Low scale inflation with large number of e-foldings

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In this paper we illustrate an interesting example of low scale inflation with an extremely large number of e-foldings. This realization can be implemented easily in hybrid inflation model where usually inflation ends via phase transition. However this phase transition can be so prolong that there is a subsequent epoch of slow roll inflation governed by the dynamics of two fields. This second bout of inflation can even resolve the η problem which plagues certain kind of inflationary models. However we also notice that for extremely low scale inflation it is hard to obtain the right amplitude for the scalar density perturbations. In this regard we invoke alternative mechanisms for generating fluctuations. We also describe how to ameliorate the cosmological moduli problem in this context.

INTRODUCTION

Low scale inflation has its virtues and challenges \[\text{[1]}\]. The virtues are low reheating temperature, which can avoid thermal and non-thermal gravitino problems \[\text{\cite{2,3}}\], solving moduli problem by either invoking thermal inflation \[\text{\cite{4}}\] or via TeV scale scale inflation \[\text{\cite{5,6}}\]. Among the main challenges it is usually hard to realize successful baryogenesis scenario, especially if the reheat temperature comes out to be below the electroweak scale, however see Q-ball baryogenesis \[\text{\cite{7}}\], and for a review see Ref. \[\text{\cite{8}}\].

Usually low scale inflationary models do not give plenty of inflation compared to a simple chaotic type models, partly because the vevs of the fields involved can be less than \(O(M_p)\), where \(M_p = 2.4 \times 10^{19}\) GeV. Even hybrid \text{\cite{9}} or natural inflation type models \[\text{\cite{10}}\] give limited inflation, e.g. \(N_e \sim 100\), or, even less \[\text{\cite{11}}\]. Very Large number of e-foldings is not mandatory for structure formation, because the required number of e-foldings can be at most 60, but in any case it is an interesting academic curiosity that how large e-foldings of inflation can we obtain for low scale inflaton.

In this paper we will illustrate an example based on a simple hybrid inflation model \[\text{\cite{9}}\], where it is quite possible to tweak the parameters which can give an extremely large e-foldings of inflation. Usually in hybrid model inflation ends when the phase transition occurs and the transition can be either first order or second order \[\text{\cite{12}}\]. Particularly in the case of a second order phase transition it is often believed that it happens quite fast, on the contrary to the common dogma here we give an example where this phase transition occurs extremely slowly which can drive a second bout of inflation.

One of the most interesting outcome of our scenario is that we can get enough e-foldings of inflation even if we had a Hubble induced mass correction to the inflaton \[\text{\cite{13}}\]. Imagining that we did not have any significant inflation before the phase transition due to the break down of a slow roll inflation, nevertheless, we can still generate sufficiently large e-foldings during the slow phase transition.

This is the most interesting result of our paper, which could provide a resolution to many inflationary models, such as in \(N = 1\) supergravity where one commonly obtains such correction \[\text{\cite{12,14}}\]. In this paper we also illustrate that inspite of having large mass correction to the inflaton it is possible to generate almost scale invariant density perturbations, which is a compelling result. We will also address the issue of cosmological moduli problem in this paper.

THE MODEL AND ITS DYNAMICS

The dynamics of hybrid inflation models where there is a second period of inflation have previously been studied in Refs \[\text{\cite{5,6}}\], with particular emphasis on the production of large density perturbations, which may lead to the over-production of primordial black holes. The importance of a second bout of inflation was rediscovered in the context of inflationary models with large extra dimensions \[\text{\cite{15}}\], see also \[\text{\cite{16}}\]. Now we will discuss briefly the dynamics of a second bout of inflation and describe its rich applications.

