Simulation studies of the beam cooling process in presence of heating effects in the Extra Low ENergy Antiproton ring (ELENA)

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ABSTRACT: The Extra Low ENergy Antiproton ring (ELENA) is a small synchrotron equipped with an electron cooler, which is currently being constructed at CERN to further decelerate antiprotons from the Antiproton Decelerator (AD) from 5.3 MeV to energies as low as 100 keV. At such low energies it is very important to carefully take contributions from electron cooling and beam heating mechanisms (e.g. on the residual gas and intrabeam scattering) into account. Detailed investigations into the ion kinetics under consideration of effects from electron cooling and heating sources have been carried out, and the equilibrium phase space dimensions of the beam have been computed, based on numerical simulations using the code BETACOOL. The goal is to provide a consistent explanation of the different physical effects acting on the beam in ELENA.

KEYWORDS: Low-energy ion storage; Accelerator modelling and simulations (multi-particle dynamics; single-particle dynamics); Beam dynamics
1 Introduction

The Extra Low ENergy Antiproton ring (ELENA) [1–4] is a new facility currently under construction at CERN to provide lower energy, higher quality and more abundant antiproton beams to all the experiments working at the Antiproton Decelerator (AD) [5–9], thus enabling the production of larger quantities of antihydrogen for antimatter research.

The AD is currently the only facility in the world providing antiprotons of low energy (5.3 MeV) for antihydrogen based experiments: ASACUSA, ALPHA, ATRAP, AEGIS, BASE [10]. Most of these experiments use degrader foils to further decelerate the antiprotons from the AD ejection kinetic energy (5.3 MeV) down to the kinetic energy needed for trapping (5 keV). The beam traversing the foil material experiences energy straggling which causes the loss of more than 99% of the antiprotons produced by the AD. It is also necessary to note that there are AD experiments not using degraders, e.g. ASACUSA, where the antiprotons are decelerated using an rf quadrupole (RFQ), and where about one order of magnitude higher trapping efficiencies are achieved with respect to degrader foils based experiments.

In this context, to improve the capture efficiency of the experiments, ELENA shall further decelerate the antiprotons injected from the AD at kinetic energy 5.3 MeV (momentum 100 MeV/c) to 100 keV (momentum 13.7 MeV/c), with a beam population of ~ 10^7 cooled antiprotons. Electron cooling will be used to counteract the emittance and the momentum spread blow-up caused by the deceleration process. The deceleration and the electron cooling in ELENA will bring about a 100-fold increase in the efficiency of the experiments (traps), and a 10-fold increase in the case of ASACUSA, and will also allow the possibility to accommodate an extra experimental area to investigate gravitation with antihydrogen (GBAR) [11].
A particular challenge for low energy storage rings, such as ELENA, is the question of achievable beam lifetime and stability. To address this question, we have investigated the long-term beam dynamics in ELENA considering different effects limiting the achievable phase space dimensions obtained under electron cooling. Concretely, we have performed simulations of the electron cooling process in the presence of important heating effects, such as rest gas scattering and Intra-Beam Scattering (IBS). These simulations consist of multiparticle tracking based on a Monte-Carlo method (model beam algorithm) of the code BETACOOL [12]. This code allows us to calculate the evolution of beam distributions under the action of cooling forces (in this case electron cooling) and different scattering effects, and has been successfully benchmarked against experimental data in different rings, see [13] and references therein.

This paper is organised as follows. Section 2 briefly introduces the ELENA optics layout used in our beam dynamics simulations. The ELENA cycle is described in section 3. The key element of ELENA, the electron cooler, is described in section 4. Emittance growth rates due to heating processes (rest gas interaction and IBS) are computed in section 5. BETACOOL based simulations are performed in section 6 to investigate the beam stability during electron cooling in the presence of rest gas and IBS phenomena, and the corresponding emittances and momentum spread at the equilibrium are computed.

2 The ELENA optics layout

The ELENA facility will be placed in a space inside the AD hall, and its nominal design consists of a compact hexagonal ring with a circumference of 30.4 m. It has been designed with two slightly longer straight sections without quadrupoles: one dedicated to beam injection, and another one prepared to host the electron cooler, including the corresponding orbit correctors and the compensation solenoids. There will be an extraction with a fast electrostatic deflector towards existing experiments (ASACUSA, ALPHA, ATRAP and AEGIS) and the new BASE experiment [10], and another extraction towards a new experimental area (GBAR experiment [11]). Here we focus on studies for the ELENA ring, the ELENA transfer lines are described elsewhere (see details in [4, 14]).

The ELENA lattice configuration is based on six sector bending magnets. Each bending magnet has an effective length of 0.93 m and provides a kick angle of \( \pi/3 \). The maximum magnetic rigidity is \( B \rho \approx 0.33 \) T·m (at injection energy) and the minimum rigidity is \( B \rho \approx 0.045 \) T·m (at extraction energy). The hexagonal lattice has been designed with a periodicity of 2 (the electron cooling section plus two standard sections, and the injection section plus two standard sections), with three families of quadrupoles (each of four members) to control the orbit transversally. Two skew quadrupoles are used for \( x-y \) coupling correction, and the chromaticity is counteracted using two families of sextupoles (each of two members).

The ELENA optics presents good tunability in the range \( 2 < Q_x < 2.5 \) and \( 1 < Q_y < 1.5 \). This tune range will allow it to operate with a nominal direct space charge tune shift \( \Delta Q \approx 0.1 \). The optics configuration of the ELENA ring is shown in figure 1 for the working point \( Q_x = 2.3 \) and \( Q_y = 1.3 \), obtained using the MAD-X program [15]. This optics provides the nominal transverse acceptance \( A_\perp = 75 \pi \) mm-mrad. The maximum betatron functions have the following values: \( \beta_x,\max \approx 12 \) m and \( \beta_y,\max \approx 6 \) m. The maximum first order horizontal dispersion is \( D_x,\max \approx 1.7 \) m.
In this paper this lattice structure has been used for the simulations. A more complete description of the ELENA optical features can be found in [4, 16].

