A bound on CPT violation with sQED radiative corrections and $\pi H(\Lambda H)$ atoms

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We have studied the 1-loop corrections to the scalar-photon vertex interactions of a minimal Lorentz-violating and CPT-odd scalar electrodynamics with the photon having a sufficiently small mass. The CPT-odd term is the well known Carroll-Field-Jackiw (CFJ) which only modifies the kinetic term of the photon field. We have observed the radiative generation of the UV finite term $igw_\mu \bar{D}_\mu \phi - \phi(D_\mu \phi)$, a dimension-5 operator which behaves like an anomalous magnetic moment for the scalar particle. The current bounds for photon mass and CPT violation are used to estimate a upper-bound for the coupling $|gw_\mu| < 1 \times 10^{-13} \text{eV}^{-1}$. In order to estimate another upper-bound for the generated coupling, we have also analyzed its contribution to the pionic (kaonic) hydrogen energy. In this case, the experimental data for the 1S strong shift transitions are used to compute the upper-limit $|gw_\mu| < 1.1 \times 10^{-12} \text{eV}^{-1}$.

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I. INTRODUCTION

The possibility of Lorentz and CPT violation in quantum electrodynamics (QED) has been studied intensely in the last years. The principal motivation is the possibility of occurring the spontaneous breaking of both symmetries at very high energy, i.e., the Planck scale [1]. The standard model extension (SME) [2] is the main framework proposed to study the possible effects of Lorentz violation into the standard model of the fundamental particles and their interactions. In the minimal SME the Lorentz-violating (LV) coefficients concerning to photonic and fermionic sectors are added in such a way to preserve gauge symmetry and renormalizability and, some of they have strong experimental upper bounds [3]. Detailed studies about the LV coefficients of the fermionic sector can be founded in Refs. [4], for the ones of the CPT-odd photonic sector in Refs. [5–7] while for the CPT-even ones in Refs. [8–10]. The introduction of Lorentz violation by means of operators with dimensions higher than 4 was first considered in Ref. [11] by studying the effects on particle dispersions relation of a dimension-five operator. Others applications of such a proposal were performed in Refs. [12]. On the other hand, higher order operators are considered in the non-minimal SME, for example, the photon and fermion sectors were analyzed in Refs. [13, 14]. Some implications are studied in Refs. [15]. The introduction of Lorentz violation in systems with scalar and/or fermion and gauge fields can be made via nonminimal couplings which could introduce higher order operators terms [16–20]. Some consequences or effects of that non-renormalizable coefficients have been investigated in several distinct scenarios [21–35].

The quantum field theory describing the interaction between spinless charged particles and the electromagnetic field is known as scalar quantum electrodynamics (sQED). The Lorentz-violating extension of sQED has not received same attention like its fermionic version. The reason could be the fact do not exist in standard model a fundamental spinless charged particle or because it is not so rich phenomenologically like its fermionic version. As an example, a CPT-odd power-counting renormalizable term is only possible in photonic and fermionic sectors. Even so, some Lorentz-violating of the scalar electrodynamics has been studied in recent years. For example, aspects like causality, unitarity and spontaneous symmetry breaking of a LV and CPT-odd sQED were analyzed in Ref. [36]. The Higgs mechanism in the context of a Lorentz-violating and CPT-even sQED was studied in Ref. [37]. At classical level many studies about existence of vortices BPS in Lorentz-violating scalar electrodynamics were performed in Refs. [38].

The aim of this manuscript is to study the 1-loop radiative generation of a nonminimal Lorentz-violating and CPT-odd coupling between spinless charged particles and the electromagnetic field in a scalar electrodynamics whose gauge sector includes the Carroll-Field-Jackiw (CFJ) term [39] which breaks both the Lorentz and CPT symmetries. Our study is organized as follows: In Sec. II we present the Lorentz-violating and CPT-odd scalar electrodynamics in which our proposed is based and we establish the respective Feynman rules. In Sec. III, we have computed at 1-loop order the first-order Lorentz-violating corrections to the 3- and 4-point vertex functions composed by scalar and photon fields. The 3-point

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vertex function allows to impose theoretical upper-bound for the generated coupling constant. In Sec. IV we have established the non-relativistic limit and used the experimental data from the strong shift of the ground state of the pionic (kaonic) hydrogen to impose upper-bounds, too. Finally, we summarize our conclusions and perspectives in Sec. V.

