PRODUCTION OF $Z$ BOSON PAIRS
AT PHOTON LINEAR COLLIDERS

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The $ZZ$ pair production rate in high energy $\gamma\gamma$ collisions is evaluated with photons from laser backscattering. We find that searching for the Standard Model Higgs boson with a mass up to, or slightly larger than, 400 GeV via the $ZZ$ final state is possible via photon fusion with backscattered laser photons at a linear $e^+e^-$ collider with energies in the range $600 \text{ GeV} < \sqrt{s_{e^+e^-}} < 1000 \text{ GeV}$.
I. INTRODUCTION

In many cases, interesting physics processes can be studied with high precision at linear $e^+e^-$ colliders where the background is usually low and the signal is much cleaner than that of hadron colliders. The Next Linear Collider (NLC) is a projected linear $e^+e^-$ collider with a center of mass (CM) energy ($\sqrt{s_{e^+e^-}}$) of 500 GeV and a yearly integrated luminosity of about 10 $fb^{-1}$. In $e^+e^-$ collisions, the Higgs boson of the Standard Model (SM) with a mass, $M_H$, up to 350 GeV \cite{1,2} will be observable at the NLC. Improvements in the technology of laser backscattering have made it likely that the NLC could be run as a high energy photon collider \cite{3}-\cite{7}. Photon fusion can become a promising source to produce and study the Higgs bosons \cite{8}-\cite{12} of the SM and its extensions when the high energy $\gamma\gamma$ luminosity at linear $e^+e^-$ colliders is greatly enhanced by laser backscattering. However, it was recently found \cite{13,14}, that the transverse $Z_TZ_T$ pair produced from photon fusion can become a serious irreducible background and make the Higgs search via the $ZZ$ decay mode in $\gamma\gamma$ collisions impossible at the NLC and higher energy linear $e^+e^-$ colliders if $M_H$ is larger than about 350 GeV.

In this letter, the complete SM calculation of $\gamma\gamma \rightarrow ZZ$ is evaluated independently. A non-linear gauge is used to greatly reduce the number of diagrams and simplify the Feynman rules. The total cross section and invariant mass distribution of $ZZ$ pair at photon colliders is presented and the search for the SM Higgs boson is examined. Our cross sections of $\gamma\gamma \rightarrow ZZ$ for monochromatic photon photon collisions agree with that of Ref. \cite{13} where a different non-linear gauge was used, and Ref. \cite{14} where a linear gauge was adopted and unpolarized initial $e^+e^-$ and laser beams were considered. We have also checked the total cross section and invariant mass distribution for $ZZ$ pair production with the polarizations of initial $e^+e^-$ and laser beams as well as CM energies of $e^+e^-$ considered in Ref. \cite{13}, and have found good agreement. In addition, we have considered other CM energies of $e^+e^-$ and other polarizations of the electron positron and laser beams. Our conclusion as to a viability of a Higgs search with a realistic energy spectrum for backscattered photons is slightly more
optimistic than that of Ref. [13] or [14].

II. NON-LINEAR GAUGE FIXING AND LOOP INTEGRATION

In the SM, the lowest order $\gamma\gamma ZZ$ coupling comes from the 1-loop diagrams of the leptons ($l$), the quarks ($q$), and the physical $W$ boson ($W^\pm$) in the unitary gauge. The Higgs boson has a significant effect on the $W$ loop and the top quark loop contributions. The Nambu-Goldstone boson ($G^\pm$) and the Fadeev-Popov ghosts ($\theta^\pm, \bar{\theta}^\pm$) play important roles in a general gauge and make $W$ loop calculation unnecessarily complicated. It has been demonstrated for processes with photons that a carefully chosen non-linear gauge [15]-[19] can remove the mixed vertices of photon-$W$-$G$ ($A^\mu W^\mu G^-_\mu$) and Higgs-photon-$W$-$G$ ($H A^\mu W^\mu G^-_\mu$), reduce the number of loop diagrams and simplify the Feynman rules.

In this letter, a non-linear $R_\xi$ gauge is introduced to remove not only the mixed vertices $\gamma WG$ and $\gamma H WG$ but also the vertices $ZWG$ and $ZHWG$. The gauge fixing terms are chosen to be

$$L_{GF} = -\frac{1}{\xi_W} f^+ f^- - \frac{1}{2\xi_Z} (f^Z)^2 - \frac{1}{2\xi_A} (f^A)^2$$

