Predicting from Predictions

Celestine Mendler-Dünner¹, Frances Ding², and Yixin Wang³

¹Max-Planck Institute for Intelligent Systems, Tübingen
²University of California, Berkeley
³University of Michigan

Abstract

Predictions about people, such as their expected educational achievement or their credit risk, can be performative and shape the outcome that they aim to predict. Understanding the causal effect of these predictions on the eventual outcomes is crucial for foreseeing the implications of future predictive models and selecting which models to deploy. However, this causal estimation task poses unique challenges: model predictions are usually deterministic functions of input features and highly correlated with outcomes, which can make the causal effects of predictions impossible to disentangle from the direct effect of the covariates. We study this problem through the lens of causal identifiability, and despite the hardness of this problem in full generality, we highlight three natural scenarios where the causal effect of predictions on outcomes can be identified from observational data: randomization in predictions or prediction-based decisions, overparameterization of the predictive model deployed during data collection, and discrete prediction outputs. We show empirically that, under suitable identifiability conditions, standard variants of supervised learning that predict from predictions can find transferable functional relationships between features, predictions, and outcomes, allowing for conclusions about newly deployed prediction models. Our positive results fundamentally rely on model predictions being recorded during data collection, bringing forward the importance of rethinking standard data collection practices to enable progress towards a better understanding of social outcomes and performative feedback loops.

Keywords— Performative Feedback, Causal Inference, Distribution Shift, Feature Engineering, Social Impact

1 Introduction

Predictions can impact sentiments, alter expectations, inform actions, and thus change the course of events. Through their influence on people, predictions have the potential to change the regularities in the population they seek to describe and understand. This insight underlies the theories of performativity [MacKenzie, 2008] and reflexivity [Soros, 2015] that play an important role in modern economics and finance.

Recently, Perdomo et al. [2020] pointed out that the social theory of performativity has important implications for machine learning theory and practice. Prevailing approaches to supervised learning assume that features $X$ and labels $Y$ are sampled jointly from a fixed underlying data distribution that is unaffected by attempts to predict $Y$ from $X$. Performativity questions this assumption and suggests that the deployment of a predictive model can disrupt the relationship between $X$ and $Y$, and hence changes to the model can induce shifts in the data distribution. For example, consider a lender with a predictive model for risk of default – performativity could arise if individuals who are predicted as likely to default are given higher interest loans, which make default even more likely [Manso, 2013]. In turn, changing the predictive model and offering more low-interest loans as a result of the updated predictions could cause some individuals who previously looked risky to be able to pay the loans back, which would appear as a shift in the relationship between features $X$ and loan repayment outcomes $Y$. This performative nature of predictions poses a challenge to using historical data to predict the outcomes that will arise under the deployment of a future model.

*Correspondence to: cmendler@tuebingen.mpg.de*
1.1 Our work

In this work, we study prediction in performative environments where performative effects surface as a shift in the outcome $Y$, which is mediated by the model prediction $\hat{Y}$. Building on this structural restriction, our goal is to identify the causal relationship between the outcome $Y$ and the prediction $\hat{Y}$ in order to anticipate the effects of planned model updates on the population. More concretely, we aim to build a meta machine learning model that accurately predicts the outcome $Y$ for an individual with features $X$, subjected to a prediction $\hat{Y}$. We term this data-driven strategy predicting from predictions because it treats the predictions as an input to the meta machine learning model.

The meta model seeks to answer “what would the outcome be if we were to deploy a different prediction model?” Crucially, this “what if” question is causal in nature; it aims to understand the potential outcome of the intervention where we deploy a prediction model different from the one in the historical data; this goal is different from merely estimating the outcome variable in previously seen data. Whether such a transferable model is learnable depends on whether the training data provides causal identifiability [Pearl, 2009b], i.e. whether the causal relationship can be written as a deterministic functional of the training data distribution. Only after causal identifiability is established can we rely on data-driven supervised learning techniques to select and design optimal downstream prediction models under performative.

For our main technical results, we start by illustrating the hardness of performativity-agnostic learning: we establish a lower bound on the extrapolation error of predicting $Y$ from $X$ under the deployment of a model different from the one deployed during data collection. We then explore the feasibility of this task when predictions are observed and training data samples ($X, Y, \hat{Y}$) are available for learning. We first show that this data is not sufficient for causal inference if it was collected under the deployment of a deterministic prediction function. Then, we establish several natural conditions that are each sufficient to identify the causal effects of predictions. The first condition exploits the presence of randomness in the prediction or a prediction-based decision. This randomness could be purposely built into the prediction for fairness, differential privacy, or other considerations. The second condition exploits the property that predictive models are often over-parameterized, which leads to incongruence in functional complexity between different causal paths; such incongruence enables the effects of predictions to be separated from other variables’ effects. The third condition takes advantage of discreteness in predictions or discreteness in a downstream decision such that performative effects can be disentangled from the continuous relationship between covariates and outcomes. Taken together, these conditions reveal that natural inaccuracies and particularities of the performative prediction problem can provide identifiability. Moreover, we show that under these conditions find these transferable functional relationships and close the extrapolation gap, as long as predictions are provided as model inputs. Empirically, we observe that supervised learning succeeds in finding the causal model in finite samples even in the presence of mild violations of our assumptions, such as model misspecification.

Our positive results illustrate that it can be possible to build on data-driven techniques to foresee implications of new model deployments. However, information about predictive models deployed during data collection must be contained in machine learning datasets. We hope this adds another perspective to recent calls for rethinking data collection practices in the machine learning community.

We conclude with a discussion of limitations and extensions of our work, pointing out potential violations of the assumptions underlying our causal analysis and directions for future work, as well as highlighting that our results alone do not justify the use supervised learning to address performativity beyond the scope of our identification results.

1.2 Broader context and related work

The work by Perdomo et al. [2020], initiated the discourse of performativity in the context of supervised learning by pointing out that the deployment of a predictive model can impact the data distribution we train our models on. Existing scholarship on performative prediction [c.f., Drusvyatskiy and Xiao, 2020, Izzo et al., 2021, Jagadeesan et al., 2022, Kulynych, 2022, Mendler-Düner et al., 2020, Miller et al., 2021, Narang et al., 2022, Perdomo et al., 2020, Piliouras and Yu, 2022, Wood et al., 2022] has predominantly focused on achieving a particular solution concept with a prediction function that maps $X$ to $Y$ in the presence of performative distribution shifts. Complementary to these works we are interested in understanding the underlying causal mechanism of the performative distribution shift in order to reason about the implications of any potential new model deployment. Our work is motivated by the seemingly natural approach to lift the supervised-learning problem and incorporate the predictive model as an input feature when building a meta machine learning model for explaining the outcome $Y$. By establishing a connection to causal identifiability, our goal is to understand when such a data-driven strategy can lead to meaningful solutions.
This work focuses on the setting where predictions lead to changes in the relationship between covariates \( X \) and label \( Y \), while the marginal distribution \( P(X) \) over covariates is assumed to be fixed. This describes a subclass of problems falling under the umbrella of performative (aka. model-induced or decision-dependent) distribution shifts [Drusvyatskiy and Xiao, 2020, Liu et al., 2021, Perdomo et al., 2020]. Our assumptions are complementary to the strategic classification framework [Brückner et al., 2012, Hardt et al., 2016b] that focuses on a setting where performative effects concern \( P(X) \), while \( P(Y|X) \) is assumed to remain stable. Consequently, causal questions in strategic classification [e.g., Bechavod et al., 2020, Harris et al., 2021, Shavit et al., 2020] are concerned with identifying stable causal relationships between \( X \) and \( Y \). Since we assume \( P(Y|X) \) can change (i.e. the true underlying 'concept' determining outcomes can change), conceptually different questions emerge in our work. Similar in spirit to strategic classification, the work on algorithmic recourse and counterfactual explanations [Karimi et al., 2021, Laugel et al., 2018, Tsirtsis and Gomez Rodriguez, 2020] focuses on the link between features and predictions, whereas we focus on the down-stream effects of predictions.

There are interesting parallels between our work and related work on the offline evaluation of online policies [e.g., Li et al., 2011, 2015, Schnabel et al., 2016, Swaminathan and Joachims, 2015]. In particular, Swaminathan and Joachims [2015] explicitly emphasize the importance of logging propensities of the deployed policy during data collection to mitigate selection bias. Since in our work the deployed model can induce a concept shift, we show that additional information about the predictions of the deployed model needs to be recorded to be able to foresee the impact of a new policy on the conditional distribution \( P(Y|X) \), beyond enabling propensity weighting [Rosenbaum and Rubin, 1983]. Also driven by the observation that predictions can be performative, Wager et al. [2014] focuses on how predictions at one time step can impact predictions in future time steps. Our problem formulation is different in that we do not work with data from multiple time steps but we focus on the causal effect of \( \hat{Y} \) on \( Y \). Furthermore, complementary to these existing works we show that randomness in the predictive model is not the only way causal effects of predictions can be identified.

For our theoretical results, we focus on causal identifiability and build on classical tools from causal inference [Pearl, 2009a, Rubin, 1980, Tchetgen and VanderWeele, 2012]. In particular, we distill unique properties of the performative prediction problem to design assumptions for the identifiability of the causal effect of predictions. Finally, we connect to the literature on causal inference with interference to discuss the spill-over effects of predictions [cf. Aronow and Samii, 2017, Manski, 1993, Sobel, 2006, Tchetgen and VanderWeele, 2012], a phenomenon that has so far widely been ignored in model-based approaches to performative prediction [Hardt et al., 2016a, Harris et al., 2021, Jagadeesan et al., 2021, Miller et al., 2020, Shavit et al., 2020].

2 The causal force of prediction

Predictive models and their predictions can be performative and impact the population of individuals they aim to predict. Formalizing it in the language of causal inference [Pearl, 2009a], the deployment of a predictive model effectively performs an intervention on a causal diagram that describes the underlying data generation process of the population. We will build on this causal perspective to study a particular instance of the performative prediction problem described below.

2.1 Prediction as a partial mediator

Consider a machine learning application relying on a predictive model \( f \) that maps features \( X \) to labels \( \hat{Y} \). We assume the predictive model \( f \) is performative in that the prediction \( \hat{Y} = f(X) \) has a direct causal effect on the outcome variable \( Y \) of the individual it concerns. Thereby the prediction impacts how the outcome variable \( Y \) is generated from the features \( X \). The causal diagram illustrating this setting is below:

\[
\begin{align*}
    X & = \xi_X \quad \xi_X \sim \mathcal{D}_X \\
    \hat{Y} & = f(X, \xi_f) \quad \xi_f \sim \mathcal{D}_f \\
    Y & = g(X, \hat{Y}) + \xi_Y \quad \xi_Y \sim \mathcal{D}_Y
\end{align*}
\]

Figure 1: Performative effects through prediction for a given \( f \)
The features \( X \in \mathcal{X} \subseteq \mathbb{R}^d \) are drawn i.i.d. from a fixed underlying continuous distribution over covariates \( D_X \) with support \( \mathcal{X} \). The outcome \( Y \in \mathcal{Y} \subseteq \mathbb{R} \) is a function of \( X \), partially mediated by the prediction \( \hat{Y} \in \mathcal{Y} \). The prediction \( \hat{Y} \) is determined by the deployed predictive model \( f : \mathcal{X} \to \mathcal{Y} \). For a given prediction function \( f \), every individual is assumed to be sampled i.i.d. from the data generation process described by the causal graph in Figure 1. We assume the exogenous noise \( \xi \) is zero mean, and \( \xi \) allows the prediction function to be randomized. This setup differs from the traditional supervised learning setting by including the arrow between \( \hat{Y} \) and \( Y \) in the causal graph.

