Numerical Simulation of Vertical Pullout of Plate Anchors Embedded in Reinforced Sand

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Abstract. This paper presents a model to predict the ultimate pullout load \( P_u \) for a shallow single vertical plate anchor embedded in sand and fiber reinforced sand for depth \( (H)/\text{diameter (D)} \) ratios of 1.0, 1.5 and 2.0. The model was developed based on literature field test results and wide laboratory investigation. The numerical analysis was performed using the software ABAQUS considering elastoplasticity (Mohr Coulomb - Abaqus CAE) for modelling fiber reinforced sand. The results indicate that this type of fiber reinforced sand significantly increases the \( P_u \) of shallow anchor plates. Based on test results, critical values were discussed and recommended in order to estimate the \( P_u \) for shallow single vertical plate anchor embedded in sand and fiber reinforced sand. The proposed theory was compared against available bibliography of homogeneous material (sand) in field tests or analytical methods. The variability of the results shows that the proposed method is in the range of expected results. For fiber reinforced sand, no method for comparison has been found literature.

Keywords: elastoplasticity, fiber reinforced sand, numerical simulation, plate anchors, pullout load.

1. Introduction

A series of investigations and researches have been carried out to determine the pullout resistance of circular plate anchors placed horizontally in a homogeneous soil medium. Foundations subjected to pullout loadings rely heavily on the passive resistance developed on such elements. Anchors are known to be the best foundation elements to provide such resistance. The development of analytical models for these types of foundations depends on identifying a representative failure mechanism of the surrounding soil. This constitutes one of the major difficulties.

In the literature, the different studies were based on small-scale trials (Das & Seeley, 1975; Murray & Geddes, 1987), the limit equilibrium method (Meyerhof, 1973; Murray & Geddes, 1987; Jesmani et al., 2013), the elastoplastic finite element method (Rowe & Davis, 1982a, 1982b; Andresen et al., 2011; Jesmani et al., 2013), models of centrifugal test (Dickin, 1988; Ovesen, 1981), the stress characteristics method (Subba Rao & Kumar, 1994), the upper limit-boundary analysis (Murray & Geddes, 1989; Kumar, 2001; Merifield & Sloan, 2006; Kumar & Kouzer, 2008; Kouzer & Kumar, 2009) and the lower limit-boundary analysis (Merifield & Sloan, 2006; Merifield et al., 2006; Khatri & Kumar, 2011). Most of these studies were focused mainly on plates embedded in a homogeneous medium of a single layer of soil.

The lack of agreement between the various theories of tensile load capacity is due to the difficulty of predicting the geometry of the rupture zone. In the case of compressive load capacity tensions occur below the foundations in a continuous medium, which is assumed to be homogeneous and isotropic; thus, the zones of rupture are predicted and coherent with the Classical Soil Mechanics (Bhattacharya & Kumar, 2016).

In the majority of earlier studies, a failure mechanism was assumed and the uplift capacity was then determined by considering the equilibrium of the soil mass above the anchor and contained by the assumed failure surface. Based on the underlying assumptions, these methods of analysis are commonly referred to as the “Soil cone” method (Mors, 1959) and the “Friction cylinder” method (Downs & Chieruzzi, 1966). A similar study to that of Pearce (2000) was performed by Ilamparuthi et al. (2002) who conducted a series of laboratory pullout tests on horizontal circular plate anchors pulled vertically in loose to dense sand. A discussion of the observed failure mechanisms, load displacement response and critical embedment depth was also provided. A set of empirical equations were presented for estimating the break-out factors for circular anchors with any friction angle.

At the ultimate pullout load \( P_u \) the tensile load tensions are distributed above the base and their distribution seems to be unique and influenced by the surface of the terrain: the behavior of deep foundations has been generally distinguished from shallow foundations in studies (CIGRÉ, 2008). Various prediction formulas based on an assumed failure mechanism have also been proposed (Vermeer & Sutjjadi, 1985; Trautmann et al., 1985; Murray and Geddes, 1987). More rigorous solutions based on plasticity theory have been presented by Merifield et al. (2001) and Merifield & Sloan (2006). These various prediction methods, based on different assumed failure mechanisms, stress...
distributions and material behaviour, can lead to very different results. The solutions have a wide variation and these kinds of methods do not give a prediction for fiber reinforced sand.

