Linear Resonance Compressor for Stirling-Type Cryocoolers Activated by Piezoelectric Stack-Type Elements

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Abstract. A novel type of a PZT-based compressor operating at mechanical resonance, suitable for pneumatically-driven Stirling-type cryocoolers was developed theoretically and built practically during this research. A resonance operation at relatively low frequency was achieved by incorporating the piezo ceramics into the moving part, and by reducing the effective piezo stiffness using hydraulic amplification. The detailed concept, analytical model and the test results of the preliminary prototype were reported earlier and presented at ICC17 [2]. A fine agreement between the simulations and experiments spurred development of the current actual compressor designed to drive a miniature Pulse Tube cryocooler, particularly our MTSa model, which operates at 103 Hz and requires an average PV power of 11 W, filling pressure of 40 Bar and a pressure ratio of 1.3. The paper concentrates on design aspects and optimization of the governing parameters. The small stroke to diameter ratio (about 1:10) allows for the use of a composite diaphragm instead of a clearance-seal piston. The motivation is to create an adequate separation between the working fluid and the buffer gas of the compressor, thus preventing possible contamination in the cryocooler. Providing efficiency and power density similar to those of conventional linear compressors, the piezo compressor may serve as a good alternative for cryogenic applications requiring extreme reliability and absence of magnetic field interference.

1. Introduction
Pneumatically activated Stirling-like cryocoolers and all types of the Pulse Tube Refrigerators (PTR) are driven by a pure pressure oscillator, which is sometimes referred to as a valve-less compressor. Concerning a PTR, the pressure oscillator is a critical component, since it is the only mechanically active part subject to wear and failure, and therefore, the lifetime of the cryocooler depends mostly on the compressor reliability. Additionally, occasional contamination of the working gas originates generally from the compressor, which may be a source of wear products, outgassing, lubricant vapour, etc.

Mostly, two compressor types, rotary inductive and linear inductive, are employed in cryocoolers today. A rotary compressor generally has a shorter lifetime than a linear one due to wear of bearings, the crank shaft mechanism and the piston-cylinder interface. Moreover, a rotary compressor produces a troublesome angular momentum, which is hard to eliminate or reduce. Linear compressors possess higher reliability and are easier to balance. However, they feature lower efficiency, increased weight and volume, and require precise adaptation to each refrigerator because of their resonant nature. Additionally, any inductive compressor naturally produces magnetic and electric field emissions,
which may degrade performance or cause malfunction of the cooled detectors and neighboring instruments.

In an effort to improve the efficiency and reliability, as well as eliminate the electromagnetic emissions, it was proposed to replace the driving mechanism of the conventional linear compressor by a device activated by a piezoelectric stack-type element (PZT actuator), which is frictionless, has a high volumetric power density and extremely long lifetime [1]. To make the piezo compressor competitive with the conventional one in terms of size and weight, it was critical to force the driving system to operate at mechanical resonance, thus minimizing the electrical power required to elongate exclusively the PZT stack, which is very stiff due to its ceramic nature.

2. Theory

A concept of the proposed piezoelectric compressor shown in figure 1 was first presented at ICC17 [2]. The concept was accompanied by an appropriate mass-spring-damper model shown in figure 2, detailed development of the governing equations and experiment results. The actual drive mechanism and the corresponding experiment setup have provided successful validation of the concept and the analytical model.

![Figure 1. Schematic of the proposed PZT compressor.](image)

![Figure 2. Three-degrees-of-freedom model of the proposed PZT compressor.](image)

The main idea behind the proposed concept is to force the PZT element, which is reversible by nature, to operate in both actuator and generator modes at very low frequency relative to the natural one of the piezo stack. Otherwise, the electromechanical conversion factor of the PZT actuator at out-of-resonance is limited to an order of magnitude of ten percent only. In order to force the stack-type piezo actuator to generate the electrical charge periodically, it is essential to apply very high dynamic forces to the stack by means of the adjacent reciprocating mass.
Once the frequency is set by the cryocooler requirements, the dynamic force can be increased by increasing the oscillating mass and its amplitude. In our case the oscillating mass is increased by attaching the piezo actuator together with the PZT housing to the compressing piston; thus, the piezo stack becomes subject to both deformation and bulk motion. The PZT elongation is being increased using an amplification system attached to the PZT end to obtain the amplified bulk displacement of the piezo stack and the attached oscillating assembly. Furthermore, the aforementioned amplification system multiplies the resulting dynamic force of the assembly; thus, the PZT stack undergoes sufficient stress to enter the generator mode. Certainly, the effectiveness of the concept increases as the oscillating system enters into resonance.

