Long-term performance of packaged fiber Bragg grating sensors for strain monitoring inside creep medium

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ABSTRACT

To investigate the long-term performance of the packaged fiber Bragg grating (FBG) sensors embedded in civil infrastructure for strain monitoring, in this paper, the influence of host matrix’s creep effect on the behavior of the FBG sensors was systematically studied through theoretical, numerical, and experimental analysis. A theoretical strain transfer analysis between the optic fiber, packaging layer, and host matrix to consider the creep effect of the host matrix was performed accordingly for long-term strain monitoring. Parametric studies were carried out using numerical analysis for FBG sensors packaged with glass fiber reinforced plastic (GFRP), also known as FBG-GFRP sensors in concrete, as an example. The results show that embedded in a creep medium, an acceptable long-term performance of packaged FBG sensors requires the packaging layer to have a minimum length and maximum thickness. Laboratory long-term creep tests using epoxy resin and concrete as host matrix for FBG-GFRP sensors also clearly demonstrated that the influence of creep effect cannot be ignored for strain measurements if the host matrix has a creep potential and the developed correction model performed well to reduce measurement errors of such sensors in creep medium.

KEYWORDS

Fiber Bragg gratings (FBG) sensor; strain transfer; creep; packaging layer; epoxy resin; concrete

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1. Introduction

The service lives of civil infrastructures such as super-span bridges and other spatial structures, super-tall buildings, large-scale water conservancy projects, large-scale offshore platforms, and nuclear power plants, are expected to be decades to over 100 years. Local deteriorations of matrix materials are inevitable in long service life, among which the creep behavior plays a significant role for deterioration. Concrete is widely used in modern civil engineering construction, and its creep property leads to the development of strain, excessive deflections, and cracking which compromise the sustainability and durability of structures [1–3]. A better understanding of creep response plays an important role in evaluating the overall mechanical properties and structural safety of the infrastructure during their whole-life cycle.

Structural health monitoring (SHM), especially, real-time strain monitoring, has been widely used to identify structural mechanical properties, and evaluate the safety performance and service life of civil infrastructure [4]. To monitor the strains of these structures, there are several types of strain sensors that can be used in the field, including electrical resistance gauges, piezo-electrical sensors, and fiber optic sensors [5]. Fiber optic sensors, especially fiber Bragg grating (FBG) sensors, have attracted intensive attention among these strain sensors due to the advantages of high sensitivity, small size, and immunity to corrosion and electromagnetic interference [6–9]. However, since the bare FBG sensor is made of silicon dioxide with a diameter of 125 μm, it is difficult for the bare FBG sensors to survive during the harsh construction and service environments of civil engineering applications. To improve the robustness and durability of FBG sensors for practical applications, FBG sensors are usually packed with one or more protective packaging layers. With the packaging layer(s), the strain measured by the FBG sensors is expected to be different from the strain in the host matrix, which is known as strain transfer loss. Based on the Newton’s third Law, while the matrix drives the packaging layer to produce a shear deformation through a shear stress, the matrix will also be subjected to a shear stress in the opposite direction of the packaging layer. This will result in the opposite shear deformation of the matrix within a certain range and make the strain of the matrix outside the packaging layer less than that of the undisturbed matrix. To reduce the difference between the strain measured by the optical fiber sensor and that of the host matrix, strain transfer analysis has been conducted by researchers to establish their quantitative relationship.

The strain transfer mechanism of fiber optic sensors has been investigated since 1990’s when the relationship of strains between fiber optical sensor and concrete was researched under certain assumptions [10]. Later, an infinite elastic cylinder model consisting of a host material and an optical fiber sensor was built, and the strain transfer mechanism was studied by adopting a plane-strain theory [11] and the shear-lag theory of fiber reinforced composites [12]. Since then, strain transfer analysis of the fiber optic sensor embedded or attached to the surface of host material has been investigated under different damage status of host matrix and loading conditions [13–23]. However, most of these published studies assumed that all host materials were linearly elastic and perfectly bonded with the optical fiber at their interfaces. For instance, Huang et al. [24] proposed the strain transfer model of the embedded FBG sensors when concrete matrix has plastic deformation and validated such a model using experimental analysis. Wang et al. [25] studied the strain transfer of surface-
bonded FBG sensors when the host matrix is under fatigue loads. Zhou et al. [26] proposed the strain transfer coefficient of the embedded FBG sensors considering the creep property of concrete matrix and verified its using finite element analysis. This study shows that the creep of the host matrix decreases the strain transfer ratio. However, up to date, the influence of host materials’ creep property on long-term strain sensing using FBG sensors has not yet been validated through experimental study.

The main objective of this study is to investigate the influence of host materials’ creep effects on long-term strain sensing using FBG sensors. First, laboratory experiments were performed to illustrate the influence of creep on the strain transfer of packaged FBG sensors. To address the strain transfer error induced by creep, a theoretic strain transfer model was established using the fundamentals of engineering mechanics. Considering the creep properties of the host material, a strain transfer formula is proposed. Last but not least, parametric studies using numerical simulations were performed to investigate various sensor design considerations on the strain transfer efficiency of the axially loaded cylinder model for sensor optimization in practical applications.

