Impact of the spectral hardening of TeV cosmic rays on the prediction of the secondary positron flux

J. Lavalle⋆†

Departamento de Física Teórica (UAM) & Instituto de Física Teórica (UAM/CSIC), Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain

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ABSTRACT

The rise in the cosmic-ray positron fraction measured by the PAMELA satellite is likely due to the presence of astrophysical sources of positrons, for example, pulsars, on kpc scales around the Earth. Nevertheless, assessing the properties of these sources from the positron data requires a good knowledge of the secondary positron component generated by the interaction of cosmic rays with the interstellar gas. In this paper, we investigate the impact of the spectral hardening in the cosmic-ray proton and helium fluxes recently reported by the ATIC2 and CREAM balloon experiments on the predictions of the secondary positron flux. We show that the effect is not negligible, leading to an increase in the secondary positron flux by up to \(\sim 60\) per cent above \(\sim 100\) GeV. We provide fitting formulae that allow a straightforward utilization of our results, which can help in deriving constraints on one’s favourite primary positron source, for example, pulsars or dark matter.

Key words: cosmic rays.

1 INTRODUCTION

The increase in the positron fraction above a few GeV reported by the PAMELA Collaboration (Adriani et al. 2009) has triggered a lot of interpretation attempts, but it is now likely that it is due to positrons originating from conventional astrophysical sources, like pulsars (e.g. Shen 1970; Sturrock 1970; Harding & Ramaty 1987; Boulares 1989; Aharonian, Atoyan & Voelk 1995; Chi, Cheng & Young 1996; Profumo 2008; Malysh, Cholis & Gelfand 2009; Yüksel, Kistler & Stanev 2009; Delahaye et al. 2010) or supernova remnants (e.g. Berezhko et al. 2003; Blasi 2009), while some other spatial or solar effects might also play a role (e.g. Shaviv, Nakar & Piran 2010). These works have notably shown that a very few sources may dominate the high-energy positron flux at the Earth, opening interesting perspectives for more accurate predictions in the near future (see Delahaye et al. 2010, for a detailed analysis). Nevertheless, these perspectives are based on the assumption that the secondary positron flux prediction is under control.

Secondary positrons originate from nuclear interactions of cosmic-ray (CR) nuclei with the interstellar (IS) gas and have been investigated in detail by Moskalenko & Strong (1998), and more recently by Delahaye et al. (2009), who provided new insights into the theoretical uncertainties. Delahaye et al. (2010) have improved the predictions of the latter by including the Klein–Nishina corrections to the energy-loss treatment. All these predictions rely on CR nucleus spectra constrained by rather low-energy data (\(\lesssim 100\) GeV), which are mostly power laws. Nevertheless, two balloon experiments, ATIC2 (Panov et al. 2009) and CREAM (Ahn et al. 2010), have recently reported on a clear and almost concordant hardening in these spectra around a few TeV, with very good statistics. Since high-energy stable nuclei have quite long range propagation (e.g. Taillet & Maurin 2003), it is likely that this spectral inflection is not merely local and pertains over a few-kpc scale around the Earth. Therefore, a consistent prediction of the secondary positron flux should take it into account.

In this paper, we study in detail the impact of these new CR measurements on the secondary positron flux predictions. We provide the reader with user-friendly fitting formulae which summarize our results and which can be used to constrain any extra source of CR positrons.

