Theory of incoherent self-phase modulation of non-stationary pulses

S B Cavalcanti
Departamento de Física, Universidade Federal de Alagoas, Maceió AL, 57072-970, Brazil
E-mail: solange@lux.ufal.br

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Abstract. A theory describing propagation of partially coherent pulse trains in Kerr media is proposed. It is shown that changes in the statistical properties of chaotic modulated pulses propagating under self-phase modulation (SPM) are easily obtained within the framework of cyclostationary processes. Specific applications for laser light and thermal light are considered showing that in contrast with laser light, the thermal light spectrum on propagation suffers a significant change which smears out the characteristic peaks of the SPM spectrum.

1. Introduction

For a long time the fluctuations of light beams have been the subject of extensive investigation [1]. In the last decade, the lack of perfect coherence of optical fields in wave propagation through nonlinear Kerr media has been addressed by some authors [2]–[5]. Recently, experimental studies have demonstrated the existence of incoherent spatial solitons [6, 7] and theories to explain them have also been established [8]–[10]. However, the work done on the propagation of optical pulses that can be represented by nonstationary processes through Kerr media, does not address the problem of finding the spectra. Studies on spectral properties has been largely limited to those cases where the input field belongs to the restricted class of stationary random processes. Nonstationarity is a difficult feature to handle, and most difficult of all is to define theoretically the spectrum of nonstationary optical fields [11]. The difficulty lies in the fact that for nonstationary processes average values, such as correlation functions, are not time translation invariant and hence depend on two time variables. In such cases, the Wiener–Khintchine theorem cannot be used as it involves only one time variable. The easiest way out of this situation is to use the notion of inducing stationarity into the nonstationary process by random translation, whenever this is possible as, for example, for
periodic nonstationary processes [12, 13]. Here, by a periodic nonstationary process we mean a process for which the joint probability distribution is invariant under translations that are integer multiples of a period $T$. In fact, work in this direction has been developed in other areas of study such as electrical engineering where such processes are known as cyclostationary processes [14, 15].

The purpose of this work is to theoretically investigate self-phase modulation (SPM) of incoherent stationarizable optical beams. To this end we develop a theory of incoherent SPM in section 2 by assuming a general periodic random input for the nonlinear Kerr medium. Next, we apply the developed theory to some interesting practical cases in section 3, and finally in section 4 we conclude after discussing the results.

2. Model and formalism

The propagation of a partially coherent optical pulse through a Kerr medium, depends on the statistical properties of the input as well as on the nonlinear properties of the medium. Within the conventional model of SPM, the nonlinear properties of the medium are governed by the wave equation

$$i\frac{\partial A}{\partial z} - \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial t^2} + n_2 k |A|^2 A = 0,$$

(1)

where $A(z, t)$ is the slowly varying envelope amplitude, $k$ is the wavenumber, $n_2$ is the nonlinear Kerr coefficient and $\beta_2$ is the group velocity dispersion (GVD) parameter. A review on the theory of self-phase modulation including other effects, is provided in [16]–[18]. Here we intend to illustrate the effects of SPM on a noisy pulse beyond the usual stationary distribution approach and so we shall limit ourselves to pure SPM supposing that wave dispersion in (1) may be neglected. It should be noted that the interplay between GVD and SPM can lead to a qualitatively different behaviour from that expected from SPM alone. In particular, for anomalous dispersion ($\beta_2 < 0$) these effects balance each other so that equation (1) admits soliton solutions. In optical fibres for example, one may define the dispersion length $L_D = T_0^2 / \beta_2$ and the nonlinear length $L_{NL} = 1 / \gamma P_0$ to set the length scales over which dispersive and nonlinear effects become important for pulse propagation along a fibre of length $L$. In this way, the present theory should be applicable to fibre lengths such that $L \ll L_D$ and $L \geq L_{NL}$ which defines the nonlinearity dominant regime $L_D / L_{NL} \gg 1$. This condition is satisfied for wide pulses with $T_0 > 100$ ps and large peak powers $P_0 \gtrsim 1$W. According to [19] where experimental measurements of SPM spectra generated with picosecond laser pulses at 532 and 1064 nm propagating in optical fibres, the fibre lengths used were a few kilometres. Without the dispersion term, equation (1) is readily solved to yield the following expression for the complex amplitude:

$$A(z, t) = A(0, t) \exp\left[i n_2 k z |A(0, t)|^2 \right].$$

(2)

