The $\gamma$-ray afterglows of tidal disruption events

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ABSTRACT
A star wandering too close to a supermassive black hole (SMBH) will be tidally disrupted. Previous studies of such ‘tidal disruption event’ (TDE) mostly focus on the stellar debris that are bound to the system, because they give rise to luminous flares. On the other hand, half of the stellar debris in principle are unbound and can stream to a great distance, but so far there is no clear evidence that this ‘unbound debris stream’ (UDS) exists. Motivated by the fact that the circum-nuclear region around SMBHs is usually filled with dense molecular clouds (MCs), here we investigate the observational signatures resulting from the collision between an UDS and an MC, which is likely to happen hundreds of years after a TDE. We focus on $\gamma$-ray emission ($0.1$–$10^5$ GeV), which comes from the encounter of shock-accelerated cosmic rays with background protons and, more importantly, is not subject to extinction. We show that because of the high proton density inside an MC, the peak $\gamma$-ray luminosity, about $10^{39}$ erg s$^{-1}$, is at least 100 times greater than that in the case without an MC (only with a smooth interstellar medium). The luminosity decays on a time-scale of decades, depending on the distance of the MC, and about a dozen of these ‘TDE afterglows’ could be detected within a distance of about 16 Mpc by the future Cherenkov Telescope Array. Without careful discrimination, these sources potentially could contaminate the searches for starburst galaxies, galactic nuclei containing millisecond pulsars or dark matter annihilation signals.

Key words: acceleration of particles – cosmic rays – galaxies: active – Local Group – gamma rays: galaxies.

1 INTRODUCTION
A star wandering too close to a supermassive black hole (SMBH, with mass $M_\bullet \gtrsim 10^6 M_\odot$) would be tidally disrupted, an incident known as the ‘tidal disruption event’ (TDE; Hills 1975; Rees 1988). The disruptive process starts when the tidal force exerted by the SMBH becomes comparable to the self-gravity of the star. Given the mass $M_\star$ and radius $R_\star$ of the star, this happens at a critical distance of $R_c \simeq R_\star (M_\bullet/M_\star)^{1/3}$ from the SMBH (Hills 1975). Since this ‘tidal radius’ is tiny – it is merely $23 (M_\star/10^6 M_\odot)^{-2/3}$ times greater than the Schwarzchild radius of the SMBH – the TDE rate in a galaxy is usually low, about $\mathcal{O}(10^{-6})$ yr$^{-1}$ according to theoretical calculations (e.g. Magorrian & Tremaine 1999; Wang, Watarai & Mineshige 2004).

A fraction of the star after tidal disruption has a negative energy and remains gravitationally bound to the SMBH. Mutual collisions and the later accretion (by SMBH) of these bound stellar debris are expected to give rise to an electromagnetic outburst with a thermal spectrum, which peaks at UV and soft X-ray bands (Rees 1988). This ‘tidal flare’ provides an effective means to reveal an otherwise dormant SMBH. So far, tens of tidal disruption events (TDEs) have been discovered in this way (see Komossa 2015, for a review).

In principle, about half of the debris released from the disrupted star would gain a positive energy and escape from the system with a velocity asymptotically approaching $10^3$–$10^5$ km s$^{-1}$ (Rees 1988). Simulations showed that these unbound material will develop into an elongated ‘unbound debris stream’ (UDS; Kochanek 1994; Guillochon et al. 2009, 2015; Coughlin & Nixon 2015). Two observational signatures directly associated with the UDS have been envisaged in the literature: (i) A brightening of Hydrogen lines several days after a TDE caused by the recombination of the initially ionized plasma of the UDS (Kasen & Ramirez-Ruiz 2010), and (ii) variability of optical emission lines on a time-scale of months to years due to the illumination of the UDS by the central tidal flare (Strubbe & Quataert 2009, 2011). While broad lines have been detected from TDEs (Gezari et al. 2012), the detected lines likely originate from the circularizing disc of debris (Guillochon, Manukian & Ramirez-Ruiz 2014) rather than the aforementioned UDS signatures.

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As a result, one important element in the current TDE model – that at least half of the stellar mass becomes unbound – remains untested. A possible cause of the non-detection is attenuation of optical/UV photons, by either the interstellar medium (ISM) close to supermassive black holes (SMBHs; Donley et al. 2002; Gezari et al. 2009) or a gaseous envelope shrouding the radiative debris (Loeb & Ulmer 1997; Ulmer 1999; Strubbe & Quataert 2009; Metzger & Stone 2015; Miller et al. 2015). For this reason, it is important to look for signatures of unbound debris stream (UDSs) in an electromagnetic waveband that is less subject to extinction, such as radio, infrared, hard X-ray or γ-ray bands.

Such a signature potentially could be generated by the collision of an UDS with the surrounding ISM. It has been realized since about two decades ago (Khokhlov & Melia 1996) that the UDS–ISM interaction will create a structure similar to a supernova remnant (SNR). Recently, we started investigating the radiation from this ‘unbound debris remnant’ (UDR; Guillochon et al. 2015). We found that if it exists in an environment like the Galactic Centre (GC), the spectrum would peak in UV and soft X-ray, where extinction is still an issue. However, if a significant fraction of the collisional energy ends up in accelerating electrons, radio emission will be produced due to synchrotron radiation and the luminosity could amount to $O(10^{35})$ erg s$^{-1}$ (assuming a TDE rate of $10^{-4}$ yr$^{-1}$). Given this luminosity, it is possible to detect $O(1)$ such remnant in the GC, but not in other galaxies because the object would be too faint (Guillochon et al. 2015).

In our earlier studies of the UDR, we had not considered radiative mechanisms of hard photons, such as hard X-ray and γ-ray. It is worth mentioning that one way of generating hard X-rays/γ-rays is by forming a jet, which can up-scatter low-energy photons to higher energies by inverse Compton process. This mechanism was introduced to explain the transient hard X-ray (15–50 keV) emission detected by Swift in three TDEs (Bloom et al. 2011; Burrows et al. 2011; Levan et al. 2011; Zauderer et al. 2011; Cento et al. 2012; Brown et al. 2015). However, jets are not directly related to UDSs, therefore, jetted TDEs cannot be used for our purpose of looking for UDSs, not to mention the fact that the hard-X-ray/γ-ray radiation from a jet is highly beamed so that the probability of seeing it is small.

Recently, an alternative way of producing γ-ray emission was proposed, originally to explain the possible excess of GeV γ-rays in the GC (Cheng, Chernyshov & Dogiel 2006, 2007; Cheng et al. 2011, 2012). This type of γ-ray emission is closely linked to the UDS–ISM collision. The collision basically generates a large amount of relativistic protons, known as cosmic rays (CRs), through a process called diffusive shock acceleration (Hinton & Hofmann 2009; Treumann 2009). The γ-ray emission is produced when the CRs escape from the shock region and start bombarding the non-relativistic protons in the ISM, leading to the formation of neutral pions ($\pi^0$) which almost immediately decay into γ-rays ($\gtrsim 10$ MeV; Kafexhiu et al. 2014).

According to the calculations done for the GC (Cheng et al. 2006, 2007), the γ-ray luminosity would sustain at a level of about $10^{38}$ erg s$^{-1}$ if the TDE rate is $10^{-3}$ yr$^{-1}$. Given this luminosity, a γ-ray telescope whose sensitivity is typically $10^{-13}$ erg cm$^{-2}$ s$^{-1}$ at present (Funk et al. 2013) is able to detect the above source out to a distance of about 3 Mpc.

It is a good time to revisit this latter mechanism of γ-ray emission because of two instances of recent progress. (i) Excess of γ-ray (0.1–100 GeV) emission has been clearly detected in the GC (The Fermi-LAT Collaboration 2016), and possibly in two nearby dwarf galaxies (reported by Geringer-Sameth et al. 2015; Hooper & Linden 2015; Li et al. 2016, but see Drlica-Wagner et al. 2015 for a different result), by the Fermi Large Area Telescope (Fermi-LAT) on-board the Fermi γ-ray satellite. It is important to understand the contribution to these γ-ray emission by TDEs. (ii) Both theoretical and observational studies (see Section 2 for details) suggest that around SMBHs, the ISM is likely very clumpy, filled with long-lived molecular clouds (MCs). If UDSs collide with MCs, an enhancement of γ-ray emission is anticipated because the high proton density inside MC accelerates the production and the subsequent decay rates of CRs.

