Abstract: We have calculated the thermal partition function for the thermodynamical system of massless free bosonic higher spin fields (Fronsdal theory) by using Feynman path integral formalism. However, we have established a connection between the microscopic world (Quantum statistical) of the thermodynamical system of massless free bosonic HS fields to the thermodynamics of the macroscopic world its and also we have observed the duality between the thermodynamical system of massless free HS fields on $d$-dimensional Minkowski spacetime and thermodynamical system of Klein-Gordon scalar fields on 4-dimensional Minkowski spacetime at the thermal equilibrium condition.

Keywords: Massless Higher spin gauge fields, Thermal equilibrium, Entropy, Average energy
1 Introduction

Higher spin field theory has attracted a significant amount of attention due to its unbroken phase of a string theory. Higher spin gauge theories have attracted considerable interest during the last three decades, other than fascinating topic by itself. Fronsdal, [1] who has first found the equations of motions and the action principle for massless fields of arbitrary spin for $s = 1$ and also spin greater than two and also interacting higher spin fields can not propagate in Mikowski spacetime for which subjected to the no go theorem.

The gauge invariance of massless free bosonic higher spin field theory have to remove non-Physical polarisation and ghost from the spectrum. However, it is highly nontrivial to study the thermodynamical behavior of massless free higher spin field theory on $d$-dimensional Minkowski spacetime.

Free massless fields it has to satisfy the Klein-Gordon equation $\Box \phi(x)_{\mu_1 \mu_2 \ldots \mu_s} = 0$ here the field we would like to describe is a tensor field of arbitrary rank $s$ completely symmetric in its indexes or in other words, a field with spin $s$. Since, the field is massless it has to satisfy $\partial^{\mu_1} \phi(x)_{\mu_2 \ldots \mu_s} = 0$ transversality condition.

In this article, we have studied the thermodynamical behavior of non-linear equations for completely symmetric massless bosonic higher spin gauge fields in Minkowski spacetime of $d$-dimensional is provided and these massless free bosonic HS fields contains infinite sets of infinitely increasing spins.

Our present investigation is essential because it is motivated by the following key factors. First and foremost, we have observed the duality between massless free bosonic higher spin field theory on $d$-dimensional Minkowski spacetime and the Klein-Gordon scalar field theory on 4-dimensional Minkowski spacetime and also we have demonstrated the thermodynamical properties at the thermal equilibrium of free massless bosonic Higher spin field theory by using Feynman path integral approach. These calculations are nontrivial in order to understanding thermodynamical characteristic of free massless bosonic Higher spin fields.

We have established a connection between the microscopic world (Quantum statistical nature) of massless bosonic HS fields to the macroscopic world of its thermodynamical system. The derivations of free energy, average energy and entropy of thermodynamical system could be achieved by using path integral approach. However, the path integral method provide us with powerful mathematical tool to describe physical properties of the system of interest.

Finally, our present work is our modest first step towards our central goal of providing a theoretical generality for the existence of the duality between the two theories at the thermal equilibrium condition.

2 Preliminaries: Fronsdal formulation of massless free bosonic Higher spin fields

The Fronsdal formulation of free massless bosonic higher spin fields of gauge theories [1-7] were originally formulated in terms of completely symmetric and double traceless massless bosonic HS field $\phi(x)_{\mu_1 \mu_2 \ldots \mu_s}$ (which is analogous to the metric like formulation of gravity).
In Minkowski spacetime $R^{d-1,1}$ the spin $s$ Fronsdal action is

