Equilibrium and nonequilibrium quantum correlations between two detectors in curved space time

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ABSTRACT: We investigate the equilibrium and nonequilibrium quantum information correlations encoded in two-qubit system (near the horizon of a Kerr black hole). We study the impact of mass and the angular momentum, and further the local curvature or accelerations on the behaviors of the quantum correlations between two qubits. We show the quantum information of two qubits is encoded in the space time structure. In nonequilibrium case, the nonequilibrium can also contribute to the correlations.
1 Introduction

Quantum correlations including coherence [1], entanglement [2] [3], mutual information and the quantum discord [4] etc have been widely theoretically studied. They provide profound insights on the black hole physics and even cosmology. [5] [6] [7] [8]. They are often needed as the key resources to carry out certain quantum information processing tasks, e.g., quantum teleportation, quantum computation, quantum cryptography, quantum metrology etc. [9] [10] [11] [12] [13] One of the major obstacles to realize the quantum information technologies is the environmental induced decoherence and dissipation effect to the quantum systems, which may give rise to quantum correlations degradation. However, an external environment can also provide indirect interactions between the subsystems through the correlations that exist. A phenomenological and illuminating example is the independent atoms immersed in external quantum fields and weakly coupled to them through Unruh–DeWitt detector interaction [7] [8] [14]. The atoms can be usually treated in a non-relativistic approximation, as independent n-level systems (qubits or harmonic oscillator typically), with negligible size, while the environment can be described by a set of quantum fields in a given quantum state, typically either a thermal state or simply the vacuum state. Despite the simplified setting, this can also provide heuristic insight on the relationship between the environment and the quantum correlations. It has been shown that there are certain scenarios where the environment may create quantum correlations rather than destroy them,
in both flat space time \[15\] \[16\] \[17\] \[18\] and curved spacetime backgrounds \[7\] \[8\] \[19\] \[20\]. However, the studies of the quantum information harvesting have so far been focused on special case where two detectors couple to the environment with equal strength and under the same space time background (local curvature or the local accelerations is same, which corresponds to the equilibrium case). How the two detectors at different locations of the curved space time (corresponds to the nonequilibrium case) correlate with each other is still an unresolved problem.

Black hole, which relates gravity, quantum theory and thermodynamics, is fascinating and very challenging subject in physics. The Hawking radiation emerges at the horizon as a pure quantum effect \[21\], can also be understood by the open quantum system theory. Concretely, consider a single two level atom system near the horizon, treat the massless scalar field in the curved background as the environment and then compute the spontaneous excitation rate of the atom. This reveals that close to the horizon, the ground state detector in the vacuum would spontaneously excite with an excitation rate same as the case when there is a thermal bath around the atom. \[22\] \[23\] Therefore, the near-horizon geometry plays the key role to the character of a black hole space time. Moreover it was shown that around the horizon of a Kerr space time, the scalar field theory can be reduced to a 2-dimensional effective field theory. \[23\] \[24\] \[25\] It is then convenient to study the quantum correlations near the black hole with this insight.

In this paper, we study the quantum correlations near the horizon of the Kerr black hole by using the dimensional reduction method. Both equilibrium and nonequilibrium scenarios are discussed. For equilibrium scenario (which means two detectors coupled to the field with the same coupling at the same location near the black horizon). We start with a product state initially, an equilibrium steady state can be found at final time in certain scenario. We focus on the equilibrium steady state and study its quantum correlations. We found that the quantum correlations (including entanglement, coherence, discord and mutual information) in two-qubit system vary non-monotonically with the mass of the black hole and are amplified by the angular momentum. The Von Neumann entropy which measures the entanglement between the system and environment behaves oppositely compared to the quantum correlations with respect to the mass and the angular momentum. Moreover, we found that the curvature can suppress the quantum correlations within the system but enhance the correlation with environment.

For the nonequilibrium case (one of two qubits is weakly coupled to the field that can be viewed equivalently as isolated from the environment.). The system evolves from the maximal entangled state and the information will scramble to environment. The decay rates of the quantum correlations increase at first and then decrease with respect to the mass. The larger angular momentum suppresses the decay rates of the correlations. At a fix time, we found the behavior of the quantum correlations are very similar to the equilibrium case: the quantum correlations in the two-qubit system vary non-monotonically with the mass of black hole and are amplified by the angular momentum, while the curvature suppresses the quantum correlations within the system. We quantify the EPR (entropy production rate) of the system, and found that it decreases in time. The EPR decreases at first and then increases to a constant as the black hole mass becomes larger and increases when the
angular momentum becomes larger. Besides, the pace time curvature is found to suppress the information scrambling. The local curvature enhances the decay rate of the quantum correlations and EPR, but reduces the decay rate of the Von Neumann entropy.

Another nonequilibrium scenario is also investigated. There are two types of massless scalar fields equivalent to two different independent bathes coupled to two interacting qubits respectively. The nonequilibrium is measured by the difference in the radius separating of two qubits. We investigate the quantum correlations of the nonequilibrium steady state. On the whole, the correlations can survive and be maintained at a steady value when \( \Delta r \) is large. The entanglement, discord and the mutual information behave non-monotonically in certain parameters. This shows that the quantum correlations can be amplified by the nonequilibrium. The coherence monotonically decreases to a constant. The Von Neumann entropy decreases to a constant which indicates that the nonequilibrium is harmful to produce the correlation between the system and the environment. The flux increases to a constant as the difference in \( \Delta r \) or nonequilibrium increases. The EPR as a measure of thermodynamics cost increases as the \( \Delta r \) increases.

The organization of our paper is as follows. In section 2, we will describe the simplest model, which can be used for the later generalization. Then we review the basic formulations, including the master equation describing the system of the detector in the Born – Markov approximation. In section 3, we introduce certain quantum correlations we are interested in. In section 4, the dimensional reduction technique is used to investigate the massless scalar field in a Kerr space time, and two types of vacua are discussed. In section 5, we discuss the quantum correlations of the equilibrium steady state for the two atom detectors near Kerr black hole. Importantly, we study the nonequilibrium case. In section 6, we consider the quantum correlations in the curved space time in a specific nonequilibrium transient scenario. In section 7, we study the nonequilibrium quantum correlations in curved space time at the steady state. At last, we will draw a conclusion in section 8.

2 Master Equation for open quantum system

Our main objective in this section is to formulate the time evolution of an open quantum system and to obtain the GSKL master equation which properly describes the non-unitary behaviors and can be obtained by partial trace over the environmental baths i.e. the massless probe scalar field placed on the Kerr black hole space time background. Generally, the open quantum set up can be described by the following Hamiltonian

\[
H_{\text{total}} = H_0 + H_1 = H_{\text{sys}} + H_{\text{field}} + H_1.
\]  

Here \( H_{\text{sys}} \) is the Hamiltonian of the atom or the detector. For the single two-level atom internal dynamics will be driven by a \( 2 \times 2 \) hamiltonian matrix. In a given basis can be assumed to have the form: \( \frac{\omega}{2} \sigma_z \), where \( \sigma_z \) is the Pauli matrix, while \( \omega \) represents the gap between the two energy eigenvalues. Then, the atom Hamiltonian becomes \( H_{\text{sys}} = \frac{\omega}{2} \sigma_z \). We assume that the Hamiltonian describing the interaction between the atom and
the scalar field can be taken in the form of the Unruh – DeWitt detector interaction: 
\[ H_I = \mu (\sigma_+ + \sigma_-) \phi(x(\tau)) \], in which \( \mu \) is the coupling constant. Also, we set \( \sigma_+ (\sigma_-) \) as the atomic raising (lowering) operator, and \( \phi(x) \) corresponds to the scalar field operator in Kerr spacetime. The time evolution of the total system in the proper time \( \tau \) is governed by the VonNeumann equation

\[ \partial_\tau \rho_{\text{total}} = -i[H_{\text{total}}, \rho_{\text{total}}]. \quad (2.2) \]

For convenience, one usually performs a unitary transformation to transform the above Liouville – VonNeumann equation into the interaction picture

\[ \partial_\tau \rho_I^{\text{total}} = -i[H_I^{I}, \rho_I^{\text{total}}]. \quad (2.3) \]

The upper index \( I \) represents the operator in interaction picture, the unitary transformation reads \( \rho_I^{\text{total}}(\tau) = e^{iH_0 \tau} \rho_{\text{total}} e^{-iH_0 \tau} \) and \( H_I^{I}(\tau) = e^{iH_0 \tau} H_I e^{-iH_0 \tau} \) for \( \rho_{\text{total}} \) and \( H_I \) respectively. Integrate the above equation Eqn.(2.3), we get

\[ \rho_I^{\text{total}}(\tau) = \rho_I^{\text{total}}(0) - i \int_0^\tau ds [H_I^{I}(s), \rho_I^{\text{total}}(s)]. \quad (2.4) \]

