A Novel Single-Loop Mechanism and the Associated Cylindrical Deployable Mechanisms

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Abstract: This paper presents a new type of 2-DOF single-loop mechanism inspired by the Sarrus mechanism, and it utilizes this mechanism to construct 2-DOF cylindrical deployable mechanisms. First, the motion pattern of the single-loop mechanism is analyzed utilizing screw theory. According to the structural symmetries, the cylindrical deployable mechanisms are constructed through the linear pattern combination and circular pattern combination of the single-loop mechanisms. After the geometrical analysis and interference condition analysis, the axial, circumferential and area magnification ratios are defined and, furthermore, applied to the parameter optimization of the deployable mechanisms, forming an example surface. Finally, a simplified 1-DOF single-loop mechanism is derived from the proposed 2-DOF mechanism, which is used to construct 1-DOF cylindrical deployable mechanisms with singular free workspaces.

Keywords: single-loop mechanism; deployable mechanism; motion pattern analysis; screw theory; magnification ratio

1. Introduction

Deployable mechanisms (DMs) are usually constructed by symmetrically assembling similar mechanism units, and they are capable of large configuration changes in specific manners. This enables DMs to be widely applied in aerospace engineering, civil engineering, medical and military structures, such as space antennas and reflectors [1–5], solar panels [6,7], mobile tents and emergency shelters [8,9], retractable rooftops [10,11], reconfigurable robots and agricultural equipment [12,13]. A large-scale DM is usually constructed by assembling modules with specific connecting approaches. The smallest module of the DM is referred to as the deployable unit (DU). Typical DUs are single-loop mechanisms, such as Bennett mechanisms [3,14], Bricard mechanisms [15], Myard mechanisms [16] and Sarrus-like mechanisms [17]. Typical connecting approaches include scissor connection [18], common part-chain connection and rigid connection [19].

In recent years, researchers have developed and analyzed various DMs that can deploy in 1–3 dimensions. The authors of [20] demonstrated a coilable mast for solar sail systems with flexible rods and cables which can be extended to a large scale in the axial direction. Ding et al. [21] developed a prism deployable mechanism based on polyhedral linkages which had half-closed polyhedron characteristics. Zirbel et al. [22] proposed a rigid-foldable array inspired by the origami flasher model. Chen et al. [14–16] developed a class of single-degree-of-freedom (DOF) DMs utilizing spatial overconstrained mechanisms and the geometry tessellation method. Pellegrino et al. [23–25] conducted a detailed discussion on the design, motion analysis and application of deployable structures. Deng et al. [26,27] investigated the synthesis approach and mobility analysis of single-loop DMs and developed a series of deployable single-loop mechanisms. Lu et al. [18] utilized scissor
mechanisms to connect Bennett mechanisms and constructed Bennett networks, which can approximate cylindrical surfaces. Agrawal et al. [28] constructed a class of deployable polyhedrons with preserved shapes based on prismatic joints. Kiper et al. [29,30] discussed the synthesis methods for constructing deployable polyhedral mechanisms. Wei and Dai [31] developed a novel 2-DOF single-looped spatial eight-bar linkage with exact straight-line motion and utilized the linkage to construct deployable platonic mechanisms with radially reciprocating motion.

A large family of DMs is constructed based on scissor mechanisms. You and Pelligrino [32] conducted geometrical analysis of scissor mechanisms and developed a class of planar and spatial DMs. Zhao et al. [33] discussed the design theory of deployable structures based on scissor mechanisms, which can guarantee zero stress during the deployment process. Bai et al. [34] synthesized a family of scaling mechanisms for geometric figures with scissor mechanisms. In addition, scissor mechanisms are often used to construct kinetic architectures and structures. Hoberman [35–37] developed a series of retractable rooftops and other architectures based on angulated scissor mechanisms. Roovers and De Temmerman [38] investigated the full geometric potential of scissor networks composed of translational units through the analysis of the general principles governing the motion and shape.

A widely used method for the construction of DMs is the tessellation method [39], which utilizes a number of mechanism units to cover the target surface symmetrically. Many deployable origami structures, such as the Miura-ori pattern [40] and Resch pattern [41], are tessellations of basic unit patterns with symmetry properties in several directions. Inspired by the tessellating origami patterns, various kinds of deployable mechanisms have been developed [42,43]. Baker studied the symmetry property of the Bricard mechanism [44,45], which enabled the Bricard mechanism to be used in deployable structures [15]. Sareh and Chen exploit the symmetry scheme to analyze the flat-foldability of the developable double-corrugated surface [46,47].

