On the swimming strategies to escape a rip current: a mathematical approach

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Abstract
Rip currents represent significant hazards to swimmers all around the world. The danger arises when a misinformed swimmer uses an inadequate strategy to escape the rip, such as fighting the current directly. This can lead to fatigue, panic, and in some cases drowning. There exists a range of strategies put forward by experts (both lifeguards and scientists) to escape rip currents. However, these recommendations are based on a limited number of scientific studies and there is still much discrepancy surrounding the best strategy to escape a rip. Thus, here we present a simple, physics-based theoretical model aimed at assessing different escape strategies in terms of their associated ‘energetic cost’ (in work and power) for any given rip current and swimmer’s proficiency level. Many combinations of swimmers and rips are considered, including both idealised and a realistic rip current. Our quantitative results back the common knowledge that swimming against the rip (which is strongly discouraged by lifeguards) is almost universally the worst possible strategy, especially when compared against strategies favoured by experts, such as floating with the current before attempting to swim back to the shore. For a realistic rip, our results suggest that swimming directly against the rip can require several times more power from the swimmer than other strategies advised by lifeguards, thus lending further scientific support to experts’ recommendations. This study may help promote education around the dangers posed by rip currents and how best to address them.

Keywords Rip current · Escape · Swimming strategy · Work · Power

1 Introduction
Rip currents are near-shore, narrow, seaward directed flows of fast-moving water that pose a serious threat to swimmers. The instinctive reaction of a misinformed swimmer caught in a rip might be to fight the rapid current directly, which can lead to fatigue, panic and, in some cases, drowning (NOAA 2019). To put this danger into perspective, in Australia 89% of the 25,000 surf rescues conducted by lifeguards per year are carried out due to rip current incidents.
current accidents, with an average of 21 rip-related deaths per year (Short and Hogan 1994; Brighton et al. 2013). In the USA, the number of rip-related fatalities can exceed 100 in a given year (Brewster et al. 2019).

Much of the education surrounding rip currents and how best to escape them is based on lifeguard experience and is communicated to the public through signage on beaches, often poorly (Williamson 2006). Among the several recommendations given to swimmers, the following tend to be the most common (see e.g. RNLI: Rip currents 2020; Brander et al. 2011):

- Stay calm and draw the attention of a lifeguard if possible
- Never attempt to swim directly against the rip current
- To escape, swim parallel to the beach until you are out of the rip, then turn and swim back to shore (active response)
- If you are fatigued or a weak swimmer, float and allow yourself to be taken out by the flow past the end of the rip before swimming back to shore with the oncoming waves (passive response)

These recommendations are based on a combination of common sense, lifeguard experience and some scientific studies. However, disagreement exists within lifeguard circles over whether an active or a passive response is preferable (McCarroll et al. 2014; Drozdzewski et al. 2015). In fact, such a disagreement motivated the International Life Saving Federation (www.ilsf.org) to adopt an official position on the matter (ILSF: Lifesaving position statement - lsp 18 2016), based on scientific consensus, which is in line with the above recommendations. In any case, given the significant hazard posed by rips and the lack of a universally accepted ‘best’ escape strategy, there appears to exist at present relatively little research backing these guidelines on scientific principles. Some exceptions are the works by Castelle et al. (2016), McCarroll et al. (2014, 2015) and Miloshis and Stephenson (2011), which are discussed next.

In Miloshis and Stephenson (2011), passive human swimmers (drifters) are deployed in a rip to analyse qualitatively what the best strategy may be for a given rip scenario, without actually assessing directly those strategies. The latter caveat is remedied in McCarroll et al. (2014), where swimmers are asked to follow different strategies (active and passive responses described above), evaluating the latter quantitatively in terms of the time required for the swimmer to reach safety. In both these studies, however, a major caveat is the lack of generality arising from the need to employ only proficient swimmers and the focus on a particular site. The former point is addressed in McCarroll et al. (2015), where swimmers are simulated numerically (but a single rip, based on field observations, is still employed). Building on McCarroll et al. (2015), Castelle et al. (2016) also simulate escape strategies numerically, although considering different rip scenarios that may arise in a given site (an open beach in SW France). However, assessing an escape strategy exclusively in terms of elapsed time between starting point and reaching safety might skew the results in favour of fast (and tall) swimmers, which may arguably give preponderance to swimming proficiency over strategy adopted. In other words, considering solely elapsed time may lead to the natural conclusion that very proficient swimmers are better prepared than unskilled swimmers to escape a rip, regardless of strategy adopted (although this metric may still yield valuable insights when comparing swimmers of similar proficiency, such as those considered in the aforementioned works). Nevertheless, an important conclusion from Castelle et al. (2016) is that optimal escape strategies are dependent on the particular rip current, the structure of which tends to be highly sensitive to hydro-morphodynamic
changes (e.g. small changes in the bar/rip morphology). Indeed, Castelle et al. (2016) remark that arriving at a universally acceptable escape strategy advice may not at all be possible due to the importance of site-specific factors.

