On the Interactions of Light Gravitinos

T.E. Clark¹, Taekoon Lee², S.T. Love³, Guo-Hong Wu⁴

Department of Physics
Purdue University
West Lafayette, IN 47907-1396

Abstract

In models of spontaneously broken supersymmetry, certain light gravitino processes are governed by the coupling of its Goldstino components. The rules for constructing SUSY and gauge invariant actions involving the Goldstino couplings to matter and gauge fields are presented. The explicit operator construction is found to be at variance with some previously reported claims. A phenomenological consequence arising from light gravitino interactions in supernova is reexamined and scrutinized.

¹e-mail address: clark@physics.purdue.edu
²e-mail address: tlee@physics.purdue.edu
³e-mail address: love@purdd.physics.purdue.edu
⁴e-mail address: wu@physics.purdue.edu
In the supergravity theories obtained from gauging a spontaneously broken global $N = 1$ supersymmetry (SUSY), the Nambu-Goldstone fermion, the Goldstino [1, 2], provides the helicity $\pm \frac{1}{2}$ degrees of freedom needed to render the spin $\frac{3}{2}$ gravitino massive through the super-Higgs mechanism. For a light gravitino, the high energy (well above the gravitino mass) interactions of these helicity $\pm \frac{1}{2}$ modes with matter will be enhanced according to the supersymmetric version of the equivalence theorem [3]. The effective action describing such interactions can then be constructed using the properties of the Goldstino fields. Currently studied gauge mediated supersymmetry breaking models [4] provide a realization of this scenario as do certain no-scale supergravity models [5]. In the gauge mediated case, the SUSY is dynamically broken in a hidden sector of the theory by means of gauge interactions resulting in a hidden sector Goldstino field. The spontaneous breaking is then mediated to the minimal supersymmetric standard model (MSSM) via radiative corrections in the standard model gauge interactions involving messenger fields which carry standard model vector representations. In such models, the supergravity contributions to the SUSY breaking mass splittings are small compared to these gauge mediated contributions. Being a gauge singlet, the gravitino mass arises only from the gravitational interaction and is thus far smaller than the scale $\sqrt{F}$, where $F$ is the Goldstino decay constant. More-
over, since the gravitino is the lightest of all hidden and messenger sector
degrees of freedom, the spontaneously broken SUSY can be accurately de-
scribed via a non-linear realization. Such a non-linear realization of SUSY
on the Goldstino fields was originally constructed by Volkov and Akulov [1].

The leading term in a momentum expansion of the effective action de-
scribing the Goldstino self-dynamics at energy scales below $\sqrt{4\pi F}$ is uniquely
fixed by the Volkov-Akulov effective Lagrangian [1] which takes the form
\[ L_{AV} = -\frac{F^2}{2} \det A. \] (1)

Here the Volkov-Akulov vierbein is defined as $A_{\mu}^\nu = \delta^\nu_\mu + \frac{i}{F} \lambda^{\nu \sigma} \partial_\mu \sigma^\nu \bar{\lambda}$, with $\lambda(\bar{\lambda})$ the Goldstino Weyl spinor field. This effective Lagrangian pro-
vides a valid description of the Goldstino self interactions independent of
the particular (non-perturbative) mechanism by which the SUSY is dynam-
ically broken. The supersymmetry transformations are nonlinearly realized
on the Goldstino fields as $\delta^Q(\xi, \bar{\xi}) \lambda^\alpha = F \xi^\alpha + \Lambda^\rho \partial_\rho \lambda^\alpha$ ; $\delta^Q(\xi, \bar{\xi}) \bar{\lambda}_{\dot{\alpha}} = F \bar{\xi}_{\dot{\alpha}} + \Lambda^\rho \partial_\rho \bar{\lambda}_{\dot{\alpha}}$, where $\xi^\alpha, \bar{\xi}_{\dot{\alpha}}$ are Weyl spinor SUSY transformation param-
eters and $\Lambda^\rho \equiv -\frac{i}{F} \left( \lambda^\sigma \bar{\xi} - \xi^\sigma \bar{\lambda} \right)$ is a Goldstino field dependent translation
vector. Since the Volkov-Akulov Lagrangian transforms as the total diver-
gence $\delta^Q(\xi, \bar{\xi})L_{AV} = \partial_\rho (\Lambda^\rho L_{AV})$, the associated action $I_{AV} = \int d^4x \ L_{AV}$ is
SUSY invariant.

