The $a_0(980)$ physics in semileptonic $D^0$ and $D^+$ decays

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Abstract

The decays $D^0 \rightarrow d \bar{u} e^+\nu \rightarrow a_0^-(980) e^+\nu$ and $D^+ \rightarrow d \bar{d} e^+\nu \rightarrow a_0^0(980) e^+\nu \rightarrow \pi^0 \eta e^+\nu$ (and the charge conjugated ones) is the direct probe of the two-quark components in the $a_{0}^{\pm}(980)$ and $a_{0}^{0}(980)$ wave functions. Recent BESIII experiment is the first step in experimental study of these decays. We present a possible variant of $\eta \pi$ invariant mass distribution when $a_0(980)$ has no $q\bar{q}$ component at all.
I. INTRODUCTION

The $a_0(980)$ and $f_0(980)$ mesons are well-established parts of the proposed light scalar meson nonet \[1\]. From the beginning, the $a_0(980)$ and $f_0(980)$ mesons became one of the central problems of nonperturbative QCD, as they are important for understanding the way chiral symmetry is realized in the low-energy region and, consequently, for understanding confinement. Many experimental and theoretical papers have been devoted to this subject.

There is much evidence that supports the four-quark model of light scalar mesons \[2,3\]. The suppression of the $a_0(980)$ and $f_0(980)$ resonances in the $\gamma\gamma \rightarrow \eta\pi^0$ and $\gamma\gamma \rightarrow \pi\pi$ reactions, respectively, was predicted in 1982 \[4\], $\Gamma_{a_0} \approx \Gamma_{f_0} \approx 0.27$ keV, and confirmed by experiment \[1\]. The elucidation of the mechanisms of the $\sigma(600)$, $f_0(980)$, and $a_0(980)$ resonance production in $\gamma\gamma$ collisions confirmed their four-quark structure \[5,6\]. Light scalar mesons are produced in $\gamma\gamma$ collisions mainly via rescatterings, that is, via the four-quark transitions. As for $a_2(1320)$ and $f_2(1270)$ (the well-known $q\bar{q}$ states), they are produced mainly via the two-quark transitions (direct couplings with $\gamma\gamma$).

The argument in favor of the four-quark nature of $a_0(980)$ and $f_0(980)$ is the fact that the $\phi(1020) \rightarrow a_0^0\gamma$ and $\phi(1020) \rightarrow f_0\gamma$ decays go through the kaon loop: $\phi \rightarrow K^+K^- \rightarrow a_0^0\gamma$, $\phi \rightarrow K^+K^- \rightarrow f_0\gamma$, i.e., via the four-quark transition \[7,11\]. The kaon-loop model was suggested in Ref. \[7\] and confirmed by experiment ten years later \[12,14\].

It was shown in Ref. \[8\] that the production of $a_0^0(980)$ and $f_0(980)$ in $\phi \rightarrow a_0^0\gamma \rightarrow \eta\pi^0\gamma$ and $\phi \rightarrow f_0\gamma \rightarrow \pi^0\pi^0\gamma$ decays is caused by the four-quark transitions, resulting in strong restrictions on the large-$N_C$ expansions of the decay amplitudes. The analysis showed that these constraints give new evidence in favor of the four-quark nature of the $a_0(980)$ and $f_0(980)$ mesons.

In Refs. \[15,16\] it was shown that the description of the $\phi \rightarrow K^+K^- \rightarrow \gamma a_0^0(980)/f_0(980)$ decays requires virtual momenta of $K(\bar{K})$ greater than 2 GeV, while in the case of loose molecules with a binding energy about 20 MeV, they would have to be about 100 MeV. Besides, it should be noted that the production of scalar mesons in the pion-nucleon collisions with large momentum transfers also points to their compactness \[17\].

