Abstract—Deep learning for supervised learning has achieved astonishing performance in various machine learning applications. However, annotated data is expensive and rare. In practice, only a small portion of data samples are annotated. Pseudo-ensembling-based approaches have achieved state-of-the-art results in computer vision related tasks. However, it still relies on the quality of an initial model built by labeled data. Less labeled data may degrade model performance a lot. Domain constraint is another way to regularize the posterior but has some limitation. In this paper, we proposed a fuzzy domain-constraint-based framework which loses the requirement of traditional constraint learning and enhances the model quality for semi-supervision. Simulations results show the effectiveness of our design.

1. Introduction

In the past ten years, machine learning especially deep learning achieves a huge success in diverse tasks of supervised learning. A good predictive model of supervised learning needs a large set of annotated training examples. A training example usually contains two parts: a feature vector and a label. The vector indicates an observed event or an object. And the label represents the ground-truth outcome.

However, data annotation/labeling process is expensive and difficult. Its more desirable to use weakly supervised learning rather than supervised learning in practice. Weak supervision has three types: incomplete supervision, inexact supervision and inaccurate supervision [1]. The most common one is incomplete supervision. Incomplete supervision assumes training data are mostly unlabeled and only a small subset of them is annotated. And there are two approaches to leverage the unlabeled data. The first one is active learning. Active learning efficiently utilizes the knowledge of domain experts to selectively label a small amount of training data which are the most valuable for training model [2]. Another one is semi-supervised learning (SSL) which combines supervised learning and unsupervised learning together. Generative model, low-density separation, and graph-based model are used to analyze unlabeled data [3]. Recent studies [4]–[7] utilize heuristic model to analyze unlabeled training data and generate weak labels for them. But it relies too much on the number of labeled data. Domain knowledge is proved useful to regularize the posterior [8] [9] [10] [11]. However, these works only focus on regularizing the posterior of model output (target variable) and needs dedicated constraint design before model training.

In this paper, we proposed a new fuzzy domain-constraint-based framework which utilizes an additional penalty network to approximate unknown parameter not limited to target variable (model output) of the learned model by referring the abstract latent variables of the learned model. Section 2 discuss the related works about heuristic learning and constraint learning for semi-supervision. Section 3 illustrate the detail of our designed model and Section 4 shows the performance of our proposed model compared with previous work.

2. Related work

2.1. Heuristic models in Semi Supervision

Heuristic models used for semi-supervision have a prerequisite that the labeled samples and unlabeled samples share the same or similar data distribution. Assuming there is a classification task, [4] proposed a bootstrapping method by utilizing a neural network’s inference of unlabeled sample as the pseudo label. In other words, the class with maximum logit will be considered as the pseudo label. Then these pseudo labels are used to regularize the network in turn. So the cost function of a network is the cross-entropy losses of labeled data and unlabeled data.

\[
L_p = \frac{1}{n} \sum_{m=1}^{n} \sum_{i=1}^{C} E(y_i^m, f_i^m) + \alpha(t) \frac{1}{n'} \sum_{m=1}^{n'} \sum_{i=1}^{C} E(y_i'^m, f_i'^m) \tag{1}
\]

where \( n \) is the number of labeled training data per mini-batch if SGD is used, \( n' \) is the number of unlabeled data per mini-batch, \( C \) is the number of classes, \( E(y_i^m, f_i^m) \) is the cross-entropy between true label and inference result.
for labeled data. $E(y^m_i, f^m_i)$ is the cross-entropy between pseudo label and inference result for unlabeled data sample $m$ for class $i$ (Pseudo label and true label are both one hot vector, thus $y^m_i$ and $y^m_i$ are either 1 or 0). $\alpha(t)$ is a ramp-up function to determine the contribution of unlabeled data for the network. Since the inference result of unlabeled data is not accurate in the initial stage of training, $\alpha(t)$ is set to zero. It then increases gradually as the network learns more knowledge from labeled samples. The former part of the loss function is a supervised loss for labeled data and the latter one is the unsupervised loss for unlabeled data. However, the quality of the pseudo label relies on the quality of the network model. Meanwhile, the pseudo label will eventually impact the quality of the network as well. If wrong pseudo labels are incorporated with labeled samples during the training process, the model error will be exacerbated as the ramp-up index increases.

To make the prediction of pseudo labels more robust, self-ensembling training [5]–[7] is proposed to develop multiple child models with diverse variants from a parent model. Although child models have diverse variants, their output is expected to be consistent with each other for samples in the final. Ensemble learning is used for the fusion of child models [12]. The fusion of child models can smooth the prediction of pseudo labels. Data augmentation and diverse network configuration are two approaches to force child models having different variants [5], [6].

