The Gribov problem and its solution from a toy model point of view

Kurt Langfeld

Institut für Theoretische Physik, Universität Tübingen
D–72076 Tübingen, Germany.

Abstract

The standard Faddeev Popov gauge fixing procedure is put on solid grounds by taking into account a topological factor which corrects for the number of Gribov copies within the first Gribov horizon. Zwanziger’s stochastic approach to gauge fixed Yang-Mills theory is briefly reviewed. A simple toy model is presented which illustrates both methods. Within the toy model, I show that a stochastic drift force can be constructed with which the gauged configurations are attracted by the fundamental modular region. The toy model shows that an action which gives rise to the drift force can be found. This makes a “heat bath” simulation possible, which is seen to be superior to the Langevin approach at the numerical level.

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1 Introduction

Although lattice gauge simulations can directly address gauge invariant, physical observables of the theory of strong interactions, it might turn out fruitful for detecting basic mechanisms to remove redundant degrees of freedom by means of gauge fixing. Moreover, in the case of the perturbative approach to QCD Greenfunctions as well as in the case employing Dyson-Schwinger techniques [1, 2], gauge fixing is inevitable.

In the Faddeev Popov approach, gauge fixing is imposed by demanding that the gauge fields obey the gauge fixing condition. The degeneracy factor which determines the weight with which a representative of the gauge orbit contributes to the partition function is assumed to be given by the Faddeev Popov determinant. It was firstly pointed out that e.g. the well known Landau gauge condition is not sufficient for an unambiguous choice of the gauge field [3]. As a further refinement of the gauge condition, one might demand that the lowest eigenvalue of the Faddeev Popov matrix evaluated with gauged fields, is positive [3, 4]. These gauge fixed configurations are said to lie within the first Gribov horizon. It still turns out that more than one representative of the gauge orbit lies within the first Gribov regime in the generic (non-perturbative) case [5]. Finally, the domain of gauged configurations which have been unambiguously selected from the first Gribov regime is known as the fundamental modular region. In practice, any of the refinements of the naive Landau gauge condition can be used for a calculation of the gauge invariant observables, once the weight factor of each configuration is known. The Gribov problem emerges when this weight factor is calculated by the standard Faddeev Popov method: it was pointed out in [6, 7, 8] that the inverse Faddeev Popov determinant identically vanishes leaving us with an ill-defined partition function. There are two possibilities which are actually used in lattice gauge theories to avoid this problem: (i) gauge fixed configurations are randomly selected from the first Gribov regime, and their weight factor is exactly taken into account (see e.g. [9]-[12]); this method does not suffer from the Faddeev-Popov problem, but the selection of the configurations from the first Gribov regime is still arbitrary. This could lead to ambiguous results for gauge variant quantities, such as the gluon propagator; (ii) a Laplacian version [13] of the gauge condition is chosen which allows a unique definition of the gauged configuration in practical simulations (see e.g. [14]).

In order to avoid the Gribov problem in an ab initio continuum formulation of Yang-Mills theory, it was proposed by Zwanziger [4, 15] to select stochastically, but in a well defined manner the gauge field configurations from the configuration space. Thereby, preference is given to configurations within or close to the first Gribov regime. Using the framework of the stochastic quantization [16], the bias towards the first Gribov regime is provided by a drift force “tangent” to the gauge orbit.

In the present paper, I briefly review the gauge fixing techniques of continuum Yang-Mills theory. A simple toy model is used to illustrate the Gribov problem of the standard Faddeev Popov quantization, and its resolution in the stochastic approach. I find in the case of the toy model that a modification of the stochastic approach generates configurations which
are attracted by the fundamental modular region. I finally point out that a "potential" representation of the drift force seems possible. This paves the path to a very efficient numerical simulation using the heat bath techniques.

