Comparison at the sub-100 fW optical power level of calibrating a single-photon detector using a high-sensitive, low-noise silicon photodiode and the double attenuator technique

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Abstract
A comparison down to sub-100-fW optical power level was carried out between a low-noise Silicon photodiode and a low optical flux measurement facility based on a double attenuator technique. The comparison was carried out via a silicon single-photon avalanche diode (Si-SPAD), which acted as transfer standard. The measurements were performed at a wavelength of 770 nm using an attenuated laser as a radiation source at optical power levels between approximately 86 fW and approximately 1325 fW, corresponding to approximately 330 000 photons s⁻¹ and approximately 5.2 × 10⁶ photons s⁻¹, respectively. The mean relative deviation of the detection efficiencies of the Si-SPAD, determined by the Si-photodiode and the low optical flux measurement facility, i.e. between two completely independent traceability routes, was < 0.2%, thus well within the combined standard uncertainty of the two measurements. To our knowledge, this is the first comparison for the detection efficiency of a single photon detector using a direct optical flux measurement by a conventional Si-photodiode at such low power levels.

Keywords: detection efficiency, Si-SPAD, calibration, comparison

(Some figures may appear in colour only in the online journal)
Figure 1. Schema of the comparison of the CMI low flux standard (LOFD), and the PTB low optical flux measurement facility.

are given e.g. in Schmunk et al [3] and in López et al [4]. Rarity et al [5] were the first to compare the correlated photon technique with classical, cryogenic radiometer based radiometry. Biller et al [6] calibrated a single-photon detector traceable to cryogenic radiometry. The closest agreement thus far between the correlated photon technique and traceability to cryogenic radiometry was reported by Polyakov and Migdall [7]. With a reference detector similar to the one used in this report, Cheung et al [8] also obtained a rather good agreement between the correlated photon technique and traceability to cryogenic radiometry. Using a synchrotron as an attenuator, Müller et al [9] calibrated a single-photon detector traceable to a cryogenic radiometer. Recently, Lee et al [10] also used a switched integrator amplifier for the calibration of fibre-coupled single-photon detectors.

However, in this paper, for the first time to our knowledge, the detection efficiency of a single photon detector, i.e. a Si-SPAD, is determined by the comparison with a highly sensitive detector developed at CMI, the Czech National Metrology Institute [11], by directly measuring an optical power as low as 100 fW. Furthermore, the obtained detection efficiency is compared to the one determined by the low optical flux measurement facility based on a double attenuator technique developed at PTB, the German National Metrology Institute [4].

2. Traceability chain for the PTB low optical flux measurement facility

The idea for this comparison is depicted in figure 1. The calibration of a Si-SPAD is carried out via comparison with a calibrated Si-diode, which acts as the reference detector. The traceability chain for the spectral responsivity $s_{Si}$ of the Si-SPAD is shown in figure 2, where $\lambda_i$ is the wavelength, $\Phi$ is the optical radiant flux and $u$ is the standard measurement uncertainty. In the first step, a silicon-trap detector is calibrated against a cryogenic radiometer at specific wavelengths [12]. Using a physical model for the wavelength dependence of the spectral responsivity, the spectral responsivity at 770 nm is calculated [13] and used for the determination of the spectral responsivity of a Si-diode used in the low radiation flux facility. For the spectral responsivity of the Si-diode at 770 nm we thus obtained:

$$s_{Si} = 0.356324 \text{ A W}^{-1} \pm 131 \times 10^{-6} \text{ A W}^{-1}$$  \hspace{1cm} (1)$$

The setup and a photograph for the calibration of the detection efficiency of the Si-SPAD are shown in figure 3. This calibration is performed using the double attenuator technique, i.e. the Si-diode performs a subsequent in situ calibration of two attenuators. Then, the total attenuation is calculated by multiplying the two attenuation values. From this value and the total photon flux originating from the laser, the photon flux impinging on the Si-SPAD detector is calculated, when both attenuators are moved into the laser beam. This procedure is necessary, because it is not possible with the standard Si-diode used in the setup at PTB to measure the photon flux after the two attenuators directly, since the photon flux is too low. A detailed description of this procedure and the facility is given in [4].

