Coherent Manipulation of Thermal Transport by Tunable Electron-Photon and Electron-Phonon Interaction

Federico Paolucci,1 Giuliano Timossi,1 Paolo Solinas,2 and Francesco Giazotto1

1 NEST, Instituto Nanoscienze-CNR and Scuola Normale Superiore, I-56127 Pisa, Italy
2 SPIN-CNR, Via Dodecaneso 33, I-16146 Genova, Italy

We propose a system where coherent thermal transport between two reservoirs in non-galvanic contact is modulated by independently tuning the electron-photon and the electron-phonon coupling. The scheme is based on two gate-controlled electrodes capacitively coupled through a dc-SQUID as intermediate phase-tunable resonator. Thereby the electron-photon interaction is modulated by controlling the flux threading the dc-SQUID and the impedance of the two reservoirs, while the electron-phonon coupling is tuned by controlling the charge carrier concentration in the electrodes.

To quantitatively evaluate the behavior of the system we propose to exploit graphene reservoirs. In this case, the scheme can work at temperatures reaching 1 K, with unprecedented temperature modulations as large as 245 mK, transmittance up to 99% and energy conversion efficiency up to 50%. Finally, the accuracy of heat transport control allows to use this system as an experimental tool to determine the electron-photon coupling in two dimensional electronic systems (2DES).

Control and manipulation of thermal currents in solid-state structures is of particular interest especially at the nanoscale where heat strongly affects the physical properties of the systems. In this direction, coherent caloritronics [1–3], which takes advance of phase-coherent mastering the heat current in solid-state nanostructures, represents a crucial breakthrough in several fields of science such as quantum computing [4], ultrasensitive radiation detectors [5] and electron cooling [6, 7]. Although phase coherence plays a fundamental role in the functionalities of several nano-electronic devices, the impact of coherence in caloritronics is far to be completely understood. Despite it is smaller than galvanic thermal transport [8], electron-photon mediated heat transfer [9, 10] provides the possibility of contactless heating or cooling bodies in non-galvanic contact allowing to investigate low energy physics in small quantum devices, and put the basis of novel-concept logic elements. These approaches assume that at low temperatures the phonon modes become effectively frozen [11, 12] and the energy losses through electron-phonon coupling are minimized. Yet, the possibility of tuning the electron-phonon coupling in such systems can pave the way to new device concepts, unexplored physical effects and novel quantum-state engineering.

In this Letter, we propose a method to fully control the coherent heat transport between two bodies in non-galvanic contact by modulating their electron-photon interaction and their electron-phonon coupling. The system we study consists of a source $S$ and a drain $D$ electron reservoirs (see Fig. 1a) interacting thorough an intermediate coupling circuit (CC). The lattice phonons residing in every element of the structure are assumed to be thermalized with the substrate phonons at bath temperature $T_{bath}$, and the heat losses are exclusively due to the electron-phonon interaction. A power $P_n$ injected into the source electrode originates a heat flow described by the following system of energy balance equations:

$$\begin{cases} P_n = P_{e-\text{ph}S} + P_{R_S-R_D} \\ P_{R_S-R_D} = P_{e-\text{ph}D}. \end{cases}$$ (1)

Above $P_{e-\text{ph}S}$ and $P_{e-\text{ph}D}$ are the power losses due to electron-phonon coupling in $S$ and $D$, respectively, and $P_{R_S-R_D}$ is the electron-phonon mediated power transfer between the two reservoirs in non-galvanic contact. As a consequence, the temperatures of the source $T_S$ and drain $T_D$ electrodes differ from the phonon temperature $T_{bath}$ and follow $T_S \geq T_D \geq T_{bath}$ (in the case of positive $P_n$).

