The effects of ring-shaped porous inert media on equivalence ratio oscillations in a self-excited thermoacoustic instability

Cody Dowd and Joseph Meadows

Abstract
Gas turbine operation increasingly relies on lean premixed (LPM) combustion to reduce harmful emissions, which is susceptible to thermoacoustic instabilities. Most combustion systems are technically premixed and exhibit a degree of equivalence ratio inhomogeneity. Thermoacoustic pressure oscillations can couple with the heat release oscillations through the generation of equivalence ratio fluctuations at fuel injection sites, which are then convected to the flame front. Previous experimental studies have shown that porous inert media (PIM) can passively mitigate these instabilities by adding acoustic damping and by reducing the thermoacoustic feedback mechanism. To understand the role of PIM on these equivalence ratio oscillations, spatially resolved, phased averaged equivalence ratio fluctuations are measured using the ratio of OH*/CH* chemiluminescence. Spatial imaging of OH* or CH* radicals produce integrated line of sight intensity values and an Abel transformation is used to obtain spatially resolved values. Phase averaged images are synced with dynamic pressure measurements, and an axisymmetric atmospheric burner is used to study the effects of ring-shaped PIM on the spatially resolved equivalence ratio field with self-excited thermoacoustic instabilities. The results show that PIM significantly reduces these fluctuations, and the effects on the stability of the system are discussed.

Keywords
Thermoacoustic, porous inert media, OH*/CH* chemiluminescence

Date received: 7 January 2021; accepted: 8 January 2021

1. Introduction
Lean premixed (LPM) combustion is utilized in gas turbine combustors to reduce NOx emissions and reduce fuel consumption; however, LPM combustion is more susceptible to thermoacoustic instabilities. According to the Rayleigh criterion, a system is thermoacoustically unstable when the energy added due to an in-phase relationship between acoustic pressure and unsteady heat release rates is greater than the energy dissipated by the damping effects. These instabilities have been shown to damage hardware, limit operational ranges, and have unwanted noise emission. Prediction of these instabilities is challenging as they are a result of multiple physical interactions including, but not limited to, acoustic wave propagation, nonlinear damping effects, and turbulent flame vortex interaction.

Active and passive control methodologies have been employed to mitigate thermoacoustic instabilities. Active control methodology focuses on breaking the coupling between the pressure and the heat release oscillations. Active control strategies adjust fuel and/or air, relative to system response to reduce instabilities, but this approach adds additional complexity and cost to a system. Conversely, passive methods are normally only applicable to a narrow frequency band but do not require the additional control complexity. Passive methods often involve adding acoustic resonators, such as Helmholtz resonators and quarter wave tubes, which add acoustic damping to the system at a specific frequency. Gysling et al. and Bourquard and

Corresponding author:
Joseph Meadows, Virginia Polytechnic Institute and State University, 445 Goodwin Hall, 635 Prices Fork Rd, Blacksburg, VA 24061, USA.
Email: jwm84@vt.edu

Virginia Polytechnic Institute and State University, Blacksburg, VA, USA

International Journal of Spray and Combustion Dynamics
2021, Vol. 13(1–2) 3–19
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DOI: 10.1177/1756827721991776
journals.sagepub.com/home/scd

Original Research Article
Acoustic fluctuations within a combustor may cause variations in the local equivalence ratio, which can lead to heat release oscillations. Quantitative determination of equivalence ratio fluctuations is key to understanding thermoacoustic instabilities in technically premixed systems. A combustion chamber presents a challenging environment to determine local equivalence ratio, due to restricted access and the high temperature of the flame. Recent work by Storr et al. studied the independent effects of velocity fluctuations and equivalence ratio oscillations on flame dynamics. They found that a time delay between the velocity and equivalence ratio fluctuations was critical in determining the stability of a system. They also determined that accurate time resolved, unsteady interactions of velocity and equivalence ratio fluctuations are required for the proper prediction of thermoacoustics.

Planar laser induced fluorescence measurements (PLIF) have been used in combustion environments by Allen et al. to measure spatial radical formation at a single plane of the flame, which can offer information regarding local equivalence ratio and heat release rates. Another tool to spatially resolve radical formation is chemiluminescence imaging. Light emission from a flame can be filtered by wavelength to target specific radical formation. PLIF measurements offer the benefit of acquiring spatially and temporally resolved radical quantities at a plane in the volume of interest. Chemiluminescence captures temporally resolved line of sight measurements, which makes it challenging to determine spatial resolution at a single plane. Chemiluminescence offers a reduced complexity and ease of capturing different radicals with a simple optical filter change, whereas PLIF requires additional hardware and a new laser wavelength to analyze different radical species. Integrated line of sight values from chemiluminescence can also be used but require deconvolution or multiple viewing angles for tomography. Dribinski et al. developed a methodology to perform Abel inversions on an integrated line of sight image to determine the distribution of radicals at a single plane under the assumption that the flow/fuel distribution is symmetric about the central axis.

For chemiluminescence measurements in a perfectly premixed system, OH* or CH* radical formation is proportional to heat release, but for a technically premixed system with turbulent flames this is not true, as demonstrated by Lauer et al. Lauer et al. have shown that the turbulent structures effect flamelet generation and their subsequent heat release. Hardalupas and Orain have shown that the ratio of radicals OH* and CH*, can directly correlate to the equivalence ratio of a flame. Local equivalence ratio measurements offer quantitative insight as to how the heat release is changing in a combustion environment.
Previous experimental studies using PIM, focused on the different parameters of the PIM to understand the physics in relation to mitigation of thermoacoustic instabilities found in a variety of combustion environments. It was concluded from these works\textsuperscript{13,14} that the natural acoustic damping and the change in the flow field/vortex formation are the primary mechanisms in improving system stability when adding porous media; however, previous studies focused more on near perfectly premixed combustion rather than technically premixed combustion. In technically premixed combustion, equivalence ratio oscillations can serve as an additional source mechanism for thermoacoustic instabilities.\textsuperscript{31} To better understand how thermoacoustic instabilities are being mitigated with PIM in technically premixed systems, quantitatively resolved equivalence ratio oscillations are required. The current study analyzes an atmospheric dump combustor with chemiluminescence images to determine phase averaged, spatially resolved equivalence ratio oscillations. The experimental method for measuring phase averaged CH* and OH* chemiluminescence images and post processing using an Abel inversion are presented. Investigation of the mode shape of the acoustic perturbations in the combustor demonstrates how the acoustic damping reduces perturbations in the mixing tube which reduces the equivalence ratio oscillation. PIM reduces the magnitude acoustic pressure oscillations resulting in a reduction of equivalence ratio perturbations, while simultaneously altering the flames response. The spatially resolved equivalence ratio distribution also demonstrates how the PIM creates a locally rich and better mixed flame at the core, which is less susceptible to thermoacoustic instability.

