Ambiguity Function of Symmetric Triangular Frequency Modulated Continuous Waveform

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Abstract. The ambiguity function of Linear Frequency Modulated (LFM) pulse compression signal has been discussed widely, but the ambiguity function of LFM continuous waveform (CW) is rarely heard about. In this paper, the Single Periodic Ambiguity Function (SPAF) of Symmetric Triangular Frequency Modulated Continuous Waveform (STFMCW) is analytically derived in an explicit form, and its Periodic Ambiguity Function (PAF) is obtained by the relationship between SPAF and PAF. According to both theoretical analysis and simulated results, it is obtained that the dominant feature of the SPAF about STFMCW is the intact X-shaped diagonal ridge and two halves with 3 peaks, and a low pedestal which has a shape of the figure “8”. Using the frequency-difference of up-sweep and down-sweep respectively, the range-velocity coupling can be eliminated for a moving target. And any cuts of SPAF or PAF of STFMCW can be gotten.

1. Introduction

The radar ambiguity function can provide the insight about how different radar waveforms could be suitable for the various radar applications. The Ambiguity function of LFM pulse compression signal is well known and has been discussed commonly [1-2]. LFMCW signals are widely used in radars. The design of a Chirp CW radar is well known and has been described in most literature [3-7]. But the ambiguity function of STFMCW is lacking.

The periodic ambiguity function (PAF) is the generalization of the periodic autocorrelation function, to the case of non-zero Doppler shift, which describes the response of a correlation receiver to a continuous signal modulated by a periodic waveform [8]. In this paper, the Single Periodic Ambiguity Function (SPAF) of Symmetric Triangular Frequency Modulated Continuous Waveform (STFMCW) is analytically derived in an explicit form, and its Periodic Ambiguity Function (PAF) is obtained by the relationship between SPAF and PAF. Then the ambiguity diagram is analyzed according to the formulas. The cuts along delay axes of its SPAF and the autocorrelation function of its PAF are also analyzed.
2. Single period ambiguity function of STLFMCW

2.1. STLFMCW signal

STLFMCW complex signal envelope for single period is given by

\[ v(t) = \begin{cases} e^{\beta k t^2} & 0 < t \leq \frac{T_m}{2}, \\ e^{-\beta k (t-T_s)^2} & \frac{T_m}{2} < t \leq T_m \end{cases} \]  \hspace{1cm} (1)

where \( T_m \) is the modulation period, \( k \) is the slope of the instantaneous frequency change \( k = \frac{2B}{T_m} \), \( B \) is the modulation bandwidth. According to Equation (1) the STLFMCW complex signal envelope can be written as

\[ u(t) = \sum_{n=-\infty}^{\infty} v(t-nT_m) \]  \hspace{1cm} (2)

Instantaneous frequency of STLFMCW and its amplitude with 3-period are shown in Fig.1.

![Fig 1. STLFMCW signal: (a) instantaneous frequency vs. time; (b) STFMCW 3-period waveform](image)

2.2. SPAF of STFMCW

![Fig 2. Instantaneous frequency vs. time for transmitted waveform and its received waveforms in a period](image)

- (a) \(-T_m/2 < \tau < -T_m/2\);
- (b) \(-T_m/2 < \tau < 0\);
- (c) \(0 < \tau < T_m/2\);
- (d) \(T_m/2 < \tau < T_m\).
The traditional ambiguity function is studied for the limited energy signals. SPAF of the cycle modulated continuous wave is \[9,10\]

\[
\chi_m(\tau, \xi) = \frac{1}{T_m} \int_0^{T_m} u(t) u^*(t + \tau) \exp(j2\pi\xi t) dt
\]

(3)

Where \(\tau\) is the time delay, and \(\xi\) is the Doppler shift. For the LFMCW signal, \(\xi\) may be the algebraic sum of beat frequency caused by the round-trip delay and Doppler frequency caused by the moving target. \(u(t)\) is the complex envelope of transmitted signal; and \(u^*(t)\) is the complex conjugation. The situation with \(-T_m \leq \tau < -T_m/2\) will be considered, and the instantaneous frequency vs. time for transmitted waveform and its received waveforms in a period is shown in Fig.2 (a). In this case, substituting Equation (2) into Equation (3) yields

\[
\chi_m(\tau, \xi) = -\sin[\pi(\xi - k\tau - kT_m)(T_m/2 + \tau)]\frac{T_m/2 + \tau}{T_m}
+ 2\sqrt{k} \frac{1}{T_m} \left[\left[C(U_m^*) - C(U_m^f)\right] + j\left[S(U_m^*) - S(U_m^f)\right]\right] e^{j\frac{\pi}{2}(k\tau - k\tau^2 - 2k\xi)}
\]

(4)

