The phenomena of spin rotation and depolarization of high-energy particles in bent and straight crystals at Hadron Collider (LHC) and Future Circular Collider (FCC) energies and the possibility to measure the anomalous magnetic moments of short-lived particles (charm and beauty baryons)

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Abstract

We study the phenomena of spin rotation and depolarization of high-energy particles in crystals in the range of high energies that will be available at Hadron Collider (LHC) and Future Circular Collider (FCC). It is shown that these phenomena can be used to measure the anomalous magnetic moments of short-lived particles in this range of energies. We also demonstrate that the phenomenon of particle spin depolarization in crystals provides a unique possibility of measuring the anomalous magnetic moment of negatively-charged particles (e.g., beauty baryons), for which the channeling effect is hampered due to far more rapid dechanneling as compared to that for positively-charged particles. Channeling of particles in either straight or
bent crystals with polarized nuclei could be used for polarization and the analysis thereof of high-energy particles.

1 Introduction

The magnetic moment is an important characteristic of elementary particles, but the magnetic moments of many particles (e.g., charm and beauty baryons) are, as yet, not measured. This is because for particles with a short lifetime $\tau$ ($\tau = 2 \cdot 10^{-13}$ s for $\Lambda^+_c$ and $\tau = 3.5 \cdot 10^{-13}$ s for $\Xi^+_c$), the decay length $l \sim 3 \div 4$ cm if the energy acquired through the production reaction equals 1 TeV [1]. For this reason, the anomalous magnetic moments of short-lived particles cannot be measured with conventional methods.

The existence of the spin rotation phenomenon for high-energy particles moving in bent crystals in the channeling regime was first established in [2]. The spin rotation angle is determined by the anomalous magnetic moment $\mu'$ and, according to [2], can attain values as high as many radians. The unique feature of the effect is that for Lorentz-factors $\gamma \gg 1$, the value of the limit spin rotation angle per unit length of 1 cm in a crystal with a limit radius of curvature $R_{cr}$ at which the particle still moves in the channeling regime without being expelled from the channel by the centrifugal force is independent of particle energy.

The idea advanced in [2] was experimentally verified and confirmed for $\Sigma^+$ hyperons in Fermilab [3][4][5]. A detailed analysis of the experiment [3][4][5] given

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in [1,6,7] suggests the feasibility of using the spin rotation effect in bent crystals to measure the anomalous magnetic moment of short-lived charm baryons $\Lambda_c^+$ and $\Xi_c^+$. In [1], a thorough consideration was given to charm baryons $\Lambda_c^+$ produced by a beam of protons whose momentum in a tungsten target was 800 GeV/s. The characteristic momentum of the produced $\Lambda_c^+$ was $200 \div 300$ GeV/s (characteristic Lorentz factor $\gamma \simeq 153$). At the same time, it was stated in [7] that since the beauty quark has a negative charge, the beauty baryons have a negative charge, too, and cannot be channeled. As a result, the spin rotation phenomena for them does not exists. Positively-charged antiparticles could be channeled, but the production rates at LHC energies would be too small. It is noteworthy here that with growing particle energy the dechanneling length is increased not only for positively-charged particles, but also for their negatively charged counterparts. What is more, successful experiments recently demonstrated the possibility to deflect negatively-charged particles at GeV energies [8]. For this reason, in light of the projected increase in energy at LHC and FCC, it seems pertinent to return to the issue of using the spin rotation effect in bent crystals for measuring the magnetic moment of not only positively- but also negatively-charged particles.

The effect of spin depolarization of high-energy particles scattered by crystal axes (planes) provides another possibility of measuring the anomalous magnetic moment of short-lived, high-energy particles [9]. What is more, an important advantage of the spin depolarization effect is that it applies to both positively- and negatively-charged particles. Thus, this effect enables magnetic moment measurements of not only positively-charged baryons, but also beauty baryons, whose negative charge and rapid dechanneling make such measurements quite a challenge even in the range of high energies.
This paper considers the effects of particle spin rotation and spin depolarization in crystals at LHC and FCC energies. It is shown that these effects can be used for measuring anomalous magnetic moments of short-lived particles in this range of energies. It is also demonstrated that the effect of spin depolarization of high-energy particles scattered by crystal axes (planes) can be used effectively for measuring the anomalous magnetic moments of negatively-charged beauty baryons, too. Because the depolarization effect is possible at wider angles as compared with the angles of capture into channeling regime (determined by the Lindhard angle), the number of usable events in this case appears to be greater than in the experiments with bent crystals, thus giving us hope for measuring the anomalous magnetic moment of beauty baryons.

Before proceeding to the consideration of the possibility to use the phenomena of spin rotation and depolarization of high-energy particles moving in bent or straight crystals for measuring the anomalous magnetic moment of charm and beauty baryons, let us recall briefly the theory underlying these phenomena, as it is stated in [2,9,10].

2 Spin rotation of relativistic particles passing through a bent crystal and measurement of the magnetic moment of short-lived particles

In 1976, E. Tsyganov [11,12] demonstrated that high-energy particles may move in the channeling regime in a bent crystal, thus moving along the curved path and deviating from the initial direction. In 1979, the predicted effect was experimentally verified in JINR (Dubna, Russia) [13]: the proton beam with the kinetic energy of 8.4 GeV was rotated through the angle of 26 mrad by
means of the silicon crystal of length 2.0 cm. At present, the experiments on channeling in bent crystals are performed at the world’s largest accelerators. In these experiments a great variety of crystal optical elements are widely used to manipulate the beams of high-energy particles.

The next step forward was made in 1979 by V. Baryshevsky, who demonstrated [2] that the spin of a particle moving in a bent crystal rotates with respect to the momentum direction Figure (1).

![Figure 1. Spin rotation in bent crystal](image)

Indeed, in a bent crystal a particle moves along a curved path under the action of the electric field \( \vec{E} \) induced by the crystallographic plane. In the instantaneous rest frame of the particle, due to relativistic effects, this field produces a magnetic field \( \vec{H} \), which acts on the particle magnetic moment, thus causing spin rotation in this field (Figure 1).

Owing to an extremely large magnitude of the field \( \vec{E} \approx 10^7 \div 10^9 \) CGSE and, as a result, a large magnitude of the field \( \vec{H} \approx 10^7 \div 10^9 \) Gauss, the spin rotation angle in the crystal of several centimeters in length can reach several radians.

Now, let us proceed to a more detailed consideration of the effect of particle spin rotation in a bent crystal.
From the Bargmann-Michel-Telegdi (BMT) equation \[14\] follows that as the energy of particles increases, their spin precession frequency in the external magnetic field changes, being determined in the ultrarelativistic case by the anomalous magnetic moment. As a result, for example, in a magnetic field \(H = 10^4\) Gs, the electron (proton) spin precession frequency \(\omega = 2\mu' H/\hbar = 10^8\) s\(^{-1}\) (\(\mu'\) is the anomalous part of the magnetic moment), and the spin rotation angle per one centimeter path length \(l\) is just

\[
\vartheta = \omega \frac{l}{c} \approx 10^{-2}\text{ rad}.
\]

As mentioned above, a relativistic particle in a bent crystal "senses" a much greater magnetic field. As the particle energy is increased (particle wavelength is decreased), the quasi-classical character of particle motion enables one to apply the laws of motion determined by classical mechanics.