Due to the convenient form of equation (2), the statistical evolution is determined solely by the statistical properties of the signal input, i.e. the statistical properties of the output signal, such as the power spectrum are completely determined by the initial conditions. This formulation of the statistics has been referred to in the literature as the homogeneous approach [20]. Next, we consider that the input field is a cyclostationary process described by

$$A(0, t) = \xi(0, t) f(0, t),$$

(3)
where \( \xi(0, t) \) represents a random process and \( f(0, t) \) a periodic function with period \( T_0 \), that is
\[
f(t + T_0) = f(t).
\]

To determine the coherence properties of the transmitted field, we need to calculate its autocorrelation function defined as
\[
G(z, t_1, t_2) = \langle A^*(z, t_1)A(z, t_2) \rangle,
\]
where the angular brackets stand for an ensemble average. The evaluation of \( G(z, t_1, t_2) \) is somewhat involved even in the case where \( A(0, t) \) is a Gaussian random process, since \( A(z, t) \) is not likely to remain Gaussian on propagation because of the nonlinear nature of the Kerr medium. However, in the particular case of pure SPM described by equation (2) it turns out that \( G(z, t_1, t_2) \) can be evaluated analytically without making any approximations as follows. Inserting equations (2) and (3) into (5) we find
\[
G(z, t_1, t_2) = \langle f_1^* f_2 \exp[i n_2 k z (|\xi_1|^2 |f_1|^2 - |\xi_2|^2 |f_2|^2)] \rangle \xi_1 \xi_2.
\]

Here, \( f_i \) and \( \xi_i \) stand for \( f(0, t_i) \) and \( \xi(0, t_i) \), respectively, with \( i = 1, 2 \). To define a power spectral density for the process \( A(z, t) \), we note that it is a periodic random process with period \( T_0 \), according to equation (3). This means that a shift in \( A(t) \) by an arbitrary amount \( a(0 < a < T_0) \) takes us to the periodic process \( A(t + a) \) whose averages are different from those of the original process \( A(t) \). It is said that these two processes have different phases. The statistical properties of a periodic nonstationary process are only invariant under shifts by a multiple of a particular period \( T_0 \), that is, under shifts of \( a = n T_0 \), with \( n \) an integer. Therefore, the autocorrelation function of the process \( A(t) \) will, in general, depend on the absolute time as well as on time differences. However, the dependence on the absolute time is periodic. Hence, supposing \( a \) to be random, independent of \( A(t) \) with uniform distribution, i.e.
\[
\omega(a) = \frac{1}{T_0} \quad \text{if } 0 < a < T_0
\]
\[
\omega(a) = 0 \quad \text{otherwise}
\]
the correlation function \( G(z, t_1, t_2) \) may be averaged with respect to \( a \) to obtain the stationarized auto-correlation function \( \tilde{G}(z, \tau) \), that is
\[
\tilde{G}(z, \tau) = \frac{1}{T_0} \int_0^{T_0} G(z, t_1 + a, t_2 + a) \, da.
\]

As a result of this averaging process, the only dependence which remains is that involving the time difference \( \tau = t_1 - t_2 \), so that we have replaced a cyclostationary process by a stationary one, corresponding to a random phase spread. In this way, using equation (6) and averaging first with respect to \( \xi(t) \) and then with respect to \( a \) we find the following expression for \( \tilde{G}(z, \tau) \):
\[
\tilde{G}(z, \tau) = \frac{1}{T_0} \int_0^{T_0} \langle \xi_1^* \xi_2 f_1^*(t_1 + a) f(t_2 + a) \exp[i n_2 k z (|\xi_1|^2 |f(t_1 + a)|^2 - |\xi_2|^2 |f(t_2 + a)|^2)] \rangle \, da.
\]

