TOPICAL REVIEW

Supergravity-based inflation models: a review

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Abstract
In this review, we discuss inflation models based on supergravity. After explaining the difficulties in realizing inflation in the context of supergravity, we show how to evade such difficulties. Depending on types of inflation, we give concrete examples, particularly paying attention to chaotic inflation because the ongoing experiments like Planck might detect the tensor perturbations in the near future. We also discuss inflation models in Jordan frame supergravity, motivated by Higgs inflation.

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1. Introduction

Recent observations of the cosmic microwave background (CMB) anisotropies [1] strongly support the presence of accelerated expansion era in the early Universe, that is, inflation. Although inflation was originally introduced to solve the horizon, flatness, monopole problems, and so on [2], it can also give an explanation to the origin of primordial density fluctuations, which are responsible for the large scale structure of the present Universe. Unfortunately, however, the origin of inflation, namely what (which field) caused inflation, is still unknown and it is one of the most important mysteries of particle physics and cosmology.

Inflation can be caused by the potential energy of a scalar field. Such a potential must be relatively flat in order to guarantee long duration of inflation and small deviation of scale invariance of primordial density fluctuations. However, the flatness of the scalar potential can be easily destroyed by radiative corrections. One of the leading theories to protect a scalar field from radiative corrections is supersymmetry (SUSY), which also gives an attractive solution to the (similar) hierarchy problem of the standard model (SM) of particle physics as well as the unification of the three gauge couplings. In particular, its local version, supergravity, would govern the dynamics of the early Universe, when high-energy physics was important. Thus, it is quite natural to consider inflation in the framework of supergravity. However, it is a non-trivial task to incorporate inflation in supergravity. This is mainly because a SUSY breaking potential term, which is indispensable to inflation, generally gives a would be inflaton an additional mass, which spoils the flatness of an inflaton potential [3–5]. Specifically, the exponential factor appearing in the $F$-term is troublesome. Assuming the canonical Kähler
potential, this exponential factor generates the additional mass comparable to the Hubble parameter for a field value smaller than the reduced Planck scale \( M_p \approx 2.4 \times 10^{18} \text{ GeV} \), which makes it difficult to realize small field inflation such as new and hybrid inflations in supergravity. In addition, it prevents a scalar field from acquiring a value larger than \( M_p \). This fact implies that it is almost impossible to realize large field inflation like chaotic inflation in supergravity. In fact, after the original chaotic inflation had been proposed, almost 20 years passed until a natural model of chaotic inflation in supergravity appeared. The main purpose of this review is to explain how to circumvent these difficulties and how to realize inflation in supergravity.

While the standard model of particle physics includes only one scalar field (Higgs field), scalar fields are ubiquitous in the supersymmetric theories. The Higgs field, unfortunately, cannot be responsible for inflation as long as it has the canonical kinetic term and is minimally coupled to gravity because it predicts too large density fluctuations as well as too large tensor-to-scalar ratio. Recently, a possibility of realizing Higgs inflation with a non-minimal coupling to gravity [7] and/or a non-trivial kinetic term [8–10] has drawn much attention. We are going to mention such a possibility in the last part of this review. However, first of all, we concentrate on inflation models with an (almost) canonical kinetic term and a minimal coupling to gravity.

Finally, we would like to make a comment on the relation between supergravity and superstring. While supergravity is a field theory, actually, the low energy effective field theory of superstring, superstring is a string theory though other higher dimensional objects like D-branes are found to join it. Since inflation only needs the positive potential energy, it can be realized in the framework of an (effective) scalar field. Thus, inflation does not necessarily require superstring theory. However, superstring theory can give us concrete forms of the potentials like the Kähler potential, superpotential, and gauge kinetic function in supergravity. Therefore, once we find particular forms of such potentials suitable for inflation, it would be interesting to investigate whether such forms naturally appear in the framework of superstring. Since we do not pursue such possibility in this review, we refer recent reviews of string inflation [11] as well as other excellent reviews of supergravity inflation [12].

The organization of this review is as follows. In the next section, we first give the basics of inflation and a scalar potential in supergravity, which is composed of the \( F \)-term and \( D \)-term. Then we explain why it is difficult to incorporate inflation in the context of supergravity. In section 3, after giving a general discussion how to evade such a difficulty, we will give concrete examples of inflation based on the \( F \)-term or \( D \)-term for each inflation type. In section 4, we will discuss inflation models in the Jordan frame supergravity. Section 5 is devoted to the conclusion.

2. Basic formulae and difficulty in realizing inflation in supergravity

2.1. Basic formulae of inflation dynamics and primordial perturbations

Let us consider a single-field \( \phi \) inflation model with the canonical kinetic term and potential \( V(\phi) \), whose action is given by

\[
S = \int d^4x \sqrt{-g} \mathcal{L} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi) \right].
\] (1)

Here \( g \) is the determinant of the metric \( g_{\mu\nu} \) and the scalar field is assumed to be minimally coupled to gravity. The extension to the multi-field case is straightforward. Provided that the scalar field is homogeneous and the metric is \( ds^2 = -dt^2 + a(t)^2 dx^2 \), the equation of motion

\[1\] See [6] for other attempts to realize inflation in the context of SM and its SUSY extensions.

\[2\]
and the Friedmann equations are given by
\[
\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \\
H^2 = \frac{1}{2} \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) \right],
\]
where the dot and the dash represent the derivatives with respect to the cosmic time and the scalar field \( \phi \), respectively. Here and hereafter we set the reduced Planck scale \( M_P \) to be unity unless otherwise stated. These equations can be approximated as
\[
3H\dot{\phi} + V'(\phi) \approx 0, \\
H^2 \approx \frac{V(\phi)}{3},
\]
as long as the following two slow-roll conditions are satisfied:
\[
\epsilon \equiv \frac{1}{2} \left( \frac{V'}{V} \right)^2; \quad \epsilon \ll 1, \\
\eta \equiv \frac{V''}{V}; \quad |\eta| \ll 1.
\]
From equations (3), the e-folding number \( N \) is estimated as
\[
N = \int_{t_e}^{t} H dt \simeq \int_{\phi_e}^{\phi} \frac{d\phi}{H} \simeq \int_{\phi_e}^{\phi} \frac{V}{V'} d\phi,
\]
where \( t_e \) and \( \phi_e \) are the cosmic time and the field value at the end of inflation. Typically, a cosmologically interesting scale corresponds to \( N \sim 50 \) or 60 depending on inflationary energy scale and cosmic history after inflation.

During inflation, the curvature perturbations were generated through inflaton fluctuations and then their amplitude in the comoving gauge \( R \) [13] on the comoving scale \( 2\pi/k \) is given by
\[
R^2(k) = \frac{1}{4\pi^2} \left( \frac{H^4(t_k)}{\dot{\phi}(t_k)^2} \right) \simeq \frac{1}{24\pi^2} \frac{V}{\epsilon}. 
\]
Here \( t_k \) is the epoch when \( k \) mode left the Hubble radius during inflation [14]. The spectral index of the curvature perturbation is calculated as
\[
n_s - 1 \equiv \frac{d\ln R^2(k)}{d\ln k} \simeq -6\epsilon + 2\eta. 
\]
On the other hand, the tensor perturbations (gravitational wave) \( h \) are also generated and their amplitude on the comoving scale \( 2\pi/k \) is given by [15]
\[
h^2(k) = 8 \left( \frac{H(t_k)}{2\pi} \right)^2 \simeq \frac{2V}{3\pi^2}, 
\]
where the coefficient 8 comes from the canonical normalization and the number of polarization of tensor modes. Then, the tensor to scalar ratio \( r \) is given by
\[
r \equiv \frac{h^2(k)}{R^2(k)} \simeq 16\epsilon. 
\]
A detailed derivation of these standard formulae is given in the textbook [16], for example.

Recent observations of the Wilkinson microwave anisotropy probe (WMAP) can strongly constrain these observable quantities as [1]
\[
R^2(k_0) = 2.441^{+0.088}_{-0.092} \times 10^{-9}, \\
n_s = 0.963 \pm 0.012, \\
r < 0.24,
\]
with \( k_0 = 0.002 \) Mpc\(^{-1} \). Note that, if we allow the running of the spectral index, these constraints can be significantly relaxed.
2.2. Difficulty in realizing inflation in supergravity

The scalar part of the Lagrangian in supergravity is determined by the three functions, Kähler potential $K(\Phi_i, \Phi^*_i)$, superpotential $W(\Phi_i)$, and gauge kinetic function $f(\Phi_i)$ [17]. While the last two functions ($W$ and $f$) are holomorphic functions of complex scalar fields, the first one $K$ is not holomorphic and a real function of the scalar fields $\Phi_i$ and their conjugates $\Phi^*_i$. Note that the above three functions are originally the functions of (chiral) superfields. Since we are mainly interested in the scalar part of a superfield, we identify a superfield with its complex scalar component and write both of them by the same letter.

The action of complex scalar fields minimally coupled to gravity consists of kinetic and potential parts:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{\sqrt{-g}} L_{\text{kin}} - V(\Phi_i, \Phi^*_i) \right].$$

(14)

The kinetic terms of the scalar fields are determined by the Kähler potential $K$ and given by

$$\frac{1}{\sqrt{-g}} L_{\text{kin}} = -K_{ij} D_\mu /\Phi_i D^\mu /\Phi_j g^{\mu\nu},$$

(15)

where

$$K_{ij} = \frac{\partial^2 K}{\partial /\Phi_i \partial /\Phi_j},$$

(16)

and $D_\mu$ represents the gauge covariant derivative. Here and hereafter, the lower indices of the Kähler potential stand for the derivatives. The potential $V$ of scalar fields $\Phi_i$ is made of two terms, the $F$-term $V_F$ and the $D$-term $V_D$. The $F$-term $V_F$ is determined by the superpotential $W$ as well as the Kähler potential $K$:

$$V_F = e^K \left[ D_{\Phi_i} W K^{-1}_{ij} D_{\Phi^*_j} W^* - 3 |W|^2 \right].$$

(17)

with

$$D_{\Phi_i} W = \frac{\partial W}{\partial /\Phi_i} + \frac{\partial K}{\partial /\Phi_i} W.$$ 

(18)

On the other hand, the $D$-term $V_D$ is related to gauge symmetry and given by the gauge kinetic function as well as the Kähler potential:

$$V_D = \frac{1}{2} \sum_{a,b} |\text{Re} f_{ab}(\Phi_i)|^{-1} g_a^2 D_a^2,$$

(19)

with

$$D_a = \Phi_i (T_a)_{ij} \frac{\partial K}{\partial /\Phi_j} + \xi_a.$$

(20)

Here, the subscript $a$ represents gauge symmetries, $g_a$ is a gauge coupling constant, and $T_a$ is an associated generator. $\xi_a$ is a so-called Fayet–Iliopoulos (FI) term and can be non-zero only when the gauge symmetry is Abelian, that is, $U(1)$ symmetry. Note that only a combination

$$G \equiv K + \ln |W|^2$$

(21)

is physically relevant. Then, the kinetic and the potential terms are invariant under the following Kähler transformation:

$$\Phi_i \rightarrow e^{i\alpha} \Phi_i,$$

$$\Phi^*_i \rightarrow e^{-i\alpha} \Phi^*_i.$$
\[ K(\Phi^i, \Phi^*_i) \rightarrow K(\Phi^i, \Phi^*_i) - U(\Phi^i) - U^*(\Phi^*_i), \]
\[ W(\Phi^i) \rightarrow e^{U(\Phi^i)} W(\Phi^i), \]

where \( U(\Phi^i) \) is any holomorphic function of the fields \( \Phi^i \).

From the potential form (17), it is manifest that in order to acquire the positive energy density necessary for inflation, at least one of the terms \( D/\Phi^i W \) must be non-zero. Since these terms are the order parameters of SUSY, it turned out that inflation is always accompanied by the SUSY breaking, whose effect could be transmitted to any scalar field and generate a dangerous mass term. Specifically, taking (almost) canonical Kähler potential,

\[ K(\Phi^i, \Phi^*_i) = \sum_i |\Phi^i|^2 + \cdots, \tag{23} \]

the kinetic term of the scalar fields \( \Phi^i \) becomes (almost) canonical. Here the ellipsis stands for higher order terms. Then, the F-term potential, \( V_F \), is approximated as [4, 5]

\[ V_F = \exp \left( \sum_i |\Phi^i|^2 + \cdots \right) \times \left\{ \left[ \frac{\partial W}{\partial \Phi^*_i} + (\Phi^*_i + \cdots) W \right] \sum_{i,j} (\delta_{ij} + \cdots) \right. \\
\times \left. \left[ \frac{\partial W^*}{\partial \Phi^-j} + (\Phi^-j + \cdots) W^* \right] - 3 |W|^2 \right\} \]
\[ = V_{\text{global}} + V_{\text{global}} \sum_i |\Phi^i|^2 + \text{other terms}, \tag{24} \]

where \( V_{\text{global}} \) is the effective potential in the global SUSY limit and given by

\[ V_{\text{global}} = \sum_i \left| \frac{\partial W}{\partial \Phi^i} \right|^2. \tag{25} \]

Thus, any scalar field including a would be inflaton receives the effective mass squared \( V_{\text{global}} = 3H^2 \), which gives a contribution of order unity to the slow-roll parameter \( \eta \) and breaks one of the slow-roll conditions necessary for successful inflation:

\[ \eta = \left. \frac{V''}{V} \right|_V = 1 \quad + \quad \text{other terms}. \tag{26} \]

This is the main difficulty in incorporating inflation in supergravity and is called the \( \eta \) problem.

Several methods have been proposed to evade this problem thus far.

- Use the Kähler potential different from the (almost) canonical one. In this case, we have two possibilities to obtain flat potentials, both of which are related. When the Kähler potential is far from canonical, so is the kinetic term of the scalar field. By redefining the scalar field such that its kinetic term is canonically normalized, the effective potential could be flat even if it was originally steep. In the next section, we mainly focus on the (almost) canonical Kähler potential and consider this possibility only in chaotic inflation of F-term models.

The second possibility is to impose some conditions (or symmetries) on the Kähler potential and/or the superpotential, which guarantee the flatness of the potential. For example, the Heisenberg symmetry was used to avoid the additional scalar mass [20]. Another condition is given in [5] and, interestingly, the required form of the Kähler potential appears in weakly coupled string theory. Therefore, it is better to discuss such possibilities in the context of string theory and hence we skip them in this review.