The present paper aims to propose a simple, physics-based model that can be readily adapted to specific sites and swimming abilities, thus serving as a quick and easy-to-use tool that can inform both lifeguards and beachgoers. To this end, we adopt an energy-based approach, whereby different escape strategies are evaluated in terms of the work and power needed by any swimmer to reach the shoreline from a given starting point inside an arbitrary rip current. This allows us to explore each strategy’s performance under diverse combinations of the relevant parameters, such as rip velocity, width and swimmer’s escape speed, thus yielding insights with an enhanced degree of generality. Since, as shown later, both power and work increase with escape speed, this approach aims to remedy the perceived issue of McCarroll et al. (2014, 2015) discussed above, where swimming proficiency may overshadow the strategy adopted (our approach actually inflicts a ‘penalty’ on fast swimming). Energy-related metrics (work and power) are adopted due to their natural connection with fatigue in humans, which is one of the main causes of the hazard posed by rip currents. Nevertheless, by neglecting the time required by a swimmer to reach safety, this methodology presents some other caveats, which we discuss further, along with other limitations of our simple model, in Sect. 4.

The paper is organised as follows. We start by characterising and defining a typical rip current based on the existing literature (Sect. 2.1). Then, in Sect. 2.2 we derive a general equation for the work done by a swimmer for this particular problem. Subsequently, in Sect. 2.3 we define the escape strategies to be studied (four in total) and present the equations for estimating the work done by the swimmer under each strategy, as well as the rate at which work is done (power). The mathematical model is then employed in Sect. 3 to quantify the ‘energetic cost’ (in work and power) of each strategy under diverse rip characteristics (length, width, velocity, etc.), including a realistic (i.e. non-idealised) rip velocity field. Discussions on the main strengths and limitations of this approach can be found in Sect. 4 and final remarks are presented in Sect. 5.

### 2 The mathematical model

This section begins with a description of the rip current model employed. Then, the general equation estimating the work done by the swimmer to escape a rip is derived; the equation for the associated power follows readily. These equations are then applied to each of the four strategies studied.

#### 2.1 The rip current model

The structure and origin of rip currents have been well documented and discussed in the scientific literature; a comprehensive review can be found in e.g. Dalrymple et al. (2011), and Castelle et al. (2016). In simple terms, rip currents form as oncoming waves drive surface water up the beach, causing a rise in water level along the shore. This excess water flows back to the sea via the route of least resistance, which can be, for instance, a breach in a sandbar. There exist several types of rip currents (Dalrymple et al. 2011; Castelle et al. 2016), which are due to different mechanisms that originate them, such as nearshore morphology (Wright and Short 1984), wave-wave interactions
(Johnson and Pattiaratchi 2004) or engineered structures along the coastline such as piers and groins (Castelle et al. 2016). Adding to their hazardous nature, the appearance of rips can be highly unpredictable, which stems in part from the complex interaction between the beach morphology and wave hydrodynamics (Calvete et al. 2007). The modelling of the intricate hydrodynamics of rip currents is not, however, a goal of this paper, as discussed next.

Since our motivation is to generate a simple tool that can quickly assess different escape strategies under various scenarios (and can thus be of general use), we favour model simplicity over accuracy. Our first aim is then to describe a typical rip with as few parameters as possible. We begin by identifying the most prevalent features of these systems in order to model a typical rip. The literature surveyed in order to establish a representative rip includes review papers (MacMahan et al. 2006; Dalrymple et al. 2011; Castelle et al. 2016) and studies of rips based on field observations, numerical simulations or a combination of both (Brander and Short 2000; Brander 1999; Philip and Pang 2016; Sonu 1972; McCarrroll et al. 2014; Miloshis and Stephenson 2011; Calvete et al. 2007; Haas et al. 2003; Winter et al. 2014; Austin et al. 2013). Supported on this literature review, we propose the conceptualisation illustrated in Fig. 1, which shows the main characteristics of the rip; namely: a shoreline-normal neck of width \( w_r \) and length \( l_r \), where water flows at an offshore-directed velocity \( V_r \); left and right shoreline-parallel feeders of width \( w_f \) where water flows onto the main channel at a long-shore speed \( V_f \); a central zone where the lateral feeders meet retaining their velocity magnitude \( V_f \) (as shown later, we are solely concerned with the centreline in this zone, where the flow is directed offshore); and zones of near-null velocity at both sides of the rip neck. Beyond the neck, the rip diffuses out and its velocity progressively decays (this is sometimes called the ‘head’ of the rip). Rip currents typically present parallel channels with onshore flows, which may help the swimmer get back to the coast. We choose to disregard these onshore currents for the sake of simplicity (we aim to minimise the number of variables involved), accepting that, consequently, our estimates of work and power associated with each escape strategy will be rather conservative in most cases (by not accounting for the potential help provided by these currents to the swimmer). The velocity field is considered to be steady. A given rip is then completely determined, according to the above conceptualisation, by the parameters \( w_r, l_r, w_f, V_f, V_r \). Table 1 shows some typical values of these parameters reported in the literature surveyed. Major limitations of the model are discussed in detail in Sect. 4.

| Source                  | Location              | \( l_r \) (m) | \( w_r \) (m) | \( w_f \) (m) | \( V_f \) (m/s) | \( V_r \) (m/s) |
|-------------------------|-----------------------|---------------|---------------|---------------|----------------|----------------|
| Brander and Short (2000)| Muriwai Beach, NZ     | 400           | 150           | 75            | 1.0            | 1.4            |
| Miloshis and Stephenson (2011)| Woolamai Beach, Au. | 300           | 20–100        | –             | –              | 0.5–0.9        |
| Brander (1999)          | Palm Beach., Au.      | –             | –             | –             | 0.3            | 1.64           |
| Philip and Pang (2016)  | –                     | –             | –             | –             | 0.6–0.7        | 0.5–2.5        |
| McCarrroll et al. (2014)| Shelly Beach, Au.    | –             | 30–50         | –             | 0.4–0.6        | 0.3–1.0        |
| Sonu (1972)             | –                     | –             | 45            | 20            | 25             | 2.0            |
| Bruneau et al. (2009)   | Biscarrosse Beach, Fr | –             | 110           | 50–60         | 0.4–0.6        | 0.3–1.2        |
| MacMahan et al. (2010)  | Monterey Beach, CA, USA| 150           | 125           | 50            | 0.2–0.3        | 0.4–0.65       |
2.2 The necessary work to escape