The Wiener–Khinchine theorem may now be used to obtain the output power spectrum:
\[
S(z, \nu) = \int_{-\infty}^{+\infty} \tilde{G}(z, \tau) \exp[i 2 \pi \nu \tau] \, d\tau.
\]

It should be noted here that equation (10) is suited for systems that do not respond to the phase and instead carry out a time average. To see whether our theory works we turn to some practical applications.
3. Phase and amplitude modulation of laser light

To illustrate the above outlined procedure, let us consider an incoherent model that describes certain idealized properties of laser light. The model consists of a monochromatic oscillator of known amplitude \( A_0 \), known frequency \( \omega \) and a fluctuating phase arising from noise inherent in the output of any noise-driven nonlinear oscillator. This means that the random process may be written as

\[
\xi(t) = \sqrt{I_0} e^{i[\varphi_0 + \delta\varphi(t)]}
\]  

(12)

where \( I_0 \) and \( \varphi_0 = n_2 k I_0 z \) are the stationary values for the amplitude and phase of a field where the average beam intensity \( I_0 \). \( \delta\varphi(t) \) represents small fluctuations from the average value \( \varphi_0 \) in such a way that \( \delta\varphi \ll \varphi_0 \). Here, we are essentially using a well known model of the laser in which the optical field is represented as the sum of a constant phasor and a weak Gaussian-noise phasor whose phase varies randomly over the entire \( 2\pi \) range. In this case, the phase fluctuations of the total field \( \delta\varphi \) can be shown [21] to represent a real Gaussian random process with zero average, that is \( \langle \delta\varphi \rangle = 0 \). Furthermore, we choose a phase diffusion model in which phase fluctuations have a variance that grows linearly with time, and frequency fluctuations corresponding to white noise [22]. In this way we write the periodic input in the form

\[
A(0, t) = \sqrt{I_0} e^{i[\varphi_0 + \delta\varphi(t)]} f(t)
\]  

(13)

and evaluate the correlation function upon use of equation (6), which gives

\[
G(z, t_1, t_2) = I_0 f(t_1) f(t_2) \exp\{i n_2 k z I_0 \langle e^{i\Delta\varphi} \rangle
\]

(14)

where \( \Delta\varphi = \delta\varphi(0, t_2) - \delta\varphi(0, t_1) = \Delta\varphi(\tau) \) with \( \tau = t_2 - t_1 \). Note how the statistical average has become completely independent of the periodic process so that the second-order coherence function may be written in the following particularly simple way:

\[
\tilde{G}(z, \tau) = I_0 \langle e^{i\Delta\varphi} \rangle \frac{1}{T_0} \int_0^{T_0} f(t_1 + a) f(t_2 + a) \, da = G_c(\tau)G_p(\tau).
\]  

(15)

Here \( G_c(\tau) \) represents the chaotic autocorrelation function and \( G_p(\tau) \) the autocorrelation function of the deterministic periodic process [12]. This product form is a consequence of the fact that one is considering only phase fluctuations of the optical input field. As SPM is an amplitude-dependent effect, the coherence function becomes a product of autocorrelation functions. In the particular case we are considering here a random Gaussian process for which the average in \( \varphi \) is easily calculated via,

\[
\langle e^{-i(\Delta\varphi(\tau))} \rangle = \exp[-\frac{1}{2} \langle \Delta\varphi^2(\tau) \rangle].
\]  

(16)

