Are nuclear matter properties correlated to neutron star observables?

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We investigate properties of nuclear matter and examine possible correlations with neutron star observables for a set of microscopic nuclear equations of state derived within the Brueckner-Hartree-Fock formalism employing compatible three-body forces. We find good candidates for a realistic nuclear EOS up to high density and confirm strong correlations between neutron star radius, tidal deformability, and the pressure of betastable matter. No correlations are found with the saturation properties of nuclear matter.

I. INTRODUCTION

Neutron star (NS) observations allow us to explore the equation of state (EOS) of nuclear matter at densities well beyond the ones available in terrestrial laboratories. The nature of matter under conditions of extreme density and stability, found only in the NS core, still remains an open question. In particular, the mass and radius of NSs encode unique information on the EOS at supranuclear densities. Currently the masses of several NSs are known with good precision [8–11], but the information on their radii is not very accurate [8–10], being more elusive than NS masses. Particularly, a measurement of the radius with an error of about 1 km could discriminate between soft and stiff EOSs, as discussed in the current literature [11]. For this purpose, present observations of NICER [12, 13] could in principle achieve an accuracy of about 2% for the radius, whereas future planned missions like eXTP [14] will allow us to statistically infer their mass and radius to within a few percent. This information can be used to determine the EOS of the matter in the NS interior, and the nature of the forces between fundamental particles under such extreme conditions.

The recent detection by the Advanced LIGO and Virgo collaborations of gravitational waves emitted during the GW170817 NS merger event [15–17] has stimulated an intense research activity towards the understanding of the nuclear matter EOS. In particular, it provided important new insights on the structural properties of these objects, most prominently their masses and radii, by means of the measurement of the tidal deformability [18, 19], and allowed to deduce upper [15, 16] and lower [20] limits on it.

In this paper we compare the constraints on the nuclear EOS obtained from heavy-ion collisions (HICs) with those extracted from the analysis of the NS merger event GW170817. We also examine possible correlations among properties of nuclear matter close to saturation with the observational quantities deduced from GW170817. For this purpose we present recent calculations of NS structure and their tidal deformability using various microscopic EOSs for nuclear configurations.

The paper is organized as follows. In Sec. III we give a brief overview of the hadronic EOSs we are using, and in Sec. III we discuss their saturation properties at normal nuclear density. Constraints on the NS maximum mass are evaluated and the tidal deformability as important NS observable is introduced in Sec. IV. In Sec. V we investigate the compatibility of the EOSs with constraints obtained from HIC data and the GW merger event, and examine possible correlations between both. In Sec. VI we draw our conclusions.

II. EQUATIONS OF STATE

The theoretical description of nuclear matter under extreme density conditions is a very challenging task. Assuming that the most relevant degrees of freedom are nucleons, thus neglecting other particles such as hyperons, kaons, or quarks, the theoretical models can be either ab-initio (microscopic) or phenomenological.

In this paper we use several EOSs based on the microscopic Brueckner-Hartree-Fock (BHF) many-body theory [21, 22], which provides a density expansion [23–25] of the nuclear-matter binding energy based on the use of realistic two-body forces. It is well known that nucleonic three-body forces (TBF) are needed in order to reproduce correctly the saturation properties of nuclear matter. Currently a complete ab-initio theory of TBF is not available yet, and therefore we adopt either phenomenological or microscopic models [26–29]. The microscopic BHF EOSs employed in this paper, described in detail in Refs. [25, 30], are based on different nucleon-nucleon potentials, namely the Bonn B (BOB) [31, 32], the Nijmegen 93 (N93) [33, 34], and the Argonne V18 (V18) [35]. In the latter case, we also provide an EOS obtained with the phenomenological Urbana model for describing TBF (UX). Useful parametrizations of these EOSs are given in the Appendix.

In the same theoretical framework, we also studied an EOS based on a potential model which includes explicitly the quark-gluon degrees of freedom, named fss2 [36, 37]. This reproduces correctly the saturation point of symmetric matter and the binding energy of few-nucleon systems without the need of introducing TBF. In the following those EOSs are labelled FSS2CC and FSS2GC, indicating two different parametrizations of solving the BHF equations.

