In Search for an Optimal Authenticated Byzantine Agreement
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Abstract
In this paper, we challenge the conventional approach of state machine replication systems to design deterministic agreement protocols in the eventually synchronous communication model. We first prove that no such protocol can guarantee bounded communication cost before the global stabilization time and propose a different approach that hopes for the best (synchrony) but prepares for the worst (asynchrony). Accordingly, we design an optimistic byzantine agreement protocol that first tries an efficient deterministic algorithm that relies on synchrony for termination only, and then, only if an agreement was not reached due to asynchrony, the protocol uses a randomized asynchronous protocol for fallback that guarantees termination with probability $\frac{1}{2}$.

We formally prove that our protocol achieves optimal communication complexity under all network conditions and failure scenarios. We first prove a lower bound of $\Omega(f t + t)$ for synchronous deterministic byzantine agreement protocols, where $t$ is the failure threshold, and $f$ is the actual number of failures. Then, we present a tight upper bound and use it for the synchronous part of the optimistic protocol. Finally, for the asynchronous fallback, we use a variant of the (optimal) VABA protocol, which we reconstruct to safely combine it with the synchronous part.

We believe that our adaptive to failures synchronous byzantine agreement protocol has an independent interest since it is the first protocol we are aware of which communication complexity optimally depends on the actual number of failures.

Keywords and phrases
Byzantine agreement; Optimistic; Asynchronous fallback

1 Introduction
With the emergence of the Blockchain use case, designing efficient geo-replicated Byzantine tolerant state machine replication (SMR) systems is now one of the most challenging problems in distributed computing. The core of every Byzantine SMR system is the Byzantine agreement problem (see [3] for a survey), which was first introduced four decades ago [32] and has been intensively studied since then [11, 22, 25]. The bottleneck in geo-replicated SMR systems is the network communication, and thus a substantial effort in recent years was invested in the search for an optimal communication Byzantine agreement protocol [20, 21, 22, 23].

To circumvent the FLP [17] result that states that deterministic asynchronous agreement protocols are impossible, most SMR solutions [12, 20, 21, 23] assume eventually synchronous communication models and provide safety during asynchronous periods but can guarantee progress only after the global stabilization time (GST).

Therefore, it is quite natural that state-of-the-art authenticated Byzantine agreement protocols [20, 21, 22, 23] focus on reducing communication cost after GST, while putting up with the potentially unbounded cost beforehand. For example, Zyzzyva [23] and later SBFT [20] use threshold signatures [33] and collectors to reduce the quadratic cost induced by the all-to-all communication in each view of the PBFT [12] protocol. HotStuff [26] leverages ideas presented in Tendermint [10] to propose a linear view-change mechanism, and a few follow-up works [29, 30, 9] proposed algorithms for synchronizing parties between views.
Some \[29, 30\] proposed a synchronizer with a linear cost after GST in failure-free runs, while others \[9\] provided an implementation that guarantees bounded memory even before GST. However, none of the above algorithms bounds the number of views executed before GST, and thus none of them can guarantee a bounded total communication cost.

We argue in this paper that designing agreement algorithms in the eventually synchronous model is not the best approach to reduce the total communication complexity of SMR systems and propose an alternative approach. That is, we propose to forgo the eventually synchronous assumptions and instead optimistically consider the network to be synchronous and immediately switch to randomized asynchronous treatment if synchrony assumption does not hold. Our goal in this paper is to develop an optimistic protocol that adapts to network conditions and actual failures to guarantee termination with an optimal communication cost under all failure and network scenarios.

1.1 Contribution

Vulnerability of the eventually synchronous model. A real network consists of synchronous and asynchronous periods. From a practical point of view, if the synchronous periods are too short, no deterministic Agreement algorithm can make progress \[17\]. Therefore, to capture the assumption that eventually there will be a long enough synchronous period for a deterministic Agreement to terminate, the eventually synchronous model assumes that every execution has a point, called GST, after which the network is synchronous. In our first result, we capture the inherent vulnerability of algorithms designed in the eventually synchronous communication model. That is, we exploit the fact that GST can occur after an arbitrarily long time to prove the following lower bound:

- **Theorem 1.** There is no eventually synchronous deterministic Byzantine agreement protocol that can tolerate a single failure and guarantee bounded communication cost even in failure-free runs.

Tight bounds for synchronous Byzantine agreement. To develop an optimal optimistic protocol that achieves optimal communication under all failure and network scenarios we first establish what is the best we can achieve in synchronous settings. Dolev and Reischuk \[14\] proved that there is no deterministic protocol that solves synchronous Byzantine agreement with \(o(t^2)\) communication cost, where \(t\) is the failure threshold. We generalize their result by considering the actual number of failures \(f \leq t\) and prove the following lower bound:

- **Theorem 2.** Any synchronous deterministic Byzantine agreement protocol has \(\Omega(ft + t)\) communication complexity.

It is important to note that the lower bound holds even for deterministic protocols that are allowed to use perfect cryptographic schemes such as threshold signatures and authenticated links. Then, we present the first deterministic cryptography-based synchronous Byzantine agreement protocol that matches our lower bound for the authenticated case. That is, we prove the following:

- **Theorem 3.** There is a deterministic synchronous authenticated Byzantine agreement protocol with \(O(ft + t)\) communication complexity.

We believe these results are interesting on their own since they are the first to consider the actual number of failures, which was previously considered in the problem of early decision/stopping \[15, 21\], for communication complexity analysis of the Byzantine agreement.
Optimal optimistic Byzantine agreement. Our final contribution is an optimistic Byzantine agreement protocol that tolerates up to $t < n/3$ failures and has asymptotically optimal communication cost under all network conditions and failure scenarios. That is, we prove the following:

\begin{itemize}
  \item \textbf{Theorem 4.} There is an authenticated Byzantine agreement protocol with $O(ft + t)$ communication complexity in synchronous runs and expected $O(t^2)$ communication complexity in all other runs.
\end{itemize}

To achieve the result, we combine our optimal adaptive synchronous protocol with an asynchronous fallback, for which we use a variant of VABA \cite{1}. As we shortly explain, the combination is not trivial since we need to preserve safety even if parties decide in different parts of the protocol, and implement an efficient mechanism to prevent honest parties from moving to the fallback in synchronous runs.

### 1.2 Technical overview

The combination of our synchronous part with the asynchronous fallback introduces two main challenges. The first challenge is to design a mechanism that (1) makes sure parties do not move to the fallback unless necessary for termination, and (2) has $O(ft + t)$ communication complexity in synchronous runs. The difficulty here is twofold: first, parties cannot always distinguish between synchronous and asynchronous runs. Second, they cannot distinguish between honest parties that complain that they did not decide (due to asynchrony) in the first part and Byzantine parties that complain because they wish to increase the communication cost by moving to the asynchronous fallback. To deal with this challenge, we implement a \texttt{Help\&tryHalting} procedure. In a nutshell, parties try to avoid the fallback part by helping complaining parties learn the decision value and move to the fallback only when the number of complaints indicates that the run is not synchronous. This way, each Byzantine party in a synchronous run cannot increase the communication cost by more than $O(n) = O(t)$, where $n$ is the total number of parties.

The second challenge in the optimistic protocol is to combine both parts in a way that guarantees safety. That is, since some parties may decide in the synchronous part and others in the asynchronous fallback, we need to make sure they decide on the same value. To this end, we use the \textit{leader-based view (LBV)} abstraction, defined in \cite{35}, as a building block for both parts. The LBV abstraction captures a single view in a view-by-view agreement protocol such that one of its important properties is that a sequential composition of them preserves safety. For optimal communication cost, we adopt techniques from \cite{35} and \cite{1} to implement the LBV abstraction with an asymptotically linear cost ($O(n)$).

Our synchronous protocol operates up to $n$ sequentially composed pre-defined linear LBV instances, each with a different leader. To achieve an optimal (adaptive to the number of actual failures) cost, leaders invoke their LBVs only if they have not yet decided. In contrast to eventually synchronous protocols, the synchronous part is designed to provide termination only in synchronous runs. Therefore, parties do not need to be synchronized before views, but rather move from one LBV to the next at pre-defined times. As for the asynchronous fallback, we use the linear LBV building block to reconstruct the VABA \cite{1} protocol in a way that forms a sequential composition of LBVs, which in turn allows a convenient sequential composition with the synchronous part.
1.3 Related work

The idea of combining several agreement protocols is not new. The notion of speculative linearizability [19] allows parties to independently switch from one protocol to another, without requiring them to reach agreement to determine the change of a protocol. Aguilera and Toueg [2] presented a hybrid approach to solve asynchronous crash-fault consensus by combining randomization and unreliable failure detection. Guerraoui et al [18] defined an abstraction that captures byzantine agreement protocols and presented a framework to compose several such instances.

Some previous work on Byzantine agreement consider a fallback in the context of the number rounds required for termination [7, 26, 34]. That is, in well-behaved runs parties decide in a single communication round, whereas in all other runs they fallback to a mode that requires more rounds to reach an agreement. We, in contrast, are interested in communication complexity. To the best of our knowledge, our protocol is the first protocol that adapts its communication complexity based on the actual number of failures.

