Young’s experiment and the finiteness of information

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Young’s experiment is the quintessential quantum experiment. It is argued here that quantum interference is a consequence of the finiteness of information. The observer has the choice whether that information manifests itself as path information or in the interference pattern or in both partially to the extent defined by the finiteness of information.

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I. INTRODUCTION

Young’s experiment, originally the definitive proof of the wave nature of light, commands an essential role in the discussion of the foundations of quantum mechanics. For example, in the Bohr-Einstein-Dialoque [1], the double-slit experiment was used as a gedanken experiment with individual quanta. In that discussion, Einstein wanted to argue that quantum mechanics is inconsistent in the sense that one can have path information and observe the interference pattern at the same time, while Bohr was always able to demonstrate that Einstein’s point of view was not correct. Indeed, if one carefully analyzes any situation where it is possible to fully know the path the particle took, the interference pattern cannot be observed. Likewise, if one observes the full interference pattern, no path information is available.

Young’s experiment today is considered the most beautiful demonstration of the concept of quantum superposition [Fig. 1]. Whenever we do not know, not even in principle, which of the two paths the particle takes, the quantum state can be written as

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\text{passage through left slit}\rangle + |\text{passage through right slit}\rangle).$$

In that case, no information whatsoever is available about the slit the particle passes through. Indeed, if one asked which path the particle takes in an experiment for a specific run, one would find the particle in either slit with equal probability.

Yet, obviously, this requires the use of detectors. If one places one detector each into each slit and if one describes the detector states by quantum mechanics, then, clearly, the quantum state of the whole system becomes

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|D_L\rangle|\text{passage through left slit}\rangle + |D_R\rangle|\text{passage through right slit}\rangle).$$

The description of not only the particle considered, but also the detector by a quantum state as given in (2) only has the meaning that the property of the particle to take a definite path is related to a property of the detectors. The two detector states $|D_L\rangle$ and $|D_R\rangle$ describe the detector having registered the particle passing through the left and right slit, respectively. These can even be states of an internal degree of freedom of the interfering particle (e.g. spin or polarization states or internal atomic states).

A proposal for such an experiment has been made by Scully et al. [2]. Summhammer et al. [3] performed a neutron interference experiment and Dür et al. [4] performed an atomic interference experiment where the disappearance of the interference pattern has to be attributed to the correlations between the internal neutron or atomic states, which serve as which-path detectors, and the paths taken inside the interferometer. In these experiments the loss of interference is due to the fact that path information is available, in principle, independent of the fact whether the experimentalist cares to read it out or not.

If the two detector states are orthogonal, then the two particle states cannot interfere, as Eq. (2) describes then a maximally entangled state and thus one could determine the path of the particle by observing the detector state. Only if the two detector states are not orthogonal or if they are projected by a measurement onto a state that is orthogonal to neither one of them then path
interference of a certain contrast may reappear, as then the complete knowledge about the path is not available.

II. COHERENCE AND PATH INFORMATION IN INTERFERENCE EXPERIMENT WITH FULLERENES

Technological progress in the times since the Bohr-Einstein-Dialogue made it possible to realize quantum interference with many different particles all the way to massive molecules, like the fullerene [7–9] C-60 and C-70. It is interesting to note that in the latter experiment, the fullerene molecules are at temperatures as high as 900 K. This implies that they are not completely decoupled from the environment. On the contrary, they typically emit a few photons on their path to the source to the detector [9]. So why do interference fringes still appear [Fig. 2]? Could one not use the emitted photons to trace the path of the fullerene? The reason can easily be understood by referring to Eq. (4). The wavelength of the emitted photons is typically of the order of a few micrometers, which has to be compared to the path separation, which is much lower. Therefore, the states of the two photons emitted by a fullerene on either of the interfering paths are nearly identical, implying that the photons carry virtually no information into the environment.

The modulus of the scalar product between the two states of the photons corresponding to the emission by a fullerene on either of the interfering paths can be used to quantify the information about the path of the fullerene, which can in principle be extracted if the photons were observed. Only if the scalar product is non-zero, then an interference pattern of a certain contrast may appear, as then the path is not completely known. In general, the contrast (visibility $V$) of the interference pattern is equal to the modulus of the scalar product between the two detectors states, $V = |\langle D_R | D_L \rangle|$. We now calculate the scalar product between the two photon states which serve as detector states in the fullerene experiment.

