Sound radiated from low Mach number turbulent boundary layer flows  
(Turbulent boundary layer on a smooth plate and over a small forward facing step)

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Abstract
High-resolution numerical simulation has been performed to study aeroacoustic noise radiated from a turbulent boundary layer at freestream Mach number $Ma = 0.3$, which develops on a smooth flat plate and over a small forward-facing step. Sound waves radiated from the turbulent boundary layer on the flat plate are dominant in a very-low-frequency band and have characteristics of a linear sound source. The sound waves are observed in the far-field from the boundary propagate outside of the flow region where small hydrodynamic pressure fluctuations with a significant long-wavelength appear. The instantaneous hydrodynamic pressure fields that gradually develop or decay while moving downstream with the turbulent boundary layer are associated with the evolution of various vortical structures. The characteristics of the sound wave being a linear sound source and dominant in a very-low-frequency band are similar to those of the real high-speed train. The sound waves generated from the turbulent boundary layer over the forward-facing step with a height $(SH)$ of $SH/y^+ \approx 62$ are significantly larger in a full frequency band than those radiated from the turbulent boundary layer on the flat plate. Numerical results of the turbulent boundary layer over the forward-facing step with a height of $SH/y^+ \approx 30$ show that the sound waves are dominant in the low-frequency band, and also step-specific sound waves are generated in the high-frequency band. Further, even if the step height is $SH/y^+ \approx 7.5$, the sound waves unique to the small step are generated in the high-frequency band.

Keywords: Turbulent boundary layer, Aeroacoustic noise, Low Mach number, Forward facing step, Direct numerical simulation

1. Introduction

In recent years, noise generated from transportation such as high-speed trains and civilian aircraft has become an important problem for social environment compatibility; consequently, measures to reduce these noises are necessitated. The noise originating from trains not only causes interior noise pollution and reduces the comfort of passengers, but also seriously affects the residents living along the high-speed train line or near the airport (Yang et al., 2017). The primary noise sources radiated from the trains are the rolling sounds generated from the contact between wheels and rails, equipment noise generated by driving devices such as a primary motor fan, the sliding current noise generated by current collectors rubbing the overhead lines, and aerodynamic disturbances caused by the fluid phenomena around trains (Kitagawa, 2017).

Among these noises, aerodynamic noise is different from the noise generated by other mechanical factors and arises from the unsteady motion of a fluid such as airflow and vortex motion. The sound power of aerodynamic noise grows in proportion to sixth to eighth power of the speed (Mitumoji et al., 2014). Kitagawa (2017) has reported that aerodynamic noises are more than rolling noises and become the primary noise source for the high-speed trains. Thus, it is essential to elucidate the mechanism of aerodynamic noise to reduce the noise of high-speed trains. The cavity sound radiated from the lower part of the train (Uda et al., 2018), and the aerodynamic sound radiated from the pantographs are known as
aerodynamic noise (Kurita et al., 2011). These aerodynamic sounds have been studied for a long time. Moreover, appropriate reduction measures have also been implemented (Kitagawa et al., 2012; Shigeta et al., 2009; Kurita et al., 2011).

However, with the increasing speed of trains, “low-frequency noise” radiated from the turbulent boundary layer around trains, which has not been recognized as a target of noise in previous studies, has recently been identified as a noise source (Takami et al., 2008). “Low-frequency noise,” which is occasionally taken up as a noise problem from factories and wind-smoke facilities, refers to pressure fluctuations of approximately 1-100 Hz; this includes very low-frequency sounds with frequencies below 20 Hz. Low-frequency noise ranges from below the lower limit of the audible range to near the lower limit and generally belong to the region where the human sense threshold is high. Therefore, recognizing low-frequency noise is more complicated than recognizing high-frequency noise. However, low-frequency noise is close to the resonance frequency of buildings and fittings and may cause rattling noises, or render a feeling of oppression or tinnitus on the human body.

