Neutralino spectrum in top-down models of UHECR

Alejandro Ibarra* and Ramon Toldrà†

Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK

Abstract

We calculate the cosmic ray spectrum of ultra high energy neutralinos that one should expect provided that the observed ultra high energy cosmic rays are produced by the decay of superheavy particles $X$, $M_X > 10^{12}$ GeV, in supersymmetric models. Our calculation uses an extended DGLAP formalism. Forthcoming cosmic ray observatories should be able to detect these neutralinos.

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I. INTRODUCTION

Over one hundred cosmic ray events with energy higher than $4 \times 10^{10}$ GeV have been detected by past and present observatories. About 30 events have energy in the range $(1 - 3) \times 10^{11}$ GeV (see [1] for an experimental review). In top-down production mechanisms of Ultra High Energy Cosmic Rays (UHECR) the decay products of very massive particles $X$ with $M_X > 10^{12}$ GeV account for these events. Particle $X$ can be either a GUT scale particle produced by the decay of a cosmological network of topological defects created in a phase transition in the early universe [2–4], or a cold dark matter particle clustered in the galactic halo whose abundance was generated during or shortly after inflation [6–10]. Whatever the nature of $X$ is, it decays into partons which hadronise and give the UHECR primaries that we observe on the Earth. Therefore, in order to calculate the predicted spectrum and composition of UHECRs one has to calculate the nonperturbative Fragmentation Functions (FFs) of partons into hadrons at the energy scale $M_X$. Several approaches have been taken to solve the fragmentation process and calculate the spectrum of UHECRs: Montecarlo generators [8,11], analytical estimates (MLLA) [4,5,7,12] and DGLAP evolution [13–16] (see [8,15] for a comparison of these different approaches).

As long as supersymmetry (SUSY) is a low energy symmetry of nature, the decay of $X$ will produce supersymmetric particles. If R-parity is conserved, the Lightest Supersymmetric Particle (LSP) is stable and should be among the primary cosmic rays that reach the Earth. Theoretical motivations favour the neutralino as the LSP, hence we shall concentrate on neutralinos. In the Minimal Supersymmetric Standard Model (MSSM) with R-parity conserved, neutralinos are a mixture of higgsinos and neutral gauginos (bino, $\tilde{B}$, and neutral wino, $\tilde{W}^0$). The precise composition of the neutralinos in terms of the interaction eigenstates depends on the particular SUSY scenario. For simplicity, we shall concentrate on the Constrained MSSM, which assumes that the soft SUSY-breaking scalar masses, the gaugino masses and the trilinear parameters are all universal at the GUT scale. In this scenario, it can be shown that in the region of the parameter space where the relic density of neutralinos is of cosmological significance, the LSP is mainly a bino [18].

Weakly interacting particles like neutralinos or neutrinos have cross sections with ordinary matter too small to be detected with present day UHECR observatories. Baryons, nuclei and perhaps photons may account for all the events detected so far [19]. Forthcoming detectors will be sensitive to weakly interacting UHECRs (see Refs. [20,21] and references therein). It is therefore important to obtain a good estimate of the neutralino flux expected in top-down models.

The neutralino spectrum in top-down models has already been estimated using Montecarlo generators by Berezinsky&Kachelrieß [11,22]. In the Montecarlo approach a primary parton with energy $M_X/2$ produced in the decay of $X$ initiates a parton cascade which proceeds until a specific minimal virtuality is reached. The squarks and gluinos in the parton shower decay into the LSP and ordinary partons once they reach the scale $M_{\text{SUSY}}$, the universal mass of squarks and gluinos. All quark and gluons evolve down to the hadronization scale where a phenomenological model is employed to bind them into hadrons.

