Cosmic curvature from de Sitter equilibrium cosmology

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I show that the de Sitter Equilibrium cosmology generically predicts observable levels of curvature in the Universe today. The predicted value of the curvature, $\Omega_k$, depends only on the ratio of the density of non-relativistic matter to cosmological constant density, $\rho_m/\rho_\Lambda$, and the value of the curvature from the initial bubble that starts the inflation, $\Omega_k^B$. The result is independent of the scale of inflation, the shape of the potential during inflation, and many other details of the cosmology. Future cosmological measurements of $\rho_m/\rho_\Lambda$ and $\Omega_k$ will open up a window on the very beginning of our Universe and offer an opportunity to support or falsify the de Sitter Equilibrium cosmology.

The de Sitter Equilibrium ("dSE") cosmology is a framework for cosmology that pictures the Universe eternally fluctuating in an equilibrium state. In this picture phenomena similar to the cosmos we observe around us come about as fluctuations. The dSE framework assumes that the observed cosmic acceleration is driven by a true cosmological constant $\Lambda$ which causes the Universe to approach a "de Sitter space" at late times when the cosmological constant dominates the cosmic evolution. The de Sitter space is the equilibrium state. It has an equilibrium entropy given by $S_\Lambda = \pi(cH_\Lambda^3/l_P)^2$ which is known to be maximal [1], a temperature given by $T_\Lambda = k_B h H_\Lambda$ where $H_\Lambda = 8\pi G/3\rho_\Lambda \equiv \Lambda/3$ is the Hubble constant during the $\Lambda$ dominated phase and $l_P$ is the Planck length. Background on dSE cosmology, including how it evades the notorious “Boltzmann Brain” problem of equilibrium cosmologies may be found in [2] [3].

Cosmic inflation gives an established account of the very early history of the Universe. Inflation assumes that the early Universe was dominated by the potential energy of a scalar field, the "inflaton", which caused a period of accelerated cosmic expansion or "inflation" before decaying into ordinary matter through a process called reheating. A simple account of this inflationary epoch leads to a detailed set of predictions which so far have been born out by observations [4]. However, to understand inflation fully and make the predictions robust one must put inflation into a larger context that accounts for how inflation starts and assigns relative probabilities to different possible starts to inflation as well as other starts to the observed Universe that may not even involve inflation. dSE cosmology gives one way to do this.

All ideas for complete cosmological frameworks (including dSE and the popular “eternal inflation” picture) involve some ad hoc assumptions about how the underlying fundamental physics actually works [3]. Until we understand which assumptions about the fundamental physics are correct, the best any of these pictures can hope to provide is an opportunity for observational tests of one set of assumptions or another. This paper reports a prediction of the value of the cosmic curvature, $\Omega_k$, from the dSE picture. The predicted value depends only on the ratio of the non-relativistic matter density today $\rho_m^0$ to $\rho_\Lambda$ and is proportional to the initial curvature $(\Omega_k^B)$ provided by the bubble that started the period of cosmic inflation. The prediction is depicted in Fig. 1.

The prediction is interesting for a couple of reasons. Firstly, the result is only just consistent with current data [4], and uncertainties in $\Omega_k$, $\rho_m^0$ and $\rho_\Lambda$ will reduce substantially in the foreseeable future [5]. Future data showing a positive value for $\Omega_k$ would offer strong support for the dSE picture. Data consistent with $\Omega_k = 0$ with very small uncertainties (the cosmic variance limit of $\Delta \Omega_k \approx 0.00001$ may be achievable) would rule out the dSE picture except for extremely small values of $\Omega_k^B$.

FIG. 1. The predicted value of $\Omega_k^B$ vs. $\rho_m^0/\rho_\Lambda$ using the fiducial value $\Omega_k^B = 0.5$. Predications from other values of the bubble curvature are given by $\Omega_k = \Omega_k^B \times (\Omega_k^B/0.5)$.
Solid: The Hubble length vs. cosmic scale factor \( a \) (scaled by today’s values, \( cH_0^{-1} \) and \( a_0 \) respectively). The letters mark the times when the density of the inflaton \((I)\), relativistic matter \((r)\), non-relativistic matter \((m)\) and \((\Lambda)\) in turn start to dominate Eqn. 1. Dashed: The past horizon of observations deep in the de Sitter era. The shaded region shows events that will never be seen by the observer no matter how late the observation is made. Dot-Dashed: Maximum length scale affected by inflation.

Further study of the initial bubbles could even completely rule out the small \( \Omega_B \) dSE case, leading to the possibility of fully falsifying the dSE picture.

Secondly, the dSE prediction is interesting for its lack of dependence on many details of the cosmology. There is no dependence, for example, on the shape of the inflaton potential during inflation, the scale of inflation, the specifics and duration of the reheating and many other factors. As future data further constrain cosmological parameters, only \( \Omega_B \) will be in play, and this quantity would be subject to the sort of pressures just discussed.

