Entropy of 2+1 de Sitter space with the GUP

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Abstract

By introducing the generalized uncertainty principle (GUP) on the quantum state density, we calculate the statistical entropy of a scalar field on the background of (2+1)-dimensional de Sitter space without artificial cutoff. The desired entropy proportional to the horizon perimeter is obtained.

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I. INTRODUCTION

Three decades ago, Bekenstein suggested that the entropy of a black hole is proportional to the area of the horizon through the thermodynamic analogy [1]. Subsequently, Hawking showed that the entropy of the Schwarzschild black hole satisfies exactly the area law by means of Hawking radiation based on the quantum field theory [2]. After their works, 't Hooft investigated the statistical properties of a scalar field outside the horizon of a Schwarzschild black hole by using the brick wall method (BWM) with the Heisenberg uncertainty principle (HUP) [3]. The entropy proportional to the horizon area is obtained, but a brick wall cutoff, which was introduced to remove the divergence of the density of states, looks unnatural. This method has been used to study the statistical property of bosonic and fermionic fields in various black holes [4, 5, 6], and it is found that the general expression of the black hole entropy consists of the term, which is proportional to the area of the horizon, and the divergent logarithmic term. Although this BWM is useful for various models, some difficulties may arise because it is assumed that there exists a thermal equilibrium between the black hole and the external field even in a large spatial region. Obviously, this method cannot be applied to a non-equilibrium system such as a system of non-stationary space-time with two horizons because the two horizons have different temperatures and the thermodynamical laws are also invalid there. Solving these problems, an improved brick-wall method (IBWM) has been introduced by taking the thin-layer outside the event horizon of a black hole as the integral region [7]. In the thin-layer, local thermal equilibrium exists and the divergent term due to large distance does not appear any more. However, the IBWM does not still essentially solve the difficulties including the artificial cutoffs.

On the other hand, many efforts have been devoted to the generalized uncertainty relations [8], and its consequences, especially the effect on the density of states. Recently, in Refs. [9, 10], the authors calculated the entropy of a black hole by using the new equation of state density motivated by the generalized uncertainty principle (GUP) [8]. As a result, the serious divergence of the just vicinity near the horizon was removed.

However, most of statistical entropy calculations have been done for the asymptotically flat cases by using the BWM and IBWM with the HUP or by using the GUP. Up to now, the statistical entropy [11] of the 2+1 de Sitter (DS) space, which has a cosmological horizon and asymptotically non-flat spacetime, was only studied by using the BWM [12] and IBWM
In this paper, we study the entropy of the black hole in the 2+1 DS space. Firstly, we briefly recapitulate the previous results in BWM and IBWM with the HUP. But, we will avoid the difficulty in solving the Klein-Gordon wave equation by using the quantum statistical method. Next, we derive the free energy of a massive scalar field on the DS space background directly by using the new equation of state density motivated by the GUP \[9, 10\] in the quantum gravity. Finally, we calculate the quantum entropy of the black hole via the relation between free energy and entropy. As a result, we obtain the desired entropy proportional to the horizon perimeter without any artificial cutoff and any little mass approximation.

II. SCALAR FIELD ON 2+1 DE SITTER BACKGROUND

Let us start with the following action

$$I = \frac{1}{2\pi} \int d^3x \sqrt{-g} \left[ R - \frac{2}{l^2} \right],$$  \hspace{1cm} (1)$$

where $\Lambda = \frac{1}{l^2}$ is a cosmological constant. Then the classical equation of motion yields the DS metric as

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\theta^2,$$  \hspace{1cm} (2)$$

$$f(r) = \left(1 - \frac{r^2}{l^2}\right).$$  \hspace{1cm} (3)$$

The horizon is located at $r \equiv r_H = l$ and our spacetime is bounded by the horizon as the two-dimensional cavity within the inner space of the horizon ($0 \leq r \leq l$) in contrast to the BTZ case \[5\] where the spacetime is defined within the outer space of the horizon ($l \leq r < \infty$). The inverse of Hawking temperature is given by

$$\beta_H = 2\pi l.$$  \hspace{1cm} (4)$$

In this DS background, let us first consider a scalar field with mass $\mu$, which satisfies the Klein-Gordon equation

