Stellar physics with the ALHAMBRA photometric system

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Abstract. The ALHAMBRA photometric system was specifically designed to perform a tomography of the Universe in some selected areas. Although mainly designed for extragalactic purposes, its 20 contiguous, equal-width, medium-band photometric system in the optical wavelength range, shows a great capacity for stellar classification. In this contribution we propose a methodology for stellar classification and physical parameter estimation ($T_\text{eff}$, log $g$, [Fe/H], and color excess E($B – V$)) based on 18 independent reddening-free $Q$-values from the ALHAMBRA photometry. Based on the theoretical Spectral library BaSeL 2.2, and applied to 288 stars from the Next Generation spectral Library (NGSL), we discuss the reliability of the method and its dependence on the extinction law used.

1. Introduction
The ALHAMBRA (Advanced Large, Homogeneous Area Medium Band Redshift Astronomical) survey is a project aimed at getting a photometric data super-cube, which samples a cosmological fraction of the Universe (almost 4 square degrees in 8 discontinuous regions of the sky) with sufficient precision to draw an evolutionary track of its content and properties (see [1], for a more detailed description of the scientific objectives of the project).

The number, width and position of the filters composing an optimal filter set to accomplish
these objectives was discussed and evaluated by [2], yielding a filter set formed by a uniform system of 20 constant-width, non-overlapping, medium-band filters in the optical range [3] plus the three standard \( JHK_s \) near-infrared (NIR) bands. The capabilities of the optical ALHAMBRA photometric system to perform an accurate spectral classification and physical parameters estimation is examined making use of synthetic photometry on the theoretical spectra from BaSeL 2.2 [4], and 288 observed spectra from the Next Generation Spectral Library (NGSL) [5]. The methodology developed, \( Q \) Fit Algorithm (QFA), leads to estimates of \( T_{\text{eff}} \), log \( g \), \([\text{Fe/H}]\), and \( E(B-V) \) for a great variety of spectral and luminosity classes, from O- to M-type stars, and including hot white dwarfs and chemically peculiar stars. The method is based on reddening-free \( Q \)-parameters, as has been done with other photometric systems, such as [6] for the UPXYZVS Vilnius photometric system [7], or [8] for the UBV system.

2. Stellar classification and estimating physical parameters: \( Q \) Fit Algorithm

The methodology is mainly based on reddening-free \( Q \)-parameters, which do not depend on the distance or the reddening of the objects, but lie on the choice of an interstellar extinction law. This parameter was originally defined by [8] for the UBV system, and in the same way, for the ALHAMBRA system the definition would be:

\[
Q_{ijkl} = (m_i - m_j) - \frac{E_{ij}}{E_{kl}} (m_k - m_l),
\]

where \( m_i \) is the \( AB \)-magnitude in the ALHAMBRA band \( i \) estimated as:

\[
m_i = -2.5 \log \frac{\int f_{\nu} S_{\nu} d(\log \nu)}{\int S_{\nu} d(\log \nu)} - 48.6.
\]

\( S_{\nu} \) is the response function of the system corresponding to the atmosphere-telescope-filter-detector combination, and \( f_{\nu} \) is the flux of an object per unit frequency in erg cm\(^{-2}\) s\(^{-1}\) Hz\(^{-1}\). \( E_{ij} \) is the color excess of bands \( i \) and \( j \), where

\[
E_{ij} = A_i - A_j,
\]

and \( A_i \) the extinction in band \( i \).

2.1. Methodology

The \( Q \) Fit Algorithm has the following main steps:

(i) Generation of 18 independent \( Q \)-parameters from ALHAMBRA photometry: theoretical spectra and observed spectra.

(ii) Estimation of the main physical parameters, \( T_{\text{eff}} \), log \( g \), and \([\text{Fe/H}]\), by comparing observations to predictions from models.

(iii) Estimation of \( E(B-V) \) from 19 correlated ALHAMBRA colors.

However, other factors have to be taken into account to obtain the most accurate estimation of the parameters possible with this methodology, such as zero point corrections for the synthetic ALHAMBRA photometry of BaSeL models, or checking the dependence of the method on the interstellar extinction law used to generate the \( Q \)-values and to estimate the color excess of stars.

