Speed Approach for UAV Collision Avoidance

V D Berdonosov, A A Zivotova, Zaw Htet Naing, D O Zhuravlev

Komsomolsk-na-Amure State University (Russia); 681013, 27, Lenina Ave., Komsomolsk-on-Amur, 681013, Russia

E-mail: berd1946@gmail.com

Abstract. The article represents a new approach of defining potential collision of two or more UAVs in a common aviation area. UAVs trajectories are approximated by two or three trajectories' points obtained from the ADS-B system. In the process of defining meeting points of trajectories, two cutoff values of the critical speed range, at which a UAVs collision is possible, are calculated. As calculation expressions for meeting points and cutoff values of the critical speed are represented in the analytical form, even if an on-board computer system has limited computational capacity, the time for calculation will be far less than the time of receiving data from ADS-B. For this reason, calculations can be updated at each cycle of new data receiving, and the trajectory approximation can be bounded by straight lines. Such approach allows developing the compact algorithm of collision avoidance, even for a significant amount of UAVs (more than several dozens). To proof the research adequacy, modeling was performed using a software system developed specifically for this purpose.

1. Introduction

In the last ten years, researches connected with identification and prevention of civil unmanned aerial vehicles (UAV) collisions in a common aviation area have been actively conducted [1, 2, 3, 4]. It is natural that air traffic control systems face the similar task, but the fundamental difference is UAV complete autonomy during the accomplishment of assigned missions. To prevent a collision an intelligent calculation system shall be used. It must be able to solve the following tasks [5, 6]:

- acquisition of potentially dangerous dynamically moving objects;
- estimation of collision probability, considering the features and behavior of these objects;
- implementation of a collision avoidance algorithm.

To solve the first two tasks, information about trajectory is required, as for “friend”, as for “foe” UAVs. For these purposes some of researchers [7, 8, 9] suggest the usage of automatic dependent surveillance system (ADS-B), which can be currently installed not only in manned aircrafts, but also in UAVs. Thus, let’s point several preliminary notes.

The first one. UAV is moving in space and three coordinates (x, y, z) are required for describing its trajectory. However, calculations in space are significantly more complex than calculations on plane. For this reason, calculation of collision points is divided into two stages. At the first stage, a crossing point on plane XOY is to be defined; at the second stage, UAV attitude in the crossing point is to be checked along a Z-direction.

The second one. Each UAV can use information from ADS-B; a “friend” has complete sampled time information about coordinates of a “foe” UAV. It is suggested that its own coordinates can be accessed through the GPS/GLONASS system.
2. Calculation of trajectories and potential collision points for the multiplicity of UAVs

Let us suppose that all UAVs move along straight lines, that is completely adequate. Thus, receiving the latest coordinates, one can recalculate main parameters related to collision points. As calculation expressions are quite simple, a recalculation will be finished before the next coordinates receiving. Considering such suggestion, it is enough to have coordinates of only two points for each UAV [10, 11]: one point from the previous cycle and a new one. Relevant coordinates are stored in two arrays \( M_1 \) and \( M_2 \), where \( M_1 \) is the array of previously received coordinates and \( M_2 \) is the array of the latest coordinates. Arrays \((M_{0j})\) size is 3 x N, where 3 is defined by the number of coordinates \((x, y, z)\) and \( N \) is the total number of UAVs. The second array index \( j=0...N \), at that \( M_{0j} \) elements store coordinates of a “friend”.

At first, let us define coordinates of trajectories’ crossing for \( N \) UAVs, where crossing means the existence of a common point for two lines. Using trivial manipulations with coordinates of \( N \) lines, let us get an expression to define crossing coordinates:

\[
\begin{align*}
 x_j &= \frac{- (C_j \cdot B_j - C_j \cdot B_0)}{\sqrt{A_0 \cdot B_j - A_j \cdot B_0}} \\
 y_j &= \frac{- (A_0 \cdot C_j - A_j \cdot C_0)}{\sqrt{A_0 \cdot B_j - A_j \cdot B_0}}
\end{align*}
\]

(1)

where \( A_j = M_{2j} - M_{1j}; B_j = M_{20j} - M_{10j}; C_j = B_j \cdot M_{1j} - A_j \cdot M_{0j} \).

It should be noticed, that the crossing point will be the point of a potential collision only if considered UAVs fly together, and it will not be, if considered UAVs fly away from each other.