2 FROM THE HARDENING OF COSMIC-RAY NUCLEUS SPECTRA TO THE HARDENING OF THE SECONDARY POSITRON SPECTRUM

2.1 Generalities

Predictions of secondary positrons are usually valid in the frame of a propagation model, which specifies the way the propagation equation is solved. For more insights into CR propagation, we refer the reader to, for example, Ginzburg & Syrovatskii (1964), Berezhinskii et al. (1990) and Longair (1994). Here, we adopt the formalism described in Delahaye et al. (2010), where convection and diffusive re-acceleration are neglected, which is known to be a
good approximation in the GeV–TeV energy range (Delahaye et al. 2009) and where fully relativistic energy losses are considered. Besides energy losses, our propagation ingredients are therefore the diffusion coefficient $K(E)$, which we take to be homogeneous, and the half-thickness $L$ of the diffusion slab. We use sets of propagation parameters consistent with the analysis of the secondary-to-primary nucleus ratios performed by Maurin et al. (2001) and further used in Donato et al. (2004) to define the minimal, median and maximal sets widely used in the literature. We note that the med model, which was the best-fitting model found in Maurin et al. (2001), has properties very similar to the best-fitting model derived more recently in Putze, Derome & Maurin (2010) from a Markov chain Monte Carlo analysis. Details on the effects of these parameters on the electron and positron propagation are given in Delahaye et al. (2010). For the energy losses, we adopt the model denoted by M1 in this reference.

Our whole propagation framework can be encoded in the form of a Green function $G(E, x_O \rightarrow E_n, x_n)$ that characterizes the probability of a positron injected at any coordinate $x_n$ with energy $E_n$ to reach an observer on the Earth with energy $E \leq E_n$. This allows us to write the positron flux as the following convolution:

$$\phi(E) = \int_{\text{slab}} d^3 x \int dE_n G(E, x_O \rightarrow E_n, x_n) \phi(E_n, x_n),$$  

where $Q$ is the source term that we are going to determine in the following. For secondaries, it formally reads (e.g. Delahaye et al. 2009):

$$Q(E, x) = 4\pi \sum_{i,j} \int dE_k \phi_i(E_k) \frac{d\sigma_{ij}}{dE}(E_k \rightarrow E) n_j(x),$$  

where $\phi_i$ is the flux of a CR species of index $i$, $n_j$ is the ISD density of a gas species of index $j$ and $d\sigma_{ij}$ is the inclusive nuclear cross-section associated with the production of a positron of energy $E_k$ from an ion of kinetic energy $E_k$. For the CR nuclei and IS gas, we can safely consider the dominant species only, that is, the protons and $\alpha$ ions, on the one hand, and the hydrogen (90 per cent) and helium (10 per cent) gas, on the other hand. As in Delahaye et al. (2009) we assume an overall gas density of $n_0 = 1 \text{cm}^{-3}$ homogeneously distributed inside an infinite flat disc of half-thickness $h = 100 \text{pc}$, such that $n_{\alpha}(x) = 2h n_0 \delta(z)$, where $z$ is the coordinate perpendicular to the Galactic plane. These values are justified either by measurements of the interstellar medium (ISM) (Ferrière 2001) or by the fact that high-energy positrons have very short range propagation due to efficient energy losses – large-scale fluctuations of the gas density have almost no effect on the local high-energy positron flux.

### 2.2 The incident CR flux

In Delahaye et al. (2009), we solved equations (1) and (2), assuming the proton and $\alpha$ fluxes as fitted in Shikaze et al. (2007) to the low-energy BESS data (Sanuki et al. 2000; Wang et al. 2002; Haino et al. 2004). These fits are recalled here:

$$\phi_p^{\text{bess}}(E_k) = A \beta_p^{a_2} \left( \frac{R}{1 \text{GV}} \right)^{-a_2},$$  

$$\phi_\alpha^{\text{bess}}(E_k/n) = B \beta_\alpha^{b_2} \left( \frac{R}{1 \text{GV}} \right)^{-b_2},$$  

with $(A, a_1, a_2) = (1.94 \text{cm}^{-2} \text{s}^{-1} \text{GeV}^{-1} \text{sr}^{-1}, 0.7, 2.76)$ and $(B, b_1, b_2) = (0.71 \text{cm}^{-2} \text{s}^{-1} \text{GeV}^{-1} \text{sr}^{-1}, 0.5, 2.78)$, that is, single power laws of indices $-a_2$ and $-b_2$, respectively, at high energy. The variables $\beta$ and $R$ denote the CR velocity in units of light speed and the CR rigidity, respectively.

