The CO White Dwarf + Intermediate Mass/Massive Star Binary Evolution: Possible Merger Origins for Peculiar Type Ia and II Supernovae

Iminhaji Ablimit$^{1,2}$

ABSTRACT

Binary stellar evolution has been studied as important pathway to initiate various transient events like supernovae (SNe). Although the common envelope (CE) in a binary, outcomes of the CE and conditions for the SN explosion during the CE phase are uncertain, it has been suggested that SN explosions can be triggered during the CE phase. In this work, we explore formation and evolution routes of carbon/oxygen (CO) WDs binaries in order to investigate mergers of CO WDs and cores of non-degenerate stars during the CE phase as possible origins for SNe under the core merger detonation (CMD) scenario by considering several binary physical models. Evolution of CO WD + intermediate mass normal (non-degenerate and hydrogen-rich) star binaries lead to mergers during the CE phases still may trigger type Ia SNe (SNe Ia) interacting with circumstellar material under different models. Mergers between CO WDs and cores of He (non-degenerate and helium-rich) stars within the CE phases are rare comparing to other CE merger events. These two channels may produce peculiar SNe Ia such as over-luminous/super-chandrasekhar mass SNe Ia as certain fraction of them have combined core masses $\geq 2 M_\odot$. In the channel of CO WD + massive star ($\geq 8 M_\odot$) CE events, we find that rates of mergers between CO WDs and He cores of massive stars are $(0.85 - 12.18) \times 10^{-5} M_\odot^{-1}$ which may initiate type II superluminous SNe like SN 2006gy, and delay times from this scenario are in the range of 34 - 120 Myr. Our results based on the CMD model are comparable with observational results of peculiar SNe.

Subject headings: Unified Astronomy Thesaurus concepts: Binary stars (154); Supernovae (1668); Close binary stars (254); X-ray binary stars (1811); Common

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$^1$Key Laboratory for Optical Astronomy, National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China. iminhaji@nao.cas.cn

$^2$Department of Astronomy, Kyoto University, Kitashirakawa-Oiwake-cho, Sakyo-ku, Kyoto 606-8502, Japan.
1. Introduction

The explosion of a carbon/oxygen white dwarf (CO WD) has been commonly accepted for generating a type Ia supernova (SN Ia). In the main progenitor scenarios of SNe Ia, CO WDs accrete matter from their non-degenerate companions until its mass reaches the Chandrasekhar limit mass or merges with other degenerate ones to explode as SNe Ia (Tutukov & Yungelson 1979; Nomoto 1982; Iben & Tutukov 1984; Yungelson et al. 1994; Li & van den Heuvel 1997; Langer et al. 2000; Han & Podsiadlowski 2004; Hachisu et al. 2008; Lü et al. 2009; Ruiter et al. 2011; Toonen et al. 2012; Hamers et al. 2013; Ablimit et al. 2016; Wang et al. 2017; Livio & Mazzali 2018). The problem of the evolutionary scenarios that may lead to SNe Ia still remains uncertain. It is still not known, whether any of the suggested single-degenerate, double-degenerate, core-degenerate etc. channels may lead to explosion, whether one or several channels may contribute. In the past decades, many dozens of works have been attempting to solve the problem by using binary population synthesis (BPS) method (recent reviews, e.g., Maeda & Terada 2016; Livio & Mazzali 2018).

Binaries should survive from the CE (Paczyński 1976) phase in those scenarios if they evolve into the CE. In addition to the two main scenarios, Sparks & Stecher (1974) first proposed that the SN explosion may happen by the merger inside the CE, and this indicates that the merger inside the CE might be a possible trigger for various SNe (not just for SNe Ia). There is a type of merger model during the CE phase, the so-called core-degenerate scenario which is a merger of a WD with the hot CO core of the asymptotic giant branch (AGB) star during the CE evolution that may trigger an SN Ia, and there would be a circumstellar material (CSM) interaction in this model (e.g., Livio & Riess 2003; Kashi & Soker 2011; Soker 2011; Soker et al. 2013; Soker 2019). Although the rate of the core degenerate model is controversial (Wang et al. 2017; Canals et al. 2018), this model has been suggested as the possible progenitors for peculiar SNe Ia-CSM, i.e., SN 2014J and PTF 11kx where the large CSM mass results from the evolution of a CO WD + intermediate mass star (< 8M⊙) system (e.g., Soker 2011; Soker et al. 2013; Soker 2019). One typical observational result for supporting the merger of a WD with the AGB star core process might be the optical observations of SN 2002ic (Hamuy et al. 2003) which reveal large amounts of CSM seen as a strong hydrogen emission. Hamuy et al. (2003) suggested that the progenitor system contained a massive AGB star which lost a few solar masses of hydrogen-rich gas prior to the type Ia explosion. Briggs et al. (2015) performed the BPS calculation for the merger
of CO WDs + AGB cores, and argued that the majority of CO WDs with strong magnetic
fields might be resulted from mergers during the CE phase. It is also worth to note that the
recent detailed CO WD binary evolution study by Ablimit & Maeda (2019a) showed that
the strong magnetic field of a CO WD may play a key role in a different way during the
mass transfer, so that highly magnetized CO WD binaries may have significant contributions
to SNe Ia without merging inside the CE (see also Ablimit & Maeda 2019b). Considering
various SN Ia populations including SNe Ia-CSM, we will investigate mergers during the CE
phase as the possible origins for SNe Ia.

Unlike SNe Ia, core-collapse SNe (CCSNe) originate from massive stars. The binary evolu-
tion including mass loss and/or merger has influence stellar populations and progenitors of
type II CCSNe (e.g., Eldridge et al. 2008; Zapartas et al. 2017). Super-luminous supernovae
(SLSNe) are type of stellar explosions with the luminosity 10 or more times higher than that
of standard supernovae, and several mechanisms are proposed to generate SLSNe including
their light-curves and spectral features (e.g., Gal-Yam 2019). Type I SLSNe which is one
of the two main classes of SNe do not show hydrogen lines in their spectra (e.g., Pastorello
et al. 2010; Inserra et al. 2013; Quimby et al. 2011), while the other main class type II
SLSNe exhibit hydrogen lines (e.g., Smith et al. 2007; Ofek et al. 2007) and result from
massive stars (e.g., Woosely et al. 2007; Moriya et al. 2013). Sabach & Soker (2014) pointed
out that the CE merger-induced events may be the origins of SNe-like transients. Recently,
Jerkstrand et al. (2020) presented their observational and theoretical results for a type II
SLSN named SN 2006gy (e.g., Smith et al. 2007; Ofek et al. 2007; Agnoletto et al. 2009),
and demonstrated that the observed signature in the late-time nebular spectrum and light
curve of this transient can be reproduced by the iron-rich SN Ia ejecta and its interaction
with a massive ($\sim 10M_\odot$) CSM. Jerkstrand et al. (2020) suggested to explain SN 2006gy
with one evolution pathway proposed by Sabach & Soker (2014) which is that of a merger
of a CO WD with the core of a red supergiant (RSG) star. Based on Sabach & Soker (2014)
and Jerkstrand et al. (2020), we will study the possible progenitor model for SN 2006gy-like
SLSNe under a new model (see section 2 for the model description) with different binary
physics by considering several important evolution pathways.

