**Abstract**

Recent work has shown that optimizing the Learning Rate (LR) schedule can be a very accurate and efficient way to train deep neural networks. We observe that the rate of change (ROC) of LR has correlation with the training process, but how to use this relationship to control the training to achieve the purpose of improving accuracy? We propose a new method, k-decay, just add an extra item to the commonly used and easy LR schedule (exp, cosine and polynomial), is effectively improves the performance of these schedule, also better than the state-of-the-art algorithms of LR schedule such as SGDR, CLR and AutoLRS. In the k-decay, by adjusting the hyper-parameter $k$, to generate different LR schedule, when $k$ increases, the performance is improved. We evaluate the k-decay method on CIFAR And ImageNet datasets with different neural networks (ResNet, Wide ResNet). Our experiments show that this method can improve on most of them. The accuracy has been improved by 1.08% on the CIFAR-10 dataset and by 2.07% on the CIFAR-100 dataset. On the ImageNet, accuracy is improved by 1.25%. Our method is not only a general method to be applied other LR Shcedule, but also has no additional computational cost.

**1. Introduction**

Deep learning is widely used in image recognition, speech recognition, and many other fields. Now we have convolutional neural networks for images, recurrent neural networks for speeches, and graph neural networks for graphs. With the development of technology, our research goal is to obtain better model performance under the same resource conditions. To achieve this goal, we studied the learning rate schedule from a new perspective, and some research shows a good schedule for the learning rate can improve the training performance.

In deep neural networks, the parameters are updated by

$$w' = w - \eta \nabla L$$

where $w$ and $w'$ are parameters, $\eta$ is the learning rate, and $L$ is the loss function. The $\eta$ controls the update speed of parameters. When $\eta$ is large, the model converges very quickly but may skip local minimum values. When $\eta$ is small, local minimum values can be found, but the model converges slowly. The LR schedule $\eta(t)$ governing the decay from maximum LR $\eta_0$ to the minimum LR $\eta_e$ can solve this contradiction.

The Fig.4 shown the different polynomial of $N$ to training the ResNet on the CIFAR-10 dataset. We found that the ROC of the errors will vary with the ROC of LR, has positive correlation, especially at the end stage. So we suppose increase the ROC of the LR to increase of ROC of the errors, be equivalent to improves the model’s performance, at the end of training period, but how to increase the ROC of the LR?

In this paper, we purpose a method based on mathemat-
ical derivatives, name k-decay, which by impacting its k-th order derivative, to increase the ROC of original LR schedule, at the end stage. Denote the function of original LR schedule by $\eta(t)$ and its k-th order derivative function by $\eta^k(t)$, then $\eta^k(t)$ is updated in the following way,

$$\eta^k(t) = \eta^k(t) + \Delta f^k(t)$$

(1)

where $k \in \mathbb{N}$. The $\Delta f^k(t)$ is the increment of $\eta^k(t)$, which controls the ROC of $\eta(t)$ in the k-th order. Eventually, the solution of $\eta^k(t)'$ is the $\eta(t)'$ for new LR schedule. Obviously, the solution $\eta_o(k, t)$ must contain $k$, so named k-decay.

In order to preliminarily verify the effectiveness of this method, we simplified the equation through the following steps and tested its effect. **STEP1:** Find its k-th derivative $\eta^k(t)$. **STEP2:** Add an increment $\alpha_0$: $\eta^k(t) + \alpha_0$. **STEP3:** Solve this used for training. The Fig.2 show the 1-th derivative graph of the polynomial LR schedule at $N = 0.5$, we gradually increase $\alpha_0$, the accuracy is indeed improved by 0.5% at $\alpha_0 = 0.000005$. Also we have done many other examples and k-th derivative, have the same effect.

In Sec.3 we derive a special solution $\eta_o(k, t)$ as a standard procedure based on polynomial LR schedule and above steps. It should be pointed out that there is no particular preference for our choice of $\eta(t)$ like polynomial LR schedule, we can still choose other LR schedule for special solutions of k-decay. Then we used the solution as an additional item to other LR schedule, the performance of these LR has been significantly improved. In summary, our k-decay method is proposed with wide applicability, which can obtain different forms of variants when applied to different LR, and can greatly improve the performance of the model.

2. Related Work

Recent years have seen many optimization studies based on the Learning Rate. On the one hand, in the better LR schedule role, the global LR change with time in the optimal algorithm. Such examples include the multi-step decay, polynomial decay, Stochastic Gradient Descent with Restarts (SGDR) [15], Cyclic Learning Rates (CLR) [17], and Hyperbolic-Tangent Decay (HTD) [7]. The SGDR uses the cosine decay, combined with periodic function. The CLR is a periodic function, which uses the maximum and minimum values as a period. The HTD uses the tanh function to construct a new LR schedule. A recent work AutoLRS [9] propose a automatic LR schedule by bayesian. All these schedules can improve the performance of the model of interest. But, compared with our method, these methods have more hyperparameter settings, and the training process is extremely fluctuating. Our method has only one hyperparameter $k$, and the performance is positively correlated with $k$. Our method has wide applicability, and different forms of variants will be obtained when applied to different LR schedules.

