Implementation of quantum gates and preparation of entangled states in cavity QED with cold trapped ions

Mang Feng\textsuperscript{1,2} *, Xiaoguang Wang\textsuperscript{1} †
\textsuperscript{1} Institute for Scientific Interchange (ISI) Foundation,
Villa Gualino, Viale Settimio Severo 65, I-10133, Torino, Italy
\textsuperscript{2} Laboratory of Magnetic Resonance and Atomic and Molecular Physics,
Wuhan Institute of Physics and Mathematics, Academia Sinica, Wuhan, 430071, China
(March 31, 2022)

We propose a scheme to perform basic gates of quantum computing and prepare entangled states
in a system with cold trapped ions located in a single mode optical cavity. General quantum
computing can be made with both motional state of the trapped ion and cavity state being qubits.
We can also generate different kinds of entangled states in such a system without state reduction,
and can transfer quantum states from the ion in one trap to the ion in another trap. Experimental
requirement for achieving our scheme is discussed.

PACS numbers: 03.67.-a, 32.80.Lg. 42.50.-p

I. INTRODUCTION

Both cavity QED and cold trapped ion systems have been drawn much attention over past decades, due to not
only the fundamental interest of physics involved but also some potential applications, such as quantum information
processing [1]. To our knowledge, the entangled states between atoms and cavity [2], between ions’ internal states and
motional states [3], and between internal states of different ions [4] have been experimentally achieved. If we combine
the cavity QED technique with ion trap one, for example, placing the ion trap into an optical cavity, the problem
would be more interesting because it involves three quantum degrees of freedom, namely, the ion’s internal levels,
phonon field of the trap and photon field of the cavity [5]. It is of great importance in the field of quantum information
to study entanglement of three quantum degrees of freedom as well as information transfer between one of them to
another. In early papers, Zeng and Lin proposed a technique to generate nonclassical vibrational states of cold trapped
ions in a harmonic trap located in an optical cavity [6]. Buzek et al [7] and Luo et al [8] investigated independently
the quantum motion of a cold trapped ion interacting with a quantized light field. Parkins and his cooperators [9] also
studied the coherent transfer between motional states and quantized light in detail. More recently, we notice that a
work under similar consideration emphasized the production of Bell states between motional states of a trapped ion
and the quantized light [10]. If we consider entanglement to be important source of quantum information processing,
all the works referred to above have clearly shown that this kind of source does exist in the ion-trap-cavity system,
whereas none of them showed specifically how to use this source for processing quantum information.

Various proposals have been put forward by using internal states of cold atoms in cavities or in ion traps to be
qubits for performing quantum computing and quantum communication [11]. In some proposals, motional states of
the trapped ion are employed to be qubits [12]. Recently, two proposals [13,14] were presented, which involve
the ion-trap-cavity system for robust quantum computing against decoherence by means of large detuning between
the cavity state and internal states of trapped ions. In fact, if the cavity decay can be effectively used, entangled states

*Electronic address: feng@isiosf.isi.it
†Electronic address: xgwang@isiosf.isi.it
II. QUANTUM COMPUTING IN THE SYSTEM OF A SINGLE TRAPPED ION IN THE HIGH-Q CAVITY

Consider a single cold ion confined in the ion trap which itself is located in an optical cavity. We assume that the cavity mode, together with classical standing waves of lasers, couples to the internal and vibrational states of the ion, as discussed in Refs. [7,8,10]. The Hamiltonian in the unit of $\hbar = 1$ can be written as follows

$$H = \frac{\omega_0}{2}\sigma_z + \nu b^\dagger b + \omega_e a^\dagger a + G[\sigma_+ e^{i[\nu(b^\dagger b) - \omega_L t]} + h.c.] + g\sigma_z (a^\dagger + a) \sin[\eta_e (b^\dagger b) + \phi]$$  \hspace{1cm} (1)

