Analysis of the Influence of Tolerance Design on Unbalanced Mass of Rotating Pressure Vessel

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Abstract. The unbalanced mass will cause forced vibration of the pressure vessel when it rotates, and the magnitude of the unbalanced mass is positively correlated with the amplitude of the pressure vessel. Therefore, the study of unbalanced mass is of great significance to the safety of rotating pressure vessels. The influence of form and position tolerances and assembly tolerances on the unbalanced quality of pressure vessel parts is analyzed, and the normal distribution model is used to simulate the tolerance distribution of parts in mass production, and then the total unbalanced mass of the pressure vessel is calculated by the vector synthesis method. Through sensitivity analysis, the parts and key positions that have a greater impact on the total unbalanced mass of the pressure vessel are found, and the geometric tolerances of the parts with high process difficulty are optimized, which provides a theoretical basis for the tolerance optimization design work of the rotating pressure vessel.

1. Introduction
From the perspective of rotor dynamics, unbalanced mass will cause forced vibration of the rotating machine. If the unbalanced mass is too large, it may have a serious impact on the over-critical performance of the rotating machine and its dynamic behaviour at operating speed[1]. Therefore, it is necessary to clarify the initial unbalanced mass of each part of the rotor, and find out the main factors that produce the unbalance. This research is of great significance to the reliability and safety of the rotating pressure vessel.

Tolerance design mainly considers three types of tolerances, namely dimensional tolerances, geometric tolerances and assembly tolerances [2-3]. Dimensional tolerance refers to the gap between the actual size of the part and the design requirements, which will not have a significant impact on the unbalanced quality of a single part; the difference in shape and position in the shape and position tolerance will have a great impact on the centroid and quality of the geometry; assembly Tolerance refers to the gap between the angle or offset between the axis of each component after assembly and the design requirements. Therefore, form and position tolerances and assembly tolerances are the main factors affecting unbalanced quality. Li Xiuxia[4], Liao Zihao[5] and others established a rotor model considering manufacturing tolerances, and analyzed the influence of factors such as shape and position tolerances on rotor imbalance and vibration. Wang Bo and Wang Fayao[6] considered the influence of shape and position tolerance on the reduction factor of thin-walled cylindrical shells, and proposed a prediction method. Peng Heping[7] analyzed the design method of geometric tolerances and allocated the geometric tolerances and process tolerances in parallel.
This article mainly studies the unbalanced mass produced in the process of processing and assembling the main parts of the rotating pressure vessel. Calculate the unbalanced mass caused by geometric tolerance and assembly tolerance. Through the sensitivity analysis, the components and related dimensions that have a greater impact on the unbalanced quality are found, and the geometric tolerances of the more difficult-to-process parts are optimized to provide a reference for the unbalanced quality design and tolerance optimization design of the rotating pressure vessel.

2. Calculation method of unbalanced mass produced by geometric tolerance
The rotating pressure vessel is composed of 5 parts, as shown in the Figure 1.

![Figure 1. Schematic diagram of rotating pressure vessel parts.](image)

The shape and position tolerances mainly involved in these parts include straightness, end perpendicularity, unevenness of wall thickness, concentricity, radial runout and roundness. This chapter will introduce the unbalanced quality caused by the above-mentioned types of shape and position tolerances.

2.1. Unbalanced mass caused by coaxiality
According to the definition of coaxiality in form and position tolerance, the worst state of coaxiality of molded parts is shown in Figure 2. In Figure 2, the oblique area is the cross-sectional view of the part, and the diameter of the dashed circle is the coaxiality a. When the y-axis deviates to one side along the axis direction to a/2, the unbalanced mass produced is the largest.

