Investigation of the rotational motion stability of the AIST small spacecraft prototype according to the measurements of the Earth's magnetic field

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Abstract. The paper deals with the rotational motion stabilization of the AIST small spacecraft prototype under the influence of a magnetic moment. According to the measurements of the Earth's magnetic field by onboard measuring instruments, the angular velocity of small spacecraft rotation has significantly decreased over several years of active existence in a near-earth orbit. At the same time, attempts to actively control the angular velocity by magnetic executors were unsuccessful. In this investigation, approximate solution regions for the magnetic moment are obtained from the condition of the rotational motion stability around the center of mass of the AIST small spacecraft prototype. These regions can be further analyzed for compliance with the operation of the onboard equipment of a small spacecraft.

1. Introduction
The flight model and prototype of AIST small spacecraft are two identical in design and layout small spacecraft weighing 39 kg (Figure 1) [1, 2].

The flight model and prototype of AIST small spacecraft were launched on April 19, 2013 and December 28, 2013, respectively, and operated in a near-circular orbit with an altitude of 575 km. To control the angular velocity of AIST small spacecraft rotation around the center of mass, an information-measuring control system was provided, consisting of magnetometers and magnetic executors [3, 4]. High values of the angular velocity of small spacecraft rotation hindered the qualitative exchange of telemetric data, significantly reducing its reliability (Figure 2) [5].

At the same time, on two small spacecraft, five attempts were made to turn on the magnetic executors to reduce the angular velocity of small spacecraft rotation. One of two attempts on the flight model of AIST small spacecraft was successful, allowing an order of magnitude to reduce the angular velocity modulus [5]. The second attempt did not significantly affect the angular velocity modulus. On the AIST small spacecraft prototype all three attempts were unsuccessful. As an operation result of the magnetic executors in all three cases, the angular velocity of rotation increased (Figure 3) [6].
Figure 1. General view and main elements of the prototype of AIST small spacecraft: 1 is the main body; 2 are photoconverters of the solar battery; 3 is the antenna receiver; 4 is the antenna transmitter; 5 are scientific equipment; 6 is the user navigation equipment antenna.

Figure 2. Reliability of telemetric data (curve 2) in accordance with the dynamics of changes in the angular velocity modulus of the prototype of AIST small spacecraft rotation (curve 1).
Figure 3. Dynamics of changes in the angular velocity modulus of the prototype of AIST small spacecraft rotation (dotted lines indicate attempts to reduce the angular velocity by magnetic executors).

As can be seen from Figure 3, after the termination of attempts to reduce the angular velocity by magnetic executors, its values decreased due to external disturbances to an extremely low level. The paper is devoted to the analysis and modeling of this phenomenon.

2. Mathematical model

Let us assume that the measurement data indicate the stability of the AIST small spacecraft prototype rotation relative to the components of the angular velocity vector. Since the initial perturbations of the angular velocity significantly exceed the values to which it tends over time. Let us assume as a first approximation that magnetic perturbations significantly prevail over gravitational and aerodynamic perturbations. The magnetic moment arises from the interaction of conducting loops, which create their own magnetic field, and the magnetic field of the Earth.

Let us choose as a generalized coordinate frame the angles of a small spacecraft rotation in the main bound coordinate system: $\varphi_1, \varphi_2, \varphi_3$. We will assume that the generalized forces represent the magnetic moment of interaction: $Q_{\text{m}} = M_{\text{m}}$. Then the differential equations of the perturbed motion of a small spacecraft (dynamic Euler equations) will have the form:

$$
\begin{align*}
\ddot{\varphi}_1 &= \frac{M_{\text{m}}}{I_x} - \frac{I_z - I_y}{I_x} \dot{\varphi}_2 \dot{\varphi}_3, \\
\ddot{\varphi}_2 &= \frac{M_{\text{m}}}{I_y} - \frac{I_z - I_x}{I_y} \dot{\varphi}_1 \dot{\varphi}_3, \\
\ddot{\varphi}_3 &= \frac{M_{\text{m}}}{I_z} - \frac{I_y - I_x}{I_z} \dot{\varphi}_1 \dot{\varphi}_2.
\end{align*}
$$

(1)

where $I_x, I_y, I_z$ are tensor diagonal components in principal body-fixed axes.
We will consider the unperturbed motion: \( \dot{\phi}_1 = \omega_1; \dot{\phi}_2 = \omega_2; \dot{\phi}_3 = \omega_3 \), where \( \omega_1, \omega_2, \omega_3 \) are some constants whose absolute values are much less than one.

For the Lyapunov function, we choose the positive-definite quadratic form:

\[
V = \frac{1}{2} (\dot{\phi}_1^2 + \dot{\phi}_2^2 + \dot{\phi}_3^2).
\] (2)

Then, by the Lyapunov stability theorem, its total time derivative must be negative-definite:

\[
\dot{V} = \dot{\phi}_1 \dot{\phi}_1 + \dot{\phi}_2 \dot{\phi}_2 + \dot{\phi}_3 \dot{\phi}_3 \leq 0.
\] (3)

Taking into account (1), instead of (3):

\[
\dot{V} = \dot{\phi}_1 \left( \frac{M_{xx} - I_x - I_z}{I_x} \dot{\phi}_1 \right) + \dot{\phi}_2 \left( \frac{M_{yy} - I_y - I_z}{I_y} \dot{\phi}_2 \right) + \dot{\phi}_3 \left( \frac{M_{zz} - I_z - I_x}{I_z} \dot{\phi}_3 \right).
\] (4)