After a WD + a massive non-degenerate star formation from a primordial MS-MS
binary, the massive star may fill its Roche lobe, undergo Roche-lobe overflow (RLOF) and
start the mass transfer. Then, the WD can be engulfed by the companion’s envelope due
to the rapid and unstable mass transfer (Paczyński 1976). Then, the WD may merge with
the massive companion’s core during the CE phase, or the binary may expel the CE and
eventually may become a neutron star (NS) - WD system which is one of the promising
gravitational wave (GW) and gamma ray burst sources (e.g., van Paradijs et al. 2000).
Previous works mainly studied the formation of NS - WD systems that formed from binary
evolutions (e.g., Tutukov & Yunggelson 1993; Tauris & Sennels 2000; O’Shaughnessy & Kim 2010; Sabach & Soker 2014). In this work, we study the formation and evolution of a CO WD with a massive star ($\geq 8M_\odot$) by considering survival from or merger inside the CE. A CO WD-NS will be formed if they survive from the CE, or may produce SN 2006gy-like transients if they merge inside the CE.

By adopting BPS models including a wide range of physical ingredients, we investigate evolutionary processes in CO WD binaries by mainly focusing on the possible merger-induced detonation inside the CE, and explore the effect of different physical conditions on the possible progenitors of SNe Ia, SN 2006gy-like transients. Formation of CO WD-NS systems as GW sources is also studied in this work. In §2, we describe our method to treat the binary physical processes and parameters in the BPS simulations, such as mass transfer (decided from critical mass ratio), CE (different CE efficiency, different binding energy parameter) and metallicity. Formation and evolution of the CO WDs + intermediate mass stars including He stars as the progenitors for SNe Ia are shown in §3. We show that two channels in §3 could produce one sub-types of SNe Ia which is the over-luminous / super-Chandrasekhar mass SNe Ia, 2003fg-like SNe Ia (Howell et al. 2006; Hicken et al. 2007; Scalzo et al. 2010; Hachisu et al. 2012). The possible contribution of mergers of CO WDs with the He cores of massive stars to the SN 2006gy-like SLSNe is examined in §4. CO WD-NS systems that evolved from the CO WD + massive star binaries are also investigated in §4. The discussion and summary are in §5.

2. Binary Population Synthesis

In this work, a large number ($10^7$) of binary evolution calculations have been done by using the updated BSE population synthesis code with Monte Carlo technique (Hurley et al. 2002; Kiel & Hurley 2006; Ablimit et al. 2016; Ablimit & Maeda 2018). In this updated BPS code, the initial input parameters of the primordial binaries are set as follows. In the subroutine sample of the code, the initial mass function of Kroupa et al. (1993) is adopted for the primary (the more massive one in a binary) mass,

$$f(M_1) = \begin{cases} 
0 & M_1/M_\odot < 0.1 \\
0.29056(M_1/M_\odot)^{-1.3} & 0.1 \leq M_1/M_\odot < 0.5 \\
0.1557(M_1/M_\odot)^{-2.2} & 0.5 \leq M_1/M_\odot < 1.0 \\
0.1557(M_1/M_\odot)^{-2.7} & 1.0 \leq M_1/M_\odot \leq 150, 
\end{cases}$$

(1)
The secondary mass is decided by following the distribution of the initial mass ratio of the secondary to the primary,

\[ n(q) = \begin{cases} 
0 & q > 1 \\
\mu q^\nu & 0 \leq q < 1, 
\end{cases} \tag{2} \]

where \( q = M_2/M_1 \), \( \mu \) is the normalization factor for the assumed power law distribution with the index \( \nu \). A flat initial mass ratio distribution which are \( \nu = 0 \) and \( n(q) = \text{constant} \) are utilized in the calculations. The initial orbital separation, \( a_i \), is used as \( (\text{Davis et al. 2008}) \),

\[ n(a_i) = \begin{cases} 
0 & a_i/R_\odot < 3 \text{ or } a_i/R_\odot > 10^6 \\
0.078636(a_i/R_\odot)^{-1} & 3 \leq a_i/R_\odot \leq 10^6. 
\end{cases} \tag{3} \]

The uniform initial eccentricity distribution which has a range between 0 and 1 is taken for the calculations. Initial binary parameters \( M_1, M_2 \) and \( a \) within the limits introduced above can be set up with the \( n_x \) grid points of parameter \( \chi \) logarithmically spaced,

\[ \delta \ln \chi = \frac{1}{n_x - 1}(\ln \chi_{\text{max}} - \ln \chi_{\text{min}}), \tag{4} \]

For each set of initial parameters, we evolve the binary system to an age of the Hubble time, or until it is destroyed. Each phase of the evolution, such as tidal evolution, angular momentum changes due to mass variations etc., is followed in detail according to the algorithms described in Hurley et al.(2002). Any other main revisions in the code for this work are as follows.

For the SN remnant calculation, we adopted the rapid remnant-mass model of Fryer et al.(2012) in the subroutine \textit{HRDIAG} of the code, and note that a typo in Fryer et al. (2012) has been corrected in our code (corrected as \( a_1 = 0.25 - \frac{1.275}{M_1-M_{\text{proto}}} \) in Equation 16). The NS formation by the accretion-induced collapse of a WD and the possibility of NS formation through electron-capture SN is taken into account in the work. The wind mass loss prescription of Vink et al. (2001) is utilized for O and B stars in different stages (hot stars) in the subroutine \textit{mlwind} of the updated code. For the wind mass loss of luminous blue variable stars- \( 1.5 \times 10^{-4} M_\odot \text{ yr}^{-1} \) (Vink & de Koter 2002) is taken, while default wind mass loss prescriptions of Hurley et al.(2000) are used for other types of stars. The velocity dispersion of \( \sigma_k = 265 \text{ km s}^{-1} \) (Hobbs et al. 2005) and \( \sigma_k = 40 \text{ km s}^{-1} \) (Dessart et al. 2006) are employed in the Maxwellian distribution of the kick velocity imparted to the newborn NS formed from CCSNe and electron-capture SNe, respectively. Note that these prescriptions would not affect the formation of the CO WD + non-degenerate star systems which never experience SN explosions nor evolve through very massive stars. However, they can influence the formation of CO WD - NS systems. A CO or ONeMg WD accretes matter and grows in mass, and finally WD explodes or collapses to form an NS when the WD’s mass reach to the Chandrasekhar limit mass (1.44 \( M_\odot \) is adopted in this work). The accretion efficiency is
important, and the magnetic field also may play key role during the mass transfer process of the WD binary (e.g., Hillman et al. 2016; Ablimit & Maeda 2019a, b; Ablimit 2019). The CO WDs mildly accrete matter and grow in mass when the unstable mass transfer (CE phase) considered.

