Compositional and Abstraction-Based Approach for Synthesis of Edit Functions for Opacity Enforcement

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Abstract—This paper develops a novel compositional and abstraction-based approach to synthesize edit functions for opacity enforcement in modular discrete event systems. Edit functions alter the output of the system by erasing or inserting events in order to obfuscate the outside intruder, whose goal is to infer the secrets of the system from its observation. We synthesize edit functions to solve the opacity enforcement problem in a modular setting, which significantly reduces the computational complexity compared with the monolithic approach. Two abstraction methods called opaque observation equivalence and opaque bisimulation are first employed to abstract the individual components of the modular system and their observers. Subsequently, we propose a method to transform the synthesis of edit functions to the calculation of modular supremal nonblocking supervisors. We show that the edit functions synthesized in this manner correctly solve the opacity enforcement problem.

I. INTRODUCTION

Opacity characterizes whether the integrity of the secrets of a system can be preserved from the inference of an outside intruder, potentially with malicious purposes. The intruder is modeled as a passive observer with knowledge of the system’s structure. A system is called opaque if the intruder is unable to infer the system’s secrets from its observation.

Starting with [2], [3] in the computer science literature, opacity has been extensively studied, especially in the field of discrete event systems (DES), under multiple frameworks.

For finite state automaton models, various notions of opacity have been studied, e.g., language-based opacity [22], current-state opacity [24], initial-state opacity [26], K-step opacity [29] and infinite-step opacity [33]. Opacity has also been discussed in some other system models, like infinite state systems [6], modular systems [24] and Petri nets [42], [43]. Opacity under a special observer called Orwellian observer is discussed in [30] and opacity under powerful attackers is studied in [14]. A more recent work [51] investigates opacity for networked supervisory control systems. Furthermore, some works investigate opacity in stochastic settings, e.g., [1], [7], [21], [45]. Specifically, [52] presents a novel approach to tackle infinite-step and K-step opacity in stochastic DES. The survey paper [16] summarizes some recent results on opacity in DES.

When opacity does not hold, it is natural to study its enforcement [10]. One popular approach is supervisory control [8], [9], [35], [41], [48], where some behaviors of the system are disabled before they reveal the secrets. Another widely-applied method is sensor activation [5], [50], [53], where the observability of events is dynamically changed.

Recently, a new enforcement method called insertion function was proposed in [46], which inserts fictitious events into the output of the system to obfuscate the intruder. The authors of [18] extended the method to study opacity enforcement under the assumption that the intruder may or may not know the implementation of insertion functions, while [19] discussed opacity enforcement by insertion functions under quantitative constraints. As a following work, [47] investigates a more general method called edit functions, which manipulate the output of the system by either inserting or erasing events. Then [17], [20] considers the case when the edit function’s implementation is known to the intruder. As a summary and extension, [20] characterizes opacity enforcement by edit functions as a three-player game and proposes a novel information structure called three-player observer (TPO) to embed edit functions. A special TPO called the All Edit Structure (AES) is also introduced in [20] to characterize the edit constraints.

In this work, we elaborate the method in [20] to study opacity enforcement in a modular setting. Our motivation is as follows. To generate a three-player observer, the observer of the system needs to be calculated, which is potentially costly in computation. Furthermore, modern engineering systems usually contain multiple components that are synchronized and subject to malicious inference. In this sense, if we are to apply edit functions to enforce opacity, heavy computation is involved both from determining individual systems and synchronizing them, which may be potentially cumbersome.

To alleviate this issue, this paper applies a compositional and abstraction-based method to reduce the size of the modular system before calculating the All Edit Structure. Bisimulation and observation equivalence [25] are well-known methods to abstract the state space of an automaton, while they do not preserve opacity properties in general. As a variant, [24] proposes several innovative concepts termed opacity-preserving (bi)simulation relations to reduce the state space of the system in opacity verification. A compositional visible bisimulation equivalence method is discussed in [51] for abstraction-based opacity verification.

For abstraction, we introduce opaque observation equivalence and opaque bisimulation, which consider the secrecy status of states when merging them. In our framework, each individual system is abstracted using opaque observation equivalence. After that, the observer is calculated. Since abstraction reduces the size of the state space, the computational complexity of calculating the observer is lowered potentially. Next, opaque bisimulation is employed to the observer of each abstracted individual system, resulting in the smallest possible automaton for future discussion.
We further leverage some results from supervisor reduction and modular supervisor control theory to reduce the complexity of supervisor synthesis. There is a rich literature on both topics, see, e.g., \cite{23, 28, 37, 59, 40}. The main idea is to convert the construction of the monolithic All Edit Structure to a modular supervisory control problem. Specifically, we first transfer each individual three-player observer (without considering edit constraints) to its automaton form and view the set of interacting automata as the "plant" to be controlled. Then we put the edit constraint as the specification, also in an automaton form. Afterwards, we perform modular supervisory control to synthesize a least restrictive and nonblocking modular supervisor. It is shown that all the traces accepted by the supervisor represent valid edit decisions contained in the monolithic AES. Compared with the conventional monolithic approach for supervisor synthesis \cite{I}, our compositional approach is more efficient in computation.

The presentation of this work is organized as follows. Section \[II\] gives a brief background introduction about the system model, supervisory control theory and edit functions. The general idea of the paper is presented in Sect. [III] as a flow chart. Section [IV] explains the abstraction methods and synchronization of three-player observers. Next, Section [V] transforms the calculation of the monolithic All Edit Structure to the calculation of a modular supervisor. Finally, some concluding remarks are given in Section [VI].

A preliminary and partial version of this work appears in \cite{29}. The current work improves \cite{29} in the sense that \cite{29} only considers abstraction methods to synthesize edit functions in a monolithic setting, while this work also takes synchronous composition into consideration and the edit functions are synthesized by a modular approach.

II. MODELING FORMALISM AND BACKGROUND

A. Events, Automata and their Composition

In this work, we consider discrete event systems modeled as deterministic or nondeterministic automata.

Definition 1: A (nondeterministic) finite-state automaton is a tuple $G = (\Sigma, Q, \rightarrow, Q^0)$, where $\Sigma$ is a finite set of events, $Q$ is a finite set of states, $\rightarrow \subseteq Q \times \Sigma \times Q$ is the state transition relation, and $Q^0 \subseteq Q$ is the set of initial states. $G$ is deterministic if $|Q^0| = 1$ and if $x \rightarrow y_1$ and $x \rightarrow y_2$ always implies that $y_1 = y_2$.

When state marking is considered, the above definition is extended to $G = (\Sigma, \mathcal{Q}, \rightarrow, Q^0, Q^m)$, where $Q^m \subseteq Q$ is the set of marked states. In this paper, we identify marked states using gray shading in the figures.

We assume that the system is partially observed, thus the concepts of observable and unobservable events are introduced. Since the exact identity of unobservable events is irrelevant in our later discussion of opacity, they are uniformly represented by a special event $\tau$. The event $\tau$ is never included in the alphabet $\Sigma$, unless explicitly mentioned. For this reason, $\Sigma_\tau = \Sigma \cup \{ \tau \}$ is used to represent the whole set of observable and unobservable events. Hereafter, nondeterministic automata may contain transitions labeled by $\tau$, while deterministic automata never contain $\tau$ transitions. Moreover, $P_i : \Sigma^*_\tau \rightarrow \Sigma^*$ is the projection that removes from strings in $\Sigma^*_\tau$ all the $\tau$ events.

When automata are brought together to interact, lock-step synchronization in the style of \cite{15} is used.

Definition 2: Let $G_1 = (\Sigma_1, Q_1, \rightarrow, Q^0_1, Q^m_1)$ and $G_2 = (\Sigma_2, Q_2, \rightarrow, Q^0_2, Q^m_2)$ be two nondeterministic automata. The synchronous composition of $G_1$ and $G_2$ is defined as

$$G_1 \parallel G_2 := (\Sigma_1 \cup \Sigma_2, Q, \rightarrow, Q^0, Q^m)$$

where

$$(x_1, x_2) \rightarrow (y_1, y_2) \quad \text{if} \quad \sigma \in (\Sigma_1 \cap \Sigma_2),$$

$x_1 \rightarrow_1 y_1$, and $x_2 \rightarrow_2 y_2$;

$$(x_1, x_2) \rightarrow (y_1, y_2) \quad \text{if} \quad \sigma \in (\Sigma_1 \setminus \Sigma_2) \cup \{\tau\},$$

and $x_1 \rightarrow_1 y_1$;

$$(x_1, x_2) \rightarrow (y_1, y_2) \quad \text{if} \quad \sigma \in (\Sigma_2 \setminus \Sigma_1) \cup \{\tau\},$$

and $x_2 \rightarrow_2 y_2$.

Importantly, synchronous composition only imposes lock-step synchronization on common events from $\Sigma_1$ and $\Sigma_2$.

The transition relation of an automaton $G$ is written in infix notation $x \overset{\sigma}{\rightarrow} y$, and it is extended to strings in $\Sigma^*$ by letting $x \overset{\sigma}{\rightarrow} x$ for all $x \in Q$, and $x \overset{\sigma}{\rightarrow} z$ if $x \overset{\tau}{\rightarrow} y$ and $y \overset{\sigma}{\rightarrow} z$ for some $y \in Q$. Furthermore, $x \rightarrow y$ means that $x \overset{\tau}{\rightarrow} y$ for some $y \in Q$, and $x \rightarrow y$ means that $x \overset{\tau}{\rightarrow} y$ for some $t \in \Sigma^\tau$. These notations also apply to state sets, where $X \overset{\sigma}{\rightarrow} Y$ for $X, Y \subseteq Q$ means that $x \overset{\tau}{\rightarrow} y$ for some $x \in X$ and $y \in Y$, and to automata, where $G \overset{\sigma}{\rightarrow} \text{means that } Q^0 \overset{\sigma}{\rightarrow} (t \text{ is defined in } G)$ and $G \overset{\tau}{\rightarrow} x \text{ means } Q^0 \overset{\tau}{\rightarrow} x$.

For brevity, $p \overset{\sigma}{\rightarrow} q$ for $s \in \Sigma^*$ represents the existence of a string $t \in \Sigma^*$ such that $P_i(t) = s$ and $p \overset{\tau}{\rightarrow} q$. Thus, $q \overset{\sigma}{\rightarrow} p$ for $u \in \Sigma^*$ means a path containing exactly the events in $u$, while $q \overset{\tau}{\rightarrow} p$ for $u \in \Sigma^*$ means existence of a path between $p$ and $q$ with an arbitrary number of $\tau$ events between the observable events in $u$. Similarly, $p \overset{\tau}{\rightarrow} q$ means the existence of a string $t \in (\tau)^*$ such that $p \overset{\tau}{\rightarrow} q$.

The language of an automaton $G$ is defined as $L(G) = \{ s \in \Sigma^* \mid G \overset{\sigma}{\rightarrow} s \}$ and the language generated by $G$ from $q \in Q$ is $L(G,q) = \{ s \in \Sigma^* \mid q \overset{\sigma}{\rightarrow} s \}$, thus we do not include event $\tau$ in the language of an automaton. Moreover, we also introduce projections $P_i$ for $i = 1, 2$, which are $P_i : (\Sigma_1 \cup \Sigma_2)^* \rightarrow \Sigma_i^*$ for $i = 1, 2$.

For a nondeterministic automaton $G = (\Sigma_\tau, Q, \rightarrow, Q^0)$, the set of unobservable reached states of $B \in 2^\Sigma$, is $UR(B) = \{ C \subseteq Q \mid B \overset{\tau}{\rightarrow} C \}$. Its observer $det(G) = (\Sigma, X_{obs}, \rightarrow_{obs}, x^0_{obs})$ is a deterministic automaton, where $X^0_{obs} = UR(Q^0)$ and $X_{obs} \subseteq 2^\Sigma$, and $X \overset{\sigma}{\rightarrow}_{obs} Y$, where $X, Y \in X_{obs}$, if and only if $Y = \{ U \cap \{ s \overset{\sigma}{\rightarrow} y \mid s \in X \text{ and } y \in Q \} \}$. By convention, only reachable states from $X^0_{obs}$ under $\rightarrow_{obs}$ are considered in this paper. We also refer to the observer as the (current-state) estimator of the system while an observer state is referred to as (current-state) estimate.

A common automaton operation is the quotient modulo, which is an equivalence relation on sets of states.