These functions are displayed (solid curves) in Fig. 1 together with the $BESS$ (Sanuki et al. 2000; Wang et al. 2002; Haino et al. 2004; Shikaze et al. 2007), $CAPRICE$ (Boezio et al. 1999, 2003), $AMS-01$ (Aguilar et al. 2002), $ATIC2$ (Panov et al. 2006, 2009) and $CREAM$ (Ahn et al. 2010) data – note that these data have been corrected for solar modulation effects (demodulated) by means of the Force Field approximation (Gleeson & Axford 1968; Fisk 1971), with Fisk potentials made explicit in the plot. It is clear that though these parametrizations provide reasonably good fits to the low-energy data, they completely fail above a few tens of GeV. In particular, we can remark that the proton data (left-hand panel) are overshot between $\sim 10$ GeV and a few TeV, while the helium data (right-hand panel) are underpredicted above $\sim 100 \text{GeV/n}$. Moreover, though $CREAM$ and $ATIC2$ seem to agree in their measurements of the helium flux, there is an unequivocal discrepancy in the proton flux.

Since the functions of equation (3) were used in Delahaye et al. (2009) and Delahaye et al. (2010) to improve the predictions of

![Figure 1. Left-hand panel: CR proton data. Right-hand panel: CR $\alpha$ data.](https://academic.oup.com/mnras/article-abstract/414/2/985/976479)

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the secondary positron flux, it is worth revisiting these predictions again in light of the new CR data. For the inclusive nuclear cross-sections, we still use the numerical approach presented in Kamae et al. (2006) for the proton–proton collision, with the correction prescriptions of Norbury & Townsend (2007) for nucleus–nucleus interactions. The change in the derivation of the positron source term defined in equation (2) will therefore only come from the updated fits of the proton and helium fluxes.

To illustrate the difference in the positron source term arising from considering either the CREAM or the ATIC2 proton data, we consider two different modellings defined by the preference we put in one or the other experiment. The proton flux parametrization will be the only change between these two cases, since the helium data of the two experiments are consistent and can be simultaneously fitted by the same function.

In the following, we denote by $F_{1p}$ and $F_{2p}$ the parametrizations associated with the proton data, the former providing a good fit to the CREAM data and the latter providing a good fit to the ATIC2 data. We also define a function $F_{1He}$ that provides a good fit to both CREAM and ATIC2 helium data. Functions $F_{1p}$, $F_{2p}$ and $F_{1He}$ are given in Section 3.1 and are displayed in Fig. 1 – the first two are displayed in the left-hand panel and the last one in the right-hand panel. The functions of equation (3) will be referred to as the low-energy fit or reference fit. In contrast, the combination of $F_{1p}$ and $F_{1He}$ will be referred to as the CREAM fit, while that of $F_{2p}$ and $F_{1He}$ is referred to as the ATIC2 fit.

2.3 Impact of the CR hardening on the secondary positron source term

In the left-hand panel of Fig. 2, we derive the secondary positron source term defined in equation (2) and associated with the low-energy fit (black curve), the CREAM fit (red curve), and the ATIC2 fit (blue curve); note that all of them run close to a power law in energy of index $\sim 2.7$. The contribution coming from the CR proton is shown explicitly (dashed curves) for each modelling, as well as the $\alpha$ contribution (dotted curves – superimposed in the ATIC2 and CREAM cases). We see that the relative increase in the $\alpha$ contribution, from the low-energy fit to $F_{1He}$, is very large, reaching a factor of $\sim 3$ around a few TeV. Nevertheless, it remains subordinate with respect to the proton contribution. The difference in the proton flux modelling translates almost linearly into the positron source term, leading to a relative decrease with respect to the low-energy fit below 100 GeV (1 TeV) and a relative increase above, for the ATIC2 (CREAM) fit. This comes from the fact that the inclusive cross-section featuring in equation (2) scales like $1/E_k$ (Delahaye et al. 2009), straightforwardly leading to $Q \propto \phi_\alpha(E_k)$; this explains why all contributions almost scale like $E_k^{-2.7}$. When summing up the contributions coming from proton and $\alpha$ interactions, we see that the relative decrease apparent in the proton-only case is less prominent due to the positive yield, though modest, of the $\alpha$ interactions: the net effect is a very slight decrease below 100 GeV and a larger increase above. This is illustrated in more detail in the right-hand panel of Fig. 2, where we plot the relative difference between the CREAM (red curve) and ATIC2 (blue curve) CR-induced positron source terms with the low-energy reference case: the slight decrease below 100 GeV makes a 10 per cent difference at most with the reference case, while the increase above reaches $\sim 30$ per cent (60 per cent) above a few TeV for the CREAM (ATIC2) configuration. The impact of using these new CR data is therefore not negligible in terms of secondary positron production.

