Assessment of immersed boundary method as a tool for direct numerical simulation of aeroacoustic sound

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Received: 30 August 2019; Revised: 9 December 2019; Accepted: 7 January 2020

Abstract
An immersed boundary method of discrete type is tested as a tool for direct numerical simulation of aeroacoustic sound. The numerical method consists of the WENO scheme, the immersed boundary method by Chaudhuri et al. (J. Comp. Phys. Vol. 230, 1731–1748 (2011)), and the perfectly matched layer together with the dyadic mesh refinement and the Runge-Kutta method. The accuracy of the method is shown to be sufficient for four basic problems: propagation of acoustic waves, aeroacoustic sound generation in a flow past a fixed circular cylinder, in a flow past an oscillating square cylinder, and from a vortex pair passing through a circular cylinder. The results confirm that the developed method can deal with moving bodies and it is accurate not only for viscous flows but also for inviscid flows.

Keywords: Aeroacoustic sound, Direct numerical simulation, Immersed boundary method

1. Introduction

Noise radiated from engineering devices is a significant problem. There are increasing needs for reducing aeroacoustic noise as the moving or rotating speed of the devices increases since the acoustic power of the aeroacoustic noise is proportional to $M^5$ $- M^6$, where $M$ is the Mach number of the flow or speed of the device. In order to reduce aeroacoustic noise we need to understand its generation mechanism.

Direct numerical simulation (DNS) of aeroacoustic sound became feasible more than two decades ago (Colonius and Lele, 2004; Wang et al., 2006). Here, DNS implies that the sound pressure, which is normally order of magnitude smaller than the ambient pressure, is obtained directly as a numerical solution of the compressible Navier-Stokes equations in contrast to hybrid methods which use aeroacoustic analogy to obtain the sound pressure; a highly-accurate method is required since the magnitude of the sound pressure is usually order of magnitude smaller than the dynamic pressure due to fluid motion and the pressure at far field. DNS is useful in understanding the generation mechanism of aeroacoustic noise since it provides whole fields of physical quantities free from background noise. Unfortunately, DNS has been limited to simple low Reynolds number flows since it demands heavy numerical costs (Mitchell et al., 1995; Colonius et al., 1997; Mitchell et al., 1999; Inoue and Hattori, 1999; Freund et al., 2000; Inoue et al., 2000; Inoue and Hatakeyama, 2002; Gloerfelt et al., 2003; Inoue et al., 2006b; Nakashima, 2008; Sharma and Lele, 2011). It has been also difficult to deal with complex and/or time-varying geometry since high accuracy required to resolve sound pressure is easily lost. This has been an obstacle for practical applications as a number of practical problems of aeroacoustic noise which include wind turbines, helicopters, axial flow fans and other turbomachines have complex and/or time-varying geometry.

The immersed boundary method has developed into a standard method for dealing with complex and/or time-varying geometry. In the immersed boundary method, the surfaces of rigid bodies in a flow do not coincide with a set of grid points; the boundary conditions are introduced as forcing either before discretization (continuous approach) or after discretization (discrete approach). See Mittal and Iaccarino (2005) for a review. Although the immersed boundary method has been used...
for incompressible flows by a number of researchers, the number of applications to compressible flows is relatively small. There have been several applications to acoustic problems. Chung and Morris (1998) applied the immersed boundary method in acoustic scattering problem for the first time. Chaudhuri et al. (2011) studied shock/obstacle interactions. There are a few studies on the acoustic scattering (Seo and Mittal, 2011; Bae and Moon, 2012; Sun et al., 2012). However, there has been no DNS of the aeroacoustic sound by the immersed boundary method of discrete approach except that Khalili et al. (2019) showed limited results on the aeroacoustic sound generated from a flow past a circular cylinder. In order to capture acoustic waves of small amplitude the accuracy of the immersed boundary method should be checked carefully since it can be lost by interpolation, treatment of boundary conditions, or other reasons.

Recently, the corrected volume penalization (VP) method has been shown to be able to resolve aeroacoustic sound generated in complex and time-varying geometry with sufficient accuracy (Komatsu et al., 2016; Hattori and Komatsu, 2017). This method is an immersed boundary method of continuous approach based on the VP method for compressible flow developed by Liu and Vasilyev (2007) and modified to satisfy Galilean invariance. There are still two limitations in the corrected VP method by Komatsu et al. (2016): (i) it cannot deal with inviscid flows since the volume penalization imposes no-slip boundary conditions at the surface of the rigid bodies; and (ii) it is difficult, though not impossible (Inoue and Hattori, 1999), to resolve shock waves since the compact scheme (Lele, 1992), which is numerically unstable in the absence of viscosity, is employed.