II. THE THEORETICAL FRAMEWORK

The basic framework of our investigation is a scalar electrodynamics with a massive photon whose kinetic term possess the CPT-odd and Lorentz-violating (LV) Carroll-Field-Jackiw (CFJ) term. The LV lagrangian density representing this model is given by

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_A + \mathcal{L}_I,$$

where $\mathcal{L}_\phi$ is the charged scalar field part, $\mathcal{L}_A$ the gauge part, and $\mathcal{L}_I$ the interaction part. Explicitly:

$$\mathcal{L}_\phi = \partial_\mu \phi \partial^\mu \phi - m^2 \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2,$$

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} m^2 A^\mu A_\mu + \frac{1}{4} \epsilon^{\mu\nu\alpha\beta} (K_{AF})_\mu A_\nu F_{\alpha\beta} - \frac{1}{2 \xi} (\partial_\mu A^\mu)^2,$$

$$\mathcal{L}_I = ie A^\mu (\partial_\mu \phi^\dagger \phi - \phi^\dagger \partial_\mu \phi) + e^2 A^\mu A^\nu \phi^\dagger \phi.$$

The covariant canonical quantization of the CPT-odd and Lorentz violating massive electrodynamics given in Eq. (3) was performed in [40, 41]. The small photon mass term is included through the Stueckelberg mechanism and it is required for a consistent quantization.

It must be observed that we have not included the CPT-odd term

$$i \kappa^\mu \left[ (D_\mu \phi)^\dagger \phi - \phi^\dagger D_\mu \phi \right],$$

because it can be eliminate by means of the following field redefinition $\phi \rightarrow e^{-i \kappa^\mu \tau_\mu} \phi$ which transforms (5) in

$$(D_\mu \phi)^\dagger (D^\mu \phi) - (m^2 + \kappa^\mu \kappa_\mu) \phi^\dagger \phi,$$

It implies that LV and CPT-odd vector background $\kappa^\mu$ can be absorbed by means of a mass redefinition therefore the interaction (5) is not a true LV term.

In the remain of the manuscript we will consider only the 1-loop corrections to the vertex interactions given by the Eq. (4) produced at first-order by the CFJ vector background.

The Feynman rules, in the Feynman gauge, we will use to attain our goal, are:

- Scalar propagator

$$-i\Delta(p) = \frac{-i}{p^2 - m^2 + i\varepsilon}. \quad (7)$$

- Photon propagator

$$-i\Delta_{\mu\nu}(q) = \frac{ig_{\mu\nu}}{q^2 - m^2 + i\varepsilon} - iS_{\mu\nu}(q), \quad (8)$$

where $S_{\mu\nu}$ is the first order LV modification to photon propagator given by

$$S_{\mu\nu}(q) = \frac{i\varepsilon_{\mu\nu\delta}(K_{AF})^\delta}{(q^2 - m^2 + i\varepsilon)^2}. \quad (9)$$

- Tree level scalar-photon 3-vertex

$$-ie (p + p')^\mu. \quad (10)$$

- Tree level scalar-photon 4-vertex

$$2ie^2 g^{\mu\nu}. \quad (11)$$

III. 1-LOOP CORRECTIONS TO THE SCALAR-PHOTON VERTEX INTERACTIONS

The 1-loop radiative corrections we are interested are those containing only contributions at first-order in the Lorentz-violating background $(K_{AF})_\mu$. The antisymmetric character of the LV tensor $S_{\mu\nu}$ precludes the first-order LV corrections at 1-loop order for vacuum polarization tensor and pure scalar Green functions. However, the 3- and 4-vertex functions which mix the gauge and scalar fields have it.
A. The 3-point vertex function

The 3-vertex function receives the nonnull corrections represented by the following Feynman graphs:

![Feynman Graphs]

FIG. 1: 1-loop correction to the scalar-photon 3-vertex.

