Entangling Atomic Spins with a Strong Rydberg-Dressed Interaction

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Controlling quantum entanglement between parts of a many-body system is the key to unlocking the power of quantum information processing for applications such as quantum computation, high-precision sensing, and simulation of many-body physics. Spin degrees of freedom of ultracold neutral atoms in their ground electronic state provide a natural platform given their long coherence times and our ability to control them with magneto-optical fields, but creating strong coherent coupling between spins has been challenging. We demonstrate a Rydberg-dressed ground-state blockade that provides a strong tunable interaction energy (∼1 MHz in units of Planck’s constant) between spins of individually trapped cesium atoms. With this interaction we directly produce Bell-state entanglement between two atoms with a fidelity ≥ 81(2)%, excluding atom loss events, and ≥ 60(3)% when loss is included.

Pristine quantum control of many-body correlations is fundamental to realizing the power of quantum information processors (QIP). Steady progress has continued in various platforms ranging from solid state spintronics [1] and superconductors [2] to nanophotonics [4] and ultracold trapped atoms, both ionic [3] and neutral [8–10]. Cold neutral atoms have some particular attractions as the ability to create entanglement between atoms would allow for greatly increased precision of interferometers for applications in magnetometers [11–12], and clocks [14–17], force sensors [18, 19], and other precision measurements [20–23]. In addition cold atoms provide a natural platform for quantum-simulation of condensed matter physics [24] and scalable digital quantum computers [25–27]. Controlled entanglement of neutral atoms, however, has been challenging, particularly if one seeks tunable interactions that are strong, coherent, and long-range. Neutral atoms in their electronic ground state have short-range van der Waals interactions, which have been successfully employed as a mechanism for generating many-body correlations, but this requires contact interaction via ultracold collisions, typically in a trapped condensate [28–32], or in an optical lattice where trapped atoms tunnel and collide on-site [33–35]. In such configurations, individual addressing of atoms is not possible, or requires an atomic-gas microscope [36, 37]. Moreover, the interaction strengths are typically small, < 100 Hz [35, 38, 40], making this mechanism less than ideal for some applications.

In order to induce stronger, long-range (∼ μm) interactions, one must introduce excited states. At very high principal quantum numbers, a strong electric dipole-dipole interaction (EDDI) occurs between Rydberg atoms. This leads to the Rydberg blockade mechanism [41], which has been successfully employed for implementing controlled entangling interactions between atoms [3, 10, 42] and quantum logic gates [43]. In the standard protocol, short pulses excite the population of one atom to the Rydberg state and a second atom is blocked to optical excitation because of the EDDI [27]. This approach, however, requires maintaining phase coherence between optically excited “bright” and “dark” states; fluctuations in the optical phase seen by thermal atoms limit the entangling fidelity of the interaction [10, 44]. Control can be more robust when we require that coherence is maintained only between ground-state levels, which are associated with the phases of mw (microwave frequencies), and thus several orders of magnitude less sensitive to the thermal motion of atoms. In addition, the application of fast pulses is not fully compatible with standard Hamiltonian paradigms of many-body simulations. For this reason, many researchers have explored the use of Rydberg-dressed interactions, whereby Rydberg character is admixed into the bare ground state to form an optically-dressed ground state [15, 40]. This mechanism has been studied for a variety of applications including quantum simulation of condensed matter [46–58], spin-squeezing for metrology [45, 59], and quantum computing [44, 60].

