Uncertainty of polarized gluon distribution from prompt photon production

M. Hirai *

Radiation Laboratory, RIKEN (The Institute of Physical and Chemical Research)
Wako, Saitama 351-0198, JAPAN

Abstract

Constraint of prompt photon data on the polarized gluon distribution is discussed in terms of uncertainty estimation for polarized parton distribution functions (PDFs).

By comparing uncertainty of the double spin asymmetry $A_{LL}^\gamma$ with expected statistical errors at RHIC, we found that the gluon distribution is effectively constrained in the region $0.04 < x_T < 0.2$ with the data at transverse momentum $p_T = 10 - 20$ GeV for center-of-mass energies $\sqrt{s} = 200$ and 500 GeV.

Key words: polarized parton distribution, uncertainty estimation, prompt photon

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1 Introduction

By the recent global analyses with the experimental data for polarized deep inelastic scattering (DIS), the polarized quark and antiquark distributions are determined well [1,2,3,4]. These distributions are obtained with enough accuracy to indicate that the quark spin content is smaller than prediction of naive quark model; $\Delta \Sigma = 0.1 \sim 0.3$ ($\neq 1$). These polarized parton distribution functions (PDFs) reproduce well the experimental data; however, the polarized gluon distribution $\Delta g(x)$ cannot be constrained because of indirect and small contribution through $Q^2$ evolution and higher order correction at next-to-leading order (NLO). Furthermore, PDF uncertainty estimation indicated large uncertainty of the gluon distribution. It implies difficulty of the $\Delta g(x)$ determination with only the polarized DIS data.

* Present affiliation: Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization, 1-1, Ooho, Tsukuba, Ibaraki, 305-0801, Japan

Email address: E-mail:mhirai@rarfaxp.riken.jp (M. Hirai)
As a probe for the polarized gluon distribution, prompt photons will be measured by the longitudinally polarized proton-proton collider at RHIC [5]. The gluon distribution contributes directly in the quark-gluon compton process \((qg \rightarrow \gamma q)\) at leading order (LO), and the process dominates in the whole \(p_T\) region. The future asymmetry data contain useful information for clarifying the gluon contribution to the nucleon spin. Therefore, we are interested in the influence of the prompt photon data on the \(\Delta g(x)\) determination by the polarized PDF analysis.

In this paper, we consider constraint of prompt photon data on the polarized gluon distribution. For evaluating the data constraint, we will compare the uncertainty of the spin asymmetry with the expected statistical error by the RHIC experiments. The asymmetry uncertainty coming from the polarized PDFs is estimated by the Hessian method, and it is comparable with the measurement error. In this comparison, the statistical error of the spin asymmetry plays a role of constriction for the \(\Delta g\) uncertainty via the asymmetry. In practice, it is indicated that uncertainties of the polarized PDFs can be reduced by including new precise data for the polarized DIS in the Asymmetry Analysis Collaboration (AAC) [1]. Therefore, the same thing is expected by including the future data for prompt photon production.

2 Uncertainty of the spin asymmetry

The spin asymmetry \(A^\gamma_{LL}\) is defined as a ratio of the polarized and unpolarized cross sections: \(A^\gamma_{LL} = \Delta \sigma^\gamma / \sigma^\gamma\). By the factorization theorem, the polarized cross section \(\Delta \sigma^\gamma(p_A p_B \rightarrow \gamma X)\) as a function of the transverse momentum \(p_T\) is expressed by

\[
\frac{d \Delta \sigma^\gamma}{dp_T} = \sum_{a,b} \int_{\eta \text{-bin}} d\eta \int dx_a \int dx_b
\]

\[
\times \Delta f^a(x_a, \mu_F) \Delta f^b(x_b, \mu_F)
\]

\[
\times \frac{d \Delta \sigma^\gamma_{ab}}{dp_T d\eta}(x_a, x_b, \sqrt{s}, p_T, \eta, \mu_R, \mu_F),
\]