Moreover, a variable filter may be additionally used to achieve different calibration photon fluxes as well as to reduce the statistical uncertainty in the two subsequent attenuation measurements carried out with filters 1 and 2 (see figure 3). However, unlike in [4], an additional measurement is introduced for the calibration of the detection efficiency of the Si-SPAD; i.e. five signals, $V_1$ to $V_5$, are now measured. That is,

$$V_1 = A_1 \cdot s_{Si,1} \cdot \Phi_1$$
$$V_2 = A_2 \cdot s_{Si,2} \cdot \Phi_2 = A_2 \cdot s_{Si,2} \cdot T_{F2} \cdot \Phi_1$$
$$V_3 = A_3 \cdot s_{Si,3} \cdot \Phi_3 = A_3 \cdot s_{Si,3} \cdot T_{F3} \cdot \Phi_1$$
$$V_4 = A_4 \cdot s_{Si,4} \cdot \Phi_4 = A_4 \cdot s_{Si,4} \cdot T_{F4} \cdot \Phi_1$$
$$V_5 = \text{CR} = \frac{\Phi_5}{\hbar c / \lambda} = \eta \cdot \left( T_{F2} \cdot T_{F3} \cdot T_{F4} \right) \cdot \Phi_1$$

$$\eta = \frac{hc}{\lambda} \cdot \frac{A_2 \cdot A_3 \cdot A_4}{A_1^2} \cdot \frac{CR \cdot V_1^2}{V_2 \cdot V_3 \cdot V_4} \cdot \frac{s_{Si,2} \cdot s_{Si,3} \cdot s_{Si,4}}{s_{Si,1}^2}$$  \hspace{1cm} (2)$$

With $\eta$: detection efficiency, $h$: Planck constant, $c$: speed of light and $\lambda$: wavelength; $V_1$: signals ($V_1$; signal for the measurement of the laser power, $V_2$: signal for the measurement of the laser power, $V_2$: signal for the measurement...
of filter 2, $V_3$: signal for the measurement of filter 3, $V_4$: signal for the measurement of filter 4 (variable filter), $V_5 = CR$: count rate of the Si-SPAD, $A_i$: amplification factor of the transimpedance amplifier in the measurements 1–4, $s_{Si,i}$: spectral responsivity of the Si-diode for the different irradiation levels, $T_{Fi}$: transmissions of the filters used in the measurements 2–4, $\Phi_{1−5}$: optical radiant fluxes in the measurements 1–5.

Since the spectral responsivity of the Si-diode is practically linear for the applied optical flux levels between approximately 1 nW and 10 $\mu$W, the Si-diode responsivity is considered constant for all measured signals $V_i$. This was proven by measuring the nonlinearity of the Si-diode with the PTB’s in-house facility for the calibration of the nonlinearity. Furthermore, in order to minimize any fluctuation of the laser power, a monitor technique is applied during the measurement. Thus, the Si-SPAD detection efficiency is obtained as:

$$\eta = \frac{hc}{\lambda} \frac{A_2 A_3 A_4}{A_1} \frac{V_{\text{moni}}}{V_{\text{moni}}^{\text{ref}}} \frac{V_{\text{ref}}}{V_{\text{ref}}^{\text{ref}}} \frac{CR_{\text{ref}}^{\text{ref}}}{CR_{\text{ref}}} \frac{\Phi_{\text{ref}}}{\Phi_{\text{ref}}^{\text{ref}}} s_{\text{Si},i} F_{\text{filt}} F_{\text{stray}}$$

$$= \frac{hc}{\lambda} \frac{A_2 A_3 A_4}{A_1} \frac{Q_{\text{ref}} Q_3}{Q_{\text{ref}} Q_4} s_{\text{Si},i} F_{\text{filt}} F_{\text{stray}}$$

With $V_{\text{moni}}$: signals from the monitor detector for the measurements 1–5, $F_{\text{filt}} = 0.9998 \pm 0.0023$, uncertainty component taking into account the uncertainty in the filter transmission measurement and $F_{\text{stray}} = 1.004 \pm 0.004$, an uncertainty component taking into account the effect of stray light. This effect became significant, due to the use of a Si-diode as a standard detector and the proximity of the microscope objective. Thus back reflections into the diode occur. Using different apertures in front of the Si-diode, the uncertainty was estimated to be about 0.4%.

Table 1 shows the uncertainty budget for the Si-SPAD detection efficiency measurement carried out for an optical flux of approximately 86 fW. The obtained standard uncertainties are in the order of 0.5%. Main contributions to the measurement uncertainty are the filter transmissions due to the subsequent measurement of single filters instead of a combined measurement and the stray light arising from back reflections from the Si-diode to the microscope objective, which hit the Si-diode again. The results for the detection efficiency of the Si-SPAD detector for different optical flux levels are summarized in table 2 and shown in figure 4.