While the promise of Rydberg-dressed interactions is great, experimental demonstration has been elusive. We present here a clear measurement of this interaction between two Rydberg-dressed atoms, and show that it can be orders of magnitude larger than typically considered. Rydberg-dressing arises from the modification of the light-shift (LS) (i.e., the optical AC Stark shift) of two atoms due to the EDDI. We characterize this by the interaction strength $J \equiv \Delta E_{LS}^{(2)} - 2\Delta E_{LS}^{(1)}$, where $\Delta E_{LS}^{(2)}$ is the LS for two atoms in the presence of EDDI and $\Delta E_{LS}^{(1)}$ is the LS for a lone atom optically excited near a Rydberg level $|r\rangle$. By tuning the Rydberg excitation laser near the Rydberg resonance $|0\rangle \rightarrow |r\rangle$, two-photon coupling to $|r\rangle$ from $|0,0\rangle$ is blocked when $U_{dd} \gg h\sqrt{\Omega_L^2 + \Delta_L^2}$, where $\Omega_L$ is the laser Rabi frequency, $\Delta_L$ is the laser detuning, and $U_{dd}$ is the EDDI
energy. The resulting difference in the two-atom spectrum implies that $\Delta E_{LS}^{(2)} \neq 2\Delta E_{LS}^{(1)}$, which plateaus at $J = \frac{\hbar}{2} \left[ \Delta_L + \text{sign}(\Delta_L) \left( \sqrt{\Delta_L^2 + 2\Omega_L^2} - 2\sqrt{\Delta_L^2 + \Omega_L^2} \right) \right]$ when the blockade is perfect [42]. This can lead to a very strong interaction, $J \sim \Delta E_{LS}^{(1)}$, when $\Omega_L \sim \Delta_L$. By designing the dressing energy to be conditional on sublevels in the two-atom ground-electronic manifold, we can create a strong entangling mechanism between ground-state atoms. The key parameter that determines the interaction strength $J$ depends on the EDDI, and thus the distance between the two Rydberg-dressed atoms is a crucial experimental parameter. Ideally, we would like the atoms to be located far apart for individual addressing, and conversely, in close proximity to maximize the EDDI. Our particular implementation of an AOM (acousto-optic modulator) allows us to create two optical tweezers that trap each atom from the same laser by simultaneously driving the AOM at two frequencies (see supplementary material). By independently sweeping these frequencies, we are able to move the two traps dynamically and achieve this goal.

We employ $^{133}$Cs atoms, and encode qubits in the “clock states” $|0\rangle \equiv |6S_{1/2}, F = 4, m_F = 0\rangle$, $|1\rangle \equiv |6S_{1/2}, F = 3, m_F = 0\rangle$ of two-atoms individually trapped in optical tweezers at 937 nm and separated by a few microns [42]. By choosing $|\Delta_L|$ small compared to the 9.2 GHz hyperfine splitting, in the presence of EDDI only the two-qubit state, $|0, 0\rangle$, receives a non-negligible, two-atom dressing energy $J$. By applying 9.2 GHz radiation on the clock-state resonance (e.g., via two-photon Raman lasers) with Raman-Rabi frequency $\Omega_{mw}$, two atoms initially in the state $|1\rangle$ are blockaded from undergoing double spin-flips, $|1\rangle \rightarrow |0\rangle$ for both atoms when $J/h \gg \Omega_{mw}$. This is the dressed-ground-state analog of the Rydberg blockade.

We directly measure $J$ as a function of the interatomic distance. Our experiment is illustrated in Fig. 1. The two trapped $^{133}$Cs atoms are initially prepared in state $|1, 1\rangle$ and we dynamically translate them to be in close proximity at a targeted distance $R$. We then extinguish the tweezers for a short time to eliminate light shifts from the dipole-trap laser, and immediately apply the Raman and Rydberg-dressing lasers simultaneously. Afterward, the optical tweezers restore to recapture the falling atoms and translate them back to the original positions for independent state detection. We use a 319-nm laser for dressing the $^{133}$Cs atom, which couples atoms directly from the ground state to the Rydberg level, $6S_{1/2} \rightarrow 64F_{3/2}$, in a single photon transition [42]. This avoids unwanted population in an intermediate, short-lived excited state that arises in the typical two-photon Rydberg excitation method, which causes additional ground-state decoherence [60]. To drive spin flips, we apply the Raman laser fields to the two Rydberg-dressed atoms when they are at a desired separation. By sweeping the Raman (microwave) frequency and measuring the resulting spin flips, we obtain a two-qubit energy spectrum of
we use a best-fit of the blockade shift curve to calculate. Our experiment shows no evidence of such strengths and details of level mixing that are difficult to avoid the Rydberg blockade. The existence of such resonances could potentially lead to additional unwanted resonances that would ruin the Rydberg blockade. The existence of such resonances, however, depends on the oscillator strengths and details of level mixing that are difficult to calculate. Our experiment shows no evidence of such resonances. We use a best-fit of the blockade shift curve of $|r, r\rangle$ from our detailed model to calculate $J$ as shown in Fig. 2 (see supplementary materials and Fig. S3). With an ideal Rydberg blockade, $U_{dd} \to \infty$, we find the expected plateau in $J$ at short interatomic distances.

