Effect of pressure dependent viscosity on couple stress squeeze film lubrication between porous circular stepped plates

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Abstract: In this paper, the effect of PDV on the couple stress squeeze film lubrication between porous circular stepped plates is presented. Keeping the base of Christensen’s stochastic theory modified Reynolds equation is derived. Reynolds equation, fluid film pressure, squeeze film time and load carrying capacity are solved using standard perturbation technique. The results are tabulated and presented graphically for selected physical parameters and found that the squeeze effect is depleted in a porous bearing compared to its nonporous and increasing permeability has an adverse effect on the pressure, load carrying capacity and time of approach.

1. Introduction:

From past two decades study on porous squeeze film bearings attracted by many researchers because of wide applications in engineering namely lubrication of film elements, artificial joints, automatic transmissions and internal combustion engines etc. Porous bearings are useful because of self lubricant characteristics and low cost. Initially Morgan and Cameron [1] analyzed under steady conditions on narrow porous journal bearing. Later Prakash and Vij [2] studied on non rotating porous journal bearing. Researchers [3-8] studied on anisotropic porous rectangular plates with lubricants containing polar additives in squeeze film lubrication. Hanumagowda et al.[9] extended the work on MHD and surface roughness on circular stepped plates.

Available literature shows that not much work has been carried out on the effect of viscosity on porous circular stepped plates. We can see some of the studies on bio-lubricated joints and squeeze film
bearings by the researchers [10-18]. Considering earlier works Gould [19] noticed that viscosity and pressure play an important role especially in the high pressure squeeze films, Reddy et. al. [20], Lin and Lin [21-23], Siddangouda et al. [24] and Naduvnamani, et al. [25] worked on the variation of viscosity with pressure. Resulting the effect of viscosity-pressure dependency and couple stress fluid as a lubricant causes an increase in the squeeze film time and load-carrying capacity.

Considering the flexibility of porous bearings and their applications, in the present article, authors worked on the effects of pressure dependent viscosity (PDV) between porous circular stepped plates on the couple stress squeeze film. The obtained numerical results are found to be good agreement with available literature. Further, the results reveal many curious behaviors that warrant further study on non-Newtonian couple stress fluid in the presence of PDV.

2. Mathematical Formulation

![Physical model and geometry](image-url)

Fig-1: Physical model and geometry

The physical representation of circular stepped plates with velocity \( V (= dh/dt) \) is shown in Figure 1. The effect of pressure dependency on the fluid viscosity \( \mu \) is a function of both pressure and temperature. The pressure dependent viscosity relation was analyzed by Barus [26] and Bartz and Ether [27].

\[
\mu = \mu_0 e^{\rho p}
\]  

(1)
where $\mu$ and $\mu_0$ are the coefficients of PDV and viscosity at ambient pressure with constant temperature respectively.

The basic equations of motion with viscosity variations are

$$\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial v}{\partial y} = 0$$

(2)

$$\frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) - \eta \frac{\partial^4 u}{\partial y^4} = \frac{\partial p}{\partial r}$$

(3)

$$\frac{\partial p}{\partial y} = 0$$

(4)

where $u$ and $v$ are components of velocity in $r$ and $y$ direction respectively, pressure $p$, dynamic viscosity $\mu$ and material constant $\eta$ are parameters of couple stress fluids.

Couple stress fluid in the porous matrix is given by

$$u^* = \frac{-k}{\mu(1-\alpha)} \frac{\partial p^*}{\partial r}$$

(5)

$$v^* = \frac{-k}{\mu(1-\alpha)} \frac{\partial p^*}{\partial y}$$

(6)

BC’s are

$$u = 0 \quad , \quad v = -\frac{\partial h}{\partial t} \quad \frac{\partial^2 u}{\partial y^2} = 0$$

(7a)

i) upper region $y=h$:

$$u = 0 \quad , \quad v = -v^* \quad \frac{\partial^2 u}{\partial y^2} = 0$$

(7b)

ii) lower region $y=0$:

Use the relation (1), (7a) and (7b) in equation (3) we get

$$u = -\frac{1}{2\mu} \frac{\partial p}{\partial r} \left[ y^2 - hy + 2l^2 \left\{ 1 - \frac{Cosh\left((2y-h)/2l\right)}{Cosh(h/2l)} \right\} \right]$$

