The center of mass energy of two colliding particles in the STU black holes

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Abstract

In this paper we consider collision of two particle near the STU black hole and calculate center of mass energy. In the case of uncharged black hole we find that the maximum energy obtained near the black hole horizon which similarly happen for charged black hole. We verify that the black hole charge may be decreased or increased the center of mass energy near the black hole horizon.

Keywords: Particle acceleration; STU black hole.

1 Introduction

In the Ref. [1] the center-of-mass (CM) energy of a Kerr black hole studied and proposed that this might lead to signals from ultra high energy collisions such as dark matter physics [2, 3]. It yields to universal property of rotating black holes which is infinite CM energy of colliding particles and investigated by several papers [4-16].

Now, aim of this paper is studying the particle acceleration mechanism of STU black hole [16-22]. The STU black hole exist in special case of $D = 5$, $\mathcal{N} = 2$ gauged supergravity theory which is dual to the $\mathcal{N} = 4$ SYM theory with finite chemical potential. In this background, generally there are three electric charges. In this paper we assume that three of them will be equal and discussed about uncharged black holes. Therefore, first we introduce STU black hole and then try to obtain CM energy of two colliding particles near the STU black hole and then discuss about consequence.

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2 STU black hole

STU black hole is described by the following solution,

\[ ds^2 = -\frac{f}{\mathcal{H}^3} dt^2 + \mathcal{H}^2 \left( \frac{dr^2}{f} + \frac{r^2}{R^2} d\Omega^2 \right), \quad (1) \]

where,

\[ f = 1 - \frac{\mu}{r^2} + \frac{r^2}{R^2} \mathcal{H}, \]

\[ \mathcal{H} = \prod_{i=1}^{3} H_i, \]

\[ H_i = 1 + \frac{q_i}{r^2}, \quad i = 1, 2, 3, \quad (2) \]

and \( R \) is the constant AdS radius which relates to the coupling constant via \( R = 1/g \), and \( r \) is the radial coordinate along the black hole. The black hole horizon specified by \( r = r_h \) which is obtained from \( f = 0 \). Also \( \mu \) is called non-extremality parameter. So, for the extremal limit one can assume \( \mu = 0 \). Moreover \( d\Omega^2 \) includes \( d\theta^2 \) and \( d\phi^2 \). The Hawking temperature of STU black hole is given by,

\[ T = \frac{r_h}{2\pi R^2} \frac{2 + \frac{1}{r_h} \sum_{i=1}^{3} q_i - \frac{1}{r_h^3} \prod_{i=1}^{3} q_i}{\sqrt{\prod_{i=1}^{3} (1 + \frac{q_i}{r_h^2})}}. \quad (3) \]

So, in the case of \( q_i = 0 \) we get,

\[ r_h = \pi R^2 T. \quad (4) \]

Here it is useful to discuss horizon structure of the metric (1). The \( f = 0 \) reduced to the following equation,

\[ r^6 + \mathcal{A} r^4 - \mathcal{B} r^2 + q_1 q_2 q_3 = 0, \quad (5) \]

where,

\[ \mathcal{A} \equiv q_1 + q_2 + q_3 + R^2, \quad (6) \]

and,

\[ \mathcal{B} \equiv \mu R^2 - q_1 q_2 - q_2 q_3 - q_1 q_3. \quad (7) \]

A possible solutions of the equation (5) is given by,

\[ r_{\pm} = \pm \left( \frac{W^2 - 2\mathcal{A} W + 4(3\mathcal{B} + \mathcal{A}^2)}{6W} \right)^{\frac{1}{2}}, \quad (8) \]

where,

\[ W^3 = -36 \mathcal{A} \mathcal{B} - 108 \prod_{i=1}^{3} q_i - 8\mathcal{A}^3 \]

\[ + 12 \sqrt{-12\mathcal{B}^3 - 3\mathcal{A}^2 \mathcal{B}^2 + 54 \mathcal{A} \mathcal{B} \prod_{i=1}^{3} q_i + 81 (\prod_{i=1}^{3} q_i)^2 + 12 \mathcal{A}^3 \prod_{i=1}^{3} q_i}. \quad (9) \]
The $r_+$ and $r_-$ denote outer and inner horizons respectively.

3 Center of mass energy

In order to obtain the CM energy we consider planar motion which yields $\dot{\theta} = 0$. Other 4-velocity of the particles are $\dot{t}$, $\dot{\phi}$ and $\dot{r}$. The first two components obtained by using the following relations,

$$E = \frac{f}{\mathcal{H}^{2/3}} \dot{t},$$  \hspace{2cm} (10)

and,

$$L = \frac{R^2}{r^2} \mathcal{H}^{2/3} \dot{\phi},$$  \hspace{2cm} (11)

where $E$ and $L$ denote the test particle energy and the angular momentum parallel to the symmetry axis per unit mass respectively. In order to obtain $\dot{r}$ we assume,

$$S = \frac{1}{2} \tau - Et + L\phi + S_r(r),$$  \hspace{2cm} (12)

where $\tau$ is proper time. Then we use,

$$\frac{\partial S}{\partial \tau} = -\frac{1}{2} g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu},$$  \hspace{2cm} (13)

and,

$$\frac{dS_r(r)}{\partial r} = g_{rr} \dot{r}.$$  \hspace{2cm} (14)

