Nonfactorizable contributions in $\overline{B}^0 \rightarrow D^+_s D^-_s$ and $\overline{B}^0_s \rightarrow D^+ D^-$ decays*

J.O. Eeg$^a$, S. Fajfer$^{b,c}$, and A. Hiorth$^a$

$^a$ Department of Physics, University of Oslo, P.O. Box 1048 Blindern, N-0316 Oslo, Norway
$^b$ J. Stefan Institute, Jamova 39, P.O. Box 3000, 1001 Ljubljana, Slovenia
$^c$ Department of Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia

Abstract

The decay amplitudes for $\overline{B}^0 \rightarrow D^+_s D^-_s$ and $\overline{B}^0_s \rightarrow D^+ D^-$ have no factorizable contributions. We suggest that dominant contributions to the decay amplitudes arise from two chiral loop contributions and one soft gluon emission contribution. Then we determine branching ratios $\text{BR}(\overline{B}^0 \rightarrow D^+_s D^-_s) \simeq 7 \times 10^{-5}$ and $\text{BR}(\overline{B}^0_s \rightarrow D^+ D^-) \simeq 1 \times 10^{-3}$.

Numerous experimental data coming from Ba Bar, Belle and Tevatron on B meson decays stimulate many studies of their decay mechanism. Through decades the factorization assumption has been used in calculations of the decay amplitudes. Recently, it has been shown [1] that some classes of $B$-meson decay amplitudes exhibit QCD factorization. This means that, up to $\alpha_s/\pi$ (calculable), and $\Lambda_{QCD}/m_b$ (not calculable), their amplitudes factorize into the product of two matrix elements of weak currents. Typically, the decay amplitudes which factorize in this sense are $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$ where the energy release is big compared to the light meson masses. However, for decays where the energy release is of order 1 GeV, QCD factorization is not expected to hold. Here we discuss the dominant contributions in $\overline{B}^0 \rightarrow D^+_s D^-_s$ and $\overline{B}^0_s \rightarrow D^+ D^-$ [2]. At quark level these decays occur through the annihilation mechanism $b\bar{s} \rightarrow c\bar{c}$ and $b\bar{d} \rightarrow c\bar{c}$, respectively (Fig.1). However, within the factorized limit the annihilation mechanism will give a zero amplitude due to current conservation, as for the $D^0 \rightarrow K^0\overline{K}^0$ decay [3]. The axial part of the weak current might lead to non-zero factorized contributions if one of $D$-mesons in the final state is a vector meson $D^*$. Such contributions are proportional to the numerically small Wilson coefficient $C_1$, which we will neglect in our analysis. In contrast, the typical factorized decay modes which proceed through the spectator mechanism, say $\overline{B}^0 \rightarrow D^+_s D^-_s$, are proportional to the numerically larger Wilson coefficient $C_2$. If one or both of charm mesons

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in this decay are vector mesons, such amplitudes will give nonfactorizable chiral loop contributions to the process \( \overline{B}^0_d \rightarrow D_s^+ D_s^- \) due to \( K^0 \)-exchange. We determine these chiral loop contributions.

There are also nonfactorizable contributions due to soft gluon emission. Such contributions can be calculated in terms of the (lowest dimension) gluon condensate within a recently developed Heavy Light Chiral Quark Model (HL\( \chi \)QM) [4], which is based on Heavy Quark Effective Theory (HQEFT) [5]. This model has been applied to processes with \( B \)-mesons in [6][7]. The gluon condensate contributions is also proportional to the favorable Wilson coefficient \( C_2 \). We follow the standard approach [8] for non-leptonic decays where one constructs an effective Lagrangian \( \mathcal{L}_W \) in terms of quark operators multiplied with Wilson coefficients containing all information of the short distance (SD) loop effects above a renormalization scale \( \mu \) of order \( m_b \). Within Heavy Quark Effective Theory (HQEFT) [5], the effective Lagrangian \( \mathcal{L}_W \) can be evolved down to the scale \( \mu \sim \Lambda_\chi \sim 1 \text{ GeV} \) [9][10].