2. Experimental method

2.1 Experimental setup

Experimental measurements are conducted in an atmospheric dump combustor capable of producing lean, partially premixed, swirl stabilized flames. The dimensional break down of the combustor is detailed in Figure 1(a) with an image of the physical system setup in Figure 1(b). The system consists of an air plenum, annular mixing section, optically accessible combustion chamber, and steel combustor section. Air is injected through a choked orifice into the plenum and fuel is injected through six circumferentially spaced discrete openings. Mass flow controllers (Alicat MCR) are used to measure and control flow rates of air and methane into the system. Air flow up to 10.34 g/s (500 SLPM) and fuel flow up to 0.546 g/s (50 SLPM) are possible with these controllers, with an uncertainty of ±0.8% which correlates to 82.6 mg/s and 4.37 mg/s, respectively.

Flow initially passes through the plenum, which has two 5 kW speakers, with a frequency range of 48 to 800 Hz, located on the side walls of the rectangular plenum. These speakers are driven by a Crown XLS 1002 amplifier. The speakers can be used to introduce acoustic waves into the system at non thermoacoustically self-excited conditions to study the flame response. Atop the plenum is an annular mixing tube, which has an inner diameter of 15.75 mm, outer diameter of 19.05 mm, and is 0.51 m long. Fuel is delivered 123.75 mm upstream of the baseplate of the combustor through the fuel lance that has a 9.525 mm outer diameter. A 45° flat vane axial swirler is used to improve the mixing of reactants and produces a swirl number of 0.6. The swirler is flush mounted into the baseplate at the end of the annular mixing tube and fuel lance. The swirler has the same inner diameter as the mixing tube to maximize flow area and prevent any step in the flow path. The combustion chamber consists of a quartz cylinder with an inner diameter of 70 mm and length of 410 mm, which sets concentric to a steel combustor section that has an inner diameter of 64 mm and is 101.6 mm long, used to hold the quartz cylinder and house instrumentation. The PIM is fabricated out of silicon carbide foam with 45 pores per inch and a porosity of 85%. The PIM is ring shaped with a 65 mm OD with a 22 mm hole in the center and is 50 mm long, depicted physically and in a schematic including the baseplate in Figure 2(a) and 2(b), respectively. PIM is wrapped in a graphite sheet, to prevent...
any flow from passing between the quartz ID and PIM OD. The location of PIM on the baseplate does not interfere with the measurements by the dynamic pressure transducer, PT1; however, the PIM will reduce pressure fluctuations in the combustor due to the acoustic absorbing property of the material.

Inlet air temperature is determined by a K-type thermocouple in the air plenum. Pressure perturbation measurements are obtained via microphones (PCB 1/4" ICP Microphone System) in the mixing tube and dynamic pressure transducers (Kistler 6025 A) in the baseplate and steel combustor, mounted 50.8 mm from the exit plane, to study the pressure perturbations in the combustor. The microphones are located 62.4 mm from each other and start 247.3 mm upstream of the baseplate, as detailed in Figure 1(a). The microphones and dynamic pressure transducers have a sensitivity of 2 mV/Pa and 103 pC/bar, respectively. The dynamic transducers can operate up to 700 °C and signals are amplified with a charge amplifier with a 400 mV/pC gain. For this experiment, mass flow rate, temperature, and pressure transducers/microphones are sampled at 1 Hz, 1 Hz, and 100 kHz, respectively.

A high-speed camera (Photron FASTCAM SA5) and high-speed intensifier (SIL-25HG50D) are used to capture chemiluminescence emission intensity focused on the baseplate region through the quartz cylinder. Flame intensity is filtered with monochromatic filters, 310 nm with a FWHM of 10 nm and 430 nm with a FWHM of 10 nm for OH* and CH*, respectively. The images are captured at 10 kHz with an 896 × 848-pixel resolution. The spatial resolution of the images captured was 0.175 mm per pixel. A gain of 9, exposure time of 85 μs, and delay of 55 ns are used for the intensifier to properly amplify the images. The camera, microphones, and pressure transducers are configured for synchronized data collection.

2.2 Acoustic mode shape prediction
An acoustic analysis can be performed on a combustion system using reduced order modeling to predict its natural frequencies and the mode shapes. \(^\text{4}\) A 1D reduced order acoustic model was previously developed by Dowd and Meadows, \(^\text{22}\) and it is capable of solving the acoustic response for a system with PIM. The PIM modeling is accomplished with an updated governing equation including PIM acoustic parameters. The governing equation for a one-dimensional inhomogeneous damped acoustic wave equation with porous media shown in equation (1).

\[
s_r \rho_0 \frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x^2} + \sigma h \frac{\partial p'}{\partial t} = 0
\]

where \(s_r\) is the structure factor, \(\rho\) is density, \(\kappa\) is the effective bulk modulus, \(\sigma\) is the flow resistivity, and \(h\) is the porosity of the PIM. \(p'\) is the acoustic pressure fluctuations. Physically, the structure factor represents the ratio of the effective bulk fluid density of the PIM and the air to the freestream density, the effective bulk modulus is representative of the compressibility of the bulk fluid, and the flow resistivity is a representative of pressure drop. Equation (1) governs the evolution of acoustic pressure as a function of space and time with porous media and unsteady heat release. A full derivation from the conservation equations and explanation of equation (1) can be found in Dowd and Meadows. \(^\text{22}\) Equation (1) has been modified to include the ring-shaped PIM effects by integrating the acoustic governing equations radially and circumferentially. The full form of that equation is shown in equation (2).

