Driven conductance of an irradiated semi-Dirac material

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We theoretically investigate the electronic and transport properties of a semi-Dirac material under the influence of an external time dependent periodic driving field (irradiation) by means of Floquet theory. We explore the inelastic scattering mechanism between different side-bands, induced by irradiation, by using Floquet scattering matrix approach. The scattering probabilities between two nearest side-bands depend monotonically on the strength of the amplitude of the irradiation. The external irradiation induces gap into the band dispersion which is strongly dependent on the angular orientation of momentum. Although, the high frequency limit indicates that the gap opening does not occur in an irradiated semi-Dirac material. However, a careful analysis of the band structure beyond this limit reveals that gap opening indeed occurs for higher values of momentum. The contrasting features of irradiated semi-Dirac material, in comparison to irradiated graphene, can be probed via the behavior of conductance. The latter exhibits the appearance of non-zero conductance dips due to the gap opening in Floquet band spectrum. Moreover, by considering a nanoribbon geometry of such material, we also show that it can host a pair of edge modes which are fully decoupled from the bulk, which is in contrast to the case of graphene nanoribbon where the edge modes are coupled to the bulk. We also investigate that if the nanoribbon of this material is exposed to the external irradiation, decoupled edge modes penetrate into the bulk.

I. INTRODUCTION

In recent times, the research interests on externally irradiated two dimensional (2D) Dirac material have grown significantly in many aspects among the research community. One of the remarkable aspect of this study lies in the ability of irradiation to modulate the electronic band structure and most importantly to generate topologically protected conducting edge modes with insulating bulk in 2D electronic systems (Floquet topological insulator)$^{1–10}$. Extensive research works in this direction have been put forwarded both from the theoretical as well as experimental point of view$^{11–13}$. The importance of irradiation effects on 2D Dirac materials are not only limited to engineering the Floquet topological insulator, in fact it has also been proposed to modulate spin and valley degree of freedom in graphene$^{14}$, α-$T_3$ lattice$^{15}$ and MoS$_2$. Among the other applications of periodic driving, tuning the thermoelectric performance in MoS$_2$, 0–π phase transition in Josephson junctions of silicene and Weyl semi-metal etc. are also worth to be mentioned.

Along the same line, several attempts have also been made to look into the electronic properties of semi-Dirac material under the influence of a circularly polarized light, in the form of irradiation$^{7,20}$. The Semi-Dirac materials exhibit quadratic as well as linear band dispersion along two orthogonal momentum directions$^{21,22}$. The lattice structure of this material mimics the monolayer graphene with two different hopping parameters $t_1$ and $t_2$, as shown in Fig. 1. This kind of lattice has been fabricated into the three unit cell slab of VO$_2$ confined within insulating TiO$_2$. A special interest has been shown to the case of $t_2 = 2t_1$ which manifests the semi-Dirac band dispersion in the vicinity of single Dirac cone. By using an effective low energy Hamiltonian, a series of theoretical studies have been performed like collective excitations (plasmon)$^{23}$, irradiation effects on electronic bulk band structure$^{7,20}$, optical properties$^{24}$, disorder effects$^{25}$ and Landau level formation in some specific geometry$^{26}$ etc.

Unlike the graphene case where external irradiation can open up a gap$^3$, the Floquet band structure of the semi-Dirac material remains gapless within high frequency limit as reported in Ref. [20]. On the other hand, a topological phase transition can take place under the consideration of a Haldane type mass term in such material, as predicted in Ref. [7]. These are the two works which have recently addressed the issue of how irradiation affects the electronic band structure, which are subsequently followed by the study of optical conductivity in this material under the influence of irradiation.

