Neutron-neutral particle mixing in partial compositeness model and its observable consequences

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In this work, we explore the possibility that the mixing between neutron (\(n\)) and elementary neutral particle (\(\eta\)), which violates both the baryon number (\(B\)) and the lepton number (\(L\)) by one unit but conserves their difference (\(B - L\), may give rise to non-trivial effects that are different from the Standard Model predictions. We focus on two different scenarios, namely the pure and impure oscillation, roughly corresponding to whether the decay products of neutron are responsible for the \(n-\bar{n}\) oscillation. In the scenario where the neutral particle oscillation is pure oscillation, the Majorana phase, which leads to CP-violating effects, can be observable. In the scenario where the neutral particle oscillation is impure, we analyze the testable implications on the masses and lifetimes of the mass eigenstates, arising from the \(n-\bar{n}\) oscillation mediated by \(\eta\). In this scenario, we also suggest a unified interpretation of the neutron lifetime anomaly and the \(n-\bar{n}\) oscillation measurements based on the \(n-\eta\) mixing. In both scenarios, we present the lower bounds imposed by the results of the searches for \(n-\bar{n}\) oscillations on the masses of the color triplet bosons and point out that they could be within the reach of a direct detection at the LHC or future high-energy experiments. Furthermore, we discuss about the observability of the geometric phase associated with the \(n-\eta\) mixing. The measurement of such a geometric phase may provide another opportunity for the study of new physical effects.

I. INTRODUCTION

New physical phenomena beyond the Standard Model (SM) have been mathematically predicted by many new physics models and intensively explored in a wide variety of experiments over the past decades [1]. Since many new physics models are featured with particle mixing and oscillation, the phenomena of particle mixing and oscillation play a critical role in the construction of the extensions to the SM. For example, the neutrino flavor oscillation has been confirmed in various scenarios [2] and indicates that at least one type of neutrino has a non-zero mass, which contradicts the basic assumption of the SM and suggests that the SM is not perfect and thus new physics models need to be constructed [1]. Furthermore, particle mixing and oscillation are indispensable to understand CP-violating effects, which have been confirmed by many experiments (see e.g. Refs. [8–12]).

Cold neutrons can serve as a rich and varied environment where many interesting processes occur [13], making it possible to search for new physical phenomena in a smaller experiment, comparing with the ones at the LHC. The neutron lifetime and the neutron-antineutron (\(n-\bar{n}\)) oscillation time are the two key observables that are investigated intensively [14–23]. Theoretical investigations on the properties of neutron not only can help develop an efficient measurement strategy for new physical phenomena, but also can help interpret the results more correctly after the measurements. In this work, we focus on the theoretical aspects associated with the measurements of the neutron lifetime and the \(n-\bar{n}\) oscillation.

The \(n-\bar{n}\) oscillation, which violates baryon number (\(B\)) by two units, has attracted an enormous level of attention both theoretically and experimentally [24]. The searches for the \(n-\bar{n}\) oscillation have been performed in various mediums [24], including bound states, field-free vacuum, and etc. Up to date, no significant signal for the \(n-\bar{n}\) oscillation has been found. In field-free vacuum, the lower limit on the \(n-\bar{n}\) oscillation time presented by the Institut Laue-Langevin (ILL) experiment is approximately 0.86 \times 10^8 \text{ s} [14]. In bound states, the \(n-\bar{n}\) oscillations have been searched for by various experiments, such as Irvine-Michigan-Brookhaven (IMB) [15], Kamionkande (KM) [16], Frejus [17], Soudan-2 (SD-2) [18], Sudbury Neutrino Observatory (SNO) [19], Super-Kamionkande (Super-K) [20, 21], and etc. Among them, the most stringent constraint on the \(n-\bar{n}\) oscillation time is imposed by the Super-K experiment with the value of 4.7 \times 10^8 \text{ s} [21], when converting to the field-free vacuum values. Although the measurements on neutrons bound in nuclei provide relatively tighter limits, such limits depend heavily on the details of the nuclear models [14], whereas the limits imposed by the measurements on free neutrons are model independent.

From the theoretical aspect, the \(n-\bar{n}\) oscillation can be predicted by many new physics models (see e.g. Ref. [25]), such as left-right symmetry model [20–27], grand unified symmetry model [28–29], super-symmetry model [29–32], extra dimension model [25–33], mirror world model [34–36], and etc. Among such models, the mirror
world model was initially proposed for the understanding of parity violation [37, 39]. As a special case of the mirror world model, the mixing between neutron \((n)\) and mirror neutron \((n')\) has been studied [40, 41]. Such a mixing may give rise to new physical phenomena (see e.g. Ref. [13]), such as neutron disappearance \((n-n')\) [36, 46, 47], neutron regeneration \((n-n'(\bar{n})-n)\) [48, 50], and neutron-antineutron oscillation \([n-n'(\bar{n})-n]\) [35, 41, 42, 61]. Instead of direct mass mixing terms, the \(n-n'\) oscillation can be achieved indirectly through mirror particles as intermediate states [39, 61]. Recently, high-sensitivity measurement schemes for the \(n-n'\) oscillation has been designed and demonstrated [45, 52].

The neutron lifetime anomaly, which refers to the discrepancy in the measured neutron lifetime between two different experimental approaches, has attracted great attention recently (see e.g. Ref. [1]). For example, the trap experiment reports a neutron lifetime \(\tau_n = 877.79^{+0.26}_{-0.28}\text{(stat.)}^{+0.14}_{-0.16}\text{(syst.)}\) s [25] through the observation of neutron disappearance. However, the beam experiment reports a neutron lifetime \(\tau_n = 887.7^{+1.2}_{-1.1}(\text{stat.})^{+1.9}_{-1.9}\) s [53] through the neutron \(\beta\)-decay products, such as protons and electrons. The results presented by the two different approaches provide an approximately 4 \(\sigma\) deviation [22], which may imply a signal for new physics. It has been shown that the interpretation of neutron lifetime anomaly based on exotic dark decay channels can be excluded through the analysis of the neutron decay \(\beta\) asymmetry [54]. This analysis along with the neutron lifetime anomaly has continuously been discussed in the literature (see e.g. Refs. [35, 56, 62]). Therefore, the neutron lifetime anomaly is still far from being fully conclusive. A specific dark decay channel: \(n \rightarrow \chi^+\chi^-\) has been ruled out by the PERKEO II experiment [66], where the limits on the corresponding branching ratios and the mass scales of dark matter particles has also been provided. A new measurement scheme, which can verify the explanation of neutron lifetime anomaly via neutron-mirror neutron oscillations, has also been designed [60]. To summarize, we consider that the neutron lifetime anomaly remains a puzzle and a reasonable theoretical explanation needs to be constructed.

The hierarchy of the mixing angles and family masses can be explained by the partial compositeness model [67], where the Lagrangian can be divided into three sectors, such as the elementary, composite, and mixing sectors [68]. According to this model, both the composite and the elementary particles of the SM may not necessarily be mass eigenstates, whereas their superpositions could be mass eigenstates [68]. The partial compositeness model predicts various new phenomena, such as proton-positron oscillation [69], neutron-neutrino oscillation [69], and etc. As a special case of the partial compositeness model, the mixing between neutron \((n)\) and elementary neutral particle \((\eta)\), which violates \(B\) and \(L\) by one unit while conserving their difference \((B - L)\), may give rise to many interesting and observable consequences.

In what follows, we focus on the \(n-\eta\) mixing, which is a special case of the partial compositeness model [67]. We analyze the physical consequences arising from the \(n-\eta\) mixing, such as the \(n-\bar{n}\) oscillation, and discuss about the expected signal observability at the present and future experiments.

II. NEUTRON-NEUTRAL PARTICLE MIXING

The mixing between \(n\) and \(\eta\) can be mediated by color triplet bosons with hypercharge \(Y \equiv \pm 4/3\) and the corresponding operator can be given by [32, 70, 73]

\[
\hat{O}_1 \equiv \lambda_1 \bar{u}d^c\phi + \lambda_2 \bar{\eta}d^c\phi + \frac{1}{2} m_\eta \bar{\eta} \eta^c + m_\phi \phi^c \phi + \text{H.c.} \tag{1}
\]

Here, \(\lambda_1\) and \(\lambda_2\) are two dimensionless coupling constants. The superscript \(c\) denotes charge conjugation. \(m_\eta\) is the mass of the elementary neutral particle \((\eta)\). \(m_\phi\) is the mass of the color triplet bosons and thus it is associated with the new physics energy scale. The mass terms for neutron is not written down explicitly. \(\eta\) may have a non-zero lepton number \((L = 1)\) and may intertwine with dark matter or else its decay products could be dark matter candidates [70]. Furthermore, \(\eta\) has to be neutral as required by the charge conservation law. When written down in terms of the neutron field, the relevant operator that accounts for the \(n-\eta\) mixing can be given by [73]

\[
\hat{O}_2 \equiv \frac{\lambda_1 \lambda_2}{m_\phi^2} \bar{u_2}d^c_2 \eta + \text{H.c.} \equiv \frac{\lambda_1 \lambda_2 |\psi_4(0)|^2}{m_\phi^2} \bar{n_1} \eta + \text{H.c.} \tag{2}
\]

with

\[
\delta \equiv \frac{\lambda_1 \lambda_2 |\psi_4(0)|^2}{m_\phi^2} \tag{3}
\]

Here, the color indices are omitted for simplicity of notation. The following substitutions [67, 73]: \(u_2d^c_2 \rightarrow \psi_4(0)^2n, u^c_2d^c_2 \rightarrow |\psi_4(0)|^2n\) have been made. \(\psi_4(0)\) is the overlap factor of quarks. Lattice QCD calculations give the value \(|\psi_4(0)|^2 = 0.0144(3)(21)\) GeV\(^3\) [74], where the numbers in the parentheses are the statistical and systematic uncertainties, respectively. We have assumed that antineutron has the same overlap factor of quarks as neutron does.

