An important role of the scalar-isoscalar $\sigma$-meson in the low-energy physics is discussed. The behavior of the $\sigma$-meson in the hot and dense medium is studied. It is shown that in the vicinity of the critical values of the temperature($T$) and the chemical potential($\mu$) the $\sigma$-meson can become a sharp resonance. This effect can lead to a strong enhancement of the processes $\pi\pi \rightarrow \gamma\gamma$ and $\pi\pi \rightarrow \pi\pi$ near the two-pion threshold. Experimental observation of this phenomenon can be interpreted as a signal of approaching the domain where the chiral symmetry restoration and the phase transition of hadron matter into quark-gluon plasma take place.
1. INTRODUCTION

In the last years, the problem of studying the scalar-isoscalar $\sigma$-meson properties has attracted great attention of many authors [1, 2]. The subjects of investigations are internal properties of the $\sigma$-meson and its role as an intermediate particle in various processes both in vacuum and in hot and dense matter. The latter problem is especially important, because many experiments on heavy ion collision are performed and planned (CERN, Brookhaven, DESY, Darmstadt). One of the aims of those experiments is to study the problem of phase transition of hadron matter into quark-gluon plasma. Special workshops were dedicated to the study of the $\sigma$-meson properties, in Japan, June 2000 [1] and in France, September 2001 [2]. Very interesting talks on this topic were given by T. Kunihiro [3, 4, 5]. In this article, we will follow these reports. However, here we will use the results obtained only in our previous works.

Let us start with the experimental status of the $\sigma$-meson. The experimental value of the $\sigma$-meson mass is not accurately determined and lies in a wide interval [6]:

$$M_\sigma = 400 - 1200\, MeV$$  \hspace{1cm} (1)

It is explained by large values of the decay width of this meson into two pions [6]:

$$\Gamma_\sigma = 600 - 1000\, MeV$$  \hspace{1cm} (2)

However, at high a temperature and density $M_\sigma$ can become lower than the mass of two pions. Therefore, the decay $\sigma \to \pi\pi$ is closed, and the $\sigma$-meson becomes a stable particle. As a result, the $\sigma$-meson can give sharp resonance when it participates in processes as an intermediate state.

Now let us describe the theoretical status of the $\sigma$-meson. The $\sigma$-meson is a chiral partner of the $\pi$-meson in different linear $SU(2) \times SU(2)$ $\sigma$-models [7]. On the other hand, it is a scalar-isoscalar singlet in a $U(3) \times U(3)$ symmetric quark model of the Nambu-Jona-Lasinio (NJL) type [8].

In the NJL model, the $\sigma$-meson mass can be expressed via the pion mass $M_\pi$ and the constituent quark mass $m$ [2]:

$$M_\sigma^2 = M_\pi^2 + 4m^2$$  \hspace{1cm} (3)

Here we assume that the constituent masses of up and down quarks are equal to each other $m_u \approx m_d = m$.

Formula (3) plays a very important role for the description of different processes in hot and dense matter where the $\sigma$-meson participates as an intermediate particle. Indeed, from (3) it follows that in vacuum $M_\sigma$ is larger than $M_\pi$ because $m \approx 280\, MeV$, $M_\pi \approx 140\, MeV$, $M_\sigma \approx 580\, MeV$ [8]. A different situation occurs in hot and dense matter in the vicinity of the critical

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1) Note that for a long time the information about the $\sigma$-meson was absent in PDG and only in 1998 it appeared.

2) We would like to notice that for a more accurate description of the mass spectra of scalar mesons it is necessary to take into account the singlet-octet mixing of scalar-isoscalar mesons with each other and with the scalar glueball [3, 10].
values of the temperature and chemical potential, where \( m \to m_0 \approx 0 \) and \( M_\sigma \to M_\pi \). This corresponds to the restoration of the chiral symmetry. As will be shown below, this behavior of \( M_\sigma \) can lead to the resonant enhancement of some processes where the \( \sigma \)-meson participates as an intermediate particle (for example \( \pi \pi \to \gamma \gamma \), \( \pi \pi \to \pi \pi \)). Observation of such effects, for instance in heavy ion collisions, could indicate approaching the domain of \( T, \mu \) values where the phase transition from hadron matter into quark-gluon plasma appears. The possibility of such a phase transition and the chiral symmetry restoration is a subject of the intensive investigation at present.

The paper is organized as follows. In the next section we demonstrate the important role of the \( \sigma \)-meson for the correct description of different processes in vacuum. In sec.3 we compare the behavior of the \( \sigma \)-meson propagator in vacuum and hot dense matter. We show that in hot and dense matter the \( \sigma \)-meson propagator can be a sharp resonance. Processes \( \pi^+ \pi^- \to \gamma \gamma \), \( \pi^0 \pi^0 \to \gamma \gamma \) in vacuum and hot dense matter are investigated in sec.4. Noticeable enhancement of these cross-sections near the two-pion threshold in the vicinity of the critical \( T, \mu \) values is found. In the last section a short discussion of the results is given. Theoretical and experimental results concerning \( \pi \pi \)-scattering process in hot and dense matter are discussed.

