Embedded Split Ring Resonator Tunable Notch Band Filter in Microstrip Transmission Lines

FARHAD FARZAMI (Member, IEEE), SEIRAN KHALEDIAN (Member, IEEE), ALEX C. STUTTS (Student Member, IEEE), BESMA SMIDA (Senior Member, IEEE), AND DANIL0 ERRICOLO (Fellow, IEEE)

Department of Electrical and Computer Engineering, University of Illinois Chicago, Chicago, IL 60607, USA

Corresponding author: Danilo Erricolo (derric1@uic.edu)

This work was supported in part by the U.S. National Science Foundation CAREER Award 1620902, in part by the U.S. Department of Defense Award FA9550-12-1-0174, and in part by the U.S. Department of Defense Award W911NF-17-1-0525.

ABSTRACT Novel design techniques for tunable and fixed frequency notch band filters are presented and validated with experiments. A notch band filter based on an Embedded Split Ring Resonator (ESRR) in a Microstrip Transmission Line (MTL) is presented. The physical dimensions of the ESRR determine the filter resonance frequency, which, when combined with a Varactor (Varactor Loaded ESRRs or VLESRRs), provides a continuously tunable resonance frequency. Tunable notch bands centered from 1.7 GHz to 4.8 GHz are obtained with resonators having an unprecedented electric length of 0.05 \( \lambda_0 \) at the lowest frequency, corresponding to a physical length of 7 mm and a total occupied area of 8.5 mm\(^2\). Two or more ESRRs are proposed to provide a deeper single notch band, a wider notch band, and/or multi notch bands.

INDEX TERMS Embedded ring resonator (ESRR), harmonics suppression, low pass filters (LPFs), microstrip transmission line (MTL), tunable notch band, wide band antenna.

I. INTRODUCTION

This article describes a novel design method to obtain a narrow and short stop band filter in a Microstrip Transmission Line (MTL) using embedded split ring resonators (ESRRs). The ESRR can be used standalone, or it can be miniaturized by loading it with a fixed external capacitor or, if tunability is desired, with a Varactor diode. Preliminary results were presented in [1].

Filters are fundamental components in many microwave circuits [2]–[6]. In particular, an example, stop band filters (SBF) are in increasingly high demand for wide-band applications where they are used to suppress parasitic frequencies to reduce interference to and from other systems.

There are different methods to obtain a stop band filter. These include the standard half- or quarter- wavelength resonator based stop-band filters [7], others based on Ferromagnetic Resonance (FMR) absorption [8], [9], or Electromagnetic Band-Gap (EBG) structures, which have appeared in recent years [10]–[13].

Furthermore, reconfigurable filters are a key component in low profile frequency agile communication systems where different frequency bands can be selected by using a single tunable filter with embedded tuning elements. Those elements can be electrical switches such as PIN or varactor diodes, mechanical switches such as MEMS or even use variable bias voltage to change the resonance frequency of the ferromagnetic materials [14].

Each of these methods have their advantages and disadvantages. For example, an advantage of standard SBFs is that they have a well-known design procedure, while a disadvantage is the size of the resulting filter, which is particularly large at lower frequencies. Ferrite based SBFs need a magnetic bias to determine the resonance frequency of the ferromagnetic layer, resulting in an intrinsic tunable stopband without altering any physical dimensions. The main
disadvantages of FMR based SBFs include high insertion loss and extra room required for the magnetic bias. Split Ring Resonator (SRR) or Complementary SRR (CSRR) stop- band filters, which are based on EBG structures, have various shapes and can be used in a variety of transmission lines to establish a stop band. These resonators can be modeled as RLC circuits coupled to the host transmission line and with tunable frequency response obtained by loading them with varactor diodes to change the amount of capacitance of the resonators [15], [16]. Embedded SRRs (ESRRs) are non-planar versions of SRRs. The ESRRs are utilized in Substrate Integrated Waveguides (SIWs) to provide negative effective uniaxial permeability [17]–[19] and in microstrip antenna substrates to provide magneto-dielectric characteristics [20]. Negative effective uniaxial permeability is artificially produced in the material and effectively allows it to reject wave propagation at its resonant frequency. Thus, in a microstrip transmission line, the signal passing through the ESRR is attenuated at an arbitrary resonant frequency determined by the composition of the design as outlined in Section II.