The use of fiber reinforced sand has also attracted considerable interest in recent years in the research area (Silva dos Santos et al., 2010; Consoli et al., 2007a, b; Consoli et al., 2009a, b). Despite that, no work has focused on determining the ultimate pullout load for a plate anchor embedded in fiber reinforced sand.

The studies available in the literature are mainly focused on the capacity of vertical anchors as well as the deformation mechanisms at the soil-plate interface. However, the effect of soil improvement with the use of fiber reinforced sand on the behaviour of a plate anchor has not yet been investigated. Therefore, the aim of this study is to develop more understanding about the behavior of loaded vertical anchor plates embedded in sand and fiber reinforced sand. This is achieved by studying the influence of the reinforcing element parameters, such as its length and diameter on the pullout resistance of the anchor plate. Also, the effect of these parameters is investigated on anchor plates with different geometry, embedment depth and the kind of soil (sand or fiber reinforced sand).

2. Objectives

The purpose of this research is to present the validation of a simple limit equilibrium solution for the vertical pullout resistance of plate anchors embedded in sand and fiber reinforced sand. The limit equilibrium solution is inspired by the formulation of Rowe & Davis (1982b) and Merifield et al., 2003. The use of elastoplasticity (Mohr Coulomb - Abaqus CAE) in the analysis by the Finite Element Method (FEM) for fiber reinforced sand was adopted, following Hibbit et al., 2006, in order to establish the geometry of the mechanism, and the resistance mobilized on the failure planes. The solution is shown to provide agreement with a large database of model test results that have been assembled from the published literature (the solution shows the variation of the results that exist in the literature). Simple design charts are presented. It is shown that plasticity solutions for an ideal frictional material can be conservative. Finite element solutions with a non-associated flow rule can give closer predictions. However, finite element analysis is not routinely used in practice, and simple analytical idealisations of the kind described in this paper remain the principal tool used by designers.

3. Problem Definition

The experimental program was carried out in two parts. First, all the description of the pull of circular anchor plates embedded in sand and fiber reinforced sand test was developed in the experimental program by Consoli et al. (2012, 2013) such as the characterization of the backfill material (the configuration made for the test follows the instructions of ASTM D 1194, 1994). The foundation consists of a steel cable and a rigid circular steel plate 300.0 mm in diameter and 25.4 mm thick. Then a numerical 2D study was conducted using the source code of the commercial finite-element package ABAQUS, with further development to simulate the deformation analysis.

A basic explanation of the problem is now provided. The anchor plate is placed at a distance H measured from the top of the backfill and the diameter of the anchor plate is B as shown in Fig. 1. The thickness of the anchor is assumed to be negligible compared to its width. The soil mass
is perfectly plastic and is assumed to obey an associated flow rule. The Mohr-Coulomb failure criterion is applicable for the analysis. The objective of this work is to find the ultimate pullout load \( (P_u) \) per unit of length of the plate anchor for different values of \( H/D \) (1.0, 1.5 and 2.0) where the direction of the pullout is perpendicular to the anchor plate. The ultimate pullout capacity of the plate anchor is determined for the sand and fiber reinforced sand.

4. Elastoplasticity (Mohr Coulomb - Abaqus Cae)

The non-associative Drucker-Prager model (Drucker & Prager, 1952) was implemented in this work. Since ABAQUS (Hibbit et al., 1998) requires parameters in the \( p-t \) coordinate system, calculations need to be performed to transfer soil parameters from the \( t-s \) to the \( p-t \) coordinate system. The following explanations will summarize references (Drucker & Prager, 1952 and Chen, 1982) which provide detailed explanation for obtaining parameters for the Drucker-Prager soil model.

The Mohr-Coulomb failure criterion is an elastoplastic constitutive model that is controlled by the laws of yield, flow, and expansion (hardening). The Mohr-Coulomb constitutive model, which can be provided by the Abaqus program, has some requirements that are important when studying or using the model in geotechnical problems (Hibbit et al., 2006). The requirements are as follows:

- Stresses and deformations do not depend on time.
- The material studied should be isotropic.
- The material is expanding (hardening) or softening isotopically. The model does not use an equation that controls the expansion of the material, but this is controlled by the user through the control of the cohesion with the plastic deformation.
- Cohesion has two functions, i) first, the yield function where the resistance parameter of the material is known by the traditional Mohr-Coulomb constitutive model; ii) second, the function of the plastic potential where the stress is controlled when plastic deformations called Cohesion yield stress are generated.

