Blackfold as a Conformal Fluid

J. Sadeghi,* Z. Amoozad*

*Department of Physics, University of Mazandaran, Babolsar, Iran

E-mail: pouriya@ipm.ir, z.amoozad@stu.umz.ac.ir

Abstract: In this paper we review some properties of higher dimensional black holes. In that case we take advantages of fluid/gravity duality and obtain the fluid properties of higher dimensional black holes on the boundary. So we consider two natural and charged blackfold cases and extract the Brown-York stress energy tensor (tensor of fluid) on the boundary. As we know, the compactification of some directions in any asymptotically AdS black branes corresponds to some kind of Ricci-flat spacetime. For example the neutral black hole’s spacetime is Ricci-flat. So by calculating the AAdS form of that metric the dual renormalized holographic stress tensor has been extracted. This stress tensor is conserved and traceless, also it is same as perfect fluid.
1 Introduction

Recently a nice relation between fluid and gravity has been emerged. This duality comes from the gauge/gravity duality which gives an equivalence between D dimensional gravitational theory and D-1dimensional quantum field theory. It connects asymptotically AdS geometry to the states in the field theory side. The pure AdS bulk geometry corresponds to the vacuum state of the gauge theory. But deformed bulk geometry in the way that AdS asymptotic preserves can be correspond to exited state. The perturbed metric on the gravity side corresponds to the expectation value of stress energy tensor on the CFT side. In general this is the holographic principle that first has been proposed by Gerard ’t Hooft in about 1993, which is one of the most important subject that is extracted from the study of quantum gravity and string theory [1–5].

Near thermal equilibrium quantum field theory describes the existence of a hydrodynamic regime that show the long wavelength behavior of the microscopic degrees of freedom. So by acceptance such holography, a hydrodynamic can be reproduced and also the existence of fluid/gravity is proven [6–8].

One of the main difference of higher dimensional black holes by the 4-dimensional ones is that the horizons of higher dimensional black holes can have at least two very different size lengths. By definition, if black holes regarded as a black brane (possibly boosted locally) which folded in to multiple dimension a blackfold is shaped. One reason that blackfolds can have at least two very different size horizon is that there is no bound on the angular momentum of them, although in 4-dimension the Kerr bound \( J \leq GM^2 \) don’t admit higher momentum. So in blackfolds, there is two length scale wich are given by,

\[
l_M \sim (GM)^{1/D-3} , \quad l_J \sim \frac{J}{M}
\]
In situations which $l_J \gg l_M$, this separation suggest an effective description of long wavelength dynamics \cite{9}. Also it may cause to the Gregory-Laflamme instability of blackfolds. One of the simplest example of this theory effectively defines a thin black ring (as a boosted black string) \cite{10}.

As a theory of classical brane dynamics \cite{11} explains a long wavelength effective theory, it can take the form of the dynamics of a fluid that lives on a dynamical worldvolume. But if higher dimensional black holes have regarded instead of 4-dimensional cases, how can extend the famous fluid/gravity correspondence? The holographic duality for Ricci-flat spacetime is one of the interesting duality that recently some of people work on.

Many different subjects have been investigated in these grounds. For example, AdS black brane solutions can be correspond to conformal and non conformal fluid at thermal equilibrium \cite{12, 13} and also some relativistic expansion of fluid/gravity is in \cite{14, 15}. Extension of fluid /gravity to spherical horizon topologies, de sitter spacetime and charged fluid calculated by \cite{16-22}. All above information of fluid/gravity gives us motivation to work this subject.

By using the AdS/Ricci-flat correspondence and natural blackfold as a Ricci-flat one, we extract AAdS form of that and in Fefferman-Graham coordinate the dual renormalized stress tensor which is properties of boundary have been obtained. This paper is organized as follows. In the next section some properties and metric of the natural and charged blackfolds have reviewed. In section 3 the Brown-York stress energy tensor of black folds on time-like hypersurface is extracted. Section 4 is belong to calculation of stress tensor of dual theory. In final section we conclude and note some points.

