Investigation of Mixed Symmetry States in 170-178Yb isotopes

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Abstract: low-lying positive parity states, dynamic symmetries, mixed symmetry states (MSS), reducing electric quadrupole transition probabilities B(E2), branching ratio, reducing magnetic dipole transition probabilities B(M1), and mixing ratios δ(E2/M1) for 170-178Yb have been investigated by applied IBM-1 and IBM-2 program package. The software package IBM-1 and IBMMT computer code for IBM-1 and Neutron Proton Boson NPBOS and Neutron Proton Boson Electromagnetic NPBCM software package have been used. The Ytterbium nuclei with the (170 – 178) mass number considered as rotational nuclei after applying the first and second interacting boson model with studying dynamic symmetry and energy ratios (E4+/E2+), (E6+/E2+) and (E8+/E2+) that approximated to (3.33, 7 and 12). The reducing transition probability for electric quadrupole B(E2) and branching ratios that state the small values of the ratios R’ and R” as having rotational characteristics. A study of the mixed symmetry state of these nuclei shows that the lower energy mixing level is the first 1+ level, which distinguishes the rotational determination nuclei. The energy levels values for the states 2+, 3+, 4+, 1+ have a clear mixed symmetry state (MSS) directly proportional with Majorana parameters ζ2, while 2+, and 2++ lead to be approximately more conservative.

1. Introduction

The interacting bosons model is an effective method to describe the behavior of nuclei and knowing their characteristics, as it is considered one of the most flexible models in nuclear structure to describe a wide range of nuclei have the medium and heavy mass number, with its three rotational, vibratory, unstable, γ unstable limits and transitional regions. The medium heavy rare-earth Ytterbium nuclei with the neutron number between 100 and 108 have decay scheme characterized by small and nearby energy levels in the ground band which generally as rotational nuclei. 170-178Yb isotopes have Z=70, then 6 hole bosons the number of protons and neutrons lying between 50, 82 and 82, 126 magic shells, respectively. The 170,172Yb isotopes have 100,102 neutrons which mean (9,10) particle neutron bosons with total boson numbers 15,16 respectively. The 176,178Yb isotopes have 106 and 108 neutrons which mean (10,9) hole neutron bosons, the nucleons, and 16, 15 boson numbers respectively. There are many studies and research that attempted to understand and explain the behavior of 170-178Yb nuclei by using different models[1-5], in the present research the low –lying positive parity states, dynamic symmetries, mixed symmetry states (MSS), and reducing transition probabilities for electric quadrupole B(E2), branching ratio, magnetic dipole B(M1), and mixing ratios δ(E2/M1) 170-178Yb have been investigated by applying IBM – 1 and IBM – 2 program package.
2. Theoretical Part

The IBM is built on a closed shell i.e., the total number of bosons \((N)\) depends on the number of active nucleon particle (or hole) pairs outside a closed shell. Each type of bosons, the s- and d-boson, has its binding energy with regard to the closed shell [6]. The IBM does not distinguish between proton and neutron bosons; the total number of bosons \((N = n_p + n_n)\) is finite and conserved in a given nucleus. The \((N = 0)\) and \((N = 2)\) bosons of the IBM have six sub-states; therefore, it defines a six-dimensional space so that one can describe it in terms of the unitary group in six dimensions, \(U(6)\). This leads to drive many of the characteristic properties of the IBM-1 by group theoretical methods and express it analytically[7-9]. The Hamiltonian operator according to IBM-1 describes the system of s and d boson can be written as [10-13]:

\[
H = \epsilon n_d + a_0 \rho^T + a_1 L . L + a_2 Q . Q + a_3 T_3 T_3 + a_4 T_4 T_4
\]

(1)

where \(\epsilon = \epsilon_s - \epsilon_d\) is the boson energy, for simplicity \(\epsilon_d\) is set equal to zero only \(\epsilon = \epsilon_s\) appears \(a_0, a_1, a_2, a_3, a_4\) designate the strengths of the quadrupole, angular momentum, pairing, octupole and hexadecapole interacting between bosons respectively. The electromagnetic transition operators has been given as [6-13]:

\[
T_m^d = \alpha_2 \delta_{l2} [d^T s + s^T d]^{(2)} + \beta_1 [d^T d]^{(1)} + \gamma_0 \delta_{l0} \delta_{m0} [s^T s]^{(0)}
\]

(2)

where \(\alpha_2, \beta_1, \gamma_0\) are the coefficients of the various terms in the operators. The \(M1\) operator has been given by [12,14]:

\[
T_{M1}^d = \beta_1 [d^T d]^{(1)}
\]

