MODELING ACOUSTIC EMISSION IN MICROFRACTURING PHENOMENA

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ABSTRACT

It has been recently observed that synthetic materials subjected to an external elastic stress give rise to scaling phenomena in the acoustic emission signal. Motivated by this experimental finding we develop a mesoscopic model in order to clarify the nature of this phenomenon. We model the synthetic material by an array of resistors with random failure thresholds. The failure of a resistor produces a decrease in the conductivity and a redistribution of the disorder. By increasing the applied voltage the system organizes itself in a stationary state. The acoustic emission signal is associated with the failure events. We find scaling behavior in the amplitude of these events and in the times between different events. The model allows us to study the geometrical and topological properties of the micro-fracturing process that drives the system to the self-organized stationary state.

INTRODUCTION

Acoustic Emission (AE) is produced by sudden movements in stressed systems. Several experiments have recently observed this phenomenon on very different length scales (from the largest scale of an earthquake to the smallest one of dislocation motions) [1, 2, 3]. Unfortunately, the AE analysis is a rather delicate technique since each external stress is unique and tests the whole sample. For instance, it is very difficult to obtain in this way insight on the microscopic dynamics of the fracturing phenomena. The statistical analysis, however, gives rise to the hypothesis that AE is generated by fracturing phenomena which are similar to critical points. Correlations develop leading to cascade events which drive the systems into a critical stationary state. For this reason, as a working hypothesis, the mechanism of Self-Organized Criticality (SOC) [4] has been invoked.

The understanding of this statistical behavior calls for a model that can simulate the fracturing phenomenon. Unfortunately, the models usually considered describe the formation of a macroscopic crack [5]. These can therefore model AE of fracturing phenomena on mesoscopic scale culminating in a large event that change the system’s properties dramatically [6]. This is very different from the stationary state generated by stressing the sample below the breaking threshold of the system. In fact, in this case the fracturing phenomenon produces an energy release that changes the physical properties in a non-destructive way (like the passage to a different metastable state).

Here we show a statistical model for fracturing phenomena, where the rupture burst and the following energy release changes the properties but does not destroy the system. This
allows us to obtain a stationary state for AE of which we can investigate the statistical properties in space, time and magnitude.

**THE MODEL**

The mesoscopic description of an elastic disordered medium is obtained discretizing macroscopic elastic equations. In the theory of linear elasticity, these equations relate the stress tensor \( s_{\alpha\beta} \) to the strain tensor \( \epsilon_{\gamma\delta} \) via the Hooke tensor \( C_{\alpha\beta\gamma\delta} \). The full tensorial formalism is quite heavy to handle numerically. A compromise is obtained by considering scalar elastic equations. In fact the phenomenology of fractures in scalar models captures many essential features of more complex tensorial models. Scalar elasticity is formally equivalent to electricity, provided one identifies the current \( I \) with the stress, the voltage \( V \) with the strain and the conductivity \( \sigma \) with the Hooke tensor.

The discretization scheme we use corresponds to the study a resistor network. For symmetry reason we will consider a rotated square lattice. The disorder, due to the inhomogeneity in the synthetic material, is introduced in the model in the failure thresholds \( I_c \) of the resistors. For simplicity we will use an uniform distribution. The crucial part of the model is the breaking criterion, which describe the dynamics of the micro-fracturing process. Typically the breaking criterion is chosen so that if the current flowing in a resistor exceed the failure threshold the conductivity of the bond drops abruptly to zero. In this way the system develops a macroscopic crack and the lattice breaks apart. To describe the micro-fracturing phenomena in the stage preceding the onset of the macroscopic crack it is useful to consider the concept of damage. For a macroscopic elastic material, in which micro-fracturing processes are taking place, the damage \( D \) is a tensor relating the effective Hooke tensor \( \hat{C} \) to the Hook tensor of the undamaged material. The damage is defined as \( D = I - C\hat{C}^{-1} \). For scalar elasticity the damage is just a constant relating the effective resistance of the damaged material to that of the undamaged one. We generalize the concept of damage from the macroscopic to the mesoscopic description, using it in the breaking criterion. When the current in a bond exceeds the threshold we impose a permanent damage to the bond. In other words, the conductivity of the bond drops by a factor \( a = (1 - D) \).

In the synthetic materials we are describing, after a micro-fracturing event, local rearrangements take place. We model this effect by changing at random the breaking threshold of the damaged bond as well as those of the neighboring bonds. This rearrangement of the disorder emphasize the probability of breaking successively neighboring bonds. In crack models this process is enforced by imposing the connectivity of the crack or by similar rules.

**SIMULATION RESULTS**

To simulate the model we start from an undamaged lattice where the conductivities are equal to one for all the bonds. The breaking threshold are chosen at random between zero and one. We then impose an external voltage between two edges of the lattice and we use periodic boundary conditions in the other direction.
The voltage is increased until the current in some bond exceed the threshold. When this happens we apply the breaking rule and we check if subsequent failures occurs. In fact due to the long range elastic interactions combined with the redistribution of the disorder, a single failure can be followed by other similar events, thus generating an avalanche. We consider the number of bonds that break in an avalanche to be proportional to the amplitude of the emitted acoustic signal. In the early stage of the process only small avalanches occur and the current carried by the material steadily increases. In this stage the damage is scattered through the lattice in a random uniform way. After some time when some damage has been accumulated into the system the activity starts to increase. Eventually the system reaches a stationary state where the current does not increase anymore. In other words the increase of the voltage is exactly balanced by the damage, in such a way that the current is kept constant. In this state the damage is no longer homogeneously scattered, but tends to be localized along lines. We are modeling an ideal case in which a bond can suffer a very big damage without breaking completely. In fact we can slightly modify the model by introducing a threshold in the conductivity after which the bond is considered broken (i.e. the conductivity drops to zero). One can then easily understand that the regions of localized damage are those where the macroscopic crack will eventually form and this is indeed what we observe in our simulations.

The activity in this stationary state is highly fluctuating and the distribution of amplitude follows a power law. The scaling region increases with the system size and the fact is a signature of an underlying critical state (see figure 1). Another sign of the criticality of the system is provided by the distribution of time intervals between subsequent avalanches. Since the voltage is increased linearly in time, this corresponds to consider the distribution of voltage increases $\Delta V$. The power law with slope close to $x = -1$ is reminiscent of the Omori [7] law for fore-shocks in earthquakes.

CONCLUSIONS

In summary we have introduced a statistical model for fracturing phenomena. This model describes on a mesoscopic scale the energy release and the following rearrangements in the material produced by fracturing events. The simulations on the model show that the system organizes itself in a stationary state where we can relate the AE signal with the rupture events. We investigate the statistical properties of rupture sequences during fracturing and their correlation properties. We find that the stationary state develops critical correlations and scaling behavior through a self-organization process. This kind of analysis is usually considered in the study of experimentally detected AE signals and the proposed model could give important clues in the understanding of the general scale invariance of the fracturing phenomena. The Center for Polymer studies is supported by NSF.

References
Figure 1: The avalanche distribution in the stationary state for different system sizes.

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