CKM Matrix and Standard-Model CP Violation

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The currently favored model of CP violation is based on phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix describing the weak charge-changing couplings of quarks. The present status of parameters of this matrix is described. Tests of the theory, with particular emphasis on the study of B meson decays, are then noted. Some remarks are made regarding the possible origin of the baryon asymmetry of the universe; the corresponding coupling pattern of the leptons could shed light on the question. Some possibilities for non-standard physics are discussed.

1. INTRODUCTION

For more than thirty years, the neutral kaon system has been the only direct place in which CP violation has been seen. The currently favored theory of this phenomenon involves complex phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix describing the charge-changing weak transitions of quarks. It is very likely that a number of experiments will be able to test this theory in the next few years. The present talk is meant as an overview of this activity, with particular emphasis on $B$ meson decays. A number of related subjects are covered in more detail by other speakers at this Workshop.

We begin in Section 2 with a survey of the patterns of quark and lepton masses and couplings. The latter are described by parameters of the CKM matrix whose current status we review in Section 3. We then turn in Section 4 to some tests of this picture based on $B$ meson decays. The baryon asymmetry of the Universe, described briefly in Section 5, provides indirect evidence for CP violation, though the CKM pattern alone is probably insufficient for understanding it. Some possibilities for non-standard physics are discussed in Section 6, while Section 7 concludes.

2. QUARK AND LEPTON PATTERNS

The present status of quark and lepton masses and couplings is summarized in Figure 1. The top quark is the heaviest known, but the fractional error on its mass, $m_t = 175 \pm 6$ GeV/$c^2$, is now the smallest for any quark! A detailed pattern of charge-changing couplings among quarks occurs; in addition to the dominant couplings $u \leftrightarrow d$, $c \leftrightarrow s$, and $t \leftrightarrow b$, all the others are allowed, but with diminished strengths, as shown in Table 1.
Table 1
Relative strengths of charge-changing weak transitions.

| Relative ampl. | Transition | Source of information (example) |
|----------------|------------|----------------------------------|
| ~ 1            | u ↔ d     | Nuclear β-decay                  |
| ~ 1            | c ↔ s     | Charm decays                     |
| ~ 0.22         | u ↔ s     | Strange particle decays          |
| ~ 0.22         | c ↔ d     | Neutrino charm prod.            |
| ~ 0.04         | c ↔ b     | b decays                         |
| ~ 0.003        | u ↔ b     | Charmless b decays               |
| ~ 1            | t ↔ b     | Dominance of t → Wb              |
| ~ 0.04         | t ↔ s     | Only indirect evidence           |
| ~ 0.01         | t ↔ d     | Only indirect evidence           |

The couplings in Table 1 are encoded in the unitary 3 × 3 Cabibbo-Kobayashi-Maskawa (CKM) matrix. We now explore the current status of its parameters. More complete discussions may be found in [5], [6], and [7].

3. CKM MATRIX AND PARAMETERS

The CKM matrix $V$ describing the charge-changing weak transitions of left-handed quarks may be written as

$$V \approx \begin{pmatrix} 1 - \lambda^2/2 & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$  

The quantity $\lambda = 0.2205 \pm 0.0018 = \sin \theta_c$ expresses the suppression of $s \to u$ decays with respect to $d \to u$ decays [11]. This parameter is sufficient to describe the $u$, $d$, $s$, and $c$ couplings via the upper left $2 \times 2$ submatrix of $V$ [12].

When a third family of quarks is added, three more parameters are needed. One may express them in terms of (1) the strength characterizing $b \to c$ decays, $A\lambda^2 = 0.0393 \pm 0.0028$ [13] so that $A = 0.808 \pm 0.058$ (see [1], [11] for slightly different values); (2) the magnitude of the $b \to u$ transition element measured in charmless $b$ decays, $V_{ub}/V_{cb} = 0.08 \pm 0.016$ [3] (see also [13]) so that

$$(\rho^2 + \eta^2)^{1/2} = 0.363 \pm 0.073.$$  

(3) the phase of $V_{ub} = A\lambda^3(\rho - i\eta)$. The unitarity of the CKM matrix implies that the scalar product of the complex conjugate of any row with any other row should vanish, e.g.,

$$V_{ud}^*V_{td} + V_{us}^*V_{ts} + V_{ub}^*V_{tb} = 0.$$  

Since $V_{ud} \approx 1$, $V_{us}^* \approx \lambda$, $V_{ts} \approx -A\lambda^2$, and $V_{tb} \approx 1$ we have $V_{td} + V_{ub}^* = A\lambda^3$, expressing the least-known CKM elements in terms of relatively well-known parameters. This result can be visualized as a triangle in the complex plane. Dividing $V_{ub}^*$ by $A\lambda^3$, since $V_{ub}^*/A\lambda^3 = \rho + i\eta$, $V_{td}/A\lambda^3 = 1 - \rho - i\eta$, one obtains a triangle of the form shown in Fig. 2. In this figure the angles $\alpha$, $\beta$, and $\gamma$ are defined as in [10]. Each can be measured using $B$ meson decays.

The value of $V_{ub}^*/A\lambda^3$ may then be depicted as a point in the $(\rho, \eta)$ plane. The major remaining ambiguity in the determination of the CKM matrix elements concerns the phase of $V_{ub}$, or the shape of the unitarity triangle. The answer depends on the value of $V_{td}$. In order to learn about this one must resort to indirect means, which involve loop diagrams.

