How Large Fraction of A Population Must be Vaccinated before A Disease is Controlled?

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Short Report

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Abstract

The ongoing Covid-19 pandemic has already caused more than 5 million casualties despite hard restrictions and relatively high vaccine coverage in many countries. The crucial question is therefore, how large vaccination rate and how severe restrictions are required to terminate the spread of the decease, assuming that the vaccine efficiency and the basic reproduction ratio ($R_0$) are known? To answer this question, a simple mathematical equation was developed to visualize the required vaccination level as function of vaccine efficiency, restriction efficiency and basic reproduction ratio ($R_0$). In addition to the modelling study, Covid-19 data from Europe was collected during 19/11-26/11 (2021) to assess the relation between vaccination rate and incidence. The analysis indicates that a vaccination rate of ~92% (2 doses) is currently required to stop Delta (B.1.617.2) without severe restrictions, using the vaccines that are most common in Europe today. A third vaccine dose, improved vaccines, higher vaccination rates and/or stronger restrictions will be required to force Omicron (B.1.1.529) to expire without infecting a large fraction of the population.

Introduction

The Covid-19 (SARS-CoV-2) pandemic has caused more than 5 million casualties during the years 2020-2021, despite huge global efforts to dampen the spread of the virus [1]. Several vaccines have been developed and more than 9 billion vaccine doses have already been distributed [1]. Unfortunately the virus is gradually evolving, resulting in new Covid-19 variants under monitoring (VUM), variants of interest (VOI) and variants of concern (VOC), including the Alpha (B.1.1.7), Beta (B.1.351), Gamma (P.1), Delta (B.1.617.2) and Omicron (B.1.1.529) variants. These variants have higher basic reproduction number ($R_0$) and higher resistance against today’s vaccines as compared to the original Covid-19 virus.

Governments and other authorities all around the world are constantly trying to identify ideal strategies for counteracting the pandemic with minimal impact on health and economy. The aim of this study was to develop an improved generic epidemiological equilibrium transmission model and utilize it (using empirical input data) to predict required vaccination rates and restriction strengths for controlling different Covid-19 variants, including the Delta and Omicron variants. The model results can be used to optimize the preventive measures against Covid-19 variants.

Models And Methods

Compartmental models, like the SIR (Susceptible, Infectious or Recovered) model [2], are used in epidemiology to capture time-dependent phenomena, but the aim of this study is only to capture steady-state properties. Therefore, the mathematical model developed in this study was instead based on a well-known epidemiological equilibrium equation [3]:

$$R_t(t) = R_0 \times (1 - I(t)) \times \left(1 - E_R(t)\right)$$
where $R_t(t)$ is the time-dependent reproduction ratio at time $t$, $R_0$ is the initial reproduction ratio without restrictions, $\lambda(t)$ is the relative proportion of the population that is immune to the disease, and $E_R(t)$ is the relative efficiency all preventive measures against the spread (e.g. social distancing, increased hygiene rules, face-masks, quarantines etc.). Both $\lambda(t)$ and $E_R(t)$ are limited to the interval 0 to 1. For $\lambda(t)$, 0 and 1 correspond to no immunity and complete immunity in the population, respectively. For $E_R(t)$, 0 and 1 correspond to no preventive measures and complete isolation of every single individual in the population, respectively.

In this study, the original equation (Eq. 1) was updated by postulating that the immunity fraction equals the fraction of vaccinated persons $\phi_v(t)$ times the efficiency of the vaccine $E_v(t)$, plus the fraction of recovered (non-vaccinated) persons $\phi_r(t)$ times the preventive effect of recovering $E_r(t)$:

$$I(t) = \phi_v(t) E_v(t) + \phi_r(t) E_r(t)$$

For simplicity and for pedagogical reasons, we counterfactually assume that $E_r(t)=E_v$ even though $E_r$ in reality often is higher than $E_v$, especially after longer periods of time. This assumption gives $\phi = \phi_v + \phi_r$, resulting in:

$$I(t) = \phi(t) E_v(t)$$

The condition for a decreasing infection rate is $R_t(t)<1$ and the equilibrium level is $R_t(t)=1$. If the latter condition is inserted in Eq. 4, an equation for the required equilibrium fraction of vaccinated and recovered persons $\phi_{Eq}$ can be derived:

$$1 = R_0 \left( 1 - \phi_{Eq}(t) E_v(t) \right) \left( 1 - E_R(t) \right) \rightarrow$$

$$\phi_{Eq}(t) = \left( 1 - \frac{1}{R_0 \left( 1 - E_R(t) \right)} \right) \frac{1}{E_v(t)}$$
Even though this expression is relatively uncomplicated, it is useful for rapidly predicting the required fraction of vaccination in a population, both for Covid-19 and other pandemic infections. Since the model uses an average $R$-number for the whole population, it is conservative and rather overestimates than underestimates the required vaccination rates [ref].

As a complement to the modelling, data for Covid-19 incidence as function of vaccination fraction was collected from public sources for different European countries and regions during the time period 19-26 November 2021.