Note that a structural assumption implicit in our causal graph is that performative effects are entirely mediated by the prediction, such that \( Y \perp f \mid \hat{Y} \). This is not meant to describe performativity in its full generality (which includes other ways \( f \) may affect \( P(X, Y) \)), but highlights an interesting and important class of performative feedback problems that we focus on in this work.

**Application examples.** The phenomenon described by the causal graph in Figure 1, where predictions or expectations about an event impact the outcome of the event, has been documented in various contexts: A bank’s prediction about the client (e.g., his or her creditworthiness in applying for a loan) determines the interest rate assigned to them, which in turn changes a client’s financial situation [Manso, 2013]. Mathematical models that predict stock prices inform the actions of traders and thus heavily shape financial markets and economic realities [MacKenzie, 2008]. Zillow’s housing price predictions directly impact sales prices [Malik, 2020]. Predictions about the severity of an illness play an important role in treatment decisions and hence the very chance of survival of the patient [Levin et al., 2018]. Another prominent example from psychology is the Pygmalion effect [Rosenthal and Jacobson, 1968]. It refers to the phenomenon that high expectations lead to improved performance, which is widely documented in the context of education [Bezuijen et al., 2009], sports [Solomon et al., 1996], and organizations [Eden, 1992]. Examples of such performativity abound, and we hope to have convinced the reader that the performative effects studied in this work should be taken into account in algorithmic prediction.

### 2.2 Hardness of performativity-agnostic learning

Begin with considering the classical supervised learning task of learning a model \( h : \mathcal{X} \to \mathcal{Y} \) for predicting the label \( Y \) from the features \( X \), without taking performativity into account. To understand the inherent challenge of classical prediction under performativity, we investigate the relationship between \( X \) and \( Y \) more closely. Specifically, the structural causal model (Figure 1) that describes the data generation process implies that

\[
P(Y | X) = \int P(Y \mid \hat{Y}, X) P(\hat{Y} | X) d\hat{Y}.
\]  

(4)

This expression makes explicit how the relationship between \( X \) and \( Y \) that we aim to learn depends on the predictive model governing \( P(\hat{Y} | X) \). As a consequence, when the deployed predictive model at test time differs from the model deployed during training data collection, our model will no longer be accurate because performative effects surface as concept shift [Gama et al., 2014]. Such distribution shift problems are known to be intractable without structural knowledge about the shift, implying that we can not expect \( h \) to generalize to distributions induced by future model deployments. Let us inspect the resulting extrapolation gap in more detail and put existing positive results for prediction under performativity into perspective.

**Extrapolation loss.** We illustrate the effect of performativity on predictive performance using a simple instantiation of the structural causal model from Figure 1. Therefore, assume a linear performative effect of strength \( \alpha > 0 \) and a base function \( g_1 : \mathcal{X} \to \mathcal{Y} \)

\[
g(X, \hat{Y}) := g_1(X) + \alpha \hat{Y}.
\]

(5)

Now, assume we collect training data under the deployment of a predictive model \( f_0 \) and validate our model under the deployment of \( f_\alpha \). Using our running example of a lender predicting the risk of default, the lender may have historical data about individuals who defaulted or not. Given this data the lender aims to learn a model to predict whether similar individuals will default in the future. However, in the time between data collection and model validation, the predictive model might have been updated and hence the interest rates assigned to individuals are different. If not accounted for, the resulting effects of the change in the interest rate on an individual’s default risk will surface to the lender as extrapolation loss.
To formalize this phenomenon, we adopt the notion of a distribution map from Perdomo et al. [2020] and write \( D_{XY}(f) \) for the joint distribution over \((X, Y)\) surfacing from the deployment of a model \( f \). We assess the quality of our predictive model \( h : \mathcal{X} \to Y \) over a distribution \( D_{XY}(f) \) induced by \( f \) via the loss function \( \ell : Y \times Y \to \mathbb{R} \) and write \( R_f(h) := \mathbb{E}_{x,y \sim D_{XY}(f)} \ell(h(x), y) \) for the risk, \( h_{opt}(f) \) for the risk minimizer \( h_{opt}(f) := \arg\min_{h \in \mathcal{H}} R_f(h) \), and \( \mathcal{H} \) for the hypothesis class we optimize over. The following result shows that the extrapolation loss of a model optimized over \( D_{XY}(f_0) \) and evaluated on \( D_{XY}(f_\phi) \) grows with the strength of performativity and the distance between \( f_0 \) and \( f_\phi \) as measured in prediction space. Proposition 1 can be viewed as a concrete instantiation of the more general extrapolation bounds for performative prediction discussed in Liu et al. [2021] within the feedback model from Figure 1.

**Proposition 1** (Extrapolation gap). Consider the data generation process in Figure 1 with \( g \) given in (5) and \( f_0, f_\phi \) being deterministic functions. Take a loss function \( \ell : Y \times Y \to \mathbb{R} \) that is \( \gamma \)-smooth and \( \mu \)-strongly convex in its second argument. Let \( h_{opt}(f_0) \) be the risk minimizer over the training distribution and assume the problem is realizable, i.e., \( h_{opt}(f_0) \in \mathcal{H} \). Then, we can bound the extrapolation loss of \( h_{opt}(f_0) \) on the distribution induced by \( f_\phi \) as

\[
\frac{\gamma}{2} \alpha^2 d_{D_X}^2(f_0, f_\phi) \geq \Delta R_{f_0 \to f_\phi}(h_{opt}(f_0)) \geq \frac{\mu}{2} \alpha^2 d_{D_X}^2(f_\theta, f_\phi)
\]

where \( d_{D_X}^2(f_0, f_\phi) := \mathbb{E}_{x \sim D_X} (f_0(x) - f_\phi(x))^2 \) and \( \Delta R_{f_0 \to f_\phi}(f) := R_{f_\phi}(f) - R_{f_0}(f) \).

The extrapolation loss \( \Delta R_{f_0 \to f_\phi}(h_{opt}(f_0)) \) is zero if and only if either the strength of performativity tends to zero (\( \alpha \to 0 \)), or the predictions of the two predictors \( f_0 \) and \( f_\phi \) are identical over the support of \( D_X \). If this is not the case, an extrapolation gap is inevitable. This elucidates the fundamental hardness of performative prediction from feature, label pairs \((X, Y)\) when performative effects disrupt the causal relationship between \( X \) and \( Y \).

The special case where \( \alpha = 0 \) aligns with the assumption of classical supervised learning, in which there is no performativity. This may hold in practice if the predictive model is solely used for descriptive purposes, or if the agent making the prediction does not enjoy any economic power [Hardt et al., 2022]. However, the strength of performative effects is not a parameter we can influence as machine learning practitioners and thus we wish to anticipate the effects of any prediction that is likely to be performative.

The second special case where the extrapolation error is small is when \( d_{D_X}^2(f_\theta, f_\phi) = 0 \). Given our causal model, this implies that \( D_{XY}(f_0) \) and \( D_{XY}(f_\phi) \) are equal in distribution and hence exhibit the same risk minimizer. Such a scenario where \( f_0 \) and \( f_\phi \) are similar can happen, for example, if the model \( f_\phi \) is obtained by retraining \( f_\theta \) on observational data and \( f_\phi \) is locally optimal: \( f_\phi = h_{opt}(f_\theta) \) (a.k.a. performative stability [Perdomo et al., 2020]). The convergence of different policy optimization strategies to stable points has been studied in prior work [e.g., Drusvyatskiy and Xiao, 2020, Mendler-Dünn et al., 2020, Perdomo et al., 2020] and enabled positive results that show zero extrapolation loss even in the presence of performative concept shifts, relying on \( f_\phi \) not being chosen arbitrarily, but based on a pre-specified update strategy.

### 3 Prediction in the presence of performativity

Having illustrated the hardness of performativity-agnostic learning, we explore under what conditions incorporating the presence of performative predictions into the learning task enables us to close the extrapolation gap. Therefore we need to build a causal, transferable understanding for how the output \( Y \) is generated in order to predict the impact of a new model deployment on the population. Towards this goal, a necessary first step is to assume that the mediator \( \hat{Y} \) in Figure 1 is observed—\( \hat{Y} \) takes on the role of the treatment in our causal analysis and we can not possibly hope to estimate the treatment effect of a treatment that is unobserved.

#### 3.1 Problem setup

Assume we are given access to data points \((x, \hat{y}, y)\) generated i.i.d. from the structural causal model in Figure 1 under the deployment of a prediction function \( f_\theta \). From this observational data, we wish to estimate the expected potential outcome of an individual under the deployment of an unseen (but known) predictive model \( f_\phi \). Towards this goal we
Lemma 2. Consider the data generation process in Figure 1. If \( \hat{d} \) determines \( h \) The main challenge for identification of \( h^* \) will allow us to make inferences about the potential outcome surfacing from planned model deployments beyond explaining historical data. In particular, we can evaluate \( h^* \) for any \( x \) at \( y = f_\theta(x) \) for \( f_\theta \) being the model of interest. For simplicity of notation, we will write \( D(f_\theta) \) to denote the joint distribution over \( (X, \hat{Y}, Y) \) of the observed data collected under the deployment of the predictive model \( f_\theta \). We say \( h^* : \mathcal{X} \times \mathcal{Y} \to \mathcal{Y} \) can be identified, if it can uniquely be expressed as function of \( D(f_\theta) \).

Identification with supervised learning. Identifiability guarantees of \( h^* \) from samples of \( D(f_\theta) \) imply that the historical data collected under the deployment of \( f_\theta \) contains sufficient information to recover the invariant relationship (7). Consider the following standard variant of supervised learning that takes in samples \((x, \hat{y}, y)\) and builds a meta-model that predicts \( Y \) from \( X, \hat{Y} \) by solving the following risk minimization problem

\[
h_{SL} := \arg\min_{h \in H} E_{(x,\hat{y},y) \sim D(f_\theta)} [(h(x,\hat{y}) - y)]^2.
\]  

(8)

where \( H \) denotes the hypothesis class, which we assume contains the target function \( h^* \). We consider the squared loss for risk minimization because the exogeneous noise \( \xi_Y \) in (3) is additive and zero mean.

Lemma 2. Consider the data generation process in Figure 1. If \( h^* \) is causally identifiable from data collected under the deployment of \( f_\theta \), then the risk minimizer \( h_{SL} \) in (8) will coincide with \( h^* \) as long as \( h^* \in H \), and all functions in \( H \) satisfy the identifiability conditions.

Lemma 2 provides a sufficient condition for the supervised learning solution \( h_{SL} \) to recover the invariant causal quantity \( h^* \) in a realizable setting. In contrast, without causal identifiability, there might be other models \( h' \in H \) that explain the training distribution equally well but do not transfer to the distribution induced by the deployment of a new model. Such a meta model would not be useful for policy evaluation (evaluating the effects of a new prediction rule when deployed on a population) nor for policy optimization (as a decision maker, finding the prediction rule that minimizes a desirable risk function under performativity).

3.2 Challenges for identifiability

The main challenge for identification of \( h^* \) from data samples \((X, \hat{Y}, Y)\) is that in general, the prediction rule \( f_\theta \) determining \( \hat{Y} \) is a deterministic function of the covariates \( X \). This means that, for any realization of \( X \), we only get access to one particular \( \hat{Y} = f_\theta(X) \) in the training distribution, which makes it challenging to disentangle the direct and the indirect effects of \( X \) on \( Y \). To illustrate this challenge, consider the function \( \hat{h}'(x, \hat{y}) := h^*(x, f_\theta(x)) \) that ignores the input parameter \( \hat{y} \) and only relies on \( x \) for explaining the outcome. This function explains the data equally well and can not be differentiated from \( h^* \). The problem is akin to fitting a linear regression model to two perfectly correlated covariates. More broadly, this ambiguity is due to what is known as a lack of overlap (or lack of positivity) in the literature of causal inference [Imbens and Rubin, 2015, Pearl, 1995]. It persists as long as \( P(X|Y = \hat{y}) \) and \( P(X|Y = \hat{y}') \) in the observed distribution do not have common support for pairs of predictions \( \hat{y}, \hat{y}' \) we are potentially interested in. In the covariate shift literature, the lack of overlap surfaces when the covariate distribution violates the common support assumption and the propensity scores are not well-defined (see e.g., Pan and Yang [2010]). This problem renders causal identification and thus data-driven learning of performatve effects of deterministic prediction functions fundamentally challenging.

