Superhumps in Binary Systems and Their Connection to Precessional Spiral Density Waves

P.V. Kaygorodov¹, D.V. Bisikalo¹, O.A. Kuznetsov¹,² and A. A. Boyarchuk¹

¹ Institute of Astronomy, Moscow, Russia
² Keldysh Institute of Applied Mathematics, Moscow, Russia

Abstract

We consider a mechanism for the formation of superhumps in the TV Col system, based on the possible existence of a precessional spiral wave in the accretion disk of the system. This mechanism can act in binaries with arbitrary component-mass ratios, and our precessional spiral wave model can be applied to explain observed superhumps of all types.

1 Introduction

Superhumps are modulations of the light curves of binary systems with periods that differ from the orbital periods by several percent, and are observed mainly during superoutbursts in SU UMa systems. The main observational features of superhumps are described in [1]. Various authors have put forward models to explain the superhump phenomenon (a brief description of various models and their problems are given, for instance, in [1]). Currently, the most popular model explains these light variations as an effect of precession of the outer regions of the accretion disk. The presence of the Lindblad 3:1 resonance in the disk results in an instability that leads to precession of the outer part of the disk, with a period that is appreciably longer than the orbital period. The beating of the orbital and precessional periods gives rise to the periodic variations that are identified with superhumps. This model has several shortcomings, the most important being that it implies a limit on the maximum component mass ratio. In order for the Lindblad 3:1 resonance to be located inside the accretion disk, the ratio of the donor and accretor masses $q$ must be lower than $\sim 0.33$ [2], while there are several observed systems with superhumps that have appreciably higher $q$ values, such as TV Col.

TV Col is a “permanent superhumper”, or one of a class of systems in which superhumps are always observed. It was suggested in [3] that cataclysmic variables with high mass-transfer rates (TV Col is such a system) may persist in a superhump regime that is not interrupted by quiescent periods. Among stars with superhumps, TV Col has an unusually long orbital period, $P_{\text{orb}} \sim 5.5$ h [4]. Since there is a direct correlation between the orbital period of a system and the donor mass (see, for instance, [3]), this may be as high as $\sim 0.56 M_\odot$, while the component-mass ratio is probably in the range $q \sim 0.6 \div 0.9$ [5]. It is obvious that the Lindblad 3:1 resonance cannot be located inside the
accretion disk with such a $q$ value. This casts doubt on the popular superhump model indicated above.

In 2004, a new mechanism for the formation of superhumps in SU UMa systems was suggested in [6,7]. The basis of this mechanism is the idea that a precessional-type density wave can form in the accretion disk.

The precessional wave in the accretion disks forms as a result of interactions between elliptical streamlines. The asphericity of the gravitational field of the binary system forces the semimajor axes of the streamlines to precess opposite to the direction in which the matter flows. The rate of this precession is proportional to the semimajor axis of the corresponding streamline. Since the streamlines in the disk cannot intersect, their interaction results in the establishment of a certain equilibrium precession rate of all the streamlines, with their apastrons lining up so that they form a spiral pattern. Because the matter velocity is minimum at the apastrons, the density of the matter at these points grows, and a precessional spiral density wave is formed in the disk.

After the formation of the precessional density wave in the disk, the rate of accretion grows sharply (by up to an order of magnitude). Matter approaches the surface of the accretor along the precessional wave, and the region of accretion is localized in azimuth, and, hence, forms a radiating spot at the surface of accretor. The increase in the accretion rate due to the density wave explains both the development of a superoutburst and the amplitude of the superhump. The wave slowly precesses in the stationary (observer's) coordinate frame, leading to a shift of the region of enhanced accretion (i.e., of the superhump) with each rotation of the system. The beating of the orbital period and the precessional period of the wave result in the superhump period, which is slightly larger than the orbital period.

This superoutburst model based on the presence of a precessional spiral wave in the accretion disk [7] made it possible for the first time to explain all important observational manifestations of superoutbursts and superhumps in SU UMa systems, including their period, duration, and energy, the anticorrelation of the brightness and color temperature at the maximum of an ordinary superhump, the late-superhump phenomenon, etc. Moreover, this model does not place strict constraints on the component mass ratio, and so can be applied to the superhumps in the TV Col system.

The main aim of the present paper is to investigate the possible formation of precessional spiral waves in accretion disks in systems with $q > 0.33$. We will also consider the possibility of explaining all types of superhumps with this model.

