Localized averaging kernels for probing the deep meridional flow with data from GONG, MDI and HMI

H.-P. Doerr and M. Roth
Kiepenheuer-Institut für Sonnenphysik, Schöneckstraße 6, 79104 Freiburg, Germany
E-mail: doerr@kis.uni-freiburg.de

Abstract. The Fourier-Legendre decomposition (FLD) method to study wave absorption in sunspots is a helioseismic technique which is also suited for the measurement of the meridional flow. Because the FLD method is sensible to low-degree oscillation modes, it bears the potential to be used to probe the average meridional flow in much deeper layers as it is currently possible with other methods. In order to rate the suitability of the available data for inversions of the deep flow, we compare localized averaging kernels as computed with data from the Global Oscillation Network Group (GONG), the Michelson Doppler Imager (MDI) aboard the SOHO spacecraft and also with preliminary data from the Helioseismic and Magnetic Imager (HMI) aboard the SDO spacecraft.

1. Introduction
The solar meridional circulation has a mainly poleward directed surface flow of about 15 m/s (eg. [1–3]) and plays a key role in the theoretical description of the dynamics of the differential rotation and the overall solar convection zone [4–6]. Moreover the motion is important for some type of solar dynamo models, where the revolution time-scale of the motion might define the length of the the solar cycle [7–10]. In contrast to the surface measurements, knowledge about the structure of the meridional flow in the solar interior is marginal at best. Local helioseismology techniques as ring-diagram analysis and time-distance helioseismology have investigated the flow down to a depth of 15–20 Mm below the solar surface [11–13]. In these layers the measurements show a mainly poleward, highly variable flow with amplitudes up to 40 ms\(^{-1}\).

The Fourier-Legendre decomposition (FLD) originally developed by [14] for studying wave absorption by sunspots can also find application for determining the subsurface structure of the meridional flow [15]. First comparisons of the meridional flow measurements obtained by ring-diagram analysis and the FLD [16] in the solar subsurface layers seem to be in qualitatively good agreement. In order to decide whether the method is also capable to probe layers below 20 Mm depth we study in this contribution the localization of inversion kernels as they are obtained from a SOLA (Subtractive Optimally Localized Averages) inversion method. The input data that forms the basis of our analysis were recorded by the instruments of GONG (Global Oscillation Network Group), SOHO/MDI (Michelson Doppler Imager aboard the Solar and Heliospheric Observatory; [17]) and SDO/HMI (Helioseismic and Magnetic Imager aboard the Solar Dynamics Observatory).
2. Methods

The time dependent, two-dimensional oscillation signal
\[
\delta V(\theta, \phi, t) = \sum_{l,m,\nu} \left[ A_{lm\nu} X_{lm}(\theta) + B_{lm\nu} (X_{lm})^* (\theta) \right] e^{i(m\phi + 2\pi \nu t)},
\]  
(1)
on the solar surface can be decomposed into the mode amplitudes \( A_{lm\nu} \) and \( B_{lm\nu} \) of two wavefields traveling in poleward and equatorward direction respectively. Here \( \theta \) denotes the co-latitude and \( \phi \) the longitude on the solar surface, \( \nu \) is the temporal frequency, \( l \) the harmonic degree and \( m \) the azimuthal order. The basis functions \( X_{lm} \) and its complex conjugate \( X_{lm}^* \) are superpositions of the associated Legendre functions \( P_{lm}(\cos \theta) \) and \( Q_{lm}(\cos \theta) \):
\[
X_{lm}(\theta) = N_{lm} \left[ P_{lm}(\cos \theta) - \frac{2i}{\pi} Q_{lm}(\cos \theta) \right],
\]  
(2)where \( N_{lm} \) is a normalization factor. A meridional flow will result in a slight frequency shift \( \delta \nu \) between the poleward and equatorward components. This frequency shift can be measured by fitting Lorentzian profiles to the single peaks in the power spectra obtained for each solar hemisphere according to [18]. The guess frequencies for the fits are obtained from the standard solar ‘Model S’ [19]. According to [15, 20] this frequency shift is a result of the advection due to the meridional flow
\[
\Delta \nu_{nl} = \frac{l}{\pi R_\odot} \int \bar{U}_{mer}(r) K_{nl}(r) \, dr,
\]  
(3)where \( \bar{U}_{mer}(r) \) is the averaged meridional flow over the observed region of interest as a function of the position \( r \) inside the Sun. The sensitivity kernel \( K_{nl}(r) \) is the energy density of a given mode which is also a function of the position \( r \) inside the Sun. We then employ the SOLA inversion technique [21] to construct localized average inversions kernels at a given depth \( r \).