Definition 3: Let $Z$ be a set. A relation $\sim \subseteq Z \times Z$ is called an equivalence relation on $Z$ if it is reflexive, symmetric, and transitive. Given an equivalence relation $\sim$ on $Z$, the
equivalence class of $z \in Z$ is $[z] = \{ z' \in Z \mid z \sim z' \}$, and $Z = \{ [z] \mid z \in Z \}$ is the set of all equivalence classes modulo $\sim$.

Definition 4: Let $G = (\Sigma, Q, \to, Q^0)$ be an automaton and let $x \subseteq Q \times Q$ be an equivalence relation. The quotient automaton of $G$ modulo $\sim$ is $G_\sim = (\Sigma, Q, \to/\sim, Q^0/\sim)$, where $\to/\sim = \{(x, \sigma, y) \mid x \sim y \}$ and $Q^0/\sim = \{ [x^0] \mid x^0 \in Q^0 \}$.

In order to compare automata structurally, we say that an automaton is a subautomaton of another automaton if all states and transitions in the first automaton are contained in the second one. Formally, we have the following definition:

Definition 5: Let $G_1 = (\Sigma_1, Q_1, \to_1, Q^0_1)$ and $G_2 = (\Sigma_2, Q_2, \to_2, Q^0_2)$ be two automata. $G_1$ is a subautomaton of $G_2$, denoted by $G_1 \subseteq G_2$, if $Q_1 \subseteq Q_2$, $\to_1 \subseteq \to_2$, $Q^0_1 \subseteq Q^0_2$, and $Q^0_1 \cap Q^0_2 = \emptyset$.

B. Supervisory Control Theory

Considering plant $G$ and specification $K$, supervisory control theory provides a method to synthesize a supervisor to restrict the behavior of the plant such that the given specification is always fulfilled. The supervisor $S$ is responsible for controlling the plant $G$ and restricting the final automaton to the remaining states and their associated transitions, for more details please see [4], [13], [44].

In this paper, we assume that the modular system has a set of interacting components $\{G_1, \ldots, G_n\}$, and there is also a set of supervisors in a modular structure, i.e., $\mathcal{S} = \{S_1, \ldots, S_n\}$. Here supervisor $S_i$ is responsible for controlling $G_i$. The set of modular supervisors may be synchronized as $\|_{i=1}^n S_i$.

C. Opacity and Edit Functions

In this work, we suppose system $G$ has certain secret information which is characterized by the set of states. Thus the state space is partitioned into two disjoint subsets: $Q = Q^S \cup Q^{NS}$ where $Q^S$ is the set of secret states capturing the secrets of the system, while $Q^{NS}$ is the set of non-secret states.

If the system $\mathcal{S}$ is modular, $\mathcal{S} = \{S_1, \ldots, S_2\}$, the set of secret states of the system, $Q_S$, is $Q^S = \{ (x_1, \ldots, x_n) \mid \exists x_i \in Q_i^S \}$.

Suppose there is an external intruder modeled as the observer of the system, which intends to infer the secrets of the system from its observation. Then a system is called opaque if the intruder is unable to determine unambiguously if the system has entered a secret state or not. Different notions of opacity have been introduced in literature and we focus on current-state opacity in this work.

Definition 9: A nondeterministic automaton $G$ with a set of secret states $Q^S$ is current-state opaque w.r.t. $Q^S$ if \( \forall s \in L(G, d^0) : Q^0 \not\Rightarrow Q^S \)

The system is current-state opaque if for any string reaching a secret state there is string with the same sequence of observable events reaching a non-secret state. It is known that current-state opacity can be verified by building the standard observer automaton.

Theorem 1: Let $G = (\Sigma, Q, \to, Q^0)$ be a nondeterministic automaton with set of secret states $Q^S$. Let $det(G) = (\Sigma, X_{obs}, \to_{obs}, X^0_{obs})$ be the current-state estimator of $G$. Then $G$ is current-state opaque w.r.t. $Q^S$ if and only if $[det(G) \xrightarrow{\epsilon} X]$ implies that $X \not\subseteq Q^S$.

If all states violating current-state opacity are removed from the observer $det(G)$, then the accessible part of the remaining structure is called the desired observer, denoted by $det_d(G) = (\Sigma, X_{obs}, \to_{obs}, X^0_{obs})$. The language generated by the desired observer is referred to as the safe language, $L_{safe} = L(det_d(G))$. Accordingly, we also define the unsafe language, $L_{unsafe} = L(G) \setminus L_{safe}$.

If a system is not current state opaque then an interface based approach called edit function [20], [47] may be applied to enforce it. An edit function may insert events into the output of the system or erase events from the output of the system. It is assumed that the intruder fails to distinguish between an inserted event and its genuine counterpart. Let $\Sigma' = \{ \sigma \to \epsilon : \sigma \in \Sigma \}$ be the set of “event erasure” events.

Definition 10: A deterministic edit function is defined as $f_e : \Sigma' \times X \to \Sigma'$. Given $s \in L(G), \sigma \in \Sigma,$

$$f_e(s, \sigma) = \begin{cases} s_l \sigma & \text{if } s_l \text{ is inserted before } \sigma \\ \epsilon & \text{nothing is inserted and } \sigma \text{ is erased} \\ s_l & \text{if } s_l \text{ is inserted and } \sigma \text{ is erased} \end{cases}$$

With an abuse of notation, we also define a string-based edit function $f_e$ recursively as: $f_e(\epsilon) = \epsilon$, $f_e(s \sigma) = f_e(s) f_e(s, \sigma)$.
for $s \in \Sigma^*$ and $\sigma \in \Sigma$. In the sequel, to ease the notational burden, we will drop the "$\hat{\cdot}\$" in $\hat{f}_e$ and it will be clear from the argument(s) of $f_e$ which function we are referring to (incremental single-event one or string-based one).

Two notions termed public safety and private safety were defined in [20] to characterize the behavior of edit functions. In this paper, we consider private safety alone under the assumption that the intruder does not know about the implementation of an edit function.

Definition 11 (Private Safety): Given $G$ and its observer $\det(G)$, an edit function $f_e$ is privately safe if $\forall s \in \mathcal{L}(\det(G))$, $f_e(s) \in L_{\text{safe}}$.

Recently a three-player game structure called three-player observer (TPO) w.r.t. the system was defined in [20] to embed edit functions. For the sake of completeness, we recall this definition (more details are available in [20]).

Definition 12 (Three-player Observer): Given a system $G$ with its observer $\det(G)$ and desired observer $\det_d(G)$, let $I \subseteq \mathcal{N}_{\text{obsd}} \times \mathcal{N}_{\text{obs}}$ be the set of state changes. A three-player observer w.r.t. $G$ is a tuple of the form $T = (Q_I, Q_W, \Sigma', \Theta, \theta, z_{\rightarrow y}, z'_{\rightarrow z}, z_{\rightarrow w}, y_0)$, where:

- $Q_I \subseteq I$ is the set of Y states.
- $Q_Z \subseteq I \times \Sigma$ is the set of Z states. Let $I(z), E(z)$ denote the information state component and observable event component of a $Z$ state $z$ respectively, so that $z = (I(z), E(z))$.
- $Q_W \subseteq I \times (\Sigma \cup \Theta')$ is the set of W-states. Let $I(w), A(w)$ denote the information state component and action component of a W state $w$ respectively, so that $w = (I(w), A(w))$.
- $\Sigma'$ is the set of event observables.
- $\Theta \subseteq \Sigma \cup \{e\} \cup \Theta'$ is the set of edit decisions at Z states.

(i) $\rightarrow_{yz}: Q_Y \times \Sigma \times Q_Z$ is the transition function from Y states to Z states. For $y = (x_d, x_f) \in Q_Y$, $e_o \in \Sigma$, we have: $y^{x_d x_f}_{e_o} \rightarrow \{y^{x_d x_f}_{e_o}\} \cap \{I(z) = y\} \cap \{E(z) = e_o\}$.

(ii) $\rightarrow_{z}: Q_Z \times \Theta \times Q_Z$ is the transition function from Z states to Z states. For $z = (x_d, x_f, e_o) \in Q_Z$, $e_o \in \Theta$, we have: $z^{x_d x_f}_{e_o} \rightarrow \{z^{x_d x_f}_{e_o}\} \cap \{I(z') = (x_d', x_f')\} \cap \{x_d' \rightarrow_{de,y} x_d\} \cap \{E(z') = e_o\}$.

(iii) $\rightarrow_{zw1}: Q_Z \times \Theta \times Q_W$ is the $e$ insertion transition function from Z states to W states. For $z = (x_d, x_f, e_o) \in Q_Z$, $e_o \in \Theta$, we have: $z^{x_d x_f}_{e_o} \rightarrow \{z^{x_d x_f}_{e_o}\} \cap \{I(z) = y\} \cap \{A(w) = e_o\} \cap \{x_d' \rightarrow_{de,y} x_d\}$.

(iv) $\rightarrow_{zw2}: Q_Z \times \Theta \times Q_W$ is the event erasure transition function from Z states to W states. For $z = (x_d, x_f, e_o) \in Q_Z$, $e_o \in \Theta$, we have: $z^{x_d x_f}_{e_o} \rightarrow \{z^{x_d x_f}_{e_o}\} \cap \{I(z) = y\} \cap \{A(w) = e_o\} \cap \{x_f' \rightarrow_{obs,y} x_f\}$.

(v) $\rightarrow_{wy1}: Q_W \times \Sigma \times Q_Y$ is the transition function from W states whose action component is in $\Sigma \to Y$ states. For $w = (x_d, x_f, e_o) \in Q_W$, we have: $w^{x_d x_f}_{e_o} \rightarrow \{w^{x_d x_f}_{e_o}\} \cap \{y = (x_d', x_f')\} \cap \{x_d' \rightarrow_{de,y} x_d\} \cap \{x_f' \rightarrow_{obs,y} x_f\}$.

(vi) $\rightarrow_{wy2}: Q_W \times \Sigma \times Q_Y$ is the transition function from W states whose action component is in $\Sigma' \to Y$ states. For $w = (x_d, x_f, e_o) \in Q_W$, we have: $w^{x_d x_f}_{e_o} \rightarrow \{w^{x_d x_f}_{e_o}\} \cap \{y = (x_d', x_f')\} \cap \{x_f' \rightarrow_{obs,y} x_f\}$.

$y_0 = (x_{\text{obsd}, 0}, x_{\text{obs}, 0}) \in Q_Y$ is the initial state of $T$, where $x_{\text{obsd}, 0}$ and $x_{\text{obs}, 0}$ are initial states of $\det_d(G)$ and $\det(G)$, respectively.

In general, a three-player observer characterizes a game between a dummy player, the edit function and the environment (system). The state space of a TPO is partitioned as: $Q_I$ states ($Y$ states) where the dummy player plays; $Q_Z$ states ($Z$ states) where the edit function plays; $Q_W$ states ($W$ states) where the environment plays. A $Y$ state contains the intruder’s estimate (left component) as well as the system’s true state estimate (right component). A $\rightarrow_{y,z}$ transition is defined out of a $Y$ state, indicating that an observable event may occur and thus is received by the edit function. Then the TPO transits to a $Z$ state and the turn of the game is passed to the edit function. Notice that the observable event does not really occur and this dummy player is only introduced to help determine the decisions of edit functions.

At a $Z$ state, the edit function may choose to insert certain events (including $e$) or erase its last observed event. If a non-$e$ event is inserted, a $\rightarrow_{z}$ transition leads the TPO to another $Z$ state, which means the edit function still has the turn to insert more events until it decides to stop insertion by inserting $e$ or by erasing the last observed event. There may be multiple transitions defined out of a $Z$ state, i.e., multiple edit decisions; we write $\Theta(z)$ to denote the set of edit decisions defined at $z \in Q_Z$ in a TPO.

If the edit function inserts nothing (respectively erases the event it receives from the dummy player), then a $\rightarrow_{zw1}$ (respectively $\rightarrow_{zw2}$) transition is defined and the TPO is at a $W$ state. Then the environment plays by letting the observable event executed from its preceding $Y$ state occur. Correspondingly, there are also two types of $\rightarrow_{wy1}$ transitions, where $\rightarrow_{wy1}$ indicates that the executed observable event will be observed by the intruder while $\rightarrow_{wy2}$ indicates that the executed observable event will not be observed by the intruder since it has been erased by the edit function.

When the three players take turns to play, the components of each player’s states also get updated. From Def. [12] a $\rightarrow_{y,z}$ transition does not change the state estimates for the intruder or the system since the player at $Y$ states is dummy and the observable events from $Y$ states do not really occur. With a $\rightarrow_{zw1}$ transition only $e_o$ is updated since $x_d$ is the estimate of the intruder and event insertion only alters the observation of the intruder. For $\rightarrow_{zw}$ transitions, we only require the observable event to be defined at $x_d$ or $x_f$. Finally, a $\rightarrow_{wy1}$ transition updates both $x_d$ and $x_f$ while a $\rightarrow_{wy2}$ transition only updates $x_f$ as the intruder does not observe the erased event. To characterize the information flow in a TPO, the notion of run is defined in [20].