Takami et al. (2009) conducted an experimental study on low-frequency noise generated by high-speed trains and observed that pressure fluctuations, including low-frequency components below 100 Hz, radiated from the turbulent boundary layer of a train moving at approximately 300 km/h. When this pressure fluctuation propagates as a sound wave, its wavelength reaches 3 m or more, and a sufficient reduction effect cannot be expected even if appropriate measures on the ground side are taken, such as raising the soundproof wall. With the increasing speed of trains, the magnitude of aeroacoustic noise generated from the Shinkansen increases in a proportion that ranges from the sixth to eighth power of speed, and the frequency of the generated sound could shift to higher frequency bands. Therefore, if the train speed further increases in the future, the sound generated from the turbulent boundary layer sufficiently enters the human audible range; therefore, measures against trains that act as sources of pressure fluctuations are necessary (Uda et al., 2019).

Our study aims at providing a deep insight into the noise radiation characteristics by analyzing spatial-developing turbulent boundary layers at low Mach number of 0.3. The turbulent boundary layer developing on a smooth plate (Case A) and over a small forward-facing step (Case B) was investigated using direct numerical simulation (DNS).

2. Numerical Method
2.1 Governing equations and discretization method

In direct numerical simulations, the non-dimensional form of three-dimensional, time-dependent compressible Navier–Stokes equations; mass, momentum, and energy conservation, governing the flow of a viscous compressible ideal gas, are given as

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0,$$

(1)

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} + \frac{\partial p}{\partial x_i} = \frac{\partial \tau_{ij}}{\partial x_j},$$

(2)

$$\frac{\partial E_T}{\partial t} + \frac{\partial (E_T + p) u_j}{\partial x_j} = \frac{\partial}{\partial x_j} \left( u_i \tau_{ij} \right) - \frac{\partial q_j}{\partial x_j},$$

(3)

where $t$ is the time, $x_i$ is a Cartesian coordinate system, $\rho$ is the density, $u$ is the velocity vector, $p$ is the thermodynamic pressure, related to the density and temperature $T$ by the dimensionless form of the perfect gas law, and is the total energy which is defined as (Tokura et al., 2011)

$$E_T = \frac{p}{\gamma - 1} + \rho \frac{u_i u_i}{2},$$

(4)

The components of the viscous stress tensor $\tau_{ij}$ are
\[
\tau_{ij} = \frac{\mu}{Re_{\delta_{\text{in}}}^*} \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] - \frac{2}{3} \delta_{ij} \frac{\partial u_i}{\partial x_i},
\]

(5)

where \( Re_{\delta_{\text{in}}}^* \) is the Reynolds number based on the displacement thickness at the inlet boundary layer and the freestream velocity \( U_{\infty} \), and \( \delta_{ij} \) denotes the Kronecker delta function, which takes the value 1 if \( i = j \) and 0 otherwise.

\[
Re_{\delta_{\text{in}}}^* = \frac{U_{\infty}\delta_{\text{in}}}{v_e} = 1250
\]

(6)

At the inflow boundary, the Reynolds number \( Re_x(= U_{\infty} x / \nu) \) based on the distance from the leading edge of the plate is \( 5.28 \times 10^5 \).

The Mach number \( Ma \) is based on the mainstream velocity \( U_{\infty} \) and the speed of sound \( c_{\infty} \).

\[
Ma = \frac{U_{\infty}}{c_{\infty}} = 0.3.
\]

(7)

A power law approximates the non-dimensional, temperature-dependent dynamic viscosity as \( \mu = T^{2/3} \). By assuming a unity Prandtl number \( Pr = 0.71 \) and a constant ratio of specific heats \( \gamma = 1.4 \), the heat flux vector is given by

\[
q_i = -k \frac{\partial T}{\partial x_i}
\]

