Synthesis of Algorithm for Range Measurement Equipment to Track Maneuvering Aircraft Using Data on Its Dynamic and Kinematic Parameters

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Abstract. The problem of improving automated air traffic control systems is considered through the example of the operation algorithm synthesis for a range measurement channel to track the aircraft, using its kinematic and dynamic parameters. The choice of the state and observation models has been justified, the computer simulations have been performed and the results of the investigated algorithms have been obtained.

1. Introduction

The problem of accuracy, stability and noise immunity of tracking devices for automated air traffic control (ATC) systems is related to the fact that the aircraft movement in the aerodrome area does not correspond to the existing classical traffic models. In some aircraft maneuvers (turns, sharp turns, etc.), classical Kalman filters tend to diverge, resulting in an increase in errors when estimating the range ($D$), velocity ($V$), acceleration ($a$) because target’s kinematic parameters $V$ and $a$ have complex time dependences [1].

In modern ATC systems, the model of the state of a range measurement channel is based on the hypothesis of a constant-speed maneuvering aircraft. The studies showed [2, 3] that big errors in range estimation might occur for maneuvering targets; this does not meet modern requirements for the ATC systems in terms of increasing the throughput and maintaining a given level of air traffic safety.

The preferred solution to this problem is the algorithm synthesis for ATC tracking devices to estimate the aircraft’s trajectory using information on its dynamic and kinematic parameters.

The information on dynamic and kinematic parameters of the aircraft can be retrieved from discrete address information transmission systems via an individual address request. This information will allow selecting a model of the range measurement channel more accurately with regard to aircraft maneuvers compared to the existing system models.

2. Selection and rationale for system models

In the synthesis of radio-electronic tracking systems, the Singer tracking model is frequently used, where acceleration is simulated using a stationary process with respect to the ensemble of all possible trajectories of the aircraft [2, 4]. The Singer tracking model for the range measurement system has the following form:
In expressions (1) and (2): $\alpha_j$ is time constant of maneuver; $\xi_a(k-1)$ is centered Gaussian noise with known variance $R_{\xi_a}$; $T$ is a sampling interval; $k$ is a discrete time number; $\xi_{D_1}$ is discrete centered Gaussian noise of distance measurements with known variance $R_{D_1}$.

As in models (1) and (2), the acceleration is given with regard to the ensemble of all possible trajectories, the filter being formed is optimal with respect to the ensemble of trajectories, but it is not optimal with respect to a single trajectory. The studies showed [2] that with this model, acceptable estimates for the range, velocity, acceleration take place for a constant-speed maneuvering aircraft or an evenly accelerated one. For aircraft maneuvers, the mean square error of the range estimate for models (1) and (2) increases by 2 ... 2.5 times, which is unacceptable in automated ATC systems. For a radar station, the aircraft’s maneuvers are a nonstationary process since the trajectory consists of the sections where the acceleration varies depending on the type and phase of the maneuver. It is quite challenging to give a statistically accurate description of the acceleration variation patterns for a maneuvering aircraft, using the Singer model. The improved principles for the automated ATC systems require the estimation of the current acceleration of an aircraft [3, 5, 6]. The aircraft maneuvers are characterized by a variation in their kinematic parameters ($\theta$ is a path inclination angle; $\varphi$ is a path turning angle; $\gamma_v$ is a banking angle; $\epsilon_a$ is an azimuth; $\epsilon_s$ is an elevation angle) and dynamic parameters ($n_z$ is a longitudinal acceleration; $n_y$ is a normal acceleration; $n_z$ is a lateral acceleration). This information can be used to create a range measurement model for determining the range of an aircraft relative to the ATC radar:

$$D(k+1) = D(k) + D(k)T + 0.5a(k)T^2;$$

$$V(k+1) = V(k) + a(k)T;$$

$$a(k) = (1 - \alpha_j T)a(k-1) + \xi_a(k-1),$$

$$D_i(k+1) = D_i(k+1) + \xi_{D_1}(k+1).$$

In equations (3) – (7): $\xi_{D_1}$ is discrete centered Gaussian noise acceleration measurements with known variance $R_{D_1}$; $g$ is the acceleration of gravity.

3. **Operation algorithm for range measurement device**

The algorithm for generating an optimal estimate of the phase coordinates by the filtration criterion of minimum error variance is of the form [4, 7, 8, 9]:

$$x_{es}(k) = x_e(k) + K_{PE}(k)\xi_e(k);$$

$$x_e(k) = F(k)x_e(k-1), \quad x_e(0) = x_{es}(0);$$

$$P_e(k) = F(k)P_{es}(k-1)F^T(k) + Q_e(k), \quad P_e(0) = P_{es}(0);$$

$$x_{es}(k) = x_e(k) + K_{PE}(k)\xi_e(k);$$

$$x_e(k) = F(k)x_e(k-1), \quad x_e(0) = x_{es}(0);$$

$$P_e(k) = F(k)P_{es}(k-1)F^T(k) + Q_e(k), \quad P_e(0) = P_{es}(0);$$
\[
K_F(k) = P_e(k) \cdot H^T(k) \{ H(k) \cdot P_e(k) \cdot H^T(k) + R(k) \}^{-1}; \quad (11)
\]

\[
P_e(k) = P_e(k) - K_F(k) \cdot H(k) \cdot P_e(k). \quad (12)
\]

In equations (8) – (12): \( x_e(k) \) is the prognosis estimation (extrapolation) of state vector \( x(k) \); \( K_F(k) \) is a matrix of optimal transmission coefficients of digital filter dimensionality; \( F(k) \) is a transition matrix; \( P_e(k) \) is a posteriori covariance filter error matrix; \( P_{es}(k) \) is a priori covariance filter error matrix; \( R(k) \) is a measurement variance matrix; \( Q_{es}(k) \) is a disturbance variance matrix; \( H(k) \) is a measurement matrix.

