Far-field subwavelength imaging by harnessing the single-mode resonance and sparsity

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Abstract
Although far-field superlenses and resonant metalenses provide a way to obtain the far-field subwavelength imaging through the resonant excitation of multiple surface wave modes, the high sensitivity of these resonant modes to the absorption loss hinders their practical applications. To break this limitation, only a single resonant mode which survives the absorption loss is chosen for imaging, where a reconstruction algorithm based on the sparsity is adopted to compensate for the reduction of the number of degrees of freedom. An experiment is carried out to verify a far-field subwavelength imaging of two home-made sources, and it is found that the two imaged sources can be well resolved by using multiple frequencies near the fifth resonant mode.

1. Introduction
To break the imaging resolution limitation constrained by half the wavelength of bulk waves, evanescent waves have been picked up by point-by-point scanning [1] or amplified in amplitudes by a superlens [2]. Owing to evanescent waves being bounded at the interface, a probing sensor should be in the near-field of the imaged object or superlens. Therefore, various far-field subwavelength imaging schemes have been developed to overcome this shortcoming.

The anisotropic metamaterial device named as ‘magnifying hyperlens’ was fabricated to magnify subwavelength objects by gradually converting evanescent components into propagating waves [3–7], resulting super-resolution imaging directly measured by a probing sensor. The scattering of evanescent waves can also be used to unravel the subwavelength world [8–10], in which how to realize a high efficient conversion of evanescent waves into propagating waves is quite critical. Usually, a strategy is to excite the surface wave modes which are being converted into propagating waves through scattering. The far-field superlenses [11–14] were proposed to measure the spatial spectrum of imaged objects by using multiple lenses working at a single frequency or multiple surface wave modes excited at different frequencies. The time reversal technique provided another way to achieve the far-field subwavelength imaging by manipulating the polychromatic information through resonant metalenses [15–18]. In addition, other approaches [19–23] based on the conversion of evanescent waves into propagating waves were also proposed, for example, the imaging by deep learning.

As is stated above, a super-resolution imaging can be achieved if the evanescent components of imaged objects are picked up. However, by the superoscillatory metamaterial [24, 25] or the sparsity of imaged objects [26–28], a super-resolution imaging can also be obtained without evanescent waves. And the sparsity based computational imaging via compressed sensing has been developed with the aid of metamaterials to reduce the number of sensors [29–33], where the metamaterial plays a role in controlling the radiation and reception pattern of sensors. Note that the performance of super-resolution imaging without evanescent waves will be degraded when there exists noise and random errors, and it was shown...
that the conversion of evanescent waves into propagating waves can enhance the robustness of far-field subwavelength imaging against the white noise [34].

Although far-field superlenses and resonant metalenses provide a way to convert evanescent waves into propagating waves through the resonant excitation of multiple surface wave modes, the high sensitivity of these resonant modes to the absorption loss hinders their practical applications. Fortunately, we can always find a resonant surface mode, which survives the absorption loss. When only a single mode is adopted for imaging, a challenge is met due to the reduction of the number of degrees of freedom, however, this shortcoming can be overcome by using the sparsity of imaged objects. Therefore, we demonstrate an experiment of far-field subwavelength imaging by harnessing the single-mode resonance and sparsity. Owing to the sensitivity of surface wave mode to the thermoviscous loss and the imperfectness of the fabricated superlens, here, we provide an easy way to choose a robust surface wave mode in practice, and confirm that the far-field super-resolution imaging can be realized by manipulating only a single surface wave mode.

In section 2, we discuss how to design an acoustic superlens of finite length and propose an incoherent processing by compressed sensing. In section 3, we investigate how the white noise, the absorption loss and the mode shape affect the imaging performance (resolvability), and reveal the role of evanescent waves in the far-field subwavelength imaging. Then, in section 4, an easy way is proposed to select a robust surface wave mode in experiment, and the imaging performance is confirmed. Finally, in section 4, the present work is concluded.
2. Acoustic superlens and compressed sensing

2.1. The design of acoustic superlens of finite length

A two-dimensional scheme of far-field subwavelength imaging through an acoustic superlens of finite length is given in figure 1. The acoustic superlens is made up of multiple Helmholtz resonators, where each Helmholtz resonator consists of one cavity and two throats. A receiving array having 31 elements at equidistant locations on a semi-circle is adopted to measure sound waves, where the radius is large enough to attenuate the evanescent components of imaged objects. By the semi-circular array, the received signals mainly come from the transmitted waves due to the imaged sources being located in the near-field of superlens, resulting in the far-field sound strongly manipulated by surface wave modes which are supported by the superlens. In order to confirm the proposed two-dimensional scheme of far-field subwavelength imaging, the resonance frequencies of surface wave modes should be lower than the cutoff frequency, which is determined by the distance between two parallel plates in experiment. On the other hand, the parameters of Helmholtz resonator should be suitably chosen, that the resonant mode adopted for imaging can survive the absorption loss (see in section 3.3). More detailedly, the dispersion curve of surface wave $f_{\text{sur}}(k_x)$ is first calculated for an acoustic superlens of infinite length, and then a reference frequency and horizontal wavenumber $(f_0, k_{x0})$ on the dispersion curve $f_{\text{sur}}(k_x)$ are selected. It is noted that the reference horizontal wavenumber $k_{x0}$ should be away from the wavenumber $\omega/c_0$ in free space to improve the imaging resolution, and on the other hand, $k_{x0}$ should be away from the edge of the first Brillouin zone $\pi/\Lambda$ to reduce the effect of the absorption loss. For the given $(f_0, k_{x0})$, multiple superlenses of different length can have a surface wave mode near the reference frequency $f_0$. Usually, a higher conversion efficiency of evanescent waves into propagating waves can be obtained when the length of superlens is about one wavelength [17]. By this principle, a superlens made up of nine Helmholtz resonators is designed in experiment. Note that since the evanescent waves play a role in enhancing the robustness of imaging against the white noise (see in figure 5), it is possible to design a superlens with an optimal number of resonators, if a detailed comparison of the imaging performance among superlenses of different length is made under different signal-to-noise ratios (SNR).

