Escaping stars from young low-$N$ clusters

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ABSTRACT

With the use of $N$-body calculations, the numbers and properties of escaping stars from low-$N$ ($N = 100$ and $1000$) young embedded star clusters prior to gas expulsion are studied over the first $5$ Myr of their existence. Besides the numbers of stars, different initial radii and binary populations are also examined as well as virialized and collapsing clusters. It is found that these clusters can lose substantial amounts (up to 20 per cent) of stars within $5$ Myr, with considerable velocities of up to more than $100 \text{ km s}^{-1}$. Even with their mean velocities between $2$ and $8 \text{ km s}^{-1}$, these stars will still travel between $2$ and $30 \text{ pc}$ during the $5$ Myr. Therefore large numbers of distributed stars in star-forming regions cannot necessarily be counted as evidence for the isolated formation of stars.

Key words: methods: numerical – binaries: general – stars: formation – open clusters and associations: general.

1 INTRODUCTION

Stars do not form in absolute isolation, but rather in groups or embedded clusters in dense molecular cloud cores (Lada & Lada 1995, 2003; Adams & Myers 2001; Allen et al. 2007). Theoretical and observational results indicate that these groups and the majority of the embedded clusters dissolve quickly due to gas expulsion after about $10$ Myr and release their stars into the Galactic field (Adams & Myers 2001; Kroupa, Aarseth & Hurley 2001; Kroupa & Bouvier 2003; Lada & Lada 2003; Baumgardt & Kroupa 2007; Weidner et al. 2007). However, the ratio of the distributed mode of star formation to the clustered one is not fully understood. Observations show a distributed fraction of stars in the Orion molecular clouds of about $20$ per cent (Allen et al. 2007). It is generally assumed that this fraction is primordial, as dynamical evolution of clusters is considered to slow in order to reproduce the observed fraction of isolated stars at such an early stage. For a full physical understanding of the formation of stars, however, it is vital to know all modes of star formation in as much detail as possible. The purpose of this work is therefore to assess whether or not the observed amount of distributed stars outside embedded clusters could have a dynamical origin, and henceforth imply initial formation in young, tight embedded clusters, or whether there is intrinsically isolated star formation. In star-forming regions that do not show massive star clusters, in particular, the number of stars released through the dynamical evolution of low-$N$ clusters is not well known. In order to study this question, a large series of $N$-body calculations is performed and the number, velocity and mass spectrum of stars ejected due to dynamical interactions within $5$ Myr from these clusters are studied.

2 THE CLUSTER SET-UP

With the use of the $N$-body6 code (Aarseth 1999a,b, 2003, 2008), the evolution of numerical star clusters with small numbers of stars is examined. Two different numbers of stars, $N$, are used: 100 and 1000. As the actual number of escaped stars can be rather low, each set of initial conditions is simulated 100 times using different random number seeds to set up the masses, velocities and positions of the stars in order to improve the statistical significance of the results. All stars in the clusters follow the canonical initial mass function (IMF; Kroupa 2002).

Besides the initial number of stars, several other initial parameters are studied, each with 100 calculations. The following set-ups are used:

(i) all stars are initially single stars;
(ii) all stars reside in binaries;
(iii) 50 per cent of the stars are in binaries;
(iv) the cluster is initially in virial equilibrium;
(v) the cluster is initially subvirial (collapsing with a kinetic energy one-tenth the potential energy);
(vi) all star clusters are set up as Plummer spheres (Plummer 1911) with different initial half-mass radii of 0.1, 0.25 or 0.5 pc.

Each of these set-ups is then numerically evolved with $N$-body6 for $5$ Myr.1 The evolution time of $5$ Myr is chosen to minimize mass

1 Depending on the initial radius of the cluster, $5$ Myr is between about 10 and 150 crossing times. See equation (2) for how to calculate the crossing time.
loss due to stellar evolution, to avoid supernovae and primarily to have a setting of a star cluster still embedded in its parental cloud, prior to gas expulsion. Still, initially clusters probably form from clumpy structures (Williams, Blitz & McKee 2000; Carpenter & Hodapp 2008). Simulations of subvirial clumpy initial conditions (Allison et al. 2009) have shown that such clusters mass-segregate faster than virial clusters due to the formation of short-lived very dense cores. However, these calculations did not include a background gas potential.

