$T$–odd Asymmetry 
in Chargino Pair Production Processes

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Abstract

$T$–violating asymmetry in chargino pair production processes is studied in the Minimal Supersymmetric Standard Model. The asymmetry emerges at the tree level in the production of two different charginos, and could be as large as of order $10^{-2}$ in unpolarized electron beam experiments and $10^{-1}$ in polarized electron beam experiments. In the pair production of the same charginos, the asymmetry emerges through the electric and the weak “electric” dipole moments of the charginos at the loop level. Its magnitude is of order $10^{-4}$. The consistency with the electric dipole moments of the neutron and the electron is also discussed.

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1. Introduction

Supersymmetry reduces the number of the parameters contained in a model. However, once it is violated, the number of the parameters proliferates. In an extension of the standard model with the exact supersymmetry, we have only the Kobayashi-Maskawa phase as a source of $CP$–violation. We know that the breakdown of the supersymmetry must occur at least at the electro-weak energy scale to prevent the supersymmetric partners of the known particles from appearing under the LEP and Tevatron energy scale. In realistic extensions of the standard model with the broken supersymmetry, we have new complex parameters as sources of $CP$–violation in addition to the Kobayashi-Maskawa phase.

It is well-known that these new complex parameters give large contributions to the electric dipole moments (EDMs) of the neutron [1-3] and the electron [4], which would be taken as a troublesome problem of the supersymmetric standard model. From the present experimental upper bounds of the EDMs of the neutron [5] and the electron [6], the phases of the complex parameters should be less than order of $10^{-2}$ or the masses of the super-partners, especially squarks and sleptons, should be larger than order of 1 TeV [7]. We take the latter position in this report, i.e. the phases of the complex parameters of the model are of order 1, but super-partners are heavy enough to clear the EDM constraints.

These new $CP$–violating phases cause various $CP$– and $T$–violating phenomena [8-12]. Amongst them $T$–odd phenomena may be possible to be observed in high energy collider experiments, especially in $e^+e^-$ collisions at 500 GeV. In the previous report [11] we considered, as an effect of $CP$–violation, the $T$–odd asymmetry in the neutralino production processes by $e^+e^-$ annihilation. We are now to discuss the chargino case.

2. $CP$–violating Interactions

The Lagrangian of the minimal supersymmetric extension of the standard model (MSSM) [13] is given by

$$L = L_{kin} + L_{gauge} + L_F + L_S;$$

$$L_F = [(E^c Y_E L) H_1 + (D^c Y_D Q) H_1 + (U^c Y_U Q) H_2 + m_H H_1 H_2]_F + \text{h.c.},$$

$$-L_S = (\bar{E}^c \eta_E \bar{L}) \bar{H}_1 + (\bar{D}^c \eta_D \bar{Q}) \bar{H}_1 + (\bar{U}^c \eta_U \bar{Q}) \bar{H}_2 + \bar{M}^2 \bar{H}_1 \bar{H}_2$$

$$+ \frac{1}{2} \sum_{i=1}^{3} \bar{m}_i \bar{\lambda}_{iR} \lambda_i L + \frac{1}{2} \sum_{a,b} \bar{M}_{ab} \phi_a^* \phi_b + \text{h.c.}$$
If one assumes $N=1$ supergravity and the grand unification of the fundamental interactions, some of these parameters in (1) could be related to each other at the unification scale,

\[
\tilde{M}_{ab}^2 = |m_{3/2}|^2 \delta_{ab}, \\
\eta_f = A m_{3/2} Y_f \quad (f = E, D, U), \\
\tilde{M}_H^2 = B m_{3/2} m_H, \\
\bar{m}_i = M_\lambda \quad (i = 1, 2, 3).
\]  

Besides the Yukawa coupling constants $Y_f$ and the vacuum expectation values of the two neutral Higgs scalar fields, the complex parameters involved in the model are $A m_{3/2}$, $B m_{3/2}$, $m_H$, and $M_\lambda$.

