Two-step orthogonal-state-based protocol of quantum secure direct communication with the help of order-rearrangement technique

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Abstract The Goldenberg–Vaidman (GV) protocol for quantum key distribution uses orthogonal encoding states of a particle. Its security arises because operations accessible to Eve are insufficient to distinguish the two states encoding the secret bit. We propose a two-particle cryptographic protocol for quantum secure direct communication, wherein orthogonal states encode the secret, and security arises from restricting Eve from accessing any two-particle operations. However, there is a non-trivial difference between the two cases. While the encoding states are perfectly indistinguishable in GV, they are partially distinguishable in the bipartite case, leading to a qualitatively different kind of information-versus-disturbance trade-off and also options for Eve in the two cases.

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1 Introduction

Recent advances in device-independent quantum cryptography [1] have brought to the fore the relevance of multi-particle systems in quantum cryptography, following a line of thought first initiated by Ekert [2]. In response to this work, Ref. [3] proposed a quantum key distribution (QKD) scheme based on Einstein–Podolsky–Rosen (EPR) correlations, which was equivalent to the original Bennett and Brassard [4] protocol for QKD, but uses separable particles instead of entangled ones. The argument of Ref. [3] would suggest that the security features of EPR were reflected in Bennett and Brassard [4]. A similar relation exists arguably between the Boström and Felbinger [5] on the one hand, Deng and Long [6] and QSDC or Lucamarini and Mancini [7] protocols, on the other, in the sense that the former may be considered as the entangled version of the latter.

All the separable-state protocols discussed above, Bennett and Brassard [4], Deng and Long [6]-QSDC and Lucamarini and Mancini [7], employ non-orthogonal states, whose perfect indistinguishability lies at the heart of their security. Further, perfect indistinguishability of non-orthogonal states also provides security to many other protocols of QKD, such as Bennett [8] and Deng and Long [9]-QKD protocols. By contrast, Goldenberg and Vaidman [10] proposed a protocol (GV), demonstrating that secure cryptography can be accomplished even with orthogonal states. The key point was that they were superpositions of geographically separated wave packets. Secrecy in this case arises because of the set of operations Eve can apply are restricted by quantum mechanics.

Most of the early protocols [2,4,8] of quantum cryptography were limited to QKD. Specifically, these quantum cryptographic protocols are designed to generate an unconditionally secure random key by quantum means, and subsequently classical cryptographic procedures are used to encode the message using the key generated by these protocols. Interestingly, later protocols for secure communication [11,12] were proposed that allow to either generate a deterministic key or to circumvent the prior generation of key. These protocols of secure direct quantum communication can broadly be divided into three subclasses: (i) deterministic QKD protocols [5,7,10]; (ii) protocols for deterministic secure quantum communication (DSQC) [11–13]; and finally, (iii) protocols for quantum secure direct communication (QSDC) [6].

In deterministic QKD and DSQC, there is some information leakage of classical data prior to detection of eavesdropping by Eve. Deterministic QKD solves this problem by transmitting a random key, rather than the secret message. In DSQC, the receiver (Bob) can decode the message only after receipt of an encoding key, which is some additional classical information (at least one bit for each qubit transmitted by the sender (Alice)). Thus, in the event of leakage detection, the encoding key is not published, in order to protect the message.

In contrast to DSQC, when no such additional classical information is required, a direct secure quantum communication of the message can be achieved, which happens
in a QSDC protocol. Protocols of DSQC and QSDC are interesting for various reasons. Firstly, a conventional QKD-based quantum communication protocol uses a classical intermediate step to transmit the message, but no such classical intermediary is required in DSQC and QSDC. Further, a QSDC or DSQC protocol can always be turned into a protocol of QKD as the sender who is capable of communicating a meaningful message can also choose to communicate a random string of bits to convert the protocol into a protocol of QKD. However, the converse is not true (i.e., a QKD protocol cannot be used as a protocol of QSDC or DSQC).

