Dynamical Properties of the 1/r²-Type Supersymmetric t-J Model in a Magnetic Field: Manifestation of Spin-Charge Separation

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Quasi-particle picture in a magnetic field is pursued for dynamical spin and charge correlation functions of the one-dimensional supersymmetric t-J model with inverse-square interaction. With use of exact diagonalization and the asymptotic Bethe-ansatz equations for finite systems, excitation contents of relevant excited states are identified which are valid in the thermodynamic limit. The excitation contents are composed of spinons, antispinons, holons and antiholons obeying fractional statistics. Both longitudinal and transverse components of the dynamical spin structure factor are independent of the electron density in the region where only quasi-particles with spin degrees of freedom (spinons and antispinons) contribute. The dynamical charge structure factor does not depend on the spin-polarization density in the region where only quasi-particles with charge (holons and antiholons) are excited. These features indicate the strong spin-charge separation in dynamics, reflecting the high symmetry of the model.

KEYWORDS: dynamical spin structure factor, dynamical charge structure factor, supersymmetric t-J model, inverse-square interaction, magnetic field, spin-charge separation, exact diagonalization, recursion method

§1. Introduction

The concept of spin-charge separation is no longer a purely academic idea. In various low-dimensional electron systems, angle-resolved photoemission measurements have revealed the existence of spin-charge separation. In quasi-one-dimensional (1D) Mott insulators such as SrCuO3 and Sr2CuO3Cl, angle-resolved photoemission data show two distinct dispersions in the energy-momentum plane. The two correspond to the spinon band and the holon band, respectively. This is an indication of the spin-charge separation. It has been reported that the spin-charge separation in Sr2CuO3 appears also in dielectric response.

Recently magnetic-field-induced dimensional crossover has been observed in the normal state of YBa2Cu3O6[5]. The CuO chains of this material play the critical role in magnetoconductivity. The following question may arise: When a 1D electron system is under a magnetic field, how is the spin-charge separation affected by the field? The magnetic field controls spin degrees of freedom, while the hole doping controls charge degrees of freedom. However, realistic situation should be rather complex due to convoluted of both degrees of freedom in a strongly correlated system.

It is well-known that interacting electron systems in one dimension are described by the Tomonaga-Luttinger liquid[6][7], but the information is restricted to that in the long-wavelength and low-energy limit. One can adopt the Hubbard model and the supersymmetric t-J model as Bethe-ansatz solvable models. With use of the exact wave function of the U → ∞ Hubbard model, critical exponents and global features of static spin structure factors have been investigated in a magnetic field[8][9]. The exponents depend only on the magnetization per electron and are independent of the electron density. For the supersymmetric t-J model with nearest-neighbor interaction, the exponents depend both on the magnetization and the electron density[10]. In order to obtain deeper understanding of spin-charge coupling, it is imperative to derive theoretically not only the critical exponents but also the overall spectral weight in dynamics.

The supersymmetric t-J model with inverse-square interaction[11] reveals the spin-charge separation in a typical form. The spin velocity is independent of the electron density m, and the charge velocity is independent of the spin-polarization density m. At low temperature, the spin susceptibility is independent of m, and the charge susceptibility is independent of m. It has been found from the exact thermodynamics that elementary excitations consist of four types of quasi-particles: spinons, antispinons, holons and antiholons. In zero magnetic field, the regions of nonvanishing spectral weight in the momentum-energy plane, which are called the “compact support”, have been obtained for various dynamical correlation functions at zero temperature[12]. The excitation contents are composed of spinons, holons and antiholons. In particular, the weight of the dynamical spin structure factor does not depend on m in the region where only two spinons contribute[13]. At half filling with m = 1, i.e., in the Haldane-Shastry model[14], magnetic-field dynamics has been investigated[15]. So far, study of full dynamics are lacking at arbitrary filling and spin-polarization density.
In this paper, we extend our previous work[25] to the
case of finite magnetic field for the $1/r^2$-type supersym-
metric $t$-$J$ model. We investigate how the spin-charge
separation appears in the spin and charge dynamics in
nonzero field. To this end, exact diagonalization up to
16 sites is used. In order to analyze the results of dynamical
quantities, we employ the asymptotic Bethe-ansatz equa-
tions[3,4,23] and the skew Young diagram. In addition, we examine the static structure factors in
the $1/r^2$ model as well as the $t$-$J$ model with realistic param-
eters.

This paper is organized as follows: Section 2 describes
the model and definitions of dynamical quantities. In
§3 we derive the supports for dynamical spin and charge
structure factors. In §4 we discuss the spectral weights
themselves, and clarify manifestation of the strong spin-
charge separation in dynamics. In §5 we show the results
of the static structure factors. Finally we give the sum-
mary of this study in §6. A brief account of this work has
been presented in ref. 25.

§2. Model and Dynamical Structure Factors

We consider the following Hamiltonian:

$$\mathcal{H} = \sum_{i<j}^{N} \left[ - \sum_{\sigma} t_{ij} \left( \hat{c}_{i\sigma}^{\dagger} \hat{c}_{j\sigma} + \hat{c}_{j\sigma}^{\dagger} \hat{c}_{i\sigma} \right) \right] + J I_{ij} \left( S_i \cdot S_j - \frac{1}{4} n_i n_j \right) - g \mu_{B} H \sum_{i=1}^{N} S_{i}^{z}, \quad (2.1)$$

with even number $N$ of sites and average electron number $\bar{n}$ per site. Here $\hat{c}_{i\sigma} = c_{i\sigma}(1 - n_{i\sigma})$ is the annihilation
operator of an electron with spin $\sigma$ at site $i$ with the
constraint of no double occupation. The Zeeman term
couples the system to a uniform magnetic field $H$ in the $z$
direction. A dimensionless parameter $h$ is defined as $h = g \mu_{B} H/t$. We assume the periodic boundary condition.

Then $I_{ij}$ is given by $I_{ij} = \left[ (N/\pi) \sin(\pi(i - j)/N) \right]^{-2}$
for the $1/r^2$ type, and by $I_{ij} = \delta_{i,j+1}$ for the nearest-
neighbor type. In this paper we mainly discuss the $1/r^2$-
type supersymmetric $t$-$J$ model ($J/t = 2$).

