Frequency Diverse Array with Cubic Sinusoidal Frequency Offset

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Abstract. Frequency diversity array radar (FDA) has range-angle dependent beampattern, providing enormous potential for the application of this new type of radar. However, the beampattern of conventional FDA (CFDA) has two problems: range-angle coupling and range periodicity, limiting the application of FDA. How to solve the problems is the premise of realizing beam accurate control and target accurate positioning. In this paper, we propose a new FDA transmitter structure with cubic sinusoidal frequency offset (Cubic sin-FDA). Simulation results show that this FDA beampattern eliminates coupling and forms a spot-shaped beampattern. Compared with the existing nonlinear frequency offset FDA beampattern, this method has a narrower half power bandwidth and lower side lobe level.

1. Introduction
Frequency Diverse Array (FDA), which was first put forward in 2006 [1], has recently aroused wide interest among scientists. At present, phased array radar is widely used and its elements work at a same frequency, different from which, there is a slim frequency offset between the elements of the conventional FDA (CFDA), forming range-angle dependent beampattern. This feature of CFDA provides many advantages for application in interference suppression and active sensing [2-4].

However, the range and angle of a CFDA’s beampattern are coupled, which results in the beampattern has an “S”-shape. The application of CFDA in targets locating and tracking accurately can be affected by this feature. Several experiments have been conducted to solve this problem. In Ref. [5], the double pulse FDA is introduced to decouple by transmitting two pulse with non-zero and zero frequency offset respectively, but the transmission rate decreases dramatically. The array segmentation is an alternative, that is, a frequency diverse array is segmented into several sub-arrays, each of which emits diverse signals [6]. However, in the case of multiple targets, false targets can occur.

In addition to the above methods, some researches have been reported recently, which changes frequency offset of the array to form a decoupled beam. In Khan et al, the liner frequency offset employed in the linear array was replaced by the logarithmic frequency offset (Log-FDA) [7]. Log-FDA can only form an extreme point for the target of interest, but it has a large beam width in both range and angle, which easily leads to the fuzziness of target positioning or tracking. Besides, the square and cubic frequency offsets [8] were also put forward by Gao et al for decoupling. But multiple maxima may occur when searching a wide area in this scheme. Sinusoidal frequency array (sin-FDA) was proposed in 2017, which can form a narrower main lobe and search targets in the specified range [9].
This paper presents a new frequency array transmitter structure which is called Cubic sin-FDA. The results show that the FDA beampattern eliminates coupling and forms a spot-shaped beampattern. Compared with the existing nonlinear FDA-beam diagrams, this method not only has narrower half power bandwidth but also lower side lobe level under the same maximum bandwidth criterion.

The remainder is organized as follows. In Section 2, the design and beampattern of Cubic sin-FDA are presented. Simulation and analysis are exhibited in Section 3, and conclusion is made in Section 4.

2. Proposed Cubic sin-FDA Scheme

2.1. Cubic sin-FDA Design

Assuming an array of selfsame transmit elements whose space is \( d \), a cubic sinusoidal frequency offset \( \Delta f_m \) is introduced to each antenna element:

\[
\begin{align*}
    f_m &= f_0 + \Delta f_m \\
    \Delta f_m &= \sin^3(m \pi / 3) \delta, \quad m = 0, 1, \ldots, M - 1
\end{align*}
\]

where \( f_m \) is the carrier frequency of the \( m \)th element; \( f_0 \) represents the reference frequency meeting \( f_0 \gg \Delta f_m \); \( \delta \) represents the fixed frequency offset parameter in unit Hz and \( M \) is the number of elements. The structure of the proposed Cubic sin-FDA is presented in figure 1.

\[
\begin{align*}
    \sin (\pi/3) = \sin (2\pi/3) = \sin (\pi) = 0
\end{align*}
\]

Figure 1. FDA with cubic sinusoidal frequency offset.

2.2. Cubic sin-FDA Transmit Beampattern

In the proposed Cubic sin-FDA, the transmitting signal of the \( m \)th element is

\[
x_m(t) = a_m \exp(j2\pi f_m t), \quad 0 < t < T
\]

where \( a_m \) is the weight related to \( m \)th element and \( T \) is the array transmits pulses of duration. The signal reaching the target far from the array can be expressed as

\[
x(t; R_0, \theta) = \sum_{m=0}^{M-1} x_m(t - R_m / c) = \sum_{m=0}^{M-1} a_m \exp\left[ j2\pi f_m \left( t - \frac{R_m - md \sin \theta}{c} \right) \right]
\]

where \( c \) represents the speed of light and \( R_m \) is the distance between the target and the \( m \)th element. Substituting (2) into (4), we can get
\[
x(t; R_0, \theta) = \sum_{m=0}^{M-1} a_m \exp \left\{ j2\pi \left[ \sin^3 \left( \frac{m}{\pi} \right) \delta \left( t - \frac{R_0}{c} \right) \right] + \left[ f_0 + \sin^3 \left( \frac{m}{\pi} \right) \delta \left( \frac{md \sin \theta}{c} \right) \right] \right\}
\times \exp \left\{ j2\pi f_0 \left( t - \frac{R_0}{c} \right) \right\}
\]

