MORPHOLOGY AND DYNAMICS OF SOLAR PROMINENCES FROM 3D MHD SIMULATIONS

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ABSTRACT

In this paper we present a numerical study of the time evolution of solar prominences embedded in sheared magnetic arcades. The prominence is represented by a density enhancement in a background-stratified atmosphere and is connected to the photosphere through the magnetic field. By solving the ideal magnetohydrodynamic equations in three dimensions, we study the dynamics for a range of parameters representative of real prominences. Depending on the parameters considered, we find prominences that are suspended above the photosphere, i.e., detached prominences, but also configurations resembling curtain or hedgerow prominences whose material continuously connects to the photosphere. The plasma-$\beta$ is an important parameter that determines the shape of the structure. In many cases magnetic Rayleigh–Taylor instabilities and oscillatory phenomena develop. Fingers and plumes are generated, affecting the whole prominence body and producing vertical structures in an essentially horizontal magnetic field. However, magnetic shear is able to reduce or even to suppress this instability.

Key words: magnetic fields – magnetohydrodynamics (MHD) – Sun: corona

Supporting material: animations

1. INTRODUCTION

Quiescent prominences are large structures of cool and dense plasma suspended in quiet regions of the solar corona. These structures can have lifetimes of weeks although they have a highly dynamic nature. It is generally believed that the mass in a quiescent prominence is supported against gravity by the magnetic Lorentz force. Different models of the magnetic structure of prominences have been proposed in the past (see the review of Mackay et al. 2010), and magnetic dips, i.e., sites where the magnetic field lines are locally horizontal and curved in the upward direction, are thought to play a relevant role. Examples of such configurations are the Kippenhahn & Schlüter (1957) model, hereafter referred to as KS, or the Hood & Anzer (1990) model, where dips are self-consistently created by the weight of the heavy prominence. An alternative is that dips already exist before the dense material is deposited in them. In this context, there are models suggesting that prominences are supported by flux ropes that are essentially horizontal and lie above the polarity inversion line (see Sakurai 1976; Low 1981; Rust & Kumar 1994, 1996; Low & Zhang 2004). Another possibility is that instead of a flux rope, it is a sheared arcade that is responsible for the dips (see Antiochos et al. 1994; Aulanier et al. 2002; DeVore et al. 2005; Karpen et al. 2005; Luna et al. 2012). However, the interplay between magnetic field and the dense prominence is not addressed in these works. In the present paper we investigate in a consistent way a magnetic shear arcade model together with a dense prominence, leaving the analysis of flux rope models for future studies.

The Hinode satellite has provided unprecedented high-resolution images of quiescent prominences, allowing a detailed study of the dynamics of these structures. Prominences show changes in morphology, irregular motions on different spatial and temporal scales, downflows, upflows, vortices, raising bubbles, plumes, etc. Berger et al. (2008, 2010) reported dark upflows that formed at the base of some quiescent prominences. Berger et al. (2010) proposed that these observed upflows were caused by the Rayleigh–Taylor instability (see Rayleigh 1883; Taylor 1950). Ryutova et al. (2010) showed how many fundamental plasma instabilities can be linked to dynamical processes occurring in prominences. In particular, they showed that the development of regular series of plumes and spikes taking place at the interface between the prominence and corona is most likely related to the development of magnetic Rayleigh–Taylor (MRT) instabilities. Hillier et al. (2011) described how upflows can be the consequence of the MRT instability acting on the boundary between the KS prominence model and a tube inserted in the structure. Later, Hillier et al. (2012a) performed a detailed description of the dynamics of the system using the KS model for a wide range of model parameters. Hillier et al. (2012b) have extended the previous works to include interchange reconnection in the models. Dudić et al. (2012) have proposed that the existence of prominence bubbles is connected to a pair of magnetic null points associated with the prominence feet, and that a separator-reconnection scenario, which is very different from the Rayleigh–Taylor hypothesis, may naturally explain the bubble and plume formation. The role of neutrals on the MRT instability was investigated (in the linear regime) by Díaz et al. (2012), while Khomenko et al. (2014) have considered the non-linear regime and have found that the configuration is always unstable on small scales. The models presented by these authors are based on local models of prominences, and the effect of the corona is not included. Moreover, the magnetic field does not connect to the photosphere, which is an important boundary condition for prominence support. In this paper these points are properly addressed.

Another example of dynamical phenomena reported in quiescent prominences are oscillations. These oscillations are either of global nature, producing motions of the whole prominence (see Tripathi et al. 2009; Li & Zhang 2012; Luna et al. 2014; Shen et al. 2014), or local. In the latter case, the small-amplitude periodic motions are mostly detected in Doppler shifts of spectral lines; see, for example, Tsubaki (1988), Oliver (1999), and Oliver & Ballester (2002) and the reviews of Mackay et al.
(2010) and Arregui et al. (2012). The theoretical understanding of the oscillations is based on the hypothesis that they correspond to magnetohydrodynamic (MHD) waves. An intense modeling of prominence oscillations has been done in recent years and has been mostly based on the determination of the normal modes of different prominence models. These equilibrium models are rather simple, and the configuration is represented by slabs or cylindrical tubes. It is important to remark that the MHD perturbations on these models are imposed to satisfy line-tying conditions at the photosphere. This is a crucial boundary condition that has a strong effect on the eigenmodes of the prominence and has important consequences regarding the stability of the structure. The importance of line-tying conditions on prominence oscillations is well known, and a self-consistent way of investigating the dynamics of prominences must involve the effect of the underlying photosphere (see Schutgens 1997; van den Oord et al. 1998).