The condition of flying together is as follows:

\[
\begin{align*}
 |x_j - M_{10j}| &> |x_j - M_{20j}| \Rightarrow |y_j - M_{10j}| > |y_j - M_{20j}| \quad &\text{(1)}
\end{align*}
\]

Having coordinates of a potential collision point, one can easily find the value of \( z \)-coordinate in the point of a potential collision for all UAV pairs, where a pair means a “friend” and one of \( N \) “foes”:

\[
\begin{align*}
 z_{1j} &= \frac{[M_{220} - M_{120}] \cdot x_j + (M_{220} - M_{120}) \cdot M_{120} - (M_{220} - M_{120}) \cdot M_{100}]}{\sqrt{(M_{200} - M_{100})}} \\
 z_{2j} &= \frac{[M_{22j} - M_{12j}] \cdot x_j + (M_{22j} - M_{12j}) \cdot M_{12j} - (M_{22j} - M_{12j}) \cdot M_{10j}]}{\sqrt{(M_{20j} - M_{10j})}}
\end{align*}
\]

(2)

It is obvious, that if \( |z_{1j} - z_{2j}| \leq \Delta \) then a collision will occur, where \( \Delta \) is defined by UAV height.

For the calculation of critical speed it is required to get information about the distance of each UAV to the point of a potential collision and calculate

\[
\cos(\alpha_j) = \frac{|A_j \cdot A_j + B_j \cdot B_j|}{\sqrt{A_j^2 + B_j^2}}
\]

(3)

Now, there is enough information to start the calculation of the critical speed range.

3. Calculation of critical speeds for the multiplicity of UAVs

Thus, let’s suppose that the previous calculations show that there are points of potential collisions between two or more UAVs. But a collision will occur only if the relative speeds of these UAVs are in the critical speed range.

**Figure 1.** Variants of relative motions of a “friend” UAV (green arrow) and a “foe” UAV (red arrow): a – same-direction; b – same-direction, strictly; c – opposite-direction; d – opposite-direction, strictly; e – collision model; P – trajectories’ meeting point, \( C_f \) and \( C_s \) – centers of UAVs (“friend”, “foe”), \( K \) – UAV touch point (collision point), \( \alpha \) – angle between UAVs trajectories.
To estimate critical speeds which lead to a collision of two or more UAVs, one shall consider different collision conditions (see figure 1).

It should be pointed out that figure 1 shows projections of “friend” and “foe” motion on X0Y plane; calculations related to Z-coordinate can be performed in the same way, but will not be considered in the present article.

Thus, let’s describe the plane model. There are two UAVs: “friend” and “foe” - each of them is characterized by a coverage radius (m), \( r_f, r_s \); UAV speed (m/s), \( V_f, V_s \); the angle of convergence (deg.), \( \alpha \); the distance between a UAV center and trajectories’ meeting point (m), \( L_f, L_s \). Coverage area - region of space, which is occupied by a UAV, is considered as a circle (on plane) and as an ellipsoid (in space).

The collision model (figure 1e) allows formulating collision conditions (speed ratio, ratio of initial (current) coordinates). UAVs will collide when the distance between their centers equals to the sum of coverage radii. Let’s introduce two functions \( L_f(t) \) and \( L_s(t) \), which are the distances between centers of respective UAVs (\( C_f \) and \( C_s \)) and trajectories’ meeting point (P): \( L_f(t) = L_f(t_0) - V_f \cdot t \) and \( L_s(t) = L_s(t_0) - V_s \cdot t \). Thus, the collision condition is as follows:

\[
[(L_f(t_0))^2 + (L_s(t_0))^2 - 2 \cdot L_f(t_0) \cdot L_s(t_0) \cdot \cos(\alpha)]^{1/2} \leq r_f + r_s \tag{4}
\]

where \( t_k \) - collision timepoint.

Plugging expressions for \( L_f(t) \) and \( L_s(t) \) in (4), one gets:

\[
[(L_f(t_0) - V_f \cdot t)^2 + (L_s(t_0) - V_s \cdot t)^2 - 2 \cdot (L_f(t_0) - V_f \cdot t) \cdot (L_s(t_0) - V_s \cdot t) \cdot \cos(\alpha)]^{1/2} \leq r_f + r_s \tag{5}
\]

In expression (5), variables \( V_f, V_s \) are of outstanding interest as the occurrence of a collision to the fullest extent depends on their ratio. In turn, critical values of these variables depend on time of “foe” UAV acquisition, i.e. on \( L_f(t_h) \) and \( L_s(t_h) \) distances between UAVs centers and meeting (crossing) point at the timepoint when the “friend” discovers the “foe”.

