Analytical and numerical solutions of the shock tube problem in a channel with a pseudo-perforated wall

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Abstract. An approximate analytical solution is obtained for the shock tube problem in a rectangular channel with an array of rectangular grooves in the lower wall. The analytical solution is based on the approximate quasi-1D shock adiabat for a shock wave that propagates in a channel with periodically located barriers. This problem is also studied numerically. It is found that the approximate analytical solution correctly predicts the propagation velocity of the leading discontinuity and the flow parameters at this discontinuity. The leading gas-dynamic discontinuity is followed by a relaxation zone in which there are long-wave and short-wave flow oscillations. These oscillations are caused by waves arising from the interaction of the flow behind the leading shock wave with the side and bottom walls of the grooves. An approximate analytical solution allows one to obtain the values of the flow parameters at the end of the relaxation zone. These values are in good agreement with the numerical calculation.

1. Introduction

The problem of the propagation of a shock wave in a channel with obstacles is of great interest. This problem is important for the study of various processes in the chemical industry and nuclear power. Attenuation of a shock wave in a channel with obstacles is investigated in problems of industrial safety. A weak shock wave is generated when modern high-speed trains are entering in the tunnel [1]. Obstacles can be created specifically for attenuation of the shock wave or for rapid development of turbulence. To attenuate the shock wave, in addition, channels with perforated or pseudo-perforated walls are used. The attenuation of the shock wave in the channel filled with foams, grids and dust suspensions was investigated in recent decades. An overview of various ways of a shock wave attenuation in a channel is given in [2, 3].

Approximate analytical models for describing the process of attenuation of a shock wave in a channel with perforated (having through holes) walls were developed in [4, 5]. If the hole in the wall is closed from one end or has a closed cavity behind it, then such walls are called pseudo-perforated. The attenuation of a shock wave in a channel with pseudo-perforated walls was studied in [6]. In [7], an oblique interaction of a shock wave with a perforated plate was investigated. The methods of approximate calculation of the velocity of the leading shock wave in perforated channels and in channels with barriers and screens are described in [8, 9, 10, 11]. Obstacles can also arise as a result of instability of the flow, if the walls of the channel are deformable (see, for example, [12]).

The influence of the shape of the obstacles on the attenuation of the shock wave was studied in [13] by numerical simulation. The interaction of a shock wave with porous obstacles of various
shapes was investigated in [14]. In [15], experimental investigations of the influence of the shape and type of the barrier on the parameters of the flow behind the barrier during the propagation of the shock wave were performed. The interaction of a shock wave with rigid barriers was studied in [16] both numerically and experimentally.

In [17] the results of numerical and experimental studies of the interaction of a shock wave with a plane barrier are presented. In [1, 18, 19, 20, 21] studies of the interaction of shock waves with various types of obstacles have been carried out. In [18, 20] the barriers were equally spaced and parallelepipedic; the interaction of a wave with fourteen barriers was considered. Experiments on the interaction of a shock wave with obstacles of exactly the same ”comb” type are described in [2, 22].

The results of the above studies dealt mainly with the specifics of the initial interaction of the shock wave with a small number of obstacles.

The present work is devoted to the study of the propagation of shock waves in a channel with grooves (a pseudo-perforated wall). The grooves have the same size and shape and are located at equal distances from each other. Such periodically located grooves are called regular or cascade [11]. We consider both the flow at a great distance from the leading gas-dynamic discontinuity, and the local flow structure in the region where the flow is determined primarily by the parameters of this gas-dynamic discontinuity.

The presence of a relatively narrow zone in which a nonmonotonic change in the flow parameters occurs to a certain constant value is characteristic of the structure of discontinuities in the presence of dispersion and dissipation [23, 27, 29, 24, 25, 26, 28, 30]. In this case, the problem of calculating the flow corresponding to the structure of the discontinuity may not have a stationary solution in the form of a traveling wave, but a solution that periodically depends on time, as shown in [25, 26, 28]. If the scale at which the flow parameters change as a whole is much larger than the width of the shock wave structure, then the average velocity and width of the structure can be considered constant during the time interval over which it travels over a distance equal to its width. In this case, it is possible to obtain relationships connecting the parameters behind the structure of the discontinuity with the structure velocity. Questions concerning the uniqueness of such a relationship are discussed in detail in [23, 27, 29, 24, 25, 26, 28, 30].