In Terradas et al. (2013) a two-dimensional (2D) prominence model embedded in a coronal quadrupolar arcade was constructed to mimic a normal polarity prominence. In that work, dense material representing the prominence was injected in an initially stable magnetic structure, increasing the gravitational energy. From the analysis of the evolution of the system it was found that in some cases the system relaxed toward a situation that was close to a stationary state. The obtained models using this method represent cool prominences supported against gravity by magnetic dips (see also the work of Hillier & van Ballegooijen 2013, who considered a 2D flux rope model instead of an arcade configuration). The formation process (see the works of Xia et al. 2011, 2012; Keppens & Xia 2014) was not addressed in this model, and the main aim was to study MHD waves on the generated models.

The main motivation in the present paper is to extend the study of the 2D model of Terradas et al. (2013) to the three-dimensional (3D) case. Again our interest is in global models of prominences rather than in the details of the internal structure. Line-tying conditions are applied at the photosphere as they are crucial to have magnetic support and they strongly affect the dynamics of the system. Since the model is 3D, it allows us to analyze perturbations along the longitudinal axis of the prominence neglected in the 2D case. This changes significantly the physics of the system because MRT unstable modes are easily excited. The development of the instabilities, together with the oscillatory behavior of the structure, generates complex motions in the prominence body. One of the main aims of this work is not only to investigate the dynamics but also to understand how the morphology of the prominence depends on the different parameters of the model.

2. INITIAL CONFIGURATION AND SETUP

The basic initial configuration is an isothermal stratified atmosphere as in Terradas et al. (2013). The same parameters have been used in the present work. The main difference is in the magnetic field configuration, which now has a component in the $y$-direction that introduces shear in the system. Again, the force-free magnetic field is based on arcade solutions. The magnetic field has to satisfy

$$\nabla^2 \mathbf{B} = 0,$$

where it has been assumed that $\nabla \times \mathbf{B} = \alpha \mathbf{B}$ and $\alpha$ is uniform.

An arcade solution of Equation (1) has the following magnetic field components:

$$B_x(x, z) = B_0 \frac{l}{k} \cos kx e^{-lz},$$

$$B_y(x, z) = B_0 \frac{\alpha}{k} \cos kx e^{-lz},$$

$$B_z(x, z) = -B_0 \sin kx e^{-lz},$$

where $B_0$ is a reference constant. The parameter $k$ is related to the lateral extension of the arcade, while $l$ is a measure of the vertical magnetic scale height. The parameter $\alpha$ is given by the expression

$$\alpha = (k^2 - l^2)^{1/2},$$

and it is related to the amount of shear in the arcade. For $l = k$, $\alpha = 0$, the magnetic field is purely potential and the $B_y$ component is zero.

Magnetic field lines in the configuration given by Equations (2)–(4) do not show any dips because the configuration is bipolar. A simple way to obtain a configuration with magnetic dips is to chose a particular superposition of two magnetic arcades representing a quadrupolar configuration

$$B_x(x, z) = B_1 \frac{l_1}{k_1} \cos k_1x e^{-l_1z} - B_2 \frac{l_2}{k_2} \cos k_2x e^{-l_2z},$$

$$B_y(x, z) = B_1 \frac{\alpha}{k_1} \cos k_1x e^{-l_1z} - B_2 \frac{\alpha}{k_2} \cos k_2x e^{-l_2z},$$

$$B_z(x, z) = -B_1 \sin k_1x e^{-l_1z} + B_2 \sin k_2x e^{-l_2z}.$$

The individual arcades, quoted with the subindexes 1 and 2, must have the same $\alpha$ to satisfy the Helmholtz equation given by Equation (1). Thus, we have, according to Equation (5), the constraint

$$k_1^2 - l_1^2 = k_2^2 - l_2^2.$$

The width of the full structure is $L_0$, and we select the wavenumbers $k_1 = \pi/2L_0$ and $k_2 = 3\pi/2L_0$ since they generate a magnetic configuration with dips at $x = 0$. The parameter $l_1/k_1$, hereafter referred to as $l/k$, is related to the amount of shear in the structure. Several examples of different sheared arcades are found in Figure 12 by imposing the previous wavenumbers and that $B_2 = B_1$. Note the location of dips and that the magnetic configuration is invariant in the $y$-direction.

The initial prominence mass and size are prescribed according to typical values reported from observations (see, e.g., Labrosse et al. 2010). Here we assume that the typical size of the density enhancement has a width of $5 \times 10^3$ km, a length of $4 \times 10^4$ km, and a height of $10^6$ km. The prominence is initially suspended above the photosphere at a height of $1.75 \times 10^4$ km. With a typical prominence density of $5.2 \times 10^{-2}$ kg km$^{-3}$ and taking into account the geometry of the prescribed prominence (see Figure 1, top panel), the total mass is $1.3 \times 10^{11}$ kg. For this case the density contrast between the corona and the core of the prominence is around 120. In some simulations we have changed the total mass of the prominence by reducing the density.
Figure 1. Time evolution of density and magnetic field lines for a typical case. In this simulation $l/k = 0.95$ (weak shear) and $v_{A0} = 20c_{s0}$, where $v_{A0} = B/\sqrt{\rho_{0}}$ is the maximum Alfvén speed in the system. The maximum magnetic field ($B$) corresponds to the points $x = \pm L_a$ and $z = 0$. Density is normalized to the coronal density ($\rho_0$). Time in minutes is shown at the top of each panel.

