Boiling of nuclear liquid in core-collapse supernova explosions

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We investigate the possibility of boiling instability of nuclear liquid in the inner core of the proto-neutron star formed in the core collapse of a type II supernova. We derive a simple criterion for boiling to occur. Using this criterion for one of best described equations of state of supernova matter, we find that boiling is quite possible under the conditions realized inside the proto-neutron star. We discuss consequences of this process such as the increase of heat transfer rate and pressure in the boiling region. We expect that taking this effect into account in the conventional neutrino-driven delayed-shock mechanism of type II supernova explosions can increase the explosion energy and reduce the mass of the neutron-star remnant.

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I. INTRODUCTION

Detection of electron antineutrinos from SN 1987A (see [1, 2, 3]) confirmed the previous theoretical ideas about neutrinos playing crucial role in the core collapse of type II supernovae. According to well established estimates, only about one percent of the gravitational binding energy (or \((1.4 \pm 0.4) \times 10^{51}\) erg for SN 1987A; see [4]) is released in the thermal and kinetic energies of the expanding ejecta; the remaining part is carried away by different types of neutrinos [5], which are known to be trapped effectively within nuclear matter during the last stages of core collapse (see [6] and references therein). Therefore, initially it did not seem hard to explain the observed values of kinetic energy of the expanding ejecta using neutrino-nucleon interaction as an effective channel of energy transfer from the emitted neutrinos to nuclear matter [7, 8]. However, concrete realization of this mechanism of supernova explosions encountered with great difficulties. Thus, it was realized that the prompt shock wave, generated during the core collapse, fades away at the time scale of several milliseconds, losing its energy on nuclear dissociation [9, 10] and on the radiation of \(\nu\bar{\nu}\)-pairs [11]. As a result, in simulations, the shock starts to move inward, pushed down by the infalling matter, leading to the formation of a black hole rather than to explosion.

Considerable efforts were made to remedy this situation. Bethe & Wilson [12] showed that the shock wave could be revived about 100 msec after the core bounce if neutrino luminosity were sufficiently enhanced. The way of such enhancement of neutrino luminosity was opened after neutrino diffusion inside the center of the proto-neutron star (PNS) was discovered [13] and found to be convectively unstable (see [14, 15] and references therein). Nevertheless, numerical simulations of the neutrino-driven shock revival still yield contradictory results. While explosions with kinetic energy up to \(1.72 \times 10^{51}\) erg are obtained in [16] (\(0.5 \times 10^{51}\) erg in [15], \(10^{50}\) erg in [17], \(0.94 \times 10^{51}\) erg in [18]), there is also a number of simulations where explosions are not observed at all [19, 20].

This inconsistency between the observational data and the results of core-collapse numerical simulations motivated some researches to look for alternative mechanisms. The magnetorotational mechanism, proposed by Bisnovatyi-Kogan [21], can produce explosions with energies up to \(0.61 \times 10^{51}\) erg at the time scale of \(\sim 0.5\) sec after the core bounce [22, 23]. The acoustic mechanism, developed by Burrows et al. [24], also produces explosion; although its energy is rather uncertain in their numerical simulations, it develops for a time
range of several hundred milliseconds. Thus, even with new mechanisms and effects taken into account, the resulting energy release is marginally short of the observed values.

One can conclude that all mechanisms of core collapse require some additional engine to provide the observable explosions. Such a new engine is the subject of the present paper. By analogy with the work of Bethe et al. [13], we consider an additional type of transport of nuclear matter in the core which is different from diffusion and convection. It occurs in the form of boiling, i.e., first order transition between different phases of nuclear matter.

We start with the observation that supernova matter at the inner core can exist in several phases and their mixtures [25, 26, 27, 28, 29, 30]. Among them are the following (in the order of increasing density):

- the spherical nuclei phase
- the phase consisting of elongated nuclei (often called pasta)
- the slab-like nuclei
- the phase with cylindrical holes
- the phase with charged microscopic bubbles, or chee ed phase (we will use this last term below not to confuse it with macroscopic bubbles of which we will speak later)
- the homogeneous supernova matter

It is natural to expect that several phase transitions can occur during the evolution of nuclear matter during as well as after the core collapse. Numerical simulations of the nuclear-matter phase transitions in supernovae were usually aimed at determining the thermodynamic properties at the pre-bounce stage of collapse, and were needed to understand the development of the prompt shock wave. Such transitions taking place in a rapidly changing environment during collapse can be called “short-term” phase transitions. In our paper, we discuss what we call “long-term” phase transitions, which occur in nuclear matter after the bounce under the condition of relative mechanical and local thermodynamic equilibrium. The main goal of this paper is to examine the potential importance of such “long-term” phase transitions on the dynamics of supernova explosion.