3 The ELENA cycle

The ELENA beam cycle is shown in figure 2. The ramp up of the ELENA cycle will take between one and a few seconds. There are three principal plateaus: injection and two cooling plateaus. A first deceleration ramp of approximately 5 s is applied from the injection beam momentum 100 MeV/c (kinetic energy 5.3 MeV) down to the intermediate momentum 35 MeV/c (kinetic energy 0.65 MeV). During the deceleration process the beam emittances and momentum spread of the beam blow up adiabatically. Therefore, to counteract this blow-up electron cooling is applied at 35 MeV/c during approximately 8 s (first e-cooling plateau). A second deceleration process is applied for 3 s from 35 MeV/c to 13.7 MeV/c (kinetic energy 100 keV). At this final beam momentum, a second electron cooling process is applied for 2 s. In both cases the cooling is applied to a coasting antiproton beam. Finally the beam is bunched prior to ejection to the transfer lines between ELENA and the different experimental areas. Further cooling applied to the bunched beams at 13.7 MeV/c (for ~0.2–0.3 s) may be needed to counteract IBS effects and reduce the momentum spread of the short bunches required by the experiments [4]. The total ELENA cycle duration is expected to be ≈20 s. The repetition rate is limited by the AD repetition rate and is ≈100 s.

In the next sections we investigate the beam dynamics at the two electron cooling plateaus for a coasting antiproton beam. Some relevant input parameters for our simulation study are summarised in table 1, adopted from ref. [4]. For the case of cooling of a bunched beam prior to ejection, detailed studies are ongoing and will be presented in [17].
Figure 2. ELENA cycle scheme. Figure reproduced from ref. [4].

Table 1. ELENA parameters for the two cooling plateaus.

| Parameter                                | 1st cooling plateau | 2nd cooling plateau |
|------------------------------------------|---------------------|---------------------|
| Beam momentum [MeV/c]                    | 35                  | 13.7                |
| Initial momentum spread $\Delta p/p$     | 0.1%                | 0.05%               |
| Initial emittances $\epsilon_x, \epsilon_y$ (1σ) [π mm-mrad] | 8, 8               | 2.5, 2.5            |
| Beam intensity                           | $2.5 \times 10^7$  | $2.5 \times 10^7$  |
| Average beta function $\langle \beta_\perp \rangle$ [m] | 3                   | 3                   |
| Average dispersion $\langle D_x \rangle$ [m] | 1.2                 | 1.2                 |
| Transverse acceptance $A_\perp$ [π mm-mrad] | 75                  | 75                  |
| Vacuum pressure [Torr]                   | $3 \times 10^{-12}$ | $3 \times 10^{-12}$ |

4 ELENA cooling process

The electron cooling is a well consolidated technique to compensate the beam dimension growth of hadronic beams after deceleration to low energies as well as counteract the beam phase space growth due to heating effects such a residual gas scattering and IBS.

The electron cooling systems employed at low-energy coolers are typically based on an electron beam immersed in the longitudinal magnetic field of a solenoid. The ELENA electron cooler is described in detail in [4, 18], and its main parameters are summarised in table 2. The electron gun of the system has been designed to provide electron beams with transverse temperature $k_B T_{e\perp} < 0.1$ eV and longitudinal temperature $k_B T_{e\parallel} \lesssim 0.001$ eV (with $k_B$ the Boltzmann constant).

Let us consider a cylindrical transverse profile of the electron beam with a uniform charge density. For this case figure 3 depicts the friction force components as a function of the antiproton velocity, computed using the Parkhomchuk’s model [19] implemented in the BETACOOL code,
Table 2. ELENA electron cooler parameters.

| Parameter                                           | Value                                |
|-----------------------------------------------------|--------------------------------------|
| Beam momentum [MeV/c]                               | 35 – 13.7                            |
| Velocity factor \( \beta = v/c \)                   | 0.037 – 0.015                        |
| Electron beam energy [eV]                           | 355 – 55                             |
| Electron current \( I_e \) [mA]                     | 5 – 2                                |
| Electron beam density \( [m^{-3}] \)                | \( 1.38 \times 10^{12} – 1.41 \times 10^{12} \) |
| Magnetic field in the gun, \( B_{\text{gun}} \) [G] | 1000                                 |
| Magnetic field in the drift, \( B_{\text{drift}} \) [G] | 100                                 |
| Expansion factor                                     | 10                                   |
| Cathode radius [mm]                                 | 8                                    |
| Electron beam radius \( r_b \) [mm]                 | 25                                   |
| Twiss parameters [m]                                | \( \beta_{x,y} = 2.103, 2.186, D_s = 1.498 \) |
| Flange-to-flange length [m]                         | 1.93                                 |
| Solenoid length [m]                                 | 1.0                                  |
| Effective length (good field region) [m]            | 0.7                                  |
| Electron beam transverse temperature, \( k_B T_{e\perp} \) [eV] | 0.01                                |
| Electron beam longitudinal temperature, \( k_B T_{e\parallel} \) [eV] | 0.001                              |
| Transverse electron beam spread \( \theta_{e\perp} \) | 0.0038 – 0.0093                      |
| Longitudinal electron beam spread \( \theta_{e\parallel} \) | 0.0012 – 0.0029                      |

which is valid for magnetised electron distributions. This model is based in an empirical generalisation of the theoretical drag force for unmagnetised electron distributions. For \( k_B T_{e\perp} = 0.01 \) eV and \( k_B T_{e\parallel} = 0.001 \) eV, the force peak values are \( F_{\parallel} \simeq -0.0095 \) eV/m and \( F_{\perp} \simeq -0.00016 \) eV/m at the antiproton velocity \( v_i = 11000 \) m/s. If, for instance, the longitudinal electron temperature is reduced by one order of magnitude from \( k_B T_{e\parallel} = 0.001 \) eV to \( k_B T_{e\parallel} = 0.0001 \) eV, the force peak moves to \( v_i = 6000 \) m/s and its absolute value increases by approximately a factor 3. Although well known, it is worth mentioning that for antiproton or ion velocities below the peak the friction force scales proportional to \( v_i \) and above the peak it scales approximately as \( 1/v_i^2 \).

