Population synthesis models including AGB stars and their ingredients

Paola Marigo

Dipartimento di Astronomia, Università di Padova, Italy

Abstract. I will briefly review the state of the art of evolutionary population synthesis (EPS) models that include the contribution from AGB stars.

1. Assembling AGB stars in EPS models: techniques

Three possible methods can be distinguished to add the AGB phase in EPS models, which are based on the use of A) stellar isochrones (e.g. Bressan et al. 1996, Bruzual & Charlot 2003, Mouhcine 2002, Marigo et al. 2003); B) stellar tracks (Groenewegen & de Jong 1993, Marigo et al. 1999, Mouhcine & Lançon 2002ab, Mouhcine & Lançon 2003); and C) the fuel consumption theorem by Renzini & Buzzoni (1986; Maraston 1998, 2005).

Both methods A) and B) need essentially the same ingredients, namely sets of AGB stellar models that provide the evolution of basic stellar parameters (i.e. total mass, core mass, luminosity, effective temperature, surface chemical composition) with a possibly wide coverage of initial stellar masses and metallicities. Owing to the heavy requirement of computing time set by the full modelling of the AGB phase, the most convenient way to build up an extended library of AGB stellar tracks is to adopt a synthetic approach, that gives a simplified but flexible and easy-to-test description of the stellar evolution on the AGB. Another positive aspect is that synthetic AGB models can be readily calibrated on the base of basic observables (e.g. the carbon star luminosity functions), hence ensuring a more realistic description of AGB properties (see Sect. [1]). Available stellar isochrones including the whole AGB phase are listed in Table [1] that also outlines a few relevant characteristics, namely: the inclusion or not of the third dredge-up and hot-bottom burning, the way \(T_{\text{eff}}\) is obtained depending on the adopted molecular opacities, the consideration or not of the effect of circumstellar dust on the emitted spectrum.

The fundamental ingredient of Method C) is the fuel matrix, \(F_{\text{AGB}}(M_{\text{TO}}, Z)\), providing the nuclear fuel burnt during the AGB phase as a function of the turn-off mass \(M_{\text{TO}}\) (or equivalently of the age), and the metallicity \(Z\) of the corresponding single stellar population.

Then, for all methods the passage from theoretical quantities to observed ones requires the adoption of suitable spectral libraries as a function of \(T_{\text{eff}}\), surface gravity and C/O ratio, or alternatively tables of bolometric corrections and \(T_{\text{eff}}\)–colour transformations. Finally, one has to specify global properties of
the simulated galaxy, like the initial mass function (IMF), the star formation rate history (SFR), and the age-metallicity relation (AMR).

Concerning the characteristics of the EPS techniques, it should be remarked that both methods A) and B) are tied, by construction, to the adopted stellar models. On one hand this brings along all the weak points that affect the theory of stellar evolution, but on the other hand it allows to get a useful feedback to input prescriptions any time a comparison between predictions and observations is performed. Method C) usually adopts an empirical calibration of the AGB nuclear fuel based on observed data of Magellanic Clouds’ clusters. On one hand this guarantees to account for the correct integrated light contribution from AGB stars at LMC and SMC metallicities, but on the other hand it makes it difficult to derive explicit indications on uncertain aspects of the underlying AGB evolution. Moreover, while for methods A) and B) the resolution element is the single star, so that they can address the study of both integrated and resolved stellar populations, for method C) the maximum resolution is given by the simple stellar population (SSP), which limits its application to the integrated properties of unresolved AGB stars.

Because of their intrinsic sensitivity to the details of AGB modelling, this review will mostly focus on EPS methods A) and B) and their applications.

2. Synthetic AGB evolution: ingredients

Synthetic AGB models are usually constructed on the base of some analytical recipe – derived from complete modelling of the AGB phase and/or observations – providing, for instance, the core mass-luminosity relation, the core mass-interpulse period relation, and the mass-loss rates as a function of stellar parameters. Examples of such calculations can be found in Groenewegen & de Jong (1993) and Izzard et al. (2004).

The purely analytical scheme can be importantly complemented with the aid of envelope integrations - from the photosphere down to the surface of the H-exhausted core – which allows us e.g. to predict the effective temperature along the AGB, as well as to follow the hot-bottom burning (HBB) nucleosynthesis by solving a network of nuclear reactions in the deepest envelope layers of the most massive AGB models. This kind of approach was pioneered by Renzini & Voli (1981), and more recently adopted by e.g. Marigo et al. (1999 and references therein), Gavilán et al. (2005), Marigo & Girardi (2006).