The Montecarlo simulations did not include the electroweak radiation of neutralinos by off-shell partons in the shower. In the next section we show how the DGLAP formalism employed to

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1In some SUSY models the gravitino, the gluino and even the sneutrino could be the LSP. An exotic hadron state containing a stable gluino has been proposed to explain the UHECR events [17].
calculate the neutrino, photon and baryon spectra of UHECR \cite{13,14} can be extended to calculate the spectrum of neutralinos. We concentrate on the electroweak radiation of neutralinos by off-shell partons to complement the Montecarlo simulations. We find that at large $x$ electroweak radiation of neutralinos is important. The large $x$ region is the most relevant for future cosmic ray observatories since in this region the neutralino flux is larger than the neutrino flux and will not be swamped by the neutrino signal.

II. EVOLUTION EQUATIONS FOR NEUTRALINO FRAGMENTATION FUNCTIONS

The main channels of neutralino production are the hadronic decays of $X$. Hence the flux of neutralinos is given by the fragmentation functions from partons, $D_a^\chi(x, M_X^2)$, where $a$ is any quark $q_k$ and squark flavour $s_k$, a gluon $g$ or a gluino $\lambda$. The flux is proportional to the sum over $a$ of $D_a^\chi(x, M_X^2)$ each one weighted with the branching ration for the decay of $X$ into parton $a$. The variable $0 < x < 1$ is the fraction of the momentum of $a$ carried off by $\chi$. These nonperturbative functions depend on the energy scale $\mu$; in our case $\mu = M_X$. The rate of change of $D_a^\chi(x, \mu^2)$ with $\mu^2$ is given by the sum of two terms. The first one takes into account the ordinary SUSY QCD branching of partons and gives rise to the so-called Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations \cite{23,24}. The second term takes into account the emission of neutralinos in the parton shower, which stems from the tree level weak SUSY coupling among quarks, squarks and neutralinos. For any (anti)quark and (anti)squark we obtain, to lowest order in the strong coupling $\alpha_S$ and the weak couplings $\alpha^W_{\chi,k}$,

$$\mu^2 \partial_\mu^2 D_{q_k}^\chi(x, \mu^2) = \frac{\alpha_S(\mu^2)}{2\pi} \left[ \sum_i P_{q_i q_k}(x) \otimes D_{q_i}^\chi(x, \mu^2) + \ldots \right] + \frac{\alpha^W_{\chi,k}(\mu^2)}{2\pi} P_{\chi q_k}(x) \otimes D_{\chi}(x, \mu^2),$$

(1)

$$\mu^2 \partial_\mu^2 D_{s_k}^\chi(x, \mu^2) = \frac{\alpha_S(\mu^2)}{2\pi} \left[ \sum_i P_{s_i s_k}(x) \otimes D_{s_i}^\chi(x, \mu^2) + \ldots \right] + \frac{\alpha^W_{\chi,k}(\mu^2)}{2\pi} P_{\chi s_k}(x) \otimes D_{\chi}(x, \mu^2),$$

(2)

where the dots inside the square brackets stand for the remaining leading order SUSY QCD terms\cite{15,25,26}. For gluons and gluinos, to lowest order in $\alpha_W$, there are no additional terms to the usual SUSY QCD ones. The FF of neutralinos from neutralinos is $D_{\chi}(x, \mu^2) = \delta(1 - x) + O(\alpha_W)$. Note that the functions $D_{q_i}^\chi(x, \mu^2)$ are of order $\alpha^W_{\chi q_i}$. The splitting functions of quarks and squarks into neutralinos are given by $P_{\chi q_k} = 1 - x$ and $P_{\chi s_k} = 1$. We make use of the convolution operator

$$A(x) \otimes B(x) \equiv \int_x^1 \frac{dz}{z} A(z)B(\frac{z}{x}).$$

(3)

Analogous equations to Eqs. \cite{11,12} employed to study photon structure or photon production by parton showers can be found in \cite{27,28}.

