Detection of mechanical resonance of a single-electron transistor by direct current

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We have suspended an Al based single-electron transistor whose island can resonate freely between the source and drain leads forming the clamps. In addition to the regular side gate, a bottom gate with a larger capacitance to the SET island is placed underneath to increase the SET coupling to mechanical motion. The device can be considered as a doubly clamped Al beam that can transduce mechanical vibrations into variations of the SET current. Our simulations based on the orthodox model, with the SET parameters estimated from the experiment, reproduce the observed transport characteristics in detail.

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Nanomechanical resonators, usually doubly clamped beams or cantilevers, offer rich physics as well as a wide range of applications [1, 2]. In order to do experiments on them, one needs a detector, called a transducer, coupled to the resonator, which converts mechanical displacement into an electrical signal. A number of techniques have been applied to measure mechanical motion at the micro and nanoscales. For the past decade in the quest for higher sensitivity and speed, the dimensions of the resonators were scaled down, pushing their resonance frequency to above 1 GHz [3]. At the same time the requirements to the transducers become more stringent in terms of sensitivity to the mechanical displacement, thus narrowing the choice of possible detectors. Among various detection techniques described elsewhere [4], a single-electron transistor (SET) [3, 6] proved to be an efficient transducer due to its extremely high sensitivity and a capability of detecting the motion of a mechanical resonator in the quantum limit. When capacitively coupled to the resonator and biased at a dc voltage, the SET senses the resonator’s mechanical motion due to the variations of the electrical charge induced on the SET island. Using a radiofrequency (rf) circuitry and an SET as a mixer, displacement sensitivity as good as $2 \times 10^{-15} \text{m/Hz}^{1/2}$ was achieved for a GaAs mechanical resonator [3]. In the later experiment, an rf version of the SET was used to detect the thermal motion of the SiN resonator demonstrating position resolution only a factor of 4.3 above the quantum limit [3].

In this letter, we describe an aluminum structure that combines both the doubly clamped beam and SET, and therefore can be referred to as a two-in-one device. Our fabrication process allows easy integration of metallic nanomechanical resonators into the electronic circuits such as SETs or SQUIDs. We show experimentally that a conventional SET in the dc regime can detect flexu-
between the bottom gate and the island can be made several times larger as compared to the one-side gate configuration implemented in a single layer. This makes the SET more sensitive to mechanical motion. Second, the gap between the bottom gate and the island depends on the thickness of the corresponding polymer used as a sacrificial layer and therefore can be controlled accurately.

Third, a high dc voltage $V_{dc}$ and slowly varying voltage $V_g$ can be applied to different gates simplifying the measurement process. The voltage applied to the bottom gate has two components: $V_{gb} = V_{dc} + V_{rf}$. The former is used to control coupling between the mechanical motion and SET transport while the latter drives the beam. All the dc voltages are supplied to the sample by the filtered dc wires. The rf signal is delivered through a coaxial line with a 20 dB attenuator at 4.2 K. The measurements are done in a dilution refrigerator with a mixing chamber temperature of about 25 mK.

To model SET transport in the presence of mechanical oscillations, we perform simulations based on the orthodox theory \[12\] with the mechanical degree of freedom taken into account. We consider the classical dynamics of the SET island and solve the equation of motion for the coordinate $x$, which is the displacement of the beam center from the equilibrium position:

$$\ddot{x} + \frac{\omega_0}{Q} \dot{x} + \omega_0^2 x = F/m,$$  \tag{1}

where $F$ is the driving force acting on the SET island, $m$ is the beam effective mass, $Q$ is the quality factor and $\omega_0 = 2 \pi f_0$ is the angular resonance frequency. The force can be found as a displacement derivative of the total energy stored in the SET island. Assuming $C_{gb} \ll C$, where $C$ is the total capacitance of the SET island, we obtain $F \approx -\frac{\partial C_{gb}}{\partial x} \frac{1}{2} V_{gb}^2 - e \frac{C_{gb}}{C} (n + C_{gb} V_{gb})$, where $n$ is the instant number of electrons on the SET island.

We then further assume that the tunneling of electrons is much faster than the mechanical oscillations, so that the tunneling rate for each event can be calculated for constant $x$. The rates are calculated using the golden rule approach assuming both SET junctions are equal.

We first characterize the transistor by measuring the SET current $I$ as a function of the bias voltage $V$ and two gate voltages: side gate voltage $V_g$ and bottom gate voltage $V_{dc}$. From these measurements we estimate the following parameters of the device: total tunnel resistance $R = 140 \, \text{k}\Omega$, charging energy $E_c = e^2/2C = 0.235 \, \text{meV}$ ($E_c/k_B = 2.7 \, \text{K}$) corresponding to $C = 3.4 \times 10^{-16} \, \text{F}$, and side gate capacitance $C_g = 1.2 \times 10^{-18} \, \text{F}$. By sweeping the voltage applied to the bottom gate, we obtain its capacitance $C_{gb} = 5.4 \times 10^{-17} \, \text{F}$. This value is in agreement with the naive estimation $2.4 \times 10^{-17} \, \text{F}$ from the parallel-plate geometry, which underestimates the capacitance due to not taking into account the fringing effects as well as additional bottom gate - island coupling through the tunnel junctions. In this estimation, the island width $w = 92 \, \text{nm}$ and length $1500 \, \text{nm}$, and the vacuum ($\varepsilon = 1$) gap $d = 50 \, \text{nm}$ between the island and bottom gate, measured in the scanning electron microscope, were used. With the given dimensions of the 38.6 nm-thick island, and Al material parameters, such as the mass density $2700 \, \text{kg/m}^3$ and Young’s modulus 70 GPa, we estimate the unstressed beam resonance frequency for the out-of-plane fundamental flexural mode to be about 90 MHz. However, as expected, the resonance frequency measured in the experiment is higher due to the tensile stress produced by the difference in thermal expansion coefficients of Al and Si \[11\].