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) - \mu^2 \Phi = 0.$$  \hspace{1cm} (5)$$
Substituting the wave function $\Phi(r, \theta, t) = e^{-i\omega t}\Psi(r, \theta)$, we find that this Klein-Gordon equation becomes

$$\frac{\partial^2 \Psi}{\partial r^2} + \left(\frac{1}{f} \frac{\partial f}{\partial r} + \frac{1}{r}\right) \frac{\partial \Psi}{\partial r} + \frac{1}{f} \left(\frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \theta^2} + \frac{\omega^2}{f} - \mu^2\right) \Psi = 0.$$  

(6)

By using the Wenzel-Kramers-Brillouin (WKB) approximation [3] with $\Psi \sim \exp[iS(r, \theta)]$, we have

$$p_r^2 = \frac{1}{f} \left(\frac{\omega^2}{f} - \mu^2 - \frac{p^2}{r^2}\right),$$

(7)

where

$$p_r = \frac{\partial S}{\partial r}, \quad p_\theta = \frac{\partial S}{\partial \theta}.$$

(8)

On the other hand, we also obtain the square module momentum

$$p^2 = p_rp^i = g^{rr}p_r^2 + g^{\theta\theta}p_\theta^2 = \frac{\omega^2}{f} - \mu^2.$$

(9)

Then, the area in the momentum phase space is given by

$$A_p(r) = \int dp_r dp_\theta = \pi \sqrt{\frac{1}{f} \left(\frac{\omega^2}{f} - \mu^2\right)} \cdot \sqrt{\frac{r^2(\omega^2}{f} - \mu^2)}.$$ 

(10)

$$= \pi \frac{r}{\sqrt{f}} \left(\frac{\omega^2}{f} - \mu^2\right)^{1/2}.$$ 

(11)

with the condition $\omega \geq \mu \sqrt{f}$.

III. BRICK WALL MODELS WITH HUP

A. Original Brick Wall Model

According to the BWM with the HUP, let us briefly recapitulate the previous work [12] in the 2+1 de Sitter space. However, we would like to avoid the difficulty of solving the wave equation by using the quantum statistical method. The usual position-momentum uncertainty relation followed by the HUP is given by

$$\Delta x \Delta p \geq \frac{\hbar}{2}.$$ 

(12)

From now on we take the units $G = c = \hbar = k_B \equiv 1$. When gravity is ignored, the number of quantum states in a volume element in phase cell space based on the HUP in the 2+1 de Sitter space is given by

$$dn = \left(\frac{dr dp_r}{2\pi}\right) \left(\frac{d\theta dp_\theta}{2\pi}\right) = \frac{d^2x d^2p}{(2\pi)^2},$$

(13)
where one quantum state corresponds to a cell of volume \((2\pi)^2\) in the phase space \([9, 10]\). Then, the number of quantum states with energy less than \(\omega\) is given by

\[
n_{O}(\omega) = \frac{1}{(2\pi)^2} \int dr d\theta dp_{r} dp_{\theta} = \frac{1}{(2\pi)^2} \int dr d\theta A_{p}(r) = \frac{1}{2} \int_{L}^{r_{H} - \epsilon} dr \frac{r}{\sqrt{f}} \left( \frac{\omega^2}{f} - \mu^2 \right).
\]

(14)

Note that \(\epsilon\) and \(L\) are ultraviolet and infrared regulators, respectively, where \(\epsilon > 0\) and \(0 \leq L < r_{H} - \epsilon\).