If a crystal is nonmagnetic, the BMT equation for the spin polarization vector \(\vec{\zeta} = \vec{s}/s\) can be written in the form \[2,9,10\]

\[
\frac{d\vec{\zeta}}{dt} = \frac{2\mu'}{\hbar} \left[ \vec{E}(\vec{\zeta}\vec{n}) - \vec{n}(\vec{\zeta}\vec{E}) \right],
\]

where \(\vec{E}\) is the electric field at the point of particle location and \(\vec{n} = \vec{v}/c\) (where \(\vec{v}\) is the particle velocity). Let us recall here that vector \(\vec{\zeta}\) characterizes the polarization of a particle in its "instantaneous" rest frame.

Intracrystalline fields \(\vec{E}\) are large, reaching the values of \(10^7\) CGSE and even greater. Therefore from (1) follows that for constant intracrystalline fields, the spin precession frequency could reach \(10^{11}\) s\(^{-1}\) and the angle \(\vartheta\) could be of the order of \(10\) rad/ cm.
However, when a particle moves through a crystal in arbitrary direction, the field $\vec{E}$, in a similar manner as in the amorphous medium, takes on random values at the particle location point. As a consequence, such a field causes spin depolarization.

Under channeling conditions, the situation is basically different. If the crystal bending radius is $\rho_0$, then, for example, a beam of high-energy protons will change its direction following the crystal bend \cite{11, 12}, i.e., the particle will move along a curved path. The stated motion is due to a constant mean electric field acting on a particle in a bent crystal.

Equation (1) for a particle moving in a crystal, for example, in a planar channel bent to a radius of curvature $\rho_0$ around the $y$-axis, has a form ($v_y = 0, E_y = 0$, the trajectory lies in the $x, z$ plane)

$$\frac{d\zeta_{x(z)}}{dt} = \pm \frac{2\mu'}{\hbar} (E_x n_z - E_z n_x) \zeta_{x(z)}.$$  

The position vector $\vec{\rho} = (x, z)$ of a particle in such a channel rotates about the $y$-axis with the frequency $\Omega = c/\rho_0$. Its magnitude oscillates about the particle equilibrium position $\rho'_0$ in the channel with the frequency $\Omega_k$, the amplitude $a$, and the initial phase $\delta$. In the explicit form $x = \rho(t) \cos \Omega t, z = \rho(t) \sin \Omega t$, and $\rho(t) = \rho'_0 + a \cos(\Omega_k t + \delta)$. We point out that, due to the presence of centrifugal forces in a bent crystal, the equilibrium point $\rho'_0$ does not coincide with the position $\rho_0$ of the minimum of the electrostatic potential $\varphi(\rho)$ of the channel. For example, for a particle moving in a harmonic well

$$\varphi = -\frac{k}{e} \frac{(\rho - \rho_0)^2}{2},$$

where the constant $k$ is determined by the crystal properties.
Integration of (2) in the polar coordinate system gives ($|\vec{\zeta}| = 1$, $\vec{E} = -\vec{\nabla}\varphi$):

$$\zeta_z(x) = \cos \left\{ \frac{2\mu'\Omega}{\hbar c} \int_0^t \rho \frac{d\varphi}{d\rho} dt' + \arctan \frac{\zeta_x(0)}{\zeta_z(0)} \right\}.$$ \hspace{1cm} (3)

For a harmonic well we can write (3) accurate up to the terms of the order $(\rho_0' - \rho_0)/\rho_0$ and $a \rho_0^{-1} \ll 1$ in the form

$$\zeta_z(x)(t) = \cos \left\{ \omega t + \beta [\sin(\Omega_k t + \delta) - \sin \delta] + \arctan \frac{\zeta_x(0)}{\zeta_z(0)} \right\},$$ \hspace{1cm} (4)

where

$$\omega = \frac{2\mu'}{\hbar} E(\rho_0')$$ \hspace{1cm} (5)

and

$$E(\rho_0') = -\frac{k}{e}(\rho_0' - \rho_0)$$ \hspace{1cm} (6)

is the electric field at the location point of the particle center of equilibrium in the bent crystal;

$$\beta = -\frac{2\mu'ka}{\hbar e \Omega_k}.$$ \hspace{1cm} (7)

The coefficient $\beta$ in (4) is small (for Si, the coefficient $k = 4 \cdot 10^{17} \text{eV/cm}^2$, $\Omega_k \simeq 10^{13} \text{s}^{-1}$ for protons with the energy $W \sim 100 \text{GeV}$, as a result, $\beta \simeq 10^{-2}$). Neglecting the term containing $\beta$, we obtain that the spin rotates with frequency $\omega$ (with growing energy $\Omega_k \sim 1/\sqrt{W}$, the coefficient $\beta \sim \sqrt{W}$.
increases, and the spin rotation turns into oscillations at frequency \( \omega \) and the frequencies multiple of \( \Omega_k \). Due to a large magnitude of the field \( E(\rho_0) \) curving the particle trajectory \( (E(\rho_0) \sim 10^7 - 10^9 \) CGSE), the frequency \( \omega \approx 10^{11} - 10^{13} \) s\(^{-1} \), and the rotation angle \( \vartheta \approx 10 - 10^3 \) rad/cm.

If the radius of curvature \( \rho_0 \to \infty \) (a straight channel), then only spin oscillations due to the term containing \( \beta \) remain. In this case, a significant spin rotation occurs only at high energies: for 100 TeV protons, for example, \( \beta \approx 0.3 \).

Thus, according to (11), the spin precession frequency of a particle moving in a bent crystal is

\[
\omega = \frac{2\mu' E}{\hbar}.
\]  

(8)

As stated above, this expression for the frequency follows immediately if we consider what field acts on the spin in the particle’s instantaneous rest frame. As a result of relativistic transformations, the electric field that is transverse relative to the particle velocity in the particle rest frame generates a magnetic field \( H = \gamma E \) that is orthogonal to \( E \) and has a magnitude equal to \( \gamma E \). The spin precession frequency associated with the anomalous magnetic moment in the particle rest frame is

\[
\omega' = \frac{2\mu' H}{\hbar} = \frac{2\mu' \gamma E}{\hbar}.
\]  

(9)

The spin precession frequency in the laboratory frame is

\[
\omega = \frac{\omega'}{\gamma} = \frac{2\mu' E}{\hbar}.
\]  

(10)

It is noteworthy that with zero particle’s anomalous magnetic moment \( \mu' \), the
particle spin precession frequency would equal the orbital rotation frequency of the momentum, and the particle spin direction would follow that of the momentum. If $\mu' \neq 0$, the angle between the polarization and momentum direction vectors will change. The angle $\vartheta_s$ of particle spin rotation relative to the particle momentum direction is $\vartheta_s = \omega T = \omega \frac{L}{c}$, where the time $T = \frac{L}{c}$, with $L$ being the path length traveled by the channeled particle in the bent crystal. From this follows that the rotation angle per unit path length

$$\vartheta_{s1} = \frac{\omega}{c} = \frac{2\mu' E}{hc}.$$  \hfill (11)

