Model of Saturn’s protoplanetary disk forming in-situ its regular satellites and innermost rings before the planet is formed

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ABSTRACT

We fit an isothermal oscillatory density model of Saturn’s protoplanetary disk to the present-day major satellites and innermost rings D/C and we determine the radial scale length of the disk, the equation of state and the central density of the primordial gas, and the rotational state of the Saturnian nebula. This disk does not look like the Jovian and Uranian disks that we modeled previously. Its power-law index is extremely steep (κ = −4.5) and its radial extent is very narrow (ΔR ≤ 0.9 Gm), its rotation parameter that measures centrifugal support against self-gravity is somewhat larger (β0 = 0.0431), as is its radial scale length (395 km); but, as was expected, the size of the Saturnian disk, Rmax = 3.6 Gm, takes just an intermediate value. On the other hand, the central density of the compact Saturnian core and its angular velocity are both comparable to that of Jupiter’s core (density of ≈ 0.3 cm−3 in both cases, and rotation period of 5.0 d versus 6.8 d); and significantly less than the corresponding parameters of Uranus’ core. As with the other primordial nebulae, this rotation is sufficiently slow to guarantee the disk’s long-term stability against self-gravity induced instabilities for millions of years of evolution.

Keywords. planets and satellites: dynamical evolution and stability—planets and satellites: formation—protoplanetary disks

1. Introduction

In previous work (Christodoulou & Kazanas 2019a,b,c), we presented isothermal models of the solar, Jovian, and Uranian primordial nebulae capable of forming protoplanets and, respectively, protosatellites long before the central object is actually formed by accretion processes. This entirely new “bottom-up” formation scenario is currently observed in real time by the latest high-resolution (~1-5 AU) observations of many protostellar disks by the ALMA telescope (ALMA Partnership 2018; Andrews et al. 2018; Guzmán et al. 2018; Isella et al. 2018; Keppler et al. 2018; Long et al. 2018; Pérez et al. 2018). These disks have the same characteristics as Saturn’s extended outer rings, which are the remnants of the Saturnian disk; the disk scale length of the protoplanetary disk is 395 km versus 368 km and the core density is 0.27 g cm−3 versus 0.31 g cm−3. The maximum size of the Saturnian disk, Rmax = 3.6 Gm is intermediate to those of the other two protoplanets (12 Gm and 0.60 Gm for Jupiter and Uranus, respectively). In addition, Saturn’s uniform core size is slightly larger than Jupiter’s (R1 = 0.321 Gm versus 0.220 Gm). This is necessary because Saturn’s core hosts five closely packed density maxima as opposed to Jupiter’s core that hosts only two widespread density maxima. On the other hand, the outer flat-density region of Saturn’s disk R2 = 1.21 Gm is a lot smaller than R2 = 5.37 Gm of Jupiter’s disk. Finally, the Saturnian disk exhibits significantly higher rotational support against self-gravity (Saturn’s β0 is about 50% larger than Jupiter’s), but still this value is sufficiently low to guarantee long-term dynamical stability. Just as in the case of Jupiter’s primordial disk, the high central densities and the mild differential rotation speeds of Saturn’s nebula signify that its major equatorial moons and its rings were formed in-situ long before Saturn was actually fully formed.

The analytic (intrinsic) and numerical (oscillatory) solutions of the isothermal Lane-Emden equation and the resulting model of the gaseous nebula have been described in detail in Christodoulou & Kazanas (2019a,b) for the primordial disks of Jupiter and Uranus and we will not repeat these descriptions here. In what follows, we apply in §2 our model nebula to the major moons and the inner rings D/C of Saturn and we compare the best-fit results to Jupiter’s extended Model 2 and Uranus’ best-fit model. In §3 we summarize and discuss our results.

2. Physical Model of Saturn’s Protoplanetary Disk

2.1. Best-Fit Saturnian disk model

The numerical integrations that produce oscillatory density profiles were performed with the MATLAB ode15s integrator (Shampine & Reichelt 1997; Shampine et al. 1999) and the optimization used the Nelder-Mead simplex algorithm as implemented by Lagarias et al. (1998). This method (MATLAB routine...
The error of the fit is 6.7%, as they improve the fit considerably. (Key: D-r:D-ring, P:Pan, M:Mimas, E:Enceladus, T:Tethys, D:Dione, R:Rhea, T:Titan, I:Iapetus.) The best-fit model needs an excessively large core to host 4 moons and the D-ring as well; none of the previous models of disks around solar-system protoplanets required so many moons inside the uniform core. The parameter $R_2$ is also necessary in order to fit simultaneously the orbits of the outer moons Titan and Iapetus.

