Gravitational Lorentz Violation
and Superluminality
via
AdS/CFT Duality

Raman Sundrum
Department of Physics and Astronomy, Johns Hopkins University
3400 North Charles St., Baltimore, MD 21218 USA

Abstract

A weak quantum mechanical coupling is constructed permitting superluminal communication within a preferred region of a gravitating $AdS_5$ spacetime. This is achieved by adding a spatially non-local perturbation of a special kind to the Hamiltonian of a four-dimensional conformal field theory with a weakly-coupled $AdS_5$ dual, such as maximally supersymmetric Yang-Mills theory. In particular, two issues are given careful treatment: (1) the UV-completeness of our deformed CFT, guaranteeing the existence of a “deformed string theory” $AdS$ dual, and (2) the demonstration that superluminal effects can take place in $AdS$, both on its boundary as well as in the bulk. Exotic Lorentz-violating properties such as these may have implications for tests of General Relativity, addressing the cosmological constant problem, or probing “behind” horizons. Our construction may give insight into the interpretation of wormhole solutions in Euclidean $AdS$ gravity.
1 Introduction

Relativistic invariance is a pillar of the fundamental laws of physics. It is worth questioning whether this structure is exact or just a (very good) approximation. The issue is subtle in the context of General Relativity which promotes Poincare invariance to a local symmetry, whose breaking therefore requires some sort of Higgs mechanism. While low-energy effective field theories with partial Higgsing of General Relativity \cite{1,2}, consistent with observation, have been constructed, their incorporation into UV-complete theories of quantum gravity such as string theory has not been demonstrated.

In the description in terms of a Higgs mechanism, relativistic invariance is respected by the dynamics and broken only by the state of some “Higgs” degrees of freedom. However, such a broad categorization encompasses some rather familiar and unremarkable cases. For example, the preferred frame occupied by the cosmic microwave background can formally be thought of as spontaneously breaking Lorentz invariance, and by going to co-moving coordinates general coordinate invariance is effectively “Higgsed”. But there may also be exotic Higgs phases, breaking relativistic invariance with much more dramatic implications. There is of course the possible phenomenology of measurable quantitative deviations from standard expectations of General Relativity. See Refs. \cite{3} for examples. But there may be important qualitative effects as well. In Lorentz invariant theories, superluminal propagation and interactions in one reference frame would imply acausal effects in other frames. But this need not be the case for Lorentz-violating interactions, which may have a preferred frame in which causal unitary evolution is defined. Superluminal interactions would be liberating in our vast universe, and might also allow us to probe “behind” black hole and cosmological horizons, normally off limits by relativistic causality. If Lorentz violation is significant, it can go beyond being merely a probe of horizons, it can modify their character \cite{4}. The observation that some apparently “innocent” effective field theories display superluminal behavior \cite{5} would no longer immediately be a disqualification. Lorentz-violation in General Relativity may also help us understand some of gravity’s other mysteries. For example, in Ref. \cite{6} it was shown that large Standard Model quantum contributions to dark energy can be canceled by a symmetry, “Energy-Parity”, but the longevity of flat empty space then requires a Lorentz-violating short-distance breakdown of General Relativity. Finally, if relativistic invariance is an approximation, it may well be an emergent (accidental) symmetry, simple examples of which occur in the long-wavelength approximation of some condensed matter systems. The question then arises, what more fundamental structure or symmetry underlies Relativity.

In this paper, we exploit the powerful approach to quantum gravity offered by the AdS/CFT correspondence \cite{7} (reviewed in Ref. \cite{8}), to engineer UV-complete gravitational dynamics exhibiting weak breaking of (local) Poincare invariance and superluminal action-at-a-distance. The construction is made on the CFT side of the correspondence, specifically by perturbing strongly-coupled large-$N_{\text{color}}$ $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory by suitably chosen spatially non-local operators. The advantage of working in terms of these holographic degrees of freedom is that it finesse the tricky issues of breaking gauge symmetries, such as general coordinate invariance, that appear in the dual description of
AdS gravity. In particular, the correspondence relates the breaking of gauge symmetry in AdS to breaking of global symmetry in the CFT, which is much easier to understand. This feature is illustrated by two well-known examples (in which, however, the relevant Higgs dynamics in AdS takes quite familiar and unexotic forms). The first example is given by simply adding a spacetime-dependent mass term for some SYM scalars, \( m^2(x) \text{Tr} A^2(x) \), to the CFT Lagrangian, thereby explicitly breaking the global Poincare invariance (or conformal invariance for that matter). In the dual AdS picture the perturbation is reflected by gravitating spacetime-dependent fields turned on in the bulk, sourced from the AdS boundary. But even away from the boundary, these fields break AdS isometries. This is a physical effect, apparent to a bulk observer. Theoretically, one can phrase this as the turned-on fields “Higgsing” the bulk general coordinate invariance, although we usually do not adopt this language. The second example is provided in Ref. [9] which studied a supersymmetric field theory with a global \( U(1)_R \) symmetry, explicitly broken by an anomaly. The field theory has a supergravity dual in which the global \( U(1)_R \) is mapped to an AdS gauge field. The explicit breaking in the field theory must map to a Higgsing of the AdS gauge field. Therefore we know on general grounds that the requisite Higgs condensate must appear on the AdS side of the dual, and Ref. [9] shows in more detail that this is the case.

Our final construction has the following properties:

(i) The deformation of SYM takes the form of a manifestly hermitian perturbation to the SYM Hamiltonian, thereby guaranteeing unitary and time-local quantum mechanical evolution.

(ii) The deformation explicitly violates the Poincare invariance of SYM. It mediates superluminal processes at first order.

(iii) To all orders in perturbation theory, the deformed theory is UV finite (no new divergences beyond the renormalized SYM CFT). The finiteness properties are related to the spatial non-locality of our perturbation Hamiltonian. In particular, the deformed Hamiltonian is well-defined.

(iv) The deformed SYM theory is indeed a weak perturbation of all SYM processes, viewed for a finite period of time. That is, perturbation theory can be trusted. This implies that there must be some gravity dual of our deformed theory, including the dual of (ii).

(v) Some degree of superluminality in the gravity dual is taking place in the AdS bulk, not just at the AdS boundary. Since the bulk spacetime gravitates, the necessary Higgs set-up must appear, but we are unable to give its explicit form.

(vi) Our deformed Hamiltonian is a sum of squares of hermitian operators and therefore has a energy bounded from below. But we cannot prove that the SYM vacuum is a ground state of the deformed theory. For instance, Ref. [10] proves a positive energy theorem in
the gravitational dual based on the assumption of relativistic causality of the CFT, but our deformation violates this assumption. Instead, it is likely that the SYM vacuum corresponds to an excited state (not even an energy eigenstate), and therefore can decay into the true ground state of the full Hamiltonian. At first sight this appears problematic since we wish to consider propagation of simple objects in the “recognizable” AdS vacuum, with weak superluminal corrections. The decay of the SYM vacuum implies the decay of the AdS vacuum into an unknown state. In such a state we would not necessarily know what spacetime metric to use to even define superluminality.

(vii) Fortunately, the vacuum decay rate can be controlled by the weakness of our perturbation, and we can allow the perturbation to be turned on for only a finite duration, thereby ensuring that most regions of space do not experience vacuum decay. This still leaves a small but non-zero amplitude for superluminal propagation and interaction of bulk quanta in this (approximate) AdS(CFT) vacuum. In this way, we engineer rare breakdowns of the general relativistic approximation, with long-range superluminal consequences.

The simultaneously weak and long-range character of the superluminal Lorentz-violating interactions distinguishes our construction from earlier Lorentz-violating deformations of SYM and their gravitational duals [11], and points to how such striking effects might be compatible with real world gravity. It is possible that the puzzling CFT interpretation of wormhole solutions in Euclidean AdS gravity [12] [13] [14] is related to constructions similar in spirit to ours. However in this paper we work in Lorentzian signature spacetime. Several Lorentzian aspects of the AdS/CFT correspondence are discussed in Ref. [15].