On the other hand, for the bosonic case the free energy at inverse temperature \(\beta\) is given by

\[
e^{-\beta F} = \prod_{K} \left[ 1 - e^{-\beta \omega_{K}} \right]^{-1},
\]

(15)

where \(K\) represents the set of quantum numbers. By using Eq. (14), the free energy can be rewritten as

\[
F_{O} = \frac{1}{\beta} \sum_{K} \ln \left[ 1 - e^{-\beta \omega_{K}} \right] \approx \frac{1}{\beta} \int dn_{O}(\omega) \ln \left[ 1 - e^{-\beta \omega} \right]
\]

\[
= - \int_{\mu \sqrt{f}}^{\infty} d\omega \frac{n_{O}(\omega)}{e^{\beta \omega} - 1}
\]

\[
= - \frac{1}{2} \int_{L}^{r_{H} - \epsilon} dr \frac{r}{\sqrt{f}} \int_{\mu \sqrt{f}}^{\infty} d\omega \frac{\omega^2}{f} \frac{(e^{\beta \omega} - \mu)^2}{(e^{\beta \omega} - 1)^2}.
\]

(16)

Here we have taken the continuum limit in the first line and integrated by parts in the second line.

Now, let us evaluate the entropy for the scalar field, which can be obtained from the free energy (16) at the Hawking temperature; then the entropy is

\[
S_{O} = \beta^2 \frac{\partial F_{O}}{\partial \beta}
\]

\[
= \frac{\beta^2}{2} \int_{L}^{r_{H} - \epsilon} dr \frac{r}{\sqrt{f}} \int_{\mu \sqrt{f}}^{\infty} d\omega \frac{\omega e^{\beta \omega} (\omega^2 - \mu^2)}{(e^{\beta \omega} - 1)^2}.
\]

(17)

Note that at this stage it is difficult to carry out the analytic integral about \(\omega\) because the value of \(\mu \sqrt{f}\) varies depending on \(r\) in the wide range \((L, r_{H} - \epsilon)\).

For the case of the massless limit, the entropy becomes

\[
S_{O} = \frac{\beta^{-2}}{2} \int_{L}^{r_{H} - \epsilon} dr \frac{r}{\sqrt{f}} \int_{0}^{\infty} \frac{e^{x^2} dx}{(e^x - 1)^2}
\]

\[
= \frac{1}{2} \pi a \left( \frac{l}{\sqrt{l^2 - (r_{H} - \epsilon)^2}} - \frac{l}{\sqrt{l^2 - L^2}} \right).
\]

(18)
where \( x = \beta \omega \) and the constant is defined by \( a \equiv \frac{3\zeta(3)}{2\pi^3} \). Note that this result is exactly the same as that of the previous work \([13]\), which was obtained through considering of the number of modes according to the semiclassical quantization rule. Then, when \( \epsilon \to 0 \), the dominant contribution term to the entropy is given by

\[
S_O \approx \frac{\pi a}{2\sqrt{2}} \sqrt{\frac{l}{\epsilon}}. 
\] (19)

As a result, the ultraviolet divergence of the entropy comes from near horizon \((r_H - \epsilon \leq r \leq r_H)\) as the BTZ case \((r_H \leq r \leq r_H + \epsilon)\). Moreover, the invariant distance of the brick wall from the horizon at \( r = r_H = l \) is related to the ultraviolet cutoff as

\[
\tilde{\epsilon} = \int_{r_H - \epsilon}^{r_H} \frac{dr}{\sqrt{f(r)}} = l \left( \frac{\pi}{2} - \sin^{-1} \frac{l - \epsilon}{l} \right). 
\] (20)

Then, the entropy (18), which is always positive, can be represented in terms of the invariant cutoff (20) as follows

\[
S_O = \frac{1}{2} \pi a \left( \frac{1 - \sin \tilde{\epsilon}}{\sin \frac{l}{2}} \right) \equiv \frac{1}{2} \pi a \, s(\tilde{\epsilon}, l). 
\] (21)

On the other hand, the infrared cutoff can be simply fixed as \( L = 0 \) without loss of generality because there does not exist any infrared divergence in the DS space where the spacetime is bounded within the inner space of the horizon \((0 \leq r \leq r_H)\) in contrast to the BTZ black hole case where the spacetime is defined within the outer space of the horizon \((r_H \leq r < \infty)\). Furthermore, if we choose the cutoff \( \tilde{\epsilon} \) as \( a \equiv l/s(\tilde{\epsilon}, l) \), then the entropy can be rewritten by the perimeter law \( S_O = (2\pi r_H)/4 \). Note that for \( l \gg \tilde{\epsilon} \), the invariant cutoff is simply written as \( \tilde{\epsilon} \approx a \) that does not depend explicitly on \( r_H = l \).

