Dual quark condensate in the Polyakov-loop extended Nambu–Jona-Lasinio model

Kouji Kashiwa,† Hiroaki Kouno,‡ and Masanobu Yahirot

†Department of Physics, Graduate School of Sciences, Kyushu University, Fukuoka 812-8581, Japan
‡Department of Physics, Saga University, Saga 840-8502, Japan

(Dated: December 9, 2009)

The dual quark condensate $\Sigma^{(n)}$ proposed recently as a new order parameter of the spontaneous breaking of the $Z_3$ symmetry is evaluated by the Polyakov-loop extended Nambu–Jona-Lasinio model, where $n$ are winding numbers. The Polyakov-loop extended Nambu–Jona-Lasinio model well reproduces lattice QCD data on $\Sigma^{(1)}$ measured very lately. The dual quark condensate relates the chiral condensate to the quark confinement and hence a good quantity to determine the strength.

PACS numbers: 11.30.Rd, 12.40.-y

Recently, a new order parameter of the $Z_3$ center symmetry was proposed by using the chiral condensate $\sigma$ and evaluated by quenched [1, 2] and full lattice QCD [3]; the new order parameter is called the dual quark condensate. This makes it possible to discuss the connection between the quark confinement and the chiral symmetry. The relation can be discussed also by the Polyakov-loop extended NJL (PNJL) model [4] in which the Polyakov loop $\Phi$ is approximately treated as a classical variable. Actually, extensive studies are made on the relation; for example, see Refs. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. The dual quark condensate is calculated also in the color $SU(2)$ system by the Dyson-Schwinger equation [16], but this approach does not treat the confinement mechanism dynamically. Very recently, the functional renormalization-group method was also applied to evaluate the dual quark condensate in the color $SU(3)$ system [17].

In this paper, we evaluate the dual quark condensate by using the PNJL model and show that the PNJL result can reproduce full lattice QCD (LQCD) data [3] on the dual quark condensate.

We consider the quark field $q$ that obeys a twisted temporal boundary condition

$$q(x, \beta) = e^{-i\beta \varphi} q(x, 0),$$

where $\varphi$ is a twisted angle. Now we define the $\varphi$-dependent chiral condensate $\sigma_{\varphi}$ by

$$\sigma_{\varphi} = -\frac{1}{V} \langle \text{Tr}[(m + D_{\varphi})^{-1}] \rangle,$$

where the twisted boundary condition [1] is imposed on the Dirac operator $D_{\varphi}$, but the bracket $\langle \ldots \rangle$ keeps the anti-periodic boundary condition $\varphi = \pi [1, 2, 3]$. The dual quark condensate $\Sigma^{(n)}$ is defined by

$$\Sigma^{(n)} = -\int_0^{2\pi} d\varphi \frac{d\varphi}{2\pi} e^{-i n \varphi} \sigma_{\varphi}$$

with winding numbers $n [1, 2, 3]$. In particular, $\Sigma^{(1)}$ is called the dressed Polyakov loop. The winding number is 1 in both $\Sigma^{(1)}$ and $\Phi$. In full LQCD calculations of Ref [3], the dressed Polyakov-loop is evaluated with $m/T = 0.032$. We can expect that the two have similar $T$ dependence to each other. Thus, the dual quark condensate relates the chiral condensate to the quark confinement $[1, 2, 3, 16, 17, 18].$

We start with the two-flavor PNJL Lagrangian with the vector-type four-quark and the scalar-type eight-quark interactions; see Ref. [8] for the details. The scalar-type eight-quark interaction makes the chiral phase transition stronger. Hence, the PNJL result becomes consistent with LQCD data at finite $T$ [10, 19]. Furthermore, both the scalar-type eight-quark and the vector-type four-quark interaction are necessary for the PNJL model to reproduce LQCD data [20, 21, 22] at imaginary chemical potential [10].