Note that discontinuities whose structure is not stationary (not a traveling wave), but time-dependent periodically, are considered, according to [28], to be admissible discontinuities.

In the previous paper, [32], we analyzed the possibility of the formation of slowly changing (with time) quasistationary wave structures, which consist of a gas-dynamic discontinuity, in which the parameters of the gas change abruptly, and the zone of the nonequilibrium flow following it, in which the pressure is relaxed to the new equilibrium value. In this case, the instantaneous velocity of the leading discontinuity may not be constant and vary periodically due to the periodic location of obstacles, but the average velocity changes on scales of a much larger scale of one obstacle, and uniquely determines the value of the pressure at the end of the relaxation zone. It was found in [32] that, from the point of view of large spatial scales, a complex consisting of a leading gas-dynamic discontinuity and a relaxation zone can be considered as a discontinuity with the structure, and an effective shock adiabat for such a discontinuity is obtained.

2. Statement of the problem
The computational model is shown in Figure 1. We consider a rectangular channel. The channel has a height of $H$. The channel consists of two sections. On the right side of the channel, for $x_0 < x < \infty$, there are grooves in the lower channel wall. The width of the rectangular grooves is $l$, the height is $h$, the distance between the grooves is $L$. In the left part of the channel, as $-\infty < x < x_0$, the wall is smooth.

At the initial instant of time, for $x < 0$, the gas has parameters $P_3, v_3 = 0$ and $\rho_3$ and the
parameters \( P_0 \), \( v_0 = 0 \), \( \rho_0 \) for \( x \geq 0 \). Here \( P \) is the pressure, \( v \) is the velocity, and \( \rho \) is the gas density. The normal component of the velocity of the gas is zero at the gas-channel-wall and gas-groove-wall boundaries. Adhesion on the walls is absent.

Similarly [20, 32], the effects of viscosity, the heat diffusion, and body force is neglected. We assume that the gas behaves like a perfect gas and \( \gamma = 1.4 \) is the gas specific heat ratio. So the basic equations governing the propagation of a shock wave along a pseudo-perforated tube are as follows

\[
\frac{\partial}{\partial t} \begin{bmatrix} \rho \\ \rho v_i \\ \rho E \end{bmatrix} + \frac{\partial}{\partial x_j} \begin{bmatrix} \rho v_i \\ \rho v_i v_j \\ (\rho E + P) v_j \end{bmatrix} = 0, \quad E = e + \frac{|v|^2}{2}, \quad e = \frac{P}{(\gamma - 1)\rho},
\]

where \( E \) is the total specific energy of the gas and \( e \) is the specific internal energy.

The gas velocity has only two nonzero components (longitudinal \( v_x \) and vertical \( v_z \)), since the groove width is equal to the channel width.

**3. Approximate analytical solution of shock tube problem**

In our previous paper [32] we have demonstrated that the shock wave propagating in the right part of the channel represents a discontinuity with the structure.

According to [32], behind the leading gas-dynamic discontinuity there is a relaxation zone in which the flow parameters change rapidly and nonmonotonically. After some time after the passage of the leading wave, the gas velocity in the grooves is close to zero and the gas moves only in the region above the grooves. The resulting tangential velocity discontinuity is unstable, but the development of this instability occurs in a much longer time than that corresponding to the relaxation zone.

Behind the zone of relaxation is a zone in which flow parameters change slowly. A diagram of the simplifying separation of the flow into zones and regions is shown in Fig. 2. Averaging of the gas parameters over grooves in volumes equal to \( hl \) is performed, and the parameters of the gas in the space inside the grooves are over the corresponding volumes \( HL \). For the average values of the gas parameters at the end of the relaxation zone, we use the index 1 if the gas is outside the groove and the index is 2 if the gas is inside the groove.

