Inverse Neutrino-less Double Beta Decay Revisited: Neutrinos, Higgs Triplets and a Muon Collider

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Abstract

We revisit the process of inverse neutrino-less double beta decay ($e^-e^- \rightarrow W^-W^-$) at future linear colliders. The cases of Majorana neutrino and Higgs triplet exchange are considered. We also discuss the processes $e^-\mu^- \rightarrow W^-W^-$ and $\mu^-\mu^- \rightarrow W^-W^-$, which are motivated by the possibility of muon colliders. For heavy neutrino exchange and center-of-mass energies larger than 1 TeV, we show that masses up to $10^6$ ($10^5$) GeV could be probed for $ee$ and $e\mu$ machines, respectively. The stringent limits for mixing of heavy neutrinos with muons render $\mu^-\mu^- \rightarrow W^-W^-$ less promising, even though this process is not constrained by limits from neutrino-less double beta decay. If Higgs triplets are responsible for inverse neutrino-less double beta decay, observable signals are only possible if a very narrow resonance is met. We also consider unitarity aspects of the process in case both Higgs triplets and neutrinos are exchanged. An exact see-saw relation connecting low energy data with heavy neutrino and triplet parameters is found.

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1 Introduction

Observation of Lepton Number Violation (LNV) would show that neutrinos are Majorana particles [1] and would add most interesting information on the origin of neutrino masses. In particular, the process

$$e^-e^- \rightarrow W^- W^-,$$

(1)
called “inverse neutrino-less double beta decay”, has frequently been proposed as a probe of LNV and new physics in general [2–7]. Running a future linear collider in an $e^-e^-$ mode would allow looking for this and other lepton number violating and conserving processes [8].

We study inverse neutrino-less double beta decay here in the presence of Majorana neutrinos and a Higgs triplet. We also point out that the currently discussed muon colliders [9,10] would allow to search for lepton number (and flavor) violating processes like

$$\mu^-\mu^- \rightarrow W^- W^- \quad \text{(and} \quad e^-\mu^- \rightarrow W^- W^- \text{)},$$

(2)

if the machines are run in a like-sign lepton mode. Physics potential of like-sign muon collisions has also been discussed in Refs. [11], and is mentioned as a possibility in Refs. [9], where the prospects and technology of muon colliders are outlined (see [12] for a recent review on the current status of muon collider research). We summarize the model-independent limits on heavy neutrino and Higgs triplet parameters which are relevant to these processes and give the corresponding values for the cross sections. We show in which situations the processes are observable. For heavy neutrino exchange, we show that in electron-electron collisions masses up to $10^6 \text{ GeV}$ could be probed, while like-sign $e\mu$ machines can reach $10^5 \text{ GeV}$. The process $\mu^-\mu^- \rightarrow W^- W^-$ is less promising, due to strong constraints on mixing of heavy neutrinos with muons (note that this process is not constrained by limits from neutrino-less double beta decay). If Higgs triplets are exchanged in inverse neutrino-less double beta decay, observable signals are unlikely unless a very narrow resonance is met. We stress already here that we do not consider situations in which there are cancellations. This means we assume in the processes only the exchange of one heavy neutrino or of one Higgs triplet, and not the cases in which several neutrinos are present or both a triplet and a heavy neutrino are present. However, we note that if both terms are present (as in the type I + II seesaw mechanism) there is an exact seesaw relation, which uses low energy data coming from neutrino-less double beta decay and other neutrino data to constrain a particular combination of high energy (i.e., heavy neutrino and Higgs triplet) parameters. It generalizes a previously discussed formula for the type I seesaw [13,14]. We also comment on unitarity of the cross section $e^-e^- \rightarrow W^-W^-$ in case of heavy neutrinos and triplets being simultaneously present. An exact seesaw relation turns out to be helpful there.

The paper is built up as follows: in Section 2 we discuss the present limits on parameters relevant for inverse neutrino-less double beta decay. Those come from neutrino-less double beta decay, global fits and other data, in particular lepton flavor violation. Section 3 discusses the process of inverse neutrino-less double beta decay for heavy Majorana neutrinos,
Figure 1: Diagrams for $e^-e^- \rightarrow W^-W^-$ with Majorana neutrinos $N$ and a doubly charged Higgs scalar $\Delta^{--}$. Diagram (a) is the $t$-channel, (b) the $u$-channel and (c) the $s$-channel.

while Section 4 discusses the situation with a Higgs triplet. In Section 5 we argue that unitarity of the cross section is automatically fulfilled in case a triplet and heavy neutrinos are present. We conclude in Section 6.

2 Neutrino-less Double Beta Decay, its Inverse, and Limits on Neutrino and Triplet Parameters

Fig. 1 shows the three different possible diagrams for inverse neutrino-less double beta decay in the presence of Majorana neutrinos and a Higgs triplet. The diagrams (a) and (b) are connected to the diagram of neutrino-less double beta decay ($0\nu\beta\beta$). This is the process $(A, Z) \rightarrow (A, Z+2) + 2e^-$, for which the electrons are outgoing and the $W^-$ couple to incoming $u$ and outgoing $d$ quarks. Indeed, as each vertex receives a factor $U_{ei}$ and the propagator of the neutrino introduces a term $m_i/(q^2 - m_i^2)$, the dependence on neutrino parameters is the same as for neutrino-less double beta decay.

