Nonlinear forced vibration of the magnetic gear system with the internal resonance

Xiuhong Hao¹, Wenbin Hao and Deng Pan
School of Mechanical Engineering, Yanshan University, Qinhuangdao 066004, Hebei, China

¹E-mail: hxlhong@ysu.edu.cn

Abstract. Considering the nonlinear magnetic coupling stiffness among components of the magnetic gear with intersecting axes and a straight SP (MGIASS), the nonlinear dynamic model and differential equations of the MGIASS system are established. Then, the expression of the forced response is obtained when the excited frequency is close to the torsional mode frequency of the input rotor. The forced responses analysis of the example system shows that: the response frequencies include the excited frequency and the double frequency. Multiple proximity between the natural frequencies directly affects the stability and resonance amplitude.

1. Introduction
The magnetic gears (MG) can transmit motion and power transmission through magnetic field coupling. MG have the advantages of no contact, no wear, no lubrication, and overload protection, and overcome the mechanical gear issue of gear teeth breakage due to contact fatigue, making it one of the best alternatives to mechanical gear transmission [1].

The field modulated magnetic gear (FMMG) proposed by Atallah provides some advantages including a high utilization of permanent magnets (PMs), larger output torque, higher torque density, and transmission capacity comparable to that of the conventional MG [2-3]. A magnetic gear with intersecting axes and bend stationary pole-pieces (SP) was presented by Liu YL [4], and a magnetic gear with intersecting axes and a straight SP (MGIASS) is proposed by Hao XH, which has a greater torque density [5]. Because of the weak magnetic coupling stiffnesses (MCS), the lower natural frequency will result in the low-frequency resonance and the slow transient resonant attenuation when there are external sources of excitation [6-7].

Considering the modulation effect of magnetic field, the MCS among components present the obvious nonlinearity. When there is the multiple relationship between the natural frequencies, the internal resonances will occur. The internal resonances will lead to the energy transfer between modes and the slower transient resonant attenuation, and worsen the dynamic behavior of the system to a certain extent. In this paper, the nonlinear dynamic model and nonlinear differential equations of magnetic gear system are established. The forced response of the MGIASS system with 1:2 internal resonance was analyzed and these will lay a foundation for the further exploration of dynamic behavior of the MGIASS system.

2. The nonlinear dynamic model
MGIASS shown in Figure 1 is composed of the input and output rotors, and the SP. PMs with N poles and S poles, are layered at regular intervals on the end-faces of the input and output rotors,
respectively. Number of pole-pairs of the PMs on the input and output rotors are \( N_1 \) and \( N_3 \). SP with the elliptical section is composed of the non-permeable and permeable magnetic strips layered at different intervals. The angle between the two end-faces of the SP is usually \( 90^\circ \). The two end-faces of the SP are circular rings, which is same as the back sections of the input and output rotors.

![Figure 1. Structure diagram of the MGIASS system.](image)

When the input rotor runs, the magnetic field with the main harmonic \( N_1 \) will be formed in the air-gap between the input rotor and the SP. The magnetic field with the main harmonic \( N_3 \) will be got after the magnet modulation of the SP, and is consistent with the number of the PMs pole-pairs on output rotor. Then equal magnetic pole coupling is achieved.

The dynamic model of the MGIASS system shown in Figure 2 consists of two subsystems, that is, the input rotor/FP subsystem and the SP/output rotor subsystem. Considering that the transverse supporting stiffnesses is much larger than the torsional MCS, only torsional vibrations of components are considered.

![Figure 2. Nonlinear dynamic model of the MGIASS system.](image)

In the MGIASS system, the input and output torques can be expressed as

\[
\begin{align*}
T_1 &= T_{c1} \sin(N_1 \theta_{12} + \theta_{10}) \\
T_3 &= T_{c3} \sin(N_3 \theta_{23} + \theta_{30})
\end{align*}
\]

(1)

where \( T_1 \) and \( T_3 \) are the instantaneous torque on the input and output rotors, respectively; \( T_{c1} \) and \( T_{c3} \) are the average torque on the input and output rotors, respectively; \( \theta_{10} \) and \( \theta_{12} \) are the initial relative rotation angles and relative torsional angular displacement between the input rotor and SP, respectively; \( \theta_{30} \) and \( \theta_{23} \) are the initial relative rotation angles and relative torsional angular displacement between the output rotor and SP, respectively.

