One of the approaches for the analytical description of the interaction of an elastic wheel with a solid surface

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Abstract. The design of branched transmissions of multi-axle vehicles, multistage steering systems and automated workflow control systems requires their preliminary dynamic calculation. These calculations are based on mathematical models of structural elements corresponding to the modern level of development of system theory, i.e. taking into account dynamic interrelations between the system links. The aim of the work is to build a mathematical model of the automobile wheeled propulsor intended for use in the dynamic model of the "car-road" system or models of separate automobile systems. The approach to mathematical modeling of the automobile wheeled propulsor is offered, consisting in its description by methods of the control theory as an element of dynamic system. The method of construction and analysis of the complete system of equations of kinematic bonds superimposed in the contact zone on the movement of points of the external surface of the tire of linear elastic wheel at its perturbed rectilinear rolling on a solid uneven support surface is developed. The method of construction of a transfer matrix of automobile linear elastic wheel at which its structure practically does not depend on features of the settlement scheme of the tyres is developed too in this study. The method of obtaining the tire characteristics necessary for the calculation of the transfer matrix elements by the generalized method of successive approximations, which gives the qualitatively correct results in zero approximation, is proposed. The method of qualitative theoretical analysis of the features of rectilinear stationary rolling of the automobile wheel is given in this paper. The results allow to carry out researches and calculations of dynamic characteristics of automobile systems, units and assemblies directly interacting with the wheel; they can serve as a basis for further improvement of mathematical models of automobile ruts by methods of system theory.

1. Introduction

The scientific literature devoted to the study of the elastic wheel in the general dynamic model of a car, the processes occurring during the rolling of elastic bodies and the development of their mathematical models is very extensive [1–27], therefore, when analyzing it is advisable to describe the whole scientific directions on the example of the most complete and complete works. In the context of this work, the scope of modeling tasks is limited to the case of rolling the wheel on a hard road, as it is the most widespread, although the mathematical model of the wheel of high cross-country vehicles should allow for the possibility of generalization in the event of interaction with pliable soil without disrupting its structure or principal processing.

The following principal approaches to the modeling of wheeled propulsion properties can be distinguished: 1) the use of "conceptual models" based on hypotheses about the processes occurring during rolling; 2) the use of simplified tire models; 3) adaptation of already known solutions of similar
problems, most often - the classical contact problem of the elasticity theory; 4) the creation of special mathematical models.

The main limitation of the first direction of work (the hypothesis of "apparent sliding" and lateral withdrawal) is the stationarity of the problem. The peculiarity of conceptual models is that their equations contain unknown in advance parameters of interaction of tyres about the road, and for their calculation various simplified models of tyres are offered. The current high level of solution of internal tire mechanics problems allows to develop methods of calculation of parameters of conceptual models with full account of the peculiarities of tire construction. The more general results are obtained on the basis of the classical contact problem of elasticity theory, developing it into the theory of rolling elastic bodies. Such models differ in their distraction from the design of real tyres, stationary mode of rolling and difficulty in spreading the results to the spatial motion of the wheel. In general, in the study of stationary rolling of the wheel on a solid road a high level of theoretical generalization and accumulated significant experimental material. Otherwise, it is the case with the study of unsteady rolling of the wheel - here the course of research was determined by the works based on a priori hypotheses about the interaction of the tyre with the road and requiring experimental determination of its characteristics. The need for experiments with real tires is a serious inconvenience that makes it difficult to use the results at the design stage, especially in cases where the experimental determination of tire parameters is difficult or impossible, - for example, for large-size tires of heavy vehicles.