The DGLAP equations do not include soft gluon emission coherence. Coherence is important at low $x$. Our results will only hold for $x$ much larger than the position of the Gaussian peak produced by coherence, at $x_p \approx \sqrt{\Lambda_{QCD}/M_X}$. As pointed out in Sec. \ref{sec:1}, we are only interested in the large $x$ region. The DGLAP equations can be modified to include coherence, see for example \cite{13}.
It is useful to introduce the following evolution variable

\[ \tau \equiv \frac{1}{2\pi b} \ln \frac{\alpha_S(\mu_0^2)}{\alpha_S(\mu^2)}, \]  

(4)

\( b \) being the coefficient in the leading order \( \beta \)-function governing the running of the strong coupling: \( \beta(\alpha_S) = -b\alpha_S^2 \).

For simplicity we shall only consider gaugino production. We shall argue later that the final results are not substantially altered if higgsinos are included in the calculation. Also for clarity we shall concentrate on the evolution of quark and squark singlet functions, defined as the following

\[ D_q^X \equiv \sum_k D^X_{q_k} + D^X_{\bar{q}_k}, \]  

(5)

\[ D_s^X \equiv \sum_k D^X_{s_k} + D^X_{\bar{s}_k}. \]  

(6)

The singlet functions are coupled to the gluons and gluinos by means of the following \( 4 \times 4 \) matrix integro-differential equation

\[
\partial_{\tau} \begin{pmatrix} D_q^X \\ D_g^X \\ D_s^X \\ D_\lambda^X \end{pmatrix} = \begin{pmatrix} P_{qq} & 2n_F P_{qg} & P_{sq} & 2n_F P_{q\lambda q} \\ P_{gg} & P_{gg} & P_{sg} & P_{gg} \\ P_{qs} & 2n_F P_{gs} & P_{ss} & 2n_F P_{s\lambda s} \\ P_{\lambda q} & P_{g\lambda} & P_{s\lambda} & P_{\lambda\lambda} \end{pmatrix} \otimes \begin{pmatrix} D_q^X \\ D_g^X \\ D_s^X \\ D_\lambda^X \end{pmatrix} + \alpha_W^X \frac{\alpha_S}{\alpha_S} \begin{pmatrix} P_{\chi q} \\ 0 \\ P_{\chi s} \\ 0 \end{pmatrix},
\]  

(7)

where \( \alpha_W^X = \sum_k \alpha_W^{X,k} \) and \( n_F \) is the number of flavours. These will be the only relevant equations if one assumes flavour universality in the decay of \( X \). If one wishes to study the evolution of each flavour separately then one needs the complete set of DGLAP equations as given in \([15,25,26]\) to which one has to add the neutralino radiation terms on the right hand side of Eqs. (1-3).

We can write Eq. (7) in the simple matrix form

\[
\partial_{\tau} xD^X = xP \otimes xD^X + xX^X,
\]  

(8)

where \( xD^X \) and \( xX^X \) are \( 4 \times 4 \) matrices and \( xP \) is a \( 4 \times 4 \) matrix whose elements are all the leading order SUSY QCD splitting functions \( P_{ab} \) times \( x \). These functions were calculated in \([25,26]\). We have multiplied Eq. (7) by \( x \) to improve numerical stability.

Equation (8) is a linear inhomogeneous equation. Its associated homogeneous equation is the usual DGLAP equation for the coupled evolution of the quark and squark singlets, gluons and gluinos. The inhomogeneous or source term arises from the neutralino radiation by partons. The general solution to Eq. (8) is the sum of the general solution to the associated homogeneous equation plus a particular solution

\[
xD^X(x, \tau) = E(x, \tau - \tau_0) \otimes xD^X(x, \tau_0) + \int_{\tau_0}^\tau d\tau' E(x, \tau - \tau') \otimes xX^X(x, \tau').
\]  

(9)

The evolution operator \( E(x, \tau) \) is the solution to the associated homogeneous equation with the initial condition \( E(x, 0) = \delta(1 - x)I_4 \) (see \([14,29]\))

\[
E(x, \tau) = e^{xP(x)\tau} \equiv \delta(1 - x)I_4 + xP(x)\tau + \frac{1}{2!} xP(x) \otimes xP(x)\tau^2 + \ldots
\]  

(10)
We take as initial energy scale $\mu_0 = M_{\text{SUSY}}$, $\tau_0 \equiv \tau(\mu_0) = 0$, the typical scale for all sparticle masses.

