (7) and achieve the inverted quadratic potential during inflation;

$$V_{IH} = V_0 - \frac{1}{2}m^2_\phi \phi^2. \quad (11)$$

From here on we may analyse the two models simultaneously.

The spectral index, $n$, in terms of the slow-roll flatness parameters, $\epsilon$ and $\eta$ evaluated when cosmological scales leave the horizon $N$ e-folds before the end of inflation, is given, to lowest order, by

$$n - 1 = -6\epsilon + 2\eta, \quad (12)$$
where
\[ \epsilon \equiv \frac{M_P^2}{16\pi} \left( \frac{V'}{V} \right)^2; \quad \eta \equiv \frac{M_P^2}{8\pi} \left( \frac{V''}{V} \right), \quad (13) \]

and for slow-rollover to be a valid approximation, the flatness conditions, \( \epsilon \ll 1, |\eta| \ll 1 \) are to be satisfied.

In our models,
\[ \epsilon = \frac{1}{16\pi} \frac{M_P^2 m_N^4 \phi_N^2}{V_0^2}, \quad (14) \]
\[ |\eta| = \frac{M_P^2 m_N^2}{8\pi} \frac{\phi}{V_0}, \quad (15) \]

where \( \phi_N \) is the slow-roll value of the field \( N \) e-folds before the end of inflation, and \( |\eta| = \eta_H = -\eta_{IH} \).

Assuming that \( V_0 \) dominates the potential during inflation, \( \phi_N \) is given by
\[ \phi_N = \phi_c e^{\eta N}, \quad (16) \]

obtained using \( N = 8\pi M_P^{-2} \int_{\phi_c}^{\phi_N} \frac{V}{V'} \, d\phi \). Although \( N \) is a function of the inflationary scale, for all calculational purposes, this variation is insignificant and we may treat \( N \) as constant taking a value between 40 and 60.

From these results we obtain;
\[ \frac{\epsilon}{|\eta|} < \frac{m_N^2 \phi_c^2}{2V_0} \ll 1, \quad (17) \]

since \( V_0 \) dominates the vacuum energy. Thus \( \epsilon \) is negligible compared to \( |\eta| \) and we may write,
\[ (n - 1)_H = (1 - n)_{IH} \approx 2|\eta| = \frac{M_P^2 m_N^2}{4\pi} \frac{\phi}{V_0}. \quad (18) \]

Further, the relative contribution of gravitational waves to the CMB anisotropy given by \( R \approx 12\epsilon \) is therefore negligible. So we are concerned only with the scalar perturbations and the COBE normalisation can be written;
\[ \left( \frac{\Delta T}{T} \right)_Q^2 = \frac{32\pi}{45} \frac{V^3}{V'^2 M_P^6}, \quad (19) \]

where the rhs is to be evaluated \( N \) e-folds before the end of inflation (in this model triggered by \( \phi \) reaching \( \phi_c \)).
The COBE constraint becomes

\[
\left( \frac{\Delta T}{T} \right)_Q^2 = \frac{32\pi}{45} \frac{V_0^3}{m_\phi^4 M_p^6 \phi_c^2} e^{-2\eta_N},
\]

(20)

which is valid for both hybrid and inverted hybrid cases providing we remember that the sign of \( \eta \) changes between the two cases.

To obtain an order of magnitude estimate of the problem let us take \( \phi \sim \phi_N \sim \phi_c \). Then as in ref. \[4\] we find:

\[
\left( \frac{M}{5.5 \times 10^{11}\text{GeV}} \right) = (\lambda)^{-\frac{1}{10}} (\lambda_\psi)^{-\frac{1}{5}} \left( \frac{m_\phi}{1\text{TeV}} \right)^{2/5}.
\]

(21)

Thus in SUSY inspired models with \( m_\phi \sim 1\text{ TeV} \) the typical scale \( M \) must be of the order \( 10^{11}\text{GeV} \), and assuming all dimensionless parameters to be of order unity we see from Eq.\[3\] that we also have \( \phi_c \sim 10^{11} \text{ GeV} \). Now in hybrid inflation this presents no particular problem, but in inverted hybrid inflation we see that \( \phi_m \sim 1\text{ TeV} \) and the condition \( \phi_c < \phi_m \) is clearly violated, as discussed in the introduction. The only way to satisfy this condition is if we relax our assumption that the dimensionless couplings are of order unity, as discussed in the next section.