**Mass transfer stability and critical mass ratio:** The stability of mass transfer has a decisive role in the binary evolution, and a binary would evolve into a CE phase if the mass transfer becomes dynamically unstable. It is not an easy task to simulate the mass transfer in the BPS or detailed evolution codes (e.g. de Mink et al. 2007; Pavlovskii & Ivanova 2015). The mass transfer in the BPS code is usually determined with parameterized and simplified stellar models. In this study, the critical mass ratio is seen as a key parameter in the binary evolution which has a crucial role in the stability of mass transfer. The CE evolution may start if the mass transfer becomes dynamically unstable when the mass ratio of two stars is larger than $q_{\text{crit}}$, and the CE phase is very important for the formation of various celestial objects. Hjellming & Webbink (1987) showed that $q_{\text{crit}}$ varies with the evolutionary state of the donor star at the onset of RLOF. We adopt two different prescriptions for $q_{\text{crit}}$ when the donor star is on MS or crossing the Hertzsprung Gap (HG) in the subroutine *evolv2* (for more details see Ablimit et al. (2016), Ablimit & Maeda (2018), and Table 1). In the first model which is referred to as $q_{\text{crit}} = q_{\text{const}}$, it is taken as $q_{\text{crit}} = q_{\text{const}} = 4.0$ (e.g., Hurley et al. 2002). We adopted another prescription of $q_{\text{crit}} = q_{\text{cs}}$ which is derived by considering the spin of the accretor during the mass transfer process (e.g., Ablimit et al. 2016; note that here the mass ratio $q_{\text{cs}}$ is the primary to secondary stars). In the mass transfer phase, even a small amount of mass accretion onto the secondary star from primary star will make the secondary spin faster and closer to its critical value (Packet 1981). In this model, the spin of the accretor may reduce the accretion rate onto the rotating star by a factor of $(1 - \Omega/\Omega_{\text{cr}})$, where $\Omega$ is the angular velocity of the star and $\Omega_{\text{cr}}$ is its critical value (e.g., Stancliffe & Eldridge 2009). Thus, this rotation-dependent mass transfer makes the mass transfer accretion rate very low, and most of the transferred matter can be escaped from the binary. We assume that the material is ejected with the specific angular momentum of the accretor (e.g., Hurley et al. 2002). When we consider the non-conservative mass transfer with highly mass loss, the maximal initial mass ratio of the primary to the secondary stars in primordial binaries will be $\sim 5-6$ (the mass ratio increases with the spin of the accretor), and a larger number of the primordial binaries will not have the contact phase and can experience stable mass transfer phases until the primary’s envelope is completely exhausted (see Abdusalam et al. (2020) for similar discussions and results). If the primordial primary is on the first giant branch (FGB) or the AGB, we use

$$q_{\text{crit}} = 0.362 + \frac{1}{3(1 - M_c/M)^{1/3}}.$$  \hfill (5)
where \( M \) and \( M_c \) are the whole mass and core mass of the donor star (Hjellming & Webbink1987). If the mass donors are stripped helium-rich MS or helium-rich giants, \( q_{\text{crit}} = 3.0 \) or 0.784 (e.g., Hurley et al. 2002), respectively.

**Common envelope model:** The energy conservation model (e.g., De Marco et al. 2011; Ivanova et al. 2013) has been applied for the CE evolution and there are two important but unknown physical parameters in the model which have been set as constants. They are the CE efficiency (\( \alpha_{\text{CE}} \)) and binding energy (\( \lambda \)) parameters. Whether the two stars merged to one or survived from the CE to continue their evolution depend on these two parameters, and the relation is,

\[
\frac{GM_{1,i}M_{1,\text{env}}}{\lambda R_{\text{RL}}} = \alpha_{\text{CE}} \left( \frac{GM_{1,f}M_{2,f}}{2a_f} - \frac{GM_{1,i}M_{2,i}}{2a_i} \right),
\]

where \( G \) is the gravitational constant; \( M_{1,i}, M_{1,\text{env}} \) and \( M_{1,f} \) are the initial, envelope and final masses of the primary star, respectively; \( M_{2,i}, M_{2,\text{env}} \) and \( M_{2,f} \) are the initial, envelope and final masses of the secondary star, respectively; \( R_{\text{RL}} \) is the Roche lobe radius of the donor star at the onset of RLOF. We first take a constant case \( \lambda = 0.5 \) for the binding energy parameter. However, it has been ascertained that the binding energy parameter changes with the mass and evolutionary stage of the star (e.g., Dewi & Tauris 2000). We also use variable \( \lambda = \lambda_w \) in the subroutine \( \text{comenv} \) of the updated code by following Wang et al. (2016; see also Klenck et al. (2020)) in which they estimated lambda based on detailed stellar evolution models. For the CE efficiency, we take \( \alpha_{\text{CE}} = 1.0 \) (e.g., Klenck et al. 2020) in the simulations, and \( \alpha_{\text{CE}} = 0.1 \) is taken as a pessimistic one for the comparison.

In order to test the effect of the metallicity on the model outcomes, the initial metallicity of stars is set to be 0.02 and 0.001. Default values of other parameters (Hurley et al. 2002) are used in the calculation. In total, we have five models which cover results of other parameter combinations, and see Table 1 for the main prescriptions taken for the BPS simulations.

**Description of the key merger model:** The thermonuclear explosion of a CO WD following an initial He detonation on the WD surface (the double detonation scenario for SNe Ia) has been discussed as a popular scenario for SNe Ia (e.g. Livne 1990; Iben & Tutukov 1991; Woosley & Weaver 1994; Fink et al. 2010; Townsley et al. 2012; Jiang et al. 2017; De et al. 2019). A CO WD merger with another CO WD with the He-rich shell or merger between a CO WD and a He WD has been suggested for triggering the SN Ia explosion through the He detonation (Shen et al. 2018; Tanikawa et al. 2019). In this work, we assume that a merger of a CO WD with a He core of the companion star during the

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1There are some other different mechanisms to change fate of the CE evolution, such as jets (e.g., Shiber et al. 2019)
CE phase could initiate the SN explosion, which we term as the core merger detonation (CMD) model. In this hypothesis, a thick He layer would be accumulated on the surface of the CO WD following the rapid accretion during the spiral-in process between the CO WD and He core of the H-rich non-degenerate star after the CE event, then conditions for the initial He detonation and stronger second detonation inside the WD could be achieved. Due to the short timescale and lack of direct observational evidences for the CE phase, what happens inside the CE and merger outcomes from the CE are poorly known. Besides, the exact conditions (mass of the He layer, temperature, density of the CO WD, etc.) for the He detonation for the SN explosion are also not well established. Thus, we simply assume that at least a $\geq 0.6M_\odot$ core of a star (e.g., Bloecker 1995; Soker 2015) and a CO WD with a critical mass $\geq 0.9M_\odot$ or $\geq 1.0M_\odot$ inside the CE would be needed to trigger the SN (e.g., Shigeyama et al. 1992; Sim et al. 2010; Ruiter et al. 2011). We set a reasonable critical mass for the CO WD for the He detonation-triggered SN explosion, because a massive enough WD is helpful for the accreting He materials to experience the stronger shock heating, and a massive enough WD is dense enough to have stronger detonation.

