Design of Fractional-Order PID for Stabilized Sight System via Internal Model Control Approach

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Abstract—This paper aims at the problem of accuracy reduction of stabilized sight system caused by uncertain factors and disturbances under complex working conditions. Here, a fractional-order PID (FOPID) controller design method based on internal model control (IMC) approach is proposed for stabilized sight system. First, the IMC-PID is constructed by IMC approach, and then the fractional order is added to its differential and integral terms to obtain the FOPID. The traditional FOPID requires five parameters for tuned, while the proposed FOPID has only three tuning parameters, which can be obtained through the CRONE control technology. The optimal rational approximation algorithm (ORAA) with a high degree of fit is used to approximate the fractional-order calculus operator. Matlab simulation experiments show that the proposed controller outperforms other controllers and can obtain good robustness, disturbance rejection and tracking performance.

1. Introduction
Stabilized sight system is an important component in fire control system, and plays a key role in the precise strike of the target. At present, it is extensively used in various vehicular, shipborne and airborne dynamic weapon platforms [1]. In practical engineering, the traditional PID is mainly implemented to this system because of its simplicity in designing and small amount of computation. However, under the effect of parameter variations, torque fluctuations and external disturbances, the traditional PID is difficult to achieve the desires of control accuracy and anti-interference.

In recent years, as the research on fractional calculus becoming more and more mature, Fractional-order PID (FOPID) has been widely used in various fields [2-4]. While a variety of formats of FOPID have been suggested, this paper uses the format first suggested by professor Podlubny who saw it as $PI^{\lambda}D^{\mu}$, where $\lambda$ and $\mu$ can be any real numbers, and the traditional PID is a special case of $\lambda=1$ and $\mu=1$ [5]. FOPID has the ability to deal with parameter uncertainties and good disturbance rejection, at the same time it can minimize the steady-state error. At present, FOPID controller has been designed by many methods, such as optimal control, adaptive control and intelligent control. [6]-[9]. But most of these algorithms require complex mathematical calculations. Although the five parameters of the such controller improve the control flexibility compared with the traditional PID, its complicated and lengthy...

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design brings difficulties to implement controller. Therefore, this paper is aiming to propose a FOPID controller with simple mathematics and excellent performance.

Internal model control (IMC) is a new control strategy based on mathematical models, it has high engineering application value because of its simpleness to implement in PLC, intelligent instrument and bus control system. Researchers have shown an increased interest in this strategy, because of its computational simplicity along with improved performance. Research in this field has also achieved certain results. Using IMC strategy to design controller, the designed controller has better tuning quality, good robustness and disturbance rejection. Ref. [10] gives a FOPID controller based on IMC, this controller is very simple in mathematics with only two tuning parameters and the control effect is excellent. However, this method is only applicable to the second-order system and is not suitable for the stabilized sight system. Therefore, this paper draws on outstanding achievements of the predecessors, and proposes a design method of FOPID controller based on IMC which is suitable for stabilized sight system.

Firstly, the mathematical model of the stabilized sight system is established, and then FOPID controller is designed according to this model. Different from the traditional FOPID which demands five tuning parameters, the FOPID controller designed in this paper has only three parameters which can be obtained by the CRONE control method, this method simplifies the complex tuning task of FOPID. Through the simulation of MATLAB, the effectiveness of the proposed controller is verified.

The essay has been organized in the following way: Section II describes the structure and mathematical model of the stabilized sight system. Section III introduces the basic knowledge of FOPID theory, IMC approach and CRONE control technique. The design method of the controller is proposed in section IV. Then, the performance of the proposed controller is compared and analyzed with other controllers in section V, and the conclusion is given in the last section.

