R-parity violation and CP-violating and CP-conserving spin asymmetries in
\( \ell^+\ell^- \rightarrow \tilde{\nu} \rightarrow \tau^+\tau^- \): probing sneutrino mixing at LEP2, NLC and \( \mu\mu \) colliders

S. Bar-Shalom, G. Eilam\(^1\)
Physics Department, University of California, Riverside, CA 92521, USA

and A. Soni
Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

We consider the sneutrino resonance reaction \( \ell^+\ell^- \rightarrow \tilde{\nu} \rightarrow \tau^+\tau^- \) in the MSSM without R-parity. We introduce new CP-violating and CP-conserving \( \tau \)-spin asymmetries which are generated already at the tree-level if there is \( \tilde{\nu} - \bar{\tilde{\nu}} \) mixing and that are forbidden in the SM. It is remarkable that these spin asymmetries can reach \( \sim 75\% \) around resonance for a sneutrino mass splitting of \( \Delta m \sim \Gamma \) and \( \sim 10\% \) for a splitting as low as \( \Delta m \sim 0.1\Gamma \), where \( \Gamma \) is the \( \tilde{\nu} \) width. We show that they are easily detectable already at LEP2 so long as the beam energy is within \( \sim 10 \) GeV range around the \( \tilde{\nu}_\mu \) masses and may therefore serve as extremely powerful probes of sneutrino mixing phenomena.

\(^1\)On leave from: Physics Department, Technion-Institute of Technology, Haifa 32000, Israel.
The MSSM lagrangian, which conserves $R$-parity, possesses a very distinct prediction: superpartners must be produced in pairs and, as a consequence, the lightest supersymmetric particle is stable. This implies that direct production of sparticles is restricted to colliders with c.m. energy at least twice the typical sparticle mass. This may pose a serious limitation in performing a detailed study of the supersymmetric free-parameter space in present and future leptonic colliders. However, it is well known that if $R$-parity is not conserved (in a way that keeps the proton lifetime within its experimental limit), then one cannot distinguish between the supermultiplets of the lepton-doublet $\hat{L}$ and that of the down-Higgs doublet $\hat{H}_d$. Thus, there is no good theoretical reason that prevents the superpotential from having additional Yukawa couplings constructed by $\hat{H}_d \to \hat{L}$ \cite{1}. In this paper we are interested only in the pure leptonic $R$-parity violating ($R/P$) operator:

$$\mathcal{L}_E = \frac{1}{2} \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k^c,$$

(1)

that violates lepton number $L$, but not baryon number. $\hat{E}_c$ is the charged lepton-singlet superfield, $i$ and $j$ are flavor indices such that $i \neq j$ because of the anti-commuting superfields. $\mathcal{L}_E$ drastically changes the phenomenology of the supersymmetric leptonic sector since it gives rise to the possibility of having $s$-channel slepton resonant formation in scattering processes, thus enabling the detection of slepton with masses up to the collider c.m. energy. This fact was observed already about 10 years ago \cite{2} and is recently gaining more interest for sneutrino resonance searches \cite{3}.

In this work we focus specifically on the effects of $R/P$ interactions on the process $\ell^+ \ell^- \to \tau^+ \tau^-$; the existence of other possible $R/P$ operators is irrelevant for that process at tree-level. In fact, the situation is rather economical for $e^+ e^- \to \tau^+ \tau^-$, in the sense that the fewest new parameters have to be considered. That is, since $i \neq j$ in (1), the couplings $e\tilde{\nu}_e e$ and $\tau\tilde{\nu}_\tau\tau$ are absent and only the $s$-channel $\tilde{\nu}_\mu$ exchange contributes with the couplings $\lambda_{121}$ and $\lambda_{323}$ for $e\tilde{\nu}_\mu e$ and $\tau\tilde{\nu}_\mu\tau$, respectively \cite{4}. We will explore two new aspects of $\tilde{\nu}_\mu$ resonance at LEP2: the detection of $\tilde{\nu}_\mu - \tilde{\nu}_\mu$ mixing and CP-violation. Both phenomena may exist once $\lambda_{121}, \lambda_{323} \neq 0$ in (1).
Sneutrino mixing has been the subject of several recent papers [5, 6]. The question of whether the sneutrinos mix or not is of fundamental importance since this mixing is closely related to the generation of neutrino masses [5, 6]. In fact, it was found in [6] that \( \Delta m_{\tilde{\nu}_i}/m_{\nu_i} \lesssim \text{few} \times 10^3 \) is required in order for \( m_{\nu_i} \) to be within the present experimental upper limits.

