Gyroscope Fault Pattern Recognition Based on Wavelet Packet Decomposition and Fuzzy Clustering

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Abstract. Aiming at the problem that the sensor component's inertial navigation gyro failure symptom is not obvious in a certain aircraft flight control system, this paper proposes a fault cluster recognition method based on fuzzy clustering. First, use wavelet packet decomposition to perform three-layer wavelet packet decomposition on the gyro output data to obtain the feature extraction of the gyro fault signal; then use the fuzzy clustering method to classify the extracted fault signal to achieve gyro fault pattern recognition in the sensor assembly the goal of.

1. Introduction
A certain type of aircraft flight control system is the “brain” of the aircraft to realize flight control, and it is a comprehensive system integrating software and hardware. As an important component of the flight control system, the sensor component plays an important role in transmitting and obtaining various flight parameter information. The research on aircraft fault diagnosis system has been started since the 1980s. The maintenance diagnosis expert system developed by GM based on F-15 platform involves the fault diagnosis research of the relevant components of the flight control system [1]; With the continuous development of in-flight self-checking technology, the flight control system has been upgraded from system-level fault diagnosis to replaceable component level, with F/A-18 and A320 aircraft as the representatives to realize in-flight digital fault detection [2].

How much of the above literature is aimed at the flight control system level faults, and there are few related researches on the failure mode recognition of key components. This article takes the gyro in the sensor component of the flight control system as an example, first introduces the principle of wavelet packet decomposition, and then clarifies the fuzzy clustering algorithm Composition and classification mechanism, and finally combined with specific examples for gyro failure pattern recognition.

2. Wavelet Transform and Wavelet Packet Decomposition
Wavelet transform is a variable-scale time-frequency analysis method, which contains basic elements such as wavelet function and scale function [3].

Definition 2.1: Let $\psi(t) \in L^2(R)$, when the admissible condition $\hat{\psi}(\omega)$ is met:

$$C_\psi = \int_\mathbb{R} \left| \frac{\hat{\psi}(\omega)}{\omega} \right|^2 d\omega < \infty$$

(1)

Where $\psi(t)$ is called a basic wavelet, which can be obtained by stretching and panning:
\[ \psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right) \quad a,b \in \mathbb{R}; a \neq 0 \]  

(2)

It is called a wavelet sequence. The \( a \) is a scaling factor. And the \( b \) is a translation factor. The continuous wavelet transform for any function \( f(t) \in L^2(\mathbb{R}) \) can be defined as:

\[ W_f(a,b) = \langle f, \psi_{a,b} \rangle = \left| a \right|^{1/2} \int_{\mathbb{R}} f(t) \psi\left(\frac{t-b}{a}\right) dt \]  

(3)

From Equation 2, the time-frequency resolution of the wavelet transform \( \psi_{a,b}(t) \) is determined by the time-frequency width of the wavelet sequence.

Compared with wavelet transform, wavelet packet transform can adaptively select the corresponding frequency band according to the characteristics of the analyzed signal to match it with the signal spectrum and improve the time-frequency resolution. For a detailed understanding of wavelet packet analysis, you can use a two-layer decomposition in the following figure to illustrate:

![Wavelet Packet Decomposition Tree](image)

Figure 1. Wavelet Packet Decomposition Tree.

Among them, A represents low frequency, D represents high frequency, and the serial number at the end represents the scale number of wavelet decomposition.

Definition 2.2: Let that \( \psi(t) \) and \( \varphi(t) \) are the wavelet function and scale function, respectively. \( h(k) \) is the low-pass filter coefficients. \( g(k) \) is the high-pass filter coefficients, and if there is \( g(k) = (-1)^k h(1-k) \), then there are:

\[
\begin{align*}
    u_{2n}(t) &= \sqrt{2} \sum_{k \in \mathbb{Z}} h(k) u_n(2t-k) \\
    u_{2n+1}(t) &= \sqrt{2} \sum_{k \in \mathbb{Z}} g(k) u_n(2t-k)
\end{align*}
\]

(4)

The set \( \{u_n(t)\}_{n \in \mathbb{N}} \) is the orthogonal wavelet packet.

3. Fuzzy Clustering
Cluster analysis belongs to the category of unsupervised classification, and is one of the important contents in the field of pattern recognition. It refers to when data samples are classified according to a specified rule, the classification of categories is adjusted to make the similarity between samples in the same class as much as possible. Large, the similarity between samples of different classes is as small as possible. The method of fuzzy clustering is proposed on the basis of clustering, which integrates the relevant theoretical research of fuzzy mathematics, and is one of the more popular clustering algorithms.
Fuzzy C-Means (FCM) clustering algorithm was proposed by Bezdek in 1974. It is improved on the basis of the hard C-means clustering algorithm, which can effectively reduce the number of clustering iterations. Reducing clustering time has obvious advantages for online fault status monitoring [4].

