Refinement of the Use of Inhomogeneous Background Error Covariance Estimated from Historical Forecast Error Samples and its Impact on Short-Term Regional Numerical Weather Prediction

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Abstract

Background error covariance (BEC) is one of the key components in data assimilation systems for numerical weather prediction. Recently, a scheme of using an inhomogeneous and anisotropic BEC estimated from historical forecast error samples has been tested by utilizing the extended alpha control variable approach (BEC-CVA) in the framework of the variational Data Assimilation system for the Weather Research and Forecasting model (WRFDA). In this paper, the BEC-CVA approach is further examined by conducting single observation assimilation experiments and continuous-cycling data assimilation and forecasting experiments covering a 3-week period. Additional benefits of using a blending approach (BEC-BLD), which combines a static, homogeneous BEC and an inhomogeneous and anisotropic BEC, are also assessed.

Single observation experiments indicate that the noise in the increments in BEC-CVA can be somehow reduced by using BEC-BLD, while the inhomogeneous and multivariable correlations from BEC-CVA are still taken into account. The impact of BEC-CVA and BEC-BLD on short-term weather forecasts is compared with the three-dimensional variational data assimilation scheme (3DVar) and also compared with the hybrid ensemble transform Kalman filter and 3DVar (ETKF-3DVar) in WRFDA. The results show that BEC-CVA and BEC-BLD outperform the use of 3DVar. BEC-CVA and BEC-BLD underperform ETKF-3DVar, as expected. However, the computational cost of BEC-CVA and BEC-BLD is considerably less expensive because no ensemble forecasts are required.
1. Introduction

Accurate numerical weather forecasts depend on the accuracy of the initial conditions used by numerical weather prediction (NWP) models, which are usually estimated and optimized using data assimilation techniques. Among several factors, the background error covariance (BEC) plays a key role in data assimilation systems such as 3/4-dimensional variational data assimilation systems (3/4DVar), ensemble-based Kalman filters, and hybrid ensemble variational data assimilation systems. This is because BEC spatially spreads the observation information around it and defines the correlations among control variables. When BEC was introduced into 3/4DVar, a control variable transform along with the assumption of spatial homogeneity and isotropy in static error covariance estimation was usually used, which simplify the error modeling process and increase computational efficiency (Wu et al. 2002; Barker et al. 2004). However, 4DVar uses a forecast model as a dynamic constraint of the analysis, and thus the flow-dependent BEC is implicitly included (e.g., Sun and Crook 1997; Rabier et al. 2000; Honda et al. 2005; Rawlins et al. 2007; Huang et al. 2009; Wang et al. 2013; Lorenc et al. 2015). The establishment and maintenance of the adjoint model, however, involve tremendous effort, and the computational cost of 4DVar remains expensive for mesoscale and convective-scale data assimilation.

In addition to 4DVar, ensemble-based Kalman filters are other implementations that estimate flow-dependent BEC in an ensemble subspace (e.g., Evensen 1994; Anderson 2001; Bishop et al. 2001; Whitaker and Hamill 2002; Hunt et al. 2007). However, BEC calculated from an ensemble with a limited number of members might have sampling errors and require localizations to suppress long-distance spurious correlations and inflation techniques to enlarge the ensemble spread (Houtekamer and Mitchell 1998; Hamill and Snyder 2000; Bannister 2007; Kondo et al. 2016).

A hybrid data assimilation approach has been developed to take advantage of an existing variational data assimilation system and a flow-dependent estimation of BEC provided by an ensemble-based Kalman filter or other ensemble generation methods (Hamill and Snyder 2000; Lorenc 2003b; Buehner et al. 2010; Wang et al. 2007b; Clayton et al. 2013; Lorenc et al. 2015; Bowler et al. 2017). Hamill and Snyder (2000) constructed a hybrid scheme that directly replaces the BEC term in the cost function by a linear combination of a static BEC and an ensemble-based flow-dependent covariance. Lorenc (2003b) proposed another form of the hybrid variational scheme where the control variables in the cost function were augmented by another set of control variables, preconditioned on the square root of the ensemble covariance (Wang et al. 2007b). In general, the hybrid data assimilation method combines a flow-dependent BEC estimated from an ensemble and a static BEC in the variational framework.