Our present interest here is in the detection of sneutrino mixing rather than its origins. We will therefore not assume any specific model for it to occur. Instead, we write \( \tilde{\nu}_\mu = (\tilde{\nu}_+ + i\tilde{\nu}_-)/\sqrt{2} \) and simply assume that, due to some new short distance physics, there exists a mass splitting between the new CP-even and CP-odd muon-sneutrino mass eigenstates \( \tilde{\nu}_+ \) and \( \tilde{\nu}_- \), respectively (we assume CP-conservation in the mixing), such that \( \Delta m/m_\pm \ll 1 \), where \( \Delta m \equiv m_+ - m_- \) and \( m_\pm \equiv m_{\tilde{\nu}_\pm} \). In particular, we take \( \Delta m \lesssim \Gamma \) and \( \Gamma \equiv \Gamma_- \simeq \Gamma_+ = 10^{-2}m_- \). Indeed, if \( m_\pm > m_{\tilde{\chi}^\pm} \), \( m_{\tilde{\chi}^0} \) (\( \tilde{\chi}^\pm \) and \( \tilde{\chi}^0 \) are the charginos and neutralinos, respectively), then the two-body decays \( \tilde{\nu}_\pm \rightarrow \tilde{\chi}^\pm \ell \); \( \tilde{\chi}^0 \nu \) are open and the corresponding partial widths are given by: \( \Gamma(\tilde{\nu}_\pm \rightarrow \tilde{\chi}^\pm \ell) \); \( \Gamma(\tilde{\nu}_\pm \rightarrow \tilde{\chi}^0 \nu) \sim \mathcal{O}\left[10^{-2}m_\pm \times \left(1 - m_{\tilde{\chi}^\pm}/m_\pm^2\right)^2; \left(1 - m_{\tilde{\chi}^0}/m_\pm^2\right)^2\right] \) (see Barger et al. in [2]). Therefore, for \( m_\pm \gtrsim 200 \text{ GeV} \), \( \Gamma = 10^{-2}m_- \) serves our purpose as it is a viable estimate even without taking into account the new \( R/P \) two-body decay modes which, when summed, can also form a significant fraction of the total sneutrino width.

Apart from the rough theoretical requirement that \( \Delta m/m_- \ll 1 \), imposed by neutrino masses [3], there is another important reason why we wish to consider the limit \( \Delta m \lesssim \Gamma \). The reason is that in that case the \( \tilde{\nu}_+ \) and \( \tilde{\nu}_- \) resonances will overlap and distinguishing between the two peaks becomes a non-trivial experimental task. In such an event, in order to observe the small \( \tilde{\nu}_+ - \tilde{\nu}_- \) mass splitting one would have to search for the effects of flavor oscillations in sneutrino decays in analogy to the \( B^0 - \bar{B}^0 \) system, for example in \( \tilde{\nu}\)-pair production, \( e^+e^- \rightarrow \tilde{\nu}_+\tilde{\nu}_- \) [3]. However, for \( m_\pm \gtrsim 200 \text{ GeV} \), such a \( \tilde{\nu} \)-pair production awaits the next generation of lepton colliders. In what follows, we will show that an alternative for a detection of \( \Delta m \neq 0 \) for \( m_\pm \gtrsim 200 \text{ GeV} \) is to measure appropriate CP-even and CP-odd \( \tau \)-spin asymmetries in \( e^+e^- \rightarrow \tau^+\tau^- \) which are proportional to \( \Delta m \). We show that these asymmetries may reach tens of percents in a relatively
wide sneutrino mass range around the sneutrino resonance even for a small splitting of \(\Delta m \lesssim \Gamma/4\). As a consequence, such a small mass splitting may be detectable already at LEP2 to many standard deviations (SD). Note that spin asymmetries in \(\ell^+\ell^- \rightarrow f\bar{f}\) can be measured only for \(f = \tau\) or \(t\). However, in sneutrino resonant formation they may apply only to the \(\tau\) since the \(t\bar{\nu}t\) coupling is forbidden by gauge invariance.

