Adapting the explicit time integration scheme for modeling of the hydraulic fracturing within the Planar3D approach

E B Starobinskii and A D Stepanov
Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russia

E-mail: st.eb@ailurus.ru

Abstract. This work is devoted to the model of hydraulic fracture propagation in a layered medium based on the Planar3D approach. The key features of the proposed model are to reduce the system of partial differential equations to a dynamic system, as well as apply universal asymptotes for determining the position of the crack front. We compare calculations with published results of ILSA and EP3D models and discuss methods for accelerating calculations and the possibility of taking into account additional effects (proppant transfer, elastic modulus contrast).

1. Introduction
In world practice, hydraulic fracturing is widely used to intensify the flow of oil as the technology for creating a highly conductive fracture in an oil-bearing formation. The unavailability of direct observation for hydraulic fracture leads to the need for computer experiments that take into account many physical effects. Optimization of hydraulic fracturing requires a large number of calculations with different parameters for a limited time [1]. The development of modern computing systems and new approaches allows the use of more complex models of hydraulic fracturing, while maintaining a balance between the number of effects taken into account and the computation speed. The need for the development of existing approaches is shown in [1-7]. Today, commercial simulators use the Pseudo3D and Planar3D models as the main ones [7]. The Pseudo3D model uses the assumption that the crack length is much greater than its height. Unlike Pseudo3D, the Planar3D model has no such limitation and takes into account the two-dimensional flow of a fluid. Such a formulation allows one to simulate cracks of complex shape and solve the problem of proppant transfer in the body of a crack more accurately, but leads to a significant increase in the computational complexity of the calculation algorithms. An example of a numerical implementation of this approach is the Implicit Level Set Algorithm (ILSA) [8, 9], which uses an implicit time integration scheme. The implicit scheme allows using a sufficiently large time step, but it requires solving a system of nonlinear equations at each step. The possibility of practical application of the explicit scheme is shown by A. M. Linkov [10]. This paper proposes the implementation of the Planar3D model, the distinctive features of which are an explicit time integration scheme, used asymptotic solutions, and a front tracking method without evaluation of the norm vector. The described method can be effectively used to conduct parallel calculations on multi-core computing systems. The aim of the work is to create a software module for predicting crack growth in a layered medium in the variable injection mode of a non-Newtonian fluid.
2. Basic assumptions

The Planar3D model commonly uses the following assumptions:

- the reservoir is represented as a set of layers, each of which is homogeneous and isotropic in mechanical properties;
- each layer is characterized by its own values of minimum compressive stress, elastic modulus, toughness, Carter leak-off coefficient;
- the crack propagates in a plane perpendicular to the direction of minimum compressive stresses;
- fluid is power-law and incompressible;
- fluid transfer in a crack is described by the lubrication theory equation, gravity is not taken into account;
- deformation is described by the linear theory of elasticity;
- there is no lag of the fluid front from the crack front;
- the well is a point source.

In the present work, it is assumed that the elastic properties of the reservoir are uniform and are described by the effective flat Young modulus, which is calculated as the weighted arithmetic mean over all layers. There are more complex approaches, within which the contrasts of elastic modulus are taken into account explicitly. In [11], an algorithm is given that can be used to solve such a problem.

3. Solvable equations and discretization

In the case of a medium that is homogeneous in elastic modulus, the relationship between pressure and crack opening is described by the hypersingular integral of the elastic theory [12, 13] (1):

$$ p(x, y, t) = \sigma(y) + \frac{E'}{8\pi} \int \frac{w(x', y', t)}{(x-x')^2 + (y-y')^2} \, dx' \, dy', $$

where $p(x, y, t)$ is the pressure at the point with coordinates $(x, y)$ at the time moment $t$, $\sigma$ is compressive reservoir stresses in the direction perpendicular to the plane of the crack, $w$ is the crack opening, $E' = \frac{E}{1-\nu^2}$ is flat Young’s modulus, $E$ is Young’s modulus, $\nu$ is Poisson’s ratio.

We assume that the flow of a fluid can be described by a Poiseuille-type equation, the fluid leaks off into the reservoir following Carter’s law. In that case the mass balance for the fluid takes the form (2):

$$ \frac{\partial w(x, y, t)}{\partial t} = \nabla \cdot \left( \frac{(w(x, y, t)^{2n+1})^{1/\mu'}}{\mu'} \nabla p(x, y, t) \right) + Q(0, 0, t) - \frac{2C_L(y)}{\sqrt{t-t_0(x, y)}} $$

where $Q$ is a term describing the injection of fluid into the crack, $t_0$ is the activation time corresponding to the moment of passage of the crack front through the point $(x, y)$, $n$ is the behavior index, $\mu' = 2 \left(4 + \frac{n}{2}\right)$ is the consistency index, $\mu$ is the dynamic viscosity, $C_L$ is Carter’s leak-off coefficient.

