Implications of the HyperCP Data on $B$ and $\tau$ decays

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Abstract

If the HyperCP three events for the decay of $\Sigma^+ \rightarrow p\mu^+\mu^-$ are explained by a new pseudoscalar (axial-vector) boson $X_{P(A)}$ with a mass of 214.3 MeV, we study the constraints on the couplings between $X_{P(A)}$ and fermions from the experimental data in $K$ and $B$ processes. Some implications of the new particle on flavor changing $B$ and $\tau$ decays are given. Explicitly, we show that the decay branching ratios of $B_s \rightarrow \phi X_{P(A)} \rightarrow \phi \mu^+\mu^-$, $B_d \rightarrow K^*_0(1430)P(A) \rightarrow K^*_0(1430)\mu^+\mu^-$ and $\tau \rightarrow \mu X_{P(A)} \rightarrow \mu \mu^+\mu^-$ can be as large as $2.7 \ (2.8) \times 10^{-6}$, $7.4 \ (7.5) \times 10^{-7}$ and $1.7 \ (0.14) \times 10^{-7}$, respectively.

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The HyperCP collaboration \cite{1} has presented the branching ratio of $\Sigma^+ \rightarrow p\mu^+\mu^-$ to be $(8.6^{+6.6}_{-5.4}\pm5.5) \times 10^{-8}$, which is hardly explained within the Standard Model \cite{1,2,3}, and suggested a new boson $X$ with a mass of $214.3\pm0.5$ MeV to induce the flavor changing transition of $s \rightarrow d\mu^+\mu^-$. It has been demonstrated \cite{4,5,6} that to explain the data the new particle cannot be a scalar (vector) but pseudoscalar (axial-vector) boson $X_P(A)$ based on the direct constraints from $K^+ \rightarrow \pi^+\mu^+\mu^-$ and $K_L \rightarrow \mu^+\mu^-$. A possible candidate with a light sgoldstino in spontaneously local supersymmetry breaking theories has been extensively discussed in the literature \cite{7}. Recently, He, Tandean and Valencia \cite{8} have also shown that the light pseudoscalar Higgs boson in the next-to-minimal supersymmetric standard model can be identified as $X_P$. In addition, searching for the new light boson at colliders has been studied by Ref. \cite{9}.

In this paper, we will explore the implications of the HyperCP Data on flavor changing $B$ and $\tau$ decays. In particular, we will examine constraints on the effective interactions induced by $X_P(A)$ from the experimental data in $B$ processes and study the possibility of having large effects in semileptonic $B_{d,s}$ decays. For the tau decays, we will concentrate on the decays of $\tau \rightarrow \ell\mu^+\mu^-$. We start by writing the effective interactions for the new pseudoscalar ($X_P$) or axial-vector ($X_A$) particle coupling to quarks and leptons to be \cite{4}

$$\begin{align*}
-\mathcal{L}_P &= \left(-i g_{Pij}^Q \bar{q}_i \gamma_5 q_j - i g_{Pij}^L \bar{\ell}_i \gamma_5 \ell_j + H.c.\right) X_P, \\
-\mathcal{L}_A &= \left(g_{Aij}^Q \bar{q}_i \gamma_\mu \gamma_5 q_j + g_{Aij}^L \bar{\ell}_i \gamma_\mu \gamma_5 \ell_j + H.c.\right) X^\mu_A,
\end{align*}$$

where $g_{Pij}^F(g_{Aij}^F) (F = Q, L)$ denote the couplings of $X_P(X_A^\mu)$ to quarks and leptons, respectively, and the indices $i,j$ stand for the quark or lepton flavors. Although the exotic events observed in the HyperCP are associated with the flavor changing neutral current (FCNC) in the first two generations of quark flavors and lepton flavor (LF) conservation, to study the effects of the new particle on $B$ and $\tau$ decays, we will include all FCNCs in both quark and lepton sectors. It has been studied and known that the constraint on the $s-d-X$ coupling from the decay branching ratio (BR) of $K_L \rightarrow \mu^+\mu^-$ is more strict than that from the $K-\bar{K}$ mixing \cite{4}. In order to search for the most strict bounds on $X$-mediated $B$-meson decay processes, we will examine those measured well in experiments, such as the $B_d(s) - \bar{B}_d(s)$ mixings and the decays of $B_{d,s} \rightarrow \mu^+\mu^-$ and $B \rightarrow K^*\mu^+\mu^-$. To study the $B_q$ related processes, in terms of Eq. (1), we explicitly write the relevant
interactions to be

$$- \mathcal{L} = -ig_{Pq}^Q \bar{b} \gamma_5 q X_P - ig_{Pq}^L \bar{b} \gamma_\mu \gamma_5 q X_P + g_{Aq}^Q \bar{b} \gamma_\mu \gamma_5 q X_A^\mu + g_{Aq}^L \bar{b} \gamma_\mu \gamma_5 q X_A^\mu + \text{H.c.}. \quad (2)$$

