Abstract

For a neutral element \( n \in L \), [III] have introduced the concept of \( n \)-distributive lattices which is a generalization of both 0-distributive and 1-distributive lattices. For a central element \( n \) of a nearlattice \( S \), we have discussed \( n \)-distributive nearlattices which is a generalization of both 0-distributive semilattices and \( n \)-distributive lattices. For an element \( n \) of nearlattice \( S \), a convex subnearlattice of \( S \) containing \( n \) is called an \( n \)-ideal of \( S \). In this paper, we have given some properties of \( n \)-distributivenearlattices. Finally, we have included a generalization of prime Separation Theorem in terms of annihilator \( n \)-ideal.

**Keywords:** Central element, 0-distributive lattice, \( n \)-distributive lattice, \( n \)-annihilator, annihilator \( n \)-ideal, prime \( n \)-ideal, \( n \)-distributive nearlattice.
In this paper, we generalize the concept of 0-distributive lattice and \(n\)-distributive lattice and give the notion of \(n\)-distributive nearlattice where \(n\) is a central element of this nearlattice.

A nearlattice \(S\) is a meet semilattice with the property that, any two elements possessing a common upper bound, have a supremum. Nearlattice \(S\) is distributive if for all \(x, y, z \in S\), \(x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)\) provided \(y \vee z\) exists. For detailed literature on nearlattices, we refer the reader to consult [V] and [VIII]. An element \(n\) of a nearlattice \(S\) is called medial if \(m(x, n, y) = (x \wedge y) \vee (x \wedge n) \vee (y \wedge n)\) exists in \(S\) for all \(x, y \in S\). A nearlattice \(S\) is called a medial nearlattice if \(m(x, y, z)\) exists for all \(x, y, z \in S\).

An element \(s\) of a nearlattice \(S\) is called standard if for all \(t, x, y \in S, t \wedge [(x \wedge y) \vee (x \wedge s)] = (t \wedge x \wedge y) \vee (t \wedge x \wedge s)\). The element \(s\) is called neutral if (i) \(s\) is standard and (ii) for all \(x, y, z \in S, s \wedge [(x \wedge y) \vee (x \wedge z)] = (s \wedge x \wedge y) \vee (s \wedge x \wedge z)\).

In a distributive nearlattice, every element is neutral and hence standard. An element \(n\) in a nearlattice \(S\) is called sesquimedial if for all \(x, y, z \in S\), \(((x \wedge n) \vee (y \wedge n)) \wedge [(y \wedge n) \vee (z \wedge n)] \vee (x \wedge y) \vee (y \wedge z)\) exists in \(S\).

An element \(n\) of a nearlattice \(S\) is called a upper element if \(x \vee n\) exists for all \(x \in S\). Every upper element is of course a sesquimedial element. An element \(n\) is called a central element of \(S\) if it is neutral, upper and complemented in each interval containing it.

Let \(S\) be a nearlattice and \(n \in S\). Any convex subnearlattice of \(S\) containing \(n\) is called an \(n\)-ideal of \(S\). For two \(n\)-ideals \(I\) and \(J\) of a nearlattice \(S\), [V] has given a description of \(I \vee J\) while the set theoretic intersection is the infimum. Hence, the set of all \(n\)-ideals of a nearlattice \(S\) is a lattice which is denoted by \(I_n(S)\). \(\{n\}\) and \(S\) are the smallest and largest elements of \(I_n(S)\).

An \(n\)-ideal generated by a finite number of elements \(a_1, a_2, \ldots, a_m\) is called a finitely generated \(n\)-ideal and it is denoted by \(< a_1, a_2, \ldots, a_m >_n\). The set of all finitely generated \(n\)-ideals is denoted by \(F_n(S)\). Clearly, \(< a_1, a_2, \ldots, a_m >_n = < a_1 >_n \vee < a_2 >_n \vee \ldots \vee < a_m >_n\). An \(n\)-ideal generated by a single element \(a\) is called a principal \(n\)-ideal denoted by \(< a >_n\). The set of principal \(n\)-ideals is denoted by \(P_n(S)\).

