INFRARED OBSERVATIONAL MANIFESTATIONS OF YOUNG DUSTY SUPER STAR CLUSTERS

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ABSTRACT

The growing evidence pointing at core-collapse supernovae as large dust producers makes young massive stellar clusters ideal laboratories to study the evolution of dust immersed in a hot plasma. We address the stochastic injection of dust by supernovae, and follow its evolution due to thermal sputtering within the hot and dense plasma generated by young stellar clusters. Under these considerations, dust grains are heated by means of random collisions with gas particles which result in the appearance of infrared spectral signatures. We present time-dependent infrared spectral energy distributions that are to be expected from young stellar clusters. Our results are based on hydrodynamic calculations that account for the stochastic injection of dust by supernovae. These also consider gas and dust radiative cooling, stochastic dust temperature fluctuations, the exit of dust grains out of the cluster volume due to the cluster wind, and a time-dependent grain size distribution.

Key words: dust, extinction – galaxies: star clusters: general – hydrodynamics

1. INTRODUCTION

The idea of core-collapse supernovae as major dust producers was first envisaged in the pioneering work of Cernuschi et al. (1967). They showed that the effective condensation of refractory elements due to the large variation of temperature in the ejecta of core-collapse supernovae can lead to the formation of massive quantities of dust. However, it took more than two decades until SN1987A provided the first direct evidence for the condensation of iron into dust grains (Moseley et al. 1989; Suntzeff & Bouchet 1990; Wooden et al. 1993; Bautista et al. 1995, and references therein) in the SN ejecta. According to Todini & Ferrara (2001) and Nozawa et al. (2003), one can expect the formation of (0.1–1) M⊙ of dust in the first decades after an SN II event, during which a dust mass fraction between 0.2 and 1.0 would be destroyed by the supernova reverse shock before being injected into the ISM (Nozawa et al. 2007). Also, the dust composition consists mostly of silicates and carbon (see conflicting interpretations of which composition is dominant by Matsuura et al. 2015 and Dwek & Arendt 2015).

These predictions find strong support in recent Herschel and ALMA observations of nearby supernova remnants like the Crab Nebula, Cassiopeia A, and SN1987A. Gomez et al. (2012) found evidence for the presence of 0.1–0.25 M⊙ of ejected dust in the Crab Nebula, a value that is orders of magnitude larger than what was obtained with Spitzer data (Temim et al. 2012). Barlow et al. (2010) estimated 0.075 M⊙ of cool dust (∼35 K) in the ejecta of Cassiopeia A; however, due to high cirrus contamination along the line of sight, they were not able to identify the presence of cold dust (∼20 K) which could increase the content of dust in the ejecta to values in the range of 0.5–1.0 M⊙ (Gomez 2013). More recently, Arendt et al. (2014) estimated the total mass of dust in the shocked ISM and ejecta regions of Cassiopeia A to be 0.04 M⊙, and ≤0.1 M⊙ in the unshocked ejecta.

Indebetouw et al. (2014) and Matsuura et al. (2015) fitted the spectral energy distribution (SED) of SN1987A and derived ~0.8 M⊙ of newly formed dust in the ejecta of the supernova with ~0.3 M⊙ of amorphous carbon and ~0.5 M⊙ of silicates. On the other hand, Dwek & Arendt (2015) derived a total mass of dust after day 8500 after the explosion of SN1987A consisting of ~0.4 M⊙ of silicates and ~0.05 M⊙ of amorphous carbon.

The large SN rate expected in super star clusters (SSCs) (a few thousand SN events during the SN II era for a 107 M⊙ star cluster), together with the large production and injection of dust, implies a frequent replenishment of dust inside the star cluster volume (Tenorio-Tagle et al. 2013). In such clusters, the thermalization of the matter reinserted by massive stars and SNe inside young and massive star clusters leads to a large central overpressure and the launching of hot (∼107 K) and dense (∼(1–1000) cm−3) star cluster winds (Chevalier & Clegg 1985; Tenorio-Tagle et al. 2007).

These considerations make SSCs ideal places to heat newly injected dust grains due to the transfer of thermal energy from the gas via stochastic collisions with electrons and nuclei, as discussed by Dwek (1986).

Dust grains then cool down in a short timescale and re-emit the obtained energy in the infrared regime. This is a very effective cooling mechanism for the hot and dusty gas which can surpass the cooling from a gas in collisional ionization equilibrium by several orders of magnitude (Ostriker & Silk 1973; Dwek & Werner 1981; Dwek 1987; Smith et al. 1996; Guillard et al. 2009). Infrared excesses have been observed in a considerable number of star clusters in the low-metallicity blue compact dwarf galaxies, e.g.: SBS 0335-052E, Haro 11, Mrk 930, and I Zw18 (Vanzo et al. 2000; Adamo et al. 2010a, 2010b, 2011; Fisher et al. 2014; Izotov et al. 2014). From these studies, Vanzo et al. (2000), Reines et al. (2008), and Izotov et al. (2014) have invoked a hot dust component (∼800 K) in order to explain the near-infrared SEDs observed in the bright SSCs 1 and 2 in SBS 0335-052.

Here we combine Dwek’s (1986, 1987) stochastic dust heating and cooling prescriptions with our steady-state wind hydrodynamic model to propose several scenarios of dust injection and its influence on the SEDs from starburst regions. To follow the evolution of the dust size distribution, and therefore the evolution of the SEDs, it is crucial to notice that the stellar winds are steady but the rate of supernova makes the dust injection a stochastic process. Dust is injected into the intracluster medium by stochastic supernova events, collisionally heated, and eroded before the next injection episode.
Moreover, we consider the exit of dust grains as they stream out, coupled to the gas, from the starburst region.

