Making two dysprosium atoms rotate —Einstein-de Haas effect revisited

Wojciech Górecki and Kazimierz Rzązewski

Center for Theoretical Physics, Polish Academy of Sciences - Al. Lotników 32/46, 02-668 Warsaw, Poland

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Abstract – We present a numerical study of the behaviour of two magnetic dipolar atoms trapped in a harmonic potential and exhibiting the standard Einstein-de Haas effect while subject to a time-dependent homogeneous magnetic field. Using a simplified description of the short-range interaction and the full expression for the dipole-dipole forces we show that under experimentally realisable conditions two dysprosium atoms may be pumped to a high (l > 20) value of the relative orbital angular momentum.

Introduction. – The Einstein-de Haas effect (EdH) [1] is a classic physical phenomenon discovered more than 100 years ago. The magnetism of a ferromagnet is associated with the electronic spins. Due to the angular-momentum conservation, freely suspended piece of ferromagnet starts to rotate when its magnetization is changed because of the changing external magnetic field. The effect has been experimentally demonstrated by the authors placing a ferromagnetic cylinder inside a coil and driving a burst of electric current through this coil.

As the angular-momentum conservation is universal, the effect occurs in micro-scale as well. Recently the first experimental observations of the EdH effect were reported for a single molecule [2]. Simultaneously, since the successful condensation of chromium [3,4], due to its large magnetic dipole moment, the interest in the EdH was also extended to gaseous Bose-Einstein condensates [5,6].

As in the original idea of Einstein and de Haas, the magnetic dipole interaction couples the atomic spins to the orbital angular momentum. Such a coupling is the essential ingredient of the effect. With the condensation of erbium [7] and dysprosium [8,9] even stronger dipolar forces became available. For chromium [10] and rubidium [11–13] condensates the EdH effect was also extended to gaseous Bose-Einstein condensates [5,6].

In this letter we analyze the EdH effect for two dysprosium atoms placed in a spherical harmonic trap. The dysprosium fermionic isotope has its spin equal to (21/2)ℏ. Hence a significant computational complexity of the problem at hand. Here we focus on the time evolution of the ground state while the value of the homogeneous magnetic field is gradually changed. In contrast to [16,17] we do not use the perturbation theory; the calculation method is very similar to the one from [18]. It turns out that the large effect could be reached for the available magnetic-field strength and the realistic time of changing this field. A lot of orbital angular momenta may be generated in the system.

Theoretical model. – Let us consider a system of two identical dipolar atoms in an isotropic harmonic trap with external homogeneous magnetic field B. The Hamiltonian is given as

\[ H = H_{OSC}(\mathbf{r}_1, \mathbf{r}_2) + H_{INT}(\mathbf{r}_1 - \mathbf{r}_2, \mathbf{F}_1, \mathbf{F}_2) + H_B(\mathbf{F}_1 + \mathbf{F}_2), \]

where \( \mathbf{r}_1, \mathbf{r}_2 \) are the position vectors of the atoms and \( \mathbf{F}_1, \mathbf{F}_2 \) are their spins (total internal angular momenta). Here

\[ H_{OSC} = -\frac{\hbar}{2m} \nabla_1^2 + \frac{1}{2} m \omega^2 \mathbf{r}_1^2 - \frac{\hbar}{2m} \nabla_2^2 + \frac{1}{2} m \omega^2 \mathbf{r}_2^2, \]

\[ H_{INT} \]

is an interaction energy (which contains both short-range (\( H_{SR} \)) and long-range dipole-dipole (\( H_{DD} \)) parts) and \( H_B \) is the energy of the magnetic dipoles in an external magnetic field.
In $H_{OSC}$ one can separate the part governing the mass center motion $H_{CM}$ and the part describing a relative motion of atoms $H_{REL}$ by introducing $r = \frac{1}{\sqrt{2}}(r_1 - r_2)$, 

$$R = \frac{1}{\sqrt{2}}(r_1 + r_2):$$

$$H_{OSC} = H_{CM} + H_{REL} = \left(-\frac{\hbar}{2m} \nabla^2 + \frac{1}{2} m \omega^2 R^2\right) + \left(-\frac{\hbar}{2m} \nabla^2 + \frac{1}{2} m \omega^2 r^2\right).$$

