Passive Channel Gain Estimation Between Primary Transceivers in Cognitive Radio Networks

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Abstract—In cognitive radio networks, the channel gain between primary transceivers is crucial for a cognitive transmitter (CT) to control the transmit power and achieve the coexistence. To obtain the channel gain, a backhaul between the primary system and the CT is needed. However, the backhaul is usually unavailable in practice. To deal with this issue, two passive estimators are proposed for the CT to obtain the channel gain between primary transceivers in this paper. In particular, by sensing primary signals, a maximum likelihood (ML) estimator is developed for the CT to obtain the channel gain between primary transceivers. After demonstrating the high computational complexity of the ML estimator, a median based (MB) estimator with low complexity is proposed. Furthermore, the estimation accuracy of the MB estimation is theoretically characterized. By comparing the ML estimator and the MB estimator from the aspect of computational complexity as well as the estimation accuracy, both advantages and disadvantages of two estimators are analyzed. Numerical results show that the estimation errors of the MB estimator and the ML estimator can be as small as 0.4 dB and 6 dB, respectively. Otherwise, the MB estimator is superior to the ML estimator in terms of the estimation error if the signal to noise ratio (SNR) of the sensed primary signals at the CT is no smaller than 4 dB. Therefore, compared with the OSA, the MB estimator outperforms the other one in two ways. One is the opportunistic spectrum access (OSA) [3] and the other one is the spectrum sharing (SS) [4], [5]. In the OSA, cognitive users are allowed to access the spectrum only if the spectrum is idle, and have to withdraw from it as soon as possible once the spectrum is re-occupied. In the SS, cognitive users are able to access the spectrum even when the spectrum is occupied, provided that the co-channel interference inflicted to the primary receiver (PR) does not violate the maximum interference power constraint, namely, interference temperature. Therefore, compared with the OSA, the SS is able to exploit more spectrum opportunities, obtain higher spectrum utilization efficiency, and thus achieve higher cognitive throughput [6], [7], [8].

To achieve the SS between primary and cognitive users, the information of the interference temperature is crucial for the cognitive transmitter (CT) to control the transmit power and protect primary transmissions. Briefly, the interference temperature is highly related to the channel gain between the primary transceivers. Within a quality of service (QoS) guaranteed primary system, the primary transmitter (PT) automatically adapts its transmit power to satisfy a target signal to interference plus noise ratio (SINR or SNR) at the PR or equivalently a target transmission rate. A large channel gain between the primary transceivers means that the target SINR between the primary transceivers can be easily satisfied, even when the transmit power of the primary signals is small. Under the maximum transmit power constraint at the PT, the primary transmission is able to tolerate a strong interference signal meanwhile achieving the target QoS by increasing the transmit power of the primary signals. This leads to a large interference temperature and contributes to a high cognitive throughput. On the contrary, if the channel gain between the primary transceivers is small, a large transmit power of the primary signals is required to satisfy the target QoS. Then, to achieve the target QoS of the primary transmission, only a weak interference signal can be tolerated, even when the PT works with the maximum transmit power. This leads to a small interference temperature and compromises the cognitive throughput.

In fact, the interference temperature is calculated in the primary system with the channel gain between the primary transceivers. Consider the scenario that the PT is transmitting data to the PR with a target SNR $\gamma_T$. If the PR is interfered by cognitive signals and the target SNR $\gamma_T$ at the PR cannot be satisfied, even when the PT works with the maximum transmit power $P_{\text{max}}$, an outage of the primary transmission is claimed. In general, a specific $\gamma_T$ corresponds to a certain wireless service in the primary system and requires a preset maximum outage probability $\Theta$. Thus, an interference temperature $p_I$ is imposed on the transmit power of the CT to protect the primary transmission. Mathematically, we have

$$\Pr \left\{ \frac{P_{\text{max}} g_0 |h_0|^2}{\sigma^2 + p_I} < \gamma_T \right\} = \Theta, \quad (1)$$

I. INTRODUCTION

Cognitive radio technique is a promising candidate to deal with the spectrum shortage problem in the wireless communication [1], [2]. By coexisting with the primary users on the under-utilized licensed spectrum band, cognitive users enhances the utilization efficiency of the spectrum band meanwhile leverages cognitive throughput. In general, cognitive users are able to achieve coexistence with the primary users in two ways. One is the opportunistic spectrum access (OSA) [3] and the other one is the spectrum sharing (SS) [4], [5]. In the OSA, cognitive users are allowed to access the spectrum only if the spectrum is idle, and have to withdraw from it as soon as possible once the spectrum is re-occupied. In the SS, cognitive users are able to access the spectrum even when the spectrum is occupied, provided that the co-channel interference inflicted to the primary receiver (PR) does not violate the maximum interference power constraint, namely, interference temperature. Therefore, compared with the OSA, the SS is able to exploit more spectrum opportunities, obtain higher spectrum utilization efficiency, and thus achieve higher cognitive throughput [6], [7], [8].
where \( g_0 \) is the large scale channel gain between the primary transceivers, \( h_0 \) denotes the small scale fading coefficient and follows a Rayleigh distribution with unit mean, and \( \sigma^2 \) represents the power of the additive white Gaussian noise (AWGN). From (1), the interference temperature \( p_I \) can be calculated as

\[
p_I = \frac{-p_{\text{max}} g_0 \ln(1 - \Theta)}{\gamma_T} - \sigma^2,
\]

which is a function of the channel gain \( g_0 \) between the primary transceivers.

In the recent years, SS has been studied extensively \[9\], \[10\], \[11\], where the interference temperature of the primary system is usually assumed to be known to the CT for the power control. However, to obtain the interference temperature from the primary system, a backhaul between the primary system and the CT is needed. Then, the primary users can transmit the information of the interference temperature to the CT through the backhaul. In practice, there may not exist any backhaul between the two systems. Thus, it is challenging for the CT to obtain the interference temperature and achieve SS in such situation.

To deal with this issue, we intend to estimate the channel gain \( g_0 \) between the primary transceivers, such that the CT can calculate the interference temperature with (2), in which \( p_{\text{max}} \) is a typical value of a PT and can be known as a prior knowledge at the CT. \( \gamma_T \) can be known at the CT by observing the modular and coding scheme (MCS) of the primary signal \[13\], \( \Theta \) corresponds to a specific \( \gamma_T \) and can be known by the CT once \( \gamma_T \) is obtained, and \( \sigma^2 \) is the power of the AWGN and is also available at the CT. After being aware of the interference temperature, the CT is able to control its transmit power to achieve the SS without any backhaul from the primary system. In principle, within the QoS guaranteed primary system, the primary signal is carefully designed based on the channel gain between the primary transceivers. In particular, if the channel gain between the primary transceivers is strong, the PT is able to satisfy the target QoS with a small transmit power of the primary signal. Otherwise, the PT increases its transmit power to compensate for the target QoS. In other words, the primary signal contains some information of the channel gain between the primary transceivers. Thus, it is possible for the CT to estimate the channel gain between the primary transceivers by sensing the primary signals.