Here, we focus initially our attention on $P_{R_S-R_D}$. The heat transport between two remote bodies mediated by the electron-photon interaction has been studied within a nonequilibrium Green’s function formalism [9] or a circuital approach [10]. For simplicity, our analysis is based on the circuital approach, since the two methods give equivalent results [10]. The thermal current between $S$ and $D$ can be calculated as the difference between the power emitted from the drain and the source reservoirs, and it can be expressed as:

$$P_{R_S-R_D} = \int_0^\infty \frac{d\omega}{2\pi} \omega \tau(\omega) \left[ N_D(\omega) - N_S(\omega) \right],$$ (2)

where $\tau(\omega)$ is the photonic transfer function of the system, while $N_S$ and $N_D$ are the Bose-Einstein distributions of the source and drain electrodes respectively. The frequency-dependent transfer function is defined as [10]:

$$\tau(\omega) = \frac{4\Re[Z_{\text{TOT}}(\omega)]\Re[Z_D(\omega)]}{|Z_{\text{TOT}}(\omega)|^2} = \frac{4R_S R_D}{(R_S + Z_{\text{CC}}(\omega) + R_D)^2};$$ (3)
where $Z_{CC}(\omega)$ is the frequency-dependent impedance of the coupling circuit. In order to phase coherently modulate the electron-photon interaction, it is necessary to design an appropriate electronic circuit with a tunable $Z_{CC}$. A typical intermediate quantum circuit is represented by a flux-controlled dc-SQUID (superconducting quantum interference device) [11] [10] [12], which is modeled as a LC resonator with a magnetic flux-dependent inductance $L_{SQUID}(\Phi) = L_0/|\cos(\Phi/\Phi_0)|$ [11] [8], where $L_0 \propto 1/I_C$ is the Josephson inductance arising from the device critical current $I_C$, $\Phi$ is the flux threading the loop and $\Phi_0 = 2.067 \times 10^{-15}$ Wb is the flux quantum. Therefore, the SQUID acts as a phase-dependent thermal modulator.

The implementation that we have chosen in order to ensure phase-coherent heat modulation in non-galvanic contacts is depicted in Fig. 1b, where a dc-SQUID is capacitively-coupled with the electron reservoirs. Then the total series impedance of the coupling circuit is

$$Z_{CC} = 2Z_C + Z_{SQUID},$$

where $Z_C = 1/\omega C$ and $Z_{SQUID} = \omega L_{SQUID} + 1/\omega C_{SQUID}$ are the impedances of the capacitor and the dc-SQUID, respectively. Contrary to the high frequency narrow-band pass filter characteristic of an inductive coupling [9] [10], the broad-band pass filter behavior of capacitive coupling ensures the maximum power transfer across the device, because the term $[N_D(\omega) - N_S(\omega)]$ of Eq. 2 shows the maximum value at low photon frequency (see Fig. 1c).

From Eq. 3 it is clear that the transfer function $T(\omega)$ also depends on the values of the source and drain impedances, and it is maximized in the case of a matched circuit ($Z_S = Z_D$). Therefore, the electron-photon interaction can also be modulated by controlling the value of $R_S$ and $R_D$. We propose the use of a two dimensional electron system (2DES) as material implementing the $S$ and $D$ electrodes. By employing two gates ($V_{GS}$ and $V_{G_D}$ in Fig. 1b) it is possible to independently control the charge carrier concentration $n$ of the two reservoirs. As a consequence, the modulation of their resistance ($R_S$, $R_D \propto 1/n$) is reflected in a change of $P_{R_S-R_D}$ and thereby in thermal transport across the system.

We now turn on the analysis of $P_{e-ph}$. In the following we impose a constant $T_S$, thereby we will consider only the impact of $P_{e-ph}$ on thermal transport. Equation 1 shows that the thermal efficiency of the system strongly depends on the electron-photon coupling of the source and drain electrodes. At temperatures lower than the Debye temperature, the power dissipated to the lattice in a clean metal takes the form $P_{e-ph} = \Sigma V(T^5 - T^5_{bath})$ where $\Sigma$ is the electron-phonon coupling constant and $V$ is the volume [12]. The coupling constant is defined as $\Sigma = 12\zeta(5)\hbar^3 k_B^3 |M|^2/\pi\hbar^2 v_s^3$ where $\zeta(5) \approx 1.0369$, $M$ is the deformation potential and $v_s$ is the speed of sound in the material. In the dirty limit (diffusive regime) $P_{e-ph}$ scales with $T^4$ or $T^6$ depending on the nature of disorder [13] [14]. In conventional caloritronic devices the reservoirs are usually made of metals, and $\Sigma$ is fixed by the choice of material. As mentioned above, we propose to use gated 2DESs as source and drain electrodes. In two dimensional electronic systems the electron phonon coupling constant depends on the charge carrier concentration [15] [17]. Therefore, our system allows to in-situ modulate $\Sigma$ and, therefore, the temperature of the drain electrode $T_D$ by tuning the gate voltage applied to the reservoirs.

