A tool to estimate the critical dynamics and thickness of superconducting films and interfaces

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We demonstrate that the magnetic field dependence of the conductivity measured at the transition temperature allows the dynamical critical exponent, the thickness of thin superconducting films and interfaces, and the limiting lateral length to be determined. The resulting tool is applied to the conductivity data of an amorphous Nb0.15Si0.85 film and a LaMO3/SrTiO3 interface.

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In a phase transition, sufficiently close to the transition temperature $T_c$, critical fluctuations are expected to dominate. The closer one gets to $T_c$, the longer these fluctuations will last, and the larger the relevant length scale becomes. In a superconductor the relevant length scale is the correlation length $\xi$. Without loss of generality we can assume that the lifetime of the fluctuations, $\tau$, varies as $\tau \propto \xi^z$ which defines $z$, the dynamical critical exponent. As we approach the critical region, all the physics that really matters is associated with the diverging length and time scales.

Using experimentally accessible quantities, voltage $V$ and current $I$, dynamic scaling predicts for superconducting films and interfaces the relationship

$$V = I \xi^{-z} g_\pm \left(\frac{I}{T_c}\right).$$  (1)

$g_\pm (x)$ is a scaling function of its argument above (+) and below (-) $T_c$. Above $T_c$, in the limit $x \to 0$, $g_+(x)$ tends to a constant and the conductivity to

$$\sigma = \frac{I}{V} \propto \xi^z.$$  (2)

On the other hand, at $T_c$ in the limit $x \to \infty$, $g_\pm (x)$ tends to $x^z$ so that

$$V \propto I^{a(T_c)}, \ a(T_c) = z + 1.$$  (3)

In practice $I - V$ data exhibit resistive tails revealing finite size induced free vortices which make it difficult to estimate the transition temperature $T_c$ and the dynamical scaling exponent $z$.

Alternatively, the application of the conductivity relation requires the explicit form of the correlation length. Since superconducting thin films and interfaces are expected to undergo a Berezinski-Kosterlitz-Thouless (BKT) transition from the superconducting to the normal state the correlation length adopts for $T \geq T_c$ the characteristic form

$$\xi(T) = \xi_0 \exp \left(\frac{2\pi}{bl^{1/2}}\right), \ t = \frac{T}{T_c} - 1.$$  (4)

$\xi_0$ is related to the vortex core radius and $b$ to the energy needed to create a vortex. Accordingly the analysis of conductivity or resistivity data in zero magnetic field provide in terms of $\sigma \propto \xi^z$ estimates for $T_c$, $\xi_0$ and $z$.

In this context it is important to recognize that the existence of the BKT-transition (vortex-antivortex dissociation instability) in $^4$He films is intimately connected with the fact that the interaction energy between vortex pairs depends logarithmic on the separation between them. As shown by Pearl, vortex pairs in thin superconducting films (charged superfluid) have a logarithmic interaction energy out to the characteristic length $\lambda_{2D} = \lambda^z/d$, beyond which the interaction energy falls off as $1/r$. Here $\lambda$ is the magnetic penetration depth of the bulk. As $\lambda_{2D}$ increases the diamagnetism of the superconductor becomes less important and the vortices in a thin superconducting film become progressively like those in $^4$He films. Accordingly the $\lambda_{2D} \gg \min |W, L|$ is required, where $W$ and $L$ denote the width and the length of the perfect sample. Invoking the Nelson-Kosterlitz relation $\lambda_{2D} (T_c) = \lambda^2 (T_c) / d = \Phi_0^2 / (32\pi^2k_BT_c)$ it is readily seen that for sufficiently low $T_c$’s and min $|W, L| < < 1$ cm this condition is well satisfied. As a result any rounding of the transition due to finite size effects should be more important than that due to the finite magnetic "screening length" $\lambda_{2D}$.