In this paper, we develop two passive estimators, namely, a high-complexity maximum likelihood (ML) estimator and a low-complexity median based (MB) estimator, for the CT to obtain the channel gain between the primary transceivers. Numerical results show that the estimation errors of the ML estimator and the MB estimator can reach as small as 0.6 dB and 0.7 dB, respectively. Meanwhile, the ML estimator outperforms the MB estimator in terms of the estimation error if the SNR of the received primary signals at the CT is no smaller than 4 dB. Otherwise, the MB estimator is superior to the ML estimator from the aspect of both the computational complexity and the estimation accuracy. To our best knowledge, this is the first work considering unknown interference temperature at the CT and estimating the channel gain between primary transceivers for the SS. For clarity, we summarize the contributions of this paper as follows.

- By enabling the CT to sense primary signals and adopting the maximum likelihood criterion, we develop a ML estimator for the CT to obtain the channel gain between primary transceivers. In particular, the ML estimator is obtained by solving a nonlinear equation. To shed more light on the estimator design, we study the property of the nonlinear equation and develop a bisection method to solve it. In addition, we analyze the computational complexity of the ML estimator.

- After demonstrating the high computational complexity of the ML estimator, we develop a MB estimator with proved low complexity. By denoting \( K \) as the number of the sensed primary signals, we derive both the lower and upper bounds of an estimation under a certain probability. In particular, the probability is expressed as a function of \( K \) and monotonously increases as \( K \) grows. We also study the performance limit of the MB estimator by approaching \( K \) to the infinity. Furthermore, we analyze the computational complexity of the MB estimator.

- By comparing the ML estimator and the MB estimator from the aspect of computational complexity as well as the estimation accuracy, both advantages and disadvantages of two estimators are analyzed. Numerical results verify our theoretical analysis.

II. System Model

Fig. 1 provides the system model, which consists of a PT, a PR, and a CT. In particular, the PT is transmitting data to the PR on a wireless channel. Meanwhile, the CT intends to estimate the channel gain between the primary transceivers for SS. In what follows, we will present the channel model and signal model in the considered system, respectively.

A. Channel Model

Block fading channels are considered among three users. Specifically, if we denote \( h_0 (h_1) \) and \( g_0 (g_1) \) as the block fading and the path-loss coefficients between the PT and the PR (CT), the channel between the PT and the PR (CT) is


\[ h_0 \sqrt{g_0} (h_1 \sqrt{g_0}) \]. On one hand, the block fading coefficient \( h_i \) (i = 0, 1) follows a Rayleigh distribution with unit mean. In particular, \( h_i \) (i = 0, 1) remains constant within each block and varies independently among different blocks. On the other hand, the path-loss coefficient \( g_i \) (i = 0, 1) is determined by the path-loss model \[14]\]

\[ P_i(d_i) = 128 + 37.6 \log_{10}(d_i), \quad \text{for} \quad d_i \geq 0.035 \text{ km}, \quad (3) \]

where \( d_i \) is the distance between two transceivers. Then, \( g_i \) (i = 0, 1) remains constant all the time after given a distance \( d_i \).

Thus, the CT needs to estimate the channel gain \( g_0 \) from the PT to the PR for the SS. Since we focus on the sensing phase for the CT to estimate the channel gain between primary transceivers, we will not discuss the transmission phase in the rest of this paper.

\section*{B. Signal Model}

1) Signal model from the PT to the PR: Denote \( x_p \) as the primary signal with unit power, i.e., \( |x_p|^2 = 1 \). If the PT transmits the primary signal with power \( p_0 \), the received signal at the PR in a block \( k \) is

\[ y_p(k) = h_0(k) \sqrt{g_0p_0(k)}x_p(k) + n_p(k), \quad (4) \]

where \( n_p \) represents the AWGN at the PR with zero mean and variance \( \sigma^2 \). Then, the SNR of the received primary signal at the PR is

\[ \gamma_p(k) = \frac{|h_0(k)|^2g_0p_0(k)}{\sigma^2}. \quad (5) \]

We further consider that the PT and the PR adopt close loop power control (CLPC) to provide QoS guaranteed wireless communication \[15], [16], [17], [18]. That is, the PT automatically adjusts its transmit power to meet a certain target SNR \( \gamma_T \) at the PR. Then, PT’s transmit power satisfies

\[ p_0(k) = \frac{\gamma_T\sigma^2}{|h_0(k)|^2g_0}, \quad (6) \]

2) Signal model from the PT to the CT: In the meantime, the received primary signal at the CT from the PT in block \( k \) is

\[ y_c(k) = h_1(k) \sqrt{g_1p_0(k)}x_p(k) + n_c(k), \quad (7) \]

where \( n_c \) is the AWGN at the CT with zero mean and variance \( \sigma^2 \). Then, the SNR of the received primary signal at the CT is

\[ \gamma_c(k) = \frac{|h_1(k)|^2g_1p_0(k)}{\sigma^2}. \quad (8) \]

By Substituting (6) into (8), \( \gamma_c(k) \) in (8) can be rewritten as

\[ \gamma_c(k) = \gamma_T g_1 \left| \frac{|h_1(k)|^2}{|h_0(k)|^2} \right| \]

\[ \quad (9) \]

\section*{III. Maximum Likelihood (ML) Estimator}

In this section, we develop a ML estimator to obtain the channel gain \( g_0 \) between the PT and the PR. In what follows, we will provide the basic principle of the estimator followed by the estimator design and analysis.

\section*{A. Basic Principle}

From (9), the SNR of the received primary signal at the CT is highly related to the channel gain \( g_0 \). Then, it is possible for the CT to measure the SNR of the received primary signal and estimate \( g_0 \). Meanwhile, the maximum likelihood criterion is able to extract the common information from multiple data with the largest reliability and performs well in the parameter estimations \[19]. Thus, we adopt the maximum likelihood criterion and develop a ML estimator to obtain the channel gain \( g_0 \) between the PT and the PR.

\section*{B. Estimator Design}

By removing the block index \( k \) in (9) for brevity and taking the logarithm operation on both sides of (9), we have

\[ \gamma_{c,dB} = \gamma_{T,dB} + 10\log_{10}(g_1) - 10\log_{10}(g_0) + 10\log_{10}\phi \quad (10) \]

where the random variable \( \phi \) is defined as \( \phi = |h_1|^2/|h_0|^2 \).