It was also shown in Refs. \[18,19\] that the linear $S_L(2) \times S_R(2) \sigma$ model \[20\] reflects all of the main features of low-energy $\pi\pi \rightarrow \pi\pi$ and $\gamma\gamma \rightarrow \pi\pi$ reactions up to energy 0.8 GeV and agrees with the four-quark nature of $\sigma$ meson. This allowed for the development of a
phenomenological model with the right analytical properties in the complex $s$ plane that took into account the linear $\sigma$ model, $\sigma(600) - f_0(980)$ mixing and the background \[21\]. This background has a left cut inspired by crossing symmetry, and the resulting amplitude agrees with results obtained using the chiral expansion, dispersion relations, and the Roy equation \[22\], and with the four-quark nature of the $\sigma(600)$ and $f_0(980)$ mesons as well. This model well describes the experimental data on $\pi\pi \rightarrow \pi\pi$ scattering up to 1.2 GeV.

Moreover, the absence of $J/\psi \rightarrow \gamma f_0(980), \rho a_0(980), \omega f_0(980)$ decays in the presence of intense $J/\psi \rightarrow \gamma f_2(1270), \gamma f_2'(1525), \rho a_2(1320), \omega f_2(1270)$ decays is at variance with the $P$-wave two-quark, $q\bar{q}$, structure of $a_0(980)$ and $f_0(980)$ resonances \[23\].

It is shown in Ref. \[24\] that the recent data on the $K_S^0 K^+$ correlation in Pb-Pb interactions Ref. \[25\] agree with the data on the $\gamma\gamma \rightarrow \eta\pi^0$ and $\phi \rightarrow \eta\pi^0\gamma$ reactions and support the four-quark model of the $a_0(980)$ meson. It is shown that the data does not contradict the validity of the Gaussian assumption.

In Refs. \[26, 27\] it was suggested the program of studying light scalars in semileptonic $D$ and $B$ decays. We studied production of scalars $\sigma(600)$ and $f_0(980)$ in the $D^+_s \rightarrow \pi^+\pi^- e^+\nu$ decays, the conclusion was that the percentage of the $q\bar{q}$ components in $\sigma(600)$ and $f_0(980)$ is small. This is the direct evidence in favor of exotic nature of these particles. Unfortunately, at the moment the statistics is rather poor, and thus new high-statistics data are highly desirable.

It was noted in Refs. \[26, 27\] that no less interesting is the study of semileptonic decays of $D^0$ and $D^+$ mesons $- D^+ \rightarrow d\bar{u} e^+\nu \rightarrow [\sigma(600) + f_0(980)] e^+\nu \rightarrow \pi^+\pi^- e^+\nu$, $D^0 \rightarrow d\bar{u} e^+\nu \rightarrow a_0 e^+\nu \rightarrow \pi^-\eta e^+\nu$ and $D^+ \rightarrow d\bar{u} e^+\nu \rightarrow a_0 e^+\nu \rightarrow \pi^0\eta e^+\nu$ (or the charged-conjugated ones) which had not been investigated. It is very tempting to study light scalar mesons in semileptonic decays of $B$ mesons \[27\]: $B^0 \rightarrow d\bar{u} e^+\nu \rightarrow a_0 e^+\nu \rightarrow \pi^-\eta e^+\nu$, $B^+ \rightarrow u\bar{u} e^+\nu \rightarrow a_0 e^+\nu \rightarrow \pi^0\eta e^+\nu$, $B^+ \rightarrow u\bar{u} e^+\nu \rightarrow [\sigma(600) + f_0(980)] e^+\nu \rightarrow \pi^+\pi^- e^+\nu$.

