Power counting renormalization of Hořava gravity at the kinetic conformal point

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1. Introduction

Hořava’s proposal for a gravity theory assumes an anisotropic behavior of space and time. Consequently, the Lorentz symmetry is manifestly broken in this theory. The benefit would be to obtain a renormalizable theory of gravity at the UV regime. In addition, one may demand that at low energies the relativistic scale should be recovered together with a theory of gravity very close to General Relativity (GR), in order to satisfy the very good agreement of GR to the known experimental data.

The geometrical formulation of the theory is in terms of a foliation of spacelike leaves indexed by a time coordinate. The theory should be invariant under diffeomorphisms on the spacelike leaves (F Diff) together with reparametrization of the time coordinate. Although the use of ADM variables are very adequate to describe the theory, one should have in mind that there is no space-time in Hořava’s formulation of gravity. The fields are a spacelike metric denoted $g_{ij}$, a scalar and a vector under diffeomorphisms of the leaves which transform as densities under time reparametrization, they are denoted $N$ and $N^i$ respectively (the lapse and the shift in the ADM nomenclature).

The theory allows as potential terms in the Hamiltonian higher order spacelike covariant derivatives of the three dimensional Riemann tensor as well as higher order products of the Riemann tensor, which are only allowed to exist in a relativistic theory provided higher order time derivatives terms are also included. In the latest case, generically, there appear ghost fields which ruins the unitary behavior of the theory. Together with the introduction of higher order spacelike covariant derivatives or products of the Riemann tensor in the potential one can include covariant derivatives or products of the vector field $a_i \equiv \frac{\partial N}{\partial x^i}$.

The goal of the Hořava theory of gravity, which was from the beginning to obtain a renormalizable theory of gravity, has not yet been fulfilled. It has been claimed that the theory is power counting renormalizable, but even this point has been argued in terms of analogies with scalar field theories. Gravity theories contain constraints, of the first and second class in Hořava theories, consequently the argument of power counting renormalization has to be proven...
by solving the constraints and showing that the correct powers of derivatives are involved in the resolution. This point has been recently proven in [1] where a complete argument showing the power counting renormalization of the Horava gravity at the kinetic conformal point has been obtained. In this work we discuss furthermore several aspects of this proof.

The Horava theories contain a kinetic term which depends on a dimensionless coupling constant \( \lambda \). The so called non-projectable Horava gravities depend crucially on the value of this coupling constant. For \( \lambda \neq \frac{1}{3} \) the theory contains one additional propagating degree of freedom compared to GR. There has been a discussion whether the theory presents problems of strong coupling or not, associated to this propagating mode. For \( \lambda = \frac{1}{3} \) the theory contains exactly the same propagating degrees of freedom as GR, the transverse traceless components of the metric on the spacelike leaves [2].

The kinetic term of the theory at \( \lambda = \frac{1}{3} \) is invariant under a local conformal transformation. For this reason we have called this theory “Horava gravity at the kinetic conformal (KC) point”. The complete theory is not invariant under that symmetry, only its kinetic term.

2. The action
The action of the non-projectable Horava gravity for any value of \( \lambda \) and including the Blass et al. interacting terms is given by [3, 4]

\[
S = \int dt d^3x \sqrt{g} N \left( \frac{1}{2\kappa} G^{ijkl} K_{ij} K_{kl} - \mathcal{V} \right),
\]

where

\[
K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - 2\nabla_i N_j),
\]

\[
G^{ijkl} = \frac{1}{2} \left( g^{ik} g^{jl} + g^{il} g^{jk} \right) - \lambda g^{ij} g^{kl}
\]

and \( \lambda \) is a dimensionless constant. Two comments are in order: first, if \( z = d = 3 \), \( \kappa \) becomes a dimensionless coupling constant [3]. Second, in a relativistic theory, we would have \( \lambda = 1 \), \( z = 1 \) and \( \kappa \) would be dimensionful. We do not put a constant in front of the potential \( \mathcal{V} \) because we are going to include an independent coupling constant for each one of its terms.