(3)

The magnetic transition will occur only for d-boson since the s-d interaction will vanish. The angular momentum operator and \(g\) – factor have been written as [13,15]

\[
T_{m}^{M1} = \sqrt{3/4\pi} g_B L, g_B = \beta_1 / 10 \sqrt{4\pi/3} L
\]

(4)

where \(g_B\) is the effective boson \(g\) – factor, the factor \(\sqrt{3/4\pi}\) has been introduced to conform with standard notation since the operator \(L\) is diagonal in any basis \(M1\) transition can occur in this approximation, the diagonal matrix element depends only on \(L\) and is the same for all three cases they given by[12,16]:

\[
\langle L' | T^{M1} | L \rangle = \sqrt{3/4\pi} g_B \sqrt{[L(L + 1)(2L + 1)]}
\]

(5)

In terms of magnetic moment \((\mu L)\) [14,17]

\[
\eta_B = \mu_L / L
\]

(6)

The states \(g\) – factor defined as [18]:

\[
g_B = \mu_L / L, \text{ where } g_1 = g_B
\]

(7)

The magnetic dipole transition probability can be given by the form [18]:

\[
B(M1; L' \rightarrow L) = \frac{1}{2L + 1} |\langle L' | T^{M1} | L \rangle|^2
\]

(8)

The electric quadrupole \(T(E2)\) operator can be taken from equation (2) as [6]:

\[
T_{m}^{E2} = \alpha_2 [d^T s + s^T d]^{(2)} + \beta_2 [d^T d]^{(2)}
\]

(9)

where \(\alpha_2\) is the effective charge of s and d bosons, \(\beta_2\) is the effective charge of d- boson. The selection rules of the electric quadrupole operator for three dynamical symmetries (Chain I, II, and III) are as follows [6-13].

Chain I; SU(5): \(\Delta n d = 0, \pm 1\);

Chain II; SU(3): \(\Delta l = 0, \Delta \mu = 3\);
Chain III: O(6) : \( \Delta \sigma = 0, \Delta \tau = \pm 1 \).

The electric quadrupole transition rates are governed by \( B(E2) \) values, defined as [8]

\[
B(E2; L_i \rightarrow L_f) = \frac{1}{2L_i+1} |\langle L_f | J^M_1 | L_i \rangle|^2
\]

(10)

where \( L_i \) is the initial angular momentum and \( L_f \) is the final angular momentum. The (Branching Ratios) of the reduced transition probabilities for electric quadrupole transition of the dynamical symmetry SU(5), SU(3), and O(6) obey the following relations [6-10]:

\[
R = \frac{\langle E2; A^+ \rightarrow A^+ \rangle}{\langle E2; A^+ \rightarrow A^+ \rangle}
\]

(11)

\[
R' = \frac{\langle E2; A^+ \rightarrow A^+ \rangle}{\langle E2; A^+ \rightarrow A^+ \rangle}
\]

(12)

\[
R'' = \frac{B(E2; A^+ \rightarrow A^+)}{B(E2; A^+ \rightarrow A^+)}
\]

(13)

they can be written in terms of the total boson number \( N \) [13]:

\[
R = R' = R'' < \frac{2(N-1)}{\nu} < 2 \; ; \; SU(5)
\]

(14)

\[
R \approx \frac{10}{\varrho} ; R' = R'' \approx 0 \; ; \; SU(3)
\]

(15)

\[
R = R' < \frac{10}{\varrho} \; ; \; R'' = 0 \; ; \; O(6)
\]

(16)

The Hamiltonian operator in \( \{BM-\frac{1}{2}\} \) will have three parts: one part for each of proton and neutron bosons and a third part for describing the proton-neutron interaction, i.e.

\[
H = H_p + H_\nu + V_{p\nu}
\]

(17)

A simple schematic Hamiltonian guided by microscopic consideration is given by [12]:

\[
H = \varepsilon (n_p + n_\nu) + \mu \sigma \tau + Q_\nu + V_{p\nu} + V_{\nu\nu} + M_{p\nu}
\]

(18)

where

\[
Q_\rho = (d^+_\rho s^+_{\rho} + s^+_{\rho} d^+_{\rho})^2 + \chi_\rho (d^+_{\rho} d^+_{\rho})^2 \; \rho = p, \nu
\]

(19)

\[
V_{\rho\rho} = \sum_{\sigma=0,2,4} \frac{1}{2} \left( 2L + \frac{1}{2} \right) E_L (d^+_{\sigma\rho} (d^+_{\rho})^0(L) (d^+_{\rho})^0(L))^0
\]