Equation (3) gives the total neutralino FF as the sum of two terms. The first one, the solution to the homogeneous equation, is the contribution from partons that decay on-shell into neutralinos (assuming a universal mass for all squarks and gluinos $M_{\text{SUSY}}$). This contribution was studied with Montecarlo generators in Refs. [11,22]. The second term is the contribution stemming from the electroweak radiation of neutralinos by partons. From now on, we concentrate on this second term and take

$$xD^\chi(x, \tau) = \int_0^\tau d\tau' E(x, \tau - \tau') \otimes xX^\chi(x, \tau').$$

(11)

This expression is our main equation. It will allow us to estimate the relevance of electroweak radiative emission of neutralinos.

### III. NUMERICAL CALCULATION AND RESULTS

In order to calculate numerically $xD^\chi(x, \tau)$ as given in Eq. (11) we expand the evolution operator in Laguerre polynomials$^3$ $L_n(x)$

$$E(x, \tau) = \sum_{n=0}^{\infty} E_n(\tau) L_n(-\ln x),$$

(12)

$$E_n(\tau) = \sum_{i=1}^{4} e^{\lambda_i \tau} \sum_{k=0}^{n} \frac{\tau^k}{k!} B_{i,n}^k.$$  

(13)

The scalars $\lambda_i$ are the eigenvalues of $xP_0$, the first coefficient in the Laguerre expansion of the matrix $xP(x)$. Energy conservation gives $\lambda_1 = 0$. All other eigenvalues are negative. The $4 \times 4$ matrices $B_{i,n}^k$ are given by recursive relations that were calculated in [16]. Likewise we expand the source term in Laguerre polynomials

$$xX^\chi(x, \tau) = \frac{\alpha_W^\chi(\tau)}{\alpha_S(\tau)} f(x),$$

(14)

$$f(x) = \sum_{n=0}^{\infty} f_n L_n(-\ln x),$$

(15)

$$f_n^T = \left( \left( \frac{1}{2} \right)^{n+1} - \frac{1}{3} \left( \frac{2}{3} \right)^n, 0, \left( \frac{1}{2} \right)^{n+1}, 0 \right).$$

(16)

The Laguerre expansions for the neutralino FFs are

$$xD^\chi(x, \tau) = \sum_{N=0}^{\infty} \tilde{C}_N^\chi(\tau) L_N(-\ln x),$$

(17)

where $\tilde{C}_0^\chi \equiv C_0^\chi$ and $\tilde{C}_N^\chi \equiv C_N^\chi - C_{N-1}^\chi$ for $N > 0$. The 4–vectors $C_N^\chi$ are calculated substituting all Laguerre expansions in Eq. (13).

$^3$Laguerre polynomial expansions to study scaling violations in QCD were first introduced in [29].
\[ C_N^\chi(\tau) = \sum_{n=0}^{N} \sum_{k=0}^{n} \sum_{i=1}^{4} \frac{B_{i,n} f_{N-n}^{k}}{k!} \int_0^\tau d\tau' e^{\lambda_i(\tau-\tau')(\tau - \tau')^k} \frac{\alpha_W^\chi(\tau')}{\alpha_S(\tau')}. \]  

(18)

