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Author
Gorsky, E

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THE EQUIVARIANT EULER CHARACTERISTIC OF MODULI SPACES OF CURVES.

EUGENE GORSKY

ABSTRACT. We derive a formula for the $S_n$-equivariant Euler characteristic of the moduli space $\mathcal{M}_{g,n}$ of genus $g$ curves with $n$ marked points.

1. INTRODUCTION

Consider the moduli space $\mathcal{M}_{g,n}$ of algebraic curves of genus $g$ with $n$ marked points. The symmetric group $S_n$ acts naturally on this space. Let $V_\lambda$ denote the irreducible representation of $S_n$ corresponding to a Young diagram $\lambda$, then one can decompose the cohomology of $\mathcal{M}_{g,n}$ into isotypic components:

$$H^i(\mathcal{M}_{g,n}) = \bigoplus_\lambda a_{i,\lambda} V_\lambda.$$

The $S_n$-equivariant Euler characteristic of $\mathcal{M}_{g,n}$ is defined by the formula

$$\chi_{S_n}(\mathcal{M}_{g,n}) = \sum_{i,\lambda} (-1)^i a_{i,\lambda} s_\lambda,$$

where $s_\lambda$ denotes the Schur polynomial labeled by the diagram $\lambda$. We calculate these equivariant Euler characteristics for all $g \geq 2$ and $n$.

Theorem 1.1. The generating function for the $S_n$-equivariant Euler characteristics of $\mathcal{M}_{g,n}$ has the form

$$\sum_{n=0}^{\infty} t^n \chi_{S_n}(\mathcal{M}_{g,n}) = \sum_k c_{k_1,\ldots,k_r} \prod_{j=1}^r (1 + p_j t^j)^{k_j},$$

where $p_j$ are power sums and the coefficients $c_{k_1,\ldots,k_r}$ are defined by the equation (6).

Consider the moduli space $\mathcal{M}_g(k_1,\ldots,k_r)$ of pairs $(C, \tau)$ where $C$ is a genus $g$ curve and $\tau$ is an automorphism of $C$ such that for all $i$ the Euler characteristic of the set of points in $C$ having the orbit of length $i$ under the action of $\tau$ equals $ik_i$. The coefficient $c_{k_1,\ldots,k_r}$ can be also defined as the orbifold Euler characteristic of $\mathcal{M}_g(k_1,\ldots,k_r)$.

This moduli space can be defined for any tuple of integers $(k_1,\ldots,k_r)$ of arbitrary size $r$, but we prove that (for a fixed genus $g$) it is non-empty only for a finite number of such tuples. In particular, $r$ cannot exceed $4g+2$.

Corollary 1.2. The generating function $\sum_{n=0}^{\infty} t^n \chi_{S_n}(\mathcal{M}_{g,n})$ is a rational function in $t$. Furthermore, for any $n$,

$$\chi_{S_n}(\mathcal{M}_{g,n}) \in \mathbb{Z}[p_1,\ldots,p_{4g+2}].$$

Theorem 1.1 can be compared with the computations of [4], [5], [8] and [10] in genus 2 and with the computations of [1], [2], [9], [17] and [18] in genus 3. A similar generating function for the moduli spaces of hyperelliptic curves was previously obtained in [11]. The non-equivariant Euler characteristics of moduli spaces of curves were computed by Bini and Harer in [3].

The paper is organized as follows. In Section 2 we consider a complex quasi-projective variety $X$ with an action of a finite group $G$. Theorem 2.5 provides a formula for the $S_n$-equivariant
Euler characteristic of quotients $F(X, n)/G$, where $F(X, n)$ is a configuration space of $n$ labeled distinct points on $X$. This theorem was previously proved in [10] using the results of Getzler [6, 7] concerning Adams operations over the equivariant motivic rings (see also [12]). The alternative proof presented here uses only the basic properties of Euler characteristic and seems to be more geometric. It also makes the proof of the main result self-contained.

