Measurements of nanoresonator-qubit interactions in a hybrid quantum electromechanical system

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Abstract
Experiments to probe the basic quantum properties of motional degrees of freedom of mechanical systems have developed rapidly over the last decade. One promising approach is to use hybrid electromechanical systems incorporating superconducting qubits and microwave circuitry. However, a critical challenge facing the development of these systems is to achieve strong coupling between mechanics and qubits while simultaneously reducing coupling of both the qubit and mechanical mode to the environment. Here we report measurements of a qubit-coupled mechanical resonator system consisting of an ultra-high-frequency nanoresonator and a long coherence-time superconducting transmon qubit, embedded in a superconducting coplanar waveguide cavity. It is demonstrated that the nanoresonator and transmon have commensurate energies and transmon coherence times are one order of magnitude larger than for all previously reported qubit-coupled nanoresonators. Moreover, we show that numerical simulations of this new hybrid quantum system are in good agreement with spectroscopic measurements and suggest that the nanoresonator in our device resides at low thermal occupation number, near its ground state, acting as a dissipative bath seen by the qubit. We also outline how this system could soon be developed as a platform for implementing more advanced experiments with direct relevance to quantum information processing and quantum thermodynamics, including the study of nanoresonator quantum noise properties, reservoir engineering, and nanomechanical quantum state generation and detection.

Keywords: hybrid quantum systems, nanomechanics, superconducting qubits, quantum information

(Some figures may appear in colour only in the online journal)
Quantum electromechanical systems, in which superconducting qubits are integrated with nanomechanical devices, present a potentially powerful platform for all of these applications [1, 5, 21–23]. In these hybrid systems, a Josephson-junction-based qubit [24] provides a strong non-linearity that could be utilized for engineering and measuring a wide range of non-classical mechanical states [19, 25–29]. It has thus been anticipated that these systems could enable investigations of fundamental aspects of quantum behavior at the nanoscale that are relevant to current research in quantum information, including decoherence [27, 28], entanglement [27, 28], and non-locality [29]. Additionally, because of the compact size of nanomechanical devices, qubit-coupled mechanical elements have strong prospects for integration with superconducting quantum processing architectures for use as quantum circuit elements, such as coherent switches [30] and quantum memory and bus elements [31].

However, the experimental realization of qubit-coupled mechanical devices is relatively underdeveloped (in comparison with the more experimentally active area of cavity mechanics [32]), with just a handful of results published demonstrating basic interactions between the two systems [1, 21–23]. The central challenge facing further development of qubit-mechanics for more advanced experiments or applications is to achieve strong coupling between mechanical and qubit degrees of freedom (generically denoted as $\lambda$) while simultaneously engineering weak environmental coupling to both the mechanical mode and qubit (generically denoted as $\kappa$ and $\gamma$ respectively) [33]. In a general sense, $\lambda$ sets the characteristic time scale for the dynamics of the coupled system and thus determines how quickly a state can be measured, prepared, or transferred between devices using the mutual interaction. It is therefore essential to achieve $\lambda > \kappa, \gamma$ (the strong-coupling regime) during a particular operation to ensure high fidelity interaction protocols.

Here we report and analyze measurements of a qubit-coupled nanomechanical device that consists of an ultra-high-frequency (UHF) nanoresonator [34] and superconducting transmon qubit [35] embedded in a circuit QED (cQED) architecture [36]. We demonstrate that the nanoresonator and transmon have commensurate energies and that the transmon coherence times are an order-of-magnitude larger than any previously reported qubit-coupled nanoresonators [1, 21, 22], thus presenting a viable new path toward accessing the strong-coupling regime of qubit-mechanics. We detail the design, fabrication and measurement of this novel three-body hybrid quantum electromechanical system. Moreover, we show that spectroscopic measurements of its behavior can be modeled using numerical simulations based upon a Lindblad master equation with generalized Jayne–Cummings Hamiltonians for the intra-coupled components. We further discuss how the results are consistent with the nanoresonator mode residing at low thermal occupancy ($k_B T / \hbar \omega < 1$) and acting as dissipative bath for the transmon when the two systems are tuned near resonance, providing prospects for future use of this system to study the influence of controlled environments on transmon’s dynamics. Finally, we discuss how realistic improvements could be implemented to develop the system for more advanced experiments in quantum measurement and state-generation with the nanoresonator.

2. Methods

2.1. Experimental device

Figure 1 displays a schematic (figure 1(a)) and images of the main components of the hybrid quantum electromechanical system that we report on here: a T-filtered superconducting coplanar waveguide (CPW) microwave cavity [37] (figure 1(b)), a Josephson-junction-based transmon-style
nanoresonator (figure 1(d)). Important parameters characterizing the properties of this system are summarized in table 1.

At the heart of the device is a suspended aluminum nanostructure (figure 1(d)), with nominal dimensions of 700 nm x 45 nm x 100 nm. Finite element simulations conducted with the commercial software package COMSOL [38] (figure 1(d), inset) were used to engineer the lowest in-plane flexural modes to have frequencies in the UHF regime. In particular, the third in-plane mode was designed to have a resonant frequency $\omega_{NR}/2\pi = 3.4$ GHz, which was commensurate with the transition energy of the qubit (see below). For the remainder of the text, this mode will be denoted as the nanoresonator. While the nanoresonator’s properties were not probed independently of the qubit in this work, microwave spectroscopy of the coupled system (see section 3) indicated a nanoresonator frequency $\omega_{NR,\text{mean}}/2\pi = 3.47$ GHz, which is in good agreement with the COMSOL simulations and an estimate of the mechanical quality factor $Q_{NR} = 150$ ($\kappa_{NR}/2\pi = 24$ MHz) that is in reasonable agreement with analytical calculations based upon clamping loss through the nanostructure’s supports ($Q_{NR,\text{calc}} = 280$) [39].

