Crack initiation mechanisms and threshold values of very high cycle fatigue failure of high strength steels

Daniel Spriestersbach\textsuperscript{a}\textsuperscript{*}, Patrick Grad\textsuperscript{a}, Eberhard Kerscher\textsuperscript{a}

\textsuperscript{a}University of Kaiserslautern, Materials Testing, Gottlieb-Daimler-Str., 67663 Kaiserslautern, Germany

Abstract

Fatigue failure of high-strength steels still occurs beyond $10^7$ cycles in the very high cycle fatigue (VHCF) regime \cite{1}. The reason for this late failure is that the fatigue properties in the long life region are strongly affected by flaws like non-metallic inclusions inside the material \cite{2}. In the case of VHCF failure a characteristic fine granular area (FGA) which is responsible for subsequent crack initiation can be observed in the vicinity of the inclusion on the fracture surface \cite{3}. It is still unclear how different inclusion types affect the initiation mechanism. Our study aims to clarify the influence of different inclusion types on the crack initiation in the very high cycle fatigue regime. Additionally, the threshold value for crack initiation as a result of FGA formation shall be revealed. For this purpose ultrasonic fatigue tests ($R = -1$) of the high-strength steel 100Cr6 were carried out until an ultimate number of cycles of $10^9$. Additionally, runout specimens were tested repeatedly with higher stress amplitudes until failure occurred. By this method an inclusion/flaw type dependent threshold value for the formation of the FGA was derived from fracture surfaces and stress amplitudes. The lowest threshold value indicates an absolute threshold value for VHCF crack initiation due to FGA formation.

© 2014 The Authors. Published by Elsevier Ltd. Selection and peer-review under responsibility of the Politecnico di Milano, Dipartimento di Meccanica.

Keywords: Very high cycle fatigue; crack initiation; non-metallic inclusions; threshold value; high strenght steels, artificial flaws

* Corresponding author. Tel.: +49-(0)631-205-5536; fax: +49-(0)631-205-5261.
E-mail address: spriestersbach@mv.uni-kl.de
1. Introduction

Fatigue tests of different high-strength steels show that for these materials no classical fatigue limit exists and that failure still occurs beyond an ultimate number of cycles of $10^7$ even below the classic fatigue limit for this ultimate number of cycles. As a result of requirement for increasing fatigue lives of structural components especially in the transportation and energy sectors where high reliability and low weight are important in recent years more and more research focused on the so called very high cycle fatigue (VHCF) for numbers of cycles larger $10^7$ [1, 2]. The reason for this late failure are typically non-metallic inclusions which act as stress raisers inside the matrix [3]. To estimate this effect the maximum stress intensity factor (SIF) $K_{\text{max},i}$ at an inclusion can be used [4]. The SIF is calculated by the cross section area of the initiating inclusion measured on the fracture surface, the applied maximum tensile stress $\sigma_o$ and a constant $C$ ($C = 0.65$ for surface inclusions and $C = 0.5$ for subsurface inclusions) as follows [4]:

$$K_{\text{max},i} = C\sigma_o\sqrt{\pi \text{area}}$$  \hspace{1cm} (1)

If a fatigue crack initiates at an inner inclusion under the surface normally a ring-like smooth fracture surface in the vicinity of the inclusion is build, the so called fisheye. In the VHCF regime the fracture always takes place at inner inclusions with a SIF smaller than the respective threshold value for propagation of a long crack $K_{\text{th}}$ and fracture is accompanied by an additional characteristic fine granular area (so called FGA) on the fracture surface in the vicinity of the initiating inclusion [5, 6]. The crack within the FGA grows until the SIF at the edge of the FGA reaches the above mentioned threshold value $K_{\text{th}}$ [5-8]. Then, the crack grows by forming a fish-eye. Thus, the formation of the FGA in the vicinity of an inclusion represents the initiation of a propagable long crack and is responsible for late crack initiation in the VHCF regime.