3. Traceability chain and measurement uncertainty for the CMI reference detector for low photon fluxes

The CMI reference detector for low photon fluxes comprises a small area Si-photodiode S1227 33BQ in conjunction with a custom made switched integrator amplifier (SIA) [5]. The SIA proved to be a convenient solution for traceable measurements of low photocurrents at sub-pA levels. Compared to a traditional trans-impedance amplifier, it intrinsically shows lower noise and furthermore significantly simplifies the calibration of the conversion factor for sub-pA currents, as described below. A simplified schema of the low optical flux detector (LOFD) is shown in figure 5. The SIA feedback integration capacitor $C_{\text{int}}$ is a Mica dielectric 1 pF that provides a current to voltage conversion of $10^{12}$ with just 1 s of integration time. For this comparison the SIA needs to convert photocurrents as low as 30 fA on top of a thermally generated photodiode current.
The same order of magnitude (8 fA) at room temperature. Since a measurement can last tens of seconds as described later, it is therefore important to minimize and stabilize the photodiode dark current. For this purpose the photodiode housing is surrounded by a Peltier cooler that keeps its temperature at 12 °C, just above the dew point at a relative humidity of 40%, with a stability of about 0.05 °C h⁻¹.

The output voltage \( V_{\text{out}} \) of the LOFD is related to the photocurrent \( I_{\text{ph}} \) generated by the photodiode [5] according to the equation:

\[
V_{\text{out}}(t) = \frac{I_{\text{int}}}{C_{\text{int}}} I_{\text{ph}}.
\]

where \( I_{\text{int}} \) is the SIA integration time and \( C_{\text{int}} \) is the SIA integration capacitor. The relation between \( I_{\text{ph}} \) and the optical power \( P \) impacting the photodiode is

\[
P = \frac{I_{\text{ph}} - I_{\text{dark}}}{s_{\text{Si}}(\lambda)} ,
\]

where \( I_{\text{dark}} \) is the photocurrent generated by the photodiode when the photons from the source to be measured do not reach the photodiode and \( s_{\text{Si}}(\lambda) \) is the photodiode’s spectral responsivity at wavelength \( \lambda \) expressed in A W⁻¹. Finally, the number of photons per second \( N_{\text{ph}} \) can be calculated from the optical flux \( P \) using

\[
N_{\text{ph}} = \frac{P}{h c \lambda} ,
\]
\[ N_{\text{ph/s}} = \frac{P}{e_{\text{ph}}(\lambda)}, \quad (6) \]

where \( e_{\text{ph}}(\lambda) \) is the energy of one photon at the wavelength \( \lambda \). After combining equations (4)–(6), the LOFD measurement equation is:

\[ N_{\text{ph/s}} = \frac{V_{\text{out}} - V_{\text{dark}}}{s_{\text{Si}}(\lambda)} \frac{C_{\text{int}}}{e_{\text{ph}}(\lambda)} \frac{1}{I_{\text{int}}}, \quad (7) \]

where \( V_{\text{out}} \) and \( V_{\text{dark}} \) are measured with a calibrated digital voltmeter (DVM) with an uncertainty of less than 20 ppm. The spectral responsivity of the photodiode \( s_{\text{Si}}(\lambda) \) is traceable to the CMI cryogenic radiometer with the traceability chain schema depicted in figure 6. The \( s_{\text{Si}}(\lambda) \) value at 770 nm is: 0.3306 ± 0.0006 A W\(^{-1}\).

Finally, the detection efficiency of the Si-SPAD detector obtained from the photon rate \( N_{\text{ph/s}} \) is given by

\[ \eta = \frac{CR}{N_{\text{ph/s}}} = \frac{CRs_{\text{Si}}(\lambda)}{V_{\text{out}}} \frac{e_{\text{ph}}(\lambda)}{C_{\text{int}}} \frac{1}{I_{\text{int}}} \quad (8) \]

