Adaptive sliding mode control of electro-hydraulic servo system based on RBF network

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Abstract. In order to solve the steady-state error in position tracking control for electro-hydraulic servo universal testing machine, caused by uncertain parameters in the system model, an adaptive sliding mode control strategy based on RBF neural network is proposed for this situation. This paper utilizes the adaptive ability of RBF neural network to improve the control quality of the electro-hydraulic position servo system. The strategy has three parts: the equivalent control, the reaching law control and the compensation control based on RBF network. Simulations verify that the control system can track the reference curve well with unknown parameters.

1. Introduction
Electro-hydraulic position servo systems are widely used in testing machines because of their good position capabilities, high power-to-weight ratio, high stiffness, fast response. However, the actual servo systems have highly nonlinear characteristics and many uncertain parameters, such as elastic stiffness of elastic load, hydraulic fluid bulk modulus. Sliding mode control is robust with respect to system uncertainties through the use of switching control. However, the previous sliding mode controller design requires the boundary of the system model uncertainty parameters to be known. In practical engineering, the boundary of the system uncertainty parameters is likely to be unknown, which leads to the control effect of the designed sliding mode controller is not ideal.

In order to solve this issue, a lot of researchers have proposed some advanced algorithms. Paper [1] uses adaptive algorithm and the principle of fuzzy approximation to estimate the unknown parameters of the system. Paper [2] proposes an adaptive output feedback control scheme for the hydraulic position servo system with uncertain parameters and unmeasurable states. Paper [3] constructs a discontinuous projection-based adaptive control law to handle parametric uncertainties. Paper [4] investigates a fast speed adaptive law to estimate the variant perturbation. Neural networks have received much attention because of their adaptability, and many studies have combined them with SMC. In paper [5], an adaptive particle swarm optimization wavelet neural network with double sliding modes controller is proposed to address the complex nonlinearities and uncertainties in the electric load simulator. Paper [6] introduces a recurrent type 2 fuzzy wavelet neural network to approximate the unknown nonlinear functions of the dynamic systems through tuning by the desired adaptive law. In paper [7] and paper [8], RBF neural network is used to approximate the unknown system parameters. Paper [9] and [10] use RBF neural network to compensate the model uncertainties.

This paper focuses on the position control system of electro-hydraulic servo fatigue testing machine and proposes an adaptive sliding mode robust tracking control method which combining the sliding
mode control based on the approach law with the RBF neural network. The method is easy to implement, and the simulation research on the position control system model of the fatigue testing machine with elastic load has achieved satisfactory results.

2. System Description

The hydraulic system is shown in Figure 1 comprises a double-rod cylinder, a 4-way servo valve and a load force. In the following, the dynamic model of the system will be given.

![Figure 1. Schematic diagram of the hydraulic system](image)

2.1. Dynamic models

The mathematical model of the electro-hydraulic servo system actuator is as follows:

\[ Q = K_q x_v - K_p P_l \]
\[ Q = A_p Y_s + \frac{V_t}{4 \beta_s} \cdot P_l s + C_u P_l \]
\[ A_p P_l = m Y_s^2 + B_p Y_s + KY + F_l \]  \hspace{1cm} (1)

Where \( Q \) is flow through the valve orifice, \( K_q \) is flow gain, \( x_v \) is valve opening, \( K_p \) is pressure coefficient, pressure difference between the two cylinder chambers is \( P_l \), \( \beta_e \) is the cylinder effective area, \( Y \) is the position of the cylinder measured by displacement sensor, \( V_t \) is the effective cylinder volume, \( C_u \) is leakage, \( m \) is total load mass, \( B_p \) is damping, \( K \) is equivalent spring coefficient of the elastic load, \( F_l \) is disturb.

In order to simplify the analysis, the servo valve is considered as a proportional link, which means control valve dynamics is neglected and system input is the valve opening \( x_v = u \); the load of the fatigue testing machine is equivalent to the elastic load. When the elastic test piece is clamped, the open-loop transfer function of the system is:

\[ G(S) = \frac{U_s}{\Delta U} = \frac{K_w K_4 K_5 A_p}{s (s / \omega_o + 1)(s^2 / \omega_o^2 + 2 \xi_0 s / \omega_o + 1)} \]  \hspace{1cm} (2)

Where \( K_w \) is magnification factor for servo valve, \( K_i \) is gain of servo amplifier, \( K_i \) is gain of displacement sensor. Where:

\[ K_{ce} = K_c + C_u, \omega_o = \left( \frac{4 \beta_e A_p^2}{m V_t} + \frac{K}{m} \right)^{1/2}, \alpha_t = \frac{K_e K}{A_p^2 (1 + KV_t / 4 \beta_e A_p^3)}, \xi_0 = \frac{1}{2 \omega_o} \left[ \frac{4 \beta_e K_{ce}}{V_t (1 + KV_t / 4 \beta_e A_p^3)} + \frac{B_p}{m} \right]. \]

