String-like Theory of Quantum Hall Interfaces

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We derive the effective theories for quantum hall droplets with attractive interaction among the constituent particles. In the absence of confining potentials such droplets are defined by their freely moving interfaces (or boundaries) with the vacuum. We demonstrate the effective theories take forms similar to string theories. Generalization to interfaces between different quantum Hall liquids is discussed.

I. INTRODUCTION

Quantum Hall (QH) effect is one of the most active research fields in condensed matter physics[1]. There are many different approaches to describe QH physics [2–8]. In the bulk, the low energy effective theory of a QH liquid is the Chern-Simons (CS) theory [9]. At edge, the low energy effective theory consists of two pieces, the topological and the dynamical piece [9]. The topological piece is dictated by the bulk CS theory (known as bulk-edge correspondence) so that the overall theory describing the bulk and the edge is gauge-invariant. Just like the bulk CS theory it describes the degrees of freedom at the edge (usually chiral bosonic and Majorana fermionic modes), but contains no dynamical information. Dynamics at the edge comes from external confining potential, which determines the location of the edge and dictates the edge Hamiltonian[1, 9].

Recently, one of us considered a different type of boundaries [10], namely the quantum Hall interfaces that separate different QH phases, whose locations are not determined (or pinned) by any external potential. A special class of such interfaces separate a QH liquid from vacuum, which (superficially) look very much like edges. Instead of repulsive interaction between the electrons in the system, Ref. [10] invoked attractive interactions to hold the QH liquid together, thus eliminating the necessity of the external confining potential. Accordingly, such a quantum Hall droplet (QHD) is formed spontaneously and can move freely. In the absence of confining potential the interface behaves like a string, whose dynamics is dominated by the string tension[10].

Ref. [10] focused on the low-energy excitation spectra of various interfaces, which can be accessed easily by modifying the Hamiltonian of the corresponding edge theories. To describe the general interface dynamics however, we need to study large distortions of the interface configurations from the ground state. This is the task of our present work. As we will see this requires a reformulation of the entire theory (including the topological term), which leads to a string-like theory for the interfaces. At low-energy this allows for improvement of the results of Ref. [10] by including curvature effects in a systematic way, which accounts for the (relatively small but definitely noticeable) discrepancy between theory prediction and numerical data. More importantly we demonstrate QH interfaces realize certain non-relativistic versions of both bosonic and superstrings; in the latter case there are Majorana fermion modes that live along the strings. This facilitates synergy between condensed matter and high-energy physics.

The rest of the paper is organized as follows. In Sec. II we start by including the curvature effects in the energy functional for the interface between fermionic $\nu = 1$ QH liquid and vacuum, and demonstrate this leads to significant improvement in the comparison between first-principle prediction of the low-energy excitation spectrum and numerical results. In Sec. III we show similar improvement for the Pfaffian-vacuum interface. Sec. IV is devoted to the derivation of the topological term in the action when the interface configuration deviates significantly from that of the ground state. Some concluding remarks are offered in Sec. V, where we discuss possible generalization of our approach to interfaces between different quantum Hall liquids.

II. INTERFACE BETWEEN FERMIONIC $\nu = 1$ QUANTUM HALL LIQUID AND VACUUM

We start with the simplest possible QH interface separating the fermionic $\nu = 1$ QH liquid and vacuum, with the former stabilized by an attractive Haldane $V_1$ pseudopotential[1] and the Pauli principle. Due to the simplicity of this state parameters of the string-like theory can be determined from microscopic calculations.

The effective action of a droplet with $N$ fermions and radius $r_0 = \sqrt{2N}$ (we set the magnetic length, $\ell$, to unity) in the low-energy (or linear) regime is a slight modification of the edge theory with a single chiral bosonic field[10]:

$$S = -\frac{1}{4\pi} \int dt \int_0^{2\pi r_0} dx (\dot{\phi}^2 - \frac{T_0}{2} \int dt \int_0^{2\pi r_0} dx (u')^2),$$

where $T_0$, is the string tension, $x$ is the coordinate along the unperturbed string, $t$ is time, prime is $\partial_x$, dot is $\partial_t$ and $u$, the transversal displacement from equilibrium (see Fig. 1-a), is given by

$$\dot{\phi} = u,$$
where $\phi$ is a compact scalar with identification
\[ \phi \sim \phi + 2\pi. \] (3)
The first term in Eq. (1) is the aforementioned topological piece which emerges from the bulk CS and the second term, the dynamical piece, is the length $[11]$ times the tension in the limit of $|u'| \ll 1$ and $r_0 \to \infty$. We will use the term “linear regime” for such approximation, which is valid when the deviation to the ground state configuration is small and slowly varying.

