SPIN, ACCRETION, AND THE COSMOLOGICAL GROWTH OF SUPERMASSIVE BLACK HOLES

STUART L. SHAPIRO

ABSTRACT

If supermassive black holes (SMBHs) are the energy sources that power quasars and active galactic nuclei (AGNs), then QSO SDSS 1148+5251, the quasar with the highest redshift ($z_{QSO} = 6.43$), hosts an SMBH formed within $\sim 0.9$ Gyr after the big bang. This requirement places constraints on the cosmological formation of SMBHs, believed to grow from smaller initial seeds by a combination of accretion and mergers. We focus on gas accretion onto seeds produced by the collapse of Population III stars at high redshift. We incorporate the results of recent relativistic, MHD simulations of disk accretion onto Kerr black holes to track the coupled evolution of the masses and spins of the holes. We allow for an additional amplification of $\sim 10^4$ in the mass of a typical seed due to mergers, consistent with recent Monte Carlo simulations of hierarchical mergers of cold dark matter halos containing black hole seeds. We find that the growth of Population III black hole remnants to $\sim 10^9 M_\odot$ by $z_{QSO} \geq 6.43$ favors MHD accretion disks over standard thin disks. MHD disks tend to drive the holes to a submaximal equilibrium spin rate $a/M \sim 0.95$ and radiation efficiency $\epsilon_M \sim 0.2$, while standard thin disks drive them to maximal spin ($a/M = 1$) and efficiency ($\epsilon_M = 0.42$). This small difference in efficiency results in a huge difference in mass amplification by accretion at the Eddington limit. The MHD equilibrium efficiency is consistent with the observed ratio of the QSO plus AGN luminosity density to the local SMBH mass density. Our prototype analysis is designed to stimulate the incorporation of results from relativistic stellar collapse and MHD accretion simulations in future Monte Carlo simulations of hierarchical structure formation to better determine the cosmological role of SMBHs and their mass and spin distributions.

1. INTRODUCTION

There is substantial evidence that supermassive black holes (SMBHs) with masses in the range $10^6-10^{10} M_\odot$ exist and are the engines that power active galactic nuclei (AGNs) and quasars (Rees 1984, 1998, 2001; Macchetto 1999). There is also ample evidence that SMBHs reside at the centers of many, and perhaps most, galaxies (Richstone et al. 1998; Ho 1999), including the Milky Way (Genzel et al. 1997; Ghez et al. 2000, 2003; Schödel et al. 2002).

The highest redshift of a quasar discovered to date is $z_{QSO} = 6.43$, corresponding to QSO SDSS 1148+5251 (Fan et al. 2003). Accordingly, if they are the energy sources in quasars ($QSO$), the first SMBHs must have formed prior to $z_{QSO} = 6.43$ or within $t = 0.87$ Gyr after the big bang in the concordance $\Lambda$CDM cosmological model. This requirement sets a significant constraint on black hole seed formation and growth mechanisms in the early universe. Once formed, black holes grow by a combination of mergers and gas accretion. An important clue to the growth process is provided by the ratio $R$ of the observed QSO plus AGN luminosity density to the local SMBH mass density, since $R$ is related to the mean radiative efficiency $\epsilon_M$ of accretion onto black holes (Soltan 1982): $\epsilon_M \geq R$. Recent measurements suggest $0.1 \leq R \leq 0.2$ (Yu & Tremaine 2002; Elvis et al. 2002), a range consistent with disk accretion onto black holes. Another clue is provided by the estimated ratio $\epsilon_L$ of the bolometric-to-Eddington luminosities of the broad-line quasars in a Sloan Digital Sky Survey sample of 12,698 quasars in the redshift interval $0.1 \leq z \leq 2.1$. This survey supports the value $\epsilon_L \approx 1$ as a physical upper limit (McLure & Dunlop 2004). Barring an extraordinary coincidence, the range of values inferred from observations for both $\epsilon_M$ and $\epsilon_L$ suggest that a significant fraction of the mass of SMBHs is acquired by gas accretion. Together, these two parameters control the rate of growth of black holes by accretion and are crucial in determining whether initial seed black holes formed at high redshift have sufficient time to grow to SMBHs to explain quasars and AGNs.

The more massive the initial seed, the less time is required for it to grow to SMBH scale and the easier it is to have an SMBH in place by $z \geq 6.43$. One possible progenitor that readily produces an SMBH is a supermassive star (SMS) with $M \gtrsim 10^5 M_\odot$ (see, e.g., Shapiro 2004a for a recent review and references). SMSs can form when gaseous structures build up sufficient radiation pressure to inhibit fragmentation and prevent normal star formation; plausible cosmological scenarios have been proposed that can lead to this occurrence (Gnedin 2001; Bromm & Loeb 2003). SMSs supported by radiation pressure will evolve in a quasi-stationary manner to the point of onset of dynamical collapse due to a general relativistic radial instability (Chandrasekhar 1964a, 1964b, 1964c; R. Feynman 1964, unpublished, as quoted in Fowler 1964). The collapse of a nonrotating, marginally unstable, spherical SMS of mass $M$ leads directly to the formation of a nonrotating Schwarzschild black hole of the same total mass (Shapiro & Teukolsky 1979). But like most stars formed in nature, SMSs will be rotating. In fact, in the event that viscosity and/or turbulent magnetic fields are present to drive these stars to uniform rotation, they are likely to be maximally rotating (i.e., at the mass-shedding limit) by the time they reach the onset of collapse (Baumgarte & Shapiro 1999). Recent relativistic hydrodynamical simulations have shown that unstable, maximally rotating SMSs of arbitrary mass $M$ inevitably collapse to SMBHs of mass $\sim 0.9 M_\odot$ and spin...
parameter $a/M \sim 0.75$; the rest of the mass goes into an ambient disk about the hole (Shibata & Shapiro 2002; Shapiro & Shibata 2002; Shapiro 2004b).

But the fact remains that SMSs have yet to be observed, and there is no concrete evidence that they actually form in the early universe. Moreover, simulations of cosmological structure formation performed to date indicate that the first generation of stars are more likely to be zero-metallicity Population III stars in the range $10^2 - 10^3 M_\odot$ (Bromm et al. 1999, 2002; Abel et al. 2000, 2002; but note that Norman [2004] reports preliminary indications that supermassive stellar objects with $M \gtrsim 10^4 M_\odot$ may be forming as second-generation stars at $10 \lesssim z \lesssim 15$ in his latest simulations). So the most conservative hypothesis is that the seed black holes that later grow to become SMBHs originate from the collapse of Population III stars (Madau & Rees 2001) and not SMSs, and this is the Ansatz we explore here. Newtonian simulations suggest that Population III stars with masses in the range $M \sim 60 - 140 M_\odot$ and $M \gtrsim 260 M_\odot$ collapse directly to black holes, while stars with $M \sim 140 - 260 M_\odot$ undergo explosive annihilation via pair-creation processes (Heger et al. 2003). The upper limit to the mass of a Population III star is set by accretion over a stellar lifetime, yielding $M \lesssim 600 M_\odot$ (Omukai & Palla 2003; Yoshida et al. 2003). Abel et al. (2002) argue that first-generation stars significantly larger than $100 M_\odot$ are likely to explode before they have time to accrete to larger masses. The Pistol star is an example of an existing star believed to have a mass $\gtrsim 200 M_\odot$, although it has high metallicity (Figer et al. 1998). Very massive stars are dominated by thermal radiation pressure, so the catastrophic collapse of those that do not explode will be hydrodynamically similar to the collapse of SMSs, producing black holes with masses comparable to those of their progenitors. The higher the mass of the black hole seed and the earlier it forms in the universe, the easier it is for it to grow to an SMBH, and, hence, the more conservative will be any constraints imposed on the cosmological black hole growth rate by the existence of an SMBH by $z_{\text{QSO}} = 6.43$. Accordingly, we take the highest range of plausible values for the masses of black hole seeds to establish the most conservative (robust) constraints, adopting $100 \lesssim M/M_\odot \lesssim 600$ for the range of black hole seeds arising from the collapse of Population III stars at $z \lesssim 40$.

