On Classical Equivalence Between Noncritical and Einstein Gravity: The AdS/CFT Perspectives

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ABSTRACT: We find that noncritical gravity, a special class of higher derivative gravity, is classically equivalent to Einstein gravity at the full nonlinear level. We obtain the viscosity-to-entropy ratio and the second order transport coefficients of the dual fluid of noncritical gravity to all orders in the coupling of higher derivative terms. We also compute the holographic entanglement entropy in the dual CFT of noncritical gravity. All these results confirm the nonlinear equivalence between noncritical gravity and Einstein gravity at the classical level.

KEYWORDS: Higher Curvature Gravity, Fluid-Gravity Correspondence, Holographic Entanglement Entropy.
1. Introduction

Recently gravity with higher derivative terms on Anti-de Sitter (AdS) spacetime has attracted much attention. Typically, higher derivative gravity theories contain ghost-like massive modes. An interesting observation in some class of higher derivative gravity on AdS spacetime is that these ghost-like massive modes fall off more slowly than massless modes as they approach the boundary of AdS spacetime. Then we may consistently truncate ghost-like massive modes by imposing appropriate boundary conditions, leaving massless modes only.

The first example which incorporated this idea was provided in [1], which is four-dimensional conformal gravity on AdS spacetime. The action is given by the conformally invariant Weyl-squared term only. The quantum fluctuations of conformal gravity on AdS spacetime include massless and ghost-like massive modes. It was claimed that the ghosts can be removed by Neumann boundary conditions on the metric at the boundary and thus solutions of Einstein gravity can be chosen as a consistent subset among whole solutions of conformal gravity. Based on this observation, it was claimed that four-dimensional conformal gravity on AdS spacetime may describe the same low energy physics as four-dimensional pure Einstein gravity.

Another interesting example which incorporated this idea, so-called four-dimensional noncritical Einstein-Weyl gravity, was proposed in [2] as an extension of critical gravity [3]. These theories include Weyl-squared term with a specific value of coupling in addition to the usual Einstein-Hilbert and negative cosmological constant terms. They admit AdS spacetime as the vacuum solution. The quantum fluctuations of the metric on this vacuum solution contain massless and ghost-like massive/logarithmic modes. These ghosts can be truncated by the boundary conditions as they fall off more slowly than massless modes...
toward the AdS boundary. In the case of critical gravity, the theory after the truncation might be trivial as the excitation energy of massless modes is zero. In the case of four-dimensional noncritical Einstein-Weyl gravity on (asymptotically) AdS spacetime, it was anticipated that the theory might be consistent after the truncation, which we call NEW gravity, with better UV behavior than usual Einstein gravity. This was extended in [4] to $\mathcal{N} = 1$ NEW supergravity.

Though these studies are mostly based on linearized equations of motion, they suggest that NEW gravity may be well-defined at low energy. Furthermore they indicate that the theory may describe essentially the same physics as Einstein gravity at the tree level. Indeed, the classical equivalence between NEW gravity and Einstein gravity on AdS background at the full nonlinear level was established in [5]. This was achieved by showing that the effective Lagrangian of NEW gravity becomes identical with the one of Einstein gravity up to the rescaling of Newton’s constant. It was also shown in [5] the classical equivalence between $\mathcal{N} = 1$ NEW supergravity and $\mathcal{N} = 1$ AdS supergravity.

From the perspectives of the AdS/CFT correspondence, the full classical equivalence between NEW gravity and Einstein gravity on AdS background implies the equivalence between the corresponding dual conformal field theories (CFT) in the large $N$ limit. Indeed it was shown in [5] that the boundary action of NEW gravity after using equations of motion is the same as the one of Einstein gravity up to the same overall rescaling of Newton’s constant. As a result, it was found that $n$-point correlation functions of energy-momentum tensor in the dual CFT of NEW gravity can be read off from those from Einstein gravity. It was also shown that $n$-point correlation functions among energy-momentum tensors with two supercurrents in the dual CFT of $\mathcal{N} = 1$ NEW supergravity is identical to those of the dual CFT of $\mathcal{N} = 1$ AdS supergravity.

In this paper we explore the implications of this classical equivalence further. Especially we are interested in the phenomena which reveal nonlinear aspects of the equivalence through the AdS/CFT correspondence.

First of all, we generalize the classical, but nonlinear equivalence between NEW gravity and Einstein gravity to arbitrary dimensions. In four dimensions, higher derivative terms in the action of critical gravity and NEW gravity are given by Weyl-squared term modulo Gauss-Bonnet terms. In higher dimensions, higher derivative terms of critical gravity are given by a specific combination of Ricci tensor-squared and Ricci scalar-squared terms with a special value of the coupling constant [6]. In this paper, we present the generalization of NEW gravity in higher dimensions, which we call noncritical gravity, whose action includes the same combination of the higher derivative terms with more general value of the coupling constant. Just like NEW gravity, noncritical gravity is defined on (asymptotically) AdS spacetime with the truncation of ghost-like massive modes of the metric by boundary conditions. We establish the classical equivalence between noncritical gravity and Einstein gravity on AdS spacetime in arbitrary dimensions. All these are presented in section 2.

One of the most interesting arena which involves classical but nonlinear aspects on gravity is the fluid/gravity correspondence. In section 3, we study the fluid/gravity correspondence for noncritical gravity. Among others, we obtain the linear and nonlinear transport coefficients for the dual fluid of noncritical gravity. We find the exact viscosity-to-
entropy ratio, all orders in higher derivative coupling, which saturates the KSS bound in accord with the previous perturbative result in the leading order of higher derivative coupling.

Another interesting example which reveals nonperturbative aspects with respect to the higher derivative coupling is a holographic entanglement entropy (HEE) of the dual CFT. In section 4, we propose two holographic methods which give results consistent with expected nature of entanglement entropy. Once again we find identical results for the HEE from noncritical gravity with those from Einstein gravity.

In section 5, we draw our conclusions.

2. Noncritical Gravity in Various Dimensions

In this section we generalize NEW gravity to arbitrary dimensions and show the classical, nonlinear equivalence between a class of the higher derivative gravity theories, so-called noncritical gravity, and Einstein gravity on AdS spacetime.

