Theory for Non-Fermi Liquid Temperature Dependence in Resistivity of Ce$_x$La$_{1-x}$Cu$_{5.62}$Au$_{0.38}$ ($x = 0.02 - 0.10$) on the Local Quantum Valence Criticality of Ce Impurities

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It was reported by Shiino et al. in J. Phys. Soc. Jpn. 86, 123705 (2017) that Ce$_x$La$_{1-x}$Cu$_{5.62}$Au$_{0.38}$ ($x = 0.02 - 0.10$) exhibits a new type of quantum criticality in both magnetic and thermal properties, which is the same as that observed in a series of materials exhibiting quantum critical valence fluctuations (QCVF), such as $\beta$-YbAlB$_4$, Yb$_{12}$Al$_{13}$Au$_{51}$, and so on. However, the temperature ($T$) dependence in the resistivity $\rho(T)$ for $T < 0.5$ K is quite anomalous, i.e., $\rho(T) \propto (\text{const.} - T^n)$ with $n \approx 0.75$ at $x = 0.05$. We find that this anomalous exponent is given by $n = 2(1 - \zeta)$, with $\zeta$ being the weakly temperature dependent ($0.5 \leq \zeta \leq 0.7$) critical exponent for the QCVF. The observed critical exponent $n = 0.75$ at $x = 0.05$ is reproduced by choosing $\zeta = 0.63$ which is consistent with the divergent behavior observed in the uniform magnetic susceptibility $\chi \propto T^{-\delta}$ with $\delta = 0.67$ at $x = 0.02$.

KEYWORDS: local quantum valence criticality, Ce$_x$La$_{1-x}$Cu$_{5.62}$Au$_{0.38}$ ($x = 0.02 - 0.10$), non-Fermi liquid resistivity

1. Introduction

It has been pointed out in Refs. [1, 2] that CeCu$_6$ exhibits a rather sharp crossover in the valence of Ce ion under pressure [3] though it is a bit milder than the case of CeCu$_2$(Si,Ge)$_2$ [4, 5]. Recently, a symptom of magnetic field and pressure induced quantum critical valence transition (QCVT) was reported [6, 7], which had been predicted in Refs. [8, 9]. In this sense, it may be reasonable to explore a possibility that the QCVT is realized if the magnetic order in CeCu$_{5.62}$Au$_{0.38}$ ($x \geq 0.1$) is suppressed by partly replacing Ce by La because the QCVT is an essentially local phenomenon [10, 11]. Indeed, recently, it was reported in Ref. [12] that Ce$_x$La$_{1-x}$Cu$_{5.62}$Au$_{0.38}$ ($x = 0.02 - 0.10$) exhibits non-Fermi liquid properties in temperature ($T$) dependence of the magnetic susceptibility and the specific heat, and the $T/B$ scaling of the magnetization, which are the same as those expected at around the QCVT in periodic [11] or aperiodic lattice systems [13, 14].

While one might think that the system with $x = 0.02$ can be described by the single impurity model, the mean distance between Ce ions is only about 4×(lattice constant) which is not large enough compared to the size of cloud of the Kondo-Yosida singlet state discussed in Ref. [15], so that the collective effect of Ce impurities should be taken into account. In other words, the theory for QCVT can be applied through averaging process over the random distribution of Ce ions. It is also reasonable to expect as noted above [1, 2] that the quantum criticality of valence transition is induced by diluting Ce by La which has a larger atomic or ionic radius than Ce as reported in, e.g., Ref. [16], because Ce ions feel positive pressure locally from surrounding and expanding medium which includes La ions with larger ionic radius, if the concentration of Ce is dilute enough. This positive
pressure effect changes the system from the Kondo regime to the valence fluctuations one with higher Kondo temperature, where the sharp valence crossover occurs as reported in Ref. [3].

On the other hand, the resistivity $\rho(T)$ exhibits the $T$ dependence like $\rho(T) \propto (\text{const.} - T^n)$ with $n \approx 0.75$ which is quite anomalous even from the view point of the QCVT in the lattice systems. The purpose of the present paper is to derive this anomalous $T$ dependence on the basis of the QCVT scenario.

