Multiquark Correlations in Light Mesons and Baryons from Holographic QCD

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Abstract. A hadron’s multiquark content reflects itself in the quark composition of the interpolator with which it has maximal overlap. The AdS/CFT dictionary translates the anomalous dimension of this interpolator into a mass correction for the corresponding dual mode. Hence such bulk-mass corrections can carry holographic information on multiquark correlations. Two prominent examples are studied by implementing this robust and universal mechanism into AdS/QCD gravity duals. In the baryon sector bulk-mass corrections are used to describe systematic good (i.e. maximally attractive) diquark effects. The baryon sizes are predicted to decrease with increasing good-diquark content, and the masses of all 48 observed light-quark baryon states are reproduced with unprecedented accuracy. Our approach further provides the first holographic description of a dominant tetraquark component in the lowest-lying scalar mesons. The tetraquark ground state emerges naturally as the lightest scalar nonet whereas higher excitations become heavier than their quark–antiquark counterparts and are thus likely to dissolve into the multiparticle continuum.

Keywords: Gauge/string correspondence, baryons, light scalar mesons, diquarks, tetraquarks

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INTRODUCTION

The main objective of the AdS/QCD program [1, 2] is to describe strong-interaction and especially vacuum and hadron physics holographically by means of a gravitational dual dynamics in 5D spacetime (the “bulk”). The latter has an IR deformed AdS$_5$ geometry

$$ds^2 = g_{MN} dx^M dx^N = e^{2A(z) R^2/R^4} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

($R$ is the AdS$_5$ curvature radius, $z$ parametrizes the fifth dimension, $A(z)$ is a function of $z$ that approaches zero as $z \to 0$) and generally contains other fields (e.g. dilatons) as well. Essential long-term goals of this “bottom-up” approach are to construct an at least reasonably close approximation to the QCD dual and to supply top-down string theory approaches with specific ideas for the relevant dynamics. Over the last years, this program has made significant progress [3].

In the present talk we discuss multiquark correlations in mesons and baryons from a holographic perspective. In particular, we review a systematic and universal extension of the AdS/QCD framework which describes such correlations holographically. To motivate our approach, we recall that the bulk mode $\phi$ dual to a given hadron is associated with the gauge-theory interpolator with which it has maximal overlap. The scaling (or twist) dimension $\Delta$ of this interpolator sets the UV boundary condition [4]

$$\phi(x; z) \xrightarrow{z \to 0} \phi^{(0)}(x) \sim e^{f(\Delta m z R)}$$
where \( f \) is a known, hadron-dependent function. The condition (2) is imposed on the (at small \( z \) leading) solutions of the bulk field equations by adjusting the mass term. As a result, the scaling dimension \( \Delta (m_5 R) \) becomes a function of the 5D bulk mass \( m_5 \).

In general, there exist several gauge invariant interpolators \( \eta_i \) with equal quantum numbers and the same classical dimension \( \Delta_{cl} \). Those combine the fundamental fields in different ways but can couple to the same hadron state \( |h\rangle \). Hence the overlaps \( \langle 0 | \eta_i | h \rangle \) contain subtle information on the hadron’s structure. In particular, the field composition of the interpolator with maximal overlap is expected to most closely represent the coupling among the valence partons of the hadron. In our context it is crucial to note that such structural differences manifest themselves also in different anomalous dimensions \( \gamma_i (\mu) \) of the \( \eta_i \). The boundary condition (2) then translates the \( \gamma (\mu) \) of a given interpolator \( \eta \) into a bulk mass correction \( \Delta m_5 (z) \) for the corresponding dual modes. (The \( z \) dependence is inherited from the scale dependence of \( \gamma (\mu) \) since \( \mu \sim 1/z \).) Hence such corrections can encode multiquark correlation effects [6, 7] inside hadrons\(^2\).

The practical implementation of the above corrections into bottom-up duals is not without challenges, however. For example, it is not \textit{a priori} obvious how to assign unique hadron states to the interpolators. QCD information on anomalous dimensions of hadronic interpolators in the infrared is still scarce, furthermore. A particular challenge is the necessarily naive AdS/QCD extrapolation of the multiquark physics with its pronounced \( N_c \) dependence from large \( N_c \) to \( N_c = 3 \) (cf. Ref. [6]). Major benefits, on the other hand, are that the outlined mechanism will work in any of the current AdS/QCD duals, both non-dynamical and dynamical (i.e. backreacted), and that the results will be rather independent of the chosen dual. In the following we invoke this mechanism to describe diquark effects in the light-quark baryon spectrum [7] and tetraquark correlations inside the lightest scalar mesons [6].

