Study of different geostatistical methods to model formation porosity (Cast study of Zubair formation in Luhais oil field)

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Abstract: This study is concerned with making comparison in using different geostatistical methods for porosity distribution of upper shale member - Zubair formation in Luhais oil field which was chosen to study. Kriging, Gaussian random function simulation and sequential Gaussian simulation geostatistical methods were adopted in this study. After preparing all needed data which are contour map, well heads of 12 wells, well tops and porosity from CPI log. Petrel software 2009 was used for porosity distribution of mentioned formation in methods that are showed above. Comparisons were made among these three methods in order to choose the best one, the comparing criteria was according to different statistical information and variograms analysis of entered porosity which was scaled up and modeled. The best method gave porosity distribution in model closest to enter porosity that represent real porosity of Zubair formation. The comparison proved that sequential Gaussian simulation is the best model of porosity distribution followed by Gaussian random function simulation and kriging methods respectively. Based on the results obtained, it was concluded that the distribution accuracy of the porosity or other petrophysics properties is one of the main factors that affect building correct geological model and also pore volume (Oil reserve) value was affected by geostatistical methods variation.

1. Introduction
Geostatistical modeling is accepted in petroleum industry as a technique of heterogeneous reservoir characterization, it preserve variance data instead of just the mean value, as in interpolation [1]. Geostatistical modeling deals with large number of different methods of properties estimation such as different Gaussian and kriging Techniques [2]. Kriging techniques are the estimation methods of petrophysical properties (i.e. porosity) that use variogram for showing the spatial variability [3], so simple kriging (SK) was used as one of the methods that utilized in this case study. Another method of petrophysical properties distribution is sequential Gaussian simulation (SGS) that beginning with defining values univariate distribution e.g., Evaluation of grade values, performing an original score transformation of values to a standard normal distribution, and assuming multi normality of normal scores [4]. Gaussian random function simulation (GRFS) is subsequently differ from sequential Gaussian simulation in accuracy of distribution, it is faster with additional efficiencies due to its parallel architecture [5].
For many years, variogram had been used for the determination of the spatial phenomena variability, sometimes variogram calculation, modeling and interpretation were performed hastily and the wrong must be skipped with taking in account more attention to establish a durable model of spatial variability (variogram) before the beginning to make numerical reservoir models [6].

By using these methods, porosity value at a location in space was determined, firstly by getting the probability distribution at that location, and then plotting data randomly (i.e. simulated porosity values) from this distribution. The simulated values of porosity reproduced statistical and spatial patterns of the input data [7].

Porosity values at some locations of Zubair formation in Luhais field was entered, scaled up and distributed in all of the above geostatistical methods. The comparison among these models is accomplished according to statistical and variograms analysis.

1.1 Case study area

Luhais oil field that located a way 100 km from north-west Basrah is producing from two reservoirs (Nahr omer and Zubair), Zubair formation of said field was chosen to this study because of the availability of data.

Zubair formation represent the main reservoir that contains high amounts of hydrocarbons in southern Iraq which is deposited in lower cretaceous, generally it is a sequence of sandstone, shale and siltstone layers. Mainly it is divided into three members, upper shale member, sandstone and lower shale members. The current study is for upper shale member, where the oil cumulated, while the rest of the members are water bearing [8] [9].

1.2 Geostatistical review

Geostatics: is a continuous development branch of math and statistics applications, it offers numbers of tools for spatial variability modeling and its understanding. Geostatisics was found previously in mining industry, new estimation method was developed by D.G krige, a South African engineer, and H.S sichel, a statistician at early of 1950s where classical statistics became inappropriate for the estimation of disseminated ore reserves, Krige’s concept was expanded and formalized within a single framework, and coined the word “kriging” in recognition of Krige’s work, by The French engineer Georges Matheron.

Kriging technique was developed to solve problems of ore-reserve estimation, in 1970 where the high-speed computer was turn up, Kriging technique was spread to many other areas of geology. Although geotatistics techniques have been developed and grown steadily and significant, it did not use in petroleum industry before 1980s. Engineers and geoscientists are facing complex relationship and huge geographical extension of geological phenomena with scientific requirements for preparing geological model involving the measurement of interest minute volume or area [10]. So, the main purpose of geostatistics is spatial geological systems characterization that are incompletely known [11].