The CO WD + He-rich non-degenerate star systems are descendants of CO WD + H-rich normal star binaries, and they also may evolve into the CE phase. In the CO WD + He star binary, the core of He star may not or may further evolve (it depends on evolution conditions/stellar evolution stage when the CE occurs), thus the CO WD may have rapid accretion of helium or carbon during the merger process in a CE phase. We simply assume that the rapid accretion of a helium-rich or carbon-rich material or He + C material as in the hybrid double degenerate model (Perets et al. 2019) onto a CO WD in the merger process may trigger SNe Ia. Regarding the uncertain issues in the explosion, evolution mechanisms, and limitations in the one-dimensional (1D) BPS simulation, we just consider that the thermonuclear SN may happen in the CO WD + He star binary during the CE phase if the CO WD mass satisfies the critical value showed above.

**Uncertainties:** We developed the BPS code with some new physical ingredients as introduced above, which can be helpful to our understanding of the binary evolution as origins of peculiar transients. However, there are still some common issues in this work comparing to similar previous works, such as not-well constrained parameters/processes (i.e., $\alpha_{\text{CE}}$) in the BPS and limitations on the explosion mechanisms of SNe (i.e., exact mass of the CO WD to explode). The commonly accepted and simplified/paramitized ways are adopted in the BPS, which may different from more detailed modeling (e.g., MESA modeling), and the predicted rate from the systematic study with large populations is always one of the important/meaningful outcomes of such studies. Another important question is whether the merger process is a ”fast merger” or a ”slow merger”. The idea of the ”fast merger” is that the spiral-in occurs on a dynamical timescale and the companion is dynamically disrupted and
may lead to rapid accretion onto the compact core and trigger the detonation. In the "slow merger" case, the He core may fills its RL and starts mass transfer onto the compact core during the spiral-in process, and this would have a longer delay time to trigger the detonation. It still remains unclear that what would happen inside the CE. More observational constraints and theoretical works need to be placed for binary evolutionary scenarios and explosion mechanisms for SNe in the future.

3. Mergers of CO WDs with Cores of Intermediate Mass Stars and He Stars within the CE Phase: Possible Explosions of SNe Ia

There are 467918 CE events evolved from $1 \times 10^7$ initial MS-MS binaries in our standard BPS simulation (model 1), and this CE event number is consistent with results of similar works (e.g., Howitt et al. 2020). Among these CE events, CO WD CE events with H-rich normal stars and He stars are investigated in this work. From interacting MS-MS binaries, there are several main evolutionary pathways to produce a primary CO WD with a non-degenerate companion system. We briefly introduce the one main formation channel of CO WD + evolved intermediate-mass stars ($M_2 < 8.0 M_\odot$): The primary star with relatively higher mass in a MS-MS binary evolves first, loses its mass via RLOF to the secondary star and becomes a CO WD. The secondary starts its evolution after the CO WD formed, and the unstable RLOF mass transfer may occur when it becomes a HG star, a red giant (RG) star or an AGB star. The envelope of the secondary engulfs the CO WD due to the unstable and rapid RLOF mass transfer, then a system composed of the CO WD + the core of the secondary inside the CE is formed. Figure 1 shows the mass distributions of all CO WDs + H-rich stars and CO WDs + He stars just before entering the CE evolutions, and also shows initial masses which cause those CE events under our standard model. The CO WD + H-rich star CE event channel is realized for the following typical ranges in the initial binary parameters; the initial primary mass $M_{1,i} \sim 1.0 - 10.0 M_\odot$, the initial mass ratio $q = M_{2,i}/M_{1,i} \sim 0.15 - 1.0$; and the initial orbital period $P_{\text{orb},i} \sim 0.01 - 93.6$ days. The mass range of the WD in this channel is between $\sim 0.5$ and $\sim 8.0 M_\odot$ for the companion star (see Figure 1).

Event rates and delay time ranges of the CE systems which contain CO WDs + non-degenerate H-rich stars ($M_2 < 8.0 M_\odot$) under different conditions and five different models are summarized in Table 2. The rates of the all CE events (CO WDs + non-degenerate

\footnote{Note that the WD binary evolution with the dynamically stable mass transfer and WD+MS binaries (see Willems & Kolb (2004) for more details) are out of scope of this work.}
H-rich stars \((M_2 < 8.0M_\odot)\) clearly change with the \(\alpha\) parameter, metallicity and mass transfer model. The pessimistic value of \(\alpha = 0.1\) in model 2 causes more mergers during the CE events of non-degenerate stars and non-degenerate stars, therefore it reduces the number of CO WD binaries with CE phases from 0.44\% to 0.27\%. The central temperatures and densities are higher, and more hydrogen burns stably for the low-metallicity MS stars, and it leads to larger core masses and different lifetimes during the evolution. Thus, more CO WD binaries can be formed with the low metallicity condition. The rotation-dependent (highly nonconservative) mass transfer model adopted in this work during the primordial binary evolution allows a large amount of the primordial binaries to experience stable mass transfer, thus this reduces the number of CO WD binaries with CE events. About 0.08\% - 0.12\% CO WDs merge with cores of H-rich stars at different evolutionary stages inside the CE, and mergers (numbers include delay times) at different evolutionary stages such as HG, RG or AGB \(^3\) are affected by the mass transfer (critical mass ratio), \(\alpha\) and \(\lambda\) parameters as these physical parameters are related to stellar structure and evolution. It is worth to note that at least 20\% - 40\% mergers of CO WDs and cores of intermediate mass stars have combined core masses larger than \(2M_\odot\) (Figure 2). Thus, they might be the progenitors of over-luminous SNe Ia.