It is assumed that \( P_1 = P_2, v_2 = 0, \rho_1 \neq \rho_2 \).

If the wave has traveled a distance that is much greater than the width of the structure, then on the shock wave for the flow averaged over the cross-section the relations obtained in [32] are
Figure 2. Division of the flow into zones used to obtain approximate formulas for the discontinuity structure.

satisfied proceeding from the conservation laws and physically grounded assumptions about the nature of the flow in the relaxation zone behind the shock discontinuity. These relations have the form

\[
\begin{align*}
\rho_0 D_0 + \alpha \rho_0 D_0 + v_1 \rho_1 - \rho_1 D_0 - \alpha \rho_2 D_0 &= 0 \\
\rho_1 v_1 D_0 + \alpha v_1 (\rho_2 - \rho_0) D_0 + P_0 - \rho_1 v_1^2 - P_1 &= 0 \\
\frac{(1+\alpha)(P_1-P_0)}{\gamma-1} D_0 - \frac{\rho_1 v_1^2}{2} (v_1 - D_0) - \gamma P_1 v_1 \frac{v_1}{\gamma-1} &= 0 \\
\rho_2 &= \rho_0 + (1 - \frac{\rho_0}{\rho_1}) P_1 \left( \frac{P_1}{P_3} + \frac{(\gamma-1) v_1^2}{2} \right) 
\end{align*}
\]

(2)

where \( \alpha = h l / (H L) \).

The system (2) consists of four equations for five unknown parameters \( D, P_1, v_1, \rho_1 \) and \( \rho_2 \). Hence, the system of algebraic equations implicitly determines the dependence of \( P_1 \) on \( v_1 \). This dependence is an analogue of the effective shock adiabat for a shock wave with a structure propagating in a channel with pseudo-perforated walls.

A rarefaction wave propagates to the left along the smooth part of the channel. The pressure and velocity of the gas in this wave are related by the well-known relation

\[
v_1 = 2 \sqrt{\frac{\gamma P_3}{\gamma - 1}} \sqrt{\frac{P_1}{P_3}} \left[ 1 - \left( \frac{P_1}{P_3} \right)^\frac{\gamma}{\gamma-1} \right], \quad \chi = \frac{\gamma - 1}{2 \gamma}
\]

(3)

Parameters of the flow \( P_1, \rho_1, \rho_2, v_1, D \) can be found by solving five algebraic equations (2) (3). Let \( H = 10 \) m, \( L = 2 \) m, \( h = 10 \) m and \( l = 0.8 \) m. For \( P_0 = 100 \) kPa, \( \rho_0 = 1.17 \) kg/m\(^3\), \( P_3 = 1 \) MPa and \( \rho_3 = 11.7 \) kg/m\(^3\) the system of equations (2) and (3) has the solution \( P_1 = 235 \) kPa, \( v_1 = 290 \) m/s, \( \rho_1 = 2.23 \) kg/m\(^3\), \( \rho_2 = 2.10 \) kg/m\(^3\) and \( D = 451 \) m/s.

The pressure profiles at the time \( t = 0.96 \) s are shown in Figure 3 and Figure 4 by lines 4 and 5. If the right-hand side of the channel is smooth, then the shock wave has parameters \( P_s = 285 \) kPa, \( v_s = 284 \) m/c, \( \rho_s = 2.04 \) kg/m\(^3\) and \( D_s = 556 \) m/s. Consequently, in the case under consideration, the presence of the grooves leads to a decrease in the average velocity of the shock wave by 19\% and the overpressure decreases by 17\%.

