Recent Developments in Gluon Fusion Higgs Calculations

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During recent years perturbative fixed order and resummation calculations have decreased uncertainties on predictions for gluon fusion Higgs production cross sections tremendously. Most exciting results have been published just this year. In these proceedings I present an overview of recent and most recent developments of these calculations that allow theory predictions to compete with the experimental precision reached by future collider upgrades.

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Introduction

Over the run-time of the LHC and its associated experiments, many people’s expectations in terms of reached precision have been surpassed. Although true percent level or sub-percent level precision physics is not within its scope, even the currently reached and near future reached precision poses a challenge for the theory community calculating Standard Model (SM) predictions. This is especially true for gluon fusion Higgs production, which is one of the cornerstone Higgs production mechanisms by virtue of its large cross section compared to the other channels.

Recent experimental results from ATLAS and CMS predominated by the gluon fusion production mechanism are widely about to crack the 10% level in statistical uncertainties [1–5], [6–8]. Broadly speaking, provided that methods of data analysis and calculational techniques advance as expected, we can expect single digit uncertainties for the inclusive gluon fusion Higgs cross section and about 10% uncertainties on differential quantities at a 3000 fb$^{-1}$ high luminosity upgraded LHC [9]. A large part of that burden is carried by the uncertainties of theory predictions which ideally should be significantly smaller than experimental statistical and systematic uncertainties.

An important result this year for experiment and theory is the observation of the $H \rightarrow bb$ decay [10] through the combination of many individual analysis channels. The gluon fusion production mechanism only plays a small role in this result due to large experimental, but also large theory uncertainties [11]. Its analysis was performed for a highly boosted Higgs boson and relied on Higgs+jet predictions which at that time had large uncertainties associated with the top-quark mass. This uncertainty has been mostly eliminated meanwhile as I will also highlight further below.

Although pushing gluon fusion Higgs production uncertainties to the percent level will be a task of future LHC upgrades and future colliders, theory predictions should match experimental ones, and be ideally significantly lower. The current goal is thus to provide theory predictions at the single percent level uncertainty, which is an extraordinarily difficult task given the many ingredients that enter a full Standard Model calculation at a hadron collider. The low hanging fruits are gone and it requires the effort of larger and larger groups and collaborations to stem this problem.

Reach for precision

The prediction of the inclusive gluon fusion Higgs production cross section made a big step forward a few years ago through the calculation of the N$^3$LO QCD corrections [12]. This was made possible by using the limit of an infinitely heavy top-quark (EFT) and an expansion around the Higgs production threshold. An assessment of all
uncertainties was soon-after performed [13], estimating the residual theory uncertainty and uncertainties in PDFs and $\alpha_s$ to about less than 10%. This estimate is based on the aforementioned N$^3$LO calculation in the EFT approximation augmented with finite top-quark and bottom-quark mass effects, threshold resummation and mixed QCD-electroweak correction factors.

For differential gluon fusion Higgs production with a recoiling jet, NNLO predictions in the EFT have uncertainties of 10% just from the truncation of higher order QCD effects, which has been known for some time now [14–16]. Of high importance is the Higgs transverse momentum ($p_T$) distribution, especially in the boosted high $p_T$ regime. Further effects with associated uncertainties like transverse momentum resummation, electroweak corrections and finite bottom and top-quark masses need to be taken into account and increase the uncertainties each. At this point PDF and $\alpha_s$ uncertainties were not even mentioned yet. For example in the case of the gluon fusion $H \to b\bar{b}$ analysis [11], an uncertainty of 30% is assigned to the theory prediction.

Going from these stacks of uncertainties in inclusive and differential Higgs production to a combined one percent uncertainty is a task that at the very least must be reached by each individual ingredient. At the very least we want to sufficiently surpass the current and near-future experimental precision. The consistent combination of pieces to truly reach one percent uncertainty seems like a task that is too difficult to even think about. Yet the theory community must work towards this goal and has made important progress just this year, of which I will give a summarizing excerpt of most important results in the following paragraphs.

I will limit myself to present only recent results of Standard Model gluon fusion Higgs production in the inclusive and differential case. I will also mostly just refer to studies presenting phenomenological results, and not to the equally important long tail of technical publications that paved the road. For further results see for example the recent Standard Model working group report [17] and a most recent review article on Higgs physics [18].

**Recent developments**

A direct improvement of the inclusive gluon fusion Higgs cross section has been achieved through an exact calculation at N$^3$LO in the EFT removing the need for a threshold expansion [19]. While the associated threshold expansion uncertainty was estimated to be $\approx 0.5\%$ and turned out to be a little less, it is an important step not just for the reduction of the uncertainty, but also for the experience gained from the occurrence of elliptic integrals. More specifically, dedicated progress has been made this year to classify elliptic polylogarithms and to develop them to the same degree as
Figure 1: Normalized NNLO rapidity (left) and transverse momentum (right) distributions for the Higgs boson in the threshold expansion $(1 - x)^n$ and as a full result. The normalization and full result is given as the blue line. Higher orders in the threshold expansion are reweighted such that their cumulant reproduces the unexpanded NNLO cross section. Plot taken from ref. [22].

non-elliptic polylogarithms [20, 21, 21].

