Vibration field estimation of a plate in time domain using Kalman filter

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Abstract. The modal expansion method is a technique of visualizing the vibration field of the structure by estimating the MPF (modal participation factor) from the signals from a limited number of sensors. This paper will propose a method to obtain the reliable vibration field of the whole structure by using noise-contaminated signals. In this paper, the error of the predicted vibration field due to the noise signal is reduced by Kalman filter. The Kalman filter predicts more precise state variables from the noise-contaminated signal. With this characteristic of the Kalman Filter, a technique is introduced to predict the vibration field of the structure by using the acceleration vibration signals and the MPF of the structure, used as the state variables and the measured value, respectively. In this paper, the studied analytical model is a plate with 20 sensors evenly placed on the plate. The analytical model is excited by an aperiodic force, and MCS. Nastran was used for the vibration analysis. To confirm the accuracy of the predicted vibration field, the predicted vibration signals are compared with the exact vibration signals. The result shows that the proposed method of the vibration field visualization using Kalman Filter gives reliable results.

1. Introduction
One of the vibration field prediction methods for monitoring the vibration state of the structure is the modal expansion method. The modal expansion method is a technique of visualizing the vibration field of the structure by estimating the MPF (modal participation factor) from the signals from a limited number of sensors. As an example of a previous study, Jung [1] predicted the vibration field of the washing machine using the modal expansion method. Usually, the modal expansion method is applied in frequency domain. This paper, on the other hand, will propose a method to obtain the reliable vibration field of the whole structure by using noise-contaminated signals.

2. Theory of Kalman filter for vibration field estimation
To analyze the system in the time domain, a state space model representing the vibration system was designed. The state space is represented as equation (1) and (2).

\[ X_{k+1} = AX_k + BF_k \]  
\[ Z_k = CX_k + DF_k \]
In the equation (1) and (2), \(X\) is the state variable vector, \(Z\) is a measurement vector, and \(F\) is a force vector in the physical coordinate system. The state variable vector consists of the velocity MPF of the structure and the displacement MPF. The Measurement Value \(Z\) consist of the noise-contaminated acceleration sensor signals. The subscript \(k\) represents time. In here, the matrix \(A\), \(B\), \(C\), \(D\) is represented as equation (3), (4), (5) and (6).

\[
A = I_{2m \times 2m} + \Delta t \begin{bmatrix} [\theta]_{m \times m} & [I]_{m \times m} \\ -[\omega]^2_{m \times m} & [\theta]_{m \times m} \end{bmatrix} \\
B = \begin{bmatrix} [\theta]_{m \times N} \\ \Delta t [\Phi]^T_{m \times N} \end{bmatrix} \\
C = [\tilde{\Phi}]_{m \times m} [\alpha]_{m \times m} [\theta]_{m \times m} \\
D = [\tilde{\Phi}]_{m \times m} [\Phi]^T_{m \times N} 
\]

In the equation (3), (4), (5) and (6), \(\Phi\) is modal matrix, \(\tilde{\Phi}\) is reconstructed modal matrix, and \([\alpha]\) is the natural frequency matrix. We can estimate the MPF selected by the state variable with Kalman filter using the designed state space. With these estimated MPF, it is possible to predict not only the acceleration vibration field but also the displacement and velocity vibration field.

3. Analysis and Result

3.1. Analysis Model

The analytical model covered in this paper is a finite element model in the form of a plate structure with six degrees of freedom constraint at the four vertexes. In this case, the thickness of the plate is 5 mm, and the material of the structure is steel.

![Figure 1. Aperiodic force.](image_url)
3.2. Vibration Field Prediction for Aperiodic Excitation.

The vibration field was predicted when the structure is subjected to an aperiodic excitation force as shown in Figure 1. The measured signal from the sensors are assumed to be a signal obtained by mixing the Gaussian noise with the acceleration vibration response obtained from the transient response analysis. To verify the vibration signal estimation performance for the acceleration, velocity, and displacement of the Kalman filter, Figure 2 shows the vibration response graph at the non-measurement point. From the graph a) in Figure 2, the measurement noise of the acceleration sensor is reduced in the estimation signal from which the noise is removed. Next, From the graphs b) and c), 'the velocity signal and the displacement signal affected by the noise' show a phenomenon in which the error due to noise accumulates. On the other hand, the estimated velocity signal and the displacement signal are well estimated for the response vibration response signal.

To investigate the prediction performance of the Kalman filter, the acceleration vibration field graph at a randomly selected time of 0.101[s] is shown in Figure 3. From the Figure 3, when the vibration field is estimated using the Kalman filter, the noise effect of the sensor signal is reduced, and acceleration vibration field is estimated relatively accurately.

![Figure 2. Vibration signals estimation subjected to the Aperiodic force](image)
4. Conclusion

This paper proposed a method of vibration field estimation in time domain using Kalman filter. By using Kalman filter, it was possible to obtain the reliable vibration field from the noise-contaminated signal. In this paper, a simple model, the plate structure, was studied, but it is expected that it will be useful for monitoring the vibration response of complicated shaped structure in the further studies.

References

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