The objective of the present paper is to establish a method for DNS of aeroacoustic sound that can deal with complex and/or time-varying geometry and is applicable not only to viscous but also to inviscid flows which may involve shock waves. It should be emphasized that keeping high accuracy with an immersed boundary method requires careful choice of methods and validation. To this end we combine existing methods among which the most important one is that of Chaudhuri et al. (2011), who developed an immersed boundary method with the WENO scheme to investigate shock/obstacle interactions. Combined with other methods, it will be shown that aeroacoustic sound of small amplitude can be resolved with sufficient accuracy. Although there are other schemes suitable for subsonic aeroacoustic problems, we employ the WENO scheme aiming at future applications to problems including shock waves.

The paper is organized as follows. In Sec. 2 the numerical method is presented. In Sec. 3 four problems are considered to show the accuracy of the method. Finally, we conclude in Sec. 4.

2. Numerical methods

As described in the previous section, our objective is to obtain directly the aeroacoustic waves of small amplitude generated in a flow in complex/time-varying geometry. To this end, we employ combination of the following methods:

1. the WENO scheme (Liu et al., 1994; Jiang and Shu, 1996) to achieve high precision in space;
2. dyadic mesh refinement to deal with large numerical domain;
3. the fourth-order Runge-Kutta method to achieve high precision in time;
4. the immersed boundary method developed by Chaudhuri et al. (2011) to deal with complex/time-varying geometry; and
5. the perfectly matched layer (PML) by Hu et al. (2008) to remove artificial reflection at the far-field boundaries.

We just summarize a few modifications of the above methods made in the present study after showing the governing equations since the above methods are well documented in the literature.

2.1. Governing equations

We consider compressible flows of an ideal gas in two dimensions except in Sec. 3.1 where propagation of one-dimensional acoustic waves is considered. The governing equations are the compressible Euler/Navier-Stokes equations

\[ \frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = S, \quad (1) \]

where

\[ Q = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix}, \quad E = \begin{bmatrix} \rho u \\ \rho u^2 + p - \tau_{xx} \\ \rho uv - \tau_{xy} \\ (e + p)u - \tau_{xx}u - \tau_{xy}v - kT_x \end{bmatrix} \]
Here \( \rho \) is the density, \( u \) and \( v \) are the \( x \) and \( y \) components of the velocity, \( e \) is the total energy, \( p \) is the pressure, \( T \) is the temperature, \( (T_x, T_y) = \left( \frac{\partial T}{\partial x}, \frac{\partial T}{\partial y} \right) \), and \( a = (c_x, c_y) \) is the rate of acceleration of the frame. Note that the source term \( S \) due to the inertial force is included to deal with a co-moving frame in the problem of aeroacoustic sound generation in a flow past an oscillating square cylinder, although \( S = 0 \) in most of the cases. The components of the viscous stress tensor are

\[
\tau_{xx} = \mu \left( \frac{\partial u}{\partial x} - \frac{2}{3} \frac{\partial v}{\partial y} \right), \quad \tau_{yy} = \mu \left( \frac{\partial v}{\partial y} - \frac{2}{3} \frac{\partial u}{\partial x} \right), \\
\tau_{xy} = \tau_{yx} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right). 
\]

The viscosity \( \mu \) and the thermal conductivity \( \kappa \) are assumed to be constant for simplicity; we set \( \mu = \kappa = 0 \) for inviscid flows. The above equations are closed by the thermodynamic relations for an ideal gas

\[
p = \rho RT, \quad e = \frac{\rho}{2} (u^2 + v^2) + \rho c_v T, 
\]

where \( R \) and \( c_v \) are the gas constant and the specific heat at constant volume, respectively. The Prandtl number is set to \( Pr = \mu c_p / \kappa = 0.74 \), where \( c_p = c_v + R \) is the specific heat at constant pressure. The ratio of specific heats is set to \( \gamma = c_p / c_v = 1.4 \). The variables are non-dimensionalized by using the density \( \rho_0 \) and the sound speed \( c_0 \) at infinity and a characteristic length scale which depends on the problem.

2.2. WENO scheme and dyadic mesh refinement

In the WENO scheme (Jiang and Shu, 1996), the flux is split as \( E = E^+ + E^- \) by the Lax-Friedrich splitting. Then the fluxes at points \( (i - r + 1, \cdots, i + r - 1) \) are weighted to give WENO interpolation at \( i + 1/2 \)

\[
E_{i+1/2}^+ = \sum_{k=0}^{r-1} \omega_k \left( \sum_{l=0}^{r-1} \alpha_{kl} E_{i-r+k+1}^+ \right), 
\]

where the optimal values of the coefficients \( \omega_k \) and \( \alpha_{kl} \) are given in Jiang and Shu (1996). We set \( r = 3 \) in the present paper.

Dyadic mesh refinement is employed in each direction independently: \( \Delta x = 2^n \Delta x_0 \), where the integer \( n \) is chosen to capture the phenomena of interest in each region. When the stencils of the WENO scheme involve two regions of different grid size, they are chosen as in Fig. 1 to take advantage of the dyadic mesh refinement: at the grid point \( i - 1/2 \) in Fig. 1 the flux \( E_{i+1/2}^+ \) at \( i + 1/2 \) is replaced by \( \tilde{E}_{i+1/2}^+ \), which can be evaluated on the stencils with larger grid size.