### DIQUARK CORRELATIONS IN LIGHT-QUARK BARYONS

The arguably most prominent pattern in measured hadron spectra consists of (approximately) linear trajectories with universal slopes on which the square masses \( M^2 \) of excited states organize themselves as a function of both angular momentum \( L \) (or alternatively total spin \( J \)) and radial excitation level \( n \). A rather minimal “metric soft-wall” (ms) gravity dual [9] generates such trajectories in the light-quark meson and baryon spectra. In the baryon sector, in particular, it predicts

\[
M_{n,L}^{(\text{ms})2} = 4\lambda^2 \left(n + L + \frac{3}{2} \right).
\]  

While Eq. (3) describes all experimental data for the delta resonance masses within errors [10], systematic deviations remain noticeable in the nucleon sector. In Ref. [7] the latter were related to the fraction \( \kappa \) of “good” (i.e. maximally attractive) diquarks in

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1. This is borne out in perturbative-QCD calculations of \( \gamma \), e.g. for diquark operators [5].
2. To a reasonable approximation those are probably negligible in quite a few hadrons, but not in all [6]. Anomalous-dimension induced corrections in a different context were explored in Ref. [8].
the baryons’ quark-model wave function (for recent related work see e.g. [11]). These diquark correlations are expected to generate the mass correction
\[ \Delta M^2_\kappa = -2 \left( M^2_\Delta - M^2_N \right) \kappa \]
which solely depends on the resonances’ diquark content (\(\kappa = 0\) for deltas). In Ref. [7] we have analyzed whether the anomalous-dimension-based implementation of multiquark effects is able to generate such corrections in the metric soft-wall dual. We start from the two independent leading-twist nucleon interpolators
\[ \eta_{pd} = \epsilon_{abc} \left( u^T_a C d_b \right) \gamma^5 u_c, \quad \eta_{sd} = \epsilon_{abc} \left( u^T_a C \gamma d_b \right) u_c \]
of QCD with \(\Delta = 9/2\) which contain in \(\eta_{pd}\) a pseudoscalar and in \(\eta_{sd}\) a “good” scalar diquark (and for \(L > 0\) additional covariant derivatives). The interpolators (5) are expected to have enhanced overlap with nucleon states of equivalent diquark content. Hence we introduce three bulk spinor fields \(\Psi^{(\kappa)}\) dual to the linear combinations of the interpolators (5) with \(\kappa = 0, 1/4\) and \(1/2\), and thus to nucleons with the same diquark substructure. Following our above reasoning, the \(\Psi^{(\kappa)}\) are solutions of the bulk Dirac equation
\[ \left[ ie^M_A \Gamma^A \left( \partial_M + 2 \partial_M A^{(ms)} \right) - m^{(\kappa)}_5 \right] \Psi(x, z) = 0 \]
(where \(\Gamma^A\) are 5D Dirac matrices and \(e^A_M = \delta^A_M \exp \left( A^{(ms)} \text{ fünfbeins} \right)\)) in the geometry (1) with the warp factor \(A^{(ms)}(z) = \ln \left( (R/z) \left( 1 + \lambda^2 z^2 / m^{(ms)}_5 / R \right) \right)\) [9]. The bulk masses
\[ m^{(\kappa)}_5 = m^{(ms)}_5 + \Delta m^{(\kappa)}_5 = \frac{L + \Delta m^{(\kappa)}_5 R + 1}{R}, \quad \kappa \in \{0, 1/4, 1/2\} \]
originate from \(m^{(ms)}_5 = (L + 1) / R\) [9], which accounts for the classical twist dimension \(\tilde{\tau} = L + 3\) of the baryon interpolators (5), and from their anomalous dimensions \(\gamma^{(\kappa)} = \Delta m^{(\kappa)}_5\). Equation (7) ensures that the chirally-odd components of \(\Psi^{(\kappa)}\) satisfy the AdS/CFT boundary condition (2). Although no QCD information on the nonperturbative \(\gamma(\mu)\) is currently available in the IR (for \(\mu < 2\) GeV [12]) one can check whether the phenomenologically successful modification (4) may be obtained from a judiciously chosen bulk-mass correction \(\Delta m^{(\kappa)}_5\). This indeed turns out to be the case [7]. In fact, with
\[ \Delta m^{(\kappa)}_5 = \frac{\Delta M^2_\kappa}{4\lambda^2 R} \]
the normalizable solutions of Eq. (6) generate the desired eigenvalue spectrum
\[ M^2_{n,L} = 4\lambda^2 \left( n + L + \frac{3}{2} \right) - 2 \left( M^2_\Delta - M^2_N \right) \kappa. \]
The lightest scalar meson nonet [13] has long been suggested to contain a dominant tetraquark component [14]. This at present arguably favored interpretation [15] requires an exceptionally strong four-quark binding. The underlying dynamics, able to push the tetraquark ground-state mass below the mass of the lightest scalar $\bar{q} q$ state, must similarly be of exceptional origin. This is reflected in the holographic description of spin-0 mesons. Indeed, a straightforward extension of the conventional approach, based on quark–antiquark interpolators [16], to $\bar{q}^2 q^2$ interpolators would result in four-quark states which are heavier, not lighter than the $\bar{q} q$ ground state. This is because the mass $m_5$ of the scalar bulk-mode solutions satisfies

