Testing Time Dilation on Fast Ion Beams

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Abstract. We report the status of an experimental test of time dilation in Special Relativity. This is accomplished by simultaneously measuring the forward and backward Doppler shifts of an electronic transition of fast moving ions, using high-precision laser spectroscopy. From these two Doppler shifts both the ion velocity $\beta = v/c$ and the time dilation factor $\gamma = \gamma_{SR}(1 + \hat{\alpha}\beta^2 + \hat{\alpha}_2\beta^4)$ can be derived. From measurements based on saturation spectroscopy on lithium ions stored at $\beta = 0.03$ and $\beta = 0.06$ in the TSR heavy-ion storage ring, we achieved an upper limit for a $O(\beta^2)$ deviation from Special Relativity of $|\hat{\alpha}| \leq 8 \times 10^{-8}$. In recent measurements on a $\beta = 0.34$ Li$^+$ beam in the ESR storage ring we used optical-optical double-resonance spectroscopy which, in combination with the TSR result, gives improved sensitivity on the $O(\beta^4)$ term of $|\hat{\alpha}_2| \leq 1.2 \times 10^{-5}$. We discuss current limitations and possible improvements that promise an enhancement of the sensitivity by at least one order of magnitude in the future.

1. Introduction

Shortly after the publication of his paper on Special Relativity (SR) in 1905 [1], Einstein proposed to experimentally detect time dilation by observing the transverse Doppler shift of light emitted from moving atoms produced in canal ray tubes [2]. According to the relativistic Doppler formula

$$\nu = \nu_0(1 - \beta \cos \theta)$$

the classical cosine term vanishes at an observation angle $\theta = \pi/2$, and the transverse Doppler shift between the rest frame frequency $\nu_0$ of the emission line and its frequency $\nu$ in the
laboratory frame is solely caused by time dilation, governed by the Lorentz factor $\gamma$. However as an observation angle of exactly $\pi/2$ turned out to be difficult to realize and even small misalignments would cause large residual first-order Doppler shifts due to the slope of $\cos(\theta)$ at $\pi/2$, the first observation of time dilation was only reported in 1938. Ives and Stilwell [3] had taken up Einstein’s idea of using the Doppler effect, but chose a different measurement geometry. Rather than observing transversely, they measured the longitudinal Doppler shifts both in forward ($\theta = 0$) and in backward direction ($\theta = \pi$). Longitudinal overlap is more easily achieved and misalignments lead to smaller residual first-order Doppler shifts. Moreover from the two measured Doppler shifts

$$\nu_0 = \nu_p \gamma(1 - \beta)$$
$$\nu_0 = \nu_a \gamma(1 + \beta),$$

both $\gamma$ and $\beta$ can be derived simultaneously. This makes an additional, usually less accurate measurement of $\beta$ obsolete and thus allows a more sensitive test of the time dilation relation. The reduction of the test to the measurement of only frequencies becomes obvious in the product of Eqs. 1

$$\nu_0^2 = \nu_a \nu_p$$

which, within SR, is independent of $\beta$ and provides the observable in Ives-Stilwell-type experiments. This scheme can easily be generalized to the case where two different transitions with rest frame frequencies $\nu_1$ and $\nu_2$ are used for the forward and backward Doppler shifts: $\nu_1 \nu_2 = \nu_a \nu_p$. The kinematic test theory of Robertson, and Mansouri and Sexl (RMS) [4] provides a $\beta$-dependent figure of merit $\hat{a}(\beta^2)$ for deviations from time dilation. It relates to our observable as

$$\sqrt{\frac{\nu_1 \nu_2}{\nu_0^2}} = \hat{a}(\beta^2),$$

and becomes unity in case SR holds. For $\beta \to 1$, the test function $\hat{a}$ can be expanded for $\beta^2 \ll 1$ into a power series in $\beta^2$, with expansion parameters $\hat{\alpha}_i$: $\hat{a}(\beta^2) = 1 + \hat{\alpha} \beta^2 + \hat{\alpha}_2 \beta^4 + O[\beta^6].$

A more rigorous description within the dynamic Lorentz-violating test theory called Standard Model Extension (SME) [5], which is necessary to compare different experiments, results in sensitivity of our observable to various SME parameters and is discussed elsewhere [6, 7].

2. Laser Spectroscopy at Storage Rings

In their original experiment, Ives and Stilwell used hydrogen atoms in canal rays as moving light emitters and measured the Doppler shifted wavelengths with a grating spectrometer. With particle velocities up to $\beta = 0.0005$ they confirmed time dilation for the first time. In terms of the RMS framework, this constrains deviations from SR to $|\hat{\alpha}| < 1 \times 10^{-2}$.

To improve the sensitivity, faster particles and more accurate Doppler shift measurements are desirable. The latter is accomplished by modern laser spectroscopy methods [8]. In our experiment we tackle the former requirement by using beams of atomic ions from accelerators. $^7\text{Li}^+$ ions turn out to be the clocks of choice as these light charged particles can be accelerated to high velocities. Moreover they exhibit a suitable optical transition at 548.5 nm (547 THz) which is accessible by lasers even for strong red- and blue shifts.