2. Long-term axial compression test

2.1. Test setup

To indicate the strain differences between the FBG sensor and a creep medium, three epoxy resin cylinders and three concrete cylinders were tested under long-term constant loads. To improve the robustness of the FBG sensors during the installation and testing process, glass fiber reinforced polymer (GFRP) was used to package the sensors. Figure 1 (a) shows the FBG-GFRP sensor used in this study. The outer diameter of the GFRP rods was 5 mm for both epoxy resin and concrete cylinders. The length of the Bragg grating used for the GFRP-FBG sensors was 10 mm. In addition, as shown in Figure 1(b)

![Figure 1. FBG-FRP sensor.](image)

![Figure 2. Test specimen design and specimen instrumentation.](image)
Figure 2(a), both ends of the sensor rod were wrapped with rubber tubes to protect the optical fiber connection line and isolate the sensor end from the matrix materials for eliminating the end effects. The distance between the two rubber tubes was 50 mm as shown in Figure 1(b). The FBG-GFRP sensors were then placed in the center of the cylindrical molds, followed by pouring epoxy resin or concrete into the molds to prepare the testing samples. The prepared epoxy and concrete samples can be seen in Figure 2(b, c). After initially setting off concrete, the upper and lower surfaces of the specimens were polished to level off. Three specimens were prepared for each material type, which were designated as #E1, #E2 and #E3 for epoxy samples and #C1, #C2, and #C3 for concrete samples. Before loading, the elastic modulus, Poisson’s ratio, diameter, and height of epoxy resin and concrete specimens were measured and Tables 1 and 2 show the detailed material and geometric properties of all the epoxy and concrete testing samples. An additional temperature FBG sensor was installed on the side to compensate temperature effects of the FBG sensors.

In addition, as shown in Figure 3(a), on the side face of each epoxy cylinder, four strain gauges were installed at the middle height to measure axial strains of the cylinders. Two micrometer gauges were placed at the diagonal of the platform to measure the creep deformation of epoxy resin during the load-holding process. In the concrete samples, as illustrated in Figure 3(b), four pairs of copper nails of the hand-held strain gauge were attached to the side face of the concrete cylinders with durable two-component super-glue at 90-degree angles from each other. The long-term axial deformation of concrete specimens was determined by measuring the distance between a pair of copper nails using a hand-held strain gauge with 150 mm gage length as shown in Figure 4(c). The readings from strain gauges were recorded using the Resistance Strain Interrogator at a 1.0 Hz sampling frequency. Data from the FBG sensor were collected using the FBG Interrogator (Model: Zx-fp-c32-3) supplied by Smart Technology Nantong Co., LTD., with a sampling frequency of 3 Hz. A force sensor was used to monitor the loading force during the experiment in real time.

To stabilize the loads during the loading process, leverage was used for both the epoxy and concrete cylinders with the stable load of 10kN and 80kN, respectively. Figure 4(a, b) show the loading set-up for epoxy and concrete specimens, respectively.

| Table 1. Geometrical and material properties of epoxy resin specimens. |
|---------------------------------------------------------------|
| Title | Elasticity modulus | Poisson’s ratio | Diameter | Height |
|-------|-------------------|----------------|----------|--------|
| E1    | 2.67 GPa          | 0.456          | 92.16 mm | 189.6 mm |
| E2    |                   |                | 92.00 mm | 192.8 mm |
| E3    |                   |                | 92.02 mm | 192.5 mm |

| Table 2. Geometrical and material properties of concrete specimens. |
|---------------------------------------------------------------|
| Title | Elasticity modulus | Poisson’s ratio | Diameter | Height |
|-------|-------------------|----------------|----------|--------|
| C1    | 24.45 GPa         | 0.249          | 102.90 mm | 299.50 mm |
| C2    |                   |                | 103.02 mm | 300.12 mm |
| C3    |                   |                | 102.96 mm | 299.76 mm |
2.2. Experimental results and analysis

Figure 5(a–f) show the load-time variation curves of all the epoxy and concrete specimens during the loading time. It can be seen that the loads of the three specimens remained constant during the test period, with the fluctuation range within 5%. The average load of
the epoxy specimen #E1, #E2 and #E3 were 10.2kN, 10.6kN, and 10.5kN, respectively, and the average load of concrete specimen #C1, #C2 and #C3 were 80.49kN, 80.98kN, and 79.67kN, respectively.