Recently BES Collaboration measured the decays $D^0 \rightarrow d\bar{u} e^+\nu \rightarrow a_0 e^+\nu \rightarrow \pi^-\eta e^+\nu$ and $D^+ \rightarrow d\bar{d} e^+\nu \rightarrow a_0 e^+\nu \rightarrow \pi^0\eta e^+\nu$ for the first time \[28\]. In the given paper we discuss the Ref. \[26\] program in light of these measurements taking into account contribution of $a_0'$ meson with mass about 1400 MeV. A variant when $a_0(980)$ has no $q\bar{q}$ component at all is presented.
II. \textit{D-DECAYS INVOLVING SCALARS AND PSEUDOSCALARS}

The amplitude of the $D^0 \rightarrow S(\text{scalar}) e^+ \nu$ decay is of similar form to $D_s^+ \rightarrow S$ decay \cite{26}

\begin{align}
M[D^0(p) \rightarrow S(p_1) W^+(q) \rightarrow S(p_1) e^+ \nu] = \frac{G_F}{\sqrt{2}} V_{cd} A_\alpha L^\alpha, \tag{1}
\end{align}

where $G_F$ is the Fermi constant, $V_{cd}$ is the Cabibbo-Kobayashi-Maskava matrix element,

\begin{align}
A_\alpha &= f_+^S(q^2)(p + p_1)_\alpha + f_-^S(q^2)(p - p_1)_\alpha, \\
L_\alpha &= \bar{\nu} \gamma_\alpha (1 + \gamma_5) e, \\ q &= (p - p_1). \tag{2}
\end{align}

The influence of the $f_-^S(q^2)$ form factor is negligible because of the small mass of the positron.

The decay rate into the stable $S$ state is

\begin{align}
\frac{d\Gamma(D^0 \rightarrow Se^+\nu)}{dq^2} &= \frac{G_F^2 |V_{cd}|^2}{24\pi^3} p^2_1(q^2)|f_+^S(q^2)|^2, \tag{3}
\end{align}

\begin{align}
p_1(q^2) &= \sqrt{m^4_D - 2m^2_{D^0}(q^2 + m^2_S) + (q^2 - m^2_S)^2} \\ &= \frac{2m_{D^0}}{2m^2_{D^0}} \tag{4}
\end{align}

For the $f_+^S(q^2)$ form factor we use the vector dominance model

\begin{align}
f_+^S(q^2) = f_+^S(0) \frac{m_A^2}{m_A^2 - q^2} = f_+^S(0) f_A(q^2), \tag{5}
\end{align}

where $A = D_1(2420)^\pm$ \cite{1}.

\begin{itemize}
  \item For the $f_+^S(q^2)$ form factor we use the vector dominance model.
\end{itemize}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Model of the $D^0 \rightarrow (a_0^-, a_0'^-) e^+ \nu$ decay.}
\end{figure}

Following Fig. \ref{fig:1} we write $f_+^S(0)$ in the form

\begin{align}
f_+^S(0) = g_{D^0cd} F_S g_{D^0S}, \tag{6}
\end{align}
FIG. 2: Model of the $D^+ \rightarrow (a_0^0, a'_0^0) e^+ \nu$ decay.

FIG. 3: Experimental data on (a) $D^0 \rightarrow (a_0^-, a'_0^-) e^+ \nu \rightarrow \eta \pi^- e^+ \nu$ and (c) $D^+ \rightarrow (a_0^0, a'_0^0) e^+ \nu \rightarrow \eta \pi^0 e^+ \nu$ decays. Direct copy of Fig. 2 (a) and (c) in Ref. [28]. Green curves are signal, red ones represent total contribution, other ones represent backgrounds.

where $g_{D^0 c \bar{u}}$ is the $D^0 \rightarrow c \bar{u}$ coupling constant, $g_{d \bar{u} S}$ is the $d \bar{u} \rightarrow S$ coupling constant, $F_S$ is the loop integral assuming to be constant in the region of interest.

