$N = 1$ Supersymmetric Cosmic Strings

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Abstract

We investigate the microphysics of supersymmetric cosmic strings. In particular we focus on the vortices admitted by $N = 1$ supersymmetric abelian Higgs models. We find the vortex solutions and demonstrate that the two simplest supersymmetric cosmic string models admit fermionic superconductivity. Further, by using supersymmetry transformations, we show how to solve for the fermion zero modes giving rise to string superconductivity in terms of the background string fields.

DAMTP-96-107
MIT-CTP-2612

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I. INTRODUCTION

In the past twenty years it has become clear that topological defects in quantum field theories may play an essential role in the evolution of the early universe. One important effect of these topological solitons is gravitational. The evolution of a network of cosmic strings produced at a grand unified (GUT) phase transition provides a possible origin for the seed density perturbations which became the large scale structure of the universe observed today [1].

However, important early universe physics may also arise from the microphysics of topological defects. If a network of defects is produced just prior to the electroweak phase transition, their interactions with the fields of the standard electroweak theory form the basis for an electroweak baryogenesis scenario which is insensitive to the details of the phase transition [2]. Further, the spectrum of radiation from strings produced during a Peccei-Quinn [3] symmetry breaking provides important bounds on the allowed values of any axion mass [4]. Finally, cosmology in the presence of topological defects is qualitatively altered if the strings carry superconducting currents, as first suggested by Witten [5]. In particular, if a network of cosmic strings becomes superconducting, then the possibility of producing massive stable remnants (vortons) allows one to constrain the underlying particle physics theory by cosmological considerations (for a recent analysis see [6]).

In this paper, we investigate the microphysics of cosmic string solutions admitted by supersymmetric (SUSY) field theories. This is important for at least two reasons. First, SUSY field theories include many popular candidate theories of physics above the electroweak scale. Second, the recent successes of duality in SUSY Yang-Mills theories may mean that the physics of nonperturbative solutions such as topological solitons may be easier to understand than in non-supersymmetric theories. As in early studies of non-SUSY defects [7], we work in the context of the simplest models and in particular with versions of the abelian Higgs model obeying the supersymmetry algebra with one SUSY generator ($N = 1$). We demonstrate that the particle content and interactions dictated by SUSY naturally give rise to cosmic
string superconductivity in these models. Further, by using SUSY transformations, we are able to find solutions for the fermion zero modes responsible for superconductivity in terms of the background string fields. Ours is not the first analysis of superconducting SUSY cosmic strings. However, whereas earlier analyses have focussed on the complicated structure of the supersymmetrized $U(1) \times U(1)$ Witten model, here we demonstrate the presence of superconductivity in even the simplest SUSY cosmic string theories. A special case of the solutions discussed in this paper has been obtained in a similar model by other authors using different techniques.

The structure of the paper is as follows. In the next section we present the $N = 1$ SUSY abelian Higgs models. Such simple SUSY models are well-known in particle physics (for example see Ref. [10]). However, we believe the cosmological relevance of the solutions we explore here to be new. In order to make contact with both the supersymmetry and cosmology literature, we employ both the superfield and component formalisms and repeat a number of well-established facts and conventions for the sake of clarity. Spontaneous symmetry breaking (SSB) in these models can be implemented in two distinct ways, leading to different theories with different particle content. We call these distinct models theory F and theory D respectively to refer to the origin of the SSB term in the Higgs potential. In section three we focus on theory F. We demonstrate how the cosmic string solution can be constructed in the bosonic sector and derive the equations of motion for the fermionic zero modes. We then employ SUSY transformations to solve these equations in terms of the background string fields. In section four we repeat the analysis for theory D. The type of symmetry breaking in theory D is peculiar to theories with an abelian gauge group and we therefore expect theory F to be more representative of models with nonabelian gauge groups such as grand unified theories. In section five we check our results for the special case discussed in Ref. [9]. In fact, for theory D, the solutions are already of this special form. Finally, in section six, we comment on the possible implications of our findings.
Let us begin by defining our conventions. Throughout this paper we use the Minkowski metric with signature $-2$, the antisymmetric 2-tensor $\varepsilon_{21} = \varepsilon^{12} = 1$, $\varepsilon_{12} = \varepsilon^{21} = -1$ and the Dirac gamma matrices in the representation

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix},$$

(2.1)

with $\sigma^\mu = (-1, \sigma^i)$, $\bar{\sigma}^\mu = (-1, -\sigma^i)$ and where $\sigma^i$ are the Pauli matrices.