2. CONTROL PROBLEM DESCRIPTION

The control goal of the stabilized sight system is to ensure the aiming line is accurately located in the center of the target by controlling motor torque, and cope with external disturbances in dynamic environment [11]. In practical engineering, the stabilized sight system is generally composed of horizontal and vertical stabilization systems. Because the cross coupling between horizontal and vertical stabilization systems is very small, the principle and architecture of the two systems are the same, this paper only studies the horizontal stabilization system. Stabilized sight system is composed of gyroscope, controller, power amplifier, DC motor and load as shown in Fig.1(R represents input, \(M_f\) represents disturbance). Gyroscope is the key component in the stabilized sight system. When the moving carrier or base deflects relative to the inertial space, namely when there is a disturbance torque \(M_f\) input, it will output the corresponding angular velocity or angular displacement signal. The signal is corrected by the controller and amplified by power amplifier and then sent to the stabilized motor of the platform, the stabilized motor generates corresponding torque to drive the platform to rotate in the opposite direction relative to the base until the gyroscope signal is zero. When the system is in aiming condition, namely when there is an input signal \(R\), the gyroscope will produce an angular velocity (angular displacement) signal in the relative inertial space. Similarly, this signal also needs to go through the above steps to make the line of sight accurately follow the aiming signal.

The system adopts fiber optic gyroscope (FOG) with good stability. The transfer function of the gyroscope is

\[
G_e(s) = \frac{K_e}{2\pi B_\omega s^2 + 1} e^{-T_d s}
\]

where, \(K_e\) represents the gain, \(B_\omega\) represents the closed loop bandwidth, and \(T_d\) represents the delay time constant.

The task of power amplifier is to amplify the control voltage, so that it has enough power for the motor to generate the required torque. Power amplifier can be simplified as a proportional gain \(K_{PWM}\).
Fig. 2 is the DC motor simplified model. $U_a$ and $\theta$ separately represent input voltage and output angle, $E_a$ is opposing electromotive force, $M_m$ and $M_f$ are output torque and disturbance torque respectively, $L_a$ and $R_a$ are the total inductance and the total resistance respectively, $C_m$ and $C_e$ separately represent torque coefficient and opposing electromotive force coefficient. $J$ is the motor moment of inertia.

Mechanical time constant $T_m$ and electrical time constant $T_e$ are the key constant of DC motor. The engineering calculation method is as follows:

$$T_m = \frac{I R_a}{C_m E_a}$$
$$T_e = \frac{I_a}{R_a}$$

(2)

The transfer function of motor output torque/control voltage is obtained as follows

$$\frac{M_m(s)}{U_a(s)} = \frac{J s / C_e}{T_m s^2 + T_m T_e + 1}$$
$$\frac{M_m(s)}{U_a(s)} = \frac{J s / C_e}{(T_m + 1)(T_e + 1)}$$

(3)

This paper studies the horizontal stabilization system of an armor vehicle. The parameters of the system are as follows:

| Parameter | Value       |
|-----------|-------------|
| $K_e$     | 171         |
| $R_a$     | 400 Hz      |
| $T_d$     | 0.001 s     |
| $K_{pwm}$ | 48          |
| $R_a$     | 10.71 $\Omega$ |
| $C_e$     | 2.5 V/(rad/s) |
| $C_m$     | 2500 g.cm/A |
| $L_a$     | 21.4 mH     |
| $J$       | 1400 g.cm.s$^2$ |

According to Fig. 1, the mathematical model of the stabilized sighting system is established, and then brings the parameters in Table 1 into it, system model as shown in Fig. 3, and $M_f$ is the disturbance torque.
According to Fig.3, the gyroscope transfer function is obtained as
\[ G_e(s) = \frac{171e^{-0.001s}}{0.0004s+1} \] (4)
and plant model is
\[ P(s) = \frac{96}{5s(0.002s+1)(2.4s+1)} \] (5)

3. MATHEMATICAL PRELIMINARIES

3.1 Fractional-order System
In the control system, the more the number of integrators, the better the tracking effect, but the worse the stability of the closed-loop system. On the contrary, if the number of differentiators is more, the stability margin of the closed-loop system will be improved, but the noise suppression ability will be weakened [12]. Therefore, FOPID can be a proper trade-off controller between some integer-order PID (IO-PID) controllers, such as \( PI^2D \), \( PID^2 \), \( PI^2D^2 \). In other words, FOPID can make a better trade-off between accuracy and stability. The transfer function of FOPID is
\[ C_FOPID(s) = k_p + k_i \frac{1}{s^\lambda} + k_d s^\mu \] (6)
where, fractional-order calculus operator \( s^\mu \) and \( 1/s^\lambda \) are irrational operators with infinite dimension. \( \lambda \) and \( \mu \) belong to the range \((0,2)\). If \( \lambda \geq 2 \) or \( \mu \geq 2 \), the controller adopts a higher-order structure different from the traditional PID [3].

