Hydrodynamics of composite droplet with high-viscosity shell subjected to shear flow in the narrow channel

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Abstract. In this work, the deformation and breakup behavior of the single-core composite and homogeneous droplets subjected to a simple shear flow between two parallel walls have been investigated numerically. The attention was focused on the studying of the high-viscosity droplets, which can be noticeably deformed or broken only in narrow channels. The influence of a low-viscosity core on hydrodynamics of the composite droplet with a high-viscosity shell has been studied for the first time at different capillary numbers and confinement degrees (the ratio of a droplet diameter to a channel thickness). It was found that a composite droplet is more stable as compared to a similar high-viscosity homogeneous droplet and breaks up at a higher capillary number.

1. Introduction
The processing of emulsions and polymer mixtures as well as transportation of multiphase liquids are accompanied by deformation and disintegration of the dispersed components [1–4]. These processes depend on the competition between viscous and capillary forces. The analysis of a huge array of experimental data for the two-component immiscible liquid blends made it possible to establish the dependence of the critical capillary number corresponding to the breakup of a homogeneous droplet on the ratio $m$ of droplet viscosity to that of continuous phase [5]. In particular, in a wide channel, the high-viscosity homogeneous droplets with $m \geq 4$ do not break up under the shear flow even at very high capillary numbers. However, such droplets can break up in a narrow gap between two solid plates at relatively small capillary numbers [6–8]. It is worth noting that at the same viscosity of the droplet and continuous medium ($m = 1$), the critical capillary number depends weakly on the channel thickness. The influence of the channel walls on the dynamic behavior of droplets should be taken into account when designing microfluidic devices [9, 10].

The use of liquid particles as micro-containers or micro-reactors [11, 12] has generated an interest to the hydrodynamics behavior of the composite droplets. In the simplest case, such droplets include a single liquid core surrounded by a viscous shell of another fluid. It is evident that such an inclusion should influence on deformation and breakup conditions of the droplet. Particularly, it was shown [13, 14] that increase in the relative size of the core results...
in the nonmonotonic behavior of the steady-state deformation of a composite droplet, which is associated with the “sustaining” effect due to the core [13]. On the other hand, deformation of the droplet increases with the decrease in the channel thickness [13, 15]. At the same time, the effect of the shell relative viscosity on deformation and ultimate properties of 3D composite droplet at different ratios of droplet diameter to a channel thickness (confinement parameter) remained open.

In this paper we investigated the hydrodynamic behavior of the confined composite droplet with a high-viscosity shell subjected to the simple shear flow between parallel plates. It was found that the low-viscosity core exerts a stabilizing effect on the deformation behavior of the composite droplet while the homogeneous high-viscosity droplet breaks up at the same flow conditions. It demonstrated that the steady-state deformation of the composite droplet in a narrow channel exceeds that of the corresponding homogeneous droplet.

2. Model and methods

The simple shear flow of the incompressible viscous medium containing 3D composite or homogeneous droplet placed between two parallel rigid walls moving in the opposite directions with velocities \( U \) was considered (figure 1). The basic shear rate is equal to \( \dot{\gamma} = 2U/h \). To avoid longitudinal displacement, the droplets were placed at the center of the computational domain. The distance between the walls (channel thickness) is \( h \), while the initial radii of the shell and core of the composite droplet are denoted as \( a \) and \( b \), respectively. The viscosity of the core \( \eta_3 \) was assumed to equal to the viscosity of the continuous medium \( \eta_1 \), whereas the viscosity of the shell \( \eta_2 \) is much larger. To exclude gravitation effects, densities of all components were taken equal to each other, \( \rho_1 = \rho_2 = \rho_3 \). The interfacial tensions between the liquid components are denoted as \( \sigma_{12}, \sigma_{23} \) and \( \sigma_{13} \). Their values should satisfy conditions of the thermodynamic stability of the composite droplet. In this case the spreading coefficients \( S_i = \sigma_{jk} - (\sigma_{ij} - \sigma_{ik}) \) have to meet the following inequalities \( S_1 < 0, S_2 > 0 \) and \( S_3 < 0 \) under the condition \( \sigma_{13} > \sigma_{23} \) [16].

The hydrodynamic behavior of the droplets considered depends on three parameters: the viscosity ratio \( m = \eta_2/\eta_1 \), capillary number \( Ca = \eta_1 \dot{\gamma} a/\sigma_{12} \), and confinement parameter \( n = 2a/h \) which characterizes the droplets constraint degree. The relative elongation was determined by the ratio \( L/a \), where \( L \) is the maximum distance from the droplet center to its outer boundary.