As an initial condition, we use the self-similar solution for the opening when a crack propagates in a homogeneous medium. The time for constructing the solution will be chosen in such a way that the crack front does not reach the interfaces of the layer in which the crack is initiated.

To solve the system of equations, we will use spatial discretization with a fixed cell size $(\Delta x, \Delta y = \Delta x)$. Let us number the cells from 1 to $N$ and write the pressure and expansion in the form of vectors $\hat{p}$ and $\hat{w}$, each of length $N$, connected by a constant matrix of influence factors $A$. Equation (1) can be written as (3):

$$ \frac{\partial w(x, y, t)}{\partial t} = \nabla \cdot \left( \frac{(w(x, y, t)^{2n+1})^{1/\mu'}}{\mu'} \nabla p(x, y, t) \right) + Q(0, 0, t) - \frac{2C_L(y)}{\sqrt{t-t_0(x, y)}} $$
\[ \dot{\sigma}(t) = \ddot{\sigma} + \frac{\partial^l}{\partial t^l} A \cdot \dot{\omega}(t), \]  

where \( \dot{\sigma} \) is a vector of compressive stress values.

Influence matrix \( A \) is written in the form (4):

\[ A = \begin{pmatrix} U(1, 1) & \cdots & U(1, N) \\ \vdots & \ddots & \vdots \\ U(N, 1) & \cdots & U(N, N) \end{pmatrix}. \]  

Place the collocation points at the vertices of each cell, numbering them from one from the bottom left corner in a clockwise direction. Let \( c_{kl} \) denote the distance from the center of the cell \( k \) to \( i \)-th point of the collocation of the cell \( l \), \( a_{kl} \) and \( b_{kl} \) are the difference of coordinates between the collocation point and the center of the cell, respectively, on the abscissa and ordinate. Then the function \( U \) is calculated as the sum of the four terms (5):

\[ U(k, l) = \sum_{i=1}^{4} (1 - 2(i \% 2)) \frac{c_{kl}}{a_{kl} b_{kl}}, \]

where \( i \% 2 \) denotes the remainder of dividing \( i \) by 2.

There are various mathematical and computer methods that can reduce the number of operations in the calculation of the matrix-vector product, as well as achieve high efficiency of parallelization. Thus, when modeling hydraulic fracturing using 4 parallel streams, even with the direct multiplication method, the computation speed is increased by about 2.5 times. This result was obtained using OpenMP to calculate pressure.

To track the crack front, we will use the statistical method proposed by A. D. Stepanov in [14] and not requiring evolution of the normal. The method is based on constructing tracking circles around the boundary elements—the selected cells of the computational domain located behind the propagating front. The radius of the circle built around the element is equal to the distance from the center of the element to the front of the crack. The definition of new boundary elements then occurs according to a geometric criterion.

The distance to the front can be related to the crack opening and the speed of its propagation using asymptotic formulas [15-17]. The general form of the universal asymptotic umbrella (UAU), described in the works of A. M. Linkov [16, 17], can be written in the form (6):

\[ w = A_w(v) r^\alpha, \]

where \( v \) is the front velocity, \( r \) is the distance to the front, \( A_w \) and \( \alpha \) are values determined by the propagation regime and given in [16].

Relation (6) allows us to determine the front velocity from the opening values and the distance to the front at the previous step. The new distance to the front will then be recalculated from the velocity by the Euler method.

UAU is applicable to Newtonian and non-Newtonian fluids, as well as different regimes of crack propagation (dominant toughness, dominant viscosity, dominant leak-off). With the growth of cracks in a layered medium, different propagation modes can be implemented in different front areas; to take into account this feature, the shape of the UAU is selected in each cell independently.

4. Examples of calculations

In [18], the results of calculations are presented using an enhanced Pseudo3D model (EP3D) and an ILSA model for a three-layer symmetric reservoir, in which a flat crack is initiated by uniform injection of a Newtonian fluid. Numerical experiment parameters: central layer thickness is 5 cm,
Compressive stresses are 4.3 MPa, pumping rate is 1.7 mm$^3$/s, $E = 3.3$ GPa, $v = 0.4$, $\mu = 30.2$ Pa·s, $n = 1$, $C_s = 0$, modeling time is 604 s. Figure 1 shows the opening and pressure profiles from paper [18] in the vertical and horizontal sections of a crack passing through the source. Additionally, the black dots show profiles calculated using the described Planar3D model. With the exception of a slight deviation of pressure in the crack front region, the results of the proposed model coincide with those obtained using ILSA.

During the simulation of crack growth in a layered medium, the fracture geometry is affected by the fluid rheology. Figure 2 shows the calculation results for a three-layer formation with a given stress contrast. The fluid volume and injection rate were maintained, the rheology power $n$ for the injected fluid was chosen in the range from 0.5 to 1 ($n = 1$ corresponds to the Newtonian fluid). As $n$ increases, the ratio of the crack length to its height (aspect ratio) also increases, the crack opening in the layer with compressive stresses decreases. This dependence shows the importance of taking into account the rheology of the fluid, since the production efficiency after fracturing is determined, among other things, by the form of hydraulic fracturing.