We leverage the strength of this Rydberg-dressed ground-state blockade, occurring coherently and solely in the dressed ground manifold, to create entanglement between atomic spin qubits. By driving a resonant Raman-pulse $|1\rangle \to |0\rangle$ simultaneously on the two atoms, we cause Rabi oscillations between $|1, 1\rangle$ and the entangled Bell state, $|\Psi^+\rangle = (|0, 1\rangle + |1, 0\rangle)/\sqrt{2}$, as the experimental data show in Fig. 3. A signature of this entanglement is the characteristic increase in the Raman-Rabi frequency to $\sqrt{2} \Omega_{\text{mw}}$. We can generate $|\Phi^+\rangle = (|0, 0\rangle + |1, 1\rangle)/\sqrt{2}$, from $|\Psi^+\rangle$ by subsequently applying a global $\pi/2$ rotation on the qubits. Alternatively, while the Rydberg dressing laser is still on, a two-microwave-photon $\pi/2$-pulse at a shifted frequency, resonant with the “anti-blockade” transition from $|1, 1\rangle \to |0, 0\rangle$, also generates $|\Phi^+\rangle$. This method has a lower fidelity because the two-photon microwave Rabi rate must be small in order to avoid excitation to the off-resonant $|\Psi^+\rangle$ state, and decoherence is more likely on this long time scale.

With large values of $J$, we can employ the dressed Rydberg-blockade to routinely generate two-qubit entanglement. For a given $J$, the Raman pulse duration $T$ is chosen so that $\sqrt{2} \Omega_{\text{mw}} T = \pi$ with $\Omega_{\text{mw}} \ll J/h$. Following this procedure, we check whether both atoms are still present in the traps; if so, we count it as a “valid” operation. We determine the lower bound of the entanglement fidelity by measuring the off-diagonal coherence between the two-qubit logical basis states, $(x', y')|\rho| x, y\rangle$. For this, we apply a global $\pi/2$ pulse with phase $\phi$ to the entangled state. As a function of $\phi$, we measure the expected value of the parity $Q(\phi) = \langle \sigma_z \otimes \sigma_z \rangle \phi = \langle P_{1,1} + P_{0,0} – (P_{1,0} + P_{0,1}) \rangle(\phi)$, where $P_{xy}(\phi)$ is the population in the logical state $|x, y\rangle$ after application of the $\pi/2$ pulse. For qubits prepared in the $|\Psi^+\rangle$ state, $Q$ is independent of $\phi$ and always a positive number: $Q = 1$ when the entanglement is perfect. For qubits prepared in the $|\Phi^+\rangle$ state, $Q(\phi)$ is an oscillating function of $\phi$. In this case, perfect entanglement corresponds to perfect oscillation visibility. The entanglement fidelity is $\geq 2 (|x', y'|\rho|x, y\rangle)\geq 2%$ for generating $|\Psi^+\rangle$ and $|\Phi^+\rangle$ when both atoms are recaptured in the trap.

Given that we measure the two-atom survival probability after the procedure to be $74 \pm 2\%$, the success rate of deterministically generating an entangled qubit pair is therefore $\geq 60 \pm 3\%$. With our current experimental data rate ($\approx 10$ s$^{-1}$), on average we generate 6 pairs of entangled qubits per second. The current factors limiting the entanglement fidelity given a valid procedure are: the optical pumping efficiency, decay of the Raman Rabi oscillation of the Rydberg-dressed states (Fig. 3), and the strength of $J$. Optical pumping efficiency determines how well we can prepare the atoms into the initial qubit states (computation space), which can be improved with a more careful pumping scheme. Decay of the Rabi oscillation impacts the fidelity of the $\pi$ pulse; we can

![Fig. 3. Generating entanglement directly. Top panel: Rabi oscillations of a single Rydberg-dressed Cs qubit. Lower three panels: two-atom data with Rydberg-dressed ground-state blockade ($J/h \approx 750$ kHz). There is a $\sqrt{2}$ enhancement of the microwave Rabi rate $\Omega_{\text{mw}}$ and excitation to state $|0, 0\rangle$ is strongly suppressed due to the transition blockade. Simulated density matrices are shown for various pulse times. The maximum Bell state $|\Psi^+\rangle$ entanglement is generated at around 2 μs.](image-url)
FIG. 4. Entanglement verification. A global \( \pi/2 \) pulse is applied to the undressed system after the entangled state is prepared. The data show that both Bell states generated from our experiment have a fidelity \( \geq 81(2)\% \). Here, \( P \) represents the two-atom density matrix. The insets show the simulated density-matrix populations using conditions similar to the experiment. The parity measurement, \( P_{\|1} + P_{\perp0} - (P_{\perp1} + P_{\|0}) \), allows direct determination of the amplitudes of the off-diagonal elements for both entangled states.

