Befriending The Byzantines Through Reputation Scores

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Abstract

We propose two novel stochastic gradient descent algorithms, ByGARS and ByGARS++, for distributed machine learning in the presence of Byzantine adversaries. In these algorithms, reputation score of workers are computed using an auxiliary dataset with a larger stepsize. This reputation score is then used for aggregating the gradients for stochastic gradient descent with a smaller stepsize. We show that using these reputation scores for gradient aggregation is robust to any number of Byzantine adversaries. In contrast to prior works targeting any number of adversaries, we improve the generalization performance by making use of some adversarial workers along with the benign ones. The computational complexity of ByGARS++ is the same as the usual stochastic gradient descent method with only an additional inner product computation. We establish its convergence for strongly convex loss functions and demonstrate the effectiveness of the algorithms for non-convex learning problems using MNIST and CIFAR-10 datasets.

1 Introduction

With increasing data size and model complexity, the preferred method for training machine learning models at scale is to use a distributed training setting. This involves a parameter server that coordinates the training with multiple worker machines by communicating gradients and parameters. Despite the speed up in computation due to the distributed setting, it suffers from issues such as straggling workers, while also posing a significant risk to the privacy if the data is collected at a central location. Federated Learning [1], [2] addresses these issues where the central server only has access to the model parameters/gradients computed by the workers (or independent data owners). However, this setting is prone to fail in the presence of dishonest workers or non-malicious failed workers [3].

Thus, there has been a significant interest in devising distributed machine learning schemes in the presence of Byzantine adversaries [4], [5], [6], [7]. A certain fraction of the workers are assumed to be adversarial; instead of sending the actual gradients computed using a randomly sampled mini batch to the server, the adversarial workers send arbitrary or potentially adversarial gradients that could derail the optimization at the server. Several techniques have been proposed to secure the gradient aggregation against adversarial attack under different settings such as gradient encoding [8], asynchronous updates [9], [10], decentralized learning [11], [12], [13] and Federated Learning [14], [15]. Attacks that break existing defences against Byzantine adversaries have been developed in [14], [16], [17].

One of the main assumptions in past studies about Byzantine problems is that the number of adversarial workers is less than half of the total number of workers. These approaches relied on

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techniques like majority voting, geometric median, median of means, coordinate wise median, coordinate wise trimmed mean, etc. to aggregate gradients at the server. The fundamental reason for this assumption was that the underlying concept of geometric median (often used for robust aggregation) has a breakdown point of 0.5 [6]. In other words, it yields a robust estimator as long as less than half of the data (used for aggregation) is corrupted.

The assumption that less than half of the workers are adversarial might not be practical. The more challenging problem is to ensure convergence even in the presence of a large number of adversaries. Some prior works that address this case are [10], [18], [19], [20], among several others. In these works, the server has some auxiliary data, which is used to identify adversarial workers and the gradients obtained from such workers are discarded at the server. In this paper, we allow arbitrary number of adversaries by allowing the server to use auxiliary dataset to compute the reputation score of each worker that is used for gradient aggregation. As we will see, we achieve robustness to any number of adversaries by letting the reputation score of the workers take negative values.

The key insight that allows us to train under any number of adversaries is that (a) the gradients are being computed for a specific objective function, and (b) the correct gradients would make a small angle with respect to the gradient computed using the auxiliary data with high probability (when the current parameters are far from the optimal ones). To see this, note that geometric median is a robust aggregator under any objective function. Thus, it necessarily requires a more stringent assumption on the number of adversaries to compute a reliable estimate. We explicitly use an unbiased estimate of the objective function (through the auxiliary data) in our algorithm, which eases the computation and allows us to not use general purpose robust aggregator like geometric median. Further, since auxiliary dataset has (roughly) the same distribution as the original dataset, we expect that two stochastic gradients would make a small angle with each other with high probability. On the other hand, a random vector will be almost orthogonal to the correct stochastic gradients due to the intrinsic properties of random high dimensional vectors. This intuition allows us to compute the reputation score for each worker reliably early on in the training process. Indeed, we show that our algorithm enjoys a remarkable convergence guarantee under reasonable assumptions on the objective function and the nature of the adversaries.

Our contributions In this work, we do not attempt to identify the adversarial workers and discard their gradients; instead, we use the auxiliary data at the server to compute a reputation score for the workers, and use the reputation scores to weigh the gradients of the workers to carry out the weight update. Our primary contributions are:

1. We use auxiliary dataset to learn a reputation score vector to aggregate the gradients. We show that our algorithm is Byzantine tolerant (in the sense of [16]) to an arbitrary number of attackers.
2. Our work applies to situations when the adversaries send gradients perturbed by random multiplicative noise. Thus, instead of discarding the gradients computed by these adversaries, we befriend them, by learning their reputation score and using it for robust aggregation at the server.
3. We use two time-scale stochastic approximation theory to establish the convergence of the proposed algorithm under reasonable assumptions (with strongly convex loss function).
4. In the previous works, the algorithms filtered out the adversaries and reduced the effective data size for training, which affected the test error and the generalization ability of the trained model. Our algorithm makes use of the data available at some adversarial workers, thus improving the test error and generalization of trained model. In fact, we show through our experiments that our algorithm converges to an acceptable solution even when all the workers are adversaries.

