Radial distributions of coronal electron densities and temperatures linked to solar wind streams

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Abstract

This paper is a follow up of the article where Lemaire and Stegen (2016) introduced the novel method to calculate coronal temperature distribution when the Solar Corona is not assumed to be in hydrostatic equilibrium as it has been assumed until 1957. In their study as well as in the present paper it is considered that the corona plasma is expanding with supersonic speeds $u_E$, and with electron densities $n_E$, at 1AU is given by the average values determined from the statistical study of the Solar Wind parameters reported by Ebert et al. (2009). In inner coronal altitudes $n_e(r)$ is taken from Saito et al.’s (1970) empirical electron density model.

Our dyn-method for the calculation of coronal electron temperature distribution, $T_e(r)$, considers that the Corona expands outwards, and that its density distribution $n_e(r)$ is determined at low altitudes by the Saito et al. (1970)’s empirical electron density model. The radial distribution of the temperature, $T_e(r)$, is then obtained as a definite integral of the hydrodynamic momentum transport equation.

It is found that, at high altitudes, the radial profile of the dyn-temperature distributions differ significantly from those obtained by the scale-height method shm-method generally used in the past.

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Using the expanded average coronal electron density distributions from Saito et al. (1970)’s empirical model it is also found that, at the base of the Corona, the dyn-temperature is smaller over the polar regions (and CHs) than in the equatorial plane.

The temperature gradient $\frac{\partial T_e}{\partial r}$ has very small and positive values at altitudes above the transition region, between 0.001 $R_s$ and 0.02 $R_s$, while that derived from the critical solutions of the hydrodynamical (one fluid or Multi-fluid) transport equations, is largely negative at the base of the Corona.

We confirm also that larger Solar Wind (SW) velocities, $u(r)$, observed in fast speed SW streams imply larger temperatures in the solar Corona. Furthermore, the maximum temperature $T_{e,max}$ is always located significantly above the altitude of the transition region. This is not the case for the critical hydrodynamical solutions of the SW expansion.

Furthermore, the density gradients derived from eclipse observations have steeper slopes (negative density gradients) than those derived from the usual critical solutions of the hydrodynamical transport equations of the SW expansion.

**Keywords:** Corona; Corona, Models; Corona, Quiet; Heating; Heating, Coronal; Solar Wind; Solar Wind, Theory; Velocity Fields, Solar Wind

1. Introduction

Measurements of White Light (WL) brightnesses and polarisation ($pB$) during solar eclipses have often been used in the past to determine $n_e(r)$, the radial distribution of coronal electrons densities. Assuming approximate cylindrical symmetry of the Corona around its axis of rotation, Saito et al. (1970, hereafter S70) constructed an two-dimensional model of $n_e(r,\phi)$ as a function of the heliographic distance, $r$, and latitude $\phi$. They determined this empirical model from a set of eclipse observations at epochs of minimum solar activity. S70’s model has been confirmed by a few observations since, with the most recent being fig. 1b of Howard, R.A. et al. (2019).

Postulating that the coronal density decreases exponentially with $r$, the density scale heights of $n_e(r)$ have generally been used to calculate coronal electron temperatures, $T_e(r)$, at given altitudes ($h$) and latitudes. This habitual methods postulate, however, that the corona is isothermal and in hydrostatic equilibrium; (shm-method and hst-method). These usual methods were mainly promoted by van de Hulst (1950, 1953). Now we know that the corona is not isothermal (Chapman and Zirin, 1957) nor in hydrostatic equilibrium, but expanding continuously outwards (Parker, 1958). More recently, Scudder (2019) applied the verification of mechanism method on four different case studies (including Alfvén (1941)’s corona temperature profile and Parker (1958)’s thermal wind model) and found out serious self-consistency problems. Therefore an alternative method has to be employed to computed $T_e(r)$ from $n_e(r)$ that overcomes the limitations of previous work.