4. Calculation results

The system of equations (1) was solved by the Godunov-Kolgan method [33]. A exact solution of the Riemann problem was used to calculate the fluxes across the boundaries of the cell. A structured grid was used. The cells of the difference grid are parallelepips of the same size. The calculation was carried out using the three-dimensional gas-dynamics code Avangard / SafetyFor [32] developed in NRNU MEPhI. The grid contains 12800x6x100 cells. The cell sizes
in dimensionless units $\Delta x/L, \Delta y/L, \Delta z/H$ are 0.05, 0.05 and 0.02, respectively. The time step was determined atomically for $CFL = 0.1$, which certainly provides the necessary accuracy and stability of the calculation results for the Godunov-Kolgan method.

Figure 3 and Figure 4 shows the distribution of the average pressure $\overline{P}$ as a function of the coordinate $x$, and the average velocity profile $\overline{v}$ is presented in Figure 5. The average pressure is calculated over the entire section of the channel, including the grooves, according to the formula

$$\overline{P}(x,t) = \int_{-h}^{H} P(x,z,t) dz / (H + h),$$

and the average velocity is calculated only in the part of the channel that is above the grooves:

$$\overline{v_x}(x,t) = \int_{0}^{H} v_x(x,z,t) dz / H.$$

The pressure and velocity profiles are shown for the times 0.28 s, 0.69 s and 0.96 s, when the shock wave passed 54, 147 and 270 grooves, respectively.

The rightmost characteristic of the centered rarefaction wave runs along the smooth part of the tube with a low velocity of 5 m/s, therefore for $x = x_0$ the pressure varies insignificantly.
A shock wave propagates to the right. This shock wave consists of a leading gas-dynamic discontinuity and a relaxation zone in which long-wavelength oscillations are damped (see Figure 4). The parameters on the leading gas-dynamic discontinuity and the structure of the flow behind it vary little over time. The pressure at the end of the relaxation zone remains constant. This assumption was used in [32] in deriving the equations (2). The results of gas dynamic calculations confirm this assumption.

The flow parameters averaged over the channel cross section have long-wave and short-wave oscillations. These oscillations are not introduced numerically, but are inherent in the solution of the system (1). The amplitude of the oscillations decays with distance from the leading gas-dynamic discontinuity. In the relaxation zone, long-wave pressure oscillations occur near the mean value, which agrees well with the value of $P_1$ (line 5, in Figure 4) obtained using the analytical approach described in Section 3.

The appearance of long-wavelength and short-wave oscillations of the flow parameters in the relaxation zone is associated with reflection of the waves from the lower and lateral walls of the groove. Figure 6 shows the pressure field at time $t = 0.96$ s. It can be seen that there are compression and rarefraction waves in the gas behind the leading discontinuity. Short-wave perturbations arise due to the interaction of the leading shock with the side surface of the groove, and long-wave oscillations are caused by the circulation of compression and rarefaction waves between the lower boundary of the groove and the upper channel wall. The first maximum of pressure is caused by the leading gas-dynamic discontinuity propagating along the upper part of the channel. The average speed of this discontinuity is 451 m/s. The pressure in a plane shock wave propagating at this velocity is 182 kPa and agrees well with the first maximum of pressure on Figure 4. Then the pressure increases as a result of the arrival of waves arising from the interaction of the leading discontinuity with the walls of the grooves.

5. Conclusions
An analytical solution is obtained for the shock tube problem for the decay of the initial pressure and density discontinuity in a rectangular channel with grooves in the wall. The analytical solution is based on the effective shock adiabat for a channel with periodically located barriers [32]. This problem is also solved by numerical modeling. It is shown that the approximate analytical solution correctly predicts the propagation velocity of the leading discontinuity and the flow parameters at this discontinuity. The leading gas-dynamic discontinuity is followed by a relaxation zone in which there are long-wave and short-wave flow oscillations. These oscillations are caused by waves arising from the interaction of the flow behind the leading shock wave with the side and bottom walls of the groove. An approximate analytical solution allows one to obtain

![Figure 6. Pressure field at time $t = 0.96$ s.](image-url)
the values of the flow parameters at the end of the relaxation zone. These values are in good agreement with the numerical calculation.

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