(8)

where $l = \sqrt{\eta/\mu_0}$ is the couple stress parameter.
Considering porous layer thickness $\delta$ is very small and pressure continuity condition \( p = p^* \) at the porous interface \( (y = 0) \), is reduces to

\[
\frac{\partial p^*}{\partial y} \bigg|_{y=0} = -\delta \frac{\partial^2 p}{\partial r^2}
\]

volume flow rate is

\[
Q = 2\pi r \int_0^h u dy
\]  
(9)

Substituting equation (8) in equation (9) gives volume flux,

\[
Q = \frac{\pi r}{6\mu_0} \frac{\partial p}{\partial r} A(h, l, \beta, p, \psi)
\]  
(10)

Substituting the equation (8) in (2) and integrate across film thickness using (7a) and (7b) obtain

\[
\frac{1}{r} \frac{\partial}{\partial r} \left\{ A(h, l, \beta, p, \psi) r \frac{\partial p}{\partial r} \right\} = 12\mu_0 \frac{dh}{dt}
\]  
(11)

\[
A(h, l, \beta, p, \psi) = h^3 e^{-\beta p} - 12l^2 h e^{-2\beta p} + 24l^3 e^{-2.5\beta p} \tanh(e^{-0.5\beta p} h / 2l) + \frac{12\delta e^{-\beta p}}{(1 - \alpha)}
\]  
(12)

the non-dimensional quantities are

\[
r^* = r / R, \quad p^* = \frac{ph^3}{\mu_0 R^2 (-dh/dt)}, \quad h^* = h / h_2, \quad l^* = l / h_2, \quad \kappa = \frac{\beta \mu_0 R^2 (-dh/dt)}{h^3}, \quad Q^* = \frac{Q_0 R^2}{h^3}, \quad \psi = k\delta
\]

Use the non-dimensional quantities in equations (10) and (11) we get in the form

\[
\frac{\partial}{\partial r} \left\{ A^* (h^*, l^*, p^*, \kappa, \psi), r^* \frac{\partial p^*}{\partial r} \right\} = -12r^*
\]  
(13)

\[
Q^* = \frac{\pi r^*}{6} \frac{\partial p^*}{\partial \chi} A^* (h^*, l^*, \kappa, p^*, \psi)
\]  
(14)

where

\[
A^* (h^*, l^*, \kappa, p^*, \psi) = e^{-\kappa p^*} h^* - 12l^2 e^{-2\kappa p^*} h^* + 24l^3 e^{-2.5\kappa p^*} \tanh(e^{-0.5\kappa p^*} h^* / 2l^*) + \frac{12\psi e^{\kappa p^*}}{(1 - \alpha)}
\]  
(15)
The non-dimensional Reynolds equation (13) is highly non-linear. Perturbation method for the film pressure is

\[ p^* = p_{0i}^* + \kappa p_{1i}^*, \quad i = 1, 2 \] \hspace{1cm} (16)

in region I: \[ p^*_i = P_0^* = P_{01}^* + \kappa p_{11}^* \], \[ h_i^* = h_i^* \]; \( (0 \leq r^* \leq K) \)

in region II: \[ p^*_i = P_2^* = p_{02}^* + \kappa p_{12}^* \], \[ h_i^* = 1 \]; \( (K \leq r^* \leq 1) \)

the equation (13) by neglecting second and higher order of \( \kappa \), we get pressure \( P_{0i}^* \) and \( p_{1i}^* \)

\[ \frac{\partial}{\partial r} \left( r \frac{\partial p_{0i}^*}{\partial r} \right) = \frac{-12r^*}{A_{0i}^* (h_i^*, I^*, \psi)} \] \hspace{1cm} (17)

\[ \frac{\partial}{\partial r} \left( r \frac{\partial p_{1i}^*}{\partial r} \right) = \frac{A_{1i}^* (h_i^*, I^*, \psi)}{A_{0i}^* (h_i^*, I^*, \psi)} \frac{\partial}{\partial r} \left( \frac{\partial p_{0i}^*}{\partial r} \right) \] \hspace{1cm} (18)

