Twist-3 contribution to the $\gamma^*\gamma \to \pi\pi$ amplitude in the Wandzura-Wilczek approximation

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Abstract

We have calculated the Wandzura-Wilczek contribution to the twist-3 part of $\gamma^*\gamma \to 2\pi$ amplitude. It describes interaction of the longitudinally polarized virtual photon with the real one, and it is suppressed by $1/Q^2$, where $Q^2$ is the virtuality of the $\gamma^*$, as compared to the twist-2 contribution. We have found that, in the Wandzura-Wilczek approximation, factorization applies to the twist-3 amplitude.

1 Introduction

Hadron production in the reaction $\gamma^*(q)\gamma(q') \to \mathrm{hadron}(s)$, $q^2 = -Q^2$, has been a subject of considerable interest in the QCD community from both experimental [1, 2] and theoretical [3, 4, 5] points of view. Recently it has been proposed [6, 7] to investigate a process $\gamma^*\gamma \to \pi\pi$ when the two pion state has a small invariant mass. Thanks to the QCD factorization theorem [8], in the leading twist approximation the amplitude can be represented as a convolution of perturbatively calculable Wilson coefficients and new non-perturbative objects, the so-called two-pion distribution amplitudes (2$\pi$DA’s). They are given by matrix elements of twist-2 QCD string operators between vacuum and the two-pion state [9, 8]. Moreover, 2$\pi$DA’s can be related by the crossing symmetry to skewed parton distributions [10, 11] which recently have been subject of considerable interest. Later, NLO corrections to the leading twist amplitude have been calculated [12] and the recent thorough phenomenological analysis [13] have shown that experimental studies of $2\pi$ production cross-section are possible with existing $e^+e^-$ facilities.

In general, parity invariance restricts the number of independent helicity amplitudes in the $\gamma^*\gamma \to \pi\pi$ process to three, which correspond to different possible projections of the total angular momentum on the $\gamma^*\gamma$ collision axis, $J_z = 0, \pm 1, \pm 2$. $J_z = 0$ and $J_z = \pm 2$ amplitudes scale as $Q^2 \to \infty$, and their corresponding QCD representation has been

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discussed in \[7, 9, 12\]. The amplitude corresponding to \(J_z = \pm 1\) is suppressed as \(1/Q\) and has not been discussed so far. On the other hand, as it has been demonstrated in \[13\], it can be extracted from the experimental data by a suitable weighting procedure.

The purpose of this paper is to fill this gap in the QCD description of the \(\gamma^*\gamma \to \pi\pi\) amplitude. As it can be anticipated from its \(1/Q\) suppression, the leading contribution to the \(J_z = \pm 1\) amplitude is given by matrix elements of twist-3 quark-quark and quark-gluon operators. In this paper we restrict ourselves to the Wandzura-Wilczek (WW) approximation \[14\] i.e., we consider only the contribution from quark operators. Recall that the recent experimental analysis \[15\] of the polarized nucleon structure function \(g_T(x, Q^2)\) indicates that the WW approximation can account rather well for the experimental data.

A similar conclusion is reached in the instanton model of the QCD vacuum \[16\], where quark-gluon correlations in a nucleon are suppressed by a small parameter given by the packing fraction of instantons. Our results can be therefore considered as an estimate of the order of magnitude of the \(J_z = \pm 1\) amplitude.

The problem of twist-3 contributions to the hard exclusive amplitudes has recently acquired a considerable attention \[17, 18, 19, 20, 21, 22\]. In particular, the approach presented here is rather similar in spirit to the analysis of \[21\], which presented a detailed consideration of the problem of the DVCS amplitude on a pion.

2 General definitions

Kinematics of the reaction \(\gamma^*(q)\gamma(q') \to \pi(k_1)\pi(k_2)\) can conveniently be described in terms of a pair of light-like vectors \(p, z\) which obey

\[ p^2 = z^2 = 0, \quad p \cdot z \neq 0 \] (1)

and define longitudinal directions. Here \(p \cdot z = p^\mu z^\mu\). Let \(P\) and \(k\) denote total and relative momenta of the \(\pi\) meson pair, respectively,

\[ P^2 = (k_1 + k_2)^2 = W^2, \quad k^2 = (k_1 - k_2)^2 = 4 m_\pi^2 - W^2, \quad P \cdot k = 0 \] (2)