In the “normal” case of mechanical and local thermodynamic equilibrium, different phases occupy the corresponding radial shells of the PNS, and this spatial phase picture evolves
continuously and relatively slowly in time. However, if heating of the nuclear matter (which is mainly due to diffusion of neutrinos) is sufficiently strong and inhomogeneous, it can lead to a condition similar to that in an ordinary teakettle. Specifically, a bulk of particular phase can overheat and become unstable with respect to phase transition, and small bubbles of the lighter phase can spontaneously appear in its volume. The bubbles will grow and, due to the Archimedean force, rise upwards. Henceforth, such a process is called boiling by analogy. In this paper, we argue that this process can provide more efficient mechanism of heating the outer parts of the PNS (compared with the neutrino-diffusion and convection mechanisms) and generate additional pressure wave.

The paper is organized as follows. In Sec. II we give the basic thermodynamic description of the co-existence of various phases in supernova matter after the bounce. In Sec. III we derive the criterion of boiling to occur, and, in Sec. IV we provide our numerical estimates for the values involved in this criterion using the tabulated equation of state from [31]. In Sec. V we consider a simple model of the boiling mechanism and derive numerical estimates characterizing the efficiency of this process. We summarize and discuss our results in Sec. VI.

II. PHASE EQUILIBRIUM

Just after the core bounce, the inward movement of the inner core significantly decreases, and the prompt shock wave moves outwards losing its energy mostly on nuclear dissociation and radiation of $\nu\bar{\nu}$-pairs. The material within and around the PNS approaches a state of mechanical equilibrium (while the velocities of the convective motion are much smaller than the appropriate first cosmic velocities). In contrast with the rapid collapse during the pre-bounce stage of contraction, local thermal equilibrium is a good approximation for the post-bounce stage.

Because of the radial density gradient, the PNS at this stage has onion-like structure. The density of its inner core is higher than the nuclear saturation density, and the homogeneous matter phase is thermodynamically preferred. In the outer layers of the PNS, the density is much less than the nuclear saturation density, and matter exists in the form of ordinary nuclei. A number of intermediate phases can exist between these two shells. Adjacent phases are separated by the surfaces of coexistence. Here we derive a simple condition of coexistence of phases applied to the situation under consideration.
We can consider a phase transition characterized by fixed pressure $p$, temperature $T$, baryon number $B$, electric charge $C$ and lepton number $L$. The conservation laws of the last three quantities imply the existence of the corresponding chemical potentials: $\mu_B$, $\mu_C$ and $\mu_L$. We can determine their values from the chemical potentials of the supernova matter components (we suppose that the supernova matter consists only of neutrons, protons, electrons and electron neutrinos):

$$
\mu_n = \mu_B, \quad \mu_p = \mu_B + \mu_C, \quad \mu_e = \mu_L - \mu_C, \quad \mu_\nu = \mu_L.
$$

(1)

Since the number of particle species is greater than the number of independent charges, we have one more relation on the chemical potentials (the so-called beta-equilibrium condition):

$$
\mu_p + \mu_e = \mu_n + \mu_\nu.
$$

(2)

For fixed values of $p$ and $T$, our thermodynamic system tends to a state with a minimum value of the Gibbs free energy (see, e.g., [32])

$$
\Phi = \sum_i \mu_i N_i = \mu_B B + \mu_C C + \mu_L L.
$$

(3)

Since we are interested only in electrically neutral phases, we have

$$
C \equiv 0,
$$

(4)

and the second term in (3) vanishes. The third term, $\mu_L L$, does not change during the phase transition because the electron neutrinos interact with nuclear matter very weakly.

Finally, the condition of equilibrium between phases 1 and 2 is

$$
\mu_{1n} (p_0, T_0, \mu_\nu_0) = \mu_{2n} (p_0, T_0, \mu_\nu_0),
$$

(5)

where the subscript “0” marks the values of the parameters right on the interface between the two phases.