Let us consider the following action

\[ S = \frac{\sigma}{2\kappa^2} \int d^Dx \sqrt{-g} \left[ R + \frac{(D-1)(D-2)}{\ell^2} - \frac{1}{m^2(D-2)} \left( R^{MN}R_{MN} - \frac{D}{4(D-1)}R^2 \right) \right], \]

where the overall sign \( \sigma \) can take the value \( \pm 1 \) and \( \ell^2 \) and \( m^2 \) take real values whose range will be determined later. For a special value for \( m^2 \), it becomes the action of critical gravity in arbitrary dimensions. The higher derivative terms can be rewritten as the combination of the Weyl-squared term and the Gauss-Bonnet term

\[ R^{MN}R_{MN} - \frac{D}{4(D-1)}R^2 = \frac{(D-2)}{4(D-3)} \left( C^{MNPQ}C_{MNPQ} - E \right). \]

The Euler-Lagrange equations of motion for the metric \( g_{MN} \) are given by

\[ G_{MN} - \frac{(D-1)(D-2)}{2\ell^2}g_{MN} + E_{MN} = 0, \]

where

\[ G_{MN} = R_{MN} - \frac{1}{2}Rg_{MN}, \]

\[ E_{MN} = \frac{1}{(D-2)m^2} \left[ -2(R_{MP}R^P_N - \frac{1}{4}R^{PQ}R_{PQ}g_{MN}) + \frac{D}{2(D-1)}R(R_{MN} - \frac{1}{4}Rg_{MN}) \right. \]

\[ -\left( \nabla^2 R_{MN} + \frac{1}{2}\nabla^2 R_{gMN} - 2\nabla_P \nabla_{(M}R_{N)} + \frac{D}{2(D-1)}(g_{MN}\nabla^2 R - \nabla_M \nabla_N R) \right). \]
It is convenient to introduce an auxiliary field $f_{MN}$ and rewrite the action as

$$S = \frac{\sigma}{2K^2} \int d^Dx \sqrt{-g} \left[ R + \frac{(D-1)(D-2)}{\ell^2} - \frac{1}{(D-2)} f^{MN} G_{MN} + \frac{m^2}{4(D-2)} (f^{MN} f_{MN} - f^2) \right],$$

(2.5)

which was also used in [10] in the context of critical gravity. The equation of motion for the auxiliary field $f_{MN}$ is given by

$$f_{MN} = 2 m^2 \left[ R_{MN} - \frac{1}{2(D-1)} R g_{MN} \right], \quad f = \frac{(D-2)}{m^2(D-1)} R.$$  

(2.6)

After plugging this equation of motion back in the action, we recover the original action (2.1). For appropriate ranges of $\ell^2$ and $m^2$, it admits AdS spacetime as vacuum solution,

$$\bar{R}_{MN} = - \frac{(D-1)}{L^2} g_{MN}, \quad \bar{R} = - \frac{D(D-1)}{L^2}, \quad \bar{G}_{MN} = \frac{(D-1)(D-2)}{2L^2} g_{MN},$$

(2.7)

and

$$\bar{f}_{MN} = - \frac{(D-2)}{m^2 L^2} g_{MN},$$

(2.8)

where $g_{MN}$ denotes the AdS metric and the AdS curvature radius $L$ is given by the relation,

$$\frac{1}{\ell^2} = \frac{1}{L^2} \left[ 1 - \frac{(D-4)}{4m^2 L^2} \right].$$

(2.9)

The quantum fluctuations of the metric $g_{MN}$ around AdS spacetime, generically, consist of the massless and massive modes, one of which turns out to be ghost-like. A clever choice of the expansion is given by

$$g_{MN} = \bar{g}_{MN} + h_{MN} + \phi_{MN},$$

(2.10)

$$f_{MN} = - \frac{(D-2)}{m^2 L^2} (\bar{g}_{MN} + h_{MN}) - (D-2) \left( \frac{2L^2}{\ell^2} - 1 \right) \phi_{MN},$$

which leads to the natural decoupling between massless and massive modes, denoted as $h_{MN}$ and $\phi_{MN}$, respectively.

Indeed, through these expansions, the quadratic Lagrangian $\mathcal{L}_2$, modulo the total derivative terms, is given by

$$\mathcal{L}_2 = - q h^{MN} \mathcal{G}_{MN}(h) + q \phi^{MN} \mathcal{G}_{MN}(\phi) + \sigma \frac{(D-2)}{4} m^2 q^2 (\phi^{MN} \phi_{MN} - \phi^2),$$

(2.11)

where

$$q = \sigma \left[ 1 - \frac{(D-2)}{2m^2 L^2} \right].$$

(2.12)

Here $\mathcal{G}_{MN}$ denotes the linearized Einstein operator including the cosmological constant term,

$$\mathcal{G}_{MN}(h) = R_{MN}^{(1)}(h) - \frac{1}{2} \bar{g}_{MN} \bar{g}^{PQ} R^{(1)}_{PQ}(h) + \frac{(D-1)}{L^2} h_{MN} - \frac{(D-1)}{2L^2} \bar{h} \bar{g}_{MN},$$

(2.13)
The linearized Ricci tensor, $R^{(1)}_{MN}$, is given by

$$R^{(1)}_{MN}(h) = \nabla_P \nabla_{(M} h_{N)} - \frac{1}{2} \nabla_N \nabla_M h - \frac{1}{2} \nabla^2 h_{MN},$$

(2.14)

where $\nabla$ denotes a covariant derivative with respect to the background metric $\bar{g}_{MN}$.