We compare these BHF EOSs with the often-used results of the Dirac-BHF method (DBHF) [38], which employs the Bonn A potential, and the APR EOS [39] based on the vari-
TABLE I. Saturation properties and NS observables predicted by the considered EOSs. See text for details.

| EOS    | \(\rho_0 [\text{fm}^{-3}]\) | \(-E_0[\text{MeV}]\) | \(K_0[\text{MeV}]\) | \(S_0[\text{MeV}]\) | \(L[\text{MeV}]\) | \(M_{\text{max}}[M_\odot]\) | \(A_{1.4}\) | \(R_{1.4}[\text{km}]\) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------------|----------------|
| BOB    | 0.170           | 15.4            | 238             | 33.7            | 70              | 2.50            | 570            | 12.9            |
| V18    | 0.178           | 13.9            | 207             | 32.3            | 67              | 2.36            | 442            | 12.3            |
| N93    | 0.185           | 16.1            | 229             | 36.5            | 77              | 2.25            | 473            | 12.7            |
| UIX    | 0.171           | 14.9            | 171             | 33.5            | 61              | 1.96            | 309            | 11.8            |
| FSS2CC | 0.157           | 16.3            | 219             | 31.8            | 52              | 1.94            | 295            | 11.8            |
| FSS2GC | 0.170           | 15.6            | 185             | 31.0            | 51              | 2.08            | 262            | 11.5            |
| DBHF   | 0.181           | 16.2            | 218             | 34.4            | 69              | 2.31            | 681            | 13.1            |
| APR    | 0.159           | 15.9            | 233             | 33.4            | 51              | 2.20            | 274            | 11.6            |
| LS220  | 0.155           | 15.8            | 219             | 27.8            | 68              | 2.04            | 542            | 12.9            |
| SFHO   | 0.157           | 16.2            | 244             | 32.8            | 53              | 2.06            | 334            | 11.9            |

Exp. ~ 0.14–0.17 ~ 15–16 220–260 28.5–34.9 30–87 > 2.17 ± 0.1 300–580 11.1–13.4

Ref. [36] [36] [37, 38] [39, 40] [39, 40] [7] [16, 41] [42]

III. EOS PROPERTIES AT SATURATION

It is very important that any property of the adopted EOS can be tested at the saturation density \(\rho_0 \approx 0.17 \text{ fm}^{-3}\) of symmetric nuclear matter (SNM) \([N = Z\), being \(N(Z)\) the neutron (proton) number\], where information from laboratory data on finite nuclei is available. In general, in the vicinity of the saturation point the binding energy per nucleon can be expressed in terms of the density parameter \(\varepsilon \equiv (\rho - \rho_0)/\rho_0\) and the asymmetry parameter \(\delta \equiv (N - Z)/(N + Z)\) as

\[
E(\rho, \delta) = E_{\text{SNM}}(\rho) + E_{\text{sym}}(\rho) \delta^2, \tag{1}
\]

\[
E_{\text{SNM}}(\rho) = E_0 + \frac{K_0}{18} \varepsilon^2, \tag{2}
\]

\[
E_{\text{sym}}(\rho) = S_0 + \frac{L}{3} \varepsilon + \frac{K_{\text{sym}}}{18} \varepsilon^2, \tag{3}
\]

where \(K_0\) is the incompressibility at the saturation point, \(S_0 \equiv E_{\text{sym}}(\rho_0)\) is the symmetry energy coefficient at saturation, and the parameters \(L\) and \(K_{\text{sym}}\) characterize the density dependence of the symmetry energy around saturation. The incompressibility \(K_0\) gives the curvature of \(E(\rho)\) at \(\rho = \rho_0\), whereas \(S_0\) determines the increase of the energy per nucleon due to a small asymmetry \(\delta\). These parameters are defined as

\[
K_0 \equiv 9 \rho_0^2 \frac{\partial^2 E_{\text{SNM}}}{\partial \rho^2}(\rho_0), \tag{4}
\]

\[
S_0 \equiv \frac{1}{2} \frac{\partial^2 E(\rho_0,0)}{\partial \varepsilon^2} \approx E_{\text{PNM}}(\rho_0) - E_{\text{SNM}}(\rho_0), \tag{5}
\]

\[
L \equiv 3 \rho_0 \frac{dE_{\text{sym}}}{d\rho}(\rho_0), \tag{6}
\]

\[
K_{\text{sym}} \equiv 9 \rho_0^2 \frac{d^2 E_{\text{sym}}}{d\rho^2}(\rho_0). \tag{7}
\]