where \(\Delta f_a(x)\) is the polarized PDF of the parton \(a\). We choose the AAC03 PDF set [1]. \(\Delta \sigma^\gamma_{ab}\) is the partonic cross section \((a + b \rightarrow \gamma + X)\). In order to reduce theoretical uncertainty from the scale dependence of the cross section, the NLO corrections are taken into account. The NLO partonic cross sections for prompt photon production are completely known [6]. The renormalization and factorization scales are chosen the scale \(\mu_F = \mu_R = p_T\). In addition, the cross section is integrated over the rapidity bin \(|\eta| < 0.35\), which corresponds to the acceptance of the PHENIX detector. The unpolarized cross section \(\sigma^\gamma\) is similarly computed with unpolarized PDFs and partonic cross sections [6].
We choose the GRV98 unpolarized PDF set [7], which is also used in the AAC03 analysis. These cross sections are numerically calculated at center-of-mass (c.m.) energy $\sqrt{s} = 200$ and 500 GeV, respectively.

In this study, the contribution from fragmentation is neglected. The contribution is associated with the collinear process for a scattered parton into a photon, and it can be diminished by using an isolation cut on the measured photon [8]. And so we should consider the isolation cut in this analysis [9]. For examining an effect on the polarized PDF uncertainties, we calculate the cross section for inclusive direct photon production process.

The asymmetry uncertainty is obtained from uncertainty of the polarized cross section: $\delta A^{LL}_{LL} = \delta \Delta \sigma^\gamma / \sigma^\gamma$. The uncertainty $\delta \Delta \sigma^\gamma$ comes from the polarized PDF uncertainties, and can be estimated by the Hessian method. The method based on eigenvectors of the diagonalized Hessian matrix is developed by CTEQ collaboration [10], and it is applied to estimate uncertainties of unpolarized PDFs [11]. We used the basic method, which is generally used by the polarized PDF analyses [1,2,3]. The uncertainty is given by

$$[\delta \Delta \sigma^\gamma]^2 = \Delta \chi^2 \sum_{i,j} \left( \frac{\partial \Delta \sigma^\gamma(p_T)}{\partial a_i} \right) H_{ij}^{-1} \left( \frac{\partial \Delta \sigma^\gamma(p_T)}{\partial a_j} \right),$$

where $a_i$ are optimized parameters in the polarized PDFs. $H_{ij}$ is the Hessian matrix which has information on the parameter errors and correlation between each parameter. The gradient terms of the cross section $\partial \Delta \sigma^\gamma(p_T)/\partial a_i$ are obtained as follows:

$$\frac{\partial \Delta \sigma^\gamma}{\partial a_i} = \sum_{a,b} \int_{b_{en}} d\eta \int dx_a \int dx_b \int dx_a \int dx_b \left[ \frac{\partial f_a^A(x_a)}{\partial a_i} \Delta f_b^B(x_b) + \Delta f_a^A(x_a) \frac{\partial f_b^B(x_b)}{\partial a_i} \right] \times \frac{d\Delta \sigma^\gamma_{ab}}{dp_T d\eta} (x_a, x_b, \sqrt{s}, p_T, \eta, \mu_R, \mu_F).$$

The gradient terms of the polarized PDFs can be derived analytically at initial scale, and these terms are numerically evolved to arbitrary scale $Q^2 (= p_T^2)$ by the DGLAP equation. Furthermore, the value of $\Delta \chi^2$ determines a confidence level of the uncertainty. It is obtained so that the level corresponds to 1σ error of the normal distribution [1]. The uncertainty therefore can be directly compared with the statistical error of experimental data.

For comparison with the asymmetry uncertainty, the statistical error of the spin asymmetry is estimated by

$$\delta A^{exp}_{LL} = \frac{1}{P^2 \sqrt{\mathcal{L}_{int} \sigma}},$$
where $P$ is the beam polarization, $\mathcal{L}_{\text{int}}$ is the integrated luminosity, and $\sigma$ is the unpolarized cross section integrated over the $p_T$ bin. In this study, the $\sigma$ is computed by the bin size of 5 GeV interval, and it is taken the same bin size for both c.m. energies $\sqrt{s} = 200$ and 500 GeV. Furthermore, other values are chosen as design values at RHIC [5]: $P = 0.7$ and $\mathcal{L}_{\text{int}} = 320$ (800) pb$^{-1}$ for $\sqrt{s} = 200$ (500) GeV.