Figures 6 and 7 compare the strain-time curves measured by micrometer gauges, hand-held strain gauges, and FBG-GFRP sensors. The strains measured by the FBG sensors are significantly smaller than the actual strains measured by the micrometer gauges, and the difference between them increases with the increase in load-holding time. The strain transfer rates of FBG sensors are obtained by dividing the measured strains from FBG sensors to that from the external surface strains of the cylinders from the strain gauges, as shown in Figure 8. The strain transfer rates of both epoxy and concrete samples decrease with time due to the matrix creep. The strain transfer rates of the epoxy resin cylinders are lower than that of the concrete cylinders according to different modulus and creep rate. The discrepancy in the strain transfer rate-time curves of the three epoxy cylinders is due to the difference in temperature when, they are being loaded. The loading temperatures of specimen #E1, E2, E3 were 32°C, 26°C, and 23°C, respectively. At higher temperatures it would introduce a more significant creep effect of epoxy resin 27, the strain transfer rates

![Figure 5. Load-time variation curves.](image)

![Figure 6. Strain-time curve comparisons between the FBG and micrometer gauges for the epoxy samples (a) Specimen #E1 (b) Specimen #E2 (c) Specimen #E3.](image)
of the three epoxy specimens are much more different than the concrete samples as shown in Figure 8(a). The experimental results clearly demonstrate that the creep in the host matrix will significantly influence the long-term measurement accuracy of the FBG sensors since the creep will reduce the strain transfer rate of the embedded FBG sensors as time goes by. Thus, it cannot be ignored for long-term strain monitoring using FBG sensors as embedded sensors in the host matrix with creep effects, and it is necessary to develop appropriate theoretical approaches to improve the long-term performance and measurement accuracy of FBG sensors when they are embedded in the host medium with creep effect.

### 3. Theoretic analysis through strain transfer theory to improve the long-term performance of FBGs in creep mediums

Single-mode fibers including core, cladding, and coatings, are commonly used in the monitoring field. The core and cladding are both mainly made of silicon dioxide with diameters of 9 µm and 125 µm, respectively. While the coating layer, made of polymer materials with an outer diameter of 250 µm, is generally used to enhance the flexibility, mechanical strength and aging resistance of optical fibers, as shown in Figure 9. The core
and cladding are generally regarded as the core in the analysis since they share the same main material. In the packaging process of the fiber optic sensor, the coating layer is generally removed, so only the core is left in the bare fiber. Thus, the strain transfer model of an FBG sensor can be simplified into a three-layer cylinder structure including the optical fiber core, the packaging layer, and the host material, as shown in Figure 10, in which the length of the sensor is designated as $2L$, and the radius of the core, packaging layer and host material are designated as $r_f$, $r_p$, and $r_m$ respectively.

### 3.1. Assumptions

To develop a theoretic analysis for the embedded FBGs in creep mediums, the following assumptions were made according to the application conditions to simplify the model, including:

1. It is known that optical fiber is a quite uniform and elastic material. For GFRP, in general, it shows excellent creep resistance in terms of creep strain, specifically, the creep strain of GFRP at room temperature and normal humidity remains under 1.0% after 3000 hours (125 days) at a tensile stress of even 80% of the tensile strength [27]. Thus, in this study, both the optical fiber and packaging layer are assumed to remain elastic while the host matrix may exist creep effect;
(2) The host material, packaging layer, and optical fiber are perfectly bonded at their interfaces without relative movement such as debonding or delamination;
(3) The host material is subjected to a uniform stress parallel to the axial direction of the FBG sensor and the strain transfer from the host material to the fiber core is realized by the shear strain of the intermediate layer. Thus, neither the intermediate layer nor the fiber core are directly subjected to external loads.

3.2. Theoretical analyses

Figure 11 illustrates the vertical coordinate system and the free body diagram of the infinitesimal element dx in each of the three layers, with the axial direction of the cylinder as the x-coordinate axis, the radial direction of the cylinder as the r-coordinate axis, and the positive center of the cylinder as the origin of coordinates.

A micro-section of the optical fiber is taken out to establish the static equilibrium equation as:

\[ \tau_p(x, r, t) = -\frac{r_f}{2} \frac{d\sigma_f(x, t)}{dx} \]

where, \( \tau_p(x, r, t) \) is the shear stress of the packaging layer, and \( \sigma_f(x, t) \) is the axial normal stress of the optical fiber.

Similarly, considering the equilibrium equation of the packaging layer and replacing the shear stress \( \tau_p(x, r, t) \) by Equation (1), the shear stress in the packaging layer can be expressed as:

\[ \tau_p(x, r, t) = -\frac{r_f^2}{2r} \frac{d\sigma_f(x, t)}{dx} - \frac{r_f^2 - r_p^2}{2r} \frac{d\sigma_p(x, t)}{dx} \]

in which, \( \sigma_p(x, t) \) is the axial normal stress of the packaging layer.

The strain gradient of the optical fiber is similar to that of the packaging layer and can be obtained as:

\[ \frac{d\varepsilon_p(x, t)}{dx} \approx \frac{d\varepsilon_f(x, t)}{dx} \]

Substituting Equation (3) into Equation (2) and introducing Hooke’s law, Equation (2) can be rewritten as:

![Figure 11](image.png)

Figure 11. The force diagram of an infinitesimal element in each of the three layers.
\[
\tau_p(x, r, t) = -\frac{r_f^2}{2r} E_f + \frac{r_p^2 - r_f^2}{2r} E_p \frac{d\varepsilon_f(x, t)}{dx}
\]  

(4)

where \(E_f\) and \(E_p\) are the elastic modulus of optical fiber and packaging layer, respectively.

