A note on extremal results on directed acyclic graphs

Álvaro Martínez-Pérez∗
Departamento de Análisis Económico y Finanzas, Universidad de Castilla-La Mancha, Avenida Real Fábrica de Seda, s/n. 45600 Talavera de la Reina, Toledo, Spain

Luis Montejano†, Deborah Oliveros‡
Instituto de Matemáticas, Universidad Nacional Autónoma de México, Área de la Investigación Científica, Circuito Exterior, C.U., Coyoacán 04510, México D.F., México

Abstract: This paper studies the maximum number of edges of a Directed Acyclic Graph (DAG) with \(n\) vertices in terms of its longest path \(\ell\). We prove that in general this number is the Turán number \(t(n, \ell + 1)\), the maximum number of edges in a graph with \(n\) vertices without a clique of size \(\ell + 2\). Furthermore, we find the maximum number of edges in a DAG which is either reduced, strongly reduced or extremely reduced and we relate this extremal result with the family of intersection graphs of families of boxes with transverse intersection.

Keywords: Directed graphs, Turán numbers, intersection graphs of families of boxes.

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E-mail addresses: alvaro.martinezperez@uclm.es (Álvaro Martínez-Pérez), luis@matem.unam.mx (Luis Montejano), dolivero@matem.unam.mx (Deborah Oliveros).
Opomba o ekstremalnih rezultatih o usmerjenih acikličnih grafih

Álvaro Martínez-Pérez∗

Departamento de Análisis Económico y Finanzas, Universidad de Castilla-La Mancha,
Avda. Real Fábrica de Seda, s/n. 45600 Talavera de la Reina, Toledo, Spain

Luis Montejano†, Deborah Oliveros‡

Instituto de Matemáticas, Universidad Nacional Autónoma de México,
Área de la Investigación Científica, Circuito Exterior, C.U., Coyoacán 04510,
México D.F., México

Povzetek: Ta članek obravnava maksimalno število povezav usmerjenega acikličnega grafa (UAG) z n vozlišči glede na njegovo najdaljšo pot ℓ. Dokažemo, da je v splošnem to število Turánovo število \( t(n, \ell + 1) \), maksimalno število povezav v grafu z n vozlišči brez klike velikosti \( \ell + 2 \). Nadalje, poiščemo maksimalno število povezav v UAG, ki je bodisi reduciran, krepko reduciran ali ekstremno reduciran. Ta ekstremalni rezultat povežemo z družino presečnih grafov družin škatel s transverzalnim presekom.

Ključne besede: Usmerjeni grafi, Turánova števila, presečni grafi družin škatel.

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e-poštni naslovi: alvaro.martinezperez@uclm.es (Álvaro Martínez-Pérez), luis@matem.unam.mx (Luis Montejano), dolivero@matem.unam.mx (Deborah Oliveros).