BUCKLING ANALYSIS OF TWO-DIRECTIONAL FUNCTIONALLY GRADED SANDWICH PLATES BASED ON A QUASI-3D SHEAR DEFORMATION Q4 ELEMENT

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Abstract. Quasi-3D shear deformation theory taking the thickness stretching effect into account is employed to develop a Q4 plate element for buckling analysis of two-directional functionally graded sandwich (2D-FGSW) plates. The plates consist of three layers, a homogeneous core and two functionally graded face layers with material properties varying in both the thickness and length directions by power gradation laws. The element with nine degrees of freedom per node is derived from energy expression and employed in computing the buckling loads. The accuracy of the element is confirmed by comparing the obtained result with published data. A parametric study is carried out to illustrate the effect of the material gradation and the layer thickness ratio on the buckling loads. The influence of the length to thickness ratio on the buckling loads of the plates is also examined and discussed.

Keywords: 2D-FGWS plate; power law; quasi-3D theory; Q4 element; buckling analysis.

Classification numbers: 2.9.4, 5.4.2, 5.4.3.

1. INTRODUCTION

Functionally graded materials (FGMs) invented by Japanese researchers in mid-1980 have many advanced properties, including high strength and good performance in a high-temperature environment. These materials are widely used in engineering applications, for instance, aerospace technology, medicine, and defense industry. Investigations on mechanical behaviors of FGM structures attract much attention from scientists all over the world.

Investigations on behavior of FGM plates have been reported by several authors in recent years. Zenkour [1-4] studied the static bending, free vibration, buckling, and thermal buckling of fully simply supported rectangular functionally graded sandwich (FGSW) plates without micro holes. The material gradation which has been shown by the author plays an important role on the
mechanical behavior of the plates. Bateni et al. [5] studied instability of a thick FGM rectangular plates under the practical cases of thermal and mechanical loading conditions. Neves et al. [6-8] adopted the higher-order shear deformation theory with hyperbolic and sinusoidal quasi-3D functions to investigate the static bending, free vibration and buckling of FGSW plates. Thai et al. [9] proposed a 3D theory with five unknowns for bending of fully simply supported rectangular FGM plates. The finite element formulation are then derived for computing deflections and stresses of the plates. In another work, Thai and his co-workers [10] employed the new third-order shear deformation theory to study the static bending, free vibration and buckling of the rectangular FGSW plates. Farzam-Rad et al. [11] employed the quasi-3D shear deformation theory and isogeometric method to analyze the static bending and free vibration of FGSW plate in consideration of physical plane. Vafakhah and Neya [12] presented an exact solution for bending of a thick plate. The solution is derived based on a fully simply supported 3D rectangular FG plate model. 

In this paper, the quasi-3D shear deformation theory is adopted to study buckling of a rectangular two-directional functionally graded sandwich (2D-FGSW) plate. The plate consists of three layer, a homogeneous ceramic core and two FGM face layers with material properties vary in both thickness and length directions by power gradation laws. A Q4 plate element is derived and employed in computing critical loads and buckling mode shapes of the plate. The effect of the material distribution and aspect ratio on the buckling behavior of the plate is examined and highlighted.

2. PROBLEM FORMULATIONS

A rectangular 2D-FGSW plate with the dimensions in x-, y- and z-directions are, respectively a, b and h, as shown in Figure 1 is considered. The plate is made from two constituent materials, a ceramic and a metal, with the material properties are denoted as $P_c$ and $P_m$ respectively. The plate consists three layers, a homologous core and two FGM face layers with material properties vary in both the x- and z-directions.