In this article, we study the quantum transport properties of a semi-Dirac material, exposed to a circularly polarized light. By using Floquet scattering matrix formalism,$^{29}$ we compute different scattering amplitudes and show that the transmission probabilities through different
Floquet side bands are monotonically increasing with the strength of the irradiation instead of frequency. Although it was pointed out earlier that the irradiation cannot open a gap in semi-Dirac material\textsuperscript{20} within the high frequency limit, we reveal in our work that beyond this limit a gap indeed opens up in the Floquet quasi-energy spectrum for different values of momentum. Moreover, unlike the graphene case where the gap parameter is a constant mass term\textsuperscript{1}, it is not constant in semi-Dirac material. Rather, it depends on the angular orientation of the momentum. Furthermore, several dips appear in the conductance spectrum which are in fact due to the effect of irradiation originating from the higher order correction to the Floquet spectrum. The earlier prediction that no gap appears under the influence of an irradiation within high frequency limit, is true only for particular angular orientation of momentum which is also confirmed through non-zero conductance dip at undoped situation. This is in contrast to the case of graphene where zero conductance sharp dip appears. This is due to the fact that external irradiation can induce a momentum independent constant mass term in the graphene, which yields a sharp conductance dip\textsuperscript{27}.

We also show that the zigzag nanoribbon of this material, without any irradiation, exhibits a pair of decoupled edge modes which is unlikely to appear in monolayer graphene where edge modes merge into the bulk at two valleys. However, if the ribbon is exposed to the irradiation these edge modes no longer remain fully separated from the bulk. Finally, we also carry out the analysis of conductance behavior through the nanoribbon geometry of an irradiated semi-Dirac material. We observe similar conductance dips. However, the overall amplitude of the conductance remains less sensitive to the location of the chemical potential, except at the dips. This arises due to the very slow variation of the number of transverse edge modes crossing through the chemical potential.

The remainder of the paper is organized as follows. In Sec. II, we introduce the low energy effective Hamiltonian and corresponding band structure with and without the presence of external irradiation. The Floquet scattering formalism and conductance calculation for the bulk semi-Dirac material are presented in Sec. III. Sec. IV is devoted to the discussion of band spectrum and conductance features of a nanoribbon geometry both in absence and presence of the periodic driving. Finally, we summarize and conclude in Sec. V.

II. MODEL HAMILTONIAN AND BAND DISPERSION

A. Non-irradiated case

In this subsection, we discuss the basic features of the band structure when the system is not exposed to the external irradiation. We begin with the single particle low energy effective model Hamiltonian, based on the tight binding model description\textsuperscript{7,21,22}, as

\[ H_0 = \frac{p_x^2}{2m^*} \sigma_x + v_F p_y \sigma_y, \]

where the effective mass term $m^*$ and the velocity $v_F$ can be expressed explicitly in terms of tight-binding model parameters as $v_F = 3ta/h$ and $m^* = 2h/(3ta^2)$. Here, $a$ and $t = t_1$ are the lattice constant and one of the hopping parameters, respectively. The Pauli matrices $\sigma \equiv \{\sigma_x, \sigma_y\}$ denote pseudo sublattice spin index. The 2D momentum operator is denoted by $p \equiv \{p_x, p_y\}$. The energy band dispersion of this semi-Dirac material can be obtained as

\[ E_{k_x,k_y} = \lambda \sqrt{(hv_F k_x)^2 + \left(\frac{h^2 k_y^2}{2m^*}\right)^2}, \]

where $\lambda = \pm$ is the band index and the 2D momentum vector is given by $k = \{k_x, k_y\}$. Note that, the energy dispersion is quadratic along $k_y$ direction and linear along $k_x$ direction, justifying the nomenclature of such materials as semi-Dirac material. A contour plot of the band dispersion is demonstrated in Fig. 2 which indicates the anisotropic band structure of such material.

B. Irradiated case

This subsection is devoted to address the effect of irradiation on the band structure of the semi-Dirac material via the low energy effective Hamiltonian. We consider a time dependent periodic perturbation as irradiation under which the dynamics of the charge carriers can be described by the Floquet theory\textsuperscript{28}. This is analogous to the Bloch’s theorem about the electronic motion in presence of a space dependent periodic potential. The irradiation is considered to be a circularly polarized light which can be expressed as a vector potential $\mathbf{A} = A_0[\cos(\omega t)x + \sin(\omega t)y]$. Here, $\omega$ and $A_0$
are the frequency and amplitude of the irradiation, respectively. To include this vector potential into the low energy effective Hamiltonian, we use the Peier’s substitution \( p \rightarrow (p + eA) \) and obtain the irradiated Hamiltonian as

\[ H_T = H_0 + \mathcal{H}_1 \sigma_x + \mathcal{H}_2 \sigma_y \]

where

\[ \mathcal{H}_1 = [Fk_y \cos(\omega t) + G(1 - \cos(2\omega t))] \]

and

\[ \mathcal{H}_2 = R \sin(\omega t) \]

with \( F = eA_0/m^* \), \( G = (eA_0/2m^*)^2 \) and \( R = eA_0v_F \).

