Thermo-Mechanical and Low Cycle Fatigues
of Single Crystal Ni-Base Superalloys;
Importance of Microstructure for Life Prediction*

Motoki SAKAGUCHI** and Masakazu OKAZAKI***

Behavior of thermo-mechanical fatigue (TMF) of a single crystal Ni-base superalloy, CMSX-4, was studied, compared with that of isothermal low-cycle fatigue (LCF). Strain-controlled TMF and LCF tests of CMSX-4 were carried out under various test conditions, where the experimental variables were strain rates, strain ratio, temperature range, and strain/temperature phase angle. At first it was shown experimentally that the TMF and LCF failures took places, associated with some noteworthy characteristics which were rarely seen in the traditional polycrystalline heat resistant alloys. They could not be explained reasonably, based on the historical approaches. A new micromechanics model was proposed to predict the TMF and LCF lives, applying the Eshelby’s theory and the Mori-Tanaka’s averaging approximation. The model presented in this paper enabled us to successfully estimate not only the unique characteristics in the TMF and LCF failures but also the effect of $\gamma'$ geometry on the LCF lives.

Key Words: Low Cycle Fatigue, Life Prediction, Micromechanics, Single Crystal Ni-Base Superalloy, Thermo-Mechanical Fatigue, $\gamma'/\gamma$ Microstructure, Rafting, Mori-Tanaka’s Approximation

1. Introduction

Ni-based superalloys, especially single crystal superalloys, have been received special interests for blades and vanes in advanced industrial gas turbines(1),(2), since they have superior creep strength at elevated temperatures. One of the most attractive properties of superalloys is their unique temperature dependence in strength under monotonic and creep loadings, i.e., the strength increases with increasing temperature up to a certain temperature. In general Ni-based superalloys have naturally-developed composite microstructure consisting of a solid solution matrix, $\gamma$, with Ni$_3$(Al,Ti) precipitates, $\gamma'$, ordered and coherent with matrix(1),(2). Many researchers have been pointed out that this type of microstructure may play an essential role in the above attractive properties(1)–(10). During the gas turbine operation period, on the other hand, the hot section components are subjected to many types of damages. Nowadays, thermo-mechanical fatigue (TMF) failure has been one of critical issues to be concerned. Many types of efforts have been made to understand the TMF failure mechanisms and the life criteria for design, reliability and long term durability(1)–(15). One of traditional methods from an engineering point of view is a trial to estimate the TMF failure life from an empirical correlation with the isothermal low cycle fatigue (LCF) life(13),(13). This type of approach has been successful for many kinds of polycrystalline alloys. However, as will be shown in this work, superalloys, especially single crystal superalloys, exhibit many types of inconceivable TMF failure behaviors which are not well understood from these historical understandings. For an example, the TMF failure life under the out-of-phase condition often exhibits a life significantly lower than supposed(3). The effect of strain ratio on LCF lives is also a well-understood problem(14). More or less, a series of these unique characteristics in super-alloys may be related to the $\gamma'/\gamma$ composite microstructures(1)–(4),(9),(10). Nevertheless, no quantitative models have been established to provide reasonable explanations.

On the other hand, it has been well confirmed that
when single crystal Ni-base superalloys are subjected to external loads at elevated temperature, severe directional coarsening; so-called rafting, of the initially cuboidal γ′ particles to a plate-like or needle-like structure can occur(1),(2),(6). It has shown that the raft structures often developed in high-temperature creep and also in TMF tests(7). While much attentions have been paid to the rafted γ/γ′ microstructure during creep, less attention on the effect of rafting on the high temperature fatigue properties. One of a few attempts has been made by Ott and Mughrabi(15), who have evaluated experimentally the effect of the raft structures on the fatigue strength. However, no reasonable underlying explanations have been made.

