We study the constraints on non-flavour-blind soft supersymmetry breaking terms coming from flavour and CP violating processes in the presence of hierarchical Yukawa couplings, and quantify how much these constraints are weakened in the regions of the MSSM parameter space characterized by heavy gauginos and multi-TeV sfermion masses, respectively. We also study the inverted sfermion mass hierarchy scenario in the context of D-term supersymmetry breaking, and show that generic hierarchical Yukawa couplings with arbitrary phases require first generation squarks in the few 10 TeV range.

1 Introduction

In contrast with the successful predictions of the Standard Model, supersymmetry does not guarantee suppressed flavour changing neutral current (FCNC) and CP violating processes. Instead the predictions of supersymmetric models depend on two sets of unknown parameters, the soft supersymmetry breaking terms and the Yukawa coupling matrices. Arbitrary phases and flavour structures in these parameters would lead to unacceptably large contributions to FCNC and CP violating observables: this is the well-known supersymmetric flavour problem. At the phenomenological level, there are well-known radical possibilities that make the above mentioned dependence trivial, and the Standard Model predictions are recovered: flavour universal soft terms\(\uparrow\), or alignment of the sfermion and fermion mass matrices\(\downarrow\). However, the mechanism of spontaneous supersymmetry breaking and its transmission to the visible sector is unknown and ultimately may not be consistent with any of these options (measurements of the supersymmetric mass spectrum at the LHC may tell us).

It is therefore useful to study the dependence of the FCNC and CP constraints on the supersymmetric mass spectrum and on the pattern of Yukawa couplings. In this work, we

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\(\downarrow\)Unité de Recherche associée au CNRS (URA 2306).
perform such an analysis in the framework of supergravity-mediated supersymmetry breaking and in the presence of hierarchical Yukawa couplings, without insisting on keeping a moderate fine-tuning in the Higgs potential. In particular, we quantify and compare the FCNC and CP problems in two regions of the supersymmetric parameter space where they are expected to be less stringent: the region with sub-TeV (GUT-scale) sfermion masses but possibly heavy gauginos, on the one hand, and the region with sfermion masses in the multi-TeV range, on the other hand. We also study the so-called “inverted hierarchy” scenario, with heavy first two generation sfermions and lighter third generation sfermions, which has been put forward as a way to suppress the most dangerous flavour and CP violating processes while keeping the fine-tuning at an acceptable level. We investigate the predictions of this scenario for FCNC processes in a specific inverted hierarchy model with $D$-term supersymmetry breaking.

2 FCNC and CP constraints: heavy gauginos versus heavy sfermions

The goal of this section is to quantify and compare the FCNC and CP problems in the two regions of the supersymmetric parameter space where the (GUT-scale) sfermion masses are in the sub-TeV range but the gauginos are relatively heavy, and where the sfermion masses are in the multi-TeV range, respectively.

Let us first specify our assumptions about soft sfermion masses and Yukawa couplings. In the Yukawa sector, we consider matrices of the hierarchical type, for which the smallness of the charged fermion mass ratios result from a hierarchy among Yukawa couplings rather than from cancellations between large entries in the Yukawa matrices. For definiteness, we shall use in our subsequent numerical study one of the quark and lepton Yukawa textures that were analyzed in Ref.

These matrices are associated with the spontaneous breaking, close to the GUT scale, of a horizontal $U(1)$ symmetry and have the following structure: $Y_u^{AB} = C_u^{AB} \epsilon^{q_A + \bar{q}_B + h_u}$, $Y_d^{AB} = C_d^{AB} \epsilon^{q_A + \bar{d}_B + h_d}$ and $Y_e^{AB} = C_e^{AB} \epsilon^{q_A + \bar{e}_B + h_d}$, where the $C_{u,d,e}^{AB}$ are arbitrary complex coefficients of order one, $\epsilon \ll 1$ is a small parameter associated with the spontaneous breaking of the horizontal symmetry, and $q_A, \bar{u}_A, \bar{d}_A, l_A, \bar{e}_A, h_u, h_d$ stand for the horizontal charges (assumed to be positive) of the MSSM superfields $Q_A, U_A^c, D_A^c, L_A, E_A^c, H_u, H_d$. Specifically, we choose model 1 from Ref.

