Whispering Vortices

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Experiments indicating the excitation of whispering gallery type electromagnetic modes by a vortex moving in an annular Josephson junction are reported. At relativistic velocities the Josephson vortex interacts with the modes of the superconducting stripline resonator giving rise to novel resonances on the current-voltage characteristic of the junction. The experimental data are in good agreement with analysis and numerical calculations based on the two-dimensional sine–Gordon model.

Whispering gallery modes are universal linear excitations of circular and annular resonators. They have first been observed in form of a sound wave traveling along the outer wall of a walkway in the circular dome of St. Paul’s Cathedral in London and were investigated by Lord Rayleigh and others. In the 2 meter wide walkway, which forms a circular gallery of 38 meter diameter about 40 meters above the ground of the cathedral, the whispering of a person can be transmitted along the wall to another person listening to the sound on the other side of the dome. The investigations by Rayleigh led to the conclusion that the whisper of a person does excite acoustic eigenmodes of the circular dome which can be described using high order Bessel functions. This acoustic phenomenon lends its name “whispering gallery mode” to a number of similar, mostly electromagnetic excitations in circular resonators. Whispering gallery modes are of strong interest in micro-resonators with small deformations, in which chaotic whispering gallery modes were observed, attracted a lot of attention. Here we describe the experimental observation of electromagnetic whispering gallery modes excited by a vortex moving in an annular Josephson junction of diameter less than 100 µm.

A long Josephson junction is an intriguing nonlinear wave propagation medium for the experimental study of the interaction between linear waves and solitons. In this letter we report the excitation of whispering gallery type electromagnetic modes by a topological soliton (Josephson vortex) moving at relativistic velocities in a wide annular Josephson junction. We make use of the same Josephson vortex for both exciting and detecting the whispering gallery mode. These modes are manifested by their resonant interaction with the moving vortex, which results in a novel fine structure on the current-voltage characteristic of the junction. Our experiments are consistent with the recently published theory based on the two-dimensional sine–Gordon model. We present numerical calculations based on this model, which show good agreement with experiments.

Electromagnetic waves in an annular Josephson junction are described by the perturbed sine-Gordon equation (PSGE) for the superconducting phase difference \( \phi \) between the top and bottom superconducting electrodes of the junction. The Josephson vortex, often also called fluxon, corresponds to a twist over \( 2\pi \) in \( \phi \). It carries a magnetic flux equal to the magnetic flux quantum \( \Phi_0 = h/2e = 2.07 \times 10^{-15} \text{Vs} \). Physically, this flux is induced by a vortex of the screening current flowing across the junction barrier. Linear excitations in this system are Josephson plasma waves that account for small amplitude oscillations in \( \phi \). The maximum phase velocity of electromagnetic waves in such a junction is the Swihart velocity given by \( c_0 = \lambda_J \omega_p \), where \( \lambda_J \) is the Josephson length and \( \omega_p \) the plasma frequency. In zero external magnetic field the PSGE for an annular Josephson junction of width \( w < \lambda_J \) can be written as

\[
\left( \nabla^2 - \frac{\partial^2}{\partial t^2} \right) \phi - \sin \phi = \gamma + \alpha \frac{\partial \phi}{\partial t} - \beta \nabla^2 \frac{\partial \phi}{\partial t},
\]

where space and time are normalized by \( \lambda_J \) and \( \omega_p^{-1} \), respectively. In Eq. (1) \( \nabla^2 - \partial^2 / \partial t^2 \) is the D’Alembert wave operator, \( \sin \phi \) is the nonlinear term due to the phase-dependent Josephson current and \( \gamma \) is the normalized bias current. The damping terms \( \alpha \partial \phi / \partial t \) and \( \beta \nabla^2 \partial \phi / \partial t \) are inversely proportional to the quasiparticle resistance across the junction barrier and to the quasiparticle impedance of the electrodes, respectively. For the junctions of width \( w < \lambda_J \) considered in this paper, a homogeneously distributed bias current \( \gamma \) as in Eq. (1) is justified. In contrast, for junctions with \( w > \lambda_J \) the
bias current may contribute to the boundary conditions of Eq. (3).