Several studies have suggested that the hybrid method yields better forecasts than the 3DVar method (which does not incorporate the flow-dependent BEC) and is also more robust than ensemble-based Kalman filters (e.g., Wang et al. 2008a, b; Wang 2011; Kuhl et al. 2013; Wang et al. 2013; Zhang et al. 2013; Kleist and Ide 2015a, b). The hybrid method requires a set of ensemble forecasts to provide a flow-dependent BEC during each data assimilation cycle. The computational cost is significantly higher than that of 3DVar, where no ensemble forecasts are involved. The computational cost is still a burden for some operational centers and research communities with limited computational resources. In recent studies, some weight to static climatological BEC improved forecasts using operational global models (Clayton et al. 2013; Kleist and Ide 2015a, b; Lorenc et al. 2015; Bowler et al. 2017). The value of climatological BEC information in ensemble-based Kalman filters has also been demonstrated by Kretschmer et al. (2015). Their research indicates that the use of climatological error perturbations has significant potential for improving analyses and forecasts.

The static BEC estimation used in the hybrid method requires a set of historical forecasting error samples. They can be obtained from either historical ensemble perturbations or forecast differences obtained via the National Meteorological Center (NMC) method or a combination of both. Although some significant work has been carried out to improve the anisotropic and inhomogeneous BEC of 3D/4DVar using recursive filters and wavelets (Deckmyn and Berre 2005; Bannister 2007; Oliveira 2009), in most
studies, the static BEC usually assumes spatial homogeneity and isotropy in error covariance estimation and uses various control variable transforms (Bannister 2008). This assumption ignores the most valuable inhomogeneous and anisotropic information and somehow weakens multivariate correlations (Wang et al. 2014) and even introduces questionable solutions in wind analysis through control variable transform (Xie and MacDonald 2012; Wang et al. 2014; Sun et al. 2016).

To take advantage of BEC in the historical forecast error samples, Wang et al. (2014) explored a scheme that uses the extended alpha control variable approach (Lorenc 2003a) to introduce an inhomogeneous and anisotropic historical BEC (BEC-CVA) for WRFDA (Data Assimilation system for the Weather Research and Forecasting model). The final goal of Wang et al.’s (2014) work was to make best use of both historical forecast error samples and error samples of the day (e.g., from a short-term ensemble forecast at an analysis time) to produce the best estimation of BEC for data assimilation. As a first step, they introduced the BEC-CVA approach for better use of historical forecast error samples. Their study shows that the BEC-CVA approach is capable of extracting inhomogeneous and anisotropic climatological information from historical forecast differences obtained via the NMC method (Parrish and Derber 1992).

However, similar to ensemble-based Kalman filter approaches, the BEC-CVA approach has sampling errors (Wang et al. 2014). In addition, Wang et al. (2014) conducted a case study to examine the impact of the BEC-CVA method on short-term weather forecasts where radar data were one of the main data resources. More studies will help in understanding the implementations of the method and evaluating its performance. This research sheds light on ideas for the better use of static BEC and its potential role in a hybrid BEC data assimilation system.

This paper further examines the BEC-CVA approach by conducting single observation assimilation experiments and various real data assimilation and forecasting experiments. Additional benefits of considering the contribution from a static homogeneous BEC, which helps to reduce some noise in analysis increments, are also assessed. Based on the fact that the noise in the analysis increments in the BEC-CVA approach can be somehow reduced by combining contributions of a static, homogeneous BEC, the use of the blended BEC (BEC-BLD) that combines a static, homogeneous BEC and an anisotropic and inhomogeneous BEC on short-term regional weather forecasts is tested. Continuous-cycling data assimilation and forecasting experiments over a 3-week period were performed to examine the impact of a few variations of BEC such as BEC-CVA and BEC-BLD on short-term weather forecasts for heavy rainfall events occurring in east China. We also compared the BEC-CVA and BEC-BLD approaches with the traditional hybrid data assimilation scheme in WRFDA.

The rest of this paper is structured as follows. In Section 2, the method used to refine BEC is described. Section 3 introduces the experimental design and the rainfall event examined for the study. The single observation experiments are discussed in Section 4. In Section 5, the computational costs and verification scores for the continuous-cycling data assimilation and forecasting experiments over a 3-week period are compared. The diagnosis for a rainfall event using different methods is presented in Section 6. Finally, the conclusions are given in Section 7.

