User Pairing and Power Allocation for IRS-Assisted NOMA Systems with Imperfect Phase Compensation

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Abstract—In this letter, we analyze the performance of the intelligent reflecting surface (IRS) assisted downlink non-orthogonal multiple access (NOMA) systems in the presence of imperfect phase compensation. We derive an upper bound on the imperfect phase compensation to achieve minimum required data rates for each user. Using this bound, we propose an adaptive user pairing algorithm to maximize the network throughput. We then derive bounds on the power allocation factors and propose power allocation algorithms for the paired users to achieve the maximum sum rate or ensure fairness. Through extensive simulations, we show that the proposed algorithms significantly outperform the state-of-the-art algorithms.

Index Terms—Intelligent-reflecting surfaces (IRS), non-orthogonal multiple access (NOMA), power allocation, spectral efficiency, user pairing.

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) is considered as a key radio access technique for fifth-generation (5G) and beyond 5G networks [1]. In NOMA, the users are allocated the same time and frequency resources but are multiplexed across the power domain to achieve multi-fold improvement in the network capacity. At the receiver side of NOMA systems, the successive interference cancellation is employed to decode the transmitted data. Similar to NOMA, intelligent reflecting surface (IRS) is another key technology to improve coverage for the beyond-5G networks [2]. An IRS consists of a large number of passive antenna elements where the reflection from each antenna is controlled to direct the signal towards a particular user. Motivated by the benefits from both technologies, IRS has been analyzed along with NOMA to achieve better network capacity and enhanced coverage [3].

The practical IRS systems have imperfections in the phase control because of hardware limitations and channel estimation errors [4]. These imperfections in the phase compensation have a significant impact on the data rates observed by the users. However, only a few works in the literature consider these imperfections while analyzing the network performance [3], [4]. In [3], the authors have proposed a novel design for IRS-assisted NOMA transmissions and have analyzed the impact of hardware impairments. In [4], the authors have evaluated the performance of orthogonal multiple access (OMA) systems in the presence of imperfect phase compensation. In NOMA, the network capacity is heavily dependent on the user pairing [5]–[7], and hence, IRS-assisted NOMA systems have to consider the imperfections in the phase compensation while pairing the users. Otherwise, the enhanced network throughputs will not be realized in practice. To the best of our knowledge, none of the existing works in the literature have proposed user pairing and power allocation for IRS-assisted NOMA systems with imperfect phase compensation.

In view of the aforementioned details, this letter presents the first work that discusses the following contributions.

- We derive bounds on the imperfection phase compensation to achieve minimum required data rates.
- We propose an adaptive user pairing algorithm for IRS-assisted downlink NOMA systems.
- We derive various bounds on the power allocation factors for the paired users.
- Using the derived bounds, we propose power allocation algorithms to maximize the achievable sum rate (ASR) or ensure fairness.

The organization of the paper is as follows. We present the system model in Section II. In Section III, we derive various bounds on the imperfect phase compensation and power allocation factors. We propose adaptive user pairing and power allocation algorithms in Section IV. In Section V, we present numerical results for various scenarios. We then provide concluding remarks and directions for future work in Section VI.

II. SYSTEM MODEL

We consider a base station (BS) with \( M \) antennae and an IRS with \( N \) antennae, where IRS is activated by a controller connected to the BS as shown in Fig. 1. The channel coefficients between the BS to IRS and \( i^{th} \) user to IRS are denoted by \( h_R \) and \( h_i \), respectively, and are defined as follows [4]:

\[
\begin{align*}
    h_R &= \beta_I a_N(\phi_I, \phi_I) a_R^H(\phi_B, \psi_B), \\
    h_i &= \beta_i a_N(\psi_i, \psi_i),
\end{align*}
\]