The formulation of the Mohr-Coulomb failure criterion that is implemented in Abaqus is a function of three stress invariants and state parameters presented in the following equations. The Mohr-Coulomb yield criterion is associated with the flow function by forming a yield surface in Haigh-Westergaard co-ordinates, expressed in terms of principal stresses (Hibbit et al., 1998) by.

\[
F = Rmc \times q - p \tan(\phi) - c \tag{1}
\]

where \( F \) is the yield surface; \( Rmc \) is the Mohr-Coulomb deviatoric stress measure that is a function of the angle of Lode \( (\theta) \) and the internal friction angle of the material \( (\phi) \) according to Eq. 2; \( q \) is the second stress invariant that Abaqus calls equivalent Von Mises stress presented in Eq. 6; \( p \) is the hydrostatic stress being the first stress invariant that is in Eq. 4; \( c \) is the cohesion of the material; and \( \phi \) is the internal friction angle of the material.

\[
Rmc(\theta, \phi) = \frac{1}{\sqrt{3}} \cos \phi \left( \frac{\pi}{3} \right) + \frac{1}{3} \cos \left( \frac{\pi}{3} + \tan \phi \right) \tan \phi \tag{2}
\]

where \( \theta \) is the angle between the stress path of the material and the main stress present in Eq. 3 and \( r \) is the third stress invariant presented in Eq. 7.

\[
\cos(\theta) = \frac{r^3}{q} \tag{3}
\]

\[
p = \frac{-(\sigma_{11} + \sigma_{22} + \sigma_{33})}{3} \tag{4}
\]

The Von Mises equivalent stress \( (q) \) and the third invariant \( (r) \) are in function of the deviatoric stress \( (S) \). In Eq. 5 the variables in terms of stress tensors are expressed

\[
\sigma = S - pl \tag{5}
\]

\[
q = \sqrt{\frac{3}{2} (S : S)} \tag{6}
\]

\[
r = \frac{\sqrt{9/2 (S : S)^2 - p \tan \psi}}{\tan \psi} \tag{7}
\]

The constitutive model is characterized by a non-associated plasticity where there is no equality between the yield function \( (F) \) and that of the plastic potential \( (G) \). The plastic potential function of Eq. 8 defines the direction of the plastic deformations that are parallel to the surface of the plastic potential.

\[
G_1 = \sqrt{(E \cdot c_s \tan \psi)^2 + (Rmw \times q)^2 - p \tan \psi} \tag{8}
\]

where, \( E \) is the eccentricity that controls the deformability of the function \( (G) \) in the meridional plane \( (Rmw-q) \) and approaches the asymptotic line. The Abaqus software defines the default \( (E = 0.1) \). The meridional plane represents a cut of the surface of the plastic potential in which the directions of the plastic deformations are perpendicular to the surface.

The \( c_s \) is the initial cohesion yield stress, \( \psi \) is the angle of dilatancy that relates the volumetric and shear deformation in the plastic range, different to the angle of friction due to the selection of the non-associated flow (Houlisby, 1991). \( Rmw \) is the elliptic function presented by Menêtre & Willam (1995) that generates the concave shape to the function of the plastic potential by means of the Lode angle \( (\theta) \) and the variable \( (e) \) called the out-of-roundedness parameter. The variable \( (e) \) allows smoothing the function that governs the surface of the plastic potential.

\[
e = \frac{3 - \sin \phi}{3 + \sin \phi} \tag{9}
\]
The model hardening law is controlled by the cohesion parameter under confinement pressure and the load level of the test.

5. Finite Element Simulation

Numerical simulations have been often used to analyse various types of geotechnical models in pullout testing using the finite element approach (Susila et al., 2003; Song et al., 2008; Bhattacharya & Kumar, 2014 and Bhattacharya & Kumar, 2016). However, there are not models that have been used to investigate the pull out of circular anchor plates embedded in fiber reinforced sand.

Steel plates with tensile stresses can be solved in axissimetry, planar or three-dimensional, but in this research they will be solved in axissimetry (Fig. 2a) due to the ease provided and the type of model. Loading by prescribed displacement was chosen (due to being a facilitator while the model converged), with subsequent response to the reaction force.

