String bits and the Myers effect

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Abstract. Based on the non-abelian effective action for D1-branes, a new action for matrix string theory in non-trivial backgrounds is proposed. Once the background fields are included, new interactions bring the possibility of non-commutative solutions i.e. The Myers effect for “string bits”.

MATRIX STRINGS

Matrix string theory [1, 2, 3] is one of the most interesting outcomes of the different dualities in M-theory. Perhaps a simple way to define it is by looking at Matrix theory [4] with an extra compactified dimension. For example, following Dijkgraaf et. al. [1], if we consider M-theory on a torus of radii $A$ and $B$, by first reducing on $A$ and then making an infinite boost on $B$ we get type IIA string theory on the discrete light cone (DLC) with D0-brane particles. If, on the other hand, we reduce on $B$ first, boost and then consider t-duality on $A$, we get (1+1) super Yang-Mills (SYM) theory with fundamental string charge on the world-volume i.e. the low energy theory of D1-branes. One finds therefore, that Matrix string theory is a non-perturbative definition of string theory built in terms of a two dimensional SYM theory and a collection of scalar fields in the adjoint representation of the gauge group (see the original papers for an extended discussion of this derivation). Although we have discussed only type IIA string theory, there are other constructions similar to the one sketched before, where the other four superstring theories are written in terms of two dimensional SYM theory1.

The Matrix string theory conjecture was originally formulated on flat backgrounds. Lately, using some techniques developed by Taylor and Raamsdonk [5] a generalization for closed strings on non-trivial weak backgrounds has appeared [6]. This talk is based on the paper [7], a further generalization of the original Matrix string theory to non-trivial weak backgrounds based on the non-abelian D1-brane action proposed by Myers.

In [6, 7], the possibility of non-abelian configurations of fundamental strings was pointed out. In particular, the appearance of a Myers-like effect was computed explicitly (By now the Myers effect [8] is a well known phenomenon where $N$ D-branes adopt a non-abelian configuration that can be understood as a higher dimensional abelian D-brane). These configurations come about as the result of new interaction terms that appear in the non-abelian effective actions.

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1 Actually for the cases of type IIB and type I string theory there is an additional construction in terms of a three dimensional theory [3]
The appearance of strings describing D-branes is not new. There are computations of Dp-branes collapsing into fundamental strings [9] and fundamental strings blowing up into Dp-branes [10], always in terms of the abelian Born-Infeld actions of the corresponding D-branes. What is new in the matrix string formulation is that we have a formalism in which a two-dimensional action naturally includes matrix degrees of freedom representing the “string bits”\(^2\), which also incorporate the description of higher dimensional objects of M-theory using non-commutative configurations.

One of the important properties of this new theoretical framework lies in the similarity of the mathematical language used to describe the fundamental objects of M-theory, bringing for first time the possibility of describing strings and D-branes in a unified framework, a “democracy of p-branes” [12].

**MATRIX STRING AND NON-ABELIAN D1-BRANES**

In the previous section we talked about a theoretical framework that describes fundamental strings in terms of matrix degrees of freedom. For example, in type IIA this action is a two dimensional supersymmetric gauge theory that contains DLC string theory and has extra degrees of freedom representing non-perturbative objects of string theory. Also, it is a second quantized theory as it is built from many strings.

We know that by means of different dualities the five superstring theories are described in the neighborhood of a 1+1 dimensional orbifold conformal field theory. In this language the strings are free in the conformal field theory limit, representing DLC string theory. The interactions between the strings are turned on by operators describing the splitting and joining of fundamental strings. These operators deform the theory away from the conformal fixed point.

To further clarify these ideas, let us follow a sketch of the derivation for the case of type IIA string theory. Consider type IIA strings in the DLC frame with string mass \(m_A\), string coupling \(g_A\) and a null compact direction of radius \(R_A\) (where we identify the null coordinate as \(x^- \approx x^- + R_A\)).