The combination of synchronous and asynchronous runs in the context of Byzantine agreement was previously studied by Blum et al. [5]. Their result is complementary to ours since they deal with optimal resilience rather than optimal communication. They showed lower and upper bounds on the number of failures that both (synchronous and asynchronous) parts can tolerate. For the lower bound, they showed that \( t_a + 2t_s < n \), where \( t_a \) and \( t_s \) is the threshold failure in asynchronous and synchronous runs, respectively. In our protocol \( t_a = t_s < n/3 \), which means that the protocol is optimal in the sense that neither \( t_a \) or \( t_s \) can be increased without decreasing the other. For the upper bound, they present a matching algorithm for any \( t_a \) and \( t_s \) that satisfy the weak validity condition. Our protocol, in contrast, satisfy the more practical external validity condition (see more details in the next section) with an optimal communication cost.

As for asynchronous Byzantine agreement, the lower bound in [1] shows that there is no protocol with optimal resilience and \( o(n^2) \) communication complexity. Two recent works by Cohen et al. [13] and Blum et al. [4], circumvent this lower bound by trading optimal resilience. That is, their protocols tolerate \( f < (1 - \epsilon)n/3 \) Byzantine faults. We consider in this paper optimal resilience and thus our protocol achieves optimal communication complexity in asynchronous runs.

The use of cryptographic tools (e.g. PKI and threshold signatures schemes) is very common in distributed computing to reduce round and communication complexity. To be able to focus on the distributed aspect of the problem, many previous algorithms assume ideal cryptographic tools to avoid the analysis of the small error probability induced by the security parameter. This includes the pioneer protocols for Byzantine broadcast [16, 14] and binary asynchronous Byzantine agreement [6], recent works on synchronous Byzantine agreement [28, 31], and most of the exciting practical algorithms [23, 12] including the state-of-the-art communication efficient ones [12, 36, 20, 10]. We follow this approach and assume ideal threshold signatures schemes for better readability.

2 Model

Following practical solutions [12, 20, 36, 23, 27], we consider a Byzantine message passing peer to peer model with a set \( \Pi \) of \( n \) parties and a computationally bounded adversary that corrupts up to \( t < n/3 \) of them, \( \Omega(t) = O(n) \). Parties corrupted by the adversary are called Byzantine and may arbitrarily deviate from the protocol. Other parties are honest. To strengthen the result we consider an adaptive adversary for the upper bound and static
adversary for the lower bound. The difference is that a static adversary must decide what parties to corrupt at the beginning of every execution, whereas an adaptive adversary can choose during the executions.

**Communication and runs.** The communication links are reliable but controlled by the adversary, i.e., all messages sent among honest parties are eventually delivered, but the adversary controls the delivery time. We assume a known to all parameter \(\Delta\) and say that a run of a protocol is *eventually synchronous* if there is a global stabilization time (GST) after which all message sent among honest parties are delivered within \(\Delta\) time. A run is *synchronous* if GST occurs at time 0, and *asynchronous* if GST never occurs.

**The Agreement problem.** Each party get an input value from the adversary from some domain \(V\) and the Agreement problem exposes an API to propose a value and to output a decision. We are interested in protocols that never compromise safety and thus require the following property to be satisfied in all runs:

- Agreement: All honest parties that decide, decide on the same value.

Due to the FLP result [17], no deterministic agreement protocol can provide safety and liveness properties in all asynchronous runs. Therefore, in this paper, we consider protocols that guarantee (deterministic) termination in all synchronous and eventually synchronous runs, and provides a probabilistic termination in asynchronous ones:

- Termination: All honest parties eventually decide.
- Probabilistic-Termination: All honest parties decide with probability 1.

As for validity, honest parties must decide only on values from some domain \(V\). For the lower bounds, to strengthen them as much as possible, we consider the binary case, which is the weakest possible definition:

- Binary validity: The domain of valid values \(V = \{0, 1\}\), and if all honest parties propose the same value \(v \in V\), than no honest party decides on a value other than \(v\).

For the upper bounds, we are interested in practical multi-valued protocols. In contrast to binary validity, in a multi-valued Byzantine agreement we need also to define what is a valid decision in the case that not all parties a priori agree (i.e., propose different values). One option is Weak Validity [32, 5], which allows parties to agree on a pre-defined \(\perp\) in that case. This definition is well defined and makes sense for some use cases. When Pease et al. [32] originally defined it, they had in mind a spaceship cockpit with 4 sensors that try to agree even if one is broken (measures a wrong value). However, as Cachin et al, explain in their paper [11] and book [24], this definition is useless for SMR (and Blockchains) since if parties do not a priori agree, then they can keep agreeing on \(\perp\) forever leaving the SMR with no "real" progress.

To solve the limitation of being able to agree on \(\perp\), we consider the external validity property that was first defined by Cachin et al. [11], which is implicitly or explicitly considered in most practical Byzantine agreement solutions we are aware of [1 12 36 20 23]. Intuitively, with external validity, parties are allowed to decide on a value proposed by any party (honest and Byzantine) as long as it is valid by some external predicate (e.g., all transaction are valid in the block). To capture the above, we give a formal definition below.

- External validity: The domain of valid values \(V\) is unknown to honest parties. At the beginning of every run, each honest party gets a value \(v\) with a proof \(\sigma\) that \(v \in V\) such that all other honest parties can verify.
Note that our definition rules out trivial solutions such as simply deciding on some
pre-defined externally valid value because the parties do not know what is externally valid
unless they see a proof.

We define an optimistic Agreement protocol to be a protocol that guarantees Agreement
and External validity in all runs, Termination in all synchronous and eventually synchronous
runs, and Probabilistic-Termination in asynchronous runs.

Cryptographic assumptions. We assume a computationally bounded adversary and a
trusted dealer that equips parties with cryptographic schemes. Following a common standard
in distributed computing and for simplicity of presentation (avoid the analysis of security
parameters and negligible error probabilities), we assume that the following cryptographic
tools are perfect:

- **Authenticated link.** If an honest party $p_i$ delivers a messages $m$ from an honest party $p_j$, then $p_j$ previously sent $m$ to $p_i$.
- **Threshold signatures scheme.** We assume that each party $p_i$ has a private function $\text{share-sign}_i$, and we assume 3 public functions: $\text{share-validate}$, $\text{threshold-sign}$, and $\text{threshold-validate}$. Informally, given “enough” valid shares, the function $\text{threshold-sign}$ returns a valid threshold signature. For our algorithm, we sometimes require “enough” to be $t + 1$ and sometimes $n - t$. A formal definition is given in Appendix B.

We note that perfect cryptographic schemes do not exist in practice. However, since in
real-world systems they often treated as such, we believe that they capture just enough in
order to be able to focus on the distributed aspect of the problem. Moreover, all the lower
bounds in this paper hold even if protocols can use perfect cryptographic schemes. Thus, the
upper bounds are tight in this aspect.

**Communication complexity.** We denote by $f$ the actual number of corrupted parties
in a given run and we are interested in optimistic protocols that utilize $f$ and the network
condition to reduce communication cost. Similarly to [11], we say that a word contains a
constant number of signatures and values, and each message contains at least 1 word. The communication cost of a run $r$ is the number of words sent in messages by honest parties
in $r$. For every $0 \leq f \leq t$, let $R^s_f$ and $R^e_f$ be the sets of all synchronous and eventually
synchronous runs with $f$ corrupted parties, respectively. The synchronous and eventually
synchronous communication cost with $f$ failures is the maximal communication cost of runs
in $R^s_f$ and $R^e_f$, respectively. We say that the synchronous communication cost of a protocol $A$
is $G(f, t)$ if for every $0 \leq f \leq t$, its synchronous communication cost with $f$ failures is $G(f, t)$. The asynchronous communication cost of a protocol $A$ is the expected communication cost of
an asynchronous run of $A$.

3 Lower Bounds

In this section, we present two lower bounds on the communication complexity of deterministic
Byzantine agreement protocols in synchronous and eventually synchronous runs.

3.1 Eventually synchronous runs

The following theorem exemplifies the inherent vulnerability of the eventually synchronous
approach.
The following Lemma shows that if honest parties send $o(ft)$ messages, then Byzantine parties can prevent honest parties from getting any of them.
Lemma 6. Assume that there is a Byzantine agreement algorithm $A$, which synchronous communication cost with $f$ failures is $o(ft)$ for some $1 \leq f \leq [t/2]$. Then, for every set $S \subseteq \Pi$ of $f$ parties and every set of values proposed by honest parties, there is a synchronous run $r'$ s.t. some honest party $p \in S$ does not get any messages in $r'$.

Proof. Let $r \in R_f^t$ be a run in which all parties in $S$ are Byzantine that (1) do not send messages among themselves, and (2) ignore all messages they receive and act like honest parties that get no messages. By the assumption, there is a party $p \in S$ that receives less than $t/2$ messages from honest parties in $r$. Denote the set of (honest) parties outside $S$ that send messages to $p$ in $r$ by $P \subseteq \Pi \setminus S$ and consider the following run $r'$:

- Parties in $S \setminus \{p\}$ are Byzantine that act like in $r$.
- Parties in $P$ are Byzantine. They do not send messages to $p$, but other than that act as honest parties.
- All other parties, including $p$, are honest.

