Attracting the Electroweak Scale to a Tachyonic Trap

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We propose a new mechanism to dynamically select the electroweak scale during inflation. An axion-like field $\phi$ that couples quadratically to the Higgs with a large initial velocity towards a critical point $\phi_c$, where the Higgs becomes massless. When $\phi$ crosses this point, it enters a region where the Higgs mass is tachyonic and this results into an explosive production of Higgs particles. Consequently, a back-reaction potential is generated and the field $\phi$ is attracted back to $\phi_c$. After a series of oscillations around this point it is eventually trapped in its vicinity due to the periodic term of the potential. The model avoids transplanckian field excursions, requires very few e-folds of inflation and it is compatible with inflation scales up to $10^9$ GeV. The mass of $\phi$ lies in the range of hundreds of GeV to a few TeV and it can be potentially probed in future colliders.

I. INTRODUCTION

In recent years, the idea that the electroweak scale could be dynamically determined by the cosmological evolution of a (pseudo-)scalar field sparked a paradigm shift in theories of naturalness. The first model of this kind [5] features an axion-like field $\phi$, called the relaxion, which couples to the Higgs $H$ via a term of the type $g\phi H^2$ with tiny $g$. The relaxion slows-down during inflation and scans the Higgs mass $m_H^2(\phi) = -\Lambda^2 + g\Lambda \phi$, where $\Lambda$ is the scale that New Physics (NP) is expected to appear. Electroweak symmetry breaking occurs after the field crosses the critical point $\phi_c = \Lambda/g$ and a periodic back-reaction potential for $\phi$ is generated via non-perturbative effects of a confining sector at scale $\Lambda$. The height of the potential barriers grows with the increasing Higgs vacuum expectation value (VEV), eventually stopping the relaxion and trapping it into a local minimum at the electroweak scale $v_{EW}$. No new degrees of freedom at the TeV scale charged under the Standard Model (SM) are required and as a result experimental strategies motivated by naturalness are radically different in this framework.

The original proposal was not without some theoretical shortcomings such as the requirement $M \lesssim v_{EW}$, which implies that the confining sector is hidden (i.e. not charged under the SM symmetries) and its scale coincides with the electroweak scale without any a priori reason. Moreover, transplanckian field excursions of the relaxion $\Delta \phi \sim \Lambda/g$ are necessary as well as an enormous number of e-folds that have to be produced by low-scale inflation, which raises concerns of cosmological fine-tuning [6, 7]. Various model-building attempts to address these issues have appeared in the literature [6, 8–25], albeit at the price of introducing non-minimal setups. Beyond the relaxion framework, recent works [26–28] have considered scenarios in which the electroweak scale is also determined due to the interplay between a scalar and the Higgs, but instead of a dynamical relaxation there is environmental and anthropical selection related to the vacuum energy in different patches of the inflationary universe.

In this letter we present a model of cosmological relaxation of the electroweak scale which is free of the above-mentioned pathologies while at the same time remains economical introducing only one new field at the effective theory level. In particular, it utilizes a stopping mechanism that relies on the extremely rapid production of excitations of a scalar field, in our case the Higgs field, that couples quadratically to another (pseudo-)scalar field $\phi$. The particle production takes place when the Higgs becomes massless at a critical point of the classical trajectory of $\phi$, i.e. the symmetry breaking point (SBP) $\phi_c$. The produced particles generate an effective back-reaction potential that attracts the field $\phi$, which we will call attraxion, back to the SBP. If the production is strong enough, the global minimum of the potential is now $\phi = \phi_c$ and the field starts to oscillate around it. Hubble expansion causes a decrease of the oscillation amplitude and eventually the field is trapped in the vicinity of the SBP. A similar trapping mechanism was first envisioned as a possible solution to the cosmological moduli problem [29] and then exploited in models of trapped inflation [30, 31] as a method to obtain slow-rolling conditions for the inflaton even in a non-flat potential. More recently it has been used in the context of quintessential inflation in order to freeze the inflaton dynamics until later times [32, 33].