Clearly, the massless and massive modes are decoupled in the above quadratic Lagrangian. The novel aspect of the decoupling is that the bulk-to-boundary propagators in the AdS/CFT correspondence do not mix those two kinds of modes. One may also note that the signatures of the kinetic terms of the massless and massive modes are opposite, signaling one of them is ghost-like. By taking $q$ positive, the massless gravitons, $h_{MN}$, remain physical, while the massive modes, $\phi_{MN}$, become ghost-like. The corresponding linearized equations of motion are given by

$$G_{MN}(h) = 0,$$

(2.15)

$$G_{MN}(\phi) + \sigma \frac{(D-2)}{2} m^2 q (\phi_{MN} - \phi \bar{g}_{MN}) = 0.$$

If we choose the transverse traceless gauge, which is consistent with the equations of motion, as

$$\nabla^M h_{MN} = 0, \quad \bar{g}^{MN} h_{MN} = 0, \quad \nabla^M \phi_{MN} = 0, \quad \bar{g}^{MN} \phi_{MN} = 0,$$

(2.16)

the linearized equation of motion for a massless graviton $h_{MN}$ reduces to

$$\left( \nabla^2 + \frac{2}{L^2} \right) h_{MN} = 0,$$

(2.17)

and the one for a ghost-like massive graviton $\phi_{MN}$, with mass $M$, becomes

$$\left( \nabla^2 + \frac{2}{L^2} - M^2 \right) \phi_{MN} = 0, \quad M^2 = (D-2)\sigma q m^2.$$

(2.18)

AdS spacetime has the time-like boundary on which the boundary conditions should be specified. A salient feature for the case when $M^2 < 0$ or $m^2 < \frac{D-2}{2L^2}$ is that ghost-like massive modes, $\phi_{MN}$, fall off more slowly than massless modes, $h_{MN}$, as they approach the boundary. Therefore, in this case, these ghosts can be consistently truncated by imposing appropriate boundary conditions. The BF bound which comes from the tachyon-free condition is given by $M^2 > -\frac{(D-1)^2}{4L^2}$, which is $m^2 > \frac{D^2-6D+7}{4(D-2)L^2}$. Therefore we consider the action with the coupling range

$$\frac{D^2 - 6D + 7}{4(D-2)L^2} < m^2 < \frac{D - 2}{2L^2}.$$

(2.19)

One may note that $m^2$ is always positive for $D \geq 5$ in contrast to the case $D = 4$ where negative $m^2$ is allowed\(^1\). Since $q$ should be positive to have physical massless modes, the overall sign of the action, $\sigma$, should be negative for $D \geq 5$ while in four dimensions $\sigma$ can

\(^1\)In three dimensions, analogous study has been carried out in [12].
take both signs. If \( m^2 \) saturates the upper bound, \( m^2 = \frac{D-2}{2L^2} \), the massive modes turn into the logarithmic ones, and the corresponding theory is called critical gravity.

Now comes the key point. As alluded earlier, ghost-like massive modes, \( \phi_{MN} \), are decoupled from massless modes at the quadratic Lagrangian and therefore do not mix in the bulk-to-boundary propagator. Once ghosts, \( \phi_{MN} \), are truncated at the boundary, they remain suppressed even deep in the bulk at the tree level. After the truncation of ghosts, we have a linearized relation (2.10) in which \( f_{MN} \) is proportional to \( g_{MN} \). Without ghosts, this linearized relation can be consistently lifted to the full non-linear level as

\[
 f_{MN} = -\frac{D-2}{m^2L^2}g_{MN} .
\]  
(2.20)

This means that solutions of Einstein gravity can be chosen as a consistent subset among whole solutions of our class of higher derivative gravity. We call this higher derivative gravity with the above consistent truncations of ghosts as noncritical gravity.

By plugging this in the action (2.5), we obtain the effective action:

\[
 S_{\text{eff}} = \frac{\sigma}{2\kappa^2} \int d^Dx \sqrt{-g} \left[ R + \left( \frac{D-1}{D-2} \right) \frac{1}{L^2} \right] ,
\]  
(2.21)

which is nothing but the ordinary Einstein action with Newton’s constant \( \frac{\kappa^2}{\sigma} \) and the cosmological constant \( -\frac{(D-1)(D-2)}{2L^2} \). In particular this strongly indicates that the dual conformal field theory of noncritical gravity would be identical to the one dual to Einstein gravity in the large \( N \) limit.

Let us turn into boundary actions and show that similar arguments can be applied. The boundary action, which we use in the AdS/CFT correspondence, consists of the generalized Gibbons-Hawking terms and boundary terms from the on-shell bulk action. We introduce the bulk metric in the ADM decomposed form as

\[
 ds^2 = N^2dr^2 + \gamma_{\mu\nu}(dx^\mu + N^\mu dr)(dx^\nu + N^\nu dr) ,
\]  
(2.22)

and define the boundary fields \( \tilde{f}^{\mu\nu} \) by

\[
 \tilde{f}^{\mu\nu} = f^{\mu\nu} + 2h^{(\mu}N^{\nu)} + sN^{\mu}N^{\nu} , \quad \hat{f} = \gamma_{\mu\nu}\tilde{f}^{\mu\nu} ,
\]  
(2.23)

from the bulk auxiliary fields \( f^{MN} \),

\[
 \tilde{f}^{MN} = \begin{pmatrix} s & h^\nu \\ h^\mu & f^{\mu\nu} \end{pmatrix} .
\]

Let us consider the generalized Gibbons-Hawking terms [13] at the boundary which is given by

\[
 S_{\text{GGH}} = \frac{\sigma}{2\kappa^2} \int_{\partial M} d^{D-1}x \sqrt{-\gamma} \left[ -2K + \frac{1}{(D-2)} \left( \tilde{f}^{\mu\nu}K_{\mu\nu} - \hat{f}K \right) \right] ,
\]  
(2.24)

where the extrinsic curvature \( K_{\mu\nu} \) on the boundary surface is defined by

\[
 K_{\mu\nu} = -\frac{1}{2N}(\partial_{\rho}\gamma_{\mu\nu} - \nabla_\mu N_\nu - \nabla_\nu N_\mu) .
\]  
(2.25)
After turning off ghost-like massive modes of the boundary fields we can use the similar arguments for the boundary fields $\gamma_{\mu\nu}, \hat{f}_{\mu\nu}$ as for the bulk fields $g_{MN}, f_{MN}$ and give the same relation,

$$\hat{f}_{\mu\nu} = -\frac{(D-2)}{m^2 L^2} \gamma_{\mu\nu}. \quad (2.26)$$

By plugging this relation into the generalized Gibbons-Hawking terms, we obtain the effective Gibbons-Hawking term which is given by the usual Gibbons-Hawking term of the Einstein gravity with rescaled Newton’s constant as

$$S_{\text{EHG}} = -\frac{q}{\kappa^2} \int_{\partial M} d^{D-1}x \sqrt{-\gamma} K. \quad (2.27)$$