2 Brief review of gauge fixing

2.1 The standard gauge fixing procedure

The task boils down to select a single gauge field configuration of \( \{ A^\Omega_{\mu}(x) \} \), where \( \{ A^\Omega_{\mu}(x) \} \) is the set of gauge fields \( A^\Omega_{\mu}(x) \) which have been generated from a representative \( A^\mu_{\mu}(x) \) of the gauge orbit by applying all possible gauge transformations \( \Omega(x) \). Usually, a gauge fixing condition, e.g. such as Landau gauge

\[
\Omega(x) : \quad \partial_{\mu}A^\Omega_{\mu}(x) = 0 ,
\]

is chosen to accomplish this task. Unfortunately, this procedure is plagued by the so-called Gribov problem \[3\]: background fields \( A^\mu_{\mu}(x) \) which give rise to “large” field strength generically admit several solutions to the gauge fixing condition (for an illustration see \[19\]). This implies that the topological field theory, which generates to the inverse Faddeev-Popov determinant, vanishes identically \[6, 7, 8\]

\[
\Delta_{FP}^{-1} = \int D\Omega \delta \left( \partial_{\mu}A^\Omega_{\mu}(x) \right) \equiv 0 .
\]

In the present paper, a lattice regularization as well as a finite space-time volume is implicitly understood in order to render functional integrals well defined. In order to select a gauge configuration \( A^\Omega_{\mu}(x) \) from the possible solutions of (1), one might demand

\[
f[A^\Omega] := \partial_{\mu}A^\Omega_{\mu}(x) = 0 , \quad M := \frac{\delta f[A^\Omega]}{\delta \Omega} , \quad \lambda_0 > 0 ,
\]

where \( \lambda_0 \) is lowest eigenvalue of the operator \( M \). The configurations \( A^\Omega_{\mu}(x) \) emerging from the latter equation are said to be restricted to the first Gribov regime. It turns out, that even the refinement \[3\] of gauge fixing does not yield a unique representative \( A^\Omega_{\mu}(x) \) of the gauge orbit. In order to count the number \( n[A] \) of configurations lying within the first Gribov horizon, one evaluates the functional integral, i.e.,

\[
n[A] = \int D\Omega \delta \left( \partial_{\mu}A^\Omega_{\mu}(x) \right) \text{Det} \left( \frac{\delta f[A^\Omega]}{\delta \Omega} \right) \theta \left[ \lambda_0 \right] ,
\]

where the modulus of the functional determinant has been replaced by the determinant in view of the constraint to positive eigenvalues only. Since the gauge condition \[3\] can be installed using the variational condition

\[
\int d^4x A^2_{\mu}(x) \xrightarrow{\Omega} \text{minimum} ,
\]
it is clear that at least one representative can be found on the gauge orbit implying that $n[A] \geq 1$. Hence, the partition function of gauged fixed Yang-Mills theory is well defined and is given by

$$Z = \int \mathcal{D}A \, n^{-1}[A] \, \delta \left( \partial_{\mu} A_{\mu} \right) \, \text{Det} \left( \frac{\delta f[A]}{\delta \Omega} \right) \, \theta \left[ \lambda_0 \right] \, \exp \left\{ -S_{YM} \right\} . \quad (4)$$

The determinant in the latter equation can be represented by ghost fields as usual. At the current stage of investigations present in the literature the topological factor $n[A]$ is set to unity. A perturbative treatment of the partition function is consistent with the approximation $n[A] = 1$, since only “small” field strengths are involved by construction. The influence of the factor $n[A]$ on a genuine non-perturbative approach is not known. To my knowledge, an analytic expression of the integer $n[A]$ is not available.

In order to circumvent these difficulties, Baulieu and Schaden proposed [6]-[8] to perform subsequent steps of gauge fixing where each of them avoids the topological obstruction (2).

2.2 The stochastic approach to gauge fixed YM-theory

It was pointed by Zwanziger [4,15] that the Gribov problem can be avoided when stochastic quantization [16] is used in a modified form: a Langevin simulation generates a series of configurations $\{A_\mu(x)\}$ where each configuration possesses a certain bias concerning its position on the gauge orbit. At the same time, the correct result for gauge invariant observables is recovered when the observable is calculated from the Langevin “time” series. In order to be specific, let me firstly point out that Langevin time series is generated by

$$\frac{\partial}{\partial \tau} A_\mu(x) = - \frac{\delta S_{YM}}{\delta A_\mu(x)} + K_\mu(x) + \text{[noise]} , \quad (5)$$

where $S_{YM}$ is the Yang-Mills action, and $K_\mu(x)$ is a drift force, which will be specified later. Expectations values and Green functions are evaluated using [16]

$$\int \mathcal{D}A_\mu \, G[A] \, P[A] \propto \lim_{T \to \infty} \frac{1}{T} \int_0^T d\tau \, G \left[ A_\mu(x; \tau) \right] . \quad (6)$$