The tangential coupling force between the components can be expressed as

\[
\begin{align*}
F_{ls} &= T_1 / R_1 = T_{c1} \sin[N_1 (\theta_1 - \theta_2) + \theta_{10}] / R_1 \\
F_{os} &= T_3 / R_3 = T_{c3} \sin[N_3 (\theta_2 - \theta_3) + \theta_{30}] / R_3
\end{align*}
\]

(2)

where \( F_{ls} \) and \( F_{os} \) are the tangential coupling forces among the input rotor and SP, SP and output rotor, respectively; \( R_1 \) and \( R_3 \) are the equivalent rotational radius of the input and output rotors.
respectively; \( \theta_i \) is the torsional vibration displacements of the input rotor, SP and the output rotor, respectively.

The torsional angular displacements of components are replaced with the corresponding torsional linear displacement, that is

\[
x = [u_1 \quad u_2 \quad u_3]^T
\]  

(3)

The components of the SP in the torsional vibration of the input and output rotor is \( u'_2 \), and \( u'_3 = u_2 \cos \alpha \). Where \( \alpha \) is the angle between the two end-faces of the SP, that is, the angle between the input and output rotors.

The magnetic coupling forces are the function of the relative torsional displacement between components, and are expanded into a Taylor series at time \( t=0 \), i.e. \( u_t=0 \). Based on the Lagrange equation, the nonlinear dynamic differential equations of the MGIASS system is

\[
M \ddot{u}_1 + c_1 \dot{u}_1 + a_1 u_1 + b_1 u_2 + c_1 u_1^2 + d_1 u_2^2 + e_1 u_1 u_2 = \Delta T \cos \omega_t R_i
\]

\[
M \ddot{u}_2 + c_2 \dot{u}_2 + a_2 u_2 + b_2 u_3 + c_2 u_2^2 + d_2 u_3^2 + e_2 u_2 u_3 = \Delta T \cos \omega_t R_i + u_t \cos \alpha
\]

(4)

where \( M_i \) are the equivalent masses of the input rotor, the output rotor, and the FP, respectively, when rotating about their axes, and \( M_i = J_i / R_i^2 \); \( J_i \) is the rotational inertia of the input rotor, the output rotor, and the FP, respectively; \( m_i \) are the masses of the input rotor, the output rotor, and the FP, respectively; \( k_i \) is the torsional support stiffness of the FP; \( c_i \), \( c_i' \) and \( c_o \) are the damping coefficients among the input rotor, the output rotor, the FP and the foundation support, respectively; \( \Delta T \) and \( \omega_c \) are the fluctuation torque and wave frequency on the input rotor.

The coefficients of the torsional displacements in Equation (4) can be expressed as

\[
a_1 = T_c N_1 \cos \theta_0 / R_1^2 \quad b_1 = -T_c N_1 \cos \theta_0 / R_1 R_2 \cos \alpha \quad c_1 = T_c N_1 \cos \theta_0 / 2R_1^2
\]

\[
d_1 = T_c N_1 \cos \theta_0 / 2R_1 R_2 \cos \alpha \quad e_1 = -T_c N_1 \cos \theta_0 / R_1 R_2 \cos \alpha \quad a_2 = T_c N_2 \cos \theta_0 / R_1 R_2 \cos \alpha
\]

\[
b_2 = -T_c N_2 \cos \theta_0 / R_1 \quad c_2 = T_c N_2 \cos \theta_0 / 2R_1 R_2 \quad d_2 = T_c N_2 \cos \theta_0 / 2R_1 R_2 \cos \alpha
\]

\[
e_2 = -T_c N_2 \cos \theta_0 / R_1 R_2 \cos \alpha
\]

The differential equations of the FMMGIA system in the matrix form can be expressed as

\[
M \ddot{x} + c \dot{x} + kx = F + \Delta F
\]  

