Hierarchical Meta Learning

Yingtian Zou, Jiashi Feng
National University of Singapore
{elezouy,elefjia}@nus.edu.sg

Abstract
Meta learning is a promising solution to few-shot learning problems. However, existing meta learning methods are restricted to the scenarios where training and application tasks share the same output structure. To obtain a meta model applicable to the tasks with new structures, it is required to collect new training data and repeat the time-consuming meta training procedure. This makes them inefficient or even inapplicable in learning to solve heterogeneous few-shot learning tasks. We thus develop a novel and principled Hierarchical Meta Learning (HML) method. Different from existing methods that only focus on optimizing the adaptability of a meta model to similar tasks, HML also explicitly optimizes its generalizability across heterogeneous tasks. To this end, HML first factorizes a set of similar training tasks into heterogeneous ones and trains the meta model over them at two levels to maximize adaptation and generalization performance respectively. The resultant model can then directly generalize to new tasks. Extensive experiments on few-shot classification and regression problems clearly demonstrate the superiority of HML over fine-tuning and state-of-the-art meta learning approaches in terms of generalization across heterogeneous tasks.

1 Introduction
Learning quickly to solve various tasks is a hallmark of general artificial intelligence, such as learning new skills from limited experience or learning to recognize new objects from a few examples. To achieve this, an artificial agent needs to effectively grasp generic and specific knowledge from different tasks. As a promising solution, meta learning aims to learn a task-general meta model from multiple training tasks, serving as an inductive bias to improve the learning efficiency for new tasks [Thrun and Pratt, 2012]. Recently it has received growing attention and achieved noticeable success in some machine learning applications, like model hyper-parameter optimization [Maclaurin et al., 2015; Feurer et al., 2015], reinforcement learning [Duan et al., 2016; Finn et al., 2017b] and few-shot learning [Ravi and Larochelle, 2017; Vinyals et al., 2016; Finn et al., 2017a; Santoro et al., 2016].

Existing meta learning approaches mostly assume that the training and application tasks share the same structure. The learned meta model therefore cannot be directly applied to learning to solve new tasks of different structures. For example, a meta model trained for $N$-variable few-shot regression tasks is inapplicable to the new tasks of $N'$ variables. To obtain a new meta model that is compatible with the new task structure, the meta training process has to be repeated on tasks of new structures, which is highly inefficient. A desired efficient meta model should be able to solve new tasks with different structures quickly by learning broadly suitable prior knowledge in training phase, rather than confined to the tasks only with a familiar structure. Such a fundamental limitation hinders the application of meta learning in many realistic scenarios such as few-shot classification with a varying number of categories, or the task of learning a robot agent in a non-stationary environment where the set of feasible actions continuously changes. However, to our best knowledge, such a problem of meta-learning for heterogeneous tasks has not been studied and solutions are still absent.

In this work, we propose a novel meta learning approach, termed Hierarchical Meta Learning (HML), to address the above problem and generalize meta learning to heterogeneous tasks. Inspired by the state-of-the-art meta learning approach [Finn et al., 2017a], HML follows a new hierarchical meta learning scheme, in which a meta model is trained to have maximal performance on homogeneous new tasks, and meanwhile the generalizability is optimized over the tasks of novel structures. To make the generalization performance explicitly optimizable, HML constructs factorized task distributions from available training tasks and trains the meta model on them hierarchically.

Specifically, HML synchronously factorizes the tasks and training regime at two levels. At the bottom level, following [Finn et al., 2017a], HML optimizes sensitiveness of the meta model $\theta$ to similar tasks for fast adaptation. At the top, HML takes as surrogate the performance of applying the model trained with one task distribution to the other different ones, and explicitly maximizes the generalizability across heterogeneous tasks. In this way, the obtained meta model learns broadly applicable meta knowledge and can fast learn heterogeneous tasks of novel structures directly, without re-
peating the expensive meta training. In addition, when applying to novel tasks with different structures, previous methods need to change the meta model architecture for compatibility. To alleviate such learning difficulty, HML adopts a learnable transformation for internal representation of the meta model to enable fast adaption to new architectures.