Here we present a tool to determine the dynamical critical exponent $z$, the thickness $d$, and the limiting length $\hat{L}$, associated with the resistive tail in zero magnetic field, from conductivity measurements taken at $T_c$ and in magnetic fields applied parallel and perpendicular to the film or interface. Traditionally the thickness of superconducting films is estimated from the angular dependence of the upper critical field $H_{c2}$. Noting that $H_{c2}$ is an artifact of the mean-field approximation this approach becomes questionable in two dimensions where thermal fluctuations are enhanced. The crucial component of the tool stems from the magnetic field induced finite size effect. For $T \geq T_c$ and nonzero magnetic field the mean distance between the vortex lines ($\Phi_0 / H^{1/2}$) is another characteristic length, preventing the correlation length to diverge at $T_c$ and $H > 0$. The resulting magnetic field induced finite size effect can be described by relating the zero field...
and finite field correlation length in terms of
\[ \xi_x (T, H_z) \xi_y (T, H_z) = \xi_x (T, 0) \xi_y (T, 0) G (x), \]
where
\[ x = \frac{aH_z \xi_x (T, 0) \xi_y (T, 0)}{\Phi_0} = \frac{\xi_x (T, 0) \xi_y (T, 0)}{L^2_{H_z}}, \]
\[ L^2_{H_z} = \frac{\Phi_0}{aH_z}. \] (6)

$L_{H_z}$ is the limiting magnetic length and $G (x)$ denotes the finite size scaling function with the limiting behavior
\[ G (x) = \begin{cases} 1 : x = 0 \\ 1/x : x \to \infty \end{cases}. \] (7)

Indeed, in zero field the limiting magnetic length $L_{H_z}$ is infinite and the growth of the correlation length $\xi$ is unlimited, while in finite fields the divergence of $\xi$ at $T_c$ is removed and its value is given by
\[ \xi_x (T_c, H_z) \xi_y (T_c, H_z) = \frac{L^2_{H_z} = \Phi_0}{aH_z}. \] (8)

where $a$ fixes the mean distance between vortices. The equivalence to the standard finite size effect in a film of dimensions $L \times L$ is readily established by noting that in this case the correlation length scales as $\xi (T, L) = \xi (T, L = \infty) G (\xi (T, L = \infty) / L)$.[18]

More generally in magnetic fields $H_{\perp, \parallel}$, applied perpendicular ($\perp$) or parallel ($\parallel$) to the film or interface, the divergence of $\xi (T)$ at $T_c$ is then removed because $\xi (T_c)$ cannot grow beyond
\[ \xi \xi_0 < x < x \hat{=} aH_z, \]
\[ x_0 \hat{=} aH_z \]
\[ \xi \xi_0 < x \hat{=} aH_z. \]

Here we included the limiting length $\hat{L}$ arising from the ohmic tail in zero field, e.g. due to the system size or the finite lateral extent of the homogenous domains. The expressions for the magnetic field induced limiting lengths $L_{H_{\perp}}$ and $L_{H_{\parallel}}$ follow from Eq. [3] and by noting that the correlation lengths of fluctuations which are transverse to the applied magnetic field are bounded according to $\xi_x \xi_y \leq \Phi_0 / (aH_z)$, $x \neq y \neq z$, where $\xi_x = d$, $H_{\perp} = H_z$, $H_{\parallel} = H_{\parallel}$, and accordingly $\xi_x \xi_y = \xi_h^2 \leq L^2_{H_{\perp}} = \Phi_0 / aH_{\perp}$ and $\xi_x \xi_z = \xi_{d} \leq L_{H_{\parallel}} = \Phi_0 / aH_{\parallel}$, where $d$ denotes the film thickness.

These limiting lengths prevent the divergence of the conductivity at $T_c$. In zero field it adopts according to Eqs. [2] and [9] the form
\[ \sigma (T_c, H_{\perp, \parallel}) = 0 = f \hat{L}^2, \] (10)

