Improved Force Identification Techniques using Curvature Sensors: application to Damage Detection

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Abstract. This study deals with the localization of damage in a beam, using methods which are insensitive to environmental variability. For this purpose, it considers the problem of adapting force identification techniques to the requirements of damage localization. Damage, when considered as a set of forces, creates a singularity in the structure, which can appear as a discontinuity in the spatial derivative of its transverse displacements. Damage or force localization methods attempt to locate and quantify such discontinuities. However, the damage effects are very small, when compared to those arising from forces, and can disappear due to the influence of low-level measurement noise. These considerations have led us to improve the conventional force identification techniques, in order to reduce their sensitivity to noise. The RIFF method ("Résolution Inversée Fenêtrée Filtrée"), which uses finite difference methods to compute highly noise sensitive spatial derivatives, is used to localize the forces. One major improvement proposed here is the use of curvature sensors (in the form of PVDF films), instead of displacements sensors, thereby avoiding the determination of two spatial derivatives and significantly increasing the noise robustness. Moreover, the derivatives are computed using the finite element method, rather than finite differences, which also improves the localization accuracy. When using these new sensors and calculus, the force identification techniques need to be rewritten, to enable damage to be localized, and new phenomena are detected. The aim of this study is to understand how force identification techniques can be used to locate damage, and to determine the improvements that could be made, in particular when using piezoelectric sensors as curvature sensors. Various numerical simulations of academic case studies illustrate the limitations and advantages of this approach.

1. Introduction
Loading, environmental factors, and accidental events are all possible sources of damage to a mechanical structure. When the defect is not visible, many non-destructive testing tools exist, such as ultrasound monitoring, eddy current testing, or radiography. However, these techniques can only be used locally, and therefore require knowledge of the approximate position of the damage. Regular inspection of the entire structure (generally in civil or aerospace engineering) is thus required, in order to ensure the reliability of such monitoring techniques. This problem has led researchers to focus on more holistic approaches, including those based on the measurement of vibrations and, if possible, those which are insensitive to environmental variability [1]. The aim of the present study is to develop a method for the localization and quantification of defaults that can affect the behavior of a vibrating structure. Defaults are modeled in two different ways, according to which of two phenomena they correspond. The first of these is the stiffness defect, which may for example be caused by the appearance of a crack, a localized phenomenon, or by corrosion, which is a widespread
phenomenon. We thus consider mass defects, and excitation sources in the form of a load (a force or a moment), which is either localized or distributed. Appearance of a default implies the presence of equivalent loads, related to sharp variations in curvature. In the present paper, we propose to identify this load using a large network of sensors, in order to localize any structural anomalies. Despite the presence of a displacement, which is the conventionally acquired parameter, this approach makes use of curvature measurements. We show that the measurement of curvature reduces the effects of noise amplification resulting from numerical derivation. Filtering techniques developed for force identification (such as RIFF [2,3,4], or SVD [5,6,7]) are then used to improve the quality of the results.

2. Principles of damage detection
The appearance of defaults in a structure can generate bending moments or shear force discontinuities, which can be observed as internal forces. Our aim is to acquire information related to these forces, and to interpret their meaning through the use of many sensors.

2.1. Default modeling
When the equation governing flexural vibrations in a beam is considered, it can be shown that a singularity such as stiffness reduction generates internal forces:

\[(EI)\ast(1+j\eta)\frac{\partial^4w}{\partial x^4}(x)-(\rho S)\ast\omega^2w(x) = F(x)\]  

(1)

where \(w(x)\) is the transverse displacement, and \((\rho S)\ast\) and \((EI)\ast\) are respectively the linear mass and the stiffness of the damaged structure, which are a function of the abscissa \(x\). The latter can be written as:

\[(EI)\ast = EI\left(1-\sum_{i=1}^{n_e}K_i\delta(x-x_i)\right)\]  

(2)

\[(\rho S)\ast = \rho S\left(1-\sum_{i=1}^{n_e}M_i\delta(x-x_i)\right)\]  