The probabilistic weight $P[A]$ satisfies a Fokker-Planck equation

$$H_{FP} \, P[A] = \int d^4x \, \frac{\delta}{\delta A_\mu(x)} \left[ - \frac{\delta P[A]}{\delta A_\mu(x)} + \left( - \frac{\delta S_{YM}}{\delta A_\mu(x)} + K_\mu(x) \right) P[A] \right] = 0 . \quad (7)$$

For a vanishing Drift force, i.e., $K_\mu(x) = 0$, one recovers the desired result

$$P[A] \propto \exp \left\{ -S_{YM} \right\} .$$

The key observation [15] made by Zwanziger is that if one chooses a drift force “tangent” to the gauge orbit, the gauge fields are generated with preference to a certain region of the
gauge orbit, and, hence, are interpreted as gauge fields of a certain gauge. In the context of Landau gauge, one chooses
\[ \mathcal{K}_\mu = D_\mu v(x), \]
where \( D_\mu \) is the gauge covariant derivative and \( v(x) \) is an arbitrary auxiliary field \[15\]. On the other hand, gauge invariant observables are independent of \( \mathcal{K}_\mu(x) \), and, therefore, the right hand side of \[6\] reproduces the correct Yang-Mills result for gauge invariant quantities. The goal of the work \[15\] is that a theoretical, Dyson-Schwinger type framework is developed for the stochastic approach. A numerical analysis using the stochastic approach can be found in \[17, 18\].

3 The toy model

3.1 Settings

Instead of a functional integral over the gauge fields \( A_\mu(x) \), I will study a simple integral of two variables \( x_1 = x \), \( x_2 = y \). Gauge invariance is replaced by the rotational invariance in two dimensions. The “gauge invariant” action is given by
\[ S_{YM} = x^2 + y^2 = x_k^2. \] (8)

In this very simple toy model, a separation of the degrees of freedom into gauge invariant and gauge dependent ones is done by introducing polar coordinates \((r, \varphi)\). “Gauge invariant observables” are functions of \( r \) only. We are interested in expectation values of “gauge invariant” observables:
\[ \langle f(r) \rangle = \frac{1}{N} \int dx \, dy \, f(r) \exp\left\{ -S_{YM} \right\}, \] (9)

where \( r := \sqrt{x^2 + y^2} \). If \( \vec{x}_0 \) denotes a representative of a gauge orbit, the orbit \( \{\vec{x}^\Omega\} \) is generated by
\[ \vec{x}^\Omega = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \vec{x}_0, \quad \theta \in [0, 2\pi]. \] (10)

3.2 Standard gauge fixing

In order to remove the rotational degree of freedom, we introduce a gauge fixing condition
\[ y(\theta) = 0. \] (11)

One easily verifies that the procedure suffers from the topological obstruction \[2\], i.e.,
\[ \Delta_F^{-1} = \int_0^{2\pi} d\theta \, \delta\left( y(\theta) \right) = \int_0^{2\pi} d\theta \, \delta\left( \sin \theta \, x_0 + \cos \theta \, y_0 \right) = 0 \] (12)
independent of the gauge orbit specified by \((x_0, y_0)\). To simplify the discussion below, we choose \(x_0 \geq 0, y_0 = 0\) without a loss of generality. In order to refine the gauge fixing procedure according to (11), we calculate

\[
\text{Det} \left( \frac{\delta f[A]}{\delta \Omega} \right) \rightarrow dy(\theta) = \cos \theta x_0.
\]

From

\[
\frac{dy(\theta)}{d\theta} = \cos \theta x_0 \geq 0,
\]

we conclude that the first Gribov regime consists of the positive half-space \(x \geq 0\). Consequently, the topological quantity \(n[A]\) becomes

\[
n[A] \rightarrow n[\vec{x}] = \int_{-\pi/2}^{\pi/2} d\theta \cos \theta \delta \left( \sin \theta x_0 \right) = 1.
\]

In the case of the example (11), there is only one configuration within the first Gribov horizon. In this case, the first Gribov regime coincides with the fundamental modular region.