Adding random noise is the most common way for data augmentation. The diversity between child models’ variants can be considered as the difference between features extracted from data without noise and features extracted from data with noise. Fusion of child models has to make sure the views of data without noise and data with noise are consistent with each other. In this case, it will force the network to denoise data samples and extract more features than a network without data augmentation. [5] proposed ladder networks (a nested autoencoder) to get better prediction for unlabeled data. It utilizes two same nested encoder networks. The clean encoder has clean input and the corrupted encoder always adds random noise to its encoder layers. The cost function retains the supervised loss of Eq. (1) for clean encoder but considers the consistency cost (denoising cost) of each autoencoder layers of a corrupted encoder as the unsupervised loss. However, the computation of the denoising cost of every autoencoder layers is too heavy in the training process.

[5] [7] further introduce diverse network configurations to produce diverse child models on training samples along with data augmentation. Network regularization techniques like Dropout [13] and DropConnect [14] dynamically mask a subset of the parent model during training. In this case, the dynamic parent model produces diverse child models in each epoch or step. $\pi$ model utilizes two parallel same parent models with Dropout and stochastic data augmentation. So for every step (every batch), their child models produce two views for unlabeled data [6]. The fusion of child models is to minimize the consistency cost, which is the difference between the inferences of two child models.

The loss function of $\pi$ model is

$$L_\pi = \frac{1}{n} \sum_{m=1}^{n} \sum_{i=1}^{C} E(y^m_i, f^m_i) + \alpha(t) \frac{1}{n+n'} \sum_{m=1}^{n+n'} \sum_{i=1}^{C} ||f^m_i - f'_i||^2$$

(2)

where $f$ and $f'$ are the two parallel models sharing the same network structure with dropout and stochastic data augmentation. The supervised loss remains the same as Eq. (1). The drawback of $\pi$ model is that two parallel models in the training process cost too many resources which will decrease its scalability. Temporal Ensembling utilizes only one parent model in every epoch but it also produces a diverse child model due to Dropout in different epoch [6]. In this case, the consistency cost in every step is the difference between the ensemble prediction/inference and the inference of the current model. Ensemble prediction is the exponential moving average (EMA) of predictions in previous epochs. The loss of a temporal model is

$$L_t = \frac{1}{n} \sum_{m=1}^{n} \sum_{i=1}^{C} E(y^m_i, f^m_i) + \alpha(t) \frac{1}{n+n'} \sum_{m=1}^{n+n'} \sum_{i=1}^{C} ||f^m_i - z^m_i||^2$$

(3)

where $z$ is the ensemble prediction. What’s more, mean teacher further optimizes the ensembling procedure of Temporal Ensembling [7]. Firstly, it updates ensemble prediction in each step rather than each epoch. The second optimization is that instead ofensembling previous predictions, mean teacher ensembles the weights of previous models. So the consistency cost in each step becomes the distance of prediction of the current model (student model) and prediction of the ensemble model (teacher model). The loss function of the mean teacher is

$$L_{mt} = \frac{1}{n} \sum_{m=1}^{C} \sum_{i=1}^{C} E(y^m_i, f^m_i(x, \theta)) + \alpha(t) \frac{1}{n+n'} \sum_{m=1}^{n+n'} \sum_{i=1}^{C} ||f^m_i(x, \theta') - f^m_i(x, \theta)||^2$$

(4)

where $f^m_i(x, \theta)$ is the student model with weight($\theta$) and $f^m_i(x, \theta')$ is the teacher model with weight($\theta'$). $\theta'$. And the EMA of weight in every step $t$ is computed as follows:

$$\theta'^t \leftarrow \beta \theta'^{t-1} + (1-\beta) \theta_t$$

(5)

In addition, mean-only batch normalization and weight normalization are used in convolutional and softmax layers. However, these heuristic approaches cannot guarantee the quality of the learned model since limited labeled data are probably insufficient to initialize models to produce reliable pseudo labels for unlabeled data.
2.2. Domain constraint for semi supervision

To solve the problem of limited labeled data, domain knowledge constraints are imposed to train the learned model. Domain knowledge constraints could be structured functions such as structural constraints [8] or logic rules [9]. Instead of using direct labels to regularize learned model, domain knowledge constraints provide another way to regularize the distribution of model posterior. Assume $Q$ is a set of distribution of domain constraints with respect to the posterior of learned model.