3.1. $B^0 - \bar{B}^0$ mixing

Box diagrams involving $u$, $c$, and $t$ in loops contribute to the virtual $bd \leftrightarrow \bar{d}b$ transitions which mix $B^0$ and $\bar{B}^0$. The leading contribution at high internal momentum in these diagrams cancels as a consequence of [2]. The remaining contribution is dominated by the top quark since all products...
of CKM elements $V_{td}^*V_{td}$ are of order $\lambda^3$ while $m_t \gg m_c, m_u$. The expression for the splitting between mass eigenstates is then [7]

$$\Delta m = \frac{G_F^2}{6\pi^2} |V_{ud}|^2 M_W^2 m_B f_B^2 B \eta B S \left( \frac{m_t^2}{M_W} \right) ,$$  \tag{3}

where

$$S(x) \equiv \frac{x}{4} \left[ 1 + \frac{3 - 9x}{(x-1)^2} + \frac{6x^2 \ln x}{(x-1)^2} \right] .$$  \tag{4}

We take $m_t = 175 \pm 6 \text{ GeV}/c^2$, $M_W = 80.34 \pm 0.10 \text{ GeV}/c^2$, $m_B = 5.279 \text{ GeV}/c^2$ (see [10]), and $f_B / B = 200 \pm 40 \text{ MeV}$ [8]. Here $f_B$ is the $B$ meson decay constant, defined so that the matrix element of the weak axial-vector current $A_\mu = b_\mu \gamma_\mu d$ between a $B^0$ meson and the vacuum is $\langle 0 | A_\mu | B^0(p) \rangle = i f_B \rho B$. With this normalization, the decay constants of the light pseudoscalar mesons are $f_\pi = 131 \text{ MeV}$ and $f_K = 160 \text{ MeV}$. The decay constants express the amplitude for the corresponding quark and antiquark (e.g., $b$ and $d$) to be found at a point, as they must in order to participate in the short-distance process associated with the box diagrams.

The factor $B_B$ expresses the degree to which the box diagrams provide the contribution to $B - \bar{B}$ mixing. An estimate in lattice gauge theories [13,20] is $B_B = 1.16 \pm 0.08$. Finally, $\eta_B = 0.55$ is a QCD correction. All quantities are quoted in the same consistent renormalization scheme [13].

The first evidence for mixing of nonstrange $B$’s was obtained by the ARGUS Collaboration [20]. The large mixing amplitude, $\Delta m / \Gamma \simeq 0.7$ (where $\Delta m$ is the mass difference between mass eigenstates and $\Gamma$ is the $B$ meson decay rate), was one early indication of a very heavy top quark. With the rapidly moving $B$ mesons and the fine vertex information now available at LEP [13,21], CDF [22], and SLD [23] (see also [24]), it has become possible to directly observe time-dependent $B^0 - \bar{B}^0$ oscillations with a modulating factor $\sin(\Delta m t)$ (where $t$ is the proper decay time). The current world average [23] is $\Delta m_B = 0.470 \pm 0.017 \text{ ps}^{-1}$, where the subscript refers to the mixing between $B^0 \equiv bd$ and $\bar{B}^0 \equiv \bar{b}d$. Using [3] and the parameters mentioned above, we can then obtain an estimate of $|V_{td}|$, which leads, once we factor out a term $A \lambda^3$, to the constraint

$$|1 - \rho - \eta| = 1.01 \pm 0.22 .$$  \tag{5}

This result can be plotted in the $(\rho, \eta)$ plane as a band bounded by circles with centers at $(1, 0)$.

Now that the top quark mass is known so precisely, the dominant source of error in Eq. (3) is uncertainty in the $B$ meson decay constant $f_B$. Although the desired quantity $f_B$ has not yet been measured directly, a number of results on the decay constant $f_{D_s}$ have appeared over the past few years [26–30]; these serve to check calculations of $f_B$. The values of $f_{D_s}$ obtained by measuring the rates for $D_s \to \mu \nu$ and/or $\tau \nu$ are summarized in Table 2. The errors are statistical, systematic, and (where shown) branching ratio of calibrating modes. These are added in quadrature to obtain the value of $\sigma$ in the last column. One can also estimate $f_{D_s}$ by assuming factorization in certain $B$ decays in which the charged weak current produces a $D_s$. The results are consistent with the above average.

One $D \to \mu \nu$ event has been detected by the BES group, leading to $f_{D_s} = 300^{+150 +80} _{-150 -40} \text{ MeV}$ [31]. This result is consistent with the previous upper bound of 290 MeV (90% c.l.) obtained by Mark III [32]. Flavor SU(3)-breaking estimates in lattice gauge theories [33] and quark models [34] imply $f_{D_s}/f_{D_s} = 0.8$ to 0.9.

Lattice gauge theories and quark models are both are converging on the range $f_{D_s} \simeq 180 \pm 40 \text{ MeV}$, implying branching ratios $B(B \to \tau \nu) \simeq (1/2) \times 10^{-4}$ and $B(B \to \mu \nu) \simeq 2 \times 10^{-7}$ [31]. These small rates pose a challenge to $B$ factories. The same SU(3) estimates for the ratio of $f_D/f_{D_s}$
also give a very similar ratio of \( f_B/f_{B_s} \). Equality of these two ratios is expected within a few percent [30].

### 3.2. CP-violating \( K^0 - \bar{K}^0 \) mixing

The \( K^0 \) and \( \bar{K}^0 \) are strong-interaction eigenstates of opposite strangeness. However, since the weak interactions do not conserve strangeness, they pick out linear combinations of \( K^0 \) and \( \bar{K}^0 \) in decay processes [37]. As of 1957, when the weak interactions do not conserve strangeness, states of opposite strangeness. However, since the short-lived \( K^0 \) would have to decay to three pions or a pion and two pions. The orthogonal linear combination denoted by CP invariance to decay to two pions and a lepton-neutrino pair.

In 1964 J. Christenson, J. Cronin, V. Fitch, and R. Turlay reported that in fact the long-lived neutral kaon \( d \bar{d} \) decay to two pions, with an amplitude whose magnitude is about \( 2 \times 10^{-3} \) that for the short-lived \( K \to 2\pi \) decay [38]. One then can parametrize the mass eigenstates as

\[
K_S \simeq K_1 + \epsilon K_2 , \quad K_L \simeq K_2 + \epsilon K_1 ,
\]

where \( |\epsilon| \simeq 2 \times 10^{-3} \) and the phase of \( \epsilon \) turns out to be about \( \pi/4 \). The parameter \( \epsilon \) encodes all current knowledge about CP violation in the neutral kaon system. Where does it come from?

One possibility, proposed [40] immediately after the discovery and still not excluded, is a “superweak” CP-violating interaction which directly mixes \( K^0 = d\bar{s} \) and \( \bar{K}^0 = s\bar{d} \). This interaction would have no other observable consequences since the \( K^0 - \bar{K}^0 \) system is so sensitive to it!