2. Model Hamiltonian

A canonical model for describing the valence transition is the extended periodic Anderson model that takes into account the Coulomb repulsion $U_{\text{fc}}$ between f and conduction electrons [10, 11, 13, 14]. To discuss the present situation, we have to revise the model as follows:

$$H_{\text{EPAM}} = \sum_{\mathbf{k},\sigma} e_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \varepsilon_f \sum_{\mathbf{l},\sigma} f_{\mathbf{l}\sigma}^\dagger f_{\mathbf{l}\sigma} + \frac{1}{V N_L} \sum_{\mathbf{i},\mathbf{r},\sigma} \left( V_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} c_{\mathbf{i}\sigma}^\dagger f_{\mathbf{i}\sigma} + \text{h.c.} \right) + U_{\text{fc}} \sum_{\mathbf{i},\sigma} n^f_{\mathbf{i}\sigma} n^f_{\mathbf{i}\sigma},$$

where f electrons occupy the $i$-sites which are randomly distributed in diluted systems, and conduction electrons are described by wave vector $\mathbf{k}$ defined on the periodic lattice sites of $N_L$ and have energy dispersion $e_{\mathbf{k}}$. The label $\sigma$ in Eq. (1) stands for the Kramers doublet state of the ground Crystalline-Electric-Field (CEF) level. Hereafter, the c-f hybridization $V_{\mathbf{k},\mathbf{i}}$ at $i$-site is assumed to be constant $V$.

The type of Hamiltonian [Eq. (1)] well describes phenomena associated with quantum critical valence fluctuations (QCVF) not only in periodic lattice systems such as $\beta$-YbAlB$_4$ [11, 17, 18] but also in aperiodic systems, such as clusters forming Tsai-type quasicrystal Yb$_{15}$Al$_{34}$Au$_{51}$ [13, 19], and approximant Yb$_{14}$Al$_{35}$Au$_{51}$ [14, 20], all of which exhibit the unconventional non-Fermi liquid behaviors of the same universality class different from those in the quantum criticality associated with itinerant magnetic transitions [21]. The point is that the periodicity is not essential for such unconventional universality class to appear because the QCVT is quite local in character.

3. Recipe for Average over Random Distribution of Ce Ions

A basic idea for taking the effect of Ce ions in diluted system is that the 4f electrons at Ce sites acquire the wave vector $\mathbf{q}$ dependence through the average over the impurities distribution. By this process, there arises two contributions to the scattering process of the conduction electrons, i.e., a single site effect of localized f electrons and the lattice effect due to the wave-number dependent collective valence fluctuations (VF). Hereafter, we consider the case in which the concentration $c_{\text{imp}} \equiv N_f/N_L (x)$ of Ce ions in Ce$_x$La$_{1-x}$Cu$_{5.62}$Au$_{0.38}$ is considerably smaller than 1 but is non-vanishing in the thermodynamic limit, $N_f \to \infty$ and $N_L \to \infty$.

To estimate the effect of scattering due to the random distribution of Ce ions, we have to take an average over the distribution. Before taking the average, the one-particle Green function of f electron depends on two positions as $G_{\uparrow}(\mathbf{r}_i, \mathbf{r}_j, \tau; \tau')$, which becomes a function of the relative coordinate $(\mathbf{r}_i - \mathbf{r}_j)$ after the average as in the usual case discussing impurity scattering effect [22]. Namely, by taking this average, the wave-vector dependent Green function is defined as

$$G_{\uparrow}(\mathbf{p}, \tau - \tau') \equiv \frac{1}{N_f} \sum_{(i,j)} e^{-i\mathbf{p}(\mathbf{r}_i - \mathbf{r}_j)} \langle \langle G_{\uparrow}(\mathbf{r}_i, \mathbf{r}_j; \tau - \tau') \rangle \rangle, \quad (2)$$

where $\langle \langle \cdots \rangle \rangle$ denotes the average over the random distribution of Ce ions, and $N_f$ is the number of virtual lattice sites occupied by f electrons as schematically shown in Fig. 1(b). Note here that the $N_f$
appears instead of the number $N_L$ of the original lattice sites. In other words, the lattice constant of the virtual periodic lattice system of Ce is enlarged by a factor $c^{1/3}$.

Hereafter, $\mathbf{p}$’s are used for the wave vector in the virtual lattice obtained after the random average of Ce ions, and distinguished from wave vectors $\mathbf{k}$’s in the original lattice shown in Fig. 1(a).