where \( \cdot^H \) is the Hermitian of the matrix, \( \beta_I \) and \( \beta_i \) are the distance dependent losses of BS to IRS link and IRS to \( i^{th} \) user.
link, respectively. \( \phi_\theta^a \) and \( \phi_\theta^b \) are the angle of arrival (AoA) in azimuth and elevation at the IRS, respectively, \( \psi_\theta^a \) and \( \psi_\theta^b \) are the angle of departure (AoD) in azimuth and elevation at the BS, respectively, \( \phi_\theta^i \) and \( \phi_\theta^j \) are the AoD in the azimuth and elevation at the IRS, respectively, and \( a_X(v^a, v^e) \) is the array factor that captures the beamforming gain. For a planar array with \( X \) antenna elements, we assume \( \sqrt{X} \) elements in the horizontal and vertical direction of the planar array, and thus, define the array factor as follows [4]:

\[
a_X(v^a, v^e) = \begin{bmatrix}
1 \\
e^{j \frac{2\pi}{X} (x\sin v^a \sin y \cos v^e)} \\
\vdots \\
e^{j \frac{2\pi}{X} ((\sqrt{X} - 1) \sin v^a \sin y \cos v^e)}
\end{bmatrix}
\]

where \( 0 \leq x, y \leq (\sqrt{X} - 1) \) are the indices of antenna elements in the planar array, \( d \) is the spacing between antenna elements, \( \lambda \) is the wavelength, \( v^a \) and \( v^e \) are the desired directions in azimuth and elevation, respectively.

We denote the diagonal matrix that captures the reflection of the IRS as \( \Theta \) and define each diagonal element of \( \Theta \) as \( e^{j\theta_k} \), where \( k \in [1, N] \) is the antenna index and \( \theta_k \in [0, 2\pi) \) is the phase reflection coefficient. Further, practical IRS have finite resolution while applying phase shifters, and hence, result in imperfect phase compensation. Thus, we consider the actual reflection matrix to be \( \hat{\Theta} \) with each diagonal element defined as \( e^{j\theta_k} \), where \( \theta = \theta_k + \delta \), \( \delta \) being the phase noise. We consider \( \delta \) to be uniformly distributed over \([-\delta, \delta]\) with \( \delta \in [0, \pi) \). With all this information, the signal received by the \( i \)-th user in an OMA is formulated as [3]

\[
y_i^{\text{OMA}} = h_i^H \hat{\Theta} h_R P_i s_i + n,
\]

where \( s_i \) is the data transmitted to \( i \)-th user, \( n \) denotes the noise, and \( P_i \) is the available transmit power at the BS. The signal-to-interference-plus-noise ratio (SINR) of the \( i \)-th user in the OMA system is formulated as

\[
\gamma_i^{\text{OMA}} = \frac{P_i |h_i^H \hat{\Theta} h_R|^2}{I + \sigma^2},
\]

where \( \sigma^2 \) is the noise variance and \( I \) is the interference power received at the user. In the case of an IRS-assisted NOMA system, we consider that the BS transmits \( P_i (s_1 + \alpha_1 s_2) \), where \( \alpha_1 \) and \( \alpha_2 \) are the fractions of power allocated to strong and weak user, respectively, and \( s_1 \) and \( s_2 \) denote the data to be transmitted to the strong and weak user, respectively. Further, \( 0 < \alpha_1, \alpha_2 < 1 \) and \( \alpha_1 + \alpha_2 = 1 \). The signal received by the \( i \)-th user in a NOMA system is formulated as [3]

\[
y_i^{\text{NOMA}} = h_i^H \hat{\Theta} h_R P_i (\alpha_1 s_2 + \alpha_2 s_2) + n, \forall i \in 1, 2.
\]

Thus, we define the SINR of strong and weak users in a NOMA system as follows:

\[
\gamma_1^{\text{NOMA}} = \frac{\alpha_1 P_i |h_1^H \hat{\Theta} h_R|^2}{I + \sigma^2},
\]

\[
\gamma_2^{\text{NOMA}} = \frac{\alpha_2 P_i |h_2^H \hat{\Theta} h_R|^2}{\alpha_1 P_i |h_1^H \hat{\Theta} h_R|^2 + I + \sigma^2}.
\]