The most detailed studies of SMBH formation to date involve detailed, Monte Carlo simulations that follow the cosmological growth of a distribution of black hole seeds by a combination of discrete, stochastic mergers as well as gas accretion (see, e.g., Haehnelt & Kauffmann 2000; Volonteri et al. 2003; Bromley et al. 2004; Haehnelt 2004; Haiman 2004; Yoo & Miralda-Escude 2004; and references therein). Both processes are assumed to take place in the context of the cold dark matter (CDM) model, where dark matter halos merge hierarchically and black holes are assumed to settle, merge, and accrete in their gaseous centers. Typically, the stellar dynamical processes that lead to mergers, as well as the hydrodynamical processes that fuel accretion, are modeled in these analyses by implementing simple, physically plausible, rules rather than by detailed integrations of the governing dynamical and hydrodynamical equations of motion. Performing such “first principles” integrations would prove prohibitive in this context.

In this paper we focus on SMBH cosmological growth by accretion. We identify and explore the main variables that govern this process and ultimately influence the outcome of detailed Monte Carlo simulations that track SMBH growth. We incorporate some of the most recent findings of relativistic magnetohydrodynamical (MHD) simulations of gas accretion onto black holes and explore their cosmological implications for SMBH evolution. We show how the evolution and amplification of black hole mass by accretion is intimately tied to the evolution of black hole spin, probing some of the implications of our earlier survey and analysis of black hole spin evolution (Gammie et al. 2004) for cosmology. We demonstrate how, in principle, the very existence of a quasar at redshift $z_{\text{QSO}} = 6.43$ can help constrain the formation epoch and/or size of black hole seeds and select among competing models of accretion.

Specifically, recent relativistic MHD simulations predict the radiation efficiency $\epsilon_{\text{rad}}$ as a function of the black hole spin parameter $a/M$; they also predict the spin-up rate as a function of $a/M$. Monte Carlo simulations that determine cosmological SMBH growth must integrate coupled mass and spin evolution prescriptions versus time for each accreting black hole to produce reliable growth histories and final SMBH masses and spins. This paper provides prototype integrations of the coupled evolution equations, focusing on the history of the progenitor of SDSS 1148+5251 and demonstrating the significant differences in the outcomes for standard thin disk accretion models versus recent MHD models.

Our discussion is simplified and illustrative at best; much of the input physics involving black hole seed formation, accretion flows, and mergers is still being developed. Our main goal is to isolate some of the underlying local physical issues and parameters pertaining to accretion to better understand their role in determining the global outcome of the cosmological Monte Carlo simulations of SMBH buildup during hierarchical structure formation. Excellent overviews of the input physics have appeared elsewhere (see, e.g., Haiman & Quataert 2005 and references therein). Also, earlier treatments have considered some of the constraints on cosmological growth imposed by the recent discoveries of luminous quasars at high redshift (e.g., Haiman & Loeb 2001; Haiman 2004; Yoo & Miralda-Escudé 2004). But here we specifically want to illustrate in the simplest fashion how the most recent findings related to relativistic, MHD accretion flows onto spinning black holes have important implications for evolutionary models of the growth of SMBHs in the early universe. We emphasize by concrete example the point made in Gammie et al. (2004) that tracking the spin as well as the mass of a black holes is necessary to determine its growth (see also Volonteri et al. 2005). We also show that the value of accretion radiation efficiency, $\epsilon_{\text{rad}} \sim 0.1$, adopted in many Monte Carlo simulations may not be entirely consistent with the latest MHD accretion disk modeling. Determining this parameter, on which the outcome of cosmological simulations of SMBH growth depends very sensitively (exponentially!), is coupled to the spin evolution of the hole; both may now be within our grasp via detailed relativistic MHD simulations of black hole accretion.

The calculations performed here are prototypical only; our main aim is to motivate the incorporation of these parameters in more detailed Monte Carlo studies and thereby sharpen some of the rules that enter these simulations. We also hope to provide those members of the computational relativistic MHD community who may not be SMBH Monte Carlo cognoscenti a simple means of extracting the essence of the Monte Carlo simulations, particularly the evolutionary consequences of different accretion models.

In § 2 we set out the basic equations that describe the coupled evolution of black hole mass and spin by accretion. We also summarize in this section the relations we require from the concordance $\Lambda$CDM cosmological model. In § 3 we integrate the coupled evolution equations to track the evolution of black
hole mass and spin as functions of time. In §4 we apply the results to the cosmological problem and identify some constraints imposed by the existence of an SMBH at \( z \geq 6.43 \). In §5 we summarize briefly and discuss some caveats and areas for further study.

2. BASIC EQUATIONS

Here we assemble the fundamental black hole accretion evolution equations and review the underlying assumptions on which they are based. We then specify a background cosmological model in which the growth of the black hole to supermassive size occurs.

2.1. Black Hole Mass and Spin Evolution

Define \( \epsilon_M \), the efficiency of conversion of rest-mass energy to luminous energy by accretion onto a black hole of mass \( M \), according to

\[
\epsilon_M \equiv \frac{L}{\dot{M}c^2},
\]

where \( \dot{M} \) is the rate of rest-mass accretion and \( L \) is the luminosity. Define \( \epsilon_L \), the efficiency of accretion luminosity, according to

\[
\epsilon_L \equiv \frac{L}{L_E},
\]

where \( L_E \) is the Eddington luminosity, given by

\[
L_E = \frac{4\pi M \mu_em_p c}{\sigma_T} \approx 1.3 \times 10^{46} \mu_e M_8 \text{ ergs s}^{-1}.
\]

Here we assume that the accretion is dominated by normal, baryonic matter, and we ignore any contribution of collisionless or self-interacting dark matter (but see, e.g., Ostriker [2000] and Balberg & Shapiro [2002] for alternative scenarios). In particular, we assume that the accreting gas consists of fully ionized atoms and that the principal opacity source is Thomson scattering. The quantity \( \mu_e \) is the mean molecular weight per electron, and \( m_p \) is the proton mass. The black hole growth rate must account for the loss of accretion mass energy in the form of outgoing radiation according to

\[
\frac{dM}{dt} = (1 - \epsilon_M) \dot{M}_0.
\]

Combining equations (1)–(4), we obtain the black hole growth rate,

\[
\frac{dM}{dt} = \frac{\epsilon_L(1 - \epsilon_M) M}{\epsilon_M \tau},
\]

where \( \tau \) is the characteristic accretion timescale

\[
\tau \equiv \frac{Mc^2}{L_E} \approx 0.45 \mu_e^{-1} \text{ Gyr}
\]

and is independent of \( M \).