### A. Action beyond the linear regime

When we consider larger deformations, higher order terms in $u$ and its derivatives become important, and Eq. (1) needs to be modified. As a first step we have to replace the approximated arc length in Eq. (1) (that renders it quadratic) with the actual arc length. However, at this point the energy of the system is completely determined by the length of the interface. Thus classically energy conservation means conservation of the string length. This together with the area conservation constraint (which comes from the incompressibility of the QHD) implies that the classical equation of motion of the string would be trivial, namely the only allowed motions are rigid spinning and motion of the center of mass. Thus, as we will see below such a modified action would only depend on time, and it enforces area conservation. If we set $r_0 \to \infty$ and expand the Eq. (5) up to quadratic order and set $T \to T_0$ we will get Eq. (1) back.

### B. Extracting the curvature-dependent tension

We can find $T(\kappa)$ by extending the method of Ref. [10] that determined $T(\kappa = 0)$. In units of $|V_1|$ (set to be one from now on) the ground state energy of the droplet in terms of number of electrons was found to be [10]
\[ \langle V \rangle = \frac{1}{2N} \left( 4 - \frac{4^2 - N(2N-1)!}{(N-1)!N!} \right). \] (6)
Expanding at large $N$ yields
\[ \langle V \rangle = -2N + \frac{4\sqrt{N}}{\sqrt{\pi}} - \frac{\sqrt{\pi}}{2\sqrt{N}} + O \left( \left( \frac{1}{N} \right)^{3/2} \right). \] (7)
The first term is the bulk contribution, and the rest is the surface contribution that can be compared with our low energy effective theory Eq. (5). According to Eq. (5)
and Eq. (4), the energy of a perfectly circular interface is just $2 \pi r_0 T(1/r_0)$. As we show below this with Eq. (7) determines $T(\kappa)$ up to third order in $\kappa$: 

$$T(\kappa) = T_0 + T_1 \kappa + T_2 \kappa^2 + O(\kappa^4),$$

(8) 
where $T_0$, $T_1$ and $T_2$ are constants. Comparing Eq. (8) with Eq. (7) yields

$$T(\kappa = \frac{1}{r_0}) = \frac{1}{\sqrt{2N}} \frac{\sqrt{4\pi N}}{\sqrt{2\pi}} - \frac{\sqrt{2}}{2\sqrt{\pi}},$$

(9) 
which implies $T_1 = 0$ and

$$T_2 = -\frac{T_0}{4} = -\frac{1}{8\pi},$$

(10) 
Finally, we can substitute Eq. (10) to Eq. (8) and find for an arbitrary local curvature $\kappa$, 

$$T(\kappa) = \frac{4 - \kappa^2}{\sqrt{8\pi}}, + O(\kappa^4).$$

(11) 
Note our finding $T_1 = 0$ is not an accident; in fact all odd terms in the expansion Eq. (8) are zero, because if we exchange the regions of QH liquid and the vacuum the sign of the curvature flips, but the surface contribution to the energy remains same due to the particle-hole symmetry of the two-body Hamiltonian.

C. Effects of the curvature dependent tension on the low-energy excitation spectrum

We substitute Eq. (8) to Eq. (5) and expand it up to quadratic order in $u$ which gives

$$S = -\frac{1}{4\pi} \int d^2x \phi' \phi'' + \int d^2x \lambda (\phi' - u)$$

$$- \int d^2x \left( T_0 \frac{u'^2}{2} + T_2 \left( \frac{u^2}{r_0^2} + \frac{4u'u'' + \frac{3}{2}u''^2}{r_0^2} + u''^2 \right) \right)$$

$$+ \int dw \left( \frac{1}{2} \int dx r_0 \left( \frac{u}{r_0} + 1 \right)^2 - \pi r_0^2 \right) + O(u^3),$$