Let us also remember that the magnetic moment of the particle with spin $S$ is related to the gyromagnetic (Landé) factor $g$ (see, e.g., [14]) as

$$\mu = \frac{e\hbar}{2mc} gS,$$  \hfill (12)

where $m$ is the particle mass. When $S = \frac{1}{2}$ we can write

$$\mu = \frac{e\hbar}{2mc} + \frac{e\hbar}{2mc} \frac{g - 2}{2} = \mu_B + \mu', \quad \mu' = \frac{g - 2}{2} \mu_B,$$  \hfill (13)

where $\mu_B$ is the Bohr magneton. Consequently, we have

$$\vartheta_s = \frac{2\mu' E}{hc} L = \frac{2e\hbar}{hc \cdot 2mc} \frac{g - 2}{2} EL = \frac{g - 2 eEL}{2 mc^2}.$$  \hfill (14)

Here $eE$ is the force responsible for rotating the particle momentum that should be equal to the centrifugal force $f_c = \frac{m\gamma c^2}{R}$; $eE = \frac{m\gamma c^2}{R}$, where $R$ is the radius of curvature of the channel. From this we have

$$\vartheta_s = \frac{g - 2}{2} \frac{m\gamma c^2}{mc^2R} L = \frac{g - 2}{2} \gamma \frac{L}{R}.$$  \hfill (15)

We shall take into account that $\frac{L}{R}$ equals the value of the momentum’s angle
of rotation $\vartheta_p$. Hence,

$$\vartheta_s = \frac{g - 2}{2} \gamma \vartheta_p.$$  \hspace{1cm} (16)

Equation (16) was derived by V. Lyuboshitz using the BMT equation \[15\]. It follows from (15) that the spin rotation angle per unit path length (the angle of rotation per 1 cm) is

$$\vartheta_{s1} = \frac{g - 2 \gamma}{2 R}.$$  \hspace{1cm} (17)

On the other hand, the equality $eE = \frac{mc^2 \gamma}{R}$ yields $R = \frac{mc^2 \gamma}{eE}$.

The quantity $|eE| = |e \frac{d\phi}{d\rho}| = u'$, where $u$ is the particle’s potential energy in the channel. As a consequence, we have

$$R = \frac{mc^2}{u' \gamma}.$$  

The minimum value of the radius of curvature (the critical radius) $R_{cr}$ required for the motion in a bent channel to be still possible for ceratin $\gamma$ is achieved at maximum $u'_{max}$. Consequently, the maximum rotation angle per unit length is

$$\vartheta_{s1}^{\text{max}} = \frac{g - 2 \gamma}{2 R_{cr}} = \frac{g - 2 u'_{max}}{2 mc^2}.$$  

As is seen, the maximum spin rotation angle $\vartheta_{s1}^{\text{max}}$ is energy-independent. However, at $R = R_{\text{max}}$, the capturing acceptance (the fraction of particles captured into channeling regime) vanishes \[16\]. For certain $R > R_{cr}$, the spin rotation angle can be written in the form

$$\vartheta_{s1} = \frac{g - 2 \gamma}{2 R_{cr} R} R_{cr} = \frac{\vartheta_{s1}^{\text{max}} R_{cr}}{R} = \frac{g - 2 u'_{max}}{2 mc^2} R_{cr} R.$$  \hspace{1cm} (18)
As a result, we can see that for the given ratio $\frac{R_c}{R}$, the angle $\vartheta_{s1}$ is energy-independent, too. The dechanneling length increases as the energy is increased $^{[16]}$. This enables us to increase the rotation angle by increasing the length of the bent crystal channel

$$\vartheta_s = \vartheta_{s1}L, \quad L \leq L_D,$$

where $L_D$ is the dechanneling length. For protons with energies higher than $150 \div 200$ GeV, the length $L_D > 10$ cm. According to $^{[16]}$, channeling of protons in 15-cm-long Si crystals is currently available. As reported in $^{[16]}$, for Si, $u' = 5$ GeV cm$^{-1}$, and hence the rotation angle per 1 cm path length in a Si crystal is

$$\vartheta_{s1} = \frac{g - 2}{2} \frac{5 \text{ GeV cm}^{-1}}{mc^2} \frac{R_{cr}}{R}.$$

According to the estimates $^{[1]}$, for a particle with mass $\Lambda_c^+ \simeq 2.29$ GeV, the $g$-factor lies in the range $1.36 \div 2.45$. From this we get the minimum value of the ratio $\frac{g - 2}{2} \simeq 0.32$. Taking $\frac{R_c}{R} = \frac{1}{3}$ (in which case the dechanneling length is maximum $^{[16]}$), we obtain that for $\Lambda_c^+$, the rotation angle per unit length in a bent silicon crystal equals

$$\vartheta_{s1} \simeq 0.46 \text{ rad}.$$

For $\frac{R_c}{R} = \frac{1}{10}$, we have $\vartheta_{s1} \simeq 0.1$ rad; for $L = 10$ cm, we have $\vartheta_{s1} \simeq 1$ rad.

Thus, if the ratio $\frac{R_c}{R}$ remains constant, the value of the spin rotation angle appears to be constant and rather large in a wide range of energies. The decrease in the Lindhard angle $\vartheta_L \sim \frac{1}{\sqrt{\gamma}}$ with increasing energy seemingly impairs the observation conditions. However, as the energy is increased, the
angular width $\delta \vartheta$ in which the produced particles move is $\delta \vartheta \sim \frac{1}{\gamma}$, and hence
the fraction of particles captured into the channel is increased with energy as
$\frac{\delta \vartheta}{\delta \vartheta} \sim \sqrt{\gamma}$. According to [17,18], as the energy is increased, the probability of $\Lambda_c^+$ production also increases. For this reason, the increase in energy leads to an increasing number of usable events, i.e., the number of deflected $\Lambda_c^+$ entering the detector. For example, if the energy of $\Lambda_c^+$ is increased from 300 GeV to 3 TeV, the number of produced $\Lambda_c^+$ increases tenfold if the linear growth of the production cross section for $\Lambda_c^+$, suggested in [17,18], continues as the energy of the incident protons rises. Moreover, the amount of $\Lambda_c^+$ captured into channeling regime is increased threefold. As a result, the time required to observe the effect is reduced.

### 3 Spin depolarization of relativistic particles traveling through a crystal

When a particle travels through a crystal, the field $\vec{E}$ at the point of particle location takes on random values. This is due to fluctuations of the position of the nucleus (electron) in the crystal (in particular, under thermal vibrations) and to fluctuations of the electric field that come from particle scattering by various atomic chains. The fluctuations of the field $\vec{E}$ lead to the stochastic spin precession frequency in the electric field.

Owing to the presence of fluctuations of the spin rotation frequency, the spin rotation angle of any particle that has traveled through a crystal takes on different values. As a result, after passing through a target, every particle has a different direction of the polarization vector $\vec{\zeta}$, and the distribution of the particle spin direction in the beam may even become isotropic, i.e., the
beam polarization vector goes to zero. The stochasticity of \( \omega \) leads to spin depolarization of particles moving in the crystal \[9\].

According to \[9\], depolarization of particles in the case of scattering by crystal planes (axes) is determined by the intensity of multiple scattering by crystal planes (axes).

It was shown that depolarization has practically no effect on the possibility of measuring the magnetic moment of particles moving in bent crystals in channeling regime.