For the purpose of illustration let us consider the following form of the potential

\[V(\phi, \Phi) = \lambda^2 \left( \frac{M_*}{M_p} \right)^2 \frac{\phi^2}{2} - \left( \frac{M_p}{M_*} \right)^2 \Phi^2 + g^2 \left( \frac{M_*}{M_p} \right)^2 \phi^2 \Phi^2 + \frac{1}{2} m_\phi^2 \phi^2. \quad (1)\]

where we have assumed \(\lambda, g \sim O(1)\). Here \(M_*\) is an intermediate scale. The global minimum of the potential is

\[\Phi = \sqrt{2} \left( \frac{M_p}{M_*} \right) \Phi_0, \quad \phi = 0. \quad (2)\]

If \(M_* = \Phi_0\), then the natural global minimum is \(\Phi = \sqrt{2} M_p\), and \(\phi = 0\) as before. Naturally our couplings are small and for such small couplings we will show that phase transition is indeed very slow. Furthermore if
\( \Phi_0 \gg m_\phi \) and for \( \Phi \ll (M_p \Phi_0/M_*) \) the false vacuum term in the potential dominates and the Hubble parameter remains constant with
\[
H = \sqrt{\frac{8\pi}{3} \frac{\Phi_0^2}{M_*}}. \tag{3}
\]
During the first phase of inflation when \( \phi > \phi_c \), \( \Phi = 0 \), the number of e-foldings of inflation is limited by
\[
\frac{\phi}{\phi_c} = e^{\frac{n-1}{2} N_*} \tag{4}
\]
where \( n \) is the spectral index, and it is given by
\[
n - 1 = 2\eta = \frac{2m_\phi^2}{3H^2}. \tag{5}
\]
Note that if \( m_\phi \sim O(H) \), the tilt in the spectral index will be significantly large. This is exactly what happens in the context of a Hubble induced mass correction to the inflaton in \( F \)-term supergravity inflation \([12, 14]\). In the above expression \( \phi_c \) is the critical vev which is determined by
\[
\phi_c = \frac{\lambda \Phi_0 M_p}{g M_*}. \tag{6}
\]
This is the point where usually the slow roll inflation comes to an end with a phase transition. For the time being let us concentrate upon the dynamics around this vev.

Note that the slope of the potential in the \( \phi \) direction can be written as
\[
\frac{dV}{d\phi} = \left[ 2g^2 \left( \frac{M_*}{M_p} \right)^2 \Phi^2 + m_\phi^2 \right] \phi. \tag{7}
\]
When the second term in the right hand side of the above equation dominates then the motion of \( \phi \) evolves away from \( \phi_c \). By solving the equation of motion for \( \phi \), we obtain
\[
\phi = \phi_c \exp \left[ -\frac{1}{24\pi} \frac{1}{\lambda} \left( \frac{m_\phi}{\Phi_0} \right)^2 (M_* t) \right], \tag{8}
\]
where we have now taken \( t = 0 \) when \( \phi = \phi_c \). For small vevs the quantum fluctuations in the \( \Phi \) field become important. This happens when
\[
\phi^2 < \phi_c^2 + \frac{4\pi}{3} \frac{\lambda \Phi_0^2 M_p}{M_*} \tag{9}
\]
then the Fokker-Planck equation \([17]\) is usually employed to study the dynamics of the \( \Phi \) field \([\bar{R}], [R], [15]\). It is possible to study the random walk of \( \Phi \) field by assuming that the field has a delta-function distribution at some initial time, say when \( \phi \gg \phi_c \). The average quantum diffusion per Hubble volume per Hubble time is \( \approx H/2\pi \).

It was found in Ref. \([K]\) that the typical value of the \( \Phi \) field when \( \phi = \phi_c \) is \( \Phi \sim O(10)H/2\pi \), which has also been numerically verified in Ref. \([13]\).