We consider supersymmetric versions of the spontaneously broken gauged $U(1)$ abelian Higgs model. In superfield notation, such a theory consists of a vector superfield $V$ and $m$ chiral superfields $\Phi_i$, $(i = 1 \ldots m)$, with $U(1)$ charges $q_i$. In Wess-Zumino gauge these may be expressed in component notation as

$$V(x, \theta, \bar{\theta}) = -(\theta \sigma^\mu \bar{\theta}) A_\mu(x) + i \theta^2 \bar{\theta} \lambda(x) - i \bar{\theta}^2 \theta \lambda(x) + \frac{1}{2} \theta^2 \bar{\theta}^2 D(x),$$

(2.2)

$$\Phi_i(x, \theta, \bar{\theta}) = \phi_i(y) + \sqrt{2} \psi_i(y) + \theta^2 F_i(y),$$

(2.3)

where $y^\mu = x^\mu + i \theta \sigma^\mu \bar{\theta}$. Here, $\phi_i$ are complex scalar fields and $A_\mu$ is a vector field. These correspond to the familiar bosonic fields of the abelian Higgs model. The fermions $\psi_{i\alpha}$, $\bar{\lambda}_\alpha$ and $\lambda_\alpha$ are Weyl spinors and the complex bosonic fields, $F_i$, and real bosonic field, $D$, are auxiliary fields. Finally, $\theta$ and $\bar{\theta}$ are anticommuting superspace coordinates. In the component formulation of the theory one eliminates $F_i$ and $D$ via their equations of motion and performs a Grassmann integration over $\theta$ and $\bar{\theta}$. Now define

$$D_\alpha = \frac{\partial}{\partial \theta_\alpha} + i \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^\dot{\alpha} \partial_\mu,$$

$$\bar{D}_{\dot{\alpha}} = - \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} - i \theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu,$$

$$W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V,$$

(2.4)

where $D_\alpha$ and $\bar{D}_{\dot{\alpha}}$ are the supersymmetric covariant derivatives and $W_\alpha$ is the field strength chiral superfield. Then the superspace Lagrangian density for the theory is given by
\[ \tilde{\mathcal{L}} = \frac{1}{4} \left( W^\alpha W_\alpha |_{g^2} + \dot{W}^\dot{\alpha} |_{g^2} \right) + \left( \Phi_i e^{\bar{g}_i V} \Phi_i \right) |_{g^2} + W(\Phi_i) |_{g^2} + \dot{W}(\bar{\Phi}_i) |_{g^2} + \kappa D . \] (2.5)

In this expression \( W \) is the superpotential, a holomorphic function of the chiral superfields (i.e. a function of \( \Phi_i \) only and not \( \bar{\Phi}_i \)) and \( W |_{g^2} \) indicates the \( \theta^2 \) component of \( W \). The term linear in \( D \) is known as the Fayet-Iliopoulos term [12]. Such a term can only be present in a \( U(1) \) theory, since it is not invariant under more general gauge transformations.

For a renormalizable theory, the most general superpotential is

\[ W(\Phi_i) = a_i \Phi_i + \frac{1}{2} b_{ij} \Phi_i \Phi_j + \frac{1}{3} c_{ijk} \Phi_i \Phi_j \Phi_k , \] (2.6)

with the constants \( b_{ij}, c_{ijk} \) symmetric in their indices. This can be written in component form as

\[ W(\phi_i, \psi_j, F_k) = a_i F_i + b_{ij} \left( F_i \phi_j - \frac{1}{2} \psi_i \psi_j \right) + c_{ijk} \left( F_i \phi_j \phi_k - \psi_i \psi_j \phi_k \right) \] (2.7)

and the Lagrangian (2.7) can then be expanded in Wess-Zumino gauge in terms of its component fields using (2.3, 2.2). The equations of motion for the auxiliary fields are

\[ F_i^* + a_i + b_{ij} \phi_j + c_{ijk} \phi_j \phi_k = 0 \] (2.8)

and

\[ D + \kappa + \frac{g}{2} q_i \bar{\phi}_i \phi_i = 0 . \] (2.9)

