A New Method for Constructing the Basic Probability Assignment Function in D-S Evidence Theory

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Abstract. The basic probability assignment function is the basis of D-S evidence theory and the premise to obtain accurate evaluation conclusions. A new method to construct the basic probability assignment function is proposed in this paper, which uses nonlinear function to construct the basic probability assignment function, and takes into account the relationship between the change trend of index and the change trend of the basic probability assignment value. This construction method can be applied to the construction of the basic probability assignment of the index, with the decrease of whose value, the condition will degrade gradually.

1. Introduction
D-S evidence theory is an effective tool to fuse multi-source information, and has been widely used in information fusion, fault diagnosis, risk assessment, condition assessment, pattern classification and other fields[1-3]. At present, many literatures mainly propose corresponding improvement methods to solve the problems of serious conflicts in evidence synthesis, but there are few discussions on the most basic the basic probability assignment (BPA) function in evidence theory[4-9]. Many literatures give the BPA values of each evidence, and a few literatures propose to use trigonometric functions or Gaussian distributions to generate the BPA[10]. If the BPA value of evidence is not appropriate, even if the combination method in the evidence theory is completely correct, the result of multi-evidence combination may lead to the conclusion of wrong decision.

The simplest BPA function is the trigonometric function, which is linear simplicity. However, in some occasions, such as equipment condition assessment, if a certain index is within the ideal range, the probability that the equipment is at an excellent level is relatively high, the index changes within the ideal range, and the probability value of the equipment in the excellent level changes slowly. As the index gets worse, the condition will change from excellent level to good level, and the probability that the equipment is at the excellent level will decline faster at the same time. Therefore, for a certain index, the probability of the equipment in the excellent level is not linear, but a kind of nonlinear change. Thus, a new BPA function needs to be adopted. This paper takes equipment condition assessment as an example to illustrate the construction of the BPA function based on the evaluation index. It is assumed that there are 4 levels of equipment condition, namely excellent, good, general and poor.

There are two main steps to build the BPA function:
(1) Construct the preliminary BPA function;
(2) Modify the preliminary BPA value.
2. Construct the Preliminary Basic Probability Assignment Function

The equipment condition gradually degrades as the index value decreases, which is used for an example to illustrate the construction of the preliminary BPA function in this paper. For some indexes, such as the speed or power of a diesel engine, as the operating time of the equipment increases, the maximum speed of the equipment gradually decreases, and the equipment condition is also gradually degraded. For such indexes, the rated value is assumed to be $b$, and three key thresholds of indicators that degenerate with the condition are defined, namely $c, d, e$, and $b > c > d > e$. Generally, $c$ represents a good level threshold, $d$ represents a general level threshold, and $e$ represents a poor level threshold. For description, this paper assumes that $b = 100, c = 90, d = 80, e = 70$.

According to the basic properties of the BPA function and the general trend of the condition level changing with the index, this paper proposes a preliminary BPA function for constructing excellent level, good level, general level and poor level.

2.1. The Preliminary Basic Probability Assignment Function of Excellent Level

When the index value is greater than or close to the rated value, the probability that the equipment is excellent is relatively large. In this interval, the probability change rate is small. As the index value gradually becomes smaller from the rated value, the probability of the equipment being in excellent condition gradually decreases. When the index value changes from the rated value to the good threshold value, the probability that the equipment is in excellent decreases rapidly and the probability change rate is relatively large. At the average of the rated value and the good threshold, the probability that the equipment is in excellent level is approximately 0.5. When the index value changes from good threshold to general threshold, the probability that the equipment is in excellent level is very small, so the probability change rate is also small.

According to the above analysis, the preliminary BPA function of excellent level can be expressed by equation (1).

$$m_1 = \frac{1}{1 + 1.5 \left( \frac{b_{rel}}{c} \right)} \quad (1)$$

The preliminary BPA function curve of excellent level is shown in Figure 1.

![Figure 1. The preliminary basic probability assignment function of excellent level.](image)

2.2. The Preliminary Basic Probability Assignment Function of Good Level

When the index value is greater than or close to the rated value, the probability of the equipment being excellent is high, but the probability of being good is very small. In this interval, the rate of change in the probability of the equipment being good is also small. When the index value gradually decreases from the rated value to the good threshold, the probability that the equipment is excellent decreases rapidly and the probability change rate is relatively large. At the average value of the rated value and the good threshold, the probability that the equipment is excellent is approximately 0.5. As the index value continues to decrease from the average value of the rated value and the good threshold to the good threshold, the probability that the equipment is at a good level continues to
increase, but the increase rate is gradually reduced. When the index value reaches the good threshold, the probability that the equipment is at a good level is the largest, and the rate of change of the probability is also small. In the same way, when the index value gradually decreases from the good threshold to the general threshold and the bad threshold, the probability that the equipment is in good condition gradually decreases. In summary, the curve of the preliminary BPA function of good level: first grows slowly, then grows quickly, grows slowly when approaching the highest point, and after reaching the maximum, decreases slowly, and then quickly decreases, and then slowly decreases when approaching the lowest point. Based on this feature, this project chose a power function to describe the preliminary BPA function of good level.