Besides, the possibility of having tree-level CP-violation, of the order of tens of percent in \(\tau\) pair production at LEP2, stands out as an extremely interesting issue by itself. Previous studies of CP-violating effects in \(e^+e^- \rightarrow \tau^+\tau^-\), that can emanate from models beyond the SM, such as multi-Higgs doublet model, SUSY, leptoquark and Majorana \(\nu\), all involve one-loop exchanges of the new particles which generate a CP-violating electric dipole moment for the \(\tau\) (see [7] and references therein). These CP-odd effects are therefore much smaller than our tree-level effect, which may exist at a level of tens of percents around the \(\tilde{\nu}_\pm\) resonance.

Let us now construct the \(\tau^+\tau^-\) double-polarization asymmetries. In the rest frame of \(\tau^-\) we define the basis vectors: \(\vec{e}_z \propto -(\vec{p}_{e^+} + \vec{p}_{e^-})\), \(\vec{e}_y \propto \vec{p}_{e^+} \times \vec{p}_{e^-}\) and \(\vec{e}_x = \vec{e}_y \times \vec{e}_z\). For the \(\tau^+\) we use a similar set of definitions such that \(\vec{e}_{\bar{x}}, \vec{e}_{\bar{y}}, \vec{e}_{\bar{z}}\) are related to \(\vec{e}_x, \vec{e}_y, \vec{e}_z\) by charge conjugation. We then define the following \(\tau^+\tau^-\) double-polarization operator with respect to their corresponding rest frames defined above:

\[
\Pi_{ij} \equiv \frac{N(\uparrow_i \uparrow_j) - N(\uparrow_i \downarrow_j) - N(\downarrow_i \uparrow_j) + N(\downarrow_i \downarrow_j)}{N(\uparrow_i \uparrow_j) + N(\uparrow_i \downarrow_j) + N(\downarrow_i \uparrow_j) + N(\downarrow_i \downarrow_j)},
\]

(2)

where \(i, j = x, y, z\). For example, \(N(\uparrow_x \uparrow_y)\) stands for the number of events in which \(\tau^+\) has spin +1 in the direction \(x\) in its rest frame and \(\tau^-\) has spin -1 in the direction \(y\) in its rest frame. The spin vectors of \(\tau^+\) and \(\tau^-\) are therefore defined in their respective rest frames as: \(\vec{s}^+ = (s_x, s_y, s_z)\) and \(\vec{s}^- = (s_x, s_y, s_z)\) and \(\Pi_{ij}\) is calculated in the \(e^+e^-\) c.m. frame by boosting \(\vec{s}^+\) and \(\vec{s}^-\) from the \(\tau^+\) and \(\tau^-\) rest frames to the \(e^+e^-\) c.m. frame.

It is then easy to verify that \(\Pi_{ij}\) possesses the following transformation properties under the operation of CP and of the naive time reversal \(T_N\): \(CP(\Pi_{ij}) = \Pi_{ji}\) for all
\(i, j, T_N(\Pi_{ij}) = -\Pi_{ij}\) for \(i\) or \(j = y\) and \(i \neq j\) and \(T_N(\Pi_{ij}) = \Pi_{ij}\) for \(i, j \neq y\) and for \(i = j\).