Definition 13 (Run): In a three-player observer $T$, a run is defined as: $\omega = y_0 \rightarrow_{\theta_1} y_1 \rightarrow_{\theta_2} y_2 \rightarrow_{\theta_3} \cdots \rightarrow_{\theta_{m-1}} y_{m-1} \rightarrow_{\theta_m} y_m \rightarrow_{\theta_0} \cdots \rightarrow_{\theta_{n-1}} y_{n+1}$, where $y_0$ is the initial state of $T$, $e_i \in \Sigma$, $\theta_j \in \Theta(z_j)$, $0 \leq i \leq n$, $1 \leq j \leq m$, and $n, m \in \mathbb{N}^+$. We let $\Omega_T$ be the set of all runs in a TPO $T$. For simplicity, similar notations as for automata are defined for three-player observers and thus $T \rightarrow x$ denotes the existence of a run in $T$. 


three-player observer. We also review the concepts of string generated by a run and edit projection defined in \[20\].

\textbf{Definition 14 (String Generated by a Run)}: Given a run \(\omega\) as in Definition \[13\] the string generated by \(\omega\) is defined as:
\[
 l(\omega) = \theta_0 \theta_0 \cdots \theta_{n-1} \theta_0 e_0 \theta_1 \cdots \theta_{m-1} e_1 \cdots e_n = \theta_0 e_0 \cdots e_n,
\]
where \(\forall i \leq n, \theta_i = e_i \iff e_i = \varepsilon\).

\textbf{Definition 15 (Edit Projection)}: Given TPO \(T\) and run \(\omega_T\) as in Def. \[13\] the edit projection \(P_\omega : \Omega \rightarrow \mathcal{L}(G)\) is defined such that \(P_\omega(\omega_T) = e_0 e_1 \cdots e_n\).

In a TPO, \(y \in \Theta_T\) is a terminating state if \(\exists e_0 \in \Sigma, \text{s.t. } y \xrightarrow{e_0} z\). And \(w \in Q_w\) is a deadlockin state if \(\exists e_0 \in \Sigma, \text{s.t. } w \xrightarrow{e_0} y\). Also \(z \in \mathcal{L}(G)\) is a deadlockin state if \(\exists \theta \in \Theta, \text{s.t. } z \xrightarrow{\theta} z' \text{ or } z \xrightarrow{\theta} w\).

We call a TPO \(T\) complete \[20\] if there are no deadlockin \(W\) or \(Z\) states in \(T\) and \(\forall s \in \mathcal{L}(G), \exists \omega \in \Omega_T\), s.t. \(P_\omega(\omega) = s\). The edit constraint, denoted by \(\Phi\), requires that the edit function should not make \(n+1\) consecutive erasures where \(n \in \mathbb{N}\).

Finally, we define the \textit{All Edit Structure (AES)} \[20\] by considering the edit constraint. A synthesis procedure was also presented in \[20\] to construct the AES. Notice that the following definition is slightly different from the AES in the preliminary version of this work \[26\] since edit constraints are not considered in \[26\].

\textbf{Definition 16 (Edit Function Embedded in TPO)}: Given a TPO \(T\), a deterministic edit function \(f_e\) is embedded in \(T\) if \(\forall s \in \mathcal{L}(G), \exists \omega \in \Omega_T\), s.t. \(P_\omega(\omega) = s\) and \(l(\omega) = f_e(s)\).

Next, we construct the largest three-player observer in the sense that all the other three-player observers are subautomata of it. Such a notion is well defined by considering all admissible transitions at every state of the TPO, according to the respective conditions in Def. \[12\].

Edit functions are designed to erase genuine events or insert fictitious ones to mislead the intruder. In theory, it is possible to design an edit function that erases all the events of the system, although this is not desirable. To avoid this situation usually the user provides some constraints on the edit functions. The constraint that is considered in this paper is to limit the number of consecutive erasures.

\textbf{Definition 17 (Edit Constraint)}: The edit constraint, denoted by \(\Phi\), requires that the edit function should not make \(n + 1\) consecutive erasures where \(n \in \mathbb{N}\).

Finally, we define the \textit{All Edit Structure (AES)} \[20\] by considering the edit constraint. A synthesis procedure was also presented in \[20\] to construct the AES. Notice that the following definition is slightly different from the AES in the preliminary version of this work \[26\] since edit constraints are not considered in \[26\].

\textbf{Definition 18 (All Edit Structure)}: Given system \(G\), observer \(\det(G)\) and desired estimator \(\det_H(G)\), the All Edit Structure is defined to be the largest complete three-player observer w.r.t. \(G\), which satisfies the edit constraint.

From results in \[20\], private safety is achievable when the AES is not empty by construction. However, we assume that the AES is non-empty in the following discussion; if it is empty, then opacity cannot be enforced by the mechanism of edit functions. It was also proven in \[20\] that all privately safe edit functions satisfying edit constraints are embedded in the AES. Formally speaking, the following result holds.

\textit{Theorem 2}: Given a system \(G\) and its corresponding AES under edit constraint \(\Phi\), an edit function \(f_e\) is privately safe and satisfies \(\Phi\) if and only if \(f_e \in \text{AES}\).

We end this section by briefly reviewing the pruning process discussed in \[20\] to construct the AES. The presence of edit constraints may preclude some undesired states from the AES, thus leaving some states without outgoing transitions, i.e., “deadlock” \(Z\) or \(W\) states. Those states reflect the inability of the edit function to issue a valid edit decision (for insertion or erasure) while still maintaining opacity for all possible future behaviors, thus should be removed in the pruning process. Moreover, \(Y\) states that have transitions to a deadlock \(Z\) state need to be pruned as well, since \(Y\)-states are the states where the system issues an output event and the edit function is not allowed to prevent their occurrence.

The construction of the AES may also be interpreted as the calculation of a supervisor where the “plant” is the largest three-player observer in terms of subautomaton, including all potentially feasible edit decisions without considering edit constraints. The \(Y\) states are considered as marked states. The events labeling transitions from \(Y\) states to \(Z\) states and from \(W\) states to \(Y\) states are considered as uncontrollable, while the events labeling transitions from \(Z\) states to \(Z\) states and \(Z\) states to \(W\) states are viewed as controllable. We also define the proper specification by considering edit constraints, deleting states that violate them, and taking the trim of the resulting structure. The goal is to calculate the least restrictive, controllable and nonblocking supervisor based on the plant and this specification. Similar processes of pruning game structures akin to TPOs were discussed in prior work, e.g., \[18\], \[20\], \[46\]. We will leverage this approach in the following discussion, but in the framework of \textit{modular supervisory control}.

III. COMPOSITIONAL ABSTRACTION-BASED METHODOLOGY

This section presents our novel compositional and abstraction-based methodology for synthesizing modular form edit functions based on individual three-player observers after abstracting the original system. For simplicity, we call this methodology the CA-AES (Composition Abstraction-All Edit Structure) Algorithm hereafter. The input of the algorithm is a set of nondeterministic automata, \(\mathcal{F} = \{G_1, \ldots, G_n\}\) and the output is a modular representation of edit functions, which is called \textit{Modular Edit Structure}. The algorithm is summarized in Figure \[1\] and its steps are as follows. We will explain how to interpret the modular representation of edit functions later.

(i) The algorithm first abstracts each individual automaton, \(G_i\), using opaque observation equivalence. This results in \(\tilde{G}_i\), which has fewer states and transitions compared to the original automaton.

(ii) Next, we abstract the observer of \(\tilde{G}_i\), i.e., \(\det(\tilde{G}_i)\), by opaque bisimulation and bisimulation, resulting in two abstracted deterministic automata \(H_{i,obd}\) and \(H_{i,b}\).

(iii) Then we calculate the abstracted desired observer of \(G_i\) from \(H_{i,obd}\), which is denoted by \(H_{i,obd}\).

(iv) Afterward, the largest (abstracted) three-player observer of each individual component \(G_i\) is calculated from the abstracted observer \(H_{i,b}\) and the abstracted desired observer \(H_{i,obd}\), and it is denoted by \(TPO_i\).

(v) The final step is to calculate a modular nonblocking and controllable supervisor, then obtain a set of modular edit functions. This is done by transforming the largest three-player observers and the edit constraint to a set of automata, i.e., \(G_T\) and \(K\), respectively. This modular approach is in contrast to calculating monolithic edit functions embedded in the monolithic AES \[20\].
The presented approach relies heavily on the use of three-player observers. We present an example to better understand the structure of such observers.

**Example 1:** Consider the nondeterministic automaton $G_1$ with secret states set $Q_1 = \{q_3\}$, shown in Fig. 2. To generate the three-player observer of $G_1$, first the observer of $G_1$ needs to be built, which is shown as $\text{det}(G_1)$ in Fig. 2. Then we follow the procedures in [20] to build the TPO w.r.t. $\text{det}(G_1)$ in Fig. 2 (labeled as $T_1^i$). As is discussed, the game on the TPO is initiated from $Y$-state $(q_0, q_0)$ where the dummy player executes the observable event $\gamma$ (the only event defined at $q_0$ in $\text{det}(G_1)$). Then the edit function takes the turn to play at the $Z$ state $(q_0, q_0, \gamma)$ where it has two choices: insert nothing or erase $\gamma$. If $\gamma$ is erased, then the $W$ state $(q_0, q_0, \gamma \rightarrow \varepsilon)$ is reached where the environment plays by executing $\gamma$. Then the turn is passed back to the dummy player and the rest of the structure is interpreted similarly.

The compositional abstraction-based approach is explained in more details in the following sections. First, in Section IV we discuss abstractions at the component level and synchronization of individual three-player observers, formalizing steps (i)-(iv) of Algorithm CA-AES. Then, in Section V we discuss the last step of Algorithm CA-AES.

**IV. SYNCHRONIZATION AND ABSTRACTION OPERATIONS**

This section presents results on abstraction and composition that support steps (i)-(iv) of Algorithm CA-CAS. First, Sect. IV-A describes the methods to abstract nondeterministic automata and their observers. Next, Sect. IV-B describes the process of transforming every individual three-player observer to an automaton form and shows that the automaton representation is a substructure (in the sense of subgraph) of the largest monolithic three-player observer.

### A. Opaque observation equivalence

The first strategy used in Algorithm CA-AES to alleviate state space explosion is abstraction of system components. This subsection contains a collection of abstraction methods that can be used to abstract nondeterministic automata and their observers such that the abstracted observers and the desired observers are bisimilar to their original counterparts. The abstraction methods are based on bisimulation and observation equivalence, which are computationally efficient and can be calculated in polynomial-time [12]. We will prove in Theorem 5 that if we build the largest three-player observer based on the abstracted observer and the desired observer, we obtain the same runs, consequently the same edit functions as we do from the largest three-player observer based on the original observer and desired observer.

#### Bisimulation

Bisimulation is a widely-used notion of abstraction that merges states with the same future behavior.

**Definition 19:** Let $G = (\Sigma, Q, \rightarrow, Q^0)$ be a nondeterministic automaton. An equivalence relation $\approx \subseteq Q \times Q$ is called a bisimulation on $G$, if the following holds for all

1. $q_1 \approx q_2 \Rightarrow q_1 \rightarrow a q_3 \Rightarrow \exists q_4 \in Q : q_2 \approx q_4 \land q_3 \rightarrow a q_4$
2. $q_1 \approx q_2 \Rightarrow q_2 \rightarrow a q_3 \Rightarrow \exists q_4 \in Q : q_1 \approx q_4 \land q_3 \rightarrow a q_4$
3. $q_1 \approx q_2 \Rightarrow \forall a \in \Sigma : q_1 \rightarrow a q_3 \Rightarrow q_2 \rightarrow a q_4$
x₁, x₂ ∈ Q such that x₁ ≈ x₂; if x₁ ↠ y₁ for some σ ∈ Σ, then there exists y₂ ∈ Q such that x₂ ↠ y₂, and y₁ ≈ y₂.

Bisimulation seeks to merge states with the same outgoing transitions to equivalent states including unobservable events, i.e., τ events. If the unobservable events are disregarded, a more general abstraction method called weak bisimulation or observation equivalence naturally comes [25].

