Stressed deformed condition of orthotropic spherical shell, lowered by two circular hole

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Abstract. Methods of calculating the shell structures from composite materials and studying the concentration of strains near the holes are devoted to a rather large number of works. However, most of the research was carried out within the framework of the classical Kirchhoff-Love hypothesis, which does not take into account the interlayer and lateral shifts characteristic of composite materials. Application of finite elements method to solve the problems on orthotropic shells of composites, weakened by several holes is reported within the improved theory of Timoshenko type. A numerical algorithm has been developed using the finite element method and a software package has been implemented on a computer that makes it possible to solve problems of stress concentration near two apertures in shells of composite materials. Shearing parameter has been studied for its influence on stress concentration near two circular holes internal pressure for orthotropic spherical shells. Specific results are presented for the cases of large and small holes, taking into account the rigidity of the reinforcing elements. Thus, this software package can be applied to the calculation of the elements of shell structures from orthotropic composite materials weakened by several holes.

1. Introduction
A large number of works have been devoted to methods for calculating shell structures made of composite materials and studying the concentric stresses near the holes. However, most of the studies were performed within the framework of the classical Kirchhoff-love hypothesis, which does not take into account the interlayer and transverse shifts characteristic of composite materials. Aim of the work. In this paper, we study the stress distribution around two unsupported circular holes in an orthotropic spherical shell made of composite material. Using the refined theory of Timoshenko-type shells, we will take into account the effects of cross-shear deformations for the entire package of the shell as a whole. In this case, the composite material will be considered as homogeneous with the given characteristics.
2. Problem statement

Multi-linked shells made of composite materials have been considered by few authors. The main results are given in [3,4,6]. For a transversally isotropic spherical shell with several holes, a solution was constructed by KI Sherenenko in work [14]. Moreover, infinite systems of algebraic equations for multiply connected domains with determinants of a normal type are obtained. On the example of a shell with two circular holes, the influence of shear parameters and the distance between the centers of the holes on the maximum stresses is studied. In this case, only the first approximation in the asymptotic expansion was taken into account in the solution, which, with a sufficiently close arrangement of the holes, can lead to significant errors. In the works of VN Chekhov [9], the question of the distribution of stresses in a cylindrical shell, weakened by two circular holes, in the classical formulation is considered. In the works of A. N. Guzya And K. I. Shnerenko [8,15], the stress-strain state of a transversally isotropic shell with two circular holes is studied, taking into account the shear stiffness. As follows from the above review of works, at present, the study of the stress-strain state of shells made of composite materials with closely spaced holes is insufficient. The need to solve such problems in connection with requests for engineering practice confirms the relevance of the topic of the article.

3. Methods

We consider an orthotropic spherical shell of radius $R$, thickness $h$, weakened by two circular holes of radii $r_0$, loaded with internal pressure of intensity.

Under the action of a surface load in the shell without a hole (far from the hole), the main stress state arises, which can be described by the equations of the momentless theory and later considered known. The perturbed stress state in the region of the holes is described by the homogeneous equations of the shallow shells [1, 2]. Due to the linearity of the problem, the desired stress state is represented as the sum of the primary and secondary caused by the presence of holes. In the middle surface of the shell, select the orthogonal curvilinear coordinate system $(x_1, x_2)$; the third coordinate $x_3$ reports in the direction of the external normal to the median surface of the shell.

We will proceed from the variational equation [2]:

$$
\int_\Omega (T_1\delta\varepsilon_1 + T_2\delta\varepsilon_{12} + S_{12}\delta\varepsilon_{12} + G_1\delta\kappa_1 + G_2\delta\kappa_2 + 2H_{12}\delta\kappa_{12} + Q_1\delta\varepsilon_{13} + Q_2\delta\varepsilon_{23})d\Omega + \int \left[ T_0^\rho \delta u_\rho + s_0^\rho \delta u_0 + G_0^\rho \delta y_\rho + H_0^\rho \delta y_\theta + Q_0^\rho \delta \omega \right] d\Gamma = 0, \quad (1)
$$

where $\Gamma = \Gamma_1 \cup \Gamma_2; \Gamma_1, \Gamma_2$ - contours of holes; $\Omega$ is the region whose boundary is sufficiently far from the contours of the holes, and the damping conditions are satisfied on it; $T_1, \ldots, Q_2$ - components of the disturbed stressed state; $T_0^\rho, \ldots, Q_0^\rho$ - components of the main stress state.