Finally, a static equilibrium equation is established by taking the host material micro-elements with radius \(r\) to \(r + dr\), as shown in Figure 12, and can be calculated as:

\[
\tau_m(x, r, t) + r \cdot \frac{\tau_m(x, r, t)}{\partial r} = r \cdot \frac{d\sigma_m(x, r, t)}{dx}
\]  

(5)

where \(\tau_m(x, r, t)\) and \(\sigma_m(x, r, t)\) represent the interface shear stress and axial normal stress of the host material, respectively.

At the end of the sensor, the axial strain of the host material is uniformly distributed along the radial direction. While at the middle position of the sensor, the closer it is to the sensor, the smaller the matrix strain becomes. It is therefore considered that the axial strain gradient of the matrix is approximately inversely proportional to the distance from the central axis of the fiber sensor. The strain gradient of the host matrix can be expressed as:

\[
\frac{d\varepsilon_m(x, r, t)}{dx} \approx \frac{d\varepsilon_m(x, r_p, t)}{dx} \approx \frac{d\varepsilon_m(x, r_m, t)}{dx} r_m
\]  

(6)

Substitute Equation (6) into Equation (5) and integrate at both ends of the equation yielding:

\[
r \cdot \tau_m(x, r, t) - r \cdot r_p \frac{d\sigma_m(x, r_p, t)}{dx} + \alpha = 0
\]  

(7)

where \(\alpha\) is a constant.

Introducing the boundary conditions at \(r = r_m\), \(\tau_m(x, r_m, t) = 0\), Equation (7) can be rewritten as:

\[
\tau_m(x, r, t) = \frac{r_m(r_m - r)}{r} \cdot \frac{d\sigma_m(x, r_m, t)}{dx}
\]  

(8)

According to \(\tau_p(x, r_p, t) = -\tau_m(x, r_p, t)\), Equation (9) can be obtained by Equations (4) and (8) as:

\[
\frac{d\sigma_m(x, r_m, t)}{dx} = -\frac{r_f^2 E_f + (r_p^2 - r_f^2) E_p d\varepsilon_f(x, t)}{2r_m(r_m - r_p)}
\]  

(9)

Figure 12. The force diagram of an infinitesimal element in host matrix.
Substituting Equation (9) into Equation (8), equation (8) can be rewritten as:

$$\tau_m(x, r, t) = -\frac{(r_m - r)}{r} \cdot \frac{r^2 E_f + (r_p^2 - r^2)E_p}{2(r_m - r_p)} \frac{d\varepsilon_f(x, t)}{dx}$$  \hspace{1cm} (10)$$

According to assumption (1), the constitutive equation of the host material can be expressed as:

$$\sigma_m(x, r, t) = E_m(t) \cdot d\varepsilon_m(x, r, t) \hspace{1cm} E_m(t) = \frac{E_m}{1 + \varphi(t)}$$  \hspace{1cm} (11)$$

where $E_m$ is the elastic modulus of the host material with no creep or damages, and $\varphi(t)$ is the creep coefficient.

The equivalent elastic modulus and equivalent shear modulus of the host material are, respectively, expressed as:

$$\bar{E}_m(t) = \frac{\sigma_m(x, r, t)}{\varepsilon_m(x, r, t)} \hspace{1cm} \bar{G}_m(t) = \frac{E_m(t)}{2(1 + \nu_m)}$$  \hspace{1cm} (12)$$

Due to the existence of the packaging layer, there is a certain difference between the displacement of the optical fiber and that of the host material, as shown in Figure 13.

Because the influence of the Poisson effect on the radial displacement is very small compared to the axial displacement, the radial displacement gradient can be ignored. Thus, the Hooke’s law is reduced to be:

$$\tau_i(x, r, t) = \bar{G}_i \frac{\partial u}{\partial r}, \hspace{1cm} i = p, m$$  \hspace{1cm} (13)$$

Replaced $\tau_p(x,r,t)$ and $\tau_m(x,r,t)$ with Equations (4) and (10), and taking the integration of Equation (13) separately from $r_f$ to $r_p$ and $r_p$ to $r_m$, we can obtained:

$$\int_0^x \varepsilon_m(x, r_m, t)dx = \int_0^x \varepsilon_f(x, t)dx + \Delta_p + \Delta_m$$  \hspace{1cm} (14)$$

Figure 13. The diagram of displacement balance of various layers.
\[ \Delta_p = - \left[ \frac{\rho^2 (E_f - E_p)}{E_p} \ln \left( \frac{\rho_p}{r_f} \right) \right] + \frac{\rho_f^2 - \rho_p^2}{2} \left( 1 + \nu_p \right) \frac{d\varepsilon_f(x,t)}{dx} \]

\[ \Delta_m = - \left[ r_m \ln \left( \frac{r_m}{\rho_p} \right) - (r_m - r_p) \right] \frac{\rho_p^2 - \rho_f^2}{2} \left( 1 + \nu_p \right) \frac{d\varepsilon_f(x,t)}{dx} \]

where \( \Delta_p \) and \( \Delta_m \) stand for the displacement differences.