In Section 3 we apply this theorem to the universal family over $\mathcal{M}_g$, the moduli space of genus $g$ curves. This allows us to prove in Theorem 3.3 that the coefficient $c_{k_1, \ldots, k_r}$ is equal to the orbifold Euler characteristic of $\mathcal{M}_g(k_1, \ldots, k_r)$. These Euler characteristics are then computed in Theorem 3.8 using the results of Harer and Zagier.

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2. **EQUIVARIANT EULER CHARACTERISTICS**

Let $X$ be a complex quasi-projective variety with an action of a finite group $G$. Let us denote by $F(X, n)$ the configuration space of ordered $n$-tuples of distinct points on $X$. For each $n$, the action of the group $G$ on $X$ can be naturally extended to the action of $G$ on $F(X, n)$, commuting with the natural action of $S_n$.

In the computations below we will use the additivity and multiplicativity of the Euler characteristic, as well as the Fubini formula for the integration with respect to the Euler characteristic ([15, 19], see also [16]).

**Lemma 2.1.** The following equation holds: $\sum_{n=0}^{\infty} \frac{t^n}{n!} \chi(F(X, n)) = (1 + t)^{\chi(X)}$.

**Proof.** The map $\pi_n : F(X, n) \to F(X, n - 1)$, which forgets the last point in the $n$-tuple, has fibers isomorphic to $X$ without $n - 1$ points. Therefore $\chi(F(X, n)) = (\chi(X) - n + 1) \cdot \chi(F(X, n - 1))$, and $\chi(F(X, n)) = \chi(X) \cdot (\chi(X) - 1) \cdot \cdots \cdot (\chi(X) - n + 1)$. □

Let $p_k$ denote the $k$th power sum and let $V_\lambda$ denote the irreducible representation of $S_n$ labelled by the Young diagram $\lambda$. We define the $S_n$-equivariant Euler characteristic of $F(X, n)/G$ by the equation

$$\chi^{S_n}(F(X, n)/G) = \sum_{\lambda} (-1)^{\ell(\lambda)} a_{i, \lambda} s_\lambda,$$

where $H^i(F(X, n)/G) = \bigoplus_\lambda a_{i, \lambda} V_\lambda$ and $s_\lambda$ is the Schur polynomial.

**Lemma 2.2.** The following equation holds:

$$\chi^{S_n}(F(X, n)/G) = \frac{1}{n!} \sum_{\sigma \in S_n} p_1^{k_1(\sigma)} \cdots p_n^{k_n(\sigma)} \cdot \chi([F(X, n)/G]^\sigma),$$

where $k_i(\sigma)$ is the number of cycles of length $i$ in a permutation $\sigma$.

**Proof.** It is well known that for every $i$

$$\sum_\lambda a_{i, \lambda} s_\lambda = \frac{1}{n!} \sum_{\sigma \in S_n} p_1^{k_1(\sigma)} \cdots p_n^{k_n(\sigma)} \cdot \text{Tr}(\sigma)|_{H^i(F(X, n)/G)},$$

hence

$$\chi^{S_n}(F(X, n)/G) = \frac{1}{n!} \sum_i (-1)^i \sum_{\sigma \in S_n} p_1^{k_1(\sigma)} \cdots p_n^{k_n(\sigma)} \cdot \text{Tr}(\sigma)|_{H^i(F(X, n)/G)}$$

Now the statement follows from the Lefschetz fixed point theorem. □
Lemma 2.3. Let \( \sigma \in S_n \). Then
\[
\chi \left( [F(X, n)/G]^\sigma \right) = \frac{1}{|G|} \sum_{g \in G} \chi \left( F(X, n)^{g^{-1}\sigma} \right). 
\]

Proof. For a point \( y \in F(X, n) \) whose projection on \( F(X, n)/G \) is \( \sigma \)-invariant there exists an element \( g \in G \) such that \( \sigma y = g y \). Consider the set of pairs
\[
S = \{(g, y) | g \in G, y \in F(X, n) | \sigma y = g y \}
\]
and its two-step projection \( S \to F(X, n) \to F(X, n)/G \). The fiber of the first projection over a point \( y \) is isomorphic to \( G \)-stabiliser of \( y \) or empty, the fiber of the second projection containing \( y \) is exactly the orbit of \( y \). Therefore the cardinality of every fiber of the composition is equal to \( |G| \). \( \square \)