First, note that the number of Byzantine parties in $r'$ is $|S| - 1 + |P| \leq f - 1 + t/2 \leq t$. Also, since $p$ acts in $r$ as an honest party that does not receive messages, and all Byzantine parties in $r'$ act towards honest parties in $r'$ ($\Pi \setminus (S \cup P)$) in exactly the same way as they do in $r$, then honest parties in $r'$ cannot distinguish between $r$ and $r'$. Thus, since they do not send messages to $p$ in $r$ they do not send in $r'$ as well. Therefore, $p$ does not get any message in $r'$.

The next Lemma is proven by showing that honest parties that do not get messages cannot safely decide. Not that the case of $f > t/2$ is not required to conclude Theorem 2 since in this case $o(ft) = o(t^2)$.

Lemma 7. For any $1 \leq f \leq [t/2]$, there is no optimistic Byzantine agreement algorithm which synchronous communication cost with $f$ failures is $o(ft)$.

Proof. Assume by a way of contradiction such protocol $A$ which synchronous communication cost with $f$ failures is $o(ft)$ for some $1 \leq f \leq [t/2]$. Pick a set of $S_1 \subset \Pi$ of $f$ parties and let $V$ be the set of values that honest parties propose. By Lemma 6, there is a run $r_1$ of $A$ in which honest parties propose values from $V$ s.t. some honest party $p_1 \in S$ does not get any messages. Now let $S_2 = \{p\} \cup S_1 \setminus \{p_1\}$ s.t. $p \in \Pi \setminus S_1$. By Lemma 6 again, there is a run $r_2$ of $A$ in which honest parties propose values from $V$ s.t. some honest party $p_2 \neq p_1$ does not get any messages. Since $f \leq [t/2]$, we can repeat the above $2t + 1$ times by each time replacing the honest party in $S_1$ that get no messages with a party not in $S_1 \cup \{p_1, p_2, \ldots, p_i\}$. Thus, we get that for every possible set of inputs $V$ (values proposed by honest parties) there is a set $T$ of $2t + 1$ parties s.t. for every party $p \in T$ there is a run of $A$ in which honest parties propose values from $V$, $p$ is honest, and $p$ does not get any messages. In particular, there exist such set $T_0$ for the case in which all honest party input 0 and a set $T_1$ for the case in which all honest parties input 1. Since $|T_0| = |T_1| = 2t + 1$, there is a party $p \in T_1 \cap T_2$. Therefore, by the Termination and Binary validity properties, there is a run $r$ in which $p$ does not get any messages and decides 0 and a run $r'$ in which $p$ does not any messages and decides 1. However, since $r$ and $r'$ are indistinguishable to $p$ we get a contradiction.

The following Theorem follows directly from Lemma 7 and Claim 5.

Theorem 2 (restated). Any synchronous deterministic Byzantine agreement protocol has a communication cost of $\Omega(ft + t)$. 
Our optimistic Byzantine agreement protocol safely combines synchronous and asynchronous protocols. Our synchronous protocol, which is interesting on its own, matches the lower bound proven in Theorem 2. That is, its communication complexity is $O(ft + t)$. The asynchronous protocol we use has a worst-case optimal quadratic communication complexity. For ease of exposition, we construct our protocol in steps. First, in Section 4.1, we present the local state each party maintains, define the leader-based view (LBV) building block, which is used by both protocols, and present an implementation with $O(n)$ communication complexity. Then, in Section 4.2 we describe our synchronous protocol, and in Section 4.3 we use the LBV building block to reconstruct VABA \[1\] - an asynchronous Byzantine agreement protocol with expected $O(n^2)$ communication cost and $O(1)$ running time. Finally, in section 4.4 we safely combine both protocols to prove the following:

$\blacktriangleright$ Theorem 4 (restated). There is an authenticated Byzantine agreement protocol with $O(ft + t)$ communication complexity in synchronous runs and expected $O(t^2)$ communication complexity in all other runs.

A formal correctness proof and communication analysis of the protocol appear in Appendix A.

## 4.1 General structure

The protocol uses many instances of the LBV building block, each of which is parametrized with a sequence number and a leader. We denote an LBV instance that is parametrized with sequence number $sq$ and a leader $p_l$ as $LBV(sq, p_l)$. Each party in the protocol maintains a local state, which is used by all LBVs and is updated according to their returned values. Section 4.1.1 presents the local state and Section 4.1.2 describes a linear communication LBV implementation. Section 4.1.3 discusses the properties guaranteed by a sequential composition of several LBV instances.

### 4.1.1 Local state

The local state each party maintains is presented in Algorithm 1. For every possible sequence number $sq$, $LEADER[\{sq\}]$ stores the party that is chosen (a priori or in retrospect) to be the leader associated with $sq$. The $COMMIT$ variable is a tuple that consists of a value $val$, a sequence number $sq$ s.t. $val$ was committed in $LBV(sq, LEADERS[\{sq\}])$, and a threshold signature that is used as a proof of it. The $VALUE$ variable contains a safe value to propose and the $KEY$ variable is used as proof that $VALUE$ is indeed safe. $KEY$ contains a sequence number $sq$ and a threshold signature that proves that no value other than $VALUE$ could be committed in $LBV(sq, LEADERS[\{sq\}])$. The $LOCK$ variable stores a sequence number $sq$, which is used to determine what keys are up-to-date and what are obsolete – a key is up-to-date if it contains a sequence number that is greater than or equal to $LOCK$.

**Algorithm 1** Local state initialization.

\[
\begin{align*}
LOCK & \in \mathbb{N} \cup \{\bot\}, \text{ initially } \bot \\
KEY & \in (\mathbb{N} \times \{0, 1\}^*) \cup \{\bot\}, \text{ with selectors } sq \text{ and proof, initially } \bot \\
VALUE & \in \mathbb{V} \cup \{\bot\}, \text{ initially } \bot \\
COMMIT & \in (V \times \mathbb{N} \times \{0, 1\}^*) \cup \{\bot\}, \text{ with selectors } val, sq \text{ and proof, initially } \bot
\end{align*}
\]

\[\text{for every } sq \in \mathbb{N}, \ LEADER[\{sq\}] \in \Pi \cup \{\bot\}, \text{ initially } \bot\]
4.1.2 Linear leader-based view

Detailed pseudocode of the linear implementation of the LBV building block is given in Algorithms 2 and 3. An illustration appears in figure 1. The LBV building block supports an API to start the view and wedge the view. Upon a startView((sq, p)) invocation, the invoking party starts processing messages associated with LBV(sq,p). When the leader p invokes startView((sq, p)) it initiates 3 steps of leader-to-all and all-to-leader communication, named PreKeyStep, KeyStep, and LockStep. In each step, the leader sends its VALUE together with a threshold signature that proves the safety of the value for the current step and then waits to collect n − t valid replies. A party that gets a message from the leader, validates that the received value and proof are valid for the current step, then produces its signature share on a message that contains the value and the step’s name, and sends the share back to the leader. When the leader gets n − t valid shares, it combines them into a threshold signature and continues to the next step. After successfully generating the threshold signature at the end of the third step (LockStep), the leader has a commit certificate which he sends together with its VALUE to all parties.

Algorithm 2 A linear implementation of LBV(sq,leader): API for a party pi.

Local variables initialization:
$S_{key} = S_{lock} = S_{commit} = \{\}$
$\text{keyProof, lockProof, commitProof} \in (V \times \{0, 1\}^*) \cup \{\perp\}$
val and proof, initially $\perp$
active $\leftarrow$ true; done $\leftarrow$ false

1: upon wedgeView(sq, leader) invocation do
2: active $\leftarrow$ false
3: return (keyProof, lockProof, commitProof)

4: upon startView(sq, leader) invocation do
5: start processing received messages associated with sq and leader
6: if leader $= p_i$ then
7: **/first step/**
8: send “PREKEYSTEP, sq, leader, VALUE, KEY” to all parties
9: wait until $|S_{key}| = n − t$
10: $\nu_k \leftarrow \text{threshold-sign}(S_{key})$
11: **/second step/**
12: send “KEYSTEP, sq, leader, VALUE, $\nu_k$” to all parties
13: wait until $|S_{lock}| = n − t$
14: $\nu_l \leftarrow \text{threshold-sign}(S_{lock})$
15: **/third step/**
16: send “LOCKSTEP, sq, leader, VALUE, $\nu_l$” to all parties
17: wait until $|S_{commit}| = n − t$
18: $\nu_c \leftarrow \text{threshold-sign}(S_{commit})$
19: **/broadcast the commit/**
20: send “COMMITH, sq, leader, VALUE, $\nu_c$” to all parties
21: wait for done $= true$
22: return (keyProof, lockProof, commitProof)

In addition to validating and share-signing messages, parties also store the values and proofs they receive. The keyProof and lockProof variables store tuples consisting of the values and the threshold signatures received from the leader in the KeyStep, and LockStep steps, respectively. The commitProof variable stores the received value and the commit certificate. When a party receives a valid commit certificate from the leader it returns.