The mass accretion efficiency \( \epsilon_M \) is typically a function of the black hole spin parameter \( a/M = J/M^2 \). It changes with time as the spin evolves. It is convenient to express the spin evolution in terms of the nondimensional quantity \( s = s(a/M) \), defined by

\[
s \equiv \frac{d(a/M)}{dt} \frac{M}{\dot{M} \dot{M}_0}.
\]

Inserting equations (4) and (5) into equation (7) yields the evolution equation for the black hole spin,

\[
\frac{d(a/M)}{dt} = \frac{\epsilon_L s}{\epsilon_M \tau}.
\]

In general, equations (5) and (8) must be integrated simultaneously to determine the mass and spin evolution of the black hole.

Determining \( \epsilon_M(a/M) \) and \( s(a/M) \), which are needed to integrate equations (5) and (8), requires a gasdynamical model for black hole accretion. We assume that the gas has sufficient angular momentum to form a disk about the hole, and we consider two different accretion disk models: (1) a standard, relativistic, Keplerian “thin disk” with “no-torque boundary conditions” at the innermost stable circular orbit (ISCO; Pringle & Rees 1973; Novikov & Thorne 1973; see Shapiro & Teukolsky 1983 for review and references) and (2) a relativistic, MHD accretion disk that accounts for the presence of a frozen-in magnetic field in a perfectly conducting plasma (De Villiers & Hawley 2003, De Villiers et al. 2003, Gammie et al. 2004, McKinney & Gammie 2004, and references therein). In the MHD model the magnetorotational instability (MRI; Balbus & Hawley 1991) drives magnetic turbulence and provides the necessary torque to remove angular momentum from the gas and drive the inflow. The MHD model is arguably the most realistic model for disk accretion of magnetized plasma onto a black hole. The standard thin disk model provides a simple, analytic, limiting case that is useful as a point of comparison.

In a standard thin disk corotating with the black hole, the energy and angular momentum per unit rest mass accreted by a black hole are the energy and angular momentum of a unit mass at the ISCO, immediately prior to its rapid plunge and capture by the hole:

\[
\hat{E}_{\text{ISCO}} = \frac{r_{\text{ms}}^2 - 2Mr_{\text{ms}} + a\sqrt{Mr_{\text{ms}}}}{r_{\text{ms}}(r_{\text{ms}}^2 - 2Mr_{\text{ms}} + 2a\sqrt{Mr_{\text{ms}}})^{1/2}},
\]

\[
\hat{l}_{\text{ISCO}} = \frac{\sqrt{Mr_{\text{ms}}(r_{\text{ms}}^2 - 2a\sqrt{Mr_{\text{ms}}} + a^2)}}{r_{\text{ms}}(r_{\text{ms}}^2 - 2Mr_{\text{ms}} + 2a\sqrt{Mr_{\text{ms}}})^{1/2}},
\]

where the ISCO radius \( r_{\text{ms}} \) is given by

\[
r_{\text{ms}} = M\{3 + Z_2 - [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2}\},
\]

where

\[
Z_1 \equiv 1 + \left(1 - \frac{a^2}{M^2}\right)^{1/3}\left[\left(1 + \frac{a}{M}\right)^{1/3} + \left(1 - \frac{a}{M}\right)^{1/3}\right],
\]

and

\[
Z_2 \equiv \left(3\frac{a^2}{M^2} + Z_1^2\right)^{1/2}
\]

(see, e.g., Shapiro & Teukolsky 1983, eqs. [12.7.17]–[12.7.18] and [12.7.24]). The mass accretion efficiency and spin evolution parameters corresponding to the thin disk model are then given by

\[
\epsilon_M = 1 - \hat{E}_{\text{ISCO}},
\]

\[
s = \frac{\hat{l}_{\text{ISCO}}^2 - 2 \frac{a}{M} \hat{E}_{\text{ISCO}} \hat{l}_{\text{ISCO}}}{\hat{l}_{\text{ISCO}}^2} \quad \text{(standard thin disk)}.
\]
The MHD disk accretion model of Gammie et al. (2004) and McKinney & Gammie (2004) is based on a fully relativistic, axisymmetric simulation of a nonradiative, magnetized plasma onto a Kerr-Schild black hole within the MHD approximation. The initial gas configuration is a torus with an inner radius at $r/M = 6$; in the absence of a magnetic field the torus is constructed to be in equilibrium about the hole (Fishbone & Moncrief 1976). The torus in threaded with a poloidal magnetic field initially and evolves with an adiabatic equation of state (EOS) with an adiabatic index $\Gamma = 4/3$ (to model a radiation-dominated, inner-disk EOS). The source of viscosity is MHD turbulence driven by the MRI instability. The simulations are performed for various black hole spin parameters $a/M$, holding the value of the spin parameter fixed during the simulation. The evolution proceeds for many dynamical timescales $M$, until a crude steady state, with fluctuations, is achieved.

The results of the numerical simulations suggest that in steady state the radiation efficiency parameter $\epsilon_M$ as a function of $a/M$ is remarkably close to the function characterizing the standard thin disk (eq. [13]), even though there is no sharp transition in the surface density at or near the ISCO. However, the spin evolution parameter $s(a/M)$ is different and can be represented reasonably well by the least-squares linear fit

$$s = 3.14 - 3.30 \frac{a}{M} \quad \text{(MHD disk)} \quad (15)$$

(see McKinney & Gammie 2004, Table 2). The numerical simulations demonstrate that the above parameters describing steady-state, MHD accretion-disk behavior are not particularly sensitive to the initial conditions in the disk (e.g., the initial $B$-field). As McKinney & Gammie (2004) discuss, the key results are also quite comparable to those found by De Villiers et al. (2004), who used a different numerical method and took results are also quite comparable to those found by De Villiers et al. (2004), who used a different numerical method and took the adiabatic index of the gas to be $\Gamma = 5/3$ instead of $\Gamma = 4/3$. We therefore model a relativistic MHD accretion disk by adopting equations (13) and (15) in our evolution equations.

The $s$ versus $a/M$ curve for the MHD disk is roughly parallel to, but somewhat below, the curve for the standard disk (see Gammie et al. 2004, Fig. 4, for a comparison). In particular, the parameter $s$ never falls to zero for a standard thin disk until the hole is maximally rotating at $a/M = 1$, while for an MHD disk $s$ crosses zero at $a/M \approx 0.95$. The crucial physical consequence is that steady accretion via a standard thin disk is robust (least stringent) constraints on any gas accretion scenario black hole from accretion (see eq. [5]) and, as a result, the most robust (least stringent) constraints on any gas accretion scenario for the growth of an SMBH from a smaller initial seed. To establish these constraints, we therefore set $\epsilon_L = 1$ in many of our numerical estimates below.

