Analysis of the vector meson transitions among the heavy quarkonium states

Zhi-Gang Wang

Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In this article, we study the vector meson transitions among the charmonium and bottomonium states with the heavy quark effective theory in an systematic way, and make predictions for the ratios among the vector meson decay widths of a special multiplet to another multiplet. The predictions can be confronted with the experimental data in the future.

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1 Introduction

In 2003, the CLEO collaboration observed a significant signal for the transition $\Upsilon(3S) \rightarrow \gamma \omega \Upsilon(1S)$, which is consistent with the radiative decays $\Upsilon(3S) \rightarrow \gamma \chi_{b1.2}(2P)$ followed by the hadronic decays $\chi_{b1.2}(2P) \rightarrow \omega \Upsilon(1S)$. The branching ratios are $\text{Br}(\chi_{b1}(2P) \rightarrow \omega \Upsilon(1S)) = (1.63^{+0.35+0.16}_{-0.31-0.15})\%$ and $\text{Br}(\chi_{b2}(2P) \rightarrow \omega \Upsilon(1S)) = (1.10^{+0.32+0.11}_{-0.28-0.10})\%$, respectively [1].

In 2004, the Belle collaboration observed a strong near-threshold enhancement in the $\omega J/\psi$ invariant mass distribution in the exclusive $B \rightarrow K \omega J/\psi$ decays, the enhancement has a mass of $(3943 \pm 11 \pm 13)$ MeV and a total width of $(87 \pm 22 \pm 26)$ MeV [2]. Later, the Babar collaboration confirmed the $Y(3940)$ in the exclusive decays $B^{0,+} \rightarrow J/\psi K^{0,+}$, the measured mass and width are $(3914.6^{+3.8}_{-3.2} \pm 2.0)$ MeV and $(34_{-8}^{+12} \pm 5)$ MeV, respectively [3]. In 2009, the Belle collaboration reported the observation of a significant enhancement with the mass $(3915 \pm 3 \pm 2)$ MeV and total width $(17 \pm 10 \pm 3)$ MeV respectively in the process $\gamma \gamma \rightarrow \omega J/\psi$ [4], these values are consistent with that of the $Y(3940)$. The updated values of the mass $(3919.1^{+3.5}_{-3.5} \pm 2.0)$ MeV and total width $(31^{+10}_{-8} \pm 5)$ MeV from the Babar collaboration are also consistent with the old ones [5].

In 2009, the CDF collaboration observed a narrow structure $Y(4140)$ near the $J/\psi \phi$ threshold with a statistical significance in excess of 3.8 $\sigma$ in the exclusive $B \rightarrow J/\psi \phi K$ decays produced in $p \bar{p}$ collisions [6]. The measured mass and width are $(4143.0 \pm 2.9 \pm 1.2)$ MeV and $(11.7^{+8.3}_{-5.0} \pm 3.7)$ MeV, respectively [6]. The Belle collaboration measured the process $\gamma \gamma \rightarrow \phi J/\psi$ for the $J/\psi \phi$ invariant mass distributions, and observed a narrow peak $X(4350)$ with a significance of $3.2 \sigma$, and no signal for the $Y(4140) \rightarrow J/\psi \phi$ structure was observed [7]. Recently, the CDF collaboration confirmed the $Y(4140)$ in the $B^{\pm} \rightarrow J/\psi \phi K^{\pm}$ decays with a statistical significance greater than 5 $\sigma$, the measured mass and width are $(4143.4^{+2.9}_{-2.0} \pm 0.6)$ MeV and $(15.3^{+10.4}_{-6.0} \pm 2.5)$ MeV, respectively [8].

We can take the $Y(4140)$ as an exotic hybrid charmonium [9] and the $Y(3940)$ as the $\chi_{c1}(2P)$ state [10] tentatively. The $\chi_{c1}(2P)$ state has the dominant decay mode $D\bar{D}^{*}$ and a predicted width of 140 MeV, which is consistent with that of the $Y(3940)$ within the theoretical and experimental uncertainties [10]. On the other hand, the decay $Y(3940) \rightarrow D\bar{D}^{*}$ has not been observed yet, which disfavors such identification. There have been several other identifications for the $Y(4140)$ (the molecular state [11, 12], the tetraquark state [13], the re-scattering effect [14], etc) and the $Y(3940)$ (the tetraquark state [15], the molecular state [12, 16], etc).

If the $Y(3940)$ is really the $\chi_{c1}(2P)$ state, the $2P$ charmonium and bottomonium states have similar Okubo-Zweig-Iizuka suppressed decays. Experimentally, there is another $\chi_{c1}(2P)$ candidate, the $X(3872)$ [17], which was observed in the $J/\psi \pi^{+}\pi^{-}$ invariant mass distribution by the Belle collaboration [18], and confirmed by the D0, CDF and Babar collaborations [19]. If we take the $Y(3940)$ as the $\chi_{c1}(2P)$ state, the $X(3872)$ has to be assigned to the molecular state [20, 21], the hybrid state [20], (not) the tetraquark state (22, 23, 24, (not) the $\chi_{c1}(2P)$ state with some $DD^{*} + DD^{*}$ component (25, 26, etc.

In the past years, a number of charmonium-like states besides the $Y(3940), Y(4140), X(3872)$ have been discovered, and many possible assignments for those states have been suggested, such as the conventional charmonium states, the multiquark states (irrespective of the molecule type
and the diquark-antidiquark type), the hybrid states, the baryonium states, the threshold effects, etc [27]. In this article, we focus on the traditional charmonium and bottomonium scenario, and do not mean such assignments are correct and exclude other possibilities.

In Ref. [28], Voloshin assumes that the hadronic decays $\chi_{b1,2}(2P) \to \omega \Upsilon(1S)$ take place through the chromo-electric gluon fields $\vec{E}^a$,

$$\langle \omega(\vec{\epsilon})d_{abc} \int f(q_1, q_2, q_3) \vec{E}^a(q_1) \cdot \vec{E}^b(q_2) \vec{E}^c(q_3) \delta^4(q_1 + q_2 + q_3 - p)dq_1dq_2dq_3|0\rangle = A_\omega \vec{\epsilon}^*,$$  \hspace{1cm} (1)

where $p = (m_\omega, \vec{0})$, the $\vec{\epsilon}$ stands for the polarization vector of the $\omega$ meson, the $d_{abc}$ are the symmetric $SU(3)$ constants, and the $f(q_1, q_2, q_3)$ is a totally symmetric form-factor.