Since \( \phi \) is a random variable, \( \gamma_{c,dB} \) in (10) is also a random variable. Then, the cumulative density function (CDF) of \( \gamma_{c,dB} \) can be expressed as

\[ F_{\gamma_{c,dB}}(\gamma_{c,dB}) = \Pr\{\gamma_{T,dB} + 10\log_{10}(g_1) - 10\log_{10}(g_0) + 10\log_{10}\phi \leq \gamma_{c,dB}\} \]

\[ = \Pr\left\{\phi \leq 10^{\frac{\gamma_{c,dB} - \gamma_{T,dB} + 10\log_{10}(g_1) + 10\log_{10}(g_0)}{10}}\right\} \]

\[ = F_{\phi}\left(10^{\frac{\gamma_{c,dB} - \gamma_{T,dB} - 10\log_{10}(g_1) - 10\log_{10}(g_0)}}{10}\right), \quad (11) \]

where \( F_{\phi}(\cdot) \) denotes the CDF of \( \phi \).

Since \( h_i \) (i = 0, 1) follows a Rayleigh distribution with unit mean, the CDF of \( \phi = |h_1|^2/|h_0|^2 \) is \[20]

\[ F_{\phi}(\phi) = \frac{\phi}{1+\phi}. \quad (12) \]

Substituting (12) into (13), we have the CDF of \( \gamma_{c,dB} \) as

\[ F_{\gamma_{c,dB}}(\gamma_{c,dB}) = \frac{10^{\gamma_{c,dB} - \gamma_{T,dB} - 10\log_{10}(g_1) - 10\log_{10}(g_0)}}{1+10^{\gamma_{c,dB} - \gamma_{T,dB} - 10\log_{10}(g_1) - 10\log_{10}(g_0)}}. \quad (13) \]

By taking the derivation of \( F_{\gamma_{c,dB}}(\gamma_{c,dB}) \) in terms of \( \gamma_{c,dB} \), we have the probability density function (PDF) of \( \gamma_{c,dB} \) as

\[ f(\gamma_{c,dB}) = \frac{\partial F_{\gamma_{c,dB}}(\gamma_{c,dB})}{\partial \gamma_{c,dB}} \]

\[ = \frac{\ln 10}{10^{\gamma_{c,dB} - \gamma_{T,dB} - 10\log_{10}(g_1) - 10\log_{10}(g_0) - \gamma_{c,dB}}}
\]

\[ \cdot \left(1+10^{\gamma_{c,dB} - \gamma_{T,dB} - 10\log_{10}(g_1) - 10\log_{10}(g_0) - \gamma_{c,dB}}\right)^2. \quad (14) \]

For \( K \) independent blocks, the CT is able to measure \( K \) independent values of \( \gamma_{c,dB} \), namely, \( \gamma_{c,dB}(k) \) (1 ≤ k ≤ K).
Then, the joint PDF of \(g_{c,dB}(k)\) (1 \(\leq k \leq K\)) is
\[
  f(g_{c,dB}(1), g_{c,dB}(2), \ldots, g_{c,dB}(K)) = \prod_{k=1}^{K} \frac{\ln 10 \cdot \frac{10^{\gamma_{T,dB} + 10\log_{10}(g_{T,dB} - 10\log_{10}(g_{T,dB})) - 1}}{10^{\gamma_{c,dB} + 10\log_{10}(g_{c,dB} - 10\log_{10}(g_{c,dB}))}}}{2}.
\] (15)

Based on the maximum likelihood criterion, \(g_0\) can be approximated with the largest probability by the optimal \(g_0^*\) maximizing the joint PDF \(f(g_{c,dB}(1), g_{c,dB}(2), \ldots, g_{c,dB}(K))\). Thus, we shall find the optimal \(g_0^*\) in the following.

Taking the logarithm on both sides of (15), we have
\[
  \log_{10} f(g_{c,dB}(1), g_{c,dB}(2), \ldots, g_{c,dB}(K)) = \sum_{k=1}^{K} \left[ \log_{10} \left( \frac{10^{\gamma_{T,dB} + 10\log_{10}(g_{T,dB} - 10\log_{10}(g_{T,dB})) - 1}}{10^{\gamma_{c,dB} + 10\log_{10}(g_{c,dB} - 10\log_{10}(g_{c,dB}))}} \right) \right] - 1.
\] (16)

Taking the derivative of (16) in terms of \(g_0\), we obtain
\[
  \frac{\partial}{\partial g_0} \log_{10} f(g_{c,dB}(1), g_{c,dB}(2), \ldots, g_{c,dB}(K)) = \sum_{k=1}^{K} \left( \frac{10^{\gamma_{T,dB} + 10\log_{10}(g_{T,dB} - 10\log_{10}(g_{T,dB})) - 1}}{10^{\gamma_{c,dB} + 10\log_{10}(g_{c,dB} - 10\log_{10}(g_{c,dB}))}} \right) - 1.
\] (17)

Thus, we can find the optimal \(g^*\) by solving
\[
  \frac{\partial}{\partial g_0} \log_{10} f(g_{c,dB}(1), g_{c,dB}(2), \ldots, g_{c,dB}(K)) = 0, \text{ i.e.,}
\]
\[
  \sum_{k=1}^{K} \left( \frac{10^{\gamma_{T,dB} + 10\log_{10}(g_{T,dB} - 10\log_{10}(g_{T,dB})) - 1}}{10^{\gamma_{c,dB} + 10\log_{10}(g_{c,dB} - 10\log_{10}(g_{c,dB}))}} \right) - 1 = 0.
\] (18)

After obtaining the optimal \(g_0^*\), we have the ML estimator as
\[
  \hat{g}_0 = g_0^*.
\] (19)

From (18), the ML estimator \(\hat{g}_0 \) (or the optimal \(g_0^*\)) is determined by the target SNR \(\gamma_{T,dB}\) at the PR, the channel gain \(g_1\) and the PT and the CT, and the SNR \(\gamma_{c,dB}\) of the primary signal at the CT. Note that \(\gamma_{T,dB}\) can be obtained by the CT through observing the MCS of the primary signal [13]. \(g_1\) is a deterministic function of the distance \(d_1\) between the PT and the CT, and thus can be calculated at the CT. \(\gamma_{c,dB}\) is measured at the CT and also known to the CT. Therefore, the CT is able to solve (18) and obtain the ML estimator (19).

Note that (18) is a non-linear equation of \(g_0\). It is difficult to solve the equation directly. To deal with this issue, we will develop a bisection method to solve (18) in the next part.

**C. Bisection Method to Solve (18)**

In this part, we will first study the property of (18) and demonstrate that it is possible to solve (18) with a bisection method. Then, we develop the bisection method [21] to solve (18) and obtain the optimal \(g_0^*\).

