Stability of the Kink State in a Stack of Intrinsic Josephson Junctions

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Abstract

A new dynamic state characterized by $(2m_1 + 1)\pi$ static phase kink with integers \{m\} is proposed recently in a stack of inductively coupled Josephson junctions. In the present paper, the stability of the phase kink state is investigated against many perturbations and it is shown that the kink state is stable. It is also discussed that the suppression of the amplitude of superconducting order parameter caused by the kink is weak.

Key words: intrinsic Josephson junctions, kink state, stability, terahertz radiation

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1. Introduction

The experimental breakthrough\cite{1,2} on the terahertz radiation from a mesa of Bi\textsubscript{2}Sr\textsubscript{2}CaCu\textsubscript{2}O\textsubscript{8+\delta} (BSCCO) single crystal has triggered tremendous emission. Soon after the experiments, it is proposed that a new dynamic state in a stack of inductively coupled Josephson junctions is probably responsible for the experimental observations\cite{3,4}. The new dynamic state is characterized by $(2m_1 + 1)\pi$ phase kinks that are stacked along the c direction and are localized at the nodes of electric field, where \{m\} are integers. To date, this kink state is the only solution that can successfully explain the key observations in the experiments\cite{1,2}: first, the radiation frequency and voltage obey the ac Josephson relation; secondly, strong and coherent emission appears at the cavity resonances. In the present paper, we investigate the stability of the kink state.

2. Results and discussion

The dynamics of the gauge invariant phase difference in BSCCO is properly described by the inductively coupled sine-Gordon equations\cite{5,6}

\[ \partial^2_t \Phi_l = (1 - \zeta \Delta^2) [\sin \Phi_l + \partial^2_x \Phi_l + \beta \partial_x \Phi_l - J_{\text{ext}}], \]

where the first is the rotating phase with voltage $\omega$, the second term the static phase kink and the last term the plasma oscillation with $k_1 \equiv \pi/L \gg 1$ with $L$ being the length of junction. Here we have considered the voltage near the first cavity mode. Because of the huge inductive coupling, $P_i^l$ runs sharply from 0 to $(2m_1 + 1)\pi$ in the narrowed region of width $1/\sqrt{\epsilon}$. Approximating $P_i^l$ as a step function, we obtain $A = 4/\pi(\omega^2 - k_1^2 - \beta \omega)$. In the following, we consider the region where $A < 1$.

To check the stability of the kink state Eq. (2), we add a small perturbation to the solution, $P_i^l = P_i^l + \theta_i$ with $\theta_i \ll 1$. The kink state is stable if the perturbation dies out and is unstable if it increases when $t \to +\infty$. Substituting $P_i^l$ into Eq. (1), we obtain the equation governing the evolution of the perturbation

\[ \partial^2_t \theta_l = (1 - \zeta \Delta^2) [\exp(i\omega t + P_i^l) + \exp(-i\omega t + P_i^l)] - \frac{\beta}{\epsilon} \cos(\ell k_1 x) \exp(-iP_i^l) \theta_l + \beta \partial_t \theta_l + \partial_x^2 \theta_l]. \]

Here we investigate the region that $\omega \gg 1$, $A \ll 1$ and the variation of $\theta_i$ is much faster than that of $P_i^l$ along the c direction in the following calculation. The solution of $\theta_i$ can be expressed as\cite{9}

\[ \theta_i(x, t) = \sum_q \left[ \theta_{q+} \exp(i\omega t) + \theta_{q-} \exp(-i\omega t) \right] \exp(i q l) \exp(-i \Omega t). \]

Here the complex eigenfrequency $\Omega$ is assumed to be small $\Omega \ll 1$. The higher frequency harmonics $\Omega \pm m \omega$ with $m > 1$ in Eq. (4) are small in the region of $\omega \gg 1$ and are neglected. Periodic boundary condition is imposed along the c direction, and hence $q = 2m\pi/N$ with $m$ being an integer and $N$ the total number of junctions. The kink state is stable if and only if $\text{Im} \Omega < 0$ for all $q$.

With the condition that $P_i^l$ has slow variation along the c axis, we have

\[ (1 - \zeta \Delta^2) \exp(iP_i^l) \theta_l \approx \exp(iP_i^l) \sum_q a_q \theta_q \exp(i q l), \]
where \( a_q \equiv 1 + 2\zeta(1 - \cos q) \). Furthermore, because \( P^\prime \) is almost a step function, \( \exp(iP^\prime) \approx \cos P^\prime \equiv 1 - 2\Theta(x - L/2) \) where \( \Theta(x) \) is the Heaviside step function. Substituting Eq. (4) into Eq. (3) and using Eq. (5), we have for the frequency components \( \omega \pm \Omega \) and \( \Omega \)

\[
\begin{align*}
\tilde{\omega}_q^2 \theta_q &= a_q \left[ \frac{\cos P^\prime}{2} (\theta_{q+} + \theta_{q-}) - \frac{\cos P^\prime}{2} A \cos(k_1 x) \tilde{\theta}_q - \Omega^2 \tilde{\theta}_q \right] \\
\tilde{\omega}_q \theta_{q+k} &= a_q \left[ \frac{\cos P^\prime}{2} \theta_q - \omega^2 \theta_{q+k} \right],
\end{align*}
\]

(6)

where a small term proportional to \( A \) in the coefficient of \( \theta_{q+k} \) in Eq. 7 is neglected and

\[
\tilde{\omega}_q^2 \equiv \Omega^2 + \beta \Omega; \quad \omega^2 \equiv (\pm \omega - \Omega)^2 - \beta i(\pm \omega - \Omega).
\]

(8)

By inspecting Eq. (6), the length scale over which \( \theta_q \) varies is of order of \( 1/(\Omega \sqrt{\alpha_0}) \gg 1 \). To solve Eq. (7), we resort to multimodes expansion

\[
\cos P^\prime = \sum_{n=0} a_n \cos(k_n x), \quad \theta_{q+k} = \sum_{n=0} b_{n+k} \cos(k_n x),
\]

(9)

with \( a_n = \frac{1}{n} \sin(\frac{n\pi}{2}). \) Substituting Eq. (9) into Eq. (7) and neglecting the coordinate dependence of \( \theta_q \), we obtain

\[
b_{n+k} = \frac{a_n \theta_q}{\omega^2 - k_n^2}
\]

(10)

Inserting Eq. (9) into Eq. (6), we obtain the equation for \( \tilde{\theta}_q \)

\[
\partial_t \tilde{\theta}_q + a_q \tilde{\omega}_q \tilde{\theta}_q
\]

\[
= a_q \left[ \frac{\cos P^\prime}{2} \sum_n (b_{n+k} + b_n) \cos(k_n x) - \frac{\cos P^\prime}{2} A \cos(k_1 x) \theta_q \right].
\]

(11)

In short, the kink state is stable and has negligibly small effect on the superconductivity.

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