To emulate the gas in which the stars are embedded during this time, the cluster is set up with an additional Plummer potential (Plummer 1911; Binney & Tremaine 1987) of the same half-mass radius and the same mass as the cluster, thus assuming a star-formation efficiency (SFE) of 50 per cent. However, global SFEs are observationally around 30 per cent (Lada & Lada 2003), and locally within the region where the cluster actually forms the value is most likely higher (Moeckel & Bate 2010). A SFE of 50 per cent has therefore been chosen. A large difference in results between 30 and 50 per cent SFE is not to be expected. The gas potential is in all cases kept constant during the whole calculation time. As the results are qualitatively similar to $N$-body studies without a gas background potential (e.g. Kroupa 1998; Kroupa & Bouvier 2003), the exact shape of the potential does not seem to be overly important. However, detailed further studies are needed to clarify the full impact of the shape and depth of the potential on the properties of escaping stars.

The decision to include primordial binaries into the study is based on the observational result that open clusters host a large number of binaries (Sana, Gosset & Evans 2009; Sollima et al. 2010). The initial set-up of the binary properties increases the total parameter space of the calculations significantly. For a cluster of the richness of the Orion Nebula, cluster stellar dynamics may change the mass function and binary properties in the core significantly in less than $10^6$ yr (Goodwin & Bastian 2006; Pfalzner-Altenburg & Kroupa 2006; Pfalzner & Olczak 2007; Allison et al. 2009). As the majority of the present-day field binaries are being processed through star clusters, their properties are not suited to being used for the numerical set-up (Duquennoy & Mayor 1991; Kroupa 1995a,b,c,d; Kroupa & Bouvier 2003). Therefore, the results of the inverse dynamical population synthesis of Kroupa (1995b) are used, which predict a flat (thermal) period distribution (resulting in a uniform logarithmic semimajor axis distribution) and random pairing of the stars in systems of low-mass ($\leq 1 M_\odot$) stars (Kroupa 1995b,c). While there are indications for non-random pairing of massive stars (Kobulnicky & Fryer 2007; Weidner, Kroupa & Maschberger 2009), the low-mass results are also used for massive binaries in this study, first because the mass at which a change of properties might occur is not determined yet, and it is not clear whether mass ratios near to unity for massive stars are primordial or already a sign of dynamical evolution (Pfalzner & Olczak 2007), and secondly because, due to the choice of $N = 100$ and 1000, only a relatively small number of massive stars is to be expected anyway. For example, there should be only three stars above $8 M_\odot$ for $N = 1000$ and a Kroupa IMF.

It should be noted here that rapid changes in the binary properties might also be at least partly due to the initial conditions of the $N$-body calculations and do not really reflect the star formation process, i.e. unstable binaries that are easily destroyed in early cluster dynamics may not even form in the first place (Moeckel & Bate 2010).

In the $N = 1000$ cases, for the IMF a maximal mass, $m_{\text{max}}$, of $25 M_\odot$ is chosen, in accordance with the $m_{\text{max}} - M_{\text{ecl}}$ relation of Weidner, Kroupa & Bonnell (2010). In the $N = 100$ case this relation yields a $m_{\text{max}}$ of $7 M_\odot$. Comparison calculations are also made using $m_{\text{max}} = 25 M_\odot$ for the $N = 100$ case.

3 RESULTS

The results of the $N$-body6 calculations after 5 Myr are presented in Figs 1–8. There the mean number of lost stars ($\langle N_{\text{lost}} \rangle$), the mean lost
fraction of mass $\langle M_{\text{lost}} \rangle / \langle M_{\text{tot}} \rangle$, the mean escape velocity $\langle v_{\text{esc}} \rangle$, the mean escape velocity dispersion $\langle \sigma \rangle$ and the mean mass of the lost stars $\langle M_{\text{lost}} \rangle$ are studied as functions of the initial cluster relaxation time, $t_{\text{relax}}$. The latter is calculated by using the following equations from Binney & Tremaine (1987):

$$t_{\text{relax}} \approx \frac{N}{8 \ln N} t_{\text{cross}},$$

with $N$ being the number of stars and $t_{\text{cross}}$ the crossing time a star needs to travel through the cluster, which is given by

$$t_{\text{cross}} = 2 \sqrt{\frac{R_{\text{ecl}}^3}{G M_{\text{tot}}}},$$

with $G$ being Newton’s gravitational constant, $R_{\text{ecl}}$ the cluster radius and $M_{\text{tot}}$ its mass.