The presence of three quark families [34] poses another opportunity for explaining CP violation through box diagrams involving \( u, c, \) and \( t \) quarks. With three quark families, phases in complex coupling coefficients cannot be removed by redefinition of quark phases. Within some approximations [33], the parameter \( \epsilon \) is directly proportional to the imaginary part of the mixing amplitude. Its magnitude (see [11] or [12] for a calculation in the limit of \( m_t \ll M_W \)) is

\[
|\epsilon| \approx \frac{G_F^2 m_K f_K^2 B_K M_W^2}{\sqrt{2}(12\pi^2)\Delta m_K} \times \left[ \eta_1 S(x_c) I_{cc} + \eta_2 S(x_t) I_{tt} + 2\eta_3 S(x_c, x_t) I_{ct} \right] ,
\]

where \( I_{ij} \equiv \text{Im}(V_{ei}^* V_{is} V_{jd}^* V_{js}) \). In order to evaluate these expressions we need to work to sufficiently high order in small parameters in \( V \). The application of the unitarity relation to the first and second rows tells us that \( V_{td} = -\lambda - A^2 \lambda^5 (\rho + i\eta) \). We then find

\[
I_{cc} = -2A^2 \lambda^6 \eta, \quad I_{tt} = A^2 \lambda^6 \eta, \quad \text{and} \quad I_{ct} = 2A^2 \lambda^6 \eta(1 - \rho) \].

The factors \( \eta_1 = 1.38, \quad \eta_2 = 0.57, \quad \eta_3 = 0.47 \) are QCD corrections [35], while \( x_c \equiv m_c^2 / M_W^2 \). The function \( S(x) \) was defined in Eq. (5), while

\[
S(x, y) \equiv xy \left\{ \frac{3}{4} + \frac{3}{2(1-x)} \ln \frac{y}{y-x} \right\} .
\]

Eq. (7) may then be rewritten (cf. [14]) as

\[
|\epsilon| = 4.39 A^2 B_K \eta \times \left[ \eta_3 S(x_c, x_t) - \eta_1 S(x_c) + \eta_2 A^2 \lambda^4 (1 - \rho) S(x_t) \right] .
\]

Using the experimental values [10] \( |\epsilon| = (2.28 \pm 0.02) \times 10^{-3} \), \( f_K = 160 \text{ MeV} \), \( \Delta m_K = 3.49 \times 10^{-15} \text{ GeV} \), and \( m_K = 0.4977 \text{ GeV} \), the value \( B_K = 0.75 \pm 0.15 \) [33], and the top quark mass \( m_t(M_W) = 165 \pm 6 \text{ GeV}/c^2 \) appropriate for the loop calculation [5], we find that CP-violating \( K - \bar{K} \) mixing leads to the constraint

\[
\eta(1 - \rho + 0.44) = 0.51 \pm 0.18 ,
\]

where the term \( 1 - \rho \) corresponds to the loop diagram with two top quarks, and the term 0.44 corresponds to the additional contribution of charmed quarks. The major source of error on the right-hand side is the uncertainty in the parameter \( A \equiv V_{cb} / A^2 \). Eq. (9) can be plotted in the \( (\rho, \eta) \) plane as a band bounded by hyperbolae with foci at (1.44,0).

The constraints [3], (8), and (9) define the allowed region of parameters shown in Fig. 3. The boundaries shown are 1σ errors, but are dominated by theoretical uncertainties in each case.
Figure 3. Region in the $(\rho, \eta)$ plane allowed by constraints on $|V_{td}/V_{cb}|$ (dotted semicircles), $B^0 - \bar{B}^0$ mixing (dashed semicircles), and CP-violating $K - \bar{K}$ mixing (solid hyperbolae).

A large region centered about $\rho \simeq 0$, $\eta \simeq 0.35$ is permitted. Nonetheless, the CP violation seen in kaons could be due to an entirely different source, such as a superweak mixing of kaons permitted. Nonetheless, the CP violation seen in kaons is aimed for.

4. CKM TESTS WITH $B$ MESON DECAYS

A number of experimental facilities can or will be able to make incisive tests of the CKM picture by studying hadrons containing the $b$ quark. LEP has finished productive years of running on the $Z^0$, in which millions of $b\bar{b}$ pairs were produced. SLD has the advantage of electron polarizability which partially compensates for a lower luminosity than LEP. CESR at Cornell continues to set luminosity records and is aiming for $L > 10^{33}$ cm$^{-2}$ s$^{-1}$ at the $\Upsilon(4S)$, a copious source of $B\bar{B}$ pairs. The collider detectors at Fermilab, D0 and particularly CDF, have shown the utility of $B$ studies in 1.8 TeV $\bar{p}p$ collisions. Under construction are experiments at HERA-B, PEP-II, and KEK-B, the first to study fixed-target $b$ production by 800 GeV protons and the latter two to study asymmetric $e^+e^-$ collisions at the $\Upsilon(4S)$. In the farther future lie projects at LHC-B and possibly Fermilab.

We have already mentioned the importance of $B^0 - \bar{B}^0$ mixing as a validation of the CKM description of couplings. In this section we discuss some other aspects of $B$ decays crucial to testing the CKM picture.

Decays to CP eigenstates such as $B^0 \rightarrow J/\psi K_S$, $K^0\rightarrow J/\psi K$, and $B^+ \rightarrow J/\psi K^+$ form the core of the program for discovering CP violation in the $B$ system. A key feature of such studies is the ability to distinguish an initially produced $B^0$ from a $B^0$. Progress has been made by the CDF Collaboration in its study of methods for “tagging” the flavor of a produced $B$.

The strange $B$ mesons $B_s \equiv b\bar{s}$ and $\bar{B}_s \equiv b\bar{b}$ are expected to mix with one another with a large amplitude, such that $\Delta m/\Gamma \gg 1$. The mass eigenstates are expected to be approximate eigenstates of CP: $CP B_S^{(\pm)} = \pm 1$. We shall discuss one method for separating such eigenstates on the basis of angular distributions in $B_s^{(\pm)} \rightarrow J/\psi \phi \rightarrow e^+e^- K^+ K^-$. If $\Delta m$ is large for strange $B$’s, so is $\Delta \Gamma$, with the CP-even state expected to have a 10 or 20% more rapid decay rate than the CP-odd one.

