Double copy of spontaneously broken Abelian gauge theory

Euro Spallucci\textsuperscript{1, 2}

\textit{INFN, Sezione di Trieste, Trieste, Italy}

Anais Smailagic\textsuperscript{3, 4}

\textit{INFN, Sezione di Trieste, Italy}

Abstract

Similarity in the structure of scattering amplitudes in Yang-Mills theories and General Relativity led to the idea that graviton could be described as the double copy of a vector gauge field. In this letter we discuss a realization of this idea emerging directly from solutions of equations of General Relativity. A general form of the energy momentum tensor for the electric field is derived that leads to the metric tensor in terms of the double copy of a corresponding gauge potential. We then use this general property to find the double copy of spontaneously broken scalar electrodynamics. The result is a screened Reissner-Nordström-like metric. When the horizon radius and the Compton wavelength of the massive photon become comparable the black hole becomes a quantum object. An exact solution of the horizon wave equation is found and the corresponding energy spectrum is described. It turns out that highly excited states show a characteristic string-like behavior.

1. Introduction

Expressing gravity in the language analogous to that of gauge theories is still an uncompleted task. On this path, there are at least three main obsta-
cles to be surmounted. The most evident one derives from the fact that gauge field is a one-index Lorentz vector while graviton is a two-index symmetric tensor. The first difference is the respective geometric meanings of these fields. Gauge fields are connections appearing inside covariant derivatives of Yang Mills theory while it is Christoffel symbol which is the connection in the gravitational covariant derivatives. The gravitational connection is a composite function of metric and its derivatives and not a fundamental field. On the other hand, graviton field describes deviation from flatness in the metric tensor.

The second difference is that Lagrangians for gauge theories are always quadratic in the field strength while the Hilbert-Palatini action in General Relativity is linear in the curvature of the metric. Attempts to gauge Lorentz, Poincaré or other groups do not lead straightforwardly to General Relativity but to more general theories like Riemann-Cartan geometry.

Finally, the gauge coupling constant is dimensionless (in natural units $c = 1, \hbar = 1$) while the Newton constant has dimension of length squared $G_N = \frac{L_{Pl}^2}{2}$.

In spite of many attempts, starting with the seminal papers by Utiyama [1] and Kibble [2] up to the conjectured duality between strings in $AdS_5$ and boundary SUSY $SU(N)$ gauge theories [3], the satisfactory unified description of gauge theories and gravity has not been reached.

A new approach to the connection between gravity and gauge theories was recently proposed based on the structural similarity among perturbative scattering amplitudes in Yang-Mills (YM) theory and General Relativity. We postpone technical detail of this approach to the next section and only mention here the final conclusions which can be summarized in the statement “gravity is the square of YM theory” [4, 5, 6, 7, 8, 9, 10] or, equivalently, that graviton is the double copy of a gauge field. Extending this conjecture beyond the scattering amplitude level leads to the picture in which the neutral Schwarzschild field can be described as the double copy (DC) of the Coulomb potential [11, 12, 13, 14, 15, 16, 17]. In other words, a neutral black hole solution has been designed as the DC of a point-like Coulomb electric field once correspondence between respective coupling constants is assumed. However, this type of correspondence is not compatible with other solutions of General Relativity equations. It is known that the Einstein-Maxwell equations with a massive charged particle as the source admit the Reissner-Nordström metric as an exact solution. In this solution, however, mass and charge are distinct, unrelated parameters.
In this letter we would like to go beyond DC scheme and describe Reissner-Nordström graviton as a "new" double copy (NDC) of the Coulomb potential. The paper is organized as follows: in Section(2) we review the distinctive properties of the Kerr-Schild gauge and briefly explain how it helps to describe the graviton as the DC of a gauge field. To pave the way towards a NDC relation between the gauge potential and the graviton, we first describe a derivation of the Reissner-Nordström solution in this framework.