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2 If inflation can be realized in the strongly dissipative system [18], which we do not deal with in this review, the large effective masses coming from the supergravity corrections would be harmless in such a dynamical system [19].
• Use quantum corrections. The potential given in equation (17) is a classical one. In the case that an inflaton has (large) Yukawa and/or gauge interactions, quantum corrections significantly modify the potential so that the inflaton mass runs with scale [21]. We mention this possibility in hybrid inflation of F-term models.

• Use a special form of superpotential with the (almost) canonical Kähler potential. Actually, as shown below, in the case that the superpotential is linear in an inflaton, the inflaton effective mass becomes negligible so that the η problem is avoided [4, 5, 22]. Such a form of superpotential can be easily realized by imposing the $R$ symmetry with the $R$ charge of the inflaton to be two and the others to be zeros. In the next section, we will give concrete examples of inflation with such types of superpotential.

• Use the $D$-term potential. The η problem is peculiar to the $F$-term potential. Therefore, if we can get the positive energy in the $D$-term potential, inflation is easily realized [5]. In the next section, we will give concrete examples of such $D$-term inflation models.

Before closing this section, we note that the solutions to the η problem is not sufficient for large field inflation such as chaotic and topological inflations. Though they need the field value of an inflaton larger than unity, the exponential factor of the $F$-term potential prevents the inflaton from taking such a large value as long as the Kähler potential is (almost) canonical. Thus, we need another prescription to realize large field inflation, which will be given later in the corresponding models.

3. Inflation models in supergravity

In this section, we give concrete examples of successful inflation model for each type. In the former subsection, inflation models supported by the $F$-term potential are given, and in the latter subsection, $D$-term models will be discussed. Recent observations are so precise that the original models may be disfavored in some cases and hence some extended models are proposed. In addition, the gravitino problem is another important issue in constructing inflation models in supergravity. The gravitino is a superpartner of graviton and is a fermion with spin $3/2$. During the reheating stage of inflation, gravitinos are copiously produced so that they may easily destroy light elements synthesized during the big bang nucleosynthesis (BBN) or overclose the universe, depending on its mass and lifetime determined by the SUSY breaking mechanism. Recently, a new mechanism to produce gravitinos during reheating [23, 24] was found in addition to the conventional production mechanism from thermal plasma [25]. Thus, a strong constraint on reheating temperature is imposed on inflation models, which may also rule out the original models for some range of gravitino masses. However, in this review, we stick to the original or the simplest models simply because one can easily understand the essence of each model. See the references for more elaborated models to fit the observed results well and to avoid the gravitino problem.

3.1. $F$-term inflation

As discussed in the previous section, the exponential factor appearing in the $F$-term potential can give a would be inflaton an additional mass and rule it out as an inflaton. One of the methods to circumvent this difficulty is to adopt a superpotential linear in the inflaton $\Phi$ [4, 5, 22],

$$W = \Phi f(\chi_i),$$  
(27)

where $\chi_i$ is a field other than the inflaton and $f$ is a holomorphic function of $\chi_i$. This type of superpotential can be easily realized by imposing the $R$ symmetry, under which they are
transformed as $\Phi(\theta) \rightarrow e^{\text{i}2\alpha} \Phi(\theta e^{\text{i}2\alpha})$, $f(\chi_i)(\theta) \rightarrow f(\chi_i)(\theta e^{\text{i}2\alpha})$. Note that the canonical Kähler potential given by
\begin{equation}
K = |\Phi|^2 + \sum_i |\chi_i|^2
\end{equation}
respects this $R$ symmetry. In this case, the $F$-term potential is given and approximated as
\begin{equation}
V_F = e^K \left[ |f|^2 (1 - |\Phi|^2 + |\Phi|^4) + |\Phi|^2 \left| \frac{\partial f}{\partial \chi_i} + \chi_i^* f \right|^2 \right]
\end{equation}
\begin{equation}
\simeq V_0 \left( 1 + \frac{|\Phi|^4}{2} + |\chi_i|^2 \right) + |\Phi|^2 \left| \frac{\partial f}{\partial \chi_i} + \chi_i^* f \right|^2,
\end{equation}
where $V_0 = |f|^2 = |\partial W/\partial \Phi|^2$ and we have expanded the exponential factor for $|\Phi|, |\chi_i| \ll 1$.

It is found that there is no inflaton mass associated with $V_0$, while the other fields $\chi_i$ acquire the additional masses squared $V_0 \simeq 3H^2$, which usually drive $\chi_i$ to zeros. Though the second term on the right-hand side of the last equation is the mass term of the inflaton, it is typically very small and hence the $\eta$ problem is evaded. In particular, in the case that every $\chi_i$ goes to zero and $\partial f/\partial \chi_i = 0$ at the origin of $\chi_i$, the inflaton is exactly massless.

Now, we are ready to give concrete examples of successful inflation models for each type, that is, new inflation, hybrid inflation, chaotic inflation, and topological inflation, though additional tricks are necessary for the last two types.

### 3.1.1. New inflation

New inflation was proposed as the first slow-rolling inflation model [26]. As a concrete model of new inflation, which uses the $F$-term potential, we consider the model proposed by Izawa and Yanagida [27].

A chiral superfield $\Phi_1$ with the $R$ charge $2/(n+1)$ is introduced as an inflaton. In this model, the $U(1)_R$ symmetry is assumed to be dynamically broken to a discrete $Z_{2n} R$ symmetry at a scale $v \ll 1$. Then, the superpotential is given by
\begin{equation}
W = v^2 \Phi_1 - \frac{g}{n+1} \Phi_1^{n+1},
\end{equation}
where $g$ is a coupling constant of order unity. We assume that both $g$ and $v$ are real and positive in addition to $n \geqslant 3$ for simplicity. The $R$-invariant Kähler potential is given by
\begin{equation}
K = |\Phi|^2 + \cdots,
\end{equation}
where the ellipsis stands for higher order terms, which we ignore for a while.

The scalar potential is obtained from equations (30) and (31) by the use of the formula given in equation (17) and reads
\begin{equation}
V(\Phi) = e^{\text{i}|\Phi|^2} \left[ (1 + |\Phi|^2) v^2 - \left( 1 + \frac{|\Phi|^2}{n+1} \right) g \Phi^2 - 3|\Phi|^2 \left| v^2 - \frac{g}{n+1} \Phi^2 \right|^2 \right].
\end{equation}
It has a minimum at
\begin{equation}
|\Phi|_{\text{min}} \simeq \left( \frac{v^2}{g} \right)^{\frac{1}{2}} \quad \text{and} \quad \text{Im} \Phi_{\text{min}}'' = 0,
\end{equation}
with negative energy density
\begin{equation}
V(\Phi_{\text{min}}) \simeq -3e^{\text{i}|\Phi|^2}|W(\Phi_{\text{min}})|^2 \simeq -3 \left( \frac{n}{n+1} \right)^2 v^4 |\Phi_{\text{min}}|^2.
\end{equation}
You may wonder if this negative value may be troublesome. However, in the context of SUSY, we may interpret that such negative potential energy is almost canceled by positive...
contribution due to the supersymmetry breaking, $A^4_{\text{SUSY}}$, and that the residual positive energy density is responsible for the present dark energy. Then, we can relate the energy scale of this model with the gravitino mass $m_{3/2}$ as

$$m_{3/2} \simeq \frac{n}{n+1} \left( \frac{v^2}{g} \right)^{\frac{1}{2}} v^2.$$  

(35)

Identifying the real part of $\Phi$ with the inflaton $\phi \equiv \sqrt{2} \text{Re} \Phi$, the dynamics of the inflaton is governed by the following potential:

$$V(\phi) \simeq v^4 - \frac{2g}{2n/2} v^2 \phi^n + \frac{g^2}{2} \phi^{2n}.$$  

(36)

You can easily find that the first constant term dominates the potential energy and that the last term is negligible during inflation. Then, the Hubble parameter during inflation is given by $H = v^7/\sqrt{3}$. On the other hand, the slow-roll equation of motion reads

$$3H \dot{\phi} \simeq -V'(\phi) = \frac{ng}{2n^2} v^2 \phi^{n-1},$$  

(37)

and the slow-roll parameters are given by

$$\epsilon \simeq \frac{n^2 g^2 \phi^{2(n-1)}}{2n^2} \frac{v^4}{v^4}, \quad \eta \simeq -\frac{n(n-1)g}{2} \frac{\phi^{n-2}}{v^2}.$$  

(38)

Thus, inflation lasts as long as $\phi$ is small enough, and it ends at

$$\phi = \sqrt{2} \left( \frac{v^2}{g(n/2-1)} \right)^{\frac{1}{n}} \equiv \phi_c,$$  

(39)

when $|\eta|$ becomes as large as unity. The $e$-folding number of new inflation is estimated as

$$N = \int_{\phi_0}^{\phi_1} \frac{2\pi^2 v^2}{ng\phi^{n-1}} d\phi = \frac{2\pi^2 v^2}{ng(n/2-2)} \frac{1}{\phi_N^{n-2}} - \frac{n-1}{n-2},$$  

(40)

where $\phi_N$ is a field value corresponding to the $e$-folds equal to $N$. The amplitude and the spectral index of primordial density fluctuations are expressed in terms of $N$ as

$$R^2 \simeq \frac{1}{24\pi^2} \epsilon \simeq \frac{2^{n-4}}{3\pi^2 n g^2 \phi_N^{2(n-1)}} \simeq \frac{1}{24\pi^2} \frac{1}{(n-2)N} \frac{2n-1}{n-2} v^4 \phi_N^{2n-2},$$  

(41)

$$n_s - 1 = -6 \epsilon + 2 \eta \simeq 2 \eta \simeq -\frac{n-1}{2} \frac{g}{v^2} \phi_N^{n-2} \simeq -\frac{n-1}{n-2} N.$$  

Inserting $R^2 = 2.441 \times 10^{-9}$ gives $\epsilon \simeq 6.9 \times 10^{-7}$ for $n = 4$, $g = 0.3$, $N = 50$. In this case, there are no significant tensor fluctuations and the spectral index becomes $n_s \simeq 0.94$, which is on the edge of the observationally allowed region. This situation can be improved if we take into account a higher order term $c_{11} |\Phi|^4$ ($c_{11} = \mathcal{O}(0.01)$) in the Kähler potential, which gives the inflaton the additional mass slightly less than the Hubble parameter. In this model, the gravitino mass becomes $m_{3/2} \simeq 4.3 \times 10^{-10} \simeq 1.0$ TeV.

After the inflation ends, an inflaton starts to oscillate around its minimum $\phi_{\text{min}}$ and then decays into the SM particles to reheat the universe. Such inflaton decay can occur, for example, by introducing higher order terms $\sum_i c_i |\Phi|^2 |\psi_i|^2$ in the Kähler potential, which are invariant under the $R$ symmetry. Here $\psi_i$ represent the SM particles and the couplings $c_i$ are of the order of unity. Then, the decay rate of the inflaton becomes $\Gamma \simeq \sum_i c_i^2 \phi_{\text{min}}^2 m_{\psi}^3$, where
\[ m_\phi \simeq n g^{1/2} v^{3/2} \] is the inflaton mass at its minimum \( \phi_{\text{min}} \). Thus, the reheating temperature \( T_R \) is estimated as
\[ T_R \simeq \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{\Gamma} \sim n^2 g^{1/2} v^{3/2}, \]
(42)
where \( g_* \simeq 200 \) is the number of relativistic degrees of freedom. For the above parameters, \( T_R \simeq 6.2 \times 10^{-17} \simeq 1.9 \times 10^2 \text{ GeV} \).

Finally, we would like to comment on the initial value problem of new inflation. In order to have a sufficiently long period of inflation, the initial value of the inflaton must be close to the origin, that is, the local maximum of the potential. However, since the potential is almost flat, there is no natural reason why the inflaton initially sits close to the origin. A couple of solutions to this initial condition problem have been proposed so far. One of the methods is to consider another inflation preceding new inflation [28, 29]. During the pre-inflation, the inflaton of new inflation acquires the aforementioned mass comparable to the Hubble parameter. This additional mass dynamically drives it to the local minimum of the effective potential at that time, which in turn determines initial value of new inflation. Such a double inflation model is interesting in that it can generate non-trivial features of primordial fluctuations, which were studied in the context of large scale structure and primordial black holes [30]. Another method is to use the interaction with the SM particles through the Kähler potential [31]. Also in this case, the inflaton of new inflation acquires the effective mass comparable to the Hubble parameter through the interaction with the SM particles, which are assumed to be in thermal equilibrium. This additional mass dynamically gives adequate initial value of new inflation again.

3.1.2. Hybrid inflation. Hybrid inflation [32] is attractive in that it often occurs in the context of grand unified theory (GUT) [33] since its potential can be easily associated with the spontaneous symmetry breaking [4]. Then, some other variants called mutated hybrid [34], smooth hybrid [35], shifted hybrid [36], and tribrid inflation [37] are also proposed.