Figure 2. (a) Mean escape velocity, $\langle v_{\text{esc}} \rangle$, versus $t_{\text{relax}}$ and (b) mean velocity dispersion, $\langle \sigma \rangle$, versus $t_{\text{relax}}$. The solid line shows the case without any initial binaries, the dotted line is the 50 per cent initial binary case, the dashed line is the 100 per cent initial binaries case and the dash–dotted line marks the subvirial case.

Figure 3. (a) Mean mass of the escaped stars versus $t_{\text{relax}}$. The solid line shows the case without any initial binaries, the dotted line is the 50 per cent initial binary case, the dashed line is the 100 per cent initial binaries case and the dash–dotted line marks the subvirial case. (b) Total number of lost stars versus number of relaxation times the cluster experienced during the 5-Myr simulation time. The solid line shows the case without any initial binaries, the dotted line is the 50 per cent initial binary case, the dashed line is the 100 per cent initial binaries case and the dash–dotted line marks the subvirial case. The solid black dots are the predicted numbers from equation (4), the open squares from equation (6) and the open triangles from equation (8).
It is important to note that the relaxation time of a cluster is not a constant but also evolves with time. However, for the clusters that start in virial equilibrium the change is negligible over the short period of time studied here. The clusters expand about \( \sim 10 \) per cent due to evolutionary and dynamical mass loss. This leads to a \( \sim 10 \) per cent increase in \( t_{\text{relax}} \) but, as the subvirial clusters contract significantly during the calculation time span, their \( t_{\text{relax}} \) decreases and becomes comparable to that of a cluster with half their half-mass radius.

In Figs 3(b) and 7(b), several literature descriptions of star loss are additionally shown. The first one is the analytical description of Binney & Tremaine (1987, equation 8–85) based on the Fokker–Planck approximation for mass loss by evaporation of stars from a cluster (with large \( N \)):

\[
M(t) = M_0 \left( 1 - \frac{7k_e t}{2t_0^{\text{relax}}} \right)^{2/7},
\]

with \( M_0 \) being the initial cluster mass, \( k_e \approx 0.003 \) and \( t_0^{\text{relax}} \) the initial relaxation time (equation 1). With the use of the mean mass \( m_{\text{mean}} \) of the IMF, which is \( m_{\text{mean}} \approx 0.36 M_\odot \) for a Kroupa IMF, the number of stars lost over time can be approximated:

\[
N(t) = \frac{M(t)}{m_{\text{mean}}}.
\]

This equation is indicated with solid black dots in Figs 3 and 7.

This description is strictly valid only for large \( N \) and does not seem to provide a good characterization of the \( N \)-body calculations. Heggie (1974) assumed that in low-\( N \) clusters binary interactions should dominate over the evaporation of stars. He formulated the following dissolution time-scale for low-\( N \) clusters:

\[
t_{\text{diss}} = N^2 \frac{100}{t_{\text{cross}}},
\]

which can also be used to estimate the star loss from a cluster in the following way:

\[
N_{\text{diss}} = N_0 e^{-\left[\ln(N_0)/t_{\text{diss}}\right]t},
\]

where \( N_0 \) is the initial number of stars. This is included as open squares in Figs 3 and 7.
Escapees from low-N clusters

Figure 6. (a) Mean escape velocity versus $t_{\text{relax}}$ and (b) mean velocity dispersion versus $t_{\text{relax}}$. The solid line shows the case without any initial binaries, the dotted line is the 50 per cent initial binary case, the long-dashed line is the 100 per cent initial binaries case, the dash–dotted line marks the subvirial case and the short-dashed line the case with $m_{\text{max}} = 25 \, M_\odot$.

Figure 7. (a) Mean mass of escaped stars versus $t_{\text{relax}}$. The solid line shows the case without any initial binaries, the dotted line is the 50 per cent initial binary case, the long-dashed line is the 100 per cent initial binaries case, the dash–dotted line marks the subvirial case and the short-dashed line the case with $m_{\text{max}} = 25 \, M_\odot$. (b) Total number of lost stars versus number of relaxation times the cluster experienced during the 5-Myr simulation time. The solid line shows the case without any initial binaries, the dotted line is the 50 per cent initial binary case, the long-dashed line is the 100 per cent initial binaries case, the dash–dotted line marks the subvirial case and the short-dashed line the case with $m_{\text{max}} = 25 \, M_\odot$. The solid black dots are the predicted numbers from equation (4), the open squares from equation (6) and the open triangles from equation (8).

