Quenched Light Hadron Spectrum and Decay Constants using Improved Wilson Fermion Actions

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1. LATTICE PARAMETERS

The use of a tadpole-improved coefficient \( \frac{1}{2} \) for the clover term in the SW fermion action,

\[
S_F = \sum_{xy} \bar{q}(x) \left\{ \left[ 1 - \frac{i c \kappa}{2} \sigma_{\mu \nu} F_{\mu \nu}(x) \right] \delta_{xy} - \kappa \left[ (1 - \gamma_{\mu}) U_{\mu}(x) \delta_{x + \hat{\mu}, y} \right] + (1 + \gamma_{\mu}) U_{\mu}^\dagger(y) \delta_{x - \hat{\mu}, y} \right\} q(y),
\]

where \( c = 1/u_0^3 \), \( u_0 = (\frac{4}{3} \text{Tr}(C))^1/4 \), can be regarded as a step towards full \( O(a) \) improvement \( \frac{1}{2} \). We compare the resulting light hadron quantities with those obtained using the tree-level coefficient, \( c = 1 \), for which discretisation errors are \( O(g^2 a) \) in perturbation theory \( \frac{1}{2} \), and also with data from GF11 \( \frac{1}{2} \) corresponding to \( c = 0 \), at a range of \( \beta \) values, to seek indications of better scaling behaviour.

Our data set comprises: at \( \beta = 5.7 \), Jacobismeared quark propagators at two \( \kappa \) values with \( c = 1/u_0^3 \) and \( c = 1 \) on 142 \( 16^3 \times 32 \) configurations; at \( \beta = 6.0 \), fuzzed propagators \( \frac{1}{2} \) at three \( \kappa \) values with \( c = 1/u_0^3 \) on 499 \( 16^4 \times 48 \) configurations; and, at \( \beta = 6.2 \), fuzzed (local) propagators at three (two) \( \kappa \) values with \( c = 1/u_0^3 \) \( (c = 1) \) on 130 (60) \( 24^3 \times 48 \) configurations. We construct meson correlators from all \( \kappa \) combinations, but baryon correlators only from degenerate combinations. Consequently, we do not have enough baryon data to perform a reliable chiral extrapolation. Hadron masses are obtained from multi-exponential fits to various combinations of smeared and local correlators. The results presented are from a preliminary analysis of our current data (see also \( \frac{1}{2} \)); higher statistics at \( \beta = 6.2 \) will be available soon.

2. HADRON SPECTRUM

The ratio of the nucleon to vector meson mass, \( m_N/m_V, \) at a fixed ratio of the pseudoscalar to vector meson mass, \( m_{PS}/m_V, \) has a noticeable dependence on \( c \) at our largest lattice spacing. The trend is towards improved scaling for this ratio as \( c \) is increased to the tadpole-improved value. Evidence for the latter is given in the Edinburgh plot in Figure 1.

Linear extrapolation of data for the pseudoscalar meson mass to \( m_{PS}(\kappa_c, \kappa_c) = 0 \) for the tadpole-improved action gives \( u_0\kappa_c = 0.12347(3), 0.12224(1), \) and \( 0.12208(2), \) for \( \beta = 5.7, 6.0 \) and \( 6.2 \) respectively, in reasonable agreement with the tree-level value of 0.125. We define \( \kappa_{ad}, \) corresponding to degenerate \( u \) and \( d \) quarks, at \( m_{PS}/m_V = m_\pi/m_\rho. \) Then, as shown in Figure 2, our estimates for \( m_\rho \) in units of the square root of the string tension, \( \sqrt{F}, \) have decreasing dependence on the lattice spacing, \( a, \) as \( c \) is increased from 0 to 1/\( u_0^3 \), although the dependence is not removed entirely.

The \( \kappa \) value corresponding to the strange quark mass, \( \kappa_s, \) may be fixed from any one of the ratios \( m_K/m_\rho, m_\phi/m_\rho, \) and \( m_{K^*}/m_\rho. \) In Figure 3 we show the results of fixing one of the first two ratios and calculating the third. Evidently, the \( K^* \) mass scales with the \( \rho \) mass in both cases, independently of whether \( c = 1 \) or 1/\( u_0^3 \) but diff-
ficient values are obtained for \( m_{K^*}/m_\rho \). This is an indication that the strange quark mass cannot be determined consistently in the quenched approximation \([7]\). We also find that \( J \) \([8]\) is not changed by tadpole improvement, and scales at a value inconsistent with experiment.

Finally, we observe that, at all three \( \beta \) values, the magnitude of the meson spin splitting, \( m_\rho^2 - m_{PS}^2 \) in units of \( m_K \) or \( m_\rho \), is insensitive to whether \( c = 1 \) or \( 1/u_3^0 \).

3. DECAY CONSTANTS

Our quark propagators are not ‘rotated’, so \( O(a) \)-improved matrix elements are constructed from improved quark fields:

\[
q^I = \sqrt{2\kappa u_0} \left[ 1 - \frac{\vec{D}}{2u_0} \right] q
\]

\[
= \sqrt{2\kappa u_0} \left[ 1 + \frac{1}{2u_0} \left( \frac{1}{2\kappa} - \frac{1}{2\kappa_c} \right) \right] q
\]

(2)

where we have used the equations of motion and employed the tadpole-improvement prescription \([8]\). Thus, we take the tadpole-improved pion decay matrix element to be \([8]\):

\[
\langle 0 | \bar{q} \gamma_4 \gamma_5 q | \pi \rangle = Z_A \sqrt{2\kappa u_0} \left[ 1 + \frac{1}{2u_0} \left( \frac{1}{2\kappa} - \frac{1}{2\kappa_c} \right) \right]
\]

(3)

where we obtain \( \kappa_c \) from \( m_{PS}(\kappa_c, \kappa_c) = 0 \) and the current normalisation, \( Z_A \), from tadpole-improved one-loop perturbation theory.

Comparison of the pion decay constant values obtained with different values of \( c \) is complicated by systematic effects inherent in the current normalisation prescription. Given this caveat, we show our tadpole-improved results along with those of GF11 \([4]\) in Figure 4. Both sets of data suggest scaling may set in above \( \beta = 6.0 \).

The ratio of decay constants, \( f_K/f_\pi \), is independent of the current normalisation and so may be determined more reliably. Our results, shown in Figure 5, are insensitive to whether \( c = 1 \) or \( 1/u_3^0 \). For \( \beta \geq 6.0 \) the ratio agrees with experiment, but a weak lattice spacing dependence remains below this \( \beta \) value.

4. CONCLUSIONS

Using a tadpole-improved SW fermion action in quenched QCD, we conclude the following.

1. For \( \beta \geq 5.7 \), \( m_\rho/\sqrt{\kappa} \) has a weaker dependence on the lattice spacing (Figure 2), \( m_N/m_\rho \) and \( m_{K^*}/m_\rho \) scale (Figures 1 and 3), although
the latter clearly shows that the strange quark mass cannot be determined consistently.

2. Some quantities, such as $m_{K^*}/m_\rho$, $m^2_V - m^2_{PS}$, $J$ and $f_K/f_\pi$, are insensitive to whether $c$ has its tree-level or tadpole-improved value (Figures 3 and 5).

3. Provided tadpole-improved perturbation theory for $Z_A$ is reliable, there is an indication that $f_\pi/m_\rho$ may scale for $\beta \geq 6.0$ (Figure 4).

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Figure 5. $f_K/f_\pi$ versus lattice spacing.