2 Problem setup

We consider distributed machine learning with a parameter server - worker setup. The parameter server maintains the model parameters, and updates the parameters with gradients received from the workers. We denote the model parameters by $w \in \mathcal{W} \subset \mathbb{R}^d$, and the number of workers by $m$. We assume that each worker $j$ has access to dataset, $D_j := \{x_i^j, y_i^j\}_{i=1}^{n_j} \sim \mathcal{D}$, where $N = \sum_j n_j$ is the total number of data points. In the Federated Learning scenario, this translates to each worker having its own dataset, which is not shared with anyone. In the distributed machine learning scenario,
We start with an initial reputation score of $q$. We denote the true gradient of the population loss at $w$ by $\nabla f(w, x, y)$. The problem now is to compute a good reputation score, without the knowledge of $q$. The good workers communicate the stochastic gradient $\tilde{h}_{t,j}$ to the server, where as adversarial workers inject a random multiplicative noise $\kappa$ and send $h_{t,j} := \tilde{\kappa}_t q_t, h_{t,j}$, where $\tilde{\kappa}_t$ is a random variable that the adversary draws at each time step from a fixed attack distribution. We assume that this attack distribution remains fixed for the adversary throughout the training. We denote the set of gradients received by the server as $H_t = \{h_{t,1}, \cdots, h_{t,m}\} \in \mathbb{R}^{d \times m}$. Note that we assume a synchronous setting here, i.e., all the workers communicate the gradients at the same time to the server. We assume that the server has access to an auxiliary dataset $D_{aux} := \{x_{i}, y_{i}\}_{i=1}^{n} \sim D$. The server can sample a subset $\xi_{aux,t}$ of the auxiliary dataset and compute auxiliary loss $L_{t}(w) = \frac{1}{|\xi_{aux,t}|} \sum_{(x,y) \in \xi_{aux,t}} f(w, x, y)$, such that $\mathbb{E}[\nabla L_{t}(w)] = \nabla F(w_t)$.

3 Algorithm

In this section, we motivate the importance of using a reputation score for gradient aggregation. In an ideal environment, where all the workers are benign, the gradient aggregation function simply averages the received stochastic gradients and uses the averaged gradient to update the parameters. However, our problem setup is far from ideal, it involves an arbitrary number of workers that act as Byzantine adversaries (potentially all of them can be adversarial).

To compute a meaningful estimate of the gradient, the server maintains a reputation score $q_{t,j}$ for each worker $j$. Since the adversary can be of any type, the reputation scores can take any real value. Suppose, at time $t$, the reputation score vector is $q_t = [q_{t,1}, \cdots, q_{t,m}]^T$ and the received gradients are $H_t$, then the weighted aggregation of the gradients with the reputation score is $H_t^T q_t = \sum_{i=1}^{m} q_{t,i} h_{t,i}$. If we have a good reputation score $q^*_t$ (say we know $\kappa$ and set $q^*_t = -\kappa$) at each time $t$, that is, $-H_t^T q^*_t$ is a descent direction, then no adversary can affect the training provided that sufficiently small step size is used and the adversaries satisfy some assumptions.

The problem now is to compute a good reputation score, without the knowledge of $\kappa$, for the workers using only the gradients sent to the server. Making use of our two key assumptions – availability of an auxiliary dataset and the stationary behavior of the workers, we propose two algorithms to compute the reputation score of the workers.

3.1 ByGARS

We start with an initial reputation score of $q_0 = 0 \in \mathbb{R}^m$, and iteratively improve the estimate of the reputation score. We perform a pseudo update to $w_t$ ($\gamma_t$ is a step size parameter) as $w_{t+1} \leftarrow w_t - \gamma_t H_t^T q_t$. If $q_t$ is a good reputation score and $\gamma_t$ is sufficiently small, then $-H_t^T q_t$ is a descent direction and thus $F(w_{t+1})$ must be lower in value than $F(w_t)$ or other points in its neighborhood. However, we neither have access to the true function $F$ nor the data from the workers. Instead, we have a small auxiliary dataset that is drawn from the same distribution as the data at the workers. This auxiliary dataset allows us to construct the loss function $L_t(\cdot)$ (see Section 2), and we can solve the following optimization problem to compute a good reputation score $q_{t+1}^*$:

$$q_{t+1}^* = \arg\min_{q \in \mathbb{R}^m} L_t(w_t - \gamma_t H_t^T q)$$

Using the current estimate $q_t$, we use an iterative update rule. We compute the loss on a random mini-batch of the auxiliary dataset $D_{aux}$ using $w_t$, which is denoted as $L_t(w_t) = L_t(w_t - \gamma_t H_t^T q_t)$. The objective of the meta-update is to minimize this loss with respect to $q_t$, given $H_t, w_t$. We perform a first order update to $q_t$ ($\alpha_t$ is a step size parameter) as follows
We propose Algorithm 2 (ByGARS++), in which we avoid computing a pseudo update \( \hat{q} \). Algorithm 1 has an additional computational overhead due to multiple parameter updates and multiple meta updates. Instead, we perform an update to \( q \) directly, which takes \( \mathcal{O}(md) \) time. This increased computation at the server keeps the workers idle and waiting, thus negating the computational speed-up achieved from distributed learning. In order to overcome this limitation, and driven by the motivation of ByGARS, we propose a variant which is computationally cheaper yet efficient.

We propose Algorithm 2 (ByGARS++), in which we avoid computing a pseudo update \( \hat{w} \) used for performing meta updates. Instead, we perform an update to \( q \) using the auxiliary gradients evaluated at \( w_t \) (and not at \( w_t \)). The corresponding update equation for \( q_t \) is

\[
q_{t+1} + \alpha_t H_t \nabla L_t(w_t) = q_t - \alpha_t H_t \nabla L_t(w_t) - \alpha_t (\gamma_t H_t \nabla L_t(w_t) - \gamma_t H_t q_t). 
\]

The updated reputation score is used to find the updated gradient aggregation \( H_t^T q_t \). This updated gradient is again used to compute a pseudo update to \( w_t \), and the new pseudo \( w_t \) is used to perform a meta update to \( q_t \). This is continued for a fixed number of meta iterations (or until a stopping criteria is reached, such as sufficient descent). Now \( w_t \) is updated with the updated gradient \( H_t q_{t+1} \).

