FAST TRACK COMMUNICATION

Cooling a charged mechanical resonator with time-dependent bias gate voltages

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Abstract

We show a purely electronic cooling scheme to cool a charged mechanical resonator (MR) down to nearly the vibrational ground state by elaborately tuning bias gate voltages on the electrodes, which couple the MR by the Coulomb interaction. The key step is the modification of the time-dependent effective eigen-frequency of the MR based on the Lewis–Riesenfeld invariant. With respect to a relevant idea proposed previously (Li et al 2011 Phys. Rev. A 83 043803), our scheme is simpler, more practical and completely within the reach of current technology.

(Some figures may appear in colour only in the online journal)

Micro- and nano-mechanical resonators (MRs) have attracted much research interest due to their combination of both classical and quantum properties [1, 2] together with their broad impact on fundamental research and applications [3], e.g. in ultra-sensitive measurements approaching the quantum limit [3, 4], observations of continuous-variable entanglement with mesoscopic objects [5, 6], quantum information processing [7] and biological sensing [8].

The prerequisite of the research in these aspects is to cool the MRs down to their ground states in order to suppress detrimental influence from thermal fluctuations. Up to now, there have been many proposals for cooling the MRs [9–21] in either optomechanical or electromechanical systems, in which the resolved sideband cooling method enables the vibrational ground-state cooling of MRs. So far, the ground-state cooling of MRs has been achieved experimentally [22–25].

Very recently, a different approach to fast ground-state cooling of MRs has been proposed [16] with time-dependent optical driving in a three-mirror cavity optomechanical system, in which the effective frequency of the MR can be changed like an ‘optical spring’ and the MR can be cooled down to nearly its ground state under the control of time-dependent external optical driving fields. But the scheme seems challenging experimentally due to the requirement for both adiabatic evolution under the Born–Oppenheimer approximation and very strong optical powers.

We present in this work an alternative method using a purely electronic method, which is simpler but more practical, for cooling a charged MR by two electrodes via Coulomb interaction. Under the control of bias gate voltages, the MR behaves as a single-mode harmonic oscillator with its effective frequency tunable like an ‘electrical spring’. Our method is somewhat similar to that given in [16] but without involvement of the Born–Oppenheimer approximation. Moreover, in contrast to [16], our idea is more feasible using current experimental techniques due to the manipulation of bias gate voltages. Furthermore, different from the traditional cooling schemes in electromechanical systems based on superconductor qubits [17, 18] or microwave photons [11, 20], our scheme can cool an MR down to its vibrational ground state by simply tuning the bias gate voltages. To the best of our knowledge, this is the first cooling scheme with a purely electronic way of cooling the MR system without any assistance from auxiliary qubits or additional photons.
As schematically shown in figure 1, we consider a system where a charged MR is placed in the middle of two identical electrodes and couples electrostatically to these two electrodes. Each electrode takes the charge \( Q = C_0 U \) with \( C_0 \) and \( U \) being the capacitance and voltage of the bias gate, respectively. The Coulomb force between the MR and each electrode varies with the bias gate voltage and the Coulomb potential is written as

\[
V_c = kC_0 U Q_{MR} \left( \frac{1}{d + x} + \frac{1}{d - x} \right),
\]

where \( d \) is the equilibrium separation between the MR center-of-mass position and the electrodes with \( x \) the deviation of the MR from the equilibrium position. \( k \) is the Coulomb constant in vacuum.

Figure 1. Schematic diagram of the system, where a charged MR with (positive) charge \( Q_{MR} \) couples to two identical electrodes via the Coulomb interaction. The two electrodes are distant by \( 2d \) and \( x \) is the deviation of the MR from the equilibrium position. \( C_0 \) is the capacitance of the gate and \( U \) is the tunable bias gate voltage.

Generally speaking, the dynamics of a harmonic oscillator (e.g. MR) governed by equation (4) can be obtained by the method of Lewis–Riesenfeld invariants [26–28]. In particular, for some special intermediate trajectories, the instantaneous quantum state of the harmonic oscillator at the initial time instant can be the same as that at the final time instant, although the effective eigen-frequency of the harmonic oscillator has been changed significantly [28]. That is to say, we may have the final mean phonon number of the harmonic oscillator to be the same as the initial one, but with a significant change of the effective temperature. This idea holds for arbitrary initial state and has been used to cool the spatial motion of atoms [26] and to cool the MR in cavity optomechanical systems [16].

Our present idea is to achieve cooling of the MR by modifying the time-dependent eigen-frequency via tuning the bias gate voltages on external electrodes. We note that the effective frequency \( \omega_{\text{eff}} \) of the MR can be easily changed by tuning the bias gate voltages \( U \). In fact, \( \omega_{\text{eff}} \) can be much smaller than \( \omega_m \) under negative \( U \). Moreover, \( \omega_{\text{eff}} \) can even be imaginary in order to accelerate the cooling process [16, 26].

