Parametrically-induced tunable coupling between flux qubits — dependence on the coupler bias

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Abstract. We have studied a tunable coupling scheme of two superconducting flux qubits biased at the degeneracy point. The qubits are coupled parametrically under microwave driving, via nonlinear inductance of a third qubit biased slightly away from the degeneracy point. The observed on-state coupling as well as the off-state residual coupling depend non-monotonically on the coupler bias point and agree quantitatively with calculations based on a three-qubit Hamiltonian. This simple model is useful to optimize the parameters for a large on/off ratio.

1. Introduction

The recent progress on Josephson-junction qubits has shown that they are among the promising candidates for the implementation of the quantum information processing and brought simple demonstrations of quantum algorithms in to the scope of experiments. For that, it is crucial to control the coupling between qubits. It is also required to switch on and off the coupling very fast, without degrading the coherence time. Moreover, it is desired to operate the qubits at the optimal bias points where the coherence time is maximum. Recently, a few different types of dynamically-controlled tunable coupling were successfully implemented in experiments [1, 2, 3].

A tunable coupling scheme was demonstrated in time domain in Ref.[1], where two flux qubits biased at the degeneracy points were inductively coupled via a third qubit. The Hamiltonian of the three inductively-coupled qubits is written as

\[ H = -\frac{1}{2} \sum_{j=1}^{3} (\Delta_j \sigma_j^z + \varepsilon_j \sigma_j^y) + \sum_{k \neq l}^{3} J_{kl} \sigma_k^x \sigma_l^x, \]  

where \( \Delta_j \) and \( \varepsilon_j = 2 I_{pj} \delta \Phi_j \) are respectively the splitting and bias energies between the two persistent-current states of qubit \( j \), and \( \sigma_j^y \) and \( \sigma_j^y \) are the Pauli matrices operating on qubit \( j \). The persistent current is \( I_{pj} \), and \( \delta \Phi_j = \Phi_j - \Phi_0/2 \) (\( \Phi_0 = \frac{\hbar}{2e} \)) is the flux bias with respect to the degeneracy point. The coupling energy is given by \( J_{kl} = M_{kl} I_{pk} I_{pl} \) where \( M_{kl} \) is the mutual inductance between qubit \( k \) and \( l \). When qubits 1 and 2 are biased at the degeneracy points \( \delta \Phi_1 = \delta \Phi_2 = 0 \), the inductive coupling is solely off-diagonal. Moreover, if the condition \( \Delta_1 < \Delta_2 \ll \Delta_3 \) is fulfilled, an adiabatic approximation on qubit 3, or the coupler, results in an
effective coupling in the form of $+J_{12}^{eff}\sigma_1^x\sigma_2^x$ between qubits 1 and 2 [4]. The coupling strength is given by

$$J_{12}^{eff} = J_{12} - \frac{2J_{23}J_{13}\Delta_3^2}{(\Delta_3^2 + \varepsilon_3^2)^{3/2}},$$  \hspace{1cm} (2)$$

where the first and second terms can be understood respectively as the direct and indirect inductive interaction. If qubits 1 and 2 are far detuned, i.e., $|J_{12}^{eff}|/|\Delta_1 - \Delta_2| \ll 1$, the effect of the off-diagonal coupling becomes negligible.

To switch on the coupling, microwave modulation of $\delta\Phi_3$ is applied to the coupler. Thanks to Eq. (2), the matrix element $\langle 00 | \partial H / \partial \Phi_3 | 11 \rangle \simeq \partial J_{12}^{eff} / \partial \Phi_3$ is finite. When the microwave pulse is at the sum or difference frequency of the two qubits, it parametrically induces transition $|00\rangle \leftrightarrow |11\rangle$ or $|10\rangle \leftrightarrow |01\rangle$ and effectively switches on the coupling in the form of $\sigma_1^x\sigma_2^x \pm \sigma_1^y\sigma_2^y$, respectively.