2.1. Data

We use the GONG merged full-disk Dopplergrams from January 2006. There are no GONG Dopplergrams available for 2010 yet and the GONG duty cycle was comparably high for this month (40704 out of 44640 Dopplergrams available). This data set also was used previously for FLD-based flow measurements [16]. The GONG Dopplergrams have a resolution of 1 k×1 k and the cadence is 60 seconds.

In the case of MDI we use the level 1.8 full-disk Dopplergrams from May 2010. Up to now this is the best data set from MDI that overlaps with HMI, however it has not a particularly high duty cycling and only 31175 out of 44640 Dopplergrams are available. The MDI Dopplergrams have a resolution of 1 k×1 k and the cadence is 60 seconds also.

The final calibration of the HMI data products is not yet published so we use the preliminary data of May 2010 from the hmi_test.v_45s dataseries [22]. The HMI full-disk Dopplergrams come with a 4 k×4 k resolution at a cadence of only 45 seconds. At this point we must emphasize that the preliminary status of the HMI data series does not allow us to draw any conclusions about the quality of the final calibration.

Data is processed in chunks of one hour. To end up with only the net surface oscillation signal the mean Dopplergram of each chunk is removed from each Dopplergram in the chunk. The Dopplergrams are then interpolated to an equidistant heliographic grid and the region of interest is cut out. In order to increase the SNR for low-degree modes we use a rather large region of interest of \( w \times h = 112 \times 56 \) degrees centered on the central meridian and solar latitudes of ±38 degrees for the northern and southern hemispheres.

The Dopplergrams prepared in such a way are then fed into the Fourier-Legendre decomposition procedure as described above. The result of the Fourier-Legendre decomposition
are time series of the coefficients $A_{lm\nu}$ and $B_{lm\nu}$ from Equation (1) for both hemispheres. From these time series of the equatorward and poleward traveling waves the power spectra are computed and averaged for modes with identical harmonic degree $l$ and azimuthal order of $m = -25, \ldots, 25$.

3. Results

The theoretical positions of the power-ridges and the frequencies fitted on the three data sets are shown in Figure 1. Only fits with an error of $\leq 1.5\mu$Hz are regarded. For GONG we obtain 4067 successful frequency fits, for MDI 3746, and for HMI 4707.

![Figure 1. Mode frequencies (asterisk) obtained from one month long time series of Dopplergrams from GONG (upper left), MDI (upper right) and HMI (left). For better visibility the error bars are enlarged by a factor of 50. The dotted lines mark the positions of the $f$ and $p$-mode ridges according to Model ‘S’.](image)

For the construction of the averaging kernels we use oscillation eigenfunctions which were obtained with the ADIPAK code [23]. The target kernels have Gaussian shapes with a width that is proportional to the sound speed $c$ at the target location $r_0$ [24].

Figure 2 shows averaging kernels obtained for a target depth of $r_0 = 20$ Mm and $r_0 = 70$ Mm. The kernel units are given by the normalization

$$R_\odot \int_0^{R_\odot} K(r_0, r) \, dr = 1 \quad (4)$$

of the kernels, where $K(r_0, r)$ is the averaging kernel at target depth $r_0$ as a function of $r$.

The input parameters and results of the average kernels are shown in Table 1. For a target depth of 20 Mm the resulting average kernels for all three data sets are located at 20 Mm with a FWHM of about 20 Mm. The kernels from the different data sets are virtually identical. At
Figure 2. Averaging kernels obtained with a SOLA method for the modes displayed in Figure 1. The kernels constructed for HMI, MDI, and GONG are plotted for two target depths at \( r_0 = 20 \text{ Mm} \) and \( r_0 = 70 \text{ Mm} \) (indicated by the vertical dotted lines).

A target depth of 70 Mm the resulting average kernels for GONG and MDI are located at about 53 Mm with a FWHM of about 33 Mm while the one for HMI is a bit broader and located at a depth of 60 Mm.

Table 1. Input parameters and results for the averaging kernels. All numbers are in Mm.

| Data set | Target depth | Kernel depth | Target width | Kernel width |
|----------|--------------|--------------|--------------|--------------|
| GONG     | 20           | 19.8         | 20.9         | 19.9         |
| MDI      | 20           | 19.8         | 20.9         | 20.3         |
| HMI      | 20           | 19.8         | 20.9         | 19.9         |
| GONG     | 70           | 53.3         | 43.8         | 32.7         |
| MDI      | 70           | 53.3         | 43.8         | 33.8         |
| HMI      | 70           | 60.0         | 43.8         | 39.2         |

4. Discussion & Conclusion
We have constructed localized averaging kernels for probing the meridional circulation within the upper third of the convection zone. The eigenmodes used for the construction of the kernels were obtained by Fourier-Legendre decomposition of one month long series of Dopplergrams from the GONG, MDI, and HMI instruments.