Using the threshold expansion for Higgs differential distributions has been followed in ref. [22]. It was found that in principle a threshold expansion is sufficient to determine the analytic Higgs rapidity distribution at N$^3$LO at the few percent level. This was estimated from a comparison between the threshold expansion and exact results both at NNLO. Less inclusive quantities like transverse momentum can not be approximated sufficiently by the expansion, see fig. 1. Even if not used directly, the threshold expansion can provide valuable boundary information for the computation of master integrals using differential equations.

The N$^3$LO cross section has meanwhile also been obtained in the $q_T$ subtraction formalism [23]. This result relies on the previously calculated N$^3$LO cross section though to match and numerically extract a $\delta(q_T^2)$ piece. Using this, the authors were able to obtain the Higgs rapidity distribution at N$^3$LO, which comes with a significant reduction of scale uncertainty compared to NNLO, see fig. 2.

A public implementation of inclusive gluon fusion Higgs production at N$^3$LO is available with the code iHixs [24] which also includes some effects beyond the fixed order EFT result. The competing code SusHi, while still relying on the threshold expansion and a matching to the high energy limit, is more focused towards physics beyond the Standard Model as it can also provide predictions for a CP odd scalar and dimension five Higgs-gluon coupling operators.

The first double resummed prediction for inclusive gluon fusion Higgs production is presented in ref. [25]. Both logarithmic corrections at production threshold and in the high energy limit are simultaneously resummed, the former through N$^2$LL, the latter at LL, and matched to the N$^3$LO fixed order result. Additionally the effect of
Figure 2: Higgs rapidity distribution up to N^3LO with scale variation uncertainties. At N^3LO also the $q_T^{cut}$ variation uncertainty and uncertainty from the numerical extraction of a $\delta(q_T^2)$ piece is taken into account. For details see [23], where this plot is taken from.
Figure 3: Inclusive gluon fusion Higgs cross sections at fixed orders NLO, NNLO and N^3LO with added threshold resummation and double resummation. For details regarding the uncertainties see ref. [25], where this plot is taken from.

resummed PDFs is included [26]. The implementation is based on publicly available codes for threshold and high-energy resummation, and includes some top-quark mass effects. The authors find a correction of 2% on the fixed order cross section and provide most recent and accurate cross section predictions. It is important to note that this calculation opened up a new source of uncertainty, and that as such the total uncertainty increases due to missing subleading high energy logarithms, see fig. 3.

Progress has also been made in re-evaluating the size of electroweak effects in gluon fusion [27, 28]. Electroweak (EW) effects enhance the pure LO QCD Higgs cross section by about 5% [29, 30]. A calculation of electroweak corrections at NLO in QCD at the three-loop level is important to assess the factorization approximation of QCD x electroweak corrections. This allows one to apply EW correction factors to higher order QCD cross sections. Previous results estimated those EW corrections in an approximation valid for m_H < m_W [31] and found an equal enhancement of about 5% with respect to the NLO QCD result. This clearly supports the QCD x EW factorization. The new study presented in ref. [27] employs a soft-gluon approximation for the real emission contributions but otherwise takes analytical results for the three loop virtual corrections to confirm those earlier estimates.

**Higgs in association with a jet.** I will now focus on improvements on the Higgs+jet cross sections, where the transverse momentum (p_T) distribution of the Higgs boson is of particular interest. Soft gluon resummation has to be accounted for at low p_T as well as light quark masses, while at large p_T top-quark mass effects play the biggest role.
Figure 4: Higgs transverse momentum distribution at different orders with a central renormalization scale of $m_H/2$ and a resummation matching at 30 GeV. The uncertainties are obtained by scale variation and varying the resummation matching. Taken from ref. [34].

Transverse momentum resummation for Higgs+jet production at NNLO has meanwhile been performed at $N^3LL$ on top of the NNLO fixed order EFT result. This has been achieved in two ways. It has first been calculated in a Monte-Carlo approach to resum directly in $p_T$ space [32, 33] and has then been evaluated analytically for an improved resolution in a SCET framework [34]. The analytically resummed $p_T$ distribution at different perturbative orders is shown in fig. 4.