with horizontal charges $q_A = \bar{u}_A = \bar{d}_A = (3, 2, 0)$, $l_A = \bar{d}_A = (4, 2, 2)$ and $h_u = h_d = 0$. Since the symmetry breaking parameter $\epsilon$ turns out to be very close numerically to the Cabibbo angle $\lambda \simeq 0.22$, we set $\epsilon = \lambda$. The associated quark and charged lepton Yukawa matrices then read:

$$Y_u \sim y_t \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d \sim y_b \begin{pmatrix} \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \end{pmatrix}, \quad Y_e \sim y_t \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (1)$$

where the symbol $\sim$ reminds us that an order one complex factor is understood in each entry of the matrices $Y_{u,d,e}$. In practice, we shall use randomly generated sets of order one coefficients $\{C_u^{AB}, C_d^{AB}, C_e^{AB}\}$ that fit the measured values of the quark and lepton masses and mixings, with renormalization group evolution of the Yukawa couplings between the GUT scale and the weak scale taken into account.

In the neutrino sector, this charge assignment also yields correct agreement with oscillation data upon adjusting the relevant order one parameters.

In the supersymmetry breaking sector, we do not rely on any specific model and make only mild assumptions about the flavour structure of the soft terms. In particular, we consider soft sfermion masses with both splittings among entries on the diagonal and non-vanishing off-diagonal entries (suppressed by some power of $\lambda$):

$$m_{Q_i}^2, m_{U_i}^2, m_{D_i}^2, m_{L_i}^2, m_{E_i}^2 = \begin{pmatrix} m_1^2 & O(m_2^2 \lambda^n) & O(m_2^2 \lambda^n) \\ O(m_2^2 \lambda^n) & m_2^2 & O(m_2^2 \lambda^n) \\ O(m_2^2 \lambda^n) & O(m_2^2 \lambda^n) & m_3^2 \end{pmatrix}, \quad (2)$$
where, unless otherwise stated, $m_1^2, m_2^2, m_3^2 \sim m^2$, and we have assumed the same level of suppression for all off-diagonal entries. For the A-terms, we assume the following flavour structure:

$$A^{AB}_u = a^{AB}_u \lambda^{a+\bar{a}+b +h_A} A_0, \quad A^{AB}_d = a^{AB}_d \lambda^{a+\bar{a}+\bar{b}+h_d} A_0, \quad A^{AB}_e = a^{AB}_e \lambda^{a+\bar{e}+b +h_d} A_0,$$

(3)

where $A_0$ is a mass scale and the $a^{AB}_{u,d,e}$ are complex order one coefficients. While compatible with the horizontal symmetry manifest in the fermion sector, this ansatz departs from the standard proportionality assumption $A^{AB} = A_0 Y^{AB}$, which would correspond to $a^{AB}_{u,d,e} = C^{AB}_{u,d,e}$.

To estimate the contributions to flavour and CP violating processes due to the hierarchical mass matrices $\mathbf{U}$, $\mathbf{D}$, we use the standard mass insertion parameters in (here in matrix form):

$$\delta_{LL}^d \equiv \frac{D_L m_d^2}{m_d^2}, \quad \delta_{RR}^d \equiv \frac{D_R m_d^2}{m_d^2}, \quad \delta_{LR}^d \equiv \frac{D_L \bar{m}_d^2}{m_d^2} D_R^\dagger,$$

(4)

where $D_L$ (resp. $D_R$) is the rotation that brings the left-handed (resp. right-handed) down quarks to their mass eigenstate basis, $\bar{m}_d^2 = (A_d - \mu \tan \beta Y_d) \nu_d$, and $m_d$ is an average down quark mass. The diagonal terms, irrelevant to flavour violation, have been omitted in $\delta_{LL}^d$ and $\delta_{RR}^d$. Off-diagonal entries of order $\lambda^n$ in the squark soft mass matrices will give contributions of order $\lambda^n$ to the corresponding $(\delta_{LL}^d)^{AB}$'s and $(\delta_{RR}^d)^{AB}$'s, while the contribution of splittings among diagonal entries strongly depends on the magnitude of the mixing in the Yukawa textures. For example, $m_1^2 \neq m_2^2$ in $m_d^2$ gives a contribution $D_L^{12} D_R^{2*} (m_2^2 - m_1^2)/m_d^2 \sim \lambda (m_2^2 - m_1^2)/m_d^2$ to $(\delta_{LL}^d)^{12}$. Finally, Eq. 3 gives $(\delta_{LR}^d)^{AB} \sim (A_0 v_d/m_2^2) \lambda^{a+\bar{a}+\bar{b}+h_d} (A \neq B)$ and $d |\text{Im}(\delta_{LR}^d)^{AB}| \lesssim |A_0| m_{d_A}/m^2$. Analogous quantities $(\delta_{MN}^q)^AB$ and $(\delta_{MN}^d)^AB$, where $M,N = L$ or $R$ and $\delta_{LL}^d$ and $\delta_{RR}^d$ are defined in the up squark and slepton sectors, respectively.