Here we consider the hybrid inflation model in supergravity proposed by Linde and Riotto [38]. (See also [39].) The superpotential is given by
\[ W = \lambda S \overline{\Psi}_1 \Psi_1 - \mu^2 S, \]
(43)
where \( S \) is a gauge-singlet superfield, while \( \Psi \) and \( \overline{\Psi} \) are a conjugate pair of superfields transforming as non-trivial representations of some gauge group \( G \). Note that, though gauge symmetry is not always needed, this model can be easily embedded in GUT by introducing such a gauge symmetry. \( \lambda \) and \( \mu \) are positive parameters smaller than unity. This superpotential is linear in the inflaton \( S \) and possesses the \( R \) symmetry, under which they are transformed as
\[ S(\theta) \longrightarrow e^{2i\alpha} S(\theta e^{i\alpha}), \quad \Psi(\theta) \longrightarrow e^{2i\alpha} \Psi(\theta e^{i\alpha}), \quad \overline{\Psi}(\theta) \longrightarrow e^{-2i\alpha} \overline{\Psi}(\theta e^{i\alpha}). \]
(44)
Taking the canonical (\( R \)-invariant) Kähler potential:
\[ K = |S|^2 + |\Psi|^2 + |\overline{\Psi}|^2, \]
(45)
the scalar potential is given by the standard formulae (17) and (19):
\[ V(S, \Psi, \overline{\Psi}) = e^{(|S|^2 + |\Psi|^2 + |\overline{\Psi}|^2)}[(1 - |S|^2 + |S|^4)|\lambda \overline{\Psi} \Psi - \mu^2|^2 + |S|^2[|\lambda(1 + |\Psi|^2)\overline{\Psi} - \mu^2 \overline{\Psi}^*|^2 + |\lambda(1 + |\overline{\Psi}|^2)\Psi - \mu^2 \Psi^*|^2]] + V_D. \]
(46)
Since the above potential does not depend on the phase of the complex scalar field \( S \), we identify its real part \( \sigma \equiv \sqrt{2} \text{Re} S \) with the inflaton without loss of generality. Assuming
$\sigma \ll 1$, the mass matrix of $\Psi$ and $\overline{\Psi}$ is given by
\[ V_{\text{mass}} \simeq -\lambda \mu^2 (\overline{\Psi}\Psi + \overline{\Psi}\Psi^*) + \frac{1}{2}(\lambda^2 + \mu^4)\sigma^2 (|\Psi|^2 + |\overline{\Psi}|^2) = [(\lambda^2 + \mu^4)|S|^2 + \lambda \mu^2]|\Phi|^2 + [(\lambda^2 + \mu^4)|S|^2 - \lambda \mu^2]|\overline{\Psi}|^2, \] (47)
where $\Phi \equiv (\Psi - \overline{\Psi})/\sqrt{2}$ and $\overline{\Phi} \equiv (\Psi + \overline{\Psi})/\sqrt{2}$.

Thus, we find that the eigenvalues for the corresponding eigenstates are given by
\[ M_\pm^2 = (\lambda^2 + \mu^4)|S|^2 \pm \lambda \mu^2 = \frac{1}{2}(\lambda^2 + \mu^4)\sigma^2 \pm \lambda \mu^2 \quad \text{for} \quad \Psi = \mp \overline{\Psi}. \] (48)

Since $M_\pm^2$ is always positive, we can safely set $\Phi = 0$, that is, $\Psi = \overline{\Psi}$. Therefore, we have only to consider the dynamics of $S$ and $\overline{\Phi}$. When $\sigma$ becomes smaller than the critical value $\sigma_c \equiv \sqrt{2}\mu/\sqrt{\lambda}$, $M_\pm^2$ becomes negative and hence $\overline{\Phi}$ quickly rolls down to the global minimum, which ends inflation. This feature is typical of hybrid inflation. Here and hereafter, we assume $\mu \ll \lambda$ for simplicity since the extension to the other cases is straightforward.

Since $\overline{\Phi}$ is the $D$-flat direction, the effective potential under the condition $\Psi = \overline{\Psi}$ becomes
\[ V = (\lambda |\Psi|^2 - \mu^2)^2 + \lambda^2 \sigma^2 |\Psi|^2 + \frac{1}{8} \mu^4 \sigma^4 + \cdots. \] (49)

For $\sigma > \sigma_c$, the potential is minimized at $\Psi = \overline{\Psi} = 0$ and inflation is driven by the false vacuum energy density $\mu^4$.

As a result of the positive energy density due to the SUSY breaking during inflation, the mass split is induced between the scalar fields composed of $\Psi$ and $\overline{\Psi}$ with mass squared $M_\pm^2 (\simeq \lambda^2 \sigma^2 / 2 \pm \lambda \mu^2)$ and their superpartner fermions with mass $M = \lambda \sigma / \sqrt{2}$ because these scalar fields different from the inflaton receive the Hubble-induced masses. Such a mass split generates the radiative correction to the potential. By the standard formula [40], the one-loop correction is given by
\[ V_{1L} = \frac{\lambda^2 N}{128\pi^2} \left[ (\lambda \sigma^2 + 2 \mu^2)^2 \ln \frac{\lambda \sigma^2 + 2 \mu^2}{\Lambda^2} + (\lambda \sigma^2 - 2 \mu^2)^2 \ln \frac{\lambda \sigma^2 - 2 \mu^2}{\Lambda^2} - 2 \lambda^2 \sigma^4 \ln \frac{\lambda \sigma^2}{\Lambda^2} \right], \] (50)
where $\Lambda$ is some renormalization scale and $N$ stands for the dimensionality of the representation of the gauge group $G$ to which the fields $\Psi$ and $\overline{\Psi}$ belong. When $\sigma \gg \sigma_c$, it is approximated as
\[ V_{1L} \simeq \frac{\lambda^2 N \mu^4}{8\pi^2} \ln \frac{\sigma}{\sigma_c}. \] (51)

Thus, the effective potential of the inflaton $\sigma$ during inflation is given by
\[ V(\sigma) \simeq \mu^4 \left( 1 + \frac{\lambda^2}{8\pi^2} \ln \frac{\sigma}{\sigma_c} + \frac{1}{8} \sigma^4 \right), \] (52)
with $\tilde{\lambda} \equiv \lambda \sqrt{N}$. This effective potential is dominated by the false vacuum energy $\mu^4$ for $\sigma \ll 1$. On the other hand, the dynamics of $\sigma$ is determined by the competition between the second and the third terms of the right-hand side. Comparing their first derivatives, we find that the dynamics of the inflaton $\sigma$ is dominated by the radiative correction for $\sigma < \sqrt{\tilde{\lambda}/(2\pi)} \equiv \sigma_d$ and by the non-renormalizable term for $\sigma > \sigma_d$. Note that $\sigma_c \ll \sigma_d \ll 1$ for $\mu \ll \tilde{\lambda} \ll 1$.

Then, the total number of $e$-folds during inflation, $N_{\text{total}}$, is given by
\[ N_{\text{total}} = \int_{\sigma_c}^{\sigma_d} \frac{V}{V'} d\sigma \simeq \int_{\sigma_c}^{\sigma_d} \frac{8\pi^2}{\lambda^2} \sigma d\sigma + \int_{\sigma_d}^{\sigma} \frac{2}{3} \sigma^3 d\sigma \simeq \frac{2\pi}{\lambda} + \frac{2\pi}{\lambda} = \frac{4\pi}{\lambda}, \] (53)

3 Strictly speaking, this formula is derived in the Minkowski background. Therefore, we need to modify it for the de Sitter background, where inflation happens.
where \(\sigma_i\) is the initial value of hybrid inflation. We find that the amount of inflation during \(\sigma > \sigma_d\) and that during \(\sigma < \sigma_d\) are about the same with the e-folding number \(\geq 2\pi/\lambda\). Thus in order to achieve sufficiently long inflation \(N_{\text{total}} \gtrsim 60\) to solve the horizon and flatness problems, \(\lambda\) should be rather small: \(\lambda \lesssim 4\pi/60 \simeq 0.2\). In particular, in the case that \(\lambda \lesssim 2\pi/60 \simeq 0.1\), the dynamics of the inflaton \(\sigma\) for the observable universe is determined only by the radiative correction. First of all, we concentrate on this case. Then, the e-folding number is estimated as

\[
N = \int_{\sigma_i}^{\sigma_N} \frac{V}{V'} d\sigma \simeq \int_{\sigma_i}^{\sigma_N} \frac{8\pi^2}{\lambda^2} \sigma d\sigma \simeq \frac{4\pi^2}{\lambda^2} \sigma_N^2,
\]

which yields \(\sigma_N \simeq \lambda\sqrt{N/(2\pi)}\). Here \(\sigma_N\) is a field value corresponding to the e-folding number equal to \(N\). The slow-roll parameters are estimated as

\[
\epsilon \simeq \frac{\lambda^4}{128\pi^4 \sigma_N^4} \simeq \frac{\lambda^2}{32\pi^2 N}, \quad \eta \simeq -\frac{\lambda^2}{8\pi^2 \sigma_N^2} \simeq -\frac{1}{2N}.
\]

The amplitude and the spectral index of primordial fluctuations are given by

\[
R^2 = \frac{1}{24\pi^2} \left( \frac{e^{\nu}}{\epsilon} - \frac{16\pi^2 \mu^2 \sigma_N^2}{3} \right) \simeq \frac{4\mu^4}{3\lambda^4} N, \quad n_s - 1 = -6\epsilon + 2\eta \simeq 2\eta \simeq -\frac{1}{N},
\]

\[
r = 16\epsilon = \frac{\lambda^2}{2\pi^2 N}.
\]

Taking into account \(R^2 = 2.441 \times 10^{-9}\), we obtain \(\mu \simeq 2.4 \times 10^{-3}\sqrt{\lambda}\) for \(N = 60\) and have negligible tensor perturbations. In addition, the spectral index becomes \(n_s \simeq 0.98\) for \(N \simeq 60\) and is just outside the observationally allowed values. However, numerical calculations by using the full one-loop potential (50) are necessary for more accurate estimates because the approximate form of the one-loop potential (51) is justified only for \(\sigma \gg \sigma_c\). Such detailed calculations gave the similar results \(n_s \gtrsim 0.985\). Therefore, more elaborated models to obtain a lower spectral index were proposed in [41].

In fact, we also need to take into account the formation of topological defects at the end of inflation, which is associated with the gauge symmetry breaking. It was pointed out that if we try to embed this model of hybrid inflation into a realistic model of supersymmetric GUT, the formation of cosmic strings is inevitable [42, 43]. Such cosmic strings can contribute to the CMB anisotropies, which gives a severe constraint on the model parameters. Although the prediction of the contribution of cosmic strings to the CMB anisotropies still have some uncertainties, it cannot exceed \(\sim 10\%\) [44, 45], which gives the constraint on the coupling constant \(\lambda\) [43]:

\[
\lambda \lesssim 7 \times 10^{-7} \times \frac{126}{N}.
\]

For the \(SO(10)\) GUT model with \(N = 126\), \(\lambda \lesssim 7 \times 10^{-7}\).

In the case that \(\lambda \lesssim 2\pi/60 \simeq 0.1\), we need to consider both contributions coming from the radiative correction and the non-renormalizable term. Then, the e-folding number is estimated as

\[
N = \int_{\sigma_i}^{\sigma_N} \frac{V}{V'} d\sigma \simeq \int_{\sigma_i}^{\sigma_N} \frac{8\pi^2}{\lambda^2} \sigma d\sigma + \int_{\sigma_d}^{\sigma_N} \frac{2}{\sigma^3} d\sigma \simeq \frac{4\pi}{\lambda} - \frac{1}{\sigma_N^2},
\]

where \(\sigma_N\) is a field value corresponding to the e-folding number equal to \(N\). Calculating the slow-roll parameters \(\epsilon\) and \(\eta\) for the potential (52), we find

\[
\epsilon = \frac{1}{8\sigma^2} \left( \sigma_d^4 + \sigma_N^4 \right)^2, \quad \eta = \frac{1}{2\sigma^2} \left( -\sigma_d^4 + 3\sigma_N^4 \right)^2.
\]
The amplitude of primordial density fluctuations becomes
\[ R^2 = \frac{1}{24\pi^2} \epsilon \left( \frac{\mu^2}{3\pi^2} \frac{\sigma^2}{(\sigma_1^2 + \sigma_4^2)} \right)^2. \] (60)
Inserting \( R^2 \simeq 2.441 \times 10^{-9} \) yields \( \mu \simeq 1.4 \times 10^{-3} \simeq 3.3 \times 10^{15} \) GeV and \( \sigma_N \simeq 0.17 \) for \( N = 60 \) and \( \lambda = 0.13 \). The total \( \epsilon \)-folding number is \( N_{\text{total}} \simeq 97 \) though more correct values need numerical calculations. On the other hand, the spectral index of scalar perturbation is given by
\[ n_s - 1 = -6\epsilon + 2\eta \simeq 2\eta = 3\sigma^2 - \frac{\sigma_4^2}{\sigma_1^2}. \] (61)
Interestingly, the spectral index crosses unity at \( \sigma = \sigma_d/3^{1/4} \sim 0.8\sigma_d \). This is mainly because the spectral index is smaller than unity in the region where the radiative correction dominates, while it is larger than unity in the region where the non-renormalizable term dominates. Such a feature also suggests that this model can generate the large running of the spectral index. Such a large running of the spectral index was first suggested by the WMAP first year result [46] and is slightly preferred even by the WMAP 7 year result with the ACT 2008 data [47], which yields \( n_s = 1.032 \pm 0.039 \) and \( d\eta/d\ln k = -0.034 \pm 0.018 \) at the pivot scale \( k_0 = 0.002 \) Mpc\(^{-1}\) with 68\% CL. The running of the spectral index can be evaluated by using the slow-roll parameters
\[ \frac{dn_s}{d\ln k} = 16\epsilon \eta - 24\epsilon^2 - 2\xi, \quad \xi \equiv \frac{V'''}{V''}. \] (62)
For \( \lambda = 0.13 \), \( d\eta/d\ln k \simeq -4.2 \times 10^{-3} \), whose magnitude is too small to explain the suggested running. As \( \lambda \) becomes larger, the running gets bigger. But large \( \lambda \) leads to a small number of the total \( \epsilon \)-folds \( N_{\text{total}} \). In fact, it is found that \( N_{\text{total}} \) is at most 20 in order to accommodate the running with \( d\eta/d\ln k = \mathcal{O}(10^{-2}) \), which needs another inflation after hybrid inflation to solve the flatness and horizon problems [48]. See [49] for other attempts to explain such large running.

After the inflation ends, the inflaton and the field \( \Phi \) start to oscillate around their minima and then decay into the SM particles to reheat the universe. Since the field \( \Phi \) acquires a non-zero vacuum expectation value (VEV), such decay happens by introducing the higher order terms \( \sum_i c_i |\Phi|^2 |\psi_i|^2 \) in the Kähler potential, which are invariant under the \( R \) symmetry. Here \( \psi_i \) represent the SM particles and the couplings \( c_i \) are of the order of unity. The decay rate of the inflaton becomes \( \Gamma \simeq \sum_i c_i^2 |\Phi_{\min}^2| m_{\psi_i} \), where \( \Phi_{\min} \simeq \sqrt{2}\mu/\sqrt{\lambda} \) and \( m_{\psi_i} \simeq \sqrt{2}\lambda_{\mu} \). Then, the reheating temperature \( T_R \) is evaluated as
\[ T_R \simeq \left( \frac{90}{\chi^2 g_*} \right)^{1/4} \sqrt{\lambda}, \] (63)
with \( C \equiv \sqrt{\sum c_i^2} \). For the above parameters, \( T_R \simeq 4.2 C \times 10^{-8} \simeq 1.0 C \times 10^{11} \) GeV, which is relatively high.

