Evaluation of measurement uncertainty in a static tensile test

Marcin Graba*

Abstract: This article presents a scheme of the procedure for assessing the measurement uncertainty of material constants determined in the static tensile test. The key analysis was preceded by a discussion of the concept of uncertainty of measurement, equated with absolute error, the concept of relative error and methods of assessment of these errors, depending on the type of mutual dependence of the analyzed quantity and the measured quantities. The next step in this article shows examples of stress–strain curves, which were used to calculate errors, as well as specific material constants. By importing the spreadsheets of the Mathcad package, which was used in the analysis, the procedure for assessing the measurement uncertainty was specified.

Keywords: static tensile test, measurement uncertainty, absolute error, relative error, material constant, mechanical properties

1 Introduction

The basic test in assessing the strength of construction materials is a static tensile test [1,2], which provides basic information about the material constants required to describe the properties of the material or structural elements during strength analyzes, carried out either by analytical or numerical method. These parameters include Young’s modulus $E$, Poisson’s ratio $\nu$, yield strength $\sigma_y$, tensile strength $\sigma_t$, breaking stress $\sigma_b$, strain hardening constant $a$ and strain hardening exponent $n$ in Ramberg–Osgood (RO) or other power law used to describe the stress–strain curve in its actual approach, in the range from the yield strength from refs. [1,2]. The previous studies [3–5] give various aspects of the use of material constants determined during a static tensile test. Having the yield point and tensile strength specified in the right way and with accurate accuracy, we can determine the power exponent in the R–O law and for materials with a clear yield point (i.e., the yield point elongation or discontinues the yield point) the length of the plasticity stop (i.e., lower yield or Luder’s strain) [3–5]. Procedures [3–5] allow estimating almost the entire stress–strain curve, based only on the yield strength, using the empirical formulas given in these procedures.

However, none of these documents recommended for use during static tensile tests [1,2] or used for assessing the strength of various structural elements [3–5] that mention about the measurement uncertainty. As we know, each measurement is made with certain accuracy. In the first definition, measurement inaccuracy is resulting from a specific measuring scale of a measuring tool, and it can be considered as measurement uncertainty. This uncertainty is reduced by using more accurate methods of measuring specific quantities, but it can never be eliminated completely. Therefore, in the first approximation, referring to direct measurements, the measurement uncertainty $\Delta W$ of the measured value denoted by $W$ can be understood as the accuracy of the measuring tool. In many previous studies refs. [6–8], the measurement uncertainty is referred to as the absolute error. While in the case of measuring a single quantity, there is no problem with the interpretation of the absolute error, in the case of measuring the size $W$, which is the algebraic sum of other approximate quantities $W_i$:

$$W = W_1 + W_2 + \ldots + W_n,$$  \hspace{1cm} (1)

Absolute error is the sum of the absolute errors of its individual components:

$$\Delta W = \Delta W_1 + \Delta W_2 + \ldots + \Delta W_n.$$  \hspace{1cm} (2)

However, this method may exaggerate the importance of individual measurement errors. Therefore, when calculating the absolute error, partial error compensation of different signs should be taken into account:

$$\Delta W = \sqrt{(\Delta W_1)^2 + (\Delta W_2)^2 + \ldots + (\Delta W_n)^2}.$$  \hspace{1cm} (3)
The basic science of measurement introduces the concept of relative uncertainty \( \delta W \), which is the quotient of the measurement uncertainty \( \Delta W \) and the value of the measured value \( W \):

\[
\delta W = \frac{\Delta W}{W}. \tag{4}
\]

From the previous studies refs. [6–8], one can introduce the concept of the relative error of the approximate value of \( W \), which is the ratio of the absolute error \( \Delta W \) to the absolute value of \( W \):

\[
\delta W = \frac{\Delta W}{|W|}. \tag{5}
\]

It should be noted that the relative error of the difference in positive numbers is greater than the relative errors of these numbers, especially when these numbers differ very little from each other [6–8].

There is often a situation in the laboratory where the measurement result is a function of independent variables that are multiplied or divided. Then, the relative errors of these numbers should be added up. If we are dealing with an expression that allows calculating the quantity \( f \)

\[
f = \frac{W_1 \cdot W_2 \ldots W_m}{Z_1 \cdot Z_2 \ldots Z_n}, \tag{6}
\]

and the relative error of the expression should be estimated as follows:

\[
\delta f = \delta W_1 + \delta W_2 + \ldots + \delta W_m + \delta Z_1 + \delta Z_2 + \ldots + \delta Z_n. \tag{7}
\]

In the case of such a large number of variables, we must also compensate for errors with different signs, using the relationship:

\[
\delta f = \sqrt{(\delta W_1)^2 + (\delta W_2)^2 + \ldots + (\delta W_m)^2 + (\delta Z_1)^2 + (\delta Z_2)^2 + \ldots + (\delta Z_n)^2}. \tag{8}
\]