Inserting Eqn.(2.4) back to Eqn.(2.3) and tracing out the field (or environmental) degrees of freedoms, we arrive at

\[ \frac{d\rho_I^{\text{sys}}(\tau)}{d\tau} = -\int_0^\tau Tr_{\text{field}}[H_I^{I}(\tau), [H_I^{I}(s), \rho_I^{\text{total}}(s)]]]. \quad (2.5) \]

where we have taken \( Tr_{\text{bath}}[H_I^{I}(\tau), \rho_I^{\text{total}}(0)] = 0 \), meaning that initially the interaction does not create any dynamics in the bath. Eqn.(2.5) still contains the density matrix of the total system \( \rho_I^{\text{total}}(\tau) \) on its right-hand side. In order to eliminate \( \rho_I^{\text{total}}(\tau) \) from the equation of motion, on can perform a first approximation, known as the Born approximation: the coupling between the system and the bath is weak such that the influence of the bath is small. Thus one can consider the bath as almost unchanged and then the state of the total system at time \( \tau \) may be approximately characterized by a tensor product

\[ \rho_I^{\text{total}}(\tau) = \rho_{\text{sys}}^{I}(\tau) \otimes \rho_{\text{field}}. \quad (2.6) \]

Inserting the tensor product into the exact equation of motion Eqn.(2.5), one obtains a closed integral-differential equation for the reduced density matrix

\[ \frac{d\rho_{\text{sys}}^{I}(\tau)}{d\tau} = -\int_0^\tau Tr_{\text{field}}[H_I^{I}(\tau), [H_I^{I}(s), \rho_{\text{sys}}^{I}(s) \otimes \rho_{\text{field}}]]. \quad (2.7) \]

In order to simplify the above equation further one can perform the Markov approximation, in which the integrand \( \rho_{\text{sys}}^{I}(s) \) is firstly replaced by \( \rho_{\text{sys}}^{I}(\tau) \). In this way one can obtain an equation of motion for the reduced system’s density matrix in which the time development of the state of the system at time \( \tau \) only depends on the present state.

\[ \frac{d\rho_{\text{sys}}^{I}(\tau)}{d\tau} = -\int_0^\tau Tr_{\text{field}}[H_I^{I}(\tau), [H_I^{I}(s), \rho_{\text{sys}}^{I}(\tau) \otimes \rho_{\text{field}}]]. \quad (2.8) \]
The Markov approximation grouped together with the Born approximation is often regarded as the Born – Markov approximation. However, under this approximation alone the resulting master equation does not guarantee to generate a quantum dynamical semi-group. One therefore performs a further secular approximation which involves an averaging over and discard the rapidly oscillating terms in the master equation [26]. With the aid of all these approximations, one can go back to the Schrödinger picture where we obtain the following Markovian master equation in Kossakowski – Lindblad form [27]:

\[
\frac{d\rho_{\text{sys}}(\tau)}{d\tau} = -i[H_{\text{eff}}, \rho_{\text{sys}}(\tau)] + \mathcal{L}[\rho_{\text{sys}}(\tau)] \\
= -i[H_{\text{eff}}, \rho_{\text{sys}}(\tau)] + \sum_{j=1}^{3} [2L_{j}\rho_{\text{sys}}L_{j}^\dagger - L_{j}^\dagger L_{j}\rho_{\text{sys}} - \rho_{\text{sys}}L_{j}^\dagger L_{j}] 
\]

where \(H_{\text{eff}}\) and \(L_{j}\) are given as

\[
H_{\text{eff}} = \frac{\Omega}{2}\sigma_z = \frac{\omega + i(\mathcal{K}(\omega) - \mathcal{K}(-\omega))}{2}\sigma_z \\
L_{1} = \sqrt{\gamma_{-}}\sigma_{-}, L_{2} = \sqrt{\gamma_{+}}\sigma_{+}, L_{3} = \sqrt{\gamma_{0}}\sigma_{z} 
\]

where \(\gamma_{\pm} = \mu^{2}\int_{-\infty}^{+\infty} e^{\pm i\omega s} G^{+}(s - i\epsilon)ds\) and \(\gamma_{0} = 0\), \(G^{+}(s - i\epsilon) = \langle 0 | \phi(x)\phi(x') | 0 \rangle\) is the Wightman function of the massless scalar field \((s = \tau - \tau'\) here). And \(\mathcal{K}(\lambda) = \frac{P}{i\pi}\int \frac{\mathcal{G}(\omega)}{\omega - \lambda}d\omega\) (\(P\) denotes principal value) where \(\mathcal{G}(\omega)\) is the Fourier transformation of \(G^{+}\).

3 Measures of quantum correlations

In this section, we introduce certain important measures which are required for the quantification of the quantum correlations. Coherence, being at the heart of interference phenomena, plays a central role in physics as it enables applications that are impossible within classical mechanics or ray optics and can be measured as [1]

\[
\mathcal{G}_{i_1} = \sum_{i \neq j} |\rho_{ij}| 
\]

Quantum entanglement has remained a major resource for accomplishing quantum information processing tasks such as teleportation [10], quantum key distribution [11], and quantum computing [2] etc.

Among many measures of entanglement of a two-qubit system, concurrence is extensively used so far in many contexts. The concurrence of a two-qubit mixed state \(\rho\) is defined as [3]

\[
\mathcal{C} = \text{Max}(0, \lambda_{1} - \lambda_{2} - \lambda_{3} - \lambda_{4}) 
\]

where \(\lambda_{i}\) represents the square root of the \(i\)th eigenvalue, in descending order of the matrix \(\tilde{\rho}\) with \(\tilde{\rho} = (\sigma_{2} \otimes \sigma_{2})\rho^T(\sigma_{2} \otimes \sigma_{2})\), while \(T\) denotes transposition.
A bipartite quantum state contains both classical and quantum correlations which are quantified jointly by their quantum mutual information, an information-theoretic measure of the total correlation in a bipartite quantum state. In particular, if $\rho_{AB}$ denotes the density operator of a composite bipartite system $AB$, and $\rho_A$ ($\rho_B$) denotes the density operator of part A(B), respectively, then the quantum mutual information is defined as [4]

$$I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$ (3.3)

where $S(\rho) = -\text{tr}(\rho \log_2 \rho)$ is the Von Neumann entropy. The whole system is a pure state at any time because of the unitary evolution, and $S(\rho_{\text{total}}) = 0$. When we trace out the degrees of freedoms of the field, and only consider $S(\rho_{AB})$ which is non-vanishing, this measures the entanglement between the system and the environment.

Quantum discord is a measure of non classical correlation between two subsystems of a quantum system. It includes correlations that are due to quantum physical effects, but do not necessarily involve the concept of quantum entanglement. In fact it is a different type of quantum correlation than the entanglement because separable mixed states (that is, with no entanglement) can have non-zero quantum discord. Sometimes it is also identified as the measure of quantumness of the correlation functions. It is defined as [4] [28]

$$\mathcal{D}(\rho_{AB}) = I(\rho_{AB}) - \mathcal{C}\mathcal{C}(\rho_{AB})$$ (3.4)

$\mathcal{C}\mathcal{C}(\rho_{AB})$ is the classical correlation which depends on the projection operator and we use the maximum in computing discord. For a general state, quantum discord is hard to compute and only for $X$-type state there is an exact expression. For any two qubit state the density matrix is given by the following expression:

$$\rho_{AB} = \frac{1}{4}(I_a \otimes I_b + \sum_{i=1}^{3}(a_i \sigma_i \otimes I_b + I_a \otimes b_i \sigma_i) + \sum_{i,j=1}^{3}C_{ij} \sigma_i \otimes \sigma_j)$$ (3.5)

For the class of a ”X” state, the Bloch vector is along the z-axis, the above expression can be simplified as

$$\rho_{AB} = \frac{1}{4}(I_a \otimes I_b + (a \sigma_z \otimes I_b + I_a \otimes b \sigma_z) + \sum_{i,j=1}^{3}C_{ij} \sigma_i \otimes \sigma_j)$$ (3.6)

The quantum discord is invariant under the local unitary transformations. It has been shown that the $\rho_{AB}$ can be further simplified as

$$\rho_{AB} = \frac{1}{4}(I_a \otimes I_b + (a \sigma_z \otimes I_b + I_a \otimes b \sigma_z) + \sum_{i=1}^{3}C_i \sigma_i \otimes \sigma_i)$$ (3.7)

with the local unitary transformations. [29] The $\mathcal{C}\mathcal{C}$ can be measured as [30]

$$\mathcal{C}\mathcal{C} = S(\rho_A) - \min S_1, S_2$$ (3.8)
where

\[
S_1 = -\frac{1 + a + b + c_3}{4} \log_2 \frac{1 + a + b + c_3}{2(1 + b)} - \frac{1 - a + b - c_3}{4} \log_2 \frac{1 - a + b - c_3}{2(1 + b)} - \frac{1 + a - b - c_3}{4} \log_2 \frac{1 + a - b - c_3}{2(1 - b)} - \frac{1 - a - b + c_3}{4} \log_2 \frac{1 - a - b + c_3}{2(1 - b)}
\]

(3.9)

and

\[
S_2 = 1 + f(\sqrt{a^2 + C_1^2})
\]

(3.10)

The \( f(t) \) is defined as \( f(t) = -\frac{1-t}{2} \log_2(1-t) - \frac{1+t}{2} \log_2(1+t) \). Finally, the quantum discord is given as \( \mathcal{D}(\rho_{AB}) = I(\rho_{AB}) - \mathcal{E}(\rho_{AB}) \).