Using the relation between the null compactification and a space-like compactification a la Seiberg-Sen [13], we get type IIA string theory on a space-like circle of radius \(R\) in the sector with momentum \(N\), string mass \(m\) and string coupling \(g\). The relation between these two heterotic string theories is given by

\[
m^2 R = m_A^2 R_A, \quad g = g_A, \quad R \to 0.
\]

Next, we perform a t-duality transformation on \(R\), so that the new constants of the string theory \((m', g', R')\) are given by

\[
m' = m, \quad g' = \frac{g}{mR}, \quad R' = \frac{1}{m^2 R}.
\]

\(^2\) The idea is that the string can be seen as a chain of partonic degrees of freedom [11]
Finally, we perform a s-duality transformation to obtain type IIB string theory with $N$ D1-strings and constants $(m_b, g_b, R_b)$ given by the following expressions,

$$m_b = \frac{m'}{g'^{1/2}}, \quad g_b = \frac{1}{g'}, \quad R_b = R'.$$

(3)

In terms of the initial type IIA theory and $R$ we get

$$m_b = \left[ \frac{m_A R^3}{g_A^2 R} \right]^{1/4} \to \infty,$$

$$g_b = \left[ \frac{m_A^2 R_A R}{g_A^2} \right]^{1/2} \to 0,$$

$$R_b = \frac{1}{m_A^2 R}.$$  (4)

Therefore, we get the low energy theory of $N$ D1-branes at weak coupling, where the gauge coupling constant $g_{YM}$ is given by,

$$g_{YM} \propto \frac{m_A^2 R_A / g_A}{g_A}. (5)$$

This is the 1+1 dimensional SYM theory with eight scalars in the adjoint representation of the gauge group. This effective action is obtained by the dimensional reduction of $N = 1$ supersymmetry Yang-Mills theory in ten dimensions down to two dimensions.

To define the type IIB case, a possible route to take is to start with type IIB strings in the DLC frame, then perform a t-duality transformation on the null circle taking us to type IIA in the DLC. This is similar to the previous situation with the difference that winding modes are exchanged for momentum modes. The relation between the corresponding meaningful constants is,

$$m_b = \left[ \frac{m_B^2}{g_B^2 R_B R} \right]^{1/4} \to \infty,$$

$$g_b = \left[ \frac{m_B^2 R_B R}{g_B^2} \right]^{1/2} \to 0,$$

$$R_b = R_B.$$  (6)

where $(m_B, g_B, R_B)$ are the initial type IIB string parameters and $(m_b, g_b, R_b)$ are the final (also type IIB) string theory parameters. Again, we get a low energy weakly coupled string theory with $N$ D1-branes. The gauge coupling constant $g_{YM}$ is given by

$$g_{YM} = \frac{m_B}{g_B}.$$  (7)

The heterotic case is similar but some care has to be taken with the inclusion of Wilson lines [14]. On the other hand, type I theory is more subtle and is related to the low energy limit of type IA theory in the presence of D8-branes and D0-branes plus winding modes.
on the orbifold. Therefore it is a quantum mechanics system but with an infinite tower of winding modes.

In order to obtain the relevant action for one of the five matrix string theories, we start with the world-volume gauge theory of N D1-branes, and then go back along the chain of dualities until we reach the desired DLC string theory. For example, consider first an s-duality transformation on the D1-brane effective action, then a t-duality transformation and finally the boost relations of Seiberg-Sen. As a result we get type IIA matrix string theory. This can be written as

$$L_{IIA} = B \circ T \circ S [L_{D1}].$$

(8)

Other matrix string theories Lagrangians can be obtained by similar procedures. For example, $$L_{IIB}^{IB} \equiv T \circ B \circ T \circ S [L_{D1}].$$

As we mentioned in the introduction, there are generalizations of the matrix string action which include weak backgrounds. This time the calculations are based on the relation between matrix string and the matrix theory proposal. In particular, previous works of Taylor and Van Raamsdonk [5] are used to support these results. One of the positive outcomes of the above work is a proposal for the transformation of the D1-brane world-volume fields under s-duality. Thus, based on these different proposals we are able to actually construct the matrix actions using maps like the one in equation (8).