Motivated by the latest progress, we will analyse in this paper the γ-ray emission from the UDS–MC collision and investigate the prospect of detecting them in other galaxies. The paper is organized as follows. In Section 2, we briefly review the evidences supporting the ubiquity of MCs around SMBHs, and then we summarize the typical properties of such MCs. Then in Section 3, we describe the characteristics of a typical UDS, so that in Section 4 we can use them to evaluate the time-scales and the energetics related to the UDS–MC collision. Based on these results, we discuss in Section 5 the properties of the CRs produced by the collision. In Section 6, we calculate the luminosity and spectral energy distribution of the γ-ray emission resulting from the cooling of the above CRs inside the MCs of our interest. Using these results, we evaluate in Section 7 the number of point sources that can be detected by γ-ray telescopes. Three telescopes are considered here, namely, Fermi-LAT (Atwood et al. 2009), the High Energy Stereoscopic System (H.E.S.S.; Aharonian et al. 2006) and the Cherenkov Telescope Array (CTA) currently under planning (Acharya et al. 2013). For completeness, in Section 8, we also calculate the γ-ray emission resulting from the interaction of UDSs with a smooth ISM, from which we infer the additional number of detectable sources. Finally, in Section 9, we discuss the possibility of detecting similar γ-ray sources in dwarf galaxies, due to the existence of intermediate massive black holes (IMBHs), and we also outline possible methods of separating our objects from other γ-ray sources.

2 MOLECULAR CLOUDS

There are at least three pieces of evidence suggesting that the ISM around SMBHs is very clumpy.

(i) It is now widely accepted that SMBHs grew mainly during the phase of active galactic nucleus (AGN). In the unified model of type-I/type-II AGNs, the most essential ingredient is a torus-like structure of several parsecs in size composed of dusty gas clumps surrounding the central SMBH (Krolik & Begelman 1988; Antonucci 1993). Given the ubiquity of this structure in AGNs, it is natural to suspect that many SMBHs still have relic tori around them even though they are no longer active today.

(ii) Another evidence comes from a special type of galaxies only recently discovered (Komossa et al. 2008, 2009; Wang et al. 2012; Yang et al. 2013). These galaxies show peculiar iron and oxygen emission lines, which are atypical of a normal AGN but more consistent with a model in which an ionizing spectrum similar to that of a TDE is reflected by dense MCs (e.g. Wang et al. 2012). Besides, the line strength decays on a time-scale of several years, indicating that the reflectors reside within several parsecs from the central SMBHs (e.g. Komossa et al. 2008).

(iii) The most direct evidence is from the GC. Radio observations revealed tens of MCs at a distance of 0.5–2 pc from Sgr A*, the SMBH in the GC (Mezger, Duschl & Zylka 1996). The densities of
these MCs lie in a wide range of \( n_H \sim (10^6-10^8) \text{ cm}^{-3} \) (Christopher et al. 2005), similar to what has been inferred for the MCs in those galaxies with peculiar line ratios (Wang et al. 2012). The spatial distribution and kinematics of the MCs in the GC are suggestive of a torus-like structure with a vertical thickness of about 0.5 pc at the inner edge (Christopher et al. 2005). Given these observed properties, the torus has a half-opening angle of about 27° and covers in total \( f_c \sim 40 \) per cent of the sky when viewed from the central SMBH. It has been pointed out that such a geometry resembles that of a normal AGN torus (Mezger et al. 1996; Ponti et al. 2013).

Based on these evidences, it seems likely that after a TDE, the UDS would collide with one of the MCs around the central SMBH. This collision differs in two ways from the interaction with a smooth ISM. (i) Because of the much higher proton density inside MC, the UDS loses its kinetic energy more rapidly (Guillochon et al. 2015), which implies a higher production rate of CRs. (ii) The CRs, after escaping from the shock region, are still inside a very dense environment (the MC), a condition favourable to \( \pi^0 \) formation. The second difference is also the main reason that those MCs close to supernova remnants (SNRs) often show an enhanced \( \gamma \)-ray emission (Aharonian & Atoyan 1996; Fatuzzo, Adams & Melia 2006; Gabici et al. 2015; H. E. S. Collaboration et al. 2015).

It is important to note that the MCs close to SMBHs are not the same as those seen in the Galactic plane. The former are orders of magnitude denser than the latter, and the reason is as follows. Given a typical distance of \( D \sim 1 \) pc between our MC and an SMBH, the Roche limit, which is a criterion for the MC to remain gravitationally bound, requires that

\[
n_H \gtrsim M_\star/(m_p D^3) \simeq 4.1 \times 10^7 m_p D_6^{-3} \text{ cm}^{-3},
\]

where \( m_p \) is the proton mass, \( m_0 = M_\star/(10^6 M_\odot) \) and \( D_1 = D/(1 \text{ pc}) \). Therefore, when \( D \sim 1 \) pc and \( M_\star \gtrsim 10^6 M_\odot \), we have \( n_H \gtrsim 4 \times 10^7 \text{ cm}^{-3} \). This density is orders of magnitude higher than most MCs in the Galactic plane \( n_H \sim (10^2-10^3) \text{ cm}^{-3} \), and in fact, even denser than the cores of normal MCs \( n_H \sim 10^3 \text{ cm}^{-3} \); Bodenheimer 2011).

Despite such a high density, the MCs around SMBHs do not collapse to form stars (Jeans instability), probably because they are not self-gravitating but confined mainly by an external agent, such as the surrounding hot and turbulent ISM, or the compressive tides of the central SMBHs (e.g. Chandrasekhar 1963; Shu 1992). In equilibrium with an external force, an MC could have a density significantly lower than what is required by the Roche limit (Chen, Amaro-Seoane & Cuadra 2016). For this reason, we chose \( n_H \sim 10^5 \text{ cm}^{-3} \) as our fiducial value. This value agrees within a factor of 2–3 with the mean density \( 4 \times 10^5 \text{ cm}^{-3} \) of those MCs in the GC, inferred from the brightness of molecular emission lines (optical-thin density from Christopher et al. 2005).

To complete our description of an MC, we take \( R_c = 0.25 \) pc as the fiducial radius, which is typical of the MCs in the GC as well (Christopher et al. 2005). Given this size and assuming that our MC is spherical and homogeneous, we can derive a mass of \( M_c = 4 \pi m_p n_H R_c^3/3 \simeq 1.6 \times 10^7 n_3 M_\odot \) for the cloud, where \( n_3 = n_H/(10^3 \text{ cm}^{-3}) \).

### 3 Unbound Debris Streams

The evolution of an UDS in vacuum constitutes an important part of the initial conditions of our problem. Since a series of earlier works have studied this topic (Rees 1988; Kochanek 1994; Khokhlov & Melia 1996; Strubbe & Quataert 2009; Kasen & Ramirez-Ruiz 2010), we only summarize here the main results that are most relevant to our work, including the kinetic energy, internal velocity dispersion and the opening angle with respect to the central SMBH.

An UDS originates from the part of a star which at the time of tidal disruption has a positive energy. This unbound part contains about half of the mass of the star if the star initially approaches the SMBH along a parabolic orbit with a pericentre distance \( R_p \) comparable to \( R_c \) – an improbable case in real galaxies (Rees 1988; Magorrian & Tremaine 1999). In this case, the unbound material distribute in a range of specific energy between \( E_0 = 0 \) and \( E_\gamma = G M_\star R_c / R_c^2 \) (Guillochon & Ramirez-Ruiz 2013; Stone, Sari & Loeb 2013). As a result, the total kinetic energy \( E_k \) of the UDS is

\[
E_k = \int_0^{E_0} \frac{dM}{dE} dE \simeq \frac{M_\star E_0}{4} \sim 10^{50} \text{ erg} \left( \frac{m_0}{m_\odot} \right)^{1/3} \left( \frac{M_\odot}{M_\star} \right)^{5/3} \left( \frac{R_c}{10^2 \text{ pc}} \right)^{-1},
\]

where \( m_\odot = M_\odot/M_\odot \), \( r_\odot = R_c/R_\odot \). Here we have assumed an even distribution of unbound material in specific energy, i.e. \( dM/dE = M_\star/(2E_0) \), which was proposed by Rees (1988) and later confirmed by smooth-particle hydrodynamics simulations (Evans & Kochanek 1989).