The spectral lineshape of most lasers is suitably represented by a Lorentzian profile [21]–[23] corresponding to white noise. For such lasers the spectral density of phase fluctuations may be written as

\[
\langle \Delta\varphi^2(\tau) \rangle = 2\pi \Delta\nu \tau
\]

where \( \Delta\nu \) is the full width at half maximum (FHWM) of the Lorentzian spectral lineshape. By using periodic functions such as \( f_j = e^{i\Lambda \sin(\omega_0 t_j)} \) the average of the periodic process yields

\[
\tilde{g_p}(\tau) = \frac{1}{T_0} \int_0^{T_0} e^{-i\Lambda \sin(\omega_0 t_1 + \alpha)} e^{i\Lambda \sin(\omega_0 t_2 + \alpha)} \, da = J_0 \left[ 2\Lambda \sin\left(\frac{\omega_0 \tau}{2}\right) \right],
\]  

(17)
where $J_0$ represents a Bessel function of zeroth order and where we have introduced the second-order coherence function $\tilde{g}_p(\tau)$ defined as $\tilde{G}(z, \tau)/G(z, 0)$. Therefore, for a phase diffusion model equation (15) reads

$$\tilde{g}(z, \tau) = \exp(-\pi \Delta \nu^2) J_0 \left[ 2\Lambda \sin \left( \frac{\omega_0^0 \tau}{2} \right) \right] = g(0, \tau).$$

Note that in this phase modulated case we find the trivial result that the coherence function is not influenced at all by propagation. This fact is also a consequence of the intensity-dependent effect condition, so that

$$A(0, t) = \sqrt{T_0} e^{i[\rho_0 + \delta \theta(t)]} \exp(\Lambda \sin \omega_0 t) \cong \sqrt{T_0} e^{i[\rho_0 + \delta \theta(t)]} [1 - \Lambda (\cos \omega_0 t)]$$

which gives the following coherence function:

$$\tilde{g}(z, \tau) = \exp(-\pi \Delta \nu^2) J_0 \left[ 4n_2 k z \Lambda \sin \left( \frac{\omega_0^0 \tau}{2} \right) \right].$$

Equation (21) describes how the oscillating coherence function of the periodic process is attenuated by the linewidth of the incoherent process.

Let us now consider the case of a Gaussian pulse train which is a more realistic model for the output of a mode-locked laser. We represent the Gaussian pulse train by the convolution of a Gaussian function with a comb function, defined as $\text{comb}(x) = \sum_{n=-\infty}^{\infty} \delta(x - n)$:

$$f(t) = e^{-t^2/2T_p^2} \ast \text{comb} \left( \frac{t}{T_0} \right)$$

where $T_p$ stands for the Gaussian width and $T_0$ represents the period under the single-pulse effect condition, so that $T_p \ll T_0$. Substituting equation (21) in (10) and evaluating its Fourier transform, one may study the evolution of the spectrum of the Gaussian pulse train through the nonlinear medium. Considering essentially single-pulse effects where $T_p \ll T_0$ ($\Delta \nu^{-1} \ll T_0$), we illustrate the changes in the spectrum in figures 1 and 2 where we have plotted the Fourier transform for the input spectrum ($\gamma = n_2 k z I_0 = 0$ in figures 1(a) and 2(a)) and the output broadened spectrum ($\gamma = n_2 k z I_0 = 2.5\pi$ in figures 1(b) and 2(b)) for different values of the normalized input linewidth $\Delta \nu' = \Delta \nu T_p$, so that figure 1 corresponds to $\Delta \nu' = 0.02$ and figure 2 to $\Delta \nu' = 0.5$. It should be noted here, that we have dropped the primes in the figures. Note that, as expected from laser light, the typical SPM spectrum [24] for an initial small linewidth is not influenced significantly by incoherence although some degree of incoherence is present due to the fact that total intra-pulse destructive interference is not possible and the oscillations in the spectrum do not touch the horizontal axes. Furthermore, one can see that the relative partial coherence linewidth does influence the output spectrum in the sense that these interference effects are almost completely suppressed.