Region I \( (0 \leq r^* \leq K) \); Reynolds equations is given by

\[ \frac{\partial}{\partial r} \left( r \frac{\partial p_{01}^*}{\partial r} \right) = \frac{-12r^*}{A_{01}^* (h_i^*, I^*, \psi)} \] \hspace{1cm} (19)

\[ \frac{\partial}{\partial r} \left( r \frac{\partial p_{11}^*}{\partial r} \right) = \frac{-A_{1i}^* (h_i^*, I^*, \psi)}{A_{01}^* (h_i^*, I^*, \psi)} \frac{\partial}{\partial r} \left( \frac{\partial p_{01}^*}{\partial r} \right) \] \hspace{1cm} (20)

where

\[ A_{01}^* (h_i^*, I^*, \psi) = h_i^* - 12I^2h_i^* + 24I^3 \tanh \left( h_i^* / 2I^* \right) + \frac{12\psi}{(1-\alpha)} \] \hspace{1cm} (21)

\[ A_{1i}^* (h_i^*, I^*, \psi) = -h_i^* + 6I^2h_i^* \left\{ 4 + \text{sech}^2 \left( h_i^* / 2I^* \right) \right\} - 60I^3 \tanh \left( h_i^* / 2I^* \right) - \frac{12\psi}{(1-\alpha)^2} \] \hspace{1cm} (22)

Region II \( (K \leq r^* \leq 1) \); Reynolds equations is

\[ \frac{\partial}{\partial r} \left( r \frac{\partial p_{02}^*}{\partial r} \right) = \frac{-12r^*}{A_{02}^* (I^*, \psi)} \] \hspace{1cm} (23)
\[
\frac{\partial}{\partial r} \left\{ r^* \frac{\partial p_{12}^{*}}{\partial r^*} \right\} = -\frac{A_{12}^{*}}{A_{02}^{*}} \left( I^*, \psi \right) \frac{\partial}{\partial r^*} \left\{ r^* \frac{\partial p_{02}^{*}}{\partial r^*} \right\}
\]

(24)

where

\[
A_{12}^{*} \left( I^*, \psi \right) = 1 - 12I^{*2} + 24I^{*3} \tanh \left( \frac{1}{2} I^{*} \right) + \frac{12\psi}{(1-\alpha)}
\]

(25)

\[
A_{02}^{*} \left( I^*, \psi \right) = -1 + 6I^{*2} \left\{ 4 + \csc^2 \left( \frac{1}{2} I^{*} \right) \right\} - 60I^{*3} \tanh \left( \frac{1}{2} I^{*} \right) - \frac{12\psi}{(1-\alpha)^2}
\]

(26)

The boundary conditions for the pressure

\[
dP_1^{*} \frac{dr^*}{dr} = 0 \quad \text{at} \quad r^* = 0
\]

(27)

\[
P_2^{*} = 0 \quad \text{at} \quad r^* = 1
\]

(28)

\[
P_1^{*} = P_2^{*} \quad \text{at} \quad r^* = K
\]

(29)

\[
Q_1^{*} = Q_2^{*} \quad \text{at} \quad r^* = K
\]

(30)

where \(Q_1^{*}\) and \(Q_2^{*}\) are non-dimensional volume flow rate in region I and II.

Solve equations (19), (20), (21) and (22) using the BC’s (27), (28), (29) and (30) gives

Region I, \(0 \leq r^* \leq K\); Pressure is

\[
P_1^{*} = \frac{3(K^2 - r^{*2})}{A_{00}^{*} \left( h_{1}^{*}, I^{*}, \psi \right)} + \frac{3(1-K^2)}{A_{01}^{*} \left( I^{*}, \psi \right)} - \kappa \left[ \frac{9A_{10}^{*} \left( h_{1}^{*}, I^{*}, \psi \right)}{2A_{00}^{*} \left( h_{1}^{*}, I^{*}, \psi \right)} \left\{ \left( K^2 - r^{*2} \right)^2 + \frac{2(1-K^2)(K^2 - r^{*2})}{A_{01}^{*} \left( I^{*}, \psi \right)} + \frac{9A_{11}^{*} \left( I^{*}, \psi \right) (1-K^2)^2}{2A_{01}^{*} \left( I^{*}, \psi \right)} \right\} \right]
\]

(31)

Region II, \(K \leq r^* \leq 1\); Pressure is

\[
P_2^{*} = \frac{3(1-r^{*2})}{A_{11}^{*} \left( I^{*}, \psi \right)} - \kappa \left[ \frac{9A_{11}^{*} \left( I^{*}, \psi \right)(1-r^{*2})^2}{2A_{11}^{*} \left( I^{*}, \psi \right)} \right]
\]