Let us discuss in greater detail the expected magnitude of the FCNC processes induced by splittings among diagonal entries of the sfermion soft mass matrices. The diagonalization of the Yukawa matrices yields the following hierarchical structures for the rotations that bring the quarks and charged leptons to their mass eigenstate basis:

$$U_L, D_L, U_R, E_R \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad D_R, E_L \sim \begin{pmatrix} 1 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{pmatrix}.$$

(5)

Let us first consider the implications of Eq. 3 in the down squark sector. Since $D_L$ has the same hierarchical structure as the CKM matrix $V = U_L^T D_L$, the order of magnitude of the dominant contribution to $(\delta_{LL}^d)^{AB}$ is:

$$(\delta_{LL}^d)^{AB} \sim V^{AB} (m_2^2 - m_A^2)/m^2,$$

(6)

Hence $m_A^2 \neq m_B^2$ induces a large $(\delta_{LL}^d)^{12}$, but a smaller $(\delta_{LL}^{12})^{13}$ or $(\delta_{LL}^{23})^{13}$. Furthermore, $(\delta_{RR}^d)^{12}$ and $(\delta_{RR}^d)^{13}$ are of order $\lambda^2 (m_2^2 - m_A^2)/m^2$, while $(\delta_{RR}^{12})^{23} \sim (m_2^2 - m_3^2)/m^2$. We therefore expect large contributions from splittings among diagonal entries in the squark mass matrices to $\Delta m_K$ and to $\epsilon_K$ (since large phases are generally present in the rotation matrices $D_L$ and $D_R$), while the contributions to $B^0_d - \bar{B}^0_d$ mixing should be suppressed. Large effects are also expected in $b \to s$ transitions, due to $(\delta_{RR}^d)^{23}$, as well as in $D^0 - \bar{D}^0$ mixing (including CP violating effects, which are absent in the Standard Model) and in $\mu \to e\gamma$, since Eq. 5 implies $U_L^{12} \sim U_L^{12} \sim E_R^{12} \sim D_L^{12} \sim \lambda$, as well as $E_L^{12} \sim D_R^{12} \sim \lambda^2$. Finally, since $E_R^{23} \sim D_R^{23} \sim 1$, $\tau \to \mu\gamma$ may also receive sizeable contributions from splittings among the diagonal entries of $m_E^2$.

\(^{\text{a}}\text{Needless to say, a non-vanishing } (1, 3)\text{ entry in } m_3^2, \text{ for instance, also gives a contribution to } (\delta_{LL}^d)^{12}, \text{ but with an additional suppression by the rotation angle } D_L^{23}.\)

\(^{\text{b}}\text{We assume here that the phases of the common gaugino mass and of the } \mu \text{ parameter can be simultaneously rotated away.}\)
Figure 1: $\Delta m_K$ (top left), $\text{BR}(\mu \to e\gamma)$ (top right), $\epsilon_K$ (bottom left) and $\Delta m_D$ (bottom right), as a function of the GUT-scale sfermion mass parameter $m$, assuming $m_1 = m_2 = m_3 \equiv m$, off-diagonal entries of order $\lambda^3$ in the sfermion mass matrices, $A_0 = 0$ and (a) $M_{1/2} = 400$ GeV; (b) $M_{1/2} = 1000$ GeV. Order one coefficients in the Yukawa matrices and in the off-diagonal entries of the sfermion mass matrices are varied in the range $[0, 3]$.