From the AdS/CFT correspondence, the boundary fields $\gamma_{\mu\nu}, \hat{f}_{\mu\nu}$ act as sources of the corresponding operators in the dual CFT. As far as we can consistently turn off the ghost-like massive modes by boundary conditions, the form of the effective actions in the bulk and the boundary, in (2.21) and (2.27), respectively, become exactly those of Einstein gravity. As a result, the effective boundary action for noncritical gravity becomes exactly the one for Einstein gravity up to an overall rescaling. This means that, as far as we remain at the tree level, noncritical gravity reduces to Einstein gravity on AdS spacetime and thus the dual CFT of noncritical gravity should be identical to the one of pure Einstein gravity in the large $N$ limit. In the rest of the paper, we study various aspects of the dual CFT of noncritical gravity and confirm this statement.

3. The Fluid/Gravity Correspondence for Noncritical Gravity

Fluid dynamics describes collective motions of huge number of constituent particles as continuum, of which construction is a predecessor of various modern field theories. Basic assumption in this coarse-grained description is that each small portion of fluid behaves as an entity. In other words, the length scale in the system is much larger than the ‘mean free path’ of constituent particles, so the system is well approximated by continuum field variables. The relevant variables to describe fluid dynamics are slowly varying local temperature $T(x)$ and fluid velocity $u^\mu(x)$. In the case of the neutral fluid, fluid dynamics is governed by the conservation of energy-momentum tensor. The main theme of fluid dynamics is to determine the energy-momentum tensor as derivative expansions of $T(x)$ and $u^\mu(x)$ and find coefficients of dissipation terms, so-called, transport coefficients.

A universal feature of fluid dynamics tells us that CFT also has a regime described by fluid dynamics. On top of this, CFT has a holographic dual description by gravity (or string theory) on AdS space according to the AdS/CFT correspondence. As a result, it is intriguing and natural problem to construct fluid dynamics holographically, which was achieved in [14][15] and afterwards named as the fluid/gravity correspondence.

In this section we use the fluid/gravity correspondence to determine transport coefficients of the dual fluid of noncritical gravity. As will be shown later in this section, all the transport coefficients for noncritical gravity turn out to be the same as those for Einstein gravity, up to an overall factor.
3.1 Fluid dynamics and dual geometry

In this subsection, we summarize some basics on fluid dynamics, partly to set up the notations. The usual formulation of fluid dynamics consists of two elements. One is the Navier-Stokes equation, which is based on the Newton’s second law of motion, and the other is the continuity equation, which is just the macroscopic incarnation of microscopic conservation law. In the relativistic setup, these two equations are simply represented by a single conservation equation of the energy-momentum tensor as

$$\nabla_\mu T^{\mu \nu} = 0,$$

(3.1)

which is the main dynamical equation in the following.

The construction of fluid dynamics reveals its statistical nature and so it appears as the low energy description of any quantum field theory at sufficiently long wavelength limit. Because of such statistical nature, relevant variables in relativistic fluids are local thermodynamic variables: temperature $T(x)$ and velocity $u^\mu(x)$. See [16] for the review and details.

Now we summarize some important formulae to fix our conventions for $d$-dimensional neutral fluids. Denoting density and pressure as $\rho(x)$ and $P(x)$, respectively, the energy-momentum tensor of fluids can be written as the derivative expansion:

$$T^{\mu \nu} = \left[ P(x) + \rho(x) \right] u^\mu(x) u^\nu(x) + P(x) \gamma^{\mu \nu} + \Pi^{\mu \nu}(x),$$

(3.2)

where $\Pi^{\mu \nu} = \sum_{n=1}^{\infty} \Pi^{\mu \nu}_{(n)}$ represents dissipation effects and the contribution of $n$-derivatives of $T(x)$ and $u^\mu(x)$.

We choose the, so-called, Landau frame, in which the dissipation part is transverse to $u^\mu$,

$$\Pi^{\mu \nu} u_\nu = 0.$$

It is convenient to introduce the projection operator $P^{\mu \nu} = \gamma^{\mu \nu} + u^\mu u^\nu$. The first order term $\Pi^{\mu \nu}_{(1)}$ can be decomposed as the traceless and the trace part as

$$\Pi^{\mu \nu}_{(1)} = -2\eta \sigma^{\mu \nu} - \zeta \theta P^{\mu \nu},$$

(3.3)

where $\eta$ denotes shear viscosity, $\zeta$ does bulk viscosity and

$$\theta \equiv P^{\mu \nu} \nabla_\mu u_\nu,$$

$$\sigma^{\mu \nu} \equiv P^{\mu \alpha} P^{\nu \beta} \nabla_{(\alpha} u_{\beta)} - \frac{1}{d-1} \theta P^{\mu \nu}.$$

In the context of the fluid/gravity correspondence, relevant fluids are conformal ones which have additional constraints by underlying conformal symmetry. For example, the bulk viscosity vanishes, $\zeta = 0$, in the case of conformal fluids. In the following we consider conformal fluids to study the fluid/gravity correspondence for noncritical gravity.
In the Weyl covariant formalism, the second order dissipation part for conformal fluids consists of the following five terms:

\[ I_{\mu\nu}^1 \equiv 2u^\alpha \nabla_\alpha \sigma^{\mu\nu} + \frac{1}{d - 1} \sigma^{\mu\nu} \nabla^\alpha u_\alpha - 2u^{(\mu} \sigma^{\nu)} \delta^\alpha_\beta u^\alpha \nabla_\beta u_\beta, \]

(3.4)

\[ I_{\mu\nu}^2 \equiv C^{\mu\nu\beta} u_\alpha u_\beta, \quad I_{\mu\nu}^3 \equiv 4\sigma^{\alpha (\mu} \sigma^{\nu)}_\alpha, \]

\[ I_{\mu\nu}^4 \equiv 2\sigma^{\alpha (\mu} \omega^{\nu)}_\alpha, \quad I_{\mu\nu}^5 \equiv \omega^{\alpha (\mu} \omega^{\nu)}_\alpha, \]

where the bracket \( \langle \cdot \rangle \) around the indices makes a tensor traceless and transverse to \( u^\mu \), i.e.