\[
\frac{\partial^2 p'}{\partial t^2} + \frac{\partial^2 p'}{\partial x^2} + \frac{\partial p'}{\partial t} = 0
\]

where

\[
a = \frac{\kappa R_i^2 + \gamma P_0 (R_o^2 - R_i^2)}{\kappa \gamma P_0}
\]

\[
b = R_o^2 \left[ \frac{h R_i^2 + R_o^2 - R_i^2}{h \rho_0 R_i^2 + s_r \rho_0 (R_o^2 - R_i^2)} \right]
\]

\[
c = \sigma (R_o^2 - R_i^2) \left[ \frac{h R_i^2 + R_o^2 - R_i^2}{h \rho_0 R_i^2 + s_r \rho_0 (R_o^2 - R_i^2)} \right] \left[ \frac{h k R_i^2 + h \gamma P_0 (R_o^2 - R_i^2)}{\kappa \gamma P_0 (R_i^2 h + R_o^2 - R_i^2)} \right]
\]

\[
d = \sigma (R_o^2 - R_i^2) \left[ \frac{h R_i^2 + R_o^2 - R_i^2}{h \rho_0 R_i^2 + s_r \rho_0 (R_o^2 - R_i^2)} \right]
\]
where $R_i$ and $R_o$ are the inner and outer radius, respectively, and $P_o$ is the atmospheric pressure. This equation simplifies back to equation (1) when $R_i$ is set to zero, representing a solid PIM section. Acoustic mode shapes are determined assuming the modes are purely longitudinal, there are no mean flow effects, and a system can be reduced to a set of constant area tubes or channels. A representative combustor can be constructed using the interior geometries, lengths, and boundary conditions at the exit, inlet, and interfaces within the test section, to close the problem.

### 2.3 Abel inverted phase-averaged chemiluminescence

In the presence of a strong thermoacoustic instability, there is a dominant frequency at which the system is fluctuating. From the instability frequency, a period can be defined over which a thermoacoustic cycle takes place. This cycle is then divided into phases, which represent the same temporal location in the instability from one period to the next. Phase averaged values of OH* and CH* chemiluminescence will be acquired based on the synchronization with the pressure measurements. The approach will elucidate the coupling of the chemiluminescence intensities with the thermoacoustic pressure fluctuations. In this way, simultaneous imaging is not required. To obtain intensity values of radicals in a center plane, the Abel inversion method presented by Dribinski et al. is used to reconstruct the line of sight OH* and CH* images. The method relies on the inverse Abel transform reproduced in equation (3), from Dribinski et al., which relates a two-dimensional projected image on a detector plane to the original three-dimensional light emission. The equation relates intensities from the detector coordinates, $x$ and $z$, to cylindrical 3D image coordinates $r$ and $z$. To find a solution to equation (3), numerical singularities exist when $x = r$, that are addressed by separating the problem into a summation of a set of functions which approximate the integral term. Also, the accurate resolution of the derivative term when used on a noisy image produces error, to solve these issues a basis expansion method is applied.

$$I(r,z) = -\frac{1}{\pi} \int_r^\infty \frac{dP(x,z)}{\sqrt{x^2 - r^2}} \, dx$$

where $I$ is the intensity of the original 3D source, $r$ is the radius, and $P$ is the intensity on the detector plane.

The reconstructions presented in the current work use 500 Gaussian basis set functions to reconstruct each image. The number of Gaussian functions was chosen such that halving the number of functions used showed no significant change in the radical intensity or spatial distribution of radical intensities. This signifies that further increasing the number of functions would not improve the quality of the reconstruction. The method assumes axisymmetric flow. For the current investigation, the flow is axisymmetric with the exception for slight variations in the circumferential equivalence ratio due to the fuel injection taking place at discrete locations. The authors expect this to have a minimal effect in comparison to the axial fluctuation induced by the thermoacoustic mode.

### 2.4 Equivalence ratio calculations

To determine the equivalence ratio in a gaseous combustion environment, Hardapulas et al. studied counter flow flames to understand the influence of strain rate and equivalence ratio on the production of OH*, CH*, and C2* radicals. This study showed a monotonic dependence of the equivalence ratio on the ratio of OH* to CH*. This is represented as an empirical relationship, reproduced in equation (4), which is valid for lean gaseous, methane combustion environments with up to 5% error. Also, the work showed that the ratio was independent of strain rate. As the combustion environment in the present study is similar to those presented in deriving the relationship for equation (4), it will be used in this study to relate the phase averaged, Abel inverted CH* and OH* intensity values to equivalence ratio fluctuations.

$$\phi = 0.7 - 0.26 \ln \left( \frac{\text{OH}^*}{\text{CH}^*} - 0.497 \right)$$

where $\phi$ is the equivalence ratio. Equation (4) is applied to each individual pixel from the phase averaged Abel inverted images, to calculate equivalence ratio. Due to the way equation (4) is formulated, there are points at which the resulting equivalence ratio would be undefined. For this reason, some filtering of the data is required to avoid unrealistic values. For example, in the absence of the flame, CH* intensity is approximately zero and produce unreasonable equivalence ratio values, thus these values will be filtered. Equation (4) from Hardapulas et al. is based on measurements of equivalence ratios ranging from 0.7 to 1.3 and extrapolation beyond this region will likely introduce more error. For this reason, only ratios of...
OH* to CH* will be used that produce equivalence ratios from 0.7 to 1.3.

