On the Origin of Majorana Neutrino Masses

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Abstract

In the introductory part we briefly consider basics of neutrino oscillations and convenient phenomenology of neutrino oscillations in vacuum. Main part of this report is dedicated to a discussion of a plausible BSM scenarios of neutrino mass generation, based on assumptions of massless left-handed SM neutrinos and violation of the total lepton number. It is stressed that search for light sterile neutrinos and neutrinoless double $\beta$-decay could provide a crucial tests of this scenarios.

1 Introduction

One of the most important recent discoveries in particle physics was discovery of neutrino oscillations in the atmospheric Super-Kamiokande [1], solar SNO [2] and the reactor KamLAND experiment [3] (1998-2002). In 2015 the Nobel Prize was awarded to T. Kajita and A. McDonald “for the discovery of neutrino oscillations, which shows that neutrinos have masses”. Small neutrino masses, driving neutrino oscillations, is the only evidence (in particle physics) of an existence of a new beyond the Standard Model physics.

Origin of neutrino masses, which are many orders of magnitude smaller than quark and lepton masses, is the major open problem of neutrino physics.

In the first part of this talk I will consider phenomenon of neutrino oscillations. In the second part I will discuss a possible (and plausible) origin of neutrino masses and mixing.

2 Basics of Neutrino Oscillations

Idea of neutrino oscillations was put forward by B.Pontecorvo in Dubna in 1957-58 [4]. This idea was further developed by B.Pontecorvo and V.Gribov

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1A talk at the 5-th International Conference on Particle Physics and Astrophysics, Moscow, Russia, 5-9 October 2020
(1969) and B. Pontecorvo and S. Bilenky (1975-1989) (see, for example, [6]). Idea of flavor neutrino mixing was proposed in [7].

We know from experiments on the investigation of the "invisible" decay $Z^0 \to \nu_l + \bar{\nu}_l$ (LEP, SLC) that three flavor left-handed neutrinos $\nu_e, \nu_\mu, \nu_\tau$ (and right-handed antineutrinos) exist in nature. The left-handed fields of flavor neutrinos $\nu_l L (l = e, \mu, \tau)$ enter into the Standard Model CC and NC weak interaction

$$
L^CC_T = -\frac{g}{2\sqrt{2}} j^CC_\alpha W^\alpha + \text{h.c.}, \quad j^CC_\alpha = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_l \gamma_\alpha l_L
$$

(1)

and

$$
L^{NC}_T = -\frac{g}{2\cos \theta_W} j^{NC}_\alpha Z^\alpha, \quad j^{NC}_\alpha = \sum_{l=e,\mu,\tau} \bar{\nu}_L \gamma_\alpha \nu_L.
$$

(2)

From the observation of neutrino oscillations follows that flavor neutrino fields are mixed

$$
\nu_l(x) = \sum_{i=1}^{3} U_{li} \nu_i L(x).
$$

(3)

Here $\nu_i(x)$ is the field of neutrino (Dirac or Majorana) with mass $m_i$ and $U$ is $3 \times 3$ PMNS mixing matrix.

It follows from (3) that if all neutrino mass-squared differences are small, states of the flavor neutrinos with definite momentum, produced in the decays $\pi^+ \to \mu^+ + \nu_\mu$ (accelerator and atmospheric neutrinos), $(A, Z) \to (A, Z + 1) + e^- + \bar{\nu}_e$ (reactor antineutrinos) etc are given by

$$
|\nu_l\rangle = \sum_{i=1}^{3} U_{li}^* |\nu_i L\rangle \quad (l = e, \mu, \tau),
$$

(4)

where $|\nu_i\rangle$ is the state of neutrino with mass $m_i$, momentum $\vec{p}$ and energy $E_i \simeq p + \frac{m_i^2}{2E}$ ($p^2 \gg m_i^2$).

If at $t = 0$ flavor neutrino $\nu_l$ was produced, the state of neutrino at the time $t$ will be a coherent superposition

$$
|\nu_l\rangle_t = \sum_{i} U_{li}^* e^{-iE_i t} |\nu_i L\rangle = \sum_{\nu'} A(\nu_l \to \nu_{\nu'}) |\nu_{\nu'}\rangle.
$$

(5)

Here

$$
A(\nu_l \to \nu_{\nu'}) = e^{-i p_L t} \sum_{i} U_{\nu' i} e^{-\frac{m_i^2}{2E} t} U_{li}^*
$$

(6)
is the amplitude of transition $\nu_l \rightarrow \nu_{l'}$ during time $t$, $L \simeq t$ is a source-detector distance.

