Next-to-Leading-Order QCD Corrections to $e^+e^- \rightarrow J/\psi gg$ at the B Factories

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We calculate the next-to-leading-order (NLO) QCD corrections to $e^+e^- \rightarrow J/\psi gg$ via color singlet $J/\psi(S_1)$ at the B factories. The result shows that the cross section is enhanced to 0.373 pb by a K factor (NLO/LO) of about 1.21. By considering its dependence on the charm quark mass and renormalization scale, the NLO cross section can range from 0.294 to 0.409 pb. Further including the $\psi'$ feed-down, $\sigma(e^+e^- \rightarrow J/\psi X(\text{non } c\bar{c}))$ is enhanced by another factor of about 1.29 and reaches 0.482 pb. In addition, the momentum distributions of $J/\psi$ production and polarization are presented. Recent measurements from Belle agree well with our prediction for the cross section and momentum distribution. It is expected that this process can serve as a very good channel to clarify the $J/\psi$ polarization puzzle by performing further experimental measurements.

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Perturbative quantum chromodynamics are successful to describe large momentum transfer processes at quark level due to its asymptotic freedom property. But it falls into the nonperturbative range for quark hadronization to form the final state hadrons which are measured in experiments. Therefore the quark hadronization is usually described by phenomenological models and quite far away from the first principle QCD Theory. However, in the heavy quarkonium case, a naive perturbative QCD and nonrelativistic treatment for the bound state is applied straightforwardly to the related decay or production processes. It is called color-singlet mechanism. To describe the huge discrepancy of the high-$p_T$ $J/\psi$ production between the theoretical calculation based on color-singlet mechanism and the experimental measurement at Tevatron, a color-octet mechanism was proposed based on the non-relativistic QCD (NRQCD) formalism. It allows consistent theoretical predictions to be made and to be improved systematically in the QCD coupling constant $\alpha_s$ and the heavy-quark relative velocity $v$. In recent years, there is a huge data collection in the B factory experiments. Based on that, many $J/\psi$ production processes were observed in the past. Now the integrated luminosity is more than 850 fb$^{-1}$ at the Belle detector at the KEKB and it is about 20 times larger than the integrated luminosity 32.4 fb$^{-1}$, based on which the inclusive $J/\psi$ production was measured. Therefore it supplies a very important chance to perform systematical study on $J/\psi$ production both theoretically and experimentally.

The measurements for exclusive $J/\psi$ productions $e^+e^- \rightarrow J/\psi\eta_c$, $J/\psi J/\psi$, $J/\psi \chi_cJ$ at the B factories have shown that there are large discrepancies between the leading-order (LO) theoretical predictions in NRQCD and the experimental measurements. It seems that such discrepancies can be resolved by introducing higher order corrections: next-to-leading-order (NLO) QCD corrections and relativistic corrections.

The cross section for inclusive $J/\psi$ production in $e^+e^-$ annihilation was measured by BABAR as $2.54 \pm 0.21 \pm 0.21$ pb and Belle as $1.45 \pm 0.10 \pm 0.13$ pb. These measurements infer that $\sigma(e^+e^- \rightarrow J/\psi + (\text{non } c\bar{c})) = 0.6$ pb. For this part, the contributions from the color-singlet and color-octet contributions for the processes, $e^+e^- \rightarrow J/\psi(S_1)(S_1)gg$, $J/\psi(S_1)(S_0^2P_J)g$, are about 0.2 pb and 0.27 pb, respectively, at the LO in NRQCD. However, the signal of the color octet was not found in the experiment. Therefore, the experimental measurement by Belle is about 3 times larger than the theoretical prediction. However, this large discrepancy was partially resolved by considering both NLO correction and feed-down from higher excited states. The above measurements infer that $\sigma(e^+e^- \rightarrow J/\psi X(\text{non } c\bar{c})) = 0.6$ pb. For this part, the contributions from the color-singlet and color-octet contributions for the processes, $e^+e^- \rightarrow J/\psi(J_1)(S_1)gg$, $J/\psi(S_0^2P_J)g$, are about 0.2 pb and 0.27 pb, respectively, at the LO in NRQCD. However, the signal of the color octet was not found in the experiment. Therefore, the experimental measurement by Belle is about 3 times larger than the theoretical prediction from color singlet at LO, and can be much more than 3 times by BABAR. Recently many studies have shown that higher order corrections within NRQCD framework are very important. To achieve a reasonable theoretical prediction for the process $e^+e^- \rightarrow J/\psi gg$, in this letter, we present a NLO QCD calculation to this process.