As the magnetic field increases this behavior applies as long as $\hat{L} < L_{H_{\perp, \parallel}}$, while for $\hat{L} > L_{H_{\perp, \parallel}}$ the magnetic field sets the limiting length and the conductivity approaches according to Eqs. [2] and [10] the form
\[ \sigma (T_c, H_{\perp, \parallel}) = \sigma_n + \begin{cases} f \downarrow H^{−2/3}, & f = (\Phi_0 / \alpha H)^{2/3} \\ f \downarrow H^{−2/3}, & f = (\Phi_0 / \alpha H)^{2/3} \end{cases}, \] (11)

where $\sigma_n$ is the normal state conductivity, determined from the magnetic field dependence of the conductivity at $T_c$ (Eq. [11]). So far we concentrated on temperatures at and above the BKT-transition. Below $T_c$ the correlation length diverges: $\xi \to \infty$. This implies that $\xi$ will be cut off by a limiting length and with that are Eqs. [10] and [11] expected to apply for $0 < T \leq T_c$. Since the low-temperature phase in the BKT scenario is described by a line of fixed points, each temperature $T < T_c$ may be characterized by its own $f (T)$.

An essential assumption of the outlined approach is that around $T_c$ thermal phase fluctuations dominate. There is considerable evidence for a critical magnetic field $H_{\perp, \parallel}$, emerging from a nearly temperature independent crossing point in the resistance-magnetic field plane.[24,25,26,27,28,29,30,31] It can be identified as the critical field of the quantum superconductor to insulator (QSI) transition and the resistance is predicted to scale as $R (H_{\perp, \parallel}) = R_c f \left( H_{\perp, \parallel} - H_{\perp, \parallel} \right) / T^{1/3}$. In Fig. 1 we depicted the temperature dependence of the sheet resistance in zero field to estimate $T_c$ and to uncover a rounded transition attributable to a finite size effect. Evidence for characteristic BKT-behavior emerges from the inset showing $(d (\ln R) / d T)^{−2/3}$ vs. $T$ in terms of the consistency with $(d (\ln R) / d T)^{−2/3} = (2 / b) \xi^{2/3}$ ($T - T_c$) in an intermediate temperature regime above $T_c$. The resulting estimates for $b$ and $T_c$ are then used to obtain the BKT-resistance, $R = R_0 \exp \left( -b (T - T_c)^{1/2} \right)$, by adjusting $R_0$ in this intermediate regime. The comparison between the resulting solid BKT-line and the data reveals a rounded transition and with that a finite size effect generating free vortices at and below $T_c = 0.224$ K.
In this context we note that according to the Harris criterion weak randomness in the local $T_c$, pairing interaction, etc. does not change the critical BKT-behavior. Nevertheless, inhomogeneities due to local strain or a heat current appear to be likely in both, superconducting films and interfaces. A nonzero heat current drives the system away from equilibrium. A temperature gradient is created which implies that the temperature is space dependent.

Given the estimate of the BKT-transition temperature $T_c$ and the evidence for a zero field limiting length we turn to the effects of an applied magnetic field, inducing additional free vortices. In Fig. 2 we show the sheet conductivity $\sigma_{\|}(T_c)$ vs. $H_{\perp}$ derived from the resistivity data. Above $H_{\perp}^* = 1.75$ kOe we observe for $z \simeq 2$, 

$$\sigma_{\perp}(T_c, H_{\perp}) = \sigma_n + f_{\perp} H_{\perp}^{-z/2} \quad (\text{Eq. (11)})$$

consistency with $\sigma(T_c, H_{\perp}) = \sigma_n + f_{\perp} H_{\perp}^{-z/2}$ (Eq. (11)) and therewith evidence for diffusive dynamics.1 In the low field limit deviations from Eq. (11) are expected because for sufficiently low $H_{\perp}$ the magnetic length $L_{H_{\perp}} = (\Phi_0/ aH_{\perp})^{1/2}$ is no longer large compared to $\hat{L}$, the zero field limiting length.

According to Fig. 2 depicting $\sigma_{\perp}(T_c, H_{\parallel})$ vs. $H_{\parallel}$ of the same sample, agreement with $\sigma(T_c, H_{\parallel}) = \sigma_{res} + f_{\parallel} H_{\parallel}^{-z}$ (Eq. (11)) is obtained above $H_{\parallel}^* = 6$ kOe for $z \simeq 2$. So this value is consistent with both the perpendicular and parallel magnetic field dependence. Given then the evidence for $z = 2$ and the estimates for $f_{\perp}$ and $f_{\parallel}$ we obtain with the nominal thickness of the film, $d \simeq 125 \AA$, and Eq. (12) for $a$, fixing the mean distance between vortices, the estimate