In Section(3) we concentrate on the core part of the paper and describe the gravitational field as double copy of the Abelian-Higgs model. The resulting metric has a Reissner-Nordström-like form except that the charge term has a dumping Yukawa-like factor. Below the screening length induced by the photon mass, quantum effects become dominant inducing horizon uncertainty fluctuations. We solve the horizon wave equation and determine the energy spectrum. Highly excited states reveal an interesting relationship with quantum strings.

2. Graviton as the double copy of the photon

Asymptotically flat static/stationary metrics can be cast in a simple form in the so called Kerr-Schild gauge. In this gauge, the tensor structure is determined by a null vector $l^\mu$ and a single scalar potential $\phi$. The resulting line element reads

$$ds^2 = ds^2_{\text{Mink}} - 2\phi l^\mu l_\nu dx^\mu dx^\nu, \quad l^\mu \equiv \left(1, \frac{x}{|x|}\right).$$  

(1)

In the spherical gauge the line element (1) takes the usual form

$$ds^2 = -(1 + 2\phi) dt^2 + (1 + 2\phi)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$  

(2)

The reason why we referred to $\phi$ as a scalar "potential" becomes evident in the simplest case of a point-like mass $m$ where $\phi = -mG_N/r$ is nothing else but the Newtonian potential.

The metric tensor in (1) defines the graviton field $h_{\mu\nu}$ as

---

$^5l^\mu$ is null both respect the Minkowski background metric and the full line element $\eta_{\mu\nu}l^\mu l^\nu = g_{\mu\nu}l^\mu l^\nu = 0$
\[ \kappa h_{\mu\nu} \equiv -2\phi l_\mu l_\nu, \quad \kappa \equiv \sqrt{8\pi G_N}. \quad (3) \]

The constant \( \kappa \) is introduced in order to provide canonical dimension to the bosonic field \( h_{\mu\nu} \). From equation (3) we see that the tensor structure of the graviton is given by the product of two copies of the null vector \( l_\mu \). In this description the graviton is the double copy of \( l_\mu \).

The "miracle" of the Kerr-Schild gauge is that the Einstein field equations reduce to the single Poisson equation for the unknown field \( \phi \) in flat space:

\[ R_{\mu\nu} - \frac{1}{2} \delta_{\mu\nu} R = 8\pi G_N T_{\mu\nu} \longrightarrow \nabla^2 \phi = -8\pi G_N \left( T_{00} - \frac{1}{2} T_{\mu\mu} \right). \quad (4) \]

We stress that this is an exact result and no weak field expansion is involved.

In this letter we are interested in solving equation (4) with the energy momentum tensor for an Abelian gauge theory.

Perturbative gluon-gluon and graviton-graviton scattering amplitudes show a surprising structural similarity suggesting that the graviton is the double copy of the gauge vector field or, alternatively, the gauge field is the single copy of the graviton:

\[ h_{\mu\nu} \sim \phi l_\mu l_\nu, \quad A_\mu \sim \phi l_\mu. \quad (5) \]

Following this line of reasoning one finds a duality DC relationship between the Newton and Coulomb potential by matching the respective coupling constants

\[ \phi = -\frac{G_N m}{r} \quad \leftrightarrow \quad \phi = -\frac{e}{4\pi r}, \quad (6) \]

\[ G_N m \quad \leftrightarrow \quad \frac{e}{4\pi}. \quad (7) \]

Translating the above relations in a relativistic language one concludes that the Schwarzschild metric is the DC of the Coulomb potential, provided the electric charge is replaced with the effective gravitational coupling as shown above.

On the other hand, one also knows that the metric produced by a static, massive point-like charge is the Reissner-Nordström space-time.

In DC approach the Reissner-Nordstrom geometry has been only reconstructed perturbatively up to \( O(G_N^2) \) by introducing a uniform ball of charged
dust as a source in gravity equations [18]. However, a smooth limit to a point source requires a redefinition of a physical mass in terms of the electric charge and the radius of the ball. In alternative to the DC approach we shall find that one can solve the Einstein equations leading to the general result which is quadratic in the gauge potential and linear in the Newton one. In our approach, mass and charge remain distinct parameters but the graviton can still be described as a NDC of the photon. We shall summarize our results in the concluding Sect. [4].