The initial and final states momenta can be decomposed as

\[ q = p - \frac{Q^2}{2(p \cdot z)} z, \quad q^2 = -Q^2, \quad q' = \frac{Q^2 + W^2}{2(p \cdot z)} z, \quad q'^2 = 0 \]

\[ P = q + q' = p + \frac{W^2}{2(p \cdot z)} z, \quad P^2 = W^2 \] (3)

\[ k = \xi p - \frac{\xi W^2}{2(p \cdot z)} z + k_\perp \]

The longitudinal momentum distribution between pions is described by the variable \(\xi = (k \cdot z)/(p \cdot z)\). Alternatively,

\[ \xi = \beta \cos \theta_{\text{cm}}, \]

where \(\theta_{\text{cm}}\) is the polar angle of the pion momentum in the CM frame with respect to the direction of the total momentum \(P\) and \(\beta\) is the velocity of produced pions in the center-of-mass frame

\[ \beta = \sqrt{1 - \frac{4m_\pi^2}{W^2}}. \]
The amplitude of hard photo-production of two pions is defined by the following matrix element between vacuum and two pions state:

$$T^{\mu\nu} = i \int d^4x e^{-ix \cdot \vec{q}} \langle 2\pi(P, k) | T J^\mu(x/2) J^{\nu}(-x/2) | 0 \rangle, \quad \vec{q} = \frac{1}{2}(q - q')$$  \hspace{1cm} (4)

where $J^\mu(x)$ denotes quark electromagnetic current. Hard photo-production corresponds to the limit $Q^2 \gg W^2 \geq \Lambda^2_{QCD}$ where the amplitude can be represented as an expansion in terms of powers of $1/Q$. According to the factorization theorem the leading twist term in the expansion can be written as a convolution of hard and soft blocks. The coefficient is terms of powers of $1/Q$.

According to the analysis of Ref. [12] the amplitude $T^{\mu\nu}$ is a sum of three terms

$$T^{\mu\nu}(q, q', P, k) = \frac{i}{2} (-g^{\mu\nu})_T T^\gamma_{0\pi}(q, q', P, k) + i \frac{1}{Q^2} k_\perp^{\mu}(P + q')^{\mu} T^\gamma_{1\pi}(q, q', P, k) + \frac{i}{2} \frac{k_\perp^{(\mu} k_\perp^{\nu)}}{W^2} T^\gamma_{2\pi}(q, q', P, k)$$  \hspace{1cm} (5)

where $(-g^{\mu\nu})_T = (\frac{p^\mu p^\nu - p^\nu p^\mu}{p \cdot z} - g^{\mu\nu})$ is the metric tensor in the transverse space and $k_\perp^{(\mu} k_\perp^{\nu)}$ denotes traceless, symmetric tensor product of relative transverse momenta.

Amplitudes $T^\gamma_{0\pi}$ and $T^\gamma_{2\pi}$ correspond to $J_z = 0$ and $J_z = \pm 2$, respectively. They have been discussed at length in Refs. [7, 8, 9, 12]. In this paper we consider the $J_z = \pm 1$ amplitude $T^\gamma_{1\pi}$. It arises when the (real) transverse photon collides with the (virtual) longitudinal one and produces a pair of pions in $L_z = \pm 1$ state. It is easy to see that such a process requires helicity flip along the quark line, and hence it vanishes in the leading twist approximation.

Consider now a frame in which the pion pair is moving in the positive \( \hat{z} \) direction so that $p^z$ and $z$ are the only nonzero components of $p$ and $z$, respectively. In an infinite momentum frame $p^z \sim Q \to \infty$ with fixed $(p \cdot z) \sim 1$. From (3) it follows that in this frame $(k \cdot z) \sim 1$ and $k_\perp \sim Q^0$. Hence, the contribution due to $T^\gamma_{1\pi}$ is suppressed by $1/Q$ as compared to the leading one.