We present below a more detailed description of each step.
2.2. Generation of 18 independent Q-parameters

Since the optical photometric system is formed by 20 bands, we can generate 18 independent Q-parameters adopting for their definition three correlated bands instead of four different bands. Therefore, if we enumerate the ALHAMBRA magnitudes with the 20 different bands as $m_i$, for $i = 1 : 20$, the Q-values are defined by:

$$Q_{ijk} = (m_i - m_j) - \frac{E_{ij}}{E_{jk}} (m_j - m_k), i = 1 : 18, j = i + 1, k = i + 2.$$  \hspace{1cm} (4)

We generate these values for theoretical spectra from the BaSeL 2.2 library. Since the expected number of stars with metallicity below $-3$ in the ALHAMBRA Project is very small, we select BaSeL models with [Fe/H] above or equal to $-3$ to optimize the algorithm. For the white dwarfs, we compute the Q-values from synthetic ALHAMBRA colors of the white dwarf atmosphere models of [9].

On the other hand, the same 18 Q-parameters are generated for a subset of primary standard stars of the ALHAMBRA photometric system formed by 288 stars from the NGSL [5]. They are observed spectral energy distributions (SEDs) with synthetic AB ALHAMBRA magnitudes that form the basis of the zero-point calibration of the photometric system [3]. The subset of stars presents a wide range of $T_{\text{eff}}$, log $g$, and metallicity values ($3440 \text{ K} \leq T_{\text{eff}} \leq 44500 \text{ K}$, $0.45 \leq \log g \leq 7.5$, and $-3 \leq [\text{Fe/H}] \leq 0.7$), and a range of E$(B - V)$-values between 0 and 0.6. They include several hot white dwarfs, and a large number of peculiar stars of different spectral types. The physical parameters of the 288 stars are obtained from the literature. Indeed, most of them are well-known stars and so one can easily find several values of their $T_{\text{eff}}$, log $g$, [Fe/H], and color excess. We choose the mean of these distributions of parameters to compare with the values we estimate in the algorithm.

2.3. Estimates of main physical parameters, $T_{\text{eff}}$, log $g$, and [Fe/H]

Considering the 18-dimensional euclidean space formed by 18 independent Q-parameters for each object, we then search for the model that minimizes the euclidean distance between both the star and the model. The physical parameters of the model ($T_{\text{eff}}$, log $g$, and [Fe/H]) in case
of BaSeL models, and $T_{\text{eff}}$ and $\log g$ in case of the white dwarf models) are directly associated to the star.

Figure 1 shows an example of the fit. The star we try to classify is HD 284248. The figure shows the 18 $Q$-values of the star with red crosses, and the $Q$-values of the model in black circles. The values are generated using the extinction law of [10]. The x-axis shows the number of the $Q$-parameters in increasing order. The top labels of the graph list the physical parameters of the star from the literature at the left-hand side. The right-hand side labels provide the model parameters associated to the star by the algorithm. The root mean square error (RMSE) of the fit is also listed.

### 2.4. Estimates of $E(B-V)$

For the estimation of $E(B-V)$ we proceed as follows. From the synthetic photometry of the model we find 19 unreddened ALHAMBRA colors:

$$m_{0i} - m_{0(i+1)}, \ i = 1 : 19.$$  

(5)

Hence, with the reddened colors of the star, we can determine the color excess in each band as the difference between the color of the star and the unreddened color of the model:

$$E(m_i - m_{i+1}) = (m_{1i} - m_{1(i+1)}) - (m_{0i} - m_{0(i+1)}), \ i = 1 : 19.$$  

(6)

We can then generate 19 values of $E(B-V)$ from each different color excess by adopting an extinction law. The $E(B-V)$ associated to the star would be the median of these 19 values:

$$E(B-V) = \text{median}_i (\alpha_i \cdot E(m_i - m_{i+1})), \ i = 1 : 19.$$  

(7)

Returning to the former example, Fig. 2 shows the colors fit for the same star, HD 284248. The graph shows with red crosses the color 19 values of the star already dereddened with the color excess (obtained as we explain above). The black circles show the unreddened 19 colors of the associated model. The top label also lists the value of $E(B-V)$ from the literature, and the $E(B-V)$-value obtained with our method plus the error. The latter error value is the MAD of
the distribution of color excesses, divided by the square root of the number of elements (19 in this case). The RMSE of the fit is also shown.

