Students’ conceptual understanding on inverse function concept

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Abstract. The success of students on solving question about inverse function concept is supported by conceptual understanding which they have. This article has a purpose to describe students’ conceptual understanding on inverse function concept. This research is a descriptive research with a qualitative approach. The participants of the research are 20 students, who are in the fourth semester of their study. They are the students of a university which located in Malang. The result of the research shows that when the participants answer questions about inverse function, they apply their conceptual understanding. The characteristics of students who have conceptual understanding on inverse function concept are they are able to explain and draw inverse function concept, explain steps on determining inverse of a function, and give explanation why a function which has an inverse has to be bijective. There are some students or participants who as if they solve the question using conceptual understanding. However, it turns out that they have misconception on inverse function concept after a series of investigation by the researchers.

1. Introduction

Inverse function is one of the concepts which obligatory to be learned in Calculus. Inverse function is a function that reverses original function. For example, a function \( f \) maps domain \( A \) to range \( B \), inverse function (symbolized with \( f^{-1} \)) is a function which maps \( B \) to \( A \) [1]. To master the concept of inverse function, students have to, firstly, understand the concept of a function. A bijective function is the basic requirement which has to be fulfilled in determining an inverse of a function. However, the students sometimes do not pay a careful attention to that requirement when solving a question about inverse function. It takes conceptual understanding when solving a question about inverse function.

In a broad meaning, conceptual understanding as “concept understanding, operation, and mathematical relation” [2]. Conceptual understanding involves relation between related concepts [3], understanding about why the procedure works [4] and if the procedure is legitimate [5]. According to those experts, therefore, conceptual understanding emphasizes on the connection between different concepts by treating a concept as an entity which related on certain procedure.
Conceptual understanding (knowing why) supports the understanding on mathematics principals which considered as a product of a process which connects established knowledge with new knowledge [6]. A student is considered solving question using conceptual understanding if the student is able to explain the concept, explain how to do the procedure, and finally explain why the procedure is logic [7-9]. In this research, the students are considered showing conceptual understanding when they understand inverse function concept, are able to determine inverse of a function, and know why they have to pay attention to bijective function before determining inverse of a function.

In teaching the concept of inverse function, the lecturer tends to focus on algorithmic skill and procedural rules [10]. Algorithmic skill and procedural direction are considered very important. However, those skills do not help students understand concept in various situations meaningfully. Some students find the difficulty in reaching a meaningful understanding on duties related to the concept of inverse function. This problem makes the researchers interested in describing students’ conceptual understanding when solving the question about inverse function concept.

2. Method
This research is a descriptive – qualitative research. The research participants are 20 students who are in the fourth semester of their study. Their major is Mathematics in one of the universities which located in Malang. The students are given a question about inverse function, and then they are interviewed to dig their conceptual understanding when solving the question. The main instrument of the research is the researchers themselves and helped by an instrument in the form of question. The question given to the participants is true / false statement, and then the participants are required to give the argument of their answer. The form of the question is shown in Figure 1.

![Figure 1](image1.png)

Put a tick (✓) in True box, if you consider that the statement is correct; or in False box if you consider that the statement is false, then give your argument.

If it is known function \( f(x) = 5x + 2 \); \( f: \mathbb{Z} \rightarrow \mathbb{Z} \), then function \( f \) does not have inverse.

| □ True | □ False |
|--------|--------|
| Argument:……………………|

3. Results and discussion
The focus of this research is students’ conceptual understanding on solving question related to inverse function concept. Out of 20 students, there are 8 students who answer incorrectly and there are 12 students who are able to answer the question correctly. Figure 2 is one of the answers from students who solve the question using his/her conceptual understanding.
Figure 2. The answer form student one (S1).

Figure 2 shows that Student One (S1) is able to solve the question correctly. From Figure 2, S1 agrees to the statement “if it is known $f(x) = 5x + 2; f: Z \rightarrow Z$, then function $f$ does not have inverse”. S1 gives argument that function $f(x)$ does not have inverse because $f(x)$ is not a bijective function. It means that S1 knows there is a connection between inverse function and bijective function (injective function and surjective function). To dig S1’s conceptual understanding, then, the researcher (R) conducts an interview to S1. Here is the script of the interview.