Let \(S\) be a nearlattice and \(n \in S\). For any \(a \in S\),

\[< a >_n = \{ y \in S : a \wedge n \leq y = (y \wedge a) \vee (y \wedge n) \}\]
\[
\{ y \in S : y = (y \land a) \lor (y \land n) \lor (a \land n) \} \text{whenever } n \text{ is standard element in } S.
\]

If \( n \) is an upper element in a nearlattice \( S \), then \( < a >_n = [a \land n, a \lor n] \).

We know that when \( n \) is standard and medial, the set of all principal \( n \)-ideals \( P_n(S) \) is a meet semilattice and \( < a >_n \cap < b >_n = < m(a, n, b) >_n \) for all \( a, b \in S \). Also, when \( n \) is neutral and sesquimedial, then \( P_n(S) \) is a nearlattice. By \( [V] \) if \( S \) is medial nearlattice and \( n \) is a neutral element of \( S \), then \( P_n(S) \) is also a medial nearlattice.

For a distributive nearlattice with an upper element \( n \), \( P_n(S) \) is a distributive nearlattice with the smallest element \( \{ n \} \).

A proper convex subnearlattice \( M \) of a nearlattice \( S \) is called a maximal convex subnearlattice if for any convex subnearlattice \( Q \) with \( Q \supseteq M \) implies either \( Q = M \) or \( Q = S \). A proper convex subnearlattice \( M \) of a medial nearlattice \( S \) is called a prime convex subnearlattice if for any \( t \in M, m(a, t, b) \in M \) implies either \( a \in M \) or \( b \in M \). For a medial element \( n \), an \( n \)-ideal \( P \) of a nearlattice \( S \) is a prime \( n \)-ideal if \( P \neq S \) and \( m(x, n, y) \in P \) \( (x, y \in S) \) implies either \( x \in P \) or \( y \in P \). Equivalently, \( P \) is prime if and only if \( < a >_n \cap < b >_n \subseteq P \) implies either \( < a >_n \subseteq P \) or \( < b >_n \subseteq P \).

Let \( n \) be a central element of a nearlattice \( S \). For \( a \in S \), we define \( \{ a \}^{\perp n} = \{ x \in S : m(x, n, a) = n \} \), known as an \( n \)-annihilator of \( \{ a \} \). Also for \( A \subseteq S \), we define \( A^{\perp n} = \{ x \in S : m(x, n, a) = n \text{ for all } a \in A \} \). \( A^{\perp n} \) is always a convex subnearlattice containing \( n \). If \( S \) is a distributive nearlattice, then it is easy to check \( \{ a \}^{\perp n} \) and \( A^{\perp n} \) are \( n \)-ideals. Moreover, \( A^{\perp n} = \cap_{a \in A} \{ a \}^{\perp n} \). If \( A \) is an \( n \)-ideal, then \( A^{\perp n} \) is called an annihilator-\( n \)-ideal which is obviously the pseudo-complement of \( A \) in \( I_n(S) \). Therefore, for a distributive nearlattice \( S \) with central element \( n \), \( I_n(S) \) is pseudocomplemented.

A nearlattice \( S \) with central element \( n \), is called an \( n \)-distributive nearlattice if for all \( a, b, c \in S, < a >_n \cap < b >_n = \{ n \} \) and \( < a >_n \cap < c >_n = \{ n \} \) imply \( < a >_n \cap < b >_n \cap < c >_n = \{ n \} \). Equivalently, \( S \) is called \( n \)-distributive nearlattice if \( a \lor b \leq n \leq a \lor b \) and \( a \land c \leq n \leq a \lor c \) imply \( a \land (b \lor c) \leq n \leq a \lor (b \land c) \).

**II. Main results**

To obtain the main results of this paper we need to prove the following lemmas.

**Lemma (2.1):** Every convex subnearlattice not containing \( n \) is contained in a maximal convex subnearlattice not containing \( n \).