We work up to our construction in the following stages. In Section 2, we illustrate the long-wavelength emergence of relativistic invariance, in the absence of gravity, in a simple quantum lattice model which fundamentally has a preferred reference frame. We then add a perturbation that leads to instantaneous action-at-a-distance in what would otherwise have been the continuum relativistic regime. In Section 3, we generalize this action-at-a-distance within a long-wavelength effective field theory containing gravity. The notion of “instantaneous” is ill-defined in a general relativistic context, but is replaced by superluminality. The requisite combination of Higgs effects is described. In Section 4, we review the emergent nature of (higher-dimensional) quantum general relativity, via the AdS/CFT correspondence, from $\mathcal{N} = 4$ SYM. We then describe the generalization of Section 2 to SYM. This section is primarily intended for contrast with Section 5, where our main results appear. The AdS dual set-up contains superluminality in a general relativistic context, but localized to the $AdS_5$ boundary. The deformation also leads to UV divergences, that can however be treated by renormalization. We explain how AdS/SYM vacuum decay can be suppressed by making the SYM deformation act for a finite duration. In Section 5, we describe a perturbation of the SYM Hamiltonian whose AdS dual contains superluminality in the AdS “bulk”. We show UV-finiteness and perturbativity of our deformation to SYM, and again indicate how SYM vacuum decay can be suppressed by making the deformation act temporarily. Section 6 provides our conclusions.
2 Emergent Special Relativity and Action-at-a-distance

Consider a very simple example. We start with an underlying Hamiltonian for a theory without Poincare invariance living on a spatial cubic lattice (continuous time),

\[ H = \frac{1}{2} \sum_{\vec{n}} \{ \Pi_{\vec{n}}^2 + \sum_{i=1}^{3} (\phi_{\vec{n}+\hat{i}} - \phi_{\vec{n}})^2 \}, \]  

(1)

where \( \hat{i} \) are spatial unit vectors, and \( \vec{n} \) are lattice points. In familiar fashion, for long wavelength modes of this system we arrive at the approximately relativistic theory of a massless free scalar field,

\[ H \approx \frac{1}{2} \int d^3\vec{x} \{ \Pi^2(\vec{x}) + (\partial_i \phi(\vec{x}))^2 \}. \]  

(2)

Quantization of both Hamiltonians, written above in terms of Schrodinger picture operators, is of course straightforward. Even in this simplest of examples, the underlying theory contains couplings, \( \phi_{\vec{n}+\hat{i}} \phi_{\vec{n}} \), which instantaneously connect two points at finite spatial separations.

But we can arrange for a more drastic breakdown of Poincare invariance right in the midst of the relativistic regime, for example,

\[ H = \frac{1}{2} \sum_{\vec{n}} \{ \Pi_{\vec{n}}^2 + \sum_{i=1}^{3} (\phi_{\vec{n}+\hat{i}} - \phi_{\vec{n}})^2 \} + \epsilon \left( \sum_{\vec{n}} J_{\vec{n}} \phi_{\vec{n}} \right)^2, \]  

(3)

where \( J \geq 0 \) is a lattice function with finite support, and normalized to

\[ \sum_{\vec{n}} J_{\vec{n}} = 1. \]  

(4)

The entire perturbation to the Hamiltonian is obviously also \( \geq 0 \) and minimized at the vacuum \( \langle \phi \rangle = 0 \). If the support of \( J \) has a typical size \( L \gg \) lattice-spacing \( \equiv 1 \) and \( J \) is smooth on that scale, then the perturbation can be made weak by taking \( \epsilon \ll L \). In the support region of \( J \), a lattice quantum can be absorbed by the perturbation in one location \( \vec{n}_1 \) and be instantaneously emitted at a distant location \( \vec{n}_2 \).

Note that because energy is conserved and because \( J \) is smooth on the lattice scale, soft incoming quanta necessarily scatter (non-locally in space) to soft quanta, so that the continuum long-wavelength approximation is not broken by the perturbation. There is therefore a good continuum approximation to this model,

\[ H \approx \frac{1}{2} \int d^3\vec{x} \{ \Pi^2(\vec{x}) + (\partial_i \phi(\vec{x}))^2 \} + \epsilon \left( \int d^3\vec{x} J(\vec{x}) \phi(\vec{x}) \right)^2, \]  

(5)
where $J$ is normalized as

$$
\int d^3 \vec{x} J(\vec{x}) = 1.
$$

(6)

Again, the weak $\epsilon$ coupling can absorb relativistic quanta and instantaneously emits such quanta far away. Such effects can appear acausal in another relativistic reference frame, but that frame is not co-equal to the defining one above. We see that the lattice structure is irrelevant to the question of this type of long-range interaction. Once one declares there to be a preferred frame, all that is required is a causal unitary theory in that frame. It may have a relativistic approximation when some (in this case, long-range) interactions are neglected, but this relativity is not an exact principle that disqualifies non-relativistic perturbations.

While the original lattice model, being just a discrete set of quantum mechanical degrees of freedom is manifestly UV-finite, we should check that this is the case for the continuum approximation (that is check that we can truly decouple the lattice structure and have a continuum limit). Thinking of $\epsilon$ as a perturbation quadratic in fields, we see that the only (potentially divergent) loop diagrams are to vacuum energy. Since there is no gravity in this model, vacuum energy is physically irrelevant and we can ignore these diagrams.

3 Effective Gravity and Superluminality

Are superluminal long-range effects consistent with General Relativity? Let us try to construct a long-wavelength continuum description of such a combination, generalizing (5). In this section we will not worry about the issue of UV completeness. To simplify our task a little, let us first aim for the limit in which $J$ is supported on just two points between which we want to arrange for superluminal interactions, $J(\vec{x}) = \frac{1}{2} \delta^3(\vec{x} - \vec{x}_1) + \frac{1}{2} \delta^3(\vec{x} - \vec{x}_2)$, temporarily turning a blind eye to the loss of smoothness and the product of coincident $\delta$-functions. We will add smoother sources at the end of this section. Since spacetime symmetries are gauged by General Relativity, we will realize the breaking of Poincare invariance as a Higgs effect. Really, two Higgs effects are required: one to pick out the two special locations, $\vec{x}_1, \vec{x}_2$, and one to define “simultaneous” times on these two locations at which the long-range interactions occur.

We take the first Higgs effect to be of a familiar type: we add to our theory a new species of particle, $\psi$, with mass $m$ much greater than the UV cutoff of our effective description, but smaller than the Planck scale. We also assign it a $\mathbb{Z}_2$ charge so that it can only be destroyed or created in pairs. Heavy pairs cannot be created within the effective description from light gravitational and $\phi$ quanta much softer than $m$, but the light particles can interact with a pre-existing $\psi$ pair. We take this pair of heavy particles to be so distantly separated at some initial time, that they cannot annihilate for a very long time to come. Since light and soft quanta cannot appreciably accelerate the massive $\psi$, we can take the $\psi$ pair to be approximately at rest with respect to an asymptotic Minkowski frame. Because of their large inertia and small Compton wavelength with respect to the UV cutoff of the light quanta, the pre-existing $\psi$ particles will act as effectively fixed pointlike locations. By this means, the “preferred” locations of the $\psi$-pair effectively spontaneously breaks (local) Poincare invariance. Their two locations will generalize the fixed locations $\vec{x}_{1,2}$ of our
long-range interaction.

We want an interaction between $\psi$ and $\phi$ so that a $\phi$ quantum that propagates to the location of one of the pre-existing $\psi$’s can “instantly” jump to the location of the other $\psi$. But this requires an identification of time on the worldline of one of the $\psi$’s with time on the worldline of the other distant $\psi$. Such a preferred pairing of times further breaks general coordinate invariance and necessitates the second Higgs effect. Minimally, this Higgs effect can be localised to the $\psi$ worldlines, defining the preferred times as the proper time along each $\psi$ worldline since their pair-creation in the distant past. However, it is convenient to use a Higgs effect already in the literature that defines a preferred time everywhere in space, namely the “Ghost-condensate” \cite{2}. In a generally covariant and consistent, but unusual, effective field theory a new scalar field, $\chi$, is coupled to gravity so as to admit a non-trivial stable solution, namely Minkowski spacetime metric with

$$\chi(x) = k x^0,$$

where $k$ is a fixed constant parameter from the $\chi$ action. This time dependence arises spontaneously and partially Higgses general coordinate invariance. Small $\chi$ fluctuations about (7) are “eaten” by the metric fields. Therefore in unitary gauge (7) is exact while the gravitational action is modified. Nevertheless, over a large regime this effective field theory reproduces standard General Relativity. The field (7) then gives us a global “clock”.

