Modeling and prediction of gear flank twist for generating gear grinding

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Abstract
Gear flank twist has a significant effect on the load-carrying capacity. However, most prediction models of gear flank twist cannot be applicable for gears with profile modification. A new method for predicting gear flank twist for the generating grinding process is developed in this paper, taking into consideration the effect of lead modifications, profile modifications, and position errors of CNC machine tools on gear flank twist. And the influences of four gear parameters on gear flank twist were studied using orthogonal experiments, which prove that the change of module has the greatest effect on gear flank twist. And the maximal profile modification value has minimal effect on gear flank twist. The experimental results proved that the model can accurately predict the gear flank twist. Besides, this paper described a gear parameter optimization study to reduce gear flank twist based on nine well-planned orthogonal experiments.

Keywords Gear flank twist · Gear generating grinding · Mathematical model · Gear modification

1 Introduction

Gears are widely used in ships, automobiles, and other fields. Gear generating grinding is important in finishing manufacturing processes for gears [1]. In order to reduce gear noise and mesh impact, it is necessary to modify gear profile and gear lead slope.

Gear modification includes profile modification and lead modification [2]. Gear profile modification can be realized by dressing the grinding wheel into a specific shape [3]. Lead modification can be realized by changing the center distance between the gear and the grinding wheel during axial feeding [3]. The gear flank twist occurs in this processing. Gear flank twist has a negative effect on gear, reduces transmission accuracy, increases mesh impact, and thus reduces gear life. Thus, it is important to eliminate gear flank twist.

Various methods have been proposed to reduce the flank twist and can be divided into three groups. Some research proposed gear processing parameters’ modification method, some researchers proposed machining tool reshaping methods, and others adopted both methods above. Tian et al. [4] proposed a method by establishing a coordinate transformation matrix between gear blank and tool coordinate system, obtaining the relationship between coordinates of gear flank and master–slave axis movement amount and then reducing gear flank twist by compensating for master–slave axis movement amount. Tran et al. [5] proposed an additional rotation angle for the gear blank during its manufacturing process. A nonlinear function was proposed and supplemented to this additional rotation angle of work gear. The twist of the helical tooth flank is reduced greatly. Shih et al. [6] proposed a tooth flank modification method in the five-axis gear profile grinding machine. Each axis of the grinding machine is formulated as a polynomial, and the tooth flank can be approximated to the theoretical tooth flank by adjusting the coefficients of the polynomials based on their sensitivity. Fong et al. [7] proposed a tooth flank crowning method for helical gears, which uses a diagonal feed on a grinding machine with a variable lead grinding worm, and this method can reduce gear flank twist. Hsu et al. [8]...
proposed a methodology to reduce the tooth flank twist by applying a modified variable tooth thickness tool and having a diagonal feed without varying the center distance. Jiang et al. [9] proposed a new mathematical model for the grinding wheel of the ZC1 worm, which can reduce the tooth errors of the ZC1 worm.

In order to improve the measuring efficiency of machined gear, it is necessary to establish a model of gear surface topology. So far, various gear surface topology models have been proposed. Vedmar [10] proposed a method to calculate the roughness of the gear surface by establishing the model of the tool in arbitrary directions, obtaining the geometry of the three-dimensional surface of the manufactured gear, and comparing this surface with the surface of an ideal gear. Chen et al. [11] proposed a new method for predicting gear surface roughness with the generating grinding process by an algorithm for geometrical analysis of the grooves on the gear surface left by idea conic grains. Zhou et al. [12] proposed a new approach to modeling the tooth surface topology. So far, various gear surface topology models have been proposed. Vedmar [10] proposed a method to calculate the surface topography by the Boolean operation, and obtaining the surface topography of the grinding worm, obtaining the tooth surface topology by the Boolean operation, and obtaining the gear surface topology by comparing this surface with the surface of an ideal gear. Yan et al. and Yu et al. [13, 14] proposed a methodology to reduce the tooth flank twist by applying a modified variable tooth thickness tool and having a diagonal feed without varying the center distance. Jiang et al. [9] proposed a new mathematical model for the grinding wheel of the ZC1 worm, which can reduce the tooth errors of the ZC1 worm.