Posterior Constraints Set: $Q = \{ q(y) : E_{q}(G(x,y)) \leq c \}$

(6)

where $G$ is a set of constraint functions bounded by $c$. The constraint set implies extra information on posterior’s distribution and narrows down the searching space of posterior. In this case, a penalty term is defined to push posterior to the desired implicit distributions of domain knowledge constraints where KL divergence is used to measure the distance between two distributions.

$$ KL(Q|p(x,y)) = \min_{q \in Q} KL(q(y)|p(x,y)) $$

(7)

However, this approach relies on the exact information of domain knowledge. The domain constraint feature set $G$ is carefully specified by domain experts prior to the network training. So $G$ is only suitable for particular models which makes it difficult to be used in other applications of machine learning. What’s more, $G$ is only related to the input $x$ and output $y$ of learned model, which is not realistic since most physical models have multiple parameters except $x$ and $y$. To solve the second issue, we extend the work of constraints learning and designed a penalty network to approximate latent variables in constraint features.

3. Our model

In this section, a domain-constraint-based model is proposed for semi supervision which encapsulates domain constraint knowledge in a neural network. A penalty network is implemented to emulate the approximate physical parameters of a physical system and the approximated values are optimized within a parameter space restricted by domain knowledge. During the training process, both the penalty network and primary network are updated recursively and the penalty network will restrict the search space of the primary network.

3.1. Domain-constraint-based posterior regularization

Unlike previous work doing posterior regularization of network output, domain knowledge is referred to regularize the latent variables of the last hidden layer in the primary network. However, these variables are abstract and cannot be utilized with domain knowledge constraint directly. To solve this problem, we introduce a penalty network to approximate the unknown physical parameters of domain knowledge. The penalty network transfers the latent variables to physical variables which can be regularized by domain knowledge directly.

We use a $k$-class classification model as an example to explain the structure of our model. Assume primary network has input variable $x \in \mathcal{X}$ and output variable $y \in \mathcal{Y}$ where $y$ is $k$-dimension one hot vector. Primary network learns a conditional probability $p_{\theta_{p}}(y|x)$ and output a soft prediction vector $f_{\theta_{p}}(x)$ where $\theta_{p}$ represents the weights of primary network. Consider $\theta'_{p}$ as the weights of the primary network except those of the last layer, the latent variables of last hidden layer in the primary network is

$$ l = f_{\theta'_{p}}(x) $$

(8)

Since $l$ is the latent variables of the last hidden layer, it usually has a dimension greater or equal to the dimension of output. We consider it contains more feature information of input variable $x$ than soft prediction vector $f_{\theta_{p}}(x)$. Thus, instead of regularizing the output posterior of the primary network, we optimize the latent variable $l$ to better understand the conditional probability $p_{\theta_{q}}(l|x)$. Further, we implement domain knowledge to regularize the posterior $l$.

However, $l$ is an abstract vector. It is impossible to collect prior knowledge of $l$. To address this problem, we introduce a penalty network to translate the abstract variables to physical parameters according to domain knowledge. The output of the penalty network is denoted as $f_{\theta_{q}}(l)$ where $\theta_{q}$ represents the weights of the penalty network. Domain knowledge constraints define a function set of $x$ and $f_{\theta_{q}}(l)$

$$ G(x,f_{\theta_{q}}(l),z) \leq c $$

(9)

where $c$ is a boundary vector of domain knowledge constraint, $z$ denotes a vector of known parameters and $G$ is a set of constraint equations or inequalities.

The posterior regularization learns an auxiliary variational probability $q_{\theta_{q}}(l)$ based on the domain knowledge constraints above. $q_{\theta_{q}}(l)$ is optimized by imitating the conditional probability $p_{\theta_{q}}(l|x)$. And the loss of posterior regularization is

$$ L_{pr}(\theta'_{p}, \theta_{q}) = KL(q_{\theta_{q}}(l)||p_{\theta_{p}}(l|x)) + \gamma \cdot E_{\theta_{q}}[G(x,f_{\theta_{q}}(l)) - c] $$

(10)

where the first term indicates KL divergence is used to push model posterior $p_{\theta_{p}}(l|x)$ to desired distribution space $q_{\theta_{q}}(l)$ based on domain knowledge. The second term indicates the expectations of domain constraint are bounded by a constant $c$. And $\gamma$ is set to adjust the weight of the second term.

3.2. consistency cost between child models

To fusing the features extracted from the primary network and secondary network, consistency cost is utilized to minimize the feature difference between these two models.

$$ L_{con}(\theta_{p}) = E_{\theta_{p}}(f_{\theta_{p}}(x), f_{\theta_{p}}(x)) $$

(11)
where output posteriors of two models are compared to compute the consistency cost. In our design, we use the exponential moving average of historical network weights of the primary network to construct the secondary network [7].