In this paper we will consider quark $2\pi$DA’s, defined as matrix elements of the light-cone quark string operator:

$$\langle \pi\pi(P, k) \mid \frac{1}{N_f} \sum_q \bar{q}(z) [-z] \bar{q}(-z) \mid 0 \rangle = (p \cdot z) \int_0^1 du \Phi^Q(u, \xi, W^2) e^{i(2u-1)(p \cdot z)},$$

$$= (p \cdot z) \int_{-1}^1 dv H^Q(v, \xi, W^2) e^{iv(p \cdot z)},$$  \hspace{1cm} (6)

with $\hat{x} = \gamma^\mu x_\mu$ and

$$H^Q(v, \xi, W^2) = \frac{1}{2} \Phi^Q(\frac{1 + v}{2}, \xi, W^2).$$  \hspace{1cm} (7)

$x, y$ denotes a path-ordered exponential $[x, y] = P \exp[i \int_0^1 dt (x - y)_\mu A^\mu(tx + (1 - t)y)]$ which ensures gauge-invariance of the matrix element. In (6) we have introduced both types of single-variable distributions which can be found in the literature - the distribution amplitude $\Phi^Q(u, \xi, W^2)$ and the symmetric form $H^Q(u, \xi, W^2)$. As we shall see later,
calculations in the coordinate-space naturally require to introduce a parametrisation in terms of a double-distribution function introduced in [23]:

\[
\langle 2\pi(P,k)|\bar{\psi}(x) [x,-x] \bar{\psi}(-x)|0\rangle = (P \cdot x) \int d[\alpha, \beta] F(\alpha, \beta, W^2)e^{i\alpha(P \cdot x) + i\beta(k \cdot x)} + (k \cdot x) \int_{-1}^{1} d\beta D(\beta, W^2)e^{i\beta(k \cdot x)} + O(x^2),
\]

where

\[
\int d[\alpha, \beta] \equiv \int_{-1}^{1} d\alpha \int_{-1-|\alpha|}^{1-|\alpha|} d\beta.
\]

The second term in (8) corresponds to the so-called “D-term” introduced in [24]. Comparing matrix element (8) evaluated on the light-cone, \(x^2 = 0\), with parametrisation (6) one easily obtains a relation between double and 2\(\pi\) distribution (7):

\[
H^{Q}(u, \xi, W^2) = \int d[\alpha, \beta] F(\alpha, \beta, W^2)\delta(\alpha + \xi \beta - u) + \theta(|u| < |\xi|) \text{sign}(\xi) D(u/\xi).
\]

Note that the matrix element of the pseudovector quark string operator

\[
\langle \pi\pi(P,k)|\bar{\psi}(z) [z,-z] \bar{\psi}(-z)|0\rangle = 0
\]

vanishes because of the positive parity of the final pion pair.

Finally, we recall that all distributions introduced here depend also on a normalization scale \(\mu\). This dependence is not relevant for our discussion and has been neglected. For a sake of simplicity from now on we will also neglect the gauge factors \([x,y]\) and, wherever possible, the explicit \(W\) - dependence.

3 Angular momentum analysis of \(\gamma^*\gamma \to \pi\pi\) amplitude

Our goal here is to identify the leading contribution to the amplitude \(T_1^{\gamma\gamma\pi\pi}\). For simplicity we will consider the massless quark degrees of freedom only. Taking into account quark masses and gluonic degrees of freedom does not change our conclusions qualitatively. A convenient coordinate frame is obtained by identifying the null vectors \(p\) and \(z\) introduced in (3) with a pair \(n, n^*\) which obeys

\[
\begin{align*}
n^2 = n^*2 &= 0, \quad n \cdot n^* = 2, \\
n &= (1, 0, 0, -1), \quad n^* = (1, 0, 0, 1).
\end{align*}
\]

Taking

\[
z = n, \quad p = \frac{Q}{2} n^*.
\]

one obtains a collinear collision of \(\gamma^*\) and \(\gamma\), with the final pion pair moving in the virtual photon direction. The virtual photon can carry both transverse and longitudinal polarizations. For asymptotically large \(Q^2\) the interaction between photons occurs at a light-like separation.

Let us consider first the tree-level contribution. Corresponding space-time configuration is depicted on Fig.2. The real photon and the final quark-antiquark pair move along light
rays defined by $x_\pm = x_0 - x_3 = 0$ and $x_+ = x_0 + x_3 = 0$, respectively. If both photons have the same helicities, the projection of the angular momentum onto the collision axis $J_z = 0$. Vector couplings create the $q\bar{q}$ pair with opposite polarizations, and the angular momentum conservation requires that the pair has the orbital angular momentum $L_z = 0$. Transition of such a pair into a 2 pions with $L_z = 0$ is described by twist-2 matrix element (13).