The related Feynman diagrams which contribute to the LO amplitude of the process $e^+(p_1) + e^-(p_2) \rightarrow J/\psi(p_{3}) + g(p_{4}) + g(p_{5})$ are shown in Fig. 1(a), while the others can be obtained by permuting the places of the virtual photon and gluons. In the nonrelativistic limit, the NRQCD factorization formalism is used to obtain the total cross section in $n = 4 − 2c$ dimensions as
by introducing a dimensionless kinematic variable \( \hat{s} = s/(2m_c)^2 \), where \( s \) is the squared center-of-mass energy, \( e_c \) and \( m_c \) are the electric charge and mass of the charm quark, respectively, and \( \beta = \sqrt{s}\hat{s} - 1 \). \( R_s(0) \) is the radial wave function at the origin of \( J/\psi \). The approximation \( M_{J/\psi} = 2m_c \) is taken.

At NLO in \( \alpha_s \), there are virtual corrections which arise from loop diagrams. Dimensional regularization has been adopted for isolating the ultraviolet (UV) and infrared (IR) singularities. UV-divergences from self-energy and triangle diagrams are canceled upon the renormalization of QCD. Here we adopt the same renormalization scheme as Ref. [12]. There are 111 NLO diagrams in total, including counter-term diagrams. They are shown in Fig. (d), and divided into 8 groups. Diagrams of group (d) contain Coulomb singularities, which can be isolated and mapped into the \( c\bar{c} \) wave function. Although the Feynman diagrams are similar, the calculation of tensor and scalar integrals is much more complicated than that in Ref. [25], because there is one more variable, which is the mass of the virtual photon, in this calculation. Again, the calculation was done automatically with our Feynman Diagram Calculation package (FDC) [26].

The real corrections arise from two processes, \( e^+e^- \rightarrow J/\psi gq\bar{q} \) and \( e^+e^- \rightarrow J/\psi ggg \). The related Feynman diagrams for these two processes are shown in Fig. (b) and (c). The phase space integration for them will generate IR singularities, which are either soft or collinear and can be conveniently isolated by slicing the phase space into different regions. We adopt the two-cutoff phase space slicing method [20] to decompose the phase space into three parts by introducing two small cutoffs, \( \delta_c \) and \( \delta_s \). And then the real cross section can be written as

\[
\sigma_R = \sigma^S + \sigma^{HC} + \sigma^{H\bar{C}},
\]

where \( \sigma^S \) from the soft regions contains soft singularities and is calculated analytically under soft approximation. It is easy to find that the soft singularities for a gluon emitted from the charm quark pair in the S-wave color singlet \( J/\psi \) are canceled by each other. And we have

\[
d\sigma^S = d\sigma^{(0)} \frac{\alpha_s \Gamma(1-\epsilon)}{2\pi \Gamma(1-2\epsilon)} \left( \frac{4\pi \mu^2}{s} \right)^\epsilon \left( \frac{A_s^H}{\epsilon^2} + \frac{A_s^0}{\epsilon} \right),
\]

\[
A_s^H = 6, \quad A_1^H = -12 \ln \delta_s - 6 \ln \left( \sin^2 \frac{\theta_p}{2} \right),
\]

\[
A_0^H = \frac{(A_s^0)^2}{12} + 6 \ln \left( \frac{\cos^2 \frac{\theta_p}{2}}{2} \right),
\]

