Masses and Mixings from Neutrino Beams pointing to Neutrino Telescopes*

K. Dick\textsuperscript{b}, M. Freund\textsuperscript{c}, P. Huber\textsuperscript{d} and M. Lindner\textsuperscript{e}

\textsuperscript{b,c,d,e} Theoretische Physik, Physik Department, Technische Universität München, James–Franck–Strasse, D–85748 Garching, Germany
\textsuperscript{a} Max-Planck-Institut für Physik, Postfach 401212, D–80805 München, Germany

Abstract

We discuss the potential to determine leading oscillation parameters, the value and the sign of $\Delta m_{31}^2$, as well as the magnitude of $\sin^2 2\theta_{13}$ using a conventional wide band neutrino beam pointing to water or ice Cherenkov neutrino detectors known as “Neutrino Telescopes”. We find that precision measurements of $\Delta m_{31}^2$ and $\theta_{23}$ are possible and that, even though it is not possible to discriminate between charges in the detector, there is a remarkably good sensitivity to the mixing angle $\theta_{13}$ and the sign of $\Delta m_{31}^2$.

*Work supported by “Sonderforschungsbereich 375 für Astro-Teilchenphysik” der Deutschen Forschungsgemeinschaft.
\textsuperscript{a}Email: Karin.Dick@physik.tu-muenchen.de
\textsuperscript{c}Email: Martin.Freund@physik.tu-muenchen.de
\textsuperscript{d}Email: Patrick.Huber@physik.tu–muenchen.de
\textsuperscript{e}Email: Manfred.Lindner@physik.tu–muenchen.de
1 Introduction

Recent studies of precision measurements of neutrino mixing parameters and neutrino masses focused on high intensity beams at neutrino factories, these offer unique advantages compared to conventional wide band neutrino beams. The development of a neutrino factory is however very cost intensive and still in its initial stage. For this reason it was recently emphasized that further studies of conventional wide band neutrino beams should be pursued [1]. One of the major advantages of a neutrino factory oscillation experiment would be the possibility to measure the appearance oscillation channel $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ (wrong sign muons) in very long baseline experiments. Using appearance rates enables the determination of the small mixing angle $\theta_{13}$, $|\Delta m^2_{31}|$ as well as the sign of $\Delta m^2_{31}$, which discriminates between the two possible mass ordering schemes [2, 3, 4]. An analysis of the appearance rates requires that the detector can discriminate the charges of $\mu^+$ and $\mu^-$ at a sufficient level in order to separate wrong sign muons from the large amount of right sign muons produced in the disappearance channel $\nu_\mu \rightarrow \nu_\mu$. Therefore, large magnetized iron detectors usually are the detection system of choice. In [5] it was however shown that the ability to determine $\theta_{13}$ and the sign of $\Delta m^2_{31}$ does not necessarily depend on the capability of charge identification. The information on $\theta_{13}$ and the sign of $\Delta m^2_{31}$ is also contained in the disappearance channel $\nu_\mu \rightarrow \nu_\mu$ and can thus be extracted from the $\nu_\mu$ rates without charge identification. If $\theta_{13}$ is not too small a measurement is possible in this channel with a precision which is comparable to the appearance channel. Furthermore, such a measurement would even be possible with conventional wide band neutrino beams, if a sufficiently high neutrino event rate is achieved, which is big enough to limit the statistical error, and if the systematic errors on the beam flux are under control. We show in this paper that conventional neutrino beams (consisting only of $\nu_\mu$) in combination with large water or ice Cherenkov detectors (Neutrino Telescopes without charge identification) can be used and give remarkable results. In particular we use as a prototype scenario a CNGS-type beam [6] and an AMANDA-like detector [7], for which the neutrino event rates are comparable to those of proposed neutrino factory experiments. We discuss the problems which arise in measurements of neutrinos in the energy threshold region of Neutrino Telescopes and suggest using beam pulse timing information and neutrino direction information to reduce the background from atmospheric muons. Finally, we perform a numerical analysis of the physics potential of this type of experiment and show that precision measurements of the leadings oscillation parameters as well as the determination of $\sin^2 2\theta_{13}$, the test of the MSW-effect [8] and the determination of the sign of $\Delta m^2$ are possible.