Longitudinal polarization of the virtual photon leads to a configuration with $J_z = \pm 1$, see Fig.3. Due to the angular momentum conservation, the quark-antiquark pair is created with the intrinsic orbital angular momentum $L_z = J_z$. Subsequently, it evolves into a pion pair with $L_z = \pm 1$. The amplitude must be therefore proportional to $k_\perp$. On the other hand, non-zero quark orbital angular momentum requires that the transition occurs through matrix element of a quark twist-3 operator, see the next section, which results in a suppression $k_\perp/Q$ as compared with the amplitude corresponding to $J_z = 0$.

So far we have considered only a point-like interaction of the real photon with the virtual one. Another contribution arises when the real photon splits into its quark-antiquark component long before its interaction with $\gamma^*$. Configuration arising if the transition is described by twist-2, $L_z = 0$ photon distribution amplitude is shown in Fig.4. From dimensional considerations it follows that twist-2 matrix element between a real photon and the vacuum is parametrized by a constant carrying dimension one. It is usually defined as $f_\gamma = \chi \langle \bar{\psi} \psi \rangle$, where $\chi$ is the vacuum magnetic susceptibility, and $\langle \bar{\psi} \psi \rangle$ is the quark condensate [25]. Such a contribution would therefore provide a term in $T_{1\gamma}^{\pi\pi}$ of the order of $f_\gamma/Q$. Note, however, that the angular momentum conservation requires that the $q\bar{q}$ pair has to acquire in the hard collision two units of orbital angular momentum. It means that twist-2 contribution to this amplitude vanishes to all orders in perturbation theory. The situation here is similar to the computation of the amplitude of hard photoproduction of transverse vector mesons by longitudinal photon, discussed in [27, 28]. Possible higher-twist corrections to this mechanism will be suppressed by at least one power of $Q$.

We conclude that the process described in the previous paragraph provides the leading contribution to $T_{1\gamma}^{\pi\pi}$.

4 Twist-3 amplitude $T_{1\gamma}^{\pi\pi}$ in the Wandzura-Wilczek approximation

In this section we employ technique developed in [26] to obtain twist-3 contribution to the amplitude $T_{1\gamma}^{\pi\pi}$ in the Wandzura-Wilczek approximation i.e., neglecting all quark-gluon operators. Our stating object is the amplitude (4) written in the form:

$$T^{\mu\nu} = 16i \int d^4x e^{-ix(q-q')} \langle 2\pi(P,k) | T J^\mu(x) J^\nu(-x) | 0 \rangle .$$  (14)

The main contribution to the above integral arises from distances in the vicinity of the light cone, $x^2 \sim 1/Q^2$.

Let us first consider a situation where both photons are virtual and their momenta are spacelike:

$$-q^2 = Q^2 \gg \Lambda_{QCD}, -q'^2 = Q'^2 \gg \Lambda_{QCD}$$  (15)

To extract the part of the amplitude which corresponds to the case when the one of the two photons is longitudinal we have to separate contribution which is proportional to transverse momentum $k_\perp$. An analogous problem in DVCS on a pion has been considered recently...
Let us consider the $T$-product of two electromagnetic currents. In the vicinity of the light cone $x^2 \approx 0$ the dominant terms are given by diagrams depicted on Fig. 1). Their contribution can be written as:

\[
T\{J^\mu(x)J^\nu(-x)\} = \frac{1}{16\pi^2 x^4} \left\{ t^\mu_\lambda x_\sigma [\bar{\psi}(x)\gamma_\sigma \psi(-x) - \bar{\psi}(-x)\gamma_\sigma \psi(x)] + 
+ i\epsilon_{\mu\nu\lambda\sigma} x_\lambda [\bar{\psi}(x)\gamma_\sigma \gamma_5 \psi(-x) + \bar{\psi}(-x)\gamma_\sigma \gamma_5 \psi(x)] \right\}.
\]

Here we have introduced a shorthand notation:

\[
t^\mu_\lambda = g_{\mu\lambda} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\lambda} - g_{\mu\nu} g_{\lambda\sigma}.
\]