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Proof: Let $F$ be a convex subnearlattice such that $n \notin F$. Let $\mathcal{F}$ be the set of all convex subnearlattice containing $F$ but not containing $n$. $\mathcal{F}$ is non-empty as $F \in \mathcal{F}$. Let $C$ be a chain in $\mathcal{F}$ and $M = \bigcup (X | X \in C)$. Let $x, y \in M$. Then $x \in X$ and $y \in Y$ for some $X, Y \in C$. Since $C$ is a chain, so either $X \subseteq Y$ or $Y \subseteq X$. Suppose $X \subseteq Y$. Then $x, y \in Y$. Hence $x \wedge y, x \vee y \in M$. Thus $M$ is a subnearlattice of a nearlattice containing $F$. Also it is convex as each $X \in C$ is convex. Clearly $n \notin M$. Hence $M$ is a maximal element of $C$. Therefore, by Zorn’s Lemma, $\mathcal{F}$ has a maximal element, say $Q$ with $F \subseteq Q$.

Lemma (2.2): Let $S$ be a nearlattice with a central element $n$. A convex subnearlattice $M$ not containing $n$ is maximal if and only if for all $a \notin M$ there exists $b \in M$ such that $m(a, n, b) = n$.

Proof: Suppose $M$ is a maximal convex subnearlattice and $n \notin M$. Also let $a \notin M$. Suppose for all $b \in M$, $m(a, n, b) \neq n$. Set $M_1 = \{y \in L: y \wedge n \leq (a \wedge b) \leq (a \vee b) \vee n \leq y \vee n\}$. Obviously, $M_1$ is convex subnearlattice as $n$ is central. Moreover, $n \notin M_1$. For otherwise $n \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee n \leq n \vee n$ implies $m(a, n, b) = n$ which gives a contradiction to the assumption. For $b \in M$, $b \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee n \leq b \vee n$ implies $b \in M_1$ and so $M \subseteq M_1$. Also, $a \wedge n \leq (a \vee b) \wedge n \leq (a \wedge b) \vee n \leq a \vee n$ implies $a \in M_1$ but $a \notin M$ so $M \subset M_1$. Therefore, we have a contradiction to the maximality of $M$ and so there exists some $b \in M$ such that $m(a, n, b) = n$. Conversely, if $M$ is not maximal and $n \notin M$, then by Lemma (2.1), $M$ properly contained in a maximal convex subnearlattice $N$ not containing $n$. Then for any element $a \in N - M$ there exists an element $b \in M$ such that $m(a, n, b) = n$. Thus, by convexity $a, b \in N$ and $a \wedge b \leq n \leq a \vee b$ imply $n \in N$ which is a contradiction. Hence $M$ must be maximal.

Following two lemmas are due to [VII]

Lemma (2.3): A proper subset $I$ of a join semilattice $S$ is a maximal ideal if and only if $S - I$ is a minimal prime up set (filter).

Lemma (2.4): Let $I$ be an ideal of a join semilattice $S$ with $1$. Then there exists a maximal ideal containing $I$.

Theorem (2.5): For a medial element $n$, any prime ideal $P$ containing $n$ of a nearlattice $S$ is a prime $n$-ideal.

Proof: Since every ideal $P$ is a convex subnearlattice, so any ideal $P$ containing $n$ is an $n$-ideal. To show the primeness, let $m(a, n, b) \in P$. Then $a \wedge b \leq m(a, n, b)$ implies $a \wedge b \in P$. Since $P$ is prime ideal so either $a \in P$ or $b \in P$. Hence $P$ is a prime $n$-ideal.

Following lemma is due to [VI]
Lemma (2.6): Every ideal disjoint from a filter \( F \) is contained in a maximal ideal disjoint from \( F \).

Theorem (2.7): Let \( S \) be a nearlattice with a center element \( n \). If the intersection of all prime \( n \)-ideals of \( S \) is \( \{n\} \), then \( S \) is \( n \)-distributive.

Proof: Let \( < a >_n \cap < b >_n = \{n\} \) and \( < a >_n \cap < c >_n = \{n\} \). Let \( P \) be any prime \( n \)-ideal. If \( a \in P \), then \( < a >_n \subseteq P \) and so \( < a >_n \cap < b >_n \cap < c >_n \subseteq P \). If \( a \notin P \), then \( < b >_n \cap < c >_n \subseteq P \) as \( P \) is prime \( n \)-ideal. Hence \( < b >_n \cup < c >_n \subseteq P \). Therefore, \( < a >_n \cap < b >_n \cup < c >_n \subseteq P \) for all prime \( n \)-ideals \( P \). Therefore, \( < a >_n \cap < b >_n \cup < c >_n \subseteq \{n\} \) and so \( S \) is \( n \)-distributive.