![Figure 2. Schematic diagram of the influence of coaxiality on parts.](image)

Calculate the eccentricity $x_e$ and unbalanced mass of the component according to the formula (1~2):

$$x_e = \frac{M_{all} \times 0 - M_{blank} \times x}{M_{all} - M_{blank}}$$

$$m = \rho \pi \left( R_{out}^2 - R_{in}^2 \right) l$$

Substituting the calculation result $m$ of the formula (2) into the formula (3) to calculate the unbalanced mass $\Delta m$ converted to $D/2$ from the centroid.

$$\Delta m = \frac{m / 2 \cdot x_e}{D / 2}$$
2.2. Unbalanced mass caused by uneven wall thickness

According to the definition of the unevenness of the wall thickness in the form and position tolerance, it can be seen that the worst state of the unevenness of the wall thickness of the formed part is shown in Figure 3. The wall thickness of the oblique area in Figure 3 meets the design requirements, and the wall thickness of the dot area on the right is \( \Delta \delta \). The unbalanced mass in this state is the maximum value of the unbalanced mass produced by the unevenness of the wall thickness.

![Figure 3. Schematic diagram of uneven wall thickness.](image)

In Figure 3, O is the centroid, and O’ is the centroid. First, use the formula (4) to calculate the eccentricity \( x_c \) of the part:

\[
x_c = \frac{\int x dm}{\int dm} = \frac{\rho \int y dx}{M}
\]

Then calculate the mass with thickness \( \Delta \delta \) according to formula (5):

\[
m = \rho \pi \left( \left( R + \Delta \delta \right)^2 - R^2 \right) l
\]

Substituting the calculation result \( m \) of formula (5) into formula (3) to obtain the unbalanced mass \( \Delta m \) at \( D/2 \) from the centroid.

2.3. Unbalanced mass caused by radial circular runout

According to the definition of radial runout in the form and position tolerance, it can be seen that the worst state of the radial runout of the formed part is similar to that shown in Fig. 3.

In order to simplify the calculation, the size offset of the radial circle runout is taken as the center of mass offset, that is, \( x_c = \Delta \delta \).

Use formula (6) to calculate the mass \( m \) of radial runout or roundness deviation to one side:

\[
m = \rho \pi \left( R_{out}^2 - R_{in}^2 \right) l
\]

Substitute \( m \) into equation (3) to calculate the unbalanced mass \( \Delta m \) at the distance \( D/2 \) from the centroid.

2.4. Unbalanced mass caused by roundness

Since the effect of roundness on the geometry of the part's cross-section is similar to that in Figure 4, the same method as the radial runout is used when calculating the unbalanced mass produced by the roundness.

The size offset of the roundness is taken as the center of mass offset, that is, \( x_c = a \).

Calculate the mass of the roundness to one side according to the formula (7):

\[
m = \rho \pi \left( R_{out}^2 - R_{in}^2 \right) l
\]

Substituting the calculation result \( m \) of formula (7) into formula (3), calculate the unbalanced mass \( \Delta m \) converted to \( D/2 \) from the centroid.
2.5. Unbalanced mass caused by straightness

According to the definition of straightness in the form and position tolerance, the state of the worst straightness of the molded part is shown in Figure 4. The unbalanced mass in this state is the maximum unbalanced mass produced by the straightness. In order to simplify the calculation, the bending state in Figure 4-a is simplified to the shape of Figure 4-b.

![Figure 4. Schematic diagram of axis bending deformation.](image)

Since this part is symmetrical from top to bottom, the lower half of the calculation is the object of study. Use a plane perpendicular to z to divide the components in Figure 1 into several sections, each section having a length of dz. Then the centroid and mass of each segment can be calculated by the following formula (8~9):

$$x_i = \frac{1}{2} z_i \alpha = \frac{1}{2} z_i \alpha l \left( \frac{L}{2} \right)$$  \hspace{1cm} (8)

$$\left( dm_i \right) = \frac{M dz}{L} = M \frac{dz}{L}$$  \hspace{1cm} (9)

At this time, the mass eccentricity of the lower half of the part is the position of its center of mass on the X-axis, which can be calculated according to formula (10). The eccentricity is introduced into the formula (4), and the unbalanced mass of the lower half of the part is calculated:

$$x_c = \frac{1}{dm_i} \int x_i dm_i = \frac{L}{2} \frac{x_i dz}{L / 2}$$  \hspace{1cm} (10)

Since the bending of the axis in Figure 4 is in the same direction, the unbalanced mass of the upper half is in phase with the lower half, so the total unbalanced mass converted to D/2 from the centroid is $2 \triangle m$.