The flow of the two- and three-component systems under consideration is governed by the Navier–Stokes system of equations along with the incompressibility conditions:

\[
\rho_i \left( \frac{\partial \mathbf{u}_i}{\partial t} + (\mathbf{u}_i \cdot \nabla) \mathbf{u}_i \right) = -\nabla p_i + \nabla \left( \eta_i \left( \nabla \mathbf{u}_i + (\nabla \mathbf{u}_i)^T \right) \right) + \sum_j \mathbf{F}_{ij}, \quad \nabla \cdot \mathbf{u}_i = 0. \tag{1}
\]

Here \( i, j = 1, 2, 3 \) at \( i \neq j \); \( \mathbf{u}_i \) and \( p_i \) are the velocity and pressure fields of the \( i \)-th component, respectively; \( \mathbf{F}_{ij} \) represents the Laplace capillary force density localized on the interfaces \( S_{ij} \) between the adjacent fluid components \( i \) and \( j \). It is represented as [17]

\[
\mathbf{F}_{ij} = \sigma_{ij} \zeta_{ij}(x_{S_{ij}}) \mathbf{n}_j(x_{S_{ij}}) \delta(x - x_{S_{ij}}) \quad (i \neq j), \tag{2}
\]

where \( \delta(x - x_{S_{ij}}) \) is the Dirac delta function with support on the interface \( S_{ij} \), \( \zeta_{ij} \) and \( \mathbf{n}_j \) are the curvature and unit normal at the point \( x_{S_{ij}} \) of the interface. The periodic boundary conditions for the fluid velocity and pressure are imposed on the side faces of the computational domain (see figure 1).

The Navier–Stokes equations (1) were solved numerically based on the finite-volume method of the OpenFoam package [17]. The velocity and pressure fields were calculated by means of the PISO algorithm. The current position of interfaces of the droplets was followed by the volume of fluid method (VOF) [18]. The 3D mesh for the numerical calculations was adapted dynamically in such a way to correspond to the moving interface with the highest resolution (figure 2).
Figure 1. Computational domain with the deformed composite (a) and homogeneous (b) droplets.

Figure 2. The adaptive mesh of high resolution at the interface between the fluids.

This approach allows a significant reduction of the computing time as well as high accuracy in calculation of the capillary forces at any point of the interface between fluid components.

3. Results and discussions
In this paper, we consider the equal viscosities of the continuous fluid and the core of composite droplet, $\eta_1 = \eta_3$, and the high relative viscosity of the shell, $m = \eta_2/\eta_3 = 5$. The same viscosity ratio was taken for the homogeneous droplet. The ratio of core to shell radii is equal to $R = b/a = 0.6$. Such models allow us to conduct the comparative study of effects of confinement
Figure 3. (a) The evolution of the relative elongation of the homogeneous (solid lines) and composite (dashed lines) droplets with time under the simple shear flow in the channel with $n = 0.4$ at $Ca = 0.4$ (i, ii) and 0.6 (iii, iv), respectively. (b) The shapes of the homogeneous and composite droplets corresponding to the steady-state elongations.

and shear flow intensity on the steady-state deformation and breakup conditions of the composite and homogeneous droplets. The confinement parameter $n$ was ranged between 0.3 and 0.8 while capillary number $Ca$ took three values 0.4, 0.6, and 0.8.

Figure 3(a) shows evolution of the relative elongations of the homogeneous and composite droplets with the dimensionless time $t'\dot{\gamma}$ during shear flow in the sufficiently wide channel ($n = 0.4$) at capillary numbers $Ca = 0.4$ and 0.6. It can be seen that under these conditions the homogeneous and composite droplets do not breakup attaining the steady-state elongation. Deformation of the homogeneous droplet at $Ca = 0.4$ is slightly higher than that of the composite droplet, while the steady-state deformations of both droplets coincide at $Ca = 0.6$. It is important to note that presence of the low-viscosity core leads to oscillations in elongation of the composite droplet. At the same time, deformation of the homogeneous droplet passes through a pronounced maximum after which the stationary state is reached quickly. The deformation oscillations were also observed experimentally in work [8] for the strongly confined homogeneous droplet ($n = 1$).

