Extragalactic jets are visualized as dynamic eruptive events modeled by time-dependent magnetohydrodynamic (MHD) equations. The jet structure comes from the temporally self-similar solutions in two-dimensional axisymmetric spherical geometry. The two-dimensional magnetic field is solved in the finite plasma pressure regime, or finite-$\beta$ regime, and it is described by an equation where plasma pressure plays the role of an eigenvalue. This allows a structure of magnetic lobes in space, among which the polar axis lobe is strongly peaked in intensity and collimated in angular spread compared to the others. For this reason, the polar lobe overwhelms the other lobes, and a jet structure naturally arises in the polar direction. Furthermore, within each magnetic lobe in space, there are small secondary regions with closed two-dimensional field lines embedded along this primary lobe. In these embedded magnetic toroids, plasma pressure and mass density are accordingly much higher. These are termed secondary plasmoids. The magnetic field lines in these secondary plasmoids circle in alternating sequence such that adjacent plasmoids have opposite field lines. In particular, along the polar primary lobe, such periodic plasmoid structure happens to be compatible with radio observations in which islands of high radio intensities are mapped.

Subject headings: accretion, accretion disks — MHD

1. INTRODUCTION

Collimated jets with high terminal velocities appear to be universal phenomena in astrophysics. They are often associated with young stellar objects, compact galactic objects, and active galactic nuclei (Livio 1997). These jets are always accompanied by accretion disks in the equatorial plane. The magnetosphere of the accretion disk contains a plasma that follows the accretion of the materials in the disk. The plasma density and pressure get higher as the accretion approaches the center, which drives the magnetic field. The dynamics of this system was first described in an MHD model by Blandford & Payne (1982). In this landmark paper, jets were pioneered by Low (1982a, 1982b, 1984a, 1984b) for astrophysical applications and solar corona mass ejections with pure radial plasma velocity flow. Variants of these solutions in cylindrical geometry to simulate time evolution of the jets (Ustyugova et al. 1995; Ouyed & Pudritz 1997; Krasnopolski et al. 1999) and the disk-jet system (Matsumoto et al. 1996; Kato et al. 2002).

Here, instead of a steady state model, we take the view that jets are a time-dependent spatial structure. What we see is only a snapshot of their state at this particular moment. Due to their galactic dimensions, the timescale of these structures is believed to be extraordinarily large, which gives the impression of a steady state structure. This implies that jets are the result of an eruption originating from the galactic nucleus. They could dissipate in time before another eruption takes place due to pressure built up from accretion. One eruption could be superimposed on an earlier event. To model the jet system, we do a self-similar analysis of the full time-dependent ideal MHD equations in spherical coordinates ($r, \theta, \phi$) with a mass $M$ at the nucleus. In particular, we consider axisymmetric solutions. This type of self-similar solutions were pioneered by Low (1982a, 1982b, 1984a, 1984b) for astrophysical applications and solar corona mass ejections with pure radial plasma velocity flow. Variants of these solutions include cases in which the plasma domain lies outside the mass $M$, such as interplanetary magnetic ropes in one- (Osherovich et al. 1993, 1995) and two-dimensional (Tsui & Tavares 2005) cylindrical geometry, interplanetary magnetic clouds (Tsui 2007), and atmospheric ball lightning (Tsui 2006) in spherical geometry. In these descriptions in which $M$ is outside the spherical domain of interest, the magnetic field is axisymmetric force-free and contains regions of closed field lines, while the plasma is spherically symmetric and decoupled from the magnetic field.

For the present case of extragalactic jets described by the mechanism of accretion-ejection, we follow the self-similar solutions of Low (1982a, 1982b, 1984a, 1984b) with the polytropic index $\gamma = 4/3$, but with particular emphasis on the finite plasma pressure. This current approach differs from the Keplerian disk plasma in that the radial flow is not tied to the Alfvén velocity a priori. In
this dynamic model, we consider jets as a manifestation of mass ejection on a galactic scale. The plasma pressure, in this self-similar MHD model, proves to have an important role in collimating the magnetic fields and the jet plasmas. The time evolution function gives a dynamic description of the high radial flow, especially in the jets. The self-similar solutions that converge at the center and at infinity give small regions of closed axisymmetric two-dimensional magnetic field lines where plasma density and pressure are much higher. These regions along the jets could correspond to the high-intensity islands in radio-frequency maps.

2. SELF-SIMILAR MHD

Following the accretion-ejection classical model, we also use the MHD equations to describe the plasma. Nevertheless, we retain the time dependence to write

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

\[
\rho \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{J} \times \mathbf{B} - \nabla \mathbf{p} - \rho \frac{GM}{r^3} \mathbf{r}, \tag{2}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B}), \tag{3}
\]

\[
\nabla \times \mathbf{B} = \mu \mathbf{J}, \tag{4}
\]

\[
\nabla \cdot \mathbf{B} = 0, \tag{5}
\]

\[
\frac{\partial}{\partial t} \left( \frac{p}{\rho^3} \right) + (\mathbf{v} \cdot \nabla) \left( \frac{p}{\rho^3} \right) = 0. \tag{6}
\]

Here, \( \rho \) is the mass density, \( \mathbf{v} \) is the bulk velocity, \( \mathbf{J} \) is the current density, \( \mathbf{B} \) is the magnetic field, \( p \) is the plasma pressure, \( \mu \) is the free-space permeability, and \( \gamma \) is the polytropic index. The bulk velocity consists of a radial and an azimuthal component in order to model the plasma outflow and the disk rotation. To be compatible with the physical situation, the meridian velocity is taken to be zero. For the radial component \( \mathbf{v} \mathbf{r} \), we seek self-similar solutions where the time evolution is described by the dimensionless evolution function \( \gamma(t) \). The radial profile is time invariant in terms of the radial label \( r = r(t) \gamma(t) \), which has the dimension of \( r \), such that \( \eta \) is independent of time, and corresponds to the Lagrangian radial position attached to a fixed fluid element. The label \( \eta \) corresponds to the radial position of the initial self-similar configuration that expands in time with radial Eulerian positions \( r(t) = \eta \gamma(t) \). The velocity can then be written as

\[
\mathbf{v} = \left( \eta \frac{dy}{dt}, 0, v_\phi \right). \tag{7}
\]

We consider a two-dimensional case with azimuthal symmetry in \( \phi \). In this case, the magnetic field, through the vector potential \( \mathbf{A} \), can be expressed in terms of two scalar functions \( P \) and \( Q \),

\[
\mathbf{B} = \frac{1}{r \sin \theta} \left\{ \frac{1}{r} \frac{\partial}{\partial \theta} (rA_\phi \sin \theta) + \frac{\partial}{\partial r} \left[ rA_\phi \sin \theta \left( \frac{\partial}{\partial r} (rA_\theta) - \frac{\partial}{\partial \theta} (A_\phi) \right) \right] \right\}
\]

\[
= \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (P), - \frac{\partial}{\partial r} (P), + Q \right]
\]

\[
= \nabla P \times \nabla \phi + Q \nabla \phi. \tag{8}
\]

We remark that the velocity field in the cylindrical steady state accretion-ejection model is a three-component field. This is necessary because the jets are generated by magnetocentrifugal motion of the planar disk plasma to the axial direction. Here, in this spherical dynamic model, the velocity field is a two-component field with \( v_\theta = 0 \), because the jets are generated by first pulling the disk plasma to the center and then redirecting it radially through eruptions.