Because a hypercharge $Y = 2$ Higgs triplet contains a doubly charged member ($\Delta^{--}$), diagram (c) in Fig. 1 is possible. The $\Delta^{--}$ can also lead to $0\nu\beta\beta$ [15]. One should note that the Higgs triplet contains also a singly charged scalar $\Delta^-$, which can contribute to $0\nu\beta\beta$ as well (these are diagrams in which one $W^-$ is replaced by a $\Delta^-$). However, its coupling to quarks is suppressed by $v_L/v$, where $v_L$ is the vev of the neutral component of the triplet, and $v$ the vev of the SM doublet. Moreover, the triplets are presumably heavier than the $W$. Hence, the diagrams for $0\nu\beta\beta$ containing $\Delta^-$ are suppressed with respect to the diagram containing $\Delta^{--}$ and consequently there is a direct connection between inverse neutrino-less
double beta decay and neutrino-less double beta decay also in scenarios with Higgs triplets.

We begin by studying constraints on light and heavy neutrino, as well as on Higgs triplet parameters from lepton number and flavor violation.

The most commonly assumed mechanism of $0\nu\beta\beta$ is light neutrino exchange, for which the “effective mass” $|m_{ee}|$ is constrained as follows [16]:

$$|m_{ee}| \equiv \sum N_{ei}^{2} (m_{\nu})_{i} \lesssim 1 \text{ eV}.$$  \(3\)

We have introduced here the notation that light neutrino masses are called $(m_{\nu})_{i}$ and their mixing matrix is $N$. The above limit takes generously nuclear matrix element uncertainties into account. The most model-independent neutrino mass limit is 2.3 eV from the Mainz and Troitsk experiments [17], and $|m_{ee}|$ cannot exceed this value. Hence, the above upper value of 1 eV is of the same order as the “theoretical” upper value of 2.3 eV, which is valid in case of quasi-degenerate light neutrinos (i.e., $(m_{\nu})_{1} \simeq (m_{\nu})_{2} \simeq (m_{\nu})_{3} \equiv m_{0}$).

In case heavy neutrinos are exchanged in neutrino-less double beta decay, the following quantity is constrained [18]

$$\left| \frac{1}{M_{ee}} \right| = \left| \sum S_{ei}^{2} \frac{1}{M_{i}} \right| \lesssim 5 \cdot 10^{-8} \text{ GeV}^{-1}.$$  \(4\)

Here heavy neutrino masses are called $M_{i}$ and the matrix describing their mixing with leptons is called $S$. Note that at the current stage we have not discussed any see-saw mechanism connected to light and heavy neutrino masses, which would link $|m_{ee}|$ and $\left| \frac{1}{M_{ee}} \right|$. In what regards the mixing of electrons and muons with heavy neutral fermions, there are upper limits of [19]

$$\sum |S_{ei}|^{2} \leq 0.0052, \quad \sum |S_{\mu i}|^{2} \leq 0.0001,$$  \(5\)

respectively, obtained from global fits, in particular of LEP data. Note that the limit on $|S_{\mu i}|^{2}$ is more stringent. Comparing with the $0\nu\beta\beta$-limit in Eq. (4), the global one on $|S_{ei}|^{2}$ is stronger for masses $M_{i} \gtrsim 10^{5}$ GeV.

The origin of the difference between $|m_{ee}|$ and $\left| \frac{1}{M_{ee}} \right|$ is nothing but the two extreme limits of the fermion propagator of the Majorana neutrinos, which is central to the Feynman diagram of $0\nu\beta\beta$:

$$\frac{q + m}{q^{2} - m^{2}} \propto \begin{cases} \frac{m}{q} & \text{for } q^{2} \gg m^{2} \\ \frac{1}{m} & \text{for } q^{2} \ll m^{2} \end{cases}.$$  \(6\)

Here $q$ denotes the momentum transfer in the process, which is around 100 MeV and corresponds to $1/r$, where $r$ is the average distance of the two decaying nuclei. This helps us to understand roughly the numerical value of the limit on $|m_{ee}|$ and $\left| \frac{1}{M_{ee}} \right|$: the amplitude for light neutrino exchange is proportional to