(8)

where \( T \) is the static temperature. The thermal conductivity \( k \) is proportional to the viscosity expressed by

\[
k = \frac{\mu}{(\gamma - 1)Ma^2Re_{\delta_{\text{in}}}^*Pr}.
\]

(9)

Assuming constant specific heats, we write the perfect gas equation of state as

\[
p = \frac{1}{\gamma Ma^2} \rho T.
\]

(10)

To discretize spatial derivative terms, a fifth-order dissipative compact scheme (Deng et al., 1996) for the advection terms and a sixth-order centered compact scheme (Lele 1992) for the viscous terms are employed. The Navier-Stokes equations are advanced in time using a Total Variation Diminishing Runge-Kutta method (Gottlieb et al., 1998). For the streamwise (\( x \)) and vertical (\( y \)) boundary conditions, the Navier-Stokes Characteristic Boundary Condition (Poinsot et al., 1992) is adopted. The lateral (\( z \)) boundary condition is periodic.

### 2.2 Computational domain

The computational domain is bounded by inflow and outflow boundaries, a wall boundary, an upper far flow boundary, and two boundaries in the periodic spanwise direction, as shown in Fig. 1. The sponge region (Freund, 1997) is also added to inflow and outflow boundaries to suppress the reflection of waves generated through the numerical treatment at the boundaries back into the computational domain. For the calculation of a flat plate boundary layer, we use a boundary layer thickness at the inflow boundary to avoid simulating the flow around the plate leading edge. A self-similar solution of the compressible laminar boundary layer equations obtained by a shooting method is adopted for the initial flow velocity and temperature profiles of three dimensional DNS. In order to induce laminar-to-turbulent transition, a circular region of injection by a jet from the wall is introduced near the inflow, referring to the method used by Tadokoro et al. (2018). The grid resolutions with the main parameters at the location in which reaches an equilibrium state of turbulent boundary layer are given in wall units in Table 1, where \( \Delta y_{\text{min}}^+ \) indicates the minimum spacing in the wall-
normal direction. Note that the kinematic viscosity $v_w$ at the wall and the friction velocity $u_\tau$ at the location of $x = -25\delta^{*}_{in}$ are used to calculate $y^+$ in Table 1. In the $x$ and $y$ directions, a grid stretching technique is used. The grid is compressed near the wall and the step.

Numerical simulations for the boundary layer developing on a smooth plate (Case A) and over a small forward-facing step (Case B) have been performed. The step also exists at the center of calculation area ($x = 0.0\delta^{*}_{in}$). There are four step heights, as shown in Table 2.

### Table 1 Computational grids and grid width in wall units

| $2L_x$ | $L_y$ | $L_z$ | $N_x$ | $N_y$ | $N_z$ | $\Delta x^+$ | $\Delta y^+$ | $\Delta z^+$ |
|--------|-------|-------|-------|-------|-------|-------------|-------------|-------------|
| $240\delta^{*}_{in}$ | $120\delta^{*}_{in}$ | $13\delta^{*}_{in}$ | 700 | 200 | 130 | 3.9 | 0.82 | 6.0 |

### Table 2 Step height for each Case.

| Case | Step height $(SH)$ $\delta^{*}_{in}$ | $SH$ wall scale $y^+$ |
|------|------------------------------------|------------------------|
| A    | 0.00                               | 0.0                    |
| B − 1| 1.00                               | 62                     |
| B − 2| 0.48                               | 30                     |
| B − 3| 0.25                               | 16                     |
| B − 4| 0.12                               | 7.5                    |