In this work, we consider an orthogonal-state-based quantum cryptography protocol that uses two-particle entanglement. By transmitting the two particles separately, we obtain security because the set of states distinguishable via the accessible operations to Eve fails to distinguish the encoding states. As in GV, our protocol requires delayed measurement on the first particle in order to work. (Therefore, from a practical perspective, quantum memory, an expensive resource, is required.) By contrast, for non-orthogonal-state-based protocols, delayed measurement is replaced by random measurement choice, but can be used to improve efficiency [14]. An important difference with the single-particle case is the degree of distinguishability, thus making the proof of security quite different in the two-particle case. Further, our use of block transmission and an order-rearrangement technique makes the protocol suitable for QSDC, while GV is a protocol for QKD.

In GV, both the encoding states as well as error-checking states involve only orthogonal states, while in Bennett and Brassard [4], both types of states are non-orthogonal. More generally, one may consider cryptography protocols that involve orthogonal-state encoding but allow conjugate coding for error-checking [15–19]. Using the strategy adopted for eavesdropping checking in the protocol proposed in the present paper, it is possible to modify these protocols [15–19] into equivalent completely orthogonal-state-based protocols.

2 The GV protocol and its security

We briefly review GV. Alice and Bob are located at two ends of a large Mach–Zehnder interferometer. Let \(|U(t)|\) and \(|L(t)|\) be two localized wave packets of Alice’s particle \(S\), traveling by the upper and lower arm of the interferometer, respectively. Classical bit \(j (= 0, 1)\) is encoded as follows:

\[
|\Psi_j\rangle = \frac{1}{\sqrt{2}} \left( |U(t_s)\rangle + (-1)^j |L(t_s)\rangle \right),
\]

where it is assumed that there is no overlap between the supports of \(|U(t)|\) and \(|L(t)|\). Alice sends Bob either \(|\Psi_0\rangle\) or \(|\Psi_1\rangle\) by delaying packet \(L\) by time \(\Delta\) to ensure that \(|U\rangle\) and \(|L\rangle\) are not present in the channel at the same time. In his station, Bob receives \(|U(t_s + \tau)|\), where \(\tau\) is the travel time of the pulse from Alice’s to Bob’s station. Bob puts the pulse on hold for time \(\Delta\) (where \(\tau < \Delta\), before combining it with \(|L(t_s + \Delta + \tau)|\), to recreate the superposition state \(|\Psi_j'\rangle\), which is the same as \(|\Psi_j\rangle\), apart from an inconsequential global phase. Bob then decodes bit \(j\) deterministically from his interferometric output.
Alice and Bob perform the following two tests to detect Eve’s possible malicious eavesdropping: (1) They compare the sending time \( t_s \) with the receiving time \( t_r \) for each wave packet. We must have \( t_r = t_s + \tau + \Delta \). This ensures that Eve cannot delay the upper packet until also having lower packet simultaneously, which would allow her to decode the states. Even so, she may replace both wave packets with a corresponding dummy. To avoid such an attack, the timing of transmission of particles is kept random. To facilitate this, Alice and Bob discretize their sending times into a sequence of time bins. (2) Alice selects a fraction of particles and announces their time coordinates. Bob announces his measurement outcomes on them. Alice ensures that his received bits are consistent with her transmitted bits.

The security of GV can be understood in terms of an extended no-cloning theorem applicable to orthogonal states, when Eve’s operations are restricted by the fact that she can physically access only one of the pieces \( |U(t)\rangle \) and \( |L(t)\rangle \) at a given time [20]. We present a slightly different version, amenable to subsequent generalization.