$$\mathcal{A}_{\text{light}} \simeq G_{F}^{2} \frac{|m_{ee}|}{q^{2}},$$  \(7\)
while for heavy neutrinos it is proportional to
\[ A_{\text{heavy}} \simeq G_F \frac{1}{M_{ee}}. \] (8)
Therefore, a limit of 1 eV on \(|m_{ee}|\) corresponds to a limit on \(\frac{1}{M_{ee}}\) of \(10^{-7} \text{ GeV}^{-1}\). This rather crude estimate is surprisingly close the actual limit in Eq. (4), which takes the complicated nuclear physics into account. In the same approximation, we can estimate that the contribution of the doubly charged Higgs triplet to \(0\nu\beta\beta\) has an amplitude proportional to
\[ A_{\text{triplet}} \simeq G_F \frac{h_{ee} v_L}{m_{\Delta}}. \] (9)
where the factor \(h_{ee}\) stems from the coupling of the triplet with the electrons, \(v_L\) from the coupling of the triplet to the two \(W\) and \(1/m_{\Delta}^2\) is its propagator for \(m_{\Delta}^2 \gg q^2\). Hence, we estimate the following limit on the triplet parameters from \(0\nu\beta\beta\):
\[ \left| \frac{h_{ee} v_L}{m_{\Delta}} \right| \lesssim 10^{-7(7...8)} \text{ GeV}^{-1}. \] (10)
The triplet may be connected to neutrino mass because of the following term in the Lagrangian:
\[ L = h_{\alpha\beta} L_{\alpha} i\tau_2 \Delta L_{\beta}^c + h.c. \] (11)
Here \(h\) is a symmetric matrix, \(\tau_2\) is a Pauli matrix, \(L_{\alpha}\) a lepton doublet of flavor \(\alpha = e, \mu, \tau\), and
\[ \Delta = \begin{pmatrix} \Delta^+ / \sqrt{2} & \Delta^0 \\ \Delta^0 & -\Delta^+ / \sqrt{2} \end{pmatrix} \] (12)
contains the neutral, singly and doubly charged members of the Higgs triplet. After the SM Higgs and the neutral component of the triplet obtain a vev (\(\langle \Phi \rangle = (0, v/\sqrt{2})^T\) and \(\langle \Delta^0 \rangle = v_L/\sqrt{2}\)) a direct contribution to the neutrino mass \(m_{\nu_L} = \sqrt{2} v_L h\) arises. The electroweak \(\rho\) parameter is modified to \(\rho \simeq 1 - 2 v_L^2/v^2\), which leads to the constraint \(v_L \lesssim 8 \text{ GeV}\). Direct and model independent collider limits on the mass of the doubly charged triplet are \(m_{\Delta} \gtrsim 100 \text{ GeV}\) [20]. It is interesting to compare this limit to limits stemming from searches for lepton flavor violation (see e.g. [21, 22]):
\[
\begin{align*}
|h_{ee}|^2 |h_{e\mu}|^2 \left( \frac{250 \text{ GeV}}{m_{\Delta}} \right)^4 & < 2.1 \cdot 10^{-12}, \\
|h_{ee}|^2 |h_{e\mu}|^2 \left( \frac{250 \text{ GeV}}{m_{\Delta}} \right)^4 & < 2.5 \cdot 10^{-7}, \\
|h_{e\mu}|^2 |h_{\tau\mu}|^2 \left( \frac{250 \text{ GeV}}{m_{\Delta}} \right)^4 & < 1.3 \cdot 10^{-7}, \\
|h_{\mu\mu}|^2 |h_{\tau\mu}|^2 \left( \frac{250 \text{ GeV}}{m_{\Delta}} \right)^4 & < 4.0 \cdot 10^{-7}, \\
|(hh^\dagger)_{e\mu}| \left( \frac{250 \text{ GeV}}{m_{\Delta}} \right)^4 & < 6.5 \cdot 10^{-9},
\end{align*}
\] (13)
Here we have used the current limits on the processes $\mu \to 3e$, $\tau \to \mu 2e$, $\tau \to e 2\mu$, $\tau \to 3\mu$, $\mu \to e \gamma$, $\tau \to e \gamma$ and $\tau \to \mu \gamma$ [20]. Constraints from $(g - 2)_\mu$ (the anomalous magnetic moment of the muon, constraining $|h_\mu\mu|^2/m_\Delta^4$) and muonium-antimuonium conversion (constraining $|h_\mu\mu|^2|h_{ee}|^2/m_\Delta^4$) are very weak.

3 Inverse Neutrino-less Double Beta Decay with Majorana Neutrinos

3.1 $e^-e^-$ Collider

For the process of inverse $0\nu\beta\beta$ with Majorana neutrinos, diagrams (a) and (b) in Fig. 1 apply. In the Appendix the lengthy formulae for the cross section including the mass of the $W$ can be found. In the useful and appropriate limit of negligible mass of the $W$ one has

$$\frac{d\sigma}{d\cos \theta} = \frac{G_F^2}{32 \pi} \left\{ \sum m_i U_{ei}^2 \left( \frac{t}{t - m_i^2} + \frac{u}{u - m_i^2} \right) \right\}^2.$$  (14)

Here $U_{ai} = \{N_{a1}, N_{a2}, N_{a3}, S_{a1}, S_{a2}, \ldots, S_{an}\}$ is in our notation the general mixing matrix for the coupling of charged leptons with light and heavy neutrinos, whose masses are given by $m_i = \{(m_\nu)_1, (m_\nu)_2, (m_\nu)_3, M_1, M_2, \ldots, M_n\}$. The extreme limits of the cross section are

$$\sigma(e^-e^- \to W^-W^-) = \begin{cases} \frac{G_F^2}{4 \pi} \left( U_{ei}^2 m_i \right)^2 & \text{for } s \gg m_i^2, \\ \frac{G_F^2}{16 \pi} s^2 \left( \frac{U_{ei}^2}{m_i} \right)^2 & \text{for } s \ll m_i^2. \end{cases}$$  (15)

We will comment below in Section 5 on the apparent violation of unitarity in the limit of $s \to \infty$. There are two interesting special cases for the cross section [6]:

- if only light active Majorana neutrinos contribute to the process, then we can bound the cross section as

$$\sigma(e^-e^- \to W^-W^-) = \frac{G_F^2}{4 \pi} |m_{ee}|^2 \leq 4.2 \cdot 10^{-18} \left( \frac{|m_{ee}|}{1 \text{ eV}} \right)^2 \text{ fb}.\quad (16)$$

- if only heavy Majorana neutrinos contribute to the process, then we can bound the
cross section using the $0\nu\beta\beta$-limit from Eq. (11) as

$$
\sigma(e^-e^- \rightarrow W^-W^-) = \frac{G_F^2}{16 \pi} s^2 \left| U_{ei} \right|^2 \leq 2.6 \cdot 10^{-3} \left( \frac{\sqrt{s}}{\text{TeV}} \right)^4 \left( \frac{1}{M_{ee}} \right)^2 \left( \frac{1}{5 \cdot 10^{-8} \text{ GeV}^{-1}} \right)^2 \text{fb}.
$$

