Generation of powerful terahertz emission in a beam-driven strong plasma turbulence

A V Arzhannikov\textsuperscript{1,2} and I V Timofeev\textsuperscript{1,2}

\textsuperscript{1} Budker Institute of Nuclear Physics SB RAS, 630090, Novosibirsk, Russia
\textsuperscript{2} Department of Physics, Novosibirsk State University, 630090, Novosibirsk, Russia

E-mail: timofeev@ngs.ru

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Abstract
The generation of terahertz electromagnetic radiation due to the coalescence of upper-hybrid waves in the long-wavelength region of strong plasma turbulence driven by a high-current relativistic electron beam in a magnetized plasma is investigated. The width of the frequency spectrum and the angular characteristics of this radiation for various values of plasma density and turbulence energy are calculated using the simple theoretical model adequately describing beam–plasma experiments at mirror traps. It is shown that the power density of electromagnetic emission at the second harmonic of plasma frequency in the terahertz range for these laboratory experiments can reach the level of 1 MW cm\textsuperscript{-3} with 1\% conversion efficiency of beam energy losses to electromagnetic emission.

(Some figures may appear in colour only in the online journal)

1. Introduction

It is well known that a turbulent plasma is a source of electromagnetic emission at the fundamental plasma frequency $\omega_p$ and its second harmonic. These emission mechanisms have been found to play an important role in different space phenomena such as type III solar radio bursts \cite{1–5} and radiations of planet’s magnetospheres \cite{6, 7}. Plasma emissions at $\omega_p$ and $2\omega_p$ have also been observed in laboratory beam–plasma experiments \cite{8–11}. In contrast to the problem of type III radio bursts, in which the density of wave energy $W$ is saturated at weakly turbulent levels $W/nT \sim 10^{-5}$ ($n$ is the plasma density, $T$ is the temperature of plasma electrons), we focus our attention on the regime of strong plasma turbulence $W/nT \sim 10^{-2}–10^{-1}$, which is more appropriate for laboratory experiments with powerful electron beams \cite{12–14}. A plasma column in these experiments is created in a vacuum chamber with longitudinal magnetic field (\figurename\ 1). An electron beam is injected in the plasma through one of the ends of the column, and pumps strong plasma turbulence in the beam–plasma interaction area with a length of a few meters.

Beam–plasma experiments at mirror traps \cite{14, 15} have demonstrated that the relativistic electron beam with typical energy $E_b = 1$ MeV and current density $j_b = 1–3$ kA cm\textsuperscript{-2} injected into a plasma with $n = 2 \times 10^{14}$ cm\textsuperscript{-3} loses about $\Delta E/E_b = 30–40\%$ of its energy over a length of $L = 1$ m. This means that the averaged power density pumped by the beam to the plasma turbulence can be estimated as

$$P_b = \frac{\nu_b(0)n_bE_b(0) - \nu_b(L)n_bE_b(L)}{L} \approx \frac{j_b\Delta E}{\varepsilon L}. \quad (1)$$

Here, we neglect the change in the beam velocity ($\nu_b(0) \approx \nu_b(L)$) during its passage through the plasma and express the beam density $n_b$ in terms of current density $j_b = en_b\nu_b(0)$ ($e$ is the elementary charge). Thus, for given experimental parameters, the averaged level of power density is estimated as $\sim 10$ MW cm\textsuperscript{-3}. The rate of beam energy losses, however, is not uniform over this length. The estimate for the peak pumping power can be derived assuming that a significant part of beam energy goes to the excitation of the large amplitude coherent wave-packet at the entrance of the plasma column. Since the beam is decelerated over the packet length $l \sim \nu_b/\Gamma$ by $\Delta v \sim \nu_b\Gamma/\omega_p$ \cite{16}, where $\Gamma \approx (0.7\omega_p/j_b)(n_b/n)^{1/3}$ is the growth rate of the two-stream instability and $j_b$ is the relativistic factor, the maximum power of beam energy losses reaches a value of $\sim 100$ MW cm\textsuperscript{-3}. This result can be obtained...
from formula (1) by replacing ΔE and L by γk3m_ν Δν and L, respectively. Thus, if only 1% of pumping power is able to convert to electromagnetic radiation, the maximum power density of plasma emission produced in beam–plasma experiments at mirror traps can reach the range of 1 MW cm−3. The same level of conversion has been observed recently in laboratory experiments [11] with a low plasma density n = 2 × 1014 cm−3, that is why it allows us to suppose that, in a denser plasma, a beam excited turbulence can also be considered as an efficient source of powerful electromagnetic radiation. The aim of this paper is to calculate the conversion efficiency as well as absolute values of emission power for radiation. The aim of this paper is to calculate the conversion efficiency as well as absolute values of emission power for the case when second harmonic plasma emission falls in the terahertz frequency range.