(5)

where \( M \) is the mass matrix, \( c \) is damping matrix, \( k \) is the stiffness matrix, \( F \) is the equivalent load vector matrix, and \( \Delta F \) is the load vector matrix, individually defined as follows

\[
M = \text{diag}([M_1 \quad M_2 \quad M_3]) \quad x = [x_1 \quad x_2 \quad x_3]^T \quad c = \text{diag}([c_1 \quad c_i' \quad c_o])
\]

\[
k = \begin{bmatrix} a_1 & b_1 & 0 \\ -a_1 & a_2 - b_1 + k_o & b_2 \\ 0 & -a_2 & -b_2 \end{bmatrix} \quad F = \begin{bmatrix} F_1 \quad F_2 \quad F_3 \end{bmatrix}^T \quad \Delta F = \begin{bmatrix} \Delta T \cos \omega_t R_1 \quad 0 \quad 0 \end{bmatrix}^T
\]

\[
F_1 = -\left( c_1 u_1^2 + d_1 u_2^2 + e_1 u_1 u_2 \right) \quad F_2 = c_2 u_2^2 + d_2 u_3^2 + e_2 u_2 u_3 - \left( c_2 u_2^2 + d_2 u_3^2 + e_2 u_2 u_3 \right) \quad F_3 = c_2 u_2^2 + d_2 u_3^2 + e_2 u_2 u_3
\]

Equation (5) can be regularized based on the linearly derived system

\[
\dot{u}_N + c_N \dot{u}_N + k_N u_N = F_N
\]  

(6)

In Equation (6), \( x_N \), \( k_N \), \( c_N \) and \( F_N \) are the normal displacement matrix, the normal stiffness matrix, the normal damping matrix, the normal load vector matrix, respectively. These matrices can be expressed as
\[ k_N = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix}^T, \quad F_N = \begin{bmatrix} F_{N1} & F_{N2} & F_{N3} \end{bmatrix}^T, \quad F_{Ni} = \Delta F_i \cos \omega_i t + A_{Ni} F_i + A_{N2i} F_2 + A_{N3i} F_3, \]

where \( \omega_i \) is the natural frequencies of the drive derived system of the MGIASS system; \( A_{Nij} \) is the element in the \( i \)th row and \( j \)th column of the normal modal matrix \( A_N \).

Considering damping matrix \( \varepsilon \) is the diagonal matrix, the elements on the primary diagonal of the normal damping matrix \( c_N \) are much bigger than other elements. Therefore, \( c_N \) can be simplified into the following diagonal matrix

\[
c_N = \text{diag}\left[ \begin{bmatrix} c_{N1} & c_{N2} & c_{N3} \end{bmatrix} \right]
\]

3. **Main resonances when \( \omega_e \approx \omega_1 \) and \( \omega_e \approx 2\omega_1 \)**

The internal resonances will occur when the natural frequency of the derived systems of the MGIASS system is the integral multiple or close to integral multiple of another natural frequency. When the torsional modal frequency of the output rotor is twice as the torsional modal frequency of the input rotor, the following detuning parameters are introduced

\[
\begin{align*}
\omega_3 &= 2\omega_1 + \varepsilon \sigma \\
\omega_2 &= \omega_1 + \varepsilon \sigma t
\end{align*}
\]

Equation (7) can be solved by the multi-scale method. In order to balance the effects of the damping forces and the nonlinear force, the following assumptions are introduced

\[
\begin{align*}
\Delta F_i &= \varepsilon f \\
 c_k &= \varepsilon c_k' \\
u_N(t) &= \varepsilon u_{N10}(T_0, T_1) + \varepsilon^2 u_{N11}(T_0, T_1) + \ldots
\end{align*}
\]

By substituting Equation (7) and Equation (8) into Equation (6), and equating coefficients of like powers of \( \varepsilon \), we obtain

Order \( \varepsilon \)

\[
\begin{align*}
D_0^2 \Delta u_{N10} + \omega_1^2 \Delta u_{N10} &= 0 \\
D_0^2 \Delta u_{N20} + \omega_1^2 \Delta u_{N20} &= \Delta F_2 \cos \omega_e t \\
D_0^2 \Delta u_{N30} + \omega_1^2 \Delta u_{N30} &= \Delta F_3 \cos \omega_e t
\end{align*}
\]