The common case is a random angle of convergence (\( \alpha \)). Let us introduce a function defining change of the distance between UAVs centers in relation to time:

\[
R(t) = [(L_f(t_0) - V_f \cdot t)^2 + (L_s(t_0) - V_s \cdot t)^2 - 2 \cdot (L_f(t_0) - V_f \cdot t) \cdot (L_s(t_0) - V_s \cdot t) \cdot \cos(\alpha)]^{1/2} \tag{6}
\]

The collision will occur if the minimal value of this function is less (or equal) than the sum of coverage radii. Using the standard method of minimum definition through the setting of the derived function equal to zero, one gets an expression for \( t_{\text{min}} \):

\[
t_{\text{min}} = \frac{L_f(t_0) \cdot V_f + L_s(t_0) \cdot V_s - (L_f(t_0) \cdot V_s + L_s(t_0) \cdot V_f) \cdot \cos(\alpha)}{V_f^2 + V_s^2 - 2 \cdot V_f \cdot V_s \cdot \cos(\alpha)} \tag{7}
\]

Plugging (7) in (6) and considering the conditions defined above, let us get the dependence of the collision condition from four variables: \( L_f(t_h), V_f, L_s(t_h), V_s \):

\[
R(t_{\text{min}}) = [(L_f(t_h) - V_f \cdot t)^2 + (L_s(t_h) - V_s \cdot t)^2 - 2 \cdot (L_f(t_h) - V_f \cdot t) \cdot (L_s(t_h) - V_s \cdot t) \cdot \cos(\alpha)]^{1/2} + \\
+ (L_f(t_h) - V_f \cdot t) \cdot \frac{L_f(t_h) \cdot V_f + L_s(t_h) \cdot V_s - (L_f(t_h) \cdot V_s + L_s(t_h) \cdot V_f) \cdot \cos(\alpha)}{V_f^2 + V_s^2 - 2 \cdot V_f \cdot V_s \cdot \cos(\alpha)} - \\
- 2 \cdot (L_f(t_h) - V_f \cdot t) \cdot \frac{L_f(t_h) \cdot V_f + L_s(t_h) \cdot V_s - (L_f(t_h) \cdot V_s + L_s(t_h) \cdot V_f) \cdot \cos(\alpha)}{V_f^2 + V_s^2 - 2 \cdot V_f \cdot V_s \cdot \cos(\alpha)} \cdot \cos(\alpha)]^{1/2} \leq r_f + r_s \tag{8}
\]

Performing some modifications and introducing relative variables: \( L_{tf} = L_f(t_h)/(r_f + r_s) \), \( L_{ts} = L_s(t_h)/(r_f + r_s) \) and \( V_{tf} = V_f/V_s \) let us solve inequation for \( V_{tf} \). Thus, one gets expressions for
\[ V_f^{\text{min}} \text{ and } V_f^{\text{max}}, \]
\[ V_f^{\text{min}}(L_{f/z}, L_{s/z}, \alpha) = \frac{-BV(L_{f/z}, L_{s/z}, \alpha) + \sqrt{BV(L_{f/z}, L_{s/z}, \alpha)^2 - 4 \cdot AV(L_{s/z}, \alpha) \cdot CV(L_{f/z}, L_{s/z}, \alpha)}}{2 \cdot AV(L_{s/z}, \alpha)} \]  
\[ V_f^{\text{max}}(L_{f/z}, L_{s/z}, \alpha) = \frac{-BV(L_{f/z}, L_{s/z}, \alpha) - \sqrt{BV(L_{f/z}, L_{s/z}, \alpha)^2 - 4 \cdot AV(L_{s/z}, \alpha) \cdot CV(L_{f/z}, L_{s/z}, \alpha)}}{2 \cdot AV(L_{s/z}, \alpha)} \]

where \( V_f^{\text{min}} \) – minimal relative speed of a “friend” UAV, at which a collision will not occur, because the “friend” is able to fly ahead of a “foe” in time;

\( V_f^{\text{max}} \) – maximal relative speed of a “friend” UAV, at which a collision will not occur, because a “foe” is able to fly ahead of the “friend” in time:

\[ AV(L_{s/z}, \alpha) = L_{s/z}^2 \cdot [1 - \cos(\alpha)^2] - 1; \]
\[ BV(L_{f/z}, L_{s/z}, \alpha) = 2 \cdot [\cos(\alpha) - L_{f/z} \cdot L_{s/z} \cdot [1 - \cos(\alpha)^2]]; \]
\[ CV(L_{f/z}, L_{s/z}, \alpha) = L_{f/z}^2 \cdot [1 - \cos(\alpha)^2]. \]

Obtained solutions in the analytical form allow analyzing how the convergence angle and the timepoint of “foe” UAV acquisition influence values of critical speed. Let us graph corresponding dependencies.