In this article, we propose ESRRs to provide single-band or multi-band stop-notches. In addition, these can be either fixed or tunable and if the ESRRs’ resonance frequencies are sufficiently close, they can form a wider notch band. The advantages of the proposed design include:

1) The geometry, unlike the CSRRs implemented in [12] and [13], provides an opportunity to load the resonator with a capacitor or a varactor diode to have lower and tunable resonance frequency, respectively.

2) The geometry is not as complex as similar filters found in [21]–[27]. The inclusion of vias simplifies the design; there are fewer physical parameters to tune and the filter’s width does not exceed that of the microstrip line itself allowing for easier implementation in space constrained designs.

3) Implementation in curved or bent microstrip lines is easy since the resonators are embedded in the middle of the line and can always be excited by the magnetic field, thus providing more freedom in design procedures of a circuit and leading to further reduction in size.

4) The occupied area is significantly smaller than that required by other designs demonstrated in [23]–[30]. The width is just a small portion of the microstrip line width, and the length is a small fraction of the wavelength.

5) The tunable resonant frequency range is considerably larger than those found in designs from [23]–[31]

6) When microstrip lines loaded with ESRRs are placed closely together, there is no disruptive behavior due to mutual coupling of adjacent resonators.

7) Virtually, any microwave device using microstrip or grounded coplanar waveguide (GCPW) transmission lines could take advantage of these resonators to obtain a notch band with almost arbitrary frequency selectivity and response.

Thus, the overall impact of this work is that robust, comprehensive, and widely tunable stop band filters utilizing split ring resonators are provided in compact sizes not found in any current literature and therefore can be more easily adapted for use in practical devices involving antennas, sensors, signal processing, and more.

The transmission line model (TL model) proposed for the ESRR is accurate to compute the frequency response by circuit simulators instead of full wave software. Thus, simulation, optimization or integration of them with other parts of a circuit can be performed very fast and accurately.

This manuscript is organized as follows. Section II discusses the proposed resonators embedded in a MTL with comparisons to previous similar work. The TL model and simplified Electromagnetic Model (EM Model) are discussed. In addition, reconfigurable, multi or wide notch band are investigated in this section. In section III, the advantages of Capacitor Loaded ESRRs (CLESRRs) are described for an application to the design of a wide-band antenna with single or dual notch bands. Section IV discusses using ESRRs to improve Low Pass Filter (LPF) performance in terms of eliminating harmonics and increasing the sharpness of rejection bands. Conclusions are provided in section V.

II. MICROSTRIP TRANSMISSION LINES WITH ESRRS

A. THE PROPOSED ESRR

FIGURE 1. The proposed microstrip line with the ESRR. Dimensions are: $w_m = 1.8$ mm, $w_s = 1$ mm, $w_f = 0.3$ mm, $l_s = 8.5$ mm, $l_r = 7$ mm, $g = 0.05$ mm, $c_{via} = 0.3$ mm.

Fig. 1 shows a microstrip line with an Embedded SRR. The ESRR is coupled to the microstrip line in the following way. Initially, a slot of length $l_s$ and width $w_s$ is removed from the original microstrip line thus creating a three section transmission line. Going from left to right, there is a section with the original width $w_m$, followed by another section with two narrow and parallel transmission lines with width $(w_m - w_f)/2$, and then another section with the original width $w_m$. Then, the ESRR is symmetrically placed into the slot between the two narrow transmission lines and connected at its extremes to the ground plane with two vias.

One should notice that:

1) The width $w_f$ of the slot is determined so that the resulting defected microstrip line can be modeled as two parallel microstrip lines with characteristic impedance of 100 $\Omega$. The substrate is Rogers 4003 with dielectric permittivity constant $\varepsilon_r = 3.55$ and thickness $h = 0.813$ mm (32 mil).
2) The distance between the ESRR and the two narrow transmission lines affects the coupling level between the ESRR and the overall transmission line.

3) One advantage of the proposed design is that the length \( l_s \) of the ESRR is very short (\( l_s \approx 0.05\lambda_0 \)) at the lowest frequency.

4) Another advantage is that the ESRR may be loaded with a variable load, which results in a tunable resonance frequency for the resulting stop-band filter.

**B. ESRR TRANSMISSION LINE MODEL**

The ESRR is described using a Transmission Line (TL) model because it allows to perform very fast and accurate analyses, designs, and optimizations. This is in contrast to the RLC model, which would require additional simulations each time the geometry is modified. The frequency-domain analytical model uses the Hammerstad et al. formula to calculate the characteristic impedance \( Z_0 \) and the effective dielectric constant \( \epsilon_{\text{eff}} \) of the microstrip line [32].

