Nonlocal effects in the shot noise of diffusive superconductor - normal-metal systems

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A cross-shaped diffusive system with two superconducting and two normal electrodes is considered. A voltage $eV < \Delta$ is applied between the normal leads. Even in the absence of average current through the superconducting electrodes their presence increases the shot noise at the normal electrodes and doubles it in the case of a strong coupling to the superconductors. The nonequilibrium noise at the superconducting electrodes remains finite even in the case of a vanishingly small transport current due to the absence of energy transfer into the superconductors. This noise is suppressed by electron-electron scattering at sufficiently high voltages.

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Recently, the noise properties of hybrid systems involving superconducting (S) and normal (N) metals became a subject of intensive studies. A key effect in these properties is the Andreev reflection, in which electrons incident from the normal metal are reflected from the NS interface as holes. In particular, it was found that in the zero-voltage limit, the shot noise in diffusive NS contacts with phase-coherent transport is doubled with respect to its value in a normal contact with the same resistance. This doubling was interpreted as an effective doubling of electron charge and has been experimentally confirmed in a number of papers.

Quite recently, it was shown that the doubled shot noise in diffusive NS contacts survives at finite voltages of the order of the energy gap. Moreover, this noise does not require a phase coherence and may be described in terms of a semiclassical Boltzmann - Langevin equation. In this approach, the increased noise in NS systems is due to an excess heating of electron gas rather than to the doubling of the effective charge.

Along with studying the shot noise in two-terminal systems, a considerable attention received the noise in multiterminal structures. It was found that the noise of normal-metal diffusive structures with more than two electrodes may exhibit exchange effects and nonlocal effects. The latter imply that the noise in the system is affected by a presence of open contacts even in the absence of average current flow through them.

Several authors also calculated current correlations in single-channel multiterminal NS systems. In particular, it was found that the noise measured at the two normal electrodes in a four-terminal system depends on the phase difference between the two superconducting electrodes. A current noise was also calculated at a tip placed on a multimode quantum-coherent NS structure.

In this paper, we consider nonlocal effects in multiterminal mesoscopic diffusive SN systems using the semiclassical description of noise. In particular, we report on a semiclassical "proximity" effect in the noise where the current noise in a normal conductor is increased by its contact with a superconductor. Another finding is that under certain conditions, a large noise may be induced in an SNS structure even by a small transverse transport current.

Consider a cross-shaped contact with two superconducting and two normal-metal electrodes (see Fig. 1). The normal-metal electrodes are kept at potentials $\pm V/2$, and the superconducting electrodes are kept at zero potential. It is also assumed that the cross is symmetric, i.e. it consists of two identical resistances $R$ connecting the crossing point with the normal electrodes and two identical resistances $r$ connecting it with the superconductors. Each resistor presents a long and narrow diffusive wire with the Thouless energy much smaller than the energy gap of the superconductor. The resistance of the crossing point is assumed to be negligible. The applied voltage $eV$

FIG. 1. A four-terminal NS system. Normal-metal electrodes 1 and 3 are kept at potentials $\pm V/2$, while the superconducting electrodes 2 and 4 are kept at zero potential.
is assumed to be larger than the Thouless energy but smaller than the energy gap of the superconductors. Note that in this geometry, there is now electrical current flow through the superconducting ends of the cross.

Under the above conditions, the kinetics of fluctuations may be described using a semiclassical Langevin equation

$$\delta j_i = -D \frac{\partial}{\partial r} \delta \rho - \sigma \frac{\partial}{\partial r} \delta \phi + \delta j_i^{\text{ext}},$$  \hspace{1cm} (1)

where $D$ is the diffusion coefficient, $\sigma$ is the electric conductivity, $\delta \rho(r)$ is the charge-density fluctuation, $\delta \phi(r)$ is the local fluctuation of the electric potential, and the correlator of extraneous currents $\delta j_i^{\text{ext}}$ is given by

$$\langle \delta j_i^{\text{ext}}(r_1) \delta j_i^{\text{ext}}(r_2) \rangle = 4\sigma \delta \rho \delta (r_1 - r_2) T_N(r_1),$$

$$T_N(r) = \int dr f(\varepsilon, r)[1 - f(\varepsilon, r)].$$ \hspace{1cm} (2)