(3)

where \(\delta(x)\) is the Dirac distribution, and \(K_i\) and \(M_i\) are coefficients which translate the influence of the stiffness or the mass defect, respectively. Equation (1) can be rearranged in the following form:

\[EI(1+j\eta)\frac{\partial^4w}{\partial x^4}(x)-\rho S\omega^2w(x) = F(x) + F(x)\ast\]  

(4)

where \(F(x)\ast\) is the term related to the presence of anomalies in the beam’s integrity, which will be referred to as ‘residual forces’ in the following. When the mechanical characteristics of the healthy structure and its loading are known, it is possible to determine the location of any singularity.

2.2. Using curvature
The equation of motion (1) shows that the displacement and its fourth derivative are necessary. The use of measured displacements, requiring four spatial derivations to access the result, amplifies the impact of noise due to the harmonic nature of the solution. This can be avoided through the use of curvatures. In beam theory, curvature can be estimated by means of piezoelectric sensors (PVDF). Indeed, it can be shown that piezoelectric sensors bonded to the structure can (under certain conditions) generate a signal, which is proportional to its curvature [8]. The required data can then be retrieved from this signal, using only a double spatial derivative and a double integration.
The second derivative of the curvature is obtained by means of a conventional second order finite difference method. In order to access displacement information, two different methods were tested: the first is based on the inversion of a finite difference method, and the second uses a finite element model of the structure.

2.3. Numerical simulations
We consider a simple steel cantilever beam (dimensions 1x0.1x0.02 m), on which 25 sensors have been bonded, and which is excited at its free boundary by a transverse harmonic force \( F \) (cf. figure 1). Two defects are considered: a stiffness defect located at \( x=0.4 \), with a 5% impact on the EI coefficient, and a distributed mass defect, between \( x=0.75 \) and 0.85, of magnitude 5kg/m, with the additional mass representing 3.2% of the beam’s total mass.

![Figure 1. Drawing of the simulated experiment. Stiffness default at: \( x=0.4m \); distributed mass defect between: \( x=0.75-0.85m \).](image)

The sensor data are based on the results of the finite element calculation (200 Timoshenko beam elements) and the curvature is computed from the displacement, by means of a second order finite difference method.

If it is assumed that there is no noise on the data acquired from the 25 sensors, the residual force calculation leads to the following results, for the methods based on the finite difference (F.D.) and finite element (F.E.) methods:

![Figure 2. Comparison of damage localization results computed using 25 sensors, without noise. (F.D. : Finite difference method | F.E. : Finite element method)](image)

Both methods produce efficient results, since each singularity appears at the expected location. The strongest response amplitudes are logically obtained at specific eigenfrequencies \( f_1=16.4Hz \).
Nevertheless, at this stage of the study, the nature and/or the significance of the singularities can not be clearly determined. Nevertheless, as shown in the following, distributed mass defects such as the one detected here, can be difficult if not impossible to localize in the case of noisy measurements.

2.4. Influence of noise

We now introduce noise into the 25 measured curvature values. The following mathematical formulation is used to represent the noise:

\[ c_{\text{noisy}}(x_i) = (1 + \Delta_{\text{mult}}) c_{\text{exact}}(x_i) + \Delta_{\text{add}} \]  \hspace{1cm} (5)

where \(\Delta_i\) are random variables. If one now considers the curvature data to be degraded by a 5% additive and multiplicative noise, the resulting spectra are found to be considerably less explicit.