On the other hand, initialization parameters are changed based on the history of the gradient. For example, some adaptive learning rates algorithm: RMSprob [16], AdaDelta [20], Adam [10]. These methods improve the stochastic gradient descent based on the momentum. It can accelerate the convergence and reduce the number of steps of the LR. However, such a method is not contradiction with the LR schedule, with better performance when they are combined.

3. k-decay For Learning Rate schedule

In the section, we derivation a special solution of k-decay based on the linear polynomial function and then generalize to other LR schedule. The linear polynomial function is

$$\eta(t) = (\eta_0 - \eta_e)(1 - \frac{t}{T}) + \eta_e.$$  

(2)

For simplicity in calculation, we consider here the $\Delta f^k(t)$ in the k-decay equation (eq.1) being constant with time:

$$\eta^k(t)' = \eta^k(t) + \Delta f^k_0,$$  

(3)

So the new linear polynomial function can be

$$\eta(t)' = (\eta_0 - \eta_e)(1 - \frac{t}{T}) + \eta_e + \eta_o(k, t),$$  

(4)

$\eta_o(t)$ is additional terms raised by the $\Delta f^k_0$. And the boundary condition is

$$\begin{cases} 
\eta(t + 1)' \leq \eta(t)' \\
\eta(0)' = \eta_0, \eta(T)' = \eta_e 
\end{cases}$$  

(5)
Series expansion of $\eta_0(k, t)$ leads to,
\[
\eta_0(k, t) = a_k t^k + \ldots + a_1 t + a_0,
\]
Without loss of generality, let $a_{k-1} = 0, \ldots, a_0 = 0$, we have
\[
\eta(t)' = (\eta_0 - \eta_e)(1 - \frac{t}{T}) + \eta_e + a_k t^k + a_1 t.  \tag{6}
\]
Then $k$ order of eq. (6) is given by,
\[
\eta^k(t)' = \eta^k(t) + k!a_k = 0 + k!a_k = k!a_k, \tag{7}
\]
so
\[
\Delta f^k_0 = k!a_k. \tag{8}
\]
We find the $k$-th order of the function $f(t) = (\eta_0 - \eta_e)(1 - \frac{t}{T})^n + \eta_e$ is constant with time at $n = k$:
\[
f^k(t) = (\eta_0 - \eta_e)k!(-\frac{1}{T})^k t^{n-k} = (\eta_0 - \eta_e)k!(-\frac{1}{T})^k. \tag{9}
\]
It can be used as a special solution of the eq. (6). Let $\Delta f^k_0 = f^k(t)$, we have
\[
a_k = \pm(\eta_0 - \eta_e)\frac{1}{T^k}. \tag{10}
\]
Consider the boundary conditions, substitute into the eq. (6), and then simplified to
\[
\eta_o(k, t) = (\eta_0 - \eta_e)(\frac{T^k}{T_0^k} - \frac{t}{T_0}). \tag{11}
\]
The special solution can be used as an additional term, added to other LR schedule, build a new LR schedule of the k-decay. In the LR schedule, we control the increment $\Delta f^k(t)$ by $k$. In fact, as you can see in Fig.2, adjusting the current time, $T_0$ is our additional item, $\eta(t)$ is the original LR schedule, $t$ is the current time, $T_0$ is the total time.

For instance, the polynomial (POL) of k-decay is,
\[
\eta(t)' = (\eta_0 - \eta_e)(1 - \frac{t}{T_0})^n + \eta_e + \eta_o(k, t). \tag{12}
\]
The cosine (COS) of k-decay is:
\[
\eta(t)' = \frac{1}{2}(\eta_0 - \eta_e)(1 + \cos(\frac{t}{T_0} \pi)) + \eta_e + \eta_o(k, t). \tag{13}
\]

The exp (EXP) of k-decay is:
\[
\eta(t)' = (\eta_0 - \eta_e)\exp(-\frac{t}{T_0}) + \eta_o(k, t). \tag{14}
\]

Fig. 3 shows the difference between the original LR schedule and the new LR schedule of the k-decay. The ROC of the LR in the late training period increases with the increase of $k$ in the new LR schedule. Applying the k-decay factors to different functions is equivalent to changing the ROC of the LR with $k$.

### 4. Experiments

**Datasets** We choose the accuracy of classification tasks on CIFAR [12] and ImageNet [2] as the standard to measure performance. The CIFAR datasets, including 10 categories of CIFAR-10 and 100 categories of CIFAR-100, which are respectively composed of 50,000 training sets and 10,000 test sets. The ImageNet data set includes 1000 classified. Here use the ILSVRC2012 classification data set, which has 1.28 million training pictures and 50k validation sets.

**Baseline** Usually for convenience, we will choose Step Decay, EXP, COS and POL as the learning rate schedule, and of course the latest SOAT algorithms such as SGDR [7] and CLR [17]. We will k-decay item is added to the commonly used LR schedule EXP, COS and POL, to compare the performance difference with original LR schedule, to illustrate the versatility of our method, is also compared with SDGR and CLR to illustrate the superiority of our method.