Let us study the more advanced example of a gauge fixing condition:

\[
f(\theta) = x(\theta) y(\theta) = 0, \rightarrow x_0^2 \sin \theta \cos \theta = \frac{1}{2} x_0^2 \sin(2\theta) = 0.
\]

In this case, four gauge copies exist. The first Gribov regime is determined from

\[
\frac{df(\theta)}{d\theta} = x_0^2 \cos(2\theta) \geq 0, \quad \theta \in [0, 2\pi[.
\]

The space where (16) holds is shaded in figure 1. Hence, the first Gribov regime consists of the complete \(x\)-axis. A direct calculation of \(n[\vec{x}]\) shows that there are still two gauge copies within the first Gribov horizon (see figure 1). Nevertheless, we arrived at a stage where calculations of “gauge invariant” quantities are feasible: using \(n[\vec{x}] = 2\) and adapting (11), we find

\[
Z = 2\pi \int dx \ dy \frac{1}{2} \delta (xy) \theta \left[ x^2 - y^2 \right] \exp \{-x^2 - y^2\}.
\]

The result that \(n[\vec{x}]\) is independent of \(x_0\) is due to an oversimplification by the toy model. One can easily construct examples for the generic case, i.e., \(n[\vec{x}]\) is a function of \(x_0\).

### 3.3 The stochastic approach

In the present case of only two degrees of freedom, the gauge invariant part of the Fokker-Planck Hamiltonian (7) is given by a partial differential equation

\[
H_{\text{inv}} = \frac{\partial}{\partial \vec{x}_i} \left[ -\frac{\partial}{\partial \vec{x}_i} + \left( -\frac{\partial S_{YM}}{\partial \vec{x}_i} \right) \right].
\]
Infinitesimal rotations are generated by the operator

\[ G = x_i \epsilon_{ik} \frac{\partial}{\partial x_k}, \]  

(19)

where \( \epsilon_{ik} \) is the total anti-symmetric tensor in 2-dimensions. It is easy to check that the operator \( G \) commutes with \( H_{\text{inv}} \). Following the procedure suggested by Zwanziger, the drift force “tangent to the gauge orbit” is chosen as \( K_i = \epsilon_{ik} x_k v(x) \), where the scalar function \( v(x) \) will be specified below. The Fokker-Planck equation (which determines the weight factor \( P_v \)) with the restoring drift force included is given by

\[ H_{FPP}P_v = \frac{\partial}{\partial x_i} \left[ \frac{\partial P_v}{\partial x_i} + \left( -\frac{\partial S_{YM}}{\partial x_i} + \epsilon_{ik} x_k v(x) \right) P_v \right] = 0. \]

(20)

Case study I: The trivial case.
Let us study the “gauge invariant” scalar function \( v(x) = a^{-1}/r^2 \). Using this particular choice, the Fokker-Planck equation is given by

\[ \left[ -\partial^2 - 2 \partial_k x_k + \frac{a^{-1}}{r^2} \left( x_2 \partial_1 - x_1 \partial_2 \right) \right] P_v = 0. \]

(21)

To shed light onto the solution of (21), we introduce polar coordinates:

\[ x_1 = r \cos \varphi, \quad x_2 = r \sin \varphi, \quad \varphi \in [0, 2\pi]. \]

(22)

One finds:

\[ \left[ -\partial^2_r - \frac{1}{r} \partial_r - \frac{1}{r^2} L^2 - 2r \partial_r - 4 + \frac{1}{ar^2} L_3 \right] P_v = 0, \]

(23)
Figure 2: The expectation values \( \langle r^2 \rangle \) and \( \langle y^2 \rangle \) as function of the Landau parameter \( a^{-1} \) (left panel). The probability distribution of \( x_n \) for \( a^{-1} = 5 \) (right panel). Landau type drift force, see (27).

where

\[
L^2 = \partial^2_\varphi \quad L_3 = \partial_\varphi .
\]  

As in quantum mechanics, the wave function \( P_v \) factorizes (for this particular choice of \( v(r) \)):

\[
P_v = e^{im\varphi} P_v(r) , \quad m^2 + ia^{-1}m = 0 .
\]  

The only solution obeying periodic boundary conditions, i.e., \( P_v(\varphi+2\pi) = P_v(\varphi) \), is \( m = 0 \). The Fokker-Planck solution is hence given by

\[
P_v = \exp\{-r^2\} .
\]

One recovers the trivial result of the “un-fixed” action.