In the \( \Phi \) direction the slope of the potential is given by
\[
\frac{dV}{d\Phi} = [\lambda^2 \Phi^2 + 2g^2 (\Phi^2 - \phi_c^2)] \Phi \left( \frac{M_*}{M_p} \right)^2. \tag{10}
\]
If \( \Phi \) is sufficiently large then there is a small period where the first term dominates and \( \Phi(\hat{t}) \sim \Phi \). As \( \phi \) evolves away from \( \phi_c \), however, the second term soon comes to dominate, so that for small \( \hat{t} \), using the first order expansion of Eq. \([8]\)
\[
\phi - \phi_c \sim -\frac{1}{\sqrt{24\pi} g} \left( \frac{m_\phi}{\Phi_0} \right) \left( \frac{M_*}{M_p} \right) \Phi^2 \sim m_\phi^2, \tag{11}
\]
the \( \Phi \) field grows exponentially
\[
\Phi = \tilde{\Phi} \exp \left[ \frac{1}{12\pi} \frac{1}{\Phi_0} \left( \frac{m_\phi}{\Phi_0} \right)^2 (M_* t)^2 \right], \tag{12}
\]
where we have neglected the initial period where \( \Phi \sim \tilde{\Phi} \) because its duration is negligible compared with that of the subsequent exponential growth. During this period \( \Phi \) field grows exponentially and \( \phi \) moves slowly away from \( \phi_c \). However, once
\[
2g^2 \left( \frac{M_*}{M_p} \right)^2 \Phi^2 \sim m_\phi^2, \tag{13}
\]
the first term in Eq. \([11]\), which is growing exponentially, comes to dominate the evolution of the \( \phi \) field then \( \phi \) evolves rapidly away from \( \phi_c \). At this point \( \Phi \sim (\phi_c - \phi) \) so that \( dV/d\Phi \sim dV/d\phi \) and both \( \Phi \) and \( (\phi_c - \phi) \) grow rapidly, and inflation comes to an end shortly afterwards, with both fields subsequently oscillating about the global minimum of the potential. This is the end of the slow phase transition and also the era of the end of inflation. The duration of the second phase of inflation, \( \Delta t \), lasts \([13]\)
\[
\Delta t \approx \sqrt{\frac{6\pi \Phi_0}{m_\phi M_*}} \left\{ \ln \left[ \frac{1}{2g^2} \left( \frac{M_p}{M_*} \right)^2 \left( \frac{m_\phi}{\Phi_0} \right)^2 \right] \right\}^{1/2}. \tag{14}
\]
Since the Hubble parameter remains constant until very close to the end of the second phase of inflation, the estimate of the total number of e-foldings can be given by \([12]\)
\[
N_e \approx H \Delta t = 4\pi \lambda \Phi_0^3 \frac{M_p^2}{M_*^2 m_\phi} \times \left\{ \ln \left[ 2\sqrt{\pi} \left( \frac{M_p}{M_*} \right)^2 \left( \frac{m_\phi}{\Phi_0} \right)^2 \right] \right\}^{1/2}. \tag{15}
\]
Let us consider some examples, for \( \Phi_0 \sim M_* \sim 10^{18} \) GeV, \( m_\phi \sim 1 \) MeV and \( \lambda \sim g \sim O(1) \), we obtain \( N_e \sim 10^{28} \).
an extremely large number of e-foldings of inflation. This reiterates the point that for $\Phi_0 \sim M_*$ the phase transition is extremely slow. However note that the Hubble expansion is fairly large, $H \sim 10^{17}$ GeV. Now suppose we consider $\Phi_0 \sim M_\ast \sim 10^6$ GeV and $m_\phi \sim 10$ GeV, for which $H \sim 10^6$ GeV, we still get $N_e \sim 10^{10}$, still quite large number of e-foldings of inflation. If we would like to stretch our parameters to $m_\phi \sim 10^{-3}$ eV and $\Phi_0 \sim M_\ast \sim 10^3$ GeV, we obtain $N_e \sim 10^{19}$ e-foldings of inflation. Now we turn our attention to the virtues of the slow phase transition.

A POSSIBLE SOLUTION TO THE $\eta \sim \mathcal{O}(1)$ PROBLEM

It is quite tempting to push this limit to solve the $\eta$ problem, which plagues $F$-term supersymmetric inflationary models and other models. In supergravity, the idea is that even for a minimal Kähler potential, the inflaton mass obtains a Hubble induced correction during and after inflation, e.g. $m_\phi \sim H$, provided the source for the inflaton potential belongs to the chiral sector. For $D$-term inflation it is possible to avoid this potential problem, see [10]. An another example for Hubble induced mass correction is if the inflaton, $\phi$, couples non-minimally to the gravity. The coupling $(\xi/2) R \phi^2$ gives rise to the inflaton mass correction, $12\xi H^2$ during inflation, without altering the rate of expansion of the Universe. Whateover be the main cause for obtaining large mass correction, the main result is that we immediately obtain a large tilt in the spectral index, see Eq. (5). The inflaton with a Hubble induced mass simply rolls down to its minimum within one Hubble time, therefore, ending slow roll inflation in usual sense.