(20)

and \( \epsilon_p, \epsilon_\nu \) are proton and neutron energies respectively and they assumed equal \( \epsilon_p = \epsilon_\nu = \epsilon \). The last term in Eq.(18) contains the Majorana operator \( M_{p\nu} \) and it is usually added to remove states of mixed proton-neutron symmetry. This term can be written as [13,18]

\[
M_{p\nu} = \xi_2 (s^+_{\nu} d^+_{\pi} - d^+_{\nu} s^+_{\pi})^0(L) \left( (s^+_{\nu} d^+_{\pi} - d^+_{\nu} s^+_{\pi})^0(L) + \sum_{k=1,3} \xi_k (d^+_{\nu} d^+_{\pi})^k - (d^+_{\nu} d^+_{\pi})^k \right)
\]

(21)

If there is an experimental evidence for the so-called "mixed symmetry" state, then the Majorana parameters are varied to fix the location of these states in the spectrum. The energy levels are obtained by diagonalizing the Hamiltonian Eq.(18) and allowing the parameters \( \varepsilon, \kappa, \chi_\pi, \chi_\nu \) and \( \xi \) to vary until one obtains the best fit to the experimental spectrum.

Using Eq. (18), it is possible to obtain spectra that are similar to those of the IBM-1 with only one kind of boson [18]. The \( U(5) \) limit when \( \varepsilon \gg \kappa \), the \( SU(3) \) limit when \( \varepsilon \ll \kappa \) and \( \chi_\pi = \chi_\nu = - \frac{7}{2} \), and \( O(6) \) limit when \( \varepsilon \ll \kappa \) and \( \chi_\pi = \chi_\nu \). Most nuclei do not strictly belong to any of these three limiting cases, but are somewhere between two of them. In the IBM, it is possible to make a smooth transition between the limiting cases for a series of isotopes. The general single boson transition operator of
angular momentum $\ell$ has the same form as in Eq. (2) in IBM-1 except for the fact that in each term one has to consider $\pi, \nu$ degree of freedom and this can be written as [12]

$$T^{(\ell)} = \alpha_{2\rho} \delta_{\rho 2} [d^+ s + s^+ d(\ell)] + \beta_{\rho 3} [d^- s(\ell)] + \gamma_{\rho 4} \delta_{\rho 0} s^+ s(\ell) ... \rho = \pi \text{ or } \nu \quad (22)$$

This equation yield transition operators for $E1, M1, E2, M3,$ and $E4$. For $E2$ operator [13]

$$T^{E2} = e_\pi Q_\pi + e_\nu Q_\nu$$

Where $Q_\pi$, $Q_\nu$ is the same as in Eq. (19) and $e_\pi$, $e_\nu$ are boson effective charges depending on the boson number $N$ and they can take any value to fit the experimental result. The values of effective charge are sensitive to the nuclear structure while the ratio $e_\pi/e_\nu$ depends only on the assumption of maximal $F$ spin for $0^+_\pi$ and $1^+_\nu$ states. The two effective charges $e_\pi$ and $e_\nu$ can obtained [19] by using boson number, and experimental $(2^{-}; 2^{-} \rightarrow 0^{+})$ values for a series of isotopes to produce a plot between $\gamma$ and $n$ where defined as [18-21]

$$M = N_\pi [\ln B(E2; 2^+_\pi \rightarrow 0^+_\pi)]^{1/2} \quad (eb \text{ in } U(5)) \quad (24)$$

$$M = N_\pi \ln (2N + 1)^{-1} \quad B(E2; 2^+_\pi \rightarrow 0^+_\pi)]^{1/2} \quad (eb \text{ in } U(3)) \quad (25)$$

$$M = N_\pi \ln (2N + 1)^{-1} \quad B(E2; 2^+_\pi \rightarrow 1^+_\pi)]^{1/2} \quad (eb \text{ in } O(6)) \quad (26)$$

The best straight line through the experimental point is obtained with slope $e_\pi$ and intercept $e_\nu$. The $M1$ operator obtained by letting $\ell = 1$ in Eq. (22) [18,19] as:

$$T^{(M1)} = \left[ \frac{3\ell + 1}{4\ell + 1} \right] (g_\pi L^{(1)}_{\pi} + g_\nu L^{(1)}_{\nu}) \quad (27)$$