To sum up, we make following contributions:

- We are the first to propose the problem of meta learning for heterogeneous tasks. This problem features the critical limitation of existing meta learning approaches. Solutions to this problem would substantially extend application of meta learning.

- We propose a novel hierarchical meta learning approach to solve the problem. HML explicitly optimizes both the adaptability to homogeneous tasks and generalizability across heterogeneous tasks. Moreover, we introduce a meta transformation function to further enhance the adaptability of the meta model to model architecture variation.

- We evaluate the HML with few-shot classification and regression experiments. Compared with state-of-the-arts, our HML significantly improves generalization performance to new tasks of different structures.

2 Related Work

Meta Learning Recently, meta learning has drawn increasing attention, with which automatic learning schemes are devised to improve learning efficiency of existing learning methods or to learn (induce) the algorithms directly [Pfahringer et al., 2000; Lemke et al., 2015]. For example, [Finn et al., 2017a] targets at learning how to initialize a model such that it can adapt to different tasks quickly through simple gradient descent fine-tuning. [Vinyals et al., 2016; Snell et al., 2017] learn to match the query sample with the support ones based on metric learning in the embedded space. Analogously, [Rusu et al., 2018] tries to learn to initialize distribution parameters. [Zhou et al., 2018] uses deep meta learning to learn the conception matching of categories in the conception space. In our paper, we aim to obtain a meta model capable of generalizing across heterogeneous tasks.

Few Shot Learning The few shot learning (FSL) problem was first introduced by [Fei-Fei et al., 2006] and has received much attention since then. To address this problem, some works [Koch et al., 2015; Snell et al., 2017; Garcia and Bruna, 2018; Vinyals et al., 2016] develop models to learn discriminative data representations and data matching rules. For instance, [Koch et al., 2015] introduces a Siamese network to match the representation of support and query data which effectively circumvents the difficulty of learning a complex parametric classification model. [Vinyals et al., 2016] introduces the Matching Networks to learn to construct a support set to build a weighted nearest neighbor based classifier. As a follow-up, prototypical networks proposed by [Snell et al., 2017] learn to embed data into a metric space where classification can be performed by associating the query with the nearest prototype representations of each class. Different from establishing a simple inductive bias in previous works, prototypical networks specifically model assumptions about the class conditional data distribution in the embedding space. Furthermore, some recent works use the bidirectional LSTM [Graves and Schmidhuber, 2005] to embed the images and match the full context embedding by metric learning. But all of them only target their models at the general few-shot classification setting and cannot be applied to regression problems. This work instead aims at developing a principled method that is applicable for both classification and regression.

3 Hierarchical Meta Learning

3.1 Preliminaries

The goal of meta learning is to train a meta model that can quickly adapt to a new task using only a few samples and training iterations [Pfahringer et al., 2000; Lemke et al., 2015]. Formally, let \( f_0(\cdot) : X \rightarrow Y \) denote the meta model with parameter \( \theta \). Meta learning aims to optimize \( \theta \) over a set of training tasks \( T \sim p(T) \) with

\[
T \triangleq \{(x_1, y_1), \ldots, (x_n, y_n), (x_t, y_t), \ell\},
\]

such that the model \( f_0 \) is ready to solve new tasks \( T' = \{(x'_1, y'_1), \ldots, (x'_n, y'_n), \ell\} \sim p(T) \) by updating \( \theta \) w.r.t. \( \{(x_1, y_1), \ldots, (x_n, y_n)\} \) within a few steps. Here \( (x_j, y_j), j = 1, \ldots, n \) are task-specific training data, \( (x_t, y_t) \) are reserved for training task evaluation and \( \ell \) is the associated loss. An example of meta learning problems is the \( N \)-category few-shot classification, where each task is to fast learn a classifier from a small number of \( k \) annotated samples per category (e.g., \( k \leq 5 \)).

As a state-of-the-art method, MAML [Finn et al., 2017a] solves meta learning problems by optimizing the fast adaptability of the meta model \( f_0 \) such that it can solve a new task rapidly via a few gradient descent steps on new tasks. To this end, in the meta model training phase, given a set of training tasks \( T = \{T_1, \ldots, T_m\} \), MAML fine-tunes the meta model \( f_0 \) to a particular task \( T_i \) by gradient descent at first:

\[
\theta'_i \leftarrow \theta - \alpha \nabla \mathcal{L}_{T_i}(f_0)
\]

where \( \mathcal{L}_{T_i}(f_0) = \frac{1}{n} \sum_{j=1}^{n} \ell(f_0(x^{(i)}), y_j) \) is the task-related training loss and \( \alpha \) is a universal learning rate.