**Case study II. The Landau gauge.**

Let us now investigate a drift force which is not constant along the gauge orbit, i.e., the “Landau type gauge” is given by the condition \( x^1 x^2 = 0 \) where the vector \( x^\theta_k \) is generated from the vector \( x_k \) by a rotation by an angle \( \theta \) (see (10)). Here we choose the scalar function of the drift force according \( v(x, y) = a^{-1} xy \), i.e.,

\[
K_i = \epsilon_{ik} x_k v(x) , \quad v(x) = a^{-1} xy .
\]  

The direction of this drift force tangent to the gauge orbit is illustrated in figure (right panel).

Rather than seeking the solution of the Fokker Planck equation for the case (27), I employ the corresponding Langevin equation to generate a “time” history \( x_k^{(n)} \) of the vectors and
to calculate expectation values. The “time” history is obtained by the recursion
\[\begin{align*}
x^{(n+1)} &= x^{(n)} + dt \left(-\frac{\partial S_{YM}}{\partial x} + a^{-1} x y^2\right) + \eta_x, \\
y^{(n+1)} &= y^{(n)} + dt \left(-\frac{\partial S_{YM}}{\partial y} - a^{-1} x^2 y\right) + \eta_y,
\end{align*}\]
(28)
(29)
where \(\eta_x/y\) is a Gaussian noise with width \(\sqrt{4dt}\). An extrapolation \(dt \to 0\) must be performed when expectations values are calculated. For the present example, \(N = 10^7\) pairs \((x_n, y_n)\) are created for \(dt_1 = 0.01\) and \(dt_2 = 0.005\), respectively. The estimator, \(s(dt)\), of a desired observable is calculated and the corresponding statistical error \(\delta s(dt)\) is estimated. The final error \(\epsilon\) comprises statistical as well as systematic errors due to the extrapolation \(dt \to 0\). I used the following procedure:

\[
s = \frac{1}{2} \left(s(dt_1) + s(dt_2)\right), \quad \delta s^2 = \delta s^2(dt_1) + \delta s^2(dt_2) + (s(dt_1) - s(dt_2))^2.
\]

For a smaller value of \(dt\), one must choose a larger value of \(N\) of configurations in order to explore the complete “configuration space”. Generically, the Langevin approach is by far inferior as e.g. the standard heat bath approach.

Let us firstly study the “gauge invariant” operator \(\langle r^2 \rangle (a^{-1})\) as function of the Landau parameter \(a\). The numerical result is shown in figure 2. The error bars comprise statistical as well as the systematic error from the extrapolation \(dt \to 0\). One indeed finds that \(\langle r^2 \rangle\) is independent of \(a\) within numerical accuracy. I also checked that \(\langle r^4 \rangle\) is independent of of \(a\).

Secondly, I study the “gauge variant” expectation value \(\langle y^2 \rangle\). The result is also shown in the above figure. One finds that \(\langle y^2 \rangle = \langle r^2 \rangle / 2\) for \(a^{-1} = 0\) as it should be. Increasing \(a^{-1}\) decreases fluctuations around \(y = 0\): the configurations \((x, y)\) are pushed to the first Gribov regime specified by \(y = 0\). I then calculated the expectation value of \(\langle x \rangle\). It turns out that it vanishes for the investigated range of \(a \in [0, 10]\). This shows that there is still an average over the complete first Gribov regime, which consists of the positive and negative half \(x\)-axis. The probability distribution of \(x\) is also shown in figure 2 for \(a^{-1} = 5\). The values \(x^{(n)}\) are symmetrically distributed around \(x_1 = x = 0\). Hence, the configurations are sampled over the complete first Gribov regime (rather than the fundamental modular region, i.e., the positive half \(x\)-axis).