By self-similar solutions in time, we mean a special class of time-dependent solutions where the time and space parts of the physical quantities are in a separable form. The time part is described by the evolution function \( \gamma(t) \), and the space part is solved self-consistently by separation of variables. The concept of self-similar dynamics is closely related to self-organized states that often have minimum energy under given constraints. Having this in mind, we now transform the independent variables from \((r, \theta, t)\) to \((\eta, \theta, \tau)\) and proceed to determine the explicit dependence of \( \gamma \) in each one of the physical quantities with this radial velocity. First, making use of equation (7), equation (1) becomes

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \nu) = \left( \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial r} \right) + \rho \left( \frac{\partial \nu}{\partial r} + \frac{2 \nu}{r} \right) = 0. \tag{9a}
\]

Considering the second equality, the first parentheses is the convective time derivative in Euler fluid coordinates, and this amounts to the time derivative in Lagrangian fluid coordinates. We, therefore, have

\[
\frac{\partial \rho}{\partial \eta} \frac{dy}{dt} + \rho \left( \frac{\partial \nu}{\partial \eta} + \frac{2 \nu}{r} \right) = \left( \frac{\partial \rho}{\partial \eta} + \frac{3 \rho}{y} \right) \frac{dy}{dt} = 0.
\]

Solving this equation for the \( \eta \) dependence gives

\[
\rho(r, t) = \frac{1}{y} \bar{\rho}(\eta, \theta). \tag{9b}
\]

As for equation (6), with \( F = (p/\rho^3) \) it follows that

\[
\frac{\partial F}{\partial t} + \nu \frac{\partial F}{\partial r} = \frac{\partial F}{\partial \eta} \frac{dy}{dt} = 0, \tag{10a}
\]

\[
\left( \frac{p}{\rho^3} \right) = F(r, t) = \frac{1}{y^3} \bar{F}(\eta, \theta). \tag{10b}
\]

Using the representation of equation (8), equation (3) is represented by the following two equations:

\[
\frac{\partial P}{\partial t} + \nu \frac{\partial P}{\partial r} = \frac{\partial P}{\partial \eta} \frac{dy}{dt} = 0, \tag{11a}
\]

\[
\frac{\partial Q}{\partial t} + \nu \frac{\partial Q}{\partial r} = \frac{\partial Q}{\partial \eta} \frac{dy}{dt} + \sin \theta \frac{\partial}{\partial \theta} \left( v_\phi \frac{1}{r \sin \theta} \frac{\partial P}{\partial r} \right). \tag{11b}
\]

The first equation gives the function \( P \) as

\[
P(r, t) = \frac{1}{y^3} \bar{P}(\eta, \theta). \tag{12}
\]

As for the function \( Q \), the right-hand side of the second equation vanishes with

\[
v_\phi = \omega_0 r \sin \theta. \tag{13}
\]
This amounts to a constant plasma rotation, which has to be distinguished from the Keplerian disk rotation of the solid material. Compared to the eruptive timescale of our dynamic approach of jets, this rotation rate is negligible, and we take \( \omega_0 = 0 \). As a result, we have

\[
\frac{\partial Q}{\partial t} + \frac{\partial}{\partial r} (r Q) = \frac{\partial Q}{\partial y} \frac{dy}{dt} + \frac{Q}{y} \frac{dy}{dt} = \left( \frac{\partial Q}{\partial y} + \frac{Q}{y} \right) \frac{dy}{dt} = 0,
\]

(14a)

\[
Q(r, t) = \frac{1}{y^3} \tilde{Q}(r, \theta).
\]

(14b)

As for the magnetic field components, they are given by

\[
B_r = \frac{1}{y^2} \tilde{B}_r(\eta, \theta) = \frac{1}{y^2} \frac{1}{\eta} \frac{\partial \tilde{P}}{\partial \eta},
\]

\[
B_\theta = \frac{1}{y^2} \tilde{B}_\theta(\eta, \theta) = \frac{1}{y^2} \frac{1}{\eta} \frac{\partial \tilde{P}}{\partial \eta},
\]

\[
B_\phi = \frac{1}{y^2} \tilde{B}_\phi(\eta, \theta) = \frac{1}{y^2} \frac{1}{\eta} \frac{\partial \tilde{P}}{\partial \eta}.
\]

3. SELF-ORGANIZATION

Before proceeding with the analysis, let us recall the fundamentals of self-organization, in particular, in MHD systems. We note that MHD equations, like Navier-Stokes equations, have quadratic invariants in the absence of dissipations. In MHD systems, there are three such invariants. They are the total energy (plasma and magnetic), the magnetic helicity, and the cross-helicity. Because of the existence of multiple invariants, the system tends to develop self-organized and self-similar states through dissipative processes, regardless of the details of the initial conditions (Hasegawa 1985). A simple example in fluid mechanics is the development of shock waves from an initial explosion. Another example is that fluid vortex rings (solitons) in air are often formed in fast upward drafts of smoke. Should we consider a thin layer of oil heated from below, a grid of complex highly organized hexagonal convective cells would develop regardless of the details of the initial conditions. We note that, for self-organized states, the memory of the initial conditions is lost. In other words, we cannot trace a self-organized state backward in time to its initial conditions. They are lost in the dissipative processes that lead to organization. The underlying physical arguments for self-similar solutions are also discussed in detail by Low (1982a). For these fundamental reasons, although complex self-similar solutions are only a subset of general time-dependent MHD solutions, where most of them are not self-similar, they are prone to develop in nature with simple initial configurations.

Numerically, starting from MHD equations with any fluctuations in a given initial configuration, self-organization could be reached since it is insensitive to the details in the initial conditions. Analytically, the nature of self-similarity implies that the dependent variables \((r, t)\) of a physical variable appear in separable form, as we have done in equations (9a)–(14b). As a consequence, the solutions are obtained by the method of separation of variables. Naturally, this imposes severe restrictions of the physical system for which such a procedure is feasible, such as the dimensionality and symmetry. Most self-similar solutions are established in axisymmetric systems, either cylindrical or spherical. Nevertheless, Gibson & Low (1998) have made a great leap in establishing a three-dimensional spherical solution. Under the framework of separation of variables, different kinds of solutions can be obtained for the same system, depending on the choice of constants. Likewise in MHD systems, we can have different self-similar solutions to account for different phenomena, depending on how we separate the constants. In \( \S 4 \), we choose to solve the system with an oscillating radial solution because this solution gives magnetic toroids along the jet that match observations. A monotonically decreasing or increasing radial solution in power-law form is also possible (Lynden-Bell & Boily 1994). Although this monotonic solution is not relevant for galactic jets, it could be useful for other natural phenomena such as in two-dimensional interplanetary magnetic ropes (Tsui & Tavares 2005) and some other astrophysical objects.