2 Gear flank twist prediction model via homogeneous transformation

2.1 Homogeneous transformation matrix between coordinate systems

Figure 1 shows the spatial meshing coordinate system of the gear and worm grinding wheel. Four spatial coordinate systems, $S_1 \{x_1, y_1, z_1\}$, $S_2 \{x_2, y_2, z_2\}$, $S \{x, y, z\}$, and $S_p \{x_p, y_p, z_p\}$, are created to describe the generation mechanism of helical gear with longitudinal and profile correction. $S_1$ and $S_2$ are rigidly connected to the worm grinding wheel and gear. The coordinate system $S$ and $S_p$ are fixed in space.

The coordinates of an arbitrary point $P$ in coordinate systems $S_m$, $S_p$, $S_1$, and $S_2$ are $(x_p, y_p, z_p)$, $(x_1, y_1, z_1)$, and $(x_2, y_2, z_2)$, respectively. When the coordinate system $S_p$ is translated to coordinate system $S_2$, the relationship between $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ of point $P$ can be represented as follows.

\[
\begin{bmatrix}
    x_2 \\
    y_2 \\
    z_2
\end{bmatrix}
= M_{21}
\begin{bmatrix}
    x_1 \\
    y_1 \\
    z_1
\end{bmatrix}
\]

(1)

\[
M_{21} =
\begin{bmatrix}
    a_{11} & a_{12} & a_{13} & a_{14} \\
    a_{21} & a_{22} & a_{23} & a_{24} \\
    a_{31} & a_{32} & a_{33} & a_{34} \\
    a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}
\]

(2)

\[
\begin{align*}
    a_{11} &= \cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2 \cos Q; \\
    a_{12} &= -\sin \phi_1 \cos \phi_2 + \cos \phi_1 \sin \phi_2 \cos Q; \\
    a_{13} &= -\sin \phi_2; \\
    a_{14} &= \cos \phi_2; \\
    a_{21} &= -\cos \phi_1 \sin \phi_2 + \sin \phi_1 \cos \phi_2 \sin Q; \\
    a_{22} &= \sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2 \sin Q; \\
    a_{23} &= -\cos \phi_1 \sin Q; \\
    a_{24} &= -a \sin \phi_2; \\
    a_{31} &= \sin \phi_1 \sin Q; \\
    a_{32} &= \cos \phi_1 \sin Q; \\
    a_{33} &= \cos Q; \\
    a_{34} &= 0; \\
    a_{41} &= 0; \\
    a_{42} &= 0; \\
    a_{43} &= 0; \\
    a_{44} &= 1; \\
\end{align*}
\]

where $Q$ is the setting angle of the wheel. $\phi_1$ is the worm grinding wheel’s rotation angle. $\phi_2$ is the work gear’s rotation angle.

2.2 Surface equation of gear

The transverse profile of the involute helical gear in coordinate system $S_2$ can be expressed as follows:

\[
\begin{bmatrix}
    x_0 \\
    y_0
\end{bmatrix}
= \begin{bmatrix}
    r_b \cos u + r_b \sin u \\
    r_b \sin u - r_b \cos u
\end{bmatrix}
\]

(3)

where $r_b$ is the radius of the base circle, and $u$ is the sum of the spread angle.
The surface equation of the involute helical gear in coordinate system $S_2$ can be expressed as follows:

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} r_b \cos(u + \theta) + r_b \sin(u + \theta) \\ r_b \sin(u + \theta) - r_b \cos(u + \theta) \\ p \theta \\ 1 \end{bmatrix}$$

(4)

$$\begin{bmatrix} n_xp \\ n_yp \\ n_zp \\ 1 \end{bmatrix} = \begin{bmatrix} p[r_b u + H(u)] \sin(u + \theta + \varphi_1) - p H'(u) \cos(u + \theta + \varphi_1) \\ -p[r_b u + H(u)] \cos(u + \theta + \varphi_1) - p H'(u) \sin(u + \theta + \varphi_1) \\ [r_b u + H(u)] [r_b + H'(u)] \end{bmatrix}$$

(5)

Assuming profile modification curve equation is as follows

$$H(u) = \lambda \left( \frac{r_b}{2} u^2 - \frac{l_2}{2} \right) + h_2$$

(6)

where $l_2$ is the involute rolling length, and $h_2$ is the maximal modification value of the gear tooth profile. $\lambda = \frac{-d_2}{\varepsilon}$. 

