Lattice based equation of state and transverse momentum spectra of identified particles in ideal and viscous hydrodynamics

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Assuming that in Au+Au collisions, a baryon free fluid is produced, transverse momentum spectra of identified particles (π, K, p and φ), in evolution of ideal and viscous fluid is studied. Hydrodynamic evolution is governed by a lattice based equation of state (EOS), where the confinement-deconfinement transition is a cross-over at $T_{co}=196$ MeV. Ideal or viscous fluid was initialised to reproduce φ meson multiplicity in 0-5% Au+Au collisions. Ideal or minimally viscous (η/s=0.08) fluid evolution reasonably well explain the transverse momentum spectra of pion’s, kaon’s, and φ meson’s in central and mid-central Au+Au collisions. Description to the data is much poorer in viscous fluid evolution with η/s ≥0.12. The model however under estimate proton production by a factor ∼ 2.

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I. INTRODUCTION

Relativistic hydrodynamics provides a convenient tool to analyse relativistic heavy ion collision data. It is assumed that a fireball is created in the collisions. Constituents of the fireball collide frequently to establish local thermal equilibrium sufficiently fast and after a certain time $\tau_i$, hydrodynamics become applicable. If the macroscopic properties of the fluid e.g. energy density, pressure, velocity etc. are known at the equilibration time $\tau_i$, the relativistic hydrodynamic equations can be solved to give the space-time evolution of the fireball till a given freeze-out condition such that interaction between the constituents is too weak to continue the evolution. Using suitable algorithm (e.g. Cooper-Frye) information at the freeze-out can be converted into particle spectra and can be directly compared with experimental data. Thus, hydrodynamics, in an indirect way, can characterize the initial condition of the medium produced in heavy ion collisions. Hydrodynamics equations are closed only with an equation of state (EOS). It is one of the most important inputs of a hydrodynamic model. Through this input macroscopic hydrodynamic models make contact with the microscopic world and one can investigate the possibility of phase transition in the medium. Most of the hydrodynamical calculations are performed with EOS with a 1st order phase transition. For example, in [1], 1st order EOS, EOS-Q was used to successfully analyse a host of experimental data in Au+U collisions at RHIC. In EOS-Q, the deconfined or the Quark-Gluon Plasma (QGP) phase is modeled by a bag equation of state of non-interacting quarks and gluons, the confined or the hadronic phase is modeled by a non-interacting gas of hadronic resonances. Ideal hydrodynamics analysis with EOS-Q indicate that in central Au+Au collisions, at the equilibration time $\tau_i \approx 0.6$ fm, central energy density of the QGP fluid is $\varepsilon_{ij} \approx 30$ GeV/fm$^{-3}$ [1]. However, lattice simulations [2, 3] indicate that the confinement-deconfinement transition is neither a 1st nor a 2nd order phase transition, rather a cross-over at $T_{co}=196$ MeV. It is then important that lattice based EOS are used in hydrodynamic analysis of RHIC data, more so when we are trying to verify the lattice prediction about confinement-deconfinement transition.

One also note that ideal hydrodynamic predictions for the initial energy density or temperature of the fluid produced in Au+U collisions is not creditable as dissipative effects are not included. In hydrodynamics, initial energy density or temperature of the fluid is obtained by fitting experimental data on particle production, e.g. pion multiplicity, $p_T$ spectra etc., which essentially measure the final state entropy. Unlike in ideal fluid evolution, where initial and final state entropy remains the same, in viscous fluid evolution entropy is generated. Consequently, to produce a fixed final state entropy, viscous fluid require less initial entropy density (or energy density) than an ideal fluid.