We find the following physical parameters from the best-fit model: $k = -4.5$, $\beta_0 = 0.0431$, $R_1 = 0.321$ Gm (close to the orbit of the inner moon Tethys), and $R_2 = 1.21$ Gm (nearly co-incident to the orbit of Saturn’s largest moon Titan). The radial scale of the model was determined by fitting the density peak that corresponds to the orbit of Titan to its distance of 1.222 Gm, and the scale length of the disk then turns out to be $R_0 = 395$ km. The best-fit model is certainly stable to nonaxisymmetric self-gravitating instabilities because of the low value of $\beta_0$ (the critical value for the onset of dynamical instabilities is $\beta = 0.50$; Christodoulou et al. 1995).

The model disk extends out to 7.4 Gm (ln $R = 2$ in Fig. 1), but its validity ends around the distance of the outermost major moon Iapetus ($R_{\text{max}} \approx 3.56$ Gm). The next outer density peak lies at a distance of 5.84 Gm around which no moon is known. The disk of Saturn must have been small ($< 4.5$ Gm in radial extent) because the next outer irregular moon, Kiviuq, has a semimajor axis of 11.3 Gm and Phoebe, the largest and most important irregular moon, lies even farther out at 12.9 Gm. The gap between Iapetus and Kiviuq (or Phoebe) is enormous, and no moons or moonlets are found in this region, so it must have been empty from the very beginning of the formation of the system.

2.2. Physical parameters from the best-fit Saturnian model

Using the scale length of the disk $R_0$ and the definition $R_0^2 = c_1^2/(4\pi G \rho_0)$, we write the equation of state for the Saturnian circumplanetary gas as

$$c_0^2 = 4\pi G R_0^2 = 1.31 \times 10^9 \, \text{cm}^2 \, \text{s}^{-2},$$

where $c_0$ and $\rho_0$ are the local sound speed and the local density in the inner disk, respectively, and $G$ is the gravitational constant. For an isothermal gas at temperature $T$, $c_1^2 = \rho T / \mu$, where $\mu$ is the mean molecular weight and $R$ is the universal gas constant. Hence, eq. (1) can be rewritten as

$$\rho_0 = 0.0637 \left(\frac{T}{\mu}\right) \, \text{g cm}^{-3},$$

where $T$ and $\mu$ are measured in degrees Kelvin and g mol$^{-1}$, respectively.

For the coldest gas with $T \geq 10$ K and $\mu = 2.34$ g mol$^{-1}$ (molecular hydrogen and neutral helium with fractional abundances $X = 0.70$ and $Y = 0.28$ by mass, respectively), we find that

$$\rho_0 \geq 0.27 \, \text{g cm}^{-3}.$$

This high value implies that the conditions for protosatellite formation were already in place during the early isothermal phase (Tohline 2002) of the Saturnian nebula.

Using the above characteristic density $\rho_0$ of the inner disk in the definition of $\Omega_J \equiv \sqrt{2\pi G \rho_0}$, we determine the Jeans frequency of the disk:

$$\Omega_J = 3.4 \times 10^{-4} \, \text{rad s}^{-1}.$$

Then, using the model’s value $\beta_0 = 0.0431$ in the definition of $\beta_0 \equiv \Omega_0 / \Omega_J$, we determine the angular velocity of the uniformly-rotating core ($R_1 \leq 0.321$ Gm), viz.

$$\Omega_0 = 1.5 \times 10^{-5} \, \text{rad s}^{-1}.$$
For reference, this value of $\Omega_0$ for the core of the Uranian nebula corresponds to an orbital period of $P_0 = 5.0$ d. This value is close to the present-day orbital period of Rhea (4.5 d), but it is not near the orbital period of the largest moon Titan (16 d). This is a deviation from what we found for the solar system and for Jupiter and more consistent with the disk model of Uranus: the angular velocity of the core of the primordial Saturnian nebula is comparable to the present-day angular velocity of the second largest regular satellite Rhea, but it still lands in a region where large moons were formed in the Saturnian nebula. All our protoplanetary models of gaseous-giant disks so far support this property. It remains to be examined in the case of Neptune whose regular satellites are orbiting in a closely packed configuration, challenging all the nebular models constructed so far.

2.3. Comparison between all best-fit models

We show a comparison between the physical parameters of the best-fit models of Saturn, Uranus, and Jupiter in Table 1. It is obvious that the Saturnian nebula shares more common characteristics with the Jovian nebula than with the Uranian nebula. This is not surprising, given the present-day size and orbit of Saturn. Despite being 3.3 times smaller ($R_{\text{max}}$), the primordial disk of Saturn appears to be about as heavy as the Jovian disk ($\Omega_J$ and $P_0$) and just about as compact ($R_0$). Furthermore, the inner uniform core ($R_1$) appears to rotate slower than the extremely compact core of Uranus at an angular velocity ($\Omega_0$) very much comparable to that of the Jovian core.