(An animation of this figure is available.)

contrast. Between the core of the prominence and the corona we have included in the initial density profile a prominence–corona transition region (PCTR) with a typical width of 30%–15% of the characteristic length in each direction. The mass deposition is produced just at $t = 0$, and contrary to the situation in Terradas et al. (2013), the deposition is instantaneous.

Note that there is no initial velocity perturbation introduced in the system. However, because of the mass deposition at $t = 0$ the system immediately reacts to the presence of the enhanced mass, which is pushed down by the gravity force, and therefore velocity perturbations, especially in the vertical component, will be generated.

3. NUMERICAL TOOLS

The code used to solve the ideal MHD equations is an evolution of the code MoLMHD already described in Terradas et al. (2013; see also Bona et al. 2009). The main novelty in the present version is the implementation of a WENO scheme (see, e.g., the review of Shu 2009) to solve some of the MHD equations (see Jiang & Wu 1999; Balsara et al. 2009). In particular, we have used a fifth-order-accurate method in space to solve the equations of continuity, momentum, and energy. This last equation has been written in terms of gas pressure instead of total energy. The WENO scheme is very robust with respect to the presence of stiff gradients in the variables and thus suitable to handle strong shocks. The density contrast between the prominence and corona can be higher than 100 and therefore difficult to treat without high-resolution numerical methods. Nevertheless, for the induction equation we have used a standard fourth-order central discretization with an artificial dissipation coefficient since the application of the WENO scheme on the magnetic field has been found to be too diffusive. The excess of diffusion produces plasma motions across the magnetic field, changing the dynamics with respect to the idealized case where plasma is frozen to the magnetic field. The inevitable numerical diffusion associated with the scheme has been significantly reduced with the method used in this paper.

The MHD equations solved in this study are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \left( \rho \mathbf{v} \mathbf{v} + p \mathbf{I} - \frac{\mathbf{B} \mathbf{B}}{\mu} + \frac{\mathbf{B}^2}{2\mu} \right) = \rho \mathbf{g},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}),$$

$$\frac{\partial p}{\partial t} + \nabla \cdot (\gamma p \mathbf{v}) = (\gamma - 1) \mathbf{v} \cdot \nabla p,$$

where $\mathbf{I}$ is the unit tensor, $\mathbf{g}$ is the gravitational acceleration, and the rest of the symbols have their usual meaning.

As in Terradas et al. (2013), the technique of background splitting for the magnetic field (see Powell et al. 1999) has been used, and this has allowed us to gain numerical accuracy. The splitting introduces source terms in the equations that have been treated using the fourth-order central scheme. Although in Powell et al. (1999) it is assumed that the initial magnetic field is potential, i.e., current free, it turns out that for a magnetic field that is not potential the corresponding source terms are exactly the same.
Regarding boundary conditions, we have applied line tying at the bottom plane, representing the photosphere. This condition means that the three components of the velocities are set to zero while the normal component of the magnetic field, $B_z$, in our Cartesian system, is fixed. For the rest of the variables at this boundary we have used simple extrapolation, i.e., that their spatial derivatives are zero. For top and lateral boundaries we expect the energy to escape the system. We have tested different conditions. Using decomposition in characteristic fields at the boundary (see Terradas et al. 2013) works fine, but it is computationally expensive. We have found that using fixed values at the boundary essentially produces the same results as the decomposition in characteristics, and it is much more simple to implement. This last approach has already been used in Török & Kliem (2003) and Török et al. (2004) in the context of ideal kink instabilities in magnetic loops.

The reference length in the simulations is $L = 10^4$ km, and the width of the arcade is $L_x = 6 L$. Velocities have been normalized to the coronal sound speed at a temperature of 1 MK, $c_{s0} = 166$ km s$^{-1}$. The reference time is $\tau = L/c_{s0} = 1$ minute. The total time of evolution is 100 minutes. The typical resolution that we have used in the simulations is $150 \times 150 \times 100$ points, with $-6 < x/L < 6$, $-6 < y/L < 6$, and $0 < z/L < 8$, with $L = 10^4$ km being the reference length. Higher resolutions have also been considered to see the effect on the results. Since we consider a global prominence model, we need, from the numerical point of view, to have enough grid points in both the coronal part and the prominence body. The typical distance between grid points is 800 km, but in some cases it has been reduced to 550 km. Structures of the size of individual threads are not resolved in this work.

4. RESULTS

4.1. A Typical Run

The results of a typical simulation are displayed in Figure 1. In this figure the density distribution and some specific magnetic field lines are plotted as a function of time (see also Movie1). During the first hour of evolution, the prominence body is able to be suspended above the photosphere owing to the force provided by the magnetic field. The structure resembles a detached prominence, referred to in the past as a suspended cloud. Although there is global support, the shape of the prominence changes with time. We can see that, for example, after 30 minutes of evolution, small scales appear in the density distribution, which was initially quite homogeneous. This is linked to the excitation of MRT instabilities. Changes in density involve variations of the magnetic field since the plasma is frozen to the magnetic field in ideal MHD. Therefore, the motion of magnetic field lines has to be consistent with plasma motions (see Movie2).