Definition 2.4. For any \( g \in G \) we denote by \( X_k(g) \) the subset of \( X \) consisting of points with \( g \)-orbits of length \( k \). For example, \( X_1(g) \) is a set of \( g \)-fixed points. Let \( \tilde{X}_k(g) = X_k(g)/g \), where \( (g) \) is a cyclic subgroup in \( G \) generated by \( g \).

The following theorem was deduced in [10] from the results of Getzler [6, 7], here we would like to present a more geometric and straightforward proof.

Theorem 2.5. The generating function for the \( S_n \)-equivariant Euler characteristics of the quotients \( F(X, n)/G \) is given by the following equation:

\[
\sum_{n=0}^{\infty} t^n \chi_{S_n}(F(X, n)/G) = \frac{1}{|G|} \sum_{g \in G} \prod_{k=1}^{\infty} (1 + p_k t^k)^{\chi(X_k(g))}. 
\]

Proof. Since all points in \( X_k(g) \) have \( g \)-orbit of length \( k \), we have \( \chi(\tilde{X}_k(g)) = \chi(X_k(g))/k \). From Lemma 2.1 one gets:
\[
(1 + p_j t^j) \chi(\tilde{X}_j(g)) = \sum_{k_j=0}^{p_j} \frac{p_j^{k_j} t^{k_j j}}{(k_j)!} \chi \left( F \left( \tilde{X}_j(g), k_j \right) \right). 
\]
Therefore the coefficient at \( t^n \) in the right hand side of (1) equals to:
\[
\frac{1}{|G|} \sum_{g \in G} \sum_{j k_j = n} \prod_{j} \frac{p_j^{k_j}}{k_j!} \chi \left( F \left( \tilde{X}_j(g), k_j \right) \right). 
\]
On the other hand, by Lemma 2.2 and Lemma 2.3, the left hand side of (1) can be rewritten as following:
\[
\frac{1}{|G|} \sum_{g \in G} \frac{1}{n!} \sum_{\sigma \in S_n} \prod_{j=1}^{n} p_j^{k_j(\sigma)} \cdot \chi([F(X, n)]^{g^{-1}\sigma}). 
\]
If for a tuple \( y \in F(X, n) \) we have \( \sigma(y) = g(y) \), the action of \( (g) \) at this tuple has \( k_j(\sigma) \) cycles of length \( j \). Every cycle of length \( j \) corresponds to a point in \( \tilde{X}_j(g) \), hence for every \( g \) we can define a map
\[
\alpha_g : \sqcup_{\sigma \in S_n} [F(X, n)]^{g^{-1}\sigma} \to \prod_j F(\tilde{X}_j(g), k_j)/S_{k_j}. 
\]
Given a \( g \)-invariant \( n \)-tuple of distinct points in \( X \), there are \( n! \) ways to label them and make an ordered tuple \( y \). Every such ordering defines a unique permutation \( \sigma \) such that \( \sigma(y) = g(y) \), therefore all fibers of \( \alpha_g \) have cardinality \( n! \) and
\[
\frac{1}{n!} \sum_{\sigma \in S_n} \chi([F(X, n)]^{g^{-1}\sigma}) = \prod_j \chi \left( F(\tilde{X}_j(g), k_j)/S_{k_j} \right) = \prod_j \frac{\chi \left( F(\tilde{X}_j(g), k_j) \right)}{k_j!}. 
\]
3. Moduli spaces of curves

Let us apply Theorem 2.5 to the study of moduli spaces of curves. Let \( \mathcal{M}_g \) denote the moduli space of genus \( g \) algebraic curves and let \( \mathcal{M}_{g,n} \) denote the moduli space of genus \( g \) algebraic curves with \( n \) parked points (we will always assume \( g \geq 2 \)). Let \( \mathcal{M}_g(k_1, \ldots, k_r) \) be the moduli space of pairs \((C, \tau)\) where \( C \) is a genus \( g \) curve and \( \tau \) is an automorphism of \( C \) such that \( \chi(C(\tau)) = ik_i \) for all \( i \). Since \( g \geq 2 \), every automorphism of \( C \) has finite order, hence one can choose \( r \) such that \( k_r \neq 0 \) and \( k_i = 0 \) for \( i > r \).