(12) 
where we have dropped unimportant total derivative terms. To obtain the mode spectrum we first integrate out the $\lambda$ and then express the action in term of $\phi$ in momentum-frequency space:

$$\phi(t, x) \propto \int d\omega \sum_k e^{-i\omega t + ikx} \phi_{k\omega},$$

(13) 
where the discrete momentum (due to finite size of QHD) is $k = n/r_0$ with $n \in \mathbb{Z}$. The action in momentum-frequency space is thus

$$S = \int \omega \sum_k |\phi|^2 \left( \frac{\omega}{4\pi} - T_0 \frac{k^3}{2} - T_2 \left( \frac{k}{r_0} - \frac{5}{2r_0^2} k^3 + k^5 \right) \right)$$

$$+ \int dt dw \left( \frac{1}{2} \int dx r_0 \left( \frac{u}{r_0} + 1 \right)^2 - \pi r_0^2 \right).$$

(14) 
We can neglect terms of order of $1/r_0$ and higher for sufficiently large $N$. The quadratic action is thus (back to space-time domain)

$$S = -\frac{1}{4\pi} \int d^2x \phi' \phi'' - \int d^2x \left[ T_0 \frac{u'^2}{2} + T_2 u''^2 \right]$$

$$+ \int d^2x \lambda (\phi' - u) + O(u^3) + O \left( \frac{1}{r_0} \right).$$

(15) 
Comparing to Eq. (1), we find a correction due to $T_2$ in this limit. The chiral mode dispersion relation can be easily extracted:

$$\omega = \sqrt{\frac{8}{\pi} \left( \frac{k^3}{k^5} - \frac{2}{3} \right)},$$

(16) 
which compares extremely well with numerical results (see Fig. 2): the agreement is significantly better than the spectrum without taking into account the curvature effect. We note if we had included in Eq. (8) higher order terms, their contribution would vanish at the present level of approximation (i.e. keeping quadratic terms only and taking the $r_0 \to \infty$ limit). This is clearer if we just express the curvature in the linear regime as

$$\kappa = u'' + O(u^2) + O \left( \frac{1}{r_0} \right).$$

(17) 
This means the spectrum Eq. (16) is exact in the large $N$ or $r_0$ limit; any further deviation with numerics (see Fig. 2) will have to be accounted for by nonlinearity (or interaction among the bosons), and/or finite-size effect.

D. General form of string action

Our discussion thus far has assumed the deviation of the interface from its ground state (circular) configuration is (still) small. As a result our action (5) is an expansion of the deviation parameterized by $u$. However since there is no confinement potential, such deviation is unbounded in the present case of interface (and unlike the edge), rendering $u$ hard to define, let alone using it for expansion in general (see Fig. 1-b). In the general case we should instead work directly with the world sheet of the interface $X(t, \sigma)$ where $\sigma$ parameterizes a point on the interface (see Fig. 1-b). The general form of the interface action is a functional of the world sheet $X(t, \sigma)$, and is a sum of a topological (or Wess-Zumino-like) term and an energy term:

$$S = S_{top}[X(t, \sigma)] - \int dt E[X(t, \sigma)],$$

(18) 
where, based on our analysis above,

$$E[X(t, \sigma)] = \int d\sigma T(K(\sigma)) / \partial X / \partial \sigma.$$

(19) 
The derivation of $S_{top}[X(t, \sigma)]$ is subtler, and will be the subject of Sec. IV. What should be clear at this point is the action (18) takes the form of some kind of (bosonic) string theory [14].
III. PFAFFIAN-VACUUM INTERFACE

In this section we consider a more complicated and interesting interface, that between a Moore-Read (Pfaffian) state formed by bosons of filling factor $\nu = 1$ with a trivial vacuum. Here again the $\nu = 1$ boson droplet is formed spontaneously by an attractive $V_0$ pseudopotential, supplemented by an infinitely strong three-body interaction that prevents the bosons from collapsing, which also makes the Moore-Read (Pfaffian) wave function the exact ground state [10]. The edge theory of the regular Moore-Read state consists of a chiral Majorana mode and a chiral bosonic mode which are decoupled and both have linear dispersions. In the present interface case however, the dispersion of the bosonic mode was found to have a similar form as in the previous section, while the dispersion of the Majorana mode remains linear [10]. The edge theory of the regular Moore-Read state consists of a chiral Majorana mode and a chiral bosonic mode which are decoupled and both have linear dispersions. In the present interface case however, the dispersion of the bosonic mode was found to have a similar form as in the previous section, while the dispersion of the Majorana mode remains linear [10]. We can still expect them to decouple, since by a simple dimensional analysis (or power counting) there is no relevant interaction term between the bosonic and fermionic modes.