3 The Unnaturally Small Parameters of Inverted Hybrid Inflation

In this section we shall determine how small the dimensionless parameters of inverted hybrid inflation must be in order to satisfy the condition \( \phi_c < \phi_m \). To do this we shall keep \( V_0 \) numerically fixed and apply the COBE constraint more carefully. The COBE constraint gives a complicated relation between \( m_\phi \) and \( \phi_c \), or equivalently between \( \eta \) (given in (15)) and \( \phi_c \) as,

\[
\phi_c = \frac{A}{|\eta|} e^{-\eta N},
\]

(22)

where

\[
A = \frac{1}{\sqrt{90\pi}} \left( \frac{\Delta T}{T} \right)_Q \frac{V_0^{1/2}}{M_p}.
\]
In the ordinary hybrid model, $\phi_c$ can take any value, but in the inverted case, where $\eta$ is negative and (22) can be written $\phi_c = A|\eta|^{-1}e^{\eta|N|}$, there is a minimum value of $\phi_c$ corresponding to

$$|\eta| = 1/N, \quad \phi_c = AeN. \quad (23)$$

This minimum value corresponds to the most optimistic value possible, if we are to ensure that $\phi_c < \phi_m$.

From (23) we see that for both potentials, the critical value $\phi_c = \sqrt{\frac{\lambda}{\psi}} M$. In the ordinary hybrid model, we may proceed as in ref. [4] and obtain a curve in parameter space relating $m_\phi$ and $M$ for a given $\lambda, \lambda_\psi$ which require no fine tuning and may be of order unity.

Unless the inflationary scale, $V_0^{1/4} \geq 10^{13}$GeV, the spectral index is indistinguishable from 1, without us fine tuning the coupling constants, $\lambda, \lambda_\psi$. For small $|\eta|$’s, the contribution to the spectral index from the exponential is negligible and we may write;

$$(n - 1)_H \simeq 2|\eta| \simeq \frac{2A}{\phi_c} \sim 10^4 \frac{V_0^{1/4}}{M_P} \sqrt{\frac{\lambda}{\lambda_\psi^{1/2}}}. \quad (24)$$

For example, if $V_0^{1/4} \sim 10^{11}$GeV, the intermediate scale, to acquire a significantly tilted spectrum relating to a spectral index given by $n - 1 \sim 0.01$ (the experimental accuracy expected to be obtained in the near future), we would require $\lambda_\psi \sim 10^{-8}\lambda^2$.

Returning to our inverted model, although we have an extra free parameter here, $\lambda_\phi$, we also have equation (23) as another constraint. However, for our convenience, in order that we can neglect the quartic term and simplify our calculations, we use the stronger condition $\phi_c \ll \phi_m$ (although the result would still follow from the weaker condition of equation (23)). This gives the restriction

$$\lambda_\phi \ll \left(\frac{m_\phi}{\phi_c}\right)^2. \quad (25)$$

Using equation (22), we may write,

$$\frac{m_\phi}{\phi_c} = B|\eta|^{3/2}e^{-|\eta|N}, \quad (26)$$

where $B = 4\pi \sqrt{45} \left(\frac{\Delta T}{T}\right)_Q$.