There are numerous findings in literature that different inclusion types show different crack initiation mechanisms. Lankford et al. [9, 10] and Furuya et al. [11, 12] for instance observed that due to local plasticity around the inclusions even below the yield strength aluminum oxide inclusions detach from the steel matrix before the crack initiation takes place at the matrix/inclusion interface. In this case the crack initiates inside the matrix and not inside the inclusion. In contrast titanium nitride inclusions show stronger cohesion forces to the matrix. Thus, they break instantly without detaching [11, 12]. Thus, if the stress is high enough to cause the fracture of the inclusion the initiation instantly takes place inside the inclusion. Afterwards the crack propagates across the matrix/inclusion interface into the matrix. Bonas et al. [13] also observed similar differences in crack initiation behavior between titanium nitride and calcium oxide inclusions. They noticed that titanium nitride inclusions break during loading while calcium oxide inclusions detach as mentioned by [9-12, 14]. These varying initiation mechanisms are not considered in SIFs calculated with equation (1). Thus, based on the fact that failure at different inclusion types is based on these different microstructural mechanisms of crack initiation, it is thinkable that this can also lead to varying thresholds and lifetimes for different inclusion types. According to these variations in crack initiation behavior Monnot et al. [15] state that beside the size the chemical composition of an inclusion plays an important role for the fatigue failure. Their research shows that titanium nitride inclusions are about as harmful as inclusions containing aluminum oxide, although the latter are several times larger. But till now it is not possible to estimate the impact of the inclusion type on the fatigue limit. Thus, for an accurate fatigue life prediction in the VHCF-regime and for the determination of a fatigue limit the influence of the inclusion type has to be clarified.

Within the scope of this work the threshold values of the stress intensity factor $K_{\text{th},\text{FGA}}$ for the basic mechanism which is the reason of the FGA formation and the resulting crack initiation shall be found in order to get a better understanding of the VHCF failure. With these values it will be possible to determine a fatigue limit for the VHCF -regime depending on the inclusions contained inside the material. In this context we also want to illuminate whether the chemical composition of the inclusion influences the crack initiation in the VHCF regime. During fatigue experiments it is not possible to predict or locate the initiating inclusion before the failure occurs. As a result it is hard to find accurate threshold values for failure at subsurface inclusions. In order to define the point of initiation artificial defects can be produced at the surface. Various researchers indicate that fatigue failure inside the volume is comparable to failure in a vacuum environment [16-18]. Thus, to simulate the conditions of subsurface failure at the surface, fatigue tests were performed in ultra-high vacuum with artificial flaws as crack initiation sites. By this method the position of the crack initiating flaw, the applied stress intensity factor, and as a result the expected life-
time can be clearly defined before each test. Thus, it might be possible to confirm and refine the threshold values found for non-metallic inclusions in the VHCF-regime. Further it would be possible to observe the fracture mechanism in the VHCF-regime in-situ previous to the final failure.

2. Experimental Procedure

2.1. Material and specimens

The material used in this study is the high carbon-chromium bearing steel 100Cr6 (material number 1.3505, similar to SAE 52100 or JIS SUJ2). The fatigue specimens have an hourglass-shape with a minimum diameter of 4 mm in the center and a stress concentration factor of 1.027 (see Fig. 1a). They were machined in annealed condition with radial and axial oversize. The specimens were heat treated to a martensitic or a bainitic condition. The martensitic specimens with a final hardness of 775 HV 10 were austenitized for 20 minutes at 840°C quenched in oil and finally tempered at 180°C for 2 hours (microstructure see Fig. 1b). For the bainitic microstructure the specimens were austenitized for 20 minutes at 855 °C, cooled down rapidly and then held for 6 hours at 220 °C in a salt bath. This treatment results in a lower bainite with an almost uniform hardness of 780 HV 10 (see Fig. 1c). After the heat treatment the specimens were manufactured into final shape by cylindrical grinding. To reduce residual stresses as a result of the grinding procedure the center of the specimen’s gauge length was polished after grinding. The remaining residual stresses of -250 MPa decline to 0 MPa at about 10 μm below the surface. Furthermore, the lower surface roughness reduced the probability of crack initiation caused by surface defects which were created during the machining process. To locate and simulate subsurface failure hemispherical artificial defects (AD) with a radius \( r \approx 25 \mu m \) are induced into the surface at the center of some specimens by laser sublimation treatment (see Fig. 1a). By the use of ultra-short laser shots this treatment makes it possible to remove material without affecting the microstructure.