To justify the use of a transmission line model, we observe that, when the ESRR is located in the slot inside the microstrip line of width \( w_m \), the magnetic field is transverse to the axis of the ESRR thus inducing resonating currents that generate an equivalent magnetic dipole moment in its loop [33], as shown in Fig. 2.

The TL model of the ESRR unit cell is shown in Fig. 3a, where the SRR strip line of width \( w_r \) and its two adjacent microstrip lines of width \((w_m - w_r)/2\) are modeled as a three-conductor transmission line [34] and the discontinuities at the junctions are not modeled for simplicity. The ESRR is modeled as one microstrip line connected in series with a capacitor, modeling the ESRR gap, and then connected in series with another microstrip line.

The dimensions of the TL model sections are exactly the same as the proposed structure. The geometrical dimensions of the ESRR affect its resonance frequency \( f_r \). Specifically, changes in length of the ring and substrate characteristics cause a shift of \( f_r \).

The frequency response of the microstrip line loaded by the ESRR and its equivalent TL model are depicted in Fig. 3b and they are in good agreement. The ESRR acts as a short-circuit at its resonance frequency \( f_r \). Specifically, changes in length of the ring and substrate characteristics cause a shift of \( f_r \).

The frequency response shown in Fig. 3b is quite symmetric around the resonance frequency and this is attributed to the symmetry of the structure in Fig. 3a. In order to explore the effect of lack of symmetry in the structure, Fig. 3c provides the frequency response when the gap is shifted by 1 mm away from its original position. We observe that the original resonance frequency is split into two new resonance frequencies \( f_{r1} \) and \( f_{r2} \). Thus, if the location of the gap changes, the longer section causes the lower resonance frequency \( f_{r1} \) and the shorter section causes the higher resonance frequency \( f_{r2} \).

By tuning the gap location of the ESRR, two different desired notch bands may be obtained. It should be noticed that, when the gap is located at the center, \( f_{r1} = f_{r2} \) and a deeper notch band is formed, as shown in Fig. 3b.

To further investigate the effect of the variation in the position of the gap, we consider the limiting case of a single-via ESRR having the same length \( l_r \) of the original ESRR unit cell. This is equivalent to moving the gap to the right in the original structure of Fig. 3a and results in the structure of Fig. 5a. Fig. 5b shows that the frequency response has a resonance at \( f_r \) and another at its odd harmonics \( 3f_r \).
C. SIMPLIFIED ESRR ELECTROMAGNETIC MODEL

The ESRR acts like an RLC resonator and the gap in the ring can be modeled as a small capacitor, \( C_{\text{gap}} \), whose associated magnetic moment is very weak. In contrast, a stronger magnetic moment is obtained by placing a capacitor (larger than \( C_{\text{gap}} \)) in the gap, thus increasing the displacement loop currents in the ESRR and causing a stronger magnetic moment. This results into the Capacitor Loaded ESRR (CLESRR), which shows very low resonance frequency compared to an ESRR of the same dimensions. The ESRR or CLESRR are excited by magnetic fields perpendicular to their surfaces. The resulting magnetic moments change the values of \( \mu_{\text{eff}}(r) \) and \( \epsilon_{\text{eff}}(r) \) of the substrate around the resonators.

The constitutive parameters may be extracted from a full wave electromagnetic model, as was done, for example, in [17]–[19], [35] and the results are shown in Fig. 6. Using the coordinate system of Fig. 6, the effective permeability (permittivity) changes along the \( x \) \((z)\) axis due to the incident magnetic (electric) field.

Thus the constitutive and loss tangent tensors are

\[
\mu_{\text{eff}} = \begin{bmatrix} \mu_{xx}(f) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}
\]

\[
\epsilon_{\text{eff}}' = \begin{bmatrix} 3.55 & 0 & 0 \\ 0 & 3.55 & 0 \\ 0 & 0 & \epsilon_{\text{eff}}''(f) \end{bmatrix} \tag{2}
\]

\[
\tan \delta_m = \begin{bmatrix} \tan \delta_m^{xx}(f) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3}
\]

\[
\tan \delta_e = \begin{bmatrix} 0.002 & 0 & 0 \\ 0 & 0.002 & 0 \\ 0 & 0 & \tan \delta_e^{zz}(f) \end{bmatrix} \tag{4}
\]

A simplified electromagnetic model of the CLESRR may be obtained by forming a structure consisting of a volume with the same dimensions of the CLESRR unit cell and material parameters \( \mu_0 \mu_{\text{eff}}(f), \epsilon_0 \epsilon_{\text{eff}}'(f), \tan \delta_m^{xx}(f) \) and \( \tan \delta_e^{zz}(f) \) corresponding to the frequency dependent values shown in Fig. 6c. The resulting behavior of the simplified structure, RLC model, and the CLESRR are shown in Fig. 6d, where it is apparent the good agreement of the prediction of the notch frequency. Since in the constitutive parameters extraction there is always a resonant behavior, the high permittivity and permeability losses cause a spurious notch in the model.