The use of factorization is illustrated in the \( \overline{B}^0 \rightarrow D^+ D_s^- \) decay:

\[
\langle D_s^- D^+ | \mathcal{L}_W | \overline{B}^0 \rangle_F = -(C_2 + \frac{1}{N_c} C_1) \langle D_s^- | \bar{s} \gamma_5 \gamma_\mu c | 0 \rangle \langle D^+ | \bar{c} \gamma_\mu b | \overline{B}^0 \rangle,
\]

(1)

The coefficients \( C_{1,2} \) are Wilson coefficients for the operators containing the product of two left-handed currents. In our notation \( C_i = -G_F^2 V_{cb} V_{cs}^* a_i \), where the \( a_i \) are dimensionless, \( G_F \) is the Fermi coupling, \( V_{ij} \) are CKM parameters. Numerically, \( a_1 \sim 10^{-1} \) and \( a_2 \sim 1 \) at the scale \( \mu = m_b \), and \( |a_1| \approx 0.4 \) and \( |a_2| \approx 1.4 \) at \( \mu \sim \Lambda_\chi \sim 1 \text{ GeV} \) [9][10]. Penguin operators may also contribute, but have rather small Wilson coefficients.

The factorized amplitude for \( \overline{B}^0 \rightarrow D_s^+ D_s^- \) is presented in Fig. 1, and is given by

\[
\langle D_s^- D_s^+ | \mathcal{L}_W | \overline{B}^0 \rangle_F = 4(C_1 + \frac{1}{N_c} C_2) \langle D_s^- D_s^+ | \overline{c} \gamma_\mu c_L | 0 \rangle \langle 0 | d_L \gamma^\mu b_L | \overline{B}^0 \rangle.
\]

(2)

Unless one or both of the \( D \)-mesons in the final state are vector mesons, this matrix element is zero due to current conservation:

\[
\langle D_s^+ D_s^- | \overline{c} \gamma_\mu c | 0 \rangle \langle 0 | d \gamma_5 \gamma_\mu b | \overline{B}^0 \rangle \sim f_B (p_D + p_B)^\mu \langle D_s^+ D_s^- | \overline{c} \gamma_\mu c | 0 \rangle = 0.
\]

(3)

Our approach is based on the use of bosinized currents [2] and by using them we first write down the amplitude for \( \overline{B}^0 \rightarrow D_s^+ D_s^- \). To calculate the chiral loop amplitudes we need the factorized amplitudes for \( \overline{B}^0 \rightarrow D_s^+ D_s^- \) and \( \overline{B}^0 \rightarrow D_s^+ D_s^- \), which proceed through the spectator mechanism as in Fig. 2. In this case the leading chiral coupling results from the coupling between a pseudoscalar meson \( H \), vector meson \( H^* \) a light pseudoscalar \( M \) (\( = \pi, K, \eta \)), denoted by \( g_A \). After use of bosonized currents [2], we obtain the following chiral loop amplitude for the process \( \overline{B}^0 \rightarrow D_s^+ D_s^- \) from the Fig. 3:

\[
A(\overline{B}^0 \rightarrow D_s^+ D_s^-)_\chi = (V_{cd}^* / V_{cs}) A(\overline{B}^0_d \rightarrow D_s^+ D_s^-)_F \cdot R^\chi,
\]

(4)

where the \( A(\overline{B}^0_d \rightarrow D_s^+ D_s^-)_F \) stands for the factorized amplitude for the process \( \overline{B}^0 \rightarrow D_s^+ D_s^- \) and the quantity \( R^\chi \) is a sum of contributions from the left and right part of Fig.
Figure 1: Factorized contribution for $\bar{B}^0 \to D^+_s D^-_s$ through the annihilation mechanism, which give zero contributions if both $D^+_s$ and $D^-_s$ are pseudoscalars. The double dashed lines represent heavy mesons, the double lines represent heavy quarks, and the single lines light quarks.

Figure 2: Factorized contribution for $\bar{B}^0 \to D^+_s D^-_s$ through the spectator mechanism, which does not exist for decay mode $\bar{B}^0 \to D^+_s D^-_s$ we consider in this paper.

