SIZE AND DURATION OF THE HIGH-FREQUENCY RADIATOR
IN THE SOURCE OF THE DECEMBER 26, 2004 SUMATRA EARTHQUAKE

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Abstract

We recover the gross space-time characteristics of high-frequency (HF) radiator of the great Sumatra-Andaman islands earthquake of Dec. 26, 2004, ($M_w=9.0-9.3$) using the inversion of parameters describing the time histories of the power of radiated HF $P$ waves. To determine these time histories we process teleseismic $P$ waves at 37 BB stations, using, in sequence: (1) band filtering in the bands 0.4-1.2, 1.2-2, 2-3 and 3-4 Hz; (2) calculation of squared attenuation-corrected acceleration wave amplitudes, making “power signal”; (3) elimination of distortion related to scattering and expressed as $P$ coda. In step (3) we employ, as an empirical Green function, the power signal determined from an aftershock, from which we construct an inverse filter, and apply it to the recorded power signal. We thus recover the source time function for HF power, with a definite end and no coda. Three parameters are extracted from such signals: full (“100%”) duration, temporal centroid, and 99% duration. Through linear inversion, station full durations deliver estimates of the rupture stopping point and stopping time. Similarly, signal temporal centroids and 99% durations can be inverted to obtain the position of the space-time centroid of HF energy radiator and of the point corresponding to the discharge of 99% of the energy. Inversion was successful for the three lower-frequency bands and resulted in the following joint estimates: source length of 1100±220 km (100%) and 800±200 km (99%), source duration of 690 s (100%) and 550 s (99%). The stopping point differs insignificantly from the northern extremity of the aftershock zone. Spatial HF radiation centroid is located at the distance of about 400 km at the azimuth N327W from the epicenter. Rupture propagation velocity estimates are 1.4-1.7 km/s for the entire rupture and 2.3 km/s for its southern, more powerful part. An interesting detail of the source is that the northernmost 300 km of the rupture radiated only 1% of the total HF energy.
INTRODUCTION

Recent giant Sumatra-Andaman Islands earthquake of Dec. 26, 2004 was a challenge for students of earthquake source in particular because it is difficult to isolate P- and S-waves when the source duration is very long. To alleviate this problem one can use P-wave signal at high frequencies (HF signal, roughly, at frequencies $f$ above 0.5 Hz) where it is relatively free from contamination by later phases (Ni et al. 2005; Lomax 2005). However, raw teleseismic HF signals are always distorted by wave scattering along the propagation path. This distortion is well known as P coda. Therefore, to analyze high-frequency (HF) signal efficiently, one has to apply an adequate technique. A key point is to treat body wave as a random signal. This approach, proposed by Gusev and Pavlov (1978), has been developed in the last decades both for teleseismic (Gusev and Pavlov 1991, 1998) and regional (Zeng et al.1993; Kakehi and Irikura 1996, Nishimura et al. 1996) data. As usual, with source studies, a number of assumptions are needed to permit a particular analysis. First of all, instead of the routine treatment of a source as an aggregate of elements localized in space-time, whose amplitudes at the receiver are additive, we assume that such elements are spots capable to produce random HF signals that are statistically independent (uncorrelated) and thus are additive in terms of their power (or mean squared amplitude). This assumption is a standard one in the study of such phenomena as common (non-laser) light, acoustic or hydroacoustic noise sources etc, but it is still somewhat exotic for source seismology. Therefore we assume that each spot of a source radiates HF seismic wave power. The entire source is naturally described in this case by space-time distribution of radiated energy per unit area and unit time, or, shortly, by its luminosity function, borrowing the term from light engineering. Therefore, space-time parameters of luminosity function are what we seek for.

For a surface-focus event of considerable magnitude, one can assume its vertical dimensions negligible, and seek for luminosity distribution in 3D ($x_1 \times x_2 \times t$). To analyze recorded signals, one must relate them to luminosity distribution over the source. For the well-known case of long-period body waves, source signal radiated along a certain ray arrives to a station with little contamination; and the corresponding Green function is approximately delta-like. For the HF signal, however, scattering and conversion result in significant P-coda energy that complicates this clear picture. In addition, it is impossible to sort out $P$, $pP$ and $sP$ contributions within the complete HF signal of an earthquake of considerable magnitude. To overcome this problem, Gusev and Pavlov (1991) proposed to employ the additivity of power and to deconvolve the recorded power signal using an empirical Green function for power estimated from the record of an aftershock. For the common case of sources of earthquakes with $M \geq 7.5$, when the vertical extent of the source is small as compared to horizontal one, the local mean source depth does not change significantly along the source length. In our case, this is particularly acceptable because in the deconvolution we use a rather big time step. The noisy spectral deconvolution of Gusev and Pavlov (1991) was radically stabilized by adding positivity constraint (Gusev and Pavlov, 1998).