By using the Feynman rules enumerated before, we mount the associated amplitude to be

\[ -ie \Gamma_\mu (p, p') = (-i)^6e^3 \int \frac{d^4r}{(2\pi)^4} (2p-r)_\alpha \Delta^{\alpha \beta} (r) (2p'-r)_\beta \Delta (p' - r) (p + p' - 2r)_\mu \Delta (p - r) \]

\[ -(-i)^4e^3 \int \frac{d^4r}{(2\pi)^4} 2g_{\mu \nu} \Delta^{\nu \alpha} (r) (2p'-r)_\alpha \Delta (p' - r) \]

\[ -(-i)^4e^3 \int \frac{d^4r}{(2\pi)^4} (2p-r)_\beta \Delta^{\beta \nu} (r) 2g_{\nu \mu} \Delta (p - r). \tag{12} \]

In the following, we extract the first-order LV contributions which have no ultraviolet divergences so a regularization process is no necessary. Thus, the relevant Lorentz-violating integrals are

\[ \Gamma_\mu^{(LV)} (p, p') = 4e^2 \int \frac{d^4r}{(2\pi)^4} \frac{p_\alpha \varepsilon^{\alpha \beta \chi \delta} (K_{AF})_\chi r_{\beta \delta} (p + p' - 2r)_\mu}{(r^2 - m^2 + i\varepsilon)^2 \left( (p' - r)^2 - m^2 + i\varepsilon \right) \left( (p - r)^2 - m^2 + i\varepsilon \right)} \]

\[ + 4e^2 \int \frac{d^4r}{(2\pi)^4} \frac{g_{\mu \nu} \varepsilon^{\nu \alpha \chi \delta} (K_{AF})_\chi r_{\beta \delta}}{(r^2 - m^2 + i\varepsilon)^2 \left( (p' - r)^2 - m^2 + i\varepsilon \right)} \]

\[ + 4e^2 \int \frac{d^4r}{(2\pi)^4} \frac{p_\beta \varepsilon^{\beta \nu \chi \delta} (K_{AF})_\chi r_{\mu \delta} g_{\nu \mu}}{(r^2 - m^2 + i\varepsilon)^2 \left( (p - r)^2 - m^2 + i\varepsilon \right)}. \tag{13} \]

We now introduce the momentum variables

\[ s = \frac{p' + p}{2}, \quad t = \frac{p' - p}{2}, \tag{14} \]

then by regarding the forward scattering process we will consider \( s^2 \approx m^2 \) and only the first-order contributions in \( t \), such that the Eq. (13) becomes

\[ \Gamma_\mu^{(LV)} (p, p') = e^2 \int \frac{d^4r}{(2\pi)^4} \frac{16s_\alpha \varepsilon^{\alpha \beta \chi \delta} (K_{AF})_\chi r_{\beta \delta} \left( t_\beta (s - r)_\mu + g_{\mu \beta} (s - r) \cdot t \right)}{(r^2 - m^2 + i\varepsilon)^2 \left( (s - r)^2 - m^2 + i\varepsilon \right)^2} \]

\[ + e^2 \int \frac{d^4r}{(2\pi)^4} \frac{8g_{\mu \alpha} \varepsilon^{\alpha \beta \chi \delta} (K_{AF})_\chi r_{\delta \beta} (t)_\mu}{(r^2 - m^2 + i\varepsilon)^2 \left( (s - r)^2 - m^2 + i\varepsilon \right)}. \tag{15} \]

After apply a set of known techniques to solve Feynman integrals, the Eq. (15) is reduced to be

\[ \Gamma_\mu^{(LV)} (p, p') = \frac{e^2 i \varepsilon_{\mu \nu \alpha \beta} (K_{AF})^\nu \epsilon_{s \beta} \left( \frac{m^2}{m} \right)}{2\pi^2 m^2} I \left( \frac{m_\gamma^2}{m} \right), \tag{16} \]
with
\[
I \left( \frac{m_\gamma}{m} \right) = \int_0^1 \frac{u (1 - u) \, du}{(1 - u)^2 + u \frac{m_\gamma^2}{m^2}}
= -1 - \frac{1}{2} \ln \frac{m_\gamma^2}{m^2} + ..., \quad (17)
\]
where due to \( m_\gamma \ll m \), we have considered only the more relevant contribution. By using it we rewrite the Eq. (16) in a simple form:
\[
e \Gamma^{(LV)}_\mu (p, p') = i \epsilon \mu \nu \beta \gamma \left( p' - p \right)^\alpha \left( p' + p \right)^\beta g \nu \nu, \quad (18)
\]
where we have introduced the coupling constant \( g \nu \nu \) defined by
\[
g \nu \nu = \frac{e^3}{8 \pi^2 m} \frac{(K_{AF})^\nu}{m} \left( 1 + \frac{1}{2} \ln \frac{m_\gamma^2}{m^2} \right). \quad (19)
\]
It is clear that in the limit where the photon has null mass appears an infrared divergence such as it was observed in a CPT-odd quantum electrodynamics [42]. Recently, in Refs. [40, 41], the consistent analysis of the Cherenkov radiation including the CFJ term necessarily requires the use of a massive photon whose mass satisfies the condition \( |m_\gamma^2| > |K_{AF}^2| \), i.e., the existence of CPT violation precludes the photon mass to be null. Such a condition is in agreement with the current bounds for photon mass and CPT violation:
\[
|m_\gamma| < 10^{-18} \text{ eV}, \quad |K_{AF}| < 10^{-34} \text{ eV}. \quad (20)
\]
By considering the mass of the complex scalar field \( m > m_\gamma \), and \( K_{AF} \) a space-like vector, we can show
\[
\left| 1 + \frac{1}{2} \ln \left( \frac{m_\gamma^2}{m^2} \right) \right| < \left| 1 + \frac{1}{2} \ln \left( - \frac{K_{AF}^2}{m^2} \right) \right|. \quad (21)
\]
So, for \( K_{AF} \) along the \( z \)-axis, we can rewrite the Eq. (19) as
\[
|g \nu z| < \left| \frac{e^3 (K_{AF})^z}{8 \pi^2 m^2} \right| \left( 1 + \frac{1}{2} \ln \frac{(K_{AF})^2}{m^2} \right). \quad (22)
\]
The maximum value of the right-hand side allows to impose a theoretical upper-bound for the constant \( g \nu \nu \),
\[
|g \nu z| < \frac{e^3 \exp(-2)}{8 \pi^2 m}, \quad (23)
\]
where \( \epsilon \) represents the magnitude of electron charge.

