Multi-period Voltage Optimization of Active Distribution Network with Distributed Power

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Abstract. A two-stage voltage optimization model is established for the distribution network which includes distributed power and takes into the daily load curve and the photovoltaic generation daily curve. In the first stage, the minimum objective function has the lowest cost of daily network losses and the minimum voltage deviation. Meanwhile, an improved NSGA2 algorithm is used to obtain a daily motions’ combination of voltage regulators and capacitor banks. In the second stage, in order to minimize the daily operation costs of the equipments, and integrate the goals of the first stage, multi-attribute fuzzy decision is used to obtain the optimal combination of the regulators and capacitor banks within 24h. Finally, the improved IEEE33 example simulations verify the effectiveness of the proposed voltage optimization method.

Introduction

At present, many countries around the world have started to implement new energy development strategies. Under such a circumstance, distributed generation (DG) such as photovoltaic and wind power has been vigorously developed and DG has achieved high permeability. Traditional distribution networks do not consider the existence of DG when planning and designing. The uncertainty of DG output and high-permeability access will inevitably affect the power quality in the distribution network and the operation indicators from related equipment [1]. Therefore, it is of great significance to deeply study the voltage optimization of active distribution network with DG to reduce the network losses and improve the power quality of the power grid.

The voltage and reactive power control in the distribution system are usually accomplished by adjusting the taps of the on-load tap-changer and the step voltage regulator (SVR), switching the capacitor banks and controlling the output of the DG [2]. The mathematical nature of the optimization problem is a nonlinear mixed integer programming (MINLP) problem [3].

To solve the problem, the literature [4] adopts a modified simulated annealing algorithm to optimize the voltage in the distribution network with distributed power supply, but it does not consider the daily load curves and the daily operation frequency limit of on-load tap-changer. The literature [5] proposed the coordination strategy of voltage and reactive power in the distribution network considering the load characteristics on the day and the day before. However, it does not consider the different rules of feeder load variation.

In this paper, a two-stage multi-objective voltage optimization on distribution network is established with the goal of reducing active network losses, reducing the number of equipment actions and improving the quality of voltage, considering the existence of DG and the typical daily curves for photovoltaic power generation and load utilization. The voltage optimization model uses the improved NSGA2 algorithm and multi-attribute fuzzy decision to solve the two phases respectively. Finally, the simulation of the IEEE33-bus system is verified. The simulation verifies the correctness and validity of the model.
Distribution Network Multi-objective Optimization Model

The First Stage Optimization Model

The two-stage optimization method is to split the objective function into two stages to solve the problem. The two stages are in a progressive relationship. In the first stage, the goal is to reduce active losses and voltage deviations by adjusting the SVR taps and switching the capacitor banks. The results of the first stage are to obtain multiple optimization solutions for the coordination of the voltage regulators and the capacitor banks within 24 hours.

**Objective Function.** Taking different electricity prices at different time periods into account, this paper uses the minimum active losses costs over a 24h period as the first objective function ($f_1$). And the objective function is shown in formula (1):

$$
\min f_1 = \min \sum_{t=1}^{24} \sum_{k=1}^{N} C(t) G_k (U_i(t)^2 + U_j(t)^2 - 2U_i(t)U_j(t)\cos \theta_{ij}(t))
$$

(1)

Where $t$ is the current time, $N$ is the total number of branches, $C(t)$ is the electricity price during $t$ period. $G_k$ is the kth branch, $U_i(t)$, $U_j(t)$ are the voltage standard values of node $i$ and node $j$ at time $t$ respectively, $\theta_{ij}(t)$ is the voltage phase difference between node $i$ and node $j$ at time $t$.

The minimum voltage deviation that reflects the power quality is the second objective function ($f_2$) at the first stage. The total voltage deviation for 24 hours is shown in formula (2):

$$
\min f_2 = \min \sum_{t=1}^{24} \sum_{i=1}^{n} |U_i(t) - U_{i,ref}|
$$

(2)

Where, $n$ is the number of nodes. $U_{i,ref}$ is the voltage reference value of node $i$, which usually takes 1.0 pu. The other physical quantity means the same as formula (1).