As for the validation of the leader’s messages, parties distinguish the PreKeyStep message from the rest. For KeyStep, LockStep and commit certificate messages, parties simply check
that the attached proof is a valid threshold signature on the leader’s value and the previous step name. The PreKeyStep message, however, is used by the Agreement protocols to safely compose many LBV instances. We describe this mechanism in more details below, but to develop some intuition let us first present the properties guaranteed by a single LBV instance:

- Commit causality: If a party gets a valid commit certificate, then at least \( t + 1 \) honest parties previously got a valid \textbf{lockProof}. 
- Lock causality: If a party gets a valid \textbf{lockProof}, then at least \( t + 1 \) honest parties previously got a valid \textbf{keyProof}. 
- Safety: All valid \textbf{keyProof}, \textbf{lockProof}, and commit certificates obtained in the same LBV have the same value.

\begin{algorithm}
\begin{enumerate}
\item \underline{upon receiving} “\textbf{PreKeyStep}, sq, leader, value, key” from leader \textbf{do}
\item if active \& (LOCK = ⊥ ∨ key, sq ≥ LOCK) then \hspace{0.5cm} ▷ key is up-to-date \\
\item if key = ⊥ ∨ threshold-validate((\textbf{PreKeyStep}, key, sq, \textbf{LEADERS}[key, sq, value], key.proof)) then \\
\item \( ρ_k \leftarrow \text{share-sign}_i((\textbf{PreKeyStep}, sq, leader, value)) \)
\item send “\textbf{keyShare}, sq, leader, \( ρ_k \)” to leader \hspace{0.5cm}
\item \underline{upon receiving} “\textbf{keyShare}, sq, leader, \( ρ_k \)” from \( p_j \) for the first time \textbf{do}
\item if leader = \( p_i \) then \hspace{0.5cm} ▷ \( \text{key} \) is up-to-date \\
\item if share-validate((\textbf{PreKeyStep}, sq, leader, VALUE), \( p_j \), \( ρ_k \)) then \\
\item \( S_{\text{key}} \leftarrow S_{\text{key}} \cup ρ_k \)
\item \underline{upon receiving} “\textbf{KeyStep}, sq, leader, value, ν_k” from leader \textbf{do}
\item if active then \\
\item if threshold-validate((\textbf{KeyStep}, sq, leader, value), \( ν_k \)) then \\
\item \( ρ_i \leftarrow \text{share-sign}_i((\textbf{KeyStep}, sq, leader, value)) \)
\item send “\textbf{lockShare}, sq, leader, \( ρ_i \)” to leader \hspace{0.5cm}
\item \underline{upon receiving} “\textbf{lockShare}, sq, leader, \( ρ_i \)” from \( p_j \) for the first time \textbf{do}
\item if leader = \( p_i \) then \hspace{0.5cm} ▷ \( \text{lock} \) is up-to-date \\
\item if share-validate((\textbf{KeyStep}, sq, leader, VALUE), \( p_j \), \( ν_i \)) then \\
\item \( S_{\text{lock}} \leftarrow S_{\text{lock}} \cup ρ_i \)
\item \underline{upon receiving} “\textbf{lockStep}, sq, leader, value, ν_i” from leader \textbf{do}
\item if active then \\
\item if threshold-validate((\textbf{lockStep}, sq, leader, value), \( ν_i \)) then \\
\item \( ρ_c \leftarrow \text{share-sign}_i((\textbf{lockStep}, sq, leader, value)) \)
\item send “\textbf{commitShare}, sq, leader, \( ρ_c \)” to leader \hspace{0.5cm}
\item \underline{upon receiving} “\textbf{commitShare}, sq, leader, \( ρ_c \)” from \( p_j \) for the first time \textbf{do}
\item if leader = \( p_i \) then \\
\item if share-validate((\textbf{lockStep}, sq, leader, VALUE), \( p_j \), \( ρ_c \)) then \\
\item \( S_{\text{commit}} \leftarrow S_{\text{commit}} \cup ρ_c \)
\item \underline{upon receiving} “\textbf{commit}, sq, leader, value, \( ν_c \)” from leader \textbf{do}
\item if active then \\
\item if threshold-validate((\textbf{lockStep}, sq, leader, value), \( ν_c \)) then \\
\item \( \textbf{commitProof} \leftarrow (value, ν_c) \)
\item done \leftarrow true
\end{enumerate}
\end{algorithm}

The validation of the PreKeyMessage in PreKeyStep makes sure that the leader’s value satisfies the safety properties of the Byzantine agreement protocol that sequentially composes and operates several LBVs. The PreKeyMessage contains the leader’s \texttt{VALUE} and \texttt{KEY},
where \( KEY \) stores the last (non-empty) \( \text{keyProof} \) returned by a previous LBV instance together with the LBV’s sequence number. When a party gets a \( \text{PreKeyMessage} \) it first validates, by checking the key’s sequence number \( sq \), that the attached key was obtained in an LBV instance that does not precede the one the party is locked on (the sequence number that is stored in the party’s \( \text{LOCK} \) variable). Then, the party checks that the threshold signature in the key (1) was generated at the end of the \( \text{PreKeyStep} \) step (it is a valid \( \text{keyProof} \)) in \( \text{LBV}(sq, \text{LEADER}[,sk]) \); and (2) it is a valid signature on a message that contains the leader’s \( \text{VALUE} \). Note that if the party is not locked (\( \text{LOCK} = \bot \)) then a key is not required.

Upon a \( \text{wedgeView}(sq, pl) \) invocation, the invoking party stops participating in \( \text{LBV}(sq, pl) \) and returns its current \( \text{keyProof} \), \( \text{lockProof} \), and \( \text{commitProof} \) values. These values are used by both synchronous and asynchronous protocols, which are built on top of LBV instances, to update the \( \text{LOCK} \), \( \text{KEY} \), \( \text{VALUE} \), and \( \text{COMMIT} \) variables in parties’ local states. Stopping participating in \( \text{LBV}(sq, pl) \) upon a \( \text{wedgeView}(sq, pl) \) invocation guarantees that the the LBVs’ causality guarantees are propagated the \( \text{KEY} \), \( \text{LOCK} \), and \( \text{COMMIT} \) variables in parties local states.

**Communication complexity.** Note that the number of messages sent among honest parties in an LBV instance is \( O(n) = O(t) \). In addition, since signatures are not accumulated – leaders use threshold signatures – each message contains a constant number of words, and thus the total communication cost of an LBV instance is \( O(t) \) words.

![Figure 1](image_url) A linear communication LBV illustration. The local state is used by and updated after each instance. The \( \text{keyProof} \), \( \text{lockProof} \), and \( \text{commitProof} \) are returned when a commit message is received from the leader or \( \text{wedgeView} \) is invoked.

### 4.1.3 Sequential composition of LBVs

As mentioned above, our optimistic Byzantine agreement protocol is built on top of the LBV building blocks. The synchronous and the asynchronous parts of the protocol use different approaches, but they both sequentially compose LBVs - the synchronous part of the protocol determines the composition in advance, whereas the asynchronous part chooses what instances are part of the composition in retrospect.

In a nutshell, a sequential composition of LBVs operates as follows: parties start an LBV instance by invoking \( \text{startView} \) and at some later time (depends on the approach) invoke \( \text{wedgeView} \) and update their local states with the returned values. Then, they exchange messages to propagate information (e.g., up-to-date keys or commit certificates), update their local states again and start the next LBV (via \( \text{startView} \) invocation). We claim that an agreement protocol that sequentially composes LBV instances and maintains the local state in Algorithm 1 has the following properties:
Agreement: all commit certificates in all LBV instances have the same value. Conditional progress: for every LBV instance, if the leader is honest, all honest parties invoke startView, and all messages among honest parties are delivered before some honest party invokes wedgeView, then all honest parties get a commit certificate.

Intuitively, by the LBV’s commit causality property, if some party returns a valid commit certificate (commitProof) with a value \( v \) in some LBV(sq,pi), then at least \( t+1 \) honest parties return a valid lockProof and thus lock on sq (LOCK \( \leftarrow \) sq). Therefore, since the leader of the next LBV needs the cooperation of \( n-t \) parties to generate threshold signatures, its PreKeyStep message must include a valid keyProof that was obtained in LBV(sq,pi).

By the LBV’s safety property, this keyProof includes the value \( v \) and thus \( v \) is the only value the leader can propose. The agreement property follows by induction.

As for conditional progress, we have to make sure that honest leaders are able to drive progress. Thus, we must ensure that all honest leaders have the most up-to-date keys. By the lock causality property, if some party gets a valid lockProof in some LBV, then at least \( t+1 \) honest parties get a valid keyProof in this LBV and thus are able to unlock all honest parties in the next LBV. Therefore, leaders can get the up-to-date key by querying a quorum of \( n-t \) parties.

From the above, any Byzantine agreement protocol that sequentially composes LBVs satisfies Agreement. The challenge, which we address in the rest of this section, is how to sequentially compose LBVs in a way that satisfies Termination with asymptotically optimal communication complexity under all network conditions and failure scenarios.