Now we consider the boson particle to be the pion, the theoretical upper-bound becomes
\[
|g \nu z| < \frac{e^3 \exp(-2)}{8 \pi^2 m_\pi} = 3.4 \times 10^{-13} \text{ eV}^{-1}, \quad (24)
\]
where \( m_\pi = 139.57018(35)10^6 \text{ eV} \) [43]. By considering the kaon, we attain a similar upper-bound,
\[
|g \nu z| < \frac{e^3 \exp(-2)}{8 \pi^2 m_K} = 9.6 \times 10^{-14} \text{ eV}^{-1}, \quad (25)
\]
where \( m_K = 493.677(16)10^6 \text{ eV} \) [43].

The Lorentz-violating contribution of the 3-point vertex function provides the following dimension-five operator to the effective action
\[
\mathcal{L}_3 = i \frac{g \nu \nu e^{\mu \rho \alpha \beta}}{8 \pi^2 m} \left( (\partial_\nu \phi) (\partial_\alpha \phi) \right), \quad (26)
\]
the CPT-odd coupling \( g \nu \nu \) plays a role analog to an anomalous magnetic moment. At first sight this operator is no gauge invariant due to the absence of the covariant derivative, however the terms to turn it gauge invariant arise from the LV contributions to the 4-point vertex function.

**B. The 4-point vertex function**

The LV contribution to the 4-point vertex function is obtained from the following Feynman graph,

![Feynman diagram](image)

\[ \text{FIG. 2: 1-loop corrections to the scalar-photon 4-vertex.} \]

It provides the amplitude
\[
i \Gamma_{\mu \nu} (p, p', q) = e^2 \int \frac{d^4 r}{(2\pi)^4} \Delta (p - q - r) 2 \Delta_{\mu \nu} (r) + (\mu \rightarrow \nu, p \rightarrow p', q \rightarrow -q). \quad (27)
\]

Similarly to the 3-point case, we take into account only the first-order LV contributions, so we have
\[
\Gamma_{\mu \nu}^{(LV)}(p, p', q) = 2e^2 \int \frac{d^4r}{(2\pi)^4} \frac{\varepsilon_{\mu \nu \chi} (K_{AF})^\chi \gamma^\delta}{((p - q - r)^2 - m^2 + i\varepsilon)^2} + (\mu \rightarrow \nu, p \rightarrow p', q \rightarrow -q). \tag{28}
\]

In order to extract the relevant contributions for our purpose we again use the variables \(s\) and \(t\) defined previously in Eq. (14) and by considering the forward scattering limit (we set \(s^2 \approx m^2\) and consider only the smaller contributions of \(t\) and \(q\), we obtain

\[
\Gamma_{\mu \nu}^{(LV)}(p, p', q) = 8e^2 \int \frac{d^4r}{(2\pi)^4} \frac{\varepsilon_{\mu \nu \chi} (K_{AF})^\chi (s - r) \cdot (t + q)}{((s - r)^2 - m^2 + i\varepsilon)^2}. \tag{29}
\]

