Radio-frequency Bloch-transistor Electrometer

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A quantum-limited electrometer based on charge modulation of the Josephson supercurrent in the Bloch transistor inserted into a superconducting ring is proposed. As this ring is inductive coupled to a high-Q resonance tank circuit, the variations of the charge on the transistor island (input signal) are converted into variations of amplitude and phase of radio-frequency oscillations in the tank. These variations are amplified and then detected. The output noise, the back-action fluctuations and their cross-correlation are computed. It is shown that our device enables measurements of the charge with a sensitivity which is determined by the energy resolution of its amplifier, that can be reduced down to the standard quantum limit of $\frac{1}{4}h$. On the basis of this setup a "back-action-evading" scheme of the charge measurements is proposed.

INTRODUCTION

The single electron transistor (SET) whose operation is based on correlated electron tunneling in small-capacitance double junctions has significantly extended the possibilities of modern experiments. This remarkable device with sub-electron sensitivity to the charge induced on its central electrode (island) has made it possible to study the electron transport and noise processes in various mesoscopic structures (see, for example, the review by Likharev [1]). In recent years, especially after the encouraging experiment by Nakamura et al. [2], the possibility of using SET electrometers for measuring the quantum state of the charge qubit (Cooper-pair box) has been extensively discussed [3]. In such measurements both the sensitivity of the detector (electrometer) to the input signals and its destructive back-action on the quantum mechanical state of the box are of prime importance. The detector’s figure of merit, which takes into account the back-action effect, is the energy resolution in the unit bandwidth $\epsilon$. According to the quantum mechanical uncertainty principle for a phase-insensitive detector, the figure $\epsilon \geq \hbar/2$. Its ultimate value of $\hbar/2$ (the so-called standard quantum limit - SQL) can be approached by a perfect (quantum-limited) device [4].

The normal-state metallic SET operating in the regime of sequential tunneling of electrons drops out of the category of perfect devices. For the usual case of a high tunneling resistance of junctions $R_t \gg R_Q$ (here $R_Q \equiv \hbar/4e^2 \approx 6.5 \text{ k}\Omega$ is the resistance quantum), the value $\epsilon \gg \hbar/2$ [5]. One can, in principle, approach SQL [6] by using SET with $R_t \sim R_Q$ and operating it in the co-tunneling regime at very low voltage bias. However, in this case the output signal of the electrometer is vanishingly small so that the regime can hardly be practical.

In contrast to the SET operating on "normal carriers", i.e. electrons, its superconducting counterpart with appreciable strength of Josephson coupling $E_J$ in the junctions, i.e. the Bloch transistor [7], can operate in the regime of a gate-controlled supercurrent at zero quasiparticle current. In this regime the charge carriers are the Cooper pairs with charge $2e$, and their transfer across the junctions occurs without power dissipation in the transistor. Reading-out of the critical current value can be performed by measuring the voltage across the resistor with $R_s \ll R_Q$, shunting the transistor [8]. Although this electrometer is a quantum-limited device, its implementation still suffers from low conversion factor [9].

In this paper we propose an electrometer with the Bloch transistor inserted into a superconducting ring which is inductively coupled to the radio-frequency-driven resonance tank circuit. In contrast to the so-called rf-SET electrometer [10] based on a charge-dependent dissipation in a resonance circuit containing a normal SET, the Bloch transistor controls the ac supercurrent in the loop and, hence, the effective reactance of the tank circuit. As a result, both the amplitude and phase of oscillations in the tank circuit depend on the island charge. This mode of electrometer operation is similar to that of a single-junction superconducting quantum interferometer device (SQUID) with small critical current (non-l hysteretic regime) [12]. Here we compute the characteristics of our electrometer and demonstrate its potential for qubit measurements.
MODEL

The equivalent electric diagram of the electrometer comprising the sources of fluctuations is presented in Fig. 1. The characteristic Josephson coupling energies in the first and second junctions of the transistor are assumed to be not very different, \( E_{J1} \approx E_{J2} \): so we will characterize both of them by the parameter \( E_J = \Phi_0 I_{d0}/2\pi \), where \( \Phi_0 = h/2e \approx 2.07 \times 10^{-15} \text{ Wb} \) is the flux quantum and \( I_{d0} \) is the nominal critical current of individual junctions.