The paper is organized as follows: in Section 2 we describe our star cluster, star cluster wind, and time-dependent dust size distribution models, and formulate our assumptions regarding the dust grain physics and composition. In Section 3, we use the hydrodynamic results together with the physics of stochastic dust heating and cooling to obtain the expected SEDs of young stellar clusters. In Section 4 we summarize our results and outline our conclusions. Complementary information about the dust cooling model and stochastic dust temperature fluctuations are presented in Appendices A.1 and A.2.

2. STAR CLUSTER WINDS AND DUST INJECTION

We consider young and massive star clusters in which massive stars follow a generalized Schuster stellar density distribution, \( \rho_r \propto \left[1 + (r/R)^2\right]^{-\beta} \), (Paloû et al. 2013; Tenorio-Tagle et al. 2013, 2015) with \( \beta = 1.5 \), where \( r \) is the distance from the cluster center and \( R_c \) is the core radius of the stellar distribution. This stellar distribution is truncated at radius \( R_{SC} \), the star cluster surface. Both \( R_c \) and \( R_{SC} \) define the degree of compactness of the star cluster, which can be measured by the radius at which half of the star cluster mass is located, \( R_{50} \). Similarly to Tenorio-Tagle et al. (2013), we consider star clusters in which the other input parameters are: the star cluster mechanical luminosity, \( L_{SC} \), and the adiabatic wind terminal speed, \( V_{AW} \), which are related to the mass deposition rate \( M = 2 L_{SC}/V_{AW}^2 \). We assume that the mechanical luminosity scales with the total mass of the star cluster, \( M_{SC} \), as \( L_{SC} = 3 \times 10^{39} (M_{SC}/10^5M_\odot) \text{ erg s}^{-1} \) (Leitherer et al. 1999).

In our approach, the supernova explosions inject dust uniformly throughout the cluster with a standard Mathis et al. (1977, hereafter MRN) grain size distribution (dust grain number density in the size interval \( a \) and \( a + \Delta a \)):

\[
\frac{\partial n_i^{\text{inj}}}{\partial a} = A_i^{(m)} a^{-\alpha}, \quad a_{\text{min}} \leq a \leq a_{\text{max}},
\]

where \( a_{\text{min}} \) and \( a_{\text{max}} \) are the minimum grain size and cut-off value of the size distribution. In this definition, the subindex \( i \) is used to distinguish between dust species, in our case graphite and silicate, and the subindex \( m \) numbers the consecutive dust injection events.

The normalization factors, \( A_i^{(m)} \) (with units cm\(^{-\alpha-4}\)) are obtained from the condition:

\[
A_i^{(m)} = \frac{f_i M_{\text{SSN}}^{(m)}}{V_{SC}} \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{4}{3} \pi \rho_i a^{-\alpha-4} da,
\]

where \( \rho_i \) is the dust grain density, \( f_i \) is the mass fraction of the silicate and graphite species, \( M_{\text{SSN}}^{(m)} \) is the total mass of dust injected in a single supernova event, and \( V_{SC} \) is the star cluster volume. Table 1 summarizes the input parameters for the injected dust size distribution and the characteristics of the dust species, in our case, graphite and silicate grains.

The dust lifetime against thermal sputtering at temperatures above \( 10^6 \) is defined as \( \tau_{\text{sput}} = a f_i \) where \( a \) is the rate at which the dust grain with radius \( a \) decreases with time \( t \) when it is immersed into a hot plasma with temperature \( T \) and density \( n \), is given by (Tsai & Mathews 1995):

\[
\dot{a} = \frac{da}{dt} = -1.4 n h \left[ \frac{T}{T_w} \right]^{w} + 1 \right]^{-1},
\]

which leads to:

\[
\tau_{\text{sput}} = 7.07 \times 10^{5} \frac{a^{(m)} \mu m}{a^{(m)} \text{cm}^{-3}} \left[ \frac{T}{T_w} \right]^{w} + 1 \text{ years},
\]

where \( h \), \( T_w \) and \( w \) are constants with values \( h = 3.2 \times 10^{-18} \text{ cm}^4 \text{s}^{-1} \), \( T_w = 2 \times 10^6 \text{ K} \), and \( w = 2.5 \). These formulas are an approximation to the detailed calculations of Draine & Salpeter (1979) and Tielens et al. (1994) for graphite and silicate grains. The continuity equation which governs the evolution of the dust size distribution due to thermal sputtering is (Laor & Draine 1993; Yamada & Kitayama 2005):

\[
\dot{a} = \frac{\partial}{\partial a} \left( \frac{\partial n_i}{\partial a} \right) + \frac{\partial}{\partial t} \left( \frac{\partial n_i}{\partial a} \right) = \begin{cases} A_i^{(m)} a^{-\alpha}/\tau_i^{(m)}, & \text{if } t \leq \tau_{i}^{(m)} + \tau_{\text{inj}}^{(m)}, \\ 0, & \text{if } t > \tau_{i}^{(m)} + \tau_{\text{inj}}^{(m)}, \end{cases}
\]

where the first case applies for a constant MRN dust injection during a timescale \( \tau_{i}^{(m)} \) after the \( m \)-supernova event has occurred (at \( t = \tau_{i}^{(m)} \)); the second case considers that the \( m \)-supernova dust injection has ceased. The solutions for Equations (5) after the \( n \)-supernova event (the last event considered) are then:

\[
\frac{\partial n_i}{\partial a} = \begin{cases} \sum_{m=1}^{n} \frac{A_i^{(m)}}{\tau_{i,j}^{(m)}} \left[ a^{\alpha+1} + \left[ a - \dot{a} (t - \tau_{i,j}^{(m)}) \right]^{-\alpha+1} \right]^{-1}, & \text{if } t \leq \tau_{i}^{(m)} + \tau_{\text{inj}}^{(m)}, \\ \sum_{m=1}^{n} \frac{A_i^{(m)}}{\tau_{i,j}^{(m)}} \left[ a^{\alpha+1} \right]^{-1}, & \text{if } t > \tau_{i}^{(m)} + \tau_{\text{inj}}^{(m)}, \end{cases}
\]