Then by treating each task as a training example, MAML optimizes \( \theta \) such that the following meta loss for the task-wise fine-tuned parameter \( \theta'_i \) over task-provided evaluation samples \( (x_i, y_i) \) can be minimized:

\[
\min_{\theta} \mathcal{L}_{MAML}(f_0) = \sum_{i=1}^{m} \ell(f_{\theta - \alpha \nabla_{\theta} \mathcal{L}_{T_i}(f_0)}(x_i^{(i)}), y_i^{(i)}).
\]

The meta parameter \( \theta \) is then updated by gradient descent \( \theta \leftarrow \theta - \beta \nabla_{\theta} \mathcal{L}_{MAML}. \) The trained meta model \( f_0 \) can be applied directly to a new similar meta learning tasks through gradient descent fine-tuning in Eqn. (2).

3.2 Problem Setting

Existing meta learning approaches require the training tasks \( T \) and new tasks \( T' \) to be from the same distribution \( p(T) \)
and share the same structure. Such restriction makes existing approaches, such as MAML, inapplicable to novel tasks of different structures.

To overcome such a limitation of meta learning, we introduce a more general problem where the meta model will be tested on tasks with different structures from the training ones. This problem is very common in practice. For example, one may wish to construct a meta model for $N'$-category few-shot classification but the training data are only sufficient for constructing several $N$-category classification tasks with $N' < N$; or one needs to build a navigation agent with only very few feasible actions in the training environments but faced with testing environments that are more complex and allow more actions. Solving these problems requires a novel meta learning scheme with stronger generalization guarantees—although the meta model only has access to tasks $T \sim p(T)$ for training but it can efficiently adapt to heterogeneous tasks $T' \sim p'(T)$ of different structures. Formally, with only similar tasks $T \sim p(T)$ available, we aim to obtain a meta model $\theta$ that can adapt to different tasks $T' \sim p'(T)$ efficiently through a few gradient-descent steps and minimize the generalization loss:

$$\min_{\theta} L_{\text{gen}} \triangleq \sum_{T' \sim p'(T)} \ell(f_{\theta - \alpha \nabla_{\theta} \nabla_{\theta'} (f_{\theta'}) (x'_t), y'_t).$$

This problem is quite challenging since the test tasks $T'$ are not available in advance. In the following, we develop a new hierarchical meta learning approach to address it.

### 3.3 Hierarchical Meta Learning

To address the above problem, we develop a novel hierarchical meta learning (HML) approach to learn a rule that is generalizable across heterogeneous tasks and meanwhile is suitable for similar tasks. HML achieves this by jointly optimizing the adaption performance to homogeneous tasks and the generalizability across heterogeneous tasks of the meta model, as depicted in the following learning objective:

$$\min_{\theta} \sum_{T \sim p(T)} \ell(f_{\theta - \alpha \nabla_{\theta} \nabla_{\theta'} (f_{\theta'}) (x_t), y_t) + \sum_{T' \sim p'(T)} L_{\theta \rightarrow T'}.$$  

Here, the second term is the generalization loss defined in Eqn. (3):

$$L_{\theta \rightarrow T'} = \sum_{T' \sim p'(T)} \ell(f_{\theta - \alpha \nabla_{\theta} \nabla_{\theta'} (f_{\theta'}) (x'_t), y'_t).$$

The challenge of optimizing the above objective lies in the fact that the tasks $T' \sim p'(T)$ are not available in the meta model training phase, making the generalization loss not directly optimizable. To alleviate this challenge, we propose to sample a set of training tasks, denoted as $T$, from the available distribution $T \sim p(T)$ and restructure these tasks into two-level hierarchies $\hat{T} = \{T_1, T_2, \ldots, T_H\}$ where $T_h = \{T_{h1}, \ldots, T_{hm}\}$ for $h = 1, \ldots, H$. Here the top-level tasks $T_1, \ldots, T_H$ have different structures and each of them consists of homogeneous tasks, i.e., $T_{h1}, \ldots, T_{hm}$ have the same structure. In particular, we choose to factorize the output space to construct the heterogeneous top-level tasks.

