Optical Nonreciprocity in Asymmetric Optomechanical Couplers

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We propose an all-optical integrated nonreciprocal device on the optomechanical platform with a large nonreciprocal bandwidth and low operating power. The device is based on an asymmetric silicon coupler consisting of two branches. One of them is a conventional strip waveguide fixed on the substrate, and the other is a freestanding nanostring suspended above a groove in the substrate. When light is launched into the coupler, the optical gradient force between the freestanding nanostring and the underlying substrate leads to the deflection of the nanostring, and finally results in destruction of the initial phase-matching condition between the two branches. The suspended branch would achieve distinct deflections when light is incident from different ports. The simulation results show a nonreciprocal bandwidth of 13.1 nm with operating power of 390 μW. With the advantages of simple structure, low power consumption and large operating bandwidth, our work provides a promising solution for on-chip passive nonreciprocal device.

Silicon photonics is deemed as a promising candidate meeting the urgent requirements for ultra-low power consumption, ultra-high speed computing and ultra-high density communication. It provides a competitive complementary metal-oxide semiconductor (CMOS) compatible platform with ultra-compact scale and low fabricating cost. In the past decade, various components have been exploited on the silicon-on-insulator (SOI) platform, such as optical filter, switch, modulator, isolator and polarization splitter. However, for the applications in nonlinear optics, silicon seems unlike to completely fulfill this role since its Kerr nonlinear refractive index $n_2$ is about only $6 \times 10^{-19}$ m$^2$ W$^{-1}$. Considering that Kerr nonlinearity is an intrinsic material property, to achieve a certain refractive index variation $\Delta n = n_2 I$, we often have to increase optical intensity $I$. High intensity would lead to some undesired results, such as high power consumption and additional insertion loss due to nonlinear absorption. To enhance the nonlinear effects in the silicon waveguide, schemes involved with thermo-optic effect and slow light effect are employed as alternative solutions. In addition, another extremely strong nonlinearity induced by optomechanical effect gets increasingly more attention in recent years.

In 2005, Povinelli et al. gives a theoretical investigation of the optical gradient force between parallel waveguides. Since then, many optomechanical structures are proposed and demonstrated, such as nanowaveguide, microcavity and photonic crystal cavity. The field of optomechanics has already been expanded to the single-photon level. Various applications are also exploited, such as frequency comb, Q-factor tunable microring resonator, optical switch and optical detector. The suspended waveguide deforms owing to optical gradient force and accordingly its effective index varies, which is called mechanical Kerr effect. Mechanical Kerr effect could be up to 7 orders of magnitude larger than the conventional Kerr effect of materials.

Among the photonic applications on SOI, optical nonreciprocal devices take a significant role. Optical nonreciprocity refers that light passes in one direction but gets blocked in the opposite direction. Generally, to realize optical nonreciprocity, Lorentz reciprocity must be broken. Various methods are reported, such as those relying on magneto-optical effect and a time-dependent refractive index. In addition, all-optical nonreciprocal approaches based on the Kerr nonlinearity and thermo-optic effect of materials have already been demonstrated. Yet they are either with high operating power, e.g. 3.1 W, or with a very narrow 10-dB nonreciprocal transmission bandwidth (NTB) and high insertion loss, e.g. 50 pm with 12 dB insertion loss. There are also several schemes based on mechanical Kerr effect in optomechanical devices. In 2009, Manipatruni et al. demonstrated a delicate nonreciprocal device based on a Fabry-Perot cavity. In 2012, Hafezi et al. proposed an intriguing theoretical scheme of nonreciprocal device consisting of a microtoroidal cavity optomechanical system. Their bandwidths are 250 pm and a few MHz, respectively, which are still too narrow for practical applications.
In this paper, we propose a novel nonreciprocal structure based on an asymmetric optomechanical coupler. One of the coupler branches is a suspended nanostring, and the other is fixed on the substrate. When light passes through the suspended branch, the waveguide bends to the substrate and exhibits mechanical Kerr nonlinearity. Since the nanostring deflects at different levels according to routings of light, it manifests optical nonreciprocity. The device possesses a large 10-dB NTB of 13.1 nm with low input power of 390 μW.