Order \( \varepsilon^2 \)

\[
\begin{align*}
D_0^2 \Delta u_{N11} + \omega_1^2 \Delta u_{N11} &= 2f_1 \cos \omega_e t + F_{N10} \\
D_0^2 \Delta u_{N21} + \omega_1^2 \Delta u_{N21} &= F_{N20} \\
D_0^2 \Delta u_{N31} + \omega_1^2 \Delta u_{N31} &= F_{N30}
\end{align*}
\]

where

\[
F_{N10} = A_{N11} F_{10} + A_{N21} F_{20} + A_{N31} F_{30}, \quad F_{10} = -\left( c'_1 D_0 \Delta u_{10} + 2D_0 D_1 \Delta u_{10} + c'_1 \Delta u_{10}^2 + d_1 \Delta u_{20}^2 + e_1 \Delta u_{10} \Delta u_{20} \right)
\]

\[
F_{20} = -c'_2 D_0 \Delta u_{20} - 2D_0 D_1 \Delta u_{20} + c_2 \Delta u_{10}^2 + d_2 \Delta u_{20}^2 + e_2 \Delta u_{10} \Delta u_{20} - \left( c_2 \Delta u_{30}^2 + d_2 \Delta u_{20}^2 + e_2 \Delta u_{20} \Delta u_{30} \right)
\]

\[
F_{30} = -c'_3 D_0 \Delta u_{30} - 2D_0 D_1 \Delta u_{30} + c_3 \Delta u_{20}^2 + d_3 \Delta u_{30}^2 + e_3 \Delta u_{20} \Delta u_{30}
\]

With the initial conditions \( u_{N1} \big|_{t=0} = A_t \) and \( \dot{u}_{N1} \big|_{t=0} = 0 \), the general solution of (9) is expressed as
where \( B_2 = \frac{\Delta F_2}{2(\omega_2^2 - \omega_x^2)} \) and \( B_3 = \frac{\Delta F_3}{2(\omega_3^2 - \omega_x^2)} \).

Under the regular coordinate system, the solutions of (9) are

\[
\begin{align*}
  u_{10} &= \alpha_x e^{i\omega_{T_1} t} + \alpha_e A_x(T_1) e^{i\omega_{T_1} t} + \alpha_2 A_2(T_1) e^{i\omega_{T_1} t} + \alpha_3 A_3(T_1) e^{i\omega_{T_1} t} + cc \\
  u_{20} &= \beta_x e^{i\omega_{T_1} t} + \beta_e A_x(T_1) e^{i\omega_{T_1} t} + \beta_2 A_2(T_1) e^{i\omega_{T_1} t} + \beta_3 A_3(T_1) e^{i\omega_{T_1} t} + cc \\
  u_{30} &= \gamma_x e^{i\omega_{T_1} t} + \gamma_e A_x(T_1) e^{i\omega_{T_1} t} + \gamma_2 A_2(T_1) e^{i\omega_{T_1} t} + \gamma_3 A_3(T_1) e^{i\omega_{T_1} t} + cc
\end{align*}
\]

where

\[
\begin{align*}
  \alpha_0 &= N_{12} B_2 + N_{13} B_3, \quad \beta_0 = N_{22} B_2 + N_{23} B_3, \quad \gamma_0 = N_{32} B_2 + N_{33} B_3, \quad \alpha_1 = A_{11}, \quad \alpha_2 = A_{12}, \quad \alpha_3 = A_{13} \\
  \beta_1 = A_{21}, \quad \beta_2 = A_{22}, \quad \beta_3 = A_{23}, \quad \gamma_1 = A_{31}, \quad \gamma_2 = A_{32}.
\end{align*}
\]