Making the mean field approximation to the PNJL Lagrangian and performing the path integration of the resultant partition function over $q$ under the twisted boundary condition (1), one can obtain the thermodynamical potential

$$\Omega = -2 N_f \int_{A} \frac{d^3 p}{(2\pi)^3} \left[ 3 E_p + \frac{1}{\beta} \ln [1 + 3(\bar{\Phi} + \Phi e^{-\beta E_p}) e^{-\beta E_p} + e^{-3 \beta E_p}] \right]$$

$$+ \frac{1}{\beta} \ln [1 + 3(\bar{\Phi} + \Phi e^{-\beta E_p}) e^{-\beta E_p} + e^{-3 \beta E_p}]$$

$$+ G_s \sigma^2 - G_v \omega^2 + 3 G_{88} \sigma^4 + U,$$

where $\sigma = \langle \bar{q}q \rangle$, $\omega = \langle \bar{q} \gamma_0 q \rangle$ and $E_p^\pm = E_p \pm i T \theta_{\varphi}$ with $E_p = \sqrt{p^2 + M^2}$, $M = m_0 - 2 G_s \sigma - 4 G_{88} \sigma^3$ and $i T \theta_{\varphi} = -2 G_v \omega - i T \theta_{\varphi}$. Here, $m_0$ is the current quark mass and we use $m_0 = 5.5$ MeV. The constants $G_s, G_v, G_{88}$ denote coupling strengths of the scalar-type four-quark, the vector-type four-quark and the scalar-type eight-quark interaction, respectively. The Polyakov-loop potential $U$ is a function of $\Phi$ and its Hermitian conjugate $\bar{\Phi}$. The 3-dimensional momentum integration is regularized by a cutoff $A$. The classical variables $X = \sigma, \omega, \bar{\Phi}, \Phi$ are determined by solving the stationary conditions $\partial \Omega / \partial X = 0$ numerically, and $\Sigma^{(n)}$ is obtained numerically from $\sigma_{\varphi}$ with [3]. Details of the PNJL model are shown in Ref. [3].

†kashiwa@phys.kyushu-u.ac.jp
‡kounoh@cc.saga-u.ac.jp
†yahiro@phys.kyushu-u.ac.jp
The Polyakov potentials of polynomial type [5] and logarithm type [6], respectively, while $s_8^2$, $s_4^4$, $s_8^3$ denote the scalar-type four-quark, the scalar-type eight-quark and the vector-type four-quark interaction, respectively. Symbols $T_c^\sigma$, $T_c^\phi$, $T_c^{\Sigma(1)}$ denote the pseudo-critical temperatures defined by peak positions of $d\sigma/dT$, $d\phi/dT$, $d\Sigma^{(1)}/dT$, respectively. Here, $T_c = 173$ MeV.

As mentioned above, the twisted boundary condition (1) is imposed on $D_\phi$. Therefore, $\Phi$ and $\overline{\Phi}$ are first obtained under the anti-periodic boundary condition $\varphi = \pi$. The quantities $\sigma(\varphi)$ and $\omega(\varphi)$ are determined by solving the stationary conditions, $\partial\Omega/\partial\sigma = 0$ and $\partial\Omega/\partial\omega = 0$, numerically under the twisted boundary condition (1) with $\Phi$ and $\overline{\Phi}$ fixed to the values determined at $\varphi = \pi$.

The pseudo-critical temperature $T_c^\phi$ of the deconfinement crossover is usually defined by a peak position of either the Polyakov-loop susceptibility or $d\phi/dT$. In the present analysis, we take the latter to compare $T_c^\phi$ with the pseudo-critical temperature $T_c^{\Sigma(1)}$ estimated by a peak position of $d\Sigma^{(1)}/dT$, because we can not define the susceptibility for the dressed Polyakov loop $\Sigma^{(1)}$. For consistency, the pseudo-critical temperature $T_c^{\Sigma(1)}$ of the chiral crossover is defined by a peak position of $d\sigma/dT$. In the PNJL calculations with the RRW06 potential, $d\sigma/dT$ has two peaks, but we take the second peak close to a peak position of the chiral susceptibility.