These complex parameters can in general become sources of $CP$–violation. The $CP$–violating interactions appear through rewriting the interactions by the mass eigenstates of the particles obtained by the diagonalization of the mass matrices. The mass matrix of our interest is the chargino one,

\[
(\lambda^- R, \psi^-_2)M^- \begin{pmatrix} \lambda^- L \\ \psi^-_1 \end{pmatrix} + \text{h.c.;}
\]

\[
M^- = \begin{pmatrix} \tilde{m}_2 & -g v_1^*/\sqrt{2} \\ -g v_2^*/\sqrt{2} & m_H \end{pmatrix} ,
\]

where $v_1$ and $v_2$ are the vacuum expectation values of $\tilde{H}_1$ and $\tilde{H}_2$, respectively. Extracting the phases of $\tilde{m}_2$, $v_1$, and $v_2$, we can write

\[
M^- = P_R M_C P_L^\dagger;
\]

\[
M_C = \begin{pmatrix} |\tilde{m}_2| & g |v_2| / \sqrt{2} \\ g |v_1| / \sqrt{2} & |m_H| e^{i \theta} \end{pmatrix},
\]

\[
P_R = \begin{pmatrix} e^{i \theta_g} & 0 \\ 0 & -e^{-i \theta_2} \end{pmatrix}, \quad P_L = \begin{pmatrix} 1 & 0 \\ 0 & -e^{i (\theta_1 + \theta_g)} \end{pmatrix},
\]

where $\theta = \theta_H + \theta_g + \theta_1 + \theta_2$, $\theta_H = \text{arg}(m_H)$, $\theta_g = \text{arg}(\tilde{m}_2)$, $\theta_1 = \text{arg}(v_1)$, and $\theta_2 = \text{arg}(v_2)$. The complex $2 \times 2$ matrix $M_C$ can be readily diagonalized by unitary matrices $U_{R,L}$ as

\[
U_R^\dagger M_C U_L = \begin{pmatrix} m_{\omega_1} & 0 \\ 0 & m_{\omega_2} \end{pmatrix};
\]

\[
U_R = \begin{pmatrix} \cos \theta_R & -e^{-i \beta_R} \sin \theta_R \\ e^{i \beta_R} \sin \theta_R & \cos \theta_R \end{pmatrix} \begin{pmatrix} e^{i \gamma_1} & 0 \\ 0 & e^{i \gamma_2} \end{pmatrix},
\]

\[
U_L = \begin{pmatrix} \cos \theta_L & -e^{-i \beta_L} \sin \theta_L \\ e^{i \beta_L} \sin \theta_L & \cos \theta_L \end{pmatrix},
\]
where the angles are calculated to be

\[
\tan 2\theta_R = \sqrt{2}g \frac{\sqrt{|v_2 m_2|^2 + |v_1 m_H|^2 + 2|v_1 v_2 m_2 m_H|^2 \cos \theta}}{|m_2|^2 - |m_H|^2 + g^2 (|v_1|^2 - |v_2|^2)^2 / 2},
\]
\[
\tan 2\theta_L = \sqrt{2}g \frac{\sqrt{|v_1 m_2|^2 + |v_2 m_H|^2 + 2|v_1 v_2 m_2 m_H|^2 \cos \theta}}{|m_2|^2 - |m_H|^2 + g^2 (|v_2|^2 - |v_1|^2)^2 / 2},
\]
\[
\tan \beta_R = \frac{\sin \theta}{\cos \theta + \frac{|v_2 m_2|}{|v_1 m_H|}}, \quad \tan \beta_L = -\frac{\sin \theta}{\cos \theta + \frac{|v_1 m_2|}{|v_2 m_H|}},
\]
\[
\tan \gamma_1 = -\frac{\sin \theta}{\cos \theta + \frac{|m_2|^2}{2(m_\omega^2 - |m_H|^2)^2} g^2 |v_1 v_2|},
\]
\[
\tan \gamma_2 = -\frac{\sin \theta}{\cos \theta + \frac{|m_2|^2}{2(m_\omega^2 - |m_2|^2)^2} g^2 |v_1 v_2|}.
\]