Some progress in understanding the role of “penguin” diagrams, which give rise to induced $b \rightarrow d$ and $b \rightarrow s$ transitions, has been made in recent years. We shall discuss penguin amplitudes with particular reference to their role in decays of $B$ mesons to pairs of light mesons, such as $B \rightarrow \pi\pi$, $\pi K$, $\eta K$, etc. From these decays it is possible to learn about phases of CKM elements.

4.1. Decays to CP eigenstates

By comparing rates of decays to CP eigenstates for a state produced as $B^0$ and one produced as a $\bar{B}^0$, one can directly measure angles in the unitarity triangle of Fig. 2. Because of the interference between direct decays (e.g., $B^0 \rightarrow J/\psi K_S$) and those which proceed via mixing (e.g., $B^0 \rightarrow \bar{B}^0 \rightarrow J/\psi K_S$), these processes
are described by time-dependent functions whose difference when integrated over all time is responsible for the rate asymmetry. Thus, if we define
\[
C_f \equiv \frac{\Gamma(B_t \to f) - \Gamma(B_s \to f)}{\Gamma(B_t \to f) + \Gamma(B_s \to f)}, \tag{11}
\]
we have, in the limit of a single direct contribution to decay amplitudes,
\[
A(J/\psi K_S, \pi^+\pi^-) = -\frac{x_d}{1 + x_d^2} \sin(2\beta, 2\alpha), \tag{12}
\]
where \(x_d \equiv \Delta m(B^0)/\Gamma(B^0)\). This limit is expected to be very good for \(J/\psi K_s\), but some correction for penguin contributions (to be discussed below) is probably needed for \(\pi^+\pi^-\).

To see this behavior in more detail, we note that the time-dependent partial rates for a state which is initially \(B^0\) (\(B^0\)) to decay to a final state \(f\) may be written as
\[
d\Gamma[B^0(\bar{B}^0) \to f]/dt \sim e^{-\Gamma t}[1 \mp \text{Im}\lambda_0 \sin(\Delta mt)], \tag{13}
\]
where we have neglected \(\Delta \Gamma/\Gamma\) in comparison with \(\Delta m/\Gamma\). This step is justified for \(B\)'s, though not for \(K\)'s. The final states to which both \(B^0\) and \(\bar{B}^0\) can decay are only a small fraction of those to which \(B^0\) or \(\bar{B}^0\) normally decay, so one should expect similar lifetimes for the two mass eigenstates. Integration of this equation gives
\[
C_f = \frac{-x_d}{1 + x_d^2} \text{Im}\lambda_0(f) \tag{13}
\]
for the total asymmetry. For the final states mentioned, \(\lambda_0(J/\psi K_S) = -e^{-2\beta}\) and \(\lambda_0(\pi^+\pi^-) = e^{2\alpha}\). The extra minus sign in the first relation is due to the odd CP of the \(J/\psi K_S\) final state.

The asymmetry (13) is suppressed both when \(\Delta m/\Gamma\) is very small and when it is very large (e.g., as is expected for \(B_s\)). For \(B_s\), in order to see an asymmetry, one must not integrate with respect to time. Experiments planned with detection of \(B_s\) as their focus will require precise vertex detection to measure mixing as a function of proper time. For \(B^0\), on the other hand, the value of \(x/(1 + x^2)\) for \(x = 0.7\) is 0.47, very close to its maximum possible value of 1/2 for \(x = 1\).

When more than one eigenchannel contributes to a decay, terms of the form \(\cos(\Delta mt)\) as well as \(\sin(\Delta mt)\) can appear. These complicate the analysis, but information can be obtained from them on the relative contributions of various channels to decays.

4.2. Neutral \(B\) flavor tagging

In searching for rate asymmetries in decays to CP eigenstates of neutral \(B\)'s one must know whether they were \(B^0\) or \(\bar{B}^0\) at the time of production, since the final state does not tell us this. Several methods are available for “tagging” the flavor of the produced \(B\).

1. In uncorrelated \(b\bar{b}\) production, as occurs in hadronic or high-energy \(e^+e^-\) collisions, one can identify the flavor of a neutral meson (e.g., a \(B^0\) meson containing a \(b\) quark) by means of the flavor of the hadrons produced in association: in this case \(B^0, B^+, B_s, \bar{A}_s\), etc. The semileptonic decays of the quarks in these hadrons will lead to a lepton (typically only \(e\) or \(\mu\) are useful) whose sign “tags” the flavor of the opposite-side hadron. Charged kaons may also have some utility in flavor tagging, through the chain \(b \to c \to s\).

2. Just above threshold in \(e^+e^-\) annihilations, a \(B^0\bar{B}^0\) pair is produced in a state of odd \(C\), so the decay products of the \(B^0\) and \(\bar{B}^0\) are highly correlated. If the initial \(B^0\) decays at time \(t\) and the initial \(\bar{B}^0\) decays at \(\bar{t}\), the decay asymmetry depends on \(\sin \Delta m(t-\bar{t})\), and hence is odd in \(t-\bar{t}\). When integrated over \(t\) and \(\bar{t}\), the asymmetry thus vanishes! Consequently, one needs to measure the time-dependence of the individual decays, or at least to be sensitive to the sign of the time difference. This requirement has spawned the asymmetric “B factories” now under construction at SLAC and KEK, where the Lorentz boost of the center-of-mass spreads out the decays so they can be resolved from one another. At the symmetric CESR machine, the necessary spatial resolution may be achievable despite the short decay path of the neutral \(B\)'s (only 30 \(\mu\)m!) using silicon vertex detectors and very flat beams.

3. A third method for tagging the flavor of neutral \(B\)'s at the time of production is to use the fact that a charged pion of a given sign is most likely to be associated with a given flavor of neutral \(B\). Thus, when a \(b\) quark fragments into a \(B^0\) containing a \(d\) quark, a \(d\) quark is available nearby.
in phase space to be incorporated into a $\pi^-$. As a result, one will expect a $B^0$ to be accompanied more often by a $\pi^-$ than by a $\pi^+$. This same correlation is expected in resonance production: $B^0$ can resonate with $\pi^-$ to form a non-exotic ($q\bar{q}$) state $b\bar{u}$, but not with a $\pi^+$. A similar argument favors $B^0\pi^+$ over $B^0\pi^-$ combinations.