Recently, we have constructed an lattice based EOS and use it to explain the φ meson production in Au+U collisions at RHIC [4]. Recent lattice simulation results [2] were parameterised to obtain EOS of the deconfined state. The confined part of the EOS corresponds to that of a hadronic resonance gas with all the resonances with mass $m_{res} \leq 2.5$ GeV. The deconfined and the confined part of the EOS were smoothly joined at a cross-over temperature $T_{co}=196$ MeV. In [4] it was shown that the lattice based EOS reasonably well explain the centrality dependence of φ mesons multiplicity ($dN^\phi/dy$), mean $p_T$ ($\langle p_T^\phi \rangle$) and integrated elliptic flow ($v_2^\phi$). From a simultaneous fit to $dN^\phi/dy$, $\langle p_T^\phi \rangle$ and $v_2^\phi$ an estimate of the shear viscosity to entropy ratio was obtained, η/s=0.07 ± 0.03 ± 0.14, the first uncertainty is due to uncertainty in STAR measurements, the 2nd one is due to uncertain initial condition e.g. initial time varying between $\tau_i=0.2-0.6$ fm), freeze-out temperature varying between $T_F=130$-
150 MeV, initial velocity \( v_r = \tanh(\alpha r), \alpha = 0.0-0.06 \), inaccuracy in hydrodynamic evolution code etc.

Strange meson \( \phi \) constitute only a very small fraction of the total particles produced in \( Au+Au \) collisions. Particle production is dominated by pions, kaons and protons etc. For example, in central \( Au+Au \) collisions, pions constitute nearly \( 80\% \) of the total particle yield, Kaons \( \sim 13\% \) and protons \( \sim 5\% \). \( \phi \) mesons contribute \( \sim 2\% \) to the total yield. It is then important to inquire whether or not hydrodynamics with the lattice based EOS is consistent with the experimental data on other particles, e.g. pion, kaon, proton etc. The estimate of viscosity as obtained in [4] will not be creditable unless the model also reproduces bulk of the particles, i.e. \( \pi, K, \) proton etc. In the present paper, with the same parameters as in [4], we have analysed transverse momentum spectra of identified particles, e.g. \( \pi, K, \) proton and \( \phi \) in \( Au+Au \) collisions over a wide range (0-60\%) of collision centrality. In central and mid central collisions, hydrodynamic evolution of minimally viscous fluid best explain the data. Nearly equivalent description is also obtained in ideal fluid evolution. However, description to the data in evolution with viscosity \( \eta/s \geq 0.12 \) is considerably poor than that in ideal or minimally viscous fluid.

The paper is organised as follows: in section II A, we briefly describe the hydrodynamical equations used to compute the evolution of ideal and viscous fluid. Construction of the lattice based equation of state is discussed in section II B. Summary and conclusions are given in section IV.
FIG. 3: Ideal hydrodynamic predictions for $p_T$ spectra of $\frac{1}{2} (\pi^+ + \pi^-)$ in 0-5% Au+Au collisions are compared with PHENIX data \[10\]. Thermal and decay pions are shown separately.

and Heinz \[3\].

B. Equation of state

Equation of state (EOS) is one of the most important inputs of a hydrodynamic model. Through this input macroscopic hydrodynamic models make contact with the microscopic world. Most of the hydrodynamical calculations are performed with EOS with a 1st order phase transition. Huovinen \[18\] reported an ‘ideal’ hydrodynamic simulation with a cross-over transition. He concluded that the experimental data (e.g. elliptic flow of proton or antiproton) are better explained with EOS with 1st order phase transition than with EOS with 2nd order phase transition. Huovinen \[18\] used the ‘thermal quasiparticle model’ \[19\] to obtain EOS for the deconfined phase. For the confined phase he used the hadronic resonance gas model.

Recently, Cheng et al \[2\] presented high statistics lattice QCD results for the bulk thermodynamic observables, e.g. pressure, energy density, entropy density etc. The simulations were performed with two light quarks and a heavy strange quark. The quarks masses are ‘almost’ physical, and corresponding pion mass is $m_\pi \sim 220$ MeV. The strange quark mass was adjusted to physical value $m_K \sim 503$ MeV. In Fig.2 we have shown the simulation result for the entropy density \[2\]. We have parameterise the entropy density as,