The first part of our discussion is aimed to clarify the sense in which the Reissner-Nordström geometry can be seen as the NDC of a Coulomb field.

2.1. NDC derivation of the Reissner-Nordström solution

In [19] and [20] the Reissner-Nordstro\"o\m metric was recovered in the framework of Weyl double copy using spinorial language. Here we shall obtain the same metric in the framework of General Relativity in Kerr-Schild gauge. Let us start from the Lagrangian for an Abelian gauge potential $A_\mu$

$$L[A] = -\frac{1}{4} \partial_{[\mu} A_{\nu]} \partial^{[\mu} A^{\nu]} - q J^\mu A_\mu .$$

The vector current $J^\mu$ is the field source and $q$ is the electric charge.

We assume the source to be vanishing everywhere except in $r = 0$. The general form of the electromagnetic field energy momentum tensor reads

$$T_{\mu \nu} = F_{\mu \lambda} F_{\nu}^{\lambda} - \frac{1}{4} \eta_{\mu \nu} F_{\alpha \beta} F^{\alpha \beta} .$$

It is immediate to see that $T^{\mu \mu}_{\mu} = 0$ and we need only $T^{0}_{0}$ in the r.h.s. of the Poisson equation.

The electric field generated by a static charge $q$ is time independent. Thus, both $\partial_0 A_0 = 0$ and $\partial_0 F_{0m} = 0$ and no magnetic field is present $F_{mn} = 0$. Furthermore, if the charge is point-like the current density reads

$$J^\mu \equiv \delta^\mu_0 \delta (\vec{r})$$

and solving Maxwell equations one finds the Coulomb potential

---

5

---

$^6$We adopt the definition:

$$T_{\mu \nu} = -2 \frac{\delta L}{\delta g^{\mu \nu}} + g_{\mu \nu} L .$$

---

5
\( A^0 = -q/4\pi r \) and the radial electric field \( E^r = -q/4\pi r^2 \). For \( T^0_0 \) we find:

\[
T^0_0 = F^0_k F^r_k - \frac{1}{2} F^0_m F^{0m} \tag{10}
\]

By definition the electric field is \( E^m \equiv F^0 m \) and one can write \( T^0_0 \) as

\[
T^0_0 = -\frac{1}{2} \vec{E} \cdot \vec{E} = -\frac{e^2}{32\pi^2 r^4} . \tag{11}
\]

In General Relativity textbooks (11) is inserted in the Einstein field equations leading to the Reissner-Nordström solution in spherical coordinates. Here we introduce a simpler and general way for solving the problem also when the electric field is not divergence free and the r.h.s. is of the form

\[
\nabla^2 \phi = 4\pi G_N \left( \vec{E} \cdot \vec{E} - A^0 \nabla \vec{E} \right) \tag{12}
\]

We notice that

\[
\nabla^2 \left( A^0 A^0 \right) = 2 \left( \nabla A^0 \cdot \nabla^0 A^0 + A^0 \nabla^2 A^0 \right) ,
\]

\[
= 2 \left( \vec{E} \cdot \vec{E} - A^0 \nabla \vec{E} \right) . \tag{13}
\]

Identity (13) allows to write (12) as

\[
\nabla^2 \phi = 2\pi G_N \nabla^2 \left( A^0 A^0 \right) . \tag{14}
\]

There is no need of further steps to solve (14) and write the general solution as

\[
\phi = \phi_0 + 2\pi G_N A^0 A^0 , \tag{15}
\]

\[
\phi_0 = c_1 + \frac{c_2}{r} . \tag{16}
\]