### 3.3. classification cost of labeled data

Meanwhile, traditional classification loss of labeled data is used to constrain the learned model

$$L_{\text{class}}(\theta_p) = \mathbb{E}_{\theta_p}(y, f_{\theta_p}(x))$$

(12)

### 3.4. learning procedure

The training procedure is similar to EM algorithm [8], [9]. In addition, we utilize a self-ensembling approach to generate pseudo labels for unlabeled data in order to reinforce the generalization of the primary model. Step 1 is

**Algorithm 1:** semi supervision via domain knowledge distillation

1. initialize network weights $\theta_p$, $\theta'_p$ and $\theta_q$ where $\theta'_p \subseteq \theta_p$;
2. while not converging do
   3. $\theta_q^{t+1} \leftarrow \arg\min_{\theta_q} L_{\text{pr}}(\theta'_p, \theta_q);$
   4. $\theta_p^{t+1} \leftarrow \arg\min_{\theta_p} L_{\text{pr}}(\theta'_p, \theta_q^{t+1});$
   5. $\theta_p^{t+1} \leftarrow \arg\min_{\theta_p} (L_{\text{class}}(\theta_p^{t+1}) + L_{\text{con}}(\theta_p^{t+1}));$
   6. $\theta_p^{t+1} \leftarrow \beta \theta_p^{t+1} + (1 - \beta) \theta_p^{t};$
7. Done;

similar to the E-step of posterior regularization. Nevertheless, it is used to regularize physical parameters not limited to the target variable $y$. Step 2 is the M-step of posterior regularization which refers to the updated desired distribution of posteriors and optimizes the weights of primary network $\theta_p$. Step 3 and 4 are the steps to ensemble primary model and secondary model.

### 4. Simulation Results

#### 4.1. Power Outage Attack

In this scenario, we simulate the power outage attack on WECC 9-bus power system. The frequencies of three power generators are recorded as the training samples to predict the location of cyber attacks/physical failures. And a simple 3-layer perceptron network is implemented to learn the training data and the network structure is shown in Fig. 1. Domain knowledge constraint in this experiment is expressed as a second-order differential equation (swing equation) and multiple logical constraints such as logical-or and logical-and.

$$M_i \dot{\omega}_i(t) = -D_i \omega_i(t) + P_{m,i} - |E_i|^2 G_{i,i}(t) - \sum_{j=1}^{N} P_{i,j}(t) \sin(\theta_i(t) - \theta_j(t) + \varphi_{i,j}(t))$$

where $\theta_i(t) = \theta_c + \int_{0}^{t} \omega_i(\tau) \Delta \tau$

$$\varphi_{i,j}(t) = \arctan\left(\frac{G_{i,j}(t)}{B_{i,j}(t)}\right)$$

(13)

Where $E(t)$, $G(t)$, $B(t)$ and $P(t)$ are the unknown parameters influenced by not only the location of a power outage but also the electrical attributes of 9-bus and the connection mapping of the power system. So we proposed an additional penalty network whose input is the latent variables of the last hidden layer of the primary network. These variables provide high-level features of input data and penalty network use them to approximate the unknown parameters within the searching space defined by the domain constraint shown in Eqn. 13. At the same time, self-ensembling-based bootstrapping is used to produce more pseudo labels for unlabeled data. Table. 1 shows that Hybrid Semi Supervision (domain-constraint-based semi supervision) outperforms semi supervision (bootstrapping-based semi supervision) and supervised learning.

![Network Architecture for Power Outage Attack](image1)

**Figure 1:** Network Architecture for Power Outage Attack

![Validation Accuracy during training process](image2)

**Figure 2:** Validation Accuracy during training process
TABLE 1: Testing accuracy of diverse models for power outage attack

| # of Labels | Supervised Learning | Semi Supervision | Hybrid Semi Supervision |
|----------------|---------------------|-------------------|-------------------------|
| 90(1.25%)     | 87.44 ± 2.98        | 90.47 ± 1.89      | 99.14 ± 0.20            |
| 180(2.5%)     | 90.15 ± 0.25        | 96.04 ± 3.28      | 99.00 ± 0.09            |
| 360(5%)       | 93.14 ± 1.24        | 99.30 ± 0.10      | 99.96 ± 0.04            |
| 720(10%)      | 98.19 ± 1.09        | 99.95 ± 0.05      | 99.98 ± 0.03            |
| 7200(100%)    | 99.98 ± 0.02        | N/A               | N/A                     |

5. Conclusion

In this paper, a fuzzy domain-knowledge-based framework of posterior regularization is proposed for semi supervision. Results show that domain constraints significantly improve the performance of semi supervision. Benefited by the penalty network, abstract latent variables are transformed to physical parameters of domain knowledge constraints which broadens the scope of model applications and enhances the performance of semi supervision significantly. In the future, we will investigate the performance on more advanced network structure like GAN combining with the domain constraints.

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