Quantum and Classical Correlations in the Production of Photon-Pairs with Nonlinear Crystals

J Lópezdurán and O Rosas-Ortiz
Physics Department, Cinvestav, A P. 14-740, 07000, México City, Mexico
E-mail: jlopez@fis.cinvestav.mx

Abstract. The spatial distribution of photon-pairs produced by nonlinear crystals in the spontaneous parametric down conversion of type-II is analyzed. The correlations induced by the phase matching conditions are calculated by assuming that the ideal spatial distributions are of Gaussian profile. Some predictions for the zones in the detection plane in which the entanglement of polarization is maximal are presented.

1. Introduction
It is well known that the quantum states are not separable in general. The latter is specially clear if the different parts of a given system are correlated in at least one (and the same) variable and such a correlation is preserved after the (non-destructive) manipulation of the involved variable. Such a conservation principle is the fingerprint of a quantum correlation usually known as entanglement [1, 2]. If the correlation is not maintained after the local transformation then it is merely classical. Nowadays, the most efficient system used to study quantum correlations is represented by the photon-pairs generated when a nonlinear crystal is illuminated by the appropriate light. In the crystal, one of the incoming photons, at the pump frequency $\omega_p$, is spontaneously annihilated at the time that two new outgoing photons are created at the frequencies $\omega_s$ and $\omega_i$. It is common to name the new photons as signal and idler, respectively. The phenomenon underlying the production of photon-pairs is known as spontaneous parametric down conversion (SPDC) and serves as a fundamental ingredient in many of the contemporary experiments of quantum control and quantum information [2–5].

In previous works [6, 7] we have reported some progress in the study of the spatial biphoton correlations generated in the type-I SPDC process for which the polarization of the new pair of photons is parallel to each other and orthogonal to the polarization of the pump photon. Then, the spatial distribution of the created light forms a cone that is aligned with the pump beam.

In this contribution we analyze some correlation properties of the photon-pairs generated in the SPDC process of type-II. The phenomenon is such that the polarization of the idler photons is orthogonal to the polarization of the signal ones. Then, the light produced by the nonlinear crystal forms two (not necessarily collinear) cones whose axes are symmetrically oriented with respect to the pump beam and share the same apex located somewhere in the crystal.
2. Phase matching conditions

In a first approach the SPDC occurs such that the energy and momentum are conserved. For a negative uniaxial crystal, these conservation laws are expressed as the phase matching conditions

\[ \omega_p = \omega_s + \omega_i, \quad k_p(e) = k_s(o) + k_i(e), \]

where \( \omega \) and \( k \) are respectively the angular frequency and the wave number vector. If \( \omega_s = \omega_i \) we say that the process is degenerated. The labels \( e \) and \( o \) stand for extraordinarily and ordinarily polarized light respectively \[8\]. To solve (1) we consider the wave number vectors \( k_p, k_s, k_i, \) and the unitary vector \( n \) shown in Figure 1(a). Hereafter we take \( n_p, n_s, \) and \( n_i \) as the unitary vectors that define the direction of propagation of each one of the beams involved in the down conversion. Moreover, we shall fix \( n = (\sin \sigma, 0, \cos \sigma) \) as the optical axis of the nonlinear crystal.

![Figure 1. (a) Graphic representation of the unitary vectors used to solve phase matching conditions in spherical coordinates (b) The rays of down converted light calculated from the numerical solution of the phase matching conditions.](image)

One can show \[9\] that, in the degenerated case, the solution of the phase matching conditions is given by the expressions

\[ \phi = \beta + \pi, \]

\[ \frac{n_e \left( \frac{\omega_0}{2}, \sigma \right)}{n_o \left( \frac{\omega_0}{2} \right)} \sin \alpha = \sqrt{1 - \left( \frac{n_e \left( \omega_0, \sigma \right) - \frac{1}{2} n_e \left( \frac{\omega_0}{2}, \delta \right) \cos \alpha}{\frac{1}{2} n_o \left( \frac{\omega_0}{2} \right)} \right)^2}, \]

In Fig. 1(b) we show the distribution of rays of light that are produced in the type-II SPDC. They have been depicted by using the unitary vectors, \( n_s \) and \( n_i \), calculated from the numerical solution of the system (2)–(3).

3. Spatial correlations

Following \[6,7\], we propose a Gaussian profile for the spatial distribution of the down converted photons. That is, on the detection plane (which is transversal to the propagation of the pump beam), the spatial distribution along each one of the circumferences is given by

\[ F(r, \tau) = \frac{1}{2\pi \sigma_r \sigma_\tau} \exp \left( -\frac{r^2}{2\sigma_r^2} - \frac{(C(\tau) - d)^2}{2\sigma_\tau^2} \right), \quad C(\tau) = \sqrt{d^2 - \tau^2}, \]
where $\sigma_r$ and $\sigma_\tau$ are the width of the distributions with $d$ and $r$ the radius and the radial coordinate of the circumference. The variable $\tau$ corresponds to a coordinate perpendicular to the radial one (in a point of interest).

Using (4) one can calculate the predictions for the distribution rates of coincidences in the counting of photons at the detection zone [10]. The idler and signal channels must be measured by photo collectors located at the positions $(x_i, y_i)$ and $(x_s, y_s)$ on the circumferences described by a transversal cut of the cones depicted in Fig. 1(b). The collecting of photons in the laboratory should be performed by displacing the idler and/or signal detectors accordingly. In Fig. 2 we show the prediction for the horizontal-horizontal $(x_i, x_s)$ displacements, they give rise to the ellipse-like contours oriented at $45^\circ$ which are also found in the type-I SPDC production of photon-pairs [6, 7].

