Firms Growth Dynamics, Competition and Power Law Scaling

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Abstract

We study the growth dynamics of the size of manufacturing firms considering competition and normal distribution of competency. We start with the fact that all components of the system struggle with each other for growth as happened in real competitive business world. The detailed quantitative agreement of the theory with empirical results of firms growth based on a large economic database spanning over 20 years is good. Further we find that this basic law of competition leads approximately a power law scaling over a wide range of parameters. The empirical data are in accordance with present theory rather than a simple power law.

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Recently, study of natural i.e. complex physical, biological and social systems has become a new area of physical investigations. In complex systems a large number of elementary interactions are taking place at the same time for large number of components and thereby make it difficult to make an exact analysis. Thus statistical methods are normally used to study these systems. Amaral et. al. [1], discussed firms growth dynamics based on the interaction between different units of a complex system, which is a normal practice in physics for simpler systems. However, we feel that in real business firms, this procedure is not adequate because a large number of elementary interactions are taking place at the same time including many with outside elements. In biological and social sciences, growth and evolution processes are normally explained qualitatively through competition. We feel that in describing complex systems, competition is more relevant parameter compared to internal interactions particularly in social sciences. It is therefore interesting to study quantitatively economical complex systems in the spirit of Darwin´s classical theory of biological evolution: Survival for the fittest. This will also provide an alternative thinking in understanding of complex systems. The present model may also be important for some other complex physical, biological and economical systems.

We start with the fact that all components of a system struggle i.e. compete with each other for growth, like all firms in economical world compete with each other to grow. The same is the case between different species or groups in biological, political, religious or social circle. The components which are better than their competitors will grow, while other shrink till reach to a equilibrium state where they are equal to an average competitor of their class. We formulate our problem for the growth of the manufacturing firms, because there is plenty of quantitative data [1-5] and the term competition and competency is well understood. The problem is also important due to considerable recent interest in economics in developing a richer theory of the growth dynamics of a firm [5-10]. The present model is interesting as it is close to real cut-throat competitive business.

In economics, what is commonly called “theory of the firm” is actually a theory of a business unit [7,10]. Thus the standard model of the firm does not yield any prediction about the distribution of the size of actual, multi-divisional firms or their growth rates. In the present paper, we explore the stochastic properties of the dynamics of firm growth in line with the work done by Simon [8] and Lucas [9]. Lucas suggests that the distribution of a firm size depends on the distribution of its managerial ability in the economy rather than on the
factors that determine the size in the conventional theory of the firm \[7,10\]. As all firms are competing with each other and political, economical, and technological factors are same for all, we also believe that in long terms, the managerial ability of a firm is a determining factor in its growth and the managerial ability is basically determined through its chief executive officer (CEO). The management ability means the person’s effective ability to run a firm. It includes the ability of a person to take (a) maximum advantages in given circumstances (b) to correctly predict future business happenings, (c) courage to make necessary changes and take risk and (d) maintain complete harmony in whole administration. We obtain the statistical probability distribution of the growth rate of firms of certain size and the fluctuation in the growth rates, measured by the width of the distribution, scale only approximately as a power law with firm size. Finally we discuss the results with available data.

A business firm is a very complex system. We make the following assumptions to have a simplified version of the whole system to study the statistical behavior of the growth rate of the firms. However for the growth of a particular industry in a particular segment, other factors like economical, political and technological will also be important.