To solve the above Hamiltonian, we employ the Floquet formalism which infers that the system under the time dependent periodic perturbation exhibits a complete set of solutions of the form \( \Psi(r,t) = \phi(r,t) \exp(-i\epsilon t/\hbar) \), where \( |\phi(r,t+T)| = |\phi(r,t)| \) is the corresponding Floquet states and \( \epsilon \) is the quasienergy.\(^{28}\) Here, \( T \) is the periodicity of the perturbation. Now, we can directly substitute the Floquet eigen states into the time dependent Schrödinger equation and obtain a time independent Schrödinger type equation \( \hat{H}_F \phi(r,t) = \epsilon \phi(r,t) \) with the Floquet Hamiltonian \( \hat{H}_F = \hat{H}_0 - i\hbar(\partial/\partial t) \). The Floquet eigen states can be further expressed in the Fourier expansion as\(^{28}\)

\[ \phi(r,t) = \sum_n \phi_n(r,t)e^{\im \epsilon t} \]

where \( n \) denotes the Floquet modes or side bands. Hereafter, we discuss the Floquet band structure of the irradiated semi-Dirac materials in two different regimes of frequency, \( \omega \).

1. **High frequency limit:**

Here, we briefly review the band structure of the semi-Dirac material within the high frequency limit which has been previously considered in Ref. \([7]\). In this limit, the irradiated Hamiltonian can be recast to an effective Hamiltonian as\(^7\)

\[ H_{\text{eff}} \approx H_0 + \frac{[H_{-1}, H_{+1}]}{\hbar \omega} \]  \( \tag{7} \)

where the second term arises due to the irradiation and

\[ H_m = \frac{1}{T} \int_0^T dt e^{-im\omega t} \hat{H}(t) \]  \( \tag{8} \)

with \( H_{-1} = H_{+1}^* = (Fk_y \sigma_x - iR \sigma_y)/2 \). After incorporating these terms in Eq.(7), the effective Hamiltonian can be found to acquire a direction dependent mass term as \(-\eta(\hat{F}/\hbar \omega)k_y \sigma_x \) with \( \eta = \pm \) denotes left (right) circularly polarized light. Note that, the mass term vanishes at \( k_y = 0 \) i.e., irradiation cannot open up a gap in the Floquet band structure. This was in fact the main claim of Ref. \([7]\). However, we emphasize here that a higher order term \((G)\) in the total irradiated Hamiltonian has been ignored in the earlier work.

2. **Beyond the high frequency limit:**

In this subsection, we investigate the full band structure of the semi-Dirac material beyond the high frequency approximation. Using the Fourier expansion of Floquet modes, we can write the time-independent eigen value problem as

\[ \sum_{n'} H_{F,nn'} \phi_n = \sum_{n'} [H_{0F,nn'} + V_{F,nn'}] \phi_n = \epsilon \phi_n \]  \( \tag{9} \)

where the diagonal part of the Floquet Hamiltonian can be defined as

\[ H_{0F,nn'} = \left[ \frac{p_x^2}{2m} + v_F p_x \sigma_y - \mu + n' \hbar \omega \right] \delta_{nn'} \]  \( \tag{10} \)
and the irradiation induced coupling Hamiltonian between different Floquet side-bands can be written as

\[
V_{F,nn'} = [H_{+1} \delta_{n,n'-1} + H_{-1} \delta_{n,n'+1}] \\
+ [H_{+2} \delta_{n,n'-2} + H_{-2} \delta_{n,n'+2}].
\]  