Definition 20: Let G = (Σ, Q, →, Q₀) be a nondeterministic automaton. An equivalence relation ∼ ⊆ Q × Q is called an observation equivalence on G if, the following holds for all x₁, x₂ ∈ Q such that x₁ ≈ x₂; if x₁ ↠ y₁ for some σ ∈ Σ*, then there exists y₂ ∈ Q such that x₂ ↠ y₂, and y₁ ≈ y₂.

In order to use observation equivalence for abstraction in the opacity setting, the set of secret states needs to be taken into account. In the following discussion, a restricted version of observation equivalence called opaque observation equivalence is employed. This notion was first defined in [27] in the context of verifying opacity.

Definition 21: Let G = (Σ, Q, →, Q₀) be a nondeterministic automaton with set of secret states Qₛ ⊆ Q and set of non-secret states Qⁿₛ = Q \ Qₛ. An equivalence relation ∼₀ ⊆ Q × Q is called an opaque observation equivalence on G with respects to Q², if the following holds for all x₁, x₂ ∈ Q such that x₁ ≈ x₂:

(i) if x₁ ↠ y₁ for some s ∈ Σ*, then there exists y₂ ∈ Q such that x₂ ↠ y₂, and y₁ ≈ y₂;

(ii) x₁ ∈ Q² if and only if x₂ ∈ Q².

We also wish to use bisimulation to abstract the observer of a nondeterministic system. Besides opaque observation equivalence, opaque bisimulation is also defined.

Definition 22: Let G = (Σ, Q, →, Q₀) be a nondeterministic automaton with set of secret states Qₛ ⊆ Q and set of non-secret states Qⁿₛ = Q \ Qₛ. Let det(G) = (Σ, X₀obs, →ₚobs, X₀obs) be the observer of G. An equivalence relation ∼ₚ₀ ⊆ X₀obs × X₀obs is called an opaque bisimulation equivalence on det(G) with respects to Q³, if the following holds for all X₁, X₂ ∈ X₀obs such that X₁ ≈ₚ X₂:

(i) if X₁ ↠ Y₁ for some s ∈ Σ*, then there exists Y₂ ∈ X₀obs such that X₂ ↠ Y₂, and Y₁ ≈ₚ Y₂;

(ii) X₁ ∈ Q³ if and only if X₂ ∈ Q³.

The first step of Algorithm CA-AES is to abstract the system using opaque observation equivalence. It has been shown in [32] that if two automata are bisimilar, then their observers are also bisimilar. In this paper this result is extended such that abstracting a nondeterministic automaton using opaque observation equivalence results in an observer and a desired observer which are bisimilar to the observer and the desired observer of the original system, respectively.

Proposition 3: Let G = (Σ, Q, →, Q₀) be a nondeterministic automaton with set of secret states Qₛ ⊆ Q and set of non-secret states Qⁿₛ = Q \ Qₛ. Let ∼₀ be an opaque observation equivalence on G resulting in  ̅G and let ∼ₚ₀ be a bisimulation. Let detₜ(G) and detₜ( ̅G) be the desired observer of G and  ̅G. Then detₜ(G) ∼ₚ₀ detₜ( ̅G) and detₜ( ̅G) ∼ₚ₀ detₜ(G).

Proof: First we prove that detₜ(G) ∼ₚ₀ detₜ( ̅G). To prove detₜ(G) ∼ₚ₀ detₜ( ̅G) it is enough to show that detₜ(G) → X if and only if detₜ( ̅G) → ̅X, which implies language equivalence between detₜ(G) and detₜ( ̅G) since detₜ(G) and detₜ( ̅G) are deterministic. This can be shown by induction. Moreover, in the induction we also show that x ∈ X if and only if there exist [x] ∈ ̅X such that x ∈ [x]. This is used for the second part of the proof, where we show detₜ( ̅G) ∼ₚ₀ detₜ(G).

It is shown by induction on n ≥ 0 that X₀ ↠ x₁ ↠ x₂ ↠ ... ↠ xₙ in det(G) if and only if 0 ↠ x₀ ↠ x₁ ↠ ... ↠ xₙ in det( ̅G) such that x ∈ Xₖ if and only if [x] ∈  ̅Xₖ, where x ∈ [x], for 1 ≤ j ≤ n.

Base case: n = 0. Let X₀ be the initial state of det(G) and  ̅X₀ be the initial state of det( ̅G). It is shown that x ∈ X₀ if and only if there exists [x] ∈  ̅X₀ such that x ∈ [x].

First, let x ∈ X₀. Then based on UR(x₀), it follows that there exists x₀ ∈ Q₀ such that x₀ → x in G. Since G ∼ₚ₀  ̅G then based on Def. 21 there exists  [x₀] ∈  ̅X₀ such that  [x₀] → [x] in  ̅G such that x ∈ [x₀] and x ∈ [x]. Then based on UR(x₀) it follows that [x] ∈  ̅X₀.

Now let [x] ∈  ̅X₀. Then based on UR(x₀), it follows that there exists [x₀] ∈ Q₀ such that [x₀] → [x] in  ̅G. Since G ∼ₚ₀  ̅G then based on Def. 21 there exists x₀ ∈ Q₀ such that x₀ → x in G such that x ∈ [x₀] and x ∈ [x]. Then based on UR(x₀) it follows that x ∈ X₀.

Inductive step: Assume the claim holds for some n ≥ 0, i.e., X₀ ↠ x₁ ↠ x₂ ↠ ... ↠ xₙ in det(G) if and only if 0 ↠ x₀ ↠ x₁ ↠ ... ↠ xₙ in det( ̅G), such that x ∈ Xₖ if and only if there exists [x] ∈  ̅Xₖ such that x ∈ [x] for all 0 ≤ k ≤ n.

It must be shown that X₀ ↠ x₂ ↠ ... ↠ xₙ in det(G) if and only if 0 ↠ x₀ ↠ x₁ ↠ ... ↠ xₙ in det( ̅G) such that x ∈ Xₖ if and only if there exists [x] such that x ∈ [x] ∈  ̅Xₖ.

First, let X₀ ↠ x₂ ↠ ... ↠ xₙ in det(G) and let x ∈ X₀. Then based on UR(x) it holds that x = x₁ ↠ x₂ ↠ ... ↠ xₙ in G, where xᵢ ∈ X for all 1 ≤ j ≤ r and y ∈ Y. Since G ∼ₚ₀  ̅G it holds that [x] = [x₁] ↠ [x₂] ↠ ... ↠ [xₙ] in  ̅G such that xᵢ ∈ [xᵢ] such that xᵢ ∈ [xᵢ] for all 1 ≤ j ≤ r and y ∈ [y]. Based on UR(x) and inductive assumption it holds that detₜ( ̅G) → xᵢ ↠ xᵢ in  ̅G such that xᵢ ∈ [xᵢ].

Now let X₀ ↠ x₂ ↠ ... ↠ xₙ in det( ̅G) and let x ∈ X₀. Then based on UR(x) it holds that [x] = [x₁] ↠ [x₂] ↠ ... ↠ [xₙ] = [y] in  ̅G, where [xᵢ] ∈ X for all 1 ≤ j ≤ r and y ∈ [y]. Since G ∼ₚ₀  ̅G it holds that xᵢ = xᵢ ↠ xᵢ ↠ ... ↠ xᵢ in  ̅G such that xᵢ ∈ [xᵢ] such that xᵢ ∈ [xᵢ] for all 1 ≤ j ≤ r and y ∈ [y]. Based on UR(x) and inductive assumption it holds that detₜ( ̅G) → xᵢ ↠ xᵢ in  ̅G such that xᵢ ∈ X.

Now we want to show that detₜ(G) ∼ₚ₀ detₜ( ̅G). It was proven above that detₜ(G) ∼ₚ₀ detₜ( ̅G), which means detₜ(G) → X if and only if detₜ( ̅G) → ̅X and x ∈ X if and only if [x] ∈  ̅X, where x ∈ [x]. Therefore, it is enough to show that X ∈ X₀obs if and only if X ∈  ̅X₀obs.

First assume X ⊆ Q₃, which means for all x ∈ X it holds that x ∈ Q₃ and X ⊆ X₀obs. Since for all x ∈ X it holds that there exist [x] ∈  ̅X such that x ∈ [x], then based on Def. 21 it holds that [x] ∈ Q₃. Thus, it can be concluded that for all [x] ∈  ̅X it holds that [x] ∈  ̅Q₃. This means that X ⊆ Q₃ and consequently X ∈  ̅X₀obs.

Since the base case of the induction is proven for n = 0, x₀ ↠ x, the inductive step is considered true for 0 ≤ k < n.
Now assume $\bar{X} \subseteq \bar{Q}^3$, which means for all $[y^e] \in \bar{X}$ it holds that $[y^e] \in \bar{Q}^3$ and $\bar{X} \not\subseteq X_{\text{obs,ad}}$. If $[\bar{y}^e] \in \bar{Q}^3$ then for all $x \in [\bar{y}^e]$ it holds that $x \in Q^3$. Moreover, it was shown above that $[\bar{y}^e] \in \bar{X}$ if and only if $x \in X$, where $x \in [\bar{y}^e]$. Thus, from $\bar{X} \subseteq \bar{Q}^3$ it follows that $X \subseteq Q^3$, which means that $X \not\subseteq X_{\text{obs,ad}}$.

Thus, it can be concluded that $\det_d(G) \approx \det_d(\bar{G})$. ■

Opaqueness equivalence seeks to merge states of a nondeterministic automaton, which are “equivalent”, before constructing the observer. After calculating the observer, it is possible to further abstract the observer using opaqueness bisimulation. This guarantees that the smallest abstracted observer generates the same language as the original observer. In the following, Proposition 4 shows that if abstracted observer is used to abstract the observer, then the abstracted desired observer is also bisimilar to the original desired observer. Proposition 4: Let $G = (\Sigma_I, Q, \rightarrow, Q^0)$ be a nondeterministic automaton with set of secret states $Q^S \subseteq Q$ and set of non-secret states $Q^{NS} = Q \setminus Q^S$. Let $\approx_o$ be an opaqueness bisimulation on $\det_d(G)$ resulting in $\det_d(G)$. Let $\det_d(G)$ and $H_d$ be the desired observers of $\det_d(G)$ and $\det_d(G)$, respectively. Then $\det_d(G) \approx H_d$, where $\approx$ is a bisimulation relation.

Proof: Since $\det_d(G) \approx_o \det_d(G)$ based on Def. 22, it holds that $\det_d(G) \not\xrightarrow{X} I$ if and only if $\det_d(G) \not\xrightarrow{X'} [X']$ and $X \in [X']$. Thus, it is enough show that $X \not\subseteq X_{\text{obs,det}_d(G)}$ if and only if $[X'] \not\subseteq X_{\text{obs,det}_d(G)}$, where $X \in [X']$.

First assume $X \subseteq Q^3$, so $X \not\subseteq X_{\text{obs,det}_d(G)}$. Then since $X \in [X']$ based on Def. 22, it holds that for all $X' \in [X']$, $X' \subseteq Q^3$. This means $[X'] \subseteq Q^3$ and consequently $X' \not\subseteq X_{\text{obs,det}_d(G)}$.

Then assume $[X'] \subseteq Q^3$, so $X \subseteq X_{\text{obs,det}_d(G)}$. Since $X \in [X']$ based on Def. 22, $X \subseteq Q^3$ holds, i.e., $X \not\subseteq X_{\text{obs,det}_d(G)}$. ■

We now present the main results of this subsection.

Theorem 5: Let $G$ be a nondeterministic automaton with secret states $Q^S \subseteq Q$ and non-secret states $Q^{NS} = Q \setminus Q^S$. Let $\det_d(G)$ and $\det_d(G)$ be the observer and the desired observer of $G$, respectively. Let $\approx_o$ be an opaqueness observation equivalence on $G$ such that $G \approx_o G$. Let $H_{\text{obs}} \approx \det_d(G)$ and $H_{\text{det}} \approx \det_d(G)$ where $\approx_o$ and $\approx$ are opaqueness bisimulation and bisimulation, respectively. Let $H_{\text{obs,ad}}$ be the desired observer of $H_{\text{obs}}$. Let $T$ be the largest three-player observer w.r.t. $\det_d(G)$ and $\det_d(G)$, also let $T'$ be the largest three-player observer w.r.t. $\det_d(G)$ and $H_{\text{obs}}$. Then $T \not\xrightarrow{\alpha} q$ if and only if $T' \not\xrightarrow{\alpha} \tilde{q}$.