The aim of this theoretical study is not to estimate the actual energy/power (in Joules/Watts) expended by a swimmer when escaping a rip current. Such a task would need to account for the significant complexity associated with the actual hydrodynamics of rips, the mechanics of human swimming (considering different swimming techniques and abilities) and the metabolism of swimmers. Instead, this paper aims to generate some simple physics-based criteria to judge a given strategy relative to another, and model simplicity is prioritised for the sake of generality in the results, as previously discussed. Just as the rip current structure has been simplified in Sect. 2.1, here we idealise a swimmer as a rigid object of arbitrary geometry subject exclusively to its self-generated propulsive force, $F_s$, and the opposing, flow-induced drag force, $F_D$. A consequence of this simplification is that we will approximate the real work done by the swimmer as the work done (by our rigid object) against the drag force. The former (the real work) is of course frame-invariant, while the latter (our approximation) is not, as we elaborate below.

By definition, work and power are physical quantities that depend on the frame of reference adopted (due to displacement, and hence velocity, being frame-dependent). Therefore, our results too will depend on the reference frame selected (see Sect. 4), and since such a selection is necessarily arbitrary, we adopt a problem-specific criterion to justify our preference, as discussed next. The local flow velocity $v_f$ is measured from a static, Cartesian frame of reference $K_0$ (say, the shore) where $x$ and $y$ are the long-shore and cross-shore coordinates, respectively, as shown in Fig. 1. However, work estimates employing this frame of reference would lead to the following apparent paradox. Consider a swimmer fighting against the rip but fixed in space with respect to $K_0$. For an observer at the shoreline ($K_0$), the swimmer’s displacement is null, and hence so is the work done. But the swimmer must be expending energy (doing work) to fight the current. (Note that such a paradox arises from our simplification of the problem; in reality, the energy expended by the swimmer comes from the work done by the limbs on the local fluid, which must naturally be frame-invariant.) Therefore, to account for this, we
adopt a second frame of reference $K_0$ that moves at the (time-independent) local flow velocity $v_f$ relative to $K_0$ (see Fig. 1), such that in the case discussed, the swimmer’s displacement as seen from $K$ is nonzero, thus ensuring a nonzero estimate of work (see end of this section). The velocity of $K$ with respect to $K_0$ (i.e. $v_f$) is selected due to its physical meaningfulness within the problem; any other velocity (yielding a nonzero displacement) would be equally valid, but would represent an additional variable necessary to close the model (we aim to minimise the number of variables involved).

The swimmer’s velocity relative to $K$ ($K_0$) is denoted $v_{sk}$ ($v_s$); and thus, $v_{sk} = v_s - v_f$. Note that by virtue of $v_f \neq v_f(t)$ (where $t$ is time), $K$, just as $K_0$, is an inertial frame of reference, such that the equations governing the motion of the swimmer are identical in both frames. Seen from $K$, the total work, $W$, done by the swimmer during the time interval $[t_i, t_e]$ is given by:

$$W = \int_{t_i}^{t_e} F_s \cdot v_{sk} dt,$$

(1)

where $F_s$ is the propulsive force exerted by the swimmer to escape at a velocity $v_{sk}$ relative to $K$ (or what is the same, at $v_s$ relative to $K_0$). We take $v_{sk}$ to be time-independent (the same is thus true for $v_f$). Consequently, the swimmer’s acceleration is zero, and so there must be a balance of forces between the swimmer’s propulsive force $F_s$ and the resistive drag force $F_D$; i.e. $F_s = -F_D$. The latter is in turn determined from its conventional parametrisation:

$$F_D \equiv \|F_D\| = \frac{1}{2} \rho AC_D V^2 = \overline{C_D} V^2,$$

(2)

where $\overline{C_D} \equiv 0.5 \rho AC_D$; $\rho$ is the density of the fluid; $A$ is some characteristic area; $C_D$ is the swimmer’s drag coefficient; and $V$ is a characteristic speed, which in this case is taken to be the swimmer’s speed relative to the flow, i.e.

$$V \equiv \|v_{sk}\| = \|v_s - v_f\|.$$

(3)

Note that, by definition, $F_D$ acts in the direction opposite to the swimmer’s velocity relative to the fluid, i.e. $v_{sk}$. Thus, the integrand in (1) can be rewritten as:

$$F_s \cdot v_{sk} = -F_D \cdot v_{sk} = \|F_D\| \|v_s - v_f\| = \overline{C_D} \|v_s - v_f\|^3.$$

Vectors $v_s$ and $v_f$ are 2-dimensional and have components $v_s = (v_{sx}, v_{sy})$ and $v_f = (v_{fx}, v_{fy})$ in the $x$-$y$ Cartesian plane, measured from $K_0$. Thus, eq. (1) becomes:

$$W = \overline{C_D} \int_{t_i}^{t_e} [(v_{sx} - v_{fx})^2 + (v_{sy} - v_{fy})^2]^{3/2} dt.$$

(4)

Note that, whereas the displacement used to estimate the work has been defined from a moving frame of reference ($K$), velocity components appearing in (4) are measured from the static frame $K_0$. The above equation clearly illustrates that $W > 0$ for the ‘non-moving’ swimmer fighting the current referred to at the beginning of this section; the integrand is positive even for $v_s = (0, 0)$. Similarly, the swimmer does no work when floating with the current (i.e. $W = 0$ when $v_s = v_f$), as expected.