The amplitude $A(\nu_l \rightarrow \nu_{l'})$ is a coherent sum of products of amplitudes (of transition $(\nu_i \rightarrow \nu_i) (U_{ii}^*)$) (of propagation in the state $\nu_i (e^{-iE_i t})$) (of transition $(\nu_i \rightarrow \nu_{l'}) (U_{li})$).

Neutrino oscillations in vacuum are the result of interference between different $i$-amplitudes. Taking into account the unitarity of the mixing matrix $U$ and the arbitrariness of a common phase from (5) and (6) we find a convenient expression for the transition probability (see [8])

$$P(\nu_l \rightarrow \nu_{l'}) = |\sum_{i=1}^{3} U_{li} e^{-i\frac{m_{l'i}^2 L}{2E}} U_{li}^*|^2 = |\delta_{l'l} - 2i \sum_{i \neq r} e^{-i\Delta m_{ri} U_{li}^* U_{li}^*} \sin \Delta_{rl}|^2. \quad (7)$$

Here

$$\Delta_{ri} = \frac{\Delta m_{r'i}^2 L}{4E}, \quad \Delta m_{ri}^2 = m_{rk}^2 - m_{ri}^2 \quad (8)$$

and $r$ is an arbitrary, fixed index.

The second term of (7) describes neutrino oscillations. We see from this expression that neutrino oscillations take place if

- at least one neutrino neutrino mass-squared difference is different from zero;
- there is a neutrino mixing ($U \neq I$).

Let us consider the simplest case of two flavors, say $\mu$ and $\tau$. Choosing $r = 1$ from (7) we have

$$P(\nu_l \rightarrow \nu_{l'}) = |\delta_{l'l} - 2ie^{-i\Delta_{12}} U_{l1}^* U_{l1}^* \sin \Delta_{12}|^2. \quad (9)$$

For two flavors the mixing matrix has the following general form

$$U = \left( \begin{array}{cc} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{array} \right) \quad (10)$$

From (9) and (10) we find the standard two-neutrino appearance and disappearance transition probabilities

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta \sin^2 \frac{\Delta m_{12}^2 L}{4E}, \quad P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m_{12}^2 L}{4E}. \quad (11)$$
Atmospheric, solar and long baseline accelerator and reactor data are described by the three-neutrino mixing. In this case transition probabilities depend on six parameters: atmospheric and solar mass-squared differences $\Delta m^2_A$ and $\Delta m^2_S$, three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one CP phase $\delta$.

Neutrino masses are usually labeled in such a way that $m_1$ and $m_2$ are connected with solar neutrinos. From the condition of the MSW resonance [9, 10] follows that $\Delta m^2_{12} > 0$. The solar mass squared difference is determined as follows $\Delta m^2_S = \Delta m^2_{12}$.

For $m_3$ there are two possibilities

1. Normal ordering (NO) $m_3 > m_2 > m_1$.

2. Inverted ordering (IO) $m_2 > m_1 > m_3$.

Atmospheric mass-squared difference can be determined as follows

$$\Delta m^2_A = \Delta m^2_{23} \text{ (NO)} \quad \Delta m^2_A = |\Delta m^2_{13}| \text{ (IO)}.$$ (12)

Using (12) from (7) we can easily find that transition probabilities of $\nu_l \rightarrow \nu_\ell$ differences depend on six parameters: atmospheric and solar mass-squared differences $\Delta m^2_A$ and $\Delta m^2_S$, and $\nu_3$ mixing described by the three-neutrino mixing. In this case transition probabilities are connected with solar neutrinos. From the condition of the MSW resonance [9, 10] follows that $\Delta m^2_{12} > 0$. The solar mass squared difference is determined as follows $\Delta m^2_S = \Delta m^2_{12}$.