We note that for simplicity, we have abbreviated $g_{Pq}^Q$ ($g_{Aq}^Q$) to denote the effective coupling for $b - q - X_{P(A)}$. From Eq. (2), the effective Hamiltonian for $|\Delta B| = 2$ could be obtained by

$$H_{\text{eff}}^{\Delta B = 2} = \frac{g_X^2}{m_{Bq}^2 - m_X^2} (\bar{b} \Gamma \gamma_5 q) (\bar{b} \Gamma \gamma_5 q), \quad (3)$$

where $g_X = (g_{Pq}^Q, g_{Aq}^Q)$, $m_X = (m_{X_P}, m_{X_A})$ and $\Gamma = (1, \gamma_\mu)$ for $X = (X_P, X_A)$. The $B_q - \bar{B}_q$ oscillations are dictated by the two physical mass differences, defined by $\Delta m_{Bq} = 2|\Delta B = 2| = 2|\langle \bar{B}_q | H_{\text{eff}}^{F = 2} | B_q \rangle|$. Explicitly, we have

$$\langle \Delta m_{Bq} \rangle_{X_P} = \frac{4g_X^2 m_{Bq} f_{Bq}^2}{3(m_{Bq}^2 - m_X^2)} |P_{1,2}^{SLL} - P_{1,2}^{LR}|, \quad (4)$$

$$\langle \Delta m_{Bq} \rangle_{X_A} = \frac{4g_X^2 m_{Bq} f_{Bq}^2}{3(m_{Bq}^2 - m_X^2)} -(P_{1,2}^{VLL} - P_{1,2}^{LR}) + \frac{(m_b + m_q)^2}{m_X^2} (P_{1,2}^{SLL} - P_{1,2}^{LR}), \quad (4)$$

where we have used the hadronic matrix elements, defined by [10]

$$\langle \bar{B}_q | (\bar{b} \gamma_\mu P_{L(R)q})(\bar{b} \gamma_\mu P_{L(R)q}) | B_q \rangle = \frac{m_{Bq} f_{Bq}^2}{3} P_{1,2}^{VLL},$$

$$\langle B_q | (\bar{b} \gamma_\mu P_{Lq})(\bar{b} \gamma_\mu P_{Rq}) | B_q \rangle = \frac{m_{Bq} f_{Bq}^2}{3} P_{1,2}^{LR},$$

$$\langle B_q | (\bar{b} P_{L(R)q})(\bar{b} P_{L(R)q}) | B_q \rangle = \frac{m_{Bq} f_{Bq}^2}{3} P_{1,2}^{SLL},$$

$$\langle B_q | (\bar{b} P_{Lq})(\bar{b} P_{Rq}) | B_q \rangle = \frac{m_{Bq} f_{Bq}^2}{3} P_{1,2}^{LR}, \quad (5)$$

with $f_{Bq}(m_{Bq})$ being the $B_q$ decay constant (mass). It is clear that by using $s(d)$ instead of $b(q)$, Eq. (5) can be applied to the $K$ system. In Table [10], we give the values of $P_{1,2}^{VLL}$, $P_{1,2}^{LR}$ and $P_{1,2}^{SLL}$ for K and B systems. Note that $P_{1,2}^{LR}$ and $P_{1,2}^{SLL}$ in the $B_q$ system are much smaller than those in the $K$ system as there is no enhancement from chiral symmetry breaking in $B_q$.

We now examine $P \rightarrow \mu^+ \mu^-$ with $P = (K_L, B_q)$. Note that the decay of $K_L \rightarrow \mu^+ \mu^-$ gives the strongest constraint on $g_{sd}^Q$. To estimate the decay BRs, we use

$$\langle 0 | \bar{q} \gamma_\mu \gamma_5 q_j | P(p) \rangle = if_{PP\mu}, \quad \langle 0 | \bar{q} \gamma_5 q_j | P(p) \rangle = -i \frac{f_{PM} m_p^2}{m_{q_i} + m_{q_j}}. \quad (6)$$

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By neglecting the mixing induced CP violation in the K system and using $K_L \approx K_2$, the X-mediated decay amplitudes for $P \rightarrow \mu^+ \mu^-$ could be summarized as

$$M(P \rightarrow \mu^+ \mu^-)_{X_P} = -ia_P \frac{g^Q_{Pij}g^L_{P\mu}}{m_P^2 - m^2_{X_P}m_{q_i} + m_{j}} f_P m_P^2 \bar{\mu} \gamma_5 \mu,$$

$$M(P \rightarrow \mu^+ \mu^-)_{X_A} = i2a_P \frac{g^Q_{Pij}g^L_{P\mu}}{m_{X_A}^2} m_P f_P \bar{\mu} \gamma_5 \mu,$$

(7)

where $q_i = (s, b), q_j = (d, q)$ and $a_P = (\sqrt{2}, 1)$ for $P = (K_L, B_q)$. The corresponding decay rates are given by

$$\Gamma(P \rightarrow \mu^+ \mu^-)_{X_P} = \frac{m_P}{m_{X_P}} \left| \frac{a_P g^Q_{Pij}}{m_P^2 - m^2_{X_P}m_{q_i} + m_{j}} f_P m_P^2 \right|^2 \frac{\sqrt{1 - 4m^2_\mu/m_P^2}}{\sqrt{1 - 4m^2_\mu/m^2_{X_P}}} \Gamma(X_P \rightarrow \mu^+ \mu^-),$$

$$\Gamma(P \rightarrow \mu^+ \mu^-)_{X_A} = \frac{3 m_P}{2 m_{X_A}} \left| \frac{2a_P g^Q_{Aij}}{m_{X_A}^2} m_P f_P \right|^2 \frac{\sqrt{1 - 4m^2_\mu/m_P^2}}{(1 - 4m^2_\mu/m_{X_A}^2)^{3/2}} \Gamma(X_A \rightarrow \mu^+ \mu^-),$$