It is assumed that the steel plate is in perfect contact with the ground at the beginning. The interaction between the plate and the soil is simulated using interface elements, with coefficient of friction of 0.30 for the interaction between the materials. This type of interface is able to reproduce the Coulomb-type frictional interaction between the surface of the plate and the ground in contact (Hellwany, 2007).

The extent of the mesh must be large enough to prevent discrepancies due to the boundary conditions. Thus, a minimum spacing of 11 times the diameter of the plate was adopted, and 3 times the length at depth (Fig. 2b). Bhattacharya et al. (2008) adopted a lateral distance and depth of 5D, relative to the center of the foundation, for their sand simulations. Consoli et al. (2007c) and Ratley et al. (2008),

\[
R_{mc}\left(\frac{\pi}{3}, \varphi\right) = \frac{(3 - \sin \varphi)}{(6 \cos \varphi)}
\]

\[
R_{mv}(\theta, e) = \left(\frac{4(1 - e^2)(\cos \theta)^2 + (2e - 1)^2}{2(1 - e^2)(\cos \theta) + (2e - 1)^2})\left(\frac{4(1 - e^2)(\cos \theta)^2}{5e^2 - 4e}\right) R_{mc}\left(\frac{\pi}{3}, \varphi\right)\right)
\]

Figure 2 - Axisymmetric analysis a) Parts of the model b) Dimensions of the model.
in their numerical simulations, adopted a radius and depth of 3D for sand-fiber backfills. The extent of the mesh is sufficiently large to avoid discrepancies with the boundary conditions according to the Fig. 2b.

After the step of applying the initial stress state, one must check the value of vertical displacements, which should tend to zero. It is suggested a tolerance around $1 \times 10^{-5}$ m.

The Fig. 2a and 2b show the finite element model for the analysis. Both the soil and the plate anchors are modelled using four node axisymmetric elements (CAX4 ABAQUS element) and axisymmetric pore pressure elements (CAX4P ABAQUS element) are also used but only in the soil, but the pore pressure will not be considered. The structured mesh of isosquare type elements in the entire model. In order to optimize the processing time and reduce the possibility of errors, the finite element mesh is more concentrated in the landfill area. This same procedure was also adopted by Mántaras (1995), Thomé (1999), Consoli et al. (2007c) and Ratley et al. (2008).

The base of the model is restricted in the X and Y directions, while the sides are restricted only in the X direction. The boundary conditions for the displacement constraints are shown in Fig. 2b.

The elasto-perfectly plastic Mohr-Coulomb constitutive model was adopted for the natural soil and sand, in which the input parameters are relatively easy to obtain, and the results would show good agreement with the field results.

In the fiber reinforced sand numerical model the Mohr-Coulomb hardening option was used; this option is used to define the linear behavior of the material in hardening/softening (Abaqus, 2010).

All analyses were performed in the isotropic condition ($K = 1.0$). According to Burd & Frydman (1997) and Thomé (1999), the consideration of different anisotropic conditions ($K \neq 1.0$) for surface foundations submitted to compression did not show differences from the results obtained with the elasto-perfectly plastic model.

Three steps were determined for the algorithm sequence: a) Initial, the numerical insertion of the initial stress state, its boundary conditions, and contact properties between the different parts is done; b) Geostatic, the geostatic control is activated, which verifies if the geostatic stresses applied in the previous step caused significant deformations; c) Loading, in this step the displacement is gradually applied in the steel tube, and the reading of reaction forces and displacements are checked.

5.1. Material properties of the Analysis

5.1.1. Sand

The sand used in this study comes from a deposit located in the municipality of Osório-Rio Grande do Sul. This material is characterized as a fine sand (NBR 6502-ABNT, 1995 and ASTM-D 2487, 1993), being uniform, fine, non-plastic, with grain specific gravity 26.5 kN/m$^3$ and average particle size 0.16 mm (Cruz, 2008 and Dalla Rosa, 2009).

The geotechnical parameters of cohesion ($c$) and the angle of friction ($\phi$) were obtained directly from the triaxial test of Festugato (2008) and Santos (2008). In terms of modulus of elasticity, Thomé (1999) defined as the secant value for a 0.1% deformation. This same criterion was adopted in this work. Alternatively, the modulus of elasticity ($E$) can be obtained by correlation with the shear modulus ($G$) by the elasticity theory relations as shown in Eq. 12, where a shear modulus of 20.0 MPa is obtained according to Consoli et al., 2013.

$$E = 2(1 + \nu)G$$  \hspace{1cm} (12)

In the preliminary analyses, the value of Poisson ratio ($\nu$), between 0.2 and 0.4, was varied for both the natural soil and the material of the landfill, and no influence was verified on the stress vs. relative displacement curves for the adopted range. Thus, an average value of 0.3 was considered for all materials involved. These considerations are consistent with the results of Rowe & Booker (1981) who verified that, for a homogeneous soil, there is no variation in the displacement for a variation of $\nu$ between the values 0.0 and 0.5. Cudmani (1994) verified the same result for his analyses of foundations subject to compression.