For NO case we have

$$P^{\text{NO}}(\nu_l \rightarrow \nu_\ell) = \delta_{l\ell} - 4|U_{l3}|^2(\delta_{l\ell} - |U_{l3}|^2) \sin^2 \Delta_A$$

$$- 4|U_{12}|^2(\delta_{1\ell} - |U_{12}|^2) \sin^2 \Delta_S - 8 \Re (U_{\nu 3} \ast U_{\nu 1}^* \ast U_{\nu 3}^* U_{\nu 1}) \cos(\Delta_A + \Delta_S)$$

$$\pm \Im (U_{\nu 3} \ast U_{\nu 1}^* \ast U_{\nu 3}^* U_{\nu 1}) \sin(\Delta_A + \Delta_S) \sin \Delta_A \sin \Delta_S,$$ (13)

For IO case we find

$$P^{\text{IO}}(\nu_l \rightarrow \nu_\ell) = \delta_{l\ell} - 4|U_{l3}|^2(\delta_{l\ell} - |U_{l3}|^2) \sin^2 \Delta_A$$

$$- 4|U_{12}|^2(\delta_{1\ell} - |U_{12}|^2) \sin^2 \Delta_S - 8 \Re (U_{\nu 3} \ast U_{\nu 1}^* \ast U_{\nu 2} \ast U_{\nu 1}) \cos(\Delta_A + \Delta_S)$$

$$\pm \Im (U_{\nu 3} \ast U_{\nu 1}^* \ast U_{\nu 2} \ast U_{\nu 1}) \sin(\Delta_A + \Delta_S) \sin \Delta_A \sin \Delta_S,$$ (14)

where $\Delta_{A,S} = \frac{\Delta m^2_{A,S} L}{4E}$. It is seen from comparison of these expressions that $P^{\text{NO}}(\nu_l \rightarrow \nu_\ell)$ and $P^{\text{IO}}(\nu_l \rightarrow \nu_\ell)$ differ by the change of $U_{\ell(\ell')1} \Rightarrow U_{\ell(\ell')2}$ and by the sign of last terms.

In conclusion we present in the Table [11] the result of the global analysis of existing neutrino oscillation data [11].

The study of neutrino oscillations enter now into high-precision era. High precision measurements (at % level) are necessary in order to solve such fundamental problems of neutrino physics as
Table 1: Values of neutrino oscillation parameters obtained from the global fit of existing data [11]

| Parameter | Normal Ordering | Inverted Ordering |
|-----------|-----------------|-------------------|
| $\sin^2 \theta_{12}$ | $0.310^{+0.013}_{-0.012}$ | $0.310^{+0.013}_{-0.012}$ |
| $\sin^2 \theta_{23}$ | $0.582^{+0.015}_{-0.019}$ | $0.582^{+0.015}_{-0.018}$ |
| $\sin^2 \theta_{13}$ | $0.02240^{+0.00060}_{-0.00066}$ | $0.02263^{+0.00065}_{-0.00066}$ |
| $\delta$ (in °) | $(217^{+40}_{-28})$ | $(280^{+29}_{-28})$ |
| $\Delta m^2_S$ | $(7.39^{+0.21}_{-0.20}) \cdot 10^{-5}$ eV$^2$ | $(7.39^{+0.21}_{-0.20}) \cdot 10^{-5}$ eV$^2$ |
| $\Delta m^2_A$ | $(2.525^{+0.034}_{-0.031}) \cdot 10^{-3}$ eV$^2$ | $(2.512^{+0.034}_{-0.031}) \cdot 10^{-3}$ eV$^2$ |

- What is the neutrino mass ordering?
- Is CP is violated in the lepton sector and what is the precise value of CP phase $\delta$?

Neutrino oscillation experiments allow to determine two neutrino mass-squared differences $\Delta m^2_S$ and $\Delta m^2_A$. The lightest neutrino mass and, correspondingly, absolute values of neutrino masses are at present unknown.

In recent tritium KATRIN experiment [12] the following bound was found

$$m_\beta = \left( \sum_i |U_{ei}|^2 m_i^2 \right)^{1/2} < 1.1 \text{ eV}. \quad (15)$$

From different recent cosmological measurements it was obtained [13]

$$\sum_i m_i < 0.12 \text{ eV}. \quad (16)$$

## 3 On the Origin of Small Neutrino Masses

Before starting the discussion of a possible origin of neutrino masses we would like to remind that particles with spin 1/2 can be Dirac or Majorana.