The potential \( \mathcal{V} \) can be, in principle, any \( F \) Diff scalar made with the spatial metric \( g_{ij} \), the vector

\[
a_i = \frac{\partial_i N}{N}
\]

and their FDiff-covariant derivatives (curvature tensors and their derivatives for \( g_{ij} \)). The potential contains no time derivatives and does not depend on \( N_i \). In particular, the \( z = 1 \) potential, which is the most relevant one for the large-distance physics, is

\[
\mathcal{V}^{(z=1)} = -\beta R - \alpha a_i a^i,
\]

where \( \beta \) and \( \alpha \) are coupling constants.

The kinetic term \( \sqrt{g} N G^{ijkl} K_{ij} K_{kl} \) has dimensions -2z where \( z \) is the scaling of time. The measure \( dt d^Dx \) has dimension \( z + D \), hence the overall coupling constant \( \kappa \) must have dimension \( -z + D \). It becomes dimensionless for \( z = D \). Consequently, the potential should contain all interaction terms compatible with the local symmetry of the theory up to 2D derivatives, for \( D = 3 \) up to six derivatives. The property of \( \kappa \) being dimensionless is crucial in order to show power counting renormalization of the theory.
We denote by $\pi^{ij}$ the momentum conjugated of $g_{ij}$ and by $P_N$ the one of $N$, whereas we regard the shift vector $N_i$ as a Lagrange multiplier. We study the asymptotically flat case, under which the canonical field variables behave asymptotically as

$$g_{ij} - \delta_{ij} = \mathcal{O}(1/r), \quad \pi^{ij} = \mathcal{O}(1/r^2), \quad N - 1 = \mathcal{O}(1/r).$$

(6)

The only local constraint associated to gauge symmetries that are homotopic to the identity, and hence of first class, is the momentum constraint $H^i$,

$$H^i \equiv -2\nabla_j \pi^{ij} + P_N \partial^i N = 0,$$

(7)

which generates the purely spatial diffeomorphisms. The second-class constraints are

$$P_N = 0,$$

(8)

$$\pi \equiv g^{ij} \pi_{ij} = 0,$$

(9)

$$\mathcal{H} \equiv \frac{2\kappa}{\sqrt{g}} \pi^{ij} \pi_{ij} + \sqrt{g} \mathcal{U} = 0,$$

(10)

$$\mathcal{C} \equiv \frac{3\kappa}{\sqrt{g}} \pi^{ij} \pi_{ij} - \sqrt{g} \mathcal{W} = 0.$$

(11)

$\mathcal{U}$ and $\mathcal{W}$ are derivatives of the potential defined by

$$\mathcal{U} \equiv \frac{1}{\sqrt{g}} \frac{\delta}{\delta N} \int d^3 y \sqrt{g} N V = V + \frac{1}{N} \sum_{r=1} (-1)^r \nabla_{i_1 \cdots i_r} \left( N \frac{\partial V}{\partial (\nabla_{i_{r+1}} g_{i_1})} \right),$$

(12)

$$\mathcal{W} \equiv g_{ij} \mathcal{W}^{ij}, \quad \mathcal{W}^{ij} \equiv \frac{1}{\sqrt{g} N} \frac{\delta}{\delta g_{ij}} \int d^3 y \sqrt{g} N V.$$ (13)

$\nabla_{i_1 \cdots k}$ stands for $\nabla_i \nabla_j \cdots \nabla_k$. Adopting the nomenclature of GR, $H^i = 0$ is called the momentum constraint and $\mathcal{H} = 0$ the Hamiltonian constraint. The $\pi = 0$ constraint is the primary constraint that emerges when the theory is formulated at the KC point. Indeed, the conjugated momentum $\pi^{ij}$ obeys the general relation

$$\frac{\pi^{ij}}{\sqrt{g}} = \frac{1}{2\kappa} G^{ijkl} K_{kl}.$$

(14)

