Value at Risk of the Exchange Rate in Southeast ASEAN-3 Based on Bayesian Markov-Switching GARCH Approach

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Abstract. This study analyzes Bayesian Markov-Switching of the single regime and the two regimes to forecast the risk of the exchange rate in three ASEAN countries, and various GARCH family and distribution are selected by DIC to find the best fitting models. This study will help governments to prevent the recurrence of events like the 1997 financial crisis. The study finds that Thailand has the best exchange rate stability and the lowest risk and is most suitable for foreign investors seeking stability.

1. Introduction

The central figure in risk control problems in the financial area is value at Risk (VaR). At present, the further development of economic globalization has caused the fluctuation of exchange rate returns all over the world, which have a significant impact on international trade [1], monetary policy [2], and so on. Financial crises are happening more frequently, which have become a timebomb of the economy. The former "tiger economies" suffered a depreciation of about 70% in the stock market and currencies in the Asian financial crisis. This international financial crisis reflects the lack of regulation in the exchange rate market, and there has been a long-term discussion on how to prevent future financial crises [3]. Jeon and Seo [4] found that the inherited currency in the four Asian countries most affected by the crisis was greatly underestimated by bivariate and multivariate cointegration estimates. Aquino [5] conducted a study of exchange rates and stock market performance in the Philippines pre- and post- the Asian financial crisis by two-factor arbitrage pricing theoretical model. However, there have been some drawbacks in previous exchange rate forecasting research. For the following reasons, firstly, the exchange rate market is in a highly volatile state and has a structural change pattern, VaR's forecast may not be reliable. Secondly, there might exist structural changes in the real financial world, which might break some assumption in their study and can be solved by Markov-Switching (MS) model. The MS model can describe data naturally and intuitively [3]. According to Engle [6] and Nikolsko-Rzhevskyy and Prodan [7], they claim that the MS-GARCH model can obtain more accurate prediction results than the standard GARCH model. However, it is difficult to apply the maximum likelihood technique to estimate the MS-GARCH model. Since the Bayes factor allows two specifications for transition probabilistic dynamics to be determined, we can use the Bayesian method as an alternative method to estimate the MS-GARCH model. Therefore, this paper uses the Bayesian MSGARCH model to analyze the risk of the exchange rate market index. This article presents new ideas for VaR in the exchange rate market. First, we focused on the three ASEAN countries currencies’ against the US dollar, which suffered a serious
disaster in the foreign exchange market during the Asian financial crisis, it could help to prevent currency crisis in Asian countries. Secondly, changes in the foreign exchange market will affect the national quality of life through commodity prices. Third, we solve the asymmetric effect problem by using various deviation distributions to convert to any single-peak normalized distribution.

The structure of this study is: Section 2 introduces the model specification, which includes Markov transformation GARCH (MS-GARCH) model, Bayesian and VaR. Section 3 data description. Section 4 displays empirical results. Section 5 makes a conclusion.

2. Methodology
We estimate Bayesian MSGARCH into a single-regime and two-regime. We use Markov chain Monte Carlo (MCMC) to simulate the first 80% of total observations 25,000 times and burn the first 10,000 simulated results, MCMC is a common integration method for estimating Bayesian posterior conditional distribution. Various different GARCH-type models are considered in this study, namely, standard ARCH (SARCH), standard GARCH (SGARCH), GJR-GARCH, Threshold GARCH (TGARCH), Exponential GARCH (EGARCH). In the GARCH type model, the most adaptable error term is Deviance Information Criterion (DIC). The smallest DIC is the best fitting model among the various models we obtained.

2.1. Markov-Switching GARCH (MS-GARCH) Model
Set $S_t$ in the traversal Markov chain of a finite set, and $S = \{1, \ldots, R\}$ has transitioned probability matrix

$$
P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} p & 1-q \\ 1-p & q \end{bmatrix}$$

(1)

where $p_{ij} = Pr(S_t = i | S_{t-1} = j)$. The value of the state variable ($S_t$) is 0 or 1 representing two states.

The study considered the lag order (1,1) model because it can collect total volatility from a series of financial data [11]. The lag (1,1) means the lag with ARCH effect of 1, the lag with moving average of 1. The GARCH (1,1) model is used in high frequency in many financial applications, because it meets the requirement of changing the conditional variance.