In the perturbed motion:

\[
\dot{\phi}_1 = \omega_1 + x_1; \dot{\phi}_2 = \omega_2 + x_2; \dot{\phi}_3 = \omega_3 + x_3.
\] (5)

Substituting (5) into (4) we have:

\[
\dot{V} = x_1 \left( \frac{M_{xx} - K\omega_1 \omega_1}{I_x} \right) + x_2 \left( \frac{M_{yy} - K\omega_2 \omega_2}{I_y} \right) + x_3 \left( \frac{M_{zz} - K\omega_3 \omega_3}{I_z} \right) - x_1 x_2 K\omega_1 - x_1 x_3 K\omega_2 - x_2 x_3 K\omega_3.
\] (6)

where

\[
K = \frac{I_z^2 (I_x - I_z) + I_x^2 (I_y - I_z) + I_y^2 (I_z - I_x)}{I_x I_y I_z}; \quad C = \omega_1 \frac{M_{xx}}{I_x} + \omega_2 \frac{M_{yy}}{I_y} + \omega_3 \frac{M_{zz}}{I_z} - \omega_1 \omega_2 \omega_3.
\]

Function (6) is not sign-definite and does not correspond to the inequation (3). Therefore, we will analyze its behavior according to the measurement data.

3. Experimental data analysis

To analyze function (6) according to the experimental data, we will use the measurements of the components of the Earth's magnetic field induction vector, carried out on board the AIST small spacecraft prototype [7]. Let us choose the measurement intervals corresponding to the increase and decrease in the angular velocity of rotation (Figure 3), as well as an attempt to decrease the angular velocity by magnetic executors. As the increase interval in the angular velocity, we will choose the measurements from February 7, 2014 (interval 1). In Figure 3, these measurements immediately follow the first attempt at decreasing the angular velocity with the magnetic executors (left dotted line). Let us consider the data from an attempt to reduce the angular velocity on February 15, 2014 (middle dotted line, interval 2). As an interval of decreasing the angular velocity, let us consider the measurements from September 5, 2014 (after 250 days from the moment of launch, interval 3).

The angular velocity components were estimated using the Boer formula [8]:

\[
\dot{\omega}_1 = \frac{\vec{B}_e \times \left( \vec{B}_e - \frac{d\vec{B}_e}{dt} \right)}{B_e^2},
\]
where $\tilde{\omega}_i$ is the angular velocity vector of the small spacecraft rotation around the center of mass at the time $t_i$; $\vec{B}_e$ is the Earth's magnetic field induction vector; $\frac{d\vec{B}_e}{dt}$ is the local derivative of the induction vector of the Earth's magnetic field in the body axis system of the small spacecraft, which is mainly due to the movement of the spacecraft around the center of mass; $\dot{\vec{B}}_e$ is the absolute rate of change in the Earth's magnetic field induction vector, which is mainly due to the movement of the mass center of the small spacecraft.

The dependences of the Lyapunov function (2) and its derivative (3) for intervals 1, 2 and 3 are shown in Figures 4 a, b and c, respectively.
Indeed, the derivative of the Lyapunov function (6) is not a sign-definite function on any of the considered intervals. This followed from its structure (6) and is confirmed by the measurement data (Figure 4). Let us carry out a comparative analysis of the data presented in Figure 4. The mean values
of $\dot{V}$ in line sections 1 and 2 are positive: $\dot{V}_1 \approx 0.069$, $\dot{V}_2 \approx 0.019$. Due to the control input, it was possible to locally reduce the average positive value of $\dot{V}$. However, this reducing had a short-term effect. At the same time, despite the control input, the mean value of the derivative remained positive. This contributed to the growth of function (1), and, consequently, of the angular velocity of AIST small spacecraft rotation. This pattern is typical for the majority of measurements performed approximately in the interval of $[0, 150]$ days from the launch moment of AIST small spacecraft prototype.

Then the pattern changes. For the majority of measurements performed approximately in the interval of $[150, 650]$ days from the start, a negative mean value of the derivative (6) is typical. For example, the mean value of the derivative for measurements from September 5, 2014 is $\dot{V}_s \approx -0.158$. This helps to reduce the angular velocity of AIST small spacecraft rotation.

Let us reduce requirements (3) to the following:

$$\dot{V} < 0. \quad (7)$$

Thus, based on the measurement data, we can come to the assumption that under conditions (7) the following statement is quite probable:

$$V \xrightarrow{p \rightarrow \infty} \frac{1}{2} \omega^2_{1} + \frac{1}{2} \omega^2_{2} + \frac{1}{2} \omega^2_{3}. \quad (8)$$

Statement (8) corresponds to the investigations of other authors on the stabilization of the motion of a small spacecraft in the Earth's magnetic field (for example, [9]).

4. Conclusions

According to the measurement data, it can be concluded that derivative (6), being an alternating function, more often had positive values than negative values in the interval of $[0, 150]$ days from the start. Therefore, in Figure 4a, an upward trend is observed in the structure of the Lyapunov function (1), which indicates an increase in the angular velocity of rotation of AIST small spacecraft prototype.

Further, the situation changes on the interval of $[150, 650]$ days from the start. Remaining alternating, derivative (6) more often takes negative values than positive ones. This is reflected in the structure of the Lyapunov function (1), which is already characterized by a downward trend (Figure 4c).

For the AIST small spacecraft prototype, according to the measurement data, it can be argued that the probability of the Lyapunov function (1) tends to unperturbed motion (statement (8)).

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