Dynamic Mott gap from holographic fermions in geometries with hyperscaling violation

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Abstract: We investigate a dynamically generated Mott gap from holographic fermions in asymptotically geometries with hyperscaling violation by employing a bulk dipole coupling for the fermion field. We find that when the coupling strength increases the spectral function first transfers to the negative frequency region but soon redistributes to the positive region. A stable gap and two bands emerges for all momentum when the coupling strength beyonds a critical value. Generally, The upper band on the positive frequency axis is much sharper than the lower band on the negative side. When the dipole coupling increases further, the gap becomes larger. The upper band keeps sharp while the lower band disperses and widens, concentrating on the small momentum region. We also find that the bands will be smoothed out gradually with the increasing of hyperscaling violation.

Keywords: AdS/CFT correspondence, gauge/gavity duality, holography and condensed-matter theory.
1. Introduction

Recent years, AdS/CFT correspondence has been widely used to study condensed-matter theory (AdS/CMT). The strongly coupled conformal theory in the boundary is mapped to weakly coupled gravity theory in the bulk. With this great advantage, people have successfully constructed holographic models of Fermi and non-Fermi liquids in kinds of geometries\cite{1, 2, 3, 4, 5, 6, 7, 8, 9} and analytically investigate the liquids properties, showing the dispersion relation and the width of the quasi-particle like excitation.

Since condensed-matter systems are usually described by non-relativistic field theories, to search more proper gravity duals people have further generalized the correspondence to non-relativistic holography, typically with anisotropic scaling behaviors for temporal and spatial coordinates\cite{10, 11, 12, 13} i.e. Lifshitz-like geometry with dynamical exponent. For realistic systems, importantly, another exponent, called hyperscaling violation will emerge and play a crucial role in low energy physics. It is certainly necessary to extend holography to this non-trivial case and it indeed comes true by employing the standard Einstein-Maxwell-dilaton action in the bulk\cite{4, 14, 15, 16, 17}. The metric behaves like:

\[ ds^2 = -\frac{dt^2}{r^{2m}} + r^{2n} dr^2 + \frac{dx_i^2}{r^2} \]  \hspace{1cm} (1.1)
where $i = 1, 2, ..., d$ is space index, $m$ and $n$ are related to dynamical exponent $z$ and hyperscaling violation exponent $\theta$ by

$$z = \frac{m + n + 1}{n + 2}, \quad \theta = \frac{n + 1}{n + 2} \cdot d \quad (1.2)$$

Note when $n = -1$, the metric reduces to the pure Lifshitz spacetime and $n = -2$ corresponds to a class of spacetime conformally related to $AdS_2 \times R_d$ with locally critical limit $z \to \infty$, $\theta \to -\infty$, while $z/\theta$ fixed to be a constant $[18]$. The metric transforms as

$$t \to \lambda^z t, \quad x_i \to \lambda x_i, \quad r \to \lambda^{(d-\theta)/d} r, \quad ds \to \lambda^{\theta/d} ds \quad (1.3)$$

Clearly, the metric is not scale invariant, unlike the pure Lifshitz case. The dual boundary theory exhibits this peculiar behavior generally below some non-trivial dimensional scale. But we won’t consider this complication, simply assuming the metric is asymptotically geometries with hyperscaling violation for our purpose.

We have found remarkable influence of hyperscaling violation on the dynamical gap by introducing a magnetic dipole coupling for bulk fermions. It was first proposed in $[19, 20]$ and further studied in $[21, 22, 23, 24]$. Similar to the published work, a gap in the spectral function was opened when the dipole coupling strength $p$ exceeds some critical value, very like a Mott insulator. The gap becomes further widened with $p$ increasing. The coupling strength $p$ plays a role similar to the dimensionless interaction strength $U/t$ in the Hubbard model of fermions. The novel feature we find is that two bands exist in the spectral function, a upper band on the positive frequency axis and a lower band on the negative side, respectively. These two bands behave qualitatively different with the increasing of the interaction $p$ and hyperscaling violation $\theta$.