Results

Principle of the optomechanical nonreciprocal device. Figure 1 shows the schematic illustration of the nonreciprocal device based on an asymmetric optomechanical coupler. The coupler comprises two branches. Branch 1 is a suspended double-clamped nanostring above a groove in the substrate. The groove is often obtained by etching the substrate with hydrofluoric acid. Branch 2 is fixed on the silicon dioxide substrate. When the incident power is very low, such as P = 1 μW, the device operates in the linear regime and the two branches satisfy the phase-matching condition. Thus the coupler shows reciprocity property. As a light beam exceeding the threshold power is launched, Branch 1 bends to the substrate and its effective index varies. Thereby the two branches no longer satisfy the initial phase-matching condition. Because the freestanding branch deforms at distinct levels according to the input ports, the device exhibits nonreciprocity. On the one hand, light incident from Port 4 gets transferred and outputs from Port 1 with very low insertion loss, and this is defined as the forward direction of light. On the other hand, light in the backward routing from Port 1 to Port 4 gets blocked.

Device structure, mode distribution and optical gradient force. As depicted in Figure 2(a), the cross section mode distribution at wavelength λ0 = 1550 nm is acquired with COMSOL Multiphysics, a commercial software based on finite element modeling method. Considering that Branch 1 is surrounded by air, it is slightly wider than Branch 2 to satisfy the phase-matching condition. The widths of the two branches are W1 = 500 nm and W2 = 455 nm, respectively. The heights of the branches are both H = 110 nm. The separation between the two waveguide S = 730 nm. The initial gap between the suspended waveguide and the substrate G0 = 100 nm. The device parameters have been demonstrated. The length of the coupling region L is set to the half-beat length L0 = π/(2λ0), where λ0 is the coupling coefficient. For a linear coupler, at the half-beat length L0 light incident from one branch would make a complete coupling to the other branch. Here in our coupler, the coupling coefficient κ = 1.468 × 104 m−1, and the corresponding length of coupling region L = L0 = 107 μm.

As power ascends, the optical gradient force between Branch 1 and the underlying substrate causes that the suspended nanostring starts to deform. The optical gradient force of unit length and unit power f could be described as the following:

\[ f = \frac{F}{LP} = \frac{1}{c} \frac{\partial n_{ef}}{\partial d} \]  (1)

where F and P represent the force and the optical power in the whole waveguide, respectively. Here the variable d stands for the actual separation between the waveguide and the substrate when light is incident. In its initial non-deflection state, d = G0 = 100 nm. As indicated by Eq. (1), there is a positive correlation between the derivative of effective index and the optical force.

As shown in Figure 2(b), the effective index increases as the gap reduces. When the suspended branch gets closer to the substrate, in other word the separation d is smaller, there will be more proportion of optical field in the substrate and thus the effective index becomes higher. Furthermore, the effective index varies more rapidly when d is smaller. As a result, the corresponding optical gradient force gets larger when Branch 1 gets nearer to the substrate, where the minus sign represents an attractive force in the –X direction. As the interval between them becomes large enough, the force falls to zero. Here for the initial state, d = G0 = 100 nm, the optical gradient force f = −1.67 pN/μm⋅mW−1. Indeed there also exists a horizontal optical gradient force between the two coupler branches. Nonetheless as indicated by Eq. (1) the optical force between two media decays almost exponentially as their separation increases. Obviously the horizontal gap S is much larger than the vertical gap G0, S >> G0. As a consequence, the horizontal optical force between Branch 1 and Branch 2 is far smaller than the vertical optical force between Branch 1 and the substrate, and thereby the effective index variation resulting from horizontal displacement is also far less than the vertical one. So for our device the horizontal gradient force could be neglected.

Optical field distribution and the deflection along the Z direction. The light propagation properties are usually described with nonlinear coupler mode equations:

\[ \frac{\partial A_1}{\partial z} = -\frac{1}{2} \alpha_1 A_1 + ik(z)A_2 + i\delta(z)A_1 + i\gamma|A_1|^2A_1 - iF_1 A_1 \]  (2)

\[ \frac{\partial A_2}{\partial z} = -\frac{1}{2} \alpha_2 A_2 + ik(z)A_1 - i\delta(z)A_2 + i\gamma|A_2|^2A_2 - iF_2 A_2 \]  (3)

where A is the slowly varying complex amplitude, and F is the term of free carrier dispersion (FCD). The subscripts 1 and 2 denote Branch 1 and Branch 2, respectively. The loss α contains the linear loss αlinear, the loss caused by two photon absorption (TPA) αTPA, and the loss caused by free carrier absorption (FCA) αFCA. Y and δ represent nonlinear coefficient and detuning, separately. The coupling coefficient κ(z) and detuning δ(z) vary as functions of Z due to the displacement distribution along the Z direction caused by the optical gradient force.