To begin with, we denote
\[
  f_1(g_0) = \frac{K}{\sum_{k=1}^{K} \left( \frac{10^{\gamma_{T,dB} + 10\log_{10}(g_{T,dB} - 10\log_{10}(g_{T,dB})) - 1}}{10^{\gamma_{c,dB} + 10\log_{10}(g_{c,dB} - 10\log_{10}(g_{c,dB}))}} \right) - 1}.
\] (20)

By taking the derivation of \(f_1(g_0)\) in terms of \(g_0\), we have
\[
  \frac{\partial f_1(g_0)}{\partial g_0} = \sum_{k=1}^{K} \left( \frac{-2g_1 \cdot \frac{10^{\gamma_{T,dB} + 10\log_{10}(g_{T,dB} - 10\log_{10}(g_{T,dB})) - 1}}{10^{\gamma_{c,dB} + 10\log_{10}(g_{c,dB} - 10\log_{10}(g_{c,dB}))}}}{10^{\gamma_{c,dB} + 10\log_{10}(g_{c,dB} - 10\log_{10}(g_{c,dB}))}} \right),
\] (21)

which is smaller than or equal to 0, i.e., \(\frac{\partial f_1(g_0)}{\partial g_0} \leq 0\). That is, \(f_1(g_0)\) monotonically decreases as \(g_0\) increases. Note that \(f_1(0) = \frac{\pi}{\epsilon_0} > 0\) and \(f_1(\infty) = -\frac{\pi}{\epsilon_0} < 0\). Therefore, (18) has a unique positive solution and can be efficiently solved by a bisection method.

We provide the detailed bisection method in Algorithm 1. In particular, \(g_0^{min}\) and \(g_0^{max}\) are the initial values of the bisection method, and denote the minimum value and the maximum value of \(g_0\), respectively. On one hand, since the PR is in the coverage of the PT, the maximum coverage radius \(d_0 = R\) of the PT can be used to calculate \(g_0^{min}\), i.e., \(g_0^{min} = 10^{-12.8(R) - 3.76}\), where the path-loss model in (3) is adopted. For instance, if PT is a base station of a cell, the radius of the cell can be known by the CT and be used to calculate \(g_0^{min}\). On the other hand, considering that the path-loss model in (3) requires that the distance between two transceivers is no less than 0.035 km, \(g_0^{max}\) can be calculated by substituting \(d_0 = 0.035\) km into the path-loss model in (3), i.e., \(g_0^{max} = 10^{-12.8(0.035) - 3.76}\). Similarly, if another path-loss model different from (3) is adopted, \(g_0^{min}\) and \(g_0^{max}\) can also be calculated with minor modifications.

**Algorithm 1 Bisection Method for \(g_0^*\)**

**Initialization**
\(g_0^{min}\), \(g_0^{max}\), \(g_0^{mid}\), and the maximum tolerance error \(\varepsilon_{g_0}\);

**Iterative:**
1. while \(|g_0^{mid} - g_0^{min}| > \varepsilon_{g_0}\) do
2. \(g_0^{mid} = \frac{g_0^{mid} + g_0^{min}}{2}\)
3. if \(f_1(g_0^{mid}) < f_1(g_0^{min}) > 0\) then
4. \(g_0^{min} = g_0^{mid}\)
5. else
6. \(g_0^{max} = g_0^{mid}\)
7. end if
8. end while
9. Return \(g_0^* = g_0^{mid}\).

**D. Complexity Analysis**

From the previous parts, the computational complexity of the ML estimator is dominated by solving (18) with the proposed bisection method in Algorithm 1. Besides, the computational complexity of the bisection method is \(O\left(\log_2 \frac{g_0^{max} - g_0^{min}}{\varepsilon_{g_0}}\right)\) [21, 22], where \(\varepsilon_{g_0}\) is the maximum tolerance error of the bisection method in Algorithm 1. Thus, the computational complexity of the ML estimator is \(O\left(\log_2 \frac{g_0^{max} - g_0^{min}}{\varepsilon_{g_0}}\right)\).
IV. MEDIAN BASED (MB) ESTIMATOR

In the previous section, we have developed a ML estimator to obtain an estimation of the channel gain $g_0$ between the primary transceivers. In particular, the ML estimator requires to solve a nonlinear equation and is computationally complicated. In this section, we will present a low complexity estimator of $g_0$. In what follows, we provide the basic principle of the estimator followed by the estimator design and performance analysis.

A. Basic Principle

To begin with, we provide the definition of the median $x_{1/2}$ of a random variable $X$ as follows,

**Definition 1**: For a random variable $X$ with CDF $F_X(x)$, $x \in \mathbb{R}$, if $x_{1/2}$ satisfies both

$$F_X(x_{1/2}) = \Pr\{X \leq x_{1/2}\} = \frac{1}{2} \quad (22)$$

and

$$1 - F_X(x_{1/2}) = \Pr\{X \geq x_{1/2}\} = \frac{1}{2}, \quad (23)$$

$x_{1/2}$ is defined as the median of the random variable $X$.

Based on Definition 1, we can obtain the median $\gamma_{c, dB, 1/2}$ of the random variable $\gamma_{c, dB}$ by letting $F_{\Gamma_{c, dB}}(\gamma_{c, dB})$ in (38) be $\frac{1}{2}$, i.e.,

$$F_{\Phi}\left(10^{-\gamma_{c, dB}} - 10^{\log_{10}(g_1)} + 10^{\log_{10}(g_0)}\right) = \frac{1}{2}. \quad (24)$$

By substituting (12) into (24), we have

$$F_{\Phi}\left(10^{-\gamma_{c, dB}} - 10^{\log_{10}(g_1)} + 10^{\log_{10}(g_0)}\right) = 10^{-\gamma_{c, dB}} - 10^{\log_{10}(g_1)} + 10^{\log_{10}(g_0)} + 1 + \frac{1}{1 - 0.5} = \frac{1}{2} \quad (25)$$

After solving (25), the median $\gamma_{c, dB, 1/2}$ of the random variable $\gamma_{c, dB}$ can be derived as

$$\gamma_{c, dB, 1/2} = \gamma_{T, dB} + 10\log_{10}(g_1) - 10\log_{10}(g_0). \quad (26)$$

From (26), the median $\gamma_{c, dB, 1/2}$ is a function of the channel gain $g_0$. Thus, if $\gamma_{c, dB, 1/2}$ is available to the CT, $g_0$ can be directly calculated with (26). However, $\gamma_{c, dB, 1/2}$ is unknown to the CT. Instead, we will first estimate $\gamma_{c, dB, 1/2}$ and then obtain the estimation of $g_0$ with (26).

B. Estimator Design

We first give the definition of the sample median $x_{s}^{*}$ of a random variable $X$ as follows,

**Definition 2**: For a random variable $X$ with samples $x_m$ $(1 \leq m \leq M)$, if $x_{1/2}^{s}$ satisfies both

$$\Pr\{x_m \leq x_{1/2}^{s}\} = \frac{1}{2} \quad (27)$$

and

$$\Pr\{x_m \geq x_{1/2}^{s}\} = \frac{1}{2}, \quad (28)$$

$x_{s}^{*}$ is defined as the sample median of the random variable $X$.