Notice finally that a similar spectral hardening is also observed in the spectra of CR nuclei heavier than He (Ahn et al. 2010). Despite their larger nuclear cross-sections with the ISM gas, their contributions to the secondary positron production are expected to be at the per cent level of the above predictions or less, because their fluxes are much lower than the proton or the helium fluxes – C and O species have fluxes $\sim 500$ times lower than the helium flux, far too small to be compensated by the more favourable cross-sections.

2.4 Updated predictions for the secondary positron flux

Because of propagation, guessing the effect of the new CR modellings on the secondary positron flux prediction might not look, a priori, as straightforward as guessing their effect on the positron source term. This is formally due to the non-trivial dependence

![Figure 2](http://example.com/figure2.png)

**Figure 2.** Left-hand panel: secondary positron source term for three different fit-based assumptions for the incident CR spectra; the contributions due to CR protons (dashed curves) and $\alpha$ particles (dotted curves) are shown explicitly besides the overall contributions (solid curves). Right-hand panel: relative difference between the obtained overall source terms. In both panels, black/blue/red lines refer to calculations performed by using the low-energy/ATIC2/CREAM fits, respectively, to characterize the incident CRs.

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of the positron Green function on energy. We refer the reader to Delahaye et al. (2010), in particular to their section 2.3, for more insights into this dependence. Nevertheless, since here we deal with a source term which is homogeneously distributed inside a thin disc of half-thickness $h$, and with an energy dependence close to a power law, we can assume that $Q(E, x) \approx 2h Q_{0\delta}(z)(E/\text{GeV})^{-\gamma}$. In that case, the positron flux at the Earth can be approximated as (Delahaye et al. 2010):

$$\phi_\odot(E) \propto \frac{c\hbar Q_0}{\sqrt{K_0\tau_i}} (E/\text{GeV})^{-\gamma},$$

(4)

where $K_0$ is the normalization of the diffusion coefficient, $\tau_i$ is the energy-loss time-scale, and $\gamma \approx \gamma + 0.5(\alpha + \delta - 1)$ is the predicted flux spectral index which depends on the injected index $\gamma$ and also on the diffusion coefficient index $\delta$ and the energy-loss index $\alpha$ ($\alpha = 2$ in the Thomson approximation, but is <2 when Klein–Nishina corrections become sizeable).

The above equation is of great interest to anticipate the upcoming results, since it tells us that the ratio of two different flux predictions scales like the ratio of the corresponding source terms at any energy. Therefore, the relative differences in flux predictions should be very close to the relative differences in the corresponding source terms, the latter being already plotted in the right-hand panel of Fig. 2. This is actually verified in the right-hand panel of Fig. 3, which can hardly be differentiated from the one previously mentioned, illustrating how efficient the approximation given in equation (4) is in this context. In the left-hand panel of Fig. 3, we show the corresponding predictions for the secondary positron flux, which are derived with the med propagation parameters and with model M1 for the energy losses – these parameters can be found in Delahaye et al. (2010). From these plots, we readily conclude that the secondary positron flux prediction is affected over the whole GeV–TeV energy range, decreasing by ~10 per cent below ~100 GeV and increasing by more than 30 per cent at TeV energies, up to 60 per cent in the case of the ATIC2 CR fit. The spectral index is therefore increased accordingly by almost two digits, from $\sim-3.5$ to $\sim-3.3$ in the med case employed here.

