Analyzing Conceptual and Procedural Knowledge of Geometry Among Prospective Teachers: Indonesian Perspective

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Abstract. It is well established that to scaffold students' learning of mathematics, teachers themselves need a substantial body of content knowledge of mathematics (Ma, 1989) to drive their lessons and design of learning experiences. In this paper, we address the above issue among prospective elementary mathematics teachers in Indonesia. Evidence suggests that elementary school Indonesian teachers tend to acquire surface-level knowledge of geometry, which does not support deep learning. This paper aims to bring a higher degree of insight into the above problem by analyzing prospective teachers' geometry knowledge along two knowledge strands: conceptual and procedural. Examples of these knowledge components are provided with suggestions for future research.

Keyword: Analyzing, conceptual, geometry, Indonesian perspective

1. Introduction

Education is the way to acquire knowledge, and it is used by human beings to face problems encountered in daily lives. Education also assists in the development of attitude and behavior. Teachers and their teaching play an influential role in the development of a mathematical world view by their students [1]. In so doing, the professional nature of their work is key to the success of learning mathematics. The professionalization of mathematics teachers is critical because this is the period in which students should be supported to develop a robust conceptual understanding of mathematics. It provides the foundation for not only further mathematical participation but equally the development of positive attitudes towards the subject.

To be a professional, mathematics teachers themselves need to acquire a substantial body of content knowledge [2]. Professional teachers are crucial to the success of Mathematics learning. Mathematics teachers must be professional because the elementary school has an important role as fundamental knowledge and skill, which is very influential in the development of students' Mathematics later.

There are three-dimensional teachers' professional knowledge: (1) the content knowledge (2) pedagogical content knowledge (3) general pedagogical knowledge [3]. Content knowledge is knowledge about teaching material and structure. Pedagogic content knowledge is knowledge of representation, analogy, examples of a mathematical concept and has the skills of how to teach mathematics to students. But then the pedagogical content knowledge developed so that it included knowing students, curriculum, educational goals, and instructional materials [4]. General pedagogic knowledge is knowledge about teaching methods and classroom management.
Especially in mathematics, there are some opinions about content knowledge. Mathematics Content Knowledge (MCK) is the knowledge of mathematics that will be taught at the school and knowledge of mathematics at the university level[5]. The basic math knowledge possessed by individuals considered to be mathematically literate [6]. Higher mathematical abilities than students make creative teachers in teaching mathematical concepts and able to make quality questions.

MCK for teachers is Common Content Knowledge; and Specialized Content Knowledge [6]–[8]. Common Content Knowledge is a mathematical knowledge commonly known by adults, including teachers. Specialized Content Knowledge emphasizes the knowledge of mathematical concepts and teaching skills to students. Shulman, Skemp, and Hiebert as cited in [9] agree that subject matter knowledge in mathematics can be classified into two types, both important, and called (in Hiebert's nomenclature) "Procedural Knowledge" and "Conceptual Knowledge".

Initially, Conceptual Knowledge (CK) defined as a network of knowledge (Hiebert and Lefevre, 1986), knowledge of core concepts for domains [10]. CK is the basic of the structure of mathematics, interconnection of ideas, explain and give meaning to mathematical procedures [11]. Star added CK is not only knowing about concepts but also how to find out the concept [12]. CK is knowledge of abstract ideas, including comprehension concepts and their interrelation [13]. The definition of conceptual understanding is extended to an understanding or structure of concepts and the relationship between concepts [14], [15]. Someone who has conceptual knowledge is not only able to memorize the definition of a concept but can explain concepts, relationships between concepts, definitions, and how to find concepts [16]. CK is described as knowledge in relationships that can be connected with knowledge, a network in which the linking is as prominent as the discrete pieces of information [17].

Procedural Knowledge (PK) refers to mastering numeracy skills in solving routine problems [11]. She added that knowledge had two aspects, format and symbolic presentation and knowledge of rules and algorithms. PK is the knowledge of the sequence of work steps to achieve the goal. [13][12] argues PK not only knows the procedure but also knows how to obtain the procedure. The procedure can be in the form of (1) algorithms — a certain sequence of steps to get a solution or (2) the series of work on the right questions for solving a problem [18].

[13] suggest that students' conceptual and procedural knowledge needs to be developed. This is in accordance with NCTM (2000) that the basic purpose of mathematics teacher education in teaching mathematics for understanding. Teachers need special knowledge to teach mathematics understanding [19], [20]. The development of both knowledges should be the main focus of teacher education [14][17] because it will improve student learning outcomes [19]. Promoting teacher competence and overcome learning difficulties and students' conceptual misconceptions [16]. It means the need to foster prospective teacher's math competencies, so they are better equipped to meet their students' needs through effective math pedagogy [6].