Our goal now is to determine matrix elements of vector and pseudovector non-local operators $\bar{\psi}(x)\gamma_\sigma \psi(-x)$ and $\bar{\psi}(x)\gamma_\sigma \gamma_5 \psi(-x)$. To this end we employ the operator identity derived in [19, 20, 21]:

\[
\bar{\psi}(x)\gamma_\mu \psi(-x) = \frac{D_\mu}{(Dx)} \bar{\psi}(x)\gamma_\mu \psi(-x) + 
\frac{1}{2} \int_0^1 d\alpha \left\{ e^{\tilde{\alpha}(xD)} + e^{-\tilde{\alpha}(xD)} \right\} \left[ \partial_\mu - \frac{D_\mu}{(xD)} (xD) \right] \bar{\psi}(\alpha x)\gamma_\mu \psi(-\alpha x) + 
\frac{1}{2} \int_0^1 d\alpha \left\{ e^{-\tilde{\alpha}(xD)} - e^{\tilde{\alpha}(xD)} \right\} i\epsilon_{\mu ijk} x_i \frac{D_j}{(Dx)} \bar{\psi}(\alpha x)\gamma_5 \psi(-\alpha x) + \ldots
\]

where $\tilde{\alpha} = 1 - \alpha$, $\partial_\mu = \partial/\partial x_\mu$, and ellipses denote contributions of three-point quark-gluon operators which are neglected in the WW approximation. Here, $D_\mu$ is a derivative with respect to translation introduced in [20]:

\[
D_\alpha \{ \bar{\psi}(tx)\Gamma[tx, -tx]\psi(-tx) \} \equiv \frac{\partial}{\partial y^\alpha} \left\{ \bar{\psi}(tx + y)\Gamma[tx + y, -tx + y]\psi(-tx + y) \right\} \bigg|_{y \to 0},
\]

with a generic Dirac matrix structure $\Gamma$. Hence, when acting on a matrix element between vacuum and the final state with a four-momentum $P_\mu$, $D_\mu \rightarrow iP_\mu$. Identity (18) allows to compute matrix element of $\bar{\psi}(x)\gamma_\sigma \psi(-x)$ in terms of the matrix element of the symmetric
operator $\tilde{\psi}(x)\hat{\psi}(-x)$, given by (9). Taking matrix elements of both sides of (18) between 2-pion state and the vacuum one obtains after simple manipulations:

$$\langle 2\pi(P, k)|\tilde{\psi}(x)\gamma_\sigma\psi(-x)|0 \rangle = P_\sigma F_1(P_x, kx) + k_\sigma D_1(P_x, kx) + i[k_\sigma(P_x) - P_\sigma(kx)][F_2(P_x, kx) + D_2(P_x, kx)]. \quad (20)$$

Here we have introduced a compact notation

$$F_1(P_x, kx) = \int d[\alpha, \beta]F(\alpha, \beta)e^{i\alpha(P_x) + i\beta(kx)},$$
$$F_2(P_x, kx) = \frac{1}{2}\int d[\alpha, \beta]F(\alpha, \beta)\int_0^1 dt [e^{i(P_x)} + e^{-i(P_x)}]e^{i[\alpha(P_x) + i\beta(kx)]},$$
$$D_1(kx) = \int_{-1}^1 d\beta D(\beta)e^{i\beta(kx)},$$
$$D_2(P_x, kx) = \frac{1}{2}\int_{-1}^1 d\beta D(\beta)\int_0^1 dt [e^{i(P_x)} - e^{-i(P_x)}]e^{i\beta(kx)}. \quad (21)$$

Consider now the pseudovector operator $\tilde{\psi}(x)\gamma_\mu\gamma_5\psi(-x)$. Using the identity [20]:

$$\tilde{\psi}(x)\gamma_\mu\gamma_5\psi(-x) = \int_0^1 dt \frac{\partial}{\partial x_\mu} \tilde{\psi}(tx)\gamma_5\psi(-tx) - i\epsilon_{\mu\nu\alpha\beta} \int_0^1 dt dt't \partial x'^\alpha \left[ \tilde{\psi}(tx)\gamma^\beta\psi(-tx) \right]$$
$$- \int_0^1 dt \int_{-t}^t dv \tilde{\psi}(tx) \left[ t g\tilde{G}_{\mu\nu}(vx) + v \gamma_5 i gG_{\mu\nu}(vx) \right] x'^\nu \hat{\psi}(-tx), \quad (22)$$