1.3 Variogram theory

Variogram is an important input for studying of geostatistecs, it represents a good tool for investigation and modeling spatial variability of petrophysical properties and lithology. Variogram is used in 90% of geostatistical reservoir characterization studies as a basic geostatical modeling methods [6].

1.3.1 Experimental variogram

It is a special function that calculated from variability measurement between pairs of points at various distances, mathematical representation is [2] [12]:
2\gamma(h) = E[(z(u) - z(u + h))^2] \quad (1)

Semivariogram: is a variogram one half and its mathematical representation is:

\gamma(h) = \frac{1}{2N(h)} \sum_{N(h)} [z(u) - z(u + h)]^2 \quad (2)

1.3.2 Variogram parameters
Sill is a total variance where the empirical variogram appears to level off.
Range is the distance or lag at which the sill is reached. (i.e. the distance after which data are no longer correlated.)
Nugget is an apparent intersection with Y-axis [13].

![Variogram parameters](image)

**Figure 1.** Variogram parameters

1.3.3 Variogram models
Experimental variogram is not adopted directly in geostatistical modeling methods, so its points must be fitted to parametric variogram model [2]. In petrel software, there are three variogram models points:
1) Spherical model
Represented by quadratic modified equation where spatial dependence spread out as the sill and range.

\gamma(h) = c\left[\frac{3h}{2a} - \frac{h^3}{2a^3}\right] \quad (3)
a \geq h \geq 0, \gamma(h) = c, h > a

2) Exponential model
It is similar to spherical model in variability with distance reaches to sill gradually. Two samples points relationship is decaying gradually, but at a distance of infinite spatial dependence dissipates.

\gamma(h) = c[1 - \exp\left(-\frac{3h}{a}\right)] \quad (4)

3) Gaussian model
It is using normal probability distribution curve.
Where phenomena are identical at short distance this model type is used because its progressive rise up the y-axis.
\[
\gamma(h) = c[1 - \exp\left(-\frac{3h^2}{a^3}\right)]
\]

(5)

So, the above three equations variables are (c is sill- nugget, h is a spatial vector, distance from central point and a is an effective range (the distance where h equal 95% of sill).

The three variograms models are existing in Petrel showed below in Fig.2 [14].

![Variogram Models](image)

**Figure 2.** Exponential, spherical and Gaussian theoretical variogram models.

1.4 Kriging

Kriging is a number of popularizing linear regression methods for decreasing a guesses variables from covariance of past model.

All kriging techniques are detailing on the logarithmic basic of linear regression generalization and corresponding estimator.

Simple kriging technique (SK) assumes any set of weight \( \lambda_\alpha (u) \) are linearizing to zero in calculation and this leads to minimize the estimation variance from solving the equations for these weights [12]. Its expression as:

\[
[Z^\text{SK}_* (u) - m(u)] = \sum_{\alpha=1}^{n} \lambda_\alpha (u) [Z(u_\alpha) - m(u_\alpha)]
\]

(6)

The classical stationary system of simple kriging:

\[
\sum_{\beta=1}^{n} \lambda_\beta (u) C(u_\beta - u_\alpha) = C(u - u_\alpha), \alpha = 1, \ldots, n
\]

(7)

1.5 Sequential Gaussian simulation

In the reservoir modeling applications, Gaussian sequential simulation (SGS) represents a common approach which was adopted in recent periods. Simply, flexibility and reasonability are main feature of this method.

The following steps show the SGS procedure:

1) Original z data must be transformed to normal distribution.
2) Place transforming data into the model.
3) Seek all neighboring and previously simulated data of random function u.
4) Get kriged guesses and the corresponding kriging by achieving kriging on these above and previously simulated data.

Variance:

\[
\Upsilon^*(u) = \sum_{\beta=1}^{n} \lambda_\beta : Y(u_\beta)
\]

(8)

\[
\sigma^2\text{sk} (u) = c (0) - \sum_{\alpha=1}^{n} Y_\alpha c(u, u_\alpha)
\]

(9)

5) Plot a random residual \( R(u) \) that follows a normal distribution with mean of zero and variance of \( \sigma^2\text{sk} \).
6) Get simulated value by adding the kriged estimate to residual:
\[ Y_s(u) = Y^*(u) + R(u) \]  
(10)

\( Y_s(u) \) must be acquired from the drawing normal distribution with mean \( Y^*(u) \) and variance \( \sigma^2_{sk}(u) \).