B. Improved Brick Wall Model

Although the BMW with the HUP has contributed a great deal to the understanding and calculating of the entropy of a black hole, there are generally some drawbacks in it, such as little mass approximation, neglecting logarithm term and artificial cutoffs. Moreover, the fundamental problem is why the entropy of fields surrounding the black hole is the entropy of the black hole itself since the event horizon is the characteristic of a black hole. Therefore, the entropy calculating of a black hole should be only related to its horizon. Due to this reason and the fact that the density of quantum states near the horizon is divergent, the BWM
have been improved to take only the entropy of a thin-layer near the event horizon of a black hole avoiding the drawbacks in the original BMW including the little mass approximation.

Now, according to the IBWM, let us summarize the improved results in the 2+1 DS space comparing with those of the original BWM. By just replacing the integral range \((L, r_H - \epsilon)\) of the BWM in the entropy \((17)\) with \((r_H - \epsilon_2, r_H - \epsilon_1)\), we have the entropy for a massive scalar field as follows

\[
S_T = \frac{\beta^2}{2} \int_{r_H - \epsilon_1}^{r_H - \epsilon_2} dr \frac{r}{\sqrt{f}} \int_0^\infty d\omega \frac{\omega e^{\beta \omega} (\frac{\omega^2}{f} - \mu^2)}{(e^{\beta \omega} - 1)^2}
\equiv \frac{\beta^2}{2} \int_{r_H - \epsilon_1}^{r_H - \epsilon_2} dr \frac{r}{\sqrt{f}} \Lambda_T,
\]

where \(\epsilon_i (i = 1, 2)\) with \(\epsilon_1 < \epsilon_2\) represent the coordinate distances from the horizon to the nearest and more distant boundary, respectively, of the thin-layer. Since \(f \to 0\) in the near horizon range of \((r_H - \epsilon_2, r_H - \epsilon_1)\), without any little mass approximation, the integral about \(\omega\) is reduced to

\[
\Lambda_T = \int_0^\infty d\omega \frac{f^{-1} \omega^3}{(1 - e^{-\beta \omega})(e^{\beta \omega} - 1)} = \int_0^\infty dx \frac{f^{-1} \beta^{-4} x^3}{(1 - e^{-x})(e^x - 1)},
\]

where \(x = \beta \omega\). Then, the integration gives explicitly the result as

\[
S_T = \frac{1}{2} \pi a \left( \frac{l}{\sqrt{l^2 - (r_H - \epsilon_1)^2}} - \frac{l}{\sqrt{l^2 - (r_H - \epsilon_2)^2}} \right).
\]

This result shows that the entropy behaves as \(1/\sqrt{\epsilon_i}\) at \(\epsilon_i \to 0\), which correspond to the ultraviolet divergences of the entropy.

On the other hand, the invariant distances of the thin-layer from the horizon at \(r = r_H = l\) are related to the ultraviolet cutoffs \(\epsilon_i\) as

\[
\tilde{\epsilon}_i = \int_{r_H - \epsilon_i}^{r_H} \frac{dr}{\sqrt{f(r)}} = l \left( \frac{\pi}{2} - \sin^{-1} \frac{l - \epsilon_i}{l} \right).
\]

Then, the entropy (24) can be represented in terms of the invariant cutoffs (25) as follows

\[
S_T = \frac{1}{2} \pi a \left( \frac{\sin \tilde{\epsilon}_i}{l} - \sin \tilde{\epsilon}_1 \right) \equiv \frac{1}{2} \pi a \ s(\tilde{\epsilon}_i, l).
\]