where $f^-$ is the Hermitian conjugate of $f^+$, $M_W$ and $M_Z$ are masses of the $W$ and $Z$ bosons, $g = e/\sin\theta_W$ and $\theta_W$ is the Weinberg angle. The ghost couplings that depend on the gauge fixing term (1); and all modified Feynman rules are given in an appendix. The gauge parameters are all taken to be unity, $\xi_W = \xi_Z = \xi_A = 1$, which corresponds to a non-linear 't Hooft-Feynman gauge. In this new gauge, there are 3 pure classes of diagrams for the $W$ boson ($W$-loop), the Nambu-Goldstone boson ($G$-loop) and the Fadeev-Popov ghosts ($\theta$-loop) with the same mass $M_W = M_G = M_\theta$. Further, the ghost loops
contribute -2 times the Nambu-Goldstone boson loops except for those loops with a $ZZ\theta\theta$ coupling. In addition to the box (4-point) and the triangle (3-point) diagrams which appear in the fermion loops, there are also bubble (2-point) diagrams in the $W$, $G$ and $\theta$ loops: 24 box, 48 triangle and 12 bubble diagrams without the Higgs boson; 8 triangle and 4 bubble diagrams with the Higgs boson; which add up to 96 diagrams in this gauge. In the linear $R_\xi$ gauge \[14\], there are 188 diagrams: 108 box, 48 triangle and 6 bubble diagrams without the Higgs boson; 20 triangle and 6 bubble diagrams with the Higgs boson. The fermion loops are obtained from an earlier calculation of $gg \to ZZ$ \[20,21\] with a modification of couplings. All loop integrations have been calculated with the computer program LOOP \[22,23\], which evaluates one loop integrals analytically and generates numerical data. The resulting numerical program is checked by replacing the polarization vector for one of the photons with its four-momentum. Gauge invariance requires that this yield a vanishing result which checks all integrals and algebra involved.

III. MONOCROMATIC $\gamma\gamma$ COLLISIONS

The amplitude of $\gamma\gamma \to ZZ$ can be written as

$$M_{\lambda_1\lambda_2\lambda_3\lambda_4} = \epsilon_1^\mu\epsilon_2^\nu\epsilon_3^\rho\epsilon_4^\sigma T_{\mu\nu\rho\sigma}(p_1, p_2, p_3, p_4)$$

where $\lambda_{1,2}$ and $\lambda_{3,4}$ are the helicities of the photons and the $Z$’s, the $p$’s are the momenta and the $\epsilon$’s are the polarization vectors.

The cross sections of $\gamma\gamma \to ZZ$ in different helicity states of $ZZ$ are presented in Figure 1 as a function of $\sqrt{s_{\gamma\gamma}}$ for both polarizations, ++ and +−, of the photons. The parameters used are $\alpha = 1/128$, $\sin^2 \theta_W = 0.230$, $M_Z = 91.17$ GeV and $M_W = M_Z \cos \theta_W$. The Higgs mass ($M_H$) is taken to be 300, 400, 500, and 800 GeV. If not mentioned, the top quark mass ($m_t$) is considered to be 140 GeV. Also shown is the ++ $LL$ cross section without the Higgs boson, which is the same as taking $M_H = \infty$. As can be easily seen, the $Z_TZ_T$ cross section dominates and almost approaches a constant as $\sqrt{s_{\gamma\gamma}} > 1$ TeV, except for $M_H < 300$ GeV.
where the $++LL$ cross section is larger. Not shown are the individual contributions from the $W$ loop and fermion loops. The $W$ loop is usually at least about 10 times larger. Only in the $++LL$ state and for large $m_t$ and high energy, can the top quark loop be comparable to the $W$ loop; and only in the $++LT$ state at low energy, can the fermion loop dominate. For $M_H = \sqrt{s_{\gamma\gamma}} > 450$ GeV, the $++TT$ cross section is almost an order of magnitude larger than that of $++LL$, which makes the Higgs search in the $ZZ$ mode via photon fusion impossible for $M_H > 450$ GeV, unless the transverse and longitudinal polarizations of the $Z$ boson can be distinguished. All our numerical data agree with those in Ref. [13], except the cross section for $++LL$ cross section with $M_H = \infty$.

The $m_t$ dependence and the interference between the $W$ loop and fermion loop for the $++LL$ helicity states are shown in Figure 2 for $m_t = 120, 160$ and 200 GeV. The $W$ loop cross section is not sensitive to the top quark mass; it depends on $m_t$ only in the Higgs width and therefore is evaluated with $m_t = 160$ GeV only. The total cross section at $\sqrt{s_{\gamma\gamma}} = M_H = 300 - 800$ GeV are also presented in Table I for $m_t = 120, 140, 160, 180$ and 200 GeV where the precise value of $m_t$ is used everywhere. Several interesting aspects can be learned from Fig. 2 and Table I: (1) The $W$ loop and the fermion loop interfere destructively. (2) For $M_H$ below 300 GeV, the total $++LL$ cross section grows with $m_t$, while for $M_H$ above 700 GeV it decreases as $m_t$ becomes larger. (3) For $M_H = 400, 500$ and 600 GeV there is a minimum which appears at about $m_t = 130, 160$ and 180 GeV, respectively. The $++LL$ cross section always depends on the $m_t$ which appears in the Yukawa coupling of the top quark to the Higgs boson. Not shown is the $TT$ cross section which become insensitive to $m_t$ for $\sqrt{s_{\gamma\gamma}} > 500$ GeV.