A much stronger depolarization is observed for the particles that are not involved in the channeling regime. These particles may undergo collisions with atomic chains as a whole. This leads to a sharp increase in the root-mean-square angle of multiple scattering and, as a consequence, to a large depolarization.

For example, according to \[9\], the longitudinal polarization \( \langle \zeta_z \rangle \) of particles undergoing scattering by the axes decreases as

\[
\langle \zeta_z(l) \rangle = \langle \zeta_z(0) \rangle e^{-Gl},
\]

\[
G = \frac{1}{2} \left( \frac{g-2}{2} + \frac{1}{\gamma+1} \right)^2 \gamma^2 \langle \theta_1^2 \rangle,
\]

where \( \langle \theta_1^2 \rangle \) is the mean square angle of multiple scattering of the particle by the crystal axes per 1 centimeter path length in the crystal. According to \[19,20\], the mean square angle of particle scattering by the crystal axes for a path length \( l \) traveled in the crystal can be written as

\[
\langle \theta^2(l) \rangle = \frac{16\pi D^2 Z^2 \hbar^2 \alpha^2 N \xi \psi_{\text{max}}}{M^2 c^2 \gamma^2} l.
\]
So we have

\[ G = \frac{1}{2} \left( \frac{g-2}{2} + \frac{1}{\gamma + 1} \right)^2 \frac{16\pi D^2 Z^2 \hbar^2 \alpha^2 N\xi \psi_{\text{max}}}{M^2 c^2} \frac{\psi}{\psi}, \]  

(22)

where \( \alpha = 1/137 \), \( Z \) is the atomic number, \( N \) is the number of atoms per \( \text{cm}^3 \), \( \xi \) is the numerical coefficient equal to 1.16 for the axial potential in the Lindhard model, and \( \psi \) is the angle between the direction of the particle momentum and the crystallographic axis;

\[ \psi < \psi_{\text{max}} = \frac{r_s}{d_1} \approx 0.1\sqrt{Z}, \]

where \( r_s \) is the screening radius, \( d_1 \) is the distance between the axes, and the \( z \)-axis is parallel to the crystallographic axis;

\[ D = \frac{e}{e_0} \]

is the ratio between the charge of a particle and that of an electron.

For example, for particles with the parameters of the proton (i.e., \( (g-2)/2 \sim 1.79, M \sim 938 \text{ MeV} \)), we have for W-type crystals the magnitude \( G \approx 10^{-4} \psi_{\text{max}}/\psi \)

and, according to the value of \( \psi \), it may reach \( G \approx 10^{-1} \div 1 \), i.e., for crystal thickness of the order of 1 cm, the depolarization may amount to a few tens of percent. Equation (22) is valid for \( \psi \), which are larger than the Lindhard angle \( \psi_L = \sqrt{2U/Mc^2\gamma} \) (\( U \) is the averaged axial potential).

It follows from (19) and (20) that

\[ \left| \frac{g-2}{2} + \frac{1}{\gamma + 1} \right| = \sqrt{\frac{2}{\gamma^2 \langle \theta^2(1) \rangle}} \ln \frac{\langle \zeta_z(0) \rangle}{\langle \zeta_z(l) \rangle}. \]  

(23)

When the particle energy corresponds to \( \gamma \gg 1 \), from (19) and (20) we may
obtain the following expression for \((g - 2)\) [9]:

\[
|g - 2| = \sqrt{\frac{8}{\gamma^2 \langle \theta^2(l) \rangle} \ln \frac{\langle \zeta_z(0) \rangle}{\langle \zeta_z(l) \rangle}}, \quad \frac{g - 2}{2} \gg \frac{1}{\gamma}
\] (24)

Let us note that for electrons and positrons, as reported in [10, 21, 22], the experiments on spin rotation in bent crystals provide a unique possibility for studying the effects of quantum electrodynamics of a strong field, namely, the dependence of the anomalous magnetic moment, i.e., \((g - 2)/2\), on the crystal electric field intensity and the particle energy. The same possibility exists in studying the depolarization of \(e^\pm\). Thus, with experimentally determined values of \(\langle \theta^2(l) \rangle\) and \(\langle \zeta_z(l) \rangle\), we can find the value of \((g - 2)\), which gives us another possibility to measure the anomalous magnetic moment of particles. It is noteworthy that strong fluctuating fields act on neutral particles moving at a small angle to the crystal axes (planes), too, leading to the depolarization of their spin, which fact can be used to measure the magnetic moment of neutral particles.

We should pay attention to the fact that theoretical considerations and experimental results both reveal that \(\Lambda_c^+\) produced through collisions between nonpolarized nucleons and nonpolarized nucleons (nuclei) are polarized, and their polarization vector is oriented orthogonally to the reaction plane, i.e., the plane where the momenta of the incident nucleon and the produced baryon lie. A similar situation would occur in production of other baryons—charm and beauty. This is why the reaction amplitude \(f\) includes the term of the form

\[
f \sim \vec{S} [\vec{p}_N \times \vec{p}_B]
\] (25)
where $\vec{S}$ is the spin of the produced particle, $\vec{p}_N$ is the momentum of the incident particle, and $\vec{p}_B$ is the momentum of the produced baryon. Consequently, the polarization component $\vec{\zeta}_\perp(0)$ of the produced baryon that is parallel to the momentum $\vec{p}_B$ equals zero. Depolarization occurs to the polarization vector component $\vec{\zeta}_\perp(0) = (\zeta_x(0), \zeta_y(0))$ lying in the plane transverse relative to vector $\vec{p}_B$.

Let the produced baryon be incident on the crystal axis at a small angle $\psi < \psi_{\text{max}} = \frac{R}{d_1}$ (according to [20], $\frac{R}{d_1} \sim 0.1 \sqrt{z}$). The baryon’s velocity component parallel to the axes $v_\parallel = v \left(1 - \frac{\psi^2}{7}\right)$, where $v$ is the particle velocity. The component $\vec{v}_{0\perp}$ of the particle’s initial velocity is orthogonal to the axis, and $v_{0\perp} = v\psi$. Let us choose the $x$-axis along vector $\vec{v}_{0\perp}$. Multiple scattering leads to appearing of the velocity’s component $v_y$ that is orthogonal to the $x$-axis, and the changes in $v_y$ are much greater than those in $v_x$ in the direction of the $x$-axis. As a consequence, multiple scattering by the axes is strongly asymmetric in the plane perpendicular to the direction of the baryon momentum [20].

This means that the depolarization is also asymmetric in this plane, i.e., $\zeta_x(l) \neq \zeta_y(l)$. For this reason, the maximum depolarization of the produced baryons can be obtained if the crystal axes are oriented in such a way that the initial vector of baryon polarization is oriented in the direction of the $y$-axis. According to [9], the component $\zeta_y(l)$ of the polarization vector relates to $l$ as

$$\zeta_y(l) = \langle \zeta_y(0) \rangle \exp \left[-\frac{1}{8}(g - 2)^2\gamma^2(\theta_y^2(l))\right].$$ (26)

Let us recall that in view of the analysis [20], the contributions to multiple scattering by the axes come not only from the averaged potential, but also
from particle scattering by the fluctuations of the potential of the axes that are due to thermal oscillations of atoms. As a result, \( \langle \theta^2_y \rangle = \langle \theta^2_{ty} \rangle + \langle \theta^2_{avy} \rangle \) [20], where \( \langle \theta^2_{ty} \rangle \) is the mean square angle of multiple scattering due to thermal oscillations and \( \langle \theta^2_{avy} \rangle \) is the mean square angle of multiple scattering due to scattering by averaged over thermal oscillations potential of the axes. Let us also note that \( \langle \theta^2_y \rangle \sim \frac{1}{\gamma^2} \), that is, \( \gamma^2 \langle \theta^2_y \rangle \) is independent of the particle energy.