We are also interested in the average rate at which work is done (i.e. average power, $P$) due to the connection of this quantity with muscle fatigue (Wan et al. 2017). The
average power follows readily from above; i.e. \( P = W / (t_e - t_i) \). Next, we derive expressions for \( W \) and \( P \) corresponding to each of the strategies investigated.

### 2.3 The escape strategies

Based on the literature review discussed in Sect. 1, four strategies are investigated; namely:

1. Swim directly back to the shore (S1)
2. Swim perpendicular to the rip before turning towards the shore (S2)
3. Swim at \( 45^\circ \) towards the shore (S3)
4. Float with the current before swimming towards the shore (S4)

These strategies, illustrated in Fig. 2, are discussed in detail next, along with their respective formulations of estimated energetic cost (in work and power).

#### 2.3.1 Strategy 1 (S1)

The trapped swimmer attempts to swim directly back to the shore against the current (see Fig. 2a). This strategy is universally discouraged by lifeguards due to the risk of exhaustion and subsequent drowning. This strategy is investigated as a baseline against which to compare other strategies. In all strategies, the swimmer is assumed to be trapped at the centreline of the rip current and escapes at the time-independent speed \( V_e \equiv \|v_s\| \) (measured from \( K_0 \)), so in this case \( v_s = (0, -V_e) \), as shown in Fig. 2a. The flow velocity, however, is not the same in the rip neck and in the central zone where the feeder channels meet, with values of \( v_s = (0, V_r) \) for the former and \( v_s = (0, V_f) \) for the latter. Therefore, we estimate the work done by the swimmer using Strategy 1 (S1) in these two zones separately, as follows (using eq. 4):

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Fig. 2 Sketch of the four escape swimming strategies investigated. See fig. 1 for reference to the conceptualisation of the rip current structure adopted. [Lengths in d) S4 are not to scale.]
Since we know the geometric characteristics of the current, we can transform the above time intervals (integral limits) to lengths relevant to the problem by use of the corresponding speeds; i.e.
\[
(t_1 - t_i) = \frac{(l - w_f)}{V_e},
\]
where \(l\) is the swimmer’s initial distance from the shore (see Fig. 2), and \((t_e - t_1) = w_f/V_e\), leading to:
\[
W_1 = \cd \int_{t_i}^{t_1} (V_e + V_f)^3 \, dt + \cd \int_{t_i}^{t_e} (V_e + V_f)^3 \, dt
\]
\[
= \cd \left[ (V_e + V_f)^3(t_1 - t_i) + (V_e + V_f)^3(t_e - t_i) \right].
\] (5)

The time required for the swimmer to travel the total distance \(l\) is \(l/V_e\), leading to the power associated with this strategy:
\[
P_1 = \frac{\cd}{l} \left[ (l - w_f)(V_e + V_f)^3 + w_f(V_e + V_f)^3 \right].
\] (6)

2.3.2 Strategy 2 (S2)

The swimmer swims a distance \(l_\alpha\) parallel to the shore to exit the rip neck before turning 90° and swimming directly to the shore. This strategy is an ‘active response’ and is one of two strategies mostly favoured by lifeguards (the other being Strategy 4 described below). We treat this strategy as being composed of four stages, as shown in Fig. 2b. Eq. (4) is then split into four integrals:
\[
W_2 = \cd \left[ \int_{t_i}^{t_1} (V_e^2 + V_r^2)^{3/2} \, dr + \int_{t_i}^{t_2} V_e^3 \, dt + \int_{t_2}^{t_3} V_e^3 \, dt + \int_{t_3}^{t_e} (V_e^2 + V_f^2)^{3/2} \, dr \right].
\] (8)

which, using the equivalences:
\[
(t_1 - t_i) = \frac{w_r}{(2V_e)};
\]
\[
(t_2 - t_1) = \frac{(l_\alpha - \frac{w_r}{2})}{V_e};
\]
\[
(t_3 - t_2) = \frac{(l - w_f)}{V_e} ; and
\]
\[
(t_e - t_3) = \frac{w_f}{V_e};
\]
leads to:
In this case, the swimmer covers a total distance of $l_x + l$, yielding:

$$W_2 = \widetilde{C}_D \left[ \frac{w_r}{2V_e} (V_e^2 + V_{r})^{3/2} + \left( l_x - \frac{w_r}{2} + l - w_f \right) V_e^2 \right. \right.$$

$$+ \left. \frac{w_f}{V_e} (V_e^2 + V_{r})^{3/2} \right]. \quad (9)$$

In this case, the swimmer covers a total distance of $l_x + l$, yielding:

$$P_2 = \frac{\widetilde{C}_D}{l_x + l} \left[ \frac{w_r}{2} (V_e^2 + V_{r})^{3/2} + \left( l_x - \frac{w_r}{2} + l - w_f \right) V_e^3 \right.$$