3 Constraint of prompt photon data on $\Delta g(x)$

First, we discuss predicted spin asymmetry and its uncertainty at $\sqrt{s} = 200$ GeV. In Fig. 1, the spin asymmetry by the AAC03 PDF set is compared to those by polarized PDF sets of BB (ISET=3) [2], GRSV01 (standard scenario)[4], and LSS (MS scheme) [3]. These analyses used almost the same experimental data sets for the polarized DIS, and they obtained good agreements with the data. However, there are significant differences of the gluon distributions among them. These differences are obviously reflected in variations of the predicted asymmetries. The prompt photon process is sensitive to the behavior of the gluon distribution. Moreover, the asymmetry uncertainty is indicated in the same figure. Dotted curves show the uncertainty which comes from the polarized PDF uncertainties obtained by the AAC analysis with the polarized DIS data. We find that these predicted asymmetries are within the large uncertainty. These variations are caused by weak constraint of the polarized DIS data on the gluon distribution. The prompt photon data therefore are required for reducing this asymmetry uncertainty.

In order to evaluate the gluon contribution to the asymmetry uncertainty,

\[1 \text{ See, for example, Ref. [1,2].}\]

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![Fig. 1. Comparison of the predicted spin asymmetries by different polarized PDF sets; AAC03(NLO), BB (ISET=3), GRSV01 (standard scenario), and LSS (MS scheme). The dotted curves show the asymmetry uncertainty from the PDF uncertainties of the AAC03 set.](image_url)
we compute the asymmetry uncertainty excluding the $\Delta g(x)$ uncertainty by assuming $\partial \Delta g(x)/\partial a_g = 0$ in eq. (3). Figure 2 shows the asymmetry uncertainties for $\sqrt{s} = 200$ GeV. The solid curves show the current uncertainty by the AAC analysis, and the dotted curves do the asymmetry uncertainty except the $\Delta g(x)$ uncertainty. The significant reduction of the uncertainty indicates that the $\Delta g(x)$ uncertainty is the dominant contribution to the current uncertainty.

In addition, the asymmetry uncertainty is compared to the expected statistical errors at RHIC. The current uncertainty is much larger than these statistical errors. If these data are included in the global analysis, the uncertainty is roughly reduced to these errors. This suggests that the $\Delta g(x)$ uncertainty is mainly improved. The prompt photon data therefore have strong constraint on the gluon distribution.

On the other hand, the data constraint on the quark and antiquark distributions is very weak. This is because that the asymmetry uncertainty without the $\Delta g(x)$ uncertainty, which is composed of the $\Delta q(x)$ and $\Delta \bar{q}(x)$ uncertainties, is significantly less than the statistical errors. Therefore, the data with such errors do not directly affect improvements of these uncertainties.

However, as far as the antiquark is concerned, the uncertainty can be indirectly reduced because the antiquark distribution is strongly correlated with the gluon distribution in the global analysis. In practice, reduction of the antiquark uncertainty via the error correlation is indicated by the analysis with the fixed $\Delta g(x) = 0$ at initial scale [1]. This fact suggests that the constraint on the gluon distribution indirectly affects the $\Delta \bar{q}(x)$ determination. In particular, it is not neglected when we perform flavor decomposition of the antiquark distributions.

Next, we estimate a constraint factor for the $\Delta g(x)$ uncertainty. By multiplying the gradient terms for the gluon $\partial \Delta g(x)/\partial a_g$ by the factor in eq. (3), the constricted uncertainty of the asymmetry is defined. From comparison of the asymmetry uncertainty with the expected statistical errors for $\sqrt{s} = 200$ in

![Figure 2](image-url)

Fig. 2. Comparison of the asymmetry uncertainty $\delta A_{\gamma LL}$ with the expected statistical errors for $\sqrt{s} = 200$ GeV. The dashed curves show the asymmetry uncertainty except the gluon uncertainty: $\delta \Delta g(x) = 0$. 
the region $10 < p_T < 20$ GeV, the obtained factor is $1/18$.