Several LEP groups have seen the expected correlations \cite{69}. The CDF Collaboration has now used this correlation to provide an independent measurement of $\Delta m$ in $B^0 - \bar{B}^0$ oscillations \cite{19}. Together with leptonic flavor tagging (the method mentioned in paragraph 1 above), this method shows promise for identifying the flavor of at least several percent of all hadronically produced neutral $B$’s, thus opening the possibility for observing a CP-violating rate asymmetry in $B^0$ or $\bar{B}^0 \to J/\psi K_S$ in the next CDF run.

\subsection*{4.3. Mixing of strange $B$’s}

The mixing between strange $B$’s due to box diagrams is considerably enhanced relative to that between nonstrange $B$’s:

$$\frac{\Delta m_s}{\Delta m_d} = \frac{f_{B_s}^2 B_{B_s}}{f_{B}^2 B_{B}} \left[ \frac{V_{ts}}{V_{td}} \right]^2 \simeq 17 - 52 \ , \quad (14)$$

where we have taken the expected ranges of decay constant and CKM element ratios, and $\Delta m_s$ refers to mixing between the $B_s = b\bar{s}$ and $\bar{B}_s = b\bar{s}$. Alternatively, we may retrace the evaluation of $\Delta m$ for nonstrange $B$’s, replacing the appropriate quantities in Eq. (3), to derive an analogous expression for $\Delta m_s$, which we then evaluate directly. For $V_{ts} = 0.040 \pm 0.004$, $m_{B_s} = 5.37$ GeV/$c^2$, $f_{B_s}/f_B = 225$ MeV, $\eta_{B_s} = 0.6 \pm 0.1$, and $\tau_{B_s} = 1/\Gamma_s = 1.55 \pm 0.10$ ps, we find $\Delta m_s/\Gamma_s = 22 \pm 6$, with an additional 40% error associated with $f_{B_s}^2 B_{B_s}$. This result implies many particle-antiparticle oscillations in a decay lifetime, requiring good vertex resolution and highly time-dilated $B_s$’s for a measurement. The present experimental bound $\Delta m_s > 9.2$ ps$^{-1}$ based on combining ALEPH and DELPHI results \cite{59} begins to restrict the parameter space in an interesting manner.

The large value of $\Delta m_s$ entails a value of $\Delta \Gamma_s$ between mass eigenstates of strange $B$’s which may be detectable. After all, the short-lived and long-lived neutral kaons differ in lifetime by a factor of 600. Strong interactions and the presence of key channels (e.g., $\pi\pi$) are a crucial effect in strange particle (e.g., $K^0$ and $\bar{K}^0$) decays. While the $b$ quark decays as if it is almost free, so that strong interactions are much less important, a corresponding difference in lifetimes for strange $B$’s of the order of 10 – 20% is not unlikely \cite{40}.

In the ratio $\Delta m_s/\Delta \Gamma_s$, uncertainties associated with the meson decay constants cancel, and in lowest order (before QCD corrections) one finds \cite{15} $\Delta m_s/\Delta \Gamma_s \simeq O(-[1/\pi](m_{B_s}^2/m_{B_s}^2)) \simeq -200$. The heavier state is expected to be the longer-lived one, as in the neutral kaon system. The top quark does not contribute to the width difference associated with the imaginary part of the box graphs, since no $t\bar{t}$ pairs are produced in $B_s$ decays.

Aside from small CP-violating effects, the mass eigenstates of strange $B$’s correspond to those $B_s^{(\pm)}$ of even and odd CP. The decay of a $B_s$ meson via the quark subprocess $b(\bar{s}) \to c\bar{c}s(\bar{s})$ gives rise to predominantly CP-even final states \cite{22}, so the CP-even eigenstate should have a greater decay rate. One calculation \cite{60} gives

$$\frac{\Gamma(B_s^{(\pm)}) - \Gamma(B_s^{(-)})}{\Gamma} \simeq 0.18 \left( \frac{f_{B_s}^2}{200 \text{ MeV}} \right)^2 , \quad (15)$$

while a more recent estimate \cite{51} is $0.16^{+0.11}_{-0.09}$. The lifetime difference between CP-even and CP-odd strange $B$’s thus can provide useful information on $f_{B_s}$, and hence indirectly on the weak interactions at short distances.

\subsection*{4.4. Isolating strange $B$ eigenstates}

One way to separate strange-$B$ CP eigenstates from one another \cite{60} is to study angular distributions in $B_s \to J/\psi \phi \to e^+e^-K^+K^-$ (or $\mu^+\mu^-K^+K^-$). The $J/\psi$ and $\phi$ are both spin-1 particles and hence can be produced in states of orbital angular momenta $L = 0$, 1, and 2 from the spinless $B_s$ decay. Suitably normalized in states of orbital angular momenta $L = 0, 1$, and 2 amplitudes $S$, $P$, $D$ can be defined such that $|S|^2 + |P|^2 + |D|^2 = 1$. $L = 1$ corresponds to $P = CP = -$, while $L = 0$ or 2 corresponds to $P = CP = +$. A simple transversality analysis \cite{63} permits one to separate the two cases.
In the $J/\psi$ rest frame, let the $x$ axis be defined by the direction of the $\phi$, the $x - y$ plane be defined by the kaons which are its decay products, and the $z$ axis be the normal to that plane. Let the $e^+$ (or $\mu^+$) make an angle $\theta$ with the $z$ axis. Then CP-even final states give rise to an angular distribution $1 + \cos^2 \theta$, while the CP-odd state gives rise to $\sin^2 \theta$. When both CP eigenstates are present in $B_s \to J/\psi \phi$, one will see a gradual increase of the $\sin^2 \theta$ component relative to the $1 + \cos^2 \theta$ component. More likely (if predictions are correct), the CP-even state will dominate, so that one will measure mainly the lifetime of this eigenstate when following the time-dependence of the decay. The average decay rate $\Gamma = (\Gamma_+ + \Gamma_-)/2$ (the subscripts denote CP eigenvalues) is measured in flavor-tagged decays of $B_s = \bar{b}s \to \bar{c} + \ldots$ or $B_s = b\bar{s} \to c + \ldots$.