where $\omega_0$ is the frequency of atomic resonance transition. $\omega_e$ and $\omega_L$ are frequencies of cavity mode and laser respectively. $a^\dagger$, $a$ and $b^\dagger$, $b$ are respectively creation and annihilation operators of photons of the cavity and phonons of the trap. $G$ and $g$ are the coupling constants proportional to the ion-laser and ion-cavity interaction respectively. $\eta_L$ and $\eta_e$ are respectively Lamb-Dicke parameters with respect to the radiation of laser and cavity. $\sigma_+\,\sigma_-\,\text{and}\,\sigma_z$ are usual Pauli operators. $\phi$ accounts for the relative position of motional state of the ion to the standing wave of the quantized cavity field. Within the Lamb-Dicke limit, $\sin[\eta_e (b^\dagger b) + \phi] \approx \eta_e (b^\dagger b) \cos \phi + \sin \phi$. Performing a unitary operator defined as $U = \exp[-it\left(\frac{\omega_0}{2}\sigma_z + \nu b^\dagger b + \omega_e a^\dagger a\right)]$ yields different cases under the rotating wave approximation, for example,

- **case 1 with $\omega_L = \omega_0$,** $H_1^L = G(\sigma_+ + \sigma_-)$;
- **case 2 with $\omega_L = \omega_0 - \nu$,** $H_2^L = iW(\sigma_+ - b^\dagger \sigma_-)$;
- **case 3 with $\omega_L = \omega_0 + \nu$,** $H_3^L = iW(b^\dagger \sigma_+ - b \sigma_-)$;
- **case 4 with $\omega_e = \omega_0 - \nu$,** $H_4^L = \Omega(a^\dagger b^\dagger \sigma_- + ab \sigma_+)$;
- **case 5 with $\omega_e = \omega_0 + \nu$,** $H_5^L = \Omega(ab^\dagger \sigma_- + a^\dagger b \sigma_+)$;
- **case 6 with $\omega_e = \omega_0$,** $H_6^L = \Omega(a^\dagger \sigma_- + a \sigma_+)$,

where $W = G\eta_L, \Omega = \eta_e g \cos \phi$ and $\Omega' = g \sin \phi$.

Therefore by adjusting the laser or cavity frequency, we can obtain different time evolution of states in the system. In what follows, we only list some of them which would be used for constructing quantum computing gates and preparing entangled states in the present work.

For case 1,

$$|g\rangle \rightarrow \cos(Gt)|g\rangle - i \sin(Gt)|e\rangle, \hspace{0.5cm} |e\rangle \rightarrow \cos(Gt)|e\rangle - i \sin(Gt)|g\rangle; \hspace{1cm} (2)$$

for case 2,

$$|g\rangle|m\rangle_b \rightarrow \cos(\sqrt{mW}t)|g\rangle|m\rangle_b + \sin(\sqrt{mW}t)|e\rangle|m-1\rangle_b,$$

$$|e\rangle|m\rangle_b \rightarrow \cos(\sqrt{m+1W}t)|e\rangle|m\rangle_b - \sin(\sqrt{m+1W}t)|g\rangle|m+1\rangle_b; \hspace{1cm} (3)$$

for case 4,

$$|g\rangle|m\rangle_a|n\rangle_b \rightarrow \cos(\sqrt{mn\Omega}t)|g\rangle|m\rangle_a|n\rangle_b - i \sin(\sqrt{mn\Omega}t)|e\rangle|m-1\rangle_a|n-1\rangle_b,$$

$$|e\rangle|m\rangle_a|n\rangle_b \rightarrow \cos(\sqrt{(m+1)(n+1)\Omega}t)|e\rangle|m\rangle_a|n\rangle_b - i \sin(\sqrt{(m+1)(n+1)\Omega}t)|g\rangle|m+1\rangle_a|n+1\rangle_b; \hspace{1cm} (4)$$

for case 6,

$$|g\rangle|m\rangle_a|n\rangle_b \rightarrow \cos(\sqrt{m(n+1)\Omega}t)|g\rangle|m\rangle_a|n\rangle_b - i \sin(\sqrt{m(n+1)\Omega}t)|e\rangle|m-1\rangle_a|n+1\rangle_b,$$

$$|e\rangle|m\rangle_a|n\rangle_b \rightarrow \cos(\sqrt{(m+1)n\Omega}t)|e\rangle|m\rangle_a|n\rangle_b - i \sin(\sqrt{(m+1)n\Omega}t)|g\rangle|m+1\rangle_a|n-1\rangle_b; \hspace{1cm} (5)$$