It is the first object in this work to investigate the role of microstructure in TMF and LCF failures of a single crystal superalloy. The second is to explore a quantitative life prediction method, taking account of microscopic failure mechanisms and the role of microstructures. At first in this paper, some experimental results are shown regarding the LCF and TMF tests in a single crystal superalloy, CMSX-4. Then, a new micromechanics model is proposed to estimate the LCF and TMF lives. Finally, the model is applied to estimate the effect of rafted structures on the LCF lives.

2. Experimental Procedures

The material tested in this work is a second generation single crystal superalloy, CMSX-4, which has the following chemical compositions in weight percent: 6.4Cr, 9.7Co, 0.6Mo, 6.4W, 1.0Ti, 6.5Ta, 2.9Re, 0.1Hf, 5.7Al and bal. Ni. For this materials the heat treatments were given as follows: the 8 stages solution treatments by 1 277°C×2h+1 288°C×2h+1 296°C×3h+1 304°C×3h+1 313°C×2h+1 316°C×2h+1 318°C×2h+1 321°C×2h in Ar atmosphere, and then the two stages aging treatments by 1 140°C×6h+870°C×20h in air. As the result of the heat treatments the CMSX-4 revealed very regular microstructure, involving of cubic γ′ precipitates (Fig. 1). The volume fraction and the size of cubic γ′ precipitates are about 0.5µm and 63%, respectively. The solid cylindrical specimens, of which gauge section is 6.5 mm in diameter and 20 mm in length, were machined for the LCF and TMF tests. The LCF and TMF tests conditions employed are shown in Table 1. Here, the symbols summarized in this table will be used to denote the experimental results under the respective test conditions. All tests were performed under strain-controlled condition in air, utilizing a servo-electro hydraulic test system. The specimens were heated by a high frequency induction heating system. It has been confirmed by a preliminary test that the temperature difference between the shoulder part and the middle point of the specimen gauge section, were within 5°C, when the specimen were heated at 1000°C. This was also the case along the specimen gauge section. These temperature distributions were compensated during the natural cooling process in the TMF test. Thus, the test temperature during the TMF and LCF tests was represented and controlled through the R type of thermocouples welded near the shoulder part. This method makes us release from a cracking from the welded point: an undesirable phenomenon but often experienced in fatigue test of superalloys. The number of cycles to failure of the TMF and LCF lives was defined by the number of strain cycles at which the tensile stress was reduced by 50% from the stationary value.

The LCF tests were carried out at either 400°C or 900°C, where the experimental variables were strain

![Fig. 1 Microstructure of CMSX-4](image-url)
range, strain wave shape and strain ratio. Two types of strain wave shapes were repeated; fast-fast type strain (denoted by f-f) in which test frequencies were 1/30 Hz, and slow-slow wave (denoted by s-s) in which frequency was 1/600 Hz. The strain ratio, $R_e$, was a factor to which an special attention was paid in this work. Its value was varied between 0 and $-\infty$, where $R_e = -1$, $R_e = 0$ and $R_e = -\infty$ represent a fully reversed tension-compression, 0-tension and 0-compression strain cycles, respectively.

The TMF tests were carried out by superimposing the mechanical strain on the thermal free expansion strain in a synchronized manner with thermal cycling. The maximum and minimum temperatures in the TMF tests were same as that in the LCF test: 400 and 900°C. In this work a special attention was paid to the effect of temperature/strain phase angle on the TMF life, which was varied by three conditions: in-phase ($\phi = 0$ degree), out-of-phase ($\phi = 180$ degree), and counterclockwise diamond phase ($\phi = 90$ degree), respectively. All the TMF tests were carried out with a loading frequency of 1/600 Hz under a strain ratio of $R_e = -1$.

Note that there are the LCF test programs to be compared under comparable conditions (see Table 1).

3. Results

3.1 Cyclic stress-strain relationships

Some of the typical stress-strain hysteresis loops in the LCF and TMF tests are illustrated in Fig. 2. Note that the horizontal axes for the TMF tests are mechanical strain, $\Delta \varepsilon_{\text{mech}}$, in which thermal expansion strain is subtracted from the total strain. This kind of expression enables us to directly compare the TMF tests with the LCF ones. In order to simply understand the change of hysteresis loops with strain cycles, the loops are given by those at the 1st cycle ($N = 1$) and at the half life ($N = N_f/2$, where $N_f$ is the number of cycles to failure).