![Fig. 1: a) ultrasonic fatigue specimen with the positioning of an artificial defect and SEM-image of a lateral cut through the defect; b) SEM-image of the martensitic microstructure; c) SEM-image of the bainitic microstructure](image)

2.2. Testing facility and procedure

Push-Pull fatigue tests (\( R = -1 \)) were carried out on an ultrasonic piezoelectric fatigue testing device at a frequency of about 20 kHz. The fatigue tests were performed in an open environment at room temperature. Tests with artificial surface defects were performed in ultra-high vacuum (\( p < 10^{-6} \) mbar). The experimental set-up of these vacuum tests is described more detailed in [19]. To limit the heat development of the specimens due to the high testing frequency to \( \Delta T < 15 \) K the specimens were tested by ultrasonic pulse-pause cycles and additionally cooled with compressed air during the tests. The temperature was controlled during the entire test by an infrared temperature sensor. Runout specimens which reached the ultimate number of load cycles of \( 10^9 \) were tested again until \( 10^9 \). In these following tests the stress amplitude was increased \( (\sigma_{a,new} = \sigma_{a,runout} + 50 \) MPa). If again no failure occurred after \( 10^9 \) cycles, this procedure was repeated until fracture occurred. The fracture surfaces of the failed specimens were analyzed and measured for fracture-mechanics analysis with a scanning electron microscope (SEM). In addition energy dispersive x-ray spectroscopy (EDX) was used to determine the chemical composition of the non-metallic inclusions at the fracture origin.
3. Results and Discussion

Fig. 2a presents the S-N data of the push-pull fatigue tests for smooth specimens with martensitic (M) and bainitic (B) microstructure and for bainitic specimen with an artificial surface defect tested in vacuum atmosphere. Resulting from the size of the artificial defects failure for these specimens occurs at lower stress amplitudes. Overall the fatigue behavior of the two variants is very similar. Failure for smooth specimen was always initiated at non-metallic inclusion. If an artificial defect was placed on the surface failure was initiated at this surface flaw at all times. The data points for smooth specimens are separated into the different groups regarding the inclusion types responsible for the failure. At high stress amplitudes and as a result low number of cycles cracks are always initiated at surface inclusions. The S-N curve shows that there is a change of crack initiation from surface inclusions to subsurface inclusions at $10^5$ cycles. The inclusions found within the scope of this work consist of TiN, AlCaO, CaO or seldom MgO. The different kinds of inclusions are denoted in this paper by their main chemical components. The used notation does not reflect their exact chemical composition. The size distribution of the inclusions found at the fracture surfaces are shown in Fig. 2b. It can be seen that the cross-section area $A$ of the inclusions in general range from 45 $\mu$m$^2$ for the smallest TiN up to 2700 $\mu$m$^2$ for the largest AlCaO. AlCaO- and MgO-inclusions are usually much bigger and provide a larger scatter than the other inclusion types. As a result fatigue fracture for those inclusions occurs at lower stress amplitudes than for TiN or CaO. The scatter of the inclusion size leads to similar scatter in fatigue lives. It can be observed that no failure at AlCaO-inclusions occurs far beyond $10^7$ cycles while TiN and CaO still cause fractures at nearly $10^9$ cycles. Thus, the fatigue lives of the specimens in the VHCF regime seem to depend on the crack initiating inclusion type.

Fig. 2: a) S-N curves of martensitic (M) and bainitic (B) specimens for failure at different inclusion types and artificial defects; b) size distribution of crack initiating inclusions and artificial defects

Fig. 3a illustrates the SIF $K_{\text{max},i}$ calculated with Eq. (1) at the crack initiating inclusions as a function of the number of cycles to failure $N_f$ derived from the fatigue tests for martensitic (M) and bainitic (B) specimens. For surface inclusions and artificial surface defects tested in vacuum according to Murakami [4] Equation (1) for surface failure was used to derive the maximum SIF. Generally, the number of cycles to failure increases if the SIF at the inclusion or defect decreases. For smooth specimen surface failure was observed if the SIF at a surface inclusion that exceeded the threshold value for the propagation of long cracks $K_{\text{th}}$ [6] (dashed line). Subsurface inclusions with a SIF higher than the specific threshold value $K_{\text{th}}$ lead to the so called fisheye fracture. If the SIF of an inclusion at the crack origin is lower than $K_{\text{th}}$ for long cracks FGA formation occurs in the vicinity of the inclusion independent of its type. But the inclusion’s type seems to have a significant influence on the number of cycles to failure on the one hand and on the crack initiation in the VHCF regime on the other hand. The fatigue tests clearly show reduced scatter in fatigue lifetime if the inclusion’s type of the crack initiating inclusion is considered for the analysis of the
fatigue behavior. Each inclusion type shows an individual correlation between the SIF and the resulting fatigue life. Thus, the number of cycles to failure at a given SIF is smaller for TiN as for AlCaO or CaO. AlCaO in general do not lead to failure in the VHCF-regime. It is furthermore obvious that the correlation for every inclusion type converges to its own threshold value in the VHCF-regime. Below these threshold values, here called $K_{th,FGA}$, no failure occurs until $10^9$.