(11)

Here, \(H_{+2} = H_{-2}^\dagger = (eA_0) \lambda / 4m^*\). Note that, to diagonalize the Floquet eigen value problem, we need to set a cut-off to the dimension of the Floquet space. For example, if we impose the cut-off to the Floquet side band at \(N\), then the size of the matrix becomes \(2(2N + 1) \times 2(2N + 1)\) dimensional including the sublattice degree of freedom 2. In our present case, we consider \(n = 2\) for the quasi-energy and transport properties. This is a valid approximation, as the higher side-bands cause vanishingly small contribution to the transport. We show the quasi energy band spectrum in Fig. 3, which is obtained by numerically diagonalizing the Floquet Hamiltonian \(H_{F,nn'}\), for three different values of angular orientation of momentum. Also note that, the eigen value equation [Eq.(9)] is normalized by \(\hbar \omega\) in order to diagonalize the Hamiltonian. This introduces two new parameters: \(\beta = e v_F A_0 / \hbar \omega\) defining the strength of the irradiation and a length scale \(L_s = v_F / \omega\). A finite value of \(\beta\) is required for gap opening in the spectrum. The quasi energy spectrum is gapless for \(\theta = 0\), whereas a finite gap appears for \(\theta \neq 0\). However, note that even for \(\theta = 0\), gap opening does not occur at \(\epsilon = 0\) for \(k = 0\). This was also predicted earlier via high frequency approximation\(^7\). Another noticeable point here is that unlike the case of irradiated monolayer graphene\(^{27}\), where gap opening appears at the momentum values \(k = \pm j \omega / 2 v_F\) \((j = 1, 2, 3, ...)\), here in semi-Dirac materials it depends on the angular orientation of the momentum.

### III. METHODOLOGY AND NUMERICAL RESULTS FOR THE BULK TRANSPORT

In this section, we discuss the quantum transport properties through the irradiated bulk region of the semi-Dirac material by using Floquet scattering matrix formalism\(^{29}\). The relevant geometry is schematically depicted in Fig. 4. The set up is composed of three different regions in which left and right regions are protected from the exposure of irradiation by placing a cover on it (as shown by grey color) while the middle region is illuminated by circularly polarized light.

#### A. Solving scattering problem using Floquet method

In order to solve the scattering problem, we employ the approach prescribed in Ref. [29] which is very recently employed in graphene\(^{27}\). The non-irradiated left region is doped in such way that the chemical potential matches with the zeroth \((n = 0)\) Floquet side band inside the central irradiated region\(^{27}\). In such case, the incoming electron from the left region will be scattered inelastically among different Floquet side-bands inside the irradiated region by absorption/emission of photon process (see the lower panel of Fig. 4). Subsequently, the reflection and transmission processes will occur from all Floquet bands. To proceed further, we need to obtain the Floquet states and momenta \((k_x)\) of electrons scattered from each side band, for a particular \(k_y\). To obtain that, we multiply \(\sigma_y\) from the left side in Eq.(9) and rearranging it we get

\[
\sum_n (Q_{0F,nn'} + Q_{VF,nn'}) \phi_n = \hbar v_F k_x \phi_n.
\]  

(12)

where

\[
Q_{0F,nn'} = \sigma_y \left[ (\mu - n' \hbar \omega) - \frac{p_y^2}{2m^*} \sigma_x \right] \delta_{nn'},
\]  

(13)

and

\[
Q_{VF,nn'} = - \frac{\sigma_y}{2} (Fk_y \sigma_x - i R \sigma_y) (\delta_{n,n'-1} + \delta_{n,n'+1}).
\]  

(14)

The numerical diagonalization of Eq. (12) yields \(2(2N + 1)\) values of \(k_x^\nu\) and eigen vectors \(\phi_n^\nu\) with \(\nu\) being the index of different eigen modes. Here, we set the quasi energy at \(\epsilon = 0\) which corresponds to the incident electron at the chemical potential of the left normal lead. The wave function for a given \(\nu\) can be expressed as

\[
\phi^\nu(x,t) \sim e^{i k_x^\nu x} \sum_{n=-N}^{N} \phi_n^\nu e^{-i \omega t}.
\]  