Detailed computation of the cooling process effects and the time cooling is usually performed by means of numerical simulations. For a rough estimate, at the order of magnitude level, of the cooling time some analytical expressions have been derived in the past. For instance, the cooling time in the transverse phases can be roughly estimated using the following approximate formula (see, e.g., [20–22] and references therein).

\[
\tau_c \approx k \frac{\pi e}{r_e r_p} \frac{A}{Z^2} \frac{\beta^4 \gamma^5}{\eta_e I_e} \frac{1}{\Lambda} \theta^3, \tag{4.1}
\]

where \( r_e \) is the classical electron radius, \( r_p \) the classical proton radius, \( A \) the atomic mass of the ion, \( Z \) the electric charge of the ion (antiproton in our case), \( r_b \) the electron beam radius, the \( \beta \) the relativistic velocity of the antiproton, and \( \gamma \) the Lorentz factor. The factor \( \eta_e \) is called duty
Figure 3. Longitudinal (left) and transverse (right) component of the cooling friction force as a function of the antiproton velocity, obtained using the code BETACOOL. The following cases of cold electrons are compared: $k_B T_{e\perp} = 0.01 \text{ eV}$, $k_B T_{e\parallel} = 0.001 \text{ eV}$; and $k_B T_{e\perp} = 0.01 \text{ eV}$, $k_B T_{e\parallel} = 0.0001 \text{ eV}$.

Table 3. Transverse cooling rates computed by means of BETACOOL simulations and analytically for the two cooling plateaus.

| Method                  | $p = 35 \text{ MeV/c}$ | $p = 13.7 \text{ MeV/c}$ |
|-------------------------|-------------------------|-------------------------|
| BETACOOL $1/\tau_x$, $1/\tau_y$ [s$^{-1}$] | -0.193, -0.184          | -1.207, -1.168          |
| Analytical $1/\tau_c$ [s$^{-1}$] | -0.386                  | -1.025                  |

cycle factor and is defined as the fraction of the circumference occupied by the cooling system ($\eta_c = 0.023$, 0.7 m over the total 30.4 m circumference at ELENA). $I_e$ represents the electron beam intensity in the cooler, and $\Lambda$ is the so-called Coulomb logarithm ($\Lambda \approx 10$ in our case). Here $k$ is a numerical factor given by the kind of distribution of the electron velocities: $k = 0.6$ for Maxwellian distributions, and $k = 0.16$ for flattened distributions. The term $\theta$ is the rms angular spread between the ions (antiprotons) and the electrons: $\theta_{\perp} = \sqrt{\theta_{i\perp}^2 + \theta_{e\perp}^2}$, where the ion angle has the contribution $\theta_{i\perp} = \sqrt{\epsilon / \beta_{i\perp}}$, with $\epsilon$ the transverse emittance (horizontal or vertical; here $\epsilon = \epsilon_x = \epsilon_y$ is assumed) of the ion (antiproton) beam and $\beta_{\perp}$ the betatron function of the corresponding transverse phase at the e-cooler position. The electron angle spread contribution is determined by $\theta_{e\perp} = \sqrt{k_B T_{e\perp} / (m_e c^2 \beta_{e\perp}^2)}$, with $k_B T_{e\perp}$ the transverse temperature of the electron beam in units of eV. It is necessary to mention that here all the magnitudes are being calculated in the laboratory reference frame. For the simulations let us assume a pessimistic case where the electron gun of the ELENA cooler produces an electron beam with temperatures $k_B T_{e\perp} \approx 0.01 \text{ eV}$ and $k_B T_{e\parallel} \approx 0.001 \text{ eV}$.

Table 3 compares the cooling rate $1/\tau_c$ calculated analytically from eq. (4.1) with the result from BETACOOL for the immediate rates (at the beginning of the cooling process), assuming a flattened velocity distribution for the electron beam of the cooler ($k = 0.16$). Here we follow the usual convention: cooling or damping rates are generally written with negative sign and heating rates are written with positive sign.
5 Heating processes

In this section the effects on the long term beam dynamics from important sources of beam heating are analysed: the interaction of the beam particles with the molecules of the residual gas and the IBS. Additional sources originated by nonlinearities of the machine optics, such as tune resonances, are not considered here.

5.1 Rest gas scattering

The emittance growth due to rest gas scattering is usually considered to be caused mainly by Multiple Coulomb Scattering (MCS). Following ref. [23], the rms transverse emittance ($1\sigma$) growth rate due to MCS on residual gas is given by:

$$\frac{d\epsilon_{\text{rms}}}{dt} = 2\pi \langle \beta_\perp \rangle n_{\text{ms}} \ln \left( \frac{280}{\alpha} \right) r_e^2 \left( \frac{m_e c^2}{\beta c p^2} \right)^2,$$  \hspace{1cm} (5.1)

with $m_e$ the electron rest mass, $r_e$ the classical electron radius, $c$ the speed of light, $\alpha$ the fine structure constant, $\beta = v/c$ the relativistic velocity factor of the beam and $p$ the beam momentum. Here $\langle \beta_\perp \rangle = 1/2 (\langle \beta_x \rangle + \langle \beta_y \rangle)$ represents the average betatron function over the ring, and $n_{\text{ms}}$ is the multiple scattering density given by the following expression:

$$n_{\text{ms}} = \sum_i n_i Z_i^2 \ln \left( \frac{280}{\alpha (AZ_i)^{1/3}} \right) \ln \left( \frac{280}{\alpha} \right),$$  \hspace{1cm} (5.2)

with $n_i$ the density of each $(i)$ gas component in the vacuum pipe.

In eq. (5.1), notice that the rate of emittance blow-up due to scattering on the residual gas is independent of the beam intensity and the initial emittance. However, it is strongly momentum dependent ($\propto 1/p^2$), and increases rapidly at lower momenta.

In BETACOOL, growth rates due to the scattering of antiprotons or ions on the residual gas are calculated by the same methods as for a thin internal target such as a gas jet, applying MCS. The residual gas model is composed of gas cell targets which are distributed along the whole circumference of the ring. Then, BETACOOL applies a Monte-Carlo method to simulate the MCS process on those cell targets and the beam distribution evolution. The rest gas heating rates are integrated over the whole lattice structure and lattice functions are used from each optical element.

To evaluate the rest gas scattering effects in ELENA, first we have performed BETACOOL simulations assuming only the heating effect of the rest gas and no cooling process. These simulations have been performed considering 1000 model particles and 10 random seeds. The vacuum pressure has been set to the nominal value $3 \times 10^{-12}$ Torr for a pessimistic case where the rest gas consists of 100% N$_2$ molecules. Figures. 4 and 5 compare the emittance growth results from BETACOOL simulations and analytical calculations using the expression (5.1) for the case of the two cooling plateaus. The results are summarised in table 4. The BETACOOL values are in good agreement with the analytical formula.