A third description of stellar loss can be derived from the evaporation time-scale when assuming the collisionless Boltzmann equation given in Binney & Tremaine (1987):

$$ t_{\text{evap}} = 136t_{\text{relax}}, $$

which can be transformed into the following number loss formula:

$$ N_{\text{evap}} = N_0 e^{-\left[\ln(N_0)/t_{\text{evap}}\right]}, $$

where $N_0$ is the initial number of stars. In Figs 3 and 7 this relation is plotted as open triangles.
Like the mass loss, the mean escape velocity \( \langle v_{\text{esc}} \rangle \) (Fig. 2a) is mainly dependent on \( t_{\text{relax}} \), though the existence of a high binary fraction raises \( \langle v_{\text{esc}} \rangle \) by about 1 km s\(^{-1}\), independent of \( t_{\text{relax}} \). Primordial binaries introduce three- and four-body interactions early on in the cluster evolution, as opposed to the mainly two-body interactions for single-star-only clusters. Such higher order interactions generally result in larger velocities.

The mean velocity of the ejected stars in the case of \( N = 1000 \) is 4–8 km s\(^{-1}\) (Fig. 2) and the escaped stars travel on average up to 10–30 pc (Fig. 4) within 5 Myr dependent on the original size of the cluster.\(^3\) However, individual stars can be up to 1 kpc away, as the highest escape velocity encountered for the \( N = 1000 \) clusters was about 260 km s\(^{-1}\), though the typical range of escape velocities is between 2 and 40 km s\(^{-1}\). These values are in the same range as the results of Kroupa (1998), who studied clusters with \( N = 400 \) in a Galactic tidal field but without a gas background potential. A comparison with observations seems to contradict the relatively large number of lost stars found here. Tobin et al. (2009) study the surroundings of the Orion Nebula cluster (ONC) and find that the majority of stars with velocity differences from the cluster larger than 10 km s\(^{-1}\) have little or no infrared (IR) excess and are therefore most likely old and not associated with the ONC. When looking only at escapees with velocities larger than 10 km s\(^{-1}\) after only 1 Myr (the age of the ONC, Hillenbrand & Hartmann 1998), only between zero and 14 are to be expected from the \( N \)-body calculations, depending on the initial conditions. Interestingly, Tobin et al. (2009) do find six stars with a velocity offset above 10 km s\(^{-1}\) that have an IR excess consistent with being young stars and therefore could have been ejected from the ONC, consistent with our predictions. As the Tobin et al. (2009) study cannot differentiate between bound and unbound cluster stars below 10 km s\(^{-1}\), it is currently not possible to test the predictions of the \( N \)-body calculations further. Also consistent with this picture is the recently discovered halo of low-mass stars around the small young cluster \( \eta \) Chamaeleontis (Murphy, Lawson & Bessell 2010). Generally, though, an IR excess is probably not a good age indicator for ejected stars, as most of these had a close encounter with another star which led to the ejection. During this encounter any disc around that star could be truncated or largely stripped, therefore reducing or eliminating the IR excess. In order to search for a distributed population of ejected young stars around clusters, the position of the stars in the Hertzsprung–Russell (HR) diagram would likely be a more robust age indicator.

For less rich clusters (Fig. 6a), smaller \( \langle v_{\text{esc}} \rangle \) are generally achieved. These are also increased by 0.5–1 km s\(^{-1}\) by the presence of binaries. Stars escaping from \( N = 100 \) clusters on average travel 3–6 pc (Fig. 8) within 5 Myr, with a typical range of escape velocities between 1 and 12 km s\(^{-1}\). For the low-\( N \) clusters, the fastest escapee had a velocity of about 40 km s\(^{-1}\) and travelled about 100 pc in total. Interestingly, the mean of the velocity dispersion of escape velocities, \( \sigma \), is more strongly affected by the presence of binaries, both in the \( N = 1000 \) (Fig. 2b) and \( N = 100 \) (Fig. 6b) cases. Therefore, the presence of a large fraction of binaries, while only slightly enlarging the mean escape velocity, adds more scatter to the velocities of the escaping stars. This increases the area around a young embedded cluster in which to expect escaping stars significantly by 50–200 per cent. The mean mass of the escaping stars, however, seems to be rather independent of the fraction of binaries in the \( N = 1000 \) clusters but is strongly correlated with \( t_{\text{relax}} \) (Fig. 3a). It

\(^{2}\) It should be noted here that all figures are evaluated after a calculation time of 5 Myr and that the abscissae in most cases is the relaxation time and therefore not a time evolution.