A 2D split-junction transmon served as the qubit in this work (figure 1(c)). The transmon was formed from two Al/AIO/Al Josephson junctions connected in parallel between

| Parameter | Value |
|-----------|-------|
| $\omega_{CPW}/2\pi$ | 4.94 GHz |
| $E_C/h$ | 0.227 GHz |
| $E_J/h$ | 15.4 GHz |
| $T_1$ | 15 $\mu$s |
| $T_2^*$ | 1.4 $\mu$s |
| $\omega_{NR}/2\pi$ | 3.4 GHz |
| $\omega_{NR,\text{mean}}/2\pi$ | $\approx 3.47$ GHz |
| $m$ | 7 f\text{g} |
| $w$ | 45 nm |
| $L$ | 700 nm |
| $t$ | 100 nm |
| $\rho$ | 2.7 g cm$^{-3}$ |
| $x_{cp}$ | 25 fm |
| $\alpha$ | 0.447 |
| $g/2\pi$ | 120 MHz |
| $\lambda/(2pV_{NR})$ | $\approx 300$ k\text{Hz V}^{-1} |
| $Q_{NR}$ | 150 |
| $Q_{NR,\text{calc}}$ | 280 |
| $\kappa_{CPW}/2\pi$ | 0.28 MHz |
| $\kappa_{NR}/2\pi$ | 24 MHz |

**Table 1.** Experimental parameters characterizing the sample. The first set of values shows the CPW and transmon’s characteristic energies, and $T_1$ and $T_2^*$ for $\omega_{01}/2\pi = 4.2$ GHz. The nanomechanical resonator mechanical properties are shown in the second set, and the last set displays the measured coupling and relaxation rates. Note that the value provided for $\kappa_{CPW}$ is for $V_{NR} = 0$.

$qubit$ [35] (figure 1(c)), and a flexural nanomechanical resonator (figure 1(d)). Important parameters characterizing the properties of this system are summarized in table 1.

At the heart of the device is a suspended aluminum nanostructure (figure 1(d)), with nominal dimensions of 700 nm x 45 nm x 100 nm. Finite element simulations conducted with the commercial software package COMSOL [38] (figure 1(d), inset) were used to engineer the lowest in-plane flexural modes to have frequencies in the UHF regime. In particular, the third in-plane mode was designed to have a resonant frequency $\omega_{NR}/2\pi = 3.4$ GHz, which was commensurate with the transition energy of the qubit (see below). For the remainder of the text, this mode will be denoted as the nanoresonator. While the nanoresonator’s properties were not probed independently of the qubit in this work, microwave spectroscopy of the coupled system (see section 3) indicated a nanoresonator frequency $\omega_{NR,\text{mean}}/2\pi = 3.47$ GHz, which is in good agreement with the COMSOL simulations and an estimate of the mechanical quality factor $Q_{NR} = 150$ ($\kappa_{NR}/2\pi = 24$ MHz) that is in reasonable agreement with analytical calculations based upon clamping loss through the nanostructure’s supports ($Q_{NR,\text{calc}} = 280$) [39].

A 2D split-junction transmon served as the qubit in this work (figure 1(c)). The transmon was formed from two Al/AIO/Al Josephson junctions connected in parallel between

Figure 2. Spectroscopy and time-domain measurements from which the parameter values of the transmon and CPW cavity mode were determined. (a) Single-tone spectroscopy of the CPW with $V_{NR} = 0$ V, as a function of flux bias $\Phi$ and spectroscopy frequency $\omega$. Gray scale represents the magnitude of the cavity’s transmitted signal. The data illustrates the dispersive pull of the CPW mode over one flux period $\Phi_0$. The red dotted line represents a simulation of the CPW’s transition energy versus $\Phi$ calculated numerically using the full CPW-transmon Hamiltonian and parameters in table 1. Insets: (top) measurement of the relaxation time $T_1$ for the transmon’s $\omega_{01}$ transition at $\omega_{01}/2\pi = 4.2$ GHz, where the CPW, transmon and nanoresonator are all far-detuned in energy; (bottom) Ramsey interference measurement of the transmon taken at $\omega_{01}/2\pi = 4.2$ GHz. (b) Two-tone spectroscopy measurements of the transmon at high and low powers, illustrating the CPW-number-state-resolved Stark shift of the transmon. The solid line is a numerical simulation including only the transmon and CPW, using $T_1$, $T_2^*$, and $\kappa_{CPW}$ from table 1, with the transmon and CPW temperatures set to $T_0 = 30$ mK and $T_{ipw} = 45$ mK (see supplementary material).
two large area (200 \mu m \times 50 \mu m) niobium pads. From spectroscopic measurements (figures 2(a) and (b)), the charging energy and maximum total Josephson energy of the transmon were determined to be \( E_C/h = 0.227 \text{ GHz} \) and \( E_R/h = 15.4 \text{ GHz} \). An on-chip niobium flux-bias trace was used to provide magnetic flux \( \Phi \) to the transmon in order to tune the transmon’s Josephson energy \( E_J = E_R \cos (\pi \Phi / Q_0) \) (figure 2(a)). Time-domain measurements of the relaxation time \( T_1 \) of the transmon’s \( \omega_{01} \) transition (i.e. \([1] \rightarrow [0]\)) conducted at a flux bias point \( \Phi = 0.824 Q_0 \), where \( \omega_{01} \) was detuned from both the nanoresonator and CPW fundamental mode (see below), yielded a maximum relaxation time of \( T_1 = 15 \mu s \) (top inset, figure 2(a)), which was consistent with estimates of radiative loss through the \( T \)-filtered CPW (see supplementary material). Ramsey interference measurements made at the same value of \( \Phi \) yielded a maximum coherence time of \( T_2^* = 1.4 \mu s \) (bottom inset, figure 2(a)), which we believe was likely limited by dephasing that arises due to the high susceptibility of our symmetric junction design to low frequency flux noise—for the measurements reported here, the transmon was operated in a regime where \( \partial \omega_{01} / \partial \Phi \) is large, leading to reduced \( T_2^* \) values in comparison with what is typically observed when the transmon is operated at the ‘sweet spot’ where \( \partial \omega_{01} / \partial \Phi = 0 \) [35]. It is important to point out that these coherence values are more than an order of magnitude larger than achieved in all previously published works [1, 21, 22] involving the integration of a nanoresonator and superconducting qubit.