### 2.4 Proportional rule for aeroacoustic wave of real train

The flow Reynolds number of original Shinkansen is $10^7$ or more, but the typical flow Reynolds number $(Re_x)$ of this study is $5.28 \times 10^5$, while the Mach number of this study is the same as that of the real train. The boundary layer length scale is much different between these flows. There is no convenient way to analyze Reynolds number effects, however, the scale of time is able to be calculated by the freestream velocity and the boundary layer representative length. Therefore, to calculate the observed real frequencies of the original Shinkansen, it is convenient to use the method of converting the frequency obtained from the boundary layer scale of this study to the frequency of real train scale, which is referred to as the proportional rule (Takami et al., 2009) or a real train conversion defined as,

$$ f_c \propto \frac{U}{\delta}. \quad (11) $$

where $f_c$ is the frequency, $U$ is the train speed, and $\delta$ is the boundary layer thickness. In this study, the real train speed is $360$ km/h (Mach number: 0.3), and the boundary layer thickness of the real train is $1.5$ m (Takami et al., 2009).
3. Results and Discussion

3.1 Turbulent transition

Turbulent intensities wave analyzed to confirm whether a fully turbulent flow was generated by the tripping technique in this study for the laminar-turbulent transition in a boundary layer. Figure 2 shows the velocity fluctuations of \(u', v', \) and \(w'\) at the location \((x = -25\delta_in)\), where the Reynolds number based on the momentum thickness \(Re_\theta\) is about 630. Here, we compared the present results with the numerical simulations (Schlatter et al., 2010) at \(Re_\theta = 677\). A comparison between the present results of velocity fluctuations and DNS results of the incompressible flow by Schlatter et al. indicates that there is a remarkable agreement for all three velocity components. Figure 3 shows the average velocity at the location \((x = -25\delta_in)\) where the influence of step is small. In Fig. 3, we can confirm that the average velocity distribution in the viscous sublayer and the logarithmic region falls on the linear sublayer and the wall law. In these results, it is confirmed that fully turbulent boundary layer flows are generated upstream of the step in this computational domain.

3.2 Calculation Results in Case A

Figure 4 shows the root-mean-square (RMS) of pressure fluctuations normalized by \(u_T^2\) and the DNS result \((Re_\theta = 677)\) of Schlatter et al. (2010). The peak pressure fluctuations are larger than Schlatter’s simulation result \((Re_\theta = 677)\) and occur at nearly the same location \((y \cong 0.1\delta)\). The value of RMS of pressure fluctuations at the wall \((y = 0.0\delta)\) and at the peak \((y \cong 0.1\delta)\) are \((p_{rms})_w/u_T^2 \cong 2.25\) and \((p_{rms})_{max}/u_T^2 \cong 2.7\), respectively.

Figure 5 shows the distribution of pressure fluctuations obtained by subtracting the average pressure at each point from the instantaneous pressure field in Case A. In Fig. 5 (bottom), large pressure fluctuations are generated in the inner
layer of the turbulent boundary layer. These large pressure fluctuations are caused by the existence of the various vortex structures in the boundary layer. The instantaneous large-scale pressure fields are associated with the time evolution of the vortical structures. As shown in Fig. 5 (top), very large-scale positive and negative pressure fields above the turbulent boundary layer also are generated. The very large-scale pressure fields expand to the freestream region \( y \approx 5.0\delta \). These positive and negative pressures are hydrodynamic pressure fluctuations that gradually develop or decay while moving downstream with the turbulent boundary layer. The large-scale pressure fluctuations existing above the outer boundary layer \( (y > \delta) \) are considered to cause these very large-scale positive and negative pressure in the freestream. The pressure fluctuations propagate outside of the flow layer, where pressure fluctuations with a significant long-wavelength appear above the boundary layer. This process causes the sound observed in the far-field. The sound generation process resembles the mechanism found in the previous study of free shear flows, such as a compressible plane wake (Watanabe and Maekawa, 2004).