In the f-f type LCF tests at 400°C (Fig. 2 (a)) the specimen deformed fully elastically, with little inelastic deformation. On the other hand, a very small (but enough to be identified) inelastic strain appeared under the f-f LCF test at 900°C (Fig. 2 (b)). These situations were almost comparable in the s-s type LCF test at 900°C, when they were carried out $R_e = -1$ (compare Fig. 2 (c) with 2 (b)).

A clear difference was found when the strain ratio, $R_e$, was changed in the LCF tests under s-s type strain wave shape at 900°C (compare among Fig. 2 (c) – (e)). This was very clear at the 1st strain cycle in which a significant difference in the mean stress level was produced at the beginning of strain cycles: it was in tensile under $R_e = 0$, about zero under $R_e = -1$, and compressive under $R_e = -\infty$, respectively. However, the mean stress levels shifted towards zero very rapidly by repeating strain cycles, and the hysteresis loops resulted in the stable shape. After being stabilized, the hysteresis loops revealed very similar shapes, so that it is too hard to identify the strain ratio dependence at $N_f/2$ (see Fig. 2 (c) – (e)).

Meanwhile the hysteresis loops in the TMF tests reveal the peculiar asymmetric shapes in which inelastic deformations predominantly occurred in tensile portion under the in-phase conditions (Fig. 2 (f)), and in compressive...
portion under the out-of-phase condition (Fig. 2 (g)), respectively. The inelastic strain ranges were low, as well as in the LCF tests at 900°C. Whereas the mean stress at the 1st cycle was almost 0 under the both phase angle conditions, they gradually shifted so that they resulted in compressive under the in-phase and in tensile under the out-of-phase condition, respectively.

3.2 LCF and TMF lives

Figure 3 summarizes the LCF and TMF lives on the basis of mechanical strain range, $\Delta \varepsilon_{\text{mech}}$, where each symbol follows Table 1. For an easily understanding, the LCF and TMF lives are compared in Fig. 4 under a fixed strain range, $\Delta \varepsilon_{\text{mech}} = 1.2\%$. To be compared, the f-f LCF life of CMSX-2 at 600°C(4) is also plotted in Figs. 3 and 4. The following characteristics should be pointed out in Figs. 3 and 4:

(i) Regarding the effects of test temperature on the LCF lives; the f-f type LCF lives at 400°C are longer than those at 900°C.

(ii) Regarding the effects of strain rate on the LCF lives; the LCF lives under f-f strain wave shape are longer than those under the s-s strain wave-shape type LCF lives at 900°C.

(iii) Regarding the effects of strain ratio on LCF lives; the CMSX-4 failed even when it was subjected to 0-compression strain cycling. Note that the LCF life under $R_\varepsilon = -\infty$ was almost comparable to that under $R_\varepsilon = 0$. Thus, the LCF lives were not significantly influenced by the strain ratio at 900°C.

(iv) Regarding the effects of temperature/strain phase angle on the TMF lives; the TMF lives under the diamond conditions were the longest of all the TMF lives. The out-of-phase TMF lives were almost comparable to the in-phase TMF lives, and they are noticeably lower than those under the diamond phase condition. These characteristics are unique to the TMF of CMSX-4. Note that out-of-phase TMF lives are in general longer than the diamond phase TMF lives in other heat resisting metallic materials(12).

(v) Regarding the relation between LCF and TMF lives; according to the empirical law which has been established by polycrystalline materials(12), (13), the TMF lives under in-phase and out-of-phase conditions are supposed to be almost comparable to the LCF lives which are measured at the maximum and at the intermediate temperatures of the TMF tests, respectively. However, it is clear from Figs. 3 and 4 that the above approximation is not satisfied in the CMSX-4. For an example, the out-of-phase TMF life is significantly shorter than the LCF life of CMSX-2 at 600°C. Thus, the above empirical law can not be recommended for CMSX-4 single crystal superalloys.