Let us now consider our potential, Eq. (1), with $m_\phi \sim \mathcal{O}(H)$. Note that in Eq. (13), if we assume $m_\phi \sim \mathcal{O}(1)H$, then we obtain $N_e \sim \mathcal{O}(10)(\Phi_0/M_\ast)$. For instance if we select $\Phi_0 \sim 10M_\ast$, we obtain a considerable e-foldings of inflation from a slow phase transition, $N_e \sim \mathcal{O}(100)$. However we immediately note that for the above parameters our approximations leading to Eq. (13) breaks down. With the above parameters phase transition occurs actually fast and therefore the problem remains.

Nevertheless it is possible to ameliorate this situation if we allow mismatching couplings, for example if we only alter

$$g = \left(\frac{M_\ast}{M_\ast}\right),$$

in which case the coupling between the fields simply reads: $\varphi^2 \Phi^2$ in Eq. (11), and similarly all other equations are modified, importantly Eq. (12), which now reads $\Phi^2 \sim m_\phi^2/2$. If we suppose $m_\phi \sim \mathcal{O}(H)$, then we obtain a vev of $\Phi \sim (\lambda/2)(m_\phi^2/M_\ast)$, when roughly inflation ends.

Note that this vev is smaller than the global minimum for $\Phi$, see Eq. (2), as long as $\Phi_0 < (2\sqrt{2}/\lambda)M_\mu$. With the above choice of parameter, $g$, we find an equivalent expression for the number of e-foldings to be

$$N_e \approx 4\pi \frac{\Phi_0}{M_\ast} \left\{ \ln \left[ \frac{1}{2} \left( \frac{m_\phi^2}{\Phi_0^2} \right) \right] \right\}^{1/2}.$$  \hspace{1cm} (17)

We note that we can obtain $\approx 60$ e-foldings of inflation if we just assume $\Phi_0 \sim 5 \times M_\ast$. The number of e-foldings can easily increase if $M_\mu \sim \Phi_0 \gg M_\ast$. We have ensured numerically that for a wide ranging parameters $m_\phi/\Phi > \sqrt{2}$. The conclusion is robust which reiterates that indeed the phase transition is sufficiently slow to have enough e-foldings of inflation even if there is a Hubble induced mass correction to the inflaton.

WHAT HAPPENS TO THE COSMOLOGICAL MODULI PROBLEM?

The moduli problem occurs with light scalar particles having mass within a range $m_{3/2} \leq 1$ TeV, oscillating with an initial vev larger than or close to $M_\mu$, with a Planck suppressed interactions to the Standard Model degrees of freedom. The small decay rate allows the field to oscillate many times before it decays, while oscillating the equation of state mimics that of a dust and soon the coherent energy density stored in the moduli dominates the Universe, i.e. $\rho \sim H^2 M_{3/2}^2$. This is known as the moduli problem. If the moduli mass is greater than $10$ TeV then there is no moduli problem because they decay before BBN, causing no harm to the predictions of the hot big bang model. For a supersymmetric moduli if there is a low scale inflation, e.g. $H \sim \mathcal{O}(\text{TeV})$, then it is possible to dilute the moduli abundance, this was the suggestion made in Refs. [1, 12, 13].

In our case it is quite possible to tune the parameters to obtain TeV scale inflation from the slow phase transition. For an example, $M_\ast \sim \Phi_0 \sim \mathcal{O}(\text{TeV})$ and $m_\phi \sim 1$ GeV can give rise to $H \sim \text{TeV}$ and $N_e \sim 100$. However in models where gravity mediated supersymmetry breaking is allowed the natural mass scale comes out to be the electroweak scale. In such a case it is rather difficult to tune parameters $M_\ast$, $\Phi_0$ to have extremely slow phase transition with a large number of e-foldings.