### 4.2 Adaptive to failures synchronous protocol

**Algorithm 4** Adaptive synchronous protocol: Procedure for a party \( p_i \).

1: \textbf{upon} Synch-propose(vi) \textbf{do}
2: \hspace{1em} VALUE \( \leftarrow \) vi
3: \hspace{1em} tryOptimistic()

4: \textbf{procedure} tryOptimistic()
5: \hspace{1em} trySynchrony(1, pi, 7Δ)
6: \hspace{1em} for \( j \leftarrow 2 \) to \( n \) \textbf{do}
7: \hspace{2em} if \( i \neq j \) \textbf{then}
8: \hspace{3em} trySynchrony(j, pj, 9Δ)
9: \hspace{2em} else if \( \text{COMMIT} = \bot \) \textbf{then}
10: \hspace{3em} send “keyRequest” to all parties
11: \hspace{2em} wait for \( 2\Delta \) time
12: \hspace{2em} trySynchrony(j, pj, 7Δ)

13: \textbf{procedure} trySynchrony(sq, leader, T)
14: \hspace{1em} invoke startView(sq, leader) \hspace{2em} \( \triangleright \) non-blocking invocation
15: \hspace{1em} wait for \( T \) time
16: \hspace{1em} (keyProof, lockProof, commitProof) \( \leftarrow \) wedgeView(sq, leader)
17: \hspace{1em} updateState(sq, leader, keyProof, lockProof, commitProof)

18: \textbf{upon} receiving “keyRequest” from party \( p_k \) for the first time \textbf{do}
19: \hspace{1em} send “keyReply, KEY, VALUE” to party \( p_k \)

20: \textbf{upon} receiving “keyReply, key, value” \textbf{do}
21: \hspace{1em} check\&updateKey(key, value)

In this section, we describe a synchronous Byzantine agreement protocol with an asymptotically optimal adaptive communication cost that matches the lower bound in Theorem 2.
Namely, we prove the following Theorem:

**Theorem 3** (restated). There is a deterministic synchronous authenticated Byzantine agreement protocol with $O(ft + t)$ communication complexity.

 ![Figure 2](image.png) Illustration of the adaptive synchronous protocol. Shaded LBVs are not executed if their leaders have previously decided.

A detailed pseudocode is given in Algorithms 4 and 5 and an illustration appears in figure 2. The protocol sequentially composes $n$ pre-defined LBV instances, each with a different leader, and parties decide $v$ whenever they get a commit certificate with $v$ in one of them. To avoid the costly view-change mechanism that is usually unavoidable in leader-based protocols, parties exploit synchrony to coordinate their actions. That is, all the `startView` and `wedgeView` invocation times are predefined, e.g., the first LBV starts at time 0 and is wedged at time $7\Delta$ simultaneously by all honest parties. In addition, to make sure honest leaders can drive progress, each leader (except the first) learns the up-to-date key, before invoking `startView`, by querying all parties and waiting for a quorum of $n - t$ parties to reply.

**Algorithm 5** Auxiliary procedures to update local state.

```plaintext
1: procedure UPDATESTATE(sq, leader, keyProof, lockProof, commitProof)
2: LEADERS[sq] ← leader
3: if keyProof ≠ ⊥ then
4: KEY ← (sq, keyProof.proof)
5: VALUE ← keyProof.val
6: if lockProof ≠ ⊥ then
7: LOCK ← sq
8: if commitProof ≠ ⊥ then
9: COMMIT ← (commitProof.val, sq, commitProof.proof)
10: decide COMMIT.val

11: procedure CHECK&UPDATEKEY(key, value)
12: if (KEY = ⊥ v key.sq > KEY.sq) then
13: if threshold-validate(∀keyStep, key.sq, LEADER[key.sq], value, key.proof) then
14: KEY ← key
15: VALUE ← value

17: procedure CHECK&UPDATECOMMIT(commit)
18: if COMMIT = ⊥ then
19: if threshold-validate(∀lockStep, commit.sq, LEADER[commit.sq], commit.val), commit.proof) then
20: COMMIT ← commit
21: decide COMMIT.val
```

Composing $n$ LBV instances may lead in the worst case to $O(t^2)$ communication complexity – $O(t)$ for every LBV instance. Therefore, to achieve the optimal adaptive complexity, honest leaders in our protocol participate (learn the up-to-date key and invoke `startView`) only in
case they have not yet decided. (Note that the communication cost of an LBV instance
in which the leader does not invoke $\text{startView}$ is 0 because other parties only reply to the
leader’s messages.) For example, if the leader of the second LBV instance is honest and has
committed a value in the first instance (its $\text{COMMIT} \neq \bot$ at time $7\Delta$), then no message is
sent among honest parties between time $7\Delta$ and time $16\Delta$.

Algorithm 6 Asynchronous fallback: Protocol for a party $p_i$.

1: upon $\text{Asynch-propose}(v_i)$ do
2: \hspace{1em} $\text{VALUE} \leftarrow v_i$
3: \hspace{1em} $\text{fallback}(1)$

4: \hspace{1em} \textbf{procedure} $\text{FALLBACK}(sq_{\text{init}})$
5: \hspace{2em} $\text{RRleader} \leftarrow 1$
6: \hspace{2em} $sq \leftarrow sq_{\text{init}} + 1$
7: \hspace{2em} \textbf{while} true do
8: \hspace{3em} $\text{wave}(sq)$
9: \hspace{3em} $\text{exchangeState}(sq)$
10: \hspace{3em} $\text{help} \& \text{tryHalting}(sq)$
11: \hspace{3em} $\text{trySynchrony}(sq + 1, \text{RRleader}, 8\Delta)$
12: \hspace{3em} $\text{exchangeState}(sq + 1)$
13: \hspace{3em} $\text{help} \& \text{tryHalting}(sq + 1)$
14: \hspace{2em} $\text{RRleader} \leftarrow \text{RRleader} + 1 \text{ mod } |\Pi|$
15: \hspace{2em} $sq \leftarrow sq + 2$

16: upon $\text{startView}(sq, p_j)$ returns do
17: \hspace{2em} send “YOUR-VIEW-DONE, $sq$” to party $p_j$

18: \hspace{1em} \textbf{procedure} $\text{WAVE}(sq)$
19: \hspace{2em} for all $p_j = p_1, \ldots, p_n$ do
20: \hspace{3em} invoke $\text{startView}(sq, p_j)$ \hspace{1em} $\triangleright$ non-blocking invocation
21: \hspace{2em} $\text{barrier-sync}(sq)$ \hspace{1em} $\triangleright$ blocking
22: \hspace{2em} $\text{leader} \leftarrow \text{elect}(sq)$
23: \hspace{2em} $(\text{keyProof}, \text{lockProof}, \text{commitProof}) \leftarrow \text{wedgeView}(sq, \text{leader})$
24: \hspace{2em} $\text{updateState}(sq, \text{leader}, \text{keyProof}, \text{lockProof}, \text{commitProof})$

25: upon receiving $n - t$ “YOUR-VIEW-DONE, $sq$” messages do
26: \hspace{2em} invoke $\text{barrier-ready}(sq)$ \hspace{1em} $\triangleright$ note that $n - t$ parties must invoke it for $\text{barrier-sync}(sq)$ to return
27: \hspace{1em} \textbf{procedure} $\text{EXCHANGESTATE}(sq)$
28: \hspace{2em} send “EXCHANGE, $sq$, KEY, VALUE, COMMIT” to all parties
29: \hspace{2em} $\text{wait}$ for $n - t$ “EXCHANGE, $sq$, $s$, $s'$” messages from different parties
30: upon receiving “EXCHANGE, $sq$, key, value, commit” do
31: \hspace{2em} $\text{check\&updateKey(key, value)}$
32: \hspace{2em} $\text{check\&updateCommit(commit)}$

Termination and communication complexity. A naive approach to guarantee
termination and avoid an infinite number of LBV instances in a leader based Byzantine
agreement protocols is to perform a costly communication phase after each LBV instance.
One common approach is to reliably broadcast commit certificates before halting, while a
complementary one is to halt unless receiving a quorum of complaints from parties that did
not decide. In both cases, the communication cost is $O(t^2)$ even in runs with at most one
failure.

The key idea of our synchronous protocol is to exploit synchrony in order to allow honest
parties to learn the decision value and at the same time help others in a small number of
messages. Instead of complaining (together) after every unsuccessful LBV instance, each
party has its own pre-defined time to “complain”, in which it learns the up-to-date key and
value and helps others decide via the LBV instance in which it acts as the leader.
By the conditional progress property and the synchrony assumption, all honest parties get a commit certificate in LBV instances with honest leaders. Therefore, the termination property is guaranteed since every honest party has its own pre-defined LBV instance, which it invokes only in case it has not yet decided. As for the protocol’s total communication cost, recall that the LBV’s communication cost is $O(t)$ in the worst case and 0 in case its leader already decided and thus does not participate. In addition, since all honest parties get a commit certificate in the first LBV instance with an honest leader, we get that the message cost of all later LBV instances with honest leaders is 0. Therefore, the total communication cost of the protocol is $O(ft + t)$ – at most $f$ LBVs with Byzantine leaders and 1 LBV with an honest one.