Putting together the ingredients, we take our model of superluminality to be given by

$$S = S_{\text{Einstein}}[g_{\mu\nu}] + S_{\text{ghost}}[g_{\mu\nu}, \chi] + \frac{1}{2} \int d^4x \sqrt{-g} \left\{ g^\mu\nu \partial_\mu \phi \partial_\nu \phi + g^\mu\nu \partial_\mu \psi \partial_\nu \psi - m^2 \psi^2 \right\} - \frac{\epsilon}{4} \int d^4y \sqrt{-g(y)} \psi^2(y) \phi(y) \int d^4z \sqrt{-g(z)} \psi^2(z) \phi(z) k \delta(\chi(y) - \chi(z)).$$

(8)

First note that this action is generally coordinate invariant. Somewhat similar non-local operators were discussed in Ref. \cite{16} as coordinate invariant observables in ordinary effective general relativity. Here, the non-local operator represents a true modification of the dynamics, not just a probe of standard gravity.

After passing to the ghost condensate unitary gauge the non-local term above becomes

$$S_{\text{superluminal}} = -\frac{\epsilon}{4} \int dt \int d^3y \sqrt{-g(t, y)} \psi^2(t, y) \phi(t, y) \int d^3z \sqrt{-g(t, z)} \psi^2(t, z) \phi(t, z),$$

(9)

which is non-local in space, but local in time. The leading behavior of this system can be seen in the limit that the UV cutoff is $\ll m \ll M_{\text{Pl}}$. In the limit, with $k$ held fixed, gravity decouples from the dynamics, but $\chi$ continues to provide a global time. The pair of distant heavy $\psi$ particles become infinitely massive and point-like, with static locations $\vec{x}_{1,2}$. Therefore $\phi$ effectively has an action in this limit,

$$S_{\text{eff}} = \frac{1}{2} \int d^4x (\partial_\mu \phi)^2 - \frac{\epsilon}{4} \int dt (\phi(t, \vec{x}_1) + \phi(t, \vec{x}_2))^2,$$

(10)

which is equivalent to (5) with $J = \frac{1}{2} \delta^3(\vec{x} - \vec{x}_1) + \frac{1}{2} \delta^3(\vec{x} - \vec{x}_2)$. 

7
With $M_{Pl}, m$ large but finite, the $\epsilon$ interaction is instantaneous in the unitary gauge, but not in a fully general coordinate invariant sense. Rather, the general statement is that the $\epsilon$ interaction is \textit{superluminal} with respect to the metric $g_{\mu\nu}$.

Finally, let us discuss how to generalize this construction to allow a smooth $J(\vec{x})$. The simplest way is to replace the pair of heavy elementary particles $\psi$ with a smooth soliton and an anti-soliton. For example suppose $\psi^a$ is an isotriplet Higgs field for an $SO(3)$ gauge theory, which Higgses the symmetry down to $SO(2)$ (Georgi-Glashow model [17]). This theory supports smooth magnetic monopole solitons. We take our heavy pre-existing particles to be a distantly separated monopole + anti-monopole pair. Let us generalize our $\psi^a\bar{\psi}^a$ couplings above to $(\psi^a\bar{\psi}^a - v^2)\phi$, where $v$ is the magnitude of the $\psi^a$ VEV. Therefore the $\epsilon$ interaction turns on smoothly as one enters the cores of the monopoles where $\psi^a$ deviates appreciably from its VEV. The $J$ we have engineered has negligible support except in the two widely separated soliton cores.

It is not known if this relatively simple effective gravitational dynamics exhibiting superluminality can be UV completed, but it is a sensible low-energy effective field theory (at least over a finite but long time interval to avoid any gravitationally induced collapse) and illustrates the principles we will pursue, indirectly, in the context of AdS/CFT.

4 AdS/CFT and Boundary Superluminality

The $CFT \rightarrow AdS$ correspondence is in a very real sense a case of emergent gravity and relativity. The UV completeness of the CFT transfers to the AdS quantum gravity. Let us specialize to the CFT given by strongly-coupled large-$N_{\text{color}}$ $\mathcal{N} = 4$ SYM. There are good arguments [18] [19] [20] to suggest that this theory might itself be realizable as the IR limit of a lattice theory (continuous time) with a preferred frame for unitary quantum evolution. Such a lattice system would violate all of Poincare invariance except for time translation invariance. Poincare and conformal invariance would emerge in the continuum long-wavelength limit. The gravity dual of such a lattice system would have a “UV” boundary at which Poincare invariance is badly broken, reflecting the YM lattice structure, with IIB superstring field profiles emanating from the UV boundary, perturbing the usual $AdS_5 \times S^5$ background. But the dual of the statement that the far IR of the lattice theory is successfully approximated by continuum SYM translates to saying that, far away from the UV boundary in the IR of the bulk, the $AdS_5 \times S^5$ background and fluctuations are gradually restored (the deviating profiles damp out). In this sense, (higher dimensional) General Relativity can emerge from a quantum theory which fundamentally does not enjoy (even special) relativistic structure. The above features follow on general grounds, but details of the AdS dual of such a lattice gauge theory are not known. However, a provocative related example, with a single lattice dimension, has been studied in Ref. [19]. A general moral to keep in mind is this: if a UV complete quantum theory has a regime or approximation in which it matches a CFT which has an AdS gravity (string) dual, then the entire quantum theory must have a dual description which has a gravitational regime or approximation. This latter gravitational (string) dual must also reflect the deviations from CFT behavior, and must possess the objects and defects necessary to do so.
For most of this section we work directly in the continuum (only briefly invoking a possible lattice realization in subsection 4.4). We generalize (5) by perturbing the SYM CFT by a bilocal interaction, but now each local factor must be a $SU(N_{\text{color}})$ gauge-invariant composite operator. SYM has six “flavors” of real color-adjoint scalar fields, $A_I = 1, \ldots, 6$. Flavor-adjoint color-singlet scalar bilinears, $\text{Tr} A_I A_J - \frac{N}{6} \text{Tr} A_K A_K$, are primary operators of the SYM CFT of dimension 2 (related by extended supersymmetry to conserved currents). We will pick any of them, say $O(x) \equiv \text{Tr} A_1 A_2$, to build a bilocal perturbation to SYM:

$$H = H_{\text{CFT}} + \epsilon \left( \int d^3 \bar{x} J(\bar{x}) O(\bar{x}) \right)^2.$$  \hspace{1cm} (11)

Here the operator $O$ is in Schrodinger picture. We have chosen a very low-dimension operator so as to minimize the issues of UV divergences, studied in subsection 4.2. Local double-trace operator deformations were studied in Refs. [21] [22] [23] [24] [25].

We can also pass to the action formulation and path integral quantization:

$$S = S_{\text{CFT}} - \epsilon \int dt \left( \int d^3 \bar{x} J(\bar{x}) O(t, \bar{x}) \right)^2$$

$$= S_{\text{CFT}} - \epsilon \int d^4 x J(\bar{x}) O(x) \int d^4 y J(y) O(y) \delta(x_0 - y_0).$$ \hspace{1cm} (12)

4.1 Superluminality

We consider the reference frame of $H$ as the preferred one in which quantum time-evolution is defined. As in Section 2, the $\epsilon$ perturbation can absorb CFT excitations and instantly re-emit them far away in the support of $J$, consistent with causality in the defining frame. This effect is reflected as superluminality in the gravity dual. Without $\epsilon$, the dual vacuum configuration is of course the well known $AdS_5 \times S^5$. For the point we want to make, the $S^5$ is just a detail. We will not bother keeping track of locality on the $S^5$, just Kaluza-Klein reducing from 10 dimensions down to 5. Choose Poincare coordinates in AdS, in the same preferred frame as the perturbed CFT,

$$ds^2_{\text{AdS}} = \eta_{\mu\nu} dx^\mu dx^\nu - \frac{dz^2}{z^2}. \hspace{1cm} (13)$$