For the sake of completeness, it is interesting to derive the theoretical uncertainty bands associated with these novel predictions, which come from the uncertainties in the propagation parameters. With this aim, we proceed as in Delahaye et al. (2009) by bracketing the med with the min and max propagation configurations. We report our results for the positron flux (fraction) in the bottom panels of Fig. 4 – a solar modulation is applied, using the Force Field approximation (Gleeson & Axford 1968; Fisk 1971) with a Fisk potential of 600 MV and ignoring any charge dependence effect potentially important below a few GeV (e.g. Heber & Potgieter 2006). The positron flux data are taken from Boezio et al. (2000), DuVernois et al. (2001) and Aguilar et al. (2002), while the positron fraction data are taken from Barwick et al. (1997), Beatty et al. (2004), Aguilar et al. (2007) and Adriani et al. (2010). To calculate the positron fraction $f = \phi_p/\phi_\gamma$, we determined the denominator above 10 GeV from the Fermi-LAT data (Abdo et al. 2009; Ackermann et al. 2010) and used the AMS-01 data (Aguilar et al. 2002) to constrain the electron flux at lower energy. In all panels, we also display the result obtained by Moskalenko & Strong (1998), as fitted by Baltz & Edsjö (1998), since it is still very often used in the literature as a reference. Note that the predictions obtained from the low-energy CR fit are obviously identical to the ones derived in Delahaye et al. (2010).

From the predictions based on the low-energy CR fit to the ones based on the ATIC2 fit, the secondary positron spectrum is hardened, which translates into a slightly flatter positron fraction. Of course, we did not expect this study to be relevant to the discussion on the rise of the positron fraction itself, since the enhancement in the secondary positron flux was already known to be far too small from the incident CR nucleus data. It is instead very useful to constrain any extra source of positrons, like pulsars or dark matter, with the data. Indeed, in that case, one needs to add a secondary contribution to the primary one in a consistent manner before comparing the sum to the data. From the top panels of Fig. 4, we note incidentally that the min propagation setup already leads to a conflict with the data because of its too small value of $K_0$ [see equation (4)] (associated with a small value of $L_0$, $K_0/L$ being roughly fixed by secondary-to-primary CR nucleus ratios). This configuration is in any case obsolete, since it is no longer supported by recent secondary-to-primary analyses (e.g. Patze et al. 2010), or by the Fermi-LAT diffuse gamma-ray data (e.g. Ackermann et al. 2011) (see also a dedicated discussion in Lavalle 2010). Nevertheless, it can still be thought of as an extreme configuration and used for illustration purposes.

At first sight, the hardening effects studied in this paper seem to be completely negligible with respect to the uncertainties due to propagation – the uncertainty bands in the panels of Fig. 4 are indeed larger than 30–60 per cent. Nevertheless, we emphasize that the predicted propagated spectral index $\tilde{\gamma}$ has a half-reduced dependence on the diffusion index $\delta$ compared to that on the source index $\gamma$, since $\tilde{\gamma} \approx \gamma + 0.5(\delta + 1)$ [see discussion below equation (4)]. From the more reasonable med to max propagation configurations, $\delta$ actually goes from 0.7 to 0.46, leading to $\Delta \tilde{\gamma} \approx 0.12$, which turns out to be smaller than the approximately two-digit difference found between the predictions using the low-energy CR fit and those using the ATIC2 CR fit. Therefore, although uncertainties due to propagation lead to larger errors in terms of flux amplitude, the hardening effects analysed here are still relevant as regards their spectral impact. Note finally that the uncertainties in the propagation modelling are likely to be decreased in the near future, thanks to both theoretical and observational improvements.

We provide user-friendly empirical fitting functions associated with all the propagation models discussed above in Section 3.2.

