Comparative Study on Application of Extended Kalman Filter and Unscented Kalman Filter in Target Tracking

Xiao-Tong Hu Xiao-Fei Deng* Si-Qi Zhao Qin Guo Cui-Xia Zeng and Kai-Qing Zhou
School of Information Science and Engineering Jishou University Jishou 416000 Hunan China
Email: xiaofei0228@163com

Abstract This paper mainly describes the basic principles of extended Kalman filter and unscented Kalman filter and the application of them in target tracking based on observation distance At last the two algorithms are compared by Matlab software The simulation results show that unscented Kalman filter has better matching effect than extended Kalman filter and the former has smaller error variation and better convergence than the latter

1. Introduction

According to the measured values obtained by the motion observer such as bearing frequency distance the target tracking is a classic problem in the field of target motion analysis [1] Therefore it is of great significance to study target tracking based on distance information In 1971 Bucy et al [2] raise extended Kalman filtering EKF for nonlinear systems the basic idea of which is using Taylor to change the truncated high-order item of the nonlinear equation to first-order namely changing the system state equation and measurement equation to a first order linear equation Then the standard Kalman filter framework is used for filter processing Meanwhile in the linearization process of EKF the first-order term of the expansion is only intercepted so there is a large approximate error in the linearization process of the algorithm [3]

On this basis S Julier et al [4] propose Unscented Kalman Filter UKF in 2000 Unscented transform is adopted to approximate the Gaussian density probability density of the state space variables of the nonlinear system And it obtains a certain number of sample points through deterministic sampling Moreover it deals with the nonlinear transmission of the mean and covariance of the state equation and measurement equation of the nonlinear system under the Kalman filtering framework UKF does not need to calculate the Jacobian matrix or Hessians matrix which widens the application range of the algorithm and improves the calculation accuracy of the algorithm by retaining higher-order terms [5] Compared with EKF UKF does not ignore higher-order terms so it has a higher computational accuracy for statistics of nonlinear distribution which effectively overcomes the shortcomings of EKF with low estimation accuracy and poor stability [6]

The work of this paper is to compare UKF with EKF and apply them to target tracking based on observation distance This paper is organized as follows: The principle of EKF is represented in Section II The principle of UKF is described in Section III The detailed application of EKF and UKF in target tracking and simulation results are described in Section IV The conclusion is finally given in Section V
2. The Principle of Extended Kalman Filter and Unscented Kalman Filter

2.1. Local Linearization of Extended Kalman Filter
The dynamic equation of discrete nonlinear system is expressed as follows:
\[
X_{k+1} = f[kX_k] + G_kW_k
\]
\[
Z_k = h[kX_k] + V_k
\]
When the process noise and observation noise are constantly 0 the real solutions of nonlinear system 1 and 2 are called "true trajectory" or "true state" According to the state equation of system 1 in order to filter value \(X_k\) as the core to the nonlinear function of first order Taylor expansion
\[
X_{k+1} \approx f[k\bar{X}_k] + \frac{\partial f}{\partial X_k}[X_k - \bar{X}_k] + G[k\bar{X}_k]W_k
\]
Then the equation of state is
\[
X_{k+1} = \Phi_k + 1|kX_k + G_kW_k + \Phi_k
\]
The initial value is \(X_0 = E[X_0]\)
It is obtained in previous step filtering value \(\bar{X}_k\) the equation of state 5 joins the nonrandom effect \(\Phi_k\) From the system state equation 2 in order to filter value \(\bar{X}(k)\) nonlinear function \(f(\cdot)\) fas the core to a first-order Taylor expansion and the equation of state is
\[
Z_k = H\bar{X}k + y_k + V_k
\]

2.2. Extended Kalman Filter
After the linearized equation of state 4 and observation equation 6 are obtained the recursive relation of Kalman filtering can be extended
State transition \(\Phi_k + 1|k\) and the observation matrix \(H_k + 1\) by jacobian matrix instead of \(f\) and \(h\) If the n-dimensional state variable is assumed to be \(X[x_1 x_2 \cdots x_n]^T\) then the jacobian matrix is calculated as follows:
\[
\Phi_k + 1 = \frac{\partial f}{\partial X} = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n}
\end{bmatrix}
\]
\[
H(k + 1) = \frac{\partial h}{\partial X} = \begin{bmatrix}
\frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \cdots & \frac{\partial h_1}{\partial x_n} \\
\frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \cdots & \frac{\partial h_2}{\partial x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial h_n}{\partial x_1} & \frac{\partial h_n}{\partial x_2} & \cdots & \frac{\partial h_n}{\partial x_n}
\end{bmatrix}
\]