Because modified curve whose normal distance to involute is $H$, gear profile-modified flank equation in coordinate system $S_p$ can be obtained as follows:

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} r_b \cos(u + \theta + \varphi_2) + (r_b u + H) \sin(u + \theta + \varphi_2) \\ r_b \sin(u + \theta + \varphi_2) - (r_b u + H) \cos(u + \theta + \varphi_2) \\ p \theta \\ 1 \end{bmatrix}$$

(7)

The normal vector of crowning lead modified flank can be obtained as follows:

$$\begin{bmatrix} n_xp \\ n_yp \\ n_zp \\ 1 \end{bmatrix} = \begin{bmatrix} p[r_b u + H(u)] \sin(u + \theta + \varphi_1) - p H'(u) \cos(u + \theta + \varphi_1) \\ -p[r_b u + H(u)] \cos(u + \theta + \varphi_1) - p H'(u) \sin(u + \theta + \varphi_1) \\ [r_b u + H(u)] [r_b + H'(u)] \end{bmatrix}$$

(8)

2.3 Mesh track on the gear flank

According to the theory of gearing [15], the necessary conditions of envelope existence may be given by equations as follows:

$$r_2 = r_1 + ai$$

(9)

$$n_1 + n_2 = 0$$

(10)

$$n \cdot V^{(12)} = 0$$

(11)

where $r_1$ is the coordinate of worm grinding wheel flank in coordinate system $S_p$, $r_2$ is the coordinate of gear flank in coordinate system $S_p$, $n_1$ is the normal vector of worm grinding wheel flank in coordinate system $S_p$, $n_2$ is the normal vector of gear flank in coordinate system $S_p$, and $V^{(12)}$ is the velocity vector at the contact point.

Equations (7–11) yield Eq. (12).

$$-n_yp \left[ (1 - i_{21} \cos \theta) y_p + i_{21} z_p \sin \theta \right] + n_yp \left[ (1 - i_{21} \cos \theta) x_p - ai_{21} \cos \theta \right] + n_yp i_{21} (x_p + a) \sin \theta = 0$$

(12)

By simplifying Eq. (12), the relationship among $n$, $\theta$, and $\varphi_1$ can be obtained as follows:

$$\varepsilon = \arccos \left\{ \frac{r_b u + H(u) [r_b + H'(u)] i_{21} \sin \theta}{p (1 - i_{21} \cos \theta) \sqrt{H'(u)^2 + [r_b u + H(u)]^2}} \right\} + \arctan \left\{ \frac{H'(u)}{r_b u + H(u)} \right\}$$

(13)

where $\varepsilon = u + \theta + \varphi_1$.

Equations (12) and (13) yield Eq. (14).
Equations (7), (13), and (14) yield a mathematical model of mesh track on the gear flank.

Because of the existence of the gear tooth profile modifications, the mesh track on the gear flank is not a straight line with slight slope alterations.

2.4 Gear flank twist model

Gear flank twist is the phenomenon that transverse profile of gear is twisted along an axial direction. Gear flank twist is a principal error influenced by profile modification. Figure 3 shows the gear flank twist, where I and II refer to the gear end face. According to BS ISO 21771: 2007, gear flank twist can be calculated as follows:

\[
T = \left| C_{\text{Haf}} - C_{\text{Had}} \right| \tag{16}
\]

where \(C_{\text{Haf}}\) is the profile slope deviation on gear end plane I. \(C_{\text{Had}}\) is the profile slope deviation on gear end plane II.

When the grinding wheel’s geometric center is in the middle of the breadth of the tooth, there is an axial distance \(S_v\) between the reference point and the midpoint of the breadth of the tooth. The reference point is the intersection of the meshing track and the reference circle.

Assuming lead modification curve equation is as follows:

\[
G(z) = \frac{g_2}{l_4} (z + S_v)^2 - g_2 \tag{17}
\]

where \(l_4\) is half of the breadth of the tooth, and \(g_2\) is the maximal gear lead modification value.

Figure 4 shows the relationship between the mesh track on the gear flank and the gear flank twist. The figure shows the unfolded drawing of the gear flank. The right figure shows the lead modification curve.