As a point of reference, and for later application, it is useful to integrate equation (5) assuming that both the mass and luminosity efficiencies remain constant with time, yielding

$$M(t)/M(t_0) = \exp \left( \frac{\epsilon_L (1 - \epsilon_M) (t - t_0)}{\epsilon_M \tau} \right), \quad (\epsilon_M, \epsilon_L \text{ constant}), \quad (16)$$

where $t_0$ is the initial time at which the black hole has a mass $M(t_0)$. Note that the right-hand side of equation (16) is independent of black hole mass. This fact makes it possible, under certain plausible conditions that we specify, to disentangle and track separately the amplification of black hole mass by accretion from the amplification by discrete mergers. Let $M_p(t)$ be the mass of the black hole at time $t$ following its $n$th merger with another hole at time $t_n$, where $t_n \leq t \leq t_{n+1}$. Assume that the duration of a merger, as well as the time required for accretion to drive the merged remnant to spin equilibrium (see eq. [22] below), are both much shorter than the time interval between mergers, and that the hole continues to accrete steadily throughout this interval. Note that black hole mergers can completely eject black holes from halo centers owing to gravitational wave recoil and thereby turn off accretion altogether (see, e.g., discussions of black hole recoil in halos in Hut & Rees 1992, Merritt et al. 2004, and Madau & Quataert 2004). However, incorporating the most recent recoil calculations (Favata et al. 2004) into simple models of dark halo mergers, Yoo & Miralda-Escudé (2004) conclude that the kick velocities are not sufficiently large to impede black hole growth significantly (cf. Haiman 2004). Note also that a major merger between two black holes of comparable mass may change both the magnitude and direction of the spin of the resulting black hole remnant. Following such a merger, the orientation of the black hole spin may not be aligned with the orientation of the asymptotic gaseous disk at radii $r > 100M$ outside the hole. (Rees [1978] and Natarajan & Pringle [1998] point out that the black hole exerts a torque on the asymptotic disk, which also exerts a torque back on the hole, eventually forcing their mutual alignment but on a timescale that is still uncertain.) Moreover, the orientation of the asymptotic disk will likely fluctuate in time because of the distribution of gas following mergers of dark halo cores, galaxy mergers, and the tidal disruptions of passing stars by the central hole. However, near the black hole, at radii from $r \sim 2M$ to $20M$, where the bulk of the disk’s gravitational energy is released and the hole-disc interactions are strong, the hole’s gravitomagnetic field will exert a force on the disk that, when combined with viscous forces and magnetic fields, will drive the disk down into the hole’s equatorial plane (the “Bardeen-Petterson effect”; Bardeen & Petterson 1975). This effect will maintain the alignment between the axis of the inner disk and the spin axis of the
hole. Accretion will subsequently drive the hole to spin equilibrium and restore $\epsilon_s$ to its equilibrium value. For the situation described above, we can take $\epsilon_s$ to be constant and equal to its value at spin equilibrium throughout most of the lifetime of the hole. Assume further that $\epsilon_s$ is also constant, which should be the case if the available gas is sufficiently copious that the accretion is always Eddington-limited, whereby $\epsilon_s \approx 1$. Let $f_n$ be the mass amplification of the hole following its $n$th merger with another hole: $f_n = M_n(t_n)/M_{n-1}(t_{n-1}) > 1$. Then we may use equation (16) to calculate the total mass amplification from $t_i$ to $t_f$ according to

$$
\frac{M_f}{M_i} = \frac{M_N(t_f)}{M_0(t_i)} \frac{M_0(t_i)}{M_0(t_f)} \frac{M_0(t_f)}{M_1(t_f)} \cdots \frac{M_{N-1}(t_N)}{M_{N-1}(t_{N-1})} \frac{M_{N-1}(t_{N-1})}{M_N(t_N)} \cdots \frac{M_0(t_0)}{M_0(t_f)} \frac{M_0(t_f)}{M_N(t_f)} \frac{M_N(t_f)}{M_N(t_i)} \\
= \exp[C(t_1 - t_0)] f_1 \exp[C(t_2 - t_1)] \cdots \frac{f_1 f_2 \cdots f_N}{f_N \exp[C(t_f - t_i)]},
$$

(17)

where $C = \epsilon_s (1 - \epsilon_s)/(\epsilon_s \tau)$ and is essentially constant. Comparison of equations (16) and (17) reveals that the net mass amplification due to accretion can be treated as a single multiplicative factor that is independent of the net amplification factor due to mergers, $f_1 f_2 \cdots f_N$. Thus, for the simple scenario envisioned here, equation (16) can be applied to determine the growth of a black hole by accretion, even when steady growth by accretion is interrupted and augmented by discrete, stochastic black hole mergers.

2.2. Cosmological Model

To relate the time parameter $t$ appearing in the evolution equations to an observable timelike quantity such as the redshift $z$, we adopt the concordance $\Lambda$CDM, spatially flat ($k = 0$), cosmological model. All the free parameters in this model that we need for our computations have been measured by now, so the model is uniquely specified.

The basic evolution equation for the Friedmann-Robertson-Walker expansion parameter $a(t)$ is given by

$$
H^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_m^0 \left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda^0,
$$

(18)

where $H(t)$ is Hubble’s constant and the normalized mass density parameter $\Omega_m^0$ and cosmological constant parameter $\Omega_\Lambda^0$ satisfy the relation

$$
\Omega_m^0 + \Omega_\Lambda^0 = 1.
$$

(19)

In the above equations the sub(super)script “0” denotes the value of a quantity at the current epoch, $z = 0$. Recalling that $a/a_0 = 1 + z$ and substituting equation (19) into equation (18), we can integrate equation (18) to obtain $t = t(z)$,

$$
t(z) = \frac{2}{3H_0(1 - \Omega_m^0)^{1/2}} \sinh^{-1}\left[\left(\frac{1}{\Omega_m^0}\right)^{1/(1 + z)} \left(1 + \frac{1}{(1 + z)^2}\right)^{1/2}\right].
$$

(20)

To evaluate $t(z)$ appearing above, we adopt the Wilkinson Microwave Anisotropy Probe values for the cosmological parameters $\Omega_m^0 \approx 0.27$ and $H_0 \equiv 100h$ km s$^{-1}$ Mpc$^{-1}$ with $h \approx 0.71$ (Bennett et al. 2003; Spergel et al. 2003).

To evaluate the molecular weight $\mu_e$ appearing in equations (3) and (6), we assume that the accreting plasma is a zero-metallicity, primordial gas, for which

$$
\mu_e = \frac{1}{1 - Y/2}.
$$

(21)

We take the primordial helium abundance $Y$ to be $Y \approx 0.25$ (Cyburt et al. 2003).