Although we made a couple simplifications to derive equation (2), the result agrees well with the median value computed from a statistically representative ensemble of UDSs (Guillochon et al. 2015). Using this equation and the fact that \( r_\odot \propto m_0^{6/5} \) for main-sequence stars at \( m_\star > 1 \) (Demircan & Kahraman 2009; Gorda & Svednikov 1998), we further derive \( E_k \propto m_0^{3/2} \). This scaling relation indicates that more massive stars generally give rise to more energetic UDSs.

It is worth noting that equation (2) does not include the additional gain of kinetic energy during the compression and bouncing of the star at the orbital periapsis (Brassard & Luminet 2008, and references therein). We neglected it because the bouncing energy \( E_\beta \), about \( \beta^2 GM_\star / R_c \) per unit mass (Guillochon et al. 2009), is usually much smaller than \( E_0 \), where \( \beta = R_c/R_\odot \) is the so-called penetration factor.

Take tidal disruption of a solar-type star \( (M_\star = M_\odot \text{ and } R_c = R_\odot) \) for example. When \( M_\star = 4.3 \times 10^4 M_\odot \), which is the mass of the SMBH in the Milky Way (MW, Genzel, Eisenhauer & Gillessen 2010), the penetration factor \( \beta \) cannot be greater than 4.3; otherwise, the star will enter a radius smaller than the ‘innermost bound circular orbit’ (twice the Schwarzschild radius for a non-rotating black hole) and eventually be swallowed as a whole by the SMBH. Because of such a limit to \( \beta \), the ratio \( E_\beta/E_0 \simeq \beta^2 (M_\star / M_\odot)^{1/3} \) is small, in fact in the range (0.0063, 0.12) for the SMBHs in the mass range of \( 10^5 < M_\star / M_\odot < 10^6 \).

We notice that our \( E_0 \) differs by a factor of \( \beta^2 \) from that derived by Khokhlov & Melia (1996), Strubbe & Quataert (2009) and Kasen & Ramirez-Ruiz (2010). This is because those authors calculated \( E_0 \) using the formula \( GM_\star R_c / R_c^2 \), i.e. they assumed that the energy spread is determined at \( R_c \) instead of at \( R_\odot \). However, recent studies of the tidal-disruption process suggest that using \( R_c \) is physically more appropriate (Guillochon & Ramirez-Ruiz 2013; Stone et al. 2013). Therefore, using \( R_c \) in the calculation will result in an over-estimation of \( E_0 \) by a factor of \( \beta^2 \), which is non-negligible when \( \beta > 1 \).

Besides \( E_0 \), another important quantity of an UDS is its velocity. It determines the time of collision with an MC as well as the Mach number of the subsequent shock. It is important to realize that different parts of an UDS travel at different velocities because of the energy difference imprinted in different parts of the star at the time of tidal disruption. The fastest part has the highest specific energy, which, as we already know, is \( E_\gamma \). As a result, it has the highest
velocity. When it has travelled to a distance of $D \gg R_\ast(M_\ast/M_\odot)^{1/3}$, its velocity has asymptotically dropped to

$$v_t \simeq \sqrt{2GM_\odot} \simeq 6.2 \times 10^3 \text{ km s}^{-1} m_\odot^{-1/3} m_\ast^{-1/3} r_\odot^{-1/2}.$$  \hfill (3)

Suppose the time $t = 0$ coincides with the time of stellar disruption, then the time at which the fastest part of the UDS has traversed a distance of $D$ is

$$t_0 \simeq D/v_t \simeq 1.6 \times 10^5 \text{ yr} m_\odot^{-1/3} m_\ast^{-1/3} r_\odot^{-1/2} D_1.$$  \hfill (4)

For our interest $D \sim 1 \text{ pc} \gg R_\ast(M_\ast/M_\odot)^{1/3}$. As for the part with a lower specific energy, i.e. $E \ll E_\odot$, it follows that the velocity asymptotically approaches $\sqrt{2E}$ and it arrives at $D$ later, around the time $t \simeq D/\sqrt{2E}$.

Not only the velocities are different, the directions where different parts of an UDS are headed also differ (e.g. Kochanek 1994). In the equatorial plane – the plane defined by the initial orbit of the progenitor star – the difference is caused mainly by the energy spread $E_\odot$. The corresponding angular span relative to the central SMBH can amount to $2v_t/v_\odot$ when the self-gravity of UDS is negligible (Kokhlov & Melia 1996), where $v_\odot \simeq \sqrt{2GM_\odot}/R_\odot$ is the mean stellar velocity during the pericentre passage. In the direction perpendicular to the equatorial plane, the angular span is caused by the bouncing velocity $v_\perp \simeq \sqrt{2GM_\odot}/R_\odot$ and is about $2v_\perp/v_\odot$ (Kasen & Ramirez-Ruiz 2010). From these angles, we derive a typical solid angle of $\Omega \simeq \pi v_\odot v_\perp/v_\odot^2 = \pi [M_\ast/(2M_\odot)]^{1/3}$ for UDSs.

According to the last formula, if a solar-type star is tidally disrupted by an SMBH with a mass of $M_\odot = (10^8, 10^9) M_\odot$, we find that $\Omega = (2.2 \times 10^{-3}, 2.2 \times 10^{-4})$. These values are consistent with those from the earlier studies of UDSs, which showed that $\Omega$ generally lies in the range of $(10^{-4}, 10^{-2})$. It is known that the uncertainty is due partly to the different ways of calculating $\Omega_0$ (e.g. Kokhlov & Melia 1996; Strubbe & Quataert 2009; Kasen & Ramirez-Ruiz 2010 used $R_\odot$ to calculate $E_\odot$), and partly to the additional consideration of the effects of self-gravity (Kochanek 1994; Coughlin & Nixon 2015).

To account for these theoretical uncertainties, we parametrize $\Omega$ with $\Omega = 10^{-3}\Omega_\odot, 1$ as our fiducial value. In any case, we find that $\Omega$ is much smaller than the typical solid angle, $\pi R_\odot^2/D_\odot^2 \simeq 0.20 D_1^{-2}$, of those MCs considered in the previous section. This result indicates that an UDS interacts with only one MC at a time.

### 4 Collision Between UDS and MC

So far, we have specified the initial conditions of MCs and UDSs. Now we can proceed to study their collisions. Such a collision happens mostly likely at a distance of $O(1) \text{ pc}$ from an SMBH, where MCs exist in large amount (Section 2). An UDS arriving at this distance is in a free-expansion phase, because the amount of ISM that has been swept up by the UDS is too small to affect the kinematics. For example, if the ISM surrounding the SMBH has a constant density of $10^7 \text{ cm}^{-3}$ (as has been assumed by Khokhlov & Melia 1996; Cheng et al. 2006), the swept-up mass would be only $0.08 D_1^2 \Omega_\odot \sim 10^5 M_\odot$. As a result, the UDS behaves in the same way as in vacuum until the time of collision, at about $t \simeq t_0$.

Immediately after the collision, the UDS still continue its free expansion. But as it advances deeper into the dense MC, it soon sweeps up an amount of material that is as massive as the stream itself. When this happens, the free-expansion phase ends and a Sedov-like expansion follows. This phase transition occurs at a depth of

$$\Delta D \sim \frac{M_\odot}{n H m_\odot \Omega D^2} \simeq 0.0020 \text{ pc} m_\odot n_\odot^{-1} \Omega_\odot^{-1} D_1^{-2} \cdot \hfill (5)$$

from the surface of the MC, where $M_\odot = 0.5 M_\odot$ is the mass of the UDS. The fact that $\Delta D \ll R_\odot$ suggests that the MC is thick enough to stop the UDS completely. On the other hand, a normal MC like those in the Galactic plane cannot stop an UDS, because the density, typically of $10^{-3} \text{ cm}^{-3}$ (Bodenheimer 2011), would be too low.