Our calculations of the spectral power of electromagnetic emission produced in a turbulent magnetized plasma are based on the theoretical model [17], in which it is assumed that electromagnetic waves are generated predominantly in the source region of strong plasma turbulence due to coalescence of upper-hybrid waves. Since theoretical predictions have been found to agree with recent experimental results [11] obtained at the GOL-3 multimirror trap in the low-density regime, we can use this model to study spectral and angular characteristics of electromagnetic emission for the whole range of plasma densities and turbulence energies which can be achieved in our beam-plasma experiments.

Thus, in section 2, we formulate the main ideas of the theoretical model that takes into account not only spontaneous generation of electromagnetic waves in coalescence processes, but also contributions of induced inverse processes. In section 3, we present calculations of second harmonic emission power in the terahertz frequency range and study whether this emission can escape from the plasma. Our main results are summarized in the concluding section 4.

2. Theoretical model

To calculate the emission power from a turbulent magnetized plasma we use the model of strong plasma turbulence proposed in [17]. According to this model, we assume that most of the wave energy is concentrated in upper-hybrid modes, which occupy the long-wavelength region of the turbulent spectrum with wavenumbers k < √W/T_νcτ_D (τ_D is the Debye length). These modes turn out to be the most unstable in the beam–plasma system, but their prominence in the quasi-stationary turbulent state is now a postulate, which must be verified in our future work using particle-in-cell simulations. The saturation level of wave energy in our model is determined by the balance between the constant power P_b pumped to the turbulence by the beam and the power dissipated due to the wave collapse. It results in the following relation between the pumping power and the turbulence energy:

\[ P_b \sim \alpha \nu P \left( \frac{m}{m_e} \right)^{1/2} \left( \frac{W}{n T} \right)^2, \]  

where m is the ion mass and m_e is the electron mass.

We calculate the electromagnetic emission power P and the conversion efficiency \( \epsilon = P/P_b \) assuming that electromagnetic waves are generated due to coalescence of long-wavelength upper-hybrid waves. Since in the source region of the turbulent spectrum these waves are not trapped in collapsing caverns, nonlinear interaction between electromagnetic t- and upper-hybrid ℓ-modes can be treated in the framework of weak turbulence. In this case, the generation of electromagnetic radiation is described by the equation

\[ \frac{\partial W_\ell}{\partial t} = P_k - 2\gamma_k W_\ell, \]  

where \( P_k \) is the power of spontaneous emission produced in the coalescence process \( \ell + \ell \rightarrow t \) and \( \gamma_k \) is the nonlinear dissipation rate due to the decay \( t \rightarrow \ell + \ell \). By t-waves we mean both types of electromagnetic modes (ordinary and extraordinary) that can be excited in a magnetized plasma. In the case of a rather strong magnetic field, one more branch of high-frequency plasma oscillations that can take part in the generation of electromagnetic waves is a whistler branch. However, in the small magnetic field that is of interest in this paper, possible excitation of whistler modes can result in the generation of plasma emission in the lower frequency range (\( \omega \sim \nu_p \)). In dimensionless units \( \alpha_\nu, \omega/\omega_p, \nu c_k/\nu_p \) for time, frequency, position and wavenumber, respectively, the spectral wave energy is normalized by the condition

\[ \int W_\ell^\nu d^3k = \frac{W^\nu}{nmc^2}, \]  

where c is the speed of light. Since in the long-wavelength region of the turbulent spectrum, the phase velocities of the upper-hybrid modes are much larger than the thermal velocity of plasma electrons, we can consider three-wave interaction processes in the cold plasma limit. Dimensionless values of \( P_k \) and \( \gamma_k \) in this limit take the forms

\[ P_k = \frac{2\pi}{\omega_k (\partial\Lambda/\partial\omega)_{\nu_k}} \int W_{k1}^\nu W_{k2}^\nu G_{k1,k2}^{\nu\nu} \left| G_{k1,k2}^{\ell\ell} \right|^2 \Delta_{k1,k2} \delta(k - k_1 - k_2) d^3k_1 d^3k_2, \]  

\[ \gamma_k = \frac{1}{\omega_k (\partial\Lambda/\partial\omega)_{\nu_k}} \int W_{k1}^\nu \delta(k - k_1 - k_2) \left[ -iG_{k1,k2}^{\ell\nu} G_{k1,k2}^{\ell\ell} \right] d^3k_1 d^3k_2. \]
where we introduce the following notation:

\[ \Lambda^\sigma(k, \omega) = |k \cdot e_k^\sigma|^2 - k^2 + \omega^2 (e_k^{\sigma^*} e_k^{\sigma^*})^2, \]  