After the reconstruction of the source power pulse, one can analyze signals over the focal sphere to deduce the space-time structure of the source. In the following we confine our study only to two (vectorial) source parameters: (1) rupture stopping point/moment; and (2) spatio-temporal centroid, or, more formally, the order 1 normalized spatio-temporal power moment of luminosity function. Generally speaking, the determination of the stopping point may be an ambiguous problem; however it becomes well-defined for the case of purely unilateral rupture; fortunately, this is the case for the Dec. 26, 2004 event. Source centroid, in
contrast, is a model-independent entity. In addition to these two relatively well-defined parameters, we also make inversion for the 99% duration, to obtain some additional information regarding the source.

METHOD

We give here a compressed description only; see Gusev and Pavlov (1988, 1991) for more details. Let us consider first the case of an homogeneous scattering medium. Let a finite planar source \( \Sigma \) of diameter \( D \) with area element \( d\Sigma \) be described, for a certain frequency band \([f_o-\Delta f/2, f_o+\Delta f/2]\) by the source luminosity function \( L(x, t, f_o, \Delta f) \). In the following we assume both central frequency \( f_o \) and bandwidth \( \Delta f \) fixed, and thus omit these arguments. Let the hypocenter be located within \( \Sigma \) and be the origin of the Cartesian reference system, and the zero-time reference to coincide with the source nucleation time (=origin time). Let us consider a far-field receiver located at \( y \), on a ray \( r \) (\(|r|=R>D\)). We introduce the Green function for power \( G_0(y, t|_x) \) as the power that reached the receiver at \( y \) starting from a unit source area located at \( x \), produced by a spike-like luminosity burst with unit integral, fired at \( t=0 \). We assume, further, that the dependence of \( G_0(y, t|_x) \) on the argument \( x \) is weak, and can be ignored; thus we can substitute \( G_0(y, t|_x) \) with \( G(y, t) \).

Informally, we assume that the mean scattering response of the medium is the same for any \( d\Sigma \). Now, the contribution from \( d\Sigma \) to body wave power at \( y \) can be written as a convolution over time:

\[
dW(y, t) = \int_{-\infty}^{\infty} G(y, t - \tau + x \cdot r / c) L(x, \tau) d\tau
\]  

where \( c \) is the body wave velocity. When there is no scattering, \( G(y, t) \) reduces to \((\Re / R)^2 \delta(t)\) where \( \Re \) is the point-source radiation pattern. Integrating (1) over \( \Sigma \) one obtains:

\[
W(y, t) = \int_{-\infty}^{\infty} G(y, t - \tau + x \cdot r / c) L(x, \tau) d\tau d\Sigma
\]  

The order of integration in (2) can now be changed, to result in:

\[
W(y, t) = \int_{-\infty}^{\infty} \left[ \int_{\Sigma} L(x, t - \tau + x \cdot r / c) d\Sigma \right] G(y, \tau) d\tau
\]  

The outer convolution in (3) can be inverted if one knows \( G(y, t) \) and can construct the corresponding deconvolution operator. Let its time domain representation be \( G^{-1}(y, t) \). The result of deconvolution is the power signal in a non-scattering medium:

\[
W_0(y, t) = G^{-1}(y, \cdot) * W(y, \cdot) = \int_{\Sigma} L(x, t + x \cdot r / c) d\Sigma
\]  

In our compact derivation we skipped some less important points, however some of them must be mentioned. First, the diameter of a source element \( d\Sigma \) cannot be infinitely small – it cannot be smaller than the “source correlation radius”, order of or larger than half wavelength; thus the integral representation (2) is only an approximate one. Second, when
considering \(L(),G()\) and \(W()\) functions, one must distinguish between mean (ensemble average) power time histories and observed realizations (sample functions). Equations (1-4) are written for ensemble averages and when they are used for practical inversion, one inevitably meets with a significant fluctuational noise (related to the finiteness of the bandwidth \(\Delta f\)). Third, bandwidth \(\Delta f\) cannot be too small, to provide signal correlation time \(1/\Delta f\) much less than source process duration.

With \(W_0(y,t)\) known at several rays, one can recover the parameters of the source space-time structure. In particular, it is interesting to determine the centroid of \(L(x,t)\), or 4-vector of normalized order 1 power moments \(\{M_0, M_1, M_2, M_3\}\) (Gusev and Pavlov, 1988):

\[
F(g,z,n) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,t)z^n d\Sigma d\tau;
\]

\[
M_i = F(L, t, 1)/F(L, 1, 0);
\]

\[
M_i = F(L, x/c, 1)/F(L, 1, 0)
\]

To determine the source centroid, one solves the linear system of equations:

\[
M_k - (1/c)(r_{ik}M_1 + r_{2k}M_2 + r_{3k}M_3) = e_k
\]

where \(k\) is the number of the station/ray/receiver, \(k=1,2,...,N\), and

\[
e_k = \int W_0(t)dt / \int W_0(t)dt
\]

is the temporal centroid of the observed body-wave power pulse at the \(k\)-th receiver.