Lemma (2.8): Let \( S \) be a nearlattice with a center element \( n \). Then \( p \in \{x\}^{\downarrow}n \) if and only if \( p \land x \leq n \leq p \lor x \).

Proof: \( p \in \{x\}^{\downarrow}n \) if and only if \( m(p, n, x) = n \) if and only if \( (p \land x) \lor (p \land n) \lor (x \land n) = (p \lor x) \land (p \lor n) \land (x \lor n) = n \), as \( n \) is central. This implies that \( p \land x \leq n \leq p \lor x \).

Lemma (2.9): Let \( S \) be a nearlattice with a central element \( n \). Then \( p \in \{x\}^{\downarrow}n \) if and only if \( p \lor n \in \{x \lor n\}^{\downarrow}n \) in \( [n] \) and \( p \land n \in \{x \land n\}^{\downarrow}d \) in \( (n) \).

Proof: Let \( p \in \{x\}^{\downarrow}n \). Then \( p \land x \leq n \leq p \lor x \) and so \( (p \lor n) \land (x \lor n) = (p \land x) \lor n = n \) and \( (p \land n) \lor (x \land n) = (p \lor x) \lor n = n \) as \( n \) is central element. Thus \( p \lor n \in \{x \lor n\}^{\downarrow}n \) in \( [n] \) and \( p \land n \in \{x \land n\}^{\downarrow}d \) in \( (n) \). Conversely, let \( p \lor n \in \{x \lor n\}^{\downarrow}n \) in \( [n] \) and \( p \land n \in \{x \land n\}^{\downarrow}d \) in \( (n) \). Then since \( n \) is central element, so \( (p \lor n) \land (x \lor n) = n \) and \( (p \land n) \lor (x \land n) = n \). This implies \( p \land x \leq n \). Also, \( (p \lor n) \lor (x \lor n) = n \) implies \( p \lor x \land n = n \) and so \( p \leq n \). Hence \( p \land x \leq n \leq p \lor x \). Therefore, by Lemma (2.8), \( p \in \{x\}^{\downarrow}n \).

Now, we give some characterizations of \( n \)-distributive nearlattices.

Theorem (2.10): For a nearlattice \( S \) with a central element \( n \), the following conditions are equivalent:

(i) \( S \) is \( n \)-distributive
(ii) For every \( a \in S \), \( \{a\}^{\downarrow}n \) is an \( n \)-ideal
(iii) For any \( A \subseteq S \), \( A^{\downarrow}n \) is an \( n \)-ideal
(iv) \( I_n(S) \) is pseudocomplemented.
(v) \( I_n(S) \) is 0-distributive
(vi) Every maximal convex subnearlattice not containing \( n \) is prime.
Proof: (i)⇒(ii). Let \( x, y \in \{a\}^n \). Then \( a \land x \leq n \leq a \lor x \) and \( a \land y \leq n \leq a \lor y \). Since \( S \) is distributive, so \( a \land (x \lor y) \leq n \leq a \lor (x \land y) \). Then \( a \land (x \lor y) \leq n \leq a \lor (x \land y) \) and \( a \land (x \lor y) \leq n \leq a \lor (x \land y) \) imply \( x \land y, x \lor y \in \{a\}^n \) [by Lemma (2.8)]. Since \( m(x, n, a) = n \), so \( n \in \{a\}^n \).

Again, let \( x, y \in \{a\}^n \) and \( x \leq t \leq y \). Then \( a \land x \leq n \leq a \lor x \) and \( a \land y \leq n \leq a \lor y \) so \( a \land t \leq n \leq a \lor t \) which implies that \( t \in \{a\}^n \). Hence \( \{a\}^n \) is an \( n \)-ideal.

(ii)⇒(iii). Since \( \{a\}^n \) is an \( n \)-ideal and \( A^\perp = \cap_{a \in A} \{\{a\}^n\} \), so \( A^\perp \) is an \( n \)-ideal.