Both numbers are far too small to be observable. In order to calculate the cross section for arbitrary neutrino masses, we have two limits to take into account: first, the global limit on $|S_{ei}|^2$ from (5), and the limit on $S_{ei}/M_i$ from neutrino-less double beta decay given in Eq. (4). Assuming the exchange of only one heavy neutrino results in Fig. 2, where we plot the cross section for $e^-e^- \rightarrow W^-W^-$ as a function of $M_i$ for $\sqrt{s} = 1 \text{ TeV}$ and $\sqrt{s} = 4 \text{ TeV}$. We give the curves for applying no limit, only the global one, and finally the $0\nu\beta\beta$-limit in addition to the global one. We indicate in the plot the cross section where five events for an assumed luminosity of 80 $(s/\text{TeV}^2) \text{ fb}^{-1}$ would arise. From the plot one can see that the limit from $0\nu\beta\beta$ renders the process unobservable for $\sqrt{s} = 1 \text{ TeV}$, while for $\sqrt{s} = 4 \text{ TeV}$ up to several $10^4$ events are possible. The masses for which events are observable range from $\text{TeV}$ to $10^3 \text{ TeV}$. This has to be compared with the situation at the LHC, where heavy Majorana neutrinos are observable in the range 10 to 400 GeV for 100 fb$^{-1}$ (see [23] for a review on neutrino production at colliders).

In Fig. 3 we show the differential cross section $d\sigma/d|\cos \theta|$ for three different values of the neutrino mass and the mixing $|U_{ei}|^2$ chosen such that the total cross sections are the same. We see that once $m_i \gg \sqrt{s}$ the differential cross section is essentially flat, which is also obvious from Eq. (14).
Figure 3: Differential cross section for $e^- e^- \rightarrow W^- W^-$ with $\sqrt{s} = 4$ TeV and three different values of the neutrino mass, with the mixing chosen such that the total cross sections are identical.

### 3.2 $e^- \mu^-$ Collider

The plans of building a muon collider open up the possibility of studying the lepton number and flavor violating mode $e^- \mu^- \rightarrow W^- W^-$. The differential cross section is

$$
\frac{d\sigma}{d\cos\theta} = \frac{G_F^2}{32\pi} \left\{ \sum m_i U_{ei} U_{\mu i} \left( \frac{t}{t - m_i^2} + \frac{u}{u - m_i^2} \right) \right\}^2.
$$

If there are only light active neutrinos, then the cross section is proportional to $|m_{e\mu}|^2$, which is the $e\mu$ element of the mass matrix [25][26]. As this element can not be larger than 2.3 eV either, there is no hope of seeing the process in this case. In the case of heavy neutrinos contributing to $e^- \mu^- \rightarrow W^- W^-$, the limit on $|M_{ee}|$ influences this process as well. One needs to compare its effect with the global limit of $|S_{ei} S_{\mu i}| \lesssim 0.00072$. The cross section is given in Fig. 4. One can note that the global limit can be stronger than the $0\nu\beta\beta$-limit for large part of the parameter space.

In what regards the luminosity of like-sign $e\mu$ or $\mu\mu$ machines, it is currently not clear what numbers can be achieved. Let us use the numbers of $\mu^+ \mu^-$ muon colliders as examples. According to Ref. [9], integrated luminosities of 45 $(s/\text{TeV}^2) \ \text{fb}^{-1}$, where we have assumed a year of $10^7$ s, are possible. We will take this value in the following for both $e\mu$ and $\mu\mu$ like-sign collisions. While large uncertainty is presumably associated with this value, our results are easy to modify once more realistic estimates are present.

From the plots one can see that for $\sqrt{s} = 1$ TeV there is only a tiny window around (400 - 600) GeV where a few events may happen, but for $\sqrt{s} = 4$ TeV up to a few 100 events between 100 and $10^5$ GeV are possible. The situation is thus slightly worse than for $ee$
Figure 4: Cross section for $e^-\mu^- \rightarrow W^-W^-$ with $\sqrt{s} = 1$ TeV (left) and $\sqrt{s} = 4$ TeV (right) and three limits for the mixing parameter. The dotted line corresponds to five events for an assumed luminosity of 45 ($s$/TeV$^2$) fb$^{-1}$.

collisions, even though there are strong constraints from neutrino-less double beta decay on $S_{22}/M_i$. The reason is that the global limits on $|S_{\mu i}|^2$ are significantly stronger than on $|S_{ei}|^2$.

3.3 $\mu^-\mu^-$ Collider

Finally, let us discuss the possibility of a $\mu^-\mu^-$ mode. The cross section is

$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2}{32\pi} \left( \sum m_i U_{\mu i}^2 \left( \frac{t}{t-m_i^2} + \frac{u}{u-m_i^2} \right) \right)^2 \cdot$$

The only constraint comes from the global limit in Eq. (5), which however is rather strong. Fig. 5 shows the cross section. From the plots one can see that for $\sqrt{s} = 4$ TeV there is only a smallish window between 400 and $10^4$ GeV in which up to a few 10 events are possible.

We conclude from this Section that like-sign $ee$ lepton collisions are most promising to search for heavy Majorana neutrinos, and to constrain the parameter space of mixing matrix elements and mass. However, the center-of-mass energies should exceed 1 TeV. The already rather stringent limit on the mixing of heavy neutrinos with muons renders like-sign $e\mu$ collisions a bit less promising, and $\mu\mu$ facilities show little prospects to determine LNV due to Majorana neutrinos. As a numerical example, for $M_i = 1.5$ TeV and $\sqrt{s} = 4$ TeV, a like-sign $e\mu$ or $ee$ collider would generate 5 events even for $|S|^2 = 3 \cdot 10^{-5}$, and improvement
on the present bound of $|S|$ by one to two orders of magnitude would be possible (here $|S|^2$ denotes the respective combination of mixing parameters). For $M_i = 2 \cdot 10^5$ GeV, 5 events are possible even for $|S|^2 = 7 \cdot 10^{-4}$, resulting in an improvement of the bound on the mixing by one order of magnitude.