### Algorithm 7

**Barrier synchronization and Leader-election: protocol for a party $p_i$.**

---

**Local variables for Barrier synchronization:**

1. $S_{\text{barrier}} \leftarrow \{\}$; $\text{READY} \leftarrow \text{false}$

2. **procedure** BARRIER-SYNC($sq$)

3. **wait** until $\text{READY} = \text{true}$

4. **procedure** BARRIER-READY($sq$)

5. $\rho \leftarrow \text{share-sign}_i(\langle \text{shareReady}, sq \rangle)$

6. send “shareReady, sq, $\rho$” to all parties

7. **upon receiving** “shareReady, sq, $\rho$” from a party $p_j$ do

8. if $\text{share-validate}(\langle \text{shareReady}, sq \rangle, p_j, \rho)$ then

9. $S_{\text{barrier}} \leftarrow S_{\text{barrier}} \cup \{\rho\}$

10. if $|S_{\text{barrier}}| = n - t$ then

11. $\nu \leftarrow \text{threshold-sign}(S_{\text{barrierReady}})$

12. send “barrierReady, sq, $\nu$” to all parties

13. **upon receiving** “barrierReady, sq, $\nu$” do

14. if $\text{threshold-validate}(\langle \text{barrierReady}, sq \rangle, \nu)$ then

15. send “barrierReady, sq, $\nu$” to all parties

16. $\text{READY} \leftarrow \text{true}$

---

**Local variables for Leader election:**

17. $S_{\text{coin}} \leftarrow \{\}$

18. **procedure** ELECT($sq$)

19. $\rho \leftarrow \text{share-sign}_i(sq)$

20. send “coinShare, sq, $\rho$” to all parties

21. **wait** until $|S_{\text{coin}}| = t + 1$

22. $\nu \leftarrow \text{threshold-sign}(S_{\text{coin}})$

23. return $p_j$ s.t. $j = \text{Hash}(\nu) \mod |\Pi|$

24. **upon receiving** “coinShare, sq, $\rho$” from $p_j$ do

25. if $\text{share-validate}(sq, p_j, \rho)$ then

26. $S_{\text{coin}} \leftarrow S_{\text{coin}} \cup \{\rho\}$

---

### 4.3 Asynchronous fallback

In this section, we use the LBV building block to reconstruct VABA [1]. Note that achieving an optimal asynchronous protocol is not a contribution of this paper but reconstructing the VABA protocol with our LBV building block allows us to safely combine it with our adaptive synchronous protocol to achieve an optimal optimistic one. In addition, we also improve the protocol of VABA in the following ways: first, parties in VABA [1] never halt,
meaning that even though they decide in expectation in a constant number of rounds, they
operate an unbounded number of them. We fix it by adding an auxiliary primitive, we
call \textit{help&tryHalting} in between two consecutive waves (details below). Second, VABA
guarantees probabilistic termination in all runs, whereas our version also guarantees standard
termination in eventually synchronous runs. The full detailed pseudocode of our fallback
protocol appears in Algorithms \ref{algo:lbv}, \ref{algo:lbvElection}, \ref{algo:lbvStop}, and \ref{algo:lbvHelp}.

On a high level, the idea in VABA \cite{VABA} that was later generalized in \cite{Yao} is the following:
instead of having a pre-defined leader in every “round” of the protocol as most eventually
synchronous protocols and our synchronous protocol have, they let $n$ leaders operate sim-
ultaneously and then randomly choose one in retrospect. This mechanism is implemented
inside a wave and the agreement protocol operates in a wave-by-wave manner s.t. parties
exchange their local states between every two conductive waves. To ensure halting, in our
version of the protocol, parties also invoke the \textit{help&tryHalting} procedure after each wave.
See the \textit{tryPessimistic} procedure in Algorithm \ref{algo:lbv} for pseudocode (ignore gray lines at this
point) and Figure \ref{fig:lbv} for an illustration.

\textbf{Wave-by-wave approach.} To implement the wave mechanism (Algorithm \ref{algo:lbv}) we use our
LBV and two auxiliary primitives: Leader-election and Barrier-synchronization (Algorithm \ref{algo:lbvElection}).
At the beginning of every wave, parties invoke, via \textit{startView}, $n$ different LBV instances, each
with a different leader. Then, parties are blocked in the Barrier-synchronization primitive
until at least $n - 2t$ LBV instances complete. (An LBV completes when $t + 1$ honest parties
get a commit certificate.) Finally, parties use the Leader-election primitive to elect a unique
LBV instance, wedge it (via \textit{wedgeView}), and ignore the rest. With a probability of $1/3$
parties choose a completed LBV, which guarantees that after the state exchange phase all
honest parties get a commit certificate, decide, and halt in the \textit{help&tryHalting} procedure.
Otherwise, parties update their local state and continue to the next wave. An illustration
appears in figure \ref{fig:lbvWave}.

\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{lbvWave.png}
\caption{Asynchronous fallback. Using linear LBV to reconstruct the VABA \cite{VABA} protocol.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\linewidth]{lbvWave.png}
\caption{An illustration of a single wave. The returned \texttt{keyProof}, \texttt{lockProof}, and \texttt{commitProof} are taken from the elected LBV.}
\end{figure}

Since every wave has a probability of $1/3$ to choose a completed LBV instance, the
protocol guarantees probabilistic termination – in expectation, all honest parties decide
after 3 waves. To also satisfy standard termination in eventually synchronous runs, we “try synchrony” after each unsuccessful wave. See the gray lines in Algorithm 6. Between every two conjunctive waves parties deterministically try to commit a value in a pre-defined LBV instance. The preceding help&tryHalting procedure guarantees that after GST all honest parties invoke startView in the pre-defined LBV instance with at most $1\Delta$ from each other and thus setting a timeout to $8\Delta$ is enough for an honest leader to drive progress. We describe the help&tryHalting procedure in the next section. The description of the Barrier-synchronization and Leader-election primitives (Algorithm 7) can be found in [35].

**Communication complexity.** The communication cost of the Barrier-synchronization and Leader-election primitives, as well as that of $n$ LBV instances, is $O(n^2)$, which brings us to a total of $O(n^2)$ cost per wave. Since every wave have a probability of $1/3$ to choose a completed LBV, the protocol operates 3 waves in expectation. Therefore, since the communication cost of state exchange and help&tryHalting is $O(n^2)$, we get that the total cost, in expectation, is $O(n^2)$.

### 4.4 Optimal optimistic protocol: combine the pieces

At a high level, parties first optimistically try the synchronous protocol (of section 4.2), then invoke help&tryHalting and continue to the asynchronous fallback (of section 4.3) in case a decision has not been reached. Pseudocode is given in Algorithm 8 and an illustration appears in Figure 5. The parameters passed in Algorithm 8 synchronize the LBV sequence numbers across the different parts of the protocol.

**Algorithm 8** Optimistic byzantine agreement: protocol for a party $p_i$.

```plaintext
1: upon Optimistic-propose($v_i$) do
2:   VALUE $\leftarrow v_i$
3:   tryOptimistic()
4:   help&tryHalting($n$)
5:   fallback($n$)  ▷ Blocking invocation
```

One of the biggest challenges in designing an agreement protocol as a combination of other protocols is to make sure safety is preserved across them. Meaning that parties must never decide differently even if they decide in different parts of the protocol. In our protocol, however, this is inherently not a concern. Since both parts use LBV as a building block, we get safety for free. That is, if we look at an execution of our protocol in retrospect, i.e., ignore all LBVs that were not elected in the asynchronous part. Then the LBV instances in the synchronous part together with the elected ones in the asynchronous part form a sequential composition, which satisfies the Agreement property.

On the other hand, satisfying termination without sacrificing optimal adaptive complexity is a non-trivial challenge. Parties start the protocol by optimistically trying the synchronous part, but unfortunately, at the end of the synchronous part they cannot distinguish between
the case in which the communication was indeed synchronous and all honest parties decided
and the case in which some honest parties did not decide due to asynchrony. Moreover,
honest parties cannot distinguish between honest parties that did not decide and thus wish
to continue to the asynchronous fallback part and Byzantine parties that want to move to
the fallback part to increase the communication cost.

To this end, we implement the `help&tryHalting` procedure, which stops honest parties from
moving to the fallback part in synchronous runs. The communication cost of `help&tryHalting`
is $O(ft)$. The idea is to help parties learn the decision value and move to the fallback part
only when the number of help request indicates that the run is asynchronous.

**Algorithm 9 Help and try halting: Procedure for a party $p_i$.**

```
Local variables initialization:
$S_{help} = \emptyset$; $HALT \leftarrow true$

1: procedure help&tryHalting($sq$)
2:   if COMMIT = ⊥ then
3:     $\rho \leftarrow share\text{-}sign_i(⟨helpRequest, sq⟩)$
4:     send "helpRequest, sq, $\rho$" to all parties
5:   wait until $HALT = false$
6: upon receiving "helpReply, sq, COMMIT" do
7:     check&updateCommit(commit)
8: upon receiving "helpRequest, sq, $\rho$" from a party $p_j$ do
9:   if share-validate(⟨helpRequest, sq⟩, $p_j$, $\rho$) then
10:    $S_{help} \leftarrow S_{help} \cup \{\rho\}$
11:    send "helpReply, sq, COMMIT" to $p_j$
12:   if $|S_{help}| = t + 1$ then
13:     $\nu \leftarrow threshold\text{-}sign(S_{help})$
14:    send "complain, sq, $\nu$" to all parties
15: upon receiving "complain, sq, $\nu$" do
16:   if threshold-validate(⟨helpRequset, sq⟩, $\nu$) then
17:     send "complain, sq, $\nu$" to all parties
18:   HALT $\leftarrow false$
```

The pseudocode of `help&tryHalting` is given in Algorithm 9 and an illustration appears in
Figure 6. Each honest party that has not yet decided sends a share signed `helpRequest` to
all other parties. When an honest party gets an `helpRequest`, the party replies with its
`COMMIT` value, but if it gets $t + 1$ `helpRequest` messages, the party combines the shares
to a threshold signature and sends it in a `complain` message to all. When an honest party
gets a `complain` message for the first time, it echos the message to all parties and continues
to the fallback part.