2 Three Neutrino Oscillations in Matter

The basic mechanism which allows the extraction of the sign of $\Delta m^2_{31}$ comes from coherent forward scattering of electron neutrinos in matter (MSW-effect) which leads to effective masses and mixings different from vacuum. In the approximation where the solar $\Delta m^2$ is ignored compared to the atmospheric $\Delta m^2$, i.e. $\Delta m^2_{21}$ = 0 they are given by $\Delta m^2_{31,m} = \Delta m^2 C_\pm$, $\Delta m^2_{32,m} = \Delta m^2 [(C_\pm + 1) + A]/2$, $\Delta m^2_{21,m} = \Delta m^2 [(C_\pm - 1) - A]/2$ and $\sin^2 2\theta_{13,m} = \sin^2 2\theta_{13}/C^2_\pm$ where $C_\pm = [(A/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta]^{-1/2}$ and $A = 2EV =
\[ \pm 2\sqrt{2} G_F Y \rho E / m_n, \]  
\[ \theta_{23} \] is not changed in the approximation \( \Delta m^2_{21} = 0 \) (we follow in this work the notation of [3]). The size of the matter corrections for a given neutrino species depends on the sign of \( \Delta m^2_{13} \) which enters into \( C_\pm \). The resulting modification of the muon disappearance probability in matter therefore depends on the sign of \( \Delta m^2_{13} \):

\[
P(\nu_\mu \leftrightarrow \nu_\mu) = 1 - \sin^2 2\theta_{23} \sin^2 \theta_{13,m} \sin^2(\Delta_{31,m}) 
- \sin^4 \theta_{23} \sin^2 2\theta_{13,m} \sin^2(\Delta_{31,m}) 
- \sin^2 2\theta_{23} \cos^2 \theta_{13,m} \sin^2(\Delta_{31,m}).
\]  

For antineutrinos, the matter induced modifications of the disappearance probability correspond to opposite sign of \( \Delta m^2_{31} \) for neutrinos. This can be used to amplify the signature of matter effects if \( \nu_\mu \) and \( \bar{\nu}_\mu \) beams can be compared. The biggest effect from the sign of \( \Delta m^2_{13} \) will be seen at the MSW resonance energy (in the Earth \( \approx 10^{-15} \text{ GeV} \)) and the sensitivity to matter effects will thus be best when the maximum of the neutrino spectrum is suitably adjusted and the detector is sensitive in this energy region. For a more detailed treatment see [3].

The differential muon event rates in the detector are:

\[
\frac{dn_\mu}{dE} = 10^9 N_A N_{KT} \epsilon_\mu \left( \frac{L_{\text{near}}}{L_{\text{far}}} \right)^2 f_{\text{near}}(E) \sigma(E) P_{\nu_\mu \rightarrow \nu_\mu}(E). \tag{2}
\]

Here \( 10^9 N_A \) is the number of nucleons per kiloton in the detector, \( N_{KT} \) is the detector size in kilotons, \( \epsilon_\mu \) is the detection efficiency and \( \sigma \) is the charged current neutrino cross section per nucleon. The neutrino beam flux \( f_{\text{near}} \), which we assume to be monitored at the near detector, must be corrected by the geometrical suppression factor \( (L_{\text{near}} / L_{\text{far}})^2 \), where \( L_{\text{near}} \) and \( L_{\text{far}} \) are the distances to the near and far detectors. Total rates are then obtained by integrating these differential rates from the threshold to the maximum neutrino energy.