Recall each task $T \in \mathcal{T}$ is to learn a mapping function $f_0 : X \rightarrow Y$. We factorize $Y$ into $Y_1, \ldots, Y_H$ and require $Y_1 \subset Y_2 \subset \ldots \subset Y_H = Y$. Then the task $T_i$ corresponds to learning a mapping function $f_0 : X \rightarrow Y_i$, and $T_H = T_i$. Take the $N$-category few-shot learning problem as an example. Each task is to learn to solve an $N$-category classification problem. One thus can extract heterogeneous tasks of 2-category, 3-category to $N$-category few-shot learning from the available tasks to form the task hierarchy.

Instead of optimizing the meta model $f_0$ on tasks $T$ directly as conventional meta learning approaches, HML performs meta learning on the hierarchy of re-structured tasks that demonstrate task heterogeneity. The purpose of extracting and aggregating tasks in this way is to facilitate the hierarchical meta learning, which offers heterogeneous tasks systematically and makes optimization of the generalization meta objective feasible. The meta learning objective of HML becomes

$$\min_{\theta} L_{\text{HML}} \triangleq \sum_{T \in \mathcal{T}} \ell(f_{\theta - \alpha \nabla_{\theta} \nabla_{\theta'} (f_{\theta'}) (x_t), y_t) + \sum_{h=1}^{H-1} \sum_{h'=T_h} L_{\theta \rightarrow T_{h' + 1}}.$$  

The loss has two components. The first one requires the meta model $\theta$ to fast adapt to the training tasks quickly, which is similar to MAML. The second one requires the meta model trained on tasks $T_{h-1}$ to be able to quickly solve new tasks $T_h$ with a different structure. Through minimizing this hierarchical loss, the meta model is exposed to multiple meta learning tasks and more importantly, the cross-task learning scenarios. During meta training phase, HML is trying to get the optimal solution for every low-level task $T_h$. In this way, the low-level meta model learns the generalized solution on the task set. High-level meta learner asynchronously learns the global generalized solution over all the heterogeneous tasks. Thus, besides learning the rule for few-shot classification, the HML model also learns the rule of generalizing to heterogeneous tasks. The resulted meta model gains the ability to solve homogeneous and heterogeneous tasks.

For optimization, we first sample a batch of low-level tasks $\{T_i, i = 1, \ldots, m\}$. Then, we compute meta gradients $\nabla L_{\text{HML}, T_i}$ and update the meta model parameter as

$$\theta \leftarrow \theta - \beta \sum_{i=1}^{m} \nabla \theta \nabla L_{\text{HML}, T_i},$$

where $\beta$ is the meta learning rate.

**Meta Model Transformation** When applied to another task $T' \sim p'(T)$ with a different structure, the meta model $\theta$ from vanilla meta learning is not directly applicable, because $\theta$ cannot adapt to such unseen tasks. But with HML, the meta model parameter $\theta$ will converge to a more generalized position in the representation space which easily transfers to various task distribution and adapts to new tasks quickly.

HML gives the optimal meta model parameter $\theta$ that can adapt to different task distributions. When applying to a new task, the architecture of the output needs to change from the trained tasks due to the task structure difference. This brings
Secondly, the transformation function adapted to a specific task between the meta model transformation feature maps. \( \omega \) the transformation function that does not change the size of input directly (e.g., the output layer of a neural network model). We propose to add an intermediate transformation function \( \omega \) between the meta model \( \theta \) and the output function \( \varphi \) to transfer smoothly to different tasks.