On the other hand, conduction electrons described by wave vector $\mathbf{p}$ are essentially unaltered by the effect of scattering by Ce impurities except for some broadening of the dispersion due to impurities scattering [22]. Namely, for example, the density of states (DOS) of conduction electrons at the Fermi level are essentially unaltered. However, since the size of the Brillouin zone (BZ) of the virtual lattice is shortened by a factor $c^{1/3}$, the band of conduction electrons splits into multibands in the shortened and reduced BZ. However, we use an extended zone scheme for conduction electrons.

Thus, the virtual Hamiltonian $H_{EPAM}^v$ is given explicitly as follows:

$$H_{EPAM}^v \equiv \sum_{\mathbf{p},\sigma} \epsilon_{\mathbf{p}\sigma} \tilde{c}_{\mathbf{p}\sigma} \tilde{c}^\dagger_{\mathbf{p}\sigma} + \sum_{\mathbf{i},\sigma} u_{\text{imp}} P_i \tilde{n}^\dagger_{\mathbf{i}\sigma} \tilde{n}_{\mathbf{i}\sigma} + \epsilon_{\mathbf{f}} \sum_{\mathbf{i},\sigma} \tilde{f}_{\mathbf{i}\sigma} \tilde{f}^\dagger_{\mathbf{i}\sigma} + \sqrt{c_{\text{imp}}} \sum_{\mathbf{p},\sigma} (\mathcal{V}_{\mathbf{p}\sigma} \tilde{f}_{\mathbf{i}\sigma} \tilde{f}^\dagger_{\mathbf{i}\sigma} + \text{h.c.}) + U_{\text{ff}} \sum_{\mathbf{i}} n_{\mathbf{i}\uparrow} n_{\mathbf{i}\downarrow} + U_{\text{fc}} \sum_{\mathbf{i},\sigma\sigma'} n_{\mathbf{i}\sigma} n_{\mathbf{i}\sigma'},$$

where $\tilde{c}_{\mathbf{p}\sigma}$ and $\tilde{f}_{\mathbf{p}\sigma}$ are the annihilation operators of conduction- and f-electrons in the virtual periodic lattice system. Note that the factor $\sqrt{c_{\text{imp}}} \equiv \sqrt{N_f/N_L}$ in the fourth term of Eq. (3) reflects the fact that the f electrons are located on the periodic virtual lattice points of $N_f$ while the conduction electrons are hopping among the original lattice points of $N_L$ including the points $i \neq \tilde{i}$, as discussed in detail elsewhere. This factor manifests the diluteness of Ce ions in the original lattice [Fig. 1(a)]. The random variables $P_i$ in the second term of Eq. (3) represent the effect of impurity scattering in the original lattice shown in Fig. 1(a). With the use of this replica Hamiltonian, the theoretical framework discussing QCVF can be applied as it is because the QCVF are essentially the local phenomena [11].

Since the the virtual Hamiltonian [Eq. (3)] is essentially the same as that for the periodic lattice system (except the random variable and the factor $\sqrt{c_{\text{imp}}}$ weakening the c-f hybridization), the condition for the QCVT is not altered seriously as will be discussed more quantitatively elsewhere. This is because the valence transition is essentially the local phenomenon or valence susceptibility has extremely weak wave-vector dependence [10, 11].
4. Two Contributions to Resistivity

After taking the average over the random distribution of Ce ions, both the Green function and the self-energy of conduction electrons become the function of the relative coordinate \((\mathbf{r}_i - \mathbf{r}_j)\)'s and acquire the wave vector representation. Namely, the Green function of conduction electrons defined as 

\[
\tilde{G}_c(\mathbf{p}, \epsilon_{\text{f}}) = \left(1/N_L \right) \sum_{i(j)} e^{-i\mathbf{p} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \langle \langle \tilde{G}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle \]

satisfies the Dyson equation as

\[
\tilde{G}_c(\mathbf{p}, \epsilon_{\text{f}}) = G_c^{(0)}(\mathbf{p}, \epsilon_{\text{f}}) + G_c^{(0)}(\mathbf{p}, \epsilon_{\text{f}}) \tilde{\Sigma}_c(\mathbf{p}, \epsilon_{\text{f}}) \tilde{G}_c(\mathbf{p}, \epsilon_{\text{f}}), \tag{4}
\]

where \(G_c^{(0)}(\mathbf{p}, \epsilon_{\text{f}})\) is the Green function of free band electrons on the original lattice, and \(\tilde{\Sigma}_c(\mathbf{p}, \epsilon_{\text{f}})\) is defined by

\[
\tilde{\Sigma}_c(\mathbf{p}, \epsilon_{\text{f}}) \equiv \frac{1}{N_L} \sum_{i(j)} e^{-i\mathbf{p} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \langle \langle \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle. \tag{5}
\]