In the absence of external (magnetic) fields, the entire effective Lagrangian including kinetic terms can be written as (see e.g. Ref. [75])

\[
\mathcal{L}^1_{\text{eff}} \equiv \bar{\eta} \eta \bar{\theta} \eta + \frac{1}{2} m_\eta \bar{\eta} \eta^c + \bar{n}(i\partial - m_n)n + \delta \bar{n_1} \eta + \text{H.c.} \tag{4}
\]

Here, \(m_n\) is the mass of neutron. Without external magnetic fields, the mixing angle for the \(n-\eta\) mixing can be
where the CP-even phases \( \phi \) have the value: \( g_n \simeq -3.826 \) \(^{76}\) and the nuclear magneton has the value: \( \mu_N \simeq 3.152 \times 10^{-8} \) eV.T\(^{-1} \) \(^{76}\). In the ILL experiment, the magnetic field in the neutron propagation region can be as low as \( B \lesssim 1 \times 10^{-8} \) T \(^{14}\) and to be conservative we can choose the maximum value: \( B = 1 \times 10^{-8} \) T in our analysis. In Eq. \([5]\), the minus and plus sign corresponds to the \( n-\eta \) and \( \bar{n}-\bar{\eta} \) mixing, respectively. The matrix elements \( M_{12} \) and \( \Gamma_{12} \) describe the dispersive and absorptive amplitudes of the effective mass matrix, respectively.

In the presence of the \( n-\eta \) mixing, the mass eigenstates \( |n\rangle \) and \( |\eta\rangle \) can be expressed as linear superposition of the interaction eigenstates \( |\tilde{n}\rangle \) and \( |\tilde{\eta}\rangle \):

\[
|n\rangle = c_1 |\tilde{n}\rangle + c_2 |\tilde{\eta}\rangle, \quad (7)
\]
\[
|\eta\rangle = c_1 |\tilde{n}\rangle + c_2 |\tilde{\eta}\rangle. \quad (8)
\]

Here, \( c_{1,2} \) and \( \epsilon_{1,2} \) represent the mixing coefficients, which are the elements of the transformation matrix \( T \) that is used to diagonalize the effective mass matrix \( W \) (see Appendix \( A \) for more details).

In the presence of the external magnetic fields, the transformation matrix associated with the \( n-\eta \) mixing \((T_1)\) is different from the one associated with the \( \bar{n}-\bar{\eta} \) mixing \((T_2)\):

\[
T_1 = \begin{bmatrix} \cos \theta_1 & \sin \theta_1 \\ -\sin \theta_1 & \cos \theta_1 \end{bmatrix}, \quad (9)
\]
\[
T_2 = \begin{bmatrix} \cos \theta_2 & \sin \theta_2 \\ -\sin \theta_2 & \cos \theta_2 \end{bmatrix}. \quad (10)
\]

Here, \( \theta_1 \) and \( \theta_2 \) are the mixing angles associated with the \( n-\eta \) mixing and the \( \bar{n}-\bar{\eta} \) mixing, respectively, and they satisfy the following expressions:

\[
\theta_1 \equiv \arctan \left( \frac{2\delta}{m_n + |\mu_n B| - m_\eta + \Delta W_1} \right), \quad (11)
\]
\[
\theta_2 \equiv \arctan \left( \frac{2\delta}{m_n + |\mu_n B| - m_\eta + \Delta W_2} \right), \quad (12)
\]

with

\[
\Delta W_{1,2} = \left( m_n + |\mu_n B| - m_\eta \right)^2 + 4\delta^2. \quad (13)
\]

Based on Eq. \([9]\) and \([10]\), the probability of the \( n-\eta \) and \( \bar{n}-\bar{\eta} \) oscillations in the same external magnetic field can be respectively given by \([25, 47, 77]\):

\[
P_{n\rightarrow\eta} = P_{n\rightarrow\eta} = \frac{4\delta^2}{2} \frac{\sin^2 \left( \frac{\theta}{2} \right)}{\left( m_n + |\mu_n B| - m_\eta \right)^2 + 4\delta^2}, \quad (14)
\]

\[
P_{\bar{n}\rightarrow\bar{\eta}} = P_{\bar{n}\rightarrow\bar{\eta}} = \frac{4\delta^2}{2} \frac{\sin^2 \left( \frac{\theta}{2} \right)}{\left( m_n + |\mu_n B| - m_\eta \right)^2 + 4\delta^2}, \quad (15)
\]

where the CP-even phases \( \phi_1 \) and \( \phi_2 \) are defined by

\[
\phi_{1,2} \equiv \frac{\left( m_n + |\mu_n B| - m_\eta \right)^2 + 4\delta^2}{4\delta^2} t. \quad (16)
\]

As can be seen, without external magnetic fields, the mixing angles, the CP-even phases, the transformation matrices, and the oscillation probabilities in Eq. \([14]\) and \([15]\) take the same value for the particle and anti-particle sectors, i.e. \( \theta \equiv \theta_1 = \theta_2, \phi \equiv \phi_1 = \phi_2, T \equiv T_1 = T_2, P_{n\rightarrow\eta} = P_{\bar{n}\rightarrow\bar{\eta}} \).

The time evolution of the interaction eigenstates can be given by

\[
|n(t)\rangle = \frac{1}{c^2 + \epsilon^2} \left( \left( c^2 e^{-i\omega_1 t - \Gamma_1 t} + \epsilon^2 e^{-i\omega_2 t - \Gamma_2 t} \right) |n\rangle \right.
\]
\[
+ \left( \epsilon c e^{-i\omega_1 t - \Gamma_1 t} - \epsilon c e^{-i\omega_2 t - \Gamma_2 t} \right) |\eta\rangle \bigg), \quad (17)
\]

\[
|\eta(t)\rangle = \frac{1}{c^2 + \epsilon^2} \left( \left( c^2 e^{-i\omega_1 t - \Gamma_1 t} + \epsilon c e^{-i\omega_2 t - \Gamma_2 t} \right) |n\rangle \right.
\]
\[
+ \left( \epsilon^2 e^{-i\omega_1 t - \Gamma_1 t} + c^2 e^{-i\omega_2 t - \Gamma_2 t} \right) |\eta\rangle \bigg). \quad (18)
\]

Here, \( \omega_{1,2} \) and \( \Gamma_{1,2} \) are the masses and widths of the mass eigenstates \( |n_1\rangle \) and \( |n_2\rangle \), respectively. The mixing coefficients \( c \) and \( \epsilon \) are defined by Eq. \([58]\) and \([59]\) in Appendix \( A \). As can be seen, a neutron that is created at the beginning \( (t = 0) \) can later be detected to be an \( \eta \) particle with a specific probability. The above-mentioned \( n-\eta \) mixing may lead to many interesting and observable consequences, one of which is the \( n-\bar{n} \) oscillation.

### A. \( n-\bar{n} \) oscillation

Fig. \( \text{F2} \) shows that the \( n-\bar{n} \) oscillation can be achieved at the tree level indirectly through elementary neutral
FIG. 1: The $n\bar{n}$ oscillation can be induced at the tree level through the mixing between neutron and the elementary particle $\eta$ [29, 72, 80, 81] in partial compositeness model.

In previous studies (see e.g. Refs [70, 73, 93–95]), the following restriction is imposed on $m_\eta$ to make the proton decay kinetically forbidden:

$$m_p - m_e \lesssim m_\eta \lesssim m_p + m_e.$$  \hspace{1cm} (19)

Here, $m_p$ and $m_e$ are the proton and electron mass respectively. An even more stringent restriction $m_\eta > 937.9$ MeV can be derived from the stability of $^9$Be [75, 76]. Such restrictions are very stringent and lack of experimental support. Another possibility that has not been excluded is that $m_\eta$ may lie outside this narrow range while the stability of proton and nuclei (e.g. $^9$Be) can be guaranteed by imposing additional assumptions or symmetries [32, 70, 71, 96]. In this work, we loosen the restriction on $m_\eta$ to the whole range where the decay of neutron into $\eta$ is kinetically allowed:

$$m_\eta \lesssim m_n.$$  \hspace{1cm} (20)

The masses $m_n$ and $m_\eta$ definitely refer to the masses of the mass eigenstates $|n_1\rangle$ and $|n_2\rangle$.

