Matter and interactions: a particle physics perspective

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Abstract
In classical mechanics, matter and fields are completely separated; matter interacts with fields. For particle physicists this is not the case; both matter and fields are represented by particles. Fundamental interactions are mediated by particles exchanged between matter particles. In this article we explain why particle physicists believe in such a picture, introducing the technique of Feynman diagrams starting from very basic and popular analogies with classical mechanics and making the physics of elementary particles comprehensible even to high school students, the only prerequisite being a knowledge of the conservation of mechanical energy.

Introduction
In this paper we present the outline of a lecture in which we try to make particle physics popular, even including those mathematical details usually skipped when teaching modern physics to degree students of disciplines other than physics, such as engineering or informatics. The content of the lecture proved to be effective for medical students, too. With some effort, it can even be understood by high school students. The prerequisites, in fact, are just a knowledge of the concept of energy, the principle of its conservation and some basics of mathematics.

Particles and fields
Particle physics is a very ambitious field, aiming to describe what the Universe is made of as well as how its most fundamental constituents interact. In other words, it is the science of elementary particles and fundamental forces. Because of its name, particle physics is often thought to be related to matter only. Actually, forces are the subject of particle physics, too, because of quantum mechanics. The latter predicts that matter behaves like waves or fields under certain circumstances, as well as fields and waves behaving as particles. Matter and fields, then, are tightly bound to each other in quantum mechanics.

In particle physics, only fundamental forces are considered, i.e. those forces that can be described in terms of fields. For example, gravity is the force felt by a body of mass $m$ (such as a planet) immersed in a gravitational field $\mathcal{G}$, in such a way that $F = mg$. The field produced by the Sun exists irrespective of the presence of the planets and depends only on the Sun’s mass and on the distance with respect to it. Conversely, there is no field for friction or elastic forces. Those, actually, are nothing but manifestations of fundamental forces in complex configurations. For example, the atoms of a spring are bound together because of Coulomb-like forces. There is an equilibrium between repulsive forces and attracting ones. Extending the spring, the atoms become further from each other and at a given point attractive forces dominate, causing the spring...
to develop a force that is, in fact, the sum of elementary, individual electrostatic forces.

We say that mass is the source of the gravitational field. Electric charge is the source of the electric field. Currents are the sources of the magnetic field. Each of these sources provides a field, i.e. modifies the space around it in such a way that, putting another source of the field in that space produces a force between them. For example, electric charges generate electric fields. Putting another electric charge in such a field produces a force. No force is expected by putting a neutral body in the electrostatic field, since such a body is not in turn a source of electric field. A magnetic field develops around an electric current flowing in a conductor. Currents are moving electric charges. If we put any body within this field, it does not show any force unless it has an electric charge and has non-zero speed.

Besides gravity and electromagnetism, we know about the existence of two further forces: the strong force, responsible for holding protons together in the nucleus, against electrostatic repulsion, and the weak force, responsible for nuclear decays.

**Dynamics**

The equation of motion of particles in a field, whatever their nature, can be written simply as

$$\Delta U = 0.$$  

That is quite trivial. Such an equation, in fact, can describe the whole Universe because any physics law can be written as $f(x) = 0$. For example, one can write $F = ma$ as $f_1 = F - ma = 0$, or $V = RI$ as $f_2 = V - RI = 0$. Then, summing up all the known physics laws we can define $\Delta U = \sum f_i = 0$. That is not very useful. In contrast, if we were able to give a simple enough definition for $U$, it is possible that all the dynamics of particles could be described by what is called a variational principle. The equation above states, in fact, that $U$ is a constant. A variation in the value of $U$ in an isolated system (i.e. a system that cannot exchange matter nor energy with other systems) must be zero. From this principle comes the laws of physics.

Students should already know at least an example of this principle. Consider a mechanical system composed of a particle of mass $m$ in a uniform gravitational field with acceleration $g$. The particle energy $U$ is the sum of its potential energy $V = mgh$ and its kinetic energy $K = \frac{1}{2}mv^2$. The height $h$ of the particle is measured with respect to a conventional level, for which we define $U = 0$. The energy of the particle is conserved, i.e. $U = \text{const}$ at any time. That, of course, does not mean that nothing changes. In fact, leaving the particle falling, its speed increases, while its position varies with time. The variational principle states that a variation of $U$ must be zero. Imposing $U = \text{const}$ corresponds to taking its derivative and putting it equal to zero.

