CP VIOLATION FROM DIMENSIONAL REDUCTION: A SIMPLE EXAMPLE.

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CP is a symmetry of pure gauge theories, that is without scalar interactions. Actually its violation originates in the Standard Model from the completely arbitrary Yukawa couplings. Thus, as the unification principle would wash out the arbitrariness of these couplings possibly relating them to the gauge interactions, the resulting theory should be expected as CP-invariant. According to this, a breaking mechanism of CP should then take place. We will explore here the possibility of CP breaking through the dimensional reduction process.

Firstly, we resume briefly the issue of discrete symmetries in the context of dimensionaly extended space or more particulary of parity in an even number of spatial dimensions. In 3 spatial dimensions, the central inversion $\vec{x} \rightarrow -\vec{x}$ and the specular reflexion, say $x_1 \rightarrow -x_1$, are two equivalent definitions of parity modulo a spatial rotation. However, for an even number of spatial dimensions, the central inversion becomes simply an element of the rotation group, with $\text{det} = +1$. The specular reflexion remains a discrete symmetry and the equivalence does not hold anymore. Actually, the best generalisation of parity is the specular reflexion which leads to a generalisation of the CPT theorem. This leads to a universal definition of parity whatever the number of dimensions: let take the reflexion of the last spatial coordinate ($x_{d-1} \rightarrow -x_{d-1}$ for $d$ dimensions).

As a consequence, the scalar term $\bar{\psi}\psi$ in 4+1D is $P$-violating. Indeed, in 3+1D, it is easy to go from $\bar{\psi}\psi$ to $\bar{\psi}i\gamma_5\psi$ by a transformation on the semi-spinor and only the simultaneous presence of both couplings achieve $P$-violation. In 4+1D, since the last component of a vector has to be identified with $\bar{\psi}i\gamma_5\psi$ (with $i\gamma_5 = \gamma_4$ to get a right extention of the Clifford algebra), the transformation cannot be done and this results in $\bar{\psi}\psi$ to be $P$-violating while $CP$-conserving. This is at the origin of $CP$ violation in the dimensional reduction process.
To the aim of getting those two components in a reduced theory, we consider a gauge theory in 4+1D:

$$\mathcal{L} = i\bar{\psi} D\psi - M \bar{\psi}\psi,$$  \hspace{1cm} (1)

with $D_B = \partial_B - ieA_B$ ($B = 0, 1, 2, 3, 4$). We observe directly that the reduction to 3+1D could introduce effective complex mass term through non-vanishing contribution arising from the last component of the kinetic term, which we denote by $X_4$: $\psi(M + i\gamma_5 X_4)\bar{\psi}$.

Such a structure could lead to CP violation even if in the minimally coupled $U(1)$ case the complex phase can be removed in 3+1D. CP violation can be achieved, first as it was considered by Thirring using the $\partial_4$ term, identifying $X_4$ to the Kaluza-Klein mass $\frac{n}{R}$ and getting a non-minimally coupled $U(1)$ from the reduction of gravity; or, as it is addressed here, taking an "expectation value" for the last component of the gauge field $\langle A_4 \rangle \neq 0$ and extending the gauge group. This choice enables us to clearly separate CP violation from the use of exited KK states.

Let first comment on the possibility of a non-vanishing expectation value for $A_4$. As such, $\langle A_4 \rangle$ is not gauge invariant since the value of $A_4$ can be rotated away at any given point. A gauge invariant formulation is provided by a loop integral over a path which conserve 3+1D Lorentz invariance, i.e.:

$$X_4 = \int dy A_4,$$  \hspace{1cm} (2)

assuming $X_4$ to be time and $\vec{x}$ independent ($y = x^4$). This quantity thus depends on the compactification scheme.

We turn now to a concrete example based on the $SU(2)$ group minimally coupled through the $W^a_A$ gauge bosons to fermions with a mass, and assuming:

$$\langle W_4 \rangle = \int dy W_4(y) = \begin{pmatrix} w & -w \end{pmatrix}.$$  \hspace{1cm} (3)

This results in the effective 3+1D Lagrangian:

$$\left( \begin{array}{c} \bar{\psi}_1 & \bar{\psi}_2 \end{array} \right) i(\partial^\mu - iW^\mu_3 \gamma^\alpha)\gamma_\mu \left( \begin{array}{c} \psi_1 \\ \psi_2 \end{array} \right) + \left( \begin{array}{cc} \bar{\psi}_1 \\ \bar{\psi}_2 \end{array} \right) M \left( \begin{array}{c} \psi_1 \\ \psi_2 \end{array} \right),$$  \hspace{1cm} (4)

where

$$M = \begin{pmatrix} M + iw\gamma_5 & M - iw\gamma_5 \\ M - iw\gamma_5 & M + iw\gamma_5 \end{pmatrix}.$$  \hspace{1cm} (5)

The mass matrix can be made real generally by a bi-unitary transformation, $M' = U_R^\dagger M U_L$. In the present case, we can choose ($\alpha = \gamma_5 \arctan \frac{w}{M}$):

$$U_R = I, \quad U_L = \begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}.$$  \hspace{1cm} (6)