$$+ \left. w_f (V_e^2 + V_{r})^{3/2} \right]. \quad (10)$$

### 2.3.3 Strategy 3 (S3)

The swimmer swims at a $45^\circ$ angle to the shoreline. This strategy, also assessed in McCa- rroll et al. (2015), is explored here as an alternative to Strategy 2, which has the disadvantage of being sensitive to the parallel distance $l_x$ and is underpinned by the crucial assumption that the swimmer starts beyond the feeders ($l > w_f$). The latter assumption has been previously identified as a major weakness of S2 by Miloshis and Stephenson (2011). Strategy 3 is divided into three stages as shown in Fig. 2c. The procedure is similar to that followed in previous strategies, leading to:

$$W_3 = \widetilde{C}_D \left\{ \frac{V_2}{2V_e} \left[ \frac{V_e^2}{2} + \left( \frac{V_e}{\sqrt{2}} + V_r \right) \right]^{3/2} + \sqrt{2} \left( l - w_f - \frac{w_r}{2} \right) V_e^2 \right\}$$

$$+ \frac{\sqrt{2}w_f}{V_e} \left[ \left( \frac{V_e}{\sqrt{2}} + V_f \right)^2 \frac{V_e^2}{2} \right]^{3/2} \}. \quad (11)$$

Here, the swimmer travels a total distance of $\sqrt{2}l$, leading to:

$$P_3 = \frac{\widetilde{C}_D}{l} \left\{ \frac{w_r}{2} \left[ \frac{V_e^2}{2} + \left( \frac{V_e}{\sqrt{2}} + V_r \right) \right]^{3/2} + \left( l - w_f - \frac{w_r}{2} \right) V_e^3 \right\}$$

$$+ w_f \left[ \left( \frac{V_e}{\sqrt{2}} + V_f \right)^2 \frac{V_e^2}{2} \right]^{3/2} \}. \quad (12)$$

Note that this strategy implicitly requires that $l \geq w_f + \frac{w_r}{2}$, for otherwise the swimmer would not go through the null-velocity zone as assumed, but rather through the central zone where the feeders meet (see Fig. 2c).
2.3.4 Strategy 4 (S4)

The swimmer floats with the current (doing no work) until swept out to a point either side of the rip neck, where he/she can return to the shore swimming parallel to the rip. This strategy, also known as ‘passive response’, is, along with S2, a popular advice given by lifeguards. In the conceptualisation adopted here (see Fig. 2d), this strategy has two stages. In the first stage, it is assumed that the swimmer starts swimming back to the shore from a distance equal to two rip lengths from the feeder, or \(2l_r + w_f\) from the shoreline (this distance is purely based on visual inspection of typical rips), leading to the following formulation for the necessary work:

\[
W_4 = \frac{C_D}{2l_r + w_f} \left[2l_r V_e^2 + \frac{w_f}{V_e} \left(V_e^2 + V_f^2\right)^{3/2}\right],
\] (13)

and associated power:

\[
P_4 = \frac{C_D}{2l_r + w_f} \left[2l_r V_e^3 + \frac{w_f}{V_e} \left(V_e^2 + V_f^2\right)^{3/2}\right].
\] (14)

Note that the expressions derived above for each of the strategies do not all depend on the same variables, and some variables are exclusive to a given strategy. In particular, we have: \(W_1\) and \(P_1 = f(C_D, V_e, V_r, V_f, l, w_f)\); \(W_2\) and \(P_2 = f(C_D, V_e, V_r, V_f, l, w_f, w_r, l_x)\); \(W_3\) and \(P_3 = f(C_D, V_e, V_r, V_f, l, w_f, w_r)\); and \(W_4\) and \(P_4 = f(C_D, V_e, V_r, l, w_f)\). We consider the swimmer to be trapped inside the rip when found beyond the feeders’ width; i.e. we take \(l \geq w_f\), except for Strategy 3, where \(l \geq w_f + \frac{w_r}{2}\) is required. The starting distance \(l\) is also upper-bounded by the geometry of the rip, such that \(l \leq w_f + l_r\).

Our focus is on the performance of each strategy relative to one another, rather than the absolute values of work and power required to escape, which would be beyond the scope of the simple model proposed here (see Sects. 2.2, 4). In other words, we are interested in the comparison of the ratios \(W_i/W_1\) and \(P_i/P_1\) (for \(i = 2, 3, 4\)), and their reciprocals, such that the parameter \(C_D\) becomes irrelevant. This assumption is discussed next, along with additional comments on the swimmer-associated variables.

2.4 Swimmer-related parameters

Variables characterising a given rip (i.e. \(V_r, V_f, l_r, w_r\) and \(w_f\)) have been discussed in Sect. 1 (see Table 1). Here, we turn our attention to the parameters associated with the swimmer; i.e. \(V_e, l, l_x\) and the drag coefficient.