for case 7,

$$|g\rangle|m\rangle_a \rightarrow \cos(\sqrt{m\Omega'}t)|g\rangle|m\rangle_a - i \sin(\sqrt{m\Omega'}t)|e\rangle|m-1\rangle_a,$$

$$|e\rangle|m\rangle_a \rightarrow \cos(\sqrt{m+1\Omega'}t)|e\rangle|m\rangle_a - i \sin(\sqrt{m+1\Omega'}t)|g\rangle|m+1\rangle_a.$$  \hspace{1cm} (6)
where the subscripts $a$ and $b$ denote the states of photon and phonon respectively. $|g\rangle$ and $|e\rangle$ are ground and excited internal states of the ion respectively, and $m, n = 0, 1, \cdots$.

With above time evolutions of states for different cases, we can construct Hadamard and CNOT gates in the system, which are basic elements of a general quantum computing [18]. We first try to implement CNOT gate between $|I\rangle_a$ and $|\langle I|_b$. If the internal state of the ion is prepared to the ground state, by performing operations $R^{I}_I(\frac{\pi}{2})$, $R^{I}_I(\frac{3\pi}{2})$ and $R^{I}_I(\frac{\pi}{2})$ sequentially, where $R^{I}_I(\frac{\pi}{2})$ means the time evolution of the case 4 with $\Omega = \frac{\pi}{2}$ and $R^{I}_I(\frac{3\pi}{2})$ is the time evolution of the case 7 with $\Omega t = \frac{3\pi}{2}$, we have

$$
|g\rangle|0\rangle_{ab} \rightarrow |g\rangle|0\rangle_{ab} \rightarrow |g\rangle|1\rangle_{ab} \rightarrow |g\rangle|1\rangle_{ab},
$$

$$
|g\rangle|01\rangle_{ab} \rightarrow |g\rangle|01\rangle_{ab} \rightarrow |g\rangle|01\rangle_{ab},
$$

$$
|g\rangle|11\rangle_{ab} \rightarrow -i|e\rangle|00\rangle_{ab} \rightarrow |g\rangle|10\rangle_{ab} \rightarrow |g\rangle|10\rangle_{ab},
$$

$$
|g\rangle|00\rangle_{ab} \rightarrow |g\rangle|00\rangle_{ab} \rightarrow |g\rangle|00\rangle_{ab} \rightarrow |g\rangle|00\rangle_{ab}.
$$

(7)

which is the CNOT gate with $|I\rangle_a$ and $|\langle I|_b$ being control and target states respectively. Similarly, to perform the CNOT gate with $|\langle I|_b$ and $|I\rangle_a$ being control and target states respectively, we combine $H^{I}_I$ and $H^{I}_I$. With operations of $R^{I}_I(\frac{3\pi}{2})$, $R^{I}_I(\frac{\pi}{2})$ and $R^{I}_I(\frac{3\pi}{2})$ sequentially, we have

$$
|g\rangle|10\rangle_{ab} \rightarrow |g\rangle|10\rangle_{ab} \rightarrow |g\rangle|10\rangle_{ab} \rightarrow |g\rangle|10\rangle_{ab},
$$

$$
|g\rangle|01\rangle_{ab} \rightarrow |g\rangle|01\rangle_{ab} \rightarrow -|e\rangle|00\rangle_{ab} \rightarrow i|g\rangle|11\rangle_{ab},
$$

$$
|g\rangle|11\rangle_{ab} \rightarrow i|e\rangle|00\rangle_{ab} \rightarrow i|g\rangle|01\rangle_{ab} \rightarrow i|g\rangle|01\rangle_{ab},
$$

$$
|g\rangle|00\rangle_{ab} \rightarrow |g\rangle|00\rangle_{ab} \rightarrow |g\rangle|00\rangle_{ab} \rightarrow |g\rangle|00\rangle_{ab}.
$$

(8)

So further single qubit rotation is needed for eliminating the prefactor $i$ when the final result of motional state of ion is $|1\rangle_{b}$.