Specifically, considering some experimentally achievable parameters in charged MR systems [10, 29–31], such as \( k = 8.988 \times 10^9 \text{N}\cdot\text{m}^{-2}\cdot\text{C}^{-2} \), \( \omega_m = 2\pi \times 134 \text{kHz} \), \( m = 40 \text{pg} \), \( d = 3.15 \mu\text{m} \), \( U_0 = 7.00 \text{ V} \), \( Q_{MR} = |e|\sigma_{MR} \times s \), \( \sigma_{MR} = 1.25 \times 10^3 \text{ cm}^{-2} \), \( C_0 = 27.5 \text{ nF} \) and \( s = 0.08 \mu\text{m}^2 \), we have \( \eta \approx 1.25 \times 10^7 \), meeting the approximate condition in equation (2).

Similar to [16], our protocol for cooling MR comprises the following two steps.

Step I: Decreasing the mean phonon number with the increase of the MR effective frequency.

In the absence of bias gate voltages (i.e. \( f(t_o) = 0 \)), we assume the MR with eigen-frequency \( \omega_{\text{eff}}(t_o) = \omega_m \) initially in a thermal equilibrium state

\[
\rho(t_o) = e^{-\frac{h\omega_m}{k_B T}}/\text{Tr}(e^{-\frac{h\omega_m}{k_B T}})
\]

at temperature \( T = 20 \text{ mK} \). So the corresponding mean thermal phonon number is given by

\[
\tilde{n}(t_o) = 1/ \{ \exp[\hbar\omega_m/(k_B T)] - 1 \} \approx 3100.
\]

With an arbitrary trajectory of \( \omega_{\text{eff}}(t) \) [or \( f(t) \)], the MR is assumed to be in a new thermal state

\[
\rho(t) = e^{-\frac{h\omega(t)}{k_B T}}/\text{Tr}\left[e^{-\frac{h\omega(t)}{k_B T}}\right],
\]

at a later time \( t_l \) with \( f(t) = 1 \), where the effective frequency is \( \omega_{\text{eff}}(t_l) \approx 3500 \omega_m \) and the mean thermal phonon number is

\[
\tilde{n}(t_l) = \frac{1}{\exp[h\omega_{\text{eff}}(t_l)/(k_B T)] - 1} \approx 0.47.
\]

It is clear that the temperature of the MR at the time \( t_l \) remains unchanged compared with the initial one, while the mean thermal phonon number of the MR is much less than that at the initial time \( t_o \) due to enlargement of the effective
frequency $\omega_{\text{eff}}$. The fact $\bar{n}(t_1) \simeq 0.47 < 1$ means that the MR with enlarged effective frequency has been cooled down to nearly the vibrational ground state.

**Step II: Decreasing the MR effective frequency by keeping the low mean phonon number**

This step is accomplished via a special trajectory of $f(t)$ evolving to the final time $t_f$ under the control of bias gate voltages satisfying two conditions: (i) the final effective frequency is equal to the bare one (i.e. $\omega_{\text{eff}}(t_f) = \omega_{\text{m}}$), implying that the bias gate voltages are absent again at the final time $t_f$ (i.e. $f(t_f) = 0$); (ii) during the cooling process, the mean phonon number remains unchanged compared with that at time $t_i$ (i.e. $\bar{n}(t_i) = \bar{n}(t_f) \simeq 0.47$). Consequently, we have the MR with the bare frequency cooled to nearly the vibrational ground state.

The solution to such a special trajectory of $f(t)$ between $t_i$ and $t_f$ governed by equation (4) follows the Lewis–Riesenfeld invariant [26–28]. Using the inverse-invariant method [28, 32] in the appendix, we obtain the control function of the trajectory of $f(t)$ by tracing the Lewis–Riesenfeld invariant,

$$ f(t) = \frac{\omega_{\text{eff}}^2(t) - \omega_{\text{m}}^2}{\eta \omega_{\text{m}}^2} = \frac{\omega_0^2 - b(t)^2 \bar{b}(t) - \omega_{\text{m}}^2 b(t)^4}{\eta b(t)^4 \omega_{\text{m}}^2}, \quad (9) $$

where the dimensionless function $b(t)$ is defined in the appendix.

The trajectory of $f(t)$, starting from $f(t_i) = 0$ and ending with $f(t_f) = 0$, corresponds to the effective frequency of the MR decreasing from the end frequency in the first step ($\omega_{\text{eff}}(t_1) \equiv \omega_0 \simeq 3500 \omega_{\text{m}}$) to the final frequency $\omega_{\text{eff}}(t_f) = \omega_{\text{m}}$. We plot in figure 2 the control function $f(t)$, the corresponding instantaneous eigen-frequency $\omega_{\text{eff}}(t)$ and the effective temperature $T_{\text{eff}}$.

Using equation (5), we know that our cooling enables the MR from the initial temperature $T_{\text{eff}}(t_i) = T_{\text{eff}}(t_f) = T = 20$ mK down to the final temperature $T_{\text{eff}}(t_f) = 6$ µK, as plotted in figure 2(c). Here the effective temperature is defined through $\bar{n}(t) \equiv 1/[\exp[h\omega_{\text{eff}}(t)/k_B T_{\text{eff}}(t)] - 1]$.