2 The Model

The model used for our numerical simulations of mass flows in binary systems is described in [8]. The flow is described by a three-dimensional system of gravitational
gas-dynamics equations, including nonadiabatic radiative heating and cooling of the gas:

\[
\left\{ \begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0, \\
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) + \nabla P &= -\rho \nabla \Phi - 2[\Omega_{\text{bin}} \times \mathbf{v}]\rho, \\
\frac{\partial \rho (\varepsilon + |\mathbf{v}|^2/2)}{\partial t} + \nabla \cdot (\rho \mathbf{v} (\varepsilon + P/\rho + |\mathbf{v}|^2/2)) &= -\rho \mathbf{v} \nabla \Phi + \rho \varepsilon m_p^{-2} (\Gamma(T, T_{wd}) - \Lambda(T)).
\end{aligned} \right. \tag{1}
\]

Here, \( \rho \) is the density, \( \mathbf{v} = (u, v, w) \) the velocity vector, \( P \) the pressure, \( \varepsilon \) the internal energy, \( \Phi \) the Roche potential, \( m_p \) the proton mass, \( \Omega_{\text{bin}} \) the orbital angular velocity of the system, and \( \Gamma(T, T_{wd}) \) and \( \Lambda(T) \) the radiative heating and cooling functions. The system of gas-dynamical equations was closed with the ideal gas equation of state, \( P = (\gamma - 1)\rho\varepsilon \), where \( \gamma \) is the adiabatic index, which was specified to be \( 5/3 \).

This system of equations was solved numerically using the Roe-Osher method \([9–11] \) adapted for a multiprocessor computer. A three-dimensional, noninertial, Cartesian coordinate system corotating with the binary was used for the modeling.

Only half of the space occupied by the disk was modeled, due to the symmetry of the problem about the equatorial plane. The computational domain was specified to contain both the disk and stream of matter from \( L_1 \), and had a size of \( 1.12A \times 1.14A \times 0.17A \) (where \( A \) is the binary separation). The computational grid contained \( 121 \times 121 \times 62 \) cells and was distributed among 81 processors in a twodimensional \((9 \times 9)\) matrix. The grid became more dense closer to the equatorial plane, making it possible to resolve the vertical structure of the disk.

The boundary conditions at the outer edges of the computational domain were specified as follows. In all cells except those in the vicinity of \( L_1 \), we assumed a constant density, \( \rho_b = 10^{-8}\rho_{L_1} \), where \( \rho_{L_1} \) is the density at \( L_1 \), a temperature \( 13600^\circ \text{K} \), and zero velocity. The accretor was specified as a hemisphere with a radius of \( 10^{-2}A \). Matter entering the cells forming the accretor was taken to fall onto the star. The stream was defined as a boundary condition: matter with a temperature of \( 5800^\circ \text{K} \) and an \( x \)-velocity \( v_x = 6.3 \text{ km/s} \) was injected into a region with radius \( 0.014A \) centered on \( L_1 \). The density of the matter at \( L_1 \) was specified so that the mass-transfer rate corresponded to the observed rate in TV Col.

We made three computational runs with different parameters of the system. Table 1 gives the accretor \((M_1)\) and donor \((M_2)\) masses and the binary separation \((A)\) for the three models. The binary separation was adjusted to be consistent with the observed orbital period of the TV Col system of 5.5 h. Since the component mass ratio for TV Col is not known precisely, we used the two extreme values \( q = 0.93 \) and \( q = 0.3 \) (which

| No. | \( q \) | \( M_1(M_\odot) \) | \( M_2(M_\odot) \) | \( A(R_\odot) \) |
|-----|-----|-------|-------|--------|
| 1   | 0.3 | 1.4   | 0.42  | 1.923  |
| 2   | 0.7 | 0.8   | 0.56  | 1.745  |
| 3   | 0.93| 0.6   | 0.56  | 1.655  |

Table 1: Model parameters \( q, M_1, M_2, A \)
still allow the disk to contain the 3:1 Lindblad resonance), as well as the intermediate value \( q = 0.7 \). For the given rate of mass inflow into the system, the accretion rate in the model corresponded to \( \approx 10^{-8} M_\odot \). The modeling covered large intervals of time (up to several dozen orbital periods), making it possible to reach a quasistationary accretion regime with a constant mass of the disk.

