The Multifractal Characteristics of Seismic Activities in China

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Abstract. In recent years, with the frequent occurrence of geological disasters, the prediction and analysis of earthquakes has gradually become a hot topic. This paper studies the multifractal spatio-temporal characteristics of China seismic energy. The study data derives from the seismic activities in the Mainland China from 1970 to 2013. In this study, it comes to a comprehensive conclusion of spatial and temporal multifractal characteristics of earthquake activities in China. The study area is divided into five regions: The Xinjiang region, the Qinghai-Tibetan Plateau region, the Northern China, the Northeast China and the Southern China. In this study, we analyze the spectrum curves of the seismic releasing energy time series to each region, and compare the differences. Then the differences of the multifractal characteristics in the time dimension can be pointed out based on the curve, and the law of earthquake occurrence is analyzed from the sequence diagrams of time intervals among various tectonic areas. In the spatial dimension, it discrete seismic data points for each region through rectangular square grid at different resolutions as the input, then derive the multifractal spectrum curve of spatial characteristic to each region. By analyzing the multifractal dimension, it explores the similarities and differences of the spatial and temporal characteristics of the earthquake among five regions.

1. Introduction

The seismic activities [1] like other natural disasters [2-4] even worse capture world attention with the disastrous losses of lives and properties. Lots of scholars have been exploring the secret of earthquake for centuries. However, the quantification of spatio-temporal dynamic process for the seismic activities still presents difficulties [5,6]. And the spatiotemporal characteristics of seismic activities should be assessed for the further understanding of the spatio-temporal dynamic properties [7,8]. The spatiotemporal distribution of earthquake disaster is complicated and irregular [9,10]. The classic physical model [11] has a great difficulty in reflecting the seismic actual situation accurately. These have not blocked the pace of scientists. In recent years, with the development of machine learning [12,13] and deep learning [14,15], they have been applied to various fields [16-21], and they have gradually penetrated into seismic research and achieved good results. At the same time, statistics study [22] also further promoted the development of seismology. The putting forward and development of the nonlinear systems theory [23,24] offers new methods of spatio-temporal dynamic analysis for study of seismic activities [25,26], mathematicising engineering applications [27]. Thus, based on the nonlinear systems theory, using multiple fractal model, we analyze the spatio-temporal
characteristics on seismic activities in China, and aim to explain the spatio-temporal dynamic properties of seismic activities in this study.

According to the self-similarity process feature of the complex system [8], the earthquake has been regarded as an irregular and self-organized criticality process with the scale-invariable fractal characteristic [28-32]. Xiaodong Zhang and others [33,34] present the statistical characteristics between distribution and magnitude based on the self-affine fractal model, and define the characteristic earthquake as a bound of the distribution relationship between earthquake frequency and magnitude. Some researchers say that the spatio-temporal characteristics of earthquake is not a single process, and they suggest that use the multifractal method to further explore the seismic dynamic process. Roy [35] analyzes the earthquake in the Himalayas by multifractal theory, and points out the formation process of high stress area, using fractal aggregation model. Telesca et al. [36] studies the earthquake by the method of spatio-temporal multifractal distribution characteristics, and prove that multifractal theory can objectively reflect the uniform change of stress field in the tectonic system. F. Masci [37] studied application of multifractal algorithm in the earthquake, through the analysis of the characteristics of the multifractal spectrum. Combined with the geological structure of the study area, it concluded that the spatio-temporal characteristic parameters of the seismic activities can be effectively reflected.

In this study, we analyzed and compared the spectral curves of the time series of seismic energy released in different regions. On this basis, we point out the difference of the multifractal characteristics in the time dimension of each tectonic region, and analyze the occurrence law of earthquakes from the time interval sequence diagram of each tectonic region. In spatial dimension, the seismic data points of each region are discretized as inputs by rectangular grid with different resolutions, and the multifractal spectral curves of spatial characteristics of each region are derived. Through multi-fractal dimension analysis, the similarities and differences of spatial and temporal characteristics of earthquakes in five regions are discussed.

2. Multifractal analysis model

Multifractal analysis is a model to describe the dynamics process of a multifractal system using a continuous spectrum of exponents called singularity spectrum [38,39]. Multifractal analysis model is generally used to intricate natural systems, which has been proposed in various fields such as fluid dynamics, finance [40], image recognition [41-44], meteorology [45], and geophysics [5,46]. Based on the singularity of the fractal measure, the study area is decomposed into a series of sub-regions. Each of the sub-regions also has the singularity of the fractal measure and a series of the fractal dimensions [47].

