Theoretical analysis of the sound-absorption coefficient and transmission loss for longitudinal clearances among the close-packed cylinders
(Three kinds of estimation and experiment)

Shuichi SAKAMOTO*, Kouta SUTOU**, Arata NAKANO**, Hirohiko TANIKAWA** and Takanari AZAMI**

* Department of Mechanical and Production Engineering, Niigata University
2-8050 Ikarashi, Nishi-ku, Niigata, 950-2181, Japan
E-mail: sakamoto@eng.niigata-u.ac.jp

** Graduate School of Science and Technology, Niigata University
2-8050 Ikarashi, Nishi-ku, Niigata, 950-2181, Japan

Received 11 December 2014

Abstract
This study estimates the sound-absorption coefficient and transmission loss for the dimensions of clearances among the close-packed cylinders by theoretical analysis and compares these estimates with experimental values. In the analysis, we performed an elemental breakdown of the clearance configuration, and then approached each element as the clearance between two planes. Considering the effects of sound wave attenuation in each element of clearance, we found the propagation constant and characteristic acoustic impedance using three-dimensional analysis. By connecting these elements in parallel, we treated them as a one-dimensional transfer matrix. We then calculated the sound-absorption coefficient and transmission loss through the transfer matrix method. No sound-absorption coefficient could be obtained with calculations that did not account for attenuation. In addition, calculations that approached the clearance as an equivalent circle and a triangle did not match experimental values well. On the other hand, calculations using the method that accounted for clearance shape, that is the method used in this study, closely matched experimental values. Estimations of transmission loss expressed all clearances by connecting their transfer matrices in parallel. In calculations, these estimations did not account for attenuation, and the falling curve for transmission loss greatly diverged from experimental values. In the three calculations that accounted for attenuation, the falling segment of transmission loss showed frequency characteristics that were close to experimental values. Calculations through the methods that accounted for clearance shapes and that analyzed clearance as an equivalent triangle also had peak transmission loss values that closely matched experimental values.

Key words: Sound and acoustics, Noise control, Sound absorption coefficient, Sound transmission loss, Silencer design

1. Introduction

Bundles of narrow holes, such as catalysts and heat exchangers within tubes, result in widespread sound-absorption and transmission loss, even when these outcomes are not intended (Ohno, et al., 2004), (Katayama, et al., 1999). When cylindrical shafts and tubes are closely packed, longitudinal clearances occur, surrounded by three cylindrical faces. It is useful in engineering to quantify the acoustic characteristics, such as sound-absorption coefficient and transmission loss, based solely on the geometrical dimensions of these narrow tubes.

Analyses of sound-wave propagation in narrow tubes has been performed for cylinders, equilateral triangles, and parallel planes (Tijdenman, 1975), (Stinson and Champou, 1992). In case of clearances among the close-packed cylinders, the theoretical values of the complex density and complex bulk modulus have been estimated (Matsuzawa, 1963). However, theoretical values of the absorption coefficient and transmission loss have not estimated so far.
Consequently, the comparison between experimental value and calculated value also has not performed. In a previous study, the authors performed a theoretical analysis of bundles of cylindrical holes accounting for attenuation within the tube, found the sound-absorption coefficient and transmission loss, and compared these values with experimental results (Sakamoto, et al., 2013a). There has also been research on sound-absorption characteristics in the tubule structures of naturally occurring biomass such as barley straw (McGinnes, et al., 2005) and rice straw. The authors clarified experimentally that absorption characteristics of cylindrical clearances of bundled rice straws are significant (Sakamoto, et al., 2011). We also reported the experimental results and theoretical analyses for the absorption coefficient of sound-absorbing structures with layered clearances between two planes (Sakamoto, et al., 2013b), (Sakamoto, et al., 2014a). Moreover, the idea of the transfer matrix approach applied to the parallel assembly of sound absorption material with different shapes has been introduced by Verdière et al (Verdière, et al., 2013).

In this study, we report the results of our theoretical analysis on bundles of narrow tubes surrounded by three cylindrical structures that account for attenuation within the tube. We find the sound-absorption coefficient and transmission loss, and compare the obtained values with experimental results. Moreover, to verify the effectiveness of this analysis method, calculated values for approximating a clearance to an equivalent circle and a triangle were also shown.

In the experiment, we measured the sound-absorption coefficient and transmission loss of a test sample constructed using acoustic impedance tubes with two and four microphones.

Sound wave attenuation within tube-shaped clearances, as in porous sound-absorbing materials having continuous pores, occurs mainly as a result of friction due to viscosity in the boundary layer of the inner-wall of the clearance. In this study, we performed an elemental breakdown of the clearance configuration, and then approached each element as the clearance between two planes. To account for sound wave attenuation caused by viscosity of air within the clearance between two planes, we used Navier–Stokes equations, continuity equation, gas state equation, energy equation, and dissipation function representing the heat transfer due to internal friction to derive the propagation constant and characteristic acoustic impedance (Stinson and Champou, 1992). In the analysis of sound wave attenuation, we performed a three-dimensional analysis using a Cartesian coordinate system. By connecting these elements in parallel, we treated them as a one-dimensional transfer matrix. We then calculated the sound-absorption coefficient and transmission loss through the transfer matrix method (Sakamoto, et al., 2013a). This allowed us to apply the propagation constant and characteristic acoustic impedance of the clearance from the three-dimensional analysis to a one-dimensional transfer matrix, such as is generally used in the design of silencers. We attempted simple, high-precision estimation through this method.