The Higgs boson contributes only to the states with the same photon helicities and the same $Z$ helicities. In order to improve the ratio of signal to background while saving most of the $LL$ signal, we consider a cut on the CM scattering angle $|\cos(\theta^*)| = |z| < 0.8$ which reduces about 30% of the $++TT$, and more than 45% of the $+-TT$ background while saves about 80% of the $++LL$ signal. For the total cross section, the efficiency of this
angular cut and one with $|z| < \cos(30^\circ)$ are presented in Table II for $\sqrt{s_{\gamma\gamma}} = M_H = 300, 400$ and 500 GeV.

IV. BACKSCATTERED LASER $\gamma\gamma$ COLLISIONS

It has been shown that $\gamma\gamma ZZ$ can hardly be observed with the Weizsäcker-Williams photons [13], because the $\gamma\gamma$ luminosity falls rapidly as the $\gamma\gamma$ invariant mass increases. Fortunately, Compton laser backscattering can produce high energy photons with high luminosity. The total cross section of $ZZ$ pair production at linear $e^+e^-$ colliders with backscattered laser photons is evaluated from the differential cross section of the photon fusion subprocess $\gamma\gamma \rightarrow ZZ$ with the convolution of photon spectrum.

\[
d\sigma_{\lambda_3\lambda_4} = \kappa \int_{4m_{Z}^2/s}^{y_m^2} d\tau \frac{dL_{\gamma\gamma}}{d\tau} \left[ \frac{1+<\xi_1\xi_2>}{2} d\hat{\sigma}_{++\lambda_3\lambda_4} + \frac{1-<\xi_1\xi_2>}{2} d\hat{\sigma}_{+-\lambda_3\lambda_4} \right], 
\]

\[
dL_{\gamma\gamma}/d\tau = \int_{\tau/y_m}^{\tau/y_m} dy f_{\gamma/e}(y/x) f_{\gamma/e}(\tau/y, x), 
\]

\[
r = M_{ZZ}/\sqrt{s}, 
\]

\[
\tau = \hat{s}/s = r^2, 
\]

\[
y = E_\gamma/E_e, 
\]

\[
y_m = \frac{x}{x+1}, 
\]

\[
x = 4E_e\omega_0/m_e^2 
\]

where $f_{\gamma/e}$ is the photon energy distribution function, $M_{ZZ}$ is the invariant mass of the $ZZ$ pair, $E_e$ is the initial electron energy, $E_\gamma$ is backscattered photon energy, $\omega_0$ is the laser photon energy, $\kappa$ is the number of high energy photons per one electron, and $\xi_{1,2}$ is the mean helicities of the photon beams. The maximal energy available in the CM frame of $\gamma\gamma$ is $E_{MAX} = y_m\sqrt{s_{e^+e^-}}$. We have taken $\kappa = 1$, and $x = 4.8$ which gives $y_m = 0.83$. As noted in Ref. [3], if $x > 4.8$, number of high energy photons will be reduced by unwanted $e^+e^-$ pair production. The $f_{\gamma/e}$ and $\xi_i$ are taken from equations (4), (12) and (17) of Ref. [4].

The energy spectrum of photons from Compton laser backscattering depends on the product $2\lambda_e\lambda_\gamma$ [4], where $\lambda_e$ is the degree of polarization (mean helicity) of the initial electron.
(positron) and $\lambda_\gamma$ is the degree of circular polarization or mean helicity of the laser beam.
The number of high energy photons increases while the number of soft photons decreases when $-2\lambda_e\lambda_\gamma$ becomes larger. We have studied the photon energy spectrum with $x = 4.8$ for three combinations of polarizations of the initial $e^+e^-$ and laser beams: (a) $\lambda_e = 0.5$ and $\lambda_\gamma = -1.0$, polarized $e^+e^-$ and laser beams with $2\lambda_e\lambda_\gamma = -1$; (b) $\lambda_e = 0.5$ and $\lambda_\gamma = 1.0$, polarized $e^+e^-$ and laser beams with $2\lambda_e\lambda_\gamma = +1$; and (c) $\lambda_e = 0$ and $\lambda_\gamma = 0$, unpolarized $e^+e^-$ and laser beams with $2\lambda_e\lambda_\gamma = 0$. Several interesting aspects have been found: (1) All of them produce about the same number of photons at an energy fraction $y_0 = E_\gamma/E_e = 0.7$. (2) Below $y_0$, the photon luminosity of case (b) is slightly larger than the others. However, it falls off rapidly for $y > y_0$. Case (a) rises sharply for $y > y_0$, but yields the smallest number of photons below $y_0$. (3) In case (c), $2\lambda_e\lambda_\gamma = 0$, the spectrum is almost flat below $y_0$, and the photon luminosity rises significantly as $y > y_0$. (4) In $\gamma\gamma$ collisions, the energy fractions $y_1$ and $y_2$ are related by $y_1y_2 = \tau$. With $\lambda_{e_1} = \lambda_{e_2} = \lambda_e$ and $\lambda_{\gamma_1} = \lambda_{\gamma_2} = \lambda_\gamma$, case (a) has the highest photon photon luminosity for $r = M_{ZZ}/\sqrt{s_{e^+e^-}} > 0.7$ while case (b) dominates for $r < 0.6$. Case (c) produces a larger number of photons in a broad range of energies.