Figure 6 shows the spectrum analysis results of the pressure fluctuations at \( x = 0.0\delta \). The horizontal axis of the graph in Fig. 6a shows the Strouhal number \( St = f\delta_{in}/U_\infty \) and the frequency \( f_c \) converted to the real train scale. The vertical axis shows the sound pressure level (SPL). Figure 6b shows the distance attenuation from the wall of the specific frequency (Strouhal number) and the root-mean-square of pressure fluctuations \( (p_{rms}) \) in the vertical direction at \( x = 0.0\delta \). In Fig. 6a, large pressure fluctuations occur in a wide frequency range due to the vortex structure existing inside the boundary layer in the region \( y < \delta \). In the far-field \( (y > \delta) \), The pressure fluctuations around \( St = 0.01 \) are predominant, and the pressure fluctuations become smaller as the frequency becomes higher than \( St = 0.01 \). We also confirm that the large-scale pressure fluctuations occurred around \( f_c = 25 \) at the frequency-converted to the real train scale, and the result is qualitatively equivalent to the model experiment by Takami et al. (2009). According to Fig. 5b, in the far-field \( (y > \delta) \), the pressure fluctuations of the Strouhal number in the low-frequency \( (St = 0.008, 0.02) \) range and the root-mean-square of pressure fluctuations tend to attenuate slowly and coincide with the \(-0.5\) power of distance. Hence, the pressure fluctuations radiated from the turbulent boundary layer are acoustically close to the characteristics of a linear source. The pressure fluctuations of the Strouhal number in the high-frequency range \( (St = 0.30 \) and 0.90) also decrease rapidly around \( y = 1.0\delta \). The obtained attenuation rate of pressure fluctuations and the pressure fluctuations in the low-frequency region in the far-field are qualitatively consistent with the results of experiments (Takami, et al., 2009, Kikuchi, et al., 2005) and numerical calculations (Watanabe, 2008).

![Instantaneous snapshots of the pressure fluctuations in the longitudinal median plane. The color contour levels for \( p' \) are displayed from -1 Pa to +1 Pa (top) and from -10 Pa to +10 Pa (bottom).](image)

Figure 7 shows the time-series pressure fluctuations in the \( y \)-direction at \( x = 0.0\delta \). The dotted black lines in Fig. 6 show the characteristic curve of the speed of sound. Since the characteristic curve of sound velocity and the pressure
fluctuations radiated from the boundary layer have the same slope whose orientation is calculated from the sound speed in the time developing pressure, the pressure fluctuations radiated from the turbulent boundary layer are acoustic pressure fluctuations (sound waves).

In the inner layer of the turbulent boundary layer \((y < 1.0\delta)\), larger hydrodynamic fluctuations occur by various existing vortex structures. We can confirm that very large-scale positive and negative pressure hydrodynamic pressure fluctuations occur near the outer layer of the boundary layer \((y < 5.0\delta)\), and that long-wavelength sound waves radiated from the turbulent boundary layer to regions outside the boundary \((y > 5.0\delta)\).

(a) Sound pressure spectra at \(x = 0.0\delta^*_{in}\) in Case A. The horizontal axis shows the Strouhal number \(f\delta^*_{in}/U_\infty\) and the frequency \(f_c\) converted to the real train scale.

(b) The pressure fluctuations of representative frequencies inside and outside the turbulent boundary layer.

Fig. 6 Frequency characteristics of pressure fluctuations radiated from the turbulent boundary layer.

3.3 Calculation Results in Case B

Figure 8 shows the distribution of the average mean streamwise velocity, vertical velocity, and the turbulent kinetic energy related to the acoustic source in Cases B – 1, 2, and 4. In Case B – 1 (Fig. 8 a), the separation flow occurs at the front of the step \((x < 0)\) and downstream from the corner of the step \((x > 0)\). In Case B – 2 (Fig. 8 b), the separation flow occurs at the front of the step but hardly downstream from the corner of the step. In Case B – 4 (Fig. 8 c), there is a backward flow region of the mean streamwise velocity in front of the step, but the vertical velocity is almost vertically upward. In Cases B – 1 and 2, the turbulent kinetic energy has a peak near the downstream from the corner of the step.
Fig. 8  Iso-contours of mean streamwise velocity (left), vertical velocity (middle) and turbulent kinetic energy $k^* = (u'^2 + v'^2 + w'^2)/(2U_∞^2)$ (right) in Case B – 1: (a), Case B – 2: (b), and Case B – 4: (c). 12 contours levels from -0.1 to 1.0 are plotted in the visualization diagram of the mean streamwise velocity. 5 contours levels from -0.05 to 0.2 are plotted in the visualization diagram of vertical velocity. 11 contours levels from 0.0 to 0.04 are plotted in the visualization diagram of turbulent kinetic energy.