2. Theoretical analysis
2.1 Analytical models for close-packed cylindrical clearances

We analyzed the clearance created by the three cylindrical surfaces (hereafter referred to as clearances) using the transfer matrix method for sound pressure and volume velocity, based on a one-dimensional wave equation. The colored area of Fig. 1 shows a cross-section of clearance. A portion of the dashed line that trisects the clearance in Fig. 1 is one analysis unit. To find the propagation constant and characteristic acoustic impedance of this analysis unit, we approach the clearance in Fig. 2(a), as shown in Fig. 2(b). In other words, we perform an analysis for n clearances between two parallel planes for which clearance thickness b is changed to n steps. Figures 2 and 3 show schematics for n = 5. Figure 3 shows an elemental breakdown in the analysis unit. When dividing the analysis unit into n, each element has equal cross-sectional area. Using element thickness b and Eqs. (2) and (5) below, we find the transfer matrix for each element. Here, the element breakdown number n = 40. This is a number where the calculated values for the sound-absorption coefficient and transmission loss are sufficiently converged.

Figure 4 shows the cross-section that approached the clearance as an equivalent circle and a triangle. Figure 5 shows an equivalent circuit corresponds to one clearance. By connecting the n = 40 divided elements T_{A1} - T_{A40} in parallel (Sakamoto, et al., 2014a) (Verdière, et al., 2013), we calculated the transfer matrix for each analysis unit T_{All} - T_{C40} (using Eqs. (10) and (11)). Next, by connecting three analysis units in parallel, we calculated the transfer matrix for one clearance. In this case, the dimensions of clearance is smaller than the wave length of the sound wave. Thus, it is considered that the sound wave propagates as a plane wave. Therefore, we neglected diffusion between adjacent elements (Sakamoto, et al., 2014a). By substituting this transfer matrix in Eqs. (6) and (10), we obtain theoretical values for the sound-absorption coefficient and transmission loss.
Using the element cross-sectional area $S$, length $l$, characteristic acoustic impedance $Z_c$, and propagation constant $\gamma$, the transfer matrix $T_{A1} - T_{A40}$ are expressed Eq. (1).

$$
T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_c}{S} \sinh(\gamma l) \\ S \sinh(\gamma l) \cosh(\gamma l) \end{bmatrix}
$$

Ignoring attenuation within the tube, characteristic acoustic impedance $Z_c$ is expressed by the product of the speed of sound in air $c_0$ and the density of air $\rho_0$, and the propagation constant $\gamma$ is expressed by the wave number $k$.

Fig. 1  Cross section of clearance among close-packed cylindrical rods

Each clearance is divided by chain lines to three analysis units. (Assume that sound wave propagates in a direction vertical to the paper surface)

Fig. 2  Approximation of clearance

Actual clearance is divided by $n$ same cross-sectional areas. Each divided element is transformed to parallel space surrounding by a pair of planes.

Fig. 3  Division fashion for one third of clearance

This is an example for division number $n = 5$ as five same areas. Actually, 40 is used for division number, cause of convergence of the calculation result.
(a) Equivalent circular estimation
(Tijdeman, 1975), (Sakamoto, et al., 2013a)

(b) Equivalent triangle estimation
(Stinson and Champou, 1992)

Fig. 4 Two additional approximation models for comparing
(Assume that sound wave propagates in a direction vertical to the paper surface)

Fig. 5 Equivalent circuit for one clearance as transfer matrices
(Elements $T_{A1}$ - $T_{A40}$ in an analysis unit and three analysis units $T_{Aall}$ - $T_{Call}$)
2.2 Propagation constant and characteristic acoustic impedance-considering attenuation

For the tubes with internal diameters greater than 20 mm, the attenuation constant inside the tube has been experimentally found (Suyama and Hirata, 1979).

Research on propagation constants and characteristic acoustic impedances with consideration for the viscosity of air within a tube has been performed by Stinson (Stinson and Champou, 1992), which deals with equilateral triangles. The propagation constant for clearance between two parallel plates has been derived by Stinson (Stinson and Champou, 1992) and Beltman (Beltman, et al., 1998).

In this section, we perform an analysis on sound wave attenuation within the clearance between two planes for which we performed an elemental breakdown in the previous section. We applied the Stinson method (Stinson and Champou, 1992) to express the individual elements of the clearance between two planes. Taking a Cartesian coordinate plane as shown in Fig. 6, we performed a three-dimensional analysis using Navier–Stokes equations, continuity equation, gas state equation, energy equation, and dissipation function representing the heat transfer due to internal friction to derive the propagation constant and characteristic acoustic impedance (Stinson and Champou, 1992).

The ideal condition of the method (Stinson and Champoux, 1992), y-direction width should be larger enough than the z-direction clearance. In this case, the summation of y-direction width is larger than severalfold of clearance b which varies continuously (Sakamoto, et al., 2014a). Thus, it is considered that it will be able to adopt the method (Stinson and Champou, 1992) to this case as an acceptable approximation.