Properties of the various considered EOSs are listed in Table I, namely, the value of the saturation density \(\rho_0\), the binding energy per particle \(E_0\), the incompressibility \(K_0\), the symmetry energy \(S_0\) (note that we use the second definition involving the energy of pure neutron matter (PNM) for the values in the table), and its derivative \(L\) at \(\rho_0\). The curvature of the symmetry energy \(K_{\text{sym}}\) is only loosely known to be in the range of \(-400 \text{ MeV} \leq K_{\text{sym}} \leq 100 \text{ MeV}\) [49, 50], and therefore will not be examined in this paper. We notice that all the adopted EOSs agree fairly well with the empirical values. Marginal cases are the slightly too low \(E_0\) and \(K_0\) for V18, too small/large \(S_0\) for LS220/N93, and too low \(K_0\) for UIX and FSS2GC. The \(L\) parameter does not exclude any of the EOSs.

IV. NEUTRON STAR STRUCTURE AND TIDAL DEFORMABILITY

A very important constraint to be fulfilled by the different EOSs (assuming a purely nucleonic composition of NS matter) is the value of the maximum NS mass, which has to be compatible with the observational data [4, 6], in particular the recent lower limit \(M_{\text{max}} > 2.17 \pm 0.1\) [7]. As shown in Table I, many models give compatible values of the maximum mass, apart from UIX, FSS2CC, and LS220. We notice that recent analyses of the GW170817 event indicate also an upper limit of the maximum mass of about \(2.2 - 2.3 M_\odot\) [51, 53], with which several of the microscopic EOSs (V18, N93, DBHF, and APR) would also be compatible.

The tidal deformability \(\lambda\), or equivalently the tidal Love number \(k_2\) of a NS [54, 55], has recently been acknowledged to provide valuable information and constraints on the related EOS. In general relativity, it can be calculated along with the Tolman-Oppenheimer-Volkoff (TOV) equations for pressure \(p\) and enclosed mass \(m\) of a static NS configuration. The only input required is the EOS. More specifically, the Love number
can be obtained by solving the equations

\[
k_{2} = \frac{3}{2} \frac{\lambda}{R^{5}} + \frac{3}{5} \beta^{5} \Lambda = \frac{8}{5} \frac{\beta^{5} \zeta}{F},
\]

\[
z = (1 - 2 \beta^{2})(2 - y_{R} + 2 \beta(y_{R} - 1)),
\]

\[
F = 6 \beta(2 - y_{R}) + 6 \beta^{2}(5y_{R} - 8) + 4 \beta^{3}(13 - 11y_{R})
\]
\[+ 4 \beta^{4}(3y_{R} - 2) + 8 \beta^{5}(1 + y_{R}) + 3z \ln(1 - 2 \beta),
\]

(with \(\Lambda \equiv \lambda/M^{2}\) and \(\beta \equiv M/R\) being the compactness) along with a system of three coupled first-order differential equations \([57]\), namely

\[
\frac{dp}{dr} = \frac{m e (1 + p/\epsilon) (1 + 4\pi r^{3} p/m)}{1 - 2m/r},
\]

\[
\frac{dm}{dr} = 4\pi r^{2} \epsilon,
\]

\[
\frac{dy}{dr} = \frac{y^{2}}{r} - \frac{y - 6}{r - 2m} - rQ,
\]

\[
Q = 4\pi \left( \frac{(5 - y)e + (9 + y)p + (\epsilon + p)/c^{2}}{1 - 2m/r} \right)
\]
\[- \left[ \frac{2(m + 4\pi r^{3} p)}{r(r - 2m)} \right],
\]

with the EOS \(\epsilon(p)\) as input, \(c^{2} = d\epsilon/dp\) the speed of sound, and boundary conditions given by

\[
[p, m, y](r = 0) = [p_{c}, 0, 2],
\]

being \(y_{R} \equiv y(R)\), and the mass-radius relation \(M(R)\) provided by the condition \(p(R) = 0\) for varying \(p_{c}\).