$$m_5^2 R^2 = \Delta (\Delta - 4)$$

(10)

to meet the boundary condition (2). Hence the larger classical dimension $\Delta$ of interpolators with a larger quark-field content generates larger bulk-mode and meson masses.

To put these and the following considerations into an explicit dynamical context, we adopt the popular dilaton-soft-wall dual of Ref. [17] (which is based on the AdS$_5$ metric (1) with $A \equiv 0$ and a quadratic dilaton $\Phi(z) = \lambda^2 z^2$). The bulk modes dual to scalar mesons in this background solve the radial Klein-Gordon equation

$$[-\partial_z^2 + V(z)] \phi(q,z) = q^2 \phi(q,z),$$

(11)

which we have cast into Sturm-Liouville form, with the potential

$$V(z) = \left(\frac{15}{4} + m_5^2 R^2\right)\frac{1}{z^2} + \lambda^2 \left(\lambda^2 z^2 + 2\right).$$

(12)

For constant $m_5$ the solutions of this eigenvalue problem can be found analytically [6]. Using Eq. (10), the resulting discrete square-masses $M_n^2 = q_n^2$ of the $n$-th radial meson excitation follow the linear trajectories

$$M_n^2 = 4 \left(n + \frac{\Delta}{2}\right) \lambda^2$$

(13)

with universal slope $4 \lambda^2$, which indeed grow with $\Delta$ [6]. While the (up to a factor) unique quark-antiquark interpolator $J_{\bar{q} q}^{A} = \bar{q}^A t^A q^A$ with $\Delta_{\bar{q} q} = 3$ corresponds to ordinary scalar mesons, there exist several four-quark interpolators with $\Delta_{\bar{q}^2 q^2} = 6$ [15]. Those do not have an a priori unique assignment to specific meson states with a dominant tetraquark component. In our case, a suggestive choice may be $J_{\bar{q}^2 q^2}^{A} = \epsilon^{abc} \epsilon^{ade} \bar{q}^A t^A C \Gamma^A q^c t^d C \Gamma^d q^e$ which contains a good diquark and antidiquark. For our purposes it suffices, however, to specify the tetraquark interpolator’s quantum numbers, its scaling dimension and the defining property of maximal overlap with the tetraquark ground state.

As anticipated, Eq. (13) with $\Delta_{\bar{q}^2 q^2} = 6$ (i.e. no anomalous dimension) generates a substantially larger square mass $M_{\Delta=6}^2 = 2M_{\bar{q} q,0}^2$ than the $\Delta_{\bar{q} q} = 3$ ground state. Hence it misses the exceptionally strong binding required for a light tetraquark state. In view of our above discussion of the holographic multiquark representation this is not
surprising. Indeed, if the exceptional lightness of the tetraquark ground state originates from multiquark correlations (e.g. in the good diquark channel), it should be encoded in the anomalous dimension \( \gamma \) of the tetraquark interpolator which is neglected in Eq. (13).