Since the collisional velocity initially is about $v_t \sim v_\odot$, which is orders of magnitude higher than the typical (turbulent) sound speed $c_S \sim 20 \text{ km s}^{-1}$ in our MC (Genzel et al. 2010), a strong shock will be produced. The shock efficiently dissipates the kinetic energy of the UDS, converting it into heat. To estimate the heating rate, we first recall that mass is injected into the collisional region at a rate of

$$\frac{dM}{dt} = \frac{dM}{de} \frac{de}{dt} \simeq \frac{M_\odot}{t_0} \left( \frac{t}{t_0} \right)^{-3}.$$  \hfill (6)

where $t \geq t_0$. The steep dependence on $t$ implies that UDS is head-heavy: from the arrival time of the fastest debris at $t = t_0$ to the time $1.4 t_0$, already half of the mass of the UDS (50 per cent of $M_\odot$) has been deposited into the MC, and in the following period of 0.6$t_0$, much less matter (25 per cent of $M_\odot$) arrives. Knowing the mass-injection rate $dM/dt$, we can calculate the energy-injection/heating rate with

$$\frac{dE_k}{dt} = \frac{dM}{dt} \frac{t_0}{t}^5.$$  \hfill (7)

The even steeper dependence on $t$ indicates that kinetic energy is more concentrated at the head of the UDS than is mass: From $t = t_0$ to about 1.2$t_0$, already 50 per cent of the total kinetic energy has been injected, and by $t = 2t_0$ the injection of kinetic energy is about 94 per cent complete.

The material immediately after the shock front has a temperature of about $T_{\text{sh}} \simeq 3 n H c_S^2/(16 k_0)$ (Inoue et al. 2012; Pan, Patnaude & Loeb 2013, $k_0$ is the Boltzmann constant), which is of order $10^9 \text{ K}$ for our system (also see Guillot et al. 2015). Such a hot medium would emit X-rays, but it would be difficult to see directly this emission, because the MC of our interest has a column density of $n H R_\odot \sim 6 \times 10^{22} n_\odot \text{ cm}^{-2}$, i.e. it is Compton thick. Consequently, the seed X-ray photons are likely absorbed inside the MC, and what could be seen are mostly the reprocessed, thermalized low-energy photons emerging from the shocked region (hotspot) at the surface of the cloud.

To estimate the energies of the reprocessed photons, we note that the heating rate at the hotspot is about $E_k/t_0 \sim 2 \times 10^{40} \text{ erg s}^{-1}$ at the beginning of the collision, and the radiative cooling rate is $\sigma_\odot T^4 \Omega_\odot D^2$, where $\sigma_\odot$ is the Stefan–Boltzmann constant and $T$ the effective temperature. The heating and cooling rates are comparable when $T = 420 \text{ K}$. The corresponding blackbody spectrum peaks at about 7 $\mu$m in mid-infrared.

Because of this ability of radiative cooling, our MC would not expand significantly even though the total energy injected by the UDS, $E_k$, greatly exceeds the gravitational binding energy of the cloud, which is about $E_k \sim GM_\odot^2/R_\odot \sim 2.4 \times 10^{49} n_\odot^2 \text{ erg}$. On the other hand, in the (unlikely) case of insufficient cooling, the time-scale for our MC to expand is about $R_\odot/c_S \sim 2.4 \times 10^4 \text{ yr}$, much longer than the collisional time-scale $t_0$. Therefore, we can in any case neglect the dynamical evolution of the MC during
the collision, which significantly simplifies our later analysis of the CR and γ-ray productions.

5 Cosmic Rays

It is well known that a strong shock like what we have just described in the previous section produces CRs. The mechanism, known as the diffusive shock acceleration, is well established due to the studies of SNRs (Hinton & Hofmann 2009). These earlier studies suggested that typically ε = 10 per cent of the kinetic energy injected into the shock region can be tapped to accelerate CRs, and those CRs escaping from the shock region follow a power-law distribution in the momentum space with a power-law index of γ0 ≃ 2 (the ‘universal power law’, see review by Treumann 2009).

Applying these earlier results, we find that a total amount of εE0 ∼ 10^59 erg of CRs would be produced by the UDS–MC collision, and in the space of kinetic energy Eπ, which is a more convenient frame for the later calculation of π^0 and γ-ray production, the CRs initially follow a spectrum of

\[ dN_\pi/dE_\pi \propto (E_\pi + 1)^{-(\nu + 1)/2} \]  

(also see Cheng et al. 2006, for the CR spectrum).

If γ0 ≤ 2, the CR spectrum must cut-off at a maximum, \( T_{p,\text{max}} \), to avoid divergence of the total kinetic energy. This maximum energy in principle depends on the power of the shock, the duration of the particle-acceleration process and the strength of the magnetic field in the shock region. In practice, we will calculate \( T_{p,\text{max}} \) using the formula \( 10^5 \nu^2 \beta B_{\text{MC}}/E_\pi \text{TeV} \) (Hinton & Hofmann 2009), where \( \nu = v_\gamma/(10^3 \text{km s}^{-1}) \) characterizes the shock strength, \( t_0 = t_0/(10^3 \text{yr}) \) is the shock-acceleration time-scale (\( t_0 \)) in unit of 100 yr, and \( B_{\text{MC}} \) is the magnetic field strength in unit of mG. This formula is derived under the assumption of Bohn diffusion, which is the slowest diffusion process for particles to cross shock front. Correspondingly, it determines the lower limit of \( T_{p,\text{max}} \).

We now quantify the typical value of \( T_{p,\text{max}} \) in our model. Since the shock velocity initially is about \( v_\gamma \) and decays with time as (\( t/t_0 \))^−1 (Section 3), it follows that \( v_\gamma = (6000, 3800) \text{ km s}^{-1} \) when \( t = t_0 = (6, 100) \text{ yr} \), where we have assumed \( m_0 = m_\pi = r_\pi = D_\pi = 1 \). For \( B_{\text{MC}} \), observations of the GC region showed that the magnetic field varies from 2 to 4 mG around the edge of the MCs (Yusef-Zadeh et al. 1996), and is about 0.2 mG in the field between those MCs (Crocker et al. 2005). Moreover, the empirical scaling relation between the magnetic field strength and the density of MC (Crutcher 1999) suggests that \( B_{\text{MC}} \sim \Omega(1) \) when \( n_{\text{H}} \sim \Omega(10^5) \text{ cm}^{-3} \). For these reasons we assume \( B_{\text{MC}} = 1 \) in our model, and finally we find that \( T_{p,\text{max}} \sim (0.2, 1.4) \text{ PeV} \) when \( t = t_0 = (6, 100) \text{ yr} \). It is interesting to note that the \( T_{p,\text{max}} \) results from the interaction between UDSs and a smooth ISM also lies in the PeV range (Cheng et al. 2012). Therefore, UDSs are effective accelerators of PeV CRs.

The CRs escaping from the shock region will bombard the non-relativistic protons in the MC. A significant fraction of the subsequent proton–proton (pp) collisions are inelastic, so that the CRs gradually lose their kinetic energies and cool down. The pp-collision time-scale can be calculated with \( (\sigma_{\text{pp}}n_\pi c)^{-1} \), where \( \sigma_{\text{pp}} \sim 40 \text{ mbar} \) is the total collisional cross-section and \( c \) denotes the speed of light. Taking into account the inelasticity of the collision, usually parametrized by \( k \) which has a value of about 0.45 (Fatuzzo et al. 2006), we find a cooling time-scale of

\[ \tau_{\text{pp}} = (k \sigma_{\text{pp}}n_\pi c)^{-1} \sim 5.9 \text{ yr} n_\gamma^{-1}. \]  

During this period of \( \tau_{\text{pp}} \), a CR would have diffused in the MC by a length of \( d \sim (D\tau_{\text{pp}})^{1/2} \), where \( D \) is the diffusion coefficient. Inside an MC where the density of neutral gas is high, the magnetohydrodynamics waves which conventionally determine the diffusion of CRs would be damped by neutral-ion friction, and the diffusion is more likely due to magnetic fluctuations excited by turbulent gas (Dogiel et al. 2015). The corresponding diffusion coefficient is proportional to \( n_{\text{H}}^{-1/2} \) and is about \( D = 3 \times 10^{27}n_{\text{H}}^{-1/2} \text{ cm}^2 \text{ s}^{-1} \), which is obtained by scaling the diffusion coefficient \( 3 \times 10^{27} \text{ cm}^2 \text{ s}^{-1} \) derived by Dogiel et al. (2015) for Sgr B2, which is an MC near the GC with a density of \( 10^7 \text{ cm}^{-3} \) and a magnetic field of 0.55 mG.

It follows that

\[ d \sim \sqrt{D\tau_{\text{pp}}} \sim 0.08 \text{ pe} n_\gamma^{-3/4}. \]  

(10)

Now it is clear that \( d < R_s \). This means that a CR would be trapped inside our MC and completely lose its kinetic energy. In this case, about 10 per cent of the kinetic energy of the CR will be trapped to produce \( \pi^0 \) (Crocker et al. 2005).