Nevertheless it is possible to have intermediate scale inflation which helps diluting the moduli by decreasing their initial amplitude from $\mathcal{O}(M_\mu)$, but it actually does not solve problem, because the moduli vev changes during and after inflation. However if there is an enhanced symmetry, which allows local minimum during inflation and the true minimum of the moduli coinciding, then the moduli field can be dynamically relaxed to its minimum during inflation [18]. In this paper we will consider
moduli induced isocurvature density perturbations later on.

**DENSITY PERTURBATIONS**

The amplitude for the density perturbations towards the end of the first period of inflation, when $\phi \geq \phi_0$ and $\Phi \sim 0$, can be calculated easily

$$\delta_H \sim 8.2\lambda^2 \frac{\Phi_0^2}{M^2 \phi_0^2 M_P}. \quad (18)$$

However as we have shown that number of foldings could be fairly large if the phase transition is extremely slow, in which case the above estimation will not hold any significance. One has to invoke the dynamics of both the fields, we therefore need to employ the formula for multiple slow-rolling scalar fields [17]:

$$\delta_H^2 = \frac{1}{75\pi^2} \left( \frac{8\pi}{M_p^2} \right)^3 V^3 \left[ \left( \frac{dV}{d\phi} \right)^2 + \left( \frac{dV}{d\Phi} \right)^2 \right]^{-1}. \quad (19)$$

The scales that we are interested in leave the Hubble radius very close to the end of the second period of inflation when both fields are evolving rapidly. However note that after the beginning of the phase transition, but far from the end of inflation, $dV/d\phi \approx m^2 \phi \gg dV/d\Phi$ so that $\delta_H$ has the same value as prior to the phase transition, given by Eq. (18). The fields begin evolving more rapidly during the last couple of e-foldings or so with $dV/d\Phi$ and $dV/d\phi$, which are of the same order of magnitude, increasing significantly, so that $\delta_H$ decreases.

We can make an order of magnitude estimate of $\delta_H$ at the end of inflation, $\delta_H(\epsilon = 1)$, by pretending that only one of the fields is dynamically important and utilizing the single-field expression for $\delta_H$ in terms of the first slow-roll parameter $\epsilon_\phi \equiv (M_p^2/16\pi)(V'/V)^2$:

$$\delta_H = \frac{32}{75M_p^2} \epsilon. \quad (20)$$

Inflation ends when $\epsilon = 1$, so that

$$\delta_H(\epsilon = 1) < \frac{\lambda \Phi_0^2}{2M_\phi^2 M_P}. \quad (21)$$

We can obtain an amplitude $\delta_H \sim 10^{-5}$ for density perturbations if we assume $M_\phi \sim \Phi_0$ and $M_\phi \sim 10^{15}$ GeV, for $\lambda \sim 1$. Nevertheless we warn that the above estimation is an upper bound only. In fact we can imagine a situation where we have large number of e-foldings with small parameters, $M_\phi$, $\Phi_0$, such that we do not obtain the right amplitude, e.g. $\Phi_0 < M_\phi < \lambda \Phi_0^2$, $\sim 10^6$ GeV and $m_\phi < 10$ GeV, we obtain $\delta_H \lesssim 10^{-12}$. In such cases we will need to invoke alternative mechanisms for generating adiabatic density perturbations.

**ALTERNATIVE MECHANISMS FOR GENERATING ADIABATIC DENSITY PERTURBATIONS**

**Curvaton Scenario**

It is possible to obtain adiabatic perturbations on comoving scales larger than the size of the horizon by some other light degrees of freedom which does not necessarily belong to the inflaton sector. This is the main motive behind the curvaton scenario [20, 21, 22, 23, 24, 25]. Especially the last reference, [25], describes the curvaton scenario in a low scale inflation models. Let us briefly recall here the outline of the curvaton scenario. The perturbations in the light degrees of freedom can be regarded as isocurvature in nature. On large scales isocurvature perturbations feed the adiabatic fluctuations. The isocurvature perturbations can be converted into the adiabatic ones when the light degree of freedom decays into SM radiation. Such a conversion takes place when the contribution of the curvaton energy density $\rho$ to the total energy density in the universe grows, i.e., with the increase of

$$r = \frac{3\rho}{4\rho_\gamma + 3\rho}. \quad (22)$$

Here on $\rho_\gamma$ is the energy in the radiation bath from inflaton decay. Non-Gaussianity of the produced adiabatic perturbations requires the curvaton to contribute more than 1% to the energy density of the universe at the time of decay, that is $r_{\text{dec}} > 0.01$ [21, 26].