(8)

with

$$\Gamma(X_P \rightarrow \mu^+ \mu^-) = \frac{|g^L_{P\mu}|^2 m_{X_P}}{8\pi} \sqrt{1 - \frac{4m^2_\mu}{m_{X_P}^2}},$$

$$\Gamma(X_A \rightarrow \mu^+ \mu^-) = \frac{|g^L_{A\mu}|^2 m_{X_A}}{12\pi} \left(1 - \frac{4m^2_\mu}{m_{X_A}^2}\right)^{3/2}.$$

(9)

After introducing $|\Delta F| = 2$ ($F = S, B$) and purely rare dileptonic decays of $K_L$ and $B_q$, we investigate other X-mediated rare semileptonic processes, such as $B_q \rightarrow M X \rightarrow M \mu^+ \mu^-$ with $M$ being a light meson. For the s-wave states in the processes, $M$ has to be a vector-meson ($V$) due to parity. On the other hand, for the production of p-wave states in the decays, $M$ can be either scalar ($S$) or axial-vector ($A$) mesons, such as $f_0(980), a_0(980), \kappa, f_0(1500), K^*_0(1430)$ and $K_1$. To study the decay rates, we illustrate the formulas in the case with $M = V$ and $X = X_A^\mu$. We write the corresponding decay amplitude to be

$$M = \langle V \ell^+ \ell^- | H_{\text{eff}} | B_q \rangle = \frac{-g^{\mu
u} + q^\mu q^\nu/m_{X_A}^2}{q^2 - m_{X_A}^2 + i\Gamma_{X_A} m_{X_A}} \bar{\mu} \gamma_\nu \gamma_5 \mu,$$

(10)
where \( h_\mu = \langle V|\bar{b}\gamma_\mu\gamma_5 q'|B_q \rangle \) denotes the hadronic transition matrix element and \( \Gamma_{X_A} \) is the total decay width of \( X_A^\mu \). In terms of the narrow width approximation, given by

\[
|q^2 - m_{X_A}^2| \approx \frac{\pi}{\Gamma_{X_A} m_{X_A}} \delta(q^2 - m_{X_A}^2),
\]

(11)

the squared decay amplitude can be written as

\[
|M|^2 = \left| \sum_{\lambda=0,\pm} h_\mu \varepsilon_{X_A}^\mu(\lambda) \varepsilon_{X_A}^\nu(\lambda) \bar{\mu} \gamma_\nu \gamma_5 \mu \right|^2 \frac{\pi}{\Gamma_{X_A} m_{X_A}} \delta(q^2 - m_{X_A}^2),
\]

(12)

and the differential decay rates for the semileptonic \( B_q \) decays are given by

\[
d\Gamma(B_q \rightarrow V \mu^+ \mu^-) = \frac{1}{2m_{B_q}} \left[ \sum_{\lambda=0,\pm} |h_\mu \varepsilon_{X_A}^\mu(\lambda)|^2 \right] d\Gamma(B_d \rightarrow VX_A)
\]

\[
\times \frac{1}{2m_{X_A} \Gamma_{X_A}} \left[ \sum_{\lambda=0,\pm} |\varepsilon_{X_A}^\nu(\lambda) \bar{\mu} \gamma_\nu \gamma_5 \mu|^2 \right] d\Gamma(X_A \rightarrow \mu^+ \mu^-)
\]

(13)

where the factor 3 is due to the spin-degree of freedom from \( X_A \). The formula in Eq. (13) could be applied to any decaying chain of \( P \rightarrow MX \rightarrow M \mu^+ \mu^- \). As a result, we find that

\[
\Gamma(B_q \rightarrow MX \rightarrow M \mu^+ \mu^-) = f_X \Gamma(B_q \rightarrow MX) \times BR(X \rightarrow \mu^+ \mu^-),
\]

(14)

where \( f_X = (1, 3) \) for \( X = (X_P, X_A) \), representing the spin degree of freedom.

In our following analysis, we will focus on the decays of \( B_q \rightarrow MX \). It is clear that \( X \) is emitted and the meson in the final state owns the same spectator quark as \( B_q \). The hadronic effects for the decays are only related to \( B_q \rightarrow M \) transition form factors. The relevant form factors for various mesons are parametrized by

\[
\langle V(p, \varepsilon_V)|\bar{b}\gamma_\mu\gamma_5 q'|B_q(p_B)\rangle = i \left[ (m_{B_q} - m_V) \left( \varepsilon_{V}^\mu(\lambda) - \frac{\varepsilon_V^\nu(\lambda) \cdot P}{q^2} q^\nu \right) A_{1}^{B_q} \right.
\]

\[
- \frac{\varepsilon_V^\nu \cdot P}{m_{B_q} + m_V} \left( P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) A_{2}^{B_q}(q^2)
\]

\[
+ 2m_V \frac{\varepsilon_V^\nu \cdot P}{q^2} q^\mu A_{0}^{B_q V}(q^2) \right],
\]

\[
\langle A(p, \varepsilon_A)|\bar{b}\gamma_\mu\gamma_5 q'|B_q(p_B)\rangle = -\frac{A_{B_q A}(q^2)}{m_{B_q} - m_A} \varepsilon_{\mu\nu\rho\sigma} \varepsilon_{X}^\rho(\lambda) P^\nu q^\sigma,
\]

\[
\langle S(p)|\bar{b}\gamma_\mu\gamma_5 q'|B_q(p_B)\rangle = -i \left[ \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) F_1^{B_q S}(q^2) + \frac{P \cdot q}{q^2} q_\mu F_0^{B_q S}(q^2) \right],
\]