In the above model of hybrid inflation, quantum correction such as the one-loop correction plays an important role in the dynamics. Then, you may wonder whether quantum correction could make the effective potential flat even if the classical potential is steep. Such a possibility is actually what was pointed out in [21] and is called running mass inflation. This is another solution to the \( \eta \) problem. Let us consider a potential of hybrid inflation type:
\[ V(\sigma, \psi) = V_0 f(\psi) + \frac{1}{2} m^2 \sigma^2 + \frac{1}{2} g^2 \sigma^2 \psi^2, \] (64)
where \( \sigma \) is an inflaton and \( \psi \) is a waterfall field staying at the origin during inflation. \( f(\psi) \) is a function of \( \psi \) and takes a maximum value \( f(\psi = 0) = 1 \) at \( \psi = 0 \). Below some critical value
\( \sigma_e, \psi \) is destabilized and quickly rolls down to the global minimum of \( f(\psi) \). \( g \) is a coupling constant. \( m \) is a tree-level mass of the inflaton \( \sigma \) and could be as large as the Hubble parameter, which would rule out \( \sigma \) as an inflaton. However, suppose that the SUSY be explicitly (softly) broken during inflation, quantum correction generates an additional mass squared:

\[
m^2(\sigma) = m^2 + \frac{\lambda}{32\pi^2} \tilde{m}^2 \ln \frac{\sigma}{\Lambda}.
\]  

(65)

Here \( \tilde{m}^2 \) is the soft mass squared, \( \lambda \) is a Yukawa or gauge coupling constant, which is not too small, and \( \Lambda \) is the renormalization scale. Then, the potential near \( m^2(\sigma) = 0 \) is flat enough to support inflation even if the tree-level mass \( m \) is too large. This is the essential idea of the running mass inflation. Making adequate changes of the variables and parameters, the effective potential of the inflaton \( \sigma \) can be recast into [50]

\[
V(\sigma) = V_0 + \frac{1}{2} m^2(\sigma) \sigma^2 = V_0 \left[ 1 + \frac{1}{2} \eta_0 \sigma^2 \left( \ln \frac{\sigma}{\sigma_e} - \frac{1}{2} \right) \right].
\]  

(66)

We can easily find that \( m^2(\sigma_0) = 0 \) at \( \ln \sigma_0/\sigma_e = 1/2 \) and

\[
\frac{V'}{V_0} = \eta_0 \sigma \ln \frac{\sigma}{\sigma_e},
\]  

(67)

which implies that the potential takes a maximum value at \( \sigma = \sigma_e \) by assuming \( \eta_0 < 0 \). The slow-roll parameters are given by

\[
\begin{align*}
\epsilon & \simeq \frac{1}{2} \left( \eta_0 \sigma \ln \frac{\sigma}{\sigma_e} \right)^2 = \mathcal{O} \left( \eta_0^2 \sigma_e^2 \right), \\
\eta & \simeq \eta_0 \left( 1 + \ln \frac{\sigma}{\sigma_e} \right) = \mathcal{O}(\eta_0), \\
\xi & \simeq \eta_0^2 \ln \frac{\sigma}{\sigma_e} = \mathcal{O}(\eta_0^2).
\end{align*}
\]  

(68)

Then, the spectral index and its running are calculated as

\[
\frac{d n_s}{d \ln k} = 16\epsilon - 24\epsilon^2 - 2\xi \simeq -2\xi \simeq -2\eta_0^2 \ln \frac{\sigma}{\sigma_e}.
\]  

(69)

From the last equation, we find that the negative running is obtained for the region with \( \sigma > \sigma_e \). Taking \( |\eta_0| = \mathcal{O}(0.01) \) gives \( n_s - 1 = \mathcal{O}(0.01) \) with the negligible running, which is compatible with the observational results. On the other hand, if we take \( |\eta_0| = \mathcal{O}(0.1) \), \( n_s - 1 = \mathcal{O}(0.1) \) and \( d n_s/d \ln k = \mathcal{O}(10^{-2}) \) as suggested in [47]. The e-folding number is given by

\[
N \simeq \int_{\sigma_0}^{\sigma_*} \frac{V}{V'} d\sigma = \frac{1}{\eta_0} \int_{\sigma_0}^{\sigma_*} \frac{d\sigma}{\sigma \ln \frac{\sigma}{\sigma_e}} = -\frac{1}{|\eta_0|} \left[ \ln \left( \frac{\sigma_*}{\sigma_e} \right) \right].
\]  

(70)

Here \( \sigma_* \) is the field value of \( \sigma \) at the end of inflation, determined either by the critical value \( \sigma_c \) or by the violation of the slow-roll conditions. Then, the following condition must be satisfied at least:

\[
|\eta_*| = |\eta_0| \left( 1 + \ln \frac{\sigma_*}{\sigma_e} \right) \simeq |\eta_0| \ln \frac{\sigma_*}{\sigma_e} \lesssim 1.
\]  

(71)

Inserting equation (70) into this condition yields

\[
\ln \frac{\sigma_N}{\sigma_e} = e^{-N|\eta_0|} \ln \frac{\sigma_e}{\sigma_*} \lesssim \frac{1}{|\eta_0|} e^{-N|\eta_0|} \lesssim 1 \quad \text{for} \quad |\eta_0| \sim 0.1.
\]  

(72)
which shows that $\sigma_N \sim \sigma_*$ for $|\eta_0| \sim 0.1$. The amplitude of the primordial density fluctuations becomes

$$R^2 \simeq \frac{1}{24\pi^2} \left( \frac{V_0}{\eta_0 \sigma_N \ln \frac{\sigma_N}{\sigma_*}} \right)^2.$$  \hspace{1cm} (73)

Inserting $R^2 \simeq 2.441 \times 10^{-6}$ gives the relation $V_0^{1/4} \simeq 2.8 \times 10^{-2} \sqrt{\eta_0 \sigma_N \ln \frac{\sigma_N}{\sigma_*}}$. For $|\eta_0| \sim 0.1$ and $\sigma_N \sim \sigma_*$, $V_0^{1/4} \sim 8.7 \times 10^{-3} \sigma_0^{1/2}$. 

Like the new inflation, hybrid inflation may have severe initial value problem, which states that only very narrow range of initial field values (less than unity) can lead to successful inflation [51]. Though the recent paper claims the opposite result [52], there is still another initial value problem, that is, why an inflaton is homogeneous over the Hubble horizon scale at the onset of inflation. The initial value problem may be solved by considering pre-inflation preceding hybrid inflation [53] in the same way as new inflation. Such double inflation scenario can produce non-trivial features of primordial fluctuations such as the break of the spectral index and the formation of the primordial black holes.

### 3.1.3. Chaotic inflation.

Chaotic inflation is the most natural inflation in that it does not suffer from any initial condition problem simply because it can start around the Planck time or the Planck energy density scale. Since the other inflations occur at later time or lower scale, the universe would recollapse before inflation starts, unless the universe is open at the beginning. In addition, the other models suffer from the initial value problem, that is, why the inflaton field is homogeneous over the horizon scale and takes a value which can lead to successful inflation.

A simple power-law potential $V(\phi) = \lambda_n \phi^n/n$ ($\lambda_n \ll 1$ : a coupling constant) can accommodate chaotic inflation. The slow-roll parameters are given by

$$\epsilon \simeq \frac{n^2}{2\phi^2}, \hspace{1cm} \eta \simeq \frac{n(n-1)}{\phi^2},$$  \hspace{1cm} (74)

which requires the field value of the inflaton to be larger than unity for successful inflation. That is, chaotic inflation started around the Planck scale, where the field value is much larger than unity (and also is stochastic), and then it ended around $\phi_e \sim n$. The e-folding number becomes $N \simeq \frac{\phi_e^2}{2n}$ and the observable quantities are expressed in terms of $N$ as

$$R^2 = \frac{1}{24\pi^2} \epsilon = \frac{\lambda_n}{12\pi^2 n^2} \phi_N^{n+2} \simeq \frac{\lambda_n}{12\pi^2 n^3} (2nN)^n, \hspace{1cm} (75)$$

$$n_s - 1 = -6\epsilon + 2\eta \simeq -\frac{n(n+2)}{\phi_N^2} \simeq - \frac{n+2}{2N},$$

$$r = 16\epsilon = \frac{8n^2}{\phi_N^2} \simeq \frac{4n}{N}.$$  \hspace{1cm} (76)

The constraint on the tensor-to-scalar ratio $r < 0.24$ leads to $n < 0.06N \lesssim 3.6$ for $N \lesssim 60$, which rules out the case with $n \geq 4$ if the primordial density fluctuations are mainly generated by the inflaton. Though another source of the primordial density fluctuations like the curvaton and the modulated reheating mechanism can save the case with $n \geq 4$ [54], we do not consider such a case in this review.

As shown above, a large field value of a would be inflaton is required to cause chaotic inflation. However, the exponential factor appearing in the F-term potential prevents any scalar field from taking a value larger than unity, provided that the Kähler potential is almost canonical. Thus, it is extremely difficult to incorporate chaotic inflation in supergravity, even if
we can solve the η problem somehow. Though chaotic inflation was proposed in some models [55], most of them used a rather specific Kähler potential, which was fine-tuned without the symmetry reason.

In this review, following [56], we introduce the Nambu–Goldstone-like shift symmetry of the inflaton superfield Φ in order to naturally realize chaotic inflation in supergravity. We require the Kähler potential $K(\Phi, \Phi^*)$ to be invariant under the shift of Φ:

$$\Phi \rightarrow \Phi + iC,$$

(76)

where C is a real parameter. Then, the Kähler potential is a function of $\Phi + \Phi^*$:

$$K(\Phi + \Phi^*) = K(\Phi, \Phi^*),$$

which implies that the imaginary part of the scalar components of Φ does not appear in the exponential factor of the F-term and hence can take a value larger than unity. Note that this model solves the η problem as well. However, as long as the shift symmetry is exact, the potential is completely flat along the inflaton direction. Therefore, in order to cause inflation, a small breaking term of the shift symmetry must be introduced.

As such a breaking term, we consider a small mass term in the superpotential by introducing another superfield $X$:

$$W = mX \Phi.$$

(77)

Note that this model is natural in 't Hooft's sense [57] because we have an enhanced symmetry (the shift symmetry) in the limit $m \rightarrow 0$. Neglecting the induced breaking terms such as $K \simeq |m\Phi|^2 + \cdots$, which are irrelevant for the dynamics of the inflaton, we consider the following Kähler potential:

$$K = \frac{1}{2}(\Phi + \Phi^*)^2 + XX^*,$$

(78)

which gives the canonical kinetic terms for Φ and X. This model possesses the $R$ symmetry under which

$$X(\theta) \rightarrow e^{2i\alpha} X(\theta e^{i\alpha}), \quad \Phi(\theta) \rightarrow \Phi(\theta e^{i\alpha}),$$

(79)

and $Z_2$ symmetry under which

$$X(\theta) \rightarrow -X(\theta), \quad \Phi(\theta) \rightarrow -\Phi(\theta).$$

(80)

By inserting the forms of the Kähler potential and the superpotential into the formulae (15) and (17), the Lagrangian density $L(\zeta, \varphi, X)$ becomes

$$L(\zeta, \varphi, X) = -\frac{1}{2} \partial_\mu \zeta \partial^\mu \zeta - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \partial_\mu X \partial^\mu X^* - V(\zeta, \varphi, X),$$

(81)

with the potential $V(\zeta, \varphi, X)$ given by

$$V(\zeta, \varphi, X) = m^2 \exp(\zeta^2 + |X|^2) \left[ \frac{1}{2}(\zeta^2 + \varphi^2)(1 + |X|^4) + |X|^2 \right] \times \left[ 1 - \frac{1}{2}(\zeta^2 + \varphi^2) + 2\zeta^2 (1 + \frac{1}{2}(\zeta^2 + \varphi^2)) \right].$$

(82)

Here we have decomposed Φ into a real part $\zeta$ and an imaginary one $\varphi$:

$$\Phi = \frac{1}{\sqrt{2}}(\zeta + i\varphi),$$

(83)

and identify $\varphi$ with the inflaton. While the inflaton $\varphi$ can have a value much larger than unity, $|\zeta|, |X| \lesssim 1$ because of the presence of $e^K$ factor. Then, the potential is rewritten as

$$V(\zeta, \varphi, X) \simeq \frac{1}{2} m^2 \zeta^2 (1 + \zeta^2) + m^2 |X|^2.$$

(84)

The effective mass of $\zeta$ is larger than the Hubble parameter during inflation; it quickly rolls down to zero. On the other hand, though the effective mass of $X$ is not larger than the Hubble
parameter, we can easily show that it is irrelevant for the dynamics and the primordial density fluctuation. Then, \( X \) can be safely set to zero, which yields, together with \( \zeta = 0 \),

\[
V(\varphi) \simeq \frac{1}{2} m^2 \varphi^2. \tag{85}
\]

Thus, a simple power-law potential can be naturally obtained in the context of supergravity.

After the inflation ends, an inflaton starts to oscillate around the origin and then decays into the SM particles to reheat the universe. Such inflaton decay can occur, for example, by introducing the following superpotential:

\[
W = g X H_u H_d, \tag{86}
\]

where \( H_u \) and \( H_d \) are a pair of Higgs doublets, whose \( R \) charge's are assumed to be zeros, and \( g \) is a coupling constant. Then, we have a coupling of the inflaton \( \varphi \) to the Higgs doublets as

\[
L \sim g m \varphi H_u H_d, \tag{87}
\]

which gives the reheating temperature

\[
T_R \sim 10^9 \text{ GeV} \left( \frac{g}{10^{-5}} \right) \left( \frac{m}{10^{13} \text{ GeV}} \right)^{1/2}. \tag{88}
\]

By inserting \( n = 2 \) and \( N = 60 \) to formulae (75), we obtain the spectral index \( n_s \simeq 0.967 \) and the inflaton mass \( m = \sqrt{2} \simeq 6.3 \times 10^{-6} = 1.5 \times 10^{13} \text{ GeV} \). The large tensor-to-scalar ratio is also predicted to be \( r \simeq 0.13 \), which is still compatible with the present observations and will be detected or ruled out. Then, it may be interesting to consider the chaotic inflation model with a lower \( n \) and even a fractional one. In fact, such a chaotic inflation model with a lower power-law index was proposed in the context of superstring [58].

