Evaluations of freeze-out parameters from $\frac{dE_T}{d\eta} / \frac{dN_{ch}}{d\eta}$ ratio measured at RHIC and SPS

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In the presented paper curves of constant $\varepsilon_T/n_{\text{charged}}$ are calculated in $T-\mu_B$ plane, in the framework of a single-freeze-out thermal hadron gas model. The ratio is a theoretical equivalent of $\frac{dE_T}{d\eta} |_{\eta=0} / \frac{dN_{ch}}{d\eta} |_{\eta=0}$ measured at RHIC and SPS. In both $\varepsilon_T$ and $n_{\text{charged}}$ decays of hadron resonances are taken into account. The freeze-out temperature $T_{f.o.} = 156^{+14}_{-11}$ MeV is obtained for RHIC, whereas $T_{f.o.} = 134 - 140$ MeV is evaluated for SPS.

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In this letter, the allowed region of freeze-out parameters in $T-\mu_B$ plane is established on the basis of $\frac{dE_T}{d\eta} |_{\eta=0} / \frac{dN_{ch}}{d\eta} |_{\eta=0}$ ratio measured for Au-Au collisions at $\sqrt{s_{NN}} = 130$ GeV (RHIC) [1] and Pb-Pb collisions at $\sqrt{s_{NN}} = 17.2$ GeV (SPS) [2]. Surprisingly, the ratio is roughly constant as a function of the impact parameter (besides the narrow region of peripheral collisions) and equals about 0.8 GeV in both experiments.

The concept of the presented paper is similar to the idea of [3], but here the information about freeze-out conditions is extracted straightforward from the experimentally measured quantity. In [3], a curve of the freeze-out in $T-\mu_B$ plane is obtained from the observation that the average energy per hadron, calculated in the framework of a thermal model, equals 1 GeV at the chemical freeze-outs determined for SPS, AGS and SIS. In the presented paper, the thermal model with single freeze-out is used [4–7]. The model has been applied successfully to reproduce ratios and $p_T$ spectra of particles observed at RHIC [4–6]. The main assumption of the model is the simultaneous occurrence of chemical and thermal freeze-outs. The new data on $K^*(892)^0$ production revealed by the STAR Collaboration [8] support strongly this assumption. Since $\frac{dE_T}{d\eta} |_{\eta=0} / \frac{dN_{ch}}{d\eta} |_{\eta=0}$ measurement has been done at midrapidity, the presented estimations of freeze-out parameters are valid for the Central Rapidity Region (CRR) of heavy-ion collisions under consideration.

Therefore, it is assumed that a noninteracting gas of stable hadrons and resonances at chemical and thermal equilibrium is present at the CRR. As a first approximation, a static fireball is considered (the incorporation of the expansion into the model will be the subject of the subsequent paper). Then the distributions of various species of primordial particles are given by usual Bose-Einstein and Fermi-Dirac formulae. Only baryon number $\mu_B$ and strangeness $\mu_S$ chemical potentials are taken into account, here. The isospin chemical potential $\mu_I$ has very low value in collision cases considered [4,9]) and therefore can be neglected. For given $T$ and $\mu_B$, $\mu_S$ is determined from the requirement that the overall strangeness of the gas equals zero. In this way, the temperature $T$ and the baryon chemical potential $\mu_B$ are the only independent parameters of the model.

At the temperature $T$ and the baryon chemical potential $\mu_B$, the transverse energy density $\varepsilon_T$ of specie $i$ could be defined as ($h = c = 1$ always)

$$\varepsilon_T = (2s_i + 1) \int_{-\infty}^{\infty} dp_z \int_{0}^{\infty} dp_T \ p_T \sqrt{m_i^2 + p_T^2} \ f_i(p; T, \mu_B),$$