4. LOW MODEL

Comparing the equation of \( F = (p/\rho) \) and the equation of \( P \), we conclude that \( F = F(P) \) is a functional of \( P \), or \( F = F(\tilde{P}) \). Usually we cannot make the above statement just based on the similarity of the governing equations. It is only possible when we are under the framework of self-similarity. We can, therefore, write the \( \theta \)-dependence in \( \tilde{P} \) and \( \tilde{P} \) in terms of \( P \) to get

\[
\tilde{\rho} = \tilde{\rho}(\tilde{P}),
\]

(15a)

\[
\tilde{\rho}(\tilde{P}) = \tilde{\rho}(\tilde{P}),
\]

(15b)

Furthermore, the \( \eta \)- and \( \tilde{P} \)-dependences should be in a separable form in both

\[
\tilde{\rho}(\tilde{P}), \tilde{P} = \tilde{\rho}_{1}(\tilde{P})\tilde{P}_{2}(\tilde{P}),
\]

so that, with an adequate \( \gamma \), \( \tilde{F} \) could come out as a functional of \( \tilde{P} \) only. Making use of equation (4) to eliminate the current density in equation (2), we get the momentum equation, which has three components. First, we examine the \( \phi \)-component, which contains only the magnetic force,

\[
\frac{\partial P}{\partial r} \frac{\partial Q}{\partial \theta} - \frac{\partial P}{\partial \theta} \frac{\partial Q}{\partial r} = \frac{1}{y^2} \left( \frac{\partial \tilde{P}}{\partial \eta} \frac{\partial \tilde{Q}}{\partial \theta} - \frac{\partial \tilde{P}}{\partial \theta} \frac{\partial \tilde{Q}}{\partial \eta} \right) = 0,
\]

(16a)

\[
\tilde{Q}(\tilde{P}).
\]

(16b)

The vanishing of the magnetic force in the azimuthal direction described by equation (16a) implies the functional relationship given by equation (16b). As for the \( \theta \)-component, with \( p = \tilde{\rho}(\tilde{P}, \tilde{P})y^{3\gamma} \), it reads

\[
\frac{\partial P}{\partial \theta} + \frac{1}{y^2} \frac{\partial \tilde{P}}{\partial \theta} \frac{1}{\sin \theta} \frac{\partial P}{\partial \theta} = \frac{1}{y^2} \left( \frac{\partial \tilde{P}}{\partial \eta} \right) \frac{1}{\eta^2} \frac{\partial \tilde{P}}{\partial \theta} + \frac{1}{y^2} \frac{\partial \tilde{Q}}{\partial \theta} \frac{1}{\eta^2} \frac{\partial \tilde{Q}}{\partial \theta} + \frac{\partial \tilde{P}}{\partial \theta} \frac{\partial \tilde{Q}}{\partial \eta} \frac{1}{\eta^2} \frac{\partial \tilde{P}}{\partial \theta} = 0.
\]

(17)

We remark that the first three terms of this equation represent the force-free field equation for \( J \times B = 0 \) (Aly 1984; Low 1986; Low & Lou 1990; Lynden-Bell & Boily 1994). The last term is the plasma pressure. This equation would be independent of the
evolution function \( y \) should we consider the \( \gamma = 4/3 \) case pioneered by Low.

We note that a nonlinear equation like equation (17) has to be solved subject to the boundary conditions of a given physical problem. For example, should the problem on hand have an infinite external domain and the power-law radial solutions be physically reasonable, then we use fractional powers of \( P \) for the functionals of \( \bar{Q} \) and \( \bar{p} \) (Lynden-Bell & Boily 1994). In our present case, we are interested in decaying oscillations in \( \eta \). We, therefore, write

\[
\bar{Q}(\bar{P}) = a\bar{P},
\]

\[
\bar{p}(\eta, \bar{P}) = \bar{p}_1(\eta)\bar{p}_2(\bar{P}) = (\eta^{-4})(b^2\bar{P}^2 + \bar{C}),
\]

where \( \bar{C} \) is a positive constant independent of coordinates and the functional \( P \). This is the simplest representation in which the first equation gives a linear dependence of \( \bar{P} \) and the second equation reflects the positive definite nature of the plasma pressure. This choice of constant \( \bar{C} \) in the plasma pressure profile differs from the spherically symmetric, radial coordinate-dependent aditive term in equations (21) and (22) of Low (1984b). This choice of representation stems from the view that Low considers equation (17) as an equation that solves for the plasma pressure under a given \( \bar{P} \). For this reason, the homogeneous solution corresponds to the spherically symmetric gasdynamical solution. We regard equation (17) as an equation that solves for \( \bar{P} \) under a given plasma pressure that has a positive definite separable form, as in equation (18b). The choice of equation (18a) gives \( \partial \bar{Q}/\partial \bar{P} = a \), a constant. With \( b^2 = 2a^2b^2 \), equation (17) now reads

\[
\eta^2 \frac{\partial^2 \bar{P}}{\partial \eta^2} + \sin \theta \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial \bar{P}}{\partial \theta} \right) + \eta^2 a^2 \bar{P} + b^2 \sin^2 \theta \bar{P} = 0.
\]

(19a)

Writing \( \bar{P}(\eta, \theta) = R(\eta)\Theta(\theta), x = \cos \theta, \) and with \( n(n+1) \) as a separation constant, equation (19a) becomes

\[
\eta^2 \frac{\partial^2 R}{\partial \eta^2} + [a^2 \eta^2 - n(n+1)] R = 0,
\]

\[
(1 - x^2) \frac{d^2 \Theta(x)}{dx^2} + [n(n+1) + b^2 (1 - x^2)] \Theta(x) = 0.
\]

The \( \Theta(\eta) \) equation can be solved readily to give

\[
R(\eta) = (ar)^{1/2} J_{n+1/2}(ar).
\]

(19b)

Such a spherical Bessel functional was used by Low (1984b) in equation (28) of his paper as one of the numerical examples in spherical two-dimensional self-similar MHD to model coronal mass ejections.