The application of a large DC voltage (on the order of Volts) between the nanoresonator and the transmon \( V_{NR} \) served to establish coupling between the nanoresonator’s flexural motion and the electrostatic energy of transmon. To lowest order, mechanical displacement \( x \) of the nanoresonator linearly modulates the transmon’s polarization charge through the devices’ mutual capacitance \( C_{NR} \) (figure 1(a)), resulting in an interaction that is characterized by the coupling strength \[ \lambda = -\frac{4E_C}{h} \frac{dC_{NR}}{dx} V_{NR} x_{zp}, \] (1)

where \( x_{zp} = \sqrt{\hbar / 2m_{NR}} \) are the rms zero-point fluctuations of the mode, with \( m = \alpha / \omega_L L \) as the effective mass of the resonator, and the parameters \( w, L, \) and \( t \) are the geometrical width, length and (out-of-plane) thickness of the structure respectively. Estimates of these parameters, along with the effective mass ratio factor \( \alpha \), are provided in table 1. From spectroscopic measurements of the coupled-device (see section 3), we estimate \( \lambda / 2\pi V_{NR} \approx 300 \text{ kHz} \) \( V^{-1} \), which is the value we use to perform numerical simulations of the coupled-device (section 3). For this magnitude of coupling, the strong-coupling regime of the transmon and nanoresonator with respect to qubit decoherence (i.e. \( \lambda > \frac{2\pi}{T_1} \)) is accessed for \( \left| V_{NR} \right| > 2 \text{ V} \). However, the system remains in the weak coupling regime in regard to nanoresonator dissipation, with \( \lambda < \lambda_{NR} \) for all values of \( V_{NR} \) explored (\( \left| V_{NR} \right| \lesssim 8 \text{ V} \)). It should be noted that the value of \( \lambda \) extracted from measurements and used in our simulations is within the right range of the values of \( \lambda \) predicted by equation (1) using the parameters in table 1 and the approximation \( \frac{dC_{NR}}{dx} \approx \frac{C_{NR}}{d} \), where \( d = 35 \text{ nm} \) is the spacing between nanostructure and the transmon’s coupling electrode; but it exceeds by a factor of 10 estimates based upon simple numerical simulations of \( \frac{dC_{NR}}{dx} \) using finite element simulations (see supplementary material)\(^6\). The source of the discrepancy remains a subject of ongoing investigations.

In order to isolate and measure the coupled transmon and nanoresonator, these devices were embedded in a cQED architecture (figure 1(b)) [36]. In this configuration, the transmon and nanoresonator were located in a pocket of the ground plane of a superconducting CPW near a voltage antinode of the CPW’s fundamental mode, which had a frequency of \( \omega_{cpw}/2\pi = 4.94 \text{ GHz} \), and a loaded quality factor \( Q_{cpw} = 20 \times 10^3 (\kappa_{cpw}/2\pi = 280 \text{ kHz} \), when far detuned in energy from the transmon. A superconducting stub, which extended into the ground plane pocket from the center trace of the CPW, provided capacitive coupling \( C_p \) between the CPW’s fundamental mode and the transmon. The resulting coupling \( g \) was given by the standard expression used in cQED [35]

\[ g = 2\beta e V_{cpw} / \hbar, \] (2)

where \( \beta = C_{eff} / C_p \) is the ratio of CPW-transmon mutual capacitance \( C_{eff} \) (figure 1(a)) to the transmon’s total capacitance \( C_p \), and \( V_{cpw} = \sqrt{\hbar \omega_{cpw} / 2C_{cpw}} \) are the rms zero-point fluctuations of the CPW cavity, with total capacitance \( C_{cpw} \). For this device, the engineered parameters yielded \( g = 2\pi/250 \approx 120 \text{ MHz} \), providing access to the strong dispersive coupling limit between the transmon and CPW [40], where the effective dispersive coupling strength \( \chi \) exceeded the linewidths of both the CPW and transmon (i.e. \( \chi / 2\pi > \left( \frac{2\pi}{T_1}, \frac{2\pi}{T_2}, \kappa_{cpw} \right) \)). This enabled measurements of the number-state-resolved AC Stark shift of transmon’s 0–1 transition energy that arises due to the dispersive coupling to the CPW mode (figure 2(c)). Note that the resonant limit between the transmon and CPW cavity was inaccessible because \( \omega_{cpw} > \omega_{01} \) for all values of \( \Phi \).