7) Correction of future predictions is tested by adding \( Y_s(u) \) to data set to ensure that covariance with this value.

8) Revise the location of random order.

9) When model is populated, all data values and simulated values are backed transform.

10) By repeating the above steps with different random numbers seeds, you can create any number of realization [2].

1.6 Gaussian random function simulation

Petrel software introduced a new Gaussian simulation algorithm, geostatistical simulation library and user’s guide (GSLIB) showed substantially difference’s from the sequential Gaussian simulation:

1) Typically, It is faster than SGS.
2) It is not a sequential algorithm.
3) It has been parallelized.
4) It has a fast co-located co-simulation option.

As showed above, other than SGS, it is not a sequential algorithm and this has permitted to be parallelized algorithm.

It is based on well-known decomposition which stated:

\[ \text{Conditional simulation} = \text{Kriging} + \text{Unconditional Simulation} \]  
(11)

So, new parallel kriging algorithm will use as a base.

When correct algorithm is used, unconditional simulations can be very fast. Fourier Transform is a basic of a faster algorithm that is used. By is parallelizability it is showing a good variogram reproduction for a wide class of variograms [3].

2 Methodology

This case study work was started with preparing data of contour map, well heads, well tops, and CPI porosity, and finished with making Zubair surfaced that had distributed porosity in three showed methods.

2.1 Structural modeling

Structural modeling of Zubair formation was built by applying different statistic algorithms in Petrel software such as convergent interpolation on the contour map, inserted well tops and the plotted polygon as a boundary.

Fig.3. is a structural contour map of Zubair formation top, it represents a clear image of formation that has four anticline folds with axis trending NW-SE, the length of structure is 16.079 km and width is 15.218 km.
2.2 3D grid construction
Zubair formation was represented by three dimensional grid system of 53 cells along the x-axis and 51 cells along the y-axis and 75 layers along the Z-axis. The size of grids was 300 m in width, 300 m in length and the total number of cells is 202725 cells as shown in fig.4.

2.3 Layering
The average thickness of the mentioned formation in 12 Luhais wells is approximately 60 m. Previous reservoirs studies were divided this formation to seven zones according to the formation evaluation process, but in this case study, it was taken as single zone and divided to 75 layers for distribution porosity at each layer and that for easy interpretation to results of statistics and variograms analysis. Top layer was selected as a sample in the illustrative figures.

2.4 Scale up of well log porosity
Scale up is the process of averaging porosity by any selected algorithm to produce one porosity value that will fall in each grid cell of skeleton. Each grid cell in skeleton will be used as start point for porosity modeling.
Before entering into the grid, porosity (or any petrophysical property) of well log must be scaled up, this process is also called blocking of well logs as shown in Fig.5. Many statistical methods are used for scale up such as (arithmetic mean, harmonic, geometric mean method and etc…), where arithmetic average is always utilizing for numbers of properties such as porosity, water saturation, and net/gross, so arithmetic average method was used for porosity scale up.
Figure 5. Porosity scale up of Zubair formation

2.5 Histograms

Histogram is a corresponding graph of data sets and its frequency, a frequency records values is showing how often observed values fall within certain intervals or classes [16]. Histograms of entered porosity, scale up and the distribution porosity in all layers by three methods recalled above were shown in Fig.6, 7 and 8, pinky column is well log porosity while the scale up porosity is represented by green column and blue column is the modeling porosity.

Figure 6. Histogram of entered, scale up and porosity modeled by sequential Gaussian.

Figure 7. Histogram of entered, scale up and porosity modeled by kriging.

Figure 8. Histogram of entered, scale up and porosity modeled by Gaussian random function.
2.6 Variogram map and directions

Variogram is calculated for a number of directions and distances in reservoir and its values posted on a map called variogram map, the center of this map is lag distance (zero), so the variogram map takes the subject of calculating the variogram in a number of directions and distances, so determination major and minor directions in continuity is the main target of variogram map. Fig.9. shows the 2-D variogram maps that calculated from porosity. Geometric anisotropy that occurs in 2-D areal direction is clear by elliptic shape [17].