For the reason of simplicity we consider the fullerene experiment as a double-slit experiment. Suppose that the interfering fullerene emits $N$ photons at the moment it reaches the screen with the two slits. That is, the photons are emitted by the fullerene either at the left slit or at the right slit. Then the visibility $V$ of the fullerene interference pattern at the observation screen is equal to the modulus of the following scalar product

$$ V = |\langle N \text{ photons from left slit} | N \text{ photons from right slit}\rangle|. $$

Because the two possible states are the same for every of the $N$ photons, one can transform Eq. (3) into

$$ V = \left| \int d\vec{r} \phi(\vec{r}, \vec{r}_L) \phi^*(\vec{r}, \vec{r}_R) \right|^N, $$

where

$$ \phi(\vec{r}, \vec{r}_L) = \frac{e^{iK|\vec{r}-\vec{r}_L|}}{|\vec{r}-\vec{r}_L|} \quad \text{and} \quad \phi(\vec{r}, \vec{r}_R) = \frac{e^{iK|\vec{r}-\vec{r}_R|}}{|\vec{r}-\vec{r}_R|} $$

are the two amplitudes (spherical waves) of a photon at observation point $\vec{r}$, which are emitted from the point source localized at the position $\vec{r}_L$ of the left slit and $\vec{r}_R$ of the right slit, respectively. Here $K$ is the wave-number of the photon.

To calculate the integral in Eq. (4) we use the substitution $\xi f = \frac{|\vec{r}-\vec{r}_R|+|\vec{r}-\vec{r}_L|}{2}$ and $\eta f = \frac{|\vec{r}-\vec{r}_R|-|\vec{r}-\vec{r}_L|}{2}$ and perform an integration over prolate spheroidal coordinates within the intervals: $1 \leq \xi < \infty$, $-1 \leq \eta \leq 1$ and $0 \leq \phi \leq 2\pi$. The integration volume is $d\vec{r} = d\eta d\xi d\phi |f^3(\eta^2 - \xi^2)|$. Using straightforward algebra one obtains

$$ V \propto \left| \frac{\sin(Kd)}{Kd} \right|^N, $$

where $2f = d$ and $d$ is the separation between two slits.

Such dependence of the visibility on the number $N$ of emitted photons and their wave-number $K$ is in agreement with decoherence observed in an atom interferometry [10]. It is now clear from Eq. (5) that in the extreme case of the wave length much smaller then the slit separation and/or sufficiently large number of emitted photons the visibility $V$ vanishes. Yet, in the fullerene experiment another extreme case is reached. There the slit separation $d = 1\mu m$, the photons wave length is of the order of $10\mu m$, and the estimated number of photons emitted during the entire time of flight of the fullerene are 1-2. Therefore $\left| \frac{\sin(Kd)}{Kd} \right|^N \approx 1$ and the high visibility remains preserved.

![Interference pattern of C-60 molecules behind a 100 nm grating](image)
III. INFORMATION AND COMPLEMENTARITY IN A QUANTUM INTERFERENCE EXPERIMENT

The possible choice between path information and the observability of interference patterns is one of the most basic manifestations of quantum complementarity, as introduced by Niels Bohr. Following our discussion, it is clear that it is the experimentalist who decides which observable to measure. He can decide, for example, whether to put a detector into the interfering paths or not. This role of the observer has led to numerous misunderstandings about the Copenhagen interpretation of quantum mechanics. Very often, and erroneously, a strong subjective element is brought into the discussion, implying that even the consciousness of the observer has a role in the quantum measurement process. One has to be very careful at this point.

Just to follow our example, the observer can decide whether or not to put detectors into the interfering path. That way, by deciding whether or not to determine the path through the two-slit experiment, he can decide which property can become reality. If he chooses not to put the detectors there, then the interference pattern will become reality; if he does put the detectors there, then the beam path will become reality. Yet, most importantly, the observer has no influence on the specific element of the world which becomes reality. Specifically, if he chooses to determine the path, he has no influence whatsoever which of the two paths, the left one or the right one, Nature will tell him is the one where the particle is found. Likewise, if he chooses to observe the interference pattern he has no influence whatsoever where in the observation plane he will observe a specific particle. Both outcomes are completely random.

We therefore argue that the observer has a qualitative influence on Nature by deciding via his choice of apparatus which quality can manifest itself as reality, but he has no quantitative influence in the sense of which specific result will be the outcome. It therefore appears that the objective randomness of quantum measurement provides a limit to the control any experimentalist has. Bohr [11] writes succinctly: "... a subsequent measurement to a certain degree deprives the information given by a previous measurement of its significance for predicting the future course of phenomena. Obviously, these facts not only set a limit to the extent of the information obtainable by measurement, but they also set a limit to the meaning which we may attribute to such information. We meet here in a new light the old truth that in our description of nature the purpose is not to disclose the real essence of the phenomena but only to track down, so far as it is possible, relations between the manifold aspects of our experience."