3. Results and discussion

The following results section details the effect that PIM has on the sound pressure level (SPL), which demonstrate the effectiveness of PIM at mitigating an instability. Acoustic modes shapes are calculated to determine the locations of nodes/anti-nodes within the combustor and mixing tube. To better understand how the final equivalence ratio results are determined, interim results of the post processing are presented. These results detail the Abel inverted phase averaged radical intensity values. To detail the effects of PIM in the test section, spatially averaged equivalence ratios showing the variation with phase through a thermoacoustic cycle are presented. The flame response is studied via acoustic forcing and the effects of PIM addition are shown. Finally, the section concludes by presenting the phase-averaged spatial distributions of equivalence ratio without and with PIM.

3.1 Instability mitigation with PIM

The experimental apparatus has a self-excited thermoacoustic instability corresponding to an air flow rate of 4.86 g/s (235 SLPM) and a global equivalence ratio of 0.85, which resulted in an instability frequency of 318 Hz and sound pressure level (SPL) of 158 dB. PIM was then added to the test section which caused a shift in the instability frequency to 330 Hz and a significant reduction of 38 dB in SPL. Figure 3 shows the Fast Fourier Transform (FFT) of pressure perturbations from the dynamic pressure transducer, mounted in the baseplate, which illustrates the effectiveness of PIM on both peak amplitude at the instability frequency and broadband combustion noise. The SPL was calculated using a reference pressure of 20 μPa.

The larger peak in the no PIM case corresponds to the instability frequency at 318 Hz, also included is the second harmonic at 636 Hz. The excited frequency at 318 Hz in Figure 3, is similar to the system resonance frequency for a longitudinal acoustic wave, and as such, transverse and circumferential acoustic waves were neglected for this analysis. The reduction in SPL and improved stability is due to PIM and not the effect of the variation in flame location between the two cases. Johnson and Agrawal,18 studied PIM and a similarly sized solid body in a combustion environment and determined that PIM can reduce SPL in all cases while the solid body was not as effective or would amplify the instability. Therefore, the instability mitigation is reliant on the acoustic damping properties of the porous media and/or the flow through the porous structure. Figure 4 presents the temporal traces of the respective pressure and spatially averaged OH* intensity for the no PIM and PIM case. The sum of the acoustic pressure and static pressure are normalized by the static pressure and the OH* intensity is normalized by a mean intensity calculated from the data set.

The results for the no PIM case show the peaks of the acoustic pressure and OH* intensity occur with a
slight phase shift but are considered in-phase and representative of an instability. The PIM case is plotted over a shorter duration to highlight that there is little coupling between the OH* intensity and the acoustic pressure. The fluctuation of OH* with PIM indicates that radical field is dominated by random turbulence fluctuations as opposed to coherent structures coupling with the instability. This conclusion is also supported by Meadows and Agrawal, where the velocity field during an instability was studied with and without PIM. The work used a proper orthogonal decomposition to study the energy distribution between no PIM and PIM. They found that the PIM distributed the energy to different turbulent structures instead of a single, prominent structure in the no PIM case.

3.2 Standing wave – acoustic response

The acoustic mode shape is reconstructed for the combustor utilizing methods detailed in previous work. The geometric lengths and cross-sectional areas are selected to match the physical geometry of the experimental setup. Additional length was added to the exit of the combustor in the analytical model to account for entrainment of quiescent air as detailed in Ref. Acoustic parameters for PIM were calculated based on previous analysis shown in Ref. Porosity, flow resistivity, structure factor, and effective bulk modulus were assigned values of 0.85, 1740 kg m\(^{-3}\)s\(^{-1}\), 9, and 181500 Pa, respectively. For the no PIM case these values are 1, 0, 1, and \(\gamma P_0\), for porosity, flow resistivity, structure factor, and bulk modulus, respectively, where \(\gamma\) is defined as the ratio of specific heats and \(P_0\) is the atmospheric pressure, which will reduce equation (1) to the traditional acoustic wave equation presented in Poinset and Veynante. No flame was modelled in the current model as inclusion of flame has been shown to only cause a small shift in the predicted frequency with minimal effect on mode shape prediction.

The pressure and velocity mode shapes are reconstructed using the numerical model developed by Dowd and Meadows, in the mixing tube and the combustor. From this analysis the frequency response and mode shapes are determined, which are used to find the node/antinode locations and relative acoustic fluctuations within the mixing tube and combustor. The interface between the mixing tube, combustor, and PIM preserves continuity such that the acoustic pressure is constant across the interface, and a velocity jump occurs at area changes from the mixing tube and PIM. Both the inlet to the mixing tube and exit of the combustor are treated as open boundaries, due to the large area change at the inlet to the mixing tube and the exit of the combustor. Figures 5 and 6 show the mode shapes of the acoustic velocity and pressure, respectively. The results are presented along the combustor normalized by the total length for the no PIM and PIM results. The different lines represent different phases during the instability period. The different phases show how the pressure or velocity will vary with time and allow for better identification of node and antinode locations.