3 Results of The Modeling

Figure 1 shows density contours and velocity vectors in the equatorial plane for model nos. 2 and 3 from Table II.

Places where the density contours become closer together correspond to stationary shocks in the disk. A dense, round accretion disk and a compact circumdisk halo form in the system. At the edge of the circumdisk halo, which is formed by matter that collides with the stream when the former moves around the accretor, the density rapidly falls to the background values. The interaction of the circumdisk halo gas and the stream results in the formation of a shock (hot line; see [6–9,12–16]) outside the disk. A two-armed tidal spiral shock forms in the disk, with both the arms located in the outer regions of the disk. A precessional spiral density shock is clearly visible in the inner regions of the disk. The two-armed spiral shock is at rest in the noninertial coordinate frame fixed to the binary, while it moves with a slow velocity in the inertial observer’s frame. The period of the precessional wave in the noninertial coordinate frame is slightly longer than the orbital period.

To find the rate of precession, we determined the phases at which the maximum den-
Figure 2: Dependence of the angular velocity of the density peaks at three different distances from the accretor in models with $q = 0.7$ (left) and $q = 0.93$ (right). Angular velocities are shown for a noninertial coordinate frame rotating with the orbital angular velocity of the system. The horizontal dashed line corresponds to the orbital velocity of the binary.

Density is observed for various distances from the accretor. The precession of the wave results in a shift of these phases with time. The typical time dependences of the angular displacements of the density wave for three distances from the accretor (the beginning, middle, and end of the wave) are shown in Fig. 2. These plots demonstrate that the velocities of the streamlines in the disk level out, and the wave moves with some equilibrium velocity $\Omega_{\text{wav}}$ in the coordinate frame fixed to the binary.

Table 2 lists the precession rates $\Omega_{\text{wav}}$ found via the numerical modeling for systems with various component mass ratios $q$. Table 2 also presents the period excesses $\epsilon^+ = \frac{P_{\text{sh}}^+ - P_{\text{orb}}}{P_{\text{orb}}}$, where $P_{\text{sh}}^+$ is the superhump period.

## 4 Connection Between Superhumps and Precessional Waves

As a rule, five types of superhumps observed in cataclysmic variables can be distinguished [17].

1. Positive (or ordinary) superhumps have a period that is a few percent longer than the orbital period. Such superhumps were first observed in SU UMa during a

| No. | $q$ | $\Omega_{\text{wav}}/\Omega_{\text{bin}}$ | $\epsilon^+$ |
|-----|-----|--------------------------------|
| 1   | 0.3 | -0.07 | 7 % |
| 2   | 0.7 | -0.1  | 11% |
| 3   | 0.93 | -0.05 | 5% |

Table 2: Rate of precession in the three models
superoutburst.

2. **Orbital** superhumps represent a modulation of luminosity with the orbital period.

3. **Late** superhumps are observed after the end of a superoutburst, and have the same velocity as the positive superhump, but shifted by half a period relative to the latter.

4. **Permanent** superhumps have the same features as positive superhumps, but are observed in stars that lack superoutbursts.

5. **Negative** superhumps have a period that is several percent shorter than the orbital period. Such superhumps were first discovered during monitoring of systems with permanent superhumps.

The precessional spiral wave model is able to explain all types of observed superhumps. The main observational features of positive and late superhumps are a direct consequence of the formation of a precessional spiral wave in the disk, as is explained in [7]. The orbital superhump can be explained by the release of energy in stationary shocks in the disk and circumdisk halo, such as the hot-line and tidal shocks; the existence of tidal shocks is not precluded in the precessional wave model. Permanent superhumps can also be explained, if the mass-transfer rate in the system is high enough to sustain a high accretion rate due to prolonged existence of the precessional spiral wave. The existence of luminosity modulations whose period is shorter than the orbital period, i.e., of negative superhumps, can also be plausibly explained in this model.

In the model, the superhump radiation arises from a relatively compact region near the surface of the accretor, into which matter flows along the precessional wave. If the bright spot arising in this region is located above the accretor surface (i.e., the rotation of accretor does not influence the motion of the spot) and has a tendency to spread out due to diffusion, the leading edge of the spot may be observed somewhat earlier after each rotation of the system, creating a modulation with a period shorter than the
orbital period. The fact that the observed periods of negative superhumps do not display significant scatter can be taken to justify the assumption that the energy release occurs above the surface of the accretor.