The simplest operation accessible is a projective measurement onto the basis \( \{ |U(t)\rangle, |L(t)\rangle \} \), where we may ignore the time-dependence for convenience. If Eve measures projectively in this basis, she merely disrupts the coherence between the wave packets and is detected, but obtains no information about the secret bit \( j \). More generally, Eve can introduce a probe \( P \) that interacts with Alice’s particle according to the following:

\[
U \equiv |U\rangle\langle U| \otimes C_U + |L\rangle\langle L| \otimes C_L,
\]

where \( C_U \) and \( C_L \) are unitaries acting on the ancilla alone. Because the two packets are never together on the channel, causality demands that Eve’s attack cannot unitarily mix the \( U \) and \( L \) pieces. An implication is that no attack by Eve, which is confined to the form (2), can extract secret bit \( j \), because this is stored as the phase information between the two wave packets, and cannot be accessed even probabilistically. We prove this below.

Let \( |R\rangle \) be the initial ‘ready’ state of the probe. Acting on the particle-probe system, Eq. (2) transforms an initial state \( |\Psi_j\rangle \otimes |R\rangle \), after they are recombined by Bob, to the state

\[
\rho_{SP} = \frac{1}{4} \begin{pmatrix}
|u\rangle\langle u| & (1)^j|d\rangle\langle u| \\
(1)^j|u\rangle\langle d| & |d\rangle\langle d|
\end{pmatrix},
\]

where \( |u\rangle \equiv C_0|R\rangle \) and \( |d\rangle \equiv C_1|R\rangle \). The probe is now left in the state:

\[
\rho'_P = \text{Tr}_S(\rho_{SP}) = \frac{1}{2}(|d\rangle\langle d| + |u\rangle\langle u|),
\]

which, as with the case of projective measurements, yields no information to Eve about the secret bit \( j \). In other words, Eve gains nothing by attacking in the case of individual attacks. It is not difficult to see that this is also true for Eve’s collective and joint attacks.
Assuming ideal single-photon sources and detectors with Alice and Bob, the only way for Eve to attack the GV protocol is that she substitutes dummies by blocking fraction $f$ of the genuine particles. Suppose that Alice and Bob agree to discretize the random sending time. In each sequential block of $\gamma$ (an integer) number of time steps, one particle is transmitted by Alice in a randomly chosen time cell within the block. Eve’s strategy would be to fully blockade a fraction $f$ of randomly chosen blocks and transmit a dummy prepared by her in a randomly chosen time within the block. The probability that she gets a match with Alice’s transmission cell is $1/\gamma$.

To calculate the error rate Eve generates, we note that even when Eve gets the timing right, she will be wrong half the time about the encoded state. Thus, Eve generates error rate

$$e = f \times \left[ \frac{1}{2} + \left( 1 - \frac{1}{\gamma} \right) \right] = f \times \left( 1 - \frac{1}{2\gamma} \right),$$

(5)

where $\gamma$ is a publicly known number. Bob’s average information on the sifted bits is given $I_B = I(A : B) = 1 - h(e)$, where $h(\cdot)$ is the binary Shannon information. On the dummies whose timing is right, Eve has full information, i.e.,

$$I(A : E) = I(B : E) \equiv I_E = \frac{f}{\gamma} = \frac{2e}{2\gamma - 1},$$

(6)

where the last equation follows from Eq. (5) and the $I(A : E)$ and $I(B : E)$ denote Eve’s mutual information on Alice and Bob. She knows when she got it right when Alice and Bob perform the equivalent of basis reconciliation for the sending times.

The corresponding data is plotted in Fig. 1. The requirement for positive secret key rate is determined by [21]

$$K \equiv I_B - \min(I(A : E), I(A : B)) = I_B - I_E,$$

(7)

from which the maximum tolerable error is found to be $e_{\text{max}} \approx 0.26$.