At $\lambda = 1/3$ the hypermatrix $G^{ijkl}$ becomes degenerated, $g_{ij} G^{ijkl} = 0$, which leads directly to the $\pi = 0$ constraint. As a consequence, the secondary constraint $\mathcal{C} = 0$ emerges when the preservation in time of $\pi = 0$ is demanded. Thus, $\pi$ and $\mathcal{C}$ are the two second-class constraints that emerge at the KC point. Unlike GR, in the nonprojectable Hořava theory the Hamiltonian constraint $\mathcal{H}$ is of second-class behavior, which is associated to the fact that it lacks its role as generator of gauge symmetry. Finally, the $P_N = 0$ constraint must be added since in this theory (with $\lambda = 1/3$ or not) we are forced to included the lapse function $N$ as part of the canonical variables.

In addition, the boundary term corresponding to the ADM energy [5],

$$E_{ADM} \equiv \oint d\Sigma_i (\partial_j g_{ij} - \partial_i g_{jj}),$$

(15)

1 We have modified the original definition of $\mathcal{C}$ given in Ref. [2] by dividing it by $N$. 


must be incorporated because it is needed for the differentiability of the Hamiltonian under the most general asymptotic variations compatible with asymptotic flatness [6, 7].

By incorporating the constraints $P_N$ and $\pi$, we finally cast the classical Hamiltonian in the form

$$H = \int d^3x \left( \frac{2\kappa N}{\sqrt{g}} \pi^{ij} \pi_{ij} + \sqrt{g} \mathcal{N} \mathcal{V} + N_i \mathcal{H}^i + \sigma P_N + \mu \pi \right) + \beta E_{ADM},$$

where $N_i$, $\sigma$ and $\mu$ are Lagrange multipliers. This classical Hamiltonian is subject to the constraints (10) and (11), which have not been added with Lagrange multipliers.

In the counting of the independent degrees of freedom we have 14 non-reduced canonical variables in the set $\{(g_{ij}, \pi^{ij}), (N, P_N)\}$, three components of the first-class constraint $\mathcal{H}^i$ and four second-class constraints in the set $\{P_N, \pi, \mathcal{H}, \mathcal{C}\}$. The number of independent degrees of freedom is given by

$$(14 \text{ can. var.}) - [2 \times (3 \text{ first-cls. c.}) + (4 \text{ second-cls. c.})] = 4 \text{ indep. can. var.} \quad (17)$$

Thus, there are two even physical modes in the theory; that is, two modes that propagate themselves with a complete pair of canonical variables. This is the same number of degrees of freedom of GR; there are no extra modes in this theory. This property naturally raises the question whether the dynamics of this theory is able to reproduce the dynamics of GR for suitable large distances, i.e., at least in a perturbative regime for both theories. This was analyzed for the perturbatively linearized theory in Ref. [2].

3. The solution of the constraints at the linearized level

We present in this section the perturbative approach around Minkowski space-time and the solution of the constraints at the linearized level.

The complete set of non-equivalent terms which contribute to the propagator of the theory is [8],

$$-\mathcal{V}^{(z=1)} = \beta R + \alpha_i a^i,$$
$$-\mathcal{V}^{(z=2)} = \alpha_1 R \nabla_i a^i + \alpha_2 \nabla_i \nabla^i a^j + \beta_1 R_{ij} R^{ij} + \beta_2 R^2,$$
$$-\mathcal{V}^{(z=3)} = \alpha_3 \nabla^2 R \nabla_i a^i + \alpha_4 \nabla^2 a_i \nabla^2 a^j + \beta_3 \nabla_i R_{jk} \nabla^i R^{jk} + \beta_4 \nabla_i R \nabla^i R,$$

where $\nabla^2 \equiv \nabla_i \nabla^i$ and all the alphas and betas are coupling constants.