2. Method

WRFDA is used to explore the impact of variations in BEC formulations on NWP. The cost function that combines two different BECs in this paper can be written as a traditional hybrid method (Wang et al. 2008a; Lorenc 2003b):

$$J(\delta x, \alpha) = \beta \frac{1}{2} \delta x^T B^{-1} \delta x + \beta + \frac{1}{2} \alpha^T A^{-1} \alpha$$
$$+ \frac{1}{2} (d - H \delta x)^T R^{-1} (d - H \delta x).$$  (1)

In Eq. (1), $d = y - H(x^k)$ is the innovation vector, where $y$ is the observation and $H$ is the nonlinear observation operator whose linearized observation operator is denoted by $H$. The $R$ matrix represents the observation error covariance.

The first term on the right-hand side of Eq. (1) is the traditional background term, and $\delta x^k$ is the increment associated with the static BEC. In WRFDA, $B$ is a homogeneous and isotropic BEC typically estimated via the NMC method by taking the difference between forecast samples of different lead times to be valid at the same time (Parrish and Derber 1992). The second term on the right-hand side is the background term associated with the inhomogeneous BEC, where $\alpha$ is the extended control variable and $A$ is the correlation matrix of the extended control variable. $\delta x = \delta x^k + \sum_{k=1}^{K} (\alpha_k x_k^k)$ is the final analysis increment, and the vectors $\alpha_k (k = 1, \ldots, K)$ denote the extended control variables for each ensemble member. $\alpha_k$ is a vector formed by concatenating by $K$ vectors $\alpha_k$. In other
words, \( \alpha^r = (\alpha_1^r, \alpha_2^r, \ldots, \alpha_n^r) \). The coefficients \( \beta_1 \) and \( \beta_2 \) represent the weights applied for the static homogeneous BEC and the inhomogeneous BEC, respectively, and \( 1/\beta_1 + 1/\beta_2 = 1 \).

Recently, Wang et al. (2014) used the second term on the right-hand side of Eq. (1) to incorporate the inhomogeneous and anisotropic BEC by using historical forecast error samples, which can be obtained from a time series of ensemble perturbations and/or NMC-style forecast differences that are defined as the difference between forecasts with different forecast time lengths but valid at the same time.

In this paper, the historical NMC-type forecast differences are used. \( x_k \) is redefined as

\[
x^e_k = \frac{(x^\text{diff}_k - \bar{x})}{\sqrt{2M}},
\]

where \( M \) is the total number of forecast differences, \( x^\text{diff}_k \) is the forecast difference defined by Eq. (3), \( T1 \) and \( T2 \) are the forecast lead times, and \( \bar{x} \) is the time-averaged bias of forecast differences. It is noted that \( x^e_k \) here needs to be scaled for a specific application because \( (x^e)^T x^e \) is a combination of true forecast error covariances at the forecast lead times \( T1 \) and \( T2 \), and their covariances (Bannister 2008; Wang et al. 2014).

Compared with Wang et al.’s (2014) formation, the contribution from a static BEC is considered. As shown in Wang et al. (2014) and single observation assimilation experiments that will be depicted in Section 3, the analysis increments from BEC-CVA include small-scale noise. Addition of the static BEC term will reduce the weight of the inhomogeneous and anisotropic BEC and add smooth analysis increment components into the final analysis at the same time. Another benefit is that it expands the analysis increment in solution subspace. In practice, the BEC in Eq. (1) can combine a static \( B \) from a longer period of NMC-type forecast differences and an anisotropic and inhomogeneous BEC from another shorter period of forecast error samples. If the latter is replaced by ensemble forecast perturbations at an analysis time, then it is the standard hybrid scheme defined by Wang et al. (2008a).

3. Experimental setup and rainfall event

3.1 Experimental setup

The Advanced Research Weather Research and Forecasting (ARW-WRF) Model (V3.6.1) and WRFDA were used to investigate the impacts of BEC. All experiments were conducted over a single domain that covered the Yangtze–Huaihe area and its surrounding areas with a 181 × 151 horizontal mesh grid utilizing 12-km spacing and 41 vertical levels up to 50 hPa. The initial and boundary fields were interpolated from NCEP’s Global Forecast System (GFS) 0.5° × 0.5° analyses and forecasts. The WRF physics components are the WRF single-moment five-class (WSM) microphysics scheme (Hong et al. 2004), the Yonsei University (YSU) boundary layer scheme (Hong et al. 2006), the Kain–Fritsch cumulus parameterization scheme (Kain and Fritsch 1990), the Rapid Radiative Transfer Model (RRTM) long-wave radiation scheme (Mlawer et al. 1997), and the Dudhia short-wave radiation scheme (Dudhia 1989).