The \( \Theta(x) \) equation with finite plasma pressure, \( b^2 \neq 0 \), can be solved by a power series,

\[
\Theta(x) = \sum m a_m x^m,
\]

\[
(m + 2)(m + 1)a_{m+2} = [m(m - 1) - n(n+1) - b^2]a_m + b^2 a_{m-2},
\]

\[
a_2 = -[n(n+1) + b^2]a_1,
\]

\[
a_1 = -[n(n+1) + b^2]a_0,
\]

where the infinite sum in equation (19c) starts from \( m = 0 \). There are two independent solutions. The first one has \( a_0 = \Theta(0) \neq 0 \) and \( a_1 = 0 \), and the second one has \( a_1 = 0 \) and \( a_0 = d\Theta(0)/dx \neq 0 \). They correspond to even and odd powers of the series, respectively,

\[
\Theta_{\text{even}}(x) = \sum m a_{2m} x^{2m},
\]

\[
\Theta_{\text{odd}}(x) = \sum m a_{2m+1} x^{2m+1}.
\]

In the absence of plasma pressure with \( b^2 = 0 \), the above solution reduces to

\[
\Theta(x) = (1 - x^2) \frac{d P_n(x)}{dx} = -n(n+1) \int_1^x P_n(x) dx,
\]

(19c')

where \( P_n(x) \) is the Legendre polynomial. The even solution corresponds to \( n \) odd, and the odd solution is otherwise.

5. SELF-SIMILAR MAGNETIC FIELD

With the solution \( \bar{P}(\eta, \theta) = R(\eta)\Theta(\theta) \) established by equations (19b)–(19c'), the magnetic field components are

\[
B_r = -\frac{1}{y^2 \eta^2} R(\eta) \frac{d \Theta(x)}{dx},
\]

\[
B_\theta = -\frac{1}{y^2 \eta} R(\eta) \frac{1}{(1 - x^2)^{1/2}} \Theta(x),
\]

\[
B_\phi = -\frac{1}{y^2 \eta} R(\eta) \frac{1}{(1 - x^2)^{1/2}} \Theta(x).
\]

We note that the self-similar evolution of the MHD plasma distorts the dipole-like magnetic field by generating an azimuthal component of the magnetic field with finite \( a_2 \). To grasp the magnetic structure given by equations (19b) and (19c), we first examine the case with \( b^2 = 0 \). In this special case, the even and odd power series of equation (19c) will terminate at a finite number of terms when \( m = n + 1 \) to give equation (19c') and \( \Theta(x) = 0 \) at the poles \( x = \pm 1 \).

The self-similar radial structure \( R(\eta) \) given by equation (19b) allows oscillations in \( \eta \) if \( ar \) is sufficiently large, which means that the azimuthal magnetic field is sufficiently large. The meridian structure \( \Theta(x) \) given by equation (19c') also oscillates in \( x \). Let us denote \( \eta_1 \) and \( x_j \) as where \( R(\eta) \) and \( \Theta(x) \) vanish, respectively. We remark that \( \eta_1 \) are circles of constant \( \eta \) and \( x_j \) are spokes of constant \( x = \cos \theta \). Consequently, \((\eta_1, x_j)\) divide the \((\eta, x)\) plane into many smaller regions. On \( \eta_1 \), we have \( B_r = 0 \) and \( B_\theta = 0 \), with \( B_r \) and \( B_\phi \) changing signs across \( \eta_1 \). The only component that does not vanish completely on \( \eta_1 \) is \( B_\phi \). Referring to the complete expression of \( B_\phi \) above, this magnetic component is modulated by \( \Theta(x) \) so that it changes sign on the circle of constant \( \eta \) on crossing each spoke region of \( x_j \). On \( x_j \), we have \( B_r = 0 \) and \( B_\phi = 0 \), with \( B_r \) and \( B_\phi \) changing signs across \( x_j \). The only component that does not vanish completely on \( x_j \) is \( B_r \). Referring to the complete expression of \( B_r \) above, this magnetic component is modulated by \( R(\eta) \) so that it changes sign on the spoke of constant \( x \) on crossing each circular region of \( \eta \). As a result, axisymmetric closed magnetic field lines are formed in these regions, generating toroidal belt plasmoids circumscribing the \( z \)-axis of symmetry. We call these secondary plasmoids and call the larger self-similar plasmoid surrounding all the secondary plasmoids the primary plasmoid. Neighboring plasmoids have field lines circling in the opposite sense. If one plasmoid has field lines in the clockwise direction,
the adjacent one has them in the counterclockwise direction. The azimuthal components also rotate opposite each other. In each region bounded by \((\eta_0, \eta_{i+1})\) and \((x_j, x_{j+1})\), the topological center defined by \(dR(\eta)/d\eta = 0\) and \(d\Theta(x)/dx = 0\) has \(B_r = 0\) and \(B_\theta = 0\). This is the magnetic axis of each toroid. The field lines about this center are given by

\[
\frac{B_r}{dr} = \frac{B_\theta}{r d\theta} = -\frac{B_\phi}{r \sin \theta d\phi}. \tag{20a}
\]

By axisymmetry, the magnetic field components are independent of \(\phi\). For this reason, the third group of the above equation is decoupled from the first two groups. The \(B_\phi\) field circles about the \(z\)-axis of symmetry without twisting. It is simply superimposed on the \(B_rB_\theta\) field lines. In terms of Fourier components \(e^{im\phi}\), this means \(m = 0\). For the field lines on an \((r, \theta)\) plane, we consider the first equality between \(B_r\) and \(B_\theta\), which gives \(P = R(\eta)\Theta(x)\) equal to a constant, or

\[
P(\eta, x) = (an)^{1/2} J_{n+1/2}(an) \Theta(x) = C. \tag{20b}
\]

In other words, the nested field lines are given by the contours of \(P(\eta, x)\) on the \((r, \theta)\) plane.

As an example, the field lines for \(n = 4\) and \(b^2 = 0\) are shown in Figure 1 for axisymmetric secondary plasmas. We have taken \(a\eta_0 = 8.2\) at the first zero of the spherical Bessel function, where \(\eta_0\) is the radial label of the plasma boundary. The horizontal axis is \(x = 0\), or \(\theta = \pi/2\), and the vertical axis is \(x = +1\), or \(\theta = 0\). For \(n = 4\), \(\Theta(x)\) vanishes at \(x = 0, \pm(3/7)^{1/2}\), and \(\pm 1\), and \(R(\eta)\) vanishes at \(\eta = 0.0\) and 8.2. These are locations where \(B_\theta = 0\) and \(B_r = 0\), respectively, dividing the quadrant in two regions, as indicated in Figure 1. These dividing lines are obtained by solving the contours with \(C = 0\) numerically. The radial grids are equally spaced in \(\eta\). The meridian grids are equally spaced in \(x\). By converting to \(\theta\) through \(x = \cos \theta\), it generates an uneven grid distribution in \(\theta\) that appears in Figure 1. Closed field lines are also shown in each region. Negative-value contours of \(C = -0.3\), \(-0.5\), and \(-0.7\) show the field lines in the region adjacent to the axis of \(x = 0\), and positive-value contours of \(C = +0.3\) and +0.5 show the field lines in the region adjacent to the axis of \(x = +1\). At the topological center of each region, we have \(\Theta(x)\) maximum and \(R(\eta)\) maximum, thereby giving \(B_\theta = 0\) and \(B_r = 0\) at the same location. Since

\[
2\pi r \sin \theta B_\phi = 2\pi a P,
\]

the center has the maximum of the line integral of \(B_\phi\) about the axis of symmetry. Adjacent secondary plasmas have opposite signs of \(B_\phi\). Should we take \(a\eta_0 = 11.7\) as the second zero of the spherical Bessel function, one additional shell of secondary plasmas would be added to Figure 1. Furthermore, because of the \((1 - x^2)^2\) factor in equation (19c), \(\Theta(x)\) is zero at the poles, with \(x = \pm 1\) where the magnetic field is purely radial. If \(n \gg 1\), the null at the poles is tightly surrounded by a polar lobe. Along the polar lobe, the embedded belt plasmas are tightly wound about the polar axis, and we name them secondary plasmas.