The bottomonium state can emit three gluons in the $1^{--}$ channels, then the three gluons hadronize to the vector meson $\omega$ or $\phi$, and the bottomonium state translates to another bottomonium state subsequently. The chromo-electric and chromo-magnetic gluon fields $\vec{E}^a$ and $\vec{B}^a$ have the quantum numbers $J^{PC} = 1^{--}$ and $1^{+-}$, respectively. The chromo-magnetic gluon fields $\vec{B}^a$ are related to the heavy quark spin-flipped transitions, and suppressed by the factors $1/m_Q^2$ with $n \geq 1$. The dominant contributions of the three gluons to the light vector mesons come from the three chromo-electric fields $\vec{E}^a$ with the special configuration $d_{abc}\vec{E}^a \cdot \vec{E}^b \vec{E}^c$. We can integrate out the intermediate gluons, and obtain the effective Lagrangians, which should obey the heavy quark symmetry.

If the initial and final heavy quarkonium states have large energy gaps, the emissions of three energetic gluons are greatly facilitated in the phase space and the corresponding decay widths may be large, although such processes are Okubo-Zweig-Iizuka suppressed in the flavor space and the strong coupling constant $|g_s|$ has smaller value due to the larger energy scale. On the other hand, if the $\omega$ and $\phi$ transitions are kinematically suppressed in the phase space, the branching ratios may be very small. The gluons are electro-neutral, the couplings of the special configuration $d_{abc}\vec{E}^a \cdot \vec{E}^b \vec{E}^c$ to the photons are supposed to be very small and can be neglected. The radiative transitions among the heavy quarkonium states can take place through the emissions of photons from the heavy quarks directly. The dynamics which govern the transitions to the light vector mesons (through three gluons) and to the photons are different. The hadronic decays $\chi_{b1,2}(2P) \to \omega \Upsilon(1S)$ also receive contributions from the virtual photons through the vector meson dominance mechanism.

In the effective Lagrangians at the hadronic level, we do not need to distinguish the virtual photon and three-gluon contributions, which means the coupling constants in the effective Lagrangians contain contributions from both the electromagnetic and strong interactions.

The Okubo-Zweig-Iizuka suppressed decays can take place through the final-state re-scattering mechanism, for example,

$$\chi_{b2}(2P) + q\bar{q} \to B^*(B) + \bar{B}^*(\bar{B}) \to \Upsilon(1S) + \omega,$$

$$\chi_{b1}(2P) + q\bar{q} \to B^*(B) + \bar{B}(\bar{B}^*) \to \Upsilon(1S) + \omega.$$  \hspace{1cm} (2)

If the masses of the initial heavy quarkonium states are above or near the thresholds of the open-charmed or open-bottom meson pairs, the transitions to the intermediate open-heavy mesons are facilitated, there are additional contributions from the final-state re-scattering mechanism. The contributions from the intermediate open-charmed or open-bottom meson loops are not necessary large compared with the tree level contributions. In Ref. [29], Guo et al perform systematic studies about the effects of the intermediate charmed meson loops in the $\pi^0$ and $\eta$ transitions among the charmonia. On the other hand, the intermediate meson loops can result in mass shifts, continuum components and mixing amplitudes for the conventional heavy quarkonium states [30], the $B^*\bar{B}^*$, $BB$, $B^*\bar{B}^*$ and $B\bar{B}^*$ components can also result in the final-state $\Upsilon(1S)\omega$ through the exchange of the intermediate $B$ (or $B^*$) meson. In such processes, the fusion of the open-heavy meson pairs can result in a light vector meson or a photon, the corresponding contributions are related with each other through the approximated vector-meson dominance.

We can carry out the integral of the intermediate meson-loops firstly, then parameterize the net effects by some momentum-dependent couplings of the $\chi_b^0\bar{\Upsilon}\omega$ or $\chi_c^0J/\psi\omega$. Here we prefer phenomenological analysis, and do not intend to obtain effective field theory, and do not separate the energy scales of the relevant degrees of freedom explicitly so as to integrate out some of them in a systematic way. The momenta of the final states in the center of mass coordinate are about $10\text{ MeV}$ and $40\text{ MeV}$, the decay width $\Upsilon(1S)\omega$ is about $10^{-8}\text{ GeV}^2$. We consider the vector-meson dominance
vector meson $\omega$ is not energetic. If we smear the momentum-dependence of the coupling constants, we can obtain an simple Lagrangian, which describes the Okubo-Zweig-Iizuka suppressed $\omega$ or $\phi$ transitions among the heavy quarkonium states, and should obey the heavy quark symmetry.

In Ref. [31], we focus on the traditional charmonium and bottomonium scenario and study the radiative transitions among the charmonium and bottomonium states with the heavy quark effective theory systematically. The charmonium and bottomonium states are heavy quarkonium states, the heavy quark symmetry can put powerful constraints in diagnosing their natures. In this article, we extend our previous works to study the vector-meson transitions among the heavy quarkonium states based on the heavy quark effective theory [32, 33].

There are a number of unknown parameters which determine the magnitudes of the scattering amplitudes in the multipole expansion in QCD and the final-state re-scatterings mechanism. It is very difficult to distinguish the contributions of the three-gluon emissions from that of the open-heavy mesons re-scatterings quantitatively without enough precise experimental data to fitting. In this article, we smear the underlying dynamical details, introduce momentum-independent coupling constants, and make estimations based on the heavy quark symmetry.

The article is arranged as follows: we study the vector meson transitions among the heavy quarkonium states with the heavy quark effective theory in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

### 2 The vector meson transitions among the heavy quarkonium states

The heavy quarkonium states can be classified according to the notation $n^{2s+1}L_j$, where the $n$ is the radial quantum number, the $L$ is the orbital angular momentum, the $s$ is the spin, and the $j$ is the total angular momentum. They have the parity and charge conjugation $P = (-1)^{L+1}$ and $C = (-1)^{L+s}$, respectively. In the non-relativistic potential quark models, the wave-functions $\psi(r, \theta, \varphi)$ of the heavy quarkonium states can be written as $R_{nL}(r)Y_{LM}(\theta, \varphi)$ in the spherical coordinates, where the $R_{nL}(r)$ are the radial wave-functions and the $Y_{LM}(\theta, \varphi)$ are the spherical harmonic functions. The states have the same radial quantum number $n$ and orbital momentum $L$ can be expressed by the superfields $J(n)$, $J^\mu(n)$, $J^{\mu\nu}(n)$, etc [34],

\[
J = \frac{1 + j}{2} \left\{ \gamma_{\mu}^{\mu} - \eta_{\nu\gamma} \right\} \frac{1 - j}{2},
\]