4. Pulses of chaotic light: chopped thermal light source

Let us now study the specific case of thermal chopped light by considering the following input:

$$A(0, t) = [u(t) + iv(t)] f(t)$$

where $u(t)$ and $v(t)$ are respectively, the real and imaginary parts of a complex Gaussian stationary process and $f(t)$ is a periodic function. The processes $u$ and $v$ satisfy the following relations:

$$\langle u \rangle = \langle v \rangle = 0 \quad \langle u^2 \rangle = \langle v^2 \rangle = \sigma^2 \quad \langle u(t_1) v(t_2) \rangle = \Gamma(\tau) \sigma^2$$

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where $\sigma^2$ is the variance and $\Gamma(\tau)$ the normalized autocorrelation function associated with the processes $u$ and $v$. Introducing the variables $u_1 = u(t_1)$, $v_1 = v(t_1)$, $u_2 = u(t_2)$ and $v_2 = v(t_2)$ and using equation (6), one may evaluate the autocorrelation function:

$$
\tilde{G}(z, t_1 + a, t_2 + a) = \int dv_1 \int du_1 \int dv_2 \int du_2 (u_1 - iv_1)(u_2 + iv_2)
\times p(u_1, u_2)p(v_1, v_2)f^*(t_1 + a)f(t_2 + a)
\times e^{i\nu_2 k z (u_2^2 + v_2^2)}f(t_2 + a)^2 - (u_1^2 + v_1^2)/f(t_1 + a)^2
$$

(24)

Figure 1. Spectral evolution of a laser field with normalized linewidth $\Delta \nu = 0.02$ at the (a) input ($\gamma = 2\pi k z \sigma^2 = 0$) and (b) output ($\gamma = 2\pi k z \sigma^2 = 2.5\pi$).
with the joint probability function

\[
p(u_1, u_2) = \frac{1}{2\pi\sigma^2(1-\Gamma^2)} \exp\left[-\frac{u_1^2 + u_2^2 - 2\Gamma u_1 u_2}{2\sigma^2(1-\Gamma^2)}\right].
\]  

(25)

The probability density \(p(v_1, v_2)\) is obtained from equation (25) after replacing \(u\) by \(v\). The four-fold integration in equation (24) can be performed in a closed form and the final result for

**Figure 2.** The same as in figure 1, except for a different linewidth, i.e. \(\Delta\nu = 0.5\).
the second-order nonstationary autocorrelation function is given by

\[ G(z, t_1 + a, t_2 + a) = \frac{\Gamma(1 - \Gamma^2)\sigma^2}{[(1 + \beta_2(a))(1 - \beta_1(a)) - \Gamma^2]^2}, \]  

(26)

where

\[ \beta_i(a) = 2i n^2 k z |f(t_i + a)|^2 (1 - \Gamma^2)\sigma^2. \]  

(27)

Averaging in \( a \) we finally obtain the stationary autocorrelation function of the cyclostationary process as

\[ \tilde{g}(z, \tau) = \frac{\Gamma}{T_0} \int_0^{T_0} f_1^* f_2 \, da \left[ 1 + 2i n^2 k z \sigma^2 (|f_2|^2 - |f_1|^2) + 4(n^2 k z)^2 \sigma^4 (1 - \Gamma^2)|f_1|^2|f_2|^2 \right]. \]  

(28)

Equation (28) represents an analytical expression for the output coherence function in terms of the input optical field and shows the effect of SPM on the coherence properties of a chaotic pulse train such as a pulse train originating from a laser light which is first passed through a moving diffuser. To illustrate the effect of self-phase modulation we proceed to study specific examples of chaotic modulated pulses. Consider an input field \( A(0, t) \) obtained after a CW random signal has gone through a frequency modulator. Supposing that we may write it as

\[ A(0, t) = (\xi + i\eta)e^{i\Lambda \sin(\omega_0 t)}. \]  