We are now ready to examine the general case of \(b^2 \neq 0\). This case of \(b^2 \neq 0\) is known as the finite-\(b\) case, where \(b\) is the ratio of plasma pressure to magnetic pressure. The presence of the plasma pressure with finite \(b^2\) distorts the linear solution of \(\Theta(x)\). With finite \(b\), the series usually has a finite value at \(x = \pm 1\) that leads to singular magnetic fields there because of the \((1 - x^2)^{1/2}\) factor in the denominator. Constrained by nature to regular solutions, \(b^2\) has to be the eigenvalues such that \(\Theta(x)\) remains zero at \(x = \pm 1\). To search for these eigenvalues, we evaluate \(\Theta(x)\), equation (19c), at the boundary \(x = \pm 1\) for different values of \(b^2\) with a given \(n\). The eigenvalues are shown in Figure 2 as the intercepts of \(\Theta(+1) = 0\) at 119, 261, and 425 for the even series with \(n = 13\). For \(n = 23\), they are 200 and 422. The even series has \(\Theta(0) \neq 0\) and \(d\Theta(0)/dx \neq 0\), and the odd series has \(\Theta(0) = 0\) and \(d\Theta(0)/dx \neq 0\). The even eigenfrequencies with \(n = 13\) and \(b^2 = 119\), and with \(n = 23\) and \(b^2 = 200\), are shown in Figure 3. With \(n = 13\) and \(b^2 = 119\), there are seven nodes in the interval \((0, 1)\). Should we take the second eigenvalue \(b^2 = 261\), one more node would be added. With \(n = 23\) and \(b^2 = 200\), there are 12 nodes in \((0, 1)\). The next eigenvalue \(b^2 = 422\) would add one more node as well.

The fact that the plasma pressure has to be as such that it is the eigenvalue of the \(\Theta(x)\) equation appears to be a very restrictive constraint for the self-similar solutions. Nevertheless, we note that the plasma pressure appears in the \(\Theta(x)\) equation, where the separation constant \(n(n + 1)\) is an as yet unspecified free parameter. As a result, there will be an adequate plasma pressure for almost any given plasma pressure. For example, with \(b^2 = 422\), instead of being the second eigenvalue of \(n = 23\), it could be the first eigenvalue.
of some different $n$ that is larger than 23. The important point is that for any plasma pressure $b^2$, it will coincide with one of the eigenvalues of some $n$. The chances are that $n$ is a large integer, which, as we show in § 6, leads to good jet collimation.

6. JET COLLIMATION AND VORTEX STRUCTURE

One important point we should note is that, although we have taken a spherically symmetric, radial expansion in the plasma velocity, the plasma density and pressure need not be symmetric, much less the magnetic fields. The plasma profiles are discussed below. Here, we discuss the magnetic fields that are given in terms of $R(\eta)$ and $\Theta(x)$, as listed in § 5. The maximum of $\Theta(x)$ at $x = 0$ gives $B_r = 0$ in the equatorial accretion disk plasma where the magnetic field has only a $B_\theta$ meridian and a $B_\phi$ azimuthal component. Off the disk, due to the nodes in Figure 3, there are also maxima in space, so that the magnetic fields are structured in lobes other than the equatorial lobe.

To consider the collimation of polar jets, we examine the magnetic field components. For the $B_\theta$ and $B_\phi$ components, we plot the function $\Theta(x)/(1 - x^2)^{1/2}$ in Figure 4 with the corresponding eigenvalue $b^2 = 119$ for $n = 13$ according to Figure 2. It peaks up off the scale to $-3.0$ as $x$ approaches unity but plunges to zero at $x = +1$. As $n$ increases, the peak edges closer to $x = +1$. This is clearly shown in Figure 4 with $b^2 = 200$ for $n = 23$. The peak then goes to $+3.8$ in this case. As a consequence, the peak rises in amplitude but narrows in width. To examine $B_r$, we plot the function $d\Theta(x)/dx$ in Figure 5, which shows that the magnetic field is purely radial on the polar axis. As we have said, this radial magnetic field changes sign as each region of $\eta_1$ is crossed. With $n = 13$ and $n = 23$, the peak on the polar axis is $+71$ and $-155$, respectively. The magnetic energy is the quadratic quantity of this, and it peaks even more with respect to the off-axis lobes. The detailed structures near the polar axis are shown in Figure 6 for $B_\theta$ and $B_\phi$, and in Figure 7 for $B_r$. The magnetic field, therefore, converges to the polar axis as $n$ increases, leading to a jet structure. Comparing the equatorial lobe of the magnetic field with the polar lobe, it is clear that the polar lobe is much narrower than the equatorial one. This lobe pattern is similar to directional antenna arrays. Due to the oscillating nature of equation (19b), there are spherical toroids, or secondary plasmoids, formed by closed magnetic field lines along this narrow peak. The number of secondary plasmoids embedded in this peak measures the number of zeros of equation (19b) contained within $ar_0$. The magnitude of $ar_0$ also indicates the large amplitude of $B_r$ through $Q = aP$ of equation (18a), which is itself consistent with the magnetic field convergence onto the polar axis.

To understand the magnetic structure of the jet, we plot the contours of $P$ given by equation (20b), which amount to closed field lines of embedded secondary plasmoids. A contour plot in
Figure 8, similar to Figure 1 but with \( n = 13 \) and \( b^2 = 119 \), shows plasmoids along the polar cone that goes from \( x = 0.96 \) to 1, as in Figure 7, which corresponds to a cone angle of about 14°. The radial label goes from \( aj_0 = 0 \) to 40, and this interval is divided into seven regions set by the zeros of the spherical Bessel function. The first zero is at \( aj_1 = 18.40 \), and the subsequent zeros are approximately equally spaced, as indicated in Figure 8. Negative contours of \( C = -0.010 \) are plotted in the first, third, fifth, and seventh regions only. Likewise, positive contours of \( C = +0.005 \) are plotted in the second, fourth, and sixth regions. To avoid overcrowding, the \( C = -0.005 \) contours are not plotted in the odd-numbered regions, and the \( C = +0.010 \) ones are not shown in the even-numbered regions. Once more, we recall that the magnetic field lines of these secondary plasmoids circle in alternating sequence. The boundary of the jet is given by the root of \( \Theta(x_j) \), with \( x_j = 0.96 \). At this boundary, the magnetic field is purely radial but alternates in sign. The other boundary is at \( x_j = 1.0 \), where the magnetic field is also radial and alternates in sign opposite to that of \( x_j = 0.96 \). Together with the roots of \( R(\eta) \) at \( \eta \) where the magnetic field is meridian, the plasmoids are magnetic vortices bounded by closed field lines at the border of each region. By axisymmetry, these poloidal magnetic contours rotate about the polar axis to generate tightly wound toroidal. Furthermore, by axisymmetry, the toroidal azimuthal field lines are decoupled from the bipolar poloidal field lines. The field lines are, therefore, two-dimensional. As in the discussions concerning Figure 1, this amounts to the \( m = 0 \) mode in the Fourier transform \( e^{im\phi} \) of the toroidal dependence.