**Restrictions.** Equality constraints
Each node's active power and reactive power balance equation constraints are shown in equation (3).

$$
P_{DG_i} - P_{Li} - U_i \sum_{j=1}^{n} U_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0
$$

$$
Q_{DG_i} + Q_{Ci} - U_i \sum_{j=1}^{n} U_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0
$$

(3)

Inequality constraints
(1) Voltage magnitude limits for all nodes

$$
U_{i,\text{min}} \leq U_i(t) \leq U_{i,\text{max}}
$$

(4)

Where, $U_{i,\text{min}}$ and $U_{i,\text{max}}$ are the upper and lower voltage limits of node $i$, and $U_i(t)$ is the voltage of node $i$ at time $t$.

(2) Limits on tap position of SVR

$$
T_{k,\text{min}} \leq T_k(t) \leq T_{k,\text{max}}
$$

(5)

Where, $T_k(t)$ is the regulator taps of branch $k$ at time $t$, and $T_{k,\text{max}}$ and $T_{k,\text{min}}$ are the max and min tap position of regulator on the branch $k$.

(3) Discrete capacity limits on capacitor banks
0 \leq m_i(t)Q_{ci} \leq Q_{cimax} \quad (6)

Where, $m_{ci}(t)$ is the number of capacitors of node $i$ at time $t$, $Q_{ci}$ is the capacity of single capacitor bank of node $i$, $Q_{cimax}$ is the maximum compensation capacity of node $i$ capacitor banks.

(4) Limits of switching operations for regulators and capacitors

$$N_{Tk} \leq N_{Tkmax} \quad N_{ci} \leq N_{cimax} \quad (7)$$

Where, $N_{Tk}$ is the total number of operations per day for regulator of the branch $k$, $N_{Tkmax}$ is the maximum daily operation limit for the regulator of branch $k$, $N_{ci}$ is the total number of operations per day for capacitor banks of node $i$, and $N_{cimax}$ is the maximum daily operation limit for the capacitor banks of node $i$.

The Second Stage Decision Model

The pareto solutions which obtained in the first stage are the optimization space of the second stage. Considering the optimization goal in the first stage, the second stage aims to reduce the operation costs of devices. Finally the optimal decision for the operation of the regulators and capacitor banks is obtained within one day.

The second stage objective function ($f_3$) is

$$\min f_3 = \min \left( \sum_{t=1}^{T_3} \left( \sum_{k=1}^{n_T} C_T [T_k(t) + 1 - T_k(t)] + \sum_{i=1}^{n_c} C_c [m_i(t) + 1 - m_i(t)] \right) \right) \quad (8)$$

Where, $n_T$ is the installation quantity of the voltage regulator, $n_c$ is the installation quantity of the capacitor banks, $C_T$ is the operating cost of the regulator once, and $C_c$ is the operating cost of the capacitor banks switching once.

Model Solving

NSGA2 Algorithm and its Improvement Strategy

The first stage optimization problem is the problem of multi-objective nonlinear integer programming. In order to obtain better overall results on the condition that multiple goals interact or even conflict under multiple constraints, this paper uses the fast non-dominated sorting genetic algorithm (NSGA2) [6] with elite strategy to solve this problem and makes some improvements based on the traditional NSGA2 algorithm.

For the NSGA2 algorithm to solve multi-period multi-target voltage optimization problems, the steps are the same as reference 6 except for the encoding method. The encoding method is as follows:

- Daily operation position encoding form of the regulator on branch $k$:

  $$T_{k1} \rightarrow T_{k2} \rightarrow \ldots \rightarrow T_{kn} \quad (9)$$

- Switching number encoding forms of capacitor of node $i$:

  $$m_{i1} \rightarrow m_{i2} \rightarrow \ldots \rightarrow m_{in} \quad (10)$$

- Chromosome encoding: $[T_{k1} \rightarrow T_{k2} \rightarrow \ldots \rightarrow m_{i1} \rightarrow m_{i2} \rightarrow \ldots]$.