The integration capacitor value \( C_{\text{int}} \) is calibrated with a measurement facility and depicted in figure 7 and described in more detail in [5]. It comprises a voltage calibrator, a standard resistor \( R_{\text{std}} \), a calibrated DVM and a calibrated counter (CC). The input current \( I_{\text{in}} \) for the SIA is generated by the voltage...
calibrator in conjunction with the standard resistor. The \( I_{in} \) value can be calculated using the equation:

\[
I_{in} = \frac{V_c}{R_{std}} \tag{9}
\]

The DVM measures alternately \( V_{out} \) and \( V_s \). The measurement is performed with the SIA integration time \( t_{int} \) set to 20 ms to minimize noise pick up at 50 Hz [5] and using 20 different levels of \( I_{in} \) to cover the entire SIA dynamic range. The slope \( G \) of the linear fit of the measured \( I_{in}, V_{out} \) is calculated. The measurement is performed 70 times to assess the statistical variance. The maximum absolute residual value is used to estimate the LOFD nonlinearity and it is taken into account in the uncertainty budget for the measurement of the number of photons per second (table 3). The integration capacitor value is then calculated using the equation:

\[
C_{int} = \frac{t_{int}}{G} \tag{10}
\]

The integration time \( t_{int} \) is measured using the SIA integration timing signal with the calibrated counter. The standard resistor \( R_{std} \) value 100008900 ± 10000 Ω is traceable to

![Figure 7. CMI calibration facility to measure the integration capacitor of the LOFD’s front end electronics (SIA).](image)

**Table 3.** Uncertainty budget for the determination of the detection efficiency of the Si-SPAD detector using the LOFD and the low optical flux facility at PTB.

| Measurand  | Unit | Description                                  | Value       | Standard uncertainty | Distribution | Sensitivity coefficients | Contribution  |
|------------|------|----------------------------------------------|-------------|----------------------|--------------|-------------------------|--------------|
| \( V_{out} \) | V    | LOFD output voltage                          | 2.712 \times 10^{-2} | 5.435 \times 10^{-4} | Standard     | -23.163                 | -1.259 \times 10^{-3} |
| \( C_{int} \) | F    | LOFD integration capacitor                   | 1.045 \times 10^{-12} | 3.731 \times 10^{-16} | Standard     | -6.018 \times 10^{11}   | -8.211 \times 10^{-5} |
| \( s_{Si} \) | A W^{-1} | LOFD photodiode responsivity                   | 0.33065    | 6.612 \times 10^{-4} | Standard     | 1.904                   | 1.259 \times 10^{-3} |
| \( e_{ph} \) | J    | Energy of photon at 700 nm                    | 2.580 \times 10^{-19} | 1.116 \times 10^{-23} | Standard     | 2.440 \times 10^{18}    | 2.723 \times 10^{-5} |
| LOFD non linearity | 1 | LOFD non linearity corr. factor                | 0.9994     | 5.000 \times 10^{-4} | Rectangular  | 0.62982                | 3.149 \times 10^{-4} |
| CR         | 1    | Si-SPAD-counts                                | 209668.81  | 37.21                | Standard     | 3.002 \times 10^{-6}    | 1.117 \times 10^{-4} |
| \( \eta \) | 1    | Detection efficiency                          | 0.6295     | 1.82 \times 10^{-3}  | Standard     |                         |              |

**Table 4.** Uncertainty budget for the calibration of the LOFD integration capacitor value in CMI.

| Measurand  | Unit | Description                                  | Value       | Standard uncertainty | Distribution | Sensitivity coefficient | Contribution  |
|------------|------|----------------------------------------------|-------------|----------------------|--------------|-------------------------|--------------|
| \( R_{std} \) | Ω s  | Standard resistor                            | 1.0000 \times 10^{8} | 1.00 \times 10^{4}  | Standard     | 191.36                  | 1.914 \times 10^{6} |
| \( V_{out} \) | V    | LOFD output voltage                          | 1.2441     | 2.48 \times 10^{-5}  | Standard     | 1.5432 \times 10^{10}   | 382716 |
| \( V_s \) | V    | Calibrator voltage output                    | 6.4811     | 5.19 \times 10^{-7}  | Standard     | -2.953 \times 10^{12}   | -1.533 \times 10^{6} |
| \( G \) | V A^{-1} | Linear fit                                  | 1.9110     | 2.48 \times 10^{-6}  | Standard     |                         |              |
| \( t_{int} \) | s    | LOFD integration time                        | 2.0010     | 2.00 \times 10^{-7}  | Standard     | 5.4823 \times 10^{-23}   | -1.3604 \times 10^{-16} |
| \( C_{int} \) | F    | LOFD integration capacitor                   | 1.046 \times 10^{-12} | 1.36 \times 10^{-16} | Standard     |                         |              |
the quantum Hall Effect resistance standard. The calibration of the DVM is traceable to the Josephson standard and the instrument stability is better than 40 ppm yr$^{-1}$. The $C_{int}$ value is $1.046 \pm 0.001$ pF, the uncertainty budget is reported in table 4.

