Paper

Behavioral modeling of Class-E switching circuits via Weierstrass canonical form

Yuichi Tanji¹a) and Hiroto Kamei²

¹ Dept. of Electronics and Information Engineering, Kagawa University
  2217–20 Hayashi-cho, Takamatsu, Kagawa 761–0396, Japan

² Mitsubishi Electric Building Techno-Service
  Co., Ltd., Tokyo, Japan

a) tanji@eng.kagawa-u.ac.jp

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Abstract: With the behavioral models of Class-E switching-mode circuits, we can simulate the steady state behaviors of original circuits efficiently. However, if the dynamical systems, which approximate the behaviors of the original circuits, include impulse modes, the behavioral models cannot be easily obtained. In this paper, we show that the behavioral models are given by using Weierstrass canonical form, even if impulse modes happen. Since the proposed method is quite general, this procedures would be widely applied to behavioral modeling of various kinds of power converters. We demonstrate effectiveness of the proposed models with some examples.

Key Words: descriptor system, impulse mode, steady-state response, resonant power converter

1. Introduction

The Class-E switching-mode circuits have become increasingly valuable components in many applications such as radio transmitters, switching mode-DC power supplies, and medical devices. Because of the Class-E switching, namely, both zero-voltage switching (ZVS) and zero-derivative switching (ZDS), the Class-E switching circuits can achieve high power-conversion efficiency at high frequencies. However, the design of the Class-E circuits (amplifiers) is difficult because two switching conditions should be satisfied simultaneously in the steady state of the circuits.

Since invention of Class-E amplifier, many analytical descriptions have been presented [1–7]. Early designs assumed ideal switch, infinite output network Q, and RF choke in the DC supply [1, 2]. Later work allowed finite output Q [3–5], drain current fall time [5], and nonlinear parasitic capacitance on the active device [6, 7]. Although these treatments give useful guidance in the design of Class-E amplifiers, the behaviors in the steady state cannot be fully known.

To analyze the behaviors of Class-E amplifiers, general-purpose circuit simulators such as SPICE are useful. The steady state behaviors of nonlinear circuits can also be analyzed by using a family of SPICE. However, simulations need to be run repeatedly to know the performances such as power conversion efficiency, ripple, and ZVS or ZDS condition depending on the design parameters. Moreover, direct use of such simulators is not suitable for design schemes based on an optimization algorithm [8,
Therefore, an effective model that simulates the steady state behavior of Class-E amplifier is required. The behavioral models [12, 13] fulfill this purpose. The behavioral models are obtained via the simplified circuits that are given by replacing MOSFET including in a Class-E amplifier with an ideal switch. Then, the dynamics of the simplified circuits can be analyzed by using knowledge of control community [12, 13]. Moreover, in [13], the simplified circuit is formulated by modified nodal analysis (MNA) [14] that makes the formulation easy for any modification of circuit configuration. Then, using singular value decomposition (SVD), the behavioral model is obtained.

A system represented by MNA equations is known as a descriptor system in control community [15]. The response may include impulses, which are called impulse modes. If an impulse mode happens, we cannot apply SVD to the analysis of the system. Even worse, the conventional numerical integration cannot be always applied to the system [18]. Unfortunately, we find that an impulse mode happens in the simplified circuit of Class-E amplifier, which means that SVD based behavioral modeling [13] is not complete.

In this paper, the behavioral modeling of Class-E switching-mode circuits is presented, where the simplified circuits have impulse modes. The proposed method is based on Weierstrass canonical form [19] which expresses a descriptor system into invariant subspaces corresponding to finite and infinite eigen values associated with the generalized eigen value problem. Using the canonical form, we can express the time domain response separating from impulses. Even if impulse modes happen in a system, the system can be simulated stably. Hence, we can provide the behavioral models of Class-E amplifiers using Weierstrass canonical form. Although we focus on Class-E amplifiers, the proposed method can be applied to other power converters. Since replacing MOSFET with an ideal switch is widely used for analyzing power converters, the proposed method would be also useful for these cases.

This paper is organized as follows. In Sect. 2, we review the behavioral modeling of Class-E switching-mode circuits. In Sect. 3, Weierstrass canonical form is provided. In Sect. 4, the behavioral models using Weierstrass canonical form are given, where two input waveforms are considered. In Sect. 5, we provide some illustrative examples. Section 6 is conclusions.

