Anchor-based Nearest Class Mean Loss for Convolutional Neural Networks
Fusheng Hao1,2, Jun Cheng3, Lei Wang3, Xinchao Wang3, Jianzhong Cao1, Xiping Hu3, Dapeng Tao3
1 Xi’an Institute of Optics and Precision Mechanics, CAS, 2 University of Chinese Academy of Sciences
3 Shenzhen Key Laboratory of Virtual Reality and Human Interaction Technology, Shenzhen Institutes of Advanced Technology, CAS
4 Department of Computer Science, Stevens Institute of Technology
haofusheng@opt.cn, {jun.cheng, lei.wang1, xp.hu}@siat.ac.cn, xinchao.wang@stevens.edu
cjz@opt.ac.cn, dapeng.tao@gmail.com

Abstract

Discriminative features are critical for machine learning applications. Most existing deep learning approaches, however, rely on convolutional neural networks (CNNs) for learning features, whose discriminant power is not explicitly enforced. In this paper, we propose a novel approach to train deep CNNs by imposing the intra-class compactness and the inter-class separability, so as to enhance the learned features’ discriminant power. To this end, we introduce anchors, which are predefined vectors regarded as the centers for each class and fixed during training. Discriminative features are obtained by constraining the deep CNNs to map training samples to the corresponding anchors as close as possible. We propose two principles to select the anchors, and measure the proximity of two points using the Euclidean and cosine distance metric functions, which results in two novel loss functions. These loss functions require no sample pairs or triplets and can be efficiently optimized by batch stochastic gradient descent. We test the proposed method on three benchmark image classification datasets and demonstrate its promising results.

1 Introduction

Deep learning methods [Krizhevsky et al., 2012; Simonyan and Zisserman, 2014; Szegedy et al., 2015] have demonstrated superior results in many computer vision tasks, including image recognition even on the very-large-scale benchmark such as ImageNet [Deng et al., 2009]. Most deep learning methods exploit the strong representation capacity of convolutional neural networks (CNNs) and are often supervised by the softmax loss (SoftMax). This loss function, however, does not explicitly drive CNNs to learn discriminative features.

One recent trend to make better use of the strong representation capacity of CNNs is to explicitly encode the discriminant into the model. For example, Hadsell et al. proposed a Siamese network with a contrastive loss for dimensionality reduction [Hadsell et al., 2006], which learned a mapping such that similar points in the input space were mapped to nearby points in the low dimensional manifold. Schroff et al. extended the contrastive loss to a triplet loss by introducing a third anchor point which was sampled from the training set, and minimized the distance between an anchor and a positive of the same identity and meanwhile maximized the distance between the anchor and a negative of different identity [Schroff et al., 2015]. However, training samples needed to be carefully selected, as the number of training sample grows at the rate of $O(N^2)$, where $N$ is the number of training samples.

In this paper, we propose a novel approach to train deep CNNs by enforcing unequivocal intra-class compactness and inter-class separability. In contrast to prior approaches, ours explicitly encourages CNNs to learn discriminative features without relying on sample pairs or triplets, and can be efficiently optimized with batch stochastic gradient descent (SGD). This is achieved by introducing to our objective function the notion of anchors, based on which a Nearest Class Mean (NCM) loss is raised. Here, anchors refer to predefined vectors that are fixed during training. The anchors are regarded as class centers, each of which is assigned to a particular class. We foster the learned features to gather around the anchors of the same label, and meanwhile expect the anchors to be as separate as possible. Specifically, we derive anchors using the following two principals:

- The anchors have the unit norm, i.e., they are sampled from a unit hypersphere;
- The angle between any two anchors is large.

Our objective function, which will be discussed in detail in Section 3, involves computing the distance measure between the two points, for which the only requirement is it being differentiable. We consider two common and simple distances, Euclidean and cosine distances, which further leads to two novel loss functions. One toy example is shown in Figure 1 highlighting the effect of anchor-based cosine and Euclidean distances on the random deep features. Note that, the proposed method is not limited to the Euclidean or cosine distance and any differentiable distance metric functions can be readily adopted.