4 Massless scalar field quantized near Kerr horizon and the two vacua

In order to find out how the reduced density matrix evolves with proper time from Eqn.(2.9), we will review the scalar wave equation of the Kerr black hole space time by following Liu et al. [23]. The metric of Kerr spacetime in Boyer–Lindquist coordinates is given as

\[
ds^2 = -\frac{\Delta}{R^2} (dt - a \sin^2 \theta d\varphi)^2 + \frac{\sin^2 \theta}{R^2} [(r^2 + a^2)d\varphi - adt]^2 + \frac{R^2}{\Delta} dr^2 + R^2 d\theta^2
\]

(4.1)

where \( \Delta = (r - r_+)(r - r_-) \), \( R^2 = r^2 + a^2 \cos^2 \theta \) and \( r_\pm = M \pm \sqrt{M^2 - a^2} \). \( M \) and \( a \) represent the mass and the angular momentum per unit mass of the black hole, respectively. The event horizon of the Kerr black hole is located at \( r = r_+ \). Then, Liu et al show that the scalar field theory in the background Eqn.(4.1) can be reduced to a 2-dimensional field theory in the near-horizon region with the dimensional reduction technique. [23] This technique firstly has been employed for the Kerr black hole by Murata and Soda [24] and developed with a more general technique by Iso et al. [25]

First of all, we write further the action of the massless scalar field as

\[
S[\phi] = \frac{1}{2} \int dx^4 \sqrt{-g} \partial_\mu \phi \partial^\mu \phi
\]

(4.2)

By substituting Eqn.(4.1) into Eqn.(4.2), and then transform the radial coordinate \( r \) into the tortoise coordinate \( r_* \) defined by

\[
\frac{dr_*}{dr} = \frac{1}{F(r)} = \frac{r^2 + a^2}{\Delta}
\]

(4.3)
Now the action reads
\[
S[\phi] = -\frac{1}{2} \int dr_* dt d\theta d\varphi \sin \theta \phi \\
\times \left[ -(r^2 + a^2) - \frac{F(r) a^2 \sin^2 \theta}{r^2} \partial_r^2 - 2a (1 - F(r)) \partial_t \partial_r + \left( \frac{F(r)}{\sin^2 \theta} - \frac{a^2}{r^2 + a^2} \right) \partial_\varphi^2 \\
+ \partial_r (r^2 + a^2) \partial_r + \frac{F(r)}{\sin \theta} \partial_\theta \sin \theta \partial_\theta \right] \phi
\]
(4.4)

We only consider the region near Kerr horizon. Since \( F(r_+) \to 0 \) when \( r \to r_+ \), we can only consider dominant terms in Eqn.(4.4).

\[
S[\phi] = -\frac{1}{2} \int dr_* dt d\theta d\varphi \sin \theta \phi \\
\times \left[ -(r^2 + a^2) \partial_r^2 - 2a \partial_t \partial_r - \frac{a^2}{r^2 + a^2} \partial_\varphi^2 + \partial_r (r^2 + a^2) \partial_r \right] \phi
\]
(4.5)

And then one returns to the \( r \) coordinate system and uses a globally corotating coordinate system as
\[
\psi = \varphi - \frac{a}{a^2 + r^2} t \\
\xi = t
\]
(4.6)

In the new coordinates, we can rewrite Eqn.(4.5) as
\[
S[\phi] = \frac{1}{2} \int dr_* dt d\theta d\varphi \sin \theta \phi \\
\times \left[ (r^2 + a^2) \sin \theta \phi \left( -\frac{1}{F(r)} \partial_\xi^2 + \partial_r F(r) \partial_r \right) \phi \right]
\]
(4.7)

Therefore the angular terms disappear completely. Using the spherical harmonics expansion \( \phi = \sum_{l,m} \phi_{lm}(\xi, r) Y_{lm}(\theta, \psi) \), we obtain the effective 2-dimensional action
\[
S[\phi] = \sum_{l,m} -\frac{1}{2} \int (r^2 + a^2) dr d\xi \phi_{lm} \times \left( -\frac{1}{F(r)} \partial_\xi^2 + \partial_r F(r) \partial_r \right) \phi_{lm}
\]
(4.8)

where we have used the orthonormal condition for the spherical harmonics as follows:
\[
\int d\psi d\theta \sin \theta Y_{lm}^* Y_{lm'} = \delta_{ll'} \delta_{mm'}
\]
(4.9)

From the action Eqn.(4.8), it is obvious to find that \( \phi \) can be considered as a (1+1)-dimensional massless scalar field in the backgrounds of the dilaton \( \Phi \). The effective 2-dimensional metric near the horizon and the dilaton can be written as
\begin{equation}
\Phi = r^2 + a^2
\end{equation}

Hence, we have reduced the 4-dimensional field theory to a 2-dimensional case. This is consistent with [24] [25]. From Eqn.(4.10), we can define two types of vacua: the \textit{Boulware} Vacuum and the \textit{Unruh} Vacuum.

According to Eqn.(4.3), the effective 2-dimensional metric Eqn.(4.10) changes to

\begin{equation}
\begin{align*}
ds^2 &= F(r)(-d\xi^2 + dr^2) \\
\end{align*}
\end{equation}

We can see Eqn.(4.11) is exactly conformal to \textit{Minkowski} metric form, hence, the scalar field equation reads

\begin{equation}
(\partial^2_{\xi} - \partial^2_{r_\ast})\phi(\xi, r_\ast) = 0
\end{equation}

We can derive the standard ingoing and outgoing orthonormal mode solutions of Eqn.(4.12): \( \phi(\xi, r_\ast) \sim (e^{-i\omega(\xi + r_\ast)}, e^{-i\omega(\xi - r_\ast)}) \). The particle can be suitably defined: the modes are positive frequency modes with respect to the killing vector field \( \partial / \partial \xi \) for \( \omega > 0 \). Near the horizon, one only considers the outgoing modes \( \phi(\xi, r_\ast) = \frac{1}{\sqrt{4\pi\omega}}e^{-i\omega(\xi - r_\ast)} \), so the massless scalar field near horizon can be quantized as

\begin{equation}
\phi^B = \sum_{\omega} \left[ a^B_\omega \phi(\xi, r_\ast) + a^{B\dagger}_\omega \phi(\xi, r_\ast) \right]
\end{equation}

where \( a^B_\omega \) and \( a^{B\dagger}_\omega \) are the the annihilation and creation operators acting on the \textit{Boulware} vacuum state. The \textit{Fock} vacuum state corresponds to \( a^B_\omega |0\rangle = 0 \). The \textit{Wightman} function of \textit{Boulware} vacuum state can be showen as

\begin{equation}
G^B+(x, x') = -\frac{1}{4\pi^2} \frac{1}{(\Delta \xi - i\epsilon)^2}
\end{equation}

with the proper \( ie \) prescription. Its Fourier transform with respect to the proper time \( g^{B+}(\omega) = 0 \). In fact, the \textit{Boulware} vacuum corresponds to our familiar notion of a vacuum state.