Even in the context of supergravity, it is easy to realize such a model of chaotic inflation by extending the Nambu–Goldstone-like shift symmetry, which is named running kinetic inflation. According to [59], we impose the following type of shift symmetry on a composite field \( \Phi \):

\[
\Phi^2 \rightarrow \Phi^2 + C, \tag{89}
\]

where \( \Phi \neq 0 \) and \( C \) is a real parameter. Then, the Kähler potential is a function of \( (\Phi^2 - \Phi^* \Phi^2) \), that is,

\[
K(\Phi, \Phi^*) = i c (\Phi^2 - \Phi^2) - \frac{1}{4} (\Phi^2 - \Phi^* \Phi^2)^2 + \cdots, \tag{90}
\]

where \( c \) is a real parameter of the order of unity. As long as this shift symmetry is exact, the potential is completely flat along the direction of the shift and the kinetic term is singular at the origin. Then, small breaking terms of the shift symmetry are necessary for the Kähler potential as well as the superpotential. We add the following breaking term to the Kähler potential:

\[
K = \kappa |\Phi|^2, \tag{91}
\]

with \( \kappa \ll 1 \). This term cures the singular behavior of the kinetic term of \( \Phi \) at the origin. On the other hand, we consider a small mass term as a breaking term in the superpotential:

\[
W = m X \Phi, \tag{92}
\]

Here \( m \ll 1 \) and we have introduced another superfield \( X \), which is assumed to have the canonical Kähler potential. Note that, though we have introduced the breaking terms, this model is still natural in ’t Hooft’s sense [57] because we have an enhanced symmetry (the shift

4 If we take into account a higher order term \(-c|X|^4 \) \( (c \gg 1) \) positive constant) in the Kähler potential, it gives \( X \) an additional mass larger than the Hubble parameter, which quickly drives \( X \) to zero.
symmetry) in the limit \( m \to 0 \) and \( \kappa \to 0 \). Then, the total Kähler potential we consider here is given by

\[
K = \kappa |\Phi|^2 + ic(\Phi^2 - \Phi^{*2}) - \frac{1}{4} (\Phi^2 - \Phi^{*2})^2 + |X|^2.
\]

(93)

which yields the following kinetic terms:

\[
L_{\text{kin}} = \frac{1}{2} \kappa (\Phi^2 + \phi^2 + \chi^2) \left( \partial_\mu \Phi \partial^\mu \Phi - \partial_\mu X \partial^\mu X^* \right) + \partial_\mu X \partial^\mu X^*.
\]

(94)

Here we have decomposed \( \Phi \) into a real part \( \phi \) and an imaginary one \( \chi \),

\[
\Phi = \frac{1}{\sqrt{2}} (\phi + i\chi).
\]

(95)

and identify \( \phi \) with the inflaton. Though the potential term is a bit complicated, we can safely set \( X = 0 \) in the same way as the original chaotic inflation model in supergravity. Then, the potential at \( X = 0 \) reads

\[
V = e^{\kappa} m^2 |\Phi|^2 = \frac{1}{2} \exp \left[ \frac{\kappa}{2} (\phi^2 + \chi^2) - 2c\phi\chi + \phi^2 \chi^2 \right] m^2 (\phi^2 + \chi^2).
\]

(96)

You can find the flat direction \( \phi\chi = \text{constant} \) reflecting the shift symmetry (89). For a large value of \( |\phi| \), \( \chi \) is determined such that the Kähler potential takes a minimum, that is,

\[
\chi \simeq \frac{c}{\phi}.
\]

(97)

In this limit \( |\phi| \gg 1 \), the potential reduces to

\[
V \simeq \exp \left[ \frac{\kappa}{2} \left( \phi^2 + \frac{c^2}{\phi^2} \right) - c^2 \right] m^2 \left( \phi^2 + \frac{c^2}{\phi^2} \right).
\]

(98)

Since \( \kappa \ll 1 \), the exponential factor is at most of the order of unity for \( |\phi| \lesssim 1/\sqrt{\kappa} \). Then, for \( 1 \ll \phi \ll 1/\sqrt{\kappa} \), the effective Lagrangian is approximated as

\[
L \simeq -\frac{1}{2} \phi^2 \delta_\mu \phi \delta^\mu \phi - \frac{1}{2} \overline{m}^2 \phi^2
\]

with \( \overline{m}^2 \equiv e^{-c^2} m^2 \). By taking the canonically normalized field \( \varphi \equiv \phi^2/2 \), the effective Lagrangian is rewritten as

\[
L \simeq -\frac{1}{2} \delta_\mu \varphi \delta^\mu \varphi - \overline{m}^2 \varphi
\]

(100)

for \( 1 \ll \varphi \ll 1/\kappa \). Thus, chaotic inflation with the linear potential is realized in supergravity. If we introduce another type of shift symmetry, under which

\[
\Phi^a \longrightarrow \Phi^a + C,
\]

(101)

and the superpotential \( W \propto \Phi^a X \), we have the chaotic inflation with the effective potential \( V \propto \varphi^{2m/n} \). Thus, this model can even possess a fractional power.

Although it is not usually stressed, the fact is crucially important that the superpotential depends on another field \( X \) other than the inflaton and is linear in it, as given in the previous examples. This superpotential vanishes at \( X = 0 \), which guarantees the positivity of the potential. In fact, a lot of attempts to realize chaotic inflation in supergravity have been hindered by the dangerous negative term \( -3 |W|^2 \) of the potential in supergravity. On the other hand, the fact that \( \partial W/\partial X \) does not depend on \( X \) allows the inflaton to acquire the potential depending only on the inflaton. Recently, by using this type of superpotential, Kallosh et al gave the prescription to construct arbitrary potential in supergravity [60]. According to [60], we give such a prescription and the criterion for the stability of the given potential.

First of all, we consider the superpotential linear in \( X \),

\[
W = X f (\Phi),
\]

(102)
where \( f(\Phi) \) is a real holomorphic function, that is, \( f^*(\Phi) = f(\Phi) \). This type of superpotential can be easily obtained by imposing the \( R \) symmetry with the \( R \) charges of \( X \) and \( \Phi \) to be 2 and 0, respectively. The real part of \( \Phi \) will be identified with the inflaton. On the other hand, \( \text{Im} \Phi \) and \( X \) will be set to zeros during inflation.

We assume that the Kähler potential \( K(\Phi, \Phi^*, X, X^*) \) is separately invariant under the following transformations:

\[
X \mapsto -X, \quad \Phi \mapsto \Phi^*, \quad \Phi \mapsto \Phi + C,
\]

where \( C \) is a real parameter. This \( Z_2 \) symmetry on \( X \) guarantees \( K_X = K_{X^*} = K_{X^*\Phi} = K_{X^*\Phi^*} = 0 \) and \( \partial V/\partial X = \partial V/\partial X^* = 0 \) at \( X = 0 \). Since \( D_\Phi W = 0 \) and \( D_X W = f(\Phi) \) at \( X = 0 \), the potential at \( X = 0 \) becomes

\[
V = e^{S(\Phi, \Phi^*, 0, 0)} f^2(\Phi) K^{-1}_{XX}(\Phi, \Phi^*, 0, 0).
\]

By virtue of the shift symmetry, the Kähler potential only depends on \( \text{Im} \Phi \) and \( X \) become positive and larger than the Hubble parameter squared under some conditions, which will be discussed later. In this case, we can safely set \( \text{Im} \Phi \) and \( X \) to be zeros. Then, the potential at \( \text{Im} \Phi = X = 0 \) is rewritten as

\[
V = e^{S(\Phi, \Phi^*, 0, 0)} f^2(\text{Re} \Phi) K^{-1}_{XX}(0, 0, 0, 0).
\]

By use of the Kähler transformation, we can always set \( K(d, 0, 0, 0, 0) = 0 \), corresponding to the rescaling of \( f(\Phi) \). By rescaling of the fields, we can also set \( K^{-1}_{XX}(0, 0, 0, 0) = K_{\Phi\Phi}(0, 0, 0, 0) = 1 \), which, together with \( K_{X\Phi^*} = K_{X^*\Phi} = 0 \) at \( X = 0 \), means the canonical kinetic terms of \( X \) and \( \Phi \) at the origin. Thus, the potential reduces to

\[
V = f^2(\text{Re} \Phi) \geq 0.
\]

By decomposing the complex scalar field \( \Phi \) into a real part \( \phi \) and an imaginary one \( \chi \),

\[
\Phi = \frac{1}{\sqrt{2}} (\phi + i\chi),
\]

the potential is rewritten in terms of the inflaton \( \phi \) as

\[
V = f^2 \left( \frac{\phi}{\sqrt{2}} \right) \geq 0.
\]

In order to investigate the stability conditions during inflation, after some calculations, we obtain the effective masses squared at \( X = 0 \) (namely the inflationary trajectory \( \text{Im} \Phi = X = 0 \)),

\[
m^2_X = 2 (1 - K_{\Phi\Phi\cdot XX^*}) f^2 + \left( \frac{df}{d\Phi} \right)^2 - f \frac{d^2f}{d\Phi^2} \simeq 3H^2 \left[ 2 (1 - K_{\Phi\Phi\cdot XX^*}) 2\epsilon - \eta \right],
\]

\[
m^2_X = -K_{XX\cdot XX^*} f^2 + \left( \frac{df}{d\Phi} \right)^2 \simeq 3H^2 \left[ -K_{XX\cdot XX^*} + \epsilon \right],
\]

where the slow-roll parameters are given by \( \epsilon = (d \ln f/d\Phi)^2 \), \( \eta = d^2 \ln f/d\Phi^2 + 2\epsilon \). Here and hereafter, we assume that the Kähler potential only depends on the combination \( XX^* \), though the more general case makes \( X \) less stable. The effective mass squared along the inflationary trajectory is positive if the following conditions are satisfied:

\[
K_{\Phi\Phi\cdot XX^*} + \frac{\eta}{2} - \epsilon \leq 1, \quad K_{XX\cdot XX^*} - \epsilon \leq 0.
\]

More stringent conditions that the effective mass squared along the inflationary trajectory is larger than the Hubble parameter squared, which quickly drive \( \text{Im} \Phi \) and \( X \) to zeros, require that

\[
K_{\Phi\Phi\cdot XX^*} \lesssim \frac{5}{6}, \quad K_{XX\cdot XX^*} \lesssim -\frac{1}{3}.
\]

Thus, as long as we have the Kähler potential satisfying the above conditions, we can construct an arbitrary potential from the function \( f \) in supergravity.
3.1.4. Topological inflation. As another type of large field inflation, we consider topological inflation [61] in this subsection. Though topological inflation has a similar form of the potential as that of new inflation, it has interesting features. First of all, different from other low scale inflations, it is free from the initial value problem, that is, why the initial field is homogeneous over the horizon scale and is fine-tuned to the small region. Therefore, as long as the universe is open at the beginning, topological inflation can occur without fine-tuning. Observationally, it predicts a significant amount of the tensor-to-scalar ratio, which will be confirmed or ruled out in the near future. Thus, topological inflation is still attractive.

Topological inflation is realized by the symmetry-breaking potential such as

\[
V(\phi_i) = \lambda (\phi_i^2 - \langle \phi_i \rangle^2)^2, \tag{112}
\]

where \( \phi_i^2 = \sum_{n=1}^N \phi_i^n \) and \( \lambda \) is a coupling constant. Domain walls, strings, and monopoles would be formed for \( n = 1, 2, 3 \) unless inflation happens. In order to support inflation, the VEV of \( \langle \phi_i \rangle \), must be larger than unity. This can be understood from the following simple discussion. The typical radius of topological defect \( R \sim 1/(\sqrt{\lambda} \langle \phi \rangle) \) is determined by equating the gradient energy density \( (\langle \phi \rangle / R)^2 \) and the potential energy density \( \lambda \langle \phi \rangle^4 \). For topological inflation to occur, the typical radius \( R \) must be larger than the Hubble radius given by \( H^{-1} \sim 1/(\sqrt{\lambda} \langle \phi \rangle)^3 \), which leads to the condition \( \langle \phi \rangle \gtrsim 1 \). In fact, numerical calculations give more precise conditions \( \langle \phi \rangle \gtrsim 1.7 \) [62] for a double-well potential and \( \langle \phi \rangle \gtrsim 0.95 \) [63] for the supergravity model [64]. Therefore, the exponential factor in the \( F \)-term potential is again problematic in constructing the topological inflation model in supergravity. Then, we give a model given in [65], in which the Nambu–Goldstone-like shift symmetry is used to avoid the above problem.

