CP Violation and Prospects at B Factories and Hadron Colliders

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We review the information on the CKM matrix elements, unitarity triangle and CP-violating phases $\alpha$, $\beta$ and $\gamma$ in the standard model which will be measured in the forthcoming experiments at B factories, HERA-B and hadron colliders. We also discuss two-body non-leptonic decays $B \rightarrow h_1 h_2$, with $h_i$ being light mesons, which are interesting from the point of view of CP violation and measurements of these phases. Partial rate CP asymmetries are presented in a number of decay modes using factorization for the matrix elements of the operators in the effective weak Hamiltonian. Estimates of the branching ratios in this framework are compared with existing data on $B \rightarrow K \pi, \eta' K, K^* \pi, \rho \pi$ decays from the CLEO collaboration.

1 Introduction

We shall review the following three topics in quark flavour physics:

- An update of the Cabibbo-Kobayashi-Maskawa CKM matrix.

Here, the results of a global fit of the CKM parameters yielding present profiles of the unitarity triangle and CP-violating phases $\alpha$, $\beta$ and $\gamma$ and their correlations in the standard model (SM) are summarized.

- Estimates of the CP-violating partial rate asymmetries for charmless non-leptonic decays $B \rightarrow h_1 h_2$, where $h_1$ and $h_2$ are light mesons, based on next-to-leading-order perturbative QCD and the factorization approximation in calculating the matrix elements of the operators in the effective Hamiltonian approach.

Here, we first discuss a general classification of the CP-violating asymmetries in non-leptonic $B$ decays and then give updated numerical estimates for a fairly large number of two-body decays involving penguin- and tree-transitions. Most of the decays considered here have branching ratios which are estimated to be in excess of $10^{-6}$ (and some in excess of $10^{-5}$) and many have measurable CP asymmetries. Hence, they are of interest for experiments at $B$ factories and hadron machines.

- Comparison of the branching ratios for $B \rightarrow h_1 h_2$ decays measured by the CLEO collaboration with the factorization-based theoretical estimates of the same.

The interest in these decays lies in that they provide first information on the QCD penguin-amplitudes and the CKM-suppressed non-leptonic $b \rightarrow u$ transitions. Hence, they will provide complementary information on the CKM matrix elements. It is argued that present data supports the factorization approach though it is not conclusive.
Within the standard model (SM), CP violation is due to the presence of a nonzero complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix $V$. We shall use the parametrization of the CKM matrix due to Wolfenstein:

$$V \simeq \begin{pmatrix}
1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3 (\rho - i\eta) \\
-\lambda(1 + iA^2\lambda^4\eta) & 1 - \frac{1}{4}\lambda^2 & A\lambda^2 \\
A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix},$$

(1)

which has four a priori unknown parameters $A$, $\lambda$, $\rho$ and $\eta$, where $\lambda$ is the Cabibbo angle and $\eta$ represents the Kobayashi-Maskawa phase. The allowed region in $\rho-\eta$ space can be elegantly displayed using the so-called unitarity triangle (UT). While one has six such relations, resulting from the unitarity of the CKM matrix, the one written below has received particular attention:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$$  

(2)

Using the form of the CKM matrix in Eq. (1), this can be recast as

$$\frac{V_{ub}^*}{AV_{cb}} + \frac{V_{td}}{AV_{cb}} = 1,$$

(3)

which is a triangle relation in the complex plane (i.e. $\rho-\eta$ space). Thus, allowed values of $\rho$ and $\eta$ translate into allowed shapes of the unitarity triangle.

The interior CP-violating angles $\alpha$, $\beta$ and $\gamma$ can be measured through CP asymmetries in $B$ decays. Likewise, some of these these angles can also be measured through the decay rates. Additional constraints come from CP violation in the kaon system ($|\epsilon|$), as well as $B_s^0-\bar{B}_s^0$ mixing. In future, the decays $B \rightarrow (X_s,X_d)\gamma$, $B \rightarrow (X_s,X_d)\ell^+\ell^-$ and $K \rightarrow \pi\nu\bar{\nu}$ will further constrain the CKM matrix.

### 2.1 Input Data

The experimental and theoretical data which presently constrain the CKM parameters $\lambda$, $A$, $\rho$ and $\eta$ are summarized below.

- $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}/V_{cb}|$:

We recall that $|V_{us}|$ has been extracted with good accuracy from $K \rightarrow \pi\ell\nu$ and hyperon decays to be $|V_{us}| = \lambda = 0.2196 \pm 0.0023$. The determination of $|V_{cb}|$ is based on the combined analysis of the inclusive and exclusive $B$ decays. $|V_{cb}| = 0.0395 \pm 0.0017$, yielding $A = 0.819 \pm 0.035$. The knowledge of the CKM matrix element ratio $|V_{ub}/V_{cb}|$ is based on the analysis of the end-point lepton energy spectrum in semileptonic decays $B \rightarrow X_u\ell\nu_\ell$ and the measurement of the exclusive semileptonic decays $B \rightarrow (\pi,\rho)\ell\nu_\ell$. Present measurements in both the inclusive and exclusive modes are compatible with $|V_{ub}/V_{cb}| = 0.093 \pm 0.014$. This gives $\sqrt{\rho^2 + \eta^2} = 0.423 \pm 0.064$. 