Now one can define the \textit{Unruh} vacuum state following the method as mentioned above. First of all, we write down the Kerr space time line element according to \textit{Kruskal – like} coordinates

\begin{equation}
ds^2 = C(r)(-dT^2 + dR^2)
\end{equation}
where \( T = \kappa^{-1} e^{\kappa r} \sinh \kappa \xi \), \( R = \kappa^{-1} e^{\kappa r} \cosh \kappa \xi \) and \( \kappa = \frac{r_+ - r_-}{2(r_+ + a^2)} \), and \( C(r) = e^{-2\kappa r} F(r) \) is a finite constant near horizon. As seen from Eqn.(4.15), we can see that Eqn.(4.15) is exactly conformal to Minkowski metric form, hence, the scalar field equation reads

\[
(\partial_T^2 - \partial_R^2) \phi(T, R) = 0 \tag{4.16}
\]

Similar to the previous proceeding, we can derive the outgoing wave equation as \( \phi(\xi, r_\ast) \sim e^{-i\omega(T-R)} \). The particle can be suitably defined: the modes are positive frequency modes with respect to the killing vector field \( \frac{\partial}{\partial T} \) for \( \omega > 0 \). Near the horizon, we only consider the outgoing modes \( \phi(T, R) = \frac{1}{\sqrt{4\pi \omega}} e^{-i\omega(T-R)} \), so the massless scalar field near horizon can be quantized as

\[
\phi^U = \sum_\omega [a_\omega^U \phi(T, R) + a_\omega^U \dagger \phi(T, R)] \tag{4.17}
\]

where \( a_\omega^U \) and \( a_\omega^U \dagger \) are the the annihilation and creation operators acting on the Unruch vacuum state. The Fock vacuum state corresponds to \( a_\omega^U |0\rangle = 0 \). The Wightman function of Unruch vacuum state can be shown as

\[
G^U_{TT}(x, x') = -\frac{1}{16\pi^2 \kappa^2 \sinh^2[\frac{\Delta \xi}{\kappa} - i\epsilon]} \tag{4.18}
\]

with the proper \( i\epsilon \) prescription. Its Fourier transform with respect to the proper time is given as \( G^U_{TT}(\omega) = \frac{1}{2\pi} \frac{1}{1 - e^{-2\kappa r} \frac{\omega}{\kappa}} \), where \( \kappa_r = \frac{\kappa}{\sqrt{F(r)}} \). It is found that the detector in the Unruh vacuum can spontaneously get excited with a nonvanishing probability, in the same way as the thermal radiation with Hawking temperature from a Kerr black hole. Hawking – Unruh effect of a Kerr spacetime can be understood as a manifestation of thermalization behavior in an open quantum system. [23] We will only consider the Unruh vacuum in the following studies. In fact, the local \( \kappa_r \) plays an essential role when we study the characteristics of the quantum correlations. This not only reflects the local curvature of the space time, but also embodies thermal nature of the black hole. In Fig.1, at fixed angular momentum, the local acceleration \( \kappa_r \) decreases to a steady value as the mass increases. The local acceleration only shows non-monotonic behavior when the mass is close to the angular momentum per mass. At fixed mass, the local acceleration keeps a steady value when the angular momentum per mass is less than and away from the mass and only significantly decreases when the angular momentum per mass is close to the mass.

The proper acceleration of the stationary detector near the horizon is divergent, therefore one can define a renormalized value termed as surface gravity. The surface gravity is generally the local proper acceleration multiplied by the gravitational time dilation factor.
(which goes to zero at the horizon). It corresponds to the Newtonian gravitational value in the non-relativistic limit. For a asymptotic observer, we can use Newtonian gravity to obtain the surface gravity at Schwarzschild black hole horizon $\kappa = \frac{M}{r^2} |_{r=r_0} = \frac{1}{\sqrt{4M}}$. \[31\] For a $3 + 1$ dimensional asymptotically flat Kerr black hole with angular speed $\Omega_+ = \frac{a}{r_+ + a^2}$, one can use it to define an effective spring constant $k = M\Omega_+^2$. The surface gravity of Kerr black hole can be formulated as $\kappa_{Kerr} = \frac{1}{4M} - k$ which decreases when $M$ and angular speed $\Omega_+$ increase. \[32\] One can naively consider that the reduction of the Schwarzschild black hole surface gravity compensates as a centripetal force for the detector co-rotating with the black hole. The behaviors of $\kappa$ with respect to the mass and the angular momentum are very similar to those of $\kappa_r$ except that the $\kappa$ decreases monotonically with respect to the mass. In our derivation steps, the effective surface gravity is changed as $\kappa_r = \frac{\kappa}{\sqrt{F(r)}}$ due to the use of dimensional reduction. Finally it leads to non-trivial behaviors of $\kappa_r$ on the mass and the angular momentum.

5 Equilibrium quantum correlations in curved space time

We consider that both two detectors are coupled to the field. Generalizing Eqn.(2.9) from one atom to two atoms, generalizing free hamiltonian as $H_0 = \frac{\omega}{2} \hat{n} \cdot \hat{\sigma}$. The interaction hamiltonian becomes $H_I = \sum_{\mu=0}^{3} [\sigma_\mu^0 \Phi_\mu(x_\alpha) \] where $\Phi_\mu(x) = \sum_{i=1}^{N} \chi_\mu^i \phi^-(x) + \chi_\mu^i \phi^+(x)$. $\phi^\pm(x)$ are positive and negative energy field operators of the massless scalar field, and $\chi$ are the corresponding complex coefficients. After assuming $\sum_{i=1}^{N} \chi_\mu^i \chi_\mu^{i*,} = \delta_{\mu\nu}$ the master equation reads \[15\] \[16\]
\[
\frac{d\rho_{\text{sys}}(\tau)}{d\tau} = -i[H_{\text{eff}}, \rho_{\text{sys}}(\tau)] + \mathcal{L}[\rho_{\text{sys}}(\tau)]
\]

\[
H_{\text{eff}} = \frac{\omega}{2} \sum_{i=1}^{2} \sum_{\alpha=1}^{2} n_i^{\alpha} \sigma_i^{\alpha\alpha} - \frac{i}{2} \sum_{\alpha,\beta=1}^{2} \sum_{i,j=1}^{3} H_{ij}^{\alpha\beta} \sigma_i^{\alpha} \sigma_j^{\beta}
\]

\[
\mathcal{L}[\rho_{\text{sys}}] = \frac{1}{2} \sum_{\alpha,\beta=1}^{2} \sum_{i,j=1}^{3} \varrho_{ij}^{\alpha\beta} [\sigma_j^{\beta} \rho_{\text{sys}}^{\alpha} - \{\sigma_j^{\beta} \sigma_j^{\beta} ; \rho_{\text{sys}}\}]
\]

where \(\sigma_i^1 = \sigma_i \otimes \sigma_0\) and \(\sigma_i^2 = \sigma_0 \otimes \sigma_i\). The GSKL matrix \(\varrho_{ij}^{\alpha\beta}\) is given by the following expression

\[
\varrho_{ij}^{\alpha\beta} = A^{\alpha\beta} \delta_{ij} - iB^{\alpha\beta} \epsilon_{ijk} n_k + C^{\alpha\beta} n_i n_j
\]

where the \(A^{\alpha\beta}\) and \(B^{\alpha\beta}\) and \(C^{\alpha\beta}\) for the two atomic system are defined as:

\[
A^{\alpha\beta} = \frac{\mu^2}{4} (G^{\alpha\beta}(\omega) + G^{\alpha\beta}(-\omega))
\]

\[
B^{\alpha\beta} = \frac{\mu^2}{4} (G^{\alpha\beta}(\omega) - G^{\alpha\beta}(-\omega))
\]

and \(C^{\alpha\beta}\) is given as \(G(0) - A^{\alpha\beta}\). Similarly, the coefficients of \(H_{ij}^{\alpha\beta}\) can be obtained by replacing \(G^{\alpha\beta}(\omega)\) with \(K^{\alpha\beta}(\omega)\) in the above equations where \(K^{\alpha\beta}(\lambda) = \frac{P}{i \pi} \int_{-\infty}^{\infty} \frac{G^{\alpha\beta}(\omega)}{\omega - \lambda} d\omega\). [15]

In the following we set \(\mu_\alpha = \mu_\beta = 0.01\).

These results for the Hamiltonian contributions require some further comments. The \(K^{11}\) can be splitted (similar results hold also for \(K^{12}\)):

\[
K^{11}(\lambda) = \frac{1}{2\pi^2 i} \left[ P \int_0^{\infty} d\omega \frac{\omega}{\omega - \lambda} + P \int_0^{\infty} d\omega \frac{\omega}{1 - e^{2\pi \kappa_r^{-1} \omega}} \left( \frac{1}{\omega + \lambda} - \frac{1}{\omega - \lambda} \right) \right]
\]
curvature effect, [15] [16] [19], by disregarding the Hamiltonian contribution in Eqn.(2.9) and only concentrate on the study of the effects induced by the dissipative part. We consider a situation that there is no real distance between two qubits, which means the system in an equilibrium common environment. The presence of an equilibrium state $\rho^{\infty}$ can be in general determined by setting $\mathcal{L}[\rho_{\text{sys}}(\tau)] = 0$. Consider a general density matrix of the two-atom system in the form of $\rho(\tau) = \frac{1}{4}[\mathbb{1} \otimes \mathbb{1} + \rho_{0i}(\tau)\sigma_i \otimes \sigma_i + \rho_{0j}(\tau)\sigma_j \otimes \sigma_j + \rho_{ij}(\tau)\sigma_i \otimes \sigma_j]$, inserting it into $\mathcal{L}[\rho_{\text{sys}}(\tau)] = 0$, one derives the following result [15]:

$$
\rho^{\infty}_{ii} = \rho^{\infty}_{i0} = -\frac{R}{3 + R^2}(\tau_* + 3)n_i
$$

$$
\rho^{\infty}_{ij} = \frac{1}{3 + R^2}[\left(\tau_* - R^2\right)\delta_{ij} + R^2(\tau_* + 3)n_i n_j]
$$

where $R = B/A$, $\tau_*$ is the trace of the density matrix $\tau_* = \sum_{i=1}^{3}\rho_{ii}$, which is actually a constant of motion, and the positivity of $\rho(0)$ requires that $-3 \leq \tau_* \leq 1$. At the initial state, consider the direct product of two pure states: $\rho(0) = \rho_{\tilde{a}} \otimes \rho_{\tilde{b}}$, where $\rho_{\tilde{a}} = \frac{1}{2}(1 + \tilde{a} \cdot \vec{\mathbf{σ}})$, $\rho_{\tilde{b}} = \frac{1}{2}(1 + \tilde{b} \cdot \vec{\mathbf{σ}})$, and $\tilde{a}$ and $\tilde{b}$ are two unit vectors. In this case, one easily finds that $\tau = \tilde{a} \cdot \tilde{b}$. In this paper, we set $\tilde{a} = (0, 0, 1)$ and $\tilde{b} = (0, 0, -1)$, the hamiltonian for the single atom is $H_0 = \frac{\omega_2}{2}\sigma_z$.