In what follows, we analyze the physical consequences (i.e. the $n\bar{n}$ oscillation) arising from the $n$-$\eta$ mixing and discuss about their expected signal observability at the present and future experiments. We focus on two different scenarios, depending on whether the neutral particle oscillation is the type-I or type-II oscillation. In the scenario where the neutral particle oscillation is type-I, we evaluate the $n\bar{n}$ oscillation probability and analyze the observability of the Majorana phase and CP-violating effects. In the scenario where the neutral particle oscillation is type-II, we analyze the testable implications on the masses and lifetimes of the mass eigenstates that are resulted from the $n\bar{n}$ oscillation mediated by $\eta$. In both scenarios, the lower limits imposed by the results of the searches for $n\bar{n}$ oscillations are presented.

B. Majorana phase, CP-violation, and oscillation probability

In this subsection, we focus on the type-I oscillation, where the decay products are not responsible for the $n\bar{n}$ oscillation. We evaluate the $n\bar{n}$ oscillation probability and analyze the observability of the Majorana phase and the CP-violating effects. Furthermore, we derive the lower limits imposed by the results of the searches for $n\bar{n}$ oscillations on the masses of the color triplet bosons.

The connection between Majorana phases and CP-violation has been studied intensively in the neutrino sector [97, 100]. In this case, the Majorana phases may have non-trivial impact on the observable effects, such
as the decay rate of the neutrino-less double $\beta$ decay [100], the antineutrino-neutrino oscillations [97–101], and etc. If neutrinos are Majorana-type particles, there could be more CP-violating phases, possibly leading to more observable consequences [100]. However, in neutrino-neutrino oscillations, the Majorana phase may not have observable effects since it is unable to produce a coherent state for different types of neutrinos, which does not correlate to the charged leptons [102]. Even though the Majorana phases can lead to non-trivial effects on the decay rate of the neutrino-less double $\beta$ decay [100], it does not necessarily manifest as CP-violation [103].

The $n^-\bar{n}$ oscillation process can be considered as a potential signal of CP-violation based on CPT and Lorentz symmetries [104]. However, such a process may not necessarily lead to observable CP-violating effects [105]. It was pointed out that if there is an interaction or an interference between amplitudes, the CP-violating effects can be observable [24]. Although a single neutron is not a Majorana particle, if there is a direct mixing between neutron and antineutron, their superposition could be a Majorana particle [106]. In this case, since the CP-violating phase in the $n^-\bar{n}$ mixing matrix can be absorbed into the definition of the neutron field [100], there is no observable CP-violating effect. Nevertheless, the situation could be different, if a neutron mixes with a Majorana particle $\eta$.

We assume that the neutral elementary particle $\eta$ satisfies the following Majorana condition [99]:

$$\eta^c \equiv \eta e^{i\xi},$$

(21)

where $\xi$ is the Majorana phase, which comes from the Majorana nature of $\eta$ and cannot be eliminated by a field redefinition (i.e. a rephrasing transformation). In what follows, we will discuss the observable consequences of the Majorana phase $\exp(i\xi)$ contained in the $n^-\eta$ mixing matrix and reveal its CP-violating nature.

Without loss of generality, in the following discussions we assume the initial neutron is right handed. We can read from Eq. (1) and (2) that the relevant operator responsible for the $n^-\eta$ mixing takes the following form [103]:

$$\hat{O} = \frac{\lambda_1\lambda_2}{m_0^2} \bar{\eta} n L \eta + \text{H.c.}$$

$$= \frac{\lambda_1\lambda_2}{m_0^2} e^{-i\xi} \bar{\eta} n L \eta + \text{H.c.},$$

(22)

where in the last step we have used the condition in Eq. (21). $n_{R/L}$ stands for the right and left handed spinors which are defined by $n_{R/L} \equiv P_{R/L} n$ with $P_{R/L} \equiv (1 \pm \gamma^5)/2$. In the following discussions, we assume that the external magnetic fields are absent and thus the mixing angles $\theta_1$ and $\theta_2$ take the same value, i.e. $\theta \equiv \theta_1 = \theta_2$, $T \equiv T_1 = T_2$. The results can be generalized to the case where the external magnetic fields are present, simply by substituting $\theta_1$ for $\theta$ in the $n^-\eta$ mixing and by substituting $\theta_1$ for $\theta$ in the $n^-\bar{n}$ mixing. The phase factor $\exp(i\xi)$ can be arranged into the off-diagonal elements of the transformation matrix $T$ [99]:

$$T = \begin{bmatrix} \cos \theta & e^{i\xi} \sin \theta \\ -e^{-i\xi} \sin \theta & \cos \theta \end{bmatrix}. \tag{23}$$

Note the phase factors can be equivalently moved to the elements in the second column of the matrix $T$ without triggering additional measurable effects [102].

Previous studies have shown that the CP-violating effects induced by the Majorana phase may be observable in the neutrino-antineutrino (or antineutrino-neutrino) oscillations [99–101, 103]. Possible measurement schemes for the Majorana phase have been proposed for the antineutrino-neutrino ($\bar{\nu}\nu$) oscillations [99, 101]. As an example, such a measurement can be accomplished by an oscillation process and two sequences of scattering processes [101]: (1) $\mu^+ n \rightarrow \bar{\nu}_\mu p$, (2) $\bar{\nu}_\mu \rightarrow \nu_\mu$, (3) $\nu_\mu n \rightarrow \mu^- p$. Here, CP-violation can be achieved through the $\bar{\nu}_\mu$-$\nu_\mu$ oscillation process. Similar to the $\bar{\nu}_\mu$-$\nu_\mu$ oscillation process, CP-violation can also be achieved through the $\eta^-\bar{\eta}$ oscillation process contained in the entire $n^-\bar{n}$ oscillation process. In this case, the $n^-\bar{n}$ oscillation process induced by the intermediate state $\eta$ can be divided into three sub-processes: (1) $n^-\eta_R$ oscillation, (2) $\eta_R^-\bar{\eta}_L$ oscillation, (3) $\bar{\eta}_L^-\eta_L$ oscillation. Note in the second sub-process, there is a chirality flip due to the Majorana mass term. The probability of the entire process can be expressed as

$$P_{n^-\rightarrow \bar{n}} \equiv P_{n^-\rightarrow \eta_R} P_{\eta_R^-\rightarrow \bar{\eta}_L} P_{\bar{\eta}_L^-\rightarrow \eta_L}.$$ \tag{24}

The $\eta_R^-\bar{\eta}_L$ oscillation amplitude can be given in the same manner as described in the antineutrino-neutrino oscillation case [99–101, 103]:

$$A_{\eta_R^-\rightarrow \bar{\eta}_L} = \sum_{i=1}^{2 \lambda_i} \left[ K_1 m_i \left( \frac{T}{m_i} \right)^2 e^{-i\omega_i t} \right]. \tag{25}$$

Here, $m_i$ are the masses of $n$ and $\eta$, respectively. $K_1$ is a kinetic factor and up to a trivial phase factor satisfies the expression: $K_1 = 1/\omega_i$ [99–101, 103], where $\omega_i$ is the energy of the initial neutron. $T_{\eta_R}$ represents the elements in the second row of the matrix $T$. The $\eta_R^-\bar{\eta}_L$ oscillation probability is then given by [99–101, 103]:

$$P_{\eta_R^-\rightarrow \bar{\eta}_L} = |K_1|^2 \left[ m_\eta^2 \cos^4 \theta + m_\eta^2 \sin^4 \theta \right]$$

$$+ \frac{1}{2} m_n m_\eta \sin^2(2\theta) \cos(\phi + 2\xi). \tag{26}$$

Here, we have assumed that the external magnetic fields are absent and thus the CP-even phase $\phi$ satisfies the condition: $\phi \equiv \phi_1 \simeq \sqrt{(m_n - m_\eta)^2 + 4\delta^2 t}$. Analogously, the antineutrino-neutron ($\bar{n}^-n$) oscillation, which is the CP-conjugate process of the $n^-\eta$ oscillation, can also be divided into three distinct sub-processes: (1) $\bar{n}^-\eta_L$ oscillation, (2) $\eta_L^-\bar{\eta}_R$ oscillation, (3) $\bar{\eta}_R^-n_R$ oscillation. The corresponding probability of the entire process can be written as

$$P_{\bar{n}^-\rightarrow n} \equiv P_{\bar{n}^-\rightarrow \eta_R} P_{\eta_R^-\rightarrow \bar{\eta}_L} P_{\bar{\eta}_L^-\rightarrow \eta_L}.$$ \tag{27}
Here, the $\bar{n}_L$-$\eta_R$ oscillation amplitude can be given by

$$A_{\bar{n}_L \to \eta_R} = \sum_{i=1}^{2} \left[ K_2 m_i (T_{\bar{n}}^i)^2 e^{-i\omega_i t} \right].$$

