Photoproduction of $h_c$

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Abstract

Using the NRQCD factorization formalism, we calculate the total cross section for the photoproduction of $h_c$ mesons. We include color-octet and color-singlet mechanisms as well as next-to-leading order perturbative QCD corrections. The theoretical prediction depends on two nonperturbative matrix elements that are not well determined from existing data on charmonium production. For reasonable values of these matrix elements, the cross section is large enough that the $h_c$ may be observable at the E831 experiment and at the HERA experiments.

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Of all the charm-anticharm quark boundstates lying below the threshold of open charm production, the most elusive to experimental investigation is the $h_c$ meson. To date this particle has been observed by only a few experiments, which measure an average mass of $M_{h_c} = 3.52614 \pm 0.00024 \text{ GeV}$ [1]. Studies of the $h_c$ are difficult because it has quantum numbers $J^{PC} = 1^{+-}$, and thus cannot be produced resonantly in $e^+e^-$ annihilation, or appear in the decay of a $J^{PC} = 1^{--}$ charmonium state via an electric dipole transition.

Since charm quarks are heavy compared to $\Lambda_{QCD}$, it is natural to view charmonium as a nonrelativistic system, where the $h_c$ is a spin-singlet P-wave state of a $c$ and $\bar{c}$. This approach is taken in the color-singlet model [2] (CSM) of quarkonium production and decay. In the CSM, it is assumed that the $c\bar{c}$ must be produced in a color-singlet state with the same angular-momentum quantum numbers as the charmonium meson which is eventually observed. However, the CSM has serious deficiencies. It is well known that perturbative QCD calculations of production and decay of P-wave quarkonia within the CSM are plagued by infrared divergences [3]. More recently, measurements made at the Fermilab Collider Detector Facility (CDF) show that the CSM also fails to accurately predict the production cross sections of S-wave quarkonia ($J/\psi$ and $\psi'$) [4].

The naive CSM has been supplanted by the NRQCD factorization formalism of Bodwin, Braaten, and Lepage [5]. This formalism allows the infrared safe calculation of inclusive charmonium production and decay rates. It also predicts new mechanisms in which a $c\bar{c}$ pair is produced at short distances in a color-octet state, and hadronizes into a final state charmonium nonperturbatively. These color-octet mechanisms can naturally account for the CDF data on $J/\psi$ and $\psi'$ production.

In this paper, we examine photoproduction of $h_c$ within the NRQCD factorization formalism. Color-octet mechanisms play an essential role. By itself the CSM yields infrared divergent expressions for the $h_c$ cross section; however, once color-octet contributions are included we obtain sensible predictions. The resulting expression then depends on two undetermined nonperturbative matrix elements. Using heavy-quark spin symmetry these can, in principle, be estimated from similar matrix elements extracted from data on the production and decay of $\chi_{cJ}$ mesons. Currently, theoretical and experimental uncertainties in the determination of the $\chi_{cJ}$ matrix elements preclude us from making definitive predictions for the $h_c$ photoproduction cross section. This will be discussed in greater detail below. However, for some choices of the matrix elements which are consistent with current data, we find a large enough cross section that the $h_c$ may be observed in current photoproduction experiments at the DESY HERA collider and at the Fermilab fixed target experiment E831.

In the NRQCD factorization formalism, inclusive quarkonium production cross sections have the form of a sum of products of short-distance coefficients and NRQCD matrix elements. The short-distance coefficients are associated with the production of a heavy quark-antiquark pair in specific color and angular-momentum states. They can be calculated using ordinary perturbative techniques, and are thus an expansion in the strong coupling constant $\alpha_s$. The NRQCD matrix elements parameterize the hadronization of the quark-antiquark pair, and each scales as a power of the average relative velocity $v$ of the heavy quark and antiquark as determined by the NRQCD velocity-scaling rules [3].

The NRQCD factorization formula for $h_c$ photoproduction, at leading order in $v$, is

$$
\sigma(\gamma + N \rightarrow h_c + X) = \int dx \sum_i f_{i/N}(x) \left[ \hat{\sigma} (\gamma + i \rightarrow c\bar{c}(8, 1 S_0) + X) \langle O^{h_c}_{8}(1 S_0) \rangle^{(\mu)} + \right]
$$
\[
\hat{\sigma}(\gamma + i \to c\bar{c}(1, P_1) + X; \mu) \langle O_{h^c}(1 P_1) \rangle,
\]