Where, $n_{Tk}1$, $n_{Tk}2$, and $n_{Tk}n$ are the number of periods during which the taps $T_{k1}$, $T_{k2}$ and $T_{kn}$ of regulator $k$ put into operation respectively. $n_{ci}1$, $n_{ci}2$, and $n_{ci}n$ are the number of periods during which the number $m_{i1}$, $m_{i2}$ and $m_{in}$ of the capacitor banks $i$ respectively. The following relationship is satisfied between variables.
\[
\begin{align*}
\begin{cases}
T_{11} + T_{12} + \ldots + T_{k_{10}} = 24 \\
n_{C1} + n_{C2} + \ldots + n_{C_n} = 24 \\
T_{k_{\min}} \leq T_{k_i} \leq T_{k_{\max}} \quad (t = 1, 2, \ldots n) \\
0 \leq m_{k_i} \leq m_{k_{\max}} \quad (t = 1, 2, \ldots n)
\end{cases}
\end{align*}
\] (11)

For the traditional NSGA2 algorithm, it has the disadvantages of slow convergence and easy to fall into precocity. This paper makes the following improvements to the NSGA2 algorithm:

The tabu search (TS) strategy [7] is embedded in the genetic algorithm to avoid invalid repeated searches. At the same time, the genetic algorithm can make up for the disadvantages of the tabu search which is more sensitive to the initial value.

The flow chart of the TS-NSGA2 voltage optimization algorithm is shown in Figure 1.

**Multi-attribute Fuzzy Decision**

According to the pareto solutions which obtained in the first stage, the objective function with the lowest operation cost of equipment is introduced, which takes into account the minimum network losses and voltage deviation. The optimal compromise solution is obtained from the fuzzy decision based on these three objective functions. The fuzzy membership function is defined as follows:

\[
h_i = \begin{cases} 
0 & f_i \leq f_{i_{\min}} \\
1 & f_{i_{\min}} < f_i \leq f_{i_{\max}} \\
\left(\frac{f_{i_{\max}} - f_i}{f_{i_{\max}} - f_{i_{\min}}}\right) & f_{i_{\min}} < f_i < f_{i_{\max}} 
\end{cases}
\] (12)

Where, \( f_i \) is the i-th objective function, \( f_{i_{\max}} \) is the maximum value of the individual objective function \( i \), and \( f_{i_{\min}} \) is the minimum value of the individual objective function \( i \).

Finally, the one with the largest value of the sum of \( h_i \) is taken as the compromised optimal solution.
Case Study

To verify the effectiveness of the proposed optimization strategy, the voltage optimization program was developed on Matlab and Opendss platform.

This article is based on IEEE33 node power distribution system for simulation verification. The system structure diagram is shown in Figure 2, and the system parameters are given in reference [8]. The system is equipped with a voltage regulator in the branch 4-5, the capacity is 2500kVA, the short circuit impedance value is 5%, and the voltage regulation range is -10% -10%, the step length is 0.625%.

Photovoltaic power supply is installed on node 11 and node 20, photovoltaic power supply capacity is 500kW, power factor is 1.0, irradiation is 1kW/m2, maximum power tracking point is 500kW. The capacitors are equipped on the node 6, 24 and 31, each node is equipped with 10 sets of capacitor banks and each capacitor bank has a capacity of 50kVar. The electricity prices for different periods can be found in reference [9]. The operating costs of the regulator and capacitor banks are $C_r=400$ yuan/time, and $C_c=300$ yuan/time respectively. The maximum number of daily operations of the regulator and capacitor banks is limited to 7 times. The daily load curves of distribution feeders are shown in Figure 3, and the daily curve of photovoltaic generation is shown in Figure 4.