The system finally reaches the equilibrium steady state since the two subsystems are at the same temperature. More concretely speaking, the space time curvature leads to Hawking temperature due to the Unruh effect, i.e., there is a common thermal bath around the two-qubits system which is near the black hole horizon and the temperature depends on the curvature.

On the above basis, we study the quantum correlations of two-qubit system near the Kerr black hole with a global corotating coordinate and as shown in Fig.2.

In our setting, the initial state is a separable state, i.e. there is no any quantum correlation initially. After evolution, it has been shown that the system reaches a steady state. More remarkably, the system harvests the quantum correlation from the Unruh vacuum in Fig.2, consistent with the suggestions made in Ref [17] [18] [19]. The initial state of the system is a pure state where the Von Neumann entropy vanishes and the final state is a mixed state which the Von Neumann entropy is nonvanishing. Apparently, the system has experienced a nonequilibrium process and finally forms a steady state with quantum correlations under the parameters of the black hole. We focus on this steady state. The quantum correlation is derived from nonunitary evolution which is caused by the interaction between the field and the system. In the Fig.2(a)(b), the concurrence, mutual information, discord and coherence all decrease first and then increase to a constant as the mass of black hole increases while the angular momentum per mass is unchanged. On the contrary, the Von Neumann entropy of the system which measures the entanglement between the system and the environment increases initially and then decreases to a constant. For the black hole with larger mass, the quantum correlations are neither more sensitive to the change of the angular momentum per mass, nor to the mass. The angular momentum per mass has significant effect on the quantum correlations only when it is comparable to the mass. This
Figure 2: Quantum correlations at equilibrium state vs mass or angular momentum per mass. The system is located at $1.01r_+$, the angular momentum per mass $a$ is set up 10 and the mass is changed from 10 in Fig.2(a)(b). The system is located at $1.01r_+$, the mass $a$ is set up as 10.01 and the angular momentum per mass is changed from 0.1 in Fig.2(c)(d). $\tau = -1$. The eigenfrequencies of the qubits are $\omega_1 = \omega_2 = 0.1$

is also demonstrated in Fig.2(c)(d). The quantum correlations are boosted by the angular momentum per mass except that the Von Neumann entropy decreases.

The above non-trivial result comes from the fact that the dependence of the quantum correlations on the local acceleration $\kappa_r$. The $\kappa_r$ is directly related to the curvature of space time. All the quantum information between the two qubits are reduced by larger curvature. On the contrary the Von Neumann entropy which measures the entanglement between the system and the environment increases as shown in Fig.3. At fixed angular momentum, the $\kappa_r$ decreases to a steady value as the mass increases, and only shows non-monotonously behavior when the mass is close to the angular momentum per mass. At fixed mass, it keeps a steady value when the angular momentum per mass is away and less than mass and only significantly decreases when the angular momentum per mass is close to the mass. Thus the behavior of the $\kappa_r$ with respect to the mass and angular
momentum determines the behavior of the correlations. Moreover, the $\kappa_r$ appears to be inversely related to the quantum correlations in the system and similarly to the system-environment entanglement. The larger $\kappa_r$ correspond to the higher space time curvature and higher effective temperature. This makes the system more classical and weakens the quantum nature. On the contrary, the higher temperature leads to strengthening of the interaction between the system and the field. Therefore this makes more easily for them to correlate. Here we see the impact of the space time structure on the quantum correlations.

![Graph](image)

**Figure 3**: Quantum correlations at equilibrium state vs $\kappa_r$. Other parameters are the same as those of the Fig.2

6 Nonequilibrium transient quantum correlations in curved space time

To study the quantum correlation in the Kerr black hole space time, we introduce an auxiliary system (the same two-level atom) which is isolated from the environment, meaning the coupling between the atom and environment is extremely weak. [33] [34] The schematic diagram is shown in Fig.4. The qubit coupled to the field is called A while the qubit free of the field coupling is called B. The field is represented by $E$. Initially, A and B is maximally entangled, only the A interacts with $E$. After initial time, the initial quantum correlations between A and B is expected to transferred to the correlations between A and $E$. We can expand any general density matrix for the bipartite two-level atom system as follows

$$\rho = \sum_{i,j=0}^{3} \rho_{ij} \sigma_i \otimes \sigma_j$$  \hspace{1cm} (6.1)

where $\{\sigma_i \otimes \sigma_j | i, j \in 0, ..., 3\}$ forms sixteen linearly independent complete vector basis and we choose

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$  \hspace{1cm} (6.2)
and $\sigma_0 = 1$. A good property about this choice of basis is that the expansion coefficients $\rho_{ij}$ are real and satisfy $\rho^\dagger = \rho$ and $\text{Tr}\rho = 1$. Furthermore the expansion coefficients can be computed directly using $\rho_{ij} = \text{Tr}\{\rho \sigma_i \otimes \sigma_j\}$.

Substituting Eqn.(6.1) into Eqn.(2.9), we derive

$$\sum_{i,j=0}^3 \frac{d\rho_{ij}(\tau)}{d\tau} \sigma_i \otimes \sigma_j = -i[H_{eff}, \sum_{i,j=0}^3 \rho_{ij}(\tau)\sigma_i \otimes \sigma_j]$$

$$+ \sum_{i,j=0}^3 \sum_{m=1}^3 \rho_{ij}[2L_m\sigma_i L_m^\dagger - L_m^\dagger L_m\sigma_i - \sigma_i L_m^\dagger L_m] \otimes \sigma_j.$$  

We comment more about the above equation. The auxiliary atom is isolated from the environment meaning the environment has no interaction or dissipative effect on it. During the evolution, the system and the environment exchange the energy and the information. In the transient process, the interior of system is unbalanced.

From Eqn.(5.4), we can derive the time dependent density matrix elements

$$\rho_{ij}(\tau) = \rho_{ij}(0),$$
$$\rho_{1j}(\tau) = \rho_{1j}(0)e^{-\frac{A\tau}{2}} \cos(\Omega \tau) - \rho_{2j}(0)e^{-\frac{A\tau}{2}} \sin(\Omega \tau),$$
$$\rho_{2j}(\tau) = \rho_{1j}(0)e^{-\frac{A\tau}{2}} \sin(\Omega \tau) + \rho_{2j}(0)e^{-\frac{A\tau}{2}} \cos(\Omega \tau),$$
$$\rho_{3j}(\tau) = \rho_{1j}(0)e^{-A\tau} - \frac{B}{A} \rho_{0j}(0)(1 - e^{-A\tau}).$$  

where $A = \gamma_+ + \gamma_-$ and $B = \gamma_+ - \gamma_-$. In the following we consider that two atoms initially share a maximally entangled state, i.e, $\rho_{00} = \rho_{11} = -\rho_{22} = \rho_{33} = \frac{1}{4}$, while the rest of $\rho_{ij}$ vanish.

In Fig.5 we plot various quantum correlations vs time in the unit of $\frac{1}{\mu_2}$ at $a = 0.01$, $M = 10$ and $a = 9.9$, $M = 10$ respectively. In Fig.5, all quantum correlations between two qubits decrease to zero due to the dissipative effect of the environment but the Von Neumann entropy increases to a constant in time. Comparing Fig.5(a)(b) and Fig.5(c)(d), for a black hole with a larger mass relative to angular momentum, the quantum correlations decrease faster and reach a larger entropy. In the meanwhile, we study the dependence of the quantum correlations on the mass in Fig.6(a)(b) and on the angular momentum per mass in Fig.6(c)(d). The behaviors of the quantum correlations are very similar to the equilibrium case: the concurrence, the mutual information, the discord and the coherence all decrease.
Figure 5: Nonequilibrium quantum correlation evolutions vs time with the different angular momentum per mass in the unit of $\frac{1}{\mu^2}$. Other parameters are the same as those of the Fig.2

first and then increase to a constant as the mass of black hole increases while the angular momentum per mass is unchanged. On the contrary, the entropy increases initially and then decreases to a constant. The angular momentum can amplify quantum correlations, especially when the angular momentum becomes larger. Also the quantum correlations vs the local curvature $\kappa_r$ is plotted in Fig.7(6a)(6b). All the quantum correlations between the two atoms decrease by larger curvature. On the contrary the Von Neumann entropy increases. These imply that the quantum correlations in this nonequilibrium model is also determined by the local curvature or acceleration $\kappa_r$. The larger $\kappa_r$ makes the system more classical and this weakens the quantum nature. On the contrary, the larger $\kappa_r$ leads to strengthening of the interaction between the system and the field, therefore makes them more easily correlate.

