Detecting Seasonal Changes in the Fundamental Constants

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We show that if one or more of the ‘constants’ of Nature can vary then their values, as measured in the laboratory, should oscillate over the year in a very particular way. These seasonal changes in the constants could well be detected, in the near future, with ground-based atomic clocks.

Recently, there has been a great deal of interest in the possibility that some, or all, of the traditional constants of Nature are actually dynamical and change slowly in space and time.

In this Letter, we show that if the ‘constants’ of Nature do vary, then their values, as measured by laboratory experiments, should display annual variation as the Earth moves around the Sun. We calculate the magnitude of this effect and find that, although it is expected to be very small, the continually increasing precision and stability of atomic frequency standards mean the prospects for detecting it in the near future are very good.

Theoretical and experimental interest in variation of the constants has a long history, but the recent renaissance in the field has, to a great extent, been motivated by the recent observations of relativistic fine structure in the absorption lines formed in dust clouds around quasars carried out by Webb et al. [1].

Using a data set of 128 objects at redshifts 0.5 < z < 3, Webb et al. found the absorption spectra to be consistent with a shift in the fine structure constant, \( \alpha = e^2/4\pi\varepsilon_0\hbar c \), between those redshifts and the present day (a period of about 10 Gyrs): \( \Delta\alpha/\alpha \approx (\alpha(z) - \alpha(0))/\alpha(0) = -0.57 \pm 0.10 \times 10^{-5} \). A smaller study of 23 absorption systems between 0.4 ≤ z ≤ 2.3 by Chand et al. [2] found a result consistent with no variation: \( \Delta\alpha/\alpha = -0.6 \pm 0.6 \times 10^{-6} \). However, a recent reanalysis of the same data by Murphy et al. [3] was unable to confirm the conclusions of Chand et al., and suggested the revised result: \( \Delta\alpha/\alpha = -0.44 \pm 0.16 \times 10^{-5} \). Reinhold et al. [4] found a 3.5\( \sigma \) indication of a cosmological variation in another ‘constant’, the proton-electron mass ratio \( \mu = m_p/m_e \), in their analysis of the vibrational levels of H2 in the absorption spectra of two quasars at redshifts \( z \approx 2.6 \) and \( z \approx 3.0 \); they found \( \Delta\mu/\mu = 2.0 \pm 0.6 \times 10^{-5} \) over the last 12 Gyrs.

Despite these hints of a variation in \( \alpha \) and \( \mu \), few would be prepared to conclude that \( \alpha \) and \( \mu \) definitely change with time until either the precision of the astronomical studies is greatly increased, or a variation in one of these constants can be directly detected in the more controlled environment of the laboratory. A significant improvement in the precision of astronomical studies would most likely require new instrumentation such as the proposed CODEX spectrograph [5], which is not expected to be operational before 2017. A firmer understanding of the potential systematic errors would also be needed. In contrast, the prospects for a significant improvement, in the near future, in the precision of laboratory-based varying-constant searches seem much better.

Laboratory constraints on the variation of the constants are generally found by comparing different atomic frequency standards over a period of many months or several years. The most stringent bound on the temporal variation of \( \alpha \) published thus far made use of 6 years of data and is: \( \dot{\alpha}/\alpha = (-2.6 \pm 3.9) \times 10^{-16} \text{yr}^{-1} \); if \( \alpha \) has varied at a constant rate over the last 10 Gyrs, then the findings of Webb et al. suggest: \( \dot{\alpha}/\alpha = (6.4 \pm 1.4) \times 10^{-16} \text{yr}^{-1} \). An important motivation for this Letter is that the ability of laboratory tests to measure changes in \( \alpha \) seems likely to improve markedly in the near future. The ACES (Atomic Clock Ensemble in Space) project, currently projected to fly on the International Space Station in 2010, will be able to constrain \( \dot{\alpha}/\alpha \) at the \( 10^{-17} \text{yr}^{-1} \) level. Recently, Cingöz et al. [10] reported a new limit on \( \dot{\alpha}/\alpha \) found by monitoring the transition frequencies between two nearly degenerate, opposite-parity levels in two isotopes of atomic Dysprosium (Dy) over 8 months. These energy levels are particularly sensitive to changes in \( \alpha \). Cingöz et al. found that \( \dot{\alpha}/\alpha = -(2.7 \pm 2.6) \times 10^{-15} \text{yr}^{-1} \), but importantly they estimate that an ultimate sensitivity to changes, \( \delta\alpha \), in \( \alpha \) of one part in \( 10^{18} \) is feasible. Moreover, Flambaum [11] has recently noted that an even greater improvement in precision could be achieved by making use of the enhanced effect of \( \alpha \) variation on the very narrow UV transition between two levels of the \( ^{299}\text{Th} \) nucleus. The corresponding experiment could potentially detect a non-zero \( |\delta\alpha/\alpha| \) as small as \( 10^{-23} \), 7 orders better than current bounds. Despite the expected increase in the precision of laboratory tests, any such experiment must still run for many months, or even several years, if it is to place tight constraints on any time variation. In this Letter we note that if a ‘constant’, \( C \), can vary, then its value, as measured in the laboratory, should vary during the year. This variation will have a very distinctive shape and be correlated with the Earth’s distance from the Sun. These two properties should make it easier to separate any signal from noise. Additionally, an experiment would only need to run for six months to constrain any fluctuation in the constants.
due to this effect.