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2 SM Fits of the CKM Parameters and the CP-Violating Phases $\alpha$, $\beta$ and $\gamma$

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• $|\epsilon|, \hat{B}_K$:

The experimental value of $|\epsilon|$ is

$$|\epsilon| = (2.280 \pm 0.013) \times 10^{-3}.$$  \hspace{1cm} (4)

In the standard model, $|\epsilon|$ is essentially proportional to the imaginary part of the box diagram for $K^0\bar{K}^0$ mixing and is given by

$$|\epsilon| = \frac{G_F^2 f_K^2 M_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} \hat{B}_K \left( A^2 \lambda^6 \eta \right) \left( y_c \left\{ \hat{\eta}_ct f_3(y_c, y_t) - \hat{\eta}_{ct} \right\} + \hat{\eta}_{tt} f_2(y_t) A^2 \lambda^4 (1 - \rho) \right),$$  \hspace{1cm} (5)

where $y_i \equiv m_i^2/M_W^2$, and the functions $f_2$ and $f_3$ are the Inami-Lim function. Here, the $\hat{\eta}_i$ are QCD correction factors, calculated at next-to-leading order: $(\hat{\eta}_{ee})$, $(\hat{\eta}_{tt})$, and $(\hat{\eta}_{ct})$. The theoretical uncertainty in the expression for $|\epsilon|$ is in the renormalization-scale independent parameter $\hat{B}_K$, which represents our ignorance of the hadronic matrix element $\langle K^0 | (\bar{d} \gamma^\mu (1 - \gamma_5) s) | K^0 \rangle$. Recent calculations of $\hat{B}_K$ using lattice QCD methods are summarized at the 1998 summer conferences by Draper and Sharpe, yielding $\hat{B}_K = 0.94 \pm 0.15$.  \hspace{1cm} (6)

• $\Delta M_d, f_{B_d}^2 \hat{B}_{B_d}$:

The present world average for $\Delta M_d$ is

$$\Delta M_d = 0.471 \pm 0.016 \ (ps)^{-1}.$$  \hspace{1cm} (7)

The mass difference $\Delta M_d$ is calculated from the $B_d^0 - \bar{B}_d^0$ box diagram, dominated by $t$-quark exchange:

$$\Delta M_d = \frac{G_F^2 M_V^2 M_B}{6\pi^2} \left( f_{B_d}^2 \hat{B}_{B_d} \right) \left( \hat{\eta}_B y_t f_2(y_t) \right) |V_{td}^* V_{tb}|^2,$$  \hspace{1cm} (8)

where, using Eq. (1), $|V_{td}^* V_{tb}|^2 = A^2 \lambda^6 \left[ (1 - \rho)^2 + \eta^2 \right]$. Here, $\hat{\eta}_B$ is the QCD correction, which has the value $\hat{\eta}_B = 0.55$, calculated in the $\overline{MS}$ scheme.

For the $B$ system, the hadronic uncertainty is given by $f_{B_d}^2 \hat{B}_{B_d}$. Present estimates of this quantity using lattice QCD yield $f_{B_d} \sqrt{\hat{B}_{B_d}} = (190 \pm 23)$ MeV in the quenched approximation. The effect of unquenching is not yet understood completely. Taking the MILC collaboration estimates of unquenching would increase the central value of $f_{B_d} \sqrt{\hat{B}_{B_d}}$ by 21 MeV. In the fits discussed here, the following range has been used

$$f_{B_d} \sqrt{\hat{B}_{B_d}} = 215 \pm 40 \text{ MeV}.$$  \hspace{1cm} (9)

• $\Delta M_s, f_{B_s}^2 \hat{B}_{B_s}$:
Table 1. Data used in the CKM fits.

| Parameter | Value |
|-----------|-------|
| $\lambda$ | 0.2196 |
| $|V_{cb}|$ | 0.0395 ± 0.0017 |
| $|V_{ub}/V_{cb}|$ | 0.093 ± 0.014 |
| $|\epsilon|$ | (2.280 ± 0.013) $\times 10^{-3}$ |
| $\Delta M_d$ | (0.471 ± 0.016) (ps)$^{-1}$ |
| $\Delta M_s$ | > 12.4 (ps)$^{-1}$ |
| $\bar{m}_t(m_t(pole))$ | (165 ± 5) GeV |
| $\bar{m}_c(m_c(pole))$ | 1.25 ± 0.05 GeV |
| $\tilde{\eta}_B$ | 0.55 |
| $\tilde{\eta}_{cc}$ | 1.38 ± 0.53 |
| $\tilde{\eta}_{ct}$ | 0.47 ± 0.04 |
| $\tilde{\eta}_{tt}$ | 0.57 |
| $\tilde{B}_K$ | 0.94 ± 0.15 |
| $f_{B_d} \sqrt{\bar{B}_{B_d}}$ | 215 ± 40 MeV |
| $\xi_s$ | 1.14 ± 0.06 |

The $B^0_s$-$\bar{B}^0_s$ box diagram is again dominated by $t$-quark exchange, and the mass difference between the mass eigenstates $\Delta M_s$ is given by a formula analogous to that of Eq. (8):

$$\Delta M_s = \frac{G_F^2}{6\pi^2} M_{B_s}^2 \left( f_{B_s}^2 \tilde{B}_{B_s} \right) \tilde{\eta}_{B_s} y_t f_2(y_t) |V_{ts}^* V_{tb}|^2 .$$

(10)

Using the fact that $|V_{cb}| = |V_{ts}|$ (Eq. (4)), it is clear that one of the sides of the unitarity triangle, $|V_{td}|/\lambda V_{cb}$, can be obtained from the ratio of $\Delta M_d$ and $\Delta M_s$:

$$\frac{\Delta M_s}{\Delta M_d} = \frac{\tilde{\eta}_{B_s} M_{B_s} \left( f_{B_s}^2 \tilde{B}_{B_s} \right) |V_{ts}^* V_{tb}|^2}{\tilde{\eta}_{B_d} M_{B_d} \left( f_{B_d}^2 \tilde{B}_{B_d} \right) |V_{ts}^* V_{tb}|^2} .$$

(11)

The only real uncertainty in this quantity is the ratio of hadronic matrix elements $f_{B_s}^2 \tilde{B}_{B_s} / f_{B_d}^2 \tilde{B}_{B_d}$. Present estimate of this quantity is 1.14 ± 0.06.