By substituting Equation (12) into Equation (10), the secular terms in the right-side of Equation (10) can be eliminated to obtain

\[
\begin{align*}
  -iP_2 \dot{A}_x - Q \omega_{T_1} \dot{x}_i + R_i A_x + S_i \dot{A}_x A_x e^{i\omega_{T_1} t} + f_i e^{i\omega_{T_1} t} = 0 \\
  P_2 A_x + R_2 A_x = 0 \\
  -iP_3 \dot{A}_x - iR_i A_x + S_i A_x^2 e^{i\omega_{T_1} t} = 0
\end{align*}
\]

where

\[
\begin{align*}
  P_1 &= -2\omega I (A_{11} \alpha_1 + A_{22} \beta_1 + A_{33} \gamma_1), \quad Q_i = \omega \left( A_{11} \nu_i \alpha_0 + A_{22} \nu_i \beta_0 + A_{33} \nu_i \gamma_0 \right) \\
  R_i &= \omega \left( A_{11} \nu_i \alpha_0 + A_{22} \nu_i \beta_0 + A_{33} \nu_i \gamma_0 \right) \\
  S_i &= -A_{11} \left[ 2c_1 \alpha_0 \beta \alpha + 2d_1 \beta_1 \beta_1 + e_1 \left( \alpha_3 \beta_1 + \alpha_1 \beta_1 \right) \right] + A_{33} \left[ 2c_2 \beta \beta_1 + 2d_1 \gamma \gamma_1 + e_2 \left( \beta_3 \gamma_1 + \beta_1 \gamma_1 \right) \right] \\
  P_2 &= 2c_2 \alpha_2 A_{11} + 2d_2 \beta_2 A_{12} + 2c_3 \nu_3 A_{22}, \quad R_2 = c_1 \omega_i \alpha_i A_{11} + c_2 \omega_i \beta_i A_{12} + c_3 \omega_i \gamma_i A_{22} \\
  P_3 &= 2c_1 \omega_i \alpha_i A_{11} + 2d_1 \beta_1 \beta_1 + e_1 \left( \alpha_3 \beta_1 + \alpha_1 \beta_1 \right) \\
  S_3 &= -A_{11} \left[ c_1 \alpha_1 \beta_i + d_1 \beta_1 \beta_1 + e_1 \alpha_1 \beta_i \right] + A_{22} \left[ c_1 \alpha_1 \beta_i + d_1 \beta_1 \beta_1 + e_1 \alpha_1 \beta_i - c_1 \beta_1 \beta_i - d_1 \gamma \gamma_1 - e_1 \beta_1 \gamma_1 \right] \\
  &+ A_{33} \left[ c_2 \beta_i \beta_i + d_2 \gamma \gamma_1 + e_2 \beta_i \gamma_1 \right]
\end{align*}
\]

The solution of the second expression in Equation (10) will decrease to zero because of the damping, and can be expressed as

\[
A_2 = E_2 e^{-Q_2 t/1}
\]

The solutions of the first and third equations are written as

\[
A_1 = E_1(T_1) e^{i\phi(T_1)}
\]

Substitute Equation (15) into Equation (13) and separate the real and imaginary parts, the following expression can be got

\[
\begin{align*}
  P_1 E_1 \dot{\gamma}_1 - Q \sin \gamma_3 + S_1 E_1 E_3 \cos \gamma_3 + f_1 \sin \gamma_3 &= 0 \\
  P_2 E_1 \dot{\gamma}_3 + S_2 E_1 E_3 \cos \gamma_3 &= 0 \\
  -P_1 E_1 \sin \gamma_3 - R_1 E_1 + S_1 E_1 E_3 \sin \gamma_3 - f_1 \sin \gamma_3 &= 0 \\
  -P_2 E_1 - R_2 E_1 + S_2 E_1 E_3 \sin \gamma_3 &= 0
\end{align*}
\]

(16)
where $\gamma_i = \theta_i - 2\theta_i + \sigma T_i$, $\gamma_i = \theta_i - \sigma T_i$.