In contrast to the slow-rolling relaxion, the mechanism is effective in the high initial velocity regime of the parameter space, which additionally enables a fast scanning of the Higgs mass requiring only very few e-folds of inflation. The attraxion potential also has a periodic term which is initially not interfering with the fast rolling, but after the kinetic energy is depleted, the field is eventually

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trapped in one of its valleys. The process occurs before the Higgs number density is diluted due to inflation or the Higgs bosons decay removing the back-reaction term. It is worth noticing that the periodic potential does not depend on the Higgs VEV disentangling in principle the scale of the confining sector from the electroweak scale. Furthermore, the size of the coupling $g$ required by the mechanism is much larger than the one in relaxation models which implies that field excursions are always smaller than the Planck scale.

II. THE ATTRAXION MODEL

Effective potential - The effective potential at tree-level reads

$$V_{\text{tree}}(H, \phi) = g^2 \phi^2 - \frac{\phi^2}{2} |H|^2 + \frac{\lambda}{4} |H|^4 + V_\phi(\phi),$$ (1)

where $\phi_c \equiv \Lambda/g$. The potential has two SBPs at $\phi = \pm \phi_c$. In this letter, we study the case of a quadratic attraxion-Higgs coupling (e.g. see Ref. [9]).

We assume that $\phi$ does not couple at tree level to the NP at scale $\Lambda$. Despite that, closing the Higgs loop provides the leading loop-level correction

$$V_{\text{loop}}(\phi) \sim \frac{g^2 \phi^2}{16\pi^2} \phi^2 = \frac{g^2 \Lambda^2}{16\pi^2} \phi^2.$$ (2)

Finally, we assume that $\phi$ obeys a shift symmetry broken at scale $f$ and couples to a hidden confining sector at scale $M$, which yields the periodic potential

$$V_\phi(\phi) = M^4 \cos \frac{\phi}{f}.$$ (3)

Unlike in the traditional relaxation model, this term does not depend on the Higgs VEV and is present even before the stopping mechanism is triggered. This term allows for the existence of local minima close to the SBPs when

$$M^4 \gtrsim \frac{g \Lambda^3 f}{8\pi^2}.$$ (4)

A concrete ultraviolet (UV) completion is beyond the scope of this letter, but we mention that variations of the constructions laid out in Refs. [5, 9, 34] and in particular the clockwork framework of Ref. [10, 11] could match to our model in the low-energy limit.

The electroweak symmetry is broken in the region $\phi < \phi_c$, where the minimum of the potential in the Higgs direction is situated at

$$v_H^2(\phi) = \frac{g^2}{\lambda} (\phi_c^2 - \phi^2).$$ (5)

The minimum of the potential in the attraxion direction is at $\phi = 0$.

Trapping mechanism - The rolling of the attraxion starts during the inflation era at large negative field values $\phi_i < -\phi_c$ (the choice of the sign is free) and with a large initial velocity towards the origin. As the attraxion comes close to the first SBP $\phi = -\phi_c$ with velocity $\dot{\phi}_c$, the Higgs becomes massless. The Higgs modes with momentum $k$ and frequency $\omega_k = \sqrt{k^2 + m_H^2(\phi)}$ for which the non-adiabatic parameter $\omega_k/\omega_k^2$ becomes large, are excited and resonant particle production takes place [35]. After it crosses the SBP, the mass parameter becomes negative and the modes with $k^2 < m_H^2(\phi)$| will be exponentially amplified via a process called tachyonic resonance [33, 36–39]. The particle production occurs throughout the non-adiabatic region between the two SBPs $|\phi| < \phi_c$ and it peaks at $\phi = 0$, where the maximal number of modes become tachyonic.