The amplitude of the $D^0 \rightarrow d \bar{u} e^+ \nu \rightarrow [a_0^-(980) + a'_0^-] e^+ \nu \rightarrow \eta \pi^- e^+ \nu$ decay is

$$M(D^0 \rightarrow d \bar{u} e^+ \nu \rightarrow \eta \pi^- e^+ \nu) = \frac{G_F}{\sqrt{2}} V_{cd} L^\alpha (p + p_1)_\alpha g_{D^0 c \bar{u}} f_A(q^2)$$

$$\frac{1}{\Delta(m)} \left( F_{a_0^-} g_{d \bar{u} a_0^-} D_{a_0^-}(m) g_{a_0 \eta \pi} + F_{a_0^-} g_{d \bar{u} a_0^-} \Pi_{a_0^-} (m) g_{a_0' \eta \pi} \right)$$
FIG. 4: Results of the global fit, see Tables I and II, on a) the Belle data on $\gamma\gamma \rightarrow \eta\pi^0$ cross-section [29], and b) the KLOE data on the $\phi \rightarrow \eta\pi^0\gamma$ decay [14], cross points are omitted in fitting, $m$ is the invariant $\eta\pi^0$ mass.

FIG. 5: $K_0^0K^+$ correlation, solid line represents our fit, points are experimental data [25].

\[ + F_{a_0^-}g_{d_{a_0^-}}\Pi_{a_0^- a_0^-}(m)g_{a_0\eta\pi} + F_{a_0^-}g_{d_{a_0^-}}D_{a_0^-}(m)g_{a_0\eta\pi} \right), \]

where $m$ is the invariant mass of the $\eta\pi^-$ system, $\Delta(m) = D_{a_0^-}(m)D_{a_0^-}(m) - \Pi_{a_0^- a_0^-}(m)\Pi_{a_0^- a_0^-}(m)$, $D_{a_0^-}(m)$ and $D_{a_0^-}(m)$ are the inverted propagators of the $a_0^-$ and $a_0'^-$ mesons, $\Pi_{a_0^- a_0^-}(m) = \Pi_{a_0^- a_0^-}(m)$ is the off-diagonal element of the polarization operator, which mixes the $a_0^-$ and $a_0'^-$ mesons. All the details could be found in Appendix I.

The double differential rate of the $D^0 \rightarrow d\bar{u} e^+\nu \rightarrow [a_0^- (980) + a_0'^-] e^+\nu \rightarrow \eta\pi^- e^+\nu$ decay
FIG. 6: The plot of $D^0 \rightarrow (a^0_0, a'^0_0) e^+\nu \rightarrow \eta\pi^- e^+\nu$ spectrum with parameters of the Fit (with $g_{d\bar{u}a^-} = 0$). Solid line is the total contribution, dotted line is the term $\sim \frac{F_{a^0_0} - g_{d\bar{u}a^0_0} \cdot \Pi_{a^0_0} a^- (m) g_{a_0\eta\pi}}{\sqrt{2}}$ contribution, dashed line is the term $\sim \frac{F_{a'^0_0} - g_{d\bar{u}a'^0_0} \cdot \Pi_{a'^0_0} a^- (m) g_{a_0\eta\pi}}{\sqrt{2}}$ contribution, see Eq. (8).

Taking into account the $a'_0$ scalar meson is

$$\frac{d^2\Gamma(D^0 \rightarrow \eta\pi^- e^+\nu)}{dq^2 dm} = \frac{G_F^2 |V_{cd}|^2 g_f^2 |V_{cd}| F_A(q^2)|^2 p_1^2(q^2, m)}{24\pi^3}$$

$$\times \frac{1}{8\pi^2} m \rho_{\eta\pi^-}(m) \left| \Delta(m) \right|^2 \left| F_{a_0} g_{d\bar{u}a_0} D_{a_0^-}(m) g_{a_0\eta\pi} + F_{a_0} g_{d\bar{u}a_0} \Pi_{a_0^-} a^- (m) g_{a_0\eta\pi} \right|^2,$$

where $\rho_{\eta\pi^-}(m) = \sqrt{(1 - (m_\eta + m_\pi^-)/m^2)(1 - (m_\eta - m_\pi^-)/m^2)}$.