Figure 3(b) represents the corresponding steady-state shapes of the longitudinal sections of the homogeneous droplet (i, iii) and composite droplet (ii, iv). The comparison indicates that at $Ca = 0.4$ elongation of the composite droplet is smaller than that of the homogeneous droplet. This can be explained by the “sustaining” effect due to the internal core which was first discussed in work [13] with an example of 2D composite droplet. An increase in the capillary number to 0.6 leads to the visible deformation and orientation of the low-viscosity core. This neutralizes the “sustaining” effect resulting in coincidence of the steady-state deformations of the homogeneous and composite droplets.

Figure 4(a) demonstrates variations of elongations of the homogeneous and composite droplet with time in a narrower channel at $n = 0.6$ for the same capillary numbers as in figure 3. It is seen that at $Ca = 0.4$, the elongations of the homogeneous and composite droplets also reach their steady-state values. During this process, elongation of the composite droplet undergoes noticeable oscillations and deforms less than the homogeneous droplet as in the case
Figure 4. (a) The evolution of the relative elongation of the homogeneous (solid lines) and composite (dashed lines) droplets with time under the simple shear flow in the channel with $n = 0.6$ at $Ca = 0.4$ (i, ii) and 0.6 (iii, iv), respectively. (b) The shapes of the homogeneous and composite droplets corresponding to the steady-state and ultimate elongations.

of $n = 0.4$ (figure 4(a), curve ii). However, at $Ca = 0.6$, the homogeneous droplet breaks up, while the composite droplet reaches the steady-state elongation (figure 4(a), curves iii and iv). The corresponding images of the droplets are given in figure 4(b). The breakup of the high-viscosity homogeneous droplet at $n = 0.6$ and $Ca = 0.6$ agrees with the earlier result [6]. The breakup of the homogeneous droplet is accompanied by the sharp increase in the deformation rate (figure 4(a), curve iii), which is associated with the appearance and development of the narrowing sites (necks) in the droplet. The further increase in the confinement parameter up to $n = 0.8$ leads to the breakage of the homogeneous droplet at smaller capillary number, $Ca = 0.4$ (figure 5(b), (i)) whereas the composite droplet keeps stable and reaches its steady-state elongation (figure 5(b), (ii)). On the other hand, both droplets break up at $Ca = 0.6$ (figure 5(b), (iii) and (iv)). The disintegration of the composite droplet occurs due to separation of the side parts of the shell while the low-viscosity core is localized in the mid part and then breaks up into three parts. The extension rate of the composite droplet is higher than that of the homogeneous one (figure 5(a), curves iii and iv).

Thus, the obtained results show that the low-viscosity core stabilizes deformation process of the composite droplet with the high-viscosity shell. It is disintegrated at the higher capillary number than the homogeneous droplet.

Figure 6 shows dependences of the steady state and ultimate elongations of the homogeneous and composite droplets on the confinement parameter $n$ for different capillary numbers. The ultimate elongations just before the droplets breakup are indicated by the asterisks. It can be seen that for small $n$, the elongations of the homogeneous and composite droplets are rather small and almost independent of capillary number. However, the increase in $n$ results in the sharp growth of the relative elongations of both droplets terminating by their breakage. The larger is the capillary number, the smaller is the confinement parameter at which the droplets break up. A comparison of the obtained results shows that the disruption of the composite droplet occurs in narrower channels than in the case of the homogeneous droplets. This proves the stabilizing influence of the low-viscosity core.
Figure 5. (a) The evolution of the relative elongation of the homogeneous (solid lines) and composite (dashed lines) droplets with time under the simple shear flow in the channel with $n = 0.8$ at $\text{Ca} = 0.4$ (i, ii) and 0.6 (iii, iv), respectively. (b) The shapes of the homogeneous and composite droplets corresponding to the steady-state and ultimate elongations.

Figure 6. Dependence of the steady-state and ultimate elongations of the homogeneous (solid lines) and composite (dashed lines) droplets on the confinement parameter $n$ for different capillary numbers: $\text{Ca} = 0.4$ (i), 0.6 (ii), and 0.8 (iii).

4. Conclusions
The effect of a low-viscosity core on deformation and breakup behavior of the confined composite droplet with the high-viscosity shell subjected to the simple shear flow between two parallel
plates was investigated. It was found that the narrowing of the channel results in the breakup of both homogeneous and composite drops whereas they keep stability in the wide channel at the same capillary numbers. The increase in the confinement parameter leads to the decrease in the critical capillary number. The low-viscosity core is deformed along with the high-viscosity shell. Such a character of deformation process results in stabilization of the composite droplet while its disintegration occurs at higher capillary numbers than in the case of the high-viscous homogeneous droplet.

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