Such an alternative method was developed by Lemaire and Stegen (2016, hereafter LS16), who called it the dyn-method (where dyn stands for ‘hydrodynamical’). It can be applied to any observed, empirical or theoretical distributions
Coronal electron temperature and polarization

Figure 1. Expanded coronal electron density distributions for the equatorial (blue-dashed curve, Seq), and polar regions described by LS16 (black curve, Spv), taken from S70 (red dotted curve, Spv/nE=0), and from Pottasch (1960) (green-dash-dotted curve, P). Pottasch’s density distribution was also included in Fig. 6.5 of Parker (1963).

In the following we apply the dyn-method to S70’s electron density distributions that have been expanded in LS16, as recalled in the following section.

2. Empirical model of the density distribution in the corona

The red-dotted curve (Spv/nE = 0) corresponds to Saito’s original polar density model for φ = 90°. Note that a logarithmic scale is used here for h, the altitude above the photosphere; it is ranging between 0.01 ≤ h ≤ 10R_s, where R_s is the solar radius. The black curve (Spv) corresponds to Saito’s polar density distribution expanded by the Solar Wind density at large distance which is determined by:

\[ n_e(r) = n_E \left( \frac{r_E}{r} \right)^2 \]  

(1)

where \( n_E \) corresponds to the SW electron density at Earth’s orbit, and \( r_E \) is 215 R_s.

The difference between the red curve and the black curve in fig. I could be regarded as the partial density of the escaping SW electrons - compared to the ballistic and trapped particles - in the exospheric models of Lemaire and Scherer.
In these collisionless models, the escaping electrons are the only one contributing to the net outward flux of particles. Therefore the red-dotted curve ($Spv/nE = 0$) might be regarded as corresponding to the sum of densities of the ballistic and trapped electrons which have not high enough kinetic energies to escape out the coronal electrostatic potential determined in the early kinetic SW models built by Lemaire and Scherer (1971, 1973).

The blue-dashed curve ($S_{eq}$) in Fig.1 corresponds to S70’s expanded equatorial density model (i.e. $\phi = 0$).

The difference between the black and red curves in fig. 1 might be viewed as the density of the fraction of escaping electrons contributing the evaporation flux in the collisional kinetic models of the SW, like the Fokker-Planck models of Lie-Svendsen, Hansteen, and Leer (1997); Pierrard, Maksimovic, and Lemaire (1999, 2001). These Fokker-Planck models belong to the fourth generation of SW kinetic models also described in Sect. 4.6 of Echim, Lemaire, and Lie-Svendsen (2011).

Let us also point out here that this special distinction between escaping, ballistic, trapped and incoming electrons is not in order in these latter collisional kinetic models, nor is it, of course, in any standard hydrodynamical and MHD one of the SW.

The green curve ($P$) in fig. 1 corresponds to a fit of the equatorial electron density distribution derived by Pottasch (1964) from WL brightness and polarization measurements during a solar eclipse of 1952. It should be noted here that the temperature of the ions is assumed to be equal to that of the electrons.

Fig. 1 indicates that the coronal densities are significantly smaller over the poles (and in CHs) when compared to that over the equatorial regions. This characteristic difference between the polar and equatorial density distributions is well-known since the observations made with soft X-ray telescope, S-056 on board of the SKYLAB missions (https://solarscience.msfc.nasa.gov/Skylab.shtml).

### 3. Electron temperature distributions in the Corona

The equatorial and polar density distributions shown in Fig.1 have been employed to calculate the electron temperatures by using all three methods ($shm$, $hst$, and $dyn$) described by LS16. These temperatures profiles are shown in Fig.2 for the polar regions. Three similar temperature profiles had been displayed in Fig. 3 of LS16, but for the equatorial regions of the Corona.

Note that in both figures the maximum of the $shm$-temperature is located at higher altitudes than the maximum of the $hst$- temperatures. Interestingly enough these two maxima have almost the same value whatever the method of calculation ($shm$-method or $hst$-method) is used to determine $T_{e,max}$. Nevertheless, significantly different $T_{e}(r)$ distributions obtained when the $dyn$-method is used (red-dotted curve $Spv/dyn$) instead of the $hst$-method (black curve $Spv/hst$).