(32)

Load carrying capacity \(W\) is

\[
W = 2\pi \int_{0}^{KR} rP_1^{*} dr + 2\pi \int_{KR}^{1} rP_2^{*} dr
\]

The non-dimensional form is

\[
W^* = \frac{Wh_0^3}{\mu KR^4 (-dh/\,dt)} = 2\pi \int_{0}^{K} rP_1^{*} dr^* + 2\pi \int_{K}^{1} rP_2^{*} dr^*
\]
\[ W^* = \frac{3\pi K^4}{2A_0^{\prime 0}(h^*_i, l^*, \psi)} + \frac{3\pi(1 - K^4)}{2A_0^{\prime 0}(l', \psi)} - \kappa \left[ \frac{3\pi A_0^{\prime 0}(h^*_i, l^*, \psi)}{2A_0^{\prime 0}(h^*_i, l^*, \psi)} \left( \frac{K^6}{A_0^{\prime 0}(h^*_i, l^*, \psi)} + \frac{3(1 - K^2)K^4}{A_0^{\prime 0}(l', \psi)} \right) \right] + \frac{3\pi A_0^{\prime 0}(l', \psi)(2K^6 - 3K^4 + 1)}{2A_0^{\prime 0}(l', \psi)} \]  

The squeezing time for reducing the film thickness from an initial value \( h^*_2 = 1 \) to a final value \( h^*_f \) is given by

\[ T^* = \frac{Wh_0^2}{\mu_i R^2} = \int_{h^*_2}^{h^*_f} \left[ \frac{3\pi K^4}{2A_0^{\prime 0}(h^*_2, h^*_j, l^*, \psi)} + \frac{3\pi(1 - K^4)}{2A_0^{\prime 0}(h^*_2, l^*, \psi)} - \kappa \left[ \frac{3\pi A_0^{\prime 0}(h^*_2, h^*_j, l^*, \psi)}{2A_0^{\prime 0}(h^*_2, l^*, \psi)} \left( \frac{K^6}{A_0^{\prime 0}(h^*_2, l^*, \psi)} + \frac{3(1 - K^2)K^4}{A_0^{\prime 0}(l^*, \psi)} \right) \right] + \frac{3\pi A_0^{\prime 0}(l^*, \psi)(2K^6 - 3K^4 + 1)}{2A_0^{\prime 0}(l^*, \psi)} \right] dh^*_2 \]  

where

\[ A_0^{\prime 0}(h^*_2, h^*_j, l^*, \psi) = \left( h^*_2 + h^*_j \right)^3 - 12l^2\left( h^*_2 + h^*_j \right) + 24l^3 \tanh \{ \left( h^*_2 + h^*_j \right)/2l^* \} + \frac{12\psi}{(1 - \alpha)}\]  

\[ A_0^{\prime 0}(h^*_2, l^*, \psi) = \left( h^*_2 + h^*_j \right)^3 + 6l^2\left( h^*_2 + h^*_j \right) \left[ 4 + \text{sech}^2 \{ \left( h^*_2 + h^*_j \right)/2l^* \} \right] - 60l^3 \tanh \{ \left( h^*_2 + h^*_j \right)/2l^* \} - \frac{12\psi}{(1 - \alpha)^2} \]  

\[ A_0^{\prime 0}(l^*, \psi) = h^*_2 - 12l^2h^*_2 + 24l^3 \tanh \left( h^*_2/2l^* \right) + \frac{12\psi}{(1 - \alpha)} \]  

\[ A_0^{\prime 0}(l^*, \psi) = -h^*_2^3 + 6l^2h^*_2 \left( 4 + \text{sech}^2 \{ h^*_2/2l^* \} \right) - 60l^3 \tanh \left( h^*_2/2l^* \right) - \frac{12\psi}{(1 - \alpha)^2} \]  

where \( h^*_2 = h_2, h^*_j = h_j, l^* = l/l_0 \).

**Limiting case:** The limiting cases of the present study give excellent agreements of results with the previous contributions.