2. Sample characterization

The three-qubit sample was fabricated by electron-beam lithography and shadow evaporation technique using Al films (Fig. 1(a)). Each flux qubit is a superconducting loop intersected by four Josephson junctions, among which one has a size $\alpha_j$ times smaller compare to others. The $\alpha_j$ values were designed to satisfy the condition $\Delta_1 < \Delta_2 \ll \Delta_3$. Two qubits (qubits 1 and 2)
are individually integrated into a DC-SQUID loop. The third one (qubit 3), which is placed in between and shares a part of the loop with qubits 1 and 2, serves as a coupler. The two DC-SQUIDs are used to read out the state of each qubit. An external superconducting coil is used to apply a global magnetic flux. Together with two local dc-bias lines (lines 1 and 2), it allows us to control the flux bias of each qubit, $\delta\Phi_j$, independently.

The readout was performed using the switching event of the SQUIDs from the supercurrent state to the voltage state under a current pulse [5]. To avoid readout crosstalk, only one SQUID was operated at a time, and the bias current of the second one was kept at zero. For qubit manipulations, we applied resonant microwave pulses with controllable durations. The relaxation times of the two qubits were around 300 ns. Using SQUIDs 1 and 2 individually, we performed spectroscopy measurements to characterize the qubits. Figure 1(b) shows a wide-range spectrum taken with SQUID 2. Because of the geometry, signals from qubit 2 and 3 are mainly seen. The spectrum is fitted well with the eight-level Hamiltonian (Eq. (1)), and the qubit parameters are deduced reliably.

3. Results and discussion
As a measure of the on-state coupling, we use the Rabi frequency $\Omega_{12}/\hbar$ of $|00\rangle \leftrightarrow |11\rangle$ transition which works equivalently as an iSWAP gate after a duration of $\hbar/(2\Omega_{12})$. The Rabi frequency is naturally driving-power dependent; therefore, we used the maximum driving power allowed in the measurement setup. It turned out that the driving amplitude was still in the linear regime (see below). Figure 2(a) shows $\Omega_{12}/\hbar$ as a function of $\delta\Phi_3$. The flux biases of qubits 1 and 2

![Figure 2](image.png)

**Figure 2.** On-state and off-state coupling as functions of the coupler flux bias $\delta\Phi_3$. (a) Rabi frequency $\Omega_{12}/\hbar$ of the sum-frequency transition. The solid line is the calculated value assuming $\delta\Phi_3^{ac} = 0.52\ m\Phi_0$. (b) Residual coupling $J_{zz}^{res}$ in the off state. The solid line is the calculated value. (c)(d) Calculated $\Omega_{12}/\hbar$ for $\delta\Phi_3^{ac} = 1\ m\Phi_0$ and $J_{12} = 0$ (solid line; right axis) and $J_{zz}^{res}$ for different values of $J_{12}$ (others; left axis). $\Delta_3/\hbar = 10\ GHz$ in (c) and $\Delta_3/\hbar = 15\ GHz$ in (d). Other parameters used here are the same as those in the caption of Fig. 1.
are kept at the degeneracy points. Note that the degeneracy points are slightly shifted from \(\delta\Phi_1 = \delta\Phi_2 = 0\) due to the interaction with qubit 3. We find that the non-monotonic dependence is well fitted with the calculated value \(\langle 00|\partial H/\partial\Phi_3|11\rangle\delta\Phi_3^{ac}\) by assuming the driving flux amplitude of \(\delta\Phi_3^{ac} = 0.52\ m\Phi_0\). The driving is still in the linear regime as the amplitude is small compared to the peak width in Fig. 2(a).