We try to quantify the nonequilibrium by considering entropy production ($EP$) and entropy production rate ($EPR$) for our setting. The initial state of the correlated system $AB$ is denoted by $\rho_{AB}^i$ and the initial state of the field is denoted by $\rho_E^i$. In our setup, we
Figure 6: Fig.6(a)(b): Quantum correlations at nonequilibrium transient state vs mass when $t = \frac{100}{\mu^2}$ and $a = 10$. Fig.6(c)(d): Quantum correlations at nonequilibrium transient state vs the angular momentum per mass when $t = \frac{100}{\mu^2}$ and $m = 10$. Other parameters are the same as those of the Fig.2

assume there is no correlation between the system $AB$ and $E$ initially, so the initial state of total system reads $\rho^{i}_{ABE} = \rho^{i}_{AB} \otimes \rho^{i}_{E}$. Although the system $AB$ experiences a nonunitary evolution, the total system is isolated and follows unitary evolution. At the finial state of $ABE$, the density matrix is given by

$$\rho^{f}_{ABE} = U_{AE}\rho^{i}_{AB} \otimes \rho^{i}_{E} U_{AE}^{\dagger}$$ (6.5)

and the evolution of $AE$ is given as

$$\rho^{f}_{AE} = U_{AE}\rho_{AE} U_{AE}^{\dagger}$$ (6.6)

where $U_{AE}$ is associated to the unitary transformation. The entropy production of the system $A$ about the evolution $U_{AE}$ can be given as [35] [36]
\[ \Sigma_A(t_i : t_f) := I_{A:E}(t_f) + S(\rho_E^f || \rho_E^i) \] (6.7)

where \( S(\rho || \sigma) = \text{Tr} \rho \ln \rho - \rho \ln \sigma \) is the relative entropy.

The entropy production of the system AB about the evolution \( U_{AE} \) is

\[ \Sigma_{AB}(t_i : t_f) = I_{AB:E}(t_f) + S(\rho_E^f || \rho_E^i) \] (6.8)

The entropy production can be rewritten as \( \Sigma_{AB}(t_i : t_f) \)

\[ \Sigma_{AB}(t_i : t_f) := \Delta I_{A:B}(t_i : t_f) + \Sigma_A(t_i : t_f) \] (6.9)

where \( \Delta I_{A:B}(t_i : t_f) := I_{A:B}(t_i) - I_{A:B}(t_f) \), \( I_{A:B} \) is mutual information between \( A \) and \( B \). For the detailed derivation, see appendix A. In our early setup, we have assumed a Born approximation: the coupling between the system and the bath is weak such that the influence of the bath is small. Thus we can consider the bath is almost unchanged. Then the state of the total system at time \( \tau \) may be approximately characterized by a tensor product as Eqn.(2.6), so that \( U_{AE} \rho_{AB}(\tau^i) \otimes \rho_E U_{AE}^\dagger \approx \rho_{AB}(\tau^f) \otimes \rho_E \). Hence \( \Sigma_{AB}(t_i : t_f) \geq 0 \) are due to the positivity of \( I_{A:B}(t_i : t_f) \) and the quantum relative entropy. Consider the Born approximation, we further derive a lower bound of the entropy production:

\[ \Delta I_{A:B}(t_i : t_f) \geq I_{A:E}(t_f) = S_A^f + S_E^f - S_{AE}^f \approx 0 \] (6.10)

\[ S(\rho_E^f || \rho_E^i) \approx 0 \]
The first greater than or equal to sign comes from the fact that the scrambling information between $A$ and $B$ is greater than the increased information between $A$ and $E$. The later approximately equal sign is due to the Born approximation.

Figure 8: a) EPR at nonequilibrium state varying with time $t$ at angular momentum $a = 9.9$ and the mass $M = 10$. b) EPR versus mass when $a = 10$ and $t = \frac{100}{\mu^2}$. c) EPR versus the angular momentum per mass when $m = 10$ and $t = \frac{100}{\mu^2}$. d) EPR versus $\kappa_r$ when $t = \frac{100}{\mu^2}$. Other parameters are the same as the Fig.2.

For a nonequilibrium state the EPR quantifying the dissipative cost is always larger than zero, the Born – Markov approximation is assumed such that the two positive parts are omitted, hence the EPR in our paper is actually a lower bound. The EPR of the system is plotted in Fig.8. The EPR decreases in time, and finally vanishes. At fixed time, the EPR varies non-monotonically with respect to the mass. The increase in angular momentum can amplify EPR. The non-trivial behavior of EPR on the mass and the angular momentum also comes from the fact that the mass and the angular momentum are directly related to the local curvature or acceleration $\kappa_r$. The EPR varying with $\kappa_r$ is plotted in Fig.8d. The EPR decreases monotonically with the local curvature. Hence, we see that the increase of
the local curvature reduces the dissipative cost of information scrambling.

\[ QC(t) = QC(0) - \frac{QC(0) - QC(t)}{t} \]

where \( QC \) is quantum correlation. The

\( \text{Figure 9:} \) The decay rates of the quantum correlations with respect to the mass when \( a = 10, \ t = \frac{100}{\mu} \) or w.r.t. the angular momentum per mass when \( m = 10, \ t = \frac{100}{\mu^2} \). Other parameters are the same as those of the Fig.2
Figure 10: The decay rates of the quantum correlations with respect to the mass when $a = 10$, $t = \frac{100}{\mu^2}$, or w.r.t. the angular momentum per mass when $m = 10$, $t = \frac{100}{\mu^2}$. Other parameters are the same as those of the Fig. 2.
Figure 11: The decay rates of the quantum correlations with respect to the local acceleration or curvature $\kappa_r$, when $t = \frac{100}{\mu^2}$. Other parameters are the same as those of the Fig.2.
the eigenstates of two qubits system are:

\[ H_{\text{total}} = H_0 + H_1 = H_{\text{sys}} + H_{\text{field}} + H_1 \]

\[ H_{\text{sys}} = \frac{\omega_1}{2} \sigma_{z1} + \frac{\omega_2}{2} \sigma_{z2} + K(\sigma_{1+}\sigma_{2-} + \sigma_{1-}\sigma_{2+}) \]  

\[ H_1 = \mu_1(\sigma_{1+} + \sigma_{1-})\phi(x_1(\tau_1)) + \mu_2(\sigma_{2+} + \sigma_{2-})\psi(x_2(\tau_1)) \]  

(7.1)

where \( K \) is the coupling of the inter-qubits and \( \mu_1 = \mu_2 = 0.01 \). The eigen energy and the eigenstates of two qubits system are: \( E_1 = -\frac{\omega_1 + \omega_2}{2}, |\lambda_1\rangle = |0, 0\rangle; E_2 = \frac{\omega_1 - \omega_2}{2}, |\lambda_2\rangle = |1, 1\rangle; E_3 = \kappa, |\lambda_3\rangle = \cos(\theta/2)|1, 0\rangle + \sin(\theta/2)|0, 1\rangle; E_4 = -\kappa, |\lambda_4\rangle = -\sin(\theta/2)|1, 0\rangle + \cos(\theta/2)|0, 1\rangle. \) We define \( \kappa = \sqrt{K^2 + (\omega_1 - \omega_2)^2/4} \) and \( \theta = \arctan(2K/(\omega_1 - \omega_2)) \). In this paper, we only consider symmetric case which means \( \omega_1 = \omega_2 \). In the eigen energy basis, we can define two groups of transitions operators:

\[ V_{1,1} = \cos(\theta/2)(|\lambda_1\rangle\langle\lambda_1| + |\lambda_4\rangle\langle\lambda_2|) \]

\[ V_{1,2} = \sin(\theta/2)(|\lambda_3\rangle\langle\lambda_2| - |\lambda_1\rangle\langle\lambda_4|) \]  

\[ V_{2,1} = \sin(\theta/2)(|\lambda_1\rangle\langle\lambda_3| - |\lambda_4\rangle\langle\lambda_2|) \]

\[ V_{2,2} = \cos(\theta/2)(|\lambda_3\rangle\langle\lambda_2| + |\lambda_1\rangle\langle\lambda_4|) \]  

(7.2)

with transition frequency \( \Omega_1 = E_2 - E_3 \) and \( \Omega_2 = E_2 + E_3 \).