The necessary rate of diffusional spreading can be estimated using the observed period. Let us denote the observed “period excess” for a negative superhump $\epsilon^- = \frac{P_{sh}^- - P_{sh}}{P_{orb}}$, where $P_{orb}$ is the orbital period of the system and $P_{sh}^-$ is the observed period of the negative superhump. For TV Col, $P_{sh}^- \sim 5.2$ h [4], so $\epsilon^- \sim -0.055$. If the observed period is the result that of beating between the orbital period and rotational period of the leading edge of the spot $P_{lead}$, this period can be found using the formula

$$P_{lead} = \frac{P_{sh}^- P_{orb}}{P_{orb} - P_{sh}^-} = -P_{orb} \frac{\epsilon^- + 1}{\epsilon^-} \approx 95.3$$ h.

The linear velocity of the spot edge will be

$$v_{lead} = \frac{2\pi r_*}{P_{lead}} = \frac{2\pi r_*}{P_{orb}} \frac{\epsilon^-}{\epsilon^- + 1} \approx 0.18$$ km/s,

if the radius of the accretor is $r_* \approx 10^9$ cm. The velocity of the leading edge has two components: the rate of diffusional spreading $v_{diff}$ and the (negative) velocity of the spot center, relative to the retrograde precession of the wave, $v_{wav}$. Knowing the period of the precessional wave $P_{wav}$ (or the period excess of the positive superhump $\epsilon^+$), we can find the linear velocity of the wave, $v_{wav}$:

$$v_{wav} = -\frac{2\pi r_*}{P_{wav}} = -\frac{2\pi r_*}{P_{orb}} \frac{P_{sh}^+ - P_{orb}}{P_{sh}^+ P_{orb}} = -\frac{2\pi r_*}{P_{orb}} \frac{\epsilon^+}{\epsilon^+ + 1} \approx -0.4$$ km/s.

This velocity is negative, since the direction of the wave precession is retrograde. Accordingly, $v_{diff}$ will be given by

$$v_{diff} = v_{lead} - v_{wav} \approx 0.58$$ km/s.

Let find a relation between the diffusion rate derived from observations and the parameters of the disk gas. The time scale for diffusion over a distance $L$ is $\tau = L^2 / D$, where $D$ is the diffusion coefficient [18]. If $\tau = P_{orb}$, the linear diffusion rate is

$$v_{diff} = \frac{L}{\tau} = \sqrt{\frac{D}{P_{orb}}}$$.

In this case, the diffusion coefficient for the TV Col system will be

$$D = P_{orb} \cdot v_{diff}^2 \sim 1.1 \cdot 10^{16} \frac{L^2}{P_{orb}} \left(1 + \frac{\epsilon^-}{\epsilon^- + 1 + \epsilon^+} + \frac{\epsilon^+}{1 + \epsilon^+} \right)^2 \approx 6.8 \cdot 10^{13}$$ cm$^2$/s

The diffusion coefficient for an ionized gas is defined as $D = l \cdot V_{rms}$, and depends on the mean free path $l$ and rms velocity $V_{rms}$. For ionized gas, $l = \frac{3.2 \cdot 10^6 \cdot T^2}{n \ln \Lambda}$, where $T$ is the temperature of the gas, $n$ the number density of particles, and $\Lambda = \frac{1.3 \cdot 10^4 T^{3/2}}{\sqrt{n}}$. 7
The rms velocity is \( \left( \frac{3kT}{m} \right)^{1/2} \), where \( k \) is Boltzmann’s constant and \( m \) is the mass of the gas particles. Accordingly, the diffusion coefficient is a function of the temperature and number density of the matter: \( D = f(n, T) \). Thus, if we know the value of \( D \) from observations, it is possible to find \( n \) and \( T \).

Figure 4 shows the dependence \( D(T) \) for various \( n \). The dashed line shows the value of \( D \) that corresponds to the observed diffusion for TV Col. If we can estimate the typical temperature from observations of the superhump, Fig. 4 can be used to find the density of matter in the spot. Let us now assume that there is pressure equilibrium in the accretion disk, so that \( nT = n_0T_0 \), where \( n_0 \) and \( T_0 \) are the number density and temperature of the outer parts of the disk. Assuming that \( T_0 \sim 10^4K \), we can find the typical number density in the disk, \( n_0 \). Figure 5 shows a set of solutions for various \( n_0 \) (for \( T_0 = 10^4K \)). For convenience, we also show \( |e^-(T)| \) for TV Col in this figure. If we suppose the temperature of the spot to be \( T \sim 10^6K \) and use the observed value of \( |e^-| \) (dashed line), then \( n_0 \approx 10^{13} \text{cm}^{-3} \). The estimated values of \( T \) and \( n_0 \) are within the range typical for accretion disks in close binary systems [8]. Thus, this model with the diffusional spreading of the spot can adequately explain the observed negative superhumps.