2. Mathematical model of the automobile wheel

2.1. Requirements for the mathematical model

In general, there is a tendency to create a structure of mathematical model of the automobile wheel, which would allow to combine the calculations on the basis of the classical contact problem of the theory of elasticity with a detailed description of the design of the tire and would cover the non-stationary modes of motion. This avoids the use of a priori hypotheses about the rolling mechanism, and the results are sought in a form suitable for the use of the model in the dynamic system "vehicle-road". In addition, there is a gradual separation of two main directions: the study of internal processes (i.e., those occurring in the tire and depending on the peculiarities of its construction, materials, etc.) and the study of the interaction of the elastic wheel with the road and its features (i.e., affecting the operation of systems and units connected to the wheel). In other words, the tasks of external and internal mechanics of the automobile wheel are distinguished. It should be noted that the solutions of external problems are important in the design of automobiles. Hence the desire to create such a model of elastic wheel, the structure of which would depend as less as possible on the peculiarities of the design scheme of the tyres, as the design of tyres are diverse and the internal processes in the tyres are very complex.

Thus, the level of development of the car theory, as well as the logic and trend of development of the car wheel theory require the construction of a mathematical model of the wheel that meets the following conditions:

- model should be based on a limited number of general hypotheses that allow for direct experimental testing (axiomaticity),
- model should be describe the spatial non-stationary movement of the wheel on an uneven road,
- model should be universal in relation to different tyres design schemes and be consistent with their possible mathematical description,
- model should be provide for the possibility of generalization for more complex cases of movement (on the ground, etc.),
- structure of the model should be convenient for use in the calculation of control systems.

In our paper we consider an insulated wheel consisting of a rigid rim and an elastic tyre fixed to it. The wheel rolls on a smooth, hard, uneven support surface. The reactions at the points where the wheel rim is connected to the suspension, transmission and steering are led to the main vector and the main torque, which are set by six scalar time functions. The movement of the wheel rim is also
described by six scalar time functions, which form the vectors of its linear speed and angular velocity.

2.2. Theoretical background
In the general case, the model of the automobile wheel that meets the above requirements is considered a nonlinear operator $A(\bullet;\bullet)$, who connected the space of vectors of the input parameters of the model $R[\vec{u}]$ and the space of vectors of the state of the model $R[\vec{u}]$ with the space of vectors of its output parameters $R[\vec{y}]$:

$$\vec{y}(t) = A(\vec{u}(t_0);\vec{u}(t,t_0))$$  \hspace{1cm} (1)

In this paper we use two equivalent definitions of the vector of input parameters - the main ("power input") $\vec{u}\left(f^{(n)};\vec{M};\vec{F}\right)'$ and auxiliary ("kinematic input") $\vec{u}\left(f^{(n)};\vec{V};\vec{\Omega}\right)'$, which are most often found in the applied tasks of the vehicle’s theory. They correspond to the vectors of output parameters $\vec{y} = (\vec{V};\vec{\Omega})'$ and $\vec{y} = (\vec{M};\vec{F})'$. In these formulas $f^{(n)}$ — derivatives from the function describing the shape of the road; $\vec{F}$ — the main vector of external forces acting on the wheel rim, $\vec{M}$ — the main moment of these forces; $\vec{V}$ — the linear speed of the wheel rim; $\vec{\Omega}$ — its angular speed.

Because of the extreme complexity of the nonlinear problem in the general formulation, it is advisable to limit oneself to the case of perturbed rectilinear motion of the wheel, defined as a set of independent basic motion (evolution) and oscillation near the evolutionary trajectory. For simplification purposes, only rectilinear stationary evolutionary motion is considered, and the solution is performed in the space of images of the two-way Laplace transformation, therefore (1) disintegrates into two matrix equations - evolutionary

$$\vec{Y} = A(\vec{u}_0;\vec{u})$$  \hspace{1cm} (2)

and outraged movement

$$\vec{y}(p_1) = \vec{A}(\vec{u}_0;\vec{Y}) . \vec{u}(p_1)$$  \hspace{1cm} (3)

The solution of the problem can be considered achieved if the matrix structure $\vec{A}$ ("transfer matrix of linear elastic wheel") corresponding to the above mentioned conditions is found and described in terms of system theory, the possibility of using for construction of operators $A$ and $\vec{A}$ any of the existing tire models is proved, and the identification of mathematical model with the real prototype of the automobile wheel is performed.

