INSTABILITIES IN THE GRAVITATIONAL BACKGROUND AND STRING THEORY

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ABSTRACT

We indicate the tentative source of instability in the two-dimensional black hole background. There are relevant operators among the tachyon and the higher level vertex operators in the conformal field theory. Connection of this instability with Hawking radiation is not obvious. The situation is somewhat analogous to fields in the background of a negative mass Euclidean Schwarzschild solution (in four dimensions). Speculation is made about decay of the Minkowski black hole into finite temperature flat space.

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1. Introduction

Recently there has been some interest in the classical gravitational background consistent with string theory in a two-dimensional target space\cite{1,2,3,4,5,6,7}. If one looks at the graviton-dilaton sector, one finds a black hole solution (or one of its relatives). It has been conjectured \cite{2} that the Hawking radiation would transform this black hole to an extreme Reissner-Nordstrom-like solution, which would then represent the $c = 1$ matrix model \cite{8,20,9,10,11,12} and the particle absorption and re-emission by this terminal black hole would be the tachyon scattering near the wall \cite{13,14,15,16,17,18,12,19}, in the matrix model picture.

In any case, it is of some importance to understand how the various classical solutions in 2-d string theory are related to each other. The $c = 1$ matrix model seems to be a solution with a nontrivial dilaton and tachyon background but with a flat metric \cite{9,10,11,12}. The black hole solution has nontrivial dilaton-graviton background in which we can introduce tachyon perturbations \cite{1,2,3,4}. We suggest that the second solution is an unstable stationary point. This instability would make it go over to a different spacetime geometry, may be a flat spacetime with one compact direction.

However, it is not clear that this instability has something to do with Hawking radiation. Usually, in static black hole background, there is no particle production by the black hole. The situation is more like instabilities in the background of a negative mass Schwarzschild solution in 4-dimensions, as we will indicate later.
2. The unstable modes

We will investigate the question of instability in the \((SL(2, R)/U(1))_k\) formulation of the Euclidean black hole background \([2]\). We search for Virasoro primaries of this CFT which are relevant operators. If we have the dimensions \(h = \bar{h} < 1\) for a primary, it corresponds to a negative eigenvalue off-shell mode, \(i.e.\) a small deviation from the classical solution which lowers the string field action. This solution, then, is not a local minimum. Calculation of one-loop correction around such a stationary point is divergent and has to be defined by analytic continuation. In that process the contribution might become imaginary.\(^\star\) This indicates that the classical solution is unstable under quantum fluctuations and most of the probability would be concentrated around some other solution which is a true minimum of the action. In other words, a quantum state peaked around this classical solution will decay into some other more stable state.

To obtain the Euclidean black hole solution, we look at an \(SL(2, R)\) WZW-model with level \(k = 9/4\), and gauge an \(SO(2)\) (\(i.e.\) \(U(1)\)) subgroup. The stress tensor of the gauged theory is given by

\[
T(z) = T^{SL(2)}(z) - T^{U(1)}(z)
\]  

(2.1)

Hence

\[
L_0 = L_0^{SL(2)} - L_0^{U(1)}
\]  

(2.2)

For a chiral primary \(|j, m\rangle\) characterized by \(SL(2)\) isospin \(j\) and \(J_3^0\) (generator of the \(U(1)\) subgroup) eigenvalue \(m\)

\[
L_0^{SL(2)}|j, m\rangle = \frac{j(j + 1)}{k - 2}|j, m\rangle
\]  

(2.3)

\(^\star\) This piece is a volume independent contribution to the one-loop partition function. The bulk contribution is the same as that of Liouville coupled to a compact boson with radius three halves the self-dual radius.
The scaling dimension therefore is

$$h_{j,m} = -\frac{j(j+1)}{k-2} + \frac{m^2}{k}$$

(2.5)

When $j$ corresponds to the principal continuous series (i.e. $j = -\frac{1}{2} + i\lambda, \lambda \in \mathbb{R}$), $h_{j,m} \geq 1$. So, there are no relevant operators for these $j$'s. For real $j$ however it is possible to get low enough scaling dimensions which will make the operator relevant.