The dynamical spin and charge structure factors are
given by

$$S^{\alpha\alpha}(q, \omega) = \sum_{\nu} \left| \langle \nu | s_{\nu}^{\alpha} | 0 \rangle \right|^{2} \delta(\omega - E_{\nu} + E_{0}), \quad (2.2)$$

$$N(q, \omega) = \sum_{\nu} \left| \langle \nu | n_{\nu} | 0 \rangle \right|^{2} \delta(\omega - E_{\nu} + E_{0}), \quad (2.3)$$

where $s_{\nu}^{\alpha} = N^{-1/2} \sum_{\ell} s_{\nu}^{\alpha} e^{-i\ell \cdot q}$ ($\alpha = x, y, z$), $n_{q} = N^{-1/2} \sum_{\ell} (n_{\ell} - \bar{n}) e^{-i\ell \cdot q}$, and $|\nu\rangle$ denotes an eigenstate of $\mathcal{H}$ with energy $E_{\nu}$ ($E_{0}$ being the ground-state energy).
The transverse component $S^{\alpha\alpha}(q, \omega)$ is decomposed into two parts:

$$S^{\alpha\alpha}(q, \omega) = S^{yy}(q, \omega) = \frac{1}{4} \left[ S^{++}(q, \omega) + S^{+-}(q, \omega) \right]. \quad (2.4)$$

We calculate $S^{++}(q, \omega)$ and $S^{+-}(q, \omega)$ instead of
$S^{xx}(q, \omega)$ and $S^{yy}(q, \omega)$. These dynamical quantities can be written in the form of a continued fraction.[25] The coefficients of the continued fraction are obtained from

the Lanczos algorithm. This method is called the recur-
sion method. We truncate the continued fraction be-
tween 100 and 200 iterations and take the Lorentzian
width of $O(10^{-5}t)$. In our calculation, energy levels and
intensities have accuracies of 7 digits and 4-5 digits, re-
spectively.

We assume $\bar{n} \geq 0$ without loss of generality. In the
thermodynamic limit, $\bar{n}$ is determined by $h$ as follows:

$$\bar{n} = \left\{ \begin{array}{ll}
1 - \sqrt{1 - 2h/\pi^{2}}, & \text{for } 0 \leq h \leq h_{c}, \\
\bar{h}, & \text{for } h \geq h_{c},
\end{array} \right. \quad (2.5)$$

where $h_{c} = \bar{n}(2 - \bar{n})\pi^{2}/2.[25]$ For finite systems, $\bar{n} = (N - Q - 2M)/N$ and $\bar{n} = (N - Q)/N$. Here $Q$ and
$M$ denote the numbers of holes and down-spins, respec-
tively. For $\bar{n} < 1$ we consider the case of $Q = \text{even}$ and
$M = \text{odd}$ under the periodic boundary condition.

§3. Spectra of Quasi-Particles and Supports

In this section we determine the regions of nonvan-
ishing spectral weight in the energy-momentum plane
(i.e., the compact supports) for various dynamical cor-
relation functions. To achieve this goal, the following
points should be clarified:

- Dispersion relations of the relevant quasi-particles.
- Excitation contents of the excited states.
- Selection rules for the excited states.

Let us begin with the dispersion relations.

3.1 Dispersion relations of quasi-particles

The spectra of quasi-particles in the model are most
easily obtained by the asymptotic Bethe ansatz method.
It is known[12,23] that the method gives a part of the
effect eigenstates corresponding to the Yangian highest-
weight states (YHWS), but that other eigenstates are
missed. Since the YHWS contain all kinds of quasi-
particles, the asymptotic Bethe ansatz is sufficient to
drive the dispersion relations. The Yangian multiplet
structure of eigenstates is then derived by the skew
Young diagram.[3]

We write the energy of a spinon with spin $\sigma$ as $\epsilon^{\sigma}_{\bar{h}}$, while
that of an antispinon as $\epsilon^{\dagger \bar{h}}$. Note that an antispinon can have only a down spin with $\bar{n} \geq 0$. The energies of
a holon and an antiholon are written as $\tilde{\epsilon}_{h}$ and $\tilde{\epsilon}_{\bar{h}}$, respectively. Details of the derivation of the spectra are
given in Appendix. The final results are given by

$$\epsilon^{\dagger \bar{h}}/t = -k(k - \pi) - \frac{h}{2}, \quad (3.1)$$

$$\epsilon^{\bar{h}}/t = -k(k - \pi) + \frac{h}{2}, \quad (3.2)$$

$$\epsilon^{\dagger \bar{h}}/t = \frac{1}{2}k(k - 2\pi) + h, \quad (3.3)$$

$$\epsilon_{h}/t = \left( k - \frac{\pi}{2} \right)^{2} + \frac{h^{2}}{12}, \quad (3.4)$$

$$\epsilon_{\bar{h}}/t = \frac{1}{2} \left( k - \pi \right)^{2} - \frac{h^{2}}{6}, \quad (3.5)$$

where $h = \bar{n}(2 - \bar{n})\pi^{2}/2$.

We note that only a part of the momentum space is
relevant as shown in Fig. 1. For classification of ex-
cited states, it is convenient to define the right and left
branches for each kind of quasi-particles. For example, the right \([\text{left}]\) up and down spinons are allowed only in the range \(\bar{m}\pi/2 \leq k \leq \bar{n}\pi/2 \ [(1 - \bar{n}/2)\pi \leq k \leq (1 - \bar{m}/2)\pi]\); the right \([\text{left}]\) holons are allowed only for \((1 - \bar{n}/2)\pi \leq k \leq (1 - \bar{m}/2)\pi \ [ar{n}\pi/2 \leq k \leq \bar{n}\pi/2]\).

The range of momentum for the right \([\text{left}]\) antispinons is \((2 - \bar{n})\pi \leq k \leq 2\pi \ [0 \leq k \leq \bar{n}\pi]\), while the antiholons propagate in the region \(\bar{n}\pi \leq k \leq (2 - \bar{n})\pi\).

By shifting the origins of momentum and energy for each kind of quasi-particles, we can write the dispersion relations for right \([\text{left}]\) spinons \(\epsilon_{\bar{R}(\bar{L})}\), right \([\text{left}]\) antispinons \(\epsilon_{\bar{S}(\bar{L})}\), right \([\text{left}]\) holons \(\epsilon_{\bar{h}(\bar{L})}\) and antiholons \(\epsilon_{\bar{h}^\ast}\) in the following form:

\[
\epsilon_{\bar{R}(\bar{L})}/t = -k(k \mp v_0^0), \tag{3.6}
\]

\[
\epsilon_{\bar{S}(\bar{L})}/t = \frac{1}{2}k(k \pm 2v_0^0), \tag{3.7}
\]

\[
\epsilon_{\bar{h}(\bar{L})}/t = k(k \mp v_c^0), \tag{3.8}
\]

\[
\epsilon_{\bar{h}^\ast}/t = \frac{1}{2}\left[ (v_0^0)^2 - k^2 \right]. \tag{3.9}
\]

where \(v_0^0 = (1 - \bar{m})\pi\) and \(v_c^0 = (1 - \bar{n})\pi\). In the shifted momentum space, the right \([\text{left}]\) spinons and holons are allowed in the range \(0 \leq k \leq (\bar{n} - \bar{m})\pi/2 \ [-(\bar{n} - \bar{m})\pi/2 \leq k \leq 0]\). The allowed region of momentum for the right \([\text{left}]\) antispinons is \(0 \leq k \leq \bar{m}\pi \ [-(\bar{n} - \bar{m})\pi \leq k \leq 0]\), while the antiholons propagate in the region \(-v_0^0 \leq k \leq v_c^0\).

In the case of \(\bar{m} = 0\), these dispersion relations and the range of momentum reduce to those given by Ha and Haldane.

3.2 Determination of the supports

The remaining task to determine the supports is to derive excitation contents and selection rules for each dynamical correlation function.