Proposition 3 (Hardness of identifiability from deterministic predictions). Consider the structural causal model in Figure 1 where \( X, \hat{Y}, Y \) are observed. For a deterministic prediction function \( f_\theta \), the causal quantity \( h^* \) is not identifiable from observational data collected under the deployment of \( f = f_\theta \).

\(^1\)Instead of aiming for the conditional expectation of \( Y \) we could similarly aim for \( E[\kappa(X, Y, \hat{Y})|X = x, do(\hat{Y} = \hat{y})] \) where \( \kappa \) is any continuous function that measures social welfare, individual improvement, and other goals for socially beneficial machine learning. The criteria for identifiability would remain the same but the target in the supervised learning objective (8) would need to be adapted accordingly. See Appendix A for more discussion.
The identifiability issue persists as long as the two variables $X, \hat{Y}$ are deterministically bound and there is no additional structure that can be exploited to disentangle the direct effect of $X$ on $Y$ from the indirect effect mediated by $\hat{Y}$. In the following, we focus on particularities of prediction problems and show how they lead to conditions that render the causal effect of the mediator $\hat{Y}$ on $Y$ in Figure 1 identifiable.

3.3 Identifiability from randomization

We start with the most natural setting that provides identifiability guarantees: randomness in the prediction function $f_\theta$. Using standard arguments about overlap we can identify $h^*(x, \hat{y})$ for any pair $x, \hat{y}$ with positive probability in the data distribution $\mathcal{D}(f_\theta)$ from which the training data is sampled. To relate this to our goal of identifying the outcome under the deployment of an unseen model $f_\theta$ we introduce the following definition:

**Definition 1** (output overlap). Given two predictive models $f_\theta, f_\phi$, the model $f_\phi$ is said to satisfy output overlap with $f_\theta$, if for all $x \in \mathcal{X}$ and any subset $\mathcal{Y}' \subseteq \mathcal{Y}$ with positive measure, it holds that

$$\frac{P[f_\phi(x) \in \mathcal{Y}']}{P[f_\theta(x) \in \mathcal{Y}']} > 0. \tag{9}$$

In particular, output overlap requires the support of the new model’s predictions $f_\phi(x)$ to be contained on the support of $f_\theta(x)$ for every potential $x$ in the population. The following proposition takes advantage of the fact that the joint distribution over $(X, Y)$ is fully determined by the deployed model’s predictions to relate output overlap to identification:

**Proposition 4.** Given the causal graph in Figure 1, the causal quantity $h^*(x, \hat{y})$ is identifiable from observational data generated under the deployment of $f_\theta$ for pairs $x, \hat{y}$ with $\hat{y} = f_\phi(x)$ for $f_\phi$ being a prediction function that satisfies output overlap with $f_\theta$.

Proposition 4 allows us to pinpoint the models $f_\phi$ we can extrapolate from data collected under $f_\theta$. Furthermore, it makes explicit that for learning about performative effects, the ideal prediction function $f_\theta$ is a randomized prediction function whose output has full support over $\mathcal{Y}$ for any $x$. Such a model would generate a dataset that guarantees global identification of $h^*$ over $\mathcal{X} \times \mathcal{Y}$ and thus robust conclusions about any future deployable model $f_\phi$. One interesting and relevant setting that satisfies this property is the differentially private release of predictions through an additive noise mechanism applied to the output of the prediction function [Dwork et al., 2006]. Here, instead of $\hat{Y}_{\text{orig}} = f_\theta(X)$ a noisy version $\hat{Y} = \hat{Y}_{\text{orig}} + \eta$ with $\eta \sim \text{Lap}(0, b)$ for an appropriately chosen $b > 0$ is released. Since the Laplace noise has full support output overlap and identification is guaranteed by Proposition 4 for any $f_\phi$. In contrast, noise with bounded support would only allow for extrapolation to models that are sufficiently similar in prediction space.

**Side-information and alternate sources of randomness.** If additional side-information, observations, or more fine grained knowledge about the causal graph structure is available, then identification can also be achieved from other sources of randomness. For example, consider the causal graph in Figure 2 where the performative effect of predictions is mediated by a down-stream decision $T \in \{0, 1\}$, such that $Y \perp \!\!\!\perp \hat{Y} | T, X$. In this case, randomness in the discrete decision function $T$ (instead of the continuous prediction $\hat{Y}$) is sufficient for identification of the causal graph.

**Proposition 5.** Consider the data generation process in Figure 2 where $T$ is a discrete random variable and $T$ is observed. Then, as long as $0 < P(T = t | X = x) < 1$ in the training dataset for every $t \in T$ and $x \in \text{supp}(\mathcal{D}_X)$, the causal relationship $h^*$ can be identified.
Randomness in prediction-based decisions can be a deliberate part of an algorithmic system for a number of reasons, including designing individually fair decision rules [Dwork et al., 2012].

**Example 3.1** (randomized resource allocation). A practical strategy to achieve individual fairness through randomization is discussed in Berger et al. [2020a,b]. They propose a two-step procedure for resource allocation: first, select the instances that satisfy a basic first step qualification criteria and then assign resources among the qualified ones by means of a lottery. Here the deterministic first step limits the extrapolation capabilities from the observed data in that output overlap with \( f_0 \) only holds for models \( f \) that apply the same or stricter first step criteria than \( f_0 \). However, among those functions, randomization in the allocation decision enables extrapolation even if \( f_0 \) is deterministic.

Another natural source of randomness in performative prediction is noise in the measurement of the covariates \( X \), representing the unobserved true underlying attributes \( U \). In this case, side-information about the structure of the measurement noise enables identification even for deterministic prediction functions \( f_0 \). The intuition is that the attributes \( U \) that are causal for the outcome \( Y \) enter the prediction through the noisy measurements \( X \), which adds independent variation to the indirect causal path. We present this identification result in Appendix B. However, incorporating such side-information requires going beyond standard empirical risk minimization (ERM) as an identification strategy, and thus they are not the main focus of this work.

More broadly, the results in this section demonstrate that there are natural sources of randomness and algorithmic requirements on prediction functions deployed in production, such as differential privacy [Dwork et al., 2006] and algorithmic fairness [Dwork et al., 2012], that help provide identifiability. However, a caveat of these identification strategies is that accurate side-information is not always available to an analyst, and that there are several reasons a decision-maker may choose not to deploy a randomized prediction function in performative environments, including negative externalities and concerns about user welfare, but also business interests to preserve consumer value of the prediction-based service offered. This suggests a tension between the accuracy of the model deployed and the informativeness of the data it induces for learning performative effects. In the context of our credit scoring example, random predictions would imply that interest rates are randomly assigned to applicants in order to learn how the rates impact their probability of paying back. This is a scenario we can not presently observe, given regulatory requirements for lending institutions. In the following, we will discuss two additional settings where we can achieve identifiability without requiring randomization of the predictive model \( f_0 \) or relying on additional side-information.

### 3.4 Identifiability through overparameterization

This section considers a situation where we can achieve identification, without overlap, from data collected under a deterministic \( f_0 \). Our result exploits incongruences in functional complexity arising from machine learning models that are overparameterized, which is common in modern machine learning applications [e.g. Krizhevsky et al., 2012]. By overparameterization, we refer to the fact that the representational complexity of the model is larger than the underlying concept that needs to be described. We formalize this as follows:

**Assumption 1** (overparameterization). We say a function \( f \) is overparameterized with respect to \( \mathcal{G} \) over \( \mathcal{X} \) if there is no function \( g' \in \mathcal{G} \) and \( c \in \mathbb{R} \) such that \( f(x) = c \cdot g'(x) \).

For the purpose of this section, assume the structural equation for how \( Y \) is generated is separable and has the following form \( g(X, \hat{Y}) = g_1(X) + \beta \hat{Y} \), where \( \hat{Y} \) is the output of the prediction function \( f_0 \) mapping \( X \) to \( \hat{Y} \), and \( \beta \geq 0 \) is a constant. As we have emphasized earlier, the challenge for identification is that for deterministic \( f_0 \) the prediction can be reconstructed from \( X \) without relying on \( \hat{Y} \) and thus the function \( h'(x, \hat{y}) = g_1(x) + \beta f_0(x) \) can not be differentiated from the true \( h^* \) based on observational data. However, note that this ambiguity relies on there being an \( h \in \mathcal{H} \) such that \( h(\cdot, \hat{y}) \) for a fixed \( \hat{y} \) can represent \( f_0 \). Thus, for prediction functions \( f_0 \not\in \mathcal{H} \) the solution \( h_{SL} \) (for a well specified \( \mathcal{H} \)) will necessarily rely on \( \hat{Y} \) to explain the effect of the prediction. To make this intuition more concrete, consider the following example:

**Example 3.2.** Assume \( g(x, \hat{y}) = \alpha x + \beta \hat{y} \) for some unknown \( \alpha, \beta \) and let \( \mathcal{H} \) be the class of linear functions. Now consider prediction functions \( f_0 \) of the following form \( f_0(x) = \gamma x^2 + \xi x \) for some \( \gamma, \xi \geq 0 \). Then, any consistent parameter estimate \( \alpha', \beta' \) needs to satisfy the following two equations: \( \alpha' + \beta' \xi = \beta \xi \) and \( \beta' \gamma = \beta \gamma \) matching the multipliers of the base polynomials. This system of equations has a unique solution as long as \( \gamma > 0 \) which corresponds to the case where \( f_0 \) is overparameterized with respect to the class of linear functions. In contrast, for \( \gamma = 0 \) where there is no incongruence in the functional complexity any combination of parameters \( \alpha', \beta' \) such that \( \alpha' + \beta' \xi = \beta \xi \) would be a consistent estimate.
The following result makes this argument more formal:

**Proposition 6.** Consider the structural causal model in Figure 1 where \( f_0 \) is a deterministic function. Assume that \( g \) can be decomposed as \( g(X, \hat{Y}) = g_1(X) + \alpha \hat{Y} \) for some \( \alpha > 0 \) and \( g_1 \in \mathcal{G} \), where the function class \( \mathcal{G} \) is closed under addition (i.e. \( g_1, g_2 \in \mathcal{G} \Rightarrow a_1 \cdot g_1 + a_2 \cdot g_2 \in \mathcal{G} \ \forall a_1, a_2 \in \mathbb{R} \)). Let \( \mathcal{H} \) contain functions that are separable in \( X \) and \( \hat{Y} \), linear in \( \hat{Y} \), and \( \forall h \in \mathcal{H} \) it holds that \( h(\cdot, \hat{y}) \in \mathcal{G} \) for a fixed \( \hat{y} \). Then, if \( f_0 \) is overparameterized with respect to \( \mathcal{G} \) over the support of \( D_X \), \( h^* \) is identifiable from observational data.

The above result can be extended to more general structural causal models of the form \( g(X, \hat{Y}) = g_1(X) + g_2(\hat{Y}) \). In this case linear independence between \( g_1 \) and \( g_2 \circ f_0 \) is needed for identification. This is achieved if the model is overparameterized, and, in addition, we can ensure that \( g_2 \circ f_0 \) remains sufficiently complex. As a concrete instantiation where this is the case, we could have \( g_1, g_2 \in \mathcal{G} \) with \( \mathcal{G} \) being the class of degree \( k \) polynomials, and \( f_0 \) being of degree \( k' > k \). More generally, in practical settings with overparameterized models, we expect incongruence to persist beyond the linear setting. In particular, because there is no reason to believe that there is any structural similarity in the direct link describing the structural relationship between features and label and the nature of performative effects, it is reasonable to assume that \( g_2 \circ f_0 \) inherits the complexity of \( f_0 \).