As mentioned in the previous section, for $K$ independent blocks, the CT is able to measure $K$ independent samples of $\gamma_{c, dB}$, namely, $\gamma_{c, dB}(k)$ $(1 \leq k \leq K)$. In what follows, we will approximate the median $\gamma_{c, dB, 1/2}$ with the sample median $\gamma_{c, dB, 1/2}$ of these $K$ samples. With the approximated $\gamma_{c, dB, 1/2}$, $g_0$ can be estimated by calculating (26).

To begin with, by sorting the $K$ samples in ascending order, the $K$ samples can be relabelled as $\gamma_{c, dB}(k)$ $(1 \leq k \leq K)$, i.e., $\gamma_{c, dB}(i) \leq \gamma_{c, dB}(j)$ for $1 \leq i \leq j \leq K$. Since the sample medians $\gamma_{c, dB, 1/2}$ of these $K$ samples for odd and even $K$ can be different, we will discuss the two sample medians for odd and even $K$ separately.

1) For the case that $K$ is odd: When $K$ is odd, the sample median is $\gamma_{c, dB, 1/2} = \gamma_{c, dB}\left(\frac{K+1}{2}\right)$. Then, the median of $\gamma_{c, dB}$ can be approximated as

$$\gamma_{c, dB, 1/2} \approx \gamma_{c, dB}\left(\frac{K+1}{2}\right). \quad (29)$$

By substituting (29) into (26), we have the MB estimator as

$$\hat{g}_0 = 10^{\frac{\gamma_{T, dB} + 10\log_{10}(g_1) - \gamma_{c, dB}\left(\frac{K+1}{2}\right)}{10}}. \quad (30)$$

2) For the case that $K$ is even: When $K$ is even, the sample median is between $\gamma_{c, dB}\left(\frac{K}{2}\right)$ and $\gamma_{c, dB}\left(\frac{K}{2}+1\right)$. Then, the median of $\gamma_{c, dB}$ can be approximated as

$$\gamma_{c, dB, 1/2} \approx \gamma_{c, dB}\left(\frac{K}{2}\right) + \gamma_{c, dB}\left(\frac{K}{2}+1\right). \quad (31)$$

By substituting (31) into (26), we have the MB estimator as

$$\hat{g}_0 = 10^{\frac{\gamma_{T, dB} + 10\log_{10}(g_1) - \gamma_{c, dB}\left(\frac{K}{2}\right)+\gamma_{c, dB}\left(\frac{K}{2}+1\right)}{10}}. \quad (32)$$

Consequently, the MB estimator can be summarized as

$$\hat{g}_0 = \begin{cases} 10^{\frac{\gamma_{T, dB} + 10\log_{10}(g_1) - \gamma_{c, dB}\left(\frac{K+1}{2}\right)}{10}}, & \text{for } K \text{ is odd}, \\ 10^{\frac{\gamma_{T, dB} + 10\log_{10}(g_1) - \gamma_{c, dB}\left(\frac{K}{2}\right)+\gamma_{c, dB}\left(\frac{K}{2}+1\right)}{10}}, & \text{for } K \text{ is even}. \end{cases} \quad (33)$$

From (33), the MB estimator is determined by the target SNR $\gamma_{T, dB}$, the channel gain $g_1$ from the PT to the CT, and the measured SNRs at the CT, all of which are available to the CT. Thus, the estimation of $g_0$ can be directly calculated with (33). In other words, the computational complexity of the MB estimator in (33) is $O(1)$.

C. Estimation Accuracy Analysis

In this part, we present the estimation accuracy analysis of the MB estimator in Theorem 1 and Theorem 2. In particular, Theorem 1 studies the estimation accuracy including both the upper bound and the lower bound of an estimation for a given number $K$ of the sensed primary signals. Theorem 2 investigates the performance limit by approaching $K$ to the infinity.
we provide the true value of the channel gain $g_0$ converges to the true value of the channel gain $g_0$ as $K$ increases. This certifies the results in Theorem 2.

**D. Comparison between of the ML Estimator and the MB Estimator**

In this part, we compare the ML estimator and the MB estimator from two aspects, namely, computational complexity and estimation accuracy.

1) **Computational complexity comparison**: As mentioned above, the computational complexity of the ML estimator and the MB estimator are $O\left(\frac{\gamma_c, dB}{\log_2 \frac{20}{\sigma_{\text{awgn}}}}\right)$ and $O(1)$, respectively. Thus, the MB estimator is much simpler than the ML estimator.

2) **Estimation error comparison**: In principle, the ML estimator utilizes all the available samples of $\gamma_{c, dB}$, i.e., $\gamma_{c, dB}(k)$ ($1 \leq k \leq K$), and outputs an estimation of $g_0$. This is different from the MB estimator, which only utilizes the sample median to estimate $g_0$. Ideally, the more samples one estimator utilizes, the more accurate the estimation is. In fact, each sample of $\gamma_{c, dB}$ is physically measured at the CT and thus disturbed by the noise. Thus, each sample contains both the information of $g_0$ and the noise. In particular, if each measured SNR sample of $\gamma_{c, dB}$ is large, i.e., the conveyed information of each sample is much more than the contained noise, estimators are able to extract more knowledge of $g_0$ from more samples, and obtain more accurate estimations. Otherwise, estimators will be more confused by more samples, and thus output less accurate estimations. Therefore, the ML estimator is expected to outperform the MB estimation in terms of the estimation accuracy when the measured SNRs at the CT are larger. Otherwise, the MB estimator is superior to the ML estimator. This is verified through numerical results.

**V. Numerical results**

In this section, we provide the numerical results to demonstrate the performance of the proposed ML estimator and MB estimator. Here, we adopt the system model as in Section II, where the radius of the PT’s coverage is $R = 0.5$ km, the power of the AWGN $\sigma^2 = -114$ dBm, the target SNR of the PR is $\gamma_T = 10$ dB, and the number of samples to measure a SNR at the CT within each block is $J = 100$. Furthermore, $10^4$ Monte Carlo trails are conducted for each curve.

To begin with, we define the estimation error as

$$\epsilon = \left| 10 \log_{10} \left( \hat{g}_0 \right) - 10 \log_{10} (g_0) \right|.$$  \hfill (36)
Fig. 3. Estimation errors with the distance \(d_1\) between the PT and the CT grows from 0.1 km to 0.5 km. In particular, the distance \(d_0\) between the PT and the PR is 0.25 km.

Fig. 4. Average measured SNR at the CT with the distance \(d_1\) between the PT and the CT grows from 0.1 km to 0.5 km. In particular, the distance \(d_0\) between the PT and the PR is 0.25 km.