Similarly, for the case \(-T_m/2 \leq \tau < 0\) shown in Fig.2 (b) can be carried out. In this case

\[
\chi_m(\tau, \xi) = 2\sqrt{k} \frac{1}{T_m} \left[\left[C(U_m^*) - C(U_m^f)\right] + j\left[S(U_m^*) - S(U_m^f)\right]\right] e^{j\frac{\pi}{2}(k\tau - k\tau^2 - 2k\xi)}
+ \sin[\pi(\xi - k\tau)(T_m/2 + \tau)]\frac{T_m/2 + \tau}{T_m}
+ 2\sqrt{k} \frac{1}{T_m} \left[\left[C(U_m^*) - C(U_m^f)\right] - j\left[S(U_m^*) - S(U_m^f)\right]\right] e^{-j\frac{\pi}{2}(k\tau + k\tau^2 + 2k\xi)}
+ \sin[\pi(\xi + k\tau)(T_m/2 + \tau)]\frac{T_m/2 + \tau}{T_m}
\]

(5)

The case \(0 \leq \tau < T_m/2\) shown in Fig.2 (c) yields

\[
\chi_m(\tau, \xi) = \sin[\pi(\xi - k\tau)(T_m/2 - \tau)]\frac{T_m/2 - \tau}{T_m} e^{j\pi(3k\tau/2 - k\tau^2 + 2k\xi)}
+ 2\sqrt{k} \frac{1}{T_m} \left[\left[C(U_m^*) - C(U_m^f)\right] + j\left[S(U_m^*) - S(U_m^f)\right]\right] e^{j\frac{\pi}{2}(k\tau - k\tau^2 - 2k\xi)}
\]

(6)

The case \(T_m/2 \leq \tau < T_m\) shown in Fig.2 (d) yields
For Equation (4) to (7), $C(U)$ and $S(U)$ are the Fresnel integrals, which are defined as

$$C(U) = \int_0^U \cos(\frac{\pi x^2}{2})dx,$$

$$S(U) = \int_0^U \sin(\frac{\pi x^2}{2})dx; \quad \text{and} \quad U^u = \sqrt{k(T_\text{u} + T_\text{w}) + \xi/\sqrt{k}}, \quad U^d = \sqrt{k(T_\text{w} - T_\text{w}) - \xi/\sqrt{k}};$$

$$U^u = \sqrt{k(T_\text{u} + T_\text{w}) - \xi/\sqrt{k}}, \quad U^d = \sqrt{k(T_\text{w} - T_\text{w}) - \xi/\sqrt{k}};$$

$$U^u = \sqrt{k(T_\text{u} - T_\text{w}) + \xi/\sqrt{k}}, \quad U^d = \sqrt{k(T_\text{w} - T_\text{w}) - \xi/\sqrt{k}};$$

$$U^u = \sqrt{k(T_\text{u} - T_\text{w}) - \xi/\sqrt{k}}, \quad U^d = \sqrt{k(T_\text{w} - T_\text{w}) - \xi/\sqrt{k}}.$$
corresponding to two frequency-differences which have the same absolute value. The positive frequency-difference is generated in the up-sweep of STLFMCW and the minus is in the down-sweep. Since each branch of X-shaped diagonal ridge has a certain thickness, there will be a little difference between them for a moving target. Using the frequency-difference of up-sweep and down-sweep, the range-velocity coupling can be eliminated.

3.2. Cuts along delay axes

Cuts along delay axes of the SPAF at \( \tau = \beta T_w \) (\( \beta = 0, 0.1, 0.2, 0.3 \)) are presented. Fig. 4 covers the Doppler shift from \(-50\)Hz to \(+50\)Hz. The cut along time delay at \( \tau = 0 \) is \( \{\sin(\pi \zeta T_w)\} \). Here \( \text{Sa}(x) = \sin(x)/x \) as commonly defined. It can be seen that the peaks of cuts along delay axes become wider and their peak-values get smaller as time delay approaches the value of \( T_w/2 \).

4. PAF of STFMCW

4.1. The relationship between PAF and SPAF

Using the definition of SPAF in Equation (3), the relationship between PAF and SPAF of a CW signal with periodic complex envelope is obtained with the same idea in Equation [8].

\[
|X_{N\zeta m}(\tau, \xi)| = |X_{m}(\tau, \xi)| \frac{\sin(\pi \xi N T_w)}{N \sin(\pi \xi T_w)} \tag{8}
\]

It can be see that PAF \( |X_{N\zeta m}(\tau, \xi)| \) of a continuous signal modulated by a periodic waveform can be obtained by multiplying SPAF \( |X_{m}(\tau, \xi)| \) and \( \left| \frac{\sin(\pi \xi N T_w)}{N \sin(\pi \xi T_w)} \right| \) together, where \( T_w \) is the period and \( N \) is the number of the period, respectively.