So far we have neglected the energy loss of CRs due to collisional ionization. This type of energy loss is relatively unimportant because the corresponding time-scale, \( \tau_{\text{ion}} \sim 20 n_{\text{H}}/(T_\gamma/1 \text{ GeV}) \text{ yr} \) (Berezinskii et al. 1990), is longer than \( \tau_{\text{pp}} \), if we consider only the energy range relevant to \( \pi^0 \) production which is \( T_\gamma \gtrsim 3 \text{ GeV} \).

6 γ Rays

6.1 Luminosity

Having characterized the CRs, we can now investigate the production of \( \pi^0 \) by pp collisions and calculate the subsequent γ-ray luminosity \( L_\gamma \). We start from the formula (Fatuzzo et al. 2006)

\[ L_\gamma = \eta(\sigma_{\text{pp}}n_\pi c)E_{\text{CR}}(t), \]  

(11)

where \( E_{\text{CR}}(t) \) is the total energy of CRs inside an MC and \( (\sigma_{\text{pp}}n_\pi c) \) is the collisional rate between a CR and the background protons. The factor \( \eta \) denotes the efficiency of \( \pi^0 \) production, and when \( \gamma_0 \) increases from 2 to 2.6, \( \eta \) decreases from 0.18 to 0.04 (Crocker et al. 2005).

In equation (11), we deliberately wrote \( E_{\text{CR}} \) as a function of \( t \), so as to draw attention to its time dependence, which we now elaborate. From the time \( t = t_0 \) when the UDS first hits the MC, to about \( t_0 + \tau_{pp} \), the newly produced CRs do not yet have time to cool down, so \( E_{\text{CR}}(t) \) increases monotonically. For this reason, we derive \( E_{\text{CR}}(t) \) by integrating \( dE_{\text{CR}}/dt \), where \( dE_{\text{CR}}/dt \propto (t/t_0)^{-3} \) is the energy injection rate coming from equation (7). The result is that \( E_{\text{CR}}(t) = \epsilon\bar{E}_t(\gamma_0), \) where the function \( \gamma_0(1 - t/t_0)^{-4} \) comes from an integration of \( (t/t_0)^{-3} \). Therefore, we know that the luminosity evolves as

\[ L_\gamma = \eta(\epsilon/\nu)\bar{E}_tC(t)/\tau_{\text{pp}} \]  

(when \( t_0 < t < t_0 + \tau_{\text{pp}} \)).

Afterwards (\( t > t_0 + \tau_{\text{pp}} \)) cooling becomes relevant. Consider now a time interval \( (t, t + \tau_{\text{pp}}) \) in this later stage. During this interval, the CRs produced earlier than \( t \) would have cooled down completely and meanwhile a fresh amount of \( d(\epsilon dE_{\text{CR}}/dt) \) of CRs are created. This consideration suggests that \( E_{\text{CR}}(t) \sim \epsilon dE_{\text{CR}}/dt \) at any moment of \( t > t_0 + \tau_{\text{pp}}. \) Now using equation (11), we find that the corresponding luminosity is \( L_\gamma \sim (\epsilon/\nu) \) \( d\bar{E}_t/dt \propto (t/t_0)^{-5} \).

Fig. 1 illustrates the evolution of \( L_\gamma \) as we have just described. We can see that (i) the γ-ray emission appears hundreds of years (depending on \( t_0 \)) after the initial TDE, and (ii) the luminosity evolves on a time-scale of several decades (also depend on \( t_0 \)). This behaviour is very different from that of a tidal flare, which already appears days after the moment of stellar disruption and lasts at most a couple of years (e.g. Rees 1988; Komossa 2015). To highlight


\[ L_\gamma \propto 1 - \left( \frac{t}{t_0} \right)^{-\frac{1}{2}} \]

\[ L_\gamma \propto \left( \frac{t}{t_0} \right)^{-5} \]

this difference, in the following, we will refer to the \( \gamma \)-ray signature resulting from the UDS–MC collision as the ‘afterglow’ of TDE.

Fig. 1 also shows that for a large range of \( n_{\text{MC}} \), the peak luminosity of the TDE afterglow is more or less constant, about \( 10^{39} \) erg s\(^{-1}\). This result can be understood as a compromise between a higher \( \pi^0 \) production rate and a shorter CR cooling time when \( n_{\text{MC}} \) increases. A more stringent proof of this insensitivity to \( n_{\text{MC}} \) can be performed by evaluating \( L_\gamma \) at the moment \( t = t_0 + t_{\text{pp}} \). Noticing that \( r_{\text{pp}} \ll t_0 \) when \( n_{\text{MC}} > 4 \times 10^{20} \text{ cm}^{-3} \), we can re-write \( C(t_0 + t_{\text{pp}}) \) as \( 4r_{\text{pp}}/t_0 \), accurate to first order, and consequently equation (11) reduces to

\[ L_{\text{peak}} = (\epsilon \eta/\kappa)E_{\chi}/t_0 \]

\[ \simeq 1.6 \times 10^{39} \text{ erg s}^{-1} \epsilon_{-1}\eta_{-1}m_{\chi}^{1/2}m_{\nu}^{1/2}r_{\text{MC}}^{3/2}D_{1}^{-1}, \]

where we have normalized \( \epsilon \) and \( \eta \) using their typical values such that \( \epsilon_{-1} = \epsilon/0.1 \) and \( \eta_{-1} = \eta/0.1 \). It is now clear that \( L_{\text{peak}} \) does not depend on \( n_{\text{MC}} \).

Although insensitive to \( n_{\text{MC}} \), \( L_{\text{peak}} \) does depend on different model parameters. To see more clearly the dependence, we adopt the relation \( r_{\text{MC}} \propto m_{\chi}^{3/4} \) for main-sequence stars with \( m_{\chi} > 1 \) (see references in Section 3) and derive \( t_0 \propto m_{\nu}^{-1/6}m_{\chi}^{-1/2}m_{e}^{1/6}D_{1}^{1} \) and \( L_\gamma \propto m_{\nu}^{3/2}m_{e}^{1/4}D_{1}^{-1} \). These scaling relations indicate that a closer MC, a more massive SMBH or tidal disruption of a more massive star generally leads to a brighter afterglow, but the time delay between the initial tidal flare and the later afterglow depends only on the location of the MC.

An additional, albeit small, contribution to the total \( \gamma \)-ray luminosity is from the bremsstrahlung radiation of relativistic electrons and positrons. These leptons are expected to be produced by the decay of charged pions (\( \pi^\pm \)) – for every \( \pi^0 \) created by the pp-collision, one \( \pi^+ \) and one \( \pi^- \) will also be produced. The total energy of these secondary leptons is about 3 per cent of the initial CR energy and the distribution peaks at an energy of about \( E_e \sim 60 \text{ MeV} \) (Fatuzzo et al. 2006).

Since the radiative time-scale is about \( 4t_0 \) yr due to bremsstrahlung radiation and about \( 15B_{\text{MC}}^{-2}E_{\text{MC}}^{-1} \) yr due to synchrotron radiation (Crocker et al. 2007), where \( E_{\gamma} = E_\gamma/(1 \text{ TeV}) \), we expect bremsstrahlung to be the dominant radiative mechanism for the secondary leptons with \( E_e \lesssim 1 \text{ TeV} \). The resulting \( \gamma \)-ray spectrum peaks at about \( 0(10) \text{ MeV} \) and the luminosity is \( 0.03\epsilon(dE_\gamma/dt) \), about \( \eta/0.03\epsilon \simeq 7.4 \text{ times smaller than the } \gamma \text{-ray luminosity due to } \pi^0 \text{ decay.} \)

Synchrotron radiation dominates only when \( E_e > 3 \text{ TeV} \). Using the relation \( E_{\gamma} \simeq 41B_{\text{MC}}E_{\text{MC}}\epsilon_{\gamma} \text{ eV} \) (Crocker et al. 2007) between the energies of synchrotron photons (\( E_{\gamma} \)) and their parent leptons (\( E_{\gamma} \)), we find that the synchrotron radiation produces mainly soft X-ray photons, which are likely absorbed by the dense molecular gas.

\[ (n_{\chi}, m_{\chi}, \gamma_{\chi}) = (1, 1, 2) \]

\[ m_{\chi} = 10 \]

\[ \gamma_{\chi} = 2.4 \]

\[ m_{\chi} = 10 \]

\[ \gamma_{\chi} = 10 \]

\[ H.E.S.S. \ (100 \text{ hrs}) \]

\[ CTA \ (10^5 \text{ hrs}) \]

\[ \text{Fermi (10 yr)} \]

\[ \text{without MC} \]

\[ E_\gamma \text{ (GeV)} \]

\[ L_\gamma \text{ (erg cm}^{-2} \text{ s}^{-1}) \]

\[ E_\gamma^2 \Phi_\gamma \text{ (eV cm}^{-2} \text{ s}^{-1}) \]

\[ \text{CTA (10^5 hrs)} \]

\[ \text{Fermi-LAT, H.E.S.S. and CTA} \]
about 3 Mpc. A more thorough study of the effect of distance will be presented in the next section.