Note that there does not exist any infrared divergence even though we consider a massive scalar field, and the entropy \(S_T\) is always positive since \(\tilde{\epsilon}_1 < \tilde{\epsilon}_2\). Furthermore, if we choose the cutoffs as \(a = l/s(\tilde{\epsilon}_i, l)\), then the entropy can be also rewritten by the perimeter law \(S_T = (2\pi r_H)/4\) as the BMW case.
It seems to be appropriate to comment on the entropy relation between the BWM and IBWM. For \( l \gg \bar{\epsilon}_i \), we could choose the cutoffs in the IBWM as \( \bar{\epsilon}_2 \equiv 2\bar{\epsilon} \) and \( \bar{\epsilon}_1 \equiv \bar{\epsilon} \) without loss of generality. Then, the value \( a \) satisfying the perimeter law \( S_T \) becomes \( 2\bar{\epsilon} \), and this value does not also depend explicitly on \( r_H = l \) as the original BWM case. Furthermore, the entropy \( S_O \) with \( a = 2\bar{\epsilon} \) in Eq. (21) becomes \( S_O \approx 2S_T \). This means that the entropy contribution of the whole rest range \((0, r_H - 2\epsilon)\) is equal to that of the near horizon range \((r_H - 2\epsilon, r_H - \epsilon)\).

IV. ENTROPY WITH GENERALIZED UNCERTAINTY PRINCIPLE

Recently, many efforts have been devoted to the generalized uncertainty relation given by
\[
\Delta x \Delta p \geq \frac{1}{2} \left(1 + \lambda (\Delta p)^2\right).
\] (27)

Then, since one can easily get \( \Delta x \geq \sqrt{\lambda} \), which gives the minimal length, it can be defined to be the thickness of the thin-layer near horizon, which naturally plays a role of the brick wall cutoff. Furthermore, based on the generalized uncertainty relation, the volume of a phase cell in the de Sitter space is changed from \((2\pi)^2\) into
\[
(2\pi)^2(1 + \lambda p^2)^2,
\] (28)

where \( p^2 = p_i^2 p_i \) \((i = r, \theta)\).

From Eq. (28), the number of quantum states with energy less than \( \omega \) is given by
\[
n_I(\omega) = \frac{1}{(2\pi)^2} \int dr d\theta dp_r dp_\theta \frac{1}{\left(1 + \lambda \left(\frac{\omega^2}{f} - \mu^2\right)\right)^2} A_p(r)
\]
\[
= \frac{1}{(2\pi)^2} \int dr d\theta \frac{1}{\left(1 + \lambda \left(\frac{\omega^2}{f} - \mu^2\right)\right)^2} A_p(r)
\]
\[
= \frac{1}{2} \int dr \frac{r}{\sqrt{f}} \frac{\left(\frac{\omega^2}{f} - \mu^2\right)}{\left(1 + \lambda \left(\frac{\omega^2}{f} - \mu^2\right)\right)^2}.
\] (29)

Note that it is convergent at the horizon without any artificial cutoff due to the existence of the suppressing \( \lambda \) term in the denominator induced from the GUP. Then, by using Eq. (29), the free energy can be obtained as
\[
F_I = -\int_{\mu \sqrt{f}}^{\infty} d\omega \frac{n_I(\omega)}{e^{\beta \omega} - 1}.
\]
\[
-\frac{1}{2} \int dr \frac{r}{\sqrt{f}} \int_{\mu/\sqrt{f}}^{\infty} d\omega \frac{\left(\frac{\omega^2}{T} - \mu^2\right)}{(e^{\beta \omega} - 1) \left(1 + \lambda \left(\frac{\omega^2}{T} - \mu^2\right)\right)^2}.
\]

From this free energy, the entropy for the massive scalar field is given by

\[
S_I = \beta^2 \frac{\partial F_I}{\partial \beta} = \frac{\beta^2}{2} \int dr \frac{r}{\sqrt{f}} \int_{\mu/\sqrt{f}}^{\infty} d\omega \frac{\omega e^{\beta \omega} \left(\frac{\omega^2}{T} - \mu^2\right)}{(e^{\omega} - 1)^2 \left(1 + \lambda \left(\frac{\omega^2}{T} - \mu^2\right)\right)^2} \equiv \frac{\beta^2}{2} \int dr \frac{r}{\sqrt{f}} \Lambda_I.
\]