Suppose we have the tasks \( T_1, \ldots, T_H \) constructed as above and let \( \varphi_h \) denote the output function compatible with tasks \( T \in T_h \). The transformation function \( \omega \) is trained to optimize the following objective. Firstly, the meta model \( \theta \) is adapted to a specific task \( T^h \) by

\[
\theta^h \leftarrow \theta - \nabla_\theta L_{T^h}(f_{\theta, \varphi^h}).
\]

(6)

Secondly, the transformation function \( \omega \) should map the meta model \( \theta \) to a different task \( T^{h+1} \) as follows:

\[
\theta^{h+1} \leftarrow \theta^h - \nabla_\theta L_{T^{h+1}} f(\omega(\theta^h), \varphi^{h+1}).
\]

(7)

Thus the loss for optimizing \( \omega \) is computed as

\[
L_\omega = \sum_h L_{T^h}(f_{\theta^h, \varphi^h}(x^{h+1}_t), y^{h+1}_t).
\]

(8)

We optimize \( \omega \) through gradient descent. To enable the transformation function with parameter \( \omega \) to fully learn the generalization rule, we just update the parameter of \( \omega \) while fixing other parameters \( \theta \) and \( \varphi^h \).

**Training Algorithms** HML optimizes the meta model \( \theta \) and transformation function \( \omega \) alternatively. The details are summarized in Algorithm 1.

### 4 Experiments

We first investigate whether the proposed HML can generalize well to new tasks with different structures from the training ones through experiments with few-shot classification on three benchmark datasets and few-shot regression. We also evaluate the effectiveness of the proposed hierarchical meta learning and meta model transformation. Finally, we present experiments to explain how the meta model from HML obtains the ability of generalization across heterogeneous tasks.

#### 4.1 Datasets

We use three datasets for few-shot classification evaluation. The Omniglot dataset [Lake et al., 2011] consists of 1,623 characters (categories) from 50 different alphabets and each character is drawn by 20 different subjects. Following [Lake et al., 2015], we form the training set with 30 alphabets and use the rest alphabets for testing. The second dataset is the miniImageNet [Vinyals et al., 2016] which consists of 60,000 color images of size 84×84 from 100 categories. Each category has 600 examples. In our experiment, we split the 100 categories into 64, 12, and 24 randomly. The first 64 categories are used for making training tasks, and the 12 categories are used for validation and 24 for testing. The third dataset is the SUN2012 [Xiao et al., 2010] containing 899 categories and 130,519 images in total. To construct few-shot learning tasks, we remove the categories whose example images are less than 6. We obtain 441 training classes and 65 testing classes which contain 11,226 images and 613 images respectively. We randomly select categories from the training set and test set to form the training and test learning tasks respectively.

#### 4.2 Evaluation Protocol

Under the conventional few-shot meta learning evaluation setting, the training tasks share the same structure as the test tasks. More specifically, a meta model is trained on a collection of \( N \)-way \( k \)-shot learning tasks from the training set and then evaluated on multiple \( N \)-way \( k \)-shot tasks sampled from the test set. Here \( N \) is fixed for training and test.

In this work, we aim to develop a meta model capable of fast solving new tasks of different structures. To evaluate its generalization performance, we construct the following evaluation protocol. During the meta model training phase, similar to the conventional setting, a collection of \( N \)-way \( k \)-shot learning tasks from the training set are provided. Across all the experiments, we set \( N = 5 \). During testing, we randomly sample \( N' \)-way \( k \)-shot tasks from the test set. Here the values of \( N' \) are varied and different from \( N \) of the training tasks. We are particularly interested in the model gener-
alizability from simpler classification tasks to more complex ones. Therefore, we set \( N' \geq N \) across all the evaluations. In particular, we adopt \( N' = 5, 10, 20 \) and 50 on the Omniglot dataset. On the miniImageNet and SUN datasets, we evaluate the models for \( N' = 5, 6, \ldots, 10 \). We report the classification averaged accuracy for each type of test tasks over 10 random train/test splits.

4.3 Baselines and Model Architecture

To our best knowledge, there is no solution ready for coping with heterogeneous tasks without requiring to repeat meta training. For evaluation and comparison, we construct following competitive baselines. The first is fine-tuning. The model is trained on the meta-training tasks and then fine-tuned on heterogeneous test tasks. The second one is MAML [Finn et al., 2017a] which has been shown to excel at adapting to new tasks swiftly across a variety of meta learning problems. We use MAML to obtain a meta model and fine-tune it on the test tasks by a few steps of gradient descent. We replicate MAML following source code\(^2\) and achieve the results reported in [Finn et al., 2017a]. The third is Meta-SGD [Li et al., 2017].