(12)

The present lower bound on $\Delta M_s$ is: $\Delta M_s > 12.4$ (ps)$^{-1}$ (at 95% C.L.)

There are two other measurements which should be mentioned here. First, the KTEV collaboration has recently reported a measurement of direct CP violation in the $K$ sector through the ratio $\epsilon'/\epsilon$, with

$$\text{Re}(\epsilon'/\epsilon) = (28.0 \pm 3.0\text{(stat)} \pm 2.6\text{(syst)} \pm 1.0\text{(MC stat)}) \times 10^{-4} ,$$

in agreement with the earlier measurement by the CERN experiment NA31, which reported a value of $(23 \pm 6.5) \times 10^{-4}$ for the same quantity. The present
world average is $\text{Re}(\epsilon'/\epsilon) = (21.8 \pm 3.0) \times 10^{-4}$. This combined result excludes the superweak model by more than 7σ.

A great deal of theoretical effort has gone into calculating this quantity at next-to-leading order accuracy in the SM. The result of this calculation can be summarized in the following form due to Buras and Silvestrini:

$$\text{Re}(\epsilon'/\epsilon) = \text{Im}\lambda_t \left[ -1.35 + R_s \left( 1.1|r_{Z}^{(8)}|B_6^{(1/2)} + (1.0 - 0.67|r_{Z}^{(8)}|)B_8^{(3/2)} \right) \right].$$

(14)

Here $\lambda_t = V_{td}V_{ts}^* = A^2\lambda^4\eta$ and $r_{Z}^{(8)}$ represents the short-distance contribution, which at the NLO precision is estimated to lie in the range $6.5 \leq |r_{Z}^{(8)}| \leq 8.5$. The quantities $B_6^{(1/2)} = B_6^{(1/2)}(m_c)$ and $B_8^{(3/2)} = B_8^{(3/2)}(m_c)$ are the matrix elements of the $\Delta I = 1/2$ and $\Delta I = 3/2$ operators $O_6$ and $O_8$, respectively, calculated at the scale $\mu = m_c$. Lattice-QCD and the $1/N_c$ expansion yield:

$$0.8 \leq B_6^{(1/2)} \leq 1.3, \quad 0.6 \leq B_8^{(3/2)} \leq 1.0.$$

(15)

Finally, the quantity $R_s$ in Eq. (14) is defined as:

$$R_s \equiv \left( \frac{150 \text{ MeV}}{m_s(m_c) + m_d(m_c)} \right)^2,$$

(16)

essentially reflecting the s-quark mass dependence. The present uncertainty on the CKM matrix element is ±23%, which is already substantial. However, the theoretical uncertainties related to the other quantities discussed above are considerably larger. For example, the ranges $\epsilon'/\epsilon = (5.3 \pm 3.8) \times 10^{-4}$ and $\epsilon'/\epsilon = (8.5 \pm 5.9) \times 10^{-4}$, assuming $m_s(m_c) = 150 \pm 20$ MeV and $m_s(m_c) = 125 \pm 20$ MeV, respectively, have been quoted as the best representation of the status of $\epsilon'/\epsilon$ in the SM. These estimates are somewhat on the lower side compared to the data but not inconsistent.

Thus, whereas $\epsilon'/\epsilon$ represents a landmark measurement, establishing for the first time direct CP-violation in decay amplitudes, and hence removing the superweak model of Wolfenstein and its various incarnations from further consideration, its impact on CKM phenomenology, particularly in constraining the CKM parameters, is marginal. For this reason, the measurement of $\epsilon'/\epsilon$ is not included in the CKM fits summarized here.

Second, the CDF collaboration has recently made a measurement of $\sin 2\beta$. In the Wolfenstein parametrization, $-\beta$ is the phase of the CKM matrix element $V_{td}$. From Eq. (1) one can readily find that

$$\sin(2\beta) = \frac{2\eta(1-\rho)}{(1-\rho)^2 + \eta^2}.$$

(17)

Thus, a measurement of $\sin 2\beta$ would put a strong constraint on the parameters $\rho$ and $\eta$. However, the CDF measurement gives

$$\sin 2\beta = 0.79^{+0.41}_{-0.44},$$

(18)

or $\sin 2\beta > 0$ at 93% C.L. This constraint is quite weak – the indirect measurements already constrain $0.52 \leq \sin 2\beta \leq 0.94$ at the 95% C.L. in the SM. (The CKM fits reported recently in the literature yield similar ranges.) In light of this, this measurement is not included in the fits. The data used in the CKM fits are summarized in Table [1].
Figure 1. Allowed region in $\rho$–$\eta$ space in the SM, from a fit to the ten parameters discussed in the text and given in Table 1. The limit on $\Delta M$ is included using the amplitude method. The theoretical errors on $f_{B_d}\sqrt{\hat{B}_{B_d}}$, $\hat{B}_K$ and $\xi_s$ are treated as Gaussian. The solid line represents the region with $\chi^2 = \chi^2_{\text{min}} + 6$ corresponding to the 95% C.L. region. The triangle shows the best fit. (From Ref. 2.)