The approach of Wen et al. [Wen et al., 2016] also relies on center-based Euclidean distance to learn discriminant features. Unlike our method, it performed updates of class centers based on mini-batch. However, the number of training samples for each class in a training batch is random and
thus the centers of any two different class may be very close
to each other or have very large difference in norm. These
two factors result in large perturbations on the class centers
and further lead to slow convergence. Furthermore, it alone
cannot learn meaningful features due to the trivial solutions
like all deep features being mapped to the origin. By contrast,
our method fixes the anchors and avoids such pertur-
bations, yielding discriminative features as well as promis-
ting results. Also, our method requires no tunable hyper-
parameters, making it easy to be deployed in practice. The
approach of Liu et al. [Liu, 2016], on the other hand,
introduces a large-margin softmax loss (L-Softmax), which
requires the angle between different classes to exceed a mar-
gin. It requires a huge amount of hyper-parameters that are
difficult to tune. Also, its backward propagation becomes very
demanding when the angle margin is large as the triangle
formula must be expanded. Our method however directly
constrains the angle between the deep features and anchors,
which avoids the tricky strategy of updating the deep features
to accelerate convergence. Further, our anchors are selected
from the deep feature space to ensure minimal inter-class an-
gular margin between classes.

Our contribution is therefore, introducing the novel anchor-
based loss functions into the deep neural works, which does
not require sample pairs or triplets and can be efficiently op-
timized with batch SGD. It is free of tunable parameters and
thus very easy to be deployed. Our method leads to very
promising results on several standard benchmarks, demonstrat-
ing the effectiveness of the proposed method.

2 Related Works

Previous works that focus on encouraging CNNs to learn dis-
criminative features fall into two main categories: the ones
relying on distance metric learning and those adopting the
class centers. Distance metric learning learns a mapping for
the input space of data from a given collection of pair of sim-
ilar/dissimilar points that preserves the distance relationships
in the training data [Liu, 2006]. A strong feature representa-
tion is critical for distance metric learning performance. In-
spired by the success of the contrastive loss [Hadsell et al.,
2006], several works have adopted this loss function to ad-
dress related problems. For example, Sun et al. combined the
contrastive loss with softmax to reduce intra-personal vari-
ations while enlarging inter-personal differences, achieving
impressive face verification accuracy [Sun et al., 2014]. Very
deep CNNs result in superb representation capacity, further
improving face verification accuracy [Sun et al., 2015]. Li et
al. proposed a more general contrastive loss function to ob-
tain strong feature representations for classification [Li et al.,
2017].

The other category of algorithms adopt the notion of class
center, defined as the mean feature vector of its elements.
Weinberger et al. proposed a large-margin nearest neigh-
bror approach, its goal being that the k-nearest neighbors al-
ways belonged to the same class while examples from dif-
ferent classes were separated by a large margin [Weinberger
and Saul, 2009]. Further, some subsequent works utilized the
strong representation capacity of CNNs and introduced loss
functions to constrain CNNs to map the deep features from
the same identity as closely as possible and from the different
identity as far as possible. For example, Liu et al. extended
the center loss [Wen et al., 2016] and proposed a congen-
erous cosine loss, which cooperatively increased similarity
within classes and enlarged variation across classes [Liu
et al., 2017]. Snell et al. adopted the class center concept for
few-shot learning [Snell et al., 2017].

Our method differs from the existing methods in that, we
introduce the concept of anchors, which are regarded as class
centers and fixed during training, and propose two novel loss
functions that directly constrain CNNs to map each sample
to the corresponding class center as close as possible, where
these centers are as isolated as possible.

3 The Proposed Method

Firstly, we briefly review the basic nearest class mean classi-
fier and describe its neural version, and then analyze the lim-
itations of the neural version of the nearest class mean classi-
fier and detail our proposed anchor-based nearest class mean
loss. Finally, we explore its relationship to existing methods.

3.1 Nearest Class Mean Classifier

Given $N$ training images from $C$ classes $\{(x_i, y_i)\}_{i=1}^N$, where
$y_i$ is the ground-truth label of image $x_i$, the trivial nearest
class mean classifier represents classes by their mean feature

![Diagram](https://via.placeholder.com/150)

Figure 1: One toy example of the anchor-based NCM loss. The deep features of the two classes are denoted by purple squares and blue
circles respectively. The corresponding anchors for each class are represented by the Hexagonal star of the same color. The deep features are
constrained to be as close as possible to the anchors, measured by the cosine or Euclidean distance.
vector of its elements. A test image \( x \) is assigned to the class \( c^* \in \{1, \ldots, C\} \) with the closest mean, which can be formulated in the following form:

\[
e^* = \arg \min_{c \in \{1, \ldots, C\}} M(x, \mu_c) \tag{1}
\]

\[
\mu_c = \frac{1}{N_c} \sum_{i:y_i=c} x_i \tag{2}
\]

where \( M \) is the distance metric between the test image \( x \) and the class mean \( \mu_c \), and \( N_c \) is the number of training images in class \( c \).