We write the effective edge theory for such system as

$$S = \int d^2 x (i \dot{\psi} \gamma^i - v_i \psi \gamma^i \dot{\psi'}) + S_B,$$

where $S_B$ is the action given in Eq. (5), with different coefficients for $T_0$ etc. which will be estimated by fitting to numerical data Fig. 3. Accordingly, we find that the dispersion relation for pure bosonic excitations to be

$$\omega \approx 0.2k^3 - 0.04k^5,$$

from which we extract $T_0 \approx 0.031$ and $T_2 \approx -0.0031$. The fermionic excitation dispersion relation remains linear with $v \approx 0.98$, all in units of $|V_0|$. Since there is no particle-hole symmetry for bosons, $T_1$ is not expected to be zero in this case. However it does not enter the quadratic action (12), as a result of which it has no effect on the dispersion. It does, however, has effects on the action beyond the quadratic level, which gives rise to boson-boson interaction.

Due to the presence of the fermionic mode, the interface here is more like a "superstring" [14], although there is no supersymmetry here. It has been shown [15], however, the Moore-Read (Pfaffian) edge can be made supersymmetric, and this supersymmetry can even be broken spontaneously, resulting in a novel Goldstino mode at low energy. It is thus worthwhile to explore the analogy between the dynamics of this interface and superstring in future work.

IV. TOPOLOGICAL TERM OF THE STRING THEORY VIA CHERN-SIMONS THEORY WITH FLUCTUATING BOUNDARY

So far we have treated interfaces as a string-like objects such that the energy of the system is a functional of its length and local curvature. We showed that such
identification enabled us to derive an effective low energy interface theory, that gives a low-energy dispersion relation that agrees with the numerical calculations of the microscopic models extremely well. We have achieved this success by combining the energy functional with a topological term in the action that was derived earlier for edge theory, which, as we discussed in Sec. II.D, is only appropriate for small fluctuations of the interface. In this section we attempt to derive the topological action that is appropriate for large fluctuations, for the simplest case of interface between the fermionic $\nu = 1$ QHD and vacuum.

Let us recall how the topological term in the edge theory, namely the first term of Eq. (1) was obtained [9]. One starts with the bulk Chern-Simons theory, and place it on a 2D manifold with fixed (or time-independent) boundary. One finds that the Chern-Simons theory is not gauge-invariant, as a gauge transformation generates an additional boundary term to the action. This indicates there must be boundary degrees of freedom not captured by the bulk Chern-Simons theory, which are precisely the edge states. To describe such edge states, one impose certain "gauge fixing" condition at the boundary, which turns certain gauge "degree of freedom" there to physical degree of freedom, whose dynamics is actually sensitive to the gauge fixing condition. One thus has to carefully choose the gauge fixing condition (in an ad hoc way) to get the correct edge dynamics.

The situation we are facing here has three important differences. (i) The interface, as parameterized by $X(t,\sigma)$, is fluctuating and time-dependent. (ii) $X(t,\sigma)$ is the explicit physical boundary degree of freedom, so no new degree of freedom needs to be introduced. (iii) Perhaps most importantly, the dynamics of $X(t,\sigma)$ comes from the energy functional $E[X(t,\sigma)]$ in (18). We thus should not get (additional) dynamical term from gauge-fixing the Chern-Simons theory. As we show below, these allow us to derive the appropriate topological term in the action (18).

To proceed, we go back to the bulk Chern-Simons theory and now explicitly assume a kinematically time-dependent boundary, and discuss its gauge properties. The theory is given as

$$S_{CS} = \int_{-\infty}^{\infty} dt \int_{\partial D(t)} d^2x \left[ -\frac{1}{4\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda \right]$$

where $A$ is the background gauge field and $D$ is time-dependent that the droplet lives. Under the gauge transformation $a_\mu \rightarrow a_\mu + \partial_\mu \Lambda$ the total change of the action is

$$\Delta S_{CS} = \int d\tau \int_{\partial D(t)} d^2x \epsilon^{ij} \partial_i (\Lambda \partial_j a_j)$$

$$+ \int d\tau \int_{\partial D(t)} d^2x \Lambda (\partial_i a_j + \partial_j a_i)$$

$$= \int d\tau \int_{\partial D(t)} d^2x \left[ \partial^i (\Lambda \partial_i a_j) - dx^j (\partial_i a_j + \partial_j a_i) \right]$$