2 General properties of blackfolds

2.1 Natural blackfolds

As we know 4-dimensional black holes only have short scale dynamics ($\sim r_0$). But for higher dimension black holes we see some more physical information and have at least two very different size horizons $r_0 \ll R$. This properties predicts an existence of long distance worldvolume effective theory. So for higher dimension ones new methods are needed to capture the long distance $\sim R \gg r_0$ dynamics \cite{9}. In general brane like objects are sources of energy momentum tensor \cite{23}. The requirement of consistent coupling of source to gravity overtake,

$$\nabla_\mu T^{\mu\nu} = 0 \quad \rightarrow \quad T^{\mu\nu} K_{\mu\nu} = 0 \quad (2.1)$$

where $\mu, \nu$ are spacetime coordinates ($\mu, \nu = 0, ..., D - 1$) and $\rho$ is the orthogonal one. $K_{\mu\nu}$ is the extrinsic curvature tensor of the blackfold embedding. On scales $r \ll R$, the blackfold is equivalent to a black p-brane up to a Lorentz- transformation which can depend on position. The effective stress tensor of a static natural flat black brane with orthonormal worldvolume coordinates (if the velocity field is $u^a$ with $u^a u^b \eta_{ab} = -1$ and $\sigma^a = (\sigma^0, \sigma^i)$) is given by \cite{9}:

$$T^{ab} = r_0^n (nu^a u^b - \eta^{ab}) \quad (2.2)$$
where $\sigma^a$ is the worldvolume coordinates and $a, b = 0, \ldots, p$. As mentioned before the blackfolds can have $\frac{\Delta}{\gamma} \to 0$, so flat black brane produced and effective theory describes the collective dynamics of a black p-brane. It’s geometry in $D = n + p + 3$ spacetime dimension is \cite{11},

$$
\text{ds}^2_{p-\text{brane}} = -(1 - \frac{r_0^n}{r^n}) dt^2 + \sum_{i=1}^p dz_i^2 + (1 - \frac{r_0^n}{r^n})^{-1} dr^2 + r^2 d\Omega_{n+1}^2 \quad (2.3)
$$

where $\sigma^a = (t, z^i)$ is related to brane worldvolume. By boosting it along the worldvolume, one can easily find the following relation,

$$
\text{ds}^2_{p-\text{brane}} = (\eta_{ab} + \frac{r_0^n}{r^n} u_a u_b) d\sigma^a d\sigma^b + (1 - \frac{r_0^n}{r^n})^{-1} dr^2 + r^2 d\Omega_{n+1}^2 \quad (2.4)
$$

where $r_0$ is the thickness of horizon. Here $(p + 1)$ coordinates are on the worldvolume of the blackfold and $(D - p - 1)$ coordinates are transverse directions to the worldvolume. By taking long distance effective theory, the effective Einstein’s equation can be written as,

$$
R_{\mu\nu}^{(\text{long})} - \frac{1}{2} R^{(\text{long})} g_{\mu\nu}^{(\text{long})} = 8\pi G T_{\mu\nu}^{\text{eff}}. \quad (2.5)
$$

So the effective worldvolume stress tensor will be following,

$$
T_{\mu\nu}^{\text{eff}} = -\frac{2}{\sqrt{-g^{(\text{long})}}} \frac{\delta I_{\text{eff}}}{\delta g^{\mu\nu}^{(\text{long})}} \big|_{W_{p+1}}. \quad (2.6)
$$