The core of our algorithm lies at the auxiliary dataset available to the worker. It is a reasonable assumption to make since in practical scenarios, it is not difficult to procure a small amount of clean auxiliary data without violating the privacy of the worker’s data. This data can be taken from publicly available datasets (that match the distribution of the data available at the workers), from prior data leaks (that is now publicly available), or data given voluntarily by workers. In supplementary material, we provide more analysis on the effect of auxiliary dataset size on the performance of our algorithms.

### 3.2 ByGARS++: faster ByGARS

Algorithm 1 has an additional computational overhead due to multiple parameter updates and multiple gradient computations to update the reputation score in the meta updates. This increased computation at the server keeps the workers idle and waiting, thus negating the computational speed-up achieved from distributed learning. In order to overcome this limitation, and driven by the motivation of ByGARS, we propose a variant which is computationally cheaper yet efficient.

We propose Algorithm 2 (ByGARS++), in which we avoid computing a pseudo update \( \hat{w} \) used for performing meta updates. Instead, we perform an update to \( q_t \) using the auxiliary gradients evaluated at \( w_t \) (and not at \( w_t \)). The corresponding update equation for \( q_t \) is

\[
q_{t+1} = q_t - \alpha_t \nabla L_t(w_t) - \alpha_t (\gamma_t H_t \nabla L_t(w_t) - \gamma_t H_t q_t). 
\]

In this case, the reputation score of each worker is updated using only the inner product between the gradient sent by the worker, and the auxiliary gradient, both evaluated at \( w_t \). The only additional computation as compared to traditional distributed SGD is the udpate of \( q_t \) which takes \( \mathcal{O}(md) \) time. However, the server can update \( q_t \) when the workers are computing the gradients for next time step, therefore ByGARS++ has the same computational complexity as traditional distributed SGD.

If a worker consistently sends the gradient scaled with a negative value, then the reputation score accumulates negative values, since the inner product is negative in expectation. Therefore by multiplying the received gradient with \( q_{t,j} \), we can recover the actual direction. When the parameters are far away from the optima, we compute the reputation score of each stochastic gradient by taking an inner product with the stochastic gradient computed on the auxiliary data. When the parameters
are closer to the optima, the inner product value is random \cite{21} (since the directions of the stochastic gradients are random), and hence does not contribute to the reputation score. If unheeded, this phenomenon can destroy the reputation of good workers/boosts that of adversaries, therefore we employ a decaying learning rate schedule for both $\gamma_t$ and $\alpha_t$. Thus, by the time the parameters are close enough to the optima or a flat region (in non-convex settings), the learning rates would have decayed significantly. This enables the reputation score to accumulate over time and converge; therefore, the score is robust to the noisy inner products near the optima.

**Algorithm 2 ByGARS++**

1: $w_0$ initialized randomly and sent to workers
2: $q_0 = 0$
3: for $t = 1, \cdots, T$ do
4: \hspace{1em} Receive $H_t^T = [h_{t,1}, \cdots, h_{t,m}]$ from workers
5: \hspace{1em} Compute $\nabla L_t(w_t)$ using a subset of the auxiliary data
6: \hspace{1em} $w_{t+1} \leftarrow w_t - \gamma_t H_t^T q_t$
7: \hspace{1em} Send $w_{t+1}$ to workers
8: \hspace{1em} $q_{t+1} \leftarrow (1 - \alpha_t)q_t + \alpha_t H_t \nabla L_t(w_t)$ \hspace{1em} $\triangleright$ Computed when workers compute gradients
9: end for
10: Return $w_{T+1}$

4 Convergence of ByGARS++

We now analyze the convergence of ByGARS++, which has the following update equation

$$w_{t+1} \leftarrow w_t - \gamma_t H_t^T q_t, \quad q_{t+1} \leftarrow (1 - \alpha_t)q_t + \alpha_t H_t \nabla L_t(w_t) \quad (5)$$

**Assumption 1.** The population loss $F$ is $c$-strongly convex, with $w_*$ as the unique global minimum, such that $\nabla F(w_*) = 0$. Further, $\nabla F$ is a locally Lipschitz function with bounded gradients.

**Assumption 2.** The Byzantine adversaries corrupt the gradients using multiplicative noise. If the worker $i$ computes a stochastic gradient $h_{t,i}$ which is an unbiased estimate of $\nabla F(w_t)$, the worker sends $h_{t,i} := \tilde{\kappa}_t,i h_{t,i}$ to the parameter server, where $\tilde{\kappa}_t,i$ is an iid multiplicative noise with mean $\kappa_i$ and finite second moment. The random noise satisfies $|\tilde{\kappa}_t,i| \leq \kappa_{\text{max}}$ almost surely for all the workers.

The workers have the following types:

1. Benign worker: $\mathbb{E}h_{t,i} = \nabla F(w_t)$ with $\kappa_i = 1$;
2. Scaled adversary: $\mathbb{E}h_{t,i} = \kappa_i \nabla F(w_t)$, where $\kappa_i$ is a real number (negative or positive);
3. Random adversary: $\mathbb{E}h_{t,i} = 0$, where adversary sends random gradients with mean 0 (i.e. $\kappa_i = 0$).