Let us focus on the propagation of the AdS scalar, $\phi(x,z)$, dual to the operator $O$. It is a “good” tachyon with 5D mass-squared of $-4$, saturating the Breitenlohner-Freedman stability bound [26]. Consider two spacelike-separated events in the AdS bulk spacetime, $(0, \vec{0}, z)$ and $(t > 0, \vec{x}, z)$, with $2z < t \ll |\vec{x}|$, so that causal communication between them is ordinarily ($\epsilon = 0$) impossible. However, let us now suppose that $\vec{0}$ and $\vec{x}$ are both within the support of $J$. To first order in $\epsilon$, perturbation theory pulls down from the action $\epsilon \int dt' \left( \int d^3 \bar{x}' J(\bar{x}') O(t', \bar{x}') \right)^2$. The AdS dual of this perturbation at leading order in large $N_{\text{color}}$ is that each $O(t', \bar{x}')$ maps to a bulk-boundary free-field AdS propagation of the $\phi$ scalar, with the boundary point being $(t', \vec{x}', 0)$. Denoting the free-field bulk-boundary propagator between $(x, z)$ and $x'$ by $K(x - x', z)$, we see that our leading correction to the
bulk-to-bulk propagator between \((0, \vec{0}, z)\) and \((t > 0, \vec{x}, z)\) is

\[ \epsilon \int dt' \int d^3 \vec{x}' d^3 \vec{y}' J(\vec{x}')J(\vec{y}')K(t', \vec{x}', z)K(t - t', \vec{x} - \vec{y}', z). \tag{14} \]

Now for example, boundary points such as \((t' \approx t/2, \vec{x}' \approx \vec{0})\) and \((t' \approx t/2, \vec{y}' \approx \vec{x})\) contribute to this integral. In AdS, such boundary points are causally connected to our bulk points \((0, \vec{0}, z)\) and \((t > 0, \vec{x}, z)\) respectively. That is each \(K\) factor allows causal communication (they have imaginary parts) and hence so does the entire bulk-to-bulk correction. This result is also a limiting case of that of Section 5, which provides a more formal derivation.

(This leading order superluminality is UV-finite in the continuum. Therefore it is also a good approximation to a lattice realization of the deformed SYM theory, as long as all the relevant length scales are much larger than the lattice spacing, in particular \(t, |\vec{x}|, z\) above and the dominant wavelengths of \(J\).)

A quantum gravitational theory thereby admits superluminal propagation, although in this case the “magic” is localized to the AdS boundary. (A brief comment to similar effect is made in the discussion of Ref. [13].) Still, the AdS string theory must possess the necessary boundary defects that allow this to occur, as long as our CFT deformation is UV complete. But we cannot argue that this takes the form of a Higgs mechanism on the AdS boundary since in a sense gravity and general coordinate invariance end there.

### 4.2 Renormalizability

We must take care to understand what divergences emerge due to the multiple operator products of \(\mathcal{O}\) appear in \(\epsilon\) perturbation theory. The reader may wish to follow this section by using the free SYM field theory as a simple example. Although we are primarily interested in strongly coupled SYM so that the AdS dual is weakly coupled, the operator \(\mathcal{O}\) has very similar divergence properties at arbitrary coupling because of its supersymmetry-protected dimension.

We will power-count \(\epsilon\) perturbation theory to identify the superficial divergences. The bi-local nature of our perturbation makes this somewhat unfamiliar. We can massage it a little to make it more amenable to standard power-counting. We attribute our perturbation to one that is linear in \(\mathcal{O}\),

\[ \Delta S = \int d^4 x \epsilon^{1/2} J(\vec{x}) \mathcal{O}(x)\sigma(x), \tag{15} \]

where \(\sigma\) is an auxiliary field with a “propagator”, for \(\epsilon = 0\),

\[ G_0(x, y) \equiv -2i\delta(x_0 - y_0). \tag{16} \]

We can use this propagator to “integrate out” \(\sigma\) and return to the original perturbation. This is a formal device in that \(G_0\) does not follow from some quadratic \(\sigma\) action, but it is useful for power-counting purposes. Since \(\mathcal{O}\) has scale dimension 2, for power-counting purposes the background field \(\epsilon^{1/2} J(\vec{x})\) has dimension 3/2, and from its propagator it is clear that \(\sigma(x)\) has power-counting dimension 1/2.
Let us consider what superficial divergences there can be in the $\epsilon$ perturbation expansion. These must be local products of CFT operators multiplied by powers of $\sigma(x)$ and $\epsilon^{1/2}J$ (and derivatives), with total dimension $\leq 4$. Since a single operator insertion of $\mathcal{O}$ in the CFT is not divergent, the divergences can only begin at quadratic order in $\epsilon^{1/2}J$, already “costing” dimension 3. At most this could be multiplied by powers of $\sigma$, since all CFT gauge invariant operators have scaling dimension $> 1$. By $\sigma \rightarrow -\sigma, J \rightarrow -J$ symmetry, the only divergent structures can be $\epsilon J^2(\vec{x})$ and $\epsilon J^2(\vec{x})\sigma^2(x)$. The first of these divergent structures can only arise from the leading order VEV of the perturbation Hamiltonian, a physically irrelevant real $c$-number constant, that can be renormalized away by simply subtracting it from our Hamiltonian (or action).

We are thus only left to contend with $\epsilon J^2(\vec{x})\sigma^2(x)$, that is, a divergence in the $\sigma$ self-energy. Indeed there really is a logarithmic divergence of this form, and it is coupled to the rest of the CFT because each $\sigma(x)$ field in it can be contracted with $\sigma$’s elsewhere in the perturbative expansion, so we do have to address this divergence. The leading correction to the $\sigma$ self-energy takes the form

$$
\epsilon \int d^4 x J(\vec{x})\sigma(x) \int d^4 y J(\vec{y})\sigma(y) \langle 0|T\{\mathcal{O}(x)\mathcal{O}(y)\}|0\rangle = \epsilon c \ln(\mu a) \int d^4 x J^2(\vec{x})\sigma^2(x) + \text{finite},
$$

where $a$ is a short-distance cutoff and $\mu$ is an arbitrary renormalization scale put in to separate out the UV divergence, and $c$ is an unimportant constant. We have used that $\mathcal{O}$ is a dimension 2 primary operator in constraining the form of its correlator in the usual way.

This divergence is removed by renormalization of $\epsilon$. To see this let us resum the $\sigma$ self-energy contributions to its propagator arising from integrating out the CFT. Define the self-energy correction

$$
\epsilon \Pi(x, y) = \epsilon J(\vec{x})J(\vec{y}) \langle 0|T\{\mathcal{O}(x)\mathcal{O}(y)\}|0\rangle.
$$

Log divergences are removed by the subtraction

$$
\epsilon \Pi_{\text{sub}}(x, y) = \epsilon \Pi(x, y) - \epsilon \Pi_{\text{div}}(x, y),
$$

where by (17)

$$
\epsilon \Pi_{\text{div}}(x, y) = \epsilon c \ln(\mu a) J^2(\vec{x})\delta^4(x - y).
$$

Resumming this self-energy in the $\sigma$ propagator gives, in an obvious matrix notation,

$$
\epsilon G = \epsilon G_0(1 - \epsilon \Pi G_0)^{-1} = \epsilon G_0(1 - \epsilon \Pi_{\text{sub}} G_0 - \epsilon \Pi_{\text{div}} G_0)^{-1}.
$$

This is the only combination in which $\epsilon$ and $G_0$ appear in the perturbative expansion, and all powers of $\epsilon$ are explicitly shown. Note that in the expansion of this expression $\Pi_{\text{div}}$ always appears in the sandwich $G_0\Pi_{\text{div}}G_0$, for which it is easy to prove,

$$
G_0\Pi_{\text{div}}G_0 = \ln(\mu a)\mathcal{J}G_0,
$$

11
where $\mathcal{J}$ is just the finite constant

$$
\mathcal{J} \equiv c \int d^3 \vec{x} J^2(\vec{x}).
$$

(23)

Using this relation, we can rewrite the resummed $\sigma$ propagator as

$$
\epsilon G \begin{array}{c}
\epsilon G_0 (I - \epsilon \Pi_{sub} G_0 - \epsilon \mathcal{J} \ln(\mu a) I)^{-1} \\
= \epsilon_R(\mu) G_0 (I - \epsilon_R(\mu) \Pi_{sub} G_0)^{-1}
\end{array}
$$

(24)

where we define a renormalized coupling at $\mu$,

$$
\epsilon_R(\mu) \equiv \frac{\epsilon}{1 - \epsilon \mathcal{J} \ln(\mu a)}.
$$

(25)

Since there are no more superficial divergences other than the real divergence subtracted by this renormalization, the perturbative expansion is now finite in terms of $\epsilon_R$ as the short-distance cutoff $a \to 0$.