## 2. \( \sigma \)-MESON IN VACUUM

Before describing the \( \sigma \)-meson properties in hot and dense matter, we show a very important role of the \( \sigma \)-meson in a set of processes going in vacuum. Let us consider here some of them: \( \pi \pi \)-scattering, \( \pi \pi \to \gamma \gamma \), the rule \( \Delta I = 1/2 \) (where \( I \) is the isospin of the meson system) in kaon decays and the calculation of the pion-nucleon \( \Sigma \)-term in \( \pi \)-nucleon scattering. We give only a qualitative picture of these processes. Details can be found in the original works [11, 12, 13, 14, 15, 16].

Let us start with \( \pi \pi \)-scattering. In the \( SU(2) \times SU(2) \) chiral NJL model, this process can be described by the lagrangian:

\[
L(q, \bar{q}, \sigma, \pi) = \bar{q}(x) \left( i \gamma_\mu - M + g \left( \sigma(x) + i \gamma_5 \tau \pi(x) \right) \right) q(x)
\]  \hspace{1cm} (4)

where \( \bar{q}(x) = (\bar{u}(x), \bar{d}(x)) \) is the quark field, \( \sigma(x), \pi(x) \) are the \( \sigma, \pi \) meson fields, \( M \) is the diagonal mass matrix of the constituent quarks, \( \tau \) are the Pauli matrices, \( g \) is the quark-meson strong coupling constant

\[
g = (4I_2)^{-1/2}
\]  \hspace{1cm} (5)

where \( I_2 \) is a logarithmical divergent integral that appears in the quark loop, \( I_n(n=1,2) \) in the Euclidean metric is equal to

\[
I_n = \frac{N_c}{(2\pi)^4} \int d^4k \frac{\Theta(\Lambda^2 - k^2)}{(m^2 + k^2)^n}
\]  \hspace{1cm} (6)

\(^3\)For simplicity, we do not take into account \( \pi - a_1 \) transitions (\( a_1 \) is the axial-vector meson)
where $\Lambda$ is a cut-off parameter ($\Lambda = 1.25 GeV$) \[8\].

Diagrams describing $\pi\pi$-scattering are given in Fig.1. Then using lagrangian (4) we obtain the following expression for the amplitude $A_{\pi\pi}$ \[8, 11\]

$$A_{\pi\pi} = -4g^2 + \frac{(4mg)^2}{M^2_\sigma - s} = \frac{s - M^2_\pi}{F^2_\pi},$$ \hspace{1cm} (7)

where $F_\pi = 93 MeV$ is the pion weak decay constant and $s = (p_1 + p_2)^2$, $p_1$ and $p_2$ are the momenta of the incoming pions.

The final expression for $A_{\pi\pi}$ is the famous formula describing the $\pi\pi$-scattering amplitude at low energies. This formula was first obtained by Weinberg in the sixties years of the last century. It was one of the basic formulas demonstrating the chiral symmetry of strong interaction \[7\]. The following relations were used to derive this formula:

1. The Goldberger- Treiman identity $g = m/F_\pi$
2. Formula for the $\sigma$-meson mass (3).
3. The amplitude for the decay $\sigma \rightarrow \pi^+\pi^- A_{\sigma \rightarrow \pi^+\pi^-} = 4mg$.

We can see from eq.(7) that the constant part of the $\sigma$ pole diagram cancels the contribution of the box diagram. The remaining part of the $\sigma$-pole diagram determines the $s$-dependence of the $\pi\pi$-scattering amplitude in agreement with the chiral symmetry requirements \[4\].

The $\sigma$-pole diagram plays an important role for describing the polarizability of the pion which is a significant characteristic of its electromagnetic structure \[8, 17, 18\]. The $\sigma$-meson is also necessary for description of the processes $\pi\pi \rightarrow \gamma\gamma$ and $\gamma\pi \rightarrow \gamma\pi$ in hot and dense matter \[12, 13\]. Below we will discuss the process $\pi\pi \rightarrow \gamma\gamma$ in detail.