There is a natural forgetful map \( \pi_{g,k} : \mathcal{M}_g(k_1, \ldots, k_r) \to \mathcal{M}_g \) sending \((C, \tau)\) to \( C \). For a curve \( C \) we define \( \operatorname{Aut}_g(C) = \pi_{g,k}^{-1}(C) \subset \operatorname{Aut}(C) \).

**Proposition 3.1.** Suppose that \( \mathcal{M}_g(k_1, \ldots, k_r) \) is not empty. Then \( k_r < 0, k_i = 0 \) for \( i \mid r \) and \( k_i \geq 0 \) for \( i < r \). Moreover, we have the following bounds on \( r \) and \( k_i \):

\[
r \leq 4g + 2, \quad |k_r| \leq 2g, \quad \sum_{i=1}^{r-1} k_i \leq 2g + 2.
\]

**Proof.** Let \( \tau \) be an automorphism of a genus \( g \) curve \( C \) such that \( \chi(C(\tau)) = ik_i \) for all \( i \). Note that \( C_i(\tau) \) are finite sets for \( i < r \) and

\[
\chi(C) = 2 - 2g = \sum_{i=1}^{r-1} ik_i - r|k_r|.
\]

The quotient \( C_1 = C/\tau \) is a smooth curve of some genus \( h \), and the Riemann-Hurwitz formula yields its Euler characteristic:

\[
\chi(C_1) = 2 - 2h = \sum_{i=1}^{r-1} k_i - |k_r|.
\]

The projection of \( C \) to \( C_1 \) is a ramified covering of order \( r \) with \( s = \sum_{j=1}^{r-1} k_j \) ramification points. The automorphism \( \tau \) has order \( r \), so \( i \mid r \), if \( k_i \neq 0 \). By a theorem of Wiman ([20], see also [14]), the maximal order for an automorphism of a genus \( g \) curve equals \( 4g + 2 \), hence \( r \leq 4g + 2 \).

Since proper divisors of \( r \) cannot exceed \( r/2 \), equation (3) implies:

\[
\sum_{i=1}^{r-1} ik_i \leq \frac{r}{2} \sum_{i=1}^{r-1} k_i = \frac{r}{2}(2 - 2h + |k_r|),
\]

hence by (2):

\[
2g - 2 = r|k_r| - \sum_{i=1}^{r-1} ik_i \geq \frac{r}{2}(2h + |k_r| - 2).
\]

Therefore \( |k_r| - 2 \leq 2g - 2 \) and \( |k_i| \leq 2g \). Finally, \( \sum_{i=1}^{r-1} k_i = |k_r| + 2 - 2h \leq 2g + 2 \).

**Remark 3.2.** The bounds on \( r \) and on \( k_i \) are sharp. Indeed, consider a hyperelliptic curve \( P \) covering \( \mathbb{CP}^1 \) with ramifications at the vertices of a regular \((2g + 1)\)-gon and at its center. The covering can be chosen such that the automorphism of \( P \) induced by the rotation of this polygon acts nontrivially in the fibers and hence has order \( r = 2(2g + 1) = 4g + 2 \).