Note that such limits and considerations apply most probably not for heavy neutrinos of the type I see-saw mechanism. In its natural form there is a clash between production of colliders and TeV scale masses of the heavy neutrinos: the mixing of the heavy neutrinos with the SM fermions is of order $|S| \sim m_D/M_R$, and the contribution to neutrino mass is $m_\nu \sim m_D^2/M_R$. Since $m_\nu \lesssim$ eV, TeV-scale $M_R$ implies MeV-scale $m_D$, and hence $|S|$ is of order $10^{-6}$. However, the see-saw mechanism involves matrices, and highly fine-tuned scenarios in which the contributions of several heavy neutrinos compensate each other are possible, though they seem rather unnatural and in particular unstable. We continue by studying inverse neutrino-less double beta decay in an often studied extension of the Higgs sector.
4 Inverse Neutrino-less Double Beta Decay with a Higgs Triplet

The production of Higgs triplets in like-sign lepton collisions has been discussed also in Ref. [2,3]. The cross section for \( \alpha^{-\beta^-} \to W^-W^- \) is

\[
\sigma = 2 \frac{d\sigma}{d\cos\theta} = \frac{G_F^2 v_L^2 |h_{\alpha\beta}|^2}{2\pi} \frac{(s - 2 m_W^2)^2 + 8 m_W^4 \sqrt{1 - 4 m_W^2}}{(s - m_\Delta^2)^2 + m_\Delta^2 \Gamma_\Delta^2} \cdot \frac{1}{s^2} \left( m_\Delta^2 \right)
\]

where \( \Gamma_\Delta \) is the width of the \( \Delta^{-\neg} \). We note that, in case only a Higgs triplet contributes to neutrino mass, the process \( \alpha^{-\beta^-} \to W^-W^- \) can not take place if the entry \( (m_\nu)_{\alpha\beta} \) vanishes. Recall that \( v_L h = m_L/\sqrt{2} \), where \( m_L \) is the triplet contribution to neutrino mass. Hence \( |v_L h_{\alpha\beta}| \) can not exceed 1 eV, unless there are cancellations between the triplet and another contribution to neutrino mass, e.g., a type I seesaw term. Neglecting this unnatural possibility, \( v_L h \) can be at most \( m_\nu/\sqrt{2} \), and the order of \( \frac{G_F^2}{4\pi} |(m_\nu)_{\alpha\beta}|^2 \) is \( 10^{-18} \) fb for \( m_\nu \approx eV \). It is therefore clear that the resonance needs to be met in order to see an observable signal. On resonance \( (s = m_\Delta^2) \) we find

\[
\sigma^{res} = \frac{G_F^2}{2\pi} v_L^2 |h_{\alpha\beta}|^2 \frac{m_\Delta^2}{\Gamma_\Delta^2}.
\]

Assuming 40 inverse femtobarn of luminosity and asking for more than 5 events gives the requirement \( m_\Delta/\Gamma_\Delta \gtrsim 10^8 \).

We need to discuss the width of the triplet. Since the mass of the \( \Delta^{-\neg} \) is very close to the mass of the \( \Delta^- \) and \( \Delta^0 \), decays in final states containing the other members of the triplet are very much suppressed. The other decays of interest are into like-sign lepton pairs

\[
\Gamma_{\ell} \equiv \Gamma(\Delta^{-\neg} \to \alpha^{-\beta^-}) = \frac{|h_{\alpha\beta}|^2}{4\pi(1 + \delta_{\alpha\beta})} m_\Delta \simeq 19.9 \frac{|h_{\alpha\beta}|^2}{(1 + \delta_{\alpha\beta})} \left( \frac{m_\Delta}{250 \text{ GeV}} \right) \text{ GeV},
\]

and into a pair of \( W \):

\[
\Gamma_W \equiv \Gamma(\Delta^{-\neg} \to W^-W^-) = \frac{v_L g^4}{16\pi m_\Delta^2} \sqrt{m_\Delta^2 - 4 m_W^2} \left( 2 + \left( \frac{m_\Delta^2 - 2 m_W^2}{4 m_W^2} \right) \right) \simeq \frac{G_F^2}{2\pi} v_L^2 m_\Delta^2 \simeq 3.4 \cdot 10^{-10} \left( \frac{v_L}{\text{MeV}} \right)^2 \left( \frac{m_\Delta}{250 \text{ GeV}} \right)^3 \text{ GeV}.
\]

We have neglected the mass of the \( W \) in the last row. Summing \( \Gamma_{\ell}^{\alpha\beta} \) over all leptons and taking for simplicity \( h_{\alpha\beta} = h \) gives \( m_\Delta/\sum \Gamma_{\ell}^{\alpha\beta} \simeq 1/|h|^2 \). Thus, for \( |h| \approx 10^{-4} \) the

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\[ \text{The mass splitting due to electroweak corrections between the doubly and singly charged members (if they have initially the same mass) is of order } G_F m_W^{29} \text{ and therefore too small to allow for decays such as } \Delta^{-\neg} \to \Delta^- W^- \].
condition $m_\Delta/\Gamma_\Delta \simeq 10^8$ can be met. These order of magnitude estimates imply $v_L \simeq 10$ keV if $m_\nu \simeq 1$ eV. Indeed, choosing for instance $m_\Delta = 500$ GeV, for such values of the triplet vev the width in a $W$ pair is of order $10^{-13}$ GeV, while $\sum \Gamma^{\alpha\beta}_\ell \simeq 10^{-6}$ GeV. We have therefore found a consistent scenario. Hence, the resonance condition is obtainable in cases in which the decay into charged lepton pairs is favored. Interestingly, these cases are the ones frequently studied in the literature \cite{27, 28}. Pairs of $\Delta^{--}$ and $\Delta^{++}$ are produced mainly in Drell-Yan processes, and the cross section \cite{28} between 250 and 800 GeV can approximately be written as $\sigma \simeq 30 (250 \text{GeV}/m_\Delta)^4$ fb, so that 100 fb$^{-1}$ of luminosity can generate a sizable amount of triplet pairs. This in turn would motivate the study of $\alpha^-\beta^- \to W^-W^-$ at a lepton collider, and to scan the center-of-mass energy to make precision tests at resonance.