So far our discussion has been qualitative and has ignored the effect of the renormalization group evolution on soft supersymmetry breaking terms, as well as the dependence of the flavour and CP violation observables on the superparticle mass spectrum. We now move to the quantitative study of a few observables in the low ($\lesssim 1$ TeV) and the high ($5-50$ TeV) sfermion mass regions for various values of the GUT-scale parameters $A_0$ and $M_{1/2}$, $|\mu|$ and $B\mu$ being determined from radiative electroweak symmetry breaking. Our goal is to compare the FCNC and CP problems in these two regions for the hierarchical Yukawa couplings $\lambda^3$. In order to do this, we scan over the different flavour and CP violating (GUT-scale) parameters in the sfermion sector: (i) the splittings $m_2^2 - m_1^2$ among diagonal entries of the soft sfermion mass matrices; (ii) the off-diagonal entries of the soft sfermion mass matrices (in practice the complex order one coefficients multiplying some fixed power of $\lambda$); (iii) the complex order one coefficients $a_{u,d,e}^{AB}$ in the $A$-terms, Eq. (3). Order one parameters are scanned in the range $[0, 3]$. We also vary the Yukawa couplings by using 100 randomly generated sets of the order one coefficients $C_{u,d,e}^{A,B}$ fitting the measured values of the quark and lepton masses and mixing angles.

Let us first consider off-diagonal entries of order $\lambda^3$. Fig. 1 shows $\Delta m_K$, $\epsilon_K$, $\text{BR}(\mu \to e\gamma)$ and $\Delta m_D$ as a function of the GUT-scale sfermion mass parameter $m$, assuming $m_1 = m_2 = m_3 \equiv m$, off-diagonal entries of order $\lambda^3$ in the sfermion mass matrices, $A_0 = 0$ and $M_{1/2} = 400$ GeV and 1000 GeV, respectively. Not surprisingly, the most severe constraints come from $\epsilon_K$ and $\mu \to e\gamma$. For $M_{1/2} = 400$ GeV, the scatter plots for $\Delta m_K$, $\epsilon_K$ and $\Delta m_D$ look approximately symmetric around $m = 1$ TeV, where these observables reach a maximum. Above 1 TeV, the effect of decoupling the squarks running in the loop is clearly visible: $\Delta m_K$, $\epsilon_K$ and $\Delta m_D$ decrease as $1/m^2$, as expected. This effect is practically insensitive to the value of $M_{1/2}$. Below 1 TeV, on the other hand, one can see the “aligning effect” of the gluino mass on the squark mass matrices$^9$ (for a recent discussion of this effect, see Ref. 10). Indeed, the one-loop renormalization

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$^6$We do not include in this list the right-handed neutrino couplings needed for the seesaw mechanism, which are known to induce large flavour violations in the slepton sector through radiative corrections$^3$. In fact our charge assignment suppresses the corresponding contribution to the $l_i \to l_j \gamma$ amplitude by a factor $\lambda_i^{l_j + l_j + 2\bar{n}_3}$, where $\bar{n}_3$ is the horizontal charge of the third generation right-handed neutrino; for $\mu \to e\gamma$, this is already $\lambda^6 \sim 10^{-4}$ for $\bar{n}_3 = 0$, which can safely be neglected.
group equations of squark masses receive a large and negative contribution from the gauginos, whose effect is to enhance the diagonal entries of the squark mass matrices by a universal piece. Schematically, one has:

$$16\pi^2 \frac{dm_{2Q,U,D}^2}{d\ln \mu} = (\text{Yukawas}) - \sum_i c_i g_i^2 M_i^2 \mathbb{1}_{3\times3} + \cdots$$  \hspace{1cm} (7)$$

For the first two generations, the Yukawa couplings can be neglected, implying $m_{2Q}(M_Z) \approx m_{A}^2(M_{\text{GUT}}) + c M_{1/2}^2 (A = 1, 2)$, with $c \approx (6 - 7)$. Hence, $(\delta_{LL}^{d})^{12}$ is suppressed with respect to its “GUT-scale value” $\lambda^3$: $(\delta_{LL}^{d})^{12} \approx m^2 \lambda^3/(m^2 + c M_{1/2}^2)$, and this effect is more important for larger values of the ratio $M_{1/2}/m$. This is illustrated in the two sets of plots of Fig. 1, which only differ by the value of $M_{1/2}$: the supersymmetric contributions to $\Delta m_K$ and $\epsilon_K$ are under much better control, in the low $m$ region, for $M_{1/2} = 1000$ GeV than for $M_{1/2} = 400$ GeV; in both case, $\Delta m_K$, $\epsilon_K$ and $\Delta m_D$ decrease quickly when $m$ drops below 1 TeV. Note that $m = 200$ GeV and $M_{1/2} = 400$ (1000) TeV correspond to $m_{31/2} \approx 1$ (2.5) TeV and $M_{3} \approx 1.2$ (3) TeV; thus a GUT-scale sfermion mass parameter $m$ in the few 100 GeV range does not necessarily mean light squarks. In the lepton sector, the aligning effect is much milder, since $c \approx 0.5$ (0.15) for $m_{1E}^2$ ($m_{2E}^2$). It follows that, for moderate values of $M_{1/2}$, the $\mu \rightarrow e\gamma$ constraint becomes more and more severe when $m$ decreases; for large values of $M_{1/2}$, however, $\text{BR}(\mu \rightarrow e\gamma)$ start decreasing when $m$ drops below 1 TeV.