The deflection of the freestanding nanostring is determined by Euler Bernoulli beam theory:

\[ \frac{d^4u}{dz^4} = \frac{12}{EW^3} \frac{F}{LP} \frac{P}{a} \]  (4)

where u, usually a negative value, is the deflection of the nanostring, E = 131 × 10^10 Pa is the Young’s modulus of silicon, and a = W1 × H is the cross-section area of Branch 1. Here in our device, d = G0 + u = 100 [nm] + u. Substituting Eq. (1) into Eq. (4), we obtain the following equation:

Figure 1 | Schematic illustration of the asymmetric optomechanical coupler. Branch 1 is a freestanding silicon waveguide in close proximity to the underlying silicon dioxide substrate. Branch 2 is a conventional stripe waveguide fixed on the substrate. The backward transmission light is incident from Port 1, and emits from Port 4. The forward transmission route is just opposite.
nanostring. The mechanical Kerr coefficient remains in the linear regime, and shows nonlinear switching character. As mentioned, the detuning $\delta$ between the two branches due to the nanostring deflection is power dependent, and therefore the transmittance of the device is also power dependent. Taking the black curve $T_{14}$ in Fig. 3(b) for instance, when the light is incident from Port 1, the transmittance at Port 4 falls as power rises, from 100% at $P \sim 1 \mu W$ down to approximate to zero at $P = 390 \mu W$. Then as the power increases continuously, some properties such as the period of coupling changes and the zero-output condition is not satisfied any more. Just like the nonlinear optical switching based on conventional Kerr nonlinear effect, the transmittance bounces off zero as power continues to rise.

Likewise, when light is incident from Port 4 on the fixed branch, there is also a power dependent switching characteristic. However, because the power proportions in the suspended waveguide are different, the deflections of the suspended branch and the effective index variations achieved are distinct. The maximum deflections of the freestanding nanostring are 15 nm and 29 nm for the forward and the backward transmission, respectively. The distinction finally leads to the difference in their switching thresholds, $\sim 400 \mu W$ for the forward transmission and $\sim 280 \mu W$ for the opposite direction. As indicated in Fig. 3(b), the black curve of the backward transmission doesn’t overlap the blue curve of the forward transmission, in other word $T_{14} \neq T_{41}$. The shadow region in Fig. 3(b), manifests a nonreciprocal transmission window.

\[
\frac{d^2u}{dz^2} = \frac{12}{E W^2} \frac{P}{L a} = \frac{12}{E W^2} \left( \frac{1}{c} \frac{dn_{\text{eff}}}{dP} \right) \frac{P}{a}
\]

The double-clamped nanostring obeys the boundary conditions of $u(0) = u'(0) = u(L) = u'(L) = 0$.

**Optical switching characteristic and nonreciprocity.** The conventional Kerr nonlinear effect of materials is defined as $n = n_0 + n_1 I$. Similarly, in the optomechanical device, the deflection of the suspended nanostring and the corresponding variation of effective index show positive correlation to the incident power. The mechanical Kerr effect could be evaluated by effective index changes at the point where the maximum deflection is achieved, $n_{\text{eff}}(u_{\text{max}}) = n_0 + \Delta n_{\text{max}}$, where $u_{\text{max}}$, usually a negative value, is the maximum deflection of the nanostring. The mechanical Kerr coefficient $Y_{\text{om}}$ and the mechanical Kerr index $n_{\text{om}}$ are defined as the follow: $Y_{\text{om}} = k_0 \Delta n_{\text{max}} / P$, $n_{\text{om}} = A_{\text{eff}} \Delta n_{\text{max}} / P$, where $k_0$ is the wavenumber in vacuum and $A_{\text{eff}}$ is the effective area. As indicated in Fig. 3(a), the point where the maximum deflection is achieved, the effective index variation is the largest. Here the incident power $P = 390 \mu W$, the maximum deflection $|u_{\text{max}}| = 29$ nm, and its corresponding effective index variation is 0.0174. Thus it could be derived that $n_{\text{om}} = 6.1 \times 10^{-12}$ m$^2$W$^{-1}$ and $Y_{\text{om}} = 1.8 \times 10^8$ m$^{-1}$ W$^{-1}$.

As the incident power continues to rise, the device doesn’t remain in the linear regime, and shows nonlinear switching characteristic and nonreciprocity.

\[
T_{ij} = 1 - \frac{1}{2} \delta_{ij} \left( 1 - \frac{2 |u_{\text{max}}|}{\lambda W} \right) \left( 1 - \frac{2 |u_{\text{max}}|}{\lambda W} \right)
\]

*Figure 2* | (a) Mode profile of the cross section. (b) Effective index and optical gradient force of Branch 1 versus the separation $d$ between Branch 1 and the substrate.