The absolute error of the functions of the variables \( f \) (\( W_i \)) is calculated a little differently, \( i = 1,2,\ldots, n \), which is differentiable in some area, assuming that this error is caused by small argument errors. Then, the total differential method should be used, according to the scheme (9):

\[
\Delta W = \left[ \frac{\partial W}{\partial W_1} \right] \Delta W_1 + \left[ \frac{\partial W}{\partial W_2} \right] \Delta W_2 + \ldots + \left[ \frac{\partial W}{\partial W_n} \right] \Delta W_n. \tag{9}
\]

However, the aforementioned method exaggerates the importance of individual measurement errors. Ref. [7] proposes a method of considering compensation of individual errors. Based on the relationships given earlier, the value of the absolute error should be calculated as follows [6–8]:

\[
\Delta W = \sqrt{\left( \frac{\partial W}{\partial W_1} \cdot \Delta W_1 \right)^2 + \left( \frac{\partial W}{\partial W_2} \cdot \Delta W_2 \right)^2 + \ldots + \left( \frac{\partial W}{\partial W_n} \cdot \Delta W_n \right)^2}. \tag{10}
\]

The introduction of error calculus to engineering issues requires an appropriate record of the measured quantities, which should be given taking into account the absolute error value \( \Delta W \) and a commentary on the relative error made (given in percentage).

2 Experimental data used in the paper, determination of material constants in a static tensile test

Returning to the first paragraph of this article in Section 1, this article will familiarize the reader with the subject of estimating measurement errors for quantities determined in a static tensile test. The study will use research material collected and presented in refs. [9–12], which will concern the five different stress–strain curves. Figure 1 presents curves recorded during the experimental tests, illustrating the change of force acting on the specimen as a function of extensometer displacement, and Figure 2 presents determined engineering tensile curves in the
stress system as a function of deformation. The same measurement base \( I_0 = 50 \text{ mm} \) was used for all tests carried out. Table 1 presents the selected quantities necessary to determine selected material constants measured during experimental tests [9–12], and Table 2 presents the selected mechanical quantities determined in accordance with applicable standards.

Determination of Young’s modulus \( E \), Poisson’s ratio \( \nu \), yield stress \( \sigma_0 \) and corresponding strains \( \epsilon_0 \), tensile strength \( \sigma_m \) and corresponding strains \( \epsilon_m \), breaking stress \( \sigma_u \) and corresponding strains \( \epsilon_u \), or elongation at break \( A_e \) raises no doubts, and it is described in detail in the standards [1,2]. Due to the simplicity of these analyzes, this will not be discussed in this article. However, due to the fact that the stress–strain curve sometimes requires that it be described by appropriate constitutive relationships, which is required in the case of analysis of strength of materials based on appropriate hypotheses, here the method of estimation of the strain hardening exponent \( n \) in the Ramberg–Osgood law will be indicated. In a general form, Ramberg–Osgood law is expressed as follows:

\[
\frac{\epsilon}{\epsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \cdot (\frac{\sigma}{\sigma_0})^n.
\]  

(11)

where \( \sigma_0 \) is the yield point \( (R_c \text{ or } R_{0.2}) \); \( \epsilon_0 \) is the strain corresponding to the yield point \( (\epsilon_0 = \sigma_0/E) \); \( E \) is Young’s module; \( \alpha \) is a constant, referred to as the strain hardening constant; \( n \) is the power exponent, referred to as the strain hardening factor (i.e., strengthening factor) or strain hardening exponent (strengthening exponent).

The full analysis of stress distribution and strains near the front of crack in a nonlinear material, carried out by Hutchinson [13], was based on the constitutive relationship, which is a three-dimensional generalization of equation (11). However, in a number of articles on fracture mechanics and analysis of stress and strain fields near the crack tip [14–18], the elastic–plastic material is described by another version of equation (11):

\[
\begin{align*}
\frac{\epsilon}{\epsilon_0} &= \frac{\sigma}{\sigma_0} & \text{for } & \sigma \leq \sigma_0 \\
\frac{\epsilon}{\epsilon_0} &= \alpha \cdot (\frac{\sigma}{\sigma_0})^n & \text{for } & \sigma > \sigma_0.
\end{align*}
\]

(12)

In the aforementioned power laws (11) and (12), often in engineering calculations, the strain hardening constant \( \alpha \) is taken as equal to unity. The degree of material hardening is then determined only on the basis of the strain hardening exponent \( n \). Figure 3 shows the

---

**Table 1: The quantities necessary to determine selected material constants measured during experimental tests [9–12] (selected results)**

| Measured value | Steel symbol |
|----------------|--------------|
| \( \sigma \) (MPa) | \( \epsilon \) (mm/mm) |
| \( \nu \) | \( \alpha \) |
| \( \epsilon_0 \) (MPa) | 0.00451 |
| \( \epsilon_m \) (MPa) | 0.99588 |
| \( \sigma_m \) (MPa) | 1002.71 |
| \( \epsilon_u \) | 0.10628 |
| \( \sigma_u \) (MPa) | 1.0 |
| \( \sigma_0 \) (MPa) | 1.0 |
| \( \sigma_0 \) (mm) | 2.0 |
| \( \epsilon_0 \) (mm) | 20.0 |
| \( \alpha \) | 1.0 |
| \( \alpha \) | 1.0 |
| \( \alpha \) | 1.0 |