There is at least one benign or scaled adversary with $\kappa_i \neq 0$ among the workers. Further, we assume the adversaries' noise distributions do not change with time.

This is a reasonable assumption as the goal of the attacker is to derail the training progress by corrupting the aggregate gradient so that it is not a descent direction. This adversary model is used in several works including \cite{4}, \cite{9}, \cite{10}, \cite{22}, but it does not contain the label flip adversary and attacks proposed in \cite{14}, \cite{17}.

The following theorems capture the main result of this paper.

**Theorem 1.** If Assumption 2 is satisfied, then ByGARS++ is DSSGD-Byzantine Tolerant \cite{16} Def. 4, that is, $\mathbb{E}[\langle \nabla F(w_t), H_t^T q_t \rangle] \geq 0$.

**Proof.** The proof follows by applying the law of iterated expectations. We refer the reader to the supplementary material for a detailed proof. \hfill $\square$

Thus, by accruing reputation scores using the inner products for every worker, the sign of $q_{t,i}$ is the same as that of $\kappa_i$. Due to this reason, even gradients of adversaries can be helpful in training if multiplied with an appropriate reputation score. When the loss function is strongly convex and smooth, we can prove an even stronger result, which is captured below.
Theorem 2. Suppose that \( \{\alpha_t\}, \{\gamma_t\} \) are diminishing stepsizes, that is, \( \sum \alpha_t = \infty, \sum \gamma_t = \infty, \sum \alpha_t^2 < \infty, \sum \gamma_t^2 < \infty \), with \( \gamma_t/\alpha_t \to 0 \) as \( t \to \infty \). If Assumptions 1 and 2 are satisfied and \( \sup_t \|w_t\|, \sup_t \|q_t\| < \infty \), then \( \{w_t\} \) generated by ByGARS++ converges almost surely \( w* \).

The proof leverages two timescale stochastic approximation to establish the above result. The assumption of \( \sup_t \|w_t\|, \sup_t \|q_t\| < \infty \) is typical in stochastic approximation literature; see, for instance, [23, 24, 25]. To avoid making this assumption, a typical workaround is to project the iterates back to a very large set in case the iterates go outside the set [26, 24]. For single timescale stochastic approximation, [27] derives some sufficient conditions under which the iterates would be automatically bounded almost surely. However, we are unable to leverage this result since this method has not been extended to two timescale stochastic approximation. We now prove Theorem 2.

Proof of Theorem 2. We use \( \kappa = [\kappa_1, \ldots, \kappa_m]^T \) to denote the mean vector of the random multiplicative noises of the workers. We show that all the hypotheses of Theorem 2 of [25] is satisfied by ByGARS++, which leads to the desired convergence result. Consider the two functions and the corresponding differential equations (here \( w, q \) are functions of continuous time and \( \bar{w}(t) \) denotes differentiation with respect to time):

\[
G_1(w, q) = -(\kappa^T q) \nabla F(w), \quad \bar{w}(t) = G_1(w(t), \bar{q}(t)) \tag{6}
\]

\[
G_2(w, q) = -q + \kappa \|\nabla F(w)\|^2, \quad \bar{q}(t) = G_2(w, q(t)) \tag{7}
\]

Lemma 1. The differential equation in (7) has a unique globally asymptotically stable equilibrium, which is denoted by \( \phi(w) \) and is given by \( \phi(w) = \kappa \|\nabla F(w)\|^2 \). The differential equation in (6) with \( \bar{q}(t) = \phi(w(t)) \) has a unique globally asymptotically stable equilibrium \( w* \).

Proof. To see the first result, not that for any \( w \), the solution to the differential equation is \( \bar{q}(t) = (q(0) - \kappa \|\nabla F(w)\|^2) \exp(-t) + \kappa \|\nabla F(w)\|^2 \), which is globally asymptotically stable with \( \phi(w) = q(\infty) = \kappa \|\nabla F(w)\|^2 \). We now use Lyapunov stability theory to establish the second statement. Let us substitute into (6) \( \bar{q}(t) = \phi(w(t)) \). Now define the Lyapunov function \( V(w) := F(w) - F(w*) \), which is a valid Lyapunov function since \( F \) is strongly convex by Assumption 1. We get

\[
\nabla V(w(t)) = -\|\kappa\|^2 \|\nabla F(w(t))\|^4 < 0 \quad \text{for all} \quad w(t) \neq w*.
\]

Thus, the differential equation has a unique globally asymptotically stable equilibrium where the Lyapunov function is 0, that is, at \( w* \). \( \square \)

Define \( u_t = -G_1(w_t, q_t) - H_t^T q_t \) and \( v_t = H_t \nabla L_t(w_t) - \kappa \|\nabla F(w_t)\|^2 \). Using the definition of \( u_t \) and \( v_t \), the algorithm ByGARS++ is rewritten as

\[
w_{t+1} = w_t - \gamma_t H_t^T q_t = w_t + \gamma_t G_1(w_t, q_t) + \gamma_t u_t \tag{8}
\]

\[
v_{t+1} = (1 - \alpha_t) q_t + \alpha_t H_t \nabla L_t(w_t) = q_t + \alpha_t G_2(w_t, q_t) + \alpha_t v_t \tag{9}
\]

We show that \( u_t \) and \( v_t \) are martingale difference stochastic processes. Let \( (\Omega, \mathcal{F}, \mathbb{P}) \) be a standard probability space and let \( \mathcal{F}_t \) be the \( \sigma \)-algebra generated by all the randomness realized up to time \( t \):

\[\mathcal{F}_t = \sigma\{w_0, h_{1,0}, \ldots, h_{m,0}, q_0, \ldots, w_{t-1}, h_{1,t-1}, \ldots, h_{m,t-1}, q_{t-1}, w_t, q_t\}.\]