![Variogram Map](image)

**Figure 9.** 2D variogram map from porosity data.

Variogram two horizontal direction below in Fig.10 are according to 2D variogram map in Fig.9, the number of lags was determined as 17 lags with 909.1 m distances. Porosity variability with different scale distances and directions is explained by variogram calculation and interpretation as be shown in results.

![Location Map](image)

**Figure 10.** Location map in two horizontal direction

2.7 Reservoir modeling

As clarified above, porosity was distributed in three methods, these distributions were in 75 layers that subdivision of formation. Fig 11, 12 and 13 below show the porosity distribution of Zubair formation top in three mentioned methods and the color scale is also shown in these figures. According to this scale, red and yellow represent higher porosity values, while blue and purple are lower ones.
3. Results and Discussion
The results of this case study were obtained from Petrel software after entered all mentioned data and the made model.

The results were statistical information, variograms and pores volumes calculation, Comparison has been made between the results and finished with choosing the best distribution of porosity model that is closest to real reservoir and pores volume calculation in each method.

3.1 Statistical analysis
Statistical analyses were performed for entered porosity log of 12 well, scale up and the distribution porosity by sequential Gaussian, kriging and Gaussian random function models on Zubair formation layers. Tables 1, 2 and 3 involved the estimations of basic univariate statistics including the mean, standard deviation and variance, and these estimation values gave an indication for closest results of sequential Gaussian and Gaussian random function techniques to scale up and entered porosity. While kriging results were further away from them, but these results do not make a decision for what is the best method, so the variogram analysis is a key for determination of the best porosity distribution model.
Table 1. Statistical information results of sequential Gaussian modeling.

| Porosity type          | Mean  | Std.  | Variance |
|------------------------|-------|-------|----------|
| Well log porosity      | 0.0814| 0.0871| 0.0076   |
| Up scaled porosity     | 0.0953| 0.0897| 0.0081   |
| Porosity modeling      | 0.1095| 0.0558| 0.0031   |

Table 2. Statistical information results of Kriging modeling.

| Porosity type          | Mean  | Std.  | Variance |
|------------------------|-------|-------|----------|
| Well log porosity      | 0.0814| 0.0871| 0.0076   |
| Up scaled porosity     | 0.0953| 0.0897| 0.0081   |
| Porosity modeling      | 0.0963| 0.0225| 0.0005   |

Table 3. Statistical information results of Gaussian random function modeling.

| Porosity type          | Mean  | Std.  | Variance |
|------------------------|-------|-------|----------|
| Well log porosity      | 0.0814| 0.0871| 0.0076   |
| Up scaled porosity     | 0.0953| 0.0897| 0.0081   |
| Porosity modeling      | 0.1194| 0.0660| 0.0044   |

As shown in Fig. 6, 7 and 8 the scale up histograms and distributed porosity by random and sequential Gaussian models histograms were showed convergence to entered porosity rather than kriging model.

3.2 Variograms analysis
For the variogram analysis of 12 wells porosity producing from Zubair formation that distributed in three listed methods, was utilized to calculate corresponding experimental variograms.

Firstly, average of experimental variogram was calculated for scale up of entered porosity that represent the real date and reference for comparing the modeling methods, Fig.14 shows the experimental variogram result that fitted to exponential variogram model, there are small error in porosity values that measured by well logging because of the zero-nugget effect except the value nugget (0.251) in major direction (The area of most wells) that somewhat was acceptable. The horizontal experimental variogram changed from positive correlations to negative correlations at a length scale that indicate geologic cycles, these cycles according to varies in rocks properties along formation. In major direction there is some deviation because of the experimental variogram reach to sill before exponential model that happen because of large distance between data (i.e. distance between LU 13 and LU4). The presence of a number of layers extend along large distance of reservoir and the porosity difference of two separated wells by large distance (i.e. LU19 and LU4) made the minor horizontal and vertical Variograms did not reached or reached near to sill and that called areal trends and layering. [6] The exponential variogram model results is shown in table 4.
Figure 14. Average experimental variogram of scale up porosity in three direction.