Let us take this derivative and impose that it must vanish. Since $U$ is a function of both $h$ and $v$, the variation $dU$ of $U$ is

$$dU = \frac{\partial U}{\partial h} dh + \frac{\partial U}{\partial v} dv = m g dh + m v dv.$$  

If $dU = 0$, then $dU/dt = 0$, and

$$\frac{dU}{dt} = m g \frac{dh}{dt} + m v \frac{dv}{dt} = m g v + m a v dt,$$

where $a = dv/dt$ is the acceleration of the particle. The same result can be obtained just using differences, without the need for a knowledge of the concept of derivatives, as shown in many high school textbooks.

In the end, we have $-mg = F = ma$, a statement of Newton’s second law. The dynamics of particles in uniform gravitational fields, then, comes from a variational principle. The fundamental law is the one expressing energy conservation. Newton’s law is a consequence of energy conservation.

The same is true in non-uniform gravitational fields. In fact, the definition $V = mgh$ is just an approximation. Everyone knows that gravity is not uniform, but scales as $1/r^2$, where $r = r_e + h$ and $r_e$ is the Earth’s radius. However, if $h \ll r_e$ it can be considered as uniform. In fact, $M$ being the Earth mass and remembering that $GM/r_e^2 = g$, the gravitational potential at distance $r_e + h$ from the Earth’s centre is

$$V = -G \frac{M}{r_e + h} = -G \frac{M}{r_e (1 + \frac{h}{r_e})}$$

$$\simeq -G \frac{M}{r_e} \left(1 - \frac{h}{r_e}\right) = -G \frac{M}{r_e} + gh.$$  

(1)

The potential energy of a body of mass $m$ is just $V$ multiplied by $m$. The first term is constant.
and we can arbitrarily set it as zero, since adding
constants to or subtracting them from $U$ does not
affect the results. Generally speaking, any function
$V = V(r)$ can be written as a power series
$V(r) \sim V(r_e) + V'(r_e)h + \frac{1}{2}V''(r_e)h^2 + \cdots$,
where $r = r_e + h$. This technique is called Taylor
expansion. In the case of gravity, its potential can
be expanded in a power series like

$$
V \simeq -\frac{GM}{r_e} + \frac{M}{r_e} \left( \frac{r - r_e}{r_e} \right) - \frac{GM}{r_e} \left( \frac{r - r_e}{r_e} \right)^2 + \cdots.
$$

It is straightforward to recognize the first two
terms in the expansion as those written in
equation (1).

**The equation of the Universe**

Given the ingredients outlined above, we can now
proceed to write the fundamental equation for the
particles and fields of which the Universe is made.
Remember that, in quantum mechanics, particles
and fields are almost the same object. Despite it
not being an ordinary number, for our purposes we
can treat a field almost like a number.

Our equation of the Universe is always of the
form $\Delta U = 0$, where $U = K + V$ is now a
function of the fields. The simplest expression for
$V$, which generates the dynamics, is just a product
of all the independent variables determining the
energy of the Universe, such that the physical
dimensions of the product are those of the energy.
There must be at least two particles to give rise to
some interaction. Let us call $\bar{\psi}$ and $\psi$ the fields
of those two particles. $\psi$ and $\bar{\psi}$ are some function
of the state of each particle and depend on mass,
position, momentum, angular momentum and any
other number characterizing the particles.

Calling $A$ the field representing the inter-
action, a possible expression for the interaction
potential $V$ is $V = \alpha \bar{\psi} A \psi$, where $\alpha$ is some
coupling constant, defining the strength of the
interaction. The higher $\alpha$ is, the stronger the
interaction.

The term $\alpha \bar{\psi} A \psi$ must be analogous to the
first term of the expansion of the potential energy
of a body in a gravitational field, $mg\hbar$. Then it
can be conceived as a term of the Taylor expansion
of the function $V$ representing the energy of the
Universe. The higher order terms are given by
powers of $\alpha \bar{\psi} A \psi$:

$$
V \simeq V_0 \left( 1 - \alpha \bar{\psi} A \psi + \frac{1}{2} \alpha^2 \left( \bar{\psi} A \psi \right)^2 + \cdots \right).
$$

The constant term is irrelevant because we can
add or subtract arbitrary values to $V$ without
changing the dynamics. Despite the fact that the
product $\bar{\psi} A \psi$ is not an ordinary product between
numbers, a theorem known as Wick’s theorem [1]
provides simple rules about how to compute the
various terms of $V$. The theorem states that the
product must be taken in such a way that terms
of order $\alpha^n$ must be obtained by evaluating the
fields at $n$ different points. Richard Feynman [2]
observed that each term in Wick’s theorem can
be graphically represented in such a way that
identifying the relevant terms for a given process
is as simple as writing lines on a piece of paper
forming connected graphs. For this reason, these
graphs are called Feynman diagrams.

