Control of vehicle active suspensions by using PD+PI type fuzzy logic with sliding surface

Yuksel Hacioglu, Nurkan Yagiz

Department of Mechanical Engineering, Faculty of Engineering, Istanbul University, 34320 Avcilar, Istanbul, Turkey
E-mail: yukselh@istanbul.edu.tr

Abstract. A PD+PI type fuzzy logic controller with sliding surface is presented in this study. This controller consists of two parts which are PD type and PI type fuzzy logic units. Inputs to those fuzzy logic units are the sliding surface functions and their derivatives. The integrated controller is applied to two degrees of freedom vehicle active suspension model. Both time and frequency domain analysis are evaluated. Numerical results demonstrate that the proposed controller improves the vibration isolation of the vehicle body, without causing a suspension degeneration problem and without degrading road holding very much.

1. Introduction
Semi-active suspensions in which electrorheological (ER) and magnetorheological (MR) fluid dampers are frequently preferred provide considerable improvements in ride comfort of passengers especially with very small power requirements [1]. On the other hand active suspensions offer significant improvements in ride comfort if compared with passive and semi-active suspensions. Therefore, extensive studies concerning active suspensions have been carried out during last decades. Hrovat [2] applied the optimal control laws on quarter car, half car and full car models and compared their performances with their passive counterparts. D’Amato and Viassolo [3] proposed a fuzzy logic controller (FLC) for a quarter car active suspension system.

By using fuzzy logic (FL) the knowledge coming from experts can be expressed by fuzzy rules. FLC is applicable to systems with uncertain mathematical model and there are two types of FLCs. The first one is the PD type FLC in which the error and its derivative are used as inputs and the control signal is the output. The second one is the PI type FLC in which error and its derivative are used as inputs and the incremental change in control signal is the output. Transient response of the PD type FLC is better than the PI type FLC, but in some cases steady state error can not be removed for PD type FL controlled system [4].

2. Design of the controller
In this section, a PD+PI type FLC with sliding surface is presented (Figure 1). The sliding function $\sigma_v$ and its derivative $\dot{\sigma_v}$ are used as inputs. Here the sliding function (surface) is defined as $\sigma = \alpha e + \dot{e}$ where $e$ is the error, $\dot{e}$ is derivative of error and $\alpha$ is the negative value of the slope of the sliding surface. In sliding mode control (SMC), by changing the control
input according to certain predefined rules, system states are driven to the sliding surface and then forced to remain there. Also it is known that conventional FLCs operate like SMCs with a boundary layer [5]. This is why sliding function and its derivative were chosen as inputs in this study. The outputs of the PD+PI type FLC are control signal $u_N$ and incremental change in control signal $\Delta u_N$.

Figure 1 Block diagram of the PD+PI type FLC with sliding surface

Triangular membership functions for the input and output variables are presented in Figure 2. Here NB, NM, NS, Z, PS, PM and PB denote negative big, negative medium, negative small, zero, positive small, positive medium and positive big, respectively. SF$_i$ (i=1,2,3,4) are the input scaling factors and SF$_u$ and SF$_{\Delta u}$ are the output scaling factors of the PD+PI type FLC.

Figure 2 Membership functions for the a) inputs variables b) output variables

Table 1 Rule table for computing $u_N$ and $\Delta u_N$

| $\sigma_N$ | NB | NS | Z | PS | PB |
|------------|----|----|---|----|----|
| NB         | NB | NB | NM | NS | Z  |
| NS         | NB | NM | NS | Z  | PS |
| Z          | NM | NS | Z  | PS | PM |
| PS         | NS | Z  | PS | PM | PB |
| PB         | Z  | PS | PM | PB |    |

For the PD part output of the FLC it is thought that if the states are far from the sliding surface then control input should be big and vice versa in order to bring the states of the system to the that surface. Similarly, for the PI part output of the FLC it is thought that if the states are far from the sliding surface then incremental change in control input should be increased and vice versa. The rule table for the PD+PI type FLC is constructed by using this manner and it is given in Table 1.

3. Vehicle model
Quarter car model, given in Figure 3, has two degrees of freedom which are body bounce $y_2$ and displacement of the wheel $y_1$ that are both in vertical directions. In order to have a realistic model, the spring and damper elements used in the suspension and the tire spring have nonlinear characteristics as seen in Figure 3 and the corresponding spring and damper forces are defined as [6]:

![Figure 3 Vehicle model](image-url)
Here $F_t$ is the tire force, $F_s$ and $F_d$ are the spring and damper forces of the suspension, $\xi_t = y_{1r} - y_0$, $\xi_s = y_2 - y_1$, and $\xi_d = \dot{y}_2 - \dot{y}_1$ are the tire deflection, suspension deflection and suspension deflection velocity, respectively. In the spring and damper force equations $k_t, k_s, b$ and $k_t, k_s, b_d$ are the coefficients for the linear and nonlinear terms, respectively. The actuator, producing the control input $u$, is placed between the sprung mass $m_2$ and unsprung mass $m_1$ as parallel to the suspension elements. $V$ is the velocity of the vehicle and $y_0$ is the road surface input. Numerical values of the vehicle parameters are given in Appendix. Equations of motion of the nonlinear quarter vehicle suspension model are as