After some algebraic manipulations to solve the Feynman integrals, the LV contribution to the 4-point vertex function is

\[
\Gamma_{\mu \nu}^{(LV)}(p, p', q) = \frac{i\varepsilon_{\mu \nu \alpha \beta} (K_{AF})^\alpha (\gamma^\beta q^\alpha)^2}{4\pi^2 m^2} I \left( \frac{m\gamma}{m} \right). \tag{30}
\]

By considering only the relevant contribution (17) of \(I (m\gamma/m)\) we can rewrite the expression (30)

\[
e^2\Gamma_{\mu \nu}^{(LV)}(p, p', q) = -2i\varepsilon_{\mu \nu \alpha \beta} (\gamma^\alpha g_w)^\beta, \tag{31}
\]

where we have using the coupling constant (19). It provides the following contribution to the effective action:

\[
\mathcal{L}_{(4)} = -egw_\mu \epsilon^{\mu \nu \alpha \beta} F_{\alpha \beta} A_\nu \phi^\dagger \phi. \tag{32}
\]

Here it is necessary to make an observation about the absence of terms proportional to \(t\) in Eq. (30), the major reason is that in despite of they be non null their contributions to the effective action vanish identically.

The gauge invariant contribution to the effective action is obtained by summing the terms (26) and (32), i.e., \(\mathcal{L}_{eff} = \mathcal{L}_{(3)} + \mathcal{L}_{(4)}\),

\[
\mathcal{L}_{eff} = \frac{i}{2} g_\pi \epsilon^{\mu \nu \alpha \beta} F_{\alpha \beta} \left[(D_\nu \phi)^\dagger \phi - \phi^\dagger (D_\nu \phi)\right]. \tag{33}
\]

This interaction could be useful in the study of Lorentz and CPT violation in scalar charged boson systems despite of it be a nonrenormalizable one.

### IV. THE MESONIC HYDROGEN ATOMS

The pionic hydrogen is a system where the electron is replaced by a pion. While the standard hydrogen atom can be described by Dirac’s equation, the pionic hydrogen can be in principle treated in the context of scalar electrodynamics because the spinless character of the pion. The crucial difference is the fact the pion is an unstable composite particle constituted by a quark-antiquark pair with mean lifetime around \(3.95 \times 10^{16}\) eV\(^{-1}\). Many properties of the pionic hydrogen were studied in Ref. [44] by considering only QED effects. However, there are quantum chromodynamics (QCD) contributions to the binding energies and level widths of the atomic levels whose most notorious effect is presented by the 1S state. It is measured by means of the X-ray transitions by comparing with the pure electromagnetically bound state.

In the context of Lorentz and CPT violation some aspects of low-energy QCD are analyzed in Ref. [45] and for pions and nucleons in Ref. [46]. Both references perform their analysis within the formalism of chiral perturbation theory [47], an effective quantum field theory useful to describe some low-energy aspects of QCD. Nevertheless, because of its large mass, the pionic hydrogen can be considered as a nonrelativistic system such that we use the transition 2P–1S to impose an upper-bound for the coupling constant \(g_w\) of the CPT-odd and Lorentz-violating interaction obtained in Eq. (33).

The relevant contribution at nonrelativistic limit of the term (33) is

\[
\mathcal{L}_{eff} = g_\pi \vec{w} \cdot \vec{B} \phi^\dagger \phi + ..., \tag{34}
\]

which contributes to the Hamiltonian with the following perturbation,

\[
\Delta H = -g_\pi \vec{w} \cdot \vec{B}. \tag{35}
\]

such an interaction is not predicted by pure quantum electrodynamics so it can be used to impose an upper-bound for the coupling \(g_\pi \vec{w}\). The LV coupling \(g_\pi \vec{w}\) is playing the role of a magnetic moment for the pion allowing to study it as a background-orbit interaction. Then, the corresspondent correction to pionic hydrogen Hamiltonian is given by

\[
\Delta H = \frac{-eg_\pi \vec{w} \cdot \vec{L}}{4\pi \mu r^3}, \tag{36}
\]

where \(\mu\) is the reduced mass of the proton-pion system and the magnetic field in Eq. (35) was set to be

\[
\vec{B} = \frac{e\vec{L}}{4\pi \mu r^3}, \tag{37}
\]