An oversimplified version of the rules to write
a Feynman diagram are as follows: for each $\bar{\psi}$
write an arrow, entering in one point of the space-
time; an arrow must be written also for $\psi$, but
it has the opposite direction, i.e. it comes out of
that point. A translates into a wavy line. The
points in which all the lines come together are
those foreseen by Wick’s theorem and are called
vertices. Of course, real Feynman rules are much
more complex than the ones we are illustrating
here.

The first term of the expansion, then,
translates into a diagram like the one in figure 1.
If $\bar{\psi}$ represents, for example, an electron, and $A$
another particle, $\psi$ is an electron, too, moving in
the opposite direction with respect to the vertex
(the point in which the three lines merge together).
There is just one vertex for this term, since it
is of order $\alpha$ ($n = 1$). The Feynman diagram
in figure 1 can be thought of as the visual
representation of a process in which an electron
coming from the bottom left, emits a particle
represented by $A$ and, consequently, modifies
its direction towards the top left, to conserve
momentum. It is important to understand that this
is not necessarily what happens at the microscopic
level. The reality is described by the equation
of motion, not by the Feynman diagram. The
latter is just another, funny way to write $\alpha \bar{\psi} A \psi$.
What is important for physics are the predictions.
Figure 1. A very simple Feynman diagram showing the interaction between two particles and a field.

Figure 2. A Feynman diagram generated by taking the second term of the Taylor expansion of the variation of the energy.

made by the equations. As far as these predictions conform to experimental results, we are then free to interpret the diagram as exactly what happens at the microscopic level.

Quantum mechanics allows the computation of the probability of a given process. Computing the first term of the $V$ expansion (the one given by $\alpha \bar{\psi} A \psi$) gives zero, i.e. such a process is forbidden. In fact, there is no way for an electron to undergo such a process, since it does not conserve energy and momentum. That is encouraging, in fact.

The dynamics, then, must come from the second term of the expansion, whose Feynman diagram is the product of two diagrams like those discussed above. Take two of them and join them together. In this way we have two vertices, as prescribed by Wick’s theorem. A possible way is joining the two curly lines, by which one obtains the diagram in figure 2.

The associated term in the expansion contains four particle fields and represents an interaction with two electrons in the initial state and two electrons in the final state. It then gives the probability of interaction of two electrons. Computing this term we see that it is non-zero and it is incredibly similar to the expectations from experiments.

We can interpret this diagram as follows: two electrons moving from left to right come close to each other. One emits a particle that we call a photon, which is absorbed by the other. Both, then, change their momentum and proceed in their motion. The net result is the scattering of two electrons, initially coming close to each other, then moving apart. In other words, we are describing the electrostatic repulsion between the two electrons in terms of the exchange of a photon. In this respect, the photon can be seen as the mediator of the electromagnetic field. The electrostatic force is nothing but the result of the exchange of a photon, which is not visible in the final state, but carries the electric field.

This is why particle physics includes particles and fields as the same object. Both are particles: some of them are matter particles, some others are force mediator particles. We do not use Feynman diagrams because we believe a priori that particles interact between themselves exchanging particles, so we can write the interaction in terms of products of the corresponding fields. It is just the opposite: since the terms that enter into the equation of motion can be graphically represented by a Feynman diagram and the results are compatible with experiments, we are free to interpret Feynman diagrams as what happens at the microscopic level and we are led to the conclusion that interactions can be represented by the exchange of mediators, even if we cannot see them.

Another possibility is to write a Feynman diagram composed of the two terms of order $\alpha$ joined by the particle leg, instead of the photon, like in figure 3. This diagram is a valid diagram and its computation gives rise to a non-vanishing probability. In fact such a diagram can be interpreted as the interaction of an electron in an external electromagnetic field. The field is modelled by the photons. In both the initial and final state there is one electron and one photon. This same diagram represents
the Compton scattering, where a photon interacts with a free electron and emerges with a different wavelength, while the electron acquires some energy.

In summary, thanks to Feynman diagrams, we can graphically represent an equation and, in turn, we can interpret these diagrams as what really happens microscopically. In the end, we can interpret the fields as particles acting as force mediators.

**Higher order corrections**

As said in the previous section, the computation of the probability for a process to happen, according to the theory outlined above (called quantum electrodynamics or QED), is very close to that experimentally measured.

The predictions from QED differ from experiments only by a few per cent. However, this is not the end of the story. In fact, in the computation we just took the first non-vanishing term of the expansion. We can write other diagrams, with a higher number of vertices, corresponding to higher order terms in the expansion.