2.1. Luminosity and effective temperature

The recent availability of high-accuracy formalisms based on full evolutionary calculations (e.g. Wagenhuber & Groenewegen 1998, Izzard et al. 2004) has allowed synthetic TP-AGB models to account for the complex behaviour of the luminosity due to the occurrence of thermal pulses and HBB. An example of the nice performance of these formalisms is presented in Fig. 1, showing the flash-driven luminosity variations in models with different masses, and the over-luminosity effect due to HBB in the \((M_1 = 4.0 \, M_\odot, Z = 0.008)\) model.

As to the effective temperature, several analytic fits to the results of full AGB evolutionary calculations are available in the literature, yielding \(T_{\text{eff}} = T_{\text{eff}}(M, L, Z)\) (e.g. Vassiliadis & Wood 1993). However, it should be mentioned
Population synthesis with AGB stars

Table 1. Available stellar isochrones including the AGB phase

| Reference                  | 3rd-D-up and HBB(b) | Teff(c)         | Dust(d)         | available at(e) |
|----------------------------|---------------------|-----------------|-----------------|-----------------|
| Bertelli et al. 1994, A&AS, 106, 275 | NO                  | solar-scaled comp. | NO              | [http://pleiadi.pd.astro.it](http://pleiadi.pd.astro.it) |
| Girardi et al. 2000, A&AS, 141, 371   | NO                  | solar-scaled comp. | NO              | [http://pleiadi.pd.astro.it](http://pleiadi.pd.astro.it) |
| Marigo & Girardi 2001, A&A, 377, 132 | YES                 | solar-scaled comp. | NO              | [http://pleiadi.pd.astro.it](http://pleiadi.pd.astro.it) |
| Pietrinferni et al. 2004, ApJ, 612, 168 | NO                  | solar scaled comp. | NO              | [http://astro.ensc-rennes.fr/basti](http://astro.ensc-rennes.fr/basti) |
| Cioni et al. 2006, A&A, 448, 77      | YES                 | O-/ C-rich comp.  | NO              | [http://pleiadi.pd.astro.it](http://pleiadi.pd.astro.it) |
| Bressan et al. 1998, A&A, 332, 135   | NO                  | solar scaled comp. | YES             | upon request    |
| Mouhcine 2002, A&A, 394, 125         | YES                 | solar scaled comp. | YES             | upon request    |
| Piovan et al. 2003, A&A, 408, 559    | NO                  | solar scaled comp. | YES             | upon request    |
| Kim et al. 2006, PASP, 118, 62       | NO                  | solar scaled comp. | YES             | upon request    |

(a): bibliographic reference  
(b): explicit inclusion or not of AGB nucleosynthesis, i.e. third dredge-up and hot-bottom burning, in the underlying AGB models  
(c): choice of low-temperature opacities affecting the predicted effective temperature, i.e. for solar-scaled or variable (O- or C-rich) chemical mixtures  
(d): inclusion or not of circumstellar dust in the computation of the emitted spectrum  
(e): public www address for retrieval

Table 2. Available chemical yields from low- and intermediate-mass stars

| Reference                  | model type(b) | HBB(c) | isotopes(d) | M/M⊙(c) | Z(f)  |
|----------------------------|---------------|--------|-------------|---------|------|
| Iben & Truran 1978, ApJ, 220, 980 | synthetic     | NO     | C, N, Ne   | 1.0 – 8.0 | 0.02 |
| Renzini & Voli 1981, A&A, 94, 175   | synthetic+env. | network | He, C, O   | 1.0 – 8.0 | 0.004 – 0.02 |
| v.d. Hoek & Groenewegen 1997, A&AS, 123, 305 | synthetic     | analytic | He, C, O    | 0.8 – 8.0 | 0.001 – 0.04 |
| Forestini & Charbonnel 1997, A&AS, 123, 241 | synthetic/full | network/extr. | many       | 3.0 – 7.0 | 0.005 – 0.02 |
| Marigo 2001, A&A, 370, 194        | synthetic+env. | network | He, C, O   | 0.8 – 5.0 | 0.004 – 0.019 |
| Chieffi et al. 2001, ApJ, 554, 1159 | full       | network | many       | 4.0 – 7.0 | 0    |
| Ventura et al. 2002, A&A, 393, 215   | full        | network | He, Li, C, O | 2.5 – 5.5 | 2.10^-4 – 0.01 |
| Karakas 2003, PhD thesis, Monasch Univ. | full      | network | many       | 1.0 – 8.0 | 0.004 – 0.02 |
| Dray et al. 2003, MNRAS, 338, 973   | full        | network | C, N, O     | 1.25 – 6.0 | 0.02 |
| Herwig 2004, ApJS, 155, 651          | full      | network | He, C, N    | 2.0 – 6.0 | 10^-4 |
| Izzard et al. 2004, MNRAS, 350, 407 | synthetic   | analytic | He, C, O, Ne | 0.8 – 8.0 | 10^-4 – 0.03 |
| Gavilán et al. 2005, A&A, 432, 861   | synthetic+env. | network | He, C, O     | 0.8 – 8.0 | 0.013 – 0.032 |