Variation of some, or all, of the constants of Nature is fairly generic prediction of most modern proposals for fundamental physics beyond the standard model. Indeed, it is one of the few low-energy signatures of such theories. At the low energies appropriate to classical physics, the values of the constants are determined by the vacuum expectation of a scalar field, or dilaton, $\phi$ and $\alpha = \alpha(\phi)$, $\mu = \mu(\phi)$. Henceforth, we use units in which $c = \hbar = 1$. It is generally assumed that the scalar field theory associated with $\phi$ has a canonical kinetic structure, and that variations in $\phi$ conserve energy and momentum, as well as contributing to the curvature of space-time in the usual way. These considerations imply that $\phi$ satisfies a non-linear wave equation:

$$\Box \phi = \frac{1}{\omega} \left( \sum_i \frac{\partial C_i(\phi)}{\partial \phi} \frac{\delta L_m}{\delta C_i} + \frac{\partial V(\phi)}{\partial \phi} \right), \quad (1)$$

where $L_m$ is the Lagrangian density for the matter fields, and the $C_i(\phi)$ represent different, $\phi$-dependent, ‘constants’ of Nature: $V(\phi)$ is some self-interaction potential for the dilaton, and $\omega$ is a constant with units of (mass)$^2$ which sets the strength with which $\phi$ couples to matter. General expectations from string theory suggest that $4\pi G\omega \sim O(1)$ and that if a constant, $C_i$, is dynamical then $\partial \ln C_i/\partial \phi \sim O(1)$. Ultimately, it is our goal to determine, or bound, the parameters of such theories experimentally. Solar-system tests of gravity currently seem to prefer $4\pi G\omega \gg 1$ [12]. A similar equation to Eq. (1) applies when variations in the constants are driven by multiple scalar fields. Multiple fields might complicate the cosmological evolution of the ‘constants’ but, over solar system scales, the dynamics of each field are wellapproximated by a wave equation with the form of Eq. (1).

At the present time, the matter to which $\phi$ couples is non-relativistic. Defining $\rho_j$ to be the energy-density of the $j$th matter species, we then have $L_m \approx \sum_j \rho_j$ and Eq. (1) reduces to:

$$\Box \phi = \frac{1}{\omega} \left( 2 \sum_j \zeta_j(\phi) \rho_j + \frac{\partial V(\phi)}{\partial \phi} \right) = \frac{2\zeta(\phi)\rho + V_\phi}{\omega}, \quad (2)$$

where $2\zeta_j(\phi) = \sum_i C_{ij} \delta(\ln \rho_j)/\delta C_i$, $\rho = \sum_j \rho_j$ is the total energy density of matter, and $\zeta(\phi) = \sum_j \zeta_j \rho_j/\rho$. The value of the $\zeta$ for a body, or a system of bodies, generally depends on its composition. The effective mass-squared of the scalar field is given by $m^2_\phi = V_{\phi\phi}/\omega$. Generally, if time variations in a constant occur at anything approaching a detectable level then $m_\phi \lesssim 10^{-63} \text{g} \sim H_0$, where $H_0$ is the Hubble parameter today. If $V(\phi)$ is highly non-linear, and $\zeta/\omega$ and $V_\phi$ have opposite signs, then $\phi$ may behave as a chameleon field (see refs. [13]). The mass of $\phi$ would then depend heavily on $\rho$. Whilst chameleon field theories are very interesting, late-time variation of the fundamental constants is negligible in all known, experimentally viable, chameleon models. For this reason, we do not consider such theories here, and henceforth assume that $\phi$ is not a chameleon field.