The amplification system performances are critical for the compressor operation, since it must fulfil two important functions: to transform the actuator elongation into amplified displacement of the oscillating assembly, and to apply the resulting dynamic force on the PZT stack with additional amplification. Since the objective amplification ratio should be about 20 and very high forces should be transferred, the only practically effective implementation can be the hydraulic one based on the area ratio \( A_1/A_2 \), as shown in figure 1. A mechanical amplification system, such as a lever used in the model in figure 2 with ratio \( L_1/L_2 \), would comprise larger losses due to the structure and contact deformations. Additionally, any mechanical amplifier implies increased weight and volume of the compressor.

The three-degrees-of-freedom model of the proposed piezo compressor appearing in figure 2 is given by set (1) of the motion equations; the PZT electric current is calculated using (2):

\[
\begin{bmatrix}
    m_1 & 0 & 0 \\
    0 & m_2 + m_3 & m_3 \\
    0 & m_3 & 3L_3
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_1 \\
    \ddot{x}_2 \\
    \ddot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
    b_1 + b_p & -b_p & 0 \\
    -b_p & b_p + b_3 & b_3 \\
    0 & b_3 & b_3
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_1 \\
    \dot{x}_2 \\
    \dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
    k_p + k_s + k_g & -(k_p + k_s) & -k_s \\
    -(k_p + k_s) & k_p + k_s + k_f & k_s - (a-1)k_f \\
    -k_s & k_s - (a-1)k_f & k_s + (a-1)^2k_f
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    \theta
\end{bmatrix}
= \begin{bmatrix}
    NV_e \\
    -NV_e \\
    0
\end{bmatrix}
\]

(1)

\[ I = N (\dot{x}_1 - \dot{x}_2) + C_0 \frac{dV_e}{dt} \]

(2)

where the model parameters are: \( m_1 \) – mass of the right-hand side assembly relative to the section line in figure 1, \( m_2 \) – mass of the left-hand half of the PZT stack together with the attached piston \( A_1 \), \( m_3 \) – mass of the left-hand part of the PZT housing, \( b_1 \) – damping coefficient due to the load (cryocooler), \( b_3 \) – damping coefficient of the \( A_2 \) piston-cylinder interaction, \( b_p \) – self-damping coefficient of the PZT actuator, \( k_s \) – structure stiffness substituting for the section of the PZT housing in figure 1, \( k_g \) – cryocooler gas stiffness, \( k_p \) – stiffness of the piezo stack, \( k_f \) – stiffness of the amplification fluid measured on piston \( A_2 \), \( a \) – ideal amplification ratio equal to \( A_1/A_2 \) in figure 1 or \( L_1/L_2 \) in the model, \( N \) – electromechanical conversion coefficient of the piezo actuator, \( C_0 \) – PZT capacitance.

The model variables are: \( x_1 \) – driving piston displacement, \( x_2 \) – displacement of the \( A_1 \) piston, \( x_3 \) – displacement of the left-hand side of the PZT housing given by (3), \( \dot{x}_4 \) – figure of merit of the liquid compressibility given by (4), also given by displacement of the upper point of mechanical lever in figure 2, \( \theta \) – virtual angle of the mechanical lever in the model, \( V_e \) – driving voltage applied on the PZT actuator, \( I \) – PZT current, \( F_e \) – driving electromechanical force, \( t \) – time.

\[ x_3 = x_2 + L_2 \theta \]

(3)

\[ x_4 \equiv \frac{\Delta V_e}{A_2} = x_2 - (a-1)L_2 \theta \]

(4)

Pressure fluctuations within the cooling gas and hydraulic space, \( \Delta P_g \) and \( \Delta P_f \) respectively, can be calculated using (5) and (6):
\[
\Delta P_g = \frac{k_g x_1}{A_1 \sin \phi} \\
\Delta P_f = -\frac{k_f x_4}{A_2}
\]

(5) Where \( \phi \) is the flow-to-pressure phase on the driving piston boundary.