3.2 Internal Model Control
IMC is a design approach in which a system model is embedded in a controller to form a high-precision feedback control system [13]. The controller designed by this method has better disturbance rejection and robust towards parameter variations. In this paper, the FOPID controller is designed by IMC to simplify the complex parameter tuning problem of \( PI^2D^2 \) controller.

Fig. 4 is the strategy of IMC, where \( P(s) \) is the practical plant model, \( M(s) \) is the process model, \( Q(s) \) is a controller. \( Q(s) \) and \( M(s) \) are combined to form the IMC controller \( C_{IMC}(s) \). The equivalent feedback control structure is shown in Fig.5.
\[ C_{IMC}(s) = \frac{Q(s)}{1-Q(s)M(s)} \] (7)
where,
\[ M(s) = M_-(s)M_+(s) \] (8)
\[ Q(s) = M_-^{-1}(s)f(s) \] (9)

In (8), \( M_-(s) \) is the minimum phase part of the original system model and \( M_+(s) \) is the non-minimum phase part of the system mode. In (9), \( f(s) \) is a low-pass filter, the role of the filter is to ensure that \( Q(s) \) is rational, and its transfer function is \( f(s) = \frac{1}{(1+\eta s^m)} \).
3.3 CRONE Control
CRONE (Commande Robuste d’Ordre Non Entier) control technique is a frequency domain method that can design fractional-order systems. According to the Bode’s ideal transfer function and robustness, the frequency domain indexes of the system are designed, such as system bandwidth, phase margin, etc. [14].

Assume that the gain crossover frequency is given by \( \omega_c \), and phase margin is specified by \( \phi_m \). The open-loop transfer function \( L(s) \) of system should satisfy the following constraints [15]:

**Gain crossover frequency constraint**

\[
|L(j\omega_c)|_{dB} = 0
\]

(10)

**Phase margin constraint**

\[
\text{Arg}[L(j\omega_c)] = -\pi + \phi_m
\]

(11)

**Robust constraint against gain variation**

\[
\frac{d(\text{Arg}(L(j\omega)))}{d\omega} \bigg|_{\omega=\omega_c} = 0
\]

(12)

4. Suggested controller design method for stabilized sight system

4.1 Construction of FOPID by IMC
In fact, the method proposed in this paper is to use the internal model control PID (IMC-PID) to construct FOPID, so as to reduce the tuning parameters. The design steps of the controller are as follows:

1. Factorization of process model
   From section III, we know that \( M(s) = P(s) \). Because plant model \( P(s) \) in (5) is a minimum phase system, the process model \( M(s) \) can be factorized as

\[
M_-(s) = \frac{96}{5s(0.002s+1)(2.4s+1)}, M_+(s) = 1
\]

(13)

2. Design of controller \( Q(s) \)
   The format of the filter is given in \( f(s) = \frac{1}{(1+\eta s^2)} \), then substituting \( f(s) \) and \( M_-(s) \) into (9), we get

\[
Q(s) = \frac{5s(0.002s+1)(2.4s+1)}{96} \times \frac{1}{(1+\eta s^2)}
\]

(14)

where, \( \eta \) is the filter time parameter, \( \eta \in (0,2) \).

3. Design of FOPID
Since \( M(s) = P(s) \), substituting (14) and (5) into (7), the IMC-PID controller can be obtained as
\[
C_{\text{IMC}}(s) = \frac{0.1251}{\eta} \left( \frac{0.0521}{\eta} \right) + \frac{1}{s^2 + \frac{2.5 \times 10^{-4}}{\eta}} \quad (15)
\]

Obviously, this is a traditional PID format with a parameter \( \eta \) to be tuned. Add the fractional order to the integral and differential terms of the controller, we get
\[
C_{\text{proposed}}(s) = \frac{0.1251}{\eta} \left( \frac{0.0521}{\eta} \right) + \frac{1}{s^\mu} + \frac{2.5 \times 10^{-4}}{\eta} \quad (16)
\]
Hence, the final FOPID controller is obtained, which has three tuning parameters \( \eta, \lambda, \mu \).