**Fig. 1** Bob’s information ($I_B$, falling dashed curve) and Eve’s information ($I_E$, rising line), respectively, as a function of eavesdropping parameters observed error $e$. For $e \geq e_{\text{max}} \equiv 0.26$, $I_E > I_B$. The falling line corresponds to the positive key rate (7).
3 Toward a two-particle orthogonal-state-based protocol

In seeking a protocol that extends GV to a two-particle (or multi-particle) scenario, we are naturally led to consider cryptographic adaption of quantum dense coding to cryptography (cf. the protocol of Ref. [22] for dense coding based secure direct communication.) On the analogy of GV, one might expect that Alice should transmit the two entangled particles one after another at random timings and such that both are not found on the open channel. Surprisingly, this can be completely insecure against Eve, whose strategy would be as follows. When the first particle comes, she holds it and transmits her own half of a Bell state toward Bob. She can in principle find out the position of the randomly sent second particle, measure it jointly with Alice’s first particle, determine their joint state, and then transmit a dummy particle appropriately entangled with her first dummy particle. Here we have assumed that Eve’s measurements take negligible time.

To avoid this attack, such bipartite cryptographic protocols may add multi-partite non-orthogonal states either to the coding or in the checking step (as in Bennett and Brassard [4]). However, if we remain restricted to orthogonal states, then the order of particles needs to be scrambled, via the permutation of particle (PoP) action, an idea first introduced by Deng and Long [23] in a pioneering work on the “controlled order rearrangement encryption” (CORE) QKD protocol. In the present work, a two-particle QSDC protocol inspired by GV, which is referred to as 2GV, is presented along these lines in the next section.

Now suppose Eve does not launch the dummy particle attack. Assuming ideal sources and detectors, GV is secure. Interestingly, a bipartite generalization of GV (without PoP) is not. The reason is interesting and highlights a difference between single- and bipartite nonlocality: while Eve gets no information on the encoded bits when the two packets are de-synchronized, in the bipartite case, partial information can be obtained, as detailed below.

Alice and Bob employ a key distribution protocol where the key is shared via a dense coding strategy and must test for Eve after the transmissions are completed. Alice and Bob need to model Eve’s attack strategy and estimate whether Eve’s information on their secret bits, as a function of observed noise, is too high to be eliminated by subsequent classical post-processing. If it is, only then do they abort the protocol run. We furnish a security proof of the protocol, assuming individual attacks by Eve on each of the two coding particles. From this, we extract an information-vs-disturbance trade-off and hence determine the largest tolerable error rate.

As a specific example of the attack employed by Eve, we consider a model given in Ref. [24], which is based on one proposed by Niu and Griffiths [25]. Probes $E_0$ an $E_1$ interact with each transmitted qubit, being subjected to the interaction:

$$
|0\rangle_E \rightarrow \sqrt{\frac{1 + \cos \theta}{2}} |0\rangle_0 + \sqrt{\frac{1 - \cos \theta}{2}} |1\rangle_0,
$$

$$
|1\rangle_E \rightarrow \sqrt{\frac{1 + \cos \theta}{2}} |1\rangle_1 + \sqrt{\frac{1 - \cos \theta}{2}} |0\rangle_0,
$$

(8)
where, furthermore $\langle \epsilon_0 | \epsilon_1 \rangle = \langle E_0 | E_1 \rangle = \cos \theta$ by virtue of symmetry in the attack strategy. For simplicity, the same attack parameter $\theta$ is assumed to characterize the attack on both particles. This results in the initial state $\rho_{AB}$, which is a Bell state in 2GV, evolving into a joint state of the particles and probes, $\rho_{ABE_1E_2}$.

After some straightforward calculation, the above attack can be shown to produce the reduced density operator

$$\rho_{AB}'' = \text{Tr}_{E_1E_2} \left( \rho_{ABE_1E_2}'' \right)$$

$$= \left( \begin{array}{cccc}
\frac{1}{2}(1 + \cos^2 \theta) & 0 & 0 & \frac{1}{2}(1 + \cos^2 \theta) \\
0 & \frac{1}{2} \sin^2 \theta & \frac{1}{2} \sin^2 \theta \cos^2 \theta & 0 \\
0 & \frac{1}{2} \sin^2 \theta \cos^2 \theta & \frac{1}{2} \sin^2 \theta & 0 \\
\frac{1}{2}(1 + \cos^2 \theta) \cos^2 \theta & 0 & 0 & \frac{1}{2}(1 + \cos^2 \theta)
\end{array} \right).$$