The propagation constant $\gamma$ between two planes with consideration for attenuation is given below (Stinson and Champou, 1992). Here, $x$ is the position of the longitudinal axis on the coordinate plane, $y$ and $z$ are the planes within the clearance cross-section, $j$ represents the imaginary units, $\kappa$ is the ratio of specific heat, $C_P$ is the specific heat at constant pressure, $\lambda$ is thermal conductivity, $\mu$ is viscosity, $\omega$ is angular frequency, and $\varepsilon$ and $\varepsilon'$ are constants for convenience.

$$
\gamma = kj \sqrt{\frac{\kappa - (\kappa - 1) \left(1 - \frac{1}{\varepsilon'}, \tanh \varepsilon'\right)}{1 - \frac{1}{\varepsilon} \tanh \varepsilon}} \quad \cdot \quad , \quad \varepsilon^2 = \frac{j \rho_0 \omega b^2}{4\mu} \quad , \quad \varepsilon'^2 = \frac{j C_P \rho_0 \omega b^2}{4\lambda}
$$

(2)

Fig. 6 Cartesian coordinate system for parallel space surrounding by a pair of planes (Assume that sound wave propagates along x-direction. In other words, sound wave propagates in a direction vertical to the paper surface)

Average particle velocity $\vec{u}$ and sound pressure $p'$ for a traveling wave component in the clearance with attenuation are shown in Eqs. (3) and (4) (Stinson and Champou, 1992). Because particle velocity is a function including $y$ and $z$, which express the cross-section of the clearance, after taking the surface integral, we averaged the element cross-section by dividing by the area of the element of clearance. Because sound pressure is a plane wave, it did not need to be averaged. Here, $\beta$ is an arbitrary constant, $P_a$ is atmospheric pressure, and $t$ is time.
\[
\overline{u}^+ = -\frac{P_+ j \omega}{j \rho_0 \omega} \left( 1 - \frac{1}{\varepsilon} \tanh \varepsilon \right) (-\beta e^{-\gamma x}) e^{j \omega x}
\]

(3)

\[
p^+ = P_+ \beta e^{-\gamma x} e^{j \omega x}
\]

(4)

Next, we show specific acoustic impedance \( Z_c \) in a clearance between two planes with consideration for attenuation.

\[
Z_c = \frac{p^+}{\overline{u}^+}
\]

(5)

Following the above steps, we found the propagation constant \( \gamma \) and specific acoustic impedance \( Z_c \) for a clearance with consideration for attenuation.

Substituting these values for the \( \gamma \) and \( Z_c \) in Eq. (1), we can import the analysis result of two-dimensional of the clearance between two planes to one-dimensional transfer matrix.

For the Fig. 3 in the section 2.1, in case of the division number \( n = 5 \), by naming the colored area divided at an angle of 17.90° to "a1", divided areas are named "a1, a2, a3, a4, a5" from bottom. In the theoretical analysis, areas are named "a1, a2, ..., a39, a40" from bottom because we set the division number \( n = 40 \). In this section, the calculated values for the propagation constant \( \gamma \) and characteristic impedance \( Z_c \) in the each region of a1, a5, a20, in case of Fig. 11 (b) are shown in Figs. 7 to 10. The calculated results of analyzing a clearance as an equivalent circle and a triangle are also shown. Here, for the characteristic impedance, the real and imaginary parts are normalized by characteristic impedance of air, respectively. For the propagation constant, the real and imaginary parts are converted to the attenuation constant and phase velocity, each.

The calculated results in Fig. 7 to 10 show well agreement with the trend of the porous materials that solid borne sound can be ignored (Koshiroi and Tateishi, 2012) and the case of a small clearance between two planes (Sakamoto, et al., 2014a). Besides, comparing to larger clearance between two planes, the attenuation constant and characteristic impedance are larger, speed of sound is smaller. In case of clearance between two planes (Sakamoto, et al., 2014b), we obtain calculated values correspond to thickness of clearances \( a1, a5, a20 \). On the other hand, we can obtain each only one curve in approximation of an equivalent circle (Sakamoto, et al., 2013a), (Tijdeman, 1975) and a triangle (Stinson and Champou, 1992).

These results closely matched the results of converting Tijdeman’s method to two planes (Sakamoto, et al., 2014a).
3. Verification experiment

3.1 Test sample used for measurement

Figure 11 and Table 1 show a photograph and the details of the measured sample, respectively. To find how length of narrow tube, and size of cross-section resulting from the clearance affected the sound-absorption coefficient and transmission loss, we compared them with the length and diameter of round bar of the sample. Diameters $d = 0.75$ mm and $8.0$ mm for a round bar, given in Table 1, are constants used in the calculations, the results of which are given in section 4.

The test sample was constructed of stainless round bars (hereafter referred to as “round bars”), and the cut surfaces were evenly grinded. The round bars in the sample were rigidly bound with fiberglass tape. This was to ensure that the clearance among the three connecting round bars would form an ideal cross-sectional configuration. In other words, this was to prevent the separation of the curving connections of the round bars. Next, we created a cylindrical frame with a diameter of $29$ mm from a sheet of polypropylene, inserted the bundle of round bars into the frame, and filled the clearance between the round bars and frame with modeling clay.