CNNs have a strong feature representation capacity and can be regarded as a feature extractor \( f_W : x \rightarrow \mathbb{R}^d \), where \( W \) is learnable parameters of CNNs and \( d \) is the dimension of the output deep features. By replacing the image \( x \) or \( x_i \) in equations (1) and (2) with \( f_W(x) \) or \( f_W(x_i) \), we obtain the neural version of the nearest class mean classifier.

### 3.2 Anchor-based Nearest Class Mean Loss

The standard optimization algorithm for training CNNs is SGD or its variants, such as Adam [Kingma and Ba, 2015]. In each iteration, a batch of training images are sampled randomly from the training sets to update the learnable parameters of CNNs. Therefore, the number of training images for a specific category in a batch may be any number from zero to batch size. This raises an issue that how to update the class mean vectors with limited and uncertain number of training samples. We need to carefully design an update rule for the mean vector of each class, especially when \( N_c (\forall c \in \{1, \ldots, C\}) \) is much larger than the batch size.

In order to avoid a tricky updating rule for the class mean vector, we replace the class mean vector with a set of anchors. These anchors are a set of predefined vectors \( \{a_c\}_{c=1}^{C} (a_c \in \mathbb{R}^d) \), each of which is randomly assigned to one category and regarded as the class center for that class. An image \( x \) is assigned to the class \( c^* \in \{1, \ldots, C\} \) with the closest mean to the corresponding anchor:

\[
e^* = \arg \min_{c \in \{1, \ldots, C\}} M(f_W(x), a_c) \tag{3}
\]

Further, to interpret the anchor-based nearest class mean loss from the perspective of probability, we adopt softmax function to represent the categorical distribution of \( x \) over \( C \) classes. Given an image \( x \), the probability for a class \( c \) is defined as:

\[
p(c|x) = \frac{\exp(-M(f_W(x), a_c))}{\sum_{c'=1}^{C} \exp(-M(f_W(x), a_{c'}))} \tag{4}
\]

To train CNNs with the proposed anchor-based nearest class mean loss, we directly minimize the negative log-likelihood of the correct predictions of the training images:

\[
L = -\frac{1}{N} \sum_{i=1}^{N} \sum_{c=1}^{C} I(y_i = c) \ln(p(y_i|x_i)) \tag{5}
\]

where \( I(y_i = c) \) is the indicator function that equals one if its arguments is true and zero otherwise. For optimization, we need the derivative of loss function (5) with respect to \( W \):

\[
\nabla_W L = -\frac{1}{N} \sum_{i=1}^{N} \sum_{c=1}^{C} I(y_i = c) \{ \nabla_W (-M(f_W(x), a_c)) - \sum_{c'=1}^{C} \exp(-M(f_W(x), a_{c'})) \nabla_W (-M(f_W(x), a_{c'})) \} \tag{6}
\]

Considering that CNNs can be viewed as a potent universal function approximator, we choose two simple distance metric functions: Euclidean and cosine, although any differentiable function approximator, we choose two simple distance metric functions: Euclidean and cosine, although any differentiable function approximator.
Given training set \((x_i, y_i)_{i=1}^N\), CNNs are constrained to map each sample to the corresponding anchor as close as possible. Ideally, all deep features are mapped to the corresponding anchors, which inspires us to design a rational way to select the anchor set to ensure inter-class dispersion and enhance intra-class compactness. To preserve the deep feature balance, we expect the norms of these anchors to be equal. Further, these anchors should be as dispersed as possible to maximize the inter-class dispersion and minimize intra-class compactness of deep features. These two principles can be mathematically formulated as:

\[
\|a_c\|_2 = 1, \forall c \in \{1, \ldots, C\} \quad (9)
\]

\[
\frac{a_c \cdot a_{c'}}{\|a_c\|_2 \|a_{c'}\|_2} \geq \cos(\theta_M), \forall c \neq c' \quad (10)
\]

where \(\theta_M\) is the predefined angle margin. We use polar coordinates to represent points on the hypersphere, and obtain the anchors by meshgridding the angle coordinates and computing the coordinate values. For example, in the two-dimensional space, the points on the unit circle are represented by \((\cos \theta, \sin \theta)\); by meshgridding \(2\pi\) with interval \(\frac{2\pi}{C}\) and computing the coordinate values, we obtain \(C\) anchors in this space. We summarize the steps for our proposed approach in Algorithm 1.