(6)

We note that only one combination of the four complex phases, \(\theta\), acts as a physical \(CP\)-violation source. Finally the chargino mass eigenstates \(\omega_i\) and the unitary matrices \(C_{R,L}\) diagonalizing (3) are obtained from \(U_{R,L}\) and \(P_{R,L}\) as

\[
\begin{pmatrix}
\omega_1 \quad \omega_2
\end{pmatrix}_{R(L)} = C_L^R \begin{pmatrix}
\omega_1 \\
\omega_2
\end{pmatrix}_L, \quad \begin{pmatrix}
\omega_1 \quad \omega_2
\end{pmatrix}_{R(L)} = C_R^L \begin{pmatrix}
\lambda \quad \psi
\end{pmatrix}_R,
\]

(7)

\(C_{R,(L)} = P_{R,(L)} U_{R,(L)}\).

The neutral current of the charged gaugino \(\lambda^-\) and the charged Higgsinos \(\psi_{1,2}\),

\[
J_\mu^Z = -e \cot \theta_W \bar{\lambda}^- \gamma_\mu \lambda^- - e \cot 2\theta_W \bar{\psi}_1 \gamma_\mu \psi_1 - e \cot 2\theta_W \bar{\psi}_2 \gamma_\mu \psi_2,
\]

(8)

is now rewritten by the mass eigenstates \(\omega_i\) as

\[
J_\mu^Z = e\omega_1 \gamma_\mu [G^1_L P_- + G^1_R P_+] \omega_1 + e\omega_2 \gamma_\mu [G^2_L P_- + G^2_R P_+] \omega_2
\]
\[
+ e\omega_1 \gamma_\mu [G^1_L P_+ + G^1_R P_-] \omega_2 + e\omega_2 \gamma_\mu [G^2_L P_+ + G^2_R P_-] \omega_1,
\]

(9)

where \(P_\pm = (1 \pm \gamma_5)/2\), and

\[
G^1_{L,R} = -\cot \theta_W |C^1_{L,R}|^2 - \cot 2\theta_W |C^{21}_{L,R}|^2,
\]
\[
G^2_{L,R} = -\cot \theta_W |C^{12}_{L,R}|^2 - \cot 2\theta_W |C^{22}_{L,R}|^2,
\]
\[
G^{11}_{L,R} = -\cot \theta_W C^{11}_{L,R} C^{11}_{L,R} - \cot 2\theta_W C^{21*}_{L,R} C^{21}_{L,R},
\]
\[
G^{21}_{L,R} = -\cot \theta_W C^{12}_{L,R} C^{11}_{L,R} - \cot 2\theta_W C^{22*}_{L,R} C^{21}_{L,R}.
\]

(10)
The coupling constants $G_{L,R}^{21}$ of the off-diagonal current can be explicitly given as

$$G_{L}^{21} = \frac{1}{\sin 2\theta_{W}} e^{i\beta_{L}} \cos \theta_{L} \sin \theta_{L},$$

$$G_{R}^{21} = \frac{1}{\sin 2\theta_{W}} e^{i(\beta_{R}+\gamma_{1}-\gamma_{2})} \cos \theta_{R} \sin \theta_{R},$$

and their relative phase is not zero in general. Thus we can see that the off-diagonal interaction of the charginos with $Z$ breaks $CP$ at the tree level.