2.3. Immersed boundary method

In an immersed boundary method, how to impose the boundary conditions is the most important issue. Although we employ the same method as Chaudhuri et al. (2011), essential points are briefly described below for completeness. The method is based on the ghost-point/cell approach. It is known that conservation laws are not satisfied rigorously by this
Fig. 2 Points defined in immersed boundary method. GP: ghost point, IP: image point, BP: boundary point, NP: neighboring point. (Left) The values of the flow-field variables at the image point are interpolated from the neighboring points; then the values at the ghost point are determined to satisfy the boundary conditions at the boundary point. (Middle) Same as in the left figure but the interpolation is performed with three neighboring points in the fluid region $\Omega_f$. (Right) The values at the freshly created point are interpolated using the values at the two image points and the boundary conditions at the boundary point.

approach and there is a method to recover conservation laws (Hu and Khoo, 2006); however, the error can be reduced by using sufficiently small grid size (Hu and Khoo, 2004).

The grid points belong either to the fluid region $\Omega_f$ or to the solid region $\Omega_s$. The grid points in $\Omega_s$ which are close to the boundary are called ghost points (GPs). The values of flow-field variables at a ghost point are required to calculate flux at interfaces near the boundary; they are determined so that the boundary conditions are satisfied at the boundary point (BP) which is the intersection point of the boundary and the line from the ghost point normal to the boundary (Fig. 2, left). In order to express the boundary condition at the boundary point, an image point (IP) is defined so that the mid-point of the image point and the ghost point be the boundary point. The flow-field variables at the image point are obtained by interpolation as

$$Q_{IP} = \sum_k \delta_k Q_{NP_k}, \quad \delta_k = \eta_k \left( \sum_k \eta_k^{-1} \right)^{-1}, \quad \eta_k = \begin{cases} d_k^{-2} & (NP_k \in \Omega_f), \\ 0 & (NP_k \in \Omega_s), \end{cases}$$

where $NP_k$ ($k = 1, \ldots, 4$) are neighboring points of the image point which are the edge points of the cell including the image point and $d_k$ is the distance between the image point and the neighboring point. If a neighboring point belongs to $\Omega_s$, it is excluded from the interpolation and the weight $\eta_k$ is set to zero since the values of the flow-field variables are not available (Fig. 2, middle).

When the boundary moves, there is another issue called fresh cell problem (Fig. 2, right). The point marked by an open circle belongs to $\Omega_s$ at the $(n - 1)$-th time step and enters $\Omega_f$ at the next step. Thus the values of the flow-field variables at this freshly created point should be determined properly, for which the boundary conditions are also used. The values at the two image points and the boundary conditions at BP are combined to give the values at the fresh point by second-order interpolation. The boundary condition for pressure is modified as

$$\frac{\partial p}{\partial n} = -\rho a \cdot n,$$

where $\frac{\partial}{\partial n} = n \cdot \nabla$, $n$ is the unit normal vector at the boundary, taking account of the inertial force due to acceleration.

3. Numerical assessment of the method

In this section we show that the combination of the methods listed in the previous section can capture aeroacoustic sound not only in viscous flows but also in inviscid flows. The following four problems are chosen:

1. propagation of one-dimensional acoustic waves;
2. aeroacoustic sound generation in a flow past a fixed circular cylinder;
3. aeroacoustic sound generation in a flow past an oscillating square cylinder;
4. vortex sound generation from a vortex pair passing a circular cylinder.

The first problem is a basic test; the numerical error due to dyadic mesh refinement is checked; a criterion for the mesh size is also given. The second problem has been studied by several authors (Inoue and Hatakeyama, 2002; Marsden et al., 2005; Lysenkoa et al., 2014; Komatsu et al., 2016); the numerical results obtained by the present method is compared to those obtained by a standard method. The third problem is a test for moving boundaries as the cylinder moves in a static grid system; the results are also compared to those obtained by the present method and a standard method in a frame co-moving with the cylinder. The last problem checks applicability to aeroacoustic sound in inviscid flows. For the second and third problems the results by a standard method are available since they are also considered in Komatsu et al. (2016).
3.1. Propagation of one-dimensional acoustic waves

Here we consider propagation of a one-dimensional acoustic pulse wave in quiescent inviscid fluid. The initial pressure distribution is

\[ \Delta p = p - p_0 = \begin{cases} p_a \sin^9 \left( \frac{x}{\lambda} \right) & 0 \leq x \leq \lambda, \\ 0 & \text{otherwise}, \end{cases} \]

where the amplitude \( p_a \) is set to \( 10^{-5} \) and we set \( \lambda = 4 \). The other variables are given by the relations for acoustic waves which propagate in the positive \( x \) direction: \( u = \Delta p \) and \( \rho^f = \gamma \rho \) in the present scaling. Since the amplitude is sufficiently small, the above pulse wave propagates with the sound speed \( c_0 \) in the \( +x \) direction without changing its shape. The one-dimensional compressible Euler equations are solved by the present method.