To estimate the imperfection in the trajectory of $f(t)$, which corresponds to the experimental error in tunable bias gate voltages on the electrodes, we give the trajectory of $f(t)$ a 10% fluctuation, i.e. $(1 \pm 10\%) f(t)$, as in figure 3(a), and simulate in figures 3(b) and (c) the changes of the instantaneous eigen-frequency $\omega_{\text{eff}}(t)$ of the MR and the corresponding effective temperature of the MR. We find that, although the experimental deviation is as large as 10%, the effective eigen-frequency can still reach 1.23$\omega_{\text{m}}$ for $+10\% f(t) (0.84 \omega_{\text{m}}$ for $-10\% f(t)$ deviation with the corresponding effective temperature 7 µK for $+10\% f(t)$ (5 µK for $-10\% f(t)$ deviation. In other words, the MR in such cases can still be cooled down to nearly its vibrational ground state. Moreover, we note that the final effective temperature 5 µK for $-10\% f(t)$ deviation (see figure 3(c)) seems lower than 6 µK under the ideal condition, but this is the case of fluctuation, which is uncontrollable. In addition, in this deviating case, the effective frequency is 0.83$\omega_{\text{m}}$, rather than the desired effective frequency $\omega_{\text{m}}$. Therefore, our choice of trajectory in figure 3(b) is definitely optimal.

![Figure 2](image_url)
Figure 3. (a) Time evolution of \( f(t) \) with 10% error (i.e. \( 0.9f(t) = (1 - 10\%)f(t) \) and \( 1.1f(t) = (1 + 10\%)f(t) \); (b) time evolution of the effective frequency \( \omega_{\text{eff}} \) under the different trajectory of \( f(t) \); (c) time evolution of the effective temperature of the MR under the different trajectory of \( f(t) \). In each plot, we consider two kinds: \( 0.9f(t) \) (thick line) and \( 0.9f(t) \) (thin line). Except for \( f(t) \), the other parameters are the same as in figure 2.

We would like to point out that our scheme is only for an instantaneous cooling, which is different from the usual sideband cooling scheme achievable in the steady state. This is because the MR is decoupled from electrodes at the end of our cooling process. So the MR cannot be kept at low temperature for a very long time but will be heated again by environment.

However, after the cooling, the MR would not return to the bath temperature suddenly due to its high \( Q \) factor (>10⁵) [22–25] which ensures the small decay of the MR. As a result, there should be a long enough time to finish the scheduled experimental works, such as quantum information processing [7], before the MR is heated to the bath temperature. Even if the MR is thermalized to the bath temperature, we may repeat the same cooling process to put the MR down to the vibrational ground state again. Therefore, both the cooling process and the experimental work should be done alternately in the implementation of the scheduled experimental works. This is similar to that in an ion trap system, wherein one carries out the sideband cooling and the operations of quantum algorithms alternately [33].

In summary, we have proposed a practical protocol for cooling an MR near to its vibrational ground state by controlling the bias gate voltages on two nearby electrodes. Using the achievable experimental parameters and considering experimental imperfection, we have shown the feasibility of the proposal using currently available technology. As a result, our work is not only a practical cooling scheme, but is also very promising to verify the Lewis–Riesenfeld invariant method experimentally.

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Appendix

Here we present how to obtain the special trajectory of \( f(t) \) between \( t = t_i (= 0) \) and \( t = t_f \) by means of the Lewis–Riesenfeld invariant. Based on the inverse-invariant method [28, 32], equation (4) satisfies the time-dependent invariant [27],

\[
I = \frac{1}{2} \left[ \frac{x^2}{b(t)^2 m \dot{b}(t)^2} + \frac{\Lambda^2}{m} \right], \tag{A.1}
\]

where \( \omega_0 = \omega_{\text{eff}}(t) \) for simplicity, \( \Lambda \equiv b(t)p - \dot{b}(t)x \) takes the role of the momentum conjugate to \( x/b(t) \) and the dimensionless real function \( \tilde{b}(t) \) follows

\[
\ddot{b}(t) + \omega_{\text{eff}}^2(t)b(t) = \frac{\omega_0^2}{b(t)^2}. \tag{A.2}
\]

According to the boundary conditions at time \( t_i \) and the final time \( t_f \), we have

\[
b(t_i) = 1, \quad \dot{b}(t_i) = 0, \quad \ddot{b}(t_i) = 0, \quad b(t_f) = \chi, \quad \dot{b}(t_f) = 0, \quad \ddot{b}(t_f) = 0, \tag{A.3}
\]
with $\chi = \sqrt{\omega_0/\omega_{\text{eff}}(t_f)}$. So the simplest polynomial $b(t)$ between $t_i (= 0)$ and $t_f$ is given by

$$b(t) = 6(\chi - 1)s^5 - 15(\chi - 1)s^4 + 10(\chi - 1)s^3 + 1 \quad (A.4)$$

with $s = t/t_f$.

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