The velocity translation in the game of ”Life”

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Abstract

The velocity translation law for the patterns in the game of ’Life’ is discovered. It is found for an arbitrary angle between the velocities in the moving reference frames. The formula differs from its physical prototype of special relativity but admits gentle reduction to the Galilean limit and provides absence of extra-light speeds.

1 Introduction

The game of LIFE is the most famous cellular automata introduced by J. Conway [1, 2]. In 1970 it appeared as a mere recreation puzzle, now it has found applications in various scopes of science, from pure game theory to biology, chemistry, economics etc. The natural and simple rules make this game popular for the wider community also. In the initial formulation of Conway it acts on a two-dimensional square lattice where the cells exist in two states – dead (0) or alive (1). The evolution of each cell depends on the status of its eight neighbors.

A dead cell comes to live, if it has exactly three living neighbors.

A cell remains living if it has two or three living neighbors; otherwise (without support) it will die.

The name of this cellular automata reflects a colony of primitive one-cell microorganisms: both rare and dense population has no chance to survive,
while the intermediate densities may admit intensive growth and development. This seeming simplicity is accompanied by extremely complicated dynamics, investigated in the recent years.

One of the most striking features of this game of Life is the existence of cell societies endowed with definite regularity in their evolution. The so-called spaceships form the most important class of such objects. Spaceship is the pattern which reproduces itself after $P$ moves ('period') and appears replaced in several squares, vertical or horizontal, or in both directions. If $n$ is the magnitude of this replacement the quantity $v = n/P$ implies the velocity of the spaceship. 'Glider (below) is the smallest spaceship

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It travels as a bishop in chess and in four moves it is shifted one cell forward and down and its velocity is $v = 1/4$.

No object of the game of Life can move faster than a chess king – one square per one move. Being called as the speed of light, this quantity is taken as the universal unit $c \equiv 1$. But the fact of 'motion' in discrete space-time of Life never obeys to direct physical laws. The process of motion is based on the intrinsic nature of the cellular automata and the relevant velocity translation law (if any) should be derived from it. The law which governs the relative motion in the game of Life should be derived from the principle of motion in discrete space-time.

The spaceships of the game of Life have direct resemblance with chess pawns because they move at a discrete speed and strictly linear (straight or diagonal) motion. The spaceship’s volatility is originated from the nature of cell unit which has only two possibilities – either to survive or to die. The
process of motion follows from the death and birth that periodically occurs in the cell society.

The existence of spaceships is extremely important fact in the game of Life as it can be applied to construction of mathematical (heuristic) models of the physical laws. Particularly, the observer flying in the spaceship launches a 'bullet', and some external party is monitoring the events in his laboratory reference frame. The latter must record the 'bullet' whose velocity is not coinciding with that measured in the reference frame co-moving the 'astronaut'.

The recent attempt to derive special relativity from the cellular automata [3] inspired us to solve this vital problem: what is the formula of velocity translation in the reference frame co-moving the spaceship? Either it is the Galilean summation or the Lorentz relativistic formula? Although there is no spaceship flying faster than \( v = 1/2 \), it does not imply validity of the plain Galilean summation. The attempts to derive special relativity from the cellular automata [3] may hint to some answer. However, the standard relativistic formula is also unmotivated because the motion in discrete space-time of Life is not a continuous process. It is composed of two primitive acts: elementary jump to the neighboring square and staying at rest.

Is there any law to govern the relative motion between the moving reference frames? Its importance cannot be overestimated. Its knowledge will turn the cellular automata game of Life towards realities of the material world.

2 Motion on chessboard

Consider a chess king on the chess board. Let the king always moves forward and straight (never goes back or diagonal). Or, we can consider a pawn instead (without the initial double step). Let after \( P \) moves it appears shifted in \( n \) squares forward. We define its velocity as \( v = n/P \). What has happened within this time period? It is clear that the pawn was remaining at rest during \( P - n \) moves, while during the rest \( n \) moves it was jumping one square forward. In fact, the pawn has only two states: either to move or to stay at rest. On the other hand, each move can be explained in terms of the game of Life as follows: the old pawn dies and the new pawn appears on a free square.

Now consider two pawns going on the same file of the chessboard to met
each other.

