ABSTRACT

It is pointed out that the universality might seriously be violated by models with several fixed points.

1. Multiple fixed points

The dependence of the fundamental equations and their parameters on the observational length scale is described by the renormalization group equations. In the vicinities of the fixed points the linearized version of the equations allow us to classify the relevant parameters in a systematic manner. This classification leads to renormalizable field theories as the largest class of models to consider.

Unfortunately the situation is more involved when the description of different interactions is sought in some unified theory. Consider for the sake of definiteness a Grand Unified Theory, GUT, defined at the energy range of $10^{15}$GeV. The interaction vertices of the lower energy effective theories, such as the electro-weak theory below 100GeV, QCD around few GeV, nucleon-meson vertices of nuclear physics and even the phenomenological coupling constants of condensed matter physics in the range of MeV and eV, respectively, are composite operators in terms of the elementary constituents of the GUT. The true coupling constant space where the renormalized trajectory generated by the Wilson-Kadanoff blocking procedure, [1], is given includes all of these parameters. The renormalized trajectory starts at the cut-off, $\Lambda > O(10^{15}GeV)$, at the GUT ultraviolet fixed point and arrives at the vicinity of the ultraviolet fixed point of the Standard Model for $\Lambda = O(10^{3}GeV)$. In fact, the theory exhibits asymptotic scaling in this energy range because the super heavy particles of the GUT scale have already decoupled and the high energy scaling of the Standard Model is manifest. When the energy is further decreased, at $\Lambda = O(1GeV)$, the heavy intermediate bosons and the Higgs particles decouple leaving behind the non-renormalizable Fermi contact interaction. The QCD evolution equations dominate the scale dependence in this range. Thus the renormalized trajectory is in the vicinity of the QCD fixed point at these energies.

This pattern repeats itself as we move towards lower energies. The higher energy (effective) theories generate effective vertices. The renormalizable vertices govern the renormalized trajectory at the infrared side of the lower energy fixed point. At the other, ultraviolet side of the low energy fixed point the relevant coupling constants are small and the irrelevant ones are dominant. The non-renormalizable, i.e. irrelevant vertices arising at the level of the lower energy effective theory are suppressed according to the decoupling theorem [2]. These vertices are nevertheless crucial for they are the seeds of the ”new physics” as seen from the low energy effective theory. In fact,
these are just the coupling constants which grow as we move towards high energy and deflect the renormalized trajectory from the ultraviolet fixed point of the low energy effective theory. In our case it is the Fermi type contact interaction which prevents the physics of few GeV energies to "fall" into the ultraviolet fixed point of QCD as the energy is raised. These fixed points which are in the vicinity of the renormalized trajectory generate cross-over phenomena when their characteristic scale is reached. It is worthwhile noting that there are additional fixed points as well which do not belong to either asymptotical regime of the effective theories. These correspond to critical phenomena occurring under special circumstances, such as finite temperature or densities. Finally there is an infrared fixed point, too.

The universality is the statement that the physics is insensitive to the initial, UV values of the irrelevant coupling constants. This is certainly true at the vicinity of the fixed point, more precisely in the infrared end of the renormalized trajectory within the range of the linearizability. But in the presence of several fixed points it is no longer clear that the irrelevant operators of a fixed point remain irrelevant at other fixed point. Thus we are left only with the "islands" of universality in the vicinities of different fixed points. The manner the sets of relevant coupling constants of different fixed points match is not at all described by universality.

There are two different ways to establish the connection between the coupling constants of the different fixed points. These will be briefly described below.

2. Scaling at the infrared fixed point

The straightforward procedure is to simply follow the evolution of the renormalized trajectory. If there are no further fixed points as we move towards lower energies then the irrelevance of the coupling constants at the last fixed point remain valid down to zero energy. But if there is a chain of fixed point in the vicinity of the renormalized trajectory as we seen before then an vertex which was irrelevant at high energy may become relevant at a lower energy fixed point. In this case the physics at the lower energy fixed point may in principle depend on the initial value of this coupling constant. Thus one has to introduce manifestly non-renormalizable coupling constants at the high energy fixed point in order to parametrize the physics at low energies.

In order to understand such a multi-fixed point situation better one should first consider a theory with two fixed points, one at the ultraviolet and the other at the infrared. The infrared fixed point is trivial, i.e. the its relevant operators are Gaussian in the presence of a mass gap. In fact, when the block size grows beyond of the Compton wavelength of the particle the fluctuations are suppressed and the evolution of the coupling constants slows down. In order to study the importance of a non-renormalizable coupling constant we need non-Gaussian relevant operator. This might be found in theories with massless excitations.

It has been argued in [3] by the help of the functional renormalization group equation [4] that the infrared fixed point of the linear O(N) model in the symmetry broken phase has nontrivial relevant operators which contain non-renormalizable vertices. The infrared divergences of the Goldstone modes pile up and make the beta function negative and divergent for the odd vertices of the massive particle. How many new important parameters are generated by this strong coupling vacuum of the Goldstone modes ? Unfortunately the linearization and the diagonalization of the renormalization group equation has not yet been done and the actual form of the scaling operators is not known. But it seems plausible to assume that the effect the new coupling constants generate is mainly in the deep infrared limit. This is because if they start with an O(1) value in the ultraviolet they are first
suppressed as we leave the vicinity of the ultraviolet fixed point and are amplified only in the asym-
totical infrared scaling regime. The non-perturbative infrared phenomenon which surely takes place
in the vacuum is the spontaneous breakdown of the symmetry and its parameter is the magnitude of
the condensate, $|\langle \phi_a \rangle|$. Thus it is reasonable to assume that the only impact these new coupling
constants have is that they influence the magnitude of the condensate. This influence is "hidden"
for the observations made at intermediate energy where these coupling constants are extremely small.
The value of the condensate naturally influences the dynamics of higher energies indirectly since it
determines the mass gap in the massive sector. Thus the complete set of parameters of the theory
should consists of the usual renormalizable coupling constants given at the ultraviolet regime and
the values of the nontrivial condensates which are manifest in the infrared. By the help of these two
sets of parameters we can parametrize the physics of the O(N) model.