4. Calibration of the Si-SPAD detector with the CMI standard detector

The calibration of the Si-SPAD detector was carried out at PTB. The setup used was essentially the same as the one used for the Si-SPAD detection efficiency measurement described above. However, the standard Si-diode was replaced by the LOFD and the movements of the transmission filter packages 2 and 3 are not necessary, because of the direct optical flux measurement with the LOFD. Hence, the filter packages remained in the beam path. Only the variable filter in front of the laser was moved for adjusting the optical flux level approximately to the levels obtained in the measurement with the PTB low flux measurement facility. The LOFD and the Si-SPAD from PTB were mounted on the translation stage, allowing the detectors to be moved into the beam path, see figure 8.

For both measurements, the dark current, respectively the dark counts, were deducted from the signal. The main contribution to the measurement uncertainty for the LOFD is the statistical uncertainty, caused by the detector noise at these extremely low optical powers of even less than 100 fW. In order to improve the signal to noise ratio, a numerical moving average of 20 samples has been applied to the LOFD data during the data analysis. The results for the detection efficiency of the Si-SPAD detector are summarized in table 2 and shown in figure 4.

5. Results and analysis

For the Si-SPAD transfer detector used in this comparison, a Perkin-Elmer-SPCM-AQR-16 (SN: 19743), an intrinsic small signal detection efficiency at a power level of approximately 86 fW, corresponding to approximately 330 000 photons s$^{-1}$, of $\eta_{0,\text{SPAD,PTB}} = 0.629 \pm 0.003$ and $\eta_{0,\text{SPAD,CMI}} = 0.630 \pm 0.002$, respectively, were measured. It should be noted that the detection efficiency is defined here purely as the number of counts (corrected for dark counts) divided by the number of impinging photons, i.e. other effects, e.g. after-pulsing, are not considered.

For higher optical fluxes, the apparent detection efficiency $\eta_k, \text{SPAD}$ remains first nearly constant. However, for optical fluxes higher than approximately 450 fW, it drops drastically due to the combined influence of the dead time and reset time of the detector as well as the photon statistics effects caused by the Poissonian photon statistics of the attenuated laser. It should be noted that in this case the dead time and reset time of the detector are fixed by the manufacturer, so that it is not

Figure 8. Schema of the setup for the calibration of the Si-SPAD detector with the LOFD CMI detector.

Figure 9. Deviations between the detection efficiency calibrations of the Si-SPAD detector for different count rates.
possible to adjust them in order to reduce their influence on the apparent detection efficiency of the detector.

For all measured optical flux levels up to approximately 1300 fW, the deviation between the determined detection efficiencies is below 1.3%, see figure 9. In addition, the ‘normalized error’ $E_n$ was calculated according to [14],

$$E_n = \frac{\eta_{SPAD, PTB} - \eta_{SPAD, CMI}}{\sqrt{(U(\eta_{SPAD, CMI})^2 + U(\eta_{SPAD, PTB})^2)}}$$  (11)

to determine the grade of agreement of the comparison results. An $E_n$-value < 1 means that the measurement results are metrologically equivalent within the measurement uncertainty. In this comparison, the obtained $E_n$-values are smaller than 0.6 for all measured photon levels, see table 2, indicating the consistency of the measurements. Furthermore, the low $E_n$-values indicate that a reduction of the measurement uncertainty should be possible, in principle.

6. Summary and outlook

To our knowledge, the first comparison for the detection efficiency of a free space single photon detector using a direct flux measurement by a conventional Si-photodiode at low power levels corresponding to single photon level was carried out and presented in this paper. The results between the calibration of a Si-SPAD single photon detector with the double attenuator technique, carried out at the low optical flux measurement facility at PTB, and with the LOFD are consistent over a wide power range between approximately 86 fW and 1.3 pW. The deviations in the whole power range are less than 1.3% and the $E_n$-values are below 0.6, which means that both results are metrologically equivalent within the uncertainty.

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