The charge-sensitive element of the device is the transistor island. We assume that the total capacitance of the island \( C = C_1 + C_2 + C_g \) (here \( C_{1,2} \) and \( C_g \) are the capacitances of the tunnel junctions and the coupling capacitance to the gate, respectively) is sufficiently small. The corresponding charging energy \( E_c = \pi^2/2C \), the Josephson energy \( E_J \) and the energy gap \( \Delta \) of the superconducting material the transistor made of should obey the condition:

\[
\Delta > E_c \sim E_J \gg k_B T,
\]

where \( T \) is the temperature. The leftmost inequality ensures blockade of quasiparticle tunneling across the junctions owing to the even-odd parity effect on the superconducting island \([5, 6]\).

The relation chosen, \( E_J/E_c \equiv \lambda \sim 1 \), first ensures substantial modulation of the supercurrent in the whole range of variation of the polarization charge on the island, \(-e \le q \le e [3, 13]\). Secondly, the width of the forbidden band in the energy spectrum (\( \approx E_J/\lambda \) \([13]\)) is large enough to prevent thermal excitation of higher Bloch bands leading to a reduction of the resultant critical current and of the depth of its modulation by the gate. For \( \lambda \approx 1 \) the critical current of the transistor \( I_c(q) = \alpha(q) I_{d0} \), with the value \( \alpha \) varied in the range from 0.24 \((q = 0)\) to 0.56 \((q = \pm e) [7]\). The supercurrent-charge-phase relation is then approximated by the formula \( I_c = I_c(q) \sin \varphi [4] \).

Due to finite Josephson coupling the effective capacitance of the electrometer island becomes non-linear \([5, 13]\). For \(-\frac{1}{e} \le q \le \frac{1}{e} \) its value is \( C'(q) = \beta(q) C \), where the factor \( \beta(q) \gtrsim 1 \) can be assumed to be constant for moderate \( \lambda \lesssim 1 \).

The inductance \( L \) of the superconducting ring incorporating the Bloch transistor should obey two conditions:

\[
\ell = 2\pi L I_c(q)/\Phi_0 < 1 \quad \text{and} \quad \Phi_0^2/2L \gg k_B T.
\]

The first relation ensures the single-valued dependence of the total flux \( \Phi = \Phi_c - LI_c(q) \sin(2\pi \Phi/\Phi_0) \) threading the loop on the external flux \( \Phi_e = \Phi_{dc} + \Phi_{et} \) applied to the loop (see, e.g., Ref. [11]). The constant flux \( \Phi_{dc} \) can be induced by dc current through an auxiliary coil (not shown in Fig. 1), while flux \( \Phi_{et} \) is induced by the tank circuit. For sufficiently small values of \( \ell \), the flux \( \Phi \approx \Phi_e \) and the Josephson phase \( \varphi = 2\pi \Phi/\Phi_0 \approx 2\pi \Phi_{et}/\Phi_0 \). The second relation in Eq. (2) ensures an exponential smallness of thermodynamic fluctuations of flux \( \Phi \). Thus, the Josephson phase \( \varphi \) across the transistor behaves almost as a classical variable whose value and (small) fluctuations are determined by current in the tank circuit.

The eigenfrequency of the tank circuit \( \omega_0 = (L TC_T)^{-\frac{1}{2}} \) and the frequency \( \omega \approx \omega_0 \) of the rf drive \( V_{et} = V_0 \cos \omega t \) should be sufficiently low, i.e. \( \omega \ll \omega_0 \equiv E_J/h \sim E_c/h \), and, therefore, not excite the Bloch transistor by means of an alternative Josephson phase \( \varphi(t) \). In our model, \( \varphi \) is considered a slowly-varied parameter in the Hamiltonian \([3]\) of the transistor system. The quality factor is

\[
Q = \omega L/R_2 = (\omega C_T R_2)^{-1} \gg 1,
\]

where \( R_2 = R_T + R_A \) is the total series resistance of the tank circuit. The dimensionless coefficient is \( \kappa = M/(LL_T)^{\frac{3}{2}} \ll 1 \), where \( M \) is the mutual inductance, so that the product

\[
k^2 Q \ell > 1.
\]

A similar regime of operation of single-junction (rf) SQUIDs, proposed by Danilov and Likharev \([17]\), offers a significant experimental advantage in the sense of a large transfer coefficient \([13]\).