First, let us consider excitation contents. For a given magnetic field, the magnetization of the ground state is fixed. In the subspace of relevant excited states, we use the solution of total energy and total momentum from the asymptotic Bethe-ansatz equations. We then obtain the motif corresponding to the ground state and excited states with finite intensity obtained by the recursion method. The motif of the ground state with finite magnetization is given by \(0000, 0010, 0101, 0111, 1111, 01010, 0000\) \(\bar{I} = \bar{I} = \bar{II} = \bar{III} = \bar{IV} = \bar{V}\).

This ground-state motif turns out to be divided into five segments (I-V). The segment III can be regarded as a holon condensate, and the segments II and V can be regarded as a up-spinon condensate. As a perturbation from the magnetized ground state, spinons \([00]\) and holons \([11]\) are excited in the segments II and/or IV; Antiholons \([00]\) are created in the segment III, and antispinons \([11]\) are created in the segments I and/or V. Spinons and holons have semiionic statistics, while antispinons and antiholons have Bose statistics.

Let \(S^z\) and \(C\) denote the \(z\)-components of spin and the charge of each quasi-particle, respectively. It is natural to assign \((S^z, C) = (1/2, 0)\) to the up spinon, \((-1/2, 0)\) to the down spinon, \((-1, 0)\) to the down antispinon \((0, +e)\) to the holon and \((0, -2e)\) to the antiholon. However, this assignment leads to cases where some states with excitation contents decided above do not fulfill the spin and/or charge conservations for the excited states. Moreover, those states do not fulfill the statistics of quasi-particles, either. This situation has occurred also in zero magnetic field.

In ref. 18, we introduced the skew Young diagram in the \(1/r^2\)-type supersymmetric \(t-J\) model. By using this diagram, one can reproduce the supermultiplet structure, i.e., nontrivial degeneracies in the excitation spectrum. In addition, excitation contents for a finite system can be extracted by the skew Young diagram. This idea can be expanded into the magnetized case. We first determine the five segments of a diagram from the corresponding motif. Note that the segment III consists of \(Q\) boxes, and that each of the segments I and V consists of \((N - Q - 2M)/2\) boxes. Next, we put an index for the internal degrees of freedom on each box of a skew Young diagram, according to the rules presented in ref. 18. We represent an up-spin by 1, a down-spin by 2, and a hole by \(c\). One can read quasi-particles from a diagram of an excited state as follows:

(i) We regard \(c\) in the segments II and IV as a holon.

(ii) We regard a row of a 1-2 pair in the segment III as an antiholon.

(iii) In the segments II and IV, single 1 or 2 in a column, both of which do not make a pair, is regarded as a spinon; it does not matter whether the column include \(c\) or not. Single 1 corresponds to an up-spinon, and single 2 to a down spinon.

(iv) A column of a 1-2 pair in the segments I and V is regarded as an antispinon.

Because an antiholon or an antispinon consists of two boxes, there exist cases where only a part of a 1-2 pair is present in the segment defined by (ii) and (iv). We shall call such a part of the 1-2 pair a “half-antiholon (\(\bar{a}\))” for the former, and a “half-antispinon (\(\bar{s}(\bar{L})\))” for the latter. The half-antiholon [half-antispinon] is assumed to have \((S^z, C) = (0, -c) \ [-(1/2, 0)]\) and semiionic statistics. Thereby the excitation contents from the diagram satisfies both charge and spin conservations and the statistical feature.

Let us give an example of the skew Young diagram. For \(S^+ (q, \omega)\) from the initial state with \((N, Q, M) = (12, 2, 3)\), the excitation energy with \((q/\pi, \omega/t) = (0.5, 2.05617)\) has finite intensity. Its motif is \(01010|11010|10100\), and the corresponding skew Young diagram is shown in Fig. 2. The excitation content reads \(\bar{s}_1 + (\bar{s}_1^\ast + \bar{h}_L^\ast) + \bar{h}^\ast + \bar{s}_2\). Then the change of the \(z\)-component of spin becomes \(-1\) and there is no change of charge, which is consistent with the changes between the initial state and the final one.

As the number of sites increases with electron density and spin-polarization density kept constant, the effect of the edges in the holon condensate and that in the up-spinon condensate should be negligible. This means that both the half-antiholon and the half-antispinon have zero energy. In fact, for \(N(q, \omega)\) at arbitrary filling and zero field, the energy levels with contents \(\bar{h}^\ast + \bar{h}_R\) are almost along the holon dispersion for \(0 \leq q \leq \bar{n}\pi/2\).
in the compact support. For $S^{zz}(q, \omega)$ at $\tilde{n} = 1$ and arbitrary magnetization, the energy levels with contents $s + \tilde{s}_R$ are almost along the spinon dispersion for $1 \leq q \leq (1 - \tilde{m})\pi$ in the support by Talstra and Haldane. Thus half-antiholons and half-antispinons should not survive in the thermodynamic limit. In this way, the excitation contents in the thermodynamic limit are identified as shown in Table I. The $z$-component of spin ($1/2$ or $-1/2$) of spinons is assigned so that the spin conservation is satisfied.

When we draw the support using the excitation contents (quasi-particles are identified as free particles), the estimated region is much larger than the region where the poles with finite intensity are present. This implies the importance of selection rules. Ha and Haldane finding the empirical rule in zero magnetic field: for a given spinon-holon pair $(\tilde{s}_R, \tilde{h}_R)$, $|v_{\tilde{m}}| \geq |v_{\tilde{h}}|$, and the same to the left-going pair. Here $v$ is given by $\partial \varepsilon/\partial q$. We find that this rule applies also to the case of nonzero magnetic field. Then the estimated region is found to be the same as the region expected from the recursion method.

3.3 Characteristic features of the supports

Figures 3-9 show the regions of compact support for $S^+(q, \omega)$, $S^{zz}(q, \omega)$, $S^-(q, \omega)$ and $N(q, \omega)$ with finite magnetization. The solid lines mean either the spectrum of a quasi-particle or that of a pair moving with the same velocity. The presence of other quasi-particles excited at particular shifts in momentum and energy. Outermost dispersion lines correspond to the boundary of the compact support. Namely, there is no intensity outside the outermost lines.

The number of zero modes is increased by the external magnetic field. Namely for $S^+(q, \omega)$ and $S^-(q, \omega)$, $\omega = 0$ is allowed at $q = m\pi, (\tilde{n} - \tilde{m})\pi, (2 - \tilde{n} - \tilde{m})\pi$. For $S^{zz}(q, \omega)$, on the other hand, $\omega = 0$ is allowed at $q = 0 [2\pi], 2\tilde{m}\pi, (\tilde{n} - \tilde{m})\pi \equiv [v^2 - v^0], (2 - \tilde{n} - \tilde{m})\pi \equiv [v^0 + v^2]$, $(\tilde{n} + \tilde{m})\pi, (2 - \tilde{n} + \tilde{m})\pi$. For $N(q, \omega)$, $\omega = 0$ is allowed at $q = 0 [2\pi], (2 - 2\tilde{n})\pi, (\tilde{n} - \tilde{m})\pi, (\tilde{n} + \tilde{m})\pi, (2 - \tilde{n} + \tilde{m})\pi$.