3.3 Relation to Existing Methods

Relation to Gaussian mixture model. If we choose the Euclidean distance metric for anchor-based nearest class mean loss, the definition in (4) may also be interpreted as giving the posterior probabilities of a generative model where

\[
p(c|x) = N(x_i; a_c, \Sigma) \]

is a Gaussian with mean \(a_c\), and covariance matrix \(\Sigma\) [Mensink et al., 2013]. The class probabilities \(p(c)\) are set to be uniform over all classes. Our approach differs significantly from the traditional Gaussian mixture model [Bishop, 2006] in that the mean vectors are predefined and independent of the actual feature representations. We utilize the strong representation capacity of CNNs and constrain CNNs to simulate the simplified Gaussian mixture model.

Relation to SoftMax. The proposed anchor-based nearest class mean loss differs from SoftMax in two aspects: 1) the deep feature dimensions for anchor-based nearest class mean loss are not limited, while the softmax loss requires the deep feature dimensions to be equal to the class number; 2) with the same dimension, the proposed anchor-based nearest class mean loss constrains CNNs to pull deep features of the same identity to the corresponding anchors, while the softmax loss constrains CNNs to pull the elements of deep features corresponding to classes far greater than the others.

Relation to L-Softmax. L-Softmax is defined as the combination of a softmax function and the last fully connected layer with the requirement that the angle between different classes exceeds a margin. This additional condition complicates the backward propagation of L-Softmax, especially when the angle margin is large. The proposed C-NCM loss function simplifies the backward propagation by pulling the deep features to the fixed anchors. Further, the anchors are selected to ensure minimal inter-class angular margin between classes.
4 Experiments

In this section, we present our experimental validations. We first introduce the datasets we used and then show the experiment setup. We finally show comparative results and provide discussions.

4.1 Datasets

The proposed E-NCM and C-NCM loss functions are evaluated in the context of the image classification task. We conduct experiment using three standard benchmark datasets: MNIST [Lecun et al., 1998], CIFAR10 [Krizhevsky, 2009], and CIFAR100 [Krizhevsky, 2009]. MNIST consists of a training set of 60000 28 × 28 gray-scale handwritten digits of 10 classes and a test set of 10000 samples. CIFAR10 consists of 60000 32 × 32 color images from 10 classes, of which 50000 are used for training and 10000 for test. CIFAR100 consists of 60000 32 × 32 color images from 100 classes. Each class of CIFAR100 has only 500 images for training and 100 images for test. If not specified, no data augmentation is applied and the only preprocessing is a global normalization to zero mean and unit variance. In addition, we also test the performance of both loss functions on the CIFAR1 with augmentation in [Liu et al., 2016]. Considering the strong representation capacity of CNNs, the deep features may be pulled to the corresponding anchors easily. Therefore, we also introduce dropout layers in the net architectures to alleviate the issue of overfitting. Specifically, dropout layer is added after the first two max pooling layers. For convolution layers, the kernel size is 3 × 3 and 1 padding (if not specified) to keep the feature map size unchanged. The details of the net architecture for each dataset are described in Table 1.

4.2 Experimental Setup

Net architectures. SoftMax is one of the most commonly used loss function for training CNNs, which is regarded as the baseline in all experiments. We also compare the proposed E-NCM and C-NCM loss functions with Hinge Loss and L-Softmax. For fair comparisons, we use the net architectures as described in [Liu et al., 2016]. Considering the strong representation capacity of CNNs, the deep features may be pulled to the corresponding anchors easily. Therefore, we also introduce dropout layers in the net architectures to alleviate the issue of overfitting. Specifically, dropout layer is added after the first two max pooling layers. For convolution layers, the kernel size is 3 × 3 and 1 padding (if not specified) to keep the feature map size unchanged. The details of the net architecture for each dataset are described in Table 1.