Three grid systems are considered: (Case 1) dyadic grid \( \Delta x = 0.01 \times 2^n \) in \( 4n \leq x \leq 4(n + 1) \), where \( n = 0, 1, \ldots, 4 \); (Case 2) fine uniform grid with \( \Delta x = 0.01 \); and (Case 3) coarse uniform grid with \( \Delta x = 0.16 \). The numerical domain is \( 0 \leq x \leq 20 \).

![Figure 3](image1)

**Figure 3** Propagation of one-dimensional acoustic waves. (a) Initial condition. Sound pressure \( \Delta p \). (b) Sound pressure at \( t = 16 \). Cases 1–3 are compared to the exact solution shown by the solid line.

Figure 3 shows the initial profile and the final profile at \( t = 16 \) of the sound pressure \( \Delta p \). It is observed that the final profile is nearly the same as the initial one shifted by 16 in the \( x \) direction for all three cases showing that the pulse moves with the sound speed without changing its shape.

![Figure 4](image2)

**Figure 4** Propagation of acoustic waves. (a) Sound pressure at \( t = 0, 4, 8, 12 \) and 16. Case 1. (b) Relative error or difference from the exact solution plotted against time \( t \). Cases 1, 2 and 3.

Figure 4(a) shows propagation of the pulse wave on the dyadic grid (Case 1). The symbols show the values at the grid points; the grid spacings are seen to be doubled on the dyadic grid of Case 1 as time proceeds. More detailed analysis of the error or difference from the exact solution is shown in Fig. 4(b). The relative error defined by

\[ \text{error} = \left[ \frac{\sum_i (\Delta p_{\text{num},i} - \Delta p_{\text{exact},i})^2}{\sum_i (\Delta p_{\text{exact},i})^2} \right]^{1/2} \]

is shown, where \( \Delta p_{\text{num},i} \) and \( \Delta p_{\text{exact},i} \) are the sound pressure at the \( i \)-th grid point obtained numerically and that of the exact solution, respectively. The relative error is \( O(10^{-4}) \) on the fine grid (Case 2), while it is \( O(10^{-2}) \) on the coarse grid (Case 3). On the dyadic grid (Case 1) it increases from \( O(10^{-5}) \) to \( O(10^{-2}) \) as the grid spacings become large. However, the error on the dyadic grid is smaller than that on the coarse grid. Therefore, it is confirmed that the dyadic increment of the grid spacings does not add significant error. In addition, the grid spacings should be small to capture the acoustic waves.
correctly. For example, if we allow $O(10^{-3})$ error, the grid size should be equal to or smaller than 0.08, which implies $\Delta x/\lambda \leq 0.08/4 = 0.02$; this criterion is used in the following simulations.

### 3.2. Aeroacoustic sound generation in a flow past a fixed circular cylinder

![Flow past a fixed circular cylinder. Schematic diagram.](image)

Next, we consider the aeroacoustic sound generated in a flow past a fixed circular cylinder located at the origin (Fig. 5). This problem has been studied by several researchers as a benchmark problem in computational aeroacoustics (Inoue and Hatakeyama, 2002; Marsden et al., 2005; Lysenko et al., 2014; Komatsu et al., 2016). The 2D compressible Navier-Stokes equations are solved by the present method. In order to show the accuracy of our method we compare the results obtained by our method to those obtained in Komatsu et al. (2016) by the standard method which consists of the compact scheme, non-reflecting boundary conditions at the far field, and the polar coordinate system.

![Computational grids. Flow past a fixed circular cylinder. (a) Computational grids near the cylinder shown by every tenth grid. (b) grid spacing plotted against grid index. solid line: $\Delta x$, dashed line: $\Delta y$. Note that $\Delta x = \Delta y$ for index $j \leq 809$ so that the two lines collapse.](image)

The grid size is chosen so that the relevant structures are resolved with sufficient accuracy (Fig. 6): $\Delta x = 0.01D$ in $|x| \leq 2D$ which includes the boundary layer region $|x|, |y| \leq 2D$ where thin boundary layers develop; $\Delta x = 0.08D$ in the vortex region $2D \leq x \leq 18.6D$ where vortices are shed into the wake of the cylinder and $\Delta x = 0.16D$ in $18.6D \leq x \leq 100D$; and $\Delta x = 0.32D$ in $-100D \leq x \leq -2D$ which is included in the sound region $|x|, |y| \leq 100D$, where acoustic waves propagate. It is pointed out that: (i) the grid size in the $y$ direction is chosen in a similar way; (ii) the grid spacings in the $x$ ($y$) direction are independent of $y$ ($x$) so that there are no hanging nodes; and (iii) the grid size does not jump e.g. from $\Delta x = 0.01D$ to $\Delta x = 0.08D$ at the boundary of the first and second regions; there are regions with $\Delta x = 0.02D$ and $\Delta x = 0.04D$ so that the ratio of adjacent grid spacings is always one of $1/2, 1, and 2$; the number of grid points in each region is greater than or equal to 10 so that the stencil for the WENO scheme does not extend over three regions of different grid size. The number of grid points is $1500 \times 1160$ including those in the PML region. See Fig. 6 and table 1 for the details. It is pointed out that the computational grids for the other cases are prepared in a similar way.