The geometric relations between the components of the deformations and the generalized displacements have the form [3]

$$
\varepsilon_1 = \frac{\partial u_x}{\partial x} + \frac{\omega}{R}; \quad \varepsilon_2 = \frac{\partial u_y}{\partial y} + \frac{\omega}{R}; \quad \varepsilon_{12} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}; \\
\varepsilon_{13} = \gamma_x + \frac{\partial \omega}{\partial y}; \quad \varepsilon_{23} = \gamma_y + \frac{\partial \omega}{\partial x}; \quad \kappa_1 = \frac{\partial y_\rho}{\partial x}; \quad \kappa_2 = \frac{\partial y_\theta}{\partial y}; \\
2\kappa_{12} = \frac{\partial y_\gamma}{\partial x} + \frac{\partial y_x}{\partial y}.
$$

(2)

The elasticity relations for the orthotropic shell are written in the form [3,4]
\[ T_1 = B_{11} \varepsilon_1 + B_{12} \varepsilon_2; \quad T_2 = B_{12} \varepsilon_1 + B_{22} \varepsilon_2; \quad s_{12} = B_{33} \varepsilon_{12}; \quad G_1 = D_{11} \kappa_1 + D_{12} \kappa_2 \]

\[ G_2 = D_{12} \kappa_1 + D_{22} \kappa_2; \quad H_{12} = D_{13} \kappa_{12}; \quad Q_1 = K_1 \varepsilon_{13}; \quad Q_2 = K_2 \varepsilon_{23}, \]

Where \( B_{ij} = C_{ij}, D_{ij} = h^3/12C_{ij}, \) \( i, j = 1, 3; \)

\[ G_{33} = G_{12}, \quad C_{22} = E_2(1-v_1v_2), \quad C_{12} = E_1v_2/(1-v_1v_2), \]

\[ C_{13} = C_{23} = 0, \mu_1 = \mu_2 = 5/6. \]

We assume that the boundary of the domain \( \Omega \) is at least two to three diameters distant from the holes.

We consider the case when one of the anisotropy axes coincides in direction with the \( x_1 \) axis passing through the line of centers of the holes. Due to the symmetry of the problem, we restrict ourselves to considering only a quarter of the region \( (0 \leq \theta \leq \pi/2) \), setting the boundary conditions on the axes of symmetry:

\[ u_x = v_x = S_{xy} = H_{xy} = Q_x = 0 \quad \text{at} \quad (x = 0), \]

\[ u_y = v_y = S_{xy} = H_{xy} = Q_y = 0 \quad \text{at} \quad (y = 0). \]

We solve the problem using the finite element method. As a finite element, we choose quadrangular isoparametric elements.

We introduce in each element a local coordinate system \( (\alpha, \beta) \) that satisfies the conditions [6]

\[ -1 \leq |\alpha| \leq 1, -1 \leq |\beta| \leq 1. \]

In an isoparametric quadratic element, the form functions are selected in the form [7]

\[ \varphi_i = \frac{1}{4} (1 + \alpha \alpha_i) (1 + \beta \beta_i) (\alpha \alpha_i + \beta \beta_i - 1) \quad (i = 1, 3, 5, 7), \]

\[ \varphi_i = \frac{1}{2} (1 - \alpha^2) (1 + \beta \beta_i) \quad (i = 2, 6), \]

\[ \varphi_i = \frac{1}{2} (1 + \alpha \alpha_i) (1 + \beta^2) \quad (i = 2, 8), \]

where \( \alpha_i, \beta_i \) i-coordinates of the ith node of the element in the local coordinate system.

The desired displacements for each node of the finite element are defined as [6]

\[ u_x = \sum_{i=1}^{8} u_x^i \varphi_i, \quad u_y = \sum_{i=1}^{8} u_y^i \varphi_i. \]

Where \( u_x^i, u_y^i \) are the i-sought values of displacements in the ith node of the element.

Realizing the variational equation (1) taking into account formulas (3), (6) and integrating numerically using Gaussian quadrature formulas, we obtain a stiffness matrix for each element. To systematically apply this method, we reduce the integration on each element to the calculation of the integral on the element, called the standard element, which will always be the same. The coordinate functions and coefficients given in the formula are determined once and are used for all elements. Further, summing over all elements and collecting coefficients for the same variations, we arrive at a system of linear algebraic equations for determining unknowns.
Here $N$ is the number of nodes of finite elements; $u_{x_i}^d$, ..., $\gamma_{x_i}^d$ – the desired values of displacements in the $i$-th node of the region. The values $A_{ij}$, ..., $E_{ij}$ determine the stiffness matrix of each element.

The matrix is symmetric, positive definite, has a tape structure, the width of which depends on the numbering of nodes. When the shell is loaded with internal pressure, it is assumed that the hole is closed by a lid that transfers to the circuit only by the action of a cutting force.

The boundary conditions in this case will take the form [4]

$$
T_p = -\frac{pR}{2}; \; Q_p = -\frac{pr_0}{2}; \; G_p = S_{p\theta} = H_{p\theta} = 0.
$$

(9)

A numerical algorithm was developed by the method of finite elements and a computer package of applied programs was implemented that allows solving stress concentration problems near two holes in shells made of composite materials. The calculation algorithm using the finite element method in the form of a displacement method includes the following steps:

- Breakdown of the structure into finite elements and preparation of topological, geometric and physical information;
- Construction for the selected finite elements of the corresponding stiffness matrices that determine the relationship between stresses and displacements in the nodes of the element;
- Formation of a resolving system of linear algebraic equations;
- Processing of the resulting information and its analysis.