A variety of conventional and satellite observations can be assimilated by the WRFDA (Barker et al. 2012). The data assimilated include surface and upper-air observations of temperature, wind, surface pressure, and specific humidity in addition to aircraft reports of temperature and wind. Furthermore, satellite-tracked wind (SATOP) and aircraft reports (AIREP) observations are assimilated. Surface observations from surface synoptic observation (SYNOP), and aviation routine weather report (METAR) platforms are also assimilated. Observations taken within a 6-h window of each analysis are assimilated. All observations are assumed to be valid at the analysis time for each experiment.

The NMC-type forecast differences were used to derive BECs in all the experiments. For the isotropic and homogeneous BEC as denoted by \( B \) in Eq. (1), WRF 12-h and 24-h forecast differences valid at the same time during a 2-month period (1 June to 31 July 2010) were utilized as inputs for generating an isotropic and homogeneous BEC using the “GEN-BE” code in the WRFDA system with Control Variables Transform option CV5 (Chen et al. 2013). The forecasts were initiated at 00 and 12 UTC each day.

The anisotropic and inhomogeneous BEC presented by the alpha control variable in Eq. (1) was estimated using the NMC-type forecast differences as well, but with a different sampling period that starts from 00 UTC 15 June and ends at 18 UTC 14 July 2011. The forecast differences are generated every 6 h, so there are 118 forecast error samples in total (Fig. 1). Here, the two BECs from different sample periods are blended in BEC-BLD to better capture the error statistics and increase the freedom of analysis! solutions.

To examine the differences in the BEC-CVA and BEC-BLD methods from the traditional hybrid approach, a hybrid experiment based on the hybrid ensemble transform Kalman filter (ETKF) and 3DVar
(ETKF-3DVar) in WRFDA was also carried out. The hybrid experiment used the ETKF scheme to initiate ensemble forecasts that provide flow-dependent BEC; a 32-member ensemble was used. After 6 h of spin-up, the initial ensemble at the very beginning of the data assimilation cycles and the lateral boundary condition ensembles during the cycles were generated by adding 32 perturbations to GFS analyses. These perturbations were sampled from a static background error covariance (named “RandomCV” in WRFDA) (Wang et al. 2008a, b). Then, the ensembles in the following cycle were updated by ETKF. The weighting coefficient of ensemble-based BEC is 75 %; the horizontal covariance localization length scale is 200 km. The static BEC is used as described before. More detailed descriptions on the ETKF-3DVar method are given in papers by Wang et al. (2008a, b).

3.2 Rainfall events

Heavy rainfall events in the Yangtze–Huaihe River Basin in China during the summer season usually caused economical and life losses in east China. The experimental period starting on 17 July and ending on 9 August 2011 covered a few precipitation events. Figure 2a displays the 24-h accumulated precipitation reported by the China Hourly Merged Precipitation Analysis (CHMPA) (Shen et al. 2014), starting from 12 UTC 18 to 12 UTC 19 July. The 850 hPa large-scale horizontal winds and precipitable water at 12 UTC of 18 July 2011 are shown in Fig. 2b, which are derived from final global tropospheric analyses produced by NCEP’s GFS (Rutledge et al. 2006). The 24-h accumulated precipitation distribution shows a south–north (S–N) rainfall belt, and the heaviest rainfall center (exceeding 100 mm) is located at 32.5–34.5°N along 117.5°E (Fig. 2a). The southeastern flow associated with the cyclonic vortex occurs around (27°N, 119°E) (Fig. 2b), and strong water vapor flux convergence (Fig. 2c) provide a favorable environment for the heavy rainfall event.

