A Market of Inhomogeneous Threshold Cellular Automata

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This article summarizes some physics aspects of the market model of Weisbuch and Stauffer, Physica A (2003): How do demand and quality expectation adjust to each other in a buyer dominated market like cinema visits? And how can business cycles be modelled?

“Social percolation” [1] applied percolation theory to a marketing problem: How to quality expectations of potential buyers and the quality of a product adjust to each other? The model lead to self-organized criticality at the percolation threshold. Thus fluctuations were large, some products were a big hit, most were flops and lots of products were somewhere in the middle. Fig.1a [2] shows one of the simulations, and Fig.1b a corresponding result from a real market (cinema films; see also [3]). There is qualitative agreement; note that the simulation rank plot extends over four decades, the one with real data only over two decades, in vertical direction.

For this “threshold” conference, instead of this percolation model we apply threshold cellular automata to this marketing problem: How many potential buyers actually buy? Different people have different opinions, and thus our $N$ automata have different thresholds $\theta_i$, $i = 1, 2, \ldots, N$. This is what is meant with “inhomogeneous”. Each site $i$ on a large square lattice carries besides the threshold $\theta_i$ a spin $S_i$ which is +1 for buying and −1 for not buying. The spin orientation at the next time step is fully determined by the sum $Q = \sum_k S_k$ of the four neighbour spins $k$ of $i$: If this “quality” $Q$ is larger than the threshold $\theta_i$, site $i$ buys; otherwise it does not buy. The magnetization $M = \sum_i S_i/N$ is the normalized difference between the numbers of buyers and non-buyers. We start with all spins up and then use random sequential updating of the spins, with one time step meaning that on average every site was updated once.

As long as the thresholds do not vary with time, the above problem is similar to a zero-temperature simulation of a random-field Ising model.
However, after going to a movie you may not go immediately to another one except when it is very good; in contrast, when you have not seen a good movie since a long time you may visit even a mediocre one. Thus $Q$ is moved up if $S_i$ now is $+1$, and moves down if $S_i = -1$. Since the quality $Q$ as reported from the neighbours can take only the values $-4, -2, 0, 2, 4$, a threshold $2.1$ is equivalent to a threshold $3$ or $3.9$. Thus for simplicity we take our thresholds as odd integers $-5, -3, -1, 1, 2, 3$, initially distributed randomly, and change them at each time step by $2S_i = \pm 2$. When the threshold has reached $+5$ the next updating of this spin will change $S_i$ to $-1$ and $\theta_i$ to $3$, since $Q$ cannot reach $5$. (If one allows the thresholds the change by only a small amount, the dynamics gets slow and is described by a scaling law \[4\]; we ignore this complication here.)

This model thus describes a market dominated by demand of buyers and assuming sufficient supply of sellers. The judgment of the buyers is determined by what their neighbours do; if a sufficient number of neighbours buy, then also the central person buys. Which number of buying neighbours is sufficient in this sense depends on the time-dependent threshold $\theta_i(t)$. This threshold increases by $2$ (i.e. by one more neighbour) whenever the person buys, and decreases by the same amount when the person does not buy.

**Results:**

Fig. 2 shows that in a stationary equilibrium the buyers are not randomly distributed but form small clusters. The cluster size is much smaller than the lattice size (usually one million sites); in contrast to social percolation \[1\], we do not observe self-organized criticality. The fluctuations in the stationary magnetization decrease as $1/\sqrt{N}$ as usual: Fig. 3. The magnetization ($M = 1$ initially) decays to zero with damped oscillations, in one (Fig. 4), two (Fig. 5), three (Fig. 6) and four (Fig. 7) dimensions (hypercubic lattices). (Also the triangular lattice behaves similarly; not shown).

When you have drunken one bottle of wine, you should not immediately buy and drink the next one. Thus a site which just bought should wait $\tau$ time steps before considering to buy again. During this waiting time its spin stays at its value and counts as buyer. Analogously, after a decision not to buy, the spin stays at $-1$ for $\tau$ iterations and counts as not buying, to preserve up-down symmetry. The previous description thus is the special case $\tau = 0$. Then the oscillations are damped much less, as can be seen from Figs. 4-6. The amplitude of the oscillations decays exponentially with time at intermediate time scales (not shown).
Durable goods like houses are often bought only once in life. Setting in some sense $\tau = \infty$ but sacrificing the up-down symmetry kept until now, we assume that once a site has bought in never buys again. We still start with all $S_i = 1$ and $\theta_i$ random. Fig.8 then shows the numbers of actually still buying people; this number initially may vary non-monotonically with time but later decays exponentially with time until the whole market is saturated forever.

When we go back to the above reversible model with $\tau = 0$ and start with randomly oriented spins instead of all spins parallel, then the various clusters oscillate with different phases, showing oscillations for the overall magnetization $M(t)$ only for small lattices. In that case it is better to observe the autocorrelation function $\langle M(T + t)M(T) \rangle$ versus $t$ for large times $T$ such that equilibrium is established; this function then looks similar to the $M(t)$ of Fig.3. More information, including Fourier transforms, on this way of initialization are given in [4].

At the conference, many suggestions were made by the audience, like putting this model on a Barabási-Albert or small-world network, allowing for quenched or annealed solution, tune the model to the percolation threshold. J. Kertész suggested to put in a fraction $p$ of antiferromagnetic bonds, i.e. mistrust of neighbours: If this neighbour likes the movie it must be bad. We found that for $\tau = 0$ the oscillations vanish if at least half of the bonds are antiferromagnetic, while for $\tau \geq 2$ one overshooting takes place even if all bonds are antiferromagnetic.

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References

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Figure 1: a) Social percolation simulation, with small advertising [2]. We show a rank plot, i.e. the most successful “movie” is placed on rank 1, the second-most successful one on rank 2, etc. b) Rank plot of real movies, similar to [3], from IMDb.com.
Figure 2: Example of $100 \times 100$ square lattice, $\tau = 0$ after 300 iterations. Only the up spins are shown.
Figure 3: Fluctuations in $M$ versus linear dimension $L$ of square lattice, for $\tau = 0$ (+) and = 5 (x). The straight line in this log-log plot has the slope $-1$.

Figure 4: Magnetisation versus time for one dimension, $\tau = 0$ to 4 as shown in headline.
Figure 5: As Fig. 4 but for two dimensions.

Figure 6: As Fig. 4 but for three dimensions.
Figure 7: As Fig. 4 but for four dimensions.

Figure 8: Fraction of sales for 380 million irreversible automata who buy only once.