**Results And Discussion**

Data for Covid-19 incidence and vaccination rate was collected during the time-period 19/11 to 26/11 (2021) for countries in Europe, regions in Germany and Check Republic and subregions in Sachsen, Sachsen-Anhalt, and Thüringen. The data was mainly retrieved before the partial lockdown in Germany, which started 24/11. As observed in Fig. 1, the incidence decreased with increasing vaccination rate. The data indicates that a vaccination rate around 92% (2 doses) is required to halt the Delta (B.1.617.2) variant of concern, which was dominant in Europe in November 2021. Several factors, including the choice of vaccine [4], the average time since the last vaccine dose and the average number of doses, strongly affect the required vaccination rate.

In December 2021, during the partial lockdown in Germany, the German incidence decreased rapidly, indicating that those restrictions were sufficient to halt the Delta-virus at a vaccination level around 70% (2 doses). The currently dominating Omicron (B.1.1.529) variant of concern is however more infectious and requires stronger measures to quench.

The original Covid-19 virus has $R_0$ around 2-3 [5,6], Delta $R_0 \approx 5$ [6] and Omicron $R_0 \approx 7.5$ [6,7]. The basic reproduction number increases with increasing population density and it becomes higher during the winter (on the northern hemisphere), probably because of lower humidity, less sunshine and more indoor-activities.

The efficacy of today’s vaccines is lower against Omicron than against Delta and the ancestral Covid-19 virus. For Pfizer (BNT162b2), the efficacy is 88.2% and 88.0% against Delta and Omicron, respectively, 2-9 weeks after dose 2 [8]. 20-24 weeks after dose 2, the efficacy drop to 64.8% and 36. %. 2 weeks after dose 3 the efficacy increases to 92.6% and 75.5%, respectively. Similar trends were found for Moderna (mRNA-1273) [9].

In order to predict how large fraction of the population that must be vaccinated and how strong restriction that must be maintained to halt the current pandemic wave, equation 6 was plotted as heat-maps as function of basic reproduction number $R_0$, vaccine efficacy, and restriction efficiency (Fig. 2).
For the initial Covid-19 version, with basic reproduction number $R_0$ around 2-3 and vaccine efficiency around 80-90% (2 doses), the heat-maps indicate that medium-strong restrictions (50-70%) or a vaccination rate around 70% are sufficient to halt the pandemic. This has been confirmed empirically in many countries, both for the original Covid-19 virus and e.g. for the Alpha (B.1.1.7) and Beta (B.1.351) variants of concern.

For the Delta version, with $R_0 \approx 5$ and a vaccine efficiency around 65-90% (2 doses), very high vaccination rates (~90%), very strong restrictions (80%), or a combination of medium high restrictions (50-60%) and vaccination rates (70%) are required. This prediction is confirmed by the data in Fig. 1 and by e.g. the observation that the partial lockdown in Germany December 2021 succeeded to stop the Delta wave under these conditions.

For the Omicron version, with $R_0 \approx 7.5$ and vaccine efficiency around 0-88% (2 doses) or 54.6-75.5% (3 doses), extremely strong restrictions (90%) or a combination of strong restrictions (70-80%) and a high vaccination rate with three doses (70-85%) is required.

Note that the required vaccination rates in these graphs are the combined fraction of vaccinated or recovered persons. Even if the vaccination/restriction strategy fails, the virus wave will ultimately stop when a sufficient number of people has been infected and recovered, but to a larger cost in human lives and suffering. A high vaccination rate is strongly desired because it reduces both the virus transmission and the risk for serious health effects for the infected individual. Although the hospitalization probability for Omicron is 40-45% lower than for Delta, it is still high [10].

**Conclusions**

An epidemiological equilibrium model has been derived that can predict required vaccination rate and/or required amounts of restrictions as function of (empirically known) vaccine efficiencies and basic reproduction numbers ($R_0$). The model indicate that for the Omicron (B.1.1.529) variant of concern, a combination of strong restrictions (70-80%) and high vaccination rates (70-90%) with three doses of e.g. the Pfizer (BNT162b2) vaccine will be required, otherwise a large proportion of the population will be infected. Tailor-made updates of the vaccine will be needed to stop the pandemic completely without maintaining relatively severe restrictions or without accepting that large fractions of the population become infected. It is strongly recommended to rapidly encourage a high (3-dose) vaccination rate, because that will decrease both the virus transmission and the severity of the infections. For the Delta (B.1.617.2) variant of concern, our data (empirical and model predictions) indicate that a vaccination (and/or recovery) rate of ~92 % (2 doses) is sufficient for stopping the pandemic without maintaining restrictions.

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**Competing Interests**

We have no competing interests

**Author Contributions**

All authors contributed to study design. Halamicek and Schubert analyzed empirical data, Nilsson modelled+wrote.

**Ethics approval**

Not required (modelling study)

**Consent to participate**

No participants

**Consent to publish**

No participants

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Figures
Figure 1

Covid-19 incidence (per 100 000) versus vaccinate rate (2 doses) for 6 geographical regions, showing that a vaccination rate of ca. 92 % is required to cease the Delta variant with limited restrictions. The choice of vaccine, the number of doses, the time since the last dose and the currently dominating Covid-version strongly affect the number.

Figure 2

Heat maps showing required vaccination rate as function of restriction efficiency and vaccine efficiency for different basic reproduction ratios $R_0$. The color indicate required vaccine rate, with black lines in steps of 10 %. The original Covid-19 has $R_0=2-3$, Delta $R_0\approx 5$ and Omicron $R_0\approx 7.5$. 