\(K_1\) is the length of the mesh track on the gear flank in the axial direction from the involute starting point reference circle. \(K_2\) is the length of the mesh track on the gear flank in the axial direction from the reference circle to the involute end point. According to Yan [13], when the center of the grinding wheel is in the middle of the tooth width, there is a height difference \(S_v\) between the intersection of the mesh track and the reference circle and the midpoint of the tooth width. \(B\) is the facewidth.
3 Influence of machine tool position error on tooth flank twist

The machine tool position errors have a great influence on gear flank twist. So, it is necessary to study the influence of machine tool position errors on gear flank twist.

The machine tool position errors can be simplified into three position errors: relative position errors between worm grinding wheel and gear in $X_p$ direction, $Y_p$ direction, and $Z_p$ direction. The $X_p$, $Y_p$, and $Z_p$ directions can be seen in Fig. 1.

The relative position error in the $X_p$ direction is equivalent to the additional lead modification value, and the influence of it on the lead modification curve can be expressed as follows.

$$G(z)' = G(z) + \Delta G$$  \hspace{1cm} (20)

$$\Delta G = \Delta x \sin \alpha$$  \hspace{1cm} (21)

where $\alpha$ is the normal pressure angle of gear, $\Delta x$ is the relative position error in the $X_p$ direction, and $\Delta G$ is the additional lead modification value. $G(z)'$ is the lead modification curve equation considering the influence of the relative position error.

The relative position error in the $Y_p$ direction is equivalent to the flee cutter of the worm grinding wheel in the axial direction, so the influence of relative position errors in the $Y_p$ direction can be ignored.

The relative position error in the $Z_p$ direction is equivalent to the translation of the tooth lead modification curve in the $Z_p$ direction, which causes the change of $S_v$. The influence of relative position error in the $Z_p$ direction on $S_v$ can be expressed as follows.

$$S_v' = S_v \pm \Delta z$$  \hspace{1cm} (22)

where $\Delta z$ is the relative position error in the $Z$ direction, and $S_v'$ is the $S_v$ value considering the influence of the relative position error in the $Z$ direction. When the tooth surface was analyzed, the relative position error is in the $z$ direction, and the $\Delta z$ is a positive one.

4 Analyses of gear flank twist by orthogonal experiment

An orthogonal experiment is a statistical tool adopted to investigate the influence of module (factor A), number of teeth (factor B), maximal profile modification value (factor C), and helix angle (factor D) on the gear flank twist and to select the optimum parameters for gear.

4.1 Experimental conditions

The orthogonal experiments are based on an orthogonal array experimental design matrix, which is shown in Table 1.

The gears were manufactured by a worm wheel gear grinding machine. The gear flank twist was calculated by the method described above. The lead crowning of the gear tooth flank was accomplished by varying the center distance between the wheel and work gear.

In order to cover all the levels in the present study, four levels of module, number of teeth, maximal profile modification value, and helix angle were employed. Other necessary gear parameters are shown in Table 2.

In total, 9 experiments were designed by the orthogonal method, which is shown in Table 3. Table 3 gives the various gear parameters for each experiment. The different units used

| Table 1 | Levels and factors of gear flank surface |
|---------|------------------------------------------|
| Level   | Module (A) | Number of teeth (B) | Maximal profile modification value (C) | Helix angle (D) |
| 1       | 4 mm       | 17                  | 4 $\mu$m                                | 14°            |
| 2       | 5 mm       | 21                  | 6 $\mu$m                                | 17°            |
| 3       | 6 mm       | 25                  | 8 $\mu$m                                | 20°            |

| Table 2 | Basic parameters                          |
|---------|-------------------------------------------|
| Conditions | Values   |
| Normal pressure angle | 19° |
| Facewidth | 46 mm |
| Tip diameter of worm grinding wheel | 280 mm |
| Maximal gear lead modification value | 35 $\mu$m |
| Wheel grinding worm's speed | 63 rpm |

| Table 3 | Orthogonal experiment design               |
|---------|-------------------------------------------|
| Experiment number | Module | Number of teeth | Maximal profile modification value | Helix angle |
| 1       | 4     | 17       | 4                                       | 14          |
| 2       | 4     | 21       | 6                                       | 17          |
| 3       | 4     | 25       | 8                                       | 20          |
| 4       | 5     | 17       | 6                                       | 20          |
| 5       | 5     | 21       | 8                                       | 14          |
| 6       | 5     | 25       | 4                                       | 17          |
| 7       | 6     | 17       | 8                                       | 17          |
| 8       | 6     | 21       | 4                                       | 20          |
| 9       | 6     | 25       | 6                                       | 14          |
here are module – mm, maximal profile modification value –µm, and helix angle – °.