This paper is organized as follows: In section 2, we briefly review the effective gravity model i.e. Einstein-Maxwell-dilaton theory for geometries with hyperscaling violation. In section 3, we study the bulk fermions with a dipole interaction, deriving the equations of motion for the retarded correlator. In section 4, we numerically solve the equations of motion under proper boundary conditions and extract the main results of the emergence of the gap. Finally, we present a conclusion in section 5.

2. Effective Gravity Model

The standard Einstein-Maxwell-dilaton (EMD) action reads

$$S = \int d^{d+2}x \sqrt{-g} \left[ R - 2(\partial \phi)^2 - V(\phi) - \frac{\kappa^2}{2} Z(\phi) F^2 - \frac{\kappa^2}{2} H^2 \right] \quad (2.1)$$

where the AdS radius has been set to 1. The solutions with hyperscaling violation are listed in the following:

\[ ds^2 = -r^{-2m} h(r) dt^2 + r^{2n} h^{-1}(r) dr^2 + \frac{dx_i^2}{r^2}, \quad h(r) = 1 - \left( \frac{r}{r_h} \right)^\delta \] (2.2)

\[ F^{rt} = F_0 r^{(m-n+d)} Z^{-1}(\phi), \quad H^{rt} = H_0 r^{(m-n+d)} \] (2.3)

\[ \phi = k_0 \log r, \quad k_0 = \sqrt{\frac{d}{2(m-n-2)}} \] (2.4)

\[ V(\phi) = -V_0 e^{-\beta \phi}, \quad V_0 = \delta (m + d - 1), \quad \beta = \frac{2(n+1)}{k_0} \] (2.5)

\[ Z^{-1}(\phi) = Z_0 e^{-\alpha \phi} + Z_1, \quad \alpha = \frac{2(n+d+1)}{k_0}, \quad Z_0 = \frac{\delta (m-1)}{\kappa^2 F_0^2}, \quad Z_1 = -\frac{H_0^2}{F_0^2} \] (2.6)

where \( \delta = m + n + d + 1, r_h \) is the location of the horizon, \( F_0, H_0 \) are constants which are proportional to the conserved charges carried by the black brane. The Hawking temperature and the entropy density of the black brane are give by

\[ T = \frac{\delta}{4\pi r_h^{m+n+1}}, \quad s = \frac{1}{8\kappa^2 r_h^d} \] (2.7)

In the zero temperature limit \( (r_h \to \infty) \), the entropy density approaches to zero, qualitatively differing from the black holes in RN background and more important for realistic systems with degenerate ground states.

In order to admit a stable theory, the dilaton solution is required to be real, leading to \( m \geq n + 2 \) or equivalently \( z \geq 1 + \theta/d, \theta < d \). Moreover, in the boundary limit \( r \to 0 \), the field strength \( F^{\mu\nu} \) diverges such that the dual chemical potential cannot be well defined. Therefore we introduce another gauge field \( H = dB \) to obtain a proper definition for finite density

\[ B(r) = \mu (1 - \frac{r^{(d-m+n+1)}}{r_h^{(d-m+n+1)}}) dt \] (2.8)

where \( \mu \) is the chemical potential. The constrained condition satisfying \( B \) and \( H \) finite in the UV limit is

\[ 2 \leq m - n \leq d, \quad d \geq 3 \] (2.9)
The divergent behavior of the field $F_{\mu\nu}$ certainly needs to be treated properly in a holographic renormalization procedure which we won’t discuss in detail in this paper. Since the bulk fermions we consider don’t couple to the dilaton and $F$ fields directly, the results we obtain are still credible, in the absence of a full treatment of the holographic EMD theory.