The Higgs quartic self-interaction $\lambda h^4$ reintroduces an effective mass term $m_H^2 + 3\lambda H^2$, which suppresses the particle production. Taking this effect into consideration, in Ref. [33] the authors derive an analytic approximation for the total particle number density after the exit from the non-adiabatic region at the second SBP $\phi = \phi_c$,

$$n_H \approx \left( \frac{\sqrt{g\phi_c}}{2\pi} \right)^3 \frac{\pi \Lambda^2}{e^{|\phi_c|} \times \frac{3\pi \lambda(\Lambda)/(H^{(0)}_{\text{eff}})}{e^{\frac{3\pi \lambda(\Lambda)/(H^{(0)}_{\text{eff}})}}},$$ (6)

where

$$\langle H^2 \rangle_{\text{eff}} = \frac{g\phi_c}{2\pi^3} \sqrt{\frac{\pi/2}{1 - Q/2}} e^{\pi Q/2 - 1}, Q \approx \frac{\pi \Lambda^2}{g\phi_c}.$$ (7)

The production is favored for smaller values of the quartic. Notice that $\lambda$ is evaluated at scale $\Lambda$, because this is the relevant energy scale of the Higgs potential at the point of maximum production. In the following and unless explicitly mentioned otherwise, we will abbreviate $\lambda = \lambda(\Lambda)$ and consider it as a free parameter.

The corresponding energy density stored in Higgs excitations is [29]

$$\rho_H \approx n_H |m_H(\phi)| \approx \left\{ \frac{\sqrt{2} g n_H |\Delta \phi|}{g n_H} |\Delta \phi| \gg \phi_c \right\} \left\{ \frac{g n_H \sqrt{2\phi_c |\Delta \phi|}}{g n_H} |\Delta \phi| \ll \phi_c \right\},$$ (8)

where $\Delta \phi = \phi - \phi_c$.

For a wide range of model parameters we have

$$V(0) > V(\phi_c) \Rightarrow n_H |m(0)| - \frac{\Lambda^4}{4\pi^2} > \frac{\Lambda^4}{16\pi^2},$$ (9)

which implies that $\rho_H$ acts as a back-reaction potential and the SBP $\phi = \phi_c$ becomes the new global minimum attracting $\phi$ back to it. In fact, as the attraxion moves away from the SBP its kinetic energy is transferred to the Higgs energy density and when $\rho_H \sim \dot{\phi}_h^2/2$ at $\Delta \phi = A$ with

$$A \equiv \left\{ \frac{\sqrt{\pi/2}}{8g n_H}, |\Delta \phi| \gg \phi_c \right\} \left\{ \frac{\sqrt{\pi/2}}{8g n_H \Lambda}, |\Delta \phi| \ll \phi_c \right\},$$ (10)

it stops and returns back to the SBP. As it crosses this point again (with practically the same velocity) it triggers a second burst of particle production and the newly
created Higgs bosons are added to the total bath. The attraction dynamics enter a phase characterized by fast oscillations around the SBP. The Hubble friction dilutes the Higgs number density and dissipates the kinetic energy. As a consequence the amplitude of each oscillation $A(t)$ and the velocity $\dot{\phi}_c(t)$ at the SBP both decrease with time (see Appendix A).

Eventually, the kinetic energy of the attraction drops enough so that the periodic potential (see Eq. (3)) becomes relevant. The oscillations will stop in the local minimum closest to the SBP $\phi_{\text{min}} \sim \phi_c - f$. The Higgs field which was initially anchored at the origin $(H) = 0$ now acquires the VEV $\langle H \rangle = v_{\text{EW}}$.