2.2 SM Fits

In the fit presented here, ten parameters are allowed to vary: $\rho$, $\eta$, $A$, $m_t$, $m_c$, $\eta_{cc}$, $\eta_{ct}$, $f_{B_d}\sqrt{\hat{B}_{B_d}}$, $\hat{B}_K$, and $\xi_s$. The $\Delta M_s$ constraint is included using the amplitude method. The rest of the parameters are fixed to their central values. The allowed (95% C.L.) $\rho$–$\eta$ region is shown in Fig. 1. The best fit has $(\rho, \eta) = (0.20, 0.37)$.

The CP angles $\alpha$, $\beta$ and $\gamma$ can be measured in CP-violating rate asymmetries in $B$ decays. These angles can be expressed in terms of $\rho$ and $\eta$. Thus, different shapes of the unitarity triangle are equivalent to different values of the CP angles. Referring to Fig. 1 we note that the preferred (central) values of these angles are $(\alpha, \beta, \gamma) = (93^\circ, 25^\circ, 62^\circ)$. The allowed ranges at 95% C.L. are

\[
65^\circ \leq \alpha \leq 123^\circ \\
16^\circ \leq \beta \leq 35^\circ \\
36^\circ \leq \gamma \leq 97^\circ
\] (19)

Of course, the values of $\alpha$, $\beta$ and $\gamma$ are correlated, i.e. they are not all allowed simultaneously. We illustrate these correlations in Figs. 2 and 3. Fig. 2 shows the allowed region in $\sin 2\alpha$–$\sin 2\beta$ space allowed by the data. And Fig. 3 shows the allowed (correlated) values of the CP angles $\alpha$ and $\gamma$. This correlation is roughly linear, due to the relatively small allowed range of $\beta$ (Eq. (19)).

We remark that the correlations shown are specific to the SM and are expected to be different, in general, in non-SM scenarios. A comparative study for some
Figure 2. Allowed 95% C.L. region of the CP-violating quantities $\sin 2\alpha$ and $\sin 2\beta$ in the SM, from a fit to the data given in Table 1.

$$\sqrt{\mathcal{L}} = 21.5 \pm 40 \text{ MeV}, \quad B_k = 0.94 \pm 0.15$$

variants of the minimal supersymmetric model (MSSM) has been presented recently, underlying the importance of measuring the angles $\alpha$, $\beta$, and $\gamma$ precisely. One expects almost similar constraints on $\beta$ from the CKM fits in the SM and MSSM, but $\alpha$ and $\gamma$ may provide a discrimination.

Figure 3. Allowed 95% C.L. region of the CP-violating quantities $\alpha$ and $\gamma$ in the SM, from a fit to the data given in Table 1.

$$\sqrt{\mathcal{L}} = 21.5 \pm 40 \text{ MeV}, \quad B_k = 0.94 \pm 0.15$$
3 CP-Violating Asymmetries in $B^\pm \to (h_1 h_2)^\pm$ Decays

Apart from the decay modes $B \to J/\psi K^0_s$ and $B \to \pi \pi$, discussed at great length in the literature, there are many interesting two-body decays $B \to h_1 h_2$ which are expected to have large CP asymmetries in their partial decay rates. Recent measurements by the CLEO collaboration of $B$-decays into final state such as $h_1 h_2 = \pi K, \eta' K, \pi \rho, \pi K^*$ have rekindled theoretical interest in these decays. A completely quantitative description of these and related two-body decays is a challenging enterprise, as this requires knowledge of the four-quark-operator matrix elements in the decays $B \to h_1 h_2$, for which the QCD technology is not yet ripe. Hence, calculations of the decay amplitudes from first principles in QCD are difficult and a certain amount of model-building is unavoidable. Here, we shall summarize the work done in estimating the rates and CP asymmetries in some selective decay modes, based on perturbative QCD and factorization.

For charged $B^\pm$ decays the CP-violating rate-asymmetries in partial decay rates are defined as follows:

$$A_{CP} \equiv \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)},$$

(20)

where $f^\pm = (h_1 h_2)^\pm$. To be non-zero, these asymmetries require both weak and strong phase differences in interfering amplitudes. The weak phase difference arises from the superposition of amplitudes from the various tree- and penguin-diagrams, with the former involving $b \to u$ and the latter $b \to s$ or $b \to d$ transitions. The strong phases, which are needed to obtain non-zero values for $A_{CP}$ in (20), are generated by final state interactions. This is modeled using perturbative QCD by taking into account the NLO corrections, following earlier suggestions along these lines. It should be stressed that this formalism includes not only the so-called charm penguins but all penguins (as well as the tree-contribution) in the framework of an effective Hamiltonian.

3.1 CP-violating Asymmetries Involving $b \to s$ Transitions

For the $b \to s$, and the charge conjugated $\bar{b} \to \bar{s}$, transitions, the respective decay amplitudes $\mathcal{M}$ and $\mathcal{M}$, including the weak and strong phases, can be generically written as:

$$\mathcal{M} = T \xi_u - P_{tc} \xi_t e^{i \delta_{tc}} - P_{uc} \xi_u e^{i \delta_{uc}},$$

$$\mathcal{M}^* = T \xi_u^* - P_{tc} \xi_t^* e^{i \delta_{tc}} - P_{uc} \xi_u^* e^{i \delta_{uc}},$$

(21)

where we define

$$P_{tc} e^{i \delta_{tc}} \equiv P_t e^{i \delta_t} - P_c e^{i \delta_c},$$

$$P_{uc} e^{i \delta_{uc}} \equiv P_u e^{i \delta_u} - P_c e^{i \delta_c}.$$ 

(22)

Here $\xi_t = V_{tb} V_{ts}^*$ and use has been made of the unitarity relation $\xi_c = -\xi_u - \xi_t$. In the above expressions $T$ denotes the contributions from the current-current operators; $P_t$, $P_c$ and $P_u$ denote the contributions from penguin operators proportional
to the product of the CKM matrix elements $\xi_t$, $\xi_c$ and $\xi_u$, and the corresponding strong phases are denoted by $\delta_t$, $\delta_c$ and $\delta_u$, respectively. The explicit expressions for the CP-violating asymmetry $A_{CP}$ are, in general, not very illuminating. However, as the amplitudes involve several small parameters, much simplified forms for $A^-$ and $A^+$, and hence $A_{CP}$, can be obtained in specific decays by keeping only the leading terms.