one can express its matrix elements in the WW approximation through matrix elements of the vector operator $\psi\gamma_\sigma\psi$. In this way one obtains

$$\langle 2\pi(P, k)|\tilde{\psi}(x)\gamma_\sigma\gamma_5\psi(-x)|0 \rangle = \epsilon_{\sigma\lambda\mu\delta} x^\lambda P^\mu k^\delta [F_3(P_x, kx) + D_3(P_x, kx)] \quad (23)$$

where

$$F_3(P_x, kx) = \frac{1}{2}\int d[\alpha, \beta]F(\alpha, \beta)\int_0^1 dt [e^{i(P_x)} - e^{-i(P_x)}]e^{i[\alpha(P_x) + i\beta(kx)]},$$
$$D_3(P_x, kx) = \frac{1}{2}\int_{-1}^1 d\beta D(\beta)\int_0^1 dt [e^{i(P_x)} + e^{-i(P_x)}]e^{i\beta(kx)}. \quad (24)$$

Now, combining (24) and (23) with (14) and (14) one finally finds

$$T^{\mu\nu} = \frac{i}{\pi^2} \int d^4xe^{ixq}x^4 \left\{ \frac{1}{x^4} \left[ T^{\mu\nu}_{\lambda\sigma} P_\sigma \{ F_1(P_x, kx) - \bar{F}_1(P_x, kx) \} + ight. \right.$$
$$\left. + ix_\mu x_\sigma [k_\perp^{\nu}, P^\mu] \{ F_2(P_x, kx) - \bar{F}_2(P_x, kx) \} + \bar{F}_2(P_x, kx) + \bar{F}_3(P_x, kx) \} + ight.$$
$$\left. + ix_\mu x_\nu [k_\perp^{\mu}, P^{\nu}] \{ F_2(P_x, kx) + \bar{F}_3(P_x, kx) \} + \bar{F}_2(P_x, kx) - \bar{F}_3(P_x, kx) \} + ight.$$
$$\left. + ix^2 [k_\perp^{\mu}, P^{\nu}] \{ F_3(P_x, kx) - \bar{F}_3(P_x, kx) \} + \bar{F}_3(P_x, kx) \} + \cdots \right\}, \quad (25)$$

where for simplicity we have denoted by ellipses all D-type contributions, $\bar{F}_i(P_x, kx) = F_i(-P_x, -kx)$ and

$$[k_\perp^{\mu}, P^{\nu}] = k_\perp^{\mu}P^{\nu} - k_\perp^{\nu}P^{\mu} = k^{\mu}P^{\nu} - k^{\nu}P^{\mu} + O(W^2/Q^2). \quad (26)$$
The last line follows from the light-cone expansion [3].

Calculation of the Fourier integrals is now straightforward. Note that at this point the advantage of using the double-distribution representation for the matrix element [8] becomes clear, as all dependence on the momentum $k^\mu$ is in the exponents. Separating the linear in $k_\perp$ term one obtains after simple manipulations:

$$T_{1F}^{\mu\nu} = i \sum e^2 q^2 \int d[\alpha, \beta] F(\alpha, \beta) \left[ k_\perp^\nu (P + \omega q')^\mu \partial_\xi C(\alpha + \xi \beta, w) + k_\perp^\mu (P - \omega q)^\nu \partial_\xi C(\alpha + \xi \beta, -w) \right],$$

where

$$\omega = \frac{q^2 - q'^2}{q^2 + q'^2}, \quad -1 \leq \omega \leq 1,$$

and

$$C(u, w) = \frac{2}{\omega^2(1 - u)} \ln \left(\frac{1 + \omega u}{1 + \omega}\right) - \frac{2}{\omega^2(1 + u)} \ln \left(\frac{1 - \omega u}{1 + \omega}\right),$$