For $S^-(q, \omega)$, the excitation contents are the same as those for $S(q, \omega)$ in zero magnetic field (see Table I). Note that no antispinons contribute to $S^-(q, \omega)$. At arbitrary filling, the support turns out to be essentially a squeezed version of $S^+(q, \omega)$ in zero field.

For $S^{zz}(q, \omega)$ and $N(q, \omega)$, the region of $(\tilde{s}_L, \tilde{h}_L)$ shrinks and splits into three directions by a magnetic field: the two of them is the parallel shift by $\pm \tilde{m}\pi$ along the momentum axis (horizontal axis), and the remaining one is the parallel shift by $+h$ along the energy axis (vertical axis). This is due to the presence of the antispinon.

Finally we note that the support of $S^{zz}(q, \omega)$ over $h$ agrees with that of $S^{zz}(q, \omega)$ for the same filling and magnetization.

4.1 $S^{(q, \omega)}$

First of all, we discuss the results of $S^{(q, \omega)}$ at $\tilde{n} = 1$. Figures 3(a) and 3(b) show the results in the 16-site chain for $\tilde{m} = 0.25$ (10 up-spins and 6 down-spins) and $\tilde{m} = 0.5$ (12 up-spins and 4 down-spins), respectively. With use of $h = \tilde{m}(2 - \tilde{m})\pi^2/2$, the values of the magnetic field are taken as $h = 2.15898$ for $\tilde{m} = 0.25$ and $h = 3.70110$ for $\tilde{m} = 0.5$. We find that for finite systems the momentum $q$, the excitation energy $\omega$, and the form factor $M^{q}_\nu$ have strong correspondence with those for $S(q, \omega)$ in zero field. Namely, for $0 \leq c_2 \leq c_1 \leq M$, with $c_1$ and $c_2$ being integers, it is known that

$$ q = m\pi + \frac{2\pi}{N} (c_1 + c_2), $$

$$ \omega = \frac{t}{2} \left( \frac{2\pi}{N} \right)^2 \frac{1}{2M - 2c_1 - 1} \frac{1}{2M - 2c_1 - 1} \prod_{i = c_2 + 1}^{c_1 - 1} \left( \frac{2i}{2i - 1} \frac{2M - 2c_1 - 1}{2M - 2c_1 - 1} \right), $$

$$ M^{q}_\nu = \left\{ \begin{array}{ll}
\frac{1}{(2c_1 - 1) (2M - 2c_1 + 1)}, & \text{for } c_2 = c_1 - 1,
\end{array} \right.$$

Here $M$ denotes the number of down-spins in the initial state. The number of poles with finite intensity is $M(M + 1)/2$. We have checked the validity of eqs. (4.1)-(4.3) by comparison with numerical results up to $N = 16$.

Taking the thermodynamic limit, we obtain the explicit formula

$$ S^{+(q, \omega)} = \frac{1}{2} \frac{\Theta (\varepsilon_U(q - \omega)) \Theta (\omega - \varepsilon_{L_\omega}(q)) \Theta (\omega - \varepsilon_{L+}(q))}{\sqrt{(\omega - \varepsilon_{L_\omega}(q)) (\omega - \varepsilon_{L+}(q))}}, $$

where $\Theta (\omega)$ is the step function, and

$$ \varepsilon_{L-}(q) = t(q - m\pi)(\pi - q), $$

$$ \varepsilon_{L+}(q) = t(q - 2\pi + m\pi)(\pi - q), $$

$$ \varepsilon_U(q) = \frac{t}{2} (q - 2\pi + m\pi)(\pi - q). $$

In the zero-field limit ($\tilde{m} = 0$), this expression reduces to the result obtained by Haldane and Zirnbauer.

Next we consider the case of $\tilde{n} < 1$. Figures 4(a) and 4(b) show the results of $S^{+(q, \omega)}$ in the 16-site chain with $\tilde{n} = 0.875$ (2 holes) for $\tilde{m} = 0.25$ (9 up-spins and 5 down-spins) and $\tilde{m} = 0.5$ (11 up-spins and 3 down-spins), respectively. Like the zero-field case ($\tilde{m} = 0$), at fixed $\tilde{m} > 0$ energy levels and intensities in the two-spinon region (i.e., the hatched area in Fig. 4) agree with those for $\tilde{n} = 1$ within the numerical accuracy. We have checked this fact for $N \leq 16$ with various $\tilde{n}$ and $\tilde{m}$.

From this fact, it is conjectured that analytical expression of $S^{+(q, \omega)}$ in the 2$\tilde{n}$ and 2$\tilde{m}$ is identically equal to eq. (4.4). This feature is an indication of the strong spin-charge separation, which should be due to supersymmetric Yangian symmetry of the present model. For the

4. Spectral Weights

In this section we focus on spectral weights in the compact supports.
nearest-neighbor supersymmetric $t$-$J$ model, such strong separation does not occur. This statement holds also for $S_{zz}(q, \omega)$, $S_{xx}(q, \omega)$ and $N(q, \omega)$ to be discussed later.

For finite systems, the momentum, the excitation energy and the form factor are expressed as follows:

$$q = \bar{m}\pi + \frac{2\pi}{N}c,$$  

$$\omega = \frac{t}{2} \left( \frac{2\pi}{N} \right)^2 f_\nu(Q, M),$$  

$$M^g_\nu = g_\nu(Q, M).$$

Here $M$ and $Q$ denote the numbers of down-spins and holes in the initial state, respectively, and $c$ is an integer which satisfies $0 < c < Q + 2M$. We note that $f_\nu(Q, M)$ and $g_\nu(Q, M)$ are independent of the size of system $N$. This implies that $S^{-+}(q, \omega)$ for the present model is equivalent to the hole propagator, where one boson is removed, for the SU(1,1) Calogero-Sutherland model with $Q$ fermions and $M$ bosons.

4.2 $S_{zz}(q, \omega)$

Figures 5(a) and 5(b) show the results of $S_{zz}(q, \omega)$ for the 16-site Haldane-Shastry model with magnetization $\bar{m} = 0.25$ and 0.5, respectively. The intensity at $(q, \omega) = (0, 0)$ is given by $N\bar{m}^2/4$. It is found that dominant intensity lies in the region defined by four lines: A, B, C and D in Fig. 5. Analytical expression of $S_{zz}(q, \omega)$ in the region $q \leq \bar{m}\pi$ has been computed with use of strong coupling limit of the spin Calogero-Sutherland model.

In the 16-site chain with $\bar{n} = 0.875$, the results of $S_{zz}(q, \omega)$ for $\bar{m} = 0.25$ and 0.5 are shown in Figs. 6(a) and 6(b), respectively. The compact support reveals the region where only quasi-particles with spin degrees of freedom (i.e., spinon and antispinon) contribute. In this region, energy levels and intensities agree with those for $\bar{n} = 1$ with the same $\bar{m}$. This is an indication of the strong spin-charge separation in the longitudinal component of spin dynamics.