To exemplify this, we note that $|\xi_u| \ll |\xi_t| \simeq |\xi_c|$, with an upper bound $|\xi_u|/|\xi_t| \leq 0.025$. In some channels, such as $B^\pm \to K^\pm \pi^0$, $K^{*\pm} \pi^0$, $K^{*\pm} \rho^0$, typical value of the ratio $|P_{tc}/T|$ is of $O(0.1)$, with both $P_{uc}$ and $P_{tc}$ comparable with typically $|P_{uc}/P_{tc}| = O(0.3)$. Using these approximations, the CP-violating asymmetry in $b \to s$ transitions can be expressed as

$$A_{CP} \simeq \frac{2z_2 \sin \delta_{tc} \sin \gamma}{1 + 2z_2 \cos \delta_{tc} \cos \gamma + z_2^2}, \quad (23)$$

where $z_2 = |\xi_u/\xi_t| \times T/P_{tc}$. Note that $A_{CP}$ is approximately proportional to $\sin \gamma$, as pointed out by Fleischer and Mannel in the context of the decay $B \to K \pi$. Due to the circumstance that the suppression due to $|\xi_u/\xi_t|$ is stronger than the enhancement due to $T/P_{tc}$, restricting the value of $z_2$, the CP-violating asymmetry for these kinds of decays are expected to be $O(10\%)$. Explicit calculations in model estimates confirm this pattern.

There are also decay modes with vanishing tree contributions, such as $B^\pm \to \pi^\pm K_S^0$, $\pi^\pm K^{*0}$, $\rho^\pm K^{*0}$. With $T = 0$ and $|\xi_u| \ll |\xi_t|$, the CP-violating asymmetry can now be expressed as

$$A_{CP} \simeq 2P_{uc}/P_{tc} \left| \frac{\xi_u}{\xi_t} \right| \sin(\delta_{uc} - \delta_{tc}) \sin \gamma. \quad (24)$$

As $P_{uc}/P_{tc} \ll 1$, and also $|\xi_u/\xi_t| \ll 1$, the CP-violating asymmetries are expected to be small. Some representative estimates are: $A_{CP}(\pi^\pm K_S^0) = -1.5\%$, $A_{CP}(\pi^\pm K^{*0}) = -1.7\%$, $A_{CP}(\rho^\pm K^{*0}) = -1.7\%$. In scenarios with additional CP-violating phases, these CP asymmetries can be greatly enhanced and hence they are of interest in searching for non-SM CP-violation effects in $B$ decays.

### 3.2 CP-violating Asymmetries Involving $b \to d$ Transitions

For $b \to d$ transitions, the decay amplitudes can be expressed as

$$\mathcal{M} = T\zeta_u - P_{tc}\zeta_te^{i\delta_{tc}} - P_{uc}\zeta_u e^{i\delta_{uc}},$$

$$\overline{\mathcal{M}} = T\zeta_u^* - P_{tc}\zeta_t^* e^{i\delta_{tc}} - P_{uc}\zeta_u^* e^{i\delta_{uc}}, \quad (25)$$

where $\zeta_i = V_{ib}V_{id}^*$, and again the CKM unitarity has been used in the form $\zeta_c = -\zeta_t - \zeta_u$. For the tree-dominated decays involving $b \to d$ transitions, such as

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*The smallness of these quantities reflects the CKM-suppression and/or QCD dynamics calculated in perturbation theory.*
The CP asymmetries involving the neutral $B^\pm \rightarrow \pi^\pm \eta(0)$, $\rho^\pm \eta(0)$, $\rho^\pm \omega$, the relation $P_{uc} < P_{tc} \ll T$ holds, and the CP-violating asymmetry is approximately given by

$$A_{CP} \simeq \frac{-2z_1 \sin \delta_{tc} \sin \alpha}{1 + 2z_1 \cos \delta_{tc} \cos \alpha},$$

with $z_1 = |\xi_t/\xi_u| \times T_{P_{tc}}/T'^2$ and $T'^2 \equiv T^2 - 2TP_{uc} \cos \delta_{uc}$. Note, the CP-violating asymmetry is approximately proportional to $\sin \alpha$ in this case. Concerning $z_1$, we note that the suppression due to $P_{tc}T/T'^2 \ll 1$ is accompanied with some enhancement from $|\xi_t/\xi_u|$ (the central value of this quantity is about 2.1), making the CP-violating asymmetry in this kind of decays to have a value $A_{CP} = (5-10)\%$.

For the decays with vanishing tree contribution, such as $B^\pm \rightarrow K^\pm K_S^0$, $K^\pm K^{*0}$, $K^{*+}K^{*0}$, the CP-violating asymmetry is approximately proportional to $\sin \alpha$ again,

$$A_{CP} = \frac{-2z_3 \sin(\delta_{uc} - \delta_{tc}) \sin \alpha}{1 - 2z_3 \cos(\delta_{uc} - \delta_{tc}) \cos \alpha + z_3^2},$$

with $z_3 = |\xi_u/\xi_t| \times P_{uc}/P_{tc}$. As the suppression from $|\xi_u/\xi_t|$ and $|P_{uc}/P_{tc}|$ is not very strong, the CP-violating asymmetry are typically of order (10-20)\%.