\( \phi_0 \) is the solution of the homogeneous Poisson equation \( \nabla^2 \phi_0 = 0 \). The two integration constants \( c_1, c_2 \) are fixed by the physical conditions defining the problem we are considering:

i) asymptotic flatness implies \( c_1 = 0 \);

ii) the source is a static charge which must be supported by a massive particle. The mass of the particle generates its own gravitational field, thus \( c_2 = \)
The complete solution and the graviton field read

\[ \phi = -\frac{G_N m}{r} + 2\pi G_N A^0 A^0, \quad (17) \]

\[ A_\mu \equiv A^0 l_\mu, \quad A^0 = -\frac{e}{4\pi r}, \quad (18) \]

\[ \kappa h_{\mu\nu} = \frac{2G_N m}{r} l_\mu l_\nu - 4\pi G_N A_\mu A_\nu. \quad (19) \]

Equation (19) displays the Newtonian and Coulombic components of the graviton. It is also evident that each contribution is again the double copy of \( l_\mu \) but each term has a different potential function in front. The gravitational potential appears once in the neutral part of graviton while the electric potential enters twice in the charged term.

The important conclusion of the first part of this paper is that every time the energy density can be written in the form (11) the Einstein-Maxwell equations are solved by (17).

3. Higgs electrodynamics

The Higgs mechanism is a milestone of spontaneously broken gauge theories. It is, therefore, interesting to look for the corresponding gravitational double copy. In the true vacuum the expectation value of the Higgs field is non-vanishing, i.e. \( v_0 \neq 0 \), and the photon field acquires mass. The simplest model exhibiting Higgs mechanism is scalar QED with a symmetry breaking potential. String theory derived effective quantum field theories involving additional dilaton and axion fields are beyond the purpose of this letter. Here we shall study the simplest case.

The dynamics of the heavy photon is described by the effective non-local Lagrangian:

7It may be interesting to note that the non-local Lagrangian (20) can be derived in the path integral formalism. By expanding the original Lagrangian around the true vacuum up to quadratic terms in modulus of the complex scalar field one obtains a Gaussian integral for the phase (i.e. the Goldstone boson) which can be integrated away. The resulting one-loop effective Lagrangian for \( A_\mu \) is (20). It is worth to note that integrating away the Goldstone boson, instead of imposing the unitary gauge, preserves gauge invariance even in the presence of the photon mass.
\[ L[A] = -\frac{1}{4} F_{\mu\nu} \left( 1 - \frac{\mu^2}{\partial^2} \right) F^{\mu\nu} - q J^\mu A_\mu, \quad \mu^2 = e^2 v_0^2. \]  

(20)

The current density \( J^\mu = \delta^\mu_0 \delta \left( \vec{r} \right) \) leads to the single field equation

\[
\left( 1 - \frac{\mu^2}{\nabla^2} \right) \nabla^2 A^0 = q \delta \left( \vec{r} \right)
\]

(21)

which is solved by a Yukawa-type potential

\[ A^0 = -\frac{q}{4\pi r} e^{-\mu r}. \]

(22)

The corresponding electric field is

\[ E^r = -\frac{q}{4\pi r^2} (1 + \mu r) e^{-\mu r}. \]

(23)

In the non-trivial Higgs vacuum electrostatic interaction is short-range and the wavelength of the massive photon represents the screening length. By proceeding as in the previous section we find the following energy momentum tensor

\[ T_{\mu\nu} = F_{\mu\lambda} \left( 1 - \frac{\mu^2}{\partial^2} \right) F^{\nu\lambda} + \eta_{\mu\nu} L. \]

(24)

The resulting electrostatic energy density reads

\[ T^0_0 = -\frac{1}{2} \vec{E} \left( 1 - \frac{\mu^2}{\nabla^2} \right) \cdot \vec{E}. \]

(25)

Using the identity

\[ \vec{E} \frac{1}{\nabla^2} \vec{E} = -A^0 A^0, \]

(26)

(27)

we see that the non-local term in (25) can be written as \( \mu^2 A^0 A^0 \). As \( A^0 \) is a Yukawa type potential satisfying a massive Klein-Gordon type equation we have

\[ \mu^2 A^0 A^0 = A^0 \nabla^2 A^0 = -A^0 \nabla \vec{E} \]

(28)
The general discussion of the previous section allows us to write the gravitational potential as

$$\phi = -\frac{m G_N}{r} + \frac{q^2 G_N}{8\pi^2} \frac{e^{-2\mu r}}{r^2},$$  \hspace{1cm} (29)$$

where $m$ is the mass of the test charge $q$ not to be confused with the photon mass $\mu$.