(15)
where \( \varepsilon_{V(A)}(\lambda) \) denotes the polarization of \( V(A) \) with the helicity state \( \lambda \), \( P_\mu = (p_B + p)_\mu \) and \( q_\mu = (p_B - p)_\mu \). Consequently, the decay amplitudes for the decays \( B_q \to (V, S)X_P \) can be written as

\[
A(B_q \to VX_P) = -g_{P_Bq}^Q \frac{2m_V}{m_b + m_q} \varepsilon^*_V \cdot q A_0^{B_qV}(m_{X_P}^2),
\]

\[
A(B_q \to SX_P) = g_{P_Bq}^Q \frac{m_{B_q}^2 - m_S^2}{m_b + m_q} F_0^{B_qS}(m_{X_P}^2). \tag{16}
\]

Since \( m_X \) is as light as 0.214 GeV, it is a good approximation to adopt \( F(0) \approx F(m_X^2) \) for the various form factors. From Eq. (16), we obtain

\[
\Gamma(B_q \to VX_P) \approx \frac{m_{B_q}^3 (A_{0}^{B_qV}(0))^2}{16\pi (m_b + m_q)^2} \left( 1 - \frac{m_{B_q}}{m_{B_q}^2} \right)^3 \left| g_{P_Bq}^Q \right|^2 ,
\]

\[
\Gamma(B_q \to SX_P) \approx \frac{m_{B_q}^3 (F_0^{B_qS}(0))^2}{16\pi (m_b + m_q)^2} \left( 1 - \frac{m_{B_q}}{m_{B_q}^2} \right)^3 \left| g_{P_Bq}^Q \right|^2 . \tag{17}
\]

Similarly, the decay amplitudes for \( B_q \to (V, A, S)X_A \) are given by

\[
A(B_q \to VX_A) = i g_{A_Bq}^Q \left[ (m_{B_q} + m_V) A_1^{BV}(0) \delta^{*}_V(\lambda) \cdot \delta^{*}_X_A(\lambda) \right. \\
- \frac{2 A_2^{BV}(0)}{m_{B_q} + m_V} \varepsilon^*_V(\lambda) \cdot p_B \varepsilon^{*}_X_A(\lambda) \cdot p_B \bigg] ,
\]

\[
A(B_q \to AX_A) = -2 g_{A_Bq}^Q A_3^{VA}(0) \epsilon_{\mu\nu\rho\sigma} \varepsilon^{*}_{X_A}(\lambda) \varepsilon^{\mu\nu}(\lambda) p_B^\rho q^\sigma ,
\]

\[
A(B_q \to SX_A) = -2 i g_{A_Bq}^Q \varepsilon^{*}_{X_A}(\lambda) \cdot p_B F_1^{B_qS}(0) . \tag{18}
\]

To get the unified formulas for the decay rates with two spin-1 mesons in the final states, we can write the general decay amplitude in terms of the helicity basis as

\[
A_\lambda = \varepsilon^{*}_{1\mu}(\lambda) \varepsilon^{*}_{2\nu}(\lambda) \left[ a g^{\mu\nu} + \frac{b}{m_1 m_2} p_B^{\mu} p_B^{\nu} + i \frac{c}{m_1 m_2} \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta} \right] \tag{19}
\]

where \( m_{1,2} \) stand for the meson masses. Then, the longitudinal and transverse components are given by \( H_0 = -ax - b(x^2 - 1) \) and \( H_\pm = a \pm c \sqrt{x^2 - 1} \), respectively, with \( x = (m_B^2 - m_1^2 - m_2^2)/(2m_1 m_2) \). Since \( B_q \) is a spinless particle, both spin-1 mesons in the final states should have the same helicity. Hence, the decay rates are given by \[12,13\]