D. VARACTOR-LOADED ESRR (VLESRR)

A tunable notch band may be obtained with a varactor loaded ESRR, i.e. by replacing the constant capacitor with an adjustable capacitor, through an appropriate reverse bias. In this section, we will show also how to use VLESRRs to increase the notch depth as well as to obtain a wideband notch.

A possible configuration of a VLESRR is shown in Fig. 7, where a DC bias capacitor \( C_b \) is introduced to avoid short circuits. First, we examine the ideal lossless case to understand the benefits of the VLESRR. Referring to Fig. 8, the simulations show that it is possible to obtain a notch band deeper than \(-30\) dB. For all the values of the capacitors considered, a VLESRR with the length of 7 mm and a varactor diode with capacitance \( C_v \) ranging between 0.25 pF to 2.25 pF can provide a notch band between 1.7 GHz to 4.8 GHz. One should notice that at the lowest resonance frequency (longest wavelength), the resonator length is just about 0.05\( \lambda_0 \). The Quality factor of notch filter \( Q_n \) can be derived from [36],

\[
A_{\text{dB}} = 20 \log \left( \frac{Q_n \Delta f_{3\text{dB}}}{f_r} + 1 \right) \tag{5}
\]

where \( A_{\text{dB}}, f_{3\text{dB}}, \) and \( f_r \) represent rejection, 3 dB bandwidth, and notch frequency of the filter, respectively.

Actual varactors have losses that reduce the depth of the notch. Therefore, we introduce a series resistor of value \( r_s \) to
model the losses and the simulated results for the corresponding depth of the notch are shown in Fig. 9, where $r_s \approx 1 \Omega$ for varactor diode SMV2019 [17].

In order to increase the depth of the notch in the presence of losses, one approach consists of cascading two VLESRRs with the same resonant frequency, as it is shown in Fig. 10, where single unit and double unit resonators are compared. In this figure, for example, measurement results indicate that a notch depth below $-30$ dB is achieved with two cascaded VLESRR separated by a distance $s = 10$ mm (equivalent to $0.07\lambda_0$ at the lowest frequency), which is an improvement of at least $-17$ dB over the value of approximately $-14$ dB obtained with a single VLESRR.

In order to widen the notch band in the presence of losses, one approach consists of cascading some VLESRRs with different, but close, resonant frequencies. As an example, Fig. 11 shows measurement results for three cascaded VLESRRs and compares them with full wave simulation results. The optimal values obtained by Ansoft Designer were: resonance frequency $f_r = 2.5$ GHz, $-3$ dB and $-10$ dB bandwidth 400 MHz and 600 MHz respectively, separation distances between consecutive VLESRRs $s_1 = 7.3$ mm and $s_2 = 8.1$ mm, capacitance of middle varactor 0.9 pF and of the other two 1.1 pF and 1.2 pF.

Another approach to obtain a wider band notch is to obtain a multi-notch band by cascading many VLESRRs with different, and not close, resonant frequencies. Figs. 12a and 12b show measurement and simulation results for five cascaded VLESRRs. These require five capacitor values and four separation distance values, for a total of nine parameters to optimize. However it was assumed that the separation distances were fixed, since they cannot be changed after fabrication, unlike the values of the capacitors that can be adjusted by changing the bias. Therefore, only five values were optimized and the final values are given in Table 1. In all cases the separation distance was 7.5 mm. The results of the simulation in Ansoft Designer and HFSS are in good agreement with the measurements in both cases.

Table 2 displays a summary of results for the proposed design and previous related work found in [23]–[31]. In comparison, it is shown that the proposed filter has a larger tuning range and provides a similar rejection level but with a much smaller occupied area. Also, the complexity level of each
FIGURE 9. Variation of $S_{21}$ at the resonance frequency versus the series resistance of the VLESRR loaded microstrip line.