### 3 USER-FRIENDLY FITTING FORMULAE

In this section, we provide the reader (i) with the parametric functions that we used to fit the CR data; and also (ii) with the ones that fit our prediction results for the secondary positron flux.

#### 3.1 CR proton and helium IS fluxes

The proton data can be accommodated by two different empirical functions, depending on whether one favours the CREAM (F1p) or ATIC2 (F2p) data at high energy. Both these functions are characterized by the following parametrization:

$$F_{p}(E_{k}) = \phi_{0}^{p} \left[1 - e^{-\left(E_{k}/E_{p1}\right)^{p_{1}}}ight] \left(\frac{E_{k}}{10\text{GeV}/m}\right)^{-\gamma_{p}} \times \left(1 + \frac{E_{k}}{E_{p2}}\right)^{p_{2}} \left(1 + \frac{E_{k}}{E_{p3}}\right)^{p_{3}}.$$  

(5)

One recognizes a standard power law of the main index $\gamma_{p}$ associated with an exponential attenuation factor active at low energy, which clearly improves the low-energy fit with respect to equation (3), and a double spectral correction of indices $p_{2}$ and $p_{3}$ operating above kinetic energies $E_{p2}$ and $E_{p3}$, respectively. Note that...

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Figure 3. Left-hand panel: secondary positron flux predictions associated with the three different fit-based assumptions considered for the incident CR spectra; the *med* propagation setup has been used. Right-hand panel: relative difference between the predictions shown in the left-hand panel. In both panels, black/blue/red lines refer to calculations performed by using the low-energy/ATIC2/CREAM fits, respectively, to characterize the incident CRs.

Figure 4. Top panels: secondary positron flux predictions, both IS and on top of atmosphere (ToA), assuming a Fisk potential of 600 MV for the solar modulation. Bottom panels: associated positron fractions. These predictions correspond to source terms calculated from the low-energy CR fit (left-hand panels), the CREAM CR fit (middle panels) or the ATIC2 CR fit (right-hand panels).

function \( F_p \) behaves asymptotically as a power law of index \( \gamma_\infty = -\gamma + p_2 + p_3 \).

In contrast, a single function (F1He) is enough in the case of helium ions, since both sets of high-energy data are in agreement.

In that case, a slight correction to equation (3) is enough, so that

\[
F_{\text{He}}(E_k/n) = \phi^{\text{bess}}_\alpha(E_k/n) \left( 1 + \frac{R}{R_{p2}} \right)^{p_2} \left( 1 + \frac{R}{R_{p3}} \right)^{p_3},
\]

(6)
\[ \phi_{\text{bess}}(E) = \exp \left\{ \sum_{i=0}^{4} c_i \left[ \ln \left( \frac{E}{\text{GeV}} \right) \right]^i \right\}. \]  

The parameters associated with all the configurations discussed in Section 2.4 are given in Table 2. We emphasize that these parameters are valid only in the frame of the propagation models discussed throughout this paper. The former (latter) case led to a 30 per cent (60 per cent) increase in the production rate of TeV positrons.

Then, we have propagated these positrons to the Earth, using the propagation framework described in Delahaye et al. (2010), which includes a fully relativistic treatment of the energy losses, with spatial diffusion parameters as constrained in Maurin et al. (2001), still consistent with the more recent analysis performed in Putze et al. (2010). We have notably explained why, in this context, the relative differences in the differential flux predictions were almost equal to the relative differences in the energy-dependent positron production rate, and consequently to the relative differences in the considered incident CR fluxes. This led us to establish a robust estimate of the effect: as for the positron production rate, the secondary positron flux is increased by up to 30 per cent (60 per cent) at TeV energies when constraining the CR nucleus fluxes from the CREAM (ATIC2) data. Therefore, these predictions differ from the ones performed in Moskalenko & Strong (1998), Delahaye et al. (2009) and Delahaye et al. (2010), resulting in harder secondary positron spectra. We have also derived a quite conservative estimate of the theoretical uncertainties coming from propagation effects, notably by using the extreme min propagation model (see the discussion at the end of Section 2.4). Although the spectral hardening effects discussed throughout this paper seem to be swamped by these uncertainties at first sight – so they are in terms of flux amplitude – we have shown that both are in fact of the same order of magnitude in terms of spectral index. These results are complementary to the recent analysis of Donato & Serpico (2011) on the secondary antiproton and diffuse gamma-ray fluxes.