Note. \({}^{a}\) Hirashita & Nozawa (2013).

| Symbol | Silicate | Graphite | Definition |
|--------|---------|---------|------------|
| \( \rho_{gr} \) | 3.3 | 2.26 | Dust grain density (g cm\(^{-3}\)) |
| \( \alpha \) | 3.5 | 3.5 | Power index of the MRN distribution |
| \( f_i \) | 0.5 | 0.5 | Mass fraction of the grain species |
with the conditions that $A^{(m)} = 0$ until the $m$-supernova event occurs and the mass of dust at $t = \tau_{SN}^{(1)} = 0$ equals zero. These general solutions take into account the residual mass of dust from the previous injections and the evolved dust size distribution associated with them. Note that the asymptotic behavior of these solutions is similar to that derived by Dwarka et al. (2008) for the case of grain destruction with continuous injection.

The total mass of dust for each dust species as a function of time is then:

$$M_d(t) = \frac{4\pi}{3} \rho_{\text{gas}} V_{\text{SC}} \int_{a_{\text{min}}}^{a_{\text{max}}} a^3 \frac{\partial n_i}{\partial a} da,$$

which implies a time-dependent dust-to-gas mass ratio given by:

$$Z_d(t) = \frac{4\pi}{3} \frac{\rho_{\text{gas}}}{\rho} \int_{a_{\text{min}}}^{a_{\text{max}}} a^3 \frac{\partial n_i}{\partial a} da,$$

where $\rho = 1.4 m_H n$ is the gas mass density and $m_H$ is the hydrogen mass.

The above equations do not take into account that dust, independent of its size, is expelled out from the cluster and thus $A^{(m)}$ is no longer a constant. The rate at which dust is ejected from the cluster is $M_d(t) = 4\pi R_{\text{SC}} \rho_c Z_d(t)$, where $c_s$ is the outflow’s local sound speed obtained from the wind hydrodynamical calculations.

In order to consider the dust outflowing from the cluster, we have taken a finite differences approach described as follows: (1) calculate $M_d(t)$ with the original value of $A^{(m)}$ at $t = \tau_{SN}^{(m)} + \Delta t$; (2) at the next time step, $t = \tau_{SN}^{(m)} + 2\Delta t$, subtract $M_d(t)\Delta t$ to $M_d(t)$ and with this mass, replace $A^{(m)}$ with

$$A^{(m)}_i = \frac{f_i \left[ M_d(t) - M_d(t) \Delta t \right]}{\int_{a_{\text{min}}}^{a_{\text{max}}} \frac{4\pi}{3} \rho_{\text{gas}} a^3 \frac{\partial n_i}{\partial a} da},$$

(3) repeat the procedure for every time step and for the normalization constants associated with each supernova dust injection. In our calculations, we have taken $\Delta t = 100$ years and $\tau_{SN}^{(m)} = \tau_{\text{min}} = 1000$ years (the same timescale for every dust injection). We tested this method against the analytic solution, Equations (1)–(8), in the case when $M_d(t) = 0$, and both methods agree very well.

We have selected normally distributed pseudo-random values for $M_{\text{dSN}}^{(m)}$ (except for the first supernova, in which $M_{\text{dSN}}^{(m)}$ was chosen so that $Z_d = 10^{-3}$ at $\tau_{\text{min}}$) and $\Delta \tau_{SN} = \tau_{\text{SN}}^{(n+1)} = \tau_{\text{SN}}^{(m)}$, the interval between the supernova events, with a mean 0.5 $M_\odot$ and standard deviation 0.15 $M_\odot$ for $M_{\text{dSN}}^{(m)}$; and a mean interval between the supernova explosions ($\sim 17,000$ years for a $10^4 M_\odot$ cluster, see Figure 1) obtained from the supernova rate output of the Starburst99 synthesis model (Leitherer et al. 1999) with a standard Kroupa initial mass function with lower and upper cut-off masses 0.1 $M_\odot$ and 100 $M_\odot$, respectively. The standard deviation for $\Delta \tau_{SN}$ was taken to be 10% of the mean value.

These considerations imply the presence of a time-dependent reservoir of dust grains embedded into the high-temperature ($\sim 10^6$–$10^7 K$) thermalized gas inside the star cluster volume.

We calculate the gas number density and temperature inside the star cluster by making use of our hydrodynamical model (thoroughly discussed in Silich et al. 2004, 2011; Palouš et al. 2013). Our models are quasi-adiabatic; however, they include the effects of gas (Raymond et al. 1976) and dust radiative cooling (Dwek 1987) (see Appendix A.1 for the complete description of the dust cooling calculation).

