Study on bonded slip of reinforced concrete based on extended finite element

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Abstract. In this paper, for the tensile test of reinforced concrete specimens, the numerical simulation model is established to simulate the whole process of concrete specimen from crack generation and extension to final fracture based on the extended finite element method. Based on the numerical simulation results and the tensile test results of concrete specimens, the differences of the numerical simulation results with the bond slip relationship and those without the bond slip relationship are analysed. The results show that the numerical analysis results of bearing ratio of steel bar with bond slip relationship are close to those of the test, and its numerical analysis results of crack width are also closer to the measured values of the test. And there is great deviation between the numerical analysis results without bond slip relationship and the test results, at the same time, the feasibility of solving the continuous - discontinuous problem based on extended finite element method is verified.

1. Introduction

The bond and slip characteristics between reinforced concrete and concrete are one of the hot issues in the numerical simulation of reinforced concrete structures. With the continuous development of numerical methods, numerical simulation has become an important mean to study the reinforced concrete structure. For a long time, the simulation of the bond relationship between the reinforcement and the concrete is often simplified, in other words, both are considered to be well bonded [1]. In the study on numerical simulation of reinforced concrete, it is very few to take both the cracking of concrete and the bond slip relationship between the reinforcement and the concrete into account. Therefore, based on the extended finite element method, the numerical analysis model is established to simulate the bearing process of reinforced concrete specimen, the stress and the crack width of the reinforced concrete specimen are compared with the results of the test, and the differences of the results with the bond slip relationship and those without the bond slip relationship are analysed, at the same time, the feasibility of solving the continuous - discontinuous problem based on extended finite element method is verified.

2. Bond slip characteristics of reinforced concrete

In the reinforced concrete components, these two materials of different properties can be effectively combined to force together. The main reason is that reinforced concrete has a bond slip characteristics, and it produces cohesive force between the reinforcement and the concrete. Through the interaction, the mutual transmission of force and compatible deformation can occur between the two materials.
2.1. Bond-slip constitutive relations

The bond-slip constitutive relation used in this paper is the bond-slip relationship proposed by CEB-FIP [2]. The constitutive relation is expressed as follows:

\[
\begin{align*}
\text{Nonlinear section:} & \quad \tau = \tau_{\max} \left( \frac{s}{s_1} \right)^\alpha \quad 0 \leq s \leq s_1 \\
\text{Platform section:} & \quad \tau = \tau_{\max} \quad s_1 < s \leq s_2 \\
\text{Descent section:} & \quad \tau = \tau_{\max} - \left( \tau_{\max} - \tau_f \right) \left( \frac{s - s_2}{s_3 - s_2} \right) \quad s_2 \leq s \leq s_3 \\
\text{Residual segment:} & \quad \tau = \tau_f \quad s_3 < s
\end{align*}
\]

Where, \( \tau \) is the bond stress between the concrete and steel bar; \( s \) is the bond-slip between the concrete and steel bar; \( \alpha \) is the bond residual stress between the concrete and steel bar.

In consideration of the bond between concrete and steel bar is better in test specimen, the relevant parameters are as follows:

\( s_1 = s_2 = 0.6 \text{mm} \), \( s_3 = 1 \text{mm} \), \( \alpha = 0.4 \text{mm} \), \( \tau_f = 0.15 \tau_{\max} \), \( \tau_{\max} = 2.0 \sqrt{f_c} \), \( f_c = 0.8 f_{cu} \), \( f_{cu} \) is the compressive strength of the concrete cube.

2.2. Bond slip simulation method

In the extended finite element simulation of reinforced concrete structures, the way of establishing models of steel bar can be divided into three types: integral type model, composite type model and separate type model [3]. In the monolithic model, the reinforcement is dispersed throughout the elements, and the element is treated as a continuous and uniform material. The contribution of steel bars in the whole structure is reflected by adjusting the mechanical properties of the elements. This method can only get the overall results of the structure under load, and the effect of reinforcement and concrete cannot be obtained separately. When it is assumed that the bond between the reinforcement and the concrete in the model is intact and there is no slip between the two, the modular model will be used. In a separate model, the reinforcement and concrete are treated as different units, and they are divided into different units separately. In the plane, the concrete can be divided into triangular or quadrilateral elements, steel can also be divided into triangular or quadrilateral unit. Taking the reinforced as an elongated material into account, we usually ignore its lateral shear effect, so that the steel can be regarded as a linear unit. This method can greatly reduce the number of units, and can avoid the interface in the steel and concrete interface with a lot of transition unit due to the unit is too dense [4]. Therefore, when using the extended finite element method to analyse the bond slip relationship, we should use the separate type model.