Where $g_\pi$, $g_\nu$ are the boson $g$ factor in unit $\mu N$ and $t^{(1)} = \sqrt{11/12} (d^+ d)^{(1)}$, then

$$T^{(M1)} = \left[ \frac{3\ell + 1}{4\ell + 1} \right] (g_\pi + g_\nu) (L^{(1)}_{\pi} + L_{\nu}^{(1)}) + \frac{1}{2} (g_\pi - g_\nu) (u^{(1)}_{\pi} - u^{(1)}_{\nu}) \quad (28)$$

The first term on the right hand side of this equation is diagonal; therefore, for $M1$ transition the previous equation may be written as

$$T^{(M1)} = 0.77 [(d^+ d)^{(1)}_{\pi} - (d^+ d)^{(1)}_{\nu}] \quad (g_\pi - g_\nu) \quad (29)$$

Few experimental absolute $B(M1)$ transition are available to test IBM predictions, the total $g$ factor is defined by [21,22]:

$$g = \frac{g_\pi n_{\pi}}{n_{\pi} + n_{\nu}} + \frac{g_\nu n_{\nu}}{n_{\pi} + n_{\nu}} \Rightarrow g = \frac{(g_\pi n_{\pi} + g_\nu n_{\nu})}{(n_{\pi} + n_{\nu})} \quad (30)$$

where $Z$ is the atomic number, and $A$ is the mass number, $g_\pi$ and $g_\nu$ are obtained by using the same method which used to calculate the effective charges $e_\pi$ and $e_\nu$. The $M1$ strength many be expressed in terms of the multipole mixing

$$\delta \left( \frac{E2}{M1} \right) = 0.35 E_\nu (Me \text{. A, } B = \frac{[u^{(1)}_{\pi} - u^{(1)}_{\nu}]}{[u^{(1)}_{\pi} - u^{(1)}_{\nu}]}) \quad (31)$$

by having fitted $E2$ matrix elements, one can then use them with $\delta (E2/M1)$ to obtain $M1$ matrix element and compare them with the matrix elements predicted by the model using the operator eq. (31). An important property of this new version is that the proton-neutron symmetry character of each state is specified in terms of a new quantum number called $F-$spin [23]. The $F-$spin raising, lowering, and $F-$component operators given in [24] as:

$$F_+ = d^{+}_\pi d_{\nu} + s^{+}_\pi s_{\nu} \quad F_- = d^{+}_\nu d_{\pi} + s^{+}_\nu s_{\pi} \quad (32)$$
Proton boson have an intrinsic quantity, $F - 1/2$ which can be introduced in a formalism similar to that used for isotopic spin [2,24]. Then proton bosons will have $F$ projection $F_L = +1/2$ while neutron bosons have $F_L = -1/2$, thus one can write an $F$ spin multiple as

$$|\pi\rangle = \frac{1}{2}(+1\rangle + \frac{1}{2}\langle -1\rangle, \quad |\nu\rangle = \frac{1}{2}(+1\rangle - \frac{1}{2}\langle -1\rangle$$

The total symmetric states have the maximum value $F$ spin $F_{max} = (N_\pi + N_\nu)/2$ while the mixed symmetry states characterized by decreasing $F$ spin value $F_{max} = |N_\pi + N_\nu|/2 - |N_\pi|/2$ (2). $F = F_{max} - 2, \ldots, F_{min} = |N_\pi - N_\nu|/2$ bosons is fully symmetric under the interchange of neutron and proton bosons if $\chi$ has maximal $F$ spin $F_{max} = N/2$. A state with only $s$-boson is naturally fully symmetric and has $F = N/2$ it should be noted that the state of maximum $F – 2$ spin are in one to one correspondence with the states of IBM-1 while the states with $F – 2$ spinless than the maximum value of $N/2$ having no counterparts in IBM-1. They have mixed proton-neutron symmetry character, thus they are called mixed symmetry states. These states are considered as the main interest in studying the IBM-2 [23]. Mixed symmetry states have been observed in many even-even nuclei between the mass numbers 50 and 240. This urges when the movement of protons and neutrons forming collective excitation are not in one phase, it has been observed in the three limits of collective excitations in vibrational rotational and $\chi$-unstable nuclei the lowest energy to mixed symmetry states is first $1^+$ level at about 3MeV in rotational nuclei while lowest energy mixed symmetry states in vibrational and $\chi$-unstable is around 2 MeV [23].