$$S(\phi) = \frac{1}{2} \int d^d x \left( \partial_\nu \phi(x)_{\mu_1 \mu_2 \ldots \mu_s} \partial^\nu \phi(x)_{\mu_1 \mu_2 \ldots \mu_s} ight) - \frac{s(s-1)}{2} \partial_\nu \phi(x)_{\lambda \mu_3 \ldots \mu_s} \partial^\nu \phi(x)_{\rho \mu_3 \ldots \mu_s}
+ s(s-1) \partial_\nu \phi(x)_{\lambda \mu_3 \ldots \mu_s} \partial_\rho \phi(x)_{\nu \rho \mu_3 \ldots \mu_s}
- s \partial_\nu \phi(x)_{\mu_2 \ldots \mu_s} \partial_\rho \phi(x)_{\rho \mu_2 \ldots \mu_s}
- \frac{s(s-1)(s-2)}{4} \partial_\nu \phi(x)_{\nu \rho \mu_2 \ldots \mu_s} \partial_\lambda \phi(x)_{\lambda \sigma \mu_2 \ldots \mu_s} \right)$$

where the metric like field is double traceless ($\eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4} \phi(x)_{\mu_1 \mu_2 \ldots \mu_s} = 0$) (here spin $s$ massless bosonic HS fields satisfy the traceless conditions i.e. $\phi(x)_{\lambda \mu_3 \ldots \mu_s} = 0$ and $\phi(x)_{\nu \rho \mu_2 \ldots \mu_s} = 0$) and has the dimension of $(\text{length})^{1-d/2}$. This action is invariant under Abelian HS gauge transformations

$$\delta \phi_{\mu_1 \mu_2 \ldots \mu_s} = \partial_\mu_1 \epsilon_{\mu_2 \mu_3 \ldots \mu_s} \right)$$

where gauge parameter $\epsilon$ is completely symmetric and traceless rank $(s-1)$ tensor

$$\eta^{\mu_1 \mu_2} \epsilon_{\mu_1 \mu_2 \ldots \mu_{s-1}} = 0.$$ 

By using the above gauge symmetry transformations, double traceless condition of massless bosonic HS field and traceless condition of gauge parameter we could obtained the Euler-Lagrange equations of motion of massless free bosonic HS fields which is nothing but a Klein-Gordon equations

$$\Box \phi_{\mu_1 \mu_2 \ldots \mu_s} = 0.$$ 

The massless bosonic HS field we would like to describe is a tensor field of arbitrary rank $s$, which is completely symmetric in its indexes. However, the field is massless it has to satisfy the transversality condition. Furthermore, in order not to have propagation of states with negative norms ghosts the massless free bosonic HS fields should satisfy the transversality condition i.e.

$$\partial_\mu \phi_{\mu_1 \mu_2 \ldots \mu_s}(x) = 0.$$ 

Since, $\eta^{\mu \nu}$ are the components of the Minkowski spacetime metric tensor, $\partial_\mu = \frac{\partial}{\partial x^\mu}$ and

$$\partial^2 = \eta^{\mu \nu} \partial_\mu \partial_\nu = \partial_\mu \partial^\mu = \Box.$$ 

### 3 Thermal partition function of massless free bosonic Higher spin fields

The canonical ensemble provides us with a model of matter in thermal equilibrium and allows us to calculate all the thermodynamical quantities in terms of quantities characterizing the microscopic structure of massless free bosonic higher spin fields.
By using the path integral formulation[9-10], thermal partition function [8] of canonical ensemble of free massless bosonic Higher spin fields is to be calculated as follows. Moreover, path integrals have revealed the deep mathematical relationship between the massless free higher spin fields and the thermodynamical properties of its. These relations have played a major role in our understanding of a thermodynamical properties of massless free higher spin field theory at the thermal equilibrium condition.

Indeed, the path integral representation are used for the canonical partition function of thermodynamical system of a massless free HS fields with action $S(\phi)$ for massless free bosonic higher spin fields at inverse temperature $\beta = \frac{1}{K_B T}$ and we integrating over all massless free bosonic higher spin fields

$$Z = \int \mathcal{D}\phi \exp\left(-\beta S(\phi)\right) = \text{Tr}(\exp(-\beta S(\phi)))$$ (6)

$$Z = \int \mathcal{D}\phi \exp\left(-\frac{1}{2} \beta s \int d^d x (\partial^\nu \phi^\rho \partial_\nu \phi^\rho)\right)$$ (7)

$$Z = \frac{1}{\beta s} (\text{det} \partial_\nu \partial_\rho)^{-\frac{d}{2}} = \frac{1}{\beta s} (\text{det} \Box)^{-\frac{d}{2}}$$ (8)