In order to decide the question of whether such real $j$'s are allowed, requires the knowledge of the full operator content. In absence of this information, we have some indirect arguments about their presence. First, the modular invariant combination that can be formed from just the $j$'s belonging to the principal continuum series only, seems to represent the physics of a flat cylindrical target space where the noncompact direction has a background charge. From the spacetime point of view, in the background of the semi-infinite cigar solution (which is how the Euclidean black hole solution looks) [2], we expect square-integrable solutions concentrated around the cigar tip, which decaying exponentially towards the flat cylinder end of the cigar. Dijkgraaf et al [6] call these offshell modes the bound states. Let $r$ be the coordinate along the length of the semi-infinite cigar and $\theta$ be the coordinate along the periodic direction. $r$ goes to infinity as one goes towards the asymptotically flat region. $\theta$ has period $2\pi$. The asymptotic form of the vertex operator in these coordinates, if chosen appropriately, is $e^{jr+im\theta}$. Hence the solutions which decay at large $r$ correspond to real $j$. Once certain real $j$ solutions are there, it seems possible to generate larger values of $j$ by fusion.

The theory at the flat cylinder end looks like a compact boson $\theta$ times a free scalar field $r$ with a background charge. The radius of the compact boson is.

and

$$L_{0}^{U(1)}|j, m\rangle = -\frac{m^2}{k}|j, m\rangle$$

(2.4)
\[ \sqrt{k}R_0 = 3R_0/2 \] where \( R_0 \) is the self dual radius. Taking this asymptotic theory seriously, we have, \textit{a la} Dijkgraaf et al.\cite{6},

\[ m = \frac{p}{2} + \frac{qk}{2} \quad p, q \in \mathbb{Z} \quad (2.6) \]

For continuous representation, as already stated, \( j = -\frac{1}{2} + i\lambda, \lambda \in \mathbb{R} \) and for discrete representation, \( j + |m| \in \mathbb{Z} \).

If we look at the vertex operators of zero spin, the modes which will be considered in string field theory, then either \( q \) is zero or \( p \) is zero. The first case corresponds to winding modes and the second to momentum modes. For these two kinds of spin zero vertex operators, the operator \( L_0 - 1 \) written as a differential operator in the target space coordinates looks like:

\[
(L_0 - 1)_{\text{mom}} = -4 \left[ \frac{\partial^2}{\partial r^2} + \left( \coth^2 \frac{r}{2} - \frac{8}{9} \right) \frac{\partial^2}{\partial \theta^2} + \frac{1}{16} \coth^2 \frac{r}{2} + \frac{1}{16} \tanh^2 \frac{r}{2} - \frac{1}{8} \right]
\]

(2.7)

for the momentum modes and

\[
(L_0 - 1)_{\text{wind}} = -4 \left[ \frac{\partial^2}{\partial r^2} + \left( \tanh^2 \frac{r}{2} - \frac{8}{9} \right) \frac{\partial^2}{\partial \theta^2} + \frac{1}{16} \tanh^2 \frac{r}{2} + \frac{1}{16} \coth^2 \frac{r}{2} - \frac{1}{8} \right]
\]

(2.8)

for the winding modes in the coordinates of the dual space-time (\( \theta \) has a different periodicity here). Note that these differential operators are being applied on the tachyon wave-function \( \tilde{T} \), where \( \tilde{T} = Te^{\Phi} \), \( T \) being the usual tachyon and \( \Phi \) is the dilaton background. If we take \( \tilde{T} = R_{j,m}(r)e^{im\theta} \) the ‘radial’ operator would be obtained from the above mentioned by replacing \( \frac{\partial^2}{\partial \theta^2} \) by \(-m^2\). These operators are one dimensional Schrödinger operators on half-line, since \( r \) goes from zero to infinity. The potential in the Schrödinger operator, for the winding mode, has an attractive \( 1/r^2 \). For the tachyon mode at non-zero \( m \), that is not the case. When \( m = 0 \), the solutions which are smooth at the tip of the cigar, are not square-integrable. Therefore, it seems that the square-integrable modes come only from the winding sector. They can have arbitrarily negative values of \( L_0 - 1 \).
It is interesting to look at the same problem from the dual space-time. The Euclidean black hole solution goes to a solution with a naked singularity. The cigar becomes a trumpet with an infinitely large rim [7,6]. Since under duality transformation the momentum and the winding modes get exchanged, we now have particle-like modes giving rise to instability.

Something very similar happens in the 4-d Schwarzschild solution with negative mass. The Laplacian operator in this background has negative modes. It becomes obvious if one goes to the equation for the radial part of the eigenvalue problem. By changing to some new variable \( r_* = r_*(r) \), the problem can be made equivalent to a half line Schrödinger problem with an attractive potential going to \(-\infty\) near the origin.