2.5. Zero-point corrections

To improve the parameter estimation method, we correct the synthetic photometry of the theoretical spectra from BaSeL with synthetic photometry of the primary standard stars. We run the algorithm for the 288 NGSL stars and BaSeL models to compute the offsets. Next, we only select stars with RMSE-values in the $Q$ fit of less or equal than 0.03, thereby rejecting white dwarfs. The resulting average of 190 selected stars depends on the extinction law used by the algorithm. In particular, 191 stars are selected by using [10], 193 stars with [11], and 187 stars with [12]. For each of these stars and their associated model, and for each of the 19 colors, we then determine the differences between the colors of both. The zero-point corrections are the mean of these differences. The parameter determination of the algorithm becomes, in most cases, more accurate with the corrections, and yields smaller average RMSE-values for fits to all different spectral types. In most cases we obtain more accurate physical parameters compared to the literature values (they generally improve after correcting), which directly results from our estimates of the color excess. In particular, we also compute smaller RMSE-values for fits to the coolest stars (of G- to M-type) we expect to observe in the ALHAMBRA fields. The RMSE-values provide important information about the fit quality for estimating the parameters, which also depend on the grid density in the employed library of models. The RMSE-values are also related to the stellar spectral type. Cool stars have on average larger RMSEs because they show more complex $Q$ distributions or SEDs. Their range of $Q$-values or colors is also wider than for the hottest stars (see Figure 3).

3. Choice of the interstellar extinction law and results

The $Q$-parameters are very sensitive to the employed extinction law. Our fit method is suited to assess how the results vary with different extinction laws. In particular, we checked the validity of the method with three extinction laws of [11], [12], and [10]. For all extinction laws we use the value of $R_V = 3.1$ for Vega. The results of the $Q$ Fit Algorithm using different extinction laws

Figure 3. Example of $Q$s fit (upper panel) and colors fit (lower panel) of the K-type star HD 030834. Symbols are as in Fig. 2. The RMSE-values of both fits are larger compared to HD 284248, although the spectral classification of HD 030834 appears to be precise. The larger RMSEs result from the complexity of the $Q$-distribution and color values.
Table 1. Statistical indicators computed with the QFA results for 288 NGSL stars and BaSeL models, corrected for the photometric zero-points, and using three different extinction laws of [10], [11], and [12]. The errors are the standard deviations of the differences divided by the square root of the number of elements (see text).

|                     | Nandy et al | Cardelli et al | Fitzpatrick |
|---------------------|-------------|----------------|-------------|
| Mean(MAD(E(B − V))/\sqrt{19}) | 0.018       | 0.019          | 0.019       |
| Std(MAD(E(B − V))/\sqrt{19})   | 0.016       | 0.016          | 0.016       |
| | 0.069±0.005  | 0.062±0.007  | 0.062±0.006  |
| δ(E(B − V))         | −0.043±0.006| −0.037±0.008  | −0.038±0.006|
| δ(T_{\text{eff}})  | 583.20±93   | 454.63±61     | 511.74±69   |
| δ(\text{log } g)   | −505.32±94  | −386.29±63    | −422.23±71  |
| δ(\text{log } g)   | 0.57±0.03   | 0.51±0.03     | 0.51±0.03   |
| δ([Fe/H])           | −0.17±0.05  | −0.09±0.04    | −0.10±0.04  |
| δ([Fe/H])           | 0.49±0.02   | 0.44±0.03     | 0.49±0.03   |
| δ([Fe/H])           | −0.32±0.03  | −0.27±0.03    | −0.25±0.04  |

and BaSeL photometry, and corrected for the corresponding zero-points, are listed in Table 1 for various statistical indicators. The color excesses determined by the algorithm are the median of the 19 E(B − V)-values from the colors fit, which allows us to also calculate the MAD of the distribution. The mean of the 288 MADs is given in the first row, and the standard deviation in the second row. The first row is an estimate of the internal precision of the extinction law for computing the color excess. Based on this parameter we conclude that any of the three extinction laws can be adopted by the algorithm using our data because we find that the Nandy et al extinction law has the smallest value for this estimator, while the differences with Fitzpatrick and Cardelli et al are not significant. The third row lists the mean differences (using absolute values) between the color excess from the literature and our algorithm. The next row provides the same mean differences, however without using absolute values. Rows 5 and 6 are the same as the former two, but computed using the effective temperature-, log g-, and [Fe/H]-values. The differences using these values are not significant and do not allow us to conclude if one extinction law should be preferred over the other. Table 2 lists row Nos. 4, 6, 8, and 10 of Table 1 separately for hot, intermediate, and cool stars.