![Fig 9](image1.png) Attenuation constant of dB/m

$(20 \log_{10} \exp(\text{Re}(\gamma)) = 8.6859 \text{ Re}(\gamma))$

(For dimension of Fig. 11 (b))

![Fig 10](image2.png) Phase velocity $(\omega / \text{Im}(\gamma))$

(For dimension of Fig. 11 (b))

![Fig 11](image3.png) Test sample elements in measurement tube (outer diameter: approximately 29 mm)
Table 1  Typical specifications of test sample elements

| Length $l$ [mm] | Diameter of the cylindrical rod $d$ [mm] | Diameter of the equivalent circle [mm] | Length of one side of the equivalent triangle [mm] | Number of cylindrical rods | Number of clearances $N$ | Aperture ratio $S/S_0$ |
|-----------------|------------------------------------------|----------------------------------------|-----------------------------------------------|---------------------------|-------------------------|---------------------|
| 0.75 *          | 0.17                                     | 0.229                                  | ---                                           | 1456                      | 372                     | 0.0500              |
| 1.5             | 0.34                                     | 0.458                                  | 211                                           | 372                       | 130                     | 0.0511              |
| 2.5             | 0.566                                    | 0.763                                  | 80                                            | 54                        | 0.0527                  |
| 4.0             | 0.906                                    | 1.22                                   | 37                                            | 54                        | 0.0508                  |
| 8.0 *           | 1.812                                    | 2.441                                  | ---                                           | 13                        | 0.0508                  |

* Only for calculation

3.2 Apparatus used for measurement of the sound-absorption coefficient and transmission loss

The composition of the sound-absorption coefficient measuring apparatus is shown in Fig. 12. The test sample was enclosed within a Brüel & Kjær type 4206 2-microphone impedance measuring tube. Sinusoidal signals were output from a fast Fourier transform (FFT) internal signal generator, and the transfer function between the sound pressure signals of the two microphones attached to the impedance-measuring tube were measured using the analyzer. Using the measured transfer function, we calculated the normal incident sound-absorption coefficient based on ISO 10534-2. The threshold wavelength for plane waves differs based on the internal diameter of an acoustic tube. Because the test sample used in this study did not have a high sound-absorption coefficient in the low frequency range, we used a small tube with an internal diameter of 29 mm. Therefore, our measurement range was 500 Hz–6400 Hz.

The composition of the transmission loss measuring apparatus is shown in Fig. 13. The test sample was enclosed within a Brüel & Kjær type 4206T 4-microphone impedance measuring tube. Two microphones were placed on each of the wall surfaces in front of and behind the test sample. Reference signals were generated by a signal generator, and the sound pressure in front of and behind transmission through the test sample was measured using the microphones on the FFT analyzer. Then, we calculated the normal incident transmission loss based on ASTM E2611-09. A small tube with an internal diameter of 29 mm was also used in the transmission loss measurement for the same reasons as in the sound-absorption coefficient measurement apparatus.

![Diagram of apparatus for sound-absorption coefficient measurement](image1)

![Diagram of apparatus for transmission loss measurement](image2)
4. Comparison of calculated results and measured results

4.1 Deriving the sound-absorption coefficient

For the entrance and exit of one clearance, we define the respective sound pressures \( p_1 \) and \( p_2 \), and the particle velocities \( u_1 \) and \( u_2 \). For the boundary condition, the terminal end of the clearance is a rigid wall, the particle velocity is \( u_2 = 0 \), the transfer matrix is given by Eq. (6). In open-end correction, we add 0.4 times the internal radius of a circle with the same area as the clearance to the edge of the entrance of the element of clearance (Bolt, et al., 1949), (Sakamoto, et al., 2013a).

\[
\begin{bmatrix}
  p_1 \\
  Su_1 
\end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix}
  p_2 \\
  Su_2 
\end{bmatrix} = \begin{bmatrix} Ap_2 \\ Cp_2 \end{bmatrix}
\] (6)

Figure 14 shows a schematic of the thick portion around one clearance. Here \( S/S_0 \) is equal to the aperture ratio of the test sample, as given in Table 1. In other words, \( S_0 \) is the unit area for one clearance including the non-aperture portion (bars and modeling clay), and this area is calculated by dividing the cross-sectional area of the test sample, (29 mm in diameter) by the number of clearances \( N \).

If we define the sound pressure and particle velocity of just left hand side of Plane 1 in Fig. 14 as \( p_0 \) and \( u_0 \), respectively, from \( p_0 = p_1 \), \( S_0 u_0 = Su_1 \) and Eq. (6), the specific acoustic impedance \( Z_0 \) on the interior of the test sample from the incident face is given by the following equation:

![Diagram of clearance with labeled parts](image)

Fig. 14 Unit area \( S_0 \) for each narrow clearance
(Assume that sound wave propagates along \( x \)-direction)
Here, the relationship between the specific acoustic impedance \( Z_0 \) and the reflection coefficient \( R \) is expressed by the following equation:

\[
\frac{Z_0}{\rho_c c_0} = \frac{1 + R}{1 - R}
\]  

From the sound-absorption coefficient and reflection coefficient in Eq. (9) below and Eq. (8), we can calculate the theoretical value for the sound-absorption coefficient \( \alpha \).

\[
\alpha = 1 - |R|^2
\]  

### 4.2 Sound-absorption coefficient

For aperture ratio is \( S/S_0 = 0.05 \), we compared the calculated and experimental values of the sound-absorption coefficient when changing the round bar diameter \( d \), and sample length \( l \). The results are shown in Figs. 15–20. In Figs. 15 and 16, \( d = 1.5 \) mm, \( S/S_0 = 0.0511 \), and \( l = 25 \) mm and 50 mm, respectively. In the same manner, Figs. 17, 18 and 19, 20 show values for \( d = 2.5 \) mm and \( d = 4.0 \) mm.