Note that the conduction electrons are defined on the original lattice points as shown in Fig. 1(a) so that the factor \((1/N_L)\) appears in Eq. (5) instead of the factor \((1/N_l)\) appearing in the definition of \(G_T(\mathbf{p}, \tau - \tau')\) [Eq. (2)].

The \(T\) dependence of the resistivity is essentially given by the imaginary part of the retarded function of \(\tilde{\Sigma}_c(\mathbf{p}, \epsilon_{\text{f}})\) [Eq. (5)], in which \(\langle \langle \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle\) consists of two parts as

\[
\langle \langle \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle = \delta_{i,j} \langle \langle \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle + \langle \langle \Delta \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle, \tag{6}
\]

where \(\mathbf{r}_i\) is the site of Ce ions and \(\langle \langle \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle\) is independent of \(\mathbf{r}_i\), and the term with \(i = j = \ell\) is excluded in \(\langle \langle \Delta \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle\). The first part in Eq. (6) represents the damping effect arising from independent but dynamical scattering by localized f electron at site \(\mathbf{r}_i\), and the second part arises from the scattering by the collective VF described by the virtual Hamiltonian [Eq. (3)]. Namely, the Green function of conduction electrons \(G_c(\mathbf{p}, \epsilon_{\text{f}})\) has the following structure:

\[
\tilde{G}_c(\mathbf{p}, \epsilon_{\text{f}}) = \left[ \epsilon_{\text{f}} - \xi_{\text{f}} - c_{\text{imp}} \langle \langle \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle - \langle \langle \Delta \tilde{\Sigma}_c(\mathbf{p}, \epsilon_{\text{f}}) \rangle \rangle \right]^{-1}, \tag{7}
\]

where \(c_{\text{imp}}\) is the concentration of Ce ions on the original lattice shown in Fig. 1(a), and \(\langle \langle \Delta \tilde{\Sigma}_c(\mathbf{p}, \epsilon_{\text{f}}) \rangle \rangle\) is the wave-vector representation of \(\langle \langle \Delta \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle\) defined by

\[
\langle \langle \Delta \tilde{\Sigma}_c(\mathbf{p}, \epsilon_{\text{f}}) \rangle \rangle \equiv \frac{1}{N_L} \sum_{i(j)} e^{-i\mathbf{p} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \langle \langle \Delta \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle. \tag{8}
\]

Here, it is crucial to note that \(\langle \langle \Delta \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle\) can be replaced by that of \(\langle \langle \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle\) described by the virtual Hamiltonian [Eq. (3)] in which the terms with \(i = j = \ell\) are retained. This is because the difference arising from the inclusion of this term gives only the effect of the order of \(O(1/N_l)\) which is negligible in the bulk limit. This contribution gives the same \(T\) dependence as the resistivity of lattice system, \(\rho_{\text{lattice}}(T) \propto T\), except the factor \(c_{\text{imp}}\) arising from the scaling of the hybridization, from \(V\) to \(\sqrt{c_{\text{imp}}}V\), which is consistent with the physical picture that the conduction electrons are scattered by collective VF of f electrons with concentration \(c_{\text{imp}}\). This \(T\) dependence is weaker than that arising from \(\langle \langle \tilde{\Sigma}_c(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}}) \rangle \rangle\) in Eq. (7) as discussed in the next section, so that it can be neglected in the low \(T\) limit.