(15)
which is a two component vectors composed of two sublattices. Note that, the momentum \((k_x^0)\) can be real or imaginary depending on the location of the chemical potential in the left region. The imaginary momentum corresponds to the situation when the chemical potential lies inside the gap. This leads to the emergence of evanescent mode. On the other hand, the spectrum in the left/right non-irradiated region can be obtained in similar fashion but with zero strength of the irradiation as those regions are assumed to be free from the external light. However, the electrons which are scattered from the irradiated central region to the normal regions via different fictitious Floquet side-bands of the normal regions, are governed by the following eigen value equation

\[
\frac{p_y^2}{2m^* \sigma_x} + v_F p_x \sigma_y \phi_n = \left[ \mu(\theta) - n \hbar \omega \right] \phi_n .
\]

As the side bands are decoupled from each other in the normal regions due to the absence of irradiation, the momentum of the scattered electrons in each artificial side band remains identical. Furthermore, the wave function of the incident as well as the reflected/transmitted electrons are given by

\[
\phi_{n,\lambda}(x) = \frac{1}{\sqrt{2}} \left[ \frac{\lambda}{ie^{-i\theta}} \right] e^{i k_x x} ,
\]

where \(\tan \theta = \left[ \hbar k/(2m^* v_F) \right] \tan \theta \sec \theta \) with \(\theta = \tan^{-1}(k_y/k_x)\). After substituting the tight-binding model parameters\(^7\) for mass and Fermi velocity as \(m^* = 2\hbar/(av_F)\) and \(v_F = 3at\hbar\), we obtain \(\tan \theta = k\tan \theta/(4\cos \theta)\) with \(k = ka\). Note that, the chemical potential in the non-irradiated region becomes direction dependent as

\[
\mu(\theta) = \hbar \omega + \lambda \hbar v_F k \left[ \cos^2 \theta + \left( \frac{ka}{4} \right)^2 \sin^4 \theta \right]^{1/2} .
\]

\[\text{FIG. 5. (Color online) The behavior of transmission probabilities through different Floquet side bands are illustrated as a function of angle of incidence in the left normal region (non-irradiated).}\]

\[\text{B. Boundary conditions at the interfaces}\]

To solve the scattering problem we match the wave functions at the two interfaces \(x = 0\) and \(x = L\). Here, \(L\) is the length of the irradiated region. We assume that the electron is incident only from the \(n = 0\) Floquet band of the left non-irradiated region and after encountering inelastic scattering processes inside the irradiated region, it is either reflected or transmitted through all possible virtual Floquet side bands. Thus, there will be \((2N + 1)\) reflected channels in the left region in which the wave function is given by

\[
\Psi_L(x, t) = \left[ \phi_i e^{ik_x x} + \sum_{n=-N}^{N} r_{n0} \phi_r e^{-i(k_x x + n\omega t)} \right] e^{-i\epsilon t} ,
\]

where the Floquet coefficients of the incident and reflected waves are \(\phi_i = \left[ \frac{\lambda}{ie^{-i\theta}} \right]\) and \(\phi_r = \left[ \frac{\lambda}{ie^{-i\theta}} \right]\) respectively. Here, \(\tan \theta_r = \tan \theta (\theta \Rightarrow \pi - \theta)\). The reflection amplitude corresponding to the \(n\)-th Floquet band is denoted by \(r_{n0}\). The Floquet eigen states inside the irradiated region can be written as

\[
\Psi_I(x, t) = \sum_{\nu} a_{\nu} \phi_{\nu}(x, t) e^{-i\epsilon t} = \left[ \sum_{\nu} a_{\nu} e^{ik_x x} \sum_{n=-N}^{N} \phi_{\nu} e^{-i\epsilon t} \right] .
\]

On the other hand, the wave function in the right non-irradiated region is composed of only transmitted waves and can be written as

\[
\Psi_R(x, t) = \left\{ \sum_{n=-N}^{N} t_{n0} \phi_r e^{i(k_x x - L + n\omega t)} \right\} e^{-i\epsilon t} .
\]