It can be concluded from above characteristics that some essential behaviors in the TMF and LCF failures of the CMSX-4, especially in TMF failures, were too hard to be reasonably understood, so far as we relied on traditional macroscopic approaches.

3.3 Fracture surfaces

The SEM micrographs of fracture surfaces are given in Figs. 5 and 6. The fracture surface under the f-f type LCF tests at 400°C was composed of two types of regions (Fig. 5 (a)); Region I and II. The former is almost perpendicular to the stress axis, and the latter, significantly inclined to the loading axis, is covered with the $\{111\}$ crystallographic planes. As shown in the previous work(4), the Region I was the area of the early growth process of small cracks which were nucleated from a cast micro-pore. Note that there is a micro-pore at the core part of Region I (Fig. 5 (b)). It is also interesting that a regular mark of which size is almost comparable to the $\gamma'$ precipitates; about 0.5 μm, is seen in the Region I. The Region II is the area formed during the final fracture failure. While the
majority of the fracture surface is occupied by the Region II, the majority of fatigue life has been consumed by the Region I process(4).

Meanwhile, the fracture surface of specimen failed under the in-phase type TMF test is composed of many facets, associated with little (111) crystallographic planes (Fig. 6 (a)). Note that in this case the micro-defects are also found at the center part of each facet. It seems that a final TMF fracture might be taken place by the coalescence of many facets, which might be formed during the small crack propagation process on [001] planes.

The fracture surface of the specimens failed by the s-s type LCF tests under $R_e = -\infty$ at 900$^\circ$C, and under $R_e = -1$ at 400$^\circ$C, were quite similar to Fig. 5. On the other hand, those fractured by the s-s type LCF tests under $R_e = 0$ and $= -1$ at 900$^\circ$C, and by the out-of-phase TMF, were almost comparable to Fig. 6.

Summarizing these, while there were some apparent differences depending on the test conditions, the following is common: the preferential crack nucleation and propagation site was inside the γ matrix or γ/γ′ interface.

3.4 γ/γ′ microstructure after the TMF and LCF tests

As shown in Fig. 1, the CMSX-4 revealed the microstructure consisting of regular or cuboidal γ′ precipitates before the fatigue tests. Figure 7 shows the SEM micrograph of the CMSX-4 which were subjected to the TMF loading under the in-phase condition. It is found that the γ/γ′ microstructure almost remains the cuboidal γ′. It was also the case in all specimens after the TMF and LCF tests in this work. It may be due to that the maximum test temperatures were not so high (900°C in this work) to develop the raft structure, while the rafting has been confirmed to be pronounced in the LCF and TMF tests at higher than 1050°C(7).

4. Discussion

4.1 A life prediction model

One of main reason(s) why the traditional approaches could not be always applied to the present results, must be attributed to such a matter that there is an internal stress resulting from a lattice misfit between γ and γ′ precipitates in the single crystal Ni-based superalloys. In order to consider the effect of internal stress, the microstructure of the single crystal Ni-base superalloy is modeled by a composite material system in which γ′ precipitates distribute uniformly with a volume fraction of $V_f$ in the γ matrix. An important factor to be taken into account in the model is that the γ matrix and the γ′ precipitates have different elastic constants in general. The other important factor is a misfit strain. For the latter, it is assumed in this work, there is an isotropic lattice misfit strain, $\delta$, between them, which is defined by

$$\delta = 2 \left( \frac{\alpha_{\gamma'} - \alpha_{\gamma}}{\alpha_{\gamma'} + \alpha_{\gamma}} \right)$$

