Combined Plate-Pile Foundations Settlement Calculation Under Cyclic Loading

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Abstract. The purpose of the study is to develop a method for calculating the settlement of a combined plate-pile foundation, with taking into account the effect from the repetitive cyclic loads in the process of construction and operation of buildings and structures. Experimental researches on plate-pile foundations were conducted in laboratory tanks and the field in order to find basic laws of such foundations behavior at cyclic loading. A method has been developed for calculating the estimated subsidence of the soil base of slabs-piles, taking into account the complex stress-strain state in the system of “plate grillage - ground between the piles - ground base” under cyclic loading. The improved method for calculating the settlement of plate-pile foundations under the cyclic load action allows to increase reliability, design bearing capacity, reduce the settlement, and as a result to obtain more cost-effective design solutions by saving materials and time during the construction of this type foundations.

Keywords: cyclic loads, plate-pile foundation, weak soil, base settlement, efforts, deformations.

1 Introduction
In the modern world, the compacted urban building development has formed a trend to the construct buildings and structures with increased number of storeys and to develop construction sites with weak physical and mechanical characteristics of soils that were previously not suitable for construction. In such cases, using the combined plate-pile foundations is one of the most effective ways to increase the bearing capacity and reduce the foundations ground base settlement of buildings and structures [1-6].

Based on this, there is a need to develop a method for calculating the settlement and bearing capacity of combined plate-pile foundations taking into account the complicated stress-strain state between the elements of the foundation and the ground base during cyclic loading [7-11].

Existing methods for calculating the settlement of the plate-pile foundations ground base and the bearing capacity are mainly developed for short-term static loads. [12-16] At the same time, the influence of cyclic loading on plate-pile foundations during the construction and operation of buildings and structures is insufficiently explored [17-22].

2 Materials and methods
The conducted experimental researches have made it possible to set major model changes in forces and deformations in the system “plate grillage – ground between the piles – ground base”.

During the researches, a settlement increase of the ground base under the cyclic loads action was observed throughout the tests. In the graphs of settlement S from load F and settlement S from the number of cycles N (figure 1 a, b), the settlement develops intensively during the first 500 cycles of cyclic loading, after which the settlement growth is significantly reduced. An analysis of the change in
the base settlement shows that after a different number of loading cycles, an increase in settlement occurs due to the increase in their residual part (figure 1a). Moreover, during one cycle, the settlement value varies slightly.

![Figure 1](image1.png)

**Figure 1.** Foundation settlement development during cyclic loading: a) settlement from load; b) settlement from number of cycles.

However, as the number of loading cycles increases during testing, changes in these (“elastic”) settlements are recorded. These settlements decrease during the first 20-50 cycles. The decrease in the value of “elastic” settlements caused due to the fact of reduction in pore volume soil compaction occurs. In development speed it is faster than the decrease in shear deformations, the shear modulus between piles and the ground in the between-pile space.

3 Results
From the experimental researches results, it can be seen that in the initial period of cyclic loads the greatest ground compaction occurs. After that the “elastic” settlements of the bases begin to increase after the first 50-100 loading cycles. If the limit state of the base is not reached, relative stabilization occurs at the time of 1000 loading cycles (figure 1a).

Ground base settlement changes during cyclic loading, the deformation of the ground between the piles changes as well. At the same time, an increase in ground base settlement compared to the first loading cycle can be up to 30%.

Figure 2 shows the force changes in piles. As the number of cycles increases, the efforts in the piles increase due to the fact that there is redistribution of deformations and stresses from the ground in the between-pile space into the piles. It should be noted that the greatest efforts occur in ordinary and corner piles, and the least in central ones. This is explained by the fact that in the middle zone of the
conditional foundation, the central piles compress the most compacted soil, and the corner and ordinary piles interact with areas of less compacted ground outside the grillage plate.

Figure 2. The efforts in piles after a different number of cycles, N ($P_{\text{min}}=12.5$ tons, $P_{\text{max}}=25$ tons).

At plate-pile foundation cyclic loading, deformations in the ground of the between-pile space decrease throughout the test (figure 3). Thus, a significant decrease in general deformations is observed under the plate part of the plate-pile foundation (up to 2 times) [23].

Figure 3. Ground deformations in the between pile space after different number of cycles, kPa ($P_{\text{min}}=12.5$ tons, $P_{\text{max}}=25$ tons).

4 Discussion

For an analytical description of the vibrocreep ground deformation influence on the plate-pile foundations settlement increase, additional stresses in the ground and in piles in accordance with [24] are determined.