The power-law index of the Saturnian nebular model is $k \approx -4.5$ (surface density $\Sigma \propto R^{-4.5}$), much steeper than the other two nebular models. Such an extreme value of $k$ has never been observed in studies of young circumstellar disks in the pre-ALMA era (Andrews & Williams 2007; Hung et al. 2010; Lee et al. 2013 and references within). We believe that this could be a characteristic of some protoplanetary disks only, whereas large-scale protostellar disks should not exhibit such extremely steep density profiles. Furthermore, the Saturnian disk hosts high enough densities to ensure that a “bottom-up” hierarchical formation occurred around this protoplanet as well. As we have found for the other gaseous giants in the solar system, protosatellites are seeded early inside their nebular disks and long before the protoplanets are fully formed; these compact moon/ring systems come to be in $< 0.1$ Myr (Harsono et al. 2018) and long before the central star becomes fully formed (see also Greaves & Rice 2014).

3. Summary and Discussion

We have constructed isothermal differentially-rotating protoplanetary models of the Saturnian nebula, the primordial disk in which the regular moons and inner rings were formed (§ 2). The best-fit model is shown in Fig. 1 and its physical parameters are listed in Table 1. In the optimization, we retained also the innermost ring D and the minor moon Pan (nearly coincident with ring A) in order to fit the two density maxima near the center that form deep inside the large uniform core of the model. These additions allowed us to fit a much better model for the seven major moons of Saturn. The mean relative error of 6.7% in the best-fit model stems entirely from the inaccuracy in the position of the D-ring.

We have compared this model to the best-fit models of Jupiter and Uranus (Christodoulou & Kazanas 2019b,c) (§ 2.3). Saturn’s disk appears to be closer to the larger disk of Jupiter and very different than the smaller disk of Uranus; although it also exhibits a unique feature, an extremely steep power-law index $k = -4.5$ extending over a very narrow region of size $\Delta R \approx 0.9$ Gm. All of these models appear to be stable and long-lived, so it seems that their regular moons and ring structures could form early in the evolution of each nebula and long before the protoplanets managed to pull their gaseous envelopes on to their solid cores. Once again, the results support strongly a “bottom-up” scenario in which regular satellites form first, followed by their planets, and then by the central star.

Neptune’s regular moons have an arrangement that is very different than the moons of the gaseous giants that we modeled so far (Jacobson & Owen 2004). The six regular moons cover an annular region of only 0.07 Gm, and this extremely compact configuration represents a challenge in modeling its primordial disk. This challenge will be addressed in a forthcoming modeling effort.

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Table 1. Comparison of the protoplanetary disks of Jupiter, Uranus, and Saturn

| Property                                      | Symbol (Unit)       | Jupiter’s Model 2 | Uranus’ Model 2 | Saturn’s Model 2 |
|-----------------------------------------------|---------------------|-------------------|-----------------|-----------------|
| Density power-law index                       | $k$                 | −1.4              | −0.96           | −4.5            |
| Rotational parameter                         | $\beta$             | 0.0295            | 0.00507         | 0.0431          |
| Inner core radius                             | $R_1$ (Gm)          | 0.220             | 0.0967          | 0.321           |
| Outer flat-density radius                     | $R_2$ (Gm)          | 5.37              | ...             | 1.21            |
| Radial extent of the density power law       | $\Delta R$ (Gm)     | 5                 | ...             | 0.9             |
| Scale length                                  | $R_0$ (km)          | 368               | 27.6            | 395             |
| Equation of state                             | $c_0^2/\rho_0$ (cm$^2$ g$^{-1}$ s$^{-2}$) | $1.14 \times 10^9$ | $6.39 \times 10^6$ | $1.31 \times 10^9$ |
| Minimum core density for $T = 10$ K, $\mu = 2.34$ | $\rho_0$ (g cm$^{-3}$) | 0.31             | 55.6            | 0.27            |
| Isothermal sound speed for $T = 10$ K, $\mu = 2.34$ | $c_0$ (m s$^{-1}$) | 188               | 188             | 188             |
| Jeans gravitational frequency                 | $\Omega_J$ (rad s$^{-1}$) | $3.6 \times 10^{-4}$ | $4.8 \times 10^{-3}$ | $3.4 \times 10^{-4}$ |
| Core angular velocity                         | $\Omega_0$ (rad s$^{-1}$) | $1.1 \times 10^{-5}$ | $2.5 \times 10^{-5}$ | $1.5 \times 10^{-5}$ |
| Core rotation period                          | $P_0$ (d)           | 6.8               | 3.0             | 5.0             |
| Maximum disk size                             | $R_{\text{max}}$ (Gm) | 12                | 0.60            | 3.6             |