The effective properties of the FGM layers evaluated by Voigt’s model is

$$P(x, z) = V_c(x, z)P_c + V_m(x, z)P_m$$

where $V_c, V_m$ are the volume fraction of ceramic and metal, and they are defined as

$$V_m(x, z) = \begin{cases} \left(\frac{z - z_2}{z_3 - z_2}\right)^{p_z}\left(1 - \frac{x}{2a}\right)^{p_x} & \text{for } z \in [z_2, z_3] \\ 0 & \text{for } z \in [z_1, z_2] \\ \left(\frac{z - z_1}{z_0 - z_1}\right)^{p_z}\left(1 - \frac{x}{2a}\right)^{p_x} & \text{for } z \in [z_0, z_1] \end{cases}$$

and

$$V_c(x, z) = 1 - V_m(x, z)$$

in which $p_x, p_z$ are the positive grading indexes. When $p_x = 0$ the plate becomes an unidirectional FGSW (1D-FGSW) plate with the material property varying in z-direction only.

Substituting Eqs. (2) and (3) into Eq. (1) we get
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\[ P(x, z) = \begin{cases} 
  P_{m} - P_c \left( \frac{z - z_1}{z_0 - z_1} \right)^{p_z} \left( 1 - \frac{x}{2a} \right)^{p_x} + P_c & \text{for } z \in [z_0, z_1] \\
  P_c & \text{for } z \in [z_1, z_2] \\
  P_{m} - P_c \left( \frac{z - z_2}{z_3 - z_2} \right)^{p_z} \left( 1 - \frac{x}{2a} \right)^{p_x} + P_c & \text{for } z \in [z_2, z_3] 
\end{cases} \] (4)

\[ u(x, y, z) = u_0(x, y, 0) - z w_{p,x} - f(z) w_{s,x} \\
v(x, y, z) = v_0(x, y, 0) - z w_{p,y} - f(z) w_{s,y} \\
w(x, y, z) = w_b + w_s + g(z) w_z \]

(6)

where \( u_0, v_0, w_b, w_s \) and \( w_z \) are the unknown components in the neutral plane, and

\[ f(z) = \frac{4z^3}{3h^2}, \quad g(z) = 1 - \frac{4z^2}{h^2} \]

(7)

Figure 1. Model of 2D-FGSW plate under uniform compression load.

The 2D-FGSW plate considered herein is assumed under uniform compression loads, acting on the neutral plane at all edges. In \( x \)- and \( y \)-directions, the compression loads are, respectively, \( q_x \) and \( q_y \), as in lower part of Figure 1. The compression loads \( q_x \) and \( q_y \) are defined as follows

\[ q_x = \gamma_1 P_{cr}, \quad q_y = \gamma_2 P_{cr} \]

(5)

where \( \gamma_1, \gamma_2 \) are two values 0 and 1; \( P_{cr} \) is the critical load.

Based on the quasi-3D shear deformation theory [9], the displacement field at any points is defined as follows
In Eq. (6) and hereafter, a lower subscript comma is used to denote the derivative with respect the followed variable. The strain field resulted from (6) is given by
\[ \varepsilon_x = u_{0,x} - z w_{b,xx} - f w_{s,xx}; \varepsilon_y = v_{0,y} - z w_{b,yy} - f w_{s,yy} \]
\[ \varepsilon_z = g_{,z} w_z; \gamma_{xy} = (u_{0,y} + v_{0,x}) - 2 zw_{b,xy} - 2fw_{s,xy} \]
\[ \gamma_{xz} = g(w_{z,x} + w_{s,x}); \gamma_{yz} = g(w_{z,y} + w_{s,y}) \]

The stress field of the structure is expressed as follows
\[ \sigma_x = E(z)[a_1 \varepsilon_x + a_2 \varepsilon_y + a_3 \varepsilon_z], \quad \tau_{xy} = E'(z)a_3 \gamma_{xy} \]
\[ \sigma_y = E(z)[a_2 \varepsilon_x + a_1 \varepsilon_y + a_3 \varepsilon_z], \quad \tau_{yz} = E'(z)a_3 \gamma_{yz} \]
\[ \sigma_z = E(z)[a_3 \varepsilon_x + a_2 \varepsilon_y + a_1 \varepsilon_z], \quad \tau_{xz} = E'(z)a_3 \gamma_{xz} \]

where \( a_1, a_2, a_3 \) are calculated as [8]
\[ a_1 = \frac{1-v^2}{1-3v^2-2v^3}; a_2 = \frac{v(1+v)}{1-3v^2-2v^3}; a_3 = \frac{1}{2(1+v)} \]