(23)

where we have used the Reynolds-Leibniz rule for differentiation under integral [16] and $R$ is $[-\infty, \infty]$, $\nu(X,t)$ is the velocity vector of the boundary at point $X$. Note in earlier work considering fixed boundary, $\nu = 0$ and only the first term above is present, and it is the existence of such boundary term that ruins the gauge invariance of the Chern-Simons theory in manifolds with boundaries. We note, however, in the present case, we can make $\Delta S_{CS}$ zero, and hence the theory gauge invariant, if we impose the following local boundary condition:

$$J_n(X,t) = J_0 n(X,t),$$

(24)

where we used the fact that the bulk current [9]

$$J^\mu = \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} \partial_\nu a_\lambda,$$

(25)

and the subscript $n$ stands for the normal component of a (spatial) vector at the boundary point $X$. In particular $J_0 = \frac{1}{2\pi}$ is the density of the $\nu = 1$ QH liquid which is a constant when the back ground magnetic field $B = \epsilon^{ij} \partial_i A_j$ is uniform, although everything we say apply to non-uniform $B$ as well.

The boundary condition (Eq. (24)) has a very clear physical meaning: when there is bulk current $J_0$ flowing toward the boundary, the boundary must recede with speed $v_n$ to accommodate the excess charge. As a result charge conservation is preserved, hence gauge invariance. This contrasts with the usual approach to the edge theory which assumes the boundary is fixed, as a result charge would not be conserved (hence lack of gauge invariance, or charge conservation becomes "anomalous" at the boundary) without the additional degree of freedom $u$ introduced by hand. It should be clear from Fig. 1 that $u$ is nothing but the fluctuation of the boundary. Our discussion above thus provides a new perspective of the traditional edge theory. A bonus of Eq. (24) is it also enforces area conservation (which is nothing but charge conservation for an incompressible liquid).

The topological term of the full string theory, $S_{top}$ of (18), is nothing but the Chern-Simons action (22), subject to the constraint, Eq. (24), which is an (implicit) functional of $X(t,\sigma)$. In other words $S_{top}$ can be obtained from integrating over the gauge fields $a$ in (22), subject to the constraints of Eq. (24) and Eq. (25).

Armed with Eq. (24) which couples the boundary (interface) with the bulk Chern-Simons field $a$, we are ready
to derive the topological action $S_{\text{top}}$ in (18). To do that we adopt the gauge-fixing condition $a_0(X) = 0$ at the interface, which is known not to generate (additional) dynamical terms[17]. Following the standard procedure, we introduce a scalar field $\phi$ such that

$$a_j = A_j + \partial_j \phi \tag{26}$$

in the bulk which solves the bulk constraint $\epsilon^{ij} \partial_i a_j = \epsilon^{ij} \partial_i A_j$ obtained from integrating over $a_0$. With (26) $S_{\text{CS}}$ reduces to a boundary term:

$$S_{\text{top}} = -\frac{1}{4\pi} \int dt \int d\sigma \dot{\phi} \dot{\phi}' \tag{27}$$

which is essentially the first term of (1), but with arbitrary parameterization along the boundary $X(t, \sigma)$, and now $\phi' = \frac{d\phi}{d\sigma}$ so $S_{\text{top}}$ as well as the entire action (18) is explicitly invariant under re-parameterization. Most crucially however, $\phi$ is constrained by (24), via the identification

$$J_n(X, t) = \frac{1}{2\pi} (\partial \phi / \partial \sigma) / (\partial X / \partial \sigma). \tag{28}$$

As a result $S_{\text{top}}$ is an (implicit) functional of $X(t, \sigma)$.

V. CONCLUDING REMARKS

In this paper we have studied string-like theories for free interfaces separating quantum Hall droplets from vacuum. The two cases we studied correspond to bosonic and superstrings respectively, in terms of the degrees of freedom associated with their interfaces. While they may seem similar to the usual quantum Hall edges, we demonstrated their dynamics is very different when such interfaces are not pinned by confining potentials, and requires string-like theories to describe. We derived such a theory for the case of bosonic string, and leave such a derivation for superstring to future work.

Physically more interesting cases are interfaces between different quantum Hall liquids. A particularly interesting example is the $\nu = 5/2$ quantum Hall liquid, where multiple energetically competitive states are present, and may form various interfaces (see [18] for a recent review). The methods we developed in this paper can be generalized to such interfaces as well. In particular, the energy term in (18) should be present in the generic case. The topological term, on the other hand, needs to be derived by combining the bulk Chern-Simons-like theories on both sides of the interface, supplemented by appropriate boundary conditions like Eq. (24).

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