The superconducting stub also provided a galvanic connection to the nanoresonator, allowing for the application of DC voltages to supply \( V_{NR} \) and establish coupling between the transmon and nanoresonator. The DC voltage was applied through a superconducting Nb \( T \)-filter [37] that was connected to the mid-point of the CPW cavity and was designed to introduce negligible loss to the CPW in comparison to intrinsic dissipation and losses through the CPW’s coupling capacitors \( C_p \).

2.2. Fabrication of the device

The device was fabricated in a series of steps involving standard micro- and nanolithographic techniques. First, a 100 nm thick layer of Nb was DC-sputtered on a high-
resistivity (>10 kΩ cm) silicon wafer, whose surface was prepared with an ion-mill etch before deposition of the Nb. Next the CPW, T-filter, ground plane, transmon shunt pads, and flux bias trace were patterned from the Nb using deep-UV photolithography followed by a reactive ion etch with gas mixture Ar:BCl₃:Cl₂.

Next the nanoresonator was defined in a lift-off process, using e-beam lithography to define the pattern, and then an aluminum deposition in a dedicated e-beam evaporation system. This was followed by a dry-etch process to free the resonator using a PMMA mask defined by e-beam lithography and a reactive ion plasma of SF₆:Ar to undercut the nanostructure. The sample was then cleaned by a soft oxygen plasma ashing process (descum) to remove any residual resist of the surface of the sample.

In the final step, the transmon’s Josephson junctions were fabricated. This involved a third layer of e-beam lithography, followed by a standard Dolan-bridge double-angle evaporation [41] of aluminum in ultra-high vacuum using an evaporator dedicated to aluminum deposition.

2.3. Milli-kelvin measurement circuitry and microwave spectroscopy

Measurements of the device were performed at milli-Kelvin temperatures using a dilution refrigerator with a base temperature between 20 and 30 mK. The devices were enclosed in a light-tight OFHC copper sample holder, which itself was situated in a home-made, lead-lined Cryoperm magnetic shield; both the sample holder and shield were anchored to the base-stage of the refrigerator. In order to minimize stray radiation and the excitation of non-equilibrium quasiparticles in the superconducting circuitry [42, 43], the inner wall of the sample holder was coated with microwave/infrared absorbing foam and the input and output lines into the sample holder were heavily filtered and attenuated at various stages of the dilution refrigerator as illustrated in the circuit diagram in figure 3.

The results reported below were obtained from both single-tone and two-tone spectroscopy measurements of the coupled CPW, transmon and nanoresonator. Single-tone spectroscopy was accomplished using pulsed transmission measurements of the CPW at frequencies ω in the vicinity of ω_{CPW} for fixed values of Φ and V_{SR}. The transmitted pulses were measured with a first-stage cryogenic HEMT amplifier, followed by additional room temperature amplifiers and a home-made superheterodyne circuit that mixed the signal to an IF frequency of 10.7 MHz; the IF signal was then digitized with a high-speed ADC, and the phase and amplitude were extracted numerically using a digital homodyne technique. Measurement pulse lengths of ~1 ms were chosen so that the averaging time greatly exceed the relaxation rates of the CPW, transmon and nanoresonator, and thus the data reflected the steady-state behavior of the system.

Two-tone spectroscopy was performed using two different pulses: first a long (~40 μs) microwave pulse with variable frequency, ω_{P}, tuned near the transmon transition frequency ω_{trans}, was applied to the cavity to excite the transmon; then a second pulse of 4 μs was applied at ω_{CPW} to perform a dispersive measurement [44] of the transmon’s state. The phase and amplitude of the cavity’s transmitted signal were then recovered using the same home-made superheterodyne/digital-homodyne setup used for the single-tone spectroscopy.

2.4. Model and numerical simulations

In order to simulate the single-tone spectroscopy measurements, we follow standard approaches for modeling each part of our hybrid system and their respective interactions. Here, the transmon is considered as a multi-level atom [35], represented by the Hamiltonian

\[ \hat{H}_T = \sum_m \hbar \omega_0 \left| m \right\rangle \langle m \right|, \]  

(3)

where the eigenenergy differences \( \hbar \omega_0 \), which are dependent on \( E_0, E_C \) and the applied magnetic flux are determined using the circuit model theory [35, 45] for the bare device (see supplementary material).

The CPW cavity and the nanoresonator are modeled as single mode harmonic oscillators, represented by the canonical bosonic operators \( \hat{a}(\hat{a}^\dagger) \) and \( \hat{b}(\hat{b}^\dagger) \), respectively. It is

\xrightarrow{\text{we did not perform simulations of the two-tone spectroscopy of the complete device, due to the complexity of such simulations, and leave this for future work.}}
worth noting that their natural frequencies $\omega_{\text{CPW}}$ and $\omega_{\text{NR}}$ are, by design, out of resonance. Moreover their direct coupling is relatively weak. As a result the CPW-nanoresonator interaction has negligible affect on the unitary evolution of the entire system and thus is omitted from the system Hamiltonian.