(7)

\[ G_{k, k, 1}^{\sigma \sigma'^{\prime}} = \frac{\alpha_{k1}}{\alpha_{kk}} \left( e_k^{\sigma^*} \tilde{T}_{k1} e_{k1}^{\sigma'^{\prime}} \right) + \left( \frac{\alpha_{kk}}{\alpha_{k1}} \right) \left( e_k^{\sigma^*} \tilde{T}_{k1} e_{k1}^{\sigma'^{\prime}} \right) + e_k^{\sigma^*} \tilde{T}^* g. \]  

(8)

\[ g = \left( k_2 \tilde{T}_{k2} e_{k2} \right) \left[ \tilde{T}_{k1} e_{k1}^{\sigma'^{\prime}} \right] - \frac{1}{\omega_j^2} e_{k1} \]  

(9)

\[ \tilde{T}_{k} = \left( \epsilon_0 \epsilon_{kk} \right) \left( \tilde{I} - \tilde{\epsilon}_k \right), \quad \omega_{\alpha} = \omega_{\alpha k} + \omega_{\alpha k}. \]  

(10)

Here, eigenfrequencies \( \omega_{\alpha k} \) and eigenvectors \( e_k^\sigma \) of the linear plasma modes are determined by the standard dispersion equation with the dielectric tensor \( \tilde{\epsilon}_k \) (\( \tilde{I} \) is the unit matrix).

In contrast to the similar calculations of second harmonic emission [18, 19] based on the standard weak turbulence theory, we take into account model damping of two time correlation functions, which is used to describe the effect of finite life time of upper-hybrid plasma modes due to their scattering off density fluctuations with the typical dimensionless frequency \( \nu = W/\nu T \). This results in correlation broadening of the resonance \( \omega_{k1} - \omega_{k1} - \omega_{k2} = 0 \), which is described in (5) by the function

\[ \Delta_{k, k, 1} = \frac{2 \nu / \pi}{\left( \omega_{k1} - \omega_{k1} - \omega_{k2} \right)^2 + 4 \nu^2}. \]  

(11)

This function shows that the width of the frequency spectrum of the second harmonic electromagnetic emission depends essentially on nonlinear effects as well as on thermal and magnetic corrections to the linear dispersion of upper-hybrid modes. In order to minimize the frequency width of radiation, we should consider the case when the ratio of the electron cyclotron frequency to the plasma frequency becomes low \( \Omega = \omega_{e0} / \omega_{p0} = 0.2 \) resulting in comparable contributions of magnetic and thermal effects for the typical plasma temperature \( T = 1 \) keV in mirror traps. To account for the effect of finite temperature, we modify the eigenfrequencies and eigenvectors of the linear plasma modes, but we neglect modifications in the nonlinear current \( G_{k, k, 1}^{\sigma \sigma'^{\prime}} \), which is much less sensitive to thermal corrections than \( \Delta_{k, k, 1} \).

This approach allows us to describe qualitatively the influence of finite temperature on the width of the radiation spectrum without extra complications in the calculations of integral power, which are dependent on the turbulence model and have an order-of-magnitude accuracy. Thus, to calculate \( \omega_{\alpha k} \), we use the fluid approximation, in which thermal effects are described by the pressure gradient, and solve the linear dispersion relation with the dielectric tensor:

\[ \epsilon_{xx} = 1 - \frac{k_1^2 T_F}{\omega^2}, \]  

(12)

\[ \epsilon_{xy} = -\epsilon_{yx} = \frac{\Omega}{\omega} \left( 1 - \frac{k_1^2 T_F}{\omega^2} \right), \]  

(13)

\[ \epsilon_{yy} = 1 - \frac{k_2^2 T_F}{\omega^2}, \]  

(14)

\[ \epsilon_{xz} = -\epsilon_{zx} = -\frac{k_1^2 T_F}{\omega^2}, \]  

(15)

\[ \epsilon_{yz} = \frac{\Omega}{\omega} \left( 1 - \frac{k_1^2 T_F}{\omega^2} \right), \]  

(16)

\[ \epsilon_{zz} = 1 - \frac{k_2^2 T_F}{\omega^2} + \frac{\Omega^2}{\omega^2}, \]  

(17)

\[ A = \left( \omega^2 - \omega^2 \right)^2 + \frac{\Omega^2}{\omega^2} \]  

where \( V_F^2 = 3T/(m_e c^2) \), magnetic field is directed along the \( z \)-axis and \( k = (k_\perp, k_\parallel) \) is the wave vector with length \( k = (k_\perp^2 + k_\parallel^2)^{1/2} \).