In the absence of the microwave drive, the coupling should be ideally zero. However, the effective coupling (Eq.(2)) is not really zero, as the compensation between the direct and indirect coupling terms is not complete. Indeed, \(J_{12}\) is supposed to be negligible in the design. Therefore, the indirect coupling term would give a residual coupling in the form of \(+ J_{xx}^{zz}\sigma_1^x\sigma_z^2\). This off-diagonal coupling, for example, shifts the eigenstates |01\rangle and |10\rangle by \(\pm (J_{xx}^{zz})^2 / |\Delta_1 - \Delta_2| \approx 7.5\ MHz\), respectively. The energy shift is relatively large as \(|\Delta_1 - \Delta_2|\) is rather small in this sample. However, the effect can be considered as a renormalization of single-qubit frequencies in the case of two-qubit experiments. For a larger system, we have to carefully suppress the \(J_{xx}^{zz}\) term [6].

We also found that there existed a residual coupling term in the form of \(- J_{xx}^{zz}\sigma_1^x\sigma_z^2\). The residual coupling strength is determined by \(J_{xx}^{zz} = (E_{300-11} - E_{300-01} - E_{000-10})/2\). Using spectroscopy in the weak power limit, we measured the resonant frequencies \(E_{300-10}/h\) of qubit 1 by using SQUID 1 and \(E_{000-01}/h\) of qubit 2 by using SQUID 2. The sum frequency \(E_{300-01}/h\) was measured as well by either SQUID 1 or 2. The obtained \(J_{xx}^{zz}\) is plotted in Fig. 2(b). It also shows non-monotonic dependence on \(\delta\Phi_3\) and is reproduced well by diagonalizing Eq.(1) without any free parameters. Note that here we had to refine the parameters in the caption of Fig. 1 within the error bars by doing finer spectroscopy around the degeneracy point.

The on/off ratio defined as \(|\Omega_{12}/J_{xx}^{zz}|\) is at best \(\sim 10\) in this sample. To improve the on/off ratio, sample parameters have to be optimized. In Figs. 2(c) and (d) we plot simulated \(\Omega_{12}\) and \(J_{xx}^{zz}\) for larger \(\Delta_3\), while keeping other parameters same as in the present sample. The residual coupling \(J_{xx}^{zz}\) are also plotted for different values of \(J_{12}\), while \(\Omega_{12}\) is insensitive to \(J_{12}\). With increasing \(\Delta_3\), the on/off ratio is rapidly increased. Additional direct coupling \(J_{12}\) also helps to reduce \(|J_{xx}^{zz}|\) at the coupler bias where \(\Omega_{12}\) has a maximum. As an expense of the large \(\Delta_3\), the maximum \(\Omega_{12}\) decreases. To achieve reasonably fast two-qubit gates, it has to be enhanced by using larger \(J_{kl}\) and/or larger driving amplitude \(\delta\Phi_{3}^{ac}\). Indeed, a larger \(\Delta_3\) results in a less nonlinearity in the coupler and allows larger \(\delta\Phi_{3}^{ac}\), as seen in the broad peak of \(\Omega_{12}\) in Fig. 2(d).

4. Conclusion
In conclusion, we have studied the parametrically-induced tunable coupling scheme between two flux qubits biased at their degeneracy points. The on-state and off-state couplings depended on the flux bias of a coupler qubit. The dependence agreed quantitatively with the calculations based on the three-qubit Hamiltonian. This assured us the validity of the model, which led us to an expectation of a large on/off ratio with a reasonable assumption of parameters.

References
[1] Niskanen A O, Harrabi K, Yoshihara F, Nakamura Y, Lloyd S and Tsai J S 2007 Science 316 723
[2] Sillanpää M A, Park J I and Simmonds R W, Nature 2007 (London) 449 438
[3] Majer J et al. Nature 2007 (London) 449 443
[4] Niskanen A O, Nakamura Y and Tsai J S 2006 Phys. Rev. B 2006 73 094506
[5] Chiorescu I, Nakamura Y, Harmans C J P M and Mooij J E 2003 Science 299 1869
[6] Ashhab S, Niskanen A O, Harrabi K, Nakamura Y, Picot T, de Groot P C, Harmans C J P M, Mooij J E and Nori F 2008 Phys. Rev. B 77 014510