The basic principle of fuzzy clustering algorithm is as follows.

Suppose that the data sample set \( x = \{x_1, x_2, x_3, \ldots, x_n\} \) contains samples, and the sample set can be divided into \( c \) classes, marked \( V \) as the centre of clusters, and let \( U \) be the membership matrix. \( \mu_{ij} \) indicates that the \( j \) sample belongs to the membership degree of the category \( i \), which satisfies \( \sum_{i=1}^{c} \mu_{ij} = 1, \mu_{ij} \in [0,1] \). Define the objective function as:

\[
J_m(U,V) = \sum_{j=1}^{n} \sum_{i=1}^{c} (\mu_{ij})^m (d_{ij})^2
\] (5)

Among them, \( m \) is the weighted index; \( d_{ij} \) is the distance between the sample \( x_j \) and the \( v_i \) cluster centre of the category \( i \). The clustering criterion of FCM is to determine \( U \) and \( V \) to make the minimum \( J_m(U,V) \).

When considering that the constraint is \( \sum_{i=1}^{c} \mu_{ij} = 1, \mu_{ij} \in [0,1] \), the Lagrangian multiplier method is used to solve the process, as follows:

\[
F = \sum_{j=1}^{n} (\mu_{ij})^m (d_{ij})^2 + \lambda \left( \sum_{j=1}^{n} \mu_{ij} - 1 \right)
\] (6)

Find the partial differential is that:

\[
\begin{align*}
\frac{\partial F}{\partial \lambda} &= \sum_{i=1}^{c} \mu_{ij} - 1 = 0 \\
\frac{\partial F}{\partial \mu_{it}} &= m \cdot (\mu_{it})^{m-1} \cdot d_{it}^2 - \lambda = 0
\end{align*}
\] (7)

Then

\[
\mu_{it} = \left[ \frac{\lambda}{m \cdot d_{it}^2} \right]^{1/(m-1)}
\] (8)

The final result after iteration is:
The traditional fuzzy clustering method also has the problems of improper processing and large number of iterations in the processing of fuzzy categories. The following introduces a method to improve fuzzy clustering, based on kernel function fuzzy C-means algorithm (KFCM) [5].

The objective function of KFCM is the nuclear space form of FCM, namely

$$J_m(U, V) = \sum_{j=1}^{c} \sum_{i=1}^{n} (\mu_{ij})^m \| \phi(x_j) - \phi(v_j) \|^2$$  \hspace{1cm} (10)

Among them, $\phi$ is the non-linear mapping, the sample set is $x = \{x_1, x_2, x_3, \ldots, x_n\}$, $U = [u_{ij}]_{c \times n}$ is the fuzzy classification matrix, $V = \{v_1, v_2, v_3, \ldots, v_c\}$ is the cluster center vector set of class C, $m$ is the fuzzy weighted index, and $\| \phi(x_j) - \phi(v_j) \|^2 = K(x_j, x_j) + K(v_j, v_j) - 2K(x_j, v_j)$, 

$K(x, y) = \phi(x)^T \phi(y)$ is the kernel function.

The final objective optimization function is obtained as follows:

$$v_j = \frac{\sum_{i=1}^{n} u_{ij}^m K(x_j, v_i) x_i}{\sum_{i=1}^{n} u_{ij}^m K(x_j, v_i)}$$

$$u_{ij} = \frac{1/[K(x_j, x_j) + K(v_i, v_i) - 2K(x_j, v_i)]} {\sum_{j=1}^{c} [1/[K(x_j, x_j) + K(v_i, v_i) - 2K(x_j, v_i)]}^{m-1}$$  \hspace{1cm} (11)

4. Example Analysis of Gyro Failure Pattern Recognition

4.1. Wavelet Packet Decomposition of Gyro Fault

For this type of aircraft, the gyro failure in the sensor assembly is often dominated by the output signal failure, that is, the difference between the sensor measurement output and the real data is large. According to this characteristic, the gyro data output value can be collected, and the collection process continues for 100 seconds, the period of the gyro output data transmitted by the sensor subsystem is 10 milliseconds, and a total of 1000 points of sample data are obtained. Through the analysis of the collected data, it can be known that there are three main manifestations of gyro faults. It is set to use $y_{ouj}$ to indicate the gyro measurement output and $y_{ouj}$ to indicate the output under the normal state of the gyro. The signal model under the three failure modes is:

(1) Gyro constant drift

The signal mode is:
\[ y_{out}(t) = y_{nrot}(t) + k \quad t_f < t < t_f + \Delta t_f \]  

(12)

Where \( t_f \) is the starting time of the fault, \( y_{out} \) is the duration of the fault, and \( k \) is the constant drift value.