The famous rule $\Delta I = 1/2$ is connected with an experimentally observable enhancement of the decay $K_S \rightarrow \pi\pi$ as compared with kaon decays with $\Delta I = 3/2$. This effect can be explained by the presence of the channel with the intermediate $\sigma$-meson (see Fig.2) in this process. Indeed, the $\sigma$-pole diagram in the process $K_S \rightarrow \pi\pi$ leads to the appearance of a resonance factor in this amplitude. This factor takes the form

$$\frac{1}{M^2_\sigma - M^2_{K_S} - iM_\sigma \Gamma_\sigma},$$ \hspace{1cm} (8)

where $\Gamma_\sigma$ is the decay width of the $\sigma$ meson. The kaon mass $M_{K_S}$ is close to the $\sigma$-meson mass. That gives a noticeable enhancement of this channel as compared with the channels with $\Delta I = 3/2$ where the $\sigma$-pole diagram cannot exist. Then pions are emitted directly from quark loops containing a weak vertex \[14\].

We would also like to emphasize an important role of the $\sigma$-pole diagram in the calculation of the pion-nucleon $\Sigma$-term \[15, 16\]. The value of the $\Sigma$-term is determined by diagrams in the Fig.3. These diagrams lead to the following expression(see \[14\])

$$\langle \pi^+(0)|\bar{u}u + \bar{d}d|\pi^+(0)\rangle = 4m[1 + (I_1 - 2m^2I_2) \frac{8g^2}{M^2_\sigma}]$$ \hspace{1cm} (9)

In this formula the first term corresponds to the triangle quark diagram (Fig.3a), and the second term corresponds to the $\sigma$-pole diagram (Fig.3b).
Taking into account the relations \( g = (4I_2)^{-\frac{1}{2}} = m/F_\pi \) and \( M_\sigma \approx 2m \), we see that the first term is cancelled by the part of the second term containing \( I_2 \). The remaining part of eq.(9) takes the form

\[
\langle \pi^+(0)|\bar{u}u + \bar{d}d|\pi^+(0)\rangle \approx 4m \cdot \left[ 2I_1 \frac{g^2}{m^2} = 2 \frac{I_1}{F_\pi^2} \approx 5.8 \right]
\]

It is easy to see that the contribution from the sum of diagrams 3a and 3b is 5.8 times as large as the contribution from diagram 3a. As a result, in our work \([15]\) for \( \pi - N \Sigma \) term was obtained \( \Sigma_{\pi N} = 50 \pm 10 MeV \). Notice that in \([19, 20, 21]\), where the \( \pi-N \Sigma \)-term was calculated in the framework of nonlinear chiral models, the authors obtained a small value for \( \Sigma \)-term because they did not take into account the contribution from the scalar \( \sigma \)-meson.

3. \( \sigma \)-MESON PROPAGATOR IN VACUUM AND HOT DENSE MATTER

Up to now we have considered the above-mentioned processes only in vacuum. Now let us study the \( \sigma \)-meson properties in hot and dense matter. It is especially interesting to investigate the behavior of the \( \sigma \)-meson propagator for the process \( \pi\pi \rightarrow \pi\pi, \pi\pi \rightarrow \gamma\gamma \)

\[
\Delta_\sigma(s) = \frac{1}{M_\sigma^2 - s - iM_\sigma \Gamma_\sigma(s)}
\]

where \( s = (p_\pi^1 + p_\pi^2)^2, p_1 \) and \( p_2 \) are the momenta of the incoming pions. The decay width \( \Gamma_\sigma \) in the NJL model has the form

\[
\Gamma_\sigma(s) = \frac{3m^4}{2\pi M_\sigma F_\pi^2} \sqrt{1 - \frac{4M_\pi^2}{s}}
\]

In vacuum, when \( T = 0 \) and \( \mu = 0 \) and the constituent quark mass is \( m = 280 MeV \), we can consider two extreme cases:

1. \( s \approx M_\sigma^2 \). In this case the real part of the denominator of \( \Delta_\sigma(s) \) \([11]\) is equal to zero but the imaginary part is large: \( M_\sigma \Gamma_\sigma \approx 0.3 GeV^2 \)

2. \( s \approx 4M_\pi^2 \). In this case the imaginary part of the denominator is close to zero, however its real part is large \( M_\sigma^2 - 4M_\pi^2 \approx 0.25 GeV^2 \)

Therefore, in vacuum the real and imaginary parts of the denominator cannot be close to zero simultaneously, and the \( \sigma \)-pole diagram cannot give a sharp resonance in the whole domain of energy.

