Design of multiplierless minimum-phase FIR filters using palindromic polynomials and sharpening

G Jovanovic Dolecek and SA Salguero Luna
Department of Electronics
Institute INAOE, E Erro 1 72740 Tonantzintla, Puebla, Mexico

E-mail: gordana@ieee.org

Abstract. Minimum phase (MP) filters have all zeros inside the unit circle and have a minimum group delay and a minimum energy delay properties, especially useful in communication systems. The special class of MP filters has all zeros inside and on the unit circle. This paper presents a method for design of this class of MP FIR (Finite Impulse Response) digital filters without multipliers. The method is based on palindromic polynomials and sharpening technique. The coefficients of palindromic polynomials are presented as a sum of powers of two, and consequently can be implemented by using only adders and shifts, resulting in a multiplierless design. Two classes of palindromic polynomials, having all zeros on the unit circle, are considered. Next, the sharpening technique was applied to improve the magnitude characteristic and to move zeros inside of unit circle. The choice of the design parameters is also presented. All zeros, of the designed MP filter, are inside or on the unit circle. Method is illustrated with one example.

1. Introduction
Minimum phase (MP) filters have all zeros inside the unit circle. The MP filter can be either FIR (Finite Impulse Response) or IIR (Infinite Impulse Response) filters. We consider special class of MP filters, with the finite impulse response and all zeros either inside or on unit circle. MP filters have minimum group delay and minimum energy properties useful in communication systems. It is well known that unit sample response of a FIR filter with linear phase (LP), exhibits symmetry. The zeros of the filter are zeros of the palindromic polynomial and occur in complex-conjugate reciprocal quadruplets [1]. Each zero inside the unit circle has its reciprocal pair outside the unit circle. Each complex zero inside the unit circle has its complex conjugate zero inside the unit circle, and both zeros have the corresponding reciprocal zeros outside the unit circle. Each complex zero on the unit circle has only its complex conjugate pair. Therefore, in order to get a minimum phase filter from designed FIR filter it is necessary to move all zeros outside the unit circle to their reciprocal positions inside the unit circle.

The sharpening technique [2] is well known technique to simultaneously improve magnitude characteristic in both passband and stopband. However, it was shown in [3] that the sharpening technique can be also useful in design of minimum phase filters. We consider here method for MP filter design starting from the design of LP FIR multiplierless filters and using sharpening technique.
2. Designing multiplierless filters having all zeros on the unit circle

We consider two kinds of filters: comb filters and modified comb filters.

2.1. Comb filters

Comb filter has all its coefficients equal to unity. Its transfer function is given as

\[ H(z) = \left[ \sum_{n=1}^{M-1} z^{-n} \right]^{K} = \left[ \frac{1 - z^{-M}}{1 - z^{-1}} \right]^{K}, \]  

where \( K \) is number of the cascaded combs and \( M \) is comb length.

The magnitude response is given as:

\[ |H(e^{j\omega})| = \left| \frac{1}{M \sin(\omega/2)} \right|^K, \]  

where the scaling factor \( M \) is needed to get the magnitude characteristic \( H(e^{j0})=1 \).

Figure 1 shows the magnitude characteristic and pole-zero plot for comb with \( M=8, K=3 \).

![Pole-zero plot and magnitude response](image)

**Figure 1.** Comb filter.

Note that comb filter has low attenuation in stopband and the passband droop. In next subsection we introduce modified combs to improve stopband characteristic.

2.2. Modified combs

We consider the following modified combs:

\[ H_1(z) = 1 + 3/4 \sum_{n=1}^{M-2} z^{-n} + z^{-(M-1)}; \quad H_2(z) = 1 + 5/4 \sum_{n=1}^{M-2} z^{-n} + z^{-(M-1)}; \]  
\[ H_3(z) = 1 + 3/4 \sum_{n=1}^{M/2-2} z^{-n} + z^{-(M/2-1)}; \quad H_4(z) = 1 + 5/4 \sum_{n=1}^{M/2-2} z^{-n} + z^{-(M/2-1)}. \]  

The modified comb filters (3)-(4) are considered as multiplierless filters since the coefficients 3/4 and 5/4 can be presented using adders and shifts. All zeros of the filters (3) and (4) are on the unit circle. Figure 2 shows the pole zero-plots of the cascaded filters (3)-(4) and the magnitude response taking \( M=8 \).
The comb filter with $K=1$ (1) is cascaded with modified combs (3)-(4) with the same value of $M$:
\[ H_5(z) = H(z) \prod_{k=1}^{M} H_k(z). \]  
(5)

Figure 3 shows the pole-zero plot and magnitude response of filter (5).

Note that the attenuation in the stopband is increased. Further increasing can be obtained by increasing the comb parameter $K$. As expected, all zeros of the filter (5) are on the unit circle, and consequently the filter $H_5(z)$ is not a minimum phase filter. This problem is tackled in next subsection.

2.3. Using sharpening technique

We use simple sharpening polynomial $S_h\{ X \} = 2X^2 - X$ to improve the magnitude response of the filter $H_5(z)$:
\[ H_6(z) = S_h\{ H_5(z) \} = 2H_5^2(z) - H_5(z) = H_4(z) [ 2 - H_5(z) ]. \]  
(6)

As a difference to sharpening LP filter, where to the coefficient 2, in parenthesis of (6), is appropriately delayed, here no delay is applied. In that way all zeros of the expression in parenthesis are inside of unit circle, as shown in [3]

Figure 4 shows the pole-zero pattern of the filter $H_6(z)$ taking $M=8$ and $K=1$.

The resulting filter is narrowband MP filter and with a minimum attenuation of about 55 dB.
3. Proposed filter

Using the results from the previous section the transfer function of the proposed filter is given as

\[ H_6(z) = H_5(z) = 5h(5(z)) = 2H_5(z) - H_5(z) = H_5(z)[2 - H_5(z)], \]

where \( H_6(z) \) and \( H_5(z) \) are given in (6) and (1), (3) - (5), respectively.

The normalised stopband and passband edge frequencies are related with \( M \) as:

\[ \omega_s = 0.8 \times 2 / M, \quad \omega_p = 0.2 \omega_s. \]

From here we found the value of \( M \). If the result from (8) is not an integer, then the closest integer for \( M \) is chosen. The stopband edge frequency from (8) is chosen for a minimum stopband attenuation.

Method is illustrated in the following example:

Example 1: We design the MP filter with the normalized stopband frequency for \( \omega_s = 0.1 \). From (8) we get \( M = 16 \). Passband edge frequency is 0.02. Figure 5a shows pole-zero plot, while figure 5b presents magnitude response with the zooms in the passband and stopband.

References

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