Here, \(t_{n0}\) represents the transmission amplitude via \(n\)-th band into the right normal region. The reflection and transmission amplitudes can be evaluated by matching the wave functions at the two interfaces which yields a set of linear equations as

\[
\phi_i \delta_{n0} + r_{n0} \phi_r = \sum_{\nu} a_{\nu} \phi_{\nu} ,
\]

\[
\sum_{\nu} a_{\nu} \phi_{\nu} e^{i k_x L} = t_{n0} \phi_l .
\]

where \(\phi_l = \phi_i\).

\[\text{C. Scattering amplitudes and Conductance}\]

In this subsection, we present the numerical results for ballistic transport through the irradiated semi-Dirac
material for different strengths of driving. As mentioned earlier, we assume that the chemical potential in the left lead is set at such a position that the electrons can only be incident from the virtual $n = 0$ Floquet band of the left lead and subsequently after entering into the irradiated region it encounters scattering into different Floquet bands via the inelastic scattering processes (absorption and emission). The reflection and transmission probabilities for a particular angle of incidence and energy of the incident electron are the sum over all reflection and transmission amplitudes via different Floquet bands. This can be written as

$$T(\theta, \mu) = \sum_{n=-N}^{n=N} |t_{n0}(\theta, \mu)|^2,$$  \hfill (24)

and

$$R(\theta, \mu) = \sum_{n=-N}^{n=N} |r_{n0}(\theta, \mu)|^2,$$  \hfill (25)

which satisfies the unitarity condition as $R(\theta) + T(\theta) = 1$. Once the transmission amplitudes are evaluated, one can readily employ the Landauer-Buttiker formula to obtain the conductance by averaging over all possible angle of incidence as

$$G = \frac{\alpha^2}{h} \int_{-\pi/2}^{\pi/2} d\theta \sum_{n=-N}^{N} |t_n(\theta, \mu)|^2.$$  \hfill (26)

We present the behavior of transmission probabilities via different Floquet side bands in Fig. 5. Here, the incident electron is considered to be from the zeroth Floquet band ($n = 0$) of the non-irradiated left lead. We consider three Floquet side bands inside the central irradiated region which is sufficient to preserve the probability conservation. The transmission probability via $n = 1$ side band is shown for different strengths of irradiation $\beta$ (see Fig. 5). Note that, $\beta$ couples the two nearest side bands for which the transmission probability increases via $n = 1$ side band as we enhance $\beta$. This happens due to one photon absorption process. As a result, this reduces the same in $n = 0$ side band. The incident electron from the left lead at $n = 0$ band can be transmitted/reflected through the same band $n = 0$ which we call intra-band scattering whereas transmission/reflection through $n = 1$ side band is termed as inter-side band scattering. The intra-band transmission probability can be $100\%$ at normal angle of incidence ($\theta = 0$) for $\beta = 0$ due to the Klein tunneling. However, with the increase of $\beta$ the Klein process becomes suppressed resulting in enhancement of inter-band transmission probability. The reason can be attributed to the fact that, as the amplitude of the irradiation increases the coupling strength between the two nearest Floquet side bands increases, causing enhancement of the inter-band transmission probability.

The driven conductance of the bulk semi-Dirac material is demonstrated in Fig. 6. Here we use Eq.(26) to compute the conductance for two different lengths of the irradiated region. In Fig. 6(a), the conductance is shown for different strengths of the periodic driving around chemical potential $\mu = 0$ and $\hbar \omega/2$. These correspond to the gaps opening of the band dispersion, as shown Fig. 3. Note that, although the Floquet spectrum is gapless at $k = 0$ which is even confirmed in Ref. [20] within the high frequency approximation, the band gap indeed appears for other values of $k$ which yields a conductance dip at $\mu = 0$. We would like to emphasize here that in case of graphene\cite{27}, the location of gap appears to be at $k = \pm v_F/\omega$. On the contrary, in semi-Dirac material it depends on the angular orientation of the moemtum as shown in Fig. 3. Some other conductance dips also arise at $\mu = \hbar \omega/2$ which correspond to the dynamical gap between two Floquet side bands. The qualitative features of the driven conductance remain similar as we
choose a different length of the central irradiated region (see Fig. 6(b)). However, the conductance dips become more deeper due to enhanced probability of Floquet sub-band scattering.