The preliminary BPA function of good level is expressed by equation (2).

\[
m_2 = e^{-\left[\frac{1}{2} \left( \frac{x-c}{0.55(b-c)} \right)^2 \right]}
\]

(2)

The preliminary BPA function curve of good level is shown in Figure 2.

2.3. The Preliminary Basic Probability Assignment Function of General Level

The preliminary BPA function of general level is similar to the preliminary BPA function of good level, and can also be described by a power function, but the parameters are different. After selection and adjustment, the preliminary BPA function form of general level is shown in equation (3).

\[
m_3 = e^{-\left[\frac{1}{2} \left( \frac{x-d}{0.55(c-d)} \right)^2 \right]}
\]

(3)

The preliminary BPA function curve of general level is shown in Figure 3.
2.4. The Preliminary Basic Probability Assignment Function of Poor Level

The preliminary BPA function of poor level, in the process of gradually decreasing the performance index parameters, the probability value of the poor level gradually increases from small. The changing trend of the function curve is exactly opposite to the changing trend of the preliminary BPA function of excellent level. Therefore, referring to the form of the preliminary BPA function of excellent level, the preliminary BPA function of poor level is shown in equation (4).

\[ m_i = \frac{1}{1 + 1.5 \left( \frac{d + e}{f} \right)} \]  

(4)

The preliminary BPA function curve of poor level is shown in Figure 4.

3. Modify the Preliminary Basic Probability Assignment Function

According to the evaluation index, the different preliminary BPA functions are constructed. Under the condition of inputting specific index values, the preliminary BPA function values based on the index are obtained, respectively: \( m_1, m_2, m_3, m_4 \). Note that the sum of the preliminary BPA function values may be greater than 1, so it needs to be corrected.

Suppose the preliminary uncertainty of the index evidence is \( \beta \) and the revised BPA values are \( m_1^*, m_2^*, m_3^*, m_4^* \). According to the principle that the sum of the BPA value and the uncertainty value of each evidence in D-S evidence theory is 1, the preliminary BPA value is modified, and the modified model is shown as follows:

\[ m_1^* = \frac{m_1}{m_1 + m_2 + m_3 + m_4} \times (1 - \beta) \]  

(5)

\[ m_2^* = \frac{m_2}{m_1 + m_2 + m_3 + m_4} \times (1 - \beta) \]  

(6)

\[ m_3^* = \frac{m_3}{m_1 + m_2 + m_3 + m_4} \times (1 - \beta) \]  

(7)

\[ m_4^* = \frac{m_4}{m_1 + m_2 + m_3 + m_4} \times (1 - \beta) \]  

(8)

4. Example Analysis

Suppose the rated value of a certain index is 100, and its good level threshold, general level threshold and bad level threshold are 90, 80, 70 respectively. Thus, the basic parameters of the preliminary BPA function are \( b = 100, c = 90, d = 80, e = 70 \) respectively.
Assuming that the actual value of the measured index is 88, the preliminary BPA value obtained is shown in Table 1.

**Table 1.** The preliminary basic probability assignment value.

|   | $m_1$   | $m_2$   | $m_3$   | $m_4$   |
|---|---------|---------|---------|---------|
| Excellent probability | 0.05529 | 0.9802  | 0.7261  | 0.0051  |
| Good probability       |         |         |         |         |
| General probability    |         |         |         |         |
| Poor probability       |         |         |         |         |

Assuming that the preliminary uncertainty of the index evaluation is 0.2, that is $\beta = 0.2$, then according to the method of modifying the preliminary BPA function, the revised BPA value based on the index is shown in Table 2.

**Table 2.** The basic probability assignment value.

|   | $m_1$   | $m_2$   | $m_3$   | $m_4$   | $\beta$ |
|---|---------|---------|---------|---------|---------|
| Excellent probability | 0.0250  | 0.4439  | 0.3288  | 0.0023  | 0.2     |
| Good probability       |         |         |         |         |         |
| General probability    |         |         |         |         |         |
| Poor probability       |         |         |         |         |         |

5. Conclusion

According to the relationship between index and condition, this paper proposes a new method to construct the BPA function. This method avoids the absolute linearity of the BPA function based on the trigonometric function, and extends the BPA to the nonlinear domain, so that the BPA is more in line with engineering and reality. The method proposed in this paper has been applied to a certain type of diesel engine technical condition assessment case, which is worthy of reference.

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