2. Class-E switching-mode circuits

The basic circuit topology of Class-E switching-mode circuit (amplifier) is shown in Fig. 1. The circuit consists of DC-supply voltage $V_D$, DC-feed inductor $L_C$, n-channel MOSFET $S$, shunt capacitor $C_S$, and series resonant circuit composed of inductor $L_0$, capacitor $C_0$, and output resistor $R$. When the input voltage $v_G$ is a pulse waveform, the DC voltage $V_D$ is converted to the AC voltage over the output resistor. On the other hand, when the input voltage $v_G$ has a sinusoidal waveform, the voltage is converted to the AC output voltage. Therefore, DC/AC and AC/AC converters are obtained by the same structure. Additionally, if the Class-E rectifier is connected backward to a Class-E amplifier with a pulse waveform $v_G$, DC/DC converter is obtained [10].

We can use a circuit simulator to analyze Class-E amplifiers. Circuit simulator, however, is not always suitable for every purpose. For example, circuit simulator is expensive for optimization of passive elements [8, 9] which requires repeated simulations to evaluate fitness of design variables. Monte Carlo simulation using circuit simulator is also expensive [11]. For these cases, we need an efficient model of Class-E amplifier. Hence, MOSFET is replaced with an idealized switch, which gives the simplified circuits shown in Fig. 2. The circuit model of Fig. 2(a) is suitable for DC/AC converter. However, it is not for AC/AC converter since the input affects the on and off state of switch $S$ only and the waveform does not influence the circuit behavior completely. The model of Fig. 2(b) is suitable for both converters. The input pulse or sinusoidal waveform can be reflected for the circuit via gate-to-drain capacitance.

MNA formulation is the most flexible for design automation tools. Hence, circuits are expressed by MNA equations in almost circuit simulators. We also use MNA formulation to represent the circuits of Fig. 2 as follows:

$$
\begin{bmatrix}
G & FT \\
-F & 0
\end{bmatrix}
\begin{bmatrix}
v(t) \\
i(t)
\end{bmatrix}
+ \begin{bmatrix}
C & 0 \\
0 & L
\end{bmatrix}
\frac{d}{dt}
\begin{bmatrix}
v(t) \\
i(t)
\end{bmatrix}
= \begin{bmatrix}
J(t) \\
E(t)
\end{bmatrix},
$$

(1)
where \( v(t), i(t), J(t), \) and \( E(t) \) are vectors of node voltages, currents flowing through inductors or independent voltage sources, currents of independent current sources, and voltages of independent voltage sources, respectively. \( G, C, L, \) and \( F \) are conductance, capacitance, inductance, and incident matrices, respectively.

In control community, the system (1) is known as a descriptor system [15]. Following notation of control community, we rewrite (1) into

\[
\dot{\tilde{E}} \ddot{\tilde{y}}(t) = \tilde{A}\tilde{y}(t) + \tilde{B}u(t),
\]

(2)

where dimensions of the variables are defined as \( \dot{\tilde{y}}(t) \in \mathbb{R}^n \) and \( u(t) \in \mathbb{R}^m \). Since the matrix \( \tilde{E} \) is singular, (2) cannot be converted into a set of ordinary differential equations directly. Thus, the matrix \( \tilde{E} \) is decomposed by SVD as

\[
\tilde{E} = V \begin{bmatrix} E_{11} & 0 \\ 0 & 0 \end{bmatrix} V^T,
\]

(3)
where the rank of $\tilde{E}$ is assumed to be $r$, and $V$ is the orthonormal matrix. Since the matrix $\tilde{E}$ is symmetric, it is expressed in the form of congruence transform.

Using the linear transform $\tilde{y} = Vy$, (2) is converted into

$$
\begin{bmatrix}
E_{11} & 0 \\
0 & 0
\end{bmatrix} \frac{d}{dt} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\
A_{12} & A_{22} \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t),
$$

(4)

where $y = [y_1^T, y_2^T]^T$ and $\tilde{y} = [\tilde{y}_1^T, \tilde{y}_2^T]^T$. Dimensions of $y_1$ and $y_2$ are identical to $\tilde{y}_1$ and $\tilde{y}_2$, respectively.