5. Resistivity \(\rho_{\text{imp}}\) from Incoherent Scattering by Localized f Electrons

In this section, we derive the \(T\) dependence in the resistivity \(\rho_{\text{imp}}(T)\) which is given with the use of the retarded function \(-c_{\text{imp}} \text{Im}(\langle \langle \tilde{\Sigma}_c^R(\mathbf{r}_i, \mathbf{r}_j, \epsilon_{\text{f}} + i\delta) \rangle \rangle)\) in Eq. (7). Then, the leading \(T\) dependence of \(\rho_{\text{imp}}(T)\) is proportional to \(\text{Im}(\langle \langle \tilde{\Sigma}_c^R(\mathbf{r}_i, \mathbf{r}_j, i\delta) \rangle \rangle)\) as

\[
\rho_{\text{imp}}(T) \propto -c_{\text{imp}} \text{Im}(\langle \langle \tilde{\Sigma}_c^R(\mathbf{r}_i, \mathbf{r}_j, i\delta) \rangle \rangle), \tag{9}
\]
because \( \text{Im}\langle \Sigma^R_\ell(\mathbf{r}_\ell, \mathbf{r}_f, \varepsilon + i\delta) \rangle \) is proportional to \((\text{const.} - \varepsilon^2)\) in the region, \( \varepsilon \sim 0 \), so that it gives only a conventional \( T \) dependence as \((\text{const.} - T^2)\) of the conventional local Fermi liquid. The Feynman diagram for \( \langle \Sigma^R_\ell(\mathbf{r}_\ell, \mathbf{r}_f, \varepsilon + i\delta) \rangle \) is given by Fig. 2(a), leading to

\[
\langle \Sigma^R_\ell(\mathbf{r}_\ell, \mathbf{r}_f, \varepsilon + i\delta) \rangle = \frac{V^2}{N_\ell} \sum_\mathbf{p} G^R_\ell(\mathbf{p}, \varepsilon + i\delta) \equiv V^2 \langle \Sigma^R_\ell(\varepsilon + i\delta) \rangle. \tag{10}
\]

The Green function \( \langle \langle G^R_\ell(\varepsilon + i\delta) \rangle \rangle \) in Eq. (10) is expressed as

\[
\langle \langle G^R_\ell(\varepsilon + i\delta) \rangle \rangle^{-1} = [G^R_\ell(\varepsilon + i\delta)]^{-1} - \langle \langle \Sigma^R_\ell(\varepsilon + i\delta) \rangle \rangle. \tag{11}
\]

where \( \tilde{G}_\ell(\varepsilon) \equiv \tilde{z}_\ell(\varepsilon + i\tilde{\Delta}) \) is the Green function of a localized \( f \) electron which is renormalized only by local correlation due to \( U_{gf} \). Here, \( \tilde{z}_\ell \) is the renormalization amplitude and \( \tilde{\Delta} \equiv \tilde{z}_\ell \pi V^2 N_{cF} \), with \( N_{cF} \) being the DOS of conduction electrons per electron at the Fermi level, is the broadening width of the local 4f level due to the c-f hybridization effect. Then, \( \rho_{\text{imp}}(T) \) [Eq. (9)] is given as follows:

\[
\rho_{\text{imp}}(T) \propto c_{\text{imp}} V^2 \left[ \frac{\tilde{\Delta}}{\tilde{z}_\ell} - \text{Im} \langle \langle \Sigma^R_\ell(i\delta) \rangle \rangle \right]^{-1}. \tag{12}
\]

Since \( \tilde{\Delta}/\tilde{z}_\ell = \pi V^2 N_{cF} \) is essentially \( T \) independent in the wide region \( T \ll T^* \), the \( T \) dependence of \( \rho_{\text{imp}}(T) \) in the region \( T \ll T^*_F \) arises from that of \( \text{Im} \langle \langle \Sigma^R_\ell(i\delta) \rangle \rangle \).

Fig. 2. The Feynman diagrams for (a) \( \langle \Sigma^R_\ell(\mathbf{r}_\ell, \mathbf{r}_f, i\varepsilon_n) \rangle \) and (b) the selfenergy due to the VF. The solid and dashed lines represent the Green functions of \( f \) renormalized only by local correlations and conduction electrons at the Ce site \( \mathbf{r}_\ell \), respectively. The squares represent the c-f hybridization at the site \( \mathbf{r}_f \), so that the summation over \( \mathbf{p} \) and \( \mathbf{p}' \) should be taken. The wavy line represents the VF propagator \( \chi_v \) [Eq. (14)] and \( g \) the coupling constant between the localized 4f electron and the VF mode, respectively.