As we know the quasilocal stress energy tensor is introduced by Brown and York \cite{24}. It appropriately match the coupling of the short wavelength and long wavelength degrees of freedom. If $D_a$ be the covariant derivative with respect to the boundary metric $\gamma_{ab}$, the equation of conservation of the quasi local stress tensor will be following form,

$$
D_a T_{ab}^{\text{quasilocal}} = 0 \quad (2.7)
$$

This is the dynamics of effective fluid that lives on the worldvolume and spanned by the brane. For isotropic worldvolume theory in case of lowest derivative order the stress tensor will be same as the isotropic perfect fluid, which is given by,

$$
T_{ab} = (\varepsilon + p) u^a u^b + p \gamma^{ab} \quad (2.8)
$$

where $\varepsilon$ is the energy density and $p$ is the pressure. If it dos’t be perfect fluid then the stress tensor will have dissipative term proportional to gradients of $\ln r_0, u^a, \gamma^{ab}$. But for stationary ones their effects vanish. By definition $\nabla_\mu$ the background covariant derivative and $h^{\mu\nu} = \partial_a X^\mu \partial_b X^\nu \gamma^{ab}$, where $X^\mu$ is the embedding function, and $\nabla_\mu = h_\mu^\nu \nabla_\nu$, because of the consistent coupling of worldvolume to the long wavelength gravitational field $g_{\mu\nu}$ the stress tensor must obey,

$$
\nabla_\mu T^{\mu\rho} = 0 \quad (2.9)
$$

The $D$ equation of (2.9) separate into $D - p - 1$ equation in directions orthogonal to $W_{p+1}$ and $p + 1$ equation parallel to $W_{p+1}$, which are given by,

$$
T^{\mu\nu} K_{\mu\nu}^{\rho} = 0 \quad \text{extrinsic equation},
D_a T^{ab} = 0 \quad \text{intrinsic equation}. \quad (2.10)
$$
2.2 Charged blackfolds

In many supergravity and low energy limits of string theory black branes have charges. So for higher dimensional cases effective worldvolume theory for such a charged branes is regarded. One of the best choice is black p-branes that carry charges of Ramond-Ramond field strength \( F_{(p+2)} \). The charged dilatonic black p-branes solution of the action is [25],

\[
I = \frac{1}{16\pi G} \int dx^D \sqrt{-g} \left( R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2(p+2)!} \exp a \phi F_{(p+2)}^2 \right)
\]

where

\[
a^2 = \frac{4}{N} - \frac{2(p+1)n}{D-2}, \quad n = D - p - 3
\]

The flat black p-brane solution reads,

\[
ds^2 = H^{\frac{N}{D-2}} (-f dt^2 + \sum_{i=1}^p dz_i^2) + H^{\frac{N(p+1)}{D-2}} (f^{-1} dr^2 + r^2 d\Omega_{n+1}^2)
\]

\[
e^{2\phi} = H^{aN}, \quad A_{p+1} = \sqrt{N} \coth \alpha(H^{-1} - 1) dt \wedge dz_1 \wedge dz_2 \ldots \wedge dz_p,
\]

\[
H = 1 + \frac{r_0^n \sinh^2 \alpha}{r^n}, \quad f = 1 - \frac{r_0^n}{r^n}
\]

where \( \sinh \alpha \) relates to the boost of blackfold that is Lorentz transformation of some of directions of p-brane. As must \( a^2 \geq 0 \) the parameter \( N \) is not arbitrary and there is,

\[
N \leq 2 \left( \frac{1}{n} + \frac{1}{p+1} \right)
\]

In string/M theory, \( N \) is an integer up to 3 (when \( p \geq 1 \)) that corresponds to the number of different types of branes in an intersection. In this case the effective stress tensor can be written as,

\[
T^{ab} = \tau s (u^a u^b - \frac{a}{b} \gamma^{ab}) - \Phi_p Q_p \gamma_{ab}
\]

where \( \tau \) is temperature and \( s \) is entropy. \( \Phi_p \) is the potential that measured from the difference between the values of \( A_{p+1} \) at the horizon at \( r \rightarrow \infty \) in (2.14). As every one can see the relation (2.16) surprisingly has a brane tension component \( -\Phi_p Q_p \) and a thermal component \( \tau s \). It must not that, blackfolds with p-branes charges do not have open boundaries as the charge \( Q_p \) would not be conserved at them. So it is possible to consider that blackfolds end on another brane that carries appropriate charge.