Let us consider the following \( \text{K"{a}hler} \) potential:

\[
K = -\frac{1}{2}(\Phi - \Phi^*)^2 + |X|^2, \tag{113}
\]

which is invariant under the shift symmetry

\[
\Phi \rightarrow \Phi + C \tag{114}
\]

with \( C \) a real parameter. As small breaking of the shift symmetry, we introduce the following superpotential:

\[
W = X(v - u \Phi^2) = v X (1 - g \Phi^2), \tag{115}
\]

which is invariant under the \( R \) symmetry with \( R \) charges of \( X \) and \( \Phi \) to be 2 and 0, and under the \( \mathbb{Z}_2 \) symmetry with \( X \) and \( \Phi \) to be even and odd, respectively. While \( v \) is of the order of unity, \( u \ll 1 \) and \( g \equiv u/v \ll 1 \), representing the small breaking of the shift symmetry. The Lagrangian density \( \mathcal{L}(\Phi, X) \) for the scalar fields \( \Phi \) and \( X \) is given by

\[
\mathcal{L}(\Phi, X) = -\partial_\mu \Phi \partial^\mu \Phi^* - \partial_\mu X \partial^\mu X^* - V(\Phi, X), \tag{116}
\]

with the scalar potential \( V \) given by

\[
V = v^2 e^K [(1 - g \Phi^2)^2(1 - |X|^2 + |X|^2) + |X|^2(2g \Phi^2 + (\Phi - \Phi^*)(1 - g \Phi^2)^2)]. \tag{117}
\]

By decomposing the scalar field \( \Phi \) into the real component \( \phi \) and the imaginary one \( \chi \),

\[
\Phi = \frac{1}{\sqrt{2}}(\phi + i \chi), \tag{118}
\]

we can easily show that the effective mass squared of \( \chi \) becomes \( m_\chi^2 \simeq 6 H^2 \), which quickly drives \( \chi \) to zero. On the other hand, though the effective mass of \( X \) is not larger than the Hubble parameter\(^5\), it is clear that \( X \) is irrelevant for the dynamics and the primordial density

\(^5\) Considering higher order terms in the \( \text{K"{a}hler} \) potential can give \( X \) an additional mass larger than the Hubble parameter, which makes \( X \) go to zero quickly.
fluctuation, and hence \( X \) can be safely set to zero. The potential for the inflaton \( \phi \) reduces to

\[
V = v^2 \left( 1 - \frac{g}{2} \phi^2 \right)^2
\]

\[
\simeq v^2 (1 - g\phi^2)
\text{ for } \phi \lesssim 1.
\]

(119)

Then, the e-folding number \( N \) is given by

\[
N \simeq \int_{\phi_0}^{\phi_f} \frac{V}{V} = \frac{1}{2g} \ln \left( \frac{\phi_f}{\phi_N} \right),
\]

(120)

which gives \( \phi_N \sim \phi_f e^{-2eN} \sim \sqrt{\frac{2}{g}} e^{-2eN} \). Here \( \phi_f \sim \sqrt{\frac{2}{g}} \) is the value of \( \phi \) at the end of topological inflation. On the other hand, the slow-roll parameters are estimated as

\[
e \simeq 2g^2 \phi_N^2 \simeq 4g e^{-4eN}, \quad \eta \simeq -2g.
\]

(121)

Then, the observable quantities are given by

\[
R^2 = \frac{1}{24\pi^2} \frac{V}{\epsilon} = \frac{v^2}{48\pi^2 g^2 \phi_N^2} \simeq \frac{v^2}{96\pi^2 g} e^{4eN},
\]

\[
n_s - 1 = -6\epsilon + 2\eta \simeq 2\eta = -4g,
\]

\[
r = 16\epsilon = 32g^2 \phi_N^2 \simeq 64g e^{-4eN}.
\]

For \( g = 0.01 \) and \( N = 60, n_s = 0.96, v \geq 8.3 \times 10^{-5} \), and \( \phi_N \sim 4.3 \). This model also predicts a significant amount of the tensor perturbation \( r \simeq 0.06 \) for \( g = 0.01 \), which will be confirmed or ruled out in the near future.

After topological inflation ends, the inflaton rapidly oscillates around the global minimum \( \langle \phi \rangle \equiv \pm \sqrt{2/g} \) and decays into the standard particles to reheat the universe. The decay of the inflaton can take place if we consider higher order terms \( u' (\Phi^2 + \Phi^2 \eta) \) in the Kähler potential. Here \( u' \) is a constant associated with the breaking of the shift symmetry with \( O(u) = O(u') \) and \( \psi_i \) are the standard particles. The decay rate of the inflaton becomes \( \Gamma \sim u^2 \langle \phi \rangle^2 m_\phi^2 \sim u^2 \sqrt{g} v^5 \) with \( \langle \phi \rangle^2 = 2/g \) and \( m_\phi \simeq 2 \sqrt{g} v \). Then, the reheating temperature is given by

\[
T_R \simeq \left( \frac{90}{\pi^2 g_*} \right)^{\frac{1}{4}} \sqrt{\Gamma} \sim u' \frac{g^2}{4} v^\frac{7}{2} \sim \frac{g^4}{4} v^\frac{7}{2},
\]

(123)

where we have used \( u' \sim u = g v \). For \( g = 0.01, T_R \sim 2.0 \times 10^{-13} \sim 4.8 \times 10^5 \text{ GeV} \).

Finally, we comment on natural inflation as another type of large field inflation, which was proposed in [66]. It is attractive in that it can predict a significant amount of tensor fluctuations and the initial value problem is less severe. Such a model in the context of supergravity is discussed in [67], for example.

### 3.2. D-term inflation

In the previous subsection, we have discussed inflation models based on the \( F \)-term potential. The main obstacle to construct the inflation model in supergravity comes from the \( F \)-term, specifically, the exponential factor appearing in it. Therefore, if we can obtain the positive potential energy in the \( D \)-term, it can lead to successful inflation without the \( \eta \) problem, which was first pointed out by Stewart [5]. In this subsection, we give concrete examples of inflation model supported by the \( D \)-term.

---

6 Once the inflaton acquires a non-vanishing VEV, it can decay into the SM particles through supergravity effects [24], which predict higher reheating temperature in this model.
3.2.1. Hybrid inflation. Let us consider a $D$-term model of hybrid inflation proposed in [68]. (See also [69].) We introduce the following superpotential:

$$W = \lambda S\Phi_+\Phi_-,$$

(124)

where $S$, $\Phi_+$, and $\Phi_-$ are three (chiral) superfields, and $\lambda$ is a coupling constant. This superpotential is invariant under a $U(1)$ gauge symmetry, whose charges are assigned to be $0, +1, -1$ for the fields $S$, $\Phi_+$, $\Phi_-$, respectively\footnote{Strictly speaking, we need to modify the assignment of the charges $q_+$ and $q_-$ for $\Phi_+$ and $\Phi_-$ such that $q_+ = 1 - \xi/2$ and $q_- = -1 - \xi/2$ for the non-vanishing FI term $\xi$ in supergravity [70]. However, since $\xi \ll 1$ as shown later, $q_+$ and $q_-$ are approximated as $q_+ \simeq 1$ and $q_- \simeq -1$.}. It also possesses the $R$ symmetry, under which they are transformed as

$$S(\theta) \longrightarrow e^{2i\theta} S(\theta e^{3i\theta}), \quad \Phi_+ \Phi_-(\theta) \longrightarrow \Phi_+ \Phi_-(\theta e^{i\theta}).$$

(125)

We take the canonical Kähler potential invariant under the gauge and the $R$ symmetries,

$$K = |S|^2 + |\Phi_+|^2 + |\Phi_-|^2.$$

(126)

The tree level scalar potential is given by the standard formulae (17) and (19),

$$V(S, \Phi_+, \Phi_-) = \lambda^2 e^{S^2} |\Phi_+|^2 + |\Phi_-|^2 + |S\Phi_+|^2 + |S\Phi_-|^2 + (|S|^2 + |\Phi_+|^2 + |\Phi_-|^2 + 3)|S\Phi_+\Phi_-|^2 + \frac{g^2}{2}(|\Phi_+|^2 - |\Phi_-|^2 + \xi)^2,$$

(127)

where $g$ is a gauge coupling constant, $\xi > 0$ is a non-vanishing FI term, and we have taken a minimal gauge kinetic function. This potential possesses the unique global minimum $V = 0$ at

$$S = \Phi_+ = 0, \quad \Phi_- = \sqrt{\xi}.$$  

(128)

However, for a large value of $|S|$, the potential has a local minimum with positive energy density at

$$\Phi_+ = \Phi_- = 0.$$  

(129)

In order to find the critical value $S_c$ of $|S|$, we calculate the mass matrix of $\Phi_+$ and $\Phi_-$ at the inflationary trajectory $\Phi_+ = \Phi_- = 0$, which is given by

$$V_{\text{mass}} = m_+^2 |\Phi_+|^2 + m_-^2 |\Phi_-|^2,$$

(130)

with

$$m_+^2 = \lambda^2 |S|^2 e^{S^2} + g^2 \xi, \quad m_-^2 = \lambda^2 |S|^2 e^{S^2} - g^2 \xi.$$  

(131)

Thus, as long as $m_-^2 > 0$, which is equivalent to $|S| > S_c \simeq g \sqrt{\xi}/\lambda$ for $S_c \ll 1$, the local minimum $\Phi_+ = \Phi_- = 0$ is stable so that inflation is driven by the positive potential energy density $g^2 \xi^2/2$. In addition, such a mass split generates quantum correction calculated by the standard formula [40]

$$V_{\text{IL}} = \frac{1}{32\pi^2} \left[\left(\lambda^2 |S|^2 e^{S^2} + g^2 \xi\right)^2 \ln\left(\frac{\lambda^2 |S|^2 e^{S^2} + g^2 \xi}{\Lambda^2}\right)ight]$$

$$+ \left(\lambda^2 |S|^2 e^{S^2} - g^2 \xi\right)^2 \ln\left(\frac{\lambda^2 |S|^2 e^{S^2} - g^2 \xi}{\Lambda^2}\right)$$

$$- 2\lambda^4 |S|^4 e^{2S^2} \ln\left(\frac{\lambda^2 |S|^2 e^{S^2}}{\Lambda^2}\right).$$

(132)
where $\Lambda$ is some renormalization scale. When $|S| \gg S_c$, it is approximated as

$$V_{1L} \approx \frac{g^4 \xi^2}{16\pi^2} \ln \left( \ln \frac{\lambda^2 |S|^2 e^{iS^2}}{\Lambda^2} \right) + \frac{3}{2} \ln \frac{\lambda^2 |S|^2 e^{iS^2}}{\Lambda^2}.$$  \hfill (133)

Thus, the effective potential of $S$ during inflation is given by

$$V(S) \approx \frac{g^2 \xi^2}{2} \left[ 1 + \frac{g^2}{8\pi^2} \ln \left( \ln \frac{\lambda^2 |S|^2 e^{iS^2}}{\Lambda^2} \right) \right].$$  \hfill (134)

Since the above potential does not depend on the phase of the complex scalar field $S$, we identify its real part $\sigma \equiv \sqrt{2} \text{Re } S$ with the inflaton without loss of generality. Then, for $\sigma_c \ll \sigma \ll 1$, the effective potential of the inflaton $\sigma$ during inflation is given by

$$V(\sigma) \approx \frac{g^2 \xi^2}{2} \left[ 1 + \frac{g^2}{8\pi^2} \ln \left( \frac{\lambda^2 \sigma^2}{2\Lambda^2} \right) \right].$$  \hfill (135)

The slow-roll parameters are estimated as

$$\epsilon \approx \frac{g^4}{32\pi^4 \sigma^2}, \quad \eta \approx -\frac{g^2}{4\pi^2 \sigma^2}.$$  \hfill (136)

The inflation ends if the inflaton $\sigma$ reach $\sigma_c \equiv \sqrt{S_c}$ or $\sigma_f \equiv g/(2\pi)$ corresponding to $|\eta| = 1$. The e-folding number is given by

$$N \approx \int_{\sigma_n}^{\sigma_f} \frac{4\pi^2 \sigma}{g^2} d\sigma \approx \frac{2\pi^2}{g^2} \left( \sigma_f^2 - \sigma_n^2 \right),$$  \hfill (137)

where $\sigma_n = \max(\sigma_c, \sigma_f)$. In the case that the coupling $\lambda$ is not too small and $\lambda > 2\pi \sqrt{2\xi} \sim 0.01$, $\sigma_n = \sigma_f$. In this case, $\sigma_n^2 \approx g^2 N/(2\pi^2)$, $\epsilon \approx g^2/(16\pi^2 N)$, and $\eta \approx -1/(2N)$. Then, the observable quantities are expressed as

$$R^2 = \frac{1}{24\pi^2} \frac{V}{\epsilon} \approx \frac{N}{3} \xi^2,$$

$$n_s - 1 = -6\epsilon + 2\eta \approx -\frac{1}{N},$$

$$r = 16\epsilon \approx \frac{g^2}{\pi^2 N}.$$  \hfill (138)

Inserting $R^2 \approx 2.441 \times 10^{-9}$ and $N = 60$ yields $\xi \approx 1.1 \times 10^{-5}$, namely $\sqrt{\xi} \approx 3.3 \times 10^{-3} \approx 8.0 \times 10^{15}$ GeV. On the other hand, the spectral index becomes $n_s \approx 0.98$ for $N = 60$ and hence is just outside the observed values. However, cosmic strings are always formed at the end of inflation because the $U(1)$ gauge symmetry is broken. Such cosmic strings can contribute to the CMB anisotropies, but their contributions cannot exceed $\sim 10\%$ [44, 45]. Thus, the model parameters are severely constrained. The detailed calculations [43, 71] give the following constraints:

$$g \lesssim 2 \times 10^{-2} \quad \text{and} \quad \lambda \lesssim 3 \times 10^{-5},$$  \hfill (139)

which is equivalent to $\sqrt{\xi} \lesssim 2 \times 10^{15}$ GeV.\footnote{Recent results give a bit stronger constraint [72].} This constraint can be relaxed by introducing a non-minimal Kähler potential [73] or curvaton mechanism [45], for example.
3.2.2. Chaotic inflation. Though almost all of the inflation models based on the D-term potential belong to the hybrid inflation type, it was recently pointed out that chaotic inflation can be realized by the D-term potential as well [74]. Here, following [75], we show how to accommodate chaotic inflation in the D-term potential.

Let us consider the following superpotential:

\[ W = \lambda S(X\overline{X} - \mu^2), \]

where we have introduced three superfields \( S, X, \overline{X} \) charged under \( U(1) \) gauge symmetry and (global) \( U(1)_R \) symmetry. The \( U(1) \) gauge charges of \( S, X, \overline{X} \) are \( 0, +1, -1 \) and their \( R \) charges are \( +2, 0, 0 \), respectively. We set the constants \( \lambda \) and \( \mu \) to be real and positive for simplicity. Taking the canonical Kähler potential \( K = |S|^2 + |X|^2 + |\overline{X}|^2 \) and the minimal gauge kinetic function \( f_{ab}(\Phi_i) = 1 \), the scalar potential reads

\[ V = V_F + V_D, \]

where the phase \( \theta \) is the coupling constant of the \( U(1) \) gauge symmetry. Note that we do not need to introduce a non-vanishing FI term. The minima of the \( F \)-term (the \( F \)-flat condition, \( V_F = 0 \)) are given by

\[ X\overline{X} - \mu^2 = 0 \quad \text{and} \quad S = 0, \]

and the minima of the \( D \)-term (the \( D \)-flat condition, \( V_D = 0 \)) are given by

\[ |X| = |\overline{X}|, \]

Then, the potential takes the global minima at

\[ S = 0, \quad X = \mu e^{i\theta}, \quad \overline{X} = \mu e^{-i\theta}, \]

where the phase \( \theta \) can be set to zero by the \( U(1) \) gauge transformation.