It is important to note that recently Myers wrote a non-abelian action of N Dp-branes in general backgrounds [8] which is fully covariant under t-duality. This action incorporates (in the limit of weak backgrounds), all the couplings derived previously by Taylor et. al. and also introduces some new ones. If we believe this effective action for the D1-branes, we are forced to conjecture that:

**Matrix string theory is defined by Myers D1-brane world-volume action plus the web of dualities needed.**

Note that since the non-abelian D1-brane action proposed by Myers does not capture the full physics of the infrared limit, we can only trust its expansion up to sixth-order in the field strength [15], and this problems is inherited by the above conjecture for the matrix theory action. Another technical problem comes from the chain of dualities, since it makes it difficult to give an explicit closed form for the final Lagrangian. In particular, the t-duality map mixes RR fields and NS fields. Nevertheless, we only have to use the Buscher rules [16] on the supergravity background fields as t-duality (once we have s-dualized), leaves the world-volume fields invariant. At last the action of Myers only tells us about the bosonic degrees of freedom, therefore the fermionic counterpart has to be calculated using supersymmetry.

For example, let us consider the type IIA case. Following equation (8), the action for the Matrix string is given in two parts, the first corresponding to the original Born-Infeld term of the D1-brane action,

$$S_{F1}^I = \frac{1}{\lambda} \int d\xi^2 \text{Str} \left\{ \sqrt{-\text{det}(P|\tilde{E} + \tilde{E}(\tilde{Q}^{-1} - \delta)\tilde{E}) + \lambda e^\phi \tilde{g} F \text{det}(\tilde{Q})} \right\}$$

(9)

where

$$\tilde{E}_{AB} = \tilde{G}_{AB} - e^\phi \tilde{C}^{(2)}_{AB}.$$
\[
\tilde{Q}^i_j = \delta^i_j + i\lambda[\Phi^i, \Phi^k]\tilde{E}_{kj}(\tilde{g}e^{\tilde{\phi}})^{-1},
\] (10)

and the tilde represents the t-dual transformation of the background fields. For example the form of \(\tilde{C}_{AB}\) is

\[
\tilde{C}_{AB} = \begin{pmatrix}
C_{\alpha\beta_y} + 2C_{[\alpha B\beta]y} - 2C_{y[\alpha G\beta]y} & C_\alpha - C_y G_{\alpha y}/G_{yy} \\
-C_\beta + C_y G_{\beta y}/G_{yy} & 0
\end{pmatrix},
\] (11)

where the space-time index \(A\) has been divided into the t-dualized direction \(y\) and the other directions \(\alpha\).

The second part, corresponding to the original Chern-Simons term of the D1-brane action is

\[
S_{F_1}^2 = \frac{1}{\lambda} \int d\xi^2 \text{StTr} \left\{ P \left[ e^{i\tilde{g}^{-1}i\Phi^{(1)}} \left[ (-\tilde{B} + \tilde{C}^{(4)}) e^{-\tilde{C}^{(2)}} \right] e^{\lambda \tilde{g}F} \right] \right\}.
\] (12)

This action contains the action of the matrix string theory of Dijkgraaf et. al. [1], since by construction in trivial backgrounds the D1-brane action of Myers gives the 1+1 SYM theory corresponding the dimensional reduction on \(\text{N}=1\) SYM in ten dimensions down to two dimensions. Hence, by taking all of the background fields to be trivial, we recover the standard form of type IIA matrix string theory,

\[
S_{F_1}^1 = \lambda \int d\xi^2 \text{Tr} \left\{ \frac{\partial\Phi^2}{2} + \frac{1}{4g^2} [\Phi, \Phi]^2 + \frac{g^2}{4} F^2 \right\}.
\] (13)

Also, all of the linear couplings obtained by Schiappa [6] for the weak field case, are derivable from the action of equation (9) and (12). It has been checked that the D1-branes linear couplings found by Taylor et. al. are included in the non-abelian action of Myers and the t-duality and s-duality relations are the same as the ones used by Schiappa. Nevertheless, we have to keep in mind that there are new couplings not considered before.

Once the relevant action is obtained, we can search for non-commutative classical solutions. Given the similarity of the mathematical structure with D-brane physics, we expect to find relevant physical situations where these types of solutions appear. Nevertheless, in this framework the building blocks that make the higher dimensional objects are the “string bits” of the DLC. Remember that, the matrix-value scalar in the action represent a large number of “long strings” and these are the basic objects that form the higher dimensional branes. Some examples of non-commutative configurations of strings can be found in [6, 7, 17, 18, 19].

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