![Grid spacings of the computational grids for flow past a fixed circular cylinder.](image)

| $\Delta x/D, \Delta y/D$ | 0.01 | 0.02 | 0.04 | 0.08 | 0.16 | 0.32 |
|------------------------|------|------|------|------|------|------|
| $x$                    | $[-2, 2]$ | $[-2, 2]$ | $[-2, 2]$ | $[-3.4, -2.6]$ | $[-19.4, -3.4]$ | $[-99.4, -3.4]$ |
| $y$                    | $[-2, 2]$ | $[-2, 2]$ | $[-2, 2]$ | $[-3.4, -2.6]$ | $[-19.4, -3.4]$ | $[-99.4, -3.4]$ |

The Mach number of the incoming uniform flow is $M_0 = 0.2$, while the Reynolds number is $Re = \rho_0 U_0 D/\mu = 150$. The observation points are located at $r/D = 20, 30, \cdots, 80$. In the following the length is non-dimensionalized by $D$. 

[DOI: 10.1299/jfst.2020jfst0004] © 2020 The Japan Society of Mechanical Engineers
Figure 7 shows the vorticity and sound pressure fields at $t = 200$. Figure 7(a) shows that vortices having negative/positive vorticity are shed downstream from the upper/lower side of the cylinder. Figure 7(b) shows that acoustic waves with positive and negative sound pressure propagate cylindrically from the origin with dipolar directivity. These features are the same as in previous works (Inoue and Hatakeyama, 2002; Marsden et al., 2005; Lysenkoa et al., 2014; Komatsu et al., 2016).

Figure 8 compares the time histories of sound pressure obtained by the present method, the standard method, and the corrected VP method (Komatsu et al., 2016). They are observed at $\theta = 90^\circ$ and $270^\circ$ at which the amplitude of the sound pressure is nearly maximum, while the distance $r$ is set to 80 for which the sound pressure is almost unaffected by pseudo sound. They are in good agreement, confirming the accuracy of the present method. The difference of the amplitude at $\theta = 90^\circ$ between the three methods is less than 3.6%, while that of the time period is less than 1.9%. It is emphasized that the sound pressure of which magnitude is quite small $\Delta p = O(10^{-4})$ is resolved with high accuracy.

Figure 9 shows dependence of the sound pressure on (a) direction and (b) distance from the origin. Flow past a fixed circular cylinder. (a) The solid and dashed lines show the results obtained by the present and standard method, respectively. $r = 80$. (b) The dotted line is proportional to $r^{-1/2}$.

Figure 9 shows dependence of the sound pressure on the direction at $r = 80$ and the distance from the origin at $\theta = 90^\circ$. Figure 9(a) shows that the sound exhibits dipolar directivity with small shift due to the Doppler effect as in the
previous studies (Inoue and Hatakeyama, 2002; Marsden et al., 2005; Lysenkoa et al., 2014; Komatsu et al., 2016). It is well known that in two dimensions the amplitude of the cylindrical linear acoustic waves is proportional to $r^{-1/2}$, which is confirmed in Fig. 9(b). In both figures the results obtained by the present and standard methods are shown to nearly coincide. These results also confirm the accuracy of the present method.

The spatial resolution is checked by changing the grid spacings. Additional numerical simulation is performed on three coarse grids: the minimum grid spacings in the boundary layer region are set to $\Delta x_{\text{min}} = 0.02$, 0.04, and 0.08; The grid spacings in the other regions are also multiplied by the ratio of the minimum grid spacings to the original grid spacing which are 2, 4, and 8, respectively, except that the minimum number of the grid points in each region is fixed to 10 to prevent the stencil from extending over three regions of different grid size. Figure 10 compares the results obtained with different spatial resolution. In Fig. 10(a) time histories of the sound pressure at $r = 80$, $\theta = 90^\circ$ are shown. The time history for $\Delta x_{\text{min}} = 0.02$ is close to that for $\Delta x_{\text{min}} = 0.01$ except that there are small wiggles near the local minima. The time history for $\Delta x_{\text{min}} = 0.04$ has a larger amplitude than those for $\Delta x_{\text{min}} = 0.01$ and 0.02, while the time history for $\Delta x_{\text{min}} = 0.08$ has a smaller amplitude and a longer time period. The error of the amplitude is plotted against the minimum grid spacings in Fig. 10(b); here, the error is defined as the difference from the result of the standard method. It confirms that the error decreases with the grid spacings and the results obtained with the original computational grids with $\Delta x_{\text{min}} = 0.01$ are sufficiently accurate, while the accuracy of those with the coarse grids is not sufficient.