It is easy to see that \( \mathcal{F}_t \subset \mathcal{F}_{t+1} \), and thus, \( \{\mathcal{F}_t\}_{t \in \mathbb{N}} \) is a filtration. It is straightforward now to observe that \( \mathbb{E}[H_t | \mathcal{F}_t] = \kappa \nabla F(w_t)^T \), which implies that \( \mathbb{E}[u_t | \mathcal{F}_t] = 0 \). Further, it is easy to deduce that \( \mathbb{E}[\|u_t\|^2 | \mathcal{F}_t] \leq M(1 + \|q_t\|^2) \) for a large \( M > 0 \) that is dependent on the bounds on \( \|\nabla F(w_t)\| \) and \( \|\kappa\| \). Thus, \( u_t \) is a martingale difference noise. Next, observe that \( H_t \nabla L_t(w_t) \) are independent given \( w_t, q_t \). Therefore \( \mathbb{E}[H_t \nabla L_t(w_t) | \mathcal{F}_t] = \mathbb{E}[H_t | \mathcal{F}_t] \mathbb{E}[\nabla L_t(w_t) | \mathcal{F}_t] = \kappa \|\nabla F(w_t)\|^2 \). Again, it is easy to show that \( \mathbb{E}[\|v_t\|^2 | \mathcal{F}_t] \leq M(1 + \|q_t\|^2) \). This implies \( \{v_t\} \) is also a martingale difference noise.

It is clear from the expressions that since \( \nabla F \) is locally Lipschitz, \( G_1 \) and \( G_2 \) are locally Lipschitz maps. In addition, \( \phi(w) \) is also a locally Lipschitz map. Theorem 2 now follows from the result in Lemma 1 and Theorem 2 of [25].

5 Simulations

In this section, we discuss the experimental setup used to evaluate the proposed algorithms.
Dataset and models  We present the results of our algorithms on MNIST [28], CIFAR-10 [29] for multi-class classification using supervised learning. We used a LeNet architecture [30] for MNIST, and a VGG like network for CIFAR-10 [31]. For each dataset, we set aside a small auxiliary dataset of size 100 (sampled randomly from the training data) at the server, and the remaining training data is distributed uniformly to the workers. More details on the hyperparameters, models, size of the auxiliary dataset, and results on a synthetic dataset are provided in the supplementary material.

Figure 1: (Top) Experiments with MNIST data using 6 workers (all adversarial). (a) top-1 accuracy, (b) cross entropy loss, (c) Worker reputation scores. (Bottom) Experiments with CIFAR-10 data using 4 workers (all adversarial). (a) top-1 accuracy, (b) cross entropy loss, (c) Worker reputation scores.

Attack mechanism and baselines  Our attack model constitutes a random multiplicative noise injected to the stochastic gradients by the adversaries. If the worker is benign, it sends the computed stochastic gradient to the server. If the worker is adversarial, the worker multiplies the computed stochastic gradient with a random variable (with positive or negative mean) and scales it with a large constant (in our experiments we used 100 and 250). The resultant vector is sent to the server for aggregation. Some adversaries also send a zero mean random vector scaled by a large constant. To avoid computational issues, we clipped the gradients uniformly with a maximum value chosen independent of the multiplicative noise.

The generalization performance of SGD improves with the size of the dataset [32]. Since existing techniques for arbitrary number of Byzantines filter out the gradients that are sent by the adversarial workers, the generalization performance of those techniques is limited by the number of benign workers and the amount of data available with them. In the case where all workers are adversaries, the only available truthful data is the auxiliary dataset. Hence, we consider as baselines the algorithms that are trained only on the auxiliary data and the benign workers (if any). We evaluate the performance of the algorithms by comparing the top-1 classification accuracy on the test dataset.

Discussion  The experiments are carried out 3 times (for a fair comparison each algorithm compared in the experiment starts with the same random seed) and the averaged results are presented. We can observe from Figure 1 that both ByGARS and ByGARS++ achieve Byzantine robustness for the threat model used. They significantly outperform the baseline for both datasets. We can observe from Figures 1c and 1f that the reputation scores of the adversaries are correctly estimated to be negative or close to zero).

Since the goal of these experiments is to illustrate the effectiveness of using reputation scores in the presence of byzantine adversaries, we did not perform a hyper-parameter tuning to achieve the best test accuracy. Note that, in Algorithm 1, \(q_t\) update involves both \(\alpha_t\), \(\gamma_t\), whereas Algorithm 2, \(q_t\) update involves only \(\alpha_t\). Hence a larger \(\alpha_t\) is required for ByGARS++ to make parameter updates of
the same scale. And for this reason we use different hyperparameters for ByGARS and ByGARS++ (provided in the supplementary material). Despite this, we compare them in the same plot in Figure 1 to illustrate their Byzantine resilience. ByGARS has several intermediate gradient computations and meta updates, so comparing it based on the number of epochs till convergence is not fair. However, we observe that ByGARS has better test accuracy, where as ByGARS++ is computationally cheaper, while also performing as good as ByGARS. Also, training using ByGARS appears to be stable for non-convex problems compared to ByGARS++ which frequently gets stuck in a local minima. This can be attributed to the reputation scores being very small near local minima and they do not change in magnitude. We believe this is because ByGARS uses multiple meta updates before an actual update is performed on the parameters, where as ByGARS++ performs one noisy meta update to \( q_t \).