We can therefore define:

\[
A_{ij} = \frac{1}{2}(\Pi_{ij} - \Pi_{ji}) , \quad B_{ij} = \frac{1}{2}(\Pi_{ij} + \Pi_{ji}) .
\]

(3)

Evidently, \(A_{ij}\) are CP-odd (\(A_{ii} = 0\) by definition) and \(B_{ij}\) are CP-even. Also, \(A_{xy}, A_{zy}, B_{xy}\) and \(B_{zy}\) are \(T_N\)-odd while \(A_{xx}, B_{xz}, B_{yx}, B_{yy}\) and \(B_{zz}\) are \(T_N\)-even.

To calculate the various asymmetries defined in (3) we need the cross-sections for the \(s\)-channel sneutrino exchange and the SM \(s\)-channel \(\gamma, Z\) exchanges. The interferences between the SM diagrams and the \(s\)-channel \(\tilde{\nu}_\pm\) diagrams as well as between the \(\tilde{\nu}_+\) and the \(\tilde{\nu}_-\) \(s\)-channel diagrams are proportional to the electron mass and are therefore being neglected. The SM and \(\tilde{\nu}_\pm\) cross sections can be subdivided as: 

\[
\sigma^0_{SM,\tilde{\nu}_\pm} \equiv \sigma^0_{SM;\tilde{\nu}_\pm}/4 + \sigma^{\tilde{\nu}^+ - \tilde{\nu}^+}_{SM;\tilde{\nu}_\pm},
\]

where \(\sigma^0_{SM}\) and \(\sigma^0_{\tilde{\nu}_\pm}\) are the SM and \(\tilde{\nu}_\pm\) total cross sections, respectively, summed over the \(\tau^+\) and \(\tau^-\) spins; \(\sigma_{SM,\tilde{\nu}_\pm}^{\tilde{\nu}^+ - \tilde{\nu}^+}\) are the spin dependent parts. The total spin dependent cross-section for \(e^+e^- \rightarrow \tau^+\tau^-\) is then simply given by the sum \(\sigma^T = \sigma_{SM} + \sigma_{\tilde{\nu}_\pm}\). For the SM we find (assuming \(m_{\tau} = 0\) and \(\Gamma_Z/m_Z = 0\)):

\[
\begin{align*}
\sigma_{SM}^0 &= \frac{\pi \alpha^2}{3s} \left( 4 + 2\omega (g_L + g_R)^2 + \omega^2 (g_L^2 + g_R^2)^2 \right) , \\
\sigma_{SM}^{\tilde{\nu}^+ - \tilde{\nu}^+} &= \frac{\pi \alpha^2}{12s} \left\{ s_z s_x (4 + 2\omega (g_L + g_R)^2 + \omega^2 (g_L^2 + g_R^2)^2) + \\
&\quad (s_z s_x - s_y s_y) \left( 2 + \omega (g_L + g_R)^2 + \omega^2 g_L g_R (g_L^2 + g_R^2) \right) + \\
&\quad (s_z + s_x) \left( 2\omega (g_R^2 - g_L^2) + \omega^2 (g_R^4 - g_L^4) \right) \right\} ,
\end{align*}
\]

(4)

(5)

where \(s = (p_{e^+} + p_{e^-})^2\), \(\omega \equiv (\sin^2 \theta_W \cos^2 \theta_W (1 - m_Z^2/s))^{-1}\), \(g_L = \sin^2 \theta_W - 1/2\), \(g_R = \sin^2 \theta_W\) and \(\theta_W\) is the weak mixing angle.