**Table 2: The quantities necessary to determine selected material constants measured during experimental tests [9–12] (selected results)**

| Measured value | Steel symbol |
|----------------|--------------|
| \( E \) (GPa) | \( \nu \) |
| \( \sigma_0 \) (MPa) | \( \epsilon_0 \) |
| \( \epsilon_m \) (MPa) | \( \sigma_m \) |
| \( \epsilon_u \) | \( \sigma_u \) |
| \( \sigma_0 \) (MPa) | \( \epsilon_u \) |
| \( \sigma_0 \) (mm) | \( \epsilon_u \) |
| \( \sigma_0 \) (mm) | \( \epsilon_u \) |

---

**Figure 2: Engineering tensile diagrams \( \sigma = f(\epsilon) \) for materials used in the analysis (based on refs. [9–12]).**
Figure 3: Model stress--strain curves based on formulas (11) and (12) (based on ref. [19]).

Figure 4: Comparison of the selected stress values determined in static tensile test.

Among the quantities that were analyzed and for which it was decided to estimate the absolute error and the relative error include:

- Yield stress: \( \sigma_0 = F_0 / S_0 \).
- Ultimate strength: \( \sigma_m = F_m / S_0 \).
- Breaking stress: \( \sigma_u = F_u / S_0 \).
- Strain corresponding to: yield stress \( \varepsilon_0 \), ultimate strength \( \varepsilon_m \) and breaking stress \( \varepsilon_u \) (the general deformation formula

3 Analysis of error calculus of quantities determined in a static tensile test

Section 3 presents a method of assessing relative and absolute errors for material constants and characteristic points determined in a static tensile test (Figure 5).
3.1 The scheme for evaluating the measurement uncertainty for a quantity being stress

Estimation of the measurement uncertainty (absolute and relative error) of the stress value – a rectangular specimen (a part of the Mathcad spreadsheet):

Basic equations:
\[ S_0 = a_0 \cdot b_0 \quad \sigma = \frac{F}{S_0} \]
\[ \delta \sigma_F = \frac{d}{dF} \delta \sigma_f, \quad \delta \sigma_{a_0} = \frac{d}{da_0} \delta \sigma_{a_0}, \quad \delta \sigma_{b_0} = \frac{d}{db_0} \delta \sigma_{b_0}. \]

Absolute error \( \Delta \sigma \) according to equation (9):
\[ \Delta \sigma = |\delta \sigma_F| \cdot |\Delta F| + |\delta \sigma_{a_0}| \cdot |\Delta a_0| + |\delta \sigma_{b_0}| \cdot |\Delta b_0| \]
\[ \Delta \sigma = \frac{\Delta F}{|a_0| \cdot |b_0|} + \frac{\Delta a_0 \cdot |F|}{(|a_0|^2 \cdot |b_0|)} + \frac{\Delta b_0 \cdot |F|}{(|a_0| \cdot |b_0|^2)}. \]

Relative error \( \delta \sigma \) according to equation (5):
\[ \delta \sigma = \frac{|\delta \sigma_F|}{|\sigma|} + \frac{|\delta \sigma_{a_0}|}{|a_0|} \frac{|F|}{|b_0|} + \frac{|\delta \sigma_{b_0}|}{|b_0|} \frac{|F|}{|a_0|}. \]

Absolute error \( \Delta \sigma \) including compensation of individual errors of component quantities according to the formula (10):
\[ \Delta \sigma = \sqrt{(\delta \sigma_F \cdot \Delta F)^2 + (\delta \sigma_{a_0} \cdot \Delta a_0)^2 + (\delta \sigma_{b_0} \cdot \Delta b_0)^2} \]
\[ \Delta \sigma = \sqrt{\frac{\Delta F^2}{a_0^2 \cdot b_0^2} + \frac{\Delta a_0^2}{a_0^4 \cdot b_0^2} + \frac{\Delta b_0^2}{a_0^2 \cdot b_0^4}}. \]

Relative error \( \delta \sigma \) using equation (5) on taking into account the compensation of individual component errors (based on equation (10)):
\[ \delta \sigma = \frac{|\delta \sigma_F|}{|\sigma|} \cdot \sqrt{\frac{\Delta F^2}{a_0^2 \cdot b_0^2} + \frac{\Delta a_0^2}{a_0^4 \cdot b_0^2} + \frac{\Delta b_0^2}{a_0^2 \cdot b_0^4}}. \]

3.2 Results of the measurement errors analysis

Uncertainty of the performed measurements (see data given in Tables 1 and 2) – estimated according to the
The measurement error generally does not exceed 0.25%. The analysis of the obtained calculation results is left to the reader of this article. As can be seen, the error values calculated using the compensation method are slightly lower than the error values calculated using the total differential method. The largest relative error was recorded in the case of estimation of deformations corresponding to the yield point – in three cases, its level exceeds 4% of the value. The error in estimating the type of stress in all cases is generally less than 0.6% of the experimentally measured value. Considering the rather low measuring accuracy of the workshop caliper (±0.01 mm), a relatively small value of the measurement error should be noted in the case of specimen elongation. The measurement error generally does not exceed 0.25%.