2.3. Unscented Kalman Filter
Unscented Transform UT is a core step in UKF and it is also a method of probability statistics of random variables after nonlinear transformation

(1) Calculate 2n + 1 Sigma sampling points The formula is as follows:
\[
\begin{cases}
X^{(0)} = \bar{X}i = 0 \\
X^{(i)} = \bar{X} + (\sqrt{n + \lambda}P)_{i = 1\sim n} \\
X^{(i)} = \bar{X} - (\sqrt{(n + \lambda)P})_{i = n + 1\sim 2n}
\end{cases}
\]
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where \( n \) is the dimension in the current state \( (\sqrt{P})_i \) represents the \( i \) column converted to the square root of the matrix

2. The corresponding weights of sigma sampling points are calculated as follows:

\[
\begin{align*}
\omega_{in}^{(0)} &= \frac{\lambda}{n+\lambda} \\
\omega_{ic}^{(0)} &= \frac{\lambda}{n+\lambda} + 1 - a^2 + \beta \\
\omega_{ic}^{(i)} &= \frac{\lambda}{2(n+\lambda)} \quad i = 1 \sim 2n
\end{align*}
\]

where \( \lambda = a^2(n + k) - n \) is the scaling parameter which can be used to reduce the prediction error. The value of \( a \) can control the distribution state of the sampling points.

When UKF deals with non-linear system problems, the sigma sampling points can be obtained by directly executing UT near the prediction points by comparing the mean and covariance of the sampling points with the original statistical features and directly performing non-linear mapping to obtain an approximate probability density function.

3. The application of Extended Kalman Filter and Unscented Kalman Filter in Target Tracking

3.1. Mathematical modeling of target tracking algorithm

Assume that the target moves uniformly and in a straight line with a velocity of \( v \) At the moment of \( k \) the position of the target is set as \( sk \) After sampling time \( T \) the position of the target is \( sk + 1 = sk + vT \) In the process of target motion the random disturbance is set as \( \sigma \) which is expressed as follows[1]:

\[
\begin{align*}
\begin{cases}
x_{k+1} = x_k + v_xtk + \frac{1}{2}u_xtk^2 \\
v_{x,k} + 1 = v_xk + u_xkT \\
y_{k} + 1 = y{k} + v_yT + \frac{1}{2}u_ykT^2 \\
v_{y,k} + 1 = v_yk + u_ykT
\end{cases}
\end{align*}
\]

The equation of state of the system is

\[ k + 1 = \Phi x_k + \Gamma uk \]

Among them

\[
\Phi = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \Gamma = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix}
\]

If the position of radar station is \( (x_0,y_0) \) the position of the target at moment \( k \) is \( (xky_k) \) and the distance between target and radar is measured by the radar emission and reflected wave then the observation equation \( Zk \) is:

\[
Z_k = \sqrt{(x_k-x_0)^2 + (y_k-y_0)^2} + V_k
\]

where the measurement error of the radar itself is \( V_k \) and the covariance variance is \( R \)

3.2. Target tracking of Extended Kalman Filter

According to the previous section the mean square deviation of process noise \( U_k \) is:

\[
Q = \begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix}
\]
where \( w \) is an adjustable parameter \( w \ll 1 \)

Covariance matrix \( R \) of measurement noise \( V_k \)

Nonlinear equation 13 is linearized according to the local linearization method and the corresponding Jacobian matrix is obtained from equation 7

\[
H = \begin{bmatrix}
    x_k - x_o & 0 & 0 \\
    \frac{\sqrt{x_k - x_o}}{x_k - x_o} & \frac{\sqrt{y_k - y_o}}{y_k - y_o} & 0 \\
    -\frac{\sqrt{y_k - y_o}}{y_k - y_o} & \frac{\sqrt{x_k - x_o}}{x_k - x_o} & 0
\end{bmatrix}
\]