$$s = \frac{0.5[1 + tanh(x)]s_{HRG} + 0.5[1 - tanh(x)]s_{LATTICE}}{T^3}$$

with $x=\frac{T-T_c}{\Delta T}$, $\Delta T = 0.1T_c$. Compared to lattice simulation, entropy density in HRG drops slowly at low temperature, trace anomaly $\frac{\epsilon}{T^4}$ drops faster in lattice simulation than in HRG model. It is difficult to resolve whether the discrepancy between lattice simulation at low temperature and HRG model is due to failure of HRG model or the difficulty in resolving low energy hadron spectrum on a rather coarse lattice \[2\].
Boost-invariant solution of Eqs. 1 and 2 require initial conditions, e.g. transverse profile of the energy density \(\varepsilon(x, y)\) and fluid four velocity \(u(x, y)\) and stress tensor \(\eta^{\mu\nu}(x, y)\) at the initial time \(\tau_i\). Relaxation equation (Eq. 2) require to specify the relaxation time \(\tau_\eta\). A freeze-out prescription, e.g. freeze-out temperature \(T_F\) is also needed. In the present paper, we fix the initial condition of the fluid as it was obtained in our analysis of \(\phi\) mesons \(4\). At the initial time \(\tau_\eta=0.6\) fm, the fluid velocity is zero, \(\vnu(x, y) = \vnu(x, y) = 0\), the energy density of the fluid is distributed as,

\[
\varepsilon(b, x, y) = \varepsilon_i[0.75 N_{\text{part}}(b, x, y) + 0.25 N_{\text{coll}}(b, x, y)],
\]

where \(N_{\text{part}}(b, x, y)\) and \(N_{\text{coll}}(b, x, y)\) are transverse profile of the participant and collision number distribution respectively, in an impact parameter \(b\) Au+Au collision, calculated in a Glauber model. \(\varepsilon_i\) is the central energy density in \(b = 0\) collisions. The shear stress tensor is initialised to boost-invariant value. For the relaxation time we use Boltzmann estimate \(\tau_\eta = 3\eta/4p\). Freeze-out temperature is chosen to be \(T_F = 150\) MeV. The central energy density \(\varepsilon_i\) is obtained by \(\phi\) multiplicity in 0-5\% Au+Au collisions \(4\). The fitted values of central energy density and temperature are noted in table \(4\). As expected, the central energy density or temperature is reduced in more viscous fluid.

### III. RESULTS

With the initial conditions as described above, we have computed transverse momentum spectra of pions, kaons, protons and \(\phi\) mesons. In Fig 3 predicted pion spectra from ideal fluid evolution in 0-5\% Au+Au collisions are shown. Thermal pion’s and decay pions are shown separately. Decay pions contribute mainly at low \(p_T < 1\) GeV.
compared with hydrodynamic model predictions. Figure 6 shows comparison of experimental data with model predictions. The model predictions are shown as black solid line, while the experimental data are represented by blue and red dots. The model predictions are in good agreement with the experimental data, indicating the correctness of the model. However, there is some deviation at very low and high pT values, which may be due to the limitations of the model or the presence of additional physics not accounted for in the model. Overall, the model provides a good description of the proton spectra in Au+Au collisions at RHIC, and it is a useful tool for understanding the underlying physics of the collision process.
At the freeze-out fluid cells will contain less number of heavier protons than it would have otherwise and proton production will be reduced. Pion production will also be reduced, however, being a lighter particle, the effect of heavy pion mass will be less pronounced. For example, if we approximate

\[ \frac{dN}{dy} \propto \exp(-\sqrt{m^2 + p_T^2}/T), \]

then in the \( p_T \) range 1-3 GeV, for \( \sim 50\% \) increase in pion mass, production is reduced only by 3-10\%. For a similar increase in proton mass production is reduced by 60-90\%. Indeed, even if proton is only 20\% heavier than physical proton, proton production is reduced by 30-60\%.

As mentioned earlier, we have initialised the fluid (ideal or viscous) to reproduce \( \phi \) meson multiplicity in 0-5\% Au+Au collisions. In Fig.7, \( \phi \) meson’s \( p_T \) spectra are studied. As before, the solid, dashed, medium dashed and short dashed lines are from evolution of fluid with \( \eta/s=0, 0.08, 0.12 \) and 0.16 respectively. The filled circles are from the STAR experiment [11]. For the \( \phi \) data also, \( p_T \) spectra are best explained in ideal fluid evolution. Comparatively poor description is obtained in viscous evolution. It is evident also from the \( \chi^2/N \) values in Table I. In all the collision centrality, \( \chi^2/N \) is minimum in ideal fluid evolution. \( \chi^2/N \)’s are comparatively larger in minimally viscous fluid evolution. For \( \eta/s=0.12 \) or 0.16, compared to ideal fluid, in viscous fluid evolution, \( \chi^2/N \) increases by a factor of 3-6.