One can see that the center-of-mass motion separates and is independent of the relative motion. We assumed $H_{SR}$ to be spherically symmetric $H_{SR}(r) = H_{SR}(r)$ (more about that in the “Results” section). $H_{DD}$ is given as

$$H_{DD} = \frac{\mu_0 (\mu B) J}{4\pi |r|^3} [F_1 \cdot F_2 - 3(F_1 \cdot n)(F_2 \cdot n)],$$

where $n = \frac{r}{|r|}$, $\mu_0$ is the vacuum magnetic permeability, $\mu_B$ is the Bohr magneton, $g_j$ is Lande $g$-factor and $F_1, F_2$ are spin operators. Spin-statistic theorem says that in a free oscillator potential takes a simple form:

$$H_{DD} = -g_j \mu_B B \cdot F = -g_j \mu_B B F_z,$$

where $F_z = F_{1z} + F_{2z}$ (the external field is directed along the z-axis). $[H_B, J_z] = 0$, but generally $[H_B, J] \neq 0$. Thus, the eigenstates of the system have a well-defined $m_j$ and parity of $L$ and $F$ (which will be denoted by $p = \sigma/e$ in the superscript). The eigenstate can be written as

$$\Psi_{nmjl} = \sum_{f,m,l} \phi_{nmjl}(r)|f, m_f, l, m_l = m_j - m_f\rangle,$$

where $\phi_{nmjl}(r)$ are radial functions. $f$ is limited because $f \leq |f_1 + f_2|$, but $l$ is not (the only restriction is $l \geq |m_j - m_f|$, so the sum has infinitely many terms. Let us remind the reader that, for a pure harmonic oscillator Hamiltonian, the eigenvalues are given as $E_n = \hbar \omega (2n + l + \frac{1}{2})$, so the energy grows linearly with $l$. As we expect that eigenstates of our system are combinations of functions, which are quite similar to the eigenstates of a free oscillator and we are interested in a few lowest states, it is reasonable to cut the sum over $l$ at a suitable value $l_{max}$. This value is determined by numerical tests:

$$\Psi_{nmjl} = \sum_{f,m_f,l} \phi_{nmjl}(r)|f, m_f, l, m_l = m_j - m_f\rangle.$$  

Thus we arrive at a finite system of equations for radial functions $\phi_{nmjl}(r)$, which we can write down using harmonic oscillator units $\hbar = \hbar = \omega = 1$ and introducing dimensionless $g_{dd} = \frac{\mu_0 (\mu B) J}{8\sqrt{2\pi}} \sqrt{\frac{m_0 \omega}{\hbar}}$, $b = \frac{g_j \mu_B B}{\hbar}$,

$$\chi_{nmj}^{m_f,m_l}(q) = q \phi^{m_j,m_f,m_l}_{nmj}^{m_f,m_l}(q):$$

$$-1 \frac{d^2}{dq^2} \chi_{nmj}^{m_f,m_l}(q) + \frac{1}{2} g^2 \chi_{nmj}^{m_f,m_l}(q)$$

$$+ l(l+1) \chi_{nmj}^{m_f,m_l}(q) + H_{SR}(q).$$

$$+ \frac{g_{dd}}{q} \sum_{f,m_f,l} \alpha_{f,m_f,l} \chi_{nmj}^{m_f,m_l}(q)$$

$$b n_f \chi_{nmj}^{m_f,m_l}(q) = E_{nmjl}^{m_f,m_l} \chi_{nmj}^{m_f,m_l}(q),$$

where $\alpha_{f,m_f,l}^{m_f,m_l}$ coefficients are the matrix elements

$$\alpha_{f,m_f,l}^{m_f,m_l} = \langle f,m_f,l' | \sum_{F_1, F_2} | F_1 \cdot F_2 - 3(F_1 \cdot n)(F_2 \cdot n) | f,m_f,l \rangle.$$  