where $\alpha_{\gamma'}$ and $\alpha_{\gamma}$ are the lattice constants of γ′ and γ, respectively. In order to discuss the effect of rafting on fatigue strength, some typical ellipsoidal γ′ shapes are considered (Fig. 8). Here, the spherical geometry (Fig. 8 (a)) is taken as an approximation for cuboidal particles in superalloys. The flat ellipsoid (Fig. 8 (b)) and the needle-like geometry (Fig. 8 (c)) are simulating the rafted γ′ geometry perpendicular and parallel to external stress axis (Axis 3 in Fig. 8), respectively. These geometries can be characterized by their aspect ratios, $\alpha$, defined by $a_2/a_1$ (Fig. 8). $\alpha = 1$ and $\alpha = \infty$ for spherical and for needle-like geometries, respectively. For flat ellipsoidal geometry (Fig. 8 (b)), two kinds of aspect ratios, $a = 0$ and $\alpha = 1/3$ are considered in this work.

Misfit strain, $\delta$, produces misfit stress, $\langle \sigma_{ij} (\delta) \rangle_{\gamma}$ in the γ matrix and $\langle \sigma_{ij} (\delta) \rangle_{\gamma'}$ in the γ′ precipitates, even under no external stress(16). When the geometry of precipitates is a type of ellipsoids, these stresses can be estimated by applying both the Eshelby’s equivalent inclusion theory(16),(17), and the averaging stress field approximation.
by Mori and Tanaka\textsuperscript{(18)}. Here, note that the internal stress level is significantly influenced by the geometry of precipitates, which can be expressed in terms of so-called Eshelby’s tensor (Table 2).

When the composite material system is subjected to external stresses, $\sigma_{ij}^{A}$, additional stresses, $\langle \sigma_{ij} \rangle_{\gamma}$, $\langle \sigma_{ij} \rangle_{\gamma'}$, should be produced in each material component (i.e., $\gamma$ and $\gamma'$ in this work) to compensate an inhomogeneous elastic deformation\textsuperscript{(16), (17)}. Furthermore, when the material system undergoes plastic deformation, new additional internal stresses, $\langle \sigma_{ij} \rangle_{\gamma}$, $\langle \sigma_{ij} \rangle_{\gamma'}$, are produced by inhomogeneous plastic strain between $\gamma$ and $\gamma'$\textsuperscript{(16)}. In this work, a magnitude of the plastic strain inhomogeneity between $\gamma$ and $\gamma'$ will be denoted by a variable $p$.

When the material system undergoes a plastic deformation, the $\gamma$ and $\gamma'$ material components are supposed to carry the sum of each stress component as follows:

\begin{align}
\langle \sigma_{ij} \rangle_{\gamma} &= \sigma_{ij}^{A} + \langle \sigma_{ij} \rangle_{\gamma} + \langle \sigma_{ij} \rangle_{\gamma'}(2.a) \\
\langle \sigma_{ij} \rangle_{\gamma'} &= \sigma_{ij}^{A} + \langle \sigma_{ij} \rangle_{\gamma} + \langle \sigma_{ij} \rangle_{\gamma'}(2.b)
\end{align}

Each stress term in Eq. (2) is calculated by solving the simultaneous equations of the Eshelby’s theory\textsuperscript{(16), (17)}, and the averaging stress field approximation\textsuperscript{(18)}.

The elastic moduli of the composite material system under the fully elastic regime and the stress-strain relationships under the plastic regime can be calculated by combining the energy theorem and the energy equilibrium condition with respect to the increment of the plastic strain difference between $\gamma$ and $\gamma'$, $p$\textsuperscript{(16)--(18)}, which are given by

\begin{align}
\frac{\partial E_{el}}{\partial \sigma_{ij}^{A}} &= -\varepsilon_{ij}^{\gamma}\quad (3.a) \\
\frac{\partial E_{el}}{\partial p} &= 0, \quad \frac{\partial^{2} E_{el}}{\partial p^{2}} \geq 0. \quad (3.b)
\end{align}

Here, $E_{el}$ and $\varepsilon_{ij}^{\gamma}$ are the elastic potential energy and the macroscopic strain of the composite material system. The details of the numerical calculation procedure have been given in Refs. (16) and (18).