(9)

The quality of state received by Bob can be quantified by the fidelity $\langle \Phi^+ | \rho_{AB}'' | \Phi^+ \rangle = (1 + \cos^2 \theta)^2$ (where we assume $\rho_{AB} = | \Phi^+ \rangle \langle \Phi^+ |$). It follows that in order to produce no errors, Eve must ensure that $\theta = 0$, which by virtue of Eq. (8), implies that no entanglement is generated, and in fact $| \epsilon_0 \rangle = | \epsilon_1 \rangle$, implying a trivial interaction of the probe with Alice’s qubit. Thus, if no errors are generated, then Eve gains no information. More generally, suppose finite errors are observed.

The error rate observed by Alice and Bob is given by the following:

$$e = 1 - \langle \Phi | \rho_{AB}' | \Phi \rangle$$

(10)

where $| \Phi \rangle \in \{ \Phi^\pm \}, | \Psi^\pm \}$ and

$$\rho_{AB}'(\theta, \lambda) = (1 - \lambda) | \Phi \rangle \langle \Phi | + \lambda \rho_{AB}''$$

(11)

is the corresponding two-particle state obtained assuming Eve attacks fraction $\lambda$ of the incoming particle pairs with eavesdropping parameter $\theta$ as defined in Eq. (8). Bob’s information $I_B$ is quantified as the Alice–Bob mutual information $I_B \equiv I(A : B)$ when Bob measures the incoming states in the Bell basis. As a function of $\theta, \lambda$, it is:

$$I_B(\theta, \lambda) = H(A) - H(A' | B = \Phi),$$

(12)

where $H(A)$ is Alice’s preparation entropy and $H(A' | B = \Phi)$ is the conditional entropy of $\rho_{AB}'(\theta, \lambda)$ when Bob measures in the Bell basis. The quantity is presented in Fig. 2a as a function of Eve’s attack parameters.

Eve’s information $I_E \equiv I(A : E) = I(A : B)$ is upper-bounded by the Holevo bound $\chi$ of the reduced density operator of the two probes:

$$\chi = S \left( \sum_j p_j \rho_{E_1E_2}^{(j)} \right) - \sum_j S \left( \rho_{E_1E_2}^{(j)} \right) \geq I_E(\theta, \lambda).$$

(13)
where $\rho^{(j)}$ $(j = 0, 1, 2, 3)$ is the noisy version of the density operator corresponding to the four Bell states $|\Phi^\pm\rangle, |\Psi^\pm\rangle$, respectively, being sent by Alice. This bound on Eve’s information is depicted in Fig. 2b. Using the following notation: $c \equiv \cos(\theta), s \equiv \sin(\theta), K \equiv \frac{1}{2}(1 + c), A \equiv K^2c^2s^2, B \equiv K^2c^3s^3, C \equiv K^2s^3c, D \equiv \frac{1}{4}s^3c^3, E \equiv \frac{1}{4}s^4c^2, F \equiv \frac{1}{4}s^3c^2, H \equiv K^2(1 + c^4), I \equiv \frac{1}{4}(1 + c^2)s^2c, J \equiv \frac{1}{2}s^4c, L \equiv \frac{1}{2}s^2(1 + c^4), M \equiv \frac{1}{4}s^3, N \equiv \frac{1}{4}s^5c, P \equiv (1 - K)^2cs, Q \equiv (1 - K)^2s^2, R \equiv 2(1 - K)^2c^2$, we find that if Alice transmits states $|\Phi^\pm\rangle$, then the corresponding probe states of Eve are:

$$
\rho_{E_1E_2}^\pm = 
\begin{pmatrix}
H & B & 0 & 0 & B & A & 0 & 0 & 0 & 0 & \pm I & \pm M & 0 & 0 & \pm F & 0 \\
B & A & 0 & 0 & A & C & 0 & 0 & 0 & 0 & \pm F & 0 & 0 & \pm J & 0 \\
0 & 0 & L & D & 0 & 0 & D & E & \pm I & \pm M & 0 & 0 & \pm F & 0 & 0 \\
0 & 0 & D & E & 0 & 0 & E & N & \pm F & 0 & 0 & \pm J & 0 & 0 & 0 \\
B & A & 0 & 0 & A & C & 0 & 0 & 0 & 0 & \pm F & 0 & 0 & \pm J & 0 \\
A & C & 0 & 0 & C & K^2s^4 & 0 & 0 & 0 & 0 & \pm J & 0 & 0 & 0 & \pm \frac{1}{2}s^5 \\
0 & 0 & D & E & 0 & 0 & E & N & \pm F & 0 & 0 & \pm J & 0 & 0 & 0 \\
0 & 0 & E & N & 0 & 0 & N & \frac{1}{2}s^6 & \pm J & 0 & 0 & 0 & \pm \frac{1}{2}s^5 & 0 & 0 \\
0 & 0 & \pm I & \pm F & 0 & 0 & \pm F & \pm J & \frac{1}{2}s^2c^2 & M & 0 & 0 & M & 0 & 0 & 0 \\
0 & 0 & \pm M & 0 & 0 & 0 & 0 & 0 & M & \frac{1}{2}s^4 & 0 & 0 & 0 & 0 & 0 & 0 \\
\pm I & \pm F & 0 & 0 & \pm F & \pm J & 0 & 0 & 0 & 0 & R & P & 0 & 0 & P & 0 \\
\pm M & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P & Q & 0 & 0 & 0 & 0 \\
0 & 0 & \pm F & \pm J & 0 & 0 & \pm J & \pm \frac{1}{2}s^5 & M & 0 & 0 & 0 & \frac{1}{2}s^4 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q & 0 & 0 & 0 & 0 & 0 \\
\pm F & \pm J & 0 & 0 & \pm J & \pm \frac{1}{2}s^5 & 0 & 0 & 0 & 0 & P & 0 & 0 & Q & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

(14)

It is immediately seen that $\rho_{E_1E_2}^+$ and $\rho_{E_1E_2}^-$ are not identical, implying that Eve can gain some information about Alice’s transmission by distinguishing $\rho_{E_1E_2}^\pm$. This encoding dependence of Eve’s probe state in 2GV is in stark contrast to the general probe state (4) obtained when Eve attacks GV. Thus, Eve’s attack in 2GV can obtain partial information about the encoding, whereas she obtains none in the case of GV, even when no dummy states are used. Therefore, unlike with GV, in the case there is an information-vs-disturbance trade-off even when Eve employs no dummy particles.
Fig. 3  Plot of error (10) for which a positive key rate (7) exists according to the data of Fig. 2. This is the plot of $e = e(\theta, \lambda)$, which is truncated over the region where $I_{AE} > I_{AB}$. The maximum tolerable error is the minimum across the ‘cliff’ (cf. Eq. (15)).

The tolerable error rate is computed as follows:

$$e_0 = \min_{I_B - \chi = 0} e,$$

the smallest error for which $\chi$ just exceeds $I_B$. It may be considered as the problem of minimizing $e$ subject to the constraint that Eve’s information has zero excess over Bob’s. Numerically, applying the criterion (15) to the information-vs-disturbance trade-off data in Fig. 2, we found the tolerable error rate $e_0 = 26.7\%$, as plotted in Fig. 3. This rather large tolerance can be attributed to the limited power of Eve’s attack here.

4 Two-particle orthogonal-state-based protocol

Instead of random transmission, Alice transmits multiple halves of Bell states and scrambles the order of the second halves. In realistic protocols, there will be inevitable noise. As is usually done, any error observed by Alice and Bob is attributed to a putative Eve’s intervention, though errors can arise also due to channel noise, too. The presented scheme enumerated below uses this idea of re-ordering or permutation of particle order. An illustration of the protocol is given in Fig. 4.