\[ A_{\langle \mu \nu \rangle} \equiv \frac{1}{D - 1} P^{\mu\alpha} P^{\nu\beta} A_{\alpha\beta} \quad \text{and} \quad \omega_{\langle \mu \nu \rangle} = \frac{1}{D - 1} P^{\mu\alpha} P^{\nu\beta} \nabla_\beta u_\alpha \]

denotes the vorticity tensor. The second order dissipation part of conformal fluids is given by

\[ \Pi_{(2)}^{\mu\nu} = \tau_{\eta} I_{(2)}^{\mu\nu} + \kappa I_{(2)}^{\mu\nu} + \lambda_1 I_{(2)}^{\mu\nu} + \lambda_2 I_{(2)}^{\mu\nu} + \lambda_3 I_{(2)}^{\mu\nu}, \]

(3.5)

where \( \tau_{\eta}, \kappa \) and \( \lambda_i \) are called as second order transport coefficients.

Since fluid dynamics describes a thermodynamic system by construction, the holographic dual geometry should be a finite temperature system, like black holes/branes. It turns out that the relevant dual geometry is a planar-type black brane. Moreover, the temperature throughout fluids is not a constant but the local, slow-varying, function of the position, which represents the nature of fluids at local thermal equilibrium. This is realized in the gravity side as the local position dependence of black brane horizon.

One of the novel aspects of \( D \)-dimensional noncritical gravity is the existence of the planar black brane solution,

\[ ds^2 = L^2 \left[ \frac{dr^2}{r^2 f(br)} - r^2 f(br) dt^2 + r^2 d\mathbf{x}_{D-2}^2 \right]. \]

(3.6)

where

\[ f(r) = 1 - \frac{1}{r^{D-1}}. \]

One can easily confirm that the metric \( g_{MN} \) in (3.6) satisfies the equation of motion (2.3) by noting that the corresponding geometry is an Einstein manifold where the Ricci tensor is given by

\[ R_{MN} = -\frac{(D - 1)}{L^2} g_{MN}. \]

(3.7)

The Hawking temperature of the planar black brane is proportional to the horizon radius and is given by

\[ T = \frac{D - 1}{4\pi b L}. \]

In what follows, we use this black brane solution to study the dual fluid dynamics of noncritical gravity. The Bekenstein-Hawking entropy density \( s \) of the planar black brane can be obtained by using the Wald formula with the original action \( S \) in (2.1) and is given by

\[ s = -2\pi \sqrt{\gamma} \frac{\partial L}{\partial R_{abcd}} \epsilon_{ab} \epsilon_{cd} \bigg|_{\text{horizon}} = \frac{2\pi}{\kappa^2} \left( \frac{1}{b} \right)^{D - 2} \sigma \left[ 1 - \frac{(D - 2)}{2m^2 L^2} \right]. \]

(3.8)

One may note that the same result can be obtained directly from the effective action \( S_{\text{eff}} \) in (2.21),

\[ s = -2\pi \sqrt{\gamma} \frac{\partial L_{\text{eff}}}{\partial R_{abcd}} \epsilon_{ab} \epsilon_{cd} \bigg|_{\text{horizon}} = \frac{2\pi \eta}{\kappa^2} \left( \frac{1}{b} \right)^{D - 2}, \]

(3.9)
where \( q \) is given by (2.12). This gives an evidence of the nonlinear, classical equivalence between noncritical gravity and Einstein gravity on this AdS planar black brane background.

One may boost, with boost parameter \( \beta_i \), the AdS planar black brane solution along the translationally invariant spatial coordinates \( x^i \) and write the metric using ingoing Eddington-Finkelstein coordinate as

\[
d s^2 = L^2 \left[ -2 u^\mu dx^\mu dr - r^2 f(br) u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu \right],
\]

where

\[
u^\mu = \left( \frac{1}{\sqrt{1 - \beta^2}}, \frac{\beta_i}{\sqrt{1 - \beta^2}} \right),
\]

may correspond to the globally uniform fluid velocity and \( P_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu \). Starting from this boosted black brane metric, we may implement slowly varying fluid velocity and temperature by giving the position dependence as \( u_\mu = u_\mu(x) \) and \( T = T(x) \). Accordingly, we consider the metric,

\[
d s^2 = L^2 \left[ -2 u_\mu(x) dx^\mu dr - r^2 f(b(x)r) u_\mu(x) u_\nu(x) dx^\mu dx^\nu + r^2 P_{\mu\nu}(x) dx^\mu dx^\nu \right],
\]

which is, surely, not the solution of the equations of motion for arbitrary \( \beta_i(x) \) and \( b(x) \). To satisfy the equations of motion we need to add correction terms in \( g_{MN}, \beta_i(x) \) and \( b(x) \) which can be done by systematic perturbation. Once the gravity solution is obtained perturbatively, one can determine the boundary energy-momentum tensor as will be described in the next section.

### 3.2 The Fluid/Gravity Correspondence with Higher Curvature Terms

Now we turn to the holographic description of the \( d \)-dimensional fluid dynamics corresponding to noncritical gravity in \( D = d + 1 \) dimensions. The fluid/gravity correspondence gives us the computational tool for transport coefficients in the strong coupling regime. According to the standard AdS/CFT prescription\(^1\), the energy-momentum tensor of the \( d \)-dimensional CFT is identified with Brown-York tensor\(^2\), up to counter terms, for the ADM-decomposed metric, (2.22), of \( D \)-dimensional asymptotically AdS space\(^2\).