![Figure 1](image1.png)

**Figure 1.** Profiles of opening and pressure in horizontal (left) and vertical (right) sections. The blue dashed lines show the results of EP3D, the blue solid lines show ILSA, and the black dots show the results of the given Planar3D model. The vertical line shows the interface between the layers.

5. **Conclusion**

The paper describes a model of the flat crack propagation in a layered medium based on the Planar3D method. The key features of the proposed model are to reduce the system of partial differential equations to the dynamic system, and also to apply universal asymptotes for determining the position of the crack front. The proposed approach allows to simulate the flow of non-Newtonian fluids and add new effects: transfer of the proppant, contrasts of elastic modulus, changes in the rheology of the fluid, etc.
Figure 2. The effect of the fluid rheology on the opening profiles in horizontal (top) and vertical (bottom) sections. The vertical line shows the interface between the layers.

Acknowledgments
This work was supported by Ministry of Science and Higher Education of the Russian Federation within the framework of the Federal Program "Research and development in priority areas for the development of the scientific and technological complex of Russia for 2014 – 2020" (activity 1.2), grant No. 14.575.21.0146 of September 26, 2017, unique identifier: RFMEFI57517X0146. The industrial partner of the grant is LLC "Gazpromneft Science & Technology Centre".

References
[1] Pitakbunkate T et al 2011 Hydraulic Fracture Optimization with a P-3D Model (SPE Production and Operations Symp./Society of Petroleum Engineers)
[2] Adachi J et al 2007 Computer Simulation of Hydraulic Fractures (International Journal of Rock Mechanics and Mining Sciences) 44 739–757
[3] Peirce A 2016 Implicit Level Set Algorithms for Modelling Hydraulic Fracture Propagation (Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Science) 374 20150423
[4] Mack M G and Warpinski N R 2000 Mechanics of Hydraulic Fracturing (Reservoir
6-1

[5] Linkov A M et al 2017 Modified Formulation, $\varepsilon$-Regularization and the Efficient Solution of Hydraulic Fracture Problems (ISRM International Conf. for Effective and Sustainable Hydraulic Fracturing/International Society for Rock Mechanics and Rock Engineering)

[6] Osipov A A 2017 Fluid Mechanics of Hydraulic Fracturing: a Review (Journal of Petroleum Science and Engineering) 156 513–535

[7] Khasanov M M et al 2018 Scientific Engineering as the Basis of Modeling Processes in Field Development (Georesources) 20 142–148

[8] Peirce A 2015 Modeling Multi-scale Processes in Hydraulic Fracture Propagation Using the Implicit Level Set Algorithm (Computer Methods in Applied Mechanics and Engineering) 283 881–908

[9] Dontsov E V et al 2016 Implementing a Universal Tip Asymptotic Solution into an Implicit Level Set Algorithm (ILSA) for Multiple Parallel Hydraulic Fractures (50th US Rock Mechanics/Geomechanics Symp./American Rock Mechanics Association)

[10] Stepanov A D and Linkov A M 2016 On Increasing Efficiency of Hydraulic Fracture Simulation by Using Dynamic Approach of Modified Theory (Proc.s of Summer School-Conf. Advanced Problems in Mechanics 2016) 393–403

[11] Markov N S and Linkov A M 2017 An Effective Method to Find Green's Functions for Layered Media (Materials Physics and Mechanics) 32 133–143

[12] Peirce A and Detournay E 2008 An Implicit Level Set Method for Modeling Hydraulically Driven Fractures (Computer Methods in Applied Mechanics and Engineering) 197 2858–2885

[13] Hills D A et al 2013 Solution of Crack Problems: the Distributed Dislocation Technique (Springer Science & Business Media) 44

[14] Stepanov A D 2018 Statistical Method for Tracing Hydraulic Fracture Front Without Evaluation of the Normal (International Journal of Engineering & Technology) 7 274–278

[15] Garagash D I, Detournay E and Adachi J I 2011 Multiscale Tip Asymptotics in Hydraulic Fracture with Leak-off (Journal of Fluid Mechanics) 669 260–297

[16] Linkov A M 2015 The Particle Velocity, Speed Equation and Universal Asymptotics for the Efficient Modelling of Hydraulic Fractures (Journal of Applied Mathematics and Mechanics) 79 54–63

[17] Linkov A M 2014 Universal Asymptotic Umbrella for Hydraulic Fracture Modeling (arXiv preprint 1404.4165)

[18] Dontsov E V, Peirce A P 2015 An Enhanced Pseudo-3D Model for Hydraulic Fracturing Accounting for Viscous Height Growth, Non-local Elasticity, and Lateral Toughness (Engineering Fracture Mechanics) 142 116–139