This partition function result is correspondence with the partition function of the thermodynamical system of Klein-Gordon massless free scalar field theory (action for Klein-Gordon massless free scalar field theory [11] on 4-dimensional Minkowski spacetime is $S_0(\phi) = \int d^4 x (\frac{1}{2} (\partial_\mu \phi)^2)$. Therefore, massless free higher spin scalar field theory on d-dimensional system dual to the Klein-Gordon massless free scalar field theory on $d = 4$ dimensional spacetime. The Green function is the inverse of the operator $\Box$ on a $d$-dimensional Minkowski spacetime

$$G(x - y) = \int \frac{d^dk}{(2\pi)^d} \exp(ik \cdot (x - y))\zeta(k)$$ (9)

where $\zeta(k) = \frac{1}{\eta_{\mu\nu} k^2}$ and the Green function is also called as Co relation function of thermodynamical system of HS fields which could explain thermal characteristics of a thermodynamical system near the phase transition regime.

However, we have interested to calculate the thermodynamical properties of massless free bosonic HS field at the thermal equilibrium condition, because thermodynamical laws are valid only when system will be at the the thermal equilibrium. Therefore, the free energy of a thermodynamical massless free bosonic Higher spin fields is as follows

$$F = -\frac{1}{\beta} \log Z.$$ (10)

Since, thermal equilibrium state of a free massless bosonic higher spin fields is realized as a minimization of the free energy, we consider

$$\frac{\partial F}{\partial \phi} \bigg|_{\phi_0} = 0$$ (11)
where $\phi_0$ as a thermal equilibrium state of a massless free bosonic Higher spin field. The average energy associated with the system is

$$E = -\frac{\partial}{\partial \beta} \log Z$$

$$= F + \beta \frac{\partial F}{\partial \beta}.$$  \hspace{1cm} (12)

The entropy of a massless free bosonic Higher spin fields is connect with the microscopic nature (Quantum statistical) of massless free bosonic HS fields and macroscopic nature of thermodynamical system of its

$$S = -\frac{\partial F}{\partial T}.$$  \hspace{1cm} (13)

Since, we are interested in the entropy as a function of average energy, because entropy is most important thermodynamical property in order to understand the thermal behavior of the thermodynamical system

$$S = \log Z + \beta E.$$  \hspace{1cm} (14)

The maximization of the entropy of thermodynamical massless free bosonic higher spin fields could be achieved when the thermodynamical equilibrium prevail in the system i.e.

$$\frac{\partial S}{\partial E} \bigg|_{E_o} = 0$$  \hspace{1cm} (15)

where $E_o$ is thermal equilibrium energy of a thermodynamical system.

4 Conclusion

We have studied the massless free bosonic higher spin field equations of motion (Fronsdal theory), we have done the two things 1) Imposing double traceless conditions on bosonic HS field and traceless condition gauge parameter which is completely symmetric 2) Performing gauge symmetry transformations procedure.

In our present investigation, we calculate that the thermal partition function for the thermodynamical system of massless free bosonic higher spin fields (Fronsdal theory) by using Feynman path integral formalism. We have established a connection between the microscopic world of thermodynamical system of massless free bosonic HS fields to the thermodynamics of macroscopic world of its. Therefore, the path integral representation is a powerful mathematical tool to estimate the thermal behaviour of the system.

In this paper, we have shown that the duality between the thermodynamical system of massless free HS fields on $d$-dimensional Minkowski spacetime and thermodynamical system of Klein-Gordan scalar fields on 4-dimensional Minkowski spacetime at the thermal equilibrium condition. However, the results of this paper suggest that the study of thermodynamical system of massless free bosonic HS fields has been much more important in order to understand the thermal properties of the system.
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