The same Equation (6) holds for the space-time position (4-vector) of any common feature discernible on many records and caused by a certain individual event of the fault motion, if this event is definitely localized in space-time. In this case, \(e_k\) must be replaced by the delay of the feature with respect to the first arrival. Traditionally, this approach has been widely applied to "starting phases", "stopping phases" and "subevents". In the following, we shall use total signal duration \(T_{fin}\) to determine in this way the 4-vector of the stopping point of the rupture; we denote its space-time location as \(\{F_t F_1 F_2 F_3\}\). Additionally, we determine, for each \(W_i(t)\) signal, the "99% point" \(T_{99}\), such that the integral of \(W_i(t)\) over the interval \([0, T_{99}]\) makes 99% of the total energy of the deconvolved signal \(W_i(t)\). This parameter may indicate the space-time point where/when practically all the energy has already been emitted. For the case of elongated, narrow fault area and a relatively small rise time, this parameter can be considered as well-defined. The related 4-vector is denoted \(\{P99_t P99_1 P99_2 P99_3\}\).

To pass to the case of the real Earth, we introduce the coordinates \(\{x,y,z\}\) along N, E, Z directions. In the following we shall use the constraint \(M_z=M_3=0\), reducing the number of unknowns in (6) to three. This is justified by the relatively small source size along the z direction. In the case of a small source size it would be sufficient to replace the first two \(r_{ik}/c\) coefficients in (6) by the corresponding travel time derivatives \(dT/dx = \cos(Az)dT/d\Delta; \ dT/dy = \sin(Az)dT/d\Delta; \ dT/dz\). However, the actual size of the spatial part of the discussed space-time vectors is too large to rely on linearization. Therefore, an appropriate non-linear least-squares procedure has been employed.
For the case of a homogeneous scattering medium, we could believe that the Green function \( G_0(y, t \mid x) \) is merely a combination of the direct body wave and scattered coda. For the real Earth, however, \( G_0(y, t \mid x) \) also includes deterministic \( pP, sP \), and similar phases. This means that to extract the “clean” \( G_0(y, t \mid x) \) from observations we must use an aftershock with moment tensor orientation and depth preferably similar to that of the main shock. Possible biases related to non-identical moment tensor orientation of mainshock and aftershock (that, in theory, may result in variations of relative amplitudes of \( P, pP \) and \( sP \) phases) were ignored, in particular because of the known inefficiency of standard radiation pattern factors to predict the mentioned relative amplitudes for HF \( P \)-waves of large earthquakes. The aftershock used was selected to have the hypocenter depth near to the mainshock centroid depth (30 km), and to represent a low-angle thrust whose strike is only 35° different from that of the mainshock. These choices are however not very critical in our case since to stabilize the deconvolution we shall use a rather large time step, of 25 s. With such a time step, all differences in time delays of \( P, pP \), and \( sP \) among source elements located at various depths are negligible because the entire \( P \)-wave group is represented by a single sample of the time history.

According to the described approach, the following processing algorithm has been developed. A \( P \)-wave record is corrected for instrument and, approximately, for path attenuation, and converted to acceleration signal whose amplitude spectrum is nearly flat. This property is desirable as it provides the widest signal effective bandwidth and thus the maximum suppression of fluctuational noise. The acceleration signal is passed through four pass-band filters 0.4-1.2, 1.2-2, 2-3 and 3-4 Hz, identified in the following by their central frequencies 0.8, 1.6, 2.5 and 3.5 Hz. In each band, the “raw” signal power \( A_{rs}(t) = a_{rs}^2(t) \) is estimated as a squared modulus of the analytical signal: \( a_{rs}^2(t) = u^2(t) + (H[u(t)])^2 \) (where \( H[\cdot] \) denotes Hilbert transform and \( u(t) \) is the filtered acceleration). A similar operation is applied to a segment of noise before \( P \)-wave arrival, and the average power of noise, \( A_{n,av} = \bar{A}_n(t) \) is subtracted from the samples of “raw” signal power, to obtain an initial signal power \( W(t) \). The samples of \( W(t) \) are then averaged twice, over intermediate and large time bins. In the first averaging we use a bin size \( dt_1 = 1 \) s; this averaging generates a jagged envelope signal that is used to set, interactively, the \( P \)-wave onset time. In the second averaging, with bin size \( dt_2 = 25 \) s, a signal representation is obtained that is suitable for the deconvolution procedure. This \( dt_2 \) value has been selected by trial and error to provide a tradeoff between a detailed representation of the source signal, on one side, and a stable result of deconvolution, on the other. Note that smoothing was not applied to data, only binning and averaging.