It is not hard to extend the above process to cyclic loading. In order to combine the stress/strain response estimated by the above procedure with fatigue failure, it is necessary to express “fatigue damage” by an appropriate parameter. Since “damage” is irreversible process, it should be represented by a parameter which can reasonably denote energy dissipation process. Plastic work density, $W_p$, should be such a candidate parameter. Here, $W_p$ is defined by

\begin{equation}
W_p = \int \sigma_{ij} \cdot d\varepsilon_{p,ij} \quad (4.a)
\end{equation}

where $d\varepsilon_{p,ij}$ is a plastic strain increment\textsuperscript{(20)}. The $W_p$ under a cyclic loading should be represented by a value per one cycle.

\begin{equation}
W_p = \oint \sigma_{ij} \cdot d\varepsilon_{p,ij} \quad (4.b)
\end{equation}

It is worth noting that the Manson-Coffin’s law has been derived in terms of $W_p$ by this type of concept\textsuperscript{(20)}. From these backgrounds, the following relations are assumed:

\begin{align}
W_p(\gamma) \cdot N_f(\gamma) = C(\gamma) \quad (5.a) \\
W_p(\gamma') \cdot N_f(\gamma') = C(\gamma') \quad (5.b)
\end{align}

where $N_f(\gamma)$ and $N_f(\gamma')$ are fatigue life of $\gamma$ and $\gamma'$ phases, and $C(\gamma)$ and $C(\gamma')$ are the material constants. The total fatigue life of the material system is supposed to be governed by a shorter value; either $N_f(\gamma)$ or $N_f(\gamma')$, according to the weakest link model. Thus, we can qualitatively discuss the effects of both the test conditions and the rafting on the LCF or TMF lives by comparing the values of $W_p(\gamma)$ or $W_p(\gamma')$, even when the values of $C(\gamma)$ and $C(\gamma')$ are undetermined.
Table 2  Summary of the Eshelby’s tensor for some typical types of ellipsoidal inclusions

| Aspect ratio, $\alpha = \gamma$ | Sphere | Flat ellipsoid | Needle |
|-------------------------------|---------|---------------|--------|
| $S_{1111}$                   | $\frac{7 - 5\nu}{15(1 - \nu)}$ | $0$ | $\frac{2(-7\nu - 5 + 16\nu^2)}{16(1 - \nu)}$ | $0$ |
| $S_{1122}$                   | $\frac{7 - 5\nu}{15(1 - \nu)}$ | $0$ | $\frac{-6 + (1 + 13 - 64\nu^2\gamma)}{128(1 - \nu)}$ | $\frac{5 - 4\nu}{8(1 - \nu)}$ |
| $S_{1233}$                   | $\frac{7 - 5\nu}{15(1 - \nu)}$ | $1$ | $\frac{2(-7\nu - 5 + 16\nu^2\gamma)}{16(1 - \nu)}$ | $\frac{3 - 4\nu}{8(1 - \nu)}$ |
| $S_{1133}$                   | $\frac{5\nu - 1}{15(1 - \nu)}$ | $0$ | $\frac{-2 - (5 - 64\nu^2\gamma)}{128(1 - \nu)}$ | $\frac{4\nu - 1}{8(1 - \nu)}$ |
| $S_{2233}$                   | $\frac{5\nu - 1}{15(1 - \nu)}$ | $0$ | $\frac{-2 - (11 - 16\nu^2\gamma)}{32(1 - \nu)}$ | $\frac{\nu}{2(1 - \nu)}$ |
| $S_{2211}$                   | $\frac{5\nu - 1}{15(1 - \nu)}$ | $0$ | $\frac{-2 - (5 - 64\nu^2\gamma)}{128(1 - \nu)}$ | $\frac{4\nu - 1}{8(1 - \nu)}$ |
| $S_{2222}$                   | $\frac{5\nu - 1}{15(1 - \nu)}$ | $0$ | $\frac{-2 - (11 - 16\nu^2\gamma)}{32(1 - \nu)}$ | $\frac{\nu}{2(1 - \nu)}$ |
| $S_{3311}$                   | $\frac{5\nu - 1}{15(1 - \nu)}$ | $\frac{\nu}{1 - \nu}$ | $\frac{2(1 + 16\nu - 1(1 + 32\nu^2\gamma)}{32(1 - \nu)}$ | $0$ |
| $S_{3322}$                   | $\frac{5\nu - 1}{15(1 - \nu)}$ | $\frac{\nu}{1 - \nu}$ | $\frac{2(1 + 16\nu - 1(1 + 32\nu^2\gamma)}{32(1 - \nu)}$ | $0$ |
| $S_{3333}$                   | $\frac{4 - 5\nu}{15(1 - \nu)}$ | $0$ | $\frac{-2 + (5 - 64\nu^2\gamma)}{128(1 - \nu)}$ | $\frac{3 - 4\nu}{8(1 - \nu)}$ |
| $S_{3111}$                   | $\frac{4 - 5\nu}{15(1 - \nu)}$ | $\frac{1}{2}$ | $\frac{2(9 - 8\nu + 1(9 - 8\nu^2\gamma)}{32(1 - \nu)}$ | $1/4$ |
| $S_{3133}$                   | $\frac{4 - 5\nu}{15(1 - \nu)}$ | $\frac{1}{2}$ | $\frac{2(9 - 8\nu + 1(9 - 8\nu^2\gamma)}{32(1 - \nu)}$ | $1/4$ |