For an asymmetric binary NS system, \((M, R)_{1} + (M, R)_{2}\), with mass asymmetry \(q = M_{2}/M_{1}\), and known chirp mass

\[
M_{c} = \frac{(M_{1}M_{2})^{3/5}}{(M_{1} + M_{2})^{1/5}},
\]

the average tidal deformability is defined by

\[
\tilde{\Lambda} = \frac{16}{13} \frac{(1 + 12q)\Lambda_{1} + (q + 12)\Lambda_{2}}{(1 + q)^{5}}
\]

with

\[
\frac{[M_{1}, M_{2}]}{M_{c}} = \frac{297}{250} (1 + q)^{1/5} \left[ q^{-3/5}, q^{2/5} \right].
\]

From the analysis of the GW170817 event \([15]\), a value of \(M_{c}/M_{0} = 1.188^{+0.004}_{-0.002}\) was obtained, corresponding to \(M_{1} = M_{2} = 1.365 M_{0}\) for a symmetric binary system, \(q = 0.7 - 1\) and \(\tilde{\Lambda} < 800\) from the phase-shift analysis of the observed signal. A lower limit, \(\tilde{\Lambda} > 400\), was deduced from a multi-messenger analysis of the GW170817 event combined with an analysis of the UV/optical/infrared counterpart with kilonova models \([20]\). Recently, those values were updated to \(300 \lesssim \tilde{\Lambda} \lesssim 580\) \([16, 41, 42]\), although the lower bound has been disputed \([58]\). The corresponding column in Table I shows the values of the tidal deformability for a \(1.4 M_{0}\) NS for all the adopted EOSs, which lie in the above interval with the exception of FSS2GC, DBHF, and APR.

V. CONSTRAINTS AND CORRELATIONS

A. Constraints on the symmetry energy

An important test for the EOS has to do with the symmetry energy, for which the experimental constraints are abundant at saturation density (see, e.g., \([53, 61]\)). We show in Fig. 1 a set of different experimental constraints together with the values of \((S_{0}, L)\) predicted by the different theoretical models considered in this paper. More in detail,

- the label “HIC” (blue region) corresponds to the constraints inferred from the study of isospin diffusion in HICs \([62]\);
- the label “polarizability” (violet region) represents the constraints on the electric dipole polarizability deduced in \([63]\);
- the label “Sn neutron skin” (grey region) indicates the constraints deduced from the analysis of neutron skin thickness in Sn isotopes \([64]\);
- the label “FRDM” (rectangle) corresponds to the values of \((S_{0}, L)\) inferred from finite-range droplet mass model calculations \([65]\);
- the label “IAS + \(\Delta r_{np}\)” (green diagonal region) indicates the isobaric-analog-state (IAS) phenomenology combined with the skin-width data, and represents simultaneous constraints of Skyrme-Hartree-Fock calculations of the IAS and the \(^{208}\)Pb neutron-skin thickness \([66]\);
- the horizontal band (in red colour) labelled “neutron stars” is obtained from a Bayesian analysis of mass and radius measurements of NSs by considering the 68% confidence values for \(L\) \([67]\);
- the dashed curve is the unitary gas bound on symmetry energy parameters derived in Ref. \([49]\); only values of \((S_{0}, L)\) to the right of the curve are permitted.

All considered constraints are not simultaneously fulfilled in any area of the parameter space, and this is probably due to
the model dependencies that influence the derivation of constraints from the raw data, besides the current uncertainties in the experimental measurements. Given this uncertainty, at the moment no definitive conclusion can be drawn and, except for models predicting values of the symmetry energy parameters outside the limits given in Table 1 like the LS220 or the N93 EOS), no theoretical models can be ruled out a priori on this basis.

A further crucial point in the understanding of the nuclear symmetry energy is its high-density behavior, which is among the most uncertain properties of dense neutron-rich matter. Its accurate determination has significant consequences in understanding not only the reaction dynamics of heavy-ion reactions, but also many interesting phenomena in astrophysics, such as the explosion mechanism of supernovae and the properties of NSs. In fact several aspects of the NS structure and dynamics depend crucially on the symmetry energy, e.g., the composition and the onset of the direct Urca cooling reaction, which is a threshold process dependent on the proton fraction, and therefore on the symmetry energy.

A big experimental effort has been devoted during the last few years to constrain the high-density symmetry energy using various probes in HICs at relativistic energies. Fig. 2 displays some constraints deduced for the density dependence of the symmetry energy from the ASY-EOS data (green band) and the FOPI-LAND result of (blue band) as a function of the density. The results of Ref. 63 are reported in the grey area (HIC Sn+Sn), whereas the dashed contour labelled by IAS shows the results of Ref. 66. We observe that the experimental results exhibit a monotonically increasing behavior with increasing density, and that several microscopic EOSs turn out to be compatible with experiments, except LS220 around saturation density, whereas N93, FSS2CC, and FSS2GC above the saturation density are only marginally compatible with the data.