Tests of gravity constrain any variations in $\phi$ in the solar system to be very small [12]. We can therefore linearise Eq. (2) about the background value of the field, $\phi_b(t)$, which will track the cosmological value of $\phi$, $\phi(t)$, i.e. we take $\phi \approx \phi_b(t) + \delta \phi$. The majority of recent laboratory-based searches for varying-constants have looked for changes in $\alpha$. We therefore focus our attention on theories in which $\alpha$ varies, and scale $\phi$ and $\omega$ so that $\partial \ln \alpha(\phi)/\partial \phi = 2$ today. Although we primarily consider the annual variations of $\alpha$, similar variations should be expected in all varying constants.

In the solar system, any temporal-gradients in $\delta \phi$ are expected to be small compared to the spatial ones (see Ref. [5]) and so, at leading order, we may replace the wave-operator, $\Box$, by the Laplacian, $\nabla^2$, giving:

$$\nabla^2 \delta \phi = \frac{2\zeta}{\omega} \rho + m_\phi^2 \delta \phi, \quad (3)$$

where $m_\phi$ and $\zeta$ independent of $\delta \phi$, although $\zeta$ is still composition dependent. Solving this equation, we find that the Sun induces the following contribution to $\delta \phi$:

$$\delta \phi_\odot = \frac{\zeta_\odot}{4\pi G\omega} \frac{2GM_\odot}{r} e^{-m_\phi r}, \quad (4)$$

where $r$ is the distance to the Sun’s centre of mass, $M_\odot$ is the Sun’s mass, and $\zeta_\odot$ is the value of $\zeta$ for the Sun. The Earth’s orbit around the Sun is not perfectly circular and so the distance of the Earth from the Sun changes slightly over the year, fluctuating by about 3% from aphelion to perihelion. As a result, the value of $\delta \phi_\odot$, and hence also $\alpha$, will oscillate annually as $r$ changes. The values of any other constants of Nature that depend on $\phi$ will also vary throughout the year. Experimental searches for a time-variation in $\alpha$ generally assume an approximately linear drift in the $\alpha$; however, these seasonal changes are clearly oscillatory. Attempting to fit a linear drift to an annual oscillation would lead to inaccurate conclusions. The change in $\alpha$ from aphelion to perihelion due this annual fluctuation is given by $\delta \alpha/\alpha \approx 2(\delta \phi_\odot(r_{ap}) - \delta \phi_\odot(r_{per}))$. If $m_\phi(r_{ap} - r_{per})/2 \ll 1$, we predict:

$$\delta \alpha \bigg|_{\text{seasonal}} = -1.32 \times 10^{-9} \frac{\zeta_\odot}{4\pi G\omega} e^{-m_\phi a}, \quad (5)$$

where $a = 1.496 \times 10^8 \text{km}$ is the radius of the Earth’s semi-major axis. If measurements are taken throughout the year then seasonal changes in $\alpha$ should be distinguishable from noise, or any linear drift, by their distinctive shape, which is shown in FIG. [11]. It is also important to note that, whereas cosmological changes in the constants will virtually non-existent if $m_\phi \lesssim 10^{-63} \text{g} \sim H_0$, the seasonal fluctuations noted here only require $m_\phi \lesssim 10^{-51} \text{g} \sim 1 \text{AU}^{-1}$ to be potentially detectable.
The value of $\alpha$ is proportional to $\zeta$, consistent with no variation at the $2 \times 10^{-15}$ level $^{3}$ $^{10}$. This corresponds to a bound on $\zeta/4\pi G \omega$ of:

$$\left| \frac{\zeta}{4\pi G \omega} \right| < 2 \times 10^{-6}.\tag{5}$$

The quantity $\zeta/4\pi G \omega$ also sets the magnitude of violations of local position invariance (LPI) $^{6}$. Currently, the best bounds on the validity of LPI only give a limit of $|\zeta/4\pi G \omega| < 10^{-4}$ $^{12}$; two orders of magnitude worse than is already achievable by making use of the Earth’s motion around the Sun. If $\delta \alpha/\alpha$ can be measured at the $10^{-18}$ level then this would correspond to a measure of $\zeta/4\pi G \omega$, and any violation of LPI, at the $10^{-9}$ level. Similarly, if a $^{239}$Th-based experiment can be carried out, and $\delta \alpha/\alpha$ constrained to within one part in $10^{23}$, $\zeta/4\pi G \omega$ and the validity of LPI can be measured to within one part in $10^{14}$. Schiller et al. $^{13}$ recently noted that space-based atomic clocks could utilise the altitude dependence of the Earth’s gravitational potential to provide a similar increase in the precision to which violations of LPI can be measured. Whilst this is an excellent idea, the potential improvement in precision they find is no better than can be achieved, much more cheaply, on the ground by making use of the eccentricity of the Earth’s orbit.

In general, the $\zeta$ parameter is composition dependent and, as a result, the weak equivalence principle (WEP) is violated at some level. The magnitude of the expected WEP violations over distance of 1 AU will also depend on $\zeta/4\pi G \omega$. We now use bounds on any WEP violation to estimate the precision to which $\delta \alpha/\alpha$ must be measured before any annual fluctuations in $\alpha$ could potentially be detected. Over distance of about 1AU, the best current constraint on WEP violations was reported in Ref. $^{15}$. Using a modified torsion balance, the differential acceleration towards the Sun of two test bodies was measured and found to be:

$$\eta = \frac{2|a_1 - a_2|}{|a_1 + a_2|} = 0.1 \pm 2.7 \pm 1.7 \times 10^{-13},\tag{6}$$

which is consistent with no violation of WEP; here $a_1$ and $a_2$ are respectively the accelerations of the first and second test body towards the Sun. The Eötvos parameter, $\eta$, measures the strength of WEP violations. The reasonably general varying-constant model used throughout this Letter predicts:

$$\eta = 4|\zeta_1 - \zeta_2| \left| \frac{\zeta}{4\pi G \omega} \right|.\tag{6}$$

In some varying-constant theories, such as the varying-speed of light (VSL) model considered in Ref. $^{6}$, $\zeta_1 = \zeta_2$ for all bodies and so WEP is not violated. For such theories, WEP violation searches cannot constrain $\zeta/4\pi G \omega$. It is more generally the case, however, that WEP is violated in theories where some or all ‘constants’ can vary, and that $|\zeta_1 - \zeta_2| \sim O(10^{-1}|\zeta|)$ for such experiments. For varying-$\alpha$ theories, we additionally expect $10^{-4} \lesssim |\zeta| \lesssim 1$ for baryonic matter. The smallest values of $\zeta$ are expected for theories in which $\alpha$ is the only $\phi$-dependent constant. If $\Lambda_{QCD}$ is also $\phi$-dependent then generally $\zeta \sim O(1)$. Therefore, in a scalar field theory, which describes a variation in $\alpha$ and violates WEP, we expect:

$$\left| \frac{\delta \alpha}{\alpha} \right|_{seasonal} \lesssim 10^{-21} - 10^{-17},$$

where larger values of $|\delta \alpha/\alpha|$ are more feasible if $\zeta$ is smaller. If, as seems likely, $\delta \alpha/\alpha$ can be measured to a precision of one part in $10^{18}$ in the near future, then constraints on annual oscillations of $\alpha$ could well be detected or constraints on some varying-$\alpha$ theories improved by at least an order of magnitude. If $\delta \alpha/\alpha$ can be constrained at the $10^{-23}$ level, as suggested by Flambaum $^{11}$, then the prospects for detecting the predicted seasonal variation in $\alpha$ are even better, and, in almost all cases, direct laboratory bounds on $\delta \alpha/\alpha$ would more tightly constrain varying-$\alpha$ theories than WEP violation tests currently do. Indeed, if the $10^{-23}$ precision can be reached, then we could derive a model-dependent constraint on WEP violations due to varying-$\alpha$ at the one part in $10^{14} - 10^{19}$ level; the greatest precision is for theories with small $\zeta$. For comparison, proposed satellite-based EP experiments such as MICROSCOPE, GG and STEP promise a model-independent measure of any WEP violation at the 1 part in $10^{15}$, $10^{17}$ and $10^{18}$ levels respectively $^{10}$.