It is well known, even from classical physics, that when electrons accelerate they produce an electromagnetic field. In terms of QED the field is modelled by photons. In electron scattering, represented by the diagram in figure 2, at least one electron may be accelerated enough to produce an electromagnetic field, i.e. a photon.

The next term in the expansion of \( V \) reproduces exactly this situation, as can be seen from figure 4. This diagram, as expected, has three vertices, and can be built by joining three basic diagrams, like those in figure 1. It represents a process where two electrons come close to each other, interact and are scattered away, with the emission of some electromagnetic field.

Moreover, there are processes modelled by higher order terms, for which the final state is still composed of two electrons. In evaluating the probability for electron scattering, we must take into account these terms, since they are part of the expansion. One of these terms is represented by the Feynman diagram shown in figure 5, which has four vertices, representing a term of order \( \alpha^4 \), and is obtained by joining four basic diagrams.

The associated probability is small (smaller than the one associated with the main diagram). However, adding the corresponding probability to the one obtained by computing the diagram in figure 2 gives a result much closer to the experimental value. If we add more terms we obtain results closer and closer to those experimentally measured. In certain cases we are able to compute these kind of processes with a precision up to a few parts per million [3]. No other process in physics is known to this level of accuracy.

Another, very nice, accurate, yet popular, description of the Feynman diagrams technique is given in [4]. Feynman himself wrote a popular book on the subject [5].
Antiparticles
At the beginning of the 20th century, Dirac [6] noticed that the equation of motion of free electrons moving back in time can be regarded as the equation of motion of a free positron, i.e. a free electron with positive electric charge. Positrons are called the antiparticles of the electrons. Generally speaking antiparticles are just ordinary particles with opposite charges. Then, antiprotons are negatively charged protons.

Positrons were in fact discovered in cosmic rays in 1932 by Anderson [7]. However, positrons are not usually found in ordinary matter. So, how is it possible to observe them?

Feynman diagrams, together with the observation from Dirac, give us the answer. If we take the diagram in figure 2 and rotate it clockwise by 90°, assuming time flowing from left to right, we notice that there are two particles moving back in time: one in the initial state and one in the final state. They can be interpreted, according to Dirac, as antiparticles moving forward, as in the diagram shown in figure 6.

In this diagram the two legs moving from left towards the vertex are a particle and its antiparticle. When they come together they are said to annihilate into a photon. The photon, then, is said to materialize in a particle–antiparticle pair. In fact, photons with enough energy, however they are produced, can materialize in particle–antiparticle pairs. This explains how it is possible to observe antiparticles: they are the products of materialization, described by our theory that, in turn, can be outlined in terms of Feynman diagrams.

Needless to say, the probability of this process is predicted by the theory and is found to be exactly the size of the experimentally observed probability.

By the way, this is also the process used in colliders to produce new particles. Colliders make particles and antiparticles collide, producing new particle–antiparticle pairs that can be detected. The energy $E$ at which the initial state particles must collide to produce two particles of mass $m$ must be at least $E = 2 \times mc^2$ in the centre of mass, as predicted by Einstein’s theory of relativity.

Other interactions
Using Feynman diagrams we explained the phenomenology of electromagnetic interactions, identifying the photon as the electromagnetic force mediator. It turns out that the same theory can be made for other interactions. For example, if $A$ in the expansion terms of the energy variation is taken to be the field of strong or weak fields, and $\alpha$ is replaced by the corresponding coupling constants, the same technique can be applied and we can write down Feynman diagrams for which the curly lines represent the mediators of those forces rather than a photon. The diagrams appear exactly the same, but the values of the fields are different and the computation of the probability differs in size, but not in principle.

Other mediators, then, should exist for weak and strong forces; we believe the same applies for gravitation. In fact, using colliders, we can produce such mediators and observe their decay products, i.e. the particle–antiparticle pairs in which they materialize.

In 1983 CERN’s experiment [8, 9] UA1 discovered three new particles called $Z$, $W^+$ and $W^-$, predicted by the theory of weak interactions to be the mediators of the weak force. The mediators were produced by the annihilation between quarks and antiquarks and materialize in pairs of matter particles with a predictable topology. In fact, weak forces need three mediators to be explained, but they behave like photons with respect to Feynman diagrams.

The existence of the mediators of strong forces, the gluons, was proven in 1979 by looking at annihilations between electrons and positrons [10].

Conclusion
We introduced the technique of Feynman diagrams using very basic and popular concepts, explaining why particle physicists describe the interaction between matter particles as the exchange of mediators.
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