\[
J^\mu = \frac{1 + j}{2} \left\{ \chi_{\alpha}^{\mu\nu} \gamma_{\nu} + \frac{1}{\sqrt{2}} \epsilon^{\alpha\beta\lambda \gamma} v_{\alpha} \gamma_{\beta} \chi_{\lambda}^{\mu} + \frac{1}{\sqrt{3}} (\gamma_{\alpha}^{\mu} - \eta_{\mu\alpha}) \chi_{0} + h_{\nu}^{\mu} \right\} \frac{1 - j}{2},
\]

\[
J^{\mu\nu} = \frac{1 + j}{2} \left\{ \gamma_{\alpha}^{\mu\nu} + \frac{1}{\sqrt{6}} \left[ \epsilon^{\mu\alpha\beta\lambda} v_{\alpha} \gamma_{\beta} g_{\gamma \lambda}^{\nu} + \epsilon^{\nu\alpha\beta\lambda} v_{\alpha} \gamma_{\beta} g_{\gamma \lambda}^{\mu} \right] \gamma_{\tau \lambda}^{2} + \left[ \frac{3}{20} [(\gamma_{\mu}^{\mu} - \eta_{\nu\mu}) g_{\nu\alpha}^{\alpha} + (\gamma_{\nu}^{\nu} - \eta_{\nu\mu}) g_{\mu\alpha}^{\mu}] - \frac{1}{\sqrt{15}} (g_{\mu\nu}^{\mu\nu} - \eta_{\mu\nu}^{\mu\nu}) \right] \gamma_{\alpha} + \eta_{\nu}^{\mu\nu} \gamma_{5} \right\} \frac{1 - j}{2},
\]

where the $v^\mu$ denotes the four velocity associated to the superfields. The superfields $J$, $J^\mu$, $J^{\mu\nu}$ are functions of the radial quantum numbers $n$, the fields in a definite superfield have the same $n$, and form a multiplet. Here (and subsequential) we write down the bottomonium states explicitly, and smear the radial numbers $n$ in the fields for simplicity, the corresponding ones for the charmonium states are obtained with an simple replacement. We multiply the bottomonium fields $\Upsilon_3^{\mu\nu}$, $\Upsilon_2^{\mu\nu}$, $\Upsilon_1^{\mu\nu}$, $\chi_{\mu\nu}, \cdots$ with a factor $\sqrt{M_{\Upsilon_3}}, \sqrt{M_{\Upsilon_2}}, \sqrt{M_{\Upsilon_1}}, \sqrt{M_{\chi_2}}, \cdots$, and they have dimension of mass $\frac{1}{2}$.

The superfields $J^{\mu_1\cdots\mu_L}$ are completely symmetric, traceless and orthogonal to the velocity, furthermore, they have the following properties under the parity, charge conjunction, heavy quark
spin transformations,

\[ J^{\mu_1...\mu_L} \rightarrow P \gamma^0 J^{\mu_1...\mu_L} \gamma^0, \]

\[ J^{\mu_1...\mu_L} \rightarrow (-1)^{L+1} C [J^{\mu_1...\mu_L}]^T C, \]

\[ J^{\mu_1...\mu_L} \rightarrow S J^{\mu_1...\mu_L} S^T, \]

\[ v^\mu \rightarrow v^\mu, \tag{4} \]

where \( S, S' \in SU(2) \) heavy quark spin symmetry groups, and \( [S, \gamma] = [S', \gamma] = 0 \).

The vector meson transitions between the \( m \) and \( n \) heavy quarkonium states can be described by the following Lagrangians,

\[ \mathcal{L}_{SS} = \sum_{m,n} \delta(m, n) \text{Tr} \left[ \vec{J}(m) \sigma_{\mu\nu} J(n) \right] F^{\mu\nu}, \]

\[ \mathcal{L}_{SP} = \sum_{m,n} \delta(m, n) \text{Tr} \left[ \vec{J}(m) J^\mu(n) + \vec{J}(n) J^\mu(m) \right] V^\mu, \]

\[ \mathcal{L}_{PD} = \sum_{m,n} \delta(m, n) \text{Tr} \left[ \vec{J}_{\mu\nu}(m) J_{\mu\nu}(n) + \vec{J}_{\mu\nu}(n) J_{\mu\nu}(m) \right] V^\mu, \tag{5} \]

where \( \vec{J}_{\mu\nu} = \gamma^0 J_{\mu\nu} \gamma^0, V^\mu = F^{\mu\nu} v_\nu, F^{\mu\nu} = \sqrt{2} \left( \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \right) + (\partial_\mu \phi_\nu - \partial_\nu \phi_\mu), \) and the \( \delta(m, n) \) is the coupling constant. The present Lagrangians are analogous to the Lagrangians which describe the radiative transitions between the \( m \) and \( n \) heavy quarkonium states \(^{32, 31, 35}\).

The Lagrangians \( \mathcal{L}_{SP} \) and \( \mathcal{L}_{PD} \) preserve parity, charge conjugation, gauge invariance and heavy quark spin symmetry, while the Lagrangian \( \mathcal{L}_{SS} \) violates the heavy quark symmetry. The effective Lagrangians \( \mathcal{L}_{SP} \) and \( \mathcal{L}_{PD} \) describing the electric dipole \( E_1 \)-like transitions can be realized in the leading order \( \mathcal{O}(1) \), while the heavy quark spin violation effective Lagrangian \( \mathcal{L}_{SS} \) describing the magnetic dipole \( M_1 \)-like transitions can be realized in the next-to-leading order \( \mathcal{O}(1/m_Q) \). In the heavy quark limit, the contributions of the order \( \mathcal{O}(1/m_Q) \) are greatly suppressed, and we expect that the flavor and spin violation corrections of the order \( \mathcal{O}(1/m_Q) \) to the effective Lagrangians \( \mathcal{L}_{SP} \) and \( \mathcal{L}_{PD} \) are smaller than (or not as large as) the leading order contributions.