When the separate type model is used, the reinforcement and the concrete in the component are mutually constrained and produce a relative slip after the member is stressed. In order to simulate this effect, it is necessary to add a coupling element in the reinforcement and concrete. The coupling unit includes spring unit [5], no thickness quadrilateral bond unit [6], bond area unit [7]. In this paper, spring element is used to simulate the bond slip behavior between steel bar and concrete.
3. Study on Simulation of Bonded Slip of Reinforcement and Concrete

In this paper, the extended finite element method is used to simulate the axial tensile test of reinforced concrete specimen [8] which has been completed by our team. The size of the specimen is shown in the figure below, and a reinforcement which is a diameter of 10 mm is located at the center of the section.

Concrete strength grade is C25, and steel strength grade is HRB400 under test. Concrete elastic modulus, ultimate tensile strength and fracture energy are the experimental measured values. The material parameters are shown in the following table.

| Name   | Bulk density (KN/m3) | Elastic modulus (GPa) | Poisson ratio | Ultimate tensile strength (MPa) | Fracture energy (N/m) |
|--------|----------------------|-----------------------|---------------|---------------------------------|----------------------|
| Steel bar | 78.0                | 200                   | 0.3           | —                               | —                    |
| Concrete | 25.0                 | 28.59                 | 0.167         | 1.54                            | 116.3                |

3.1. A brief introduction of models

In this paper, the models are established by the separate type model, and reinforcement and concrete are divided into grids separately. The cracking criterion of concrete is the maximum principal stress criterion. The concrete model adopts the eight-node isoparametric element, and steel bar model adopts two-node bar element. The specimen model has a total of 6485 elements, including 6396 concrete elements and 89 steel bar elements.

During the test, the specimen was clamped by the fixture. When the clamp is clamped, the displacement of the clamping end of the specimen is almost the same as that of the clamp. In order to simulate the constraint and force of the specimen, two reference points are established in the model and a coupling constraint is established between the clamping end and the reference point. At the side of the reference point, another reference point is applied a full constraint, and at the other side,
concentrated force load is applied, as shown in figure 4. In order to compare the numerical analysis results and test results, the concentrated force is step loading.

![Figure 4. Boundary condition of reinforced concrete specimen.](image)

In this paper, under the condition of not including the bond slip relationship of reinforced concrete, the steel bar is embedded in the concrete. According to the elastic modulus of steel and concrete and cross-sectional size to share the quantity of the load, and it makes them coordinated deformation. But, under the condition of including the bond slip relationship of reinforced concrete, the three-spring model is established, and the three springs elements which are orthogonal to each other are built between corresponding reinforcement node and concrete node. The spring which is perpendicular to the axis of the steel bar is used to simulate the vertical force between the reinforcement and the concrete. Since the mutual compression between the reinforcement and the concrete is not taken into account in this paper, its stiffness take the maximum value is $1.0 \times 10^9$N/m; and the spring which is parallel to the axial direction of reinforcement simulate the bonded slip relationship between the reinforcement and concrete, and the relationship is decided by formula 1.

3.2. Bearing Analysis of Reinforcement in Reinforced Concrete Specimen

The results of steel stress of the specimen model under the loading are shown in the following figure, and the feasibility of the numerical simulation method is analyzed by comparing the steel stress of the model with that of the test under similar load.

![Figure 5. The curve of steel stress and load.](image)

From the above figure, the results of steel stress of the model including bonded slip, the model excluding bonded slip and the test are close under different grade before crack. After the concrete cracking, the results of the model including bonded slip and those of the test are approximately equal. However, there is a great deviation between the numerical simulation results without bond slip and the test.
Table 2. Bearing ratio of steel bar (%).

| Load (N) | Numerical simulation | Test |
|----------|----------------------|------|
|          | Bearing ratio of steel bar with bond slip relationship | Bearing ratio of steel bar without bond slip relationship | Load (N) | Bearing ratio of steel bar |
| 18000    | 71.3                 | 26.5 | 17682 | 63.9 |
| 17000    | 52.4                 | 20.4 | 16938 | 51.4 |
| 16000    | 31.5                 | 13.5 | 15924 | 29.5 |
| 15000    | 7.3                  | 5.7  | 14855 | 6.5  |
| 14000    | 4.6                  | 4.6  | 14030 | 5.7  |
| 13000    | 4.5                  | 4.5  | 13045 | 5.8  |
| 12000    | 4.5                  | 4.5  | 12599 | 5.4  |
| 11000    | 4.5                  | 4.5  | 11391 | 4.4  |
| 10000    | 4.5                  | 4.5  | 10066 | 4.6  |

From the above figure, the bearing ratio of steel bar of the model with bonded slip, the model without bonded slip and the test are close under different grade before crack. After the concrete cracking, the results of the model including bonded slip and those of the test are approximately equal. However, there is a great deviation between the numerical simulation results without bond slip and the test. Before the cracking of the concrete, the concrete in the specimen takes most of the load. When the tensile load of the specimen is increased to 15000N, the cracks are generated at both sides of the concrete specimen. As the load increases, the crack gradually extends to the axial of the specimen. The quantity of bearing of the specimen is gradually reduced, and the numerical analysis results with bond slip are in accordance with those of the test. Therefore, the numerical results with bond slip are closer to those of the test.