Four categories of calculated values are shown: those with no consideration for attenuation within the clearance, those analyzing the clearance cross-section as a circular hole and a triangular hole having the same area (Sakamoto, et al., 2013a), (Stinson and Champou, 1992), and those analyzing the clearance broken down (using the analysis given in section 2.1).

First, some comments about the overall experimental results. Even in shapes and configurations that are not designed for sound-absorption, densely packed round bar clearances show significant sound-absorption characteristics.

The experimental values of sample length \( l \) are compared in Figs. 15 and 16, Figs. 17 and 18, Figs. 19 and 20. We observe that frequencies at which the sample length is mostly 1/4 times the wavelength of the sound wave, the first sound-absorption peak frequency occurs. Thus, the sound-absorption peak frequency is about half at \( l = 50 \) mm, shifting to the low frequency side.

The experimental values of the round bar diameter \( d \) are compared in Figs. 15, 17, and 19, Figs. 16, 18, and 20. We discuss below the fact that the sound-absorption coefficient becomes lower as \( d \) becomes smaller. The thickness of the laminar boundary layer generated by the sound wave particle velocity is said to be tens of micrometers at 1 kHz, though it depends on the frequency (Wesley, 1958). Here, we examine the diameter of an equivalent circle in Table 1. The diameter of the equivalent circle for the 3 clearances used in this experiment was less than 1 mm. If we consider the cross-section shape of the clearance, it is clear that sound-wave attenuation within the clearance was larger than that within a circular tube. In our previous study (Sakamoto, et al., 2013a), the sound-absorption coefficient peak value rose as the circular hole diameter became smaller (8, 4, and 2 mm). However, simulating a diameter shrinking to 0.25 mm in the calculated values showed that the sound-absorption coefficient shrinks. This is because as the clearance shrinks too far, with respect to the thickness of the boundary layer, the characteristic acoustic impedance of the air within the clearance increases and the reflectance increases (thus, the sound-absorption coefficient decreases). Similarly, when the sound wave enters the clearance between two planes and when the clearance is smaller than the boundary layer, the sound-absorption coefficient becomes lower as the size of the clearance becomes smaller (Sakamoto, et al., 2013b). This indicates that for clearances addressed in this study, because the cross-section has a small area and clearances that narrow locally, there are many regions that are small with respect to boundary layer thickness. Thus, it is thought that as \( d \) becomes smaller, the sound-absorption coefficient becomes lower.
Comparison between experiment and calculations

- Fig. 15: Comparison between experiment and calculations
  \((d = 1.5 \text{ mm}, l = 25 \text{ mm}, S/S_0 = 0.0511)\)

- Fig. 16: Comparison between experiment and calculations
  \((d = 1.5 \text{ mm}, l = 50 \text{ mm}, S/S_0 = 0.0511)\)

- Fig. 17: Comparison between experiment and calculations
  \((d = 2.5 \text{ mm}, l = 25 \text{ mm}, S/S_0 = 0.0496)\)

- Fig. 18: Comparison between experiment and calculations
  \((d = 2.5 \text{ mm}, l = 50 \text{ mm}, S/S_0 = 0.0496)\)

- Fig. 19: Comparison between experiment and calculations
  \((d = 4.0 \text{ mm}, l = 25 \text{ mm}, S/S_0 = 0.0527)\)

- Fig. 20: Comparison between experiment and calculations
  \((d = 4.0 \text{ mm}, l = 50 \text{ mm}, S/S_0 = 0.0527)\)
Next, we discuss the comparison of the calculated and experimental values. First, because the reflection coefficient reaches 1 in the calculated values without consideration for attenuation, the sound-absorption coefficient could not be obtained. In all the cases in Figs. 15–20, the calculated values were closer to the experimental values when obtained through analysis in which the clearance was broken down, rather than through analyzing it as a circular hole and a triangular hole. This is because analysis that considers shape allows for suitable consideration of regions that are close to the connections between the round bars.

Figures 21 and 22 show three categories of round bar diameter $d$ at $l = 25$ and 50 mm, respectively, to facilitate the comparison of the experimental values. In general, a phenomenon is seen in which sound velocity decreases due to attenuation within the tube, and the sound-absorption peak frequency also falls. In addition, in Figs. 21 and 22, as $d$ becomes smaller (in other words, as the clearance becomes smaller), the sound-absorption peak frequency falls.