in a similar way occurring in the spin-orbit case.
By considering, the background \( \vec{u} \) along the \( z \)-axis, the correspondent shift to the pionic hydrogen energy is
\[
\Delta E = \langle \Delta H \rangle = -\frac{\varepsilon g_w z}{4\pi\mu} \left( \frac{L_z}{r^3} \right), \tag{38}
\]
Only states with nonnull angular momentum projection \( L_z \) will receive energy corrections so the energy of the \( 1S \) state remains unaltered. All other states gain the following energy correction:
\[
\Delta E = -g_w z \frac{\mu^2 e^7}{4\pi n^3 (\ell + 1) (\ell + 1/2) \ell}, \tag{39}
\]
We observe the states with lower values of \( n \) and \( \ell \) receive the most significant Lorentz violating corrections. Consequently, the \( 2P \) state is the more affected while \( 1S \) no receive LV correction.

In the pionic hydrogen the study of the transitions from excited levels \( 4P \), \( 3P \) and \( 2P \) to the fundamental state \( 1S \) are used to measure the effects of the strong interactions [48]. The difference between standard QED predictions and experimental data are usually considered to compute the shift produced in the \( 1S \) state by QCD effects (see Fig. (3)). The experimental value of the shift is
\[
\varepsilon_{1S} \approx (7.086 \pm 0.007(stat) \pm 0.006(sys)) eV, \tag{40}
\]
to low. 

Our purpose is to consider such a energy shift by supposing the LV corrections are smaller than strong corrections error. Thus, we attain the following upper-bound,
\[
|g_w z| < 1.1 \times 10^{-12} \text{ eV}^{-1}, \tag{41}
\]
where we use the proton mass to be 938.272081(6)\( \times 10^6 \) eV [43]. Such a bound is in according with the theoretical one obtained in Eq. (23).

Similar procedure can be used with the kaonic hydrogen whose measured strong shift [49] is,
\[
\varepsilon_{1S} \approx -283 \pm 36(stat) \pm 6(sys) \text{eV}, \tag{42}
\]
such that it provides the upper-bound
\[
|g_K z| < 5.2 \times 10^{-10} \text{ eV}^{-1}, \tag{43}
\]
which is not better than the one provided by the pionic hydrogen.

V. REMARKS AND CONCLUSIONS

We have studied the radiative generation of a dimension-5 operator, \((i/2)gw_w e^\nu_\alpha \rho_\beta \phi^\dagger D_\nu \phi\), in a scalar electrodynamics endowed with the Carroll-Field-Jackiw term breaking both the Lorentz and CPT symmetries. Such a term coupling the charged scalar and electromagnetic fields is ultraviolet finite but it could generate infrared divergences for massless photons. The presence of the CFJ background precludes the photon becomes massless because of the constraint \( |m_\phi^2| > |K_{AF}^2| \) compatible with current bounds for photon mass and CPT violation [40, 41]. Such bounds allow to impose theoretical upper-bounds for the coupling constant \( gw_w \) by considering the charged scalar particle to be a meson (pion or Kaon), the best one is obtained when consider the Kaon (see Table I).

By regarding the non relativistic contribution of (33) for the energy of a hadronic atom (pionic or kaonic hydrogen) it is observed that the background vector \( gw_w \) interacts with the magnetic field (see Eq. (35)) such that it looks like playing the role of a magnetic moment for scalar particle. By considering it like a spin-orbit interaction, the resulting Hamiltonian (see Eq. (36)) provides the energy correction given in Eq. (38). Such energy correction is compared with the experimental error of the measured 1S strong-shift of the hadronic atom (pionic or kaonic hydrogen) to obtain a upper-bound for the coupling \( gw_w \). This time, the strong-shift of the pionic hydrogen gives a better upper-bound (see Table I).

| Meson/atom | theoretical \( gw_w (\text{eV}^{-1}) \) | 1S shift \( gw_w (\text{eV}^{-1}) \) |
|-----------|-----------------|-----------------|
| \( \pi/\pi \text{H} \) | \( 3.4 \times 10^{-13} \) | \( 1.1 \times 10^{-12} \) |
| \( K/\text{K} \) | \( 9.6 \times 10^{-14} \) | \( 5.2 \times 10^{-10} \) |

TABLE I: Comparative upper-bounds for the coupling \( gw_w \).

Finally, we point out the study of other exotic atoms in Lorentz-violating frameworks with the aim of the impose more restrictive bounds for other Lorentz-violating parameters. Advances in this direction will be reported elsewhere.

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