### 3. Holographic fermion with magnetic dipole coupling

In order to explore the effects of magnetic dipole coupling on the spectral function of fermions, we start from the following action

$$ S_f[\Psi] = i \int d^{d+2}x \sqrt{-g} \bar{\Psi} (\Gamma^a D_a - M - ip\mathcal{H}) \Psi + S_{bdy}[\Psi] \quad (3.1) $$

$$ S_{bdy}[\Psi] = i \int d^{d+1}x \sqrt{-g} \gamma^r \bar{\Psi} \gamma^r \Psi \quad (3.2) $$

where $S_{bdy}$ is a boundary action to ensure a well defined variational principle\cite{25} for the total fermion action. And $\bar{\Psi} = \Psi \Gamma^t$, $D_a = (e_a)^\mu D_\mu$, with $D_\mu = \partial_\mu - iqB_\mu + \frac{i}{4}\omega_{\mu ab}\Gamma^{ab}$, $\Gamma^{ab} = \frac{1}{2}[\Gamma^a, \Gamma^b]$. $\omega_{\mu ab}$ is the spin connection 1-form and $\mathcal{H} = \frac{1}{2}\Gamma^{ab}(e_a)^\mu(e_b)^\nu H_{\mu\nu}$. $\Gamma^a$ are the $d+2$ dimensional gamma matrices and $(e_a)^\mu$ are the vielbeins and $M$ is the mass of the probe fermion. Furthermore, $g_\epsilon$ is the determinant of the induced metric on the constant $r$ slice, $r = \epsilon$. $\Psi_\pm$ is defined by

$$ \Psi_\pm = \frac{1}{2}(1 \pm \Gamma^r)\Psi, \quad \Gamma^r \Psi_\pm = \pm \Psi_\pm \quad (3.3) $$

The Dirac equation derived from the action reads

$$ (\Gamma^a D_a - M - ip\mathcal{H})\Psi = 0 \quad (3.4) $$

Taking a Fourier transformation

$$ \Psi(r, x_\mu) = (-g g^{rr})^{-\frac{1}{4}} e^{-i\omega t + i k_1 x_1} \psi(r, k_\mu), \quad k_\mu = (-\omega, \vec{k}) \quad (3.5) $$

where the prefactor was introduced to remove the spin connection in the equations of motion. Since the theory is rotational invariant, we can choose the momentum along the $x_1$ direction. The Gamma matrices are chosen as follows

$$ \Gamma^r = \begin{pmatrix} -\sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}, \quad \Gamma^t = \begin{pmatrix} i\sigma^1 & 0 \\ 0 & i\sigma^1 \end{pmatrix}, \quad \Gamma^{x_1} = \begin{pmatrix} -\sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} \quad (3.6) $$

where $\sigma$ are Pauli matrices. We further set
\[ \psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \quad \psi_\pm = \begin{pmatrix} u_\pm \\ d_\pm \end{pmatrix} \quad (3.7) \]

Since the Dirac equation is first order, there exists some relation between \( \psi_+ \) and \( \psi_- \). Assuming \( \psi_+(r, k_\mu) = -i\xi(r, k_\mu)\psi_-(r, k_\mu) \), we can derive an elegant equation to extract correlators

\[ \sqrt{g^{rr}} \partial_r \xi_\pm + 2M \xi_\pm = (v_- \pm k\sqrt{g^{zz}})\xi_\pm^2 + (v_+ \mp k\sqrt{g^{zz}}) \quad (3.8) \]

where \( \xi_+ = iu_-/u_+ \), \( \xi_- = id_-/d_+ \), \( \xi_\pm \) are the eigenvalues of the matrices \( \xi \). And

\[ v_\pm = \sqrt{-g^{tt}(\omega + qB_t \pm p\sqrt{g^{rr}}\partial_t B_t)} \quad (3.9) \]

The corresponding retarded functions can be readily obtained as follows

\[ G_O(k_\mu) = \lim_{r \to 0} \xi(r, k_\mu) \quad (3.10) \]

At the event horizon, we impose in-falling boundary conditions

\[ \xi(r_h, k_\mu) = i, \quad \text{for } \omega \neq 0 \quad (3.11) \]

We emphasize that the dimension of the fermionic operator \( O \) is \( \Delta = (m + d)/2 \) which leads to the unitarity bound was automatically satisfied with \( m \geq 0 \) given by the null energy conditions. The fermion mass decouples from the operator UV dimension, contributing only to the IR physics which is peculiar in the asymptotical geometries with hyperscaling violation.

