Comment on "Teleportation of Three-Qubit State via Six-qubit Cluster State"

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Abstract

Recently Yu and Sun \textsuperscript{1} have presented probabilistic teleportation of 3-qubit cat state via 6-qubit cluster state. The success probability depends on absolute value of only two of the coefficients of cluster state i.e. \(|a|^2 + |b|^2\). We have demonstrated the feasibility to teleport 3-qubit cat state via 2-qubit non maximally entangled cluster state. In this comment we have prescribed an optimal protocol for teleportation of \(n\)-qubit state of the form \(\langle \alpha|x\rangle + \beta|\bar{x}\rangle_n\) via non maximally entangled Bell state \(a|00\rangle + b|11\rangle\) henceforth allowing teleportation of \(n\)-qubit state via 2-qubit Bell state. The success probability of the prescribed protocol is \(|b|^2\).

Yu and Sun \textsuperscript{1} have recently presented a scheme of probabilistic teleportation where they have teleported 3-qubit cat state \(|000\rangle + |y|111\rangle\) via 6-qubit non maximally entangled cluster state \(a|000000\rangle + b|000111\rangle + c|111000\rangle + d|111111\rangle\) as the quantum channel. Pathak and Banerjee \textsuperscript{2} have reported that an \(n\)-qubit state \(\langle \alpha|x\rangle + \beta|\bar{x}\rangle_n\) where \(x \in \{0,1\}\) can be deterministically teleported via a Bell state. We can therefore teleport an \(n\)-qubit state with certain probability via a 2-qubit non maximally entangled Bell state.

Alice and Bob are spatially separated parties. Alice wish to teleport an unknown \(n\)-qubit state in Z-basis of the form as presented in equation (1). The quantum circuit of the proposed protocol is also presented in Figure 1.

\[
|\psi_{\text{unknown}}\rangle = \alpha|x_1x_2 \ldots x_n\rangle + \beta|\bar{x}_1\bar{x}_2 \ldots \bar{x}_n\rangle, \tag{1}
\]

where \(x \in \{0,1\}\) and \(\alpha\) and \(\beta\) are real such that \(|\alpha|^2 + |\beta|^2 = 1\) and \(|\alpha| < |\beta|\). Let all the particles from 2\(^{nd}\) to the \(n\)th be denoted by \(|e\rangle\) and only the first qubit be denoted by \(x\). Therefore, \(|e\rangle = |x_2 \ldots x_n\rangle\) and \(|\bar{e}\rangle = |\bar{x}_2 \ldots \bar{x}_n\rangle\). Finally, we can write the unknown \(n\)-qubit as

\[
|\psi_{\text{unknown}}\rangle = \alpha|x\rangle_n + \beta|\bar{x}\rangle_n. \tag{2}
\]

Alice and Bob possess prior shared non maximally entangled Bell state

\[
|\psi_{\text{channel}}\rangle = a|00\rangle + b|11\rangle, \tag{3}
\]

where \(|a|^2 + |b|^2 = 1\). The final state of the system is given by

\[
|\psi_1\rangle = |\psi_{\text{unknown}}\rangle \otimes |\psi_{\text{channel}}\rangle = (\alpha|x\rangle + \beta|\bar{x}\rangle) \otimes (a|00\rangle + b|11\rangle) = \alpha a|x00\rangle + \beta b|x11\rangle + \beta a|\bar{x}00\rangle + \beta b|\bar{x}11\rangle. \tag{4}
\]

Alice applies controlled-NOT (CNOT) \textsuperscript{2} operations on all her qubits keeping the first qubit as control. The transformed state is denoted by

\[
|\psi_2\rangle = \alpha a|x \oplus e\rangle (x \oplus 0)0\rangle + \alpha b|x \oplus e\rangle (x \oplus 1)1\rangle + \beta a|\bar{x} \oplus \bar{e}\rangle (\bar{x} \oplus 0)0\rangle + \beta b|\bar{x} \oplus \bar{e}\rangle (\bar{x} \oplus 1)1\rangle. \tag{5}
\]

We will use the following identities in equation (5).

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\textsuperscript{2}CNOT is a two qubit gate, it operates on two qubits \(|x\rangle\) and \(|y\rangle\) such that the output is given by \(|x\rangle\) and \(|x\rangle \oplus |y\rangle\) respectively.
Bob measures the ancilla in the computational basis. If he gets $|\bar{e}\rangle$, then the protocol is successful and if he gets $|1\rangle$ then the protocol fails. Iff, he gets $|0\rangle$ then with $b^2$ probability he has obtained the state

$$|\psi_6\rangle = \alpha|0\rangle + \beta|1\rangle.$$  

Bob will apply $U_2$ on his shared qubit and obtain the unknown qubit which in this case is Identity operator. For other results of Alice (01/10/11), we refer to the standard teleportation protocol for which Bob needs to apply suitable operator($X/Z/iY$) . Now, he has to obtain all the $n$-qubits in his lab thus, he will produce the separable bits $|e\rangle^{\otimes n-1}$. Thus, the resultant state is

$$|\psi_7\rangle = \alpha|0e\rangle + \beta|1e\rangle = \alpha|0(x \oplus x_2, \ldots, x \oplus x_n)\rangle + \beta|1(x \oplus x_2, \ldots, x \oplus x_n)\rangle.$$
Bob applies CNOT operations on $|e^\prime\rangle^\otimes n-1$ and obtains the $n$-qubit state as shown in equation (11).

$$|\psi\rangle = \alpha|x_1 x_2 ... x_n\rangle + \beta|\bar{x}_1 \bar{x}_2 ...... \bar{x}_n\rangle$$

(11)

Therefore in this letter we have shown that an $n$-qubit state $(\alpha|x\rangle + \beta|\bar{x}\rangle)_n$ where $x \in \{0, 1\}$ can be teleported via a 2-qubit Bell state. This can be done with success probability $b^2$.

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References

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