For the concordance model, the age of the universe is $T = t(0) = 13.7$ Gyr, while the age at redshift $z = 6.43$, the highest known quasar redshift (SDSS 1148+525; Fan et al. 2003), is only $t_{QSO} = 0.87$ Gyr. Hence, $t_{QSO}$ is the upper limit to the time available for accretion to occur onto the initial seed black hole that powers this quasar. In fact, assuming that the black hole seed forms from the collapse of a first-generation, Population III star at redshift $z \leq 40$ and $t \geq t(40) = 0.067$ Gyr, the available time for accretion is reduced to $t_{accret} \leq 0.80$ Gyr. We note that the stellar evolution (hydrogen-burning) lifetime of a massive Population III progenitor, $t_{\star} \sim 0.003$ Gyr (Wagoner 1969; Omukai & Palla 2003), is much smaller than $t(40)$, so the delay between stellar formation and collapse is of little consequence for determining the total time available for accretion growth. Most important, the exponential accretion growth timescale is given by equations (5), (6), and (21) to be

$$
\tau_{growth} = \frac{\epsilon_m}{1 - \epsilon_m} \frac{1}{\epsilon_L} \tau = 0.0394 \left(\frac{\epsilon_m/0.1}{1 - \epsilon_m}\right) \frac{1}{\epsilon_L} \text{ Gyr},
$$

(22)

which is considerably smaller than $t_{accret}$. It is the availability of many exponential growth timescales from the time of black hole seed formation to the birth of a quasar that makes it possible for the black hole to grow from stellar to supermassive size by gas accretion (aided by mergers) in the early universe.

3. BLACK HOLE GROWTH AND SPIN-UP

Here we integrate the coupled mass and spin evolution equations (5) and (8) to study black hole growth and spin-up by gas accretion. In Figure 1 we show the increase in mass by accretion at the Eddington limit ($\epsilon_s = 1$) as a function of time. In Figure 1a the accretion disk is a standard thin disk with a no-torque boundary condition at the ISCO (eq. [14]); in Figure 1b the disk is an MHD disk as calculated by McKinney & Gammie (2004; eq. [15]). We consider two different initial values for the black hole spin parameter, $a/M$. The solid lines show the case in which the initial black hole is nonrotating with $a/M = 0$. The dotted lines show the case in which the black hole is spinning with $a/M = 0.75$. The latter is the value calculated for a black hole formed from the catastrophic collapse of a massive, radiation-dominated star spinning uniformly at the mass-shedding limit that has evolved to the onset of relativistic radial instability just prior to collapse (Shibata & Shapiro 2002; Shapiro & Shibata 2002; Shapiro 2004b). For each disk model, the solid dots show the black hole growth that would occur if the accretion were maintained at the asymptotic black hole spin and efficiency from the beginning. Time is plotted in units of $\tau$ given by equations (6) and (21) for a zero-metallicity, cosmological abundance of H and He. The total duration plotted corresponds to the age difference between redshift $z_1 = 40$ (the earliest plausible birth date of a black hole from the collapse of a Population III star) and redshift $z_f = 6.43$ (the highest known quasar redshift, corresponding to SDSS 1148+5251; Fan et al. 2003) in the adopted $\Lambda$CDM cosmology.
The key difference between the two disk models is that the standard thin disk drives the black hole to maximal spin and efficiency \((a/M = 1\) and \(\epsilon_{M} = 0.42);\) Bardeen (1970), while the MHD disk drives the black hole to spin equilibrium at lower values \((a/M \approx 0.95\) and \(\epsilon_{M} \approx 0.19)).\) A moderately lower radiation efficiency \(\epsilon_{M}\) for Eddington-limited accretion results in substantially larger black hole growth. As is evident from the figure, apart from an initial transient lasting \(\sim 0.1\tau \sim \tau_{\text{growth}}\) (see Fig. 1, inset), the asymptotic evolution and final black hole mass amplification factor is little affected by the initial black hole spin.

In Figure 2 we show the spin evolution during Eddington-limited accretion for the cases shown in Figure 1. The large figure plots the spin parameter versus time, while the inset plots the spin parameter as a function of black hole mass amplification. After the initial transient lasting \(\sim 0.1\tau\), during which time the black hole grows by a factor of \(\sim 2\), the black hole spin and efficiency approach their asymptotic values. An analytic integration of equation (8) with equation (15), crudely holding \(\epsilon_{M}\) fixed, yields an exponential spin-up timescale \(\tau_{\text{spin}} \approx (1 - \epsilon_{M})\tau_{\text{growth}}/3.30\) which explains the rapid spin-up rate. This result also explains why the asymptotic black hole evolution and mass amplification are little affected by the initial black hole spin and depend only on the equilibrium spin rate of the accreting Kerr black hole. Thus, for all but the initial transient, it is adequate to adopt the equilibrium spin rate and use equation (16) with the corresponding constant efficiency \(\epsilon_{M}(a/M)_{\text{eq}}\) to calculate the cosmological black hole mass amplification in lieu of a detailed integration of coupled evolution equations.

4. COSMOLOGICAL IMPLICATIONS

Next we consider some of the cosmological implications of the results of the previous section for the growth of SMBHs in the early universe. In Figure 3 we employ equations (16) and (20) to study the black hole mass amplification factor \(M_{f}/M_{i}\) as a function of the redshift \(z_{f}\) of the initial seed black hole. Here we plot the final amplification factor achieved by redshift \(z_{f} = 6.43\), the highest known quasar redshift. Each solid curve is labeled by the adopted constant radiation efficiency \(\epsilon_{M}\); the luminosity is assumed to be the Eddington value \(\epsilon_{L} = 1\). The horizontal dashed lines indicate the range of amplification factors required for accretion alone to grow an SMBH of mass \(10^{8} M_{\odot}\) from an initial seed black hole of mass \(100 \leq M_{i}/M_{\odot} \leq 600\) formed from the collapse of a Population III star. A mass of \(10^{8} M_{\odot}\) is the value inferred for typical quasars, including 1148+5251 (Fan et al. 2003). The lower the mass of the initial seed, the larger the required mass amplification. The horizontal dotted lines indicate the required range of accretion-driven mass amplification assuming that mergers assist the growth process by accounting for an amplification of \(f \sim 10^{4}\) in black hole mass, with the remaining amplification provided by gas accretion (Yoo & Miralda-Escudé 2004).

Some important cosmological implications can be inferred from Figure 3. Consider a seed black hole that forms sometime after redshift \(z_{f} \leq 40\), by which time the earliest stars have formed and collapsed. In the absence of mergers, steady accretion cannot by itself achieve the required growth to explain quasars at \(z_{f} = 6.43\) unless the efficiency satisfies \(\epsilon_{M} \leq 0.13\). For disk accretion, this constraint requires the steady-state black hole spin parameter to be below \(a/M \approx 0.83\). This value is below (1) the maximal black hole spin \(a/M = 1\), the asymptotic equilibrium value for a standard thin disk; (2) \(a/M = 0.998\) (for which \(\epsilon_{M} = 0.32\)), the equilibrium value of a standard thin disk accounting for the recapture of some of the emitted photons by the black hole.
(Thorne 1974); and (3) $a/M \approx 0.95$, the equilibrium value of a typical MHD disk. Therefore, it is likely that mergers are required to assist accretion to achieve black hole growth to supermassive size by $z_f = 6.43$. As may be inferred from the figure, this conclusion holds even in the (unlikely) event that the black hole seed forms much earlier than $z_i \approx 40$.