For implementing a Hadamard gate of $|I\rangle_a$, we also need to initially prepare the internal state of the ion to the ground state. By performing operations $R^{I}_I(\frac{\pi}{2})$, $R^{I}_I(\frac{3\pi}{2})$ and $R^{I}_I(\frac{\pi}{2})$, we obtain

$$
|g\rangle|1\rangle_a \rightarrow -i|e\rangle|0\rangle_a \rightarrow -\frac{1}{\sqrt{2}} i(|e\rangle + i|g\rangle)|0\rangle_a \rightarrow \frac{1}{\sqrt{2}} |g\rangle(|0\rangle_a - |1\rangle_a),
$$

$$
|g\rangle|0\rangle_a \rightarrow |g\rangle|0\rangle_a \rightarrow \frac{1}{\sqrt{2}} (|g\rangle + i|e\rangle)|0\rangle_a \rightarrow \frac{1}{\sqrt{2}} |g\rangle(|0\rangle_a + |1\rangle_a).
$$

(9)

The Hadamard gate of $|\langle I|_b$ can be obtained similarly by replacing $R^{I}_I(\frac{\pi}{2})$ with $R^{I}_I(\frac{3\pi}{2})$.

Therefore, the general quantum computing can be made with the bosonic qubits, i.e., the motional state of the trapped ion and the cavity state being qubits. However, it would be more meaningful if we consider a system with more trapped ions in a cavity. Therefore in next section, we will discuss how to use above quantum computing gates to transfer states and prepare entangled states in the case that two trapped ions are coupled to the same cavity state.

III. QUANTUM COMMUNICATION IN THE SYSTEM OF TWO TRAPPED IONS IN A HIGH-Q CAVITY

Suppose that there are two separate traps placed in the same high-Q optical cavity with a single ion confined in each trap. If Coulomb interaction between the two ions is weak enough, and the two subsystems of trapped ions are identical, we can obtain Eqs.(2)-(9) for the two subsystems respectively, where the cavity state $|\langle I|_a$ plays the role of connection between these two subsystems. Although we only focused our attention on the two bosonic qubits in last section, we will investigate in this section how to make entangled states and state transfer with all quantum degrees of freedom involved.

Application 1: State transfer between two trapped ions
Suppose that the two trapped ions are respectively prepared to \( C|g\rangle_1 + D|e\rangle_1 \) and \( |g\rangle_2 \) with \( |C|^2 + |D|^2 = 1 \) and subscripts being labeling of the ions. Both motional states of the two ions and the cavity state are prepared to ground states. So we have \( (C|g\rangle_1 + D|e\rangle_1)\|b_1\|g\rangle_2\|b_2\|0\rangle_a \) with subscripts \( b_1 \) and \( b_2 \) labeling motional states of ions 1 and 2 respectively. By performing operations \( R_{I_1}^L(\frac{\pi}{2}) \) and then \( R_{I_2}^L(\frac{\pi}{2}) \), where \( R_{I_1}^L \) and \( R_{I_2}^L \) are time evolutions of the case 7 for ions 1 and 2 respectively, we have

\[
(C|g\rangle + D|e\rangle)_1|0\rangle_{b_1}|g\rangle_2|0\rangle_{b_2}|0\rangle_a \rightarrow |g\rangle_1|0\rangle_{b_1}|g\rangle_2|0\rangle_{b_2}(C|0\rangle - iD|1\rangle)_a
\]

\[
\rightarrow |g\rangle_1|g\rangle_2(C|0\rangle + D|1\rangle)_{b_1}b_2|0\rangle_a.
\]

The state transfer shown above is made through coupling to the cavity state, instead of the direct interaction between the two trapped ions. So if there are many trapped ions placed in the cavity, we can transfer the state from one trapped ion to another even if they are not neighboring. In fact, we can also perform swapping operation above between two trapped ions. To this end, we have to define \( S_{ab} = CNOT_{ab} \cdot CNOT_{ba} \cdot CNOT_{ab} \) where \( S_{ab} \) is the operation of state exchange between \( |b\rangle \) and \( |a\rangle \), and \( CNOT_{ij} \) the CNOT gate with \( |i\rangle \) and \( |j\rangle \) being control and target states respectively. If we start our process from the initial state of \( (C|g\rangle + D|e\rangle)_1|0\rangle_{b_1}(E|g\rangle + F|e\rangle)_2|0\rangle_{b_2}|0\rangle_a \) where \( |C|^2 + |D|^2 = 1 \) and \( |E|^2 + |F|^2 = 1 \), performing \( R_{I_1}^L(\frac{\pi}{2}) \), \( R_{I_2}^L(\frac{\pi}{2}) \), \( S_{ab_1} \), \( S_{ab_2} \), \( S_{ab_1} \), \( R_{I_1}^L(\frac{\pi}{2}) \), and \( R_{I_2}^L(\frac{\pi}{2}) \) sequentially will yield