3 Brown-York stress energy tensor

3.1 Charged blackfolds

In the fluid/gravity duality essential physical properties of field theory side are mentioned by boundary stress tensor. As we know the Brown-York stress energy tensor [26], which is induced by the bulk geometry, can be obtained by finding an appropriate hypersurface in the bulk. The bulk metric that is the case is (2.3). Hypersurfaces can be obtained
by taking a constraint equation between independent coordinates of spacetime. The time-like hypersurface play important role for causality relations between different coordinates. These hypersurfaces can be find by the following restriction equation,

\[ r = r_c, \]  \hspace{1cm} (3.1)

As we know, the perpendicular vectors on time-like hypersurface are spacelike and connects time-like hypersurfaces. In that case, the Brown-York stress energy tensor [22] is,

\[ T_{\mu\nu} = 2(K_{\gamma\mu} - K_{\nu\mu}) \]  \hspace{1cm} (3.2)

where \( \gamma_{ab} \) is the induced metric on the time-like hypersurface and \( K_{ab} \) is the extrinsic curvature of the bulk. This can be determined by,

\[ K_{\mu\nu} = -n_{\mu}a_\nu - n_{\nu;\mu} \]  \hspace{1cm} (3.3)

\( n_\lambda \) is the unit normal vector to the hypersurface,

\[ n_\lambda = \frac{\phi_\lambda}{|g^{\mu\nu}\phi_\mu\phi_\nu|^{\frac{1}{2}}} \]  \hspace{1cm} (3.4)

\[ a_\nu = n^\theta n_{\nu;\theta} \]  \hspace{1cm} (3.5)

and \( \phi \) in the (3.4) is the equation of hypersurface. In case of \( r = r_c \), \( \phi \) just depends on \( r \).

The charged dilatonic blackfold from (2.13), will be given as

\[ ds^2_{p\text{-brane}} = Adt^2 + Bdz_1^2 + Cdr^2 + F_\alpha d\psi^2_\alpha \]  \hspace{1cm} (3.6)

and

\[ A = -(1 + \frac{r_0^n \sinh^2 \alpha}{r^n}) \frac{n_\alpha}{\sqrt{C}} (1 - \frac{r_0^n}{r^n}) \]  ,

\[ B = (1 + \frac{r_0^n \sinh^2 \alpha}{r^n}) \frac{n_\alpha}{\sqrt{C}} \]  ,

\[ C = (1 + \frac{r_0^n \sinh^2 \alpha}{r^n}) ^{\frac{N(\mu+1)}{\beta-2}} (1 - \frac{r_0^n}{r^n})^{-1} \]  ,

\[ F_\alpha = F_\alpha (r, \psi_\alpha) \]  .

This metric is diagonal, so the inverse matrix can be fined easily. For such a metric tried to determine the extrinsic curvature (3.3). In order to obtain the extrinsic curvature, one can arrange the \( n_\lambda \), \( n^\theta \) and \( a_\nu \) as

\[ n_\lambda = \sqrt{C} \delta_\lambda^r \]  ,

\[ n^\theta = \sqrt{C} g^{\theta r} \]  ,

\[ a_\nu = n^\theta (n_{\nu;\theta} - \Gamma_{\nu;\theta}^r \sqrt{C}) \]  .

So, by using the equation (3.8) in to equation (3.3) one can obtain the corresponding extrinsic curvature as,

\[ K_{\mu\nu} = -\delta_\nu^r \delta_\mu^r \frac{C_\nu}{2\sqrt{C}C} + C\sqrt{C} \delta_\mu^r g^{\nu\tau} \Gamma_{\nu\tau}^r - \frac{C_\mu}{2\sqrt{C}} \delta_\nu^r + \sqrt{C} \Gamma_{\mu\nu}^r \]  \hspace{1cm} (3.9)
By calculating the Christoffel symbols the nonvanishing components of extrinsic curvature for that geometry are given by,

\[ K_{tt} = -\frac{A_r}{2\sqrt{C}} , \quad K_{ii} = -\frac{B_r}{2\sqrt{C}} , \quad K_{\psi,\psi} = -\frac{F_{\alpha,r}}{2\sqrt{C}} \] (3.10)