The Lagrangian variation principle is used to derive the equations of motion, the labour intensity of which is justified by the possibility of successively opening up all the necessary simplifications and obtaining the conditions for determining the boundary of the contact zone; the reactions of all the bonds are explicitly introduced, as it allows the simplest way to obtain a closed system of equations of the elastic wheel

$$M \frac{d\vec{V}}{dt} - \vec{F} - \int_{S_k} \vec{Q} dS_e = 0$$  \hspace{1cm} (4)

$$\left\{ J_x \frac{d\Omega_x}{dt} \right\} - \left[ \Omega_{\omega\omega}; \left\{ J_x \Omega_x^{\omega\omega} \right\} \right] - \vec{M} - \int_{S_k} \left[ \vec{Q}; \vec{F} + \vec{\xi} \right] dS_e = 0$$  \hspace{1cm} (5)

$$L[\vec{\xi}] = L_0[\vec{\xi}] + L_x[\vec{\xi}] - \chi(S_k) \vec{Q} = 0$$  \hspace{1cm} (6)
In formulas (4.7) $M$ — total weight of a wheel; $V_{06}$ — linear speed of a rim; $\bar{\Omega}_{06} = \{ \Omega_{1_{06}} \}$ — its angular speed; $J^k_s$ — components of a tensor of inertia of the deformed wheel; $F^k$ — main vector of external forces acting on the rim; $M$ — main moment of these forces; $\bar{Q}$ — distributed reaction in the area of contact of the tire with the road; $r^k_s$ — radius-vector, drawn from the center of the wheel and describing the position of the free external surface of the tire relative to the rim of the wheel; $\hat{\zeta}^k$ — vector of moving the points of the outer surface of the tire from the initial position to the actual one; $S_x$ — display of the area of contact of the tire with the road on the initial outer surface of the tire; $\Gamma_x^k$ — boundary of the area $S_x^k$; $\chi(S_x^k)$ — characteristic function of the area; $L[ ]$ — linear differential operator; $s; \beta$ — coordinates on the outer surface of the free tire; $t$ — time.

The equations (4.7) are derived under the following assumptions:
- the rim and tire are symmetrical with respect to the rolling plane to the values of the same order of magnitude with the square of motion of the points of the outer surface of the tire,
- the starting point of the rim and tire center of gravity and inertia coincide with the center of the wheel,
- the material characteristics and design of the tire are independent of changes in the temperature in the vicinity of the tire of a steady state,
- the tire material is incompressible,
- the initial state of the bus does not depend on time, it is physically realizable and unambiguously determined by the finite number of constructive parameters;
- the motion of the tire elements can be studied within the framework of a geometrically linear (infinitesimal) model;
- the properties of the tire are described by differential equations, and the internal connections in the tire are known and allow for the calculation of deformations of the inner areas of the tire at known displacements and deformations of its outer surface;
- The tire temperature does not change significantly in the course of perturbation.

Instead of equation (6), it is proposed that it be treated in a mutually unambiguous manner, which, if it is not possible to travel on a ledge, to leave the road, etc., will take the form of

$$
\lim_{t_0 \to -\infty} \bar{z}(s;\beta; t; t_0) = \int_{-\infty}^\infty G(s;\beta; t; s_0; \beta_0; t_0; r) \chi(S_{x0}) \bar{Q}(s_0; \beta_0; \tau) dS_{x0} d\tau
$$

where $G(\ )$ — operator’s Green function $L[ ]$. In this case, the initial conditions are related to the point $t_0 \to -\infty$ and are excluded from consideration.