### III. CRITERION OF BOILING

Local thermal equilibrium described in the previous section is continuously disturbed by the process of diffusion of electron neutrinos away from the central region of the PNS. This causes inhomogeneous heating of the bulk of nuclear matter. If this heating is sufficiently
strong, it can lead to an overheat of a particular phase, which may result in its boiling — emergence of bubbles of the adjacent lighter phase in its bulk. These bubbles will then raise and grow, effectively transferring heat and lepton number. Boiling is a particular case of a non-equilibrium first order phase transition. In this section, we derive a necessary condition of boiling in the form of a relation on the parameters of the supernova matter.

According to Eq. (5), in the process of external heating, a heavier phase 1 becomes metastable with respect to its transition to a lighter phase 2 if the condition is reached such that

$$\mu_{1n}(p_0 + \delta p, T_0 + \delta T, \mu_{\nu 0} + \delta \mu_{\nu}) > \mu_{2n}(p_0 + \delta p, T_0 + \delta T, \mu_{\nu 0} + \delta \mu_{\nu}),$$

where the positive increments of the thermodynamic variables correspond to their radial gradients. From this, we obtain the condition

$$\left[ \frac{\partial \mu_{2n}}{\partial T} - \frac{\partial \mu_{1n}}{\partial T} \right]_{p, \mu_{\nu}}dT_{\mu_{\nu}} + \left[ \frac{\partial \mu_{2n}}{\partial \mu_{\nu}} - \frac{\partial \mu_{1n}}{\partial \mu_{\nu}} \right]_{p, T}d\mu_{\nu} + \left[ \frac{\partial \mu_{2n}}{\partial p} - \frac{\partial \mu_{1n}}{\partial p} \right]_{T, \mu_{\nu}}d\rho_{m} > 0.$$  

(7)

In the important particular case of hydrostatic equilibrium, we have

$$\frac{dp}{dr} = -\rho_{m}g,$$

where \(\rho_{m}\) is the mean density, and \(g\) is the local acceleration of free fall. Substituting this into (7), we obtain

$$\left[ \frac{\partial \mu_{2n}}{\partial T} - \frac{\partial \mu_{1n}}{\partial T} \right]_{p, \mu_{\nu}}dT_{\mu_{\nu}} + \left[ \frac{\partial \mu_{2n}}{\partial \mu_{\nu}} - \frac{\partial \mu_{1n}}{\partial \mu_{\nu}} \right]_{p, T}d\mu_{\nu} + \rho_{m}g > 0.$$  

(9)

IV. NUMERICAL ESTIMATES

It remains to check whether the condition of boiling derived in the previous section can be realized in the usual supernova core-collapse. For this purpose, we estimate the values of the partial derivatives in (9) using the numerical simulations of the EoS of supernova matter given in [31].
TABLE I: The parameters from [31] used in the estimate of the partial derivatives in (9).

The authors of [31] tabulate all essential parameters describing the supernova matter in three phases: nuclei, cheesed phase and homogeneous matter. In Table I, we show the parameters which are required in our analysis. Because the required values of derivatives are not listed in [31], we try to obtain them from interpolation. Namely, we use the finite differences:

$$\mu_n(p_2, T_2, \mu_\nu_2) - \mu_n(p_1, T_1, \mu_\nu_1) \approx \frac{\partial \mu_n}{\partial p}(p_2 - p_1) + \frac{\partial \mu_n}{\partial T}(T_2 - T_1) + \frac{\partial \mu_n}{\partial \mu_\nu}(\mu_\nu_2 - \mu_\nu_1),$$

(10)

neglecting the higher derivatives. For each phase, we solve a system of three equations for three unknown variables. The numerical values obtained in such a manner should be considered as estimates. They are presented in Table II.

To check whether the conditions of boiling are realized, we use the conventional values of the other parameters. Numerical simulations (see, e.g., [19]) give the following estimates:

$$g \approx 1.0 \times 10^{14} \text{ cm} \cdot \text{sec}^{-2}, \quad \frac{d\mu_\nu}{dr} \approx -(10-20) \text{ MeV} \cdot \text{km}^{-1}, \quad \frac{dT}{dr} < 0.$$  

(11)

Using this data together with Table II we obtain the following conditions for boiling:
Phase \( \left( \frac{\partial n}{\partial p} \right)_{T,\mu \nu}, \text{fm}^3 \), \( \left( \frac{\partial \mu_v}{\partial T} \right)_{p,\mu \nu}, \text{MeV} \), \( \left( \frac{\partial \mu_v}{\partial \mu_v} \right)_{p,T} \)  

| Phase       | \( \left( \frac{\partial n}{\partial p} \right)_{T,\mu \nu}, \text{fm}^3 \) | \( \left( \frac{\partial \mu_v}{\partial T} \right)_{p,\mu \nu} \) | \( \left( \frac{\partial \mu_v}{\partial \mu_v} \right)_{p,T} \) |
|-------------|-------------------------------------------------|-----------------------------|-----------------------------|
| Nuclei      | -1.353                                          | -2.626                      | 0.125                       |
| Cheesed     | 0.666                                           | 5.182                       | -0.201                      |
| Homogeneous | 2.87                                            | 1.289                       | -0.0317                     |

TABLE II: The results for the partial derivatives.