**Case study III. The fundamental modular region.**

In this section, I will study a drift force which pushes the configurations \((x_n, y_n)\) towards the fundamental modular region \(y = 0, x \geq 0\). The drift force is given by

\[
K_i = \epsilon_{ik} x_k v(x), \quad v(x) = -a^{-1} y/r^3, \quad r = \sqrt{x^2 + y^2}.
\]

(30)

The numerical simulation reveals that the “gauge invariant” observable \(\langle r^2 \rangle\) is independent of the gauge fixing parameter \(a^{-1}\). In addition, the points \((x^{(n)}, y^{(n)})\) are nicely attracted by the positive half \(x\)-axis (see figure 3).
The crucial observation in deriving the function $P_v(x)$ is that the drift force can be written as the gradient of a gauge fixing action function, i.e.,

$$K_i = - \partial_i S_{\text{fix}}, \quad S_{\text{fix}} = a^{-1} x/r.$$  

(31)

Hence, the Fokker-Planck equation is given by

$$H_{FP} P_v = \frac{\partial}{\partial x_i} \left[ - \frac{\partial P_v}{\partial x_i} + \left( - \frac{\partial S_{YM}}{\partial x_i} - \frac{\partial S_{\text{fix}}}{\partial x_i} \right) P_v \right] = 0.$$  

(32)

From this representation, one can read off the desired solution, i.e.,

$$P_v(x) = \exp \left\{ -r^2 + a^{-1} \cos \varphi \right\}.$$  

(33)

Here, the angular part and the radial part factorizes. This proves that gauge invariant observables are indeed independent of the gauge fixing parameter. The maximum probability of $P_v(x)$ is obtained for $\varphi = 0$, which is the positive half $x$-axis, i.e., the fundamental modular region.

The heat-bath simulation.

Finding the gauge fixing action $S_{\text{fix}}$ which generates the drift force paves the path to an efficient simulation using the heat-bath approach. Thereby, configurations $(x, y)$ are generated according the probability

$$\exp \left\{ -S_{YM} - S_{\text{fix}} \right\},$$  

(34)
where $S_{\text{fix}}$ is given by (31). It turns out that $N_h = 10^4$ heat-bath configurations are sufficient for error bars at the 1% level (compare figure 4 with figure 3). A scatter plot of the configurations $(x_n, y_n)$ is also shown in figure (19).

In conclusion, the heat-bath approach is roughly two orders of magnitude more efficient than the corresponding Langevin-approach. It is therefore highly desirable to construct the gauge fixing function $S_{\text{fix}}$ generating the drift force.

For these purposes, the drift force $K_i$ must obey certain constraints in order to be generated by a gauge fixing action. These constraints are derived from (31) by observing that

\[- \left( \partial_k \partial_i - \partial_i \partial_k \right) S_{\text{fix}} = 0 ,
\]
\[\partial_k K_i - \partial_i K_k = 0 .\] (35) (36)

The only non-trivial information is obtained from (36) by choosing $i = 1$ and $k = 2$. Using $K_i$ from (30), we find

\[x \partial_x v(x, y) + y \partial_y v(x, y) + 2 v(x, y) = 0 .\] (37)

The last expression is most instructive using polar coordinates, and (37) becomes

\[r \partial_r \ln \bar{v}(r, \varphi) + 2 = 0 .\] (38)

Hence, the general solution of $\bar{v}(r, \varphi)$ which admit a representation in terms of a gauge fixing function $S_{\text{fix}}$ is given by

\[v(x, y) = \bar{v}(r, \varphi) = \frac{1}{r^2} f(\varphi) ,\] (39)

where the function $f(\varphi)$ is still arbitrary.
4 Conclusions

A simple toy model was designed to illustrate the Gribov problem of the standard Faddeev Popov quantization. Using this model, the resolution of the Gribov problem along the lines of Zwanziger’s version of the stochastic approach [4] has been outlined. In the Zwanziger approach, the gauged configurations are attracted by the first Gribov regime. Here, it turned out that the approach can be modified in order to obtain attraction by the fundamental modular region. Moreover, an integrable drift force towards the fundamental region was developed. It was therefore possible to construct the gauge fixing action which generates the drift force in the Langevin simulation. The latter construction was essential to perform a heat bath simulation. In the case of the toy model, it was observed that the heat bath method is more efficient than the Langevin technique.

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