where \( f_{i/N}(x) \) is the probability of finding a parton \( i \) in the nucleon with a fraction \( x \) of the nucleon momentum. \( \hat{\sigma}(\gamma + i \to c\bar{c}(8, S_0) + X) \) is the short-distance coefficient for producing a \( c\bar{c} \) pair in a color-octet \( ^1S_0 \) configuration, and \( \hat{\sigma}(\gamma + i \to c\bar{c}(1, P_1) + X; \mu) \) is the short-distance coefficient for producing a \( c\bar{c} \) pair in a color-singlet \( ^1P_1 \) configuration. The matrix element \( \langle O_{h^c}(8^{2s+1}L_J) \rangle \) describes the hadronization of a color-singlet (color-octet) \( ^{2s+1}L_J \) \( c\bar{c} \) pair into an \( h_v \). According to the \( v \)-scaling rules, both matrix elements in Eq. (1) scale as \( v^5 \). The scale \( \mu \) appearing in Eq. (1) arises from the factorization of the cross section into long-distance and short-distance contributions. As we discuss below, the \( \mu \) dependence of the color-octet matrix element is cancelled by the \( \mu \) dependence of the color-singlet short-distance coefficient so that the expression for the cross section is \( \mu \) independent. Note that in addition to the NRQCD scale \( \mu \) shown explicitly in Eq. (1) there is a renormalization scale and a factorization scale (associated with the parton distribution function \( f_{i/N}(x) \)) which have been suppressed. These three scales do not have to be the same, however, in our numerical calculations we choose them to be equal.

The short-distance coefficients in Eq. (1) can be calculated using the techniques of Ref. [7]. At leading order in \( \alpha_s \), we obtain the following expression for the color-octet short-distance coefficient:

\[
\frac{d\hat{\sigma}}{dz}(\gamma + g \to c\bar{c}(8,1 S_0)) = \frac{\pi^3 \alpha_s e_c^2}{4m_c^5} \delta(1 - z),
\]

where \( z = \rho/x, \rho = 4m_c^2/S_{\gamma N} \), and \( S_{\gamma N} \) is the photon-nucleon center-of-mass energy squared. We calculate the color-singlet cross section, regulating the infrared divergences using dimensional regularization. The result in the \( \overline{MS} \) scheme is:

\[
\frac{d\hat{\sigma}}{dz}(\gamma + g \to c\bar{c}(1,1 P_1) + g; \mu) = \frac{8\pi^2 \alpha_s^2 e_c^2}{27m_c^5} \left[ f(z) + \frac{z^4(1 + z^2)}{(1 + z)^2} \frac{1}{1 - z} \left( \ln(\frac{\mu}{2m_c}) \right) \right],
\]

where

\[
f(z) = -\frac{z^5 \ln(z)}{(1 + z)^3} + \frac{z^2(5 + 3z + 14z^2 + 2z^3 + 9z^4 - z^5) \ln(z)}{(1 - z)^3(1 + z)^5}
\]

\[
+ \frac{z^2(1 + z + 10z^2 + 4z^3 + 15z^4 - z^5 + 2z^6)}{(1 - z)^2(1 + z)^5},
\]

and the functional distribution is defined by

\[
\int_0^1 dz \ f(z) \left( \frac{1}{1 - z} \right)_\rho = \int_0^1 dz \ \frac{f(z) - f(1)}{1 - z}.
\]

The NRQCD expression for the cross section is obtained by substituting Eqs. (2) and (3) into Eq. (1). Note that a \( 1/\epsilon \) divergence in the color-singlet coefficient has been absorbed into the definition of the leading color-octet matrix element. The renormalized matrix element \( \langle O_{h^c}(1 S_0) \rangle^{(\mu)} \) can be shown to obey the renormalization group equation [5].
\[ \mu \frac{d}{d\mu} \langle O^{8c}_{S} (1S_0) \rangle^{(\mu)} = \frac{16\alpha_s}{27\pi m_{c}^2} \langle O^{hc}_{1} (1P_1) \rangle. \] (6)

Thus the logarithmic dependence on \( \mu \) of the short-distance coefficient \( \sigma(\gamma + g \rightarrow c\bar{c}(1,1P_1 + X; \mu) \) is canceled to this order by the \( \mu \) dependence of the renormalized matrix element \( \langle O^{hc}_{S} (1S_0) \rangle^{(\mu)} \).

Note that the leading color-octet coefficient is \( O(\alpha_s) \) while the leading color-singlet coefficient is \( O(\alpha_s^2) \). Therefore, next-to-leading order QCD corrections to the color-octet coefficient are of the same order as the leading color-singlet coefficient, and must be included if we are to have a complete \( O(\alpha_s^2) \) calculation of \( h_c \) photoproduction. The \( O(\alpha_s^2) \) corrections to the color-octet contribution are computed in Ref. [8]. These corrections are included in our calculation of the cross section.