Dirac field $\psi(x)$ is a complex (non hermitian) four-component field which satisfies the Dirac equation. If a Lagrangian is invariant under a global transformation $\psi(x) \rightarrow e^{iA} \psi(x)$ ($A$ is a constant) a charge is conserved and $\psi(x)$ is a field of particles and antiparticles, which have opposite charges, same masses (due to the CPT invariance) and helicities $\pm 1$. 
Majorana field $\chi(x)$ is a two-component field which satisfies the Dirac equation and the Majorana condition

$$\chi(x) = \chi^c(x) = C\chi^T(x), \quad C\gamma^T\gamma^{-1} = -\gamma, \quad C^T = -C.$$  \hfill (17)

There is no global invariance of a Lagrangian in the Majorana case. Majorana field is two-component field of truly neutral particles with helicities $\pm 1$.

Neutrino masses and mixing are generated by a neutrino mass term. The first neutrino mass term was proposed by V. Gribov and B. Pontecorvo \[14\] in 1969. At that time it was established that the Lagrangian of the weak interaction had $V - A$ current $\times$ current form

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} j^{CC}\tilde{j}^{CC}$$  \hfill (18)

where the leptonic current was given by the expression

$$j^{CC,\text{lep}} = 2(\bar{\nu}_e L \gamma_\alpha e_L + \bar{\nu}_\mu L \gamma_\alpha \mu_L).$$  \hfill (19)

Gribov and Pontecorvo put themselves the following question: is it possible to introduce neutrino masses and mixing in the case if neutrino fields are left-handed $\nu_e L, \nu_\mu L$? They understood that if the total lepton number $L = L_e + L_\mu$ is not conserved it is possible to build a neutrino mass term in the case of $\nu_e L, \nu_\mu L$ fields. In fact, taking into account that the $C$-conjugated field $\nu^c_L = C\nu^{TL}_L$ is right-handed, in the general three-flavor case (see \[15\]) we have the following unique mass term

$$\mathcal{L}^M(x) = -\frac{1}{2} \sum_{i,\ell} \bar{\nu}_\ell(x)M^M_{\ell i}\nu^c_i(x) + h.c., \quad M^M = (M^M)^T.$$  \hfill (20)

The matrix $M^M$ can be diagonalized as follows

$$M^M = U m U^T, \quad U^\dagger U = 1,$$  \hfill (21)

where $m_{ik} = m_i \delta_{ik}, \ m_i > 0$. From (20) and (21) we find

$$\mathcal{L}^M(x) = -\frac{1}{2} \sum_{i=1}^{3} m_i \bar{\nu}_i(x)\nu_i(x),$$  \hfill (22)

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2It was a common opinion at that time that left-handed neutrinos are massless.
where $\nu_i(x)$, the field of neutrino with mass $m_i$, satisfies the Majorana condition

$$\nu_i(x) = \nu_i^c(x) = C\bar{\nu}_i^T(x).$$

The flavor field $\nu_{L}(x)$ is a "mixed" field

$$\nu_{L}(x) = \sum_{i=1}^{3} U_{li} \nu_{iL}(x), \quad (l = e, \mu, \tau)$$

The mass term $\mathcal{L}^M$ is called the Majorana mass term. It is the only possible mass term in which the left-handed flavor fields $\nu_{L}$ enter. We would like to stress that in the framework of purely phenomenological approach, we discussed, neutrino masses $m_i$ (and mixing matrix $U$) are parameters. There are no any clues, why neutrino masses are much smaller then lepton and quarks masses.

Origin of neutrino masses and neutrino mixing is an open problem. Exist many different models. It is commonly suggested that the Standard Model neutrinos are massless particles.