\[4\] Here we neglect the momentum and mass of the pion and use the formula

\[
I_1 = \frac{3}{(4\pi)^2} \left( \Lambda^2 - m^2 \ln\left(\frac{\Lambda^2}{m^2} + 1\right) \right) = 0.025 GeV^2
\]
A more interesting situation can arise in hot and dense medium. The constituent quark mass decreases and the pion mass slightly increases with increasing $T$ and $\mu$. Therefore the case that $4m^2 \approx 3M^2_\pi$ is possible. Then, if we consider the above-mentioned processes near the two-pion threshold $s = 4M^2_\pi(1 + \epsilon)$ $(\epsilon \ll 1)$, we can see that the real and imaginary parts of the $\Delta_\sigma(s)$ denominator become very small simultaneously

$$
\text{Re} \left( \Delta_\sigma(s)^{-1} \right) = M^2_\sigma - s = 4m^2 - 3M^2_\pi - 4M^2_\pi \epsilon \approx -4M^2_\pi \epsilon \quad (13)
$$

$$
\text{Im} \left( \Delta_\sigma(s)^{-1} \right) = -M_\sigma \Gamma_\sigma = -\frac{3m^4}{2\pi F^2_\pi} \sqrt{\frac{\epsilon}{1 + \epsilon}} \approx -\frac{3m^4}{2\pi F^2_\pi} \sqrt{\epsilon} \quad (14)
$$

As a result, the propagator takes the form

$$
\Delta_\sigma \approx \frac{1}{-4M^2_\pi \epsilon - i\frac{3m^4}{2\pi F^2_\pi} \sqrt{\epsilon}} \quad (15)
$$

The formula shows that in hot and dense matter in the vicinity of the critical $T$ and $\mu$ values, the $\sigma$-meson propagator can become a sharp resonance. This leads to a noticeable enhancement of processes where the $\sigma$-meson participates as an intermediate particle.

In the next section we demonstrate this effect on the basis of the process $\pi\pi \rightarrow \gamma\gamma$ following the papers [12, 13].

4. PROCESS $\pi\pi \rightarrow \gamma\gamma$ IN HOT DENSE MATTER

To describe the processes $\pi^+\pi^- \rightarrow \gamma\gamma$ and $\pi^0\pi^0 \rightarrow \gamma\gamma$ it is necessary to consider quark loop diagrams of three types (see Fig.4). Diagram $4a$ exists only for charged pions. Diagrams $4a$ and $4b$ define the Born terms in a local approximation (see Fig.5). In this approximation, only divergent parts of quark diagrams are considered. As a result, the lagrangian for the photon-meson vertices

$$
L^{\text{Born}} = i e A_\mu \left[ \pi^- \partial_\mu \pi^+ - \pi^+ \partial_\mu \pi^- \right] + A^2_\mu \pi^+ \pi^- \quad (16)
$$

is obtained in this approximation. In the next $k^2$-approximation only diagrams $4b$ and $4c$ give nontrivial contributions. The lagrangians corresponding to the vertices $\pi\pi \rightarrow \gamma\gamma$ and $\sigma \rightarrow \gamma\gamma$ take the form

$$
L^{\text{box}} = \frac{\alpha}{18\pi F^2_\pi} \left[ \pi^+ \pi^- + 5\pi^0 \pi^0 \right] F_{\mu\nu}^2 
$$

$$
L^{\sigma \rightarrow \gamma\gamma} = \frac{5\alpha}{9\pi F_\pi} \sigma F^2_{\mu\nu}, \quad (17)
$$

here we use the notation $\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$ (e is the e.m. charge), $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The lagrangian describing the vertex $\sigma \rightarrow \pi\pi$ has the form

$$
L^{\sigma\pi\pi} = 2mg_\sigma \pi^2 \quad (19)
$$
These lagrangians allow us to define the total amplitude describing processes $\pi\pi \rightarrow \gamma\gamma$.

$$T^{\mu\nu}(s) = T^{\mu\nu}_{\text{Born}}(s) + T^{\mu\nu}_{k^2}(s)$$ (20)

$$T^{\mu\nu}_{\text{Born}}(s) = 2e^2[\gamma^{\mu\nu} - \frac{p_1^\mu p_2^\nu}{p_1 \cdot k_1} - \frac{p_2^\mu p_1^\nu}{p_2 \cdot k_1}]$$ (21)

$$T^{\mu\nu}_{k^2}(s) = e^2 A(s) [\gamma^{\mu\nu} k_1^\nu k_1^\mu - k_2^\mu k_2^\nu]$$ (22)

$$A(s) = \frac{1}{(6\pi F_\pi)^2} \left[ \frac{40m^2}{M_\sigma^2 - s - iM_\sigma \Gamma_\sigma} - 1 \right]$$ (23)

where $s = (p_1 + p_2)^2$, $p_i$ and $k_i$ are the momenta of the pions and photons, respectively, $\Gamma_\sigma$ is the decay width of the $\sigma$-meson (see (12)).

For the process $\pi^0\pi^0 \rightarrow \gamma\gamma$ the Born term is absent. The contribution of the box diagram increases 10 times [8].