Using these to eliminate \( F_i \) and \( D \) we obtain the Lagrangian density in component form as

\[ \mathcal{L} = \mathcal{L}_B + \mathcal{L}_F + \mathcal{L}_Y - U , \] (2.10)

with

\[ \mathcal{L}_B = (D_i^* \bar{\phi}_i)(D^\mu \phi_i) - \frac{1}{4} F^{\mu \nu} F_{\mu \nu} , \] (2.11)

\[ \mathcal{L}_F = -i \psi_i \sigma^\mu D_\mu \bar{\phi}_i - i \lambda_i \sigma^\mu \partial_\mu \bar{\lambda}_i , \] (2.12)

\[ \mathcal{L}_Y = \frac{ig}{\sqrt{2}} q_i \bar{\phi}_i \psi_i \lambda - \left( \frac{1}{2} b_{ij} + c_{ijk} \phi_k \right) \psi_i \psi_j + (\text{c.c.}) , \] (2.13)

\[ U = |F_i|^2 + \frac{1}{2} D^2 \]

\[ = |a_i + b_{ij} \phi_j + c_{ijk} \phi_j \phi_k|^2 + \frac{1}{2} \left( \kappa + \frac{g}{2} q_i \bar{\phi}_i \phi_i \right)^2 , \] (2.14)
where $D^i_\mu = \partial_\mu + \frac{1}{2} ig q_i A_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Now consider spontaneous symmetry breaking in these theories. Each term in the superpotential must be gauge invariant. This implies that $a_i \neq 0$ only if $q_i = 0$, $b_{ij} \neq 0$ only if $q_i + q_j = 0$, and $c_{ijk} \neq 0$ only if $q_i + q_j + q_k = 0$. The situation is a little more complicated than in non-SUSY theories, since anomaly cancellation in SUSY theories implies the existence of more than one chiral superfield (and hence Higgs field). In order to break the gauge symmetry, one may either induce SSB through an appropriate choice of superpotential, or, in the case of the $U(1)$ gauge group, one may rely on a non-zero Fayet-Iliopoulos term.

We shall refer to the theory with superpotential SSB (and, for simplicity, zero Fayet-Iliopoulos term) as theory F and the theory with SSB due to a non-zero Fayet-Iliopoulos term as theory D. Since the implementation of SSB in theory F can be repeated for more general gauge groups, we expect that this theory will be more representative of general defect-forming theories than theory D for which the mechanism of SSB is specific to the $U(1)$ gauge group.

### III. THEORY F: VANISHING FAYET-ILIPOULOS TERM

The simplest model with vanishing Fayet-Iliopoulos term ($\kappa = 0$) and spontaneously broken gauge symmetry contains three chiral superfields. It is not possible to construct such a model with fewer superfields which does not either leave the gauge symmetry unbroken or possess a gauge anomaly. The fields are two charged fields $\Phi_{\pm}$, with respective $U(1)$ charges $q_{\pm} = \pm 1$, and a neutral field, $\Phi_0$. A suitable superpotential is then

$$W(\Phi_i) = \mu \Phi_0 (\Phi_+ \Phi_- - \eta^2),$$

with $\eta$ and $\mu$ real. The potential $U$ is minimised when $F_i = 0$ and $D = 0$. This occurs when $\phi_0 = 0$, $\phi_+ \phi_- = \eta^2$, and $|\phi_+|^2 = |\phi_-|^2$. Thus we may write $\phi_{\pm} = \eta e^{\pm i\alpha}$, where $\alpha$ is some function. We shall now seek the Nielsen-Olesen [13] solution corresponding to an infinite straight cosmic string. We proceed in the same manner as for non-supersymmetric
theories. Consider only the bosonic fields (i.e. set the fermions to zero) and in cylindrical polar coordinates \((r, \varphi, z)\) write

\[
\begin{align*}
\phi_0 &= 0, \quad (3.2) \\
\phi_+ &= \phi_+^* = ne^{i\varphi} f(r), \quad (3.3) \\
A_\mu &= \frac{2}{g} a(r) \frac{r}{r} \delta_\mu^\varphi, \quad (3.4) \\
F_\pm &= D = 0, \quad (3.5) \\
F_0 &= \mu \eta^2 (1 - f(r)^2), \quad (3.6)
\end{align*}
\]