Masses of quarks and leptons are of the Standard Model origin. They are generated by $SU_L(2) \times U_Y(1)$ invariant Yukawa interactions. In the case of leptons the Yukawa Lagrangian has the form

$$\mathcal{L}^Y_l = -\sqrt{2} \sum_{l' \neq l} \bar{\psi}_{lL} Y_{ll'} l_R \phi + h.c.$$  

Here

$$\psi_{lL} = \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix}, \quad (l = e, \mu, \tau), \quad \phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}$$

are lepton and Higgs doublets, $l_R$ is a singlet, $Y$ is a dimensionless complex matrix. After spontaneous symmetry breaking and diagonalization of $Y$ we come to the Dirac mass term

$$\mathcal{L}^Y_l(x) = -\sum_{l=e,\mu,\tau} m_l \bar{l}(x) l(x).$$

Here $l(x) = l_L(x) + l_R(x)$ is the Dirac field of leptons $l^-$ ($Q = -1$) and antileptons $l^+$ ($Q = 1$). The lepton mass $m_l$ is given by the relation

$$m_l = y_l \nu \quad (l = e, \mu, \tau).$$
Here $y_i$ is a Yukawa coupling (eigenvalue of the matrix $Y$) and $v = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV is the Higgs vev (electroweak scale).

All SM masses (masses of quarks, leptons, $W^\pm$ and $Z^0$ bosons, the mass of the Higgs boson) are proportional to $v$.

If neutrino masses are also of the Standard Model origin in this case

- Right-handed singlets $\nu_{iR}$ enter into Lagrangian.
- The total lepton number is conserved and neutrinos with definite masses $\nu_i$ are Dirac particles.

Neutrino masses are given by the expression

$$m_i = y^\nu_i v.$$  \hspace{1cm} (29)

Yukawa constants are determined by masses. For quarks and lepton of the third family we have

$$y_t \simeq 7 \cdot 10^{-1}, \quad y_b \simeq 2 \cdot 10^{-2}, \quad y_\tau \simeq 7 \cdot 10^{-3}. \hspace{1cm} (30)$$

Absolute values of neutrino masses are not known at present. However, assuming normal ordering of neutrino masses and using a conservative cosmological bound ($\sum_i m_i < 1$ eV) for the largest neutrino mass $m_3$ and neutrino Yukawa coupling $y^\nu_3$ we find the following bounds

$$(5 \cdot 10^{-2} \lesssim m_3 \lesssim 3 \cdot 10^{-1}) \text{ eV}, \quad 2 \cdot 10^{-13} \lesssim y^\nu_3 \lesssim 10^{-12}. \hspace{1cm} (31)$$

Yukawa couplings of quarks and lepton of the same family differ by about two orders of magnitude. Neutrino Yukawa coupling $y^\nu_\tau$ is about ten orders of magnitude smaller than Yukawa couplings of top and bottom quarks and $\tau$-lepton. It is very unlikely that neutrino masses are of the same Standard Model origin as masses of leptons and quarks. The Standard Model with left-handed, massless $\nu_e, \nu_\mu, \nu_\tau$ (without right-handed neutrino fields) is a minimal theory, originally proposed by Weinberg and Salam. We come to the conclusion that in order to generate small neutrino masses, observed in neutrino oscillation experiments, we need a new beyond the Standard Model mechanism.

\textsuperscript{3}This is connected with the fact that $v$ is the only parameter of the Standard Model which has a dimension M.
A general method which allow to describe effects of a beyond the Standard Model physics is a method of the effective Lagrangian. Effective Lagrangian is a dimension five or more non renormalizable Lagrangian built from the Standard Model fields and invariant under \( SU(2)_L \times U(1)_Y \) transformations.

Effective Lagrangians are generated by beyond the Standard model interactions of SM particles with heavy particles with masses much larger than \( v \). In the electroweak region such interactions induce processes with virtual heavy particles, which are described by effective Lagrangians (fields of heavy particles are "integrated out"). Typical example is the four-fermion, dimension six, Fermi effective Lagrangian of the weak interaction.

In order to built an effective Lagrangian which generate a neutrino mass term, let us consider dimension \( M^{5/2} \) \( SU_L(2) \times U_Y(1) \) invariant

\[
(\tilde{\phi}^\dagger \psi_{lL}), \tag{32}
\]

where \( \tilde{\phi} = i\tau_2\phi^* \) is a conjugated Higgs doublet. After spontaneous symmetry breaking we have

\[
(\tilde{\phi}^\dagger \psi_{lL}) \rightarrow \frac{v}{\sqrt{2}} \nu_{lL}. \tag{33}
\]