In general, quantification of a (human) swimmer’s drag is not straightforward. The value of \(C_D\) may depend on the body’s orientation with respect to the current and on whether the propulsive parts of the swimmer’s body (legs and arms) are considered (Webb et al. 2011, 2015). Estimation of \(C_D\) further requires knowledge of the characteristic area \(A\), often taken as the swimmer’s wetted surface area, which is not trivial to quantify (see e.g. Gehan and George 1970; Bailey and Briars 1996; Lee et al. 2008). To avoid the complexity associated with the estimation of an accurate \(C_D\), the above treatment implicitly assumes that the swimmer’s drag coefficient is independent of their orientation relative to the local flow velocity, leading to its cancelling out in the equations describing the ratios \(W_{2,3,4}/W_1\) and \(P_{2,3,4}/P_1\).
Average competitive swimmers can reach velocities of 1.2 to 1.5 m/s in static flows, such as swimming pools (Pendergast et al. 1977; Thompson et al. 2000), whereas professional lifeguards have been measured to attain speeds of 0.7–0.9 m/s in the surfzone (Tippton et al. 2008). Here, we define the escape speed with respect to the static frame of reference \( K_0 \), such that \( V_e \) must account for the local flow velocity (i.e. for the fact that the swimmer may be fighting a current). Also, we are interested in the average beachgoer, who may not necessarily be a competitive swimmer. Thus, for the analysis presented in Sect. 3.1 we adopt an upper limit of 1.0 m/s, and set the lower limit to an arbitrarily small value (based on McCarroll et al. 2015), such that \( 0.1 \leq V_e \leq 1.0 \) m/s.

The value of \( l_x \) is here assumed to vary between \( w_r \) and \( 2w_r \). This range, based purely on geometrical considerations, is an estimate of the distance a swimmer would need to swim parallel to the shore in order to safely exit the rip neck before turning 90° towards the beach. The starting distance from the shore, \( l \in [w_f, w_f + l_r] \), is taken as the main independent variable to evaluate the performance of each strategy (with the exception of Strategy 4, which is independent of \( l \)).

## 3 Results

Ranges of values for all parameters involved in the quantification of work and power for each strategy have been obtained from previous studies and other considerations, as discussed in Sects. 1 and 2.4. However, these variables are not necessarily independent from one another. For example, a very wide rip neck (large \( w_r \)) may not coincide in a given rip with a very narrow feeder (small \( w_f \)); fast-flowing feeders (large \( V_f \)) are expected to lead to similarly fast-flowing rips (large \( V_r \)), which in turn might restrict how fast a swimmer can actually swim (\( V_e \)); and so on. We circumvent this complexity by first following a ‘brute force’ approach, whereby all possible combinations are considered without any sort of discrimination. Remarks obtained following this approach are then put to test by considering a realistic rip velocity field generated numerically.

### 3.1 Idealised scenarios (‘brute force’ approach)

Based on the literature reviewed (see Table 1), we use the ranges of values for each rip-related parameter shown in Table 2. As discussed in Sect. 2.4, other relevant parameters take the values: \( 0.1 \leq V_e \leq 1.0 \) m/s, \( w_r \leq l_x \leq 2w_r \), and \( w_f \leq l \leq w_f + l_r \). Figure 3 illustrates that, in general, all strategies are more efficient in terms of work than Strategy 1, for all combinations of rip- and swimmer-related parameters considered, in line with common

| Parameter                  | Symbol | Lower limit | Upper limit |
|----------------------------|--------|-------------|-------------|
| Rp width                   | \( w_r (m) \) | 10          | 200         |
| Rip length                 | \( l_x (m) \) | 25          | 400         |
| Feeders width              | \( w_f (m) \) | 25          | 75          |
| Rip velocity               | \( V_r (m/s) \) | 0.2         | 2.5         |
| Feeders velocity           | \( V_f (m/s) \) | 0.25        | 1.00        |
Fig. 3  Ratios of the work required for each strategy $W_i (i = 2, 3, 4)$ to that for Strategy 1, against normalised starting distance, $l/w_f$. All combinations of rip- and swimmer-related parameters are shown for each value of $l/w_f$ (including unrealistic combinations).

Fig. 4  Ratios of the power required for each strategy $P_i (i = 2, 3, 4)$ to that for Strategy 1, against normalised starting distance, $l/w_f$. All combinations of rip- and swimmer-related parameters are shown for each value of $l/w_f$ (including unrealistic combinations).
knowledge (Brander et al. 2011; RNLI: Rip currents 2020; ILSF: Lifesaving position statement - lsp 18 2016). Specifically, the percentages of data points complying with $W_i < W_1$ are 86, 67 and 80% for Strategies 2, 3 and 4, respectively, with these proportions increasing for larger values of $l/w_f$. Atypically large values of $W_2/W_1$ when $l \rightarrow w_f$ can be attributed to large values of $l_x$ and slow, short rip currents; under these conditions it makes sense to swim directly back to the shore rather than swim a long shore-parallel distance. As may be expected, Strategy 1 is only competitive when $l \rightarrow w_f$, as previously highlighted by Castelle et al. (2016), and McCarroll et al. (2015). Note that $l \rightarrow w_f$ represents the situation when a swimmer begins to escape at the ‘root’ of the rip neck. However, it typically takes inexperienced swimmers longer than that to realise that they are caught in a rip (Castelle et al. 2016), and so S1 should probably not be promoted as a safe escape strategy in general. The latter statement is further supported by the performance of the strategies in terms of power. Figure 4 shows that $P_i < P_1$ for all strategies when $l > 1.5w_f$, and in fact $P_{3,4} < P_1$ for all $l/w_f$.

These results begin to explain why Strategy 1 is so universally discouraged, but the specific values of the ratios here considered must be interpreted with caution. Extreme values

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**Fig. 5** Velocity field based on the rip current generated numerically by Calvete et al. (2007). Escape swimming strategies shown in red for an arbitrary starting distance
of these ratios (implying \( W_1 \gg W_i \) and \( P_1 \gg P_i \), and vice versa) may be due to unrealistic combinations of the parameters considered. For this reason, we investigate a more realistic rip next.