so that the \(z\)-axis is the axis of symmetry of the defect. The profile functions, \(f(r)\) and \(a(r)\), obey

\[
\begin{align*}
f'' + \frac{f'}{r} - n^2 \frac{(1 - a)^2}{r^2} &= \mu^2 \eta^2 (f^2 - 1) f, \quad (3.7) \\
a'' - \frac{a'}{r} &= -g^2 \eta^2 (1 - a) f^2, \quad (3.8)
\end{align*}
\]

with boundary conditions

\[
\begin{align*}
f(0) = a(0) &= 0, \\
\lim_{r \to \infty} f(r) = \lim_{r \to \infty} a(r) &= 1.
\end{align*}
\]

Note here, in passing, an interesting aspect of topological defects in SUSY theories. The ground state of the theory is supersymmetric, but spontaneously breaks the gauge symmetry while in the core of the defect the gauge symmetry is restored but, since \(|F_i|^2 \neq 0\) in the core, SUSY is spontaneously broken there.

We have constructed a cosmic string solution in the bosonic sector of the theory. Now consider the fermionic sector. With the choice of superpotential \((3.1)\) the component form of the Yukawa couplings becomes

\[
\mathcal{L}_Y = i \frac{g}{\sqrt{2}} \left( \bar{\phi}_+ \psi_+ - \bar{\phi}_- \psi_- \right) \lambda - \mu \left( \phi_0 \psi_+ \psi_- + \phi_+ \psi_0 \psi_- + \phi_- \psi_0 \psi_+ \right) + (c.c.) \quad (3.9)
\]
As with a non-supersymmetric theory, non-trivial zero energy fermion solutions can exist around the string. Consider the fermionic ansatz

$$\psi_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_i(r, \varphi) ,$$  \hspace{1cm} (3.10)

$$\lambda = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \lambda(r, \varphi) .$$  \hspace{1cm} (3.11)

If we can find solutions for the $\psi_i(r, \varphi)$ and $\lambda(r, \varphi)$ then, following Witten, we know that solutions of the form

$$\Psi_i = \psi_i(r, \varphi)e^{\chi(z+t)} , \quad \tilde{\Lambda} = \lambda(r, \varphi)e^{\chi(z+t)} ,$$  \hspace{1cm} (3.12)

with $\chi$ some function, represent left moving superconducting currents flowing along the string at the speed of light. Thus, the problem of finding the zero modes is reduced to solving for the $\psi_i(r, \varphi)$ and $\lambda(r, \varphi)$.

The fermion equations of motion derived from (2.10) are four coupled equations given by

$$e^{-i \varphi} \left( \partial_r - \frac{i}{r} \partial_\varphi \right) \bar{\lambda} - \frac{g}{\sqrt{2}} \eta f \left( e^{in \varphi} \psi_\pm - e^{-in \varphi} \psi_\pm^* \right) = 0 ,$$  \hspace{1cm} (3.13)

$$e^{-i \varphi} \left( \partial_r - \frac{i}{r} \partial_\varphi \right) \bar{\psi}_0 + i \eta \mu f \left( e^{in \varphi} \psi_\pm + e^{-in \varphi} \psi_\pm^* \right) = 0 ,$$  \hspace{1cm} (3.14)

$$e^{-i \varphi} \left( \partial_r - \frac{i}{r} \partial_\varphi \pm \frac{a}{r} \right) \bar{\psi}_\pm + \eta fe^{\mp in \varphi} \left( i \mu \psi_0 \pm \frac{g}{\sqrt{2}} \lambda \right) = 0 .$$  \hspace{1cm} (3.15)

The corresponding equations for the lower fermion components can be obtained from those for the upper components by complex conjugation, and putting $n \rightarrow -n$. The superconducting current corresponding to this solution (like (3.12), but with $\chi(t-z)$) is right moving. The angular dependence may be removed with the substitutions

$$\lambda = A(r)^* e^{i(l-1)\varphi} ,$$  \hspace{1cm} (3.16)

$$\psi_+ = B(r) e^{i(n-l)\varphi} ,$$  \hspace{1cm} (3.17)

$$\psi_- = C(r) e^{-i(n+l)\varphi} ,$$  \hspace{1cm} (3.18)