1. In absence of permeability parameter \( \psi \) and viscosity \( \kappa \to 0 \), this work reduces to Naduvanamani and Siddanagouda [8].
2. In absence of permeability and step length \( K=1 \), our study reduces to Lin et al., [22].
3. In the absence of permeability, this work is same as Hanumagowda [28].
3. Results and Discussion

In this model our focus is to analyze the characteristics with respect to dimensionless parameters, namely, couple-stress parameter $l^*$, radial parameter $r^*$, pressure $P^*$, squeeze film time $T^*$, load carrying capacity $W^*$, which are the functions of permeability parameter $\nu$ and pressure dependent viscosity $\kappa$ and the step length $K$. In order to illustrate the salient features of the permeability and viscosity phenomena, the numerical results are presented in Table 1, Table 2 and Figs. 2–13. To demonstrate the accuracy of the present method, results are compared for the available literature for a special case (see in table-1) and the comparison shows a very good agreement with the results reported by Hanumagowda [28].

![Figure 2](image-url)  

**Figure 2** Variation of non-dimensional pressure $P^*$ with $r^*$ for different values of $l^*$ with $h^* = 0.6$, $K = 0.7$, $\kappa = 0.04$, $\nu = 0.001$, $\beta = 0.1$. 
Figure 3 Variation of non-dimensional pressure $P^*$ with $r^*$ for different values of $\kappa$ with $h^* = 0.6$, $K = 0.7$, $l^* = 0.3$, $\psi = 0.001$, $\beta = 0.1$.

Figure 4 Variation of non-dimensional pressure $P^*$ with $r^*$ for different values of $\psi$ with $h^* = 0.6$, $l^* = 0.3$, $K = 0.7$, $\kappa = 0.04$, $\beta = 0.1$.
Figure 5 Variation of non-dimensional pressure $P^*$ with $K$ for different values of $\psi$ with $h^* = 0.6$, $l^* = 0.3$, $\kappa = 0.04$, $\beta = 0.1$, $r^* = 0$.

Figure 6 Variation of non-dimensional Load carrying capacity $W^*$ with $h^*$ for different values of $l^*$, with $\kappa = 0.04$, $K = 0.7$, $\psi = 0.001$, $\beta = 0.1$. 
Figure 7 Variation of non-dimensional Load carrying capacity $W^*$ with $h^*$ for different values of $\kappa$, with $\ell^* = 0.3$, $K = 0.7$, $\psi = 0.001$, $\beta = 0.1$.

Figure 8 Variation of non-dimensional Load carrying capacity $W^*$ with $h^*$ for different values of $\psi$, with $\kappa = 0.04$, $K = 0.7$, $\ell^* = 0.3$, $\beta = 0.1$. 
Figure 9 Variation of non-dimensional Load carrying capacity $\bar{W}$ with $K$ for different values of $\psi$, with $\kappa = 0.04$, $\bar{l} = 0.3$, $\beta = 0.1$, $h^* = 1.2$.

Figure 10 Variation of squeeze film time $\bar{T}$ with $h^*$ for different values of $\bar{l}$, with $G = 0.04$, $K = 0.7$, $\psi = 0.001$, $\beta = 0.1$, $h^*_1 = 0.2$. 
Figure 11 Variation of squeeze film time $T^*$ with $h_f^*$ for different values of $\kappa$, with $l^* = 0.3$, $K = 0.7$, $\psi = 0.001$, $\beta = 0.1$, $h_0^* = 0.2$.

Figure 12 Variation of squeeze film time $T^*$ with $h_f^*$ for different values of $\psi$ with $l^* = 0.3$, $\kappa = 0.04$, $\beta = 0.1$, $h_0^* = 0.2$. 
Figure 13 Variation of squeeze film time $T^*$ with $K$ for different values of $\varphi$ with $l^* = 0.3$, $\kappa = 0.04$, $\beta = 0.1$, $h^*_f = 0.4$, $h'^*_f = 0.2$.