### 3. Results and discussion

IBM-1 and IBM-2 software package have been used to estimate a set of parameters described in the Hamiltonian operator as it is shown in equations (1) and (17). The parameters estimated for the low-lying calculations of the excited energy levels for Ytterbium isotopes are given in table (1)

| Isotopes | IBM-1 parameters in MeV unless $\chi$ |
|----------|--------------------------------------|
| $^{170}$Yb | $N_{tot}$ $\tau$ $a_0$ $a_1$ $a_2$ $a_3$ $a_4$ $\chi$ |
| $^{172}$Yb | 15 0.0 0.0 0.00945 0.0122 0.0 0.0 -1.31 |
| $^{174}$Yb | 16 0.0 0.0 0.0088 0.0112 0.0 0.0 -1.31 |
| $^{176}$Yb | 16 0.0 0.0 0.0092 0.012 0.0 0.0 -1.31 |
| $^{178}$Yb | 15 0.0 0.0 0.0087 0.0142 0.0 0.0 -1.31 |

| Isotopes | IBM-2 parameters in MeV unless $\chi$, $\chi_{0} = -1.24$, $N_{s} = 6$ |
|----------|---------------------------------------------------------------|
| $^{170}$Yb | $N_{s}$ $\epsilon_{s}$ $\kappa$ $\chi_{s}$ $\zeta_{2}$ $\zeta_{1,3}$ $C_{\nu}^{L}$ $C_{\pi}^{L}$ |
| $^{172}$Yb | 9 0.618 -0.07 -1.24 0.01 0.0 0.08,0.08,0.0 0.4,0.08,0.0 |
| $^{174}$Yb | 10 0.6 -0.064 -1.24 -0.01 0.01 0.08,0.06,0.0 0.4,0.08,0.0 |
| $^{176}$Yb | 10 0.61 -0.067 -1.24 -0.01 0.01 0.08,0.08,0.0 0.4,0.08,0.0 |
| $^{178}$Yb | 9 0.61 -0.075 -1.24 -0.01 0.02 0.08,0.08,0.0 0.4,0.08,0.0 |

One of the most important tests have been carried out on the nuclei is the calculation of energy ratios that are a good indication and a guide for the determination symmetries region that the isotope belongs. $(E_{1}^{1}/E_{1}^{2})$, $(E_{9}^{1}/E_{3}^{2})$ and $(E_{9}^{1}/E_{3}^{2})$ have been calculated for $^{176,178}$Yb isotopes put in Figure (1), That leads to guessing the nearest dynamic symmetries which correspond to the characterizes of $^{170-178}$Yb isotopes [12].
Figure 1: The experimental[25-29], theoretical and standard[18] energy ratios ($E_{4+/2+}$, $E_{6+/2+}$, and $E_{8+/2+}$) respectively as a function of mass numbers for $^{170-178}$Yb isotopes. The calculated energy levels compared with experimental data [24-29] $^{170-178}$Yb isotopes have been shown in Figure (2).
The most important features of IBM-2 was the possibility of clarification of mixed symmetry states in even-even nuclei which made up from mix up of the protons and neutrons waves functions. In more vibrational and gamma soft nuclei the lowest MS states with $J = 2^+$, in the rotational nuclei observed as the $J = 1^+$ states. The effect of Majorana parameters ($\zeta$) on the calculated excitation energy levels for $^{170-178}$Yb isotopes has been investigated by vary the $\zeta$ around the best-fitted with experimental data as shown in figure (3).
Figure 3: Mixed symmetry states $M_{ss}$ as a function $\zeta_4$ for $^{170-178}\text{Yb}$ isotopes.

The effective boson charges have been estimated from equations (9) and (23) to calculate $B(E2)$ transition probabilities, theoretical branching ratios. $B(E2)$ compression with the experimental values [25-31] for $^{170-178}\text{Yb}$ isotopes were listed in tables (2) and (3).

Table 2: The used effective boson charges used in IBM-1 and IBM-2 to calculate $B(E2)$ transition probabilities for $^{170-178}\text{Yb}$ isotopes and branching ratios.