4.2 Application to estimate the LCF and TMF lives

The material constants used for the numerical calculation are summarized in Table 3. Here, the values of Young’s modulus and the misfit have been measured in Ref. (21). We assumed both $\gamma$ and $\gamma'$ are isotropic elastic-plastic solids. The values of yield stresses of $\gamma$ and $\gamma'$ in Table 3 are tentative, but they can provide a reasonable stress-strain relation under monotonic loading, as shown later.

The method presented in section 4.1 can be directly applied for the isothermal LCF tests. For the TMF test, however, it is necessary to employ some approximations, since the material temperature always varies with time or strain. In this work, the TMF test was approximated by “bi-thermal” fatigue test at two temperatures; the maximum and minimum temperatures of the TMF test. Thus, for an example, the out-of-phase TMF test is assumed by both the LCF test at minimum temperature in tension side, and the LCF test at maximum temperature in compression side. The value of $\gamma/\gamma'$ misfit and elastic constants were assumed to be independent on temperature (Table 3).

Let’s look at the calculation results to estimate the stress-strain response. Since the $\gamma/\gamma'$ microstructures have not been changed to the rafted structure even after all the TMF and LCF tests (see section 3.4), the geometry of $\gamma'$ precipitates is approximated to be held spherical (Fig. 8 (a)) in this subsection.

Figure 9 shows some of the predicted hysteresis loops under the uni-axial load to axis 3 (see Fig. 8). The experimentally measured hysteresis loops at steady cycle are also presented. In Fig. 9, the symbols, (1) through
Fig. 9  Hysteresis loops under the condition of $\Delta \varepsilon_{\text{mech}} = 1.2\%$ estimated from numerical calculation. (a) TMF in-phase ($R_e = -1$), (b) LCF at 900°C ($R_e = -1$), (c) TMF out-of-phase ($R_e = -1$)

(5), denote some key points in deformation. From 0 to (1) the material deforms elastically during the 1st monotonic loading period. During the periods between (1)→(2), (3)→(4) and (5)→(2), the material system deforms inelastically associated with plastic deformation only in the $\gamma$ matrix, and without plastic strains in the $\gamma'$ phase. While these calculations were repeated over 20 fatigue cycles, the hysteresis loops were stabilized or closed after the 2nd loading cycles, and no ratcheting behavior was estimated. It seems from these figures that both the yield stress level and the characteristics in hysteresis loops under the respective test conditions seem to be well reproduced by the present method. The deviations in average stress from the experimental result under the in-phase TMF (Fig. 9 (a)) and out-of-phase TMF (Fig. 9 (c)) would be due to an approximation in the proposed method that neglects creep or recovery process during high temperature cyclic deformation.