$$a \simeq 4.8,$$
each temperature $T < T_c$ may be characterized by its own $f(T)$. To clarify this conjecture we invoke Eq. (11) in the form $H_\perp \sigma(T, H_\perp) = H_\parallel \sigma_n + f_\perp(T)$ with $z = 2$. The data should then fall on straight lines with slope $\sigma_n$ and intercepts $f_\perp(T)$. In Fig. 3 depicting $H_\perp \sigma(T, H_\perp)$ vs. $H_\perp$, for temperatures at and below $T_c$, we observe that above $H_\perp^* = 1.75$ kOe (see Fig. 2), where the magnetic field sets the limiting length, the data falls on a single line, while below $H_\perp^*$ a crossover to the zero field limit behavior, $\sigma(T, H_\perp = 0) = f(T) \tilde{L}$ (Eq. (10)) sets in. Indeed, around $H_\perp^*$ the magnetic limiting length $L_{H_\perp}$ becomes comparable to $\tilde{L}$. From the inset, showing $\sigma_\parallel(T)$ vs. $H_\perp$, it is seen that in zero field $f(T)$ increases with reduced temperature, reflecting that by lowering the temperature the density of the finite size induced vortices is reduced and with that the conductivity increases. Thus, as conjectured, $f(T)$ in Eq. (10) depends on temperature. The agreement with Eq. (11), taking thermal fluctuations into account only, also reveals that around $T_c$ the contribution of quantum fluctuations is negligibly small, although a nearly temperature independent crossing point in the resistance-magnetic field plane occurs around $H_\perp \simeq 5.5$ kOe.

To illustrate this tool further, allowing to determine $z, d$ and $\tilde{L}$ from the magnetic field dependence of the conductivity at $T_c$ we analyze the conductivity data of Reyren et al.\textsuperscript{16} for a superconducting LaAlO$_3$/SrTiO$_3$ interface with $T_c \simeq 0.21$ K. In Fig. 5 we show the sheet conductivity $\sigma_\parallel(T_c)$ vs. $H_\parallel$ derived from the resistivity data. Above $\mu_0 H_\perp \simeq 10$ mT we observe consistency with Eq. (11) for $z \simeq 2$, in agreement with the value derived from I-V-data\textsuperscript{14} and predicted for diffusive dynamics.\textsuperscript{17} According to Fig. 6 and Eq. (11) $z \simeq 2$ also follows from $\sigma(T_c)$ vs. $H_\parallel$ above $\mu_0 H_\parallel \gtrsim 300$ mT. Given then the evidence for $z = 2$ and the estimates for $f_\perp$ and $f_\parallel$ we obtain with Eqs. (12) and (14) for the thickness of the superconducting interface the value

$$d \simeq 67 \AA,$$

in agreement with previous estimates where $z = 2$ was assumed.\textsuperscript{16} Recently, room temperature studies have also been performed to estimate the thickness of the LaAlO$_3$/SrTiO$_3$ interface grown at “high” oxygen pressures leading to a value of $70 \AA$, $100 \AA$ and $120 \AA$ at 8 K.\textsuperscript{38}

![Figure 3](image3.png)

**FIG. 3:** $\sigma_\parallel(T_c)$ vs. $H_\parallel$ for an amorphous 125 Å thick Nb$_{0.15}$Si$_{0.85}$ film and $T = 0.224$ K$\simeq T_c$ derived from Aubin et al.\textsuperscript{20} The solid line is Eq. (11) with $\sigma_n = 0.71$ kΩ$^{-1}$ and $f_\parallel = 8$ kΩ$^{-1}$kOe$^2$. The arrow marks $H_\parallel^* = 6$ kOe and the dot $H_\parallel^* = 3.85$ kOe.