Of course, the $dyn$ and $hst$ methods give closely identical temperatures distributions at the base of the corona, since at low altitude the SW expansion
Coronal electron temperature and polarization

Figure 2. The temperature profiles produced by all three methods: The scale-method (shm-method), the hydrostatic (hst-method), and the hydrodynamical (dyn-method) introduced by LS16. All three curves are calculated with the same electron density profile (Spv; the black curve in Fig. 1) for which \( n_E = 2.2 \text{ cm}^{-3} \). The red-dotted curve (Spv/dyn) is calculated by assuming that \( u_E = 329 \text{ km/s} \).

Velocity is subsonic, and tends asymptotically to zero when \( h \to 0 \), as in any atmospheres in hydrostatic equilibrium. Above the transition region (i.e. for \( h_{tr} \in [0.003 R_S, 0.1 R_S] \)), all calculated temperatures tend to almost constant values. But these nearly constant values of \( T_e(h) \) are significantly larger over the equatorial regions (1.03 MK) than over the poles (0.72 MK). This prediction of our calculations is also fully consistent with the observations reported in (https://solarscience.msfc.nasa.gov/Skylab.shtml).

At these low altitudes in the polar and equatorial regions our calculations show that the plasma is characterized by a polytropic index, \( \gamma \), which is almost equal to unity:

\[
\gamma = 1 + \frac{\partial \log(n_e)}{\partial \log(T_e)} \approx 1.1
\]

Note that the value \( \gamma = 1.0 \) corresponds to gases which are in isothermal equilibrium, i.e. for which \( p \propto \rho \).

It is important to recall that when the coronal plasma is not assumed to be in hydrostatic equilibrium but expanding steadily outwards, and when the dyn-method is used to determine \( T_e(h) \) over the poles, the maximum temperature is found at significantly higher altitude \( (h = 2 R_S) \) than when the hst-method is used with the same density profile \( (h = 0.6 R_S) \). Furthermore, the value of
Radial distribution dyn-temperature profiles $Spz$, $Spv_{100}$, $Spv$ and $Spv_{450}$ are obtained when the SW bulk velocity at 1AU is respectively assumed to be equal to $u_E = 0$ km/s, 100 km/s, 329 km/s, and 450 km/s.

$T_{e, \text{max}}$ over the poles is much larger when the dyn-method is used instead of the hst-method. This is shown by the red-dotted curve in Fig. 2. Additionally, the comparison of this curve with those in Fig. 3 of LS16 indicates that the $T_{e, \text{max}}$ is much larger over the poles (or CHs) than in the equatorial plane where it peaks at a lower altitude ($h = 0.4 R_S$).

4. Effects of the SW expansion velocity on the coronal temperature distribution

All dyn-temperature profiles shown in Fig. 3 are based on the polar electron density, $n_e(r)$, of Fig. 1 which are obtained from Saito’s expanded density model for $\phi = 90^\circ$. They are calculated by integrating numerically eqs. (16) and (15) of LS16, where $u(r)$, the SW bulk velocity is determined by:

$$u(r) = u_E A(r) n_e(r)$$

Eq. 3 is derived from the equation of conservation of particle flux; $u_E$ is the assumed value of the SW velocity at 1AU; $A(r)$, is the cross-section of flow tubes which is defined by eq. (12) in LS16, it is the same as that introduced by Kopp and Holzer (1976).

