Hadron tomography for pion and its gravitational form factors

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Outline

Generalized distribution amplitude (GDA) of pion

- Motivation
- GDA in two-photon process
- GDA analysis for Belle data
Generalized Parton Distributions (GPDs) provide information on $\Delta L$ to solve the proton puzzle!

Generalized Distribution Amplitudes (GDAs) $\leftrightarrow$ s-t crossing of GPDs

Pion GDAs are investigated.

GDA carry many important physical quantities of the hadron, such as distribution amplitudes (DAs) and timelike form factors.
Generalized distribution amplitude for pion

In the process $\gamma \gamma^* \rightarrow h \bar{h}$, an hard part describing the process $\gamma \gamma^* \rightarrow q \bar{q}$ with produced collinear and on-shell quark, and a soft part describing the production of the hadron $h$ pair from a $q \bar{q}$. This soft part is called Generalized Distribution Amplitude (GDA).

GDA is an important quantity of hadron, it is defined as

$$\Phi^q(z,\xi,W^2) = \int \frac{dx^-}{2\pi} e^{-izp^+x} \left\langle h(p)\bar{h}(p')|\bar{q}(x^-)\gamma^+q(0)|0\right\rangle$$

$$z = \frac{k^+}{p^+}, \quad \xi = \frac{p^+}{p^+}, \quad s = W^2 = (p+p')^2 = P^2$$

M. Diehl, Phys. Rep. 388 (2003) 41.
M. Diehl and P. Kroll, EPJC 73 (2013) 2397.
GDA is closely related to generalized parton distribution (GPD) by the s-t crossing, so GDA could provide another way to obtain GPD information.

\[ \Phi^q(z, \xi, W^2) \leftrightarrow H^q \left( x = \frac{1-2z}{1-2\xi}, \xi = \frac{1}{1-2\xi}, t = W^2 \right) \]

GPD can be used to study the proton spin puzzle!

\[
\int \frac{dx^-}{2\pi} e^{-iz(P^+x)} \left\langle h(p_2) | \bar{q}(x^-) \gamma^+ q(0) | h(p_1) \right\rangle \\
= \frac{1}{2\bar{P}^+} \left[ H^q(x, \xi, t) \bar{u}(p_2) \gamma^+ u(p_1) + E^q(x, \xi, t) \bar{u}(p_2) \frac{i\sigma^{+\alpha} \Delta^+}{2m} u(p_1) \right]
\]

\[ \bar{P} = (p_1 + p_2)/2, \Delta = p_2 - p_1, x = \frac{-q_1^2}{2p_1 q_1}, \xi = \frac{\Delta^+}{p_1^+ + p_2^+} \]

M. Diehl, Phys. Rep. 388 (2003), 41.
H. Kawamura and S. Kumano, PRD 89 (2014), 054007.
The cross section of process $\gamma^*\gamma \rightarrow \pi^0\pi^0$

$$d\sigma = \frac{1}{4} \frac{1}{4\sqrt{q_1^2 q_2^2 - q_1^2 q_2^2}} \sum_{\lambda_1 \lambda_2} | -iT_{\mu\nu}\epsilon^\mu(q_1)\epsilon^\nu(q_2) |^2 d\Phi_2$$

$$d\sigma = \pi\alpha^2 \sqrt{1 - \frac{4m^2}{s}} | A_{++} |^2 \sin \theta d\theta$$

$A_{\lambda_1 \lambda_2}$ is the helicity amplitude, and there are 3 independent helicity amplitudes, they are $A_{++}, A_{0+}$ and $A_{+-}$. The leading-twist amplitude $A_{++}$ has a close relation with the generalized distribution amplitude (GDA) $\Phi^q(z, \xi, W^2)$.

$$A_{\lambda_1 \lambda_2} = T_{\mu\nu}\epsilon^\mu(\lambda_1)\epsilon^\nu(\lambda_2) / e^2$$

$$A_{++} = \sum_q \frac{e^2}{2} \int_0^1 dz \frac{2z - 1}{z(1-z)} \Phi^q(z, \xi, W^2)$$

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL 81 (1998) 1782.
M. Diehl, T. Gousset and B. Pire, PRD 62 (2000) 07301.
Higher twist and higher order contributions

Higher-twist contribution $A_{0+}$ requires a helicity flip along the fermion line, and it decreases as $1/Q$. Higher-order contribution $A_{+-}$ contributes with the GDA of gluon, since $A_{+-}$ indicates the angular momentum $L_z = 2$. Therefore $A_{+-}$ is suppressed by running coupling constant $\alpha_s$.

\[ \gamma^\ast (q_1 \lambda_1) \rightarrow \pi^0 (p_1) \]
\[ \gamma (q_2 \lambda_2) \rightarrow \pi^0 (p_2) \]

Gluon GDA

M. Diehl, T. Gousset and B. Pire, PRD 62 (2000) 07301.
N. Kivel, L. Mankiewicz and M.V. Polyakov PLB 467 (1999) 263.
**GDA expression**