For the \(s\)-channel \(\tilde{\nu}_\pm\) we assume for simplicity that \(\lambda_{121}\) is real (this assumption does not change our predictions below) and define \(\lambda_{323} \equiv (a + ib)/\sqrt{2}\). The relevant couplings of the CP-even \((\tilde{\nu}_+\)) and the CP-odd \((\tilde{\nu}_-\)) sneutrino mass eigenstates are then: \(e\tilde{\nu}_+ e = i\lambda_{121}/\sqrt{2},\)
\[ e\tilde{\nu}_- e = -\lambda_{121}\gamma_5/\sqrt{2}, \quad \tau\tilde{\nu}_+ \tau = i(a - ib\gamma_5)/2, \quad \tau\tilde{\nu}_- \tau = i(b + ia\gamma_5)/2 \]

and the sneutrinos cross-section is \((m_\tau = 0)\):

\[
\sigma^0_{\tilde{\nu}_\pm} = \frac{s}{64\pi}\lambda^2_{121}|\lambda_{323}|^2 D_+ ,
\]
\[
\sigma^{\tilde{\nu}_- \tilde{\nu}_+}_{\tilde{\nu}_\pm} = -\frac{s}{512\pi}\lambda^2_{121} \left\{ s_x^2 + b^2 \right\} D_+ + \left( s_x s_x^* + s_y s_y^* \right) \left( b^2 - a^2 \right) D_+ + \frac{2ab(s_y s_x - s_x s_y)}{D_-} ,
\]

where \( D_\pm \equiv |\pi_+|^2 \pm |\pi_-|^2 \) and \( \pi_\pm = \left( s - m_\pm^2 + im_\pm \Gamma \right)^{-1} \).

The calculation of the various spin asymmetries is now straightforward. For the \( \tilde{\nu}_\pm \) exchanges, at tree-level, only \( A_{xy}, B_{xx}, B_{yy} \) and \( B_{zz} \) are non-zero:

Sneutrinos only : \( A_{xy} = \left( \frac{2ab}{a^2 + b^2} \right) \frac{D_-}{D_+} \), \( B_{xx} = B_{yy} = \left( \frac{a^2 - b^2}{a^2 + b^2} \right) \frac{D_-}{D_+} \), \( B_{zz} = -1 \). \hfill (8)

For the SM case, only the following CP-even asymmetries are non-zero at tree-level:

SM only : \( B_{xx} = -B_{yy} = \frac{2 + \omega(g_L + g_R)^2 + \omega^2 g_L g_R(g_L^2 + g_R^2)}{4 + 2\omega(g_L + g_R)^2 + \omega^2(g_L^2 + g_R^2)} \), \( B_{zz} = 1 \). \hfill (9)

As expected, a non-vanishing CP-odd asymmetry is unique to the sneutrino exchange contribution. It is clear from (4) and (8) that CP-violation in \( e^+e^- \rightarrow \tilde{\nu}_\pm \rightarrow \tau^+\tau^- \) arises already at tree-level from the interference of the scalar and pseudoscalar couplings of \( \tilde{\nu}_\pm \) to \( \tau^+\tau^- \). The possibility of generating a tree-level CP-violating effect when a scalar-fermion-antifermion coupling is of the form \((a + ib\gamma_5)\) was first observed in [9]. There it was suggested that a neutral Higgs of a two Higgs doublet model may drive such large tree-level CP-violating effects. Recently, it was shown [10] that spin correlations can trace similar scalar-pseudoscalar tree-level interference effects in the \( H^0 \rightarrow t\bar{t} \) and \( H^0 \rightarrow \tau^+\tau^- \) decay modes. However, the tree-level CP-violation effect in \( H^0 \rightarrow \tau^+\tau^- \), when applied to \( e^+e^- \rightarrow H^0 \rightarrow \tau^+\tau^- \), is only of academic interest since the neutral Higgs coupling to electrons is \( \propto m_e \).
In [10], a non-vanishing tree-level CP-even spin correlation of the form $\vec{s}^+ \cdot \vec{s}^-$ was suggested for $H^0 \to \tau^+ \tau^-$. However, we note that $\vec{s}^+ \cdot \vec{s}^-$ simply translates to the observable $\Sigma_{i=x,y,z}B_{ii}$ and it is therefore clear from (9) that in the SM, $\vec{s}^+ \cdot \vec{s}^- \propto B_{zz} = 1$. A measurement of such an observable will therefore be insensitive to the couplings $a$ and $b$ in $\lambda_{323}$. We suggest here a way out by defining the new CP-even observable: $B \equiv (B_{xx} + B_{yy})/2$. Obviously, at tree-level, $B = 0$ in the SM and $B = B_{xx} = B_{yy}$ for the sneutrino case. Thus, a measurement of $B \neq 0$ will be a strong indication for the existence of new physics in $e^+e^- \to \tau^+ \tau^-$ in the form of new non-vanishing $s$-channel scalar exchanges and will provide explicit information on the new $\tau\tilde{\nu}_\mu\tau$ coupling.