### 3.5 Identifiability for classification

A second ubiquitous source of incongruence that we can exploit for identification is the discrete nature of predictions \( \hat{Y} \) in the context of classification. The resulting discontinuity in the relationship between \( X \) and \( \hat{Y} \) enables us to disentangle the direct causal link between \( X \) and \( Y \) from the indirect link mediated by the prediction \( \hat{Y} \). This identification strategy is akin to the popular regression discontinuity design [Lee and Lemieux, 2010] and relies on the assumption that all other variables in \( X \) are continuously related to \( Y \) around the discontinuities in \( \hat{Y} \). Together with the separability of the structural causal model, we can establish the following global identifiability result:

**Proposition 7** (discrete classification). Assume that the effect of \( X \) and \( \hat{Y} \) on \( Y \) are separable \( g(X, \hat{Y}) = g_1(X) + g_2(\hat{Y}) \), \( \forall X, \hat{Y} \) for some differentiable functions \( g_1 \) and \( g_2 \). Further, suppose \( X \) is a continuous random variable and \( \hat{Y} \) is a discrete random variable that takes on at least two distinct values with non-zero probability. Then, \( h^* \) is identifiable from observational data.

Similar to Proposition 6, the separability assumption together with incongruence provides a way to separate the direct effect from the indirect effect of \( X \) on \( Y \). This is necessary in order to achieve global identification guarantees without randomness because the identification of entangled components without overlap is fundamentally hard. Thus, under violations of the separability assumptions, we can only expect the separable components of \( g \) to be correctly identified. Furthermore, a regression discontinuity design only enables the identification of the causal effect locally around the discontinuity by comparing similar units right above and right below the threshold that obtained a different prediction. This means that the resulting meta-model only allows us to confidently reason about infinitesimally small changes to the prediction function.

In general, the further from \( f_0 \) we aim to extrapolate, the more we rely on assumptions, and the more brittle our causal conclusions become to violations of that said assumptions. Akin to Section 3.3 (if one can not be confident about the overlap being satisfied on all of \( \hat{Y} \) for every \( X \)), we recommend being cautious when relying on supervised learning approaches to reason about the impact of substantial updates to the predictive model, even if we put aside concerns about data scarcity. Rather, we would recommend considering data-driven predictions as a tool to inform local updates to the predictive model in the context of gradual exploration so as to stay within a suitably chosen trust region within which \( h_{SL} \) can be assumed to remain reliable.

### 4 Empirical evaluation

The three settings studied in the previous section described several natural scenarios where we can hope to answer the causal question outlined in Section 3.1 with a model learned using tools from supervised learning that rely on observational data. The purpose of this section is to investigate empirically how well the supervised learning solution \( h_{SL} \) in (8) is able to identify a transferable functional relationship from data in practical settings with finite data.
Figure 3: Extrapolation error of supervised learning with and without access to $\hat{Y}$. (a) In the non-identifiable setting, adding $\hat{Y}$ as a feature harms generalization performance. (b)-(d) Overparameterization, randomization, and discrete predictions are each sufficient for avoiding this failure mode. Supervised learning obtains models robust to distribution shift when $\hat{Y}$ is given as a feature, while the extrapolation loss of the performativity-agnostic model grows with the strength of performativity.

Methodology. We generated semi-synthetic data for our experiments, using a Census income prediction dataset from folks-tables.org [Ding et al., 2021]. Using this dataset as a starting point, we simulate a training dataset and test dataset with distribution shift as follows: First, we choose two different predictors $f_0$ and $f_0$ to predict a target variable of interest (e.g. income) from covariates $X$ (e.g. age, occupation, education, etc.). If not specified otherwise, $f_0$ is fit to the original dataset to minimize squared error, while $f_0$ is trained on randomly shuffled labels. Next, we posit a function $g$ for simulating the performative effects. Then, we generate a training dataset of $(X, \hat{Y}, Y)$ tuples following the structural causal model in Figure 1, using the covariates $X$ from the original data, $g$, and $f_0$ to generate $\hat{Y}$ and $Y$. Similarly, we generate a test dataset of $(X, \hat{Y}, Y)$ tuples, using $X, g, f_0$. We assess how well supervised methods learn transferable functional relationships by fitting a model $h_{\text{SL}}$ to the training dataset and then evaluating the root mean squared error (RMSE) for regression and the accuracy for classification on the test dataset. In our figures, we visualize the standard error from 10 replicates with different random seeds and we compare it to an in-distribution baseline trained and evaluated on samples of $D(f_0)$.

4.1 Necessity of identification guarantees for supervised learning

We start by illustrating why our identification guarantees are crucial for supervised learning under performativity. We use the following experimental setup instantiating the structural causal model in Figure 1 as

$$g(X, \hat{Y}) = \beta^T X + \alpha \hat{Y}$$

with $\xi, Y \sim \mathcal{N}(0, 1)$. The coefficients $\beta$ are determined by linear regression on the original dataset. The hyperparameter $\alpha$ quantifies the performativity strength that we vary in our experiments. The predictions $\hat{Y}$ are generated from a linear model $f_0$ that we modify to illustrate the resulting impact on identifiability. We optimize $h_{\text{SL}}$ in (8) over $\mathcal{H}$ being the class of linear functions and assume there are plenty of training data points ($N = 200,000$) available.

We start by illustrating a failure mode of supervised learning in a non-identifiability setting falling under the umbrella of Proposition 3. Therefore, we let $f_0$ be a deterministic linear model fit to the base dataset. This results in $h^*$ not being identifiable from $D(f_0)$. In Figure 3(a) we can see that supervised learning indeed struggles to identify a transferable functional relationship from the training data. What we observe in the experiment is that $f_0(X) = \beta^T X$ and the meta model returns $h_{\text{SL}}(X, \hat{Y}) = (1 + \alpha) \hat{Y}$ as a valid model for the outcome, instead of identifying $h^* = g$ correctly. However, this relationship is not preserved for our test model $f_\vartheta$, which leads to a high extrapolation error independent of the strength of performativity. While we only show the error for one $f_\vartheta$ in Figure 3(a), the error grows with the distance $d^2_{\mathcal{H}}(f_0, f_\vartheta)$. In contrast, when the feature $\hat{Y}$ is not included, the supervised learning strategy returns $h_{\text{SL}}(X) = (1 + \alpha) \beta^T X$. The extrapolation loss of this performativity-agnostic model scales with the strength of performativity (c.f. Proposition 1) and is thus strictly smaller than the error of the model that predicts from predictions in this example.

Once we leave the non-identifiable setting and move into the regime of our identification results (Proposition 4-7), the benefit of including $\hat{Y}$ as a feature becomes apparent. To illustrate this, we reuse the same setup but modify the
way the predictions in the training data are generated. In Figure 3(b) we use additive Gaussian noise to determine the predictions as $\hat{Y} = f_\theta(X) + \eta$ with $\eta \sim \mathcal{N}(0, \sigma^2)$. In Figure 3(c) we augment the input to $f_\theta$ with second-degree polynomial features to achieve overparameterization. In Figure 3(d) we round the predictions of $f_\theta$ to obtain discrete values. In all three cases, the results show that adding $\hat{Y}$ as a feature allows the model to match in-distribution accuracy baselines and close the extrapolation gap that is inevitable if $\hat{Y}$ is not included as a feature.

### 4.2 Strength of incongruence

We next conduct an ablation study and investigate how the degree of overparameterization and the noise level for randomized $f_\theta$ impacts the extrapolation performance of supervised learning. Therefore, we consider the following instantiation of the structural equation model in Figure 1:

$$g(X, \hat{Y}) = g_1(X) + \alpha \hat{Y}$$

with $\xi_Y \sim \mathcal{N}(0, 1)$. We fix the level of performativity at $\alpha = 0.5$ for this experiment. We optimize $h_{\text{SL}}$ in (8) over $\mathcal{H}$ (which we vary) and assume there are plenty of training data points ($N = 200,000$) available.

#### Degree of overparameterization. 

To explore the effect of overparameterization we choose fully connected neural networks with a single hidden layer to represent the functions $g_1$ and $f_\theta$. For the function $g_1$ we take a neural network with $m_\theta = 3$ units in the hidden layer and fit it to the original dataset. This model is kept fixed and the hypothesis class $\mathcal{H}$ are also neural networks with 3 units in the hidden layer. Then, to simulate the degree of overparameterization of $f_\theta$, we vary the number of neurons in the hidden layer of $f_\theta$, denoted $m_\theta$. The resulting extrapolation performance of $h_{\text{SL}}$ on the test distribution is shown in Figure 4(a). We can see how the extrapolation error of the learned model decreases with the complexity of $f_\theta$. In particular, as soon as $m_\theta > m_\phi$ there is a significant benefit to adding $\hat{Y}$ as a feature to the meta model. In this regime, $h^*$ becomes identifiable and $h_{\text{SL}}$ successfully recovers the transferable functional relationship in (11) as Proposition 6 suggests. In turn, without adding $\hat{Y}$ as a feature the model suffers an inevitable extrapolation gap due to a concept shift that is independent of the properties of $f_\theta$.

#### Magnitude of noise.

In our second experiment on incongruence, we investigate the effect of the magnitude of additive noise added to the predictions in the linear model setting shown in Figure 3(b). Here $\mathcal{H}$ and $g_1$ are linear functions and we vary the level of noise added to the predictions $f_\theta$. More specifically, we have $\hat{Y} = f_\theta(X) + \beta \eta$ with $\eta \in \mathcal{N}(0, 1)$ where we vary $\beta$. The corresponding results can be found in Figure 4(b). We see that even small amounts of noise are sufficient for identification and adding $\hat{Y}$ as a feature to our meta-machine learning model is effective as soon as the noise in $f_\theta$ is non-zero.

### 4.3 Learning with finite data

Recall that causal identification results are feasibility guarantees. They imply that $h^*$ can be recovered from observational data in the limit of infinite data. However, in practical settings, we only get access to a finite set of data points.
from the training distribution. In the following, we show that supervised learning can successfully learn transferable functions $h_{SL}$ with only a few training data points, given that our identifiability conditions are satisfied.

In Figure 5(a)-(b) we consider the same setup as in Section 4.1; we fix performativity strength at $\alpha = 0.5$, and vary training set size. We find that only moderate dataset sizes are necessary for $h_{SL}$ to identify an accurate model that is robust to performative distribution shifts in our identifiable settings.

In Figure 5(c) we choose $N = 5000$ and investigate the performance of supervised learning as we vary the distance between predictions from $f_\theta$ and $f_\phi$, i.e. the distribution shift between train and test set. We achieve this by interpolating the parameters of the predictive model in the test set between $f_\theta$ and $f_\phi$, where the latter is trained on randomized labels as before. We observe that the error and variance of $h_{SL}$ grow with the magnitude of the distribution shift. This can be explained as failures in the meta model to identify the transferable causal model $h^*$ become more pronounced as distribution shifts get larger. In addition, the variance is expected to grow further away from the mean of $f_\theta(x)$ due to data scarcity implied by the shape of the noise distribution in the randomized $f_\theta$. This observation supports our recommendation to explore the parameter space gradually for policy optimization under performativity, instead of directly extrapolating to models $f_\phi$ that are substantially different from $f_\theta$.

5 Discussion

This paper examined a supervised learning approach to tackle the problem of predicting under performativity. We have focused on a setting where performative effects are mediated by an individual’s prediction and highlighted fundamental challenges of learning performative effects from observational data. Then, within the scope of our modeling assumptions we have identified natural properties of prediction functions that lead to training data that is sufficiently informative to allow for conclusions about the potential outcome of an individual under the deployment of an unseen predictive model. By drawing a connection between the feasibility of prediction under performativity and causal identifiability, this paper contributes to a better understanding of the suitability of data-driven approaches for explaining social effects arising from the deployment of predictive models in economically and socially relevant applications.