$$\psi_0 = E(r)^* e^{i(l-1)\varphi} .$$  \hspace{1cm} (3.19)
For large $r$ the four solutions have the asymptotic forms

$$A(r) \sim B(r) - C(r) \sim e^{\pm gr}, \quad (3.20)$$

$$E(r) \sim B(r) + C(r) \sim e^{\pm \sqrt{2}gr}. \quad (3.21)$$

To be physically significant, solutions must be normalisable \cite{14}, and so must be well behaved at $r = 0$ and decay sufficiently rapidly as $r \to \infty$. At small $r$ the least well behaved parts of the four solutions are

$$A(r) \sim r^{l-1}, \quad (3.22)$$

$$B(r) \sim r^{n-l}, \quad (3.23)$$

$$C(r) \sim r^{-n-l}, \quad (3.24)$$

$$E(r) \sim r^{l-1}. \quad (3.25)$$

Thus in order to match up with some combination of the two normalisable solutions at large $r$, at least three of the small $r$ solutions must be well behaved at $r = 0$. This occurs when $1 \leq l \leq |n|$, giving a total of $|n|$ independent solutions. Similarly, solutions for the lower components of the fields also have $|n|$ independent solutions. In terms of the superconducting solution \cite{3.12}, these two sets of solutions correspond to currents flowing in opposite directions along the string. Note that the zero modes may also be enumerated using index theorems \cite{14}.

In general, in non-supersymmetric theories, it is difficult to find solutions for fermion zero modes in string backgrounds. However, in the supersymmetric case, SUSY transformations relate the fermionic components of the superfields to the bosonic ones and we may use this to obtain the fermion solutions in terms of the background string fields. A SUSY transformation is implemented by the operator $G = e^{\xi Q + \bar{\xi}Q}$, where $\xi_\alpha$ are Grassmann parameters and $Q_\alpha$ are the generators of the SUSY algebra which we may represent by

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu, \quad (3.26)$$

$$\bar{Q}^{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} - i\bar{\sigma}^\mu_{\dot{\alpha}\alpha} \theta_\alpha \partial_\mu. \quad (3.27)$$
In general such a transformation will induce a change of gauge. It is then necessary to perform an additional gauge transformation to return to the Wess-Zumino gauge in order to easily interpret the solutions. For an abelian theory, supersymmetric gauge transformations are of the form

\[ \Phi_i \rightarrow e^{-i\Lambda q_i} \Phi_i , \]  
\[ \bar{\Phi}_i \rightarrow e^{i\bar{\Lambda} q_i} \bar{\Phi}_i , \]  
\[ V \rightarrow V + \frac{i}{g} \left( \Lambda - \bar{\Lambda} \right) , \]

where \( \Lambda \) is some chiral superfield.

Consider performing an infinitesimal SUSY transformation on (3.6), using \( \partial_\mu A^\mu = 0 \). The appropriate \( \Lambda \) to return to Wess-Zumino gauge is

\[ \Lambda = i g \bar{\xi} \sigma^\mu \theta A_\mu (y) \]

The component fields then transform in the following way

\[ \phi_\pm (y) \rightarrow \phi_\pm (y) + 2i \theta \sigma^\mu \bar{\xi} D_\mu \phi_\pm (y) , \]  
\[ \theta^2 F_0 (y) \rightarrow \theta^2 F_0 (y) + 2 \theta \xi F_0 (y) , \]  
\[ - \theta \sigma^\mu \bar{\theta} A_\mu (x) \rightarrow - \theta \sigma^\mu \bar{\theta} A_\mu (x) \]
\[ + i \theta^2 \bar{\theta} \frac{1}{2} \sigma^\mu \sigma^\nu \xi F_{\mu \nu} (x) - i \theta^2 \theta \frac{1}{2} \sigma^\mu \sigma^\nu \xi F_{\mu \nu} (x) . \]

Writing everything in terms of the background string fields, only the fermion fields are affected to first order by the transformation. These are given by

\[ \lambda_\alpha \rightarrow \frac{2na'}{gr} i (\sigma^z)^{\alpha} \xi_{\alpha} , \]  
\[ (\psi_\pm)_\alpha \rightarrow \sqrt{2} \left( if' \sigma^r \mp \frac{n}{r} (1 - a) f \sigma^r \right) a_\alpha \xi^a \eta e^{\pm in_\varphi} , \]  
\[ (\psi_0)_\alpha \rightarrow \sqrt{2} \mu \eta^2 (1 - f^2) \xi_\alpha , \]

where we have defined
\[
\sigma^\varphi = \begin{pmatrix} 0 & -ie^{-i\varphi} \\ ie^{i\varphi} & 0 \end{pmatrix},
\]
(3.38)
\[
\sigma^r = \begin{pmatrix} 0 & e^{-i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix}.
\]
(3.39)