Note that this superpotential \((140)\) and the corresponding scalar potential \((141)\) are the same as those of \((43)\) and \((46)\) in the \( F \)-term hybrid inflation model with the gauge symmetry \( G \) to be Abelian, which was discussed in the previous subsection. Here, the letters are changed from \( \Psi, \overline{\Psi} \) to \( S, X, \overline{X} \) and \( \mu^2 \) is rescaled. In the \( F \)-term hybrid inflation model, the gauge singlet field \( S \ll 1 \) plays the role of an inflaton while \( X \) and \( \overline{X} \) remain zero due to the heavy masses during the inflation, which satisfy the \( D \)-flat condition. After inflation ends, they are destabilized and roll down to the global minima. In order for this hybrid inflation to start, the initial condition is such that the field \( S \) has to be relatively large but smaller than unity due to the exponential factor in the \( F \)-term while \( X \) and \( \overline{X} \) need to almost vanish. On the other hand, in this \( D \)-term chaotic inflation model, we consider another initial condition given by \( |X| \gtrsim 1 \) or \( |\overline{X}| \gtrsim 1 \) with \( S \sim 0 \) and \( X\overline{X} \sim \mu^2 \), which almost satisfies the \( F \)-flat condition. When the universe starts around the Planck scale, the potential energy as well as the kinetic energy is expected to be of the order of the Planck energy density. Then, the almost \( F \)-flat direction is naturally realized around the Planck scale due to the exponential factor of the \( F \)-term, which leads to the domination of the \( D \)-term potential so that chaotic inflation takes place.

As explained above, the inflationary trajectory is expected to be given by the (almost) \( F \)-flat direction, \( S = 0 \) and \( X\overline{X} = \mu^2 \). In fact, the effective mass squared of \( S \) along this direction becomes \( \lambda^2 e^K(|X|^2 + |\overline{X}|^2) \) and hence is much larger than the Hubble parameter squared \( H^2 \approx g^2 |X|^4/2 \). Thus, we can safely set \( S \) to be zero and the potential reduces to

\[ V = \frac{\lambda^2}{4} e^{\frac{1}{2}(|X|^2 + |\overline{X}|^2)}(X\overline{X} - \mu^2)^2 + \frac{g^2}{8} (X^2 - \overline{X}^2)^2, \]

then the domination of the \( D \)-term potential so that chaotic inflation takes place.
where $\mu' = \sqrt{2}\mu$ and we have redefined the fields $X \equiv \sqrt{2}\text{Re}X$, $\overline{X} \equiv \sqrt{2}\text{Re}\overline{X}$ (we take both $X$ and $\overline{X}$ to be positive for definiteness). The kinetic terms of the fields $X$ and $\overline{X}$ are still canonical. Though the trajectory is along the valley of the two-dimensional configuration and is a bit complicated, the $F$-flat condition $X\overline{X} - \mu'^2 = 0$ is satisfied for $X \gg 1$ because of the exponential factor. By inserting this condition into equation (145), we obtain the reduced potential $V(X)$,

$$V(X) \simeq \frac{\mu'^2}{8} X^4.$$  \hspace{1cm} (146)

As explicitly shown in [76], when there is only one massless mode and the other modes are massive, the generation of adiabatic density fluctuations as well as the dynamics of the homogeneous mode is completely determined by this reduced potential $V(X)$. Thus, this model of chaotic inflation has a quartic potential.

As given in equation (75), this constraint on the tensor-to-scalar ratio $r < 0.24$ rules out the case with $n \geq 4$ if the primordial density fluctuation is mainly generated by the inflaton. However, if we take the non-minimal gauge kinetic function such as a form $f = 1 + dX/|X|^2 + d\overline{X}/|\overline{X}|^2$ ($dX, d\overline{X}$: constants), chaotic inflation with a quadratic potential is realized. Another $D$-term chaotic inflation with a quadratic potential is also considered by the use of the FI field [77]. Note that, in the model with the potential (141), no cosmic string is formed after inflation because the $U(1)$ gauge symmetry is already broken during inflation, while the formation of cosmic strings severely constrains both the $D$-term and the $F$-term hybrid inflation models.

After the inflation, the inflaton starts to oscillate around the global minimum and decays into standard particles. In this model, the inflaton can decay into the right-handed neutrino $N$, which quickly decays into the standard particles through the Yukawa coupling. By introducing the following superpotential:

$$W = \alpha X\overline{X}NN,$$  \hspace{1cm} (147)

the decay rate of the inflaton to the right-handed neutrino is then given by

$$\Gamma \simeq \frac{1}{32\pi} \alpha^2 \langle X \rangle^2 m \sim \frac{1}{32\pi} \alpha^2 \lambda \mu'^3$$

$$\sim 10^{-3}\text{GeV} \left(\frac{\alpha}{0.1}\right)^2 \left(\frac{\lambda}{10^{-3}}\right) \left(\frac{\mu'}{10^{14}\text{GeV}}\right)^3,$$  \hspace{1cm} (148)

where $\alpha$ is the coupling constant of order unity, $\langle X \rangle \simeq \mu'$ is the VEV of $X$, and $m \simeq \lambda \mu'$ is the mass around the minimum. Then, the reheating temperature $T_R$ becomes

$$T_R \simeq \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \sqrt{\Gamma} \sim 10^3\text{GeV} \left(\frac{\alpha}{0.1}\right)^{1/4} \left(\frac{\lambda}{10^{-3}}\right)^{1/4} \left(\frac{\mu'}{10^{14}\text{GeV}}\right)^{1/2}.$$  \hspace{1cm} (149)

In this model, we can also show that the decay rate asymmetry of the right-handed neutrino to the standard particles generates lepton asymmetry [78], which is converted to baryon asymmetry through sphaleron effects.

4. Higgs inflation in Jordan frame supergravity

In this section, we discuss inflation models in Jordan frame supergravity, focusing on Higgs chaotic inflation proposed a couple of years ago. The (classical) potential of the physical Higgs field $h$ is given by $V(h) = (\lambda/4)(h^2 - v^2)^2 \sim \lambda h^4/4$ for $h \gg v$. Then, this quartic type of potential can cause chaotic inflation for $h \gg 1$. Unfortunately, as shown in the previous
section, the coupling $\lambda$ of order unity predicts too large density fluctuation and also the large tensor-to-scalar ratio prohibits a potential with quartic power. However, these constraints rely on three important assumptions: (i) the Higgs field minimally couples to gravity, (ii) the kinetic term of the Higgs field is canonical, and (iii) the primordial curvature perturbation is dominantly produced by the Higgs field. If we could relax one of these three conditions, the Higgs field may be responsible for inflation. Recently, an interesting possibility relaxing the assumption (i) was proposed by Bezrukov and Shaposhnikov, in which a non-minimal coupling of the Higgs field to gravity is considered [7]. Motivated by this work, several attempts were made to realize Higgs chaotic inflation in Jordan frame supergravity [79–82]. In this section, we first give the basic formulae of inflation where the inflaton is non-minimally coupled to gravity and explain how Higgs inflation with quartic potential can circumvent the above constraints in the Jordan frame. Then, we give the formulation of a scalar field in Jordan frame supergravity and show how to accommodate Higgs chaotic inflation in this framework.

4.1. Inflation in the Jordan frame

In this subsection, following [83], we derive the slow-roll conditions for inflation in the Jordan frame and express observational quantities in terms of the slow-roll parameters. The action in the Jordan frame with the metric $g_{\mu\nu}$ is given by

$$ S = \int d^4x \sqrt{-g} \left( \frac{\Omega(\phi)}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right), \quad \Omega(\phi) \equiv 1 - 2 F(\phi), \quad (150) $$

where $\phi$ is an inflaton, $\Omega(\phi)$ is a conformal factor, and $F(\phi)R$ stands for a non-minimal coupling of the inflaton to gravity. The case with $F = 0$ ($\Omega = 1$) corresponds to a minimal coupling and the case with $F(\phi) = \phi^2/12$ corresponds to a conformal coupling. Then, the equation of motion and the Friedmann equation are given by

$$ \Omega(\phi + 3H\phi + V') + \frac{3\Omega'}{2} (\Omega + 3H\Omega) + \Omega' \left( \frac{1}{2} \phi^2 - 2V \right) = 0, \quad (151) $$

$$ H^2\Omega + H\Omega = \frac{1}{3} \left[ \frac{1}{2} \phi^2 + V(\phi) \right], \quad (152) $$

where we have assumed the flat Friedmann background and the homogeneity of the scalar field. These equations are approximated as

$$ 3H\phi \simeq \frac{\Omega^2}{f} \left( \frac{V}{\Omega^2} \right)' = -V'_\text{eff}, \quad f(\phi) \equiv 1 + \frac{3\Omega^2(\phi)}{2\Omega(\phi)}, \quad (153) $$

as long as the following three slow-roll conditions are satisfied$^{10}$:

$$ \epsilon_J \equiv \frac{\Omega V'_\text{eff}}{2V^2}; \quad \epsilon_J \ll 1, \quad (154) $$

$$ \eta_J \equiv \frac{\Omega V''_\text{eff}}{V}; \quad |\eta_J| \ll 1, \quad (155) $$

$$ \delta_J \equiv \frac{\Omega' V'_\text{eff}}{V}; \quad |\delta_J| \ll 1. \quad (156) $$

$^9$ See references for other possibilities without the assumption (ii) [8–10] or (iii) [54].

$^{10}$ Strictly speaking, one subsidiary condition $|V'_\text{eff}/V| = O(1)$ is also necessary.
Before going to observational quantities, we comment on the relation between the Jordan frame and the Einstein frame. Introducing the Einstein metric $\hat{g}_{\mu\nu}$ by conformal transformation with a conformal factor $\Omega_1(\phi)$,

$$\hat{g}_{\mu\nu} = \Omega_1(\phi) g_{\mu\nu}, \quad (157)$$

action (150) in the Jordan frame can be rewritten in the Einstein frame as

$$S = \int d^4x \sqrt{\hat{g}} \left( \frac{\hat{R}}{2} - \frac{1}{2} \hat{g}^{\mu\nu} \partial_{\mu} \hat{\phi} \partial_{\nu} \hat{\phi} - \hat{V}(\hat{\phi}) \right), \quad (158)$$

where we have defined a scalar field $\hat{\phi}$ with the canonical kinetic term and its potential $\hat{V}$:

$$d\hat{\phi}^2 = \frac{f(\phi)}{\Omega_1(\phi)} d\phi^2, \quad \hat{V}(\hat{\phi}) = \frac{V(\phi)}{\Omega_1^2(\phi)}. \quad (159)$$

Note that, in this section, the physical quantities in the Einstein frame are characterized with hats while those in the Jordan frame without hats. Then, it is easy to show that, as long as the slow-roll conditions are satisfied, the slow-roll parameters in both frames are related as

$$\hat{\epsilon} \equiv \frac{1}{2} \frac{d\hat{V}}{\hat{V}} \simeq \epsilon_J f, \quad \hat{\eta} \equiv \frac{1}{2} \frac{d^2\hat{V}}{\hat{V} d\hat{\phi}^2} \simeq \eta_J - \frac{3}{2} \delta_J + \frac{1}{2} \frac{f'}{f} \frac{\Omega_1}{\Omega_1'} \delta_J, \quad (160)$$

and that the curvature perturbation $\mathcal{R}$ in the comoving gauge is invariant under the conformal transformation, that is, $\hat{\mathcal{R}} = \mathcal{R}$. Then, the observable quantities are expressed as [84]

$$R^2 = \frac{1}{24\pi^2 \Omega_2^2 \epsilon_J f} \left( = \frac{1}{24\pi^2 \hat{\epsilon}} \hat{R}^2 \right),$$

$$n_s - 1 = -6\epsilon_J f + 2\eta_J - 3\delta_J + \frac{f'}{f} \frac{\Omega_1}{\Omega_1'} \delta_J \left( = -6\hat{\epsilon} + 2\hat{\eta} = n_s - 1 \right),$$

$$r = 16\epsilon_J f \left( = 16\hat{\epsilon} = \hat{r} \right).$$

The $e$-folding number $N$ is calculated as

$$N = \int_{t_0}^{t_e} H dt \simeq \int_{\phi_e}^{\phi_0} \frac{V}{\Omega V_{\text{eff}}'} d\phi \left( = \int_{\hat{\phi}_e}^{\hat{\phi}_0} \frac{\hat{V}}{\hat{V}_{\text{eff}}'} d\hat{\phi} = \hat{N} \right). \quad (162)$$

Now, we are ready for concrete examples. Let us consider chaotic inflation with the power-law potential $V(\phi) = \lambda_n \phi^n/n$, in which the inflaton has a non-minimal coupling to gravity $F(\phi) = \xi \phi^2/2$ ($\xi$ : a dimensionless parameter, $\Omega(\phi) = 1 - \xi \phi^2$) [85]. Then, the action is given by

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} (1 - \xi \phi^2) R - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - \frac{\lambda_n}{n} \phi^3 \right]. \quad (163)$$

For $|\xi| \phi^2 \ll 1$, the slow-roll parameters and the $e$-folding number are given by

$$\epsilon_J \simeq \frac{n^2}{2 \phi^2} \simeq -\frac{n}{4N}, \quad \eta_J \simeq \frac{n(n - 1)}{\phi^2} \simeq \frac{n - 1}{2N}, \quad \delta_J = -2n\xi, \quad (164)$$

and

$$N = \int_{\phi_e}^{\phi_0} \frac{V}{\Omega V_{\text{eff}}'} d\phi \simeq \frac{\phi_0^3}{2n}. \quad (165)$$
Thus, $|\xi| \ll 1$ and $\phi^2 \gg n$ are necessary for slow roll. Then, the observable quantities are evaluated as
\[
R^2 = \frac{1}{24\pi^2} \frac{V}{\Omega_{eff}^2} \frac{1}{f} \sim \frac{\lambda_n}{12\pi^2} \frac{\phi_n^{n+2}}{n^3},
\]
\[
n_s - 1 \simeq -6\epsilon f + 2\eta - 3\delta \simeq -\frac{n + 2}{2N} + 6n\xi,
\]
\[
r = 16\epsilon f \simeq \frac{4n}{N},
\]
where we have used $f \simeq 1$. The tensor-to-scalar ratio $r$ coincides with that in the minimal coupling case. Thus, this case still excludes chaotic inflation with $n \geq 4$ power-law index.