The magnetic structure of this spherical temporal self-similar model, which shows a sequence of magnetic toroids along the jet, differs from the continuous helical field line structure in the cylindrical spatial self-similar model, complemented by numerical simulations (Casse 2004). To understand the differences, the spatial cylindrical model has an imposed initial magnetic configuration everywhere in space. Jets are formed by transporting the disk plasma from the disk plane to the axis through magnetocentrifugal action. They are maintained along the axis by bringing disk plasma continuously in steady state to overcome transport losses. In this description, spatially self-similar jets in \( z/r \) were formed in their present position in the distant past and are maintained there continuously at the present and in the future.

As for the temporal self-similar spherical model, the disk plasma is first accreted to the galactic nucleus, which builds up the plasma pressure there. While the plasma is in this bounded region, self-organization is nourished through dissipations to

7. EVOLUTION FUNCTION

The \( r \)-component of the momentum equation reads

\[
\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} \right) + \frac{d}{dr} p(r, P(r, \theta)) + \rho \frac{GM}{r^2} = -\frac{1}{\mu} \left( 1 \frac{1}{r \sin \theta} \right)^2 \frac{\partial P}{\partial r} \left( \frac{\partial^2 P}{\partial r^2} + \frac{1}{r^2 \sin \theta} \frac{\partial P}{\partial \theta} \left( \frac{\partial P}{\partial \theta} \right) \right) + \frac{Q}{\rho \partial P}.
\]

(21)

The term \( dp/dr \) on the left-hand side refers to radial derivatives for the explicit dependence and the implicit dependence in \( P \). Making use of the \( \theta \)-component, equation (17), the equation above becomes

\[
\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} \right) + \frac{d}{dr} p(r, P(r, \theta)) + \rho \frac{GM}{r^2} = -\frac{1}{\mu} \left( 1 \frac{1}{r \sin \theta} \right)^2 \frac{\partial P}{\partial r} \left( -\mu r^2 \sin^2 \theta \frac{\partial P}{\partial P} \right) = + \frac{\partial P}{\partial r} \frac{\partial P}{\partial \theta}.
\]

The right-hand side is just the radial derivative of plasma pressure on the implicit dependence in \( P \). This cancels the corresponding term on the left-hand side, leaving only the explicit radial derivative

\[
\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} \right) + \frac{\partial p(r, P)}{\partial r} + \rho \frac{GM}{r^2} = 0.
\]
In terms of self-similar parameters, this equation reads
\[ y^2 \frac{d^2 y}{dt^2} + \frac{1}{\eta} \left( y^{4-3\alpha} \frac{1}{\rho} \frac{\partial \rho}{\partial y} + \frac{GM}{\eta^2} \right) = y^2 \frac{d^2 y}{dt^2} + \frac{1}{\eta} \left( \frac{1}{\rho} \frac{\partial \rho}{\partial y} + \frac{GM}{\eta^2} \right) = 0. \]  
(22)

With \( \alpha \) as the separation constant and \( H \) as an integration constant, the evolution function is, therefore, described by
\[ y^2 \frac{d^2 y}{dt^2} = +\alpha, \]
\[ \frac{1}{2} \left( \frac{dy}{dt} \right)^2 + \frac{\alpha}{y} = H, \]  
(23a)
\[ \frac{dy}{dt} = \pm \left( 2 \left( H - \frac{\alpha}{y} \right) \right)^{1/2}. \]  
(23b)

To understand the meaning of \( \alpha \), we note that plasma acceleration in Lagrangian coordinates is
\[ \frac{dv}{dt} = \frac{d^2 y}{dt^2} = \frac{\alpha \eta}{y^2} - \frac{\alpha \eta^3}{r^2}. \]

A negative \( \alpha \) means an outward decelerating flow or an inward accelerating flow. The deceleration gets smaller as \( y \) or as \( r \) gets larger; the acceleration gets larger as \( y \) or as \( r \) gets smaller. As for the meaning of \( H \), we take the limit \( y = \infty \), which gives \( H = 4\pi = (v^2/2)/4\pi \eta^2 \) by using \( r = \eta y \). Physically, this is the asymptotic radial kinetic energy per unit mass per unit area of a Lagrangian fluid element. For circular orbits, we would have \( H = 0 \). Furthermore, we can rewrite equation (23a) making use of the above physical expression of \( \alpha \) to get
\[ \rho v^2 = \frac{1}{2} \eta v^2 + (\vec{r} \cdot \vec{F})y, \]
where \( \vec{F} \) denotes all the forces on the right-hand side of equation (2). It is clear that \( H \) measures the total energy of the fluid element.

To consider the accretion disk magnetospheric plasma, we take \( \alpha = -|\alpha| < 0 \) for an inward accelerating flow. With \( H = 0 \) as the boundary condition at infinity, we get
\[ \frac{dy}{dt} = \pm \left( \frac{2|\alpha|}{y} \right)^{1/2} = - \left( \frac{2|\alpha|}{y} \right)^{1/2}. \]  
(24a)

The two signs for the square root correspond to the outward and inward flows. Taking the negative sign for the inward flow, the equatorial accretion disk plasma starts at infinity with an asymptotically zero radial velocity and ends near the center with a large influx.