**III. Computation of Bounds**

We derive the achievable data rates by users in IRS-assisted NOMA and OMA systems as follows. From (1)-2, we get

\[
h_i^H \hat{\Theta} h_R = \beta_i \beta_j \sum_{n=1}^{N} e^{j\theta_n} a_M^H (\psi_\theta^a, \psi_\theta^b),
\]

\[
||a_M^H (\psi_\theta^a, \psi_\theta^b)||^2 = M,
\]

\[
||h_i^H \hat{\Theta} h_R||^2 = ||\beta_i \beta_j||^2 \sum_{n=1}^{N} e^{j\theta_n} ||^2 M.
\]

We define channel state information (CSI) of \( i \)-th user \( (\gamma_i^{\text{CSI}}) \) as

\[
\gamma_i^{\text{CSI}} = \frac{P_i |h_i^H \hat{\Theta} h_R|^2}{I + \sigma^2} = \frac{P_i |\beta_i \beta_j||^2 N^2 M}{I + \sigma^2}.
\]

**Lemma 1.** The normalized achievable data rates in an IRS-assisted OMA and NOMA systems with 2 users are as follows:

\[
R_i^{\text{OMA}} = \frac{1}{2} \log_2 \left( 1 + \gamma_i^{\text{CSI}} \sin^2(\delta) \right), \forall i \in 1, 2,
\]

\[
R_i^{\text{NOMA}} = \log_2 \left( 1 + \alpha_1 \gamma_1^{\text{CSI}} \sin^2(\delta) \right),
\]

\[
R_2^{\text{NOMA}} = \log_2 \left( 1 + \frac{\alpha_2 \gamma_2^{\text{CSI}} \sin^2(\delta)}{\alpha_1 \gamma_1^{\text{CSI}} \sin^2(\delta) + 1} \right).
\]

**Proof.** We adopt the SINR approximation formulated in [3] and define the following:

\[
\left| \frac{1}{N} \sum_{n=1}^{N} e^{j\theta_n} \right|^2 \xrightarrow{(a)} \left| \mathbb{E}[e^{j\theta_n}] \right|^2 \xrightarrow{(b)} \left| \mathbb{E}[\cos(\theta_n)] \right|^2 \xrightarrow{(c)} \sin^2(\delta),
\]

where \( (a) \) is based on law of large numbers [4], \( (b) \) is obtained by integrating the odd symmetrical function \( \sin(\theta_n) \) for \( \theta_n \in [-\delta, \delta] \), and \( (c) \) uses the probability density function of \( \theta_n \) which is defined as \( f(\theta_n) = 1/2\delta, \forall \theta_n \in [-\delta, \delta] \) and \( \sin(\delta) = \sin(\delta)/\delta \). Substituting (6)-(9) and (13) in (3)-(5), we get

\[
\gamma_1^{\text{OMA}} = \frac{\alpha_1 P_i |\beta_1 \beta_j||^2 \sum_{n=1}^{N} e^{j\theta_n} ||^2 M}{I + \sigma^2} = \gamma_1^{\text{CSI}} \sin^2(\delta), \forall i \in 1, 2,
\]

\[
\gamma_1 = \frac{\alpha_1 P_i |\beta_1 \beta_j||^2 \sum_{n=1}^{N} e^{j\theta_n} ||^2 M}{I + \sigma^2} = \alpha_1 \gamma_1^{\text{CSI}} \sin^2(\delta),
\]

\[
\gamma_2^{\text{NOMA}} = \frac{\alpha_2 P_i |\beta_2 \beta_j||^2 \sum_{n=1}^{N} e^{j\theta_n} ||^2 M + I + \sigma^2}{I + \sigma^2}.
\]

\[
= \frac{\alpha_2 \gamma_2^{\text{CSI}} \sin^2(\delta)}{\alpha_1 \gamma_1^{\text{CSI}} \sin^2(\delta) + 1}.
\]

Assuming the full bandwidth allocation for the two users in case of NOMA and half bandwidth allocation for each user in OMA, and substituting (14)-(16) while calculating the normalized data rates, we complete the proof of Lemma 1. 