Let us now consider a splitting between the first two diagonal entries of the sfermion mass matrices. Fig. 2 shows $\Delta m_K$, $\epsilon_K$, $\text{BR}(\mu \rightarrow e\gamma)$ and $\Delta m_D$ as a function of the GUT-scale sfermion mass parameter $m$, assuming $m_2 = m_3 \equiv m$ and scanning over $m_1$ in the range $[0.5m, m]$, with vanishing off-diagonal entries in the sfermion mass matrices, $A_0 = 0$ and $M_{1/2} = 400$ GeV and 800 GeV, respectively. One can observe qualitatively similar effects to the case of non-vanishing off-diagonal entries in the low and high $m$ regions. However, as already mentioned above, there is a substantial difference between the two sources of flavour violations. While the contribution of off-diagonal entries to FCNC processes only weakly depends on the Yukawa couplings, especially if the latter are of the hierarchical type, this is no so for the contribution of splittings between the diagonal entries. For the “generic” hierarchical case considered in this paper, $U_L$ and $D_L$ have the same hierarchical structure as the CKM matrix. In particular, $D^{12}_L$ is of order the
Cabibbo angle and gives a large $(\delta_{LL}^{d})^{12}$; furthermore, $D_{R}^{12}$ is only suppressed by an additional factor of $\lambda$ relative to $D_{L}^{12}$ (remember that the strongest constraints from the kaon sector are on the combination $\frac{\lambda_{2}}{g_{1}}(\delta_{LL}^{d})^{12}(\delta_{RR}^{d})^{12}$). Yukawa textures with suppressed mixing among down-type quarks (either in the left-handed or in the right-handed sector, or in both) would yield a much smaller contribution to $\Delta m_{K}$ and $\epsilon_{K}$, thus softening the constraint on the $m_{1}^{2} - m_{2}^{2}$ splitting in the squark sector. Similarly, a much stronger hierarchical structure for $Y_{e}$, with the large lepton mixing angles coming solely from the neutrino sector, would at least partially relax the strong constraints from lepton flavour violating processes such as $\mu \rightarrow e\gamma$.

Finally, non-zero $A$-terms have little impact on FCNCs, due to their proportionality to Yukawa couplings; their main effects show up in the electric dipole moments (EDMs) of the neutron and of the charged leptons. For $A_{0} \sim m$ and $M_{1/2} = 400$ GeV, the contribution of $|\text{Im}(\delta_{LR}^{d})|^{11}$ to the neutron EDM is under control, while the contribution of $|\text{Im}(\delta_{LR}^{u})|^{11}$ provides a strong constraint (this is however mainly due to the fact that the charge assignment predicts $A_{u}^{11}/A_{0} \propto \lambda^{6} \gg m_{u}/m_{t}$). Still this constraint becomes weaker at high $m$ as well as for large values of $M_{1/2}$ at low $m$.

We conclude that there are two regions in the (GUT-scale) MSSM parameter space where the constraints associated with flavour and CP violating processes are significantly weakened, both in the quark and lepton sectors: (i) a low sfermion mass / high gaugino mass region, with $m \lesssim 500$ GeV and $M_{1/2} \gtrsim 800$ GeV; and (ii) a high sfermion mass region, with $m \gtrsim 10$ TeV and practically no constraint on $M_{1/2}$. It is essentially flavour violation in the lepton sector that requires high values of $M_{1/2}$ at low $m$. Unless $m$ or $M_{1/2}$ are pushed towards very large values, however, a strong suppression of off-diagonal entries in the sfermion mass matrices is still needed, especially in the 1-2 sector. Splittings among the diagonal entries are also constrained, but the allowed level of non-degeneracy strongly depends on the Yukawa structure.