In this paper, a new type of 2-DOF single-loop DU is proposed based on the Sarrus mechanism. During the deployment of the DU, the links are always in symmetry configurations. This symmetry property enables the DU to be used in the construction of large cylindrical DMs through the tessellation method. In comparison with prior cylindrical DUs and DMs, the proposed DU and DM have the following features. During the deployment and retraction, several feature points of the DU and DM are always located on the cylindrical surface. Aside from that, the fully deployed configurations form the exact cylindrical surface. This paper is organized as follows. In Section 2, the single-loop 2-DOF DU is presented, and its motion pattern is analyzed utilizing screw theory. Through the linear pattern combination and circular pattern combination, the cylindrical DM is constructed in Section 3. Based on the geometrical analysis, the magnification ratio, which is one of the most important performance indicators of DMs, is defined in Section 4, and the area magnification ratio is optimized for an example surface through interference condition analysis. In Section 5, a simplified 1-DOF 6R DU is constructed by fixing some of the links together, which has a singular-free workspace.

2. A 2-DOF Cylindrical DM Unit

Normally, a DM is composed of many identical or similar basic units. The 2-DOF unit mechanism we propose is developed from the Sarrus mechanism. The neighboring units are connected utilizing scissor elements in two directions.

2.1. The Sarrus Mechanism

The Sarrus mechanism is a classical overconstrained single-loop mechanism [44–46]. As shown in Figure 1, it contains two planar RRR limbs whose motion planes are not parallel. The mechanism uses six revolute joints to connect the base (AF), the moving platform (CD) and four links, and it generates a translation of the moving platform relative to the base. The axis of the translation is perpendicular to the axes of all revolute joints.
Due to the unique motion feature and simple structure, the Sarrus mechanism is often used to construct 1-DOF and multi-DOF DMs.

2.2. A Novel 8R Mechanism

The cylindrical deployment contains the axial deployment and circumferential deployment. To realize two-direction deployment in one DU, herein we propose a novel design based on the Sarrus mechanism. The base and moving platform of the Sarrus mechanism were replaced by two spherical RRR chains with identical parameters, and each spherical RRR chain consisted of two circular links and three rotation joints whose axes intersected at the same point; thus, we obtained an 8R mechanism. As is shown in Figure 2, the 8R mechanism contained four planar links and four circular links, which formed a closed loop with revolute joints A11, C11, A12, B12, A22, C21, A21 and B11. The axes of A11, B11 and A21 intersected at point M, while the axes of A12, B12 and A22 intersected at point N. For the initial configuration, the two spherical RRR chains were translationally symmetrical along line MN, while the two planar RRR chains were rotationally symmetrical around line MN. This symmetry property not only enabled the application of the tessellation method, but it also benefited the structure’s compactness in the fully folded configuration.

For clarity, in the derivation of the motion pattern and magnification ratio, we used $\pi_1$ and $\pi_2$ to represent the spherical surfaces formed by the circular links and used $\pi_3$ and $\pi_4$ to represent the planes formed by the planar links. According to the initial configuration of the mechanism, $\pi_3$ and $\pi_4$ were both common tangent planes of $\pi_1$ and $\pi_2$.

Although the structural design of the links and joints was irrelevant to the motion pattern of the mechanism, it may have influenced the interference conditions and furthermore influenced the magnification ratio. To improve the magnification ratio, joints $B_{11}$, $B_{12}$, $C_{11}$ and $C_{21}$ adopted the double-shear connection, while $A_{11}$, $A_{12}$, $A_{21}$ and $A_{22}$ adopted the single-shear connection as shown in Figure 3. Furthermore, the contact planes of the single shear connections were set to be $\pi_3$ and $\pi_4$. In the remainder of the paper, $A_{ij}$, $B_{ij}$ and $C_{ij}$ not only denote the axes of the joints, but they also denote the intersecting points of the
corresponding axes and corresponding surfaces (π₁, π₂, π₃ or π₄). The above structural design can reduce the coupling of folding in two directions to some extent and reduce the thickness of the fully folded mechanism.