On the other side, the induced metric \( \gamma_{ab} \) on the time-like hypersurface is,

\[ ds^2|_{r=r_c} = A_c dt^2 + B_c dz_i^2 + F_{\alpha c} d\psi_\alpha^2 \] (3.11)

The trace of the extrinsic tensor can be written by following equation,

\[ K = K_{ab} \gamma^{ab} = \frac{-1}{2\sqrt{C}} \left\{ \frac{A_r}{A_c} + p \frac{B_r}{B_c} + \sum_{\alpha=1}^{(n+1)} \frac{F_{\alpha,r}}{F_{\alpha c}} \right\} \] (3.12)

where \( p \) is the number of spatial coordinates of worldvolume of blackfold. So the Brown-York stress energy tensor can easily be constructed. For the case of D/NS-brane in type II string theory we have [23], be

\[ D = 10 \quad , \quad N = 1 \quad , \quad p = 0,...,6 \] (3.13)

by following definition,

\[ A = \frac{A_r}{A_c} , \quad B = \frac{B_r}{B_c} , \quad F_{\alpha} = \frac{F_{\alpha,r}}{F_{\alpha c}} , \] (3.14)

in case of \( p = 5 \) brane \( (n = 2) \), the nonvanishing component of Brown-York tensor are,

\[ T_{tt} = \frac{1}{\sqrt{C}} \left\{ [A + pB + F_1 + F_2]A_c + A_r \right\} \] (3.15)

\[ T_{ii} = \frac{1}{\sqrt{C}} \left\{ [A + pB + F_1 + F_2]B_c + B_r \right\} \] (3.16)

\[ T_{\alpha\alpha} = \frac{1}{\sqrt{C}} \left\{ [A + pB + F_1 + F_2]F_{\alpha c} + F_{\alpha,r} \right\} \] (3.17)

### 3.2 Natural blackfolds

The case of natural blackfold has been reviewed in the second section, so the appropriate metric is equation(2.3). The method is the same as previous section, and we just present of \( p = 5 \) brane results as,

\[ ds^2 = A dt^2 + B dz_i^2 + C dr^2 + F_{\alpha} d\psi_{\alpha}^2 \] (3.18)

and

\[ A = -(1 - \frac{r_0^2}{r^2}) , \]
\[ B = 1 , \]
\[ C = (1 - \frac{r_0^2}{r^2})^{-1} , \]
\[ F_{\alpha} = F_{\alpha}(r, \psi_\alpha) . \] (3.19)
The method is straightforward and the nonvanishing stress tensor of natural blackfolds take a following form,

\[ T_{tt} = (1 - \frac{r_0^2}{r^2})^\frac{3}{2} \left\{ - (1 - \frac{r_0^2}{r^2}) \left[ \frac{2r_0^3}{r^3(1 - \frac{r_0^2}{r^2})} + \frac{4r}{r_c^2} \right] + \frac{2r_0^2}{r^3} \right\} \] (3.20)

\[ T_{ii} = (1 - \frac{r_0^2}{r^2})^\frac{3}{2} \left\{ \left[ \frac{2r_0^3}{r^3(1 - \frac{r_0^2}{r^2})} + \frac{4r}{r_c^2} \right] - 2 \frac{r_0^2}{r^3} \right\} \] (3.21)

\[ T_{\theta\theta} = (1 - \frac{r_0^2}{r^2})^\frac{3}{2} \left\{ r^2 \left[ \frac{2r_0^3}{r^3(1 - \frac{r_0^2}{r^2})} + \frac{4r}{r_c^2} \right] + 2 \frac{r_0^2}{r^2} \right\} \] (3.22)

\[ T_{\phi\phi} = (1 - \frac{r_0^2}{r^2})^\frac{3}{2} \left\{ r^2 \sin^2 \theta \left[ \frac{2r_0^3}{r^3(1 - \frac{r_0^2}{r^2})} + \frac{4r}{r_c^2} \right] + 2r \sin \theta \right\} \] (3.23)