6 Related Work

**Byzantine SGD (fraction of adversaries < 0.5)** Blanchard et al. [4] showed that the then existing approaches of aggregation of gradients using linear combinations were inadequate to deal with Byzantine adversaries. They showed experimentally that the presence of a single Byzantine adversary is enough to derail the training. They proposed an algorithm Krum and its variants and proved the convergence for non convex functions. Chen et al. [6] proposed to use a median of means algorithm to robustly aggregate the gradients, and showed rigorous convergence guarantees for strongly convex objective functions. Similarly, Yin et al. [33] studied theoretical guarantees of coordinate wise median, and trimmed mean approaches. Alistarh et al. [5] show optimal sample complexity for convex problems by using historical information of the gradients sent by workers even in high dimensions. Yang et al. [34] use a Lipschitz inspired coordinate median algorithm to achieve good performance. Bernstein et al. [35] propose a simple, computationally cheap and efficient technique to use only the element wise sign of the gradients and perform a majority voting.

**Byzantine SGD (arbitrary number of adversaries)** By assuming availability of auxiliary data Xinyang Cao and Lifeng Lai [20] study Byzantine Gradient Descent for an arbitrary number of Byzantine workers, and show convergence guarantees for strongly convex objective functions. Under similar assumptions, Xie et al. [10] propose Zeno++ for asynchronous Byzantine SGD. They formulate a stochastic descent score akin to the Armijo condition for line search, to filter out adversarial workers. Jin et al. [18] also study the case of arbitrary number of adversaries in an asynchronous setting. Although [18] does not assume an auxiliary dataset, the algorithm requires each worker to have access to the gradients of all other workers, and using their own data the workers figure out which of the other workers are benign and use them for aggregation and parameter update. Ji et al. [19] also use an auxiliary dataset to aggregate gradients using an LSTM. In comparison, we use a simple reputation score based aggregation, with theoretical guarantees. Yao et al. [36] also compute an auxiliary gradient and use it for the update in the Federated Learning setting. However, the reputation scores of their algorithm are constant (fraction of samples) and don’t address adversaries.

7 Conclusion

We devise a novel, Byzantine resilient stochastic gradient aggregation algorithm for distributed machine learning with arbitrary number of adversarial workers. This is achieved by exploiting a small auxiliary dataset to compute a reputation score for every worker, and the scores are used to aggregate the workers’ gradients. By assuming availability of a very small auxiliary dataset, and some requirements on the behavior of the adversaries, we show that even gradients from adversarial workers can be useful to the training if multiplied with an appropriate reputation score. We showed that under reasonable assumptions, ByGARS++ converges to the optimal solution using a result from two timescale stochastic approximation theory [25]. Through simulations, we showed that the proposed algorithms exhibit remarkable robustness property even for non-convex problems — even if all the workers are adversarial, the trained model with adversaries achieves significantly better test error and generalization performance as that of the trained model under only benign workers. Although ByGARS and ByGARS++ are developed for this specific setting of Byzantine adversaries, we believe that these algorithms serve a much general purpose. This algorithm can be modified to train models in other cases such as learning from heterogeneous datasets, learning under privacy constraints, other adversarial settings (such as injecting different noises to individual dimensions of the gradient), and poisoned data attacks. We leave such analyses for a future work.
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where we used the fact that \( q_\beta \). Theorem 3. If Assumption 2 is satisfied, then ByGARS++ is DSSGD-Byzantine Tolerant [16, Def. 4], that is, this establishes the equality in (10a), (10b), and (10c).

\[ Yann LeCun, Corinna Cortes, and CJ Burges. MNIST handwritten digit database. ATT Labs [Online]. Available: http://yann.lecun.com/exdb/mnist, 2010. \]

A Proof of Theorem 1

**Theorem 3.** If Assumption 2 is satisfied, then ByGARS++ is DSSGD-Byzantine Tolerant [16, Def. 4], that is, \( \mathbb{E}[(\nabla F(w_t), H_t^T q_t)] \geq 0 \).

In order to establish this result, we first define the filtration and then prove a lemma. Let \((\Omega, \mathcal{F}, \mathbb{P})\) be a standard probability space and let \( \mathcal{F}_t \) be the \( \sigma \)-algebra generated by all the randomness realized up to time \( t \):

\[ \mathcal{F}_0 = \{w_0, q_0\}, \quad \mathcal{F}_t = \sigma\{w_0, h_1, 0, \ldots, h_m, 0, q_0, \ldots, \mathcal{W}_{t-1}, h_1, t-1, \ldots, h_m, t-1, q_{t-1}, w_t, q_t\}. \]

It is easy to see that \( \mathcal{F}_t \subseteq \mathcal{F}_{t+1} \) and, thus, \( \{\mathcal{F}_t\}_{t \in \mathbb{N}} \) is a filtration. We can now establish the following result.