2.6. Unbalanced mass caused by the verticality of the end

According to the definition of straightness in the form and position tolerances, the state of the worst verticality of the end face of the molded part is shown in Figure 5. The unbalanced mass in this state is the maximum value of the unbalanced mass produced by the verticality.

![Figure 5. Schematic diagram of the cross-sectional shape of parts.](image)
Calculate the unbalanced mass in Figure 5. First use the formula (11~12) to calculate the length $\Delta h$:

$$\frac{\Delta h}{2R} = \frac{\Delta r}{L}$$  \hspace{1cm} (11)$$

$$\Delta h = 2R \frac{\Delta r}{L}$$  \hspace{1cm} (12)

Among them, $R$ and $L$ are the average radius and length of the part respectively. Next, use the formula (13) to calculate the mass $m_{BCD}$ of the BCD segment, and then substitute it into the formula (14) to calculate the unbalanced mass $\Delta m$ of the BCD segment equivalent to $D/2$ from the centroid:

$$m_{BCD} = \frac{1}{2} \rho \pi (R_1^2 - R_2^2) \Delta h$$  \hspace{1cm} (13)$$

$$\Delta m = \frac{m}{D/2} \frac{R}{3}$$  \hspace{1cm} (14)

3. Calculation method of unbalanced mass caused by assembly tolerance

The rotating pressure vessel is composed of 5 parts. The main tolerances of these parts after assembly include end axial run-out and radial circular run-out, as shown in Figure 6.

3.1. Unbalanced mass caused by axial runout at the rear end of the assembly

The difference between the maximum value and the minimum value of the axial runout of the end plane of the part cannot exceed $a$. When the unilateral axial runout of the component is higher than the other side by $a$, the unbalanced mass produced by the assembly tolerance reaches the maximum, as shown in Figure 7.

![Figure 6. Assembly tolerance detection position.](image)

![Figure 7. Schematic diagram of assembly parts.](image)
In the figure, O is the centroid, and O' is the centroid. Calculate the eccentricity $x_c$ of the component according to the formula (15):

$$x_c = R - R \cos \alpha = R(1 - \cos \alpha)$$  \hspace{1cm} (15)$$

In the formula (15), R is the radius of the component. Substituting the mass $m$ of the top cover into equation (3), calculate the unbalanced mass at the distance $D/2$ from the centroid.

3.2. Unbalanced mass caused by radial runout after assembly

The difference between the maximum and minimum radial runout cannot exceed $a$. When the radial runout of one side of the skirt and the three joints is higher than the other side by $a$, the unbalanced mass produced reaches the maximum, as shown in Figure 8.

First, calculate the eccentricity $x_c$ of the shaded part in the figure, where $h$ is the height of the mating surface of the part.

$$x_c = \frac{h}{2} \sin \beta$$  \hspace{1cm} (16)$$

The shaded part in Figure 8 is the part where the centroid is shifted. Substitute its mass into equation (7) to calculate the unbalanced mass at $D/2$ from the centroid.

4. Calculation results of unbalanced mass of rotating pressure vessel parts

4.1. Calculation results of unbalanced mass produced by geometric tolerances

Summarize the unbalanced quality caused by the geometric tolerance. Take a cylinder as an example to illustrate the calculation method and its distribution of unbalanced mass, as shown in Table 1.