One may wonder about another process in which a triplet is exchanged in the $s$-channel, namely $\alpha^-\beta^- \to \gamma^-\delta^-$, i.e., production of two like-sign leptons $\gamma$ and $\delta$ by collisions of two like-sign leptons $\alpha$ and $\beta$. The cross section is

$$\sigma(\alpha^-\beta^- \to \gamma^-\delta^-) = \frac{|h_{\alpha\beta}|^2 |h_{\gamma\delta}|^2}{4\pi (1 + \delta_{\gamma\delta})} \frac{s}{(s - m_\Delta)^2 + m_\Delta^2 \Gamma_\Delta^2}. \quad (24)$$

The ratio of the cross sections is

$$\frac{\sigma(\alpha^-\beta^- \to W^-W^-)}{\sigma(\alpha^-\beta^- \to \gamma^-\delta^-)} = \frac{\sigma_{WW}}{\sigma_{\text{lep}}} \simeq 2 \frac{G_F^2 v_L^2 s}{|h_{\gamma\delta}|^2/(1 + \delta_{\gamma\delta})} \frac{\Gamma_W}{\Gamma_\ell}. \quad (25)$$

At resonance, the ratio of cross sections equals the ratio of decay widths. In Fig. 6 we show for two values of $m_\Delta$ the ratio of decay widths $\Gamma_W$ and $\Gamma_\ell$, as well as the ratio of cross sections (at $\sqrt{s} = 1$ TeV) as a function of $v_L$. The ratio $m_\Delta$ to $\Gamma_\Delta$ is also plotted, where $\Gamma_\Delta$ is the total width of the triplet. We demanded the neutrino mass matrix $m_L = \sqrt{2} v_L h$ to be of order 0.1 eV with $h_{\alpha\beta} = h$.

The simultaneous requirement of $\frac{\sigma(\alpha^-\beta^- \to W^-W^-)}{\sigma(\alpha^-\beta^- \to \gamma^-\delta^-)} \gg 1$ and $m_\Delta/\Gamma_\Delta \gtrsim 10^8$ implies a certain region in $m_\Delta$-$v_L$ space, see Fig. 6. A typical point is $v_L = 0.002$ GeV, leading to $h \simeq 3 \cdot 10^{-8}$ for $m_\nu = 0.1$ eV. For such small couplings the limits from lepton flavor violation given above are obeyed.

The width of the $\Delta^{--}$ is extremely small, much smaller than the beam spread, which has been estimated to be about $R = 10^{-2} \sqrt{s}$ for $ee$ colliders \cite{8} and $R = 4 \cdot 10^{-4} \sqrt{s}$ for muon colliders \cite{9}. For instance, if $m_\Delta = 600$ GeV, then for $\sqrt{s} = 600$ GeV the cross section is 50.1 fb, while for $\sqrt{s} = 599.995$ GeV the cross section is only $1.4 \cdot 10^{-10}$ fb. Picturing the spread as a box of width $R$ and convoluting the cross section over this box \cite{8} will smear out the resonance and give a $1/(R \Gamma)$ instead of a $1/\Gamma^2$ dependence of the cross section, thus reducing the cross section by several orders of magnitude. For instance, with $m_\Delta = \sqrt{s} = 600$ GeV and a spread $R$ the result is $1.3/R \cdot 10^{-6}$ pb. We conclude that observing triplet induced inverse $0\nu\beta\beta$ at like-sign lepton colliders is very unlikely.

The situation is better for $\alpha^-\beta^- \to \gamma^-\delta^-$, where with $h \simeq 0.1$ one estimates the cross section far away from resonance to be of order $\simeq h^4/(4\pi s) \simeq 3 \text{(TeV}/\sqrt{s})^2$ fb, which could lead to sizable event numbers. With maximal Yukawa couplings of $4\pi$ the cross section
Figure 6: The ratio of decay widths $\Gamma_W$ and $\Gamma_\ell$, as well as the ratio of cross sections (at $\sqrt{s} = 1$ TeV) as a function of $v_L$. The ratio $m_\Delta$ to $\Gamma_\Delta$ is also plotted, where $\Gamma_\Delta$ is the total width of the triplet.

would be $\sigma \simeq (4\pi)^3/s \simeq 0.8 \ (\text{GeV}/\Gamma_\Delta)^2 \text{ mb}$. At resonance one has again with $h \simeq 0.1$ for the cross section $\sigma \simeq 3 \ (\text{TeV}/\sqrt{s})^2 \text{ nb}$. We will study the prospects of this process elsewhere.