(1)

where $p = \sqrt{p_T^2 + p_z^2}$ and $m_i$, $s_i$ and $f_i(p; T, \mu_B)$ are the mass, spin and the momentum distribution of the specie, respectively. At the freeze-out the thermal system ceases and there are only freely escaping particles instead of the fireball. The measured $\frac{dE_T}{d\eta} |_{\eta=0}$ is fed from two sources: (a) stable hadrons which survived until catching in a detector, (b) secondaries produced by decays and sequential decays of primordial resonances after the freeze-out. Therefore, if the contribution to the transverse energy from particles (a) is described, the distribution $f_1$ in (1) is the Bose-Einstein or Fermi-Dirac distribution at the freeze-out. But if the contribution from particles (b) is considered, the distribution $f_1$ is the spectrum of the finally detected secondary and could be obtained from elementary kinematics of a many-body decay or from the superposition of two or more such decays (for details, see [4,6]; also [10,11] could be very useful). In fact, if one considers a detected specie $i$, then $f_i$ is the sum of final spectra of $i$ resulting from a single decay (cascade) over all such decays (cascades) of resonances that at least one of the final secondaries is of the kind $i$.

Since measured $\frac{dN_{ch}}{d\eta} |_{\eta=0}$ has also its origin in the above-mentioned sources (a) and (b), to properly define the density of charged particles, decays should be also taken into account. Thus, the density of measured charged particle $j$ reads
\[ n^j = n^j_{\text{primordial}} + \sum_i \alpha(j,i) n^i_{\text{primordial}} , \]  

where \( n^j_{\text{primordial}} \) is the density of specie \( i \) at the freeze-out and \( \alpha(j,i) \) is the final number of specie \( j \) which can be received from all possible simple or sequential decays of particle \( i \). The density \( n^j_{\text{primordial}} \) is given by the usual integral of the Bose-Einstein or Fermi-Dirac distribution.

Now, the theoretical equivalent of \( \frac{dE_T}{d\eta} \bigg|_{\eta=0} / \frac{dN_{\text{ch}}}{d\eta} \bigg|_{\eta=0} \) could be defined as

\[ \frac{dE_T}{d\eta} \bigg|_{\eta=0} / \frac{dN_{\text{ch}}}{d\eta} \bigg|_{\eta=0} \equiv \frac{\epsilon_T}{n_{\text{charged}}} , \]  

where the transverse energy density \( \epsilon_T \) and the density of charged particles \( n_{\text{charged}} \) are given by

\[ \epsilon_T = \sum_{i \in A} c^i_T , \]  

\[ n_{\text{charged}} = \sum_{j \in B} n^j . \]  

Note that there are two different sets of final particles, \( A \) and \( B \) \((B \subset A)\). \( B \) denotes final charged particles and these are \( \pi^+, \pi^-, K^+, K^-, p \) and \( \bar{p} \), whereas \( A \) also includes \( \gamma, K^0_L, n \) and \( \bar{n} \) \([1]\). The formula (4) is the natural application (and generalization for a thermal system) of the transverse energy definition from \([12]\).

As the first step, the case of \( \epsilon_T \) without decays included is considered. This means that the transverse energy density is given also by (4) but with the sum over all constituents of the gas and \( f_i \) in (1) is the usual Bose-Einstein or Fermi-Dirac distribution at the freeze-out. Following \([9,13]\), the excluded volume hadron gas model is used. The foundations of the model could be found in \([14–16]\) (in the following, the formulation of \([16]\) is used). The gas consists of all mesons up to \( K^* \) and baryons up to \( \Omega^- \) with antiparticles treated as a different specie (151 species all together). A hard core radius of 0.4 fm is put for all particles \([13]\) (it has been checked that for the radius equal to 0.3 fm, the results are the same). The results of calculations are presented in Fig. 1. The solid curve represents (3) equal to 0.8 GeV, whereas dashed curves are for the maximum and minimum values of (3) taken at the edges of error bars of the last two points of the PHENIX measurement (see Fig.4b of \([1]\)).