### 3.3. Aeroacoustic sound generation in a flow past an oscillating square cylinder

Next, we consider the sound generated in a flow past an oscillating square cylinder (Fig. 11). We solve this problem in a static frame in which the cylinder is oscillating; that is, the boundary between the fluid and solid regions moves in the static grid system, the surface of the cylinder passing through the grid points; thus this is a more severe test for the present method. This problem can be also solved in a frame co-moving with the cylinder adding inertial forces due to oscillation. In the following, after showing the results obtained by the present method in a static frame, we compare them with the results obtained by the present method in a co-moving frame and those obtained by the standard method in the co-moving frame (Komatsu et al., 2016). In particular, comparison between the present methods in a static and co-moving frames will show errors caused by the fresh cell problem since same methods are used except that fresh cells are generated in the static frame but not in the co-moving frame.

The 2D compressible Navier-Stokes equations are solved. The square cylinder oscillates sinusoidally in the $y$ direction. The amplitude and the frequency of the oscillating motion are set to $A = 0.2D$ and $St = fD/U_0 = 0.14$, respectively. The Mach number of the incoming uniform flow and the Reynolds number are $M_0 = 0.2$ and $Re = \rho_0U_0D/\mu = 100$, respectively. The grid system is the same as that of the previous subsection.

Figure 12 shows evolution of the vorticity field in one period of the cylinder oscillation. For the present set of parameters, the vortex motion is known to lock-in to the cylinder oscillation. It is observed that a vortex with negative
vorticity exists just behind the cylinder at $t = 8T$, where $T = 1/f$ is the time period of the cylinder oscillation; it grows and eventually shed into wakes at $t = (8 + 1/4)T$. On the other hand, a vortex with positive vorticity emerges at $t = (8 + 2/4)T$, grows, and shed into wakes at $t = (8 + 4/4)T$ at which the vorticity distribution is the same as that at $t = 8T$. Thus, the vortex motion is periodic with the time period of the cylinder oscillation.

Figure 13 shows the pressure distribution at $t = 8T$. Acoustic waves are seen to propagate from the origin cylindrically with nearly dipolar directivity as in the case of the flow past a fixed cylinder.

Figure 14 compares the time histories of sound pressure obtained by the present method in a static frame, the present method in a co-moving frame, and the standard method in a co-moving frame. The observation points are the same as in the case of the flow past a fixed circular cylinder. Again we observe good agreement between the three cases. In the following the results obtained by the standard method are taken as the reference values. The relative errors of the period are less than 0.1% for the present methods in a static and co-moving frames; this small value is due to synchronization of vortex motion to the cylinder oscillation. The relative error of the pressure amplitude is 3.7% and 5.6% in a static and co-moving frames, respectively; the amplitudes obtained by the present method are smaller than that obtained by the
standard method since interpolation introduces a small dissipation. Although this is larger than the previous case, the accuracy of the present method is satisfactory.

![Graph](image)

Fig. 15 Dependence of the sound pressure on direction. Flow past an oscillating cylinder. Comparison between the present method (solid and dotted lines) and the standard method (dashed line). $r = 80$. Solid line: present method in the static frame, dotted line: present method in the co-moving frame, dashed line: standard method.

Figure 15 compares the amplitudes of sound pressure at $r = 80$ between the present methods in the static and co-moving frames and the standard method. Good agreement is observed. The difference between the present methods in the static and co-moving frames is 2.0% showing that the possible errors due to the fresh cell problem are sufficiently small.

3.4. Vortex sound generation from a vortex pair passing a circular cylinder

![Diagram](image)

Fig. 16 Vortex sound generation from a vortex pair passing a circular cylinder. Schematic diagram.

The final problem for assessment of the present method is the sound generated by a vortex pair passing a circular cylinder (Fig. 16). This is a test for inviscid application; we solve the Euler equations with slip boundary conditions being imposed on the surface of the cylinder. In this problem it is known that pulse waves are generated as the vortex pair passes around the cylinder. The essential mechanism of the sound generation is inviscid since the deformation of the orbits of the vortices due to the cylinder generates the sound. The boundary layer at the surface of the cylinder is neglected. The resulting sound would be different from the viscous case; however, the difference would be small as far as the Reynolds number is high. At least we can compare the numerical results to Curle’s acoustic analogy consistently.

The vortex pair consists of two vortices with the same parameters but opposite sense of rotation. The initial vorticity distribution is Gaussian. The parameters are chosen as follows: The Mach number based on the initial speed of the vortex pair is $M = 0.1, 0.2,$ and $0.3$. The initial distance between the vortices is $2H = \pi D$, where $D$ is the diameter of the circular cylinder, while the core size of the vortices is $2R = 0.2\pi D$. The vortex pair is located at $y = -L = -4D$, the origin coinciding with the center of the cylinder. The observation points are located at $r = 40D$.

Figure 17 shows time evolution of vorticity distribution. The vortex pair moves in the $y$ direction. As it approaches the cylinder, the distance between the vortices becomes large; another vortex pair with weak vorticity appears as a result of separation. The original vortex pair goes around and moves past the cylinder interacting with the secondary pair.