**Results.** - We consider the case of fermionic dysprosium atoms with total internal angular momentum $F_1 = F_2 = \frac{3}{2}$ in a harmonic trap of frequency $2\pi \cdot 30$ kHz, for which $g_{dd} = 0.0018$. For calculations we assume that the $H_{SR}(q)$ potential takes a simple form:

$$H_{SR}(q) = \begin{cases} +\infty, & \text{for } q < a_0 = 89 r_0, \\ 0, & \text{for } q > a_0 = 89 r_0, \end{cases}$$

where $r_0$ is the Bohr radius and $a_0$ is the s-wave scattering length [19]; its value in dimensionless harmonic oscillator units is $a_0 = 0.12$. We do it for a few reasons: firstly, although the short-range potential is rather complicated, in cold atoms experiments only the s-wave scattering length matters so realistic radius dependence is not essential. It is known that the s-wave scattering lengths for dysprosium [19] are different in different scattering channels. With its precise values not known, we are simplifying our task by assuming only one, universal scattering length $a_0$. What is more, this approach lets us neglect the electric interactions (which are of quadrupole-quadrupole type or weaker). At last, we are interested in cases when $g_j \mu_B B \approx \hbar \omega \rightarrow B \approx \frac{\hbar}{g_j \mu_B} \approx 0.1$ G. This value of $B$ is far away from magnetic Feshbach resonances [20], so it is justified to assume that the value of $a_0$ does not depend on $B$.

Generally, the interaction energies $H_{DD}$ and $H_{SR}$ are much smaller than the others, so for most $b$ values the eigenenergies are very close to energies of sates $|n, f, m_f, l, m_l\rangle$, where $n$ is the harmonic oscillator number. For states with $l > 0$ any changes generated by $H_{SR}$ are practically negligible. For $s$-states a change of the shape of eigenfunction is significant, but the value of energy shift is much smaller than $\hbar \omega$. Therefore, the energy in a very crude approximation can be written as

$$E_{nmj}^{f,m_f,m_l} \approx \left(\frac{3}{2} + 2n + l\right) - bf_j.$$  

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Fig. 1: (Colour online) Energies of eigenstates vs. magnetic field. Anticrossings of 121 eigenstates occur for $B \approx \frac{\hbar \omega_{gj}}{g_j \mu_b} (b \approx 1)$. The strong asymmetry can be explained by the fact that the short-range interaction increases the eigenenergies of states with low $\langle L^2 \rangle$ value and nearly does not change the energy of states with high $\langle L^2 \rangle$.

Fig. 2: (Colour online) Difference between energies of the first excited states and the ground state for four cases. One can notice that the $b$ value, for which the curve starts to be flat, depends strongly on $a_0$, and the size of gap is determined by the $g_{dd}$ value.

(Note that we used this approximation only to find the value of $b$, for which interesting effects occur. Of course, in our calculations we include all interactions.) One can note that for $b = 1$, every state $|0, f, m_f, m_f+20, -(m_f+20)\rangle$ (in the absence of dipole-dipole and short-range interactions) would have the same energy $\frac{\hbar \omega_{gj}}{2}$. This means that here anticrossings [21,22] (generated by $H_{DD}$ interaction) must occur.

In fig. 1 we plotted the eigenenergies of the full Hamiltonian as a function of the dimensionless magnetic field $b$. One can note that the plot is definitely asymmetrical for reflection with respect to the vertical $b = 1$ line. The reason for this is the property of the short-range interaction. This interaction increases the energy of states with low $l$ values and almost does not change the energy of states with high $l$ — in the second case the atoms do not reach the hard sphere because of the strong centrifugal barrier. This is the reason for the observed asymmetry.

In fig. 2 we plotted the value of a difference between eigenenergies of the first excited state and the ground state (for broader range of $b$ than in the previous plot). Besides our case (blue, tiny dashed curve), we have considered here three other examples (note that the red dashed curve ($a_0 = 0.04, g_{dd} = 0.0006$) responds to dysprosium atoms...
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Fig. 3: (Colour online) Time evolution of the initial ground state for three different times of transition through the anticrossing region. Time $\Delta t$ is the time of changing the magnetic-field value $b$ from 0.99 to 1.01. On the main chart the distribution of atoms’ relative position is presented—-at the beginning (left, the same for all cases) and at the end of the evolution. On the right there are histograms of various orbital momentum contributions to the final state.