WEP violation searches are sensitive only to the magnitude of the $\zeta$ parameters and not to their sign. As noted by Magueijo, Barrow and Sandvik in Ref. $^{3}$, knowledge of the sign of $\zeta$ for baryonic matter is very important when attempting to discern between two different
varying-α models. Varying-α theories that are most naturally interpreted as a change in the fundamental electric charge $e$, $\zeta > 0$ is natural for baryonic matter. For such theories, the observations of Webb et al., [1], suggest that either $\zeta < 0$ for dark matter, or that the cosmological evolution of $\alpha$ is dominated by the potential ($V_{\phi}/\omega$) term in Eq. [2] [6]. In contrast, models that are simplest when viewed as describing a varying speed of light predict $\zeta < 0$. Unlike WEP violation searches, the annual fluctuations in $\alpha$ are sensitive to the sign of $\zeta$. If $\alpha$ does vary, and the seasonal variation could be detected, then sign of $\zeta$ would be known. Additionally, when combined with WEP violation bounds, such a measurement would also allow the values of both $\zeta$ and $\omega$ to be deduced separately. Achieving this would greatly increase our understanding of the theory that underpins any variation in $\alpha$.

We have noted, in this Letter, that if some of the constants of Nature vary, then we should expect their values on Earth to oscillate seasonally. The estimated upper bound on the magnitude of these fluctuations is such that they could well be detected in the near future by atomic-clocks. Importantly, the search for these variations would not require the creation of a totally new experimental programme, as laboratory searches for a temporal drift in the constants would also be sensitive to these yearly oscillations. It is feasible that such experiments could measure variations in $\alpha$ at the $10^{-18}$ level [10], and there have been suggestions that changes as small as one part in the $10^{23}$ could be detected [11].

It is also important to note that, in many theories, the expected magnitude of the annual oscillations in a ‘constant’, such as $\alpha$, will be comparable to, if not much larger than, the expected yearly drift in the value of the constant. Indeed, if $10^{-63} g > m_\phi \lesssim 10^{-53} g$, then any linear temporal drift in the ‘constant’ would be imperceptibly small, whereas the magnitude of the annual variations would be at the $10^{-9} \zeta_{\odot}/4\pi G \omega$ level. Furthermore, even if $\phi$ is very light ($m_\phi \lesssim 10^{-63} g$), the cosmological evolution of $\phi$ is, in some theories, primarily driven by the coupling of $\phi$ to matter (i.e. the effect of the $V_{\phi}/\omega$ in Eq. [2] is negligible). Such models predict that today $\dot{\alpha}/\alpha \approx -(3H^2_{\mathrm{U}})/(2\pi G \omega) (\zeta_{\text{dm}}\Omega_{\text{dm}} + \zeta_{b}\Omega_{b})$, where $\Omega_{\text{dm}}$ and $\Omega_{b}$ are respectively the density of the dark and baryonic matter in the Universe today as a fraction of the critical density, $t_{\text{U}}$ is the age of the Universe, and $\zeta_{\text{dm}}$ and $\zeta_{b}$ are the respective values of $\zeta$ for dark and baryonic matter. Using values for $\Omega_{\text{dm}}, \Omega_{b}, t_{\text{U}}$ and $H_0$ from WMAP, [17], we see that, over the course of a year, $\alpha$ is expected to change by a fractional amount:

$$\left. \frac{\delta \alpha}{\alpha} \right|_{\text{cosmo}} \approx \frac{(9.7 \pm 0.9) \zeta_{\text{dm}} + (1.9 \pm 0.1) \zeta_{b}}{4\pi G \omega} \times 10^{-11}.$$ 

We expect $\zeta_b \sim O(\zeta_{\odot})$ and so, unless $\zeta_{\text{dm}} \gtrsim 13 \zeta_{\odot}$, the magnitude of the annual changes in $\alpha$, identified in this Letter, will be greater than any linear temporal drift in its value. Failure to allow for these seasonal changes, could serious compromise the analysis of data found by laboratory searches for variations in $\alpha$.

In this Letter we have shown that if one or more of the ‘constants’ of Nature can vary, then the non-zero eccentricity of the Earth’s orbit will cause their values, as measured in the laboratory, to vary annually in a very particular way. Recent and expected advances in the precision and stability of frequency standards make it feasible that such a seasonal change in $\alpha$ could be detected, without the need to perform space-based tests, in the near future. Improved constraints on any such oscillation will greatly enhance our understanding of varying-constant theories and would also be used to improve current bounds on local position invariance. By both these means, we will be able to constrain the nature of fundamental physics beyond the standard model.

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