(28)

$K_2$ is another kinetic factor, which is different from $K_1$ by an irrelevant phase factor [103] and thus has the same modulus as $K_1$, i.e. $|K_1| = |K_2|$. The $\bar{n}_L$-$\eta_R$ oscillation probability is then given by [99, 101, 103]

$$P_{\bar{n}_L \to \eta_R} = |K_2|^2 \left[ m_n^2 \sin^4 \theta + m_{\eta}^2 \cos^4 \theta + \frac{1}{2} m_n m_{\eta} \sin^2(2\theta) \cos(\phi - 2\xi) \right].$$

(29)

Here, we have also assumed that the external magnetic fields are absent and thus the CP-even phase $\phi_2$ satisfies the condition: $\phi = \phi_2 \simeq \sqrt{(m_n - m_{\eta})^2 + 4\delta^2 t}$ too. Furthermore, in the absence of external magnetic fields, the following relations can be obtained from Eq. (30):

$$P_{\eta_R \to \bar{n}_L} = P_{\bar{n}_L \to \eta_R} = P_{\bar{n}_L \to \bar{n}_L} = P_{\eta_R \to \eta_R} = \sin^2(2\theta) \sin^2 \left( \frac{\phi}{2} \right).$$

(30)

In the non-relativistic scenario, i.e. $\omega_1 \simeq m_n$, Eq. (26) and (29) can be rewritten as [99, 101, 103]

$$P_{\eta_R \to \bar{n}_L} = \left[ \left( \frac{m_n}{m_{\eta}} \right)^2 \cos^4 \theta + \sin^4 \theta + \frac{1}{2} \left( \frac{m_n}{m_{\eta}} \right) \sin^2(2\theta) \cos(\phi + 2\xi) \right].$$

(31)

$$P_{\bar{n}_L \to \eta_R} = \left[ \left( \frac{m_n}{m_{\eta}} \right)^2 \cos^4 \theta + \sin^4 \theta + \frac{1}{2} \left( \frac{m_n}{m_{\eta}} \right) \sin^2(2\theta) \cos(\phi - 2\xi) \right].$$

(32)

Since the $\eta_R$-$\bar{n}_L$ and $\bar{n}_L$-$\eta_R$ oscillations are characterized by a chirality flip, the corresponding probabilities are suppressed by the mass of $\eta$ as expected. Based on Eq. (31) and (32), the probabilities for the $n$-$\bar{n}$ and $\bar{n}$-$n$ oscillation are respectively given by

$$P_{n \to \bar{n}}^{B=0} = \sin^4(2\theta) \sin^4 \left( \frac{\phi}{2} \right) \left[ \left( \frac{m_n}{m_{\eta}} \right)^2 \cos^4 \theta + \sin^4 \theta + \frac{1}{2} \left( \frac{m_n}{m_{\eta}} \right) \sin^2(2\theta) \cos(\phi + 2\xi) \right].$$

(33)

$$P_{\bar{n} \to n}^{B=0} = \sin^4(2\theta) \sin^4 \left( \frac{\phi}{2} \right) \left[ \left( \frac{m_n}{m_{\eta}} \right)^2 \cos^4 \theta + \sin^4 \theta + \frac{1}{2} \left( \frac{m_n}{m_{\eta}} \right) \sin^2(2\theta) \cos(\phi - 2\xi) \right].$$

(34)
\[ A_{CP}(B = 0) = \frac{P_{n\to\bar{n}} - P_{\bar{n}\to n}}{P_{n\to\bar{n}} + P_{\bar{n}\to n}} = \frac{m_n m_\eta \sin^2(2\theta) \left(\cos(\phi + 2\xi) - \cos(\phi - 2\xi)\right)}{4m_n^2 \sin^4\theta + 4m_\eta^2 \cos^4\theta + m_n m_\eta \sin^2(2\theta) \left(\cos(\phi + 2\xi) + \cos(\phi - 2\xi)\right)}. \quad (35) \]

\[
P_{n\to\bar{n}}^{B\neq 0} \approx \frac{1}{8} \left[ \frac{4\delta^2}{(m_n - |\mu_n B| - m_\eta)^2 + 4\delta^2} \right] \left\{ \frac{(m_n - |\mu_n B| - m_\eta)^2 + 2\delta^2}{(m_n - |\mu_n B| - m_\eta)^2 + 4\delta^2} + \sqrt{(m_n - |\mu_n B| - m_\eta)^2 + 4\delta^2} \right] \\
+ \left[ \frac{(m_n - |\mu_n B| - m_\eta)^2 + 2\delta^2}{(m_n - |\mu_n B| - m_\eta)^2 + 4\delta^2} - \sqrt{(m_n - |\mu_n B| - m_\eta)^2 + 4\delta^2} \right] \left[ \frac{4\delta^2}{(m_n + |\mu_n B| - m_\eta)^2 + 4\delta^2} \right]. \quad (36) \]

Similar to the CP-violating conditions given in Ref. 103, Eq. (33) and (34) show that, if \( \cos(\phi + 2\xi) \neq \cos(\phi - 2\xi) \) (i.e. \( \phi \neq n\pi \) and \( \xi \neq n\pi/2, n \in \mathbb{N} \)), a CP-violating effect \( \left( P_{n\to\bar{n}} \neq P_{\bar{n}\to n} \right) \) can occur due to the Majorana phase. This is different from the situation in the type-II oscillation, where the Majorana phases of the dispersive and absorptive parts cancel out and there will be no CP-violation unless some conditions are satisfied [99]. Eq. (35) gives the corresponding CP asymmetry \( A_{CP}(B = 0) \) and indicates explicitly that the \( n\bar{n} \) oscillation can accommodate CP-violating effects, which originate from the Majorana phase. Furthermore, the absolute value of the CP asymmetry \( |A_{CP}| \) has the following maximum at the points \( \phi = \pm \pi/2 \) and \( \xi = \pm \pi/4 \), where the signs are not correlated:

\[
|A_{CP}^{\max}(B = 0)| = \frac{m_n m_\eta \sin^2(2\theta)}{2m_n^2 \sin^4\theta + 2m_\eta^2 \cos^4\theta}. \quad (37) \]

\( B \)-violation and CP-violation (along with C-symmetry violation) are two of the three conditions presented by Sakharov to explain the observed matter-antimatter asymmetry in our Universe [107]. The \( n\bar{n} \) oscillation accompanied by observable CP-violating effects provides an appealing scenario and may open a promising avenue for explaining the origin of the matter-antimatter asymmetry.

At the beginning of Sec. 11, we have explained the reason why the restriction imposed on \( m_\eta \) can be chosen to be \( m_\eta \lesssim m_n \). In the ILL experiment, the measurement of the \( n\bar{n} \) oscillation time was performed with a mean propagation time of neutron \( \tau_m \equiv t \simeq 0.1 \text{ s} \) [14]. Nearly over the whole range \([m_\eta \in (0, m_n)]\), the CP-even phase \( \phi \) satisfies the condition:

\[
\phi = \sqrt{(m_n - m_\eta)^2 + 4\delta \tau_m} \gtrsim |m_n - m_\eta| \tau_m \gg 1. \quad (38) \]

Here, the CP-even phase \( \phi \) is associated with the \( n\eta \) oscillation subprocess rather than the entire \( n\bar{n} \) oscillation process. The above condition holds even in the presence of external magnetic fields as the magnetic interaction term \( |\mu_n B| \lesssim 6 \times 10^{-22} \text{ MeV} \) is very small. Furthermore, this condition does not necessarily contradict the quasi-free condition \( \Delta E t \ll 1 \) given in Refs. [14] 108, where the CP-even phase \( \Delta E t \) is associated with the entire \( n\bar{n} \) oscillation process. A rough estimation shows that even in the absence of external magnetic fields, the condition \( \phi \ll 1 \) implies that \( |m_n - m_\eta| \ll 6.6 \times 10^{-23} \text{ MeV} \). Since \( n \) and \( \eta \) are completely different particles, it is extremely unnatural to require that they have almost equal masses, i.e. \( m_n - m_\eta \ll 6.6 \times 10^{-23} \text{ MeV} \). Therefore, in the following discussions, we employ the condition given in Eq. (38). In this limit, Eq. (33) takes the following form:

\[
P_{n\to\bar{n}}^{B=0} \approx \frac{1}{8} \left[ \frac{4\delta^2}{(m_n - |\mu_n B| - m_\eta)^2 + 4\delta^2} \right] \left\{ \frac{(m_n - |\mu_n B| - m_\eta)^2 + 2\delta^2}{(m_n - |\mu_n B| - m_\eta)^2 + 4\delta^2} + \sqrt{(m_n - |\mu_n B| - m_\eta)^2 + 4\delta^2} \right] \\
+ \left[ \frac{(m_n - |\mu_n B| - m_\eta)^2 + 2\delta^2}{(m_n - |\mu_n B| - m_\eta)^2 + 4\delta^2} - \sqrt{(m_n - |\mu_n B| - m_\eta)^2 + 4\delta^2} \right] \left[ \frac{4\delta^2}{(m_n + |\mu_n B| - m_\eta)^2 + 4\delta^2} \right]. \quad (39) \]

In Eq. (33) and (34), we have assumed that the external magnetic field is absent. In reality, the external magnetic fields may not be fully shielded and an unavoidable background magnetic field needs to be considered. Under the condition expressed in Eq. (38), the \( n\bar{n} \) oscillation probability in the presence of external magnetic fields can be given by Eq. (36). In the Appendix B, the probabilities for the \( n\bar{n} \) and \( n\eta \) oscillations without making the approximation associated with the limit \( \phi_{1,2} \gg 1 \) can be given by Eq. (58) and (59), respectively.