4. Single observation experimental results

Before performing real data assimilation experiments, seven single observation assimilation experiments (Table 1) were conducted to demonstrate the analysis increment difference among 3DVar, ETKF-3DVar, BEC-CVA, and BEC-BLD when different weights were given to inhomogeneous BEC and different horizontal localization length scales were used. The experiment BEC-BLD_J0.75_L200 (BEC-BLD_J0.50_L200) means that the weighting coefficient of the inhomogeneous ensemble-based BEC is 75 % (50 %) and the horizontal localization length scale is 200 km. The experiment BEC-BLD_J0.75_L100 (BEC-BLD_J0.75_L300) means that the weighting coefficient of the inhomogeneous ensemble-based BEC is 75 % and the horizontal localization length scale is 100 km (300 km). BEC-CVA (3DVar) is a specific application of the blended method giving the weight with a value of 100 % (0) to the inhomogeneous BEC. The location of the single observation is at (32°N, 111°E) and the 21st vertical level (approximate pressure level 650 hPa).

4.1 Structures of specific humidity increments

Figure 3 shows the specific humidity increments at the 21st level as a result of assimilating a single specific humidity observation (the innovation of specific humidity is 0.001 g kg⁻¹, and the observation error is 0.001 g kg⁻¹). The pattern of the specific humidity increment in 3DVar shows characteristics of isotropy and homogeneity (Fig. 3a). In contrast, the increments from the BEC-BLD, BEC-CVA, and ETKF-3DVar experiments are clearly characterized by the aniso-
tropic and inhomogeneous features (Figs. 3b–g). The increments of specific humidity in the BEC-BLD, BEC-CVA, and ETKF-3DVar experiments are more physically reasonable than that in 3DVar because the increments are extended along the background specific humidity with large gradients. It is also seen that the amplitude of the increments in ETKF-3DVar is less than the increments of BEC-BLD and BEC-CVA, which indicates that the variance of the moisture background errors of BEC-BLD and BEC-CVA is larger than that of ETKF-3DVar.

Comparing the three BEC-BLD experiments with different horizontal localization length scales (100, 200, and 300 km) (Figs. 3c, e, f), it is obvious that the smaller horizontal localization length scale restricts the increments to a local area around the position of single observation.

4.2 Multivariate analyses

The temperature and wind increments, resulting from the assimilation of a single specific humidity observation, are shown in Figs. 4 and 5.

The temperature and wind increments in 3DVar are zero (Figs. 4a, 5a), because the control variables adopted in 3DVar for this study are the default control variable options (CV option = 5) in WRFDA that do not take into account the correlations between moisture and other control variables (Chen et al. 2013).

Figures 4b–g show that the patterns of the temperature increment by assimilating a single specific humidity observation in the BEC-BLD, BEC-CVA, and ETKF-3DVar experiments distribute roughly along the region where the background temperature has large

| Experiment name     | Weighting coefficient \((1/\beta^2)\) | Horizontal localization scaling |
|---------------------|--------------------------------------|---------------------------------|
| 3DVar               | –                                    | –                               |
| BEC-BLD J0.50_L200  | 0.50                                 | 200                             |
| BEC-BLD J0.75_L200  | 0.75                                 | 200                             |
| BEC-CVA             | 1.00                                 | 200                             |
| BEC-BLD J0.75_L100  | 0.75                                 | 100                             |
| BEC-BLD J0.75_L300  | 0.75                                 | 300                             |
| ETKF-3DVar          | 0.75                                 | 200                             |
Fig. 3. Specific humidity increments (shaded; g kg$^{-1}$) at the 21st level as a result of assimilating a single specific humidity observation: (a) 3DVar, (b) BEC-BLD J0.50_L200, (c) BEC-BLD J0.75_L200, (d) BEC-CVA, (e) BEC-BLD J0.75_L100, (f) BEC-BLD J0.75_L300, and (g) ETKF-3DVar. The black contours in (a)–(g) represent the background specific humidity (g kg$^{-1}$) at the 21st level, which is derived from GFS analyses at 00 UTC 17 July 2011.
Fig. 4. The same as Fig. 3 but for temperature increments (shaded; K). The black contours in (a)–(g) are the background temperature (K) at the 21st level, which is derived from GFS analyses at 00 UTC 17 July 2011.
Fig. 5. The same as Fig. 3 but for wind increments (vector; m s$^{-1}$). The color shades in (a)–(g) are the pressure field (hPa) at the 21st level, which is derived from GFS analyses at 00 UTC 17 July 2011.
gradients. In Figs. 5b–f, there are noticeable cyclonic wind increments near the location of the low-pressure system by assimilating a single specific humidity observation. The results show that the multivariate correlations among moisture, temperature, and wind can be modeled by the blended BEC. From Figs. 4e, 4f, 5e, and 5f, one can also conclude that the smaller horizontal localization length scale limits the increments to a local area around the single observation.