The cross-section of the process $\pi^+\pi^- \rightarrow \gamma\gamma$ consists of three parts

$$\sigma_{\pi^+\pi^- \rightarrow \gamma\gamma} = \frac{\pi\alpha^2}{4\kappa s} [\bar{\sigma}_1 + \bar{\sigma}_2 + \bar{\sigma}_3]$$ (24)

where

$$\kappa = \sqrt{1 - \frac{4M_\pi^2}{s}}$$ (25)

Here $\bar{\sigma}_1$ corresponds to the Born term, $\bar{\sigma}_3$ corresponds to contributions from the $\sigma$-pole and box diagrams, and $\bar{\sigma}_2$ is the interference term of the Born and $k^2$-approximation contributions. For the neutral pion we have only the $\bar{\sigma}_3$ term, where box the contribution is ten times larger than in the case with charge pions. Further we will consider these processes near the two-pion threshold. Variables $s$ and $\kappa$ in this domain take the form

$$s = 4M_\pi^2 (1 + \epsilon), \kappa = \sqrt{\frac{\epsilon}{1 + \epsilon}}$$ (26)

where $\epsilon \ll 1$. Then for $\bar{\sigma}_1, \bar{\sigma}_2, \bar{\sigma}_3$ we have

$$\bar{\sigma}_1 = 16 \left[ 2 - \kappa^2 - \frac{1 - \kappa^4}{2\kappa} \ln \frac{1 + \kappa}{1 - \kappa} \right]$$

$$\bar{\sigma}_2 = 4s Re A(s) \frac{1 - \kappa^2}{\kappa} \ln \frac{1 + \kappa}{1 - \kappa}$$

$$\bar{\sigma}_3 = s^2 |A(s)|^2$$ (27)

The $\bar{\sigma}_3$ term has the form

$$\bar{\sigma}_3 = \left( 1 + \epsilon \right) \left( \frac{M_\pi}{3\pi F_\pi} \right)^2 \left\{ \left[ \frac{40m^2 a(s)}{a(s)^2 + M_\sigma^2 \Gamma_\sigma^2} - 1 \right]^2 + \frac{(40m^2 M_\sigma \Gamma_\sigma)^2}{[a(s)^2 + M_\sigma^2 \Gamma_\sigma^2]^2} \right\}$$ (28)

Let us note that in the Born approximation only diagrams 4a and 4b together give a gauge-invariant expression for amplitudes. In the $k^2$-approximation, diagrams 4b and 5b give a gauge-invariant expression separately.
where \( a(s) = M^2_\pi - s = M^2_\pi - 4M^2_\pi (1 + \epsilon) \). This expression consists of two parts. The first part contains the contributions from the box diagram and from the real part of the \( \sigma \)-pole diagram (the expression in the square brackets). The second part corresponds to the contribution from the imaginary part of the \( \sigma \)-pole diagram. Both parts have the common small factor \( \delta \)

\[
\delta = \left( \frac{M_\pi}{3\pi F_\pi} \right)^4
\]

Now let us compare the behavior of \( \tilde{\sigma}_1 \) and \( \tilde{\sigma}_3 \) in vacuum and in a dense medium near the two-pion threshold \( s = 0.1 GeV^2 \). In both cases the values of \( \tilde{\sigma}_1 \) change very little:

\[
\tilde{\sigma}_1 \sim 12 - 15
\]

The opposite situation takes place for \( \tilde{\sigma}_3 \). Indeed, in vacuum we have \( \epsilon = 0.28, \delta = 6.5 \times 10^{-4} \) and the main part of \( \tilde{\sigma}_3 \) is defined by the contribution connected with the real part of the \( \sigma \)-pole diagram

\[
\left( \frac{40m^2a}{a^2 + M^2_\sigma \Gamma^2_\sigma} \right)^2 \approx 84
\]

We can see that the contribution from \( \tilde{\sigma}_3 \) is very small as compared with the contribution of the Born term in this case.

A different situation can take place in hot and dense matter. Indeed, at \( T = 100 MeV \) and \( \mu = 290 MeV \) we have \( m = 138 MeV, M_\pi = 156 MeV, M_\sigma \approx 317 MeV, F_\pi = 57 MeV \) [22]. In this case, the imaginary part of the \( \sigma \)-pole diagram gives a dominant contribution to \( \tilde{\sigma}_3 \). The parameter \( \delta \) in this case equals to \( 7 \times 10^{-3} \), and the main contribution from the imaginary part has the form \( (\epsilon = 0.02) \)

\[
\left( \frac{40m^2a}{M_\sigma \Gamma_\sigma} \right)^2 \approx 10^4, \quad (a \ll M_\sigma \Gamma_\sigma)
\]

As a result, the contribution from the \( \sigma \)-pole diagram becomes comparable with the contribution from the Born term.