(iii)⇒(iv) is trivial as for any \( n \)-ideal \( A \in I_n(S) \), \( A^\perp \) is the pseudocomplement of \( A \) in \( I_n(S) \).

(iv)⇒(v) is also trivial because every pseudocomplemented lattice is 0-distributive.

(v)⇒(vi). Suppose \( F \) is maximal convex subnearlattice not containing \( n \). Since \( F = \{F \} \cap \{F\} \) and \( n \in F \), so either \( n \notin \{F\} \) or \( n \notin \{F\} \). Hence by the maximality of \( F \), either \( F \) is an ideal or a filter. Let \( x \notin F \) and \( y \notin F \). Then byLemma (2.2), there exist \( a \in F \) and \( b \in F \) such that \( m(x, n, a) = n = m(y, n, b) \). This implies \( a \land m \leq n \leq x \lor a \) and \( y \lor b \leq n \leq y \lor b \). Hence \( x \land a \land b \leq n \), \( y \land a \land b \leq n \) and \( x \lor a \lor b \geq n \), \( y \lor a \lor b \geq n \) and so \( a \land b, a \lor b \in F \). Then \( x \lor y \lfloor_a \leq n \cap a \land b \lor y \cap a \land b \lor n \)

\[ = [n, x \land a \lor b] \land n = [n, n] = [n] \] as \( n \) is central.

Similarly, \( < y \lor n \lfloor_{n \lfloor n} < a \land b \lor n = [n] \). Since \( I_n(S) \) is a \( 0 \)-distributive, so \( < a \land b \lor y \lfloor n \cap a \land b \lor n = [n] \). This implies \( [n, (a \land b \land (x \lor y)) \lor n] = n \). Hence \( a \land b \land (x \lor y) \leq n \).

Dually, \( < x \land n \lfloor_a \leq n \cap a \land b \lor n = [n] \) and \( < y \land n \lfloor n \cap a \lor b \lor n = [n] \). Without loss of generality, suppose \( F \) is filter. If \( x \lor y \in F \), then \( a \land b \land (x \lor y) \leq n \) implies \( n \in F \) which is a contradiction. Hence \( x \lor y \notin F \). Therefore, \( F \) is a prime filter. Similarly, if \( F \) is a ideal, then it is a prime ideal.

(vi)⇒(i). Let \( a \land b \leq n \leq a \lor b \) and \( a \land c \leq n \leq a \lor c \) provided \( a \lor b \) and \( a \lor c \) exist. We need to show that \( a \land (b \lor c) \leq n \leq a \lor (b \land c) \). If not, without loss of generality, let \( a \land (b \lor c) \notin n \) and \( b \lor c \notin n \). Then by Lemma 1, there exists a maximal convex subnearlattice \( M \ni F \) and \( n \notin M \). But a convex subnearlattice containing a filter is itself a filter. Then by (vi), \( M \) is a filter.

Now, \( a \in M \) and \( b \lor c \in M \) imply \( a \land b \in M \) or \( a \land c \in M \) as \( M \) is prime. This implies \( n \in M \) which is a contradiction. Hence \( a \land (b \lor c) \leq n \leq a \lor (b \land c) \). Therefore, \( S \) is \( n \)-distributive.

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Corollary (2.11): In an n-distributive nearlattice every filter not containing n is contained in a prime filter.

Proof: This is trivial by Lemma (2.1) and Theorem (2.10).

Theorem (2.12): Let S be an n-distributive nearlattice. If A ≠ {n} and A = ∩ {I ∈ Iₙ(S): I ≠ {n}}, then A¹ⁿ = {x ∈ S: {x}¹ⁿ ≠ {n}}.

Proof: Let x ∈ A¹ⁿ. Then m(x, n, a) = n for all x ∈ A. Since A ≠ {n} so \{x\}¹ⁿ ≠ {n}. Hence x ∈ R.H.S. So A¹ⁿ ⊆ R.H.S. Conversely, let x ∈ R.H.S. Since S is n-distributive so \{x\}¹ⁿ is an n-ideal and so \{x\}¹ⁿ ≠ {n}. Then A ⊆ \{x\}¹ⁿ and so A¹ⁿ ⊇ \{x\}¹ⁿ. Therefore, A¹ⁿ = {x ∈ S: \{x\}¹ⁿ ≠ \{n\}}.