Since \( f \to 0 \) near the event horizon, \( i.e. \), in the range of \((r_H - \epsilon, r_H)\), then, without any little mass approximation, the integral about \( \omega \) is reduced to

\[
\Lambda_I = \int_{0}^{\infty} dx \frac{f^{-1} \beta^{-4} x^3}{(1 - e^{-x})(e^x - 1) \left(1 + \frac{\lambda \sqrt{T}}{\beta^2} x^2\right)^2},
\]

where \( x = \beta \omega \).

On the other hand, we are only interested in the contribution from the just vicinity near the horizon, \((r_H - \epsilon, r_H)\), which corresponds to a proper distance of order of the minimal length, \( \sqrt{\lambda} \). This is because the entropy closes to the upper bound only in this vicinity, which it is just the vicinity neglected by BWM and IBWM. We have

\[
\sqrt{\lambda} = \int_{r_H-\epsilon}^{r_H} \frac{dr}{\sqrt{f(r)}} = \int_{r_H-\epsilon}^{r_H} \frac{dr}{\sqrt{2\kappa (r_H - r)}} = \sqrt{\frac{2\epsilon}{\kappa}},
\]

where \( \kappa \) is the surface gravity at the horizon of black hole and it is identified as \( \kappa = 2\pi \beta \).

Now, let us rewrite Eq. (31) as

\[
S_I = \frac{1}{2\lambda} \int_{r_H-\epsilon}^{r_H} dr \frac{r}{\sqrt{f}} \Lambda_I,
\]

where

\[
\Lambda_I = \int_{0}^{\infty} dX \frac{b^2 X^3}{(e^{bX} - e^{-bX})^2 (1 + X^2)^2}
\]

with \( x = \beta \sqrt{\frac{T}{\lambda}} X \equiv bX \). Since \( r \to r_H \), \( b \to 0 \), removable pole becomes \((e^{bX} - e^{-bX})^2 \approx b^2 X^2 + O(b^3)\). Then, the integral equation (35) can be easily solved without the help of the complex residue theorem as follows

\[
\Lambda_I \approx \int_{0}^{\infty} \frac{X dX}{(1 + X^2)^2} = \frac{1}{2}
\]
Finally, when $r \to r_H$, we get the entropy as follows

$$S_I = \frac{1}{2\lambda} \cdot r_H \sqrt{\lambda} \cdot \frac{1}{2} = \frac{2\pi r_H}{8\pi \sqrt{\lambda}}. \quad (37)$$

Note that there is no divergence within the just vicinity near the horizon due to the effect of the generalized uncertainty relation on the quantum states. Furthermore, if we assume $\lambda = \alpha l_P^2$, where $l_P$ is Planck length, and in the system of Planck units $l_P = 1$, then the entropy can be rewritten by the desired perimeter law $S_I = \frac{1}{4}(2\pi r_H)$ with $\alpha = \frac{1}{4\pi^2}$.

It seems to be appropriate to comment on the entropy of the original BWM comparing with the entropy (37), which is effectively considered the contribution inside of the brick wall. From the Eq. (20), if we take the brick wall cutoff $\tilde{\epsilon}$ as the minimal length $\sqrt{\lambda}$ induced by the GUP, we have the relation $\epsilon = \frac{\lambda}{2\tilde{\epsilon}}$. Then, we effectively obtain the entropy contribution outside of the brick wall as follows

$$S_O \approx \frac{3\zeta(3)}{\pi^2} \frac{2\pi r_H}{8\pi \sqrt{\lambda}} = \frac{3\zeta(3)}{\pi^2} S_I. \quad (38)$$

In summary, we have investigated the massive scalar field within the just vicinity near the horizon of a static black hole in the 2+1 de Sitter space by using the generalized uncertainty principle. In contrast to the cases of the BWM and IBWM, we have obtained the desired entropy proportional to the horizon perimeter without any artificial cutoff and any little mass approximation, simultaneously.

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