Parameter settings. We implement E-NCM and C-NCM with the PyTorch deep learning framework. By stacking on the top of CNNs, the two proposed loss functions constrain CNNs to map each training sample to the corresponding anchor. We adopt the standard batch SGD for optimization. Following the settings in L-Softmax, we use a weight decay of 0.0005 and momentum of 0.9 and the batch size is 256. As we pull deep features of the same identity to the fixed anchor, more training iterations may be helpful to further improve the performance. To this end, we start with a learning rate of 0.1, divide it by 10 when the training loss plateaus. Our model requires approximately 90 epochs for the optimal performance. In addition, we extensively evaluate the effect of the dropout ratio (with $p = 0, p = 0.1, p = 0.25$ and $p = 0.5$) on the performance of the proposed E-NCM and C-NCM loss functions. If not specified, we take 10 or 100 standard orthogonal bases from the deep feature space.

4.3 Results

Visualization on MNIST. For the convenience of visualization, we set the dimension of the output deep features to 2. As SoftMax requires the dimension of input features to be equal to the number of classes, another fully connected layer with 10 outputs is added after the deep features. The proposed two loss functions are not dependent on the input features dimension. The ten anchors are sampled from the unit circle with equal angle between adjacent anchors. The net architecture for this experiment is illustrated in Table 1. The test accuracy for SoftMax, C-NCM and E-NCM are 99.32%, 99.00% and 99.53% respectively. In Figure 2 we can observe that C-NCM constrains the training samples tighter than SoftMax, which demonstrates the feasibility of pulling deep features to the predefined anchors. The vast majority of training samples are mapped close to the anchors by E-NCM. It is not straightforward to notice that, as most of the sample points reside at the same location. There are a very small amount of outliers in the test phase, some of which are far from the corresponding anchors in both E-NCM and C-NCM. One possible factor is that the training samples for a particular class cannot cover the whole space of the particular class. Based on this observation, we introduce dropout layers in the net architectures to expand sample space in the following experiments. Note that the scales of deep features between different loss functions

| Layer | MNIST (for Figure 2) | MNIST | CIFAR10/CIFAR10+ | CIFAR100 |
|-------|----------------------|-------|-----------------|----------|
| Conv0.x | [3 × 3, 64]×1 | [3 × 3, 64]×1 | [3 × 3, 64]×1 | [3 × 3, 96]×1 |
| Conv1.x | [3 × 3, 64]×3 | [3 × 3, 64]×3 | [3 × 3, 64]×4 | [3 × 3, 96]×4 |
| Pool1 | 2 × 2 Max, Stride 2 | dropout ratio $p$ | | |
| Dropout1 | | dropout ratio $p$ | | |
| Conv2.x | [3 × 3, 64]×3 | [3 × 3, 64]×3 | [3 × 3, 96]×4 | [3 × 3, 192]×4 |
| Pool2 | 2 × 2 Max, Stride 2 | | | |
| Dropout2 | | dropout ratio $p$ | | |
| Conv3.x | [3 × 3, 64]×3 | [3 × 3, 64]×3 | [3 × 3, 128]×4 | [3 × 3, 384]×4 |
| Pool3 | 2 × 2 Max, Stride 2 | | | |
| FC | 2 | 256 | 256 | 512 |
Table 2: Recognition error rate (%) on MNIST, CIFAR10/CIFAR10+ and CIFAR100 datasets. + denotes the performance with data augmentation.

| Method       | MNIST   | CIFAR10 | CIFAR10+ | CIFAR100 |
|--------------|---------|---------|----------|----------|
| Hinge Loss [Liu et al., 2016] | 0.47    | 9.91    | 6.96     | 32.90    |
| L-softmax (m=2) [Liu et al., 2016] | 0.32    | 7.73    | 6.01     | 29.95    |
| L-softmax (m=3) [Liu et al., 2016] | 0.31    | 7.66    | 5.94     | 29.87    |
| L-softmax (m=4) [Liu et al., 2016] | 0.31    | 7.58    | 5.92     | 29.53    |
| C-NCM (p=0.1) | 0.25    | 8.78    | 5.98     | 30.86    |
| E-NCM (p=0.1) | 0.26    | 8.89    | 5.67     | 28.14    |