Our main purpose is to enhance the Higgs signal as much as possible. The Stokes parameters $<\xi_1\xi_2>$ in Eq. (6) play important roles in enhancing or reducing the Higgs signal. To study the effect of $<\xi_1\xi_2>$ with $\lambda_{e_1} = \lambda_{e_2} = \lambda_e$ and $\lambda_{\gamma_1} = \lambda_{\gamma_2} = \lambda_\gamma$ in photon photon collisions, we have considered two more cases, (d) $\lambda_e = 0.5$ and $\lambda_\gamma = 0$, polarized $e^+e^-$ and unpolarized laser beams; and (e) $\lambda_e = 0$ and $\lambda_\gamma = 1.0$, unpolarized $e^+e^-$ and polarized laser beams; in addition to the three cases just considered for the photon energy spectrum. Since the Higgs appears only in the same helicity states of $\gamma\gamma ZZ$, we would like to enhance the cross section of $\hat{\sigma}^{\pm_+\lambda_3\lambda_4}$ while reducing $\hat{\sigma}^{\pm_-\lambda_3\lambda_4}$. We have found that, in case (d), $<\xi_1\xi_2>$ is always positive, and is enhanced as $M_{ZZ}$ becomes larger. Case (b) usually has positive $<\xi_1\xi_2>$ and it is the largest at low $M_{ZZ}$, however it drops rapidly for $r > 0.7$. In case (a), $<\xi_1\xi_2>$ is usually positive for $r < 0.30$ and $r > 0.63$, but usually negative in between. In Case (c), $<\xi_1\xi_2> = 0$. In case (e), $<\xi_1\xi_2>$ is usually positive for $r < 0.54$ and $r > 0.76$, but becomes negative in between. The combination of polarizations $-\lambda_e$ and
−λγ has the same product 2λeλγ as that of λe and λγ, therefore produces the same energy spectrum but it yields <ξ1ξ2> with an opposite sign. As a combined effect from energy spectrum and the Stokes parameters, case (d) seems to be the best choice for a the Higgs search over a broad range of MH, case (a) is the best for MH > 0.7√s_{e^+e^-}, and case (b) is the best for MH < 0.6√s_{e^+e^-}.

The total cross section of γγ → ZZ in high energy photon photon collisions with backscattered laser photons is presented as a function of √s_{e^+e^-} in Table III, for mt = 140 GeV and mH = 300, 400 GeV and ∞ (the background) and the five combinations of polarizations for the initial e^+e^- and laser beams used for studying the Stokes parameters. From Table III, we can find that (1) For M_H close to E_{MAX}, λ_e1 = λ_e2 = 0.5 and λ_γ1 = λ_γ2 = −1.0 produces the largest cross section; (2) For M_H much smaller than E_{MAX}, λ_e1 = λ_e2 = 0.5 and λ_γ1 = λ_γ2 = 1.0 produces the largest cross section; and (3) the unpolarized initial e^+e^- or laser beams, yield a clear Higgs signal for a broad range of energy.

To study the observability of the Higgs signal as a pronounced peak in the ZZ invariant mass distribution, we consider the total contribution without the Higgs boson as the background and show the Higgs signal with the background in Figures 3 and 4. The invariant mass distribution of ZZ for γγ → ZZ at the NLC, √s_{e^+e^-} = 500 GeV, is shown in Figure 3 for the three most promising polarizations of initial electron(positron) and laser beams: (a) λ_e1 = λ_e2 = 0.45 and λ_γ1 = λ_γ2 = −1.0, (b) λ_e1 = λ_e2 = 0 and λ_γ1 = λ_γ2 = 0, (c) λ_e1 = λ_e2 = 0.45 and λ_γ1 = λ_γ2 = 0, and also (d) λ_e1 = λ_e2 = 0 and λ_γ1 = λ_γ2 = 1.0, for completeness. The difference between λ_e’s being 0.45 and 0.5 is about 5% for case (c) and less than 3% for case (a) in the invariant mass differential cross section. At the NLC, the Higgs signal appears as a pronounced peak in the ZZ invariant mass distribution, up to M_H = 390 GeV. We can find in Fig. 3 that the ratio of Signal/Background is enhanced for, λ_e1 = λ_e2 close to +0.5 and λ_γ1 = λ_γ2 = 0 in a broad range.