This effect plays an especially important role for the neutral pion. In vacuum, the cross section \( \pi^0 \pi^0 \rightarrow \gamma \gamma \) is very small. However, in hot and dense matter the cross sections of \( \pi^+ \pi^- \rightarrow \gamma \gamma \) and \( \pi^0 \pi^0 \rightarrow \gamma \gamma \) can be comparable.

After qualitative estimations, let us give more exactly the numerical calculation for the above-considered cases.

The value \( s = 0.1 GeV^2 \) corresponds to the energy of outgoing photons \( \omega_\gamma = 160 MeV \). In vacuum \((T = 0, \mu = 0)\), we have the following values for masses \( m, M_\sigma \) and parameters \( \epsilon, \kappa \): \( m = 280 MeV, M_\sigma = 580 MeV, \epsilon = 0.28, \kappa = 0.46 \). Using these values, we obtain for the charge pions \( \tilde{\sigma}_1 \approx 12, \tilde{\sigma}_2 \approx 1.8, \tilde{\sigma}_3 \approx 0.1 \). For the neutral pion we have \( \tilde{\sigma}_3 \approx 0.04 \).

The cross-sections for processes \( \sigma_{\pi^+ \pi^- \rightarrow \gamma \gamma}, \sigma_{\pi^0 \pi^0 \rightarrow \gamma \gamma} \) are approximately

\[
\sigma_{\pi^+ \pi^- \rightarrow \gamma \gamma} \approx 4.8 \mu b, \sigma_{\pi^0 \pi^0 \rightarrow \gamma \gamma} \approx 0.015 \mu b
\]

(33)
Now let us consider these processes in hot and dense matter when $T = 100 \text{MeV}$, $\mu = 290 \text{MeV}$. Here the parameters $\epsilon, \kappa$ are equal to 0.02, 0.14, respectively. The masses $m, M_{\pi}, M_{\sigma}$ and value of $F_{\pi}$ are given above. After substituting the parameters and masses into (27), we obtain

$$\tilde{\sigma}_1 \approx 15.5, \tilde{\sigma}_2 \approx 2.83, \tilde{\sigma}_3 \approx 57.4$$

(34)

for the charge pions and

$$\tilde{\sigma}_3 \approx 57.5$$

(35)

for the neutral pions. For the cross-sections $\sigma_{\pi^+\pi^-\rightarrow\gamma\gamma}, \sigma_{\pi^0\pi^0\rightarrow\gamma\gamma}$ we have

$$\sigma_{\pi^+\pi^-\rightarrow\gamma\gamma} \approx 75.6 \mu\text{b}, \sigma_{\pi^0\pi^0\rightarrow\gamma\gamma} \approx 57.5 \mu\text{b}$$

(36)

So, we see that in this domain of $T$ and $\mu$ near the two-pion threshold, $\sigma_3$ increases dramatically. As a result, the charged pions cross-section increases approximately one and a half order. The neutral pions cross-section increases more than three orders and becomes comparable with charged pions cross-section.

The cross-sections $\sigma$ and $\sigma_1, \sigma_2, \sigma_3$ ($\sigma_i = \frac{\pi a^2}{4\kappa} \tilde{\sigma}_i, i = 1, 2, 3$) for process $\pi^+\pi^- \rightarrow \gamma\gamma$ are plotted in Fig.6 and Fig.7 as a function of $s$. Fig.6 shows the numerical results for vacuum where the main contribution comes from the Born terms. The situation changes at a finite temperature and chemical potential. As is shown in Fig.7, the $\sigma$-pole diagram gives the dominant contribution and the cross-section is strongly increased at the threshold. The behavior of the $\pi^0\pi^0 \rightarrow \gamma\gamma$ cross-section is shown in Fig.8 in vacuum and in hot and dense matter. It is easy to see from Fig.8 that the contribution from the $\sigma$-pole diagram dramatically increases near the two-pion threshold.

Here we used the approximative expression for the amplitudes where the $T$ and $\mu$ dependence of quark loops was neglected. A more careful calculation was made in works [12, 13].

5. DISCUSSION AND CONCLUSION

We have shown that the scalar $\sigma$-meson plays an important role in low-energy meson physics. The $\sigma$-pole diagram gives the main contribution to the $\pi\pi$-scattering amplitude and ensures its chiral invariance [8, 11, 23]. The intermediate $\sigma$-meson gives a dominant contribution to charge pion polarizability in the process $\gamma\pi \rightarrow \gamma\pi$ [8, 17]. Using the $\sigma$-pole diagram, we can explain the rule $\Delta I = 1/2$ in kaon decays [14, 24]. The diagram with the $\sigma$-meson determines the value of the pion-nucleon $\Sigma$-term [15, 16, 25]. It is not a complete list of significant physical results where taking the $\sigma$-meson into account allows us to correctly describe hadron properties in vacuum [17].