Next, we derive bounds on the power allocation factors.
A. Bounds on $\alpha_1$ and $\alpha_2$

We define $R_1$ and $R_2$ as the minimum rates required by the strong and the weak user, respectively. For the lower bound on $\alpha_1$, we assume that rate of weak user in NOMA ($R_1^{\text{NOMA}}$) should be greater than or equal to the minimum rate required by the weak user ($R_1$). Thus, by considering $R_1^{\text{NOMA}} \geq R_1$, we get

$$\log_2 \left(1 + \alpha_1 \gamma_1^{\text{CSI}} \sin^2(\delta)\right) \geq R_1,$$

$$\alpha_1 \geq \frac{2^{R_1} - 1}{\gamma_1^{\text{CSI}} \sin^2(\delta)} \triangleq \alpha_{1\text{lb}}. \quad (17)$$

Similarly, for the upper bound, by using $R_2^{\text{NOMA}} > R_2$, we get

$$\log_2 \left(1 + \frac{\alpha_2 \gamma_2^{\text{CSI}} \sin^2(\delta)}{\alpha_1 \gamma_1^{\text{CSI}} \sin^2(\delta) + 1}\right) \geq R_2.$$

Substituting $\alpha_2 = 1 - \alpha_1$ in (13), we obtain

$$\alpha_1 \leq \frac{\gamma_2^{\text{CSI}} \sin^2(\delta) - (2^{R_2} - 1)}{2\gamma_2^{\text{CSI}} \sin^2(\delta)} \triangleq \alpha_{1\text{ub}}. \quad (19)$$

Note that similar bounds can be achieved for the power allocation factor of the weak user by substituting $\alpha_2 = 1 - \alpha_1$ in (17)–(19).

B. Upper Bound on the Imperfect Phase Compensation ($\delta_{ub}$)

For the upper bound on $\delta$, we consider that the upper bound of $\alpha_1$ in (19) should be greater than or equal to the lower bound of $\alpha_1$ in (17). Using (17)–(19) and solving $\alpha_{1\text{ub}} \geq \alpha_{1\text{lb}}$, we get

$$\sin^2(\delta) \geq \frac{(2^{R_1} - 1)2^{R_2}}{\gamma_1^{\text{CSI}}} + \frac{(2^{R_2} - 1)}{\gamma_2^{\text{CSI}}} \triangleq \sin^2(\delta_{ub}). \quad (20)$$

Note that when we consider $R_1 = R_2^{\text{OMA}}$, $\delta_{ub}$ is computable at the base station as it is only dependent on $\gamma_1^{\text{CSI}}$. From (20), we conclude that it is beneficial to pair the users in IRS-assisted NOMA systems only when $\delta_{ub}$ with that user pair is greater than or equal to $\delta$. Otherwise, the data rates achieved by the users in NOMA will not be higher than their OMA counterparts.

IV. PROPOSED ALGORITHMS

In this section, we initially present an adaptive user pairing (AUP) algorithm for the IRS-assisted NOMA system based on the $\delta_{ub}$ derived in (20). We then propose maximum ASR achieving power allocation (MPA) and fair power allocation (FPA) algorithms for the paired users.

A. AUP

In [7–9], the authors have shown that pairing of near users with far users achieves better data rates. Motivated by this, we initially sort the users based on their SINRs and group the near users with far users. For each user, we then define the rate achievable with OMA as the minimum required rate (i.e., $R_i = R_i^{\text{OMA}}$). From (20), it is evident that pairing two users in IRS-assisted NOMA with imperfect phase compensation will not always ensure that achievable data rates are better than OMA rates. Hence, to exploit the benefits from NOMA, we pair only those users whose achievable data rates outperform the OMA counterparts. Thus, for users in each group, we check if the imperfect phase compensation ($\delta$) is less than or equal to $\delta_{ub}$ formulated in (20). If this criterion is satisfied, we consider the users in that group to be a NOMA pair. Otherwise, we consider them to be OMA users. This procedure will ensure that each user achieves at least OMA rates. All this procedure is pictorially presented in Fig. 2 for a set of 14 users with $\gamma_1^{\text{CSI}} \geq \ldots \geq \gamma_{14}^{\text{CSI}}$. Next, we present MPA procedure in detail.