From (33) it is obvious that (like in the Gribov-Pontecorvo case) we can build an effective Lagrangian which generates a neutrino mass term only if \textit{the total lepton number is not conserved}. We come to the following unique expression for the effective Lagrangian (Weinberg [16])

\[
\mathcal{L}_I^W = -\frac{1}{\Lambda} \sum_{l',l} (\tilde{\phi}^\dagger \psi_{l'L}) X_{l'l} (\tilde{\phi}^\dagger \psi_{lL})^c + \text{h.c.} \tag{34}
\]

Here \( X \) is \( 3 \times 3 \) dimensionless, symmetrical matrix and \( \Lambda \) is a parameter which has dimension \( M \) (the operator in \( \mathcal{L}_I^{\text{eff}} \) has a dimension \( M^5 \)). The parameter \( \Lambda \) characterizes a scale of a beyond the SM physics.

In connection with non conservation of \( L \) by the Lagrangian (34) we would like to make the following general remark. Global invariance and conservation of \( L \) (and \( B \)) is not a fundamental symmetry of QFT [17, 18]. Local gauge symmetry ensure conservation of \( L \) (and \( B \)) by the Standard Model Lagrangian. It is natural to expect that a beyond the Standard Model theory does not conserve \( L \) (and \( B \)).

After spontaneous symmetry breaking from (34) we come to the Majorana
mass term

\[ L^M = -\frac{1}{2} \sum_{l',l} \bar{\nu}_{l'} L \frac{v^2}{\Lambda} X_{l' l} \nu_{l L}^c + \text{h.c.} = -\frac{1}{2} \sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i. \]  

(35)

Here \( \nu_i = \nu_i^c \) is the field of the Majorana neutrino with the “seesaw mass”

\[ m_i = \frac{v^2}{\Lambda} x_i = \frac{v}{\Lambda} (x_i v), \]  

(36)

where \( x_i \) is the eigenvalue of the matrix \( X \). In (36) \( (x_i v) \) is a “typical” SM mass. Thus, the generation of neutrino masses via the effective Lagrangian mechanism leads to a suppression factor

\[ \frac{v}{\Lambda} = \frac{\text{EW scale}}{\text{scale of a new physics}}. \]  

(37)

There are two unknown parameters \( (x_i \) and \( \Lambda) \) in (36). Thus, values of neutrino masses can not be predicted. However, if \( \Lambda \gg v \) in this case Majorana neutrino masses \( m_i \) are naturally much smaller than masses of leptons and quarks.

Notice that assuming \( x_3 \simeq 1 \) (like Yukawa coupling of the top quark) for the scale of a new physics, responsible for neutrino masses, we find

\[ \Lambda \simeq (10^{14} - 10^{15}) \text{ GeV}. \]  

(38)

4 On the Origin of the Weinberg Effective Lagrangian

In this section we will briefly discuss a possible origin of the Weinberg effective Lagrangian (33). We will start with the simplest and most economical scenario. Let us assume that lepton-Higgs pairs interact with heavy Majorana leptons \( N_i = N_i^c \) \( (i = 1, 2, \ldots n) \), \( SU_L(2) \) singlets, via \( SU_L(2) \times U_Y(1) \) interaction

\[ L_I = -\sqrt{2} \sum_{l, i} (\bar{\nu}_{l L} \phi) y_{l i} N_{i R} + \text{h.c.} \]  

(39)

Here \( y_{l i} \) are dimensionless constants.
In the tree approximation for low-energy processes with virtual heavy leptons at \( Q^2 \ll M_i^2 \) we obtain the Weinberg effective Lagrangian in which

\[
\frac{1}{\Lambda} X_{\nu i} = \sum_{i=1}^{n} y_{\nu i} \frac{1}{M_i} y_{i}. \tag{40}
\]

Thus, masses of heavy Majorana leptons determine the scale \( \Lambda \).

The mechanism, we discussed, is called the type-I seesaw mechanism \([19, 20, 21, 22, 23]\). Notice, that the Weinberg effective Lagrangian can be also generated by interaction of heavy triplet scalar bosons with a Higgs pair and lepton pair (type-II seesaw mechanism) and by interaction of lepton-Higgs pairs with heavy Majorana triplet leptons (type-III seesaw mechanism).