\[
(C|g\rangle + D|e\rangle)_1|0\rangle_{b_1}(E|g\rangle + F|e\rangle)_2|0\rangle_{b_2}|0\rangle_a \rightarrow |g\rangle_1|0\rangle_{b_1}|g\rangle_2|0\rangle_{b_2}(E|0\rangle - F|1\rangle)_{b_1}b_2|0\rangle_a \rightarrow
\]

\[
|g\rangle_1|g\rangle_2(C|0\rangle - D|1\rangle)_{b_1}b_2|0\rangle_a \rightarrow (E|g\rangle + F|e\rangle)_1|0\rangle_{b_1}(C|g\rangle + D|e\rangle)_2|0\rangle_{b_2}|0\rangle_a.
\]

Briefly speaking, the above swapping involves three steps, i.e., quantum information transferring from ions' internal states to corresponding motional states, then swapping between motional states of the two ions, and then quantum information transferring from ions' motional states to corresponding internal states.

Application 2: Preparation of Greenberger-Horne-Zeilinger (GHZ) state [20]

Suppose that the two trapped ions are respectively prepared to \( D|g\rangle_1 + e|e\rangle_1 \) and \( |g\rangle_2 \) with subscripts \( b_1 \) and \( b_2 \) labeling motional states of ions 1 and 2 on above

\[
(C|g\rangle + D|e\rangle)_1|0\rangle_{b_1}|g\rangle_2|0\rangle_{b_2}|0\rangle_a \rightarrow |g\rangle_1|0\rangle_{b_1}|g\rangle_2|0\rangle_{b_2}(C|0\rangle + D|1\rangle)_a
\]

\[
\rightarrow |g\rangle_1|g\rangle_2(C|0\rangle - iD|11\rangle)_{b_1}b_2|0\rangle_a.
\]

The GHZ state prepared here is different from that in Ref.[7]. It is more interesting because we can control the entanglement of the GHZ state by preparing the initial state of the ion 1. If many trapped ions are located in the same cavity, we can obtain a GHZ state with many qubits along this idea.

Application 3: Entanglement of the two trapped ions

After obtaining the GHZ state presented in Eq.(12), we perform a CNOT \( b_1 \). Then we have

\[
|g\rangle_1|g\rangle_2|0\rangle_{b_1}b_2|0\rangle_a \rightarrow |g\rangle_1|g\rangle_2(C|0\rangle - iD|11\rangle)_{b_1}b_2|0\rangle_a.
\]

If we want to have other types of Bell states, we can implement Hadamard gates of \( |b_1\rangle \) and \( |b_2\rangle \) respectively on above state, which yields,

\[
|g\rangle_1|g\rangle_2(C|0\rangle - iD|11\rangle)_{b_1}b_2|0\rangle_a \rightarrow
\]

\[
\frac{1}{2}[|g\rangle_1|g\rangle_2((C - iD)(|00\rangle + |11\rangle))_{b_1}b_2 - (C + iD)(|01\rangle + |10\rangle))_{b_1}b_2]|0\rangle_a.
\]

By choosing values of \( C = \pm iD \), we can obtain different Bell states between motional states of two trapped ions, which would be useful in future teleportation experiments with massive particles involved. With the similar idea, we can have entanglement between internal states of the two ions. Suppose we start from \( |e\rangle_1(C|0\rangle + D|1\rangle)|b_1|g\rangle_2|0\rangle_{b_2}|0\rangle_a \) with \( |C|^2 + |D|^2 = 1 \), performing \( R_{I_1}^L(\frac{\pi}{2}) \), \( CNOT_{ab_2} \), and \( R_{I_2}^L(\frac{\pi}{2}) \) sequentially will yield