We assume a sound wave $p_{\text{inc}} = \exp\left(-j\omega t + j k_x x - j k_y (y - l)\right)$ incident from the upper half-space, with $k_x$ the horizontal wavenumber, $k_y$ the vertical wavenumber, and $k = \omega/c_0 = 2\pi/\lambda$ the wavenumber corresponding to incident wavelength $\lambda$ in air. By the boundary integral method, the sound power scattered into the lower half-space ($y < 0$) is calculated for incident evanescent waves $(k_y = j\sqrt{k_x^2 - k^2})$,

$$W_{\text{half}} = \frac{1}{2} \int_{\pi}^{2\pi} \text{Re} \left[ p_\phi^* (\vec{r}) u_\phi (\vec{r}) \right] r \, d\phi,$$

where $p_\phi (\vec{r})$ and $u_\phi (\vec{r})$ represent the pressure and radical velocity of the scattered field, respectively, and $\phi$ denotes the azimuth angle of scattered waves. Figures 2(a) and (b) show the normalized scattered power $W_{\text{half}}/N$ predicted by the theoretical method for a superlens of length $NL$, where $N$ is the number of resonators. It is illustrated in figure 2(a) that the conversion of evanescent waves into propagating waves is enhanced at the resonance frequencies of surface wave modes, which are marked by empty circles. By comparing between figures 2(a) and (b), it is found that the width of the evanescent spectrum that can be converted is decreased with the length of superlens. In addition, since the absorption loss is inevitable for the resonant excitation of surface wave modes, the viscous friction on the wall of apertures is considered using the finite element numerical simulation (the thermoviscous acoustic module in COMSOL MULTIPHYSICS, where the air has density 1.2 kg m$^{-3}$, sound velocity 343 m s$^{-1}$, dynamic viscous $\eta_0 = 1.81 \times 10^{-5}$ Pa s, heat capacity at constant pressure $C_p = 1001$ J (kg$^{-1}$ K$^{-1}$) and thermal conductivity 0.03 W (m$^{-1}$ K$^{-1}$), and the material of superlens is steel. Owing to the thermoviscous effect, the scattered power in figure 2(c) is decreased as compared with that in figure 2(a), especially for the surface wave modes having higher resonance frequencies. Concerning the absorption loss and the resolution of imaging, the fifth mode marked in figure 2(c) is adopted for imaging in experiment, which can be measured (see in figure 10). Here, the fifth mode means that this mode has five antinodes. It is worthy to evaluate the transmission coefficient, which is defined as,

$$T(k_x) = \left| \frac{\tilde{p}_{\text{total}}(k_x, y)|_{y=-0.5\text{ cm}}}{\tilde{p}_{\text{inc}}(k_x, y)|_{y=1}} \right|,$$

where $\tilde{p}_{\text{inc}}(k_x, y)|_{y=1}$ and $\tilde{p}_{\text{total}}(k_x, y)|_{y=-0.5\text{ cm}}$ are, respectively, for the Fourier transform of the incident pressure and the total pressure, being sampled along the surface of superlens. Figure 2(d) shows that a high efficient transmission of incident waves through the superlens can be achieved over a broad spatial spectrum $k_x$ at frequencies near the fifth resonant mode, where the transmission coefficient is close to one.
Figure 2. (a)–(c) Sound power (dB) scattered into the lower half-space \((y < 0)\) for evanescent waves incident from the upper half-space on the superlens made up of \(N\) Helmholtz resonators, where (a) and (b) are for the theoretical results of \(N = 9\) and \(N = 101\), respectively, and (c) for the simulated result of \(N = 9\). Note that the sound power is normalized by the number \(N\) of resonators for comparison, the same color bar range is set for subplots (a)–(c), and the discrete resonant modes of superlens are also being marked in (a) and (c). (d) The simulated result of the transmission coefficient \(T(k)\) for propagating waves and evanescent waves incident at frequencies near the fifth resonant mode marked in (c), in which a receiving array of length \(Nλ\) is located at \(y = −0.5\) cm.

for propagating waves, except for \(k_x\) near the wavenumber \(ω/c_0\) in free space, and the evanescent waves are amplified due to the resonance of the fifth mode. Therefore, the spatial spectrum of imaged objects can be delivered to the far-field by the fifth mode.