**Implementation** The implementation of the Wide ResNet-28-10 for CIFAR and [6] the ResNet-50 for ImageNet [19] is the same as the original paper. The opti-
Table 1. Testing error (%) on CIFAR-10 and CIFAR-100 datasets. The overall best results are in bold. For instance in our results denoted by $[5.26_{20}]$, 5.26 means the errors (%), subscript 2.0 means $k = 2.0$ on k-decay.

| Method       | CIFAR-10 | CIFAR-100 |
|--------------|----------|-----------|
| CLR [17]     | 4.93     | 21.58     |
| StepDecay [19] | 4.17     | 20.50     |
| SGDR [7]     | 4.03     | 19.58     |
| EXP          | 4.11     | 20.40     |
| EXP with k-decay (ours) | 3.92↑↑0.08 | 19.05↑↑1.35 |
| COS          | 3.68     | 18.68     |
| COS with k-decay (ours) | 3.82↓0.14 | 18.44↑0.24 |
| POL          | 3.95     | 19.42     |
| POL with k-decay (ours) | 3.59↑0.36 | 18.43↑0.99 |

Figure 4. Error rates (%, single-crop testing) on ImageNet validation with model ResNet-50. (a) is the learning rate curve of the new liner polynomial decay function of different $k$. (b) is the error rate curve zoomed in from 80 to 90.

Results on CIFAR Datasets The main results of tests on CIFAR-10 and CIFAR-100 are shown in Table 2. The best state-of-the-art results are marked in bold. Compared with the Multi-Step Decay baseline method, the accuracy of POL with k-decay is improved by 0.58% on the CIFAR-10 and 2.07% on the CIFAR-100. Comparing with the recent work SGDR, our results (POL with k-decay) are better than 0.44% on CIFAR-10 and 1.15% on CIFAR-100. Comparing with the CLR, our results (POL with k-decay) are better than 0.44% on CIFAR-10 and 1.15% on CIFAR-100. Compared to themselves (EXP, COS and POL), our method is better than the original function, can improve up to 1.34% on CIFAR-10 and 3.15% on CIFAR-100. But there is a performance drop in COS with k-decay on CIFAR-10, we think this because the k-decay term derived on POL does not apply to COS, the derivative of POL is a monotonic function, COS is not, we can also see that its performance improvement on POL is more obvious.

Results on ImageNet Datasets In Fig. 4 we employ the POL with k-decay to train the ResNet-50 model on the ImageNet datasets. The result indicates that the model accuracy is improved by 1.25%, when we set $k = 1.5$ than original method ($k = 1$). And training the ResNet-50 with Step Decay is 24.3% [1], and ours method is better than 1.19%. In Table 2 we also compared with recent works SGDR, AutoLRS and CLR, the accuracy improved by 1.03%, 0.96% and 0.95%.

5. Discussion

Impact of $k$ on Performance Fig 5 shows the relation between error rate and the $k$ value on residual neural networks with different depths (ResNet-47, ResNet-74, ResNet-101). The test accuracy increases with the increase of the hyperparameter $k$ (starting from $k = 1$). However, when $k > k_v$, the accuracy starts to drop. In ResNet-101, the $k_v$ is 3. In ResNet-47, $k_v$ should be 7. We can found that the threshold value $k_v$, the model will be decreasing with the increase of the model’s depth. It reflects that deeper models are more sensitive to the LR’s ROC than shallow ones. The threshold is different for different models and datasets. According to our experiments, the recommended $k$ is 1.5.

Impact of $k$ on Training According to Fig. 6, we found that $k$ can effectively affect the training process. The loss curve of large $k$ is always above the loss curve of small $k$. And also the ROC of the loss with an increase of $k$ at the later stage. Due to the ROC of the LR with the increase of $k$ on the new function. Larger $k$ makes that the ROC of the LR
at the early and intermediate stages will be smaller, which makes the ROC of the loss smaller, lead to the speed of the convergence is slowing down, resulting in a minor loss. At the later stage, the ROC of the LR will be giant, lead to the rapid convergence of the loss function to compensate for the loss initially. It is why the performance increase with $k$. However, when $k$ is too big, the intermediate stage’s loss is too low, the speed of the convergence cannot make up at the later stage. It is the reason the performance degradation at $k$ value is greater than $k_c$.

6. Conclusion

This paper proposes a new method for the learning rate schedule by using the $k$-th order derivatives to obtain a new function. A specific solution of k-decay $\eta_k(k, t)$ is derived, which can be widely used in other LR schedules. In the k-decay method, we introduce a hyper-parameter $k$ to control the LR’s ROC in the new function, which enriches the LR schedule functions. The experiments show how the accuracy improvement changes with the increase of $k$. It is proved that the k-decay method is effective and easier to use. This article studies the situation where time is not involved and finds a particular solution. For the situation when time is considered, there may even be other better forms of particular solutions for different LR schedules.

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Figure 6. The loss curves of the ResNet, Wide ResNet, DenseNet during the training period. The values of $k$ in the red line are greater than the values of $k$ in the blue line. The red line always tops the blue one in the loss curves. The red line is steeper than the blue one at the later stage because the ROC of the LR increases with $k$. 
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