Substituting equation (15) and (16) into Equation (14) and integrating over \( x \) gives as follows:

\[ \varepsilon_m(t) = \varepsilon_f(x,t) - \frac{1}{[k(t)]^2} \cdot \frac{d^2\varepsilon_f(x,t)}{dx^2} \]

\[ \frac{1}{[k(t)]^2} = \left[ \frac{\rho^2 (E_f - E_p)}{E_p} \ln \left( \frac{\rho_p}{r_f} \right) \right] + \frac{\rho_f^2 - \rho_p^2}{2} \left( 1 + \nu_p \right) + \]

\[ \left[ r_m \ln \left( \frac{r_m}{\rho_p} \right) - (r_m - r_p) \right] \frac{\rho_p^2 - \rho_f^2}{2} \left( 1 + \nu_p \right) \frac{d\varepsilon_f(x,t)}{dx} \]

Equation (17) can be rewritten as follows:

\[ \frac{d^2\varepsilon_f(x,t)}{dx^2} - [k(t)]^2 \varepsilon_f(x,t) + [k(t)]^2 \cdot \varepsilon_m(t) = 0 \]

The general solution of Equation (19) can be expressed as:

\[ \varepsilon_f(x,t) = A \cosh[k(t) \cdot x] + B \sinh[k(t) \cdot x] + \varepsilon_m(t) \]

In the boundary conditions, both ends of the fiber core and packaging layer are not loaded directly. In addition, since the host material does not contact with the fiber core at both ends of the interface between the fiber core and the packaging layer, it is assumed that the fiber core is not affected by the axial stress at both ends. The boundary conditions can then be expressed as \( \varepsilon_f(\pm L, t) = 0 \) and substituted into Equation (20) to obtain the coefficients with \( A = -\varepsilon_m(t) / \cosh[L \cdot k(t)] \) and \( B = 0 \).

The strain relationship between the host material and the fiber core can then be obtained as

\[ \varepsilon_f(x,t) = \left\{ 1 - \frac{\cosh[k(t) \cdot x]}{\cosh[k(t) \cdot L]} \right\} \varepsilon_m(t) = \eta(x,t) \cdot \varepsilon_m(t) \]

where \( \eta(x,t) \) the strain transfer rate is at time \( t \) and at a distance of \( x \) from the origin of the coordinates.

Let the length of the FBG gauge area be \( 2L_0 \), and then the average strain transfer rate of the FBG sensor is,

\[ \eta(t) = \frac{\varepsilon_f(t)}{\varepsilon_m(t)} = \frac{\int_0^{L_0} \eta(x,t) dx}{L_0} = 1 - \frac{\sinh[L_0 k(t)]}{L_0 k(t) \cosh[Lk(t)]} \]
In practical application, the strain \( \varepsilon_f(x, t) \) of the sensor itself can be directly measured, and the elastic modulus of the structure can be obtained by experiments. Set the start loading time to \( t = 0 \), the strain transfer rate of the sensor can be obtained by the following steps:

1. At \( t = 0 \), no creep occurred in the structure, and the strain transfer rate \( \eta(0) \) at this time is obtained from Equation (22).
2. The equivalent elastic modulus \( E_m(t_1) \) at time \( t = t_1 \) can be obtained from the following equation,

\[
E_m(t_1) = \frac{E_m(t_1) \cdot d\varepsilon_m(t_1)}{\varepsilon_m(t_1)} = \frac{\varepsilon_m(0) \cdot E_m(t_1 - 0) + [\varepsilon_m(t_1) - \varepsilon_m(0)] \cdot E_m(t_1 - t_1)}{\varepsilon_m(t_1)} \tag{23}
\]

where \( \varepsilon_m(0) = \eta(0) \cdot \varepsilon_f(0) \), and \( E_m(t) \) can be obtained by creep tests or creep formulas. Uniting Equations (18), (22) and (23) to form a set of simultaneous equations, \( \varepsilon_m(t_1) \) is solved. Substitute \( \varepsilon_m(t_1) \) into Equation (22) to obtain strain transfer rate \( \eta(t_1) \).