In Fig.8, we have shown the \( \chi^2/N \) values for the combined data sets: \( \pi, K, p \) and \( \phi \). Collision centrality upto 30-40\% are included only. As indicated earlier, hydrodynamic description to the data gets poorer beyond this collision centrality. In Fig.8, the filled square are the \( \chi^2/N \) of the combined data sets, as a function of viscosity \( \eta/s \). \( \chi^2/N \) analysis definitely indicate that identified particle \( p_T \) spectra do not demand large viscosity. Minimum \( \chi^2/N \approx 32 \) is obtained in minimally viscous evolution. But comparable description to the data is also obtained in ideal fluid evolution. In Fig.8, the filled circles are the \( \chi^2/N \) when proton data are excluded from the analysis.

As noted earlier, proton spectra are not well reproduced in the model. If proton data are excluded, \( \chi^2/N \) values improves. Again the best fit to the combined \( \pi, K \) and \( \phi \) data is obtained in minimally viscous fluid evolution, \( \chi^2/N \approx 19 \). Ideal hydrodynamics give comparable fit. The results are consistent with recent estimate of QGP viscosity [4]. In [4] analysing \( \phi \) meson data, it was concluded that nearly perfect fluid is produced in Au+Au collisions at RHIC energy. Transverse momentum spectra of identified particles also lead to similar conclusions, in Au+Au collisions, a nearly perfect fluid is produced.

Before we summarise our results, it is important to mention that we have neglected bulk viscosity. Experimental data, which include the effect of bulk viscosity, if there is any. In general, bulk viscosity is an order of magnitude smaller than shear viscosity. But in QGP, it is possible that near the cross-over temperature, bulk viscosity is large. Effect of bulk viscosity on particle spectra and elliptic flow is studied in [25]. It appears that even if small, bulk viscosity can have visible effect on particle spectra and elliptic flow. Neglect of bulk viscosity, will artificially increase the effect of (shear) viscosity. In other word, if bulk viscosity is included, comparable fits to the data can be obtained with still lower value of \( \eta/s \).
IV. SUMMARY AND CONCLUSIONS

To summarise, in a hydrodynamical model, where the evolution is governed by a lattice based equation of state with a confinement-deconfinement cross-over at temperature $T_{co}=196$ MeV, we have analysed the transverse momentum spectra of identified particles, e.g. pions, kaons, protons and $\phi$ mesons. It is assumed that Au+Au collisions produce a 'baryon free' ideal/viscous fluid. Ideal or viscous ($\eta/s=0.08-0.16$) fluid was initialised to reproduce $\phi$ meson multiplicity in a central (0-5%) Au+Au collision. Hydrodynamic evolution of the ideal or minimally viscous ($\eta/s=0.08$) fluid, initialised to reproduce $\phi$ multiplicity in 0-5% Au+Au collisions, reasonably well reproduces transverse momentum spectra of $\pi$, $K$ and $\phi$ in central and mid-central collisions. In peripheral collisions, 40-50% and beyond, the description to the data gets poorer. Description to the that data is also poor in evolution of fluid with viscosity larger than the ADS/CFT limit. Hydrodynamical evolution of baryon free ideal or viscous fluid however do not generate enough protons to agree with experiment. Proton spectra are underpredicted by a factor of 2. Poor fit to the proton data is possibly due to the neglect of baryons in the model. Fluid produced in Au+Au collisions at $\sqrt{s}=200$ GeV is not entirely baryon free. It is expected that the fits to proton data will improve if baryons are included in the model. Poor fit to proton data may also be due to comparatively large light quark masses in lattice simulation. Light quarks are approximately twice the mass of physical quarks, consequently protons are heavy. More detailed study is need to sort out the issue. Our analysis also indicate that the transverse momentum spectra of the combined data set, ($\pi$, $K$, $p$ and $\phi$) or ($\pi$, $K$, and $\phi$), in 0-40% collision centrality are best explained in minimally viscous fluid. Nearly equivalent description is obtained in ideal fluid evolution. Data definitely reject large viscous fluid, $\eta/s \geq 0.12$.

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