2.2. Sparse reconstruction algorithm based on compressed sensing

For the imaging we discuss here, the sound pressure at a position \(\vec{r}\) can be written in a general form as,

\[
p(\vec{r}; \omega) = \int g(\vec{r}, \vec{r}'; \omega) \chi(\vec{r}'; \omega) \, d\vec{r}',
\]

where the Green’s function \(g(\vec{r}, \vec{r}'; \omega)\) builds a connection from a point source at position \(\vec{r}'\) to an observed point \(\vec{r}\), and \(\chi(\vec{r}'; \omega)\) represents the complex amplitude of image. Note that the position \(\vec{r}'\) should be located in the near-field of superlens, that the evanescent waves can be amplified, while the position \(\vec{r}\) is located in the far-field, so that only the propagating waves are received by sensors. More detailedly, the position \(\vec{r}'\) is located in the imaging region \((-5\text{ cm} < x < 5\text{ cm}, l < y \leq l + 2\text{ cm})\), which is discretized into \(Q = 80\) points in simulation and experiment, and the sound pressure is measured at \(M = 31\) different positions on a semi-circle marked in figure 1. Then, equation (3) can be expressed in a discretized form,

\[
p = G\chi,
\]

with the pressure data \(p = [p(\vec{r}_1; \omega), p(\vec{r}_2; \omega), \ldots, p(\vec{r}_M; \omega)]^T \in \mathbb{C}^{M \times 1}\), the unknown vector of the complex image amplitudes \(\chi = [\chi(\vec{r}_1'; \omega), \chi(\vec{r}_2'; \omega), \ldots, \chi(\vec{r}_Q'; \omega)] \in \mathbb{C}^{Q \times 1}\), and the sensing matrix \(G \in \mathbb{C}^{M \times Q}\) having the element \((G)_{mq} = g(\vec{r}_m, \vec{r}_q'; \omega)\). For the number of measurements smaller than that of unknown variables \((M < Q)\), equation (4) is an underdetermined linear system, which admits many solutions. However, considering the sparsity of imaged objects, an optimized solution \(\hat{\chi}\) can be obtained in
an unconstrained form \[35\] with the use of regularization parameter \(\mu \geq 0\),

\[
\hat{x} = \arg \min_{x \in \mathbb{C}^{Q \times 1}} \left[ \| p - G x \|_2^2 + \mu \| x \|_1 \right].
\] (5)

The sparse signal reconstruction problem in equation (5) is equivalent to requiring the \(\ell_1\)-norm of the vector \(x\) to be minimized under the constraint \(\|p - Gx\|_2 \leq \varepsilon\), which have been adopted in computation imaging via compressed sensing [29–33]. It should be noted that the metamaterial adopted in computation imaging has no role in amplifying the evanescent waves of imaged objects, but plays a role in controlling the radiation and reception pattern of sensors, leading to the reduction of the number of sensors. However, for the superlens adopted here, the evanescent waves of imaged objects are converted into propagating waves through the resonance of surface wave mode. The existence of surface wave mode plays a role in enhancing the incoherence between the columns of the sensing matrix \(G\) (see in figure 8), resulting in the improvement of imaging performance.

In practice, multiple frequencies are adopted to reconstruct the image, so equation (5) is extended as,

\[
\hat{X} = \arg \min_{X \in \mathbb{C}^{Q \times \Omega}} \left[ \sum_{\omega \in \Omega} \| p(\omega) - G(\omega) \chi(\omega) \|_2^2 + \mu \sum_{q=1}^{Q} \left( \sum_{i=1}^{\Omega} | \tilde{G}_q^i(\omega_i) | \right)^2 \right],
\] (6)

with \(X = [\chi(\omega_1), \chi(\omega_2), \ldots, \chi(\omega_\Omega)]\) and \(\hat{X} = [\hat{\chi}(\omega_1), \hat{\chi}(\omega_2), \ldots, \hat{\chi}(\omega_\Omega)]\). Then, the distribution of intensity is obtained by taking an average, i.e., \(\Omega^{-1} \sum_{\omega \in \Omega} \chi(\tilde{r}_q^i; \omega)\) at position \(\tilde{r}_q^i\). The optimization problem in equation (6) is solved by a convex optimization tool (e.g., CVX [36]). In order to avoid the possible numerical overflow, for each frequency, \(p(\omega)\) is normalized by its \(\ell_2\)-norm, and the matrix \(G(\omega)\) is normalized by the maximum value, which is defined as \(\max_{1 \leq q \leq Q} \left\| G_q(\omega) \right\|_2\), where the vector \(G_q(\omega)\) denotes the \(q\)-th column of matrix \(G(\omega)\).

In a word, the reconstruction algorithm based on the sparsity plays a role in reducing the number of degrees of freedom, therefore, different from the methods based on multiple surface wave modes \([11–21, 23]\), only a single surface wave mode is used here to obtain the super-resolution.