The loop integral results in gauge symmetry breaking with two massive $W^\pm$ and one massless $W^3$. The use of such a line integral has been developed in details by Hosotani in the framework of dynamical symmetry breaking. Even if the fermion masses are transformed to be real, the coupling to $W^\pm$ is no longer vectorial, but includes a phase between the left and right-handed part resulting in a $W^3$-dipole moment at one loop level (see Figure). CP violation thus arises here via the fermion mass matrix as in the Standard Model and its origin is related to both dimensional reduction and breaking of the internal symmetry. However, strong limitations need extension to more realistic models, namely: mass degenerancy, the vectorlike coupling in the current basis, etc.
For instance, since in 4+1D only vectorlike couplings are allowed, to get chiral couplings we could consider topological defect, e.g. domain wall which selects chiral massless fermionic modes in its core. Moreover, since $CP$ violation arises here through the generation of fermion mass from the gauge bosons, we choose the defect to be part of the internal group resulting in the selection of left and right-handed fermions which are linked through the loop integral. Nevertheless, this procedure divides directly the number of fermionic components by two and thus implies for the $U(1)$ case no complex mass while for the $SU(2)$ case one single complex mass which can be safely redefined as real.

One simple example is then provided by $SU(4)$. Let us consider the vacuum configuration:

$$
\Phi = \frac{\phi(y)}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\
1 & -1 & 0 & 0 \\
-1 & -1 & 0 & 0 
\end{pmatrix}, \quad \chi = \frac{\langle \chi \rangle}{2} \begin{pmatrix} 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 
\end{pmatrix}, \quad \eta = \frac{\langle \eta \rangle}{2} \begin{pmatrix} 1 & -1 & 0 & 0 
\end{pmatrix}, \quad (7)
$$

with $\Phi$ being the domain wall from which the fermion localisation will select an $SU(2)_L \times SU(2)_R \times U(1)_A$ group between fermion zero modes: $(u^1_L, d^1_L, u^2_R, d^2_R)$; while $\eta$ and $\chi$, acquiring a constant vev, will break down respectively the $SU(2)_L$ and the $SU(2)_R$ subgroup. After that, the generation of fermion masses needs non-diagonal scalars $H^1$ and $H^2$; e.g.:

$$
H^1 = \frac{\langle H^1 \rangle}{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 
\end{pmatrix} = \langle H^1 \rangle \lambda_4, \quad H^2 = \frac{\langle H^2 \rangle}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 
\end{pmatrix} = \langle H^2 \rangle \lambda_{11}. \quad (8)
$$

Since those two breakings commute, they clearly minimise their interaction potential, but moreover allow the Hosotani term to get a component in each direction without cost of energy: $\int dy W^4 = w_4 \lambda_4 + w_{11} \lambda_{11}$. This feature provides actually two masses with two phases, i.e.:

$$
u^1_L \frac{1}{2} (m_1 \langle H^1 \rangle + iw_4 \gamma_5) \quad u^2_R + d^1_L \frac{1}{2} (m_2 \langle H^2 \rangle + iw_{11} \gamma_5) \quad d^2_R + h.c. \quad (9)
$$

It can be easily checked that both phases cannot be removed completely from the Lagrangian. However, to have $CP$ violation through the $W_L$ alone, more generations are needed. The model discussed here is not yet realistic in that charge assignments in the fundamental of $SU(4)$ are not compatible with the observed ones.

Acknowledgments

This work was done in collaboration with J.-M. Frère and L. Lopez Honorez and is supported in part by IISN, la Communauté Française de Belgique (ARC), and the belgian federal government (IUAP).
References

1. N. Cosme, J. M. Frère and L. Lopez Honorez, arXiv:hep-ph/0207024.
2. W. E. Thirring, Acta Phys. Austriaca Suppl. 9 (1972) 256.
3. G. C. Branco, A. de Gouvea and M. N. Rebelo, Phys. Lett. B 506 (2001) 115 [arXiv:hep-ph/0012289]. C. S. Huang, T. j. Li, W. Liao and Q. S. Yan, Eur. Phys. J. C 23 (2002) 195 [arXiv:hep-ph/0101002]. D. Chang and R. N. Mohapatra, Phys. Rev. Lett. 87, 211601 (2001) [arXiv:hep-ph/0103342]. D. Chang, W. Y. Keung and R. N. Mohapatra, Phys. Lett. B 515 (2001) 431 [arXiv:hep-ph/0105177]. Y. Sakamura, Nucl. Phys. B 608 (2001) 279. P. Q. Hung and M. Seco, arXiv:hep-ph/0111013. D. Dooling, D. A. Easson and K. Kang, arXiv:hep-ph/0202206. S. Ichinose, arXiv:hep-th/0206187.
4. M. Belen Gavela and R. I. Nepomechie, Class. Quant. Grav. 1 (1984) L21.
5. Y. Hosotani, Phys. Lett. B 126 (1983) 309. Y. Hosotani, Phys. Lett. B 129 (1983) 193.
6. V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 125 (1983) 136.