Monte Carlo simulations by Yoo & Miralda-Escudé (2004) of hierarchical CDM halo mergers, accompanied by mergers of their central black holes, suggest that black hole mass amplification factors of $\sim 10^4$ are achieved via mergers by $z_f = 6.43$. This result implies that $f = f_1 f_2 \cdots f_N \sim 10^4$ in equation (17). While other simulations predict lower growth factors due to mergers, we adopt $10^4$ to be conservative, noting that our conclusions below are strengthened if the smaller factors turn out to be correct. As equation (17) shows, equation (16) can be used to estimate the cumulative mass amplification by gas accretion, even when steady growth by accretion is interrupted and enhanced by stochastic black hole mergers. Minor mergers of massive black holes with smaller holes tend to spin down the massive hole (Hughes & Blandford 2003; Gammie et al. 2004). Major mergers of two black holes of comparable mass following binary inspiral drive the merged remnant to $a/m \gtrsim 0.7$. (See, e.g., Baumgarte & Shapiro [2003] for a review of binary black hole coalescence and for references; see Gammie et al. [2004] for a general discussion of black hole spin-up and spin-down mechanisms and for references.) In either case, after a short transient epoch, accretion will drive the merged remnant to the disk accretion equilibrium spin rate and corresponding mass efficiency (see §2), as assumed in employing equation (16).

Figure 3 indicates that for black hole seeds that arise from collapsed Population III stars, merger-assisted mass amplification to SMBH status is easily achieved for typical MHD accretion disks, marginally possible for a standard thin disk in spin equilibrium accounting for photon recapture, but not possible for a standard thin disk that drives the black hole to maximal rotation. However, if the black hole seed is less than $600 M_\odot$, the standard thin disk appears to be ruled out; the required amplification places the lower dotted line in the plot above the curve for $\epsilon_M = 0.32$, corresponding to $a/M = 0.998$, the value for a standard thin disk in spin equilibrium, accounting for photon recapture. These conclusions also hold even if the black hole seed forms much earlier than $z_i \approx 40$, while they may be tightened if the seed forms later. In fact, it may be likely that the seed forms later, at $z_i \lesssim 40$, given that even $4 \sigma$ peaks in the density perturbation spectrum for the progenitor halo of SDSS 1148+5251 do not collapse until $z \approx 30$ in the $\Lambda$CDM concordance cosmology (see, e.g., Barkana & Loeb 2001, Fig. 5). Moreover, the potential wells of the earliest halos are quite shallow ($\sim 1 \text{ km s}^{-1}$) and may not be able to retain enough gas to form stars. Nevertheless, the effect of altering the date of birth of the black hole seed is not very great unless $z_i \lesssim 20–25$, as is evident from Figure 3.

It is significant that the range of equilibrium accretion disk radiation efficiencies required to achieve the necessary growth of a black hole seed to supermassive size by $z_f = 6.43$ is consistent with the values inferred observationally for $R$, the ratio of the QSO plus AGN luminosity density to the mass density of SMBHs in nearby galaxies: $\epsilon_M \gtrsim R \sim 0.1–0.2$ (Soltan 1982; Yu & Tremaine 2002; Elvis et al. 2002). This consistency supports the notion that accretion plays an important role in SMBH growth and is responsible for the acquisition of the bulk of the final mass of an SMBH. The range of $\epsilon_M$, inferred from $R$ favors accretion disk models that drive the black hole to spin equilibrium in the range $0.7 \lesssim a/M \lesssim 0.95$, well below maximal rotation and consistent with the values found in recent simulations of relativistic MHD accretion disks.

How are our conclusions altered if (when) a quasar is discovered at a higher redshift, $z_{\text{QSO}} > 6.43$? We anticipate this possibility in Figure 4, where we solve equation (16) to plot the mass efficiency $\epsilon_M$ required to build an SMBH from a seed as a function of the host quasar redshift, $z_{\text{QSO}}$. We again assume that a seed black hole of mass $100 \lesssim M_0 / M_\odot \lesssim 600$, formed from the collapse of a Population III star at $10^8 M_\odot$. The horizontal dotted lines bracket the required amplification range assuming that mergers account for a growth of $10^4$ in black hole mass, the remainder being by gas accretion.
We have explored the evolution of black hole mass and spin by gas accretion in the early universe. We have illustrated how for Eddington-limited accretion, the growth of an SMBH depends sensitively on the radiative efficiency, $\epsilon_M$. For disk accretion, the mean efficiency is determined by the equilibrium black hole spin, which in turn depends on the torques acting on the gas near the black hole horizon. We have explored the consequences of the assumptions that seed black holes are the remnants of collapsed Population III stars that form at $z \sim 40$ and grow to $10^9 M_\odot$ by $z_{QSO} = 6.43$, the highest redshift discovered to date, corresponding to QSO SDSS 1148+5251. Allowing for growth by both accretion and mergers, simple theory suggests that the required mass amplification is possible provided the radiation efficiency satisfies $\epsilon_M \lesssim 0.2$, with the upper limit decreasing should a quasar be discovered at higher redshift $z_{QSO} > 6.43$. The inferred efficiency is consistent with the observed ratio $R$ of the QSO plus AGN radiation density to the mass density of local SMBHs.

When mergers are included, the upper limit to the efficiency required to build an SMBH by $z_{QSO} \approx 6.43$ increases to $\epsilon_M \sim 0.30$ (i.e., any black hole seed born at a finite redshift, $\infty > z_i > z_{QSO}$, must accrete at a lower efficiency than 0.30 to reach $10^9 M_\odot$). This upper limit is roughly consistent with the observational constraint $\epsilon_M \geq R$ and with theoretical values for accretion from an MHD disk in spin equilibrium, but only marginally consistent for accretion from a standard thin disk in spin equilibrium accounting for photon recapture, and inconsistent for accretion from a standard thin disk that drives the black hole to maximal spin. Should a quasar be discovered at $z_{QSO} > 6.43$, it would appear that accretion from a standard thin disk will be ruled out; the upper limit will fall below $\epsilon_M = 0.32$, the value for a standard thin disk in spin equilibrium accounting for photon recapture. Should a quasar be discovered at $z_{QSO} \approx 10^0$, $\epsilon_M$ would fall below 0.19 and the results would be difficult to reconcile with accretion from a typical MHD disk as modeled in recent simulations. These critical values of $z_{QSO}$ are all smaller if the initial black hole seed is smaller than $600 M_\odot$, as the top curve, which sets the limit, is lowered in the figure. Finally, should a quasar be discovered (perhaps unexpectedly) at $z_{QSO} \approx 18$, the upper limit to $\epsilon_M$ would drop below the observationally inferred value 0.1 for all $z_i \geq z_{QSO}$. This drop would not be easily explained by any of these models of accretion and might require super-Eddington accretion with $\epsilon_T > 1$ to keep $\epsilon_M \approx 0.1$. Alternatively, the value of $R$, observed for local host galaxies, might be significantly smaller at high redshift, which would relax the constraint on $\epsilon_M$. But achieving the inferred lower radiation efficiencies might then require advection-dominated accretion disks or spherical accretion.