It also follows from (26) that

\[
|g - 2| = \sqrt{\frac{8 \gamma^2 (\theta^2_y(l))}{\ln \left( \frac{\zeta_y(l)}{\zeta_y(0)} \right)}}.
\]

(27)

In our further estimations we shall consider only the contribution coming from multiple scattering by the averaged over thermal oscillations potential of the axes \( \langle \theta^2_{avy} \rangle \) and assume that \( \langle \theta^2_y \rangle \sim \langle \theta^2_{avy} \rangle \).

As a result, we have the following expression for the damping rate

\[
\zeta_y(l) = \langle \zeta_y(l) \rangle e^{-Gl},
\]

(28)

where \( G \) is defined by (22).

Thus, the depolarization rate in this case is similar to that in (19). Now let \( \Lambda_c^+ \) be incident on the crystal. For our estimate we use the value of the \( g \)-factor given in [1]: \( g \simeq 1.36 \div 2.45 \). The minimum value is estimated as \( \frac{2 - \sqrt{2}}{2} = 0.32 \).

So we have the following estimate for \( G \) in tungsten (W): \( G \simeq 10^{-1} \) (the beam is assumed to be incident on the axes at an angle of about several Lindhard angles); then for the length \( l = 10 \) cm, for example, \( Gl \sim 1 \). Hence, the polarization in this case decreases by a factor of \( e \), i.e., quite appreciably. More precise estimates require thorough simulations, but even the estimates given here indicate that this phenomenon can hopefully be used for negatively-
charged beauty baryons.

4 Spin rotation and oscillations of high-energy channeled particles in the effective nuclear potential of a crystal as one of the possibilities to obtain and analyze the polarization of high-energy particles.

Now, let us note that when moving in the crystal, such particles as baryons (antibaryons and nuclei) take part not only in electromagnetic, but also in strong interactions. As a result, the spatially periodic effective potential energy of interaction between a particle and a crystal is determined by two interactions: Coulomb and strong.

The effective periodic potential $U(\vec{r})$ can be introduced to describe the passage of a coherent wave in the crystal:

$$U(\vec{r}) = \sum_{\vec{\tau}} U(\vec{\tau}) e^{i\vec{\tau} \cdot \vec{r}},$$

(29)

where $\vec{\tau}$ is the reciprocal lattice vector of the crystal,

$$U(\vec{\tau}) = \frac{1}{V} \sum_j U_{j0}(\vec{\tau}) e^{-W_j(\vec{\tau})} e^{i\vec{\tau} \cdot \vec{r}_j}.$$  

(30)

Here $V$ is the volume of the crystal elementary cell, $\vec{r}_j$ is the coordinate of the atom (nucleus) of type $j$ in the crystal elementary cell, and the squared $e^{-W_j(\vec{\tau})}$ equals the thermal-factor (i.e., the Debye-Waller factor), well-known for X-ray scattering:

$$U_{j0}(\vec{\tau}) = -\frac{2\pi \hbar^2}{M\gamma} F_j(\vec{\tau}),$$

(31)
where $M$ is the mass of the incident particle, $\gamma$ is its Lorentz-factor, $F_j(\vec{\tau}) = F_j(\vec{k} - \vec{k} = \vec{\tau})$ is the amplitude of elastic coherent scattering of the particle by the atom, $\vec{k}$ is the wave vector of the incident wave, and $\vec{k}'$ is the wave vector of the scattered wave.

It will be recalled that in the case of a crystal, the imaginary part of the amplitude $F_j(0)$ does not contain the contribution from the total cross section of elastic coherent scattering. The imaginary part of $F_j(0)$ in a crystal is only determined by the total cross sections of inelastic processes. This occurs because in a crystal, unlike in amorphous matter, the wave elastically scattered at a nonzero angle, due to rescattering by periodically located centers, is involved in formation of a coherent wave propagating through the crystal.

Elastic coherent scattering of a particle by an atom is caused by both the Coulomb interaction of the particle with the atom’s electrons and nucleus and its nuclear interaction with the nucleus. Therefore, the scattering amplitude can be presented as a sum of two amplitudes:

$$F_j(\vec{\tau}) = F_{j}^{\text{Coul}}(\vec{\tau}) + F_{j}^{\text{nuc}}(\vec{\tau}),$$

where $F_{j}^{\text{Coul}}(\vec{\tau})$ is the amplitude of particle scattering caused by the Coulomb interaction with the atom (nucleus) (it contains contributions from the Coulomb interaction of the particle with the atom along with the spin-orbit interaction with the Coulomb field of the atom (nucleus)); $F_{j}^{\text{nuc}}(\vec{\tau})$ is the amplitude of elastic coherent scattering of the particle caused by the nuclear interaction (this amplitude contains the terms independent of the incident particle spin along with the terms depending on spin of both the incident particle and nucleus, in particular, spin-orbit interaction). Therefore, $U(\vec{r})$ and $U(\vec{\tau})$ can also
be expressed:

\[ U(\vec{r}) = U^{\text{Coul}}(\vec{r}) + U^{\text{nuc}}(\vec{r}) , \quad (33) \]

\[ U(\vec{\tau}) = U^{\text{Coul}}(\vec{\tau}) + U^{\text{nuc}}(\vec{\tau}) . \]

Let us suppose that a high energy particle moves in a crystal at a small angle to the crystallographic planes (axes) close to the Lindhard angle \( \vartheta_L \sim \sqrt{U/E} \) (in a relativistic case \( \vartheta_L \sim \sqrt{2U/E} \)), where \( E \) is the energy of the particle, \( U \) is the height of the potential barrier created by the crystallographic plane (axis). This motion is determined by the plane (axis) potential \( \hat{U}(\vec{\rho}) \), which could be derived from \( U(\vec{r}) \) by averaging over the distribution of atoms (nuclei) in the crystal plane (axis).