The mass of the attraction is given by

$$m_H^2 = \frac{-\partial^2 V}{\partial \phi^2}\bigg|_{\phi = \phi_{\text{min}}} \approx \frac{M^4}{f^2} + \frac{g^4A^2}{8\pi^2} - \frac{2g^2A^2}{\lambda(v_{\text{EW}})}.$$  

Unlike in the relaxation case, as we will see, this is typically larger than the mass of the Higgs. In this limit, we may also write the mixing angle between the two scalars as

$$\sin \theta \approx \left( \frac{\partial^2 V}{\partial H \partial \phi} / \frac{\partial^2 V}{\partial \phi^2} \right)\bigg|_{\phi = \phi_{\text{min}}} \approx \frac{2g^2}{m_H^2} \sqrt{ \frac{2A^4f}{\lambda(v_{\text{EW}})}}.$$  

### III. CONDITIONS FOR SUCCESSFUL TRAPPING

In this Section, we investigate further the details of the trapping mechanism by listing the necessary conditions for its realization in “chronological order”.

1. **Classical over quantum.** The attraction evolution must be dominated by classical rolling and not by the quantum fluctuations during inflation:

$$\dot{\phi}_c > H_{\text{inf}}^2.$$  

2. **Inflaton domination.** For inflation to occur the energy budget must be dominated by the inflaton potential energy and not by the kinetic energy of the attraction:

$$\dot{\phi}_c^2 < H_{\text{inf}}^2 M_{\text{pl}}^2.$$  

3. **Selecting the electroweak scale.** The final trapping should occur in a valley of the periodic potential close to the SBP (see Eq. (11)). In order to satisfy Eq. (11) we require then

$$f \sim \frac{\lambda(v_{\text{EW}})v_{\text{EW}}^2}{gA}.$$  

4. **Efficient trapping.** The trapping is achieved at the right scale if at time $t_{\text{EW}}$ the amplitude of the oscillation enters the region

$$A(t_{\text{EW}}) \lesssim 2\pi f.$$  

Afterwards, the Higgs number density is diluted due to inflation to the point that the periodic term in the potential takes over and the minima, previously erased by the back-reaction term, reemerge. This condition is equivalent to equating the slopes of the two terms, i.e.

$$\frac{M^4}{f} \gtrsim gn_H(t_{\text{trap}}) \sqrt{ \frac{\phi_c}{2A(t_{\text{trap}})}}.$$  

As shown in Appendix A the amplitude close to the SBP “red-shifts” as $A \propto a^{-6/5}$. If $t_1$ is the time when the attraction reaches the amplitude of the first oscillation, we can calculate the number of necessary e-folds as

$$\frac{a(t_{\text{trap}})}{a(t_1)} = \left( \frac{A(t_1)}{A(t_{\text{trap}})} \right)^{5/6} = \left( \frac{\phi_c^4}{8gn_H^2A(t_{\text{trap}})} \right)^{5/6}.$$  

The produced abundance of Higgs excitations red-shifts like matter $n_H \propto a^{-3}$. Solving now Eq. (18) for $A(t_{\text{trap}})$ and requiring that $a(t_{\text{trap}}) > a(t_{\text{EW}})$ yields the following bound for the particle production

$$n_H(t_1) > \frac{\sqrt{2}M^{2/3} \phi_c^{5/3}}{4\pi^{1/3}f^{1/2}g^{1/2}A^{1/2}}.$$  

An effect that can disturb the trapping is also the perturbative Higgs decay, which suppresses $n_H$ further. The decay can be neglected if its rate is slower than the Hubble expansion rate $\Gamma_H < H_{\text{inf}}$. The maximum Higgs decay rate

$$\Gamma^\max_H \approx \frac{g^2 |m_H(A)|}{16\pi} = \frac{g^2 \phi_c^2}{32\pi n_H},$$  

provides then a lower bound on the inflation scale.

5. **No Freezing before trapping.** In the derivation of the evolution of the oscillation amplitude in the Appendix A, we assume that the Hubble expansion is negligible during the timescale of one oscillation.

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1. In the region of the parameter space that is interesting for our setup, we know a posteriori that the maximum particle production is achieved by the first burst. Since this induces a large term $\langle H^3 \rangle_{\text{off}}$, it either stops the particle production immediately after the first oscillation or renders the rest of them subdominant.