In order to present our results we multiply FFs by \( x^3 \) since the measured spectrum is usually multiplied by \( E^3 \) to highlight its structure. We plot in Fig. (1) our numerical calculation for the bino \( x^3D_a(x, M_X^2) \) when \( M_X = 10^{12} \) GeV and \( M_{\text{SUSY}} = 400 \) GeV. The major contribution comes from squarks and quarks, which can radiate neutralinos at order \( \alpha_W \). The contribution from gluinos and gluons is much smaller since they do not have order \( \alpha_W \) coupling to neutralinos, therefore to lowest order they can only generate neutralinos through mixing with squarks and quarks, see Eq. (7). The fraction of available energy \( M_X = 10^{12} \) GeV carried by binos is \( \langle xD_a(x, M_X) \rangle = \)

![Graph showing fragmentation functions](image)

FIG. 1. \( \tilde{B} \) fragmentation functions for \( M_X = 10^{12} \) GeV and \( M_{\text{SUSY}} = 400 \) GeV. The solid line is the squark contribution, the dotted lines is the quark one, the dashed line is the gluino contribution and the dot-dashed line is the gluon one.

\( \tilde{C}_0^\chi(\tau(M_X)) = (0.047, 0.002, 0.109, 0.003) \) for \( q, g, s, \lambda \), respectively (divide by \( 2n_F \) to get fraction per quark and squark flavour). From Eqs. (11) and (14) one can check that for the quark and squarks singlets the order of magnitude is given by \( \langle xD^\chi \rangle \sim \tau \alpha_W / \alpha_S \).

In Fig. (2) we show how \( \tilde{B} \) distributions change for different values of \( M_X \). Each curve is the sum of the quark and squark singlets, gluon and gluino contributions. Note that \( \partial_x D^x > 0 \) for all \( x \) because of the inhomogeneous term in Eq. (5). If the sparticl e mass scale \( M_{\text{SUSY}} \) is around the electroweak scale, the final result depends feebly on \( M_{\text{SUSY}} \).

The decay of \( X \) will produce neutral winos as well. The \( B \) and \( \tilde{W}^0 \) curves have similar shapes, the main difference between them being that, for a common scale \( M_X \), \( \tilde{W}^0 \) FFs are always slightly larger than \( \tilde{B} \) FFs, since the coupling \( \alpha_W^\chi \) is slightly stronger for \( \tilde{W}^0 \) than for \( \tilde{B} \). The wino will eventually decay into the bino, the LSP. Its lifetime times its Lorentz factor is much smaller than
FIG. 2. $\bar{B}$ fragmentation functions for $M_{\text{SUSY}} = 400$ GeV and $M_X = 10^{14}$ GeV (solid line), $M_X = 10^{12}$ GeV (dotted line) and $M_X = 10^{10}$ GeV (dashed line).

1 kpc/c, therefore neutral winos produced in the galactic halo or further will disintegrate into the LSP before reaching the Earth. The $\bar{B}$ spectrum on the Earth will be the sum of the $\bar{B}$ contribution produced at the decay spot of $X$ plus the $\bar{B}$ contribution stemming from $\tilde{W}^0$ decay into $\tilde{B}$.

The decay sequence of a neutral wino into a bino depends on the details of the scenario considered. Even within the framework of the Constrained MSSM, the freedom is still large. Nevertheless, some benchmark points have been proposed [30] for study at the Tevatron collider, the LHC and $e^+e^-$ colliders. These points are consistent with different experimental constraints, as well as cosmology, and can be regarded as generic in the whole parameter space. We shall be only concerned with the benchmark points A-D and G-M in Ref. [30], where the LSP is mainly the bino and the next-to-LSP is the neutral wino, with some admixture of higgsinos. The two remaining points correspond to the so-called “focus-point” region at large scalar masses, that we shall not consider. In any case, our analysis covers a large region of the allowed CMSSM parameter space.