3. Simulated results

3.1. Imaging performance: resolvability

The imaging of two acoustic sources is introduced here to investigate the performance of the proposed method. If not particularly indicated, equation (6) is adopted to get the distribution of average intensity, taking an average over multiple frequencies, and the normalized distribution of average intensity is called as the reconstructed image. In order to evaluate the quality of reconstructed image, we define the two main peak intensities of reconstructed image as \(A_1\) and \(A_2\), respectively, and the maximum intensity of sidelobes as \(A_s\), where a condition \(A_1 \geq A_2 \geq A_s\) is satisfied. Then three main criterions are introduced to judge whether the two imaged sources can be discerned from the reconstructed image. (1) The ratio between two main peak intensities \(A_2/A_1\) should be larger than 0.4; (2) a limitation on the sidelobe level should be met, i.e., \(A_2/A_s > 2.5\); (3) by considering the lattice constant of superlens \(\lambda = 1.4\) cm, the distance between the
two main peaks should be larger than 2 cm, which also constrains the regularization parameter interval, i.e., discarding very large regularization parameters $\mu$. When three main criteria are met, the two imaged sources are thought to be resolvable, and the dependence of resolvability on the regularization parameter $\mu$ and the positions of imaged sources is demonstrated in the following. On the other hand, the robustness of resolvability against random errors or white noise is investigated by adding the Gaussian white noise to the pressure data $p$ in equation (4).

### 3.2. The imaging performance under no-noise condition and under the different SNR

In order to demonstrate a good performance by introducing a superlens, the imaging obtained with the aid of superlens should be compared with that achieved in free space. For a superlens made up of nine Helmholtz resonators, eight frequencies ($f = 2560 : 4 : 2588$ Hz) marked by empty circles in figure 3(a) are near the resonance frequency of the fifth mode, which can be seen from the distribution of pressure amplitude near the surface of superlens in figure 3(b). By using these frequencies near the fifth mode, figures 4(a)–(c) show the corresponding imaging performance (resolvability) on the regularization parameter $\mu$ and the distance $s$ under no-noise condition. Two imaged sources (see in figure 1(a)) are located at a distance $s$ away from the acoustic superlens, and being symmetrically separated by an interval $d$ with respect to the center of superlens, $d = 3$ cm $\approx 0.225 \lambda_c$ in (a), $d = 4$ cm $\approx 0.30 \lambda_c$ in (b), and $d = 6$ cm $\approx 0.45 \lambda_c$ in (c), where $\lambda_c$ being the central wavelength of working bandwidth ($f = 2560 : 4 : 2588$ Hz). For comparison, the imaging results without the aid of superlens are given in (d)–(f). Note that the gray lines represent that two imaged sources are chosen to be located at different distances $s$ away from the superlens, and the thick lines in (a)–(c) represent the regularization parameter intervals in which two imaged sources can be discerned from the reconstructed images by using multiple frequencies near the fifth resonant mode, while the thick lines in (d)–(f) are for the imaging in free space using the same frequencies. In addition, the dotted lines in (a) represent the imaging obtained by using only a single frequency ($f = 2572$ Hz) near the fifth mode, and the results marked by empty circles in (a) are for the imaging by using multiple frequencies ($f = 2256 : 4 : 2284$ Hz) near the third resonant mode marked in figure 3(a).
Figure 5. Dependence of simulated imaging performance (resolvability) on the regularization parameter $\mu$ and distance $s$ under the different SNR, where (a)–(c) are for the imaging results using multiple frequencies ($f = 2560 : 4 : 2588$ Hz) near the fifth mode of superlens made up of nine Helmholtz resonators, and (d)–(f) are for the imaging results in free space using the same frequencies. Note that the meaning of adopted line styles is the same as that in figure 4.

parameter $\mu$ and the distance $s$ for two imaged sources separated by a distance $d = 3$ cm, $d = 4$ cm, and $d = 6$ cm, respectively. Here, the parameter $\mu$ plays a role in controlling a trade-off between the sparsity of the reconstructed image and the least squared error, that is between the estimated and simulated pressure. Usually, a large $\mu$ means that the reconstructed image is very sparse and the estimated pressure deviates largely from the simulated. Although the fit between the estimated and simulated pressure is improved for a small $\mu$, the reconstructed image becomes less sparse. In practical applications, a very small $\mu$ should be discarded due to the inevitable random errors and noise. It is illustrated in figures 4(a)–(c) that the two imaged sources can be discerned over a broad regularization parameter interval with the aid of superlens, although the fit between the estimated and simulated pressure is not degraded even with the large regularization parameter, which means the two imaged sources can be discerned with a large least squared error. It is also seen that the resolvability changes discontinuously as a function of the regularization parameter, which will be discussed in section 4. In addition, the imaging result obtained by using a single frequency near the fifth mode is also illustrated in (a), and the decrease of regularization parameter interval is observed. For comparison, figures 4(d)–(f) show the imaging performance without the aid of superlens, where the regularization parameter interval becomes narrower (see in figures 4(d) and (e)), that is to say, the imaging performance without superlens is more sensitive to the random errors and noise. This is confirmed by the simulated imaging performance when the Gaussian white noise is added to the pressure data with the SNR equalling 20 or 10 dB. For two imaged sources symmetrically separated by a distance $d = 3$ cm, it is shown in figures 5(a) and (b) that the resolvability is robust against the white noise with the aid of superlens, however, it is almost impossible to
Figure 6. The dependence of simulated imaging performance (resolvability) on the regularization parameter $\mu$ and the distance $s$ for different values of dynamic viscous $\eta$, where (a) for $\eta = \eta_0$, (b) for $\eta = 8\eta_0$, (c) for $\eta = 16\eta_0$, and (d) for $\eta = 100\eta_0$. Note that the two imaged sources are located at $x_1 = -1.5$ cm, $x_2 = 1.5$ cm, the gray lines represent that the two imaged sources are chosen to be located at different distances $s$ away from the superlens made up of nine Helmholtz resonators, and the thick lines represent the regularization parameter intervals in which the two imaged sources can be discerned from the reconstructed images using multiple frequencies ($f = 2560 : 4 : 2588$ Hz) near the fifth mode.