(a): bibliographic reference  
(b): kind of AGB model, i.e. synthetic → purely analytic; synthetic+env. → analytic with envelope integrations;  
full → complete stellar structure calculations  
(c): treatment of HBB, i.e. analytic → based on analytic recipe; network → integration of a nuclear network;  
extr. → extrapolation of partial results up to the end of the AGB  
(d): elemental species and their main isotopes for which stellar yields are available  
(e) and (f): initial mass and metallicity intervals
that these fitting relations are strictly valid for giant stars with C/O < 1 as they stand on the adoption of low-temperature opacities for solar-scaled chemical compositions, while they are completely inappropriate to describe the atmospheric properties of carbon stars. As discussed by Marigo (2002), the adoption of molecular opacities consistently coupled to the surface chemical composition of AGB models leads to a significant decrease of $T_{\text{eff}}$ as soon as C/O > 1, which is confirmed by observations of galactic AGB stars (Bergerat et al. 2001). The aforementioned effect is clearly shown in the right panel of Fig. 1, where the behaviour of $T_{\text{eff}}$ appears to mirror somehow that of C/O.

2.2. The third dredge-up and hot-bottom burning

Extensive full AGB calculations have recently provided a more detailed picture of the third dredge-up and its dependence on stellar mass and metallicity, supplying a fitting formalism for two classical quantities: the minimum core mass $M_{\text{min}}(M, Z)$ and the efficiency $\lambda(M, Z)$ (Karakas et al. 2002). This allows to include a more realistic description of the dredge-up process in synthetic TP-AGB models (Marigo & Girardi 2006), so as to relax the assumption of constant dredge-up parameters characterizing several works in the past literature (e.g. Groenewegen & de Jong 1993, Marigo et al. 1999, Mouhcine & Lançon 2003). It is also interesting to see in Fig. 2 how much the predictions for the efficiency
A have changed over the years, e.g. passing from almost no dredge-up in Renzini & Voli (1981) to quite extreme values according to Stancliffe et al. (2005).

Taking HBB into account in synthetic TP-AGB models can be done via two possible ways, namely either analytically or with the inclusion of a nuclear network coupled to envelope integrations. The former scheme has been adopted in various models (Groenewegen & de Jong 1993, Mouhcine & Lançon 2002a, Izzard et al. 2004), but its performance is heavily affected by (too) many free parameters. The best possible approach is clearly the latter one, producing results that are closer to those of full AGB calculations (e.g. Marigo 2001, Gavilán et al. 2005).

3. Stellar yields

A direct by-product of AGB evolutionary calculations is the computation of stellar yields, which are key-ingredients in chemical evolution models of galaxies. A compilation of available sets of yields from low- and intermediate-mass stars is presented in Table 2.

An important consistency requirement to galaxy models is set by the fuel consumption theorem (Renzini & Buzzoni 1986) that states the direct proportionality between the nuclear fuel, $F$, burnt during a post main-sequence phase and its contribution to the integrated light of a SSP.

Marigo & Girardi (2001) have shown, in particular, that the nuclear fuel of the AGB phase can be conveniently split into two contributions, one corresponding to the fuel burnt to increase the mass of the C-O core and the other related to the fuel ejected from the star in the form of chemical yields (essentially helium, carbon and oxygen). In other words, this means that chemical yields are just a precise fraction of the stellar emitted light. It follows that in chemo-spectro-photometric galaxy models the basic stellar ingredients, i.e. light and chemical yields, ought come from the same set of stellar models or, at least, the reciprocal consistency of heterogeneous inputs should be verified.