According to Equation (16), it can be obtained as follows

$$\begin{cases} 
\theta_i = \gamma_i + \sigma T_i \\
\theta_i = 2\gamma_i + 2\sigma T_i - \sigma T_i
\end{cases}$$  

(17)

Equation (16) can be expressed

$$\begin{align*}
E_i E_i P_i \gamma_i &= -2E_i (Q_i \sin \gamma_i - S_i E_i \cos \gamma_i - E_i P_i \sigma_i - f_i \cos \gamma_i) - 2E_i E_i P_i \sigma_i + E_i E_i P_i \sigma - P_i S_i E_i \cos \gamma_i / P_i \\
E_i \gamma_i &= Q_i / P_i \sin \gamma_i - S_i / P_i E_i \cos \gamma_i - E_i \sigma_i - f_i / P_i \cos \gamma_i \\
-P_i \dot{E}_i - Q_i \cos \gamma_i - R_i E_i + S_i E_i \sin \gamma_i - f_i \sin \gamma_i &= 0 \\
-P_i \dot{E}_i - R_i E_i - S_i E_i^2 \sin \gamma_i &= 0
\end{align*}$$  

(18)

For the steady-state response, $E_i' = \gamma_i' = 0$. By combining Equations (17) and (18), Equation (18) can be written

$$\begin{align*}
-2E_i E_i P_i \sigma_i + E_i E_i P_i \sigma - P_i R_i E_i^3 \cos \gamma_i / P_i &= 0 \\
Q_i \sin \gamma_i - S_i E_i \cos \gamma_i - P_i E_i \sigma_i - f_i \cos \gamma_i &= 0 \\
-Q_i \cos \gamma_i - R_i E_i + S_i E_i \sin \gamma_i - f_i \sin \gamma_i &= 0 \\
-R_i E_i - S_i E_i^2 \sin \gamma_i &= 0
\end{align*}$$  

(19)

The following expression can be obtained from Equation (19)

$$\begin{align*}
S_i^2 E_i^2 = E_i^2 \left[ R_i^2 + (P_i \sigma - 2P_i \sigma_i)^2 \right] \\
S_i^2 ZE_i^3 + E_i^2 \left[ 2S_i P_i \sigma \sigma_i + 2R_i S_i Y \right] + \left( P_i \sigma_i^2 + R_i^2 \right) E_i^3 - Q_i^2 + f_i^2 &= 0
\end{align*}$$  

(20)

where

$$X = \frac{P_i \sigma - 2P_i \sigma_i}{\sqrt{Q_i^2 + (P_i \sigma - 2P_i \sigma_i)^2}}, \quad Y = \frac{Q_i}{\sqrt{Q_i^2 + (P_i \sigma - 2P_i \sigma_i)^2}}, \quad Z = \frac{R_i}{\sqrt{Q_i^2 + (P_i \sigma - 2P_i \sigma_i)^2}}.$$

In Equation (20), the negative value of $\sin \gamma_i$ is taken for the result with the same symbols of $R_i$ and $E_i$. In the opposite case, the positive value of $\sin \gamma_i$ is taken. $\gamma_i$, $E_i$ and $E_i$ can be calculated from Equation (20). $\gamma_i$ can be got by substituting $\gamma_i$, $E_i$ and $E_i$ into the second or third equation in Equation (19). Then the first-order approximate solutions in the normal coordinate system are obtained by substituting $\gamma_i$, $\gamma_i$, $E_i$ and $E_i$ into Equation (15), which will be substituted into Equation (8) and Equation (12).

$$\begin{align*}
u_n \approx \nu_{n0} &= 2\varepsilon E_i \cos \left( \omega_i T_0 + \gamma_i + \varepsilon \sigma_i T_0 \right) \\
u_{n2} &= 2\varepsilon B_2 \cos \left( \omega_i T_0 \right) \\
u_{n3} &= 2\varepsilon E_i \cos \left( \omega_i T_0 + 2\gamma_i + 2\varepsilon \sigma_i T - \varepsilon \sigma T \right)
+ 2\varepsilon B_3 \cos \left( \omega_i T_0 \right)
\end{align*}$$  

(21)

Based on Equation (21), the first-order approximate analytical solution of the FMMGIA system in the regular coordinate system can be obtained: $x = A_0 x_0$.

4. Resonance of the example MGIASS system and discussion

The MCS of the FMMGIA system is related to the output torque. The larger the output torque, the bigger the MCS. The parameters of the MGIASS system are shown in Table 1. By substituting
dynamic parameters into the forced response expression of the MGIASS system with internal resonance, the forced response curves can be obtained and shown in Figure 3. When the parameters such as the remanences of PMs on the input and output rotor are changed, the difference between and gradually changes. The frequency response curves of the MGIASS system is shown in Figure 4 with \( \sigma_1 \) increasing.