**Table 1.** The unbalanced mass of the cylinder and its distribution.

| Factors that produce unbalanced mass | Maximum unbalanced mass/mg | Normal distribution characteristic parameter |
|-------------------------------------|----------------------------|---------------------------------------------|
| Straightness                        | 45                         | (22.8,7.59²)                                |
| End face perpendicularity           | 858                        | (429.2,143.07²)                             |
| Unevenness of wall thickness         | 2884.2                     | (1443.2,481.07²)                            |
| Coaxiality                          | 2412.2                     | (1206.4,402.12²)                            |
| Total                               | 8121.2                     | (1232.1,617²)                               |

It can be seen from Table 1 that the unevenness of wall thickness and coaxiality are the main factors that produce unbalanced mass. Assuming that the unbalanced masses everywhere are in the same phase, vector synthesis is performed on them, and the maximum value of the unbalanced mass of the cylinder is 8121.2 mg.

Simulate and calculate the unbalanced mass distribution in mass production. The shape and position of the cylinder obey the normal distribution. The phase of the unbalanced mass is uniformly distributed in the range of $0$–$2\pi$. The sample size of the cylinder is taken in the calculation is $10^6$, vector synthesis is performed on the unbalanced masses everywhere, and the unbalanced mass distribution is obtained. Parameter estimation of the distribution curve with 95% confidence level shows that the unbalanced mass of the cylinder obeys a normal distribution (1232.1,617²) within the
range of \([0,812.1]\) mg, indicating that 99% of the unbalanced mass of the cylinder is below 2700mg, as shown in Figure 9.

\[\text{Figure 9. Unbalanced mass distribution of cylinder.}\]

In the same way, the unbalanced mass of other parts and their distribution can be obtained. The unbalanced mass of all parts is shown in Table 2.

| Components          | Unbalanced mass range of parts/mg | Normal distribution parameter estimation | \(\bar{x}/\text{mg}\) |
|---------------------|----------------------------------|----------------------------------------|---------------------|
| Cylinder            | \([0,8121.2]\)                   | \((1232.1,617^2)\)                     | 2700                |
| Filter plate 1      | \([0,1001.7]\)                   | \((3170.158.5^2)\)                     | 687.2               |
| Filter plate 2      | \([0,758.4]\)                    | \((232.8,116.4^2)\)                    | 505.1               |
| Top cover 1         | \([0,485.9]\)                    | \((243.2,121.6^2)\)                    | 467.2               |
| Top cover 2         | \([0,188.2]\)                    | \((83.2,41.6^2)\)                      | 173.0               |
| Total               | \([0,10298]\)                   | \((1802.3,955.1^2)\)                   | 4036.8              |

It can be seen from Table 2 that within the tolerance range, the component with the largest unbalanced mass caused by machining errors is the cylinder, followed by the filter plate 1, the filter plate 2, and the top cover 1, and the smallest is the top cover 2.

The vector synthesis obtains the total unbalanced mass of the parts, which obeys the normal distribution of \((1802.3,955.1^2)\) in the interval of \([0,10298]\)mg, and the confidence level is 95%, indicating that the total unbalanced mass of 99% of the parts is 4036.8mg the following.

4.2. Calculation result of unbalanced mass caused by assembly tolerance

Summarize the unbalanced masses caused by assembly tolerances, and perform vector synthesis on them, as shown in Table 3.

| Components            | Unbalanced mass range of parts/mg | Normal distribution parameter estimation | Confidence level 95% |
|-----------------------|----------------------------------|----------------------------------------|---------------------|
| Axial runout 1        | 0.288                            | \((0.1458,0.0486^2)\)                  |                     |
| Axial runout 2        | 0.209                            | \((0.1055,0.0351^2)\)                  |                     |
| Radial circle runout 1| 82.6                             | \((31.5,10.5^2)\)                      |                     |
| Radial circle runout 2| 67.87                            | \((34.2,11.4^2)\)                      |                     |
| Radial circle runout 3| 29.57                            | \((14.88,4.9^2)\)                      |                     |
| Radial circle runout 4| 38.33                            | \((19.63,9.35^2)\)                     |                     |

It can be seen from the above table that after assembly, the unbalanced masses produced at the radial runouts 1 and 2 are relatively large.