From the heavy quark effective Lagrangians \( \mathcal{L}_{SS}, \mathcal{L}_{SP} \) and \( \mathcal{L}_{PD} \), we can obtain the vector meson decay widths \( \Gamma \),

\[ \Gamma = \frac{1}{2j + 1} \sum \frac{k_V}{8\pi M^2} |T|^2, \tag{6} \]

where the \( T \) denotes the scattering amplitude, the \( k_V \) is the momentum of the final states in the center of mass coordinate, the \( \sum \) denotes the sum of all the polarization vectors, the \( j \) is the total angular momentum of the initial state, and the \( M \) is the mass of the initial state. The summation of the polarization vectors \( \epsilon_q(\lambda, p), \epsilon_{\mu\nu}(\lambda, p), \epsilon_{\mu\nu\rho}(\lambda, p) \) of the states with the total angular momentum \( j = 1, 2, 3 \) respectively results in the following three formulæ,

\[ \sum_{\lambda} \epsilon_\mu^q \epsilon_q = g_{\mu\nu} = g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}, \]

\[ \sum_{\lambda} \epsilon_\mu^q \epsilon_\alpha = \bar{g}_{\mu\alpha} \bar{g}_{\nu\beta} \bar{g}_{\gamma\delta} + \bar{g}_{\mu\beta} \bar{g}_{\nu\alpha} \bar{g}_{\gamma\delta} - \frac{\bar{g}_{\mu\nu} \bar{g}_{\alpha\beta}}{3}, \]

\[ \sum_{\lambda} \epsilon_{\mu\nu} \epsilon_{\alpha\beta} = \frac{1}{6} \left( \bar{g}_{\mu\alpha} \bar{g}_{\nu\beta} \bar{g}_{\rho\delta} + \bar{g}_{\mu\beta} \bar{g}_{\nu\alpha} \bar{g}_{\rho\delta} + \bar{g}_{\mu\alpha} \bar{g}_{\nu\beta} \bar{g}_{\rho\delta} + \bar{g}_{\mu\beta} \bar{g}_{\nu\alpha} \bar{g}_{\rho\delta} + \bar{g}_{\mu\alpha} \bar{g}_{\nu\beta} \bar{g}_{\rho\delta} + \bar{g}_{\mu\beta} \bar{g}_{\nu\alpha} \bar{g}_{\rho\delta} + \frac{\bar{g}_{\mu\nu} \bar{g}_{\alpha\beta} \bar{g}_{\rho\delta}}{3} \right), \tag{7} \]

and we use the FeynCalc to carry out the contractions of the Lorentz indexes.

### 3 Numerical Results

The heavy quarkonium states listed in the Review of Particle Physics are far from complete and leave a fill the spectroscopy \(^{36}\). Recently, the Belle collaboration observed the bottomonium...
The measured masses are $M_{h_8(1P)} = (9898.25 \pm 1.06^{+1.03}_{-1.07})$ MeV and $M_{h_8(2P)} = (10259.76 \pm 0.64^{+1.43}_{-1.03})$ MeV, respectively. There have been several theoretical works on the spectroscopy of the charmonium and bottomonium states, such as the relativized potential model (Godfrey-Isgur model) [38, 39], the Cornell potential model, the logarithmic potential model, the power-law potential model, the QCD-motivated potential model [40, 41], the relativistic quark model based on a quasipotential approach in QCD [42], the Cornell potential model combined with heavy quark mass expansion [43], the screened potential model [44, 45], the potential non-relativistic QCD model [46], the confining potential model with the Bethe-Salpeter equation [47], etc.

In Tables 1-2, we list the experimental values of the charmonium and bottomonium states compared with some theoretical predictions [36, 37, 38, 39, 40, 41]. For the newly-observed charmonium-like states, there are hot controversies about their natures, and we focus on the traditional charmonium scenario, although such identifications are not superior to others. One can consult Refs. [27, 31] for detailed discussions.

We calculate the vector meson decay widths $\Gamma$ using the FeynCalc to carry out the contractions of the Lorentz indexes in the summation of the polarization vectors. In calculations, the masses of the charmonium and bottomonium states are taken as the experimental values from the Particle Data Group [36], see Tables 1-2; for the unobserved charmonium and bottomonium states, we take the values from the screened potential model [44, 45].

The numerical values of the vector meson transition widths are presented in Tables 3-11, where we retain the unknown coupling constants $\delta(m, n)$ among the multiplets of the radial quantum numbers $m$ and $n$. In general, we expect to fit the parameters $\delta(m, n)$ to the precise experimental data, however, in the present time the experimental data are rare. In Tables 4, 12-18, we present the ratios of the vector meson decay widths among the charmonium (and bottomonium) states.

In Ref. [28], Voloshin observes that the ratio of the vector meson decay widths $\chi_{b1,2}(2P) \to \Upsilon(1S)\omega$ can be approximated by the ratio of the $S$-wave phase factor,

$$\frac{\Gamma(\chi_{b2}(2P) \to \Upsilon(1S)\omega)}{\Gamma(\chi_{b1}(2P) \to \Upsilon(1S)\omega)} = 1.4 (1.467),$$

where the value in the bracket is the theoretical prediction from the heavy quark effective theory. The agreement between the approximated experimental data and the theoretical calculation based on the heavy quark effective theory is rather good, and the heavy quark effective theory works rather well. The ratios presented in Tables 3-4, 12-18 can be confronted with the experimental data in the future at the BESIII, KEK-B, RHIC, PANDA and LHCb, and put powerful constraints in identifying the $X, Y, Z$ charmonium-like (or bottomonium-like) mesons.

In this article, we do not distinguish between the contributions of the three-gluon emissions and the open-heavy mesons re-scatterings, smear the underlying dynamical details, and introduce momentum-independent coupling constants, which can be fitted to the precise experimental data in the future. There is a relative $P$-wave between the two final-state mesons. In calculations, we assume the decay widths $\Gamma \propto k_V$, the uncertainties originate from the masses can be estimated as