The perturbations around Minkowski spacetime are defined by introducing the variables $h_{ij}$, $p_{ij}$ and $n$ in the following way

$$g_{ij} = \delta_{ij} + \epsilon h_{ij}, \quad \pi^{ij} = \epsilon p_{ij}, \quad N = 1 + \epsilon n. \quad (21)$$

We use the orthogonal transverse/longitudinal decomposition

$$h_{ij} = h_{ij}^{TT} + \frac{1}{2} (\delta_{ij} - \partial_i \partial_j \partial^{-2}) h^T + \partial_i (h_{ij}^L),$$

where $\partial_{ij-k}$ stands for $\partial_\nu \partial_\nu \cdots \partial_\nu$ with $\nu = 2$ and $\partial^{-2} = 1/\partial^2$. $h_{ij}^{TT}$ is subject to $\partial_i h_{ij}^{TT} = h_{ij}^{TT} = 0$. We make an analogous decomposition on $p_{ij}$. We impose the transverse gauge

$$\partial_i h_{ij} = 0,$$  

under which all the longitudinal sector of the metric is eliminated.
We study the constraints (7 - 11) of the theory at linear order in perturbations in terms of the potential defined in (18 - 20). The momentum constraint (7), simplified by using $P_N = 0$ explicitly, eliminates the longitudinal sector of $p_{ij}$,

$$\partial_t p_{ij} = 0,$$

whereas the $\pi = 0$ constraint dictates that $p_{ij}$ is traceless, hence $p^T = 0$. So far we are left with the set $\{h^{TT}_{ij}, p^{TT}_{ij}, h^T, n\}$ as the set of remaining canonical variables.

Now we consider the $\mathcal{H}$ and $\mathcal{C}$ constraints. To present the results in a compact form, we introduce the vector $\phi$ of scalars and the functional matrix $\mathcal{M}$ in the way

$$\phi = \begin{pmatrix} h^T \\ n \end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix} \mathcal{D}_1 & \mathcal{D}_2 \\ \mathcal{D}_2 & \mathcal{D}_3 \end{pmatrix},$$

where

$$\mathcal{D}_1 \equiv \frac{1}{8} ((3\beta_3 + 8\beta_4)\partial^6 - (3\beta_1 + 8\beta_2)\partial^4 + \beta\partial^2),$$

$$\mathcal{D}_2 \equiv \frac{1}{2} (\alpha_3\partial^6 + \alpha_1\partial^4 + \beta\partial^2), \quad \mathcal{D}_3 \equiv \alpha_4\partial^6 - \alpha_2\partial^4 + \alpha\partial^2.$$

Thus, with the potential given in (18 - 20), the $\mathcal{H}$ and $\mathcal{C}$ constraints at linear order become

$$\mathcal{M}\phi = 0,$$

where the first row of this vectorial equation represents the $\mathcal{C}$ constraint and the second row the $\mathcal{H}$ constraint.

To solve the constraints (27) we start by decoupling them; that is, we want two separate equations in which $h^T$ and $n$ are not mixed. To this end we multiply Eq. (27) with

$$\begin{pmatrix} \mathcal{D}_3 & -\mathcal{D}_2 \\ -\mathcal{D}_2 & \mathcal{D}_1 \end{pmatrix}$$

from the left and get a diagonal matrix acting on $\phi$, which we write as

$$\mathbb{L}\phi = 0, \quad \mathbb{L} \equiv (\mathcal{D}_1\mathcal{D}_3 - \mathcal{D}_2^2) \mathbb{1}.$$

Equation (29) represents two decoupled equations for $h^T$ and $n$ and, moreover, the equations are the same (with the same boundary conditions).