### 3.2 A realistic scenario

The velocity field shown in Fig. 5 is based on the rip generated numerically by Calvete et al. (2007). The field illustrates the main characteristics of a typical rip. Vectors have been scaled such that the flow velocity within the rip neck is roughly 1.0 m/s (no scale is given in Calvete et al. (2007)). Work for each strategy is here estimated by using the general expression for \( W \), eq. (4), with \( P \) following readily. These results are also compared against those arising from the rip model adopted in this study (Fig. 1 and equations derived in Sect. 2.3). To this end, from visual inspection of Fig. 5, we use \( l_r = 55 \) m, \( w = 50 \) m, \( w_f = 60 \) m, \( V_r = V_f = 1.0 \) m/s, and adopt \( l_x = 60 \) m. Since this represents a relatively strong rip, a small escape speed is employed, namely, \( V_e = 0.25 \) m/s.

Results are summarised in Tables 3 and 4. The main conclusions arrived to in Sect. 3.1 are confirmed. The worst strategy tends to be S1 (though here S3 is sometimes worse), with the best strategy being S4, followed by S2, in agreement with previous research (Miloshis and Stephenson 2011; Brander et al. 2011; RNLI: Rip currents 2020). These remarks are irrespective of starting distance, energetic metric (i.e. work or power) or whether a realistic or idealised rip is employed. Although the particular values of the ratios \( W_1/W_{2,3,4} \) and \( P_1/P_{2,3,4} \) will naturally depend on the specific rip and swimmer (see also Sect. 4), a conservative reading of these results suggests that swimming directly against the current (S1) can demand several times more energy and power than strategies encouraged by experts. It is also worth noting that predictions of the work and power ratios based on the idealised rip model proposed here are generally in good agreement with those obtained from the realistic rip current field. This may be taken as a successful, albeit very coarse, validation of

### Table 3

| Starting distance | Type of rip    | \( W_1/W_2 \) | \( W_1/W_3 \) | \( W_1/W_4 \) |
|-------------------|----------------|----------------|----------------|----------------|
| \( l \approx 100 \) m | Realistic rip  | 1.50           | 0.64           | 2.41           |
|                   | Model rip      | 1.66           | –              | 2.32           |
| \( l \approx 130 \) m | Realistic rip  | 2.77           | 0.80           | 2.59           |
|                   | Model rip      | 2.28           | 1.06           | 3.19           |

### Table 4

| Starting distance | Type of rip    | \( P_1/P_2 \) | \( P_1/P_3 \) | \( P_1/P_4 \) |
|-------------------|----------------|----------------|----------------|----------------|
| \( l \approx 100 \) m | Realistic rip  | 2.47           | 0.86           | 4.89           |
|                   | Model rip      | 2.91           | –              | 4.92           |
| \( l \approx 130 \) m | Realistic rip  | 4.21           | 1.11           | 4.40           |
|                   | Model rip      | 3.52           | 1.50           | 4.92           |
the model presented in this study. Note that a proper validation would require sophisticated experiments or simulations that are currently non-existent, and which are well beyond the scope of this paper.

4 Discussion

Based on experience, lifeguards and other proficient swimmers have arrived at a set of recommendations to escape potentially-deadly rip currents. Here we show that with a simple mathematical model these recommendations can be quantitatively verified, in line with previous scientific findings (e.g. Castelle et al. 2016; McCarroll et al. 2014, 2015; Miloshis and Stephenson 2011). For example, we show that for a realistic rip, a person swimming against the current can require about 5 (2.5) times more power (energy) than floating with the current or swimming parallel to the shore before returning to the beach (S4 and S2, respectively, which are commonly recommended by experts). In general, our results: (i) back the common knowledge that swimming against the rip is the worst possible strategy; and (ii) suggest that the most energy-efficient strategy is usually S4, followed by S2. A main strength of the model here proposed is its simplicity, which enables a quick exploration of a wide range of scenarios involving different rips and swimmer proficiency levels. However, even though the general conclusions stemming from this study are unlikely to depend on the model’s conceptual limitations, it is important to highlight the latter, which may lead to this work’s future refinement. The main limitations of this theoretical work, along with their potential implications and solutions, are discussed below:

1. By design, our idealised rip model (Fig. 1) is very simple, for the reasons just discussed (ease of exploration of a wide variety of scenarios). However, the model adopted deviates significantly from real rip currents. We consider a symmetrical, shore-normal rip (of varying size and strength) with a time-independent flow. The complex hydrodynamics of typical rips, which include wave-current interactions and infragravity pulsations (MacMahan et al. 2004), are thus not considered. This may have an influence on the optimal escape strategy. In particular, the symmetry assumption is likely to have a strong impact on the escape strategy deemed optimal. For example, Castelle et al. (2016) point out that the ‘swim parallel’ approach (S2) can have a very variable success rate in natural, non-symmetrical rips, depending on the direction chosen by the swimmer to escape. As discussed in Sect. 2.1, we also neglect the onshore flow, which makes our work/power estimates rather conservative. At least some of these limitations may be readily addressed in future versions of this model; for example, by adding the onshore flows parallel to the rip and/or by including a variable angle between the rip and the shoreline. The model may also be adapted to study different types of rips, such as those described in Castelle et al. (2016) (e.g. a rip due to a coastal structure).

