Exchange Enhancement of the Electron-Phonon Pair Interaction

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The critical temperature of high-$T_c$ superconductors is determined, at least in part, by the electron-phonon coupling. We include the effect of an exchange interaction between the electrons and calculate the renormalization of the bare phonon frequencies and the electron-phonon vertices in a random phase approximation and obtain a strongly enhanced attractive phonon-induced electron-electron interaction. Using Fast Fourier Transform techniques, the weak-coupling selfconsistency equation for the order parameter is solved in the 2D first Brillouin zone for the Emery tight-binding band with different band fillings. The enhancement of $T_c$ arises primarily from the softening of the phonon frequencies rather than the vertex renormalization.

1. INTRODUCTION

Although it is widely believed that the phonon mechanism alone cannot yield $T_c$ of 90K or more, the role of phonons in the high-$T_c$ superconductors is still a subject of debate. Even if electronic correlations are primarily responsible for the superconductivity, the phonon mediated part of the pair interaction may still make a significant contribution and it is important to estimate the effect of electronic correlations on the phonons[1]. Since strong antiferromagnetic (AF) spin correlations are known to be present in the cuprate systems, we consider a simple two dimensional RPA-type model for the exchange-enhanced electron system, and calculate the effect of the AF spin fluctuations on the phonon frequencies and the electron-phonon interaction vertex. We then use the resulting renormalized phonon mediated interaction in the gap equation with a 2D tight-binding-structure.

2. RENORMALIZATION OF THE PHONONS

We employ the usual model Hamiltonian consisting of bare electrons interacting through a 2D Coulomb interaction $v(q) = 2\pi e^2/q$, bare phonons of frequency $\Omega(q)$, and a bare electron-phonon vertex with coupling constant $\alpha(q)$. For simplicity we take the jellium approximation. The screening of the Coulomb interaction, including an effective constant exchange interaction $U$, leads to the renormalized phonon frequency[2]:

$$\omega_q^2 = \Omega_q^2 - 2\Omega_q^2 | \alpha(q) |^2 \frac{\tilde{\chi}(q, \omega)}{1 + v(q)\tilde{\chi}(q, \omega)}$$

where

$$\tilde{\chi}(q, \omega) = \frac{2\chi_0(q, \omega)}{1 - U\chi_0(q, \omega)}$$

with the non-interacting susceptibility given by

$$\chi_0(q, \omega) = \sum_k \frac{f(\epsilon_{k+q}) - f(\epsilon_k)}{\omega - (\epsilon_{k+q} - \epsilon_k)}$$

The screened electron-phonon vertex is given by

$$\tilde{\alpha}(q, \omega) = \alpha(q, \omega)/\epsilon(q, \omega)$$

with the dielectric function

$$\epsilon(q, \omega) = 1 + (2v(q) - U)\chi_0(q, \omega)$$

In $\chi_0$ we use the 2D tight-binding Emery band[3]

$$\epsilon_k = 2t[2 - \cos(k_x a) - \cos(k_y a) - \mu]$$

with $t = 170\text{meV}$. At $\mu = 2$ (half filling), $\chi_0$ has a peak at the nesting wave vector[4], $Q = (\pi, \pi)$. This gives rise to a softening of $\omega_q$ with increasing $U$ for $q$ near $Q$. This effect is clearly seen in Fig.(1) where $\omega_q$ is shown along the line $\Gamma M X$. We have solved Eq.(1) with $\omega_q = 0$ in $\tilde{\chi}$. This should be a good approximation near the AF transition ($U = U_c$) for $q \approx Q$. 

\[1\]

\[2\]

\[3\]

\[4\]

\[5\]

\[6\]
Figure 1. Renormalized phonon frequency.

It is interesting to note that, for a Hubbard model, the diagrams considered here lead to an \( \omega_\mathbf{q} \) that increases with increasing \( U \):

\[
\omega_\mathbf{q}^2 = \Omega_\mathbf{q}^2 - 2\Omega_\mathbf{q} |\alpha(\mathbf{q})|^2 \frac{2\chi_0(\mathbf{q}, \omega)}{1 + U\chi_0(\mathbf{q}, \omega)}.
\]

It is however not clear whether one double counts the screening diagrams in this case.

3. SUPERCONDUCTIVITY

\( T_c \) is obtained by solving the linearized version of the s-wave weak coupling gap equation in the first Brillouin zone using Eq.(6):

\[
\Delta(k) = \int \frac{d^2 k'}{(2\pi)^2} V(k-k') \frac{\tanh(E(k')/2T)}{2E(k')} \Delta(k')
\]

with

\[
E(k) = \sqrt{\epsilon_k^2 + \Delta^2(k)}
\]

We take for the phonon mediated electron-electron interaction in weak coupling

\[
V(\mathbf{q}) = -|\alpha(\mathbf{q})|^2 D(\mathbf{q}, \omega = 0)
\]

where \( D \) is the renormalized phonon propagator. The results for \( T_c \) as a function of the exchange parameter \( U \) are shown in Fig.2 for various band fillings. The enhancement of \( T_c \) arises almost entirely from the softening of the phonon; the small electronic vertex correction seems to be typical of 2D AF spin fluctuation systems[5].

Figure 2. \( T_c \), including exchange effects.

4. CONCLUSIONS

We conclude that the phonon-softening effect of the 2D AF spin fluctuations can considerably raise \( T_c \). In order to make reliable predictions for the oxide superconductors it seems however important to do a strong coupling Eliashberg calculation since changes in the phonon coupling function \( \alpha^2 F(\omega) \) can have subtle effects on \( T_c \) [6]. Also one should go beyond the jellium model and consider optical phonons. We have not found any solutions of the gap equation for d-wave symmetry. If strong AF spin fluctuations lead to d-wave pairing, the corrections to the phonon-mediated part of the interaction considered here should still be relevant.

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