We investigate the possible CMD explosions triggered by mergers between CO WDs and He cores of non-degenerate intermediate stars during CE events by considering the critical masses introduced above. They would be the origins for SNe Ia-CSM or/and overluminous SNe Ia, and their rates are given in Table 2. This channel includes products of the CD model when the WD CE core merges with an AGB star. Figure 2 displays property distributions of CO WD binaries which lead to the CMD event. The mass transfer model (model 5) clearly affects all CE event numbers, mass and orbital period distributions due to the mass accretion, mass loss and corresponding angular momentum evolution. In the mass transfer simulation of model 5, the larger mass loss (corresponding angular momentum loss) of the primordial binary due to the mass ratio (see section 2) makes the mass transfer stable, thus more primordial binaries can avoid CE phases (and have different orbital periods) comparing to the other model. The \(\alpha\) parameter (model 2) causes different event numbers and distributions including delay times (Table 2), because the less energy budget contributed for expelling the CE in the model 2 with \(\alpha = 0.1\) comparing to model 1 with \(\alpha = 1.0\). Heavier cores are formed with the low metallicity (model 3) due to the same reason discussed above. Two different prescriptions of \(\lambda\) (model 1 and model 4) give similar mass distributions (Figure 2)

\(^3\)The initial conditions, such as initial masses and initial separations, are important in determining at which evolutionary phase of the companion the WD enters its envelope (e.g., Hurley et al. 2002). These conditions are extensively studied in the literature and we do not give the details here.
while CE event numbers are different. \( \lambda \) is not the physical parameter for the single stellar evolution, so it does not affect the stellar features such as mass of the star. It is a parameter in the CE evolution of the binary. The more energy (i.e. internal energy) that is available in the \( \lambda \) prescription of model 4 for the CE ejection (see above section for more details) implies more successful CE ejections in the primordial binaries. This in turn reduces the number of CO WD CE systems in later evolution phases.

Comparing with the observed/time-integrated rate of SNe Ia which is about \( 10^{-3}M_\odot^{-1} \) (e.g. Maoz et al. 2014; Maoz & Graur 2017; Frohmaier et al. 2018), all predicted rates from this channel (Table 2) are lower than the observational rate. However, the rate of model 4 is consistent with the predicted SNe Ia-CSM rate by Soker (2019) and the fraction of known SNe Ia-CSM (roughly about few\( \times 10^{-5}M_\odot^{-1} \)) among all observed SNe Ia available to date (e.g., Li et al. 2011; Silverman et al. 2013), and it implies that the treatment of the binding energy in this work is physically motivated one. Besides, there are a few robust candidates for the He-detonation triggered SNe Ia reported so far, and the observationally inferred rate of such events which is estimated as 0.5% of the SN Ia rate (Jiang et al. 2017; De et al. 2019) can be reproduced by this work. Delay times under conditions 1 and 2 (see the caption of Table 2 for two conditions) for mergers tend to have the shorter delay times than \( \sim 2 \) Gyr in our results (see Table 2) due to the evolution timescales, and they can be longer if the ”slow merger” inside the CE is considered.

The H-rich envelope can be stripped away if a binary system survives from the CE phase, and a He star with a CO WD can be formed. If the He star begins unstable RLOF mass transfer, the core of the He star and CO WD enter the CE. The CO WD + He star CE event channel is realized for the following ranges in the initial binary parameters: the initial primary mass \( M_{1,i} \sim 1.2 - 7.2M_\odot \), the initial mass ratio \( q = M_{2,i}/M_{1,i} \sim 0.25 - 1.0 \) and the initial orbital period \( P_{\text{orb},i} \sim 0.02 - 47.1 \) days. The mass range of the WD in this channel is between \( \sim 0.5 \) and \( \sim 1.35M_\odot \) while the He stars’ mass is distributed between \( \sim 0.4 \) and \( \sim 3.4M_\odot \) (see Figure 1). In this channel, we consider mergers between cores of He stars and CO WDs with masses 0.9 or 1.0 \( M_\odot \) with the CE event that produce SNe Ia. From the rates (0.0 - 1.61 \times 10^{-5}M_\odot^{-1}) shown in Table 2, we can see that only at most \( \sim 10\% \) of all CE events of CO WDs + He stars end up with mergers under the condition \( M_{\text{WD}} \geq 0.9M_\odot \). In Figure 3 we demonstrate mass relations for CO WDs, He stars and their cores for model 1 which has the highest rate, and results show that mass distributions have relatively narrow ranges. These results suggest that the CO WD that merges with the core of the He star companion may not play an important role as much as mergers with other different companions. However, this might be the origin for some rare transients (peculiar SNe Ia like over-luminous SNe Ia or SNe Iax (see Foley et al. (2013) for the subclass SNe Iax)) which need further investigations. Our result suggests that the He accretion and
thermonuclear explosion, the so-called double detonation scenario in SD systems, from CO WDs + He stars that survive from the CE can contribute to SNe Ia more than those in the merger case.

4. CO WD and Massive Star Binaries with the CE Evolution

In our standard model, there are 1232 CO WD + massive star (> 8\(M_\odot\)) systems which experience the CE phase. The primordial MS/MS binaries of these systems have the following typical initial parameters: the initial primary mass \(M_{1,i} \sim 5.3 - 10.0M_\odot\), the initial secondary mass \(M_{2,i} \sim 2.6 - 7.3M_\odot\) (Figure 4) and the initial orbital period in a range of \(P_{\text{orb},i} \sim 0.1 - 40.5\) days. The mass relation of 1232 CO WDs and companion stars at the onset of the CE are given in Figure 4.

4.1. Mergers of CO WDs with Cores of Massive Stars inside the heavy CE: Possible Progenitors for SLSNe like SN 2006gy

In the standard model (model 1), 505 and 209 CO WDs merge with the He cores of the massive star companions inside the CE under the condition of the critical CO WD mass of 0.9 and 1.0\(M_\odot\) (Table 3), respectively. The mass relations between CO WDs and massive stars, and relations between CO WDs + cores of companions and massive stars’ envelopes for the mergers with \(M_{\text{WD}} \geq 0.9M_\odot\) are also shown in Figure 4. It can be seen from the figure that a system can indeed have the envelope as massive as > 6.0\(M_\odot\). If the CO WD merges with the non-degenerate He core of the massive star inside the CE, the accreted helium-rich material onto the CO WD ignited and started the detonation, then thermonuclear explosion is triggered following the core merger, thus the detonation process is different from the CD model which is suggested as mergers of two degenerate cores for the SNe Ia. Two main evolutionary pathways are demonstrated in Figures 5 and 6. Initially, most secondary stars in these evolutionary pathways are not massive stars. They become O/B type massive stars due to mass accretion from the primary stars, and the angular momentum transported with the accreted mass may spin the secondary up to be fast rotating Be stars (Waters et al. 1988; Pols et al. 1991; Willems & Kolb 2004; Ablimit & Lü 2013). The increasing number of candidates by the recent X-ray binary observations (e.g., SWIFT and eROSITA) (Coe et al. 2020; Haberl et al. 2020a,b) show that this kind of WD binaries may not be so rare as suggested by previous works (e.g., Li et al. 2012; Cracco et al. 2018). After the CO WDs are formed by going through two RLOF mass transfers, the massive star companions evolve and fill their RL at the evolution stages of the HG and core helium burning (CHeB), and the
mass transfers will be unstable as the mass ratios of CO WDs/massive companions are low. Thus, the massive giant branch stars’ envelopes engulf CO WDs and He cores, and a merger would happen inside the CE if the CE ejection failed. The delay times are relatively short which is between 34 and \( \sim 120 \) Myr. The mergers with HG stars are much more common than mergers with CHeB stage stars. With a standard BPS simulation, we have shown that the created CSM (Figure 4) and the rate (Table 3) of CMD SN explosions in CO WD + massive star systems can be comparable with the observational constraints of SN 2006gy (Jerkstrand et al. 2020; see below for the rate comparison).