I now derive the result shown in Fig. 2. The Friedmann equation

\[
H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left( \rho_I + \rho_r + \rho_m + \rho_k + \rho_\Lambda \right) \tag{1}
\]

relates the expansion rate \( H \) to the (effective) energy densities of, in order of appearance, the inflaton, relativistic matter, non-relativistic matter, curvature, and cosmological constant in a homogeneous and isotropic Universe. The scale factor \( a \) tracks the cosmic expansion. For the solutions I consider \( a \) is monotonic in time and I use it as a time variable in what follows. The “Hubble length” (\( \equiv cH^{-1} \)) is shown by the solid curve in Fig. 2 for the entire history of the Universe in a standard cosmological picture.

The dashed curve in Fig. 2 is the “past horizon”

\[
h_p(a_1) = a_1 \int_{a_1}^{a_\Lambda} \frac{da}{a^2 H} \tag{2}
\]
scenarios. According to the dSE bound the early part of inflation adjacent to the shaded region of Fig. 2 is excluded by the breakdown of the field theory description. The earliest that inflation, and thus the $cH^{-1}(a)$ curve, is allowed to start is right on the past horizon (dashed) curve. Since other factors exponentially favor inflation starting as early as possible the dSE bound is saturated in what follows: I start all inflation scenarios “on the past horizon”, giving $cH^{-1} = h_p(a)$ where superscripts designate the start of inflation.

Figure 3 is similar to Fig. 2 except that a multitude of different inflation models (not resolved on this plot) represent a wide range of inflation scales, inflaton potentials and reheating rates (but use the same values of $\rho_m^0$ and $\rho_\Lambda$). Also shown is the curvature length given by $cH_k^{-1}$ (triangles). The fact that the past horizon (which defines the dSE bound) tracks $cH_k^{-1}$ in such a simple manner over most of the history of the Universe leads to the simple predictions for $\Omega_k$, independently of many details of the cosmology.

There are actually 47 different solid curves (unresolved on the plot, and discussed in detail in [13]) which correspond to changing the inflation potential, the scale of inflation, and the rate of reheating (in the slowest cases, the reheating only completes at around $T = 10^{10}K$, just in time for Big Bang Nucleosynthesis). Each scenario starts right on the past horizon (dashed line) thus saturating the past horizon bound.

Figure 3 also shows the curvature radius $cH_k^{-1}$ ($H_k^2 \equiv 8\pi G/3\rho_k$), displaying information about $\rho_k$ on the plot and helping to illustrate how the simple dSE prediction comes about. The initial value $H_i^2$ is related to the bubble curvature and the initial Hubble parameter $H^i$ by $H_i^2 \equiv (H_i^2/H^i)^2$. Because by definition $\rho_k \propto 1/a^2$, $H_i^{-1} \propto a$, so $cH_k^{-1}$ runs parallel to the past horizon in Fig. 3 for most of the history of the Universe. This parallel behavior is crucial to my result. According to the dSE picture, the curve $cH^{-1}$ must start on the past horizon curve. The $cH_k^{-1}$ and $h_p$ curves evolve linearly together until near the current epoch (shown in Fig. 4), allowing a simple relationship to be established between $\Omega_k$ and $\Omega_k^B$.

Figure 4 illustrates how the crucial ingredients needed to compute $\Omega_k \equiv (H_k^2/H)^2$ are all contained in the curves and asymptotic behaviors discussed above. To the extent that we know $\rho_m^0$, $\rho_K$ and $\rho_\Lambda$ we know the shape of $H(a)$ around today, since $\rho_m$ and $\rho_\Lambda$ (and to a much lesser extent $\rho_K$) completely dominate Eqn. 1 during the current era. The quantity $H(a)$ appears in $\Omega_k$ as well the expression for $h_p(a)$ (Eqn. 2). Since $h_p(a)$ only deviates from its asymptotic values around the current era, only the $\rho$’s listed here are needed to determine this curve as well. The quantitative expressions for these various ingredients (all given above) can be combined to produce the main result:

$$\Omega_k = \frac{1}{g^2} \frac{\Omega_k^B}{\rho_\Lambda} = \frac{\Omega_k^B}{\rho_\Lambda + \rho_k}$$

where

$$g \left( \frac{\rho_m^0}{\rho_\Lambda}, \frac{\rho_k^0}{\rho_\Lambda} \right) = \int_0^\infty \frac{dx}{x^2 \sqrt{x^2 - 3\rho_m^0/\rho_\Lambda - x^2 - 2\rho_k^0/\rho_\Lambda + 1}}$$

Due to the appearance of $\rho_k^0$ on the right hand side,
Eqn. 3 give an implicit equation for \( \Omega_k \left( \rho_m^0 / \rho_\Lambda, \Omega_k^B \right) \) which can easily be solved numerically to give Fig. 4.