If the distribution of movements in the contact zone and its boundary are known, the reaction of the road in the space of images of the two-way Laplace transformation is determined by the formula (9), which allows to solve the problem, if the boundary $\Gamma_x^*\kappa_e$ of the contact zone $S_x^*\kappa_e$ and the distribution of movements of points of the outer surface of the tire in this zone will be determined.

$$
\bar{Q}(s;\beta; t) \chi(S_x^*) = \int_{S_x^*} G^{-1}(s;\beta^*; s_0; \beta_0; p_i) \bar{\zeta}(s_0; \beta_0; p_i) dS_e^* - \chi(S_x^*) \bar{Q}_1(s;\beta^*; p_i)
$$

The points on the external surface of the tire lying inside the contact area are in contact with the road, so their movements before leaving contact are dependent on changes in the position of the rim of the wheel relative to the stationary coordinate system associated with the road. The vector equation shall be accurate to the values of the second order of lowness, taking into account possible slippage in the contact zone.
\[ \bar{V}_0 + \bar{V} + \left[ \Omega_{\text{sd}}, \hat{r} \right] + \frac{\partial \bar{E}}{\partial x} V_i x + \frac{\partial \bar{E}}{\partial t} = \bar{V}_{yx}; \quad i = 1;2. \] (10)

where \( \bar{V}_0 \) — velocity of evolutionary motion of the wheel rim center; \( \bar{V} \) — velocity of perturbed motion of the wheel rim center; \( \Omega_{\text{sd}} \) — angular velocity of the wheel rim; \( \hat{r} \) — radius-vector describing the outer surface of the deformed tire in the coordinate system connected with the wheel rim; \( \bar{V}_{yx} \) — velocity of the tire points in the contact zone relative to the supporting surface (slip rate).

If we consider that the shape of the reference surface is known, we can proceed to the vector-matrix form of equation (10)

\[
P_i \left\{ V_i^0 + V_i - V_{ix}^x \right\} + \bar{P}_i \omega_0^i \left[ + P_i \omega_i^i \right] + P_i \left[ \Phi_i + P_i \xi_i^i + P_i \xi_i'^i \right] + P_i \left[ \xi_i'^i \right] + P_i \left[ \xi_i'^i \right] = 0
\] (11)

The coefficient \( P_i \) — nonlinear matrix-functions of evolutionary motion parameters, the shape of the external surface of the free tire and the parameters of the reference surface. If we consider the angles of the initial installation of the wheel to be small (\( \leq 10^\circ \)) and assume that the height of unevenness of the hard roads has the same order of smallness with the movements of the tire, then within the contact zone, these coefficients become matrices-functions linear to the coordinates of points on the outer surface of the free tire. Then equation (11) is translated into the space of images of two-sided Laplace transformation.

By analyzing the dependence of the change in the distance between the two adjacent points of the outer surface of the tire in contact with the road on the sliding speeds of these points, it is possible to obtain, to the accuracy of the values of the second order of smallness, three more equations of connection in the images on the Laplace

\[
\begin{align*}
-\frac{\partial \bar{E}_1}{\partial \beta^*} \xi^2 \cos \phi + \bar{E}_3 \sin \phi & = \frac{1}{p_i} \left\{ -\frac{\partial \bar{V}_1^{xz}}{\partial \beta^*} \cos \beta^* \sin \beta^* \right\}; \\
-\frac{\partial \bar{E}_2}{\partial \beta^*} - r \frac{\partial \bar{E}_1}{\partial \beta^*} \xi_1 \sin \phi & = -\sin \phi \frac{\partial \bar{V}_2^{xz}}{\partial \beta^*} - r \sin \beta^* \frac{\partial \bar{V}_1^{xz}}{\partial \beta^*} - \frac{\partial \bar{V}_1^{xz}}{\partial \beta^*} \frac{\partial \bar{V}_1^{xz}}{\partial \beta^*} \\
- \frac{r \cos \phi \cos \beta^* \bar{V}_1^{xz}}{p_i} \frac{\partial \bar{V}_1^{xz}}{\partial \beta^*} & = \cos \phi \sin \beta^* \frac{\partial \bar{V}_1^{xz}}{\partial \beta^*} \frac{\partial \bar{V}_1^{xz}}{\partial \beta^*} \\
- \frac{\partial \bar{E}_3}{\partial \beta^*} \frac{1}{\rho} \xi^3 & = \frac{1}{p_i} \left\{ -\frac{\partial \bar{V}_3^{xz}}{\partial \beta^*} \cos \beta^* \sin \beta^* - \frac{\partial \bar{V}_3^{xz}}{\partial \beta^*} \sin \phi + \frac{\partial \bar{V}_3^{xz}}{\partial \beta^*} \cos \phi \sin \beta^* \right\}
\end{align*}
\] (12)