Table 3: The effect of dropout layers and data augmentation on the performance of SoftMax, C-NCM and E-NCM. Recognition error rate (%) is reported on CIFAR10, CIFAR10+ and CIFAR100 datasets. + denotes the performance with data augmentation.

| Dataset | Method       | Dropout ratio |
|---------|--------------|---------------|
|         | p=0 | p=0.1 | p=0.25 | p=0.5 |
| CIFAR10 | SoftMax | 10.50 | 10.22 | 9.81 | 10.13 |
|         | C-NCM | 9.98 | 8.78 | 8.97 | 8.52 |
|         | E-NCM | 9.94 | 8.89 | 8.43 | 8.94 |
| CIFAR10+| SoftMax | 6.60 | 6.61 | 6.75 | 7.12 |
|         | C-NCM | 6.10 | 5.98 | 6.04 | 6.46 |
|         | E-NCM | 5.90 | 5.67 | 5.77 | 6.32 |
| CIFAR100| SoftMax | 35.37 | 37.26 | 37.26 | 40.25 |
|         | C-NCM | 31.92 | 30.86 | 30.57 | 31.31 |
|         | E-NCM | 29.41 | 28.14 | 28.11 | 28.68 |

Ablation study. We constrain CNNs to map limited training samples to the corresponding anchors as close as possible, which may result in overfitting and weak generalization. Thus, we introduce data augmentation and dropout to alleviate this issue. Without data augmentation and dropout, C-NCM and E-NCM already achieve better performance than the baseline SoftMax on CIFAR10 and CIFAR100 as shown in Table 2. The proposed two loss functions benefit even more from the data augmentation and dropout than SoftMax. E-NCM achieves slightly better performance than C-NCM on CIFAR10 and CIFAR10+, but roughly 2% lower error rate than C-NCM on CIFAR100. Note that, E-NCM not only enforces the angle between training samples and the corresponding anchors as small as possible, but also restricts the norm of deep features to be equal to the anchors. This observation may imply that with limited training samples, stricter constraints could lead to better performance.

Comparison with the-state-of-the-art. We also compare the proposed two loss functions with the-state-of-the-art method L-Softmax. C-NCM explicitly increases the angle margin between different categories, which shares the same goal with L-Softmax. E-NCM imposes stronger constraints on CNNs than C-NCM does. The E-NCM loss function enforces CNNs to map training samples to fall on the corresponding anchors exactly. C-NCM achieves an improvement of 0.06% over L-Softmax on MNIST as shown in Table 2. Both C-NCM and E-NCM achieve worse performance than L-Softmax on CIFAR10. However, with data augmentation and dropout, E-NCM achieves the best performance with an improvement of 0.25% over L-Softmax while the C-NCM achieves the comparable performance with L-Softmax. The proposed E-NCM loss function achieves the lowest error rate of 28.14%, which is 1.39% lower than that of L-Softmax on CIFAR100. For L-Softmax, the hyper-parameters, including base, γ and λ need to be set empirically. We also conduct extensive experiments using L-Softmax under many parameter configurations with the net architectures in Table 1. With dropout layers in the architectures, we do not get better results than the ones reported in the original paper. We therefore take directly the results reported in the paper. Note that, the proposed two loss functions have no hyper-parameters to be specified in advance.

5 Conclusions

In this paper, we introduce the notion of anchors, which are predefined vectors and fixed during training, into the objective function of deep neural networks. We propose a novel way to utilize the strong representation capability of CNNs, which is to constrain CNNs to map training samples to corresponding anchors as close as possible, and meanwhile isolate the anchors as much as possible. We apply two commonly-used distance metric functions to measure the distance between deep feature of a training sample and the corresponding anchor, which leads to two novel loss functions: E-NCM and C-NCM. Both loss functions require no hyper-parameters and are easy to be implemented. Extensive experiments are conducted on three benchmark datasets. The experimental results demonstrate the feasibility of enforcing CNNs to map training samples to the corresponding anchors, as well as the effectiveness of the proposed loss functions.
References

[Bishop, 2006] Christopher Bishop. Pattern Recognition and Machine Learning. Springer, 2006.

[Deng et al., 2009] Jia Deng, Wei Dong, Richard Socher, Li-Jia Li, Kai Li, and Fei-Fei Li. Imagenet: A large-scale hierarchical image database. In IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pages 248–255, June 2009.