2. Issue and related literature

Some researches on CK and PK of prospective mathematics teachers have been conducted in some countries. Among others, [21] found that 17% respondents can correctly answer the question “Explain why the following statement is true: A right triangle cannot be equilateral”, 29% correctly answers the question “For each of the triangles, draw a height on the figure,” and 49% is correct in solving the problems of rectangular. [22] reports that students of teaching school experience difficulty in understanding the concept of diagonal in spatial and diagonal characteristics.

[23] concludes that prospective teachers of Elementary Schools experience misunderstanding in the concept mastery of measuring angles, the definition of two perpendicular lines, and wrong procedures in obtaining straight angles. Another finding shows a misunderstanding of the concept in Elementary School prospective teachers in determining characteristics of spaces [24]. The misconception are: 1) a square has 4 vertices, 4 sides of the same length and can be cut into two triangles, and has 2 pairs of parallel sides; 2) a triangle has 3 sides of equal length, sometimes the length of its sides differs, has 3 vertices, its three sides are not parallel, and the total of vertices is 180°; 3) a trapezoid has a long side and a short side, two sides facing each other, two hypotenuses with perpendicular lines.
Some reasons might cause the weaknesses of prospective Elementary School teachers in CK and PK. Firstly, the lecturer transfers knowledge to the students. In this case, the lecturer explained in detail the subject matter to prospective teachers, then question and answer session. In this learning process, prospective teachers passive derived knowledge. The effect of this learning is the prospective teacher’s lack of development and enriches their understanding. Bethel (Sousa, 2005) expressed student’s absorption of the material depends on the learning process experienced by students. Prospective teachers only absorb 5% because they only listen. Secondly, the lecturer divide subject matter to prospective teachers and they present the subject matter in a group. This learning process causes them only mastering the subject that they presented and lack the quality and accuracy of knowledge. The lecturer should clarify the information and complete the report. Thirdly, prospective teachers need a concrete model in some mathematics topics. For example, prospective teachers define a trapezoid has two parallel sides and two hypotenuses, a rectangular has a characteristic of four sides, upper, lower, left, and right [24]. The misconception happened because learning rarely used concrete, just image. When using the image as learning media, the position of objects will be the same. On the contrary, when using concrete models, the location of objects is relative. The fourth, activities of prospective teachers dominant in the form of paper and pencil. [3] uses the terminology of Subject Matter Knowledge (SMK) for knowledge and understanding of facts, concepts, principles, and relationships between concepts/principles. It means CK and PK conclude to SMK, although they are not explicitly stated. Researchers have found that insufficient SMK among teachers leads their students to develop misconceptions, misunderstanding, and misinterpretation regarding the subject matter during instruction [4]. Concerning Mathematics education, SMK is a core component of teacher professional competence to make mathematics comprehensible for students (e.g., [7], [25]). A teacher’s strategy, mathematical representation, and explanation in the teaching process really depend on his broad and deep conceptual knowledge [5].

3. Conceptual and procedural knowledge of geometry: Examples
Geometry plays a significant role in primary schools' mathematics, and it provides a rich source of visualization for arithmetical, algebraic, and statistical concepts [26]. Geometry is a branch of mathematics that deals with the study of different shapes or figures and their properties.

Conceptual knowledge is understanding the definition, nature, relationships between concepts, and how to figure out a concept. Conceptual knowledge in geometry includes: 1) Understanding the meaning of geometric objects such as: parallel lines, angles, plane shapes and solid shaped, etc; 2) Categorizing planes based on their geometric properties; 3) Understanding the relationship between concepts such as the relationship between the equilateral triangle with isosceles triangle, rectangular relationship with parallelogram, etc; 4) Knowing how to figure out formulas, such as: perimeter and area, etc. Procedural knowledge is almost always about solving problems, and the answers or procedures must be precise. Conceptual knowledge in geometry includes: 1) Calculate the perimeter and area; 2) Determine the area of a square if the perimeter is the same as the perimeter of a triangle; 3) Determine the size of the largest rectangle that can be formed from a piece of string; Calculate the volume of the sphere.

4. Enhancing CK and PK of Prospective Teachers
In some countries, Elementary School teachers teach all subjects except when there are specialty teachers such as sports and religion. In Indonesia, the prospective teachers of Elementary Schools teach Mathematics, Natural Sciences, Social Sciences, Indonesian Language, and Citizenship. Also, these prospective Elementary School teachers spend most of their time in the lectures of teaching methods and other pedagogical knowledge. So, they do not get an in-depth understanding of subject matter knowledge, especially CK and PK in mathematics.