Nevertheless, it is necessary to mention that the ELENA cycle will last approximately 20 s, and during this time the interaction rate with residual gas molecules has been computed to be approximately one interaction with a rest gas molecule on the intermediate plateau (first cooling), and significantly less than one interaction along the low energy plateau (second cooling) [24].
Figure 4. Horizontal (left) and vertical (right) emittance evolution due to rest gas scattering for an ELENA antiproton beam of $p = 35 \text{ MeV}/c$. The blue line represents the average curve over 10 realisations simulated with BETACOOL. The red line is the analytical estimate. No cooling is applied.

Figure 5. Horizontal (left) and vertical (right) emittance evolution due to rest gas scattering for an ELENA antiproton beam of $p = 13.7 \text{ MeV}/c$. The blue line represents the average curve over 10 realisations simulated with BETACOOL. The red line is the analytical estimate. No cooling is applied.

Table 4. Emittance growth rate results for ELENA due to rest gas scattering with vacuum pressure $3 \times 10^{-12}$ Torr and assuming 100% N$_2$ residual gas.

| Model                     | $d\epsilon_{rms}/dt$ [\text{\mu m/s}] (at $p = 35 \text{ MeV}/c$) | $d\epsilon_{rms}/dt$ [\text{\mu m/s}] (at $p = 13.7 \text{ MeV}/c$) |
|---------------------------|---------------------------------------------------------------|---------------------------------------------------------------|
| BETACOOL                  | 0.015                                                         | 0.21                                                          |
| Analytical formula, eq. (5.1) | 0.013                                                         | 0.21                                                          |

Therefore, it has recently been pointed out in [24] that in the nominal ELENA cycle the scattering on rest gas molecules is not a multiple scattering phenomenon, but rather a single scattering one. However, although the MCS formalism implemented in BETACOOL may be overestimating the emittance increase due to rest gas, the BETACOOL simulations are still appropriate for estimates in this case, since they will result in safer conservative vacuum pressure tolerable limits. At this point, for a better understanding of the rest gas scattering effects in ELENA, further studies are needed and future benchmarking with experimental measurements will be extremely useful.
Intrabeam scattering

Intra-Beam Scattering (IBS) is a beam heating effect produced by multiple small-angle Coulomb scatterings of charged particles within the accelerator beam itself. It causes an exchange of energy between the transverse and longitudinal degree of freedom, leading to the growth of the beam phase space dimensions. This phenomenon places fundamental limitations to the achieving of ultra-small beam emittances in storage rings.

The theory of IBS is extensively described in several publications [25–30]. Piwinski [25] developed the IBS theory of weak focusing machines (smooth lattice approximation) where the lattice functions are constant. Later this theory was extended for strong focusing machines, taking into account the variation of the betatron functions and the dispersion function along the lattice, and is described in reports by Martini [27] and Piwinski [28]. A different approach to IBS was followed by Bjorken and Mtingwa [26] to evaluate emittance growth rates using the scattering matrix formalism from quantum electrodynamics, but making approximations valid for ultrarelativistic beams only. Both the Bjorken-Mtingwa’s and Matini’s model require the evaluation of the formulas at each element of the lattice along the ring. It is worth mentioning that in order to save computational time, some other approximations have been formulated in the limit of high beam energy, see, e.g., refs. [29, 30]. For low and medium energies, the Bjorken-Mtingwa theory was adapted to include non-ultrarelativistic corrections, resulting in the so-called Martini-Conte formulae [31].

Many of the IBS models mentioned above are implemented in the code BETACOOL, and have been benchmarked with one another and with experimental results in the past. For instance, measurements of IBS in hadron machines at high energy agree with accuracy better than 50% with the Bjorken-Mtingwa’s and Matini’s models [32]. In the case of hadron machines at low and medium energy, reasonable agreements have been found between the Martini’s model (extended Piwinski) and the Martini-Conte model with experimental measurements (see, e.g., [31]).

In our BETACOOL simulations we use the Martini’s model, thus taking into account the lattice derivatives. The IBS growth rates are then calculated in accordance with this model using the ring lattice functions imported from an output file of the ELENA lattice in the format of the MAD-X program.

Let us introduce briefly the IBS theory to illustrate the growth rate dependences on the beam parameters. Following both Piwinski’s and Martini’s models, the growth rates are defined by:

\[
\frac{1}{\tau_x} = \frac{1}{2\epsilon_x} \frac{d\epsilon_x}{dt} = A_0 \left\langle f \left( \frac{1}{\bar{a}} \frac{\hat{b}}{\bar{a}} \frac{\hat{c}}{\bar{a}} \right) \rightangle + D_x^2 \sigma_h^2 \frac{\beta_x}{\beta_y} f(\bar{a}, \hat{b}, \hat{c}) \right\rangle,
\]

\[
\frac{1}{\tau_y} = \frac{1}{2\epsilon_y} \frac{d\epsilon_y}{dt} = A_0 \left\langle f \left( \frac{1}{\bar{b}} \frac{\hat{a}}{\bar{b}} \frac{\hat{c}}{\bar{a}} \right) \rightangle + D_y^2 \sigma_h^2 \frac{\beta_y}{\beta_x} f(\bar{a}, \hat{b}, \hat{c}) \right\rangle,
\]

\[
\frac{1}{\tau_p} = \frac{1}{2\sigma_p^2} \frac{d\sigma_p^2}{dt} = n A_0 \left\langle \frac{\sigma_h^2}{\sigma_p^2} f(\bar{a}, \hat{b}, \hat{c}) \right\rangle,
\]

where \( n = 1 \) for bunched beams and \( n = 2 \) for coasting beams; \( \beta_{x,y} \) indicates the lattice betatron functions and \( D_{x,y} \) the first order dispersion functions; \( \sigma_p \) is the relative momentum spread assu-
ming a Gaussian momentum distribution; the term $\sigma_h$ is given by:

$$\frac{1}{\sigma_h^2} = \frac{1}{\sigma_p^2} + \frac{D_x^2}{\beta_x \epsilon_x} + \frac{D_y^2}{\beta_y \epsilon_y}. \quad (5.4)$$