\[
\Gamma(B_q \to M_1 M_2) = \frac{|\vec{p}|}{8\pi m_{B_q}^2} \left( |H_0|^2 + |H_+|^2 + |H_-|^2 \right) \tag{20}
\]
with \(|p| = m_1 m_2 \sqrt{x^2 - 1/m_{Bq}^2}|. By comparing Eq. (18) with Eq. (19), we find that
\[
a = ig_A^{Q}(m_{Bq} + m_{V})A_{1}^{B_{q} V}(0), \quad b = -ig_A^{Q} m_{V} m_{X_{A}} \frac{2A_{2}^{B_{q} V}(0)}{m_{Bq} + m_{V}}, \quad c = 0, \quad (21)
\]
for \(B_q \to V X_A\) and
\[
a = 0, \quad b = 0, \quad c = ig_A^{Q} m_{X_{A}} \frac{2A_{2}^{B_{q} A}(0)}{m_{Bq} - m_{A}}, \quad (22)
\]
for \(B_q \to AX_A\). The decay rates for \(B_q \to SX_A\) are given by
\[
\Gamma(B_q \to SX_A) \approx \frac{m_{Bq}^3}{16\pi} \left(\frac{F_{1}^{S}(0)}{m_{X_A}}\right)^2 \left(1 - \frac{m_{S}^2}{m_{Bq}^2}\right) \left|g_A^{Q}\right|^2. \quad (23)
\]
Finally, we study the rare decays of \(B_q \to \gamma X_A\). Although \(\gamma\) is a vector boson, unlike an ordinary vector meson, it is massless and only has transverse degrees of freedom. With the transition form factor for \(B_q \to \gamma\), defined by
\[
\langle \gamma(k) | \bar{b} \gamma_{\mu} \gamma_5 q | B_q(p_B) \rangle = i e \frac{F_A(q^2)}{m_B} \left[\varepsilon_{\gamma}\ast(\lambda)p_B \cdot k - \varepsilon_{\gamma}\ast(\lambda) \cdot p_B k_{\mu}\right], \quad (24)
\]
the decay amplitudes are given by
\[
A(B_q \to \gamma X_A) = i e \frac{F_A(q^2)}{m_{Bq}} p_B \cdot k \varepsilon_{\gamma}\ast(\lambda) \cdot \varepsilon_{X_A}\ast(\lambda), \quad (25)
\]
leading to
\[
\Gamma(B_q \to \gamma X_A) = \frac{\alpha_{em}}{8} m_{Bq} |F_A(0)|^2 \left|g_A^{Q}\right|^2 \quad (26)
\]
with \(\alpha_{em} = e^2/4\pi\).

After the \(B_q\) processes, we now discuss the pure leptonic decays. As the new particle of \(X_P\) or \(X_{P}^\mu\) couples to the muon, it is natural to speculate that it will also couple to other leptons such as \(e\) and \(\tau\). Moreover, the new particle could also give rise to the LFV, like the FCNCs in the quark sector, such as \(\mu \to e\gamma, \mu \to 3e\) and \(\tau \to \ell\mu^+\mu^-\). Recently, the LFV in \(\tau\) decays has been improved up to \(O(10^{-7})\) \([16, 17]\). It should be interesting to explore the LFV due to \(X_{P,A}\). To illustrate the effects of the LFV, we will concentrate on the processes related to \(\tau\). We write the relevant effective interactions as
\[
-\mathcal{L} = -i g_{P_{\ell}}^{F} \bar{\ell} \gamma_5 \tau X_P + g_{A_{\ell}}^{F} \bar{\ell} \gamma_{\mu} \gamma_5 \tau X_{A}^\mu + H.c. \quad (27)
\]
The rates for $\tau \rightarrow \ell X \rightarrow \ell \mu^+\mu^-$ are described by $\Gamma(\tau \rightarrow \ell X \rightarrow \ell \mu^+\mu^-) = f_X \Gamma(\tau \rightarrow \ell X)BR(X \rightarrow \mu^+\mu^-)$. Similar to the $B_q$ decays, we only discuss $\tau \rightarrow \ell X$. In terms of the interactions in Eq. (27), the decay rates with $X_P$ and $X_A$ are given by

$$\Gamma(\tau \rightarrow \ell X_P) \approx \frac{m_\tau}{16\pi} |g_{P\ell\tau}|^2, \quad \Gamma(\tau \rightarrow \ell X_A) \approx \frac{m_\tau^3}{32\pi m_{X_A}^2} |g_{P\ell\tau}|^2, \quad (28)$$

respectively, where we have neglected the masses of $\ell$ and $X_{P,A}$ due to $m_\tau \gg m_{X_{P,A}} > m_\ell$.

| Table II: The input values of parameters in units of $GeV$ |
|----------------------------------------------------------|
| $f_K$ | $f_{B_d}$ | $f_{B_s}$ | $m_X$ | $m_{K^0}$ | $m_{B_d}$ | $m_{B_s}$ |
| 0.16 | 0.20 | 0.22 | 0.214 | 0.497 | 5.28 | 5.37 |
| $m_s$ | $m_d$ | $m_b$ | $\tau_{KL}$ | $\tau_{B_d}$ | $\tau_{B_s}$ |
| 0.15 | 0.01 | 4.4 | $7.87 \times 10^{16}$ | $2.33 \times 10^{12}$ | $2.22 \times 10^{12}$ |

In order to do the numerical estimations, the input values for the various parameters are presented in Table II. To see the effects of the new particle on low energy physics, we first consider its contributions to $\Delta F = 2$ processes. From the current experimental data, the mass differences in the $K$ and $B_q$ systems are given by $\Delta m_K = (3.483 \pm 0.006) \times 10^{-15}$, $\Delta m_{B_d} = (3.337 \pm 0.033) \times 10^{-13}$ and $\Delta m_{B_s} = (11.45^{+0.20}_{-0.13}) \times 10^{-12}$ GeV. By utilizing these values and those inputs in Tables II and III from Eq. (4) the direct constraints on the couplings are found to be

$$|g_{Psd}^Q|^2 < 2.3 \times 10^{-15}, \quad |g_{Pbd}^Q|^2 < 2.2 \times 10^{-10}, \quad |g_{Pbs}^Q|^2 < 6.4 \times 10^{-9},$$

$$|g_{Asd}^Q|^2 < 0.67 \times 10^{-15}, \quad |g_{A bd}^Q|^2 < 5.1 \times 10^{-13}, \quad |g_{Abs}^Q|^2 < 1.4 \times 10^{-11}. \quad (29)$$