(i) We consider the sales of a firm to define its size, although other parameters like number of employee or assets can also be used \[3\]. The observed sales distributions of firms are very close to log-normal distribution \[5,6\] except for very large firms. In general, we expect a normal distribution of a variable. We therefore, define size of a firm equal to natural logarithm of its sales \(S\).

\[
M = \ln(S)|_{\text{Equilibrium}}
\]  

(iii) When the management ability \(M\) of a CEO is more than what is needed to handle the firm under his control \(\ln(S)\), the firm grows. As per common experience of growth rate of a firm under same CEO for long time \[11\] and observed probabilistic distribution of the
growth rate of the firms, we consider for mathematical simplicity that the growth rate \( (\gamma) \) of the firm is directly proportional to the relative management ability \( (R) \) of its CEO, i.e.

\[
\gamma = \ln\left(\frac{S_1}{S}ight) = kR
\]  

The relative management ability of a CEO, is the difference of his management ability and management ability needed to handle the firm:

\[
R = |M - \ln(S)|
\]  

\( k \) is a constant of the system. \( S_1 \) is the sales of the firm in the following year. Under this assumption, the optimum size of a firm is determined through management ability of its CEO \( (S_{optimum} = \exp(M)) \) and the firm approaches to this size [11]. In Figure 1, we have shown the variation of annual sell of a firm with initial sell \( 10^6 \) and is under a CEO, who can handle satisfactorily a firm of annual sell \( 10^8 \).

(iii) We consider that the management ability of people in general \( (M_G) \) is having a normal distribution [12] with mean \( \overline{M_G} \) and standard deviation \( \sigma_G \). Thus:

\[
P_G(M) = \frac{1}{\sqrt{2\pi}\sigma_G} \exp\left(-\frac{(M - \overline{M_G})^2}{2\sigma_G^2}\right)
\]  

where \( P_G(M) \) is the probability of a person in general to have management ability equal to \( M \). We have shown it in Figure 2a.

(iv) Let us consider firms of annual sales \( S_0 \). In general, they have their CEO of different management ability. The firms under CEO with management ability \( M_0 \), where \( M_0 = \ln(S_0) \) will be in equilibrium. We consider that this is also means of the management ability of CEO’s of these firms. Basically a person is hired as CEO of a firm because he (a) is competent i.e. his management ability is either equal or better than average of his group, (b) is having strong influence on the share holders. We consider that political factors are the same for all and all competent people have equal chance to become CEO, thus the distribution of management ability of CEO of these firms for positive segment is in same way as people are available.. For negative segment \( (M < M_0) \), we consider the distribution as mirror image of the positive segment as most of the people will be rejected on merit ground. This will
give a symmetrical growth dynamics at zero growth rate. Further the over all distributions of firm size is invariant with time, which is more or less true. The probability of a person to be selected as CEO must decrease with his incompetency. For big firms, this probability decreases more rapidly as they are more mature and exigent. This will give truncated normal distribution with mean management ability equal to $M_0$. In Figures 2b, we have drawn distribution of relative management ability of the CEO through Equation (3).

Under these assumptions and after normalization, the probability distribution of management ability ($M$) of CEO of firms of the size $S_0$ ($P(M/S_0)$) for $M \geq M_0$ is given by:

$$P(M/S_0) = \frac{P_G(M)}{2 \int_{M_0}^{\infty} P_G(M) dM}$$

(5)

The probability distribution for relative management ability ($P(R/S_0)$) is thus given by:

$$P(R/S_0) = \frac{P_G(M_0 + R)}{2 \int_{0}^{\infty} P_G(M_0 + R) dR}$$

(6)

Using equations (3), (6) and normalization condition, the probability distribution of the growth rate of firms of size $S_0$ ($P(\gamma/S_0)$) and standard deviation of the growth rate of these firms ($\sigma(S_0)$) are given by:

$$P(\gamma/S_0) = \frac{P_G[M_0 + \frac{\gamma}{k}]}{2k \int_{0}^{\infty} P_G(M_0 + R) dR}$$

(7)

$$\sigma^2(S_0) = \frac{\int_{0}^{\infty} P\left(\frac{\gamma}{S_0}\right)\gamma^2 d\gamma}{\int_{0}^{\infty} P\left(\frac{\gamma}{S_0}\right)d\gamma} = k^2 \frac{\int_{0}^{\infty} P_G(M_0 + R).R^2 dR}{\int_{0}^{\infty} P_G(M_0 + R)dR}$$

(8)

In Figure 2c, we have shown distribution of growth rate using Equation (7).