| Isotopes | IBM-1 | IBM-2 |
|----------|-------|-------|
| $E2SD$   | $E2DD$| $e_\pi$| $e_\nu$|
| $^{170}\text{Yb}$ | 0.107 | -0.1398 | 0.06 | 0.22 |
| $^{172}\text{Yb}$ | 0.104 | -0.137 | 0.061 | 0.22 |
| $^{176}\text{Yb}$ | 0.099 | -0.127 | 0.049 | 0.22 |
| $^{178}\text{Yb}$ | 0.11 | -0.143 | 0.06 | 0.22 |

| Isotopes | R | IBM-1 | IBM-2 | R' | IBM-1 | IBM-2 | R'' | IBM-1 | IBM-2 |
|----------|---|-------|-------|----|-------|-------|-----|-------|-------|
| $^{170}\text{Yb}$ | -- | 1.141 | 1.428 | -- | 8.5x10^{-6} | 0.0014 | -- | 0 | 0.0022 |
| $^{172}\text{Yb}$ | 1.42 | 1.412 | 1.41 | -- | 1.9x10^{-6} | 0.0009 | -- | 0 | 0.0019 |
| $^{176}\text{Yb}$ | 1.445 | 1.412 | 1.41 | -- | 3.5x10^{-5} | 0.0029 | -- | 2.7x10^{-7} | 0.0014 |
| $^{178}\text{Yb}$ | -- | 1.41 | 1.407 | -- | 1.4x10^{-5} | 0.0011 | -- | 3.3x10^{-5} | 0.00138 |

* $E2SD$ and $E2DD$ are IBM7 parameters where $\alpha$ and $\beta$ are the boson effective charge for IBM1, $E2SD = \alpha_2$, $E2DD = \sqrt{5}\beta_2$ where $\beta_2 = -\frac{\sqrt{5}}{\sqrt{2}}\alpha_2$, $\beta_2 = \sigma$ in $U(5) SU(3)$, and $O(6)$ limits respectively.
Table 3: Calculated reduced electric quadrupole transitions probability $B(E2)$ values in ($e^2b^2$) unit compression with experimental data [24-30] for $^{170-178}$Yb isotopes.

| Isotopes  | $^{170}$Yb | IBM-1 | IBM-2 | $^{172}$Yb | IBM-1 | IBM-2 |
|-----------|------------|-------|-------|------------|-------|-------|
| 2$^+ \rightarrow 0^+$ | 1.124     | 1.124 | 1.126 | 1.204     | 1.208 | 1.203 |
| 4$^+ \rightarrow 2^+$ | --        | 1.59  | 1.6   | 1.71      | 1.71  | 1.697 |
| 6$^+ \rightarrow 4^+$ | --        | 1.719 | 1.74  | 1.818     | 1.853 | 1.843 |
| 8$^+ \rightarrow 6^+$ | 1.697     | 1.75  | 1.693 | 2.27      | 1.893 | 1.896 |
| 10$^+ \rightarrow 8^+$ | 1.678     | 1.73  | 1.68  | 2.1       | 1.883 | 1.958 |
| 0$^+ \rightarrow 2^+$ | --        | 0.0   | 0.0025| --        | 0.0   | 0.0022|
| 2$^+ \rightarrow 0^+$ | --        | 0.0076| 0.201 | --        | 0.0072| 0.265 |
| 2$^+ \rightarrow 2^+$ | --        | 0.0000096| 0.00163| --        | 0.0000023| 0.0011 |
| 4$^+ \rightarrow 2^+$ | --        | 0.542 | 0.57  | --        | 0.59  | 0.609 |
| 6$^+ \rightarrow 4^+$ | --        | 1.048 | 1.147 | --        | 1.144 | 1.385 |
| 8$^+ \rightarrow 6^+$ | --        | 1.229 | 1.272 | --        | 1.346 | 1.385 |
| 10$^+ \rightarrow 8^+$ | --        | 1.285 | 1.331 | --        | 1.415 | 1.478 |
| 3$^+ \rightarrow 2^+$ | --        | 1.64  | 1.446 | --        | 1.786 | 1.444 |
| 5$^+ \rightarrow 3^+$ | --        | 0.859 | 0.982 | --        | 0.937 | 1.032 |
| 7$^+ \rightarrow 5^+$ | --        | 1.154 | 1.168 | --        | 1.263 | 1.232 |
| 9$^+ \rightarrow 7^+$ | --        | 1.253 | 1.287 | --        | 1.379 | 1.373 |
| 4$^+ \rightarrow 2^+$ | --        | 1.293 | 0.994 | --        | 1.411 | 1.306 |
| 6$^+ \rightarrow 4^+$ | --        | 1.388 | 1.267 | --        | 1.519 | 1.404 |
| 8$^+ \rightarrow 6^+$ | --        | 1.397 | 1.423 | --        | 1.538 | 1.5   |
| 10$^+ \rightarrow 8^+$ | --        | 1.36  | 1.517 | --        | 1.509 | 1.622 |
| 3$^+ \rightarrow 1^+$ | --        | 0.842 | --    | --        | --    | 0.875 |
| 1$^+ \rightarrow 2^+$ | --        | 0.032 | --    | --        | --    | 0.033 |
| 2$^+ \rightarrow 0^+$ | --        | 1.0001| --    | --        | --    | 0.989 |