The rates and delay times of CO WDs + massive star core mergers under different models are given in Table 3. Figure 7 shows the property distributions of CO WD + massive star CE mergers with \( M_{\text{WD}} \geq 0.9M_\odot \) under different models. Comparing the effect of physical parameters on properties of CO WD + intermediate star CE mergers, the metallicity with the massive stars case has a very clear influence on the mass distribution of cores due to the fact that a heavier core can be formed with low metallicity (see above sections for the reason). Of course, the mass transfer and \( \lambda \) also play an important role in the formation of CO WD/ Massive star binaries with the CE phase due to the reasons as discussed before. In any case, the combined masses of CO WDs and He cores of massive stars and H-rich envelope masses are always larger than \( 2.5M_\odot \) and \( 6.0M_\odot \), respectively. Derived envelope masses, core masses and rates imply that the CO WD + massive star core mergers could be progenitors of SN 2006gy-like type II SLSNe. Observationally, there is possibility of mixing the SNe Ia and type II SLSNe, and we assumed that mergers between CO WDs and He core of massive stars can trigger type II SLSNe. Based on Quimby et al. (2013), we roughly take the rate of type II SLSNe as \( \sim 10^{-5}\text{yr}^{-1} \), but note that the rate of type II SLSNe is not observationally well-determined yet. If we adopt the star formation rate of \( 2\ M_\odot\text{yr}^{-1} \) and binary fraction of 0.7, predicted rates (\( \sim 1.19 - 7.7 \times 10^{-5}\text{yr}^{-1} \)) of models (mode 1, 4 and 5) are comparable with the observation.

4.2. Survivors from the heavy CE: Formation of CO WD-NS Systems as the Gravitational Wave Sources

If the energy budget of the system is enough to expel the CE in the CO WD + massive star CE systems introduced in the above section, the CO WD + non-degenerate star binary will be formed in a significantly reduced orbit, and no clear accretion occurs during the CE (Ivanova et al. 2013). Given the reduced orbit, the secondary fills its RL and starts the mass transfer, and the accretion onto the WD depends on the retention efficiency and mass transfer rate (e.g. Bours et al. 2013). After a series of evolutions, the secondary may undergo
a SN explosion and collapse to an NS. The eccentricities of the post-SN orbits span the full range of (0-1) with natal kicks in our simulations (see Section 2). Formed CO WD - NS systems can be merge in a Hubble time due to the very small orbital period reduced by the CE ejection. Table 3 shows the formation rates of CO WD - NS systems through the channel introduced above. Excluding the model with a pessimistic value of α (all cores merge inside the CE), the rate changes from 0.54 to $2.83 \times 10^{-5} M_\odot^{-1}$ under different models. Note that here we focused on the reverse formation pathway that evolved from CO WD + massive star CE systems, while there are several pathways to form NS-WD systems (e.g., Sabach & Soker 2014). More CO WD - NS systems can be formed in our model 4 and 5, because more primordial binaries can avoid the CE evolution due to the highly non-conservative mass transfer model (model 5), and the λ prescription used in model 4/5 contributes more energy to expel the CE (more systems survive with reduced close orbits).

The merger of NS-WD systems would have various observable outcomes, and the GW signal from it is one of the good sources for the next generation GW detectors. In a CO WD- NS system, the orbital motion and angular momentum evolution with the GW emission (according to general relativity) will lead to merger of the two objects at the final stage of the inspiraling process. The merger timescale (Peters & Mathews 1963; Peters 1964; Lorimer 2008; Kuerban et al. 2020) is,

$$t = 9.88 \times 10^6 \left( \frac{P_{\text{orb}}}{1\text{hr}} \right)^{8/3} \left( \frac{\mu}{1 M_\odot} \right)^{-1} \left( \frac{M}{1 M_\odot} \right)^{-2/3} \text{yr},$$

(7)

where $\mu = M_1 M_2 / (M_1 + M_2)$ and $M = M_1 + M_2$ are the reduced mass and total mass of the system ($M_1$ and $M_2$ are the masses of two objects in a binary system), respectively. The GW strain will increase with time during the merging process. If the distance of the binary system with respect to us is $d$, then the strain amplitude of the GW (Peters & Mathews 1963) is,

$$h = 5.1 \times 10^{-23} \left( \frac{P_{\text{orb}}}{1\text{hr}} \right)^{-2/3} \left( \frac{M_{\text{ch}}}{1 M_\odot} \right)^{5/3} \left( \frac{d}{10 \text{kpc}} \right)^{-1},$$

(8)

where $M_{\text{ch}} = (M_1 M_2)^{3/5} / (M_1 + M_2)^{1/5}$ is the chirp mass. The GW frequency $f$ is twice the orbital motion frequency. The derived GW strain and frequency from merger signals of CO WD - NS systems formed in models 1 (lowest rate) and 5 (highest rate) are shown in Figure 8. About 30% and 75% of the whole CO WD - NS populations from models 1 and 5 are detectable by the upcoming GW detector LISA.
5. Discussion and Summary

We explored the formation and end products of binary CO WDs with non-degenerate companion stars which evolve into the CE phase by considering a number of physical elements in this BPS simulation. In particular, we investigated mergers between CO WDs and cores of non-degenerate stars inside the CE as possible progenitors of peculiar SNe with the proposed CMD scenario. We examined rates, delay times, properties of progenitors of these events, and their dependence on mass transfer, metallicity and CE evolution. Different BPS models coupled with Monte Carlo method give relative statistical errors (Ablimit et al. 2016; Ablimit & Maeda 2018). The difference in results of this work is mainly caused by the CE and mass transfer models. Future observational results may provide more direct evidences to constrain the CE evolution and mass transfer in the binary evolution. Predicted rates from the CMD model in this work (especially results of our model 4) are basically consistent with observational results of special transients.