In order to ensure as accurately as possible the longitudinal measurement geometry inherent to the Ives-Stilwell-type experiment, as well as the stability of the particle velocity, we employ heavy ion storage rings equipped with electron cooling [9]. In this cooling technique, a constantly renewed cold electron beam is overlapped with the ion beam in one section of the storage ring.
Coulomb interaction between the electrons and the ions then leads to cooling of the ions which manifests itself in a narrow longitudinal velocity spread and transverse divergence. The latter is indispensable to avoid large contributions from residual first-order Doppler shifts to the measured frequencies.

To reduce the residual first-order Doppler shift further, we optimized the alignment of the laser beams to 0° and 180° with respect to the ion beam. We achieved alignment uncertainties as low as 70 µrad, which lead to frequency shifts of the order of 50 kHz at β = 0.06.

A further source of systematic uncertainty comes from the Gaussian phase structure of the laser beams. Along its axis z, a Gaussian laser beam shows a phase deviation from a plane wave of the same frequency of ξ = \arctan(z/z_R), where the Rayleigh range z_R quantifies the collimation. This z-dependent phase does not affect the frequency probed by a particle at rest. However ions traveling along z at high speed v experience a change of this additional phase and thus a frequency shift γv(∂ξ/∂z), which is maximal close to the focus (z = 0) and can reach up to a MHz for v = 0.06c. The Rayleigh ranges and positions of the laser foci were thus chosen to place the interaction outside the focused region which reduced the shifts to below 100 kHz. We used a Monte-Carlo simulation to estimate the residual effect and were able to push the corresponding uncertainty to the 10 kHz level at β = 0.06.

Comparably large frequency shifts of the order of MHz/Gauss may be caused by the Zeeman effect in magnetic stray fields. We thus use linearly polarized laser beams so that, depending on the direction of the magnetic field, we only drive unshifted ∆m = 0 transitions or an equal number of ∆m = −1 and ∆m = +1 transitions. For unpolarized ion beams, i.e. equal distribution over the m sublevels, the latter only lead to broadening but not to net shifts.
Test measurements with circularly polarized light give insight in the strength of the magnetic stray fields present in the experimental section during the beam times. Measurements with linearly polarized light in stronger externally applied fields allow to detect a possible ion beam polarization. From such test measurements, upper limits on residual Zeeman shifts of the order of 50 kHz were derived. dc-Stark shifts due to motional electric fields are below 1 kHz. The ac-Stark effect for typical laser intensities of the order of 30 mW/cm$^2$ (saturation intensity: 6.7 mW/cm$^2$) mostly leads to symmetric broadening of the order of few hundred kHz. Measurements showed that net residual shifts caused by a possible asymmetry in the velocity distribution around the resonance are negligible compared to the dominating frequency uncertainties.

To overcome the Doppler broadening associated with the residual velocity spread, we employ either saturation spectroscopy [10] or optical-optical double resonance spectroscopy [11] which both effectively select a narrow velocity class of the order of the natural linewidth, on which the Doppler shift measurements are performed. In saturation spectroscopy, the $^7$Li$^+$ $^3S_1(F = 5/2) \rightarrow ^3P_2(F = 7/2)$ two-level transition at $\nu_0$ (see Fig. 1b) is addressed by two Doppler-shifted counterpropagating lasers with intensities equal or larger than the saturation intensity, and the laser-induced fluorescence is observed with photomultipliers. The frequency of one of the lasers is kept fixed in resonance with the ions at the center $\beta_0$ of the velocity distribution. The other laser is tuned across the resonance. Due to saturation, a Lamb dip occurs in the Doppler fluorescence background, when the tuning laser is resonant with the $\beta_0$ ions already addressed by the fixed-frequency laser. The experimental signature is thus a small dip on a large background. As the fluorescence background reflects the velocity distribution it is sensitive to its modification due to the strong laser forces. Contrary to saturation spectroscopy at high sample density in gas cells where collisions quickly flatten the velocity distribution again, saturation spectroscopy on beams suffers from laser-intensity dependent distortions of the fluorescence background leading to apparent shifts of the Lamb dip. We have thus chopped the lasers at frequencies up to 200 kHz and employed a lock-in scheme to quasi-simultaneously measure the distorted fluorescence background and then subtract it from the Lamb dip trace [10].