| Isotopes  | $^{170}$Yb | IBM-1 | IBM-2 | $^{171}$Yb | IBM-1 | IBM-2 |
|-----------|------------|-------|-------|------------|-------|-------|
| 2$^+ \rightarrow 0^+$ | 1.072     | 1.076 | 1.074 | --         | 1.184 | 1.182 |
| 4$^+ \rightarrow 2^+$ | 1.55      | 1.524 | 1.565 | --         | 1.675 | 1.66  |
| 6$^+ \rightarrow 4^+$ | 1.71      | 1.651 | 1.706 | --         | 1.811 | 1.796 |
| 8$^+ \rightarrow 6^+$ | 1.72      | 1.687 | 1.723 | --         | 1.844 | 1.839 |
| 10$^+ \rightarrow 8^+$ | --        | 1.677 | 1.74  | --         | 1.825 | 1.882 |
| 0$^+ \rightarrow 2^+$ | --        | 0.0   | 0.0016| --         | 0.0   | 0.0016|
| 2$^+ \rightarrow 0^+$ | --        | 0.0064| 0.198 | --         | 0.008 | 0.267 |
| 2$^+ \rightarrow 2^+$ | --        | 0.000038| 0.0031| --         | 0.000017| 0.00137|
| 4$^+ \rightarrow 2^+$ | --        | 0.526 | 0.531 | --         | 0.571 | 0.585 |
| 6$^+ \rightarrow 4^+$ | --        | 1.021 | 1.202 | --         | 1.021 | 1.188 |
| 8$^+ \rightarrow 6^+$ | --        | 1.201 | 1.225 | --         | 1.201 | 1.235 |
| 10$^+ \rightarrow 8^+$ | --        | 1.263 | 1.309 | --         | 1.263 | 1.298 |
| 3$^+ \rightarrow 2^+$ | --        | 1.593 | 1.176 | --         | 1.593 | 1.427 |
| 5$^+ \rightarrow 3^+$ | --        | 0.836 | 0.909 | --         | 0.836 | 1.01  |
| 7$^+ \rightarrow 5^+$ | --        | 1.127 | 1.221 | --         | 1.127 | 1.36  |
| 9$^+ \rightarrow 7^+$ | --        | 1.23  | 1.34  | --         | 1.23  | 1.329 |
Magnetic properties of nuclear states are effective probes of nuclear wave functions. The nuclei near closed shells mainly determine the factors by single particle motion and configuration mixing. In the deformation nuclei, the collective motion of protons and neutrons is responsible for the factors values. $B(M1)$ transition probability has been calculated using the effective factors for proton $\pi$ and neutron $\nu$, for $^{170,178}$Yb isotopes $\pi = (0.51\mu_N)$ and $\nu = (0.26\mu_N)$. Eq. (3) have been used in IBM-2 to calculate the $B(M1)$ transition probabilities and $\delta(E2/M1)$ as shown in table (4). The calculated values for $B(M1)$ and the mixing ratio $\delta(E2/M1)$ have been compared with the available experiments data[25-29].

### Table 4: Comparison between calculated magnetic transitions in unit $\mu_N$ and mixing ratio $\delta(E2/M1)$ with the experimental data [25-29,31] $^{178}$Yb isotopes.