Once we know the conditions prevailing inside the star cluster volume (i.e., average values for the gas density and temperature), we can apply Dwek (1986) prescriptions to calculate the temperature distribution, $G(a, T_j)$, of dust grains that follow different dust size distributions. The infrared flux per unit wavelength, produced by a population of dust grains with the same chemical composition, from a star cluster located at distance $D_{\text{SC}}$, can then be calculated as (Dwek & Arendt 1992):

$$f_\lambda = \left( \frac{1.4 m_H Z_d N_H}{\rho_d} \int_{a_{\text{min}}}^{a_{\text{max}}} \int_0^\infty a^2 \frac{\partial n_i}{\partial a} Q_{\text{abs}}(\lambda, a) B_\lambda(T_d) G(a, T_d) dT_d da \right) \pi \Omega_{\text{SC}}$$

in units erg s$^{-1}$ cm$^{-2}$ Å$^{-1}$, or alternatively, in units erg s$^{-1}$ cm$^{-2}$ Hz$^{-1}$ 1 Jansky, if one is interested in the flux per unit frequency, $f_\nu$, where $f_\lambda = c^2 f_\nu / \lambda^2$. Since both quantities, $f_\lambda$ and $f_\nu$, are widely used by different authors (e.g., Reines et al. 2008; Adamo et al. 2010b; Fisher et al. 2014; Izotov et al. 2014), we present them both in all our figures, taking into account the contribution from graphite and silicate grains. In the above equation, $N_H$ is the hydrogen column density throughout the star cluster volume ($=4/3 n_{\text{RSC}}$), $a$ is the dust grain radius, $\rho_d = 4\pi \int_{a_{\text{min}}}^{a_{\text{max}}} a^2 \frac{\partial n_i}{\partial a} da$ is the size-averaged dust density, and $\Omega_{\text{SC}} = \pi (R_{\text{SC}}/D_{\text{SC}})^2$ is the solid angle subtended by the star cluster. Additionally, $T_d$ is the dust temperature, $G(a, T_d)$ is the dust temperature distribution resulting from stochastic temperature fluctuations, $Q_{\text{abs}}(\lambda, a)$ is the dust absorption efficiency, and $B_\lambda(T_d)$ is the Planck function in terms of the wavelength, $\lambda$. In our models we have set the distance to the star cluster as $D_{\text{SC}} = 10$ Mpc. A complete discussion of the stochastic dust temperature fluctuations is presented in Appendix A.2.

In all our calculations we neglected the charge of the dust grains (Smith et al. 1996) as well as the contribution to the infrared flux from dust grains outside the star cluster volume. We are also not dealing with the possible effects related to the absorption of ionizing photons by dust grains, which could be important as long as the ionizing absorption of ionizing photons by dust grains, which could be important as long as the ionizing photons are absorbed before they can ionize the gas. However, coeval clusters suffer a substantial reduction of their ionizing photon flux as soon as they enter the SN era. The number of emitted UV photons falls with time, as $t^{-5}$ (Beltrametti et al. 1982) and thus after 5–6 Myr the number of UV photons is almost two orders of magnitude smaller than that at the start of the evolution. This fall in the ionizing flux reduces the time during which the UV radiation may be more important than gas–dust collisions, which could be important during the SN II era (from ~3 to 40 Myr). The hydrodynamical model assumes that the dust grains move with the same velocity as the injected gas and thus we have not considered the effects of kinetic sputtering in the intracluster medium.
3. INFRARED SPECTRAL ENERGY DISTRIBUTIONS

To assess the impact that the collisional heating of dust grains has on the expected infrared SEDs from young and massive dusty star clusters, we have run several models with the input parameters described in the previous section \((L_\text{SC}, V_\infty, R_c, R_{\text{SC}}, a_{\text{min}}, a_{\text{max}}, \text{ and } t)\). Our reference model \(A\), consists of a star cluster with a total mass of \(10^5 M_\odot\) (which corresponds to the mechanical luminosity \(3 \times 10^{39} \text{ erg s}^{-1}\)), \(R_{\text{hm}} = 3.92 \text{ pc}\) (obtained from values \(R_c = 4 \text{ pc}\) and \(R_{\text{SC}} = 5 \text{ pc}\)), an adiabatic wind terminal speed \(V_\infty = 1000 \text{ km s}^{-1}\), lower and upper limits for the injected dust size distribution, \(a_{\text{min}} = 0.001 \mu\text{m}\), and \(a_{\text{max}} = 0.5 \mu\text{m}\), respectively; and an equal mixture of graphite and silicate grains. The other models vary one or more of the input parameters with respect to model \(A\). Models \(B\)–\(C\) explore different values of the mechanical luminosity, models \(D\) and \(E\) vary the adiabatic wind terminal speed, and models \(F\) and \(G\) differ in the compactness of the star cluster. The reference model \(A\), as well as models \(B\)–\(G\), are evaluated at the end of the first injection event \((t = 1000 \text{ years})\) where, as pointed out before, \(M_\text{dSN}^{(1)}\) is set to allow \(Z_d(t_{\text{hm}})\) to be equal to \(10^{-3}\). Models \(A1\)–\(A4\) are evaluated at later times. Table 2 presents the input parameters for our 11 models.

3.1. The Reference Model

The outcomes for our reference model \(A\) are displayed in Figure 2. In this case, the prevailing conditions inside the star cluster are: an average value for the gas density \(\sim 10^{-3} \text{ cm}^{-3}\), and an average gas temperature \(\sim 1.35 \times 10^3 \text{ K}\). From these conditions we computed the dust temperature distributions, \(G(a, T_d)\), for different dust sizes, and calculated the resultant flux, and averaged it by the size distribution of the graphite and silicate grains (see Appendix A.1). In order to quantify the contribution to the total flux, we display separate fluxes for the graphite and silicate grains. As shown in panels (e) and (f), small grains \((\lesssim 0.05 \mu\text{m})\) are more likely to undergo strong temperature fluctuations and therefore span a wide range of temperatures (from a few \(\sim 10 \text{ K}\) to a few \(\sim 1000 \text{ K}\) for grains with \(a = 0.001 \mu\text{m}\), causing them all to strongly emit in near-infrared (NIR), mid-infrared (MIR), and far-infrared (FIR) wavelengths) due to their low heat capacities (which scale as \(\sim a^3\)) and small cross sections.