3.3. Target tracking of Unscented Kalman Filter

The mathematical model refers to 31 of this section Suppose that the sampling interval \( T=1s \) and the total running time is \( N=60s \) then the process-driven matrix and noise-driven matrix are constant matrices as shown below:

\[
\Phi = \begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix} \quad \Gamma = \begin{bmatrix}
0 & 5 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
1 & 0 & 1
\end{bmatrix}
\]

\( W_k \) of mean square deviation of \( Q = \sigma_w \times \text{diag}[11] \) \( \sigma_w \) is an adjustable parameter \( \sigma_w \ll 1 \)

Set the correlation coefficient of UT transformation \( a = 001 \) \( K = 0 \) \( \beta = 2 \) and dimension \( n = 9 \)

Assume that the real motion state of the moving target object is as follows

\[
X_{\text{real}}(k) = [x_{\text{real}}(k) \dot{x}_{\text{real}}(k) y_{\text{real}}(k) \dot{y}_{\text{real}}(k)]^T
\]

Target tracking status obtained by using UKF algorithm is shown in the following formula

\[
X_{\text{UKF}}(k) = [x_{\text{UKF}}(k) \dot{x}_{\text{UKF}}(k) y_{\text{UKF}}(k) \dot{y}_{\text{UKF}}(k)]^T
\]

Find the root mean square error as follows:

\[
\text{RMSE}(k) = \sqrt{\dot{x}_{\text{UKF}}(k) - x_{\text{real}}(k))^2 + \dot{y}_{\text{UKF}}(k) - y_{\text{real}}(k))^2}
\]

3.4. Simulation results

As is known to all EKF produces better results in signal processing whereas UKF is better in econometrics Therefore these two algorithms are compared using target tracking case study in this paper

As shown in Figure 1 the black track represents the real track the red track represents the simulation result of EKF and the green track represents the simulation result of UKF It can be seen from the figure that the simulation effect of EKF on the real trajectory is not as good as UKF This is precisely caused by the linearization error caused by retaining the first-order approximation term and omitting the second-order term and the above term in the Taylor expansion UKF uses the unscented transform so the simulation result of UKF is superior to EKF in this case

Figure 2 shows that there is a significant gap between EKF and UKF on tracking error algorithm On the one hand EKF changes dramatically while UKF’s tracking error is good On the other hand UKF’s mean square error changes little and its convergence is better than EKF indicating that UKF has a good tracking effect

To further clarify the tracking results their statistical properties include the mean absolute deviation \( E[|\epsilon|] \) the standard deviation of the tracking error \( \sigma_\epsilon \) and the Correlation CoefficientsCC between the actual and the tracking value are used for comparison The statistical results are summarized in Table 1

Table 1 shows that all \( E[|\epsilon|] \) and \( \sigma_\epsilon \) results of the UKF algorithm are smaller than that of the EKF algorithm These statistical properties show that the tracking of the UKF algorithm results in a smaller error And the outstanding performance of the UKF algorithm is validated by the result that the value of the CC is much closer to 10 than that of the EKF algorithm
Figure 1 The track of moving targets by EKF and UKF

Figure 2 EKF and UKF tracking errors

| Table 1 Statistical comparisons of the two algorithms |
|-----------------------------------------------|
| The algorithms | $E[|\epsilon|]$ | $\sigma_\epsilon$ | Correlation Coefficients |
|----------------|----------------|----------------|-------------------------|
| EKF            | 1077089751     | 852608688      | 0.99876456              |
| UKF            | 323592617      | 163276416      | 0.99987096              |

4. Conclusion

Through EKF and UKF's target tracking it can be found that EKF linearizes the nonlinear function equation by taking its partial derivatives high order derivatives and Jacobian matrix and then obtains an approximate linearized function equation for target tracking. UKF directly approximates the probability density of the nonlinear function equation through Gaussian distribution and uses an unscented transformation to obtain Sigma sampling points which are nonlinearly mapped with the previously estimated equation of state to obtain the accurate target tracking equation. Therefore, UKF has better accuracy and convergence than EKF.

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