The $D^+ \rightarrow d\bar{d} e^+\nu \rightarrow S e^+\nu$ and $D^+ \rightarrow \eta\pi^0 e^+\nu$ decays are described the same way, see Fig. 2. It is enough to substitute in Eqs. (1-8) $D^0 \rightarrow D^+, d\bar{u} \rightarrow d\bar{d}, a^0_0, a'^0_0, a^0_0 \rightarrow a^0_0, a'^0_0 \rightarrow a^0_0$ and $\pi^- \rightarrow \pi^0$. Note $g_{d\bar{u}a^0_0} = g_{d\bar{u}a'^0_0}/\sqrt{2}$ with corresponding effect on matrix element and branching.

The key question is the size of $a'_0$ contribution. In Ref. 28 fits take into account only the $a_0(980)$ contribution, but the data prefer bigger signal contribution in the interval $m \equiv M_{\eta\pi} = 1.1 \div 1.3$ GeV, i.e., on the right end of the distribution, see Fig. 3. It can be a manifestation of sizeable $a'_0$ contribution.
In Ref. [24] we simultaneously described the data on $\gamma\gamma \rightarrow \eta\pi^0$ reaction Ref. [29], $\phi \rightarrow \eta\pi^0\gamma$ decay [14] and the recent data on the $K_S^0K^+$ correlation in Pb-Pb interactions Ref. [25].

For the first time we present a variant of data description when $a_0(980)$ has no $q\bar{q}$ component at all: the $a_0^0(980)$ direct coupling to $\gamma\gamma$ channel $g_{a_0^0\gamma\gamma} = 0$ and $g_{d\bar{u}a_0^0} = g_{d\bar{d}a_0^0} = 0$. The results are shown on Figs. 4 in and in Tables I and II. The corresponding prediction of $dBr(D^0 \rightarrow d\bar{u} e^+ \nu \rightarrow (a_0^-, a_0^{-0}) e^+ \nu \rightarrow \pi^- \eta e^+ \nu)/dm$ curve is shown on Fig. 6. The line shape of this curve differs from the signal curve on Fig. 3, though does not contradict the data on Fig. 3. The normalization is on total branching $Br(D^0 \rightarrow d\bar{u} e^+ \nu \rightarrow (a_0^-, a_0^{-0}) e^+ \nu \rightarrow \pi^- \eta e^+ \nu) = 1.33 \times 10^{-4}$ - the middle value of the experimental result $(1.33^{+0.33}_{-0.29}(stat) \pm 0.09) \times 10^{-4}$. This normalization is quite arbitrary here since the large contribution in the region $m > 1.05$ GeV. Note also that the experimenters could obtain different results for total branching using the form of curve presented on Fig. 6.

In Table I $\lambda$ and $R$ are the parameters of the two-kaon correlation description, see Ref. [24]. Some details and parameters of the fit are placed in Appendix II and Table II therein.

| $m_{a_0}$, MeV | $m_{a_0'}$, MeV | 1439.4 | $\chi^2_{sp} / 24$ points | 25.3 |
|----------------|----------------|---------|--------------------------|------|
| $g_{a_0^0K^+K^-}$, GeV | 3.50 | $g_{a_0^0K^+K^-}$, GeV | 4.45 | $\chi^2_{corr} / 29$ points | 17.2 |
| $g_{a_0\pi\pi}$, GeV | 3.42 | $g_{a_0'\pi\pi}$, GeV | -0.20 | $\chi^2_{\gamma\gamma} / 36$ points | 10.9 |
| $g_{a_0\pi'\pi'}$, GeV | -3.64 | $g_{a_0'\pi'\pi'}$, GeV | 0.41 | $(\chi^2_{\gamma\gamma} + \chi^2_{sp} + \chi^2_{corr})/n.d.f.$ | 55.5/75 |
| $g_{a_0^{(0)}0\gamma\gamma}$ | 0 | $g_{a_0^{(0)}0\gamma\gamma}$, $10^{-3}$ GeV$^{-1}$ | -14.62 | |
| $C_{a_0a_0'}$, GeV$^2$ | -0.127 | $\lambda$ | 1 | $R$, fm | 6.3 |

### III. CONCLUSION

The first measurement of $D^0 \rightarrow d\bar{u} e^+ \nu \rightarrow [a_0^-(980) + a_0^{-0}] e^+ \nu \rightarrow \pi^- \eta e^+ \nu$ and $D^+ \rightarrow \bar{d}\bar{d} e^+ \nu \rightarrow [a_0^0(980) + a_0^0] e^+ \nu \rightarrow \pi^0 \eta e^+ \nu$ decays [28] is an important step for the investigation of light scalar mesons nature.