Let us choose \(\xi_\alpha\) so that only one component is nonzero. Taking \(\xi_2 = 0\) and \(\xi_1 = -i\delta/(\sqrt{2}\eta)\), where \(\delta\) is a complex constant, the fermions become

\[
\lambda_1 = \delta \frac{n\sqrt{2}a'}{g\eta} \frac{a'}{r},
\]
(3.40)
\[
(\psi_+)_1 = \delta^* \left[ f' + \frac{n}{r} (1 - a) f \right] e^{i(n-1)\varphi},
\]
(3.41)
\[
(\psi_0)_1 = -i\delta \mu \eta (1 - f^2),
\]
(3.42)
\[
(\psi_-)_1 = \delta^* \left[ f' - \frac{n}{r} (1 - a) f \right] e^{-i(n+1)\varphi}.
\]
(3.43)

It is these fermion solutions which are responsible for the string superconductivity. Similar expressions can be found when \(\xi_1 = 0\). It is clear from these results that the string is not invariant under supersymmetry, and therefore breaks it. However, since \(f'(r), a'(r), 1 - a(r)\) and \(1 - f^2(r)\) are all approximately zero outside of the string core, the SUSY breaking and the zero modes are confined to the string. We note that this method gives us two zero mode solutions. Thus, for a winding number one string, we obtain the full spectrum, whereas for strings of higher winding number, only a partial spectrum is obtained.

**IV. THEORY D: NONVANISHING FAYET-ILIPOULOS TERM**

Now consider theory D in which there is just one primary charged chiral superfield involved in the symmetry breaking and a non-zero Fayet-Iliopoulos term. In order to avoid gauge anomalies, the model must contain other charged superfields. These are coupled to the primary superfield through terms in the superpotential such that the expectation values of the secondary chiral superfields are dynamically zero. The secondary superfields have no effect on SSB and are invariant under SUSY transformations. Therefore, for the rest of this section we shall concentrate on the primary chiral superfield which mediates the gauge symmetry breaking.
Choosing $\kappa = -\frac{1}{2}g\eta^2$, the theory is spontaneously broken and there exists a string solution obtained from the ansatz

\begin{align*}
\phi &= \eta e^{i\nu f(r)}, \\
A_\mu &= -\frac{2}{g} n a(r) \frac{\delta^\nu}{r} f(r), \\
D &= \frac{1}{2} g \eta^2 (1 - f^2), \\
F &= 0.
\end{align*}

The profile functions $f(r)$ and $a(r)$ then obey the first order equations

\begin{align*}
f' &= n \frac{(1 - a)}{r} f, \\
na' &= \frac{1}{4} g^2 \eta^2 (1 - f^2).
\end{align*}

Now consider the fermionic sector of the theory and perform a SUSY transformation, again using $\Lambda$ as the gauge function to return to Wess-Zumino gauge. To first order this gives

\begin{align*}
\lambda_\alpha &\rightarrow \frac{1}{2} g \eta^2 (1 - f^2) i (I + \sigma^z)_{\alpha\beta} \xi^\beta, \\
\psi_\alpha &\rightarrow \sqrt{2} \frac{n}{r} (1 - a) f (i \sigma^r - \sigma^\varphi)_{\alpha\beta} \xi^\beta \eta e^{i\varphi}.
\end{align*}

If $\xi_1 = 0$ both these expressions are zero. The same is true of all higher order terms, and so the string is invariant under the corresponding transformation. For other $\xi$, taking $\xi_1 = -i\delta/\eta$ gives

\begin{align*}
\lambda_1 &= \delta g \eta (1 - f^2), \\
\psi_1 &= 2\sqrt{2} \delta^* n \frac{r}{(1 - a) f} e^{i(n-1)\varphi}.
\end{align*}