Proof: From Propositions 3 and 4 it holds that $\det_d(G) \approx H_{\text{obs}}$ and $\det_d(G) \approx H_{\text{obs,ad}}$. Thus, we need to show that a transition is defined in $T$ if and only if the same transition is defined in $T'$. It is shown by induction on $n \geq 0$ that $\gamma_{\alpha} \xrightarrow{T} q_n$ in $T$ if and only if $\gamma_{\alpha} \xrightarrow{T'} \tilde{q}_n$ in $T'$.

Base case: $(\Rightarrow)$ First assume $\gamma_{\alpha} \xrightarrow{T} \bar{q}$ in $T$, where $\gamma_{\alpha} = (X_{\text{obs,ad}}^0, X_{\text{obs}}^0)$. Based on Def. 12, it holds that $X_{\text{obs,ad}}^0 \approx_{\text{det}_d(G)} \bar{X}$ and $I(\bar{X}) = \gamma_{\alpha}$ and $E(\bar{X}) = \gamma_{\alpha}$. From $X_{\text{obs,ad}}^0 \approx_{\text{det}_d(G)} \bar{X}$ and since $\det_d(G) \approx H_{\text{obs}}$ it holds that $X_{\text{obs,ad}}^0 \approx_{\text{det}_d(G)} \bar{X}$. Thus, $\gamma_{\alpha} = (X_{\text{obs,ad}}^0, X_{\text{obs}}^0)$ and $\bar{X} \approx_{\text{det}_d(G)} H_{\text{obs}}$, $I(\bar{X}) = \gamma_{\alpha}$ and $E(\bar{X}) = \gamma_{\alpha}$. This means $\gamma_{\alpha} \xrightarrow{T'} \bar{q}$ in $T'$.

$(\Leftarrow)$ Now assume $\gamma_{\alpha} \xrightarrow{T'} \bar{q}$ in $T'$, where $\gamma_{\alpha} = (X_{\text{obs,ad}}^0, X_{\text{obs}}^0)$. The same argument as $(\Rightarrow)$ holds.

Inductive step: Assume the claim holds for some $n \geq 0$, i.e., $n \geq 0$ that $\gamma_{\alpha} \xrightarrow{T} q_n$ in $T$ if and only if $\gamma_{\alpha} \xrightarrow{T'} \tilde{q}_n$ in $T'$.
Since \( \text{det}(G) \approx \mathcal{H}_p \) and from \( x_f \xrightarrow{e} x'_f \) in \( \text{det}(G) \) it holds that \( \tilde{x}_f \xrightarrow{e} \tilde{x}'_f \) in \( H_b \). Moreover, based on the inductive assumption it holds that there exists \( \tilde{w} = (\tilde{x}_f, \tilde{x}'_f), e_0 \xrightarrow{e} \) such that \( w \xrightarrow{e} \tilde{w} \) in \( T' \). Thus, \( w \xrightarrow{e} \gamma \) in \( T' \), where based on Def. 12 it holds that \( \gamma = (\tilde{x}_f, \tilde{x}'_f) \) and \( \tilde{x}_f \xrightarrow{e} \tilde{x}'_f \) in \( H_b \).

(\( \Rightarrow \)) It must be shown that if \( q_n \xrightarrow{e} p_n \) in \( T' \) then \( q_n \xrightarrow{e} p_n \) in \( T \). The same argument as (\( \Rightarrow \)) holds.

Theorem 3 proves that the largest three-player observer obtained from the abstracted system (using opaque observation equivalence and opaque bisimulation) has the same set of runs with that obtained from the original system. This result is essential for the correctness of Algorithm CA-AES.

Remark 1: The abstractions in the worst case scenario fail to merge any states. However, as pointed out in the paper the complexity of the abstraction methods is polynomial, while the complexity of calculating the observer is exponential in the number of states. Thus, if the abstraction results in merging even few states, it can potentially reduce the complexity of calculating the observer significantly. Therefore, it is worth applying the abstraction algorithm before calculating the observers.

Example 2: Consider the nondeterministic system \( \mathcal{G} = \{G_1, G_2\} \), shown in Fig. 2 with secret states sets \( Q^i_1 = \{q_3\} \) and \( Q^i_2 = \{s_3\} \) where all the events are observable except event \( \tau \). In \( G_1 \) states \( q_1 \) and \( q_2 \) are opaque observation equivalent as they both have the same secrecy status and equivalent states can be reached from both, \( q_1 \xrightarrow{\alpha} q_3 \) and \( q_2 \xrightarrow{\alpha} q_3 \), and \( q_1 \xrightarrow{\beta} q_2 \) and \( q_2 \xrightarrow{\beta} q_2 \). Merging \( q_1 \) and \( q_2 \) results in the abstract automaton \( \tilde{G}_1 \) shown in Fig. 2. Moreover, states \( s_1 \) and \( s_2 \) are also opaque observation equivalent and merging them results in automaton \( \tilde{G}_2 \) shown Fig. 2. After abstracting the automata, the system becomes a deterministic system. Moreover, the observers as of \( \tilde{G}_1 \) and \( \tilde{G}_2 \) are bisimilar to \( \text{det}(G_1) \) and \( \text{det}(G_2) \), respectively. The same is also true for the desired observer of \( \tilde{G}_1 \) and \( \tilde{G}_2 \). Fig. 2 shows the largest three-player observers of \( G_1 \) and \( G_2 \), respectively.

B. Synchronous composition of TPOs

The second strategy used in Algorithm CA-AES to reduce computation complexity is synchronous composition of individual systems. In this work, the main advantage of our compositional approach is to build the largest three-player observer of each component individually, instead of synchronizing individual components and then building the largest monolithic three-player observer. Before synchronization, we first transfer each individual TPO to an automaton using Def. 23. Next, the individual automata are transformed to a set of interacting automata based on Def. 24. It is shown in Theorem 8 that the set of modular three-player observers form a subsystem of their monolithic counterpart, in the sense that some runs are omitted after synchronization. Before Theorem 8 Lemmas 6 and 7 establish that synchronization of individual observers (respectively, desired observers) is isomorphic to the observer (respectively, desired observers) of the synchronized system.

Definition 23: Let \( T = (Q_Y, Q_Z, Q_W, \Sigma, \Sigma^X, \Theta, \rightarrow_{yz}, \rightarrow_{zw}, \rightarrow_{wy}, \rightarrow_{yz}, \rightarrow_{zw}, \rightarrow_{zw}, \rightarrow_{wy}, \rightarrow_{yz}) \) be a three-player observer. Automaton

\[
\begin{align*}
\end{align*}
\]

Fig. 2. System \( \mathcal{G} = \{G_1, G_2\} \) and its abstraction \( \{\tilde{G}_1, \tilde{G}_2\} \). The figure also shows the largest three-player observers \( T'_1 \) and \( T'_2 \) of the abstracted components and their automata transformations, denoted by \( G'_1 \) and \( G'_2 \). The uncontrollable events are marked by \( ! \).
The table shows the link between the events of a TPO, its transformed automaton and the corresponding renaming.

| TPO $T$ | Automaton $G'_1$ | Renaming $\rho$ |
|---------|-----------------|-----------------|
| $y \xrightarrow{\alpha} z$ | $y \xrightarrow{\alpha} z'$ | $\rho(\alpha) = \alpha$ |
| $z \xrightarrow{\alpha_1} w$ | $z \xrightarrow{\alpha_1} w'$ | $\rho(\alpha_1) = \alpha$ |
| $w' \xrightarrow{\gamma} \gamma(w)$ | $w \xrightarrow{\gamma} \gamma(w')$ | $\rho(\gamma(w)) = \alpha$ |

Since some shared events in the transformed automaton become local after incorporating the extra state information, they need to be added in the alphabet of the transformed automaton, $\bigcup_{\alpha \in (\Sigma')_{'\alpha} \cup (\Sigma')_{\alpha \rightarrow w} \cup (\Sigma')_{\alpha \rightarrow x}}$ in Def. 23. Moreover, the events not defined from $Y$ states of certain TPO $T$ but defined from $Y$ states of some other TPO $T'$ are added as self-loops at the corresponding states in the transformed automaton of $T$, $\{(p, \alpha, p) \mid p \in Q'_y \text{ and } \alpha \in \bigcup_{j \neq i} (\Sigma' \setminus \Sigma^j)\}$. To create a map between the events of a TPO $T$ and its transformed automaton, renaming of events is necessary. Note, when the transformation of a single automaton is considered Def. 23 and Def. 24 produce the same results. Thus, in the following wherever a transformed automaton is discussed we refer to Def. 24.

Renaming $\rho$ simply removes the extra information from the events of the transformed automaton and maps them back to the original events in the TPO. To be more specific, $\rho$ is a map such that $\rho(\alpha_0) = \alpha$ and $\rho(\alpha) = \alpha$. Table 1 shows how the events in a transformed automaton are linked to the original events of a TPO, while the third column shows how renaming works. Specifically, in the case where events label $\gamma_{yz}$ transitions, renaming does not change events names.

Example 4: Consider the abstracted system $\mathcal{G} = \{G_1, G_2\}$, shown in Fig. 3. The sets of secret states are $\hat{Q}_1^S = \{q_3\}$ and $\hat{Q}_2^S = \{s_3\}$ where all the events are observable. $T_1'$ and $T_2'$ are the largest three-player observers of $\hat{G}_1$, $\hat{G}_2$, respectively. In Example 3 the monolithic transformed automata of $T_1'$ and $T_2'$ were generated. The three-player observer system $\{T_1', T_2'\}$ is transformed to automata system $\mathcal{G} = \{G_1', G_2'\}$, shown in Fig. 2 by adding self-loops at the marked states. Event $\beta$ is not in the alphabet of $T_1'$ so it appears as a self-loop at all marked states in $G_1'$, which correspond to $Y$ states in $T_1'$. Similarly, $\gamma$ is added as a self-loop at marked states in $G_2'$ since $\gamma$ is not in the alphabet of $T_1'$.
that are omitted in the modular structure. Before Theorem 8, Lemma 6 and Lemma 7 establish that the modular observer and desired observer are isomorphic to their monolithic counterparts.

**Lemma 6:** Let $G_1 = (\Sigma_1, Q_1, q_1, T_1)$ and $G_2 = (\Sigma_2, Q_2, q_2, T_2)$ be two nondeterministic automata. Then $det(G_1) \parallel G_2$ is isomorphic to $det(G_1) \parallel det(G_2)$.

**Lemma 7:** Let $G_1 = (Q_1, \Sigma_1, q_1, T_1)$ and $G_2 = (Q_2, \Sigma_2, q_2, T_2)$ be two nondeterministic automata with sets of secret states $Q_1^d$ and $Q_2^d$, respectively. Then $det_2(G_1) \parallel det_2(G_2)$ is isomorphic to $det_2(G_1) \parallel G_2$.

**Proof:** From $det(G_1) \parallel G_2$ is isomorphic to $det(G_1) \parallel det(G_2)$ if and only if $det(G_2) \rightarrow X$ if and only if $det(G_2) \rightarrow X$, which implies $det(G_1) \parallel G_2$ if and only if $(x_1, x_2) \in X \times X$, respectively. Now we need to show that $X \notin X_{obsd}$ if and only if $(x_1, x_2) \notin X_{obsd} \times X_{obsd}$.

First assume $X \notin X_{obsd}$, which means $X \in Q_2^d$. This further means that for all $(x_1, x_2) \in X$, either $x_1 \in Q_1^d$ or $x_2 \in Q_2^d$, which implies either $x_1 \notin X_{obsd}$ or $x_2 \notin X_{obsd}$. Thus, $(X_1, X_2) \notin X_{obsd} \times X_{obsd}$.

Now assume $(x_1, x_2) \notin X_{obsd} \times X_{obsd}$. This means either $x_1 \notin X_{obsd}$ or $x_2 \notin X_{obsd}$, which implies either $x_1 \notin Q_1^d$ or $x_2 \notin Q_2^d$. Hence for all $(x_1, x_2) \in X_{obsd}$, either $x_1 \in Q_1^d$ or $x_2 \in Q_2^d$, which implies $X \in Q_2^d$. Thus, $X \notin X_{obsd}$.