![Graph](image_url)  

**FIG. 1.** Curves of constant \( \epsilon_T/n_{\text{charged}} \) for \( \epsilon_T/n_{\text{charged}} = 0.8 \) GeV (solid) and \( \epsilon_T/n_{\text{charged}} = 0.74, 0.88 \) GeV (dashed). Calculations have been done for the excluded volume hadron gas model with a hard core radius of 0.4 fm and decays have not been included in \( \epsilon_T \). Also estimates of freeze-out parameters from \([13]\) (square), \([4]\) (circle), \([7]\) (star) and \([9]\) (triangle) are depicted.
For comparison with previous estimates of the freeze-out conditions, those results are also depicted as separate points. They were obtained from thermal model fitting to the measured particle ratios. The square denotes $T = 175 \pm 7$ MeV and $\mu_B = 51 \pm 6$ MeV [13], the circle $T = 165 \pm 7$ MeV and $\mu_B = 41 \pm 5$ MeV [4] and both are for RHIC. The star is at $T = 164 \pm 3$ MeV and $\mu_B = 234 \pm 7$ MeV [7], the triangle at $T = 168 \pm 2.4$ MeV and $\mu_B = 266 \pm 5$ MeV [9] and both represent SPS conditions.

Since the aim of the presented paper is to calculate curves of constant ratio (3) with decays taken into account in both $\epsilon_T$ and $n_{\text{charged}}$, some simplifications are necessary. This is because the complete treatment of resonance decays in $\epsilon_T$ is complex and consuming a lot of computer working time in numerical calculations. Therefore the initial set of resonances should be as small as possible. The lifetime of at least 10 fm is chosen as the necessary condition, here. This reduces constituents of the hadron gas to 36 species. Of course, the condition is arbitrary but makes sense because: the first, most neglected resonances have the lifetime of the order of a few fm, so they could be thought of as decaying already at the pre-equilibrium stage; the second, they have masses of the order of 1 GeV or more and because of the damping factor proportional to $\exp(-m/T)$ their contribution to $\epsilon_T$ is negligible in comparison with the lighter particles in the temperature range considered here. The results of calculations of curves of constant $\epsilon_T/n_{\text{charged}}$ ratio with no decays included in the numerator and for the reduced gas case are depicted in Fig. 2. To compare with the previous case of 151 species, also the solid curve from Fig. 1 is repeated as the short-dashed one. It can be seen that the reduction in the number of species has not changed the results substantially. Actually, the temperature has increased from $T = 137.6$ MeV to $T = 141.9$ MeV (at $\mu_B = 1$ MeV), i.e. about 3% for $\epsilon_T/n_{\text{charged}} = 0.8$ GeV.

![FIG. 2. Same as Fig. 1 but for the gas consisting of 36 species only. For comparison, the solid curve from Fig. 1 is also repeated (short-dashed here).](image)

It has been also checked that the excluded volume corrections do not change any of the presented results. The curves of constant $\epsilon_T/n_{\text{charged}}$ are exactly the same for the excluded volume hadron gas model and for the corresponding point-like one. There are two reasons for that: the first, the volume corrections placed in denominators of expressions for various densities cancel each other in a ratio; the second, the eigenvolume of a hadron and the pressure in the region of $T$ and $\mu_B$ considered are so small that their product correction to the chemical potential is negligible there. Therefore, the point-like non-interacting hadron gas with 36 species will be used in final calculations.

As it has been already mentioned, the main difficulty in complete treatment of decays in numerical evaluation of $\epsilon_T$ is their complexity. Therefore some further simplifications should be done. First of all, some decays and cascades are neglected: (i) four-body decays, (ii) superpositions of two three-body decays, (iii) superpositions of two three-body and one two-body decays, (iv) superpositions of four two-body decays, (v) some decays of heavy resonances with very small branching ratios. Their maximal contribution to $\epsilon_T$ has been evaluated at 1.8%. Also some additional numerical simplifications have been done and they cause that actual $\epsilon_T$ could differ 0.5% at most from the calculated one. Thus, the real $\epsilon_T$ for the 36-specie hadron gas could be 2.3% higher maximally than its evaluation. Finally, the results of numerical calculations of curves of constant $\epsilon_T/n_{\text{charged}}$ with decays taken into account are presented in Fig. 3. The solid and dashed curves have the same meaning as in Figs. 1-2. The short-dashed curve represents the above-mentioned error, i.e. it has been obtained by simple replacement of evaluated $\epsilon_T$ with the value $1.023 \cdot \epsilon_T$. This causes the decrease of the temperature from 156.4
MeV to 153.3 MeV (at $\mu_B = 1$ MeV), i.e. about 2%. But remember that the reduction in the number of species has caused the *increase* of the temperature by about 3%. So, those two effects should cancel each other and the solid and dashed curves of Fig. 3 reflect the realistic physical conditions of the freeze-out in the RHIC and SPS range.