3 respectively [2]. In the $\overline{MS}$ scheme we obtained

$$R^\chi = \frac{m_K^2}{(4\pi f)^2} g_A^2 \left[ \left\{ \frac{(\omega + 1)}{(\omega + \lambda)} [r(-\omega) + r(-\lambda)] - 1 \right\} \ln \left( \frac{m_K^2}{\Lambda^2} \right) - 1 \right].$$

(5)

with $\omega = M_B/(2M_D)$, $\lambda = [M_B^2/(2M_D^2) - 1]$ and $f$ being the $\pi$ decay constant. The function $r(x)$ is:

$$r(x) \equiv \frac{1}{\sqrt{x^2 - 1}} \ln \left( x + \sqrt{x^2 - 1} \right), \quad r(-x) = -r(x) + \frac{i\pi}{\sqrt{x^2 - 1}},$$

(6)

which means that the amplitude gets an imaginary part. Numerically, we find [2]:

$$R^\chi \simeq 0.12 - 0.26i.$$

(7)

The genuine nonfactorizable part for $\bar{B}^0 \to D^+_s D^-_s$ can, by means of Fierz transformations and identities for the product of two color matrices, be written in terms of colored currents

$$\langle D^-_s D^+_s | \mathcal{L}_W | \bar{B}^0 \rangle_{NF} = 8 C_2 \langle D^-_s D^+_s | (\bar{d}_L \gamma^\alpha t^a b_L) (\bar{c}_L \gamma^\alpha t^a c_L) | \bar{B}^0 \rangle.$$

(8)
Figure 3: Nonfactorizable chiral loops for $B_0 \to D_s^+ D_s^-$. 

Figure 4: Nonfactorisable contribution for $B_0 \to D_s^+ D_s^-$ through the annihilation mechanism with additional soft gluon emission. The wavy lines represent soft gluons ending in vacuum to make gluon condensates.

Within our approach, this amplitude is written in a quasi-factorized way in terms of matrix elements of colored currents:

$$\langle D_s^+ D_s^- | \mathcal{L}_W | \overline{B}_0 \rangle^G_{NF} = 8 C_2 \langle D_s^+ D_s^- | \overline{c}_L \gamma^\mu t^a c_L | G \rangle \langle G | d_L \gamma^\mu t^a b_L | \overline{D}_s \rangle ,$$

where a $G$ in the bra-kets symbolizes emission of one gluon (from each current) as visualized in Fig. 4. In order to calculate the matrix elements in (9), we have used [2] the Heavy Light Chiral Quark Model (HL$\chi$QM) recently developed in [4], which incorporates emission of soft gluons modeled by a gluon condensate. Then we defined a quantity $R_G$ for the gluon condensate amplitude analogously to $R_\chi$ in (4) and (5) for chiral loops. Numerically, we determine [2] that the ratio between the two amplitudes is

$$R_G \simeq 0.055 + 0.16i ,$$

which is of order one third of the chiral loop contribution in eq. (5).

Adding the amplitudes $R_\chi$ and $R_G$ and multiplying with the Wilson coefficient [9] $a_2 \simeq 1.33 + 0.2i$, we obtain the quantity:

$$\widetilde{R}_T \equiv a_2 (R_\chi + R_G) \simeq 0.26 - 0.11i .$$

We have found that the amplitude for $\overline{B}_0 \to D_s^+ D_s^-$ is of order $15 - 20\%$ of the factorizable amplitude for $\overline{B}_0 \to D^+ D_s^-$, before the different CKM-factors are taken into account.
Finally, we predict [2] that the branching ratios are

\[ \text{BR}(B_d^0 \to D_s^+ D_s^-) \approx 7 \times 10^{-5} ; \quad \text{BR}(B_s^0 \to D^+ D^-) \approx 1 \times 10^{-3} . \] (12)

The current searches at BaBar and Belle might soon result in the limit on the rate \( B^0 \to D_s^+ D_s^- \). However, the \( B_s^0 \to D^+ D^- \) mode will be accessible at Tevatron and later at LHC.

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