In the ILL experiment, the measurement of the \( n\bar{n} \) oscillation was carried out using cold neutrons with a beam intensity of around \( 1.25 \times 10^{11} \text{ neutrons per second} \) and a neutron propagation time of around \( 0.1 \text{ s} \) [14]. In the quasi-free condition, the \( n\bar{n} \) oscillation probability can be estimated by the following expression: \( P_{n\to\bar{n}} \approx \frac{1}{2} \frac{\tau_{n\bar{n}}}{\tau_{n\eta}} \) [14], where \( \tau_{n\bar{n}} \) is the \( n\bar{n} \) oscillation time in field-free vacuum. Under the assumption that the typical propagation time of free neutrons is \( 0.1 \text{ s} \), the \( n\bar{n} \) oscillation times can be translated into the \( n\bar{n} \) oscillation probabilities. The results of the searches for \( n\bar{n} \) oscillations reported by various experiments and the corresponding estimated oscillation probabilities are shown in Tab. 1.

As an example, the lower bound on the \( n\bar{n} \) oscillation time reported by the ILL experiment is about \( 8.6 \times 10^7 \text{ s} \) [14], which, approximately, corresponds to the oscillation probability of the order of \( 10^{-18} \) (see e.g. Ref. 35).
would be more favorable to take the values in the order \(10^9\)), which roughly corresponds to the range from several TeV to several 10 TeV, the coupling constants \(|\lambda_1\lambda_2|\) would be more favorable to take the values in the order of \(10^{-2}\) and \(10^{-1}\).

Fig. 2 and 3 show the experimental constraints on the masses of the color triplet boson \(m_{\eta}\), corresponding to the coupling constants \(|\lambda_1\lambda_2| \approx 10^{-1}, 10^{-2}\) respectively. As can be seen from Fig. 2 and 3 comparing with the scenario where the coupling constant is \(|\lambda_1\lambda_2| \approx 10^{-1}\), the scenario with \(|\lambda_1\lambda_2| \approx 10^{-2}\) predicts a smaller mass of color triplet boson, i.e. a smaller new physics energy scale. Furthermore, the experimental constraints on the mass of color triplet boson varies gently from 1 to 8 TeV \((|\lambda_1\lambda_2| \approx 10^{-2}\) and from 5 to 25 TeV \((|\lambda_1\lambda_2| \approx 10^{-1}\) throughout the entire range of the allowed \(m_{\eta}\) values except in the vicinity of the neutron mass \(m_n\). This illustrates that the bounds on the new physics energy scale are insensitive to \(m_{\eta}\) unless \(m_{\eta}\) lies within the vicinity of the neutron mass.

### C. Mass and lifetime

In this subsection, we focus on the type-II oscillation, where the decay products, besides interfering with each other, are responsible for the \(n-\bar{n}\) oscillation. We analyze the \(n-\bar{n}\) oscillation originated from the \(n-\eta\) mixing and estimate its implications on masses and lifetimes. In this case, the \(n-\bar{n}\) oscillation occurs indirectly through the on-shell absorptive and off-shell dispersive processes as depicted in Fig. 4 [96]. Within the framework of the type-II oscillation, the possibility of establishing CP-violation in baryon oscillations has been discussed in Ref. 96. The results of the searches for \(n-\bar{n}\) oscillations can be employed to derive the lower bounds on the masses of the color triplet bosons. Furthermore, we analyze the compatibility between the interpretation of the neutron lifetime anomaly and the interpretation of the \(n-\bar{n}\) oscillation with regard to the \(n-\eta\) mixing.

The \(n-\eta\) mixing can lead to the \(n-\bar{n}\) oscillation, which can subsequently give rise to a mismatch between the neutron and antineutron interaction eigenstates and their mass eigenstates. Due to the \(n-\bar{n}\) oscillation, the linear superposition of the neutron and antineutron interaction eigenstates gives rise to mass eigenstates:

\[
\begin{align*}
|N_1\rangle &\equiv \epsilon_1^* |n\rangle + \epsilon_1 |\bar{n}\rangle, \quad (40) \\
|N_2\rangle &\equiv \epsilon_2^* |n\rangle + \epsilon_2 |\bar{n}\rangle. \quad (41)
\end{align*}
\]

Here, \(|N_1\rangle\) and \(|N_2\rangle\) are the two mass eigenstates arising from the \(n-\bar{n}\) oscillation and, according to Eq. (52) in Appendix A, their mass difference satisfies the condition: \(|m_{N_1} - m_{N_2}| \lesssim 2|M_{12}|. \quad \epsilon_1, \epsilon_2\) and \(\epsilon_1, \epsilon_2\) are the corresponding mixing coefficients associated with the entire \(n-\bar{n}\) oscillation process and, in general, are different from the ones associated with the \(n-\eta\) oscillation process presented in Eq. (5) and (8). Due to the condition: \(\epsilon_1, \epsilon_2 > \epsilon_1, \epsilon_2\), the \(|N_1\rangle\) state is predominantly composed of the \(|n\rangle\) state while the \(|N_2\rangle\) state is predominantly composed of the \(|\bar{n}\rangle\) state.

Fig. 4 (a) shows the main possible contribution to the absorptive amplitude, which is mainly originated from the process mediated by the on-shell \(\gamma\) and \(\eta\) [96]. Fig. 4 (b) shows that the dispersive amplitude \(M_{12}\) is mainly

![Diagram](attachment:image.png)

**FIG. 4:** The possible contributions to the \(n-\bar{n}\) oscillation [96]: (a) The absorptive amplitude \(\Gamma_{12}\) is mainly originated from the process mediated by the on-shell \(\gamma\) and \(\eta\); (b) The dispersive amplitude \(M_{12}\) is mainly originated from the process mediated by the off-shell \(\eta\) [96].

---

**TABLE I:** Results of the searches for \(n-\bar{n}\) oscillations and the corresponding estimated oscillation probabilities.

| Param. | ILL [14] | IMB [15] | KM [16] | Frejus [17] | SD-2 [18] | SNO [19] | Super-K [21] |
|--------|---------|---------|---------|-----------|----------|---------|-----------|
| Candidates \(S_0\) | 0 | 0 | 0 | 0 | 0 | 5 | 23 | 11 |
| \(\tau_{n\bar{n}}\) in matter (yr) | \(-\) | \(2.4 \times 10^{31}\) | \(4.3 \times 10^{31}\) | \(6.5 \times 10^{31}\) | \(7.2 \times 10^{31}\) | \(3.0 \times 10^{31}\) | \(3.6 \times 10^{32}\) |
| Suppression \(R (s^{-1})\) | \(-\) | \(1.0 \times 10^{23}\) | \(1.0 \times 10^{23}\) | \(1.4 \times 10^{23}\) | \(1.4 \times 10^{23}\) | \(2.5 \times 10^{22}\) | \(5.1 \times 10^{22}\) |
| \(\tau_{n\bar{n}}\) in vacuum (s) | \(8.6 \times 10^6\) | \(1.1 \times 10^8\) | \(1.2 \times 10^8\) | \(1.2 \times 10^8\) | \(1.3 \times 10^8\) | \(1.37 \times 10^8\) | \(4.7 \times 10^8\) |
| Probability \(P_{n-\bar{n}}\) | \(1.4 \times 10^{-18}\) | \(8.3 \times 10^{-19}\) | \(6.9 \times 10^{-19}\) | \(6.9 \times 10^{-19}\) | \(5.9 \times 10^{-19}\) | \(5.3 \times 10^{-19}\) | \(4.5 \times 10^{-20}\) |