At a given instant, temperature and plasma-$\beta$ distributions ($\beta$ is the coefficient between gas and magnetic pressure) are quite irregular owing to the presence of the cool and dense material representing the prominence. The distribution of these magnitudes at the central plane is plotted in Figure 2. We find that density variations produced during the evolution also involve temperature changes. Temperature is typically of the order of 10,000 K inside the prominence, while in the corona it is around 1 MK. Between these two regions we find the PCTR, clearly visible in temperature. In this configuration the plasma-$\beta$ changes mostly with height, it has a minimum around the prominence location, and it increases toward the photosphere and upward with height. At the prominence core it is below 0.02.

The dynamics of the system is quite rich, and several features need to be described in detail. For example, the prominence shows oscillations before the clear onset of the MRT instability. In Figure 3 the vertical component of the velocity at early stages of the simulation is represented at the plane passing through the prominence core. At the center of the prominence the velocity at $t = 2.88$ minutes is negative since the prominence body is moving downward. In the following frame, at $t = 5.76$ minutes the motion is in the positive vertical direction. This is a consequence of the vertical oscillatory motion that the whole prominence is experiencing during the relaxation process. Note also the deformation of the lower boundary of the prominence displaying a convex curve for $t = 8.64$ minutes. At the same time strong shear motions are generated at the lateral edges of the prominence, where we find positive/negative velocity patterns in $v_z$. The spatial scales of these patterns decrease with time, as can be appreciated in Figure 3. This has to do with the process of mode conversion and phase mixing that takes place at the inhomogeneous layers of the structure, in this case the prominence PCTR at the sides of the prominence body in the $y$-direction. Part of the attenuation of the oscillatory vertical
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Figure 3. Time evolution of density and vertical velocity at the yz-plane at \(x = 0\) during the first 12 minutes of evolution before the onset of the instability. Shear vertical motions are produced at the boundaries of the prominence during the oscillations. Velocity amplitudes are normalized to the coronal sound speed.

motion of the structure is due to this process; however, wave leakage might play a role as well. Since our aim is to concentrate on the global evolution of the system, we leave the analysis of oscillations and damping for future studies. However, it is interesting to mention that these shear motions at the edges of the prominences may lead to the development of another type of instabilities due to velocity shear, i.e., Kelvin–Helmholtz instabilities. Indeed, a closer inspection of the lateral edges of the prominence indicates the presence of very dynamic motions at the PCTR associated with this kind of instability, but with the particularity that shear changes periodically with time owing to the global oscillation of the structure. This effect is similar to the deformations produced at the boundaries of a cylindrical tube when it is oscillating (see Terradas et al. 2008).

At the beginning of the simulation there are no clear signatures of MRT unstable modes. Instabilities start to develop around \(t = 20\) minutes, as can be appreciated in Figure 4. Apart from the shear motions associated with the lateral edges of the prominence and explained before, we also find shear motions in the vertical component of the velocity that start to form at the bottom part of the prominence (see Figure 4). Descending fingers of cool and dense plasma and ascending plumes of hot and less dense plasma have been developed in the structure and modify significantly the density distribution of the prominence. For the particular simulation studied here the prominence bottom shows several arches that spread all over the prominence body. In particular, we can identify up to four clear fingers. The density and \(v_z\) distribution are further visualized in Figure 5 (see also Movie 2). The excitation of MRT unstable modes affects the prominence on a global scale, and the instability is modified by the effect of line-tying conditions. For the simulation shown in Figures 4 and 5 the growth time of the instability is of the order of 20 minutes. This estimation is based on the change in position of the finger-like structures in the prominence, but it is important to remark that during the evolution of the MRT instability the prominence is still oscillating (see Movie 2), and this complicates the estimation of growth rates. Thus, this growth time does not correspond to the growth time that one would calculate in the linear stage of development of the instability. Nevertheless, it is a useful parameter since it gives a typical timescale associated with the physical process. It is worth mentioning that in this simulation the prominence is not destroyed by the instability at least during the first 100 minutes of evolution. The eventual destruction of the prominence in a short period of time would mean that indeed a continuous supply of material is required, as some observations suggest.

Further information about the dynamics of the system is derived from the behavior of the energy. We have calculated from the simulations the integrated kinetic energy at the yz-plane \((x = 0)\). This magnitude is useful since it shows clear signatures of the development of instabilities associated with MRT unstable modes. In Figure 6 the total kinetic energy (solid line) and the energy associated with the vertical velocity component (dotted line) are plotted as a function of time. According to the plot, most of the kinetic energy is due to vertical motions that are initially induced by the gravity force. The energy decreases with time owing to the effect of leakage, while the prominence is oscillating vertically (see the fluctuations in the dotted line for \(t < 20\) minutes). Later, around \(t = 25\) minutes, the kinetic energy starts growing; this is an indication of the development of the instability and coincides with the appearance of the fingers and plumes in density (see, e.g., Figure 1, middle panel, or Movie 1). After peaking around \(t = 32\) minutes, the energy starts to decrease again, meaning that fingers do not continue.
Figure 4. Time evolution of density and vertical velocity at the $yz$-plane at $x = 0$ during the development of the MRT instability.

moving toward the photosphere at the same velocity; in fact, they are decelerating. In the energy profile (and this can also be identified in Movie1) there is a faint indication of a new development of the instability around $t = 60$ minutes. Therefore, in this particular example the instability is unable to destroy the prominence although it produces dynamic motions at the prominence body.