**Theorem 8:** Let $G_1 = (Q_1, \Sigma_1, q_1, T_1)$ and $G_2 = (Q_2, \Sigma_2, q_2, T_2)$ be two nondeterministic automata with sets of secret states $Q_1^d$ and $Q_2^d$, respectively. Let $T_1 = (Q_1^d, Q_1^d, Q_1^d, \Sigma_1, \Sigma_1, \Sigma_1, \Theta_1, \Theta_1, \Theta_1, \sigma_1, \sigma_1, \sigma_1, x, x, x, y, y, y)$ and $T_2 = (Q_2^d, Q_2^d, Q_2^d, \Sigma_2, \Sigma_2, \Sigma_2, \Theta_2, \Theta_2, \Theta_2, \sigma_2, \sigma_2, \sigma_2, x, x, x, y, y, y)$ be the largest three-player observers w.r.t. $G_1$ and $G_2$, respectively. Let $G_{1T} = (\Sigma_{1T} \cup \Sigma_{1T}, q_1, T_1)$ and $G_{2T} = (\Sigma_{2T} \cup \Sigma_{2T}, q_2, T_2)$ be the transformed automata of $T_1$ and $T_2$, respectively. Let $B$ be the largest monolithic three-player observer w.r.t. $G_1 \parallel G_2$. Then let $\rho : \Sigma_{1T} \cup \Sigma_{2T} \rightarrow (\Sigma_{1T} \cup \Sigma_{2T} \cup \Theta_1) \cup (\Sigma_{2T} \cup \Sigma_{2T} \cup \Theta_2)$ be a renaming.

We have $[Q_1^d \parallel G_1 \parallel \sigma_1, q_2] \Rightarrow [T_{\rho_1} \rightarrow q_1, q_2]$. Let $\rho : \Sigma_{1T} \cup \Sigma_{2T} \rightarrow (\Sigma_{1T} \cup \Sigma_{2T} \cup \Theta_1) \cup (\Sigma_{2T} \cup \Sigma_{2T} \cup \Theta_2)$ be a renaming. We have $[G_1^T \parallel G_1^T \rightarrow (q_1, q_2)] \Rightarrow [T_{\rho_1} \rightarrow q_1, q_2]$.

**Proof:** We need to show that a transition is defined in $G_1^T \parallel G_1^T$ if the equivalent transition is defined in $T$. It is shown by induction on $n \geq 0$ that $(y_1, y_2) = (q_1, q_2) \in G_1^T \parallel G_1^T$ implies $y_1 \rightarrow (q_1, q_2)$ in $G_1^T \parallel G_1^T$.

Let $G_1^T \parallel G_1^T$ be the initial state of $G_1^T \parallel G_1^T$. Let $y_1 \rightarrow (q_1, q_2)$ be the initial state of $T_1$ (respectively $T_2$). From Def. 10, $y_1 = (X_{obsd} \times X_{obsd})$, where $X_{obsd}$ and $X_{obsd}$ are the initial state of $det(G_1)$ and $det(G_2)$, respectively. From Lemmas 6 and 7, $det(G_1) \parallel G_2$ and $det(G_1) \parallel G_2$ are isomorphic to $det(G_1) \parallel det(G_2)$ and $det(G_1) \parallel det(G_2)$, which implies $y_1 \rightarrow (q_1, q_2)$.

Inductive step: Assume the claim holds for some $0 \leq n$, which means if $(y_1, y_2) = (q_1, q_2)$, then $(q_1, q_2)$ is isomorphic to $(q_1, q_2)$, then $y_1 \rightarrow (q_1, q_2)$ in $G_1^T \parallel G_1^T$ implies $(q_1, q_2) \in G_1^T \parallel G_1^T$.

Now we need to show that $\rho(q_1, q_2) \rightarrow (p_1, p_2)$ in $G_1^T \parallel G_1^T$ then $\rho_{\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_n} \rightarrow p$ in $T$. From $(q_1, q_2) = (q_1', q_2') \in G_1^T \parallel G_1^T$ and based on Def. 2, it holds that $q_1 \rightarrow p_i$ in $G_1^T$ for $i \in \{1, 2\}$, which means $q_1 \rightarrow p_i$ in $T_i$ for $i \in \{1, 2\}$. Consider the following four cases for all the possible transitions:

1. If $\rho(q_1, q_2) = \sigma_1$ and $\sigma_2$ and $\sigma_3$ and $\ldots$ and $\sigma_n$ then based on Def. 24, it holds that $q_1 \rightarrow p_i$ is a $\gamma$ transition in the original $T_i$ such that $E(p_i) = \sigma_1$ and $I(p_i) = q_i$. If $\sigma_1 \in \Sigma_1$ and $p_1 = \sigma_1$ otherwise for $i \in \{1, 2\}$. Let $q_i = (x_{i,d}, x_{i,f})$ for $i \in \{1, 2\}$. Based on Def. 12, this means $x_{i,f} \rightarrow q_i$ in $det(G_1)$ if $\sigma_i \in \Sigma_1$. Moreover, based on the inductive assumption there exists $y = (x_{i,f}, x_{i,f})$ such that $y \rightarrow y$ in $T$, which implies $det(G_1) \parallel G_2 \rightarrow x_{i,f}$ for $y \in \Sigma_1$. Consider the following three cases for all the possible transitions:

2. If $\rho(q_1, q_2) = \alpha$ and $\sigma_2$ and $\sigma_3$ and $\ldots$ and $\sigma_n$ then based on Def. 24, there are three possibilities for $q_1 \rightarrow p_i$ in $T_i$: it is a $\gamma$ transition or a $\gamma_1$ transition or a $\gamma_2$ transition and $e_o = E(q_1)$. Then consider the following cases:

3. If $q_1 \rightarrow p_i$ in $T_1$ is a $\gamma$ transition, then based on Def. 12, it holds that $x_{i,d} \rightarrow x_{i,d}$ in $det(G_1)$ and $x_{i,f} \rightarrow x_{i,f}$ in $det(G_1)$. If $q_1 \rightarrow p_i$ in $T_1$ is a $\gamma_1$ transition, then based on Def. 12, it holds that $x_{i,d} \rightarrow x_{i,d}$ in $det(G_2)$ and $x_{i,f} \rightarrow x_{i,f}$ in $det(G_2)$. These mean $q_1 \rightarrow p_i$ is a $\gamma$ transition if $q_1 \rightarrow p_i$ is a $\gamma_1$ transition in $T$ and $q_1 \rightarrow p_i$ is a $\gamma_2$ transition in $T$.

4. If $q_1 \rightarrow p_i$ in $T_1$ is a $\gamma_2$ transition, then based on Def. 12, it holds that $x_{i,d} \rightarrow x_{i,d}$ in $det(G_1)$ and $x_{i,f} \rightarrow x_{i,f}$ in $det(G_2)$. In all the three cases based on Lemmas 6 and 7, they show $det(G_1) \parallel G_2 = det(G_1) \parallel det(G_2)$ and $det(G_2) \parallel det(G_2)$. Moreover, it holds that $(x_{i,d}, x_{i,f}) \rightarrow (x_{i,d}, x_{i,f})$ in $det(G_1) \parallel G_2$ and $(x_{i,f}, x_{i,f}, x_{i,f}) \rightarrow (x_{i,f}, x_{i,f}, x_{i,f})$ in $det(G_1) \parallel G_2$. These mean $q_1 \rightarrow p_i$ in $T_1$ is a $\gamma_2$ transition if $q_1 \rightarrow p_i$ is a $\gamma_1$ transition in $T$ and $q_1 \rightarrow p_i$ is a $\gamma_2$ transition in $T$.
are isomorphic, it holds that \((x_{1,d}, x_{2,d}) \xrightarrow{q_i} (x'_{1,d}, x'_{2,d})\) in \(det_G \Pi \parallel G_2\) and \((x_{1,f}, x_{2,f}) \xrightarrow{q_i} \) in \(det_G \Pi \parallel G_2\). These mean \(q_i = \rho_T \xrightarrow{s} p_T\) in \(T\) is a \(zz\) transition if \(q_i \xrightarrow{a} p_i\) in \(T_1\) is a \(zz\) transition, \(q_i = \rho_T \xrightarrow{s} p_T\) is a \(zw\) transition in \(T\) if \(q_i \xrightarrow{a} p_i\) is a \(zw\) transition in \(T_1\); and \(q_i = \rho_T \xrightarrow{s} p_T\) is a \(zw\) transition in \(T\) if \(q_i \xrightarrow{a} p_i\) is a \(zw\) transition in \(T_i\) for \(i \in \{1, 2\} \).

3) \(\sigma_{\text{new}} \in \Sigma_{G_2^f} \setminus \Sigma_{G_1^f}\). The same argument as case 1.

- if \(\rho^{-1}(\sigma_n) = \sigma_{\text{new}}\), then based on Def. 2, there are again three cases:

1) \(\sigma_{\text{neu}, w} \in \Sigma_{G_2^f} \setminus \Sigma_{G_1^f}\). This means \(q_i \xrightarrow{\sigma} p_i \in G_2^f\) from \(\sigma_{\text{neu}, w} \notin \Sigma_{G_2^f}\) and \(\sigma_{\text{neu}, w} \notin \Sigma_{G_1^f}\). It follows that \(\sigma_2 \notin \Sigma_2\), which implies \(q_2 = p_2\). From \(\rho^{-1}(\sigma_n) = \sigma_{\text{neu}, w}\), it holds that \(p_1 \xrightarrow{\sigma} q_1\) in \(T_1\) is a \(ww\) transition. This means \(x_{1,d} \xrightarrow{\sigma} x'_{1,d}\) in \(det(G_1)\) and \(x_{1,f} \xrightarrow{\sigma} x'_{1,f}\) in \(det(G_1)\). Based on Lemmas 6 and 7 where they show \(det(G_1 \parallel G_2) = det(G_2) \cup det(G_1)\) and \(det(G_2) \cup det(G_1)\) are isomorphic, it holds that \((x_{1,d}, x_{2,d}) \xrightarrow{\sigma} (x'_{1,d}, x'_{2,d})\) in \(det(G_1 \parallel G_2)\) and \((x_{1,f}, x_{2,f}) \xrightarrow{\sigma} (x'_{1,f}, x'_{2,f})\) in \(det(G_1 \parallel G_2)\). This means \(q_i = \rho_T \xrightarrow{s} p_T\) is a \(ww\) transition in \(T\).

2) \(\sigma_{\text{neu}, w} \in \Sigma_{G_2^f} \cap \Sigma_{G_1^f}\). Then it follows that \(q_i \xrightarrow{\sigma} p_i \in G_2^f\) for \(i \in \{1, 2\}\). This means \(q_i \xrightarrow{\sigma} p_i \in T_1\) is a \(ww\) transition, which implies \(x_{1,d} \xrightarrow{\sigma} x'_{1,d}\) in \(det(G_1)\) and \(x_{1,f} \xrightarrow{\sigma} x'_{1,f}\) in \(det(G_1)\) for \(i \in \{1, 2\}\). Based on Lemmas 6 and 7 where they show \(det(G_1 \parallel G_2) = det(G_2) \cup det(G_1)\) and \(det(G_2) \cup det(G_1)\) are isomorphic, it holds that \((x_{1,d}, x_{2,d}) \xrightarrow{\sigma} (x'_{1,d}, x'_{2,d})\) in \(det(G_1 \parallel G_2)\) and \((x_{1,f}, x_{2,f}) \xrightarrow{\sigma} (x'_{1,f}, x'_{2,f})\) in \(det(G_1 \parallel G_2)\). This means \(q_i = \rho_T \xrightarrow{s} p_T\) is a \(ww\) transition in \(T\).

3) \(\sigma_{\text{neu}, w} \in \Sigma_{G_2^f} \setminus \Sigma_{G_1^f}\). The same argument as case 1.

- if \(\rho^{-1}(\sigma_n) = \sigma_{\text{neu}, w} \notin \Sigma_{G_2^f} \setminus \Sigma_{G_1^f}\), then the same argument as above, \(\rho^{-1}(\sigma_n) = \sigma_{\text{neu}, w}\), holds.

Theorem 8 shows that synchronization of individual transformed three-player observers is a subsystem of the largest monolithic three-player observer. Specifically, if there is a string \(s\) in \(G_1^f \parallel G_2^f\), then there always exists a corresponding path \(\rho(s)\) in \(T\). The synchronized automaton form TPOs may not always be equal to the monolithic TPO since some \(zz\) transitions may not appear in the synchronized system. This happens when a state in the synchronization of TPOs is a transition defined from a \(Y\) state to another \(Y\) state in TPOs, those transitions are missing in \(||^{\Pi_{\text{neu}}} G^f\), which implies the synchronized system \(||^{\Pi_{\text{neu}}} G^f\) may only contain a subset of edit decisions in the largest monolithic TPO.

However, the above mentioned operation may not be preferred in practice since it involves explicitly synchronizing individual TPOs in their automaton form. This is usually not feasible in modular approaches and should be avoided in our Algorithm CA-EAS, as well.