2.2 Converting the transfer function to a controllable standard state space equation

Defining state variables: \( x = [x_1 x_2 x_3] = [y \dot{y} \ddot{y}]^T \). Converting the transfer function into a controllable standard state space as follows:
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= -a_1 x_1 - a_2 x_2 - a_3 x_3 + bu \\
y &= x_1
\end{align*}
\]  \hspace{1cm} (3)

Where \( a_i = \alpha_1^2 \omega_1, a_2 = \alpha_2^2 + 2\beta_1 \omega_1 \alpha_0, a_3 = \alpha_3 + 2\beta_2 \omega_1 \alpha_0, b = \alpha_0^2 \alpha_0 K_v, K_v = K_m K_q K_c A / K_s\).

For an electro-hydraulic position servo system such as in equation (3), when parameters, such as \( K, m, K_q, K_c, \beta_c \), are uncertain, system parameter \( a_i (i=1,2,3)\) and control gain \( b \) will be different from the nominal parameters used when designing the controller. At this point, make assumptions:

- Position reference \([\hat{r}, \hat{\dot{r}}, \hat{r}, r]\) are known as bounded and continuous;
- System state \( x \) is the parameter which can be obtained.

According to the assumptions, system can be described as a linear uncertain system as follows:

\[
\dot{x} = (A_n + \Delta A)x + (b_n + \Delta b)u
\]

\( y = x_1 \)  \hspace{1cm} (4)

Where \( A_n, b_n \) are the system’s rated parameters, \( \Delta A, \Delta b \) are the uncertain parts of the system.

The goal of this paper is to compensate the disturbance caused by the system in the presence of uncertain parameters, so that the actual displacement of the actuator can follow the reference signal as much as possible within a limited time.

3. Design of sliding mode controller based on RBF neural network compensation

The design of sliding mode control is divided into two parts: the first part is to select the sliding surface, on which the system should meet the required performance; the second part is to choose the ideal sliding mode control law to make the system state can reach the sliding surface in a limited time, ensuring that the system enters the sliding mode.

3.1 Sliding mode control based on the approach law

In this paper, the design of the sliding mode controller adopts the equivalent control based on the approach law and the structure of the control input is:

\[
u_{SMC} = u_{eq} + u_{sw}
\]

(5)

Where \( u_{SMC} \) is the output of SMC, \( u_{eq} \) is equivalent control output, \( u_{sw} \) is switching control output.

Choose linear sliding surface function to design the sliding surface:

\[
s = C \cdot x
\]

(6)

Where \( C = [c_1, c_2, \cdots, c_{n+1}, 1], c > 0 \) and must be Hurwitz.

Concerning about equation (3), design the sliding surface as:

\[
\begin{align*}
eg_1 &= r - x_1 \\
eg_2 &= \dot{r} - x_2 \\
eg_3 &= \ddot{r} - x_3 \\
s &= c_1 \neg_1 + c_2 \neg_2 + \neg_3
\end{align*}
\]

(7)

Make \( \dot{s} = 0 \) to obtain the equivalent control law:

\[
\dot{s} = c_1 \neg_1 + c_2 \neg_2 + \ddot{r} - x_3 = c_1 \neg_2 + c_2 \neg_1 + \ddot{r} - (-a_1 x_1 - a_2 x_2 - a_3 x_3 + bu) = 0
\]

\[
u_{eq} = \frac{1}{b} [(c_1 \hat{r} + c_2 \hat{\dot{r}} + \hat{r}) + a_1 x_1 + (a_2 - c_1)x_2 + (a_3 - c_2)x_3]
\]

(9)

Selecting the exponential approach law as the sliding mode controller switching control:

\[
s = -q \text{sgn}(s) - ks, \quad q > 0, k > 0
\]

(10)
make $\dot{s} = c_1\dot{s} + c_2s + f - x_1 = -q\text{sgn}(s) - ks$, we can get the control law of sliding mode controller based on the approach law:

$$u_{SMC} = \frac{1}{b}(c_1s + c_2s + f + a_1\dot{x}_1 + a_2x_2 + a_3\dot{x}_3 + q\text{sgn}(s) + ks) \quad (11)$$

### 3.2 Compensation control based on RBFNN

In this paper, we use RBFNN to compensate the uncertain part of the model. The control structure of SMC based on RBFNN compensation can be expressed as:

$$u = u_{SMC} + u_{RBF} \quad (12)$$

The control structure is illustrated in Figure 2:

![Figure 2. Block diagram of RBFNN sliding mode control](image)

In this paper, the structure of the RBFNN, as illustrated in Figure 3, is 3-2-1, which means the numbers of input-layer neuron, hidden-layer neurons, linear output-layer are respectively to be 3, 2, 1.