\(^{3}\) A velocity of 1 km s\(^{-1}\) is roughly equal to 1 pc per Myr.
rises from about 0.22 M⊙ in the 10-Myr case (→ Relax = 0.5 pc) to about 0.4 M⊙ for very compact (1 Myr → Relax = 0.1 pc) clusters, while for the collapsing clusters it stays at about 0.3 M⊙. In contrast, for the low-N = 100 clusters (m_sn) roughly stays constant with Relax (Fig. 7a) but varies strongly with the binary fraction. Here the subvirial, 50 per cent binary and higher upper mass limit cases are rather similar and well separated from the only-single-stars case and the 100 per cent binaries.

In Appendix A, histograms of the mass and ⟨v_sn⟩ of the escaped stars for the different initial conditions are shown. Mass histograms are only plotted for the cases without initial binaries for N = 1000 and N = 100 (panels (a) of Figs A1 and A2). The mass histograms in the N = 1000 case look very similar to the input IMF and those for N = 100 suffer from the low number of escaped stars. Velocity histograms are only shown for the cases with 100 per cent initial binaries. All velocity histograms are also rather similar, but the cases with binaries have a larger high-velocity tail when compared with the single-star-only run.

4.1 Comparison with analytical estimates

Figs 3(b) and 7(b) show the number of stars lost per number of relaxations the clusters experiences during 5 Myr for N = 1000 and N = 100, respectively. Besides the results from the different initial conditions, the analytic estimates given by equations (4) (solid black dots), (6) (open squares) and (8) (open triangles) are also shown. In both cases equation (4) predicts far too low escape numbers, which is expected as this equation should be applied only to clusters with large N. For the N = 1000 case, the resulting star loss is between the predictions of equations (6) and (8) but somewhat closer to the binary-interaction-dominated model (equation 6). This can be seen as an indication that star loss in such clusters is not purely dominated by binary interactions (equation 6), but evaporation (equation 8) also plays a role. However, the pure evaporation model of equation (8) technically also only applies to clusters with larger N. The subvirial models are much higher above the predictions of equation (6), probably because the shorter time-scales of the collapsing clusters lead to more binary interactions compared with the virial cases.

The lower N clusters with N = 100 behave differently in the sense that the numerical number of escapees is between the estimates given by equations (4) and (8), but the binary-interaction-dominated model (equation 6) overestimates star loss strongly, while the evaporation model (equation 8) seems to give a result closer to the numerical calculations. Here it might be that the higher escape velocity due to the background potential impedes the loss of stars after low-energy encounters. As the radii for the N = 100 and N = 1000 clusters are the same, the more massive clusters have higher densities and therefore higher encounter rates. For these it is therefore more likely that they experience high-energy encounters compared with the N = 100 clusters and therefore the N = 1000 clusters might be less affected by the background potential.

It should be noted here that none of the analytical estimates takes any background potential into account and they only focus on one physical process for star loss, either energy equipartition or binary interactions.

5 CONCLUSIONS

(i) Low-mass star clusters lose between 1 and 20 per cent of their stars (between 1 and 15 per cent of their mass) within their first 5 Myr.
APPENDIX A: MASS AND VELOCITY HISTOGRAMS

In this Appendix are shown histograms of the mass and velocities of the escaped stars. Fig. A1 shows a mass histogram and velocity histogram example for $N = 1000$, while Fig. A2 shows the same histograms for $N = 100$.

Figure A1. (a) Mass function of the lost stars for the models started without initial binaries and with 1000 stars. (b) Velocity histogram for the case with 100 per cent initial binaries. All models are started without initial binaries and with 1000 stars. The solid histogram is the $R_{\text{cl}} = 0.1$ pc model, the dotted histogram is the $R_{\text{cl}} = 0.2$ pc model, the short-dashed histogram is the $R_{\text{cl}} = 0.3$ pc model, the long-dashed histogram is the $R_{\text{cl}} = 0.4$ pc model and the dash–dotted histogram is the $R_{\text{cl}} = 0.5$ pc model. The solid line in the mass function histogram is the input IMF (not to scale).
Figure A2. (a) Mass function of the lost stars for the models started without initial binaries and with 100 stars. (b) Velocity histogram for the case with 100 per cent initial binaries. The solid histogram is the $R_{\text{ ecl}} = 0.1$ pc model, the dotted histogram is the $R_{\text{ ecl}} = 0.25$ pc model and the long-dashed histogram is the $R_{\text{ ecl}} = 0.5$ pc model. The solid line in the mass function histogram is the input IMF (not to scale).

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