![Figure 2. Topology of the improved IEEE33 distribution system.](image)

![Figure 3. Daily load curves of IEEE33 distribution subarea.](image)
Establish IEEE33 node model in Opendss. When voltage regulator and capacitor banks are not installed, the system initial state voltage curves for partial moments are shown in Figure 5. From Figure 5, it can be seen that the voltage deviation is large and there is a limit violation when voltage regulators and capacitor banks are not added.

Based on the model proposed in this paper, the NSGA2 and TS-NSGA2 algorithms were used in the first stage. The convergence criteria adopted is the maximum number of generations. It can be seen from the Figure 6 that the pareto solution of the TS-NSGA2 algorithm is superior to the NSGA2 algorithm in the spatial distribution. The search ability of the TS-NSGA2 algorithm and the search for the global optimal value is better than the NSGA2 algorithm. Table 1 confirms that, too.

| The price of network losses [yuan] | Voltage deviation in 24h [pu] |
|---------------------------------|-----------------------------|
| NSGA2                           | 558.81                      | 6.92                        |
| TS-NSGA2                        | 552.63                      | 6.51                        |
The pareto solution obtained in the first stage is as the space of optimal solution, the objective function with the lowest cost is introduced in the second stage. Through the coordination between the regulator and the existing reactive compensation, the fuzzy decision is used to obtain the optimal target of the integrated equipment, which is shown in Figure 7 and Figure 8 respectively. The optimization results show that the number of daily operations of the capacitor banks and the regulator is within the limit range. The regulator adjusts the tapping position to improve the voltage distribution along the line. The capacitor banks cooperates with it to compensate for the reactive power due to the change of the node voltage, and it plays a role in peak load shifting within 24 hours.

![Figure 7. The states of regulator taps in 24h.](image)

![Figure 8. Switching results of capacitor banks in 24h.](image)

The active power losses before and after optimization is shown in Figure 9. Voltage curves before and after optimization at 8:00 and 12:00 is shown in Figure 10. The optimization results show that the proposed model and algorithm can improve the voltage distribution and quality of the network, and reduce the active losses for the distribution network with distributed power.

![Figure 9. Active power losses before and after optimization.](image)

![Figure 10. Voltage curves before and after optimization at 8:00 and 12:00.](image)
Table 2. Optimization results of improved IEEE33 distribution system.

| Active power losses in 24h [kW] | The price of network losses [yuan] | Voltage deviation in 24h [pu] | The number of limit nodes |
|--------------------------------|-----------------------------------|--------------------------------|--------------------------|
| **Initial** | **After optimization** | **Initial** | **After optimization** | **Initial** | **After optimization** |
| 1090.5 | 746.5 | 815.7 | 552.6 | 19.82 | 6.51 | 15 | 0 |
| Reduction rate | Reduction rate | Reduction rate | - | - |
| 31.5% | 32.2% | 67.1% | - | - |

Summary

In this paper, a two-stage voltage optimization model for distribution network is proposed. It considers the operation constraints in multi-period of equipment, daily curve of photovoltaic generation, and daily load curves of distribution feeder with inconsistent variation rule.

In the first stage of the model, the resources such as voltage regulators and capacitor banks are coordinated. The NSGA2 algorithm embedded in the tabu search strategy is used. The second stage uses the first stage solution as the optimization space. The multi-attribute fuzzy decision is used to obtain the optimal solution, which makes the lowest power losses, the minimum voltage deviation and the lowest operation costs of the equipment in 24h. Then get the best coordination and action strategy for the voltage regulators and capacitor banks in one day.

Simulation results of the IEEE33 example show that the improved NSGA2 algorithm has high solving efficiency and better searching ability. The two-stage optimization model can reliably obtain the optimal solution for the comprehensive effect.

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