Big grains \((\gtrsim 0.1 \mu\text{m})\) with larger cross sections (which make them subject to more frequent collisions) nearly emit as a blackbody at their equilibrium temperature. Intermediate-size grains \((0.05 \gtrsim a \gtrsim 0.1)\) exhibit a combination of both behaviors. Hence, the emission from 1 to 8 \(\mu\text{m}\) is dominated by hot and small graphite grains. Between 9 and 14 \(\mu\text{m}\), the emission is dominated by the 10 \(\mu\text{m}\) broad feature, associated with the dust absorption efficiency, \(Q_{\text{abs}}(\lambda, a)\), of silicate grains. The emission from 15 to several hundred microns peaks around \(\sim 35 \mu\text{m}\); it is dominated by big grains near their equilibrium temperature \((\sim 93 \text{ K}\) for graphite grains and \(\sim 75 \text{ K}\) for silicate grains), and by small grains with temperatures ranging from \(\sim 10 \text{ K}\) to \(\sim 100 \text{ K}\). Note that the emission from the graphite and silicate grains is almost identical for \(\lambda \gtrsim 35 \mu\text{m}\) because their dust temperature distributions are very similar. In this case, the mass of dust inside the star cluster volume is \(M_d(t = t_{\text{hm}}) = 0.19 M_\odot\).

We have evaluated our reference model \(A\) at four later times (models \(A1, A2, A3,\) and \(A4\) evaluated at \(\sim 17,000, 17,500, 25,000,\) and 33,000 years, respectively). When dust injection is not taking place (models \(A2\)–\(A3\)), the dust size distribution and

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**Figure 1.** Evolution of the dust mass and dust-to-gas mass ratio with and without the exit of dust grains from the starburst region. Panels (a) and (b) show the residual \(M_d(t)\) and dust-to-gas mass ratio, respectively, from six injection events accounting for thermal sputtering. Solid lines depict the case when dust grains are subject to thermal sputtering; dashed lines also consider the case when dust is expelled out of the star cluster. This case corresponds to a \(3 \times 10^3 M_\odot\) cluster with \(V_\infty = 1000 \text{ km s}^{-1}\), \(R_{\text{sc}} = 5 \text{ pc}\), and \(R_c = 4 \text{ pc}\). The values of \(M_\text{dSN}^{(1)}\) and the interval between consecutive supernova events \(\Delta t_{\text{SN}}\) were pseudo-randomly selected. Note that the exit of dust grains in the cluster wind leads to a more rapid depletion of dust.

**Table 2**

| Model | \(t\) (years) | \(R_c\) (pc) | \(R_{\text{SC}}\) (pc) | \(L_{\text{SC}}\) (\(10^3\) \text{ erg s}^{-1}) | \(V_\infty\) (\(\text{km s}^{-1}\)) | \(Z_d\) (\(10^{-3}\)) |
|-------|---------------|-------------|---------------------|---------------------------------|-----------------|---------|
| \(A\)  | 1000          | 4           | 5                   | 3.52                            | 3\times10^{39} | 1000    | 1.0     |
| \(A1\) | 17000         | 4           | 5                   | 3.52                            | 3\times10^{39} | 1000    | 3.1     |
| \(A2\) | 17500         | 4           | 5                   | 3.52                            | 3\times10^{39} | 1000    | 2.5     |
| \(A3\) | 25000         | 4           | 5                   | 3.52                            | 3\times10^{39} | 1000    | 0.4     |
| \(A4\) | 33000         | 4           | 5                   | 3.52                            | 3\times10^{39} | 1000    | 0.1     |
| \(B\)  | 1000          | 4           | 5                   | 3.52                            | 1\times10^{39} | 1000    | 1.0     |
| \(C\)  | 1000          | 4           | 5                   | 3.52                            | 9\times10^{39} | 1000    | 1.0     |
| \(D\)  | 1000          | 4           | 5                   | 3.52                            | 3\times10^{39} | 750     | 1.0     |
| \(E\)  | 1000          | 4           | 5                   | 3.52                            | 3\times10^{39} | 1500    | 1.0     |
| \(F\)  | 1000          | 2           | 5                   | 2.98                            | 3\times10^{39} | 1000    | 1.0     |
| \(G\)  | 1000          | 4           | 7                   | 4.59                            | 3\times10^{39} | 1000    | 1.0     |
the dust-to-gas mass ratio rapidly evolve and greatly depart from the injected dust size distribution as a consequence of the short timescale for thermal sputtering (see Equation (4)). This is reflected in a lack of small grains, and therefore the NIR excesses noted in models $A$ and $A1$ (evaluated just before the end of the first and second dust injection episodes, respectively) rapidly vanish. This situation is illustrated in Figure 3, which shows evolving SEDs for models $A$, $A1$, $A2$, $A3$, and $A4$. The