Due to the strong cancelation between $P_1^{SSL}$ and $P_1^{LR}$ in $B_q$, the constraints from $\Delta m_{B_s}$ are two orders of magnitude less than the naive expectation. Next, we discuss the decays of $P \rightarrow \mu^+\mu^-$. It is well known that the long-distance effect dominates the process of $K_L \rightarrow \mu^+\mu^-$, while the short-distance contribution usually is taken to be $BR(K_L \rightarrow \mu^+\mu^-)_{SD} < 3.6 \times 10^{-10}$ [4]. As to the dileptonic decays in $B_q$ decays, we also know the upper bounds of $BR(B_d \rightarrow \mu^+\mu^-) < 2.3 \times 10^{-8}$ and $BR(B_s \rightarrow \mu^+\mu^-) < 8 \times 10^{-8}$ [15]. With these constraints
and Eq. (8), we have

\[
\begin{align*}
|g_{Psd}^Q|^2 \Gamma(X_P \to \mu^+ \mu^-) &< 6.5 \times 10^{-20}, & |g_{Pbd}^Q|^2 \Gamma(X_P \to \mu^+ \mu^-) &< 3.8 \times 10^{-20}, \\
|g_{Pbs}^Q|^2 \Gamma(X_P \to \mu^+ \mu^-) &< 1.2 \times 10^{-19}, & |g_{Abs}^Q|^2 \Gamma(X_A \to \mu^+ \mu^-) &< 1.0 \times 10^{-20}, \\
|g_{Abd}^Q|^2 \Gamma(X_A \to \mu^+ \mu^-) &< 2.3 \times 10^{-24}, & |g_{Abs}^Q|^2 \Gamma(X_A \to \mu^+ \mu^-) &< 7.0 \times 10^{-24}. \quad (30)
\end{align*}
\]

It has been shown that the current strict bounds on $|g_{Ps}^{L}|^2$ and $|g_{A\mu}^{L}|^2$ are from muon $g-2$, given by $|g_{Ps}^{L}|^2 < 2.6 \times 10^{-7}$ and $|g_{A\mu}^{L}|^2 < 6.7 \times 10^{-8}$ \cite{4}, respectively. From Eq. (9), one gets the upper bounds on the rates as $\Gamma(X_P \to \mu^+ \mu^-) < 4.3 \times 10^{-10}$ GeV and $\Gamma(X_A \to \mu^+ \mu^-) < 2.7 \times 10^{-12}$ GeV. To illustrate the constraints on the couplings, we take $\Gamma(X_{P,A} \to \mu^+ \mu^-) \sim (10^{-10}, 10^{-12})$ GeV and we obtain

\[
\begin{align*}
|g_{Psd}^Q|^2 &< 6.5 \times 10^{-19}, & |g_{Pbd}^Q|^2 &< 3.8 \times 10^{-10}, & |g_{Pbs}^Q|^2 &< 1.2 \times 10^{-9}, \\
|g_{Abd}^Q|^2 &< 1.0 \times 10^{-17}, & |g_{Abs}^Q|^2 &< 2.3 \times 10^{-12}, & |g_{Abs}^Q|^2 &< 7.0 \times 10^{-12}, \quad (31)
\end{align*}
\]

respectively. It is clear that the constraints from $\Delta m_K$ are weaker than those from $K_L \to \mu^+ \mu^-$, which are consistent with the HyperCP data in the decay of $\Sigma^+ \to p^+ \mu^+ \mu^-$, given by \cite{4}

\[
\begin{align*}
|g_{Psd}^Q|^2 BR(X_P \to \mu^+ \mu^-) &= (8.4^{+6.5}_{-5.1} \pm 4.1) \times 10^{-20}, \\
|g_{Abs}^Q|^2 BR(X_A \to \mu^+ \mu^-) &= (4.4^{+3.4}_{-2.7} \pm 2.1) \times 10^{-20}.
\end{align*}
\]

On the other hand, the bounds from $\Delta m_{B_q}$ and $BR(B_q \to \mu^+ \mu^-)$ are similar.