**Termination.** Consider two cases. First, the parties move to the fallback part, in which
the case (standard) termination is guaranteed in eventually synchronous runs and probabilistic
termination is guaranteed in asynchronous runs. Otherwise, less than $t + 1$ parties send
`helpRequest` in `help&tryHalting`, which implies that at least $t + 1$ honest parties decided
and had a commit certificate before invoking `help&tryHalting`. Therefore, all honest parties
that did not decide before invoking `help&tryHalting` eventually get a `helpReply` message
with a commit certificate and decide as well.

Note that termination does not mean halting. In asynchronous runs, `helpRequest`
messages may be arbitrary delayed and thus parties cannot halt the protocol after deciding
in the synchronous part. However, it is well known and straightforward to prove that halting
cannot be achieved with $o(t^2)$ communication cost in asynchronous runs, and thus our protocol is optimal in this aspect.

**Round complexity.** Since in synchronous runs all parties decide at the end of an LBV instances with an honest leader, we get that the round complexity in synchronous runs is $O(f + 1)$. Since in asynchronous runs parties may go through $n$ LBV instances without deciding before starting the fallback, we get that the round complexity in asynchronous runs is $O(n + 1)$ in expectations.

**Communication complexity.** The synchronous (optimistic) part guarantees that if the run is indeed synchronous, then all honest parties decide before invoking help&tryHalting. The help&tryHalting procedure guarantees that parties continue to the fallback part only if $t + 1$ parties send an helpRequest message, which implies that they move only if at least one honest party has not decided in the synchronous part. Therefore, together they guarantee that honest parties never move to the fallback part in synchronous runs.

The communication complexity of the synchronous part is $O(ft)$, so to show that the total communication cost of the protocol in synchronous runs is $O(ft + t)$ we need to show that the cost of help&tryHalting is $O(ft + t)$ as well. Since in synchronous runs all honest parties decide in the synchronous part, they do not send helpRequest messages, and thus no party can send a valid complain message. Each Byzantine party that does send helpRequest messages can cause honest parties to send $O(t)$ replies, which implies a total communication cost of $O(ft)$ in synchronous runs.

As for all other runs, Theorem 1 states that deterministic protocols have an unbounded communication cost in the worst case. Thanks to the randomized fallback, our protocol has a communication cost of $O(t^2)$ in expectation.

## 5 Discussion and Future Directions

In this paper, we propose a new approach to design agreement algorithms for communication efficient SMR systems. Instead of designing deterministic protocols for the eventually synchronous model, which we prove cannot guarantee bounded communication cost before GST, we propose to design protocols that are optimized for the synchronous case but also have a randomized fallback to deal with asynchrony. Traditionally, most SMR solutions avoid randomized asynchronous protocols due to their high communication cost. We, in contrast, argue that this communication cost is reasonable given that the alternative is an unbounded communication cost during the wait for eventual synchrony.

We present the first authenticated optimistic protocol with $O(ft + t)$ communication complexity in synchronous runs and $O(t^2)$, in expectation, in non-synchronous runs. To strengthen our result, we prove that no deterministic protocol (even if equipped with perfect cryptographic schemes) can do better in synchronous runs. As for the asynchronous runs,
the lower bound in [1] proves that $O(t^2)$ is optimal in the worst case of $f = t$.

Future work. Note that our synchronous protocol satisfies early decision but not early stopping. That is, all honest parties decide after $O(f)$ rounds, but they terminate after $O(t)$. Therefore, a natural question to ask is whether exist an early stopping synchronous Byzantine agreement protocol with an optimal adaptive communication cost. In addition, it may be possible to improve our protocol’s complexity even further. In particular, the lower bound on communication cost in synchronous runs applies only to deterministic algorithms, so it might be possible to circumvent it via randomization [8].

Another interesting future direction is the question of optimal resilience in synchronous networks. Due to the lower bound in [5], the resilience of our protocol is optimal since the resilience in synchronous runs cannot be improved as long as the resilience in asynchronous runs is the optimal $t < n/3$. However, if we consider synchronous networks in which we do not need to worry about asynchronous runs, we know that we can tolerate up to $t < n/2$ failures. The open question is therefore the following: is there a synchronous Byzantine agreement protocol that tolerates up to $t < n/2$ failures with an optimal communication complexity of $O(ft + t)$?

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A Correctness Proof of our optimistic byzantine agreement protocol

In this section we prove the correctness of our optimistic byzantine agreement protocol from Section 4.

A.1 Safety.

We start by proving that our sequential composition of the linear LBV building block is safe. In a sequential composition parties sequentially invoke LBV instances s.t. for every LBV in the sequence they first invoke `startView` and use the local state variables, then, at some point they invoke `wedgeView` and update the local state with the returned values and move to the next LBV.

Lemma 8. If a party $p$ gets a commit certificate for value $v$ from an LBV instance with sequence number $sq$, then the local `LOCK` variable of at least $n - 2t$ honest parties is at least $sq$ when they start the LBV instance with sequence number $sq + 1$.

Proof. To generate a commit certificate, a party needs $n - t$ `COMMITSHARE` signatures. In addition, honest parties wedge an LBV instance before moving to the next one in sequential compositions. Thus, at least $n - 2t$ honest parties sent their `COMMITSHARE` in the LBV with $sq$ before starting the LBV with $sq + 1$. The lemma follows since parties set their `lockProof` before sending their `COMMITSHARE` and update their `LOCK` variable accordingly before invoking the next LBV.

The next corollary follows from Lemma 8 and the fact that the local `LOCK` variables are never decreased in our sequential compositions.

Corollary 9. If a party $p$ gets a commit certificate for value $v$ from an LBV instance with sequence number $sq$, then there at least $n - 2t$ honest parties which `LOCK` variable is at least $sq$ when they start any LBV with sequence number $sq' > sq$.

Lemma 10. It is impossible to generate two commit certificates for different values from the same LBV instance.

Proof. In every LBV instance, to generate a commit certificate for a value $v$ at least $n - t$ parties need to send a valid `COMMITSHARE` for value $v$. Therefore, to generate generate two commit certificates for different values from the same LBV instance we need at least 1 honest party need to send two contradicting `COMMITSHARE` message, which is impossible by the code.

The next lemma shows that after some party generates a commit certificate for a value $v$ it is impossible to generate a valid key with different value.

Lemma 11. Assume some honest party $p$ gets a commit certificate for value $v$ from an LBV instance with sequence number $sq$. Then no party can get a valid `keyProof` on a value other than $v$ from an LBV instance with sequence number $sq' \geq sq$ in a sequential composition.
Proof. We prove by induction on LBVs’ sequence numbers.

Base: Sequence number $sq$. To generate a commit certificate for value $v$ at least $n - 2t$ honest parties need to generate a COMMITSHARE signature. An honest party generates a COMMITSHARE signature only if it gets a valid LOCKSTEP message with value $v$, which in turn requires at least $n - 2t$ honest parties to generate LOCKSHARE signatures on $v$. An honest party generates a LOCKSHARE signature on $v$ only if it gets a valid KEYSTEP message with value $v$, which in turn requires at least $n - t$ parties to generate KEYSHARE signatures on $v$. Thus, if some party gets a commit certificate for value $v$ from an LBV instance with sequence number $sq$, then $n - t$ parties previously generated KEYSHARE signatures on $v$ in this LBV instance. Moreover, since honest parties never generate KEYSHARE signatures on different values, we get that it is impossible to generate two valid KEYSTEP messages with different values. The lemma follows.

step: Assume the lemma holds for all LBVs with sequence number $sq''$, $sq \leq sq'' \leq sq'$, we now show that it holds for $sq' + 1$ as well. Assume by a way of contradiction that some party gets a valid keyProof with value $v' \neq v$ from the LBV with sequence number $sq + 1$. Thus, at least $n - t$ parties generated KEYSHARE signatures on $v'$ in the LBV with $sq + 1$. By Lemma 8 there are at least $n - 2t$ honest parties whose local LOCK $\geq sq$ in the LBV with $sq + 1$. Thus, since $n \geq 3t + 1$, we get that at least 1 honest party $p$ whose local LOCK $\geq sq$ generated keySHARE signatures on $v'$ in the LBV with $sq + 1$. Therefore, $p$ gets a valid KEY for $v'$ with a sequence number $sq'' \geq sq$. A contradiction to the inductive assumption.

Lemma 12. Our optimistic byzantine agreement protocol, which is given in Algorithms 1, 2, 3, 4, 5, 6, 7, 8 and 9, satisfies the Agreement property.