The amplifier is characterized by the law \( I_R = \frac{\varphi}{C} \) and uncorrelated sources of voltage \( v_A \) and current \( i_A \) fluctuations \([4]\). These two sources and the source associated with losses in the tank circuit, \( v_T \), have spectral densities

\[
S_{v,1}(\omega) = \frac{2}{\pi} \theta_A R_A^{\pm 1} \quad \text{and} \quad S_T(\omega) = \frac{2}{\pi} \theta_T R_T,
\]

respectively, \( \theta_A = (\hbar \omega_0^2) \coth(\hbar \omega_0/(2k_B T_A)) \) with \( T_A \) \((T_T)\) symbolizing the noise temperature of the amplifier \((\text{temperature of the tank circuit})\) in the classical limit \( k_B T_A, T_T \gg \hbar \omega_0 \). Only these three sources of fluctuations are considered in our model \([13]\).

DYNAMICS

As long as dynamic equations describing the rf-SQUID were solved elsewhere by the method of harmonic balance \([4, 17, 20]\), we skip mathematical details and focus chiefly on the results.

Due to large \( Q \) and weak coupling \( \kappa \), the steady oscillations of the tank circuit current, \( I_T = I_a \cos(\omega t + \vartheta) \), and the Josephson phase,

\[
\varphi = a \cos(\omega t + \vartheta) + \varphi_0,
\]

are quasi-harmonic with slowly varying parameters \( a = 2\pi M I_a/\Phi_0 \) and \( \vartheta \) and constant phase \( \varphi_0 = 2\pi \Phi_{dc}/\Phi_0 \). The dependence of the dimensionless amplitude \( a \) on detuning \( \delta_0 = (\omega - \omega_0)/\omega_0 \) is shown in Fig. 2. At a sufficiently large amplitude \( V_0 \) of the driving voltage this dependence is multi-valued. This property allows high values (theoretically infinite) of the conversion coefficients
FIG. 2: Resonance curves of the tank circuit for different values of the drive amplitude $V_o$ and the value of product $\kappa^2 Q \ell$. The dotted lines correspond to a 10% increase in critical current $I_c(q)$ of the transistor.

"charge-to-amplitude" and "charge-to-phase" to be realized. Because of the shift of the resonance frequency in the tank circuit coupled to the rf loop, the effective detuning is $\xi = \xi_0 - \kappa^2 \ell(q) \cos \varphi_0 J_1(a)/a$. Here, $J_1$ is the Bessel function of the first order. These peculiar curves are typical of the rf-SQUID (see, e.g., Ref. [16]).

The coefficients $\eta_a = \frac{|\partial \xi|}{\partial \varphi}$ and $\eta_0 = I_0 \frac{\partial \varphi}{\partial \delta \varphi}$, governing the transformation of small charge variations $\delta q$ into variations of two orthogonal components of ac current in the tank, $\delta I_a$ and $I_a \delta \varphi$, are expressed as $\eta_a = |\xi| \eta_0$ and $\eta_0 = (2Q)^{-1} \eta_0$, respectively. Here, the factor

$$\eta_0 = \frac{M}{L_T} \left| \frac{\cos \varphi_0 J_1(a)}{(2Q)^{-2} + \xi^2} \right|,$$

the transfer function $\mu = \left| \frac{\partial \varphi}{\partial \delta q} \right|$ and dynamic detuning $\xi = \xi_0 - \kappa^2 \ell(q) \cos \varphi_0 J_1(a)$. Equation (6) in particular shows that zero magnetic flux $\Phi_{dc}$ giving $\varphi_0 = 0$ and, hence, $\cos \varphi_0 = 1$, ensures a maximum of either $\eta_a$ and $\eta_\varphi$. This is in contrast to the rf-SQUID operating ultimately at $|\sin \varphi_0| = 1$, i.e. at nonzero dc flux $\varphi_0$. The optimum amplitude of the rf-drive should give the value of $a \approx 1.8$ corresponding to the maximum value of $(J_1)_{\text{max}} = j_1 \approx 0.58$.