To calculate the amplitudes of the acoustic waves traveling in each duct, a reference value must be used, which the remaining amplitudes will be calculated relative to this value. The value is chosen as the max pressure fluctuation from the baseplate dynamic pressure transducer determined during testing to show the relative amplitudes between the no PIM and PIM cases. The natural frequency of the system is calculated as 322 Hz and 320 Hz, which are similar to the frequencies for no PIM case and PIM case, respectively. The resulting acoustic velocity mode shape for the no PIM and PIM cases are presented in Figure 5(a) and (b), respectively. The pressure mode shapes are presented in Figure 6(a) and (b) for the no PIM and PIM cases, respectively. The circles along the x-axis mark the location of microphones, the squares depict the locations of the dynamic pressure transducers, and the location for fuel injection is represented by a diamond.
Note the different scales in Figure 5 and 6, the no PIM mode shapes have amplitudes approximately 10 and 100 times larger than the PIM case for velocity and pressure, respectively. This is expected due to the reference pressure magnitude used for no PIM being significantly larger than the PIM case, as shown in Figure 3. It is important to note, that the model does not include a flame transfer function, and it is only scaled based on experimental data to show the relative amplitudes of the standing waves. The similarity in the mode shapes between PIM and no PIM result from a similar prediction in the real portion of the frequency. Acoustic pressure is continuous in the domain, but acoustic velocity is not. The sharp drop in acoustic velocity at a normalized length of 0.5 is due to the area change at the dump plane. The PIM addition also causes a jump in velocity at the entrance and exit, highlighted in the blown-up image of Figure 5 (b). For the PIM case, the imaginary term in the frequency response has a negative value indicating acoustic damping whereas the no PIM case does not have an imaginary term from the standing wave mode shapes. The profiles and location of the node are similar for PIM and no PIM cases. At the injection site, acoustic velocity fluctuations will affect the air flow and have an inverse relation to equivalence ratio fluctuations. Assuming constant fuel flow, increasing air velocities will cause a decreasing equivalence ratio. For acoustic pressure fluctuations, the flowrate of fuel is driven by the pressure gradient across the injection opening. Assuming a constant air flowrate, the increase in pressure in the air flow will decrease the fuel flow rate and produce a leaner mixture. The spatial mode shapes show that both acoustic velocity and pressure will affect the fuel injection, therefore, the effect on equivalence ratio will include changes in air flow rate due to velocity fluctuations and changes in fuel flow rate due to pressure fluctuations. The relationship between acoustic pressure and velocity will be 90° out of phase. When acoustic pressure node is at a maximum, the acoustic velocity will be at a node, resulting in a locally lean mixture. A graphical depiction of this argument is presented in Figure 7, where a period of instability is shown with representative acoustic pressure and velocity variation at the fuel injection site. As the pressure approaches zero and velocity approaches a local maximum at 90°, the local mixture will be lean. Similar conclusions for the remainder of the period can be made based on this rationale. The period is separated into 4 sections, every 90 deg, and each section is unshaded or shaded. The unshaded region represents a lean mixture produced by the acoustic pressure and velocity during that portion of the instability. The shaded region represents a rich mixture at the fuel injection site. Conclusions regarding the quantitative change in equivalence ratio progressing through the instability period, will be discussed in section 3.4.

Figure 7 depicts approximately half of the instability period will produce lean and the other half would produce rich equivalence ratio mixtures at the point of fuel injection. The results depict the coupling between the acoustic waves and the equivalence ratio fluctuation generation. The strength of the acoustic perturbation

![Figure 6. Acoustic pressure (Pa) mode shape for (A) no PIM and (B) PIM case.](image)

![Figure 7. Acoustic pressure and velocity over instability period. Unshaded and shaded region represents lean and rich equivalence ratio formation, respectively, at the point of fuel injection.](image)
will strongly affect the magnitude of the equivalence ratio variation during the period of instability. It should be noted that these fluctuations are present in the PIM case as well, due to the similarity in mode shape and frequency response, but the reduction in strength of the acoustic waves will significantly reduce the magnitude of the equivalence ratio fluctuation. A more quantitative discussion will take place in section 3.4.

To validate the predicted mode shapes from Figures 5 and 6, the experimental acoustic pressure data from microphones and dynamic pressure transducers are compared in Table 1. Results are only shown for the no PIM case; similar results are present in the PIM case. An FFT is performed on each set of pressure data and the phase and magnitude of each measurement, at the instability frequency, is used to calculate the phase difference and gain. The phase difference is defined as the baseplate phase subtracted from the other measurement phases. The gain is defined as the magnitude of the other measurements divided by the magnitude of the baseplate transducer. Gain results are also calculated using mode shape data from Figure 6, for comparison. The naming of each measurements corresponds to those used in Figure 1.

From Figures 5 and 6 the microphones will be out of phase with the dynamic transducers. This is confirmed by the phase difference data in Table 1. A phase difference greater than 180° between the microphones and dynamic pressure measurements, is indicative of the measurements being spatially separated by a node in the acoustic wave. This was demonstrated in the velocity mode shape in Figure 5, where the node occurs just after microphone 3. The gain data also confirms the trend demonstrated in Figure 6, where the pressure fluctuations in the mixing tube are larger than in the combustor. Finally, a similar trend is noted in the gain of the experimental and mode shape predictions. Although the absolute value of the gain does not match, the relative gains between the microphones are predicted (i.e., the M3 has the largest pressure magnitude and PT2 has the smallest pressure magnitude), which agrees with mode shape prediction form Figure 6. This analysis gives confidence in the mode shape prediction and subsequent conclusions drawn about how the equivalence ratio fluctuations are being introduced into the system.

The reduction in acoustic perturbations weaken the driving mechanism for the instability formation. The mode shape analysis shows that the fuel injection site is highly influenced by acoustic perturbations. Acoustic perturbation reduction due to PIM introduction, results in less equivalence ratio fluctuation, as the acoustic waves have lower amplitudes near the fuel injection points. Equivalence ratio oscillations at the flame lead directly to heat release oscillations. According to the Rayleigh criterion, the system will be unstable if the energy added to the system is larger than the energy dissipated. Energy will be added if the pressure fluctuations and heat release oscillations are in phase. The addition of PIM does not guarantee an out of phase relationship, but it will add additional damping to the system. Further investigation is necessary to determine if the alteration in flow field by addition of PIM has affected the reduction in acoustic perturbations or if the acoustic damping of the PIM is primarily responsible.

### Table 1. Phase difference and gain from pressure measurements relative to the dynamic pressure in the baseplate for no PIM case.

|        | M1  | M2  | M3  | PT1 | PT2 |
|--------|-----|-----|-----|-----|-----|
| Phase difference | 271 | 276 | 280 | –   | 7   |
| Gain – Exp       | 1.06| 1.26| 2.04| –   | 0.45|
| Gain – Mode shape| 4.5 | 5.72| 6.2 | –   | 0.145|

![Figure 8. Phases for a thermoacoustic instability.](image-url)
Figures 9 and 10 for no PIM and PIM case, respectively. Orientation of the flow is from the bottom to the top of the figure. Figure 9 detail the instantaneous OH* image, the 0° phase averaged image after averaging for 100 instability periods, and the Abel inverted phase averaged OH* image for the no PIM case. Figure 10 presents the final Abel inverted phase averaged OH* image for the PIM case. All images present relative intensity values calculated using the max intensity from each individual image and dividing all pixel quantities in the respective image by that value. Images presented use a weak Gaussian blending function to eliminate inconsistencies in the flame structure.