4. Tuning of controller
Take (16) as the controller of the stabilized sight system in Fig. 3, the open-loop transfer function of the system is as follows
\[
L(s) = G_e(s)C_{\text{proposed}}(s)P(s) \quad (17)
\]
The spectral transfer function of (17) is
\[
L(j\omega) = G_e(j\omega)C_{\text{proposed}}(j\omega)P(j\omega) \quad (18)
\]
According to constraints in section III, we can get the constraint equations of \( L(j\omega) \) as follows
\[
\left\{ \begin{array}{l}
|L(j\omega_c)| = |G_e(j\omega_c)| \times |C_{\text{proposed}}(j\omega_c)| \times |P(j\omega_c)| = 1 \\
\text{Arg}[L(j\omega_c)] = \text{Arg}[G_e(j\omega_c)] + \text{Arg}[C_{\text{proposed}}(j\omega_c)] + \text{Arg}[P(j\omega_c)] = -\pi + \phi_m \\
\frac{d\text{Arg}[L(j\omega)]}{d\omega} \bigg|_{\omega=\omega_c} = \frac{d\text{Arg}[G_e(j\omega)]}{d\omega} + \frac{d\text{Arg}[C_{\text{proposed}}(j\omega)]}{d\omega} + \frac{d\text{Arg}[P(j\omega)]}{d\omega} \bigg|_{\omega=\omega_c} = 0
\end{array} \right. (19)
\]
Therefore, given the gain crossover frequency \( \omega_c \) and phase margin \( \phi_m \), the tuning parameters \( \eta, \lambda, \mu \) can be obtained by (19), and \( j^\pm \mu \) can be calculated by \( j^\pm \mu = \cos \frac{\pm \pi}{2} \pm j\sin \frac{\pm \pi}{2} \). These nonlinear equations can be solved by fsolve function in MATLAB.

4.2 Approximation of Fractional-order Calculus Operator
Fractional-order calculus operator \( s^\alpha \) and \( s^{\gamma} \) are irrational operators with infinite dimensions. In order to achieve a FOPID controller, \( s^\alpha \) and \( s^{\gamma} \) need to be rationalized and approximated. At present, the commonly used method is Modified Oustaloup Filter Algorithm (MOFA) [16]. Since the filter cannot achieve the approximation of the fractional-order calculus operator at full frequency, the desired approximation frequency band \( (\omega_b, \omega_h) \) and the order of approximation \( 2N + 1 \) can be used to approximate the operator.

The transfer function of the MOFA filter is as follows
\[
s^\alpha \approx K \left( \frac{ds^2+b\omega_hs^\alpha}{d(1-\alpha)s^2+b\omega_hs^\alpha+d\alpha} \right) \prod_{k=-N}^{N} \frac{s+\omega_k}{s+\omega_k^\prime} \quad (20)
\]
Its pole, zero and gain are as follows
\[
\left\{ \begin{array}{l}
\omega_k^\prime = \left( \frac{d\omega_h}{b} \right)^{2N+1} \\
\omega_k = \left( \frac{b\omega_h}{d} \right)^{2N+1} \\
K = \left( \frac{d\omega_h}{b} \right)^{\alpha}
\end{array} \right. (21)
\]
where \( \alpha \) is the order of the fractional-order calculus operator. According to experience calculation, \( b = 10, d = 9 \). Assume that the approximate differential order is 0.5 and the frequency band is (0.01, 100), namely \( \alpha = 0.5, \omega_h = 0.01, \omega_h = 100 \). Although MOFA filter have good approximation in the phase-frequency and amplitude-frequency, it has some shortcomings. For example, the order of filter can only be odd number \( 2N + 1 \), and can only study symmetrical frequency range \( (\omega_b, \omega_h) = 1 \). Therefore, this paper uses another algorithm, Optimal Rational Approximation Algorithm (ORAA), with good
approximation characteristics, which can eliminate the limitations of MOFA without increasing the complexity of the algorithm [17], [18]. This algorithm uses the numerical optimal trajectory method to obtain the band gain, which can improve the performance of the system.