5 On Unitarity of $e^- e^- \rightarrow W^- W^-$ and the Type I + II See-Saw Mechanism

It is a useful exercise to consider the cross section of $e^- e^- \rightarrow W^- W^-$ in the presence of both the triplet and heavy neutrinos, and study the unitarity behavior of the process. Towards this, consider scenarios with fermion singlets and Higgs triplets. Such a scenario is called type I + II see-saw, while the presence of only a triplet may be called type II see-saw (sometimes denoted triplet see-saw). The presence of only fermion singlets is called type I see-saw. We can write down a coupling of lepton doublets with the triplet, a Yukawa mass term for the coupling of lepton doublets with fermion singlets, and a direct bare mass term for the singlets:

$$L = h_{\alpha\beta} \overline{\mathcal{T}}_\alpha i \sigma_2 \Delta L^c_\beta + \overline{\mathcal{T}}_\alpha (Y_D)^{\alpha i} \Phi N_R,j_i + \frac{1}{2} \overline{N^c_{R,i}} (M_R)_{ij} N_{R,j} + \text{h.c.}$$

(26)

Here $\Phi = (\phi^+, \phi^0)^T$ is the SM Higgs doublet and $M_R$ is a symmetric matrix. After the SM Higgs and the neutral component of the triplet obtain a vev, the complete mass term
containing the Dirac and Majorana masses can be written as
\[ L = \frac{1}{2} \bar{\nu}_L m_L \nu_L + \bar{\nu}_L m_D N_R + \frac{1}{2} \bar{N}_R M_R N_R + h.c. \]
\[ = \frac{1}{2} (\bar{\nu}_L, \bar{N}_R) \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R \end{pmatrix} + h.c. \equiv \frac{1}{2} (\bar{\nu}_L, \bar{N}_R) \mathcal{M} \begin{pmatrix} \nu_L \\ N_R \end{pmatrix} + h.c., \quad (27) \]
with \( m_D = Y_D v/\sqrt{2} \) and \( m_L = \sqrt{2} v_L h \). There are in general six eigenvalues,
\[ m^{\text{diag}} = \text{diag}(m_1, m_2, m_3, m_4, m_5, m_6) \quad (28) \]
arising from diagonalizing the full 6 \times 6 mass matrix \( \mathcal{M} \) by a unitary 6 \times 6 matrix
\[ U = \begin{pmatrix} N & S \\ T & V \end{pmatrix} \quad \text{with} \quad \mathcal{M} = U \begin{pmatrix} m^{\text{diag}} & 0 \\ 0 & M^{\text{diag}} \end{pmatrix} U^T. \quad (29) \]
Here \( m^{\text{diag}} = \text{diag}((m_\nu)_1, (m_\nu)_2, (m_\nu)_3) \) contains the light “active” neutrino masses and \( M^{\text{diag}} = \text{diag}(M_1, M_2, M_3) \) the heavy ones. This difference between light and heavy neutrinos is valid if \( m_L \) is much smaller than \( m_D \) and \( M_R \) is much bigger than \( m_D \). The entries of \( S \) and \( T \) are in this case of order \( m_D/M_R \), and hence one can obtain the expression
\[ N^\dagger (m_L - M_R^{-1} m_D^T) N^* \simeq m^{\text{diag}}. \quad (30) \]
Therefore, the mixing matrix in type I see-saw scenarios is strictly speaking not unitary, since \( NN^\dagger = 1 - SS^\dagger \neq 1 \). The other set of heavy eigenvalues of \( \mathcal{M} \) is obtained from \( V^\dagger M_R V^* \simeq M^{\text{diag}}_R \). We have illustrated the approximate nature of these expressions with the symbol \( \simeq \), but for the usual magnitude of \( m_L, m_D \) and \( M_R \) the implied non-unitarity of \( N \) is completely negligible. The matrix \( S \) characterizes the mixing of the light neutrinos with the heavy ones:
\[ \nu_\alpha = N_{\alpha i} \nu_i + S_{\alpha i} N_i, \quad (31) \]
where \( \nu_i \) (\( N_i \)) are the light (heavy) neutrinos with \( i = 1, 2, 3 \) and \( \alpha = e, \mu, \tau \). The masses \( (m_\nu)_i \) and \( M_i \), and the associated mixing matrix elements \( N \) and \( S \) can be constrained by neutrino-less double beta decay, see Section 2.
Note that the 11-element of Eq. (29), together with Eq. (27) reads
\[ U m^{\text{diag}} U^T = N m^{\text{diag}}_\nu N^T + S M^{\text{diag}}_R S^T = m_L. \quad (32) \]
We stress that this is an exact relation. It generalizes the relation \( N m^{\text{diag}}_\nu N^T + S M^{\text{diag}}_R S^T = 0 \), which is valid in absence of a triplet contribution to neutrino mass and which has been discussed in Ref. [13] and further studied in [14]. The relation links, in type I + II see-saw scenarios, the measurable light neutrino parameters (the PMNS matrix \( N \) and the light neutrino masses) with the heavy Majorana neutrinos and the Higgs triplet couplings and vev. In particular, Eq. (32) implies for the effective mass that
\[ |m_{ee}| = |(m_L)_{ee} - \sum S^2_{ei} M_i| \quad (33) \]
Consequently, if the triplet contribution to neutrino-less double beta decay is negligible with respect to the light and heavy neutrino exchange, the experimental limits on $|m_{ee}|$ apply directly to this combination of parameters:

$$\left| (m_L)_{ee} - \sum S_{ei}^2 M_i \right| = \left| \sqrt{2} h_{ee} v_L - \sum S_{ei}^2 M_i \right| \lesssim 1 \text{ eV}. \quad (34)$$