4.3 $S_{xx}(q, \omega)$

In Figs. 7(a) and 7(b) we show the results of $S_{xx}(q, \omega)$ in the 16-site chain at $\bar{n} = 1$ for $\bar{m} = 0.25$ and 0.5, respectively. At $q = 0$ there is a single pole; its energy level is $\omega = h$ and its intensity $\langle \nu | S_{xx}(q=0) | 0 \rangle^2 = \bar{m}$. The lower edge over $0 \leq q \leq \bar{m}\pi$ is not only the boundary of the continuum but also the dispersion line of an antispinon (i.e., magnon) excited excite.

At a fixed $\bar{n} = 0.875$ we show the results for the 16-site chain with $\bar{m} = 0.25$ and 0.5 in Figs. 8(a) and 8(b), respectively. Let us compare the present case with the half-filled case in the same magnetization. Although the structure of intensity is partially broken down by existence of holes, the $(q, \omega)$-region lies where the structure remains intact. Namely, energy levels and intensities in the region agree with those for $\bar{n} = 1$ and the same magnetization. This region corresponds to a case where holons and antiholons are never excited, in terms of the support in the thermodynamic limit. This fact indicates the strong spin-charge separation in $S_{xx}(q, \omega)$.

4.4 $N(q, \omega)$

We now turn to the charge dynamics. In ref. 18 we have discussed the features of $N(q, \omega)$ in zero magnetic field. Figures 9(a) and 9(b) show our results for electron density $\bar{n} = 0.875$ (16 sites and 2 holes) with two values of magnetization $\bar{m}$. The following remarkable feature is found: in the pure $\bar{h} + 2h$ region, the poles and intensities of $N(q, \omega)$ are independent of $\bar{m}$ within the numerical accuracy. For $q \leq (\bar{n} - \bar{m})\pi/2$, in particular, this statement has been proved analytically. The independence is a manifestation of the strong spin-charge separation in charge dynamics.

§5. Static Structure Factors

Integration of the dynamical structure factor over $\omega$ yields the static structure factor. Figures 10-12 show numerical results of $S_{zz}(q)$, $S_{xx}(q)$ and $N(q)$ for the inverse-square type.

Particularly for $S_{xx}(q)$, the analytic expression can be obtained. At $\bar{n} = 1$, by integrating eq. (4.4) over $\omega$, we obtain

$$S_{xx}^{-+}(q) = \begin{cases} 0, & \text{for } 0 \leq q \leq \bar{m}\pi, \\ -\frac{1}{2} \log \frac{1 - q/\pi}{1 - \bar{m}}, & \text{for } \bar{m}\pi \leq q \leq \pi. \end{cases}$$

The Fourier transformation leads to

$$\langle S_0^- S_0^+ \rangle = \frac{(-1)^r}{2\pi r} \left[ \text{Si} \left( (1 - \bar{m})\pi r \right) - \text{Si} \left( (1 - \bar{n})\pi r \right) \right].$$

where $r$ is an integer which satisfies $r \geq 1$, and $\text{Si}(x)$ is the sine integral.

For $\bar{n} < 1$, numerical calculations show that $S_{xx}^{-+}(q)$ are equivalent to that for $\bar{n} = 1$ if $0 \leq q \leq \bar{n}\pi$; $S_{xx}^{-+}(q)$ for $\bar{n}\pi \leq q \leq \pi$ are equal to the value for $q = \bar{n}\pi$. On the basis of these facts, we conjecture the following expression:

$$S_{xx}^{-+}(q) = \begin{cases} 0, & \text{for } 0 \leq q \leq \bar{m}\pi, \\ -\frac{1}{2} \log \frac{1 - q/\pi}{1 - \bar{n}}, & \text{for } \bar{m}\pi \leq q \leq \bar{n}\pi, \\ -\frac{1}{2} \log \frac{1 - q/\pi}{1 - \bar{m}}, & \text{for } \bar{n}\pi \leq q \leq \pi. \end{cases}$$

The Fourier transformation leads to

$$\langle S_0^- S_0^+ \rangle = \frac{(-1)^r}{2\pi r} \left[ \text{Si} \left( (1 - \bar{m})\pi r \right) - \text{Si} \left( (1 - \bar{n})\pi r \right) \right].$$

When $\bar{m} = 0$, this result reduces to the previous result.

The relation between $S_{xx}^{-+}(q)$ and $S_{xx}^{-+}(q)$ is as follows:

$$S_{xx}^{-+}(q) = \bar{m} + S_{xx}^{-+}(q),$$

irrespective of the electron density. Therefore, using the relation $S_{xx}^{zz}(q) = [S_{xx}^{-+}(q) + S_{xx}^{-+}(q)]/4$, we get analytic expression of $S_{xx}^{zz}(q)$:

$$S_{xx}^{zz}(q) = \frac{1}{4}\bar{m} + \frac{1}{2}S_{xx}^{-+}(q),$$

where $S_{xx}^{-+}(q)$ is given by eq. (5.1) or eq. (5.3).

The results of static structure factors for the inverse-square type are summarized as follows:

(i) At fixed $\bar{m}$, $S_{zz}^{zz}(q)$ is independent of $\bar{n}$ for $0 \leq q \leq \bar{n}\pi$.
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\[ (\tilde{n} - \tilde{m})\pi. \]

(ii) At fixed \( \tilde{m} \), \( S^{xx}(q) \) is independent of \( \tilde{n} \) for \( 0 \leq q \leq \tilde{n}\pi. \)

(iii) At fixed \( \tilde{n} \), \( N(q) \) is independent of \( \tilde{m} \) for \( 0 \leq q \leq (\tilde{n} - \tilde{m})\pi. \)

These features are regarded as a manifestation of the strong spin-charge separation in static structure factors.

What about the case of the nearest-neighbor interaction? Figures 13, 14 and 15 show the results of \( S^{zz}(q) \), \( S^{xx}(q) \) and \( N(q) \), respectively, for the nearest-neighbor type with two representative values of \( \bar{J}/t \). We look at the difference of \( S^{alpha}(q) \) [\( \alpha = z, x, y, z \)] between the two values of filling (\( \bar{n} = 1 \) and 0.875), and that of \( N(q) \) between the two values of magnetization (\( \bar{m} = 0 \) and 0.25). In any case, within the special momentum range described in (i)-(iii), the difference is less than 6\% for \( S^{zz}(q) \), 5\% for \( S^{xx}(q) \) (\( q = \tilde{n}\pi \) not included), and 1\% for \( N(q) \). This indicates that the 1D \( t-J \) model with realistic parameters shares the properties of static structure factors in the 1/r^2-type supersymmetric \( t-J \) model. The independence of static spin (charge) structure factors with varying filling (magnetization) spreads over considerably wide range of momentum, if the system is near half-filling and under a weak magnetic field. This behavior might be detected experimentally in a quasi-1D conductor.