In the previous studies on the multifractal analysis, the study subjects normally are some series of chronological attribute values, or spatial arrangement in certain orders [6,7]. Thus, in this study, the first step is to divide the study object, the seismic activity data, into N subsets at a wide range of scales, \( s_i (i=1,2,..., N) \). In Eq. (1), \( R_i \) represents the size of the \( i^{th} \) subset, and \( P_i \) represents the generation probability of \( S_i \), while the sum of \( P_i \) equals 1. For each subset \( S_i \), \( P_i \) could be represented by the scale index \( \alpha_i \) shown by Eq.(1). The \( \alpha_i \), called Lipschitz-Holder index (referred to as Holder index) [48], is the fractal dimension of the \( i^{th} \) subset.

\[
p_i = R^{\alpha_i} (i=1,2,..., N)
\]  

(1) [38]

For \( R_i \) approaches to zero, then Eq.(1) can be transferred into Eq.(2) [38]:

Consider a time increment \( \{ \xi(t) \} \), here \( \{ \xi(t) \} = B(t)-B(t-1) \), where B(t) is the observed value of time t (t = 1, 2, …). For any positive integer \( \tau \), define the mean sequence:

\[
\alpha_i = \lim_{\tau \to \infty} \frac{\ln P_i}{\ln R_i}
\]  

(2) [38]
Then each subset \( S_i \) is divided into a plurality of small units, to set the equal \( \alpha_i \) values for each small unit. \( N_\alpha(e) \) represents the total number of units in the subset. \( N_\alpha(e) \propto e^{-\tau(\alpha)} \), Thus:

\[
f(\alpha) = -\lim_{e \to 0} \frac{\ln N_\alpha(e)}{\ln e}
\]

(3) [38]

Where \( f(\alpha) \) indicates that the fractal dimension of the subsets when \( \alpha \) is equal. Since it is continuous allowed values for \( \alpha \geq 0 \), the spectrum of the multifractal dimension \( f(\alpha) \) derived by \( \alpha \) is formed [48].

Besides \( f(\alpha) \sim \alpha \), multifractal characteristic also be displayed by \( D_q(\alpha) \) (as Eq.(5)) [49]. Using the order \( q \), both sides of Eq. (1) are multiplied by the \( q \) power, and then calculated the sum separately as followed:

\[
\sum_{i}^{N} p_i^q = \sum_{i}^{N} (r_i)^{\alpha q} = X(q)
\]

(4) [38]

\[
D_q = \lim_{L \to 0} \frac{\ln X(q)}{\ln L} = D(q)
\]

(5) [50]

The partition function \( M \) can be defined as \( M(e,q) = \sum_{i}^{N} p_i^q \), \( q \in (-\infty, +\infty) \) is the order of partition function to indicate the inhomogeneity degree of multifractal characteristics. If \( q \) satisfies the formula \( M(e,q) \propto e^{\tau(q)} \), \( \tau(q) \) is the mass index function that reflects the characteristic of the fractal behavior[51]. Based on the curves of \( \tau(q) \), we could determine whether the fractal behavior is single fractal or multifractal. When \( \tau(q) \) curve is a straight line, it shows that the fractal behavior is single fractal. Conversely, if the \( \tau(q) \) curve is a convex curve, the slopes at every point are various, the fractal behavior is multifractal [52]. In this study, \( \tau(q) \) is used to determine whether the earthquake activities have the multifractal characteristics. \( D_q \sim q \) and \( f(\alpha) \sim \alpha \) could be combined with Legendre transform[38]:

\[
D_q = \frac{1}{q-1} \left[q\alpha - f(\alpha)\right] \text{ or } f(\alpha) = q\alpha - \tau(q)
\]

(6) [38]

where the mass index function \( \tau(q) = (q-1)D_q \), and \( \alpha(q) = \frac{d\tau(q)}{dq} = \frac{d(q-1)D_q}{dq} \) [52].

Based on Eq.(6), the multi-fractal dimension \( D_q \) was calculated in this study[51]. When \( q = 0 \), \( D_0 \) called the box dimension or the capacity dimension is \( D_0 = f(\alpha(0)) = -\tau(0) \); while \( q = 1 \), \( D_1 \) named the information dimension is \( D_1 = f(\alpha(1)) = \alpha(1) \) [14]. As \( q \gg 1 \), \( M(e,q) \) and \( \tau(q) \) reflect the fractal characteristics in the study area with the high-probability of earthquake activity[31]; On the contrary, while \( q \ll 1 \), \( M(e,q) \) and \( \tau(q) \) reflect the characteristics in the study area with the low-probability of earthquake activity [31].

3. Study Area

China lies between two major earthquake belts, the circum-Pacific seismic belt and the Alpine-Humalayan seismic zone [8]. Due to the special geographical location [53], China suffered seismic disasters frequently, and the seismicity intensity is great severity. Most of the earthquakes occurred in China are continental, accounted for about one third of global destructive continental earthquakes. It has been inflicted huge losses not only on the properties, but also on people's lives by
the numerous seismic activities in China. Accordingly, the study on the spatiotemporal characteristics of the seismic activities in China is of great importance.