The dilatation curves (stress ratio - $q$ vs. dilatation - $\delta e_v / \delta e_s$) for the triaxial sand tests are presented by Festugato (2008) and Santos (2008). The dilatancy angle $\Psi$ can be obtained directly through these curves and through Eq. 13.

$$\tan(\Psi) = \frac{\delta e_v}{\delta e_s}$$  \hspace{1cm} (13)

where $\delta e_v$ is the volumetric deformation and $\delta e_s$ is the shear deformation.

As $\varphi \neq \psi$ (non-associated flow) the stiffness matrix is non-symmetric. It is necessary to use the “Unsymmetric Matrix Storage” option in ABAQUS (Abaqus, 2010).

5.1.2. Sand with fiber

The fibers used consist of polypropylene monofilaments of 50.0 mm in length and 0.01 mm in diameter, relative density of 0.91, tensile strength of 120.0 MPa, modulus of elasticity of 3.0 GPa and deformation at rupture of 80.0%. The fiber content used was 0.50% of the weight of the dry sand.

From the results of the triaxial tests, performed under different effective confinement stresses for a sand with fiber with 50.0% of relative density, the resistance parameters of the analyzed mixtures, angle ($\varphi'$), and cohesive intercept ($c'$), are defined through their rupture envelope (Festugato, 2008) and the value adopted in the initial numerical model is presented in Table 1.
In terms of modulus of elasticity, the same criterion of the sand was adopted in this work. The internal friction angle values of the blends are not influenced by the fiber aspect ratio. The rupture envelopes are parallel. In contrast, the values of cohesive intercept of the composites are strongly influenced by the aspect ratio of reinforcements. The higher the aspect ratio, the greater the cohesive intercept (Festugato, 2008).

The same criterion of the sand was used for obtaining the value of $\sigma_{11}$ (Cudmani, 1994) and the dilation angle (Festugato, 2008).

5.1.3. Natural soil

The excavated soil that served as the base for the execution of the load tests is of the homogeneous residual type, originating from the decomposition of basaltic rocks (igneous) and sandstones (sedimentary). Standard penetration test (SPT) was performed in the experimental field by Lopes Jr. and Thomé, 2005.

Dalla Rosa et al. (2004a, b) conducted a geotechnical investigation along the profile to a depth of 5.0 m to determine the physical properties and indices along the depth (moisture, specific gravity, particle size distribution and limits of liquidity and plasticity).

Considering the geotechnical parameters presented above (granulometry and Atterberg limits), the soil of the experimental field can be classified as an A-5-7 soil (silt-clay soil) by the American Association of State Highway and Transportation Officials (AASHTO) and CL (low to high liquidity clay) by the unified soil classification system (USCS).

Dalla Rosa et al. (2004a, b) performed oedometric tests in the natural and flooded conditions, simple compression tests and Thomé et al. (2005) carried out consolidated drained (CD) triaxial tests.

In addition to the geotechnical characterization of the experimental field, Dalla Rosa et al. (2004a) performed compressive load tests on steel plates with diameters of 30, 60 and 90 cm and set at a depth of 80.0 cm. Lopes Jr. & Thomé (2005) performed six static load tests on excavated cuttings (three of which were isolated with styrofoam), with a diameter of 25.0 cm and drilled between the depths of 3.86 and 4.70 m.

5.1.4. Steel

For the steel plate, the technical characteristics of the type of steel used in its manufacture were used (Souza, 1974).

Table 1 provides a summary of the calibrated material properties that were used in the present study.

### 6. Results and Comparison

#### 6.1. Results of the model

Figure 3 shows the comparison of the results of the experimental tests with the numerical simulation of vertical pullout of plate anchors embedded in sand and fiber reinforced sand.