Figure 2. The Reference Model. Top panels (a) and (b) show the gas density and temperature radial profiles for our reference model $A$. Panels (c) and (d) present the fluxes per unit wavelength, $f_{\lambda}$, and per unit frequency, $f_{n}$, respectively. The dashed line depicts the contribution from graphite grains, the dotted line is the contribution from silicate grains, whereas the solid line comprises both contributions. Bottom panels (e) and (f) present the dust temperature distribution for the different grain sizes for graphite and silicate grains, respectively. In the bottom panels, the solid, dashed, dotted, dashed–dotted, long-dashed, and the delta-like curves correspond to sizes $0.001$, $0.002$, $0.01$, $0.05$, $0.1$, and $0.5 \, \mu m$, respectively.
Figure 3. Spectral energy distributions for models A, A1, A2, A3, and A4. Top panels (a) and (b) show the evolution of the dust mass and dust-to-gas mass ratio, respectively, during the nine injection events, taking into account both the dust sputtering and their exit out of the cluster as a wind. The times at which models A1–A4 were evaluated are marked with crosses. Bottom panels (c) and (d) present the values of the fluxes per unit wavelength, \( f_\lambda \), and per unit frequency, \( f_\nu \), for each of the evolved models, respectively. Solid, dashed, dotted, dashed–dotted, and dashed-double-dotted lines depict the SEDs for models A, A1, A2, A3, and A4, respectively. Note that the strong emission present during the dust injection rapidly vanishes when the dust injection has ceased.

The top panels show the evolution of the dust mass, panel (a), and dust-to-gas mass ratio, panel (b), during the nine injection events; the times at which these models were evaluated are marked with crosses. The bottom panels show the evolving spectral energy distributions at the end of the first dust injection (1000 years, model A1), and at the end of the second dust injection. A strong emission at NIR and MIR wavelengths is present during the dust injection; however, when the dust injection has ceased this strong emission rapidly vanishes, which is notorious just 500 years after dust injection (model A). As more massive clusters are considered, a

stronger gas density inside the star cluster, which results in more frequent gas–grain collisions leading to an increase in the infrared emission (see Equations (21) and (23) in Appendix A.2). As more massive clusters are considered, a
higher supernova rate is expected, and therefore there is less time between the supernova episodes to erode dust grains. This leads to a more persistent dust reservoir at all times, which is reflected in an enhancement of the infrared spectrum. Thus, model C surpasses the infrared emissions from models A and B. However, model C is also more affected by thermal sputtering which is noticeable by a decreased emission at $15\mu m$. The mass of the dust inside $R_{SC}$ at $t = t_{\text{inj}}$ is $6.0 \times 10^{-2} M_\odot$ and $0.57 \ M_\odot$ for models B and C, respectively.

### 3.3. Models with Different Adiabatic Terminal Speeds

Figure 5 shows the results obtained from models A, G, and E. In these models we examine different values of the adiabatic wind terminal speed. As the gas density decreases with an increasing adiabatic wind terminal speed, the characteristic time between successive electron collisions with a dust grain increases (see Equation (21) in Appendix A.2). The dust grains then are less heated and their emission decreases (model E compared to model A). The opposite situation occurs in models with a lower value of the adiabatic wind terminal speed (model D compared to model A) which also causes a decrease in the emission at $\lambda \lesssim 15\mu m$ provoked by the depletion of small grains by the action of thermal sputtering in a denser medium. The mass of the dust inside $R_{SC}$ is $0.47 \ M_\odot$ and $5.29 \times 10^{-2} \ M_\odot$ for models D and E, respectively.

### 3.4. Models for Different Cluster Sizes

We now focus on models with different values of $R_c$ and $R_{SC}$ (see Figure 6). Model F is evaluated with a smaller star cluster core radius $R_c = 2 \ pc$. Model G corresponds to a star cluster with a larger cut-off radius $R_{SC} = 7 \ pc$. A more compact cluster, as in model F compared to model A, yields an enhanced flux at all wavelengths; however, this effect is more noticeable at FIR wavelengths where the role of thermal sputtering is less important. The situation is different when one considers a less compact cluster (model G compared to model A), when the gas number density is decreased, and therefore the dust emission is diminished. In these models half of the star cluster mass is located inside $3.52, 2.98$, and $4.59 \ pc$ for models A, F, and G, respectively. The mass of the dust inside the star cluster volume is $0.21 \ M_\odot$ and $8.62 \times 10^{-2} \ M_\odot$ for models F and G, respectively.
4. CONCLUSIONS

Motivated by the abundant evidence for core-collapse supernovae as major dust producers, and the large SN rate expected in young massive star clusters, we have studied the frequent injection of dust grains into the plasma interior of SSCs, which become ideal places to heat dust grains by means of random gas–grain collisions. This has led us to combine our hydrodynamic star cluster wind model with the stochastic dust injection, heating, and cooling models to calculate the expected SEDs from super stellar clusters.

We have followed the evolution of the grain size distribution, what changes drastically, and the resultant spectrum. We have also considered the exit of the dust grains as they stream out coupled to the gas, to compose the star cluster wind. For the latter, we have used a finite difference method.

In this scenario, a certain mass of silicate and graphite dust, with an initial grain size distribution is injected into the intracluster medium. On top of this, the stellar winds are steady but the rate of supernova makes the dust injection a stochastic process. Therefore, dust is injected into the medium stochastically, and then heated and eroded before the next injection episode.

Several models were defined in order to quantify all these effects in the resultant infrared spectrum. Models which give more weight in the dust size distribution to small grains (as when dust injection is taking place), as well as models with larger values of the star cluster mechanical luminosity (in which the SN rate increases leading to more persistent dust reservoirs), lead to an enhanced dust emission. The opposite situation occurs with more extended star clusters and larger adiabatic terminal speeds, which lead to a decrease in their dust emission. When dust injection ceases, the resultant SEDs change drastically, and the emission at NIR–MIR wavelengths vanishes due to thermal sputtering acting more effectively on small grains.