\(\text{a}\) The oscillation probabilities are converted into the field-free vacuum values based on the oscillation times in vacuum.
arising from the process mediated by the off-shell $\eta$ [96]. The two processes can be described by the effective Lagrangian [75, 96, 110]:

$$\mathcal{L}_{\text{eff}}^2 \equiv \bar{\eta} (i \partial - m_\eta) \eta + \delta (\bar{n} \eta + \text{H.c.})$$

$$+ \bar{n} \left( i \partial - m_n + \frac{g_n}{2m_n} \sigma_{\mu\nu} F^{\mu\nu} \right) n. \quad (42)$$

Although there is no direct coupling between the neutral particle $\eta$ and the vector field $F^{\mu\nu}$, the following Lagrangian, which is responsible for the process depicted in the Fig. 4(a), can be obtained by the diagonalization of the mass matrix [75, 96, 110]:

$$\mathcal{L}_{\text{eff}}^2 \equiv \frac{g_n \sin \theta}{2m_n} \bar{n} \sigma_{\mu\nu} F^{\mu\nu} \eta + \text{H.c.}. \quad (43)$$

The mixing angle $\theta$, which is associated with the $n$-$\eta$ mixing, can be obtained from Eq. (11) or (12) [75, 110]:

$$\theta \simeq \frac{\lambda_1 \lambda_2 |\psi_q(0)|^2}{m_\phi^2 (m_n - m_\eta)^2} \left( 1 - \frac{m_n^2}{m_\phi^2} \right)^3. \quad (44)$$

Following Ref. [96], the absorptive amplitude $\Gamma_{12}$, which describes the $n$-$\bar{n}$ oscillation through on-shell intermediate states, can be approximately given by

$$\Gamma_{12} \simeq \frac{g_n^2 \lambda_1^2 \lambda_2^2 |\psi_q(0)|^4 m_\eta}{64 \pi m_\phi^4 (m_n - m_\eta)^2} \left( 1 - \frac{m_n^2}{m_\phi^2} \right)^3. \quad (45)$$

Note Eq. (45) differs from Eq. (11) of Ref. [96] by a factor associated with the $n$-$\eta$ mixing. The dispersive amplitude $M_{12}$, which describes the $n$-$\bar{n}$ transition through off-shell intermediate particles, can be estimated according to Fig. 4(b). Assuming that the single-particle process makes a dominating contribution, the general form of $M_{12}$ can be given by (see e.g. Refs. [96, 111])

$$M_{12} \simeq \frac{1}{2\sqrt{s}} \sum_i |A(n \rightarrow \psi_i)|^2$$

$$\simeq \frac{\lambda_1^2 \lambda_2^2 |\psi_q(0)|^4 m_\eta}{m_\phi^2 (m_n^2 - m_\eta^2)}. \quad (46)$$

In the first step, the sum runs over all the intermediate particles $\psi_i$. The Mandelstam variable $s$ is defined in the conventional way and in the rest frame of the neutron it takes the value $s = m_n^2$. $A(n \rightarrow \psi_i) \equiv \langle \eta(p') | \hat{O} | n(p) \rangle$ is the amplitude in connection with the $n$-$\eta$ oscillation. In the second step, an approximation is made based on the assumption that $M_{12}$ is predominantly contributed by the process depicted in Fig. 4 where $\eta$ is the only intermediate particle. According to Eq. (45) and (46), the ratio between $\Gamma_{12}$ and $M_{12}$ takes the form:

$$\kappa \equiv \frac{\Gamma_{12}}{M_{12}} = \frac{g_n^2 (m_n - m_\eta)^2 (m_n + m_\eta)}{64 \pi m_\phi^4 m_n^2}. \quad (47)$$

This expression shows that by taking the ratio between $\Gamma_{12}$ and $M_{12}$ a large degree of uncertainty arising from the parameters, such as $\lambda_1, \lambda_2, m_\phi$ and $|\psi_q(0)|^2$, could be eliminated.

Once $\Gamma_{12}$ and $M_{12}$ are known, the observable consequences arising from the $n$-$\bar{n}$ oscillation can be obtained based on Eq. (47). The $n$-$\bar{n}$ oscillation time reported by the ILL experiments is around $0.86 \times 10^8$ s [14], which imposes a stringent constraint: $M_{12} \leq |\delta| \lesssim 7.7 \times 10^{-30}$ MeV [35, 112]. With the help of this constraint, the impact of the $n$-$\bar{n}$ oscillation on the masses and lifetimes of the mass eigenstates $N_1$ and $N_2$ can be evaluated based on Eq. (52) and (53) in Appendix A. Furthermore, the mass of the color triplet boson can also be estimated according to Eq. (10).
Fig. 5 shows the predicted lifetime of the mass eigenstate $N_2$ based on the results of the $n$-$\bar{n}$ oscillation experiments. The solid curve in red corresponds to the predicted lifetime of $N_2$ in the scenario where $N_1$ is heavier than $N_2$. The solid curve in blue corresponds to the predicted lifetime of $N_2$ in the scenario where $N_1$ is lighter than $N_2$. The horizontal dashed line is the experimental lifetime of $N_1$ reported by the trap experiment [23]. As can be seen from Fig. 5, the difference in the lifetime between $N_1$ and $N_2$ has a maximum value ($\Delta \tau \approx 1.84 \times 10^{-3}$ s) around the point $m_{\eta} = m_{\eta}/3$. This implies that when measuring the lifetimes of $N_1$ (mainly $n$) and $N_2$ (mainly $\bar{n}$), a lifetime difference as large as $1.84 \times 10^{-3}$ s would be expected. Such a lifetime difference is probably beyond the reach of the present experiments but may lie within the detectable regions in future experiments.

Fig. 6 shows the constraints on the mass of the color triplet boson $m_{\phi}$ in the framework of impure oscillation. The regions below the solid curves have been excluded. Similarly, if we are only interested in the appealing scenario where the masses of the color triplet boson lie within the range from several TeV to several 10 TeV, the coupling constants $|\lambda_1 \lambda_2|$ are more favorable to take the values of $10^{-3}$ and $10^{-2}$. As can be seen from Fig. 6 comparing with the scenario where the coupling constant is $|\lambda_1 \lambda_2| \approx 10^{-2}$, the scenario with $|\lambda_1 \lambda_2| \approx 10^{-3}$ predicts a smaller new physics energy scale. The bounds on the masses of the color triplet boson vary gently from several TeV to several 10 TeV throughout the entire range of the allowed $m_{\eta}$ values except in the vicinity of the neutron mass $m_n$. Similar trends have also been found in subsection 11B. If we require that $m_{\phi}$ lies within the range, which is accessible to a direct detection at the LHC or future high-energy experiments [70, 113], the mass of the neutral particle $m_{\eta}$ cannot be too close to the neutron mass $m_n$.

The neutron lifetime anomaly, which refers to the discrepancy in the measured neutron lifetime between two different experimental approaches, has attracted great attention recently (see e.g. Ref. [1]). This discrepancy suggests that the branching fraction for the decay of neutron into proton through the $\beta$-decay is around 99% and thus the invisible branching fraction is around $\Gamma_a/\Gamma_n \approx 0.01$ [51, 73, 94, 110], where $\Gamma_a$ and $\Gamma_n$ are the neutron decay rate and the anomaly-induced decay rate, respectively. As discussed in Sec. 4, the neutron lifetime anomaly remains a puzzle and a reasonable theoretical explanation needs to be constructed.

Concerning the neutron lifetime anomaly, it is necessary to figure out what the manifestations of a neutron state really are in its production, interaction, and detection processes. Particularly, in the detection process, it is important to distinguish between what particles have been created and what particles have been really detected in experiments [114]. If we maintain that a pure particle should have a definite mass and a definite lifetime, only the mass eigenstate can be treated as pure particles because it has a well-defined mass and lifetime. For example, in the $K^0$-$\bar{K}^0$ mixing [8, 82, 83], the mass eigenstates $K_L$ and $K_S$, which are the mixtures of the $K^0$ and $\bar{K}^0$ mesons, can be treated as pure particles because $K_L$ and $K_S$ have well-defined masses and lifetimes, but $K^0$ and $\bar{K}^0$ cannot according to this definition.

In the SM, a commonly recognized neutron state $|\eta\rangle$, which almost exclusively decays into electron, proton, and antineutrino through the $\beta$-decay process ($n \rightarrow pe^{-}\bar{\nu}_e$) [1], is mainly created by the weak and strong interactions and may not necessarily coincide with a mass eigenstate. It is analogous to the explanation for the solar neutrino problem [4, 115], where neutrinos are produced and detected in weak interaction eigenstates rather than in mass eigenstates. In this manner, the commonly recognized neutron state would not have a well-defined lifetime and the neutron lifetime discrepancy, which lies between the disappearance of the neutrons in a trap (bottle) and what have been detected in a beam, may be resolved in a simple way. The trap and bottle experiments are performed through the detection of the neutron disappearance [22], where the measurements are associated with the mass eigenstate. Alternatively, the beam experiments are performed through the detection of the $\beta$-decay products, such as proton and electron [22], where the measurements are associated with the weak interaction eigenstate. Therefore, what have been detected in the trap (bottle) and beam experiments are two different manifestations of neutron states.