distinguish two imaged sources under SNR = 10 dB in free space (see in figure 5(e)). With the increase of interval between two imaged sources, the robustness is enhanced in free space, for example, $d = 6$ cm in figure 5(f).

3.3. The effect of dynamic viscous on imaging performance

The influence of absorption loss is considered for the design of acoustic superlens, where the main loss comes from the viscous friction on the wall of apertures. Usually, the thickness of viscous layer is estimated as $d_v = \sqrt{\frac{2\eta}{\rho_0\omega}}$, with $\eta$ being the dynamic viscous. If the size of throat satisfies $a \gg d_v$, the effect of absorption loss can be neglected. For air at room temperature, the dynamic viscous is $\eta = \eta_0 = 1.81 \times 10^{-5}$ Pa s, and the ratio $\frac{a}{d_v}$ is approximately equal to 116 at frequency $f = 2572$ Hz near the fifth mode. In order to investigate the effect of absorption loss on imaging performance, the dynamic viscous $\eta$ is increased from $\eta_0$ to $100\eta_0$, i.e., the ratio $\frac{a}{d_v}$ is decreased approximately from 116 to 11.6. It is illustrated in figure 6 that the imaging is robust against the absorption loss when the size of throat satisfies $a \gg d_v$, and the imaging performance is observed to be improved at $s = 0.5$ cm for the dynamic viscous $\eta = 8\eta_0$ (see in figure 6(b)), where the absorption loss might play a role in weakening the interference between two sources. However, for a very large dynamic viscous $\eta = 100\eta_0$, the robustness of imaging is decreased due to the attenuation of surface wave mode.

3.4. The effect of mode shape on the imaging performance

Since the mode shape affects the coupling efficiency between evanescent waves and the superlens, it is valuable to investigate its effect on the imaging performance. Considering the thermoviscous effect and the resolution of imaging, the fifth resonant mode is chosen here, and its mode shape can be manipulated by the number of Helmholtz resonators. Therefore, the performance comparison is made between the superlens consisting of eight resonators and nine resonators. By the amplitude ratio between the received signals with and without the aid of superlens (see in figures 3(a) and 7(a)), the fifth mode determined by the normalized pressure amplitude along the surface of superlens is shown in figure 7(b). It is found that the mode shape for $N = 9$ is sharper than $N = 8$ at the center of superlens. For $N = 9$, the throat of the fifth resonator is at the center of superlens, resulting in the maximum pressure amplitude at $x = 0$. However, for $N = 8$, the center of superlens is located between the throats of the fourth and the fifth
Figure 7. (a) Frequency dependence of the simulated amplitude ratio between the received signals with and without the aid of superlens ($N = 8$ resonators), where a source (see in figure 1(a)) is located at position $(x_s = 0, y_s = l + 1 \text{ cm})$ and a receiver at position $(x = 0, y = -0.5 \text{ cm})$. (b) Normalized pressure amplitude of the fifth mode along $y = -0.5 \text{ cm}$ simulated with the aid of superlens made up of $N$ Helmholtz resonators, where the resonance frequency is $f_r = 2598 \text{ Hz}$ for $N = 8$ (marked in (a)), and $f = 2554 \text{ Hz}$ for $N = 9$ (marked in figure 3(a)). (c) and (d) Give the dependence of simulated imaging performance on the regularization parameter $\mu$ and the distance $s$ for the superlens made up of $N$ resonators, where (c) for the imaging of the two symmetrical sources located at $x_1 = -1.5 \text{ cm}, x_2 = 1.5 \text{ cm}$, and (d) for the two unsymmetrical sources at $x_1 = -1 \text{ cm}, x_2 = 2 \text{ cm}$. Note that the gray lines represent that two imaged sources are chosen to be located at different distances $s$ away from the superlens, the empty right-pointing triangles in (c) and (d) represent the regularization parameter intervals in which two imaged sources can be discerned from the reconstructed images with the aid of superlens made up of eight Helmholtz resonators using multiple frequencies ($f = 2598 : 4 : 2626 \text{ Hz}$) near the fifth mode, and the thick lines for a superlens made up of nine Helmholtz resonators using multiple frequencies ($f = 2560 : 4 : 2588 \text{ Hz}$) near the fifth mode.

resonator, leading to a flatter mode shape. As a result, the interference between two symmetrical sources for $N = 9$ is stronger than $N = 8$, which can be seen from the imaging performance in figure 7(c). It is found that the superlens with eight resonators still works well when the two imaged sources are being moved towards the superlens, while failures occur in imaging by the superlens with nine resonators. In addition, figure 7(d) shows the imaging results of two unsymmetrical sources.