Motion equations (1) imply existence of three natural frequencies in the system, of which only the first one is in our interest. The upper limit of the first resonance can be obtained by omitting the damping coefficients and setting \( k_f \) and \( k_s \) to infinity. In this case the equations set reduces to the one-degree-of-freedom system with the natural frequency given by (7):

\[
f_0 = \frac{1}{2\pi} \sqrt{\frac{k_s/a^2 + k_g}{m_1 + m_3 + (1 - \frac{1}{a}) m_2}}
\]

According to (7), the influence of the stiff PZT stack on the resonance frequency can be drastically reduced by applying a sufficiently large amplification ratio. The lowest possible natural frequency in the ideal system is obtained by setting \( a \) to infinity. In this case the frequency is proportional to the square root of the gas stiffness divided by the overall moving mass.

3. Design

Optimization of the compressor parameters is strongly dependent on the design configurations and the corresponding restrictions. The main focus was on design of the three compressor pistons, since they are directly responsible for the compressor performances and reliability. Generally, we believe the diaphragm implementation of the pistons is superior, since it is frictionless, does not require high precisions, may provide perfect fluid and pressure separation, and may serve as a flexural bearing. Therefore, major effort was spent on designing the appropriate diaphragms where applicable.

Pistons \( A_1 \) and \( A_2 \) are responsible for the amplification ratio, as well as for the stiffness and effectiveness of the entire amplification system. The stroke to diameter ratio of \( A_1 \) is about 1:1000; therefore, metal diaphragm implementation of the piston is quite feasible, even at the high pressure amplitudes obtained within the liquid. Practically, the \( A_1 \) piston was constructed from a thin sheet of spring steel deformed to produce a shallow bowl with an arc-shaped cross-section, fastened between two rigid parts at the internal perimeter of the arc. The external margins of the bowl were fastened between the PZT housing and the cover. A proper optimization of the bowl geometry supplies the required stroke for the piston, as well as adequate volume stiffness, which prevents the breathing effect under the pressure alternations.

Unfortunately, the stroke to diameter ratio of \( A_2 \) is much larger than that of \( A_1 \), about 1:5, which makes it impossible to design a diaphragm with adequate properties. Instead, we relied on a gap-sealed piston-cylinder assembly, in which the fluid separation occurs on the opposite side of the piston, outside the PZT housing, by using some compliant structure, such as a soft diaphragm. In this case some wear on the piston-cylinder interface is possible; however, the wear products must remain within the liquid space and thus cannot contaminate the filling gas. According to figure 3, in our prototype the \( A_2 \) outside fluid separation is provided by a bowl-shaped rubber diaphragm. Certainly, this sort of separation is not perfect in the long term because of the helium penetration abilities, and thus should be replaced by some metal bellow structure in the future.

The driving piston \( A_3 \) is subject to much lower pressure amplitudes than \( A_1 \) and \( A_2 \) are. The stroke to diameter ratio of \( A_3 \) is about 1:10, which is marginal for diaphragm implementation, considering the conventional pressure ratios of Stirling-type cryocoolers. Thus, the driving diaphragm was implemented as a composite structure, consisting of a stack of the spring steel semi-flexural disks with spiral-like cuts filled and coated by soft silicone rubber. Such sort of diaphragm should provide
adequate separation between the cryocooler working fluid and the compressor’s buffer gas, thus preventing possible contamination in the cryocooler.

Once the piston configurations are determined, the corresponding restrictions should be considered. First, the rubberized driving diaphragm is not capable for creating a perfect helium barrier, particularly when using the soft silicone, which possesses about the highest gas permeability among thermosets. Therefore, the compressor pressure will be level with the mean gas pressure within the cryocooler regardless of the initial filling conditions. Compliant fluid separation outside the PZT housing on the gap-sealed A\textsubscript{2} piston implies the pressure levelling of the hydraulic fluid with the compressor gas. As a result, the maximum pressure amplitude within the amplification system is restricted by the value of the filling pressure.

It should be noticed that the governing equations (1) and (2) do not include the absolute dimensions of the pistons in the amplification system, only the ratio of their respective areas. However, for a given amplification ratio, given PZT actuator and given load conditions, the absolute values of the areas are primarily responsible for the pressure amplitude developing within the liquid, i.e., the larger are the pistons, the lower pressures are needed to produce the piston forces required by the system. The aforementioned limitation on the liquid pressure amplitude determines, in fact, the minimum diameter of the A\textsubscript{1} piston, which in turn defines the radial dimensions of the compressor envelope.

4. Optimization
Optimization of the actual compressor concentrated on its ability to drive a miniature Pulse Tube cryocooler, particularly our MTS\textsubscript{a} operating at 103 Hz [3], as well as our future models, which should have similar dimensions, but operate at higher frequencies. The currently available MTS\textsubscript{a} Pulse Tube requires 11 W average PV power, 40 Bar filling pressure and a pressure ratio of 1.3 to produce a net cooling of 0.4 W at 110 K.