Figure 9 shows the RMS of pressure fluctuations on the wall around the step (Sherry et al., 2010). Here $SH_{case B – 1}$ shows the step height of Case B – 1 ($SH = 1.0\delta_{in}$). Figure 10 shows typical features of the forward-facing step flow, including the two recirculating regions and the reattachment. In Fig. 9a, the pressure fluctuations on the wall larger than Case A occur near the step ($x = 0.0\delta_{in}$) in all other cases. In Case B – 1, the maximal value is located downstream from the step ($x \approx 1.7\delta_{in}$), and the other Case B groups are maximum at the front of the step ($x = 0.0\delta_{in}^*$). We consider that the separation flow occurs from the step to the downstream only in Case B – 1 and that large pressure fluctuations are generated on the wall surface by the reattachment of flow ($x = 1.7\delta_{in}$). In the other Case B groups, there is almost no separation flow from the leading edge of the step to the downstream. Thus, the wall pressure fluctuations rapidly decrease after reaching the maximal value near the leading edge of the step. In the vertical velocity distribution (Figs. 7b, 7c), there is no diffraction of flow from the corner of the step to the downstream. However, there are locations where the flow is vertically downward ($\nu < 0$). Thus, we consider that its influence caused the wall pressure fluctuations to be larger than that of Case A downstream from the corner of the step. The wall pressure fluctuation also decreases as $SH$ decreases, and becomes closer to the wall pressure fluctuation distribution of the turbulent boundary layer in Case A.

In the root-mean-square of pressure fluctuations at the front of the step (Fig. 8b), the maximal value is near the corner of the step ($y \approx SH$), which is larger than Case A in any region where the step exists ($0.0\delta_{in} < y < SH$) in the Cases B – 1, 2, and 3. We consider that the reattachment of flow caused the maximal value near the corner of the step, as shown in Fig. 9 (Sherry et al., 2009). In Cases B – 1, 2, and 3, there was also the second maximal value on the wall surface ($y = 0$). We considered that the vortex structures interfered immediately below the step because the flow generated from near the step corner ($x = 0.0\delta_{in}^*$, $y \approx SH$) to directly below the step ($y = 0.0\delta_{in}$) caused the RMS of pressure fluctuations to be smaller. In the other Case B groups, the second maximal value is the same as that of Case A ($y = 0$).
fluctuations on the wall. In Case B – 4, the RMS of the pressure fluctuations at the front of step is larger than that of Case A. This is because the vortex structures in the turbulent boundary layer collide with the front of the step.

(a) distribution in the x-direction at \( y = 0.0\delta_{in}^* ( -L_x \leq x/\delta_{in}^* < 0.0 ) \), and \( y = SH (0.0 \leq x/\delta_{in}^* \leq L_x) \).

(b) distribution in the y-direction at \( x = 0.0\delta_{in}^* \).

Fig. 9  The RMS of pressure fluctuations \( p_{rms} \) on the wall for each Case.