There exist numerous radiative neutrino mass models which lead to the Weinberg effective Lagrangian and, correspondingly, to the Majorana neutrino mass term. In these models values of neutrino masses \( m_i \) are suppressed by loop mechanisms which require existence of different beyond the Standard Model particles with masses which could be much smaller than \( 10^{15} \) GeV (see review \([24]\)).

5 Conclusion

In the first part of this report we considered a convenient phenomenology of neutrino oscillations in vacuum. In the second part we discussed a possible (and plausible) origin of neutrino masses.

The approach, we considered, is based on the following general assumptions

1. There exist a beyond Standard Model interaction(s) of SM lepton and Higgs doublets and new particles whose masses are much larger than the electroweak scale \( v \).

2. Standard Model neutrinos are massless left-handed particles.

Beyond the SM interactions (after fields of heavy particles are integrated out) generate in the electroweak region an effective Lagrangian. From 2. it follows that independently on a type of model, tree-level or radiative, the only possible effective Lagrangian is \( L \)-violating, dimension five Weinberg Lagrangian which lead to the most economical Majorana mass term.
The effective Lagrangian method of the generation of neutrino masses can explain (and, apparently, was inspired by) the smallness of neutrino masses. Values of neutrino masses $m_i$, neutrino mixing angles and $CP$ phase unknown parameters which depend on model and can not be predicted.

However, the following features are common for all models we discussed (in this sense are model independent)

1. The number of neutrinos with definite masses $\nu_i$ is equal to the number of lepton flavors (three).

2. Neutrinos with definite masses $\nu_i$ are Majorana particles.

Thus, the effective Lagrangian method of neutrino mass generation predicts that there are no transitions of flavor neutrinos into sterile states. As it is well known, indications in favor of fourth (sterile) neutrino $m_4$ with mass in the range $(10^{-1} \lesssim m_4 \lesssim 10)$ eV were obtained in different short baseline neutrino experiments. About 25 years ago in the accelerator LSND experiment indications in favor of $\bar{\nu}_\mu \to \bar{\nu}_e$ were found. Later, these indications were confirmed by the MiniBooNE accelerator experiment in which transitions $\nu_\mu \to \nu_e$ were studied. The sterile neutrino anomaly was found also by reanalysis of old reactor neutrino experiment data and by analysis of the data of GALLEX and SAGE Gallium calibration experiments (see recent review [25]).

Several new short baseline reactor, accelerator, atmospheric and source neutrino experiments are going on or in preparations at present. From existing data it is not possible to make definite conclusions on the existence of sterile neutrinos (see talks presented at the NEUTRINO2020 conference http://nu2020.fnal.gov).

Notice, however, that recent combined analysis of the data of the reactor Daya Bay and Bugey-3 experiments and accelerator MINOS+ experiment, allows to exclude at 90 % CL LSND and MiniBooNE allowed regions for $\Delta m^2_{14} < 5$ eV$^2$ [26], in new reactor DANSS experiment [27] the best-fit point in the allowed region of previous reactor experiments is excluded at $5\sigma...$ etc.

The study of neutrinoless double $\beta$-decay $(A,Z) \to (A,Z + 2) + e^- + e^-$ is the most sensitive way which could allow us to reveal the Majorana nature of neutrinos with definite masses $\nu_i$. In recent experiments the following lower limits on half-lives of the $0\nu\beta\beta$-decay of different nuclei were reached: $T_{1/2}^{(76}\text{Ge}) > 9 \cdot 10^{25}$ yr (GERDA) [28], $T_{1/2}^{(136}\text{Xe}) > 10.7 \cdot 10^{25}$ yr (KamLAND-Zen) [29], $T_{1/2}^{(130}\text{Te}) > 3.2 \cdot 10^{25}$ yr (CUORE) [30].
About one-two orders of magnitude larger half-lives are expected, if neutrinos are Majorana particles. In future $0\nu\beta\beta$-experiments such sensitivities are planned to be reached (see [31]).

Summarizing, we discussed a plausible (apparently, the most plausible) scenarios of the origin of neutrino masses, based on such fundamental hypotheses as a total lepton number violation by beyond the SM interactions. Crucial tests of this scenarios can be realized in experiments on

- The search for light sterile neutrinos.
- The search for neutrinoless double $\beta$-decay.

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