As a consequence, the potential \( \hat{U}(\vec{\rho}) \) for a particle channeled in a plane (or axis) channel or moving over the barrier at a small angle, close to the Lindhard angle, can be expressed as a sum \([10,24]\):

\[ \hat{U}(\vec{\rho}) = \hat{U}^{\text{Coul}}(\vec{\rho}) + \hat{U}^{\text{sp-orb}}(\vec{\rho}) + \hat{U}^{\text{nuc eff}}(\vec{\rho}) + \hat{U}^{\text{mag}}(\vec{\rho}) , \quad (34) \]

where \( \hat{U}^{\text{Coul}} \) is the potential energy of particle Coulomb interaction with the crystallographic plane (axis), \( \hat{U}^{\text{sp-orb}} \) is the energy of spin-orbit interaction with the Coulomb field of the plane (axis) and spin-orbit nuclear interaction with the effective nuclear field of the plane (axis), and \( \hat{U}^{\text{nuc eff}} \) is the effective potential energy of nuclear interaction of the incident particle with the crystallographic plane (axis).

\[ \hat{U}^{\text{nuc eff}}(\vec{\rho}) = -\frac{2\pi \hbar^2}{M \gamma} N(\vec{\rho}) \hat{f}(0) , \quad (35) \]

where \( N(\vec{\rho}) \) is the density of nuclei at point \( \vec{\rho} = (x, y) \) of the crystallographic
plane (axis). The z-axis is directed along the crystal axis (along the velocity component parallel to the crystallographic plane in the case of plane channeling), \( \hat{f}(0) \) is the amplitude of elastic coherent forward scattering, which depends on the incident particle spin and nucleus polarization, \( \hat{U}^{\text{mag}}(\vec{\rho}) \) is the energy of magnetic interaction of the particle with the magnetic field produced by the electrons (nuclei).

Thus in describing particle motion in crystals, the contribution of strong interactions to the formation of the effective potential acting on the particle from the crystallographic planes (axes) should be considered along with the Coulomb interaction. In the case of high energies, the particle motion in the potential \( \hat{U}(\rho) \) can be described in the quasi-classical approximation. The spin-evolution equations for a particle moving in straight and bent crystals in the presence of the contribution from \( \hat{U}^{\text{nuc}}_{\text{eff}}(\vec{\rho}) \) is discussed in [10,25].

Let us assume now that the crystal nuclei are polarized. The cross section of high-energy particle scattering by a nucleus (the amplitude of zero-angle scattering) in a crystal with polarized nuclei depends on the particle spin orientation with respect to the polarization of the target. Therefore, the absorption coefficient also depends on particle spin orientation.

Let a particle move in a crystal at a small angle to the crystallographic plane (axis) close to the Lindhard angle. In this case the coefficient of particle absorption by nuclei can differ from the absorption coefficient for the particle moving in the crystal at a large angle to the crystallographic plane (axis). Moreover, it can appear to be greater even for positively-charged particles. Let us discuss the possibility to use a polarized crystal to obtain a polarized beam of high-energy particles and to analyze the polarization state of
high-energy particles [10,24].

Let us consider a positively-charged, high-energy particle moving close to the top of the potential barrier. Therefore, it moves in the range with nuclei density higher than that of an amorphous medium. In this range, the growth is described by

$$\frac{d}{a_0} \approx 10^2,$$

here $d$ is the lattice period, $a_0$ is the amplitude of the nucleus thermal oscillations. For the over-barrier motion in the vicinity of the crystallographic axis, the growth is described by the relation

$$\left(\frac{d}{a_0}\right)^2 \approx 10^4.$$

After passing through the polarized crystal of thickness $L$, the particles moving in the region where the crystal nuclei are located acquire the polarization degree that can be described by formula

$$P = \frac{e^{-\rho\sigma_{\uparrow\uparrow}L} - e^{-\rho\sigma_{\uparrow\downarrow}L}}{e^{-\rho\sigma_{\uparrow\uparrow}L} + e^{-\rho\sigma_{\uparrow\downarrow}L}},$$  \hspace{1cm} (36)

where $\sigma_{\uparrow\uparrow}$ is the total scattering cross section for the particle whose spin is parallel to the polarization vector of nuclei and $\sigma_{\uparrow\downarrow}$ is the total scattering cross section for the particle whose spin is anti-parallel to the polarization vector of nuclei.

To avoid misunderstanding, it will be recalled that the exponential absorption law holds true if the characteristic scattering angle of a particle is much larger than the angle at which the observer from the point of target location may see the input window of the detector, registering the flux of particles transmitted
through the target. In the opposite case all of the scattered particles enter the
detector, and by $\sigma_{\uparrow\uparrow}(\sigma_{\downarrow\uparrow})$ one should mean the total cross section of absorp-
tion and reactions that remove the particle from the incident beam (causing
appreciable change in its energy). The density matrix techniques should be
used as a tool in considering the general case.

The magnitude of the effect is determined by the parameter $A = \rho(\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow})L$.
With $\rho_{av}$ being the average density of nuclei in the crystal, $A = \rho_{av} \cdot 10^2 \Delta \sigma L$
for a plane and $A = \rho_{av} \cdot 10^4 \Delta \sigma L$ for an axis.

The length $L_1$ corresponding to $A = 1$ is $L_1 = \frac{1}{\rho \Delta \sigma}$. If we suppose that
$\rho_{av} \approx 10^{22}$ and $\Delta \sigma \sim 10^{-25}$ cm$^2$, then for a plane $L_1 \approx 10$ cm and for an axis
$L_1 \approx 10^{-1}$ cm. As is seen, in this case a beam of polarized particles can be
obtained using rather small-sized polarized targets.

Let positively-charged particles move parallel to a crystallographic plane. In
this case, most of them appear close to the bottom of the potential well and
far from the nuclei, therefore, the absorption coefficient for this fraction of
particles is smaller comparing to the average crystal absorption. Nevertheless,
in this case there is also a fraction of particles that moves in the region where
the nuclei are located.

A fraction of particles moving near the barrier can be sorted out by means
of angular distribution or by using bent crystals. Moving in a bent crystal,
the fraction of channeled particles that due to Coulomb repulsion is hardly
involved in the nuclear interaction is deflected from the initial direction of
motion. Thus the particles that have undergone the interaction with the nuclei
will change the beam moving in the initial direction. Because the absorption
for this fraction of particles increases, it becomes possible to achieve quite a
significant polarization of the initially nonpolarized beam.

A bent crystal can be used to increase the interaction of a positively-charged particle with a nucleus, too. In this case, the centrifugal forces push the channeled beam to the range with high density of nuclei. A detailed analysis of the polarization state of transient particles requires a numerical simulation.

In contrast to positively charged particles, negatively charged particles (for example, $\Omega^-$ hyperons, antiprotons, and beauty hyperons), being channeled, move in the region with high density of nuclei, therefore, even a very thin polarized crystal ($L \approx 10^{-1}$ cm) can be an effective polarizer and polarization analyzer.

Moreover, for a negatively-charged channeled particle, the angle of spin rotation in the nuclear pseudo-magnetic field of the polarized crystal \cite{24} can be larger than that for amorphous matter \cite{10,24}. This is the reason for the thin polarized crystals to be used as nuclear-optical elements that can guide polarization of high-energy particle beams.

Note that because of the presence of the amplitude similar to that in \cite{25}, the spin-orbit interaction also leads to particle polarization, as well as to the left–right asymmetry in scattering of polarized particles (both charged and neutral). The density of nuclei near the axis is much higher than the density of an amorphous target. This makes the induced polarization (left–right asymmetry) for particles scattered by the crystal axes noticeably higher, as compared with amorphous matter, due to interference of the Coulomb and nuclear interactions \cite{10,24}.

In this context, intensively developing techniques for collimating and handling
the beams of high-energy particles, using the volume reflection of charged particles from bent crystal planes deserve attention [26]-[40]. In particular, according to [32], multiple volume reflection from different planes of one bent crystal becomes possible when particles move at a small angle with respect to the crystal axis. Such multiple volume reflection enables a severalfold increase in the particle deflection angle inside one crystal. This effect is experimentally revealed now, and it is noteworthy that it leads to an increasing probability of particle-nucleus interaction.