When the above processing is completed for the mainshock and the aftershock, deconvolution is performed. It consists in the numerical solution of the convolution equation:

\[
W_1(.) \ast W_a(.) = W_m(.)
\]

where \( W_m(t) \) and \( W_a(t) \) are known mainshock and aftershock time functions at a station, and \( W_1(t) \) is an unknown source signal for the mainshock. Solution is sought for in time domain using non-negative least squares. With \( W_1(t) \) time history of station \( k \) at hand, we determine: \( P \)-wave signal end time, centroid time \( e_k \), and the 99% point \( T_{99} \). Equation (8) is valid for our purpose since we assume a laterally homogeneous Earth, along each path.
DATA SELECTION AND PROCESSING

About 50 P-wave records of the mainshock (Dec. 26, 2004 at 005853, instrumental hypocenter 3.30°N, 95.98°E, depth 30 km, \(M_w(HRV)=9.00\)) on BHZ channels of GDSN stations have been acquired through IRIS DMS data centre. In selecting data we tried to avoid severely non-uniform data coverage over a focal sphere, and also to minimize the number of coastal stations with, usually, low signal-to-noise (S/N) ratio for the higher-frequency bands. An unexpectedly high number of stations have shown significantly distorted HF signal, with isolated or periodic spikes and pulses, HF noise bursts and other peculiarities of probably instrument/recorder origin. The 37 better-quality records were selected for further analysis.

More problems appeared when selecting aftershock records. Recorder problems are rare, but noise is too high in many cases, most severely at higher frequencies. Some larger-magnitude aftershocks were discarded as having too long source duration. Among the data within the optimal magnitude span, that is, of \(m_b=5.8-6.5\), no event was acceptably recorded on all stations simultaneously. Abstractly speaking, there is the possibility to combine the results from the deconvolution performed using different aftershocks for different stations; but this approach could not be followed for the following reason. We found that even in this lower-magnitude range, and in cases of matching depth and moment tensor orientation, the event duration can vary significantly from one aftershock to another. (We ascribe this observation, conceivably, to location-dependent water column resonances/Pwp phases.). Our way to overcome this difficulty is primitive. When the P-wave of our “preferable” aftershock (Feb. 26, 2005 at 125652, hypocenter 2.91°N, 95.59°E, depth 36 km, \(M_w(HRV)=6.7, m_b=6\)) at a certain station is of acceptable quality, we use it for deconvolution, as planned. If the S/N ratio is too high, we use instead the record of the same aftershock, but from another station. In all such cases, we use the record of KURK whose duration is intermediate among the various stations. To illustrate possible errors related to such a replacement, in Fig. 1 we compare \(e_k\) and \(T_{fin,k}\) values obtained using the “own” aftershock record at each station, against the case when the KURK aftershock record was used. This comparison is presented for the 25 stations where the aftershock was recorded acceptably. One can see that the distortion is limited. In numerical terms, rms relative error related to such replacement was estimated to be about 7% for \(T_{fin,k}\) and about 6% for \(e_k\). This suggests that for the other 12 stations, where we do not have usable records of the same aftershock, related distortion is, probably, tolerable.

In Fig. 2 we illustrate our processing procedure. One can see band-filtered HF signals whose duration is much longer than S-P time. Envelopes clearly vary from band to band, justifying the independent processing of bands. Accurate determination of the stopping point of the record is impossible (as usual) because of P-wave coda.

In Fig. 3 we show the example power signals of the mainshock and of the aftershock with one-second resolution. Evident is the low signal level of the mainshock after about 450 s from onset, mainly for the three highest frequency bands. In Fig. 4 we show, for each frequency band, the same signals in 25-s bins, and the results of deconvolution (\(W_1(t)\)). The positivity constraint imposed during deconvolution might result in an imperfect fit of “observed” \(W_m(t)\) and “fitted” \(W_{mf}(t)=W_1(.)\ast W_a(.)\) signals. In Fig 4a we actually plotted in the same strip of the plot both traces: observed \(W_m(t)\) and fitted \(W_{mf}(t)\); one can see that visually they are indistinguishable, showing quite acceptable fit.

One could expect a considerable dispersion of the estimates of \(T_{fin}\) among different frequency bands, especially in view of the low amplitudes in the final part of the deconvolved
signal. Actual results were encouraging: $T_{fin}$ values of various bands match well in the overwhelming majority of cases. Among 27 stations with three or four $T_{fin}$ estimates, some triple of estimates (all multiples of 25 s) coincided in 25 cases. This fact has a methodological importance; it suggests that deconvolution results are rather reliable. In addition to this general match, we must emphasize the particular match between $T_{fin}$ values for the 0.8 Hz band and for the other bands. This fact indicates that S-wave contamination of the 0.8 Hz band, real (though only marginally noticeable) has no practical effect on duration estimates.

All $W_{1}(t)$ signals for the 1.6 Hz band are shown on Fig.5. Despite considerable scatter, systematic variation of $T_{fin, k}$ and $e_k$ with azimuth is seen, indicating Doppler effect from the approximately northward rupture propagation.

Sets of $T_{fin, k}$, $e_k$ and $T_{99, k}$ values (Table 1) were processed by non-linear least squares as mentioned above, to result in vectors: $\{F_i, F_x, F_y\}$ or $F$ for stopping time-location, $\{M_{1i}, M_{1x}, M_{1y}\}$ or $M_{1}$ for centroid, and $\{P_{99i}, P_{99x}, P_{99y}\}$ or $P_{99}$ for 99% point. This last vector is well-defined for the case of a point radiator propagating along a line source (straight or curved); in such a model it is a point on the line that corresponds to (cumulative) 99% of the total radiated energy. Examples of the least squares fit can be seen in Fig 6, where, despite a considerable scatter, the Doppler effect is clearly seen.