Finally, the results of this section are formally recapped in Theorem 9, which illustrates that the synchronization of transformed (automaton form) three-player observers w.r.t. individual abstracted systems contains a subset of the transitions of the largest monolithic three-player observer. The proof follows directly from Theorem 8 and Theorem 5.

Theorem 9: Let \(G_1\) and \(G_2\) be two nondeterministic automata with sets of secret states \(Q^S_i \subseteq Q_i\) and sets of non-secret states \(Q^NS_i = Q_i \setminus Q^S_i\) for \(i = 1, 2\). Let \(det(G_i)\) and \(det(T_i)\) be the observer and the desired observer of \(G_i\), respectively. Let \(\approx_o\) be an opaque observation equivalence on \(G_i\) such that \(\approx_o \sim_{G_i} G_i\) for \(i = 1, 2\). Let \(H_{i,ob} \approx_o det(G_i)\) and \(H_{i,b} \approx det(G_i)\) for \(i \in \{1, 2\}\), where \(\approx_o\) and \(\approx\) are opaque bisimulation and bisimulation, respectively. Let \(T\) be the largest three-player observer w.r.t. \(G_1 \parallel G_2\) and let \(T' = \{Q_1', Q_2', Q_{1w}, \Sigma, \Sigma_i, \Theta_i, \rightarrow_i', \rightarrow_i', \rightarrow_i'_{zw}, \rightarrow_i'_{wy}, \rightarrow_i'_{wy}, y_i\}\) be the largest three-player observer w.r.t. \(det(H_{i,ob})\) and \(H_{i,b}\) for \(i \in \{1, 2\}\). Let \(G_{11} = \{\Sigma_{G_1}, Q_1, \rightarrow_i', \Sigma_{G_2}, \Theta_1, \rightarrow_i', \rightarrow_i'_{zw}, \rightarrow_i'_{wy}, y_i\}\) be the transformed automata of \(T'\) and let \(\rho : (\Sigma_{G_1} \cup \Sigma_{G_2}) \rightarrow (\Sigma_1 \cup \Sigma_2 \cup \Theta_1) \cup (\Sigma_1 \cup \Sigma_2 \cup \Theta_2)\) be a renaming. We have \([G_1^f \parallel G_2^f \xrightarrow{\rho} (q_1, q_2)] \approx [T\rho(s)]\).

Example 5: Consider the system \(\mathcal{G} = \{G_1, G_2\}\) shown in Fig. 2. In the first step of the compositional approach the system is abstracted by applying opaque observation equivalence, see Example 2. The abstracted system \(\mathcal{G} = \{G_1, G_2\}\) is shown in Fig. 2. Next, the three-player observer of individual components are built. As explained in Example 4, the three-player observers of \(\tilde{G}_1\) and \(\tilde{G}_2\) are \(T'\) and \(T_2\), respectively, shown in Fig. 3. Moreover, Fig. 3 also shows \(G_{11}^f\) and \(G_{21}^f\) the transformed automata of \(T'\) and \(T_2\), respectively. The largest monolithic three-player observer w.r.t. \(G_1^f \parallel G_2^f\) is denoted by \(T\) and is shown in Fig. 3. In particular
example, it can be verified that the original and abstracted three-player observers are identical (this need not be true in general); therefore, $T$ in Fig. 3 also represents the largest three-player observer w.r.t. $G_1 \parallel G_2$. In $T$, we have states: $A = \{(q_0, s_0)\}$, $B = \{(q_0, s_1), (q_0, s_2)\}$, $C = \{(q_1, s_0), (q_2, s_0)\}$, $D = \{(q_1, s_1), (q_2, s_1), (q_2, s_2), (q_1, s_2)\}$ and $E = \{(q_3, s_1)\}$.

After synchronizing $G_T^1$ with $G_T^2$, we find that there are some transitions in Fig. 3 which do not correspond to any transition in $G_T^1 \parallel G_T^2$. For example, no transition in $G_T^1$ corresponds to the $zz$ transition of $\beta$ from state $(A, B, \gamma)$ to $(B, B, \gamma)$ in Fig. 3.

V. FROM ALL EDIT STRUCTURE CALCULATION TO SUPERVISOR SYNTHESIS

So far we have shown that in our compositional and abstraction-based approach, individual components can be abstracted and each largest three-player observer w.r.t. an abstracted component can be calculated individually. Then we transfer those three-player observers (TPOs) to their automaton forms. After that, we have also shown in Theorem 9 that the synchronization of the transformed three-player observers results in a subsystem of the largest monolithic three-player observer up to the renaming of events.

Recall that the All Edit Structure (AES) is obtained after pruning deadlocking states from the largest TPO. Here the modular structure of the transformed TPO is kept and the calculation of a “Modular Edit Structure” can be done by mapping this problem to a modular nonblocking supervisory control problem under full observation.

As was discussed at the end of Section II-C, we pursue an approach to convert the pruning process (from the largest TPO to the AES) to a supervisory control problem. In this setting, the plant is a collection of automata transformed from individual largest TPOs obtained at the end of step (iv) of Algorithm CA-AES. The specification is the automaton form of the edit constraint. The constraint of having up to $n + 1$ consecutive erasures can be modeled by a specification automaton with $n$ states where transitions are labeled by the decision events and all states are marked except the last state, which is a blocking state. After $n$ consecutive event erasures, the next transition of event erasure $\alpha \rightarrow \varepsilon_\alpha$ leads the specification back to the blocking state. If the next event is a non-erasure event, it leads the specification back to the initial state, thus resetting the sequence of erasures. Since we have a modular representation of the plant, we are able to leverage computationally efficient compositional techniques for modular nonblocking supervisory control problems.

Definition 25: Let $\mathcal{T} = \{T_1, \ldots, T_n\}$ be a three-player observer system where $T_i = (Q^i, \Sigma^i, \Pi^i, \Sigma^i, \Theta^i, \rightarrow^i, i^1, i^2, i^3, i^4)$ and let $\Phi$ be the edit constraint on $\mathcal{T}$ such that there are not $n + 1$ consecutive event erasures. Then $K = (\Sigma_K, Q_K, \rightarrow_K, Q^0_K, Q^m_K)$ is the automaton form of $\Phi$ where,

- $Q_K = \{x_1, \ldots, x_n\}$
- $\rightarrow_K = \bigcup_{1 \leq i \leq n-1} \{(x_i, \alpha \rightarrow \varepsilon_{\alpha} \alpha_i+1) \mid p \rightarrow_{\alpha_i+2} q \}$ and $E(p) = \alpha$ and $E(p) = \alpha$ with $\bigcup_{1 \leq i \leq n} \{(x_{i+1}, \alpha_{i+1}, \alpha_i, x_1) \mid p \rightarrow_{\alpha_i+1} q \}$ and $E(p) = \alpha$ with $\bigcup_{1 \leq i \leq n} \{(x_{i+1}, \alpha_{i+1}, \alpha_i, x_1) \mid p \rightarrow_{\alpha_i+1} q \}$ and $E(p) = \sigma$ with $\bigcup_{1 \leq i \leq n} \{(x_{i+1}, \alpha_{i+1}, \alpha_i, x_1) \mid p \rightarrow_{\alpha_i+1} q \}$ and $E(p) = \sigma$
- $Q^0_K = x_1$
- $Q^m_K = \{x_1, \ldots, x_{n-1}\}$

Example 6: Consider the transformed system $G_T = \{G_1, G_2\}$ with $\rightarrow_K \bigcup_{1 \leq i \leq n} \{(x_{i+1}, \alpha_{i+1}, \alpha_i, x_1) \mid p \rightarrow_{\alpha_i+1} q \}$ and $E(p) = \sigma$

Fig. 3. The monolithic largest three-player observer w.r.t. $G_1 \parallel G_2$ (also same as that w.r.t. $G_1 \parallel G_2$ in this particular case) in Example 5.
\[ G_1, G_2 \] shown in Fig. 2. Assume the constraint \( \Phi \) only allows one erasure. The specification automaton of this constraint is shown in Fig. 4 as \( K \). Automaton \( K \) has three states. As there is no constraint on event insertion, the events related with event insertion just form self-loops at the initial state of \( K \). On the other hand, by executing \( \alpha \rightarrow e_\alpha \), \( \beta \rightarrow e_\beta \) or \( \gamma \rightarrow e_\gamma \) the specification transits from \( x_1 \) to \( x_2 \). Next, at \( x_2 \) if the edit decision is to certain events, then the system goes back to the initial state \( x_1 \), thus allowing more event erasures since there are no consecutive erasures. However, if another event erasure occurs from \( x_2 \), the system goes to the blocking state \( x_3 \).

The following theorem establishes that the three-player observer of the system under constraint \( \Phi \) and the transformed system synchronized with the specification \( K \) have the same runs up to a renaming of the events.

**Theorem 10:** Let \( G = (\Sigma, Q, P, F) \) be a nondeterministic automaton with the set of secret states \( Q^S \subseteq Q \). Let \( T' = (Q_T', Q_Z', Q_W', \Sigma, \Sigma_K, \Theta, \rightarrow_{yz}, \rightarrow_{zz}, \rightarrow_{zw}, \rightarrow_{wy}, y_0) \) be the largest three-player observer of \( G \) under the edit constraint \( \Phi \) which prohibits \( n + 1 \) consecutive erasures. Let \( T = (Q_T, Q_Z, Q_W, \Sigma, \Sigma_K, \Theta, \rightarrow_{yz}, \rightarrow_{zz}, \rightarrow_{zw}, \rightarrow_{wy}, y_0) \) be the transformed system of \( G \) with the set of secret states \( Q^S \subseteq Q \) and \( \Theta \) be the edit constraint which prohibits \( n + 1 \) consecutive erasures. Let \( \rho = (\Phi, Q_T, Q_Z, Q_W, \Sigma, \Sigma_K, \Theta, \rightarrow_{yz}, \rightarrow_{zz}, \rightarrow_{zw}, \rightarrow_{wy}, y_0) \) be the transformed automaton of \( G \) under the edit constraint \( \Phi \).

Let \( \rho : Q_T \rightarrow Q_Z \upharpoonright (q_1, q_2) \) be a renaming of \( \Phi \) and let \( AES = (q_1, q_2) \) be the All Edit Structure obtained from \( T \). Let \( S \) be the suprema controllable and nonblocking subautomaton of \( G \) after considering the specification introduced by \( \Phi \). Then \( S \upharpoonright (q_1, q_2) \) if and only if \( AES \upharpoonright (q_1, q_2) \).

**Proof:** The pruning process to obtain the AES and the supervisory synthesis procedure are both iterative, which remove states at each iteration. We will show by induction that, at each iteration a state is removed in \( G \) by the supervisory control synthesis procedure if and only if the corresponding state is removed by the pruning process from \( T \).

**Base case:** Clearly \( G \) and \( T \) have the same transition relation.

**Inductive step:** Assume the claim holds for some \( n > 0 \). We let \( X_G \) be the state space of \( G \) at the \( n \)-th iteration of supervisor synthesis process and \( X_T \) be the state set of \( T \) at the \( n \)-th iteration of pruning process. Then \( S \upharpoonright (q_1, q_2) \) holds for all \( q \in X_G \). Now we need to show that \( X_G \) and \( X_T \) are also equal. Assume \( G \upharpoonright (q_1, q_2) \), which implies \( T \upharpoonright (q_1, q_2) \) based on Def. 24.

\[ (\Rightarrow) \] First we show that if \( q \notin X_T \), which means \( q \) is removed by the pruning process at the \( n \)-th iteration, then \( q \notin X_G \). Then \( q \notin X_T \) if and only if \( q \) is a deadlock state and \( q \) is a \( Z \) state and there exists \( e \in E \) such that \( q \rightarrow_{e} z' \), where \( z' \) is a deadlock \( Z \) state. If \( q \) is a deadlock state then based on Def. 12 it holds that \( q \) is either a \( W \) state or a \( Z \) state. Thus, consider the following three cases:

- \( q \) is a \( W \) state. Then \( \beta_{e_\alpha} \in \Sigma \) such that \( q \rightarrow_{e_\alpha} z' \). Then based on Def. 24 it holds that \( q \rightarrow_{e_\alpha} z' \) or \( q \rightarrow_{e_\alpha} z' \) in \( G \) either, which means \( q \rightarrow_{e_\alpha} z' \) thus, \( q \) is \( W \) state in \( G \) and \( q \notin \Theta_{\text{net}}(X_G) \).
- \( q \) is a \( Z \) state. If \( q \) is a deadlock state then \( \beta_{e_\alpha} \in \Theta \) such that \( q \rightarrow_{e_\alpha} z' \). Then based on Def. 24 it holds that \( q \rightarrow_{e_\alpha} z' \) or \( q \rightarrow_{e_\alpha} z' \) does not exist in \( G \), which means \( q \rightarrow_{e_\alpha} z' \) hence, \( q \) is \( Z \) state in \( G \) and \( q \notin \Theta_{\text{net}}(X_G) \).
- \( q \) is a \( Y \) state and \( q \rightarrow_{e_\alpha} z' \), where \( z' \) is a deadlock \( Z \) state.
Then based on Def. 24, it holds that $q \xRightarrow{a} q'$ in $G^T$ and $e_o \in \Sigma_o$. As it was shown above if $z'$ is a deadlock state in $T$ then $z'$ is also a deadlock state in $G^T$, thus removed by the supervisor synthesis procedure. If $z'$ is removed, i.e. $z' \notin X^n_o$, then $q \notin \Theta^{cont}(X^n_o)$. Thus, if $q$ is removed in the prunning of $T$, then $q$ is also removed from $G^T$ in supervisor synthesis.