The ultimate pullout load ($P_u$) of plate anchors embedded in sand was 5.0 kN and in fiber reinforced sand was 4.1 kN, from the numerical modeling was 5.1 kN and 5.1 kN respectively for a displacement of 1.5 mm and 6.0 mm in the cases of a $H/D$ ratio of 1.5 and 1.0 respectively with a steel plate of 30.0 cm. There is a difference in $P_u$ of 0.1 kN that is approximately 2.0% higher in the numerical model than in the field test, which is considered a satisfactory result.

Figure 4a presents the initial stress state for an embedded sand of $H/D$ ratio of 1.5. The S, S22 (stress components at integrations points) view helps to show that points A, B and C meet the state of initial stress. Hence, it can be inferred that all the points in the model meet the state of initial stress. Figure 4b, for an embedded fiber reinforced sand with $H/D$ ratio of 1.0, shows the U, U2 (spatial displacement at nodes) view that helps check the initial displacements. In the same figure, it is shown that points A, B and C meet the state of initial displacements with displacements close to zero. Therefore, all the model satisfactorily meets the initial displacement expected conditions.

The displacement occurring during the initial step is not due to the external load, but it is due to the difference between initial stresses predicted in the computational program by the user and the converged stresses calculated by ABAQUS which are in equilibrium with the external load.
The verification of the state of initial stresses and the initial displacements helps check that the numerical model behaves close to reality.

In Figure 5 the largest displacements ($U$) are observed in the soil that is on top of the upper face of the steel plate. In this case the simulation was done with continuous elements and that is why the solid does not present cracks. However, the displacement gradient indicates the probable location of the rupture surface, which can be observed with a frustoconical shape. The generatrix of the failure surfaces in the case of fiber reinforced sand forms a larger angle with the vertical compared to the sand case. From the same figure we can see that a displacement of 2.0 mm was used for the sand model and for the fiber reinforced sand model a displacement of 6.0 mm.

The stresses and deformations are developed in several directions and one way of presenting these stresses is to combine them in the so called equivalent von Mises ($S$). In three-dimensional models the combination of the six stress components in a single equivalent stress is related to the real stress system (Abaqus, 2010). Von Mises stresses or equivalent stresses are concentrated in the landfill zone as shown in Fig. 6 and they increase as the $H/D$ ratio increases. It can be seen that in the case of sandy soil the stresses are distributed more randomly than in the case of fiber reinforced sand in which they concentrate in the direction of failure. In general, reinforced soils (sand-fiber) reach higher values of stress than in sandy soil.

The plastic deformation at the integration points ($E_p$) is a scalar variable used to present the non-elastic deformation of the material. When the variable is greater than zero, it means that the material has yielded and when the variable is less than zero it means that the material is still in elasticity (Abaqus, 2010). The blue color zone in Fig. 7 indicates that...
the material has elastic behaviour; it can be observed that
the plastification in all cases is concentrated in the elements
that are near the upper corner of the plate and follow the tra-
jectory of the rupture surface obtained experimentally.

6.2. Comparison

Figure 8a shows the ultimate pullout load ($P_u$) for
each $H/D$ ratio of the numerical models and the field tests.
It can be seen that in the comparisons of the $P_u$ for the fiber
reinforced sand, the $H/D$ ratio of 1.0 and 1.5 presents very
similar strength gains with a difference of 0.1 kN and
0.2 kN respectively. Additionally, the difference of $P_u$ for
the sand was 0.4 kN and for the fiber reinforced sand was
0.1 kN for $H/D$ ratio of 1.0. For the $H/D$ ratio of 1.5 the
difference of $P_u$ for the sand was 0.1 kN and for the fiber re-
inforced sand was 0.2 kN; nevertheless, for the $H/D$ ratio of
2.0 there is a considerable difference since for both sand
and fiber reinforced sand we have a difference of 1.4 kN. In
other words, there are variations between the numerical
model and the field test from 0.1 to 0.4 kN for a $H/D$ ratio of
1.0 and 1.5 in sand and fiber reinforced sand, but in the $H/D$
ratio of 2.0 we have a variation of 1.4 kN.