(2) Gyro random signal interference failure

The signal mode is:

\[
y_{out}(t) = \begin{cases} 
y_{nrot}(t) & 0 < t < t_f \\
y_{nrot}(t) + \text{rand} & t_f < t < t_f + \Delta t_f 
\end{cases}
\]  

(13)

Among them \( \text{rand} \) is the random noise signal.

(3) Gyro gain loss failure

The signal mode is:

\[ y_{out}(t) = \lambda y_{nrot}(t) \quad t_f < t < t_f + \Delta t_f \]  

(14)

Where, \( \lambda \) is the gain loss coefficient.

4.2. Case Analysis of Failure Pattern Recognition

The wavelet packet decomposition method is used to decompose the sampled data in the time-frequency domain, and the three-layer db4 wavelet basis function is selected for wavelet packet decomposition. The signal frequency band is divided into 8 frequency bands, and the name of each frequency band is \( y_{30}, y_{31}, y_{32}, \ldots, y_{37} \). The decomposition of the normal signal of the gyro and the wavelet packets of the three fault signals is as follows:

![Wavelet Packet Decomposition Diagram of Gyro Signal.](image)

We increase the number of experimental groups to obtain the final failure data sample. The following table lists some failure data samples.
Table 1. Fault Samples and Corresponding Fault Types.

| Sample serial number | y30    | y31    | y32    | y33    | y34    | y35    | y36    | y37    | Fault types                  |
|----------------------|--------|--------|--------|--------|--------|--------|--------|--------|-----------------------------|
| 1                    | 0.6364 | 0.3044 | 0.0205 | 0.0321 | 0.0022 | 0.0025 | 0.0007 | 0.0012 | Gyro constant drift         |
| 2                    | 0.6514 | 0.3014 | 0.0186 | 0.0223 | 0.0021 | 0.0025 | 0.0006 | 0.0011 | Gyro random signal interference failure |
| 3                    | 0.6401 | 0.3109 | 0.0123 | 0.0311 | 0.0018 | 0.0022 | 0.0007 | 0.0009 | Gyro gain loss failure       |
| 4                    | 0.3062 | 0.3429 | 0.0471 | 0.1213 | 0.0532 | 0.0412 | 0.0429 | 0.0452 | Gyro constant drift         |
| 5                    | 0.3189 | 0.3317 | 0.0462 | 0.1252 | 0.0455 | 0.0439 | 0.0456 | 0.0434 | Gyro random signal interference failure |
| 6                    | 0.3112 | 0.3448 | 0.0468 | 0.1198 | 0.0448 | 0.0439 | 0.0471 | 0.0416 | Gyro gain loss failure       |
| 7                    | 0.6151 | 0.3082 | 0.0298 | 0.0394 | 0.0021 | 0.0032 | 0.0008 | 0.0014 | Gyro gain loss failure       |
| 8                    | 0.6202 | 0.3065 | 0.0254 | 0.0374 | 0.0034 | 0.0048 | 0.0012 | 0.0011 | Gyro gain loss failure       |
| 9                    | 0.6187 | 0.3101 | 0.0274 | 0.0361 | 0.0028 | 0.0029 | 0.0011 | 0.0009 | Gyro gain loss failure       |

In Table 1, the y30 series represents the result of normalizing the energy of each frequency band after the decomposition of the three-layer wavelet packet, which is dimensionless.

Finally, after the FCM clustering algorithm and KFCM clustering algorithm are calculated, the comparison chart of the diagnosis results is shown in the following table:

Table 2. Comparison of Fault Diagnosis Results between FCM Clustering Algorithm and KFCM Clustering Algorithm.

| Fault type                          | Total number of samples | The accuracy of FCM | The accuracy of KFCM |
|-------------------------------------|-------------------------|---------------------|----------------------|
| Gyro constant drift                 | 30                      | 90.00%              | 93.33%               |
| Gyro random signal interference failure | 30                      | 93.33%              | 100.00%              |
| Gyro gain loss failure              | 30                      | 90.00%              | 96.67%               |
| Total                               | 90                      | 91.11%              | 96.67%               |

From the results of the final cluster fault diagnosis, the classification accuracy of the KFCM algorithm in a specific fault state is higher than that of the FCM algorithm, and it has a better diagnosis and classification effect for the key components of the flight control system in engineering applications.

5. Conclusion
This paper presents a method of gyro failure pattern recognition based on wavelet packet decomposition and fuzzy clustering. Firstly, the wavelet packet decomposition and its principle are analyzed. Then, the fuzzy clustering algorithm is analyzed, and the fuzzy clustering based on kernel function is also given. Finally, the above theory is applied to the gyro component of an aircraft flight control system. The experimental results show that the combination of the two methods can effectively classify the gyro failure mode and achieve a better failure mode recognition effect.

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