### 3 Experimental Issues

Next we turn to the physics potential of large water or ice Cherenkov detectors as components of very long baseline neutrino oscillation experiments. Several experiments of this type which consist of photomultipliers which are attached to vertical strings are under construction or already taking data (AMANDA [7] / ICECUBE [9], ANTARES [10], BAIKAL [11] and NESTOR [12]). For the proposed ICECUBE array, the vertical spacing is about 15 m, and the 80 strings of 1 km length are 100 m away from each other. These Cherenkov detectors allow detector masses up to \( 10^5 \) kt which is a factor 100 more than the most ambitious ideas proposed for a megaton Super-Kamiokande-like water Cherenkov detector. These detectors, which conventionally are called “Neutrino Telescopes”, are primarily built in order to search for very high energy cosmic and atmospheric neutrinos. In combination with neutrino beams, some comments are in order concerning measurement of neutrinos very near the threshold energy of the detector (20 to 100 GeV). For very long baseline oscillation experiments it is important to achieve a detector energy threshold of 10 GeV to 20 GeV.
and reasonable energy resolution of about 10 GeV. This low neutrino energy is needed to take advantage of the MSW-effects which enhances sub-leading effects from $\theta_{13}$ and from the sign of $\Delta m^2_{31}$. The usually quoted energy thresholds for the detectors mentioned above are a result of the geometry of the photo multiplier (PM) array, the optical properties of ice or sea water and the specific energy loss of muons per traveled distance ($\sim 0.2$ GeV/m). Only upward moving muons are unambiguously assigned to neutrino interactions and the ability to reconstruct the particle track with sufficient precision is crucial in order to reject the huge background from downward going muons. Typically, hits in 7 or more PMs are requested for proper track reconstruction. For AMANDA, the most simple majority trigger requests 8 to 16 hit PMs, and this sets the energy threshold to 30-50 GeV. By triggering on certain hit patterns like hits in 5 neighboring (or nearly neighboring) PMs with some of them having high amplitude, one can select muons close to a string, with lower energy threshold (and of course paying with a reduced effective volume). Still, the background of downward muons dwarfs the signal, since the tracking error obtained from 5 hits is rather large. The situation is however different, when a neutrino beam is used as a source: The background can be reduced by five to six orders of magnitude, if beam pulse timing information\(^1\) and beam direction information\(^2\) is used. Independently a lower bound on the threshold is imposed by the requirement to distinguish hadronic showers which are produced by neutral current reactions of all neutrino flavors. The length of such a shower grows as $\log E$; the track length of a muon grows as $E$; therefore a cut on the minimum track-length is used which corresponds to an energy threshold of 10 GeV to 20 GeV, depending on the optical properties of the detector medium, and yields an energy resolution in the same range.

These arguments suggest strongly that Neutrino Telescopes can be used as large mass detectors for very long baseline neutrino experiments, but this idea requires certainly further investigation. For our work we assume a prototype detector with an effective mass of 5000 kt. This seems to be a reasonable estimate for the effective volume of AMANDA and ANTARES with respect to muons in the range of 10-20 GeV. The corresponding ICECUBE volume might come close to 10 to 100 Mt. The assumed energy threshold is 15 GeV and the energy resolution is 10 GeV \[^{14}\]. To illustrate the enormous potential of such a detector, we use as source a conventional wide band neutrino beam like NuMI or CNGS. The CNGS beam consists mainly of $\nu_\mu$ with an admixture of about 2\% $\bar{\nu}_\mu$, 0.8\% $\nu_e$ and 0.05\% $\bar{\nu}_e$ and the number of additional muons expected from oscillated $\nu_e$‘s in the beam is small and negligible. The mean energy of the beam lies in the range of 20 GeV to 30 GeV, with a long tail towards higher energies. In principle it is also possible to get a $\bar{\nu}_\mu$-beam by reversing the current in the lens system; this would result however in an smaller flux of about 75\% and larger admixtures of other neutrinos.

---

\(^1\)The duty cycle of wide band neutrino beams is about 1 : $10^6$, that of neutrino factories may reach 1 : $10^3$ \[^3\]. The absolute timing accuracy of events is about 100 ns (GPS accuracy).