The data is from the USGS (United States Geological Survey) earthquake catalogue from January 1, 1970 to June 30, 2013. Figure 1 clearly shows the distribution of the epicenters in the study area.

Based on the previous studies, we further divided the study area into five regions: The Xinjiang region (XJ) (blue), the Qinghai and Tibet Plateau region (Q-T) (yellow), the Northern China (N-C) (purple), the Northeast China (NE-C) (orange) and the Southern China (S-C) (green). In this study, we analyzed respectively and compared the seismic fractal characteristics for each region.

After the integrity verification of seismic records for each region using the Gutenberg-Richter Relation law, the usable minimum magnitude is Ms (the Richter magnitude) ≥2.8 in three regions (the Xinjiang region, the Northeast China and the Southern China), as Ms≥2.9 in the Qinghai and Tibet Plateau region, and Ms≥2.7 in the Northern China.

As shown in Figure 1, the earthquakes are not distributed equally in different regions. The seismic activities are more frequent in the Western China than the Eastern China.

Figure 1. The distribution of the epicenter in China

The seismic energy release is one of the most important indexes for the intensity evaluation of seismic activity. In this study, we selected the release energy as the index to represent the intensity of earthquake. The magnitude of earthquake is used to estimate the release energy with Eq. (7) [54-56]:

\[ E = \Omega = 10^{1.5M_s - c} \]

Ms is the Richter magnitude of the earthquake; c is a constant value equals 4.8.

4. The Temporal Characteristics of Seismic Activities in China

First of all, the time intervals between each pair of earthquakes were calculated from the chronological earthquake records from 1970 to 2013 for each region [36]. Then, the energy released of the earthquake was calculated by Eq. (7). With the temporal sequence of intervals and the estimated release energy of earthquakes, the multifractal spectrum of \( \tau(q) \sim q \) and \( f(\alpha) \sim \alpha \) were calculated. Based on the previous studies, the range of parameter \( q \) in Eq. (6) is from -10 to 10 in 0.1 as a unit in this study [57].

In Figure 2, we could notice the slopes of the \( \tau(q) \sim q \) were not constant, but nonlinear curve in five regions. Hence, the earthquake activities have multifractal characteristics in all five regions. For the multifractal curve \( \tau(q) \sim q \), the more extent of the slope change, the more pronounced or evident the multifractal characteristic of the complex system is. Compared the curve slopes of five regions (shown as Figure 2(a)), we could observe the variations of the temporal multifractal characteristics. In the temporal dimension, the most distinct multifractal characteristics is in the Southern China, then followed by the Qinghai-Tibetan Plateau region, the Northeast China, the Xinjiang region and the Northern China. That is, as time goes by, the energy release process has the multifractal characteristic in all five regions based on time dimension. However, there are differences of the multifractal characteristic of the energy release processes between five regions.
Figure 2. The pattern of the $\tau(q) \sim q$

Figure 3. The multifractal spectrum curves of $f(\alpha) \sim \alpha$

Table 1. The descriptive statistics of parameter for $f(\alpha) \sim \alpha$.

|       | XJ      | Q-T     | N-C     | NE-C    | S-C     |
|-------|---------|---------|---------|---------|---------|
| $\alpha_{min}$ | 0.3039  | 0.08318 | 0.003147| 0.004246| 0.183   |
| $\alpha_{max}$ | 2.124   | 2.343   | 2.362   | 2.336   | 2.034   |
| $\alpha_{max} - \alpha_{min}$ | 1.8201  | 2.25982 | 2.35853 | 2.331754| 1.851   |
| $\bar{\alpha}$ | 1.214   | 1.208   | 1.197   | 1.189   | 1.108   |
| $D_0$     | 1.000   | 1.000   | 1.001   | 1.002   | 1.005   |

The capacity dimensions $D_0 = f(\alpha(0)) = -\tau(0)$ in the regions $D_{0,\text{Qinghai–Tibetan Plateau region}} = 1.005$. It shows that mainly the energy releases of seismic activities extend smooth-linearly along with time. In other words, the earthquake energy will have been releasing at a specific frequency in each region. And the regional plate stress fields change stably, continuously and cumulatively in the area. The energy releases of seismic activities are the most stable in the Qinghai–Tibetan Plateau region and the Xinjiang region, followed by the Northern China and the Northeast China, then the Southern China.

Based on Eq. (6), $f(\alpha) \sim \alpha$ were transformed with $\tau(q) \sim q$ for the five regions (shown in Figure 3). The spectrum curves of $f(\alpha) \sim \alpha$ show that the scale index of seismic released energy with time, according to the spectrum width and shape. The $f(\alpha) \sim \alpha$ curves generally rise to a maximum that approximates the capacity dimension $D_0$ at $q=0$, and then fall away, like bell shape. The descriptive statistics of parameter $\alpha$ and $D_0$ in multifractal spectrums $f(\alpha) \sim \alpha$ for five regions are in Table 1. The left part of the curve ($\alpha(q) \leq \alpha(0)$) represents the frequency of earthquakes occurred in cluster, otherwise the right part ($\alpha(q) \geq \alpha(0)$) shows the frequency of earthquakes occurred sparsely.