The solution we considered here is an Euclidean solution with no conical singularity. It is tempting to generalize to a solution which has a conical singularity. This allows us to take the radius of the asymptotic compact boson \( R = 3R_0/2\xi \), where \( \xi \) is an arbitrary positive number. Allowed \( m \) values would become

\[
m = \frac{p\xi}{2} + \frac{qk}{2\xi}
\]

(2.9)

Most of the qualitative physics should not change. It appears that for \( \xi < 1/2 \), i.e. \( R > R_0 \), momentum mode instabilities are also possible if one follows the arguments involving the potential in the related ‘radial’ operator.

So far we have talked of instabilities only in chiral primaries. These are the tachyon modes. To get the unstable modes of higher tensor fields which lead to instability we look for some current algebra secondaries which are Virasoro primaries. Nelson and Distler found an interesting isomorphism between different discrete representations which keeps the dimension unchanged but changes the oscillator number [21]. In the free field representation, the stress tensor is

\[
T(z) = -\frac{1}{2}\partial\sigma\partial\sigma + \frac{1}{2}a\partial^2\sigma + \frac{1}{2}\partial\phi\partial\phi
\]

(2.10)
φ is a compact boson and \( a = \sqrt{\frac{2}{k-2}} \). The chiral primaries are

\[
V_{j,m} = e^{-ja\sigma + ima\phi}
\]  

(2.11)

\[
\alpha = \sqrt{2/k}. \]

Note that \( r \sim -a\sigma \) and \( \theta \sim \alpha\phi \) in the asymptotically flat region. For the so-called \( D^+ \) representations, \( j - m < 0 \). Let \( S^- \) be the operator

\[
\oint dz \ e^{\sigma(z)/a - i\phi(z)/\alpha} \]

\[
S^- V_{j,m}(0) = \oint dz \ z^{j-m} V_{j,m}(0) e^{\sigma(z)/a - i\phi(z)/\alpha} : 
\]

(2.12)

\[
\sim [(J^+)^{m-j+1} + \text{other terms}] e^{-a(j-1/a^2)\sigma + i\alpha(j-1/\alpha^2)\phi}
\]

where \( J^+ \sim \partial \sigma e^{i\alpha\phi} \). These are higher level states in the representation they call \( \tilde{D}^- \).

If these operators are allowed in the spectrum of the conformal field theory, then they correspond to generically offshell modes of higher tensor fields. Since the \( L_0 \) eigenvalue is unchanged, if we start with a relevant tachyon operator, we end up with a relevant operator from the higher level.

3. Conclusion

We indicate the operators in the conformal field theory which correspond to unstable modes in the target space field theory. It is interesting to compare this instability with the Kosterlitz-Thouless (KT) instability [22].

For 26 dimensional bosonic string having one compact direction (one can think of this as a finite temperature theory), the vertex operator of the form \( e^{ip\cdot x} \times \) (winding mode of winding number \( n \)) has dimension

\[
h = \frac{p^2}{2} + \left(\frac{nR}{2R_0}\right)^2
\]

For \( p \to 0 \) and \( n = 1 \), this operator becomes relevant for \( R < 2R_0 \). Thus there is KT transition above a certain temperature.
In the case at hand, the situation is more tricky. Here,

\[ h = -4j(j + 1) + \left( \frac{n R}{2 R_0} \right)^2 = 1 + \left( \frac{n R}{2 R_0} \right)^2 - (2j + 1)^2 \]

If \( j = -\frac{1}{2} + i\lambda \), no matter how small \( R \) is, the operator is irrelevant. So, the instability is a consequence of \( j \) being real for discrete representations.

Let us believe that we can understand the question of metastability of Minkowsky gravity by doing a semiclassical analysis of the corresponding Euclidean path integral [23]. We would like to speculate that the string field theory path integral around the Euclidean black hole or its dual is dominated by the contribution from the region around the corresponding flat infinite cylinder solution with the same radius for the asymptotic compact boson. Presumably this space-time has a tachyon background also. This will indicate that the Minkowski black hole (or a naked singularity, which is dual to the region outside the horizon) decays into flat finite temperature space, the temperature being the same as the Hawking temperature of the curved space *. Presumably, for the flat infinite cylinder, the real \( j \) vertex operator solutions are not allowed, since they diverge at one of the ends. Exclusion of these operators would make this solution stable. It would be interesting to understand the relation between this viewpoint with the discussion of matrix model on a circle by Gross and Klebanov [20].

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* This possibility was indicated by S. Wadia in a talk given at MSRI, Berkeley, earlier this year.
Note added: After this work was finished, we received a preprint by J. Ellis, N. Mavromatos and D. Nanopoulos, CERN Preprint, CERN-TH.6309/91, ACT-53, CTP-TAMU-90/91/. However, they seem to be referring to a different source of instability.

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