4. Observational constraints to AGB stellar models

Two important constraints will be discussed here, namely: i) the counts of AGB stars, and ii) the carbon star luminosity functions (CSLF) in the Magellanic Clouds (MCs). Their simultaneous fulfillment should guarantee that AGB life-
times and luminosities are fairly estimated, that is the amount AGB fuel in EPS models is consistent with observations.

4.1. AGB stellar lifetimes

Star counts in MCs’ clusters provide quantitative information on the duration of the AGB phase as a function of stellar mass (and metallicity). Further, one can distinguish between M-type and C-type lifetimes, $\tau_M$ and $\tau_C$ respectively (Girardi & Marigo 2006). Figure 3 compares the LMC data with the predictions of a few synthetic TP-AGB models, obtained with different assumptions concerning the third dredge-up, mass-loss rates, molecular opacities, etc. We note that $\tau_C$ varies notably from author to author, being particularly sensitive to the adopted mass-loss formalism. Different is the case of $\tau_M$, which is practically independent of mass loss (for AGB stars that later become C-stars), while it is crucially affected by the third dredge-up, since its duration is limited by the transition to the C-star domain. Interestingly, the significant underestimation of $\tau_M$ at $M \approx 2.0 - 2.5 M_\odot$ by models adopting constant $M_{\text{min}}^c$ and $\lambda$ is filled in when accounting for the dependence of these parameters on mass and metallicity, as provided by Karakas et al. (2002).

4.2. Carbon star luminosity functions

Observed CSLFs in the MCs pose an important constraint to AGB models. The long-standing theoretical difficulty to account for the formation of faint C-stars – the well-known carbon star mystery designated by Iben (1981) – still affects present full AGB evolutionary calculations (e.g. Izzard et al. 2004; see however the results of Stancliffe et al. (2006) for the LMC case). Synthetic TP-AGB models indicate that to remove the discrepancy one has to invoke an earlier onset (lower $M_{\text{min}}^c$) and higher efficiency (larger $\lambda$) of the dredge-up in lower-mass models (Groenewegen & de Jong 1993, Marigo et al. 1999).
However, it should be mentioned that most simulations of CSLFs have assumed a constant metallicity (i.e. \( Z = 0.008 \) for the LMC and \( Z = 0.004 \) for the SMC), while according to a recent study (Marigo & Girardi 2006) this approximation should be dropped in favour of a proper coupling between a more realistic mass- and metallicity-dependent description of the third dredge-up and the AMR of the parent galaxy (see Fig. 4.2.). By doing so fainter C-stars tend to have lower metallicities, just where the occurrence of the third dredge-up is naturally favoured by AGB models. In this way the extent of the theoretical difficulty turns out to be not so severe as believed so far.

Recently Guandalini et al. (2006) have suggested that the carbon star mystery might be a false problem when accounting for the obscured C-stars, which would make the CSLF brighter than previously believed. However, both observations (Van loon et al. 2005) and population synthesis models (see Girardi & Marigo, this meeting) agree in estimating as \( \approx 10 - 20\% \) the fraction of dust-enshrouded C-stars in the MCs, i.e. the bulk of the C-star population consists of optically-visible objects. This implies that the consideration of the obscured C-stars would not dramatically affect the observed CSLFs, at least in these two galaxies.

![Figure 4. Carbon star luminosity functions of the LMC and SMC. Observed distributions (gray histograms) are compared to the results of calibrated synthetic TP-AGB models (Marigo & Girardi 2006), assuming either an AMR (continuous line) or a constant metallicity (dashed line) for the parent galaxies. Models include variable molecular opacities, and a proper mass- and metallicity-dependence of the third dredge-up based on Karakas et al. (2002).](image)

5. Resolved stellar populations with AGB stars

5.1. C-star populations and the C/M ratio

Recent wide-area near-IR surveys (DENIS, 2MASS) have shown that C-stars draw a striking red tail in colour-magnitude diagrams, clearly separated from the branch of oxygen-rich giant stars toward redder colours. With the aid of the isochrone synthesis approach, Marigo et al. (2003) have shown that the reproduction of such feature requires the adoption in AGB stellar models of molecular opacities consistently coupled with the surface C/O (see Sect. 2.1). It is interesting to notice in Fig. 5.1 that the C-star population is expected to be more prominent as we move from the Milky Way disk to the SMC, which is essentially driven by a metallicity effect.
As a matter of fact, whereas the C/M ratio is predicted to be primarily a function of metallicity (Mouhcine & Lançon 2003), it is expected to change also with population age, hence being sensitive to the star formation history of the host galaxy (Cioni et al. 2006a). Moreover, the mean K-band magnitude of C- and M-type stars also varies with the SFH. Cioni et al. (2006ab) take advantage of these theoretical predictions in order to map variations of mean age and metallicity across the LMC and SMC.