where \( \mu \) is the renormalization scale and \( \theta_p \) is the angle between two gluons in the \( p_1 + p_2 \) rest frame. \( \sigma^{HC} \) from the hard collinear regions contains collinear singularities and can also be factorized. Here we have

\[
d\sigma^{HC} = d\sigma^{(0)} \frac{\alpha_s \Gamma(1-\epsilon)}{2\pi \Gamma(1-2\epsilon)} \left( \frac{4\pi \mu^2}{s} \right)^\epsilon \left( \frac{A_{HC}}{\epsilon^2} + \frac{A_{0HC}}{\epsilon} \right),
\]

\[
A_{HC} = 11 + 6 \ln \delta_s + 6 \ln \delta_s - \frac{2}{3} n_{lf},
\]

\[
A_{0HC} = \frac{67}{3} - \frac{10}{9} n_{lf} - 2\pi^2 - 3 \ln^2 \delta_s - 3 \ln^2 \delta_s - \ln \delta_s A_{1HC},
\]

where \( \delta_s = \delta_s/[1 - (p_3 + p_2)^2/s] \) and \( n_{lf} \) is the number of active light quark flavors. The hard noncollinear part \( \sigma^{H\bar{C}} \) is IR finite. Finally, all the IR singularities are canceled analytically. After adding all the contribution together, the cross section at NLO can be expressed as

\[
\sigma^{(1)} = \sigma^{(0)} \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[ a(\hat{s}) + \beta_0 \ln \left( \frac{\mu}{2m_c} \right) \right] \right\},
\]

where \( \beta_0 \) is the one-loop coefficient of the QCD beta function.
To study the polarization of $J/\psi$ production, we define the angular distribution $A$ and polarization factor $\alpha$ as:

$$\frac{d^2\sigma}{d\cos \theta dp} = S(p)[1 + A(p) \cos \theta], \quad \alpha = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L},$$

where $p$ and $\theta$ are the 3-momentum and production angle of $J/\psi$ in the laboratory frame. $\sigma_T$ and $\sigma_L$ are the transverse and longitudinal polarized cross section. To calculate $\alpha$, we use the same method to represent the polarized cross section as Eqs. (8) and (9) in Ref. [23].

The scale dependence of the cross section is shown at Ref. [13]. The numerical results are showed in Table. I. J/$\psi$ of amplitude square with very good behavior in numerical production, a simplified method is used to calculate the $J/\psi$ total cross section and momentum distribution of large numerical error.

The values of $\alpha_s$ and the wave function at the origin of $J/\psi$ in the NLO calculation are taken the same as in Ref. [13]. The numerical results are showed in Table. I. The scale dependence of the cross section is shown at

| $m_c$(GeV) | $\alpha_s(\mu)$ | $\sigma^{(0)}(pb)$ | $\sigma^{(1)}(pb)$ | $\sigma^{(1)}/\sigma^{(0)}$ |
|------------|-----------------|-----------------|-----------------|-----------------|
| 1.4        | 0.267           | 0.341           | 2.35            | 0.409           |
| 1.5        | 0.259           | 0.308           | 2.57            | 0.373           |
| 1.6        | 0.262           | 0.307           | 2.89            | 0.384           |

TABLE I: Cross sections with different charm quark mass $m_c$ where the renormalization scale $\mu = 2m_c$ and $\sqrt{s} = 10.6$ GeV.