Assume that $A_{22}$ is invertible. Then, the descriptor system (4) is rewritten by

$$
\begin{align*}
\frac{dy_1(t)}{dt} &= E_{11}^{-1} (A_{11} - A_{12}A_{22}^{-1}A_{21}) y_1(t) + E_{11}^{-1} (B_1 - A_{12}A_{22}^{-1}B_2) u(t), \\
y_2(t) &= -A_{22}^{-1}A_{21} y_1(t) - A_{22}^{-1}B_2 u(t).
\end{align*}
$$

(5)

(6)

However, we can not apply the linear transform $\tilde{y} = Vy$, unless $A_{22}$ is invertible. Then, an impulse mode happens and the time domain response might include impulses. In this case, the conventional numerical integration formula is not always applied to analyzing the system [18], which means that the conventional SPICE like simulators can not also be used for analyzing the simplified circuit. In this paper, we present a method for analyzing the systems with impulse modes and finding the steady-state solutions. Impulse modes may happen for any simplified circuits to power converters, because replacing MOSFET with an ideal switch is commonly used. Although we focus on Class-E amplifiers, this method can be widely applied to analyzing other circuit structures.

### 3. Canonical form

The solution of (2) with impulse modes is obtained via Weierstrass canonical form:

$$
W \dot{E} T = \begin{bmatrix} I_d & 0 \\
0 & N \end{bmatrix}, \quad W \dot{A} T = \begin{bmatrix} \Lambda & 0 \\
0 & I_{n-d} \end{bmatrix},
$$

(7)

In (7), $W$ and $T$ are transfer matrices, and $\Lambda$ is Jordan form composed of finite eigenvalues. The degree of characteristics polynomial is assumed to be $d$, and $\Lambda$ is $d \times d$ matrix. $N$ is nilpotent, the eigen values of which are all zero. $I_l$ is $l \times l$ identity matrix. Weierstrass canonical form is numerically calculated via QZ transform to the matrices $\tilde{E}$ and $\tilde{A}$ and solving Sylvester equations [19].

Using (7), we can rewrite (2) into

$$
\begin{align*}
\dot{x}_s(t) &= \Lambda x_s(t) + B_s u(t), \\
N \dot{x}_f(t) &= x_f(t) + B_f u(t),
\end{align*}
$$

(8)

(9)

where

$$
T^{-1} x(t) = \begin{bmatrix} x_s(t) \\ x_f(t) \end{bmatrix}, \quad W \tilde{B} = \begin{bmatrix} B_s \\ B_f \end{bmatrix}.
$$

(10)

Equations (8) and (9) are associated with finite eigenvalues of $(E, A)$ and infinite ones, respectively.

The general solutions of (8) and (9) are respectively written by

$$
\begin{align*}
x_s(t) &= e^{\Lambda(t-t_0)} x_s(t_0-0) + f_s(t), \\
x_f(t) &= -B_f u(t) - \sum_{i=1}^{\mu-1} \left( N^i \delta^{(i-1)}(t-t_0) x_f(t_0-0) + N^i B_f u^{(i)}(t) \right),
\end{align*}
$$

(11)

(12)

where $e^{\Lambda t}$ is the matrix exponential of $e^{\Lambda t}$ and $\mu$ satisfies $N^\mu = 0$. The function $f_s(t)$ depends on the input $u(t)$ and is shown for DC/AC and AC/AC converters in the next section. The time-domain response of (2) has impulses unless ker $N = x_f(t_0-0)$. Moreover, if derivatives of the input $u(t)$ include impulses, the response has also impulses.

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Since the circuits of Fig. 2 are composed of passive elements except for independent voltage sources, passivity of the networks is guaranteed, and then $\mu$ is at most 2 [20]. Hence, the solutions (2) are rewritten by

$$x(t) = \alpha(t)x(t_{0-0}) + \beta(t),$$

(13)

where

$$\alpha(t) = T \begin{bmatrix} e^{\Lambda(t-t_0)} & 0 \\ 0 & -N\delta(t-t_0) \end{bmatrix} T^{-1},$$

$$\beta(t) = T \begin{bmatrix} f_s(t) \\ f_f(t) \end{bmatrix},$$

$$f_f(t) = -B_f u(t) - NB_f u(1)(t).$$

We need to find the steady state solutions to make the behavioral models of Class-E amplifier. In the next section, we provide them using the expression (13).