Figure 18 shows time evolution of sound pressure distribution. Acoustic waves of dipolar directivity are generated and propagate cylindrically. First, a positive/negative sound pressure wave is emitted in the positive/negative $y$ direction, which is followed by the wave with the opposite sign and much weaker waves.

Figure 19 compares the sound pressure obtained numerically to the prediction by Curle’s acoustic analogy. Two clear peaks correspond to the waves observed in Fig. 18. The force exerted on the cylinder, which is required in Curle’s acoustic analogy, is calculated by integrating pressure on the surface of the cylinder. We observe reasonable agreement. The difference is mostly due to pseudo sound involved in the numerical results, which is not small at $r = 40$ in two dimensions.
Fig. 17 Time evolution of vorticity field. Vortex sound generation from a vortex pair passing a circular cylinder. $M = 0.2$. $t =$ (a) 40, (b) 60, (c) 80, (d) 100.

Fig. 18 Time evolution of pressure field. Vortex sound generation from a vortex pair passing a circular cylinder. $M = 0.2$. $t =$ (a) 60, (b) 80, (c) 100, (d) 120.
Fig. 19  Time history of sound pressure. Vortex sound generation from a vortex pair passing a circular cylinder. Comparison between numerical results (solid lines) and Curle’s acoustic analogy (dashed lines). $r = 40$. $	heta = (a) 90^\circ$, (b) 270$^\circ$.

Fig. 20 Mach number dependence of the sound pressure peaks. Vortex sound generation from a vortex pair passing a circular cylinder. +: first peaks, ×: second peaks. The dotted line is proportional to $M^{5/2}$. $\theta = (a) 90^\circ$, (b) 270$^\circ$.

Figure 20 shows Mach number dependence of the sound pressure peaks observed at $\theta = 90^\circ$ and 270$^\circ$. The clear peaks of the first and second waves are chosen. In two dimensions the Curle’s acoustic analogy predicts that these peaks are proportional to $M^{5/2}$ in the low Mach number limit shown by the solid lines. The numerical results obey this scaling approximately. These results confirm that the present method gives satisfactorily accurate results also in the inviscid case.

4. Concluding remarks

An immersed boundary method of discrete type was tested as a tool for DNS of aeroacoustic sound. The numerical method was developed by combining the WENO scheme (Liu et al., 1994; Jiang and Shu, 1996), the immersed boundary method by Chaudhuri et al. (2011), and PML (Hu et al., 2008) together with the dyadic mesh refinement and the Runge-Kutta method. Numerical accuracy was shown to be sufficient for propagation of acoustic waves, aeroacoustic sound generation in a flow past a fixed circular cylinder, in a flow past an oscillating square cylinder, and from a vortex pair passing through a circular cylinder. The results confirm that the developed method can deal with moving bodies and it is accurate not only for viscous flows but also for inviscid flows as long as the grid size is sufficiently small to resolve the flows.

The results shown in the present paper have been limited to two dimensions because of limited computational resources. However, application to three-dimensional simulations is straightforward and feasible if large computational resources are available. Using the present method with large eddy simulation would give abundant possibilities of applications, although the numerical accuracy should be checked carefully. It will also be applicable to fluid-structure interaction, for which the immersed boundary methods have an advantage over other methods.

Acknowledgements

This work was supported by Kawai Foundation for Sound Technology & Music. Numerical calculations were performed on the UV1000 and UV2000 at the Institute of Fluid Science, Tohoku University.
References

Bae, Y., Moon, Y. J., On the use of Brinkman penalization method for computation of acoustic scattering from complex boundaries. Computers and Fluids, Vol. 55 (2012) pp. 48–56.

Chaudhuri, A., Hadjadj, A. Chinnayya, A., On the use of immersed boundary methods for shock/obstacle interactions. Journal of Computational Physics, Vol. 230 (2011), pp. 1731–1748.

Chung, C. Morris, P. J., Acoustic scattering from two- and three-dimensional bodies. Journal of Computational Acoustics, Vol. 6 (1998), pp. 357–375.

Colonius, T., Lele, S. K., Computational aeroacoustics: progress on nonlinear problems of sound generation. Progress in Aerospace Sciences, Vol. 40 (2004), pp. 345–416.

Colonius, T., Lele S. K., Moin, P., Sound generation in a mixing layer. Journal of Fluid Mechanics, Vol. 330 (1997), pp. 375-409.

Freund, J. B., Lele, S. K., Moin, P., Direct numerical simulation of a Mach 1.92 turbulent jet and its sound field. AIAA Journal, Vol. 38 (2000), pp. 3-31.

Gloerfelt, X., Bailly, C., Juve, D., Direct computation of the noise radiated by a subsonic cavity flow and application of integral methods. Journal of Sound and Vibration, Vol. 266 (2003), pp. 119-146.

Hattori, Y., Komatsu, R., Mechanism of aeroacoustic sound generation and reduction in a flow past oscillating and fixed cylinders. Journal of Fluid Mechanics, Vol. 832 (2017), pp. 241–268.