5.1 The message for data collection practices

The positive results in this work demonstrate the value of logging information about the state of the deployed prediction function when collecting data for the purpose of supervised learning in social settings; if predictions are observed, they can be incorporated to anticipate the performative effects of future model deployments. In contrast, if the predictions are not available, $f_\theta$ disrupts the causal relationship between $X$ and $Y$ that we aim to understand, leading to unavoidable prediction errors. Thus, observing the predictive model, or the prediction $\hat{Y}$ is crucial for an analyst hoping to understand the effects of deployed predictive models, for engineers hoping to foresee consequences of new model deployments, and for the research community hoping to study performative phenomena. To date, such data is scarcely available in benchmark datasets, hindering the progress towards a better understanding of performative effects, essential for the reliable deployment of algorithmic systems in the social world.
5.2 Limitations and Extensions

The success of supervised learning approaches is closely tied to the corresponding identifiability conditions being satisfied. We have shown that identifiability is possible in certain cases if access to predictions is given. However, our results must not be understood as a green-light to justify the use of supervised learning techniques to address performativity in full generality beyond the scope of our theoretical results. In the following, we discuss several potential violations of our assumptions that give rise to interesting questions for future work.

**Performative effects beyond predictions.** The theme of this work has been to define a sufficient statistic for the performative distribution shift and then lift the space of covariates to include this statistic into the input features to our meta model for how the outcome variable $Y$ is generated. This allowed us to transform the original performative prediction problem with concept shift into a classical supervised learning problem with covariate shift. This strategy is not limited to predictions $\hat{Y}$ as a sufficient statistic, but could as well be applied to other performativity-relevant properties of the prediction function $f_\theta$. Such properties could for example be the importance of individual features in the final decision, relevant model parameters, or information about counterfactual outcomes. Independent of how we decide to model performative effects, the validity of any causal claim will inevitably be limited to the scope of its assumptions. Extracting the relevant features to base the assumptions on requires domain knowledge—the more expert knowledge we can incorporate about how performative effects arise, the better we can pin down these statistics. This in turn simplifies the learning task and allows us to trade off assumptions with data requirements for causal identifiability.

**Performative effects through social influence.** Our causal model, proposed in Figure 1, models performative effects at an individual level. There is no possibility for interference in the sense that the prediction of one individual can impact the outcome of his or her peers. Such an individualistic perspective is not unique to our paper but prevalent in existing causal analyses and model-based approaches to performative prediction and strategic classification [e.g., Bechavod et al., 2020, Ghalme et al., 2021, Hardt et al., 2016b, Harris et al., 2021, Jagadeesan et al., 2021, Miller et al., 2020]. However, the presence of interference effects can have important implications for how causal effects should be estimated and interpreted, which is yet unexplored in the context of performative prediction. In particular, there is a crucial difference between unilateral interventions on the prediction $\hat{Y}_i$ of a single individual $i \in [n]$ and interventions performed on the entire population. This is akin to the important distinction between the individual causal effect and the overall causal effect in treatment effect estimation [e.g., Tchetgen and VanderWeele, 2012]. Concretely, for our model, interference implies that

$$E[Y_i|X_i = x, \text{do}(f_\theta = f_{\theta^*})] \neq E[Y_i|X_i = x, \text{do}(\hat{Y}_i = f_{\theta^*}(x))].$$

As a consequence, classical approaches for building microfoundation models for performative effect estimation based on simple, unilateral interventions on an individual’s prediction\(^3\) result in different causal estimates than supervised learning based methods for identification as studied in this work. While interference biases both estimates, a data-driven approach can implicitly pick up patterns of interference effects present in the data despite model-misspecifications, whereas individualistic models are blind to these effects. In Appendix C we provide an example where this is an advantage: our data-driven approach can exploit network homophily [Goldsmith-Pinkham and Imbens, 2013] to explain the total causal effect of a model change on the outcome of an individual, whereas individualistic modeling misses out on the indirect component arising from neighbors influencing each other. This raises interesting questions for future work about how to address interference in the context of performativity.

**Performativity in anti-causal prediction.** Our causal graph in Figure 1 posits that prediction is solely based on features $X$ that are causal for the outcome $Y$. This is a desirable situation in many practical applications because causal predictions disincentivize gaming of strategic individuals manipulating their features [Bechavod et al., 2020, Miller et al., 2020] and offers explanations for the outcome that persist across environments [Bühlmann, 2018, Rojas-Carulla et al., 2018]. Nevertheless, anti-causal variables are often included as input features in practical machine learning prediction tasks. Thus, establishing a better understanding for the implications of the resulting causal cycles due to performativity could be an important direction for future work.

\(^3\)See Björkegren et al. [2020] for a related field experiment.
Covariate shift due to performativity. Finally, performative prediction [Perdomo et al., 2020] in full generality allows a predictive model $f_\theta$ to impact the joint distribution $P(X, Y) = P(Y|X)P(X)$ over covariates and labels. In this work, we have assumed that the distribution over covariates is unaffected by the attempt to predict $Y$ from $X$ and performative effects only surface in $P(Y|X)$. For our theoretical results, this implied that overlap in the $X$ variable across environments is trivially satisfied, which enabled us to pinpoint the challenges of learning performative effects due to the coupling between $X$ and $Y$. For establishing identification under covariate shift (in the presence of a causal arrow $f_\theta \to X$) additional steps are required to ensure identifiability, however the challenges discussed in this work remain.

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References

Peter M. Aronow and Cyrus Samii. Estimating average causal effects under general interference, with application to a social network experiment. *The Annals of Applied Statistics*, 11(4), 2017.

Susan Athey. Machine learning and causal inference for policy evaluation. In *Proceedings of the 21th International Conference on Knowledge Discovery and Data Mining*, KDD ’15, page 5–6, 2015.

Yahav Bechavod, Katrina Ligett, Zhiwei Steven Wu, and Juba Ziani. Gaming helps! learning from strategic interactions in natural dynamics. *ArXiv*, 2020.

Joël Berger, Margit Osterloh, and Katja Rost. Focal random selection closes the gender gap in competitiveness. *Science Advances*, 6(47), 2020a.

Joël Berger, Margit Osterloh, Katja Rost, and Thomas Ehrmann. How to prevent leadership hubris? comparing competitive selections, lotteries, and their combination. *The Leadership Quarterly*, 31(5):101388, 2020b.

Xander M. Bezuijen, Peter T. van den Berg, Karen van Dam, and Henk Thierry. Pygmalion and employee learning: The role of leader behaviors. *Journal of Management*, 35(5):1248–1267, 2009.

Peter Bühlmann. Invariance, causality and robustness, 2018.

Daniel Björkegren, Joshua E. Blumenstock, and Samsun Knight. Manipulation-proof machine learning. *ArXiv:2004.03865*, 2020.

Michael Brückner, Christian Kanzow, and Tobias Scheffer. Static prediction games for adversarial learning problems. *Journal of Machine Learning Research*, 13(85):2617–2654, 2012.

Flavio Chierichetti, Silvio Lattanzi, and Alessandro Panconesi. Rumor spreading in social networks. In Susanne Albers, Alberto Marchetti-Spaccamela, Yossi Matias, Sotiris Nikoletseas, and Wolfgang Thomas, editors, *Automata, Languages and Programming*, 2009. ISBN 978-3-642-02930-1.

Mina Cikara, Matthew M. Botvinick, and Susan T. Fiske. Us versus them: Social identity shapes neural responses to intergroup competition and harm. *Psychological Science*, 22(3):306–313, 2011.

Frances Ding, Moritz Hardt, John Miller, and Ludwig Schmidt. Retiring adult: New datasets for fair machine learning. *Advances in Neural Information Processing Systems*, 34, 2021. Code available at: [https://github.com/zykls/folktables](https://github.com/zykls/folktables).

Dmitriy Drusvyatskiy and Lin Xiao. Stochastic optimization with decision-dependent distributions. *ArXiv:2011.11173*, 2020.
Cynthia Dwork, Frank McSherry, Kobbi Nissim, and Adam Smith. Calibrating noise to sensitivity in private data analysis. In Theory of Cryptography, pages 265–284, 2006.

Cynthia Dwork, Moritz Hardt, Toniann Pitassi, Omer Reingold, and Richard Zemel. Fairness through awareness. In Proceedings of the 3rd Innovations in Theoretical Computer Science Conference, ITCS ’12, page 214–226, 2012.

Dean Eckles, Nikolas Ignatiadis, Stefan Wagner, and Han Wu. Noise-induced randomization in regression discontinuity designs. ArXiv:2004.09458, 2020.

Dov Eden. Leadership and expectations: Pygmalion effects and other self-fulfilling prophecies in organizations. The Leadership Quarterly, 3(4):271–305, 1992.

João Gama, Indre Zliobaitė, Albert Bifet, Mykola Pechenizkiy, and Abdelhamid Bouchachia. A survey on concept drift adaptation. ACM Comput. Surv., 46(4), 2014.

Ganesh Ghalme, Vineet Nair, Itay Eilat, Inbal Talgam-Cohen, and Nir Rosenfeld. Strategic classification in the dark. In Proceedings of the 38th International Conference on Machine Learning, volume 139, pages 3672–3681. PMLR, 2021.

Paul Goldsmith-Pinkham and Guido W. Imbens. Social networks and the identification of peer effects. Journal of Business & Economic Statistics, 31(3):253–264, 2013.

M. Hardt, E. Price, and N. Srebo. Equality of opportunity in supervised learning. In Advances in Neural Information Processing Systems, pages 3315–3323, 2016a.

Moritz Hardt, Nimrod Megiddo, Christos Papadimitriou, and Mary Wootters. Strategic classification. In Proceedings of the 2016 ACM Conference on Innovations in Theoretical Computer Science, ITCS ’16, pages 111–122, 2016b.

Moritz Hardt, Meena Jagadeesan, and Celestine Mendler-Dünner. Performative power. ArXiv:2203.17232, 2022.

Keegan Harris, Dung Ngo, Logan Stapleton, Hoda Heidari, and Zhiwei Steven Wu. Strategic instrumental variable regression: Recovering causal relationships from strategic responses. Arxiv:2107.05762, 2021.

Guido W Imbens and Donald B Rubin. Causal inference in statistics, social, and biomedical sciences. Cambridge University Press, 2015.

Zachary Izzo, Lexing Ying, and James Zou. How to learn when data reacts to your model: Performative gradient descent. In Proceedings of the 38th International Conference on Machine Learning, volume 139, pages 4641–4650, 2021.

Meena Jagadeesan, Celestine Mendler-Dünner, and Moritz Hardt. Alternative microfoundations for strategic classification. In Proceedings of the 38th International Conference on Machine Learning, volume 139, pages 4687–4697, 2021.

Meena Jagadeesan, Tijana Zrnic, and Celestine Mendler-Dünner. Regret minimization with performative feedback. In Proceedings of the 39th International Conference on Machine Learning, volume 162, pages 9760–9785, 2022.

Kaggle. Give me some credit, 2011. URL https://www.kaggle.com/competitions/GiveMeSomeCredit/data.

Amir-Hossein Karimi, Bernhard Schölkopf, and Isabel Valera. Algorithmic recourse: From counterfactual explanations to interventions. In Proceedings of the 2021 ACM Conference on Fairness, Accountability, and Transparency, FAccT ’21, page 353–362, 2021.

Alex Krizhevsky, Ilya Sutskever, and Geoffrey E Hinton. Imagenet classification with deep convolutional neural networks. In Advances in Neural Information Processing Systems, volume 25, 2012.