Thus, the broken Abelian gauge theory has its gravitational dual copy in the form of a screened Reissner-Nordström metric

$$ds^2 = ds^2_{Mink} - 2 \left[ -\frac{m G_N}{r} + \frac{q^2 G_N}{8\pi^2} \frac{e^{-2\mu r}}{r^2} \right] l_\mu l_\nu dx^\mu dx^\nu,$$

$$= - \left[ 1 - \frac{2m G_N}{r} + \frac{q^2 G_N}{4\pi^2} \frac{e^{-2\mu r}}{r^2} \right] dt^2 + \left[ 1 - \frac{2m G_N}{r} + \frac{q^2 G_N}{4\pi^2} \frac{e^{-2\mu r}}{r^2} \right]^{-1} dr^2 + r^2 d\Omega^2.$$  \hspace{1cm} (30)$$

One observes that, in agreement with the short range nature of the electrostatic interaction in the Higgs vacuum, the charge contribution exponentially vanishes at a distance greater than the characteristic screening length ($r > 1/2\mu$) \([30]\) and the metric looks like the Schwarzschild one.

The horizon equation for this geometry is non-linear and cannot be solved in a closed form

$$r_h^2 - 2m G_N r_h + \alpha_q G_N e^{-2\mu r_h} = 0 , \quad \alpha_q \equiv \frac{q^2}{4\pi}.$$  \hspace{1cm} (31)$$

However, we can profit from the presence of two different length scales to get approximate solutions. In fact, we have:

1. the usual Schwarzschild radius $r_h = 2m G_N$

2. the Compton wavelength $\lambda_A = 1/2\mu$ of the (double copy) massive photon contribution to the graviton.

When $r_h > 1/2\mu$, or $m > 1/4G_N\mu$, \([30]\) describes a classical black hole of
radius and temperature given by

\[ r_+ \simeq 2mG_N \left( 1 - \frac{\alpha q}{4m^2G_N} e^{-4\mu mG_N} \right), \quad (32) \]

\[ T_H \simeq \frac{1}{8\pi G_N m} \left( 1 + \frac{\alpha q}{4m^2G_N} e^{-4\mu mG_N} \right). \quad (33) \]

The above results show only exponentially small deviations form the Schwarzschild black hole behavior.

In the opposite regime \( r_h << 1/2\mu (m << 1/4G_N\mu) \), there is no enough mass to produce black hole and the source remains a point-particle.

In the intermediate case \( r_h \sim 1/2\mu \) the horizon is dominated by quantum effects and becomes fuzzy. An appropriate quantum description is needed.

One among various approaches is to mimic the horizon fluctuations in terms of the equivalent oscillatory motion of a point-particle in a suitably chosen potential \([23, 24, 25]\). In recent papers we proposed a self-consistent way to obtain such a potential starting from the classical horizon equation itself\([21, 22, 26]\). In the present model we follow the latter path and obtain the following relativistic wave equation

\[ \frac{1}{r^2} \frac{d}{dr} \left[ r^2 \frac{d\psi}{dr} \right] + \left[ E^2 - \frac{r^2}{4G_N^2} \left( 1 + \alpha q \frac{G_N}{r^2} e^{-2\mu r} \right)^2 - \frac{l(l+1)}{r^2} \right] \psi(r) = 0. \quad (34) \]