5. SUMMARY

We have explored the evolution of black hole mass and spin by gas accretion in the early universe. We have illustrated how for Eddington-limited accretion, the growth of an SMBH depends sensitively on the radiative efficiency, $\epsilon_M$. For disk accretion, the mean efficiency is determined by the equilibrium black hole spin, which in turn depends on the torques acting on the gas near the black hole horizon. We have explored the consequences of the assumptions that seed black holes are the remnants of collapsed Population III stars that form at $z \sim 40$ and grow to $10^9 M_\odot$ by $z_{QSO} = 6.43$, the highest redshift discovered to date, corresponding to QSO SDSS 1148+5251. Allowing for growth by both accretion and mergers, simple theory suggests that the required mass amplification is possible provided the radiation efficiency satisfies $\epsilon_M \lesssim 0.2$, with the upper limit decreasing should a quasar be discovered at higher redshift $z_{QSO} > 6.43$. The inferred efficiency is consistent with the observed ratio $R$ of the QSO plus AGN radiation density to the mass density of local SMBHs.

When mergers are included, the upper limit to the efficiency required to build an SMBH by $z_{QSO} \approx 6.43$ increases to $\epsilon_M \sim 0.30$ (i.e., any black hole seed born at a finite redshift, $\infty > z_i > z_{QSO}$, must accrete at a lower efficiency than 0.30 to reach $10^9 M_\odot$). This upper limit is roughly consistent with the observational constraint $\epsilon_M \geq R$ and with theoretical values for accretion from an MHD disk in spin equilibrium, but only marginally consistent for accretion from a standard thin disk in spin equilibrium accounting for photon recapture, and inconsistent for accretion from a standard thin disk that drives the black hole to maximal spin. Should a quasar be discovered at $z_{QSO} > 6.43$, it would appear that accretion from a standard thin disk will be ruled out; the upper limit will fall below $\epsilon_M = 0.32$, the value for a standard thin disk in spin equilibrium accounting for photon recapture. Should a quasar be discovered at $z_{QSO} \approx 10^0$, $\epsilon_M$ would fall below 0.19 and the results would be difficult to reconcile with accretion from a typical MHD disk as modeled in recent simulations. These critical values of $z_{QSO}$ are all smaller if the initial black hole seed is smaller than $600 M_\odot$, as the top curve, which sets the limit, is lowered in the figure. Finally, should a quasar be discovered (perhaps unexpectedly) at $z_{QSO} \approx 18$, the upper limit to $\epsilon_M$ would drop below the observationally inferred value 0.1 for all $z_i \geq z_{QSO}$. This drop would not be easily explained by any of these models of accretion and might require super-Eddington accretion with $\epsilon_T > 1$ to keep $\epsilon_M \approx 0.1$. Alternatively, the value of $R$, observed for local host galaxies, might be significantly smaller at high redshift, which would relax the constraint on $\epsilon_M$. But achieving the inferred lower radiation efficiencies might then require advection-dominated accretion disks or spherical accretion.

5. SUMMARY

We have explored the evolution of black hole mass and spin by gas accretion in the early universe. We have illustrated how for Eddington-limited accretion, the growth of an SMBH depends sensitively on the radiative efficiency, $\epsilon_M$. For disk accretion, the mean efficiency is determined by the equilibrium black hole spin, which in turn depends on the torques acting on the gas near the black hole horizon. We have explored the consequences of the assumptions that seed black holes are the remnants of collapsed Population III stars that form at $z \sim 40$ and grow to $10^9 M_\odot$ by $z_{QSO} = 6.43$, the highest redshift discovered to date, corresponding to QSO SDSS 1148+5251. Allowing for growth by both accretion and mergers, simple theory suggests that the required mass amplification is possible provided the radiation efficiency satisfies $\epsilon_M \lesssim 0.2$, with the upper limit decreasing should a quasar be discovered at higher redshift $z_{QSO} > 6.43$. The inferred efficiency is consistent with the observed ratio $R$ of the QSO plus AGN radiation density to the mass density of local SMBHs.

When mergers are included, the upper limit to the efficiency required to build an SMBH by $z_{QSO} \approx 6.43$ increases to $\epsilon_M \sim 0.30$ (i.e., any black hole seed born at a finite redshift, $\infty > z_i > z_{QSO}$, must accrete at a lower efficiency than 0.30 to reach $10^9 M_\odot$). This upper limit is roughly consistent with the observational constraint $\epsilon_M \geq R$ and with theoretical values for accretion from an MHD disk in spin equilibrium, but only marginally consistent for accretion from a standard thin disk in spin equilibrium accounting for photon recapture, and inconsistent for accretion from a standard thin disk that drives the black hole to maximal spin. Should a quasar be discovered at $z_{QSO} > 6.43$, it would appear that accretion from a standard thin disk will be ruled out; the upper limit will fall below $\epsilon_M = 0.32$, the value for a standard thin disk in spin equilibrium accounting for photon recapture. Should a quasar be discovered at $z_{QSO} \approx 10^0$, $\epsilon_M$ would fall below 0.19 and the results would be difficult to reconcile with accretion from a typical MHD disk as modeled in recent simulations. These critical values of $z_{QSO}$ are all smaller if the initial black hole seed is smaller than $600 M_\odot$, as the top curve, which sets the limit, is lowered in the figure. Finally, should a quasar be discovered (perhaps unexpectedly) at $z_{QSO} \approx 18$, the upper limit to $\epsilon_M$ would drop below the observationally inferred value 0.1 for all $z_i \geq z_{QSO}$. This drop would not be easily explained by any of these models of accretion and might require super-Eddington accretion with $\epsilon_T > 1$ to keep $\epsilon_M \approx 0.1$. Alternatively, the value of $R$, observed for local host galaxies, might be significantly smaller at high redshift, which would relax the constraint on $\epsilon_M$. But achieving the inferred lower radiation efficiencies might then require advection-dominated accretion disks or spherical accretion.
We have assumed that the mass of SDSS 1148+5251 is \( \sim 10^9 \, M_\odot \); a lower value would relax many of our constraints, while a higher value would strengthen them. If the flux from this source were amplified by gravitational lensing, or beaming, then a lower mass estimate would be appropriate. However, no multiple images have been seen (Richards et al. 2004), and it has been shown that high amplification without at least two images is very improbable (Keeton et al. 2004). Strong beaming also seems unlikely, since it would reduce the line/continuum ratio (Haiman & Cen 2002), which is not observed (Willott et al. 2003). In fact, assuming that the quasar emits at the Eddington luminosity gives a mass of \( 4.6 \times 10^9 \, M_\odot \) (Fan et al. 2003; Haiman 2004), and this higher value strengthens our conclusions somewhat.