\(^2\)In the case discussed here, the neutrino beam will meet the detector under a small angle to the direction of the strings.
4 Results

For our analysis we assume a 5000 kt detector with energy threshold 15 GeV, energy resolution 10 GeV (corresponding to four bins) and $\epsilon_\mu = 100\%$ detection efficiency. As neutrino source we used the CNGS flux spectrum for $4.5 \cdot 10^{19}$ protons on target [6]. We numerically calculate the charged current rate spectrum at the detector according to eq. 2. To compute the transition probability we integrate eq. 1 over the full Stacey matter density profile of the Earth [13]. We then impose Poisson fluctuations on the rates and re-extract the oscillation parameters with a maximum likelihood method (for details see [5]). Systematic errors of the beam flux and backgrounds are assumed to be small and are not included. The calculations are performed in the approximation $\Delta m^2_{21} = 0$, i.e. the solar mass squared splitting is ignored which is here a very good approximation. We investigate two different baselines. First 6500 km (approximately Fermilab–ANTARES), to measure $\theta_{13}$ and then 11200 km (CERN–AMANDA), to test the MSW-effect and to determine the sign of $\Delta m^2_{31}$.

In fig. 1 the total event rates at a baseline of 6500 km are shown as function of the atmospheric mass squared difference $\Delta m^2_{31}$ for different values of $\theta_{13}$ and for the two possible signs of $\Delta m^2_{31}$ (labeled $\oplus$ and $\ominus$). Apparently, a nonzero $\theta_{13}$ induces a significant depletion of the total rates only in the case of a positive sign of $\Delta m^2_{31}$. The reason for this is that the beam consists only of muon neutrinos which show in matter MSW-resonant enhancement only for positive $\Delta m^2_{31}$. In this case, studies of the sub-leading oscillation parameter $\theta_{13}$ and the sign of $\Delta m^2_{31}$ give a statistically more significant result than with negative sign of $\Delta m^2_{31}$.

Figure 1: Total muon rate $\nu_\mu$ as a function of $|\Delta m^2_{31}|$ for a baseline of 6500 km and $\sin^2 2\theta_{23} = 1$. The different line types stand for different mixing angles $\theta_{13}$ (see labels). $\oplus$ and $\ominus$ indicate positive and negative mass squared difference.

of the total rates only in the case of a positive sign of $\Delta m^2_{31}$. The reason for this is that the beam consists only of muon neutrinos which show in matter MSW-resonant enhancement only for positive $\Delta m^2_{31}$. In this case, studies of the sub-leading oscillation parameter $\theta_{13}$ and the sign of $\Delta m^2_{31}$ give a statistically more significant result than with negative sign of $\Delta m^2_{31}$.
$\Delta m^2_{31}$ (see discussion below). If indeed the sign of $\Delta m^2_{31}$ is negative, better significance would be achieved with an antineutrino beam, since antineutrinos are resonant in this case. It might, however, not be easy to cope with the systematic errors in the beam fluxes which are substantially different in the charge conjugated channel.

In order to simulate the extraction of the oscillation parameters, we performed fits to simulated spectral rates. Instead of performing a global fit to all parameters we first fitted the leading oscillation parameters $\theta_{23}$ and $|\Delta m^2_{31}|$, which are, to a good approximation, independent of $\sin^2 2\theta_{13}$. As second step, the sub-leading parameters $\theta_{13}$ and the sign of $\Delta m^2_{31}$ are fitted with the leading parameters fixed to the previously obtained best fit values. The result of the fit of the leading parameters for a given sample pair of parameters ($\sin^2 2\theta_{23} = 0.6$, $\Delta m^2_{31} = 4.0 \cdot 10^{-3} \text{eV}^2$) is shown in fig. 2. The relative $3\sigma$-errors for the whole parameter region which is allowed by the Super-Kamiokande experiment are between 0.5% and 10%, depending on the value of $\Delta m^2_{31}$. With the result of the fit of the leading parameters, it is possible to proceed to the second step: the fit of the sub-leading parameters $\sin^2 2\theta_{13}$ and the sign of $\Delta m^2_{31}$. Fig. 3 shows the parameter region in the $\sin^2 2\theta_{13}$–$\Delta m^2_{31}$ plane in which the obtained simulated spectral rates are not consistent with $\theta_{13} = 0$ at 90% confidence level. The shaded area is excluded by the CHOOZ experiment [16]. As explained above, the result differs for the two possible signs of $\Delta m^2_{31}$. For positive $\Delta m^2_{31}$ (solid line), a sensitivity for mixing angles down to $\sin^2 2\theta_{13} \approx 2 \cdot 10^{-3}$ is achieved. For negative $\Delta m^2_{31}$ (dashed line), the sensitivity is worse and does not reach the $\sin^2 2\theta_{13} \approx 10^{-2}$ level. In both measurements, $\theta_{13}$ and the sign of $\Delta m^2_{31}$, the use of an antineutrino beam would increase the sensitivity to a level comparable to the case of $\Delta m^2_{31} > 0$. In fig. 3 the $\sin^2 2\theta_{13}$–$|\Delta m^2_{31}|$ parameter region in which the determination of the sign of the mass squared difference is possible at 90% C.L for a positive sign of $\Delta m^2_{31}$ is shown. This shows also the limit of $\sin^2 2\theta_{13}$ where verification of the MSW-effect is possible. Since it is very important for this measurement to include neutrinos at the MSW resonance energy ($\approx 15 \text{ GeV}$), the value of the threshold of the