4 Discussion

The error values calculated using the compensation method are generally slightly lower than the error values calculated using the total differential method. Despite the low accuracy of the selected measuring tools, the absolute and relative error values obtained can be considered acceptable. Certainly, the introduction of error calculations into engineering practice in the field of laboratory measurements may seem natural; however, as noted at the beginning of this article, this is not shown in scientific papers. Although such analyzes are found in typical papers in the field of metrology of geometrical quantities and angle, they are difficult to find in scientific papers dealing with the subject of determining physical quantities considered as material constants or material characteristics. Perhaps this is due to the fact that the entire analysis scheme is based on painstaking calculations in

![Figure 6: Comparison of the absolute errors $\Delta \sigma$ for selected stress values determined in the static tensile test using equation (9).](image)

---

**Table 3:** Determined in accordance with the method presented in paragraph 3 and absolute and relative error values of selected quantities determined during a static tensile test (selected results of the calculations)

| Measured value | Steel symbol | Measured value | Steel symbol |
|----------------|--------------|----------------|--------------|
|                | 145Cr6      | S355J2         | 41Cr4        | 145Cr6      | S355J2         | 41Cr4        |
| $\sigma_0$ (MPa) | 934.08      | 400.18         | 473.06       | $\varepsilon_0$ (Equation (9)) | 0.00451 | 0.01572 | 0.00224 |
| $\Delta \sigma_0$ (MPa) (Equation (9)) | 1.867 | 2.400 | 2.838 | $\Delta \varepsilon_0$ (Equation (9)) | $2.01 \times 10^{-4}$ | $2.03 \times 10^{-4}$ | $2.01 \times 10^{-4}$ |
| $\delta \sigma_0$ (Equations (9) and (5)) | 0.20% | 0.60% | 0.60% | $\delta \varepsilon_0$ (Equations (9) and (5)) | 4.459% | 1.294% | 8.957% |
| $\Delta \sigma_0$ (MPa) (Equation (10)) | 1.867 | 2.040 | 2.412 | $\Delta \varepsilon_0$ (Equation (10)) | $2.01 \times 10^{-4}$ | $2.03 \times 10^{-4}$ | $2.00 \times 10^{-4}$ |
| $\delta \sigma_0$ (Equations (10) and (5)) | 0.20% | 0.51% | 0.51% | $\delta \varepsilon_0$ (Equations (10) and (5)) | 4.455% | 1.292% | 8.949% |

| Measured value | Steel symbol |
|----------------|--------------|
| $A_0$ (mm) | 5.31373      | 10.37424      | 7.65000 |
| $\Delta A_0$ (mm) (Equation (9)) | $1.00 \times 10^{-2}$ | $1.00 \times 10^{-2}$ | $1.00 \times 10^{-2}$ |
| $\delta A_0$ (Equations (9) and (5)) | 0.188% | 0.096% | 0.131% |
| $\Delta A_0$ (mm) (Equation (10)) | $1.00 \times 10^{-2}$ | $1.00 \times 10^{-2}$ | $1.00 \times 10^{-2}$ |
| $\delta A_0$ (Equations (10) and (5)) | 0.188% | 0.096% | 0.131% |
the field of differential calculus, and this seems to be time consuming and labor intensive.

However, it is suggested that the research practice be consistent with the canons of science to give results of error calculus, which certainly says a lot about the accuracy of measuring physical quantities. The analysis can be simplified by using the Mathcad package or equivalent to automate calculations, as shown in this article. The authors do not exclude the development of an application that allows estimating the level of uncertainty in a static tensile test after entering the relevant data. The author of this article intends to refer the presented method of estimating measurement errors to other tests in the field of structural strength – for example, fracture mechanics or material fatigue.

Figure 7: Comparison of the absolute errors $\Delta \sigma$ including the compensation for individual errors of component quantities, for selected stress values using equation (10).

Figure 10: Comparison of the relative errors $\delta \sigma$ for selected stress values determined in the static tensile test using equations (9) and (5).

Figure 8: Comparison of the absolute errors $\Delta \varepsilon$ and $\Delta A$ for selected strain and elongation values determined in the static tensile test using equation (9).

Figure 11: Comparison of the relative errors $\delta \sigma$ including the compensation for individual errors of component quantities, for selected stress values using equations (10) and (5).
That is why, in the Appendix, the author shows the method of determining the measurement uncertainty of the hardening exponent n in the power law (equation (12)). In the future, author plans to include the impact of measurement error of the strain hardening exponent value into account on the results obtained during the FEM calculations in the field of elastic–plastic fracture mechanics.

Funding information: This research was funded by the Faculty of Mechatronics and Mechanical Engineering at the Kielce University of Technology under the order number 01.0.09.00/2.01.01.00.0000 SUBB.MKMT.21.003.