### 4.3 Suppressed Vacuum decay

As discussed in the Introduction, there is no guarantee that the SYM vacuum remains the true ground state of the deformed theory. However, whatever the true ground state, the amplitudes for the decay of the SYM vacuum follow from our renormalizable theory, giving some finite decay rates (per unit volume) perturbatively in $\epsilon_R$. Above, we did subtract a single infinite (order $\epsilon J^2$) correction to vacuum energy as part of renormalization, but this is irrelevant for vacuum decay since this divergence is real while it is the imaginary parts of “vacuum”-energy that encode vacuum decays via the optical theorem. For sufficiently small $\epsilon_R$ (renormalized at the scale typical of $J$) these decay rates, and Lorentz-violating processes in general, are suppressed, but a sufficiently long time will always overcome the weak coupling and lead to complete SYM(AdS) decay. To prevent this from happening we will consider $\epsilon(t)$ to be smoothly time-dependent with finite support. This makes the Hamiltonian time-dependent. Now we can choose $\epsilon_R$ so weak as to not lead to catastrophic vacuum decay. This also suppresses the probability of superluminal propagation, but it does not vanish and we are only seeking this qualitative fact. Our previous analysis of superluminality in subsection 4.1 is only altered in that the $\epsilon \to \epsilon(t')$ now sits inside the time integral. Superluminality continues to hold as long as the duration of non-vanishing $\epsilon$ is taken to cover the events being discussed there.

### 4.4 Comments

We have seen that the deformed CFT is renormalizable in the same sense that QED is, logarithmic UV divergences being eliminated by $\epsilon$ renormalization, but the catch is that $\epsilon$ runs in the UV to strong coupling where our power-counting breaks down. Thus, we have not yet demonstrated true UV completeness, but we are close. Of course, one possibility
is that the short-distance cutoff \( a \) may really be finite, if for example it is a lattice spacing for SYM on a spatial lattice. A spatial lattice theory would regulate the divergences of the continuum field theory by converting it into quantum mechanics of discrete lattice degrees of freedom. In that case, the renormalizability usefully translates into insensitivity of the long-wavelength theory to details of the lattice structure.

Another possibility is not to regulate the CFT itself, but rather to replace each \( O(x) \) with a “slightly” non-local operator so as to regulate the operator product divergences appearing in \( \epsilon \) perturbation theory. A seemingly separate question is whether superluminality can be realized in the gravitating bulk of AdS, rather than the AdS boundary where the gravitational dynamics ends. We really would like to test if superluminality can appear right in the midst of quantum General Relativity. Presently, bulk quanta must propagate relativistically to the AdS boundary before they see superluminal effects. As it turns out, we can indeed engineer bulk superluminality, and the trick is to replace the pair of local operators appearing in the CFT Hamiltonian by a pair of non-local operators. As an added benefit, this replacement renders the deformed theory UV finite, that is, it regulates the UV divergences discussed above. We pursue this approach next.

## 5 Superluminality in the Bulk

In this section, \( O \) denotes an arbitrary local scalar primary operator of the SYM CFT, of scaling dimension \( d \), which has single-trace limit as \( N_{\text{color}} \to \infty \). In subsection 5.4 we will restrict \( d \) to ensure perturbativity of our deformation under all circumstances. Until then we simply assume perturbation theory in the CFT deformation is to be trusted.

### 5.1 Warm-up on non-local operators and UV finiteness

Consider the simple, but time-dependent, deformation of the CFT Hamiltonian of the form

\[
H(\tau) \equiv H_{\text{CFT}} + \Delta H(\tau) \\
\equiv H_{\text{CFT}} + \int d^3 \vec{x} J(\tau, \vec{x}) O(\vec{x}).
\]

(26)

All the operators appearing here are Schrödinger operators, time-dependence appearing only in the source, \( J(\tau, \vec{x}) \).

Since time-ordering subtleties will be important in this section, we use Hamiltonian and operator methods throughout. The master formula for time-ordered perturbation theory is

\[
Te^{-i \int_{t_1}^{t_2} d\tau H(\tau)} = e^{-iH_{\text{CFT}}(t_2-t_1)} \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{t_1}^{t_2} d\tau_1 \int_{\tau_1}^{t_2} d\tau_2 \ldots \int_{\tau_{n-1}}^{t_2} d\tau_n \Delta \hat{H}(\tau_n) \ldots \Delta \hat{H}(\tau_1).
\]

(27)

We uniformly use hats to distinguish Heisenberg operators,

\[
\Delta \hat{H}(\tau) \equiv e^{iH_{\text{CFT}} \tau} \Delta H(\tau) e^{-iH_{\text{CFT}} \tau} \\
\hat{O}(\tau, \vec{x}) \equiv e^{iH_{\text{CFT}} \tau} O(\vec{x}) e^{-iH_{\text{CFT}} \tau}.
\]

(28)
Consider the example of the CFT-vacuum persistence amplitude at order $J^2$,

$$\int d^4x d^4y J(x) J(y) \langle 0 | T\{\hat{O}(x) \hat{O}(y)\} | 0 \rangle$$

$$= \int d^4x d^4y J(x) J(y) \int dm^2 \int d^3\vec{p} \frac{|\langle 0 | \hat{O}(0) | m, \vec{p} \rangle|^2}{2 \sqrt{\vec{p}^2 + m^2}} (\theta(x_0 - y_0) e^{-ip.(x-y)} + \theta(y_0 - x_0) e^{ip.(x-y)})$$

$$\propto \int d^4x d^4y J(x) J(y) \int dm^2 m^{2d-4} G(x-y; m)$$

$$= \int dm^2 m^{2d-4} \int d^4q \frac{|\tilde{J}(q)|^2}{q^2 - m^2 + i\epsilon}$$

$$= \infty. \quad (29)$$

The first equality follows in passing to the spectral representation by inserting a complete set of states, integrating over their invariant mass-squared and momentum. The proportionality follows from the fact that by Lorentz invariance the matrix element of the scalar operator $O$ can only depend on the mass $m$, not the momentum $\vec{p}$, and the mass-dependence follows by dimensional analysis in the CFT. $G(x-y; m)$ denotes a free field scalar Feynman propagator of mass $m$. The final equality arises because, whereas the smoothing due to $J$ cuts off the $q$-integral, the $m$ integral remains divergent.

This divergence is very closely related to the one of the previous section, and is rather standard when perturbing by (superpositions of) local operators. In perturbation theory, $\hat{O}(\tau, \vec{x})$ creates a pointlike disturbance at time $\tau$, which then begins to spread out. However, at second order a second $\hat{O}(\tau', \vec{x}')$ can sample the disturbance created by the first, and this creates our divergence when the two points coincide. Such divergences are avoided if the pointlike disturbances are “thickened” to finite size. One convenient way of doing this is to use a fake time evolution to spread out the pointlike disturbance created by $O$. A simple illustration is provided by the Hamiltonian,

$$H = H_{CFT} + \int d^4x J(x) e^{iH_{CFT}x_0} O(\vec{x}) e^{-iH_{CFT}x_0}. \quad (30)$$

All the operators appearing are in Schrödinger picture. One product of these operators happens to be a Heisenberg operator in form, but the whole effect of the $x_0$ “time evolution” on $O$ is to turn this spatially local Schrödinger operator into a spatially non-local one, by evolving the disturbance it creates for a finite time which causes the disturbance to spread over a finite spatial region. Note that since $x_0$ is integrated over, the Schrödinger $H$ and $\Delta H$ are time-independent. It is important to note that by specifying a Hamiltonian, the resulting dynamics is automatically local in time. But the perturbation is spatially non-local because it cannot be written as a superposition of local Schrödinger operators. If this looks unfamiliar it is because it is inconsistent with Lorentz invariance, from which we are deviating in this paper.

To contrast this with our earlier example, let us again apply (27) to calculate the CFT-
vacuum persistence amplitude at order $J^2$,
\begin{align}
\propto \int_{-\infty}^{\infty} d\tau \int d^4x d^4y J(x)J(y) \langle 0 | T \{ \hat{O}(x_0 + \tau, \vec{x}) \hat{O}(y) \} | 0 \rangle \\
= \int d\tau \int d^2m \int \frac{d^3p}{2 \sqrt{p^2 + m^2}} \{ \theta(\tau)e^{-i\sqrt{p^2 + m^2}\tau} + \theta(-\tau)e^{i\sqrt{p^2 + m^2}\tau} \} \\
\times |\tilde{J}(\sqrt{\vec{p}^2 + m^2}, \vec{p})\langle 0 | \hat{O}(0) | m, \vec{p} \rangle|^2 \\
\propto \int d\tau \int d^2m d^{d-4} \int \frac{d^3p}{2 \sqrt{p^2 + m^2}} |\tilde{J}(\sqrt{\vec{p}^2 + m^2}, \vec{p})|^2 \\
\times \{ \theta(\tau)e^{-i\sqrt{p^2 + m^2}\tau} + \theta(-\tau)e^{i\sqrt{p^2 + m^2}\tau} \} \\
< \infty. \quad (31)
\end{align}

The time-ordering is with respect to $\tau$ only. Here, we see that the smoothness of $J$ does cut off the $m$ and $\vec{p}$ integrals, and the $\tau$ integral also converges as $\tau \to 0$. This merely reflects the “thickened”, as opposed to completely local, operator that perturbs the CFT in the second example.