3.3. Analysis of Crack Width of Reinforced Concrete Specimen

In this paper, the crack width of reinforced concrete specimen under different bearing capacity is studied, and the calculated value, numerical analysis value and the measured value of crack width are compared and analyzed. Since the model is only subjected to axial tensile load, it is type I cracking. The numerical analysis values are obtained by using the displacement difference between nodes on both sides of the crack at the cracking unit, the formula calculated using the "concrete structure design specifications" (GB 50010-2010) in the calculation of the maximum crack width formula, The formula is as follows:

\[ W_{\text{max}} = a_{cr} \phi \frac{\sigma_s}{E_s} (1.9c_t + 0.08 \frac{d_{eq}}{\rho_{te}}) \]  \hspace{1cm} (2)

\[ \phi = 1.1 - 0.65 \frac{f_k}{\rho_{te} \sigma_s} \]  \hspace{1cm} (3)

Where, \( W_{\text{max}} \) is the maximum crack width, mm; \( a_{cr} \) is component force characteristic coefficient; \( \sigma_s \) is steel stress, N/mm²; \( E_s \) is elastic modulus of steel bars, N/mm²; \( c_t \) is the distance from the outer edge of the outermost longitudinal reinforcement to the bottom of the tension zone, mm; \( d_{eq} \) is equivalent diameter of longitudinal reinforcement in tension zone, mm; \( \rho_{te} \) is ratio of reinforcement.

The calculated values of crack width, numerical analysis and shaft tensile test of reinforced concrete specimen are shown in the following table.
Table 3. Crack width (mm).

| Formula calculated value | Numerical value | The measured value |
|--------------------------|-----------------|--------------------|
|                          | Bonding slip    | No bonding slip    |                  |
| 0.203                    | 0.096           | 0.019              | 0.120            |
| 0.067                    | 0.066           | 0.014              | 0.098            |
| 0.032                    | 0.035           | 0.008              | 0.051            |
| 0.007                    | 0.005           | 0.002              | 0.000            |

As can be seen from the above table data, the results of numerical analysis of crack width considering bond slip are closer to those of test and formula calculation value. The measured value of the crack width is greater than the calculated value of the bond slip and the calculated value of the formula, and the formula calculation is the smallest. Most of the fractal width numerical analysis value deviation is smaller than the calculated value of the formula. Therefore, it can be concluded from the table that the numerical value of crack width is closer to the experimental value.

4. Conclusions
In this paper, based on the extended finite element method, the numerical model of the tensile test of the reinforced concrete specimen is established to simulate the specimen from the crack to the specimen fracture damage under the action of axial tensile load. And compared with the test results, the differences of the results with the bond slip relationship and those without the bond slip relationship are analysed, and the reliability and feasibility of the numerical calculation method are verified, the results show that:

1. Before the cracking of the concrete, the steel bearing ratio of including the bond slip, excluding the bond slip, and the model test under the different level of loading is close. However, after the cracking of the concrete, the results of steel bearing ratio of numerical analysis with the bond slip relationship are close to the results of the tensile test, and there is great deviation between the numerical analysis result without bond slip relationship and the test result.

2. Before the concrete cracking, the concrete in the specimen takes up most of the load. When the tensile load of the specimen is increased to 15000N, the cracks are generated at both sides of the concrete specimen, and as the load increases, the cracks gradually expand toward the central position. Then, concrete unloading slowly, the bearing ratio is also gradually reduced, the specimen bearing gradually reduced by the steel to bear, the numerical analysis results with bond slip relationship is closer to the results of the test.

3. The numerical analysis results of the crack width with the bond slip are closer to the measured results of test and the calculated results. The relationship of the crack width of the calculated results of the formula, the numerical analysis results with the bond slip and the measured values is both the calculated results and the numerical analysis results with bond slip relationship are close to the measured value of the test, and the numerical analysis results is closer to the measured value of test.

Acknowledgements
This work was financially supported by the National Natural Science Foundation of China (No. 51379107)

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