Isolating the accretion growth of a seed black hole in the early universe from the hole’s full dynamical history and environment does not allow us to account for other important correlations that provide clues to the formation of SMBHs. Such correlations include the SMBH mass versus bulge luminosity relation, \( M_{\text{BH}} \propto L_{\text{bulge}} \) (Kormendy & Richstone 1995), and the SMBH mass versus velocity dispersion relation, \( M_{\text{BH}} \propto v_c^2 \) (Gebhardt et al. 2000; Ferrarese & Merritt 2000; Tremaine et al. 2002), inferred for nearby host galaxies. Only by performing detailed simulations that track the formation and growth of SMBHs in a cosmological setting governed by hierarchical halo mergers, black hole mergers, gas settling, star formation, and feedback can these correlations be reliably reproduced. We look forward to the next generation of simulations that incorporate the recent results of relativistic MHD accretion onto black holes, since, as demonstrated here, the outcome of these global simulations may depend sensitively on the local physics of such accretion flows.

It is a pleasure to thank T. Abel, R. Cyburt, C. Gammie, J. McKinney, J. Miralda-Escudé, and Q. Yu for valuable discussions. We also thank the referee for a critical reading of the manuscript and valuable comments. A portion of this work was performed during the summer of 2004 at the Aspen Center for Physics, whose hospitality is gratefully acknowledged. This work was supported in part by NSF grants PHY-0205155 and PHY-0345151 and NASA grant NNG04GK54G at the University of Illinois at Urbana-Champaign.

REFERENCES

Abel, T., Bryan, G. L., & Norman, M. L. 2000, ApJ, 540, 39
———. 2002, Science, 295, 93
Balberg, S., & Shapiro, S. L. 2002, Phys. Rev. Lett., 88, 1301
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Bardeen, J. M. 1970, Nature, 226, 64
Bardeen, J. M., & Petterson, J. A. 1975, ApJ, 195, L65
Barkana, R., & Loeb, A. 2001, Phys. Rep., 349, 125
Baumgarte, T. W., & Shapiro, S. L. 1999, ApJ, 527, L5
———. 2002, ApJ, 564, 23
Bromley, J. M., Somerville, R. S., & Fabian, A. C. 2004, MNRAS, 350, 456
Bromm, V., Coppi, P. S., & Larson, R. B. 1999, ApJ, 527, L5
———. 2002, ApJ, 596, 34
Chandrashekhar, S. 1964a, Phys. Rev. Lett., 12, 114
———. 1964b, Phys. Rev. Lett., 12, 437
———. 1964c, ApJ, 140, 471
Clyburt, R. H., Fields, B. D., & Olive, K. A. 2003, Phys. Lett. B, 567, 227
De Villiers, J. P., & Hawley, J. F. 2003, ApJ, 589, 458
De Villiers, J. P., Hawley, J. F., & Krolik, J. H. 2003, ApJ, 599, 1238
Favata, M., Hughes, S. A., & Holtz, D. E. 2004, ApJ, 607, L17
Genzel, R., Eckart, A., Ott, T., & Eisenhauer, F. 1997, MNRAS, 291, 219
Gebhardt, K., et al. 2000, ApJ, 540, L13
Gnedin, O. Y . 2001, Classical Quantum Gravity, 18, 3983
Haehnelt, M. G., & Kauffmann, G. 2000, MNRAS, 318, L35
Haiman, Z. 2004, ApJ, 613, 36
Haiman, Z., & Cen, R. 2002, ApJ, 578, 702
Haiman, Z., & Loeb, A. 2001, ApJ, 552, 459
Haiman, Z., & Quataert, E. 2005, in Supermassive Black Holes in the Distant Universe, ed. A. J. Barger (Dordrecht: Kluwer), in press (astro-ph/0403225)
Hege, A., Fryer, C. L., Woosley, S. E., Langer, N., & Hartmann, D. H. 2003, ApJ, 591, 288
Ho, L. C. 1999, in Observational Evidence for Black Holes in the Universe, ed. S. K. Chakrabarti (Dordrecht: Kluwer), 157
Hughes, S. A., & Blandford, R. D. 2003, ApJ, 585, L101
Hut, P., & Rees, M. J. 1992, MNRAS, 259, P27
Keeton, C. R., Kuhlen, M., & Haiman, Z. 2004, ApJ, submitted (astro-ph/0405143)
Kormendy, J., & Richstone, R. D. 1995, ARA&A, 33, 581
Macchetto, F. D. 1999, ApSS, 269, 269
Madau, P., & Quataert, E. 2004, ApJ, 606, L17
Madau, P., & Rees, M. 2001, ApJ, 551, L27
McKinney, J. C., & Gammie, C. F. 2004, ApJ, 611, 977
McLure, R. J., & Dunlop, J. S. 2004, MNRAS, 352, 1390
Merritt, D., Milosavljevic, M., Fabara, M., Hughes, S. A., & Holz, D. E. 2004, ApJ, 607, L9
Natarajan, P., & Pringle, J. E. 1998, ApJ, 506, L97
Norman, M. L. 2004, talk presented at Aspen Summer Workshop on the Formation of Supermassive Black Holes, June 2004, Aspen Center for Physics
Novikov, I. D., & Thorne, K. S. 1973, in Black Holes, Les Houches 1972, ed. C. DeWitt & B. DeWitt (New York: Gordon & Breach), 343
Omukai, K., & Palla, F. 2003, ApJ, 589, 677
Ostriker, J. P. 2000, Phys. Rev. Lett., 84, 5258
Pringle, J. E., & Rees, M. J. 1973, A&A, 29, 179
Rees, M. J. 1978, Nature, 275, 516
———. 1984, ARA&A, 22, 471
———. 1998, in Black Holes and Relativistic Stars, ed. R. M. Wald (Chicago: Univ. Chicago Press), 79
———. 2001, in Black Holes in Binaries and Galactic Nuclei, ed. L. Kaper, E. P. J. van den Heurak, & P. A. Woudt (New York: Springer), 351
Richards, G. T., et al. 2004, AJ, 127, 1305
Richstone, D., et al. 1998, Nature, 395, 14
Shapiro, S. L., & Teukolsky, S. A. 1977, ApJ, 213, 82
Soltan, A. 1982, MNRAS, 200, 115
Spergel, D. N., et al. 2003, ApJS, 148, 175
Thorne, K. S. 1974, ApJ, 191, 507
Thorne, K. S., & Fabian, A. C. 2004, MNRAS, 350, 456
Tremaine, S. 1984, ARA&A, 22, 471
Tremaine, S., & Bolte, M. 2002, ApJ, 572, L17
Tremaine, S., & Weisberg, J. M. 2004, MNRAS, 356, L97
Tremaine, S., et al. 2002, ApJ, 574, 740
Volonteri, M., Haardt, F., & Madau, P. 2003, ApJ, 582, 559
Volonteri, M., Madau, P., Quataert, E., & Rees, M. J. 2005, ApJ, 620, 69
Wagoner, R. V. 1969, ARA&A, 7, 553
Willott, C. J., McLure, R. J., & Jarvis, M. J. 2003, ApJ, 587, L15
Yoo, J., & Miralda-Escudé, J. 2004, ApJ, 614, L25
Yoshida, N., Abel, T., Hernquist, L., & Sugiyama, N. 2003, ApJ, 592, 645
Yu, Q., & Tremaine, S. 2002, MNRAS, 335, 965