![Figure 3. Structural design of the 8R mechanism.](image)

### 2.3. Motion Pattern Analysis of the 8R Mechanism

The proposed 8R mechanism is expected to realize cylindrical deployment and retraction. In other words, if point A₁₁ is fixed to the base, point A₃₂ should be able to rotate around a fixed axis and translate along this axis. It is noteworthy that the 8R mechanism can rotate around axis A₁₁ as a rigid body. In this section, line A₁₁A₁₂ is fixed to the base so that we can focus on the deploying and folding motion. Before the motion pattern analysis, two coordinate frames, \( \{F₀\} \) and \( \{Fₚ\} \), were established as shown in Figure 4a. The origin of the base frame \( \{F₀\} \) was \( M \). Its \( z₀ \)-axis was parallel to the common perpendicular of the \( A₁₁ \)-axis and the \( A₁₂ \)-axis, and the direction was from \( A₁₁ \) to \( A₁₂ \). Its \( x₀ \)-axis was collinear with the \( A₁₁ \)-axis and pointed to the outside of the circular link. The origin of the output frame \( \{Fₚ\} \) was \( N \). Its \( zₚ \)-axis was parallel to the common perpendicular of the \( A₂₁ \)-axis and the \( A₂₂ \)-axis, and the direction was from \( A₂₁ \) to \( A₂₂ \). Its \( xₚ \)-axis was collinear with the \( A₂₂ \)-axis and pointed to the outside of the circular link. Since line segment \( A₁₁M \) was parallel and equal to line segment \( A₁₂N \), \( A₁₁MNA₁₂ \) was a parallelogram, and \( MN \) was parallel to \( A₁₁A₁₂ \). Thus, \( MN \) was collinear with the \( z₀ \)-axis of frame \( \{F₀\} \). When the base frame \( \{F₀\} \) was fixed, the output frame \( \{Fₚ\} \) was expected to realize the rotation around \( MN \) and translation along \( MN \).

![Figure 4. Coordinate frames of the 8R mechanism and the equivalent mechanism.](image)

Screw theory is a widely used tool for motion pattern analysis and type synthesis of parallel mechanisms and multi-loop mechanisms [47–49]. Herein, it is used to derive the motion pattern of the 8R mechanism. In screw theory, the joints and motion are represented by twists, while the constraints are represented by wrenches.

A screw consists of two vectors: the direction vector \( S \) and position vector \( S₀ = r \times S + hS \), where \( r \) denotes the position vector any point on the axis of the screw and \( h \) is the pitch.
of the screw. The above vectors can be described in any coordinate system. A zero-pitch screw can be utilized to represent a revolute joint or a constraint force. An infinite-pitch screw \( S = (0, S) \) can be utilized to represent a prismatic joint or a constraint couple, where \( S \) is its direction vector.

For two given screws \( S = (S, S_0) \) and \( S' = (S', S_0') \), their reciprocal product is defined as

\[
S \circ S' = S \cdot S_0' + S' \cdot S_0
\]

The reciprocal product of a twist and a wrench denotes the instantaneous power. The \( \circ \) symbol denotes the reciprocal product of the two screws. The two screws are reciprocal if their reciprocal product is zero, which represents the special relationship of the joint axes \([30]\).

The 8R mechanism does not contain the traditional base and moving platform, which is inconvenient for motion pattern analysis. In the viewpoint of twist systems, the planar RRR chain in the 8R mechanism are replaced with planar RPR chains. The resultant there are four constraint wrenches for the moving platform.

For limb PRRR, the twists of the joints are expressed as

\[
\begin{align*}
S_{A11} &= (1, 0, 0, 0, 0) \\
S_{B11} &= (a_2, b_2, c_2, 0, 0) \\
S_{A21} &= (a_3, b_3, c_3, 0, 0) \\
S_{P2} &= (0, 0, 0, d_4, e_4, f_4)
\end{align*}
\]

where \((1, 0, 0)^T, (a_2, b_2, c_2)^T\) and \((a_3, b_3, c_3)^T\) are direction vectors of the \(A_{11}\)-axis, \(B_{11}\)-axis and \(A_{21}\)-axis, respectively, and \((d_4, e_4, f_4)^T\) is the direction vector of translation joint \(P_2\).