The selfenergy \( \langle \Sigma^R_\ell(\varepsilon + i\delta) \rangle \) of the localized \( \tilde{f} \) electron (averaged over the Ce sites) in Eq. (11) represents the contribution from the effect of the local VF. Its explicit form in Matsubara frequency representation is given by the Feynman diagram shown in Fig. 2(b), and its analytic form is given by

\[
\langle \Sigma_\ell(i\varepsilon_n) \rangle = g^2 T \sum_{\omega_m} \frac{1}{N_\ell} \sum_\mathbf{q} \chi_v(\mathbf{q}, i\omega_m) \tilde{G}_\ell(i\varepsilon_n - i\omega_m), \tag{13}
\]

where \( \chi_v \) is the VF propagator described by the virtual Hamiltonian [Eq. (3)], and \( g \) is the coupling constant between the localized 4f electron and the VF mode.

As discussed in Ref. [11] and argued in the Introduction, the explicit form of \( \chi_v(\mathbf{q}, i\omega_m) \) in Eq. (13) is given in parallel to the lattice system as

\[
\chi_v(\mathbf{q}, i\omega_m) = \frac{N^*_{cF}}{\eta + Aq^2 + C|\omega_m|}, \tag{14}
\]
where $N_F^c$ is the DOS of quasiparticles renormalized by the on-site correlation effect arising from $U_{\text{eff}}$ in Eq. (1), the dimensionless parameter $\eta$ ($< 1$ in general) is the measure of the distance from the criticality ($\eta = 0$), and the coefficients $A$ and $C$ parameterize the extent of QCVF in space and time, respectively. As discussed in the end of Sect. 3, the condition for the QCVT in the present impurities system is essentially given by that in the lattice system because of the \textit{locality} of QCVT, which is manifested itself as an extremely small coefficient $A$ in $\chi_{\nu}(\mathbf{q}, i\omega_m)$ [Eq. (14)] leading to extremely small energy scale $VF$, $T_0 \equiv Aq_c^2/2\pi C$ ($q_c$ being the wave vector cutoff), as shown in Ref. [11].

With the use of the fact that the relations, $\Gamma_q \ll \Lambda$ and $T \ll \Lambda$, hold near the quantum criticality, $\text{Im} \Sigma^R_i(i\omega)$ derived from Eq. (13) is approximated as

$$\text{Im} \Sigma^R_i(i\omega) \approx -\frac{2\pi}{C\Delta N_F^c} \sum_q \left[ \Phi\left( \frac{\Gamma_q}{2\pi T} \right) - \Phi\left( \frac{\Gamma_q}{\pi T} \right) \right],$$

where $\Phi(\gamma) \equiv \log \gamma - (1/2\gamma) - \psi(\gamma)$, with $\psi(\gamma)$ being the digamma function, and $\Gamma_q \equiv (\eta + Aq^2)/C$. In the wide $T$ region, $T_0 < T < T_F^c$, with $T_F^c$ being the Fermi temperature of quasiparticles, the $T$ dependence of $\text{Im} \Sigma^R_i(i\omega)$ is given as $\text{Im} \Sigma^R_i(i\omega) \propto -(T/\eta)^2 \propto -T^{2(1-\zeta)}$. where we have used the definition of scaling exponent of valence susceptibility $\eta \propto T^\zeta$. Then, the $T$ dependence in the resistivity [Eq. (12)] is given as

$$\rho_{\text{imp}}(T) \propto \text{const} - T^{2(1-\zeta)}.$$  

This $T$ dependence is to be compared with $\rho(T) \propto (\text{const} - T^n)$ with $n \approx 0.75$ reported in Ref. 12. According to the theoretical prediction, [11] the exponent $\zeta$ is weakly $T$ dependent and takes $0.5 \leq \zeta \leq 0.7$. The exponent $n \approx 0.75$ at $x = 0.05$ is reproduced by taking $\zeta = 0.63$. This value of the critical exponent $\zeta$ is actually consistent with the measured criticality in the magnetic susceptibility $\chi \sim T^{-\zeta}$ (with $\zeta \approx 0.67$) at $x = 0.02$ [12], considering some uncertainties with inevitable measurement errors and fitting to obtain the critical exponents.

6. Conclusion

On the basis of a new formulation treating the effect of a diluted system of Ce based heavy fermion metals with QCVF, we have succeeded in explaining the anomalous non-Fermi liquid $T$ dependence of the resistivity, $\rho(T) \propto (\text{const} - T^n)$ with $n \approx 0.75$, observed in Ce$_x$La$_{1-x}$Cu$_5$Au$_{0.38}$ (x = 0.02 - 0.10) in the low temperature region $T < 0.5$ K [12].

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