In order to make a prediction for the \( h_c \) photoproduction cross section, we must determine the values of the nonperturbative matrix elements \( \langle O^{hc}_{S} (1S_0) \rangle^{(\mu)} \) and \( \langle O^{hc}_{1} (1P_1) \rangle \). To eliminate large logarithms in the expression for the cross section we choose \( \mu = 2m_c \). At this time there does not exist a direct measurement of these matrix elements. However, they are related to similar matrix elements for \( \chi_{cJ} \) production and decay by the (approximate) heavy quark spin symmetry of NRQCD:

\[
\langle O^{\chi_{1}}_{1} (3P_1) \rangle = \langle O^{hc}_{1} (1P_1) \rangle (1 + O(v^2))
\]

\[
\langle O^{\chi_{1}}_{8} (3S_1) \rangle = \langle O^{hc}_{S} (1S_0) \rangle (1 + O(v^2)).
\] (7)

The size of the \( O(v^2) \) corrections in Eq (7) can be estimated by studying radiative \( \chi_c \) and \( \psi' \) decays, where predictions based on heavy quark spin symmetry agree with experiment to 20\% accuracy \[9\].

\( \langle O^{\chi_{1}}_{1} (3P_1) \rangle \) can be extracted from \( \chi_{cJ} \) decays, or from the decay \( B \rightarrow \chi_{cJ} + X \). The authors of Ref. [10] calculate inclusive hadronic \( \chi_{cJ} \) decay including next to leading order \( \alpha_s \) corrections. The result of their fit is:

\[
\frac{\langle O^{\chi_{1}}_{1} (3P_1) \rangle}{m_{c}^2} = 0.115 \pm 0.016 \text{ GeV}^3.
\] (8)

The error includes only experimental uncertainties. It is particularly important to note that Ref. [10] does not include \( O(v^2) \) relativistic corrections which are numerically of the same size as the \( O(\alpha_s) \) perturbative corrections that have been included. Relativistic corrections to the \( J/\psi \) decay rate are large \[11\]. In the case of \( \chi_{cJ} \) decay, relativistic corrections need to be analyzed before \( \langle O^{\chi_{1}}_{1} (3P_1) \rangle \) can be extracted with confidence.

The decay \( B \rightarrow \chi_{cJ} + X \) is calculated in Ref. [12]. Measurements of \( B \) decay \[13\] allow an extraction of \( \langle O^{\chi_{1}}_{1} (3P_1) \rangle \):

\[
\frac{\langle O^{\chi_{1}}_{1} (3P_1) \rangle}{m_{c}^2} = 0.42 \pm 0.16 \text{ GeV}^3.
\] (9)

Again, the error quoted above is only due to experimental uncertainties. The authors of Ref. [12] state that their calculation suffers from large theoretical uncertainties due to next to leading order QCD corrections to the Wilson coefficients and the subprocess \( b \rightarrow c\bar{c}s \).

Note that the two extractions of \( \langle O^{\chi_{1}}_{1} (3P_1) \rangle \) agree only at the 2\( \sigma \) level.
\( \langle O_8^{c1}(3S_1) \rangle \) can be extracted from the decay \( B \to \chi_{cJ} + X \), and from CDF data on \( \chi_{cJ} \) production. The result of the fit to \( B \) decay, after running from the scale \( m_b \) to \( 2m_c \) and converting from a cutoff regularization scheme to dimensional regularization, is

\[
\langle O_8^{c1}(3S_1) \rangle (2m_c) = (2.9 \pm 2.0) \times 10^{-2} \text{ GeV}^3.
\] (10)

As before only experimental errors are included. The result of two different fits to Tevatron data are

\[
\langle O_8^{c1}(3S_1) \rangle = 0.98 \pm 0.13 \times 10^{-2} \text{ GeV}^3
\] (11)

\[
\langle O_8^{c1}(3S_1) \rangle (2m_c) = 2.6 \times 10^{-2} \text{ GeV}^3,
\] (12)

where the values are taken from Refs. [14] and [15] respectively. The error of Ref. [14] includes only experimental uncertainty; Ref. [14] does not quote errors. The central values of both fits lie within the 1\( \sigma \) error of the extraction from \( B \)-decays.