The method allows to obtain well localized kernels at depths down to 60 Mm. The kernels from the three data sets are very similar at a depth of about 20 Mm, while the kernels from HMI tend to give slightly better results in greater depth. Given the much better spatial resolution of the HMI instrument and its better technical specifications in general this seems not surprising. However the inverted flow velocities do not only depend on the shape of the used averaging kernels but also on the actually measured frequency shifts. Even with identical kernels the inverted flow velocities could differ a lot between the datasets and it will be interesting to see how HMI compares to GONG and MDI once the data has undergone the final calibration.

Concerning the Fourier-Legendre decomposition based on large region of interests, we conclude that this method promises to probe greater depths in the convection zone for the meridional flow once longer time series are used for the inversion. This will increase the SNR and also allow for a more precise frequency determination which is currently limiting the depth...
that can be reached with the method. In addition it has to be studied for which harmonic degree the frequency shifts can be used for inverting for the meridional circulation [25].

Acknowledgments
M.R. thanks Doug Braun for useful discussions. The authors thank Hannah Schunker for useful information on mode fitting. This work was supported by the European Helio- and Asteroseismology Network (HELAS) which was funded as Coordination Action under the European Commission’s Sixth Framework Programme (FP6). This work utilizes data obtained by the Global Oscillation Network Group (GONG) program, managed by the National Solar Observatory, which is operated by AURA, Inc. under a cooperative agreement with the National Science Foundation. The data were acquired by instruments operated by the Big Bear Solar Observatory, High Altitude Observatory, Learmonth Solar Observatory, Udaipur Solar Observatory, Instituto de Astrofísica de Canarias, and Cerro Tololo Interamerican Observatory. Data used in this publication is provided by the SOHO/MDI consortium. SOHO is a project of international cooperation between ESA and NASA.

References
[1] Duvall Jr T L 1979 Solar Phys. 63 3–15
[2] Hathaway D H 1996 Astrophys. J. 460 1027–1033
[3] Komm R W, Howard R F and Harvey J W 1993 Solar Phys. 147 207–223
[4] Bjerknes V 1926 Astrophys. J. 64 93–121
[5] Rüdiger G 1980 Geophysical and Astrophysical Fluid Dynamics 16 239–261
[6] Kükü M and Stix M 2001 Astron. Astrophys. 366 668–675
[7] Choudhuri A R, Schüssler M and Dikpati M 1995 Astron. Astrophys. 303 L29–32
[8] Dikpati M and Charbonneau P 1999 Astrophys. J. 518 508–520
[9] Nandy D and Choudhuri A R 2002 Science 296 1671–1673
[10] Rempel M 2006 Astrophys. J. 637 1135–1142 (Preprint arXiv:astro-ph/0610133)
[11] Haber D A, Hindman B W, Toomre J, Bogart R S, Larsen R M and Hill F 2002 Astrophys. J. 570 855–864
[12] Zhao J and Kosovichev A G 2004 Astrophys. J. 603 776–784
[13] Zaatri A, Komm R, González Hernández I, Howe R and Corbard T 2006 Solar Phys. 236 227–244
[14] Braun D C, Duvall Jr T L and Labonte B J 1987 Astrophys. J. Lett. 319 L27–L31
[15] Braun D C and Fun Y 1998 ApJ 508 L105–L108
[16] Doerr H P, Roth M, Zaatri A, Krieger L and Thompson M J 2010 Astr. Nachr. (in press)
[17] Scherrer P H, Bogart R S and Bush e a 1995 Solar Phys. 162 129–188
[18] Anderson E R, Duvall T L and Jefferies S M 1990 Astrophys. J. 364 699–705
[19] Christensen-Dalsgaard J, Däppen W, Ajukov S V, Anderson E R, Antia H M, Basu S, Baturin V A, Berthomieu G, Chaboyer B, Chitre S M, Cox A N, Demarque P, Donatowicz J, Dziembowski W A, Gabriel M, Gough D O, Guenther D B, Guzik J A, Harvey J W, Hill F, Houdek G, Iglesias C A, Kosovichev A G, Leibacher J W, Morel P, Proffitt C R, Provost J, Reiter J, Rhodes E J, Rogers F J, Roxburgh I W, Thompson M J and Ulrich R K 1996 Science 272 1286–1292
[20] Gough D O and Toomre J 1983 Solar Phys. 82 401–410
[21] Pijpers F P and Thompson M J 1994 Astron. Astrophys. 281 231–240
[22] Schou J, Scherrer P H and Bush R I 2010 in prep.
[23] Christensen-Dalsgaard J 2008 Astrophys. Spa. Sci. 316 113–120 (Preprint 0710.3106)
[24] Thompson M J 1993 GONG 1992. Seismic Investigation of the Sun and Stars (Astronomical Society of the Pacific Conference Series vol 42) ed T M Brown pp 141–+
[25] Gough D and Hindman B W 2010 Astrophys. J. 714 960–970 (Preprint 0911.2013)