3. $T$–odd Asymmetry

Our interest is in what $CP$–violating phenomena appear in the chargino pair production process of $e^{+}e^{-} \rightarrow \omega_{1}^{+}\omega_{1}^{-}$ from the interaction in (9). If one sums spins, $s_1$ and $s_2$, of both $\omega_{1}^{-}$ and $\omega_{1}^{+}$, its cross section has no $CP$–violating term. In other words one has to observe at least the spin state of either of the charginos to detect $CP$–violation effects. For purposes of illustration, let us sum $s_2$, but leave $s_1$ unsummed. The spin $s_1$ can be chosen as it is perpendicular to the interaction plane. In the C.M. system of $e^{+}e^{-}$, the cross section becomes

$$\sum_{s_2} d\sigma(e^{+}e^{-} \rightarrow \omega_{1}^{+}\omega_{1}^{-})/d\cos \theta = \frac{8\pi\alpha_{em}^{2}}{\sin^{2} 2\theta_{W}} \frac{p}{\sqrt{S}} \frac{1}{\sqrt{S} (S - M_{Z}^{2})^{2} + \Gamma_{Z}^{2} M_{Z}^{2}}$$

$$\times [ (f_{L}^{2} + f_{R}^{2}) \left( (|G_{L}^{12}|^{2} + |G_{R}^{12}|^{2})(E_{1}E_{2} + p^{2} \cos^{2} \theta) + 2\Re(G_{L}^{12}G_{R}^{12*})m_{\omega_{1}}m_{\omega_{2}} \right)$$

$$+ (f_{L}^{2} - f_{R}^{2}) \left( (|G_{L}^{12}|^{2} - |G_{R}^{12}|^{2})\sqrt{S} p \cos \theta + 2\Im(G_{L}^{12}G_{R}^{12*})\text{sign}(s_{1})m_{\omega_{2}}p \sin \theta \right)],$$

where $p$ is the magnitude of the chargino momentum, $E_{1,2}$ are the energies of $\omega_{1,2}$, $f_{L,R}$ are the coupling constants of the electron neutral current, $f_{L} = 1/2 - \sin^{2} \theta_{W}$, $f_{R} = - \sin^{2} \theta_{W}$, and sign$(s_{1}) = \text{sign}(\mathbf{s}_{1} \cdot (\mathbf{p}^{-} \times \mathbf{p}_{1}))$.

The last term in (12) breaks $T$–invariance, since the final state of sign$(s_{1}) > 0$ and that of sign$(s_{1}) < 0$ are transformed to each other by time reversal. The difference of the cross section between these two states manifests $T$–violation at the first-order perturbation. The asymmetry of these two states

$$A_{T} = \frac{d\sigma(\mathbf{s}_{1} \cdot (\mathbf{p}^{-} \times \mathbf{p}_{1}) > 0) - d\sigma(\mathbf{s}_{1} \cdot (\mathbf{p}^{-} \times \mathbf{p}_{1}) < 0)}{d\sigma(\mathbf{s}_{1} \cdot (\mathbf{p}^{-} \times \mathbf{p}_{1}) > 0) + d\sigma(\mathbf{s}_{1} \cdot (\mathbf{p}^{-} \times \mathbf{p}_{1}) < 0)},$$

would quantify how large the $T$–violation is.
For a numerical example, we show the result of a parameter set of \( \tan \beta = 2 \), \( \tilde{m}_2 = 200 \text{ GeV} \), and \( m_H = 200e^{i\pi/4} \text{ GeV} \), which leads to the chargino masses 133, 275 GeV and the neutralino masses 83, 145, 203, 278 GeV. In this parameter set, if the squarks are as heavy as 3 TeV, the EDM of the neutron can be as low as \( 0.9 \times 10^{-25} \text{ e} \cdot \text{cm} \). The figure 1(a) shows \( A_T \) as a function of the scattering angle \( \theta \) at \( \sqrt{S} = 500 \) GeV. The figure 2(a) shows \( A_T \) as a function of \( \theta \) and \( \sqrt{S} \). The magnitude of \( A_T \) is of order \( 10^{-2} \). The magnitude is somewhat smaller than what one might expect as a \( T \)–violation effect at the tree level. This is due to the fact that the electron neutral current is almost pure axial, i.e. \( |f_L| \simeq |f_R| \). If one wishes a larger \( A_T \), the polarized electron beam should be utilized. The electron beam is assumed to be left-handedly polarized in Fig. 1(b) and Fig. 2(b). The asymmetry becomes as large as \( \mathcal{O}(10^{-1}) \) in this case. From Fig. 2 it is seen that the asymmetry becomes saturated as \( \sqrt{S} \) becomes large. As long as \( \sqrt{S} \) is sufficiently larger than the \( \omega_1-\omega_2 \) threshold energy, \( A_T \) is about the same in increasing the electron beam energy further.