After plugging the bulk equations of motion in the bulk action, the leftover boundary terms combine with the Gibbons-Hawking boundary terms and become the boundary action \( S_B \). The Brown-York tensor is given by the functional derivative of the boundary action \( S_B \) with respect to the boundary metric \( \gamma_{\mu\nu} \),

\[

T^{\mu\nu}_{BY} = - \frac{2}{\sqrt{-\gamma}} \frac{\delta S_B}{\delta \gamma_{\mu\nu}}.
\]

The Brown-York tensor for the bulk action (2.3) and the generalized Gibbons-Hawking terms (2.24) is found to be

\[
8\pi G T^{\mu\nu}_{BY} = \sigma \left[ 1 - \frac{1}{2(D-2)} (\hat{s} - \hat{f}) \right] (K^{\mu\nu} - K \gamma^{\mu\nu}) + \frac{\sigma}{(D-2)} \left[ \nabla^{(\mu} \hat{h}^{\nu)} - \frac{1}{2} D_r \hat{f}^{\mu\nu} + K_\rho^{(\mu} \hat{v}^{\nu)\rho} - \gamma^{\mu\nu} (\nabla_\rho \hat{h}^{\rho} - \frac{1}{2} D_r \hat{f}) \right],
\]

\(^2\)See \[13\] for some subtle issues on counter terms in higher derivative gravity.
where hatted quantities are defined by (2.23) and
\[ \hat{s} = N^2 s, \quad \hat{h}^\mu = N(h^\mu + s N^\mu), \]  
(3.15)
and the covariant derivatives along \( r \) are
\[
D_r \hat{f}^{\mu \nu} = \frac{1}{N} \left( \partial_r \hat{f}^{\mu \nu} - N^\rho \partial_\rho \hat{f}^{\mu \nu} + \hat{f}^{\rho \nu} \partial_\rho N^\mu + \hat{f}^{\mu \rho} \partial_\rho N^\nu \right),
\]
\[
D_r \hat{f} = \frac{1}{N} \left( \partial_r \hat{f} - N^\mu \partial_\mu \hat{f} \right).
\]
(3.16)

We find that the perturbative solutions starting from the metric (3.12) of noncritical gravity has exactly the same form as those of Einstein gravity. The perturbative solution satisfies
\[
R_{MN} = -\frac{(D - 1)}{L^2} g_{MN}, \quad f_{MN} = -\frac{(D - 2)}{m^2 L^2} g_{MN}.
\]
(3.17)
Inserting this solution into the expression of Brown-York tensor in (3.14), one can read off various transport coefficients holographically. By using (2.20), we can find the hatted quantities in (3.14):
\[
\hat{s} = -\frac{(D - 2)}{m^2 L^2}, \quad \hat{h}^\mu = 0,
\]
\[
\hat{f}^{\mu \nu} = -\frac{(D - 2)}{m^2 L^2} \gamma^{\mu \nu}, \quad \hat{f} = -\frac{(D - 1)(D - 2)}{m^2 L^2},
\]
(3.18)

Then the Brown-York tensor (3.14) becomes
\[
8\pi GT_{BY}^{\mu \nu} = \sigma \left[ 1 - \frac{(D - 2)}{2m^2 L^2} \right] (K^{\mu \nu} - K \gamma^{\mu \nu}) = q(K^{\mu \nu} - K \gamma^{\mu \nu}),
\]
(3.19)
which is identical with the one obtained directly from the effective action (2.21) and the effective Gibbons-Hawking term (2.27). Therefore we find
\[
T_{BY}^{\mu \nu} \text{(noncritical)} = q T_{BY}^{\mu \nu} \text{(Einstein)},
\]
(3.20)
which confirms the nonlinear, classical equivalence between noncritical gravity and Einstein gravity.

From the above one can readily read off the transport coefficients in the dual fluid of noncritical gravity from those corresponding to Einstein gravity. The viscosity is given by
\[
\eta = q \frac{L^{d-1}}{2\kappa^2} \left( \frac{4\pi}{d-1} T \right)^{d-1} = q \frac{1}{2\kappa^2 b^{d-1}},
\]
(3.21)
and the viscosity-to-entropy ratio saturates the KSS bound:
\[
\frac{\eta}{s} = \frac{1}{4\pi}.
\]
(3.22)
We stress that this is an exact result, all orders in \( 1/m^2 \).
The transport coefficients at the second order are given by

\[ \tau_\pi = \frac{d}{4\pi T} \left[ 1 + \frac{1}{d} \right. \text{Harmonic} \left( \frac{2}{d} - 1 \right), \quad \kappa = \frac{d}{2\pi (d-2) T}, \]

\[ \lambda_1 = \frac{d}{8\pi T} \eta, \quad \lambda_2 = \frac{1}{2\pi} \text{Harmonic} \left( \frac{2}{d} - 1 \right) \frac{\eta}{T}, \quad \lambda_3 = 0, \] (3.23)

where \text{Harmonic}(x) is harmonic number function.

These results are novel as they are all order expressions in \(1/m^2\). Note that \(m^2\) enters nonlinearly in the expression of \(q\), \((2.13)\), through \(L^2(m)\) given in \((2.9)\). After expanded in terms of \(1/m^2\), our results agree with the known perturbative expressions of transport coefficients in the dual fluid of our class of higher curvature gravity. Explicitly, the shear viscosity from noncritical gravity, after expanded in terms of \(1/m^2\), is given by

\[ \eta = \frac{1}{2\kappa^2 \ell^{d-1} \sigma} \left[ 1 - \frac{d-1}{2m^2 \ell^2} + \mathcal{O}\left( \frac{1}{m^4} \right) \right]. \] (3.24)

As was emphasized in the previous sections, our results go beyond those from field redefinition or other perturbative approaches like the effective action method \([20][21]\]. Though the higher curvature terms in noncritical gravity can be absorbed into the Einstein-Hilbert term via a field redefinition up to \(1/m^2\) order, this field redefinition obscures higher order contribution in \(1/m^2\). The effective action method, on the other hand, may address higher orders systematically. However, nonlinear transport coefficients, \(\lambda_i (i = 1, 2, 3)\) are out of reach in this effective action method. In contrast we found the exact expressions for the transport coefficients in the dual fluid dynamics of non critical gravity.