We will now argue that the impossibility of joint perfect observation of both path and the interference pattern is a natural consequence of the finiteness of the information content of a quantum system. On the basis of a specific measure of information we will define information content of a quantum system. That information can fully be contained either in the path or in the interference pattern. In both of them only partially to the extent defined by the fundamental limit on the information content. Therefore we will give a quantitative information-theoretic formulation of quantum complementarity in Young’s experiment.

In a double-slit experiment the path information is a dichotomic, i.e. a two-valued observable while the position in the interference pattern is a continuous one, which makes the consideration more complicated. For that reason we will modify our set-up to that of an interferometer [Fig. 3] where both path information and interference observation are dichotomic. Afterwards we will extend our analysis to a double-slit experiment. If in Fig. 3 the incoming state $\psi_1$ has amplitude $a$ and the incoming state $\psi_2$ has amplitude $b$ ($a, b \in \mathbb{R}, a^2 + b^2 = 1$), then by the usual rules of a symmetric beam splitter [12], the outgoing states $\psi_3$ and $\psi_4$ become

$$\psi_3 = \frac{1}{\sqrt{2}}(iae^{ix} + b), \quad \psi_4 = \frac{1}{\sqrt{2}}(ae^{ix} + ib),$$

where we allow for an arbitrary, but constant, phase difference $\chi$ between amplitudes $a$ and $b$. It now follows that the probabilities $p_1, p_2, p_3,$ and $p_4$ to find an individual particle in any of the four beams are:

$$p_1 = a^2, \quad p_2 = b^2, \quad p_3 = \frac{1}{2}(1 - 2ab \sin \chi), \quad p_4 = \frac{1}{2}(1 + 2ab \sin \chi).$$

Evidently, because of unitarity, $p_1 + p_2 = 1$ and $p_3 + p_4 = 1$. How can we see now the complementarity between the path information and the interference phenomenon?

It is suggestive to assume that our ability to determine which path the particle takes is related to the modulus $|p_1 - p_2|$ of the difference between the probabilities in path 1 and path 2. This difference results in the minimal value of 0 if both probabilities are equal and in the maximal value of unity if one of the probabilities is 1. In the same way as we assume the information available about the path to be proportional to the modulus of the difference $|p_1 - p_2|$, we may also assume the information in the interference pattern to be proportional to the modulus of
the difference \(|p_3 - p_4|\). There is some complementarity between \(|p_1 - p_2|\) and \(|p_3 - p_4|\), and we will now express it quantitatively such that the total information is a constant. Indeed, we find, if we introduce our new measure of information \([\mathbf{13}]\) we are led to a quantitative statement of the complementarity principle. Our new measure of information, which is suitable to define the information gain in a quantum experiment, takes probability squares as a quantitative statement of our knowledge. In \([\mathbf{14}]\) it was shown that this particular measure of information is related to the estimation of the future number of occurrences of a specific outcome in a repetition of a binary experiment with two probabilistic outcomes.

We now introduce the following quantitative amounts of information

\[
I_1 = (p_1 - p_2)^2, \quad I_2 = (p_3 - p_4)^2, \quad \text{and} \quad I_3 = (p'_3 - p'_4)^2, \quad (9)
\]

where we have introduced the probabilities \(p'_3\) and \(p'_4\) as those probabilities where we use an additional phase shifter of phase \(\frac{\pi}{2}\) in, say, beam 2, resulting in the probabilities

\[
p'_3 = \frac{1}{2}(1-2ab\cos\chi), \quad \text{and} \quad p'_4 = \frac{1}{2}(1+2ab\cos\chi). \quad (10)
\]

The reason that we consider also the probabilities \(p'_3\) and \(p'_4\) is that for any specific phase shifts \(\chi\) between the two incoming amplitudes, even without path information, our knowledge whether the particle will be found in beam 3 or 4 might not be maximal (Fig. 3). This knowledge however can then be re-established if an additional phase shift of \(\frac{\pi}{2}\) is introduced between the two amplitudes.

Now, for the sum of the three individual measures of information, we obtain

\[
I_1 + I_2 + I_3 = 1. \quad (11)
\]

Such a complementarity relation resulting in a constant is possible only if our new measure \([\mathbf{3}]\) is used and could not be obtained if, for example, Shannon’s measure of information were used \([\mathbf{4}]\). An important property of the information content of a quantum system as defined by Eq. \((\mathbf{3})\) is that it neither depends on the incoming amplitudes \(a\) and \(b\), nor on the phase factor \(\chi\) between them. This means that the total information is invariant under unitary transformations and thus equal for all possible pure incoming states. Therefore different pure incoming states might have different individual measures of information \(I_1, I_2\) and \(I_3\) but their sum is always 1 bit of information.