In this study, the factors for the $\Delta q(x)$ and $\Delta \bar{q}(x)$ uncertainties are neglected. This is simply because that these uncertainties are not directly constricted by these data. Moreover, the correlation effect on the $\Delta \bar{q}(x)$ uncertainty is not taken into account. The effect cannot be evaluated without including experimental data in the global analysis. Since the $\Delta \bar{q}(x)$ contribution to the asymmetry uncertainty is already small, the uncertainty will be slightly modified in this process.

In Fig. 3, the constricted asymmetry uncertainties are compared with the expected statistical errors for $\sqrt{s} = 200$ and 500 GeV, respectively. The solid curves show the asymmetry uncertainties which are obtained from the $\Delta g(x)$ uncertainty multiplied by the constraint factor, and involve the $\Delta q(x)$ and $\Delta \bar{q}(x)$ uncertainties. If $x_T (= 2p_T/\sqrt{s})$ can be approximated by the Bjorken $x(= x_a = x_b)$ around central rapidity region, the data for $\sqrt{s} = 200$ GeV in the region $10 < p_T < 20$ GeV constrain the gluon distribution in the range $0.1 < x < 0.2$. Although this comparison is in ideal condition that these data are put on the predicted asymmetry, this fact agrees with the results of the trial analysis including pseudo-data for $A_{LL}^\gamma$ in Ref. [12]. These data are useful in determining the gluon distribution.

In the region $p_T > 20$ GeV, these data have rather weak constraint since the unpolarized cross section rapidly decreases as $p_T$ increases. The statistical errors depend on the $p_T$ bin size computing $\sigma$ in eq. (4). We should be careful about taking the bin size for the high $p_T$ data in order to constrain equally the gluon distribution over a wide $x$ region.

For the comparison at $\sqrt{s} = 500$ GeV, we find similar behavior. In the region

![Fig. 3. Comparison of the constricted asymmetry uncertainties with the expected statistical errors for $\sqrt{s} = 200$ and 500 GeV, respectively. The solid curves show the constricted asymmetry uncertainty with the factor $1/18$ for the $\Delta g(x)$ uncertainty.](image)
10 < p_T < 20 GeV, the asymmetry uncertainty roughly corresponds to the statistical errors. This indicates that these data have the same constraint as those for $\sqrt{s} = 200$ GeV, and constrain the gluon distribution in the range 0.04 < x < 0.08. Above the region, the statistical errors are larger than the asymmetry uncertainty. It is noteworthy to mention here that the data constraint for $\sqrt{s} = 500$ GeV is weaker than that for $\sqrt{s} = 200$ GeV in the region 10 < p_T < 20 GeV in spite of covering the same x_T region.

The reason for the weak constraint is that the unpolarized cross section for $\sqrt{s} = 500$ GeV is less than that for $\sqrt{s} = 200$ GeV in the same x_T region, and the integrated luminosity is still insufficient to provide the enough constraint at high p_T. Figure 4 shows the comparison of unpolarized cross sections for both c.m. energies. These cross sections are indicated as a function of x_T. In the region x_T > 0.1, the cross section for $\sqrt{s} = 500$ GeV is below one for 200 GeV, and indicates similar behavior. In order to obtain equal constraint at the same x_T, we need more luminosity than the design value at $\sqrt{s} = 500$ GeV. Therefore, the experimental data for $\sqrt{s} = 500$ GeV are required as constraint on the gluon distribution at low x. The medium-x behavior should be determined by using the data for 200 GeV.

Finally, let us turn to the $\Delta g(x)$ uncertainty from the prompt photon data at RHIC. Figure 5 shows the polarized gluon distribution and its uncertainties at p_T = 10 GeV. The solid curves show the $\Delta g(x)$ uncertainty from the polarized DIS data, and the shaded area shows the constricted uncertainty by comparison with the expected statistical errors. The uncertainty estimated by the constraint factor is reliable in the range 0.04 < x < 0.2. As a practical estimation, the uncertainty broadens gradually in the low- and medium-x regions where data do not exist. Moreover, we note that increasing the asymmetry uncertainty with p_T is due to uncertainty of the ratio of the polarized and unpolarized gluon distributions: $\delta \Delta g(x)/g(x)$. The uncertainty significantly increases with x due to lack of precise data for polarized DIS at large x.