Because \( f_0 \gg \sin^3 \left( \frac{m}{\pi} \right) \delta, m = 1, 2, \ldots, M - 1 \), we have \( f_0 + \sin^3 \left( \frac{m}{\pi} \right) \delta \approx f_0 \), then (5) can be approximated as:

\[
x(t; R_0, \theta) = \exp \left\{ j2\pi f_0 \left( t - \frac{R_0}{c} \right) \right\} \sum_{m=0}^{M-1} a_m \exp \left\{ j2\pi \left[ \sin^3 \left( \frac{m}{\pi} \right) \delta \left( t - \frac{R_0}{c} \right) + f_0md \sin \theta \right] \right\}
\]

To focus the signal energy at the position \( (R', \theta') \) of the desired target, the weight \( a_m \) is set as

\[
a_m = \exp \left\{ j2\pi \left[ \sin^3 \left( \frac{m}{\pi} \right) \delta R' \frac{f_0md \sin \theta'}{c} \right] \right\}
\]

The steered beampattern of the proposed Cubic sin-FDA can be expressed as

\[
B(t; R_0, \theta) = \left[ \sum_{m=0}^{M-1} \exp \left\{ j2\pi \left[ \sin^3 \left( \frac{m}{\pi} \right) \delta \left( t - \frac{R - R'}{c} \right) + f_0md \left( \frac{\sin \theta - \sin \theta'}{c} \right) \right] \right\} \right]^2
\]

3. Simulation and Comparison Analysis

3.1. Simulation Parameter Configuration
In order to verify the advantage of the proposed Cubic sin-FDA scheme, we simulate the beampattern and compare it with the CFDA, square-FDA, cubic-FDA and sin-FDA schemes. The basic parameters of the simulations are listed in table 1. Especially, for impartial comparison, the signal bandwidth of all schemes is set to be extremely identical. The values of fixed frequency offset in various array forms are shown in table 2. In order to ensure that there is no cascade, the array interval \( d \) should satisfy \( d \leq \lambda_{\max} / 2 \) [10], where \( \lambda_{\max} \) denotes maximum carrier wavelength. The target position is \((500\text{km}, 30^\circ)\).

| Parameter       | Value | Number of elements | Reference frequency | Element spacing | \( R_0 \) | \( \theta_0 \) | \( t \) |
|-----------------|-------|--------------------|---------------------|-----------------|------------|----------------|------|
| Value           | 20    | 5GHz               | 0.015m              | 500km           | 30^\circ   | 0              |

Table 2. Values of fixed frequency offset in various array forms when the bandwidth is 5.787 kHz.

| Array name | CFDA | Square-FDA | Cubic-FDA | sin-FDA | Proposed |
|------------|------|------------|-----------|---------|----------|
| \( \delta \) /Hz | 304.69 | 16.04       | 0.844     | 5790    | 5790     |

3.2. Simulation Results and Analysis
Seen from figure 2e, the proposed Cubic sin-FDA forms a spot-shaped beampattern with most of energy focus on the position \((500\text{ km}, 30^\circ)\) as well as the beampattern of the sin-FDA shown in figure 2d. On the contrary, in figures 2b and 2c, the energy distributed at the target location is less concentrated than Cubic sin-FDA or sin-FDA although they show the peak of the aperiodic beam chart. In figure 2a, the beampattern of CFDA is periodic and range-angle coupled, indicating that the
location of the target cannot be get directly from the beampattern peak. Besides, multiple maximum beampattern characteristics deteriorate the anti-interference capability. Therefore, CFDA is not referred to in the following comparison.

Figures 3 and 4 show the beam characteristics in range and in angle, respectively. From figures 3 and 4, the proposed Cubic sin-FDA has the narrowest half power bandwidth (HPBW) with the same signal bandwidth, showing the superiority of the proposed frequency offset in respect of transmitting power focusing. The beampattern of square-FDA has the lowest side lobe level (SLL) but the widest HPBW. The HPBW of the proposed Cubic sin-FDA in angle is same as that of sin-FDA, because the angular resolution is related to the antenna aperture.

(a)                     (b)                    (c)
(d)                     (e)

Figure 2. Distribution of beampattern: (a) CFDA; (b) square-FDA; (c) cubic-FDA; (d) sin-FDA; (e) Proposed Cubic sin-FDA.

(a)                     (b)

(c)                     (d)

Figure 3. The beam characteristics in range: (a) square-FDA; (b) cubic-FDA; (c) sin-FDA; (d) Proposed Cubic sin-FDA.
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SLL=0.2785

HPBW in angle

8.8~58

SLL=0.2654

HPBW in angle

16.4~45.8

SLL=0.4075

HPBW in angle

24.4~36

SLL=0.3645

HPBW in angle

24.4~36

Figure 4. The beam characteristics in angle: (a) square-FDA; (b) cubic-FDA; (c) sin-FDA; (d) Proposed Cubic sin-FDA.

4. Conclusion
In this letter, we propose a new frequency diverse array structure with cubic sinusoidal frequency offset. Compared with the existing FDA models with nonlinear frequency offset, the proposed Cubic sin-FDA can form a spot-shaped beampattern, which performs better in range and angle decoupling. Besides, it has better resolution. Simulation results demonstrate the excellence of the proposed Cubic sin-FDA. In the future, we will further investigate the ability of anti-interference and clutter suppression of FDA.

Acknowledgments
This work was funded by the National Natural Science Foundation of China under Grant No. 61871174.

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