The MRT unstable interface between the prominence and corona has smooth density gradients owing to the presence of the PCTR. It is well known that density gradients have the effect of reducing the growth rate of RT instabilities, especially at short wavelengths (see, e.g., Mikaelian 1986). We have performed a run with half the size of the PCTR in the standard simulation, but the differences in the spatial scales of the instability and the growth rates are not too significant. Although we start with a given width of the PCTR, after the time evolution the eventual “quasi-equilibrium” that is obtained has a thicker transition between the core of the prominence and corona since radiation and conduction are neglected in the model. The magnetic field also has gradients, and Yang et al. (2011) have shown that they affect the linear growth rates of the MRT instability in a certain range that depends on the magnetic field strength. Thus, the excitation of the most unstable mode with a particular spatial scale (in our case we find structures with typically three cavities) will be conditioned by all these effects, and it is very difficult

Figure 5. Zoom of density and vertical velocity at a given time (see Figure 4, bottom panel). Fingers and plumes associated with the MRT instability are present and are associated with the shear motions in velocity. Some magnetic field lines are also represented.

(An animation of this figure is available.)

Figure 6. Integrated kinetic energy at the $yz$-plane at $x = 0$ as a function of time. The solid line represents the total kinetic energy, while the dotted line corresponds to the kinetic energy in the vertical direction only. The arrows show times when the MRT instability is clearly identified.
to anticipate. However, we think that the most important factors that affect the appearance of the unstable modes and the spatial distribution of the cavities in the horizontal direction are the strength of the magnetic field at the core of the prominence and the variation of the shear angle with height. This will be studied in Section 4.4. It is worth mentioning that owing to the symmetry of the system, there is a maximum in velocity at the center of the prominence (in the xz-plane at $y = 0$). This means that there is a natural excitation of symmetric modes in velocity with respect to the center of the prominence. For this reason it is logical to find structures with an odd number of cavities along the y-direction (three in the case of Figure 1).

The spatial patterns related to the MRT instability found in the simulations may have a close link with the observed arches and cavities reported in many prominences (see, e.g., Berger et al. 2010). It is worth mentioning that in Berger et al. (2010) the cavities beneath the prominence are found forming single structures instead of multiple cavities like in the present simulations. However, the vertical structuring observed in many polar crown prominences (see, e.g., Dudík et al. 2012) resembles the spatial structures found in the simulations. Ryutova et al. (2010) associated the appearance of bubbles/cavities with the development of screw-pinchof instabilities assuming helical prominence models. However, if the structure does not have a clear flux rope shape and is better represented by a sheared arcade, then the MRT instabilities found in our simulations can provide an explanation for the formation of cavities. In this regard, MRT instabilities have already been investigated using 3D simulations by Hillier et al. (2011, 2012a, 2012b). Since these authors have focused on internal motions, the connection of the field lines with the photosphere was neglected in their models, and as we have already anticipated, line-tying conditions change considerably the dynamics of the MRT instability. The MRT instability in our configuration affects the whole prominence body, instead of small internal parts. However, it must be pointed out that because of the limitation in the numerical resolution, we are unable to resolve features with the spatial scales studied by Hillier et al. (2011).

4.1.2. Effect of Numerical Resolution

The case analyzed in Section 4.1 corresponds to a particular set of parameters. It is interesting to study how the evolution of the system depends, for example, on the strength of the magnetic field. In Figure 8 the morphology of the prominence at a fixed time is shown for three different values of the plasma-$\beta$ associated with different magnetic field strengths at the core of the prominence. In the top panel we find the previous situation where the magnetic field is able to provide the support to the dense material against gravity. This case corresponds to a value of $\beta = 0.075$ at the end of the simulation. This prominence can be classified as detached. Middle and bottom panels show a rather different scenario, and they are associated with $\beta = 0.15$ and $\beta = 0.4$, respectively. In these last two cases the magnetic field is unable to provide the restoring force to hold the whole prominence suspended above the photosphere. Most of the dense plasma falls down and essentially pushes down the magnetic field near the photosphere (see the field lines crossing the center of the prominence body in Figure 8). For $\beta = 0.15$ some voids in density are appreciated at the center, resembling some hedgerow prominences reported in observations (see the classification of prominences by Menzel & Evans 1953; Jones 1958; Zirin 1998). For the case $\beta = 0.4$ the structure seems more compact, resembling mound or curtain prominences with little structure. Note that if the view of the prominence is along the y-direction, then it can resemble a pillar prominence, which is simply a curtain prominence seen edge-on.

Details of the evolution for the curtain-like prominence are found in Figures 9 and 10 (see also Movie3). In these figures we have represented the evolution of density and the x-component of the velocity, $v_x$, passing through the central plane. The other velocity components are also important at the beginning of the simulation, but at the end of the simulation $v_x$ dominates. At early stages of the evolution the behavior is quite involved, as

We have also checked the effect of the location of the lateral and upper boundaries. We have concluded that locating these boundaries at larger distances from the prominence body does not change significantly the results. For this reason, we have kept the boundaries located at the distances given in Section 3 in order to avoid a significant decrease in the numerical resolution when a fixed number of grid points is used.

4.1.1. Effect of Boundary Conditions

For the simulations presented here line-tying conditions have been imposed at the photosphere. Without this condition at $z = 0$, the core of the prominence would simply fall as a result of the effect of gravity, and the associated changes of the local magnetic field would be unable to provide magnetic support. We have performed several simulations to check this effect by removing line-tying photospheric conditions, and indeed we have not been able to find a sustained prominence. Thus, the effect of the communication between the prominence and the photosphere due to the reflecting conditions cannot be neglected since it is very relevant regarding the support of the prominence against gravity. This is an important property of the simulations performed in this work. We plan to study in the future other boundary conditions, such as mirror boundaries or boundaries using potential field extrapolations. In fact, the photospheric magnetic field may change over timescales of a few hours owing to photospheric convective motions. Nevertheless, we have decided to concentrate on the most simple boundary conditions that lead to sustained prominences, i.e., line-tying conditions.