### 4. Numerical results and Emergence of the gap

To extract the effects of bulk dipole coupling on the spectral function, we need to numerically solve the flow equation \( (3.8) \) with initial conditions \( (3.11) \). The spectral function is proportional to \( \text{Im} G(\omega, k) \), up to normalization. Due to the relation \( G_{11}(\omega, k) = G_{22}(\omega, -k) \), we will only consider \( G_{22}(\omega, k) \) and omit the subscript in the following. For convenience to perform numerical calculation, we set \( M = 0, \mu = 1, q = 2, z = 2, d = 3 \). The dipole interaction strength \( p \) and hyperscaling violation \( \theta \) (or \( n \)) remain to be free.

First, we fix hyperscaling violation, considering \( n = 0 \) case. From the plots above in figure \( \# \), we find a sharp quasi-particle like peak occurs at \( k_F \approx 1.2044 \) for \( p = 0 \), indicating the existence of a Fermi surface. The dual liquid is of Fermi type with linear dispersion relation at the maximum height of the spectral function. For larger charge \( q \),
Figure 1: The 3D and density plots of $\text{Im}G(\omega, k)$. In the plots above, $p = 0$, a sharp peak occurs at $k \approx 1.2044$. In the plots below, $p = 4$, a gap emerges around $\omega = 0$.

it has been investigated with great detail in [26] that more branches of Fermi surfaces will emerge and even a peculiar Fermi shell-like structure exists, which exactly contains many sharp and singular peaks in some narrow interval of the momentum space when $q$ is large enough.

When the interaction strength was turned on $p = 4$, a gap emerges as it is shown in the plots below of figure 1. There are two bands, located at positive frequency (we call it upper band) and negative frequency (called lower band) regions respectively.
Evidently, the lower band is stronger than the upper one, occupying the main intensity of the spectral function. More interestingly, the upper band appears very sharp. At big momentum region, the lower band is also as sharp as the upper one but disperses for relative small momentum.

From plots in figure 2\(^1\), we can see that when \( p \) increases further, the gap becomes larger. The upper band always keeps sharp for all momentum, translationally moving

\(^1\)The black part in the 3D plots are purely numerical noise.
to the higher frequency region while the lower band is deformed much by transfer of the spectral weight to relative higher momentum region.

In order to show the emergence of the gap in detail, we present the plots of spectral function in figure 3. For very small \( p \), the spectral function still has a sharp peak at \( \omega = 0 \), showing the main feature of a Fermi surface. As \( p \) increases, the intensity of the peak degrades and first transfers to negative frequency region but soon returns to the positive frequency region. Finally, at some critical interaction strength \( p_{\text{crit}} \) the

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**Figure 3:** The plots of \( \text{Im}G(\omega, k) \) \((k = 1.2)\) for \( p = 0.01, 0.2, 0.4, 1.1, 1.5, 2.0 \), from left to right and top to down, respectively.
Figure 4: The plots of the spectral function $ImG(\omega, k)$ as a function of $\omega$ for sample values of $k \in [0.5, 2.5]$ for $p = 0$ (left plot above), $p = 4$ (right plot above), $p = 6$ (left plot below) and $p = 8$ (right plot below); $k = 0.5$ (black), $k = 0.9$ (purple), $k = 1.3$ (orange), $k = 1.7$ (red), $k = 2.1$ (blue) and $k = 2.5$ (green).

The original sharp peak at $\omega = 0$ disappears and two stable bands structure emerges in both frequency regions.