### NOISE FIGURES

For the amplitude (phase) detection of a low-frequency signal ($\omega_s \ll \omega$), the output resolution in bandwidth $\Delta f$, $\delta q_x = \langle \hat{q}_x^2 \rangle^{1/2} = \eta_{a, \varphi}^{-1} (2\pi S_{a, \varphi} \Delta f)^{1/2}$, is determined by the spectral density of the in-phase (out-of-phase) fluctuations of the current flowing through the amplifier,

$$S_{a, \varphi} = \frac{g_{a, \varphi}(\xi, \dot{\xi})}{R_{\Sigma}} [S_T(\omega) + S_V(\omega)] + S_I(\omega),$$

where $g_a(\xi, \dot{\xi}) = Q^{-2} (Q^{-2} + 4\xi^2) (Q^{-2} + 4\dot{\xi}^2)^{-2} = g_\varphi(\xi, \dot{\xi})$. The output noise in the energy representation $\epsilon_I = \langle \hat{q}_x^2 \rangle/(2C'\Delta f)$ finally takes the form

$$\epsilon_I^{(a, \varphi)} = \frac{b d_a}{\kappa^2 Q \ell \omega} \left( \frac{R_T}{R_{\Sigma}} \Theta_T + \frac{R_A}{R_{\Sigma}} \Theta_A + \frac{R_S}{g_{a, \varphi} R_A} \Theta_A \right).$$

Here, the numerical factor $b = \pi I_c/(\mu_0^2 C'\kappa^2)$, while

$$d_a = \frac{Q^{-2} + 4\xi^2}{4\xi^2}$$

for the case of amplitude and phase detection, respectively. Note that, owing to the large value of product $\kappa^2 Q \ell$ (Eq. (6)), the output noise figures $\epsilon_I^{(a, \varphi)}$ (they do not include the back-action effect!) can be made smaller than $h/2$ in the limit $T_T, T_A \ll k_B T_A$.

The electrometer back-action on the source of the input charge is determined by low-frequency ($\sim \omega_s$) fluctuations of the electric potential of the transistor island $\hat{U} = \frac{2}{\pi} \mu \sin \varphi \hat{\varphi}$. Here $\hat{\varphi}$ are fluctuations of the Josephson phase Eq. (3) and $\hat{\varphi}$ denotes averaging over time $\tau$: $2\pi/\omega_s \ll \tau \ll 2\pi/\omega_s$. Finally, the input noise figure $\epsilon_U = C'/(2\Delta f)$ for either regime is given by

$$\epsilon_U^{(a, \varphi)} = \epsilon_U^{(a)} = \frac{g_a \kappa^2 Q \ell \omega}{b \omega} \left( \frac{R_T}{R_{\Sigma}} \Theta_T + \frac{R_A}{R_{\Sigma}} \Theta_A \right).$$

From Eq. (5) it follows that fluctuations $\hat{U}$ are proportional to fluctuations of amplitude $\bar{a}$; therefore, these two signals are completely correlated. Due to this fact the cross-correlation $\epsilon_U = \langle \hat{q}_U \hat{U} \rangle/2\Delta f$ in the regime of amplitude detection has the largest magnitude which is equal to the geometric mean of $\epsilon_I^{(a)}$ Eq. (1) and $\epsilon_I^{(a)}$ with the third term omitted in Eq. (3) (1). Then the energy resolution of a narrow-band signal $\epsilon_U$ [2, 20, 21]

$$\epsilon = \left[ \epsilon_U^{(a)} - \epsilon_U^{(a)} \right]^{1/2}$$

is equal to

$$\epsilon = \omega^{-1} \left[ d_s \Theta_A [(R_T/R_A) \Theta_T + \Theta_A] \right]^{1/2},$$