The wave function \( \psi(r) \) represents the probability amplitude to find the horizon at radial distance \( r \) from the origin. \( l \) is the angular momentum quantum number. At large distance \( r > 1/2\mu \) the charge is completely screened and not accessible to an asymptotic observer. In this case one recovers the wave equation for a neutral Scharzschild black hole

\[ \frac{1}{r^2} \frac{d}{dr} \left[ r^2 \frac{d\psi}{dr} \right] + \left[ E^2 - \frac{r^2}{4G_N^2} - \frac{l(l+1)}{r^2} \right] \psi(r) = 0. \quad (35) \]

Formula (35) is nothing else but the equation for a relativistic quantum 3D harmonic oscillator with energy spectrum given by

\[ E_{n,l}^2 = \frac{1}{2G_N} \left[ 4n + 2l + 3 \right], \quad n = 1, 2, 3, \ldots, \quad l \leq n - 1. \quad (36) \]

The presence of an electric charge affects only the behaviour of the system below the screening length. In fact, for \( r < 1/2\mu \) the exponential is close to one and the wave equation becomes
\[ \frac{1}{r^2} \frac{d}{dr} \left[ r^2 \frac{d\psi}{dr} \right] + \left[ E^2 - \frac{r^2}{4G_N^2} \left( 1 + \frac{\alpha_q G_N}{r^2} \right)^2 - \frac{l(l+1)}{r^2} \right] \psi(r) = 0 \] (37)

In order to understand the physical meaning of the various terms in (37) it is useful to introduce the effective potential \( U_{\text{eff}}(r) \) as

\[ \frac{1}{r^2} \frac{d}{dr} \left[ r^2 \frac{d\psi}{dr} \right] + \left[ E^2 - \frac{\alpha_q}{2G_N} - U_{\text{eff}}(r) \right] \psi(r) = 0 , \] (38)

\[ U_{\text{eff}}(r) \equiv \frac{r^2}{4G_N^2} + \frac{\alpha_q^2}{4r^2} + \frac{l(l+1)}{r^2} . \] (39)

The first term in (39) is the harmonic potential describing the Planck frequency oscillations of the quantum horizon; the second term accounts for the charge repulsive self-interaction; the third term is the well known centrifugal barrier.

The exact solution of the horizon wave-function reads:

\[ \psi(r) = N_n \left[ \frac{r^2}{2G_N} \right]^s e^{-r^2/4G_N} L_n^{2s+1/2} \left( \frac{r^2}{2G_N} \right) , \] (40)

where, \( L_n^{2s+1/2} \left( \frac{r^2}{2G_N} \right) \) are generalized Laguerre polynomials; \( N_n \) is the normalization coefficient, and the parameter \( s \equiv \left[ \sqrt{\alpha_q^2 + (2l+1)^2} - 1 \right] /2 \)

As expected, one finds a discrete energy spectrum for the quantum horizon given by

\[ E_{n,l}^2 = \frac{\alpha_q}{2G_N} + \frac{1}{2G_N} (4n + 2s + 3 + \alpha_q) , \] (41)

and the ground state energy is:

\[ E_{0,0}^2 = m_{Pl}^2 \left( 2 + \alpha_q + \sqrt{\alpha_q^2 + 1} \right) . \] (42)

We introduced the Planck mass as \( m_{Pl}^2 \equiv 1/2G_N \). In the unit charge case \( q = e, \alpha_e = e^2/4\pi \simeq 1/137 \), \( E_{0,0}^2 \) can be expanded in powers of \( \alpha_e \) and one finds

\[ E_{0,0}^2 = (3 + \alpha_e) m_{Pl}^2 + O(\alpha_e^2) . \] (43)
Let us also consider the spectrum of highly excited states with $n \gg 1$, $l \gg 1$. In this limit $s \rightarrow l$ one has

$$G_N E_{nl}^2 \simeq 2n + l + \alpha_e. \quad (44)$$

Equation (44) allows to express the angular momentum $l$ as a "function" of energy $E_n$. This behavior is characteristic of Regge trajectories used to describe strongly interacting resonances in hadronic physics. In our case we find