Fig. 5. Estimation errors with the distance \(d_0\) between the PT and the PR grows from 0.1 km to 0.5 km. In particular, the distance \(d_1\) between the PT and the CT is 0.1 km.

We analyze the variations of the estimation errors as follows:

- **In this figure** are caused by the measure errors of each SNR.
- Since the distance \(d\) between the PT and the CT grows from 0.1 km to 0.5 km. From this figure, the estimation error \(\epsilon\) of the MB estimator outperforms the MB estimator when the average SNR at the CT when the average SNR at the CT is smaller than 4 dB. Furthermore, since the estimation errors of the MB estimator in Fig. 3 remain constant for \(d_1 \leq 0.5\) km, i.e., the average SNR at the CT is accordingly no less than 4 dB. Then, we have a conclusion that the MB estimator is superior to the ML estimator when the average SNR at the CT is smaller than 4 dB. This also verified our analysis in Section IV.

Note that, estimation errors of an estimator are affected by two factors, namely, the number \(K\) of the measured SNRs at the CT and measure errors of each SNR. Since \(K\) in Fig. 3 is given by \(K = 100\) as \(d_1\) grows, the incurred estimation errors by \(K\) are fixed. Then, the variations of the estimation errors in this figure are caused by the measure errors of each SNR. We analyze the variations of the estimation errors as follows:

- **The distance** \(d_0\) between the PT and the PR is fixed at \(d_1 = 0.25\) km, the average transmit power of the PT remains constant to guarantee the target SNR at the PR. As \(d_1\) grows from 0.1 km to 0.5 km, the channel gain \(g_1\) is degraded. Then, the average SNR of the measured primary signals at the CT decreases from around 25 dB to around 0 dB as shown in Fig. 4. This increases measure errors of each SNR at the CT. By adopting these measured SNRs to estimate the channel gain \(g_0\), estimation errors are increased.

- Since the estimation errors of the ML estimator for \(d_1 \leq 0.35\) km in Fig. 3 almost remain constant, the estimation error of the ML estimator caused by the measure errors of each SNR is negligible for \(d_1 \leq 0.35\) km, i.e., the average SNR at the CT is accordingly no less than 5 dB for \(d_1 \geq 0.35\), i.e., the average SNR at the CT is accordingly smaller than 5 dB, the estimation error of the ML estimator increases as \(d_1\) grows. Thus, the estimation error of the ML estimator is dominated by the the number \(K\) of the measured SNRs at the CT when the average SNR at the CT is smaller than 5 dB. Then, we have a conclusion that the MB estimator is more robust than the ML estimator respect to the measure errors of each SNR. By comparing the estimation errors of the ML estimator and the MB estimator, the MB estimator outperforms the MB estimator when the average SNR at the CT is no less than 4 dB. And the MB estimator is superior to the ML estimator when the average SNR at the CT is smaller than 4 dB. This also verified our analysis in Section IV.

- Fig. 5 shows the estimation errors versus the distance \(d_0\) between the PT and the PR grows from 0.1 km to 0.5 km. In particular, the distance \(d_1\) between the PT and the CT is 0.1 km. From this figure, the estimation error \(\epsilon\) of the ML estimator and the MB estimator remain at around 0.7 dB.
in this figure. The trend of the estimation error estimator is below that with the MB estimator in this figure.

This enhances the average SNR of the primary signals at the CT as shown in Fig. 6. Based on Fig. 5 and Fig. 6, the average SNR at the CT with the distance $d_0$ between the PT and the PR grows from 0.1 km to 0.5 km. In particular, the distance $d_0$ between the PT and the PR is 0.1 km.

Fig. 7 provides the estimation errors of $g_0$ versus the number of the measured SNRs at the CT, i.e., $K$. In particular, the distance $d_0$ between the PT and the PR is 0.25 km and the distance $d_1$ between the PT and the CT is 0.1 km. From this figure, the estimation errors with the ML estimator and the MB estimator monotonically decrease with the growth of $K$. This is because, a larger $K$ means that the ML estimator can utilize more measured SNRs at the CT, which provide more information of the channel gain $g_0$. By using the maximum likelihood criterion, the ML estimator is able to extract more information of the channel gain $g_0$ and outputs a more accurate estimation. This leads to a smaller estimation error with the ML estimator. This verifies the results in Theorem 2. Besides, we observe that the estimation error of the ML estimator is smaller than that of the MB estimator. This is reasonable, since the average SNR of the measured primary signals is around 24 dB from Fig. 4 when $d_0 = 0.25$ km and $d_1 = 0.1$ km. From the analysis of both Fig. 5 and Fig. 6, the estimation error of the ML estimator is smaller than the MB estimator when the average SNR at the CT is no less than 4 dB. Furthermore, we observe that, the gap of the estimation errors with two different estimators is smaller than 0.1 dB, which is negligible respect to the channel gain $g_0$.

Accordingly, Fig. 8 gives the required time with the ML estimator and the MB estimator to output an estimation. From this figure, the required time to obtain an estimation with the ML estimator is almost 100 times of the required time with the MB estimator. This shows the advantages of the MB estimator over the ML estimator from the aspect of computational complexity and also verifies our analysis in the previous sections.

Fig. 9 provides the estimation errors of the ML estimator and the MB estimator with imperfect information of $\gamma_T$ or/and $g_1$, versus the distance $d_1$ between the PT and the CT. In particular, denote Case I that the error of $\gamma_T$ or $g_1$ is uniformly distributed within $[-3, 3]$ dB. Since the impacts of imperfect $\gamma_T$ and imperfect $g_1$ on the estimation of $g_0$ are symmetrical from the relations among $g_0$, $\gamma_T$, and $g_1$ in (10), the estimation errors of both estimators are the same when the error of $\gamma_T$ or $g_1$ is uniformly distributed within $[-3, 3]$ dB. Denote Case

![Figure 6](image6.png) Average measured SNR at the CT with the distance $d_0$ between the PT and the PR grows from 0.1 km to 0.5 km. In particular, the distance $d_0$ between the PT and the PR is 0.1 km.

![Figure 7](image7.png) The estimation error of the ML estimator and the Medium value estimator, where ML estimator denotes the estimator of (19) and Medium value estimator denotes the proposed optimized estimator of (33).

![Figure 8](image8.png) Comparison of computation complexity, where the elapsed time of finding the optimal $g_0$ is presented to compare the performance of two method.

![Figure 9](image9.png)
cases decreases. This indicates that, by increasing the number of the measured SNRs at the CT, the impacts of imperfect $\gamma_T$ or $g_1$ can be reduced. In this way, we demonstrate the flexibility of the proposed two estimators.