Note that in the \( \chi_{cJ} \) production calculations of Ref. [14], there is no color-singlet contribution that gives an infrared divergence which needs to be absorbed into the definition of the color-octet matrix element, as is done in our calculation. Therefore, it is not possible to relate the “bare” matrix element appearing in Eq. (11) with the renormalized matrix element needed for our calculation. However, the fragmentation calculation of \( \chi_{cJ} \) production carried out in Ref. [14] makes use of the \( g \to \chi_{cJ} \) fragmentation function. This fragmentation function includes both color-octet and color-singlet contributions, and has an infrared divergence in that is absorbed into the definition of the color-octet matrix element. This allows us to import the extracted value given in Eq. (12) into our calculation.

Not all values for \( \langle O_8^{hc}(1S_0) \rangle \) and \( \langle O_1^{hc}(1P_1) \rangle \) result in a physically sensible prediction for the cross section. Once the infrared divergence from the color-singlet contribution to the cross section is factorized, the remaining finite contribution is actually negative (for positive \( \langle O_1^{hc}(1P_1) \rangle / m_c^2 \)). If the ratio of the color-octet to color-singlet matrix elements is too small, it is possible to obtain physically meaningless results. This is the case if, for example, the central values of the color-singlet matrix element extracted from \( B \)-decays (Eqs. (14) and (15)) is used. Therefore it is impossible to put a lower bound on the cross section given our current state of ignorance concerning this matrix element. It is also important to point out that the NRQCD velocity scaling rules imply that \( \langle O_8^{hc}(1S_0) \rangle (2m_c) \) and \( \langle O_1^{hc}(1P_1) \rangle / m_c^2 \) should be roughly the same size. Thus, theory would prefer a smaller value for the matrix element \( \langle O_1^{hc}(1P_1) \rangle \), as suggested by the fit to \( \chi_{cJ} \) decay. Clearly more accurate extractions from the Tevatron, from \( B \)-decays, and from \( \chi_{cJ} \) decays are needed to clarify the situation. This may be possible once next-to-leading perturbative QCD corrections and leading relativistic corrections to these processes are calculated.

For the NRQCD matrix elements, we will use \( \langle O_1^{hc}(1P_1) \rangle / m_c^2 = 0.115 \text{ GeV}^3 \) as determined by the analysis of \( \chi_{cJ} \) decays (Eq. (14)), and we choose \( \langle O_8^{hc}(1S_0) \rangle (2m_c) = 2.6 \times 10^{-2} \text{ GeV}^3 \). We use the CTEQ 3M parton distribution function with the factorization scale chosen to be equal to the NRQCD scale \( \mu = 2m_c \). The resulting cross section is plotted as the solid line in Fig. [1]. However, our results are extremely sensitive to the uncertainty in the determination of the matrix elements. To show this, we also plot, as the dashed line, the cross section with the choice \( \langle O_1^{hc}(1P_1) \rangle / m_c^2 = 0.2 \text{ GeV}^3 \). The cross section then drops by roughly a factor of four.
FIG. 1. The total $h_c$ photoproduction cross section $\sigma(\gamma + N \to h_c + X)$ as a function of the center of mass energy $\sqrt{S_{\gamma N}}$. The solid curve is for the choice $\langle O_{1c}^{h_c} (1 P_1) \rangle/m_c^2 = 0.115$ GeV$^3$ and $\langle O_8^{hc} (1 S_0) \rangle^{(2m_c)} = 2.6 \times 10^{-2}$ GeV$^3$. The dashed line is obtained by keeping the same value for the color-octet matrix element and by changing the value of the color-singlet matrix element to $\langle O_1^{hc} (1 P_1) \rangle/m_c^2 = 0.2$ GeV$^3$. 

\[ \sigma(\gamma + N \to h_c + X) \]
We conclude with a brief discussion of the possibility of observing the $h_c$. The E831 photoproduction experiment corresponds to roughly $\sqrt{S_{\gamma N}} = 20$ GeV, while the HERA experiments take photoproduction data at approximately $\sqrt{S_{\gamma N}} = 100$ GeV. At these energies the cross section for $h_c$ production, assuming our first choice of NRQCD matrix elements, is 30 nb and 62 nb respectively. These cross sections are comparable to $J/\psi$ production. The $h_c$ can be detected either via the rare decay $h_c \rightarrow J/\psi + \pi$ with branching ratio on the order of 1% [16], or the radiative decay $h_c \rightarrow \eta_c + \gamma$ (BR = 50%) [17]. If it is possible to reconstruct these decay modes, then the possibility of observing the $h_c$ is real, and should be experimentally investigated.

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