2.1.1. MS-GARCH (1,1) model

$$\sigma_t^2 = \gamma S_t + \alpha_{1,s} y_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

(2)

where $\sigma_t^2$ is the conditional variance, $\alpha_{1,s}$ and $\beta_{S_t}$ are the coefficients of the GARCH process. It is known that $\theta_{S_t} = (\gamma_{S_t}, \alpha_{1,s}, \beta_{S_t})^T$. In order to obtain the positiveness of the conditional variance, the restriction condition $\gamma_{S_t} > 0$, $\alpha_{1,s} \geq 0$, $\beta_{S_t} \geq 0$ are posed.

2.1.2. MS-EGARCH (1,1) model

$$\ln(\sigma_t^2) = \gamma_{S_t} + \alpha_{1,s} \left( \eta_{S_{t-1}} - E(\eta_{S_{t-1}}) \right) + \alpha_{2,s} y_{t-1} + \beta_{S_t} \ln(\sigma_{t-1}^2),$$

(3)

Obtain the expected $E = (\eta_{S_{t-1}})$ in the equation under the condition of state $S_t$. Known $\theta_{S_t} = (\gamma_{S_t}, \alpha_{1,s}, \alpha_{2,s}, \beta_{S_t})^T$.

2.1.3. MS-GJR-GARCH (1,1) model

Markov Switching-GJR-GARCH (1,1) model expresses as

$$\sigma_t^2 = \gamma_{S_t} + (\alpha_{1,s} + \alpha_{2,s} I\{y_{t-1} < 0\}) y_{t-1}^2 + \beta_{S_t} \sigma_{t-1}^2, $$

(4)
The index function (I) is defined as 1 when the condition is met, otherwise it is 0. Known \( \theta_{S_t} = (y_{S_t}, \alpha_{1, S_t}, \alpha_{2, S_t}, \beta_{S_t})^T \). The restrictions \( y_{S_t} > 0, \alpha_{1, S_t} \geq 0, \alpha_{2, S_t} \geq 0, \beta_{S_t} \geq 0 \), the positiveness of the conditional variance can be guaranteed. Whether the conditional volatility is symmetrical is determined by parameter \( \alpha_{2, S_t} \).

2.1.4. MS-TGARCH (1,1) model
Markov Switching TGARCH (1,1) model expresses as

\[
\sigma_t = \gamma_y + \left( \alpha_{1, S_t} I \{y_{t-1} \geq 0 \} - \alpha_{2, S_t} I \{y_{t-1} < 0 \} \right) y_{t-1} + \beta_{S_t} \sigma_{t-1}, \tag{5}
\]

Known \( \theta_{S_t} = (y_{S_t}, \alpha_{1, S_t}, \alpha_{2, S_t}, \beta_{S_t})^T \). In order to guarantee the conditional variance \( y_{S_t} > 0 \), \( \alpha_{1, S_t} \geq 0 \), \( \beta_{S_t} \geq 0 \), we imposed the restrictions.

2.2. Distribution
Considering the research distribution, the most commonly used is the normal distribution (norm), which cause some limitation in the research. To overcome this problem, skew normal distribution (snorm), Student’s t-distribution (std), skewed Student’s t-distribution (sstd), generalized error distribution (ged), and skewed Generalized Error Distribution (sged) are considered in this study.

2.3. Bayesian Inference
The opinion of Ardia et al. [8] is instructive to the prior distribution for this study:

\[
f(P) = f(\theta_1, \xi_1) \cdots f(\theta_K, \xi_K) f(P) \\
f(\theta_1, \xi_1) \propto f(\theta_1) f(\xi_1 I\{(\theta_1, \xi_1) \in CSCS_1\}) \quad (S_1 = 1, \ldots, K) \\
f(\theta_S) \propto f_S\left(\theta_S; \mu_{\theta_S}, \text{diag}(\sigma_{\theta_S}^2)\right) f(\theta_S \in PC_S) \quad (S = 1, \ldots, K) \\
f(\xi_S) \propto f_S\left(\xi_S; \mu_{\xi_S}, \text{diag}(\sigma_{\xi_S}^2)\right) I\{\xi_{S,1} > 0, \xi_{S,2} > 2\} \quad (S = 1, \ldots, K) \\
f(P) \propto \prod_{i=1}^{K} \prod_{j=1}^{p_{i,j}} f(0 < p_{i,j} < 1), \tag{6}
\]

More details in this prior distribution are explained in Ardia’s work [8].