Porosity was transformed to normal score because the using of Gaussian technique required data transformation. Fig.15 shows the experimental variogram of porosity distributed by sequential Gaussian model. It is fitted to exponential variogram model. The comparison of these variograms with scale up entered porosity variograms gave the same behavior and very close results as shown in table 5.

Experimental variograms of porosity distributed in kriging and Gaussian random function were calculated and fitted to exponential variogram model and that clearly was shown in Fig.16 and Fig.17. Comparison between variograms of kriging porosity model with entered scale up porosity showed a spacing in behavior and results, because the variograms of this method was indicated to zonal anisotropy and large deviation as shown in Fig.16 and results in table 5.

The last comparison between porosity Gaussian random function modeling and entered porosity scale up

|                      |     |
|----------------------|-----|
| **Sill**             | 1   |
| **Nugget**           | 0   |
| **Major range (m)**  | 10606.1 |
| **Minor range (m)**  | 2185.8 |
| **Vertical range (m)** | 8.1  |

Table 4. Results from exponential variogram model analysis of scale up porosity.
variograms, Fig.17 elucidated the high nugget values that indicating a discontinuity in porosity values and that did not appear in Fig.12, also it showed zonal anisotropy. [12]

Figure 15. The average of experimental variogram of porosity modeling by sequential Gaussian method of Zubair layers in three directions.

Figure 16. Average experimental variogram of porosity modeling by kriging method of Zubair layers in three directions.

Table 5. Results from exponential variograms models analysis of porosity modeling by three methods.

|            | Sequential Gaussian modeling | Kriging modeling | Gaussian random function modeling |
|------------|------------------------------|------------------|----------------------------------|
| Major range (m) | 13636.4                      | 7123.1           | 9697                             |
| Minor range (m) | 2080.5                       | 3744.7           | 4695.5                           |
| Vertical range (m) | 6.9                           | 11               | 9.4                              |

Figure 17. Average experimental variogram of porosity modeling by Gaussian random function method of Zubair layers in three directions.

From statistical analysis, above comparisons and according to the results in tables 4 and 5, the best porosity modeling technique in upper shale member of Zubair formation is sequential Gaussian followed by Gaussian random function and kriging techniques respectively.
3.3 Pores volumes calculation
Using different geostatistical models for porosity distribution have clear effect on pore volume, where each model was given a different pore volume value as shown in table 6. That means oil reserves values are similarly changed accordingly.

Table 6. Pore volume calculation results at each modeling method.

| Modeling method       | Pore volume (m$^3$) |
|-----------------------|---------------------|
| Sequential Gaussian   | 294 $10^6$         |
| Kriging               | 267 $10^6$         |
| Gaussian random function | 283 $10^6$     |

4 Conclusions
For building porosity distributions model, contour map, well heads and tops, and CPI porosity log data must be available. Statistical parameters analysis (one-dimension parameters) is not enough for making decisions about property modeling so any reservoir modeling study must be modeled by variogram (two-dimension parameters) interpretations where the variogram is used throughout geostatistical reservoir modeling as a measure of spatial variability, so it will represent the true reservoir heterogeneity if it modeled. Sequential Gaussian simulation was the best method of porosity distribution of upper shale member in Zubair formation. Accuracy of porosity on any petrophysical modeling means building of a good geological model that represent the structure of reservoir model. Difference using of Geostatical methods in porosity modeling effect on pore volume values which means changing in oil in place. According to the three models comparisons, Pore volume value (294 e$^6$ m$^3$), that calculated by using porosity distributed in Sequential Gaussian model, absolutely it represents the real pore volume value of upper shale member of Zubair formation.

5 Nomenclatures
2 $\gamma(h)$ = Variogram.
z(u) = property value at position u.
h = lag distance.
z(u + h) = property value at position u + h.
N(h) = number of pairs for lag.
Z (u) = random function model at u location.
u = are n data locations.
mu = E{z(u)} = RV z(u) expected values that depended on location.
Z*$sk(u)$ = linear regression estimators also called simple kriging estimators.
$\lambda u(u)$ = S.K. weights.
C(uα, uβ) = covariance matrix.
CPI= computer program interpretation
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