We have also checked the effect of the location of the lateral and upper boundaries. We have concluded that locating these boundaries at larger distances from the prominence body does not change significantly the results. For this reason, we have kept the boundaries located at the distances given in Section 3 in order to avoid a significant decrease in the numerical resolution when a fixed number of grid points is used.
Figure 7. Prominence density and magnetic field for three different values of the numerical resolution, $75 \times 75 \times 50$, $150 \times 150 \times 100$, and $225 \times 225 \times 150$. Time is essentially the same for the three simulations. All the other parameters are the same as in Figure 1.

can be appreciated in the top panel of Figure 10, where velocities up to $4 \, \text{km s}^{-1}$ are found at the edges but also at the core of the prominence. Outside the prominence we identify the emission of MHD waves; this is specially clear near the top of the domain.

Figure 8. Prominence density and magnetic field for three different values of the magnetic field ($v_{A0} = 20 \, \text{c}_0$, $v_{A0} = 15 \, \text{c}_0$, and $v_{A0} = 10 \, \text{c}_0$), which are associated with the $\beta$ values of 0.075, 0.15, and 0.4 at the core of the prominence. Time is the same for the three simulations.

where several wavefronts are visible. As time progresses, the amplitudes of the motions decrease both at the prominence and in the corona. This decrease in the velocity amplitudes is mostly due to the energy leakage that takes place at the upper
and lateral boundaries of the computational domain. Numerical dissipation, although small, also contributes to the attenuation of the motions. At later stages of the evolution (see, e.g., bottom panel of Figure 10, we can still find some velocity signals at the prominence that are related, in this particular example, to a slow change in orientation of the axis of the prominence since the motions in the $x$-direction alternate sign with respect to the center of the structure.

4.3. Dependence on Prominence Mass

Another relevant parameter is the total mass of the prominence. We have imposed a fixed strength of the magnetic field at the base of the arcade, $v_{A0} = 10 \, c_{s0}$, and have decreased
the prominence mass with respect to the mound prominence studied in Figures 9 and 10. Now the total mass is four times lower, i.e., of the order of $3.25 \times 10^{10}$ kg, which is still in the range of masses calculated from observations (typically between $10^9$ and $10^{12}$ kg). Three snapshots of the density and magnetic field are plotted in Figure 11 for this case. Although the plasma-$\beta$ at the core of the prominence is around 0.3, the magnetic structure is able to sustain the prominence suspended above $z = 0$, obtaining a detached structure. Thus, the total mass loading of the prominence essentially determines, together with the plasma-$\beta$, the morphology of the structure. Note also the appearance of the MRT instability with some fingerprints visible in Figure 11.

### 4.4. Dependence on Magnetic Shear

A parameter that can also be changed in the configuration is the amount of shear in the arcade. We have performed a set of different simulations by changing this parameter, and the results are found in Figure 12. For example, the MRT instability already developed for the case $l/k = 0.95$ at $t = 29.1$ minutes (see Figure 1, middle panel) is not present for the arcade with $l/k = 0.85$. For larger values of the shear the global shape of the prominence does not change much from the side view of Figure 12. This means that shear helps to have a more stable configuration regarding the MRT instability. In order to quantify this effect, we have plotted in Figure 13 the estimated growth rates for different values of $l/k$. The unsheared arcade, the case $l/k = 1$, has the shortest growth time, which is around 20 minutes. The curve shown in Figure 13 indicates that the growth time increases with shear, meaning that this parameter helps to reduce the instability. Note that the dependence of the growth times with shear can be quite strong, and that a small amount of shear may significantly reduce the instability. For example, we see that for values of $l/k$ below 0.92 the growth times are larger than 2 hr. For this equilibrium parameter the arcade is still weakly sheared since it would be an intermediate case between the cases $l/k = 0.95$ (see Figure 1) and $l/k = 0.85$ (represented in Figure 12, top panel).

We have investigated the reasons for the stabilization effect produced by an increase in magnetic shear. We have found basically two reasons. The first one is related to the strength of the magnetic field at the prominence body. It turns out that a decrease in the parameter $l/k$ (increase in shear) produces an increase in the horizontal component of the magnetic field at the prominence. For example, for $l/k = 0.65$ the horizontal component is 1.4 times larger than for $l/k = 0.95$. Stronger horizontal magnetic fields at the core of the prominence lead to a larger magnetic tension, and this is the force that helps to stabilize the MRT unstable modes. The second reason is related to the change of the shear angle with height. The effect of shear on MRT instabilities has been recently investigated analytically by Ruderman et al. (2014) in a much simpler geometry, a 2D slab configuration (in the $xz$-plane) with a discontinuous change in the magnetic shear and including propagation in the perpendicular direction (the $y$-direction). These authors have found that the growth rate is bounded under the presence of magnetic shear. The important result is that for small shear angles the growth time of the most unstable mode is linearly proportional to the shear angle, while in the limit of large angles the growth time is essentially independent of the shear angle. In our arcade configurations the situation is more complex since the magnetic shear changes smoothly with height (see Figure 14). This must affect the growth times of the instability. Using the results of the simple slab model, we can explain the qualitative behavior in the growth times with the parameter $l/k$. For $l/k$ large, the variation of shear with height is relatively small (see in Figure 14 the region around the prominence location, i.e., around $z = 2$), meaning that the effective shear is small. Under such conditions the growth time is small (fast development of the instability). On the contrary, when $l/k$ is small, the change of shear with height is stronger (see Figure 14), meaning that...
Figure 12. Density and magnetic field distribution for three different values of the magnetic shear, \( l/k = 0.85, 0.75, 0.65 \), that correspond to typical shear angles of 32°, 41°, and 54°, respectively. The width of the domain in the \( y \)-direction is larger than in the previous simulations since an increased shear leads to longer field lines.