The listed stages lend themselves to a clear universal algorithmization, and their software implementation does not cause difficulties when there is a library of standard routines that are implemented in the calculation process. The program is modular in nature and consists of a main (control) program and a set of auxiliary routines.

A numerical algorithm was developed by the finite element method, and a computer package of applied programs was implemented that allows solving stress concentration problems near two holes in shells made of composite materials. As an example, the calculations of the orthotropic shell with the following physical and geometric parameters were carried out:

$$
\frac{r_0}{R} = 0.1; \quad \frac{h}{r_0} = 0.1; \quad \nu_1 = 0.30; \quad \frac{E_2}{E_1} = 1.4; \quad \frac{G_{12}}{E_1} = 0.18;
$$

$$
\frac{G_{23}}{E_1} = 0.06
$$

Below the results of a numerical study of the stress-strain state of a shell with two holes are given. The distance between the holes was chosen so that the holes do not affect each other. In this regard, it is possible to compare these results with the results of the analytical solution for the case of a shell with one hole. Table 1 shows the values of the concentration of ring forces $K_T = 2T_\theta/p_0R + 1$ for a different number of elements obtained by the proposed method (FEM), as well as the values $K_T$, obtained using the analytical solution [4] (exact solution).

| $\theta$ | $G_{13}/E_1$ | $G_{13}/E_1$ |
|----------|--------------|--------------|
| 1.0      | 3.14         | 3.25         |
| 0.1      | 3.17         | 3.25         |
| 0.01     | 3.25         | 3.32         |
| 0.005    | 3.32         | 0.72         |
Figure 1. The distribution of the concentration coefficients of the ring forces

Figure 2. The distribution of the concentration coefficients of the moments

Fig. 1 and Fig. 2 shows the distribution of the concentration coefficients of the ring forces $K_\theta^T$ and moments $K_\theta^M$ at various points of the contour of the holes. Graphs are given for the values $\frac{t_0}{r} =$
0.3, 0.6, 1.8. When the holes approach each other, the stress concentration $K_\theta$ characteristically increases in the region $0 \leq \theta \leq \frac{\pi}{2}$, reaching a certain maximum. At the points $\theta = \frac{\pi}{2}$ and $\theta = \frac{2\pi}{3}$, when the holes approach each other, the coefficient $K_\theta$ decreases. The coefficient $K_M$ with decreasing $\frac{l_0}{r}$ in the region $\frac{2\pi}{3} < \theta \leq \frac{5\pi}{6}$ increases, and it is in the region $\frac{5\pi}{6} < \theta$ that the influence of the second hole is most manifested and, as can be seen from the figure, reaches its maximum at the point $\theta = \pi$.

At the point $\theta = 0$, the stress concentration coefficients vary insignificantly compared with the point $\theta = \pi$. Point $\theta = 0$ is the most distant from the second hole. The coefficient $K_M$ with a decrease in the distance between the holes increases by 108%. Calculations show that the mutual influence of the holes for the shell with the above parameters begins at $\frac{l_0}{r} = 2.0$. The coefficient $K_M$ decreases as the holes approach each other first decreases and reaches a minimum in the region $0 \leq \theta \leq \frac{\pi}{3}$, then an increase in $K_M$ is observed in the region $\frac{\pi}{3} < \theta \leq \frac{\pi}{3}$. In the vicinity of the point $\theta = \frac{2\pi}{3}$, the coefficient $K_M$ decreases and at the point $\theta = \frac{5\pi}{6}$ it begins to increase, reaching its maximum at the point $\theta = \pi$.

Thus, it can be seen from the above data that with an increase in the number of elements $N$ (n-order system of equations), the results quickly converge to the exact solution.

Table 2. The distribution of the ring stress concentration coefficients

| N  | 8   | 22  | 40  | 78  | 128 | Exact solution |
|----|-----|-----|-----|-----|-----|----------------|
| n  |     |     |     |     |     |                |
| $K_T$ | 3.301 | 2.957 | 2.888 | 2.842 | 2.832 | 2.827          |

Table 2 shows the distribution of the ring stress concentration coefficients $K_T = 2T_\theta/p_0R + 1$ (upper row) and the maximum ring moments of the shell thickness maximum $K_M = 12G_\theta/p_0Rh$ (lower row) at the points of the hole contour depending on the change in the parameter value Shear $G_{13}/E_1$. Shell parameters were chosen the same as in the previous case.

4. Conclusions

According to the results of the analysis in the considered examples, taking into account the deformation of interlayer shifts leads to corrections for the maximum values of the concentration coefficients of ring forces up to 6%, and ring moments up to 47%. Thus, the proposed technique can be applied to the calculation of shell structural elements which made of orthotropic composite materials weakened by several holes.

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