The transfer function of the ORAA filter is as follows

$$s^\alpha \approx K \left( \frac{s^2 + \gamma \omega_b s}{(1-\alpha)s^2 + \gamma \omega_b s + \alpha \omega_b \omega_h} \right) \prod_{m=1}^{M} \frac{s + \omega_k'}{s + \omega_k} \quad (22)$$

Its pole, zero and gain are as follows

$$\begin{align*}
\omega_k' &= \frac{\omega_b}{\gamma} \left( \frac{\gamma^2 \omega_h}{\omega_b} \right)^{\frac{m-\alpha/2-1/2}{M}} \\
\omega_k &= \frac{\omega_b}{\gamma} \left( \frac{\gamma^2 \omega_h}{\omega_b} \right)^{\frac{m+\alpha/2-1/2}{M}} \\
K &= (\gamma \omega_h)\alpha
\end{align*} \quad (23)$$

where $\alpha$ is the order of fractional-order calculus operator, $M$ is the order of filter, $\gamma$ is the gain of frequency band, $\gamma > 1$ means the frequency band becomes wider, $0 < \gamma < 1$ means the frequency band is narrowed, when $\gamma = 1$, the frequency band does not change. Suppose that the approximate fractional differential order is 0.5 and the frequency band is (0.01,100), namely $\alpha = 0.5$, $\omega_h = 0.01$, $\omega_h = 100$. The frequency band gain is $\gamma = 35$ and the filter order is $M = 11$. Bode approximate curves of the two filters are shown in Fig. 6, and $H$ is the original curve of $s^{0.5}$.

In Fig.6, whether the phase frequency characteristics or the amplitude frequency characteristics of the fractional-order calculus operator $s^{0.5}$, the ORAA filter in the endpoint frequency approximation effect outperforms the MOFA filter.

![Figure 6. Two filter Bode diagram comparison](image)

5. SYSTEM SIMULATION RESULTS

According to the controller design method in section IV, the FOPID controller is designed by giving gain crossover frequency $\omega_c$ and phase margin $\phi_m$ of the system. Through trial and error method, the values of them are $\omega_c = 500 rad/s$, $\phi_m = 40^\circ$. Substituting these two parameters into (18), the controller parameters are given as $\eta = 0.0064$, $\lambda = 1.7645$, $\mu = 1.4251$. Take these three parameters into (16), the proposed controller is described as

$$C_{\text{proposed}}(s) = (19.5469 + 8.1406 \frac{1}{s^{1.7645}} + 0.0375s^{1.4251}) \quad (24)$$

To illustrate the efficiency of the proposed controller on stabilized sight system, integer-order PID controller (IO-PID) and fractional-order PD (FO-PD) are designed using traditional CRONE control technique. Because IMC-PID and FO-PI can’t achieve stable control of the system, there is no comparative analysis on them. The transfer function of IO-PID and FO-PD are as follows.
IO-PID controller:
\[ C_{\text{IO-PID}}(s) = 1.2355 + 19.9846 \frac{1}{s} + 0.0679s \]  \hfill (25)

FO-PD controller:
\[ C_{\text{FO-PD}}(s) = 0.4387 + 0.1418s^{0.8404} \]  \hfill (26)

5.1 Performance Analysis of Step Response

Three controllers are used to implement the closed-loop control of the stabilized sight system. Here, the reference tracking performance is as shown in Fig. 7. Dynamic performance indexes can refer to data in TABLE II. It can be seen from the figure that the proposed controller can significantly improve the response speed of the system, which is also demonstrated by frequency response of open-loop system in Fig. 8. This is because, in engineering, the gain crossover frequency \( \omega_c \) is approximately regarded as bandwidth, and higher bandwidth means faster response speed. According to TABLE II, the rise time, settling time are least for the proposed controller but the overshoot is little bit higher than FO-PD controller. It is known from [19] that 20.10% over-

![Figure 7. Comparison of step response](image)

![Figure 8. Comparison of Bode diagram](image)
TABLE II. CONTROLLERS DYNAMIC PERFORMANCE INDICATORS

| Controller type | Rise time $t_r/\text{s}$ | overshoot $\sigma\%$ | Settling time $t_s/\text{s}(\Delta = 2\%)$ |
|-----------------|--------------------------|---------------------|-------------------------------------|
| Proposed        | 0.009                    | 20.10%              | 0.071                               |
| IO-PID          | 0.014                    | 24.14%              | 0.17                                |
| FO-PD           | 0.02                     | 18.48%              | 0.13                                |

shoot meets the dynamic performance requirements of armor vehicles. So overall, the proposed controller can provide better tracking performance.