B. Constraints on the EOS from heavy-ion collisions and gravitational waves

The extraction of the gross properties of the nuclear EOS from HIC data has been one of the main objectives in terrestrial nuclear experiments in the last two decades. In fact HICs at energies ranging from few tens to several hundreds MeV per nucleon produce heavily compressed nuclear matter with subsequent emission of nucleons and fragments of different sizes. The experimental analysis has been performed using the transverse flow as an observable, since it strongly depends on the pressure developed in the interaction zone of the colliding nuclei at the moment of maximum compression. The fireball density reached during the collision can also be probed by subthreshold $K^+$ production, since this depends on its incompressibility, as shown by the data collected by the KaoS collaboration. A combined flow and kaon production analysis was presented in Ref. 72, where a region in the pressure vs. density plane was identified, through which a compatible EOS should pass.

That analysis is displayed in Fig. 3 (left panels) as a grey box for the flow data by the FOPI collaboration, and as a red box for the KaoS collaboration. Those results point in the direction of a soft EOS, with values of the incompressibility $K$ in the range $180 \leq K \leq 250$ MeV close to the saturation density. We observe that almost all considered EOSs are compatible with the experimental data, except the BOB, V18, and DBHF EOS, which are too stiff at large density, where the analysis could however be less reliable due to the possible appearance of other degrees of freedom besides nucleons. Such densities are actually never reached in HICs. For completeness, we display in the central panels (b) the pressure for the PNM case.

The EOS governs also the dynamics of NS mergers. In fact, the possible scenarios of a prompt or delayed collapse to a black hole or a single NS, following the merger, do depend on the EOS, as well as the amount of ejected matter which undergoes nucleosynthesis of heavy elements. During the inspiral phase, the EOS strongly affects the tidal polarizability $\Lambda$, Eq. 8. The first GW170817 analysis for a $1.4 M_\odot$ NS gave an upper limit of $\Lambda < 800$, which was later improved to $\Lambda = 190^{+390}_{-120}$ by assuming that both NSs feature the same EOS. In this new analysis, the values of the pressure as a function of density were extracted, and those are displayed as colored areas in Fig. 3 (c), in which the blue (green) shaded region corresponds to the 90% (50%) posterior confidence level. We notice that almost all EOSs turn out to be compatible with the GW170817 data at density $\rho > 2\rho_0$, with BOB in marginal agreement at large density. A further comparison of HIC data with GW observations can be found in Ref. 74.

A further interesting quantity to consider is the so-called symmetry pressure,

$$p_{\text{sym}}(\rho) = \rho^2 \frac{dE_{\text{sym}}(\rho, \delta = 0)}{d\rho},$$

which adds to the pressure of an isospin-symmetric system with $N = Z$. Its contribution is very important because it is related to the poorly known symmetry energy at large density,
We point out, however, that the true symmetry pressure $p_{\text{sym}} \approx p_{\text{PNM}} - p_{\text{SNM}}$ can be substantially larger than the approximation Eq. (17), if one takes properly into account the EOS for PNM, which is significantly more stiff than the beta-stable EOS for most considered models, compare Figs. 3(b) and (c).

C. Correlations between neutron star and nuclear matter observables

In order to better understand the properties of nuclear matter, it would be very interesting to find correlations between GW170817 observations and microscopic constraints from nuclear measurements, as the ones just discussed. For this purpose, the limits derived for the tidal deformability could be very valuable. We remind that a limit of a $1.4 M_\odot$ NS, $\Lambda_{1.4} < 580$, was deduced from GW170817, which rules out very stiff EOSs (DBHF). The consequence for the radius is that $R_{1.4} \lesssim 13.6$ km, as also confirmed in Refs. [75–77].