Operation algorithms for the Kalman filters to track (measure) the coordinates and parameters of the aircraft depend on the models of state and observation. Based on the state model (3) – (5) and the observation model (6), (7) and the Kalman filtering expressions (8) – (12), the following filtering range measurement algorithm for distance \( D_{es} \), velocity \( V_{es} \) and acceleration \( a_{es} \) was obtained:

\[
D_{es}(k + 1) = D_{es}(k + 1) + K_{F11}(k + 1)\Delta D(k + 1) + K_{F12}(k + 1)\Delta a(k + 1); \quad (13)
\]

\[
V_{es}(k + 1) = V_{es}(k + 1) + K_{F21}(k + 1)\Delta D(k + 1) + K_{F22}(k + 1)\Delta a(k + 1); \quad (14)
\]

\[
a_{es}(k + 1) = a_{es}(k + 1) + K_{F31}(k + 1)\Delta D(k + 1) + K_{F32}(k + 1)\Delta a(k + 1); \quad (15)
\]

\[
D_e(k + 1) = D_e(k + 1) + V_{es}(k)T + 0.5a_{es}(k)T^2; \quad (16)
\]

\[
V_e(k + 1) = V_e(k) + a_{es}(k)T; \quad (17)
\]

\[
a_e(k + 1) = (1 - a_fT)a_e(k); \quad (18)
\]

\[
\Delta D(k + 1) = D_e(k + 1) - D_e(k + 1); \quad (19)
\]

\[
\Delta a(k + 1) = a_e(k + 1) - a_e(k + 1). \quad (20)
\]

4. Simulation and investigation of the range measurement device

Using computer simulations, the real accuracy of the aircraft’s range when performing the greater box pattern maneuver was investigated [8, 10]. Computer simulation is modeling of input signals \( D_i, a_i \) and processing of these signals using algorithms (13) - (20).

The simulation of input signal \( D_i \) is the varying true relative range between the ATC radar station and tracked aircraft \( D \) (Figure 1) and observation noise \( \xi_{D_i} \). The simulation of the input signal is the sum of changing true parameter \( a \) (Figure 2) and observation noise \( \xi_{a_i} \). Observation noises \( \xi_{D_i} \) and \( \xi_{a_i} \) are simulated by random number sensors.

As can be seen from Figures 1 – 3, the dependences of range \( D \), velocity \( V \) and acceleration \( a \) along the line of sight are nonlinear, which complicates their estimation by the existing Kalman filtering methods without taking into account the kinematic and dynamic parameters of the aircraft.

The real accuracy was estimated by the mean square error of estimates \( D, V, a \) for 100 implementations by the formula [11]:

\[
\sigma_{\delta_p}(k) = \sqrt{\frac{\sum_{j=1}^{N}(x(k) - x_{oj}(k))^2}{N - 1}}. \quad (21)
\]

In formula (21): \( \sigma_{\delta_p}(k) \) is mean square error of target phase coordinates estimation; \( x(k) \) is true values of phase coordinates \( (D, V, a) \) of aircraft; \( x_{oj}(k) \) is estimated values of the target’s phase coordinates of the \( j \)-th implementation; \( N \) is the number of implementations.
Simulations were carried out for two cases. In the first case, the Singer tracking model of the aircraft’s motion relative to radar stations (1) and (2) was used in the range measurement channel. In the second case, the range measurement channel used a state model with allowance for the kinematic and dynamic parameters of the aircraft (3) – (7).

Figures 4 – 6 are the time variation graphs of the mean square errors in filtering range $D$, velocity $V$, and acceleration $a$ along the line of sight between the ATC radar and the aircraft. As can be seen from Figures 4 – 6, when the aircraft performs the greater box pattern maneuver, there is a significantly smaller amount of mean quadratic estimation errors $D$, $V$ and $a$ if the information on the kinematic and dynamic parameters of the aircraft (curve 2) is used compared with the Singer tracking model (curve 1).
Extrapolation of the aircraft acceleration using the data on the kinematic and dynamic parameters of the aircraft makes it possible to obtain very important information on the movement of the aircraft relative to the ATC radar station. In addition, the different sources of information from the discrete address data transmission system (kinematic and dynamic parameters of the aircraft) and the radar (the aircraft's sighting coordinates) make it possible to conclude that there is a possibility of a significant increase in the interference resistance of the radar.

The use of synthesized algorithms (13) – (20) will make it possible to solve the problems of the ATC system more efficiently, increasing the throughput and maintaining a given level of air traffic safety.
5. Conclusion
The analysis of the simulation results of the operation algorithms for the range measurement device showed that the accuracy of estimating the mean square errors in the filter-based estimation of aircraft’s range $D$, velocity $V$, and acceleration $a$ is 2 ... 3 times higher if its dynamical and kinematic parameters are taken into account compared with the filter of Singer’s tracking model. This is due to the fact that the use of information on the dynamic and kinematic parameters of the aircraft allows giving a more accurate description of the state models and observation models of the range measurement device.

Thus, the synthesized algorithm for tracking aircraft trajectories built with the information of airborne velocity sensors makes it possible to improve the accuracy of the tracking filters, reduce the computational costs, and consequently, increase the throughput of the ATC system and maintain a given level of safety.

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