We list the estimated CP asymmetries and branching ratios (charge-conjugate averaged) in $B^\pm \rightarrow (h_1h_2)^\pm$ decays in the upper half of Table 2, keeping only those decays which are expected to have branching ratios in excess of $10^{-6}$. While the listed $A_{CP}$ are not sensitive to the precise values of the form factors, the branching ratios are; the numbers given correspond to the BSW model [4]. We have indicated the uncertainty on $A_{CP}$ resulting from the virtuality of the gluon $g(k^2 \rightarrow q \bar{q})$, influencing the absorptive parts of the amplitudes, for $k^2 = m_b^2/2 \pm 2$ GeV$^2$.

4 CP-violating Asymmetries in $B^0 \rightarrow (h_1h_2)^0$ Decays

The CP asymmetries involving the neutral $B^0(\bar{B}^0)$ decays may require time-dependent measurements. Defining the time-dependent asymmetries as

$$A_{CP}(t) = \frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(\bar{B}^0(t) \rightarrow \bar{f})}{\Gamma(B^0(t) \rightarrow f) + \Gamma(\bar{B}^0(t) \rightarrow \bar{f})},$$

there are four cases that one encounters for neutral $B^0(\bar{B}^0)$ decays:

- case (i): $B^0 \rightarrow f$, $\bar{B}^0 \rightarrow \bar{f}$, where $f$ or $\bar{f}$ is not a common final state of $B^0$ and $\bar{B}^0$, for example $B^0 \rightarrow K^+\pi^-$ and $\bar{B}^0 \rightarrow K^-\pi^+$.

- case (ii): $B^0 \rightarrow (f = \bar{f}) \leftrightarrow \bar{B}^0$ with $f^{CP} = \pm f$, involving final states which are CP eigenstates, i.e., decays such as $B^0(\bar{B}^0) \rightarrow \pi^+\pi^-$, $\pi^0\pi^0$, $K_S^0\pi^0$ etc.

- case (iii): $B^0 \rightarrow (f = \bar{f}) \leftrightarrow \bar{B}^0$, with $f$ involving final states which are not CP eigenstates. They include decays into two vector mesons $B^0 \rightarrow (VV)^0$, as the $VV$ states are not CP-eigenstates.
Table 2. CP-rate asymmetries $A_{CP}$ and charge-conjugate-averaged branching ratios for some selected $B \to h_1 h_2$ decays, estimated in the factorization approach [1], updated for the central values of the CKM fits [2], $\rho = 0.20$, $\eta = 0.37$ and the factorization model parameters $\xi = 0.5$ and $k^2 = m_b^2/2 \pm 2\text{ GeV}^2$.

| Decay Modes | CP-class | $A_{CP}(\%)$ | $BR(\times 10^{-6})$ |
|-------------|----------|--------------|---------------------|
| $B^0 \to K^\pm \pi^\mp$ | (i) | $-7.7_{-4.4}^{+4.0}$ | 10.0 |
| $B^0 \to K^{*\pm} \pi^0$ | (i) | $-14.4_{-8.2}^{+3.9}$ | 4.3 |
| $B^0 \to K^{*\pm} \rho^0$ | (i) | $-13.5_{-6.5}^{+7.9}$ | 4.8 |
| $B^0 \to \eta \pi^\pm$ | (i) | $9.3_{-4.1}^{+1.9}$ | 5.5 |
| $B^0 \to \eta' \pi^\pm$ | (i) | $9.4_{-5.7}^{+2.5}$ | 3.7 |
| $B^0 \to \eta \rho^\pm$ | (i) | $3.1_{-1.7}^{+0.7}$ | 8.6 |
| $B^0 \to \eta' \rho^\pm$ | (i) | $3.1_{-1.8}^{+0.8}$ | 6.2 |
| $B^0 \to \rho^0 \omega$ | (i) | $7.0_{-3.4}^{+1.0}$ | 21.0 |
| $B^0 \to \eta K^\pm$ | (i) | $-4.9_{-1.1}^{+2.1}$ | 23.0 |
| $B^0 \to \eta^\prime K^\pm$ | (i) | $-3.1_{-1.0}^{+0.0}$ | 9.0 |
| $B^0 \to \eta K^{*\pm}$ | (i) | $7.0_{-2.0}^{+0.9}$ | 2.6 |
| $B^0 \to \eta K^{*\mp}$ | (i) | $-10.5_{-5.9}^{+4.3}$ | 2.1 |
| $B^0 \to \pi^\pm \omega$ | (i) | $-14.4_{-8.1}^{+1.8}$ | 3.2 |
| $B^0 \to \pi^\pm \omega$ | (i) | $7.7_{-4.7}^{+1.7}$ | 9.5 |
| $B^0 \to K^{\pm} \pi^\mp$ | (i) | $-10.3_{-5.6}^{+1.0}$ | 11.0 |
| $B^0 \to K^{*\pm} \rho^\mp$ | (i) | $-17.2_{-9.8}^{+5.5}$ | 5.4 |
| $B^0 \to K^{*\pm} \pi^\mp$ | (i) | $-8.2_{-4.3}^{+2.3}$ | 14.0 |
| $B^0 \to K^{*0} K^{*\mp}$ | (i) | $-17.2_{-9.8}^{+5.5}$ | 6.0 |
| $B^0 \to \rho^0 K^0_S$ | (ii) | $33.6_{-3.0}^{+0.2}$ | 23.0 |
| $B^0 \to \rho^0 K^0_S$ | (ii) | $25.4_{-1.0}^{+0.2}$ | 13.0 |
| $B^0 \to \eta^0 K^0_S$ | (ii) | $-45.1_{-0.8}^{+1.8}$ | 0.4 |
| $B^0 \to K^0_S \pi^0$ | (ii) | $39.4_{-0.9}^{+0.5}$ | 3.0 |
| $B^0 \to K^0_S \eta$ | (ii) | $41.2_{-1.1}^{+0.7}$ | 1.0 |
| $B^0 \to K^0_S \phi$ | (ii) | $35.2$ | 9.0 |
| $B^0 \to \rho^+ \rho^-$ | (iii) | $17.4_{-0.6}^{+0.1}$ | 24.0 |
| $B^0 \to \rho^0 \rho^0$ | (iii) | $-46.0_{-4.4}^{+1.4}$ | 1.0 |
| $B^0 \to \omega \omega$ | (iii) | $56.5_{-2.8}^{+1.3}$ | 1.1 |
| $B^0 \to \rho^+ \pi^- / \rho^- \pi^+$ | (iv) | $13.9_{-3.0}^{+0.9}$ | 7.8 |
| $B^0 \to \rho^+ \pi^- / \rho^- \pi^+$ | (iv) | $9.7_{-0.9}^{+0.6}$ | 29.0 |