Here \(I_1\) describes the path information and \(I_2\) and \(I_3\) together describe the visibility of the interference effect. We may therefore introduce the new variables \(I_{\text{path}} = I_1\) and \(I_{\text{interf}} = I_2 + I_3\), and we obtain the final result (See also \([\mathbf{13}]\))

\[
I_{\text{path}} + I_{\text{interf}} = 1. \quad (12)
\]

which is a quantitative statement of the principle of complementarity in Young’s experiment. One may reinterpret Eq. \((\mathbf{12})\) such that a single particle in Young’s experiment is just the representative of one bit of information and the experimentalist has the choice by deciding whether to determine the path or not, whether this information resides in the path or in interference or in both of them partially to the extend defined by Eq. \((\mathbf{12})\).

We will now extend our consideration to the situation of a double-slit experiment [Fig. 1]. We assume that the amplitude of the interfering particle is \(a\) in the left slit and \(b\) in the right slit \((a, b \in \mathbb{R}, a^2 + b^2 = 1)\), where again we allow for an arbitrary phase difference \(\chi\) between the two amplitudes. A typical interference pattern in the Fraunhofer limit has a sinusoidal form with a periodicity of \(Y = \frac{2\pi d}{L}\) where \(k\) is the de-Broglie wave-number, \(d\) is the separation between the two slits and \(L\) is the distance between the plane with slits and the observation plane. Consider now two pairs of points \(A_1 = y, A_2 = y + Y/2\) and \(B_1 = y + Y/4, B_2 = y + 3Y/4\) in the observation plane, as shown in Fig. 1. On the basis of our new measure of information we now introduce the amount of information \(I_A = \frac{p(A_1) - p(A_2)}{p(A_1) + p(A_2)}\) for the pairs of points \(A_1\) and \(A_2\), and similarly \(I_B = \frac{p(B_1) - p(B_2)}{p(B_1) + p(B_2)}\) for \(B_1\) and \(B_2\). Here, for example, \(p(A_1)\) is the conditional probability to detect particle at \(A_1\) given that the particle is to be found either in \(A_1\) or \(A_2\). Therefore \(I_A\) is the measure of the information that the particle will be found in the specific point \(A_1\) or in the specific point \(A_2\) given that we know it will be found at \(A_1\) or \(A_2\) anyway. The probability density to detect the particle at point \(y\) in the observation plane in the Fraunhofer limit is given by

\[
p(y) = \frac{1}{Y} \left[ 1 + 2ab\cos\left(\frac{kd}{L}y + \chi\right) \right]. \quad (13)
\]

Here the probability distribution is normalized such that the total probability to find the particle somewhere within the interval \([0, Y]\) of one period is unity. If we now use \(I_1\) for the amount of information contained in the path and \(I_A\) in the pair of observation points \(A_1, A_2\) and \(I_B\) in the pair \(B_1, B_2\), then we obtain again that

\[
I_1 + I_A + I_B = 1.
\]

We notice that the four selected points \(A_1, A_2, B_1\) and \(B_2\) for which the probability is calculated are just separated by \(Y/4\) and can be selected for any choice of \(y\). Like in the case of the interferometer, we will now summarize all individual measures of information \(I_A\) and \(I_B\) for all \(y\) and thus obtain the information contained in the full interference pattern.

We still use \(I_{\text{path}}\) as given above for the measure of information contained in the path. Yet now we suggest the information contained in the interfering path to be defined by the integral
\[ I_{\text{interf}} = 2Y \int_{0}^{Y/2} [p(y) - p(y + Y/2)]^2 dy \] (14)

Note that the integrand in Eq. (14) contains the combinations
\[ [p(y) - p(y + Y/2)]^2 + [p(y + Y/4) - p(y + 3Y/4)]^2 \]
for every \( y \) within the interval \( [0, Y/4) \), which correspond exactly to the sum \( I_A + I_B \) introduced above. One can easily calculate that \( I_{\text{interf}} = 4a^2b^2 \). Therefore we have again \( I_{\text{path}} + I_{\text{interf}} = 1 \) for the sum of the measures of information contained in the path and in the interference pattern.

The discussion presented above obviously is just one specific example of quantum complementarity at work. It is obvious that this can be extended to much more complicated situations, as for example to the notion of quantum entanglement [16]. From a fundamental perspective, this approach suggests that the most basic notion of quantum mechanics is information [17].

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