$$l \simeq \alpha' E_{nl}^2 + \beta_n, \quad (45)$$

$$\alpha' \equiv G_N = \frac{1}{2m_{Pl}^2}, \quad (46)$$

$$\beta_n = -2n - \alpha_e. \quad (47)$$

Equation (45) describes a family of trajectories $l = l(E^2)$ with angular coefficient $\alpha'$ and intercept $\beta_n$. The importance of a relation like (45) is that it naturally arises in string theory where the Regge slope $\alpha'$ is related to the string tension $\rho$ through the relation $\rho = 1/2\pi\alpha'$. Therefore, we arrive at the conclusion that higher excitations of the quantum horizon display a string-like nature. The correct counting of the black hole micro-states in agreement with the Area Law [27] qualified String Theory as a legitimate candidate to account for quantum black holes. From this perspective highly excited strings shows thermodynamical properties analogous to those of black holes [28, 29, 30]. In this paper we arrive the same conclusion following an inverse path: first we quantized the black hole horizon and then discovered that highly excited states behave like strings.

4. Discussion and conclusions

In this paper we have reviewed the double copy relationship connecting gravitons and gauge fields. In its original version this correspondence was inspired by the structural similarity between corresponding perturbative scattering amplitudes in Yang-Mills theory and General Relativity [31]. Once known in one case a simple "recipe" allows to write the amplitudes in the

---

*For a unit charge $\alpha_e$ is small but we keep it in the expansion in order to appreciate first order correction with respect to the neutral case.*
other one without the need of lengthy calculations.

In the static limit one-boson exchange amplitudes gives the corresponding classical interaction potentials. In this way, one establishes a duality relation between Coulomb and Newton potentials. By implementing the unique properties of the Kerr-Schild coordinate system one reaches the conclusion that a similar duality relation connects the Coulomb field and the Schwarzschild metric despite the absence of any electric charge in the latter. It follows that it is not immediate to accommodate into this framework charged solutions of the Einstein equations such as the Reissner-Nordström one and a non-perturbative approach is necessary. The fundamental feature of perturbative double copy is to express the tensor structure of a vector gauge potential and the graviton in terms of the same null vector $l_\mu$. The only distinction between the two interactions is encoded into different scalar potentials. In summary:

Perturbative double copy

\begin{align*}
A_\mu &= \phi_{\text{gauge}} l_\mu , \\
\kappa h_{\mu\nu} &= -2\phi_{\text{grav}} l_\mu l_\nu .
\end{align*}

General Relativity

\begin{align*}
A_\mu &= \phi_{\text{gauge}} l_\mu , \\
\kappa h_{\mu\nu} &= -2\phi_{\text{grav}} l_\mu l_\nu + 8\pi G_N \phi_{\text{gauge}}^2 l_\mu l_\nu .
\end{align*}

Notice that $\phi_{\text{grav}}$ is the same both in (49) and (51). However, in the perturbative approach it is obtained through a substitution rule between coupling constants while in General Relativity it represents the solution of the homogeneous Poisson equation. The second term in (51) is the contribution from the energy density stored in the electric field which is missing in (49). It enters equation (4) through the energy-momentum tensor.

Let us remark that the form (12) of $T^0_0$ is a sufficient condition to obtain a NDC-type solution for the gravitational field. We checked that (51) holds in Einstein-Maxwell theory even if the gauge symmetry is spontaneously broken. In this case the electric field contribution is damped due to the presence of the photon mass in the Higgs true vacuum. We considered the resulting screened Reissner-Nordström black hole both in the "large" and "small"
limits. When the horizon radius and the massive photon Compton wavelength become comparable a quantum description is needed. We determined the energy spectrum for this type of object and found interesting analogy with the behaviour of highly excited strings.