4. Discussion

We implicitly assumed in the preceding section that the spin of \( \omega_1 \) can be measured, which is possibly a difficult thing to do. What we can observe are the decay products of \( \omega_1 \). Since their momenta are affected by the spin state of \( \omega_1 \), they could substitute for \( s_1 \). Letting \( p_D \) denote a momentum of one of the decay products, we can then consider a \( T \)–odd asymmetry

\[
A'_T = \frac{d\sigma(p_D \cdot (p^- \times p_1) > 0) - d\sigma(p_D \cdot (p^- \times p_1) < 0)}{d\sigma(p_D \cdot (p^- \times p_1) > 0) + d\sigma(p_D \cdot (p^- \times p_1) < 0)}.
\]

The magnitude of \( A'_T \) would be the same order as \( A_T \). But its \( \sqrt{S} \) dependence will certainly be different from \( A_T \), because, as \( \sqrt{S} \) becomes larger, \( E_1 \) becomes larger, and \( p_D \) orients to the direction of \( p_1 \). Since the \( T \)–violation term is proportional to \( p_D \cdot (p^- \times p_1) \), \( A'_T \) would come to decrease at larger \( \sqrt{S} \), and have a peak at some value of \( \sqrt{S} \). It may even not be possible to measure the momentum of \( \omega_1 \). In this case the momentum of a decay product of \( \omega_2 \) could substitute for \( p_1 \).

Since the diagonal coupling constants of the charginos to \( Z \) are real, \( CP \)–violation does not occur at the tree level in the production of the same mass eigenstates of the charginos. However, the electric and the weak “electric” dipole moments of the charginos, \( D_{\omega_i} \), are generated at the one-loop level, which break \( T \)}
invariance. These dipole moment terms give rise to $T$-odd asymmetry in the same chargino pair-production processes. From the dimensional grounds, $A_T$ can be roughly estimated to be $A_T \sim \sqrt{S} D_{\omega_i}/e$. The main contributions to $D_{\omega_i}$ are given by the loop diagrams involving the top quarks, the top squarks, the $W$ bosons, and/or the Higgs bosons, and roughly $D_{\omega_i} \sim 10^{-20} e \cdot cm$. Thus we could expect $A_T \sim 10^{-4}$ in $e^+e^- \rightarrow \omega_i \bar{\omega}_i$, which would be too small for detection in the next $e^+e^-$ experiments.

Finally we should comment on the selectron exchange diagrams which have been entirely ignored in discussing the chargino pair production. They also contain $CP$–violating couplings, and produce $T$-odd asymmetry at the tree level in the production of the different charginos. However, the mass of the selectron should be as large as 3 TeV according to the analysis of the electron EDM if the charginos are as light as can be pair-produced at $\sqrt{S} = 500$GeV. Thus the contribution of the selectron could be neglected in this context.

In this report we have viewed $T$–odd asymmetry in the chargino pair production processes. If the imaginary phases of the supersymmetric parameters would have their natural value of $O(1)$, the $CP$ violation originating from the supersymmetric standard model could lead to measurable $CP$-violating phenomena. If the charginos are produced in $e^+e^-$ collisions, $T$-odd asymmetry would be observed in the angular distribution of the final decay products.

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Fig. 1 $T$-odd asymmetry as a function of $\cos \theta$ at $\sqrt{S} = 500$ GeV; (a) unpolarized electron beam, (b) left-handed polarized electron beam.
Fig. 2 $T$-odd asymmetry as a function of $\cos \theta$ and $\sqrt{S}$; (a) unpolarized electron beam, (b) left-handed polarized electron beam.