Table I summarizes values of $T_c^\sigma$, $T_c^\phi$, $T_c^{\Sigma(1)}$ in five types of model calculations, where the values are normalized by $T_c = 173$ MeV, i.e., the LQCD result [25] for $T_c^\sigma$ and $T_c^{\Sigma(1)}$. In PNJL-I, -II, -III with the RRW06 potential, $T_c^\sigma$ and $T_c^{\Sigma(1)}$ are close to each other and also to the LQCD data [25], although this property is not seen in PNJL-IV with the RTW05 potential. As for $T_c^\phi$, PNJL-II is more consistent with the LQCD data than PNJL-I and hence the scalar-type eight-quark interaction is necessary to reproduce the LQCD data.

![Fig. 1: Chiral condensate as a function of $T/T_c^\sigma$ and renormalized Polyakov loop as a function of $T/T_c^\phi$ in the case of $\varphi = \pi$. Here, $\sigma$ is normalized by the value $\sigma_0$ at $T = 0$. The values of $T_c^\sigma$ and $T_c^\phi$ are summarized in Table II. Definitions of lines are also shown in Table II. LQCD data shown by box and triangle symbols are taken from Ref. [26] and Ref. [27], respectively.](image-url)
Nevertheless, the interaction gives a sizable effect on $\Sigma^{(1)}$, as shown later. For the shape of $\Phi$, PNJL-IV (dotted curve) gives a larger disagreement with the LQCD data than PNJL-II. Thus, the RRW06 potential is better than the RTW05 potential also for the shape of $\Phi$.

Full LQCD simulation of Ref. [3] shows that $T_{C}^{\Sigma^{(1)}} \simeq T_{C}^{\sigma} \simeq T_{C}^{\pi} \simeq 153$ MeV, while that of Ref. [25] does $T_{C}^{\Sigma^{(1)}} \simeq T_{C}^{\Phi} \simeq 173$ MeV. The two LQCD results have a systematic error of $\sim 10\%$. Moreover, it is reported in Ref. [28] for the $2+1$-flavor system that there exists a non-negligible deviation between $T_{C}^{\sigma}$ and $T_{C}^{\Phi}$. This indicates that it is an unsettled problem whether $T_{C}^{\sigma}$ and $T_{C}^{\Phi}$ really coincide or not. In this work, however, we assume the coincidence, since the LQCD data [3] that we are analyzing has the property. In the present analysis, temperature is normalized by $T_{C}$, but it is taken to be 153 MeV for LQCD data of Ref. [3] and 173 MeV for model calculations and LQCD data of Ref. [25,26,27].

Figure 2 presents the normalized dressed Polyakov loop $\Sigma^{(1)}/\Sigma^{(0)}$ as a function of $T/T_{C}^{\Sigma^{(1)}}$, where $\Sigma^{(1)}$ is the dressed Polyakov loop at $T = T_{C}^{\Sigma^{(1)}}$. PNJL-I (thin solid line) and PNJL-II (solid line) well reproduces LQCD data [3] (box symbols), but PNJL-IV (dotted line) and NJL (dot-dashed line) do not. Near $T_{C}^{\Sigma^{(1)}}$, the scalar-type eight-quark interaction hardly affects the dressed Polyakov loop. Thus, the RRW06 potential is necessary to explain $T$ dependence of $\Phi$, $\sigma$ and $\Sigma^{(1)}$ consistently.

Figure 3 shows $\varphi$-dependence of the chiral condensate at $T = 200$ and 250 MeV. Definitions of lines are shown in Table. I. LQCD data (box symbols) are taken from Ref. [3] and 173 MeV for model calculations and LQCD data of Ref. [25,26,27].

Figure 4: $T$-dependence of the dual quark condensate. Definitions of lines are shown in Table. I.

