Influence of packet loss on wireless feedback control systems as a disturbance

Takashi Ogura\(^{1, a)}\), Kentaro Kobayashi\(^2\), Hiraku Okada\(^2\), and Masaaki Katayama\(^2\)

\(^1\) Department of Electrical Engineering and Computer Science, Graduate School of Engineering, Nagoya University, C3–1(631) Furo-cho, Chikusa-ku, Nagoya 464–8603, Japan
\(^2\) Institute of Materials and Systems for Sustainability, Nagoya University, C3–1(631) Furo-cho, Chikusa-ku, Nagoya 464–8603, Japan
\(a)\) ogura@katayama.muee.nagoya-u.ac.jp

Abstract: The knowledge of characteristics of the influence of packet loss is important to design controllers and observers which can reduce this influence on wireless feedback control systems. This paper clarifies the characteristics of the influence of packet loss as a disturbance that causes a difference between transmitted information and used information at the received side. Through the analyses of the influence of packet loss on both controller and controlled object sides, we clarify the power spectrum density of the influence on the controller side is not constant. Numerical results show an well-known approach - \(H_\infty\) controller, which can reduce the influence of white disturbances, can not reduce the influence of packet loss. Our analyses will be useful for good modeling of the influence of packet loss to design controllers and observers.

Keywords: wireless feedback control systems, packet loss, disturbance, characteristic analysis

Classification: Network

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1 Introduction

Wireless feedback control systems, in which controllers and plants (controlled objects) exchange information through wireless channels, are expected to be applied in many industry fields [1]. However, data packet loss in the wireless communication deteriorates performances of control systems. For expanding industrial applications, it is necessary to reduce the deterioration of the control performances due to packet loss.

An approach to reduce the deterioration of control performances due to packet loss is to design controllers and observers which can reduce the influence of packet loss [2, 3]. One concept which has been studied in designing such controllers and observers is to consider the influence of packet loss as a disturbance [4, 5, 6]. Imai et al. [4] and Suhara et al. [5] proposed communication disturbance observer-based compensators to reduce the deterioration of control performances due to packet loss. In addition, we previously proposed a controller design method which use the result of time series analysis of the influence of packet loss [6]. However, these papers have not discussed and clarified the characteristics of the influence of packet loss as a disturbance even though the knowledge of characteristics of the influence of packet loss is important to design better controllers and observers.

This paper analyzes the influence of packet loss on both controller and controlled object sides, and clarifies the characteristics such as power spectrum density of the influence on both sides. Numerical results show that an $H_{\infty}$ controller, which is an well-known approach to reduce the influence of white disturbances, can not reduce the influence of packet loss, and also show that the power spectrum density of the influence on the controller side is not constant.

2 Wireless feedback control system

This paper deals with a wireless feedback control system that is discretized at a sampling interval $T_s$. At time $t = kT_s$ ($k = 0, 1, 2, \cdots$), each of the state information and the control information is expressed as $x[k]$ and $u[k]$. The plant is assumed to be linear time invariant system and is modeled by state equation as follows:

\[ \begin{align*}
    x[k+1] &= A x[k] + B u[k] + D d[k] \\
    y[k] &= C x[k] + E u[k]
\end{align*} \]
\[ x[k + 1] = Ax[k] + Bu[k] + w[k], \quad (1) \]

where \( A \) and \( B \) are coefficient matrices and represent the state space model of the plant. \( \hat{u}[k] \) is received control information at the plant side. \( w[k] \) is a white Gaussian random vector that represents disturbances caused in the controlled plant. The mean vector and covariance matrix of \( w[k] \) are assumed to be \( \mathbf{0} \) and \( \Sigma \), i.e., \( w[k] \) is a white disturbance.

The controller generates control information \( u[k] \) by using a difference between a target value vector \( r[k] \) which is inputted every \( T_s \) seconds and an estimated state information \( x_e[k] \). The detail of the controller is mentioned later. The estimated state information is given as follows:

\[ x_e[k + 1] = A\hat{x}[k] + Bu[k], \quad (2) \]

where \( \hat{x}[k] \) is received state information at the controller side.

Each of the state information and the control information is transmitted via wireless channels as a packet. Packet loss is assumed to occur randomly with probability \( p \) in the channels and to be surely detectable at received sides. The number of transmitted packets of state information (or control information) per second is \( 1/T_s \).

If the plant side detects packet loss of control information \( u[k] \), the input to the plant is zero as follows:

\[ \hat{u}[k] = \begin{cases} 0 & \text{if packet loss occurs} \\ u[k] & \text{otherwise} \end{cases}. \quad (3) \]

If the controller side detects packet loss of state information \( x[k] \), the estimator uses \( x_e[k] \) instead of \( x[k] \) as follows:

\[ \hat{x}[k] = \begin{cases} x_e[k] & \text{if packet loss occurs} \\ x[k] & \text{otherwise} \end{cases}. \quad (4) \]

2.1 \( H_{\infty} \) controller
This paper uses an well-known approach - \( H_{\infty} \) controller which can reduce the influence of disturbances by considering the power norm of disturbances. Fig. 1 shows a controller design block diagram of \( H_{\infty} \) control. In this controller design, each the influence of packet loss in state and control information transmission is
considered as a white disturbance such as \( W_x \) and \( W_u \). The controller \( K(s) \) is designed to minimize an \( H_{\infty} \) norm of closed-loop transfer functions from four exogenous inputs (\( R, W, W_x, W_u \)) to evaluation outputs (\( z_1, z_2 \)).