$$\frac{\Delta \Gamma}{\Gamma} \approx \frac{\Delta k_V^3}{k_V^3} = 3 \frac{\Delta M_{\omega/\phi}}{M_{\omega/\phi}}.$$
| State | Experimental | Theoretical | Theoretical | Theoretical |
|-------|--------------|-------------|-------------|-------------|
| $J/\psi(1S_1)$ | 3096.916 | 3097 | 3090 | 3098 |
| $\eta_c(1S_0)$ | 2980.3 | 2979 | 2982 | 2975 |
| $\psi(2S_1)$ | 3686.09 | 3673 | 3672 | 3676 |
| $\eta_c(2S_0)$ | 3637 | 3623 | 3630 | 3623 |
| $\psi(3S_1)$ | 4039 | 4022 | 4072 | 4100 |
| $\eta_c(3S_0)$ | 3942 | 3991 | 4043 | 4064 |
| $\psi(4S_1)$ | 4263 | 4273 | 4406 | 4450 |
| $\eta_c(4S_0)$ | 4250 | 4384 | 4384 | 4425 |
| $\psi(5S_1)$ | ? 4421 | 4463 | 4463 | 4463 |
| $\eta_c(5S_0)$ | ? 4664 | 4608 | 4608 | 4608 |
| $\psi(6S_1)$ | ? 4664 | 4595 | 4595 | 4595 |
| $\eta_c(6S_0)$ | ? 4664 | | | |
| $\chi_c(1P_2)$ | 3556.20 | 3554 | 3556 | 3550 |
| $\chi_c(1P_1)$ | 3510.66 | 3510 | 3505 | 3510 |
| $\chi_c(1P_0)$ | 3414.75 | 3433 | 3424 | 3445 |
| $h_c(1P_1)$ | 3525.42 | 3519 | 3516 | 3517 |
| $\chi_c(2P_2)$ | 3929 | 3937 | 3972 | 3979 |
| $\chi_c(2P_1)$ | 3691.6 | 3901 | 3925 | 3935 |
| $\chi_c(2P_0)$ | 3842 | 3852 | 3916 | 3916 |
| $h_c(2P_1)$ | 3908 | 3934 | 3956 | 3956 |
| $\chi_c(3P_2)$ | ? 4156 | 4208 | 4317 | 4337 |
| $\chi_c(3P_1)$ | 4178 | 4217 | 4271 | 4317 |
| $\chi_c(3P_0)$ | 4131 | 4202 | 4292 | 4292 |
| $h_c(3P_1)$ | 4184 | 4279 | 4318 | 4318 |
| $\psi(1D_2)$ | 3799 | 3806 | 3849 | 3849 |
| $\psi(1D_1)$ | 3798 | 3800 | 3838 | 3838 |
| $\eta_c(1D_2)$ | 3787 | 3785 | 3819 | 3819 |
| $\eta_c(1D_1)$ | 3796 | 3799 | 3837 | 3837 |
| $\psi(2D_3)$ | 4103 | 4167 | 4217 | 4217 |
| $\psi(2D_2)$ | 4100 | 4158 | 4208 | 4208 |
| $\psi(2D_1)$ | 4089 | 4142 | 4194 | 4194 |
| $\eta_c(2D_2)$ | 4099 | 4158 | 4208 | 4208 |
| $\psi(3D_3)$ | ? 4361 | 4331 | 4327 | 4327 |
| $\psi(3D_2)$ | ? 4361 | 4331 | 4327 | 4327 |
| $\psi(3D_1)$ | ? 4361 | 4331 | 4327 | 4327 |
| $\eta_c(3D_2)$ | ? 4361 | 4331 | 4327 | 4327 |

Table 1: Experimental and theoretical mass spectrum of the charmonium states, where the unit is MeV.
| State | Experimental | Theoretical | Theoretical |
|-------|--------------|-------------|-------------|
|       | [36] [37]    | [43]        | [38]        |
| 1S $\Upsilon(1^3S_1)$ | 9460.30      | 9460        | 9460        |
| $\eta_b(1^1S_0)$ | 9390.9       | 9389        | 9400        |
| 2S $\Upsilon(2^3S_1)$ | 10023.26     | 10016       | 10000       |
| $\eta_b(2^1S_0)$ | 9987         | 9987        | 9980        |
| 3S $\Upsilon(3^3S_1)$ | 10355.2      | 10351       | 10350       |
| $\eta_b(3^1S_0)$ | 10330        | 10330       | 10340       |
| 4S $\Upsilon(4^3S_1)$ | 10579.4      | 10611       | 10630       |
| $\eta_b(4^1S_0)$ | 10595        | 10595       |             |
| 5S $\Upsilon(5^3S_1)$ | 10865        | 10831       | 10880       |
| $\eta_b(5^1S_0)$ | 10817        | 10817       |             |
| 6S $\Upsilon(6^3S_1)$ | 11019        | 11023       | 11100       |
| $\eta_b(6^1S_0)$ | 11011        |             |             |
| 7S $\Upsilon(7^3S_1)$ | 11193        |             |             |
| $\eta_b(7^1S_0)$ | 11183        |             |             |
| 1P $\chi_{b2}(1^3P_2)$ | 9912.21      | 9918        | 9900        |
| $\chi_{b1}(1^3P_1)$ | 9892.78      | 9897        | 9880        |
| $\chi_{b0}(1^3P_0)$ | 9859.44      | 9865        | 9850        |
| $h_b(1^1P_1)$ | 9898.25      | 9903        | 9880        |
| 2P $\chi_{b2}(2^3P_2)$ | 10268.65     | 10269       | 10260       |
| $\chi_{b1}(2^3P_1)$ | 10255.46     | 10251       | 10250       |
| $\chi_{b0}(2^3P_0)$ | 10232.5      | 10226       | 10230       |
| $h_b(2^1P_1)$ | 10259.76     | 10256       | 10250       |
| 3P $\chi_{b2}(3^3P_2)$ |             |             |             |
| $\chi_{b1}(3^3P_1)$ |             |             |             |
| $\chi_{b0}(3^3P_0)$ |             |             |             |
| $h_b(3^1P_1)$ |             |             |             |
| 4P $\chi_{b2}(4^3P_2)$ | 10767        |             |             |
| $\chi_{b1}(4^3P_1)$ | 10753        |             |             |
| $\chi_{b0}(4^3P_0)$ | 10732        |             |             |
| $h_b(4^1P_1)$ | 10757        |             |             |
| 5P $\chi_{b2}(5^3P_2)$ | 10965        |             |             |
| $\chi_{b1}(5^3P_1)$ | 10951        |             |             |
| $\chi_{b0}(5^3P_0)$ | 10933        |             |             |
| $h_b(5^1P_1)$ | 10955        |             |             |
| 1D $\Upsilon_3(1^3D_3)$ |             |             |             |
| $\Upsilon_2(1^3D_2)$ |             |             |             |
| $\Upsilon_1(1^3D_1)$ |             |             |             |
| $\eta_{b2}(1^1D_2)$ |             | 10156       | 10160       |
| 2D $\Upsilon_3(2^3D_3)$ |             |             |             |
| $\Upsilon_2(2^3D_2)$ |             |             |             |
| $\Upsilon_1(2^3D_1)$ |             |             |             |
| $\eta_{b2}(2^1D_2)$ |             |             |             |
| 3D $\Upsilon_3(3^3D_3)$ |             |             |             |
| $\Upsilon_2(3^3D_2)$ |             |             |             |
| $\Upsilon_1(3^3D_1)$ |             |             |             |
| $\eta_{b2}(3^1D_2)$ |             |             |             |
| 4D $\Upsilon_3(4^3D_3)$ |             |             |             |
| $\Upsilon_2(4^3D_2)$ |             |             |             |
| $\Upsilon_1(4^3D_1)$ |             |             |             |
| $\eta_{b2}(4^1D_2)$ |             |             |             |
| 5D $\Upsilon_3(5^3D_3)$ |             |             |             |
| $\Upsilon_2(5^3D_2)$ |             |             |             |
| $\Upsilon_1(5^3D_1)$ |             |             |             |
| $\eta_{b2}(5^1D_2)$ |             |             |             |
\[ \Gamma(\psi \rightarrow \eta_c \omega) \quad \Gamma(\eta_c \rightarrow \psi \omega) \quad \Gamma(\eta_c \rightarrow \psi \omega) \quad \Gamma(\eta_c \rightarrow \eta_c \omega) \]