Alternatively, we investigated optical-optical double resonance spectroscopy on the closed $\Lambda$-type three level system depicted in Fig. 1b. With a fixed-frequency laser resonant with one of the legs of the $\Lambda$ while scanning the second laser over the second leg, fluorescence is observed only when both lasers are resonant with the same velocity sub-ensemble. Otherwise the ions are pumped dark and fluorescence is suppressed. Like in saturation spectroscopy a narrow velocity group is selected and ideally the resulting width of the $\Lambda$ resonance is free of Doppler broadening. Nonetheless we observed residual Doppler broadening of the $\Lambda$ resonances which is caused by velocity-changing collisions. In saturation spectroscopy the Lamb dip occurs only if an ion interacts with both lasers within the lifetime of the upper level (43 ns), during which the ion velocity does not change significantly, and both Doppler shifts are measured for the same velocity $\beta_0$. In $\Lambda$ spectroscopy, however, the interaction is not required to take place simultaneously. Rather can ions, that were pumped dark by one laser, experience velocity-changing collisions during many roundtrips in the storage ring, and thus accidentally be Doppler shifted into resonance with the other laser. This can lead to significant broadening of tens of MHz. As the unwanted fluorescence comes from ions that were previously pumped dark, it can be partly measured by chopping the lasers and using the lock-in scheme used in saturation spectroscopy. The unwanted fluorescence stemming from slow enough velocity changes shows up in the time windows when only one laser is switched on and can thus be subtracted. The residual resonance is narrower as it is broadened only by velocity-changing collisions within one time window. Consequently we have observed narrower resonances for faster laser chopping [11].
This result can be interpreted within the RMS test theory by considering only the order $O[\beta^2]$ term in the expansion of $\hat{a}(\beta^2)$ (see eq. 4). The thick solid line is the limit implied by combining the TSR and ESR results; the resulting restriction for the expansion parameters $\hat{a}$ and $\hat{a}_2$ are shown in the inset.

3. Results

Experiments at 3% and 6% of the speed of light at the TSR storage ring in Heidelberg used saturation spectroscopy, where simultaneous resonance of the red- and blue-shifted lasers with the same velocity class were indicated by a Lamb dip. With relative accuracies of the Doppler shift measurements of $\Delta \nu/\nu = 2 \times 10^{-10}$ ($\Delta \nu \approx 100$ kHz) SR was confirmed on a level of [12]

$$|\hat{a}(\beta^2) - 1| = 3.4 \times 10^{-10}. \quad (5)$$

This result can be interpreted within the RMS test theory by considering only the order $O[\beta^2]$ term in the expansion of $\hat{a}(\beta^2)$ (see Eq. 4) to be relevant, implying the tacit assumption usually made in this field that “reasonable” higher-order expansion parameters $\hat{a}_i$ are only of the order of $\hat{a}$ or less. This analysis resulted in $|\hat{a}| < 8.4 \times 10^{-8}$ [12] (dashed line in Fig. 2).

To improve the sensitivity on time dilation, we are working on a measurement on a faster $^7$Li$^+$ beam at $\beta = 0.34$ stored in the ESR storage ring in Darmstadt (Fig. 1a). Due to a lower number of metastable ions and larger background from stray light caused by the blue-detuned laser, the signal-to-noise ratio is significantly lower at the present stage than it was at the TSR. For this reason we employ $\Lambda$ spectroscopy to avoid the fluorescence background. Fig. 1c shows a $\Lambda$ resonance from one of the first measurements [13]. With a frequency accuracy of $\Delta \nu/\nu = 2.6 \times 10^{-7}$ we arrived at an upper limit for the RMS time dilation function at $\beta = 0.34$ of

$$|\hat{a}(\beta^2) - 1| = 1.5 \times 10^{-7}. \quad (6)$$

When considering only the order $O[\beta^4]$ term, this results in $|\hat{a}| < 1.3 \times 10^{-6}$ still an order of magnitude short of the TSR result. Nevertheless, the higher particle velocity leads to increased sensitivity on the $O[\beta^4]$ term. This is illustrated in Fig. 2, where the dotted line shows the restriction set on $|\hat{a}(\beta^2) - 1|$ by the TSR experiment when the $O[\beta^4]$ term is not omitted.
large $\beta$, it only slightly undercuts previous high velocity results obtained from a Doppler shift measurement on hydrogen at $\beta = 0.84$ [14], and from a measurement of the muon lifetime [15] at $\beta = 0.9994$. Combining the TSR and the ESR results [13], one obtains a new limit for $|\hat{\alpha}(\beta^2)| - 1$ as shown by the thick solid line in Fig. 2. It improves the previous restriction from ref. [14] by a factor of 25. The allowed parameter space for $\hat{\alpha}$ and $\hat{\alpha}^2$ is shown in the inset of Fig. 2 and implies

$$|\hat{\alpha}_2| < 1.2 \times 10^{-5}.$$  

Improvements on the $\Lambda$ spectroscopy setup have recently allowed to reduce the frequency uncertainty by one order of magnitude to $\Delta \nu/\nu = 1.5 \times 10^{-8}$. Further improvements are expected by an enhanced ion current in the ESR that was recently observed after optimization of the ion source. The achieved signal-to-noise ratio promises to permit saturation spectroscopy and thus circumvent the residual Doppler broadening of the $\Lambda$ resonances, so that we expect an improved sensitivity on $\hat{\alpha}$ in the $10^{-9}$ range in the near future.

4. Acknowledgements

We acknowledge support from DFG (NO789/1-1), BMBF (06MZ9179I), and the Helmholtz Association of German Research Centres under contract VH-NG-148.

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