$(\Leftarrow)$ Now we show that if $q \notin \Theta^{cont}(X^n_o) \cap \Theta^{cont}(X^n_w)$, then $q \notin X^{n+1}_o$ and $q$ needs to be removed from $T$ by the pruning process. If $q \notin \Theta^{cont}(X^n_o) \cap \Theta^{cont}(X^n_w)$, there are two cases:

- $q \notin \Theta^{cont}(X^n_o)$. Then it holds that $q$ is a blocking state, which means $\pi \xrightarrow{p} q$ does not exist in $G^T$. There are three possibilities for $q$, it can be a $Y$, $Z$ or $W$ state. If $q$ is a $Y$ state in $T$ then $q \in Q^m$ in $G$, which means $q \notin \Theta^{cont}$, which contradicts the assumption. Thus, $q$ can only be a $W$ or $Z$ state in $T$. In both cases from $q \xrightarrow{a} q'$ in $G^T$ and Def. 24, it holds that $\exists p \in \Theta$ such that $q \xrightarrow{p}$ in $T$. This means $q$ is a deadlock state in $T$ and $q \notin X^{n+1}_o$.

- $q \notin \Theta^{cont}(X^n_w)$. This means that $q \xrightarrow{u} z$ in $G^T$ such that $u \in \Sigma_u$ and $z \notin X^n_w$. Based on Def. 24, this means that $q$ is a $Y$ state in $T$ and $z$ is a $Z$ state. It was shown above that if $z \notin X^n_w$ then $z \notin X^n_o$, which means $z$ is a deadlock $Z$ state. Thus $q$ will be removed by pruning process, which means $q \notin X^{n+1}_o$.

Thus, if $q$ is removed by synthesis from $G^T$ then $q$ is also removed from $T$ by pruning.

Theorem 11 proves that when it comes to imposing the edit constraint, the pruning process from the largest TPO to the AES removes equivalent states with the synthesis procedure of a suprema supervisor. Hence no information is lost when we apply the supervisory control approach to enforce the edit constraints and obtain edit functions. This result is essential to show that the transformation of the TPO to an equivalent automaton and the constraint $\Phi$ to specification K is correctly done in Def. 23 and Def. 25, respectively. The next step is to consider the modular representation of the system. In that case, we will use the transformation in Def. 24, which results in a set of automata transformations of the individual three-player observers, with necessary self-loops to capture the synchronization among the components. Finally, we combine the results about abstraction and decomposition, which results in Theorem 12.

Theorem 12: Let $\mathcal{G} = \{G_1, \ldots, G_n\}$ be a modular nondeterministic system with sets of secret states $Q_i$. Let AES be the All Edit Structure of $\mathcal{G}$ under constraint $\Phi$. Let $det(G_i)$ and $det_a(G_i)$ be the observer and the desired observer of $G_i$, respectively. Let $\sim_o$ be an opaque observation equivalence on $G_i$ such that $G_i \approx_o G_i$ for $i = 1, \ldots, n$. Let $H_{i,ob} \approx_o det(G_i)$ and $H_{i,b} \approx det_a(G_i)$ for $i = 1, \ldots, n$, where $\approx_o$ and $\approx$ are opaque bisimulation and bisimulation, respectively. Let $T^*_i$ be the largest three-player observer of $det_a(H_{i,ob})$ and $H_{i,b}$ for $i = 1, \ldots, n$ with the event set $\Sigma_i$. Let $G^*_i$ be the transformed automaton of $T^*_i$ and $K$ be the automaton specification. Let $P_e : \Omega \rightarrow \mathcal{L}(\mathcal{G})$ be an edit projection and $I(o)$ be a string generated by run (Def. 13 and Def. 14). Let $\mathcal{G}$ be the least restrictive controllable and nonblocking supervisor calculated from $\{G^*_1, \ldots, G^*_n, K\}$ and let $\rho : (\Sigma_{G^*_1} \cup \cdots \cup \Sigma_{G^*_n}) \rightarrow (\Sigma_1 \cup \cdots \cup \Sigma_n)$ be the renaming map. Then $[\forall s \in \mathcal{L}(\mathcal{G}), \exists \geq \mathcal{L}(\mathcal{G}) : P_e(\rho(t)) = s \Rightarrow [l(\rho(t)) = f_e(s)] \text{ where } f_e \in \text{AES}]$.

Proof: The proof follows directly from Theorems 11 and 10 in combination with Theorem 9.

Theorem 12 essentially shows the proof for all the steps shown in Fig. 5. The theorem shows that Algorithm CA-AES correctly synthesizes edit functions for opacity enforcement in a modular form, therefore, the algorithm is sound. It also reveals that the problem of calculating the modular representation of the All Edit Structure can be transformed to synthesizing modular supervisors. Consequently, such a transformation is that we may leverage various existing approaches for calculating a modular suprema nonblocking supervisor in the literature; see, e.g., [11], [28], [29], [37]. Therefore, we can obtain a modular representation of the All Edit Structure, which is efficiently efficient to compute. Then we may synchronize individual components in the Modular Edit Structure, which results in a subsystem of the monolithic AES. However, as was pointed out in Section IV, some edit decisions are omitted after the synchronization. In practice, it is usually desired to retain the modular structure and extract an edit function from it, much in the same way as a set of modular supervisors control a plant. The extraction process is explained next.

Each step of extracting a valid edit decision is described in Fig. 5. Here the edit function is an interface between the system's output and the outside environment. Assume that the system outputs event $\gamma$, then the edit function makes an edit decision for that event and the edited string will be output to the external observers.

Specifically, this process contains the following steps. (1) When $\gamma$ is received by the edit function, all the components of the Modular Edit Structure are in states that correspond to $Y$ states of the All Edit Structure. (2) At these states, event $\gamma$ is executed and states of all the components in the Modular Edit Structure are updated simultaneously. After the execution of $\gamma$, each component of the Modular Edit Structure is in a state that corresponds to a $Z$ state of the All Edit Structure. (3) Then assume there are multiple transitions defined out of such a current state, we need to select one common transition which corresponds to a specific edit decision and can be viewed as making a control decision from the current state. Note that as the selected transition needs to be accepted by all the components, thus it may happen spontaneously in all the components of the system. The solution of the modular supervisory control problem guarantees the existence of such a common transition out of the current $Z$ states.

Algorithm CA-AES returns the edited string $\rho(\sigma_0 \ldots \sigma_k)$ for event $\gamma$ when every component of the Modular Edit Structure reaches a new state corresponding to a $Y$ state of the AES. At that point, the Modular Edit Structure is ready to process the next event output by the system, and the above steps repeat. Meanwhile, the algorithm keeps track of the states of the Modular Edit Structure as its components evolve. Based on Theorem 11, the edited string $\rho(\sigma_0 \ldots \sigma_k)$ is accepted by the monolithic AES. This finally confirms that the extracted edit decision from the Modular Edit Structure corresponds a valid edit decision in the monolithic AES. The above process is
illustrated in the following example.

**Example 7:** Consider the nondeterministic system $\mathcal{G} = \{G_1, G_2\}$ shown in Fig. 2. As it was shown in Example 2 the system can be abstracted using opaque observation equivalence. After abstraction, the system becomes deterministic, which means there is no need to calculate the observers of $G_1$ and $G_2$. The largest three-player observers of $G_1$ and $G_2$ are $T_1^3$ and $T_2^3$, respectively, shown in Fig. 2. Next, the three-player observers are transformed to automata $G_1^3$ and $G_2^3$ shown in Fig. 2 as explained in Example 4.

Assume the user adds an edit constraint such that only one consecutive erase is allowed as in Example 6. The specification automaton of this constraint is $K$, shown in Fig. 3. Due to this constraint, the $Y$ states $(A, D)$ and $(B, E)$ are considered undesired states in $T$, shown in Fig. 3 and they should not be reached when we synthesize edit functions. Since $(A, D)$ is not allowed, its successor states $(A, D, \alpha)$, $(A, D, \alpha \rightarrow e)$, and $(A, E)$ become unreachable from the initial state $(A, A)$. Those three states together with $(A, D)$ and $(B, E)$ are drawn in dashed lines in Fig. 3 and are to be removed in the next step. Furthermore, states $(B, D, \alpha \rightarrow e)$, $(A, C, \beta \rightarrow e)$ and $(A, B, \gamma \rightarrow e)$ become deadlocking after $(A, D)$ and $(B, E)$ are removed. They are drawn in dotted lines in Fig. 3 and are also to be removed.

Following the compositional approach with supervisor reduction presented in [29], [40], we calculate a least restrictive and nonblocking supervisor for the transformed automaton, which is shown in Fig. 4 as automaton $S$. All the paths accepted by this supervisor represent valid edit decisions. Consider the accepted path $SP$ shown in Fig. 4, it corresponds to an edit function’s decisions for string $\gamma \beta \alpha$ such that $f_\alpha(\gamma \beta \alpha) = \gamma(\gamma \rightarrow e)\gamma \beta \epsilon \alpha \gamma(\alpha \rightarrow e)\alpha$, which is shown by thick lines in $T$, Fig. 3. As is seen, $\rho(SP) = f_\alpha(\gamma \beta \alpha)$. Specifically, when event $\gamma$ is output by the system, $SP$ returns $\gamma(\gamma \rightarrow \epsilon)(\gamma \beta \epsilon \alpha \gamma(\alpha \rightarrow e)\alpha)$, which means erasing the $\gamma$. Next, event $\beta$ is output by the system and it is unchanged according to $SP$. Finally, $\alpha$ is output by the system and $SP$ returns $\alpha(\gamma \alpha)(\alpha \rightarrow \epsilon)(\alpha \alpha \rightarrow \epsilon \rightarrow \omega)$, which means erasing $\alpha$ and inserting $\gamma$. Similarly, we may consider other paths accepted by the supervisor in Fig. 4 to track edit decisions on other strings, then we have a complete picture of how an edit function works.

**Remark 4:** From the result in Theorem 8 our modular algorithm CA-AES results in fewer edit decisions compared with the monolithic approach in [29], [40]. This is due to the synchronization process in Section 14. This indicates that our method may not be complete in the sense that even if Algorithm CA-AES does not return any modular form edit functions, the monolithic approach may still return valid edit functions. This may be viewed as the tradeoff of reducing computational complexity by the modular method.

VI. CONCLUSION

This paper investigated a compositional and abstraction-based approach to synthesize edit functions for opacity enforcement in a modular setting, given a set of individual systems. The edit functions modify the system’s output by inserting and erasing events, under the constraint of limited number of event erasures. The Three-Player Observer (TPO) and All Edit Structure (AES) proposed in our prior work were employed here; these discrete structures embed edit functions and reflect the constraints. The monolithic approach first synchronizes all individual systems, then calculates the monolithic AES to obtain edit functions. In contrast, the compositional approach first exploits the modular structure and builds individual TPOs. Then, it incorporates the edit constraint and calculates the Modular Edit Structure in a nonblocking modular supervisory control manner to obtain edit functions. In addition, we also applied abstraction methods to reduce the state space of the system before opacity enforcement. We showed that the abstraction processes preserve opacity. Combining system composition and abstraction, we proposed an efficient approach to enforce opacity for complex systems containing multiple components.

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System $\mathcal{G}$

$G_1 \parallel \ldots \parallel G_n$

Edit function

$\sigma_0 \ldots \sigma_{i-1}$

$\gamma$

$\rho(\sigma_0 \ldots \sigma_i) \gamma$

Fig. 5. Process of selecting an edit decision from the modular All Edit Structure.