![Figure 2: Fit to the muon neutrino spectrum (i.e. the unoscillated ($\nu_\mu \rightarrow \nu_\mu$) muon neutrinos) for $\Delta m^2_{31} > 0$ at a baseline of 6500 km. Shown are the 1σ, 2σ and 3σ contours. The rectangle denotes the parameter pair for which the data are generated ($\sin^2 2\theta_{23} = 0.6$, $\Delta m^2_{31} = 4.0 \cdot 10^{-3} \text{eV}^2$) and the star denotes the obtained best fit.](image)
Figure 3: The 90\% C.L. exclusion line for $\sin^2 2\theta_{13} \equiv 0$ at a baseline of 6500 km and with $\sin^2 2\theta_{23} = 1$. The solid line is for the case $\Delta m_{31}^2 > 0$ and the dashed line for $\Delta m_{31}^2 < 0$. The shaded area is excluded by the CHOOZ experiment at 90\% C.L.

Figure 4: Sensitivity to the sign of $\Delta m_{31}^2$ at 90\% C.L. for $\Delta m_{31}^2 > 0$. The different lines were obtained with threshold energies of 5 GeV, 10 GeV, 15 GeV and 20 GeV at a baseline of 11200 km and with $\sin^2 2\theta_{23} = 1$. The shaded area is excluded by the CHOOZ experiment at 90\% C.L.

neutrino detector has a substantial influence on the obtained sensitivity. The lines shown in the plot were obtained with threshold energies of 5 GeV, 10 GeV, 15 GeV and 20 GeV. A negative $\Delta m_{31}^2$ in combination with a $\nu_\mu$ beam does not produce MSW-enhanced effects in the rates and thus does not allow to determine the sign. If $\Delta m_{31}^2$ is indeed negative, the
determination of the sign of $\Delta m^2_{31}$ and the test of matter effects requires an anti neutrino beam.

5 Conclusions

We studied in this paper the physics potential of large water or ice Cherenkov detectors (Neutrino Telescopes) in very long baseline accelerator neutrino oscillation experiments. In particular we have shown that conventional wide band neutrino beams pointed to detectors like AMANDA/ICECUBE, ANTARES or NESTOR give neutrino event rates at the level of $10^4$ to $10^6$ events per year. This number is comparable to rates achieved in presently proposed neutrino factory experiments. Neutrino Telescopes are primarily built for the search for ultra high energetic cosmic and atmospheric neutrinos. Conventional neutrino beams provide neutrinos of a few times 10 GeV, which is roughly at the quoted threshold of Neutrino Telescopes. To cope with this problem, we suggest to use beam pulse timing information and neutrino direction information to reduce the background produced by cosmic muons in a very effective way. The ultimate threshold is then given by limitations which arise from misidentification of hadron showers which are produced by all active neutrino flavors. We estimate that an energy threshold between 10 GeV and 20 GeV and an energy resolution between 10 GeV and 20 GeV would finally be achievable. This would open the door to a wide spectrum of interesting oscillation physics.