FIGURE 10. (a) Sample microstrip line loaded with two VLESRR. (b) Simulation and measurement results of the frequency response of the microstrip line with two VLESRR (D-VLESRR, blue and black lines) and comparison with the same microstrip line with only one VLESRR (S-VLESRR, red lines).

FIGURE 11. Frequency response of the wide band notch by a three-VLESRR loaded MTL.

FIGURE 12. Frequency response of a dual notch band at (a) 1.8 GHz and 2.4 GHz, (b) 2 GHz and 3 GHz.

III. SAMPLE APPLICATION 1: WIDE BAND ANTENNA WITH FREQUENCY BAND-NOTCH

To show the advantages of the small size and integration of an ESRR, we consider its application in a microstrip line that feeds a wideband antenna, where, for example, one design requirement is to avoid interference with a given frequency band. The ESRR is integrated in the microstrip line that feeds the antenna. Therefore, the portions of the antenna structure that are responsible for the radiation (e.g. antenna patches and ground plane) are isolated from the feed section and, as a result, the radiation pattern and the frequency response of the antenna are not affected by the ESRR, similar to what occurs with an EBG structure [37]. The advantages of ESRRs are even more evident when dual or wide band notches need to be included. In fact, in using an ESRR it is sufficient to apply the techniques of Section II.

Let us now investigate the effect of a CLESRR on the frequency response of a trapezoidal antenna, which exhibits
TABLE 1. Optimized capacitor values to obtain a wider notch band using five cascaded VLESRRs for the results shown in Figs. 12a and 12b.

| a. Dual Notch Band at $f_{n1} = 1.4$ GHz & $f_{n2} = 2.4$ GHz |
|---------------------------------------------------------------|
| Desired Frequency Ranges with $S_{11} < -20$ dB: (1.75, 1.85) & (2.35, 2.45) |
| Desired Frequency Ranges with $S_{21} < -15$ dB: (0.7, 1.75), (1.75, 2.35), & (2.45, 3) |
| Parameters  | $C_{c1}$  | $C_{c2}$  | $C_{c3}$  | $C_{c4}$  | $C_{c5}$  |
| Value (pF)  | 1.93      | 2.12      | 2.05      | 1.05      | 1.1       |
| b. Dual Notch Band at $f_{n1} = 1.8$ GHz & $f_{n2} = 2.4$ GHz |
| Desired Frequency Ranges with $S_{11} < -20$ dB: (1.95, 2.05) & (2.95, 3.05) |
| Desired Frequency Ranges with $S_{21} < -15$ dB: (0.95, 1.95), (2.05, 2.95), & (3.05, 3) |
| Parameters  | $C_{c1}$  | $C_{c2}$  | $C_{c3}$  | $C_{c4}$  | $C_{c5}$  |
| Value (pF)  | 1.35      | 1.65      | 1.6       | 0.64      | 0.65      |

a wider bandwidth than a conventional wire monopole. Referring to Fig. 13a, this planar antenna consists of a trapezoidal monopole fed by a microstrip line. The ground plane on the other side of the substrate is beneath the microstrip line and can be extended as far as the base of the trapezoid. The dimensions of the antenna are determined to provide a bandwidth from 1.8 GHz to 5.8 GHz. We investigate (i) a single notch band at 2.4 GHz and (ii) a dual notch band at 2.5 GHz and 3.5 GHz. For both cases, the structure is split in two sections, shown in Fig. 13b: the antenna’s patch, which is simulated by HFSS, and the notch band section, which is modeled as a transmission line and is simulated by Ansoft Designer. The same figure shows the separation between consecutive CLESRRs, the capacitors, and the offset from the reference plane, which are the parameters to optimize in the transmission line model to achieve a desired VSWR. In addition, since the values of the capacitors are discrete and cannot be chosen arbitrarily, the length of the CLESRRs can be optimized to compensate for them. These parameters can be tuned or optimized and the results can be obtained simultaneously.

For the single notch band, these parameters have been optimized (HFSS+Ansoft Designer) and then the frequency response of the final optimized values is validated by HFSS. The results are shown in Fig. 14a and indicate that there is a shift of about 80 MHz. The amount of shift is just about 3 % from the center frequency and proves how much this procedure is reliable. For the dual notch band, we take advantage of the information about the 80 MHz shift. Accordingly, the frequencies of the notch bands in the TL model (HFSS+Ansoft Designer) are considered at 2.58 GHz and 3.58 GHz. The results shown in Fig. 14b indicate that the shift has been compensated for and that the antenna has two notch bands at 2.5 GHz and 3.5 GHz, as desired. The optimized values are gathered in Table 3. These results indicate the efficiency of the procedure to design a trapezoidal antenna with desired notches and can be easily extended to other types of planar antennas fed by a microstrip line. In addition, antennas with tunable notch bands may be designed using the same procedure and the results of Section II, but further details are not provided for space limitations.