7 Nonequilibrium steady quantum correlations

To generalize the above case to the intrinsic nonequilibrium case where the detailed balanced is not preserved, we introduce another massless scalar field \( \psi \), and assume that there is no interaction between two fields. Furthermore, we introduce the interaction between the two qubits. The Hamiltonian is generalized as

\[ H_{\text{total}} = H_0 + H_1 = H_{\text{sys}} + H_{\text{field}} + H_1 \]

\[ H_{\text{sys}} = \frac{\omega_1}{2} \sigma_{z1} + \frac{\omega_2}{2} \sigma_{z2} + K(\sigma_{1+}\sigma_{2-} + \sigma_{1-}\sigma_{2+}) \]  

\[ H_1 = \mu_1(\sigma_{1+} + \sigma_{1-})\phi(x_1(\tau_1)) + \mu_2(\sigma_{2+} + \sigma_{2-})\psi(x_2(\tau_1)) \]  

(7.1)

where \( K \) is the coupling of the inter-qubits and \( \mu_1 = \mu_2 = 0.01 \). The eigen energy and the eigenstates of two qubits system are: \( E_1 = -\frac{\omega_1 + \omega_2}{2}, |\lambda_1\rangle = |0, 0\rangle; E_2 = \frac{\omega_1 - \omega_2}{2}, |\lambda_2\rangle = |1, 1\rangle; E_3 = \kappa, |\lambda_3\rangle = \cos(\theta/2)|1, 0\rangle + \sin(\theta/2)|0, 1\rangle; E_4 = -\kappa, |\lambda_4\rangle = -\sin(\theta/2)|1, 0\rangle + \cos(\theta/2)|0, 1\rangle. \) We define \( \kappa = \sqrt{K^2 + (\omega_1 - \omega_2)^2/4} \) and \( \theta = \arctan(2K/(\omega_1 - \omega_2)) \). In this paper, we only consider symmetric case which means \( \omega_1 = \omega_2 \). In the eigen energy basis, we can define two groups of transitions operators:

\[ V_{1,1} = \cos(\theta/2)(|\lambda_1\rangle\langle\lambda_1| + |\lambda_4\rangle\langle\lambda_2|) \]

\[ V_{1,2} = \sin(\theta/2)(|\lambda_3\rangle\langle\lambda_2| - |\lambda_1\rangle\langle\lambda_4|) \]  

\[ V_{2,1} = \sin(\theta/2)(|\lambda_1\rangle\langle\lambda_3| - |\lambda_4\rangle\langle\lambda_2|) \]

\[ V_{2,2} = \cos(\theta/2)(|\lambda_3\rangle\langle\lambda_2| + |\lambda_1\rangle\langle\lambda_4|) \]  

(7.2)

with transition frequency \( \Omega_1 = E_2 - E_3 \) and \( \Omega_2 = E_2 + E_3 \).
A stationary detector near the horizon will experience a thermal bath with effective temperature \( \frac{k}{\pi r^2} \). In his viewpoint, any two different points have different temperature. Now if we separate the two-qubit with a finite distance along the radius, the system is equivalently the case being connected to two independent bath. Therefore, one can derive a nonequilibrium master equation for two separated atoms in the observer’s frame. Using the observer’s proper time, the master equation can be derived (neglect the contribution of the principal value which can only modify the energy level) \[37\] \[38\] \[39\]

\[
\frac{d\rho_{\text{sys}}(\tau)}{d\tau} = -i[H_{\text{sys}}, \rho_{\text{sys}}(\tau)] + \mathcal{L}_1[\rho_{\text{sys}}] + \mathcal{L}_2[\rho_{\text{sys}}] \\
\mathcal{L}_j[\rho_{\text{sys}}] = \sum_{\mu=1}^{2} G^i(-\omega_\mu)(2V_{j,\mu}\rho_{\text{sys}}V_{j,\mu}^\dagger - \rho_{\text{sys}}, V_{j,\mu}^\dagger V_{j,\mu}) \\
+ G^i(\omega_\mu)(2V_{j,\mu}\rho_{\text{sys}}V_{j,\mu} - \rho_{\text{sys}}, V_{j,\mu}V_{j,\mu}^\dagger)
\] (7.3)

\(G^i\) corresponds to the Fourier transform of Green function for different fields. From Eqn.(7.3), it is sufficient to obtain the steady state, and the concrete expression is in Appendix B. The frequency gap of the two identical atoms is different when the two atoms keep different separation distances from a stationary detector’s viewpoint due to the red-shift effect. However, we are not interested in the red-shift effect here and always set the frequency gap of two atoms is the same.

Under these considerations, we now investigate the quantum correlations of the nonequilibrium steady state in the bare basis. The \(\Delta r\) measures nonequilibrium. This is because the difference in radius reflects the difference in the local acceleration or the local space time curvature. This shows difference in temperatures through Unruh-Hawking effect. Thus the case is similar to the one of the two qubits couple to individual bath separately with the different temperature. Therefore the system is in nonequilibrium. However, there is an interesting question? Are the quantum correlations in curved space time back ground the same or different from the case where the system is coupled to two corresponding bathes? Addressing this issue can help us to understand whether the effects of curved space time is equivalent to the temperature on the global correlation level. If there is only one field and only qubit-field interaction as the case in 5. It has shown that the space time curvature, the acceleration and the temperature influence differently on the quantum correlations. \[7\] \[8\] Now let’s go back and look at this case. Strictly speaking, the coupling of the inter-qubits \(K\) relies on the distance and the space time curvature. This directly reflects that the space time curvature, the acceleration and the temperature influence differently on the system. However, we set \(K\) is a constant for simplicity. The quantum correlations of the system are established by the interaction between the qubits rather than the fields since they do not correlate. The effects of the space time curvature between the two point-like qubits on the quantum correlations vanish. Even so, we can still perceive the different effects of the space time curvature and the temperature on the quantum correlations in our setting up. We found the separation distance directly determines the property of the system due to the space time curvature. However, the system coupled to two corresponding bathes
Figure 12: We consider a nonequilibrium scenario: one of the two atoms is stationary at $1.006r_+$ while another is at $(1.006 + \Delta r)r_+$. Quantum correlations at nonequilibrium steady state varying with the separation distance at angular momentum $a = 10$ or the mass $M = 10$. 
Figure 13: We consider a nonequilibrium scenario: one of the two atoms is stationary at $1.006r_+\text{ while another is at } (1.006 + \Delta r)r_+$. Quantum correlations at nonequilibrium steady state varying with separation distance at angular momentum $a = 10$ or the mass $M = 10$.

is not related to the separation distance between the two qubits. Moreover, the redshift effect caused by the local curvature can modify the energy levels of the system, and further influence the quantum correlations of the system. These show the difference between the effects of the curved space time and the temperature on the global correlation level. Besides that, our system is very similar to the system coupled to two corresponding baxes. As mentioned before, for the purpose of only considering nonequilibrium effect induced by different locations, we omit the redshift effect.

It is shown that the final state contains no information about the initial state. We plot the correlations varying with $\Delta r$ under different mass or different angular momentum. On the whole, the correlations arrive at a steady value when $\Delta r$ is large. For the entan-
Figure 14: The a) b) flux and c) d) EPR at nonequilibrium steady state varying with separation distance at angular momentum $a = 10$ or the mass $M = 10$. e) f) the $\kappa_r$ varying with separation distance at angular momentum $a = 10$ or the mass $M = 10$. One of the two atoms is stationary at $1.006r_+$ while another is at $(1.006 + \Delta r)r_+$. 
tlement, the discord and the mutual information in Fig.12(a)(b)(f), Fig.13(b), they vary non-monotonously with $\Delta r$. More importantly, they can be amplified by the nonequilibrium. The coherence and the Von Neumann entropy monotonously decrease to a constant with $\Delta r$ for both different masses and the angular momentum. The nonequilibrium appears to reduce the coherence and generates the correlation between the system and the environment.

The flux measures the energy exchange between the system and the environment. The energy flux from the $i$th field to the system at the steady state is given by $I_i = Tr[\mathcal{L}_i(\rho_{sys})H_{sys}]$ (we can check $I_1 + I_2 = 0$ which satisfies flux conserved.) The flux increases to a constant in Fig.14(a)(b). This means that the energy exchange capacity of the system has an upper bound and is limited to the environment. We can also define an effective EPR: $I(\frac{1}{\kappa_1^2} - \frac{1}{\kappa_2^2})$, the temperature is related to the local curvature or the acceleration $k_r$. The EPR increases when the $\Delta r$ increases as we expect in Fig.14(c)(d).