In order to simulate the subsurface failure at non-metallic subsurface inclusions artificial defects with a morphology and size comparable to AlCaO inclusions were induced. The SIFs determined for artificial flaws are higher than those for subsurface inclusions with comparable fatigue life. Until now it is not completely clarified whether Equation (1) for surface failure lead to accurate values in this case. Apart from the numerical values from the fracture mechanical evaluation the specimen with artificial defects show in vacuum comparable fatigue behavior as specimen failing at subsurface inclusions. The fatigue tests show that if the calculated SIF undercuts a crack growth threshold value (in our case 4.5 MPam$^{1/2}$), comparable to $K_{th}$ observed for inclusions, FGA-like structures occur at the fracture surface in the vicinity of the artificial defects. Like for subsurface inclusions this fracture mechanism is accompanied with increasing numbers of cycles to failure. Additionally the failure in the VHCF-regime at artificial defects also seems to converge to a threshold value for FGA formation. Thus, even if the fracture mechanical evaluation is difficult the test with artificial defect can help to understand VHCF-failure and the responsible mechanisms.

To confirm the threshold values for FGA formation in the VHCF-regime derived from fatigue tests runout specimen were tested again at higher stress amplitude (+ 50 MPa). If they reached the ultimate number of cycles with the higher stress amplitude the stress was raised again. This procedure was repeated until fracture occurred. Then the SIF at the crack initiating inclusion could be calculated subsequently for each stress level. Fig. 3b shows the results of these runout tests. Stacked data points represent the different stress levels of one runout test.

Fig. 3: a) stress intensity factors at the crack initiating inclusion or artificial defect as a function of $N_f$; b) stress intensity factors of runout specimen (marked with an arrow) and retested runout specimen with number of cycles to failure at the last stress level ordered by the inclusion type.

The runout tests show that failure occurs as soon as the SIF at the inclusions exceeds an inclusion’s dependent threshold value like expected from fatigue test. These threshold values for the re-stressed specimens are calculated by average of the mean values of the stress intensity factor leading to failure and stress intensity factor of the highest stress level at which the specimen did not fail. Threshold values for the fatigue tests in Fig. 3a are set to the lowest stress intensity factor leading to failure. The dashed lines in Fig. 3 and Table 1 illustrate that the threshold values for the VHCF regime determined by runout test match well with the values of the fatigue test. The VHCF failures at TiN inclusions as a result of FGA formation occur at smaller stress intensity factors than those of CaO inclusions and smaller than those of AlCaO inclusions. In case of the runout test for the artificial defect the last stress level led
directly to the propagation of a long crack without formation of an FGA. Thus, the threshold for FGA-formation in the VHCF-regime in this case could not be clarified doubtlessly and additional tests are necessary.

In connection with the runout test it is necessary to discuss the influence of the earlier stress levels on the final level. Is there any pre-damage or even work hardening around the defects? In case of TiN prior to the fracture of the inclusion there is no stress increase in the inclusion’s surrounding steel matrix [20]. As a result pre-damage can be neglected for the runout test. This hypothesis is supported by the fact that the fatigue lives after runout and load increase are in line with the one step fatigue tests. For AlCaO and CaO inclusions damage accumulation cannot be neglected that easily. In the vicinity of these inclusions the stress inside the matrix increases by a factor of 2 and plasticity in the inclusion’s surrounding matrix is possible already at lower stress levels [21]. But only CaO inclusions show slightly shorter lives in restressed runout tests compared to the one step fatigue tests. Additionally, for AlCaO and CaO inclusions as well as for artificial defects it sticks out that the threshold values determined by restressed runout tests are slightly higher than expected from fatigue test. This might indicate that work hardening takes place in the matrix material around the inclusions at lower stress levels as long as the plasticity is too low for damage initiation. This seems to increase the threshold value slightly.