B. MPA

In this section, we present maximum ASR achieving power allocation procedure for the IRS-assisted NOMA systems.

**Lemma 2.** The power allocation factors for NOMA pair that maximize the ASR and also ensure each user achieves at least OMA rates are as follows, $\alpha_1 = \alpha_{1\text{ub}}$ and $\alpha_2 = 1 - \alpha_1$.  

**Proof.** We formulate ASR for a NOMA pair as $R_1^{\text{NOMA}} + R_2^{\text{NOMA}}$. 

$$d(\text{ASR}) \frac{\text{da}}{\alpha_1} = \frac{\gamma_1^{\text{OMA}} \sin^2(\delta) - \gamma_2^{\text{OMA}} \sin^2(\delta)}{(1 + \alpha_1 \gamma_1^{\text{OMA}} \sin^2(\delta))(1 + \alpha_2 \gamma_2^{\text{OMA}} \sin^2(\delta))}.$$  

(21)

Note that as per our formulation in (14–16), $\gamma_1^{\text{OMA}} \geq \gamma_2^{\text{OMA}}$, and thus, $\frac{\text{dASR}}{\text{da}} \geq 0$. Hence, ASR is a non-decreasing function and $\alpha_1 = \alpha_{1\text{ub}}$ will result in maximum ASR. This completes the proof of Lemma 2.

Thus, in MPA, we allocate $\alpha_1 = \alpha_{1\text{ub}}$ and $\alpha_2 = 1 - \alpha_1$ to strong and weak users, respectively, to achieve maximum ASR.

C. FPA

We define the $O_i$ as an event of outage for $i^{th}$ user, where $Pr(O_i) = Pr(R_i^{\text{OMA}} < R_i), \forall i = 1, 2$. Using (11), we get

$$Pr(O_i) = Pr(R_i^{\text{OMA}} < R_i),$$

$$= Pr\left(\frac{\gamma_i^{\text{OMA}}}{\alpha_1} < \frac{(2^{R_i} - 1)}{\alpha_1 \sin^2(\delta)}\right). \quad (22)$$

Similarly, using (12) for the weak user in NOMA, we get

$$Pr(O_2) = Pr(R_2^{\text{OMA}} < R_2),$$

$$= Pr\left(\frac{\gamma_2^{\text{OMA}}}{\alpha_2} < \frac{(2^{R_2} - 1)}{(\alpha_2 - \alpha_1)(2^{R_2} - 1) \sin^2(\delta)}\right). \quad (23)$$

In NOMA, with an increase in power allocation for a user, the probability of outage increases for the other paired user. Hence, to ensure fairness in power allocation, we consider...
Next, we present the simulation results.

The A UP algorithm with MP A and FP A requires sorting of the users based on their SINRs ($\gamma^{\text{CSI}}_i \geq \ldots \geq \gamma^{\text{CSI}}_G$):