We use a deep CNN architecture as backbone to implement different meta learning approaches. The network consists of three convolution layers with pooling operations and one fully-connected layer. For HML, we use a convolution layer to implement the meta transformation function \( \omega \), with stride 1 and kernel size \( 1 \times 1 \). When applying to novel test tasks, we replace the trained classifier (i.e., the output layer) with new task-compatible classifiers.

4.4 Few-shot Classification

To verify the effectiveness of our HML on obtaining well-generalizable meta models across heterogeneous tasks, we first compare with strong baselines over the few-shot classification problem.

Results on Omniglot

The results on Omniglot are given in Table 1. As observed, meta learning methods show significant superiority to the fine-tuning, benefiting from the explicitly optimized adaptability to new tasks. However, MAML and Meta-SGD perform not well when applying to new tasks with different structures (i.e., different \( N' \)). In particular, the meta model trained on 1-shot 5-way tasks performs poorly on 20-way and 50-way tasks, manifesting such heterogeneity of task structures raises significant challenges to state-of-the-art meta learning approaches. In contrast, our HML performs much better than other meta models, even though all of the models are trained on 5-way tasks. For the 20-way and 50-way 1-shot learning tasks, our HML outperforms MAML by a large margin of 5.5% and 14.4% respectively. We have similar observations for the 5-shot experiments. HML also outperforms the state-of-the-art by 3.6% and 12.9% on 20-way and 50-way learning tasks. Based on the observation that the gap grows notably as the classes number increases, it can be seen that the meta model obtained from HML has a stronger generalizability, and also that our method is indeed effective at lifting the meta models to solve heterogeneous tasks.

Table 1: Few-shot classification accuracy (%) on Omniglot. The models are trained on 5-way tasks and evaluated on tasks ranging from 5-way to 50-way.

|                 | 1-shot | 5-way | 10-way | 20-way | 50-way |
|-----------------|--------|-------|--------|--------|--------|
| Fine-tune       | 62     | 57    | 29.5   | 9.2    |
| Meta-SGD        | 99.3   | 85    | 69.5   | 52.7   |
| MAML            | 98     | 90    | 79.5   | 56.8   |
| HML (ours)      | 98.5   | 85    | 71.2   |

Results on miniImageNet

Few-shot learning on miniImageNet is a more challenging task than that on Omniglot as images from miniImageNet present much more complex contents. We train the meta models on the 5-way tasks and evaluate them on 6, 7, 8, 9, 10-way tasks with greater evaluation granularity (only 5, 10-way reported due to the limited space). As shown in the top panel of Table 2, the performance of the fine-tuned model rapidly falls below 10% as task difference becomes larger. In contrast, MAML shows superiority as it has fast adaptability: MAML is 19.2% higher than the fine-tuned model when extending to 10-way task. Even so, MAML still suffers inferior performance due to its lack of generalization power across new task structures. MAML has a conspicuous decline from 43.6% to 22.2% and 62.9% to 37.5% on 1, 5-shot respectively. Its degradation on 5-shot (25.4%) is more severe than the fine-tuned model (14.8%). Therefore, we can say HML offers convincing performance enhancement w.r.t. MAML and the fine-tuned model.

Table 2: Few-shot classification accuracy (%) on MiniImageNet and SUN2012. The models are trained on 5-way tasks and evaluated on tasks ranging from 5-way to 10-way. 10-way 1-shot task is abbreviated as “10w1s”.

|                 | 5w1s  | 10w1s | 5w5s  | 10w5s |
|-----------------|-------|-------|-------|-------|
| Fine-tune       | 29.6  | 7.0   | 28.6  | 9.2   |
| MAML            | 49.3  | 22.2  | 62.9  | 37.5  |
| Meta-SGD        | 47.6  | 22.2  | 64.01 | 32.4  |
| HML (ours)      | 46.6  | 28.8  | 60.4  | 41.8  |

|                 | 5w1s  | 10w1s | 5w5s  | 10w5s |
|-----------------|-------|-------|-------|-------|
| Fine-tune       | 32    | 14.0  | 41.2  | 24.2  |
| MAML            | 49.9  | 32.5  | 64.6  | 39.8  |
| Meta-SGD        | 51.3  | 32.9  | 66.8  | 37.7  |
| HML (ours)      | 48.6  | 39.5  | 65.4  | 45.2  |