2.4. Risk Measures
Value-at-risk is a statistical measure of the riskiness of loss in investments. It can be expressed mathematically as:

\[
VaR_{\alpha}^y = \inf \{ y_{S_t} \in R \mid F\left(y_{S_t} \mid I_T\right) = \alpha \} \tag{7}
\]

for a given confidence level \( \alpha \in (0, 1) \), the input of \( \alpha \) confidence level is provided by the minimum number of 1, so that the loss \( y_{T+1} \) exceeds 1 is not greater than \( 1 - \alpha \).

The Expected Shortfall (ES) is a risk measure which is sensitive to the shape of the tail of the distribution of returns, and it can be described as

\[
ES_{\alpha}^y = E\left[y_{S_t} \mid y_{S_t} \leq VaR_{\alpha}^y, I_T\right], \tag{8}
\]
3. Data description
We explored the exchange rate (USD / THB), (USD / SGD) and (USD / PHP) changes of Thailand, Singapore and Philippines, the data was obtained from Thomson Reuters. We used the monthly data which from January, 1999 to February, 2020, and calculated the exchange rate returns ($r_t$) of each country can be described as: $r_t = \log(P_t/P_{t-1})$. The Dickey-fuller test (ADF) results shows in Table 1. Table 1 shows the stationary tests of exchange rates and exchange rate market changes. The Bayes factor results show that the variables of exchange rate returns are stationary by Held and Ott [9].

| Variables | ADF Test | Bayes factors |
|-----------|----------|---------------|
| Thailand  | -5.7891  | 0.054         |
| Singapore | -6.4835  | 0.054         |
| Philippines | -5.2495 | 0.054       |

4. Empirical results
The MS allow structural changes in exchange rate returns, therefore, data are separated into high volatility and low volatility naturally. The high volatility regime is related to high exchange rate return deviations, while low volatility regime is related to low exchange rate return deviations.

Table 2 DIC criterion for each Bayesian two-regime MSGARCH-types with different distribution.

| GARCH-model | Distribution | sARCH | sGARCH | GJR-GARCH | TGARCH | EGARCH |
|-------------|--------------|-------|--------|-----------|--------|--------|
| Thailand    | norm         | -1112.030* | -1109.948 | -1102.798 | -1106.386 | -1108.173 |
|             | snorm        | -1103.917 | -1106.847 | -1106.992 | -1098.809 | -1106.327 |
|             | std          | -1110.615 | -1109.874 | -1106.769 | -1100.963 | -1105.608 |
|             | sstd         | -1107.442 | -1105.686 | -1108.606 | -1102.868 | -1093.635 |
|             | ged          | -1105.480 | -1109.258 | -1104.073 | -1100.356 | -1099.674 |
|             | sged         | -1104.466 | -1110.577 | -1106.049 | -1099.335 | -1108.071 |
|             | norm         | -1076.644 | -1088.670 | -1089.146 | -1090.137 | -1085.469 |
|             | snorm        | -1090.866 | -1098.059* | -1084.822 | -1084.972 | -1090.857 |
|             | std          | -1086.898 | -1091.440 | -1082.834 | -1089.047 | -1078.087 |
|             | sstd         | -1081.371 | -1095.286 | -1086.501 | -1090.561 | -1087.019 |
|             | ged          | -1065.773 | -1121.52 | -1084.557 | -1093.289 | -1092.520 |
|             | sged         | -1087.841 | -1085.111 | -1092.383 | -1079.724 | -1089.254 |
|             | norm         | -1112.912 | -1119.603 | -1114.308 | -1124.568* | -1113.001 |
|             | snorm        | -1112.230 | -1118.779 | -1115.697 | -1113.240 | -1120.985 |
|             | std          | -1109.325 | -1122.706 | -1108.768 | -1117.855 | -1121.760 |
|             | sstd         | -1103.386 | -1117.277 | -1113.231 | -1117.106 | -1118.835 |
|             | ged          | -1103.337 | -1121.521 | -1109.369 | -1119.897 | -1063.981 |
|             | sged         | -1110.581 | -1119.051 | -1121.839 | -1115.818 | -1106.401 |

Figure 1 Thailand Backtest VaR
Figure 2 Singapore Backtest VaR
Figure 3 Philippine Backtest VaR
We are using Bayesian MSGARCH (1,1) of single interval and two intervals to estimate different probability distributions. We use the first 80% of total observations returned by the log to illustrate 25,000 observations and burn the first 10,000. We obtain the best distribution and MSGARCH model by the minimum Deviation Information Criterion (DIC), DIC is a standard for evaluating the complexity of statistical models and measuring the superiority of statistical models. It is widely used in the Bayesian model selection problem of the posterior distribution simulated by MCMC.