Figures 4 shows that the Higgs signal for M_H = 400 GeV is visible in the invariant mass distribution of ZZ, at √s_{e^+e^-} = (a) 600, (b)700 and (c) 1000 GeV, in γγ collisions with photons from backscattered laser beams for λ_e1 = λ_e2 = 0.45 and λ_γ1 = λ_γ2 = 0. The cross
sections at the Higgs pole for $\lambda_{e1} = \lambda_{e2} = 0$ and $\lambda_{\gamma1} = \lambda_{\gamma2} = 0$ are about 25% smaller. The combination of $\lambda_{e1} = \lambda_{e2} = 0$ and $\lambda_{\gamma1} = \lambda_{\gamma2} = \pm 1.0$ is slightly better than unpolarized $e^+e^-$ and laser beams if $r < 0.5$ or $r > 0.8$. A more realistic study for the signal and background with the final states of $l^+l^-\nu\bar{\nu}$ and $l^+l^-q\bar{q}$ is under investigation.

V. CONCLUSIONS

In high energy $\gamma\gamma$ collisions, the $TT$ cross section of $\gamma\gamma \rightarrow ZZ$ dominates if $M_H > 350$ GeV. With $2\lambda_e\lambda_\gamma = 0$, the photon spectrum is almost flat, and it is possible to search for the Higgs signal in a broad range below $\sqrt{s_{e^+e^-}}$. The best case to search for the Higgs signal is to have $\lambda_{e1} = \lambda_{e2}$ close to +0.5 and $\lambda_{\gamma1} = \lambda_{\gamma2} = 0$, because the photon energy spectrum is almost flat and the contribution from $\hat{\sigma}_{++LL}$ is enhanced by the Stokes parameters; however, even in the best case, using polarized electrons or laser photons yields only a small advantage over the totally unpolarized case. At the NLC, a Higgs signal is possible for $M_H$ up to 390 GeV in the invariant mass distribution of $ZZ$. For larger CM energies, $600$ GeV $< \sqrt{s_{e^+e^-}} < 1000$ GeV, it is possible to find a Higgs with a mass slightly larger than 400 GeV. Thus there is not much advantage for this process in higher $e^+e^-$ energies. Furthermore, there is not much advantage in the $\gamma\gamma$ mode; $e^+e^-$ collisions at the NLC, by themselves, can search for the Higgs up to a mass of 350 GeV. The unique strength of high energy $\gamma\gamma$ collisions in the Higgs search is probably to measure the $H\gamma\gamma$ coupling with high precision beyond the intermediate Higgs mass range.

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APPENDIX A: RELEVANT FEYNMAN RULES IN THE NON-LINEAR GAUGE

In this appendix, relevant Feynman rules in the non-linear gauge described in section 2 are presented in our conventions. There are 3- and 4-point vertices among the gauge boson fields, $W_\mu^\pm$, $Z_\mu$, $A_\mu$; the Nambu-Goldstone boson fields, $G^\pm$, $G^0$; the Fadeev-Popov ghost fields, $\theta^\pm$, $\bar{\theta}^\pm$, $\theta^Z$, $\bar{\theta}^Z$, $\theta^A$, $\bar{\theta}^A$; and the Higgs boson field $H$. The gauge parameters are all taken to be unity, $\xi_W = \xi_Z = \xi_A = 1$, which corresponds to a non-linear 't Hooft-Feynman gauge. In this gauge, the $W$ boson ($W^\pm$), the Nambu-Goldstone boson ($G^\pm$), and the Fadeev-Popov ghosts ($\theta^\pm$) have the same mass $M_W = M_G = M_\theta$. The Feynman rules for relevant interactions involved in the $W$, $G$ and $\theta$ loops which appear in the reaction $\gamma\gamma \rightarrow ZZ$ are shown in Table IV.
REFERENCES

[1] V. Barger, K. Cheung, B. A. Kniehl and R. J. N. Phillips, Phys. Rev. D46 (1992) 3725.

[2] J. F. Gunion, to appear in Proceedings of the International Workshop on Physics and Experiments with Linear $e^+e^-$ Colliders, Hawaii, USA (1993), UCD-93-24, and references therein.

[3] I. F. Ginzburg, G. L. Kotkin, V. G. Serbo and V. I. Telnov, Nucl. Instrum. Methods 205, (1983) 47.

[4] I. F. Ginzburg, G. L. Kotkin, S. L. Panfil, V. G. Serbo and V. I. Telnov, Nucl. Instrum. Methods 219, (1984) 5.

[5] T. L. Barklow, SLAC-PUB-3564 (1990), to appear in The Proceedings of the 1990 DPF Summer Study on High Energy Physics, Snowmass (1990).

[6] V. I. Telnov, Nucl. Instrum. Methods A294, (1990) 72.

[7] D. L. Borden, D. A. Bauer, D. O. Caldwell, SLAC preprint SLAC-PUB-5715 (1992).

[8] J. F. Gunion and H. E. Haber, The Proceedings of the 1990 Summer Study on High Energy Physics, Snowmass (1990); J. F. Gunion and H. E. Haber, UCD-92-22 (1992).

[9] H. E. Haber, in Proceedings of the 1st International Workshop on Physics and Experiments with Linear $e^+e^-$ Colliders, Saariselkä, Finland, 1992, World Scientific Publishing, Singapore, (1992).