![Figure 3. RBFNN Structure in this paper](image)

Where $w_j$ is the hidden-layer-to-outputs interconnection weights, $j = 1, 2, 3$, $h_i$ is the hidden layer with RBF activation function, $i = 1, 2$, which can be expressed as:

$$h_j = \exp\left\{\frac{\|x_i - \theta_j\|^2}{2b^2}\right\}, \quad b > 0 \quad (13)$$

Where $x_i$ is the input of the network, $\theta_j$ is the center of RBFNN. The output of the output-layer is the linear weighted sum of the hidden layer radial basis functions output values. The output of output-layer can be defined as:

$$y = \sum_{j=1}^{3} w_j h_j \quad (14)$$

Finally, the output of RBFNN can be expressed as:

$$u_{RBF} = \frac{1}{b} W^T h \quad (15)$$

Use $x = [s(k), \dot{s}(k)]^T$ as RBF network inputs, take $E = s(k)\dot{s}(k)$ as evaluation metric, adjust the weight of RBF network by using gradient descent method, we can get the adjustment method as:

$$\Delta \omega_j(k) = -\eta_j \frac{\partial E(k)}{\partial \omega_j(k)} = -\eta_j \frac{\partial E(k)}{\partial \dot{s}(k)} \frac{\partial \dot{s}(k)}{\partial u(k)} \frac{\partial u(k)}{\partial \omega_j(k)} + u_{RBF}(k) \quad (16)$$

Where:
\[ \frac{\partial E(k)}{\partial s(k)} = s(k), \quad \frac{\partial(s(k))}{\partial u(k)} = -b, \quad \frac{\partial(u_{SMC}(k) + u_{RBF}(k))}{\partial \omega_j(k)} = \frac{\partial u_{RBF}(k)}{\partial \omega_j(k)} = \frac{1}{b} h_j(k). \]

Use momentum factor \( \alpha \) to accelerate the learning rate of the network, the weight is update as:

\[ \omega_j(k + 1) = \omega_j(k) + \eta_j s(k) h_j(k) + \alpha [\Delta \omega_j(k - 1) - \Delta \omega_j(k - 2)] \quad \text{(17)} \]

4. Simulations

The simulation environment of this paper is MATLAB R2016a, use the traditional sliding mode controller based on the approach law to compare with the controller designed in this paper. The reference signal of the system is \( s_d(t) = 10 \sin(2\pi t) \) mm, nominal parameters of fatigue testing machine model for simulation are as follows:

\[ K = 1 \times 10^5 \text{N/m}, \quad B_p = 8.82 \times 10^{-2} \text{N}\cdot\text{s/m}, \quad A_p = 1.01 \times 10^{-2} \text{m}^2, \quad V_t = 1.05 \times 10^{-3} \text{m}^3, \quad \beta_k = 700 \text{MPa}, \quad m = 150 \text{kg}, \quad K_d = 0.4175 \text{m}^3/(\text{A}\cdot\text{s}), \quad K_s = 100 \text{V/m}, \quad K_a = 4 \times 10^{-3} \text{A/V}, \quad K_{ce} = 0.5 \times 10^{-11} \text{(m}^3/\text{s})/\text{Pa}. \]

The sliding mode controller parameters are selected as follows:

\[ C_1 = 5000, \quad C_2 = 700, \quad q = 100, \quad k = 200. \]

Parameters of RBFNN are as follows:

\[ b_i = 50, \quad W(0) = 0, \quad \eta = 8, \quad \alpha = 0.3, \text{ the centers are set as follows:} \]

\[ \theta = \begin{bmatrix} -50 & -25 & 0 & 25 & 50 \end{bmatrix}^T \]

Model state at the begin of the simulation is \( x(0) = [0 \ 0 \ 0]^T \). Through simulation, it can be found that, as shown in Figure 4, when the actual parameters of the system are consistent with the rated parameters of the controller, the sliding mode control based on exponential approach law and the sliding mode control based on RBFNN compensation can track the reference signal fast, but obviously RBFNN slip the mode controller converges faster than the typical sliding mode controller.

![Figure 4. When parameters are consistent](image1)

As shown in Figure 5, Figure 6, Figure 7, when there are unknown parameters in the model, the control results of the sliding mode controller based on the exponential approach law have steady-state error. The sliding mode controller combined with RBF compensation and exponential approach law is insensitive to unknown parameters, its curve and reference signal curve almost coincide, and still have good tracking performance.
5. Conclusion

In this paper, the RBF neural network is used to compensate the interference caused by unknown parameters in the system, thus basically eliminating its influence on the electro-hydraulic servo position control. The connection weight of the RBF neural network which is used in this paper does not need offline training and the structure is simple and easy to implement. The simulation results show that the controller designed in this paper can still have excellent tracking control effect when the controller rated parameters are inconsistent with the actual system parameters.

6. References

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