Next, we compare every calculated value of the analysis in section 2.1. Calculated values for a sample length of 25 mm and 50 mm are shown in Figs. 23 and 24. Together with calculated values using the same conditions used for the sample in the experiment, Table 1 also shows calculated values for lengths of $d = 0.75$ and 8.0 mm. As with the comparison between experimental values, the calculated values for $d = 1.5, 2.5$, and 4.0 mm show that as $d$ becomes larger the sound-absorption coefficient peak value increases. Here, when $d = 0.75$ mm, the sound-absorption peak is unclear. At $d = 8.0$ mm, because the clearance is comparatively large with respect to boundary layer thickness, the sound-absorption peak is sharp.
4.3 Deriving transmission loss

Figure 25 shows an equivalent circuit including the impedance measuring tube in front of and behind the bundle of clearances. The bundle of clearances is considered to be a transfer matrix connecting parallel according to number of the holes (Sakamoto, et al., 2013a), assuming Eq. (1). For open-end correction, we add 0.4 times the internal radius of a circle with the same area as the clearance to the both edges of the element of clearance (Bolt, et al., 1949), (Sakamoto, et al., 2013a).

Shown in Fig. 25, defining “from 1st to (N-1)th clearances connected in parallel” and “the next Nth clearance to be added” as transfer matrices $T_{1\text{to}N-1}$ and $T_N$, respectively, in Eq. (10), a transfer matrix connecting these in parallel to $T_{1\text{to}N}$ can be expressed as in Eq. (11).

$$T_{1\text{to}N-1} = \begin{bmatrix} A_{1\text{to}N-1} & B_{1\text{to}N-1} \\ C_{1\text{to}N-1} & D_{1\text{to}N-1} \end{bmatrix} \quad T_N = \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix}$$

$$T_{1\text{to}N} = \begin{bmatrix} A_{1\text{to}N} & B_{1\text{to}N} \\ C_{1\text{to}N} & D_{1\text{to}N} \end{bmatrix} = \begin{bmatrix} A_N B_{1\text{to}N-1} + A_{1\text{to}N-1} B_N \\ B_N + B_{1\text{to}N-1} \\ C_N + C_{1\text{to}N-1} + \frac{(A_{1\text{to}N-1} - A_N)(D_N - D_{1\text{to}N-1})}{B_N + B_{1\text{to}N-1}} \end{bmatrix} \begin{bmatrix} B_N B_{1\text{to}N-1} \\ B_N + B_{1\text{to}N-1} \end{bmatrix}$$

Using the four-terminal constants in Eq. (11), the transmission loss of the bundle of both ends opened clearances $TL$ is given by Eq. (12). Here, the acoustic impedance measuring tube cross-sectional area $S_{\text{tube}}$ including the non-aperture portion (bars and modeling clay) reflects the aperture ratio in Eq. (12).

$$TL = 10 \log_{10} \left( \frac{A_{1\text{to}N} + \frac{S_{\text{tube}}}{Z_c} B_{1\text{to}N} + \frac{Z_c}{S_{\text{tube}}} C_{1\text{to}N} + D_{1\text{to}N}}{4} \right)^2$$

![Fig. 25](image-url) Equivalent circuit for bundle of clearances in impedance tube
4.4 Transmission loss

We compared the calculated values and experimental values for transmission loss when changing the round bar diameter $d$ and sample length $l$. The results are shown in Figs. 26–31. In Figs. 26 and 27, $d = 1.5$ mm, $S/S_0 = 0.0511$, and $l = 25$ mm and 50 mm, respectively. In the same manner, Figs. 28–31 show values for $d = 2.5$ mm and $d = 4.0$ mm when changing sample length.

Four categories of calculated values are shown: those with no consideration for attenuation within the clearance, those analyzing the clearance cross-section as a circular hole and a triangular hole with the same area (Stinson and Champou, 1992), (Sakamoto, et al., 2013a), and those analyzing the clearance broken down (using the analysis given in section 2.1).

The experimental values of sample length $l$ are compared in Figs. 26 and 27, Figs. 28 and 29, Figs. 30 and 31. This sample configuration is operated as an orifice silencer comprising an array of bundled narrow holes. Thus, at frequencies at which the sample length $l$ is nearly multiples of 1/2 times the wavelength of the sound wave, we see a fall in transmission loss. So the transmission loss curve shifts to the low frequency side at $l = 50$ mm.

The experimental values of the round bar diameter $d$ are compared in Figs. 26, 28, and 30, Figs. 27, 29, and 31. Because sound wave attenuation propagated to the clearance is larger for smaller values of $d$, transmission loss increases.

Transmission loss calculated without consideration for attenuation was calculated with the entire cross-sectional area of all clearances as an orifice silencer with one aperture. In Figs. 26–31, because the calculated values do not consider attenuation, the values are mostly lower than the experimental values. Calculated values that do not consider attenuation are also observed to diverge from the experimental values near the frequency where transmission loss falls.

The three categories of calculation results that considered attenuation obtained trends close to the experimental values. As with the sound-absorption coefficient, two analyses that considered the shape of the clearance showed values closer to the experimental values than analysis treating the clearance as an equivalent circle. Also, similar to the sound-absorption coefficient, it is thought that this trend is because the internal area of each cross-sectional area of the clearance is larger when considered as a clearance among three cylinders than in a circular hole, and the boundary layer comprises a larger proportion of space within the clearance.

In Figs. 30 and 31 where $d = 4.0$ mm, because the attenuation is small, consideration of attenuation does not have a large effect on calculation results. In Fig. 31, however, calculated values that do not consider attenuation diverge from experimental values at frequencies where transmission loss drops.