### 3 Inverted sfermion mass hierarchy and $D$-term supersymmetry breaking

However, if the supersymmetric FCNC and CP problems appear to be less stringent in the previous regions, they are less appealing from the point of view of naturalness – not even mentioning the possibility of detecting superpartners at the LHC. As is well-known indeed, in the MSSM the weak scale is determined by the following relation:

$$
\frac{1}{2} M_{Z} = -\mu^{2} + \frac{m_{H_{u}}^{2} - m_{H_{d}}^{2} \tan^{2} \beta}{\tan^{2} \beta} \simeq -\mu^{2} - m_{H_{u}}^{2},
$$

where the last equality holds for $\tan \beta \gtrsim 10$. Naturalness requires the absence of fine-tuned cancellations in the right-hand side of \[ Eq. \[\text{8}\]. Since $m_{H_{u}}$ receives large corrections from third generation sfermions and gluinos, these should be light enough to keep fine-tuning in the Higgs potential still “reasonable” (say at the level of 1%). The naturalness criterion then appears to select the region of the parameter space where the FCNC problem is maximized.

There is, however, a potential compromise, namely the so-called “inverted hierarchy” scenario, with heavy first two generation squarks and sleptons in order to suppress FCNC and CP violating processes, and light third generation sfermions and gluinos, in order to maintain the fine-tuning at an acceptable level. In this section, we study a specific realization of this scheme in the framework of gauge anomalous horizontal $U(1)$ models.

Before doing so, let us review the constraints that apply to the inverted hierarchy scenario. As first pointed out in Ref. \[14\], large two-loop renormalization group effects associated with the heavy first scalar generations could drive the squared masses of the third generation squarks and sleptons negative. Schematically, one has:

$$
16\pi^{2} \frac{d m_{3}^{2}}{d \ln \mu} = 6 y_{3}^{2} m_{3}^{2} - \sum_{i} c_{i} g_{i}^{2} M_{i}^{2} + \frac{1}{16\pi^{2}} \sum_{i} c_{i}' g_{i}^{4} m_{2}^{2} + \cdots
$$

(9)
Figure 3: Exclusion curves in the \((m, m_3/m)\) plane (assuming \(m_1 = m_2 \equiv m\)) corresponding to proper radiative electroweak symmetry breaking (solid line) and to the requirement that the physical third generation sfermion masses, as well as the Higgs mass, do not fall below the experimental limit (dashed line).

This contribution has to be compensated for by the contribution of gauginos and by the initial value of the third generation mass \(m_3\). Avoiding tachyonic scalars then implies a lower bound on the GUT-scale ratio \(m_3/m\) (where \(m_1 \approx m_2 \approx m\)) as a function of \(M_{1/2}\) and \(m\). An even stronger bound is obtained, for large \(m\) values, by requiring that the lightest Higgs mass be larger than its experimental limit. We point out that actually a more stringent constraint comes from the requirement of correct electroweak symmetry breaking. This is shown in Fig. 3, in which one can see that, for \(m \gtrsim 5\) TeV, the ratio \(m_3/m\) is constrained to be larger than \(0.2 - 0.3\) for radiative electroweak symmetry breaking to be possible. A smaller value of \(M_{1/2}\) (which has been set to 400 GeV in Fig. 3) would not change this lower bound, but it would also apply to lower values of \(m\).

Let us now consider an explicit realization\(^{15}\) of the inverted hierarchy scenario (see also Ref.\(^{16}\)) based on \(D\)-term supersymmetry breaking\(^{17}\) in the framework of gauge anomalous horizontal \(U(1)\) models.\(^{f}\) Neglecting corrections associated with non-canonical kinetic terms from the Kähler potential, we can write e.g. the squark doublet soft supersymmetry breaking masses as:

\[
(m_Q^2)^{AB} = q_A \delta^{AB} m_D^2 + C_Q^{AB} \lambda^{q_A - q_B} m_F^2, \tag{10}
\]