Note that, the irradiation (circularly polarized light) can open-up a gap in graphene, which does not depend on momentum distribution. This can result in rectangular shaped sharp conductance dips with almost zero values under the suitable strength of irradiation. On the other hand, the driven semi-Dirac material may not acquire a gap at \( k = 0 \), in the \( n = 0 \) band. However, the conductance dips still appear due to the gap opening at other \( k \) values away from zero. The conductance cannot be exactly zero in this case at \( \mu = 0 \) as the semi-Dirac material is metallic at \( k = 0 \). Nevertheless, around the dynamical gap \( \mu = \pm \hbar \omega / 2 \), the dips appear due to the gap opening between two nearest side bands.

IV. BAND DISPERSION AND TRANSPORT THROUGH A NANORIBBON GEOMETRY OF SEMI-DIRAC MATERIAL

In this section, we address the electronic and transport properties of this material for the nanoribbon geometry in presence of periodic driving. However, our discussion for the nanoribbon is restricted to zigzag case only due to its unique features over armchair one, as demonstrated in case of graphene.

A. Non-irradiated ribbon:

In order to find the energy band dispersion of a zigzag nanoribbon of semi-Dirac material, an effective difference equation can be constructed by employing an analogy to the case of an infinite one-dimensional chain. To implement this, the nanoribbon can be considered as the composition of an array of the vertical rectangular unit cells (supercells). The width of the zigzag nanoribbon (ZNR) is determined by the number of atoms \( N \) per unit cell. The effective difference equation of the ZNR takes the form as

\[
(E \mathbb{I} - \mathcal{E}) \psi_l = \mathcal{T} \psi_{l+1} + \mathcal{T}^\dagger \psi_{l-1},
\]

where

\[
\psi_l = \begin{bmatrix}
\psi_{l,1} \\
\psi_{l,2} \\
\vdots \\
\psi_{l,N}
\end{bmatrix}.
\]

Here, \( \mathcal{E} \) and \( \mathcal{T} \) are the on-site energy and nearest-neighbor hopping matrices of the unit cells, respectively.

B. Irradiated ribbon

In this subsection, we include the effect of irradiation in the band structure of the semi-Dirac material based.
To obtain the band dispersion we diagonalize the following Floquet tight-binding Hamiltonian as

\[ H_F = \left( \tilde{\mathcal{E}} + \tilde{T}_F e^{i k a} + \tilde{T}_F^* e^{-i k a} \right)_{N \times N} - m \hbar \omega \delta_{m,m} \delta_{m,m} \]

(30)

in the Floquet extended space. The size of the Floquet Hamiltonian is \( N \times m \) where \( m \) denotes the Floquet replicas (side bands), representing different number of photons. The above Hamiltonian is a block matrix where each block is of \( N \times N \) dimension. The Floquet on-site energy is now denoted by \( \tilde{\mathcal{E}} \). The effect of irradiation on the band structure can be included by modifying the hopping parameters between \( i \)th and \( j \)th sublattice as

\[ t_{ij} = t_{ij} \exp \left( \frac{2\pi i m \pi}{\phi_0} \int r_i A(t) d\mathbf{r} \right) \]

(31)

After integrating out the time part, the three nearest neighbour hopping parameters that are left, can be written as

\[ t_{12} = t_1 e^{i m \pi / 6} J_m(-z) \]

(32)

\[ t_{13} = t_1 e^{i 5 m \pi / 6} J_m(z) \]

(33)

and

\[ t_{14} = t_2 e^{-i m \pi / 2} J_{-m}(-z) \]

(34)

Here, \( J_m(z) \) represents the Bessel function of the first kind of order \( m \). The schematic of the three nearest neighbour hopping parameters, between \( i = 1 \) and \( j = 2, 3, 4 \), is shown in Fig. 8. The above mentioned emergent hopping parameters are taken into account in Eq. (30) in order to numerically diagonalize it in the extended Floquet space. The quasi energy band dispersion is obtained numerically and depicted in Fig. 9.