On the other hand, consider a hyperelliptic curve \( C \) with involution \( \tau \). We have

\[
\chi(C_1(\tau)) = 2g + 2, \quad \chi(C_2(\tau)) = 2 - 2g - (2g + 2) = -4g,
\]

hence a pair \((C, \tau)\) belongs to the moduli space \( \mathcal{M}_g(2g + 2, -2g) \).
### Theorem 3.3

The following equation holds:

\[
\sum_{n=0}^{\infty} t^n \chi_{S_n}(\mathcal{M}_{g,n}) = \sum_{k} \chi_{\text{orb}}(\mathcal{M}_g(k_1, \ldots, k_r)) \cdot \prod_{j=1}^{r} (1 + p_j t^j)^{k_j}.
\]

**Proof.** Consider the forgetful map \( \pi_{g,n} : \mathcal{M}_{g,n} \to \mathcal{M}_g \). Its fiber over a point representing a curve \( C \) is isomorphic to \( F(C, n) / \text{Aut}(C) \), hence one can apply Theorem 2.5 to compute its equivariant Euler characteristic:

\[
\frac{1}{|\text{Aut}(C)|} \sum_{\tau \in \text{Aut}(C)} \prod_{i} (1 + p_i t^i)^{\chi_{\tau}(C)} = \sum_{k} \frac{1}{|\text{Aut}(C)|} \sum_{\tau \in \text{Aut}(C)} \prod_{i} (1 + p_i t^i)^{k_i}.
\]

Therefore:

\[
\sum_{n=0}^{\infty} t^n \chi_{S_n}(\mathcal{M}_{g,n}) = \int_{\mathcal{M}_g} \sum_{n=0}^{\infty} t^n \chi_{S_n}(\pi_{g,n}^{-1}(C)) d\chi = \sum_{k} \prod_{i} (1 + p_i t^i)^{k_i} \int_{\mathcal{M}_g} \frac{|\text{Aut}_{k}(C)|}{|\text{Aut}(C)|} d\chi.
\]

On the other hand,

\[
\chi_{\text{orb}}(\mathcal{M}_g(k_1, \ldots, k_r)) = \int_{\mathcal{M}_g} \frac{|\pi_{g,k}^{-1}(C)|}{|\text{Aut}(C)|} d\chi = \int_{\mathcal{M}_g} \frac{|\text{Aut}_{k}(C)|}{|\text{Aut}(C)|} d\chi \quad \square
\]

Using the Proposition 3.1, we conclude that the sum in the right hand side of (5) is finite.

### Corollary 3.4

The generating function \( \sum_{n=0}^{\infty} t^n \chi_{S_n}(\mathcal{M}_{g,n}) \) is a rational function in \( t \). Furthermore, for any \( n \),

\[
\chi_{S_n}(\mathcal{M}_{g,n}) \in \mathbb{Z}[p_1, \ldots, p_{4g+2}].
\]

The orbifold Euler characteristic of \( \mathcal{M}_g(k_1, \ldots, k_r) \) can be computed using the combinatorial results of Harer and Zagier [13]. We will denote the greatest common divisor of integers \( a \) and \( b \) by \( (a, b) \). Let \( \varphi(n) \) and \( \mu(n) \) denote the Euler function and the Möbius function respectively. Define

\[
c(k, l, d) := \mu \left( \frac{d}{(d, l)} \right) \frac{\varphi(k/l)}{\varphi(d/(d, l))}.
\]

### Definition 3.5

Let \( \lambda = (\lambda_1, \ldots, \lambda_s) \) be a partition. We define a number

\[
N(r; \lambda) = |\{(x_1, \ldots, x_s) \in (\mathbb{Z}/r\mathbb{Z})^s : x_1 + \ldots + x_s \equiv 0 \pmod{r}, (x_i, k) = \lambda_i\}|.
\]

### Lemma 3.6

([13]) The following equation holds:

\[
N(r; \lambda) = \frac{1}{r} \sum_{d|r} \varphi(d) \prod_{i=1}^{s} c(k, \lambda_i, d).
\]