In order to match the compressor model given by (1) and (2) to the actual cryocooler, it was needed to estimate the gas stiffness, \( k_g \), the damping coefficient, \( b_0 \), and the flow-to-pressure phase, \( \phi \), at the driving piston boundary. According to the MTS\textsubscript{a} SAGE\textsuperscript{TM} model, which was employed for the cryocooler optimization and was successfully validated afterwards, for a given piston diameter the load characteristics are strongly dependent on the operational frequency and the cold end temperature as shown in figure 4. Data presented in figure 4 is relevant to the driving piston of 12 mm diameter, whereas the piston stroke is kinematically retained at 1 mm and both hot heat exchangers of the Pulse

![Figure 3. CAD model section view of the PZT compressor.](image)
Tube are kept at 300 K. In the parametric optimization loop the load coefficients are being corrected for piston diameters different from 12 mm by a factor of the diameters ratio to the fourth power.

![Figure 4. MTSa Pulse Tube characteristics estimated at Ø12 mm piston boundary.](image)

Parameters of the compressor optimization are the three piston diameters. The objective function may be maximum mechanical output, maximum efficiency or some compromise between them at a specific frequency, determined by the cryocooler. Optimization restrictions refer to the maximum allowable pressure amplitude within the amplification system, maximum allowable elongation of the PZT stack, maximum PZT voltage and maximum allowable stroke of the driving diaphragm. Additional optimization refers to increase of the structural stiffness, \( k_s \), and the hydraulic stiffness, \( k_f \), which should satisfy the following inequalities regarding the PZT stiffness: \( k_s > k_p, k_f > k_p/a^2 \).

The structural stiffness is maximized using the CAD model with numerical simulations applied to it. The hydraulic stiffness is obtained analytically by calculating the liquid volume from \( A_1 \) and \( A_2 \) areas with a 1 mm gap between them, and considering the bulk modulus of pure water, 2.2 GPa. The model masses \( m_1, m_2 \) and \( m_3 \) are also obtained from the CAD and are empirically correlated with the selected piezo actuator, compressor diameters and the required structural stiffness. Certainly, the excessive increase of the moving mass and the total compressor dimensions is disadvantageous and should be limited if possible.

According to figure 3, the moving part of the compressor is axially guided by the flexural bearing attached close to the \( A_2 \) piston, and by the driving diaphragm attached on the opposite side of the PZT housing. Both flexible supports possess non-negligible axial stiffness, which should be added to the gas spring constant, \( k_p \), in the calculations. Another spring constant that should be considered in calculation of the PZT stiffness, \( k_p \), refers to the PZT preload mechanism, which maintains the PZT stack continuously compressed and is vital for dynamic operation of the piezo actuator [4].

The piezo actuator was selected from the high voltage PICA Power series of PI (Physik Instrumente) company. These actuators are based on hard PZT multilayer ceramics, which exhibit the best electro-mechanical reversibility, minimum internal damping and high operating temperature, up to 150°C. The most suitable actuator was found to be P-016.40P, possessing \( k_p = 94 \) N/\( \mu \)m, \( C_0 = 510 \) nF, \( N = 5.6 \) N/V, \( F_e \) up to 5600 N and quasi-static elongation up to 60 \( \mu \)m. Omitting the attached caps, sensors and wires, the PZT stack dimensions are 58 mm length and 16 mm diameter. The PZT preload is supplied by four highly tensioned 0.7 mm diameter spring steel wires connecting the \( A_1 \) piston with the PZT housing plug, and adding 5.4 N/\( \mu \)m to the \( k_p \) in calculations.

Final optimized values of the compressor parameters used in the prototype construction are the following: \( d_1 = 34 \) mm, \( d_2 = 7.7 \) mm, \( d_3 = 12.5 \) mm. This set of diameters implies the following model values: \( a = 19.5, m_1 = 0.2537 \) kg, \( m_2 = 0.0888 \) kg, \( m_3 = 0.2918 \) kg, \( k_1 = 4.81 \) N/\( \mu \)m. PZT and \( A_2 \) piston damping coefficients used in the model were \( b_p = 1000 \) Ns/m and \( b_3 = 10 \) Ns/m. Addition to the gas stiffness due to the flexural suspension was 38 N/mm. Structural stiffness, \( k_s \), was calculated to be
around five times the PZT stiffness; however, a special test on the actual compressor showed much lower value, and $k_s$ was finally estimated at only 1.5 times $k_P$.