For limb PRRR, the twists of the joints are expressed as

\[
\begin{align*}
S_{P1} &= (0, 0, 0, 0, 0, 1) \\
S_{A12} &= (1, 0, 0, 0, 0, |MN|) \\
S_{B12} &= (a_7, b_7, c_7, -b_7|MN|, a_7|MN|, 0) \\
S_{A22} &= (a_8, b_8, c_8, -b_8|MN|, a_8|MN|, 0)
\end{align*}
\]

where \((1, 0, 0)^T, (a_7, b_7, c_7)^T\) and \((a_8, b_8, c_8)^T\) are direction vectors of the \(A_{12}\)-axis, \(B_{12}\)-axis and \(A_{22}\)-axis, respectively, and \((0, 0, 1)^T\) is the direction vector of translation joint \(P_1\).

Applying the reciprocal principle, the constraint wrenches for limb RRRP can be determined as follows:

\[
\begin{align*}
S'_1 &= (a'_1, b'_1, c'_1, 0, 0, 0) \\
S'_2 &= (a'_2, b'_2, c'_2, 0, 0, 0)
\end{align*}
\]

where vectors \((a'_1, b'_1, c'_1)^T\) and \((a'_2, b'_2, c'_2)^T\) are both perpendicular to vector \((d_4, e_4, f_4)^T\) and they also pass point \(M = (0, 0, 0)\). The constraint wrenches for limb PRRRR can be determined as follows:

\[
\begin{align*}
S'_3 &= (1, 0, 0, 0, |MN|, 0) \\
S'_4 &= (0, 1, 0, -|MN|, 0, 0)
\end{align*}
\]

These two wrenches are both perpendicular to the \(z_0\)-axis and pass point \(N\). Hence, there are four constraint wrenches for the moving platform.
Applying the reciprocal principle again, the twist system of the moving platform is obtained as follows:

\[
\begin{align*}
\mathbf{s}_1 &= (0, 0, 1, 0, 0)^	op \\
\mathbf{s}_2 &= (0, 0, 0, d_4, e_4, f_4)^	op
\end{align*}
\]

Equation (6) indicates that the motion pattern of the moving platform is the rotation around the \(z_0\)-axis (MN) and the translation along vector \((d_4, e_4, f_4)^	op\), and there are no overconstraints in the mechanism. Furthermore, vector \((d_4, e_4, f_4)^	op\) can be determined as follows. According to the geometrical layout of the two limbs and the initial configuration of the mechanism, plane \(\pi_3\) and plane \(\pi_4\) are always external common tangent planes of spherical surfaces \(\pi_1\) and \(\pi_2\). Therefore, \(\pi_1\) and \(\pi_2\) are located at the same side of \(\pi_3\) and \(\pi_4\), which means that the spheres fall on the same side of the plane. In the three-dimensional geometry, for two spherical surfaces with equal diameters, all the common tangent lines on the external common tangent planes are parallel to the connecting lines of their centers. Therefore, vector \((d_4, e_4, f_4)^	op\) is parallel to the \(z_0\)-axis of frame \([F_0]\). The motion pattern of the moving platform is decoupled rotation and translation along the \(z_0\)-axis, and the twist system of the moving platform can be obtained as follows:

\[
\begin{align*}
\mathbf{s}_1 &= (0, 0, 1, 0, 0)^	op \\
\mathbf{s}_2 &= (0, 0, 0, 0, 0, 1)^	op
\end{align*}
\]

For the above mechanism, a natural choice for the actuated joints is joint \(A_{11}\) and joint \(P_1\). According to [49], the selection of actuated joints should ensure that the mobility of the mechanism with the actuated joints blocked will be zero. Assuming that joint \(A_{11}\) and joint \(P_1\) are blocked, the rank of the matrix \(\left[\mathbf{s}_{A_{12}}, \mathbf{s}_{B_{11}}, \mathbf{s}_{A_{22}}, \mathbf{s}_{C_{21}}, \mathbf{s}_{A_{21}}, \mathbf{s}_{B_{11}}\right]\) can be calculated, which equals six. This suggests that there are no overconstraints in the partially blocked mechanism. According to [50,51], the mobility of the mechanism can be calculated with the following formula:

\[
\text{mobility} = 6(n - g - 1) + \sum f_i + \mu
\]

where \(n = 6\) denotes the number of links, \(g = 6\) denotes the number of joints, \(\Sigma f_i = 6\) denotes the sum of the mobility of all joints and \(\mu = 0\) denotes the number of overconstraints. As a result, the mobility of the mechanism with two blocked joints is zero. Therefore, the selection of actuated joints is feasible. Aside from that, the cable or gear transmission approach can also be used to provide appropriate actuations and constraints [52,53].