In the presence of the $n$-$\eta$ mixing, a neutron that is created at the beginning ($t = 0$) can later be detected as an $\eta$ particle with a specific probability. If we assume that $m_{\eta}$ satisfies the condition: $|m_n - m_{\eta}| \gg 6.6 \times 10^{-23}$ MeV, the CP-even phase in Eq. (14) and (15) would satisfy the condition: $\phi_{1,2} \gg 1$ and the probability that a neutron transits into an $\eta$ particle at later time when it propagates through space can be approximated as

$$\frac{\Gamma_a}{\Gamma_n} \approx P_{n \rightarrow \eta} \simeq \frac{2\delta^2}{(m_n - |\mu_n B| - m_{\eta})^2 + 4\delta^2}. \quad (48)$$

With this expression, the mass of the color triplet boson can be estimated by

$$m_{\phi} \simeq \left( \frac{2\Gamma_n - 4\Gamma_a}{\Gamma_n} \right)^{\frac{1}{4}} \left[ \frac{\lambda_1 \lambda_2 |\psi_0(0)|^2}{m_n - |\mu_n B| - m_{\eta}} \right]^\frac{1}{2}. \quad (49)$$

Similarly, we assume that the coupling constants take the typical value $|\lambda_1 \lambda_2| \approx 10^{-2}$. If we require that the mass of the color triplet boson lies within the experimentally interesting range at the LHC or future high-energy experiments, namely $1 \lesssim m_{\phi} \lesssim 10$ TeV, the mass difference should satisfy the condition: $2.0 \times 10^{-2} \lesssim |m_n - m_{\eta}| \lesssim 2.0$ MeV, which is automatically consistent with the condition: $|m_n - m_{\eta}| \gg 6.6 \times 10^{-23}$ MeV and thus justifies the approximation in Eq. (48).

Next, we analyze the compatibility between the interpretation of the neutron lifetime anomaly and the interpretation of the $n$-$\bar{n}$ oscillation experiments in connection
with the $n$-$\eta$ mixing. The $n$-$\bar{\eta}$ oscillation probability is

given by $P_{n \to \bar{\eta}} \approx P_{\bar{n} \to \eta} P_{\eta \to \bar{\eta}} P_{\bar{\eta} \to n}$ [see Eq. (24) in sub-
section 111]. Here, the chirality subscripts are omitted.

The observability of the color triplet boson at the LHC or future high energy experiments requires that the mass difference $|m_n - m_{\eta}|$ cannot be too large. In this case, the $\eta$-$\bar{\eta}$ oscillation probability approximately takes the value $P_{\eta \to \bar{\eta}} \simeq 1$ according to Eq. (21). The lower bound im-
posed by the ILL experiment on the $n$-$\bar{n}$ oscillation prob-
ability is roughly in the order of $10^{-18}$ [see e.g. Ref. [55]],
which is much smaller than the $n$-$\eta$ oscillation probability
defined in Eq. (48) and seems inconsistent with the inter-
pretation of the neutron lifetime anomaly using the $n$-$\eta$
mixing. This inconsistency can be resolved by assuming
that the neutral particle $\eta$ has a much shorter lifetime
compared with the neutron. In the $n$-$\bar{n}$ oscillation ex-
periments, a small fraction of the neutrons, which are cre-
ated at the beginning from the neutron source, can con-
vert into the $\eta$ particles with a small probability as they
propagate through space. Most of the $\eta$ particles would
decay rapidly into invisible products before they oscillate
into neutrons and thus only a small fraction of the $\eta$
particles could oscillate into neutrons. According to this
assumption, the $n$-$\bar{n}$ oscillation probability can be rewritten
as $P_{n \to \bar{n}} \equiv P_{\eta \to \bar{\eta}} P_{\eta \to \bar{\eta}} P_{\eta \to \bar{\eta}} P_{\bar{\eta} \to n}$, where $P_{\eta \to \bar{\eta}}$
and $P_{\bar{\eta} \to n}$ are the survival probability of the $\eta$ and $\bar{\eta}$ particles,
respectively. According to the CPT symmetry, $P_{\eta \to \bar{\eta}}$
and $P_{\bar{\eta} \to n}$ should be equal and satisfy the exponential decay
law: $P_{\eta \to \bar{\eta}} \equiv P_{\bar{\eta} \to n} \equiv \exp(-\Gamma t)$, where $\Gamma \equiv \Gamma_2$ is the
decay rate of $\eta$ and it is associated with its lifetime by
$\tau_\eta \equiv 1/\Gamma_\eta$. If we assume $P_{\eta \to \bar{\eta}} \equiv P_{\bar{\eta} \to n} \lesssim 10^{-7}$, the
inconsistency can be explained. This requires that the lifetime
of $\eta$ satisfies the condition:

$$\tau_\eta \equiv \frac{1}{\Lambda_\eta} \lesssim -\frac{\tau_m}{\ln \left( \frac{P_{n \to \eta}^2}{P_{\eta \to n}^2} \right)},$$

(50)

where $\tau_m \simeq 0.1$ s is the mean propagation time of neutron
in the ILL experiment [14]. If the lifetime of $\eta$ satisfies
$\tau_\eta \lesssim 2.0 \times 10^{-3}$ s, the interpretation for the measurement
of the neutron lifetime and the interpretation for the measurement of the $n$-$\bar{n}$ oscillation time with regard to the
$n$-$\eta$ mixing can be consistent and the neutron lifetime anomaly can be explained in a direct and simple way. Hence, we could have a unified interpretation of the neutron lifetime anomaly and the $n$-$\bar{n}$ oscillation measurements based on the $n$-$\eta$ mixing. Note the $\eta$ particle we are discussing here is free particle. Similar to the reason for the stability of the neutron inside nuclei, the stability of the $\eta$ particle inside nuclei can be guaranteed by imposing additional assumptions or symmetries.

D. Geometric phase

Geometric phases, which provide a powerful tool for a
unified description of the classical and quantum phenomen-
a [110], can be observed in a number of ways, such as polarized neutron interference (see e.g. Ref. [117]), vi-
brational spectroscopy (see e.g. Ref. [118]), and etc. In this
work, we, specifically, consider the geometric phase
associated with particle oscillations.

The geometric phase and its observability has been dis-
cussed in the neutrino oscillation case [119,122], where
controversy has emerged concerning the measurability of
the Majorana phase and its connection to the geo-
metric phase. The authors of Refs. [119,122] argued
that the Majorana phase can non-trivially contribute to
a special type of the geometric phase defined in Refs.
[123,124] and such a geometric phase may be measurable
in neutrino oscillations. On the contrary, the author
of Ref. [120] argued that the corresponding results pre-
sented in Ref. [119] are not gauge-invariant and the Ma-
ajorana phase can be eliminated from the geometric phase
through a non-physical field rephrasing transformation
(see also Ref. [102]), making it unlikely to be observed
in neutrino oscillations. Shortly afterwards, the author
of Ref. [120] commented on the assertions made in Refs.
[119,120] and analyzed the gauge-invariant property of the
off-diagonal geometric phase [123] in neutrino oscil-
lations. Recently, the authors of Refs. [119,122] have
replied to the comments given by the authors of Refs.
[120,121] and explained why their arguments are reason-
able. Since neutrinos have a tiny mass and only interact
with matter very weakly, they are notoriously difficult to
detect in experiments. This imposes a great challenge
for the detection of the geometric phase in the neutrino
sector and hence no evidence for such a geometric phase
has been found in neutrino oscillations so far.

In Sec. 1113 we have discussed the observability of the
Majorana phase associated with the $n$-$\bar{n}$ oscillations.
Since the observability of the geometric phase is not nec-
essarily determined by its dependence on the Majorana
phase, in this work we only focus on the observable con-
sequences of the geometric phase associated with the
$n$-$\eta$ mixing, rather than attempting to resolve the contro-
versy on the Majorana phase. Comparing with neutri-
os, neutrons have a much larger mass and interact more
strongly with matter, making it more feasible to detect the
geometric phase in the neutron sector. The measure-
ments of the geometric phase with neutrons have been
suggested and conducted over the past decades [117,126-
130]. Since the mutual transitions between $n$ and $\eta$
can be resulted from the $n$-$\eta$ mixing, a path-dependent geo-
metric phase can be induced when neutrons propagate
through space. The measurement of the geometric phase
may provide another opportunity for the study of new
physical effects.

The geometric phase can be possibly observed through
the following neutron interference experiment. A beam
of highly coherent neutrons from a neutron source can be
split into two neutron beams by a beam splitter. The two
neutron beams travel inside the vacuum cavities of two
arms. The two arms have different lengths, i.e. $L_1$ and
$L_2$, respectively. When the two neutron beams arrive at
the same point of the detector, they can be recombined to
produce interference. If the geometric phase is non-zero, any difference between the two arm lengths can give rise to interference effects between the two neutron beams.