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2 See, for example, Fig. 5b in Ref. [12].

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Fig. 4. The unpolarized cross sections for $\sqrt{s} = 200$ and 500 GeV.
Fig. 5. The polarized gluon distribution and its uncertainty at \( p_T = 10 \) GeV. Shaded area is the constricted gluon uncertainty.

The \( \Delta g(x) \) uncertainty becomes the same order of magnitude as the quark uncertainty from the DIS data. By including future asymmetry data for the prompt photon process, the gluon distribution can be obtained with sufficient accuracy.

In this study, polarization of the gluon distribution is not discussed. The current uncertainty in Fig. 5 indicates that we cannot rule out the possibility of zero or negative polarization. As another probe for the gluon distribution, the double spin asymmetry for \( \pi^0 \) production has recently been reported by the PHENIX collaboration [13]. Since \( gg \to \pi^0 X \) subprocess dominates at low \( p_T \), the cross section depends on \( (\Delta g)^2 \); therefore, the process is not sensitive to the sign of the gluon polarization. On the other hand, the prompt photon production which the \( qg \) Compton process dominates is sensitive to the sign in the whole \( p_T \) region. The gluon polarization is obviously reflected in the spin asymmetry. In this sense, the role of prompt photon data is of prime importance for determination of the gluon polarization.

4 Summary

In this paper, we have considered the uncertainty of the polarized gluon distribution for prompt photon production at RHIC. The uncertainty of the double spin asymmetry is estimated by the Hessian method. The asymmetry uncertainty mostly comes from the \( \Delta g(x) \) uncertainty. The large uncertainty implies the weak constraint of the polarized DIS data on the gluon distribution. By comparison with the expected statistical errors at RHIC, we indicate that the prompt photon data have the strong constraint on it. Furthermore, we suggest that the prompt photon data in the region \( 10 < p_T < 20 \) GeV effectively constrain the gluon distribution. The data of both c.m. energies constrain it in the following regions: \( 0.04 < x < 0.08 \) at \( \sqrt{s} = 500 \) GeV, and \( 0.1 < x < 0.2 \) at
200 GeV. For clarifying the gluon contribution \( \Delta g(\equiv \int_0^1 dx \Delta g(x)) \), the data covering a wide range of \( x \) are required. These experiments therefore play an important role in the \( \Delta g(x) \) determination.

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References

[1] M. Hirai, S. Kumano, and N. Saito (AAC), Phys. Rev. D69 (2004) 054021; [http://spin.riken.bnl.gov/aac/]
[2] J. Blümlein and H. Böttcher, Nucl. Phys. B636 (2002) 225.
[3] E. Leader, A. V. Sidorov, and D. B. Stamenov, Eur. Phys. J C23 (2002) 479.
[4] M. Glück, E. Reya, M. Stratmann, and W. Vogelsang, Phys. Rev. D63 (2001) 094004.
[5] G. Bunce, N. Saito, J. Soffer, and W. Vogelsang, Ann. Rev. Nucl. Part. Sci. 50 (2000) 525.
[6] L. E. Gordon and W. Vogelsang, Phys. Rev. D48 (1993) 3136.
[7] M. Glück, E. Reya, and A. Vogt, Eur. Phys. J C5 (1998) 461.
[8] S. Frixione, Phys. Lett. B 429 (1998) 369.
[9] S. Frixione and W. Vogelsang, Nucl. Phys. B568 (2000) 60.
[10] CTEQ collaboration, J. Pumplin et al., Phys. Rev. D65 (2001) 014013.
[11] CTEQ collaboration, J. Pumplin et al., JHEP 0207 (2002) 012; A. D. Martin, R. G. Roberts, W. J. Stirling, and R. S. Thorne, Eur. Phys. J C28 (2003) 455.
[12] M. Stratmann and W. Vogelsang, Phys. Rev. D64 (2001) 11407.
[13] PHENIX Collaboration, S.S Adler, et al., [hep-ex/0404027].