The results in Figure 9 show that the OH* intensity is prominent along paths leaving the baseplate at a ~45° angle, where the path impinges upon the quartz combustor wall and moves along the wall, located at a radius of ±35 mm. The vortices generated as the flow impinges on the wall and recirculate into the core flow, start to break down and produced axially elongated structures. Figure 10 depicts higher radical intensities near the centerline representative of the unimpeded flow passing through center of the ring shaped PIM. A study conducted by Meadows and Agrawal,14 investigated time-resolved PIV in a combustor. The investigation used a similarly sized PIM body, with lean premixed combustion in a swirl stabilized dump combustor at similar conditions. The resulting velocity fields presented in Ref.14 show high velocities exiting the core of the PIM. The current experiment is not an identical setup but similar flow field is expected; however, the velocity is higher based on bulk flow quantities. This higher velocity flow through the core of the PIM recirculates into the lower velocity flow that has passed through the PIM body. Due to the velocity leaving the PIM surface and concentrated flow closer to the centerline, the corner recirculation zones are eliminated, which was shown by Meadows and Agrawal.14

Additionally the work by Meadows and Agrawal,14 used proper orthogonal decomposition of the velocity field, to study the turbulence with and without PIM in an unstable system. The study of energy modes show without PIM a dominant turbulent structure is
contributing the majority of turbulent energy to the system. With the addition of PIM the energy is distributed across multiple turbulent structures. This redistribution of energy supports a more stable system. This was supported by Paschereit et al., where coherent structures were investigated in a thermoacoustic setting. The importance of these coherent structures on thermoacoustic instabilities was shown to have significant effects on the instability. This is represented in the results in Figures 9 and 10, where a longer elongated structure is noted in the no PIM case. In contrast, the PIM case has more compact structures at the exit of the PIM opening. Any acoustic or equivalence ratio coupling with the dominant structure of a no PIM case will have a larger effect than a fluctuation coupling with one of the smaller, less energetic, structures in the PIM case.

The dark region (Y = 0 to 50 mm) below the radical emission in Figure 10 is the porous media, through which there are no radical emissions captured by the camera. The OH* emission plots also show some partial radical formation along the top surface of the PIM. The radical formation just downstream of the PIM surface are representative of flamlets being formed by low velocity reactants making their way through the porous structure. The pore size was selected such that any reactions within the porous media will be quenched. The sizing was selected from findings in Sequera and Agrawal where the amount of burning within the porous structure could be eliminated with proper selection of pore size. From Ref., thermal radiation of the porous media in the visible spectrum was observed when pore sizes were sufficiently large enough for reactions to occur within the porous structure, which would heat the porous structure to a temperature hot enough to thermally radiate in the visible spectrum. Although some reactions may be taking place inside the core region, the high velocity in this region will be larger than the flame speed, so this effect is minimal, if any.

These figures show the importance of the Abel inversion, where the instantaneous and phase averaged line of sight results show significant radical formation at the centerline of the combustor, whereas the Abel inverted image shows a hollow cone structure is prevalent in the no PIM case.

To show how the flame changes through each phase, the Abel inverted phase averaged OH* intensity contours are presented below in Figure 11 for the no PIM case. Again, the results shown are the relative intensities for each pixel captured of the flame region and the images have been condensed to a subset of pixels to better highlight the features of the flame. Phases presented correspond to those identified in Figure 8.

For the no PIM case, starting from the 30° phase the OH* intensities decrease until the 180° phase. This movement follows the acoustic pressure trend, as seen in Figure 8, where the 180° phase corresponds to a negative acoustic pressure, which clearly demonstrates a coupling between the acoustic pressure and OH* intensities. At about the 210° deg phase a portion of the flame sheds from the core flame near the baseplate and the intensities start to grow near the base plate and then expand downstream reaching the 360° phase and then repeat the cycle.

### 3.4 Equivalence ratio investigation

After determining Abel inverted phase averaged intensity quantities from OH* and CH*, the ratio of the two quantities can be used to determine local equivalence ratio using equation 4 reproduced from Hardapulas et al. Equation (4) was evaluated for a range of equivalence ratios from 0.7 to 1.3, outside of this range.

![Figure 11. Abel inverted phase averaged OH* intensity for the 12 phases of the instability for the no PIM case. X and Y axis are distance in millimeters.](image-url)
extrapolation would increase error. Therefore, the presented results in this section are calculated such that they are restricted to an equivalence ratio range from 0.7 to 1.3. This clipping of the data will produce discrete values of 0 in the flame structure when the equivalence ratio falls outside of the specified range. A Gaussian smoothing function is used to weakly blend the equivalence ratio contours to eliminate these inconsistencies in the flames structure. The resulting phase averaged equivalence ratios are presented in Figure 12 for the no PIM case. Note that imaging was correlated to phases using the baseplate dynamic pressure transducer, so progressing through the phases described in Figure 8 correlate to changes in combustion chamber acoustic pressure.

From the equivalence ratio contours, the effect of the instability on the flame shape can again be seen, where the flame expands and contracts axially in phase with the acoustic pressure. Figure 12 clearly illustrates the coupling of the equivalence ratio oscillations with the instability frequency. From the mode shape discussion in Figure 7, the acoustic fluctuations generate equivalence ratio oscillations at the fuel injection, in both no PIM and PIM cases. The magnitude of the acoustic perturbations dictates the level of fluctuation introduced into the equivalence ratio. As the acoustic fluctuations are significantly larger in the no PIM case, as shown in Figure 3, there will be a larger magnitude of equivalence ratio fluctuation generated.

Equivalence ratio oscillations are known to generate heat release fluctuations and serve as a source mechanism for thermoacoustic instability. Starting with the 30° phase, the equivalence ratio distribution is contracting towards the base plate. The equivalence ratio gets leaner beyond $Y = 40\,\text{mm}$, causing the flame to separate as a portion extinguishes and only remains lit in the baseplate region due to the incoming rich fuel supply.