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IV. SAMPLE APPLICATION 2: HARMONIC SUPPRESSION AND REJECTION BAND IMPROVEMENT IN FILTERS

To show additional advantages of ESRRs, we examine their application to microwave filters, which are key components in RF front end circuits for modern wireless communications. Specifically, we consider ESRRs to improve the performance of microstrip filters by suppressing harmonics and improving rejection bands. Loading the filters with ESRRs has the main advantage of separating the harmonic suppression from the filter design, resulting into a time-saving design procedure. On the contrary, other methods for harmonic suppression, such as photonic band-gap, defected ground structure, step-impedance and the asymmetric structure, affect filter topology and make the design time-consuming [38]–[45].

Low Pass Filters (LPFs) are used to eliminate frequencies above their cutoff frequency ($f_c$). There are some classical structures to realize a LPF with a microstrip line. To understand some typical design challenges, we consider the stepped impedance prototype, consisting of a cascaded structure of a high ($Z_h$) and a low ($Z_l$) impedance microstrip line, where
TABLE 2. Comparison of our work with other published results.

| Reference | Tunable Range | Tuning Device | Typical Rejection Level | Quality Factor Range | Resonator Feed Type | Area | Complexity Level (Scale: 1 - Low, 2 - Moderate, 3 - High) |
|-----------|---------------|---------------|-------------------------|----------------------|---------------------|------|---------------------------------------------------------|
| [23]      | (0.7 to 1) GHz; 30% | Varactor Diode(s) | -25 dB | - | Microstrip | 316 mm$^2$ | 3 |
| [24]      | (2.8 to 3.4) GHz; 17.6% | Varactor Diode(s) | < -30 dB | - | Substrate Integrated Waveguide with Microstrip Phase Shifter | > 662 mm$^2$ | 3 |
| [25]      | (0.66 to 0.99) GHz; 33.3% | Varactor Diode(s) | -27 dB | - | Microstrip | > 2700 mm$^2$ | 2 |
| [26]      | (0.8 to 1.5) GHz; 46.7% | Varactor Diode(s) | < -15 dB | - | Microstrip | 420 mm$^2$ | 2 |
| [27]      | (11.3 to 16.5) GHz; 33.5% | Varactor Diode(s) | -30 dB | 4-80 (Diodes) | Microstrip | 440 mm$^2$ | 2 |
| [28]      | (4.5 to 6) GHz; 25% | Piezoelectric Transducer | < -50 dB | - | Microstrip | > 2310 mm$^2$ | 1 |
| [29]      | (3.5 to 4.1) GHz; 44.6% | Varactor Diode(s) | -30 dB | 24-35 (Filters, unloaded) | Coplanar Waveguide | > 1200 mm$^2$ | 1 |
| [30]      | (3.1 to 5.6) GHz; 44.6% | Varactor Diode(s) | < -15 dB | - | Coplanar Waveguide | 1729 mm$^2$ | 1 |
| [31]      | (1.13 to 1.94) GHz; 41.8% | Varactor Diode(s) | < -5 dB | - | Microstrip | > 70 mm$^2$ | 1 |
| Our Work  | (1.7 to 4.4) GHz; 64.6% | Varactor Diode(s) | -30 dB | - | Microstrip | 8.5 mm$^2$ | 1 |

FIGURE 14. (a) VSWR of the antenna with a notch band at 2.4 GHz, (b) dual notch band at 2.5 GHz and 3.5 GHz.

Z$_l$ < Z$_0$ < Z$_h$ (Z$_0$ is the source impedance, usually 50 Ω). The lower Z$_l$ provides a better approximation of a lumped-element capacitor, but the resulting line width should not allow for any transverse resonance to occur at the operation frequency. The higher Z$_h$ leads to a better approximation of a lumped-element inductor, but Z$_h$ cannot be too high to avoid fabrication difficulties or current-carrying capability limitations [7].

These considerations are even more restrictive in dielectric substrates with low permittivity. For instance, when Rogers 4003 with $\varepsilon_r = 3.55$ and height 0.812 mm (32 mil) is used to realize a stepped impedance LPF with $f_c = 1$ GHz, the highest impedance value cannot exceed about 120 Ω because the microstrip width will be less than 0.3 mm and using lower Z$_h$ creates unwanted harmonic near $f_c$. Therefore, to remove unwanted harmonics, the designer has to use a substrate with higher permittivity, which is more expensive and may cause the whole circuit to be split in low and high permittivity substrates.