and similarly for the other squark and slepton soft masses. The first term in Eq. (10) is the contribution of the anomalous \(D\)-term (with \(m_D^2 \equiv g < D >\)), and the second term the contribution of the \(F\)-terms. We assume a supergravity scenario leading to a hierarchy of the contributions \(m_F^2 \ll m_D^2\), with however \(m_F/m_D \geq 0.3\) in order to satisfy the constraints discussed above. The inverted hierarchy scenario is then straightforwardly implemented by assigning zero horizontal charges to the third generation superfields whose scalar components couple most strongly to the Higgs bosons, namely \(\tilde{t}_L, \tilde{t}_R\) and \(\tilde{b}_L\), and strictly positive horizontal charges to the other quark and lepton superfields\(^{15}\). This condition is fulfilled by the charge assignment used in section 2 to describe the quark and lepton masses and mixings.

In view of the FCNC problem, the main virtue of the \(D\)-term-induced inverted hierarchy scenario is to suppress the off-diagonal entries in sfermion mass matrices, since a suppression factor of \(m_F^2/m_D^2\) comes in addition to the power of \(\lambda\) associated with the breaking of the horizontal symmetry (this however assumes a canonical Kähler potential). The diagonal entries, however, are strongly split. Fig. 4 shows \(\Delta m_K, \epsilon_K, \text{BR}(\mu \rightarrow e\gamma)\) and \(\Delta m_D\) as a function of \(m_D\) for model 1 of Ref.\(^{4}\) assuming \(m_F/m_D = 0.3, A_0 = 0\) and \(M_{1/2} = 400\) GeV. In addition to the Yukawa couplings, the coefficients of the off-diagonal entries \(C_{Q,U,D,L,E}^{AB}\) in the sfermion mass matrices are varied randomly in the range \([0.3, 3]\). Supersymmetric contributions to \(\Delta m_K, \text{BR}(\mu \rightarrow e\gamma)\) and \(\Delta m_D\) are under control in the region \(m_D \gtrsim 5\) TeV; however, \(\epsilon_K\) still requires

\(^{f}\)The fine-tuning issue in such a scenario has been discussed in Ref.\(^{13}\).
values of $m_D$ in the several tens of TeV range, corresponding to even larger values of the sfermion masses (one has e.g. $m_{Q_1} = m_{U_1} = 1.7m_D$ and $m_{D_1} = 2m_D$). This is due to the fact that phases in the Yukawa couplings and in off-diagonal entries are taken to be random in our analysis, and can therefore be large. Also, due to the 2-loop renormalization group effect mentioned above, the stops, left sbottom and right stau are light and can give significant contributions to 1 − 2 flavour changing processes due to the flavour mixing present in the fermion and sfermion mass matrices. This also implies that some scalar superpartners could be accessible at the LHC, even though the first two generations are very heavy.

4 Discussion and conclusions

We have studied the contraints on non-flavour-blind soft supersymmetry breaking terms coming from FCNC and CP violating processes in the presence of hierarchical Yukawa couplings. These constraints are significantly weakened in the “low” sfermion mass / high gaugino mass region, with $m \lesssim 500$ GeV and $M_{1/2} \gtrsim 800$ GeV (where both $m$ and $M_{1/2}$ are GUT-scale parameters), as well as in the high sfermion mass region, with $m \gtrsim 10$ TeV and essentially no restriction on $M_{1/2}$. Unless $m$ or $M_{1/2}$ are pushed towards very large values, however, a strong suppression of off-diagonal entries in the sfermion mass matrices is still needed, especially in the 1-2 sector. Splittings among the diagonal entries are also constrained, but the allowed level of non-degeneracy strongly depends on the Yukawa structure. Quark textures for which the CKM matrix comes mainly from the up quark sector (see e.g. Ref. [19]), while the mixing in the left-handed down quark sector is strongly suppressed, would allow for larger splittings than the “generic” hierarchical textures that we have considered. A similar statement can be made about the charged lepton Yukawa texture.

We have also studied the inverted sfermion mass hierarchy scenario in the framework of gauge anomalous horizontal $U(1)$ models with $D$-term supersymmetry breaking. In such a scheme, the diagonal entries of the sfermion mass matrices are strongly split, but the off-diagonal entries are suppressed. FCNC and CP constraints can be satisfied at the price of very heavy first generation squarks, with masses in the few 10 TeV range. However, the situation could improve with different quark Yukawa textures arising e.g. from less minimalistic anomalous $U(1)$ models.
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