\[
|e\rangle_1(C|0\rangle + D|1\rangle)|b_1|g\rangle_2|0\rangle_{b_2}|0\rangle_a \rightarrow (C|e\rangle_1|00\rangle_{ab_1} - iD|g\rangle_1|10\rangle_{ab_1})|g\rangle_2|0\rangle_{b_2}
\]

\[
\rightarrow C|e\rangle_1|0\rangle_{b_1}|g\rangle_2|00\rangle_{ab_2} - iD|g\rangle_1|0\rangle_{b_1}|g\rangle_2|11\rangle_{ab_2} \rightarrow (C|e\rangle - D|g\rangle)|12\rangle_{00}\rangle_{b_1}b_2|a.
\]
IV. DISCUSSION AND CONCLUSION

To some extent, our scheme is similar to the well-known model proposed by Cirac and Zoller [21], in which some cold ions, confined in a linear ion trap and radiated by laser beams individually, interact with each other by coupling to the common vibrational motion. In contrast, in our scheme, if we perform swapping operations of $S_{ab}$ alternatively in the two traps, it is easy to obtain,

$$|g⟩_1|g⟩_2|0⟩_b_1|0⟩_b_2|0⟩_a \rightarrow |g⟩_1|g⟩_2|0⟩_b_1|0⟩_b_2|0⟩_a,$$

$$|g⟩_1|g⟩_2|0⟩_b_1|1⟩_b_2|0⟩_a \rightarrow |g⟩_1|g⟩_2|0⟩_b_1|1⟩_b_2|0⟩_a,$$

$$|g⟩_1|g⟩_2|1⟩_b_1|0⟩_b_2|0⟩_a \rightarrow |g⟩_1|g⟩_2|1⟩_b_1|0⟩_b_2|0⟩_a,$$

$$|g⟩_1|g⟩_2|1⟩_b_1|1⟩_b_2|0⟩_a \rightarrow |g⟩_1|g⟩_2|1⟩_b_1|1⟩_b_2|0⟩_a.$$

That means, by coupling the cavity state, we can have quantum computing on motional states of trapped ions. Compared with Ref.[21], we have an additional degree of freedom, i.e., internal states of the ions for storing quantum information. This quantum information storage can be easily made by state transfer from motional states of ions to corresponding internal states. To the best of our knowledge, our work is the first scheme to make quantum computing in the computational space spanned by ions’ motional states and cavity state. It would open a new way to perform quantum computing on bosonic qubits. Another distinguished character of our scheme is that only two levels of each ion are employed in our model, which is important for avoiding detrimental effect from the fluctuation of external magnetic field [22].

From above results, we know that the phase factor $\phi$ plays very important role in our scheme. If ion traps are located at the node of the standing wave of the cavity field, as in Ref.[10], then no case 7 would appear, and thereby it is impossible for us to have a general quantum computing presented above. Moreover, we have to mention that all the entangled states presented in last section are prepared without any measurement. This is important because sometimes it is difficult to make measurement [23], and sometimes we do not want to have state reduction if preparation of entangled states is in the course of quantum information processing. Furthermore, we must mention that both motional states of ions and the cavity state are more susceptible to decoherence than internal states of ions. So accomplishment of desired operations should be much faster than that in former schemes with internal states of atoms being qubits. Suppose $g = 2\pi \times 3 \times 10^7 H z$ [24], $G = 2\pi \times 5 \times 10^5 H z$ [3], $\eta_L = 0.2$ and $\phi = \frac{\pi}{4}$, the time for accomplishing a CNOT$_{ab}$ gate, a CNOT$_{ba}$ gate, a Hadamard gate of $|⟩_a$ and a Hadamard gate of $|⟩_b$ are respectively $1.5 \times 10^{-7}$ sec, $7.8 \times 10^{-6}$ sec, $4.2 \times 10^{-6}$ sec and $6.8 \times 10^{-6}$ sec. For more clarity, some numerical results are demonstrated in Fig.1, which shows the variation of implementation time of different quantum gates with respect to $g$ and $G$. Experimentally, decoherence time for motional state of a cold trapped ion is about $10^{-3}$ sec [25]. However, the decay time of the optical cavity is on microsec time scale. It means that, even if we neglect the delay time between any two adjacent operations, which is used to change the laser frequency or cavity frequency for obtaining different interactions, our scheme can not work well with current experimental techniques. Therefore, to implement our scheme, more advanced technique is highly demanded to prolong the decay time of the optical cavity, to enlarge the performance with respect to the decoherence time of a cold trapped ion [26]. Although no experimental reports in this respect have been presented so far. We also noticed the latest development of microwave cavity experiment in which the decay time of the cavity mode is as long as $0.2$ sec [27]. Moreover, to reduce the time delay, we may use different lasers with different frequencies, instead of changing the frequency of a single laser. Nevertheless, how to reduce the performance time of our scheme to be shorter than the decoherence time of the cavity would be a big challenge for achieving our scheme experimentally. If we suppose that there is no error in our operation, as lifetime of the motional state of the ion is much longer than the performance time of our quantum gate, decoherence of the cavity state is the only factor to affect the fidelity of our proposed gate. As a simple assessment, we present the numerical treatment for the fidelity of CNOT$_{ab}$ performance with respect to the decoherence time of cavity state. For simplicity, we assume that the cavity photon decay happens in the last step concerning $R^A_2(\pi/2)$. By means of the method in Ref.[28], we can find that our scheme works very well as long as the implementation time is half the time of the cavity photon decay.