Control information \( u[k] \) is calculated by a state space equation as follows:

\[
\begin{align*}
h[k + 1] &= A_c h[k] + B_c (r[k] - x_c[k]), \\
u[k] &= C_c h[k] + D_c (r[k] - x_c[k]).
\end{align*}
\]

\( h[k] \) is a state variable of the controller. \( A_c, B_c, C_c, D_c \) are coefficient matrices of the controller. These coefficient matrices can be obtained by discretizing \( K(s) \) and then transforming the discrete transfer function to the state space equation.

3 Influence of packet loss as a disturbance

3.1 Packet loss of state information

The influence of packet loss of state information (\( w_x[k] \)) can be defined as a difference between transmitted information \( x[k] \) and used information \( \hat{x}[k] \).

\[
w_x[k] = x[k] - \hat{x}[k].
\]

When we focus on occurring packet loss at time \( k \), \( w_x[k] \) can be expressed as follow:

\[
w_x[k] = x[k] - x_c[k] = Ax[k - 1] + Bu[k - 1] + w[k - 1] - (Ax[k - 1] + Bu[k - 1]) = w[k - 1].
\]

From this equation, we can see that \( w_x[k] \) is \( w[k - 1] \). This means that \( w_x[k] \) can be considered as adding \( w[k - 1] \) again outside the plant. Because \( w[k - 1] \) is the white disturbance, it can be said that the power spectrum density of \( w_x[k] (S_{w_x}(f)) \) is constant.

3.2 Packet loss of control information

The influence of packet loss of control information (\( w_u[k] \)) can be defined as a difference between transmitted information \( u[k] \) and used information \( \hat{u}[k] \).

\[
w_u[k] = u[k] - \hat{u}[k].
\]

This equation can be rewritten by using Bernoulli random variable \( P_k \in \{0, 1\} \) \( \text{Prob}(P_k = 0) = p, \text{Prob}(P_k = 1) = 1 - p \) as follows:

\[
w_u[k] = (1 - P_k) u[k].
\]

From this equation, the power spectrum density of \( w_u[k] (S_{w_u}(f)) \) is as follows:

\[
S_{w_u}(f) = p^2 S_u(f),
\]

where \( S_u(f) \) is the power spectrum density of \( u[k] \). Generally, \( u[k] \) depends on the reference signal. Considering application areas of the wireless feedback control systems, the power spectrum of the reference signal will not be constant like the white disturbances. Therefore, it can be said that the power spectrum density of \( w_u[k] \) is not constant.
Computer simulations were performed to evaluate control quality of the control system with the $H_\infty$ controller to packet loss, and to show that the power spectrum density of $w[k]$ is not constant. Then, we verify that the $H_\infty$ controller can not reduce the influence of packet loss because this influence of control information is not the white disturbance. A rotary inverted pendulum was employed as the controlled plant. The pendulum’s state space model of Eq. (1) is based on REALTEC RTC05 [7]. The sampling interval $T_s$ is $10^{-2}$ [s]. The pendulum is controlled by applying a voltage $v_{in}[k]$ on the DC-motor for rotating the arm, i.e., $u[k] = [v_{in}[k]]$. The state information $x[k]$ has the pendulum angle $\theta[k]$, velocity $\dot{\theta}[k]$, and the arm angle $\phi[k]$ and velocity $\dot{\phi}[k]$. The rotary inverted pendulum is controlled to make its arm angle $\phi[k]$ follow a target value while keeping the pendulum in an upright position. The target value of the arm angle is a rectangular wave with values of 0 and $\pi/2$, period $T = 10$ [s]. The variances of $w[k]$ are set to $\sigma^2 = 10^{-6}$ and the covariances are set to zero. The simulation trials are repeated 1000 times and each trial lasted 1000 seconds.

This paper evaluates control performances to packet loss by comparing between the $H_\infty$ controller and an LQ controller which does not care packet loss [8]. Tuning parameters in both controller designs are set to be the same step response for fair comparison of control performances. The stability performance is evaluated by the pendulum fall rate, and the tracking performance is evaluated by root mean square error (RMSE) of the arm angle to the case that exists neither packet loss nor $w[k]$. If the angle of pendulum is $|\theta[k]| > \pi/6$, the pendulum is assumed to have fallen down. Once the pendulum has fallen down, each simulation run is terminated.

The performance comparisons of stability performance and tracking performance are shown in Fig. 2(a) and Fig. 2(b), respectively. Fig. 2(a) shows that the pendulum fall appears at less than $p = 0.2$ in the case of the $H_\infty$ controller, but the pendulum fall does not appear in $p = 0.2$ in the case of the LQ controller. This shows that the stability performance of the $H_\infty$ controller is inferior to that of the LQ controller design. From Fig. 2(b), we can see that RMSE of the $H_\infty$ controller is larger than that of the LQ controller design. Therefore, the tracking performance of the $H_\infty$ controller is inferior to that of the LQ controller. These comparison
results show the $H_\infty$ controller, which considers the influences of packet loss as a white disturbance, can not reduce the influence of packet loss.

Fig. 3 shows the power spectrum density of $w_u[k]$ ($S_{w_u}(f)$) in the case of $p = 0.2$. Obviously, there are peaks in the power density spectrum, that is, it is not constant. This is why the $H_\infty$ controller can not reduce the influence of packet loss.

5 Conclusion

This paper clarified the characteristics of the influence of packet loss as a disturbance on the wireless feedback control system. The influence of packet loss of state information depends on the disturbance which occurs in the plant. On the other hand, the influence of packet loss of control information depends on signals of control information, namely, the reference signal. Therefore, the power spectrum density of the influence of packet loss of control information is not constant. This is why numerical results show that the control performances of $H_\infty$ controller is inferior to that of LQ controller. These analyses will be useful for good modeling of the influence of packet loss to design controllers and observers.

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