5.2. Long-period variables

Despite the huge observational progress achieved thanks to recent gravitational micro-lensing experiments (MACHO, EROS, OGLE), few population synthesis studies on pulsating AGB stars have been carried out so far (e.g. Groenewegen & de Jong 1994). The potentiality of such theoretical analyses is far-reaching. For instance, by fitting the observed period distributions of OGLE Miras in selected Bulge fields, Groenewegen & Blommaert (2005) could infer the presence of relatively young stellar populations, with ages $\lesssim 3$ Gyr.

The most intriguing future challenge of EPS models with AGB stars will be the reproduction of the several sequences populated by variable red giants period-luminosity diagrams, most of which are interpreted as due to different pulsation modes (Wood et al. 1999).

6. Effects of AGB stars on integrated luminosities and colours

One basic prediction of EPS models is the evolution of integrated luminosities and broad-band colours (e.g. $B - V$, $V - K$, $J - K$) produced by single stellar populations as a function of age (e.g. Tantalo et al. 1996, Maraston 1998, Marigo & Girardi 2001, Mouhcine & Lançon 2002a). The standard benchmark of any theoretical comparison has long been provided by AGB stars in star clusters in the MCs (Frogel et al. 1990). Unfortunately, due to the small number statistics of AGB stars in these clusters, stochastic fluctuations are of large entity, so that MCs’ clusters cannot be assimilated to standard SSP of given age and
Population synthesis with AGB stars

metallicity. The best meaningful way to perform a comparison with observations is to simulate a population of clusters with the aid of Monte-Carlo techniques (Bruzual & Charlot 2003).

For instance, by looking at Fig. 6, we notice that the observed dispersion of the data (top panel) is recovered with a synthetic sample of clusters (bottom panel), while limiting the comparison to a sequence of SSPs of varying age (solid line) may be even misleading.

Figure 6. Top panel: Observed \((V-K)\) vs \((B-V)\) diagram for a sample of LMC clusters of known ages (Persson et al. 1988, Bica et al. 1996; Girardi et al. 1995). Bottom panel: Predictions for a randomly generated sample of stellar clusters (empty triangles) obeying given mass and age distributions. The solid line connects the values of SSPs of increasing age, i.e. with increasing \((B-V)\). The metallicity is assumed \(Z = 0.008\) (Marigo & Girardi 2006)

AGB stars are important contributors to their integrated bolometric and near-IR luminosities with a maximum located at an age of \(\approx 1\) Gyr, accounting for about \(40 \sim 80\)\% of the total cluster’s luminosity (see Frogel et al. 1990; Maraston 2005). Interestingly, the position of the maximum reflects the peak in TP-AGB lifetimes at \(M \approx 2.0 \sim 2.5 M_\odot\), as shown in Fig. 3.

Going from younger to older ages the first appearance of more massive AGB stars at \(\approx 10^8\) Gyr produces a significant increase of colours like \(V-K\) (e.g. Maraston 1998), with a rising slope that depends much on the details of processes like mass loss and HBB (Girardi & Bertelli 1998). A further contribution to attain redder colours (e.g. \(V-K, J-K\)) is provided by C-stars, with a maximum effect at ages \(\approx 0.5 \sim 1.0 \times 10^9\) yr corresponding to turn-off masses of \(\approx 2.0 \sim 2.5 M_\odot\) (Mouhcine & Lançon 2002a).

As first shown by Bressan et al. (1998, see also Mouhcine 2002, Piovan et al. 2003) accounting for the effect of dusty envelope around AGB stars has a strong impact on the predicted spectral properties of galaxies, e.g. increasing the integrated light emission in the mid-infrared of an intermediate-age population by one order of magnitude.

A potentially powerful application of such EPS models including dust emission from AGB stars is the possibility to break the age-metallicity degeneracy that affect early-type galaxies, with the aid of a suitable combination of optical-mid-IR-near-IR pass-bands. A recent result to be mentioned is the detection of a broad silicate emission at 10\(\mu\)m in the Spitzer IRS spectra of a large sample of early-type galaxies in Virgo, which appears to be consistent with the contribution from an underlying population of unresolved dusty oxygen-rich AGB stars (Bressan et al. 2006).
Acknowledgments. This work is funded by the University of Padova (Progetto di ricerca di Ateneo CPDA052212).