- for the transition between the nuclear and cheesed phase \( (\rho_m \approx 0.8 \times 10^{14} \text{ g} \cdot \text{cm}^{-3}) \),
  \[ \frac{dT}{dr} < - (0.4 - 0.8) \text{ MeV} \cdot \text{km}^{-1}; \]  

- for the transition between homogeneous matter and cheesed phase \( (\rho_m \approx 1.6 \times 10^{14} \text{ g} \cdot \text{cm}^{-3}) \),
  \[ \frac{dT}{dr} > - (1.0 - 1.5) \text{ MeV} \cdot \text{km}^{-1}. \]  

The difference in the inequality signs in estimates (12) and (13) is connected with the difference in the signs of the coefficients of the temperature gradients in (7) or (9) for the two phase transitions under consideration. Note that conditions (12) and (13) are overlapping and complementary, so it is very likely that one of them is satisfied. We, therefore, can expect that boiling of nuclear matter can take place inside the supernova core. However, this only demonstrates the possibility of principle, calling for additional thorough investigation of this issue.

V. A MODEL

In this section, we construct a simplified model of phase transition between a heavier phase 1 and a lighter phase 2, which, for definiteness, we take to be the cheesed phase and the phase of nuclei, respectively.

We assume spherical symmetry of the PNS, so that the cheesed phase is located in a spherical shell with radial coordinate from \( R - H \) to \( R \). The free-fall acceleration on the upper boundary of this region is equal to

\[ g = \frac{GM}{R^2} = 1.33 \times 10^{14} \left( \frac{M}{M_\odot} \right) \left( \frac{10 \text{ km}}{R} \right)^2 \text{ cm} \cdot \text{sec}^{-2}, \]  

where \( M \) is the total mass inside the sphere of radius \( R \).
We can estimate $H$ using the condition of hydrostatic equilibrium:

$$\rho g H \approx \Delta p,$$

(15)

where $\Delta p$ is the dimension of the pressure interval in which the cheesed phase exists. According to Table I, this pressure interval is approximately equal to $0.83 \text{ MeV} \cdot \text{fm}^{-3}$. We thus have

$$H \approx \frac{\Delta p}{\rho g} \approx 1.0 \times \left( \frac{\Delta p}{0.83 \text{ MeV} \cdot \text{fm}^{-3}} \right) \left( \frac{10^{14} \text{ g} \cdot \text{cm}^{-3}}{\rho} \right) \left( \frac{1.33 \times 10^{14} \text{ cm} \cdot \text{sec}^{-2}}{g} \right) \text{ km}. \quad (16)$$

If we assume that boiling takes place in the whole volume of phase 1, then the total mass of the boiling matter can be estimated as

$$M_b \approx 4\pi \rho R^2 H \lesssim 6.1 \times 10^{-2} \left( \frac{\Delta p}{0.83 \text{ MeV} \cdot \text{fm}^{-3}} \right) M. \quad (17)$$

The densities of phases 1 and 2 are related by $\rho_1$ and $\rho_2 = \rho_1(1 - \epsilon)$, where $\epsilon \ll 1$. According to [34], $\epsilon$ is equal to 0.1 for the phase transition between the cheesed phase and homogeneous matter phase, and to 0.2 for the transition between nuclei and cheesed phase. Lower estimates for $\epsilon$ are present in [31]: it equals to 0.07–0.08 for both phase transitions if the entropy per baryon (which is conserved in the simulations) $S/A = 1.0$; for $S/A = 1.5$, the value $\epsilon = 0.05$ is obtained for the transition between nuclei and cheesed phase, and $\epsilon = 0.01$ for the transition between cheesed phase and homogeneous matter. To be conservative, we use the lowest estimate in this paper, namely, $\epsilon = 0.01$.