- case (iv): $B^0 \to (f \bar{f}) \leftrightarrow \bar{B}^0$ with $f^{CP} \neq f$, i.e., both $f$ and $\bar{f}$ are common final states of $B^0$ and $\bar{B}^0$, but they are not CP eigenstates. Decays $B^0/\bar{B}^0 \to \rho^+ \pi^-, \rho^- \pi^+$ and $B^0/\bar{B}^0 \to K^{*0} K^0_S, \bar{K}^{*0} K^0_S$ are two interesting examples in this class.
Here case (i) is very similar to the charged $B^\pm$ decays, discussed above. For case (ii), and (iii), $A_C(t)$ would involve $B^0 - \bar{B}^0$ mixing. Assuming $|\Delta \Gamma| \ll |\Delta m|$ and $|\Delta \Gamma/\Gamma| \ll 1$, which hold in the standard model for the mass and width differences $\Delta m$ and $\Delta \Gamma$ in the neutral $B$-sector, one can express $A_C(t)$ in a simplified form:

$$A_C(t) \simeq a_\epsilon \cos(\Delta mt) + a_{\epsilon+\epsilon'} \sin(\Delta mt).$$  \hspace{1cm} (29)

The quantities $a_\epsilon$ and $a_{\epsilon+\epsilon'}$ depend on the hadronic matrix elements:

$$a_\epsilon = \frac{1 - |\lambda_{CP}|^2}{1 + |\lambda_{CP}|^2}, \quad a_{\epsilon+\epsilon'} = \frac{-2|\lambda_{CP}|}{1 + |\lambda_{CP}|^2},$$  \hspace{1cm} (30)

where

$$\lambda_{CP} = \frac{V_{tb}^* V_{td} \langle f | H_{eff} | B^0 \rangle}{V_{tb} V_{td}^* \langle f | H_{eff} | B^0 \rangle}$$  \hspace{1cm} (31)

For case (i) decays, the coefficient $a_\epsilon$ determines $A_C(t)$, and since no mixing is involved for these decays, the CP-violating asymmetry is independent of time. We shall call these, together with the CP asymmetries in charged $B^\pm$ decays, CP-class (i) decays. For case (ii) and (iii), one has to separate the $\sin(\Delta mt)$ and $\cos(\Delta mt)$ terms to get the CP-violating asymmetry $A_C(t)$. The time-integrated asymmetries are:

$$A_C = \frac{1}{1 + x^2} a_\epsilon' + \frac{x}{1 + x^2} a_{\epsilon+\epsilon'},$$  \hspace{1cm} (32)

with $x = \Delta m/\Gamma \simeq 0.73$ for the $B^0 - \bar{B}^0$ case.

Case (iv) also involves mixing but here one has to study the four time-dependent decay widths for $B^0(t) \to f$, $\bar{B}^0(t) \to \bar{f}$, $B^0(t) \to \bar{f}$ and $\bar{B}^0(t) \to f$. These time-dependent widths can be expressed by four basic matrix elements

$$g = \langle f | H_{eff} | B^0 \rangle, \quad h = \langle f | H_{eff} | B^0 \rangle,$$

$$\bar{g} = \langle \bar{f} | H_{eff} | B^0 \rangle, \quad \bar{h} = \langle \bar{f} | H_{eff} | B^0 \rangle,$$

which determine the matrix elements of $B^0 \to f$ and $\bar{B}^0 \to \bar{f}$ at $t = 0$. By measuring the time-dependent spectrum of the decay rates of $B^0$ and $\bar{B}^0$, one can find the coefficients of the two functions $\cos(\Delta mt)$ and $\sin(\Delta mt)$ and extract the quantities $a_\epsilon$, $a_{\epsilon+\epsilon'}$, $|g|^2 + |h|^2$, $a_\epsilon'$, $a_{\epsilon+\epsilon'}$ and $|\bar{g}|^2 + |\bar{h}|^2$ as well as $\Delta m$ and $\Gamma$.