The vector synthesis obtains the unbalanced mass distribution caused by the assembly tolerance after the parts are assembled, and the distribution curve is estimated with 95% confidence. The result shows that the unbalanced mass caused by the assembly tolerance is in the range of \([0,209.53]\)mg. The
internal obey normal distribution (63.84, 24.692), indicating that the total unbalanced mass caused by the assembly tolerance of 99% of the parts is below 121.5mg.

5. Sensitivity Analysis
Perform sensitivity analysis on the influence of various dimensions on unbalanced mass. Assuming that all tolerance requirements of each part are increased by 0.1mm, calculate the unbalanced mass added by the part, analyze the contribution of each part to the total unbalanced mass, and analyze the result as shown in Figure 10.

![Figure 10. Sensitivity of unbalanced mass to various components.](image)

The height of the histogram indicates the contribution of each component to the total unbalanced mass. It can be seen from the figure that components 2 and 3 have a greater impact on the unbalanced mass. Analyze the shape and position tolerances that have a greater impact on the unbalanced quality of each part, and list them in Table 4 according to the degree of influence from large to small.

Table 4. Dimensions of various locations that have a greater impact on unbalanced mass.

| Part name | Tolerance type       | Total unbalanced mass increment/mg |
|-----------|----------------------|-----------------------------------|
| Filter 1  | Unevenness of wall thickness | 529.9                             |
| Filter 2  | Unevenness of wall thickness | 388.9                             |
| Lid 1     | Unevenness of wall thickness | 394.9                             |
| Cylinder  | Unevenness of wall thickness | 285.2                             |
| Lid 2     | Coaxiality            | 288.9                             |
| Cylinder  | Coaxiality            | 217.1                             |
| Cylinder  | Straiitness           | 151.5                             |

It can be seen from the table that the unevenness of wall thickness is more sensitive to unbalanced mass, and the concentricity of part 4 and the cylinder is also more sensitive. It is recommended to control the processing accuracy of these points.

Round runout and roundness are less sensitive to the unbalanced quality of each part, so they are not listed in Table 4. If there are certain difficulties in processing, the two types of size requirements can be slightly relaxed to improve the processing qualification rate.

6. Optimization suggestion
When processing thin-walled parts, the unevenness of the wall thickness has a greater impact on the unbalanced quality, and the control difficulty during operation is also higher. The research object pressure vessel of this study is composed of 5 thin-walled parts. Among them, the ratio of the wall thickness of the filter plate 2 to the unevenness of the wall thickness is the highest, so the processing is the most difficult. In order to save production costs and increase the qualified rate of processed parts, the unevenness of the wall thickness of the filter plate 2 should be optimized.

In this study, the sensitivity analysis shows that although the unevenness of the wall thickness of the filter plate 2 has a high degree of influence on the unbalanced quality, when the unevenness of the
wall thickness is relaxed from 0.1mm to 0.3mm, the total pressure vessel. The maximum unbalanced mass is increased to 7.232g, the unbalanced mass distribution parameter is (1.02, 0.522), the total unbalanced mass of 99% products is below 2.224g, and the increment is small. Therefore, it is recommended to optimize the wall thickness unevenness tolerance requirement of the filter plate 2 to 0.3 mm.

7. Conclusion
In this paper, the unbalanced mass produced by the geometric tolerance and assembly tolerance is calculated theoretically, and the unbalanced mass distribution in the mass production process is simulated, and the following conclusions are obtained:

(1) The result shows that the total unbalanced mass of the component obeys the normal distribution of (1802.3, 955.12) in the interval of [0, 10298]mg, the confidence level 95%, indicating that the total unbalanced mass of 99% of the parts is below 4036.8mg.

(2) The parts 2 and 3 have a greater influence on the unbalanced quality, and attention should be paid to controlling the processing accuracy. The unbalanced mass is highly sensitive to the unevenness of the wall thickness.

(3) An optimization suggestion to relax the tolerance requirements of the unevenness of the wall thickness of the filter plate 2 to 0.3 mm is proposed.

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