### 4.2 Parameters

There are two important parameters in a range analysis: $K_{ji}$ and $R_j$. $K_{ji}$ is defined as the sum of the evaluation indexes of all levels ($i = 1, 2, 3, 4$) in each factor ($j = A, B, C, D$), and $K_{ji}$ is used to determine the optimal level and the optimal combination of factors. The optimal level for each factor could be obtained when $K_{ji}$ is the smallest.

$R_j$ is defined as the range between the maximum and minimum value of $K_{ji}$ and is used for evaluating the importance of the factors; i.e., a larger $R_j$ means greater importance of the factor. For example, take the OA9 matrix. The calculation is shown below. For the factor of A

$$K_{A1} = Y_1 + Y_2 + Y_3$$ (23)

$$K_{A2} = Y_4 + Y_5 + Y_6$$ (24)

$$K_{A3} = Y_7 + Y_8 + Y_9$$ (25)

$$\overline{K_{A1}} = \frac{K_{A1}}{3}$$ (26)

$$\overline{K_{A2}} = \frac{K_{A2}}{3}$$ (27)

$$\overline{K_{A3}} = \frac{K_{A3}}{3}$$ (28)

$$R_A = \max(\overline{K_{A1}}) - \min(\overline{K_{A1}})$$ (29)

where $K_{Ai}$ is the $K$ value of the $i$ level of the factor of A, $\overline{K_{ji}}$ is the mean value of $K_{ji}$, and $Y_i$ is the value of the result of the No. $i$ experiment. Other $K$ values of the factors can be determined by the same calculation steps.

### 5 Results and discussions

According to the OA9 matrix, nine experiments were carried out, and their product yield results were shown in Table 4.

As mentioned before, for each factor, the range value ($R_j$) indicates the significance of the factor’s effect, and a larger $R_j$ means the factor has a bigger impact on the product yield. Therefore, compared with the range values of different factors ($R_j$), the factors’ levels of significance are as follows: module (3.35) > helix angle (3.24) > number of teeth (1.90) > maximal profile modification value (0.63).

The mean values of each factor were shown in Table 5a–d. Based on the change in the mean value of each factor (Table 5a–d), it can be observed that the gear twist increased from 6.87 to 8.43 µm when the module increased from 4 to 5. And the gear twist sharply decreased from 8.43 to 5.08 µm when the module increased from 5 to 6. For the number of teeth, the gear twist increased from 7.47 to 9.37 µm, with the module increasing from 17 to 25. Helix angle clearly influenced the gear flank twist. The gear twist slightly increased from 6.93 to 10.17 µm, with the helix angle increasing from 14 to 20°. And the gear twist slightly decreased from 8.90 to 8.27 µm, with the maximal profile modification value increasing from 4 to 8 µm. After the orthogonal experiments, the optimal level for each factor was determined as follows: module 6.0 mm, number of teeth 17, maximal profile modification value 8 µm, and helix angle 14°.
The KX TWIN HS gear grinding machine was adopted as the gear processing machine tool in this paper; main movement axes of it are radial feed axis X-axis of grinding wheel, longitudinal feed axis Y-axis of tool past, axial feed axis Z-axis of tool past, rotation axis A-axis of grinding wheel, and rotation axis B-axis of gear workpiece. The machining center is shown in Fig. 5.

Two case-carburized gears, named gear A and gear B, with profile and lead modification, were manufactured. Gear A has one crown in the axial direction. Gear B has two crowns distributed symmetrically at both sides of the gear in the axial direction. The parameters of gear A and gear B are listed in Table 6.