Given the values of all the coupling constants, the generic case is when the operator $\mathbb{L}$ is a sixth-order polynomial on $\partial^2$. We can always factorize it; in particular, we may write it as

$$\mathbb{L} = K(\partial^2 - z_1)P^{(5)}(\partial^2),$$

where $P^{(5)}(u)$ is a fifth-order polynomial on $u$, $z_1$ stands for any one of the roots of $\mathbb{L}$, where $K = (1/8) (\alpha_4 (3\beta_3 + 8\beta_4) - 2\alpha_3^2)$. If $K \neq 0$ by combining (30) with (29) we can write the constraints in the form

$$\partial^2 P^{(5)}(\partial^2)\phi = z_1 P^{(5)}(\partial^2)\phi.$$

The decoupled equation (31) implies that $P^{(5)}(\partial^2)\phi$ is an eigenfunction of the Laplacian $\partial^2$. Since we are studying the asymptotically flat case, the spatial domain of the problem is the whole $\mathbb{R}^3$ and the boundary condition is that $\phi$ and its derivatives are zero at spatial infinity.

Consider the operator $\partial^2 - z_1$, with $z_1 \in \mathbb{C}$, acting on the space of functions $\psi$ whose domain is the whole $\mathbb{R}^3$ and that go asymptotically to zero (see (6)). Thus, Eq. (31) can be cast as

$$\partial^2 - z_1)\psi = 0.$$
In the space of functions $\psi$, $\partial^2$ has a continuum spectrum valued in $(-\infty, 0]$; it has no eigenvalues. With the prescribed asymptotic behavior the inverse $(\partial^2 - z_1)^{-1}$ exists for any value of $z_1$, but it behaves in different ways depending on whether $z_1$ belongs to the spectrum or not. If $z_1 \notin (-\infty, 0]$ the inverse $(\partial^2 - z_1)^{-1}$ is a bounded operator. In this case Eq. (32) automatically implies $\psi = 0$. If $z_1 \in (-\infty, 0]$, $(\partial^2 - z_1)^{-1}$ still exists but it is an unbounded operator. However, the right-hand side of Eq. (32) is zero; $(\partial^2 - z_1)^{-1}$ acting on it gives zero anyway. Therefore, for any value of $z_1$, Eq. (32) has the function $\psi = 0$ as its only solution satisfying the prescribed asymptotic behavior. We then have

$$P^{(5)}(\partial^2)\phi = 0. \quad (33)$$

Coming back to Eq. (33), it turns out that it becomes another eigenfunction problem for the Laplacian since its left-hand side is another polynomial on $\partial^2$, such that we may factorize it again,

$$(\partial^2 - z_2)\ P^{(4)}(\partial^2)\phi = 0 . \quad (34)$$

Since the same arguments hold to solve this equation, we have $P^{(4)}(\partial^2)\phi = 0$ as the unique solution. We may proceed iteratively continuing with this last equation to finally show that the linear-order versions for the variables $h^T$ and $n$ are equal to zero.

We remark that it is the non-compactness of the domain and the prescribed asymptotic conditions of the problem posed in (31) that force the everywhere-vanishing function to be the unique eigenfunction.

If $L$ is a lower order polynomial ($K = 0$), an analogous eigenfunction problem for the Laplacian arises since we may factorize the given polynomial. By applying the same reasoning of above, we eventually arrive at the same zero solution. Therefore, we conclude that the unique solution of the linearized $\mathcal{H}$ and $C$ constraints, which are expressed in (27), is

$$h^T = n = 0 . \quad (35)$$

There remains a condition in the space of parameters: we require that the whole operator $L$ is not completely zero since otherwise the number of constraints effectively reduces and additional modes appear. In addition, we know that the perturbatively linearized version of the purely $z = 1$ theory is equivalent to perturbatively linearized GR [2]. To combine these two facts, we require that the fourth-order coefficient of $L$, associated to the $z = 1$ operators of the theory, is nonzero,

$$\beta(2\beta - \alpha) \neq 0 . \quad (36)$$

Following the analysis of the linearized theory, it can be shown that the Lagrange multipliers $\mu$ and $\sigma$, and $n_i$ the perturbation of $N_i = 0$, are zero at the linearized level.

We finally have that, when all the constraints have been solved and the gauge has been fixed at linear order, there remains the pair $\{h_{ij}^{TT}, p_{ij}^{TT}\}$ as the set of free canonical variables. This confirms rigorously the number of two propagating degrees of freedom that the generic and nonperturbative Hamiltonian analysis anticipated.