Channeling of particles in either straight or bent crystals with polarized nuclei could be used to polarize high-energy particles or analyze their polarization. The beam of nonpolarized particles extracted from the storage ring could be significantly polarized by applying the additional polarized bent (straight) crystal.

5 Spin oscillation and the possibility of quadrupole moment measurements of $\Omega^-$-hyperons moving in a crystal

A few more words should be said about the possibility of quadrupole moment measurements for $\Omega^-$-hyperons moving in a crystal [11]. Theoretical estimates of the quadrupole moment of short-lived particles, for instance of an $\Omega^-$-hyperon [12,13,14], based on the nonrelativistic quark model depend on the model parameters and can differ significantly. Experimental measurement of the quantity under consideration using the convenient method of passing through an inhomogeneous electromagnetic field is at present practically unrealizable because of the smallness of quadrupole interaction at attainable strengths of the electromagnetic field.
Despite the seeming simplicity, the use of the effect of spin oscillation of $\Omega^-$-hyperons channeled in crystals is a challenging task, too. This is, above all, due to the sharp increase of multiple scattering by nuclei in the case of negatively-charged particles ($\Omega^-$-hyperons). However, it should be emphasized that the difficulties can be overcome to a great extent using the effect of spin oscillation occurring in a nonchanneled state of motion (not too far above the critical angle) relative to the chosen family of crystallographic planes for particles with spin $I > 1/2$.

The equations describing the spin dynamics of a particle possessing magnetic and quadrupole moments can be derived using the results obtained in \[10\]. In consequence, the relativistic equation of spin motion in this case has the following form:

$$\frac{d\hat{I}_i}{dt} = \epsilon_{ijk}(\Omega_j\hat{I}_k + \frac{1}{3}\epsilon\varphi_{ji}\hat{Q}_{kl}),$$

where $\Omega_j$ is a component of the vector $\vec{\Omega} = \frac{1}{2}(g-2)\gamma + \gamma/(\gamma+1)\frac{v^2\vec{\Omega}_0}{(\gamma^2-1)}(\vec{E} \times \vec{v})$.

Let us consider (37) under the condition that the particle moves in a non-channeled state at an angle $\psi > \psi_L$ with respect to the chosen family of crystallographic planes, where $\psi_L$ is the Lindhard angle. Suppose that the coordinate axis $x$ is perpendicular to the given crystallographic planes. Taking into account that $d^2U/dxdy$, $d^2U/dy^2 \ll d^2U/dx^2$, we obtain

$$\frac{d\hat{I}_x}{dt} = \Omega_x\hat{I}_y - \Omega_y\hat{I}_x + \frac{\varphi_{xx}\epsilon Q}{2I(2I-1)}\hat{I}_{xy},$$

$$\frac{d\hat{I}_y}{dt} = \Omega_z\hat{I}_x - \Omega_x\hat{I}_z - \frac{\varphi_{yy}\epsilon Q}{2I(2I-1)}\hat{I}_{xx},$$

$$\frac{d\hat{I}_z}{dt} = \Omega_z\hat{I}_y - \Omega_y\hat{I}_z - \frac{\varphi_{zz}\epsilon Q}{2I(2I-1)}\hat{I}_{yy}.$$
Equations (37)–(40) enable one to analyze whether the spin oscillation of a particle moving in a nonchanneled state can be used for measuring the electric quadrupole moment of the $\Omega^-$-hyperon.

In view of (38)–(40), when a particle with a quadrupole moment moves in a crystal, not only spin oscillation and rotation appear, but also the transitions between tensor $P_{ik}$ and vector $\vec{P}$ polarizations of the particle.

As follows from (37)-(40), under the initial conditions $P_z(0) = 0$ and $P_{xy}(0) \neq 0$, there is a maximal change of the $\Omega^-$-hyperon spin projection $P_z$ due to the quadrupole interaction.

According to the estimates made in [41], in such experiments for the $\Omega^-$-hyperon beam with Lorentz factor $\gamma = 100$, intensity $N \approx 10^6$ particle/s, and the beam divergence angle $\theta_{\text{div}} < 0.4 \text{ mrad}$, it is possible to measure the quadrupole moment $Q$ of the $\Omega^-$-hyperon on the level $10^{-27}$ cm$^2$ in a tungsten crystal of length $l = 20 \text{ cm}$.

It should be noted that the measurement scheme suggested here for $\Omega^-$-hyperon quadrupole moment measurements requires neither a high quality crystal nor a monochromatic hyperon beam! For the realization of this measuring procedure it is quite sufficient to have a crystal with a mosaic spread $\chi < 0.4 \text{ mrad}$ or a set of crystals arranged with an exactness of not more than $\chi < 0.4 \text{ mrad}$ relative to the chosen family of crystallographic planes. This condition justifies the possibility of creating large crystal systems necessary for carrying out this experiment.
6 Conclusion

The analysis given in this paper shows that the phenomena of spin rotation and depolarization of high-energy particles in crystals at energies that will be available at LHC and FCC can be used to measure the anomalous magnetic moments of short-lived particles at such energies. It has also been demonstrated that for negatively-charged particles (e.g., beauty baryons), the phenomenon of spin depolarization in crystals is a promising tool allowing the anomalous magnetic moment measurements. Moreover it has been noted that the spin depolarization effect occurs for neutral particles incident at small angles to crystal axes (planes), and this opens the potential for magnetic moment measurements of such short-lived particles. Channeling of particles in either straight or bent crystals with polarized nuclei could be used for polarization or polarization analysis of high-energy particles, including neutral particles. Depolarization and spin rotation of that fraction of neutral particles which moves in the region of high concentration of nuclei can be defined by particle reactions in the crystal: if at a certain point the particle is scattered at a large angle, this means that it has undergone a collision, and hence it was in the vicinity of the nuclei. At high energies, particles move along a straight line. Particles moving between the planes do not undergo collisions, while those moving along the axis or a plane do.

References

[1] Samsonov V. M., On the possibility of measuring charm baryon magnetic moments with channeling, *Nucl. Instr. Methods B* **119**, 1–2, (1996) 271–279.
[2] Baryshevsky V. G., Spin rotation of ultrarelativistic particles passing through a crystal, Pis’ma Zh. Tekh. Fiz. 5, 3 (1979) 182–184.

[3] Baublis V. V. et al. First observation of spin precession of polarized $\Sigma^+$ hyperons channeled in bent crystals, LNPI Research Report (1990–1991) E761 Collaboration (St. Petersburg) (1992) 24–26.

[4] Chen D., Albuquerque I. F. and Baublis V. V., et al., First observation of magnetic moment precession of channeled particles in bent crystals, Phys. Rev. Lett. 69, 23 (1992) 3286–3289.

[5] Khanzadeev A. V., Samsonov V. M. Carrigan R. A. and Chen D., Experiment to observe the spin precession of channeled relativistic $\Sigma^+$ hyperons Nucl. Instr. Methods B 119, 1-2 (1996) 266–270.