\begin{tabular}{|c|c|c|c|}
\hline
 & \Gamma(\psi \rightarrow \eta_c \omega) [kV] & \Gamma(\eta_c \rightarrow \psi \omega) [kV] & \Gamma(\eta_c \rightarrow \eta_c \omega) [kV] \\
\hline
3S \rightarrow 1S & 0.500 [616] & 0.567 [283] & 1.132 \\
4S \rightarrow 1S & 0.843 [858] & 1.179 [728] & 2.347 \\
4S \rightarrow 1S & 0.386 [655] & 0.763 [461] & 1.976 \\
5S \rightarrow 1S & 1.113 [1007] & 2.970 [927] & 2.667 \\
5S \rightarrow 1S & 0.548 [844] & 1.410 [743] & 2.570 \\
6S \rightarrow 1S & 1.579 [1215] & 3.808 [1064] & 2.412 \\
6S \rightarrow 1S & 0.810 [1088] & 1.901 [911] & 2.348 \\
6S \rightarrow 2S & 0.494 [589] & 0.927 [415] & 1.878 \\
\hline
\end{tabular}

Table 3: The ratios of the vector meson transitions of the $S$-wave to the $S$-wave charmonium states, where the wide-hat denotes the corresponding $\phi$ transitions. The units of the widths and the $k_V$ are $\delta^2(m,n)$ and MeV, respectively.

\[ \Gamma(\Upsilon \rightarrow \eta_b \omega) \quad \Gamma(\eta_b \rightarrow \Upsilon \omega) \quad \Gamma(\eta_b \rightarrow \Upsilon \omega) \quad \Gamma(\eta_b \rightarrow \eta_b \omega) \]

\begin{tabular}{|c|c|c|c|c|}
\hline
 & \Gamma(\Upsilon \rightarrow \eta_b \omega) [kV] & \Gamma(\eta_b \rightarrow \Upsilon \omega) [kV] & \Gamma(\eta_b \rightarrow \eta_b \omega) [kV] \\
\hline
3S \rightarrow 1S & 0.499 [537] & 0.890 [363] & 1.785 \\
4S \rightarrow 1S & 1.036 [844] & 2.694 [777] & 2.599 \\
4S \rightarrow 1S & 0.410 [576] & 0.954 [471] & 2.326 \\
5S \rightarrow 1S & 1.940 [1164] & 4.623 [1038] & 2.383 \\
5S \rightarrow 1S & 0.924 [991] & 2.114 [838] & 2.287 \\
5S \rightarrow 2S & 0.317 [382] & 0.281 [127] & 0.886 \\
6S \rightarrow 1S & 2.547 [1321] & 6.721 [1244] & 2.639 \\
6S \rightarrow 1S & 1.248 [1174] & 3.255 [1085] & 2.609 \\
6S \rightarrow 2S & 0.655 [641] & 1.667 [575] & 2.545 \\
7S \rightarrow 1S & 3.339 [1492] & 8.931 [1415] & 2.675 \\
7S \rightarrow 1S & 1.663 [1365] & 4.422 [1280] & 2.660 \\
7S \rightarrow 2S & 1.098 [868] & 2.921 [811] & 2.660 \\
7S \rightarrow 2S & 0.445 [609] & 1.096 [524] & 2.464 \\
7S \rightarrow 3S & 0.285 [349] & 0.604 [260] & 2.119 \\
\hline
\end{tabular}

Table 4: The ratios of the vector meson transitions of the $S$-wave to the $S$-wave bottomonium states, where the wide-hat denotes the corresponding $\phi$ transitions. The units of the widths and the $k_V$ are $\delta^2(m,n)$ and MeV, respectively.

\[ \Gamma(\psi \rightarrow \chi_{2\omega}) \quad \Gamma(\psi \rightarrow \chi_{1\omega}) \quad \Gamma(\psi \rightarrow \chi_0 \omega) \quad \Gamma(\eta_c \rightarrow h_{\omega}) \]

\begin{tabular}{|c|c|c|c|c|}
\hline
 & \psi \rightarrow \chi_{2\omega} [kV] & \psi \rightarrow \chi_{1\omega} [kV] & \psi \rightarrow \chi_0 \omega [kV] & \eta_c \rightarrow h_{\omega} [kV] \\
\hline
4S \rightarrow 1P & 1.668 [293] & & & \\
5S \rightarrow 1P & 9.712 [330] & 7.669 [415] & 3.747 [558] & 24.238 [432] \\
6S \rightarrow 1P & 26.341 [688] & 17.595 [739] & 7.056 [839] & 41.966 [641] \\
6S \rightarrow 2P & 8.766 [379] & 6.705 [469] & 3.122 [620] & 11.381 [284] \\
\hline
\end{tabular}

Table 5: The widths of the vector meson transitions of the $S$-wave to the $P$-wave charmonium states, where the wide-hat denotes the corresponding $\phi$ transitions. The units of the widths and the $k_V$ are $10^{-2}\delta^2(m,n)$ and MeV, respectively.
| $\Gamma$ | $\chi_2 \to \psi \omega \ [k_\gamma]$ | $\chi_1 \to \psi \omega \ [k_\gamma]$ | $\chi_0 \to \psi \omega \ [k_\gamma]$ | $h_c \to \eta_\omega \ [k_\gamma]$ |
|---------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 2P → 1S | 4.132 [251]                      | 3.413 [211]                      | 7.827 [436]                      | 7.827 [436]                      |
| 3P → 1S | 14.918 [810]                     | 13.788 [846]                     | 12.863 [619]                     | 17.935 [778]                     |
| 3P → 1S | 5.076 [380]                      | 4.051 [310]                      | 3.183 [248]                      | 7.596 [542]                      |

Table 7: The widths of the vector meson transitions of the P-wave to the S-wave charmonium states, where the wide-hat denotes the corresponding $\phi$ transitions. The units of the widths and the $k_\gamma$ are $10^{-2} \delta^2 (m, n)$ and MeV, respectively.