We compare our theory with growth rate of all publicly traded manufacturing companies of US. in the 1994 Compustat database with standard industrial classification index of 2000-3999. The distribution represents all annual growth rates observed in the 19 year period.
We consider $k = 0.6$, $\bar{M}_G = 12$ and $\sigma_G = 1.3$ for drawing theoretical curves. $k$ is chosen to match the magnitude of the growth rate while $\bar{M}_G$ and $\sigma_G$ are chosen to give best fit in standard deviation vs. size. The value of $\bar{M}_G = 12.0$ signify that about 25% of people have capacity to run a pre-established manufacturing firm of annual sell of about $5.10^5$ dollars, after proper training and experience, which seems to be reasonable as this is the size of small family businesses. In Figure 3a, we compare the probability density of the growth rate for different initial sales while in Figure 3b, we compare the standard deviation as a function of the initial sales. The straight line is a guide for the eye with slope $\simeq 0.15$. We obtained all the four theoretical curves with a single set of two chosen parameters. The agreement is reasonably good for all the curves.

Power law scaling [13,14] is one of the main feature of complex systems and has been speculated in many physical [15-18], chemical [19], economical [20-23], biological [24,25], geophysical [26] and others [27,28] systems. It has also been speculated in firm growth dynamics. The present theory approximately leads to power law scaling. It is interesting to observe that the general behavior of the empirical results is also close to present theoretical prediction rather than any power law scaling (straight line fitting). Power law scaling is only a crude approximation for these results for a interesting range of parameters.

In the present theory, the probability of small positive or negative growth is more than zero growth for firms of annual sell less than $10^5$ dollars. Normally the firms of this size either grow to become competitive or soon get out of the market.

The standard deviation of management ability of people in general is 1.3, while the standard deviation of firms size for these data is 2.72 [5]. Thus the number of competent CEO available per firm decreases very rapidly with increase of firm size. This means that for small firms, many competent people are available and political considerations are more important for selection. As a firm grow, merit consideration become more and more important. For firms of annual sell more than $10^8$ dollars, it become almost necessary to have many quasi independent executives and decisions have to be taken collectively. This is what happened normally. The number of very large firms is less than what is given by log-normal distribution [5]. Perhaps this happened because of shortage of highly competent executive to run these firms. In view of these comments, the present theory is more appropriate for firms with annual sell below $10^8$ dollars per year. For firms with sell larger than $10^8$ dollars/year, it is necessary to consider effective management ability of the whole management board, where
perhaps theory proposed by Amaral et. al. [1] is also useful.

In Figure 4, we draw the variation of standard deviation of the growth rate as a function of initial sales for (a) various values of $M_G$ and (b) various values of $\sigma_G$. We consider $M_G = 12.0$, $\sigma_G = 1.3$, if it is not variable. The power law scaling behavior is approximately maintained in all these cases. This approximate power law scaling behavior is not very sensitive to the value of parameters under interested experimental range. This removes the necessity of self-tuning of parameters controlling the dynamics of the system to their critical values as is needed in critical behavior [29-31]. Perhaps, competition and normal distribution of competency is behind the approximate scaling behavior in diverse fields, particularly in social and biological sciences, independent of the microscopic details.
References

[1] L. A. N. Amaral, et. al., Phys. Rev. Lett. 80, 1385(1998)
Y. Lee, et. al., Phys. Rev. Lett. 81, 3275(1998)

[2] M. H. R. Stanley, Nature 379, 804(1996)

[3] L. A. N. Amaral, et. al., J. Phys. I France, 7, 621(1997)

[4] Y. Lee, et al., Phys. Rev. Lett. 80, 1385(1998)

[5] M. H. R. Stanley, et. al., Economics Letters, 49, 453(1995)

[6] R. Gibrat, Les Inégalités Economiques, (Sirey, Paris 1931)

[7] A. Golan, Adv. Econometrics, 10, 1(1994)