Author contributions: M. G.: conceptualization, methodology, software, validation, formal analysis, investigation, resources, data curation, writing – original draft preparation, writing – review and editing, visualization, supervision, project administration, funding acquisition.

Conflict of interest: The author declares no conflict of interest.

References

1. PN-EN ISO 6892-1:2016-09. Metals – Tensile test – Part 1: test method at room temperature; 2016 (Polish).
2. ASTM E8/E8M – 13a. Standard test methods for tension testing of metallic materials; 2013.
3. SINTAP – Structural Integrity Assessment Procedures for European Industry. Final procedure. Brite-Euram project no. BE95-1426. Rotherham: British Steel; 1999.
4. Kocak M, Webster S, Janosch JJ, Ainsworth RA, Koers R. FITNET report – European fitness-for-service network. Edited by contract no. G1RT-CT-2001-05071; 2006.
5. Neimitz A, Dzioba I, Graba M, Okrajni J. The assessment of the strength and safety of the operation high temperature components containing crack. Kielce: Kielce University of Technology Publishing House; 2008 (Polish).
6. Rumszyski LZ. Mathematical preparation of the results of the experiment. Warsaw: WNT; 1973 (Polish).
7. Greń J. Mathematical statistics. Warsaw: PWN; 1987 (Polish).
8. Klonecka W. Statistics for engineers. Warsaw: PWN; 1999 (in Polish).
9. Graba M. Experimental and numerical assessment of elasto-plastic parameters of fracture mechanics for 145Cr6 steel. Part I. Mech Rev. 2013;4:31–8 (Polish).
10. Graba M. About problems in determining selected mechanical properties of 41Cr4 steel. Mechanik. 2016;8–9:974–53 (Polish).
11. Graba M. Maximum crack opening stresses as a parameter controlling the fracture process. Proceedings of the XXIII Symposium of Fatigue and Fracture Mechanics. Bydgoszcz – Pieczyska. 1; 2010. p. 1–13 (Polish).
12. Neimitz A, Dzioba I, Molasy R, Graba M. The influence of constraints on the fracture toughness of brittle materials. Proceedings of the XX Symposium of Fatigue and Fracture Mechanics. Bydgoszcz-Pieczyska. 1; 2004. p. 265–72 (Polish).
13. Hutchinson JW. Singular behaviour at the end of a tensile crack in a hardening material. J Mech Phys Solids. 1968;16:33–31.
14. Graba M. Proposal of the hybrid solution to determining the selected fracture parameters for SEN(B) specimens dominated by plane strain. Bull Pol Acad Sci Technical Sci. 2017;65(4):523–32.
15. Graba M. The influence of material properties on the Q-stress value near the crack tip for elastic-plastic materials. J Theor Appl Mech. 2008;46(2):269–90.
16 Graba M. Catalogue of the numerical solutions for SEN(B) specimen assuming the large strain formulation and plane strain condition. Arch Civ Mech Eng. 2012;12(1):29–40.

17 Neimitz A, Graba M. Analytical-numerical hybrid method to determine the stress field in front of the crack in 3D elastic-plastic structural elements. Proceedings of XVII ECF. Brno – Czech Republic. Article in the electronic form, abstract – book of abstracts; 2008. p. 85.

18 Graba M. Numerical analysis of the influence of in-plane constraints on the crack tip opening displacement for SEN(B) specimens under predominantly plane strain conditions. Int J Appl Mech Eng. 2016;21(4):849–66.

19 Graba M, Neimitz A. On methods of determining material parameters in the Ramberg-Osgood law. Proceedings of the X Polish Conference of Fracture Mechanics. Opole – Wisła. 1; 2005. p. 323–32 (Polish).
Appendix A: Calculation of the uncertainty of the strain hardening exponent

It seems much more complicated to estimate the uncertainty of measurement of a power exponent in the RO law (equation (11)) or in the power law described by the formula (12), which, taking into account the assumption that the strengthening constant $\alpha = 1$, is quite often used for approximation stress–strain curve by many researchers, including the author of this publication. Generally, based on two points—the yield strength and tensile strength (as well as the corresponding deformations), the strengthening exponent $n$ assuming that the constant $\alpha = 1$ is determined using formula (13). The determined value of the strain hardening exponent $n$ determines the tensile curve used in the FEM numerical calculations, which in turn affects, as has already been mentioned, the value of the $J$-integral calculated numerically, or the level of stress near the crack tip, if you consider issues in the field of fracture mechanics, as shown in ref. [19].

In view of this fact, it was decided to include in the Appendix the calculation scheme carried out in the Mathcad package, which allows estimating the uncertainty of the measurement of the strain hardening exponent $n$ in the power law described by the formula (12).