2. Note that this phase is unique in our setup. Models that follow the slow-rolling relaxation paradigm and utilize particle production triggered at the SBP (e.g. see Ref. [16]) use the effect as a friction term, while the relevant term in our case corresponds to a restoring force towards the SBP.

3. In the text, unless otherwise explicitly mentioned, we denote as $\phi_c$ the velocity during the first passage from the SBP.
However, this approximation breaks down at time $t_{\text{freeze}}$ when the Hubble friction term is comparable with the slope of the back-reaction potential and the dynamics freeze

$$3H_{\text{inf}}\dot{\phi}_{c,t} \sim gn_H(t_{\text{freeze}})\sqrt{\frac{\phi_c}{2A(t_{\text{freeze}})}},$$

(22)

where $\dot{\phi}_{c,t}$ is the velocity at the last passage via the SBP before the time $t_{\text{freeze}}$. By requiring that $A(t_{\text{freeze}}) \lesssim 2\pi f$ and repeating the same steps that lead to Eq. (20) we find the following upper limit for the inflation scale

$$H_{\text{inf}} < \frac{\phi_c}{24\pi f}.$$  

(23)

6. Stability of the minimum during inflation: During inflation, the space-time has a de Sitter geometry, which is known to mimic thermal effects with fluctuations of order $H_{\text{inf}} \frac{\phi_c}{2\pi f}$. Those effects would destabilise the trapping minimum unless

$$H_{\text{inf}} < 4\pi^2 f.$$  

(24)

Additionally, we mention that the trapped minimum represents a metastable vacuum, which could undergo quantum tunnelling (see Appendix B for the calculation of the transition rate). However, we find that vacuum stability until today is ensured in all the relevant part of the parameter space.

IV. PARAMETER SPACE AND FUTURE PROSPECTS

Charting the viable parameter space - The model parameters are the couplings $g, \lambda, \Lambda, M$ and the initial velocity $\dot{\phi}_c$. In the analysis we fix $f$ according to Eq. (16) and express $\dot{\phi}_c$ as a function of $g$ by requiring that the Higgs particle number density $n_H$ given in Eq. (6) is maximized. We find that this happens for velocity values around $\dot{\phi}_c \approx c\Lambda^2/(10g)$, where $c$ is an $O(1)$ parameter that depends on the choice of $\lambda$. For the Higgs quartic and the the NP scale we use the benchmark $\lambda(\Lambda) = 10^{-3}$ and $\Lambda = 10^4$ GeV, respectively.

The parameter space that realizes the trapping mechanism can then be presented in a two-dimensional plane. We employ the parameters $\sin^2 \theta$ and $m_\phi$ and our results can be found in Fig. 1. The most stringent bounds are imposed by Eq. (4) (blue), Eq. (20) (green) and the expectation $M < f$ (red) from the axion-like effective theory construction.

Collider bounds and prospects - The attraction couples to the SM particles via its mixing with the Higgs which is proportional to $\sin \theta$. An upper bound

of $\sin \theta \lesssim 0.37$ (dash-dotted black) is obtained by indirect measurements of the SM-like Higgs bosons [40–42]. HL-LHC (dash-dotted magenta) and FCC-hh (dash-dotted purple) are expected to improve on this bound [43] by one and two orders of magnitude, respectively.

Direct searches are also relevant, since the attraction can be singly produced via vector boson fusion and then decay to a pair of SM gauge bosons or SM-like Higgs bosons $\phi \rightarrow ZZ$ (or $hh$). Present LHC exclusion limits [44, 45] (dashed black) are not constraining, while HL-LHC (dashed magenta) will be able to probe masses up to 500 GeV for $\sin \theta \gtrsim 0.07$. Regarding future colliders, a 14 TeV MuC (dashed orange) offers better sensitivity than FCC-hh (dashed purple) constraining all relevant masses for $\sin \theta \gtrsim 0.01$ [43, 46, 47].