Here, the dimensionless parameter \( z = 2\pi A_0 a / \phi_0 \) denotes the strength of periodic driving. In our analysis, the number of Floquet replicas (side bands) are considered to be \( m = 2 \). This is sufficient to capture the relevant features of the band structure. Any further enhancement of the number of side bands only assembles more quasi energy modes. Unlike the case of non-irradiated band structure of ZNR (see Fig 7), the edge modes (sky blue lines) in the irradiated case are not fully separated from the bulk. Also the dynamical band gap, at \( \hbar \omega / 2 \), appears to be vanishingly small (see Fig. 9(a)). Moreover, the band gap can be further reduced to be almost zero if we increase the amplitude of the irradiation, as shown in Fig. 9(b). Hence, one can infer that by applying external periodic driving the decoupled edge modes can be enforced to couple the bulk in semi-Dirac material. Note that, in graphene nanoribbon, the irradiation can give rise to topological chiral Floquet edge modes.
around the dynamical gap at $\hbar \omega/2$. However, such feature is absent in case of semi-Dirac ZNR. Nevertheless, we also check that by setting $t_2 = t_1$, the chiral Floquet edge modes of graphene can be recovered.

Finally, we also explore the transport signature of the ZNR geometry. To obtain the conductance, we extend the formalism used for the bulk by taking into account the quantized transverse wave vector, determined by the width of the ribbon as $k_y = n \pi/W$ with $n = 1, 2, 3..$ being the number of transverse edge modes. Note that, in this method the edge modes cannot be captured, rather mostly transverse modes reside inside the bulk. This approximate method is already implemented in case of graphene. In our case, as the edge modes are not chiral in nature, we use the same formalism as used for the bulk instead of the formalism described in Refs. [33 and 34]. The conductance for the ZNR is shown in Fig. 10. The behavior of the conductance exhibits the dips at $\mu = 0$ and $\pm \hbar \omega/2$. This is quite similar to the case of the bulk semi-Dirac material. However, the conductance varies very slowly with chemical potential except at the locations of the dips. In our numerical analysis, we consider the number of transverse edge modes to be $n = 50$ for which conductance remains weakly oscillatory around $50e^2/h$. A more careful analysis of the conductance for semi-Dirac ZNR geometry, based on Floquet-Landauer Green’s function approach is beyond the scope of the present work and will be presented elsewhere.

V. SUMMARY AND CONCLUSIONS

In our work, we investigate the electronic and transport properties of an irradiated semi-Dirac material. We consider the irradiation in the form of a circularly polarized light. We present our results for the semi-Dirac material in comparison to the irradiated monolayer graphene. The irradiation is unable to open up a gap in the Floquet spectrum of semi-Dirac material within the high frequency approximation. However, beyond this approximation, a full band dispersion analysis reveals that the band gap indeed appears for higher values of the momentum and between two nearest Floquet side bands. Such appearance of the band gap in the quasi energy spectrum can also be probed via the signature of the conductance spectrum. The latter manifests several dips around the gaps. We also observe that the band gap opening is strongly dependent on the angular orientation of momentum due to the anisotropic band structure of the semi-Dirac material. We also explore the band structure of nanoribbon geometry comprised of such material and show some distinct features in comparison to the monolayer graphene nanoribbon. The ZNR of the semi-Dirac material hosts a pair of edge modes which are fully detached from the bulk. Furthermore, in presence of an external irradiation, ZNR of such material does not host topologically protected Floquet chiral edge modes like graphene. Moreover, the edge modes become coupled to the bulk and the dynamical band gap in the Floquet spectrum begins to disappear with the increase of the strength of the periodic drive. Finally, we also explore the possible features of the conductance of the irradiated ZNR based on semi-Dirac material. The conductance exhibits similar dips around the same locations of the chemical potential as in the bulk except the amplitude mismatch due to the presence of finite number of channels.

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