4.3 Smooth effect

To show the noise problem caused by sampling errors in BEC-CVA more clearly, the temperature (T) increment values along the south–north (Fig. 6a) and west–east (Fig. 6b) that go through the single observation point (32°N, 111°E) by assimilating a single temperature observation (the innovation of temperature is 1.0 K, and the observation error is 1.0 K) with 3DVar, BEC-CVA, and BEC-BLD are compared. It should be noted that the weighting coefficient in BEC-BLD is 50 %, and the length scale is 200 km.

It can be seen that the increment of 3DVar is a unimodal shape with a smooth curve; the general increment structures of BEC-CVA and BEC-BLD are similar to that of 3DVar, although parts of the nonsmooth feature in BEC-CVA may be caused by sampling errors. There are still some noisy features in BEC-BLD. However, because of the blending static BEC and inhomogeneous BEC, BEC-BLD is smoother than BEC-CVA and closer to 3DVar. Particularly, the smooth effect is more obvious in the distance from the single observation point, which indicates that the noise in BEC-CVA can be reduced by BEC-BLD.

5. Continuous-cycling data assimilation and forecasting

A series of 6-h 3-week (00 UTC 17 July to 18 UTC 9 August 2011) continuous-cycling data assimilation and forecasting experiments (Table 2) were carried out using 3DVar, BEC-BLD, BEC-CVA, and ETKF-3DVar. The weighting coefficients of the ensemble-based BEC in ETKF-3DVar, BEC-BLD, and BEC-CVA are 75 %, 75 %, and 100 %, respectively. The background for each following assimilation cycle is the 6-h forecast of the previous cycle. The 24-h forecasts are made every 6 h. In this section, the computational costs, root-mean-square error (RMSE), and accumulated precipitation fractions skill score (FSS) (Roberts et al. 2008) for the 3-week cycling are discussed.

5.1 Computational cost

Compared with 3DVar, the additional computational cost of ETKF-3DVar mainly comes from (1) ensemble Kalman filter analysis, (2) ensemble forecasts, and (3)

| Experiment name | Weighting coefficient (1/β)
|-----------------|--------------------------|
| 3DVar | – |
| BEC-BLD | 0.75 | 200 |
| BEC-CVA | 1.00 | 200 |
| ETKF-3DVar | 0.75 | 200 |

Table 2. List of 3-week cycling experiments

![Fig. 6](image-url) Temperature increments (K) along two lines that go through the single-observation point (32°N, 111°E) at the 21st level as a result of assimilating a single temperature observation: (a) south–north and (b) west–east. The weighting coefficient in BEC-BLD is 50 %, and the length scale is 200 km.
computation of extended control variables in the variational cost function. The first two steps take up most of the additional cost in ETKF-3DVar, while BEC-BLD avoids the two steps.

Table 3 lists the total computational cost (wall clock time), on a Linux workstation with 32 CPU processors, used by the 3DVar, BEC-BLD, and ETKF-3DVar experiments for the 3-week cycling. The weight to ensemble-based BEC in BEC-BLD and ETKF-3DVar is 75%. It can be seen that 3DVar uses 756 min. Compared with 3DVar, the BEC-BLD experiment adds 195 min because of the use of extended control variables in the analysis step. However, ETKF-3DVar needs the ensemble forecasts and the corresponding EnKF analysis; therefore, ETKF-3DVar with 32 members uses 6792 min of wall clock time, which is about 9 times that of 3DVar and 7 times that of BEC-BLD. Such a computational cost might be attractive for real-time implementations for some operational centers and research communities with limited computational resources.

5.2 Verification score

In this subsection, the RMSE values from the continuous-cycling data assimilation and forecasting experiments over the 3-week period were calculated against GFS analyses.