3.5. The role of evanescent waves in the far-field subwavelength imaging

Usually, evanescent waves can be amplified by a superlens, resulting in the super-resolution imaging detected in the near-field of superlens. Owing to the finite length of superlens adopted here, the evanescent waves are diffracted into the propagating waves, which can be measured in the far-field. The diffraction of evanescent waves plays a role in manipulating the mutual coherence of the sensing matrix $G$ in equation (4), which determines the imaging performance of compressed sensing [37]. The mutual coherence is evaluated through the Gram matrix $\tilde{G}^H \tilde{G}$, where $\tilde{G}$ is the column-normalized version of the matrix $G$. Figures 8(a)–(c) show the Gram matrix for the potential positions of imaged sources at the different distance $s$ away from the superlens, while the Gram matrix in free space is given in (d). It is illustrated that the incoherence between the columns of the sensing matrix is enhanced when the potential positions of imaged sources are near the superlens, and thus improving the imaging performance for two imaged sources separated by a distance of subwavelength, which can be seen from figures 4(a) and (b) that the imaging performance does not degrade even with the large regularization parameter.

3.6. Distinction between a coherent processing and the proposed incoherent processing

A recent theoretical work in reference [22] has investigated the super-resolution electromagnetic wave imaging using multiple hole resonances without considering the absorption loss, and the complex image of objects is assumed to be frequency-independent, leading to a coherent processing. One challenge of this coherent processing in experiment is that the complex source amplitude should be known exactly to
Figure 8. Dependence of the absolute Gram matrix element on the row index \( m \) and column index \( n \) at frequency \( f = 2572 \) Hz near the fifth mode of superlens made up of nine Helmholtz resonators, where \( m \) and \( n \) represent the potential positions of imaged sources, and \( m \neq n \). (a) The Gram matrix \( \tilde{G}_1 \tilde{G}_1 \) obtained at the potential positions \( x_n = -5 + 0.5(n - 1) \) cm for \( n = 1, \ldots, 20 \), where the imaged sources are located at a distance \( s = 0.5 \) cm away from the superlens, (b) \( \tilde{G}_2 \tilde{G}_2 \) at \( s = 1 \) cm, and (c) \( \tilde{G}_3 \tilde{G}_3 \) at \( s = 1.5 \) cm, while the Gram matrix in (d) obtained at \( y = 5.5 \) cm without the aid of superlens.

measure the Green’s function. On the other hand, the frequency-independent image cannot always be satisfied in acoustics. To overcome the above two limitations, an incoherent processing is proposed here (see in equation (6)), where the same sparsity of imaged objects is assumed for different frequencies, without the requirement of the complex image to be frequency-independent. Different from the coherent processing, the imaging performance of the incoherent processing is mainly determined by the ability of the single resonant mode due to the lack of the relation among the complex image amplitudes at frequencies of different modes.

Figures 9(a) and (c)) show the imaging performances obtained by separately using multiple frequencies near the fifth mode and multiple frequencies near the seventh mode, while (b) and (d) are for the imaging using these frequencies together. Owing to a smaller transmitted amplitude by the resonance of the seventh mode (see in figure 3(a)), the robustness of super-resolution imaging by using multiple frequencies near the seventh mode is degraded in comparison with that by using multiple frequencies near the fifth mode, i.e., the super-resolution imaging achieved in a narrower regularization parameter interval, which can be seen from figures 9(a) and (c). It is illustrated in figures 9(b) and (d) that the imaging performance can be improved by using these frequencies together in the common range of \( s \) (marked by empty rectangles with the purple edges), where the super-resolution imaging can be obtained by using both multiple frequencies near the fifth mode and the seventh mode. However, the imaging performance may be degraded when the imaged sources are located outside of common range, for example \( s = 0.3 \) cm in figures 9(c) and (d).