Thus supersymmetry is only half broken inside the string. This is in contrast to theory F which fully breaks supersymmetry in the string core. The theories also differ in that theory D’s zero modes will only travel in one direction, while the zero modes of theory F (which has twice as many) travel in both directions. In both theories the zero modes and SUSY breaking are confined to the string core.
V. THE SUPER-BOGOMOLOVY LIMIT

In non-supersymmetric theories it is usually difficult to find solutions for fermion zero modes on cosmic string backgrounds. In such theories one can, however, often obtain solutions in the Bogomolnyi limit which, in our theory, corresponds to choosing

\[ 2\mu^2 = g . \]  

(5.1)

In this limit, the energy of the vortex saturates a topological bound, there are no static forces between vortices and the equations of motion for the string fields reduce to a pair of coupled first order differential equations. It is a useful check of the solutions obtained in the previous sections to confirm that they reduce to those already known in the Bogomolnyi limit.

Imposing (5.1), equations (3.7,3.8) become

\[ f' = nf^r(1 - a) , \]  

(5.2)

\[ n\frac{a'}{r} = \mu^2\eta^2(1 - f^2) . \]  

(5.3)

Note that these are identical to (4.5,4.6) and that therefore all solutions to theory D are automatically Bogomolnyi solutions. Imposing (5.1) on (3.13,3.14,3.15) gives the following solutions.

\[ \lambda_1 = \delta\mu\eta(1 - f^2) , \]  

(5.4)

\[ (\psi_+)_1 = 2\delta^*n\frac{f}{r}(1 - a)e^{i(n-1)\varphi} , \]  

(5.5)

\[ (\psi_0)_1 = -i\delta\mu\eta(1 - f^2) , \]  

(5.6)

\[ (\psi_-)_1 = 0 . \]  

(5.7)

This limit, with \( n = 1 \), was considered for a similar theory by Garriga and Vachaspati [9] and the above results are in agreement with theirs. This is a useful check of the techniques we use.
VI. CONCLUDING REMARKS

We have investigated the structure of cosmic string solutions to supersymmetric abelian Higgs models. For completeness we have analysed two models, differing by their method of spontaneous symmetry breaking. However, we expect theory F to be more representative of general defect forming theories, since the SSB employed there is not specific to abelian gauge groups.

We have shown that although SUSY remains unbroken outside the string, it is broken in the string core (in contrast to the gauge symmetry which is restored there). In theory F supersymmetry is broken completely in the string core by a nonzero $F$-term, while in theory D supersymmetry is partially broken by a nonzero $D$-term. We have demonstrated that, due to the particle content and couplings dictated by SUSY, the cosmic string solutions to both theories are superconducting in the Witten sense. Thus, all supersymmetric abelian cosmic strings are superconducting due to fermion zero modes.

Although explicitly solving for such zero modes is difficult in the case of non-supersymmetric theories, in the models we study it is possible to use SUSY transformations to relate the functional form of the fermionic solutions to those of the background string fields, which are well-studied. For theory D the solutions all obey the Bogomolnyi equations exactly, and for theory F we have also checked that the solutions we find reduce to those already known in the special case of the Bogomolnyi limit.

While we have performed this first analysis for the toy model of an abelian string, we expect the techniques to be quite general and in fact to be more useful in non-abelian theories for which the equations for the fermion zero modes are significantly more complicated. The question of superconductivity in non-abelian SUSY cosmic strings is under investigation.

There remain many unanswered questions concerning supersymmetric topological defects and the cosmological implications of particle physics theories which admit them. While this work was in preparation, two papers [15][16] appeared in which SUSY topological defects were considered in different settings to our work. We are currently investigating other roles that supersymmetric topological defects may play in the early universe. Clearly, there is much scope for further study.
ACKNOWLEDGMENTS

We would like to thank Sean Carroll, Dan Freedman, Ruth Gregory, Markus Luty, Hugh Osborn, Malcolm Perry, Patrick Peter, Tanmay Vachaspati and Mark Wise for helpful discussions.

A.C.D. and M.T. would like to thank the Aspen Center for Physics, where some of this work was done, for support and hospitality.

This work is supported in part by PPARC and the E.U. under the HCM program (CHRX-CT94-0423) (S.C.D. and A.C.D.), Trinity College Cambridge (S.C.D.) and by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement # DF-FC02-94ER40818 (M.T.).
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