The effective shear is larger and thus the growth time is longer. This provides a qualitative agreement with the results shown in Figure 13.

In fact, an increase in shear changes the depth of the dips, i.e., the vertical distance between the dip center and the highest point on a magnetic field line, and it also changes the width of the dips. In Figure 15 the magnetic field lines (projected in the \( xz \)-plane) crossing the initial density distribution at a given height are plotted for two different values of the shear. For the case \( l/k = 0.95 \) (solid lines) we see that the field lines are quite flat at the top of the prominence and the depth of the dip increases as we move toward the photosphere. For the configuration with \( l/k = 0.65 \) (dashed lines) with a much stronger shear the situation is different: at the top of the prominence the field lines are more curved than for \( l/k = 95 \) and potentially less favorable.
to support the dense material, but as we move to lower heights
the depth of the dips increases with respect to $l/k = 95$, thus
helping to sustain the mass. On the contrary, the width of the dips
is smaller than for the case $l/k = 0.95$. Therefore, the magnetic
support can be slightly different for the two configurations.

Although in Figure 12 (bottom panel) the prominence body
seems to keep a compact structure, a view from the top of
the configuration (not shown here) reveals a rather different
behavior. The dense material tends to move along the magnetic
field, and strong deformations appear at the edges of the
prominence. In fact, this part of the prominence eventually
descends toward the photosphere. This is in agreement with
the results of Karpen et al. (2001) and Luna et al. (2012), which
have found that in a highly sheared arcade with shallow dips the
condensation of cool material is short-lived and falls rapidly to
the chromosphere. Another effect that we find in the simulations
is that in the meantime the orientation of the main axis of the
prominence changes with time and tends to be aligned with
the direction of the magnetic field. These features, which might
be relevant regarding observations, need further investigation
but are left for future studies since a detailed analysis would
require much longer total simulation times with better spatial
resolutions.

It must be pointed out that in the analysis of the dependence
of the instability with shear ($l/k$) we have imposed that $B_2 = B_1$
in Equations (6)–(8). This condition selects a particular family
of magnetic arcade solutions. Other choices of the constants
$B_1$ and $B_2$ might lead to the presence of significant dips even
for cases where shear is strong. Hence, it might be possible to
find configurations with strong shear in which the mass of the
prominence does not fall to the photosphere.

4.5. An Irregular Prominence

Finally, it is important to remark that all the simulations
presented in the previous sections are based on the assumption
that the initial mass deposition is symmetric. The results are
thus affected by this assumption, and, among other things, only
symmetric modes are excited in the system. Since observations
show that in general the distribution of mass in the prominence
body is very inhomogeneous, it is also interesting to investigate
a more general case and see the effect of inhomogeneity on
the numerical experiments. For this reason, here we present
the results of a run with an irregular mass. The initial density
distribution and the time evolution are represented in Figure 16.
Most of the initial mass is concentrated around the left part
of the prominence, and the interface between the core and
corona is not purely horizontal. Again the prominence body
is pushing the magnetic field in the negative vertical direction
as a result of the effect of the gravity force. At later times,
around $t = 30$ minutes, we identify the initial stages of the
development of the MRT instability at the curved interface at
the bottom of the prominence. Plumes and fingers are again
quite evident at $t = 59$ minutes. The development of the
instability proceeds at a faster rate than in the purely symmetric
prominence. Interestingly, these fingers may eventually connect
to the photosphere, meaning that the morphology of the structure
might evolve from a detached prominence to a hedgerow
prominence.

In this simulation, since the system is asymmetric, the excita-
tion of other types of motions with respect to the symmetric case
is possible. For example, in Figure 17 the velocity component
in the $x$-direction and the density are represented as a function
of time. Now the point of view has been changed in the plot to

![Figure 16. Density isocontours at different times. Fingers and plumes associated with the MRT instability are present in this simulation. The difference with respect to Figure 1 is in the initial shape of the density distribution, which is now nonsymmetric; the rest of the parameters are the same.](image)
Figure 17. Time evolution of density and horizontal velocity at the \(xz\)-plane at \(y = 0\) at the initial stages of the evolution and before the onset of the MRT instability.
(An animation of this figure is available.)

better visualize the results. In this plot we identify motions in \(v_x\) that alternate sign with respect to the location of the prominence body around the center of the system. These motions are associated with the excitation of perturbations running mostly parallel to the magnetic field and therefore linked to the excitation of slow magnetoacoustic waves (see Terradas et al. 2013 for the equivalent results in 2D). As a consequence of these motions, the whole prominence shows changes in the cross section, as can be appreciated in Figure 17 (see density isocontours), and at the same time it is oscillating laterally, mostly along the \(x\)-direction (see Movie4). This can have close links with large-amplitude longitudinal oscillations reported in the literature (see Tripathi et al. 2009; Zhang et al. 2012; Luna et al. 2014).