From (8) we observe that $A_{xy}$ and $B \propto D_-/D_+$ where the proportionality factors do not depend on the absolute magnitude of the couplings $a$ and $b$ but rather on any function of their ratio $f(a/b)$. In particular, without loss of generality, we will assume that $a$ and $b$ are positive and study the asymmetries as a function of the ratio $r \equiv b/(a+b)$. $r$ can vary between $0 \leq r \leq 1$, where the lower and upper limits of $r$ are given by $b = 0$ and $a = 0$, respectively. One can immediately observe that $A_{xy}$ and $B$ complement each other as they probe opposite ranges of $r$. For $A_{xy}$ the maximal value $D_-/D_+$ is obtained when $r = 1/2$ ($a = b$) and $B = D_-/D_+$ when $r = 0$ ($b = 0$). Also, at $r = 1$ ($a = 0$), $B = -D_-/D_+$, thus reaching its maximum negative value.

In Figure 1 we plot the ratio $D_-/D_+$, i.e., the maximal values of $A_{xy}$ and $B$, as a function of the lighter muon-sneutrino mass $m_-$. We take $\Delta m = \Gamma$, $\Gamma/2$, $\Gamma/4$, $\Gamma/10$ (recall that $\Gamma = 10^{-2}m_-$). Also, here and throughout the rest of the paper we take the c.m. energy at LEP2 to be $E_{CM} = 192$ GeV. Evidently, $A_{xy}$ and $B$ can reach $\sim 75\%$ around resonance if $\Delta m = \Gamma$, and $\sim 10\%$ even for the very small splitting $\Delta m = \Gamma/10$. We also observe that the asymmetries stay large ($\gtrsim 10\%$) even $\sim 10$ GeV away from resonance. Around the narrow region of $E_{CM} \sim (m_+ + m_-)/2$, $D_-/D_+ \simeq \Delta m/m_-$ and the asymmetries become very small.

The statistical significance, $N_{SD}$, with which $A_{xy}$ or $B$ can be detected, is given by $N_{SD} = \sqrt{N|A|\sqrt{\epsilon}}$, where $A = A_{xy}$ or $B$, $N = (\sigma_{\nu_{\mu}}^0 + \sigma_{\tilde{\nu}_M}^0) \times L$ is the total number of $e^+e^- \to \tau^+\tau^-$ events and we take $L = 0.5$ fb$^{-1}$ as the total integrated luminosity at LEP2. $\epsilon$ is the combined efficiency for the simultaneous measurement of the $\tau^+$ and $\tau^-$
spins which, therefore, depends on the efficiency for the spin analysis and also on the branching ratios of the specific \( \tau^+ \) and \( \tau^- \) decay channels that are being analyzed. The simplest examples perhaps are the two-body decays \( \tau^\pm \to \pi^\pm \nu_\tau \) and \( \tau^\pm \to \rho^\pm \nu_\tau \), although 3-body decays may also be useful \[10, 11\]. When all combinations of only the above \( \tau^+, \tau^- \) two-body decay channels are taken into account one finds \( \epsilon \sim 0.03 \) \[10\]. We will adopt this conservative number henceforward.