Bogdan Kulychny. Causal prediction can induce performative stability. In ICML 2022: Workshop on Spurious Correlations, Invariance and Stability, 2022.
Thibault Laugel, Marie-Jeanne Lesot, Christophe Marsala, Xavier Renard, and Marcin Detyniecki. Comparison-based inverse classification for interpretability in machine learning. In Jesús Medina, Manuel Ojeda-Aciego, José Luis Verdegay, David A. Pelta, Inma P. Cabrera, Bernadette Bouchon-Meunier, and Ronald R. Yager, editors, Information Processing and Management of Uncertainty in Knowledge-Based Systems. Theory and Foundations, pages 100–111, 2018. ISBN 978-3-319-91473-2.

David S. Lee and Thomas Lemieux. Regression discontinuity designs in economics. Journal of Economic Literature, 48(2), 2010.

Scott Levin, Matthew Tooper, Eric Hamrock, Jeremiah S. Hinson, Sean Barnes, Heather Gardner, Andrea Dugas, Bob Linton, Tom Kirsch, and Gabor Kelen. Machine-learning-based electronic triage more accurately differentiates patients with respect to clinical outcomes compared with the emergency severity index. Annals of Emergency Medicine, 71(5):565–574.e2, 2018.

Lihong Li, Wei Chu, John Langford, and Xuanhui Wang. Unbiased offline evaluation of contextual-bandit-based news article recommendation algorithms. In Proceedings of the Fourth ACM International Conference on Web Search and Data Mining, WSDM ’11, page 297–306, 2011.

Lihong Li, Shunbao Chen, Jim Kleban, and Ankur Gupta. Counterfactual estimation and optimization of click metrics in search engines: A case study. In Proceedings of the 24th International Conference on World Wide Web, WWW ’15 Companion, page 929–934, 2015. ISBN 9781450334730.

Yang Liu, Yatong Chen, and Jiaheng Wei. Induced domain adaptation. ArXiv:2107.05911, 2021.

Donald MacKenzie. An engine, not a camera: How financial models shape markets. Mit Press, 2008.

Nikhil Malik. Does machine learning amplify pricing errors in housing market? : Economics of ml feedback loops. Information Systems & Economics eJournal, 2020.

Charles F. Manski. Identification of endogenous social effects: The reflection problem. The Review of Economic Studies, 60(3):531–542, 1993.

Gustavo Manso. Feedback effects of credit ratings. Journal of Financial Economics, 109(2):535–548, 2013.

Celestine Mendler-Dünnerr, Juan C. Perdono, Tijana Zrnic, and Moritz Hardt. Stochastic optimization for performative prediction. In Proceedings of the 34th International Conference on Neural Information Processing Systems, 2020.

John Miller, Smitha Milli, and Moritz Hardt. Strategic classification is causal modeling in disguise. In Proceedings of the 37th International Conference on Machine Learning, volume 119, pages 6917–6926, 2020.

John P Miller, Juan C Perdono, and Tijana Zrnic. Outside the echo chamber: Optimizing the performative risk. In Proceedings of the 38th International Conference on Machine Learning, volume 139 of PMLR, pages 7710–7720, 2021.

Adhyyan Narang, Evan Faulkner, Dmitriy Drusvyatskiy, Maryam Fazel, and Lillian J. Ratliff. Multiplayer performative prediction: Learning in decision-dependent games. abs/2201.03398, 2022.

Sinno Jialin Pan and Qiang Yang. A survey on transfer learning. IEEE Transactions on Knowledge and Data Engineering, 22(10):1345–1359, 2010. doi: 10.1109/TKDE.2009.191.

Judea Pearl. Causal diagrams for empirical research. Biometrika, 82(4):669–688, 1995.

Judea Pearl. Causality: Models, Reasoning and Inference. Cambridge University Press, New York, NY, USA, 2nd edition, 2009a.

Judea Pearl. Causal inference in statistics: An overview. Statist. Surv., 3:96–146, 2009b.

F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot, and E. Duchesnay. Scikit-learn: Machine learning in Python. Journal of Machine Learning Research, 12:2825–2830, 2011.
Juan Perdomo, Tijana Zrnic, Celestine Mendler-Dünner, and Moritz Hardt. Performative prediction. In *Proceedings of the 37th International Conference on Machine Learning*, pages 7599–7609, 2020.

Georgios Piliouras and Fang-Yi Yu. Multi-agent performative prediction: From global stability and optimality to chaos. *arxiv*, abs/2201.10483, 2022.

Aahlad Puli, Adler Perotte, and Rajesh Ranganath. Causal estimation with functional confounders. *Advances in neural information processing systems*, 33:5115–5125, 2020.

Mateo Rojas-Carulla, Bernhard Schölkopf, Richard Turner, and Jonas Peters. Invariant models for causal transfer learning. *JMLR*, 19:1309–1342, 2018.

Paul R. Rosenbaum and Donald B. Rubin. The central role of the propensity score in observational studies for causal effects. *Biometrika*, 70(1):41–55, 1983. ISSN 00063444.

Robert Rosenthal and Lenore Jacobson. *Pygmalion in the classroom*. The Urban Review, 1968.

Donald B. Rubin. Randomization analysis of experimental data: The fisher randomization test comment. *Journal of the American Statistical Association*, 75(371):591–593, 1980. ISSN 01621459.

Tobias Schnabel, Adith Swaminathan, Ashudeep Singh, Navin Chandak, and Thorsten Joachims. Recommendations as treatments: Debiasing learning and evaluation. In *Proceedings of the 33rd International Conference on International Conference on Machine Learning - Volume 48, ICML’16*, page 1670–1679, 2016.

Yonadav Shavit, Benjamin Edelman, and Brian Axelrod. Causal strategic linear regression. In *Proceedings of the 37th International Conference on Machine Learning*, volume 119, pages 8676–8686, 2020.

Michael E Sobel. What do randomized studies of housing mobility demonstrate? causal inference in the face of interference. *Journal of the American Statistical Association*, 101(476):1398–1407, 2006.

Gloria B. Solomon, David A. Striegel, John F. Eliot, Steve N. Heon, Jana L. Maas, and Valerie K. Wayda. The self-fulfilling prophecy in college basketball: Implications for effective coaching. *Journal of Applied Sport Psychology*, 8(1):44–59, 1996.

George Soros. *The Alchemy of Finance*. John Wiley & Sons, 2015.

Adith Swaminathan and Thorsten Joachims. Counterfactual risk minimization: Learning from logged bandit feedback. In *Proceedings of the 32nd International Conference on International Conference on Machine Learning - Volume 37, ICML’15*, page 814–823, 2015.

Eric J Tchetgen and Tyler J VanderWeele. On causal inference in the presence of interference. *Statistical Methods in Medical Research*, 21(1):55–75, 2012.

Stratis Tsirtsis and Manuel Gomez Rodriguez. Decisions, counterfactual explanations and strategic behavior. In *Advances in Neural Information Processing Systems*, volume 33, pages 16749–16760, 2020.

Stefan Wager, Nick Chamandy, Omkar Muralidharan, and Amir Najmi. Feedback detection for live predictors. In *Proceedings of the 27th International Conference on Neural Information Processing Systems - Volume 2, NIPS’14*, page 3428–3436. MIT Press, 2014.

Yixin Wang and David M. Blei. The blessings of multiple causes. *Journal of the American Statistical Association*, 114(528):1574–1596, 2019.

Killian Wood, Gianluca Bianchin, and Emiliano Dall’Anese. Online projected gradient descent for stochastic optimization with decision-dependent distributions. *IEEE Control Systems Letters*, 6:1646–1651, 2022.
A The extrapolation problem

This section provides additional motivation for the setup from Section 3.1. A general goal in the context of perfor-
mativity is to evaluate a statistic \( \kappa(X,Y) \) over the hypothetical distribution \( D(f_\phi) \) surfacing from the deployment of 
\( f_\phi \) in order to decide whether we want to deploy \( f_\phi \) or not. This can be summarized with the following extrapolation
problem:

**Extrapolation Problem** \( \text{EP}(\kappa, f_\theta, f_\phi) \)

**Input:** Sample access to a training distribution \( D_{XY}(f_\theta) \) for a known predictor \( f_\theta \).

**Test-time predictor:** \( f_\phi \), Statistic to evaluate: \( \kappa \)

**Assumption:** Data is generated from the structural causal model described in (1)-(3)

**Output:**

\[
E_{(x,y) \sim D_{XY}(f_\phi)}[\kappa(x, y)]
\]

Being able to solve this extrapolation problem will enable both policy evaluation and policy optimization. The
statistic \( \kappa \) can be a measure of accuracy, but could also be a measure of social welfare, individual improvement, and
other goals for socially beneficial machine learning. In the main body of our work, we have focused on \( \kappa(X,Y) = Y \)
for simplicity of exposition. However, the criteria for identifiability would remain the same for any continuous function \( \kappa \), only the target in the supervised learning objective (8) would need to be adapted accordingly.

We say \( \text{EP}(\kappa, f_\theta, f_\phi) \) can be solved if the output can be identified from the input data. Relating this learning prob-
lem to classical results from causal inference [Athey, 2015] reveals the following sufficient condition for solvability that we target in this work.

**Lemma 8.** For any continuous statistic \( \kappa \) and predictive model \( f_\phi \), a sufficient condition for \( \text{EP}(\kappa, f_\theta, f_\phi) \) being
solvable is identifiability of

\[
h_\kappa^*(x, \hat{y}) := E[\kappa(X,Y) | X = x, \text{do}(\hat{Y} = \hat{y})]
\]

from observational data.

**Proof.** If we can identify \( h_\kappa^*(x, \hat{y}) \) we can evaluate the output for any \( x, \hat{y} \) and hence \( \text{EP}(\kappa, f_\theta, f_\phi) \) as

\[
E_{x \sim D_X}[h_\kappa^*(x, f_\phi(x))], \text{ where the distribution over covariates } D_X \text{ is obtained by marginalizing the input distribution } D_{XY}(f_\theta) \text{ over } X.
\]

B Identification with side-information

In this section, we sketch an identification result that builds on additional side information to identify \( h^* \) from external
noise sources, adding to the results of Section 3.3 in the main body. Similarly, taking into account this side information
when learning \( h^* \) requires more evolved identification strategies, beyond ERM.

**Noisy measurement of covariates.** Assume the prediction is made based on covariates \( X \) that are noisy estimates
of the underlying quality \( U \) that actually affects the outcome \( Y \). The setup is illustrated in Figure 6. Importantly, this
underlying quantity does not need to be observed and \( P(U) \) can be unknown. For example, a student’s college
performance depends on their underlying scholastic ability, but predictions of performance (and perhaps admissions
decisions) are made based on a noisy proxy like SAT score.

![Extended causal graph where X is a noisy measurement of U.](image)

In this setting, we can achieve identification if we have information about the noise distribution of the measurement
error \( P(U|X = x) \) as the next result shows. A typical example for such a noise distribution could be \( P(X|U) =\)
$\mathcal{N}(X, \sigma^2)$ where $\sigma^2$ indicates the scale of the noise that can be estimated from the test-retest data, prior modeling, or a physical model for the measurement device.

**Proposition 9.** Given the data generation process in Figure 6. If the conditional density $P(Y | X = x)$ and $P[Y, U | \hat{Y} = \hat{y}]$ are known and $U$ satisfies the overlap condition with respect to $X$, then the function $h^*$ is identifiable from observational data.

**Proof.** The result follows as

$$E[Y | \text{do}(\hat{Y} = \hat{y}), X] = E[Y | \hat{Y} = \hat{y}, X]$$

$$= \int E[Y | \hat{Y} = \hat{y}, U, X] P(U | X, \hat{Y}) dU$$

$$= \int E[Y | \hat{Y} = \hat{y}, U] P(U | X) dU$$

$$= \int y \cdot P[Y, U | \hat{Y} = \hat{y}] \frac{P(U | X)}{P(U | \hat{Y} = \hat{y})} dU dy$$

$$= \int y \cdot P[Y, U | \hat{Y} = \hat{y}] \frac{P(U | X)}{\int P(U | X) P(X | \hat{Y} = \hat{y}) dX} dU dy$$

(13)

The identification formula (13) illustrates that incorporating side-knowledge about $P(X | U)$ for identifying $h^*$ requires a strategy beyond classical ERM. The identification formula does not merely rely on $P(Y | \hat{Y}, X, U)$ which is unavailable due to the unobservability of $U$.