Equation (1) may be integrated along the length of each arm of the cross, which gives for the fluctuations of currents flowing into each of the four electrodes

$$\delta I_i = \frac{1}{R_i} \left( \delta \phi^* + \frac{1}{e^2 N_F} \delta \rho^* \right) + \delta I_i^{\text{ext}},$$ \hspace{1cm} (3)

where $R_i$ is the resistance of the corresponding arm, $\delta \phi^*$ and $\delta \rho^*$ are the fluctuations of electrical potential and charge density at the crossing point, and the correlator of extraneous currents $\delta I_i^{\text{ext}}$ equals

$$\langle \delta I_i^{\text{ext}}(t_1) \delta I_j^{\text{ext}}(t_2) \rangle = \delta_{ij} \frac{4T_i}{R_i},$$ \hspace{1cm} (4)

where $T_i$ is obtained by averaging $\langle \delta \phi \rangle$ over the length of the corresponding arm. The system of equations (3) combined with the current-conservation condition at the crossing point

$$\sum_i \delta I_i = 0$$

is easily solved giving

$$\delta I_i = \delta I_i^{\text{ext}} - \frac{1}{R_i} \sum_j \delta I_j^{\text{ext}} \left/ \sum_j \frac{1}{R_j} \right.,$$ \hspace{1cm} (5)

and hence the cross-correlated spectral density is

$$S_{ij} \equiv \langle \delta I_i \delta I_j \rangle \omega = \delta_{ij} \frac{4T_i}{R_i} - \frac{4}{R_i R_j} \sum_k \frac{T_k}{R_k} \left/ \left( \sum_k \frac{1}{R_k} \right)^2 \right. + \frac{4}{R_i R_j} \sum_k \frac{T_k}{R_k} \left/ \left( \sum_k \frac{1}{R_k} \right)^2 \right. \left( \sum_k \frac{1}{R_k} \right)^2.$$ \hspace{1cm} (6)

In what follows, we restrict ourselves to relatively small voltages $eV < 2\Delta$ and zero temperatures. In each arm of the cross, the average distribution function $f(\varepsilon, x)$ obeys the standard diffusion equation $\nabla^2 f = 0$. Introducing the distribution function $f^\star(\varepsilon)$ at the crossing point, one can write down the distribution in each arm in the form

$$f(\varepsilon, x_i) = \left(1 - \frac{x_i}{L_i}\right) f^\star(\varepsilon) + \frac{x_i}{L_i} f_i(\varepsilon),$$ \hspace{1cm} (7)

where $L_i$ is the length of the arm and $f_i$ is the distribution at the end of it. The distribution function $f^\star$ should be determined from the balance of the diffusion fluxes at the crossing point for any energy

$$\sum_i \frac{\partial f(\varepsilon, x_i)}{\partial x_i} \bigg|_{x_i=0} = 0.$$ \hspace{1cm} (8)

The distribution functions at the normal ends of the cross $f_1 = f_0(\varepsilon - eV/2)$ and $f_3 = f_0(\varepsilon + eV/2)$ are just the equilibrium Fermi functions shifted in energy by $\pm eV/2$. Owing to the Andreev reflections from the interfaces with the superconductors, the distribution functions at these interfaces are related to the distribution function at the crossing point by a very simple expression

$$f_2(\varepsilon) = f_4(\varepsilon) = \frac{1}{2} \left[1 + f^\star(\varepsilon) - f^\star(-\varepsilon)\right].$$ \hspace{1cm} (9)

In the symmetric case where $R_1 = R_3 = R$ and $R_2 = R_4 = r$, it is easily obtained that

$$f^\star(\varepsilon) = f_2 = f_4 = \frac{1}{2}(f_1 + f_3).$$ \hspace{1cm} (10)

Substituting Eq. (1) into Eq. (2), calculating the corresponding $T_N(x_i)$ by means of (3) and averaging them over the corresponding segments, one easily obtains that $T_1 = T_3 = eV/6$ and $T_2 = T_4 = eV/4$. From this, one readily obtains the cross-correlated spectral densities by means of (4). Taking into account that the average current flowing between the normal electrodes is $J = eV/2R$, the expression for the noise at the normal ends may be written in the form

$$S_{11} = S_{33} = \frac{eI}{3} \frac{4R^2 + 7Rr + 2r^2}{(R + r)^2},$$ \hspace{1cm} (11)

The cross-correlated noise at the normal ends is

$$S_{13} = \frac{eI}{3} \frac{r(R + 2r)}{(R + r)^2}.$$ \hspace{1cm} (12)

The spectral densities of noise at the superconducting ends of the cross are given by

$$S_{22} = S_{44} = \frac{eV}{6r} \frac{3R^2 + 8Rr + 6r^2}{(R + r)^2},$$ \hspace{1cm} (13)

and

$$S_{24} = -\frac{eV}{6r} \frac{R(3R + 4r)}{(R + r)^2}.$$ \hspace{1cm} (14)
It is noteworthy that the noise in the "superconducting" arms remains finite even if $R \to \infty$ and the transport current through the system vanishes. In this case, $S_{22} = S_{44} = -S_{24} = eV/2r$. This is in contrast to a purely normal system, where $S_{22} = (2/3)eV/(R + r)$ and tends to zero as either $r$ or $R$ becomes infinitely large. The reason for this is that the electron gas in this case is confined between two interfaces with superconductors, which hinder heat transfer from it. Hence it may be strongly heated even with a small current, much like in the case of two-terminal diffusive SNS contacts it is strongly heated even by a small voltage.