Figure 4. Interaction schemes: a) plate-pile foundation with a ground base; b) single pile with a homogeneous ground mass $2A \times 2B$. 
A design scheme which consists from a pile, its surrounding ground and part of a plate grillage per one pile has been accepted. The stress-strain state main components behavior of the such a cell will correspond to the behavior of piles in the plate-pile foundation (figure 4 a, b). The cell dimensions are $2A \times 2B \times L$, the pile sizes are $2a \times 2b \times l$.

To solve the task, we use the system of forces equilibrium equations (1) necessary to determine 4 unknowns – $p_1, p_2, p_3$ and $t_0$:

$$
\begin{align*}
\sum \mathbf{F} &= \sum \mathbf{F}_a \quad \sum \mathbf{M} = \sum \mathbf{M}_a \\
p\cdot AB &= p_1(N) \cdot AB - p_2(N) \cdot AB \\
p_1(N) \cdot ab &= p_1(N) \cdot ab - 4(a + b) \cdot \frac{\tau_0(N)}{\alpha} \cdot \epsilon^{uz} + (a + b) \cdot \frac{\tau_s(N)}{\alpha} \\
p_2(N) \cdot \beta \cdot E_p \cdot L &= k_1 \cdot \tau_s(N) \cdot (A - a) + k_2 \cdot \tau_s(N) \cdot (B - b) \\
p_2(N) \cdot \beta \cdot E_p \cdot L &= \frac{3G_p(N)}{3G_p(N)} + \frac{3G_p(N)}{3G_p(N)} = \frac{G_p(N)}{G_p(N)} - \frac{G_p(N)}{G_p(N)} \\
\tau_s(N) \cdot (a + b) \cdot \epsilon^{uz} &= p_1(N) - \frac{G_p(N)}{E_p} + \frac{G_p(N)}{E_p} \\
\tau_s(N) &= \frac{G_p(N)}{E_p} \\
\tau(N) &= \tau_0(N) \cdot e^{\alpha z} \\
\alpha &= \frac{5}{l} \\

\end{align*}
$$

Here

$$
\begin{align*}
p_1(N) &= \sigma_{uz}^{max} (N) - \Delta \sigma_{uz} (N) , \\
p_2(N) &= \sigma_{uz}^{max} (N) + \Delta \sigma_{uz} (N) , \\
p_3(N) &= \sigma_{uz}^{max} (N) - \Delta \sigma_{uz} (N) , \\
\tau(N) &= \tau_0(N) \cdot e^{\alpha z} , \\
\tau(z) &= \tau_0(N) \cdot e^{\alpha z} , \\
\alpha &= \frac{5}{l}.
\end{align*}
$$

Figure 5. The ultimate shear stress and mobilized shear stress diagrams as the number of cycles increases.

The ultimate equilibrium zones are determined by the intersection point of the mobilized shear stress ($\tau(N)$) diagram with the limit shear stress diagram (Figure 5), herewith takes into account the rigidity of the pile material:

$$
\tau' = \gamma \cdot z \cdot \tan \phi + c \cdot N \quad \text{ (8)}
$$

where, $c(N)$ – is the ground specific adhesion under cyclic loading, taken in accordance with [25]:

$$
C(N) = C \cdot m(t \tau) \cdot \lambda(t \tau) \cdot \frac{k \cdot \tau}{k(t) + \frac{1}{k(t)} \cdot c(t \tau)}.
$$

(9)
In the case of increasing the pile length, its lateral surface area increases and changes the proportion of the load to the pile heel level.

Stresses in the ground under the plate grillage are determined by the formula:

\[ p_1(N) = \frac{p \cdot AB - p_2(N) \cdot ab}{AB - ab} \]  \hspace{1cm} (10)

Stresses in the upper plane of the head in the pile are represented in the form:

\[ p_1(N) = \frac{p \cdot G(N) \cdot 2ab (a+b) \cdot l \cdot \beta_2 (1 - \frac{L}{L})}{a \cdot \alpha (1-v) \cdot 4(1-\nu) \cdot \beta_2 (1 - \frac{L}{L}) \cdot E(N) \cdot L} \]

\[ + \frac{4.33 \cdot \tau_0(N)}{4(1-\nu) \cdot \beta_2 (1 - \frac{L}{L}) \cdot E(N) \cdot L} \]

\[ - 0.33 \cdot \tau_1(N) \cdot (A-a) \cdot L \cdot \beta_2 (1 - \frac{L}{L}) \cdot E(N) \cdot L \]

where, \( G = \frac{E}{2(1+\nu)} \) – ground shear modulus.