improve this fidelity by increasing \( \Omega_{\text{mw}} \) but only for sufficiently large \( J \). The major cause of the decay is likely the shorter than expected lifetime of the Rydberg-state. We measure this to be up to \( 30 \) \( \mu s \) while the expected natural lifetime due solely to vacuum and blackbody radiation is \( \approx 150 \) \( \mu s \). We attribute this discrepancy to an experimental imperfection that perturbs the Rydberg level, causing it to spontaneously decay to the ground state before the atom is lost from the trap.

Our method, nonetheless, is quite robust despite this technical imperfection and we expect more than an order of magnitude improvement in future experiments. In this experiment we use strong Rydberg dressing \( (\Delta \leq \Omega_L) \) to generate strong interactions. For example, with the experimental settings of \( \Omega_L/2\pi = 4.3 \) MHz and \( \Delta/2\pi = 1.3 \) MHz (see Fig. 2), we find the dressed state to have a strong admixture of Rydberg character, \( 0.6|r\rangle + 0.8|0\rangle \), or 64% probability in \( |0\rangle \). In principle this dressed state can be returned to the bare ground state by adiabatically ramping the dressing laser’s intensity and detuning in an optimal way [44]. Finally, minimizing the environmental effects will likely improve the Rydberg state lifetime, improve the deterministic entanglement fidelity, and open the door to more full control of complex quantum systems. This work was supported by the Laboratory Directed Research and Development program at Sandia National Laboratories.

SUPPLEMENTARY MATERIALS

Experimental platform

Our experimental platform [42] is built on laser cooled \( ^{133}\text{Cs} \) atoms, singly trapped in optical tweezers (dipole traps). We encode qubits in the clock states of the cesium \( 6S_{1/2} \) ground-state manifold with a hyperfine splitting \( \omega_{HF}/2\pi = 9.2 \) GHz. We choose as our logical basis \( |0\rangle = |S_{1/2}, F = 4, m_F = 0 \rangle \) and \( |1\rangle = |S_{1/2}, F = 3, m_F = 0 \rangle \) as shown in Fig. 5. The two atoms are loaded from a MOT (magneto-optical-trap) into the two optical dipole traps with a separation of \( 6.6 \) \( \mu m \). These dipole traps are generated by sending a 937-nm laser beam to an AOM (acousto-optic modulator), which is simultaneously modulated at two different frequencies. This creates two beams with a well-defined angular separation that depends linearly on the AOM drive frequency difference. These beams pass through a focusing lens forming two tightly-focused spots with waists of \( r_{1/2} = 1.26(1) \) \( \mu m \) and a well-defined spatial separation that depends on the angular separation of the two beams. Following polarization gradient cooling, the atom temperature is reduced to \( \approx 20 \) \( \mu K \). A bias field at 4.8 \( G \) then turns on, and we optically pump the atoms into state \( |S_{1/2}, F = 4, m_F = 0 \rangle \) (logical basis state \( |0\rangle \)) using a \( \pi \)-polarized laser at 895 nm tuned to \( |S_{1/2}, F = 4 \rangle \rightarrow |P_{3/2}, F' = 4 \rangle \) and a repump laser at 852 nm tuned to \( |S_{1/2}, F = 3 \rangle \rightarrow |P_{3/2}, F' = 4 \rangle \). The state preparation efficiency is \( \approx 95\% \), limited by the stray, fictitious magnetic field produced by vector light shifts from the dipole-trap laser. We apply a two-photon Raman laser field to perform a global \( \pi \) rotation to bring the atoms from \( |0, 0 \rangle \rightarrow |1, 1 \rangle \). The stimulated Raman transition uses the carrier and one sideband from a laser tuned 50-GHz red of the Cs D2 line \( (6S_{1/2} \rightarrow 6P_{3/2}) \) and modulated via a fiber-based EOM (electro-optic modulator).