On the other hand, for $|\xi| \phi^2 \gg 1$ and $n \neq 4$, the slow-roll parameters and the $e$-folding number are given by
\[
\epsilon = -\frac{(n - 4)^2}{2(1 - 6\xi)^2}, \quad \eta = -\frac{(n - 4)(n - 1)}{1 - 6\xi}, \quad \delta = -\frac{2(n - 4)\xi}{1 - 6\xi},
\]
and
\[
N = \int_{\phi_i}^{\phi_f} \frac{V}{\Omega_{eff}^2} \frac{d\phi}{|\phi|} \simeq \frac{1}{\Omega_{eff}^{2/3}} \frac{1}{n - 4} \int \frac{d\phi}{\phi^2}.
\]

Thus, $|\xi| \ll 1$ is required for slow-roll. Note that the positivity of $\Omega$ coming from equation (153) requires $\xi < 0$. Then, the observable quantities are evaluated as
\[
R^2 = \frac{1}{24\pi^2} \frac{V}{\Omega_{eff}^2} \frac{1}{f} \sim \frac{\lambda_n}{12\pi^2} \frac{\phi_n^{n+2}}{n(n - 4)^2},
\]
\[
n_s - 1 \simeq -6\epsilon f + 2\eta - 3\delta \simeq -\frac{|\xi|}{(1 - 6\xi)}(n - 4)^2,
\]
\[
r = 16\epsilon f \simeq \frac{8|\xi|}{(1 - 6\xi)}(n - 4)^2 \simeq 0.32 \left( \frac{1 - n_s}{0.04} \right),
\]
where we have used $f \simeq 1 - 6\xi \simeq 1$. Thus, the tensor-to-scalar ratio may be too large in this case unless $1 - n_s$ becomes smaller.

For $|\xi| \phi^2 \gg 1$ and $n = 4$, the slow-roll parameters and the $e$-folding number are calculated as
\[
\epsilon = -\frac{8}{(1 - 6\xi)^2 \phi^2} \simeq -\frac{1}{8N^2\xi^2}, \quad \eta = \frac{4}{(1 - 6\xi)\phi^2} \simeq \frac{1}{2N}, \quad \delta = \frac{8}{(1 - 6\xi)\phi^2} \simeq \frac{1}{N},
\]
and
\[
N = \int_{\phi_i}^{\phi_f} \frac{V}{\Omega_{eff}^2} \frac{d\phi}{\phi^2} \simeq \frac{1}{8} \phi_n^{2/3}.
\]

Thus, the slow-roll conditions are always satisfied irrespective of $\xi$, as long as $\xi < 0$. This can be easily understood because $V_{eff}^{\prime} \simeq 0$ and $dV/d\phi \simeq 0$ in this case. Then, the observable quantities are expressed in terms of $N$ as
\[
R^2 = \frac{1}{24\pi^2} \frac{V}{\Omega_{eff}^2} \frac{1}{f} \sim \frac{\lambda_n}{12\pi^2} N^2 \frac{1}{(1 - 6\xi)},
\]
\[
n_s - 1 \simeq -6\epsilon f + 2\eta - 3\delta \simeq -\frac{3(1 - 6\xi)}{4N^2|\xi|} - \frac{2}{N},
\]
\[
r = 16\epsilon f \simeq \frac{2N(1 - 6\xi)}{N^2|\xi|},
\]
where we have used $f \simeq 1 - 6\xi$. Inserting $R^2 \simeq 2.441 \times 10^{-9}$ and $N = 60$ yields the relation $\lambda_4 \simeq 4.8 \times 10^{-10}\xi^2$, namely $|\xi| \simeq 4.5 \times 10^4\sqrt{\lambda_4}$.\(^{11}\) In addition, $n_s \simeq 0.97$ and the tensor-to-scalar ratio becomes $r \simeq 3.3 \times 10^{-3}$ in this case. Thus, chaotic inflation with a quartic potential ($n = 4$) can be still viable for $|\xi|\phi^2 \gg 1$ in the Jordan frame, which strongly motivates us to consider Higgs inflation non-minimally coupled to gravity.

4.2. Higgs inflation in the Jordan frame supergravity

In this subsection, we will first give the scalar part of the Lagrangian in the Jordan frame supergravity and later discuss Higgs chaotic inflation in it.

Since the detailed derivation based on the superconformal supergravity by gauge-fixing is given in [81], we will only give the relevant results. The scalar part of the Lagrangian in Jordan frame supergravity is determined by the four functions, frame function $\Omega(\Phi_i, \Phi^*_i)$, Kähler potential $K(\Phi_i, \Phi^*_i)$ (independent of the frame function), superpotential $W(\Phi_i)$, and gauge kinetic function $f(\Phi_i)$ [81]. While the superpotential and the gauge kinetic function are holomorphic functions of complex scalar fields, the frame function and the Kähler potential are not holomorphic and real functions of the scalar fields $\Phi_i$ and their conjugates $\Phi^*_i$. The frame function $\Omega$ corresponds to a conformal factor and could stand for scalar-gravity coupling $\Omega R/2$. In particular, $\Omega = 1$ corresponds to the minimal coupling. Then, the action of the scalar and the gravity sectors in the Jordan frame is given by

$$S = \int d^4x \sqrt{-g} \left[ \frac{\Omega}{2} R + \frac{1}{\sqrt{-g}} L_{\text{kin}} - V(\Phi_i, \Phi^*_i) \right],$$

(173)

where the kinetic terms of the scalar fields are determined by the frame function $\Omega$ and the Kähler potential $K$ as

$$\frac{1}{\sqrt{-g}} L_{\text{kin}} = \left(-\Omega K_{ij} + \frac{3}{\Omega} \frac{\Omega_i \Omega^*_j}{\Omega} \right) \partial_\mu \Phi_i \partial_\nu \Phi^*_j g^{\mu\nu} - 3\Omega A^2_{\mu}.$$  

(174)

Here the lower indices of the frame function and the Kähler potential represent the derivatives, and $A_\mu$ is the on-shell auxiliary axial-vector field given by

$$A_\mu = -\frac{i}{2\Omega} \left( \Omega_i \tilde{\partial}_\mu \Phi_i - \Omega^*_j \tilde{\partial}_\mu \Phi^*_j \right),$$

(175)

with $\tilde{\partial}_\mu$ to be the gauge covariant derivative. On the other hand, the potential $V$ in the Jordan frame is related to the potential $\tilde{V}$ in the Einstein frame and is given by

$$V = \Omega^2 \tilde{V}, \quad \tilde{V} = e^K [D_{\Phi_i} W K^{-1}_{ij} D_{\Phi_j} W^* - 3|W|^2] + \frac{1}{2} \sum_{a,b} [\text{Re} f_{ab}(\Phi_i)]^{-1} g_{ab}^2 D^2_a.$$  

(176)

Although the frame function and the Kähler function are independent in general, we have a special class of the superconformal models, where the following relation is satisfied:

$$\Omega(\Phi_i, \Phi^*_i) = e^{-3/4 K(\Phi, \Phi^*)} \iff K(\Phi_i, \Phi^*_i) = -3 \ln \Omega(\Phi_i, \Phi^*_i).$$

(177)

Then, the kinetic terms of the scalar fields in this case reduce to

$$\frac{1}{\sqrt{-g}} L_{\text{kin}} = 3\Omega K_{ij} \partial_\mu \Phi_i \partial_\nu \Phi^*_j g^{\mu\nu} - 3\Omega A^2_{\mu}.$$  

(178)

You can easily find that the following form of the frame function, assuming $A_\mu = 0$, leads to the canonical kinetic terms of scalar fields:

$$\Omega(\Phi_i, \Phi^*_i) = 1 - \frac{1}{4} [\delta_{ij} (\Phi_i)^* + J(\Phi_i) + J^*(\Phi^*_i)],$$

(179)

\(^{11}\) In fact, we need to take into account loop effects, which are sensitive to the details of the UV completion [86].
where \( J(\Phi) \) is an arbitrary function. By taking this form of the frame function and setting \( A_\mu = 0 \), the action of the scalar and the gravity sectors in the Jordan frame reads

\[
S = \int d^4x \sqrt{-g} \left[ \frac{\Omega}{2} R - \delta_{ij} \partial_\mu \Phi_i \partial^\mu \Phi^*_j g^{\mu\nu} - V(\Phi, \Phi^*) \right].
\] (180)

Now, we give a model of Higgs inflation in the context of the Jordan frame supergravity. As explicitly shown in [79], as long as we take a superpotential with a form of \( C + \mu H_u H_d \) \((C, \mu: \text{constants})\), the inflationary trajectory is unstable and/or too steep. Then, we need to extend this superpotential by introducing another superfield \( S \), as done in the \((F\text{-term}) \) chaotic inflation models in the Einstein supergravity. Such an extension is accommodated in the next-to-minimal supersymmetric standard model (NMSSM). Therefore, chaotic inflation with NMSSM in the Jordan frame supergravity was proposed in [79] and modified in [80, 81].

We take the following frame function, Kähler potential, and the superpotential:

\[
\Omega = 1 - \frac{1}{3} (|S|^2 + H_u H_d^\dagger + H_d H_u^\dagger) - \frac{1}{2} \gamma (H_u H_d + \text{h.c.}) + \frac{\xi}{3} |S|^4,
\]

\[
K = -3 \ln \Omega,
\]

\[
W = -\lambda_S H_u H_d + \frac{\rho}{3} S^3.
\] (181)

In fact, we can safely set the charged fields \( H_u^+, H_d^- \) to be zeros, that is,

\[
H_u = \begin{pmatrix} 0 \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ 0 \end{pmatrix}.
\] (182)

With these truncations, three functions reduce to

\[
\Omega = 1 - \frac{1}{3} (|S|^2 + |H_u^0|^2 + |H_d^0|^2) - \frac{1}{2} \gamma (H_u^0 H_d^0 + H_d^0 H_u^0) + \frac{\xi}{3} |S|^4,
\]

\[
K = -3 \ln \Omega,
\]

\[
W = -\lambda_S H_u H_d + \frac{\rho}{3} S^3,
\] (183)

which yield the \(D\text{-term}) \) potential in the Jordan frame:

\[
V_D = \frac{1}{4} (g^2 + g'^2) \left( |H_u^0|^2 - |H_d^0|^2 \right)^2.
\] (184)

Here we have taken a minimal gauge kinetic function \( f_{ab} = 1 \), and \( g \) and \( g' \) are the gauge couplings of the \( SU(2)_L \) and the \( U(1)_Y \) symmetries, respectively. Decomposing the complex scalar fields into

\[
S = \frac{1}{\sqrt{2}} s e^{iu}, \quad H_u^0 = \frac{1}{\sqrt{2}} h \cos \beta e^{i\alpha_1}, \quad H_d^0 = \frac{1}{\sqrt{2}} h \sin \beta e^{i\alpha_2},
\] (185)

the \(D\text{-term}) \) potential vanishes when \( \beta = \pi/4 \). Then, we take an inflationary trajectory as \( \beta = \pi/4, \alpha_1 = 0, \) and \( s = 0 \), whose stability condition will be given below. Under these settings, the action reduces to

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \left( 1 - \frac{1}{6} - \frac{\gamma}{4} \right) h^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu h \partial^\nu h - \frac{\lambda_d^2}{16} h^4 \right].
\] (186)

This action is equivalent to action (163) with \( n = 4, \xi = 1/6 - \gamma/4, \) and \( \lambda_d = \lambda^2/4 \). Thus, chaotic inflation can be realized in the Higgs sector with the NMSSM by the use of the Jordan frame supergravity. The detailed analysis, in fact, shows that this inflationary trajectory is stable as long as \( \zeta > 2(\lambda_\rho)/\lambda^2 h^2 + 0.0327 \) [81].

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12 More general models of inflation in the Jordan frame supergravity are discussed in [82, 87]. In addition, another interesting class of inflation in the context of \( F(R) \) supergravity is considered in [88].
After the inflation ends at $h \simeq (4/3)^{\frac{1}{2}}/|\xi|$, the reheating quickly occurs because the interactions of the Higgs boson with the other SM particles are strong. Then, the reheating temperature $T_R$ is estimated as

$$T_R \sim \left( \frac{10}{\pi^2 g_*} \right)^{\frac{1}{2}} \frac{\lambda_1^{\frac{1}{4}}}{\sqrt{|\xi|}} \simeq 3 \times 10^{15} \text{ GeV},$$

(187)

where $g_* \simeq 200$ is the number of relativistic degrees of freedom and we have used the relation $|\xi| \simeq 4.5 \times 10^4 \sqrt{\lambda_4}$.

5. Conclusion

In this review, we have discussed inflation models in supergravity. After explaining why it is difficult to accommodate inflation in supergravity, we gave the prescriptions to circumvent such difficulties. Focusing on the cases with an almost canonical Kähler potential, we gave concrete examples of each type of inflation. Though it was long supposed that it was almost impossible to construct the natural model of chaotic inflation, we now have all types of inflation in supergravity. The ongoing observations would confirm or exclude a specific type of inflation. In particular, chaotic inflation generates a significant amount of primordial tensor perturbations, which may be detected in the near future. Then, the next step to develop inflation models in supergravity is to embed them in a realistic model of particle physics like GUT and/or in a superstring theory. It is interesting whether the Kähler potential and superpotential suitable for inflation naturally appear in the context of GUT and/or superstring.

We have also discussed inflation models based on Jordan frame supergravity, focusing on Higgs chaotic inflation. Since inflation models in Jordan frame supergravity appeared very recently, we need to investigate them in more detail, particularly paying attention to the difference between inflation models in the Jordan frame and those in the Einstein frame. We also should extend these models in the context of superstring because a non-minimal coupling is naturally found in it.

Although $F(R)$ inflation models were also formulated in the context of $F(R)$ supergravity recently [89], other important classes of inflation such as $k$ inflation [90], ghost inflation [91], DBI inflation [92], and $G$ inflation [93] are not yet formulated in supergravity. Supersymmetrization of these inflation models is also an important topic.

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13 See [94] for a recent attempt to supersymmetrize these higher derivative models of inflation.
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