According to the shape of the $f(\alpha) \sim \alpha$ curves, the multifractal spectrums of seismic released energy in time dimension are all right dipping in all five regions. The main characteristic of seismic energy released is the clustering in time dimension. The right part of the curves in the Northern China and the Northeast China are steeper drop than any other regions. It indicates that in two regions, the Northern China and the Northeast China, the clustering distribution in time is the most distinctive,
followed by the Qinghai-Tibetan Plateau region, then the Southern China, and the Xinjiang region. It also could be measured by the span of $\alpha = \alpha_{\text{max}} - \alpha_{\text{min}}$.

Based on the analysis above, there are some similar temporal characteristics of the seismic released energy in five regions. For example, although the temporal distribution of the released energy process possesses the characteristic of multifractal dimension, the capacity dimensions in five regions are fairly closed to 1. That is to say, due to the regional tectonic features present to be relatively stable, including the crustal structure, stress field, crustal deformation and crustal activity, it might lead to the continuously and evenly release process of energy in the study area. The main characteristic of temporal distribution is the clustering. It might be explained by that the seismic activities are related not only to the special geographical environment and tectonics condition, but also to the previous earthquakes with high magnitude around. Besides, there was few variabilities of the multifractal temporal characteristic among the five regions, such as the temporal distributions are more widely in the Northern China and the Northeast China, while more gently in the Southern China, and the Xinjiang region. It might be caused by the effect of the uniformity level on the distribution of the fault structure zones in the regions.

5. The Spatial Characteristics of Seismic Activities in China

Firstly, we dispersed the study area into grids, and established the non-overlapping grids and dividing earthquakes of each region into different grids. In the Xinjiang region, the Qinghai-Tibetan Plateau region and the Northeast China, the grid scale is $1^\circ \times 1^\circ$, while the grid scale $0.5^\circ \times 0.5^\circ$ is used in the Northern China and the Southern China. The sum of historical seismic released energy is the value of the grid. According to the spatial distribution of the historical seismic released energy, the energy matrix as followed was constructed as the input of spatial multifractal analysis for five regions [58]:

$$E = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1n} \\ E_{21} & E_{22} & \cdots & E_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ E_{n1} & E_{n2} & \cdots & E_{nn} \end{bmatrix}$$

Where $E$ is called energy matrix. $E_{ij}$ represents released energy in the grid area of $i$ row, $j$ column.

Using Eq. (6), the spatial multifractal spectrum of the five regions was shown in Figure 4. The multifractal spectrum parameters $\Delta f = f(\alpha_{\text{min}}) - f(\alpha_{\text{max}})$ of seismic energy release reflect spatial aggregation level of seismic released energy [33,59]. If $\Delta f > 0$, it means the seismic release energy is more likely to gather and vice versa [34,60]. From the multi-fractal spectrum curve, it is shown obviously that the $\Delta f$ in the Xinjiang region and the Qinghai-Tibetan Plateau region are greater than zero while north China, northeast and south China are less than zero. Thus, in the Xinjiang region and the Qinghai-Tibetan Plateau region, the spatial distribution of seismic activities is the aggregation. It might be explained that in the Xinjiang region and the Qinghai-Tibetan Plateau region, the seismic activities are generally occurred in the fault zones, and regional plate activities are intense, and stress field is complicated.

![Figure 4. T Multi-fractal spectrum of reach space](image-url)
6. Conclusion

Through the comprehensive analysis the spatiotemporal multi-fractal characteristics of earthquake of five regions of China, earthquakes are extremely complex distribution in both space and time. The multifractal analysis model is feasible in the spatiotemporal analysis of seismic activities. In China, the earthquake frequency is similar in five regions. However, the intensity of temporal distribution in the Northern China and the Northeast China is few higher than the rest. In comparison with the temporal analysis, the spatial distribution of seismic activities is more aggregate in the Xinjiang region and the Qinghai-Tibetan Plateau region than the rest. The spatial and temporal multifractal characteristics consist with the crustal stress field in five regions. The multifractal analysis model can be used as a new model towards researches on the quantitative evaluation of regional crustal stress field and crustal structure.

In this study, it uses the multifractal analysis to analyze the temporal and spatial propagation characteristics of the five regions in the Mainland China: Xinjiang region, Qinghai-Tibetan Plateau region, Northern China, Northeast China and Southern China. Compared with the seismicity temporal and spatial differences of each region, it would help to further understand the seismic dynamic process in the Mainland China.

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