3. FINITE ELEMENT MODEL AND FORMULATIONS

The plate is assumed being divided into a number of 4-node finite elements with the vector of nodal displacements of the form
\[
\mathbf{d} = \begin{bmatrix} d_1^T & d_2^T & d_3^T & d_4^T \end{bmatrix}^T
\]

where \( d_i \) (i = 1-4) is the vector of displacements at node I, and it contains 9 components as
\[
\mathbf{d}_i = \begin{bmatrix} u_{0i} & v_{0i} & w_{bi} & w_{si} & w_{zi} & \theta_{bix} & \theta_{biy} & \theta_{six} & \theta_{siy} \end{bmatrix}^T
\]

where \( \theta_{bx} = w_{b,x}, \theta_{sx} = w_{s,x}, \theta_{by} = w_{b,y}, \theta_{sy} = w_{s,y} \) are the rotations due to bending and shear deformation, respectively. The displacements are interpolated from the nodal displacements according to
\[
\begin{align*}
u_0 &= \mathbf{N}_d \mathbf{d}; & v_0 &= \mathbf{N}_v \mathbf{d}; & w_b &= \mathbf{N}_{wb} \mathbf{d}; & w_s &= \mathbf{N}_{ws} \mathbf{d}; & w_z &= \mathbf{N}_{wz} \mathbf{d}
\end{align*}
\]
where \( \mathbf{N}_d, \mathbf{N}_v, \mathbf{N}_{wb}, \mathbf{N}_{ws}, \mathbf{N}_{wz} \) are the shape functions with the following forms
\[
\begin{align*}
\mathbf{N}_d &= \begin{bmatrix} N_1^{(1)} & N_2^{(1)} & N_3^{(1)} & N_4^{(1)} \end{bmatrix}; & \mathbf{N}_v &= \begin{bmatrix} N_1^{(2)} & N_2^{(2)} & N_3^{(2)} & N_4^{(2)} \end{bmatrix}; \\
\mathbf{N}_{wb} &= \begin{bmatrix} N_1^{(3)} & N_2^{(3)} & N_3^{(3)} & N_4^{(3)} \end{bmatrix}; & \mathbf{N}_{ws} &= \begin{bmatrix} N_1^{(4)} & N_2^{(4)} & N_3^{(4)} & N_4^{(4)} \end{bmatrix}; \\
\mathbf{N}_{wz} &= \begin{bmatrix} N_1^{(5)} & N_2^{(5)} & N_3^{(5)} & N_4^{(5)} \end{bmatrix}
\end{align*}
\]

The matrices \( N_j^{(j)} \) (j = 1-5) in the above equation are defined as
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\[
\begin{bmatrix}
N_i^{(1)} = [N_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]
N_i^{(2)} = [0 \ N_i \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]
N_i^{(3)} = [0 \ 0 \ H_{3i-2} \ 0 \ 0 \ H_{3i-1} \ H_{3i} \ 0]
N_i^{(4)} = [0 \ 0 \ 0 \ H_{3i-2} \ 0 \ 0 \ 0 \ H_{3i-1} \ H_{3i}]
N_i^{(5)} = [0 \ 0 \ 0 \ 0 \ N_i \ 0 \ 0 \ 0]
\end{bmatrix}
\]  

(15)

with \( N_i (i = 1 \div 4) \) are Lagrangian functions and \( H_j (j = 1 \div 12) \) are Hermit functions.

The elastic strain energy of the element is of the form

\[
U_e = \frac{1}{2} \int_{V_e} \left( \varepsilon_x \sigma_x + \varepsilon_y \sigma_y + \varepsilon_z \sigma_z + \gamma_{xy} \tau_{xy} + \gamma_{xz} \tau_{xz} + \gamma_{yz} \tau_{yz} \right) dV
\]

(16)

where \( V \) denote the element volume. Substituting (13) into (8) and (9), then add the result into Eq. (16), one can express the above strain energy in the form

\[
U_e = \frac{1}{2} d^T K_e d
\]

(17)

where \( K_e \) the element stiffness matrix.