For transmission loss estimation, a practically sufficient estimate is obtained by using the approximation of an equivalent circle or a triangle; however, the approximation of a triangle is better than that of a circle. Naturally, the computational load of analysis for a triangle is lighter than that of section 2.1.

Figures 32 and 33 show values for $l = 25$ mm and 50 mm to facilitate comparison of the experimental values. As was stated regarding Figs. 21 and 22 in section 4.2, because of the falling sound velocity, as $d$ becomes smaller, the transmission loss curve shifts to lower frequencies. Also, we see that when $d = 1.5$ mm transmission loss clearly increases. In addition, falls in transmission loss are tempered by smaller values for $d$. This is thought to be because sound wave attenuation in the clearance suppresses standing waves.

Next, we compare values calculated through the analysis presented in section 2.1. Calculated values for a sample length of 25 mm and 50 mm are shown in Figs. 34 and 35. In these figures, as with the sound-absorption coefficient, calculated values are also shown for values of $d = 0.75$ and 8.0 mm.

As with the comparison between experimental values, calculated values for $d = 1.5$, 2.5, and 4.0 mm show that as $d$ becomes smaller transmission loss increases. Here, when $d = 0.75$ mm, transmission loss clearly increases and no falls are observed. At $d = 8.0$ mm, the fall in transmission loss is more clear than when $d = 4.0$ mm.
Fig. 26 Comparison between experiment and calculations
\( (d = 1.5 \text{ mm}, l = 25 \text{ mm}, S/S_0 = 0.0511) \)

Fig. 27 Comparison between experiment and calculations
\( (d = 1.5 \text{ mm}, l = 50 \text{ mm}, S/S_0 = 0.0511) \)

Fig. 28 Comparison between experiment and calculations
\( (d = 2.5 \text{ mm}, l = 25 \text{ mm}, S/S_0 = 0.0496) \)

Fig. 29 Comparison between experiment and calculations
\( (d = 2.5 \text{ mm}, l = 50 \text{ mm}, S/S_0 = 0.0496) \)

Fig. 30 Comparison between experiment and calculations
\( (d = 4.0 \text{ mm}, l = 25 \text{ mm}, S/S_0 = 0.0527) \)

Fig. 31 Comparison between experiment and calculations
\( (d = 4.0 \text{ mm}, l = 50 \text{ mm}, S/S_0 = 0.0527) \)
5. Conclusion

This study estimated the sound-absorption coefficient and transmission loss for the dimensions of clearances among close-packed cylinders through theoretical analysis, compared those estimates with experimental values, and obtained the following results.

To perform a simple, high-precision analysis, we performed an elemental breakdown of the clearance cross-section and approached each element as a clearance between two planes. We found the propagation constant and characteristic acoustic impedance with consideration for sound wave attenuation in each element through a three-dimensional analysis of the clearance between two planes for each element. Next, using parallel connection, we found a one-dimensional transfer matrix for the entire clearance.

No value could be found for calculated estimations of the sound-absorption coefficient without consideration for...
attenuation. Calculations that approached the clearance as an equivalent circle and a triangle did not match well with experimental values. Values calculated with the same method, but which considered the shape of the clearance, matched well with experimental values.

In estimating transmission loss, the clearance transfer matrix was connected in parallel to express the entire clearance. In calculations that did not account for attenuation, the curve of falls in transmission loss diverged greatly from experimental values. In the three calculated values that considered attenuation, frequency characteristics for falling portions of transmission loss showed a trend close to those for the experimental values. Calculations with this method, which accounted for the clearance shape and equivalent triangle, also had transmission loss peak values that closely matched experimental values.

**Nomenclature**

\( A-D \) : Four-terminal constants of transfer matrix  
\( b \) : Thickness of the clearance [m]  
\( c_0 \) : Speed of sound in air [m/s]  
\( C_p \) : Specific heat at constant pressure [J/(m\(^3\)·K)]  
\( d \) : Length of one side of equilateral triangular hole [m]  
\( j \) : Imaginary unit  
\( k \) : Wave number [1/m]  
\( l \) : Length of test sample [m]  
\( n \) : Division number of analysis units  
\( N \) : Number of holes  
\( P_0 \) : Atmospheric pressure [Pa]  
\( p_0 \) : Sound pressure outside of hole [Pa]  
\( p_1, p_2 \) : Sound pressure at each location [Pa]  
\( p^+ \) : Sound pressure of traveling wave [Pa]  
\( R \) : Reflection coefficient  
\( S \) : Cross-sectional area of hole [m\(^2\)]  
\( S_0 \) : Unit area per hole including non-opening section on test sample [m\(^2\)]  
\( S_{\text{tube}} \) : Cross-sectional area of impedance measurement tube [m\(^2\)]  
\( T \) : Transfer matrix  
\( TL \) : Transmission loss  
\( t \) : Time [s]  
\( u_0 \) : Particle velocity outside of hole [Pa]  
\( u_1, u_2 \) : Particle velocity at each location [m/s]  
\( \bar{u} \) : Average value of particle velocity of traveling wave [m/s]  
\( x, y, z \) co-ordinate [m]  
\( Z_c \) : Characteristic acoustic impedance [Ns/m\(^3\)]  
\( Z_0 \) : Specific acoustic impedance [Ns/m\(^3\)]  
\( \alpha \) : Sound absorption coefficient  
\( \beta \) : Arbitrary constant  
\( \gamma \) : Propagation constant [1/m]  
\( \epsilon, \epsilon' \) : Arbitrary constant  
\( \kappa \) : Ratio of specific heat  
\( \lambda \) : Thermal conductivity [W/(m·K)]  
\( \mu \) : Viscosity [Pa·s]  
\( \rho_0 \) : Density of air [kg/m\(^3\)]  
\( \omega \) : Angular frequency [rad/s]