The function $f(a, b, c)$ is the so-called Piwinski scattering function [33]:

$$f(\tilde{a}, \tilde{b}, \tilde{c}) = 8\pi^2 \int_0^1 \left[2\ln \left( \frac{\tilde{c}}{2} \left( \frac{1}{\sqrt{P}} + \frac{1}{\sqrt{Q}} \right) \right) - 0.5772 \right] \frac{(1-3u^2)}{\sqrt{PQ}} \mathrm{d}u, \quad (5.5)$$

where $P = \tilde{a}^2 + (1 - \tilde{a}^2)u^2$ and $Q = \tilde{b}^2 + (1 - \tilde{b}^2)u^2$. The terms $\tilde{a}, \tilde{b}$ and $\tilde{c}$ are given by:

$$\tilde{a} = \frac{\sigma_h}{\gamma \sqrt{\beta_x / \epsilon_x}},$$

$$\tilde{b} = \frac{\sigma_h}{\gamma \sqrt{\beta_y / \epsilon_y}},$$

$$\tilde{c} = \sigma_h \beta \sqrt{2d / r_p}, \quad (5.6)$$

where $d$ is the minimum of the horizontal and vertical beam radii, $r_p$ is the classical proton radius and $\beta$ is the velocity factor. In eq. (5.3) the brackets $\langle ... \rangle$ indicate average, and the factor $A_0$ is given by:

$$A_0 = \frac{r_p^2 c}{32\pi \sqrt{\pi} \beta^3 \gamma^4 \epsilon_x \epsilon_y \sigma_p} \cdot \lambda, \quad (5.7)$$

where

$$\lambda = \begin{cases} N/C & \text{for coasting beams}, \\ N_b/(2\sqrt{\pi}\sigma_s) & \text{for bunched beams}, \end{cases} \quad (5.8)$$

with $N$ the total number of particles of the beam, and $N_b$ the number of particles in a bunch (for a bunched beam), $c$ the speed of light, $\sigma_s$ the rms bunch length (relevant for bunched beams), and $C$ the circumference of the storage ring.

In the context of ELENA and to compare with the BETACOOL simulation results only switching on the IBS effect, we have also calculated the IBS growth rates using the IBS module of MAD-X [34, 35]. This module is based on the Bjorken-Mtingwa model including also non-ultrarelativistic corrections and vertical dispersion. The non-ultrarelativistic terms are crucial to estimate IBS effects in ultra-low energy machines. It is necessary to take into account the following considerations when computing IBS growth rates for the case of a coasting beam using MAD-X:

- A limitation of the MAD-X IBS module is that it calculates the rms emittance growth rates assuming Gaussian bunched beams only.

- Therefore, in order to apply it to the case of coasting beams, $\sigma_s$ must be replaced by the circumference of the storage ring $C$, and the MAD-X output growth rates must be multiplied by a factor $2\sqrt{\pi}$ (see eqs. (5.7) and (5.8)).

Table 5 compares the IBS growth rate results between the codes BETACOOL (Martini model) and MAD-X (IBS module based on the Bjorken-Mtingwa model with non-ultrarelativistic corrections), using the beam parameters of table 1 for the two cooling plateaus. Coasting antiproton
Table 5. Comparison of IBS growth rate results between the codes BETACOOL and MAD-X for the two ELENA cooling plateaus. These values are immediate growth rates, i.e. the initial growth rates before evolution.

| Codes    | $1/\tau_x$ [s$^{-1}$] | $1/\tau_y$ [s$^{-1}$] | $1/\tau_p$ [s$^{-1}$] |
|----------|------------------------|------------------------|------------------------|
| BETACOOL | $1.687 \times 10^{-4}$ | $-2.521 \times 10^{-5}$ | $8.299 \times 10^{-4}$ |
| MAD-X    | $1.295 \times 10^{-4}$ | $-4.253 \times 10^{-5}$ | $4.411 \times 10^{-4}$ |

beams are assumed. A good agreement is obtained between the two codes, including agreement on the sign of the rates. They yield a negative vertical rate $1/\tau_y$, which indicates that in the vertical plane there is a certain amount of shrinkage or damping, as energy is transferred to other degrees of freedom. Taking into account the dependence of the factor $A_0$ on the beam parameters (eq. (5.7)), the difference in order of magnitude between the two cooling plateaus is approximately consistent with the expected ratio

\[
\frac{1/\tau_{x,y,p}}{1/\tau_{x,y,p}} \approx \frac{(A_0)_{35\text{ MeV/c}}}{(A_0)_{13.7\text{ MeV/c}}} = \frac{\left(\beta^3\epsilon_x\epsilon_y\sigma_p\right)_{35\text{ MeV/c}}}{\left(\beta^3\epsilon_x\epsilon_y\sigma_p\right)_{13.7\text{ MeV/c}}} \approx 0.003, \tag{5.9}
\]

where we have used the beam parameters of table 1 for the initial growth rates before the cooling evolution, and the fact that $\gamma \approx 1$ at low energies.

6 Beam stability simulations

In this section we perform complete simulations of the ELENA beam dynamics at the two cooling plateaus using the code BETACOOL under realistic conditions and taking into account both heating and cooling effects: rest gas, IBS and the cooling process. IBS effects are computed using the formulae from the so-called Martini model [27], using the optics of figure 1 as an input. For the electron cooler the characteristics described in section 4 have been considered. The equilibrium emittance is obtained by solving the differential equation:

\[
\left(\frac{d\epsilon}{dt}\right)_{\text{gas}} + \left(\frac{d\epsilon}{dt}\right)_{\text{IBS}} + \left(\frac{d\epsilon}{dt}\right)_{\text{cool}} = 0,
\]

where $(d\epsilon/dt)_{\text{gas}}$, $(d\epsilon/dt)_{\text{IBS}}$ and $(d\epsilon/dt)_{\text{cool}}$ represent the emittance changing rates for the three processes considered: rest gas scattering, IBS and electron cooling, respectively.