If the path lengths of the spontaneously generated electromagnetic waves are larger than the typical size of the confined plasma \( l_k = v_g / k_\parallel > L \) (\( v_g \) is the group velocity of the electromagnetic wave), decay processes \( t \rightarrow t + \ell \) do not play a role, and the second term in (3) can be omitted. Thus, in the case of azimuthally symmetric turbulence, the spectral emission power in units of \( nm_e c^2 \) is given by the integral

\[ \frac{dP}{d\omega} = 2\pi \int_0^\pi \sin \theta \, d\theta \left( \frac{k_\perp^2}{d\omega/dk} P_k \right)_{k(\omega)}, \]  

(18)

where \( k(\omega) \) is the solution of \( \omega = \omega_k \) and \( \theta \) is the polar angle of \( k \).

### 3. Computation results

Let us study the spectral and angular characteristics of second harmonic plasma emission for various regimes of beam–plasma interaction, which can be realized in mirror traps. We will vary the plasma density from \( 2 \times 10^{14} \) to \( 5 \times 10^{15} \) cm\(^{-3} \) for different fixed values of turbulence energy \( W/nT = 0.01, 0.05, 0.1 \) and for the fixed parameters \( T = 1 \) keV and \( \Omega = 2 \).

We assume here that the source region of the turbulent spectrum contains the anisotropic population of resonant waves, which are directly pumped by the beam, and the isotropic population of nonresonant waves, which are the products of beam-driven waves scattering off density fluctuations. In our computations, the isotropic part uniformly occupies the spectral region \( kc/\omega_p \in (0.1, k_0c/\omega_p) \), where the upper bound corresponds to the typical wavenumber of the modulation instability \( k_n \simeq \sqrt{W/nT}/\tau_0 \) and the lower bound excludes modes with wavelengths larger than the typical plasma size. According to the model [17], the anisotropic population of beam-driven waves contains a small fraction of energy (10%) and occupies the region: \( kc/\omega_p \in [1.1, 1.3] \) and \( \theta \in (0, 0.3) \).

In the regime with \( n = 3 \times 10^{15} \) cm\(^{-3} \) and \( W/nT = 0.05 \), computational results for the spectral density of emission power \( dP/(d\omega d\theta) \) for extraordinary \( x \) and ordinary \( o \) electromagnetic modes are presented in figures 2(a) and (b), respectively. It is seen that \( x \)-mode emission is dominated by the contribution of obliquely propagating waves with \( \theta = 30^\circ \).
and $\theta = 150^\circ$, whereas the most intensive o-mode emission is concentrated near the transverse to the magnetic field direction. Calculations of the path length $l_k$ for these emissions are shown in figures 2(c) and (d). One can see that the path length reaches the minimal value 20–40 cm for longitudinally propagating modes regardless of their polarization. For transverse propagation, this length increases up to 90 cm for the o-mode and exceeds 3 m for the x-mode. This means that emissions of both electromagnetic modes, generated in the plasma column with diameter 5–6 cm typical of our beam-plasma experiments, are able to escape from the plasma and can be used for different applications.

Let us now find as to how the frequency spectrum of electromagnetic emission, described by $dP/d\omega$, depends on the plasma density.

As a function of dimensionless frequency $\omega/\omega_p$, the form of this spectrum is affected only by the level of turbulence. For density $n = 3 \times 10^{15}$ cm$^{-3}$ and different values of turbulence energy, examples of frequency spectra, derived by summation over polarizations, are shown in figure 3. In the dimensional form, the linear frequency $f$ corresponding to the maximum of the spectral power $dP/d\omega$ and the width at half-maximum $\Delta f$ for a fixed turbulence energy depend on the plasma density as follows: $f, \Delta f \propto n^{1/2}$. figures 4(a) and (b) demonstrate that $f$ and especially $\Delta f$ grow with the increase in the turbulence level. This means that the spectral width of plasma emission in the regimes of rather strong plasma turbulence is determined mainly by the correlation broadening of resonances in nonlinear three-wave interactions.

![Figure 2](image2.png)

**Figure 2.** Spectral power $dP/(d\omega d\theta)$ (in W cm$^{-3}$) for x-mode emission (a) and o-mode emission (b). The path length (in cm) for x-modes (c) and o-modes (d).