However, the behavior of the $\sigma$-meson is especially interesting in a hot and dense medium. Here the $\sigma$-meson can become a sharp resonance in the vicinity of the critical values of $T$ and

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6) It is worth noticing that to describe the decay $\eta \rightarrow \pi^0\gamma\gamma$ in agreement with experiment it is also necessary to take into account the channel with the intermediate scalar-isovector meson $a_0(980)$.
This situation leads to a strong increase of the processes where the $\sigma$-meson participates as an intermediate particle. In this work, we have demonstrated this effect on the basis of the process $\pi\pi \rightarrow \gamma\gamma$ (see [12, 13]). Similar results were obtained in works [24, 27]. An analogous situation can also occur in the $\pi\pi$-scattering process.

The spectral function of a $\pi\pi$-system in the $\sigma$-channel has been studied for finite densities in [3, 4, 5]. Characteristic enhancement of the spectral function near the two-pion threshold is found. This effect is close to our results obtained for the process $\pi\pi \rightarrow \gamma\gamma$ (see sec.4).

In work [5], T. Kunihiro pointed the first experimental support of his theoretical results. The CHAOS collaboration [28, 29] studied the differential cross-sections $M^A_{\pi\pi} = d\sigma^A/dM_{\pi\pi}$ for the $\pi A \rightarrow \pi^+\pi^\pm A'$ reaction on nuclei $A = 2, 12, 40, 208$. The observable composite ratio

$$C^A_{\pi\pi} = \frac{M^A_{\pi\pi} \cdot \sigma^N_{\text{tot}}}{M^N_{\pi\pi} \cdot \sigma^A_{\text{tot}}}$$

was chosen to disentangle the acceptance issue and because it is slightly dependent on the reaction mechanism and nuclear distortion (here $\sigma^N_{\text{tot}}$ and $\sigma^A_{\text{tot}}$ are the total cross-sections). It was found out that the $C^A_{\pi^+\pi^-}$ distribution peak at the $2m_\pi$ threshold and their yield increase with $A$. At the same time, the $C^A_{\pi^+\pi^+}$ distribution weakly depends on $A$. This means that nuclear matter weakly affects the $(\pi\pi)_{I,J=2,0}$ interactions, whereas the $(\pi\pi)_{I,J=0}$ state forms a strongly interacting system (the intermediate $\sigma$-meson). The experimental results were compared with some theoretical models, and the best agreement was found with the models that take into account the medium modifications of the scalar-isoscalar $\sigma$-meson and the partial restoration of chiral symmetry in nuclear matter.

The Crystal Ball(CB) collaboration [30] studied the reaction $\pi^- A \rightarrow \pi^0\pi^0 A'$ on $H, D, C, Al, Cu$. They reported that there was no peak near the $2m_\pi$ threshold, observed by CHAOS collaboration, but the increase in strength as a function of $A$ was also observed in the $\pi^0\pi^0$ system. Later, the CB results were reanalyzed [31] in terms of the composite ratio (37) and accounting for different acceptances of two experiments. It was shown that as far as the $(\pi\pi)_{I,J=0}$ interacting system is concerned, the results agree with each other very well. This agreement may be interpreted as an independent confirmation by the CB experiment of the modification of the $\sigma$-meson properties in nuclear matter, first reported by CHAOS.

The above-described experimental data for $\pi\pi$-scattering allow us to hope that similar results can be experimentally obtained for the processes $\pi\pi \rightarrow \gamma\gamma$ in hot and dense matter. It is especially interesting to study $\pi^0\pi^0 \rightarrow \gamma\gamma$, because the cross-section of this process can increase by several orders of magnitude near the two-pion threshold. Experimental observation of all these effects will be evidence for approaching the boundary of the domain where the partial restoration of chiral symmetry and the phase transition of hadron matter into quark-gluon plasma take place.

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REFERENCES

1. Proceedings Possible existence of the $\sigma$-meson and its implications to the hadron physics, Japan, Kyoto, Yukawa Institute, 12-14 June, 2000, ed. by S. Ishida et al., (KEK Proceedings 2000-4, December, 2000).

2. Proceedings International Workshop on Chiral Fluctuations in Hadronic Matter, IPN Orsay, France, September 26-28, 2001, in press.

3. T. Kunihiro, In [1], p.6

4. T. Hatsuda, T. Kunihiro, In [2]; nucl-th/0112027.

5. T. Kunihiro, hep-ph/0111121.

6. Review of Particle Physics, Eur. Phys. J. C 15, 1 (2000).

7. V. De Alfaro, S. Fubini, G. Furlan, C. Rossetti, Currents in Hadron Physics (American Elsevier Publishing company, New York, 1973).