The ELENA vacuum pipe will be coated with Non-Evaporable Getter (NEG) films. These type of vacuum techniques are also being used in other rings such as the Low Energy Ion Ring (LEIR) [36]. Therefore, for ELENA it is reasonable to expect a residual gas composition similar to that measured in LEIR [37]: 95% H$_2$, 2% CO, 2% CO$_2$ and 1% CH$_4$, with a total gas density (at room temperature) of $9.6 \times 10^{10}$ m$^{-3}$.

First let us consider the nominal ELENA parameters for the above realistic rest gas scenario at the nominal vacuum pressure $3 \times 10^{-12}$ Torr. In order to illustrate the cooling effectiveness,
Figure 6. Left: transverse and momentum spread distribution for different times during the first cooling of the ELENA cycle ($p = 35$ MeV/c). Right: transverse and momentum spread distribution for different times during the second cooling of the ELENA cycle ($p = 13.7$ MeV/c). In the horizontal axis the units are scaled to the initial rms size of the distribution at the beginning of the cooling process (red curve).

Figure 6 shows the transverse beam profile time evolution for both the antiproton beam momentum $p = 35$ MeV/c and $p = 13.7$ MeV/c. Initial Gaussian distributions of the antiproton beam in the three phase spaces are considered in these simulations.\(^1\)

For the case of 35 MeV/c, where the cooling starts from an rms emittance $\epsilon_{x,y} = 8 \pi$ mm-mrad, the beam distribution quickly deviates from a Gaussian profile and a very dense core appears. At $t = 4$ s the core peak is already well defined. However, the antiprotons with large amplitude are cooled slowly and the tails are highly populated even after 8 s cooling. The development of this

\(^1\)In practice, typical AD beams injected to ELENA from the AD might have a non-Gaussian distribution with a compact core and extended tails [38, 39]. However, studies in the AD have confirmed the feasibility of adjusting the electron cooling by making a small misalignment between the electron and antiproton beams to modify the beam profiles to be Gaussian-like [1].
core-tail particle distribution can be explained if we take into account the relatively large initial beam size considered in this simulation with respect to the electron cooler diameter. Figure 7 shows the distribution of modeled antiprotons in the space $x$–$\Delta p/p$ at the beginning ($t = 0$ s) and at the end ($t = 8$ s) of the first cooling plateau. The parabolic momentum spread of the electrons due to space charge is also represented in figure 7. Antiprotons in the centre of the beam distribution cool faster than those in the tails, which is due to the strength of the electron-antiproton interaction as a function of the relative velocity. In other words, because of the space charge parabolic velocity distribution of the electrons, beam particles at large amplitudes experience a weaker friction force than in the core, and hence a core-tail distribution develops. This can be also illustrated with the invariant plots in figure 8 (A), where comparing the core parameters ($\approx 68\%$ of the beam population) with the $95\%$ parameters one can see the formation of a very dense core and a long tail highly populated. This feature agrees with results obtained by previous studies in ref. [18].

It is necessary to remark that in the case of cooling with large initial beam sizes, where the distribution quickly deviates from a Gaussian, the use of the IBS Martini model is probably underestimating the IBS effect for the core, thus generating a high phase space density of the core, as observed in figure 7 (A), which looks unphysical and could be a simulation artefact. As pointed out in [40], the standard models of IBS, such as the Martini model, are based on the growth of the rms beam parameters of a Gaussian distribution. However, in the case of a non-Gaussian beam distribution, it would be more realistic to apply IBS induced kicks based on diffusion coefficients which are different for particles inside and outside of the core. At this point, further investigation is needed. For a better understanding, we are planning to repeat the simulations for ELENA at 35 MeV/$c$ applying a core-tail IBS model adapted to the rms parameters growth of bi-Gaussian distributions [40] and compare the results with the ones presented here [17].

At the end of the first cooling plateau, after cooling for 8 s, the following rms emittances have been obtained: $\epsilon_{x,y} = 1.1 \, \pi$ mm-mrad. Decelerating from 35 MeV/$c$ to 13.7 MeV/$c$ (see ELENA...
Figure 8. Invariant distribution for the ELENA phase space dimensions: longitudinal, \((\Delta p/p)^2\) (green); horizontal, \(\epsilon_x\) (red); and vertical, \(\epsilon_y\) (blue). The cases for the first cooling plateau (A) and the second cooling plateau (B) are represented.

Table 6. Transverse emittances and momentum spread of the antiproton beam before and after the electron cooling process for the two cooling plateaus.

| Step in cycle          | \(\epsilon_x, \epsilon_y \) [\(\pi\) mm-mrad] (rms) | \(\Delta p/p \) [%] (rms) |
|------------------------|-----------------------------------------------------|---------------------------|
| Start 1st plateau (35 MeV/c) | 8.0, 8.0                                             | 0.1                       |
| End 1st plateau (35 MeV/c)    | 1.1, 1.1                                             | 0.02                      |
| Start 2nd plateau (13.7 MeV/c) | 2.8, 2.8                                             | 0.05                      |
| End 2nd plateau (13.7 MeV/c)    | 0.5, 0.3                                             | 0.03                      |

cycle in section 3), the physical emittance increases by a factor \((\beta \gamma)_{35 \text{ MeV}/c}/(\beta \gamma)_{13.7 \text{ MeV}/c} \simeq 2.55\) (considering only the adiabatic increase due to deceleration, without blow-up due to IBS and other effects). Therefore, we could consistently adopt initial rms emittances \(\epsilon_{x,y} = 2.8 \, \pi \text{ mm-mrad}\) at the start of the second cooling plateau.

For the case of the second cooling plateau (at 13.7 MeV/c), after 1 s cooling the beam reaches the equilibrium, and the cooling is more homogeneous for both core and tails (figure 6 (right)). Furthermore, it is interesting to mention the asymmetry observed in the momentum spread distribution \((\Delta p/p)\) because of the influence of the space charge of the electron beam.

Table 6 summarises the values of emittances and momentum spread before and after the electron cooling. These values are the result of the average over 10 random seeds simulated for the evolution of the beam distribution in time. For example, the time evolution of \(\Delta p/p\) is illustrated in figure 9 for the two cooling plateaus. Notice that for the case of the first cooling plateau \((p = 35 \text{ MeV}/c)\), considering the initial parameters of table 1 \((\epsilon_{x,y} = 8 \, \pi \text{ mm-mrad}, \Delta p/p = 0.1\%)\), the equilibrium is not reached yet after 8 s of cooling. In these conditions, reaching the equilibrium state would require \(\sim 10\) s. However, as mentioned before, here we are using pessimistic parameters for the starting input phase space parameters and, in principle, smaller initial values \((\epsilon_{x,y} \sim 1 \, \pi \text{ mm-mrad} \text{ and } \Delta p/p \sim 0.05\%)\) are expected to be affordable in the first cooling plateau of ELENA [41], for which the equilibrium can be reached within 8 s cooling.
6.1 Vacuum pressure tolerance

As mentioned in the previous section, the ELENA vacuum system is based on the NEG-coating technology [36], and a typical outgassing profile of this kind of system is expected: 95% H₂, 2% CO, 2% CO₂ and 1% CH₄. A nominal ultra-high vacuum pressure of $P = 3 \times 10^{-12}$ Torr has been established for the ELENA design [4]. In principle, this value was selected as a safe level to guarantee a sufficiently long antiproton beam lifetime and a minimised beam phase space blow-up due to scattering of the antiprotons on the molecules of the residual gas.

Here, by means of numerical simulations, we review the rest gas effects on the ELENA beam dynamics to determine more precise tolerance limits. A scan over a wide range of vacuum pressure levels is performed, assuming the above realistic residual gas composition in the vacuum chamber.

Figure 10 shows the transverse emittances and momentum spread evolution for different values of gas pressure for the case of the first cooling plateau ($p = 35 \text{ MeV}/c$). Each curve represents the average over 10 simulated random seeds for the evolution of a distribution of 1000 modelled particles using the model beam algorithm of BETACOOL. In addition to the rest gas, electron cooling and IBS are switched on. From figure 10 one can observe that there is practically no difference when increasing the pressure by one order of magnitude from $3 \times 10^{-12}$ Torr to $1 \times 10^{-11}$ Torr. However, when increasing the pressure to $P = 5 \times 10^{-10}$, after 8 s of cooling, the emittances are around 90% larger and the momentum spread around 20% larger than for the case of the nominal pressure. It can be seen that the increase of the beam dimensions starts to be significant for $P > 5 \times 10^{-10}$ Torr, and the electron cooling is not effective anymore in reducing the emittances for $P > 1 \times 10^{-9}$ Torr.

In a similar way, figure 11 shows the beam phase space dimension evolution for different values of gas pressure for the case of the second cooling plateau ($p = 13.7 \text{ MeV}/c$). As before, practically there is no sensitivity to rest gas when increasing the pressure up to $\sim 1 \times 10^{-11}$ Torr. For $P > 2 \times 10^{-10}$ Torr the rest gas dominates over cooling and there is a dramatic increment of the emittances and momentum spread.

In view of these results, one could conclude that the ELENA vacuum pressure requirements could be relaxed from $P \sim 10^{-12}$ Torr to $P \sim 10^{-11}$ Torr.
Figure 10. Evolution of the rms horizontal (top left) and vertical (top right) emittance and the rms momentum spread (bottom) over the first cooling ($p = 35 \text{ MeV}/c$) for different values of gas pressure. Here a realistic gas composition for ELENA (95% H$_2$, 2% CO, 2% CO$_2$ and 1% CH$_4$) has been assumed.

6.2 Intrabeam scattering effects

In order to investigate the IBS effects in ELENA we compute the dependence on bunch charge of the equilibrium transverse emittances and the relative energy spread, considering the cooling process in presence of IBS. While electron cooling tends to decrease these quantities, IBS tends to increase them.

This study will provide us with scaling rules to predict the variation of the phase space dimensions with the change of beam intensity in ELENA: $\epsilon_{x,y} \propto N^{\kappa_{x,y}}$, $\Delta p/p \propto N^{\kappa_p}$, with $N$ the number of antiprotons in the beam and $\kappa_{x,y,p}$ power factors.

Figure 12 shows the equilibrium transverse rms emittances and rms momentum spread as a function of the beam intensity for the case of beam momentum $p = 35 \text{ MeV}/c$, after 12 s cooling. Each point represents the average over 40 simulation realisations with BETACOOL. The following processes have been switched on: electron cooling, rest gas and IBS. From the curve fitting the following dependences have been obtained for the case $p = 35 \text{ MeV}/c$:

$$
\epsilon_x [\pi \text{ mm} \cdot \text{mrad}] = 6.895 \times 10^{-7} \cdot N^{0.684} + 0.344,
\epsilon_y [\pi \text{ mm} \cdot \text{mrad}] = 1.038 \times 10^{-9} \cdot N^{0.958} + 0.325,
\Delta p/p = 9.708 \times 10^{-9} \cdot N^{0.501} + 1.357 \times 10^{-4}. \tag{6.1}
$$

For this case, the dependence of the equilibrium emittances is approximately linear, $\epsilon_x \propto N^{0.7}$ and $\epsilon_y \propto N$. However, notice from figure 12 that $\epsilon_y$ at the equilibrium is practically insensitive to
Figure 11. Evolution of the rms horizontal (top left) and vertical (top right) emittance and the rms momentum spread (bottom) over the second cooling \((p = 13.7 \text{ MeV}/c)\) for different values of gas pressure. Here a realistic gas composition for ELENA (95\% H\(_2\), 2\% CO, 2\% CO\(_2\) and 1\% CH\(_4\)) has been assumed.

the \(N\) variation for \(N \lesssim 1 \times 10^8\). The equilibrium momentum spread \(\Delta p/p\) increases proportionally to \(N^{1/2}\).

The IBS effect becomes important if the nominal beam intensity \((N \approx 2.5 \times 10^7)\) is increased by approximately one order of magnitude. For instance, increasing the beam intensity from \(2.5 \times 10^7\) to \(2.5 \times 10^8\) translates into a horizontal emittance growth of about 70\%, a vertical emittance growth of about 30\% and a momentum spread increase of about 60\%. In the range of interest for ELENA, expected to be operating in the range \((1–3) \times 10^7\) stored antiprotons, the equilibrium beam parameter variation is very small, and the main limitation of the beam intensity comes from the incoherent tune shift due to direct space charge forces.\(^2\)

Proceeding in a similar way for the case of beam momentum \(p = 13.7 \text{ MeV}/c\), figure 13 shows the equilibrium transverse rms emittance and rms momentum spread as a function of the antiproton beam intensity, after 2 s cooling. In this case, since we are dealing with lower energy and smaller initial beam dimensions one expects a stronger impact of the IBS effects than in the first cooling plateau (remember that the IBS growth rates \(\propto 1/(\gamma^4 \beta^3 \epsilon_x \epsilon_y \sigma_p)\) from eq. (5.7)). Indeed, figure 13 shows a significant blow-up of the beam dimensions due to IBS when increasing \(N\). For the case \(p = 13.7 \text{ MeV}/c\) the following dependences have been obtained:

\[
\epsilon_x [\pi \text{ mm} \cdot \text{mrad}] = 3.878 \times 10^{-5} \cdot N^{0.546} + 0.104,
\]

\(^2\)Assuming an optimistic case of 100\% deceleration efficiency for an extracted beam from the AD, the beam intensity stored in ELENA can be \(\approx 3 \times 10^7\). Preliminary calculations have estimated that operating in the range \((1–3) \times 10^7\) stored antiprotons will result in a space tune shift \(\Delta Q \approx 0.1\) [4].
Figure 12. Horizontal equilibrium emittance (top left), vertical equilibrium emittance (top right) and momentum spread at the equilibrium (bottom) as a function of the number of stored antiprotons. The points represent the BETACOOL simulation results averaged over 40 machine realisations and the corresponding standard deviation is indicated by error bars. The solid line is the fit to these points. This simulation corresponds to the cooling plateau at $p = 35\,\text{MeV}/c$.

$$
\begin{align*}
\epsilon_{x}[\pi\,\text{mm} \cdot \text{mrad}] &= 5.455 \times 10^{-6} \cdot N^{0.614} + 0.179, \\
\Delta p/p &= 1.565 \times 10^{-7} \cdot N^{0.4} + 2.045 \times 10^{-4}.
\end{align*}
$$

In eq. (6.2) the factors of proportionality are approximately two orders of magnitude higher than for the previous case, although the exponents are smaller: $\epsilon_{x} \propto N^{0.5}$,  $\epsilon_{y} \propto N^{0.6}$ and $\Delta p/p \propto N^{0.4}$.

During the future operation of the machine, it will be worth performing measurements to study the $\Delta p/p$ and $\epsilon_{x, y}$ variation with the beam intensity in the available range of intensity. The momentum spread can be determined by Schottky noise analysis, and the emittance can be reconstructed using a beam scraper to determine the transverse beam size.
Figure 13. Horizontal equilibrium emittance (top left), vertical equilibrium emittance (top right) and momentum spread at the equilibrium (bottom) as a function of the number of stored antiprotons. The points represent the BETACOOL simulation results averaged over 40 machine realisations and the corresponding standard deviation is indicated by error bars. The solid line is the fit to these points. This simulation corresponds the cooling plateau at $p = 13.7 \text{ MeV}/c$.

7 Summary and discussion

The ELENA decelerator ring at CERN will further decelerate antiprotons injected from the AD from the momentum 100 $\text{MeV}/c$ to 13.7 $\text{MeV}/c$. This will improve the trapping efficiency of the AD antimatter experiments by about a factor 10–100 (depending on the experiment) and also make new experiments possible, such as the research of gravitation with antihydrogen at rest (GBAR experiment) [11].

For a better understanding of the long term beam dynamics in the ELENA ring, we have started a detailed investigation into the different cooling and heating processes which determine the beam lifetime and stability of the ELENA antiproton beam. Concretely, in this paper we have put special emphasis on the study of the residual gas scattering and the IBS during the two electron cooling plateaus (at 35 $\text{MeV}/c$ and at 13.7 $\text{MeV}/c$) of the ELENA cycle for coasting antiproton beams. This study has been performed by means of multiparticle tracking simulations using the model beam algorithm of the code BETACOOL [12]. Emittance growth rates have been computed numerically for each process and compared with analytical calculations, obtaining a reasonable agreement. The ultimate validation of the results presented here must be performed by future measurements.
Furthermore, BETACOOL simulations of the time evolution of the beam during the electron cooling process in presence of rest gas and IBS effects have been presented, taking into account realistic assumptions for the residual gas composition and the so-called Martini model [27] for IBS. The results have revealed interesting features for the case of the first cooling plateau, where we have pessimistically assumed large values of initial phase space parameters. During the cooling process, the beam distribution quickly deviates from a Gaussian profile and a very dense core appears. Since here we have used the Martini model for the IBS, which is based on the growth of the rms beam parameters for a Gaussian distribution, we are probably underestimating the IBS effects for the core. At this point, further investigation is required to estimate more accurately the IBS effects on the beam core. A comparison of the results presented here with simulations using core-tail IBS models [40], which take into account different diffusion coefficients depending on the amplitude of the particle within the core and the tail of the beam distribution, would be useful.

Beam evolution studies for different levels of residual gas pressure in the vacuum pipe have shown a tolerance $P < 1 \times 10^{-10}$. In principle, according to the results, it could be possible to relax the ELENA nominal vacuum pressure requirements by one order of magnitude, from the very stringent value $3 \times 10^{-12}$ Torr to $\sim 1 \times 10^{-11}$ Torr, without affecting the cooling performance.

It is well known that during the cooling process the dependence of the beam parameters on the beam intensity is the result of the equilibrium between cooling and IBS. Therefore, in order to study the impact of IBS on the cooled beam, we have computed the beam parameters at the equilibrium as a function of the beam intensity. This has allowed us to determine scaling laws to calculate the equilibrium rms emittances and momentum spread as a function of the number of stored antiprotons in ELENA.

It is also worth mentioning that in the simulations described here Gaussian beam profiles have been assumed as input at the beginning of both cooling plateaus. However, in practice, the distribution of the beam injected from the AD (where it has been subjected to stochastic cooling and electron cooling) could be non-Gaussian, but a dense core with significant non-Gaussian tails. Indeed, this has been confirmed by beam profile measurements in the AD [38, 39]. Even the ELENA simulations presented here for 35 MeV/c and starting with initial Gaussian profiles lead to a distribution with tails, which will be decelerated to 13.7 MeV/c and then subjected to the second cooling process. Therefore, obvious directions for our future work include the use of particle distributions that take this effect into account in the simulation (based on features of AD beam profile measurements) at the beginning of the ELENA cycle.

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