The neutral wino decays mostly into sleptons and leptons, and it is followed by a decay of the slepton into the LSP. Therefore, only two-body decays are relevant. Given the generic case $A \rightarrow B + C$, we want to calculate which is the distribution of the decay product $dn_C/dx$ once the distribution for the decaying particle $dn_A/dx$ is known. From the phase space of the disintegration process we obtain

$$\frac{dn_C(x)}{dx} = \frac{m_A^2}{\sigma_2} \int_{y_{\text{min}}}^{y_{\text{max}}} \frac{dy}{y} \frac{dn_A(y)}{dy},$$

where we have defined

$$y_{\text{max}} \equiv \min \left[ \frac{\sigma_1 + \sigma_2}{2m_C^2} x, 1 \right],$$

(19)

(20)
\[
y_{\text{min}} \equiv \frac{\sigma_1 - \sigma_2}{2m^2_C} x,
\]
(21)

\[
\sigma_1 \equiv m_A^2 + m_B^2 - m_C^2,
\]
(22)

\[
\sigma_2 \equiv \sqrt{\lambda(m_A^2, m_B^2, m_C^2)},
\]
(23)

\[
\lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2ab - 2ac - 2bc.
\]
(24)

In addition there is the following kinematical constraint

\[
0 < x < \frac{2m^2_C}{\sigma_1 - \sigma_2}.
\]
(25)

The LSP flux reaching the atmosphere is the sum of the \( \bar{B} \) contribution produced directly in the decay of \( X \) plus the \( \bar{B} \) contribution produced by the decay of \( \bar{W}^0 \) on its way to the Earth. The two-body decay of winos into binos pushes the momenta to lower values of \( x \), making the direct \( \bar{B} \) component dominate over the component produced by \( \bar{W}^0 \) decay, for \( x > 0.1 \).

So far we have ignored higgsino production, so some words are now in order. The magnitude of the gaugino and higgsino couplings to partons are comparable, hence their FFs will be also similar. For the benchmark points that we have analysed, higgsinos will decay on their way to the Earth into winos and subsequently into binos. Therefore, for the same kinematical reason as the wino case discussed above, the total number of binos on Earth coming from higgsino decay will be a small correction to the direct bino contribution, for \( x > 0.1 \). For the same reason, we have ignored binos coming from the decay of the charginos that are also produced in the parton shower.

We compare in Fig. 3 the spectra of baryons, neutrinos and LSPs expected in the case that UHECRs are produced by the slow decay of a population of superheavy dark matter particles \( X \) clustered in the galactic halo. We take as dark matter particle mass \( M_X = 10^{12} \) GeV and as scale at which SUSY switches on \( M_{\text{SUSY}} = 400 \) GeV. The baryon and neutrino curves are obtained from Ref. [15]. The neutralino spectrum shown corresponds to benchmark point C in Ref. [30]. The final neutralino spectrum does not depend significantly on which benchmark point we choose, since the dominant contribution in the region of interest is given by the bino component produced directly by \( X \).

**IV. CONCLUSIONS**

We have used the DGLAP formalism to calculate the spectrum of neutralinos produced by the decay of particles with mass \( M_X > 10^{12} \) GeV. We have concentrated on the electroweak radiation of neutralinos by partons. This contribution has to be added to the on-shell contribution calculated with Montecarlo generators. We find that at large \( x \) the radiative contribution is slightly larger.

For \( x > 0.3 \), LSPs dominate over baryons, photons and neutrinos and their flux on Earth is non-negligible. If UHECR are produced by the decay of superheavy particles and SUSY is a low energy symmetry of nature, then forthcoming observatories sensitive to weakly interacting UHECRs should detect a flux of ultra high energy neutralinos.

**ADDENDUM**

As this manuscript was being finished, Ref. [31] appeared. This paper also discusses neutralino production by the decay of super-heavy dark matter particles using DGLAP evolution.
FIG. 3. Neutrino (dashed lime), baryon (solid line) and neutralino (dotted line) spectra expected from the decay of dark matter particles with $M_X = 10^{12}$ GeV clustered in the galactic halo.