4. Behavioral modeling

The time-domain responses of simplified circuits of Figs. 2(a) and 2(b) are expressed in a closed form. First, a pulse waveform is given as a driving input voltage source $v_G$ of Fig. 1. An example of waveform is shown in Fig. 3(a). For the on state of switch S in Fig. 2(a), the input $u(t)$ is expressed as $u(t) = [V_D, V_G]^T 1(t - t_0)$, where 1(t) is unit step function, and $V_D$ and $V_G$ are amplitude of DC voltage source and the on state of driving pulse voltage one, respectively. The function $f_s(t)$ of (13) on the on state is written by

$$f_s(t) = -e^{\Lambda t} \Lambda^{-1} \left(e^{\Lambda t} - e^{\Lambda t_0}\right) B_s \begin{bmatrix} V_D \\ V_G \end{bmatrix},$$

(14)

$$= -\Lambda^{-1} \left(I_d - e^{\Lambda(t-t_0)}\right) B_s \begin{bmatrix} V_D \\ V_G \end{bmatrix}.$$  

(15)

It should be noted that (14) cannot be evaluated within finite precision arithmetic operation, since the term $e^{\Lambda t}$ grows exponentially. In (15), the commutative property $e^{\Lambda t} \Lambda^{-1} = \Lambda^{-1} e^{\Lambda t}$ is used. On the other hand, the function $f_f(t)$ of (13) is expressed by

$$f_f(t) = -(B_f + NB_f \delta(t-t_0)) \begin{bmatrix} V_D \\ V_G \end{bmatrix}.$$  

(16)

It should be noted that since the time-domain response is not related to impulse functions in (13) except at $t = t_0$, it can be evaluated except at $t = t_0$. To obtain the response of the off state, $t_0$ is replaced with $t_1$ in (15) and (16), and $[V_D V_G]^T = [V_D 0]^T$.

When a sinusoidal waveform with DC bias is given as shown in Fig. 3(b), the input is expressed as $u(t) = [V_D V_{th}]^T 1(t - t_0) + [0 V_G]^T \sin \omega t$. Then, the functions $f_s(t)$ and $f_f(t)$ of (13) on the on state are written by

Fig. 3. Driving input waveforms for MOSFET switch. (a) Pulse input. (b) Sinusoidal input with DC bias.
\[ f_s(t) = -\Lambda^{-1} \left( I_d - e^{\Lambda(t-t_0)} \right) B_s \begin{bmatrix} V_D \\ V_{th} \end{bmatrix} - \left( I_d + \omega^2 \Lambda^{-2} \right)^{-1} \left\{ \Lambda^{-1} \left( I_d \sin \omega t - e^{\Lambda(t-t_0)} \sin \omega t_0 \right) + \omega \Lambda^{-2} \left( I_d \cos \omega t - e^{\Lambda(t-t_0)} \cos \omega t_0 \right) \right\} B_s \begin{bmatrix} 0 \\ V_G \end{bmatrix}, \]  
\[ f_f(t) = -(B_f + NB_f\delta(t-t_0)) \begin{bmatrix} V_D \\ V_{th} \end{bmatrix} - (B_f \sin \omega t + NB_f \omega \cos \omega t) \begin{bmatrix} 0 \\ V_G \end{bmatrix}. \]  
(17)

For the off state, \( t_0 \) is replaced with \( t_1 \) in (17) and (18).

For Class-E amplifiers, the following switching conditions must be satisfied on the steady state

\[ v_s(t_0) = 0, \]  
\[ \frac{dv_s}{dt} \bigg|_{t=t_0} = 0, \]  
(19)

Therefore, we need to provide the behavioral models in the steady state. The steady state response is obtained by finding the initial values which give the steady state response. Once the initial condition is found, the response is obtained by (13).

For the on state of switch \( S \), the solution of (13) \( (t_{0+0} \leq t \leq t_{1-0}) \) is written by

\[ x_1(t) = \alpha_1(t)x_1(t_{0-0}) + \beta_1(t). \]  
(21)

For the off state of switch \( S \), the solution of (13) \( (t_{1+0} \leq t \leq t_{2-0}) \) is written by

\[ x_2(t) = \alpha_2(t)x_2(t_{1-0}) + \beta_2(t). \]  
(22)

Therefore, the following time stepping algorithm can be applicable in order to find the initial values \( X \), where \( c_1 \) is tolerance for convergence and \( c_2 \) is maximum time stepping.