The inversion resulted in the estimates of $F$, $M_{1}$ and $P_{99}$ vectors (Table 2). Each line of estimates in Table 2 is followed by a line of their standard deviations determined from linearized least squares (and by propagation of errors when needed). In the header, $N$ is the number of stations, $e_{av}$ is the average value of $e_k$ and under $e_{av}$, the standard deviation of the set of $e_k$ is given. The letter $B$ is a token, to be understood as $F$, $M_{1}$ or $P_{99}$ depending on the particular line. The components of the vector are in columns $B_T$, $B_N$ and $B_E$. Columns $B_L$, $Bazim$ and $BV$ contain the estimates for the length of the spatial part of the vector, its azimuth, and propagation velocity. In the sideheading, a subscript denotes the frequency band. The results are more or less consistent among the three independent lower-frequency bands, but of limited accuracy as manifested in considerable error estimates. The graphical representation of the spatial part of these vectors is given in Fig 7. For the 3.5 Hz band, inversion results are dubious and omitted in Table 2; the main cause of this unsuccessful inversion is almost complete lack of data at the southern azimuths, combined with the reduced total number of usable stations (only 25).

In addition to single-band estimates, we made combined estimates based on all usable bands (see Fig 7). The combined $T_{fin}$ estimates, denoted $T_{fin, cmb}$ in Table 1, were obtained for each station as the maximum values among four $T_{fin}$ values of individual bands. We believe this approach provides the most reliable estimate of source duration. Therefore, from $T_{fin, cmb}$ we determined estimates for the $F$ vector (the final lines of Table 2). As for combined $M_{1}$ and $P_{99}$ estimates, they were obtained by averaging corresponding vectors estimated from data of three individual lower-frequency bands.

RESULTS

The most clear qualitative result is the reasonable match (quite within the error bounds) between our estimate for the stopping point of the rupture, and the northern extremity of the 25-hour aftershock cloud. Independent estimates for individual bands generally support this result. Our estimate for the source length is about 1100 km.
The radiation capability (“luminosity”) of the HF radiator, considered as a radiating-line source, is highly non-uniform along the rupture length. 99% of HF energy was radiated by the southern 800-km segment of the fault. The rest 1% was radiated by the northern 300 km low-luminosity segment; it roughly corresponds to the “Andaman segment” of Lay et al.(2005).

As for the centroid position, it is located approximately in the middle of the southern 800-km segment, suggesting no strong asymmetry of luminosity along this segment. One can notice a marked mismatch between centroids for different frequency bands. This mismatch seems to be a real feature of the HF radiator.

The temporal parameters of the HF radiator are determined with high accuracy. The estimate of full duration $F_t$ is about 690 s. The real accuracy of this value must be around 15 s, because the standard errors shown in Table 2 must be combined with the discretization error that can reach $dt/2=12.5$ s. The 99% duration is about $(550 \pm 15)$ s and the centroid delay time (1st temporal moment) $M_1t$ is about $(215 \pm 15)$ s. (True duration values are insignificantly larger because deconvolution slightly squeezed the signals, by half-duration of the aftershock).

Estimates of the apparent rupture velocity from $P99_i$ and $F_i$ vectors and for various frequencies do not differ significantly, giving the range 1.5-1.7 km/s for the joint estimate. Rupture velocity associated with centroid shows noticeable frequency dependence. Taking the average, we obtain $2.25$ km/s – a value somewhat larger than the previous one. This suggests that propagation of the rupture decelerated in the northern half of the rupture length.

DISCUSSION

Several rupture size and duration estimates have been published for the Dec. 26, 2004 event. We report, for comparison, the relevant ones in Table 3. A reasonable match for the length estimate is seen. However, our 100% duration estimate seems to be significantly larger than published ones (and the velocity estimate, in proportion, lower). On the other hand, our 99% duration estimate fits most of the other estimates quite well. Thus, the difference seems to lie in our capability to observe the very tail part of the source signal that contributes significantly to the duration but only hardly to the total energy. This explains well the mismatch with respect to the results of Ammon et al. (2005), Ni et al. (2005) and Lomax (2005) who used Doppler-effect techniques comparable to ours, but employed simpler techniques for the identification of the end point of the envelope.