In conclusion, we have considered an ideal situation of the ion-trap-cavity system and proposed an interesting scheme for performing quantum computing and quantum communication in the system with trapped ions placed in a high-Q optical cavity. The entangled states including Bell states and GHZ state can be prepared without state.
reduction. In principle, this model can be generalized to the case of many trapped ions in the same high-Q optical cavity. However, by using current cavity technique, with the increase of the cavity size for containing more trapped ions, the coupling coefficients $g$ would decrease. This is also the challenge for achieving some quantum computing schemes with semiconductor quantum dots [29]. Nevertheless, once our scheme is achieved, as the traps interact with each other through coupling to the same cavity state, we can have a network of quantum information processing. If the decaying effect of both motional states of the ion and the cavity state are considered and suitably used, the treatment would be more complicated, but more interesting. In particular, if cavities can be connected with each other by quantum wires, the system under consideration here would be a workable node of a larger quantum network. We have noticed the first experimental report in the ion-trap-cavity system with single atomic quantum bit confined [30]. With rapid development of experimental technique, we believe that our proposal would be of interest in exploration of quantum information processing.

The authors would like to thank Irene D’Amico and Paolo Zanardi for valuable discussion. The work is partly supported by the European Commission through the Research Project SQID within FET Program, and partly by the National Natural Science Foundation of China.

Note added: after finishing this work, we become aware of a recent work [31], in which maximally entangled states is generated between vibrational states of two trapped ions by illuminating the two ions simultaneously with two dispersive Raman pulses. That work is different from ours because the entanglement between vibrational states of trapped ions is generated in our scheme by the connection of cavity state.