4 Dual theory for natural blackfolds

In this section, we calculate the renormalized holographic stress energy tensor [27, 28] for the case of natural blackfolds. Although holography is estimated for non AdS asymptotics, such as nonrelativistic dualities and also it is very well understood for asymptotically locally AdS spacetimes [1]. For every AAdS spacetime the expectation value of the dual CFT stress energy tensor is defined as:

\[
<T_{CFT}^{ab}> = \frac{1}{16\pi G_N} \frac{\delta S_{ren}}{\delta g_{(0)}^{ab}}
\] (4.1)

\(g_{(0)}\) is the metric on the boundary and \(S_{ren}\) denotes the renormalized bulk action. In this case counterterms have been added to remove the volume divergences. We take advantages of Fefferman and Graham [29] and obtain an asymptotic solution of Einstein equation with conformal structure. The corresponding solution is given by,

\[ ds^2 = \rho^2 \left( \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ab}(x, \rho) dx^a dx^b \right) \] (4.2)

where \(a, b = 1, ..., D - 1\), \(d = D - 1\). And

\[ g(x, \rho) = g_{(0)} + ... + \rho^2 g_{(d)} + h_{(d)} \rho^2 \log \rho + ... \] (4.3)

the coefficient \(h_{(d)}\) is present only when \(d\) is even. We use ref. [30] and do some calculation, we determine the explicit form of the dual renormalized holographic stress tensor,

\[
<T_{CFT}^{ab}> = \frac{dl}{16\pi G_N} (g_{(d)}^{ab} + X^{(d)}_{ab})
\] (4.4)

where \(X^{(d)}_{ab}\) depends on the dimension. It’s form for all odd \(d\) is represented by [30], and for even \(d\) it vanishes,

\[ X^{(2k+1)}_{ab} = 0 \] (4.5)
Now we try to find the dual renormalized stress tensor for the natural blackfolds. As we know the metric (2.3) is Ricci-flat and there is a very nice connection between AdS black brane (which some of it’s coordinates are compactified) and Ricci-flat spacetime \[31, 32\]. So one can fined AAdS form of that metric. This is the schwarzchild black p-brane in \(D = n + p + 3\) dimension,

\[
ds^2_0 = -(1 - r_0^n/r^n)dt^2 + \sum_{i=1}^p dz_i^2 + (1 - r_0^n/r^n)^{-1}dr^2 + r^2d\Omega_{n+1}^2
\] (4.6)

So the required components are given by,

\[
ds^2_{p+2} = -(1 - r_0^n/r^n)\frac{\ell^2}{r^2}dt^2 + \sum_{i=1}^p \frac{\ell^2}{r^2} dz_i^2 + (1 - r_0^n/r^n)^{-1}\frac{\ell^2}{r^2} dr^2 + \ell^2 d\Omega_{n+1}^2, \tag{4.7}
\]

Then by changing \(n \leftrightarrow -d\), the AdS form of that which is planar AdS black brane with \(l = 1\) is (in \(d + 1\) dimnsion),

\[
ds^2_\Lambda = -\frac{1}{r^2}(1 - r^d/r_0^d)dt^2 + \frac{1}{r^2}(dz_i^2 + dy^2) + \frac{1}{r^2(1 - r^d/r_0^d)}dr^2
\] (4.8)

This metric is AdS black brane, the Fefferman-Graham coordinate(4.2) is the best form for this theory to find the dual stress tensor. In order to do that, we must extract the relation between \(r\) in (4.8) and \(\rho\) in (4.2). So we have,

\[
\frac{1}{r^2(1 - \frac{d}{r_0^d})}dr^2 = \frac{d\rho^2}{4\rho^2}
\] (4.9)

The results is :

\[
\left(\frac{\rho}{r_0}\right)^d = \frac{4\rho^\frac{d}{2}}{(\rho^\frac{d}{2} + 1)^2}
\] (4.10)