The source of GW170817 also released a short gamma-ray burst, GRB170817A, and a kilonova, AT2017gfo, generated by the mass ejected from the merger, and this was found to provide constraints on the EOS as well. In particular, the average tidal deformability given by Eq. (13) must be larger than $300 \pm 10$, thus implying $R_{1.4} = 12.2^{+0.8}_{-0.2}$ km. We remind the reader that radii smaller than these lower limits were deduced from observations of thermal emission from accreting NSs in quiescent low-mass X-ray binaries (LMXBs), suggesting a radius in the range $(9.9 - 11.2)\,\text{km}$ for stars of mass about $(1.4 - 1.5) M_\odot$, in spite of the uncertainties on the composition of the NS atmosphere. This discrepancy between

FIG. 3. Pressure vs. baryon density for the considered EOSs on a logarithmic (upper row) or linear (lower row) scale for (a) symmetric matter, (b) pure neutron matter, (c) beta-stable matter, and (d) the symmetry pressure Eq. (17). In (a) constraints derived from HIC data are reported as red band (KaoS experiment) and grey band (Flow data). In (c) the GW170817 constraints [16] are reported. The markers in (d) represent the data analysis of Ref. [70]. See text for details.

and plays a big role in the determination of the proton fraction, for instance, crucial for NS cooling simulations.

In Fig. 3(d) the symmetry pressure is displayed as a function of the baryon density. The markers with error bars are the results of the analysis performed in Ref. [70], where a subtraction procedure has been proposed between the kaon data (white dots) and flow data (blue dots) for SNM, both displayed in panel (a), and the GW170817 event constraints shown in panel (c), assuming matter in beta-stable condition. Accordingly, we report in panel (d) our values of the symmetry pressure, approximated by

$$p_{\text{sym}} \approx p_{\text{betastable}} - p_{\text{SNM}}.$$ (17)
large radii from GW170817 and small radii from quiescent LMXBs (if confirmed) might be solved in the two-families or twin-star scenarios, in which small and big stars of the same mass could coexist as hadronic and quark matter stars \cite{78-81}.

In Ref. \cite{70} it was found that the tidal deformability as well as the radius are strongly correlated with the pressure of betastable matter at a density $\rho \approx 2\rho_0$. That result was obtained using a NS model generated by about 200 Skyrme energy-density functionals to describe the inner core of the NS \cite{82}. This is also confirmed for our set of EOSs as shown in Fig. \ref{fig:tidal_deformability_radius} (left panels), whereas weaker correlations appear with the speed of sound and the incompressibility under the same conditions (displayed in the central and right panels respectively).

The green bands displayed in the upper panels represent the limits on $\Lambda_{1.4}$ derived in \cite{16,41}, in particular the lower limits, i.e., $\Lambda_{1.4} = 190^{+390}_{-120}$ \cite{16} (light green) and $\Lambda_{1.4} > 300$ \cite{41} (dark green), are important for the determination of the radius, which corresponds to $R_{1.4} = 11.9^{+1.4}_{-1.4}$ km in the former case, and $R_{1.4} = 12.2^{+1.0}_{-0.8} \pm 0.2$ km in the latter one. For completeness, we have checked that this correlation applies also to NS masses different from $1.4M_\odot$, but it becomes slightly weaker with increasing NS masses. Thus the determination of the tidal deformability and the NS radius could put constraints on the pressure and the symmetry pressure at twice the saturation density. The current limits exclude only the DBHF EOS due to its too high $\Lambda_{1.4}$ value.

Following the same philosophy, we have tried to find correlations among NS properties and properties of SNM around saturation density. Results are displayed in Fig. \ref{fig:tidal_deformability_radius}(the green bands display the same conditions as in Fig. \ref{fig:tidal_deformability_radius}, where the tidal deformability of a $1.4M_\odot$ NS is reported as a function of the symmetry energy $S_0$ (left panel), its slope $L$ (middle panel), and the incompressibility $K_0$ (right panel), all taken at saturation density. Apparently no evident correlations between the tidal deformability and those quantities do exist.

**FIG. 4.** Tidal deformability (upper panels) and radius (lower panels) of a $1.4M_\odot$ NS vs. the pressure (left panels), the speed of sound (middle panels), and the incompressibility $K$ (right panels) of betastable matter at twice the saturation density. The light and dark shaded bands in the upper row represents the limits derived in \cite{16,41}, respectively.
FIG. 5. The tidal deformability of a 1.4$M_\odot$ NS as a function of the symmetry energy (left panel), its derivative $L$ (middle panel), and the incompressibility $K_0$ at saturation density $\rho_0$ for all the considered EOSs. The shaded areas represent the limits listed in Table I.