Despite the fact that our models imply the presence from hundredths to tenths of solar masses of dust inside the star cluster volume and transient strong NIR–MIR infrared excesses, the predicted SEDs are strong enough to be considered in order to explain the infrared excesses observed in bright young clusters and other stellar systems. In those cases, the combined action of many nearby star clusters, as well as higher SN rates in more massive clusters, could lead to persistent infrared excesses. This and a detailed comparison with the observations of starburst galaxies will be addressed in a forthcoming communication.
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APPENDIX A

A.1. The Cooling Function via Gas–Grain Collisions

Following Dwek (1987), and keeping most of his notations and definitions, we calculated the cooling rate due to the gas–grain collisions in a dusty plasma with a normal chemical composition (1 He atom per 10 H atoms):

$$\Lambda_d = \frac{n_d}{n_e n} \frac{H_{\text{coll}}}{\rho_d} = \frac{1.4 m_H Z_d}{\rho_d} \left( \frac{32}{\pi m_e} \right)^{1/2}$$

$$\times \pi (k_B T)^{3/2} \left[ h_e + \frac{11}{23} \left( \frac{m_e}{m_H} \right)^{1/2} h_n \right]$$

where $n$, $n_d$ and $n_e$ are the gas, dust, and electron number density, $H_{\text{coll}}$ is the heating rate of a single grain due to collisions with incident gas particles, and $k_B$ is the Boltzmann constant. Functions $h_e$ and $h_n$ are the effective grain heating efficiencies due to impinging electrons and nuclei, respectively:

$$h_e = \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{\zeta(a, E)}{2} \frac{a^2 e^{-\lambda a}}{\lambda a} \frac{\partial n_i}{\partial a} \bigg|_{\alpha} \, da,$$

$$h_n = \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \left\{ 1 - \left( 1 + \frac{\lambda a}{2} \right) e^{-\lambda a} \right\} a^2 \frac{\partial n_i}{\partial a} \bigg|_{\alpha} \, da,$$
where \( \rho_g = \frac{4}{3} \pi \rho_{gs} \int_{a_{\text{min}}}^{a_{\text{max}}} a^{-4} \frac{\partial n_i}{\partial a} \, da \) is the size-averaged dust density, \( \rho_{gs} \) is the grain density, \( x_e = E/k_B T \), \( E \) is the energy of the impinging electron, \( \zeta(a, E) \) is the fraction of the electron kinetic energy transferred to the dust grain, \( n_\text{H} = E_\text{H}/k_B T \), \( n_\text{He} = E_\text{He}/k_B T \), and the energies from the incident hydrogen and helium nuclei are \( E_\text{H} = 133a \text{ keV} \), \( E_\text{He} = 222a \text{ keV} \), where \( a \) is measured in microns,

\[
\zeta(a, E) = \begin{cases} 0.875, & \text{if } E \leq E^* \\ 1 - \frac{E_f}{E}, & \text{otherwise} \end{cases}
\]

where \( E_f = \max\{E', 0.125E\} \), with \( E' \) and \( E' \), the critical energy at which an electron penetrates the dust grain and the final energy of the electron after penetrating the dust grain, respectively. \( E \) and \( E' \) are obtained by solving the following system of nonlinear equations based on experimental data

\[
\log R(E^*) = \log \left( \frac{4a\rho_g/3}{4a\rho_g/3} \right) = 0.146 \log E^{*2} + 0.5 \log E^* - 8.15,
\]

\[
\log R(E) = 0.146 \log E^2 + 0.5 \log E - 8.15,
\]

\[
\log R(E') = \log R(E) - R(E^*) = 0.146 \log E^2 + 0.5 \log E' - 8.15.
\]

In Figure 7, we show examples of the dust cooling curves for different cases of the MRN dust size distributions in which the value of the dust-to-gas mass ratio is set to \( Z_d = 10^{-3} \). Finally, we provide tables which contain the results of the calculations of the dust cooling function for different MRN dust size distributions as a function of the gas temperature, and normalized to the dust-to-gas mass ratio, for \( a_{\text{min}} \) set to 0.001 \( \mu \text{m} \) and different \( a_{\text{max}} \) values (Table 3), and for \( a_{\text{max}} \) set to 0.5 \( \mu \text{m} \) and different \( a_{\text{min}} \) values (Table 4).

A.2. Stochastic Dust Temperature Distribution

In order to calculate the temperature distribution of dust grains subjected to a bath of free electrons in a hot gas, and thus the emission by such dust grains, we follow the schemes proposed by Dwek (1986) and Guhathakurta & Draine (1989) with a few extra considerations. In Dwek’s scenario, a dust grain with an initial temperature, \( T_0 \), collides with a free electron with energy, \( E \), which transfers a fraction of its kinetic energy, \( \zeta(a, E) \), to the dust particle. Depending on the size and chemical composition of the dust grain (because its heat capacity, \( C(a, T_0) \), is a function of both), the dust particle will be heated to a peak temperature \( T_{\text{peak}} \), which is obtained from iterating the equation

\[
\zeta(a, E)E = \int_{T_0}^{T_{\text{peak}}} C(a, T_d) \, dT_d.
\]