The comparison of the relatively simple examples in this subsection should orient the reader in the full construction of the next.

### 5.2 Superluminality

Consider the Hamiltonian,
\begin{align}
H &= H_{CFT} + \Delta H \\
&\equiv H_{CFT} + \epsilon (\int d^4x J(x)e^{iH_{CFT}x_0}O(\vec{x})e^{-iH_{CFT}x_0})^2, \quad (32)
\end{align}

where everything has been written in terms of Schrödinger operators, and where $J$ is a smooth spacetime-dependent source of compact support. Note that $x_0$ is a dummy integration variable and that $H$ is in fact time-independent. (We will however introduce time-dependence in subsection 5.4.)

To demonstrate superluminality, we consider properties of the bulk-to-bulk propagator for the scalar $\phi$, dual to the primary operator $O$, between the two spacetime points $(y^\mu, z)$ and $(0, z)$, namely $\langle 0 | T \{ \hat{\phi}(y, z) \hat{\phi}(0, z) \} | 0 \rangle$. (This a well-defined object if we imagine having gauge-fixed general coordinate invariance.) Our approach (but not our result) shares some similarities with that of Ref. [25] studying AdS implications of double-trace but local deformations of SYM. We will take

\begin{equation}
0 < y_0 < |y|, \quad (33)
\end{equation}

so that the two bulk points are ordinarily ($\epsilon = 0$) causally disconnected (recalling that our AdS metric is (13)), implying the vanishing of the commutator,
\begin{align}
\langle 0 | [\hat{\phi}(y, z), \hat{\phi}(0, z)] | 0 \rangle &= 2i \mathrm{Im} \langle 0 | T \{ \hat{\phi}(y, z) \hat{\phi}(0, z) \} | 0 \rangle \text{sgn}(y_0). \quad (34)
\end{align}
The right-hand form shows that this information on causality is contained in the propagator.

We will further consider that

\[ y_0 < z, \]  

so that if superluminality were concentrated on the AdS boundary, there is simply no time to causally propagate through the bulk to get to the boundary to take advantage of it. Therefore we are guaranteed that if \( \Delta H \) gives a non-zero correction to (33), then it must be due to superluminal effects in the gravitating AdS bulk spacetime.

Let us check that there is indeed a non-vanishing correction to the bulk commutator

\[ [\hat{\phi}(y, z), \hat{\phi}(0, z)] = i \int_0^{\gamma_0} d\tau [\hat{\phi}(y, z), \Delta \hat{H}(\tau)], \]

where we have used that at zeroth-order (relativistic) causality implies \([\hat{\phi}(y, z), \hat{\phi}(0, z)] = 0\).

We can then calculate the VEV of this commutator in the large-\( N_{\text{color}} \) limit, where the resulting four-point VEVs factorize as \( \langle 0|\hat{\phi}\hat{\phi}\hat{O}\hat{O}|0\rangle \approx \langle 0|\hat{\phi}\hat{O}|0\rangle \langle 0|\hat{O}\hat{\phi}|0\rangle \), etc. The result is

\[
\begin{align*}
(0|\hat{\phi}(y, z), \hat{\phi}(0, z)||0) & \\
\approx & \quad 2i \int_0^{\gamma_0} d\tau \int d^4x d^4x' J(x)J(x')\langle 0|[\hat{O}(x_0 + \tau, \vec{x}), \hat{\phi}(y, z)]|0\rangle \langle 0|[\hat{O}(x_0' + \tau, \vec{x}'), \hat{\phi}(0, z)]|0\rangle \\
= & \quad -8i \int_0^{\gamma_0} d\tau \int d^4x d^4x' J(x)J(x')\text{Im}\langle 0|T\{\hat{O}(x_0 + \tau, \vec{x})\hat{\phi}(y, z)\}||0\rangle \text{sgn}(x_0 + \tau - y_0) \\
& \times \text{Im}\langle 0|T\{[\hat{O}(x_0' + \tau, \vec{x}')\hat{\phi}(0, z)]||0\rangle \text{sgn}(x_0' + \tau) \\
= & \quad -8i \int_0^{\gamma_0} d\tau \int d^4x d^4x' J(x)J(x')\text{Im}K(y_0 - x_0 - \tau, \vec{y} - \vec{x}, z)\text{Im}K(-x_0' - \tau, -\vec{x}', z) \\
& \times \text{sgn}(x_0 + \tau - y_0) \text{sgn}(x_0' + \tau),
\end{align*}
\]

finally arriving at an expression in terms of bulk-boundary propagators of the undeformed theory.
Choose some intermediate $\tau \approx y_0/2$ as an example. We see that if the support of $J$ has sufficiently positive $x_0'$ so that $(x_0' + \tau)^2 > (\vec{x})^2 + z^2$ and has sufficiently negative $x_0$ so that $(y_0 - \tau - x_0)^2 > (\vec{y} - \vec{x})^2 + z^2$, then causal communication between the bulk point $(0, z)$ and boundary point $(x_0' + \tau, \vec{x})$ is possible and causal communication between the bulk point $(y, z)$ and boundary point $(x_0 + \tau, \vec{x})$ is also possible. Therefore by the relation between commutator VEVs and propagators (the analog of (31)) each $\text{Im} K$ factor can be non-zero and so we have demonstrated causal communication between $(0, z)$ and $(y, z)$.

It is important to stress that although we have expressed this communication mathematically as a product of communication from bulk to boundary and then back in the undeformed theory, there is in fact not enough time available in our set-up for bulk to boundary communication to proceed unless there is superluminality in the bulk. This bulk superluminal communication must therefore be taking place. It is a mere convenience that we are parametrizing the requisite bulk disturbances in terms of boundary sources that could produce them given enough time.

In this, admittedly indirect, manner we have shown that superluminality is taking place in the AdS bulk, and therefore the exotic Higgs effect necessary to make this possible in a gravitating spacetime must be present. But we must still ask if the gravitating theory is UV-complete. This is guaranteed if the deformed CFT is UV complete. We now show this.

### 5.3 UV finiteness

Consider the amplitude to evolve from a state $|A\rangle$ to a state $|B\rangle$ over a time interval $t$. We consider the two states to be energy-momentum eigenstates of the undeformed CFT. For clarity, we begin with the example of the order $\epsilon^2$ contribution to this amplitude following from (37),

\[
\begin{align*}
&\propto \int_{0}^{t} d\tau' \int_{\tau'}^{t} d\tau \int d^{4}x \int d^{4}y J(0) J(x) J(y) \\
&\quad \times \langle B| \hat{O}(x_0 + \tau, \vec{x}) \hat{O}(y_0 + \tau, \vec{y}) |A\rangle \\
&\propto \int d\tau' \int_{\tau'}^{t} d\tau \int d^{4}x \int d^{4}y \int d^{4}x' \int d^{4}y' \int d^{4}x'' \int d^{4}y'' \\
&\quad \times \langle B| \hat{O}(x_0 + \tau, \vec{x}) |p\rangle \langle p| \hat{O}(y_0 + \tau, \vec{y}) |q\rangle \langle q| \hat{O}(x_0' + \tau', \vec{x}') |k\rangle \langle k| \hat{O}(y_0' + \tau', \vec{y}') |A\rangle \\
&= \int d\tau' \int_{\tau'}^{t} d\tau \int_{\tau}^{\tau'} d\tau' \int d^{4}p \int_{P_{A}}^{\infty} d^{4}q \int d^{4}k \int_{p_{B} - p}^{\infty} d^{4}q' \int d^{4}k' J(p_B - p) J(p - q) J(q - k) J(k - p_A) \\
&\quad \times e^{i(p_{B_{0}} - q_0)\tau} e^{i(q_0 - p_{A0})\tau'} \langle B| \hat{O}(0) |p\rangle \langle p| \hat{O}(0) |q\rangle \langle q| \hat{O}(0) |k\rangle \langle k| \hat{O}(0) |A\rangle. 
\end{align*}
\]

We have inserted complete sets of states between operators, explicitly summing over their possible momenta, with positive mass-squared and energy (the "$+$" subscript on the momentum integrals), and implicitly over any other labels. Such integrals represent sums over non-vacuum states related by Poincare symmetry and scale symmetry. One can also insert the vacuum state, in which case the relevant momentum integral drops out in the obvious way. We have not written these terms because they are less dangerous to finiteness.
UV divergences can only arise in the expression when the momentum integrals or $\tau$ integrals diverge. There are no UV divergences in the above expression however, because the smoothness of $J$ translates into the rapid damping of $\tilde{J}$ for large momenta, and because the $\tau, \tau'$ integrals have finite range with only well-behaved phase factor integrands. This generalizes the finiteness we saw in the second example of the last subsection. The reader can easily extend this check of finiteness to arbitrary order in $\epsilon$ perturbation theory by repeated insertion of a complete set of states between operators. The smooth $J$ factors always make each momentum integral converge.