Fig. 2 shows that when $\gamma_0 = 2$, the $\gamma$-ray spectrum is relatively flat. This flatness reflects the fact that $\gamma$ is almost constant in a large range of energy band. This constancy also causes the steepening of the $\gamma$-ray spectrum when $\gamma_0$ increases to 2.4. In the following, we will focus on the case $\gamma_0 = 2$ (the universal power law), motivated by the fact that the shock in our system has an extremely large Mach number (Section 4).

7 DETECTABILITY

To understand whether a TDE afterglow is detectable, we need first to know our instruments. Therefore, we consider here three $\gamma$-ray telescopes, namely, Fermi-LAT, H.E.S.S. and CTA, as representatives of the technology at present and in the near future.

We say that a telescope can ‘resolve’ a TDE afterglow if part of the $\gamma$-ray spectrum is above the sensitivity curve of the telescope. This afterglow mostly likely will look like a point source if it is extragalactic given a typical angular resolution of 1 arcmin for a $\gamma$-ray telescope (e.g. see Funk et al. 2013, for CTA). Otherwise, if the entire spectrum is below the lowest point of the sensitivity curve, the object is ‘unresolvable’, and will only contribute to the diffuse extragalactic $\gamma$-ray background (Ackermann et al. 2015) in the field of view of the telescope.

The sensitivity curves of the three telescopes are shown in Fig. 2. For H.E.S.S. and CTA, the sensitivity curves are adopted from Funk et al. (2013, 5$\sigma$ detection), and for Fermi-LAT from its performance webpage. It is clear that CTA, due to its planned superior sensitivity, can resolve all the representative TDE afterglows except the one with $\gamma_0 = 2.4$; it is best suited to search for the TDE afterglows.

From now on, we will quantify the number $N_\gamma$ of TDE afterglows resolvable by each of the three telescopes. At the end of this section, it will become clear that $N_\gamma$ is small for Fermi-LAT and H.E.S.S., but is large enough for CTA to make a detection likely. This is the reason that so far neither Fermi-LAT nor H.E.S.S. has make clear detection of the TDE afterglows.

We will restrict the following analysis to a single population of TDE afterglows with the same SMBH masses ($m_b$), stars ($m_s$) and MCs ($D_1$). This simplification enables us to derive basic results that can be generalized in the future to account for multiple populations of afterglows. The generalization, however, requires knowledge that are currently unavailable, such as the space density of low-mass SMBHs ($M_\ast \sim 10^3 M_\odot$; Stone & Metzger 2016) and the spatial distribution of MCs in other galaxies. For this reason, we have to postpone it to a future work. In this paper, we will circumvent these uncertainties by assigning a typical value to each of the parameters of ($m_b$, $m_s$, $D_1$), based on the current understanding of TDEs and MCs.

Given the above restriction, we derive $N_\gamma$ in three steps. (i) Estimate the maximum distance $r_\gamma(t)$ within which a source can be resolved by a given telescope. This maximum distance is a function of $t$ (same definition as before and we consider only $t > t_0$) because the $\gamma$-ray luminosity decays with time as $L_\gamma(t) \approx L_{\text{peak}}(t/t_0)^{-5}$. Since we are mostly interested in the case of $n_{\text{IF}} > 10^9 \text{cm}^{-3}$, we find that the rising phase (with a duration of $\tau_{\text{IF}}$) before the luminosity peak is relatively short ($\tau_{\text{IF}} \ll t_0$ and also see Fig. 1), so we have neglected its contribution to $N_\gamma$. (ii) Use $r_\gamma(t)$ to derive a detectable volume $V_\gamma(t) = 4\pi r_\gamma^3(t)/3$, and estimate the number $\Delta N_\gamma(t)$ of TDE afterglows that are inside this volume and are in the same time interval of $(t, t + \Delta t)$. (iii) Sum up the numbers $\Delta N_\gamma(t)$ in all intervals at $t > t_0$ to get $N_\gamma$.

We now address step (i). We know that $r_\gamma(t)$ is proportional to $L_{\text{peak}}(t/t_0)^{-5//2}$, and when $\gamma_0 = 2$, to $L_{\text{peak}}(t/t_0)^{-5//2}$. Now substituting $L_{\text{peak}}$ with equation (13) and introducing a normalization factor $r_\gamma$ that will be determined later, we derive

$$r_\gamma(t) \approx r_0 \epsilon_\gamma^{1//2} m_b^{1//4} m_s^{0.55} D_1^{1//2}(t/t_0)^{-5//2}.$$  \hspace{1cm} (14)

We note that $r(t)$ does not depend on $n_{\text{IF}}$ because neither $L_{\text{peak}}$ nor $t_0$ depends on it.

To quantify $r_0$, we recall that $r_0 = r_\gamma(t_0)$ if we adopt the following parameters for our model ($e_\gamma$, $m_b$, $m_s$, $D_1$) = (1, 1, 1, 1). Therefore, $r_0$ is the maximum distance within which a TDE afterglow with the above fiducial parameters can be resolved. We can find $r_0$ in Fig. 3, which shows four spectra of the fiducial TDE afterglow at four different distances, assuming $t = t_0$ in the calculation. A comparison of these spectra with the sensitivity curves of the telescopes suggests that $r_0 \approx 3$, 1.5 and 9 Mpc for Fermi-LAT, H.E.S.S. and CTA. Therefore, we find again that CTA is the most powerful among the instruments considered here to search for the TDE afterglows.

In particular, using $r_0 = 9$ Mpc for CTA and the scaling relation $r_\gamma(t_0) \approx m_b^{1/4}$ presented in equation (14), we find that $r_\gamma(t_0) \approx 16$ Mpc if $m_b$ increases to 10, which is the most probable black hole mass according to TDE observations (Stone & Metzger 2016). This detectable range already reaches the Virgo Cluster. In the above calculations of $r_0$, we did not consider the attenuation of $\gamma$-rays by the extragalactic background light (EBL), because for the $\gamma$-ray energy of our interest, i.e. $\lesssim$10 TeV (see Fig. 3), the EBL attenuation is important only when $r_\gamma \gtrsim 40$ Mpc (Domínguez et al. 2013).

Knowing $r_\gamma(t)$, and therefore $V_\gamma(t)$, we can move on to step (ii). Since TDEs are a random process in time, we can calculate $\Delta N_\gamma(t)$ using the formula $\Delta N_\gamma(t) = f \Gamma V_\gamma(t) \Delta t$. The factor $f$ denotes the fraction of the sky covered by MCs when viewed from a central SMBH. This covering factor determines the probability of an UDS to collide with an MC. In the following, we adopt 40 per cent as our fiducial value (see Section 1). The other parameter $\Gamma$ is the volumetric TDE rate, an observable usually given in unit of Mpc$^{-3}$ yr$^{-1}$. Later in this section, we will discuss the typical value of $\Gamma$.  

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1 http://www.slac.stanford.edu/exp/glast/groups/canda/lat_Performance. htm
In step (iii), we need to sum up all the $\Delta N_p(t)$ for $t > t_0$. In the limit $\Delta t \ll t_0$, the summation is equivalent to the integration \( N_p \approx \int f_p \Gamma \int_0^t V_p(t) \, dt \), which leads to

\[
N_p \approx 12 \epsilon_1^{1/2} m_{\pi}^{1/6} D_s^{-1/2} \Gamma_{-4} \left( \frac{f_p}{0.4} \right) \left( \frac{m_p}{10} \right)^{7/12} \left( \frac{r_0}{9 \text{ Mpc}} \right)^3,
\]

where $\Gamma_{-4} \equiv \Gamma/(10^{-4} \text{ Mpc}^{-1} \text{ yr}^{-1})$. In equation (15), we have normalized $m_p$ by 10 because, as has been mentioned before, it is the most typical value for the TDEs detected so far (Stone & Metzger 2016).