The data description with $g_{a_0^{(0)}0\gamma\gamma} = 0$ is presented for the first time, it means that $a_0(980)$ has no $q\bar{q}$ component. The data is perfectly described, the $a_0(980)$ coupling constants agree with the four-quark model scenario. The corresponding prediction of $D^0 \rightarrow d\bar{u} e^+ \nu \rightarrow [a_0^-(980) + a_0^{-0}] e^+ \nu \rightarrow \pi^- \eta e^+ \nu$ and $D^+ \rightarrow \bar{d}\bar{d} e^+ \nu \rightarrow [a_0^0(980) + a_0^0] e^+ \nu \rightarrow \pi^0 \eta e^+ \nu$ decays
IV. APPENDIX I: SCALAR PROPAGATORS AND POLARIZATION OPERATORS

The matrix of the inverse propagators is

\[
G_{SS'}(m) = \begin{pmatrix}
D_{a'}(m) & -\Pi_{a'0a}(m) \\
-\Pi_{a0a'}(m) & D_{a}(m)
\end{pmatrix},
\]

(9)

\[
\Pi_{a'0a}(m) = \sum_{a,b} \frac{g_{a'b}}{g_{a0ab}} \Pi_{ab}(m) + C_{a'0a},
\]

(10)

where \( m = \sqrt{s} \), and the constant \( C_{a'0a} \) incorporates the subtraction constant for the transition \( a_0(980) \to (0^-0^-) \to a'_0 \) and effectively takes into account the contributions of multiparticle intermediate states to the \( a_0 \leftrightarrow a'_0 \) transition. The inverse propagator of the scalar meson \( S \) \[7, 11, 30, 32\] is

\[
D_S(m) = m_S^2 - m^2 + \sum_{ab} [Re\Pi_S^{ab}(m_S^2) - \Pi_S^{ab}(m^2)],
\]

(11)

where \( \sum_{ab}[Re\Pi_S^{ab}(m_S^2) - \Pi_S^{ab}(m^2)] = Re\Pi_S(m_S^2) - \Pi_S(m^2) \) takes into account the finite-width corrections of the resonance which are the one-loop contributions to the self-energy of the \( S \) resonance from the two-particle intermediate \( ab \) states. We take into account the intermediate states \( \eta \pi^+, KK \), and \( \eta' \pi^+ \) in the \( a_0^+(980) \) and \( a_0'^+(980) \) propagators:

\[
\Pi_S = \Pi_S^{\eta \pi^+} + \Pi_S^{K^0 K^+} + \Pi_S^{K^0 K^+} + \Pi_S^{\eta' \pi^+},
\]

(12)
and $\eta\pi^0$, $KK$, and $\eta'\pi^0$ in the $a_0^0(980)$ and $a_0^{10}$ propagators.

For pseudoscalar mesons $a, b$ and $m_a \geq m_b$, $m \geq m_+$, one has

$$\Pi_{S}^{ab}(m^2) = \frac{g_{Sab}^2}{16\pi} \left[ \frac{m_+ m_-}{\pi m^2} \ln \frac{m_b}{m_a} + \right.$$

$$+ \rho_{ab} \left( i + \frac{1}{\pi} \ln \frac{\sqrt{m^2 - m_+^2} - \sqrt{m^2 - m_-^2}}{\sqrt{m^2 - m_+^2} + \sqrt{m^2 - m_-^2}} \right),$$

where $\rho_{ab}(s) = 2\rho_{ab}(s)/\sqrt{s} = \sqrt{(1 - m_+^2/s)(1 - m_-^2/s)}$, and $m_{\pm} = m_a \pm m_b$.