Thus our deformed CFT, and its deformed AdS dual, are UV complete. We can easily compare this with Section 4 by noting that we revert to that case in the limit

$$J(x) \rightarrow J(\bar{x})\delta(x_0)$$

$$\tilde{J}(q) \rightarrow \tilde{J}(\bar{q}).$$

(40)

Consider the simple case where the $|p\rangle, |k\rangle$ states are replaced by the CFT vacuum state, so there is only one momentum integral to worry about, and only one (relative) $\tau_- \equiv \tau - \tau' > 0$ integral. The relevant term is

$$\int_0^{d\tau_-} d\tau_- d^4q e^{i\epsilon q_0 \tau_-} |\langle 0|\hat{O}|q\rangle \tilde{J}(\bar{q})|^2.$$  

(41)

By scale invariance, $|\langle 0|\hat{O}|q\rangle|^2 \propto (q^2)^{d-2}$, and we see that without $\tilde{J}$ to help cut off $q_0$, the integral diverges as $\tau_- \rightarrow 0, q_0 \rightarrow \infty$.

### 5.4 Perturbativity and vacuum decay

Schematically, each order in perturbation theory in $\epsilon$ brings an expression,

$$\epsilon \int d\tau \int d^4x J(x) \int d^4y J(y) \int_+^+ d^4p \int_+^+ d^4q ... \hat{O}(x_0 + \tau, \bar{x})|p\rangle \langle p| \hat{O}(y_0 + \tau, \bar{y})|q\rangle \langle q|...$$  

(42)

The coupling combination $\epsilon J(x)J(y)$ has dimension $9-2d$ which we will ascribe completely to $\epsilon$, taking $J$ to be a smooth dimensionless function taking values of order unity in a spacetime volume of order $L^4$. We have seen in the last subsection how the intermediate state momenta are tied to the external momenta, which we characterize to be of order $E$, with the $J$ integrals providing momentum shifts of order $1/L$. We begin by considering $E \gg 1/L$. Each $J$ integral suppresses one $\int_+^+ d^4p$ integral, fixing $p_\mu \sim E$. Each $\hat{O}$ then counts as $E^d$ in its matrix elements and each $|p\rangle \langle p|$ counts as $E^{-d}$, by dimensional analysis and the fact that the hard external $E$ scale is dominant. Finally, the $\tau$-dependence in the operators turns into a phase factor $\sim e^{i\Delta E \tau}$, where $\Delta E$ is an energy change allowed by the “background” $J$, of order $1/L$. Therefore the $\int d\tau$ integral (with whatever time-ordered limits of integration) is at most of order $L$.

Putting these factors together, we find that every order in perturbation theory counts as the dimensionless combination, $\epsilon LE^{2d-8}$, for $E \gg 1/L$. If $d > 4$ then, no matter how small we take $\epsilon L$, there will be processes where perturbation theory is breaking down (although
we still do not find UV divergences perturbatively). We will avoid this by restricting the primary operator $O$ to have scaling dimension $d \leq 4$. Then the condition for perturbativity is $\epsilon L^{9-2d} \ll 1$ for $E > 1/L$.

We must still check perturbativity in the far IR, $E \ll 1/L$. Now all momentum scales above are dominated by $1/L$ in all the integrals and matrix elements, and the perturbative strength is simply $\epsilon L^{9-2d}$. So there is consistency in the perturbativity requirements for IR and UV processes, $\epsilon L^{9-2d} \ll 1$.

Our deformation of SYM is finite and perturbative. In particular, the deformed Hamiltonian requires no subtractions and is a sum of squares ($H_{CFT}$ is by supersymmetry), so that there is some well-defined ground state. The SYM vacuum is a finite and well-defined excitation above this ground state. We can make it arbitrarily long-lived by making $\epsilon$ weaker and weaker. Or we can do what we did in subsection 4.3, make $\epsilon \to \epsilon(\tau)$ and our Hamiltonian time-dependent so that the deformation is turned on for a finite duration. We then take $\epsilon$ small enough that the SYM (AdS) vacuum in most regions of space survives the period of deformation. This suppresses the amplitude for superluminality in the bulk but it can still take place.

6 Conclusions

Our central construction has been a weak Lorentz-violating deformation of the $\mathcal{N} = 4$ SYM CFT Hamiltonian by a superposition of spatially non-local operators. We checked that, at leading order in the perturbation, the standard AdS/CFT map gives a non-vanishing propagator between two spacetime points which are ordinarily causally disconnected in the AdS bulk. The two bulk points are also ordinarily out of causal contact with the AdS boundary during the time interval separating them, implying that the superluminal behavior is taking place in the gravitating bulk. Finally, we checked that our deformation was UV complete, in that there were no new sources of UV divergence outside the renormalized CFT. Therefore, there must be a complete deformed AdS gravity/string dual of the superluminal behavior.

However, given our indirect CFT approach to this conclusion, the detailed AdS description of bulk superluminality is not apparent. Indeed, the specific form of our spatially non-local deformation was chosen to demonstrate bulk superluminality in terms of the simplest object of the AdS/CFT dictionary, namely the bulk-boundary propagator. It is possible that a different non-local deformation might yield a simpler AdS spacetime description, although likely at the cost of a more complex translation from the CFT side.

We have presented a simple building block for superluminality, but there are clearly other directions to pursue. It appears straightforward that a similar deformation could be used to couple two different, otherwise decoupled CFTs, whose dual would describe bulk coupling of AdS degrees of freedom from both CFTs. Deformations by local operators (as opposed to non-local ones such as we are suggesting) connecting otherwise disconnected CFTs have been discussed in Refs. [27]. More ambitiously, we would like to understand regimes in which the Lorentz-violating or superluminal effects become important, as for example required in Ref. [6], in resolving the cosmological constant problem by Energy-Parity, or in [4] in modifying
the character of black hole horizons. We would also like to see if superluminal effects can be engineered as weak probes of ordinary horizons, as well as the interesting singularities that they can hide.

As discussed in the introduction, Lorentz violation in a gravitational context must appear formally as a type of Higgs effect. This suggests that even when the violation is explicit on the CFT side as in our case, on the AdS side it should appear as a property of a state or solution, not a modification of the gravitational dynamics itself. As remarked above, our approach does not straightforwardly give a detailed description of such states in AdS. It is intriguing however that, from the opposite direction, wormhole solutions in Euclidean AdS gravity pose a puzzle for CFT interpretation precisely because they suggest non-local interactions on the CFT side. Perhaps the resolution of this puzzle lies in non-local deformations of the CFT, at least similar in spirit to the example of Section 5 of this paper.

The example of superluminality and Lorentz-violation provided in this paper has a certain “premeditated” feel to it, and one naturally wonders whether it is too contrived to be at work in Nature. That would however be a premature conclusion, because we have only given an existence proof. It is possible that real world gravity and relativity is a rich emergent phenomenon with a more natural framework for these exotic effects. Hopefully we can understand the theoretical possibilities well enough to devise the right experimental tests to decide.

Acknowledgements

The author is grateful to Nima Arkani-Hamed, Juan Maldacena and Joe Polchinski for discussions. This research was supported by the National Science Foundation grant NSF-PHY-0401513 and by the Johns Hopkins Theoretical Interdisciplinary Physics and Astrophysics Center.

References

[1] T. Jacobson and D. Mattingly, “Gravity with a dynamical preferred frame,” Phys. Rev. D 64, 024028 (2001) [arXiv:gr-qc/0007031].