2. The results of our simple physics-based model back common knowledge and findings from previous research into swimming escape strategies. However, the actual values of the ratios $W_1/W_{2,3,4}$ and $P_1/P_{2,3,4}$ reported in Sect. 3 must be interpreted carefully. The numerical values of these ratios depend on the simplifications adopted, the more important of which are discussed next:
(a) As mentioned in Sect. 2.2, work and power are physical quantities that fundamentally depend on the frame of reference adopted. It can readily be seen that the same will be true for the ratios $\frac{W_1}{W_{2,3,4}}$ and $\frac{P_1}{P_{2,3,4}}$ estimated here. Moreover, the chosen frame of reference, $K$ (which moves at constant velocity $v_f$ with respect to $K_0$), varies in each strategy as the swimmer moves across regions in the idealised rip with different values of $v_f$ (see Fig. 2). In fact, note that application of eq. (4) to the realistic rip (Fig. 5) should strictly be invalid given the continuously space-varying $v_f$, even if such a variation (gradient) is smooth, for $K$ is then no longer an inertial frame (this is not an issue when deriving the expressions for $W_{1,2,3,4}$ discussed in Sect. 2.3, due to their piecewise construction). From a fundamental-theoretical perspective, these are major limitations of the model. However, note that the ‘brute force’ approach carried out in Sect. 3.1 serves in this regard a sort of sensitivity analysis; here testing the sensitivity of our conclusions to the reference frame adopted by investigating various values of the flow velocity (and hence $K$). The results from said analysis, corroborated in Sect. 3.2, illustrate that despite fundamental inconsistencies in the underlying theory, the model may yield useful insights. After all, the model’s aim is to assess (even if crudely) the appropriateness of one strategy relative to another for a given rip and swimmer, rather than estimating the actual values of the ratios $\frac{W_1}{W_{2,3,4}}$ and $\frac{P_1}{P_{2,3,4}}$.

(b) The swimmer’s escape speed $V_e$ is considered constant with respect to an observer at the coast, which would be unlikely due to fatigue and the different flow velocity regions that must be crossed. In other words, $V_e$ should be expected to be higher in the null-velocity region than in the rip neck, for example. Remedying this can be done with a space-dependent $V_e$, which in the future may be done by redefining $V_e$ as constant with respect to the local flow ($K$) rather than the coast ($K_0$). Note, however, that this seemingly simple refinement would complicate significantly the formulation of expressions for $W_{2,3,4}$ and $P_{2,3,4}$ (for Strategy 1 the modification is trivial).

Therefore, values of the ratios $\frac{W_1}{W_{2,3,4}}$ and $\frac{P_1}{P_{2,3,4}}$ for a given scenario should be interpreted in relative terms (e.g. $W_2 > W_3$, $W_4 \ll W_1$, etc.); focus on their specific numerical values ought to be avoided.

3. Defining the best escape strategy as that which requires the least work or power, without accounting for the time required by the trapped swimmer to reach safety, has some important caveats. A key driver of the hazard posed by rips is the onset of panic in trapped swimmers (Drozdzewski et al. 2012; 2015), which tends to be caused by longer times spent in the sea or simply by the perception of being dragged too far offshore (Drozdzewski et al. 2015). A future work combining the energy-based approach proposed here and elapsed time may yield more robust insights into the safest strategies to escape a rip current.

It is important to close this section by emphasising that the main aim of this work is not to put forward an escape strategy as the universally best, which may depend on factors beyond the scope of this work, such as the complexity of real rip systems (Castelle et al. 2016) and the psychological response of trapped swimmers (McCarroll et al. 2014). Instead, we propose a simple theoretical framework to assess the adequacy of several strategies (based on physical quantities associated with fatigue) for a given rip current and swimmer’s proficiency level. Expressions derived here for the energetic cost of each strategy given a rip
current can be used by lifeguards and beach authorities to readily obtain values of $W_1/W_i$ and $P_1/P_i$ for their particular rip systems (free codes solving these equations are part of the supplementary material). This tailored information can then be passed on to the public through appropriate means (e.g. signage), offering safety information based on science rather than just common sense. This may help raise awareness and educate the public about the hazards posed by rip currents and how best to escape them if trapped.

5 Conclusions

Rip currents represent significant hazards for swimmers around the world. Strategies to safely escape these currents have been put forward by lifeguards and other experts. However, there is a lack of consensus regarding what the best escape strategy is for a given rip, which may partly stem from the limited amount of research underpinning these recommendations. To address this issue, we present a simple, energy-based theoretical model to assess a given escape strategy relative to others, in terms of their associated ‘energetic cost’ (in work and power). For the sake of generality, we define a typical rip and characterise it by five parameters in total: rip neck’s width, length and velocity; and feeders’ width and velocity. Four escape strategies are considered, based on common recommendations. Our results: (i) back the widespread knowledge that swimming against the current is almost always the worst possible option; (ii) suggest that the most energy-efficient strategy is to float with the offshore-directed current prior to swimming back to the shore; and (iii) confirm that escaping the rip by swimming parallel to the shore is also a good option. Application of the model to a realistic rip velocity field confirms these findings. Swimming against the current can require several times more power and energy than escape strategies favoured by experts, thus supporting the latter’s recommendations. We hope that, by lending further scientific ground to the experience-based advice of lifeguards, this study helps promote education about the hazards represented by rip currents and how best to escape them if trapped.

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Code availability Supplementary codes can be downloaded from https://github.com/sergio-maldonado/rip_escape.

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