Fig. 4: (Colour online) Energies of eigenstates vs. magnetic field for two other cases. The plot for fermions with the odd parity of $l$ and $f$ is quite different from the previous one (fig. 1) —here there is no eigenstate with energy strongly increased by the short-range interaction (no state $l=0$). On the other hand, the plot for bosons is very similar to the dysprosium case.

in trap with frequency $2\pi \cdot 3$ kHz; two others are purely hypothetical. We can observe two effects: in every single case, for some range of $b$, $\Delta E$ is flat. At the beginning of this range anticrossing between state with $l=0$ and the lowest state with $l=2$ occurs. The energy gap here is really broad, so it is not significant for further considerations. One can notice that the position of this point depends strongly on the $a_0$ value —indeed, for higher $a_0$ the energy of the ground state is more up-shifted, so it will meet the first excited stated for lower $b$ value. What will be really important is the very narrow energy gap for the $b$ value close to 1.00. The size of the gap depends only on the $g_{ld}$ value. Now let us consider the evolution which leads to the EdH effect. It is fully determined by the Schrödinger equations written in the time-dependent bases [23]:

$$\Psi(t) = \sum_n c_n(t) \psi_n(t)e^{i\theta_n(t)},$$

$$\theta_n(t) = -\frac{1}{\hbar} \int_0^t E_n(t')dt',$$

$$\dot{c}_m(t) = -c_m(\psi_m|\dot{\psi}_m) - \sum_{n \neq m} c_n \frac{\langle \psi_m|H|\psi_n \rangle}{E_n - E_m} e^{i(\theta_n - \theta_m)},$$

(11)

We start with $b = -0.2$. The ground state of the whole system is $|0, 20, -20, 0, 0\rangle$, so the parity of $l$ and $f$ (even) and the value $m_j = -20$ are fixed for further evolution. If the fully adiabatic transition was possible the maximal
value of $l = 40$ could be reached. The time scale for the perfect adiabatic transition is of the order of a reciprocal value of the gap at the lowest anticrossings and it is longer than 15.5. Therefore we looked at the evolution for the realistic time scales. In fig. 3 we present the time evolution for three cases. $\Delta t$ is the time of changing the magnetic field $b$ from 0.99 to 1.01; the time of changing the magnetic-field value from $-0.2$ to $0.99$ can be much shorter, because there is no narrow anticrossing for these $b$ values (the reciprocal of the difference between the energies of the two lowest states is bigger than $0.001 \text{s}$), so the state remains nearly unchanged. In the case of $\Delta t = 1.0 \text{s}$, ($L^2$) as large as 600 is achieved. As is shown in the right panel of fig. 3, the dominant contributions to the final state in this case are for $20 \leq l \leq 30$. Hence, the relative motion in this case is confined to a relatively thin ring.

In fig. 4 we plotted the eigenenergies for the two other cases: for the odd parity of $l$ and $f$ (instead of even) and for bosons (instead of fermions). One can notice that changing the parity of $l$ and $f$ makes significant modification of the $E(b)$ relation—in case of odd parity there is no state with energy highly increased by the short-range interaction. The plot is much closer to be symmetrical for the reflection with respect to the vertical $b = 1$ line. On the other hand, when we consider two bosons instead of two fermions (with similar value of spin), the plot looks almost the same as in the previous case.

Conclusions. — We have presented a simple calculation of the Einstein-de Haas effect for a system of two dysprosium atoms trapped in a spherically symmetric harmonic potential and subject to a time-dependent magnetic field. The dipole-dipole interaction couples the spin variables to the orbital angular momentum of the relative motion. A large spin of dysprosium atoms allows to pump a significant amount of the orbital angular momentum into the system. The narrow anticrossings limit the highest achievable $l$ if we limit ourselves to realistic time scales of the magnetic-field switching.

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