A recent analysis of $B \to J/\psi K^*$ has bearing on the $B_s \to J/\psi \phi$ partial-wave structure. The two processes are related by flavor SU(3), involving a substitution $s \leftrightarrow d$ of the spectator quark. Thus, one expects the same partial waves in the two decays. The CLEO Collaboration has studied 146 $B \to J/\psi K^*$ decays in $3.36 \times 10^6 BB$ pairs produced at the Cornell Electron Storage Ring (CESR). The results for this process are $|P|^2 = 0.21 \pm 0.14$ from a fit to the transversity angle, and $|P|^2 = 0.16 \pm 0.08 \pm 0.04$ from a fit to the full angular distribution. This implies [via flavor SU(3)] that $B_s \to J/\psi \phi$ is dominated by the CP-even final state, and thus the lifetime in this state measures approximately $\tau(B_s^{\pm \mp})$. If any evidence for non-zero $|P|^2$ can be gathered in $B \to J/\psi K^*$, then $B_s \to J/\psi \phi$ should exhibit the time-variation in the transversity-angle distribution mentioned above.

### 4.5. Processes dominated by penguin diagrams

Although the unitarity of the CKM matrix implies flavor conservation for charge-preserving electroweak interactions in lowest order, we have seen that loop diagrams can induce flavor-changing charge-preserving interactions in higher order. Another example of this phenomenon is provided by the “penguin” diagram illustrated in Fig. 4. Although the penguin’s “leg” is a gluon in this illustration, it can also be a photon or $Z$. When the external quarks $x$ and $y$ have charge $-1/3$, the intermediate quarks have charge $2/3$ and can include the top quark. Because of the top quark’s large mass, such penguin diagrams can be very important.

An example of a predicted penguin effect in $s \to d$ transitions is a phase arising in the decays of neutral kaons to $\pi \pi$. This phase can lead to a “direct” contribution to the ratios for CP-violating and CP-conserving decays, in addition to that provided by the mixing parameter $\epsilon$ measured earlier.

One may define (see [40] for details)

$$
\eta_{+-} = \frac{A(K_L \to \pi^+ \pi^-)}{A(K_S \to \pi^+ \pi^-)}; \quad \eta_{00} = \frac{A(K_L \to \pi^0 \pi^0)}{A(K_S \to \pi^0 \pi^0)};
$$

the effect of “direct” decays then shows up in a parameter $\epsilon'$ which causes $\eta_{+-}$ and $\eta_{00}$ to differ from one another:

$$
\eta_{+-} = \epsilon + \epsilon' \quad ; \quad \eta_{00} = \epsilon - 2\epsilon'
$$

(16)

Since $\epsilon'$ and $\epsilon$ are expected to have approximately the same phase (see, e.g., [53]), one expects $|\eta_{+-}| \approx |\epsilon||1 + \Re(\epsilon')|$, $|\eta_{00}| \approx |\epsilon||1 - 2\Re(\epsilon')|$. 

![Figure 4. “Penguin” diagram describing transition of a quark $x$ to another quark $y$ with the same charge. The intermediate quarks have charge differing from $Q(x) = Q(y)$ by one unit. Here $q = (u, d, s)$.](image)
and hence
\[ \frac{\Gamma(K_L \to 2\pi^0)}{\Gamma(K_S \to 2\pi^0)} = 1 - 6 \text{Re} \frac{\epsilon'}{\epsilon}. \]

Present expectations \[66]\) are that \(\epsilon'/\epsilon\) could be a few parts in \(10^4\) (but in any case \(\epsilon'/\epsilon \leq 10^{-3}\)), requiring the above ratio of ratios to be measured to about one part in \(10^3\). Experiments now in progress at Fermilab and CERN should have the required sensitivity. The previous results of these experiments are:

E731 \[67\] : \(\text{Re}(\epsilon'/\epsilon) = (7.4 \pm 6.0) \times 10^{-4}\), \(17\)

NA31 \[68\] : \(\text{Re}(\epsilon'/\epsilon) = (23.0 \pm 6.5) \times 10^{-4}\). \(18\)

Because of the cancelling effects of gluonic and “electroweak” penguins (in which the curly line in Fig. 4 is a photon or \(Z\)), the actual magnitude of \(\epsilon'/\epsilon\) is difficult to estimate, so that one’s best hope is for a non-zero value within the rather large theoretical range, thereby disproving the superweak model \[44\] of CP violation.

The contribution of the (gluonic) penguin diagram to the effective weak Hamiltonian may be written (for \(x, y\) equal to quarks of charge \(-1/3\))

\[ \mathcal{H}_{\text{penguin}}^{\text{W}} \simeq \frac{G_F \alpha_s}{\sqrt{2} \pi} \left[ \xi_x \ln \frac{m_u^2}{m_d^2} + \xi_t \ln \frac{m_t^2}{m_d^2} \right] \times \left[ (\bar{u}_L \gamma^\mu \lambda^a x_L) \bar{u}_d \gamma_\mu \lambda^b d + \ldots + \text{H.c.} \right], \]

where \(\xi_x \equiv V_{ts}^* V_{tb}\), and \(\lambda^a\) are color SU(3) matrices \([a = (1, \ldots, 8)]\) normalized so that \(\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}\). The top quark is dominant in the flavor-changing processes \(b \to d\) and \(b \to s\) (with corrections due to charm which can be important in some cases \[24\]), while the charmed quark dominates the \(s \to d\) process. (The top quark plays a key role, however, in the electroweak penguin contribution to this process \[40\]) One may imitate the effect of an infrared cutoff for the gluonic penguin graph by using a constituent-quark mass \(m_u \sim 0.3\, \text{GeV}/e^2\).

One can estimate the effect of the \(b \to sq\) penguin graph; one finds it is comparable to that of the \(b \to ud\) “tree” contribution \((G_F/\sqrt{2}) V_{ub} V_{ud}^* [\bar{u}_d \gamma_\mu (1-\gamma_5) d]\delta^a_\mu m^a_\mu (1-\gamma_5) u\). The \(b \to dq\) penguin contribution and the \(b \to us\) tree contribution are both expected to be suppressed by approximately one power of the Wolfenstein parameter \(\lambda \sim 0.2\), as one can see by comparing CKM elements.