Estimates of $A_C(P(\bar{B}^0 \to h_1 h_2))$, representing the decays belonging to the CP-classes (i) to (iv), together with the branching ratios averaged over the charge-conjugated modes, are given in Table 2. They have estimated branching ratios in excess of $10^{-6}$ (except for the decay $\bar{B}^0 \to \pi^0 \pi^0$). They also include the decay modes $\bar{B}^0 \to K^\pm \pi^\mp$, $\bar{B}^0 \to \eta' K_S^0$, $\bar{B}^0 \to \pi^\pm K^\mp$, $B^0/\bar{B}^0 \to \rho^- \pi^+ / \rho^+ \pi^-$, $B^0/\bar{B}^0 \to \rho^- \pi^- / \rho^+ \pi^+$, whose branching ratios have been measured by the CLEO collaboration. The CP asymmetries in all these partial decay rates are expected to be large.
the form factors, as indicated by the two set of numbers corresponding to the use are large to be conclusive. Also, there is some dependence of the decay rates on modes are typically a factor 2 below the CLEO data, though experimental errors well-accounted for in the factorization-model, but the factorization model parameter to \( \xi = 0.5 \). Theory numbers correspond to using the BSW model [Lattice QCD/QCD sum rule] for the form factors.

5 Comparison of the Factorization Model with the CLEO Data

Before we discuss the numerical results, a technical remark on the underlying theoretical framework is in order. The estimates being discussed here [44, 45], and the earlier work along these lines [43], are all based on using the NLO virtual corrections for the matrix elements of the four-quark operators, calculated in the Landau gauge with off-shell quarks [41]. This renders the effective (phenomenological) coefficients used in these works both gauge- and quark (off-shell) mass-dependent [42]. The remedy for this unsatisfactory situation is to replace the virtual corrections with the ones calculated with on-shell quarks, which are manifestly gauge invariant. A proof that gauge- and renormalization-scheme-independent effective coefficients (and hence decay amplitudes) can be consistently obtained in perturbation theory now exists [43]. Further discussion of these aspects and alternative derivation of the gauge-invariant on-shell amplitudes in \( B \to h_1h_2 \) decays will be reported elsewhere [46].

From the phenomenological point of view, the corrections in the effective coefficients are, however, small [44]. So, the real big unknown in this approach is the influence of the neglected soft non-factorizing contributions.

We now compare the predictions of the factorization-based estimates with the CLEO data. Since no CP asymmetries in the decays \( B \to h_1h_2 \) have so far been measured, this comparison can only be done in terms of the branching ratios. We give in Table 3 eight \( B \to h_1h_2 \) decay modes, their branching ratios measured by CLEO [44] and the updated theoretical estimates of the same [4]. The entries given for \( B^0/\bar{B}^0 \to \rho^- \pi^+ \) are for the sum of the two modes, as CLEO does not distinguish between the decay products of \( B^0 \) and \( \bar{B}^0 \). Moreover, we have fixed the factorization model parameter to \( \xi = 0.5 \), which is suggested by the measured \( B \to \pi\rho \) branching ratios. Table 3 shows that all five \( K\pi \) and \( \pi\rho \) decay modes are well-accounted for in the factorization-model, but the \( \pi^\pm K^{\mp} \) and the two \( B \to \eta'K \) modes are typically a factor 2 below the CLEO data, though experimental errors are large to be conclusive. Also, there is some dependence of the decay rates on the form factors, as indicated by the two set of numbers corresponding to the use.

| Decay Mode                   | BR (Exp)  | BR (Theory) |
|------------------------------|-----------|-------------|
| \( B^0 \to K^+\pi^- \)       | 1.4 ± 0.3 ± 0.2 | 1.4 [1.7]   |
| \( B^+ \to K^+\pi^0 \)       | 1.5 ± 0.4 ± 0.3 | 1.0 [1.1]   |
| \( B^+ \to K^0\pi^- \)       | 1.4 ± 0.5 ± 0.2 | 1.6 [1.9]   |
| \( B^+ \to \eta'K^+ \)       | 7.4 ± 0.5 ± 1.0 | 2.3 [2.7]   |
| \( B^0 \to \eta'K^0 \)       | 5.9 ± 0.8 ± 0.9 | 2.3 [2.7]   |
| \( B^0 \to \pi^-K^{+*} \)    | 2.2 ± 1.0 ± 0.5 | 0.6 [0.7]   |
| \( B^+ \to \pi^+\rho^0 \)    | 1.5 ± 0.5 ± 0.4 | 0.9 [1.0]   |
| \( B^0/\bar{B}^0 \to \rho^-\pi^+ \) | 3.5 ± 1.0 ± 0.5 | 3.7 [4.3]   |

Table 3. Branching ratios measured by the CLEO collaboration and factorization-based theoretical estimates of the same (in units of \( 10^{-5} \)), updated for the central values of the CKM fits \( \rho = 0.20, \eta = 0.37 \) and the factorization model parameter \( \xi = 0.5 \). Theory numbers correspond to using the BSW model [Lattice QCD/QCD sum rule] for the form factors.
of the BSW-model and the Lattice-QCD/QCD-sum-rule-based estimates, and on $\xi$.

While the final verdict of the experimental jury is not yet in, it is fair to say that the factorization approach, embedded in a well-defined perturbative framework, provides a description of the present data which is accurate within a factor 2. This is a hint that non-factorizing soft final state interactions do not represent a dominant theme in $B$ decays. Present data gives some credibility to the idea that perturbative QCD-based methods can be used in estimating final state interactions. Non-leptonic $B \rightarrow h_1 h_2$ decays discussed here, many of which will be measured in experiments at the B factories and hadron machines, will test this quantitatively.

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