5 Conclusions

Our numerical modeling has shown that precessional density waves can form in the accretion disks of systems with large component mass ratios, up to \( q = 0.93 \).

Our derived dependence of the precession period on \( q \) is complex. Figure 6 shows the theoretical dependence \( e^+(q) \) obtained in [19] (bold dashed line). The open circles
in this figure show the period excesses found for three different sets of parameters of TV Col. We also show the model $\epsilon^+(q)$ values for OY Car and IP Peg found in [6, 7]. Despite the significant scatter, all the $\epsilon^+(q)$ values are fairly close to the theoretical line, although the values for $q = 0.49$ (IP Peg) and $q = 0.93$ (TV Col) are outliers. Note that the rotational rate of the precessional density wave depends not only on the component mass ratio, but also on the size of the region occupied by the wave. The wave can exist only in a gas-dynamically unperturbed region of the disk, whose size is limited by the depth of penetration of stationary shocks – the two arms of the tidal shocks and the hot line (extended region of interaction of the stream from $L_1$ and the circumdisk halo) – into the disk. The high masstransfer rate in TV Col may result in an increase in the extent of the hot line and, consequently, to precession rates that are lower than expected theoretically. It is interesting that, in the case of high component mass ratios, the size of the unperturbed region may be significantly limited by tidal waves, which would result in an even larger decrease of the precession rate with increasing $q$, compared to the theoretical value. Nevertheless, the dependence is close to the theoretical dependence, and has a peak in the range $q \sim 0.4 \div 0.6$.

In TV Col, superhumps with the period $P_{sh} \sim 6.3$ h [4] are observed, so that the superhump period excess is $\sim 15\%$. This value is out of the range of the theoretical estimates of the precession rate (but does not exceed the value obtained in [6] for IP Peg). The theoretical dependence for $\epsilon^+(q)$ was obtained in the model that did not take into account the gas pressure; i.e., it effectively estimates the precession rate of the semimajor axis of the orbit of a free particle. The excesses of the observed and simulated precession rates over the theoretical values may indicate that the estimated disk radius used in the simulations, $r_{disc} = \frac{0.6}{1+q}$ [2], is too low (in fact, the derivations in [2] did not take into

---

Figure 5: Theoretical dependence $|\epsilon^-(T)|$ in TV Col for various values of $n_0$. The dotted line corresponds to the $|\epsilon^-|$ observed in TV Col.
Figure 6: Dependence of the superhump period excess on the component mass ratio. The bold dashed line corresponds to the theoretical dependence given in [19]. The thin dashed line corresponds to the same dependence for a disk whose radius is 10% larger. The open circles show the period excess of the precessional density wave we have found for various possible values of $q$ for TV Col. The triangle corresponds to the period excess found for OY Car in [7], and the square to the period excess found for IP Peg in [6].

account the gas pressure). If the disk radius were 10% larger ($r_{\text{disc}} = \frac{0.6}{1 + q}$), all the derived precession rates would be below the theoretical line (shown by the thin dashed line in Fig. 6).

In summary, we conclude that the presence of a precessional density wave in the disk can lead to superhumps both in SU UMa stars and in binaries whose component mass ratios prohibit the location of the 3:1 Lindblad resonance inside the disk. The precessional wave model is able to explain all types of observed superhumps.

**Acknowledgments**

This work was supported by the Russian Foundation for Basic Research (project nos. 05-02-16123, 05-02-17070, 05-02-17874, 06-02-16097, and 06-02-16234), the Program of Support for Leading Scientific Schools of Russia, and the Basic Research Programs of the Presidium of the Russian Academy of Sciences “Origin and Evolution of Stars and Galaxies”, “Basic Problems in Informatics and Informational Technologies”, the Program of Support for Young Scientists of Russia and the Russian Science Support Foundation.
References

[1] B. Warner. *Cataclysmic Variable Stars*. Cambridge Univ. Press, Cambridge, 1995.