A vortex steadily moving at a velocity \( u \) driven by the Lorentz force due to the bias current \( \gamma \) generates a voltage \( V \propto u \) across the Josephson junction. This voltage can be monitored in experiment. The radiation associated with the time-dependent fields described by Eq. (1), i.e. the magnetic field \( H \propto \nabla \phi \) and the electric field \( E \propto \partial \phi / \partial t \), can be measured either directly (for certain junction geometries) or through its interaction with the moving vortex.

In contrast to most of the previous experiments focusing on quasi-one-dimensional annular Josephson junctions, we investigate comparatively wide, effectively two-dimensional junctions. We have fabricated a set of 5 annular Josephson junctions (A ... E) with the ratio \( \delta = r_i / r_c \) between the inner radius \( r_i \) and the fixed outer radius \( r_c = 50 \mu m \) being varied between \( \delta = 0.94 \) and \( \delta = 0.60 \) (see Table I). The junctions are made at Hypres Inc. using Nb-Al/AlO\(_x\)-Nb trilayer technology and employ the standard biasing geometry as shown in Fig. 1. Due to the fabrication technology, the junction area is surrounded by a small passive region about 2 \( \mu m \) wide, which is omitted from Fig. 1 for clarity. In the passive region the top and bottom electrodes are separated by a 200 nm thick SiO\(_2\) layer, which act as a small stripe in parallel to, but with electrical parameters different from the junction itself.

All junctions show a homogeneous bias current distribution, inferred from the large value of the vortex-free critical current at zero field, which is close to the theoretical limit. Their critical current density is \( j_c \approx 160 \text{ Acm}^{-2} \) and the London penetration depth is \( \lambda_{L0} \approx 90 \text{ nm} \) at 4.2 K. The thicknesses of the top and the bottom superconducting electrode are both well in excess of \( \lambda_{L0} \). Accordingly, the characteristic parameters are estimated as \( \lambda_{L} \approx 30 \mu m \) and \( \nu_p \equiv \nu_p / 2\pi \approx 50 \text{ GHz} \). All presented measurements were done at \( T = 4.2 \text{ K} \) using a well shielded low noise measurement setup.

We could realize single and multiple vortex states repeatedly and reproducibly in any of the junctions. Vortices were trapped by applying a small bias current during cooling down from the normal to the superconducting state. Single-vortex states are identified as the lowest quantized voltage step observed on the current-voltage characteristics. Also, a characteristic change of the critical current modulation with magnetic field accompanied by a suppression of the critical current by a factor of more than 100 at zero field, as reported earlier, was observed when a vortex was trapped in the junction.

In Figure 2 the single-vortex characteristics of the junctions A to E are shown. The current scale is normalized by the flux-free zero-field current of each junction. The voltage of each characteristic is multiplied by a factor \( \xi = c_0 / c_0 \) (see Tab. I), calculated using the approach by Lee et al., in order to subtract the effect of a small passive region. \( c_0 (c_0) \) is the wave velocity in the junction neglecting (including) the passive region.

A striking novel feature noticed in Fig. 2 is the fine structure on the vortex resonance which appears with increasing the junction width. The fine structure is most clearly visible for the widest junction E (see inset of Fig. 2). We argue that the observed fine structure can be well understood as due to the interaction of the moving vortex with the linear whispering gallery modes. The linear modes of the annular Josephson junction resonator are solutions to the inhomogeneous D’Alembert equation in the polar coordinates \( (r, \theta) \)

\[
\left( \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} - \frac{\partial^2}{\partial t^2} - 1 \right) \phi^{(\text{lin})} = 0,
\]

which is found from Eq. (1), neglecting all perturbations \( \gamma, \alpha \partial \phi / \partial t, \beta \nabla^2 \phi / \partial t \) and approximating the nonlinearity as \( \sin \phi \approx \phi \) to take into account the gap in the plasmon excitation spectrum. In zero external magnetic field the boundary conditions

\[
\frac{\partial \phi^{(\text{lin})}}{\partial r} (r = r_i, r_c) = 0
\]

have to be fulfilled. In terms of the electromagnetic waves in the junction, Eq. (3) corresponds to a total internal

| junction | A | B | C | D | E |
|----------|---|---|---|---|---|
| \( r_i \ [\mu m] \) | 47 | 45 | 42 | 35 | 30 |
| \( w \ [\mu m] \) | 3 | 5 | 8 | 15 | 20 |
| \( \delta = r_i / r_c \) | 0.94 | 0.90 | 0.84 | 0.70 | 0.60 |
| \( \xi = c_0 / c_0 \) | 0.70 | 0.78 | 0.85 | 0.91 | 0.94 |