The work done by \( q_x, q_y \) on the movements in the neutral plane can be expressed as

\[
W_e = \int_{V_e} \left[ q_x \frac{\partial w_0}{\partial x} + q_y \frac{\partial w_0}{\partial y} \right] dx dy
\]

(18)

Substituting \( w_0 = w(x, y, 0) \) in (6) and \( q_x, q_y \) in (5) into (18), one gets

\[
W_e = N_{ce} d^T K_{eg} d
\]

(19)

where \( K_{eg} \) is the geometrical stiffness matrix of the element. Gauss quadrature is employed herewith to evaluate the matrices \( K_e \), and \( K_{eg} \).

The critical load \( P_{cr} \) is determined from the following equation

\[
\det \left[ K - P_{cr} K_g \right] = 0
\]

(20)

where \( K = \sum_{n=1} K_e, K_g = \sum_{n=1} K_{eg} \) are the global stiffness matrix and geometrical stiffness matrix of the plate.

4. NUMERICAL RESULTS AND DISCUSSIONS

In this section, a rectangular 2D-FGSW plate made from Alumina (Al\(_2\)O\(_3\)) and Aluminum (Al) with the material properties are: \( E_c = 380\text{GPa}, v_c = 0.3, E_m = 70\text{GPa}, v_m = 0.3 \) is considered. Various types of boundary conditions, namely simply supported (SSSS) and clamped (CCCC) at all edges, simply supported at two edges and free at the others (SFSF), clamped at two edges and free at the others (CFCF), are considered. To facilitate the discussion,
The non-dimensional critical load is introduced as [2]: \( N_{cr} = \frac{P_{cr}a^2}{100E_0h^3} \), where \( E_0 = 1 \text{ GPa} \) and \( a \) is the plate length. Three numbers in the brackets are used herein to denote the layer thickness ratio, e.g. \((2-1-2)\) means that the ratio of the thickness of the layers from bottom to top is 2:1:2.

### 4.1. Formulation verification

The derived formulation is necessary to verify before computing the buckling loads of the 2D-FGSW plate. To this end, Table 1 compares the critical loads of SSSS plate under uniform compression loads \( q_x, q_y \) at all edges obtained in the present with that of Zenkour [2] and Reddy [13]. The result in Table 1 is obtained by using a mesh of 20×20 elements. We can see from the table that the result of the present work is good agreement with that of Refs. [2, 13], and the maximum error is just 2.84%. The critical loads based on quasi-3D theory taking the thickness stretching effect into account, as clearly seen from Table 1, are slightly lower than that using the other theories.

**Table 1.** The comparison results of critical loads \( N_{cr} \) of the square sandwich plate \( (\gamma_1 = \gamma_2 = 1) \).

| \( p_z \) | Theory     | \( N_{cr} \)          | 1-0-1 | 2-1-2 | 2-1-1 | 1-1-1 | 2-2-1 | 1-2-1 |
|----------|------------|------------------------|-------|-------|-------|-------|-------|-------|
| 0        | Reddy [13] | 6.5025, 6.5025, 6.5025 | 6.5025| 6.5025| 6.5025| 6.5025| 6.5025| 6.5025|
|          | Zenkour [2]| 6.5030, 6.5030, 6.5030 | 6.5030| 6.5030| 6.5030| 6.5030| 6.5030| 6.5030|
|          | Present    | 6.4601, 6.4601, 6.4601 | 6.4601| 6.4601| 6.4601| 6.4601| 6.4601| 6.4601|
| 0.5      | Reddy [13] | 3.6822, 3.9704, 4.1124 | 4.2182| 4.4050| 4.6084|       |       |       |
|          | Zenkour [2]| 3.6828, 3.9710, 4.1127 | 4.2186| 4.4052| 4.6084|       |       |       |
|          | Present    | 3.6796, 3.9619, 4.1102 | 4.2055| 4.3965| 4.5899|       |       |       |
| 1        | Reddy [13] | 2.5836, 2.9200, 3.0970 | 3.2324| 3.4747| 3.7533|       |       |       |
|          | Zenkour [2]| 2.5842, 2.9206, 3.0973 | 3.2327| 3.4749| 3.7531|       |       |       |
|          | Present    | 2.5682, 2.9029, 3.0957 | 3.2135| 3.4687| 3.7315|       |       |       |
| 5        | Reddy [13] | 1.3291, 1.5213, 1.7018 | 1.7899| 2.0561| 2.3673|       |       |       |
|          | Zenkour [2]| 1.3300, 1.5220, 1.7022 | 1.7903| 2.0564| 2.3674|       |       |       |
|          | Present    | 1.3205, 1.5119, 1.7249 | 1.7794| 2.0772| 2.3544|       |       |       |
| 10       | Reddy [13] | 1.2436, 1.3732, 1.5460 | 1.5974| 1.8537| 2.1400|       |       |       |
|          | Zenkour [2]| 1.2446, 1.3742, 1.5672 | 1.5973| 1.8729| 2.1909|       |       |       |
|          | Present    | 1.2338, 1.3645, 1.5696 | 1.5882| 1.8789| 2.1285|       |       |       |