VI. Conclusions

In cognitive radio networks, the channel gain between primary transceivers is crucial for a cognitive transmitter (CT) to control the transmit power and achieve the coexistence. By sensing primary signals, a maximum likelihood (ML) estimator is firstly developed for the CT to obtain the channel gain between primary transceivers. After demonstrating the high computational complexity of the ML estimator, a median based (MB) estimator with proved low complexity is proposed. Furthermore, the estimation accuracy of the MB estimation is theoretically characterized. By comparing the ML estimator and the MB estimator from the aspect of computational complexity as well as the estimation accuracy, both advantages and disadvantages of two estimators are analyzed. Numerical results show that the estimation errors of the ML estimator and the MB estimator can be as small as 0.6 dB and 0.7 dB, respectively. Besides, the ML estimator outperforms the median-based estimator in terms of the estimation error if the signal to noise ratio (SNR) of the sensed primary signal at the CT is no smaller than 4 dB. Otherwise, the MB estimator is superior to the ML estimator from the aspect of both computational complexity and estimation accuracy.

VII. Appendix

A. Proof of Theorem 1

From the CDF of $\gamma_{c, dB}$ in (38), the probability that $\gamma_{c, dB}(k)$ ($1 \leq k \leq K$) is smaller than or equal to $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$ is

$$
\Pr\{\gamma_{c, dB}(k) \leq \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\} = F_{\gamma_{c, dB}}(\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0))
$$

$$
= F_{\phi}(1) = \frac{1}{2}.
$$

(37)

Similarly, the probability that $\gamma_{c, dB}(k)$ ($1 \leq k \leq K$) is larger than or equal to $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$ is

$$
\Pr\{\gamma_{c, dB}(k) \geq \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\} = 1 - \Pr\{\gamma_{c, dB}(k) \leq \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\}
$$

$$
= 1 - F_{\phi}(1) = \frac{1}{2}.
$$

(38)

When $K$ is odd, $\bar{\gamma}_{c, dB} = \frac{K-1}{2}$. Then, the probability that $\bar{\gamma}_{c, dB}$ is larger than $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$ is equal to the probability that at least $\frac{K-1}{2}$ measured SNRs in $\bar{\gamma}_{c, dB}(k)$ ($1 \leq k \leq K$) are larger than $\gamma_{T, dB} + 10 \log_{10}(g_1) -$
From (39), we have the probability that $\hat{\gamma}_{c, dB}$ is no larger than $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$ is

$$\Pr\{\hat{\gamma}_{c, dB} \leq \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\} = 1 - \left(\frac{1}{2}\right)^{K+1}.$$  

(40)

Meanwhile, $\hat{\gamma}_{c, dB} = \gamma_{c, dB} \left(\frac{K+1}{2}\right)$ when $K$ is odd. Then, the probability that $\hat{\gamma}_{c, dB}$ is smaller $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$ is equal to the probability that at least $K+1$ measured SNRs in $\gamma_{c, dB}(k)$ $(1 \leq k \leq K)$ are smaller than $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$, i.e.,

$$\Pr\{\hat{\gamma}_{c, dB} < \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\} = \Pr\{\hat{\gamma}_{c, dB}(k) < \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\} = 1 - \left(\frac{1}{2}\right)^{K+1}.$$  

(41)

From (41), we have the probability that $\hat{\gamma}_{c, dB}$ is no smaller than $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$ is

$$\Pr\{\hat{\gamma}_{c, dB} \geq \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\} = 1 - \left(\frac{1}{2}\right)^{K+1}.$$  

(42)

Combining (40) and (42), we have

$$\Pr\{\hat{\gamma}_{c, dB} \leq \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0) \leq \gamma_{c, dB}\} = \left(1 - \left(\frac{1}{2}\right)^{K+1}\right)^2,$$  

(43)

which can be rewritten as

$$\Pr\left\{10^{\gamma_{T, dB} + 10 \log_{10}(g_1) - \gamma_{c, dB}} \leq g_0 \leq 10^{\gamma_{T, dB} + 10 \log_{10}(g_1) - \gamma_{c, dB}}\right\} = \left(1 - \left(\frac{1}{2}\right)^{K+1}\right)^2.$$  

(44)

When $K$ is even, $\hat{\gamma}_{c, dB} = \gamma_{c, dB} \left(\frac{K+1}{2}\right)$. Then, the probability that $\hat{\gamma}_{c, dB}$ is larger than $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$ is equal to the probability that at least $\frac{K}{2} + 1$ measured SNRs in $\gamma_{c, dB}(k)$ $(1 \leq k \leq K)$ are larger than $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$, i.e.,

$$\Pr\{\hat{\gamma}_{c, dB} > \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\} = \Pr\{\hat{\gamma}_{c, dB}(k) > \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\} = 1 - \left(\frac{1}{2}\right)^{K+1}.$$  

(45)

From (39), we have the probability that $\hat{\gamma}_{c, dB}$ is no larger than $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$ is

$$\Pr\{\hat{\gamma}_{c, dB} \leq \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\} = 1 - \left(\frac{1}{2}\right)^{K+1}.$$  

(46)

Meanwhile, $\hat{\gamma}_{c, dB} = \gamma_{c, dB} \left(\frac{K+1}{2}\right)$ when $K$ is even. Then, the probability that $\hat{\gamma}_{c, dB}$ is smaller $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$ is equal to the probability that at least $\frac{K}{2} + 1$ measured SNRs in $\gamma_{c, dB}(k)$ $(1 \leq k \leq K)$ are smaller than $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$, i.e.,

$$\Pr\{\hat{\gamma}_{c, dB} < \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\} = \Pr\{\hat{\gamma}_{c, dB}(k) < \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\} = 1 - \left(\frac{1}{2}\right)^{K+1}.$$  

(47)

From (41), we have the probability that $\hat{\gamma}_{c, dB}$ is no smaller than $\gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)$ is

$$\Pr\{\hat{\gamma}_{c, dB} \geq \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0)\} = 1 - \left(\frac{1}{2}\right)^{K+1}.$$  

(48)

Combining (46) and (48), we have

$$\Pr\{\hat{\gamma}_{c, dB} \leq \gamma_{T, dB} + 10 \log_{10}(g_1) - 10 \log_{10}(g_0) \leq \gamma_{c, dB}\} = \left(1 - \left(\frac{1}{2}\right)^{K+1}\right)^2.$$  

(49)

which can be rewritten as

$$\Pr\left\{10^{\gamma_{T, dB} + 10 \log_{10}(g_1) - \gamma_{c, dB}} \leq g_0 \leq 10^{\gamma_{T, dB} + 10 \log_{10}(g_1) - \gamma_{c, dB}}\right\} = \left(1 - \left(\frac{1}{2}\right)^{K+1}\right)^2.$$  

(50)

Combining (44) and (50), we complete the proof of Theorem 1.