One can then directly map the bounds derived for generic Higgs-singlet portal models on the $\sin \theta - m_\phi$ plane. In Fig. 1, we provide the current exclusion limits from LHC as well as the projections for the reach of HL-LHC, a 100 TeV FCC-hh and a 14 TeV Muon Collider (MuC) at the 95% CL. We infer that in the scenario where the NP cut-off lies at the 10 TeV direct detection of the attraction will be possible for a considerable part of the parameter space.

Inflation and New Physics scales - In Fig. 2 the inflation scale is displayed as a function of the coupling $g$. The viable range is constrained by Eqs. (22) (red), (24) (orange) and (23) (blue), while Eqs. (14) and (15) are readily satisfied. The rest of the conditions (that determine the allowed region in Fig. 1) also yield a lower bound for $g$ (green). We observe that our mechanism allows for scales significantly higher than the case of the relaxation, with a maximum of order $10^{15}$ GeV. Moreover, the completion of the trapping occurs after a modest number of e-folds $\log[A(t_1)/2\pi f] \sim \log[10^{-4}\Lambda^2/(g^2n_{H,\text{EW}})] \lesssim O(10)$. As a result, since the high-velocity regime of the
Among models of cosmological relaxation, our proposal uniquely features a rather sizeable pseudoscalar-Higgs coupling and a pseudoscalar mass heavier than the Higgs mass. The model is thus realized without the requirement of transplanckian field space. Ultimately, the most promising experimental avenue for the detection of the new state $\phi$ become again collider searches. For the case of New Physics at the $\mathcal{O}(10-100)$ TeV scale, the attracton can be directly probed at future colliders with the 14 TeV Muon Collider offering the best sensitivity.

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Appendix A: Evolution of the amplitude of the attracton oscillations

The equation of motion for the attracton during the Phase 2 of the trapping mechanism reads

$$\ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \quad (A1)$$

where $V \approx \rho_H$ (see main text Eq. (10)) and we have replaced $\Delta \phi \rightarrow \phi$ for convenience. The energy density of the attracton is

$$\rho = \frac{\dot{\phi}^2}{2} + V = \frac{\dot{\phi}^2}{2} = V(A) \quad (A2)$$

Over the time-scale of one oscillation, we can neglect the expansion of the universe and consider roughly $\rho \approx \text{const.}$ Following Ref. [48] we calculate the the average kinetic and potential energy as

$$\langle K \rangle = \frac{\gamma}{2} \rho , \quad \langle V \rangle = \frac{2 - \gamma}{2} \rho , \quad (A3)$$

where

$$\gamma \equiv 2 \left[ \int_0^A d\phi (1 - V(\phi)/\rho)^{1/2} \right]^2 = \begin{cases} 2/3 , & |\phi| \gg \phi_c \\ 2/5 , & |\phi| \ll \phi_c \end{cases} (A4)$$

Now we may consider the effect of the Hubble expansion imposed on the time evolution of the averaged quantities. Starting from Eq. (A1) and replacing Eq. (A2) (after taking the derivative) we get

$$\dot{\rho} - \dot{V} + 3H \dot{\phi}^2 + \frac{\partial V}{\partial \phi} \dot{\phi} = 0$$

$$\dot{\rho} + 3H \dot{\phi}^2 - V \frac{\dot{n_H}}{n_H} = 0 \quad (A5)$$
where we have used $\dot{V} = \frac{\partial V}{\partial \phi} \dot{\phi} + V \frac{\partial V}{\partial n_H}$. Averaging the above expression over an oscillation and using Eq. (A3) it follows
\[
\dot{\rho} + 3H\gamma\rho - \frac{2 - \gamma}{2} \frac{n_H}{n_H} \rho = 0 \quad \text{where we have used that } \dot{n}_H = -3Hn_H.
\]
We thus obtain
\[
\rho \propto a^{-3(\gamma+2)/2}.
\]
Given the definition of the amplitude according to Eq. (12) (main text) we finally infer for the evolution of the amplitude of the oscillation
\[
A \propto \begin{cases} a^{-1}, & |\phi| \gg \phi_c \\ a^{-6/5}, & |\phi| \ll \phi_c \end{cases}.
\]