The curve $Spv/dyn$ in Figs. 2 and 3 corresponding to the dyn-temperature has been obtained for $u_E = 329$ km/s and $n_E = 2.2 e^-/cm^3$. These input parameters
are respectively the SW bulk velocity and number density at 1 AU within fast speed streams reported by Ebert et al. (2009). The four profiles of Fig. 3 (Spz; Spv/dyn; Sp100/dyn; Sp450/dyn) refer to dyn temperatures respectively for \( u_E = 0, 329 \text{ km/s}, 100 \text{ km/s}, 450 \text{ km/s} \); where \( S \) stands for S70’s model; \( p \) stands for polar; \( v \) stands for \( u_E = 329 \text{ km/s} \); and \( z \) stands for zero expansion velocity, i.e. for hydrostatic equilibrium (evidently, for \( u_E = 0 \) the hydrodynamic model degenerates into an hydrostatic model).

Above the poles, the maximum temperature is located at \( h = 2 R_S \). This value increases gradually when the SW bulk speed is enhanced. When \( u_E \) changes from 329 km/s, to 450 km/s, to 600 km/s this maximum temperature increases respectively from \( T_{e,\text{max}} = 1.87 \text{ MK} \), to 2.69 MK, and eventually to 4.11 MK.

It would be most apposite to present this relationship the other way around: when the maximum coronal temperature, \( T_{e,\text{max}} \), is enhanced – as a consequence of a not yet understood coronal heating mechanism – the SW speed, \( u_E \), is enhanced accordingly at 1 AU and everywhere else in the SW. This equivalent way of saying is of course better in line with cause and effect in physics since it is generally accepted that the supersonic SW expansion is a consequence of the large coronal temperature, and not the reverse.

To our knowledge, however, the extreme coronal temperatures of 4 MK, predicted by our model calculation over the poles or in Coronal Holes have not been observed yet. We hope that the Parker Solar Probe (PSP) observations will help to settle this issue. Indeed, in the 70’s such extremely high coronal temperatures were also required at the base of Parker’s early hydrodynamical models to account for Solar Wind speeds exceeding 600 km/s as observed at 1 AU in some fast speed streams.

Albeit in the present paper we do not discuss any possible coronal heating mechanisms that should be able to account for the temperature profiles shown in fig. 3 it is worth noting that these profiles have a well-developed and widely spread maximum at coronal altitudes much above the altitude of the transition region: \( h_{tr} = 0.003 R_S \).

This implies that the altitude of the maximum of coronal heating rate is also expected well above the altitude of the transition region and above the base of the Corona, unlike what is found in popular hydrodynamical SW models proposed in the past.

All temperature distributions shown above imply that the heat conduction flux carried by the coronal electrons is directed upwards above the altitude of the electron temperature maximum, and downwards below this altitude, \( h_{\text{max}} \). Indeed this is required by the second principle of thermodynamics.

Evidently these are major challenges for the current heating mechanisms inferred in MHD and hydrodynamical models of the Corona and Solar Wind, wherein the corona is heated at its base and the maximum temperature is expected to be found in this region (i.e. close to \( h = 0 \)).

5. Temperature distribution at the base of the Corona.

At the base of corona, where \( h \to 0.003 R_S \), it can be seen that \( \partial T_e / \partial r \to 0 \), for all temperature distributions illustrated above and in LS16. The almost
isothermal temperature distribution immediately above the transition region is smaller than 0.8 MK over the polar regions, and less than 1.1 MK for \( h < 0.1 \) \( R_S \), over the equatorial regions (see Figs. 2, and 3 and Table 2 of [LS16]). This characteristic temperature behaviour should however not be extended below the transition region (\( h < 0.003 \) \( R_S \)) into the Chromosphere because additional physical processes (e.g. ionization and recombination) are taking place at these lower altitudes.

In the Corona, where the temperature distribution is determined by the dyn-method as a solution of the momentum density transport equation, \( T_e(r) \) has a totally different distribution to that of the critical solutions of the one-fluid or two-fluids hydrodynamical transport equations adopted to describe the SW expansion. Such singular solutions predict that \( \partial \log T_e / \partial r \) tends to a negative value when \( r \to 1 \) \( R_S \). Although never pointed out before, the very existence of a negative temperature gradient at the top of the chromospheric region is difficult to justify from a plain physical point of view.