Therefore, we can summarize,

$$\dot{t} = \frac{\mathcal{H}^{2/3}}{f} E,$$

$$\dot{\phi} = \frac{R^2}{r^2} \mathcal{H}^{-1/3} L,$$

$$\dot{r} = \sqrt{\frac{f}{\mathcal{H}^{1/3}}} \left(1 + \frac{\mathcal{H}^{2/3}}{f} E^2 - \frac{R^2}{r^2} \mathcal{H}^{-1/3} L^2 \right)^{1/2}.$$  \hspace{2cm} (15)

If we take the angular momentum per unit mass $L_1$, $L_2$ and energy per unit mass $E_1$, $E_2$, respectively, also $m_0$ as the rest mass of both particles, then using the following relation,

$$E_{CM} = \sqrt{2} m_0 \sqrt{1 - g_{tt} \dot{t}_1 \dot{t}_2 - g_{rr} \dot{r}_1 \dot{r}_2 - g_{\phi\phi} \dot{\phi}_1 \dot{\phi}_2},$$  \hspace{2cm} (16)

and obtain CM energy as the following,

$$\tilde{E}_{CM} = 1 + \frac{\mathcal{H}^{2/3}}{f} E_1 E_2 - \frac{R^2}{r^2} \mathcal{H}^{-1/3} L_1 L_2 - G_1 G_2,$$  \hspace{2cm} (17)
where,

\[ G_1 = \sqrt{1 + \frac{\mathcal{H}^{2/3}}{f} E_1^2 - \frac{R^2}{r^2} \mathcal{H}^{-1/3} L_1^2}, \]

\[ G_2 = \sqrt{1 + \frac{\mathcal{H}^{2/3}}{f} E_2^2 - \frac{R^2}{r^2} \mathcal{H}^{-1/3} L_2^2}, \]

(18)

and we used the following re-scaling,

\[ \tilde{E}_{CM} \equiv \left( \frac{E_{CM}}{\sqrt{2m_0}} \right)^2. \]

(19)

It is clear that near the black hole \((r \to r_+)\) we have \(f \to 0\) and CM energy will be infinite as expected.

### 4 Uncharged black hole

In order to find effect of black hole charge on the CM energy, first we study uncharged black hole. If we set \(q_i = 0\), then \(\mathcal{H} = 1\) and CM energy will be,

\[ \tilde{E}_{CM} = 1 + \frac{1}{f_0} E_1 E_2 - \frac{R^2}{r^2} L_1 L_2 - G_{01} G_{02}, \]

(20)

where,

\[ f_0 = 1 - \frac{\mu}{r^2} + \frac{r^2}{R^2}, \]

\[ G_{01} = \sqrt{1 + \frac{1}{f_0} E_1^2 - \frac{R^2}{r^2} L_1^2}, \]

\[ G_{02} = \sqrt{1 + \frac{1}{f_0} E_2^2 - \frac{R^2}{r^2} L_2^2}, \]

(21)

From the Fig. 1 we can see that the maximum energy is near the black hole horizon. Choosing \(R = 1\) and \(\mu = 1\) tells that \(r_+ = 0.78615\) which yields to \(\tilde{E}_{CM} = 2.5\), while the maximum of CM energy is about 4.8. Also we see negative energy in the Fig. 1 which shows regions far from the black hole. However at \(r \to \infty\), as we expected, the CM energy yields to zero.

### 5 Three-charged black hole

In that case we set \(q_i = q\) and find that the black hole charge restricted to have positive CM energy near the black hole. By choosing \(L_1 = L_2 = 1\), \(E_1 = 1\) and \(E_2 = 10\) we obtain \(q < 0.6\) is necessary to have positive energy near the black hole horizon. In that case we find that the black hole charge may be decreased or increased the CM energy. This point illustrated in the Fig. 3.
Figure 1: Center of mass energy of uncharged black hole with $\mu = 1$, $R = 1$. We choose $L_1 = L_2 = 1$ and $E_1 = 1$. Then solid line drawn for $E_2 = 10$ and dashed line for $E_2 = 5$.

Figure 2: Center of mass energy of three-charged black hole with $\mu = 1$, $R = 1$. We choose $L_1 = L_2 = 1$, $E_1 = 1$ and $E_2 = 10$. Then, space-dotted line drawn for $q = 0$, dashed line drawn for $q = 0.24$, solid line drawn for $q = 0.3$ and dotted line drawn for $q = 0.42$.

6 Conclusion

In this paper we considered STU black hole and calculate center of mass energy of two colliding particles near the black hole horizon. We found that the black hole charge may be increased or decreased the CM energy which depend on distance from the black hole. Also it is clear that the CM energy on the black hole horizon will be infinite. We found that far from the black hole the CM energy yields to zero and black hole charge is not important parameter. For the future work and complete this job it is interesting to discuss about CM energy in extremal limits of STU black hole, also one can consider this black hole in the flat space.
Figure 3: Center of mass energy versus black hole charge with $\mu = 1$, $R = 1$. We choose $L_1 = L_2 = 1$, $E_1 = 1$ and $E_2 = 10$.

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