Moreover, to further explore the properties of the spectral function, we show the spectral function as a function of $\omega$ for $p = 0, 4, 6, 8$ for sample values of momentum. From the left plot above in figure 4, some peaks appear at both frequency regions. Around $\omega = 0$, the peaks become sharper with its height tending to infinity, indicating that a Fermi surface exists at $k = k_F$. When $p$ is amplified, the quasiparticle-like peaks around $\omega = 0$ degrade and vanish when $p$ crossing some critical value $p_{crit}$. A gap will be opened for all momentum as the cases in literatures [19, 20, 21, 22]. Evidently, the upper band appears sharper than the lower one. For the lower band, the height of the spectral function increases monotonically with the increasing of momentum. As $p$ increases further, the gap is widened. Both of the bands appear robust. Notice that in the right plot below of figure 4, the lower band disappears (the green and blue lines) when momentum exceeds some critical value, implying that a redistribution and
deformation happens for the lower band, consistently with our 3D and density plots in figure 2.

In order to determine the critical strength $p_{\text{crit}}$, we plot the density of states $A(\omega)$, the total spectral weight, which is defined by the integral of the spectral function $\text{Im} G(\omega, k)$ over $k$. We find that the onset of the gap is at $p_{\text{crit}} \approx 1.2$ in figure 5. Notice that for small $p$ ($p < p_{\text{crit}}$) the total spectral weight mainly distributes at negative frequency region but transfers to the positive region to open a gap. When the value of $p$ is large enough, it will redistribute and backtrack to the negative region again. These results are compatible with our previous observations (figure 1, figure 2, figure 3 and figure 4).

Finally, in figure 6 we find that the width of the gap $\Delta$ increases with the increasing of the interaction strength $p$.

We now turn to vary hyperscaling violation $\theta$ with fixed $p$. Without loss of generality, we set $p = 6$. In figure 7 and 8, we show plots of the spectral function for $n = 0.5$ and $n = 1$. Clearly, the bands structure is highly suppressed as $n$ increases. The upper band disappears first. The lower band also becomes smooth gradually. As $n$ is amplified further, we may argue that in the $\theta \rightarrow d$ limit, any sharp peak of the spectral function will be completely smoothed out and the spectral density will transfer and redistribute to all frequency-momentum space homogeneously, with no explicit gap and bands structure again, probably indicating a critical phase.

5. Conclusions

In this paper, we have studied the novel features of fermions in the presence of bulk
Figure 6: The gap width $\Delta$ as a function of $p$.

Figure 7: The density plots of $\text{Im}G(\omega, k)$, $n = 0.5$ for the left plot, $n = 1$ for the right plot respectively.

dipole coupling in the geometries with hyperscaling violation. For a finite hyperscaling violation $\theta = d/2$, we observe that when the dipole interaction strength $p=0$, a sharp quasi-particle like peak occurs near $k_F \approx 1.2044$ at zero frequency, indicating a Fermi surface. As $p$ increases, the intensity of the sharp peak degrades and first transfers to negative frequency region but soon redistributes and returns to positive frequency space. When $p$ crosses a critical value $p_{\text{crit}}$, the Fermi sea disappears and a stable gap with two bands located on both positive and negative frequency axis respectively emerges for all momentum. It seems very interesting that these two bands behaves
Figure 8: The plots of spectral function $\text{Im}G(\omega, k)$ as a function of $\omega$ for sample values of $k \in [0.5, 2.5]$ for $n = 0.5$ (the left plot), $n = 1$ (the right plot); $k = 0.5$ (black), $k = 0.9$ (purple), $k = 1.3$ (orange), $k = 1.7$ (red), $k = 2.1$ (blue) and $k = 2.5$ (green).

quite different. The upper band on the positive frequency region keeps sharp for all momentum while the lower band on the negative frequency region is only sharp for high momentum but disperses in the small momentum space. As $p$ increases further, the gap becomes wider. The upper band is still sharp but the lower band redistributes the intensity of the spectral function to smaller momentum space by sacrificing the sharp region of the high momentum. We also find that the width of the gap increases with the increasing of $p$.

When we fix $p = 6$ and turn on larger hyperscaling violation $n = 0.5, 1$ respectively, the peaks and bands are substantially suppressed. More interesting, the upper band disappears first while the lower band also becomes smooth gradually. Thus, the strength of the spectral density might distribute homogeneously to all frequency-momentum space in the $\theta \to d$ limit. It is of certain interests to explore the postulated critical phase in this limit. We leave it in near future.

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