For $m_- \leq m < m_+$,

$$\Pi_{S}^{ab}(m^2) = \frac{g_{Sab}^2}{16\pi} \left[ \frac{m_+ m_-}{\pi m^2} \ln \frac{m_b}{m_a} - |\rho_{ab}(m)| + \right.$$

$$+ \frac{2}{\pi} |\rho_{ab}(m)| \arctan \frac{\sqrt{m_+^2 - m^2}}{\sqrt{m_-^2 - m^2}} \right],$$

and for $m < m_-$,

$$\Pi_{S}^{ab}(m^2) = \frac{g_{Sab}^2}{16\pi} \left[ \frac{m_+ m_-}{\pi m^2} \ln \frac{m_b}{m_a} - \right.$$

$$- \frac{1}{\pi} \rho_{ab}(m) \ln \frac{\sqrt{m_+^2 - m_+^2} - \sqrt{m_-^2 - m_-^2}}{\sqrt{m_+^2 - m_+^2} + \sqrt{m_-^2 - m_-^2}} \right].$$

The constants $g_{Sab}$ are related to the width as

$$\Gamma_S(m) = \sum_{ab} \Gamma(S \rightarrow ab, m) = \sum_{ab} \frac{g_{Sab}^2}{16\pi m} \rho_{ab}(m).$$

V. APPENDIX II: OTHER PARAMETERS AND DETAILS

For completeness, we show in Table II the parameters that are not described above. One can find all of the details in Ref. [30].

Table II. Parameters not mentioned in Table I.

| Parameter | Value  |
|-----------|--------|
| $c_0$     | -0.28  |
| $f_{K\bar{K}}$, GeV$^{-1}$ | -0.31  |
| $c_1$, GeV$^{-2}$ | -9.03  |
| $f_{\pi\gamma}$, GeV$^{-1}$ | -0.50  |
| $\delta$, $\circ$ | 44.2   |
| $c_2$, GeV$^{-4}$ | 0.15   |

In the given paper we take the formfactor $G_\omega(s, t) = G_\rho(s, t)$,

$$G_\omega(s, t) = G_\rho(s, t) = \exp[(t - m_\omega^2)b_\omega(s)],$$

(17)
differently from Refs. [6, 30]. We take
\begin{equation}
  b_\omega(s) = b_\omega^0 + \alpha'_\omega \ln[1 + (s/s_0)],
\end{equation}
and obtain $b_\omega^0 = 2.3 \times 10^{-3}$ GeV$^{-2}$, and $s_0 = 1.005$ GeV$^2$. $\alpha'_\omega = 0.8$ GeV$^{-2}$ is the same. Form factors for the $K^*$ exchange are modified the same way. Besides, we obtain $r_{a_2} = 1.2$ GeV$^{-1}$ instead of $r_{a_2} = 1.9$ GeV$^{-1}$ in Refs. [6, 30].

The $\pi\eta$ scattering length agrees with the estimates based on current algebra and chiral perturbation theory, according to which $a^1_0 \approx 0.005 - 0.01$ (in units of $m_\pi^{-1}$), see Ref. [6].