Time Stepping( )

\{ 
\[ X_p = X = 0; \]  
\[ i = 0; \]
\[ \text{while True do} \]
\[ X = \alpha_1(t_1)X + \beta_1(t_1); \]
\[ X = \alpha_2(t_2)X + \beta_2(t_2); \]
\[ \text{if } |X_p - X| < c_1 \text{ or } i > c_2 \text{ then} \]
\[ \text{break;} \]
\[ X_p = X; \]
\[ i = i + 1; \]
\[ \text{end while} \]
\}

However, Class-E amplifier has a resonant circuit, which means that transition time until it reaches the steady state is very long. Hence, the time stepping algorithm is not always efficient. Instead of the timing stepping algorithm, we can find the initial condition directly. As the boundary condition at \( t = t_{1-0} \), the following relation is given:

\[ x_2(t_{1-0}) = x_1(t_1). \]  
(23)

Moreover, the steady-state condition is given by

\[ x_1(t_{0-0}) = x_2(t_2). \]  
(24)
Using (23) and (24), we can write the initial condition as follows:

\[ x_1(t_{0-0}) = (I - \alpha_2(t_2)\alpha_1(t_1))^{-1}(\alpha_2(t_2)\beta_1(t_1) + \beta_2(t_2)). \]  

(25)

However, if \( V_{th} - V_G > 0 \), the Class-E amplifier operates on the on state only. The initial values then are obtained by

\[ x_1(t_{0-0}) = (I - \alpha_1(t_2))^{-1} \beta_1(t_2). \]  

(26)

If \( V_{th} + V_G < 0 \), the circuit is only on the off state, and the initial values are given by

\[ x_2(t_{0-0}) = (I - \alpha_2(t_2))^{-1} \beta_2(t_2). \]  

(27)

It should be noted that the steady state response is independent of impulse when the switch is on a state only.

5. Results

Some illustrative examples are provided in this section. The behavioral models for all the examples were calculated by MATLAB R2008b and evaluated on MacBook Pro with 2.3 GHz Intel Core i5 processor and 8 GB 1333 MHz DDR3 memory.

5.1 No impulse mode

We calculated the initial values (25) that give the steady state response for the model of Fig. 2(a). The parameters were given as \( f = 1\text{MHz} \), \( V_D = 5\text{V} \), \( R = 5\Omega \), \( L_C = 7.96\text{mH} \), \( L_0 = 7.96\mu\text{H} \), \( C_S = 5\text{nF} \), \( C_0 = 3\text{nF} \), and \( r_S = 0.16\Omega \). In this case, impulse mode does not happen. Thus, we can also apply SVD based method for fining the initial values [13]. Table I shows the initial values of \( v_S \) obtained by SVD based method and (25) via Weierstrass canonical form. The value at 0s is identical to one at 1.0μs. Thus, the steady state response is certainly obtained. The result of (25) completely matches to SVD based method, which validates the proposed method based on Weierstrass canonical form.

5.2 Pulse input

We calculated the steady state response of the model of Fig. 2(b) with \( f = 1\text{MHz} \), \( V_D = 5\text{V} \), \( R = 5\Omega \), \( L_C = 7.96\text{mH} \), \( L_0 = 7.96\mu\text{H} \), \( C_S = 6.19\text{nF} \), \( C_0 = 3.63\text{nF} \), \( C_{GD} = 0.178\text{nF} \), and \( r_S = 0.16\Omega \). For a comparison, the time stepping algorithm and the method based on Weierstrass canonical form provided in Sect. 4 were applied to finding the initial values that give the steady state response. Table II shows the initial values obtained by each method. For the time stepping algorithm, \( c_1 = 10^{-3} \) was given. To find the initial values, 3,234 cycles were necessary. Though it requires many cycles, precision is not compatible to the method based on Weierstrass canonical form. The computation time

| Table I. Initial values obtained by SVD based method [13] and (25) (Weierstrass). |
|---|---|---|
| time [μs] | SVD [V] | Weierstrass [V] |
| 0.0 | 25.90572 | 25.90572 |
| 1.0 | 25.39155 | 25.39155 |
| 2.0 | 30.48607 | 30.48607 |
| 3.0 | 28.90265 | 28.90265 |
| 4.0 | 21.68077 | 21.68077 |
| 5.0 | 12.00232 | 12.00232 |
| 6.0 | 90.91374 | 90.91374 |
| 7.0 | 87.85758 | 87.85758 |
| 8.0 | 67.89935 | 67.89935 |
| 9.0 | 11.26652 | 11.26652 |
| 1.0 | 25.90572 | 25.90572 |
Table II. Initial values obtained by SVD based method [13] and (25) (Weierstrass).