The white pawn f2 moves with the velocity \( v_1 \) and the black pawn f7 moves with the velocity \( v_2 \) (it approaches just from north). What is the net relative velocity? Checking it as a plain sum \( v_1 + v_2 \) is not correct because it may occur arbitrary high (the known Zeno’s aporia) in the series of consequent summation (of course, providing infinite chessboard). But it is not so in the light of the chess rules because the pawns do not move simultaneously: the black pawn turns to move when the white pawn stays at rest. If during \( P \) moves the white pawn has passed \( n \) squares, the black pawn has had a possibility to move exactly \( P - n \) times. Therefore the net velocity cannot exceed \( (n + P - n) / P \) that is the speed of light \( c = 1 \).

3 Relativistic translation of velocity

Let an arbitrary spaceship moves at velocity \( v_1 \) and a hypothetical astronaut is sitting inside. Let him have a gun and shot a bullet flying as fast as \( v_2 \) (it is also a spaceship in the game of LIFE classification). The latter \( v_2 \) is the velocity with respect to the reference frame co-moving the astronaut. The so-called 'guns' emitting spaceships are well known to the game of LIFE researches [5].

Imagine that some external party, an abstract policeman is monitoring the events from his outpost based on the ground or the laboratory reference frame. First, he records the astronaut flying in his spaceship with the velocity \( v_1 \). Second, he records the bullet whose velocity is \( v_{12} \) and we shall calculate it now.

Again the naive summation \( v_1 + v_2 \) may exceed the speed of light. The game of LIFE spaceship is a complex object composed of many cells but as a single pawn, considered above, it is involved in the motion consisting in \( n \)
pure jumps and $P - n$ stays because after $P$ moves the spaceship is replaced into $n$ squares. In the reference frame of the astronaut in the spaceship his proper time is staying at rest and it corresponds to $P - n$ moves. During his proper time he records the bullet passing the distance $s_2$ at its native velocity $v_2 = s_2 / (P - n)$.

The policeman records the spaceship passed $n$ cells (at the velocity $v_1 = n/P$) and the total path of the bullet is recorded as $s_1 + s_2 = v_1 P + v_2 (P - n)$. Therefore, the policemen has finally recorded the bullet flying with the velocity

$$v_{12} = v_1 + v_2 - v_1 v_2$$

It is a translation formula acting on the discrete cellular board of the game LIFE.

Since $v_1 < 1$ and $v_2 < 1$, the total velocity cannot exceed the speed of light. Particularly, light always propagates at its genuine speed $v_2 = c = 1$ because $v_{12} = 1$ without regard of the velocity of the reference frame $v_1$. Another sample: at $v_1 = v_2 = 1/2$ the relative velocity is $v_{12} = 3/4$ (but not the plain sum!).

Now let us have some recreation and solve the real situation. Imagine, you are rich enough to purchase any available vehicle and you have chosen a spaceship moving at velocity $2/5$. Of course, you have a gun [5] which can emit e.g. lightweight spaceships (SWSS) with velocity $v_2 = 1/2$ [7]. However... you are arrested because your gun is expected as dangerous: the bullets faster than $3/4$ of the speed of light are forbidden. Meanwhile, when flying in your spaceship you can shoot bullets at $v_{12} = v_1 + v_2 = 9/10$. The relativistic composition of velocities yields $v_{12} = (v_1 + v_2) / (1 + v_1 v_2) = 3/4$.

At any rate you cannot avoid jail. What shall you do? Of course, you smile and refer to the formula (3) resulting to $v_{12} = 7/10$. This cellular world is governed by the mathematical rules and laws and it is not derived from the real world...

Let us put our astronaut in a luxury puffer-class spaceship (or briefly puffer [5] according to the researches of the game of LIFE). It is not a spaceship in the strict sense but also a flying vehicle which, however, emits gliders. Glider [1] belongs to the category of spaceships moving oblique course but it is not a tedious problem to calculate the composition of velocities for this bullet. Again, consider a manned puffer flying at the velocity $v_1$ along the axis $x$. The proper time of the astronaut is $P (1 - v_1)$. Let a bullet is shot from the spaceship at some angle $\psi$ with respect to the axis $x$ and this angle
is defined as

$$\tan \psi = \frac{v_y^2}{v_x^2}$$

(4)

where $v_x^2$ and $v_y^2$ is the velocity of the bullet along the axes $x$ and $y$ respectively. Of course, it is possible to introduce some nominal velocity $v_2$ so that

$$v_x^2 = v_2 \cos \psi \quad v_y^2 = v_2 \sin \psi$$

(5)

however, this quantity has not real physical meaning of motion at the angle $\psi$ with the velocity $v_2$. The Pythagoras theorem is not applied here. Indeed, a glider is walking along the diagonal. Its path is one square forward and one square down in four moves, hence, yielding velocity $v_x = v_y = 1/4$ and its velocity along the diagonal is also $1/4$ rather than $\sqrt{2}/4$.