**B. Proof of Theorem 2**

When $K$ is odd, the sample median is $\hat{\gamma}_{c, dB} = \gamma_{c, dB} \left(\frac{K+1}{2}\right)$. Note that, for the $K$ measured SNRs $\hat{\gamma}_{c, dB}(k)$ $(1 \leq k \leq K)$, we have $\hat{\gamma}_{c, dB}(i) \leq \hat{\gamma}_{c, dB}(j)$ for $1 \leq i \leq j \leq K$. Then, the lower bound and the upper bound of $\hat{\gamma}_{c, dB}$ can be denoted as

$$\hat{\gamma}_{c, dB} = \overline{\gamma}_{c} \left(\frac{K-1}{2}\right)$$  

(51)

and

$$\hat{\gamma}_{c, dB} = \overline{\gamma}_{c} \left(\frac{K+1}{2} + 1\right),$$  

(52)

respectively.

Among the $K$ measured SNRs $\hat{\gamma}_{c, dB}(k)$ $(1 \leq k \leq K)$, the probabilities that $\hat{\gamma}_{c, dB}(k)$ is smaller than or equal to $\hat{\gamma}_{c, dB}$
and $\hat{\gamma}_{c,dB}$ are

$$\Pr\{\hat{\gamma}_{c,dB}(k) \leq \hat{\gamma}_{c,dB}\} = \frac{K-1}{K} = \frac{1}{2} - \frac{1}{K}$$

(53)

and

$$\Pr\{\hat{\gamma}_{c,dB}(k) \leq \hat{\gamma}_{c,dB}\} = \frac{K+1}{K} = \frac{1}{2} + \frac{3}{2K},$$

(54)

respectively.

When $K$ is even, the sample median is $\hat{\gamma}_{c,dB}(\frac{K}{2}) = \frac{\hat{\gamma}_{c,dB}(\frac{K}{2}) + \hat{\gamma}_{c,dB}(\frac{K}{2}+1)}{2}$. Then, the lower bound and the upper bound of $\hat{\gamma}_{c,dB}$ can be denoted as

$$\hat{\gamma}_{c,dB} = \hat{\gamma}_{c}\left(\frac{K}{2}\right),$$

(55)

and

$$\hat{\gamma}_{c,dB} = \hat{\gamma}_{c}\left(\frac{K}{2} + 1\right),$$

(56)

respectively.

Among the $K$ measured SNRs $\hat{\gamma}_{c,dB}(k)$ ($1 \leq k \leq K$), the probabilities that $\hat{\gamma}_{c,dB}(k)$ is smaller or equal to $\hat{\gamma}_{c,dB}$ and $\hat{\gamma}_{c,dB}$ are

$$\Pr\{\hat{\gamma}_{c,dB}(k) \leq \hat{\gamma}_{c,dB}\} = \frac{K}{K} = \frac{1}{2}$$

(57)

and

$$\Pr\{\hat{\gamma}_{c,dB}(k) \leq \hat{\gamma}_{c,dB}\} = \frac{K+1}{K} = \frac{1}{2} + \frac{1}{K},$$

(58)

respectively.

Based on (53), (54), (57), and (58), for any $K$, we have

$$\frac{1}{2} - \frac{1}{K} \leq \Pr\{\hat{\gamma}_{c,dB}(k) \leq \hat{\gamma}_{c,dB}\} \leq \frac{1}{2}$$

(59)

and

$$\frac{1}{2} + \frac{1}{K} \leq \Pr\{\hat{\gamma}_{c,dB}(k) \leq \hat{\gamma}_{c,dB}\} \leq \frac{1}{2} + \frac{3}{2K},$$

(60)

respectively.

When $K$ goes to the infinity, i.e., $K \to \infty$, (59) and (60) become

$$\frac{1}{2} - \varepsilon_0 \leq \Pr\{\hat{\gamma}_{c,dB}(k) \leq \hat{\gamma}_{c,dB}\} \leq \frac{1}{2}$$

(61)

and

$$\frac{1}{2} + \varepsilon_0 \leq \Pr\{\hat{\gamma}_{c,dB}(k) \leq \hat{\gamma}_{c,dB}\} \leq \frac{1}{2} + \frac{3}{2}\varepsilon_0,$$

(62)

where $\varepsilon_0$ is defined as $\varepsilon_0 = \lim_{K \to \infty} \frac{1}{K}$ and is an arbitrarily small and positive value.

Since the median $\gamma_{c,dB}$ satisfies $F_{\hat{\gamma}}(\gamma_{c,dB}, \frac{1}{2}) = \frac{1}{2}$, we have

$$\Pr\left\{\hat{\gamma}_{c,dB}, \frac{1}{2} - \hat{\gamma}_{c,dB} < \varepsilon_0\right\} = 1$$

(63)

and

$$\Pr\left\{\hat{\gamma}_{c,dB}, \frac{1}{2} - \hat{\gamma}_{c,dB} > \frac{3\varepsilon_0}{2}\right\} = 1,$$

(64)

respectively.

Combining (63) and (64), we obtain

$$\Pr\left\{\frac{3\varepsilon_0}{2} < F_{\hat{\gamma}}(\gamma_{c,dB}, 0) - F_{\hat{\gamma}}(\gamma_{c,dB}, \frac{1}{2})\right\} \leq F_{\hat{\gamma}}(\gamma_{c,dB}, \frac{1}{2}) - F_{\hat{\gamma}}(\gamma_{c,dB}, 0) \leq \varepsilon_0$$

(65)

and

$$\Pr\left\{\hat{\gamma}_{c,dB}, \frac{1}{2} - \hat{\gamma}_{c,dB} < \varepsilon_0\right\} = 1$$

(66)

Since $F_{\hat{\gamma}}(\gamma_{c,dB})$ is a continuous and monotonically increasing function of $\gamma_{c,dB}$, the following inequalities hold,

$$F_{\hat{\gamma}}(\gamma_{c,dB}, \frac{1}{2}) - F_{\hat{\gamma}}(\gamma_{c,dB}, 0) \leq F_{\hat{\gamma}}(\gamma_{c,dB}, \frac{1}{2}) - F_{\hat{\gamma}}(\gamma_{c,dB}, \frac{1}{2})$$

(67)

Then, we can always find an arbitrarily positive and small $\varepsilon_1$ satisfying

$$\Pr\left\{-\varepsilon_1 < \hat{\gamma}_{c,dB}, \frac{1}{2} - \gamma_{c,dB}, \frac{1}{2} < \varepsilon_1\right\} = 1.$$

(68)

Based on (68) and the relations between $g_0$ and $\gamma_{c,dB}$ in (10), we can always find an arbitrarily positive and small $\varepsilon$ satisfying

$$\Pr\left\{-\varepsilon < g_0 - \hat{g}_0 < \varepsilon\right\} = 1,$$

(69)

which can be rewritten as

$$\Pr\{|g_0 - \hat{g}_0| < \varepsilon\} = 1.$$

(70)

This completes the proof of Theorem 2.

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