As shown in section 3.3, the initial fatigue crack growth emanated in the $\gamma$ matrix or the $\gamma/\gamma'$ interface. Accordingly, it is adequate for us to compare the values of $W_p(\gamma)$, Eq. (5.a), for the purpose of relative life prediction of LCF and TME lives. Figure 10 shows a summary of the calculated result of $1/W_p(\gamma)$, a parameter corresponding to fatigue life. This figure can be directly compared with Fig. 4. It is found by the comparison that the failure characteristics (i), (iii) and (iv) in section 3.2 can be successfully estimated. It is worth noting that the features which the traditional approach failed to explain (e.g., effect of strain ratio in the LCF life, and the correlation between TMF and LCF lives) are successfully predicted in Fig. 10. The same predicted results were seen in other strain ranges. A consideration of internal stress resulting from $\gamma/\gamma'$ misfit may greatly contribute to this good result. However, there is a following discrepancy in Fig. 10: the calculation predicts that the TMF life under the in-phase condition is longer than the LCF life at 900°C. This must be attributed to some assumptions employed in this work. Additionally, our model can not explain the effect of strain rate on LCF lives, because time-dependent inelastic strain, or creep strain, is treated by the same status as time-independent plastic strain. Thus, our model should be extended further so that time-dependent effects are reasonably taken into account.

4.3 Application to estimate the effects of rafted structure on LCF life

The effect of the $\gamma'$ precipitates geometries on the LCF and TMF lives can be discussed in the present model, through a difference of the Eshelby’s tensor (Table 2) in Eq. (2), a parameter reflecting the geometry of the $\gamma'$ precipitates. Figure 11 shows the calculated $1/W_p(\gamma)$ for different $\gamma'$ geometry under the condition of the LCF at 900°C. The material system with the needle type of rafted
γ′; α = ∞) is predicted to carry the most superior fatigue life, while the system with flat ellipsoidal rafted γ′ (α = 0 and 1/3) reduces the fatigue life, compared with the system having the spherical or non-rafted γ′. It is also postulated that the aspect ratio, α, may change the fatigue life significantly (compare α = 0 and α = ∞).

Ott & Mughrabi have evaluated experimentally the effect of the rafting on the fatigue strength for commercial Ni-base superalloy CMSX-4 which had a negative misfit(15). They have prepared three types specimen which contained the pre-rafted plate like or needle like γ′ precipitates, then performed the strain controlled f-f type LCF test at 950°C. Their result showed that the LCF lives were significantly reduced in the specimens with plate like γ′ particles, and they were extended in the material with needle like γ′ particles. The specimen with the standard or cuboidal γ′ exhibited intermediate fatigue lives. Our numerical calculation in Fig. 11 predicts the result corresponding with their results.

However, our model assumed the geometry of the γ′ precipitate was fixed, or not changed, during whole loading cycle, while it can change during loading process at high temperatures in some cases. Once this type of microstructural change is advanced, it must affect both the deformation and the damage of the material system from time to time. For this situation, the present model is now being extended by a perturbation method, which will be presented near future.

5. Conclusions

It was shown from the experiments that a single crystal Ni-base superalloy, CMSX-4, revealed the unique TMF and LCF lives which were not always interpreted well by the traditional macroscopic mechanical parameters and approaches. A new micromechanics model was proposed to estimate the TMF and LCF lives, taking into account the γ/γ′ composite microstructure in superalloy. The proposed method enabled us to estimate not only some unique characteristics in the TMF and LCF failures but also the effect of the rafted structure on the LCF lives. For life prediction more exactly, it is necessary to acquire the mechanical properties of γ and γ′ phases. It is also necessary to extend the present model so that time dependence phenomena, creep or relaxation, can be taken into account.

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