Table 3. Total computational cost of 3-week cycling data assimilation and forecasting

| Experiment name | Weighting coefficient \((1/\beta)\) | Total Computational Cost (min) |
|-----------------|------------------------------------|-------------------------------|
| 3DVar           |                                     | 756                           |
| BEC-BLD         | 0.75                               | 951                           |
| ETKF-3DVar      | 0.75                               | 6792                          |

Fig. 7. Vertical RMSE profiles of averaged analysis against GFS analyses. The red line denotes 3DVar, the black line denotes BEC-CVA, the orange line denotes BEC-BLD, and the blue line denotes ETKF-3DVar. Error bars show the confidence interval of the mean RMSE for that level (95% confidence limit).
BEC-CVA are better than 3DVar but worse than ETKF-3DVar for almost all the forecast times. The RMSE difference of the four experiments becomes smaller with longer forecasting time, because the effect of the initial field decreases gradually.

Figure 9 shows the time series of the averaged FSS of 6-h accumulated precipitation over the 3 weeks (Fig. 9a) and its statistical significance at the 90% level (Fig. 9b). It can be seen that BEC-BLD and BEC-CVA are better than 3DVar but worse than ETKF-3DVar for almost all the forecast times. Furthermore, it is seen that compared with 3DVar and BEC-CVA, the refinement of BEC-BLD can improve precipitation forecasts slightly but not significantly.

6. More details for a rainfall event

The first rainfall event during 17–19 July in the 3-week period is examined in detail in this section.

6.1 Hourly accumulated precipitation FSS

The hourly accumulated precipitation FSS is shown in Fig. 10. The horizontal scale of FSS in this study is 24 km. FSS is averaged over the nine forecasts, which are initialized every 6 h (00, 06, 12, 18) during the 2-day assimilation cycles (00 UTC 17 to 00 UTC 19 July 2011).

The hourly accumulated precipitation FSSs with a threshold of 1 mm h$^{-1}$ in the BEC-BLD experiments with different weighting coefficients compared with 3DVar, BEC-CVA, and ETKF-3DVar are shown in Fig. 10a. The scores of BEC-BLD and BEC-CVA are better than that of 3DVar but worse than that of ETKF-3DVar for almost all the forecast times. For the two BEC-BLD experiments, the averaged FSS values in BEC-BLD_J0.75_L200 are clearly greater than those in BEC-BLD_J0.50_L200 when the forecast time is less than 18 h, and after 18 h BEC-BLD_J0.50_L200 is the best one.

Figure 10b presents the FSS of BEC-BLD experiments with different horizontal localized scales. It can be seen that the FSS values of the BEC-BLD experiments are greater than that of 3DVar. The FSS of BEC-BLD_J0.75_L200 is the best one before 21 h, and BEC-BLD_J0.75_L200 is worse than BEC-BLD_J0.75_L100 after 21 h. Therefore, horizontal localization 200 km and weighting coefficients 75% are appropriate for the simulation of this rainfall case.

![Fig. 8. Time series of averaged RMSE following forecast time (00, 06, 12, 24) against GFS analyses. The red line denotes 3DVar, the black line denotes BEC-CVA, the orange line denotes BEC-BLD, and the blue line denotes ETKF-3DVar. Error bars show the confidence interval of the averaged RMSE for that forecast time (95% confidence limit).]
Fig. 9. (a) Time series of averaged FSS (Roberts et al. 2008) of 6-h accumulated precipitation over the 3 weeks (with thresholds of 1 mm h$^{-1}$). The red line denotes 3DVar, the black line denotes BEC-CVA, the orange line denotes BEC-BLD, and the blue line denotes ETKF-3DVar. (b) Significance test. The horizontal line represents “zero lines” for differences between experiments based on the 90 % bootstrap. The differences between the two experiments were statistically significant at the 90 % level if the error did not include zero.

Fig. 10. Hourly accumulated precipitation FSS (averaged over the nine forecasts that are initialized every 6 h during the 2-day assimilation cycles) with thresholds of 1 mm h$^{-1}$: (a) 3DVar, BEC-CVA, ETKF-3DVar, and BEC-BLD experiments with different weighted coefficients and (b) 3DVar and BEC-BLD experiments with different localized length scales. The horizontal axis is the forecast time (h), and the vertical coordinate is the average value of FSS.
6.2 Diagnosis for the rainfall case

To better understand the performance of the different experiments, precipitation distribution and vapor flux divergence for the rainfall case are diagnosed in this section. For the sake of brevity, the results of BEC-BLD_J0.75_L200 are selected to represent BEC-BLD. The results of BEC-BLD_J0.75_L100, BEC-BLD_J0.75_L300, and BEC-BLD_J0.50_L200 will not be shown.