To compute the integral power of second harmonic emission from the unit volume of the turbulent plasma, we integrate the function $dP/d\omega$ over the frequency range $\omega/\omega_p \in (1.6, 2.6)$ and sum over polarizations. It is seen from figure 5 that this power increases as $P \propto n^{3/2}$ with the increase in plasma density for the fixed parameter $W/nT$ and reaches the value 1 MW cm$^{-3}$ in the regime with $n = 3 \times 10^{15}$ cm$^{-3}$ and $W/nT = 0.05$. It is interesting to note that these calculations adequately describe the recent experimental results obtained in the strong magnetic field $\Omega = 0.8$ [11]. In fact the 1 kW cm$^{-3}$ range of emission power, observed experimentally in the regime with $n = 2 \times 10^{14}$ cm$^{-3}$ and $W/nT = 0.01$, is also reproduced by calculations presented in figure 5. From comparison with the results of [17] we can conclude that the external magnetic field broadens the spectral width of
emission with the power density $1 \text{ MW cm}^{-3}$, can be achieved in beam–plasma experiments at the GOL-3 multimirror trap. In fact for typical beam energy 1 MeV and current density $1.5 \text{ kA cm}^{-2}$, the pumping power reaches the required level of $100 \text{ MW cm}^{-3}$ inside the region of the most intensive beam–plasma interaction, where large amplitude coherent wave-packets are excited. Since the length of this region is estimated as $l \sim v_b/\Gamma \simeq 1–3 \text{ cm}$, the total power of terahertz emission in our laboratory experiments can reach 30–100 MW.

4. Conclusion

The spectral power of second harmonic electromagnetic emission, generated in the long-wavelength region of strong plasma turbulence due to coalescence of upper-hybrid waves, was calculated for different regimes of beam–plasma interaction, which can be realized in modern experiments at mirror traps. Computation results have shown that beam energy transferred to plasma oscillations in the regime of strong plasma turbulence can be converted efficiently to electromagnetic radiation. We have studied spectral characteristics of plasma emission in wide ranges of plasma densities and turbulence energies and shown that the power of terahertz radiation in our beam–plasma experiments can reach tens of MW. Taking into account the angular distribution of this emission and possibility to change the emission frequency by varying the plasma density we come to the conclusion that terahertz radiation generated in a beam-driven strongly turbulent plasma can be attractive for different applications. In particular, this terahertz radiation has good prospects for application in medicine, information technology, energy transmission, security systems and radar installations.
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References

[1] Gurnett D A and Anderson R R 1976 Science 194 1159
[2] Kruchina E N, Sagdeev R Z and Shapiro V D 1981 JETP Lett. 32 419
[3] Robinson P A, Cairns I H and Willes A J 1994 Astrophys. J. 422 870
[4] Mel'nik V N and Kontar E P 2003 Sol. Phys. 215 335
[5] Li B, Willes A J, Robinson P A and Cairns I H 2005 Phys. Plasmas 12 012103
[6] Gurnett D A, Shawhan S D and Shaw R R 1983 J. Geophys. Res. 88 329
[7] Cairns I H and Menietti J D 2001 J. Geophys. Res. 106 29515
[8] Benford G, Tzach D, Kato K and Smith D F 1980 Phys. Rev. Lett. 45 1182
[9] Hopman H J and Janssen G C A M 1984 Phys. Rev. Lett. 52 1613
[10] Baranga A B, Benford G, Tzach D and Kato K 1985 Phys. Rev. Lett. 54 1377
[11] Arzhannikov A V, Burdakov A V, Kuznetsov S A, Makarov M A, Mekler K I, Postupaev V V, Rovenskikh A F, Sinitsky S L and Sklyarov V F 2011 Transact. Fusion Sci. Technol. 59 74
[12] Vyacheslavov L N, Burnasov V S, Kandaurov I V, Kruglyakov E P, Meshkov O I and Sanin A L 1995 Phys. Plasmas 2 2224
[13] Vyacheslavov L N, Burnasov V S, Kandaurov I V, Kruglyakov E P, Meshkov O I, Popov S S and Sanin A L 2002 Plasma Phys. Control. Fusion 44 B279
[14] Arzhannikov A V et al 2003 JETP Lett. 77 358
[15] Arzhannikov A V, Burdakov A V, Koidan V S and Vyacheslavov L N 1982 Phys. Scr. T2 303
[16] Timofeev I V and Lotov K V 2006 Phys. Plasmas 13 062312
[17] Timofeev I V 2012 Phys. Plasmas 19 044501
[18] Willes A J and Melrose D B 1997 Sol. Phys. 171 393
[19] Kuznetsov A A 2007 Plasma Phys. Rep. 33 482