Let us now consider a stepped impedance LPF loaded by CLESRRs and its transmission line model, which is shown in Fig. 15a and 15b. This filter was designed to have a fifth order Butterworth (Maximally Flat) response with $f_c = 1$ GHz, Z$_l = 10$ Ω and Z$_h = 100$ Ω. One CLESSR is adjusted so that its resonance frequency overlaps with the harmonic to be removed and the other CLESRR has its resonance frequency close to the filter cutoff frequency in order to form a deeper notch band neighboring $f_c$ when a sharper rejection band is needed in low order filters and it can save more room on the circuit board. The values of the parameters, such as the capacitors, are obtained with a procedure similar to the one discussed in Section II. The frequency response of

TABLE 3. Optimized parameters for the geometry of Fig. 13a to provide results for a single notch band (Fig. 14a) and a dual notch band (Fig. 14b).

| Parameters | l$_{1}$ | l$_{2}$ | c$_1$ | c$_2$ | k | s |
|------------|--------|--------|------|------|---|---|
| Value      | 7 mm   | 7 mm   | 1 pF | 1 pF | 12.5 mm | 7.5 mm |

| Parameters | l$_{1}$ | l$_{2}$ | c$_1$ | c$_2$ | k | s |
|------------|--------|--------|------|------|---|---|
| Value      | 6.5 mm | 4.6 mm | 1 pF | 0.7 pF | 4 mm | 6.5 mm |
As a final example, Fig. 16 shows the results of a design where three CLESRRs are used, which allows for more freedom in improving band rejection and harmonic suppression. The inset in the figure provides a magnification of the results around 1.2 GHz. In this case, the loaded LPF is similar to the former case except that there are two CLESRRs on the left side and one on the other side. The frequency response shows that one of the CLESRRs eliminates the unwanted harmonic and the other two create two notch-bands near the resonance frequency. This is equivalent to saying that the proposed LPF shows a very sharp rejection compared to the unloaded filter. This sharpness in rejection band without increasing the filter’s order can be interpreted as a miniaturization of the filter structure. As can be seen, there is good agreement between measurement and simulation results.

V. CONCLUSION

Embedded Split Ring Resonators (ESRRs) that can be loaded by a varactor diode (VLESRRs) or a capacitor (CLESRRs) for the purpose of providing fixed or tunable resonant frequency notch band filters are proposed. The device has an approximate tunable resonant frequency range from 1.7 GHz to 4.8 GHz with an electric length of 0.05 λ₀ corresponding to a total occupied area of 8.5 mm² at the lowest frequency. They can be used in any microwave circuit containing microstrip lines or grounded coplanar waveguides to transmit energy and attenuate desired frequencies. Furthermore, they can be combined to provide deeper single notch band, multi notch band, and wide band notch filters, or with a low pass filter to suppress undesired harmonics and improve band rejection. The discussed sample applications show improvements obtained in terms of performance and reduced time required for the design. It is found that the proposed design has distinct advantages in size, tunable frequency range, rejection levels, and low complexity in practical adaptation.

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and has been the Chair of IEEE Communication Chapter, Chicago Section, Chair for IEEE Women in Engineering, Chicago Section, from 2011 to 2013, in 2007, and the NSF CAREER Award, in 2015. She has served as the Chair for IEEE Women in Engineering, Chicago Section, from 2011 to 2013, and has been the Chair of IEEE Communication Chapter, Chicago Section, since 2019. She currently serves as an Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and the IEEE OPEN JOURNAL OF THE COMMUNICATIONS SOCIETY, and the Guest Editor for Sensors Open Access Journal. Previously, she served as an Associate Editor for the IEEE COMMUNICATION LETTERS, and a Guest Editor for special issues of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS.

INSIGHT Into Diversity Magazine’s 2015 100 Inspiring Women in STEM.

SEIRAN KHALEDIAN (Member, IEEE) received the Ph.D. degree in electrical and computer engineering from the University of Illinois Chicago (UIC), Chicago, IL, USA, in 2019. She was a member of the Networks Information Communications and Engineering Systems Laboratory, UIC, where she worked on full-duplex communication systems, analog self-interference cancellation techniques, reconfigurable antennas, and RF/microwave components. She joined Motorola Mobility, in 2018, where she is currently a Staff Antenna Engineer. She is also working on smart phone antenna design for products at the edge of technology, such as the first foldable smartphone and 5G products. She holds several patents and many published research articles in peer-reviewed IEEE journals.