[2] B. Paczynski. A model of accretion disks in close binaries. *Astrophys. J.*, 216:822–826, Sep. 1977.

[3] Y. Osaki. Dwarf-Nova Outbursts. *Publ. Astron. Soc. Pacific*, 108:39–+, Jan. 1996.

[4] A. Retter, C. Hellier, T. Augusteijn, T. Naylor, T. R. Bedding, C. Bembrick, J. McCormick, and F. Velthuis. A 6.3-h superhump in the cataclysmic variable TV Columbae: the longest yet seen. *Monthly Notices Roy. Astronom. Soc.*, 340:679–686, Apr. 2003.

[5] C. Hellier. The Four Periodicities of the Cataclysmic Variable Tv-Columbae. *Monthly Notices Roy. Astronom. Soc.*, 264:132–+, Sep. 1993.

[6] D. V. Bisikalo, A. A. Boyarchuk, P. V. Kaigorodov, O. A. Kuznetsov, and T. Matsuda. The Structure of Cool Accretion Disks in Semidetached Binaries. *Astron. Rep.*, 48:449–456, Jun. 2004.

[7] D. V. Bisikalo, A. A. Boyarchuk, P. V. Kaigorodov, O. A. Kuznetsov, and T. Matsuda. A Model for Superoutbursts in SU UMa-type Binaries. *Astron. Rep.*, 48:588–596, Jul. 2004.

[8] D. V. Bisikalo, A. A. Boyarchuk, P. V. Kaygorodov, and O. A. Kuznetsov. The morphology of interaction between a stream and cool accretion disk in semidetached binaries. *Astron. Rep.*, 47:809, 2003.

[9] A. A Boyarchuk, D. V. Bisikalo, O. A. Kuznetsov, and V. M. Chechetkin. *Mass transfer in close binary stars*. Taylor & Frances, London, 2002.

[10] P. L. Roe. Characteristic-based schemes for the euler equations. In *Annual review of fluid mechanics*, volume 18, pages 337–365. Eds. Annual Reviews Inc., 1986.

[11] S. R. Chakravarthy and S. Osher. A new class of high accuracy tvd schemes for hyperbolic conservation laws. In *Proceedings of the 23rd Aerospace Sci. Meeting*, number 85 in AIAA pap., page 363. AIAA, 1985.

[12] D. V. Bisikalo, A. A. Boyarchuk, O. A. Kuznetsov, and V. M. Chechetkin. The Effect of Viscosity on the Flow Morphology in Semidetached Binary Systems. Results of 3D Simulations. II. *Astron. Rep.*, 44:26–35, Jan. 2000.

[13] D. V. Bisikalo. Numerical Modeling of Mass Transfer in Close Binaries. *Astrophys. and Space Sci.*, 296:391–401, Apr. 2005.

[14] D. V. Bisikalo, A. A. Boyarchuk, P. V. Kaigorodov, O. A. Kuznetsov, and T. Matsuda. A new type of spiral waves in accretion discs. In *ASP Conf. Ser. 330: The Astrophysics of Cataclysmic Variables and Related Objects*, pages 383–+, May. 2005.
[15] D. V. Bisikalo, A. A. Boyarchuk, P. V. Kaygorodov, O. A. Kuznetsov, and T. Matsuda. Formation of the ‘precessional’ spiral wave in the cool accretion disk in semidetached binaries. In *AIP Conf. Proc. 797*: *Interacting Binaries: Accretion, Evolution, and Outcomes*, pages 295–300, Oct. 2005.

[16] D. V. Bisikalo, P. V. Kaigorodov, A. A. Boyarchuk, and O. A. Kuznetsov. The Possible Nature of Dips in the Light Curves of Semidetached Binaries with Stationary Disks. *Astron. Rep.*, 49:701–708, Sep. 2005.

[17] D. O’Donoghue. Superhumps: the state of play. *New Astronomy Review*, 44:45–50, Apr. 2000.

[18] K. R. Lang. *Astrophysical Formulae: a Compendium for the Physicist and Astrophysicist*. Springer-Verlag, Berlin, 1974.

[19] M. Hirose and Y. Osaki. Hydrodynamic simulations of accretion disks in cataclysmic variables - Superhump phenomenon in SU UMa stars. *Publ. Astron. Soc. of Japan*, 42:135–163, Feb. 1990.