### 4.2. Numerical results

Tables 2 and 3 list the non-dimensional critical loads \( N_{cr} \) of a square (1-1-1) 2D-FGSW plate with \( a = 20h \) for various values of the grading indexes and two type of boundary
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conditions, SSSS and CCC. The grading indexes, as seen from the tables, have significant influence on the critical load of the plate, and the buckling load increases by increasing the indexes \( p_x \) and \( p_z \), regardless of the boundary conditions. The increase of the critical load \( N_{cr} \) by increasing the index \( p_x \) is, however dependent of the \( p_z \), and the increase is more significant for a lower value of the \( p_z \). For example, with \( p_x=0.1 \), the critical load \( N_{cr} \) of the SSSS plate increase 2.12 times when raising \( p_z \) from 0.1 to 10, but the corresponding value is just 13.80% for \( p_x=10 \).

The situation is similar for the effect of the index \( p_z \), that is the increase of the by increasing \( p_z \) is also dependent of \( p_x \), and the lower \( p_x \) is, the more significant the \( N_{cr} \) increases.

In Figure 3 the variation of the critical load \( N_{cr} \) as a function of the length-to-height ratio of the square (1-2-1) 2D-FGSW plate is illustrated for various boundary conditions. The plate is assumed to be loaded by the uniform distributed load at all its edges, that is \( \gamma_1 = \gamma_2 = 1 \). As expected, the length-to-height ratio \( a/h \) has a strong effect on the critical load of the plate at the low aspect ratio, mainly \( a/h \) in range 10 and 40. The influence of the \( a/h \) ratio becomes weaker for \( a/h > 40 \), and the critical load \( N_{cr} \) is almost unchanged when \( a/h \) exceeds 40. Among the four boundary conditions considered herein, the effect of the \( a/h \) ratio is the most significant for the CCC plate, while it is the least significant for the SFSF plate.

Table 2. The critical loads \( N_{cr} \) of the square (1-1-1) SSSS plate with \( a = 20h \) under uniform compression load in both two directions \((\gamma_1 = \gamma_2 = 1)\).

| \( p_x \) | 0   | 0.1 | 0.5 | 1   | 2   | 5   | 10  |
|----------|-----|-----|-----|-----|-----|-----|-----|
| 0        | 1.4583 | 1.6123 | 2.1277 | 2.6396 | 3.4194 | 4.7863 | 5.7291 |
| 0.1      | 1.7377 | 1.8842 | 2.3801 | 2.8741 | 3.6229 | 4.9175 | 5.7974 |
| 0.5      | 2.5905 | 2.7130 | 3.1376 | 3.5654 | 4.2087 | 5.2862 | 5.9899 |
| 1        | 3.2966 | 3.3986 | 3.7567 | 4.1207 | 4.6669 | 5.5652 | 6.1356 |
| 2        | 4.1507 | 4.2275 | 4.5004 | 4.7803 | 5.2005 | 5.8802 | 6.2993 |
| 5        | 5.2392 | 5.2838 | 5.4437 | 5.6092 | 5.8585 | 6.2557 | 6.4929 |
| 10       | 5.8604 | 5.8865 | 5.9805 | 6.0785 | 6.2261 | 6.4599 | 6.5973 |