CK and PK improvement can be conducted through approaches used in learning mathematics [27]. [7] argued against applying practicum-based learning in teacher's education. One of the approaches in line with practicum-based learning is CPA Approach. The practicum-based approach has some criteria,
among others, is to give chances to prospective teachers to experience mathematics learning as that of Elementary School students learning mathematics.

CPA approach is a learning sequence that refers to Brunner’s theory, namely enactive, iconic, and symbolic representations of cognitive growth [28], [29]. According to Brunner, the formation of conceptual knowledge begins with physical experience using concrete objects (active), then transferred into the form of an image (iconic). Conceptual understanding based on physical experience and images abstracted using symbolic.

The implementation of the CPA Approach is based on the principle of learning in primary school teachers’ education that experience mathematically as the subject and the explicit learning as well as reflecting on mathematics in elementary schools [30]. This opinion answers the weakness of primary school teachers in the provision, selection, and use of teaching aids as a mathematical model.

The learning model for prospective teachers designed based on CPA Approach called CPA Professional. Implementing the CPA approach is considered very appropriate because 1) education of prospective teachers is more on reviewing pedagogical knowledge; 2) prospective teachers’ CK and PK are still shallow; 3) facilities and infrastructures in Elementary Schools are still minimal. CPA Approach learning process and its corresponding teaching sequence are described in Table. 1 [29].

| Learning process                                      | Teaching Actions                                                  |
|-------------------------------------------------------|-------------------------------------------------------------------|
| Enactive (Manipulation of the concrete object/an experience) | Selection of real objects to be manipulated, and Facilitation of the act of manipulating |
| Iconic (Perceptual images of forms of Mathematical idea)   | Facilitation of making the connection of iconic form and mathematical idea |
| Symbolic (abstract symbol)                              | Facilitation of making the connection to the abstract mathematical symbol |

This research applies CPA approach principles in mathematics teacher education called CPA Professional model. The application of the CPA Professional model is following the opinions of [31] and [17] that conceptual and knowledge are not dichotomies. Analogy, multi-representation (concrete and abstract) helps conceptual formation so that prospective teachers can explain the concept and how to find an idea. Besides, CPA Professional model provides an opportunity to learn mathematics instruction to Primary Students, and teacher's conceptual and procedural knowledge increases through learning assisted by teaching aids.

According to NAEP [32] in mathematics learning there are standard processes, namely connections, reasoning, communication and problem solving. Through modification of the CPA Approach to the mathematical process standard, the professional CPA principles are stated as follows: 1) Involves mathematical reasoning by using the teaching aid and pictures of surrounding objects to develop conceptual knowledge; 2) Involves mathematical connections and communication when abstraction from the phase of the visual/visual aids to the symbolic step to develop procedural abilities; 3) Requires problem-solving to implement conceptual and procedural skills. Based on the CPA Professional principles, the CPA Professional framework can be described in Figure 1.

The steps of CPA professional model are as follows:
1. Enactive: Manipulation of concrete objects / an experience. Conduct reasoning when observing congress models of geometry. Find differences and similarities between geometry models to form conceptual knowledge, including definitions, building properties
2. Iconic: Perceptual images of forms of Mathematical ideas. Facilitation of making connections of iconic types and mathematical concepts
3. Symbolic: abstract symbol. Facilitation of communication-based on making connections to abstract mathematical symbols
4. Problem-solving: Implementation. Recognize and formulate problems, Generate and modified procedures and use accumulate knowledge in new situation

![Diagram](Image)

Figure 1. CPA Professional Framework

The benefits of implementing CPA Approach are: a) Making their own teaching aids will provide insight and creativity to prospective teachers; b) Mathematical representation varies to give deeper understanding on one form of representations; c) prospective teachers have the opportunity to learn how to use these teaching aids to explain conceptual and procedural knowledge to students.

5. Conclusion

The learning Model proposed for the development of CK and PK of prospective teacher is a CPA Professional that is a modification of the CPA Approach with the process standards of mathematics. The order of its application is the enactive, iconic, symbolic, and problem-solving. CPA Professional emphasizes on the use of props in the understanding of mathematical concepts. Further learning aids also help prospective teachers to develop pedagogical content knowledge.

The application of the CPA professional model is still essential, even in the digital era. Because only a few primary schools have facilities of adequate infrastructure for the use of computer technology in learning. CPA professional model can be one of the learning models for prospective teachers, especially in geometry domain.
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