5.2 Performance Analysis of Disturbance Rejection

When $t = 0.5\text{s}$, add load disturbance at M of stabilized sight system as shown in Fig. 3, and the output response of the system is shown in Fig. 9. It can be seen from the figure that the FO-PD controller has been unable to achieve stable control in the face of disturbance, the proposed controller and the IO-PID controller can still achieve stable control. However, the former can fast reject the disturbance, and the oscillation amplitude is small. Clearly, the proposed controller has better performance in disturbance rejection.

5.3 Performance Analysis of Robustness

In practical control, the approximate model is often used to represent a system by neglecting some parameters with small influence. Because the model has uncertainty, which can be expressed as parameter perturbation [20]. The designed controller should be able to maintain stable and good control performance even when the model is uncertain. To compare the robustness of three controllers, the step response with $\pm 20\%$ gain variations is shown in Fig.10.

P(s) gain increased by 20%:

$$P_{+20}(s) = \frac{576}{25s(0.002s+1)(2.4s+1)}$$  \hspace{1cm} (27)

P(s) gain reduced by 20%:
According to the Fig. 10, each controller can obtain a stable response. When the gain of the plant P(s) increases by 20%, the overshoots of the proposed controller at least and with shortest settling time and rise time. Although the overshoot is not the smallest when the gain is reduced by 20%, the settling time and rise time are still the shortest.

In order to quantitatively analyze the robustness of each controller, the integral error criterion is introduced as an index to evaluate the robustness of each controller. The error integral criterion is a performance index expressed by the integral function of the error between the desired output and the actual output or the main feedback signal. In this paper, the three indicators, namely, Integral Square Error (ISE), Integral Absolute Error (IAE), and Integrated Time Absolute Error (ITAE) are used to measure the robustness of the system. The calculation formula of the indicator is as follows

\[
ISE = \int_{0}^{\infty} [e(t)]^2 \, dt
\]

\[
IAE = \int_{0}^{\infty} |e(t)| \, dt
\]

\[
ITAE = \int_{0}^{\infty} t |e(t)| \, dt
\]

where, \( t \) represents time, and \( e(t) \) is error between the actual output and the desired output.

Using integral error criterion index to measure system robustness, the smaller the above indexes, the better the robustness of the system. From the TABLE III, it is obvious that the proposed controller shows the most robust performance in all cases.

| Controller type | Nominal | Upper bound(120% gain) | Lower bound(80% gain) |
|-----------------|---------|------------------------|-----------------------|
|                 | ISE     | IAE       | ITAE     | ISE     | IAE       | ITAE     | ISE     | IAE       | ITAE     |
| Proposed        | 3.5214  | 7.8288    | 0.1785   | 3.1486  | 6.7046    | 0.1379   | 4.1523  | 9.7660    | 0.2508   |
| IO-PID          | 9.3939  | 23.3423   | 1.0583   | 8.2034  | 20.1435   | 0.8340   | 11.3150 | 28.4293   | 1.4561   |
| FO-PD           | 9.2487  | 21.0390   | 1.3581   | 8.4128  | 18.7329   | 1.1230   | 10.5050 | 24.4185   | 1.7178   |

It has been proved that the proposed controller is robust to gain variation. Further, we used the small gain theorem [21] to calculate the limit information about the process gain variation. According to this theorem, the closed-loop system is stable if it satisfies the following condition:

\[
P_{20}(s) = \frac{384}{25s(0.002s+1)(2.4s+1)}
\]
\[ \| T(j\omega)\Delta(j\omega) \|_\infty < 1 \]  
(32)

where \( T(j\omega) \) is the closed-loop complementary sensitivity function, and \( \Delta(j\omega) \) is the process multiplicative uncertainty. If there is an uncertain gain \( (\Delta K) \) in plant model \( P(s) = \frac{K+\Delta K}{s(T_1 s+1)(T_2 s+1)} \), then

\[ \Delta(j\omega) = \frac{(K+\Delta K)-K}{s(T_1 s+1)(T_2 s+1)} = \frac{\Delta K}{K} \]  
(33)