Table 2 shows that for the two regimes the best fit model for Thailand’s exchange rate returns is SARCh with a normal distribution, Singapore’s is SGARCH with a snorm distribution, and the Philippines is TGARCH with a norm distribution.

Table 3 The estimated results of MS-GARCH with the most fitted distribution.

| Parameters | Thailand | Singapore | Philippines |
|------------|----------|-----------|-------------|
| \( \alpha_{0,1} \) | 0.0001 | 0.0001 | 0.0001 |
| \( \alpha_{1,3} \) | 0.2133 | 0.0830 | 0.2590 |
| \( \beta_1 \) | 0.2050 | 0.4016 | 0.4229 |
| \( \xi_1 \) | - | 1.0695 | 0.9903 |
| \( \alpha_{0,2} \) | 1.1949 | 0.0008 | 0.3069 |
| \( \alpha_{1,2} \) | 0.4727 | 0.1394 | 0.6148 |
| \( \beta_2 \) | 0.1785 | 0.4047 | 0.1472 |
| \( \xi_2 \) | - | 6.5526 | 5.0145 |

There are two types of exchange rate returns, high volatility and low volatility. Table 3 shows that the two regimes have different unconditional volatility levels and volatility persistence for the exchange rate gains we studied. From the results in Table 3, the probability in the first regime for 3 countries are Thailand, Singapore, Philippines, respectively, 64.68%, 88.17%, 93.84%. The probability in the second regime for 3 countries are Thailand, Singapore, Philippines, respectively, 35.32%, 11.83%, 6.16%. We can also see that the Philippines is the most likely to be in the first regime, while Thailand is the most likely to be the second regime.
We showed the results of risk measures in table 4. We found that the exchange rate of Singapore showed the highest risk among the three countries in both VaR and ES. Thailand, on the contrary, has the lowest risk factor and the most stable foreign exchange market among the three countries. For foreign investors seeking stability, Thailand is the best choice among the three countries.

Figures 1-3 correspond to the Bayesian backtesting for Thailand, Singapore and the Philippines. We can compare VaR with actual risk values (black dots) to verify the accuracy and reliability of this risk measurement method. The red line in the figure represents the single regime prediction, and the blue line represents the two regimes prediction. Observing Figure 1-3 under the condition of 5% violation rate can prove that only Thailand and Singapore each have a black dot below the red and blue lines. This means that only the two prediction methods at this black spot do not predict the actual risk. This proves that the two methods of simultaneous prediction can have a better prediction risk.

5. Conclusion
We use Bayesian estimation to evaluate a single regime and two regimes with different GARCH types to switch the standard exchange rate model and different conditional distribution markets for Thailand, Singapore, Philippines. The results of the study show that Thailand’s exchange rate changes are relatively minimal, and the risks encountered when dealing with similar Asian financial storms in the future are relatively lower, while Singapore is the opposite. In this study, we use 2 different methods for risk prediction to have a better assessment of value at risk. Accurately predicting the trend of exchange rate changes will help the government to respond well in advance.

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References
[1] Hooper P, Kohlhagen S W. The effect of exchange rate uncertainty on the prices and volume of international trade[J]. Journal of international Economics, 1978, 8(4): 483-511.
[2] Céspedes L F, Chang R, Velasco A. Balance sheets and exchange rate policy[J]. American Economic Review, 2004, 94(4): 1183-1193.
[3] The Asian financial crisis: causes, contagion and consequences[M]. Cambridge University Press, 2006.
[4] Jeon B N, Seo B. The impact of the Asian financial crisis on foreign exchange market efficiency: The case of East Asian countries[J]. Pacific-Basin Finance Journal, 2003, 11(4): 509-525.
[5] Aquino R Q. Exchange rate risk and Philippine stock returns: before and after the Asian financial crisis[J]. Applied Financial Economics, 2005, 15(11): 765-771.
[6] Engel C. Can the Markov switching model forecast exchange rates?[J]. Journal of international economics, 1994, 36(1-2): 151-165.
[7] Nikolsko-Rzhevskyy A, Prodan R. Markov switching and exchange rate predictability[J]. International journal of forecasting, 2012, 28(2): 353-365.
[8] Ardia D, Hoogerheide L F. Bayesian estimation of the garch (1, 1) model with student-t innovations[J]. The R Journal, 2010, 2(2): 41-47.
[9] Held L, Ott M. On p-values and Bayes factors[J]. 2018.