[Hadsell et al., 2006] Raia Hadsell, Sumit Chopra, and Yann Lecun. Dimensionality reduction by learning an invariant mapping. In IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR), pages 1735–1742, June 2006.

[Kingma and Ba, 2015] Diederik P. Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In International Conference for Learning Representations (ICLR), 2015.

[Krizhevsky et al., 2012] Alex Krizhevsky, Sutskever Ilya, and Geoffrey E. Hinton. Imagenet classification with deep convolutional neural networks. In Advances in Neural Information Processing Systems (NIPS), pages 1097–1105, 2012.

[Krizhevsky, 2009] Alex Krizhevsky. Learning Multiple Layers of Features from Tiny Images. Technical Report, 2009.

[Lecun et al., 1998] Yann Lecun, Léon Bottou, Yoshua Bengio, and Patrick Haffner. Gradient-based learning applied to document recognition. Proceedings of the IEEE, 86(11):2278–2324, November 1998.

[Li et al., 2017] Ya Li, Xinmei Tian, Xu Shen, and Dacheng Tao. Classification and representation joint learning via deep networks. In Proceedings of the Twenty-Sixth International Joint Conference on Artificial Intelligence (IJCAI), pages 2215–2221, 2017.

[Liu et al., 2016] Weiyang Liu, Yandong Wen, Zhiding Yu, and Meng Yang. Large-margin softmax loss for convolutional neural networks. In Proceedings of the 33rd International Conference on International Conference on Machine Learning (ICML), pages 507–516, 2016.

[Liu et al., 2017] Yu Liu, Hongyang Li, and Xiaogang Wang. Learning Deep Features via Congenorous Cosine Loss for Person Recognition. arXiv:1702.06890, 2017.

[Liu, 2006] Yang Liu. Distance Metric Learning: A Comprehensive Survey. Michigan State University, 2006.

[Mensink et al., 2013] Thomas Mensink, Jakob Verbeek, Florent Perronnin, and Gabriela Csurka. Distance-based image classification: Generalizing to new classes at near-zero cost. IEEE Transactions on Pattern Analysis and Machine Intelligence, 35(11):2624–2637, November 2013.

[Schroff et al., 2015] Florian Schroff, Dmitry Kalenichenko, and James Philbin. Facenet: A unified embedding for face recognition and clustering. In IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pages 815–823, June 2015.

[Simonyan and Zisserman, 2014] Karen Simonyan and Andrew Zisserman. Very Deep Convolutional Networks for Large-Scale Image Recognition. arXiv:1409.1556, 2014.

[Snell et al., 2017] Jake Snell, Kevin Swersky, and Richard S. Zemel. Prototypical Networks for Few-shot Learning. arXiv:1703.05175, 2017.

[Sun et al., 2014] Yi Sun, Yuheng Chen, Xiaogang Wang, and Xiaoou Tang. Deep learning face representation by joint identification-verification. In Advances in Neural Information Processing Systems (NIPS), pages 1988–1996, 2014.

[Sun et al., 2015] Yi Sun, Ding Liang, Xiaogang Wang, and Xiaoou Tang. DeepID3: Face Recognition with Very Deep Neural Networks. arXiv:1502.00873, 2015.

[Szegedy et al., 2015] Christian Szegedy, Wei Liu, Yangqing Jia, Pierre Sermanet, Scott Reed, Dragomir Anguelov, Dumitru Erhan, Vincent Vanhoucke, and Andrew Rabinovich. Going deeper with convolutions. In IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pages 1–9, June 2015.

[Weinberger and Saul, 2009] Kilian Q. Weinberger and Lawrence K. Saul. Distance metric learning for large margin nearest neighbor classification. Journal of Machine Learning Research, 10:207–244, 2009.

[Wen et al., 2016] Yandong Wen, Kaipeng Zhang, Zhifeng Li, and Yu Qiao. A discriminative feature learning approach for deep face recognition. In European Conference on Computer Vision (ECCV), pages 499–515, October 2016.

[Sun et al., 2017] Sergej Schulter, Oleg Shapovalov, and Mario Fritz. DeepSLAM: Real-time dense 3D mapping and SLAM with a single camera. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pages 1066–1075, July 2017.

[Bradski, 2000] Gary Bradski. The OpenCV Library. Dr. Dobb’s Journal, 25(11):12–21, November 2000.