a. Precipitation distribution

The 24-h accumulated precipitation (shaded; mm) initialized at 12 UTC 18 July 2011 is shown in Fig. 11. Figure 11a shows the real 24-h accumulated precipitation from CHMPA, and Figs. 11b–e show the simulated 24-h accumulated precipitation initiated at 12 UTC 18 July 2011. The precipitation from CHMPA shows a rainfall band that extends roughly in the N–S direction, and the rainfall center is located...
at approximately 32.5–34.5°N along 117.5°E. It can be seen that the accumulated precipitation amount in 3DVar (Fig. 11b) is much lower than that in CHMPA, and BEC-CVA (Fig. 11d) is closer to that of CHMPA than 3DVar, but the precipitation distribution of BEC-CVA is still different from CHMPA. The location of the rainfall center and precipitation distribution of BEC-BLD (Fig. 11c) is closer to CHMPA than 3DVar and BEC-CVA. The location of the rainfall center in ETKF-3DVar (Fig. 11e) is closest to that of CHMPA, but the rainfall area, especially where the precipitation is over 50 mm, of ETKF-3DVar is smaller than that of CHMPA.

b. Vapor flux convergence

The vapor flux convergence is useful in diagnosing the potential intensity of the precipitation. The 6-h forecasts for vapor flux divergence in the main rainfall region at 850 hPa initialized at 12 UTC 18 July 2011 are presented in Fig. 12. The negative vapor flux divergence means water vapor convergence. It is seen that 3DVar has weak vapor flux convergence in the rainfall center (Fig. 12a), which also explains the weaker model-simulated precipitation in 3DVar (Fig. 11b). Comparing 3DVar, which is more southerly than the rainfall center located at 32.5–34.5°N along 117.5°E, the distributions of water vapor convergence in BEC-BLD and BEC-CVA (Figs. 12b, c) are consistent with the rainfall distribution (Figs. 11c, d). The convergence intensity in BEC-BLD (Fig. 12b) is larger than that in BEC-CVA (Fig. 12c), which can be one reason why the area of 24-h accumulated precipitation larger than 50 mm in BEC-BLD is larger and closer to CHMPA than BEC-CVA. For ETKF-3DVar (Fig. 12d), the location of water vapor convergence is more similar to the rainfall center, and the intensity is greater than that of other experiments.

7. Conclusions and discussion

In this paper, the inhomogeneous and anisotropic BEC from the BEC-CVA approach and the homogeneous and isotropic BEC from WRF-Var BEC...
modeling are blended within the hybrid framework of the WRFDA system. The performance of the blended BEC (BEC-BLD) was assessed by conducting single observation experiments and 3-week continuous-cycling data assimilation and forecasting experiments, and details of the heavy rainfall case in the 3-week period over the Yangtze–Huaihe River Basin in China were discussed.

Single observation assimilation experiments indicate that by using BEC-BLD with the blended BEC, the noise produced by the BEC-CVA approach caused by sample errors is reduced, and multivariate correlations between moisture and other control variables are introduced. Furthermore, the increments of the BEC-BLD experiments using the blended BEC are anisotropic and inhomogeneous.

The 3-week cycling data assimilation and forecasting experiments show that the BEC-BLD experiments perform better than the 3DVar and BEC-CVA experiments in both analyses and precipitation forecasts. The diagnostic study on the rainfall case that occurred during 17–19 July 2011 shows that, compared with 3DVar and BEC-CVA, the BEC-BLD experiments provide more physically favorable dynamical and water vapor environments for the heavy precipitation event and thus lead to an improvement in the location and intensity of the precipitation forecast.

It is noted that ETKF-3DVar produced the best forecasts among all the experiments, because ETKF-3DVar uses the flow-dependent BEC. ETKF-3DVar requires an ensemble of real-time short-term forecasts to provide the flow-dependent BEC. However, BEC-CVA or BEC-BLD requires only an ensemble of historical forecast error samples, and the computational cost is similar to that of 3DVar and notably less than that of ETKF-3DVar (hybrid). Such a computational cost might be attractive for real-time implementations for some operational centers and research communities with very limited computational resources. The experimental results are sensitive to horizontal localization scales. However, it is not easy to choose one optimal localization scale for all the experiments because forecast errors exist as multiple scale features. A better solution is to develop multiscale localization schemes, which is a subject of future work.

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