Again, holding $V_0$ numerically fixed, and differentiating this with respect to $|\eta|$, we
find the maximum value of $m_{\phi}/\phi_c$ corresponds to;

$$|\eta| = \frac{3}{2N},$$  

$$
\left(\frac{m_{\phi}}{\phi_c}\right)_{\text{max}} \simeq 35 N^{-3/2} \left(\frac{\Delta T}{T}\right)_Q.
$$

Hence we have that:

$$\lambda_{\phi} \ll \left(\frac{m_{\phi}}{\phi_c}\right)^2 \leq 1.2 \times 10^3 N^{-3} \left(\frac{\Delta T}{T}\right)^2_Q,$$  

where the rhs is given by:

$$\text{rhs} \simeq 2 \times 10^{-12} \quad \text{using } N = 40, \left(\frac{\Delta T}{T}\right)_Q = 10^{-5}$$

$$\text{rhs} \simeq 5.6 \times 10^{-13} \quad N = 60$$

Therefore $\lambda_{\phi}$ must be very small indeed in this model.

We will now proceed to show that the other coupling constants of the potential $(\mathfrak{H})$, $\lambda$ and $\lambda_{\psi}$ are necessarily even smaller than $\lambda_{\phi}$.

The global minimum of the potential $(\mathfrak{H})$ (at which $V$ does not vanish) corresponds to;

$$\langle \psi \rangle^2 = \frac{\phi_m^2 - \phi_c^2}{(\frac{\lambda_{\psi}}{\lambda} - \frac{1}{\lambda_{\phi}})}, \quad \langle \phi \rangle^2 = \frac{\lambda_{\psi} \phi_m^2 - \lambda \phi_c^2}{(\frac{\lambda_{\psi}}{\lambda} - \frac{1}{\lambda_{\phi}})}.$$  

Taking $\langle \psi \rangle^2 > 0 \Rightarrow \lambda_{\psi}/\lambda > \lambda/\lambda_{\phi}$. Also, as $\langle \psi \rangle^2 > \langle \phi \rangle^2$, we have that

$$\langle \psi \rangle^2 > \frac{\lambda_{\psi} \phi_m^2 - \lambda \phi_c^2}{(\frac{\lambda_{\psi}}{\lambda} - \frac{1}{\lambda_{\phi}})} > \frac{\lambda_{\psi} \phi_m^2 - \phi_c^2}{(\frac{\lambda_{\psi}}{\lambda} - \frac{1}{\lambda_{\phi}})} = \frac{\lambda_{\psi}}{\lambda} \langle \psi \rangle^2,$$

and thus $\lambda_{\psi}/\lambda < 1$, from which follows $\lambda/\lambda_{\phi} < 1$. i.e. $\lambda_{\phi} > \lambda > \lambda_{\psi}$.

The above results have been obtained at tree-level. If radiative corrections are included then, without SUSY, further fine-tuning will be required in order to stabilise the potential (this is just the usual hierarchy problem.) Within the framework of a supersymmetric model, radiative corrections to a hybrid inflation potential have been considered in ref. [8], and a similar analysis would be expected to apply to the inverted
hybrid case. However in the inverted hybrid case, it is more difficult to obtain the potential from a superpotential, as discussed in the next section.

4 Inverted Hybrid Superpotentials

So far we have studied a particular renormalisable potential giving inverted hybrid inflation and found it not to be viable due to its unacceptably small parameters. In this section we discuss some of the difficulties associated with obtaining a consistent inverted hybrid model from a superpotential. We have already mentioned the problem that the $\phi^4$ coupling does not fit in with the idea of flat directions, so we shall consider a different approach in which such a coupling is not required. Our starting point is the superpotential introduced in ref. [6]:

$$W = \left( \Lambda^2 + \frac{\lambda \phi^2 \psi^2}{\Lambda^2} \right) \Xi,$$

where $\Phi, \Psi$ and $\Xi$ are three complex chiral superfields. The globally supersymmetric scalar potential, writing $\Phi = \phi/\sqrt{2}$, $\Psi = \psi/\sqrt{2}$, minimising the potential and assuming $\Xi$ is held at zero, is

$$V = \left( \Lambda^2 - \frac{\lambda \phi^2 \psi^2}{4\Lambda^2} \right)^2 - \frac{1}{2}m_\phi^2 \phi^2 + \frac{1}{2}m_\psi^2 \psi^2,$$