Our numerical study was performed in the standard three neutrino scenario under the approximation $\Delta m^2_{21} = 0$ and taking into account the full Stacey Earth density model. It demonstrates that precision measurements of the leading oscillation parameters $\Delta m^2_{31}$ and $\sin^2 2\theta_{23}$ are possible. Using the CNGS beam spectrum as prototype neutrino source, the relative $3\sigma$-errors for the whole parameter region which is allowed by the Super-Kamiokande experiment are between 0.5% and 10%, depending on the value of $|\Delta m^2_{31}|$. We further have demonstrated that, even though a measurement of the appearance oscillation channel is not possible (due to the missing capability to identify charges), there is good sensitivity to the sub-leading oscillation parameters $\sin^2 2\theta_{13}$ and the sign of the mass squared difference. Taking into account only statistical errors, we calculated that a measurement of $\theta_{13}$ would be possible down to $\sin^2 2\theta_{13}$ values of $2 \cdot 10^{-3}$. In case of a negative sign of $\Delta m^2_{31}$ the sensitivity would be worse, but this could be overcome by using an antineutrino beam. The values given above were all obtained at a baseline of 6500 km. With larger baselines, a test of the MSW-effect would be possible. In particular, at 11200 km the the sign of $\Delta m^2_{31}$ (which can be revealed only through matter effects) can be determined down to $\sin^2 2\theta_{13}$ values of approximately $10^{-2}$, depending on the precise value of $|\Delta m^2_{31}|$.

An important aspect of the experimental scenario studied in this work is that a test of the MSW effect and the determination of $\sin^2 2\theta_{13}$ could be done with existing technologies. This type of high rate neutrino experiment might thus be an interesting alternative to neutrino factory experiments. Further studies of systematic errors in the beam flux and backgrounds are however necessary and we recommend that detailed simulations of the detector response should be performed. A study of the potential of $\nu$-factory beams pointing
to a km$^3$ Cherenkov detector will be published soon [17].

**Acknowledgments:** We thank M. Leuthold and C. Spiering for supplying us with crucial information on detector issues. Furthermore, we want to thank L. Oberauer for helpful discussions. This work was supported by the “Sonderforschungsbereich 375 für Astroteilchenphysik” der Deutschen Forschungsgemeinschaft.
References

[1] V. Palladino, Talk presented at the NuFACT’00 International Conference, Monterey, California.

[2] V. Barger, S. Geer and K. Whisnant, Phys. Rev. D 61 (2000).

[3] M. Freund, M. Lindner, S.T. Petcov and A. Romanino, e-Print Archive: hep-ph/9912457.

[4] A. Cervera, A. Donini, M.B. Gavela, J.J. Gomez Cadenas, P. Hernandez, O. Mena and S. Rigolin, e-Print Archive: hep-ph/0002108.

[5] M. Freund, P. Huber and M. Lindner, e-Print Archive: hep-ph/0004085.

[6] G. Acquistapace et al., CERN report CERN-98-02; R. Bailey, J.L. Baldy, A.E. Ball Baldy et al., CERN report CERN-SL/99-034(DI).

[7] E. Andres et al., Astropart. Physics 13 (2000) 1.

[8] S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. 42 (1985) 1441; Sov. J. Nucl. Phys. 42 (1985) 913; S.P. Mikheyev and A.Yu. Smirnov, Nuovo Cimento C 9 (1986) 17; L. Wolfenstein, Phys. Rev. D 17 (1978) 2369; 20 (1979) 2634.

[9] M. Leuthold, IceCube Performance Studies, Amanda report 19991102; F. Halzen et al., HE 6.3.01 (1999) Desy Library, Procs. to the 26th International Cosmic Ray Conference (ICRC 99), Salt Lake City.

[10] ANTARES proposal, (1999) astro-ph/9907432.

[11] I.A.Belolaptikov et al., Astropart. Physics 7 (1997) 263.

[12] Anassontzis et al., DFF-283-7-1997, CERN report CERN-97-06 (1997).

[13] S. Geer, Phys. Rev. D 57 (1998) 6989.

[14] M. Leuthold and C. Spiering, Private Communication.

[15] F.D. Stacey, Physics of the Earth, 2nd edition, John Wiley and Sons, New York, 1977.

[16] M. Apollonio et al. (CHOOZ collaboration), Phys. Lett. B 466 (1999) 415.

[17] K. Dick, M. Freund, P. Huber and M. Lindner, in preparation.