[10] E. E. Boos and G. V. Jikia, Phys. Lett. B275, (1992) 164.

[11] J. F. Gunion, UCD-93-8 (1993).

[12] D. Bowser-Chao and K. Cheung, Phys. Rev. D48, 89 (1993).

[13] G. V. Jikia, Phys. Lett. B298, (1993) 224; G. V. Jikia, IHEP 93-37 (1993).

[14] M. S. Berger, MAD/PH/771 (1993).
[15] K. Fujikawa, Phys. Rev. D7 (1973) 393.

[16] M. Bace and N. D. Hari Dass, Ann. Phys. 94 (1975) 349.

[17] M. Gavela, G. Girardi, C Malleville and P. Sorba, Nucl. Phys. B193 (1981) 257.

[18] N. G. Deshpande and M. Nazerimonfared, Nucl. Phys. B213 (1983) 390.

[19] F. Boudjema, Phys. Lett. B187 (1987) 362.

[20] D. A. Dicus, C. Kao, and W. W. Repko, Phys. Rev. D36 1570 (1987); D. A. Dicus, Phys. Rev. D38 394 (1988).

[21] E. W. N. Glover and J. J. van der Bij, Phys. Lett. B219, 488 (1989); Nucl. Phys. B321, 561 (1989).

[22] D. Dicus and C. Kao, LOOP, a FORTRAN program for doing loop integrals of 1, 2, 3 and 4 point functions with momenta in the numerator, unpublished, (1991).

[23] G. ’t Hooft and M. Veltman, Nucl. Phys. B153, 365 (1979). G. Passarino and M. Veltman, Nucl. Phys. B160, 151 (1979).

[24] K.-I. Aoki, et al., Prog. Theo. Phys. Suppl. No. 73 (1982).
TABLE I. The effect of $m_t$ on the cross section of $\gamma\gamma ZZ$ in fb at $\sqrt{s_{\gamma\gamma}} = M_H = 300, 400, 500, 600, 700,$ and 800 GeV, in the $++ LL$ helicity state, for $m_t = 120, 140, 160, 180$ and 200 GeV.

| $m_t$ (GeV)/ $M_H$ (GeV) | 300 | 400 | 500 | 600 | 700 | 800 |
|--------------------------|-----|-----|-----|-----|-----|-----|
| 120                      | 360 | 55  | 22  | 14  | 11  | 9.4 |
| 140                      | 660 | 57  | 13  | 8.1 | 7.1 | 6.6 |
| 160                      | 790 | 87  | 11  | 4.2 | 4.0 | 4.2 |
| 180                      | 810 | 160 | 16  | 2.6 | 1.8 | 2.4 |
| 200                      | 830 | 210 | 29  | 3.3 | 0.69| 1.1 |
TABLE II. The total cross section of $\gamma\gamma ZZ$ in $fb$ at $\sqrt{s_{\gamma\gamma}} = M_H = 300, 400$ and $500$ GeV, in each helicity states for $m_t = 140$ GeV after different cuts on the CM scattering angle $|\cos(\theta^*)| < z_0$: $z_0 = 1.0, \cos(30^\circ)$ and 0.8.

| $z_0$/Helicities | $++ LL$ | $++ TT$ | $++ LT$ | $+- LL$ | $+- TT$ | $+- LT$ |
|------------------|---------|---------|---------|---------|---------|---------|
| (a) $\sqrt{s_{\gamma\gamma}} = M_H = 300$ GeV |
| 1.0              | 660     | 160     | 0.099   | 1.4     | 47      | 8.0     |
| $\cos(30^\circ)$| 580     | 130     | 0.073   | 1.4     | 32      | 7.5     |
| 0.8              | 530     | 120     | 0.057   | 1.3     | 26      | 7.1     |
| (b) $\sqrt{s_{\gamma\gamma}} = M_H = 400$ GeV |
| 1.0              | 57      | 180     | 0.061   | 1.1     | 74      | 5.5     |
| $\cos(30^\circ)$| 49      | 150     | 0.037   | 1.1     | 45      | 5.1     |
| 0.8              | 46      | 130     | 0.027   | 1.1     | 35      | 4.8     |
| (c) $\sqrt{s_{\gamma\gamma}} = M_H = 500$ GeV |
| 1.0              | 13      | 240     | 0.034   | 0.96    | 98      | 3.9     |
| $\cos(30^\circ)$| 12      | 180     | 0.017   | 0.95    | 53      | 3.5     |
| 0.8              | 11      | 160     | 0.012   | 0.93    | 40      | 3.2     |
TABLE III. Total cross section of $\gamma\gamma \rightarrow ZZ$ in $fb$ as a function of $\sqrt{s_{e^+e^-}}$ with backscattered laser photons, for $|\cos(\theta^*)| < 0.8$, $m_t = 140$ GeV, $M_H = 300, 400$ GeV and $\infty$, and five combinations of polarizations of initial $e^+e^-$ and laser beams with $\lambda_{e_1} = \lambda_{e_2} = \lambda_e$ and $\lambda_{\gamma_1} = \lambda_{\gamma_2} = \lambda_{\gamma}$.