These holographic transport coefficients are formally related to the boundary \(n\)-point correlation functions in Euclidean space as

\[ \langle T^{\mu\nu}(0) \rangle = \langle T^{\mu\nu}(0) \rangle_0 + \frac{1}{2} \int d^d x \langle T^{\mu\nu}(0) T^{\alpha\beta}(x) \rangle_0 h_{\alpha\beta} \]

\[ + \frac{1}{8} \int d^d x d^d y \langle T^{\mu\nu}(0) T^{\alpha\beta}(x) T^{\gamma\delta}(y) \rangle_0 h_{\alpha\beta}(x) h_{\gamma\delta}(y) + \cdots. \] (3.25)

For example, the Kubo formula in linear response theory in Minkowski space is given by

\[ \eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt d\vec{x} e^{i\omega t} \langle [T_{12}(x), T_{12}(0)] \rangle_0 = - \lim_{\omega \to 0} \frac{1}{\omega} \text{Im} G^R(\omega, 0), \] (3.26)

where \(G^R\) is the retarded Green’s function. The second order transport coefficients are generically related to the three-point correlation functions. (See, for example, \([22]\) for more details.) As mentioned in the previous section, the \(n\)-point correlation functions in noncritical gravity should be identical to those from Einstein gravity up to rescaling of Newton’s constant and the cosmological constant. What we found is that the our result \((3.20)\) is consistent with all the nonlinear transport coefficients from the \(n\)-point correlation functions in noncritical gravity.
4. Holographic Entanglement Entropy

Entanglement entropy is one of the most interesting measures of how two systems, $A$ and its complement, are quantum-mechanically related. When two systems are entangled quantum-mechanically, the measurement of one system leads to some correlated information on the other system through quantum superposition. Formally, the entanglement entropy of $A$ is defined by the partial trace of von Neumann entropy over the complement of $A$. It measures how much system $A$ is entangled with its complement. (See [23] for some review.) Since the AdS/CFT correspondence may be regarded as a quantum principle, it may realize this entanglement entropy holographically, which is indeed done in [24] and coined as holographic entanglement entropy (HEE). According to this proposal in Einstein gravity, the entanglement entropy of region $A$ in the CFT side corresponds to the minimal area of codimension two hypersurface $\Sigma_A$ in AdS space homologous to the domain $A$ at the asymptotic boundary [25][26]. This proposal is extended to a certain higher curvature gravity, i.e. Lovelock gravity, in [27].

To compute the HEE for noncritical gravity whose action contains a specific combination of the higher curvature terms, one should extend or adjust the previous proposals. There is no known universal prescription to obtain the HEE for general higher derivative gravity. Luckily, the extension is rather simple because of the nature of noncritical gravity. It was shown in [27] that the Wald formula applied on $\Sigma_A$ for the HEE does not give the right result in the case of the Lovelock gravity. In contrast we show that the Wald formula for the HEE gives the correct answer in the case of noncritical gravity. See [28] for the three-dimensional case.

In this section, we confirm that the HEE from the effective action (2.27) is identical with the HEE of noncritical gravity. This supports our claim on the nonlinear equivalence between Einstein gravity and noncritical gravity at the tree level. In order to establish the classical equivalence we obtain the HEE for noncritical gravity using two independent but related methods in the following. Firstly, we use the Wald formula on the AdS background and then we use the replica method on the Euclidean AdS soliton background. We find that both of them satisfy basic properties of the HEE as much as those from Einstein gravity. We also comment on Rényi entropy.

Let us consider the Wald formula on $\Sigma_A$ for the HEE in the case of the AdS background. In this approach, we take into account the, so-called, central charge function which appears in holographic c-theorem. This function is monotonic under the renormalization group flow and coincides with a certain central charge at the conformal points. To introduce this function, let us consider the following geometry which corresponds to the renormalization group flow in the CFT side,

$$ ds^2 = L^2 \left[ dr^2 + e^{2A(r)} \left( - dt^2 + d\mathbf{x}_{d-1}^2 \right) \right], $$

where AdS spacetime is represented by $A(r) = r$.

Borrowing the results from [9], one can see that the central charge function of $d$-
The dimensional dual CFT of noncritical gravity is given by

$$a_d(r) = \frac{\sigma \pi^{d/2}}{\Gamma\left(\frac{d}{2}\right) (A'(r))^{d-1}} \frac{L^{d-1}}{\kappa^2} \left[ 1 - \frac{d-1}{2m^2 L^2} A'^2(r) \right].$$  \hfill (4.2)

We propose that the HEE, $S_A$, for the region $A$ can be obtained by using the Wald formula in noncritical gravity. If $S_W$ denotes the ‘entropy’ expression by the Wald formula for the hypersurface $\Sigma_A$ as

$$S_W = -2\pi \int_{\Sigma_A} d^{d-1}x \sqrt{\gamma} E \frac{\partial L}{\partial R_{abcd}} \epsilon_{ab} \epsilon_{cd} \bigg|_{AdS},$$  \hfill (4.3)

our proposal simply means $S_A = S_W$ for noncritical gravity.

It was shown in [9] that, in general, $S_W$ can be expressed in terms of $a^*_d \equiv a_d(r)|_{AdS}$ as

$$S_W = \frac{2\pi}{\pi^{d/2} L^{d-1}} a^*_d \int_{\Sigma_A} d^{d-1}x \sqrt{\gamma} E.$$  \hfill (4.4)

In even $d$-dimensions $a^*_d$ is nothing but the central charge for the A-type trace anomaly and in odd dimensions it corresponds to the renormalized partition function of dual CFT.

Now, after plugging $a^*_d = q \frac{\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)} \frac{1}{\kappa^2}$ in $S_W$, one can see that the HEE of region $A$ in noncritical gravity is given by

$$S_A = S_W = q \frac{2\pi}{\kappa^2} \text{Area}(\Sigma_A).$$  \hfill (4.5)

This is identical with the HEE from the effective action (2.27). This also confirm the equivalence between noncritical gravity and Einstein gravity at low energy.

Secondly, we adopt the replica method on the D-dimensional Euclidean AdS soliton which may be written as

$$ds^2 = L^2 \left[ \frac{dr^2}{r^2 f(r)} + r^2 f(r) d\theta^2 + r^2 (dt_E^2 + dx_{d-2}^2) \right],$$  \hfill (4.6)

where

$$f(r) = 1 - \left( \frac{r_0}{r} \right)^d,$$  \hfill (4.7)

and $\theta$ is periodic, $\theta \sim \theta + \frac{4\pi}{d r_0}$, so that the near horizon geometry is regular.