### Table 1. Parameters of the example FMMGIA system.

| \( M_1 (kg) \) | \( M_2 (kg) \) | \( M_3 (kg) \) | \( a_1 \) | \( b_1 \) | \( c_1 \) | \( d_1 \) | \( e_1 \) | \( a_2 \) |
|---|---|---|---|---|---|---|---|---|
| 5.92 | 2.91 | 5.92 | \( 3.07 \times 10^4 \) | \( -2.17 \times 10^4 \) | \( 3.93 \times 10^6 \) | \( 1.96 \times 10^6 \) | \( -5.56 \times 10^6 \) | \( 2.13 \times 10^4 \) |

| \( \Delta T \) | \( R (m) \) | \( b_2 \) | \( c_2 \) | \( d_2 \) | \( e_2 \) | \( k_s (N/m) \) | \( c_1/c_s (N \cdot s/m) \) | \( c_2 (N \cdot s/m) \) |
|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.25 | \( 5.12 \times 10^5 \) | \( 2.57 \times 10^8 \) | \( 1.28 \times 10^8 \) | \( -3.64 \times 10^8 \) | \( 10^6 \) | 0.02 | 0.3 |

(a) Time domain response  
(b) Frequency domain response

**Figure 3.** Main resonances of the FMMGIA system when \( \omega_e \approx \omega_1 \).

![Figure 3](image1)

**Figure 4.** Frequency response curves when \( \omega_e \approx \omega_1 \) (\( \sigma_1 > 0 \)).

![Figure 4](image2)

Figure 3 and Figure 4 indicate that the obvious resonances will occur when the excitation frequency \( \omega_e \) is close to \( \omega_1 \). Because of the weak magnetic coupling stiffness among components, the resonance amplitude of the input rotor is bigger than the amplitude of the output rotor. Meanwhile, the dominant frequency of the main resonances is the natural frequency \( \omega_1 \), or excited frequency \( \omega_e \), and
there are the modulation frequency $2\omega_1$. When $\omega_3$ is very close to $2\omega_1$, the MGIASS system will be unstable.

5. Conclusions
In this paper, the nonlinear forced vibrations and stability of the MGIASS system were discussed when the nonlinear magnetic coupling stiffness and the internal resonance were considered. There are the following conclusions:

(1) Considering the nonlinear magnetic coupling stiffness and internal resonances, the forced responses of the MGIASS system include the excited frequency and the corresponding doubling component.

(2) The resonance amplitudes and stability of the MGIASS system are directly affected by the proximity between $\omega_3$ and $2\omega_1$. There are the excited frequency and modulation frequency in the main resonances.

(3) The dynamic characteristic of the MGIASS system can be optimized by designing the structural parameters and changing the natural frequencies.

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References
[1] Jian L N and Chau K T 2010 A Coaxial Magnetic Gear With Halbach Permanent-Magnet Arrays *IEEE T Energy Convers* 25 293-302
[2] Rasmussen P O, Andersen T O, Jorgensen F T and Nielsen O 2005 Development of a high-performance magnetic gear *IEEE T Ind Appl* 41 764-770
[3] Atallah K and Howe D 2001 A novel high-performance magnetic gear *IEEE T Magn* 37 2844-46
[4] Liu Y L, Ho S L and Fu W N 2014 A Novel Magnetic Gear with Intersecting Axes *IEEE T Magn* 50 1-4
[5] Hao X H, Zhu H Q and Pan D 2018 Magnetic gear with intersecting axes and straight stationary pole-pieces *Adv Mech Eng* 11 1-10
[6] Frank N W, Pakdelian S and Toliyat H A 2011 Passive suppression of transient oscillations in the concentric planetary magnetic gear *IEEE T Energy Convers* 26 933-39
[7] Montague R, Bingham C and Atallah K 2012 Servo Control of Magnetic Gears *IEEE/ASME T Mech* 17 269-278