| $\Gamma$ | $\chi_2 \to \Upsilon_3 \omega \ [k_\gamma]$ | $\chi_2 \to \Upsilon_2 \omega \ [k_\gamma]$ | $\chi_1 \to \Upsilon_2 \omega \ [k_\gamma]$ | $\chi_1 \to \Upsilon_1 \omega \ [k_\gamma]$ | $\chi_0 \to \Upsilon_1 \omega \ [k_\gamma]$ | $h_b \to \eta_\omega \ [k_\gamma]$ |
|---------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 2P → 1S | 3.630 [194]                      | 2.475 [135]                      | 4.358 [361]                      | 22.599 [781]                     | 22.599 [781]                     | 22.599 [781]                     |
| 3P → 1S | 19.053 [705]                     | 18.105 [683]                     | 16.804 [653]                     | 22.599 [781]                     | 22.599 [781]                     | 22.599 [781]                     |
| 3P → 1S | 5.369 [337]                      | 4.509 [288]                      | 3.109 [203]                      | 8.085 [478]                      | 8.085 [478]                      | 8.085 [478]                      |
| 4P → 1S | 34.608 [982]                     | 33.565 [966]                     | 31.897 [942]                     | 39.146 [1048]                    | 39.146 [1048]                    | 39.146 [1048]                    |
| 4P → 1S | 15.382 [767]                     | 14.770 [746]                     | 13.843 [714]                     | 17.981 [851]                     | 17.981 [851]                     | 17.981 [851]                     |
| 5P → 1S | 51.706 [1196]                    | 50.475 [1182]                    | 48.572 [1163]                    | 57.098 [1237]                    | 57.098 [1237]                    | 57.098 [1237]                    |
| 5P → 2S | 24.774 [1030]                    | 24.084 [1012]                    | 23.129 [990]                     | 27.711 [1100]                    | 27.711 [1100]                    | 27.711 [1100]                    |
| 5P → 2S | 11.376 [501]                     | 10.621 [477]                     | 9.650 [444]                      | 12.787 [544]                     | 12.787 [544]                     | 12.787 [544]                     |

Table 8: The widths of the vector meson transitions of the P-wave to the S-wave bottomonium states, where the wide-hat denotes the corresponding $\phi$ transitions. The units of the widths and the $k_\gamma$ are $10^{-2} \delta^2 (m, n)$ and MeV, respectively.

| $\Gamma$ | $\nu_3 \to \chi_2 \omega \ [k_\gamma]$ | $\nu_2 \to \chi_2 \omega \ [k_\gamma]$ | $\nu_2 \to \chi_1 \omega \ [k_\gamma]$ | $\nu_2 \to \chi_0 \omega \ [k_\gamma]$ | $\nu_2 \to \chi_3 \omega \ [k_\gamma]$ | $\eta_2 \to \eta_3 \omega \ [k_\gamma]$ |
|---------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 3D → 1P | 2.603 [209]                      | 0.077 [197]                      | 2.147 [185]                      | 4.970 [472]                      | 4.970 [472]                      | 4.970 [472]                      |

Table 9: The widths of the vector meson transitions of the D-wave to the P-wave charmonium states. The units of the widths and the $k_\gamma$ are $10^{-2} \delta^2 (m, n)$ and MeV, respectively.
Table 11: The widths of the vector meson transitions of the $D$-wave to the $P$-wave bottomonium states, where the wide-hat denotes the corresponding $\phi$ transitions. The units of the widths and the $k_V$ are $10^{-2}\delta^2(m,n)$ and MeV, respectively.

| $\Gamma$ | $\Upsilon_3 \rightarrow \chi_2\omega$ | $Y_2 \rightarrow \chi_2\omega$ | $Y_2 \rightarrow \chi_1\omega$ | $Y \rightarrow \chi_2\omega$ | $Y \rightarrow \chi_1\omega$ | $Y \rightarrow \chi_0\omega$ | $\eta_2 \rightarrow h_\omega$ |
|----------|-----------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 3D $\rightarrow$ 1P | | 0.390 | | | | | 2.120 |
| 4D $\rightarrow$ 1P | 13.116 | 3.225 | 10.464 | 0.350 | 5.699 | 8.622 | 13.705 |
| [553] | [547] | [577] | [539] | [569] | [619] | [570] |
| 5D $\rightarrow$ 1P | 24.166 | 5.986 | 18.855 | 0.652 | 10.341 | 15.001 | 24.841 |
| [807] | [802] | [826] | [795] | [820] | [860] | [820] |
| 5D $\rightarrow$ 2P | 8.999 | 2.206 | 7.244 | 0.239 | 3.935 | 6.036 | 9.466 |
| [518] | [510] | [547] | [499] | [538] | [599] | [539] |

Table 12: The ratios of the vector meson transitions of the $S$-wave to the $P$-wave charmonium states, where the wide-hat denotes the corresponding $\phi$ transitions, $\tilde{\Gamma} = \frac{\Gamma}{\Gamma(\psi \rightarrow \chi_2\omega)}$.

| $\Gamma$ | $\psi \rightarrow \chi_2\omega$ | $\psi \rightarrow \chi_1\omega$ | $\psi \rightarrow \chi_0\omega$ | $\eta_c \rightarrow h_c\omega$ |
|----------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 5S $\rightarrow$ 1P | 1 | 0.790 | 0.386 | 2.496 |
| 6S $\rightarrow$ 1P | 1 | 0.668 | 0.268 | 1.593 |
| 6S $\rightarrow$ 1P | 0.333 | 0.255 | 0.119 | 0.432 |

Table 13: The ratios of the vector meson transitions of the $S$-wave to the $P$-wave bottomonium states, where the wide-hat denotes the corresponding $\phi$ transitions, $\tilde{\Gamma} = \frac{\Gamma}{\Gamma(Y \rightarrow \chi_2\omega)}$.

| $\Gamma$ | $Y \rightarrow \chi_2\omega$ | $Y \rightarrow \chi_1\omega$ | $Y \rightarrow \chi_0\omega$ | $\eta_b \rightarrow h_b\omega$ |
|----------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 5S $\rightarrow$ 1P | 1 | 0.653 | 0.248 | 1.522 |
| 6S $\rightarrow$ 1P | 1 | 0.635 | 0.232 | 1.829 |
| 6S $\rightarrow$ 1P | 0.322 | 0.219 | 0.087 | 0.604 |
| 7S $\rightarrow$ 1P | 1 | 0.627 | 0.224 | 1.811 |
| 7S $\rightarrow$ 1P | 0.436 | 0.277 | 0.101 | 0.793 |
| 7S $\rightarrow$ 2P | 1 | 0.641 | 0.237 | 1.788 |

Table 14: The ratios of the vector meson transitions of the $P$-wave to the $S$-wave charmonium states, where the wide-hat denotes the corresponding $\phi$ transitions, $\tilde{\Gamma} = \frac{\Gamma}{\Gamma(\chi_2 \rightarrow \psi\omega)}$.

| $\Gamma$ | $\chi_2 \rightarrow \psi\omega$ | $\chi_1 \rightarrow \psi\omega$ | $\chi_0 \rightarrow \psi\omega$ | $h_c \rightarrow \eta_c\omega$ |
|----------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 2P $\rightarrow$ 1S | 1 | 0.826 | | 1.894 |
| 3P $\rightarrow$ 1S | 1 | 0.924 | 0.862 | 1.202 |
| 3P $\rightarrow$ 1S | 0.340 | 0.272 | 0.213 | 0.509 |