![Graph](image-url)

**FIG. 3.** Curves of constant $\epsilon_T/n_{\text{charged}}$ for $\epsilon_T/n_{\text{charged}} = 0.8$ GeV (solid) and $\epsilon_T/n_{\text{charged}} = 0.74$, 0.88 GeV (dashed). Calculations have been done for the point-like 36-species hadron gas model with decays included in evaluations of $\epsilon_T$. The short-dashed curve represents the error resulting from the simplifications done (see the text for more details). Also estimates of freeze-out parameters from [13] (square), [4] (circle), [7] (star) and [9] (triangle) are depicted.

From the solid curve of Fig. 3 the freeze-out temperature could be found at $T_{f.o.} \approx 156$ MeV for RHIC and $T_{f.o.} \approx 134 - 140$ MeV for SPS, if one puts the corresponding baryon chemical potential values at estimates of [4,7,9,13]. Similarly, the allowed range of the freeze-out temperature could be established from dashed curves of Fig. 3 and one has $T_{f.o.} = 145 - 170$ MeV for RHIC and $T_{f.o.} = 125 - 153$ MeV for SPS. Note that for RHIC the good agreement with the prediction of [4] has been obtained, also the estimate from [13] agrees qualitatively. Unfortunately, much worse agreement with the previous predictions for SPS [7,9] has been reached. Nevertheless, these points could be just on the edge of the error bar of SPS measurement, because SPS data have an additional $\pm 20\%$ overall systematic error which is not shown in Fig.4b of [1] (and is not shown in Fig. 3, neither). If one takes this additional error into account, then the maximal possible value of $\frac{dE_T}{d\eta \mid \eta=0} / \frac{dN_{\text{ch}}}{d\eta \mid \eta=0}$ is about 1 GeV for SPS [2]. The estimate of [7] gives $\epsilon_T/n_{\text{charged}} = 0.95$ GeV (within errors the minimal possible value is 0.93 GeV), for the case of [9] $\epsilon_T/n_{\text{charged}} = 1$ GeV (within errors the minimal possible value is 0.99 GeV). Thus, the SPS data for $\frac{dE_T}{d\eta \mid \eta=0} / \frac{dN_{\text{ch}}}{d\eta \mid \eta=0}$ do not contradict the thermal model predictions done in [7,9], at least.

In conclusion, the region of the freeze-out parameters for RHIC and SPS heavy-ion collisions has been established on the basis of $\frac{dE_T}{d\eta \mid \eta=0} / \frac{dN_{\text{ch}}}{d\eta \mid \eta=0}$ measurement [1,2]. The point-like non-interacting hadron gas model with 36 species has been used in final calculations. Decays and sequential decays of constituents of the gas have been taken into account there. The good agreement with the previous estimates of freeze-out conditions at RHIC [4,13] obtained from the analysis of measured particle ratios has been found. Also predictions for SPS [7,9] can be accepted, but with much worse accuracy. However, it should be stressed that actually not the transverse masses are measured but energies times $\sin \theta$ ($\theta$ is the polar angel). Also RHIC is the opposite beam experiment whereas SPS is the fixed target one, which should be taken into account in theoretical modelling. Both effects could lower the estimated ratio substantially. The considerations of the more realistic case will be the subject of the next paper.

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