The astronaut will see the bullet flying the distance $s_x^2 = P (1 - v_1) v_x^2$ along the axis $x$ and the distance $s_y^2 = P (1 - v_1) v_y^2$ along the axis $y$ (which is orthogonal to the axis $x$).

The relevant path of the bullet in the earth reference frame along the axis $x$ is extended by the path $s_1 = n$ of the carrier spaceship. The summary boost of the bullet along the axis $x$ will be $s_x = n + P (1 - v_1) v_2 \cos \psi$ implying that its velocity is

$$v_{x_2}^x = v_1 + (1 - v_1) v_x^2$$

(6)

meanwhile, the summary replacement along the axis $y$ has not become longer than $s_y = s_y^2$ and we write

$$v_{y_2}^y = (1 - v_1) v_y^2$$

(7)

Note that this component does not coincide with $v_2^y$.

The 'bullet' flies in the direction defined in the earth reference frame by the angle $\chi$ so

$$\tan \chi = \frac{(1 - v_1) v_2 \sin \psi}{v_1 + (1 - v_1) v_2 \cos \psi}$$

(8)

Again, the total velocity cannot exceed the speed of light $c = 1$ and the magnitude of (6)-(7) is always smaller than the value (3) to which it is reduced when the bullet is shot parallel to the course of 'carrier' spaceship ($\psi = 0$) and the deviation from the special relativity

$$\Delta = v_1 v_2 \frac{1 - v_1 - v_2 + v_1 v_2}{1 + v_1 v_2}$$

(9)

never exceeds 0.05.
There exist puffers \[5\] consisting of about 1000 cells and drifting at a velocity \(v_1 = 1/4\). Let our astronaut drive one of them. There are many orthogonal spaceships (i.e. \(\psi = 0\)) with velocity \(v_2 = 1/3\). Some of them are small and good enough to be a bullet which the astronaut can shot at the velocity

\[
v_2^x = 0 \quad v_2^y = 1/3
\]

in the co-moving reference frame.

What shall see the earth observer in the earth reference frame? According to formulas (6)-(8) the external observer will record the object whose velocity is

\[
v_{12}^x = -1/4 \quad v_{12}^y = 1/4
\]

whose course is inclined at the angle

\[
\chi = 135^\circ
\]

with respect to the course of the carrier spaceship. In fact, among the puffers \[5\] drifting at a velocity \(v_1 = 1/4\) there are several patterns which emit gliders exactly in the prescribed direction (11)-(12).

What about the bullet spaceship (10)? We do not observe it explicitly if we sit on the earth. How we can grasp it? How we can interpret it? It can be recognized as a definite texture within the puffer pattern, an embryo carried by the carrier spaceship until it is pushed out as a glider of the earth reference frame. The virtual embryo is moving at the velocity (10) within the bosom of the carrier puffer and its embryonal motion is resulted from the interaction of cells. As soon as the embryo is born, it leaves the womb of its 'mother' spaceship and goes in the outer space where it becomes glider visible explicitly even to the external observers.

\section{Conclusion}

The velocity translation law of the game of Life is discovered (6)-(8). It differs from the relevant Lorentz formula of special relativity. Indeed, the motion in the game of Life is resulted from the given rules of the cellular automata and has no explicit physical nature. Besides there is evident
anisotropy of space in vertical and horizontal direction, while the orthogonal
and diagonal motion occurs at the same velocity (there is no Pythagoras
theorem).

The formulas (6)-(8) allow to operate with the moving reference frames.
Since before this possibility was not expected in the game of Life. Now
we have the approach to consider many interesting ideas, e.g. ‘the light
communication’ or the telegraph based on the spaceships (the pilot emits
signals propagating at various speed etc). The quantitative expression of
the velocity translation law (6)-(8) opens new horizon for formulation of the
‘relativistic’ problems in the game of Life.

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