Table 1. Comparison of the threshold values for different inclusion types derived from one step fatigue tests and runout tests for an ultimate number of cycles of $10^9$.

| Inclusion type | AlCaO | MgO | CaO | TiN | AD |
|----------------|-------|-----|-----|-----|----|
| $K_{th,\text{FGA}}$ (one step) in $\text{MPam}^{1/2}$ | 3.3   | -   | 2.4 | 2.2 | 4.4 |
| $K_{th,\text{FGA}}$ (runout) in $\text{MPam}^{1/2}$ | 3.55  | 3.3 | 2.62| 2.17| 4.78|

The different threshold values of each inclusion type can be explained with the different crack initiation mechanisms observed at the fracture surfaces and by other researchers as mentioned in the introduction. AlCaO or CaO inclusions in general stick loosely to one of the two fracture surfaces, break to pieces, or are missing (see Fig. 4a). They decay or detach and can then be treated like a void in the matrix. As a result the fatigue behavior and the initiation mechanisms of the inclusions should be comparable to the induced artificial surface flaws (see Fig. 4b). If the local plasticity around the inclusion or around the surface defect is high enough cracks can initiate at the voids. Then crack initiation and FGA formation take place at the void which can provide multiple initiation sites. Thus, the AlCaO or CaO inclusions mostly show multiple crack layers. TiN inclusions in contrast have a strong cohesion force to the matrix and as a result they do not detach. The stress concentration at TiN is located in side the inclusion and not in the matrix [20]. As a result TiN inclusions break at low SIFs and provide instantly a sharp crack. In accordance with this they show only one crack layer and one half of the TiN inclusion was always left on each fracture surface (Fig. 4c). After the failure of the inclusion the crack propagates across the matrix inclusion interface into the matrix and forms a FGA. This early initiation and the sharp crack tip can explain the shorter fatigue lives of TiN compared to oxide inclusions at identical SIF.

Fig. 4: a) fracture surface of an AlCaO inclusion detached from the matrix with FGA structures; b) fracture surface of an artificial defect with FGA structures in its vicinity; c) fracture surface of a broken TiN inclusion
4. Summary

The chemical composition of an inclusion has an influence on its mechanical properties and the interaction with the matrix during fatigue. This leads to different crack initiation mechanisms for different inclusion types. AlCaO and CaO detach from matrix or decay and can further be regarded as a pore in the matrix. The crack then initiates at the equator of this pore and propagates into the matrix. At TiN inclusions the crack initiates inside the inclusion. The inclusion instantly breaks if the applied load is high enough and provides a perfect sharp crack inside the material. Strong cohesive forces between inclusion and matrix enable the crack to propagate across the interface into the matrix. This differing initiation mechanisms result in different threshold values of stress intensity factors for an ultimate number of cycles to failure of $10^9$. These threshold values were verified both by one step fatigue test and under negligence of damage accumulation by restressed runout test. The fracture mechanisms also explain the differences in fatigue lives if comparable SIF are considered. TiN inclusions instantly break and provide a crack at which FGA formation can take place. At oxide inclusions in contrast the crack has to initiate at the pore after the inclusion has detached from the matrix. In accordance fatigue lives are comparatively shorter for fracture from TiN inclusions at identical SIFs. Artificial surface flaws tested in ultra-high vacuum are well suited to simulate the VHCF-fatigue mechanisms even if the fracture mechanical evaluation is not fully assured.

Acknowledgements

This research was carried out in the framework of the German Research Foundation (DFG) priority program 1466 Infinite Life. The authors would like to thank the DFG for the financial support of this work.