**Figure 2.** Graphs of the dependency of critical speed values \( V_f^{\text{min}} \) and \( V_f^{\text{max}} \) on distances to collision point \( L_{f/z}, L_{s/z} \) at different values of the convergence angle: a – \( \alpha = 90^\circ \); b – \( \alpha = 168^\circ \), and from the convergence angle at \( L_{f/z} = 10 \) and \( L_{s/z} = 10 \).

Graphs (figure 2a, 2b) show that the range of critical speed is dangerously expanded with the late acquisition of a “foe”. Graphs (figure 2b, 2c) show that the range of critical speed is significantly expanded (from a unit fraction to tens) as at narrow angle (same-direction traffic), as at wide angle (opposite-direction traffic). Thus, let us conclude that the most dangerous areas of “foe” acquisition are small angles (up to 10 deg.) at both same-direction and opposite-direction traffic.

4. **Speed maneuver of collision avoidance**

Expressions (9) and (10) allow defining maneuver characteristics on the plane (angle and direction of rotation). Objectively, if the speed of a “friend” is within the range of critical speed:

\[ V_f^{\text{min}} \geq V_f \geq V_f^{\text{max}}, \]  
\[ \text{then an over-range will be possible in the case of speed increase/reduction. Definitely, if the speed is closer to the upper range limit:} \]
\[ V_f^{\text{min}} \geq V_f \geq (V_f^{\text{min}} - V_f^{\text{max}})/2 \]
then it shall be increased; and if it is closer to the lower range limit:
\[ \frac{(V_f^{\min} - V_f^{\max})}{2} \geq V_f \geq V_f^{\max}, \]  
(13)

the speed shall be reduced. Let us determine the expression for defining UAV acceleration \( A_f \), for the first and the second cases. As \( V_f^{\min} \) linearly depends on acceleration, the condition of collision avoidance can be formulated for the case represented by expression (12):
\[ V_f + A_f \cdot t_k / 2 \geq V_f^{\min} \]  
(14)

where the left part of the inequation is an average UAV speed. Time to a projected collision can be defined using values of a distance and an average speed:
\[ t_k = L_f / V_f. \]  
(15)

Plugging expression (15) in inequation (14) and solving it for \( A_f \), one gets:
\[ A_f > 2 \cdot V_f \cdot (V_f^{\min} - V_f) / L_f. \]  
(16)

Analogously, for expression (13). Solving it for \( A_f \), one gets:
\[ A_f < 2 \cdot V_f \cdot (V_f^{\max} - V_f) / L_f. \]  
(17)

It should be observed that in accordance with (17), \( A_f \) is negative as we consider the case represented by inequation (13), where \( V_f > V_f^{\max} \).

Thus, expressions (16) and (17) allow defining acceleration values for collision avoidance. It means that the UAV is required to speed up or slow down, at that the value of acceleration can be analytically calculated.

For modeling cases described above, a software system was developed. It allows visualizing trajectories of up to 50 UAVs, at that required parameters of UAVs of minimal and average classes [12] can be set. Dimensions from 0.3 to 30 meters, speed from 0.5 to 99 m/s, acceleration from 0.0 to 10 m/s\(^2\). Trajectories are visualized on the polygon with the size of 2000 x 2000 m; the visualization area can be scaled and moved.

Figure 3 represents screens of the modeling system.

![Figure 3. Screens of two UAVs trajectories modeling (a “friend” trajectory is green, a “foe” trajectory is red): a – UAVs collision; b – collision avoidance with the acceleration increase only by 0.04 m/s\(^2\).](image)

Figure 3a shows the situation of the two UAVs collision, when the relative speed equals 0.998, the range of critical speed is from 1.02 to 0.98. In other words, it is the case represented by the inequation (12). Applying the acceleration defined in (17) allows avoiding the collision, as in figure 3b. It should be noted that the required acceleration is fairly small (0.045 m/s\(^2\)), because of significant distance to the collision point when the acceleration is inputted. Modeling results prove that the research is performed correctly.
5. Conclusion
The undertaken research allowed getting analytical expressions for the calculation of potential collision points for the multiplicity of UAVs flying in various directions with different speeds. Besides, expressions for the calculation of the critical speed range are also defined. Sufficiency of both groups of expressions is proved by mathematical simulation. Analytical expressions provide a means of developing an advanced algorithm of UAV collision avoidance by horizontal maneuvers and/or speed changes. Analytic property of expressions significantly decreases on-board computational costs. Required calculations are performed cyclic in time between coordinates receipt that eliminates the disadvantages of straight-line approximation of trajectories.

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