3. Mini-instantons

Another less direct mechanisms which may couple the short and the long distance regime is
operating in theories with non-homogenous saddle points. As an example consider a classically scale
invariant theory with instantons [5]. The changing the instanton scale leaves the renormalized
action invariant. The actual saddle point expansion is performed in the bare cut-off theory so it is
the bare rather than the renormalized lagrangian what should be used. The bare regulated action
does have a scale parameter, the cut-off itself. Thus it is not at all obvious that the action is indeed
independent of the instanton scale. By adding appropriate irrelevant operators to the lagrangian,
such as higher derivative terms, the action of the saddle point as the function of the instanton size
may develop a minimum. The size of the saddle point at this minimum is in the order of magnitude
of the only scale parameter, the cut-off. If this minimum is small enough then these "topological
defects" [6] may form a gas with divergent density.

Qualitatively similar phenomenon, namely the population of the vacuum by small localized
states has already been conjectured in strong coupling massless QED. There the $e^-e^+$ bound states
with the size of the cut-off condense and change the long distance physics by the dynamical breaking
of the chiral symmetry [7]. What is interesting in the mini-instanton gas is that it corresponds to
time dependent coherent states close to the cut-off without relying on strong coupling constant. In
fact, the mini-instantons are present in asymptotically free theories such as the O(3) model or QCD.
The important question is whether they can modify the scaling at low energy like the singular $e^-e^+$
pairs of chiral QED.

Though a single mini-instanton can cause no harm to the renormalized trajectory at a finite
i.e. cut-off independent energy, the gas of these saddle points might well modify the evolution of the
renormalized coupling constants at finite energies. This may happen because the change of the cut-off
generates two qualitatively new effects in the framework of the dilute instanton gas approximation.
A tree-level cut-off dependence arises from the saddle-point action. The one-loop dependence comes
from the small fluctuation determinant. When the instantons are well inside of the infrared regime
compared to the cut-off then this dependence agrees with the usual renormalized trajectory. But the
mini-instantons remain in the vicinity of the cut-off for a large part of the renormalized trajectory
and they continue to modify the trajectory.

The irrelevant terms of the action come from the next fixed point in the ultraviolet direction
as explained above. The higher energy theory cuts off the low energy effective model and provides
the irrelevant operators which appear as regulator in the low energy theory. The dimension of
the irrelevant coupling constants is given by the cut-off, i.e. the scale of the higher energy fixed point. The irrelevant operators represent the W and Z exchanges in QCD. Thus the mini-instantons saturate the path integral by the fluctuations of this higher energy fixed point. If their influence remain finite at low energy they represent a non-perturbative bridge between neighboring fixed points.

The ratio of the partition function in the one instanton and the flat sector, \( Z_1/Z_0 \), was computed in the two dimensional sigma model in the presence of higher order derivatives in [8]. If the higher order derivative terms are really irrelevant then this infrared quantity should be insensitive to the choice of the new coupling constants at the cut-off. This was not the case, \( Z_1/Z_0 \) displayed a clear dependence on the higher order terms in the action. Let us impose the renormalization condition that \( Z_1/Z_0 \) is cut-off independent. Then by assuming that the new coupling constants remain cut-off independent along the renormalized trajectory one can obtain the beta function for the usual coupling constant of the sigma model. The resulting beta function is negative and asymptotic freedom is preserved but its first coefficient which is given by the tree level mini-instanton action is not universal. For sufficiently large values of the higher derivative operators the mini-instanton action may even become negative. In this case the vacuum is a densely packed instanton-anti instanton gas, similarly an anti-ferromagnetic system. Saddle point expansion is unreliable and the asymptotic freedom is not guaranteed any more in this phase.

The phenomenon found in the sigma model is likely to be present in QCD, too. In the framework of the saddle point expansion one expects three new relevant parameters. These are the size of the min-instanton, its action and the second derivative of the action with respect to the instanton size. These parameters are determined at the cut-off by the dynamics of the higher fixed point. Their appearance in the beta function at finite energies represent the coupling between different fixed points.

Naturally one has to be careful and the final conclusion should be drawn only after computing Green functions in the infrared regime on the mini-instanton background. This is a formidable task in an asymptotically free model.

4. Conclusions

The nontrivial interplay of different fixed point of the renormalization group was briefly discussed in this talk from the point of view of the unified models. There might be another motivation as well to consider this problem in theories with nontrivial infrared dynamics. The taking into account such effect may provide a clue to follow the renormalized trajectory to regions where the usual perturbation expansion fails. Consider for example QCD [9]. The gauge coupling constant blows up according to the one-loop renormalization group equation at the cross-over separating the ultraviolet and the infrared scaling regime. The Wilson-Kadanoff renormalization group procedure suggests the inclusion of all coupling constant which might be relevant for the evolution into the action. This means the inclusion of the hadronic effective vertices together with the gauge coupling constant. This would be double counting in the hamiltonian formalism where we would use hadrons and quark-gluon states simultaneously. But this is not the case with the path integral and we should be able to pass the cross-over regime with this extended QCD without seeing diverging coupling constants. The onset of the hadronic world as we enter into the infrared scaling regime would be achieved by the hadronic vertices and the gauge coupling constant could remain finite.

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