To model the transmon-CPW and transmon-nanolensator directcouplings, we use for each a multi-level, 
generalized Jaynes–Cummings Hamiltonian [22, 35],
\[
\hat{H}_{T-\text{CPW}} = \sum_{l,m} g_{l,m} \left| l \right\rangle \left\langle m | (\hat{a}^\dagger + \hat{a}), \right.
\]
\[
\hat{H}_{T-\text{NR}} = \sum_{l,m} \lambda_{l,m} \left| l \right\rangle \left\langle m | (\hat{b}^\dagger + \hat{b}), \right.
\]
with $g_{l,m} = g \left| l \right\rangle \left\langle m |$ and $\lambda_{l,m} = \lambda \left| l \right\rangle \left\langle m |$ representing the coupling strength of the $| l \rangle \rightarrow | m \rangle$ transition, where $\hat{n}$ is the Cooper-number pair operator associated with the transmon.

Finally, the microwave field applied to the cavity to perform single-tone spectroscopy is represented by the term
\[
\hat{H}_{\text{Drive}} = E_d (e^{i\omega t} \hat{a} + e^{-i\omega t} \hat{a}^\dagger),
\]
where $\omega$ is the frequency of the signal and $E_d$ its amplitude.

We evaluate the system state dynamics by numerically solving the Lindblad form of the system master equation [46]
\[
\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}] + \sum_k \gamma_k \left( \hat{A}_k \rho \hat{A}_k^\dagger - \frac{1}{2} \{ \hat{A}_k^\dagger \hat{A}_k , \rho \} \right),
\]
where $\hat{\rho}$ represents the hybrid system density matrix and $[\hat{A}_k^\dagger \hat{A}_k , \rho] = \hat{A}_k^\dagger \hat{A}_k \rho + \rho \hat{A}_k^\dagger \hat{A}_k$. The operators $\hat{A}_k$ are standard Lindblad operators and are responsible for inducing relaxation and decoherence processes in the state time evolution. We use $\{ \hat{a}^\dagger, \hat{b}^\dagger , | l \rangle \langle l+1 | \}$ as the set of Lindblad operators associated with thermally induced absorption processes and $\{ \hat{a}, \hat{b}, | l \rangle \langle l+1 | \}$ as those representing relaxation processes. We include dephasing processes by adding the transmon operators $\{ | l \rangle \langle l | \}$ to our set of Lindblad operators. The respective rates $\gamma_k$ are determined from the knowledge of the CPW cavity and nanoresonator quality factors ($k_{\text{CPW}}, k_{\text{NR}}$), transmon relaxation and decoherence times ($T_1, T_2^*$) and temperature estimations. (See supplementary material for more details.) Since we are interested in characterizing quantitatively the steadystate regime, we only have to determine the solution of $\rho = 0$, which we calculate numerically assuming the hybrid system Hilbert space spanned by the three, five and four lowest eigenenergy states of the bare transmon, nanoresonator and CPW cavity, respectively.

3. Results

Figures 4–6 show the central result of this work. Displayed in the top panel of figure 4 (figures 4(a)–(c)) are measurements of single-tone transmission spectroscopy in the vicinity of $\omega_{\text{CPW}}$, versus $\Phi$, for three different values of $V_{\text{NR}}$. The corresponding results from numerical simulations, using the parameter values in table 1, are displayed in figures 4(d)–(f).

It is evident from both the data and simulations that for low coupling voltages ($| V_{\text{NR}} | \lesssim 5 \text{ V}$), the cavity response varied with transmon detuning as one would expect from the dispersive interaction between the CPW and transmon. However, as $V_{\text{NR}}$ was increased further ($5 \text{ V} \lesssim | V_{\text{NR}} | \lesssim 7.5 \text{ V}$), the transmon–nanoresonator interaction became prominent, producing an apparent gap in the CPW transmission spectrum around $\Phi = 0.32 \Phi_0$, where $\omega_{01} \approx 3.47 \text{ GHz}$ (figure 6), resonant with the simulated value of $\omega_{\text{NR}}$. For larger values of coupling ($| V_{\text{NR}} | \gtrsim 8 \text{ V}$, not shown), the cavity response broadened significantly around $0.32 \Phi_0$, and the gap was no longer observable.

The behavior in figure 4 can be explained as the interplay between two different effects. First, as $V_{\text{NR}}$ is increased from zero, the corresponding growth of $\lambda$ should lead to hybridization of the nanoresonator and transmon energy levels when $\omega_{01} \approx \omega_{\text{NR}}$, producing the well-known phenomenon of Rabi doublets [47] in the coupled-system’s energy spectrum. Of course, for the temperatures at which these transmission measurements were made, the transmon and nanoresonator should have each resided predominantly in their ground state (transmon and nanoresonator thermal occupancies should both have been $n_{0\Phi} \approx 0.004$ at $T = 30 \text{ mK}$), resulting in the joint state $|00\rangle$ and no change in the transmission response of the dispersively coupled CPW. However, the increase in $V_{\text{NR}}$ was accompanied by heating of the nanoresonator, believed to be a result of dissipation due to leakage current through the silicon substrate. Measurements of the leakage current flowing in the DC bias circuitry provided an estimate of the dissipated power on the order of nano-Watts for the range of $V_{\text{NR}}$ shown in figure 4. A simple model and COMSOL simulation of the heating effects indicates that this level of dissipation could indeed heat the nanoresonator to a temperature $T_{\text{NR}}$ that is out of equilibrium with the transmon, resulting in non-negligible thermal population of the nanoresonator ($n_{\Phi} > 0.1$; see supplementary material). Qualitatively, through the coupling $\lambda$, the thermally excited nanoresonator thus served as an effective thermal bath for the transmon, increasing the probability for the transmon to be found in its first excited state $|m = 1\rangle$ at $\Phi \approx 0.32 \Phi_0$, and leading to a thermally averaged dispersive shift of the CPW response.