VI. SUMMARY

We conclude that the BHF V18 and N93 models are good candidates for a realistic modeling of the nuclear EOS up to very high density. They fulfill nearly all current experimental and observational constraints discussed in this article, in particular the novel constraints on the tidal deformability imposed by GW170817. We would like to emphasize that these are not phenomenological EOSs, but they have been constructed in a microscopic way from nuclear two-body potentials and compatible three-body forces. The last issue imposed in fact strong conditions on their construction, due to which reason a perfect reproduction of all current constraints is not achieved, but was also not attempted.

We stress in particular that the predicted maximum mass values $\approx 2.3M_\odot$ could be close to the ‘true’ maximum mass conjectured from the GW170817 event. The two models predict then $R = 12.3, 12.7$ km for the canonical $M = 1.4M_\odot$ NS radius, respectively.

The new astrophysical constraints on maximum mass and tidal deformability exclude several models with too small maximum mass and the DBHF EOS with a too large deformability. Tightening the lower limit on $\Lambda_{1.4}$ could potentially exclude several other EOSs.

For all examined EOSs we also confirmed the correlation between the radius or deformability of a $M = 1.4M_\odot$ NS and the pressure of betastable matter at about twice normal density. Weaker correlations were found with the speed of sound and the compressibility of betastable matter at that density. On the other hand, we did not find any correlations between NS deformability and properties of symmetric or neutron matter at normal density.

| Table II. Parameters of the fit for the energy per nucleon $E$, Eq. (18), for symmetric nuclear matter (SNM) and pure neutron matter (PNM) in two different density domains and for the different EOSs used. |
|---|---|---|---|---|---|---|---|---|
| EOS | $\rho = (0.08 - 1) \text{ fm}^{-3}$ | $\rho = (0.14 - 0.21) \text{ fm}^{-3}$ |
| $\rho$ | $a$ | $b$ | $c$ | $d$ | $a$ | $b$ | $c$ | $d$ |
| BOB SNM | -65 | 498 | 2.67 | -9 | -189 | 446 | 1.83 | -0.83 |
| BOB PNM | 57 | 856 | 2.91 | 4 | 15 | 584 | 2.37 | 7.11 |
| V18 SNM | -60 | 369 | 2.66 | -8 | -82 | 487 | 2.58 | -4.96 |
| V18 PNM | 37 | 667 | 2.78 | 6 | 38 | 578 | 2.67 | 5.88 |
| N93 SNM | -42 | 298 | 2.61 | -12 | -62 | 803 | 3.20 | -8.18 |
| N93 PNM | 67 | 743 | 2.71 | 4 | 42 | 471 | 2.48 | 5.47 |
| UXI SNM | -174 | 323 | 1.61 | -4 | -46 | 926 | 3.38 | -9.29 |
| UXI PNM | 24 | 326 | 2.09 | 6 | 31 | 294 | 2.10 | 6.25 |

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APPENDIX: PARAMETRIZATIONS OF THE BHF EOS

For convenience we provide here simple parametrizations of our numerical results for the different EOSs, namely analytical fits of the energy per nucleon $E$ for SNM and PNM. We find that in both cases the following functional forms con-
stitute excellent representations of the numerical values

\[ E(\rho) = a\rho + b\rho^c + d, \tag{18} \]

where \( E \) and \( \rho \) are given in MeV and \( \text{fm}^{-3} \), respectively. The parameters of the fits are listed in Table III for the different EOSs we are using. We provide two sets of parameterizations, i.e., a first set to be used for NS structure calculations in the density range \((0.08–1)\text{ fm}^{-3}\) and a second set for the range \((0.14–0.21)\text{ fm}^{-3}\), more appropriate for a precise determination of the saturation properties. The rms deviations of fits and data are better than 1 MeV / 0.02 MeV for the two cases and for all EOSs.

For asymmetric nuclear matter, it turns out that the dependence on proton fraction can be very well approximated by a parabolic law as assumed in Eq. (11) \([27, 33]\):

\[ E(\rho, x_p) \approx E_{\text{SNM}}(\rho) + (1 - 2x_p)^2 \left[ E_{\text{PNM}}(\rho) - E_{\text{SNM}}(\rho) \right]. \tag{19} \]

Therefore, for the treatment of the asymmetric and beta-stable case, it is only necessary to provide parameterizations for SNM and PNM.

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