4.2. PAF and its autocorrelation function of STFMCW

Using Equation from (4) to (8), PAF of a STFMCW signal can be easily obtained. Making \( \tau = 0 \) and \( \xi = 0 \) respectively, its periodic range and velocity autocorrelation function of CW can be obtained. It’s not necessary to write them here.
Figure 5 shows 2-periods ambiguity function of STFMCW, and Figure 6 shows 3-periods ambiguity function, respectively. From figure 5-6, it can be observed that PAF of STFMCW has \((2N-1)\) intact X-shaped diagonal ridges (\(N\) is the number of periods) and two halves. The peaks are at the points of \((\pm nT_m, 0)\) (\(n = 0, 1, 2\ldots\)).
Three-period range autocorrelation function of STFMCW is shown in Fig.7. It is axisymmetric with respect to $\tau = 0$. The centers of the peaks are at the points of $\tau = \pm nT_m$ ($n = 0,1,2,\cdots$) and the number of peaks is $(2N+1)$ ($N$ is the number of periods). There are $(2N-1)$ intact peaks and two halves which constitute an intact peak. From Equation (8), it can be obtained $\left| \chi_{NT_m}(\tau,0) \right| = \left| \chi_{T_m}(\tau,0) \right|$ directly. Which is to say, the plot of periodic range autocorrelation function can be obtained by copying single range autocorrelation function $N$ times.

Figure 7(b) shows three-period velocity autocorrelation function of STFMCW. Periodic velocity autocorrelation function of STFMCW is the product of signal periodic velocity autocorrelation function $\left| \chi_T(0,\xi) \right|$ multiplying the function $\left| \sin(\pi\xi NT_m)/N \sin(\pi\xi T_m) \right|$. $\left| \chi_T(0,\xi) \right|$ is compressed with the increase of the period number $N$.

5. Conclusion

In this paper, the SPAF of STFMCW signal is derived in an explicit form and the feature are discussed. Both theoretical analysis and simulated results show that the dominant feature of the SPAF of STFMCW is the intact X-shaped diagonal ridge and two halves with 3 peaks, and a low pedestal which has a shape of the figure “8”. Using the frequency-difference of up-sweep and down-sweep respectively, the range-velocity coupling can be eliminated for a moving target. The peaks of cuts along delay axes become wider and their peak-value get smaller as the time delay approaches half of the modulation period. The plot of periodic range autocorrelation function can be obtained by copying single range autocorrelation function $N$ times. The periodic velocity autocorrelation function is compressed with the increase of the period number $N$.

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References
[1] Zejak, A.J.; Simic, I.S.; Zrnic, B.M.Chirp radar ambiguity function shaping. EUROCON'2001, Trends in Communications, International Conference on. Volume 2, 4-7 July 2001 Page(s):325 - 328
[2] Nadav Levanon , Eli Mozeson. Radar Signals. Hoboken, New Jersey: John Wiley & Sons, Inc. 2004
[3] Wang J , Zheng T , Lei P , et al. Hand gesture recognition method by radar based on convolutional neural network[J]. Beijing Hangkong Hangtian Xuebao/Journal of Beijing University of Aeronautics and Astronautics, 2018, 44(6):1117-1123.
[4] G. Wang, J.M. Munoz-Ferreras, C. Gu, Application of linear- frequency- modulated continuous-wave (LFMCW) radars for tracking of vital signals. IEEE Trans. Microw. Theory Tech. 2014, 62(6), 1387–1399
[5] Jian-dong Zhu,Jin-liang Li, Xiang-dong Gao,Li-Bang Ye,Huan-yao Dai. Adaptive Threshold Detection and Estimation of Linear Frequency-Modulated Continuous-Wave Signals Based on Periodic Fractional Fourier Transform. Circuits, Systems, and Signal Processing, July 2016, Volume 35, Issue7, pp 2502–2517
[6] Liu M, Yi Q, Zhang Y. Multi-target Detection of FMCW Radar Based on Width Filtering[J]. 2017.
[7] Wei Zhou, Junhao Xie, Gaopeng Li. High-precision estimation of target range, radial velocity, and azimuth in mechanical scanning LFMCW radar[J]. Iet Radar Sonar & Navigation, 2017, 11(11):1664-1672.
[8] Nadav Levanon, Avraham Freedom. Periodic Ambiguity Function of CW Signals with Perfect Periodic Autocorrelation. IEEE Transactions on Aerospace and Electronic Systems, Vol. 28, No. 2 APRIL 1992, pages: 387-395.

[9] Couch, L. W. Effects of Modulation on nonlinearity on the range response of FM radar. IEEE Transactions on Aerospace and Electronic Systems, AES-9, 4(July 1973), pages: 598-606.

[10] Nadav Levanon; Eli Mozeson. Radar Signals. Hoboken, New Jersey: John Wiley & Sons, Inc. 2004