The physics due to the thermally excited nanoresonator are captured quantitatively in the numerical simulations by increasing $T_{\text{NR}}$ simultaneously with $\lambda$. In figures 4(d)–(f), the best-fit eye to the data was found by increasing $T_{\text{NR}}$ from 30 mK for $\lambda/2\pi = 1.35 \text{ MHz}$ to 180 mK for $\lambda/2\pi = 1.95 \text{ MHz}$, which would have corresponded to an

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8 This leakage current is believed to have occurred in the DC bias $T$-filter and CPW, and was observed to increase in a nonlinear fashion with $V_{\text{SR}}$, suggesting it was related to breakdown in the silicon between the central trace of the $T$-filter’s capacitor and the ground plane.

9 For voltages larger than $\sim 8 \text{ V}$, the current increased dramatically, ultimately leading to observable heating of the dilution refrigerator’s sample stage, as measured with standard resistance bridge thermometry.
increase in thermal occupation of the nanoresonator mode from $n_{th} = 0.004$ to $n_{th} = 0.8$. While $n_{th} < 1$ for these values of $T_{NR}$, the numerical simulations show that in the steady-state the increased probability for occupation of the nanoresonator’s excited states is enough to enhance the population of the transmon’s $m = 1$ state and deplete the population of the $m = 0$ state (figures 5(a)–(d)), when the transmon and nanoresonator are near resonance ($\omega_{01} \approx \omega_{NR}$). This is reflected in numerical calculations of the transmon state probability versus $\Phi$, which show the excited state population increasing directly with $T_{NR}$ and $\lambda$, around $\Phi = 0.32 \Phi_0$, even while the transmon temperature is held fixed in the simulation at the base temperature of the refrigerator ($T_0 = 30$ mK).

It should be noted that, in two-tone spectroscopy measurements of the transmon at $V_{NR} = -5$ V, the transmon’s $\omega_{12}$ transition was not observable above the noise floor of the measurement set up for flux biases where the nanoresonator and transmon were tuned off resonance (figure 2(b)). This indicated that the transmon was not heated directly by the application of $V_{NR}$ and that its enhanced excited ($m = 1$) population was in fact due to the thermally excited nanoresonator. This conclusion is also supported by measurements of the number-state splitting of the transmon’s $\omega_{01}$ that arises due to the dispersive interaction with the CPW mode (figure 2(c)). This splitting effect was measured from $V_{NR} = 0$ V to $V_{NR} = -8$ V and at large detuning in energy from the nanoresonator; the sharp transitions exhibited no observable change in linewidth or peak height as a function of $V_{NR}$, indicating that heating of the transmon (and CPW mode) was negligible over this coupling range.

It is important to point out that we observed the CPW quality factor to degrade as $|V_{NR}|$ was increased, for values of $\Phi$ where $\omega_{01} \approx \omega_{NR}$. In this range, we observed the CPW linewidth to change from $\kappa_{cpw}/2\pi = 0.282$ MHz for $V_{NR} = -4.5$ V to 1.08 MHz for $V_{NR} = -6.5$ V. This increase in $\kappa_{cpw}$ could not be reproduced in the simulations with our model. Thus for the simulation we used the respective measured values of $\kappa_{cpw}$ for $V_{NR} = -4.5$ V, $-5.5$ V, and $-6.5$ V. We believe that this effect could be due to the direct

| $\omega/2\pi$ (GHz) | $\phi/2\pi$ (MHz) | $\phi/2\pi$ (MHz) | $\phi/2\pi$ (MHz) |
|-------------------|-----------------|-----------------|-----------------|
| 4.942             | 0.31            | 0.32            | 0.33            |
| 4.946             |                 |                 |                 |
| 4.948             |                 |                 |                 |
| 4.944             |                 |                 |                 |
| 4.942             |                 |                 |                 |