1. Alice prepares the state $|\Psi^+\rangle \otimes 3^n$ where $|\Psi^+\rangle = \frac{\left|00\right\rangle + \left|11\right\rangle}{\sqrt{2}}$. She divides them into two sets: set $S_1$ of $n$ pairs and set $S_2$ of $2n$ pairs. Let $S_j^{(a)}$ denote the first half ($jn$ qubits) of set $S_j$, and $S_j^{(b)}$ denote the second half of set $S_j$. She keeps $S_2^{(a)}$ with...
Alice Bob

Permutation of particles

Bell–state measurement

Verification

Dense coding

Fig. 4 Illustration of the quantum information processing steps of the new protocol, where we indicate the block transmission of qubits (cf. Ref. [15]). a Step 1 of the new protocol, where Alice transmits qubits of the set $S_2^{(b)}$ ($2n$ qubits) and set $S_1$ ($2n$ qubits) to Bob, after permuting them using operation $\Pi_{4n}$ (indicated as the dashed box); b Step 2 of the protocol, where Bob performs a Bell state measurement on the $S_1$ qubits, after reordering them based on coordinate information received from Alice; the particles of $S_2^{(b)}$ still remain permuted (dashed box); c Step 3, where Alice sends the $n$ qubits used for verification, and $n$ used for transmitting a random key, along with the required coordinate information. After these three steps Alice and Bob collaborate to check eavesdropping in Step 4 and finally in Step 5 Bob performs Bell measurements to obtain the shared key (Steps 4 and 5 are not shown in this figure)

1. After receiving Bob’s authenticated acknowledgment, Alice classically announces the coordinates of the $2n$ members of $S_1$ among the transmitted particles. Bob measures them in the Bell basis to determine if they are each in the state $|\Psi^+\rangle$. If the error detected by Bob is within a tolerable limit, they continue to the next step. Otherwise, they discard the protocol and restart from Step 1.

2. After receiving Bob’s authenticated acknowledgment, Alice classically announces the coordinates of the $2n$ members of $S_1$ among the transmitted particles. Bob measures them in the Bell basis to determine if they are each in the state $|\Psi^+\rangle$. If the error detected by Bob is within a tolerable limit, they continue to the next step. Otherwise, they discard the protocol and restart from Step 1.

3. Alice randomly chooses a sequence of $n$ qubits from the set $S_2^{(a)}$ in her possession to form the verification string $\Sigma_2^{(a|V)}$ for the next round of communication and encodes her key in the remaining $n$ qubits of $S_2^{(a)}$ to form the code string $\Sigma_2^{(a|C)}$. To encode a 2-bit message or key, Alice applies one of the 4 Pauli (dense coding) operations $I, X, iY, Z$ on her qubit. After the encoding operation, Alice sends all qubits in her possession (i.e., $S_2^{(a)}$) to Bob.

4. Alice discloses the coordinates of the verification qubits ($\Sigma_2^{(a|V)}$) and their partner particles after receiving authenticated acknowledgement of receipt of all the qubits from Bob. Bob performs Bell measurement on the verification qubits and their partner particles and computes the error rate as in Step 2.
5. If the error rate is tolerably low, then Alice announces the coordinates of the partner particles of $\Sigma_2^{(a|V)}$ and Bob uses that information to decode the encoded message or key via a Bell state measurement on the remaining Bell pairs, and classical post-processing.

2GV may be considered as the bipartite generalization of GV because the encoding is via orthogonal states, and security arises because the encoding states cannot be distinguished by the restricted operations available to Eve. However, there are three important differences. First is, as noted above, that randomizing the transmission schedule of Alice’s particle does not help. More importantly, whereas geographic separation forbids Eve’s attack in GV from unitarily mixing the states $|U\rangle$ and $|L\rangle$, in 2GV, where the encoding states are based on internal degrees of freedom, the attack can mix encoding states. Thus, there is no bar on Eve’s accessing the coherence between the particles, to gain partial information about the Bell state being sent even when restricted to attack on single particles. Thus, unlike in GV, there is an information versus disturbance trade-off even when Eve does not use dummy particles, which we discuss below.