[8] Y. Ijiri, H. A. Simon, Skew Distributions and the Size of business Firms (North Holland, Amsterdam 1977)

[9] R. Lucas, Bell J. Econ., 9, 508(1978)

[10] H. R. Varian, Microeconomics Analysis, (Norton, New York 1978)

[11] P. A. Samuelson and W. D. Nordhaus, Economics, 13th ed. (Mc. Grow-Hill, New York 1989)

[12] W. Feller, An Introduction to Probability Theory and Its Applications (Wiley, New York 1971)

[13] B. B. Mandelbrot, The Fractal Geometry of Nature (Freeman, New York 1982)

[14] U. Frish, M.F. Shlesinger, and G Zaslavasky, Lévy Flights and Related Phenomena in Physics (Springer-Verlag, Berlin 1994)

[15] T. H. Solomon, E. R. Weeks, and H. L. Swinney, Phys. Rev. Lett. 71, 3975(1993)

[16] B. Chabaud, et. al., Phys. Rev. Lett. 73, 3227(1994)

[17] C. Menevau, K. R. Streenivasan, J. Fluid Mech. 224, 429(1991)
[18] M. Nelkin, Adv. Phys. 43, 143 (1994)

[19] A. Ott, J. P. Bouchaud, D. Langevin, and W. Urbach, Phys. Rev. Lett. 65, 2201(1990)

[20] R. N. Mantegna, and H. E. Stanley, Nature 376, 46(1995)

[21] H. M. Gupta, J. R. Campanha, J. R., Physica A 275, 531(2000)

[22] T. Lux, M. Marchesi, Nature 397, 498(1999)

[23] G. Ghashghaie, et. al., Nature 381, 767(1996)

[24] C. K. Peng, al., Phys. Rev. Lett. 70, 1343(1993)

[25] J. B. Bassingthwaighte, L. S. Liebovitch, and B. J. West, Fractal Physiology (Oxford Univ. Press, New York 1994)

[26] Z. Olami, H. J. S. Feder, and K. Christensen, Phys. Rev. Lett. 68, 1244(1992)

[27] C. Tsallis, Brazilian Journal of Physics, 29, 1(1999)

[28] H. E. Stanley, Rev. Modern Physics, 71, S358(1999)

[29] P. Bak, C. Tang, and K. Wiesenfeld, K., Phys. Rev. Lett. 59, 381(1987)

[30] T. Vicsek, Fractal Growth Phenomena, World Scientific, Singapore 1992

[31] P. Bak, How Nature Works, Oxford University Press, Oxford (1997)
Figure Captions

Figure 1: Variation of sell of a firm with initial sell $10^6$ and is under a CEO who have management ability to run a firm of sell $10^8$.

Figure 2: Our theoretical model (a) probability density distribution of management ability ($P_G(M) \text{ vs. } M$) for public in general - a normal distribution. (b) Probability density distribution of relative management ability ($R = (M - \ln(S))$ for CEO of firms of sales $S_0$ ($P(R/S_0) \text{ vs. } R$). (c) Probability density distribution of the growth rate for firm of sales $S_0$ ($P(\gamma/S_0) \text{ vs. } \gamma$).

Figure 3: (a) Probability density distribution of the growth rate ($P(\gamma/S_0) \text{ vs. } \gamma$) for all publicly traded U.S. manufacturing firms. The solid lines are theoretical curves with $k = 0.6$, $\bar{M}_G = 12$ and $\sigma_G = 1.3$. (b) Standard deviation of the growth rates as a function of the initial sales ($\sigma(S_0) \text{ vs. } S_0$). The solid lines is the theoretical curve with same parameters, while broken line is a guide for eye with slope $\approx 0.15$.

Figure 4: Standard deviation of the growth rate as a function of the initial sales ($\sigma(S_0) \text{ vs. } S_0$) for different values of (a) mean management ability of people in general $\bar{M}_G$ (b) Standard deviation of the management ability of the people in general $(\sigma_G)$.