To estimate the BRs of semileptonic $B_q$ decays, we use the $B_q \to M$ transition form factors in Eq. (15), calculated by the light-front quark model (LFQM) and summarized in Table III \cite{11}. In the table, the states of $^1P_1$ and $^3P_1$ will be used to consist of the physical states $K_1(1270)$ and $K_1(1400)$ and their relations are parametrized by \cite{11,18},

\[
\begin{align*}
K_1(1270) &= K_{1P_1} \cos \theta + K_{3P_1} \sin \theta, \\
K_1(1400) &= -K_{1P_1} \sin \theta + K_{3P_1} \cos \theta. \quad (32)
\end{align*}
\]

| Table III: Values of form factors at $q^2 = 0$ defined in Eq. (15) and calculated by the LFQM | $A_{0}^{B_{s}K^{*}}$ | $A_{1}^{B_{s}K^{*}}$ | $A_{2}^{B_{s}K^{*}}$ | $A_{B_{s}K_{3P_{1}}}$ | $A_{B_{s}K_{1P_{1}}}$ | $F_{1}^{B_{s}K_{0}^{*}}$ | $F_{0}^{B_{s}K_{0}^{*}}$ |
|---|---|---|---|---|---|---|---|
| | 0.31 | 0.26 | 0.24 | 0.26 | 0.11 | 0.26 | 0.26 |
The decay of $B_d \rightarrow K^{*0} \mu^+ \mu^-$ has been measured at the B-factories with the world average on the decay BR being $(1.22 \pm 0.38) \times 10^{-6}$ [19]. From Eqs. (17), (18) and (20) and the values in Tables II and III, we obtain

$$BR(B_d \rightarrow K^{*0}X_P \rightarrow K^{*0} \mu^+ \mu^-) = 3.1 \times 10^{10} \left| g_{Q_{Pbs}} \right|^2 BR(X_P \rightarrow \mu^+ \mu^-),$$

$$BR(B_d \rightarrow K^{*0}X_A \rightarrow K^{*0} \mu^+ \mu^-) = 3.9 \times 10^{13} \left| g_{Q_{Abs}} \right|^2 BR(X_A \rightarrow \mu^+ \mu^-).$$

If we regard $BR(B_d \rightarrow K^{*0} \mu^+ \mu^-) = 1.22 \times 10^{-6}$ as the upper bound, we have

$$\left| g_{Q_{Pbs}} \right|^2 BR(X_P \rightarrow \mu^+ \mu^-) \leq 3.9 \times 10^{-17},$$

$$\left| g_{Q_{Abs}} \right|^2 BR(X_A \rightarrow \mu^+ \mu^-) \leq 3.1 \times 10^{-20}. \quad (33)$$

For $BR(X_{P,A} \rightarrow \mu^+ \mu^-) \sim 1$, we find that the decay $B_d \rightarrow K^{*0} \mu^+ \mu^-$ gives the strongest limits on the couplings of $g_{Q_{Pbs}}$ and $g_{Q_{Abs}}$.

From Eq. (33), we can study the contributions of $X_{P,A}$ to other $B_q$ decays. The first direct application is $B_s \rightarrow \phi X_{P,A} \rightarrow \phi \mu^+ \mu^-$. In terms of the formulas shown in Eqs. (17), (18) and (20), we get

$$BR(B_s \rightarrow \phi X_P \rightarrow \phi \mu^+ \mu^-) \leq 2.74 \times 10^{-6},$$

$$BR(B_s \rightarrow \phi X_A \rightarrow \phi \mu^+ \mu^-) \leq 2.81 \times 10^{-6}, \quad (34)$$

where we have used $A_{0,\phi}^B(0) = 0.474$, $A_{1,\phi}^B(0) = 0.311$ and $A_{2,\phi}^B(0) = 0.234$ for $B_s \rightarrow \phi$ transition form factors calculated by the light cone sum rules (LCSR) [20]. Interestingly, the bounds are just under the D0 upper limit of $BR(B_s \rightarrow \phi \mu^+ \mu^-) < 3.2 \times 10^{-6}$ [21]. If the events observed by the HyperCp collaboration [1] are indeed from the new particle, the decay of $B_s \rightarrow \phi \mu^+ \mu^-$ should be observed soon as the standard model prediction is around $1.6 \times 10^{-6}$ [22].

Next, we discuss the productions of p-wave mesons in $B_q$ decays. As mentioned before, the p-wave mesons could be $f_0(980)$, $a_0(980)$, $\kappa$, $K_0^*(1430)$ and $K_1$. However, since the quark contents for the light p-wave mesons ($<1$ GeV) are not certain, we will only focus on $K_0^*(1430)$ and $K_1$. By using the values given in Tables I and III, we directly display the predicted upper BRs in Table IV. From the table, we see clearly that only the production of $K_0^*(1430)$ is interesting, which is accessible to the current B-factories. As only the transverse degrees of freedom are involved in the decays of $B_q \rightarrow AX_A$, where the effects
are proportional to the masses of $A$ and $X_A$, one can easily understand why the BRs for $B_q \to K_1 X_A \to K_1 \mu^+ \mu^-$ are so small and negligible. In addition, according to Eq. (26), we get that $BR(B_s \to \gamma X_A \to \gamma \mu^+ \mu^-)$ is less than $5.4 \times 10^{-11}$, which is much smaller than the contributions of in the SM [23] and negligible.