References

[1] R. Utiyama, *Phys. Rev. D* **101** (1956), 1597

[2] T. W. B. Kibble, *J. Math. Phys.* **2** (1961), 212-221

[3] J. M. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998), 231-252

[4] Bern Z., Dixon L. J., Perelstein M., Dunbar D. C. and Rozowsky J. S., *Class. Quant. Grav.* **17** (2000), 979-988.

[5] Bern Z., Dennen T., Huang Y. t. and Kiermaier M., *Phys. Rev. D* **82** (2010), 065003.

[6] Bern Z., Carrasco J. J. M. and Johansson H., *Phys. Rev. Lett.* **105** (2010), 061602.

[7] Bern Z., *Subnucl. Ser.* **50** (2014), 157-175.

[8] Bern Z., *Mod. Phys. Lett. A* **29** (2014) no.32, 1430036.

[9] Bern Z., “Perturbative Quantum Gravity as a Double Copy of Gauge Theory and Implications for UV Properties,” doi:10.1142/9789814623995_0010.

[10] Bern Z., Carrasco J. J., Chen W. M., Johansson H. and Roiban R., *Phys. Rev. Lett.* **118** (2017) no.18, 181602.

[11] Monteiro R., O’Connell D. and White C. D., *JHEP* **12** (2014), 056.

[12] Monteiro R., O’Connell D. and White C. D., *Int. J. Mod. Phys. D* **24** (2015) no.09, 1542008.

[13] White C. D., *Contemp. Phys.* **59** (2018), 109.

[14] Bahjat-Abbas N., Luna A. and White C. D., *JHEP* **12** (2017), 004.
[15] Bahjat-Abbas N., R. Stark-Muchão and White C. D., *JHEP* **04** (2020), 102.

[16] Alkac G., Gumus M. K. and Tek M., “The Classical Double Copy in Curved Spacetime,” arXiv:2103.06986 [hep-th].

[17] R. Gonzo and C. Shi, Phys. Rev. D **104** (2021) no.10, 105012

[18] D. A. Sardelis, “The tree graphs of quantum gravity and the Reissner-Nordström solution,” IC/73/186.

[19] D. A. Easson, T. Manton and A. Svesko, Phys. Rev. Lett. **127** (2021) no.27, 271101

[20] H. Godazgar, M. Godazgar, R. Monteiro, D. Peinador Veiga and C. N. Pope, JHEP **11** (2021), 126

[21] Spallucci E. and Smailagic A., “Semi-classical approach to quantum black holes,” Published in: ”Advances in black holes research” p.1-26, Ed.: A. Barton, Nova Science Publisher, Inc. (2015), ISBN: 978-1-63463-168-6, e-Print: 1410.1706 [gr-qc].

[22] Spallucci E. and Smailagic A., “A particle-like description of Planckian black holes,” Published in: ”Quantum Gravity: Theory and Research”, Ed.: Mitchell B., Nova Science Pub. Inc. (2017), ISBN: 978-1-53610-798 e-Print: 1605.05911 [hep-th].

[23] Casadio R. and Scardigli F., *Eur. Phys. J. C* **74** (2014) no.1, 2685.

[24] Casadio R., Micu O. and Stojkovic D., *Phys. Lett. B* **747** (2015), 68-72.

[25] Casadio R., Micu O. and Scardigli F., *Rom. Rep. Phys.* **68** (2016) no.3, 923.

[26] Spallucci E. and Smailagic A., *Phys. Lett. B* **816** (2021), 136180.

[27] Susskind L., “Some speculations about black hole entropy in string theory,” arXiv:hep-th/9309145 [hep-th].

[28] Horowitz G. T. and Polchinski J., *Phys. Rev. D* **55** (1997), 6189-6197.

[29] Damour T. and Veneziano G., *Nucl. Phys. B* **568** (2000), 93-119.
[30] Veneziano G., *JHEP* **11** (2004), 001.

[31] Z. Bern, J. J. Carrasco, M. Chiodaroli, H. Johansson and R. Roiban, “The Duality Between Color and Kinematics and its Applications,” [arXiv:1909.01358 [hep-th]]