By using the “solve” function available in Mathcad, we automatically obtain the formula (13), using the second part of the formula (13), applicable for points above the yield point on the real stress–strain curve (below are excerpts of the calculation sheet from the MathCad package):

$$n = \frac{\varepsilon_m}{\varepsilon_0} = \left(\frac{\sigma_m}{\sigma_0}\right)^n \text{ solve, } n = \frac{\ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)}{\ln \left(\frac{\sigma_m}{\sigma_0}\right)}.$$

As we can see, the exponent $n$ is a function of four variables ($\sigma_0$, $\sigma_m$, $\varepsilon_0$, and $\varepsilon_m$). In view of this fact, material constants determined for the steels used in refs. [9–12] will be used in further analysis, as well as the corresponding errors generally indicated as $\Delta W$, which was shown earlier. In the next step, according to the theory in paragraph 1, the partial derivatives will be estimated:

$$\delta n_{e_0} = \frac{d}{d\varepsilon_0} n \delta \varepsilon_{e_0} = \frac{\varepsilon_0 \cdot \ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)}{\varepsilon_m \cdot \ln \left(\frac{\sigma_m}{\sigma_0}\right)} \quad \delta n_{\sigma_m} = \frac{\ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)}{\sigma_m \cdot \ln \left(\frac{\sigma_m}{\sigma_0}\right)^2},$$

which can be written as follows:

$$\delta n_{e_0} = -\frac{1}{\varepsilon_0 \cdot \ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)} \delta \varepsilon_{e_0} = \frac{\varepsilon_0 \cdot \ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)}{\varepsilon_m \cdot \ln \left(\frac{\sigma_m}{\sigma_0}\right)} \quad \delta n_{\sigma_m} = \frac{\ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)}{\sigma_m \cdot \ln \left(\frac{\sigma_m}{\sigma_0}\right)^2},$$

In the next step, we can save the final formulas for relative and absolute errors made in determining the exponent $n$, both without and with compensation for component errors:

- Relative error (equation (9)):

$$\delta n = \frac{\Delta n}{n}$$

$$\delta n = \left[\ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)\right]^2 \delta \varepsilon_{e_0} + \left[\ln \left(\frac{\sigma_m}{\sigma_0}\right)\right]^2 \delta \varepsilon_{\sigma_m} + \left[\ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)\right]^2 \delta \varepsilon_{\sigma_0} + \left[\ln \left(\frac{\sigma_m}{\sigma_0}\right)\right]^2 \delta \varepsilon_{\tau_m}$$

- Absolute error (equations (9) and (5)):

$$\delta n = \frac{\Delta n}{\left|\ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)\right|}$$

$$\delta n = \left[\ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)\right]^2 \delta \varepsilon_{e_0} + \left[\ln \left(\frac{\sigma_m}{\sigma_0}\right)\right]^2 \delta \varepsilon_{\sigma_m} + \left[\ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)\right]^2 \delta \varepsilon_{\sigma_0} + \left[\ln \left(\frac{\sigma_m}{\sigma_0}\right)\right]^2 \delta \varepsilon_{\tau_m}$$

- Relative error, taking into account individual errors of measured quantities (equation (10)):

$$\Delta n = \sqrt{(\delta n_{e_0} \cdot \Delta \varepsilon_{e_0})^2 + (\delta n_{\sigma_m} \cdot \Delta \varepsilon_{\sigma_m})^2 + (\delta n_{\sigma_0} \cdot \Delta \varepsilon_{\sigma_0})^2 + ((\delta n_{\tau_m} \cdot \Delta \varepsilon_{\tau_m})^2}$$

$$\Delta n = \sqrt{\frac{\Delta \varepsilon_{e_0}^2}{\varepsilon_0^2 \cdot \ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)^2} + \frac{\Delta \varepsilon_{\sigma_m}^2}{\varepsilon_m^2 \cdot \ln \left(\frac{\sigma_m}{\sigma_0}\right)^2} + \frac{\Delta \varepsilon_{\sigma_0}^2}{\varepsilon_0^2 \cdot \ln \left(\frac{\varepsilon_m}{\varepsilon_0}\right)^2} + \frac{\Delta \varepsilon_{\tau_m}^2}{\sigma_0^2 \cdot \ln \left(\frac{\sigma_m}{\sigma_0}\right)^2} + \frac{\Delta \varepsilon_{\tau_m}^2}{\sigma_0^2 \cdot \ln \left(\frac{\sigma_m}{\sigma_0}\right)^2} + \frac{\Delta \varepsilon_{\tau_m}^2}{\sigma_0^2 \cdot \ln \left(\frac{\sigma_m}{\sigma_0}\right)^2}.
- Absolute error, taking into account individual errors of measured quantities (equations (10) and (5)):

\[ \delta n = \frac{\Delta n}{|n|} \]

\[
\delta n = \left| \ln \left( \frac{\delta n}{\sigma_m} \right) \cdot \frac{\Delta \sigma_m}{\sigma_m} + \Delta \delta_n \right|,
\]

By using the algorithm shown earlier, for the steels considered in refs. [9–12], all absolute and relative error values were determined, and then, they were summarized in Table A1 and illustrated in Figure A1.