Proof. Our protocol sequential composes LBV instances and decides only on values with a commit certificate. So we need to show that it is impossible to generate two commit certificates for different values in a sequential composition of LBV instances. Let party $p$ be the first to generate a commit certificate for some value $v$ and let $sq$ be the sequence number of the LBV instance in which it was generated. By lemma 10 it is impossible to generate a commit certificate for a value other then $v$ in the LBV with sequence number $sq$. By Lemma 11 no party can get a valid keyProof on a value other than $v$ from an LBV instance with sequence number $sq' \geq sq$. By Lemma 10 the local LOCK variable of at least $n - 2t$ honest parties is at least $sq$ in any LBV after the one with sequence number $sq$. Therefore, the lemma follows from the fact that at least $n - t$ parties need to contribute signatures in order to generate a commit certificate and since an honest party whose LOCK $\geq sq$ will not generate a KEYSHARE signature on a value $v'$ without getting a valid KEY for $v'$ from an LBV with sequence number $sq' \geq sq$.

A.2 Liveness.

We now prove that our protocol satisfies termination in all synchronous and eventually synchronous runs and provide probabilistic termination in all asynchronous runs.

Lemma 13. Consider an LBV instance lbv in a sequential composition. If some honest party $p$ is locked on a sequence number $sq$ (its LOCK = $sq$) before starting lbv, then at least $n - 2t$ honest parties set their local KEY variable with a valid key and sequence number $sq$ immediately after wedging the LBV instance with $sq$. 
Proof. Since $p$ is locked on $sq$, then it got a valid lockStep message in the LBV instance with sequence number $sq$. To generate a valid lockStep, a party needs $n - t$ lockShare signatures. Thus, since honest parties first wedge an LBV instance and then update their local state with the returned values, we get that at least $n - 2t$ honest parties generated a lockShare signature before updating their local $KEY$ variable. Thus, at least $n - 2t$ honest parties got a valid keyProof before wedging and thus update their local $KEY$ variable accordingly immediately after wedging.

$\triangleright$

Lemma 14. Consider a synchronous run of our protocol, and consider an LBV instance $lbv$ in the synchronous part, which parties invoke at time $t$. If the leader of $lbv$ is honest and it have not decided before time $t$, then all honest parties decide at time $t + 7\Delta$.

Proof. First, by Lemma 13 and since parties overwrite their local $KEY$ variables only with more up-to-date keys, we get that at least $n - 2t$ honest parties has a $KEY$ variable that unlocks all honest parties (it’s sequence number is equal to or higher than all honest parties’ $LOCK$) variable). By the code, the leader query all parties for their $KEY$ and waits for $n - 2t$ replays. Thus, it gets a reply from at least 1 honest party that have a key that unlocks all honest parties. Therefore, the leader learn this key and thus gets all honest parties to participates. The lemma follows from synchrony and the fact that all honest parties start $lbv$ at the same time and and do not wedge before all honest parties get all messages.

$\triangleright$

Lemma 15. All honest parties decide in all synchronous runs of the protocol.

Proof. Assume by a way of contradiction that some honest party $p$ does not decide. Let $lbv$ be an LBV instance in the synchronous part in which $p$ is the leader. By Lemma 14 all honest parties decide at the end of $lbv$. A contradiction.

$\triangleright$

Lemma 16. If $t + 1$ honest parties decide in the synchronous part of a run of our optimistic protocol, then all honest parties eventually decide.

Proof. By the code of the help&tryHalting procedure, any party $p$ that does not decide in the synchronous part of the protocol sends an help request to all parties and waits for $n - t$ to reply. Since $t + 1$ honest parties decided in the synchronous part before invoking help&tryHalting, then $p$ gets a valid commit certificate and decides as well.

$\triangleright$

Lemma 17. If less than $t + 1$ honest parties decide in the synchronous part of a run of our optimistic protocol, then all honest parties eventually move to the asynchronous fallback part.

Proof. Since less than $t + 1$ honest parties decided before invoking help&tryHalting, than at least $t + 1$ honest parties send an HELPREQUEST message to all other parties. Thus all honest parties eventually get $t + 1$ help replay, combine them to a COMPLAIN message, send it to all other parties, and move to the fallback part.

$\triangleright$

Lemma 18. Our optimistic byzantine agreement protocol, which is given in Algorithms 1, 2, 3, 4, 5, 6, 7, 8 and 9, satisfies termination in all synchronous runs and provide probabilistic termination in all asynchronous runs.

Proof. Let $r$ be a run of the protocol and consider consider 3 cases:
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Lemma 15. A (standard) termination is guaranteed by Lemma 15.

More than $t+1$ honest parties decide in the synchronous part of $r$. The lemma follows from Lemma 16.

Less than $t+1$ honest parties decide in the synchronous part of $r$. By Lemma 17, all honest parties move to the asynchronous fallback. The Lemma follows from the termination proof in VABA [1] and [35].

A.3 Communication complexity.

In this section we prove that our protocol has an optimal adaptive synchronous communication complexity and an optimal worst case asynchronous communication.

Lemma 19. The communication cost of the synchronous part in synchronous runs of our optimistic protocol is $O(ft + t)$.

Proof. Consider a synchronous run. The communication cost of an LBV instance (with byzantine or honest leader) plus the leader-to-all all-to-leader key learning phase is at most $O(t)$ and the communication cost of an LBV instance with an honest leader that does not drive progress since it has already decided before is $0$ (honest parties only reply to leaders messages). By Lemma 14 all honest parties decide in the first LBV instance with an honest leader that drive progress. Therefore, there is at most 1 honest leader that drive progress in the LBV in which it acts as the leader. Thus, the total communication cost of all LBVs with honest leaders is $O(ft)$. Hence, since every byzantine leader can make honest parties pay at most $O(t)$ communication cost in the LBV instance in which the byzantine party is the leader, we get to a total communication cost of $O(ft + t)$.

Lemma 20. The communication cost of the help&tryHalting procedure in synchronous runs of our optimistic protocol is $O(ft)$.

Proof. Consider a synchronous run $r$. By Lemma 15, all honest parties decide in the synchronous part of $r$. Thus, no honest party sends an helpRequest message in the help&tryHalting procedure and it is impossible to generate a valid complain message. An helpRequest by a byzantine party causes all honest party to reply, which cost $O(t)$ in communication cost. Therefore, the total communication cost of the help&tryHalting procedure in synchronous runs is $O(ft)$.

Lemma 21. The synchronous communication cost of our optimistic byzantine agreement protocol is $O(ft + t)$.

Proof. By Lemma 15, all honest parties decide in the synchronous part of $r$ and thus no honest party sends an helpRequest message in the help&tryHalting procedure. Therefore, it is impossible to generate a valid complain message, and thus no honest party moves to the fallback part. The lemmas follows from Lemmas 19 and 20.

Lemma 22. The asynchronous communication cost of our optimistic byzantine agreement protocol is $O(t^2)$ in the worst case.
Proof. Consider an asynchronous run \( r \). We first prove that the worst case communication cost of the synchronous part and the help\&tryHalting procedure is \( O(t^2) \):

**Synchronous part.** The synchronous part consists of \( n \) LBV instances with a one-to-all and all-to-one communication phase in between. Since the communication cost of the LBV building block is at most \( O(t) \), we get that the total communication cost of the synchronous part is \( O(nt) = O(t^2) \).

**The help\&tryHalting procedure.** Every honest party sends at most one helpRequest message, helpReplay message, and complain message. Therefore, since each of the messages contains a constant number of words, we get that the worst case communication complexity of the help\&tryHalting procedure is \( O(t^2) \).

The Lemma follows from the communication cost analysis of the fallback algorithm, which appears in VABA \[1\]. ▶

The next corollary follows directly from Lemmas 21 and 22:

**Corollary 23.** The adaptive synchronous and worst case asynchronous communication cost of our optimistic byzantine agreement protocol, which is given in Algorithms 1, 2, 3, 4, 5, 6, 7, 8 and 9, is \( O(ft + t) \) and \( O(t^2) \), respectively.

\[ \text{CVIT 2016} \]

B Threshold signatures

At the beginning of every execution, every party \( p_i \) gets a private function \( \text{share-sign}_i(m) \) from the dealer, which gets a message \( m \) and returns a signature-share \( \sigma_i \). In addition, every party gets the following functions: (1) \( \text{share-validate}(m, i, \sigma_i) \), which gets a message \( m \), a party identification \( i \), and a signature-share \( \sigma_i \), and returns true or false; (2) \( \text{threshold-sign}(\Sigma) \), which gets a set of signature-shares \( \Sigma \), and returns a threshold signature \( \sigma \); and (3) \( \text{threshold-validate}(m, \sigma) \), which gets a message \( m \) and a threshold signature \( \sigma \), and returns true or false. We assume that the above functions satisfy the following properties:

**Share validation:** For all \( i, 1 \leq i \leq n \) and for every messages \( m \), (1) \( \text{share-validate}(m, i, \sigma) = \text{true} \) if and only if \( \sigma = \text{share-sign}_i(m) \), and (2) if \( p_i \) is honest, then it is infeasible for the adversary to compute \( \text{share-sign}_i(m) \).

**Threshold validation:** For every message \( m \), \( \text{threshold-validate}(m, \sigma) = \text{true} \) if and only if \( \sigma = \text{threshold-sign}(\Sigma) \) s.t. \( |\Sigma| \geq n - t \) and for every \( \sigma_i \in \Sigma \) there is a party \( p_i \) s.t. \( \text{share-validate}(m, i, \sigma) = \text{true} \).