| $M_H$ (GeV) / $\sqrt{s_{e^+e^-}}$ (GeV) | 240 | 300 | 400 | 500 | 600 | 700 | 1000 |
|----------------------------------------|-----|-----|-----|-----|-----|-----|------|
| (a) $\lambda_e = 0.5$, $\lambda_\gamma = -1.0$ |     |     |     |     |     |     |      |
| 300 | 0.55 | 8.9 | 62  | 48  | 58  | 67  | 76   |
| 400 | 0.47 | 7.7 | 30  | 54  | 59  | 66  | 75   |
| $\infty$ | 0.45 | 7.4 | 28  | 46  | 58  | 65  | 74   |
| (b) $\lambda_e = 0.5$, $\lambda_\gamma = +1.0$ |     |     |     |     |     |     |      |
| 300 | 2.8  | 1.5 | 18  | 44  | 61  | 78  | 104  |
| 400 | 2.7  | 1.3 | 10  | 27  | 47  | 63  | 96   |
| $\infty$ | 2.6  | 1.3 | 9.8 | 25  | 42  | 58  | 94   |
| (c) $\lambda_e = 0$, $\lambda_\gamma = 0$ |     |     |     |     |     |     |      |
| 300 | 0.39 | 4.5 | 26  | 37  | 48  | 57  | 72   |
| 400 | 0.38 | 4.3 | 16  | 30  | 42  | 52  | 70   |
| $\infty$ | 0.37 | 4.2 | 15  | 28  | 40  | 49  | 69   |
| (d) $\lambda_e = 0.5$, $\lambda_\gamma = 0$ |     |     |     |     |     |     |      |
| 300 | 0.19 | 4.1 | 38  | 49  | 62  | 72  | 89   |
| 400 | 0.17 | 3.6 | 18  | 39  | 54  | 67  | 87   |
| $\infty$ | 0.16 | 3.5 | 17  | 35  | 51  | 64  | 86   |
| (e) $\lambda_e = 0$, $\lambda_\gamma = +1.0$ |     |     |     |     |     |     |      |
| 300 | 0.29 | 4.8 | 25  | 35  | 47  | 55  | 77   |
| 400 | 0.27 | 4.4 | 16  | 30  | 40  | 51  | 71   |
| $\infty$ | 0.26 | 4.4 | 15  | 27  | 38  | 48  | 69   |
TABLE IV. The Feynman rules for relevant interactions involved in the $W$, $G$ and $\theta$ loops which appear in the reaction $\gamma\gamma \to ZZ$ as modified by the non-linear gauge condition which is described in Sec 2. These can be compared to the unmodified rules given in Ref. [24]. (Note that not all of the rules below are modified.) All momenta (e.g., $k$, $p$ and $q$) and charges are incoming to the vertices. $g_{\mu\nu} \equiv \text{diag}(+,-,-,-)$ is the metric tensor, $s_W$ is $\sin \theta_W$ and $c_W$ is $\cos \theta_W$.