III. CONCLUSION

The partial compositeness model, which is featured by the mixing between composite and elementary particles, may give rise to non-trivial observable effects that are different from the Standard Model predictions. In this work, we have explored the possibility that neutron (n) mixes with elementary neutral particle (η), which may have a non-zero lepton number (L = 1) and its decay products can be dark matter candidates. We focus on two different scenarios, i.e. the type-I and type-II oscillations, roughly corresponding to whether the decay products of the neutron are responsible for the n-η oscillation. In both scenarios, the n-η oscillation can be induced through intermediate states with η more or less being involved. The n-η mixing violates both the B and L symmetries by one unit, but conserves their difference (B - L). We have shown that such a mixing can serve as a versatile platform where many interesting phenomena occur and the investigations on such phenomena may open a promising avenue for exploring new physics beyond the SM.

In the scenario where the neutral particle oscillation is type-I, the Majorana phase, which leads to CP-violating effects, can be observable. This is different from the situation in the type-II oscillation, where the Majorana phases of the dispersive and absorptive parts cancel out and there will be no CP-violation unless some conditions are satisfied [96]. B-violation and CP-violation (along with C-symmetry violation) are two of the three conditions presented by Sakharov to explain the observed matter-antimatter asymmetry in our Universe [107]. The n-η oscillation can be featured by both B-violation and CP-violation and thus may open a promising window for future studies of matter-antimatter asymmetry. Moreover, in this scenario, the lower limits imposed by the results of the searches for n-η oscillations on the mass of the color triplet boson (i.e. the new physics energy scale) are presented. The experimental constraints on the mass of the color triplet boson varies gently respectively from 5 to 25 TeV and from 1 to 8 TeV throughout the entire range of the allowed m_η values except in the vicinity of the neutron mass m_n. The derived new physics energy scales can be accessible to a direct detection at the LHC or future high-energy experiments [70, 113]. If the n-η oscillation was observed, the corresponding new physics particles, namely the color triplet bosons, would be within the reach of direct searches at the LHC or future high-energy experiments. In this regard, the searches for the n-η oscillations can provide a complementary and economical way of searching for new physics besides the direct searches for new physics at high-energy colliders.

In the scenario where the neutral particle oscillation is type-II, we analyze the testable implications on masses and lifetimes. The n-η oscillation induced by the n-η mixing gives rise to two mass eigenstates, which are predicted to have different lifetimes. One mass eigenstate (N_1) is predominantly composed of neutron state (n) and the other one (N_2) is predominantly composed of anti-neutron state (n). The constraint imposed by the searches for the n-η oscillations on the lifetime difference is predicted to be as large as 1.84 × 10^{-3} s, which may be within the detectable regions of future experiments. In the SM, the commonly recognized neutrinos, which almost inclusively decay into electron, proton, and antineutrino through the β-decay process (n → pe^−ν_e) [1], might not be necessarily mass eigenstates in the presence of exotic interactions and thus might not be described properly in the conventional treatment of the SM. For example, in the presence of the n-η mixing, a commonly recognized neutron may not be a mass eigenstate and thus may not have a definite lifetime. In this case, we could explore the compatibility between the interpretation of the neutron lifetime anomaly and the interpretation of the n-η oscillation experiments in connection with the n-η mixing. If the lifetime of η satisfies the condition τ_η ≲ 2.0 × 10^{-3} s, a unified interpretation of the two types of experiments based on the same n-η mixing can be obtained. In this manner, the neutron lifetime anomaly can be explained in a direct and simple way.

Finally, we discussed about the observability of the geometric phase associated with the n-η mixing. The measurement of such a geometric phase may provide another opportunity for the study of the new physical effects. Comparing with neutrinos, neutrons have a much larger mass and interact more strongly with matter, making it more feasible to detect such a geometric phase through a neutron interference experiment and a possible measurement scheme has also been suggested.

Appendix A

The effective mass matrix W can be diagonalized by the transformation matrix T:

\[ T W T^{-1} = \left[ \begin{array}{cc} c_1 & \epsilon_1 \\ c_2 & \epsilon_2 \end{array} \right] \left[ \begin{array}{cc} W_{11} & W_{12} \\ W_{21} & W_{22} \end{array} \right] \left[ \begin{array}{cc} c_1 & \epsilon_1 \\ c_2 & \epsilon_2 \end{array} \right]^{-1} \]

\[ = \left[ \begin{array}{cc} \omega_1 & 0 \\ 0 & \omega_2 \end{array} \right]. \]

Here, the mass and width of the mass eigenstates |n_1\rangle and |n_2\rangle are given by

\[ \omega_{1,2} = \frac{1}{2} \left[ M_{11} + M_{22} \pm \text{Re}(\Delta W) \right], \]

\[ \Gamma_{1,2} = \frac{1}{2} \left[ \Gamma_{11} + \Gamma_{22} \pm 2 \text{Im}(\Delta W) \right], \]

with

\[ \Delta W \equiv \left( |W_{11} - W_{22}|^2 - 4 W_{12} W_{21} \right)^{1/2}. \]
In this work, unless otherwise specified, we assume that the CPT symmetry is conserved and thus particle and anti-particle have the same mass, i.e. $m_n = m_{\bar{n}}$ and $m_{\bar{n}} = m_{\bar{n}}$. If the condition: $|W_{11}| \gg |W_{22}| \gg |W_{12}| \simeq |W_{21}|$ is satisfied, the mixing coefficients will satisfy the following conditions:

$$c \equiv c_1 \simeq c_2, \quad |c| \simeq 1, \quad (55)$$

$$\epsilon \equiv c_1 \simeq -c_2, \quad |\epsilon| \ll 1. \quad (56)$$

With the above approximations, the transformation matrix $T$ can be written as

$$T = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}. \quad (57)$$

Here, $\theta$ is the mixing angle.

**Appendix B**

In the presence of external magnetic fields, the probabilities for the $n$-$\bar{n}$ and $\bar{n}$-$n$ oscillations without making the approximation associated with the limit $\phi_{1,2} \gg 1$ can be given by Eq. (58) and (59) respectively. Note the direction of the magnetic field in Eq. (59) is opposite to the direction of the magnetic field in Eq. (58). Eqs. (58) and (59) show explicitly that the Majorana phase can not only be given by Eq. (58) and (59) respectively. Note the direction of the magnetic field in Eq. (59) is opposite to the direction of the magnetic field in Eq. (58). Eqs. (58) and (59) show explicitly that the Majorana phase cannot only be observable but also give rise to a CP-violating effect.

$$P_{n \rightarrow \bar{n}}^{B \neq 0} \simeq \sin^2(2\theta_1) \sin^2 \left( \frac{\phi_1}{2} \right) \left[ \left( \frac{m_n}{m_{\bar{n}}} \right)^2 \cos^2 \theta_1 + \sin^2 \theta_1 + \frac{1}{2} \left( \frac{m_{\bar{n}}}{m_n} \right) \sin^2(2\theta_1) \cos(\phi_1 - 2\xi) \right] \sin^2(2\theta_2) \sin^2 \left( \frac{\phi_2}{2} \right)$$

$$= \frac{1}{2} \left\{ 4\delta^2 \sin^2 \left[ \sqrt{(m_n - |\mu_B| - m_{\bar{n}})^2 + 4\delta^2} \right] \right\} \left\{ \left( \frac{m_{\bar{n}}}{m_n} \right)^2 \left( m_n - |\mu_B| - m_{\bar{n}} \right)^2 + 2\delta^2 \right\} + \sqrt{(m_n - |\mu_B| - m_{\bar{n}})^2 + 4\delta^2} \right\}$$

$$P_{\bar{n} \rightarrow n}^{B \neq 0} \simeq \sin^2(2\theta_1) \sin^2 \left( \frac{\phi_1}{2} \right) \left[ \left( \frac{m_n}{m_{\bar{n}}} \right)^2 \cos^2 \theta_1 + \sin^2 \theta_1 + \frac{1}{2} \left( \frac{m_{\bar{n}}}{m_n} \right) \sin^2(2\theta_1) \cos(\phi_1 + 2\xi) \right] \sin^2(2\theta_2) \sin^2 \left( \frac{\phi_2}{2} \right)$$

$$= \frac{1}{2} \left\{ 4\delta^2 \sin^2 \left[ \sqrt{(m_n - |\mu_B| - m_{\bar{n}})^2 + 4\delta^2} \right] \right\} \left\{ \left( \frac{m_{\bar{n}}}{m_n} \right)^2 \left( m_n - |\mu_B| - m_{\bar{n}} \right)^2 + 2\delta^2 \right\} + \sqrt{(m_n - |\mu_B| - m_{\bar{n}})^2 + 4\delta^2} \right\}$$

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