ALEX C. STUTTS (Student Member, IEEE) received the B.S. degree in electrical engineering from the University of Illinois Chicago (UIC), Chicago, IL, USA, in 2020, where he is currently pursuing the Ph.D. degree in electrical and computer engineering. He is a member of the Andrew Electromagnetics Laboratory, where he works on biomedical, machine learning, and VLSI applications for electromagnetics. He currently serves as the Secretary for the IEEE Antennas and Propagation Society/Microwave Theory and Techniques Society Joint Chicago Chapter. He is a GEM Fellow and a NSF Bridge to the Doctorate Fellow. He is also a Volunteer of IEEE TryEngineering Together and International Microwave Symposium (IMS) Project Connect.

BESMA SMIDA (Senior Member, IEEE) received the M.Sc. and Ph.D. degrees from the University of Quebec (INRS), Montreal, QC, Canada. After completing her appointment as a Postdoctoral Researcher and later as a Lecturer with Harvard University, she became an Assistant Professor in electrical and computer engineering with Purdue University Northwest. She was a Research Engineer with the Technology Evolution and Standards Group, Microcell, Inc., (now Rogers Wireless), Montreal. She took part in wireless normalization committees (3GPP, T1P1). She is currently an Associate Professor in electrical and computer engineering with the University of Illinois Chicago. She was awarded the INSIGHT Into Diversity Magazine’s 2015 100 Inspiring Women in STEM. She received the Academic Gold Medal of the Governor General of Canada, in 2007, and the NSF CAREER Award, in 2015. She has served as the Chair for IEEE Women in Engineering, Chicago Section, from 2011 to 2013, and has been the Chair of IEEE Communication Chapter, Chicago Section, since 2019. She currently serves as an Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS and the IEEE OPEN JOURNAL OF THE COMMUNICATIONS SOCIETY, and the Guest Editor for Sensors Open Access Journal. Previously, she served as an Associate Editor for the IEEE COMMUNICATION LETTERS, and a Guest Editor for special issues of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS.

DANilo ERRIColo (Fellow, IEEE) received the Laurea degree of Doctor (summa cum laude) in electronics engineering from the Politecnico di Milano, Milan, Italy, in 1993, and the Ph.D. degree in electrical engineering and computer science from the University of Illinois Chicago (UIC), Chicago, IL, USA, in 1998. He is currently a Professor with the Department of Electrical and Computer Engineering, the Director of the Andrew Electromagnetics Laboratory, and an Adjunct Professor in bioengineering with UIC. In 2017, he was nominated as a University of Illinois Scholar. In summer 2009, he was an Air Force Faculty Fellow with the Air Force Research Laboratory, Wright-Patterson Air Force Base, Dayton, OH, USA. He has authored or coauthored more than 300 publications in refereed journals and international conferences. His research interests include the areas of antenna design, electromagnetic propagation and scattering, high-frequency techniques, wireless communications, electromagnetic compatibility, the computation of special functions, and magnetic resonance imaging. He was elected as a Full Member of Commissions B, C and E of the U.S. National Committee (USNC) of the International Union of Radio Science (URSI), and a Committee of the U.S. National Academies. He served as the Chair (2009–2011), the Vice Chair (2006–2008), and the Secretary of the USNC-URSI Commission E on Electromagnetic Environment and Interference (2004–2005). He was the Chair of the USNC-URSI Ernest K. Smith Student Paper Competition (2009–2014), the Vice-Chair of the Local Organizing Committee of the XXIX URSI General Assembly, Chicago, in August 2008, and a member at Large of USNC-URSI (2012–2017). He was the General Chairperson of the 2012 IEEE International Symposium on Antennas and Propagation and USNC-URSI National Radio Science Meeting, Chicago, in July 2012. He was an Elected Member of the IEEE AP-S Administrative Committee (2012–2014), the Chair of the IEEE AP-S Distinguished Lecturer Program (2015–2016), and the Chair of the Chicago Joint Chapter of the IEEE AP-S and Microwave Theory and Techniques Society (2011–2016). He has served on more than 50 conference technical program committees, chaired over 70 conference sessions, and organized more than 30 special sessions at international scientific conferences. He has been the Editor-in-Chief of the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, since August 2016.

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