The intermediate transverse momentum region $m_b \lesssim p_T \lesssim m_T$ is of particular importance for current and previous experimental analyses that have no precision reach at high $p_T$ yet. See for example fig. 5, where the $p_T$ distribution of the $H \rightarrow \gamma\gamma$ system as measured by ATLAS is shown. The large number of mass scales in this region makes it theory-wise difficult to compute it precisely. Logarithms of the kinematic ratios $p_T/m_b$ and $m_H/m_b$ can grow large, and their all-order resummation is currently out of reach.

Soft gluon resummation and bottom quark mass effects, which are most important in that region, were taken into account as far as currently possible in ref. [35]. The inclusion of NNLL soft gluon resummation and bottom quark mass effects lead to an increase of $\approx 5\%$ of the NLO cross section. The study focuses on assessing uncertainties in this important region and finds that the uncertainty on the predominant top-bottom interference has a $\approx 15\%$ scale uncertainty. An additional uncertainty of up to $\approx 20\%$ is due to the bottom quark mass renormalization scheme at small $p_T$. Combining
these with the uncertainty that comes from ambiguities in the soft gluon resummation in presence of the massive bottom quark, they conclude that generally uncertainties are about ≈ 20% in the discussed region. As such, one can expect further percent level effects from bottom quark mass effects in this intermediate energy region.

Finally, the region of large transverse momenta, becoming highly relevant through boosted Higgs analyses [11] has received a large improvement by removing the top-quark mass uncertainty at NLO. This uncertainty is due to the EFT approximation, reducing the number of loops to compute not just by one, but also removing the top-quark mass itself in the loop integrals. It is only valid in the limit $p_T^2 \ll m_T^2$. Using the two-loop virtual contributions in a large energy expansion [36, 37], and through a numerical evaluation of the exact integrals by sector decomposition [38], the top-quark mass uncertainty was eliminated at NLO.

The corrections due to a finite mass top-quark in the virtual corrections are ≈ 1 − 2% for $p_T \gg m_T$, when compared to an approximation with rescaled virtual corrections, see fig. 6. A similar comparison is performed in ref. [38] using the numerical evaluation of the exact top-mass dependent integrals. Relatively flat corrections of roughly 5% over the whole $p_T$ spectrum are found, see fig. 7.

Both the high energy approximated two-loop amplitude and the numerical evaluation of the full two-loop amplitude lead to compatible results that remove the previously unconstrained top-quark mass uncertainty at NLO at large transverse momenta.
Figure 6: Normalized NLO Higgs transverse momentum distribution with top-mass dependence in various approximations. The normalization (EFT rescaled) denotes the approximation where only the finite part of two-loop virtual correction in the EFT approximation is point wise rescaled by the full top-mass dependent one-loop amplitude; real emission and born part are exact in $m_t$. Using the high energy approximation for the missing two-loop virtual corrections in the EFT approximation is point wise rescaled by the full top-mass dependent one-loop amplitude; real emission and born part are exact in $m_t$. Using the high energy approximation for the missing two-loop virtual amplitudes results in the red line, while the other lines denote low energy approximations. For details see ref. [36], where this plot is taken from.

high energy approximate results published in ref. [36] are publicly available through the code MCFM-8.2 and are combined with an implementation of the analytical top-quark mass exact real emission amplitudes [39].

As of now no analytical expressions of the two-loop amplitudes necessary for the exact top-mass dependent virtual corrections of Higgs+jet at NLO are available. Analytical results for the necessary two-loop planar integrals in the euclidean region have been published a while ago [40]. Solutions for the non-planar integrals are not yet available but promised to be ready soon*. On the forefront of efficient analytical one loop amplitudes progress has been made for Higgs + $n$-gluons. For $n \leq 5$ very compact expressions have been derived in ref. [41] when all gluons have the same helicity. Extending this to the more complicated helicity combinations would offer the prospect of a very fast and stable numerical evaluation. This is highly important, as the high dimensional real emission integration usually takes most of the time to calculate cross sections.

*See talk by Hjalte Frellesvig at Loops & Legs 2018.
Figure 7: NLO Higgs transverse momentum distribution with exact top-mass dependence in comparison to an approximation that uses rescaled virtual corrections (FT$_{\text{approx}}$). For details see ref. [38], where this plot is taken from.

Top-mass effects in Higgs+jet production have traditionally been taken into account at low $p_T$ through a $1/m_t^n$ expansion at NLO, valid at energies $p_T \lesssim 2m_T$, see for example ref. [39]. The computation of analytic two-loop amplitudes including dimension 7 Higgs-gluon operators allows to include such effects at NNLO [42–44].

Finally, advancements are not just made in calculating cross sections, but also to analyze the data. Regarding this, modern machine learning and jet substructure techniques are being developed [45] that allow to promote the search [11] for the gluon fusion $H \rightarrow b\bar{b}$ channel from just a search into the possibility of an observation.

For further details and resources I would like to refer to the references within the cited studies, as well as to a recent Standard Model working group report [17] and a recent review article on Higgs physics [18].

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