Inclusion of this anomalous dimension generalizes Eq. (10) to the bulk mass term

\[
m_5^2(z)R^2 = [6 + \gamma(z)][2 + \gamma(z)]
\]

for the modes dual to the \( \Delta_{q^2q^2} = 6 \) interpolator. This adds the correction

\[
\Delta V(z) = \gamma(z)[\gamma(z) + 8] \frac{1}{z^2}
\]

(15)

to the potential (12) with \( m_5^2R^2 = 12 \). Due to the absence of QCD information on \( \gamma \), a physically reasonable ansatz \( \gamma(z) = -az^n + bz^k \) is then adopted and tightly constrained by consistency, stability and physics requirements. This strategy provides a range of qualitative as well as quantitative insights which we summarize in the following.

To begin with, the correction (15) is bounded by \( \Delta V(z) \geq -16/z^2 \) (for any \( \gamma \)) which prevents the collapse of the eigensolutions into the \( \text{AdS}_5 \) boundary. The bound is saturated by \( \gamma \equiv -4 \) and yields the lower bound

\[
M_{\bar{q}q^2,0} \geq M_{\Delta=2,0} = 2\lambda
\]

(16)
on the lightest tetraquark mass which Eq. (14) can generate. Moreover, for constant \( \gamma \) in the range \(-4 < \gamma < -3 \) one has \( M_{\bar{q}^2q^2,0} < M_{\bar{q}q,0} \). Tetraquark excitations with masses \( M_{\bar{q}^2q^2,n} \) at or beyond the \( M_{\bar{q}q,n} \) (for \( n > 0 \)) require a suitably running \( \gamma(z) \), however, as introduced with the above ansatz. The latter encodes an exceptionally large binding energy which drives \( M_{\bar{q}^2q^2,0} \) from \( \sim 40\% \) above down to \( \sim 20\% \) below\(^4\) the \( \bar{q}q \) ground-state mass \( M_{\bar{q}q,0} = \sqrt{6}\lambda \). Around \( n \gtrsim 2 \), on the other hand, the tetraquark masses \( M_{\bar{q}^2q^2,n} \) start to exceed the corresponding \( M_{\bar{q}q,n} \). Hence these excitations should become broad enough to prevent supernumeral scalar states. The above binding mechanism will work similarly in other AdS/QCD duals, furthermore, since \( \gamma \) enters the bulk dynamics exclusively through the mass term (14) which is model-independently fixed by the AdS/CFT dictionary and generates the universal correction (15) in any AdS/QCD dual.

**SUMMARY AND CONCLUSIONS**

We have analyzed the holographic representation of multiquark correlations in hadrons. Our approach is rooted in the observation that the multiquark content of a given hadron reflects itself in the quark composition of the QCD interpolator with which it has maximal overlap. Information on this substructure is encoded in a gauge-invariant and

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\(^3\) Anomalous dimensions of a qualitatively similar \( z \) dependence (in the region of interest) emerge in dual backgrounds of holographic RG-flow type [18].

\(^4\) Hence the phenomenological ratio \( m_{\bar{q}^2q^2,0}/m_{\bar{q}q,0} \sim 0.8/1.5 \) is not quantitatively reproduced, perhaps due to the neglect of the anomalous dimension of the \( \bar{q}q \) interpolator or couplings to additional bulk fields.
therefore holographically active way in the interpolator’s anomalous dimension which the AdS/CFT dictionary translates into a multiquark-content-dependent mass correction for the dual bulk modes. The typically gauge-dependent multiquark correlations thereby leave a gauge-invariant imprint on the holographic description.

The implementation of this robust and generic mechanism into two AdS/QCD duals has yielded the first holographic evidence for important multiquark correlations in hadrons. To start with, it reveals a strikingly systematic role of the good-diquark fraction $\kappa$ in baryon spectroscopy and predicts that the size of light-quark baryons decreases with growing $\kappa$. Moreover, it reproduces the masses of all 48 observed nucleon and $\Delta$ resonances with just one scale parameter and better accuracy than e.g. any quark model. In the light scalar meson sector, furthermore, we find holographic evidence for an exceptionally large four-quark binding energy and thus for the lightest scalar mesons to consist (mostly) of tetraquarks. Higher tetraquark excitations can become heavy enough, on the other hand, to prevent more low-lying scalar resonances than experimentally seen.

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