As there is no any useful information on $g_{P(A)bd}^Q$, one can only investigate those decays associated with $g_{P(A)bs}^Q$. In order to apply $g_{P(A)bd}^Q$ to decays related to $g_{P(A)bs}^Q$, we need some theoretical ansatz to connect them. One of the interesting ansatz is to relate the couplings with quark masses, such as

$$g_{Pij}^Q = \lambda_P \left( \frac{m_i m_j}{v^2} \right)^{\frac{3}{2}}, \quad g_{Aij}^Q = \lambda_A \left( \frac{m_i m_j}{v^2} \right)^{\frac{3}{2}}, \quad (35)$$

where $\lambda_{P(A)} = v/v_F$ denotes the ratio of electroweak scale ($v$) to the new scale ($v_F$) associated with the new particle. By using the ansatz, we find that $g_{P(A)bd}^Q/g_{P(A)bs}^Q \sim (m_d/m_s)^{1/2} \sim 0.26$ and the upper bounds on BRs for some decays associated with $g_{P(A)bd}^Q$ are given by

$$BR(B_u \to \rho^+ X_P \to \rho^+ \mu^+ \mu^-) = 6.3 \times 10^{-8},$$
$$BR(B_u \to \rho^+ X_A \to \rho^+ \mu^+ \mu^-) = 6.4 \times 10^{-8},$$
$$BR(B_s \to K^{*0} X_P \to K^{*0} \mu^+ \mu^-) = 5.1 \times 10^{-8},$$
$$BR(B_s \to K^*_0 (1430) X_P \to K^*_0 (1430) \mu^+ \mu^-) = 4.9 \times 10^{-8},$$
$$BR(B_s \to K^*_0 (1430) X_A \to K^*_0 (1430) \mu^+ \mu^-) = 5.0 \times 10^{-8}, \quad (36)$$

where the form factors are taken to be $A_{Bd}^{\rho}(0) = 0.28$, $A_{1Bd}^{\rho}(0) = 0.22$, $A_{2Bd}^{\rho} = 0.20$ [11] and $A_{BdK^*}^{B} = 0.247$ [20], respectively.

Finally, we study $\tau \to \ell \mu^+ \mu^-$. Here, we use the same ansatz as that for the quark sector in Eq. (35), i.e.,

$$g_{P\ell\tau}^L = \lambda_P \left( \frac{m_\ell m_\tau}{v^2} \right)^{\frac{3}{2}}, \quad g_{A\ell\tau}^L = \lambda_A \left( \frac{m_\ell m_\tau}{v^2} \right)^{\frac{3}{2}}. \quad (37)$$

---

### Table IV: BRs (in units of $10^{-6}$) for $B_d \to (K^*_0(1430), K_1(1270), K_1(1400)) X_{P,A}(X_{P,A} \to \mu^+ \mu^-)$.

| Mode | $B_d \to K^*_0(1430) X_P(X_P \to \mu^+ \mu^-)$ | $B_d \to K^*_0(1430) X_A(X_A \to \mu^+ \mu^-)$ |
|------|---------------------------------|---------------------------------|
| BR   | 0.74                            | 0.75                            |

| Mode | $B_d \to K_1(1270) X_A(X_A \to \mu^+ \mu^-)$ | $B_d \to K_1(1400) X_A(X_A \to \mu^+ \mu^-)$ |
|------|---------------------------------|---------------------------------|
| BR   | $6 \times 10^{-3}$             | $1.54 \times 10^{-4}$         |
By Eqs. (28), (33) and (35) and with $\tau = 4.4 \times 10^{11}$ GeV, the upper bounds on the flavor violating $\tau$ decays are estimated to be

$$
BR(\tau \to eX_P \to e\mu^+\mu^-) = 8.5 \times 10^{-10}, \\
BR(\tau \to \mu X_P \to \mu\mu^+\mu^-) = 1.7 \times 10^{-7}, \\
BR(\tau \to eX_A \to e\mu^+\mu^-) = 7.0 \times 10^{-11}, \\
BR(\tau \to \mu X_A \to \mu\mu^+\mu^-) = 1.4 \times 10^{-8}.
$$

Note that the current upper bounds for $\tau \to \ell\mu^+\mu^-$ are $2.0 \times 10^{-7}$ [19].

In sum, we have studied the implications of the HyperCP Data on flavor changing $B$ and $\tau$ decays. We have given constraints on the effective couplings due to the new pseudoscalar and axial-vector bosons of $X_{P,A}$ from experimental data in the $K$ and $B$ systems, respectively. We have pointed out that the strongest limits on $g_{P(A)bs}^Q$ are from the decay of $B_d \to K^{*0}\mu^+\mu^-$. We have shown that the decay BR of $B_s \to \phi\mu^+\mu^-$ can be as large as $2.7(2.8) \times 10^{-6}$ through $X_{P(A)}$, which are larger than the prediction of $1.6 \times 10^{-6}$ in the SM and close to the experimental upper limit of $3.2 \times 10^{-6}$. Furthermore, we have found that $BR(B_d \to K^{*0}(1430)P(A) \to K_0(1430)\mu^+\mu^-)$ is about $7.4 \ (7.5) \times 10^{-7}$, whereas other related modes are negligible. In addition, we have proposed an ansatz to relate the couplings with the fermion masses. Based on this ansatz, we have demonstrated that the decay BRs of $B_u \to \rho^+\mu^+\mu^-$, $B_s \to K^{*0}\mu^+\mu^-$ and $B_s \to K^{*0}_0(1430)\mu^+\mu^-$ can all be at the level of $10^{-8}$. In particular, we have shown that $BR(\tau \to \mu X_{P(A)} \to \mu\mu^+\mu^-) = 1.7 \ (0.14) \times 10^{-7}$.

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