In Figure 2 we plot the statistical significance, \( N_{SD} \), which corresponds to \( A_{xy} \) and \( B \) at their maximal values, at LEP2, as a function of \( m_- \). We choose the same values for \( \Delta m \) as in Figure 1. For completeness, the SM contribution to the denominator in \( \mathcal{E} \) is now being included, in which case \( A_{xy} \) and \( B = B_{xx} = B_{yy} \) in \( \mathcal{E} \) are multiplied by the factor \( \left( 1 + \sigma^0_{SM}/\sigma^0_{\tilde{\nu}_\pm} \right)^{-1} \). We calculate \( \sigma^0_{\pm} \) by setting \( \lambda_{121} \) and \( \lambda_{323} \) to their presently experimental allowed upper limits \[12\]: \( \lambda_{121} = 0.05 \times (m_-/100 \text{ GeV}) \) and \( |\lambda_{323}| = 0.06 \times (m_-/100 \text{ GeV}) \). It is remarkable that both \( A_{xy} \) and \( B \) may be detectable, under the best circumstances, with a sensitivity reaching well above \( \sim 10 \) SD. We see, for example, that for \( \Delta m = \Gamma \) these asymmetries induce beyond a 3σ effect at LEP2, around the resonance region, practically over the whole \( \sim 10 \) GeV mass range, \( 186.5 \text{ GeV} \lesssim m_- \lesssim 196 \text{ GeV} \). For \( \Delta m = \Gamma/4 \) the corresponding 3σ mass range is \( 189.5 \text{ GeV} \lesssim m_- \lesssim 194 \text{ GeV} \) and even for the very small splitting \( \Delta m = \Gamma/10 \) there is a 3σ region over about a 1 GeV interval near the resonance mass.

Figure 3 shows the dependence of \( N_{SD} \), for \( A_{xy} \) and \( B \), on the ratio \( r \), where, as in Figure 2, the SM diagrams are included and for illustration we set \( m_- = E_{CM} = 192 \) GeV (we note that this value of \( m_- \) does not maximize the effects). We see that a measurement of \( A_{xy} \) and \( B \) at LEP2 can cover a wide range of the parameter \( r \). In particular, one observes that for \( \Delta m = \Gamma \), LEP2 can have larger than 3σ sensitivity to values of \( r \) practically over the entire range of \( r \) for both \( A_{xy} \) and \( B \). Even for \( \Delta m = \Gamma/4 \), the following ranges are covered to at least a 3σ significance: for \( A_{xy} \), \( 0.27 \lesssim r \lesssim 0.73 \) and for \( B \), \( 0 \leq r \lesssim 0.32 \) and \( 0.68 \lesssim r \leq 1 \). Evidently, as stated before, the ranges being covered by \( A_{xy} \) and \( B \) complement each other such that, even for the \( \Delta m = \Gamma/4 \) case, the whole range \( 0 \leq r \leq 1 \) can be covered, to at least 3σ, with the simultaneous measurement of \( A_{xy} \) and \( B \). We note again that the \( r \) ranges being covered to at least 3σ by each of
the two asymmetries are wider if $m_-$ is slightly away from resonance, \textit{i.e.}, by about 0.5 GeV.

Finally, we have calculated the sensitivity of the NLC with c.m. energy $E_{CM} = 500$ GeV to $A_{xy}$ and $B$ for a very heavy muon-sneutrino $m_- \sim E_{CM}$. We found that the NLC will be able to probe these CP-odd and the CP-even asymmetries to at least 3$\sigma$ (the best effects are again at the $\sim 20\sigma$ level), in the muon-sneutrino mass range of $\sim 20$ GeV around resonance, \textit{i.e.}, $E_{CM} - 10$ GeV $\lesssim m_- \lesssim E_{CM} + 10$ GeV, even for a small mass splitting of $\Delta m = 1$ GeV $\simeq \Gamma/5$. Also, we found that, with $\Delta m = 1$ GeV, the NLC will have a sensitivity above 3$\sigma$ to either $A_{xy}$ or $B$ over almost the entire $r$ range, $0 \leq r \leq 1$.