FIG. 2. Experimental normalized current-voltage characteristics of single-vortex states in junctions A to E. An enlargement of the high voltage region of the resonances in junction D and E is shown in the inset.
FIG. 3. Numerically calculated current-voltage characteristics $V(\gamma)$ for junctions A to E. In the inset the characteristics of junctions D and E are shown on an enlarged scale. Arrows indicate the bias points used to obtain the phase profiles shown in Fig. 4.

Table I). We calculated current-voltage characteristics $V(\gamma) = \Phi_0 \Omega(\gamma) \omega_p/(2\pi)$ and two-dimensional phase profiles $\phi(r, \theta, t)$ for various bias points using a plasma frequency of $\omega_p/2\pi = 52.4$ GHz determined from experimental data. The damping parameter $\alpha = 0.03$ was chosen close to its estimated experimental value at $T = 4.2$ K. $\beta$ here was set to 0. The calculated characteristics for junctions A to E are plotted in Fig. 3. Clearly, the fine structure on the current-voltage characteristics of the wide junctions D and E is very well reproduced in the simulation. Figure 3 shows the phase profiles at bias points on subsequent fine structure resonances of junction E. Evidently, a clear whispering gallery structure is found here. With decreasing bias, the increase of the angular wave number of the mode from resonance to resonance is observed.

Using the resonance condition (3) and the proportionality between the angular frequency of the vortex $\Omega$ and $\omega_k = \omega_p/(2\pi)$, we found the phase profiles at bias points on subsequent fine structure resonances of junction E. Evidently, a clear whispering gallery structure is found here. With decreasing bias, the increase of the angular wave number of the mode from resonance to resonance is observed.

To confirm the above interpretation of our experimental results, we performed direct numerical simulations of Eq. (2) in polar coordinates with the boundary conditions (3) using our junction parameters (see Table I). We calculated current-voltage characteristics $V(\gamma) = \Phi_0 \Omega(\gamma) \omega_p/(2\pi)$ and two-dimensional phase profiles $\phi(r, \theta, t)$ for various bias points using a plasma frequency of $\omega_p/2\pi = 52.4$ GHz determined from experimental data. The damping parameter $\alpha = 0.03$ was chosen close to its estimated experimental value at $T = 4.2$ K. $\beta$ here was set to 0. The calculated characteristics for junctions A to E are plotted in Fig. 3. Clearly, the fine structure on the current-voltage characteristics of the wide junctions D and E is very well reproduced in the simulation. Figure 3 shows the phase profiles at bias points on subsequent fine structure resonances of junction E. Evidently, a clear whispering gallery structure is found here. With decreasing bias, the increase of the angular wave number of the mode from resonance to resonance is observed.

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of moving vortices (or vortex/anti-vortex pairs). Therefore, for $n > 1$ the fine structure gets clearly resolved in voltage and also more pronounced, because several vortices coherently pump the whispering gallery mode. No dependence of the fine structure step voltage positions on small external magnetic fields was noticed. We have also investigated more narrow annular junctions with a wide idle region both experimentally and theoretically\textsuperscript{[12]}. In this case, the spectrum of the whispering gallery modes (and, thus, of the fine structure) is strongly influenced by the geometry and the electrical properties of the passive region. The fine structure recently reported in Ref.\textsuperscript{[13]} appears to be consistent with our observations.

In summary, we have presented experimental and numerical evidences for the excitation of whispering gallery modes by vortices moving in wide annular junctions. This novel effect has been observed at sufficiently low damping for annular junctions in a wide range of electrical and geometrical parameters. It is very robust with respect to small external perturbations such as variations in bias current density, boundary conditions or junction inhomogeneities. The resonance frequencies have been calculated and quantitative agreement with experimental data and numerical simulations better than one percent has been reached. Thus, the vortices appear to whisper (generate radiation) at frequencies between 250 and 450 GHz in the annular whispering gallery of 100 $\mu$m diameter.

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