4. Experimental results

4.1. The measurement of the fifth resonant mode

Since the surface wave mode is sensitive to the thermoviscous loss and its resonance frequency deviates from the predicted value due to the imperfectness of the fabricated superlens, an experimental method is proposed here to discern the symmetrical surface wave mode, which can be resonantly excited when a home-made source is located at the center of superlens. Note that the home-made source is a speaker of dimensions 22 mm \( \times \) 9 mm \( \times \) 4 mm, and its back side is terminated in a closed cavity, which can be seen in figure 1(b). By measuring the amplitude response of sound waves transmitting through the superlens made up of nine Helmholtz resonators, the symmetrical surface wave mode can be determined. Figure 10 shows the measured result of the fifth resonant mode. To remove the effect of the frequency response of a source on transmitting waves, an amplitude ratio between the received signals with and without the aid of
Figure 9. Dependence of simulated imaging performance on the regularization parameter $\mu$ and the distance $s$, where two imaged sources separated by an interval $d = 3$ cm are located at a distance $s$ away from the superlens made up of nine Helmholtz resonators. (a) The imaging results of two symmetrical sources by separately using multiple frequencies ($f = 2560 : 4 : 2588$ Hz) near the fifth mode and multiple frequencies ($f = 2650 : 4 : 2678$ Hz) near the seventh mode, while (b) for the imaging result using these frequencies together. (c) and (d) Are for the imaging of two unsymmetrical sources. Note that the gray lines represent that two imaged sources are chosen to be located at different distances $s$ away from the superlens, and the other lines (including thick lines, empty diamonds and dotted lines) represent the regularization parameter intervals in which the two imaged sources can be discerned from the reconstructed images. In addition, four empty rectangles with the purple edges in (a)–(d) are used to illustrate the common range of $s$, in which the super-resolution imaging can be obtained by using both multiple frequencies near the fifth mode and the seventh mode.

Figure 10. (a) Frequency dependence of the measured amplitude ratio between the received signals with and without the aid of superlens made up of nine Helmholtz resonators, where the source (see in figure 1(b)) is located at position ($x_s = 0$, $y_s = l + 1$ cm) and a receiver at position ($x = 0$, $y = -0.3$ cm). (b) Normalized pressure amplitude along $y = -0.3$ cm measured with the aid of superlens at the frequency $f = 2662$ Hz marked in (a). Note that the frequencies used for imaging near the fifth resonant mode are marked by empty circles in (a). Due to the symmetrical excitation, the positions of local peaks in figure 10(a) correspond to the resonance frequencies of symmetrical modes. By the distribution of pressure amplitude in figure 10(b), the fifth mode is discerned by five antinodes. It is also seen from figure 10(a) that only the third mode and the fifth mode are obvious in experiment under the symmetrical excitation. Therefore, using the single-mode resonance to obtain the far-field subwavelength imaging is a good choice.
Figure 11. (a) and (b) The reconstructed images (normalized intensity) of two imaged sources are obtained using two different regularization parameters $\mu$ marked as A and B in figure 12(a), where the two imaged sources are separated by an interval $d = 3\,\text{cm} \approx 0.233\,\lambda_c$ and being located at a distance $s = 0.3\,\text{cm}$ away from the superlens. For $d = 4\,\text{cm} \approx 0.31\,\lambda_c$ and $s = 0.3\,\text{cm}$, the reconstructed images of the two imaged sources are given in (c) and (d) using the regularization parameters $\mu$ marked as C and D in figure 12(b).

Figure 12. Dependence of imaging performance (resolvability) on the regularization parameter $\mu$ and the distance $s$. (a) and (b) Two imaged sources are symmetrically located with respect to the center of superlens made up of nine Helmholtz resonators, and being separated by an interval $d = 3\,\text{cm} \approx 0.233\,\lambda_c$ in (a) and $d = 4\,\text{cm} \approx 0.31\,\lambda_c$ in (b), where $\lambda_c$ being the central wavelength of imaging bandwidth ($f = 2650 : 2678\,\text{Hz}$). (c) For two sources separated by an interval $d = 3\,\text{cm} \approx 0.233\,\lambda_c$ but not symmetrically located with respect to the center and (d) for a single source at center. Note that the gray lines represent that the imaged sources are chosen to be located at different distances $s$ away from the superlens in experiment, and the thick lines represent the regularization parameter intervals in which the imaged sources can be discerned from the reconstructed images by using multiple frequencies near the fifth mode. For comparison, the dotted lines in (a) and (c) represent the experimental results obtained by using only a single frequency $f = 2662\,\text{Hz}$ near the fifth resonant mode.
4.2. Experimental results of super-resolution imaging

By the measurement of the fifth mode, eight frequency points marked by empty circles in figure 10(a) are chosen for imaging, which are in the frequency range from 2650 to 2678 Hz. Owing to the size limitation of source in experiment, the matrix $G(\omega_i)$ is measured at each frequency $\omega_i/2\pi$ by the reciprocity principle, which has an advantage in discretizing the imaging region into small grids by using a stepping motor. Once
the matrix \( \mathbf{G}(\omega_i) \) is obtained, equation (6) is adopted to reconstruct the image of objects through the received pressure \( \mathbf{p}(\omega_i) \), which is obtained by an array of 31 elements at equidistant locations on a semi-circle marked by solid lines in figure 1(b). The imaging of two home-made acoustic sources in figure 1(b) is introduced here to confirm the performance of the proposed method.