[6] Carrigan R. A., Jr. and Smith V. J., Channeling spin precession as a technique for measuring charm baryon magnetic moments, in Proceedings of the Charm2000 Workshop Fermilab–Conf 94/190, (1994) 123.

[7] Baublis V.V., Carrigan R.A., Chen D., et al., Measuring the magnetic moments of short-lived particles using channeling in bent crystals, Nucl. Instrum. Methods B 90(1994) 112–118.

[8] Mazzolari A., et al., Steering of a sub-GeV electron beam through planar channeling enhanced by rechanneling Phys. Rev. Lett. 112 (2014) 135503.

[9] Baryshevsky V. G., Spin rotation and depolarization of relativistic particles traveling through a crystal Nucl. Instrum. Methods B 44, 3 (1990) 266–272.

[10] Baryshevsky V.G., High-Energy Nuclear Optics of Polarized Particles, World Scientific Publishing, Singapore, 2012.

[11] Tsyganov E. N., Some aspects of the mechanism of a charged particle penetration through a monocrystal, Tech. Rep. TM-682 (1976) Fermilab., Batavia.
[12] Tsyganov E. N., Estimates of cooling and bending process for particle penetration through a monocrystal Tech. Rep. TM-684 (1976) Fermilab., Batavia.

[13] Vodop’yanov A. S., Golovatyuk V. M., Elishev A. F., Ivanchenko I. M., et. al, Steering of charged–particle trajectories by means of a curved monocrystal, *JETP Lett.* **30**, 7, (1979) 442–446.

[14] Berestetskii V.B., Lifshitz E.M., Pitaevskii L.P. *Relativistic Quantum Theory* Vol. 4 (1st ed.). Pergamon Press, 1971.

[15] Lyuboshitz V. L., Spin rotation associated with the deflection of a relativistic charged particle in an electric field *Sov. J. Nucl. Phys.* **31**, 4 (1980) 509–512.

[16] Biryukov V.M., Chesnokov Yu.A., Kotov V.I., *Crystal Channeling and its Application at High-Energy Accelerators*, Springer, Berlin, 1997.

[17] Appel J., in *Proc. of the CHARM 2000 Workshop, The Future of High Sensitivity Charm Experiments*, D. M. Kaplan and S. Kwan, Editors,Fermilab, June 1994, FERMILAB-CONF-94/190 p.4.

[18] J.Russ, Ibid., p 111.

[19] Akhiezer A. I. and Shul’ga N. F., Radiation of relativistic particles in single crystals, *Sov. Phys. Usp.* **25**, 8 (1982) 541–564.

[20] Baryshevsky V. G. and Tikhomirov V. V., New polarization effects accompanying pair production in crystals, *Nucl. Instrum. Methods A* **234** (1985) 430–434.

[21] Baryshevsky V.G. and Grubich A.O. Radiative self-polarization of fast particles in bent crystals, *Pis’ma. Zh. Tekh. Fiz.* **5**, 24, (1979) 1527–1530.

[22] Baryshevsky V. G. and Tikhomirov V. V., Possibilities of obtaining polarized e± beams in proton accelerators, *Sov. J. Nucl. Phys.* **48**, 3, (1988) 429–434.
[23] James R. W., *The Optical Principles of the Diffraction of X-rays*, Ox Bow Press, 1982.

[24] Baryshevsky V. G., Spin rotation and polarization of charged particles moving in crystals with polarized nuclei, *Vesti Akad. Nauk BSSR*, Ser.fiz.-energ. 2(1992) 49–55.

[25] Baryshevsky V. G. and Gurinovich A. A., Spin rotation and oscillations of high energy particles in a crystal and possibility to measure the quadrupole moments and tensor polarizabilities of elementary particles and nuclei, *Nuclear Instrum. Methods B* 252, 1, (2006) 136–141.

[26] Taratin A. M. and Vorobiev S. A., ”Volume reflection” of high–energy charged particles in quasi-channeling states in bent crystals, *Phys. Lett. A*, 119, 8, (1987) 425–428.

[27] Grinenko A. A. and Shul’ga N. F., Turning a beam of high–energy charged particles by means of scattering by atomic rows of a curved crystal *JETP Lett.* 54, 9, (1991) 524–529.

[28] Fliller R. P. *et al.*, Results of bent crystal channeling and collimation at the Relativistic Heavy Ion Collider, *Phys. Rev. ST Accel. Beams* 9, 1, (2006) 013501 [12 pages].

[29] Ivanov Yu.M. *et al.*, Volume reflection of 1–GeV protons by a bent silicon crystal, *JETP Lett.* 84, 7 (2006) 372–376.

[30] Ivanov Yu.M. *et al.*, Volume reflection of a proton beam in a bent crystal, *Phys. Rev. Lett.* 97, 14 (2006) 144801 [4 pages].

[31] Tikhomirov V. V., A technique to improve crystal channeling efficiency of charged particles, *JINST* 2, 8, (2007) P08006.

[32] Tikhomirov V. V., Multiple volume reflection from different planes inside one bent crystal, *Phys. Lett. B*, 655, 5–6 (2007) 217–222.
[33] Scandale W. et al., High–efficiency volume reflection of an ultrarelativistic proton beam with a bent silicon crystal, Phys. Rev. Lett. 98, 15 (2007) 154801 [4 pages].

[34] Biryukov V. M., Coherent effects in crystal collimation, Phys. Lett. B 645, 1 (2007) 47–52.

[35] Carrigan R. A., Jr. et al., Channeling collimation studies at the Fermilab Tevatron, in Proc. SPIE 6634 (2007)66340I; doi:10.1117/12.741856.

[36] Scandale W., et al., High–efficiency deflection of high-energy protons through axial channeling in a bent crystal, Phys. Rev. Lett. 101 (2008) 164801–164805.

[37] Scandale W. et al., High–efficiency deflection of high–energy negative particles through axial channeling in a bent crystal, Phys. Lett. B 680, 4 (2009) 301–304.

[38] Guidi V., Mazzolari A. and Tikhomirov V. V., Increase in probability of ion capture into the planar channelling regime by a buried oxide layer J. Phys. D 42, 16 (2009) 165301.

[39] Scandale W. et al., First observation of multiple volume reflection by different planes in one bent silicon crystal for high-energy protons, Phys. Lett. B 682, 3 (2009) 274–277.

[40] Guidi V., Mazzolari A. and Tikhomirov V., On the observation of multiple volume reflection from different planes inside one bent crystal, J. Appl. Phys. 107 (2010) 114908 [8 pages].

[41] Baryshevsky V. G. and Shechtman A. G., Spin oscillation and the possibility of quadrupole moment measurement for Ω−–hyperons moving in a crystal, Nucl. Instr. Methods B 83, 1-2 (1993) 250–254.

[42] Gershtein S. S. and Zinoviev Y. M., Quadrupole moment of the Ω− hyperon, Sov. J. Nucl. Phys. 33, 5 (1981) 772–773
[43] Isgur N., Karl G. and Koniuk R., D-waves in the nucleon: A test of color magnetism, *Phys. Rev. D* 25, 9 (1982) 2394–2398.

[44] Leonard W. J. and Gerace W. J., Quark–meson coupling model for baryon wave functions and properties, *Phys. Rev. D* 41, 3 (1990) 924–936.