Figure 4. Measurements and simulations of single-tone spectroscopy of the cavity mode around $\omega_{cpw}$ as a function of $\Phi$ over a range where $\omega_{01} \approx \omega_{NR}$. The color scale indicates the amplitude of the cavity transmitted signal. Measured data are plotted for three different coupling voltages $V_{NR} = -4.5$ V, $-5.5$ V, $-6.5$ V (a)–(c). Simulated data is plotted for $\lambda/2\pi = 1.35, 1.65, 1.95$ MHz (d)–(f). In the simulations, the nanoresonator temperature $T_{NR}$ is also increased, with $T_{NR} = 30$ mK, 100 mK, and 180 mK in (d), (e), and (f) respectively. For all three simulation maps, the remaining fit parameters were fixed at the experimentally determined values. (a) Map of single-tone spectroscopy data that is typical of measurements made for $|V_{NR}| \lesssim 5$ V, for which the dispersive pull of $\omega_{cpw}$ due to the transmon is apparent, but no manifestation of the nanoresonator–transmon interaction is evident. (b) and (c) As $\lambda$ increases ($|V_{NR}| > 5$ V) and $T_{NR}$ increases due to heating from the leakage current, a gap becomes apparent in the spectroscopy data around $\Phi = 0.32 \Phi_0$, where $\omega_{01} = \omega_{NR}$. The location of this feature was reproducible through repeated cycling of fridge temperature and $V_{NR}$, and was observed to be periodic in $\Phi$ as one would expect given the dependence of $\omega_{01}$ on $E_j(\Phi)$.

It is important to point out that we observed the CPW quality factor to degrade as $|V_{NR}|$ was increased, for values of $\Phi$ where $\omega_{01} \approx \omega_{NR}$. In this range, we observed the CPW linewidth to change from $\kappa_{cpw}/2\pi = 0.282$ MHz for $V_{NR} = -4.5$ V to 1.08 MHz for $V_{NR} = -6.5$ V. This increase in $\kappa_{cpw}$ could not be reproduced in the simulations with our model. Thus for the simulation we used the respective measured values of $\kappa_{cpw}$ for $V_{NR} = -4.5$ V, $-5.5$ V, and $-6.5$ V. We believe that this effect could be due to the direct

10 It should be noted that for the simulations, $T_{NR}$ serves as the only free-parameter; the remaining parameters (table 1), were all determined through independent means (such as two-tone spectroscopy or single-tone spectroscopy).
nanoresonator-CPW coupling which we did not include in the model\(^\text{11}\).

The influence of the nanoresonator as a dissipative bath coupled to the transmon in the weak interaction limit explored here (i.e. \(\lambda/\kappa_{\text{NR}} \ll 1\)) is further substantiated through two-tone spectroscopy measurements of the transmon \(\omega_{01}\) transition for frequencies around \(\omega_{\text{NR}}\) (figure 6). These measurements show a clear increase in the linewidth \(\gamma\) for \(\omega_{01}\) as \(\Phi\) was tuned through \(\Phi = 0.32 \Phi_0\). The origin of the broadening of the transition can be understood through a quantum noise model [48, 49]—where the transmon–nanoresonator interaction is treated to lowest-order in perturbation theory, yielding a simple relationship between \(\gamma\) and the spectral density of the nanoresonator’s displacement fluctuations, given by (see supplementary material)

\[
\gamma = \frac{\lambda^2}{\hbar^2} (S_x(\omega) + S_x(-\omega)) + \gamma_0, \tag{8}
\]

where \(\gamma_0\) represents contributions to transmon dissipation and dephasing that are assumed to be uncorrelated with the nanoresonator’s fluctuations and constant over the narrow range of the nanoresonator’s response. Here the positive and negative frequency noise components \(S_x(\pm \omega)\) are given by the usual relations [49]

\[
S_x(\omega) = \frac{\kappa_{\text{NR}} \hbar}{\omega_{\text{NR}}} \left( \frac{\kappa_{\text{NR}} (\hbar \omega_{\text{NR}} + 1)}{\omega_{\text{NR}}^2 + \left( \kappa_{\text{NR}} / 2 \right)^2} \right) \tag{9}
\]

and

\[
S_x(-\omega) = \frac{\kappa_{\text{NR}} \hbar}{\omega_{\text{NR}}} \left( \frac{\kappa_{\text{NR}} \hbar \omega_{\text{NR}}}{\omega_{\text{NR}}^2 + \left( \kappa_{\text{NR}} / 2 \right)^2} \right). \tag{10}
\]

\(^{11}\) While the CPW-nanoresonator direct coupling is expected to be negligible for the unitary state evolution of our hybrid system, it is not necessarily negligible for dissipative processes affecting the CPW cavity. Because of the relatively high \(Q\) of the CPW, the interaction with the lossy nanoresonator may present an important dissipative channel for the CPW features. This remains the subject of future work.
For small $T_{SR}$ (i.e. $n_b \ll 1$), such as in figure 6, the nanoresonator should act primarily as a 'cold' bath, preferentially absorbing energy from the transmon. In this limit, equation (8) reduces to

$$\gamma = \frac{\chi^2}{\hbar^2} \left( \frac{\kappa_{NR}}{\omega_{NR} - \omega} \right)^2 + \gamma_0$$

indicating that the frequency dependence of $\gamma$ in this narrow frequency range should be determined by the nanoresonator’s susceptibility. Indeed, a fit of equation (11) to the data in figure 5(b), allowed us to extract $\omega_{NR}/2\pi = 3.47$ GHz, $Q_{NR} = \omega_{NR}/\kappa_{NR} = 150$, and $\lambda/2\pi V_{NR} = 300$ kHz V$^{-1}$, which we used in the numerical simulations of the single-tone spectroscopy (figure 4). Moreover, the observed increase in $\gamma$ around $\omega_{01} = \omega_{NR}$ is consistent with estimates of nanoresonator-induced radiative damping made from a quasilumped-element model of the admittance seen by the transmon (see supplementary material).