The corresponding two important functions will be as:

\[
\frac{1}{r^2} = \frac{1}{r_0^2 4\pi^\frac{d}{2}}(\rho^\frac{d}{2} + 1)^\frac{d}{2}
\] (4.11)

\[
\frac{1}{r^2}(1 - \frac{r^d}{r_0^d}) = \frac{1}{r_0^2 4\pi^\frac{d}{2}}(\rho^\frac{d}{2} + 1)^\frac{d}{2} - 2(\rho^\frac{d}{2} - 2\rho^\frac{d}{2} + 1)
\] (4.12)

So the AdS black brane in Fefferman-Graham coordinate take the following form,

\[
ds^2_\Lambda = -\frac{1}{r_0^2 4\pi^\frac{d}{2}}(\rho^\frac{d}{2} + 1)^\frac{d}{2} - 2(\rho^\frac{d}{2} - 2\rho^\frac{d}{2} + 1)dt^2 + \frac{1}{r_0^2 4\pi^\frac{d}{2}}(\rho^\frac{d}{2} + 1)^\frac{d}{2}(dz_i^2 + dy^2) + \frac{d\rho^2}{4\rho^2}
\] (4.13)

In the case of \(D = 10 \rightarrow d = 9\), the asymptotically AdS metric can be brought in to below geometry near the boundary (\(\rho = 0\),

\[
ds^2_\Lambda = -\frac{1}{r_0^2 4\pi^\frac{d}{2}}(1 - \frac{32}{9}\rho^2 + \frac{494}{81}\rho^9 + ...)dt^2 + \frac{1}{r_0^2 4\pi^\frac{d}{2}}(1 + \frac{4}{9}\rho^2 - \frac{10}{81}\rho^9 + ...)dz_i^2 + dy^2) + \frac{d\rho^2}{4\rho^2}
\] (4.14)

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By using the relation (4.3) one can obtain the explicit form of the metric coefficients which are given by,

\[ g^{(0)} = \frac{1}{r_0^{2+\frac{2}{3}}} \text{diag}(-1, 1, ..., 1) \]  
\[ g^{(9)} = \frac{1}{r_0^{2+\frac{2}{9}}} \text{diag}\left(\frac{32}{9}, \frac{4}{9}, ..., \frac{4}{9}\right) \]  

Finally the dual stress energy tensor in terms of relation (4.4) is

\[ \langle T^{CFT}_{ab} \rangle = \frac{9}{16\pi G_N} (g^{(9)}_{ab} + X^{(9)}_{ab}) = \frac{9}{16\pi G_N} g^{(9)}_{ab} \]  

This stress tensor can be checked formally as a conserved and traceless quantity. It is traceless because the conformal anomaly evaluated for global AdS vanishes. Also the boundary metric is conformally flat and the holographic stress tensor can take the conformal ideal fluid in 9-dimensional spacetime which has energy density, pressure and temperature.

5 Conclusion and outlook

In the covariant theory the local energy momentum density of the gravitational field is not prevalent. Instead, a quasilocal stress tensor can be defined on the boundary of spacetime. But resulting stress tensor may diverge when the boundary goes to infinity. In order to solve this difficulty one can add an appropriate boundary term to the action which is completely relevant.

Brown and York proposes a method to remove divergences. But it does not work for all intrinsic metric in the reference spacetime, so the Brown-York procedure is not generally well defined. Fortunately in AAdS spacetimes one can overcome to this problem. As gravitational action of the bulk corresponds to the quantum effective action of a conformal field theory on the AdS boundary so the expectation value of the stress tensor in the CFT solves the problem.

In this paper we considered the metric of charged and natural blackfold and obtained the boundary stress tensor. As the induced Brown-York stress tensor is conformally flat, the holographic stress tensor should be conformal to Brown-York stress energy tensor. For the case of natural blackfold, as it’s metric is Ricci-flat, By applying the AdS/Ricci-flat correspondence, we fined the corresponding AAdS form for it. Then for \( D = 10 \) the renormalized holographic boundary stress tensor extracted. As we expected, the results show that the corresponding stress tensor are conserved and traceless. In that case there is not any conformal anomaly for the pointed theory.

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