Table 3. The critical loads \( N_{cr} \) of the square (1-1-1) CCC plate with \( a = 20h \) under uniform compression load in both two directions \((\gamma_1 = \gamma_2 = 1)\).

| \( p_x \) | 0   | 0.1 | 0.5 | 1   | 2   | 5   | 10  |
|----------|-----|-----|-----|-----|-----|-----|-----|
| 0        | 3.8707 | 4.2729 | 5.6123 | 6.9387 | 8.9578 | 12.4936 | 14.8872 |
| 0.1      | 4.6073 | 4.9895 | 6.2783 | 7.5605 | 9.5055 | 12.8703 | 15.1093 |
| 0.5      | 6.8496 | 7.1681 | 8.2706 | 9.3827 | 11.0618 | 13.8871 | 15.6886 |
| 1        | 8.6986 | 8.9629 | 9.8911 | 10.8370 | 12.2645 | 14.6284 | 16.0941 |
| 2        | 10.9252 | 11.1236 | 11.8290 | 12.5552 | 13.6532 | 15.4459 | 16.5268 |
| 5        | 13.7473 | 13.8618 | 14.2735 | 14.7021 | 15.3524 | 16.4011 | 17.0153 |
| 10       | 15.3505 | 15.4174 | 15.6591 | 15.9121 | 16.2970 | 16.9144 | 17.2709 |
Figure 3. Variation of critical load $N_{cr}$ of the square (2-1-2) plate with aspect ratio $a/h$ and different boundary conditions $(\gamma_1 = \gamma_2 = 1)$.

Figure 4. The first four buckling mode shapes of the square 2D-FGSW plate, SSSS, type 2-1-2, $a = 20h$, $p_x = 5$, $p_z = 5$, $\gamma_1 = \gamma_2 = 1$. 
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The first four buckling mode shapes of the square 2D-FGSW plate, SSSS, type 2-1-2, \(a = 20h, p_x = 0.5, p_z = 0.5, \gamma_1 = 1, \gamma_2 = 0\).

The first four buckling mode shapes of the square (2-1-2) SSSS plate subjected to uniform compression load in 2 directions at all edges \((\gamma_1 = \gamma_2 = 1)\) are depicted in Figure 4 for an aspect ratio \(a/h = 20\) and grading indexes \(p_x = p_z = 5\). The critical loads corresponding to the mode shapes are also given in the figure. A careful examination of the figure shows that the first mode shape is no longer symmetric with respect to the plate center. This is due to the fact that the material properties of the plate vary in longitudinal direction, and this destroys the symmetry of the first mode shape as often seen in the homogeneous plates.

Finally, to illustrate the effect of the loading condition, Figure 5 shows the first four buckling mode shapes of the square 2D-FGSW plate subjected to uniform compression load in \(x\)-direction only \((\gamma_1 = 1, \gamma_2 = 0)\). The figure is also obtained for SSSS (2-1-2) with \(a/h=20\) and \(p_x = p_z = 5\). By comparing Figure 5 with Figure 4, one can clearly see the influence of the external loading on the mode shape, especially for the third and fourth modes.

5. CONCLUSIONS

In this paper, a Q4 plate element has been formulated and employed in computing critical loads of 2D-FGSW plates with various boundary conditions. The element with nine degrees of freedom per node is derived in the context of quasi-3D shear deformation theory. The obtained numerical result confirmed the accuracy of the derived formulation, and it reveals that the variation of the material properties plays an important role on the critical loads and mode shapes of the plates. Different from transverse FGM plate, the buckling mode shapes of the 2D-FGM
plates are no longer symmetric with respect to the central point. A parametric study was carried out to illustrate the effect of the material distribution, aspect ratio and the loading condition on the buckling behavior of the 2D-FGSW plates.

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