To Rydberg-dress the atoms with a strong EDDI (electric dipole-dipole interaction), we dynamically translate the two \( \text{Cs} \) atoms into close proximity to a targeted distance \( R \) by ramping the AOM modulation frequencies as shown in Fig. 4. From the initial separation of \( 6.6 \) \( \mu m \), we can continuously vary \( R \) down to a minimum value of \( 1.5 \) \( \mu m \), at which point the two traps begin to merge causing atom loss. During the Rydberg dressing period, we turn off the traps to eliminate the light shift due to the trap laser. Afterward, we restore the traps to recapture the falling atoms. We use a Rydberg dressing laser at 319 nm which drives direct, single-photon transitions from \( 6S_{1/2} \) to \( nP_{3/2} \). This Rydberg excitation laser can be tuned to cover the principal quantum numbers ranging from \( n = 30 \) to ionization. Because EDDI causes a red shift of the two-atom Rydberg state \( |r, r \rangle \), we blue detune the Rydberg excitation laser. With a strong bias
Measurement of $J$ and generating Bell states

In order to obtain a strong differential light shift, and therefore a strong dressed-interaction energy $J$ between two atoms, we require $\Delta_L \leq \Omega_L$ (the strong dressing regime). The probability amplitude of the ground-state $|0\rangle$ in this dressed state is $(\Delta_L/2\pi + \Omega_L^2 + 1)^{1/2}/\sqrt{2}$, and has a value between $1/\sqrt{2}$ for maximal dressing and 1 for no dressing at all. In our experiment we choose two example conditions: $(\Omega_L/2\pi = 4.4$ MHz, $\Delta_L/2\pi = 4$ MHz) and $(\Omega_L/2\pi = 4.3$ MHz, $\Delta_L/2\pi = 1.3$ MHz). This leads to the dressed states of 0.41$|r\rangle$ and 0.0$|r\rangle$ or 0.8$|0\rangle$ and 0.64 probability in $|0\rangle$ respectively. In this experiment we do not adiabatically transfer the dressed state back to the bare ground state, which lowers the atom detectability. When there is a probability that the atom is excited to the Rydberg state, there is a possibility that the atom will not be re-captured in the trap during the time window ($\approx 10$ $\mu$s) of efficient recapture. In this detection we only count the data with atoms that remain trapped. We track statistics concerning the lost atoms.

To measure $J$, we measure the microwave resonance frequency for the transition $|1, 1\rangle \rightarrow (|1, 0\rangle + |0, 1\rangle)/\sqrt{2}$ and for $|1, 1\rangle \rightarrow |0, 0\rangle$. $J/h$ is given by twice the frequency difference of these two resonances. The microwave Rabi rate $\Omega_{mw}$ and the pulse time of the stimulated Raman laser are properly chosen to ensure the observation of both the two-photon and the single-photon microwave resonance. The relative populations, $P_{11}, P_{10}, P_{01},$ and $P_{00}$ of the four, two-qubit computational basis states can be directly measured with the coincident bright and dark signals determined by the photon counting of the two APDs.

A straight-forward method for generating the Bell state $|\Psi_+\rangle = (|0, 1\rangle + |1, 0\rangle)/\sqrt{2}$ is illustrated in Fig. 3 of the main article. This shows typical experimental data representing Rabi flopping in the presence of Rydberg dressing with the parameters $n = 64$, $\Omega_L/2\pi = 4.3$ MHz, $\Delta_L/2\pi = 1.1$ MHz, $R = 2.9$ $\mu$m, and $J/h \approx 750$ kHz. Each data point is the average of several hundred measurements but with various Raman pulse durations. With a strong $J$, microwave excitation to the state $|0, 0\rangle$ is blocked. The microwave Rabi oscillation can only occur between $|1, 1\rangle$ and $(|0, 1\rangle + |1, 0\rangle)/\sqrt{2}$ with a very small probability exciting to $|0, 0\rangle$. We also find a factor of $\sqrt{2}$ enhancement of the microwave Rabi rate compared to the single-atom Rabi rate as secondary evidence of entanglement. A detailed computer simulation matching the data shows how the system is evolving into a $|\Psi_+\rangle$ state using a full master equation description of the density matrix of the two $^{133}$Cs atoms. The two-qubit density matrix presented here is the subset of the full density matrix in the simulation.