\begin{algorithm}
\textbf{Input} : Set of users $U$, and corresponding SINRs $\gamma^{\text{CSI}}_i$
\textbf{Variables}: $i$ is a variable representing user pair index.
\begin{enumerate}
\item Sort the users based on their SINRs ($\gamma^{\text{CSI}}_1 \geq \ldots \geq \gamma^{\text{CSI}}_G$);
\item for $i = 1 \rightarrow \frac{N}{2}$ do
\begin{enumerate}
\item $R^{\text{CSI}}_1 = \frac{1}{2} \log_2(1 + \gamma^{\text{CSI}}_1 \sin^2(\delta));$
\item $R^{\text{CSI}}_2 = \frac{1}{2} \log_2(1 + \gamma^{\text{CSI}}_G \sin^2(\delta));$
\item $\sin^2(\delta_{\text{UB}}) = \frac{\gamma^{\text{CSI}}_1}{\gamma^{\text{CSI}}_G - i + 1};$
\end{enumerate}
\item if $\sin^2(\delta_{\text{UB}}) > \sin^2(\delta)$ then
\begin{enumerate}
\item Consider the users for OMA;
\item else if MP A then
\begin{enumerate}
\item $\alpha_1 = \frac{\gamma^{\text{CSI}}_{G-i+1} \sin^2(\delta) - (2R^{\text{CSI}}_2 - 1)}{2R^{\text{CSI}}_2 \gamma^{\text{CSI}}_{G-i+1} \sin^2(\delta)};$
\item else if FP A then
\begin{enumerate}
\item $\alpha_1 = \frac{(2R^{\text{CSI}}_2 - 1)}{(2R^{\text{CSI}}_2 - 1 + 2R^{\text{CSI}}_1 (2R^{\text{CSI}}_1 - 1))};$
\end{enumerate}
\end{enumerate}
\end{enumerate}
\item $\alpha_2 = 1 - \alpha_1;$
\end{enumerate}
\end{algorithm}

Allocating power levels such that the probability of outage is same for both the users. Thus, for a given probability distribution function for $\gamma^{\text{CSI}}_i$, using (22)-(23), when the probability of outage is same for both the paired users, the following holds:

$$\frac{(2R^{\text{CSI}}_1 - 1)}{\alpha_1 \sin^2(\delta)} = \frac{(2R^{\text{CSI}}_2 - 1)}{(2R^{\text{CSI}}_2 - 1)} \sin^2(\delta). \quad (24)$$

Substituting $\alpha_2 = 1 - \alpha_1$ and solving it further, we obtain

$$\alpha_1 = \frac{(2R^{\text{CSI}}_1 - 1)}{(2R^{\text{CSI}}_2 - 1 + 2R^{\text{CSI}}_1 (2R^{\text{CSI}}_1 - 1))}. \quad (25)$$

Note that, we also obtain (24) by considering the upper bound in (19) equal to the lower bound in (25). Further, $\alpha_1$ obtained in (17) satisfies $0 < \alpha_1 < 1$. A pseud-code to implement the proposed AUP, MPA, and FPA is presented in Algorithm 1. The AUP algorithm with MPA and FPA requires sorting of the users, and hence, the complexity is of the order $O(G \log_2 G)$.

Next, we present the simulation results.

V. Numerical Results

For the evaluation, we have considered Poisson point distributed BSs and users with densities 25 BS/km$^2$ and 2000 users/km$^2$, respectively. Further, we have assumed $M = 8$, $N = 32$, $\delta = 11^\circ$, $R^{\text{OMA}}_1 = R^{\text{OMA}}_2$ and the urban cellular path loss model as presented in [10].

In Fig. 3, we present the comparison of data rates of strong and weak users for varying $\delta$. For analyzing the impact of $\delta$, we have considered a set of NOMA user pairs with $[\gamma^{\text{CSI}}_1, \gamma^{\text{CSI}}_2] = [8, 5]$ dB and $[\gamma^{\text{CSI}}_1, \gamma^{\text{CSI}}_2] = [8, 2]$ dB in Fig. 3a and Fig. 3b respectively. However, note that a similar behaviour holds for any NOMA user pair. In Fig. 3a and 3b, the minimum required rate for strong user ($R^{\text{OMA}}_1$) is same, however, its NOMA rates ($R^{\text{OMA}}_1$) vary as they depend on the SINR of the other paired user. Further, $\delta_{\text{UB}}$ is different for both the pairs as it is a function of individual SINRs of the users in the NOMA pair. Also note that, only when $\delta < \delta_{\text{UB}}$, the NOMA rates for both strong and weak users are better than the minimum required rates. Hence, for NOMA rates to be better than OMA rates, the base station should consider $\delta_{\text{UB}}$ while pairing the users.