**Lemma 2.** The following holds:

\[
\begin{align*}
\mathbb{E}[q_{t+1} | \mathcal{F}_t] &= (1 - \alpha_t)q_t + \alpha_t \|\nabla F(w_t)\|_2^2 \tag{10a} \\
\mathbb{E}[\kappa^T q_{t+1} | \mathcal{F}_t] &= (1 - \alpha_t)\kappa_t q_t + \alpha_t \|\nabla F(w_t)\|_2^2 \kappa_t^T \tag{10b} \\
\mathbb{E}[\kappa^T q_{t+1} | \mathcal{F}_t] &= (1 - \alpha_t)\kappa_t q_t + \alpha_t \|\nabla F(w_t)\|_2^2 \|\kappa_t\|_2^2 \tag{10c} 
\end{align*}
\]

**Proof.** We know that \( \mathbb{E}[H_t | \mathcal{F}_t] = \kappa \nabla F(w_t)^T \) and \( \mathbb{E}[\nabla L_t(w_t) | \mathcal{F}_t] = \nabla F(w_t) \). Using these relationships in the definition of \( q_{t+1} \) in ByGARS++, we get

\[
q_{t+1} = (1 - \alpha_t)q_t + \alpha_t H_t \nabla L_t(w_t) \implies \mathbb{E}[q_{t+1} | \mathcal{F}_t] = (1 - \alpha_t)q_t + \alpha_t \mathbb{E}[H_t \nabla L_t(w_t) | w_t, q_t] = (1 - \alpha_t)q_t + \alpha_t \|\nabla F(w_t)\|_2^2 \kappa_t.
\]

This establishes the equality in (10a), (10b), and (10c). \( \square \)

An immediate corollary to the above result is as follows. Define \( \beta_{s,t} = \prod_{k=s+1}^t (1 - \alpha_k) > 0 \) with the convention that \( \beta_{t,t} = 1 \). We can rewrite \( \mathbb{E}[\kappa^T q_{t+1} | \mathcal{F}_t] \) as

\[
\mathbb{E}[\kappa^T q_{t+1} | \mathcal{F}_t] = \sum_{s=0}^t \beta_{s,t} \alpha_s \|\kappa_s\|_2^2 \|\nabla F(w_s)\|_2^2, \tag{11}
\]

where we used the fact that \( q_0 = 0 \).
We now proceed to establish Theorem 1, which follows from the law of iterated expectation and the principle of mathematical induction. Consider the stochastic process \( \kappa^T q_t \) in (10c). At time \( t = 0 \), \( q_0 = 0 \). Thus, we have \( E[\kappa^T q_t | F_0] > 0 \) since we assumed that there is at least one \( \kappa_t \neq 0 \) and \( \nabla F(w_0) \neq 0 \). Thus, we conclude that \( E[\kappa^T q_t] > 0 \) by the law of iterated expectation. The same argument establishes the fact that \( E[\kappa^T q_t] > 0 \) for all \( t \geq 1 \), since the right side of (10c) is a convex combination of terms that are positive in expectation (until \( w_t = w_\ast \)). This can also be deduced from (11) by taking the expectation of the right side and noting that it is positive.

We now apply the same argument to \( E[(\nabla F(w_t), H_t^T q_t) | F_t] \). Clearly,

\[
E[(\nabla F(w_t), H_t^T q_t) | F_t] = (\nabla F(w_t), E[H_t^T q_t | F_t]) = (\nabla F(w_t), \nabla F(w_t) \kappa^T q_t) = \kappa^T q_t || \nabla F(w_t) ||^2
\]

(12)

Coupling the above expression with (11), we get

\[
E[(\nabla F(w_t), H_t^T q_t)] = \sum_{s=0}^{t-1} \alpha_s \beta_s |t-1| || \kappa ||^2 E[|| \nabla F(w_s) ||^2 || \nabla F(w_t) ||^2] > 0,
\]

since both terms within the expectation are positive random numbers unless \( w_t = w_\ast \). This completes the proof of Theorem 1.

**B Simulations**

**B.1 Dataset and models**

We generated the synthetic dataset as follows. Given input dimension \( E \), we draw a gaussian vector in \( \mathbb{R}^d \) with non-zero mean and denote it as \( \theta^* \). Next, given the number of datapoints \( N \), we generate \( N \) random vectors in \( \mathbb{R}^d \). For each data point, we assign \( y = x^T \theta^* + \epsilon \) where \( x \) is the datapoint, and \( \epsilon \) follows a zero mean Gaussian distribution. The objective is to find \( \theta \) such that the mean square error \( || Y - X^T \theta ||^2 \) is minimized. In the simulations, we used \( d = 20, N = 10,000 \) of which 2000 were used for testing, and the remaining 8000 is split between training data and the auxiliary data (size=100).

For MNIST dataset (60,000 training data, 10,000 test data; 10 classes), we used a LeNet like architecture (with two convolutional layers with ReLu activations and MaxPool2D, followed by three fully connected layers). The loss function used is cross entropy loss. For CIFAR-10 dataset (50,000 training and 10,000 test data; 10 classes), we used a vgg like model (five convolutional layers with increasing channels followed by average pooling and three fully connected layers with ReLu activations) trained with dropout. The loss function used is the cross entropy loss.

For each dataset, we set aside a small auxiliary dataset of size 100 or 250 (sampled randomly from the training data at the beginning) at the server, and the remaining training data is distributed uniformly to the workers. For the sake of simulation, we compute the auxiliary gradient on all of the auxiliary data (instead of sampling a mini-batch).

For comparison, we plot ByGARS, ByGARS++, baseline on the same plot where the x-axis corresponds to the number of gradient updates performed to \( w_t \) (for ByGARS, all the meta iterations do not count as gradient updates). For the baseline which computes the averaged gradient of the benign workers (and auxiliary dataset), the concept of epoch is ambiguous since the data from the adversarial workers isn’t used to update \( w_t \). Assume that the server is aware of the adversaries and discards the gradients of the adversarial workers. The x-axis corresponds to the number of gradient updates to \( w_t \) (in terms of epochs with respect to the data distributed across the workers).