[2] N. Arkani-Hamed, H. C. Cheng, M. A. Luty and S. Mukohyama, “Ghost condensation and a consistent infrared modification of gravity,” JHEP 0405, 074 (2004) [arXiv:hep-th/0312099].

[3] T. Jacobson and D. Mattingly, “Generally covariant model of a scalar field with high frequency dispersion and the cosmological horizon problem,” Phys. Rev. D 63, 041502 (2001) [arXiv:hep-th/0009052]; C. Eling and T. Jacobson, “Static post-Newtonian equivalence of GR and gravity with a dynamical preferred frame,” Phys. Rev. D 69, 064005 (2004) [arXiv:gr-qc/0310044]; V. A. Kostelecky, “Gravity, Lorentz violation, and
the standard model,” Phys. Rev. D 69, 105009 (2004) [arXiv:hep-th/0312310]; T. Jacobson and D. Mattingly, “Einstein-aether waves,” Phys. Rev. D 70, 024003 (2004) [arXiv:gr-qc/0402005]; S. M. Carroll and E. A. Lim, “Lorentz-violating vector fields slow the universe down,” Phys. Rev. D 70, 123525 (2004) [arXiv:hep-th/0407149]; E. A. Lim, “Can we see Lorentz-violating vector fields in the CMB?,” Phys. Rev. D 71, 063504 (2005) [arXiv:astro-ph/0407437]; B. M. Gripaios, “Modified gravity via spontaneous symmetry breaking,” JHEP 0410, 069 (2004) [arXiv:hep-th/0408127]; C. Eling, T. Jacobson and D. Mattingly, “Einstein-aether theory,” arXiv:gr-qc/0410001; R. Bluhm and V. A. Kostelecky, “Spontaneous Lorentz violation, Nambu-Goldstone modes, and gravity,” Phys. Rev. D 71, 065008 (2005) [arXiv:hep-th/0412320]; M. L. Graesser, A. Jenkins and M. B. Wise, “Spontaneous Lorentz violation and the long-range gravitational preferred-frame effect,” Phys. Lett. B 613, 5 (2005) [arXiv:hep-th/0501223]; J. W. Elliott, G. D. Moore and H. Stoica, “Constraining the new aether: Gravitational Cherenkov radiation,” JHEP 0508, 066 (2005) [arXiv:hep-ph/0505211].

[4] S. L. Dubovsky and S. M. Sibiryakov, “Spontaneous breaking of Lorentz invariance, black holes and perpetuum mobile of the 2nd kind,” Phys. Lett. B 638, 509 (2006) [arXiv:hep-th/0603158]; C. Eling, B. Z. Foster, T. Jacobson and A. C. Wall, Phys. Rev. D 75, 101502 (2007) [arXiv:hep-th/0702124]; S. Dubovsky, P. Tinyakov and M. Zaldarriaga, “Bumpy black holes from spontaneous Lorentz violation,” arXiv:0706.0288 [hep-th].

[5] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis and R. Rattazzi, “Causality, analyticity and an IR obstruction to UV completion,” JHEP 0610, 014 (2006) [arXiv:hep-th/0602178].

[6] D. E. Kaplan and R. Sundrum, “A symmetry for the cosmological constant,” JHEP 0607, 042 (2006) [arXiv:hep-th/0505265].

[7] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1999)] [arXiv:hep-th/9711200]; S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from non-critical string theory,” Phys. Lett. B 428, 105 (1998) [arXiv:hep-th/9802109]; E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2, 253 (1998) [arXiv:hep-th/9802150].

[8] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, “Large N field theories, string theory and gravity,” Phys. Rept. 323, 183 (2000) [arXiv:hep-th/9905111].

[9] I. R. Klebanov and E. Witten, “AdS/CFT correspondence and symmetry breaking,” Nucl. Phys. B 556, 89 (1999) [arXiv:hep-th/9905104].

[10] D. N. Page, S. Surya and E. Woolgar, “Positive mass from holographic causality,” Phys. Rev. Lett. 89, 121301 (2002) [arXiv:hep-th/0204198].
[11] J. M. Maldacena and J. G. Russo, “Large N limit of non-commutative gauge theories,” JHEP 9909, 025 (1999) [arXiv:hep-th/9908134]; J. M. Maldacena and J. G. Russo, “Large N limit of non-commutative gauge theories,” JHEP 9909, 025 (1999) [arXiv:hep-th/9908134]; R. G. Cai and N. Ohta, “On the thermodynamics of large N non-commutative super Yang-Mills theory,” Phys. Rev. D 61, 124012 (2000) [arXiv:hep-th/9910092]; A. Bergman, K. Dasgupta, O. J. Ganor, J. L. Karczmarek and G. Rajesh, “Nonlocal field theories and their gravity duals,” Phys. Rev. D 65, 066005 (2002) [arXiv:hep-th/0103090]; O. J. Ganor, “A new Lorentz violating nonlocal field theory from string-theory,” Phys. Rev. D 75, 025002 (2007) [arXiv:hep-th/0609107]; O. J. Ganor, A. Hashimoto, S. Jue, B. S. Kim and A. Ndirango, “Aspects of puff field theory,” [arXiv:hep-th/0702030].

[12] S. J. Rey, “Holographic principle and topology change in string theory,” Class. Quant. Grav. 16, L37 (1999) [arXiv:hep-th/9807241].

[13] J. M. Maldacena and L. Maoz, “Wormholes in AdS,” JHEP 0402, 053 (2004) [arXiv:hep-th/0401024].

[14] N. Arkani-Hamed, J. Orgera and J. Polchinski, “Euclidean Wormholes in String Theory,” [arXiv:0705.2768 [hep-th]].

[15] V. Balasubramanian, P. Kraus and A. E. Lawrence, “Bulk vs. boundary dynamics in anti-de Sitter spacetime,” Phys. Rev. D 59, 046003 (1999) [arXiv:hep-th/9805171]; V. Balasubramanian, P. Kraus, A. E. Lawrence and S. P. Trivedi, “Holographic probes of anti-de Sitter space-times,” Phys. Rev. D 59, 104021 (1999) [arXiv:hep-th/9808017].

[16] S. B. Giddings, D. Marolf and J. B. Hartle, “Observables in effective gravity,” Phys. Rev. D 74, 064018 (2006) [arXiv:hep-th/0512200].

[17] H. Georgi and S. L. Glashow, “Unified weak and electromagnetic interactions without neutral currents,” Phys. Rev. Lett. 28, 1494 (1972).

[18] D. B. Kaplan, E. Katz and M. Unsal, “Supersymmetry on a spatial lattice,” JHEP 0305, 037 (2003) [arXiv:hep-lat/0206019].

[19] S. Hellerman, “Lattice gauge theories have gravitational duals,” [arXiv:hep-th/0207226].

[20] M. J. Strassler, “Non-supersymmetric theories with light scalar fields and large [arXiv:hep-th/0309122].

[21] O. Aharony, M. Berkooz and E. Silverstein, “Multiple-trace operators and non-local string theories,” JHEP 0108, 006 (2001) [arXiv:hep-th/0105309].

[22] E. Witten, “Multi-trace operators, boundary conditions, and AdS/CFT correspondence,” [arXiv:hep-th/0112258].

[23] M. Berkooz, A. Sever and A. Shomer, “Double-trace deformations, boundary conditions and spacetime singularities,” JHEP 0205, 034 (2002) [arXiv:hep-th/0112264].
[24] P. Minces, “Multi-trace operators and the generalized AdS/CFT prescription,” Phys. Rev. D 68, 024027 (2003) [arXiv:hep-th/0201172].

[25] O. Aharony, M. Berkooz and B. Katz, “Non-local effects of multi-trace deformations in the AdS/CFT correspondence,” JHEP 0510, 097 (2005) [arXiv:hep-th/0504177].

[26] P. Breitenlohner and D. Z. Freedman, “Stability In Gauged Extended Supergravity,” Annals Phys. 144, 249 (1982).

[27] E. Kiritsis, “Product CFTs, gravitational cloning, massive gravitons and the space of gravitational duals,” JHEP 0611, 049 (2006) [arXiv:hep-th/0608088]; O. Aharony, A. B. Clark and A. Karch, “The CFT/AdS correspondence, massive gravitons and a connectivity index conjecture,” Phys. Rev. D 74, 086006 (2006) [arXiv:hep-th/0608089].