For $\Gamma$, a series of observational and theoretical studies have derived a wide range of values, between $10^{-2}$ and $10^{-4} \text{ Mpc}^{-1} \text{ yr}^{-1}$ (e.g. Magorrian & Tremaine 1999; Donley et al. 2002; Wang et al. 2004; Esquej et al. 2008; Gezari et al. 2009; van Velzen & Farrar 2014; Sun, Zhang & Li 2015; Hololoien et al. 2016; Stone & Metzger 2016). It is important to point out that these earlier works were studying TDEs at relatively large distances, in a redshift range of $0.01 \lesssim z \lesssim 1$ (Komossa 2015), so for their purposes the space density of SMBHs averages out to about $\mathcal{O}(10^{-2}) \text{ Mpc}^{-3}$ (e.g. Donley et al. 2002; Stone & Metzger 2016). We, however, are interested mainly in nearby SMBHs, because $r_0$ is relatively small. In the nearby universe there is apparently an overdensity of SMBHs – we already know three within a distance of 1 Mpc (the MW, M31 and M32, and see Kormendy & Ho 2013, for more SMBHs in the Local Group). To account for this overdensity, we will consider $\Gamma = 10^{-4} \text{ Mpc}^{-1} \text{ yr}^{-1}$ as our fiducial value, which corresponds to $\Gamma_{-4} = 1$.

Knowing the value of $\Gamma$, we now return to equation (15). A CTA-like telescope corresponds to $r_0 = 9 \text{ Mpc}$, so it could resolve about 12 TDE afterglows in all sky as point sources. This result also suggests that in order to detect $\mathcal{O}(10^{3})$ TDE afterglows, a telescope four times more sensitive than CTA is needed. On the other hand, for Fermi-LAT $N_p$ would be about 27 times smaller, i.e. $N_p < 0.5$, because $r_0 \simeq 3 \text{ Mpc}$ is three times smaller than before. The number $N_p$ for H.E.S.S. would be eight times even smaller because of the smaller $r_0$.

Note that these numbers do not include the contributions from dwarf galaxies. Later in Section 9, we will discuss the TDE afterglows in dwarf galaxies.

### 8 WITHOUT MCs

It is important to know what would happen if there were no MCs around SMBHs. In this case, neutral pions and $\gamma$-rays could still be generated by the interaction of UDSs with a smooth ISM. For the GC, earlier calculations showed that the corresponding $\gamma$-ray luminosity is as high as $10^{37} - 10^{40} \text{ erg s}^{-1}$ during the beginning of the interaction (Cheng et al. 2006, 2007, 2011, 2012). Here we will revisit the calculation using the updated UDS model.

For the sake of comparison, we adopt the same ISM model as in the earlier studies – the ISM has a constant proton density of $n_{H} \sim 10^3 \text{ cm}^{-3}$ (Khokhlov & Melia 1996; Cheng et al. 2006). We will discuss the dependence of our results on $n_{H}$ later in this section. Given this density, the characteristic length to stop UDS from free streaming, i.e. $\Delta D$ in equation (5), increases to about $\Delta D \approx 2 \text{ pc}$. Consequently, the Sedov-like expansion, during which most CRs are produced, starts at about $t'_{0} \approx \Delta D / v_0 \approx 300 \text{ yr}$.

Since $t'_{0}$ is much shorter than the proton cooling time-scale in the ISM, which is now about $t'_{p} \approx 5900 \text{ yr}$ according to equation (9), we are allowed to treat the CR-injection process as an instantaneous event, and separate it from the later CR-cooling process. For this reason, we take $E_{CR} = \epsilon E_{i}$ as the total energy of CRs eventually injected into the ISM.

Knowing $n_{H}$ and $E_{CR}$, we can calculate the $\gamma$-ray luminosity with

\[
L'_{\gamma} = n_{\gamma} \sigma_{T} n_{H} c E_{CR}
\]

\[
\simeq 1.2 \times 10^{37} \text{ erg s}^{-1} \times \epsilon_1 \times n_{\gamma} \times m_{\pi}^{1/3} m_{\nu}^{1/4} n_{s}^{-1},
\]

where $n_{\gamma} = n_{H}/(10^3 \text{ cm}^{-3})$. Unlike that in the case of UDS–MC interaction (equation 13), the luminosity here depends on the density $n_{H}$, because now it is limited not by the injection rate of CRs but the cooling rate. The evolution of the luminosity is also different. It rises on a time-scale of 300 yr ($t'_{p}$) and then persists for about 6000 yr ($t''_{p}$). To distinguish this $\gamma$-ray signature from the previous TDE afterglow, we call it the second-type afterglow.

As for the $\gamma$-ray spectrum, we calculate it following the same procedure described in Section 6.2, except for three modifications: (i) The total CR energy is now $E_{CR}$, (ii) the proton density in the background is set to $10^3 \text{ cm}^{-3}$ and (iii) the maximum CR energy is $E_{p, \text{max}} = 1 \text{ PeV}$, adopted from Cheng et al. (2012). The result is shown in Fig. 2 as the thick grey line. It clearly shows that without MCs, the $\gamma$-ray afterglow would be much fainter.

We note that the luminosity given by equation (17) is more than $10^2$ times smaller than the peak luminosity $10^{39} - 10^{40} \text{ erg s}^{-1}$ derived by Cheng et al. (2006, 2007), who considered a similar scenario of $\gamma$-ray emission following a TDE. This disparity is caused mainly by the much greater $E_{CR}$ used by Cheng et al. in their model, which is about $6 \times 10^{42} \text{ erg}$. Such a large energy is unlikely to be provided by an UDS (Guillochon et al. 2015), but a jet, whose formation may require special conditions not satisfied by most TDEs (De Colle et al. 2012).

Since the luminosity $L_{\gamma}'$ is about $10^2$ times lower than $L_{\text{peak}}$ in Section 6.1, we expect the maximum distance at which a source is resolvable, $r'_{\gamma}$, to be 10 times smaller and the corresponding volume, $V_{\gamma}'$, to be $10^4$ times smaller. These considerations lead to $r'_{\gamma} \approx 0.9 \epsilon_{1}^{1/2} n_{\gamma}^{-1} m_{\pi}^{1/6} m_{\nu}^{0.55} n_{s}^{-1/2} \text{ Mpc}$ for CTA. Correspondingly, the detectable volume is $V_{\gamma}' = 4 \pi r'_{\gamma}^3 / 3$. Now neither $r'_{\gamma}$ nor $V_{\gamma}'$ depend on time, because the $\gamma$-ray emission that we are considering now lasts from the time $t = t'_{0}$ to $t'_{0} + t''_{p}$ during which $L_{\gamma}'$ is mostly constant.

Knowing the detection volume $V_{\gamma}'$ and the duration $t''_{p}$ of the second type of TDE afterglows, we can calculate the number of resolvable point sources $N_{\gamma}'$ using the formula $N_{\gamma}' = \frac{V_{\gamma}'}{V'} \Gamma_{-4}$. Note that the covering factor $f_c$ does not appear here, because we assumed that ISM distributes uniformly around an SMBH. With the parameters relevant to CTA, we derive that

\[
N_{\gamma}' \approx \frac{t''_{p} \Gamma_{-4} V_{\gamma}'}{V'} \approx \frac{6 \epsilon_{1}^{2/3} n_{\gamma}^{-1} m_{\pi}^{1/6} m_{\nu}^{0.55} n_{s}^{-1/2} \Gamma_{-4}}{10} \left( \frac{m_{\pi}}{10} \right)^{1/2}.
\]

We expect these CTA sources to be relatively close to the earth, because $r'_{\gamma} \approx 1.5(m_{\pi}/10)^{1/6} \text{ Mpc}$ when SMBHs with $m_{\pi} = 10$ are considered. For the same reason given at the end of Section 7, the number of point sources resolvable by Fermi-LAT and H.E.S.S. is negligibly small.

In the above calculation, we have assumed $n_{H} = 10^3 \text{ cm}^{-3}$ for the sake of comparison. However, for the GC, theoretical models of the accretion flow on to the SMBH (Sgr A*) suggested that the ISM density gets as high as $10^4 \text{ cm}^{-3}$ only within the central $10^{-3} \text{ pc}$ (Yuan, Quataert & Narayan 2003). Moreover, observations of hot and X-ray-emitting ISM within a distance of 1 pc from Sgr A* showed that the ISM density is only $\mathcal{O}(10^3) \text{ cm}^{-3}$ on average (Baganoff et al. 2003). If the circum-nuclear media in other galaxies...
are similar to the ISM in the GC (Generozov, Stone & Metzger 2015), we would have a much lower value for \( n_B \) than what has been assumed above. Correspondingly, both \( L'_\gamma \) and \( N'_\gamma \) would be lowered, making it more difficult to detect the second type of TDE afterglows.