Using multiple frequencies (\( f = 2650 \pm 4 \) : 2678 Hz) near the fifth mode in figure 10(a), figures 11(a) and (b) show the reconstructed images of two sources separated by a distance \( d = 3 \) cm \( \approx 0.233\lambda_c \) for the regularization parameter \( \mu = 0.361 \) and \( \mu = 1.211 \), respectively, where \( \lambda_c \) being the central wavelength of working bandwidth. It is seen that the change of the reconstructed image is not obvious when the regularization parameter is increased from \( \mu = 0.361 \) to \( \mu = 1.211 \), but a gradual trend towards a smaller distance between two peaks of normalized intensity is observed in figure 11(b). Note that this trend is also observed in figures 11(c) and (d), which show the reconstructed images of two sources separated by a distance \( d = 4 \) cm \( \approx 0.31\lambda_c \) for the regularization parameter \( \mu = 0.851 \) and \( \mu = 1.771 \), respectively. By comparison, it is shown that there exists two small sidelobes in figure 11(c) for \( d = 4 \) cm, however, no sidelobes are observed in figure 11(a) for \( d = 3 \) cm. This is because that the position of imaged sources in figure 11(c) is located between two throats of neighbouring Helmholtz resonators, resulting in the main peaks of normalized intensity deviating slightly from the actual positions of imaged sources, while the position of imaged sources in figure 11(a) is close to the throat of a single resonator, leading to the main peaks appearing at the positions of imaged sources.

Figure 12(a) shows the regularization parameter intervals in which two imaged sources separated by a distance \( d = 3 \) cm can be discerned. It is seen that the resolvability changes discontinuously as a function of regularization parameter, for example, at \( s = 0.3 \) cm, however, the dependence of the intensity distribution on the regularization parameter in figure 13(a) shows that the two imaged sources can be discerned over a broad parameter interval if the criterion (1) in section 3.1 is relaxed. When the two imaged sources are located so far away from the superlens, the evanescent waves cannot be amplified obviously, resulting in the failure of discerning two imaged sources, which can be seen in figures 12(a) and 13(d) for two imaged sources located at \( s = 1.8 \) cm away from the superlens. On the other hand, if two imaged sources are separated by a larger distance \( d = 4 \) cm \( \approx 0.31\lambda_c \) in figure 12(b), the imaging performance (resolvability) at the distance \( s = 1.8 \) cm away from the superlens is improved (also see in figure 14(d)). Note that although the symmetrical surface wave mode is used to obtain the super-resolution imaging, it is shown in figure 12(c) that two sources separated by an interval \( d = 3 \) cm \( \approx 0.233\lambda_c \) but not symmetrically located with respect to the center of superlens can also be discerned. In addition, the imaging result for a single source is also presented in figure 12(d). By the dependence of the intensity distribution on the regularization parameter for \( d = 3 \) cm in figure 13 and \( d = 4 \) cm in figure 14, it is found that the two imaged sources can be discerned over a broad regularization parameter interval if three criterions in section 3.1 are relaxed, which means the imaging performance is very robust against the random errors and noise.

Note that the imaging results illustrated in figures 11–14 are obtained using multiple frequencies near the fifth mode. For comparison, the dotted lines in figures 12(a) and (c) represent the experimental results obtained by using only a single frequency near the fifth resonant mode. It is observed that the regularization parameter interval in which the two imaged sources can be discerned becomes narrower, and failures occur in the imaging for \( s = 1.3 \) cm in figure 12(a), which means that using multiple frequencies near a single resonant mode can improve the robustness of imaging.

4.3. Discussion

Figures 5 and 6 have demonstrate that the fifth mode can improve the robustness of reconstruction algorithm against the noise and the absorption loss, especially when the distance between two imaged sources is of subwavelength. Since the third mode is also obvious in experiment (see in figure 10), then, a natural question is: can this mode be adopted for imaging? By choosing multiple frequencies near the third mode (see the empty squares in figure 3(a)), the corresponding imaging results are marked by empty circles in figure 4(a), it is found that this mode is unsuitable for the super-resolution imaging, because the evanescent waves of the higher wavenumber \( k_s \) cannot be delivered to the far-field by the third mode (see in figure 2(c)). Therefore, a single resonant mode having a broad spatial spectrum is a better choice for imaging.

It is noted that in experiment two imaged sources separated by the distance \( d = 3 \) cm and \( d = 4 \) cm have not been discerned in free space, because the imaging performance for two sources separated by a subwavelength distance in free space degrades rapidly (see in figures 5(d) and (e)) when there exists noise and random errors in imaging systems.
5. Conclusion

In conclusion, a scheme of the far-field subwavelength imaging is proposed by harnessing the single-mode resonance and sparsity. By designing a superlens of finite length, a broadband spatial spectrum of imaged objects is manipulated by a single surface wave mode and being delivered to the far-field. Although the number of degrees of freedom for imaging is reduced by considering only a single surface wave mode, the reconstruction algorithm based on compressed sensing is adopted to overcome this shortcoming. To enhance the robustness of imaging, several frequencies near the fifth mode are adopted by an incoherent processing without the requirement of the complex image to be frequency-independent. Different from the imaging method based on multiple surface wave modes, for example, the far-field imaging based on the time reversal technique, the main advantage of the proposed method is that the imaging performance is promising by selecting only a single surface wave mode robust to the absorption loss.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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