$\Sigma^{(0)}$ and $\Sigma^{(1)}$ are appreciable only at $T_{C}^{\Sigma^{(1)}} < T < 2T_{C}^{\Sigma^{(1)}}$. Thus, the effect of the eight-quark interaction becomes more appreciable for $\varphi = \pi$ than for $\Sigma^{(0)}$ and $\Sigma^{(1)}$. At $T > 2T_{C}^{\Sigma^{(1)}}$, the effect becomes negligible, since $\sigma$ itself is tiny there. Meanwhile, the difference between PNJL-II (solid line) and PNJL-III (dashed line) presents an effect of the vector-type four-quark interaction. The effect on $\sigma$ is zero at $\varphi = \pi$, and appreciable at nonzero $\varphi(\neq \pi)$, as shown in Fig. 4. The effect on $\Sigma^{(0)}$ and $\Sigma^{(1)}$ is then appreciable at $T_{C}^{\Sigma^{(1)}} < T < 1.5T_{C}^{\Sigma^{(1)}}$ and sizable at $T > 1.5T_{C}^{\Sigma^{(1)}}$, as shown in Fig. 4. Thus, the effect of the vector-type four-quark interaction becomes more appreciable for $\Sigma^{(0)}$ and $\Sigma^{(1)}$ than for $\sigma$ at $\varphi = \pi$. This indicates that the dual quark condensate at $T > 1.5T_{C}^{\Sigma^{(1)}}$ is a good quantity to determine the strength of the vector-type four-quark interaction.

The present PNJL results are consistent with the LQCD
data for $\Sigma^{(1)}/\Sigma_{T_c}$ near $T_c$, but not for $\sigma(\varphi)$ itself even if both are compared at the same values of $T/T_c\Sigma^{(1)}$, although the latter is not shown explicitly in this paper. In LQCD, $\sigma(\varphi)$ and $\Sigma^{(1)}$ are not renormalized, while $\Phi$ presented in Fig. 1 is renormalized. The reasonable agreement of the PNJL result with the LQCD one for $\sigma/\sigma_0$ and $\Sigma/\Sigma(1)$ presented in Fig. 2 may imply that $\sigma/\sigma_0$ and $\Sigma^{(1)}/\Sigma^{(1)}_{T_c}$ have better renormalization properties than $\sigma(\varphi)$ itself.

Finally, we discuss the sensitivity of $\Sigma^{(1)}$ to the parameters of the PNJL model. If $\Lambda$ is varied by 10 %, the pion decay constant at $T = 0$ is changed by about 20 %. Hence, one can change $\Lambda$ by only 0.1 % in order that the calculated pion decay constant is consistent with the observed one with 0.1 % error. This is also the case for $G_v$. If $\Lambda$ and $G_v$ are varied by 1 %, $\Sigma^{(1)}$ are changed by about 10 %, but the normalized quantity $\Sigma^{(1)}/\Sigma^{(1)}_{T_c}$ hardly changed near $T_c$. Further, $\Sigma^{(1)}$ near $T_c$ is much less sensitive to $G_{gs}$ and $G_v$, although these are nearly-free parameters. Thus, we can think that $\Sigma^{(1)}/\Sigma^{(1)}_{T_c}$ near $T_c$ changes little from the present values.

K.K. is supported by the Japan Society for the Promotion of Science for Young Scientists.

[1] E. Bilgici, F. Bruckmann, C. Gattringer, and C. Hagen, Phys. Rev. D 77, 094007 (2008).
[2] F. Bruckmann, C. Hagen, E. Bilgici, and C. Gattringer, Pos Lattice2008, 262 (2008); Pos Confinements, 054 (2008).
[3] E. Bilgici, F. Bruckmann, J. Danzer, C. Gattringer, C. Hagen, E. M. Ilgenfritz, and A. Maas, arXiv:0906.3957; E. Bilgici, PhD Thesis, University of Graz, Austria, 2009 (http://physik.uni-graz.at/itp/files/bilgici/dissertation.pdf).
[4] K. Fukushima, Phys. Rev. D 77, 034019 (2008).
[5] C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D 73, 014019 (2006).
[6] S. Rößner, C. Ratti, and W. Weise, Phys. Rev. D 75, 034007 (2007).
[7] P. Costa, M. C. Ruivo, C. A. de Sousa, H. Hansen, and W. M. Alberico, Phys. Rev. D 79, 116003 (2009).
[8] K. Kashiwa, H. Kouno, M. Matsuzaki, and M. Yahiro, Phys. Lett. B 662, 26 (2008); K. Kashiwa, M. Matsuzaki, H. Kouno, Y. Sakai, and M. Yahiro, Phys. Rev. D 79, 076008 (2009).
[9] Y. Sakai, K. Kashiwa, H. Kouno, and M. Yahiro, Phys. Rev. D 77, 051901(R) (2008); Phys. Rev. D 78, 036001 (2008); Y. Sakai, K. Kashiwa, H. Kouno, M. Matsuzaki, and M. Yahiro, Phys. Rev. D 78, 076007 (2008).
[10] Y. Sakai, K. Kashiwa, H. Kouno, M. Matsuzaki, and M. Yahiro, Phys. Rev. D 79, 096001 (2009).
[11] H. Abuki, R. Anglani, R. Gatto, G. Nardulli, and M. Ruggieri, Phys. Rev. D 78, 034034 (2008); H. Abuki, M. Ciminale, R. Gatto, and M. Ruggieri, Phys. Rev. D 79, 034021 (2009).
[12] T. Hell, S. Rößner, M. Cristoforetti, and W. Weise, Phys. Rev. D 79, 014022 (2009).
[13] K. Kashiwa, M. Yahiro, H. Kouno, M. Matsuzaki, and Y. Sakai, J. Phys. G 36, 105001 (2009).
[14] H. Kouno, Y. Sakai, K. Kashiwa, and M. Yahiro, J. Phys. G 36, 115010 (2009).
[15] S. i. Nam, arXiv:hep-ph/0905.3609.
[16] C. S Fischer, Phys. Rev. Lett 103, 052003 (2009); C. S Fischer, and J. A. Mueller, Phys. Rev. D 80, 074029 (2009).
[17] J. Braun, L. M. Haas, F. Marhauser, and J. M. Pawlowski, arXiv:hep-ph/0908.0008.
[18] F. Synatschke, A. Wipf, and K. Langfeld, Phys. Rev. D 77, 114018 (2008).
[19] K. Kashiwa, H. Kouno, T. Sakaguchi, M. Matsuzaki, and M. Yahiro, Phys. Lett. B 647, 446 (2007).
[20] P. de Forcrand and O. Philipsen, Nucl. Phys. B642, 290 (2002); Nucl. Phys. B673, 170 (2003).
[21] M. D’Elia and M. P. Lombardo, Phys. Rev. D 67, 014505 (2003); Phys. Rev. D 70, 074509 (2004); M. D’Elia, F. Di Renzo and M. P. Lombardo, Phys. Rev. D 76, 114509 (2007).
[22] H. S. Chen and X. Q. Luo, Phys. Rev. D 72, 034504 (2005); arXiv:hep-lat/0702025; L. K. Wu, X. Q. Luo, and H. S. Chen, Phys. Rev. D 76, 034505 (2007).
[23] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lütgemeier, and B. Petersson, Nucl. Phys. B469, 419 (1996).
[24] O. Kaczmarek, F. Karsch, P. Petreczky, and F. Zantow, Phys. Lett. B 543, 41 (2002).
[25] F. Karsch, E. Laermann, and A. Peikert, Nucl. Phys. B605, 579 (2001).
[26] G. Boyd, S. Gupta, F. Karsch, E. Laermann, B. Petersson, and K. Redlich, Phys. Lett. B 349, 170 (1995).
[27] O. Kaczmarek, and F. Zantow, Phys. Rev. D 71, 114510 (2005).
[28] Y. Aoki, Z. Fodor, S. D. Katz, and K. K. Szabó, Phys. Lett. B 643, 46 (2006).