This equation shows that the electrometer figure of merit $\epsilon$ depends crucially on the amplifier characteristic $\Theta_A$. In particular, for $R_T \Theta_T \ll R_A \Theta_A$ and detuning $|\xi| \gg (2Q)^{-1}$, the figure $\epsilon = \Theta_A/\omega$, and its value approaches the SQL of $h/2$ at $k_B T_A < h \omega$.

### DISCUSSION

We arrive at the remarkable property of the rf-Bloch-electrometer: it converts an input charge into an output
signal introducing only insignificant noise on the stage preceding the amplifier. This is because the device operates as a parametric converter \( \omega_s \to (\omega \pm \omega_s) \to \omega_s \) (similar to the single-junction SQUID, see, e.g., Ref. [14]). In such a scheme of electrometer (in contrast to other SET electrometers) the amplifier can be optimized as a separate block. In the frequency range of 100 – 500 MHz, the state-of-the-art narrow-band dc-SQUID-based amplifiers make it possible to almost approach the SQL [22].

The set of experimental parameters for the Al transistor can be chosen as follows: \( E_J \sim E_c \sim 200 \text{ } \mu \text{eV} \) (corresponds to \( C \sim C'/2 \sim 0.2 \text{ } \text{fF}, I_c \sim 30 \text{ } \text{nA} \) and \( \omega_J/2\pi \sim 50 \text{ GHz} \gg \omega/2\pi \sim 300 \text{ MHz} \), \( L \sim 10 \text{ nH} \) (gives \( \ell \sim 0.3 \) and \( \Phi_0^2/2k_B L \sim 10 \text{ K} \gg T \sim 20 \text{ mK} \), \( Q \sim 300 \) (bandwidth \( \sim \omega/Q \sim 1 \text{ MHz} \) and \( \kappa^2 \sim 0.3 \). These parameters yield the value \( \kappa^2 Q \ell \sim 30 \) that ensures a large transfer coefficient. For the quantum-limited amplifier the charge resolution is expected to be equal to \( (C'h)^{\frac{1}{2}} \approx 2 \times 10^{-7} \text{ } e/\text{Hz}^{\frac{1}{2}} \).

Another important conclusion can be drawn regarding a possible "back-action-evading" measurement by the rf-Bloch-electrometer. Such measurement assumes that one quadrature component of internal noise is "squeezed" to less than SQL [24]. One of the ways to do so is to apply to the tank two driving signals with frequencies \( \omega_1 \) and \( \omega_2 \) which obey the relation \( \omega_s = \omega_1 - \omega_2 \) (see similar proposal for rf-SQUID in Ref. [21]). In this "degenerate" mode of operation [24] the device is sensitive to a quadrature component, say \( \hat{X}_1 \), of the input ac signal \( q = (\hat{X}_1 + i\hat{X}_2)e^{i\omega_0t} \) whose Heisenberg uncertainties obey the relation \( \delta \hat{X}_1 \times \delta \hat{X}_2 \geq C'h \). As a result, one side \((\delta \hat{X}_1)\) of the "error box" is squeezed while another \((\delta \hat{X}_2)\) is increased, with their product kept constant.

Finally, there is yet another advantage of the rf-Bloch-electrometer for qubit measurements: Its transducer (the ring with transistor) is generically superconducting, the tank circuit should preferably also be made of superconducting material. This device when positioned near qubit is, therefore, free from the normal-electron excitations which may significantly shorten a decoherence time of qubit.

The experimental work on the radio-frequency Bloch-transistor electrometer has been started at PTB.