![Figure 3. Comparison of beam damage localization results with noise](image)

Figure 3 clearly illustrates that a high level of noise has a non-negligible impact on the quality of the residual force reconstruction. In the present example, it is difficult to distinguish between real defects and noise artefacts. In the above figures, for the same level of noise, the F.E. method appears to produce better results than the F.D. one, because the stiffness defect at \(x=0.4\) is clearly identifiable, contrary to the result found using the F.D. method, for the first mode. Nevertheless, such observations will vary between test cases, depending on parameters such as boundaries, frequencies, etc. Under such conditions, it appears to be feasible, although difficult, to localize defects in a vibrating structure, on the basis of curvature measurements. However, two conditions must be satisfied for the defects to be correctly located. The first is that the sensor density must be sufficient to localize the damage, and the second is that, since the reconstruction quality is strongly dependent on the measurement quality, errors in the latter must be low. Although, when combined with the F.E. method to compute the displacements, curvature measurements can provide good results, noise remains a problem.

In order to assess the method's sensitivity to the effects of additive noise, only the eigenfrequencies will be considered in the following. As the nature of the damage can not be clearly identified, the study is limited to a conventional value of 5% in stiffness reduction. New technologies, such as optical fiber sensors, could allow a noise level of 1% (multiplicative and additive), and a high-density sensor network (100 measurement points), to be considered in these simulations. For convenience, the same structure has been maintained for this simulation. The results should thus be essentially the same for a larger structure, the key parameter being the ratio between the useful wavelength and the size of the sensor network.
Figure 4. Comparison of damage localization in the presence of measurement noise (1% multiplicative and additive) at the first 5 eigenfrequencies, for a network of 100 sensors.

Figure 4 illustrates the results of damage localization, for the first 5 eigenfrequencies, using a network of 100 sensors ($f_1=16.4\text{Hz}$, $f_2=104\text{Hz}$, $f_3=291\text{Hz}$, $f_4=567\text{Hz}$ and $f_5=928\text{Hz}$). Overall, the use of a large number of sensors does not appear to provide a clear improvement. Nevertheless, for the case of modes for which the damage is localized close to a curvature maximum, the localization approach appears to be feasible using the Finite Element Method, as shown in figure 5. This figure compares the damage localization at the third eigenmode, using the DF and FE methods. The FE method appears to be more robust under noisy conditions, and the damage is clearly localized at $x=0.4$. Without the use of a regularization step (see section 3), the DF method appears to be too sensitive to noise, whatever the frequency considered.

Figure 5. Comparison of damage localization in the presence of measurement noise (1% multiplicative and additive) at the third mode (291 Hz), for a network of 100 sensors (continues line :DF, dotted line: EF)

Since the implementation of a 100 sensor network would not be straightforward, the results to be expected with a real measurement remain uncertain. As it is impossible to totally eliminate noise, the following two methods are proposed in order to minimize its impact on damage detection.
3. Regularization methods
The aim of this section is to appraise various methods, which could solve the previously identified difficulties. For this, two numerical methods are compared: the first is referred to as R.I.F.F. R.I.F.F. (French for “Windowed Filtered Inverse Regularization” - “Résolution Inverse Filtrée Fenêtrée”), developed by C. Pezerat [2, 3, 4], and the second is referred to as S.V.D. (Singular Value Decomposition), based on the well known suppression of singular values of an inverse problem matrix [5,6,7].

3.1. R.I.F.F. method
The R.I.F.F. method is based on the removal of information contained at high wave numbers (corresponding to noise). It implements two operations: firstly, a local Hanning window $\phi$ is applied to the residual force $F$ at each measurement point, $x_i$, then, for each point $x_i$ a low-pass filter (cutoff wavenumber $k_c$) is applied to the weighted distribution. Physically, the windowing is designed to select only part of the set of measured data to which the low-pass filtering is applied, in order to "stifle" the frequency domain associated with noise. The windowing is applied initially, to avoid the Gibbs phenomenon. These operations can be written in the form:

$$F_{\text{filter}}(x_i) = \frac{1}{\lambda_c} \int_{x_i-\beta}^{x_i+\beta} F(x)H\left(\frac{x-x_i}{\lambda_c}\right)dx$$

where $\beta$ is the half-width of the window, $\lambda_c=2\pi/k_c$ is the wavelength of the cut-off filter, and $H(x)$ is the function corresponding to the total response of the filter with local windowing:

$$H(x) = \left(1 + \cos\left(\frac{\pi x}{f}\right)\right)\sin c(2x)$$

where $f=\beta/\lambda_c$ is referred to as the “shape factor” of the filter. The R.I.F.F. method can then be tuned by adjusting only two parameters: $f$ and $k_c$.