1. The equivalent elastic modulus \( E_m(t_2) \) at time \( t = t_2 \) can be obtained from the following equation,

\[
E_m(t_2) = \frac{E_m(t_2) \cdot d\varepsilon_m(t_2)}{\varepsilon_m(t_2)} = \frac{\varepsilon_m(0) \cdot E_m(t_2 - 0) + [\varepsilon_m(t_1) - \varepsilon_m(0)] \cdot E_m(t_2 - t_1) + [\varepsilon_m(t_2) - \varepsilon_m(t_1)] \cdot E_m(t_2 - t_2)}{\varepsilon_m(t_2)} \tag{24}
\]

By Substituting \( \varepsilon_m(0) \) and \( \varepsilon_m(t_1) \) into Equation (24) and uniting Equations (18), (22) and (24) to form a set of simultaneous equations, \( \varepsilon_m(t_2) \) is solved. Substitute \( \varepsilon_m(t_2) \) into Equation (22) to obtain strain transfer rate \( \eta(t_2) \).

1. Similarly, the equivalent elastic modulus \( E_m(t_n) \) at time \( t = t_n \) can be obtained from the following equation,

\[
E_m(t_n) = \frac{E_m(t_n) \cdot d\varepsilon_m(t_n)}{\varepsilon_m(t_n)} = \frac{\varepsilon_m(0) \cdot E_m(t_n - 0) + [\varepsilon_m(t_1) - \varepsilon_m(0)] \cdot E_m(t_n - t_1) + \cdots + [\varepsilon_m(t_n) - \varepsilon_m(t_{n-1})] \cdot E_m(t_n - t_n)}{\varepsilon_m(t_n)} \tag{25}
\]

By Substituting \( \varepsilon_m(0), \varepsilon_m(t_1), \ldots, \varepsilon_m(t_{n-1}) \) into Equation (25) and uniting Equations (18), (22) and (25) to form a set of simultaneous equations, \( \varepsilon_m(t_n) \) is solved. Substitute \( \varepsilon_m(t_n) \) into Equation (22) to obtain strain transfer rate \( \eta(t_n) \).

Thus, we can obtain the strain transmittance at any given moment by going through the above steps.

### 3.3 Experimental validation

To obtain the matrix modulus \( E_m(t_n) \) in Equation (25), it requires to have the constitutive law of the host material in accordance with the experimental data. This study adopts a 5-parameter model including springs and dampers, as shown in Equation (26) to describe the creep constitutive law, which is formed by a spring and two Kelvin models in series and shown in Figure 14[28].
\[ \varepsilon = \frac{\sigma_0}{E_3} + \frac{\sigma_0}{E_1} (1 - e^{-t/\tau_1}) + \frac{\sigma_0}{E_2} (1 - e^{-t/\tau_2}) \]  

(26)

\[ \tau_1 = \frac{E_1}{\eta_1}, \tau_2 = \frac{E_2}{\eta_2} \]

where \( \varepsilon \) and \( \sigma_0 \) are the strain and stress of the host material, respectively, \( E_1, E_2, E_3 \) are the stiffness coefficients of springs and \( \eta_1, \eta_2 \) are the viscosity coefficients of dampers, respectively.

where \( E_m(t) \) is the elastic modulus of the host material at the moment \( t \).

The time-dependent creep constitutive law of the host material can then be expressed as:

\[ E_m(t) = \frac{1}{1 + \frac{1}{E_3} (1 - e^{-t/\tau_1}) + \frac{1}{E_2} (1 - e^{-t/\tau_2})} \]  

(27)

Based on Figures 5, 6 and 7, the creep constitutive law parameters in Equation (26) for the epoxy and concrete specimens were obtained by fitting these test data as shown in Tables 3 and 4, respectively, and the fitted curves were also illustrated in Figures 15 and 16. Figures 15 and 16 show great fitting with \( R_2 \) of the fitted curves of 0.9996, 0.9966, 0.9988 for epoxy specimens and 0.9983, 0.9921, 0.9964 for concrete specimens, respectively. The creep constitutive law parameters are substituted into Equation (27) to obtain the elastic modulus \( E_m(t) \) of each specimen at time \( t \).

Validation of the developed theoretical analysis is performed by comparing the strain transfer rates between experimental and theoretic results as shown in Figure 17(a)–(f) for the epoxy and concrete specimens. As shown in both figures, the change of experimental and theoretical values of strain transfer rate is very similar, with the maximum differences

**Table 3.** The creep constitutive model parameters for the epoxy.

| Title | \( E_1/\) GPa | \( E_2/\) GPa | \( E_3/\) Gpa | \( \tau_1/\) s-1 | \( \tau_2/\) s-1 |
|-------|---------------|---------------|---------------|----------------|----------------|
| No.1  | 2.777         | 0.608         | 2.178         | 322            | 9389           |
| No.2  | 6.337         | 2.104         | 3.006         | 957            | 20,989         |
| No.3  | 9.095         | 2.980         | 2.831         | 694            | 23,399         |

**Table 4.** The creep constitutive model parameters for the concrete.

| Title | \( E_1/\) GPa | \( E_2/\) GPa | \( E_3/\) Gpa | \( \tau_1/\) d-1 | \( \tau_2/\) d-1 |
|-------|---------------|---------------|---------------|----------------|----------------|
| No.1  | 14.321        | 49.899        | 24.494        | 22.635         | 0.36770        |
| No.2  | 14.422        | 37.460        | 28.366        | 47.500         | 1.4591         |
| No.3  | 17.806        | 41.984        | 25.465        | 20.733         | 1.0473         |
Figure 15. Strain – time curve for epoxy specimens (a) Specimen #E1 (b) Specimen #E2 © Specimen #E3.