### 4.6 Decays of \(B\) mesons to pairs of light mesons

One testing ground for the magnitude of penguin contributions occurs in the decays of \(B\) mesons to pairs of light mesons. Thus, \(B^0 \to \pi^+\pi^-\) is expected to be dominated by the tree amplitude, \(B^0 \to K^+\pi^-\) is expected to be dominated by the penguin amplitude, and the rates of the two processes should be similar.

In Fig. 5 we show a contour plot of the significance of detection by the CLEO Collaboration \[71\] of the decays \(B^0 \to \pi^+\pi^-\) and \(B^0 \to K^+\pi^-\). Evidence exists for a combination of \(B^0 \to K^+\pi^-\) and \(\pi^+\pi^-\) decays, generically known as \(B^0 \to h^+\pi^-\). The most recent published result \[71\] is \(B(B^0 \to h^+\pi^-) = (1.8 \pm 0.6 \pm 0.2) \times 10^{-5}\). Although one still cannot conclude that either decay mode is nonzero at the 3\(\sigma\) level, the most likely solution is roughly equal branching ratios (i.e., about \(10^{-5}\)) for each mode. Only upper limits exist for other modes of two pseudoscalars \[72\] but these are consistent with predictions \[73\].

Penguin diagrams play a number of roles in \(B\) decays \[35\]. We enumerate several of them.

1. The process \(B^+ \to K^0\pi^+\) is expected to be almost completely due to the penguin graph. By comparison with \(B^0 \to K^+\pi^-\), where the penguin graph is expected to be the main contribution, one expects \(B(B^+ \to K^0\pi^+) \approx 10^{-5}\). The weak phase of the process (which changes sign under charge-conjugation) thus is expected to be \(\text{Arg}(V_{ub}^* V_{ts}) = \pi\), so that the charge-conjugate process has the same weak phase. As a result, one can separate strong final-state interaction phases from weak phases and obtain estimates of quantities like the angle \(\gamma = \text{Arg}(V_{ub})\) in Fig. 2 by comparing rates for \(B^+ \to (K^0\pi^+, K^+\pi^0, K^+\eta, K^+\eta')\) with the corresponding \(B^-\) rates \[74\]. One can also obtain this information by measuring the time-dependence in \(B^0(B^0) \to \pi^+\pi^-\) and the rates for \(B^0 \to K^+\pi^-\), \(B^+ \to K^0\pi^+\), and the charge-conjugate processes \[73\]. The weak phases of the major amplitudes...
Figure 5. Significance of detection of $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$ by the CLEO Collaboration. The solid curves are the $n\sigma$ contours and the dotted curve is the 1.28$\sigma$ contour.

Table 3

| $|\Delta S|$ | “Tree” elements | “Penguin” elements |
|---|---|---|
| 0 | $V_{ub}V_{ud}$, $\gamma$ | $V_{tb}V_{td}$, $-\beta$ |
| 1 | $V_{ub}V_{us}$, $\gamma$ | $V_{tb}V_{ts}$, $\pi$ |

Phases of amplitudes contributing to decays of $B$ mesons to $\pi\pi$ and $K\pi$. Here $\Delta S$ refers to the change of strangeness in the process.

contributing to these decays are summarized in Table 3. The relative weak phase of tree and penguin amplitudes for strangeness-preserving decays is $\gamma + \beta = \pi - \alpha$ (assuming the unitarity triangle to be valid), while the corresponding relative phase for strangeness-changing decays (aside from a sign) is just $\gamma$. As a result, one can measure both $\alpha$ and $\gamma$.

2. A number of processes (in addition to the decay $B^+ \to K^0\pi^+$ mentioned above) are dominated by penguin graphs. By comparing the rates for strangeness-preserving and strangeness-changing processes, one can measure the ratio $|V_{td}/V_{ts}|$ [76]. Examples of useful ratios are $B(B^+ \to K^{-}\phi K^+)/B(B^+ \to K^{*0}\phi K^+)$ and $B(B^+ \to K^{*0}K^+)/B(B^+ \to K^{*0}\phi K^+)$. The solid curves are the $n\sigma$ contours and the dotted curve is the 1.28$\sigma$ contour.

3. We mentioned that the time-integrated rate asymmetry in $B \to \pi^+\pi^-$ could provide information on the angle $\alpha$ of the unitarity triangle. The most direct test is based on the assumption that the tree process $b \to \bar{u}ud$ is the only direct contribution to this decay. However, "penguin pollution" (due to the $b \to d$ transition) makes the analysis less straightforward, even though the penguin amplitude is expected to be only about 0.2 of the tree amplitude. Ways to circumvent this difficulty include the detailed study of the isospin structure of the $\pi\pi$ final state [83], and the use of flavor SU(3) to estimate penguin effects using $B \to K\pi$, where they are expected to be dominant [72,73,74].

4.7. One determination of $\alpha$ and $\gamma$

As an example of a way in which rate and time-dependence measurements can shed light on both strong and weak phases, one can mention the results of [76] (noted above) in more detail.

(1) The decays $B^0$ or $B^0 \to \pi^+\pi^-$ are governed by strangeness-preserving tree and penguin amplitudes with magnitudes $|T|$ and $|P|$, relative weak phase $\alpha$, and relative strong phase $\delta$. By performing time-dependent studies one can measure three quantities: the magnitude $|A_{\pi\pi}|^2$ of the direct $B^0 \to \pi^+\pi^-$ amplitude $A_{\pi\pi}$, the magnitude $|A'_{\pi\pi}|^2$ of the direct $B^0 \to \pi^+\pi^-$ amplitude $A'_{\pi\pi}$, and an interference term $\text{Im}(e^{i\delta}A_{\pi\pi}A_{\pi\pi}^*)$.

(2) The decays $B^0 \to K^+\pi^-$ and $B^0 \to K^-\pi^+$ are governed by strangeness-changing tree and penguin amplitudes with magnitudes $|T'|$ and $|P'|$, relative weak phase $\gamma$, and relative strong phase $\delta$ [assuming flavor SU(3)]. One relates $|T'|$ to $|T|$ using flavor SU(3) but makes no such assumption about $|P'|$. Two rate measurements are possible.