As shown in Fig. 6, the mesh track on the two gear flanks can be calculated by Matlab. X-axis and Y-axis represent generating angle of gear flank surface and mesh track in $Z_2$ direction. The values of $S_v$, $K_1$, and $K_2$ can be obtained by analyzing the mesh track. For gear A, $K_1 = 2.275$ mm,

---

Table 5  (a) Relationship between module and gear flank twist. (b) Relationship between number of teeth and gear flank twist. Relationship between maximal profile modification value and gear flank twist. Relationship between helix angle and gear flank twist

| Module | Gear flank twist |
|--------|-----------------|
| 4      | 6.87            |
| 5      | 8.43            |
| 6      | 5.08            |

| Number of teeth | Gear flank twist |
|-----------------|-----------------|
| 4               | 7.47            |
| 5               | 8.63            |
| 6               | 9.37            |

| Maximal profile modification value | Gear flank twist |
|-----------------------------------|-----------------|
| 4                                 | 8.90            |
| 5                                 | 8.30            |
| 6                                 | 8.27            |

| Helix angle | Gear flank twist |
|-------------|-----------------|
| 4           | 6.93            |
| 5           | 8.37            |
| 6           | 10.17           |
$K_2 = 4.855$ mm, and $Sv = 1.02$ mm. For gear B, $K_1 = 3.61$ mm, $K_2 = 2.26$ mm, and $Sv = 0.89$ mm.

As shown in Fig. 7, to verify the mathematical model of gear flank twist, 6 sections were chosen to measure the profile slope deviations. Position 1 and position 2 are located on the flank face of gear A. Positions 3–6 are located on the flank face of gear B.

The measurement data are shown in Fig. 8. The comparisons between experimental profile deviations and simulated profile slope deviations are shown in Table 7. The data in Table 7 is obtained by taking the average profile slope deviations of teeth 1, 8, and 15 on the left flank.

It can be found that experimental results and simulated results are nearly the same.

By using Eq. 16, the gear flank twist can be obtained. For gear A, the simulated gear flank twist is 3.8 $\mu$m and the experimental gear flank twist is 7.6 $\mu$m. For gear B, the simulated gear flank twist is 1.9 $\mu$m and the experimental gear flank twist is 2.7 $\mu$m. The errors are caused by the wear of the worm grinding wheel and machine tool errors.

By measuring the profile deviations of chosen sections, compared them with the simulated results, the correctness of the model is proven.

| Conditions                                   | Symbols | Gear A | Gear B |
|----------------------------------------------|---------|--------|--------|
| Module                                       | $m_n$   | 4.25 mm| 4.4 mm |
| Teeth number of gear workpiece               | $Z_2$   | 21     | 17     |
| Outside diameter of worm grinding wheel      | $D$     | 280 mm | 98 mm  |
| Gear helix angle                             | $\beta$ | 17.81° | 14.85° |
| Spindle speed                                | $V$     | 63 m/s | 63 m/s |
| Normal pressure angle                        | $\alpha_n$ | 19°  | 21.5°  |
| Maximal modification value of gear tooth profile | $h_2$ | 6 $\mu$m | 6 $\mu$m |
| Maximal helix modification value of gear     | $g_2$   | 35 $\mu$m | 35 $\mu$m |
| Tooth width                                  | $B$     | 46 mm  | 99 mm  |

**Fig. 6** Mesh track on the gear flank

**Table 6** Parameters of gear A and gear B
Fig. 7 Measurement position of gear

(a) Measured positions in gear A

(b) Measured positions in gear B
Fig. 8 Measured data
7 Conclusions

A new prediction method for the gear flank twist for the generating grinding process is developed in this paper which is based on the mesh track calculation method. It takes into account the gear profile modifications, gear lead modifications, and machine tool position errors. The influence of four gear parameters on gear flank twist was studied using orthogonal experiments. The experiments showed that the gear flank twist increased as gear’s number of teeth and helix angle increased, and the gear flank twist decreased as the gear’s maximal profile modification value increased. When the gear’s module increased, the gear flank twist increased first and then decreased when the maximal profile modification value was more than 5 mm. The sort order of parameters’ levels of significance are as follows: module > helix angle > number of teeth > maximal profile modification value.

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Code availability The authors have no financial or proprietary interests in any code discussed in this article.

Declarations

Consent to participate All authors consent to participate in the study.

Consent for publication All authors consent to the publication of this article.

Competing interests The authors declare no competing interests.

Table 7 Comparisons between measured and calculated results

| Section | Simulated profile deviations (μm) | Experimental profile deviation (μm) |
|---------|----------------------------------|-----------------------------------|
| 1       | 6.30                             | 6.67                              |
| 2       | 10.10                            | 14.27                             |
| 3       | 5.70                             | 6.87                              |
| 4       | 3.80                             | 3.80                              |
| 5       | 5.70                             | 6.67                              |
| 6       | 3.80                             | 4.17                              |

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