4. The Hamiltonian at quadratic order in perturbations

Once we know the solutions of all the constraints in the transverse gauge, we may compute the reduced canonical Hamiltonian of the linearized theory.

It is given by

$$H_{\text{RED}} = \int d^3x \left( 2\kappa p_{ij}^{TT}p_{ij}^{TT} + \frac{1}{4} h_{ij}^{TT} \forall h_{ij}^{TT} \right) , \quad (37)$$

where

$$\forall = -\beta \partial^2 - \beta_1 \partial^4 + \beta_3 \partial^6 . \quad (38)$$
The large-distance dynamics of the perturbatively linearized theory can be obtained from the reduced Hamiltonian (37) by neglecting the higher order derivatives against the lowest order one. By doing so we obtain the effective Hamiltonian for the tensorial modes

\[ H^{\text{eff}}_{\text{RED}} = \int d^3x \left( 2\kappa p_{ij}^{TT} p_{ij}^{TT} - \frac{\beta}{4} h_{ij}^{TT} \partial^2 h_{ij}^{TT} \right). \]  

(39)

This is equivalent to taking only the \( z = 1 \) potential (5) and then linearizing it [2]. Thus, the perturbatively linearized version of the large-distance effective action is physically equivalent to linearized GR. Here one of the key features is the vanishing of the variables \( h^T \) and \( n \) at linear order in perturbations. The evolution equations arising from (39) constitute the wave equation for \( h_{ij}^{TT} \), thus the perturbative large-distance theory around Minkowski spacetime propagates gravitational waves exactly as linearized GR does. However, the nonperturbative dynamics of both theories are different, even considering only the \( z = 1 \) order in the side of the Hořava theory, since the nonperturbative field equations are different.

The requirement of positivity of the reduced Hamiltonian imposes constraints on the coupling constants \( \beta, \beta_1 \) and \( \beta_3 \) (we assume that \( \kappa \) is positive). We require that \( V \geq 0 \).

The resulting restrictions on the coupling constants needed for the continuity in the number of degrees of freedom, weakest regime approaching to GR, positivity and \( z = 3 \) behavior of the Hamiltonian are

\[ \alpha \neq 2\beta, \quad \beta > 0, \quad \beta_3 < 0, \quad \beta_1 \leq 2\sqrt{\beta|\beta_3|}. \]  

(40)

5. The propagator of the physical modes

From the expression (37), we obtain the full propagator of the physical modes, it is given by

\[ \langle h_{ij}^{TT} h_{kl}^{TT} \rangle = \frac{P_{ijkl}^{TT}}{\omega^2/2\kappa - \beta k^2 + \beta_1 k^4 + \beta_3 k^6}, \]  

(41)

where

\[ P_{ijkl}^{TT} = \frac{1}{\sqrt{2}} \left( \theta_{ik} \theta_{jl} + \theta_{il} \theta_{jk} - \theta_{ij} \theta_{kl} \right), \quad \theta_{ij} = \delta_{ij} - \frac{k_i k_j}{k^2}. \]  

(42)

Notice that only some terms of the potential (18 - 20) contribute to the propagator of the physical modes. The independent propagator (41) of this theory behaves just as was the aim in the original formulation of Hořava for having a renormalizable and unitary theory of quantum gravity [3]: for high \( \omega \) and \( \bar{k} \) it is dominated by the \( z = 3 \) mode \( (\omega^2/2\kappa + \beta_3 \bar{k}^6)^{-1} \) and there are no more independent propagators other than (41).

Besides, at low energies the dominant term is \( \left( \frac{\omega^2}{2\kappa} - \beta \bar{k}^2 \right)^{-1} \). That means, for appropriate values of the couplings we recover the propagator of the physical modes of GR.

6. Power counting renormalization

We will analyze the theory at the \( \text{UV} \) regime. This requires to go beyond the linear order in the solution of the constraints.