(3) The decays $B^\pm \to K\pi\pi^\pm$ are dominated by the penguin diagram and thus should have the same rate. They provide information on $|P'|$. One thus has 6 measurements with which to determine the 6 parameters $|T|$, $|P|$, $\alpha$, $\delta$, $|P'|$, and $\gamma$. In general one can learn all 6, with some interesting discrete ambiguities. Degeneracies re-
result in some cases, e.g., if \( \Gamma(B^0 \to K^+\pi^-) = \Gamma(B^0 \to K^-\pi^+) \). In that case one expects also \( |A_{\pi\pi}|^2 = |\bar{A}_{\pi\pi}|^2 \) and one has to assume something else, such as an SU(3) relation between \(|P|\) and \(|P'|\), a constraint \( \gamma \simeq \pi - 1.2\alpha \) satisfied approximately by the allowed region in Fig. 3, or both. Interesting measurements can begin to be made with about 100 \( \pi^+\pi^- \) decays but the full power of the method requires about 100 times more data than that.

5. Amplitude triangles and quadrangles

A number of constructions of amplitude triangles and quadrangles allow one to determine both strong and weak phases. Measurements based on the rates for \( B^\pm \to K^\pm\pi^0, K_S\pi^\pm, K^\pm\eta, \) and \( K^\pm\eta' \) are one example; earlier literature may be traced from this work. Other discussions of amplitude triangles and quadrangles and of decays involving \( \eta \) and \( \eta' \) are available [8].

6. REMARKS ON BARYOGENESIS

The following discussion is an update of one given a couple of years earlier [7]. Shortly after CP violation was discovered, Sakharov [80] proposed that it was one of three ingredients of any theory which sought to explain the preponderance of baryons over antibaryons in our Universe: (1) violation of C and CP; (2) violation of baryon number, and (3) a period in which the Universe was out of thermal equilibrium.

It was pointed out by ’t Hooft [3] that the electroweak theory contains an anomaly as a result of nonperturbative effects which conserve \( B - L \) but violate \( B + L \). If a theory leads to \( B - L = 0 \) but \( B + L \neq 0 \) at some primordial temperature \( T \), the anomaly can wipe out any \( B + L \) as \( T \) sinks below the electroweak scale [4]. Thus, many models of baryogenesis are unsuitable in practice.

Shaposhnikov and Farrar [5] have proposed that the CP violation in the CKM sector can be communicated to a baryon asymmetry directly at the electroweak scale. A recent analysis of the electroweak phase transition [6] now excludes this possibility. One proposed alternative is the generation of nonzero \( B - L \) at a high temperature, e.g., through the generation of nonzero lepton number \( L \), which is then reprocessed into nonzero baryon number by the ’t Hooft anomaly mechanism [7]. The existence of a baryon asymmetry, when combined with information on neutrinos, could provide a window to a new scale of particle physics. Large Majorana masses acquired by right-handed neutrinos would change lepton number by two units and thus would be ideal for generating a lepton asymmetry if Sakharov’s other two conditions are met.

One question in this scenario, besides the form of CP violation at the lepton-number-violating scale, is how this CP violation gets communicated to the lower mass scale at which we see CKM phases. In two recent models [86,87] this occurs through higher-dimension operators which imitate the effect of Higgs boson couplings to quarks and leptons.

7. BEYOND THE STANDARD MODEL

Many models manifest their non-standard effects first in particle-antiparticle mixing [89–90]. A number of tests for these effects make use of decays of \( B \) mesons. For example, if \( B^0 \) and \( B_s \) mixing amplitudes are rotated by respective phases \( \theta_d \) and \( \theta_s \) from their standard-model values, the rate asymmetries in \( B^0 \to J/\psi K_S \) and \( B_s \to J/\psi K \) will probe, respectively, \(- \sin(2\beta - \theta_d)\) and \( \sin(2\delta + \theta_s) \), where \( \delta \equiv \lambda^2 \eta \). Systematic prescriptions for identifying the effects of the new physics using a series of measurements are available [88–90].

Neutron and electron electric dipole moments are sensitive to effects of 2-Higgs boson models and supersymmetry. Limits on the neutron moment already provide useful constraints, while limits on the electron moment are close. For an overview of the field see [1].

A lively discussion of what kaon physics can tell us about violation of CPT (indeed, of quantum mechanics itself) is in progress [2]. It is probably time to improve the old test of CPT based on comparing 2 Re \( \epsilon \) (as measured in the rate asymmetry for \( K_L \to \pi^{\pm}l^\pm\nu \)) with what one calculates from the measured values of \( \epsilon \) and Arg \( \epsilon \).

The simplest SU(5) model [12] for unifying the
strong and electroweak interactions predicts too small an electroweak mixing angle and too short a proton lifetime. Proposed remedies include supersymmetry and extended gauge structures such as SO(10) and E\(_6\). Thus, for example, if we were to encounter new fermions in particle searches at the highest hadron and e\(^+\)e\(^-\) collider energies, we would then like to know whether they are superpartners of gauge bosons, exotic leptons, or something else.

Finally, I should mention a topic which, in the words of one referee, is “not new, though not popular”: the compositeness of quarks and leptons. The pattern of Fig. 1 certainly looks like a level structure of a composite system; one success of such a description would be its ability to predict not only masses but magnitudes and phases of CKM matrix elements. Exploratory steps in this direction have been taken \[94\].

8. CONCLUSIONS

I leave you with the following exercise in pattern recognition: What familiar pattern do you see in Fig. 6? One can re-express the pattern as shown in Fig. 7; perhaps it suggests something at this point. Finally, when one adds variety to the pattern, it becomes recognizable as the periodic table of the elements (Fig. 8).

The variety of the pattern of the elementary particles laid the foundations of the quark model and our understanding of the fundamental strong interactions. Will there be a similar advance for quarks and leptons? The pattern of quarks and leptons has been quite regular up to now, as if the periodic table of the elements consisted only of rows of equal length and were missing hydrogen, helium, the transition metals, the lanthanides, and the actinides. Whether one discovers superpartners of the known states, or variety such as predicted in extended gauge structures, the new states could help us to make sense of the pattern of the masses of the more familiar ones.

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