Table 1 Viscosity and couple stress characteristics $W^*$, $T^*$ and comparison with the case by Hanumagowda [28] with $K = 0.7$, $l^* = 0.3$.

| Hanumagowda [28] analysis | Present analysis | $\varphi = 0$ | $\varphi = 0.001$ | $\varphi = 0.01$ |
|---------------------------|------------------|----------------|------------------|------------------|
| $\kappa$ | $h_f^* = 1.2$ | $h_f^* = 1.5$ | $h_f^* = 1.2$ | $h_f^* = 1.5$ | $h_f^* = 1.2$ | $h_f^* = 1.5$ |
| 0 | 7.91298 | 7.31479 | 7.91298 | 7.31479 | 7.74759 | 7.15906 | 6.52756 | 6.01601 |
| 0.02 | 8.05137 | 7.42956 | 8.05137 | 7.42956 | 7.88249 | 7.27097 | 6.6351 | 6.10516 |
| 0.04 | 8.18976 | 7.54434 | 8.18976 | 7.54434 | 8.0174 | 7.38287 | 6.74264 | 6.19432 |
| 0.06 | 8.32815 | 7.65912 | 8.32815 | 7.65912 | 8.15231 | 7.49477 | 6.85017 | 6.28347 |

| $\kappa$ | $h_f^* = 0.5$ | $h_f^* = 0.8$ | $h_f^* = 0.5$ | $h_f^* = 0.8$ | $h_f^* = 0.5$ | $h_f^* = 0.8$ | $h_f^* = 0.5$ | $h_f^* = 0.8$ |
|-----------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
Table 2 Variation of $R_W^*$ and $R_T^*$ for different values $\psi$ and $\kappa$ with $l^* = 0.3$, $h_1^* = 1.2$, $h_2^* = 0.3$, $\beta = 0.1$, $h_f^* = 0.5$.

| $\kappa$ | $\psi$ | $R_W^*$ | $R_T^*$ | $R_{W_1}^*$ | $R_{T_1}^*$ |
|----------|--------|---------|---------|-------------|-------------|
| 0        | 0.0001 | -0.25   | -2.23153| -0.23683    | -2.14399    |
| 0        | 0.001  | -2.44481| -17.6832| -2.31683    | -17.017     |
| 0        | 0.01   | -20.013 | -62.8931| -19.0725    | -61.1077    |
| 0.04     | 0.0001 | -0.25127| -1.58637| -0.23835    | -1.6833     |
| 0.04     | 0.001  | -2.45824| -15.5404| -2.32921    | -15.5501    |
| 0.04     | 0.01   | -20.1943| -64.7321| -19.2518    | -62.7774    |
| 0.06     | 0.0001 | -0.25187| -1.33058| -0.23907    | -1.49476    |
| 0.06     | 0.001  | -2.46469| -14.691 | -2.34062    | -14.9476    |
| 0.06     | 0.01   | -20.2801| -65.4613| -19.3369    | -63.4632    |

3.1. Pressure

The numerical results for the non-dimensional pressure $P^*$ for several values of the governing parameters are presented in Figures 2-5. The variation of $P^*$ along $r^*$ for different values of $\kappa$, $\psi$ and $l^*$ are depicted keeping $h^*$ fixed. This shows $P^*$ is maximum at $r^* = 0$, addition to that for increasing values of $r^*$ pressure reduces. The $P^*$ increases for ascending values of $l^*$ and $\kappa$. Figure 4 and 5 shows that increasing value of $\psi$ reduces the pressure and $P_{\text{max}}^*$ is decreases for ascending values of $K$. 

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3.2. Load carrying capacity

Variations of $W^*$ with $h^*$ for different values of $l^*$ and $\kappa$ for fixed $K$ can be observed in Figure 6-9. $W^*$ increases with decreasing values of permeability parameter. Variation of $W^*$ with $K$ and $\kappa$ observes that $W^*$ increases with increase in the $l^*$ and $\kappa$.

3.3. Squeeze film time

Figure 10-13 shows that the variation of $T^*$ with $h_f^*$ for different values of $l^*$ and $\kappa$ keeping $K$ unchanged. It is observed that $T^*$ decreases for ascending values of $h_f^*$ and $K$. For the increasing values of $l^*$ and $\kappa$ leads to increase the squeeze film time. For larger values of $\psi$, the time taken in reducing the film thickness is short compared to an impermeable bearing.

4. Conclusions:
The results obtained are useful for material processing industries, such as automatic transmissions, combustion engines, lubrication of film elements and artificial joints etc. Some of the interesting results are as follows:

1. The approach time of bearing surfaces increases for decrease in porosity of the surfaces.
2. Load carrying capacity is higher for smaller permeability parameter.
3. $W^*, T^*$ decreases and $P^\text{max}$ increases for ascending values of $K$.
4. $P^*, W^*, T^*$ increases for increasing values of $l^*$ and $\kappa$.

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