Using an identity:

$$\int d[\alpha, \beta] F(\alpha, \beta) \partial_\xi C(\alpha + \xi \beta, w) = \partial_\xi \int_{-\infty}^{+\infty} du \int d[\alpha, \beta] F(\alpha, \beta) \delta(\alpha + \xi \beta - u) C(u, w) =$$

$$\partial_\xi \int_{-1}^{1} du H_Q^F(u, \xi) C(u, w), \quad \text{where} \quad H_Q^F(u, \xi) = \int d[\alpha, \beta] F(\alpha, \beta) \delta(\alpha + \xi \beta - u),$$

one arrives at

$$T_{1F}^{\mu\nu} = i \sum e^2 q^2 \int_{-1}^{1} du \partial_\xi H_Q^F(u, \xi) \left[ k_\perp^\nu (P + \omega q')^\mu C(u, w) + k_\perp^\mu (P - \omega q)^\nu C(u, -w) \right].$$

Calculation of terms involving D-functions is very similar. We have checked that all the D-type contributions can be incorporated together with F-type terms into

$$T_{1F}^{\mu\nu} = i \sum e^2 q^2 \int_{-1}^{1} du \partial_\xi H_Q^F(u, \xi) \left[ k_\perp^\nu (P + \omega q')^\mu C(u, w) + k_\perp^\mu (P - \omega q)^\nu C(u, -w) \right]$$

with the coefficient function $C(u, w)$ given in (29) and the $2\pi$ distribution function $H_Q^F(u, \xi)$ defined according to (10).

The coefficient function $C(u, w)$ has a smooth behaviour in the limit $w \to 0$:

$$C(u, w) = 2u + O(w),$$

In the limit when photon with momentum $q'$ becomes real, $q'^2 \to 0$ or equivalently $w \to 1$ one finds:

$$C(u, w = 1) = \frac{2}{(1 - u)} \ln \left(1 - \frac{1 - u}{2}\right) - \frac{2}{(1 + u)} \ln \left(1 - \frac{1 + u}{2}\right),$$

but

$$\lim_{w \to 1} C(u, -w) = \ln(1 - w) \frac{-4u}{(1 + w)^2} + O(1 + w),$$

(31)
such that the amplitude (32) is singular in this limit. This singularity cancels, however, when contribution to the scattering amplitude is evaluated by contraction with polarization vectors of photons with helicities \( \lambda \) and \( \lambda' \), respectively:

\[
T_1(\lambda, \lambda') = T_1^{\mu \nu} e_\mu(\lambda) e_\nu(\lambda') .
\]

The second term in (32) can give a non-zero contribution only when a transverse photon with momentum \( q \) collides with a longitudinal photon with momentum \( q' \). All other helicity combinations lead to vanishing contribution due to contraction of transverse polarisation vectors with longitudinal momenta. Note that in the c.m.s. of two photons the longitudinal polarisation vector \( e_\nu(0) \) can be written as

\[
e_\nu(0) = i \left( \frac{-q'^2}{(qq')^2 - q^2 q'^2} \right)^{1/2} \left( q - q' \frac{qq'}{q'^2} \right)_\nu .
\]

Evaluating the contraction (36) one finds that an additional factor \( \sqrt{1 - w} \) arises which cancels the logarithmic singularity (35). Hence, for the amplitude \( T_1^{\gamma \pi \pi} \) of a collision of the virtual photon with momentum \( q \) with a real photon with momentum \( q' \), as introduced in (5), one finds:

\[
T_1^{\gamma \pi \pi} = \sum e_q^2 \int_0^1 du \partial_\xi H^Q(u, \xi) \left[ \frac{2}{(1 - u)^2} \ln \left( 1 - \frac{1 - u}{2} \right) - \frac{2}{(1 + u)^2} \ln \left( 1 - \frac{1 + u}{2} \right) \right] ,
\]

or in terms of 2\( \pi \) distribution amplitude

\[
T_1^{\gamma \pi \pi} = \sum e_q^2 \int_0^1 du \partial_\xi \Phi^Q(u, \xi) \left[ \frac{\ln(1 - u)}{u} - \frac{\ln(u)}{1 - u} \right] .
\]

5 Discussion

Recall that due to the positive charge parity, the pion pair is produced in a configuration symmetric with respect to interchange of two pions. Obviously, the production amplitude (4) should exhibit the same symmetry. As the part involving \( T_1 \) is proportional to \( k_\perp \), which is odd under exchange of pions, \( T_1 \) itself must be an odd function of \( \xi \) as well. Note that although \( H^Q(u, \xi) \) and \( \Phi^Q(u, \xi) \) are symmetric functions of \( \xi \), the presence of \( \xi \)-derivative in expressions (38) and (39) indeed results in \( T_1 \) which is antisymmetric function of \( \xi \).