Figure 16. Strain – time curve for concrete specimens (a) Specimen #C1 (b) Specimen #C2 (c) Specimen #C3.

Figure 17. Experimental vs. theoretical strain transfer rate for epoxy and concrete samples (a) Specimen #E1 (b) Specimen #E2 (c) Specimen #E3 (d) Specimen #C1 (e) Specimen #C2 (f) Specimen #C3.
between the theoretical value and the actual value of specimens #E1, #E2, and #E3 were 6.56%, 4.77%, and 3.33% and that for specimens #C1, #C2, and #C3 were 6.26%, 7.18%, and 5.33%, respectively. Since concrete is an inhomogeneous material with significant amounts of pores in it, the sensor’s effective bonding length may be less than the estimated theoretical length. Since the strain transfer rate of the sensor can be influenced by the actual effective bonding length, some of the experimental results are lower than the analysis and FEM solutions. In the future, it is needed to investigate further approaches to better estimate the actual bonding lengths of the embedded FBG sensors. Thus, it is reasonable to conclude that developed theoretic analysis on strain transfer rate in creep matrix in this paper is accurate and applicable.

In addition, it can be observed from Figure 17 that both experimental and theoretical results of the strain transfer rates decrease with the increase in load-holding time, and the rate of decline decreases with the increase in load-holding time.

4 Numerical validation and parametric analysis

4.1. Finite element model

To optimize the FBG sensor design in creep matrix such as the appropriate sensor length and radius of the packaging layer, a finite element model (FEM), was established using ABAQUS. The FEM includes three layers: the optical fiber, the packaging layer, and the host material with creep behavior. To be able to compare with experimental results for validation, the same material properties, and dimensions as the concrete specimens in Section 2 were applied to the finite element models. Equation (27) was adopted as the creep constitutive law of the concrete. Table 5 shows the physical parameters used in the numerical model. To simplify the model and consider the symmetry of the model in practice, a quarter model was set up and a uniform compressive stress of 10.2 MPa was applied at both ends of the concrete cylinder for 102 days in the same way as the experiments. To obtain more accurate analysis results, the fiber layer and the packaging layer are divided into smaller non-regular mesh units. The displacement constraints were applied perpendicularly to the symmetric plane of the host material, packaging layer, and fiber core, as shown in Figure 18.

| Title                              | Label | Value        | Unit |
|------------------------------------|-------|--------------|------|
| Elastic modulus of the optical fiber | $E_f$ | $7 \times 10^{10}$ | pa   |
| Elastic modulus of the protective layer | $E_p$ | $5 \times 10^{10}$ | pa   |
| Elastic modulus of concrete        | $E_m$ | $2.45 \times 10^{10}$ | pa   |
| Poisson’s ratio of the packaging layer | $\mu_p$ | 0.25          |      |
| Poisson’s ratio of concrete        | $\mu_m$ | 0.249        |      |
| The radius of the packaging layer  | $r_p$ | 2.5          | mm   |
| The radius of the optical fiber    | $r_f$ | 125          | µm   |
| The radius of the host matrix      | $r_m$ | 50           | mm   |
| Length of the sensor               | $2L_f$ | 50           | mm   |
| Length of the host matrix          | $2L$ | 300          | mm   |
| Loading time of concrete           | $t$ | 0–102        | day  |
4.2. Validation FEM with experimental analysis

Figure 19 compares the decrease of strain transfer rate with the increase in the loading time obtained from the FEM analysis, following the theoretic analysis in Section 2 and from the experiments. As shown in both figures, the change of experimental, theoretical, and FEM values of strain transfer rate is very similar, with the maximum differences between the FEM value and the experimental value of specimens #C1, #C2, and #C3 were 7.9%, 5.61%, and 6.69%, and with the maximum differences between the FEM value and the theoretic value of specimens #C1, #C2, and #C3 were 2.9%, 3.49%, 2.69%, respectively. With the increase in load-holding time, the experimental results were

Figure 19. Comparison between experimental, theoretic and finite element values of strain transfer rate (a) Specimen #C1 (b) Specimen #C2 (c) Specimen #C3.
a little lower than the theoretical and FEM values. This is because as the increase of load holding time increases, the bonding interface between concrete and the sensor, especially the part near the end of the sensor, falls off due to high shear stress, thus reducing the effective bonding length of the sensor.

It is also clearly demonstrated in Figure 19(a)~(c) that if loads remain, the strain transfer rate decreases exponentially as the time increases before 1000 hours, which is around 1.5 months of time duration. After that, the strain transfer rate would remain mostly stable if the loading magnitude remained the same. This indicated that if the FBG sensors are embedded inside concrete for long-term strain monitoring purpose, the influence of creep effect of concrete matrix on the strain transfer rate of the FBG sensor cannot be ignored. For an accurate measurement, it is necessary to optimize the sensor design to minimize this measurement error and correct this error accordingly.