5. Experiment setup
In order to study the novel PZT compressor in details, the prototype was equipped with a transparent window and a number of sensors, implementation of which required additional design modifications, see figure 3 and figure 5. The cylinder-shaped Perspex window allows for the visible diagnostics on the driving diaphragm, as well as for the measurement of the axial displacements using a fast video camera. The PZT actuator is equipped with a full bridge strain gauge sensor and a temperature diode. In total, together with the power lines, the PZT required eight wires to be passed outside the compressor envelope. Thus, two feedthrough connectors with four pins each were embedded into the main body of the envelope.

The most complicated task was to measure the hydraulic pressure within the amplification system because of impossibility to embed a miniature pressure gauge inside the compressor. Therefore, the pressure gauge housing has been attached on the outside of the envelope cover, while the hydraulic fluid is being transferred to the gauge through an aperture in extension of the $A_2$ piston.

As evident from figure 5, the experiments were conducted while the PZT compressor was mounted vertically on a stiff test bench attached rigidly to a heavy steel basis. The load, implemented by the MTSa Pulse Tube cryocooler, was mounted above, in line with the compressor, and connected by a straight transfer line. All the measurements were taken in the first few seconds from the start of the run; thus, the test system was retained at room temperature, except for the cold heat exchanger of the cryocooler, which was a bit colder, at about 280 K.

![Figure 5. Test bench of the PZT compressor.](image)

6. Results
Figure 6 shows a summary of the experimental and calculated results on the PZT compressor loaded by the MTSa Pulse Tube cryocooler. All the presented amplitudes were obtained while the actuator was driven by a sine-shape voltage with peak-to-peak value of 600 V, which represents 60% of the
maximum allowable PZT voltage, or 36% of the maximum possible input power at any frequency. Figure 7 shows the mean input power calculated from instantaneous voltage and current measured on the PZT power supply. The output PV power is hard to measure; however, the MTSa Pulse Tube cryocooler requirements were supplied in full for operation at its nominal steady state regime. Certainly, the MTSa frequency of 103 Hz is out of the compressor optimum; thus, higher voltages are required to obtain the demanded pressure ratio of 1.3 within the cryocooler.

![Graphs showing frequency responses of PZT compressor](image)

**Figure 6.** Measured and calculated frequency responses of the PZT compressor.

Figure 8 demonstrates the influence of the structural stiffness on the compressor efficiency at different frequencies. One can see that for the estimated $k_s = 1.5k_p$, the maximum theoretical conversion factor that can be obtained is about 0.45 at a somewhat higher frequency than the driving piston’s resonance. Since the measured input power was slightly higher than predicted by the model and the driving diaphragm amplitude was slightly lower, one may estimate the maximum efficiency of the actual compressor at about 0.35. This does not mean that 65% of the input power converts to heat, since most of the unused electricity returns to the power supply as reactive power, where it can be recovered.

7. Conclusions
A PZT driven compressor for Stirling-type pneumatic cryocoolers operating in mechanical resonance was modelled, optimized, designed, constructed and tested in this research work. An experiment showed that under an actual load applied to the compressor the PZT stack amplitude increased at 110 Hz by 62% relative to the quasi-static operation, and thus proved the concept of the low frequency PZT resonance, which is the main point of the proposed compressor.
Despite some mismatching between the impedances, the compressor demonstrated the feasibility to drive our miniature Pulse Tube cryocooler MTSa [3], operating at 103 Hz and requiring an average PV power of 11 W, filling pressure of 40 Bar and a pressure ratio of 1.3. A cryocooler similar to MTSa optimized for operation at a frequency about 115 Hz should fit perfectly to our produced compressor.

A satisfactory agreement was obtained between the model simulations and experiments on the actual compressor, with the experimental results somewhat lower relative to the simulations. This can be explained by the secondary losses and non-linear effects that were not considered in the model. Electromechanical conversion factor of the PZT compressor was obtained much lower than was expected in initial calculations. The reason is the insufficient structural stiffness obtained within the actual assembly relative to the ideal CAD model. Most probably, the difference originates from the contact mechanics at the PZT cap ends and the interfaces of the PZT housing parts. According to figure 8, the compressor efficiency can be increased, on account of the reactive power, by 55% when the present structural stiffness is doubled, and by 81% when increasing the stiffness four times.

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