An interesting extension of this identification strategy by Eckles et al. [2020] relaxes the requirement of perfect knowledge about $P(U | X = x)$. They show that access to the heterogeneous noise bound $P(U | X = x) / P(U | X = x') \in [c_1(x, x'), c_2(x, x')]$ for some known functions $c_1, c_2$ is sufficient to obtain a consistent interval estimate for $E[Y | \text{do}(\hat{Y} = \hat{y}), X]$.

More broadly, we hope this section serves as an example of how state-of-the-art identification strategies from the causal inference literature have interesting applications in performative prediction.

### C Social influence

We have mentioned that the stable unit treatment value assumption (SUTVA) [Imbens and Rubin, 2015] underlying our causal analysis could be violated in certain performative prediction settings due to social influence and spillover effects. We want to use this section to discuss the simple interference patterns illustrated in Figure 7 that generalizes our causal graph from Figure 1. In particular, it allows for predictions of individual $i$ to impact the outcome of individual $j$:

$$X_i = \xi_X \quad \xi_X \sim \mathcal{D}_X$$

$$\hat{Y}_i = f_\theta(X_i)$$

$$Y_i = g' (X_i, \hat{Y}_i, (\hat{Y}_j)_{j \in [n]}) + \xi_Y \quad \xi_Y \sim \mathcal{D}_Y$$

**Figure 7:** Performative effects through prediction with interference (green arrows) for $n = 2$. 

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Such effects could arise due to information flow about predictions in the population through social media platforms [Chierichetti et al., 2009] or verbal sharing. This in turn leads to indirect exposure that can bring forward phenomena of social comparison such as envy or encouragement [Cikara et al., 2011]. In the presence of such interference effects the causal effect of intervening on the predictive model is no longer the same as the causal effect of intervening on an individual’s prediction, as stated in (12). On the left-hand side of (12) the predictions of all individuals are changed, whereas on the right-hand side only the prediction of individual \( i \) changed.

In the following, we want to highlight a setting where the data-driven strategy (8) (that builds a model based on data collected under a population intervention) is able to implicitly pick up on these interference effects present in the data, whereas this information is not available from data collected under unilateral interventions.

**Exposure modeling.** To formally reason about interference effects through predictions, let’s introduce \( G_i \) as a sufficient statistic that mediates the dependency among units, such that \( Y_i \perp \{\hat{Y}_j\}_{j \neq i} \mid G_i \) for all \( i \in [n] \). The statistic \( G_i \) could encode the exposure of the entire population, the average prediction across the population, relevant predictions of the closest neighbors in a social network, or the relative value of \( \hat{Y}_i \) compared to peers in a group. \( G_i \) is typically constructed based on domain knowledge and is often assumed to be low-dimensional, limiting the complexity of interference among units and making the problem more tractable. What is unique to the prediction setting studied in this work, compared to randomized treatment assignments, is that predictions (and hence \( G_i \)) are typically correlated with the covariates and thus inherit structures present in the population, such as network homophily.

**Homophily.** Homophily refers to the tendency for individuals to be similar to their neighbors which surfaces in our context as correlations between the features of neighboring units [Goldsmith-Pinkham and Imbens, 2013]. In the context of prediction, this further implies that a smooth prediction function \( f_0 \) will also exhibit correlations between predictions assigned to neighbors. We formalize it through the following property:

\[
|\mathbb{E}_{j \in N(i)} \hat{Y}_j - \hat{Y}_i| < \delta \text{ for every } i \in [n] \text{ and some small } \delta \geq 0. \tag{17}
\]

The interesting implication that we want to draw attention to is that if interference effects are mediated by the average prediction in the neighborhood, i.e., \( G_i = \mathbb{E}_{j \in N(i)} \hat{Y}_j \), then the outcome of individual \( i \) can (at least partially) be explained by the prediction \( \hat{Y}_i \) itself. This results in a machine learning model (8) that will implicitly pick up the interference effects from the training data in order to explain the total causal effect of \( f_0 \) on the outcome. This is helpful for prediction, despite a misspecified causal graph. We illustrate this advantageous property over microfoundation models with the following example:

**Example C.1** (linear-in-means model [Manski, 1993]). Let \( Y_i = g(X_i) + \alpha \hat{Y}_i + \beta G_i \) for some \( \alpha > \beta > 0 \), where \( G_i = \frac{1}{|N(i)|} \sum_{j \in N(i)} \hat{Y}_j \). This structural causal model proposed by Manski [1993] encodes a setting of positive interference where spillover effects are mediated by the average prediction in the neighborhood of an individual and represent a dampened version of the direct effect. Now, consider a strict homophily condition (17) with \( \delta = 0 \). Then, without explicitly measuring \( G_i \), fitting a model to explain \( Y \) as a function of \( X \) and \( \hat{Y} \) will result in \( h(x, \hat{y}) = g(x) + (\alpha + \beta) \hat{y} \), which identifies the causal quantity of interest, i.e., the effect of an intervention to the predictive model on the outcome \( Y_i \) of an individual. In contrast, an estimate based on unilateral interventions would result in \( h(x, \hat{y}) = g'(x) + \alpha \hat{y} \) which systematically underestimates the overall strength of performative effects and thus leads to a biased estimator.
D Proofs

Assumption 2 (positivity). Consider the causal graph in Figure 1. Positivity of \( \hat{Y} \) over \( \mathcal{Y} \) is satisfied if \( P[\hat{Y} \in S|X = x] > 0 \) for all \( x \in \mathcal{X} \) and all sets \( S \subseteq \mathcal{Y} \) with positive measure, i.e., \( P[S] > 0 \).

Lemma 10. If the training distribution satisfies positivity of \( \hat{Y} \) over \( \mathcal{Y}' \subseteq \mathcal{Y} \), then \( E[Y|X = x, \hat{Y} = \hat{y}] \) is identifiable from the training data for any \( \hat{y} \in \mathcal{Y}' \).

D.1 Proof of Proposition 1

From realizability it follows that \( D.1 \) Proof of Proposition 1 from the training data for any \( \hat{y} \). If the training distribution satisfies positivity of \( \hat{Y} \) over \( \mathcal{Y} \), then \( E[Y|X = x, \hat{Y} = \hat{y}] \) is identifiable from the training data for any \( \hat{y} \in \mathcal{Y}' \).

D.2 Proof of Lemma 2

Given that the risk minimization problem (8) is realizable and \( h^* \) is uniquely identifiable over \( \mathcal{H} \), the risk minimizer of the squared loss corresponds to \( E[Y|X = x, \hat{Y} = \hat{y}] \). Given the graph structure in Figure 1 there is no unobserved confounding and hence \( E[Y|X = x, \hat{Y} = \hat{y}] = E[Y|X = x, do(\hat{Y} = \hat{y})] \Rightarrow h_{SL}(x, \hat{y}) = h^*(x, \hat{y}) \).

D.3 Proof of Proposition 3

Our goal is to show that \( h^*(X, \hat{Y}) \) can not uniquely be identified from the training data for a deterministic \( f_\theta \). Therefore, we construct another function \( \tilde{h} \) that fits the training data equally well, but this function does not generalize to a new prediction function. Namely, it holds that \( \hat{Y} = f_\theta(X) \) for all \( x, \hat{y} \) in the observed data distribution. Thus, any alternative function \( \tilde{h} \) that does not depend on \( \hat{Y} \) but satisfies

\[
\tilde{h}(x, \hat{y}) = h^*(x, f_\theta(x))
\]

will be equally compatible with the observational data and achieves the same the prediction risk in (8). That is

\[
E_{Y = f_\theta(X)}E[Y|X, \hat{Y}] = h^*(X, \hat{Y}) = \tilde{h}(X, \hat{Y}).
\]

Hence, \( h^* \) can not be distinguished from \( \tilde{h} \) based on observational data. \( h^* \) is not uniquely identifiable.

Example D.1. Consider linear prediction \( f_\theta(x) = \theta^T x \) for some \( \theta \in \mathbb{R}^d \). Then, the two functions \( h(x, \hat{y}) = 1^T x + \hat{y} \) and \( \tilde{h}(x, \hat{y}) = (1 + \theta)^T x \) can not be distinguished from observational data \( (x, \hat{y}, y) \) collected under a deterministic \( f_\theta \) but lead to different estimates for \( E[Y|X, do(\hat{Y} = \hat{y})] \).

D.4 Proof of Proposition 4

Output overlap guarantees that \( P[\hat{Y} = f_\theta(x)|X = x] > 0 \) in the training distribution \( D(f_\theta) \) for any \( x \in \mathcal{X} \). Identification and extrapolation to models \( f_\theta \) that satisfy output overlap with \( f_\theta \) follows from positivity (Lemma 10) and the causal graph (Figure 1) which implies that there is no unobserved confounding and \( E[Y|X = x, \hat{Y} = \hat{y}] = E[Y|X = x, do(\hat{Y} = \hat{y})] \).
D.5 Proof of Proposition 5

The overlap condition $0 < P(T = t | X = x) < 1$ implies that $P(Y | Y = y, T = t)$ is identifiable from the training data for any $y, t$. So is $P(T = t | Y = \hat{y})$ identifiable since the decision $\hat{Y}$ is assumed to be randomized. Further, the causal graph (Figure 2) implies that both conditionals in the integration are identifiable from training data. Therefore, one can find $h^*(x, \hat{y})$ by estimating $\int E[Y | X = x, T = t] P(T = t) d$ from observation data, or equivalently solving $h^*$ from empirical risk minimization subject to the constraint that it is compatible with the observational data with $T$, i.e. $h(x, \hat{y}) = \int E[Y | X = x, T = t] P(T = t | \hat{Y} = \hat{y}) dt$ for all $h \in \mathcal{H}$.

D.6 Proof of Proposition 6

We first note that the overparameterization assumption implies that $g_1 \in \mathcal{G}$ and $f_0$ are linearly independent. (Proof by contradiction. If not, then there exists $\alpha_1 \neq \alpha'_1, \alpha_2 \neq \alpha'_2$ and functions $g_1, g'_1 \in \mathcal{G}$ such that $\alpha_1 g_1(x) + \alpha_2 f_0(x) = \alpha'_1 g'_1(x) + \alpha'_2 f_0(x)$ for all $x$; it implies that $\alpha_1 g_1(x) - \alpha'_1 g'_1(x) = (\alpha_2 - \alpha'_2) f_0(x)$ for all $x$. $\alpha_1 g_1(x) - \alpha'_1 g'_1(x) \in \mathcal{G}$ since the class $\mathcal{G}$ is closed under addition. This leads to a contradiction with the fact that $f_0(\cdot)$ is overparametrized with respect to $\mathcal{G}$, which requires there exist no function $g \in \mathcal{G}$ such that $g(x) = c f_0(x)$ for some $c > 0$.)

Next, since any $h \in \mathcal{H}$ is separable in $X$ and $\hat{Y}$, linear in $\hat{Y}$, and that $h(\cdot, \hat{y}) \in \mathcal{G}$ for any $\hat{y}$, we have that $h(x, \hat{y}) = g'_1(x) + \alpha' \hat{y}$ for some $g'_1 \in \mathcal{G}$ and some constant $\alpha' \in \mathbb{R}$. Therefore, finding $h^*$ function amounts to solve $g'_1, \alpha'$ from the observational data relationship $g_1(X) + \alpha \hat{Y} = g'_1(X) + \alpha' \hat{Y}$ subject to the constraint that $\hat{Y} = f_0(X)$. Plugging in the constraints gives $g_1(X) + \alpha f_0(X) = g'_1(X) + \alpha' f_0(X)$. This equations gives a unique solution that $g'_1 = g_1$ and $\alpha' = \alpha$ if we have observation from all values of $X$, hence the identifiability of $h^*$.