To describe the polar jets, we again take \( \alpha = -|\alpha| < 0 \). We consider \( H = \pm |H| \), which leads to
\[ \frac{dy}{dt} = \pm \left( 2 \left( \pm |H| + \frac{|\alpha|}{y} \right) \right)^{1/2}. \]  
(24b)

For \( H = +|H| > 0 \) and taking the positive sign on the square root, this gives a decelerated outward flow. The deceleration gets smaller as \( y \) or as \( r \) gets larger, with the terminal velocity given by \( v = \eta dy/dt = \eta(2H)^{1/2} \). For \( H = -|H| < 0 \), the outward flow would stop at \( y = |\alpha|/|H| \), where \( dy/dt = 0 \). This \( H = -|H| < 0 \) outward flow would be followed by an accelerated inward flow, should we take the negative sign on the square root, so that the system would be periodic and bounded. We therefore see that the system would make a transition from a bounded to an unbounded state when \( H \) goes from negative to positive. The bounded state allows us to define the boundaries of the radial label \((0, \eta_0)\) such that \( 0 < \eta < \eta_0 \). The solutions of the evolution function in equation (24b) suggest that jets are a result of an eruptive process. This process is fed by plasma accretion from the galactic disk. As pressure builds up at the center, the plasma begins to oscillate, or to pulse, as described by the bounded solution with \( H = -|H| < 0 \). We believe that it is during this phase that self-organized and self-similar structures in plasma parameters and magnetic fields would be nourished through dissipations. The dynamics is given by self-similar solutions where the magnetic fields are consistent with the plasma pressure that plays the role of the eigenvalue. The jet structure emerges along the polar axis as the eigenfunction of the magnetic field. By further pressure buildup and energy influx, \( H = +|H| > 0 \) becomes positive, and the structure eventually erupts.

8. RAYLEIGH-BENARD CELLS

Now we have presented the general structure of galactic jets. It begins with an influx of plasma from the accretion disk that drives up the plasma pressure at the galactic nucleus. The MHD plasma responses to self-organization by oscillating, or pulsating, periodically in time. As \( H \) goes from negative to positive, the periodic mode goes to an eruptive mode. The result of self-organization is a structured configuration in space. This structure is not unique. For galactic jets, we have solved the system with the method of separation of variables such that it resembles observations. This structure contains basically convective cells in space where magnetic field lines in adjacent cells rotate in the opposite sense. Although such a structured configuration with consistent evolution function is in accordance with the time-dependent MHD equations, there is no mention of the initial configuration that could lead to such self-similar states. Consequently, such a highly organized complex structure could be regarded as an artificial result due to special mathematical constructions, which might not have any relevance to the physical jet system. In order to bring more reality to this analytic result, we recall the classical case of Rayleigh-Benard fluid self-organization. Consider a thin layer of oil heated from below. Observations tell us that this simple homogeneous configuration will develop an array of identical hexagonal convective cells if the temperature gradient across the layer is sufficiently large. The velocity streamlines of adjacent cells rotate in the opposite sense. Should the initial state of this oil layer be altered by arbitrary fluctuations, the same convective cells would still appear after the initial fluctuations are dissipated. Numerically, such self-organized complex structure could be reached from the simple homogeneous initial configuration if the code is adequately pushed in the correct direction.

Guided by this fluid example, the self-similar jet structure could be regarded as the Rayleigh-Benard equivalent in the MHD system. We could choose an initial state by taking \( n = 1 \) in our solutions with plasma bounded between an outer sphere \( \eta_0 \) and an inner sphere \( \eta_r \). This would give a large, global long-wavelength structure in the spherical layer. We consider an equilibrium state with \( \alpha = -|\alpha| < 0 \) and \( H = -|H| < 0 \), with \( y = |\alpha|/|H| \) and \( dy/dt = 0 \). Let us take this moment as \( t = 0 \). An energy source in terms of pressure is supplied by the accretion disk to pump the MHD plasma at the lower boundary \( \eta_r \). At some moment, \( y \) begins to depart from its equilibrium and to fall toward the center due to perturbations. As in the oil-layer case, this could trigger
convection such that the long-wavelength global structure cascades to short-wavelength structures accepted by the system. Convective cell scaling becomes smaller as \( n \) gets larger. Unlike the flat-layer fluid case, the convective cells in this spherical MHD layer are not identical, since there is a focusing effect to the polar axis.

9. MASS DENSITY PROFILE

As for the spatial part, we have

\[
\frac{1}{\dot{\rho}} \frac{\partial \dot{\rho}}{\partial \eta} + \frac{GM}{\eta^2} = -\alpha \eta. 
\]

With \( \dot{\rho} \) given by equation (18b), the above equation gives

\[
\dot{\rho} = \left( \eta^{-3} \right) \left( \frac{4 \dot{p}_2(\dot{P})}{\alpha \eta^3 + GM} \right) = \left( \eta^{-3} \right) \left( \frac{4}{GM} \right) \left( \dot{p}_1(\eta) \dot{p}_2(\dot{P}) \right). \tag{25} 
\]

In our present case, we are considering a spherical shell domain such that the radial label \( \eta = 0 \) is excluded. The negative powers of \( \eta \) in plasma pressure and mass density do not cause singularity. We have set \( \alpha = 0 \), with a vanishing net force on the flow, to obtain the second equality in equation (25) to be compatible with the functional form of \( F = F(\dot{P}) \), with \( \gamma = 4/3 \). Nevertheless, we could relax this condition to \( \alpha \approx 0 \), so that \( \alpha \eta^3 \ll GM \). To understand the implication of such an inequality, we remark that

\[
\frac{\alpha \eta^3}{GM} = \frac{dv/dt}{GM/r^2} \ll 1. \tag{26} 
\]

Therefore, it is just the ratio of the flow acceleration to the central mass gravitational acceleration. Since \( dv/dt \) gets smaller as \( r \) gets larger with \( 1/r^2 \) scaling, this condition establishes that self-similar configurations can be organized and bounded oscillations of the \( H = -|H| < 0 \) case can take place, as long as the plasma acceleration is much less that the gravitational acceleration. To close the entire self-similar system, we now come to the functional \( F(\dot{P}) \) for the adiabatic equation of state. With the results in equations (18b) and (25), using \( \alpha \approx 0 \), we have

\[
\dot{F}(\dot{P}) = \frac{\dot{p}_2(\dot{P})}{\rho_2(\dot{P})} = \left( \frac{GM}{4} \right)^{4/3} \dot{p}_2^{-1/3}(\dot{P}) = \left( \frac{GM}{4} \right)^{4/3} \left( b'^2 \dot{P}^2 + \dot{C} \right)^{-1/3}. \tag{27} 
\]

To understand the plasma structure of the jet, we note that the plasma pressure and mass density are \( b'^2 \dot{P}^2(\eta, x) + \dot{C} \) dependent, which means positive definite \( b'^2 C^2 + \dot{C} \) dependent, according to equations (18b) and (25), respectively. The plasma pressure and mass density have their minimum at \( P(\eta, x) = (\alpha \eta)^{2} J_{n+1/2}(\alpha \eta) \Theta(x) = 0 \). This minimum is positive nonzero because of the positive integration constant \( \dot{C} \) in equation (18b). To discuss the plasma structure in the jets, we reproduce the part of Figure 3 in the range 0.94 \( < x < 1.0 \) in Figure 9. The segmented line for \( n = 13 \) has \( \Theta(x) = 0 \) at the cone boundary \( x = 0.96 \) and at the cone center \( x = 1.0 \). In between, there is a minimum at \( x = 0.985 \). Because of the quadratic dependence \( \dot{C}(x) \), these features correspond to a cavity-like structure for the plasma pressure and mass density of the jet, compatible with numerical simulations of the time-dependent dissipative MHD equations (Casse 2004; Zanni et al. 2004). Other than the zeros of \( \Theta(x) \), \( P(\eta, x) \) also vanishes at radial locations where \( J_{n+1/2}(\alpha \eta) = 0 \). With the quadratic dependence