As can be seen, absolute and relative errors, estimated taking into account the compensation of individual errors of measured quantities (formulas (10) and (5)), are almost half smaller than errors determined in accordance with formulas (9) and (5). It can be stated that the higher the value of the strain hardening exponent (i.e., the lower the level of material hardening), the greater the measurement uncertainty value. The more brittle the material – the small value of the strain hardening exponent \( n \), the smaller the uncertainty of measurement of this quantity. It can be arbitrarily stated that for the materials considered in this article, tested in the same laboratory conditions, with the same instruments (testing machine, extensometer, calliper), smaller errors in the determination of the strain hardening exponent \( n \) depending on the tensile curve obtained in the laboratory are observed for strongly hardening materials. An increase in the value of the strain hardening exponent results in a decrease in the hardening of the material and generates an increase in measurement uncertainty when determining this quantity.

The analysis of the assessment of measurement uncertainty in determining the strain hardening exponent \( n \), carried out by the author for the purposes of this article, for the materials considered in this article indicates that above the yield point, the stress–strain curve may differ by several percent (Figures A2 and A3). In the summary presented in Figures A2 and A3, the difference between the model real curve determined for the estimated value of the strain hardening exponent and the curves drawn based on the strain hardening exponent reduced or increased by the value of absolute error, corresponding to the tensile strength, is more than 2%.

Table A1: The absolute and relative error values of strain hardening exponent \( n \) for steels used in this article

| Steel  | \( n \) | \( \Delta n \) (Equations (9) and (5)) | \( \delta n \) (%) | \( \Delta n \) (Equations (10) and (5)) | \( \delta n \) (%) |
|--------|--------|-----------------------------------|----------------|-----------------------------------|----------------|
| 16Cr6  | 27.43  | 1.404                             | 5.119          | 0.803                             | 2.926          |
| S355J2 | 6.90   | 0.309                             | 4.48           | 0.163                             | 2.367          |
| 41Cr4  | 9.69   | 0.525                             | 5.415          | 0.286                             | 2.950          |
| 14MoV63| 9.16   | 0.232                             | 2.536          | 0.154                             | 1.676          |
| X30Cr13| 4.11   | 0.085                             | 2.061          | 0.054                             | 1.306          |

Figure A1: Comparison of the absolute and relative errors (denoted as \( \Delta n \) and \( \delta n \), respectively) for strain hardening exponent \( n \), for all steels used in this article.

Appendix B: A list of formulas helpful in assessing the measurement uncertainty

Appendix B presents a list of all formulas derived during the preparation of this article, which may prove helpful...
Table B1: Determination of the absolute and relative error values of strain hardening exponent $n$ for steels used in this article

Estimation of the measurement uncertainty of the stress quantity $\sigma$ for a specimen with a rectangular cross-section

Equations (9) and (5)

$$\Delta \sigma = \frac{\Delta \sigma_0}{|\sigma_0|} + \frac{\Delta \sigma_0}{|\sigma_0| - |\sigma_0|} + \frac{\Delta \sigma_0}{|\sigma_0|}$$

Equations (10) and (5)

$$\Delta \sigma = \sqrt{\frac{\Delta \sigma_0^2}{|\sigma_0|^2} + \frac{\Delta \sigma_0^2}{|\sigma_0|} + \frac{\Delta \sigma_0^2}{|\sigma_0|} + \frac{\Delta \sigma_0^2}{|\sigma_0|^2}}$$

Estimation of the measurement uncertainty of the stress quantity $\sigma$ for a specimen with a circular cross-section

Equations (9) and (5)

$$\Delta \sigma = \frac{4 \cdot \Delta \sigma_0}{n (|d|)^2} + \frac{8 \cdot \Delta \sigma_0}{|n (|d|)^2|}$$

Equations (10) and (5)

$$\Delta \sigma = 4 \cdot \sqrt{\frac{\Delta \sigma_0^2}{|n (|d|)^2|^2} + \frac{\Delta \sigma_0^2}{|n (|d|)^2|} + \frac{\Delta \sigma_0^2}{|n (|d|)^2|^2}}$$

Estimation of the measurement uncertainty of the quantity being a contraction $Z$ for a specimen with a rectangular cross-section

Equations (9) and (5)

$$\Delta Z = \Delta \sigma_0 \left( \frac{1}{a_0} + \frac{a \cdot b - a \cdot b_0}{a_0 b_0} \right) + \Delta \sigma_0 \left( \frac{1}{b_0} + \frac{a \cdot b - a \cdot b_0}{a_0 b_0} \right) + \Delta \sigma_0 \left( \frac{a \cdot b - a \cdot b_0}{a_0 b_0} \right) + \Delta \sigma_0 \left( \frac{a \cdot b - a \cdot b_0}{a_0 b_0} \right)$$

Equations (10) and (5)

$$\Delta Z = \sqrt{\Delta \sigma_0^2 \left( \frac{1}{a_0} + \frac{a \cdot b - a \cdot b_0}{a_0 b_0} \right)^2 + \Delta \sigma_0^2 \left( \frac{1}{b_0} + \frac{a \cdot b - a \cdot b_0}{a_0 b_0} \right)^2 + \Delta \sigma_0^2 \left( \frac{a \cdot b - a \cdot b_0}{a_0 b_0} \right)^2}$$

Estimation of the measurement uncertainty of the quantity being a contraction $Z$ for a specimen with a circular cross-section