In Fig. 8a the comparison of the strength gains be-
tween the sand and the fiber reinforced sand in field tests
for a $H/D$ ratio of 1.0 show 2.4 kN and 4.1 kN respectively,
which is an approximate gain of 40.0%. The numerical

Figure 4 - Initial conditions of the numerical model a) Initial stress state in an embedded sand for $H/D = 1.5$ b) Initial displacements in an embedded fiber reinforced sand for $H/D = 1.0$. 
model, with the resistance parameters adjusted for the same relation, has 2.8 kN in a sand backfill and 4.0 kN in a fiber reinforced sand backfill which is a gain of approximately 30.0%. In the same figure, the \( H/D \) ratios of 1.5 and 2.0 for a field test and numerical model show a gain resistance of approximately 30.0%. In other words, in an arithmetic mean there are resistance gains from 35.0 to 40.0% for fiber reinforced sand backfill in the field tests and in the numerical model.

Although Fig. 8a showed 35.0 to 40.0% strength gain, when the analysis considers the ratio of the \( P_s \) reinforced against \( P_f \), a tendency of strength gain decrease is observed (Fig. 8b). Figure 8a showed the gain of \( P_f \) with the use fiber reinforced, and the Fig. 8b showed that the gain rate decreased with the increase of the \( H/D \) ratio.

**Figure 5** - Displacements \( (U, \text{mm}) \) for an embedded sand and fiber reinforced sand.
7. Estimation of the Ultimate Pullout Capacity of Plate Anchors

The ultimate pullout load \( P \) of a steel plate in sand is generally expressed as a function of the landfill weight \( (\gamma) \) and the depth of the plate anchor \( (H) \) as shown in Eq. 14 (Rowe & Davis, 1982b; Merifield et al., 2003). There are not other equations in the literature that refer to a fiber reinforced sand backfill.

\[
q_u = \gamma \times H \times N_\gamma \tag{14}
\]

The pullout factor \( N_\gamma \) was obtained from numerical modeling with Abaqus for \( H/D \) ratios of 1.0, 1.5 and 2.0 and for different friction angles \( (\phi) \) and for a sand and sand-fiber backfill as shown in Figs. 9a and 9b. The presented figures can be used for steel plates of different dimensions, but between the \( H/D \) ratios of 1.0, 1.5 and 2.0 in order to de-
termine the $P_e$ value in sand and fiber reinforced sand embankments from the friction angle of the material.

7.1. Comparison

Figure 10a shows the Pullout factor values obtained by FEM and some methods presented in the literature for $H/D$ ratios of 1.0, 1.5 and 2.0 at a friction angle ($\varphi$) of 30.0° in sand backfill. The use of this friction angle was only for comparing the present numerical model with other methods in the literature; in addition, the methods used for estimating the pullout factor were not used for fiber reinforced sand backfill. The different factors $N_\gamma$ were obtained from the literature and this factor depend of the $H/D$ ratio, $\gamma$ and $\varphi$ of the material backfill.
Of the nine remaining methodologies employed in the
Fig. 10a, five methods (Balla, 1961; Meyerhof & Adams,
1968; Murray & Geddes, 1987; Saeedy, 1987; Merifield
et al., 2003) were extremely conservative in comparison
with the proposed method. Three methods (Sarac, 1989;
Ghaly & Hanna, 1994; Grenoble method in Biarez & Baraud,
1968; Martin, 1963 & 1966) presented overestimates for all
load tests. The Hanna model (Hanna et al., 2007) presented
the best estimates, with differences between 18.0% for a
$H/D$ ratio of 1.0, 7.0% for a $H/D$ ratio of 1.5 and 10.0% for a
$H/D$ ratio of 2.0 compared to the proposed model.

Also for sand backfill, Fig. 10b presents a comparison
between the values of the load tests done in the laboratory
or field by different authors where five authors (Murray &
Geddes, 1987 for a friction angle ($\varphi$) of 44.0°; Ilamparamuthi
et al., 2002 for a $\varphi$ of 43.0°; Balla, 1961 for a $\varphi$ of 38.0°;
Ghaly et al., 1991 for a $\varphi$ of 30.0°; Baker & Kondner, 1966
for a $\varphi$ of 42.0°) were extremely conservative when com-
pared with the proposed method. Two papers (Ghaly et al.,
1991 for friction angles ($\varphi$) of 35.0° and 40°; Kwasniewski
et al., 1975 for a $\varphi$ of 28.0°) presented overestimates for all
load tests. The field tests f Bemben & Kupferman (1975)
for a $\varphi$ of 46.0° and Ruver (2011) for a $\varphi$ of 39.2° presented
the best estimates, with differences of 0.40% for a $H/D$ ratio
of 2.0 in Bemben & Kupferman (1975) and the proposed
method. For the Ruver (2011) tests, there is a difference of
16.9% for a $H/D$ ratio of 1.0, 3.0% for a $H/D$ ratio of 1.5 and
16.6% for a $H/D$ ratio of 2.0 compared to the proposed
model.