In order to quantify the impact of $P_{R_S-R_D}$ and $P_{e-ph}$ on the modulation of the heat transfer we suppose to exploit $S$ and $D$ electrodes made of graphene. The thermal properties of graphene have been extensively studied [18] [21], but it has never been used in coherent caloritronic systems so far. Its small volume, versatility and the possibility of producing samples of different quality make graphene the prototype candidate of tunable material for caloritronic applications. The power exchanged between electrons and phonons in graphene depends on its charge...
carrier concentration, which can be controlled through the gates, and mobility [22]. In the clean limit (i.e., implying the mean free path \( \ell \geq 1\mu m \)) it takes the form [23]:

\[
P_{e-ph_{\text{clean}}} = A \Sigma_{\text{clean}} (T_e - T_{\text{bath}}),
\]

where \( A \) is the area of the graphene sheet, \( T_e \) is the electronic temperature, \( T_{ph} \) is the phonon temperature and \( \Sigma_{\text{clean}} = (\pi^2 D^2 [E_F k_F^3]/(15\rho_M h^5 v_F^3 \ell^3)) \) is the electron-phonon coupling constant. In the latter, \( v_F = 10^6 m/s \) is the Fermi velocity, \( D \) is the deformation potential, \( E_F = h v_F \sqrt{\pi n} \) is the Fermi energy, \( \rho_M \) is the mass density and \( s = 2 \times 10^{14}m/s \) is the speed of sound in graphene. In the dirty limit (\( \ell \leq 100 \text{ nm} \)), the power losses due to electron-phonon coupling have a \( T^3 \) dependence and are expressed [24] as follows:

\[
P_{e-ph_{\text{dirty}}} = A \Sigma_{\text{dirty}} (T_e - T_{\text{bath}}),
\]

where the electron-phonon coupling constant takes the form \( \Sigma_{\text{dirty}} = (1.2 D^2 [E_F k_F^3]/(\pi^2 \rho_M h^4 v_F^3 \ell^2)) \). In order to solve the energy balance system [Eq. (1)] and quantitatively analyze the behavior of the efficiency of the system, we use the following values for the physical quantities: \( T_{\text{bath}} = 10 \text{ mK} \), \( C_{SQUID} = 29 \text{ fF} \), \( L_0 = 1 \text{ mH} \), \( R_{SQUID} = 1 \text{ k\Omega} \), \( C = 750 \text{ fF} \) and \( A = 12.5 \mu m^2 \). Since the transfer function is maximized for a matched circuit, i.e., \( R_S = R_D \), in the following we assume \( n = n_S = n_D \) where \( n_S \) and \( n_D \) are the charge concentrations in \( S \) and \( D \), respectively. Figure 2 shows \( P_{R_S-R_D} \) (dark lines) and \( P_{e-ph} \) (light lines) at \( n = 1 \times 10^{12} \text{ cm}^{-2} \) for different values of \( T_S \). Differently from other 2DES [16, 17], in graphene the electron-phonon coupling constant \( \Sigma \propto \sqrt{n} \). As a consequence, the power adsorbed by phonons increases monotonically with source temperature and carrier concentration both in the clean and dirty limit (see Fig. 2). Notably, \( P_{e-ph} \) is always of the order of fW for our system. In metals, the electron phonon coupling is typically stronger and thermal losses are larger. For instance, at \( T = 200 \text{ mK} \) in our system \( P_{e-ph} \leq 1 \text{ fW} \), while AlMn or Cu thin film of comparable dimensions shows a dissipated power of \( \sim 30 \text{ fW} \) or \( \sim 100 \text{ fW} \) [25], respectively. On the other side, the photonic transmitted power \( P_{R_S-R_D} \) lowers with \( n \) with a steeper rate at higher values of \( T_S \) [\( \mathcal{T} \propto 1/n^2 \)]. The difference in the transmitted power between the clean and dirty limit predominantly arises from the difference in mobilities considered in the two cases (\( \mu_{\text{clean}} = 10^5 \text{ cm}^2/\text{Vs} \) and \( \mu_{\text{dirty}} = 10^4 \text{ cm}^2/\text{Vs} \)).