Howe, M. S., Iida, M., Fukuda, T., Maeda, T., Theoretical and experimental investigation of the compression wave generated by a entering a tunnel with a flared portal. Journal of Fluid Mechanics, Vol. 425 (2000), pp. 111–132.

Hu, F. Q., Li, X. D., Lin, D. K., Absorbing boundary conditions for nonlinear Euler and Navier-Stokes equations based on the perfectly matched layer technique. Journal of Computational Physics, Vol. 227 (2008), pp. 4398–4424.

Hu, X.Y., Khoo, B.C., An interface interaction method for compressible multifluids. Journal of Computational Physics, Vol. 198 (2004), pp. 35–64.

Hu, X.Y., Khoo, B.C., A conservative interface method for compressible flows. Journal of Computational Physics, Vol. 219 (2006), pp. 553–578.

Inoue, O., Hatakeyama, N., Sound generation by a two-dimensional circular cylinder in a uniform flow. Journal of Fluid Mechanics, Vol. 471 (2002), pp. 285–314.

Inoue, O., Hattori, Y., Sound Generation by Shock-Vortex Interactions. Journal of Fluid Mechanics, Vol. 380 (1999), pp. 81-116.

Inoue, O., Hattori, Y., Sasaki, T., Sound generation by coaxial collision of two vortex rings. Journal of Fluid Mechanics, Vol. 424 (2000), pp. 327-365.

Inoue, O., Iwakami, W., Hatakeyama, N., Aeolian tones radiated from flow past two square cylinders in a side-by-side arrangement. Physics of Fluids, Vol. 18 (2006), 046104.

Inoue, O., Mori, M., Hatakeyama, N., Aeolian tones radiated from flow past two square cylinders in tandem. Physics of Fluids, Vol. 18 (2006), 046101.

Jiang, G.-S., Shu, C.-W., Efficient Implementation of Weighted ENO Schemes. Journal of Computational Physics, Vol. 126 (1996), pp. 202–228.

Khalili, M. E., Larsson, M., Müller, B., High-order ghost-point immersed boundary method for viscous compressible flows based on summation-by-parts operators. International Journal for Numerical Methods in Fluids, Vol. 89 (2019), pp. 256–282.

Komatsu, R., Iwakami, W., Hattori, Y., Direct numerical simulation of aeroacoustic sound by volume penalization method. Computers and Fluids, Vol. 130 (2016), pp. 24–36.

Lele, S. K., Compact finite-difference schemes with spectral-like resolution. Journal of Computational Physics, Vol. 103 (1992), pp. 16–42.

Liu, X.-D., Osher, S., Chan, T., Weighted Essentially Non-oscillatory Schemes. Journal of Computational Physics, Vol. 115 (1994), pp. 200–212.

Liu, Q., Vasilyev, O. V., A Brinkman penalization method for compressible flows in complex geometries. Journal of Computational Physics, Vol. 227 (2007), pp. 946–966.

Lysenkoa, D. A., Ertesvåga, I. S., Rian, K. E., Towards simulation of far-field aerodynamic sound from a circular cylinder using OpenFOAM. International Journal of Aeroacoustics, Vol. 13 (2014), pp. 141–168.

Marsden, O., Bogey, C., Bailly, C., High-order curvilinear simulations of flows around non-Cartesian bodies. Journal of
Computational Acoustics, Vol. 13 (2005), pp. 731–748.
Mittal, R., Iaccarino, G., Immersed boundary methods. Annual Review of Fluid Mechanics, Vol. 37 (2005) pp. 239–261.
Mitchell, B. E., Lele, S. K., Moin, P., Direct computation of the sound from a compressible co-rotating vortex pair. Journal of Fluid Mechanics, Vol. 285 (1995), pp. 181-202.
Mitchell, B. E., Lele, S. K., Moin, P., Direct computation of the sound generated by vortex pairing in an axisymmetric jet. Journal of Fluid Mechanics, Vol. 383 (1999), pp. 113-142.
Nakashima, Y. Sound generation by head-on and oblique collisions of two vortex rings. Physics of Fluids, Vol. 20 (2008), 056102.
Seo, J. H., Mittal, R., A high-order immersed boundary method for acoustic wave scattering and low-Mach number flow-induced sound in complex geometries. Journal of Computational Physics, Vol. 230 (2011), pp. 1000–1019.
Sharma, A., Lele, S. K., Sound generation due to unsteady motion of a cylinder. Physics of Fluids, Vol. 23 (2011), 046102.
Sun, X., Jiang, Y., Liang, A., Jing, X., An immersed boundary computational model for acoustic scattering problems with complex geometries. Journal of the Acoustical Society of America, Vol. 132 (2012) pp. 3190–3199.
Wang, M., Freund, J. B., Lele, S. K., Computational prediction of flow-generated sound. Annual Review of Fluid Mechanics, Vol. 38 (2006), pp. 483–512.