The maximum acceleration which can be reached by a raising bubble (neglecting the liquid resistance) is

$$a_{\text{max}} = \frac{\epsilon GM}{R^2}. \quad (18)$$

The maximum velocity that can be reached by the bubble is then

$$v_{\text{max}} \sim \left( \frac{2\epsilon \Delta p}{\rho} \right)^{1/2} = 5100 \sqrt{\left( \frac{\epsilon}{10^{-2}} \right) \left( \frac{\Delta p}{0.83 \text{ MeV} \cdot \text{fm}^{-3}} \right) \left( \frac{10^{14} \text{ g} \cdot \text{cm}^{-3}}{\rho} \right)} \text{ km} \cdot \text{sec}^{-1}. \quad (19)$$

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1 Recent simulations [33] show that the total mass of different phases between nuclei and homogeneous nuclear matter (collectively called “pasta phases” in [33]) inside the supernova just before the bounce can amount to $0.13–0.30 M_\odot$. This estimate somewhat differs from [17] because, before the bounce, one deals with small pressure gradients in an (almost) free-falling matter, while, after the bounce, we have matter close to hydrostatic equilibrium with its large pressure gradients. It is possible, in principle, that the pre-bounce boiling, if it occurs, can affect the propagation of prompt shock wave, but we do not consider this issue here.
Matter in the boiling volume will convectively move in both directions. We can expect that roughly half of matter moves upwards (the bubbles and the surrounding matter) and the other half moves downwards with the same average velocity (from the momentum conservation). Therefore, the “convective boiling” overturn will be established. We can try to estimate the maximum efficiency of this overturn.

The first effect to be discussed is heat transfer. If the bubbles fill half of the boiling volume, the heating rate at the surface of the boiling layer is

$$\dot{Q}_{\text{max}} \approx \frac{1}{2} \times 4\pi R^2 v_{\text{max}} q$$

$$= 7.7 \times 10^{52} \left( \frac{R}{10 \text{ km}} \right)^2 \left( \frac{v_{\text{max}}}{5100 \text{ km} \cdot \text{sec}^{-1}} \right) \left( \frac{q}{0.015 \text{ MeV} \cdot \text{fm}^{-3}} \right) \text{erg} \cdot \text{sec}^{-1}.$$  \hspace{1cm} (20)

Here, $q$ is the specific volume heat of evaporation, and its value is estimated as $q = 0.015 \text{ MeV} \cdot \text{fm}^{-3}$ from [25]. The value (20) is comparable to the neutrino luminosity. This is, in principle, the upper estimate of heating rate which corresponds to maximal workload of the “boiling machine.” In reality, the boiling heat transfer should work together with diffusion and/or convection below and above the boiling shell, increasing the net heat transfer rate. This should lead to an increase in pressure behind the shock, providing more efficient conditions for shock revival.

The second effect is the momentum transfer. The maximum mechanical pressure that the bubbles can exert is estimated as

$$p_{\text{max}} \sim \rho v_{\text{max}}^2 = 2\epsilon \Delta p = 1.7 \times 10^{-2} \text{ MeV} \cdot \text{fm}^{-3},$$ \hspace{1cm} (21)

where the numerical value corresponds to the cheesed phase. This is much smaller than $\Delta p$ since $\epsilon \ll 1$; therefore, including this contribution to pressure does not seriously affect our previous calculation. But it can provide an additional barrier for the infalling matter to reach the inner core, reducing the final mass of the neutron star.

We should admit that most of the above estimates represent upper limits corresponding to the maximal workload of the “boiling machine.” In a subsequent paper, we will discuss these processes in greater detail.

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2 This can be compared to an electric circuit with a series of resistances. As one of the resistances in the series is shortened out, the total current increases.
VI. SUMMARY AND CONCLUSIONS

In this paper, we have demonstrated the possibility of boiling of nuclear liquid in the supernova core after the bounce. If it occurs, it can lead to effects increasing the efficiency of the neutrino-driven mechanism of supernova explosions. Among these effects are the following:

- The increase of heat transfer rate from the inner core to the neutrinosphere. This increases the mean neutrino energy, making the delayed-shock mechanism more efficient.

- The increase of pressure in the boiling region. It provides an additional barrier between the infalling matter and the inner core, thereby reducing the mass of the neutron-star remnant.

We expect that taking into account the effect of boiling in the conventional delayed-shock/acoustic mechanism is important and will enable one to explain the energetics of the supernova explosions in a more simple and self-consistent way.

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