References

Bergeat, J., Knapik, A., & Rutily, B. 2001, A&A, 369, 178
Bica, E., Claria, J.J., Dottori, H., Santos, J.F.C., Jr., Piatti, A.E. 1996, ApJS, 102, 57
Bressan, A., Chiosi, C., & Tantalo, R. 1996, A&A, 311, 425
Bressan, A., Granato, G.L., & Silva L. 1998, A&A, 332, 135
Bressan, A., Panuzzo, P., Buson, L., et al. 2006, ApJ, 639, L55
Bruzual, G., & Charlot, S. 2003, MNRAS, 344, 1000
Cioni, M.-R.L., Girardi, L., Marigo, P., Habing, H.J., 2006a, A&A 448, 77
Cioni, M.-R.L., Girardi, L., Marigo, P., Habing, H.J., 2006b, A&A, 452, 195
Frogel, J.A., Mould, J., & Blanco, V.M. 1990, ApJ, 352, 96
Gavilán, M., Buell, J.F., & Mollá M. 2005, A&A, 432, 861
Girardi, L., Chiosi, C., Bertelli, G., & Bressan, A., A&A, 298, 87
Girardi, L., & Bertelli, G. 1998, MNRAS, 300, 533
Girardi, L., & Marigo, P. 2006, A&A in press (astro-ph/0609626)
Girardi, L., Bressan, A., Bertelli, G., & Chiosi, C. 2000, A&AS, 141, 371
Girardi, L., Groenewegen, M.A.T., Hatziminaoglou, E., et al. 2005, A&A, 436, 895
Groenewegen, M.A.T. 2004, A&A 425, 595
Groenewegen, M.A.T., & de Jong, T. 1993, A&A, 267, 410
Groenewegen, M.A.T., et al. 1999, A&AS, 140, 197
Groenewegen, M.A.T., et al. 2002, A&A, 392, 741
Groenewegen, M.A.T., & Blommaert, J.A.D.L. 2005, A&A, 443, 143
Guandalini, M., Busso R., Ciprini, M., Silvestro, G., & Persi P. 2006, A&A, 445, 1069
Iben, I. 1981, ApJ, 246, 278
Izzard, R.G., Tout C.A., Karakas A.I., & Pols, O.R. 2004, MNRAS, 350, 407
Karakas, A. I., Lattanzio, J.C., & Pols, O.R. 2002, PASA, 19, 515
Maraston, C. 1998, MNRAS, 300, 872
Maraston, C. 2005, MNRAS, 362, 799
Marigo, P. 2001, A&A, 370, 194
Marigo, P. 2002, A&A, 387, 507
Marigo, P., Girardi, L. 2001, A&A, 377, 132
Marigo, P., Girardi, L., & Bressan, A. 1999, A&A, 344, 123
Marigo, P., Girardi, L., & Chiosi, C. 2003, A&A, 403, 225
Marigo, P., Girardi, L. 2006, A&A submitted
Mouhcine, M. 2002, A&A, 394, 125
Mouhcine, M., & Lançon, A. 2002a, A&A, 393, 149
Mouhcine, M., & Lançon, A. 2002b, A&A, 393, 167
Mouhcine, M., & Lançon, A. 2003, MNRAS, 338, 572
Persson, S.E., Aaronson, M., Cohen, J.G., et al. 1983, ApJ, 266, 105
Piovan, L., Tantalo, R., & Chiosi, C. 2003, A&A, 408, 559
Renzini, A., & Voli, M. 1981, A&A, 94, 175
Renzini, A., & Buzzoni, A. 1986, in Spectral Evolution of Galaxies, C. Chiosi and A. Renzini (eds.), Dordecht, Reidel, p. 195
Stancliffe, R.J., Izzard R.G., & Tout C.A. 2005, MNRAS, 356, L1
Tantalo, R., Bressan, A., Chiosi, C., & Fagotto, F. 1996, A&A, 311, 361
Vassiliadis, E., Wood, P.R. 1993, ApJ, 413, 641
Wagenhuber, J., Groenewegen, M.A.T., 1998, A&A, 340, 183
Wood, P.R., Alcock, C., Allsman, R.A., Alves, D., et al. 1999, in Asymptotic Giant Branch Stars, IAU Symp. n. 191, T. Le Bertre, A. Lebre, C. Waelkens (eds.), p. 151