At very high energy $Q^2$, we can have the asymptotic form of the GDA

$$\sum_q \Phi^+_q(z, \xi, W^2) = 18 n_f z (1 - z) (2z - 1) [B_{10}(W) + B_{12}(W) P_2 (2\xi - 1)]$$

$$= 18 n_f z (1 - z) (2z - 1) [\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2 (\cos \theta)]$$

The GDAs are related to the energy-momentum form factor in the timelike region.

$$\int dz (2z - 1) \Phi^+_q(z, \xi, W^2) = \frac{2}{(P^+)^2} \langle \pi^+(p_1) \pi^-(p_2) | T^{++}_q(0) | 0 \rangle$$

where the energy-momentum form factor for quarks is defined as

$$\langle \pi^0(p_1) \pi^0(p_2) | T^{\mu\nu}(0) | 0 \rangle = \frac{1}{2} \left[ (s g^{\mu\nu} - P^\mu P^\nu) \Theta_1 + \Delta^\mu \Delta^\nu \Theta_2 \right]$$

$$P = p_1 + p_2 , \Delta = p_1 - p_2$$

By using this sum rule we can obtain

$$B_{12}(0) = \frac{5 R_\pi}{9}$$

where $R_\pi$ is the momentum fraction carried by quarks in the pion.

M. V. Polyakov, NPB 555 (1999) 231.
M. V. Polyakov and C. Weiss PRD 60 (1999) 114017.
In 2016, the Belle Collaboration released the measurements of differential cross section for $\gamma^*\gamma \rightarrow \pi^0\pi^0$. The GDAs can be obtained by analyzing the Belle data.

![Graphs of differential cross section](image)

In these figures, the resonance $f_2(1270)$ is clearly seen around $W = 1.25$ GeV, however, other resonances are not clearly seen due to the large errors.

M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003.
Scale violation of GDA based on Belle data

\[
\frac{(Q^2 + s)d\sigma}{\beta d|\cos\theta|} \propto \Phi_{\pi^0\pi^0}(z, \cos\theta, W, Q)^2
\]

The scale dependence of the Belle data. We have red color for \( W = 0.525 \text{ GeV} \), blue color for \( W = 0.975 \text{ GeV} \), and green color for \( W = 1.550 \text{ GeV} \).

The scaling violation of the GDAs is not so obvious in the Belle data on account of the large errors, so that the \( Q^2 \)-independent GDAs could be used in analyzing the Belle data.
**$Q^2$-independent (asymptotic form) GDAs**

\[
\sum_q \Phi^+_q(z, \xi, W^2) = 18n_f z(1-z)(2z-1)[B_{10}(W) + B_{12}(W)P_2(2\xi-1)]
\]

\[
= 18n_f z(1-z)(2z-1)[\tilde{B}_{10}(W) + \tilde{B}_{12}(W)P_2(\cos\theta)]
\]

\[
\tilde{B}_{10}(W) = \overline{B}_{10}(W)e^{i\delta_0}, \tilde{B}_{12}(W) = \overline{B}_{12}(W)e^{i\delta_2}
\]

In the above equation $\delta_0$ and $\delta_2$ and are the $\pi\pi$ elastic scattering phase shifts in the isospin=0 channel (see the figure). Above the KK threshold, the additional phase is introduced for S-wave.

**The S wave and D-wave $\pi\pi$ scattering phase shifts.**

M. Diehl, T. Gousset and B. Pire, PRD 62 (2000) 07301.
P. Bydzovsky, R. Kamiski and V. Nazari, PRD 90 (2014), 116005; PRD 94 (2016), 116013.
Resonance effects

In the process $\gamma^*\gamma \rightarrow \pi^0\pi^0$, the $\pi^0\pi^0$ can be produced through intermediate meson state $h$. The $q \bar{q} \rightarrow h$ amplitude should be proportional to the decay constant $f_h$ or the distribution amplitude (DA), and the $h \rightarrow \pi^0\pi^0$ amplitude can be expressed by the coupling constant $g_{h\pi\pi}$. These resonance contributions read

\[ B_{12}(W) = \beta^2 \frac{10 g_{f_2\pi\pi} f_{f_2} M_{f_2}^2}{9 \sqrt{2} \sqrt{(M_{f_2}^2 - W^2)^2 - \Gamma_{f_2}^2 M_{f_2}^2}} \]

\[ B_{10}(W) = \frac{5 g_{f_0\pi\pi} f_{f_0}}{3 \sqrt{2} \sqrt{(M_{f_0}^2 - W^2)^2 - \Gamma_{f_0}^2 M_{f_0}^2}} \]

The resonance effects play an important role in the resonance regions.
We adopt a simple expression of GDA to analyze Belle data, here resonance effects of $f_0(500)$ and $f_2(1270)$ are introduced.