| variable | Weierstrass [V] | Time Stepping [V] |
|----------|-----------------|-------------------|
| $v_1$    | $5$             | $5.0$             |
| $v_2$    | $2.2383 \times 10^{-1} + j9.5071 \times 10^{-12}$ | $2.2040 \times 10^{-1} + j9.3090 \times 10^{-12}$ |
| $v_3$    | $-4.1211 \times 10^{-1} - j7.9165 \times 10^{-12}$ | $-4.0389 \times 10^{-1} - j5.4791 \times 10^{-12}$ |
| $v_4$    | $2.7110 - j1.3917 \times 10^{-12}$ | $2.6576 - j1.5124 \times 10^{-12}$ |
| $v_5$    | $0$             | $0$               |
| $i_D$    | $-5.1766 \times 10^{-1} - j3.6407 \times 10^{-13}$ | $-5.0738 \times 10^{-1} - j3.2832 \times 10^{-13}$ |
| $i_C$    | $5.1766 \times 10^{-1} + j3.6407 \times 10^{-13}$ | $5.0738 \times 10^{-1} + j3.2832 \times 10^{-13}$ |
| $i_0$    | $5.4220 \times 10^{-1} - j2.7830 \times 10^{-13}$ | $5.3151 \times 10^{-1} - j3.0244 \times 10^{-13}$ |
| $i_G$    | $-6.8599 \times 10^{-4} + j1.5415 \times 10^{-14}$ | $-6.7450 \times 10^{-4} + j1.5141 \times 10^{-14}$ |

Fig. 4. Steady state response of the circuit shown in Fig. 2(b).

for the time stepping method was 2.31 seconds, whereas direct finding (25) required 0.019 seconds. The method based on Weierstrass canonical form is much more efficient than the time stepping algorithm.

Figures 4 shows the time domain response with the initial values (25). It should be noted that impulses are generated at 0, 0.5μs, 1.0μs, 1.5μs, and 2.0μs. Since the response during the first cycle is identical to the second, the circuit reaches the steady state certainly.

5.3 Sinusoidal input

Finally, we calculated the steady state response of the model of Fig. 2(b) with sinusoidal waveform, where the parameters were given as $f = 1$MHz, $V_D = 5$V, $R = 5\Omega$, $L_C = 7.96 \mu$H, $L_0 = 7.96 \mu$H, $C_S = 6nF$, $C_0 = 3nF$, $C_{GD} = 0.178nF$, and $r_s = 0.16 \Omega$. Figure 5 shows the steady state response when the driving input source $u(t) = 3.2 + 6\sin \omega t$ was given. On the other hand, Figs. 6 and 7 are the steady state responses for the inputs of $u(t) = 8 + 6\sin \omega t$ and $u(t) = -8 + 6\sin \omega t$, respectively. Figure 6 is the response for the circuit with on state only, and Fig. 7 is off state only. Amplitude of sinusoidal waveforms of Figs. 6 and 7 is extremely small. These circuits do not behave appropriately as a Class-E amplifier. On the other hand, the circuit with both states converts the input to an AC waveform with appropriate amplitude. However, the Class-E switching conditions are not satisfied as shown in Fig. 5. Thus, the repeated simulations or optimization is necessary. Fortunately, we can find the circuit satisfied with Class-E switching conditions efficiently, since the behavioral model is efficiently obtained.
Fig. 5. Steady state response of Class-E amplifier with sinusoidal waveform.

(a) 

(b)

Fig. 6. Steady state response of Class-E amplifier with sinusoidal waveform, where the switch has the on state only.

(a) 

(b)

Fig. 7. Steady state response of Class-E amplifier with sinusoidal waveform, where the switch has the off state only.
6. Conclusions

The behavioral modeling of Class-E amplifiers have been presented via Weierstrass canonical form. Even if the simplified circuits obtained by replacing the MOSFET switch with an ideal one have impulse modes, the proposed method provides a behavioral steady state model. For modeling of switching converters, replacing the switching devices with ideal ones are commonly used. Hence, impulse modes may happen in the simplified circuits. The simplified circuits cannot be always analyzed by using a conventional numerical integration, and designers then cannot know whether the circuits behave correctly or not. Although we focus on Class-E amplifiers in this paper, the proposed method would be applicable to these cases. Therefore, the proposed method is useful for modeling of switching power converters.

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