### 9 DISCUSSIONS

In this paper, we have investigated a scenario likely to happen following a large number of TDEs (Sections 1 and 2), in which the ejecta, or ‘UDS’ (Section 3), launched from a disrupted star travels to a distance of \( O(1) \) pc and collides into an MC. We have shown that the collision, which happens hundreds of years after the initial TDE, would produce a strong shock into the MC (Section 4), giving rise to a large amount of CRs with a kinetic energy as high as 1 PeV (Section 5). These CRs, after escaping from the shock region, would collide with the non-relativistic protons in the MC, producing \( \pi^0 \) and subsequently \( \gamma \)-rays in a wide energy range of \( 0.1 \sim 10^2 \) GeV (Section 6). According to our calculation, this \( \gamma \)-ray signature, which we call ‘TDE afterglow’, would be detectable by a CTA-like telescope out to a distance of 10–20 Mpc (Section 7). On the other hand, if there were no MCs around SMBHs, there would be a second type of \( \gamma \)-ray afterglows resulting from the interaction of UDSs with a smooth ISM, which are much fainter than the first type (Section 8).

So far we have not considered IMBHs \( (10^5 \leq M_* / M_\odot < 10^6) \), which exist probably in the nuclei of many dwarf galaxies (Kormendy & Ho 2013). They produce TDE afterglows in the same way as SMBHs do, except that the typical luminosity is different. When an IMBH, \( M_* = 10^5 M_\odot \) for example, tidally disrupts a main-sequence star, the peak luminosity of the \( \gamma \)-ray afterglow would be \( 5 \times 10^{45} \text{erg s}^{-1} \) according to equation (13) (with MCs), or about \( 10^{45} n_{B,0} \text{erg s}^{-1} \) according to equation (17) (without MCs). On the other hand, IMBHs are also able to tidally disrupt white dwarfs. If such an event happens, the kinetic energy of the UDS according to equation (2) could significantly exceed \( 10^{46} \) erg, because a white dwarf is \( 10^4 \) times smaller than the sun. As a result, the \( \gamma \)-ray luminosity would be much higher.

Since dwarf galaxies have been found in large amount in the Local Universe, the number of TDE afterglows would also be large if IMBHs are ubiquitous in dwarf galaxies. Intriguingly, Geringer-Sameth et al. (2015); Hooper & Linden (2015) recently reported a possible detection of \( \gamma \)-ray excess, about \( 5 \times 10^{45} \text{erg s}^{-1} \) in the Fermi-LAT band, in a nearby dwarf galaxy Renticulum II (but see Drlica-Wagner et al. 2015, for a different result). This result immediately caused a debate about the origin of this \( \gamma \)-ray source, including dark matter annihilation (e.g. see Diemand et al. 2008, for an earlier proposal). Our scenario of TDE afterglow can also explain the \( \gamma \)-ray luminosity of Renticulum II. For example, if a TDE with \( M_* = 10^5 M_\odot \) and \( m_n = 1 \) happened 3200 yr ago, then after 500 yr the UDS would have traversed a distance of 1 pc to hit an MC there (we find \( t_0 \approx 5000 \) yr in this case), and today the \( \gamma \)-ray afterglow could have decayed to a luminosity of \( 5 \times 10^{41} \text{erg s}^{-1} \).

Since dark matter annihilation, among with other sources such as star-bursts and a population of millisecond pulsars (MSPs), could also produce \( \gamma \)-rays above 1 GeV in galaxy centres (Su, Slater & Finkbeiner 2010; Abazajian et al. 2014; Brandt & Kocsis 2015), it is critical to find a way of separating these sources from our TDE afterglows.

We find that the \( \gamma \)-ray luminosities from dark matter annihilation, probably about \( O(10^{37}) \text{erg s}^{-1} \) for a MW-like galaxy (The Fermi-LAT Collaboration 2016), and from MSPs, on average \( 10^{45} \sim 10^{45} \text{erg s}^{-1} \) for one MSP (Hooper & Mohlabeng 2015), are both much lower than the peak luminosity of the first-type TDE afterglows, which, we now know, is about \( 10^{46} \) erg s\(^{-1}\). This difference in luminosity could be used to distinguish TDE afterglows from the other two \( \gamma \)-ray sources. The limitation of this method is that it is effective for only ‘young’ TDE afterglows. Because by the time of \( t = (100)^{1/5} t_0 \approx 2.5 t_0 \), the luminosity of the afterglow would have dropped to 1 per cent of its peak value, and it is no longer distinguishable from dark matter annihilation signal or a population of \( O(10^6) \) MSPs.

On the other hand, some star-forming galaxies could be as luminous as \( 10^{39} \text{erg s}^{-1} \) in \( \gamma \)-ray (The Fermi LAT Collaboration 2012), and in this case a method other than comparing luminosities is needed. In the following, we outline two methods that could be useful to separate the first type of TDE afterglows (UDS-MC-induced) from star-forming galaxies.

(i) As we have seen in Fig. 1, a TDE afterglow could be highly variable on a time-scale of 10 yr, especially at the beginning. On the other hand, the \( \gamma \)-ray emission from a starburst (or a population of pulsars and dark matter annihilation) is normally constant for (at least) millions of years. Therefore, a variable \( \gamma \)-ray source with the characteristic luminosity \( O(10^{39}) \text{erg s}^{-1} \) and spectral index \( \gamma_0 = 2 \) would be a smoking-gun evidence for the first type of TDE afterglow.

(ii) Observations of star formation galaxies by Fermi-LAT revealed a tight correlation between the \( \gamma \)-ray luminosity (\( L_\gamma \) in the band 0.1–100 GeV) and the luminosity in far-infrared (\( L_{\text{FIR}} \); see The Fermi LAT Collaboration 2012). This correlation likely reflects a fundamental relationship between the star formation rate (SFR) and the rate of CR production. According to this relationship, \( L_\gamma \) decreases linearly from about \( 10^{39} \text{erg s}^{-1} \) when the SFR is about \( 1 M_\odot \text{yr}^{-1} \) (like the MW), to about \( 10^{36} \text{erg s}^{-1} \) where the SFR is about \( 10^{-2} M_\odot \text{yr}^{-1} \) (such as the Small Magellanic Cloud). Correspondingly, \( L_{\text{FIR}} \) decreases from about \( 2 \times 10^{41} \text{erg s}^{-1} \) to \( 2 \times 10^{41} \text{erg s}^{-1} \). Based on these luminosities, it can be seen that if a TDE afterglow with \( L_\gamma \sim 10^{39} \text{erg s}^{-1} \) appears in a galaxy whose SFR is lower than \( 1 M_\odot \text{yr}^{-1} \), then \( L_\gamma \) in total would rise to above \( 10^{39} \text{erg s}^{-1} \) but \( L_{\text{FIR}} \) is unaffected by the TDE afterglow, since the \( L_{\text{FIR}} \) of a TDE afterglow cannot exceed \( 10^{41} \text{erg s}^{-1} \) according to the cooling rate calculated in Section 4. As a result, a galaxy hosting a TDE afterglow would appear as an outlier in the \( L_\gamma - L_{\text{FIR}} \) diagram relative to the population of star-forming galaxies, if \( L_{\text{FIR}} \) of this galaxy is below \( 2 \times 10^{41} \text{erg s}^{-1} \) (i.e. the SFR is below \( 1 M_\odot \text{yr}^{-1} \)).

If CTA or more sensitive telescopes in the future do not detect any TDE afterglow, the non-detection, according to equation (15), may be due to one or a combination of the following factors. (i) There is a deficiency of MCs in other galaxies so that \( f_c \) is much smaller than 0.4. (ii) MCs on average lie much further away from SMBHs than a distance of 1 pc. (iii) The volumetric TDE rate \( \Gamma \) in the local universe is much lower than \( 10^{-7} \text{Mpc}^{-3} \text{yr}^{-1} \). (iv) Or the current theory of the UDS from a TDE may be incomplete. This knowledge would help us better understand TDEs, as well as the stellar and gaseous environments around the SMBHs or IMBHs in normal galaxies.

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