The curvaton potential is flat during inflation leading to a large VEV of the curvaton. The field amplitude remains fixed until the flaton mass becomes of the order of the Hubble constant, and the field starts oscillating in the potential. During this period of oscillations the curvaton energy density red shifts as non-relativistic matter $\rho_\phi \propto a^{-3}$, whereas the energy density in the radiation bath from inflaton decay red shifts as $\rho_\gamma \propto a^{-4}$. Therefore $r \propto a$ grows, and isocurvature perturbations are transformed into the adiabatic ones. This conversion ends when the curvaton comes to dominate the energy density, or if that never happens, when it decays.

We can imagine that moduli to be our curvaton. For simplicity let us suppose that there is no moduli problem, e.g. minimum of the moduli during and after inflation is the same, otherwise moduli could be heavier than 10 TeV. In these cases we can imagine that the dynamics of the moduli field can generate isocurvature density perturbations during inflation [22]. Depending on the mass of the moduli and the scale of inflation, the moduli can roll down slowly or faster during inflation. The perturbed equation of motion for the moduli field in a long wavelength regime, during inflation, is given by

$$\ddot{M}_k + 3H \dot{M}_k + m^2_{\lambda} \delta M_k = 0. \quad (23)$$
The perturbations in the moduli is Gaussian with a spectrum $P_{\delta M}^{1/2} \sim H_s/2\pi$, where $H_s = k/a$ denotes the epoch of horizon exit. The spectral tilt in the spectrum is given by

$$n_M = 2 \frac{H}{H^2} + \frac{2}{3} \frac{m_M^2}{H^2}. \quad (24)$$

Note that the spectral tilt depends on the moduli mass. If it were of order $H$, it would generate a wrong tilt in the power spectrum. For $m_M < H$, it is possible to have a flat spectrum.

When $m_M \sim H(t)$, the moduli starts oscillating. During this period the power spectrum can be evaluated by averaging over many oscillations which generates fluctuations which is $\chi^2$ in nature. The amplitude for the density perturbations is given by

$$\delta_H \sim \frac{2}{3} \frac{H_s}{\pi M} \quad (25)$$

Now there is a more freedom to accommodate the right amplitude for the density perturbations. Nevertheless if $H_s$ is sufficiently low, for instance $H \sim 10^9$ GeV, then in order to have the right amplitude the vev of moduli should be smaller compared to the Planck scale, e.g. $M < M_p$. This can be easily realized inspite of the fact that $m_M << H$, because the number of e-foldings can be fairly large and during slow rolling the moduli vev slides by $M(t) \sim M_p e^{-(2m_M^2/3H)}$.

Instead of moduli we can also involve the dynamics of MSSM flat directions 22. However in case one has to be careful with the flat direction potentials. Usually MSSM flat directions are gauge invariant monomials which obtains supersymmetry breaking contributions including non-renormalizable superpotential corrections.

It was found that only those MSSM flat directions which are lifted by $n = 7, 9$, non-renormalizable superpotential terms are the successful candidates 23.