| Isotopes | $f \rightarrow l$ | $^{2}_{1}$ | $^{2}_{2}$ | $^{2}_{3}$ | $^{2}_{4}$ | $^{2}_{5}$ | $^{3}_{1}$ | $^{3}_{2}$ | $^{3}_{2}$ | $^{5}_{2}$ | $^{5}_{4}$ | $^{5}_{4}$ | $^{3}_{3}$ | $^{4}_{1}$ | $^{4}_{1}$ |
|----------|------------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|---------|-----|
|          | $B(M1)$          |            |            |            |            |            |            |            |            |            |            |            |         |     |
|          | Exp.             | IBM-2      | Exp.       | IBM-2      | Exp.       | IBM-2      | Exp.       | IBM-2      | Exp.       | IBM-2      | Exp.       | IBM-2      | Exp.    | IBM-2|
| $2_{1} \rightarrow 1_{1}^+$ | 0.0175 | -- | 1.0729 | -- | -- | -- | 9.83×10⁻⁹ | -- | -- | -- | -- | -- | -- |
| $2_{2} \rightarrow 1_{1}^+$ | 0.00069 | -- | 0.03288 | -- | -- | -- | 0.0014 | -- | -- | +0.16 | -- | -- | -- |
| $2_{2} \rightarrow 2_{2}^+$ | 0.0003 | -- | 0.0073 | -- | -- | -- | 0.0009 | -- | -- | -- | -- | -- | -- |
| $2_{3} \rightarrow 2_{3}^+$ | 0.000327 | -- | 0.0207 | -- | -- | -- | 0.021 | +14.6×10⁻⁹ | -- | -- | -- | -- | -- |
| $3_{1} \rightarrow 2_{2}^+$ | 0.0193 | -- | +1.17 | -- | -- | -- | 0.019 | -2.7 | +1.15 | -- | -- | -- | -- |
| $4_{1} \rightarrow 3_{1}^+$ | 0.04559 | -- | +0.19 | -- | -- | -- | 0.045 | +13×10⁻⁹ | +1.0049 | -- | -- | -- | -- |
| $5_{1} \rightarrow 4_{1}^+$ | 0.0209 | -- | 0.0015 | -- | -- | -- | 0.021 | +1.43 | -0.078 | -- | -- | -- | -- |
| $5_{2} \rightarrow 4_{1}^+$ | 3.064E-05 | -- | 1.61×10⁻⁵ | -- | -- | -- | 2.33×10⁻⁵ | -- | -- | -- | -- | -- | -- |

| Isotopes | $f \rightarrow l$ | $^{3}_{1}$ | $^{4}_{1}$ | $^{3}_{2}$ | $^{3}_{3}$ | $^{3}_{4}$ | $^{4}_{1}$ | $^{4}_{1}$ | $^{4}_{1}$ |
|----------|------------------|------------|------------|------------|------------|------------|---------|-----|
|          | $B(M1)$          |            |            |            |            |            |         |     |
|          | Exp.             | IBM-2      | Exp.       | IBM-2      | Exp.       | IBM-2      | Exp.    | IBM-2|
| $2_{1} \rightarrow 1_{1}^+$ | 0.0179 | -- | -- | -- | 0.0177 | -- | -- |
| $2_{2} \rightarrow 1_{1}^+$ | 3.3×10⁻⁶ | -- | -- | -- | 3.76×10⁻⁶ | -- | -- |
4. Conclusion

The Ytterbium nuclei with the (170-178) mass number exhibit rotational properties. The first and second intersecting boson model have been studied dynamic symmetry and energy ratios ($E_{\text{1}}^2/E_{\text{2}}^2$), ($E_{\text{6}}^1/E_{\text{7}}^1$) and ($E_{\text{8}}^1/E_{\text{9}}^1$). The ratios approximate to (3.33,7 and 12). The reducing transition probability for electric quadrupole B(E2) and branching ratio have small values for R' and R'' as in rotational limit, because of selection rules for this limit consider that the transitions $B(E2; 2^+_2 \rightarrow 2^+_1)$. $B(E2; 0^+_2 \rightarrow 2^+_1)$ are equal to zero which means forbidden transitions as we see in tables (2&3) and equation (13). The Majorana parameter effect ($\zeta$) for $^{170-178}$Yb isotopes, has been investigated by vary $\zeta$ around the best-fitted with experimental data. The energy levels values for the states $2^+_2, 3^+_1, 5^+_1$ have clear mixed symmetry state (MSS) directly proportional with the Majorana parameters $\zeta$, while $2^+_3$ and $2^+_4$ tend to be more conservative. A study of the mixed symmetry state of these nuclei shows that the lower energy mixing level is the first $1^+$ level, that distinguishes the rotational determination nuclei. The collective states location of mixed proton-neutron symmetry is one of the most remarkable open experimental difficulties in the study of collective features of nuclei. The experimental values of the first $1^+$ for $^{170-178}$Yb isotopes are (1.634, 2.009 and 1.819) MeV respectively somewhat higher than the values in the best fitting (1.143, 1.082 and 1.132) MeV while no experimental value for $^{178}$Yb exist. The results of the reduced electrical transitions probability $B(E2)$ that obtained from the IBM-1 & IBM-2 program package have been compared with the experimental values and the results are satisfactory. The reduced magnetic transitions probability ($M1$) and the $\delta(E2/M1)$ ratio have been calculated using the second program and have been compared with the few available experimental values they are considered to be compatibility acceptable. All results have been shown that $^{170-178}$Yb nuclei as deformation nuclei they obey rotational limit perfectly.

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