We use the notation in (25). We have for the next order in perturbations

\[ \epsilon M_0 = \epsilon^2 T(p_{ij}^{TT}, h_{ij}^{TT}) \]

where \( T \) is a \( 2 \times 2 \) matrix with entries quadratic in \( p_{ij}^{TT} \) and quadratic in \( h_{ij}^{TT} \) and in its spacelike derivatives. Because of the dimensions there are no derivatives of \( p_{ij}^{TT} \). The solution at linear order for \( h^T \) and \( n \) is zero, hence \( T \) depends only on \( p_{ij}^{TT} \) and \( h_{ij}^{TT} \).
The relevant point is that the highest order in derivatives \( \partial^6 \) appears on the left hand member of the equation. It is given by a matrix which can be identified from \( M \), its components are dimensionless and its determinant is \( K \). We must assume \( K \neq 0 \). Then, when solving for \( \phi \) the solution has a maximal order on spacelike derivatives on the denominator. This ensures that the solution for \( h^T \) and \( n \), which has to be inserted on the interacting vertices, do not contribute with positive powers of momenta.

For higher order solutions for \( h^T \) and \( n \) in the iterative process, we have the same matrix \( M \) in the left hand of the constraints and a similar argument follows. At each iterative step the solution for \( h^T \) and \( n \) do not contribute with positive powers of momenta. It is crucial in the argument the assumption that the dimensionless \( K = (1/8) (\alpha_4 (3\beta_3 + 8\beta_4) - 2\alpha_3^2) \neq 0 \), otherwise the argument of power counting renormalization fails. This point shows that arguments based in analogies with scalar fields theories are only heuristic ones which are not valid in general.

We may now discuss the power-counting renormalizability guided by the superficial degree of divergence of general 1PI diagrams over the reduced phase space. For this computation we follow Refs. [9, 10]. Further developments on the renormalization of Lorentz-violating theories, in particular, studies on the behavior of the subdivergences, were made in Refs. [11]. From the propagator (41) we deduce that if \( \Lambda \) is an \( UV \) cutoff for the momenta, then \( \Lambda^z \) is the cutoff for the energy (up to some constants of proportionality that are irrelevant for our purposes), with \( z = 3 \). Therefore, for each loop in the \( UV \) regime we have the contribution

\[
\int d\omega d^d k \to \Lambda^{d+z},
\]

while for each propagator

\[
I = \Lambda^{2z}.
\]

In any vertex we can have at most a contribution of \( 2z \) powers of loop momenta coming from the vertex itself (for vertices that are of \( 2z \) order in spatial derivatives). If in a 1PI Feynman diagram \( L \) is the number of loops, \( I \) is the number of internal lines and \( V \) is the number of vertices, its superficial degree of divergence \( D \) is bounded by

\[
D \leq (d+z)L + 2z(V-I)
\]

\[
= (d-z)L + 2z(L+V-I).
\]

Now the identity \( L-1 = I-V \) for graphs is used and in addition in this theory we have \( z = d \). Therefore, the superficial degree of divergence is bounded by

\[
D \leq 2z.
\]

This is the bound (8) of Ref. [10], where Lorentz-violating theories with interactions depending on spatial derivatives were considered. This degree of divergence coincides with the highest order operators already included in the bare action (once we extend our potential to include all the \( z \leq 3 \) terms, not only the operators that contribute to the quadratic action). This leads to the conclusion that the theory is power-counting renormalizable. Unitarity and the criterion of power-counting renormalizability are safe in this theory.

7. Conclusions

We showed, under the assumption \( K = (1/8) (\alpha_4 (3\beta_3 + 8\beta_4) - 2\alpha_3^2) \neq 0 \) on the dimensionless coupling constants, that Hořava gravity at the kinetic conformal point propagates the same physical degrees of freedom as General Relativity and it is power counting renormalizable. The results are based on previous works [1, 2, 12].

Acknowledgments

A. R. and A. S. are partially supported by Project Fondecyt 1161103, Chile.
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