The above non-trivial phenomena can also be understood from the dependence of the local curvature $\kappa_r$ on the mass and the angular momentum. The difference is that the system now is determined by not only $\kappa_{r1}$ but also $\kappa_{r2}$. This leads to the different effect compared to the previous nonequilibrium model. We plot the quantum correlations, the flux and EPR varying with both $\kappa_{r1}$ and $\kappa_{r2}$ in Fig.15. These figures are symmetric along the line $\kappa_{r1} = \kappa_{r2}$. We can see that the entanglement, the discord and the mutual information in Fig.15(a)(c)(d) show the non-monotonic behaviors as the results before, while the coherence, the Von Neumann entropy, the flux and the EPR show the monotonic behaviors. The above figures explain the Fig.12, 13, 14 well according to the behaviors of $\kappa_r$ in Fig.14(e)(f). The non-monotonic behaviors of the entanglement, the discord and the mutual information can also be understood as a competition between the populations and the coherence. [39]. The concurrence in this model can be formulated as $C = \text{Max}(0, C_{l1} - \sqrt{\rho_{11}\rho_{22}})$ where $C_{l1}$ is the coherence. Thus the concurrence is directly dependent on the coherence and the population. We see that both the coherence and the population vary non-monotonically. Although the discord and mutual information can not be derived with the similar formula, we believe that this competition perspective still holds for the discord and the mutual information because they measure the quantum correlations with certain similar parts in some sense. The Von Neumann entropy is amplified by the $\kappa_r$, the reason is clear: the higher temperature leads to the strengthening of the interaction between the system and the field. Therefore this makes more easily to produce the correlation. The coherence is representation-dependent, it vanishes in the eigen-energy basis but is non-vanishing in the bare basis. The non-vanishing coherence in the bare basis is induced by the interaction of inter-qubits and proportional to $|\rho_{33} - \rho_{44}|$. One can also understand the behavior of the coherence from a competition relationship. On the one hand, when the temperature is low, only the ground state $|\lambda_1\rangle$ and the first excited state $|\lambda_4\rangle$ are significantly occupied, then the coherence is proportional to $\rho_{44}$ and increases with temperature. As the temperature increases, the second excited state $|\lambda_3\rangle$ also starts to be occupied. The coherence then is proportional to $|\rho_{33} - \rho_{44}|$ which shows a competition relationship. As long as the temperature is high enough, the coherence decreases and vanishes at the infinite temperature.

Here we see that the information of the two separate qubits is encoded in space time…
Figure 15: Quantum correlations at nonequilibrium steady state varying with $\kappa_{r1}$ and $\kappa_{r2}$. 
structure again. Unlike previous nonequilibrium case, the qubits system here involves two different locations so that the intrinsic nonequilibrium emergents where detailed balanced is explicitly broken. As a result, the energy flux and associated dissipative cost emerge. They are used to support sustaining and survival of the quantum correlations for long time (at the steady state). Therefore nonequilibrium can contribute to form the quantum correlation of the system.

8 Conclusion

In this paper, we focus on the quantum correlations of curved space time near the horizon of Kerr black hole by using the dimensional reduction and the Born–Markov master equation. We quantify the quantum correlations of the two-qubit system, and the entanglement between the system and the environment. In the equilibrium model, we can obtain a steady state which contains the initial partial information. It is possible to harvest the quantum correlations from the Unruh vacuum. We investigate how the quantum correlations vary with the mass and the angular momentum. We found that the quantum correlations in the system decrease at first and then increase to a constant as the mass increases from a value close to the angular momentum per mass. The increase of angular momentum can amplify the quantum correlations. The entanglement between the system and environment behaves oppositely to the correlations in the system. Importantly, we found the increase of the local space time curvature can reduce the correlations in the system due to the thermal Unruh effect but enhance the entanglement between the system and environment.

In the second nonequilibrium transient model, we found that the information scrambles inevitably to environment. The angular momentum weakens the scrambling but the mass relates to the correlations non-monotonously seen from the decay rates of the correlations. The entanglement between the system and environment also behaves oppositely to the correlations in the system. At the fixed time, the quantum correlations are very similar to the equilibrium case: the quantum correlations in two-qubit system vary non-monotonically with the mass of the black hole and are amplified by the angular momentum, the increase of the space time curvature will suppress the quantum correlations in the system. We quantify the EPR (entropy production rate) of the system and found that it decreases in time. The EPR decreases at first and then increases to a constant with respect to the increase of mass and increases when the angular momentum increases. Besides, the space time curvature suppresses the information scrambling. We also found the local curvature can enhance the decay rates of quantum correlations and EPR, but reduce the decay rate of the Von Neumann entropy which is negative growth. The $\kappa_r$ not only stands for the local curvature of space time but also the thermal nature of black hole. The similar behaviors of the quantum correlations in the above two scenarios are due to the fact that the system state is determined by $\kappa_r$. More profoundly speaking, the features and characteristics of the system information is encoded in the space time structure.

In the third nonequilibrium model, we investigate the quantum correlations of the nonequilibrium steady state. On the whole, the quantum correlations survive and sustain at a steady when $\Delta r$ (which measures the nonequilibrium) is large. The entanglement, discord
and the mutual information behave non-monotonically under certain parameters. This means the quantum correlations can be amplified by the nonequilibrium. The coherence monotonically decreases to a constant. The Von Nuemann entropy decreases to a constant which means that the nonequilibrium reduces the correlation between the system and the environment. The flux which measures the degree of the detailed balance breaking increases to a constant. The EPR as a nonequilibrium thermodynamics dissipative cost increases when the $\Delta r$ increases as we expect. We can qualitatively understand the above non-trivial behaviors by checking the dependence of the system on both the local curvatures or accelerations $\kappa_{r1}$ and $\kappa_{r2}$. In this model the information of the quantum correlations of two separate qubits system are not only encoded in the space time structure, but also from the nonequilibrium contribution.

\section{Derivation of the Entropy production}

The unitary transformation preserves the von-Neumann entropy, therefore

\begin{align}
I_{AE:B}^{f} &= S_{AE}^{f} + S_{B}^{f} - S_{ABE}^{f} \\
&= S_{AE}^{f} + S_{B}^{f} - S_{ABE}^{f} \\
&= S_{A}^{f} + S_{E}^{f} + S_{B}^{f} - S_{AB}^{f} - S_{E}^{f} \\
&= \tilde{f}_{A:B}
\end{align}

(A.1)

In the meanwhile

\begin{align}
I_{AE:B}^{f} &= S_{AE}^{f} + S_{B}^{f} - S_{ABE}^{f} \\
&= S_{A}^{f} + S_{E}^{f} - \tilde{f}_{AE}^{f} + S_{B}^{f} - S_{ABE}^{f} \\
&= S_{A}^{f} + S_{E}^{f} - \tilde{f}_{AE}^{f} + S_{B}^{f} - S_{AB}^{f} + S_{E}^{f} + \tilde{f}_{AB:E}^{f} \\
&= S_{A}^{f} + S_{E}^{f} - \tilde{f}_{AE}^{f} + S_{B}^{f} - S_{AB}^{f} + S_{E}^{f} + \tilde{f}_{AB:E}^{f} \\
&= \tilde{f}_{A:B}^{f} + \tilde{f}_{AB:E}^{f} - \tilde{f}_{A:E}^{f}
\end{align}

(A.2)

Combine with Eqn.(A.1), we get $\Delta I_{A:B}(t_i : t_f) = \tilde{I}_{AB:E}^{f} - \tilde{I}_{A:E}^{f}$. In the meanwhile, the above equation implies $\Delta I_{A:B}(t_i : t_f) > 0$, which is from the fact that the correlation between $AB$ and $E$ should be larger than the correlation between $A$ and $E$. Inserting Eqn.(6.7) into Eqn.(6.8)

\begin{align}
\Sigma_{AB}(t_i : t_f) &= I_{AB:E}(t_f) - I_{A:E}(t_f) + \Sigma_{A}(t_i : t_f) \\
&= \Delta I_{A:B}(t_i : t_f) + \Sigma_{A}(t_i : t_f)
\end{align}

(A.3)

\section{Steady state expression}

We give a concrete expression of the steady state matrix. The computation method was given in [38] [39] [40]. For the steady state matrix, the off-diagonal elements vanish and the diagonal elements are
\[\rho_{11} = \frac{X_i^+ Y_i^+}{X_i Y_2} \]
\[\rho_{22} = \frac{X_i^- Y_i^-}{X_i Y_2} \]
\[\rho_{33} = \frac{X_i^+ Y_i^+}{X_i Y_2} \]
\[\rho_{44} = \frac{X_i^- Y_i^-}{X_i Y_2} \]

(B.1)

where we define

\[X_i = X_i^+ + X_i^- \]
\[Y_i = Y_i^+ + Y_i^- \]
\[X_i^+ = 2 \cos^2(\theta/2)G_1^{\pm}(\pm \omega_i) + 2 \sin^2(\theta/2)G_2^{\pm}(\pm \omega_i)\]
\[Y_i^+ = 2 \sin^2(\theta/2)G_1^{\pm}(\pm \omega_i) + 2 \cos^2(\theta/2)G_2^{\pm}(\pm \omega_i)\]

(B.2)

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Note added. This is also a good position for notes added after the paper has been written.

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