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, 2000.
[2] A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J. Raimond, S. Haroche, Science, 288, 2024 (2000).
[3] C. Monroe, D. M. Meekhof, B. E. King and D. J. Wineland, Science, 272, 1131(1996).
[4] C. A. Sackett, D. Kielpinski, B. E. King, C. Langer, V. Meyer, C. J. Myatt, M. Rowe, Q. A. Turchette, W. M. Itano, D. J. Wineland and C. Monroe, Nature 404, 256 (2000).
[5] see, [http://heart−c704.ubk.ac.at/cavity_qed.html](http://heart−c704.ubk.ac.at/cavity_qed.html).
[6] H. Zeng and F. Lin, Phys. Rev. A 50, R3589 (1994).
[7] V. Buzek, G. Drobný, M. S. Kim, G. Adam, and P. L. Knight, Phys. Rev. A 56, 2352 (1998).
[8] X. Luo, X. Zhu, Y. Wu, M. Feng, and K. Gao, Phys. Lett. A 237, 354 (1998).
[9] A. S. Parkins and H. J. Kimble, J.Opt.B: Quantum Semiclassical opt. 1, 496 (1999); A. S. Parkins and E. Larsabal, Phys. Rev. A 63, 012304 (2000).
[10] F. L. Semiño, A. Vidiella-Barranco, and J. A. Roversi, Phys. Rev. A 64, 024305 (2001).
[11] See special issue for quantum computing of Fortschr. Phys. 48, (2000).
[12] For example, P. T. Cochrane, G. J. Milburn and W. J. Munro, Phys. Rev. A 59, 2631(1999).
[13] J. Pachos and H. Walther, LANL, quant-ph/0111088.
[14] E. Jane and M. B. Plenio and D. Jonathan, LANL, quant-ph/0111147.
[15] M. B. Plenio, S.F. Huelga, A. Beige, and P.L.Knight, Phys. Rev. A 59, 2468 (1999).
[16] S. Bose, P.L.Knight, M. B. Plenio, and V. Vedral, Phys. Rev. Lett., 83, 5158 (1999).
[17] S. J. van Enk, J. I. Cirac and P. Zoller, Phys. Rev. Lett., 78, 4293 (1997); S. J. van Enk, J. I. Cirac and P. Zoller, Science, 279, 205 (1998).
[18] S. Lloyd, Phys.Rev.Lett. 75, 346 (1995); D. P. DiVicenzo, Phys. Rev. A 51, 1015 (1995); A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVicenzo, N. Margolus, P. Shor, T. Sleator, J. A. Smolin and H. Weinfurter, Phys. Rev. A 52, 3456 (1995).
[19] C. A. Blockley, D. F. Walls, H. Risken, Europhys. Lett. 17, 509 (1992); J. I. Cirac, A. S. Parksins, R. Blatt, and P. Zoller, Phys. Rev. Lett. 70, 556 (1993); Opt. Commun. 97, 353(1993); J. I. Cirac, R. Blatt, A. S. Parksins, and P. Zoller, Phys. Rev. Lett. 70, 762(1993).
[20] D. M. Greenberger, M. A. Horne and A. Zeilinger, in Bell’s Theorem, Quantum Theory, and Conceptions of the Universe, edited by M. Kafatos (Kluwer, Dordrecht, 1989), P69.
[21] J. I. Cirac and P. Zoller, Phys. Rev. Lett. 74, 4091 (1995).
[22] C. Monroe, D. Leibfried, B. E. King, W. M. Itano, and D. J. Wineland, Phys. Rev. A 55, R2489 (1997).
[23] For example, in Eq.(15), if states of phonon and photon are entangled with internal states of the two ions, it is impossible to obtain the entangled internal states of the two ions by making measurement on phonon and photon states.
[24] S. J. van Enk, J. Mckeever, H. J. Kimber and J. Ye, Phys. Rev. A 64, 013407 (2001).
[25] B. E. King, C. S. Wood, C. L. Myatt, Q. A. Turchette, D. Leibfried, W. M. Itano, C. Monroe, and D. J. Wineland, Phys. Rev. Lett., 81, 1525 (1998).
[26] For example, J. I. Cirac, A. S. Parkins, R. Blatt and P. Zoller, Adv. Atom. Mole. Opt. Phys. 37, 237(1996).
The Captions of the figures

Fig.1 Time of quantum gate performance with respect to the coupling strength, where \( \phi = \pi/4, \eta_c = \eta_L = 0.2 \).
(A) Setting \( G = 2\pi \times 5 \times 10^5 Hz \), implementation time varies with values of \( g \) (in the unit of \( 2\pi \times 30 MHz \)). The dotted, dashed and solid curves denote \( CNOT_{ba}, H_a \) and \( CNOT_{ab} \) respectively. (B) Setting \( g = 2\pi \times 3 \times 10^7 Hz \), implementation time corresponds to different \( G \) (in the unit of \( \pi MHz \)). The dotted, dashed and solid curves denote \( CNOT_{ba}, H_b \) and \( H_a \) respectively.

Fig.2 Fidelity of the \( CNOT_{ba} \) gate performance with respect to \( T_{im}/T_d \), where \( T_{im} \) and \( T_d \) are respectively implementation time of \( CNOT_{ba} \) gate and decoherence time of the cavity state. For cases of Hadamard gates and \( CNOT_{ab} \), we can have similar results.
Fig. 1

(A)

(B)

Fig. 1
Fig. 2