We recognize that this simple quantum noise model neglects the influence of higher-level transmon states, which we leave as the subject of future work. Nonetheless, the quantitative agreement between the single-tone spectroscopy data and numerical simulations utilizing these extracted parameter values suggests that the influence of the higher-level states should present minor corrections to this picture.

4. Future prospects and conclusions

Based on the results presented in the previous section, we envision three future directions of research with this novel hybrid quantum electromechanical system, which are attainable with realistic improvements to the engineering of the device. First, this new device, operated in the same weak coupling regime ($\lambda/\kappa_{NR} \ll 1$) demonstrated here, offers prospects for exploring the quantum noise properties of the nanoresonator. Equations (8)–(10) illustrate how the transmon could be used as a spectrometer to resolve the asymmetry in nanoresonator’s quantum noise. With minor modifications to the present device to enable the controlled tuning of $T_{SR}$, the asymmetry between the nanoresonator’s positive and negative frequency noise could be carefully mapped through measurements of the transmon’s $T_1$ and polarization in the vicinity of $\omega_{NR}$. Such measurements could be implemented over a large range of temperatures (from deep in the quantum regime, $n_b \ll 1$, to $n_b \sim 10$)—and because they would not require the simultaneous use of sideband techniques to damp and cool the mechanical mode, would thus provide a complimentary approach to recent experiments in circuit optomechanics studying quantum noise of mechanical systems [3, 5].

A second (and related) direction is the use of the nanoresonator as an engineered reservoir to which the transmon could be controllably coupled for exploring the influence of specially tailored thermal and non-thermal baths. Structured baths that differ from the standard Ohmic form and hence display non-Markovian behavior are currently a subject of considerable theoretical interest, particularly when the environment contains some number of strongly coupled discrete modes [50–55]. Recent experiments have demonstrated the feasibility of characterizing and even actively engineering such non-Markovian environments in optomechanical [56] and cQED systems [57]. Our hybrid system, with the in situ control over component couplings and frequency detuning, would thus be an excellent candidate for further pursuit of such studies, which also can help to pave the way for implementations of controllable meso-nanoscale machines envisioned in the new field of quantum thermodynamics [58].

As a final direction, we envision the development of this system as a platform to explore quantum coherent dynamics of the coupled CPW mode, nanoresonator and transmon. The degree of tunability of both the transmon’s transition energy and the nanoresonator’s coupling energy would provide a versatile set-up for exploring both the resonant and dispersive regimes of interaction between the transmon and the nanoresonator and CPW. With realistic improvements to the transmon–nanoresonator coupling strength (discussed below), along with the strong dispersive coupling between CPW and transmon that is already realized in this device, this system could thus be utilized for exploration of fundamental topics related to quantum information and sensing [27–29], as a new hybrid-system for quantum state-generation in two-resonator cQED [19, 30], and for further development as an element for implementation in future quantum processing circuits.

The use of this hybrid quantum device as a platform for exploring coherent dynamics of coupled mechanical and circuit degrees of freedom will require several engineering upgrades to the nanoresonator and transmon that push the two components fully into the strong-coupling regime. First, improvements can be made to the engineering of the transmon to increase $T_1^*$ by at least a factor of 10. This can be accomplished through two steps: by designing the transmon’s ‘sweet-spot’ in energy, where it is insensitive at first-order to low-frequency flux noise, to be more closely tuned to the nanoresonator frequency than in the present design; and by utilizing an asymmetric junction design, which reduces the transmon’s susceptibility to dephasing due to low-frequency flux noise. Such designs should enable $T_1$, $T_1^* \sim 20 \mu s$ in 2D cQED architectures [59]. Second, the coupling strength to the nanoresonator $\lambda$ can be improved by eliminating the heating due to the DC voltage bias and allowing for application of coupling voltages $V_{NR} \geq 6$ V. This could be achieved either through the use of a sapphire substrate or thin-film silicon nitride layer in the T-filter, both materials of which have been previously used in voltage-biased qubit-mechanical devices without unwanted heating effects for $V \gtrsim 10$ V [21, 22]. Assuming parameters similar to the current device, the application of $V_{NR} = 15$ V, with modest improvement of qubit coherence time to $T_1^* = 10 \mu s$, would yield $\lambda/\gamma \sim 40$, safely within the strong-coupling regime with respect to transmon decoherence. Finally, truly reaching the strong-coupling limit will require improving the nanoresonator quality factor $Q_{NR}$ by at least a factor of 10. To accomplish this will require reducing clamping losses, either through the engineering of ‘free–free’ structures [34], or the use of

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phononic band gaps at the supports [60]. Ultimately, improving the mechanical quality factor could require integrating graphene membranes, which have demonstrated $Q$’s in excess of 100,000 for low-lying flexural modes [61].

In conclusion, we have demonstrated the operation of a new hybrid quantum system that integrates a high-quality superconducting qubit and microwave circuitry with UHF nanomechanics. We have shown through a comparison of spectroscopic measurements and numerical simulations that the system is well-described by a generalized multi-mode Jaynes–Cummings Hamiltonian, with the strongly damped nanoresonator serving as a dissipative bath to the qubit. With realistic improvement to the existing design, we believe this device could soon be compatible with state-of-the-art architecture currently being used in the development of superconducting quantum processors, as well as enable a large range of experiments to study the coherent quantum dynamics and quantum thermodynamics of this complex system.

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