Stresses arising under the lower end of the pile can be calculated by the formula:

\[ p_1(N) = \frac{p_2(N) \cdot 4ab + 4(a+b) \cdot l \cdot \frac{\tau_0(N)}{\alpha} - 4(a+b) \cdot l \cdot \frac{\tau_s(N)}{\alpha} \cdot e^{-\alpha}}{4ab} \]  \hspace{1cm} (12)

The tangential stress along the lateral surface of the piles \( \tau_0(N) \) can be determined based on the expression:

\[ \tau_0(N) = \frac{a \cdot b \cdot (p_1(N) - p_2(N))}{(a+b) \cdot l \cdot \frac{1}{\alpha} (4e^{-\alpha} - 1)} \] \hspace{1cm} (13)

After stresses determining calculated settlement. The settlement of the plate-pile foundation is calculated by the formula:

\[ S(N) = S_e(N) + \Delta S_e(N) + \Delta S_e(N) \] \hspace{1cm} (14)

The settlement of the conditional foundation is calculated based on the following design scheme:

**Figure 6.** Settlement calculation schemes:

- a) ground base stress state scheme of the conditional foundation under cyclic loading;
- b) the design scheme for determining the settlement of the ground base during cyclic loading.
Herewith the ground base volumetric stress state of the conditional foundation is accepted (figure 6a). Dividing the compressible thickness of the ground base into layers, for each layer based on design scheme (figure 6 a, b) we determine the strains corresponding to the vertical pressure value, and then the strain values within the compressible thickness are summed [26].

The conditional foundation settlement during cyclic loading, taking into account the volumetric ground stress-strain state, can be determined by the formula:

\[
S_{ef}(N) = \sum_{i=1}^{n} \left( \varepsilon_{z,i}(t,t_0) \right) h_i,
\]

where \( \varepsilon_{z,i}(t,t_0) \) is the axial strain increment of the \( i \) th layer under the cyclic load action \( t \) [27]; 
\( n \) – the number of layers which the ground base compressible thickness is divided into; 
\( t \) – is the time corresponding to the moment of observation and to the loading cycles number \( N \); 
\( t_0 \) – is the load application time corresponding to the first cycle; 
\( h_i \) – is the thickness of the \( i \) th layer.

The additional settlement value due to the pile shaft compression and due to the pile punching to the conditional foundation sole depends on the fulfillment of conditions Eq. (16), (17) and (18). These settlements occur irregularly as conditions are violated:

\[
\tau (N) \leq \tau^* (N),
\]

\[
p_1(N) \leq \sigma_{in} (N),
\]

\[
p_3(N) \leq \sigma_{in} (N)
\]

The function \( \sigma_{ud}(N) \) is taken:

\[
\sigma_{ud}(N) = 4 \left[ \sigma_{u} (t,t_0,N) \cdot A_p \cdot \cos \alpha \cdot (t,t_0,N) + \sigma_{u} (t,t_0,N) \cdot A_p \cdot \sin \alpha \cdot (t,t_0,N) \right]
\]

Settlement due to the pile shaft compression is determined by the formula:

\[
\Delta S_p(N) = \frac{\sigma_{ud}(N)(l-a)}{E_g(N)} \cdot \frac{E_p(N)(l-a)}{1 + \frac{E_p(N)}{E_g(N)} \cdot \frac{A_p}{A_p}}.
\]

The additional settlement value \( \Delta S_p \) due to punching piles is determined by the formula:

\[
\Delta S_p(N) = \alpha_p \left[ \frac{P_p(N)}{G(N)} \left( \frac{P_p(N) + 2P_p(N)\cos B}{3} \right) \frac{3K_n(N) - G(N)}{3K_n(N) - G(N)} \right]
\]

where, \( E_p(N) \) – piles concrete deformation modulus under cyclic loads; 
\( E_g(N) \) – ground deformation modulus under cyclic loading; 
\( \varepsilon_{v}^{in}(N) \) – ground vibrocreep deformation [28]; 
\( K_n(N) \) – volumetric ground deformation modulus under cyclic loading; 
\( G(N) \) – ground shear modulus under cyclic loading; 
\( \sigma_{ud}^{in}(N) \) – maximum stresses in the pile section; 
\( \alpha_p \) – the pile cross section size.

Improved methods for calculating ground base settlement and bearing capacity take into account changes in the stress-strain state of a plate-pile foundation, an increase in deformations, stresses and efforts, as well as their redistribution between the ground and piles during cyclic loading. Determining the influence of cyclic loading on plate-pile foundations will make it possible to obtain more profitable design solutions for the buildings and structures construction on foundations this type in the future.

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