D.7 Proof of Proposition 7

The proof is inspired by Puli et al. [2020], Wang and Blei [2019]. Because $\hat{Y}$ is discrete and $E[Y | do(\hat{Y} = \hat{y}), X = x]$ is separable, we have that $\frac{\partial}{\partial x} E[Y | do(\hat{Y} = \hat{y}), X = x] = \frac{\partial}{\partial x} g_1(x) = \frac{\partial}{\partial x} E[Y | \hat{Y} = \hat{y}', X = x]$ for any pair of $\hat{Y}, X$ that are observable, i.e. $\hat{y}' = f_0(x)$. This implication is due to $g_2(\hat{y})$ being a piecewise constant function (its partial derivative is zero with respect to $x$). Therefore, the function $h^*$ is identifiable

$$h^*(x, \hat{y}) = E[Y | do(\hat{Y} = \hat{y}), X = x] = E[Y | \hat{Y} = \hat{y}, X = x'] + \int_{x'}^x \frac{\partial}{\partial x} E[Y | \hat{Y} = \hat{y}', X = x] dx,$$

for any $\hat{y}' = f_0(x)$. This equation establishes the identifiability of $h^*$. It also implies that the solution of the risk minimization problem (8) must coincide with $h^*$ if $\mathcal{H}$ satisfies the identifiability condition, i.e. $\mathcal{H}$ contains only separable functions $g(x, \hat{y})$ with differentiable $g_1, g_2$; these constraints implies the uniqueness of solution to the risk minimization problem, hence the solution must coincide with $h^*$. 

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E  Experiment details and additional experiments

E.1  Data and Licenses

Data in folktables was extracted from Census Bureau databases, which collected data in standardized surveys with consent.\textsuperscript{4} The Census Bureau takes care to ensure that through their pre-processing of survey results, personally identifiable information is not included in their data releases.

The income dataset with binary outcome variables used in the main text is the ACSIncome task defined in folktables, with data from the 2018 Census from the state of California. The income dataset with continuous outcome variables is a modified version of ACSIncome that performs the same pre-processing, except it leaves the income target variable as a real number, rather than thresholding to produce a binary outcome. Additional experiments below (referred to as Census travel time) were conducted on the ACSTravelTime task defined in folktables, with data from the 2018 Census from the state of California. The features and preprocessing for these datasets can be found in the code documentation of Ding et al. [2021]. The data contains 10 features:

\begin{itemize}
  \item RevolvingUtilizationOfUnsecuredLines
  \item age
  \item NumberOfTime30-59DaysPastDueNotWorse
  \item DebtRatio
  \item MonthlyIncome
  \item NumberOfOpenCreditLinesAndLoans
  \item NumberOfTimes90DaysLate
  \item NumberOfRealEstateLoansOrLines
  \item NumberOfTime60-89DaysPastDueNotWorse
  \item NumberOfDependents
\end{itemize}

the target variable is a binary indicator for whether an individual became delinquent on a loan (SeriousDlqin2yrs).

E.2  Experimental details

Machine learning models were trained using functionalities from sklearn [Pedregosa et al., 2011] with default parameters if not specified otherwise. We use the class LinearRegression from sklearn.linear_model for the linear models and the class MLRRegressor from sklearn.neural_network for the fully connected neural network models.

\textbf{Training data.} The Census income dataset composes of 195665 datapoints, if not specified otherwise the full dataset was used for training.

\textbf{Test dataset.} We train $f_\phi$ on randomized labels. More precisely, we randomly shuffle the labels among data points in the original dataset to obtain $f_\phi$. This process leads a model that is different from $f_\theta$ which serves to test the extrapolation performance of our meta-machine learning model.

\textbf{Overparameterization.} For the experiment in Figure 3(c) second degree polynomial features were included to achieve overparameterization (using sklearn.preprocessing), no further hyperparameters were set; all second-order terms were included in the overparameterization. For the experiments in Figure 4(a) we simulate the degree of overparameterization by working with neural networks and varying the number of neurons in the hidden layers using the parameter hidden_layer_sizes.

\textbf{Randomization.} For the randomized decision experiments we use Gaussian noise. If not specified otherwise it is drawn from $\mathcal{N}(0, 1)$.

\textbf{Discretization.} To obtain discrete predictions, we round the prediction outputs of the linear model $f_\theta$ so we achieve 4 distinct discrete values.

\textbf{Finite data.} The finite data experiments were conducted on datasets with a continuous target variable $Y$ (Census income), and performance was assessed using root mean squared error (RMSE). To simulate the effect of trainingset size we randomly subsample the original data to obtain a smaller training set size for our meta machine learning model.

We simulate different amounts of distribution shift by choosing $\phi^* = \rho \theta + (1 - \rho) \phi$ where $\theta$ are the parameters of the model trained on the original data, and $\phi$ are the parameters of a model trained on randomized labels.

\textbf{Infrastructure.} Experiments were run on 4 CPU cores for a total of 200 hours.

\textbf{Baseline.} The baseline in the plots is the RMSE of a model trained on samples from the test set and evaluated on a validation set that is held out from the test set, but from the same distribution. Thus it represents a setting with no distribution shift.

\textsuperscript{4} documentation: https://www.census.gov/programs-surveys/acs/microdata/documentation.html.
E.3 Additional experiments: Robustness to model misspecifications

We investigate the robustness of supervised learning to misspecification of \( g \). We performed these experiments for the dataset used in the main body, as well as the Census travel time and Kaggle credit score datasets [Kaggle, 2011]. We found qualitatively similar results on all datasets (Figures 8, 9 and 10).

We focus on discrete classification where \( f_\theta \) and \( f_\phi \) are binary predictors. Our theoretical result in Proposition 7 depend on knowing the correct model class \( \mathcal{H} \) to optimize \( h_{SL} \) on. Here we test the resiliency to model misspecification. Therefore, we define \( \mathcal{H} \) to be the class of linear functions but construct a non-linear data generation process as follows:

\[
g(x, \hat{y}) = \hat{y} \text{ with probability } p, \text{ and } g(x, \hat{y}) = g_1(x) \text{ otherwise,} (25)
\]

where \( g_1(x) \) is a (possibly non-linear) function that maps \( x \) to its original label \( y \) in the original dataset, and \( p \in [0, 1] \) is a hyperparameter for performativity strength that we vary. Like in the finite data experiments, we also vary the distance between predictions from \( f_\theta \) and \( f_\phi \), i.e., distribution shift magnitude.

Figure 8(a) shows that despite misspecification, the meta model benefits of having access to \( \hat{y} \) and the accuracy of \( h_{SL} \) remains close to in-distribution accuracy as long as distribution shifts are not too large (specifically, until \( f_\theta \) and \( f_\phi \) become uncorrelated). Figure 8(b) highlights that the benefit of adding \( \hat{y} \) as a feature persists across almost all values for \( p \). However, \( h_{SL} \) is more prone to errors from model misspecification when performativity is very weak. This is intuitive, since \( \hat{Y} \) is correlated with the outcome \( Y \), and a misspecified \( h_{SL} \) might be best off attributing this correlation to the causal link; in such extreme cases, Figure 8(b) suggests that accuracy is slightly improved by dropping \( \hat{Y} \) as a feature. However, this improvement for \( p \to 0 \) is small, and Figure 8(c) shows that if the accuracy of \( f_\theta \) is reduced (by fitting to noisy labels), this effect disappears. But across all three experiments, including \( \hat{Y} \) as a feature outperforms not including it, even when little performativity is present.

Additional experimental details: The model misspecification experiments were conducted on datasets with a binary target variable \( Y \) (Census travel time and Kaggle credit), and performance was assessed using classification accuracy. The predictors \( f_\theta \) and \( f_\phi \) are both always gradient boosted decision trees implemented in sklearn [Pedregosa et al., 2011] with the default hyperparameters. As in the finite data experiments, \( f_\theta \) is usually fit to minimize error on the original dataset, while \( f_\phi \) is fit on a noisy version of the original dataset, where the level of noise is tunable. Denoting the noise with \( \gamma \), we generate noisy labels \( y' \) via

\[
y' = y \text{ with probability } 1 - \gamma, \text{ and } 1 - y \text{ with probability } \gamma.
\]

In other words, \( \gamma \) determines the probability that the label is flipped, with 0.5 making \( y \) and \( y' \) uncorrelated.

Hyperparameters for the experiments were set to the following unless otherwise specified:

- Training set size: 200,000 datapoints
- Magnitude of distribution shift \( \gamma \): 0.45
- Strength of performativity \( \alpha \): 0.5

For experiments where we denote that the training predictor \( f_\theta \) is given noisy labels, this means that \( f_\theta \) is fit to a random permutation of the original dataset labels.
Figure 8: **Performance degrades gracefully under model misspecification (Census income prediction dataset).** (a) Accuracy on the test distribution (higher is better) is plotted against distribution shift magnitude; supervised learning remains accurate until the train and test predictors, \( f_0 \) and \( f_0 \), are uncorrelated (shift magnitude of 0.5). (b) Accuracy is plotted against performativity strength; despite model misspecification, accuracy is higher when \( \hat{Y} \) is included as a feature, across most performativity strengths. (c) When the training set predictor \( f_0 \) is fit to random labels and is less accurate than \( f_0 \), including \( \hat{Y} \) as a feature universally improves accuracy.

Figure 9: **Performance degrades gracefully under model misspecification (Census travel time dataset).** (a) Accuracy on the test distribution (higher is better) is plotted against distribution shift magnitude; supervised learning remains accurate until the train and test predictors, \( f_0 \) and \( f_0 \), are uncorrelated (shift magnitude of 0.5). (b) Accuracy is plotted against performativity strength; despite model misspecification, accuracy is higher when \( \hat{Y} \) is included as a feature, across most performativity strengths. (c) When the training set predictor \( f_0 \) is fit to random labels and is less accurate than \( f_0 \), including \( \hat{Y} \) as a feature universally improves accuracy.

Figure 10: **Performance degrades gracefully under model misspecification (Kaggle credit score dataset).** (a) Accuracy on the test distribution (higher is better) is plotted against distribution shift magnitude; supervised learning remains accurate until the train and test predictors, \( f_0 \) and \( f_0 \), are uncorrelated (shift magnitude of 0.5). (b) Accuracy is plotted against performativity strength; despite model misspecification, accuracy is higher when \( \hat{Y} \) is included as a feature, across most performativity strengths. (c) When the training set predictor \( f_0 \) is fit to random labels and is less accurate than \( f_0 \), including \( \hat{Y} \) as a feature universally improves accuracy.
F Societal impact

The fact that predictions are performative and have an impact on the population they predict is a natural phenomenon observed in various applications. In this work, we discuss one dimension of performativity and investigate how to develop an improved causal understanding of these performative effects from data. Our intent is to develop this understanding from observational data in order to foresee potential negative consequences of a future model deployment before actually deploying it across an entire population. Typical machine learning approaches would not take these consequences into account when training a predictive model. At most, they would observe performative effects in a monitoring step after deploying a model and then decide post-hoc whether the model satisfies a given constraint. At this point, harm might already have been caused, even if unintentional. Naturally, though, any improvement in understanding can also be used with bad intent. Instead of being treated as a potential for harm to mitigate against, performative effects could also be instrumentalized by profit-maximizing firms or self-interested agencies in order to achieve their goals [Hardt et al., 2022]. These goals might not always be aligned with social welfare and if the respective firm has high performative power, i.e. ability to influence performative effects, these actions hold the potential for social harm.