Since \( P_{R_S-R_D}(n) \) decreases for increasing \( n \) (see Fig. 2), \( T_{\text{MAX}} = T_D(\Phi = 0) \) decreases for increasing \( n \) as shown in Fig. 3a in the case of \( T_S = 500 \text{ mK} \). Since the heat current is not physically measurable, we evaluate the phase coherent modulation of thermal transport by monitoring \( T_D \) as a function of the magnetic flux. The temperature modulation \( \Delta T_D = T_D(\Phi = 0) - T_D(\Phi = \Phi_0/2) \) increases with the charge carrier concentration of the electrodes for a fixed value of the source temperature both in the clean and the dirty limits.

In Fig. 3b we study \( T_D \) as a function of \( \Phi \) for different source temperatures at \( n = 1 \times 10^{12} \text{ cm}^{-2} \). The modulation is quite small for low temperatures (see lowest
In order to discuss in detail the behavior of $\eta$ with $P_{R_S-R_D}$ and $P_{e-ph}$, we show some curves at fixed values of $T_S$ (Fig. 4-b) and $n$ (Fig. 4-c). At constant source temperature the efficiency increases with the carrier concentration, because the electron-phonon coupling decreases more steeply near $\Phi = \Phi_0/2$ than near $\Phi = 0$. In general, at high source temperature the device efficiency decreases, because the electron-phonon coupling is bigger. In the clean limit at high $T_S$, the difference between the maximum (at $\Phi = 0$) and minimum (at $\Phi = \Phi_0/2$) value of $P_{R_S-R_D}$ starts to decrease and $P_{e-ph}$ rises. As a consequence, the efficiency decreases as shown by the curve at $T_S = 1$ K of the left panel of Fig. 4-b. The increase of electron-phonon coupling with temperature $[\text{Eq. (4)}$ and $\text{[5]}$] is even more evident. In fact, all the curves in Fig. 4-c show a non-monotonic behavior of the energy conversion efficiency. In the case of clean graphene all the maxima move towards low temperature by increasing carrier concentration, while for dirty graphene they shift in the opposite direction. This difference stems from the $T_4$ dependence of $P_{e-ph_{\text{clean}}}$ $[\text{see Eq. (4)}]$ and the $T_3$ of $P_{e-ph_{\text{dirty}}}$ $[\text{see Eq. (5)}]$.

Finally, we want to point out that the system we propose can be also used to experimentally determine the dependence of the electron-phonon coupling in 2D electronic system with the charge carrier concentration, the defects concentration and the temperature $[16]$. Within this approach the heat transferred to the sample under investigation can be controlled with unprecedented precision, thereby allowing to push the limit of the measurements to lower temperatures and higher accuracy.

In summary, we have studied the impact of electron-photon and electron-phonon coupling in the coherent heat transport between two bodies in non galvanic contact within a simple circuital approach. We have proposed a system consisting of a pair of gated 2DES capacitively-coupled through a dc-SQUID. This arrangement ensures both high transmissivity and large modulation of thermal transport. To prove the efficiency of the system, we have presented quantitative results for source and drain electrodes made of graphene in the clean and dirty limits. In both cases, the proposed system shows unprecedented thermal modulations (up to $\Delta T_{D_{\text{MAX}}} \sim 245$ mK), maximum transmittance $T_{\text{MAX}} \sim 0.99$ and energy conversion efficiencies reaching $\eta_{\text{MAX}} \sim 50\%$. The high modulation and control of thermal transport make this system the ideal platform for the investigation of electron-phonon coupling in 2D materials.

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* Electronic address: federico.paolucci@nano.cnr.it
† Electronic address: francesco.giazotto@sns.it

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