For the CO WD + intermediate mass star channel, \(\sim 0.27\% - 0.44\%\) of all \(10^7\) systems in a simulation have evolved toward CO WD + intermediate mass star binaries with the CE phases under different models. Among all CO WD + intermediate mass star CE events, \(\sim 14\% - 25\%\) of CO WDs merge with cores of companion stars. The model with the \(\alpha = 0.1\) fails to survive from the CE and only has mergers, because very low energy contribute to expel the CE in this case. The merger under two conditions introduced in the caption of Table 2 may trigger SNe Ia, and corresponding rates are \((0.22 - 8.77) \times 10^{-5} M_\odot^{-1}\) and \((0.0 - 4.7) \times 10^{-5} M_\odot^{-1}\) for lower and higher WD mass cases (Table 2), respectively. This CMD scenario here includes the CD model as an outcome of the core merger between a CO WD and an AGB star, and roughly 24%-55% of our rates are the rates derived from the CD scenario. Delay times from the star formation to the merger are mainly distributed in a range of \(\sim 0.1 - 2.5\) Gyr. Our results show that the merger systems with \(M_{WD} \geq 0.9 M_\odot\) could produce the CSM with mass larger than \(2 M_\odot\), and this could account for a certain fraction of observed SNe Ia interacting with CSM. \(0.002\% - 0.024\%\) of all \(10^7\) binaries are CO WD + He star CE events, and core merger rates from this channels are too low to explain the normal SNe Ia. However, the merger between CO WDs and core of He stars may initiate peculiar SN transients (variant of SNe Ia) such as SNe Iax and overluminous SNe Ia. Heavier core masses (\(> 2 M_\odot\)) of mergers imply that the two evolution pathways, that of CO WD + intermediate mass star CE events and that of CO WD + He star CE events, may be origins for overluminous (super-chandrasekhar) SNe Ia, and predicted rates for this peculiar SNe Ia have a range of \(\sim (0.02 - 3.31) \times 10^{-5} M_\odot^{-1}\) according to different models.

Another important aim of the work is to investigate the possible origins for SN 2006gy-like type II SLSNe with different binary evolution models. Observational results of Jerkstrand
et al. (2020) imply that a merger of a CO WD with a He core of a massive star (Sabach & Soker 2014) may be the progenitor of SN 2006gy. The rapid He accretion onto the CO WD during the spiral-in merger process inside the CE may trigger SN-like transients which is termed as the CMD scenario in this work. By performing BPS simulations with different physical models, we explored the binary evolutions toward CO WD + massive star CE events. In general, only $\sim 0.012\%$ primordial binaries formed CO WDs + massive star binaries with the CE, and rates of core mergers among them are different according to adopted physical parameters in the binary evolution as summarized in Table 3. They have relatively short delay times. The derived rates for SN 2006gy-like type II SLSNe in this channel are at least two orders of magnitude lower than the rate of observed CCSNe. The helium-burning core of a supergiant and the ejected envelope can be a factor of ten larger than that required for a lower mass AGB star (see results in above sections), therefore CE simulations involving massive donor stars offer challenges beyond those required for low- and intermediate-mass systems.

Survivors from CO WD + massive star CE events eventually can become CO WD - NS systems in very close orbits. The rates of CO WD - NS binaries are $(0.54 - 2.83) \times 10^{-5} M_\odot^{-1}$ under different models except model 2. About 30% - 75% of CO WDs - NS systems that formed through the CO WD + massive star CE channel would emit GW signatures during the spiral merger process, which can be detected by LISA.

Our results once again show that the stability and nature of the mass transfer, CE evolution and metallicity are very important in binary stellar evolution. Whether a binary experiences a CE evolution or not, whether the cores merge or survive from the CE, and how stellar evolution (e.g. core mass) depends on metallicity are described in this work. This is a clear demonstration that pre-CE binary interactions can play an important role in the outcome of the CE. Rates and properties are affected by these physical ingredients, especially with extreme conditions. This work can be used for future observations to improve our understanding of binary stellar evolution and the nature of transient events. In our future work, we will study ONeMg WD binaries with CE evolution as possible origins for peculiar transients and objects (Ablimit et al. 2021).

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**Data Availability:** The data underlying this article will be shared on reasonable request to the corresponding author.

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Table 1: Different models in our calculation. $\alpha_{\text{CE}}$, $\lambda$, $q_{\text{crit}}$ and $z$ are the CE efficiency parameter, binding energy parameter in the CE evolution ($\lambda_w$ follows Wang et al.(2016)), critical mass ratio and metallicity, respectively.

| Model | $\alpha_{\text{CE}}$ | $\lambda$ | $q_{\text{crit}}$ | $z$   |
|-------|----------------|----------|----------------|------|
| mod1  | 1.0            | 0.5      | $q_{\text{const}}$ | 0.02 |
| mod2  | 0.1            | 0.5      | $q_{\text{const}}$ | 0.02 |
| mod3  | 1.0            | 0.5      | $q_{\text{const}}$ | 0.001|
| mod4  | 1.0            | $\lambda_w$ | $q_{\text{const}}$ | 0.02 |
| mod5  | 1.0            | $\lambda_w$ | $q_{\text{cs}}$ | 0.02 |
Table 2: Calculated rates ($R$) of CO WD/non-degenerate star CE events under five different models (in $10^{-5} \ M_\odot^{-1}$)

| CE events of CO WDs/companion stars | $R1$   | $R2$   | $R3$   | $R4$   | $R5$   |
|-------------------------------------|--------|--------|--------|--------|--------|
| a: CO WDs/normal stars (all CE)     | 441.30 | 271.18 | 543.46 | 405.82 | 363.58 |
| b: CO WDs/HG stars (merger)        | 37.03  | 18.87  | 48.56  | 34.38  | 0.00   |
| c: CO WDs/RG stars (merger)        | 51.49  | 75.23  | 26.30  | 86.83  | 89.85  |
| d: CO WDs/AGB stars (merger)       | 0.33   | 11.17  | 2.84   | 0.00   | 0.00   |
| e: CO WDs/normal stars (merger con1) | 5.37  | 8.77   | 7.86   | 8.75   | 0.22   |
| f: CO WDs/normal stars (merger con2) | 0.22  | 4.70   | 2.00   | 2.27   | 0.00   |
| g: CO WDs/He stars (all CE)        | 13.35  | 0.20   | 24.24  | 11.87  | 13.46  |
| h: CO WDs/He stars (Merger with $M_{COWD} \geq 0.9$) | 1.61  | 0.00   | 1.20   | 0.10   | 0.69   |
| i: CO WDs/He stars (Merger with $M_{COWD} \geq 1.0$) | 0.61  | 0.00   | 0.52   | 0.03   | 0.50   |