5. DISCUSSION AND CONCLUSIONS

We have investigated the time evolution of a simple 3D density enhancement representing a prominence embedded in a sheared coronal arcade. We have studied the evolution of the prominence mass and the magnetic field in a self-consistent way. This is different from other studies of the topology of the magnetic field inferred from photospheric extrapolations since the prominence mass is not taken into account in the model. The results from our study indicate that in general the dynamics of the coupled prominence mass to the magnetic field is quite involved. We have tried to understand how the results depend on the different parameters of the configuration. A critical parameter that determines the characteristics of the evolution is the plasma-\(\beta\). For low-\(\beta\) configurations cold material can be suspended above the photosphere, and it can represent detached prominences typically reported in observations. In this case there is a balance between the prominence weight, which depends on the mass, and the Lorentz force, which changes with the plasma-\(\beta\). A rather different configuration is found when \(\beta\) is increased. In this last case we find that the dense material is essentially moving toward the photosphere and compressing the magnetic field. The overall shape of the structure is more similar to hedgerow or even curtain prominences. Hence, the global morphology of the prominence is highly determined by the plasma-\(\beta\), but it is also affected by the boundary conditions at the photosphere. Here we have focused on line-tying conditions and have found that they are crucial for magnetic support since they communicate the effect of the dense photosphere on the prominence body. The total mass loading of the prominence is also another parameter that determines the morphology and evolution of the structure.

For prominences supported against gravity in the low-\(\beta\) regime, we have found fingerprints of the excitation of MRT unstable modes. Fingers and plumes develop on a global scale, affecting the whole prominence body. Although the nature of the flows found in our numerical experiments is due to the release of gravitational potential energy, the flow pattern is more complex than in the standard MRT instability at an interface that is in equilibrium. The reason is that during the development of the instability the prominence is oscillating as a consequence of line-tying conditions, and this affects the growth rates of the modes. The spatial scales of plumes and fingers found in the simulations are much larger than those commonly reported in observations by, for example, Berger et al. (2010) and studied from the numerical point of view by Hillier et al. (2011, 2012a). In our simulations we are simply not resolving those spatial scales as a result of a lack of spatial resolution. The voids in density found in our simulations can be associated with the appearance of large-scale cavities/arches or even bubbles commonly reported in observations, especially in polar crown prominences. Other interpretations have been proposed to explain bubbles; for example, Ryutova et al. (2010) have suggested that they are the consequence of the excitation of the kink unstable mode in
a flux rope prominence configuration. Dudik et al. (2012) have proposed that the formation of cavities is due to the emergence of magnetic flux due to the appearance of parasitic bipoles at the photosphere. In any case, it is important to mention that at the prominence body we find vertical structures, linked to the appearance of fingers and plumes, in an essentially horizontal magnetic field. On the contrary, from the observational point of view it is thought that the magnetic field is quite vertical, at least in many curtain and hedgerow prominences formed by vertical threads.

We have showed that magnetic shear stabilizes the configuration. Performing a quantitative study of the dependence of the instability on this parameter, we have shown that it can significantly reduce the growth rates of the MRT unstable modes. Thus, it has important consequences regarding the stability of the magnetic configuration. In particular, the parameter \( l/k \) in this study is directly related to strength of the magnetic field at the prominence core and to the change of the direction of the horizontal field with height. Strongly sheared configurations also have a fast change of the shear angle with \( z \), which helps to stabilize the structure (see Ruderman et al. 2014).

In our numerical experiments the transition between the corona and the photosphere has been ignored and line-tying conditions have been applied at \( z = 0 \). A proper analysis of the evolution of prominences attached to the photosphere, like the ones described above (and even suspended prominences), should include the transition region and chromospheric layers. The physics required to reproduce chromospheric or prominence conditions is quite complex (partial ionization effects, radiative transport, etc.). Moreover, thermal effects such as conduction or radiation have been ignored in the present work but are thought to play a relevant role in the shape of the PCTR.

It is clear that the 3D configuration studied here shows very dynamic features and the relaxation to a purely stationary magnetohydrostatic configuration is hardly achieved, at least in the regime of parameters considered in this work. This is different from the 2D case studied by Terradas et al. (2013) using essentially the same magnetic configuration. In that case there were no MRT instabilities because the system was assumed to be invariant with respect to the longitudinal direction (the \( y \)-coordinate in our system) and modes with a component along this direction are in general the most unstable. This indicates, once more, that interpretations based on 2D models may miss important physics. The lack of simple magnetohydrostatic equilibrium also agrees with the observational fact that prominences are locally very dynamic.

Note that although we have not tried to model the prominence formation process (see Xia et al. 2011, 2012; Keppens & Xia 2014 for recent advances about condensations mechanisms with magnetic fields) and the continuous supply of material is missing, we already find a very complex dynamics. If the formation process is investigated using levitation, injection, or condensations models, it will most likely produce even more complex phenomena. This will make things difficult to interpret. For this reason, we have preferred to analyze first the evolution of an existing density enhancement in a coronal environment.

It is worth mentioning that it would be interesting to study a flux rope configuration to compare with the present shear arcade model and the embedded direct polarity prominence in order to assess whether there are significant differences in the results. In particular, it would be interesting to understand the role of magnetic twist on the development of the MRT modes, as well as on the kink instability. Also, it might be interesting to perform a detailed analysis of the modes of oscillation and to compare with observations of oscillating prominences/filaments induced, for example, by nearby flares. Finally, future studies should use high-resolution 3D simulations to resolve the fine structure associated with prominence threads, as well as turbulent phenomena related to nonlinearity, for a better comparison with observations.

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