3.2. Singular value decomposition
The Singular Value Decomposition (S.V.D.) is a well known technique, and is available in the form of a commercial software package. S.V.D. is well known for rectangular matrix inversion applications. In fact, although the residual force calculation does not require matrix inversion, it has the characteristics of an inverse problem, because the mechanical setup can be expressed as a linear relationship, i.e. $A.X=F$, where $A$ is a rectangular matrix, $X$ is the curvature vector, and $F$ is the residual force vector. The first step in this method involves computing the pseudo inverse $A^T$ of $A$, without removing the singular values. Then, a new pseudo-inverse ($A_f$) of $A^T$ is computed, after removing the smallest singular values. Finally, the residual force can be computed using this new matrix $A_f$, which is different to $A$. These operations are summarized by the following diagram:

![Figure 6. Schematic diagram of the S.V.D. method](image)

$$A \cdot X = F \quad \xrightarrow{\text{Pseudo-inversion}} \quad A^T \quad \xrightarrow{\text{S.V.D. truncation}} \quad (A^T)^\# \quad \xrightarrow{\text{Pseudo-inversion}} \quad A_f \cdot X = F'$$
The only parameter used in this method is the threshold, $r$, below which singular values are considered to be null.

### 3.3. Results

The following figures (7 and 8) compare these 2 regularizations methods, using FE or DF for the data estimations. A 100 sensor network and a 1% multiplicative and additive noise are assumed. As previously observed, the method can not be efficient at all frequencies, especially when damage is present near to a zone of zero curvature. Consequently, only the third mode is considered in the following. It clearly illustrates the improvements which can be achieved.

**Figure 7.** Comparison between the F.D. (Finite Difference) and F.E. (Finite Element) methods, using the RIFF regularization method for the third mode, as a function of cutoff wavenumber.

**Figure 8.** Comparison between the F.D. (Finite Difference) and F.E. (Finite Element) methods, using the SVD regularization method, as a function of the number of truncated singular values.
Clearly, the FE method provides a very considerable improvement to this damage localization approach, especially when a regularization step is added. Contrary to conventional force identification methods, the truncation number (for SVD), or the cut-off wave number, do not appear to be key parameters. Damage localization is still possible, independently of the values of these parameters. Nevertheless, the damage cannot be quantified when such parameters are not clearly chosen. Methods such as the ‘L-curve’ were tested, to adjust the regularization step, but did not produce satisfying results in terms of quantification of the damage severity.

4. Conclusion

The aim of this paper was to demonstrate the feasibility of a defect detection method, based on curvature measurement, for the case of a one-dimensional structure. A first localization and quantification method is proposed and theoretically studied, under the condition that the displacement field is efficiently reconstructed from the curvature field. The advantage of using curvature is a significant reduction in the influence of noise, through a reduction in the number of numerical derivations required in the computations. However, this method remains sensitive to measurement noise. Two regularization methods are then proposed: a windowed filtering (RIFF) method, and a numerical method based on singular value decomposition of the linear system matrix, which links the curvature field to the loads (SVD). Both methods are shown to be efficient, even in the presence of noise in the measured signals. The difficulty with these methods lies mainly in the appropriate adjustment of parameters, needed to quantify the damage. In practice, the curvature should be measured as accurately as possible. Overall, damage localization based on the FE method appears to provide better results than the FD method, as usually used for the RIFF method. However, these simulations also reveal the main limitations of the proposed methods, i.e. that, since they have been developed for force identification, they require a high number of sensors and accurate curvature measurements, before they can be applied to damage detection. Although a large number of sensors is clearly required, future studies will be devoted to the analysis of the limiting number of measurement points, and its consequence on damage localization.

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