Ranges of delay times (DT in Gyr) under five different models

| Same as above | DT1     | DT2     | DT3     | DT4     | DT5     |
|---------------|---------|---------|---------|---------|---------|
| a             | 0.03 - 13.7 | 0.03 - 13.7 | 0.03 - 13.7 | 0.03 - 13.7 | 0.03 - 13.7 |
| b             | 0.07 - 0.71  | 0.07 - 0.71  | 0.05 - 1.61  | 0.05 - 0.71  | 0.00 - 0.00  |
| c             | 0.07 - 13.6  | 0.07 - 13.6  | 0.07 - 13.6  | 0.09 - 13.6  | 0.09 - 13.6  |
| d             | 0.08 - 13.6  | 0.08 - 13.6  | 0.08 - 13.6  | 0.08 - 13.6  | 0.08 - 13.6  |
| e             | 0.06 - 1.78  | 0.07 - 1.52  | 0.05 - 2.52  | 0.08 - 0.61  | 0.08 - 0.61  |
| f             | 0.07 - 1.78  | 0.07 - 1.52  | 0.08 - 2.13  | 0.08 - 0.62  | 0.00 - 0.00  |
| g             | 0.05 - 1.69  | 0.04 - 0.51  | 0.05 - 1.80  | 0.04 - 1.58  | 0.04 - 1.58  |
| h             | 0.05 - 0.70  | 0.00 - 0.00  | 0.05 - 0.52  | 0.04 - 1.07  | 0.04 - 1.07  |
| i             | 0.04 - 0.50  | 0.00 - 0.00  | 0.05 - 0.42  | 0.04 - 1.06  | 0.04 - 1.06  |

Note: Normal stars (only stars with masses < 8$M_\odot$ in this table) mean sub-giant or giant branch stars with hydrogen-rich envelopes; He-rich stars are the (sub)giant helium (no hydrogen) stars. The merger con1 means that the merger with the condition of $M_{COWD} \geq 0.9$ and $M_{2C} \geq 0.6$, and merger con2 is the merger with condition of $M_{COWD} \geq 1.0$ and $M_{2C} \geq 0.6$. $M_{2C}$ is the core mass of the WD’s companion star. Event numbers can be easily calculated with $R \times 10^5$, and the birthrate can be calculated by star formation rate ($SFR$) and binary fraction ($f_{bin}$), $SFR \times f_{bin} \times R$. For mergers, the delay time is the time started from the evolution of MS-MS binary to just before the formation of WD CE event.
Table 3: Rates of merger events between CO WDs and cores of massive stars ($M_2 \geq 8.0 M_\odot$) inside the CE, and rates of CO WD - NS systems that survive from the CE phase under different models with different cases (in $10^{-5} M_\odot^{-1}$)

| Model | Merger ($M_{WD} \geq 0.9$) | Merger ($M_{WD} \geq 1.0$) | DT (in Myr) | COWD-NS |
|-------|-----------------------------|-----------------------------|-------------|---------|
| mod1  | 5.05                        | 2.09                        | 34 - 123    | 0.54    |
| mod2  | 12.18                       | 4.59                        | 34 - 123    | 0.0     |
| mod3  | 8.94                        | 2.24                        | 42 - 113    | 0.86    |
| mod4  | 3.54                        | 0.001                       | 34 - 123    | 2.72    |
| mod5  | 0.85                        | 0.0                         | 34 - 117    | 2.83    |

Note: DT means delay time for mergers in the case of $M_{WD} \geq 0.9$. DTs of merger inside the CE are all shorter with a massive star, and there is no big difference for mergers with $M_{WD} \geq 1.0$, thus we just show the range of DTs for one case.
Fig. 1.— The number density map from simulated results of model 1 (standard model). The numbers shown here are out of the all $10^7$ binaries in a simulation: the event counts of each cell is used to weight the density of each grid cell with a chosen grid cell size. For example, with the total event number of 800 resulted from the all $10^7$ binaries, it is weighted for each cell of the parameter space. (a) and (b) show the initial mass relation of primary MS $(M_{1,i})$ and secondary MS stars $(M_{2,i})$ in primordial binaries which form CO WD + normal star binaries (upper left) and CO WD + He star systems (upper right) which experience the CE. Mass relations at the beginning of the CE between the CO WD and its companion star for all CO WD + normal star systems and CO WD + He star binaries are given in (c) and (d), respectively.
Fig. 2.— The distributions of properties of merger systems between the CO WDs and He cores of the intermediate normal stars under the condition of $M_{2,C} \geq 0.6$ and $M_{WD} \geq 0.9$ just before entering the CE, and simulated results from five models are given. The upper left panel shows the secondary mass distributions, the upper right panel gives the distributions of CO WD + cores of the secondaries, the lower left one is for the distributions of secondaries’ H-rich envelopes, and the lower right panel shows the orbital period distributions.
Fig. 3.— The distributions from simulated results of model 1 (standard model) for the merger case of the CO WD and core of the He stars under the condition of $M_{2,C} \geq 0.6$ and $M_{WD} \geq 0.9$ at the onset of CE. (a) shows the mass relation of the CO WD ($M_{WD}$) and secondary He stars ($M_{He}$). Mass relations between the CO WD and core of the He star for the merger case are given in (b).
Fig. 4.— The number density distributions from simulated results of model 1 (standard model). (a) shows the initial mass relation of primary MS ($M_{1,i}$) and secondary MS stars ($M_{2,i}$) in primordial binaries which form CO WD + massive star binaries (upper left) which experience the CE. Mass relations at the beginning of the CE between the CO WD and its companion star for all CO WD + massive star systems with CE are given in (b). (c) gives the mass relation between the CO WDs and massive star companions at the onset of the CE for merger case. Mass relation of the CO WD + core of the massive star and the H-rich envelope of the massive star for the binaries which will have the core mergers inside the CE is shown in (d). Note that ‘in CE’ in the insets means that merger takes place during the CE evolution.
Fig. 5.— An example of the evolution of a system to the channel for the merger between a CO WD and He core of a massive star based on model 1. The primary (mass is M1) experiences two phases of mass transfer before forming the CO WD. Afterwards, the secondary (mass is M2) fills its Roche lobe at the core helium burning stage of the evolution (CHeB), and its envelope engulfs the CO WD and its core due to the unstable mass transfer. The abbreviations of the stellar types (k1 and k2) are defined in the text. The evolution time is in Myr, and separation (sep) between the two stars is in $R_\odot$. 
Fig. 6.— Another example of the merger between a CO WD and He core of a massive star inside the CE based on model 1. The evolution is very similar to that of Fig. 1, and the main difference compared to the first example is that the CE event happens when the secondary becomes the HG star, and the evolution time is longer due to the different initial conditions.
Fig. 7.— The distributions of properties of merger systems between the CO WDs and He cores of massive stars under the condition of $M_{\text{WD}} \geq 0.9$ at the beginning of the CE, and simulated results from five models are shown. The upper left panel shows the secondary mass distributions, the upper right panel gives the distributions of CO WD + cores of the secondaries, the lower left one is for the distributions of secondaries’ H-rich envelopes, and the lower right panel shows the orbital period distributions.
Fig. 8.— The distribution of GW-sources from CO WDs - NSs (evolved through the CE described in text) mergers in the strain-frequency space. The sources are assumed to be at distances of 10 kpc. The sensitivity curve of LISA is shown with the solid blue line. Gray scale is CO WD - NS sources in our model 1 (a) and model 5 (b) simulations.