(29)

with \( \omega_0 \) denoting the modulation frequency and \( \Lambda \) a constant, we can immediately write for the coherence function

\[ \tilde{g}(z, \tau) = \frac{\Gamma(\tau)}{2[1 + 4(n^2 k z)\sigma^2 (1 - \Gamma^2)]^2} J_0 \left( 2\Lambda \sin \left( \frac{\omega_0 \tau}{2} \right) \right) \]  

(30)

which in the case of phase fluctuations turns out, as expected, to be the product of the coherence function of the chaotic process times the periodic process:

\[ \tilde{g}(z, \tau) = g_c(z, \tau) g_p(\tau) \]

where

\[ g_c(z, \tau) = \frac{\Gamma(\tau)}{[1 + 4(n^2 k z)\sigma^2 (1 - \Gamma^2)]^2} \]

and \( g_c(z, \tau) \) is the coherence function for the particular case of a chaotic amplitude described by a stationary process, a result previously obtained in [2].

As before we now turn to the case where the periodic function is represented by a Gaussian pulse train defined in equation (21) and substitute it into equation (10). Next we evaluate its Fourier transform, to study the evolution of the spectrum of a Gaussian pulse train originating from a thermal source through the nonlinear medium. In figures 3 and 4 we plot the Fourier transform of a Gaussian pulse train for the input spectrum (\( \gamma = 2n^2 k z \sigma^2 = 0 \) in figures 3(a) and 4(a)) and output broadened spectrum (\( \gamma = 2n^2 k z \sigma^2 = 2.5\pi \) in figures 3(b) and 4(b)) for different values of the normalized input linewidth \( \Delta \nu' = \Delta \nu T_p \), so that \( \Delta \nu' = 0.02 \) in figure 3 and \( \Delta \nu' = 0.5 \) in figure 4. In contrast with the case of the laser light we see that thermal light has the characteristic spikes smeared off. This effect is even more pronounced when taking a larger initial linewidth (figure 4(a)) which completely erases all the spikes and oscillations which are characteristic of a SPM broadened spectrum (figure 4(b)).

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Figure 3. Spectral evolution of a thermal field with normalized linewidth $\Delta \nu = 0.02$ at the (a) input ($\gamma = 2n_2kz\sigma^2 = 0$) and (b) output ($\gamma = 2n_2kz\sigma^2 = 2.5\pi$).

5. Discussion and conclusions

In conclusion, a theory on the propagation of noisy pulses through Kerr media has been developed by using cyclostationary processes to model chaotic modulated pulse trains. The basic idea is to approach the difficult task of assigning a theoretical spectrum to a chaotic pulse envelope which is described as a nonstationary process. The above outlined method is general enough to describe the statistical properties not only of cyclostationary processes but also of any discrete parameter.
process where stationarity might be also be induced. As a result we have been able to generalize
the use of the Wiener–Khintchine theorem to a large class of nonstationary processes. Various
examples have been provided. We have shown that, due to the intensity-dependent character
of SPM, to see any relevant changes in the spectrum one must consider amplitude fluctuations
and/or temporal variations of the pulse shape. A frequency modulated optical field is practically
unaffected by phase fluctuations of the laser source. Considering amplitude variations of the
envelope produced by the laser light we have found that even a small degree of incoherence does
influence the spectrum along propagation. In the thermal case the effect of partial coherence is to

Figure 4. The same as in figure 3, except for a different linewidth, i.e. \( \Delta \nu = 0.5 \).
smear out the oscillating structure with various peaks characteristic of SPM spectra. Particularly for large line width input processes, the multi-peak structure is completely erased from the spectrum. This erasing phenomenon may be understood in terms of the broadening occurring in each peak of the multi-structure making them coalesce into a unique peak structure. The results described here can be easily verified experimentally by using fields originated from mode-locked lasers. Although our results are obtained in the context of temporal partially incoherent light pulses, they exhibit general features of nonlinear media and are also applicable to other fields of study.

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