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[1] C. Patrignani et al. (Particle Data Group), Chin. Phys. C 40, 100001 (2016) and 2017 update.
[2] R.L. Jaffe, Phys. Rev. D 15, 267 (1977); Phys. Rev. D 15, 281 (1977).
[3] S. Weinberg, Phys. Rev. Lett. 110, 261601 (2013).
[4] N.N. Achasov, S.A. Devyanin, and G.N. Shestakov, Phys. Lett. B 108, 134 (1982); Z. Phys. C 16, 55 (1982).
[5] N.N. Achasov and G.N. Shestakov, Z. Phys. C 41, 309 (1988).
[6] N.N. Achasov and G.N. Shestakov, Phys. Rev. D 77, 074020 (2008); Phys. Rev. D 81, 094029 (2010); Usp. Fiz. Nauk 54, 799 (2011) [Sov. Phys. Usp. 181, 827 (2011)].
[7] N.N. Achasov and V.N. Ivanchenko, Nucl. Phys. B 315, 465 (1989).
[8] N.N. Achasov, Nucl. Phys. A 728, 425 (2003).
[9] N.N. Achasov and V.V. Gubin, Phys. Rev. D 63, 094007 (2001); N.N. Achasov and A.V. Kiselev, Phys. Rev. D 73, 054029 (2006).
[10] N.N. Achasov and A.V. Kiselev, Phys. Rev. D 68, 014006 (2003).
[11] N.N. Achasov and V.V. Gubin, Phys. Rev. D 56, 4084 (1997).
[12] M.N. Achasov et al. (SND Collaboration), Phys. Lett. B 438, 441 (1998); M.N. Achasov et al., Phys. Lett. B 479, 53 (2000).
[13] M.N. Achasov et al. (SND Collaboration), Phys. Lett. B 440, 442 (1998); M.N. Achasov et al., Phys. Lett. B 485, 349 (2000); R.R. Akhmetshin et al. (CMD-2 Collaboration) Phys. Lett. B 462, 380 (1999); A. Aloisio et al. (KLOE Collaboration) Phys. Lett. B 537, 21 (2002); C. Bini, P. Gauzzi, S. Giovanella, D. Leone, and S. Miscetti, KLOE Note 173 06/02, http://www.lnf.infn.it/kloe/

[14] A. Aloisio et al. (KLOE Collaboration) Phys. Lett. B 536, 209 (2002).

[15] N.N. Achasov, V.V. Gubin, and V.I. Shevchenko, Phys. Rev. D 56, 203 (1997).

[16] N.N. Achasov and A.V. Kiselev, Phys. Rev. D 76, 077501 (2007); Phys. Rev. D 78, 058502 (2008).

[17] N.N. Achasov and G.N. Shestakov, Phys. Rev. D 58, 054011 (1998).

[18] N.N. Achasov and G.N. Shestakov, Phys. Rev. D 49, 5779 (1994).

[19] N.N. Achasov and G.N. Shestakov, Phys. Rev. Lett. 99, 072001 (2007).

[20] M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960).

[21] N.N. Achasov and A.V. Kiselev, Phys. Rev. D 83, 054008 (2011); Phys. Rev. D 85, 094016 (2012).

[22] I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96, 132001 (2006).

[23] N.N. Achasov, Usp. Fiz. Nauk 41, 1257 (1998) [Phys. Usp. 41, 1149 (1998)]; Yad. Fiz. 65, 573 (2002) [Phys. At. Nucl. 65, 546 (2002)].

[24] N.N. Achasov and A.V. Kiselev, Phys. Rev. D 97, 036015 (2018).

[25] S. Acharya et al. (ALICE Collaboration), Phys. Lett. B 774, 64 (2017).

[26] N.N. Achasov and A.V. Kiselev, Phys. Rev. D 86, 114010 (2012).

[27] N.N. Achasov and A.V. Kiselev, Int. J. Mod. Phys. Conf. Ser. 35, 1460447 (2014), http://www.worldscientific.com/doi/pdf/10.1142/S2010194514604475

[28] M. Ablikim et al. (BESIII Collaboration), arXiv:1803.02166.

[29] S. Uehara et al. (Belle Collaboration), Phys. Rev. D 80, 032001 (2009).

[30] N.N. Achasov, A.V. Kiselev, and G.N. Shestakov, Phys. At. Nucl. 79, 397 (2016).

[31] K.M Ecklund et al. CLEO Collaboration, Phys. Rev. D 80, 052009 (2009).

[32] N.N. Achasov, S.A. Devyanin, and G.N. Shestakov, Phys. Lett. B 88, 367 (1979).