R : What do you know about inverse function concept?
S1 : If function $f$, is a function which maps domain $A$ with range $B$, then inverse function from $f$ (symbolized by $f^{-1}$), has domain $B$ and range $A$.

R : Can you draw what you’ve explained?
S1 : Here is what I’m talking about… (Drawing an arrow diagram and explain it)

R : From your answer, do you mean that the first step to prove your answer is by checking if function $f(x)$ is bijective? (pointing at S1’s answer sheet)
S1 : Yes.
R : Why the function which has inverse must be bijective?
S1 : Hmm… It is because Range from function $f$ is Domain from $f^{-1}$ and Domain from $f$ is Range from $f^{-1}$, so if function $f$ is not bijective, then $f^{-1}$ is not a function.
Based on the interview and the result of S1’s work, it can be concluded that S1 uses his/her conceptual understanding when solving question about inverse function. To prove the truth of the statement “if it is known function \( f(x) = 5x + 2; f : \mathbb{Z} \rightarrow \mathbb{Z} \), then function \( f \) does not have inverse”, S1 firstly checks whether \( f(x) \) is bijective function or not. S1 is able to show that \( f(x) \) is an injective function but it is not a surjective function because there is member in the codomain which does not have pair in the domain. S1 is able to explain inverse function concept and able to draw it in the form of arrow diagram. It shows that someone who has understanding concept will be able to explain a concept and able to draw the concept in the various representation [10]. S1 is able to explain steps to determine inverse from a function and give argument on why before determining inverse it is important to pay attention to the requirement of a function, which is a function has to be bijective. In this matter, conceptual understanding is knowing what to do and why we do it [11].

The Figure below shows the answer from Student Two (S2).

![Figure 3](image)

**Figure 3.** The answer from student two (S2).

Based on S2’s answer, which is shown in Figure 3, S2 also agrees to the statement “if it is known \( f(x) = 5x + 2; f : \mathbb{Z} \rightarrow \mathbb{Z} \), then function \( f \) does not have inverse”. The result of the interview with S2 shows that S2 is able to explain inverse function concept and also draw it in an arrow diagram, just like what S1 did. S2 knows the connection between inverse function and bijective function. It can be seen from S2’s ability in showing that the function is injective but not surjective, then \( f(x) \) is not bijective. S2 knows that Range from function \( f \) is Domain from \( f^{-1} \), and Domain from \( f \) is Range from \( f^{-1} \), so if function \( f \) is not bijective, then \( f^{-1} \) is not a function. However, when determining whether the function is surjective or not there is a misconception made by S2. The misconception occurs because S2 too focuses on the definition of inverse function “If function \( f \), is a function which maps domain \( A \) to range \( B \), then inverse function from \( f \) (symbolized with \( f^{-1} \)), has domain \( B \) and range \( A \)”. S2 should have proved whether function \( f(x) \) is surjective or not.
Based on S3’s answer and Interview, which is shown in Figure 4, S3 makes mistake in the inverse writing from function \( y = f(x) \) written as \( y = f^{-1}(x) \). The miswriting is one kind of misconceptions which is caused by general conceptual mistake which occurs in determining inverse of a function [12]. S3 states that the statement “if it is known \( f(x) = 5x + 2; f: \mathbb{Z} \to \mathbb{Z} \), then function \( f \) does not have inverse” is false. S3 proves that the statement is false by directly applying procedural rule. The mistake done by S3 occurs because his/her dependence on procedure memorization without understanding related concepts which underlie it. The difficulty and misconception which is frequently done when solving a question about inverse function is because of the teaching which emphasizes on memorization and regular rule [12-14].

4. Conclusions

Based on the purpose of the research and analysis which has been done by the researchers about students’ conceptual understanding when solving question about inverse function concept, it can be concluded that students are able to answer the question about inverse function concept correctly because they apply conceptual understanding. The conceptual understanding which possessed by the students about inverse function can be marked by students’ ability in explaining and drawing inverse function concept, knowing the steps to determine inverse of a function, especially pay a careful attention to the connection between bijective function and inverse function, and giving argument about the reason why a function has an inverse if the function is bijective. Nevertheless, there are students’ misconceptions when determining inverse of a function. The misconception occurs as the result of teaching which focuses on memorization, skill in using algorithm, and procedural rule. Therefore, it is important for the lecturers to choose learning method which develop students’ conceptual understanding.

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