Fig. 2 and it does improve significantly at NLO. The final numerical result can be expressed as

$$\sigma^{(1)} = 0.373^{+0.036}_{-0.079} \text{ pb}$$

where the theoretical uncertainty is from the choices of $m_c$ and $\mu$, with $m_c = 1.4$ GeV and $\mu = 2m_c$ for the upper boundary and $m_c = 1.6$ GeV and $\mu = \sqrt{s}/2$ for the lower boundary. The momentum distribution of $J/\psi$ production are shown in Fig. 3. To include the $\psi'$ contribution in the momentum distribution with a suitable kinematic treatment, $m_c = m_{\psi'}/2$, $\mu = m_{\psi'}$, $\text{Br}(\psi' \rightarrow J/\psi + X) = 0.574$ and $\Gamma(\psi' \rightarrow e^+e^-) = 2.19$ KeV are used. In Fig. 3 the momentum distributions of the polarization factor $\alpha$ and the angular distribution coefficient $A$ of $J/\psi$ are shown. Both $\alpha$ and $A$ have slight changes at NLO. Furthermore, if the contribution from $\psi'$ is included, the two curves change very little. In addition, we find that $d\sigma/d\cos \theta$ is a constant within quite large numerical error.

To further discuss the renormalization scale $\mu$ dependence, two ways are applied with $m_c = 1.5$ GeV. In one way, to find the maximum cross section by using $d\sigma^{(1)}/d\mu = 0$, $\sigma^{(1)}_{\text{max}} = 0.41$ pb is obtained with $\mu = 1.7$ GeV, which means the calculation is right in the perturbative range and gives an uncertainty of about 0.04 pb for the default estimate. In another way, we apply the Brodsky, Lapage, Backenzie (BLM) scale setting[30] and find the unique scale choice $\mu^* = 0.74$GeV with the cross section $\sigma^{(1)}(\mu^*) = \sigma^{(0)}(\mu^*)(1 - 0.1\alpha_s(\mu^*)/\pi)$. It means that BLM way can not give good convergence behavior in this case.

In summary, we have calculated the NLO QCD correction to $e^+e^- \rightarrow J/\psi gg$ at the B factories. It increases the cross section to 0.373 pb with a K factor of about 1.21 for the default $m_c = 1.5$ GeV and $\mu = 2m_c$. By considering its dependence on the charm quark mass and renormalization scale, the NLO cross section ranges from 0.294 to 0.409 pb. Furthermore, it will be enhanced by another factor of about 1.29 and reaches 0.482 pb when the feeddown from $\psi'$ is considered. By using $d\sigma^{\text{NLO}}(\mu)/d\mu = 0$, a maximum $\sigma^\text{NLO} = 0.41$ pb is obtained when $\mu = 1.7$ GeV is chosen, which means the calculation is right in the perturbative region. Together with the K factor 1.21, it could be expected that higher order corrections are smaller. By considering theoretical and experimental uncertainty, our results are roughly consistent with the measurement 0.6 pb from Belle [3, 4], and furthermore agree well with their recent experimental measure-
FIG. 4: Polarization parameter $\alpha$ and angular distribution parameter $A$ of $J/\psi$ as functions of $p$ with $m_c = 1.5$ GeV and $\mu = 2m_c$.

and experimental measurements for $J/\psi$ polarization. It may due to the large contribution from color-octet states with large uncertainties, or other mechanism [32]. But in this process, color-singlet contribution is dominant and the convergence of perturbative QCD expansion works very well. Therefore, the prediction for its polarization distribution at NLO is well defined and should fit well with the experimental data. Our further work [33] gives various distributions for $e^+e^- \rightarrow J/\psi + X(c\bar{c})$. In order to clarify the situation, we suggest to perform further experimental analysis of the data based on nowadays huge integrated luminosity at the $B$ factories. It is desirable that the inclusive $J/\psi$ production could be separated into $e^+e^- \rightarrow J/\psi + X(c\bar{c})$ and $e^+e^- \rightarrow J/\psi + X(\text{non } c\bar{c})$ in experimental measurement, so that their angular and momentum distributions of $J/\psi$ polarization can be compared with their theoretical predictions separately. It may be worthwhile to include relativistic correction effects in order to sharpen the test of NRQCD.

While this paper is being prepared, we are informed of the same process also being considered by Ma, Zhang and Chao [34]. Our results are in agreement with theirs. In addition, we calculated the momentum distribution of $J/\psi$ polarization which is a very important issue to clarify the $J/\psi$ polarization puzzle.

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