Attempts such as dSE to construct a complete theoretical framework for cosmology are in a primitive state. There are a number of ad hoc assumptions that go into the dSE framework (spelled out in [3]). The prediction I report here should be understood in that context. Probably the most popular cosmological framework is eternal inflation. That picture has its own particular assumptions, including the validity of the semiclassical inflaton field theory coupled to Einstein gravity over an infinite time and infinite volume. These infinities are critical to the mechanisms believed to cause eternal inflation to dominate the cosmos and any breakdown of these assumptions (such as replacing either of these infinities with arbitrarily large but finite values) would undermine much of the current thinking on this subject. The measure problem of eternal inflation that has so far undermined the ability of eternal inflation to actually make predictions is related to these infinities, but many are hopeful that this problem will eventually find a resolution without removing the infinities that are considered critical to the overall picture [8, 15].

The dSE framework is a finite alternative to eternal inflation. The finiteness has its own intrinsic appeal (for example, dSE replaces assumptions about initial conditions with an equilibrium state for the Universe), and the finiteness prevents measures from being a problem. The dSE picture is based on the idea that physics operates in such a way that the physical world, at its most fundamental, is described by a finite Hilbert space of dimension \( e^{2S} \). One then has to view any field theoretic degrees of freedom such as the inflaton or those of Einstein gravity as approximate, since it would take an infinite Hilbert space to describe them fully. The dSE framework makes assumptions about when the field theoretic description is a good one and when and how it breaks down. These assumptions are chosen to give a workable cosmology. One can think of the dSE cosmology as an attempt to construct a realistic finite cosmology by exploiting uncertainties about the underlying fundamental physics.

The existence of such great uncertainties may not be satisfying, but it is the state of the art. Under these conditions one can hope that by demanding a realistic cosmology insights might be gained into the nature of the underlying physics. This project was conceived in this spirit and it is in this context that I find the result very interesting. Unlike other models that give nonzero \( \Omega_k \) [16], this result is independent of the shape and scale of the inflaton potential during inflation, the nature of reheating and many other details.

If future data reveal positive values of \( \Omega_k \) close to the current bounds, that could be seen as support for the dSE picture. Such results could be interpreted as constraining the value of \( \Omega_k^B \), giving a direct window on the tunneling event that created the Universe we observe. Further work is needed to understand how low a value of \( \Omega_k^B \) can be tolerated in this picture, but it seems unlikely that values much smaller than the current bound will make sense. If this claim is born out, the result presented here offers an opportunity to falsify the dSE picture.

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[1] G. W. Gibbons and S. W. Hawking, Phys. Rev. D15, 2738 (1977).
[2] A. Albrecht and L. Sorbo, Phys. Rev. D70, 063528 (2004), hep-th/0405270.
[3] A. Albrecht, J. Phys. Conf. Ser. 174, 012006 (2009), arXiv:0906.1047 [gr-qc].
[4] E. Komatsu et al. (WMAP), Astrophys. J. Suppl. 192, 18 (2011) arXiv:1001.4538 [astro-ph.CO].
[5] A. Albrecht et al., (2006), astro-ph/0609591.
[6] A. J. Albrecht, N. Kaloper, and Y.-S. Song, (2002), arXiv:hep-th/0211221.
[7] T. Banks, (2007), arXiv:hep-th/0701146; B. Freivogel and M. Kleban, and L. Susskind, JHEP 10, 011 (2002), hep-th/0208013.
[8] A. D. Linde, Phys. Lett. B175, 395 (1986); A. H. Guth, J. Phys. A40, 6811 (2007) arXiv:hep-th/0702178.
[9] A. J. Albrecht, N. Kaloper, and Y.-S. Song, (2002), arXiv:hep-th/0211221; N. Arkani-Hamed, S. Dubovsky, A. Nikolis, E. Trincherini, and G. Villadoro, JHEP 05, 055 (2007), arXiv:0704.1814 [hep-th]; S. Dubovsky, L. Senatore, and G. Villadoro, ibid 04, 118 (2009) arXiv:0812.2246 [hep-th].
[10] T. Banks and W. Fischler, (2003), arXiv:astro-ph/0307459 [astro-ph].
[11] Daniel Phillips, private Communication 2011.
[12] Subtleties about bubble details and the onset of inflation are absorbed into \( \Omega_k^B \) and are discussed further in [13].
[13] A. Albrecht, (2011), in Preparation.
[14] Evolving dark energy could change the shape of \( h_p(a) \) but a constant \( \Lambda \) is essential to the dSE idea.
[15] A. Linde and M. Noorbala, JCAP 1009, 008 (2010) arXiv:1006.2170 [hep-th]. B. Freivogel and M. Kleban, JHEP 12, 019 (2009) arXiv:0903.2048 [hep-th]. A. De Simone et al., Phys. Rev. D82, 063520 (2010) arXiv:0808.3778 [hep-th].
[16] J. Garriga, Phys. Rev. D54, 4764 (1996), arXiv:gr-qc/9609025; A. D. Linde, M. Sasaki, and T. Tanaka, Phys. Rev. D59, 123522 (1999) arXiv:astro-ph/9901135; M. Barnard and A. Albrecht, (2004), arXiv:hep-th/0409082; B. Freivogel et al., JHEP 03, 039 (2006) arXiv:hep-th/0505232.