To summarize, we have introduced new CP-violating and CP-conserving spin asymmetries and applied them to $e^+e^- \rightarrow \tau^+\tau^-$. We have shown that two of these asymmetries are unique in their ability to distinguish between the CP-odd and CP-even muon-sneutrino mass eigenstates in $e^+e^- \rightarrow \tilde{\nu}_\pm \rightarrow \tau^+\tau^-$. Both asymmetries arise already at the tree-level and can become extremely large, of the order of tens of percent. They may therefore be detectable with many SD’s already at LEP2 if the muon-sneutrino mass lies within $\sim 10$ GeV range around the LEP2 c.m. energy, even if the mass splitting between the two sneutrino particles is less than 1 GeV. As far as CP-violation is concerned, it is especially gratifying that such a large CP-nonconserving effect may arise in $\tau$-pair production at LEP2 and may be searched for in the very near future.

We have also found that these asymmetries will yield a significant signal at the NLC with $E_{CM} = 500$ GeV within a wider sneutrino mass range, of $\sim 20$ GeV, around resonance. Moreover, the effects reported here may be similarly applied to a future muon collider in the $\tilde{\nu}_e$ resonance channel $\mu^+\mu^- \rightarrow \tilde{\nu}_e \rightarrow \tau^+\tau^-$. However, we note that while the present limits on the $\tau\tilde{\nu}_e\tau$ and $\tau\tilde{\nu}_\mu\tau$ couplings are comparable, the limit on the coupling $\mu\tilde{\nu}_e\mu$ is more stringent then the one on $e\tilde{\nu}_\mu e$ by about an order of magnitude.

In parting we wish to remark that a measurement of the double $\tau$-spin asymmetries $A_{xy}$ and $B$ in $\tau^+\tau^-$ production at the Tevatron is another interesting possibility \cite{12}.

We acknowledge partial support from U.S. Israel BSF (G.E. and A.S.) and from the U.S. DOE contract numbers DE-AC02-76CH00016(BNL), DE-FG03-94ER40837(UCR).
S.B. thanks J. Wudka and D.P. Roy for helpful discussions. G.E. thanks the Israel Science Foundation and the Fund for the Promotion of Research at the Technion for partial support and members of the HEP group in UCR for their hospitality.
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**Figure Captions**

Fig. 1: The maximal value of $A_{xy}$ and $B$, *i.e.* $D_-/D_+$, as a function of the lighter $\tilde{\nu}_\mu$ mass $m_-$, for four mass-splitting values $\Delta m$.

Fig. 2: The statistical significance, $N_{SD}$, attainable at LEP2 for $A_{xy}$ and $B$ at their maximal values, as a function of $m_-$. See also caption to Figure 1.

Fig. 3: The attainable $N_{SD}$, for $A_{xy}$ and $B$, at LEP2, as a function of $r \equiv b/(a + b)$. The cases $\Delta m = \Gamma$ and $\Delta = \Gamma/4$ are illustrated. See also caption to Figure 1.
\[ E_{\text{CM}} = 192 \text{ GeV} \]
Figure 2

$E_{CM} = 192 \text{ GeV}$

$N_{SD}$

$\Delta m = \Gamma$

$\Delta m = \Gamma / 2$

$\Delta m = \Gamma / 4$

$\Delta m = \Gamma / 10$

$m_\pm [\text{GeV}]$
Figure 3

\[ m = E_{CM} = 192 \text{ GeV} \]

- \( A_{xy}, \Delta m = \Gamma \)
- \( A_{xy}, \Delta m = \Gamma / 4 \)
- \( B, \Delta m = \Gamma \)
- \( B, \Delta m = \Gamma / 4 \)