In order to detect the beam resonance, we drive the beam with an external force by applying an rf voltage to the bottom gate inducing out-of-plane oscillations. To increase the coupling of the SET to the mechanical oscillations, we simultaneously apply to the same gate a high, up to $\pm 4 \, \text{V}$, dc voltage. The search for the resonance is done in such a way that at constant rf amplitude and frequency, we sweep $V_g$ and measure current $I$ through the SET. Then the frequency is increased by an increment of $3 \, \text{kHz}$, which is smaller than the expected resonance width $f_0/Q$. Even in the absence of mechanical resonance the modulation peak gets slightly suppressed and broaden due to the fact that the working point shifts periodically when $V_{rf}$ is applied (see Fig. 2(a)). This effect, however, does not depend on frequency. Once the driving frequency approaches the resonance frequency of the beam, there is an additional effect on the modulation peak, which strongly depends on the frequency. The intensity plot revealing the expected resonance is presented in Fig. 2(b). The resonance is seen as a characteristic feature at about 95 MHz being most pronounced at $C_g V_{rf}/e = 0.5$: the current peak height decreases first and then suddenly increases when we go through resonance from lower to higher frequency.

A collection of the SET response curves measured at various $V_g$ and the corresponding simulated curves are shown in Fig. 3. In the simulations, we set $Q = 10^4$ and $f_0 = 95.007 \, \text{MHz}$; other parameters were taken from the experiment. The simulations capture all the essential features observed in the experiment. The current steps at about 94.93 MHz and 95.09 MHz in Fig. 3(a) are due to the jumps of the background charge, which were not accounted for in the simulations. Note that the observed response is expected in the fully linear regime; no non-linearity of the resonator was included in the model. The observed dispersive-like resonance curve at $C_g V_{rf}/e = 0.5$ instead of a Lorentzian expected for the driven harmonic oscillator in the linear regime can be qualitatively understood in the following way. When we approach the resonance from the lower frequency side, the SET modulation peak becomes more suppressed and smeared because both the driving force and the displacement effect...
act in phase. When we pass the resonance, the displacement and the driving force become shifted by 180 degrees with respect to each other. Therefore they compensate each other, and the SET current rises to almost its original value when no rf voltage was applied. At a higher frequency, the amplitude of the mechanical oscillations decreases and the modulation peak becomes almost equal to the one measured at a lower frequency.

The SET sensitivity to the mechanical motion can be estimated by assuming that the change in the island charge produced by the mechanical displacement should be equal to the SET charge equivalent noise. Though the charge noise was not measured in the device presented, we conservatively set the noise level at low frequencies, where the $1/f$ noise dominates, to $q_n = 10^{-3} e/\text{Hz}^{1/2}$ at 1 Hz, which is typical for metallic SETs [13]. The charge variations due to the beam displacement are $q_m = V_{\text{dc}} \Delta C_{gb} \equiv V_{\text{dc}} C_{gb} x/d$ assuming $x \ll d$. Thus, the displacement sensitivity is equal to $\delta x = q_n/d/V_{\text{dc}} C_{gb} \approx 2 \times 10^{-13} \text{ m/Hz}^{1/2}$ per volt of $V_{\text{dc}}$, which, in our low-frequency measurement, corresponds to the $\text{rms}$ displacement amplitude $\sim 10^{-12}\text{m}$. However, when the SET in the dc regime measures high-frequency mechanical oscillations, the conversion of the beam displacement into the charge variation of the SET island is a result of rectification of the rf signal on the SET current nonlinearity, which gives an even lower displacement sensitivity. On the other hand, in the MHz frequency range the SET charge noise is about two orders of magnitude lower [13]. Therefore, the rf type SET is able to resolve thermal motion of the nanomechanical resonator even at the mK temperature [8]. The $\text{rms}$ amplitude of the beam main flexural mode due to thermal fluctuations is estimated from the equipartition theorem according to the formula $\langle x_T^2 \rangle^{1/2} = (k_B T/m a_0^2)^{1/2}$, where $T$ is the temperature. This gives the value $\langle x_T^2 \rangle^{1/2} = 2.5 \times 10^{-13} \text{ m}$, which is not detectable in the present setup using dc measurement.

The reduced displacement sensitivity of the device in the dc regime, as compared to the rf regime ($\sim 10^{-15} \text{ m/Hz}^{1/2}$), by no means compromises performance of the SET as a detector but just a result of simplification of the measurement circuit. Besides detecting its own mechanical resonance, the device described can also be used for spectroscopy measurement of a suspended charge qubit. Another challenging experiment is the observation of the lasing effect in the circuit containing an artificial atom (charge qubit) coupled to a high-frequency mechanical resonator instead of a commonly used optical or microwave resonator.

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![FIG. 2: (Color online) (a) $I$ vs $V_g$ curves for $V_{\text{dc}} = -2.5 \text{ V}$ and the amplitude of $V_T$ equal to 0 (black curve) and 0.32 mV (red curve) (b) Intensity plot showing an SET modulation peak for the same values of $V_{\text{dc}}$ and the amplitude of $V_T$, when the frequency of the applied drive is varied around resonance.](image)

![FIG. 3: (Color online) Frequency dependence of the SET current at $V_{\text{dc}} = -2.5 \text{ V}$, amplitude 0.32 mV of $V_T$, and at several values of $C_g V_g/e$. (a) Response curves measured at $C_g V_g/e = 0.5, 0.4, 0.58, 0.77, 0.85$ and 0.95 (from top to bottom); (b) Corresponding simulated curves.](image)
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