$$
\Phi^+_q(z, \xi, W^2) = N_h z^\alpha (1 - z)^\alpha (2z - 1)[\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos \theta)]
$$

$$
\tilde{B}_{10}(W) = \left[ \frac{-3 + \beta^2}{2} \frac{5R_\pi}{9} F_h(W^2) + \frac{5 g_{f_0\pi\pi f_0}}{3\sqrt{2} \sqrt{\left( M_{f_0}^2 - W^2 \right)^2 - \Gamma_{f_0}^2 M_{f_0}^2}} \right] e^{i\delta_0}
$$

$$
\tilde{B}_{12}(W) = \left[ \beta^2 \frac{5R_\pi}{9} F_h(W^2) + \beta^2 \frac{10 g_{f_2\pi\pi f_2} M_{f_2}^2}{9\sqrt{2} \sqrt{\left( M_{f_2}^2 - W^2 \right)^2 - \Gamma_{f_2}^2 M_{f_2}^2}} \right] e^{i\delta_2}
$$

$$
F_h(W^2) = \frac{1}{\left[ 1 + \frac{W^2 - 4m_\pi^2}{\Lambda^2} \right]^{n-1}}
$$

The function $F_h(W^2)$ is the form factor of the quark part of the energy-momentum tensor, and the parameter $\Lambda$ is the momentum cutoff in the form factor. The parameter $n$ is predicted as $n = 2$ at very high energy, because we have $d\sigma/d|\cos \theta|/\sim 1/W^6$ by the counting rule. In the asymptotic limit, $\alpha = 1$. 
Results

By analyzing the Belle data, the values of parameters are obtained

|       | Set 1               | Set 2               |
|-------|--------------------|--------------------|
| $\alpha$ | 0.801±0.042       | 1.157±0.132       |
| $\Lambda$ | 1.602±0.109       | 1.928±0.213       |
| $a$    | 3.878±0.165        | 3.800±0.170        |
| $b$    | 0.382±0.040        | 0.407±0.041        |
| $f_{f_0}$ | -------           | 0.0184±0.034      |

\[
\frac{\chi^2}{NOF} = 1.22 \quad \frac{\chi^2}{NOF} = 1.09
\]

Set 1 is the analysis without the resonance effect $f_0(500)$, in Set 2 the resonance effect $f_0(500)$ is included.

S. Kumano, Qin-Tao Song and O. Teryaev, PRD 97 (2018) 014020.
The $W$ dependence of the differential cross section (in units of nb), and in comparison with Belle data.
The $W$ dependence of the differential cross section (in units of nb), and in comparison with Belle data.
By considering the following sum rule, we can also obtain the energy-momentum form factors for pion.

\[
\int dz(2z-1)\Phi^{+}_q(z,\xi,W^2) = \frac{2}{(P^+)^2} \left\langle \pi^0(p_1)\pi^0(p_2)|T^{++}_q(0)|0\right\rangle
\]

\[
\left\langle \pi^0(p_1)\pi^0(p_2)|T^{\mu\nu}(0)|0\right\rangle = \frac{1}{2} \left[ (sg^{\mu\nu} - P^\mu P^\nu)\Theta_1 + \Delta^\mu\Delta^\nu\Theta_2 \right]
\]

\[
\Theta_1 = \frac{3}{5}(\tilde{B}_{12} - 2\tilde{B}_{10}), \quad \Theta_2 = \frac{9}{5\beta^2}\tilde{B}_{12}
\]

\(\Theta_1 \rightarrow\) Mechanical (pressure and shear force)  
\(\Theta_2 \rightarrow\) Mass

M. V. Polyakov, NPB 555 (1999) 231.  
M. V. Polyakov and C. Weiss PRD 60 (1999) 114017.  

S. Kumano, Qin-Tao Song and O. Teryaev, PRD 97 (2018) 014020.
Timelike form factor $\rightarrow$ Spacelike form factor (pion radius): dispersion relation

$$F(t) = \int_{4m^2}^{\infty} \frac{ds \, \text{Im}(F(s))}{\pi \ s - t - i\varepsilon}$$

The spacelike form factors $\Theta_1$ and $\Theta_2$

Fourier Transform of $\Theta_1$ and $\Theta_2$

S. Kumano, Qin-Tao Song and O. Teryaev, PRD 97 (2018) 014020.
Radius can be obtained by the following equation

\[ \langle r^2 \rangle = 6 \int_{4m^2}^{\infty} \frac{\text{Im}(F(s))}{s^2} \]

\[ \sqrt{\langle r^2 \rangle} = 0.69 \text{ fm for } \Theta_2 \text{ Mass radius} \]

\[ \sqrt{\langle r^2 \rangle} = 1.45 \text{ fm for } \Theta_1 \text{ Mechanical radius (pressure and shear force)} \]

In our analysis we introduce the additional phase for S-wave above the KK threshold. However, the additional phase could be add to D-wave phase above the threshold, in this case we have

**Mass radius: 0.56-0.69 fm, Mechanical radius: 1.45-1.56 fm**
Summary

- By analyzing the Belle data the pion GDAs are obtained, and the obtained GDAs can also give a good description of experimental data.

- The energy-momentum form factors for pion are calculated from the GDA of pion.

- This is the first finding on gravitational radii of hadrons from actual experimental measurements: The mass radius (0.56-0.69fm) is obtained.

Thank you very much