**B.2 Hyperparameters**

We used the following hyper-parameters for different configurations. We scheduled the learning rate \( (\gamma_t) \) to decay as: \( 1/t\alpha \), and meta learning rate \( (\alpha_t) \) as: \( 1/t\alpha + \beta_m \). We used the following hyperparameters

1. CIFAR10 ByGARS: \((\gamma_0: 0.05), (\beta: 0.9), (\alpha_0: 0.05), (\beta_m: 0.5), (\lambda: 0.005))\)
2. CIFAR10 ByGARS++: \((\gamma_0: 0.05), (\beta: 0.9), (\alpha_0: 0.01), (\beta_m: 0.2), (\lambda: 0.005))\)
3. MNIST ByGARS: \((\gamma_0: 0.05), (\beta: 0.5), (\alpha_0: 0.05), (\beta_m: 0.5), (\lambda: 0))\)
4. MNIST ByGARS++: \((\gamma_0: 0.05), (\beta: 0.9), (\alpha_0: 0.0005), (\beta_m: 0.2), (\lambda: 0.005))\)
5. synthetic data ByGARS: \((\gamma_0: 0.05), (\beta: 0.5), (\alpha_0: 0.05), (\beta_m: 0.5), (\lambda: 0))\)
6. synthetic data ByGARS++: \((\gamma_0: 0.05), (\beta: 0.9), (\alpha_0: 0.0005), (\beta_m: 0.7), (\lambda: 0))\)
Since our objective is to illustrate the effectiveness of the proposed algorithms against the adversaries, we did not explicitly perform a hyperparameter tuning to obtain best test accuracy. Instead, we show that for appropriately chosen hyperparameters, the proposed algorithms converge and are Byzantine resilient.

B.3 Setup

The simulations are implemented in Pytorch. Although the algorithm is aimed at robustifying distributed machine learning, in the implementation we only use one node while processing the workers sequentially so that the results can be reproduced even on a single machine. We train the models on Nvidia RTX 6000 GPU.

B.4 Discussion

Update rules

We can observe from Fig 2, 3, 4 that both ByGARS, ByGARS++ converge to a better solution (smaller loss, higher test accuracy) as compared to the baseline. Note that ByGARS++ is noisy, since the \( q_t \) update happens with an inner product between stochastic gradients. Also, ByGARS++ tends to get stuck in some unfavorable local minima which results in a higher variance of the test accuracy of ByGARS++. In fact, ByGARS also seems to get stuck sometimes at unfavorable local minima, but the variance is much smaller than ByGARS++. Observe that \( q_t \) converges to 0 as the training progresses. This phenomenon can be observed clearly for the synthetic data in Fig 2b. For MNIST, CIFAR, the simulations weren’t run long enough to see the \( q_t \) scores converging to zero (but the trend is clear from Fig 3b, 4b).

Meta iterations

In order to study the dependence of ByGARS on the number of meta iterations, we repeated the experiments with number of meta iterations ranging from 1 to 5 (we don’t do this for ByGARS++ since there is only one meta update). We show the results in Fig 5. We can observe that for both MNIST and CIFAR10, the higher the number of meta iterations, the better is the test accuracy. However, we did not study if with increasing number of meta-iterations, we over fit to the auxiliary data set and leave it for future work. Note that the high variance of ByGARS for 5 meta iterations (plot in red) in Fig 5b is due to getting stuck in an unfavorable local minima in one trial (the experiments are repeated 4 times) which reduced the mean of the accuracy and increased the variance.

Auxiliary dataset size

The key to the effectiveness of ByGARS and ByGARS++ is the availability of an auxiliary dataset that is drawn from the same distribution as the testing and training data. We study the dependence of the algorithms on the auxiliary dataset by repeating the experiments with different sizes of the auxiliary dataset. From Fig 6, we can observe that a very small amount of auxiliary dataset is sufficient to face any number of adversarial workers. Since MNIST is a very easy dataset, we observe that there is very little variation in the performance of both the algorithms even at smaller sizes of the auxiliary dataset. The importance of auxiliary data is seen more clearly for ByGARS on CIFAR10 dataset in Fig 7a. The difference is not so prominent in Fig 7b due to getting stuck in unfavorable minima multiple times during the training.

It is crucial to maintain a uniform distribution over the labels of the auxiliary dataset. We believe that this might be one of the reasons why ByGARS/ByGARS++ get stuck in unfavorable local minima early on in the training. It might be interesting to analyze how the algorithms perform if the auxiliary dataset is skewed towards specific classes (or even missing some classes), and the minimum number of auxiliary datapoints required to reasonably converge (without getting stuck in unfavorable local minima). We leave such analysis for future work.
Figure 2: Experiments with synthetic data using 6 workers (5 adversarial), auxiliary dataset size 100, batch size 64 (a) Comparison of ByGARS, ByGARS++ and Baseline with plain averaging (b) Worker reputation scores of ByGARS++.
Figure 3: Experiments with MNIST using 6 workers (5 adversarial), auxiliary dataset size 100, batch size 64 (a) Comparison of ByGARS, ByGARS++ and Baseline with plain averaging (b) Worker reputation scores of ByGARS++.
Figure 4: Experiments with CIFAR using 4 workers (all adversarial), auxiliary dataset size 250, batch size 128 (a) Comparison of ByGARS, ByGARS++ and Baseline with plain averaging (b) Worker reputation scores of ByGARS++.
Figure 5: Experiments to compare performance of ByGARS with different number of meta updates
(a) MNIST (b) CIFAR10
Figure 6: Experiments to compare performance with different size of aux data set on MNIST (a) ByGARS (b) ByGARS++
Figure 7: Experiments to compare performance with different size of aux data set on CIFAR10 (a) ByGARS (b) ByGARS++