where $\phi$ and $\psi$ are now real scalar fields and this form breaks down when $\lambda \phi^2 \psi^2 / 4\Lambda^2 \sim \Lambda^2$. We have also added soft SUSY-breaking masses of judiciously chosen signs. This potential may be written as:

$$V = V_0 - \frac{1}{2} \lambda \phi^2 \psi^2 + \frac{\lambda^2 \phi^4 \psi^4}{16\Lambda^4} - \frac{1}{2}m_\phi^2 \phi^2 + \frac{1}{2}m_\psi^2 \psi^2.$$

where, unlike the previous case, now both $m_\phi^2$ and $m_\psi^2$ are of order 1 TeV, and $V_0 = \Lambda^4$. The inverted mechanism works as before with the same critical field value as in Eq.(3). However in this case there is no minimum $\phi_m$ at $\psi = 0$ and consequently, the restriction and its subsequent inference no longer applies.

The negative mass squared at low energies could be obtained from renormalisation group running of mass squared which is positive at high energy, for example.
On minimising the potential, we find the following conditions which the fields are supposed to satisfy at their VEVs:

\[
\phi^2 = 4\Lambda^4 \frac{(m^2 + \lambda \psi^2)}{\lambda^2 \phi^4}, \quad \psi^2 = 4\Lambda^4 \frac{(\lambda \phi^2 - m^2^\psi)}{\lambda^2 \phi^4} \quad \text{with } \phi^2 > \phi_c^2, \quad (37)
\]

From these equations we see that

\[
\langle \psi \rangle^2 = -\frac{m^2^\phi}{m^2^\psi} \langle \phi \rangle^2, \quad (38)
\]

which shows that all is not well since if \(\langle \phi \rangle^2\) is positive then it follows that \(\langle \psi \rangle^2\) must be negative, which is physically disallowed.

We also find:

\[
m^2^\phi \langle \phi \rangle^6 + \left( \frac{4\Lambda^4 m^2^\psi}{\lambda} \right) \langle \phi \rangle^2 - \left( \frac{4\Lambda^4 m^4^\psi}{\lambda^2} \right) = 0, \quad (39)
\]

\[
m^2^\psi \langle \psi \rangle^6 + \left( \frac{4\Lambda^4 m^2^\phi}{\lambda} \right) \langle \psi \rangle^2 + \left( \frac{4\Lambda^4 m^4^\phi}{\lambda^2} \right) = 0. \quad (40)
\]

Regarding these equations as cubic in the square of the VEV, it is trivial to show that while Eq.(39) has a solution with \(\langle \phi \rangle^2 < \phi_c^2\), Eq.(40) does not have a real solution for \(\langle \psi \rangle\).

In fact the potential of Eq.(35) is clearly unbounded from below corresponding to \(\frac{\lambda \phi^2 \psi^2}{4\Lambda^2} = \Lambda^2\) with \(\phi^2 \to \infty\). Thus we conclude that this is a very sick model.

5 Conclusion

We have argued that while hybrid inflation works perfectly well and fits in with the idea of flat directions in SUSY, inverted hybrid inflation faces severe challenges. We have shown that, when inverted hybrid inflation is expressed in the most straightforward way, as a simple modification of the hybrid inflation potential, the constraint that \(\phi_c < \phi_m\) can only be satisfied if the dimensionless couplings are tuned to be very small indeed, say \(\lesssim 10^{-12}\). A supersymmetric model which has been proposed and which does not involve the \(\phi^4\) term leads to a potential which is unbounded from below.

We stress that we have not presented a no-go theorem for models of inverted hybrid inflation, but merely have explored a representative set of such models which have been previously proposed, and found them to have the stated problems of which the proponents of these models were unaware. The main point of this paper is to expose
the challenges faced by such inverted hybrid inflation models. By contrast we find that the original hybrid inflation scenario is trouble free, and looks very promising.

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