All the four results based on local array analysis give estimates of about 500 s for the moment when the rupture reached its northern extremity. Low amplitude “coda”, with separate individual bursts, can be seen on T-phase records of the hydroacoustic signals shown in Fig. 2b of Tolstoy and Bohnenstiehl (2005) during the late time window (after 2650 s of their time scale), and this signal may be the manifestation of late low-energy HF signal in the T-wave signature. Nevertheless, the clear difference of about 200 s with respect to our estimate needs additional comments. Such a mismatch would be difficult to explain if one assumes that the rise time is small when compared to the propagation time. This, usually true, assumption (Heaton 1990) can however be violated in the special case of the Dec. 26, 2004 event. As discussed by Ammon et al. (2005), Lay et al. (2005) and Stein and Okal (2005), normal modes, tsunami and geodetic data indicate significant slow slip (or equivalently fast afterslip) in the northern segment of the rupture zone, with time scales considerably longer.
than 500 s. In such a case we have a good reason to speculate that the initial part of this slip (in the interval 500-800 s after origin time) might be accompanied by low-amplitude HF radiation. Lay et al. (2005, Fig 7), when they construct their tsunami source model, assume a rupture velocity of 2.0 km/s for the southern 745 km of the fault, but only as low 0.75 km/s in the northern segment. The average velocity in this case is about 1.1 km/s (even lower than our estimate), and can be translated to the stopping time value of 1130 s. Therefore, our late stopping time estimate may well reflect reality. Nevertheless, these considerations are not completely conclusive. Tsai et al. (2005) present evidence against the concept of delayed slip with unusually long time scales. Their estimate for the total slip duration is only about 550 s. However, this result was obtained using a limited frequency band (periods in the 200-500 s band only) and thus cannot be considered conclusive as well.

The estimates for the position and delay of the source centroid are compiled in Table 4. The initial Harvard CMT estimate of the seismic moment centroid suggests a systematic shift of the centroid position with increasing frequency. Our own estimates of the centroid position for various frequencies show the same tendency. For 0.8 and 2.5 Hz bands in particular, from Table 2 one can derive that the significance level for the hypothesis $M_{1L_{0.8}} < M_{1L_{2.5}}$ is about 10%. However, the revised estimate of the seismic moment centroid by Tsai et al. (2005), as well as the estimate of Stein and Okal (2005) makes this idea much less certain. One possibility to clarify this point is the application of the multiple-band analysis to the hydroacoustic data.

**CONCLUSION**

Using deconvolved time histories of the seismic high-frequency power radiated in different directions from the source of the Dec. 26, 2004 Sumatra-Andaman earthquake we successfully determined the positions of the final point, of the 99% point and of the centroid for the HF radiator in the source of this earthquake. This was done in parallel for three frequency bands, with comparable results. Our estimates do not contradict those obtained in other studies of the same event, and, in combination with these studies, provide a consistent joint picture. An interesting feature of the source is very low HF radiation level from its 300-km-length northern segment.

From a methodological viewpoint, we consider our detection of the tail part of the source signal (that would totally drown in coda without our deconvolution procedure) as a significant success of our technique of data analysis. “Cleaning” HF P-wave power signal by suppressing scattered-wave distortions proved to be a valuable tool that permits to resolve important details of the structure of the HF radiator.

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Table 1. Temporal parameters derived from power signals of individual stations

| Sta. | Az°  | ∆° | dT/dΔ | Tfin cmb | Tfin 0.8 | Tfin 1.6 | Tfin 2.5 | e0.8 | e1.6 | e2.5 | e3.5 | T99 0.8 | T99 1.6 | T99 2.5 | T99 3.5 |
|------|------|----|-------|---------|---------|---------|---------|------|------|------|------|---------|---------|---------|---------|
| TLY  | 6.5  | 48 | 7.69  | 675     | 675     | 675     | --      | 231  | 187  | 234  | --   | 583     | 531     | 547     | --      |
| TIXI | 10.5 | 71 | 6.04  | 550     | 550     | 550     | 550     | 217  | 206  | 233  | 227  | 422     | 439     | 443     | 448     |
| ULN  | 10.6 | 45 | 7.92  | 650     | 650     | 650     | 650     | 247  | 183  | 188  | 182  | 571     | 474     | 485     | 464     |
| BILL | 21.1 | 79 | 5.41  | 700     | 700     | 700     | 700     | 227  | 198  | 184  | 224  | 626     | 449     | 448     | 445     |
| BJT  | 23.8 | 40 | 8.24  | 500     | 500     | 500     | 500     | 141  | 133  | 159  | 169  | 456     | 432     | 440     | 475     |
| MA2  | 26.2 | 69 | 6.14  | 650     | 650     | 650     | 650     | 227  | 215  | 171  | 186  | 460     | 470     | 465     | 447     |
| MDJ  | 30.7 | 50 | 7.54  | 575     | 575     | 575     | 575     | --   | 161  | 180  | 201  | 533     | 524     | 506     | --      |
| PET  | 34.5 | 71 | 6.05  | 600     | 600     | 600     | 600     | --   | 182  | 137  | 141  | 531     | 473     | 476     | --      |
| YSS  | 35.4 | 59 | 6.92  | 625     | 625     | 625     | 625     | --   | 189  | 244  | 285  | 583     | 448     | 581     | --      |
| INCN | 35.6 | 44 | 8.01  | 650     | 650     | 650     | 650     | 241  | 234  | 235  | 227  | 567     | 525     | 478     | 497     |
| SSE  | 37.9 | 36 | 8.52  | 650     | 650     | 650     | 650     | --   | 262  | 228  | 236  | 617     | 531     | 538     | 528     |
| MAJO | 44.2 | 51 | 7.51  | 725     | 725     | 725     | 725     | 275  | 266  | 255  | 236  | 617     | 544     | 506     | 475     |
| TATO | 21.1 | 79 | 5.41  | 700     | 700     | 700     | 700     | 227  | 198  | 184  | 224  | 626     | 449     | 448     | 445     |
| GUMO | 26.2 | 69 | 6.14  | 650     | 650     | 650     | 650     | 227  | 198  | 184  | 182  | 571     | 474     | 485     | 464     |
| MDJ  | 30.7 | 50 | 7.54  | 575     | 575     | 575     | 575     | --   | 161  | 180  | 201  | 533     | 524     | 506     | --      |
| PET  | 34.5 | 71 | 6.05  | 600     | 600     | 600     | 600     | --   | 182  | 137  | 141  | 531     | 473     | 476     | --      |
| YSS  | 35.4 | 59 | 6.92  | 625     | 625     | 625     | 625     | --   | 189  | 244  | 285  | 583     | 448     | 581     | --      |
| INCN | 35.6 | 44 | 8.01  | 650     | 650     | 650     | 650     | 241  | 234  | 235  | 227  | 567     | 525     | 478     | 497     |
| SSE  | 37.9 | 36 | 8.52  | 650     | 650     | 650     | 650     | --   | 262  | 228  | 236  | 617     | 531     | 538     | 528     |
| MAJO | 44.2 | 51 | 7.51  | 725     | 725     | 725     | 725     | 275  | 266  | 255  | 236  | 617     | 544     | 506     | 475     |
| TATO | 21.1 | 79 | 5.41  | 700     | 700     | 700     | 700     | 227  | 198  | 184  | 224  | 626     | 449     | 448     | 445     |
| GUMO | 26.2 | 69 | 6.14  | 650     | 650     | 650     | 650     | 227  | 198  | 184  | 182  | 571     | 474     | 485     | 464     |