Finally, in the contact area, the outer surface of the tyre follows the shape of the support surface:

\[
\forall m_0 \in S_x : \Pi_p \left( \bar{r} + \bar{E} \right) = f \left( y^{1^*}; y^{2^*} \right)
\] (13)

The equations (11...13) form a complete system of equations of kinematic bonds that determine the distribution of displacements in the contact zone.

There are three possible cases: in the contact zone there is also a coupling zone and a sliding zone (general case), the coupling zone is degraded to a point and the coupling zone covers the entire contact area except for its rear edge (limiting cases). The first of the limiting cases is not of theoretical interest, and in the second case the equations of connection are reduced to a normal system, which is compatible and quite integrated with the accuracy of the initial hypothesis of geometric linearity of the model. It can be shown that to study the perturbed motion it is enough to consider only the system of equations.
\[
\begin{align*}
\frac{\partial^2 \xi^1}{\partial \beta^*} &= l_1 \left( \xi^1, \xi^2, \Delta \beta^*, \Delta \phi \right) - \gamma \frac{R_0}{\rho_0} \frac{\partial \xi^1}{\partial \phi} \\
\frac{\partial^2 \xi^2}{\partial \beta^*} &= l_2 \left( \xi^1, \xi^2, \Delta \beta^*, \Delta \phi \right) - \gamma \frac{R_0}{\rho_0} \frac{\partial \xi^2}{\partial \phi}
\end{align*}
\]

(14)

where \( l_1; l_2 \) — linear forms from the corresponding variables are designated, and the other equations are automatically satisfied with the accuracy of the initial hypothesis. The general integral of the system (14) is easily constructed in Laplace images and depends on four arbitrary functions \( C_x \) and the transformation parameter \( p_t \). The use of images has made it possible to bypass the serious difficulties associated with the solution of the boundary value problem for the equations of motion of bus elements together with the equations of communication at the previously unknown mobile boundary of the contact zone.

Further, the structure of the transfer matrix is formed, which describes the perturbed rectilinear motion of the elastic wheel on a horizontal uneven hard road. First, from condition (7) is determined in the first approximation the boundary of the contact area for evolutionary and perturbed cases by the Lighthill method. The rim motion equations are simplified by taking into account the hypothesis of geometric linearity of the model and are reduced to a dimensionless form using the following scaling values: \( G_{\text{nom}} \) — scale of force (nominal vertical wheel load; \( R_0 \) — linear, scale (equator radius of the outer surface of the free tyre); \( V_0 \) — scale of speed (speed of the main movement of the wheel rim centre).

After excluding from these equations the image of the reaction in the contact zone, a group of formulas

\[
\begin{align*}
\left\{ A_{qo} + A_{qe} C_x + f^q_{3z} \right\} &= 0; \quad q = 1, 2, 3; \quad k = 1, 2, \ldots, 4; \\
\left\{ A_{qo} + A_{qe} C_x + m^q_{3z} \right\} &= 0; \quad q = 4, 5, 6; \quad k = 1, 2, \ldots, 4.
\end{align*}
\]

(15)

for the evolutionary movement and

\[
\begin{align*}
\left\{ \tilde{a}_{qi\delta} \tilde{K}_{qi\delta} - f^q_{3z} \right\} &= 0; \quad q = 1, 2, 3; \quad i = 1, 2, \ldots, 8; \\
\left\{ \tilde{a}_{qi\delta} \tilde{K}_{qi\delta} - f^{q-3}_{3z} \right\} &= 0; \quad q = 4, 5, 6; \quad i = 1, 2, \ldots, 8.
\end{align*}
\]

(16)

for an outraged movement.