Furthermore, the electron density scale height at the base of the corona predicted by these critical hydrodynamical solutions of SW models, is always larger than the actual scale heights determined from WL eclipse observations. This is illustrated in fig. 1 of Scarf and Noble [1965] where the theoretical electron density distributions of \( n_e(r) \) determined by the critical solution of the Navier-Stokes transport equations have been compared to those derived experimentally from several eclipse measurements. This uncomfortable disagreement was pointed out by the authors of this early study of one-fluid SW hydrodynamical models (Scarf and Noble, 1965). Surprisingly this grievous disagreement between the theoretical density gradient derived from the critical solutions of hydrodynamical SW models, and the actual electron density gradients (or scale heights) has never again been pointed out in later discussions of the limitations of hydrodynamical SW models [Parker, 1958, 1963].

6. Discussion and conclusions

Having recalled [LS16]’s dyn-method used to calculate radial distributions of coronal electron temperatures (\( T_e(r) \)) from a given electron density distribution (\( n_e(r) \)) we have used this method to calculate the electron temperatures as a function of altitude in the equatorial and polar regions for different given input parameters: the solar wind density (\( n_E \)) and the expansion velocity (\( u_E \)) at 1 AU.

The calculated temperature distributions have been compared with those determined by using other methods of calculation, namely, the scale-height method (shm-method) as well as the hst-method which were used in the past by assuming that the Corona would be isothermal or in hydrostatic equilibrium. This confrontation indicated that \( T_e(r) \) depends significantly not only on the method that is used but also on the assumed electron density, \( n_E \), and the SW bulk velocity, \( u_E \), at 1 AU.

It is found that, in all cases the calculated temperature has a maximum value at an altitude, \( h_{\text{max}} \), which is significantly higher than \( h_{\text{tr}} \), the altitude of the transition region between the Chromosphere and the Corona.
At the base of the Corona (i.e. $h < 0.1R_\odot$) the calculated \textit{dyn}-temperatures approach almost constant values which are larger over the equatorial regions (circa 1 MK) than over the poles (circa 0.7 MK).

Fig. 3 indicates that the value of $T_{e,max}$ is increased when the value assumed for $u_E$ is also increased. This positive correlation implies that larger heating rates are required in the mid-corona to boost the solar wind up to the larger velocities observed in fast speed streams.

Our calculation show that the maximum value of the SW acceleration, $u \cdot du/dr$, is located at almost the same altitude as $h_{max}$, where the temperature reaches its peak value. This tells us that this maximum SW acceleration is correlated with the value of $u_E$, as easily expected.

It should finally be point out that the very large maximum coronal temperatures (i.e. $> 8$ MK) obtained over the polar regions for $u_E > 600$ km/s, may be significantly reduced when the coronal proton temperature is assumed to be larger than the electron temperature $T_{e,max}$. This can be inferred from Table 2 of LS16.

Other profiles can also be obtained by assuming that the cross-section, $A(r)$, of few tubes of coronal-SW expansion are diverging faster than $r^2$ as assumed in the calculations presented and discussed in LS16, but not in the present article.

Furthermore, LS16 has also shown that the radial distribution of the electron temperature also depends on the ratio of the concentrations of heavier coronal ions (He$^{++}$, etc) and of the proton and other ion temperature distributions. For all the results discussed above and illustrated in this paper, it has been assumed that the concentration of these heavier ions is zero or negligibly small, and that the proton and other ion temperatures are equal to the coronal electron temperatures at all altitudes. This assumption is most likely to be unsatisfactory except at low altitudes of the corona where Coulomb collisions play a much stronger role than at higher altitudes, and as such they cannot be ignored as happened with the ion-exosphere modes of Lemaire and Scherer (1971, 1973). The recent eclipse observations published by Koutchmy et al. (2019) seem to support the supposition made in this theoretical study, namely, that the proton and electron temperatures would be equal at least at the base of the corona. So far this remains a simple hypothesis that needs to be verified, possibly with data collected during the PSP and Solar Orbiter missions.

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