To explain why this flame shedding is occurring, the discussion from Figure 7 is recalled. As the discussion for Figure 7 is specific to the fuel injection site, the convection time for those fluctuations to reach the baseplate must be included. The time needed to traverse the distance from fuel injection, assuming constant mass flow, correlates to approximately 3.78 ms. Using the instability frequency of 318 Hz, the period it takes for the instability to complete a cycle is approximately 3.1 ms. This means it will take 1.25 periods for the equivalence ratio fluctuation to convect to the baseplate. This correlates to a positive shift of 90° in the regions highlighted lean and rich in Figure 7. For example, now at phases 0-90 deg, instead of a lean mixture, a rich mixture will be reaching the combustor. A phase of 90° is now an inversion point from a rich to lean mixture reaching the combustor. The leaner mixture cannot sustain an elongated flame and the downstream portion is shed in the 120° phase. The changes in equivalence ratios are not discrete as may be inferred from Figure 7, instead, there is a continuous change in the generation of equivalence ratio fluctuations. As the equivalence ratio is still rich but moving to a leaner mixture in phases 0-90. The effects of progressing to a leaner mixture are noticeable in phases 60 and 90, where areas of separation are noted in the core flame structure. The inverse of this argument explains the variation in equivalence ratio distribution in the remainder of the period.

Figure 13 is presented to show the equivalence ratio contours for the PIM case in comparison with a no PIM case. Figure 13(a) shows result for the 0° phase of the instability of the no PIM case. Figure 13(b) displays the equivalence ratio contour for the 0° phase of

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**Figure 12.** Abel inverted phase averaged equivalence ratio contours for the no PIM case for the 12 phases of the instability. X and Y axis are distance in millimeters.
the PIM case, the region below the equivalence ratio structure represents the PIM.

The equivalence ratio comparison shows the range at which the flame is burning is distributed in the lean regime for the no PIM case, while the PIM case has concentrated areas above and around equivalence ratios of 1. Specifically, for the PIM case, high equivalence ratios are noted around the exit of the core of the PIM. This is represented by two flame regions located at a radial distance of \( \pm 10 \text{ mm} \) in Figure 13 (b). It is hypothesized that since the fuel injection originates at the concentric fuel lance, there is a higher concentration of fuel near the centerline of the flow. The equivalence ratios detailed in Figure 12 for the no PIM case support that this effect is taking place. In each phase, there is a higher V-shaped contour of rich equivalence ratios near the centerline approximately 20 mm downstream of the baseplate. The PIM introduces a radial pressure gradient due to its partial blockage of flow. As a richer mixture is expected near the centerline, a richer mixture is expected to exit the PIM opening. Some of the mixture diffuses through the PIM and burns as small flamelets on the surface which has been shown by Meadows and Agrawal\(^{14}\) to improve stability of the system.

Diffuse burning of richer mixtures is shown to be less susceptible to instabilities due to the weaker hydrodynamic coupling to the acoustic pressure perturbations.\(^ {36}\) For example, the leaner mixture in the no PIM results in a flame shedding. Flame shedding was shown in Kostka et al.\(^ {37}\) to introduce additional flow fluctuations that support energy addition to the flow. Fuel fluctuations caused by acoustic perturbations have a reduced effect on a richer mixture as there will be sufficient fuel for the mixture to remain within the flammability limits, and less chances of local quenching. Therefore, the system with PIM will retain a more consistent heat release and reduces the coupling between pressure perturbations and unsteady heat release. The spatial distribution of equivalence ratio supports that there are more rich regions in the PIM case, which reduces coupling between unsteady heat release and acoustic pressure perturbations.

The radial pressure gradient induced by PIM also reduces the radial penetration and causes the flow to change direction where it impinges on the PIM. This results in more mixing which can also be attributed to the increased axial length and diffusion effects. The product of this increased mixing can be seen in Figure 14 where the standard deviation of equivalence ratio in each phase is compared for PIM and no PIM. Only equivalence ratio values between 0.7 and 1.3 were considered when computing the standard deviation.

Figure 14 shows a reduction in the standard deviation of the equivalence ratio results in the PIM case for a majority of the period. This results in improved mixing, which promotes stability because it reduces the effect that any perturbations in equivalence ratio would have on the heat release.

To study how the equivalence ratio varies with phase when adding PIM to combustor, Figure 15 shows...
presents the mean equivalence ratio from each phase for both the PIM and no PIM cases. The mean equivalence ratio is calculated by computing the volume weighted average of the equivalence ratio. The equivalence ratio is integrated over the radius and circumference and divided by the total area only at locations of valid equivalence ratio. Non burning regions are not accounted for in this average. Figure 15 shows that the equivalence ratio is responding to the pressure fluctuations in the no PIM case and the addition of PIM mitigates the equivalence ratio oscillations and consists primarily of random fluctuations. The PIM results show the similarity of the equivalence ratio for all phases of the PIM case. Since the PIM case has been determined to be a stable operating point, variations in flame shape from one phase to another are minimal. The importance of the acoustic perturbation reduction from Figure 3 is now apparent. The reduction in fluctuations means there will be a direct reduction in equivalence ratio perturbations, from the conclusions made in Figure 12. This concludes that the coupling between the equivalence ratio and acoustic perturbations has been reduced by the addition of PIM in addition to its natural acoustic damping properties.

An interesting note is that the pressure and equivalence ratio fluctuations are about 90° out of phase. The mean equivalence ratio fluctuations reach a peak at the 90° phase, while the peak for the pressure fluctuation is at the 0/360° phase, shown in Figure 8. This is supported by the conclusions drawn from Figure 12, where an out of phase relationship was also determined to have a 90 deg phase shift. As the system is still experiencing a strong thermoacoustic instability, a phase shift between acoustic pressure and equivalence ratio can result in an instability, provided the phase relationship in the Rayleigh criterion is still positively adding energy to the system. Further investigation is required to determine the relationship between the equivalence ratio and heat release rates.