### Theorem 3.7

([13]) The orbifold Euler characteristic of the moduli space \( \mathcal{M}_{h,s} \) of genus \( h \) curves with \( s \) marked points is given by the formula:

\[
\chi_{\text{orb}}(\mathcal{M}_{h,s}) = (-1)^s \frac{(2h-1)B_{2h}}{(2h)!} (2h + s - 3)!
\]

where \( B_k \) denote Bernoulli numbers.
Theorem 3.8. The generating function for the $S_n$-equivariant Euler characteristics of $\mathcal{M}_{g,n}$ has the form
\[
\sum_{n=0}^{\infty} t^n \chi^S_{n}(\mathcal{M}_{g,n}) = \sum_{k} c_{k_1,\ldots,k_r} \prod_{j=1}^{r} (1 + p_j t_j)^{k_j},
\]
where $p_j$ are power sums and the coefficients $c_{k_1,\ldots,k_r}$ are defined by the equation:
\[
(6) \quad c_{k_1,\ldots,k_r} = \chi^{orb}(\mathcal{M}_{h,s}) \prod_{p|\gamma} (1 - p^{-2h}) \cdot \frac{N(r;\lambda)}{r \prod_{i=1}^{k_i} 1!}.
\]

Here $h = \frac{1}{2}(1 - \sum_{j=1}^{r} k_j)$, $s = \sum_{j=1}^{r-1} k_j$, $\gamma = \text{GCD}(i : k_i > 0)$, $\lambda = (k_1^2 k_2 \ldots (r - 1) k_{r-1})$.

Proof. By Theorem 3.3 one has $c_{k_1,\ldots,k_r} = \chi^{orb}(\mathcal{M}_{g}(k_1,\ldots,k_r))$. Consider the moduli space $\mathcal{M}_{g}(k_1,\ldots,k_r)$ of pairs $(C,\tau)$. As in Proposition 3.1 to such a pair one can associate a genus $h$ curve $C_1 = C/\tau$. The projection from $C$ to $C_1$ is ramified in $s$ points subdivided into groups of size $k_1,\ldots,k_{n-1}$. The orbifold Euler characteristic of the moduli space of genus $h$ curves with such markings equals $\chi^{orb}(\mathcal{M}_{h,s}) / \prod_{i=1}^{r} 1!$.

The non-equivariant Euler characteristic  $\chi(\mathcal{M}_{g,n})$ of pairs $(C,\tau)$ associated to a curve $C_1$ with fixed marked points was computed in [13] pages 478–479 and equals
\[
\frac{1}{r} r^{2h} \prod_{p|\gamma} (1 - p^{-2h}) \cdot N(r;\lambda).
\]

This completes the proof. \qed

The non-equivariant Euler characteristic of $\mathcal{M}_{g,n}$ has been computed in [13, Theorem 4.3]. It can be compared with Theorem 3.8 since
\[
\chi(\mathcal{M}_{g,n}) = n! \cdot \chi^{S_n}(\mathcal{M}_{g,n})|_{p_1 = 1, p_k = 0 \text{ for } k > 1}.
\]

Example 3.9. The generating function for the $S_n$-equivariant Euler characteristics of the moduli spaces of genus 2 curves with marked points has a form [10]:
\[
\sum_{n=0}^{\infty} t^n \chi^S_{n}(\mathcal{M}_{2,n}) = \frac{1}{240} (1 + p_1 t)^{-2} - \frac{1}{240} (1 + p_1 t)^6 (1 + p_2 t^2)^{-4} + \frac{2}{5} (1 + p_1 t)^3 (1 + p_5 t^5)^{-1} + \frac{2}{5} (1 + p_1 t)(1 + p_2 t^2)(1 + p_5 t^5)(1 + p_1 t)^{-1} + \frac{1}{6} (1 + p_1 t)^2 (1 + p_2 t^2)(1 + p_6 t^6)^{-1} - \frac{1}{12} (1 + p_1 t)^4 (1 + p_3 t^3)^{-2} - \frac{1}{12} (1 + p_2 t^2)^2 (1 + p_3 t^3)^2 (1 + p_6 t^6)^{-2} + \frac{1}{12} (1 + p_1 t)^2 (1 + p_2 t^2)^2 (1 + p_4 t^4)^{-2}.
\]

These coefficients can be matched with the ones defined in Theorem 3.8.

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