In order to estimate the magnitude of \( T_1 \), we recall the 'minimal' model of the quark 2\( \pi \) DA [13, 29]:

\[
\Phi^Q(u, \xi, W^2) = -30u(1 - u) (2u - 1) \frac{1}{N_f} M^Q(\mu^2) B(\xi, W) .
\]

Here \( M^Q(\mu^2) \) is the momentum fraction carried by quarks in a pion at a scale \( \mu^2 \). Function \( B(\xi, W) \) is related to the so-called Omnès functions [31] \( f_0(W) \) and \( f_2(W) \) by

\[
B(\xi, W) = \left[ \frac{3C - \beta^2}{3} f_0(W) P_0(\cos \theta_{cm}) - \frac{2}{3} \beta^2 f_2(W) P_2(\cos \theta_{cm}) \right] ,
\]
where \( C = 1 + O(m_n^2) \). Omnès functions were analyzed in details in [31]. \( f_0(W) \) and \( f_2(W) \) can be obtained from dispersion relations derived in [9]:

\[
f_l(W) = \exp \left[ i\delta_l^0(W) + \frac{W^2}{\pi} \text{Re} \int_{4m_n^2}^\infty ds \frac{\delta_l^0(s)}{s(s-W^2-i0)} \right].
\] (42)

Here \( \delta_l^0 \) are \( \pi \pi \) scattering phase shifts in the correspondig channels.

After taking the derivative over \( \xi \), the \( f_0 \) contribution vanishes ane one finds:

\[
T_1^{\pi\pi}(W,\xi) = -\frac{10}{3} \sum e_q^2 \frac{1}{N_f} M^Q(\mu^2) f_2(W) \xi.
\] (43)

As a result, the corresponding contribution to the production amplitude (3) has in the CM system the angular dependence \( \sim \sin(\theta_{\text{cm}})\cos(\theta_{\text{cm}})e^{\pm i\phi_{\text{cm}}} \sim Y_l^m(\theta_{\text{cm}}, \phi_{\text{cm}}) \) with \( l = 2 \) and \( m = \pm 1 \). As far as the \( W \) dependence is considered, the Omnès function \( f_2(W) \) is small above the threshold, where pions are produced mostly in the S-wave [12]. Due to \( f_2(1270) \) resonance, \( f_2(W) \) has a peak at \( W = 1.275 \text{ GeV} \). This range of \( W \) is therefore most suitable to study \( T_1 \) experimentally, and the \( W \)-dependence provides an important test of the formula (43).

As pointed out in [13], by applying different weighting and averaging procedures one can directly compare the QCD predictions for different helicity amplitudes with the \( \gamma^*\gamma \rightarrow \pi\pi \) data. The analysis of \( T_1 \) presented here allows to answer the question about the magnitude of the matrix elements of quark-gluon operators, which are not accounted for in the WW approximation. By comparing prediction for the helicity-zero amplitude \( T_0 \) with the data one can constraint a model of \( 2\pi \) quark distribution amplitude. The same model can be subsequently used to predict the amplitude \( T_1 \) according to (38) and (39). Any discrepancy between such a prediction and the data would hint at a very interesting scenario where a significant contribution to \( T_1 \) arises from twist-3 quark-gluon operators.

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Figure 2: Space-time development of the $\gamma^*\gamma$ collision. Arrows denote polarizations of photons and quarks, respectively. As the final $q\bar{q}$ pair has no orbital angular momentum, its transition into the pion pair is described by a matrix element of twist-2 light-cone quark string operator.

Figure 3: Space-time development of the $\gamma^*\gamma$ collision. Arrows denote polarizations of photons and quarks, respectively. The final $q\bar{q}$ pair carries a unit of orbital angular momentum. Its transition into the pion pair is described by a matrix element of a twist-3 operator.
Figure 4: The real photon splits into a quark-antiquark pair long before a collision with the virtual one. Arrows denote polarizations of photons and quarks, respectively. The final $q\bar{q}$ pair has to carry two units of orbital angular momentum and therefore its transition into the pion pair must be described by a matrix element of a higher-twist operator.