The supersymmetry algebra can also be nonlinearly realized on the matter
(non-Goldstino) fields, generically denoted by $\phi^i$, where $i$ can represent any Lorentz or internal symmetry labels, as

$$\delta^Q(\xi, \bar{\xi})\phi^i = \Lambda^\rho \partial_\rho \phi^i .$$  \hspace{1cm} (2)

This is referred to as the standard realization \[6\]-\[9\]. It can be used, along with space-time translations, to readily establish the SUSY algebra. Under the non-linear SUSY standard realization, the derivative of a matter field transforms as $\delta^Q(\xi, \bar{\xi})(\partial_\nu \phi^i) = \Lambda^\rho \partial_\rho (\partial_\nu \phi^i) + (\partial_\nu \Lambda^\rho)(\partial_\rho \phi^i)$. In order to eliminate the second term on the right hand side and thus restore the standard SUSY realization, a SUSY covariant derivative is introduced and defined so as to transform analogously to $\phi^i$. To achieve this, we use the transformation property of the Volkov-Akulov vierbein and define the non-linearly realized SUSY covariant derivative \[9\]

$$D_\mu \phi^i = (A^{-1})_\mu^\nu \partial_\nu \phi^i ,$$  \hspace{1cm} (3)

which varies according to the standard realization of SUSY:

$$\delta^Q(\xi, \bar{\xi})(D_\mu \phi^i) = \Lambda^\rho \partial_\rho (D_\mu \phi^i).$$

Any realization of the SUSY transformations can be converted to the standard realization. In particular, consider the gauge covariant derivative,

$$(D_\mu \phi)^i \equiv \partial_\mu \phi^i + T^a_{ij} A^a_\mu \phi^j ,$$  \hspace{1cm} (4)
with \( a = 1, 2, \ldots, \text{Dim } G \). We seek a SUSY and gauge covariant derivative \((D_\mu \phi)^i\), which transforms as the SUSY standard realization. Using the Volkov-Akulov vierbein, we define

\[
(D_\mu \phi)^i \equiv (A^{-1})_\mu^\nu (D_\nu \phi)^i ,
\]

which has the desired transformation property, \( \delta^Q(\xi, \bar{\xi})(D_\mu \phi)^i = \Lambda^\rho \partial_\rho (D_\mu \phi)^i \), provided the vector potential has the SUSY transformation \( \delta^Q(\xi, \bar{\xi})A_\mu \equiv \Lambda^\rho \partial_\rho A_\mu + \partial_\mu \Lambda^\rho A_\rho \). Alternatively, we can introduce a redefined gauge field

\[
V^a_\mu \equiv (A^{-1})_\mu^\nu A^a_\nu ,
\]

which itself transforms as the standard realization, \( \delta^Q(\xi, \bar{\xi})V^a_\mu = \Lambda^\rho \partial_\rho V^a_\mu \), and in terms of which the standard realization SUSY and gauge covariant derivative then takes the form

\[
(D_\mu \phi)^i \equiv (A^{-1})_\mu^\nu \partial_\nu \phi^i + T^a_{ij} V^a_\mu \phi^j .
\]

Under gauge transformations parameterized by \( \omega^a \), the original gauge field varies as \( \delta^G(\omega)A^a_\mu = (D_\mu \omega)^a_\mu = \partial_\mu \omega^a + gf_{abc} A^b_\mu \omega^c \), while the redefined gauge field \( V^a_\mu \) has the Goldstino dependent transformation: \( \delta^G(\omega)V^a_\mu = (A^{-1})_\mu^\nu (D_\nu \omega)^a_\mu \). For all realizations, the gauge transformation and SUSY transformation commutator yields a gauge variation with a SUSY transformed value of the gauge transformation parameter,

\[
[\delta^G(\omega), \delta^Q(\xi, \bar{\xi})] = \delta^G(\Lambda^\rho \partial_\rho \omega - \delta^Q(\xi, \bar{\xi})\omega) .
\]
If we further require the local gauge transformation parameter to also transform under the standard realization so that $\delta^Q(\xi,\bar{\xi})\omega^a = \Lambda^\rho \partial_\rho \omega^a$, then the gauge and SUSY transformations commute.