### Fluctuating Inflaton Mass and Coupling

In our case when inflation ends after the phase transition then both $\Phi, \phi$ start oscillating in their respective minimum. However note that the inflaton, $\phi$, field obtains an effective mass term by virtue of the coupling with $\Phi$ field and vice versa. The frequency of oscillations for both the fields are determined by their effective masses at their global minimum

$$\tilde{m}_\phi = \sqrt{2} g \Phi_0, \quad \tilde{m}_\Phi = \sqrt{2} \lambda \Phi_0. \quad (26)$$

In fact we can imagine a situation where $g > \lambda$, in which case the inflaton oscillates with a larger frequency compared to the $\Phi$ field. Therefore the inflaton could decay into the Standard Model fermions at a faster rate compared to the $\Phi$ field (such couplings are certainly required for a successful reheating, for example see 27). We can then assume that $\Phi$ field is still rolling down the potential while the $\phi$ had already started oscillating and decaying. The effective inflaton mass due to the finite coupling obtains a vev dependent contribution

$$m^2_{\phi, eff} = 2 \left( \frac{g^2}{M_p^2} \right)^2 \Phi^2 + m_\phi^2. \quad (27)$$

As long as $\Phi \geq (m_\phi/2g)(M_p/M_s)$, the fluctuations in the decay rate of the inflaton can be written as

$$\frac{\delta \Gamma}{\Gamma} \sim \frac{\delta \Phi}{\Phi}. \quad (28)$$

Further imagining that if $\Phi$ field has a Gaussian fluctuation then this will be imprinted in the decay rate of the inflaton as well, e.g. $\delta \Gamma/\Gamma \sim (H_s/2\pi \Phi)$ 30, 31. The fluctuating decay rate will give rise to a fluctuation in the reheat temperature of the Universe and this will be transmitted to the energy density of the thermalized plasma

$$\frac{\delta \rho}{\rho} = - \frac{2 \delta \Gamma}{3 \Gamma} \equiv - \frac{2}{3} \frac{H_s}{2\pi \Phi}, \quad (29)$$

and the corresponding amplitude for the density perturbations will be given by

$$\delta_H \sim \frac{2}{9} \frac{H_s}{2\pi \Phi}. \quad (30)$$

The numerical factor $2/3$ appears in Eq. 29 due to the fact that the energy density of the inflaton oscillations goes as $a^{-3}$ while the radiation energy of the thermalized plasma falls as $a^{-4}$. The factor $2/9$ in Eq. 30 is due to the fact that the inflaton decays gives rise to the radiation dominated epoch. Interestingly note that $\Phi$ can take large vevs near the end of phase transition, as $\Phi \rightarrow (M_p/M_s)\Phi_0$, it is increasing probable to obtain the right amplitude for density perturbations, which requires $\Phi \sim 10^6 H_s$.

Another interesting possibility could be that the inflaton sector couples to the Standard Model sector via some Yukawa coupling $\kappa \phi HH + \phi (S/M_p) qq + \ldots$, where $H$ is the Standard Model Higgs, $q$ Standard Model quarks, which has a non-renormalizable contribution of the form

$$\kappa = \kappa_0 \left( 1 + \frac{S}{M_p} + \ldots \right), \quad (31)$$

where $S$ could be MSSM flat direction 28 or sneutrino 29. In which case the inflaton coupling $\kappa$ fluctuates by virtue of the fluctuation in $S$.

This gives rise to fluctuations in the energy density which is finally imprinted in CMB. The fluctuations in the energy density of the relativistic species is given by

$$\frac{\delta \rho}{\rho} = \frac{4}{3} \frac{\delta \kappa}{\kappa} = \frac{4}{3} \frac{\delta S}{S}. \quad (32)$$
Another useful way of imagining the coupling term $S$ in Eq. (31) as a fluctuating mass term for the inflaton. This completes our discussion on alternative ways of generating adiabatic density perturbations.

**CONCLUSION**

In this paper we argue that it is possible to obtain extremely large e-foldings of inflation at a fairly low scale. We exemplify this claim by imposing extremely slow second order phase transition. This realization can be implemented easily in hybrid inflation model where usually inflation ends via phase transition. We also show that a large Hubble induced inflaton mass is generically not a problem, because it is possible to arrange the parameters such that enough inflation can be obtained for galaxy formation during this phase transition. The scale of phase transition can be brought down to the TeV scale, which can ameliorate the cosmological moduli problem. We also discuss various alternatives of generating adiabatic density perturbations, especially we show that in hybrid model the orthogonal directions to the inflaton can be responsible for generating adiabatic density perturbations.

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