References

[1] M. Bacher-Hoechst, S. Issler, How to Deal with Very High Cycle Fatigue (VHCF) Effects in Practical Application, in: C. Berger, H.J. Christ (Eds.) VHCF 5, DVM, Berlin, 2011, pp. 45-50.
[2] S. Kovacs, T. Beck, L. Singheiser, Influence of mean stresses on fatigue life and damage of a turbine blade steel in the VHCF-regime, International Journal of Fatigue, 49 (2013) 90-99.
[3] T. Sakai, Review and Prospects for Current Studies on Very High Cycle Fatigue of Metallic Materials for Machine Structural Use, Journal of Solid Mechanics and Materials Engineering, 3 (2009) 425-439.
[4] Y. Murakami, S. Kodama, S. Konuma, Quantitative evaluation of effects of non-metallic inclusions on fatigue strength of high strength steels. I: Basic fatigue mechanism and evaluation of correlation between the fatigue fracture and the size and location of non-metallic inclusions, International Journal of Fatigue, 11 (1989) 291-298.
[5] P. Grad, B. Reuscher, A. Brodyanski, M. Kopnarski, E. Kerscher, Mechanism of fatigue crack initiation and propagation in the very high cycle fatigue regime of high-strength steels, Scripta Materialia, 67 (2012) 838-841.
[6] T. Sakai, Y. Sato, N. Oguma, Characteristic S–N properties of high-carbon–chromium-bearing steel under axial loading in long-life fatigue, Fatigue & Fracture of Engineering Materials & Structures, 25 (2002) 765-773.
[7] K. Shiozawa, Y. Morii, S. Nishino, L. Lu, Subsurface crack initiation and propagation mechanism in high-strength steel in a very high cycle fatigue regime, International Journal of Fatigue, 28 (2006) 1521-1532.
[8] P. Grad, E. Kerscher, Fatigue Crack Paths in the VHCF-regime of 100Cr6, in: 4th International Conference on Crack Paths Italy, 2012, pp. 401-408.
[9] J. Lankford, Initiation and early growth of fatigue cracks in high strength steel, Engineering Fracture Mechanics, 9 (1977) 617-624.
[10] J. Lankford, F.N. Kusenberger, Initiation of Fatigue Cracks in 4340 Steel Metallurgical Transactions, 4 (1973) 553-559.
[11] Y. Furuya, H. HIRUKAWA, T. KIMURA, M. HAYAISHI, Gigacycle Fatigue Properties of High-Strength Steels According to Inclusion and ODA Sizes, Metallurgical and Materials Transactions A, 38A (2007) 1722-1730.
[12] Y. Furuya, S. MATSUOKA, T. ABE, A Novel Inclusion Inspection Method Employing 20 kHz Fatigue Testing, Metallurgical and Materials Transactions A, 34A (2003) 2517-2526.
[13] H. BONAS, T. Linkewitz, P. Mayr, Analyse der Ermüdungsrisssbildung und Dauerfestigkeit des Stahles 100Cr6 im bainitischen Zustand, HTM, 57 (2002) 190-198.
[14] X. Xie, L. ZHANG, M. ZHANG, J. DONG, K. BAIN, Micro-Mechanical Behavior Study of non-metallic Inclusions in
P/M Disk Superalloy RENE’95, Superalloys, 451-458 (2004).
[15] J. Monnot, B. Heritier, J.Y. Cogne, Relationship of melting practice, inclusion type, and size with fatigue resistance of bearing steels, in: Effect of steel manufacturing processes on the quality of bearing steels, 1988, pp. 149-165.
[16] T. Billaudeau, Y.N., Support for an environmental effect on fatigue mechanisms in the long life regime, International Journal of Fatigue, 26 (2004) 839-847.
[17] T. Nakamura, H. Oguma, Y. Shinohara, The effect of vacuum-like environment inside sub-surface fatigue crack on the formation of ODA fracture surface in high strength steel, Procedia Engineering, 2 (2010) 2121-2129.
[18] J. Petit, C. Sarrazin-Baudoux, An overview on the influence of the atmosphere environment on ultra-high-cycle fatigue and ultra-slow fatigue crack propagation, International Journal of Fatigue, 28 (2006) 1471-1478.
[19] P. Grad, K. Schlick, E. Kerscher, VHCF under High Vacuum, Fortschritte in der Werkstoffprüfung für Forschung und Praxis, (2012) 237-242.
[20] S. Wölkerling, FEM-Berechnungen von Spannungen und Spannungsintensitätsfaktoren in und an Einschlüssen, in: University of Bremen, Bremen, Germany, 2007.
[21] A. Borbély, H. Mughrabi, G. Eisenmeier, H.W. Höppel, A finite element modelling study of strain localization in the vicinity of near-surface cavities as a cause of subsurface fatigue crack initiation, International Journal of Fracture, 115 (2002) 227-232.