In order to construct an invariant kinetic energy term for the gauge fields, it is convenient for the gauge covariant anti-symmetric tensor field strength to also be brought into the standard realization. The usual field strength $F^a_{\alpha\beta} = \partial_\alpha A^a_\beta - \partial_\beta A^a_\alpha + if_{abc}A^b_\alpha A^c_\beta$ varies under SUSY transformations as $\delta^Q(\xi,\bar{\xi})F^a_{\mu\nu} = \Lambda^\rho \partial_\rho F^a_{\mu\nu} + \partial_\mu \Lambda^\rho F^a_{\rho\nu} + \partial_\nu \Lambda^\rho F^a_{\mu\rho}$. A standard realization of the gauge covariant field strength tensor, $F^a_{\mu\nu}$, can be then defined as

$$\mathcal{F}^a_{\mu\nu} = (A^{-1})^\alpha_\mu (A^{-1})^\beta_\nu F^a_{\alpha\beta}, \quad (9)$$

so that $\delta^Q(\xi,\bar{\xi})\mathcal{F}^a_{\mu\nu} = \Lambda^\rho \partial_\rho \mathcal{F}^a_{\mu\nu}$.

These standard realization building blocks consisting of the gauge singlet Goldstino SUSY covariant derivatives, $\mathcal{D}_\mu \lambda, \mathcal{D}_\mu \bar{\lambda}$, the matter fields, $\phi_i$, their SUSY-gauge covariant derivatives, $\mathcal{D}_\mu \phi^i$, and the field strength tensor, $\mathcal{F}^a_{\mu\nu}$, along with their higher covariant derivatives can be combined to make SUSY and gauge invariant actions. These invariant action terms then dictate the couplings of the Goldstino which, in general, carries the residual consequences of the spontaneously broken supersymmetry.

A generic SUSY and gauge invariant action can be constructed as

$$I_{\text{eff}} = \int d^4x \det A \mathcal{L}_{\text{eff}}(\mathcal{D}_\mu \lambda, \mathcal{D}_\mu \bar{\lambda}, \phi^i, \mathcal{D}_\mu \phi^i, \mathcal{F}_{\mu\nu}) \quad (10)$$
where $\mathcal{L}_{\text{eff}}$ is any gauge invariant function of the standard realization basic building blocks. Using the nonlinear SUSY transformations $\delta^Q (\xi, \bar{\xi}) \det A = \partial_\rho (\Lambda^\rho \det A)$ and $\delta^Q (\xi, \bar{\xi}) \mathcal{L}_{\text{eff}} = \Lambda^\rho \partial_\rho \mathcal{L}_{\text{eff}}$, it follows that $\delta^Q (\xi, \bar{\xi}) I_{\text{eff}} = 0$.

It proves convenient to catalog the terms in the effective Lagrangian, $\mathcal{L}_{\text{eff}}$, by an expansion in the number of Goldstino fields which appear when covariant derivatives are replaced by ordinary derivatives and the Volkov-Akulov vierbein appearing in the standard realization field strengths are set to unity. So doing, we expand

$$
\mathcal{L}_{\text{eff}} = \left[ \mathcal{L}_{(0)} + \mathcal{L}_{(1)} + \mathcal{L}_{(2)} + \cdots \right],
$$

where the subscript $n$ on $\mathcal{L}_{(n)}$ denotes that each independent SUSY invariant operator in that set begins with $n$ Goldstino fields.

$\mathcal{L}_{(0)}$ consists of all gauge and SUSY invariant operators made only from light matter fields and their SUSY covariant derivatives. Any Goldstino field appearing in $\mathcal{L}_{(0)}$ arises only from higher dimension terms in the matter covariant derivatives and/or the field strength tensor. Taking the light non-Goldstino fields to be those of the MSSM and retaining terms through mass dimension 4, then $\mathcal{L}_{(0)}$ is well approximated by the Lagrangian of the minimal supersymmetric standard model which includes the soft SUSY breaking terms, but in which all derivatives have been replaced by SUSY covariant ones and the field strength tensor replaced by the standard realization field
strength:

\[ \mathcal{L}_{(0)} = \mathcal{L}_{\text{MSSM}}(\phi, \mathcal{D}_\mu \phi, \mathcal{F}_{\mu\nu}). \]  

(12)

Note that the coefficients of these terms are fixed by the normalization of the gauge and matter fields, their masses and self-couplings; that is, the normalization of the Goldstino independent Lagrangian.