The formulas (14...15) contain the following designations: \( f^q_{3z} \) — components of the evolutionary part of the dimensionless main vector of the external forces acting on the rim; \( m^q_{3z} \) — components of the evolutionary part of the dimensionless main vector of the main moment of the same forces; \( f^q_{3z} \) — components of the perturbed part of the main vector; \( f^{q-3}_{3z} \) — components of the perturbed part of the main moment vector, \( C_x \) — evolutionary motion parameters obtained by solving the equations of coupling and subject to exclusion from the equations; \( \tilde{K}_{qi\delta} \) — images of "dimensionless components of perturbed motion", which include dimensionless components of perturbed parts of the linear velocity vector of the rim center \( \tilde{v} \) angular speed of the rim \( \tilde{\omega}_b \) cog and the image of dimensionless functions that define the profile of the road and its derivatives; \( A_{qo}, A_{qe} \), and \( a_{qi\delta} \) — equation coefficients that depend on the parameters of the initial state of the tire and the resonance of its function Green.

The formulas (16) allow constructing a transfer matrix of elastic wheel for two variants of determination of input parameters - kinematic and power inputs.
For the kinematic input, the transfer matrix looks like

$$A(p_i) = \{a_{q,i}\}; q = 1,2,3; \; i = 1,2,..,8.$$

That is, it is composed of system coefficients (16).

In the case of the power input, the transfer matrix has a cellular structure

$$\tilde{A}_i(p_i) = \begin{pmatrix}
-\{\tilde{D}_{q,i-2}\} & \{\tilde{a}_{q,i}\} & 0 \\
0 & -\{\tilde{D}_{q,i-2}\} & \{\tilde{a}_{q,i}\} \\
\emptyset & \emptyset & \emptyset
\end{pmatrix} \tag{18}$$

where $\tilde{a}_{q,i-2}$ is the algebraic adjunct of an element $\tilde{D}_{q,i-2}$, $\tilde{A} = \det \tilde{A}$ is the determinant, $\tilde{D}_{q,i-2} = \tilde{a}_{q,i-2} \tilde{A}^{-1}$

The elements of the transfer matrix depend on the parameters of evolutionary rolling. Due to the independence of the main motion from oscillation, these parameters can be determined by any available in each case by experiments with the real tire, calculation by, simplified models, etc.

The analysis of the complete system of equations of kinematic bonds has allowed to establish that for an estimation of evolutionary process it is impossible to be limited to the decision of only the truncated system: (14). Taking into account additional kinematic equations: the equations made possible an analytical description of the relationship between the usual characteristics of evolutionary motion, such as rolling radius and torque, lateral force, lateral deflection angle and stabilizing moments; in particular, a nonlinear elastic characteristic of vertical wheel compression was constructed.

### 2.3. Results

Thus, a universal transfer matrix of linear elastic wheel is constructed; this one is universal because the structure of transfer matrix is identical in all cases and contains some function (Green's resonant), reflecting peculiarities of a concrete model of tyres.

Further, it is necessary to suggest such ways of construction of this function that they appeared to be compatible with the developed model of an automobile wheel. The first way is to generalize a special method of solving the ordinary differential equation developed by N.K. Kulikov and to construct the Green function taking into account the peculiarities of wheel rolling. So it is possible to investigate complex models of tyres, receiving qualitatively correct result already in zero approximation, however features of structure of function Green I. Its resonents remain latent. Another way is to use the method of eigenfunctions. Its main advantage is the explicit structure of the Green resonant, which allows to trace analytically the features of the transfer matrix of the elastic wheel.

### 3. Conclusions

In this work by methods of the theory of systems the indignant rectilinear movement of an automobile elastic wheel on nervous firm road is studied, the transfer matrix with universal structure in relation to settlement schemes of the tyre is constructed and ways of use of this matrix together with models of automobile systems for calculation of their general dynamic characteristics are offered; movement of a wheel are offered. The study is carried out from a single position without involving hypotheses, artificial methods or experimental determination of additional parameters.

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