\begin{equation}
\begin{aligned}
m_1\ddot{y}_1 - b(\dot{y}_2 - \dot{y}_1) - b_s(\dot{y}_2 - \dot{y}_1)^3 \operatorname{sgn}(\dot{y}_2 - \dot{y}_1) + k_t(y_1 - y_0) + k_{a_n}(y_1 - y_0) = 0 \\
k_s(\dot{y}_2 - \dot{y}_1) - k_{a_n}(\dot{y}_2 - \dot{y}_1)^3 = -u \\
m_2\ddot{y}_2 + b_s(\dot{y}_2 - \dot{y}_1)^3 + b_s(\dot{y}_2 - \dot{y}_1)^3 \operatorname{sgn}(\dot{y}_2 - \dot{y}_1) + k_s(\dot{y}_2 - \dot{y}_1) + k_{a_n}(\dot{y}_2 - \dot{y}_1) = u
\end{aligned}
\end{equation}

Figure 3 Vehicle suspension model

4. Numerical results

In order to demonstrate the efficiency of the proposed controller, numerical results both in time and frequency domains are presented in this section. For comparison purposes, the numerical results of the vehicle model with PID controller are also presented. Numerical parameters for the designed controllers are given in Appendix.

Time responses for the step input with limited ramp are presented in Figure 4. It is seen in Figure 4.a that passive, PID and fuzzy controlled vehicle body displacements settles on the road surface input height value where the fuzzy controlled one has no overshoot which indicates the efficiency of the proposed controller. The decrease in body acceleration magnitudes assures improved ride comfort as presented in Figure 4.b. If the suspension deflection is checked in Figure 4.c, it is observed that there is not any permanent offset which indicates that the proposed control method is a suitable choice for implementation. The control forces for the PID and fuzzy controllers are also given in Figure 4.d.

Since frequency responses are valid only for linear systems, the nonlinear terms in spring and damper equations are neglected. The frequency plots for the vertical body displacement and acceleration are given in Figure 5.a and 5.b, respectively. It is seen that by using the proposed fuzzy controller the magnitudes for both vertical displacement and acceleration of the vehicle body are decreased in a wide frequency range of interest while PID controller increased the magnitudes of both displacement and acceleration of the vehicle body between 2-4 Hz. These results indicate that the ride comfort of the passengers is improved greatly by using the proposed FL controller. The frequency response for the dynamic tire load is given in Figure 5.c. It seen that especially by using the proposed FLC, the road holding is also
improved for the low frequency ranges and there is not any significant deterioration for the high frequencies. On the other hand, it is seen that with the PID controller the magnitude of the dynamic tire load increased for the frequency range of 2-4 Hz.

5. Conclusion
For the vibration suppression of a vehicle active suspension a PD+PI type fuzzy logic controller was presented in this study. Performance of the controller was demonstrated using both time and frequency responses. Results have shown that proposed fuzzy controller was successful in vibration isolation of the vehicle body without degrading road holding and while preserving suspension working space.

![Time responses for step road input](image)

Figure 4 Time responses for step road input a) Bounce motion of the vehicle body b) Vertical acceleration of the vehicle body c) Suspension deflection d) Control force

![Frequency responses](image)

Figure 5 Frequency responses a) Body motion b) Vertical body acceleration c) Dynamic tire force

### Appendix

| Vehicle parameters | Controller parameters |
|--------------------|-----------------------|
| $m_1 = 36$ [kg]    | $k_1 = 160000$ [Nm$^{-1}$] | $SF_1 = 4$ | $SF_0 = 700$ | $K_p = 83400$ (proportional gain) |
| $m_2 = 240$ [kg]   | $k_2 = 16000$ [Nm$^{-1}$] | $SF_2 = 0.125$ | $SF_{02} = 20$ | $T_i = 0.13$ (integral time) |
| $b = 980$ [Ns$^{-1}$] | $k_{na} = 1600$ [Nm$^{-1}$] | $SF_3 = 2$ | $\alpha = 5$ | $T_d = 0.0325$ (derivative time) |
| $b_t = 402$ [Ns$m^{-2}$] | $k_{na} = 1600$ [Nm$^{-1}$] | $SF_4 = 0.25$ | $\tau = 0.12$ | $\tau = 0.12$ |
| $V = 72$ [km/h]    |                       |                  |                  |                  |

### References
[1] Hrovat D 1997 Survey of advanced suspension developments and related optimal control applications *Automatica* **33**(10) 1781-1817.
[2] Hrovat D 1993 Applications of optimal control to advanced automotive suspension design, *Journal of Dynamic Systems, Measurement, and Control* **115** 328-342.
[3] D’Amato FJ, and Viassolo DE 2000 Fuzzy control for active suspensions *Mechatronics* **10** 897-920.
[4] Lee J 1993 On methods for improving performance of PI-type fuzzy logic controllers, *IEEE Transactions on Fuzzy Systems* **1**(4) 298-301.
[5] Palm R 1992 Sliding mode fuzzy control, *Proceedings of the IEEE International Conference on Fuzzy Systems*, San Diego, CA, 519-526.
[6] Ando Y and Suzuki M 1996 Control of active suspension systems using the singular perturbation method *Control Engineering Practice* **4**(3) 287-293.