In summary, this figure demonstrates that the porous media case has reduced the coupling with acoustic pressure and the no PIM case is fluctuating due to a strong coupling with the acoustic pressure. The results show that the equivalence ratios are not fluctuating around the global equivalence ratio as expected. This is attributed to the clipping of OH*/CH* ratios and the optics used, being different than those used in Hardapulas et al. The optics used in Hardapulas et al. to build the relationship used to calculate equivalence ratio will likely have different light transmission curves, which can scale the results depending on the optical setup but will not change the conclusions of the present study. The clipping of equivalence ratios during post processing can also skew the mean global equivalence ratio by eliminating rich or lean values outside the range. It should be noted that the mean of the no PIM and PIM are not similar. This is likely due to the additional clipping of lean values in the PIM case, as the global equivalence ratio from mass flow measurements were the same between both cases. Also, the mean equivalence ratios are calculated using a volume averaged mean of the valid equivalence ratio measurements, whereas a mass flow averaged equivalence ratio would be more representative of the global equivalence ratio. Since there was no spatially resolved velocity and density measurements available for the current investigation, this was not possible. Finally, there is experimental uncertainty in the equivalence ratio correlations. As the same correlation was used for all results the above-mentioned deficiencies of using the expression presented by Hardapulas et al. would not alter the conclusions.

### 3.5 Flame response

The self-excited PIM cases causes a slight shift in dominant frequency which prevents a direct comparison of flame response between PIM and no PIM results. Flame response is defined as the change in the flame burning or heat release to the incoming acoustic perturbations. To better understand the flame response to PIM addition, the speakers in the plenum are used to introduce acoustic perturbations at distinct frequencies into a stable flame. The system is operated at 10.34 g/s (500 SLPM) at a global equivalence ratio of 0.6, which is a stable operating point. Acoustic fluctuations are introduced at various frequencies between 100 and 400 Hz. The amplitude of fluctuation used was ~500 Wrms, which introduced significant pressure fluctuations into the mixing tube and combustor. It should...
be noted that although similar power was applied for each frequency, more energy is needed to generate similar amplitudes at higher frequencies. For each case, the phase averaged equivalence ratio is calculated as demonstrated in sections 3.3–3.4. The volume weighted mean equivalence ratio, similar to results in Figure 15, are used to compute the flame response. The flame response is quantified by the gain, defined by the magnitude of equivalence ratio fluctuations normalized by the magnitude of acoustic velocity fluctuations. The acoustic velocity fluctuations are calculated from the acoustic pressure measurements at the base-plate divided by the speed of sound and density. A mean of ambient and adiabatic flame temperature is used for density and speed of sound calculation. The results of this study are presented in Figure 16 where the gain for PIM and no PIM are compared for different forcing frequencies.

The flame response is altered with the addition of PIM as shown by the results in Figure 16. The reduction in flame response in PIM cases is shown for most frequencies. The current flame response is being defined by equivalence ratio fluctuation and not actual heat release. The trend of increasing gain moving from no PIM to PIM results, shows that with a similar forcing acoustic perturbations, a higher equivalence ratio response is shown for the no PIM case. The PIM passively reduces the acoustic perturbation generated by the speakers which results in a decreased equivalence ratio fluctuation. This was demonstrated in Figure 7 where acoustic fluctuation was shown to generate oscillations in equivalence ratio. This quantitatively supports there is a change in flame response with the addition of PIM.

There are multiple phenomena presented here that contribute to the increase in system stability by the addition of PIM. As thermoacoustics are a multi-physics problem, this is expected. The richer equivalence ratio burning leads to a more stable system in the PIM case as a locally richer flame is less susceptible to being affected by equivalence ratio oscillations. Improved mixing with PIM, reduce the equivalence ratio fluctuations reaching the flame front. The addition of PIM also alters the flame response, reducing the equivalence ratio fluctuations when perturbed by similar acoustic waves. Finally, the addition of PIM results in a large reduction in acoustic perturbations at the fuel injection point, which effects the fuel flow fluctuations, as predicted by the thermoacoustic solver and the microphone reading from microphone M3. The coupling between acoustic and equivalence ratio was shown to be the driving mechanism for instability formation. The order of magnitude reduction of acoustic fluctuations has a large damping effect on the equivalence ratio fluctuations. This reduction in the perturbations reduces the coupling between acoustic and equivalence ratio and has the largest effect in creating a stable thermoacoustic system.

4. Conclusion

Spatially resolved, phased averaged equivalence ratio measurements were performed on a partially premixed combustor without and with porous inert media (PIM). A self-excited thermoacoustic instability is observed without PIM, and a 38 dB reduction in sound pressure level is observed with the addition of PIM. Equivalence ratios are derived using the ratio of Abel inverted phase averaged OH* and CH* radical intensities at the central plane of the combustor. The optical diagnostic method allowed for measurement of the coupling between acoustic pressure and equivalence ratio fluctuations, which were shown to be an important factor in understanding the mitigation ability of PIM. The equivalence ratio oscillations are coupled to the acoustic pressure perturbations for the case without PIM, and the PIM case shows a reduced coupling of the equivalence ratio fluctuations with the acoustic pressure oscillations. Comparison of the local equivalence ratios show an increase in fuel/air ratio in the central recirculation zone, and confines the flow leaving the swirler when PIM is added. Acoustic mode shapes of the two cases are analyzed and show how the PIM addition reduces the effect of the acoustic fluctuations on equivalence ratio fluctuations. PIM increases the acoustic damping within the system, advantageously alters the flow entering the combustor, and reduces the effect of acoustic pressure perturbations on equivalence ratio oscillations.
Declaration of Conflicting Interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD
Joseph Meadows https://orcid.org/0000-0002-4807-2746

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**Appendix**

**Notation**

- $h$ porosity
- $n$ interaction index
- $R$ radius
- $s_t$ structure factor
- $\tau$ convective time delay
- $\rho$ density
- $\kappa$ effective bulk modulus
- $\phi$ equivalence ratio
- $\sigma$ flow resistivity