In any case, as mentioned above, $N m_\nu^{\text{diag}} N^T$ cannot exceed 2.3 eV, and this is the formal maximal value of $| (m_L)_{ee} - \sum S_{ei}^2 M_i |$. Obviously, in type I + II see-saw scenarios there is the interesting potential of cancellations between terms involving neutrino and triplet parameters. The individual limits on them can thus be evaded, and interesting phenomenology can arise. In this letter, however, we have discussed only the cases in which the triplets and neutrinos dominate in $e^-e^- \rightarrow W^-W^-$, $e^-\mu^- \rightarrow W^-W^-$ and $\mu^-\mu^- \rightarrow W^-W^-$, respectively and will treat the effect of cancellations elsewhere. However, an interesting aspect regarding unitarity of the cross sections in case neutrinos and triplets contribute to inverse neutrino-less double beta decay is worth discussing: while the full expression for the cross section is given in the Appendix, taking the high energy limit of $\sqrt{s} \rightarrow \infty$ and setting $m_W$ to zero gives

$$\sigma = \frac{G_F^2}{4\pi} \left( (U_{ei}^2 m_i)^2 + 2 v_L^2 h_{ee}^2 - 2\sqrt{2} v_L h_{ee} U_{ei}^2 m_i \right)$$

$$= \frac{G_F^2}{4\pi} \left( (U_{ei}^2 m_i)^2 + (m_L)_{ee}^2 - 2 (m_L)_{ee} U_{ei}^2 m_i \right)$$

$$= \frac{G_F^2}{4\pi} \left( (U_{ei}^2 m_i) - (m_L)_{ee} \right)^2 = 0. \quad (35)$$

In the last line we have used the exact type I + II see-saw relation Eq. (32). Thus, the cross section becomes exactly zero in the high-energy limit. Recall that in case of no cancellation the cross section would be a constant, i.e., the amplitude would grow with $\sqrt{s}$, thus violating unitarity. The exact see-saw relation cures this. This observation generalizes the findings in [5,6], in which it was shown that in case of type I see-saw the cross section is $G_F^2/(4\pi) (U_{ei}^2 m_i)^2$ which is equal to zero in type I see-saw scenarios (see Eq. (32) for $m_L = 0$). Note that the requirement of vanishing $U_{ei}^2 m_i$ means that there can not be only one neutrino: there must be necessarily two or more in order to make the cross section vanish in the high energy limit. However, if a Higgs triplet is present then one neutrino is enough.

6 Conclusions

Future lepton colliders may be run in a like-sign lepton mode, thereby probing lepton number violation. Here we have studied inverse neutrino-less double beta decay, $\alpha^-\beta^- \rightarrow W^-W^-$, with $(\alpha, \beta) = (e, e), (e, \mu)$ and $(\mu, \mu)$. We have discussed two sources of lepton

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2 That the Higgs triplet restores unitarity of the process has been noted also in [5,6].

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number violation, namely heavy Majorana neutrinos and Higgs triplets. The former possibility is shown (for $ee$ and $e\mu$ collisions and center-of-mass energies larger than $1 \text{ TeV}$) to be observable for masses up to $10^6 \text{ GeV}$, which has to be compared with an LHC reach not exceeding $400 \text{ GeV}$. Triplet effects are unlikely to be seen, as a very narrow resonance has to be met. Surprisingly, even though no limits from neutrino-less double beta decay apply, like-sign muon colliders are a less promising option, because of strong constraints on heavy neutrino mixing with muons.

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A Cross Section including $m_W$

The three possibilities for $e^-(p_1) e^-(p_2) \rightarrow W^-(k_1, \mu) W^-(k_2, \nu)$ are shown in Fig. 1. Here $p_{1,2}$ and $k_{1,2}$ are the momenta of the particles and $\mu, \nu$ the Lorentz indices of the $W$ polarization vectors. The matrix element is

$$-i \mathcal{M} = -(\mathcal{M}_t + \mathcal{M}_u + \mathcal{M}_s), \quad (A1)$$

where the subscript denotes whether it is the $t$, $u$ or $s$ channel. The vertex for $\Delta W W$ is $i \sqrt{2} g^2 v_L g_{\mu\nu}$. In order to evaluate the cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \frac{1}{64\pi^2} \frac{1}{4} |\mathcal{M}|^2 \sqrt{\frac{\lambda(s, m_W^2, m_W^2)}{\lambda(s, 0, 0)}}, \quad (A2)$$

where the first $\frac{1}{2}$ is due to two identical particles in the final state and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(a b + a c + b c)$, we need

$$|\mathcal{M}|^2 = |\mathcal{M}_t|^2 + |\mathcal{M}_u|^2 + |\mathcal{M}_s|^2 + 2 \text{Re}(\mathcal{M}_t^* \mathcal{M}_u + \mathcal{M}_t^* \mathcal{M}_s + \mathcal{M}_u^* \mathcal{M}_s). \quad (A3)$$

The result is

$$|\mathcal{M}_t|^2 = \frac{g^4}{4 m_W^4} U_{ei}^2 U_{ej}^2 m_i m_j \left[ \frac{4 m_W^6 - 4 m_W^4 (t + u) - t^2 (t + u) + 2 m_W^2 (t + 2 u)}{(t - m_t^2)(t - m_j^2)} \right],$$

$$|\mathcal{M}_u|^2 = |\mathcal{M}_t|^2 (t \leftrightarrow u),$$

$$\mathcal{M}_t^* \mathcal{M}_u = \frac{g^4}{4 m_W^4} U_{ei}^2 U_{ej}^2 m_i m_j \left[ \frac{4 m_W^6 - 2 m_W^2 t u - t u (t + u)}{(u - m_t^2)(t - m_j^2)} \right],$$

$$|\mathcal{M}_s|^2 = 2 \frac{g^4}{m_W^4} v_L^2 h_{ee} \frac{s (8 m_W^4 + (s - 2 m_W^2)^2)}{(s - m_\Delta^2)^2},$$

$$\mathcal{M}_t^* \mathcal{M}_s = \sqrt{2} \frac{g^4}{m_W^4} v_L h_{ee} U_{ei}^2 m_i \frac{(2 m_W^2 - t - u) (4 m_W^4 + t (t + u))}{(t - m_t^2)(s - m_\Delta^2)},$$

$$\mathcal{M}_u^* \mathcal{M}_s = \mathcal{M}_t^* \mathcal{M}_s (t \leftrightarrow u).$$
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