Lastly, our protocol satisfies the stronger QSDC security requirement, while GV in its original form is a protocol for deterministic QKD which cannot be used for QSDC, but can be used for DSQC [26]. This can be understood clearly by considering that Alice sends a meaningful message to Bob by transmitting a sequence of $|\Psi_0\rangle$ and $|\Psi_1\rangle$ using the original GV protocol. In this situation, when Alice sends $|U\rangle$ then Eve can keep it with her and substitute it by a fake $|U\rangle$ and send that to Bob without causing any delay. Later, when $|L\rangle$ is sent by Alice, then also Eve will keep that with her and send a fake $|L\rangle$ to Bob. Eve can now appropriately superpose $|U\rangle$ and $|L\rangle$ and obtain the meaningful information (message) encoded by Alice. To prevent this, Alice randomizes her transmission schedule. Eve can still block particles and decode the random bits, but she will be eventually caught when Alice and Bob compare the sending and receiving times. The point is that GV works by streaming qubits. Thus, by the time she is caught, the encoded information will have already been leaked. This leakage is not a problem with GV protocol (QKD), because if Eve’s interference is too high, Alice and Bob will not use that key for any future encryption. In contrast to GV, our protocol uses particle-order arrangement in place of time schedule randomization, and further, we use block transmission [15] in place of stream transmission. As a result, eavesdropping does not reveal information as the coordinates of the partner particles of the information encoded qubits are announced only at the last step of the protocol, i.e., after confirming that no eavesdropping has happened in the second step of communication when $\Sigma_2^{(a|V)}$ and $\Sigma_2^{(a|C)}$ are communicated. Clearly, the proposed cryptographic protocol is suitable DSQC.

Assuming ideal sources and detectors with Alice and Bob, the PoP device makes the protocol exponentially sensitive to Eve’s intervention. Suppose Eve chooses to attack fraction $f$ of $n$ pairs of particles transmitted. Let $m \equiv \lfloor nf \rfloor$ is an integer. The probability that the $m$ particles are pair-wise closed (i.e., every particle’s twin is within the attacked group) is $p_{\text{closed}} \equiv \left( \binom{n}{m} \right) \left( \frac{2n}{2m} \right)^{-1}$ while the probability that all selected $m$ particles are correctly paired by Eve in the closed group is
\[ p_{\text{pair}} = \frac{1}{m-1} \cdot \frac{1}{m-3} \cdot \ldots \cdot \frac{1}{3}. \]

Thus, the probability Eve’s attack produces no error is \( p_{\text{closed}} \cdot p_{\text{pair}} \), which is exponentially small.

In our protocol, the efficiency of \( \frac{1}{3} \) can be improved upon in practice if the observed noise level remains stable over sufficiently many runs and thus fewer quantum resources need to be sacrificed to determine it. Our protocol as stated makes no such assumption about the noise and thus considers the worst case scenario. Consequently, in every transmission step, we have used half of the transmission qubits for error checking. The statistics of random sampling then guarantees that the probability that the fraction of errors observed in the check bits deviates from the error fraction in the code bits is exponentially low [27].

5 Conclusions and discussions

A two-particle QSDC protocol has been proposed, with the motivation of understanding the similarity and difference between the origins of security in GV and a multi-particle orthogonal-state-based cryptography scheme. It may be noted that 2GV is technically similar to CORE QKD protocol [23]—with added ideas (block transmission technique) from Ref. [15]—rather than to GV. A non-trivial difference between the two situations was noted. 2GV uses internal degrees of freedom, while GV uses the spatial degree of freedom, as a result of which the nature of the information-vs-disturbance trade-off and the options available to Eve are quite different, apart from the obvious difference due to employing different numbers of particles. The PoP technique is crucial to 2GV, while it can optionally be used to enhance security of GV. However, for GV, it suffices to increase the parameter \( \gamma \), which is experimentally easy to implement.

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