Figure 11 shows the sound pressure [Pa] at specific frequencies of Cases B – 1 and 4 obtained by fast Fourier transform. In Case B – 1 (Figs. 11 a – 1, 2), an enlarged view around the step is also shown. In case B – 1, large sound waves are radiated from the center of the step, and we can confirm that the sound directivity at \( St = 0.0480 \) differs from that at \( St = 0.326 \). In Fig. 11 a – 1, the sound at \( St = 0.0480 \) indicates that the large sound waves are observed from the front of the step, and the discontinuity occurs at \( x \equiv -3.0\delta_{in}^* \). Near the outer layer of the boundary layer, the large acoustic wave at \( St = 0.0480 \) is generated from the turbulent boundary layer around the step. The step affects the velocity fluctuation of the turbulent boundary layer, which oscillates quasi-periodically. This mechanism causes quasi-periodic pressure fluctuations inside the turbulent boundary layer and the acoustic wave from the boundary layer. This phenomenon is not observed in Fig. 11 b – 1, therefore we can find that the sound wave from the step is greatly related to the turbulent flow around the step with a different height. The enlarged view (Fig. 11 a) shows that there is a large sound source appearing in front of the step and downstream from the step. Therefore, the large sound wave generated upstream is caused by the reattachment of flow at the step front. In Fig. 11 a – 2, the large sound wave is generated concentrically from the step. The enlarged view, where the sound source exists only at the location of separated flow downstream of the step, indicates that the large sound source is different from that in Fig. 11 a – 1. In Case B – 4, The sound waves generated from the turbulent boundary layer are dominant at relatively low-frequency \( St = 0.0430 \), and the sound waves radiated from the step to the downstream locations are hardly observed. However, at \( St = 0.326 \), though the instantaneous vertical velocity fluctuates in random manner, we confirm that sound waves propagating upstream from the center of the step are generated.

Figure 12 shows the SPL of pressure fluctuations in Case A and Case B results at \( x = -8\delta, y = 8\delta \). In Case B-1, the sound waves generated are significantly larger in a full frequency band than those in Case A. In Cases B – 2 and 3, where
the step height is less than \( SH/y^+ \approx 30 \), the SPL similar to the sound radiated from the turbulent boundary layer (Case A) is observed in the low-frequency band \( (St < 0.05) \). Thus, we consider that the sound generated from the turbulent boundary layer is dominant in the low-frequency band. We also observed the SPLs in Cases B-2 and 3 larger than in Case A in a relatively high-frequency band \( (St > 0.10) \). Note that the SPL larger than in Case A exists in the frequency range \( (0.07 < St) \) even for \( y^+ \approx 7.5 \) \( (SH = 0.12\delta_{in}) \).

Since the magnitude of the acoustic dipole term is of \( O(M^{-2}) \) compared to that of the acoustic quadrupole term, the dipole term in the flow field is dominant at low Mach numbers. In Case B – 1, figure 11 (a-2) shows the distribution of the first derivative of pressure around the forward-facing step. The first derivative of pressure \( dp/dt \) can be seen to be dominant near the step, where the first derivative of pressure on the upper surface of the step varies quasi-periodically. It is noteworthy that the similar distribution of \( dp/dt \) appears occasionally with a higher appearance frequency corresponding to \( St = 0.326 \).

Fig. 11 Sound pressure fields at various Strouhal numbers.
4. Conclusions

We directly computed the noise radiated from a turbulent boundary layer on a flat plate (Case A) and over a small forward-facing step (Case B). The pressure fluctuations radiated from the turbulent boundary layer had a long wavelength, and acoustically, had the characteristics of a linear sound source. The sound waves are observed in the far-field from the boundary layer above the location where hydrodynamic pressure fluctuations with a significant long-wavelength appear. The instantaneous hydrodynamic pressure fields are associated with the evolution of the vortical structures. For $SH/y^+ \cong 62$, separation flow occurred at the front of the step and downstream from the step, and this phenomenon became the primary sound source. In addition, the sound wave generated from the step had different directivity for each frequency. For $SH/y^+ \cong 30$, almost no separation flow was observed to occur downstream from the step. However, even if the step height is $y^+ \cong 7.5$, a step-specific sound wave was observed in the high-frequency band.

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