4.3. Parametric analysis

To optimize the sensor design on sensor packaging length, FEM simulation were performed using four different sensor lengths, including 30 mm, 50 mm, 70 mm, and 100 mm. For all of these analyses, the same sensor radius used was 2.5 mm, as shown in Table 5. Figure 20(a) shows the strain transfer rate distribution of the FBG sensor along the sensor’s length direction with different packaging lengths when no creep occurred at \( t = 0 \). It is observed that a longer packaging length yielded a higher strain transfer rate. The average strain transfer rates at the sensor grating (10 mm length) were 84.77%, 93.24%, 95.78%, and 97.10% for sensor length of 30 mm, 50 mm, 70 mm, and 100 mm, respectively. When there is no creep in the concrete, the average strain transfer rates of FBG is larger than 95% when the packaging length was over 70 mm.

Figure 20(b) shows the strain transfer rate distribution of the FBG sensor along the sensor’s length, with different packaging lengths considering long-term creep effect when \( t = 102 \) days as same as in the experiment. After 102 days of loading, when the

![Figure 20](image-url)

**Figure 20.** The strain transfer rate changes of FBG sensors with different packaging lengths a) \( t = 0 \) (b) \( t = 102 \) days.
sensor is 30 mm, 50 mm, 70 mm, and 100 mm in length, the average strain transfer rates at the FBG were 61.71%, 78.68%, 86.25%, and 91.14%. Comparing Figure 20(a) and (b), it can be seen that the creep induced a decrease of 23.05%, 14.56%, 9.53%, and 5.96% of strain transfer loss, respectively. The strain transfer rate and the resistance to creep effects decrease with a decrease in the effective sensing length. For a concrete medium, if the sensor length is longer than 100 mm, the error induced by the strain transfer loss with creep effect will be less than 6%. Thus, in practice, if the GFRP packaged FBG sensors are embedded inside concrete for long-term strain monitoring, to avoid significant strain transfer loss induced by creep effect, a sensor length of 100 mm or larger is recommended.

To optimize the sensor diameter and radius, FEM analysis on various sensor radiuses was conducted using sensor radius of 1.5 mm, 2.5 mm, and 5 mm with a constant sensor length of 100 mm, as shown in Table 5. Figure 21(a) shows the strain transfer rate distribution of the FBG sensor along the sensor’s length with different packaging radius without any creep at t = 0. A larger packaging radius yielded lower strain transfer rates. For sensor radius of 1.5 mm, 2.5 mm, and 5 mm, the average strain transfer rates at the sensor grating (10 mm length) were 98.12%, 97.10%, and 92.20%, respectively. Figure 21(b) illustrates the strain transfer rate distribution of the sensor along its length with different packaging radius with creep considered at t = 102 days. After 102 days, the average strain transfer rates at the sensor grating were 95.03%, 91.14%, and 76.32%, for sensor radius of 1.5 mm, 2.5 mm, and 5 mm. The strain transfer induced 3.09%, 5.96%, and 15.87% more measurement error compared to no creep effect for each packaging radius, respectively. It is clearly shown that the strain transfer rate and the resistance to creep effects decrease with an increase in the package thickness of sensors. Thus, in practical applications, if the packaging layer can be made in a smaller radius of 1.5 mm or smaller, it will be beneficial. Based on Figures 20 and 21, it is shown that a GFRP packaging of 1.5 mm in radius and 100 mm in length, can eliminate most of the measurement error induced by the strain transfer in a creep host matrix of concrete.

![Figure 21](image_url)

*Figure 21.* The strain transfer rate change of the FBG sensor with various package radiuses (a) t = 0 (b) t = 102 days.
5. Conclusions

The long-term performance of FBG sensors in a creep medium is investigated using experiments, theoretical analysis, and numerical simulations. To minimize or even eliminate the measurement error induced by strain transfer loss with creep in the host matrix, the theoretical derivation of the strain transfer mechanism from an axially loaded cylinder to an embedded packaged FBG sensor is developed and validated through numerical and experimental analysis. Accordingly, the following conclusions can be drawn:

1) Experimental results indicated that the creep effect of host matrix can bring a strain transfer loss to the embedded FBG sensors and it can be compensated through sensor design or post-data analysis for long-term strain monitoring;

2) The developed theoretical strain transfer model based on the shear lag theory can effectively investigate the long-term performance of FBG sensors inside creep medium, which was also validated through experimental and numerical analysis;

3) The developed theoretical and finite element models can provide valuable information to guide the FBG sensor design if embedded inside a creep medium. A case study using concrete as host matrix and GFRP packaged FBG as sensor showed that the optimized sensor length was 100 mm or longer and the optimized diameter of the sensor was 3 mm or smaller.

Limited by time, only a relatively short-term creep test was carried out in the paper, and a longer creep test will be needed for future study.

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