Table 15: The ratios of the vector meson transitions of the $P$-wave to the $S$-wave bottomonium states, where the wide-hat denotes the corresponding $\phi$ transitions, $\tilde{\Gamma} = \frac{\Gamma}{\Gamma(Y \rightarrow \chi_2\omega)}$.
Table 15: The ratios of the vector meson transitions of the $P$-wave to the $S$-wave bottomonium states, where the wide-hat denotes the corresponding $\phi$ transitions, $\tilde{\Gamma} = \frac{\Gamma}{\Gamma(\chi_2 \rightarrow \Upsilon \omega)}$.

| $\Gamma$ | $\chi_2 \rightarrow \Upsilon \omega$ | $\chi_1 \rightarrow \Upsilon \omega$ | $\chi_0 \rightarrow \Upsilon \omega$ | $h_b \rightarrow \eta_2 \omega$ |
|----------|---------|---------|---------|---------------|
| $2P \rightarrow 1S$ | 1      | 0.682   |         | 2.027        |
| $3P \rightarrow 1S$ | 1      | 0.950   | 0.882   | 1.186        |
| $3P \rightarrow 1S$ | 0.282  | 0.237   | 0.163   | 0.424        |
| $4P \rightarrow 1S$ | 1      | 0.970   | 0.922   | 1.131        |
| $4P \rightarrow 1S$ | 0.444  | 0.427   | 0.400   | 0.520        |
| $5P \rightarrow 1S$ | 1      | 0.976   | 0.939   | 1.104        |
| $5P \rightarrow 1S$ | 0.479  | 0.466   | 0.447   | 0.536        |
| $5P \rightarrow 2S$ | 1      | 0.934   | 0.848   | 1.124        |

Table 16: The ratios of the vector meson transitions of the $P$-wave to the $D$-wave bottomonium states, $\tilde{\Gamma} = \frac{\Gamma}{\Gamma(\chi_2 \rightarrow \Upsilon \omega)}$.

| $\Gamma$ | $\chi_2 \rightarrow \Upsilon \omega$ | $\chi_3 \rightarrow \Upsilon \omega$ | $\chi_2 \rightarrow \chi_2 \omega$ | $\chi_1 \rightarrow \chi_2 \omega$ | $\chi_1 \rightarrow \chi_2 \omega$ | $\chi_0 \rightarrow \chi_2 \omega$ | $h_b \rightarrow \eta_2 \omega$ |
|----------|---------|---------|---------|---------|---------|---------|---------------|
| $3P \rightarrow 1D$ | 1      | 0.199   | 0.014   | 0.433   | 0.279   | 0.517   | 1.035        |

Table 17: The ratios of the vector meson transitions of the $D$-wave to the $P$-wave charmonium states, $\tilde{\Gamma} = \frac{\Gamma}{\Gamma(\psi_2 \rightarrow \chi_1 \omega)}$.

| $\Gamma$ | $\psi_3 \rightarrow \chi_2 \omega$ | $\psi_2 \rightarrow \chi_2 \omega$ | $\psi_2 \rightarrow \chi_1 \omega$ | $\psi_1 \rightarrow \chi_2 \omega$ | $\psi_1 \rightarrow \chi_2 \omega$ | $\psi_1 \rightarrow \chi_2 \omega$ | $\eta_2 \rightarrow \eta_2 \omega$ |
|----------|---------|---------|---------|---------|---------|---------|---------------|
| $3D \rightarrow 1P$ | $\frac{\Gamma}{\Gamma(\psi_3 \rightarrow \chi_2 \omega)}$ | 1      | 0.030   | 0.825   | 1.910   | 0.953   |

Table 18: The ratios of the vector meson transitions of the $D$-wave to the $P$-wave bottomonium states, where the wide-hat denotes the corresponding $\phi$ transitions, $\tilde{\Gamma} = \frac{\Gamma}{\Gamma(\chi_2 \rightarrow \Upsilon \omega)}$, while in the first line $\tilde{\Gamma} = \frac{\Gamma}{\Gamma(\chi_2 \rightarrow \chi_2 \omega)}$.

| $\Gamma$ | $\Upsilon_3 \rightarrow \chi_2 \omega$ | $\Upsilon_2 \rightarrow \chi_2 \omega$ | $\Upsilon_2 \rightarrow \chi_1 \omega$ | $\Upsilon \rightarrow \chi_2 \omega$ | $\Upsilon \rightarrow \chi_2 \omega$ | $\Upsilon \rightarrow \chi_0 \omega$ | $\eta_2 \rightarrow \eta_2 \omega$ |
|----------|---------|---------|---------|---------|---------|---------|---------------|
| $3D \rightarrow 1P$ | 1      | 0.246   | 0.798   | 0.027   | 0.434   | 0.657   | 1.045        |
| $4D \rightarrow 1P$ | 1      | 0.247   | 0.780   | 0.027   | 0.428   | 0.621   | 1.028        |
| $5D \rightarrow 1P$ | 0.372  | 0.091   | 0.300   | 0.010   | 0.163   | 0.250   | 0.392        |
| $5D \rightarrow 2P$ | 1      | 0.218   | 0.939   | 0.019   | 0.467   | 0.927   | 1.166        |
distort the $k_V$ significantly. On the other hand, we should take into account the mass uncertainties in the case of soft $k_V$ (about 200 MeV), where variations of the masses maybe result in considerable uncertainties. At the present time, the heavy quarkonium states listed in the Review of Particle Physics are far from complete and do not fill the spectroscopy, we have to take the masses from the quark models, where the uncertainties are usually neglected, detailed error analysis is beyond the present work. In this article, we have neglected the corrections from the terms $O(1/m_Q^2)$ and $O(1/m_Q)$ for the Lagrangians $L_{SS}$ (as the spin-flipped Lagrangian $L_{SS}$ is of order $O(1/m_Q)$) and $L_{SP}$, $L_{PD}$ respectively in the heavy quark effective theory, which maybe result in uncertainties larger that of the masses.

4 Conclusion

In this article, we study the vector meson transitions among the charmonium and bottomonium states with the heavy quark effective theory in a systematic way, and make predictions for ratios among the $\omega$ and $\phi$ decay widths of a special multiplet to another multiplet, where the unknown couple constants $\delta(m, n)$ are canceled out with each other. The predictions can be confronted with the experimental data in the future at the BESIII, KEK-B, RHIC, PANDA and LHCb, and put powerful constraints in identifying the $X$, $Y$, $Z$ charmonium-like (or bottomonium-like) mesons.

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