Following [29] we consider the region $A$ given by half space $x_1 > 0$ at $t_E = 0$. In order to compute the HEE $S_A$ via the replica method, we introduce a conical singularity and place a deficit angle $\delta = 2\pi (1 - n)$ on the $(d - 1)$-dimensional surface $\Sigma_A$ defined by $x_1 = t_E = 0$. Then the curvature tensors behave like [30]

$$R_{MN} = R_{MN} + 2\pi (1 - n) \left( n^E_M n^E_N + n^x_M n^x_N \right) \delta(\Sigma_A), \quad R = R + 4\pi (1 - n) \delta(\Sigma_A),$$  \hfill (4.8)

where $R_{MN}$ and $R$ denote the Ricci tensor and Ricci scalar without singularities while two orthonormal vectors $n^E_M$ and $n^x_M$ are orthogonal to the surface $\Sigma_A$. 
The HEE through the replica method is given by
\[ S_A = -\frac{\partial}{\partial n} \log \left( \frac{Z_n}{Z_1^n} \right)_{n=1} \simeq \frac{\partial I_n}{\partial n} \bigg|_{n=1} - I_1, \]
(4.9)
where \( I_n \) is the on-shell Euclidean action value for \( n \)-sheeted AdS soliton with conical singularity of deficit angle \( 2\pi(1 - n) \). As noted in [29], appropriate surface terms should be included in the action to cancel the divergent part of the entanglement entropy. We have omitted these terms since these are not so relevant in our discussion.

The contribution to the HEE from the Einstein-Hilbert and the cosmological constant terms is given by
\[ S_A^{(EH)} = \frac{2\pi\sigma}{\kappa^2} \text{Area}(\Sigma_A). \]
(4.10)
The contribution from the higher curvature terms in noncritical gravity can be computed as
\[ S_A^{(R^2)} = \frac{\partial I_n^{(R^2)}}{\partial n} \bigg|_{n=1} - I_1^{(R^2)} = -\frac{\pi\sigma(d-1)}{\kappa^2 m^2 L^2} \text{Area}(\Sigma_A). \]
(4.11)
Therefore, total HEE becomes
\[ S_A = S_A^{(EH)} + S_A^{(R^2)} = \frac{2\pi}{\kappa^2} \text{Area}(\Sigma_A), \]
(4.12)
where \( q \) is given by (2.12). Once again, this shows the nonlinear, classical equivalence between noncritical gravity and Einstein gravity.

Another interesting quantity related to the entanglement entropy is the, so-called, Rényi entropy which is more refined entropy than the entanglement entropy. The Rényi entropy is defined by
\[ S_A^n \equiv \frac{1}{1-n} \ln \text{tr} \left[ \rho_A^n \right], \]
(4.13)
where \( n \) is a positive real number. Note that this quantity becomes entanglement entropy when \( n \to 1 \). Interestingly, Rényi entropy for spherically entangled domain in the CFT side can be described by thermal entropy and then by black hole geometry in the dual gravity side [31]. Concretely, the relevant geometry for the Rényi entropy computation on the spherical domain turns out to be topological black holes whose metric is given in the form of
\[ ds^2 = -\left( \frac{r^2}{L^2} f(r) - 1 \right) dt^2 + \frac{dr^2}{f(r)} + r^2 dH_{d-1}^2, \]
(4.14)
where \( dH_{d-1}^2 \) denotes the metric for the hyperbolic space with unit radius. In noncritical gravity, we find that the function \( f(r) \) takes the same form as the one in Einstein gravity:
\[ f(r) = 1 - \frac{c}{r^d}, \]
(4.15)
where the constant \( c \) is related to the position of the horizon, \( r_H \) as \( c = r_H^d - L^2 r_H^{d-2} \). It is straightforward to compute the thermal entropy by the Wald formula. Through the relation between Rényi and thermal entropy, one can check that holographic Rényi entropy in noncritical gravity is given by
\[ S_A^n (\text{noncritical}) = q S_A^n (\text{Einstein}), \]
(4.16)
where \( \mathcal{A} \) is a spherical domain.
5. Conclusion

In this paper we studied a class of higher derivative gravity in arbitrary dimensions, named as noncritical gravity, which is defined on asymptotically AdS spacetime. We found that noncritical gravity is classically equivalent to Einstein gravity on the same type of background at the full nonlinear level. The equivalence implies that the dual CFT of noncritical gravity is the same as the one of Einstein gravity in the large $N$ limit. In particular, $n$-point correlation functions of the energy-momentum tensor in the CFT corresponding to noncritical gravity should be the same as those from Einstein gravity.

From the side of noncritical gravity, we calculated the transport coefficients in the dual fluid and obtained the same form as those from Einstein gravity at the nonlinear level. As a byproduct we found the viscosity-to-entropy ratio which saturates the KSS bound to all orders in the coupling of higher derivative terms. We also found the second order transport coefficients to all orders in the coupling.

Furthermore we studied the HEE for noncritical gravity. We found that in this case the Wald formula can be used to compute the HEE. We also used the replica method and obtained consistent results on the HEE. Once again we found the results which agree with those from Einstein gravity.

All these results support our claim on the equivalence between noncritical gravity and Einstein gravity. We have shown that noncritical gravity is well-defined at low energy and can be used to study the boundary CFT.

It would be interesting to generalize four-dimensional $\mathcal{N} = 1$ NEW supergravity to higher dimensions which may correspond to the supersymmetrization of noncritical gravity. It would be also interesting to study gravity whose action contains even higher curvature terms of Ricci scalar and Ricci tensor only.

Acknowledgments

SHY would like to thank S. Nam, J.D. Park and Y. Kwon at Kyunghee Univ. for useful discussion. SH is supported in part by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MEST) with the grant number 2009-0074518 and the grant number 2009-0085995. SH, WJ and SHY are supported by the grant number 2005-0049409 through the Center for Quantum Spacetime(CQUeST) of Sogang University. JJ is supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MEST) with the grant number 2009-0072755.
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