![Distribution rate of spatial coincidences predicted for the horizontal displacements of the photon collectors in the idler and signal channels for the type-II SPDC.](image)

**Figure 2.** Distribution rate of spatial coincidences predicted for the horizontal displacements of the photon collectors in the idler and signal channels for the type-II SPDC.

4. Quantum correlations

The photon-pairs produced in the type-II SPDC are entangled in the polarization if they are found at the intersections of the cones [11]. The distribution (4) allows to visualize the behavior of the distributions at the intersections for different distances between the axes of the cones [10]. Some results are depicted in Fig. 3 for different values of the angle $\gamma$ that measures the distance between the cones. The smallest zone of intersection is obtained at $\gamma = \pi/4$, see Fig. 3(c). Although this case is the closest one to the ideal photon-pair production, the photons collected in the intersection for the condition $\gamma = \pi/4$ could be not entangled with their counterparts in the other intersection. The latter because the zone of intersection in the ideal case consists of only one point on the detection plane.

![An intersection of the cones for different separations: (a) $\gamma = \pi/12$, (b) $\gamma = \pi/6$ and (c) $\gamma = \pi/4$.](image)

**Figure 3.** An intersection of the cones for different separations: (a) $\gamma = \pi/12$, (b) $\gamma = \pi/6$ and (c) $\gamma = \pi/4$. 

Considering that both cones are identical, the entangled photons must be distinguished from a balanced mixture of extraordinarily and ordinarily polarized photons. Hence, the following density matrices must be considered.

\[ \rho_{\text{mix}} = \frac{1}{2} (|oe\rangle \langle oe| + |eo\rangle \langle eo|), \]  
\[ \rho_{\text{spdc}} := |\psi_{\text{spdc}}\rangle \langle \psi_{\text{spdc}}| = \frac{1}{2} (|oe\rangle \langle oe| + |oe\rangle \langle oe| + |eo\rangle \langle eo|). \]

Figure 4. (Color online) Orientation of the polarizers located behind each intersection.

For the certification, a polarizer must be localized in each intersection, labelled as A and B in Fig. 4. Behind each polarizer, each one of the photodetectors will be displaced accordingly to obtain the distribution rate of spatial coincidences. The related probabilities are given by

\[ P_{\text{spdc}}^{AB} (\mu, \nu) = \frac{1}{2} \sin^2 (\mu + \nu), \quad P_{\text{mix}}^{AB} (\mu, \nu) = \frac{1}{2} \left[ \sin^2 \mu \cos^2 \nu + \cos^2 \mu \sin^2 \nu \right], \]

where \( \mu \) and \( \nu \) give the orientation of the polarizers with respect to the extraordinary polarization (for more details see [9] and [10]).

Figure 5. (Color online) Distinguishment between a statistical mixture and a pure entangled state for different polarizer configurations \((\mu, \nu)\).

The quantity \( \Pi = P_{\text{spdc}}^{AB} (\mu, \nu) - P_{\text{mix}}^{AB} (\mu, \nu) \) is shown in Fig. 5. As we can see, the probability of the pure state (6) is greater than that of the mixed state (5) for almost all the values of \( \mu \) and \( \nu \). The exceptions are presented when \( \mu \) and \( \nu \) are equal to either 0 or \( \pi/2 \), since both probabilities are equal for the combinations of such parameters. The latter means that the probabilities will be non-distinguishable for the pairs \((\mu, \nu)\) defined as either \((0, \nu)\), \((\pi/2, \nu)\), \((\mu, 0)\) or \((\mu, \pi/2)\).
5. Concluding remarks

A simple geometrical model has been used to make some predictions of the spatial distribution of the photon-pairs produced in the type-II SPDC process. It has been shown that the distribution rate of spatial coincidences predicted for the horizontal displacements of the photon collectors show the ellipse-like contours oriented at 45° which are also found in the SPDC of type-I. The same model has been used to make some predictions of the zones in the detection plane where it is most probable to find the photons that are entangled in polarization. In addition, we have proposed a mechanism to distinguish between pure (entangled) and mixed states of polarization in the generated photon-pairs.

Acknowledgment

JLD acknowledges the funding received through a CONACyT scholarship.

References

[1] Aczel A, Entanglement, Plume, NewYork, 2003
[2] Mielnik B, Rosas-Ortiz O, Quantum Mechanical Laws, in Fundamentals of Physics, Vol. 1, Morán-López J L (ed), Encyclopedia of Life Support Systems (EOLSS), Developed under the Auspices of the UNESCO, Eolss Publishers: Oxford, United Kingdom; 255?326, 2009. Available from: http://www.eolss.net.
[3] Menzel R, Photonics, Springer-Verlag, Berlin, 2007
[4] Bouwmeester D, Ekert A K, Zeilinger A (eds), The Physics of Quantum Information: Quantum Cryptography, Quantum Teleportation, Quantum Computation, Springer-Verlag, Berlin, 2000
[5] Zeilinger A, Dance of the Photons: From Einstein to Quantum Teleportation, Farrar, Straus and Giroux, New York, 2010
[6] Procopio L M, Rosas-Ortiz O and Velázquez V, AIP Conf. Proc. 1287 (2010) 1287
[7] Procopio L M, Rosas-Ortiz O and Velázquez V, Math. Meth. Appl. Sci. 38 (2015) 2053
[8] Fowles G R 1968 Introduction to Modern Optics (United States: Holt, Rinehart and Winston Inc.)
[9] López-Durán J, Quantum entanglement and classical correlations in the photon-pairs produced by nonlinear crystals, M.Sc. Thesis (in Spanish), Physics Department, Cinvestav, México City, 2015
[10] López-Durán J and Rosas-Ortiz O, in preparation
[11] Kwiat P G, Mattle K, Weinfurter H and Zeilinger A, Phys. Rev. Lett. 75 (1995) 4337