**Acknowledgment**

This work was supported by JSPS KAKENHI Grant Number 24560253.
References

Beltman, W. M., van der Hoogt, P. J. M., Spiering, R. M. E. J., Tijdeman, H., Implementation and experimental validation of new viscous thermal acoustic finite element for acousto-elastic problems, Journal of Sound and Vibration, Vol. 216, No. 1 (1998), pp. 159-185.

Bolt, H. R., Labate, S. and Ingård, U., The acoustic reactance of small circular orifices, Journal of the Acoustical Society of America, Vol. 21, No. 2 (1949), pp. 94-97.

Katayama, K., Tsuboi M., Kawaoka T., Shiraki K., Sato Y., Scale model test on inexperienced acoustic vibrations occurred in the operation of a heat exchanger, Transactions of the Japan Society of Mechanical Engineers, Series C, Vol. 65, No. 640 (1999), pp. 4626-4632 (in Japanese).

Koshiroi, T., Tateishi, S., Features of sound absorption data in porous material caused by elastic frame structure, Journal of Acoustical Society of Japan, Vol. 68, No. 9 (2012), pp. 474-479 (in Japanese).

Matsuzawa, K., Sound propagation in a tube of arbitrary cross-sectional shape, Journal of the Acoustical Society of Japan, Vol. 19, No. 1 (1963), pp. 1-8 (in Japanese).

McGinnes, C., Kleiner, M., Xiang, N., An Environmental and economical solution to sound absorption using straw, Journal of the Acoustical Society of America, Vol. 118, No. 3 (2005), p. 1869.

Ohno, Y., Tanaka, G., Hishida, M., Enhanced heat transfer during oscillatory flow in annular channels, Transactions of the Japan Society of Mechanical Engineers, Series B, Vol. 70, No. 698 (2004), pp. 2612-2619 (in Japanese).

Sakamoto, S., Takauchi, Y., Yanagimoto, K., Watanabe, S., Study for sound absorbing materials of biomass tube etc. (Measured result for rice straw, rice husks, and buckwheat husks), Journal of Environment and Engineering, Vol. 6, No. 2 (2011), pp. 352-364.

Sakamoto, S., Hoshino, A., Sutou, K., Sato, T., Estimating sound-absorption coefficient and transmission loss by the dimensions of bundle of narrow holes (Comparison between theoretical analysis and experiments), Transactions of the Japan Society of Mechanical Engineers, Series C, Vol. 79, No. 807 (2013a), pp. 4164-4176 (in Japanese).

Sakamoto, S., Saito, K., Murayama, N., Higuchi, K., Experimental studies on the sound absorbing materials by using of the bundle of narrow clearances between two planes, Transactions of the Japan Society of Mechanical Engineers, Series C, Vol. 79, No. 807 (2013b), pp. 4141-4152 (in Japanese).

Sakamoto, S., Higuchi, K., Saito, K., Koseki, S., Theoretical analysis for sound-absorbing materials using layered narrow clearances between two planes, Journal of Advanced Mechanical Design Systems and Manufacturing, Vol. 8, No. 3 (2014a), Paper No.14-00302, 16 pages.

Sakamoto, S., Sugahara, R., Nagumo, T., Nakano, A., Kawase, H., Theoretical analysis and measurement of sound transmission loss in louver elements with a sound attenuating function using a Helmholtz resonator array, Journal of Advanced Mechanical Design Systems and Manufacturing, Vol. 8, No. 3 (2014b), Paper No.14-00061, 16 pages.

Stinson, R. M.and Champou, Y., Propagation of sound and the assignment of shape factors in model porous materials having simple pore geometries, Journal of the Acoustical Society of America, Vol. 91, No. 2 (1992), pp. 685-695.

Suyama, E., Hirata, M., Attenuation constant of plane wave in a tube: Acoustic characteristic analysis of silencing systems based on assuming of plane wave propagation with frictional dissipation part 1, The Journal of the Acoustical Society of Japan, Vol. 35, No. 4 (1979), pp. 152-164 (in Japanese).

Tijdeman, H., On the propagation of sound waves in cylindrical tubes, Journal of Sound and Vibration, Vol. 39, No. 1 (1975), pp. 1-33.

Verdrië, K., Panetton, R., Elkoun, S., Dupont, T., Leclaire, P., Transfer matrix method applied to the parallel assembly of sound absorbing materials, Journal of the Acoustical Society of America, Vol. 134 (2013), pp. 4648-4658.

Wesley, L. N., Acoustic streaming near a boundary, Journal of the Acoustical Society of America, Vol. 30, No. 4 (1958), pp. 329-339.