As $R \to 0$, the spectral densities $S_{22}$ and $S_{44}$ increase to $eV/r$, while the cross-correlated spectral density $S_{24}$ tends to $-2eV/3r$, which implies that the current fluctuations at the opposite superconducting ends become only partially correlated.

Consider now the effects of electron-electron scattering on the spectral density $S_{22}$ in the case of a strong coupling. In the case of two-terminal SNS contacts, this type of scattering was shown to suppress the nonequilibrium noise at low voltages, because the quasiparticles confined between the two NS interfaces may be outscattered from the subgap energy range and escape into the superconducting electrodes thus transferring energy and effectively cooling the electron gas.

Consider the case where the length of a "normal" arm $L_R$ is much larger than the electron-electron scattering length. In this case, the electron gas may be described by a local effective temperature, which depends only on the coordinate $x$ in the direction of average current flow and is constant in the "superconducting" arms at $x = 0$. This effective temperature obeys a heat-balance equation

$$\pi^2 \frac{d^2}{dx^2} \left( T^*_{e} \right)^2 = - \left( \frac{eV}{2L_R} \right)^2 + \delta(x) \frac{2}{L_R \varepsilon_T} \frac{R}{r} J, \quad (16)$$

where $\varepsilon_T$ is the Thouless energy of a "superconducting" arm and

$$J = \alpha_{ee} \frac{\Delta^3 T^*}{\varepsilon_F} \exp \left( - \frac{\Delta}{T^*} \right)$$

is the density of flux of energy carried by electrons and holes outscattered from the subgap region by electron-electron collisions, which depends on the effective temperature in the "superconducting" arms $T^* = T_e(0)$. Making use of the boundary conditions $T_e(L_R) = T_e(-L_R) = 0$, one easily obtains a closed equation for $T^*$ in the form

$$(T^*)^2 = \frac{3}{4\pi^2} (eV)^2 - \frac{6}{\pi^2} \alpha_{ee} \frac{R}{r} \frac{\Delta^3 T^*}{\varepsilon_F \varepsilon_T} \exp \left( - \frac{\Delta}{T^*} \right). \quad (17)$$

From this equation, it is readily seen that in the case of large values of the prefactor $\lambda = \alpha_{ee} (R/r) (\Delta^2/\varepsilon_F \varepsilon_T)$ the effective temperature $T^*$ and the spectral density $S_{22} = 2T^*/r$ become suppressed at voltages $V > \Delta/(e \ln \lambda)$. 

FIG. 2. The dependences of normalized spectral density of noise at electrode 1 on the strength of coupling to the lateral electrodes 2 and 4 for a hybrid NS system (solid line) and for a normal-métal system (dashed line). To complete the description, we also present the cross-correlated spectral density at the normal and superconducting ends

$$S_{12} = -\frac{eV}{6} \frac{3R + 2r}{(R + r)^2} \quad (15)$$

Note that all the cross-correlated spectral densities $S_{12}$, $S_{34}$, and $S_{35}$ are always negative.

Consider now the most interesting case of a strong coupling to the superconductors where $r \ll R$. It is seen from Eqs. (11) and (12) that the noise measured at one of the normal ends of the cross is doubled with respect to a two-terminal normal-metal system, as it takes place in diffusive contacts where the transport current flows through an SN interface. However the physics of this effect is different because the cross-correlated spectral density of noise at the normal ends $S_{12}$ tends to zero.

The reason for the increased shot noise is that the "superconducting" arms, which do not carry any transport current, are open yet for the current fluctuations and act as noise generators that supply additional electric fluctuations into the system. In the case of normal-metal electrodes 2 and 4, the noise should be also slightly increased, but the increment would reach its 25% maximum at $r = R$ (see Fig. 2). At strong coupling, such cross would just break down into two independent contacts, each with the resistances $R$ and a voltage drop $V/2$. Hence the noise at electrodes 1 and 3 would be just $2eI/3$. However the NS interfaces, while being transparent for the fluctuations of electric current, forbid the energy transport into the superconductors and hence do not allow the distribution function at the crossing point to assume the equilibrium shape. Clearly, the doubling of shot noise should take place also in a three-terminal structure with only one superconducting electrode attached in the middle of the normal conductor.
Unlike the case of two-terminal SNS contacts, the suppression takes place at high voltages rather than at small ones.

In summary, we semiclassically considered nonequilibrium noise in diffusive multiterminal NS structures and found that nonlocal effects in them are by far more pronounced than in purely normal systems with the same geometry. In particular, they allow an observation of a doubled longitudinal noise and of a giant transverse nonequilibrium noise with respect to the direction of transport current.

The cross-shaped SN structures considered above are easily fabricated and hence the theoretical conclusions about the noise may be easily tested by experimentalists. An advantage of this system is that there is no voltage drop between the superconducting electrodes and the effects of electron heating are not obscured by an onset of ac Josephson effect, as it takes place for two-terminal SNS structures.

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