In Fig. 3, the optimal $\pi$ time for generating $|\Psi_+\rangle$ is
Interatomic distance, $R$ ($\mu m$)

Relative Energies/$\hbar$ (GHz)

- $0.5$
- $0.52$
- $0.54$
- $0.56$
- $0.58$
- $0.6$
- $0.62$
- $0.64$
- $0.66$
- $0.68$
- $0.7$

$J/\hbar$ (kHz)

- $-1000$
- $-800$
- $-600$
- $-400$
- $-200$
- $0$
- $200$
- $400$
- $600$
- $800$
- $1000$

$R$ ($\mu m$)

(a) Calculated two-atom Rydberg sublevels of $64P_{3/2}$ with $|E| = 6.4$ V/m and $|B| = 4.8$ G. The red dashed curve is the fitting result of our selected two-atom Rydberg state $|m_J = 3/2, m_J = 3/2\rangle$. (b) The calculated $J$ as a function of $R$ using the red dashed curve from the left panel. The parameters of the dressing laser are $\Omega_L/2\pi = 4.3$ MHz and $\Delta_L/2\pi = 1.3$ MHz for the blue curve and $\Delta_L/2\pi = -1.3$ MHz for the red curve.

$\approx 2$ $\mu s$. To produce a Bell state $|\Phi^+_+\rangle$, we simply apply a global $\pi/2$ pulse to $|\Psi^+_+\rangle$. By fine tuning the experimental parameters, our best entanglement fidelity is $\geq 81(2)\%$ excluding the atom loss events, and $\geq 60(3)\%$ when loss is included. If the atom recapture process were to completely filter all cases when the atom was excited to the Rydberg state, we would expect to have a lower two-atom survival probability compared to the 74% recapture probability we present here. With perfect filtering and excluding the atom loss events, the entanglement fidelity would be much higher and eventually limited by the state preparation efficiency. However, it would still be near 60% with loss included. For technical reasons, we observe that a fraction of the atoms that are excited to the Rydberg state decay back to the ground state within the recapture time window. This is only consistent with a shorter than expected Rydberg lifetime. The resulting increase in the two-atom survival probability reduces the value of the fidelity measurement that is conditional on the atom loss events. We anticipate a substantial improvement of the entanglement fidelity by achieving close to the natural Rydberg-state lifetime and also by using an adiabatic ramp of the intensity and detuning of the 319 nm Rydberg laser in order to return the dressed state back to the bare ground state.

**Calculation of Rydberg-state interactions and the ground-state interaction $J$**

Calculating $J$ as a function of the interatomic distance $R$ requires knowledge of how the two-atom Rydberg energy levels shift as a function of $R$. The mixing of atomic orbitals due to interactions leads to molecular-like energy levels. We use the same numerical codes we developed in our previous work $[42, 60]$ for the calculation. Figure 6 shows the calculated two-atom molecular energy levels for $64P_{3/2}$ for $R$ from 1 $\mu m$ to 10 $\mu m$ with $|E| = 6.4$ V/m and $|B| = 4.8$ G, parameters typical of our experiment. There is a total of 14,400 energy levels included in the calculation, which covers the principal quantum numbers from $n = 60$ to $n = 66$ and the orbitals from $l = 0$ to $l = 4$. In the calculation, the Rydberg-Rydberg interactions include EDDI, electric dipole-quadrupole, and electric quadrupole-quadrupole interactions. As one can see, there is a “spaghetti” structure at very short $R$ due to the strong mixing between all combinations of Rydberg energy levels. When excited with the 319-nm laser, the probability of exciting one of these resonances in the spaghetti depends on the oscillator strength of the different molecular levels. In the figure, we denote this oscillator strength by the darkness of the lines. We see that most molecular resonances are only weakly coupled to the Rydberg excitation laser. With limited computational recourses, it is numerically intractable to calculate the entirety of multipole interactions in the infinite atomic basis set. Hence, the calculated spaghetti fea-
ture can never be precise. We therefore use a best-fit curve for the doubly-excited Rydberg state of our interest \( (|m_1| = 3/2, m_\perp = 3/2) \) to represent \(|r, r\rangle\) as a function of \( J \) shown by the red dashed curve in Fig. 2. 

Using this curve, we calculate \( J \). In Fig. 2, we plot the two calculated \( J(R) \) curves with parameters \( \Omega_L/2\pi = 4.3 \) MHz and \( \Delta_L/2\pi = \pm 1.3 \) MHz using the result from Fig. 2. Both detunings show nearly constant \( J \) values at short \( R \). With this single-curve approximation for \(|r, r\rangle\), the red-detuned case \( (\Delta_L = -1.3 \) MHz) has a discontinuity near \( R \approx 4 \mu m \) due to the resonance with the excitation laser. In practice, there are additional influences from the other Rydberg magnetic sublevels if we use red-detuned \( \Delta_L \). To avoid this complexity, we use a blue-detuned laser for Rydberg dressing in our experiment.

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