§6. Summary and Discussion

We have investigated the dynamical and static properties of the 1D supersymmetric \( t-J \) model with 1/r^2 interaction in a magnetic field. We have determined the (\( q, \omega \))-region of nonvanishing spectral weight for \( S^{alpha}(q, \omega) \) [\( \alpha = x, y, z \)] and \( N(q, \omega) \) in nonzero magnetic field. We have found the following features about the spectral weights themselves:

1. At fixed \( \tilde{m} \), \( S^{alpha}(q, \omega) \) is independent of \( \tilde{n} \) in the region where only quasi-particles with spin degrees of freedom (spinons and antispinons) contribute.

2. At fixed \( \tilde{n} \), \( N(q, \omega) \) is independent of \( \tilde{m} \) in the region where only quasi-particles with charge (holons and anti-holons) contribute.

These features constitute further manifestation of the strong spin-charge separation in the 1/r^2-type supersymmetric \( t-J \) model.

When a system deviates from the 1/r^2-type model, elementary excitations in a magnetic field do not correspond to spinons and holons but to mixtures of them. This is related to the fact that the system loses the \( Z_2 \) symmetry between the statistical properties of up- and down-spin electrons in the presence of finite polarization. In fact we have checked for the polarized nearest-neighbor \( t-J \) model that the long-wavelength limit of the spin and charge excitations have a common velocity. Thus the spin-charge separation is not realized any longer. On the other hand, it has been shown that spinons and holons together with their antiparticles span the complete set in the Fock space of hard-core lattice fermions with any filling and polarization. The completeness is independent of whether or not spinons and holons form eigenstates of the model. Thus the absence of spin-charge separation in the polarized nearest-neighbor \( t-J \) model means that there are residual interactions between spinons and holons.

In contrast, the 1/r^2-type model maintains the \( Z_2 \) invariance even with finite polarization. This can be understood from the statistical matrix given in ref. 16. The parallel dispersion relations of up and down spinons as shown in Fig. 1(a) are also consistent with the \( Z_2 \) invariance. As we have shown in this paper, spinons and holons remain elementary excitations, and are essential to understanding dynamics for any polarization in the 1/r^2-type supersymmetric \( t-J \) model. This property entitles the model to be called a “canonical” or a kind of “free” Hamiltonian, which is nothing but another representation of the higher symmetry in the model.

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Appendix: Derivation of Dispersion Relations of Quasi-Particles

In this appendix, we derive dispersion relations of four types of quasi-particles (i.e., spinons, antispinons, holons and anti-holons) for the 1/r^2-type supersymmetric \( t-J \) model in a magnetic field. The derivation is based on the asymptotic Bethe-ansatz equations.

Let us take the completely up-polarized state as the reference state and consider \( M + Q \) pseudo-particles which represent \( M \) electrons with spin down and \( Q \) holes. The asymptotic Bethe ansatz leads to the following equation:

\[
\frac{E}{t} = \sum_{\mu=1}^{M+Q} \epsilon(p_{\mu}) + \frac{\pi^2}{3} Q \left( 1 - \frac{1}{N^2} \right) - h \left( \frac{N - Q}{2} - M \right),
\]

(A-1)

where \( \epsilon(p) = p(p - 2\pi)/2 \) for \( 0 \leq p \leq 2\pi \). For later convenience the origin of pseudomomenta is shifted by \( \pi \). Then \( \epsilon(p) \equiv \epsilon(p + \pi) = (p^2 - \pi^2)/2 \) for \( |p| \leq \pi \). When we consider the value of the physical momentum, we restore the origin of pseudomomenta. Note that eq. (A-1) includes the Zeeman term. The pseudomomenta \( p_{\mu} \) are determined by the following equations:

\[
2\pi J_{\mu} = p_{\mu}N + \pi \sum_{i=1}^{Q} \text{sgn}(p_{\mu} - q_{i}),
\]

(A-2)

\[
-\pi \sum_{\nu=1}^{M+Q} \text{sgn}(p_{\mu} - p_{\nu}) \equiv \vartheta(p_{\mu})N,
\]

(A-3)

where \( J_{\mu} \in [-M + Q, (N - M - 1)/2], \) and \( I_{i} \in [-M + Q, (N + Q)/2] \) for \( i = 1, 2, \ldots, Q \). The quantum numbers \( J_{\mu} \) and \( I_{i} \) are integers or half-integers, and are arranged in the ascending order.

Before considering the dispersion relations, we must derive the ground-state distribution functions of pseud-
domonenta. For the ground state, both \(J_\mu\) and \(I_i\) are distributed densely and symmetrically with respect to zero. Namely, in the subspace with \((N,Q,M)\), \(J_\mu = -(M + Q - 1)/2 + \mu - 1\) for \(\mu = 1, 2, \cdots, M + Q\), and \(I_i = -(Q-1)/2 + i - 1\) for \(i = 1, 2, \cdots, Q\). In the large-\(N\) limit, using \(2\pi \sigma(p) = d\epsilon(p)/dp\) and \(2\pi \rho(q) = dw(q)/dq\), we obtain

\[
\sigma_0(p) = \Theta(B_0 - |p|) \left[ \frac{1}{2\pi} + \int_{-D_0}^{B_0} dp \rho_0(q) \delta(p - q) \right]
\]

\[
\rho_0(q) = \Theta(D_0 - |q|) \int_{-B_0}^{B_0} dp \sigma_0(p) \delta(q - p),
\]

where \(\Theta(p)\) is the step function. Here we have attached the subscript 0 to quantities corresponding to the ground state. The cut-off parameters \(B_0\) and \(D_0\) \((B_0 \geq D_0)\) are determined by

\[
\frac{2 - \bar{n} - \bar{m}}{2} = \frac{M + Q}{N} = \int_{-B_0}^{B_0} \sigma_0(p) dp, \quad (A.6)
\]

\[
1 - \bar{n} = \frac{Q}{N} = \int_{-D_0}^{D_0} \rho_0(q) dq. \quad (A.7)
\]

Equations (A-4)-(A-7) lead to the ground-state distributions \(\sigma_0(p)\) and \(\rho_0(q)\).

We then obtain the ground-state energy per site

\[
\frac{\epsilon_0}{t} = \int_{-\pi}^{\pi} dp \rho_0(p) \epsilon(p) + \frac{\pi^2}{3} (1 - \bar{n}) - \frac{1}{2} \bar{m} \hbar
\]

\[
= \frac{\pi^2}{12} \left( \bar{n}^3 - 3\bar{n}^2 + 4\bar{n} + \bar{m}^3 - 3\bar{m}^2 \right) - \frac{1}{2} \bar{m} \hbar, \quad (A.10)
\]

in the thermodynamic limit. The ground-state momentum is given by

\[
q_0 = N \int_{-\pi}^{\pi} dp \sigma_0(p)(p + \pi)
\]

\[
= \frac{\pi N}{2} (2 - \bar{n} - \bar{m}). \quad (A.11)
\]

Now that we have obtained information on the ground state, the next task is to derive the dispersion relations of quasi-particles. We take the following steps:

(i) We first give the configurations of \(J_\mu\) and \(I_i\) so that the quasi-particle can be created in the pseudomomentum space.

(ii) We find distribution functions \(\sigma(p)\) and \(\rho(q)\) under the configurations given in (i).