4. Conclusions
We examine the capability of the ALHAMBRA photometric system for stellar classification and for estimating atmospheric parameters. With the methodology we develop, the Q Fit Algorithm

1 The minimum RMSE fit with the Q-parameters yield in some cases negative E(B − V)-values. In the current version of the algorithm when E(B − V) is negative, we adopt the next best fit providing a positive E(B − V)-value. It produces a bias in the E(B − V)-estimates that directly affects the value of temperature parameter.
Table 2. Statistical indicators of the QFA results for 288 NGSL stars and BaSeL models corrected for the photometric zero-points, and using three different extinction laws of [10], [11], and [12], for hot, intermediate, and cool stars. The errors are the standard deviations of the differences divided by the square root of the number of elements (see text).

| Nandy et al | O- to B- | A- to F- | G- to M- |
|------------|----------|----------|----------|
| $\delta(E(B - V))$ | 0.048±0.03 | -0.012±0.007 | -0.071±0.008 |
| $\delta(T_{\text{eff}})$ | -2505±1149 | -257.4±62 | -378.9±27 |
| $\delta(\log g)$ | -0.46±0.14 | -0.21±0.05 | -0.09±0.09 |
| $\delta([\text{Fe/H}])$ | 0.44±0.26 | 0.23±0.10 | -0.46±0.04 |

| Cardelli et al | O- to B- | A- to F- | G- to M- |
|---------------|----------|----------|----------|
| $\delta(E(B - V))$ | 0.048±0.03 | -0.007±0.007 | -0.055±0.01 |
| $\delta(T_{\text{eff}})$ | -1008±821 | -239.9±66 | -342.8±61 |
| $\delta(\log g)$ | -0.36±0.18 | -0.17±0.04 | -0.002±0.06 |
| $\delta([\text{Fe/H}])$ | 1.02±0.31 | 0.57±0.09 | -0.42±0.04 |

| Fitzpatrick | O- to B- | A- to F- | G- to M- |
|-------------|----------|----------|----------|
| $\delta(E(B - V))$ | 0.041±0.03 | -0.007±0.007 | -0.056±0.008 |
| $\delta(T_{\text{eff}})$ | -1365±812 | -254.5±68 | -326.9±36 |
| $\delta(\log g)$ | -0.41±0.18 | -0.17±0.04 | -0.02±0.06 |
| $\delta([\text{Fe/H}])$ | 0.92±0.27 | 0.54±0.10 | -0.44±0.04 |

(QFA), it is possible to reliably estimate the $T_{\text{eff}}$-, $\log g$-, $[\text{Fe/H}]$-, and $E(B - V)$-values of stars for a wide variety of spectral and luminosity classes, from O- to M-type, including hot white dwarfs and chemically peculiar stars. The QFA is based on 18 independent reddening-free $Q$-parameters that allow us to the perform parameter estimates without considering the reddening or distances of the objects. To test our method we use a set of primary standard stars of the ALHAMBRA system, consisting of 288 stars from the NGSL. The parameters of these stars are obtained from the literature and compared to the parameters computed with the algorithm. While the $T_{\text{eff}}$, $\log g$, and $[\text{Fe/H}]$-values are directly assigned by the BaSeL models, the determination of the color excess (or the value of $E(B - V)$) is based on the 19 correlated ALHAMBRA colors, which therefore depends on the employed extinction law. Finally, we test the validity of our method for estimating physical parameters with a number of statistical indicators. We also investigate the dependence of the algorithm on the adopted extinction law. We find that the computed differences are not significant and do not allow us to conclude if one extinction law should be preferred over the other. An advanced analysis of our method and a detailed description of the physical parameters of the NGSL stars will be presented in a forthcoming paper.
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