![Figure 4](image4.png)

**FIG. 4:** $H_\perp \sigma(T, H_\perp)$ vs. $H_\perp$ for an amorphous 125 Å thick Nb$_{0.15}$Si$_{0.85}$ film at $T = 0.224$ K$\simeq T_c$, $T = 0.201$ K, and $T = 0.181$ K derived from Aubin et al.\textsuperscript{20} The solid line is Eq. (11) in terms of $H_\perp \sigma(T_c, H_\perp) = \sigma_n H_\parallel + f_\perp$ with $z = 2$, $\sigma_n = 0.70$ kΩ$^{-1}$ and $f_\perp = 0.29$ kΩ. The inset shows $\sigma_\parallel(T)$ vs. $H_\perp$. The dashed line is Eq. (11) with $\sigma_n = 0.70$ kΩ$^{-1}$ and $f_\perp = 0.29$ kΩ$^{-1}$kOe

![Figure 5](image5.png)

**FIG. 5:** $\sigma_\parallel(T_c)$ vs. $H_\parallel$ for a LaAlO$_3$/SrTiO$_3$ interface with $T_c \simeq 0.21$ K derived from Reyren et al.\textsuperscript{16} The solid line is Eq. (11) with $\sigma_n = 1.94 \times 10^{-3}$Ω and $f_\parallel = 1.59 \times 10^{-3}$ ΩmT. The dot marks $\mu_0 H_\parallel^* = 3.8$ mT.

Furthermore, in analogy to the amorphous Nb$_{0.15}$Si$_{0.85}$ film (see Figs. 2 and 3) $\sigma_\parallel(T_c)$ vs. $H_\perp$ does not diverge in the zero field limit. This behavior was traced back to a standard finite size effect, presumably attributable to a finite lateral extent $\tilde{L}$ of the homogeneous domains.\textsuperscript{17} To substantiate this interpretation we
FIG. 6: $\sigma (T_c)$ vs $H^\parallel$ for a LaAlO$_3$/SrTiO$_3$ interface with $T_c \approx 0.21 \text{ K}$ derived from Reyren et al.\textsuperscript{15} The solid line is Eq. (11) with $\lambda_0 = 2.40 \times 10^{-3}$ $\Omega$ and $f_\perp = 153.42$ $\text{fm}^2$. The dot marks $\mu_0 H^\parallel_0 = 195$ $\text{mT}$.

invoke Eq. (15) and the respective estimates for $H^\parallel_0$ and $H^\perp_0$, yielding with $a = 4.8$ and $d \approx 67\text{Å}$, $\tilde{L} \approx 3.4 \times 10^{-5} \text{ cm}$ ($\mu_0 H^\parallel_0 = 3.8$ $\text{mT}$) and $\tilde{L} \approx 4.9 \times 10^{-5} \text{ cm}$ ($\mu_0 H^\perp_0 = 195$ $\text{mT}$), compared to the lateral dimensions $W \times L = 0.02 \text{ cm} \times 0.01 \text{ cm}$ of the superconducting interface. Invoking the Kosterlitz-Nelson relation $\lambda_2D (T_c) = \lambda^2 (T_c)$ yields $\Phi_0 (32\pi^2 k_BT_c)$ we obtain $\lambda_2D (T_c) \approx 4.8 \text{ cm}$ for $T_c = 0.21 \text{ K}$, whereupon $\lambda_2D >> \min [W, L]$ is well satisfied for the LaAlO$_3$/SrTiO$_3$ interface. Furthermore, because $\lambda_2D (T_c)$ is also large compared to $\tilde{L}$, the zero field limiting length appears to be set by the lateral extent of the homogenous domains. In any case due to the uncovered limiting length, not attributable a finite magnetic “screening length” $\lambda_2D$, it becomes possible for free vortices to form below $T_c$ which in turn precludes a true phase transition.

In summary, we presented and illustrated a simple promising tool to extract from the magnetic field dependence of the conductivity at $T_c$ the dynamical critical exponent $z$, the thickness $d$ of thin superconducting films and interfaces, and the limiting length $\tilde{L}$, giving rise to rounded BKT- and QSI transitions even in zero field. In fact, in the quantum case is the divergence of the zero temperature correlation length $\xi (T = 0) = \xi_0 \delta^{-\nu}$ prevented because it cannot beyond $\tilde{L}$ and with that is the attainable tuning regime bounded by $\delta > (\xi_0 / \tilde{L})^{1/\nu}$.

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