**Appendix B: Quantum tunnelling**

At late times the trapped minimum represents a metastable vacuum, which could undergo quantum tunnelling with a transition rate per unit of volume
\[
\Gamma_{\text{tunnel}} \sim m_4^4 e^{-S_4},
\]
where $S_4$ is the Euclidean action of the bounce profile controlling the tunneling. Vacuum stability until today is ensured only if $\Gamma_{\text{tunnel}} \ll 1/V_{0t0} \sim H_3^4 \Rightarrow S_4 \gtrsim 400$.

The first possibility is that the tunnelling occurs from the local trapping minimum $\phi_{\text{min}} \sim \phi_c + f$ towards the true minimum at $\phi = 0$. The Euclidean action is estimated using the triangular approximation [49, 50]
\[
S_4^{\text{tri}} = \frac{32\pi^2}{3} \left( \frac{\Delta \phi_+^4}{\Delta V_+^{1/3}} \right),
\]
where
\[
\Delta V_{\pm} \sim M^4, \quad \Delta \phi_{\pm} \sim f, \\
\Delta V_- \sim g^4 \phi_+^4, \quad \Delta \phi_- \sim \phi_c, \\
c \equiv \frac{\Delta V_-}{\Delta \phi_-} \sim \frac{g^4}{\lambda} \left( \frac{\phi_c}{m} \right) \frac{f}{\phi_c} = \frac{g \lambda^3 f}{\lambda m^2}.
\]

We denote as $\Delta \phi_{\pm}(-)$ the distance in field space between the local maximum and the false (true) minimum and $\Delta V_{\pm}(-)$ the difference of the values of the potential between those points.

In the limit $c \gg 1$, the Euclidean action becomes
\[
S_4^{\text{tri}} \approx \frac{32\pi^2}{3} \frac{\Delta \phi_+^4}{\Delta V_+^{1/3}},
\]
where
\[
\Delta \phi \sim f, \quad \Delta V_+ \sim M^4 - \frac{g_2^2 \Lambda^2(f)^2}{\lambda} - \frac{g^3 \lambda f}{\lambda m^2}, \\
\Delta V_- \sim M^4 + \frac{g_2^2 \Lambda^2 f^2}{\lambda} + \frac{g^3 \lambda f}{\lambda m^2}.
\]

Then we need to consider the case where the tunnelling occurs regionally towards another minimum in the vicinity. Due to the fact that the values of the potential between subsequent barriers are almost equal, here it is more suitable to use the rectangular approximation [49, 50], which is given by
\[
S_4^{\text{rec}} = \frac{2\pi^2}{3} \frac{\Delta \phi_+^4}{(\Delta V_+^{1/3} - \Delta V_-^{1/3})^3},
\]
where
\[
\Delta \phi \sim f, \quad \Delta V_+ \sim M^4 - \frac{g_2^2 \Lambda^2(f)^2}{\lambda} - \frac{g^3 \lambda f}{\lambda m^2}, \\
\Delta V_- \sim M^4 + \frac{g_2^2 \Lambda^2 f^2}{\lambda} + \frac{g^3 \lambda f}{\lambda m^2}.
\]

In the regime of Eq. (4) (main text), the first term in Eq. (B7) dominates and neglecting the second term, we obtain
\[
S_4^{\text{rec}} \approx 27648\pi^8 fM^8 / g^3\Lambda^3,
\]
which implies that $S_4^{\text{rec}} \gg S_4^{\text{tri}}$ and thus this transition is suppressed in comparison to the one towards the true minimum.

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