Equations (9) and (5)

$$\Delta Z = \Delta \sigma_0 \left( \frac{2}{d_0} + \frac{2 \cdot n \cdot d^2 - 2 \cdot n \cdot d_0^2}{n \cdot d_0^2} \right)$$

Equations (10) and (5)

$$\Delta Z = \sqrt{\Delta \sigma_0^2 \left( \frac{2}{d_0} + \frac{2 \cdot n \cdot d^2 - 2 \cdot n \cdot d_0^2}{n \cdot d_0^2} \right)^2 + \Delta \sigma_0^2 \left( \frac{2 \cdot n \cdot d^2 - 2 \cdot n \cdot d_0^2}{n \cdot d_0^2} \right)^2}$$

Estimation of the measurement uncertainty of the quantity which is the strain $\varepsilon$

Equations (9) and (5)

$$\Delta \varepsilon = \Delta \sigma_0 \left( \frac{1}{b_0} + \frac{1 \cdot b - 1 \cdot b_0}{b_0} \right)$$

Equations (10) and (5)

$$\Delta \varepsilon = \sqrt{\Delta \sigma_0^2 \left( \frac{1}{b_0} + \frac{1 \cdot b - 1 \cdot b_0}{b_0} \right)^2 + \Delta \sigma_0^2 \left( \frac{1 \cdot b - 1 \cdot b_0}{b_0} \right)^2}$$
Estimation of the measurement uncertainty of the length of the specimen 
Equations (9) and (5) 
\[ \Delta l = \Delta l' + \Delta l_0 \]
\[ \Delta l = \Delta l/|l_0| \]

Equations (10) and (5) 
\[ \Delta l = \sqrt{\Delta l'^2 + \Delta l_0^2} \]
\[ \Delta l = \sqrt{\Delta l/|l|} \]

Estimation of the measurement uncertainty of the strain hardening exponent \( n \) in the power law – formula (12) for the power constant \( \alpha = 1 \)
Equations (9) and (5) 
\[ \Delta n = \frac{\Delta n_0}{\ln \left( \frac{\sigma_0}{\sigma_0} \right)} + \frac{\Delta n_{\text{en}}}{\ln \left( \frac{\sigma_{\text{en}}}{\sigma_{\text{en}}} \right)} + \frac{\Delta n_{\text{en} 2}}{\ln \left( \frac{\sigma_{\text{en} 2}}{\sigma_{\text{en} 2}} \right)} + \frac{\Delta n_{\text{en} 3}}{\ln \left( \frac{\sigma_{\text{en} 3}}{\sigma_{\text{en} 3}} \right)} \]
\[ \delta n = \frac{\Delta n}{\ln \left( \frac{\sigma_0}{\sigma_0} \right)} \]

Equations (10) and (5) 
\[ \Delta n = \frac{\Delta n_0}{\ln \left( \frac{\sigma_0}{\sigma_0} \right)} + \frac{\Delta n_{\text{en}}}{\ln \left( \frac{\sigma_{\text{en}}}{\sigma_{\text{en}}} \right)} + \frac{\Delta n_{\text{en} 2}}{\ln \left( \frac{\sigma_{\text{en} 2}}{\sigma_{\text{en} 2}} \right)} + \frac{\Delta n_{\text{en} 3}}{\ln \left( \frac{\sigma_{\text{en} 3}}{\sigma_{\text{en} 3}} \right)} \]
\[ \delta n = \frac{\Delta n}{\ln \left( \frac{\sigma_0}{\sigma_0} \right)} \]

\( \sigma \) – stress; \( \sigma_0 \) – yield stress; \( \sigma_{\text{en}} \) – ultimate strength; \( \varepsilon \) – strain; \( \varepsilon_0 \) – strain corresponding to the yield stress; \( \varepsilon_{\text{en}} \) – strain corresponding to the ultimate strength; \( \alpha \) – strain hardening constant in power law equation (12) – in this calculation \( \alpha = 1 \); \( n \) – strain hardening exponent in power law equation (12); \( F \) – force; \( a_0, b_0 \) – initial dimensions of the specimen with rectangular cross-section \((a_0 \text{ and } b_0)\); \( a, b \) – final dimensions of the specimen with rectangular cross-section \((a \text{ and } b)\); \( d_0 \) – initial diameter of the specimen with circular cross-section \((\pi d_0^2/4)\); \( d \) – final diameter of the specimen with circular cross-section \((\pi d_0^2/4)\); \( A \) – elongation of the specimen; \( Z \) – contraction of the specimen.
in assessing the measurement uncertainty of the quantities determined during the uniaxial tensile test. These formulas were derived with the use of the MathCad package, using the symbolic calculation module.

**Figure A2:** Comparison of the real stress–strain curve determined for the estimated value of the strain hardening exponent and the curves drawn based on the strain hardening exponent reduced or increased by the value of absolute (example graph).

**Figure A3:** Comparison of the real stress–strain curve determined for the estimated value of the strain hardening exponent and the curves drawn based on the strain hardening exponent reduced or increased by the value of absolute (zoom of the graph).