From \( T_{\text{peak}} \), the dust particle starts to cool down and eventually, after many collisions, it acquires thermodynamic equilibrium, unless the characteristic time for the electron–grain collisions is larger than the grain cooling time, in which case the grain temperature will start to fluctuate (Dwek 1986; Dwek & Arendt 1992). The grain cooling time, \( \tau_{\text{cool}} \), between \( T_{\text{peak}} \) and some temperature, \( T_d \), is given by

\[
\tau_{\text{cool}} = \int_{T_0}^{T_{\text{peak}}} \frac{C(a, T_d) \, dT_d}{4\pi a^2 \sigma \langle Q_{\text{abs}} \rangle T_d^3}.
\]

where \( \sigma \) is the Stefan–Boltzmann constant and \( \langle Q_{\text{abs}} \rangle \) is the dust absorption efficiency, \( Q_{\text{abs}}(\lambda, a) \), averaged by the Planck function, \( B_\lambda(T_d) \), in terms of the wavelength, \( \lambda \):

\[
\langle Q_{\text{abs}} \rangle = \frac{1}{\sigma T_d^4} \int_0^\infty \pi Q_{\text{abs}}(\lambda, a) B_\lambda(T_d) \, d\lambda.
\]

The values of \( C(a, T_d) \) for the silicate and graphite grains were taken from Dwek (1986) and from Draine & Anderson (1985), while the values of \( Q_{\text{abs}}(\lambda, a) \) were obtained from the data files provided in the DustEM code\(^1\) (Compiégne et al. 2011). The characteristic time between successive electron collisions with

\(^1\) http://www.ias.u-psud.fr/DUSTEM
a dust grain, $\tau_{\text{coll}}$, is calculated as (Bocchio et al. 2013):

$$\tau_{\text{coll}}^{-1} = \pi a^2 n \sqrt{\frac{3k_B T}{m_e}}, \quad (21)$$

where $m_e$ is the mass of the electron. The fraction of time in which a dust grain can be found in the temperature interval $T_d + dT_d$ after a collision with an electron is (Purcell 1976):

$$P(a, E, T_d, T_0) dT_d = \begin{cases} C(a, T_d) e^{-\tau_{\text{coll}}/T_d} & \text{if } T_d \leq T_{\text{peak}} \\ 0 & \text{otherwise} \end{cases}$$

$$= \frac{4\pi a^2 \sigma}{Q_{\text{abs}}} \int_0^{\infty} dT_d \int_0^T dT_{\text{col}} \tau_{\text{coll}}^{-1} \tau_{\text{coll}}^2$$

Table 3

| $a_{\text{min}}$ ($\mu$m) | 0.001 | 0.002 | 0.005 | 0.01 | 0.05 | 0.1 | 0.5 |
|--------------------------|-------|-------|-------|------|------|-----|-----|
| $T$ (K)                  |       |       |       |      |      |     |     |
| $\Lambda_{\text{d}}/Z_d$ (erg s$^{-1}$ cm$^{-2}$) |       |       |       |      |      |     |     |
One can now obtain the probability, $G(a, T_d, T_0)$, that a dust grain is to be found between $T_d$ and $T_d + dT_d$ if one integrates the above quantity over all the electron energies according to the Maxwell–Boltzmann distribution

$$G(a, T_d, T_0) = \pi a^2 n_{\text{coll}} \int_0^\infty P(a, E, T_d, T_0) f(E)\nu(E)dE. \quad (23)$$
temperatures, we will employ the stochastic matrix method described by Guhathakurta & Draine (1989) and Mar- engo (2000).

Let \( A_{T_0, T_0'} \), be an \( N \times N \) stochastic matrix, which describes the probability (per unit time) of a grain to make the transition between \( T_0 \) and some temperature \( T_0' \). The entries of \( A_{T_0, T_0'} \) are obtained from evaluating Equation (23):

\[
A_{T_0, T_0'} = \begin{pmatrix}
G(a, T_0, T_0) & G(a, T_0, T_0') & \cdots & G(a, T_0', T_0') \\
G(a, T_0, T_0') & G(a, T_0', T_0) & \cdots & G(a, T_0', T_0') \\
\vdots & \vdots & \ddots & \vdots \\
G(a, T_0', T_0) & G(a, T_0', T_0') & \cdots & G(a, T_0', T_0')
\end{pmatrix}
\]

(24)

In our case, we employed a logarithmic grid for \( T_0 \) and \( T_0' \), from 1 to 1100 K, and \( N = 125 \).

Now let \( G^i_{n=0} \) be the initial temperature distribution given by a column vector that comes from evaluating Equation (23) with \( T_0 = T_{0\text{eq}} \), a trial initial temperature:

\[
G^i_{n=0} = \begin{pmatrix}
G(a, T_0, T_{0\text{eq}}) \\
G(a, T_0', T_{0\text{eq}}) \\
\vdots \\
G(a, T_0', T_{0\text{eq}})
\end{pmatrix}
\]

(25)

We apply the stochastic matrix to the initial temperature distribution to obtain a new stochastic temperature distribution, \( G^i_{n+1} = G^i_{n=0} A_{T_0, T_0'} \). We continue to iteratively apply the stochastic matrix,

\[
G^i_{n+1} = G^i_{n=0} A_{T_0, T_0'},
\]

(26)

until the condition \((I - A_{T_0, T_0'})G_{n+1} = 0\), with \( I \) as the identity matrix, is fulfilled. This condition ensures that, after many discrete heating events, the temperature distribution does not change from the application of the stochastic matrix; this is the steady state temperature distribution, \( G(a, T_0) \).

Big grains (\( \gtrsim 0.1 \mu m \)), with large cross sections and heat capacities, are more likely to reach thermodynamic equilibrium due to very frequent collisions. In that case, their temperature distribution approaches a delta function around the equilibrium temperature, \( T_{eq} \), which can be obtained by equating the heating and cooling rates:

\[
\pi a^2 n \int_0^\infty f(E)v(E)\zeta(a, E)dE = 4\pi a^2 \sigma (Q_{abs}) T_{eq}^4.
\]

(27)

Once the dust temperature distribution is known, the infrared flux can be calculated from Equation (15) of Dwek & Aren (1992).