Az - azimuth; ∆ - epicentral distance; and dT/dΔ, s° – P travel time derivative. For Tfin cmb, etc., e0.8, etc and T99 0.8, etc., see text.

(--) means lack of estimate because of insufficient S/N ratio or dirty data.
Table 2. Solutions for space-time vectors of source parameters for three frequency bands

| Parameter | $N$ | $e_{av}/\sigma_{av}$ | $BT$, s | $BN$, km | $BE$, km | $BL$, km | Bazim, $^\circ$ | $BV$, km/s |
|-----------|-----|----------------------|--------|----------|--------|--------|-------------|-----------|
| $F_{0.8}$ | 37  | 660                  | 692    | 1111     | -184   | 1127   | -9          | 1.62      |
| $\sigma(F_{0.8})$ | 56  | 11                   | 210    | 225      | 219    | 11     | 0.34        |
| $F_{1.6}$ | 36  | 654                  | 687    | 1010     | -100   | 1015   | -5          | 1.47      |
| $\sigma(F_{1.6})$ | 61  | 13                   | 255    | 248      | 252    | 14     | 0.39        |
| $F_{2.5}$ | 29  | 651                  | 688    | 978      | 8      | 978    | 0           | 1.42      |
| $\sigma(F_{2.5})$ | 65  | 17                   | 314    | 305      | 311    | 18     | 0.48        |
| $M1_{0.8}$ | 37  | 213                  | 220    | 259      | -138   | 293    | -28         | 1.33      |
| $\sigma(M1_{0.8})$ | 41  | 8                    | 153    | 172      | 164    | 31     | 0.79        |
| $M1_{1.6}$ | 36  | 197                  | 211    | 449      | -281   | 530    | -32         | 2.51      |
| $\sigma(M1_{1.6})$ | 44  | 9                    | 189    | 183      | 187    | 20     | 1.00        |
| $M1_{2.5}$ | 29  | 201                  | 219    | 507      | -382   | 636    | -37         | 2.90      |
| $\sigma(M1_{2.5})$ | 41  | 11                   | 210    | 200      | 206    | 18     | 1.08        |
| $P99_{0.8}$ | 37  | 562                  | 580    | 668      | -335   | 747    | -26         | 1.28      |
| $\sigma(P99_{0.8})$ | 61  | 12                   | 228    | 249      | 240    | 18     | 0.44        |
| $P99_{1.6}$ | 36  | 513                  | 541    | 861      | -136   | 872    | -8          | 1.61      |
| $\sigma(P99_{1.6})$ | 41  | 9                    | 174    | 168      | 172    | 11     | 0.34        |
| $P99_{2.5}$ | 29  | 505                  | 534    | 786      | -115   | 794    | -8          | 1.48      |
| $\sigma(P99_{2.5})$ | 39  | 10                   | 197    | 189      | 194    | 14     | 0.39        |
| $F_{comb}$ | 37  | 660                  | 692    | 1085     | -178   | 1100   | -9          | 1.58      |
| $\sigma(F_{comb})$ | 57  | 11                   | 214    | 230      | 223    | 11     | 0.34        |

Table 3. Various estimates for the size and duration of the rupture of the 2004.12.26 event.