(iii) In general, the distribution functions determining the excitation energy can be written in the form

\[
\sigma(p) = \sigma_0(p) + (1/N) \sigma_1(p), \quad (A.12)
\]

\[
\rho(q) = \rho_0(q) + (1/N) \rho_1(q). \quad (A.13)
\]

Thereby we get the explicit expression of \(\sigma_1(p)\).

(iv) We obtain the excitation energy and the momentum transfer by using \(\sigma_1(p)\), and then derive the dispersion relation.

A.1 Spinons

In a magnetic field, the dispersion relation of up spinons should be different from that of down spinons due to the Zeeman splitting. First, in order to obtain the up-spinon dispersion, we consider the spin excitation in the subspace with \((N,Q+1,M-1)\), where a down spin is removed. We take the numbers \(I_i\) \((i = 1, 2, \cdots, Q + 1)\) as

\[
I_i = -(Q-1)/2 + i - 1, \quad (A.14)
\]

which means that the charge excitation (holon excitation) takes the minimum energy that vanishes in the thermodynamic limit. On the other hand we choose the configuration of \(J_\mu\) \((\mu = 1, 2, \cdots, M + Q)\) with \(\mu_0\) being the location of spinon excitation as

\[
J_{\mu+1} - J_{\mu} = 1 + \delta_{\mu,\mu_0}, \quad (A.15)
\]

where \(J_\mu \in [- (M + Q)/2, (M + Q)/2]\). The position of a “hole” in the \(J_\mu\)-configuration is denoted by \(J\). When \(-(M + Q)/2 \leq J < I_1\) or \(I_{Q+1} < J < (M + Q)/2\), a spinon is well-defined in the pseudomomentum space. The distribution functions \(\sigma(p)\) and \(\rho(q)\) must satisfy the following equations:

\[
\sigma(p) = \Theta(B - |p|) \left[ \frac{1}{2\pi} + \int_{-D}^{D} dp \rho(q) \delta(p - q) \right]
\]

\[
- \int_{-B}^{B} dp \sigma(p') \delta(p - p')
\]

\[
- \frac{1}{N} \delta(p - p_h) \Theta(|p| - D) \right], \quad (A.16)
\]

\[
\rho(q) = \Theta(D - |q|) \int_{-B}^{B} dp \sigma(p) \delta(q - p), \quad (A.17)
\]

where \(p_h\) is the value of \(p\) for \(\mu = \mu_0\). Note that these equations fulfill the normalization conditions:

\[
\int_{-B}^{B} \sigma(p) dp = \frac{M + Q}{N}, \quad (A.18)
\]

\[
\int_{-D}^{D} \rho(q) dq = \frac{Q + 1}{N}. \quad (A.19)
\]

By introducing \(\sigma_1(p)\) by \(\sigma(p) = \sigma_0(p) + (1/N) \sigma_1(p)\), we find

\[
\sigma_1(p) = \frac{1}{2} \delta(p - p_h) \Theta(|p| - (1 - \bar{n})\pi)
\]

\[
\times \Theta((1 - \bar{m})\pi - |p|). \quad (A.20)
\]

We have used \(B \cong B_0\) and \(D \cong D_0\) in the large-\(N\) limit. Then the excitation energy from the chemical potential is given by

\[
\epsilon = \int_{-\pi}^{\pi} dp \sigma_1(p) \epsilon(p) - \frac{\hbar}{2}
\]
can be written in the form \( N, Q, M \). The subspace we consider is different from that for the
\( p \) for \( J = J_{M+Q+1} \) (A.1). The distribution function \( \sigma(p) \) can be written in the form
\[
\sigma(p) = \frac{1}{N} \delta(p - p_0) \Theta(|p| - (1 - \bar{m}) \pi) \\
\times \Theta(\pi - |p|).
\]
Note that this equation fulfills the normalization condition: \( \int \sigma(p) dp = (M + Q + 1)/N \). We can identify \( \sigma_1(p) \)
as follows:
\[
\sigma_1(p) = \frac{1}{2} (\Theta(|p| - (1 - \bar{m}) \pi) \\
\times \Theta(\pi - |p|)).
\]
Then the excitation energy is given by
\[
\epsilon = \frac{1}{2} (p^2 - p^2) + h,
\]
and the associated momentum transfer is \( k = p_0 + \pi \). Therefore we obtain the antispinon dispersion given by eq. (3.3).

A.3 Holons

We consider the charge excitation in the subspace with
\( N, Q + 1, M - 1 \), where a down-spin is removed from the ground state. The numbers \( J_\mu (\mu = 1, 2, \cdots, M + Q) \)
are chosen as follows:
\[
J_\mu = -(M + Q)/2 + \mu - 1,
\]
for \( \mu = 1, 2, \cdots, Q + 1 \) (A.37).

In the former case we denote \( q_p = 2\pi I_{Q+1}/N \), and in the latter case \( q_p = 2\pi I_1/N \). The distribution function \( \rho(q) \)
can be written in the form
\[
\rho(q) = \frac{1}{2\pi} \Theta((1 - \bar{n}) \pi - |q|) \\
+ \frac{1}{N} \delta(q - q_0) \Theta(|q| - (1 - \bar{n}) \pi).
\]
On the other hand, the distribution function \( \sigma(p) \) satisfies the following equation:
\[
\sigma(p) = \Theta(B - |p|) \left[ \frac{1}{2\pi} + \int_{-D}^D dq \rho(q) \delta(p - q) \\
- \int_{-B}^B dp' \sigma(p') \delta(p - p') \right].
\]
Note that these equations fulfill the normalization conditions: \( \rho(q) dq = (Q + 1)/N \) and \( \sigma(p) dp = (M + Q + 1)/N \).

By introducing \( \sigma_1(p) \) by \( \sigma(p) = \sigma_0(p) + (1/N) \sigma_1(p) \), we find
\[
\sigma_1(p) = \frac{1}{2} \delta(p - q_0) \Theta(|p| - (1 - \bar{n}) \pi) \\
\times \Theta((1 - \bar{n}) \pi - |p|).
\]
We have used \( B \cong B_0 \) in the large-\( N \) limit. Then the
The second term comes from $\pi^2Q(1-1/N^2)/3$ of eq. (A.42). The momentum transfer is given by $k = (q_p + \pi)/2$. The relation between $\epsilon$ and $k$ leads to eq. (3.4). The momentum $k$ is allowed for $\bar{n}\pi/2 \leq k \leq \bar{n}\pi/2$ and $(1 - \bar{n}/2)\pi \leq k \leq (1 - \bar{n}/2)\pi$. The lowest excitation energy, which occurs at $k = \bar{n}\pi/2$ or $(1 - \bar{n}/2)\pi$, corresponds to the absolute value of the chemical potential.\cite{Kim1996}

### A.4 Antiholes

In order to obtain the antiholes dispersion, we consider the excitation where two electrons are added. We assume that one of the two electrons has up-spin and the other has down-spin. Then the subspace becomes $(N, Q - 2, M + 1)$. The numbers of $J\mu$ ($\mu = 1, 2, \cdots, M + Q - 1$) are chosen as

$$J\mu = -(M + Q)/2 + \mu,$$

and the configurations of $I_i$ ($i = 1, 2, \cdots, Q - 2$) are

$$I_{i+1} - I_i = 1 + \delta_{i, i_0},$$

where $I_i \in \{-Q/2, (Q - 2)/2\}$. The position of a "hole" in the $I_i$-configuration is denoted by $\bar{I}$. When $-(Q - 2)/2 \ll \bar{I} < (Q - 2)/2$, an antiholes is well-defined in the pseudomomentum space. The distribution function $\rho(q)$ can be written in the form

$$\rho(q) = \left[\frac{1}{2\pi} - \frac{2}{N}\delta(q - q_0)\right] \Theta((1 - \bar{n})\pi - |q|),$$

where $q_0$ is the value of $q$ for $i = i_0$. Note that this equation fulfills the normalization condition: $\int \rho(q) dq = (Q - 2)/N$.