## 4 CONCLUSION

In this paper, we have studied the impact of the spectral hardening observed in the CR proton and helium fluxes by the ATIC2 (Panov et al. 2006, 2009) and CREAM (Ahn et al. 2010) experiments on the secondary positron flux prediction. To this end, we have revisited the calculation of the secondary positron source term, which spatially tracks the IS gas, showing that its energy distribution roughly scales like the incident CR spectrum. Because of the discrepancy in the proton fluxes observed by the CREAM and ATIC2 balloons – in contrast, both agree on the helium flux – we have considered two different CR modelling: one based on a fit to the CREAM proton data (moderate case) and another based on a fit to the ATIC2 proton data (maximal case), both using the same helium flux parametrization. The former (latter) case led to a 30 per cent (60 per cent) increase in the production rate of TeV positrons.

Table 1. Parameters used to fit the CR proton (F1p, F2p) and helium (F1He) data, according to equations (5) and (6). For function F1He, any parameter \( E_p \) (in GeV) in this table corresponds to \( R_p \) (in GV) in the associated equation.

| \( \phi_{\text{bess}}(E) \) (cm\(^{-2}\) s\(^{-1}\) GeV\(^{-1}\) sr\(^{-1}\)) | \( \alpha_p \) | \( E_p1 \) (GeV) | \( p1 \) | \( E_p2 \) (TeV) | \( p2 \) | \( E_p3 \) (TeV) | \( p3 \) |
|---|---|---|---|---|---|---|---|
| F1p | 3.09 \times 10^{-3} | 2.8 | 4 | 1.05 | 2.5 | 0.34 | 10 | -0.29 |
| F2p | 3.09 \times 10^{-3} | 2.8 | 4 | 1.05 | 1.5 | 0.4 | 10 | -0.35 |
| F1He | - | - | - | - | 1 | 0.5 | 10 | -0.5 |

Table 2. Parameters used to fit the IS secondary positron flux predictions, from equation (7).

| \( c_i \) | min | med | max | min | med | max | min | med | max |
|---|---|---|---|---|---|---|---|---|---|
| \( c_0 \) | -4.61 | -5.48 | -6.4 | -4.61 | -5.48 | -6.4 | -4.41 | -5.48 | -6.4 |
| \( c_1 \) | -3.55 | -3.486 | -3.37 | -3.6 | -3.53 | -3.41 | -3.6 | -3.53 | -3.41 |
| \( c_2 \times 10^{-2} \) | -8.59 | -8.34 | -8.2 | -9.28 | -8.99 | -8.83 | -8.98 | -8.68 | -8.52 |
| \( c_3 \times 10^{-2} \) | 2.21 | 2.16 | 2.13 | 2.73 | 2.67 | 2.64 | 2.79 | 2.73 | 2.69 |
| \( c_4 \times 10^{-3} \) | -1.41 | -1.38 | -1.37 | -1.8 | -1.77 | -1.754 | -1.88 | -1.84 | -1.82 |

where \( \phi_{\text{bess}} \) is given in equation (3) and \( R \) is the rigidity. There is also a double spectral correction as in the case of protons. As function \( F_p \), function \( F_{1\text{He}} \) behaves asymptotically as a power law of index \( \gamma_\infty = -b_2 + p_3 + p_1 \), where \( b_2 \) is defined in equation (3).

Given the spread in the available data, we did not perform a \( \chi^2 \) selection of the parameters. Indeed, this would require an a priori or expert selection of the data, which is beyond the scope of this paper. The values of the parameters used for functions F1p, F2p and F1He are listed in Table 1.
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