| L, km | Duration, s | $V_{rup}$, km/s | wave type | comment | reference |
|-------|-------------|-----------------|-----------|---------|-----------|
| 1200-1300* | 575-625s | 2.8 | $SH$, Raleigh | duration from Fig. 6 | (Ammon et al. 2005) |
| 1300* | approx. 480, 600** | 2.3-2.7 | $P$ (0.2-1 Hz) | local array analysis | (Ishi et al. 2005) |
| 1150 | 480-500 | 2.7-2.5 | $BB$ | $P$ phase, 1-100 Hz | (Krueger&Ohrnberger 2005) |
| 1235 | 515 | 2.7-2.5 | $T$ phase, 1-100 Hz | local array analysis | (Guilbert et al. 2005) |
| 1200 | 480 | 2.8-2.1 | $T$ phase, 1-100 Hz | local array analysis | (Tolstoy and Bohnenstiehl 2005) |
| 1200 | 500 | 2.5 | HF $P$(2-3Hz) | stopping point*** | (Ni et al. 2005) |
| 1100±300 | 560 | 1.5-1.7 (mean) | HF $P$(1Hz) | 90% stopping point | (Lomax 2005) |
| 1100±220 (100%) | 690 (100%) | 1.5-1.7 (mean) | HF $P$(0.8, 1.6, 2.5 Hz) | 100% and 99% stopping points | this study |
| (800 (99%)) | 550 (99%) | 2.25 (main part) | HF $P$(0.8, 1.6, 2.5 Hz) | 99% stopping points | |

* along a curved fault trace.
** 480-8 min is an approximate summary estimate; from their Fig 5b one can deduce source duration of at least 600 s.
*** end point of the envelope of HF $P$ waves, picked by eye.
Table 4. Various estimates for the spatial and temporal centroid of the 2004.12.26 event.

| Latitude, N | Delay, s | wave type | comment | reference |
|-------------|----------|-----------|---------|-----------|
| 7           | 139      | normal modes | standard CMT solution | (Stein and Okal 2005) |
| 3.1         | 214      | mantle waves | average over 5 subsources | (Tsai et al. 2005) |
| [4-5]       | [150-200]| SH, Raleigh, LF | | (Ammon et al. 2005) |
| [6]         | [250-270]| P (0.2-1 Hz) | local array analysis, | (Ishi et al. 2005) |
| [6]         | [200-270]| BB P | local array analysis | (Krueger&Ohrnberger 2005) |
| [6.5]       | [160]    | T phase, 1-100 Hz | local array analysis | (Tolstoy and Bohnenstiehl 2005) |
| [6-7]       | [150-180]| T phase, 1-100 Hz | local array analysis | (Guilbert et al. 2005) |
| 7(0.4-3Hz)  | 215      | HF P | | this study |
| 4.5(0.4-1.2 Hz) | 715    | | | |

in brackets: our rough estimates based on plotted slip and power distributions in the sources listed

Figure 1. Illustration of the limited bias introduced by the replacement, in the solution of Equation (7), of aftershock record by the same station with its record by station KURK. Abscissa: correct estimate of $T_{fin,k}$ and $e_k$; ordinate: estimate obtained through the replacement.
Figure 2. Attenuation-corrected mainshock signal at LSA: displacement (upper trace) and four band-filtered acceleration traces.

Figure 3. HF power signals (energy in 1-s bins) for band-filtered acceleration signals at LSA, for mainshock (left) and 2005.02.26 aftershock (right).
Figure 4. Deconvolution procedure for the power signals of Fig. 3 (averaged over 25-s bins and sampled each 25s).  

\(a\) – for each of the four frequency bands, a box is shown that contains three curves: upper curve - aftershock signal \(W_a(t)\), middle curve – mainshock signal \(W_m(t)\), lower curve – deconvolved “cleaned” mainshock signal \(W_1(t)\).  

\(b\) – \(W_1(t)\) signals in more detail. Circles are non-zero amplitudes, crosses are zero amplitudes. Triangle marks centroid \(e_k\) and vertical dashed line marks the joint end time \(T_{fin, cmb,k}\).

Figure 5. “Source” power signals for all stations, for the 1.6 Hz band. Triangle is centroid, diamond is the 99% point and upside down triangle is the stopping point.
Figure 6. Illustration of the quality of fit of observed delays by least squares (Equation (6)). Circles are calculated values and squares are observed ones. The vertical line marks the azimuth obtained in the inversion. Top left box - inversion for the stopping point derived jointly from all four channels. Other graphs are for 1.6 Hz band: Fin: stopping point, M1 – centroid, P99 - 99% point, respectively.

Figure 7. The positions of the stopping point (a), centroid(b), and the 99% point (c). Star is the mainshock hypocenter/nucleation point. Grey x marks are aftershocks of the first 25 hours, and the bold dash outline depicts the hypothetic source area consistent with these data. In a, the diamond is the stopping point determined from the data combined over four frequency bands. In b and c, the diamond is the average position over the three lower-frequency bands. For other markers see legend. Cross around a marker depicts 1σ error bounds obtained in the LS inversion.