By introducing $\sigma_1(p)$ by $\sigma(p) = \sigma_0(p) + (1/N)\sigma_1(p)$, we find

$$\sigma_1(p) = -\delta(p - q_0)\Theta((1 - \bar{n})\pi - |p|).$$

Then the excitation energy is given by

$$\epsilon = \frac{1}{2}(q_0^2 - \pi^2) - \frac{2}{3}\pi^2.$$
Table I. List of all the possible excitations from the ground state with finite magnetization.

Fig. 1. Dispersion relations of (a) up and down spinons, (b) anti-spinons, (c) holons and (d) antiholons as given by eqs. (3.1)-(3.5). Allowed ranges of momenta and the right and left branches are shown for each species.

Fig. 2. Skew Young diagram corresponding to the motif 010|10|110|10|100. There are other possibilities to put 1, 2 or ◦ than the one shown in this figure.

Fig. 3. $S^{-+}(q, \omega)$ of the Haldane-Shastry model with 16 sites and (a) $\bar{m} = 0.25$ and (b) $\bar{m} = 0.5$. The intensity of each pole is proportional to the area of the circle. The solid lines show the dispersion lines of quasi-particles in the thermodynamic limit. This way of representation applies to all figures to follow.

Fig. 4. $S^{-+}(q, \omega)$ of the $1/r^2$-type supersymmetric t-J model with 16 sites, 2 holes and (a) $\bar{m} = 0.25$ and (b) $\bar{m} = 0.5$. The hatched area indicates the region where only two spinons contribute.

Fig. 5. $S^{zz}(q, \omega)$ of the Haldane-Shastry model with 16 sites and (a) $\bar{m} = 0.25$ and (b) $\bar{m} = 0.5$. The broken line indicates the magnetic field $h$.

Fig. 6. $S^{zz}(q, \omega)$ of the $1/r^2$-type supersymmetric t-J model with 16 sites, 2 holes and (a) $\bar{m} = 0.25$ and (b) $\bar{m} = 0.5$. The broken line indicates the magnetic field $h$. The hatched area indicates the region where holons and antiholons do not contribute at all.

Fig. 7. $S^{+-}(q, \omega)$ of the Haldane-Shastry model with 16 sites and (a) $\bar{m} = 0.25$ and (b) $\bar{m} = 0.5$. The broken line indicates the magnetic field $h$ and its double $2h$.

Fig. 8. $S^{+-}(q, \omega)/4$ of the $1/r^2$-type supersymmetric t-J model with 16 sites, 2 holes and (a) $\bar{m} = 0.25$ and (b) $\bar{m} = 0.5$. The broken line indicates the magnetic field $h$ and its double $2h$. The hatched area indicates the region where holons and antiholons do not contribute at all.

Fig. 9. $N(q, \omega)$ of the $1/r^2$-type supersymmetric t-J model with 16 sites, 2 holes and (a) $\bar{m} = 0.25$ and (b) $\bar{m} = 0.5$. The broken line indicates the magnetic field $h$. The hatched area indicates the region where spinons and antispinons do not contribute at all.

Fig. 10. $S^{zz}(q)$ of the $1/r^2$-type supersymmetric t-J model with 16 sites, $\bar{m} = 0.25$ and two different values of filling.

Fig. 11. $S^{zz}(q)$ of the $1/r^2$-type supersymmetric t-J model with 16 sites, $\bar{m} = 0.25$ and two different values of filling.

Fig. 12. $N(q)$ of the $1/r^2$-type supersymmetric t-J model with 16 sites, $\bar{m} = 0.875$ and two different values of magnetization.

Fig. 13. $S^{zz}(q)$ of the nearest-neighbor-type t-J model with 16 sites, $\bar{m} = 0.25$ and two different values of filling. (a) $J/t = 2$; (b) $J/t = 0.5$.

Fig. 14. $S^{zz}(q)$ of the nearest-neighbor-type t-J model with 16 sites, $\bar{m} = 0.25$ and two different values of filling. (a) $J/t = 2$; (b) $J/t = 0.5$.

Fig. 15. $N(q)$ of the nearest-neighbor-type t-J model with 16 sites, $\bar{m} = 0.875$ and two different values of magnetization. (a) $J/t = 2$; (b) $J/t = 0.5$. 
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This figure "figureII.jpg" is available in "jpg" format from:

http://arxiv.org/ps/cond-mat/0009075v1
Table 1: List of all the possible excitations from the ground state with finite magnetization.

| Dynamical quantities | Excitation contents |
|----------------------|---------------------|
| $S^{-+}(q, \omega)$  | $(s_L, h_L) + \bar{h} + (h_R, s_R)$  |
|                      | $2s_R, (L \leftrightarrow R)$ |
| $S^{zz}(q, \omega)$  | $(s_L, h_L) + \bar{h} + (h_R, s_R) + \bar{s}_R, (L \leftrightarrow R)$ |
|                      | $(s_L, h_L) + \bar{h} + (h_R, s_R)$ |
|                      | $2s_R + \bar{s}_R, (L \leftrightarrow R)$ |
|                      | $2s_L + \bar{s}_R, (L \leftrightarrow R)$ |
|                      | $2s_R, (L \leftrightarrow R)$ |
| $S^{+-}(q, \omega)$  | $\bar{s}_L + (s_L, h_L) + \bar{h} + (h_R, s_R) + \bar{s}_R, (L \leftrightarrow R)$ |
|                      | $\bar{s}_L + (s_L, h_L) + \bar{h} + (h_R, s_R)$ |
|                      | $2s_R + \bar{s}_R, (L \leftrightarrow R)$ |
|                      | $2s_L + \bar{s}_R, (L \leftrightarrow R)$ |
|                      | $2s_R, (L \leftrightarrow R)$ |
|                      | $\bar{s}_L, (L \leftrightarrow R)$ |
| $N(q, \omega)$       | $(s_L, h_L) + \bar{h} + (h_R, s_R) + \bar{s}_R, (L \leftrightarrow R)$ |
|                      | $(s_L, h_L) + \bar{h} + (h_R, s_R)$ |
|                      | $\bar{h} + 2h_R, (L \leftrightarrow R)$ |