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REFERENCES

[1] M. Nagano, A.A. Watson, Rev. Mod. Phys. 72 (2000) 689.
[2] C.T. Hill, Nucl. Phys. B 224 (1983) 469.
[3] C.T. Hill, D.N. Schramm, T.P. Walker, Phys. Rev. D 36 (1987) 1007;
    P. Bhattacharjee, C.T. Hill, D.N. Schramm, Phys. Rev. Lett. 69 (1992) 567;
    P. Bhattacharjee, G. Sigl, Phys. Rev. D 51 (1995) 4079;
    V. Berezinsky, A. Vilenkin, Phys. Rev. Lett. 79 (1997) 5202.
[4] V. Berezinsky, P. Blasi, A. Vilenkin, Phys. Rev. D 58 (1998) 103515.
[5] G. Sigl, S. Lee, P. Bhattacharjee, S. Yoshida, Phys. Rev. D 59 (1999) 043504.
[6] V.A. Kuzmin, V.A. Rubakov, Phys. Atom. Nucl. 61 (1998) 1028.
[7] V. Berezinsky, M. Kachelrieß, A. Vilenkin, Phys. Rev. Lett. 79 (1997) 4302.
[8] M. Birkel, S. Sarkar, Astropart. Phys. 9 (1998) 297.
[9] D. Chung, E.W. Kolb, A. Riotto, Phys. Rev. D 59 (1999) 023501;
    V. Kuzmin, I. Tkachev, Phys. Rev. D 59 (1999) 123006;
    D. Chung, P. Crotty, E.W. Kolb, A. Riotto, Phys. Rev. D 64 (2001) 043503.
[10] D.J. Chung, E.W. Kolb, A. Riotto, Phys. Rev. D 60 (1999) 063504.
[11] V. Berezinsky and M. Kachelrieß, Phys. Rev. D 63 (2001) 034007.
[12] V. Berezinsky, M. Kachelrieß, Phys. Lett. B 434 (1998) 61.
[13] N. Rubin, M. Phil. Thesis, Cavendish Laboratory, University of Cambridge (1999)
    (http://www.stanford.edu/~nrubin/Thesis.ps).
[14] Z. Fodor and S.D. Katz, Phys. Rev. Lett. 86 (2001) 3224.
[15] S. Sarkar and R. Toldrà, Nucl. Phys. B 621 (2002) 495.
[16] R. Toldrà, Comput. Phys. Commun. 143 (2002) 287.
[17] D.J. Chung, G.R. Farrar and E.W. Kolb, Phys. Rev. D 57 (1998) 4606.
[18] J. R. Ellis, T. Falk, G. Ganis and K. A. Olive, Phys. Rev. D 62 (2000) 075010.
[19] M. Ave, J.A. Hinton, R.A. Vázquez, A.A. Watson, E. Zas, Phys. Rev. Lett. 85 (2000) 2244.
[20] F. Halzen and D. Hooper, hep-ph/0110201.
[21] G. Sigl, hep-ph/0109202.
[22] V. Berezinsky and M. Kachelrieß, Phys. Lett. B 422 (1998) 163.
[23] G. Altarelli and G. Parisi, Nucl. Phys. B 126 (1977) 298.
[24] L.N. Lipatov, Sov. J. Nucl. Phys. 20 (1975) 94;
    V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438;
    Yu.L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641.
[25] C. Kounnas and D.A. Ross, Nucl. Phys. B 214 (1983) 317.
[26] S.K. Jones and C.H. Llewellyn-Smith, Nucl. Phys. B 217 (1983) 145.
[27] R.J. DeWitt, L.M. Jones, J.D. Sullivan, D.E. Willen and H.W. Wyld, Phys. Rev. D 19 (1979)
    2046 [Erratum-ibid. D 20 (1979) 1751].
[28] A. Nicolaidis, Nucl. Phys. B 163 (1980) 156.
[29] W. Furmański and R. Petronzio, Nucl. Phys. B 195 (1982) 237.
[30] M. Battaglia et al., Eur. Phys. J. C 22 (2001) 535.
[31] C. Barbot and M. Drees, hep-ph/0202072.