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[1] K. K. Likharev, IEEE Proc. 87, 606 (1999).
[2] Y. Nakamura, Yu. A. Pashkin and J. S. Tsai, Nature 398, 786 (1999).
[3] A. Shnirman and G. Schön, Phys. Rev. B 57, 15400 (1998); Yu. Makhlin, G. Schön, and A. Shnirman, Nature 398, 305 (1999); Phys. Rev. Lett. 85, 4578 (2000).
[4] J. Weber, Rev. Mod. Phys. 31, 681 (1959); H. Heffner, Proc. IRE 50, 1604 (1962).
[5] A. N. Korotkov, D. V. Averin, K. K. Likharev, and S. A. Vassenko, in Single Electron Tunneling and Mesoscopic Devices, edited by H. Koch and H. Lübbig (Springer-Verlag, Berlin, 1992), p.45.
[6] A. Maassen van den Brink, Preprint, available at http://arXiv:cond-mat/0009163.
[7] D. V. Averin, Preprint, available at http://arXiv:cond-mat/0010052.
[8] K. K. Likharev and A. B. Zorin, Jpn. J. Appl. Phys. 26, Suppl. 3, 1407 (1987); D. V. Averin and K. K. Likharev, in Mesoscopic Phenomena in Solids, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991).
[9] A. B. Zorin, Phys. Rev. Lett. 76, 4408 (1996).
[10] S. V. Lotkhov, H. Zangerle, A. B. Zorin, Th. Weimann, H. Scherer, and J. Niemeyer, IEEE Trans. Appl. Supercond. 9, 3664 (1999).
[11] R. J. Schoelkopf, P. Wahlgren, A. A. Kozhevnikov, P. Delsing, and D. Prober, Science 280, 1238 (1998).
[12] P. K. Hansma, J. Appl. Phys. 44, 4191 (1973).
[13] D. V. Averin and Yu. V. Nazarov, Phys. Rev. Lett. 69, 1993 (1992); M. T. Tuominen, J. M. Hergenrother, T. S. Tighe, and M. Tinkham, ibid. p.1997.
[14] A. B. Zorin, IEEE Trans. Instrum. and Meas. 46, 299 (1997).
[15] K. K. Likharev and A. B. Zorin, J. Low Temp. Phys. 59, 347 (1985).
[16] K. K. Likharev, Dynamics of Josephson Junctions and Circuits (Gordon and Breach, New York, 1986), Ch. 14.
[17] V. V. Danilov and K. K. Likharev, Zh. Tekhn. Fiz. 45, 1110 (1975) [Sov. Phys. - Techn. Phys. 20, 697 (1976)].
[18] V. I. Shnyr'kov, V. A. Khlus, and G. M. Tsoi, J. Low Temp. Phys. 39, 477 (1980).
[19] Since the quasiparticle tunneling in the transistor is blocked by the parity effect [13], the only possible origin of its intrinsic noise could be the active \( \alpha \) component of the pair current \( \propto \cos \phi \). For our case of \( \hbar \omega, k_B T \ll \Delta \) this component has negligibly small effect, see A. B. Zorin, Radioteknika i Elektronika 29, 2224 (1984) [Sov. J. Commun. Technol. and Electron. 30, 135 (1985)]. The \( 1/f \) background charge noise is in our model also omitted.
[20] V. V. Danilov, K. K. Likharev and O. V. Snigirev, in SQUID’80, edited by H.-D. Hahlohn and H. Lübbig (W. de Gruyter, Berlin, 1980) p. 473.
[21] V. V. Danilov, K. K. Likharev and A. B. Zorin, IEEE Trans. Magn. 19, 572 (1983).
[22] M. Mück, M.-O. André, J. Clarke, J. Gail, and C. Heiden, Appl. Phys. Lett. 72, 2885 (1998);
M.-O. André, M. Mück, J. Clarke, J. Gail, and C. Heiden, Appl. Phys. Lett. 75, 698 (1999).
[23] H. Takahashi, Advances in Communication Systems,
edited by A. V. Balakrishan (Academic, New York, 1965), p. 227.

[24] V. I. Panov and F. Ya. Khalili, in *Proceedings of the 9-th International Conference on General Relativity* (Berlin, 1980), vol. 2, p. 397.