Figure 8 - Comparison in different $H/D$ ratios of (a) the ultimate pullout load ($P_u$) (b) the relationship $P_{u,\text{reinforced}}$ and $P_{u,\text{field}}$. 

![Graph](image-url)
8. Conclusions

The outcomes from this work can be summarized as follows:

- The present work introduces a method for designing and analyzing vertical pullout of plate anchors embedded in sand and fiber reinforced sand. There are still divergences in determining the pullout factor ($N_f/c_{f103}$). Hence, further investigations are needed to clarify this basic but important topic. The present work emphasizes the criterion for determining the $N_f/c_{f103}$ of plate anchors in sand and fiber reinforced sand. After a review of current studies on this topic, the criterion for determining $N_f/c_{f103}$ based on finite element analysis is recommended. This criterion is validated firstly by model tests and then applied to circular plate anchors with different embedment ratios in both sand and fiber reinforced sand to calculate bearing capacity factors.

- The proposed methodology conforms to two capacity factors. The first, the value of $N_f/c_{f103}$ can be obtained by using a sand backfill. The second, to be obtained by using a fiber reinforced sand backfill can be generally applied for different $H/D$ ratios. The ultimate pullout load ($P_u$) in the present analysis is definitely recommended, in which a non-associated flow ($\phi \neq \psi$) was used.

- The numerical model for a sand backfill had an approximate difference of $\pm 2.0\%$ from the ultimate pullout load ($P_u$) in the field test for a $H/D$ ratio of 1.5 which shows that the model reproduced with satisfactory accuracy the result obtained in real scale. In the same way, the numerical model calibrated for a fiber reinforced sand backfill had an approximate difference of $\pm 3.0\%$ from the $P_u$ in the field test for a $H/D$ ratio of 1.0.
By comparing the resistance gains in the numerical models of sand and fiber reinforced sand backfill with those from the field tests for a $H/D$ ratio of 1.0, 1.5 or 2.0, the average resistance gain is about 40.0% to 35.0%. The gain rate decreases with the increase of the $H/D$ ratio.

Although the $H/D$ ratio varied within a limited range of 1.0, 1.5 and 2.0, the simulations and their respective results for the ultimate pullout load ($P_u$) represent an important step towards understanding the application of this approach in sand and fiber reinforced sand backfills.

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Figure 10 - Comparison of pullout factor for embedded sand in a) methods of the literature and b) field tests.

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**List of Symbols**

\( c \): Cohesion of the material

\( c_2 \): Cohesion that controls the cohesion Yield Stress

\( CAX4 \): Four node axisymmetric elements

\( CAX4P \): Four node axisymmetric pore pressure elements

\( CD \): Consolidated drained

\( D \): Diameter of the plate anchor (m)

\( e \): Diverting eccentricity

\( E \): Modulus of elasticity

\( E_1 \): Meridian eccentricity

\( E_2 \): Plastic deformation at the integration points

\( FEM \): Finite Element Method

\( F \): Flow function

\( G \): Shear modulus

\( G_1 \): Plastic potential

\( K \): Isotropic condition

\( H \): Depth of the plate anchor (m)

\( I \): Stress invariant

\( N_y \): Pullout factor

\( P_u \): Ultimate pullout load (kN)

\( p \): Hydrostatic stress

\( q \): Equivalent Von Mises Stress

\( p-t \): Coordinate system

\( r \): Third invariant

\( Rmc \): Measure of the surface formation

\( Rmw \): Elliptic function

\( S \): Von Mises

\( S_1 \): Deflection effort

\( SPT \): Standard penetration test

\( t-s \): Coordinate system

\( U \): Displacements

\( X \): Horizontal axis

\( Y \): Vertical axis

\( \delta \): Small change of deformation

\( \varepsilon \): Shear deformation

\( \varepsilon_1 \): Volumetric deformation

\( \theta \): Lode angle

\( \nu \): Poisson Ratio

\( \sigma \): Axial stress

\( \varphi \): Internal friction angle of the material

\( \psi \): Angle of dilatancy

\( \gamma \): Unit weight