The \( \mathcal{L}_{(1)} \) terms in the effective Lagrangian begin with direct coupling of one Goldstino covariant derivative to the non-Goldstino fields. The general form of these terms, retaining operators through mass dimension 6, is given by

\[ \mathcal{L}_{(1)} = \frac{1}{F}[\mathcal{D}_\mu \lambda^\alpha Q_{\text{MSSM} \alpha}^\mu + \bar{Q}_{\text{MSSM} \dot{\alpha}}^\mu \mathcal{D}_\mu \bar{\lambda}^\dot{\alpha}], \]  

(13)

where \( Q_{\text{MSSM} \alpha}^\mu \) and \( \bar{Q}_{\text{MSSM} \dot{\alpha}}^\mu \) contain the pure MSSM field contributions to the conserved gauge invariant supersymmetry currents with once again all field derivatives being replaced by SUSY covariant derivatives and the vector field strengths in the standard realization. That is, it is this term in the effective Lagrangian which, using the Noether construction, produces the Goldstino independent piece of the conserved supersymmetry current. The Lagrangian \( \mathcal{L}_{(1)} \) describes processes involving the emission or absorption of a single helicity \( \pm \frac{1}{2} \) gravitino.

Finally the remaining terms in the effective Lagrangian all contain two or more Goldstino fields. In particular, \( \mathcal{L}_{(2)} \) begins with the coupling of two
Goldstino fields to matter or gauge fields. Retaining terms through mass dimension 8 and focusing only on the $\lambda - \bar{\lambda}$ terms, we can write

$$L^{(2)} = \frac{1}{F^2} D_\mu \lambda^\alpha D_\nu \bar{\lambda}^\dot{\alpha} M^{\mu\nu}_{\alpha\dot{\alpha}} + \frac{1}{F^2} D_\mu \lambda^\alpha \bar{D}_\rho D_\nu \bar{\lambda}^\dot{\alpha} M^{\mu\nu\rho}_{\alpha\dot{\alpha}}$$

$$+ \frac{1}{F^2} D_\rho \left[D_\mu \lambda^\alpha D_\nu \bar{\lambda}^\dot{\alpha}\right] M^{\mu\nu\rho}_{\alpha\dot{\alpha}},$$

(14)

where the standard realization composite operators that contain matter and gauge fields are denoted by the $M_i$. They can be enumerated by their operator dimension, Lorentz structure and field content. In the gauge mediated models, these terms are all generated by radiative corrections involving the standard model gauge coupling constants.

Let us now focus on the pieces of $L^{(2)}$ which contribute to a local operator containing two gravitino fields and is bilinear in a Standard Model fermion $(f, \bar{f})$. Those lowest dimension operators (which involve no derivatives on $f$ or $\bar{f}$) are all contained in the $M_1$ piece. After application of the Goldstino field equation (neglecting the gravitino mass) and making prodigious use of Fierz rearrangement identities, this set reduces to just 1 independent on-shell interaction term. In addition to this operator, there is also an operator bilinear in $f$ and $\bar{f}$ and containing 2 gravitinos which arises from the product of $det A$ with $L^{(0)}$. Combining the two independent on-shell interaction terms involving 2 gravitinos and 2 fermions, results in the effective action

$$I_{ff\bar{f}\bar{G}} = \int d^4x \left[-\frac{1}{2F^2} \left(\bar{\lambda} \tilde{\partial}_\mu \sigma^\nu \lambda\right) \left(f \tilde{\partial}_\nu \sigma^\mu \bar{f}\right)\right]$$
\[ + \frac{C_{ff}}{F^2} (f \partial^\mu \lambda) \left( f \bar{\partial}_\mu \bar{\lambda} \right) , \]  

(15)

where \( C_{ff} \) is a model dependent real coefficient. Note that the coefficient of the first operator is fixed by the normalization of the MSSM Lagrangian. This result is in accord with a recent analysis \[10\] where it was found that the fermion-Goldstino scattering amplitudes depend on only one parameter which corresponds to the coefficient \( C_{ff} \) in our notation.