In Fig. 4, we present the comparison of data rates with varying $\alpha_1$. We consider $[\gamma^{\text{CSI}}_1, \gamma^{\text{CSI}}_2] = [8, 5]$ dB with $\delta = 0^\circ$ and $11^\circ$ in Fig. 4a and 4b, respectively. Since the SINRs of the users in NOMA pair are same, the minimum required rates by the users ($R^{\text{OMA}}_1$ and $R^{\text{OMA}}_2$) are same in both the cases. However, the individual NOMA rates vary with $\delta$ and are better in case of $\delta = 0^\circ$. As shown in Fig. 4a and 4b, the power allocation $\alpha_1$ chosen by MPA is the upper bound, beyond which the data rates of weak user will be lower than the minimum required rate. Further, observe that ASR is a non-decreasing function as presented in (21). Hence, the ASR obtained at $\alpha_1_{\text{UB}}$ is always higher than $\alpha_1_{\text{LO}}$. Additionally, ASR is better when $\delta = 0^\circ$ in Fig. 4a as compared to $\delta = 11^\circ$ in Fig. 4b. In Fig. 4c and 4d, we consider $[\gamma^{\text{CSI}}_1, \gamma^{\text{CSI}}_2] = [8, 2]$ dB with $\delta = 0^\circ$ and $11^\circ$, respectively. Compared to Fig. 4a and 4b, the ASR at $\alpha_1_{\text{UB}}$ is higher in Fig. 4c and 4d. This is because, $\alpha_1_{\text{UB}}$ is comparatively less in the latter case, and hence, the interference observed by the weak user is less which results in better ASR. Note that with increasing $\delta$, the gap between the lower bound in (17) and the upper bound in (19) decreases, and thus, the gap between $\alpha_1_{\text{LO}}$ and $\alpha_1_{\text{UB}}$ also decreases.

In Fig. 5, we present the cumulative distribution function (CDF) of achievable data rates with various algorithms. With the Near-Far algorithm [9] and energy efficient (EE) algorithm EE-IRS-NOMA [2], the individual NOMA rates for some users are worse than the minimum required rates. However, when we consider the proposed AUP, the individual rates are never worse than the minimum required rates. Further, when AUP is used along with MPA, the strong users observe higher data rates and the weak users observe the minimum required data rates, as the algorithm allocates more power to the strong user. Even though Near-Far [9] has poor individual rates for some users, it has higher ASR as compared to the minimum required ASR. The EE-IRS-NOMA algorithm tries to maximize the data rates of the paired NOMA users, and thus, has higher ASR than the Near-Far algorithm. The proposed AUP with FPA and MPA algorithms have significant improvements in terms of ASR as compared to the minimum.
evaluation of Near-Far [9], we have assumed ASR, and hence, results in this unfairness. Further, in the allocation of power to the strong user to achieve higher is higher as compared to the strong user. This is because MP A

required ASR. Further, MPA has comparatively higher ASR than FPA, as it allocates more power to the strong user.

We have also evaluated the probability of outage and presented the results for the same in Fig. 6. It can be observed that AUP with FPA has similar levels of outage for both strong and weak users. In AUP with MPA, the outage for weak users is higher as compared to the strong user. This is because MPA allocates more power to the strong user to achieve higher ASR, and hence, results in this unfairness. Further, in the evaluation of Near-Far [9], we have assumed $\alpha_1 = \alpha_{1^Lh}$, and hence, it results in a higher outage for the strong user. Both Near-Far and EE-IRS-NOMA algorithms have higher overall outage than the AUP algorithms. As shown in Fig. 5 and 6, the proposed AUP with MPA and FPA outperform the state-of-the-art algorithms and provide trade-offs between maximum ASR and fairness.

VI. CONCLUSION

In this letter, we have derived bounds on the imperfection in the phase compensation and the power allocation factors for IRS-assisted NOMA systems. Using these bounds, we have proposed an adaptive user pairing algorithm to improve the achievable data rates of each user. We have then proposed power allocation algorithms to achieve maximum sum rate or ensure fairness, respectively. Through extensive simulations, we have shown that the proposed power allocation algorithms offer trade-offs between the achievable data rates and the fairness. In future, we plan to implement and validate the proposed algorithms on hardware testbeds.

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