According to Fig.3, in this system \( T(j\omega) \) is follow as

\[ T(j\omega) = \frac{G_c(j\omega)G_{\text{proposed}}(j\omega)P(j\omega)}{1+G_c(j\omega)G_{\text{proposed}}(j\omega)P(j\omega)} \]  
(34)

Substituting (33) and (34) into (32), we can get the robust stability constraint as

\[ \max_{\omega \in \Omega} \left( \frac{\Delta K}{K} \frac{G_c(j\omega)G_{\text{proposed}}(j\omega)P(j\omega)}{1+G_c(j\omega)G_{\text{proposed}}(j\omega)P(j\omega)} \right) < 1 \]  
(35)

For the system under the control of the proposed controller (24), the frequency response of \( \| T(j\omega)\Delta(j\omega) \|_\infty \) is shown in Fig. 11, it can be seen that when \( \Delta < 0.68 \), the condition (35) holds. This also means that the controller can handle gain changes of up to \( \pm 68\% \), and this conclusion can provide a certain reference for future engineering design.

![Figure 11. Frequency response of \( T(j\omega)\Delta(j\omega) \)](image)

5.4 Analyze the Effect of Fractional-order Calculus Operator on Closed-Loop Response

In order to further study the influence of fractional-order calculus operator on the closed-loop response, the controller \( \lambda \) and \( \mu \) are traversed in the range of \((0.1, 1.9)\) in 0.1 step size, and the Bode diagram of the system is obtained, as shown in Fig. 12 and Fig. 13.

It can be seen from the Bode diagram that \( \lambda \) mainly acts on the low-frequency part of the system, while \( \mu \) mainly acts on the high-frequency part of the system. It can be seen from Fig. 12 that when \( \lambda \) changes, the gain crossover frequency and phase margin of the system almost remain unchanged, but when \( \mu \) changes in Fig. 13 the gain crossover frequency and phase mar-
Figure 12. Bode diagram of open-loop system with the change of $\lambda$

Figure 13. Bode diagram of open-loop system with the change of $\mu$

gain of the system change. This is because $\omega_c = 500 \text{rad/s}$ is in the high-frequency part, so changing $\mu$ will affect the gain crossover frequency and phase margin. In addition, because the frequency transfer function of the fractional-order calculus operator $s^{\pm \mu}$ is

$$G(j\omega) = (j\omega)^{\pm \mu} = \omega^{\pm \mu}(\cos \frac{\pi \mu}{2} \pm j\sin \frac{\pi \mu}{2})$$

(36)

The magnitude and phase are:

$$|G(j\omega)|_{dB} = \pm 20\mu \log(\omega)$$

(37)

$$\text{Arg}[G(j\omega)] = \pm \mu \frac{\pi}{2}$$

(38)

From (37) and (38), the amplitude of $s^{\pm \mu}$ drops at a rate of $\pm 20 \mu dB/dec$ in the frequency domain, and the phase throughout the entire frequency domain at $\pm \mu \frac{\pi}{2}$. In contrast, the amplitude of the integer-order calculus operator $s^{\pm 1}$ changes in the frequency domain at a fixed rate of $\pm 20 dB/dec$, and the phase responds at $\pm \frac{\pi}{2}$. Therefore, the proposed fractional-order controller is more flexible than the traditional integer-order controller.

6. CONCLUSION

This paper proposes a design method of FOPID controller based on IMC for stabilized sight system. Three parameters of the proposed controller are debugged by CRONE control technique, which overcomes the blindness of parameter tuning. The fractional-operator of the controller is approximated by the ORAA which has a good approximation effect in low frequency and high frequency. The proposed controller is simple in design and easy to implement. Moreover, simulation results show that compared
with IOPID controller and FOPD controller, the proposed method has better reference tracking, disturbance rejection and robustness. However, the present method of this paper is only for the single axis stabilized sight system, and it is still in the stage of theoretical simulation. How to implement the application of dual axis and multi axis stabilized sight system and give the hardware implementation scheme is the next step.

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