\[
R^2(\eta) = (\alpha \eta) J_{n+1/2}^2(\alpha \eta), 
\]

there are ripples for plasma pressure and mass density along the radial axis. The radial function has two contributions. The Bessel function \( J_{n+1/2}(\alpha \eta) \) has an oscillating nature with decreasing amplitude, while \( \alpha \eta \) has an increasing amplitude that helps to maintain the ripple level of \( R^2(\eta) \). The peaks of these ripples are at the topological center of each plasmoid, which is the magnetic axis of the toroid where \( B_0 \) has a maximum line integral. These ripples are also seen in numerical simulations (Zanni et al. 2004). It is quite surprising that the results of this spherical temporal self-similar MHD model agree rather well qualitatively with numerical simulations of the time-dependent MHD equations for the cylindrical magnetocentrifugal model. Both of them give cavity structures in the transverse direction and ripple structures in the longitudinal direction.

The polar jets, therefore, have a periodic and approximately equally spaced concentration of plasmoids in its longitudinal direction, except the first region, which is more extensive. The closed magnetic field lines for adjacent plasmoids rotate in the opposite sense. The plasma pressure and mass density have a hollow conic structure with ripples along the radial direction. Such periodic structure happens to be compatible with radio observations in which high-intensity islands are mapped. These islands are usually thought to be periodic ejections of mass from the accretion disk. According to our model, they are rather the internal spatial arrangements of an expanding jet driven by an eruptive event.

10. DISCUSSIONS AND CONCLUSIONS

One of the main objections to self-organized plasmoid representation in free space in the absence of an adequate boundary is that it apparently violates the virial theorem, which states that (Schmidt 1966)

\[
\frac{1}{2} \frac{d^2 I}{dt^2} + \int x_k \frac{\partial G_k}{\partial t} dV = 2(T + U) + W^E + W^M - \int x_k (P_{ik} + T_{ik}) dS, \tag{28} 
\]
where $I$ is the moment of inertia of the plasmoid; $G$ is the momentum density of the electromagnetic field; $T$ and $U$ are the kinetic and thermal energies of the plasma, respectively; $W^E$ and $W^M$ are the electric and magnetic energies in the volume, respectively; and $P_a$ and $T_{\phi\phi}$ are the plasma and electromagnetic stress tensors, respectively. Taking the volume to cover the entire plasma and field, the surface term on the right-hand side vanishes. In laboratory plasmas, this surface could be the machine vessel. Should the plasmoid be in a steady state, the volume term on the left-hand side would be zero, and the moment of inertia would be accelerating since the terms on the right-hand side are all positive definite. While this statement has no conflict with the unbounded solutions, it apparently contradicts the steady state of the bounded solutions. Nevertheless, this argument has overlooked the asymptotically bounded nature of the $H = -|\dot{H}| < 0$ plasmoid state. In this asymptotic case, we have $dP_a/ dt = (dU/ dt)(dy/ dt) = 0$ so that $I$ is stationary. The fact that $d^2 I/ dt^2 > 0$ implies that $I$ is at an asymptotic minimum, and not an acceleration of $I$, which complies with the virial theorem.

The classical accretion-ejection model of Blandford & Payne (1982) is a time-independent steady state model with spatial self-similar MHD solutions in cylindrical geometry $(r, \phi, z)$ with Alfvénic plasma flow velocity plus rotation, all with Keplerian scaling. In this model, the jets are formed by convecting the magnetospheric disk plasma from the disk plane to the axis by magneto centrifugal action through the magnetic field lines with low inclination angles to the disk plane. The angular momentum of the plasma in the disk plane is focused to the axis. The jets were put in place in the distant past according to the spatial self-similar solutions in $z/r$ and are maintained there by continuously transporting disk plasma to the axis to sustain the axial outflow. Collimation to the axis is accomplished by the magnetic hoop force.

Here, we have taken a dynamic view in which jets are the consequences of eruptive events, based on time-dependent MHD equations in spherical geometry $(r, \theta, \phi)$. The disk plasma is accreted to the galactic nucleus, where plasma pressure is built up to cause an eruption. The radially symmetric expanding velocity interacts the plasma with the magnetic field self-consistently through the MHD equations to generate spatial structures. Due to the existence of multiple quadratic invariants in the absence of dissipations, MHD systems have the tendency of developing self-organized and self-similar states through dissipative processes. The force-free configuration and the vortex nature of the magnetic field are akin to the quadratic invariants of the magnetic helicity and cross-helicity of the MHD system. For these reasons, although temporally self-similar solutions are only a subset of general time-dependent MHD solutions, these self-similar configurations are prone to develop in natural phenomena.

We, therefore, describe these spatial structures by self-similar temporal solutions in $\eta = r/\gamma$. In this self-similar spherical model, consistent self-similar representations of plasma pressure, mass density, and magnetic fields are worked out for the $\gamma = 4/3$ Low model, with special emphasis on the finite-$\beta$ case. The spatial distribution of the magnetic field is described by an equation where the plasma pressure acts as the eigenvalue. The nature of this eigenvalue equation is such that the magnetic field converges to the polar axis as a response to the high plasma pressure directly related to the eigenvalue. Since the separation constant $n(n + 1)$ in this eigenvalue equation is a free parameter that has yet to be specified, there will be an adequate $n$ for almost any given plasma pressure. Although the radial plasma velocity is isotropic, the spatial structures are not. They could be highly collimated along the axial direction and expand into the previously void space as time progresses.

Although other types of solutions are permitted, we have specifically examined the solutions that bear resemblance to jet features with $n > 1$ and $\alpha_p \gg 1$. Since the plasma and the magnetic field are frozen into each other, the plasma outflow is also collimated to the polar axis. The existence of regions of closed field lines along the primary polar magnetic lobe permits secondary plasmoids to be embedded in it. These secondary plasmoids appear to be compatible with the observed islands of radio intensities. The time evolution function of the radial velocity consistent with the temporal self-similar solutions has different types of solutions according to the sign of $H$. Although the equatorial disk magnetospheric plasma is not addressed here, the accelerated accretion of plasma influx could be modeled with $H = 0$ as the boundary condition at infinity. As for the polar jets, $H < 0$ gives a bounded oscillating, or pulsating, solution. This bounded stage, we believe, nourishes the self-organized and self-similar states. With $H > 0$, it gives an unbounded expanding solution with a high terminal velocity. It is apparent that plasma pressure due to accretion is the prime driving force that determines the value of $H$. The bounded oscillation would make a transition to the unbounded expansion as $H$ goes from negative to positive. According to our model, jet structures are, therefore, considered as a spatial configuration that has been expanding continuously into space.

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