\(^2\)https://github.com/cbfinn/maml
Results on SUN2012
Similar to miniImageNet, we also set 5, 10-way tasks as the incremental classifier learning tasks. The results of $N'$-way 1, 5-shot tasks are given in Table 2. These results provide consistent evidence that HML is effective and provides a better solution to obtaining meta models that can generalize across heterogeneous tasks. Our new few-shot learning setting can serve as a standard testbed to evaluate the generalizability of meta models, which better aligns with realistic application scenarios.

4.5 Few-shot Regression
We further evaluate our proposed HML on multivariate linear regression tasks, to demonstrate the meta model obtained from HML is also efficient in learning new tasks for regression problems. Specifically, given 5 independent variables $X \in \mathbb{R}^{d_x}$ of dimensionality $d_x$ and corresponding output $Y \in \mathbb{R}^{d_y}$, the task is to fit the linear function $Y = w^\top X + b$ by learning the parameters $w, b$ from few $(X, Y)$ pairs. Here for different tasks, the dimensionality $d_y$ of $Y$ changes from 5 to 20. Due to the randomness of regression sampling, we adopt the error reduction rate defined as $r_{err} = e_k/e_0$, where $k \leq 5$ counts optimizing steps and $e_k$ denotes the error after $k$ steps, to measure the adaptability and performance. To eliminate the effect of random initialization, we report the average error reduction rate over 100 different tasks. The results are given in Table 3. As we can see, HML has a faster adaptation speed and better generalizability than MAML and fine-tuning baselines.

Table 3: Error reduction rate of multivariate linear regression tasks over different output dimensions, after 5 adaptation steps. Every task contains 5 context points $(X, Y)$ with $d_x = 10$ and randomly sampled coefficient $w, b$. The number in the bracket shows the 1 step adaptation performance.

| $d_y$ | 5   | 10  | 20  |
|-------|-----|-----|-----|
| Fine-tune | 92.7(99.7) | 92.21(99.8) | 101.5(99.8) |
| MAML   | 43.6(59.3) | 46.5(77.1) | 59.8(86.8) |
| HML (ours) | **18.9(29.3)** | **25.3(55.7)** | **42.4(75.6)** |

4.6 Performance Analysis on HML
To understand the effectiveness of HML intuitively, we conduct the following experiments on the Omniglot dataset. First, we visualize the data representation learned by the meta models from MAML and HML, via t-SNE [Maaten and Hinton, 2008] in Figure 1, for the 20-way 1-shot tasks. Here the model is trained on the 5-way 1-shot source tasks. Comparing with MAML, the representation learned by the meta model with HML presents higher intra-class compactness and inter-class separability. This implies the benefits of HML from explicitly optimizing the across-task generalizability. With hierarchical meta learning, the resulted meta model learns representations that are more broadly suitable for the tasks involving more categories to classify. As the representations of different categories are more separated, the model offers a stronger generalizability to other $N'$-way classification tasks with a different $N'$.

Second, we study the effects of the transformation function $\omega$ within HML. We compare the performance of the models from vanilla HML and from HML without the function $\omega$, denoted as HML and HML w/o Trans respectively. We plot their performance for 1-shot learning for different ways (ranging from 5 to 50) in Figure 2. We can observe that the accuracy of HML is consistently slightly higher than HML w/o Trans. Furthermore, the margin between them becomes more significant when the number of categories is larger than 5. Since the transformation function does not change the expressive capacity of the model internal representation, the performance improvement mainly comes from the transformation function $\omega$ by improving model adaptability to different task structures. Notably, even for HML w/o Trans, its generalization performance is much better than MAML. This demonstrates the meta model indeed benefits its generalization performance due to the hierarchical meta learning scheme of HML.

5 Conclusion
In this work, we present a new meta learning problem of making the meta models quickly solve new tasks of different structures from the training ones, and devise a new Hi-
erarchical Meta Learning (HML) to explicitly optimize the generalization performance of a model across heterogeneous tasks.

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