In a similar manner, the lowest mass dimension operator contributing to the effective action describing the coupling of two on-shell gravitinos to a single photon arises from the \( M_1 \) and \( M_3 \) pieces of \( \mathcal{L}_{(2)} \) and has the form

\[ I_{\gamma \bar{G} \bar{G}} = \int d^4 x \left[ \frac{C_\gamma}{F^2} \left( \partial^\mu \lambda \sigma^{\mu \nu} \bar{\lambda} \right) \partial_\mu F_{\rho \nu} \right] + h.c. , \]  

(16)

with \( C_\gamma \) a model dependent real coefficient and \( F_{\mu \nu} \) is the electromagnetic field strength. Note that the operator in the square bracket is odd under both parity (\( P \)) and charge conjugation (\( C \)). In fact any operator arising from a gauge and SUSY invariant structure which is bilinear in two on-shell gravitinos and contains only a single photon is necessarily odd in both \( P \) and \( C \). Thus the generation of any such operator requires a violation of both \( P \) and \( C \). Using the Goldstino equation of motion, the analogous term containing \( \bar{F}_{\mu \nu} \) reduces to Eq.(16) with \( C_\gamma \to -iC_\gamma \). Recently, there has appeared in the literature \[11\] the claim that there is a lower dimensional operator of the form \( \frac{M^2}{F} \partial^\nu \lambda \sigma^{\mu \nu} \bar{\lambda} F_{\mu \nu} \) which contributes to the single photon-
2 gravitino interaction. Here $\tilde{M}$ is a model dependent SUSY breaking mass parameter which is roughly an order(s) of magnitude less than $\sqrt{F}$. From our analysis, we do not find such a term to be part of a SUSY invariant action piece and thus it should not be included in the effective action. Such a term is also absent if one employs the equivalent formalism of Wess and Samuel. We have also checked that such a term does not appear via radiative corrections by an explicit graphical calculation using the correct non-linearly realized SUSY invariant action. This is also contrary to the previous claim.

There have been several recent attempts to extract a lower bound on the SUSY breaking scale using the supernova cooling rate \cite{11, 12, 13}. Unfortunately, some of these estimates \cite{11, 13} rely on the existence of the non-SUSY invariant dimension 6 operator referred to above. Using the correct low energy effective lagrangian of gravitino interactions, the leading term coupling 2 gravitinos to a single photon contains an additional supression factor of roughly $C_s \frac{\tilde{M}^2}{\sqrt{s}}$. Taking $\sqrt{s} \approx 0.1$ GeV for the processes of interest and using $\tilde{M} \sim 100$ GeV, this introduces an additional supression of at least $10^{-12}$ in the rate and obviates the previous estimates of a bound on $F$.

Assuming that the mass scales of gauginos and the superpartners of light fermions are above the core temperature of supernova, the gravitino cooling of supernova occurs mainly via gravitino pair production. It is interesting to
compare the gravitino pair production cross section to that of the neutrino pair production, which is the main supernova cooling channel. We have seen that for low energy gravitino interactions with matter, the amplitudes for gravitino pair production is proportional to $1/F^2$. A simple dimensional analysis then suggests the ratio of the cross sections is:

$$\frac{\sigma_{\chi\chi}}{\sigma_{\nu\nu}} \sim \frac{s^2}{F^4 G_F^2}$$

(17)

where $G_F$ is the Fermi coupling and $\sqrt{s}$ is the typical energy scale of the particles in a supernova. Even with the most optimistic values for $F$, the gravitino production is too small to be relevant. For example, taking $\sqrt{F} = 100\, GeV, \sqrt{s} = .1\, GeV$, the ratio is of $O(10^{-11})$. It seems, therefore, that such an astrophysical bound on the SUSY breaking scale is untenable in models where the gravitino is the only superparticle below the scale of supernova core temperature.

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