| 3-point vertices | Feynman Rules | 4-point vertices | Feynman Rules |
|------------------|---------------|------------------|---------------|
| $A_{\mu}(k)W_{\nu}^+(p)W_{\rho}^-(q)$ | $-c_{\mu\nu}(k - p)_{\rho}$ | $A_{\mu}A_{\nu}W_{\rho}^+W_{\sigma}^-$ | $-2c_{\mu\nu}g_{\rho\sigma}$ |
|                  | $+c_{\nu}(p - q)_{\mu}$           |                  |               |
|                  | $+c_{\rho}(q - k + p)_{\nu}$       |                  |               |
| $A_{\mu}(k)G^+(p)G^-(q)$ | $+c_{\mu}(p - q)_{\mu}$           | $A_{\mu}A_{\nu}G^+G^-$ | $+2c_{\mu\nu}$ |
| $A_{\mu}(k)\theta^+(p)\theta^+(q)$ | $+c_{\mu}(p - q)_{\mu}$           | $A_{\mu}A_{\nu}\theta^+\theta^+$ | $+2c_{\mu\nu}$ |
| $A_{\mu}(k)\theta^-(p)\theta^-(q)$ | $-c_{\mu}(p - q)_{\mu}$           | $A_{\mu}A_{\nu}\theta^-\theta^-$ | $+2c_{\mu\nu}$ |
| $Z_{\mu}(k)W_{\nu}^+(p)W_{\rho}^-(q)$ | $-gc_{W}(g_{\mu\nu}(k - p)_{\rho} + \frac{s^2_{W}}{c^2_{W}}q_{\rho})$ | $Z_{\mu}Z_{\nu}W_{\rho}^+W_{\sigma}^-$ | $-2c_{\nu}(2g_{\mu\nu}g_{\rho\sigma}$ |
|                  | $+c_{\nu}(p - q)_{\mu}$           |                  |               |
|                  | $+c_{\rho}(q - k + p)_{\nu}$       |                  |               |
| $Z_{\mu}(k)G^+(p)G^-(q)$ | $+\frac{1}{2}g_{\mu}(\frac{1-2s^2_{W}}{c^2_{W}})(p - q)_{\mu}$ | $Z_{\mu}Z_{\nu}G^+G^-$ | $+\frac{1}{2}g^2_{\nu}(\frac{1-2s^2_{W}}{c^2_{W}})^2g_{\mu\nu}$ |
| $Z_{\mu}(k)\theta^+(p)\theta^+(q)$ | $-gc_{W}(\frac{s^2_{W}}{c^2_{W}}p_{\mu} + q_{\mu})$ | $Z_{\mu}Z_{\nu}\theta^+\theta^+$ | $-2c_{\nu}g_{\mu\nu}$ |
| $Z_{\mu}(k)\theta^-(p)\theta^-(q)$ | $+gc_{W}(\frac{s^2_{W}}{c^2_{W}}p_{\mu} + q_{\mu})$ | $Z_{\mu}Z_{\nu}\theta^-\theta^-$ | $-2c_{\nu}g_{\mu\nu}$ |
| $HZ_{\mu}Z_{\nu}$ | $+\frac{g}{c_{W}}M_{Z}g_{\mu\nu}$ | $Z_{\mu}A_{\nu}W_{\rho}^+W_{\sigma}^-$ | $-egc_{W}(2g_{\mu\nu}g_{\rho\sigma}$ |
| $HW_{\mu}W_{\nu}^-$ | $+gM_{W}g_{\mu\nu}$ |                  | $-\frac{1}{c_{W}}(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\sigma}g_{\nu\rho})$ |
| $HG^+G^-$ | $-\frac{1}{2}g_{\mu}M_{W}^2$ | $Z_{\mu}A_{\nu}G^+G^-$ | $+eg(\frac{1-2s^2_{W}}{c^2_{W}})g_{\mu\nu}$ |
| $H\theta^+\theta^\pm$ | $-\frac{1}{2}gM_{W}$ | $Z_{\mu}A_{\nu}\theta^+\theta^\pm$ | $+eg(\frac{1-2s^2_{W}}{c^2_{W}})g_{\mu\nu}$ |
FIGURES

FIG. 1. The cross section of $\gamma\gamma \rightarrow ZZ$ as a function of $\sqrt{s_{\gamma\gamma}}$ for the $LL$ (solid), $TT$ (dash-dotted) and $LT$ (dashed) helicity states of $ZZ$ in (a) ++ and (b) +- helicity states of the photon with $m_t = 140$ GeV. The ++LL cross section is evaluated with $M_H = 300, 400, 500, 800$ GeV and $\infty$.

FIG. 2. The cross section of $\gamma\gamma \rightarrow ZZ$ as a function of $\sqrt{s_{\gamma\gamma}}$ in the ++ $LL$ state, for fermion loops alone (dotted), the $W$ loop alone (dashed) and the sum of all loops (solid), with $m_t = 120, 160$ and $200$ GeV and $M_H = 500$ GeV. The $W$ loop cross section has been evaluated with $m_t = 160$ GeV.

FIG. 3. Invariant mass distribution of $\gamma\gamma \rightarrow ZZ$ in high energy photon photon collisions from laser backscattered photons, for the NLC energy, $\sqrt{s_{e^+e^-}} = 500$ GeV, $m_t = 140$ GeV, and $M_H = 250, 300, 350$ and $390$ GeV. The polarizations of the initial $e^-e^+$ and laser beams are taken to be (a) $\lambda_{e_1} = \lambda_{e_2} = 0.45$ and $\lambda_{\gamma_1} = \lambda_{\gamma_2} = -1.0$, (b) $\lambda_{e_1} = \lambda_{e_2} = 0$ and $\lambda_{\gamma_1} = \lambda_{\gamma_2} = 0$, (c) $\lambda_{e_1} = \lambda_{e_2} = 0.45$ and $\lambda_{\gamma_1} = \lambda_{\gamma_2} = 0$ and (d) $\lambda_{e_1} = \lambda_{e_2} = 0$ and $\lambda_{\gamma_1} = \lambda_{\gamma_2} = 1.0$.

FIG. 4. Invariant mass distribution of $\gamma\gamma \rightarrow ZZ$ in high energy photon photon collisions from laser backscattered photons with polarizations of the initial $e^-e^+$ and laser beams being $\lambda_{e_1} = \lambda_{e_2} = 0.45$ and $\lambda_{\gamma_1} = \lambda_{\gamma_2} = 0$, for $m_t = 140$ GeV, $M_H = 250, 300, 350, 400, 450$, and $500$ GeV, and (a) $\sqrt{s_{e^+e^-}} = 600$ GeV (without $M_H = 500$ GeV), (b) $\sqrt{s_{e^+e^-}} = 700$ GeV, and (c) $\sqrt{s_{e^+e^-}} = 1000$ GeV.