*Figure 3* | (a) Deflection of the freestanding Branch 1 and the corresponding effective index variation along the Z direction when the light is launched into Port 1. The negative sign of deflection represents that the branch deforms towards the substrate. (b) Transmittance ($T_{ij}$) as a function of input power, where $i$ and $j$ are the numbers of input and output ports, respectively. Black and red solid curves: light is incident from the Port 1, and outputs from Port 4 of Branch 2 and Port 2 of Branch 1, respectively. Blue and magenta curve: light is incident from the Port 4, and then outputs from Port 1 of Branch 1 and Port 3 of Branch 2, respectively. $T_{14}$ and $T_{41}$ represent the backward and the forward transmission, separately.
As portrayed in Figure 4, with incident power of 390 mW, the transmission spectra of the forward and the backward are apparently different. At the wavelength of 1550.85 nm, the forward transmission is −3.7 dB and the backward transmission is only −58.2 dB. Thus the nonreciprocal transmission ratio (NTR) reaches its peak of 54.5 dB. Since resonance structures are not employed in our scheme, there is no inherent bandwidth limit and the bandwidth is relatively large. The 10-dB NTB is 13.1 nm corresponding to a wavelength range from 1542.5 nm to 1555.6 nm. In addition, there are also no nonlinear losses owing to low operating power. As a consequence, the insertion loss is less than 3.9 dB in the nonreciprocal band.

Discussion

Figures 5(a) and 5(b) show the power distribution along the Z direction for light incident from Port 1 and Port 4, respectively. Figs. 5(c) and 5(d) display the corresponding deflections of the nano-string, respectively. As indicated by the black curves in Figs. 5(c) and 5(d), ultra-low incident power such as $P = 1 \, \mu W$ is insufficient to
different power distribution would cause different deflections. In our device the power is non-uniform along the Z direction. According to Equations (4) and (5), the same force acts on the center of the nanostring leads to a larger deflection than on the point off the center.

Thermal noises would lead to deflection of the suspended nanosting even without incident light. The thermal noise arising from thermal Brownian motion is characterized by the root-mean-square displacement amplitude $u_{\text{rms}}$:

$$u_{\text{rms}} = \sqrt{\frac{k_BT}{m_{\text{eff}}} \omega_m^2}$$

(7)

where $\omega_m$ is the fundamental mechanical frequency, $m_{\text{eff}}$ is the effective mass of the beam, $k_B$ is Boltzmann constant, and $T$ is the ambient temperature (300 K). The natural frequency of the nth mechanical mode $\omega_n$ is described by the equation:

$$\omega_n^2 = \frac{EI}{\mu I_n}$$

(8)

where $\mu$ is the mass per unit length, and $I$ is the second moment of area. For a double-clamped beam, $\beta_n$ satisfies the equation cosh($\beta_n L$) = 1 = 0. For the fundamental mechanical mode, $\beta_1 L = 4.73004$. Here the fundamental mechanical frequency of our device $\omega_1$ is 4.64 × 10^4 rad/s, and the root-mean-square displacement amplitude $u_{\text{rms}} = 1.18$ nm. Since $\omega_m \propto H/L$, to increase natural mechanical frequencies, we could reduce $L$ and increase $H$. Since $u_{\text{rms}} \propto T^{1/2}L^{3/2}/H^{1/2}W^{1/2}$, in order to suppress thermal noises, we could reduce $L$, and increase $H$ and $W$. In addition as the ambient temperature $T$ falls, thermal noises would be also lower accordingly.

**Conclusion**

In summary, we proposed an asymmetric optomechanical coupler exhibiting nonreciprocity induced by mechanical Kerr effect. The device shows a large NTB, high NTR, low insertion loss, and low operating power. This work provides a promising solution for all-optical nonreciprocal device on a silicon chip.

**Methods**

The nonlinear coupled mode equations are solved by a differential method. In the equations, the parameters $\kappa$ and $\delta$ are given as the follow:

$$\kappa = \frac{1}{2} \left( \beta_1 - \beta_2 \right) - \left( \beta_1 - \beta_2 \right)$$

(9)

$$\delta = \frac{1}{2} \left( \beta_1 - \beta_2 \right)$$

(10)

where $\beta_1$ and $\beta_2$ are the propagation constants of even supermode and odd supermode, respectively. $\beta_1$ and $\beta_2$ are the propagation constants of individual guided modes in Branch 1 and Branch 2, respectively. The propagation constants of the cross section are calculated by 2D mode analysis of COMSOL Multiphysics.

Under continuous wave conditions, the final solution is a stationary solution. The coupled mode Eqs. (2) and (3), and the mechanical Eqs. (4) and (5), are jointly solved to calculate the 2D mode analysis of COMSOL Multiphysics. As shown in Fig. 6, the simulation results fit well with the results given by the self-made MATLAB code.

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Additional information

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