Theorem (2.13): Let S be a nearlattice with a central element n. Then S is distributive if and only if for a convex subnearlattice \(\mathcal{F}\) disjoint with \(\{x\}¹ⁿ(x \in S)\), there exists a prime convex subnearlattice \(P \ni \mathcal{F}\) and disjoint with \(\{x\}¹ⁿ\).

Proof: Let S be n-distributive and \(F\) be a convex subnearlattice disjoint from \(\{x\}¹ⁿ\). Then by Zorn’s Lemma, there exists a maximal convex subnearlattice \(P\) disjoint from \(\{x\}¹ⁿ\). Since \(P = (P) \cap (P)\) so either \((P) \cap (\{x\}¹ⁿ) = \phi\) or \((P) \cap (\{x\}¹ⁿ) = \phi\). Thus by the maximality of \(P\), it is either an ideal or a filter. Without loss of generality, let \(P\) be a filter. We claim that \(x \in P\). If \(P \ni [x] \neq P\). Then by the maximality of \(P\), \((P \ni [x]) \cap (\{x\}¹ⁿ) = \phi\). Let \(t \in (P \ni [x]) \cap (\{x\}¹ⁿ)\). Then \(t \geq p \land x\) for some \(p \in P\) and \(t \land x \leq n \leq t \lor x\). Thus \(p \land x \leq t \land s \leq n\). Hence \((p \lor n, n, x) = n\) which implies \(p \lor n \in (\{x\}¹ⁿ)\). But \(p \lor n \in P\) if \(P\) is a filter. This gives a contradiction to the fact that \(P \ni (\{x\}¹ⁿ) = \phi\). Therefore \(x \in P\). Let \(z \notin P\), then \((P \ni [z]) \land (\{x\}¹ⁿ) = \phi\). Let \(y \in (P \ni [z]) \land (\{x\}¹ⁿ)\). Then \(y \land x \leq n \leq y \lor x\) and \(y \geq p \land z\) for some \(p \in P\) so \(p \land x \land z \leq y \land x \leq n\). Hence \(m(z, n, (p \land x) \lor n) = n\) where \((p \land x) \lor n \in P\) as \(P\) is a filter. Then by Lemma (2.2), \(P\) is a maximal filter not containing n. Therefore, by Theorem (2.10), \(P\) is Prime.

Conversely, let \(<x >ₙ \cap <y >ₙ = \{n\}\) and \(<x >ₙ \cap <z >ₙ = \{n\}\). We need to prove that \(<x >ₙ \cap (<y >ₙ \lor <z >ₙ) = \{n\}\). That is, \(x \land (y \lor z) \not\leq n\). Then \([y \lor z] \land (\{x\}¹ⁿ) = \phi\). Otherwise \(t \in [y \lor z] \land (\{x\}¹ⁿ)\) implies \(t \land x \leq n \leq t \lor x\) and \(t \geq y \lor z\). These imply \(x \land y \lor z \leq t \land x \leq n\) is a contradiction. So there exists a prime filter \(P\) containing \([y \lor z]\) disjoint with \(\{x\}¹ⁿ\). Since \(z \in \{x\}¹ⁿ\) so \(y, z \not\in P\). Hence \(y \lor z \not\in P\) as \(P\) is prime. This implies \(P \ni (y \lor z)\) is a contradiction. Dually by taking \(x \lor (y \land z) \not\leq n\), we would have another contradiction. Therefore, \(x \land (y \lor z) \leq n \leq x \lor (y \land z)\) and so \(S\) is n-distributive.
III. Conclusion

In this paper, we generalize the concept of 0-distributive lattice and \( n \)-distributive lattice where \( n \) is a neutral element of this lattice and give the notion of \( n \)-distributive nearlattice where \( n \) is a central element of this nearlattice. We also include several nice characterizations of \( n \)-distributive nearlattices and prove some interesting results on \( n \)-distributive nearlattices.

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