8. M.K. Volkov, Sov. J. Part. Nucl. 17, 186 (1986).

9. M.K. Volkov, V.L. Yudichev, Eur. Phys. J. A 10, 109 (2001).

10. M.K. Volkov, M. Nagy, V.L. Yudichev, Nuovo Cim. A 112, 225 (1999).

11. M.K. Volkov, A.A. Osipov, Sov. J. Nucl. Phys. 39, 440 (1984).

12. E.A. Kuraev, M.K. Volkov, D. Blaschke, G. Röpke, S. Schmidt, Phys. Lett. B 424, 235 (1998).

13. E.A. Kuraev, M.K. Volkov, Phys. Atom. Nucl. 62, 128 (1999).

14. A.N. Ivanov, N.I. Troitskaya, M.K. Volkov, Sov. J. Nucl. Phys. 47, 736 (1988).

15. A.N. Ivanov, N.I. Troitskaya, M. Nagy, M.K. Volkov, Phys. Lett. B 235, 331 (1990).

16. M. Nagy, N.L. Russakovich, M.K. Volkov, Acta Physica Slovaca 51, 299 (2001).

17. M.K. Volkov, A.A. Osipov, Yad. Fiz. 41, 1027 (1985).

18. A.E. Dorokhov, M.K. Volkov, J. Hüfner, S. Klevansky, P. Rehberg, Z. Phys. C 75, 127 (1997).

19. J. Gasser, M.E. Sainio, hep-ph/0002283.

20. J. Gasser, H. Leutwyler, M.P. Locher, M.E. Sainio, Phys. Lett. B 213, 85 (1988).

21. P. Büttiker, Ulf-G. Meissner, Nucl. Phys. A 668, 97 (2000); hep-ph/9908247.
22. D. Ebert, Yu.L. Kalinovskii, L. Munchow, M.K. Volkov, Int. J. Mod. Phys. A 8, 1295 (1993).
23. E. Quack, P. Zhuang, Yu.L. Kalinovsky, S.P. Klevansky, J. H"ufner, Phys. Lett. B 348, 1 (1995).
24. T. Morozumi, C.S. Lim, A.I. Sanda, Phys. Rev. Lett. 65, 404 (1990).
25. T. Kunihiro, T. Hatsuda, Phys. Lett. B 240, 209 (1990); ibid Nucl. Phys. B 387, 715 (1992).
26. T. Kunihiro, Prog. Theor Phys. Suppl. 120, 75 (1995).
27. S. Chiku, T. Hatsuda, Phys. Rev. D 58, 076001 (1998).
28. F. Bonutti et. al., Phys. Rev. Lett. 77, 603 (1996).
29. F. Bonutti et. al., Nucl. Phys. A 677, 213 (2000).
30. A. Starostin et. al., Phys. Rev. Lett. 85, 5539 (2000).
31. P. Camerini et. al., nucl-ex/0109007.
FIGURE CAPTIONS

1. The quark diagrams describing the $\pi\pi$-scattering. All loops in Fig.1-Fig.4 consist of constituent quarks.

2. The diagram describing the $K_S \rightarrow \pi\pi$ decay with $\Delta I = 1/2$. The dot is a weak vertex.

3. The quark diagrams describing the matrix element $\langle \pi^+(0)|\bar{u}u + \bar{d}d|\pi^+(0)\rangle$. The left vertices are scalar quark vertices interacting with a nucleon.

4. The quark diagrams describing the matrix element $\pi\pi \rightarrow \gamma\gamma$.

5. The quark diagrams in the Born approximation describing the matrix element $\pi\pi \rightarrow \gamma\gamma$.

6. The total cross-section $\sigma$ (thick solid line) for process $\pi^+\pi^- \rightarrow \gamma\gamma$ and partial cross-sections $\sigma_1$ (long-dashed line), $\sigma_2$ (thin solid line), $\sigma_3$ (short-dashed line) in vacuum.

7. The total cross-section $\sigma$ (thick solid line) for process $\pi^+\pi^- \rightarrow \gamma\gamma$ and partial cross-sections $\sigma_1$ (long-dashed line), $\sigma_2$ (thin solid line), $\sigma_3$ (short-dashed line) in hot and dense matter at $T = 100MeV$, $\mu = 290MeV$.

8. The cross-section for process $\pi^0\pi^0 \rightarrow \gamma\gamma$ in vacuum (dashed line) and in hot and dense matter $T = 100MeV$, $\mu = 290MeV$ (solid line).
Figure 1:

Figure 2:
Figure 3:

Figure 4:

Figure 5:
Figure 6:

\[ \sqrt{s - 4M^2}, \text{MeV} \]

Figure 7:

\[ \sqrt{s - 4M^2}, \text{MeV} \]
Figure 8: