A Determination of the Nucleon Tensor Charge

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Exploiting an approximate phenomenological symmetry of the $J^{PC} = 1^{++}$ light axial vector mesons and using pole dominance, we calculate the flavor contributions to the nucleon tensor charge. The result depends on the decay constants of the axial vector mesons and their couplings to the nucleons.

1 Introduction

The spin composition of the nucleon has been intensely studied and has produced important and surprising insights, beginning with the revelation that the majority of its spin is carried by quark and gluonic orbital angular momenta and gluon spin rather than by quark helicity \cite{1,2}. In addition, considerable effort has gone into understanding, predicting and measuring the transversity distribution, $h_1(x)$, of the nucleon \cite{3}. Transversity, as combinations of helicity states, $|\perp/T > \sim (|+ > \pm |− > )$, for the moving nucleon is a variable introduced originally by Moravcsik and Goldstein \cite{4} to reveal an underlying simplicity in nucleon–nucleon spin dependent scattering amplitudes. In their analysis of the chiral odd distributions, Jaffe and Ji \cite{5} related the first moment of the transversity distribution to the flavor contributions of the nucleon tensor charge:

$$\int_0^1 (\delta q^a(x) − \delta q^a(x)) \, dx = \delta q^a \text{ (for flavor index } a).$$

This leading twist transversity distribution function, $\delta q^a(x)$, is as fundamental to understanding the spin structure of the nucleon as its helicity counterpart $\Delta q^a(x)$. However, while the latter in principle can be measured in hard scattering processes, the transversity distribution (and thus the tensor charge) decouple at leading twist in deep inelastic scattering since it is chiral odd. Additionally, the non-conservation of the tensor charge makes it difficult to predict. While bounds placed on the leading twist quark distributions through positivity constraints suggest that they satisfy the inequality of Soffer \cite{6}, there are no definitive theoretical predictions for the tensor charge. In contrast to the axial vector isovector charge, no sum rule has been written that enables a clear relation between the tensor charge and a low energy measurable quantity. Among the various approaches, from the QCD sum rule to lattice calculations models \cite{7}, there appears to be a range of expectations and a disagreement concerning the sign of the down quark contribution. We present a new approach to calculate the tensor charge that exploits the approximate mass degeneracy of the light axial vector mesons ($a_1(1260)$, $b_1(1235)$ and $h_1(1170)$) and uses pole dominance to calculate the tensor charge \cite{8,9}. Our motivation stems in part from the observation that the tensor charge does not mix with gluons under QCD evolution and therefore behaves as a non-singlet matrix element. In conjunction with the fact that the tensor current is charge conjugation odd (it does not mix quark-antiquark excitations of the vacuum, since the latter is charge conjugation even) suggests that the tensor charge is more amenable to a valence quark model analysis.

2 The Tensor Charge and PoleDominance

The flavor components of the nucleon tensor charge are defined from the forward nucleon matrix element of the tensor current,

$$\langle P, S_T | \bar{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi | P, S_T \rangle = 2 \delta q^a \langle P | P^{\mu} S^{\nu}_T - P^{\nu} S^{\mu}_T \rangle \delta q^a.$$  (1)
We adopt the model that the nucleon matrix element of the tensor current is dominated by the lowest lying axial vector mesons

$$\langle P, S_T \bar{\psi} \sigma^{\mu \nu} \gamma_5 \frac{\lambda^a}{2} \psi | P, S_T \rangle = \lim_{k^2 \to 0} \sum_M \frac{\langle 0 | \bar{\psi} \sigma^{\mu \nu} \gamma_5 \frac{\lambda^a}{2} \psi | M \rangle \langle M, P, S_T | P, S_T \rangle}{M^2_M - k^2}.$$  \hspace{1cm} (2)

The summation is over those mesons with quantum numbers, $J^{PC} = 1^{++}$ that couple to the nucleon via the tensor current; namely the charge conjugation odd axial vector mesons – the isoscalar $h_1(1170)$ and the isovector $b_1(1235)$. To analyze this expression in the limit $k^2 \to 0$ we require the vertex functions for the nucleon coupling to the $h_1$ and $b_1$ meson and the corresponding matrix elements of the meson decay amplitudes which are related to the meson to vacuum matrix element via the quark tensor current. The former yield the nucleon coupling constants $g_{NN}$ and the latter yield the meson decay constant $f_M$. Taking a hint from the valence interpretation of the tensor charge, we exploit the phenomenological mass symmetry among the lowest lying axial vector mesons that couple to the tensor charge; we adopt the spin-flavor symmetry characterized by an $SU(3)$ ⊗ $O(3)$ \cite{14} multiplet structure. Thus, the $1^{++}$ $h_1$ and $b_1$ mesons fall into a (35 ⊗ $L = 1$) multiplet that contains $J^{PC} = 1^{++}, 0^{++}, 1^{++}, 2^{++}$ states. This analysis enables us to relate the $a_1$ meson decay constant measured in $\tau^- \to a_1^- + \nu$ decay \cite{14}, $f_{a_1} = (0.19 \pm 0.03)$GeV$^2$, and the $a_1 NN$ coupling constant $g_{a_1 NN} = 7.49 \pm 1.0$ (as determined from $a_1$ axial vector dominance for longitudinal charge as derived in \cite{12} but using $g_A/g_V = 1.267$ \cite{13}) to the meson decay constants and coupling constants. We find

$$f_{b_1} = \frac{\sqrt{2}}{M_{b_1}} f_{a_1}, \quad g_{b_1 NN} = \frac{5}{3\sqrt{2}} g_{a_1 NN},$$  \hspace{1cm} (3)

where the $5/3$ appears from the $SU(6)$ factor $(1 + F/D)$ and the $\sqrt{2}$ arises from the $L = 1$ relation between the $1^{++}$ and $1^{-+}$ states. Our resulting value of $f_{b_1} \approx 0.21 \pm 0.03$ agrees well with a sum rule determination of $0.18 \pm 0.03$ \cite{14}. The $h_1$ couplings are related to the $b_1$ couplings via $SU(3)$ and the $SU(6)$ $F/D$ value,

$$f_{b_1} = \sqrt{3} f_{h_1}, \quad g_{b_1 NN} = \frac{5}{\sqrt{3}} g_{h_1 NN}$$  \hspace{1cm} (4)

For transverse polarized Dirac particles, $S^\mu = (0, S_T)$ these values, in turn, enable us to determine the isovector and isoscalar parts of the tensor charge,

$$\delta q^v = \frac{f_{b_1} g_{b_1 NN} \langle k_{1}^{2} \rangle}{\sqrt{2} M_N M_{b_1}^{2}}, \quad \delta q^s = \frac{f_{b_1} g_{b_1 NN} \langle k_{1}^{2} \rangle}{\sqrt{2} M_N M_{h_1}^{2}},$$  \hspace{1cm} (5)

respectively (where, $\delta q^v = (\delta u - \delta d)$, and $\delta q^s = (\delta u + \delta d)$). Transverse momentum appears in these expressions because the tensor couplings involve helicity flips that carry kinematic factors of 3-momentum transfer, as required by rotational invariance. The squared 4-momentum transfer of the external hadrons goes to zero in Eq. (4), but the quark fields carry intrinsic transverse momentum. This intrinsic $k_{1}$ of the quarks in the nucleon is determined from Drell-Yan processes and from heavy vector boson production.

3 Mixing

In relating the $b_1(1235)$ and $h_1(1170)$ couplings in Eq. \cite{4} we assumed that the latter isoscalar was a pure octet element, $h_1(8)$. Experimentally, the higher mass $h_1(1380)$ was seen in the $K + \bar{K} + \pi$'s
decay channel [3,13] while the $h_1(1170)$ was detected in the multi-pion channel [13,16]. This decay pattern indicates that the higher mass state is strangeonium and decouples from the lighter quarks – the well known mixing pattern of the vector meson nonet elements $\omega$ and $\phi$. If the $h_1$ states are mixed states of the $SU(3)$ octet $h_1(8)$ and singlet $h_1(1)$ analogously, then it follows that

$$f_{h_1(1170)} = f_{b_1}, \quad g_{h_1(1170)NN} = \frac{3}{5} g_{b_1NN},$$

(6)

with the $h_1(1380)$ not coupling to the nucleon (for $g_{h_1(1)NN} = \sqrt{2} g_{h_1(8)NN}$). These symmetry relations yield the results

$$\delta u(\mu^2) = (0.58 \text{ to } 1.01) \pm 0.20, \quad \delta d(\mu^2) = -(0.11 \text{ to } 0.20) \pm 0.20.$$  

These values are similar to several other model calculations: from the lattice; to QCD sum rules; the bag model; and quark soliton models [7]. The calculation has been carried out at the scale $\mu \approx 1$ GeV, which is set by the nucleon mass as well as being the mean mass of the axial vector meson multiplet. The appropriate evolution to higher scales (wherein the Drell-Yan processes are studied) is determined by the anomalous dimensions of the tensor charge [17] which is straightforward but slowly varying.

It is interesting to observe that the symmetry relations that connect the $b_1$ couplings to the $a_1$ couplings in Eq. (3) can be used to relate directly the isovector tensor charge to the axial vector coupling $g_A$. This is accomplished through the $a_1$ dominance expression for the isovector longitudinal charges derived in [12],

$$\Delta u - \Delta d = \frac{g_A}{g_V} = \frac{\sqrt{2} f_{a_1} g_{a_1NN}}{M_{a_1}^2},$$

(8)

Hence for $\delta q^v$ we have

$$\delta u - \delta d = \frac{5 g_A M_{a_1}^2}{6 g_V M_{b_1}^2 M_N M_{b_1}} \langle k^2 \rangle,$$

(9)

It is important to realize that this relation can hold at the scale wherein the couplings were specified, the meson masses, but will be altered at higher scales (logarithmically) by the different evolution equations for the $\Delta q$ and $\delta q$ charges. To write an analogous expression for the isoscalar charges ($\Delta u + \Delta d$) would involve the singlet mixing terms and gluon contributions, as Ref. [12] considers. However, given that the tensor charge does not involve gluon contributions (and anomalies), it is expected that the relation between the $h_1$ and $b_1$ couplings in the same $SU(3)$ multiplet will lead to a more direct result

$$\delta u + \delta d = \frac{3 M_{b_1}^2}{5 M_{h_1}^2} \delta q^v,$$

(10)

for the ideally mixed singlet-octet $h_1(1170)$. These relations are quite distinct from other predictions.

In conclusion, our axial vector dominance model with $SU(6)_W \otimes O(3)$ coupling relations provide simple formulae for the tensor charges. This simplicity obscures the considerable subtlety of the (non-perturbative) hadronic physics that is summarized in those formulae. We obtain the same order of magnitude as many other calculation schemes. These results support the view that the underlying hadronic physics, while quite difficult to formulate from first principles, is essentially a $1^{++}$ meson exchange process. Forthcoming experiments will begin to test this notion.

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