Since the planar RRR chain and planar RPR chain are equivalent in the viewpoint of the twist systems, the motion pattern of the 8R mechanism can also be represented by the twist system listed in Equation (7), and its driving links can be determined to be \(A_{11}B_{11}\) and \(A_{11}C_{11}\). However, as stated above, to avoid the rigid rotation of the mechanism around the \(A_{11}\)-axis, line \(A_{11}A_{12}\) needs to be fixed to the base.

Based on the above motion pattern analysis, it can be seen that during the deploying and folding processes, points \(A_{11}, A_{12}, A_{21}\), and \(A_{22}\) are always located on the same cylindrical surface and form a 2-DOF scalable cylindrical rectangle. Every partially deployed configuration forms a part of the same cylindrical surface, and the radial dimensions of the DM are kept at a relatively small level during deployment.

2.4. Kinematics of the 8R Mechanism

Given the motion pattern of the mechanism, two variables on plane \(\pi_3\) are defined as input variables: the angle \(\phi\) between \(A_{11}C_{11}\) and the bottom plane and the angle \(\theta\) from the projection of \(A_{11}B_{11}\) on plane \(\pi_3\) to \(A_{11}A_{12}\) \((\phi, \theta \in [0, \pi/2])\), as shown in Figure 5. Aside from that, the length of \(MN\) and the angle from \(x_0\) to \(x_f\) are selected as the output variables of the DM, which are denoted by \(h\) and \(\gamma\). The other parameter symbols are shown in Table 1.
that, the length of analysis of the corresponding twist system \{circular link $B_1$. The DM, which are denoted by symmetries. Two spherical RRR chains are translationally symmetrical on the cylindrical surface tangent to $A \alpha = E P$. \Symmetry 2021, 13, x FOR PEER REVIEW 8 of 21

Figure 5. Demonstration of variables.

Table 1. Link parameter symbols.

| Parameters                                                                 | Symbols |
|---------------------------------------------------------------------------|---------|
| Length of the planar links $(A_{11}C_{11}, C_{11}A_{12}, A_{21}C_{21}, C_{21}A_{22})$ | $l$     |
| Radius of the circular links $(A_{11}B_{11}, B_{11}A_{21}, A_{12}B_{12}, B_{12}A_{22})$ | $r$     |
| Twist angle of the circular links $(A_{11}B_{11}, B_{11}A_{21}, A_{12}B_{12}, B_{12}A_{22})$ | $\alpha$|

It can be seen from the above analysis that the proposed mechanism has apparent symmetries. Two spherical RRR chains are translationally symmetrical on the $z_0$-axis, while two planar RRR chains are rotationally symmetrical around the $z_0$-axis. For example, the side faces of circular link $A_{11}B_{11}$ are parallel to the corresponding side faces of circular link $A_{12}B_{12}$, and $A_{11}A_{12}$ and $A_{21}A_{22}$ are parallel and equal to each other. There is a cylindrical surface tangent to $\pi_1$ and $\pi_2$, which is denoted by $\pi_5$, and the radius of it equals the radius of $\pi_1$ and $\pi_2$ (denoted by $r$).

Based on the motion pattern analysis and geometrical relationship, output variable $h$ can be obtained as follows:

$$h = 2l \sin \varphi$$  \(9\)

The expression of output variable $\gamma$ is deduced as follows. The intersecting point of line $NB_{12}$ and plane $\pi_3$ is denoted by $P$, and the projection of point $P$ on the cross-section of the cylinder which passes point $N$ is denoted by $E$, as shown in Figure 6.

Figure 6. Diagram of the triangles.

By applying trigonometric functions to $\Delta NA_{12}P$, $\Delta EA_{12}P$ and $\Delta NA_{12}E$, we have

$$\tan \alpha = \frac{|A_{12}P|}{r}$$  \(10\)

$$\sin \theta = \frac{|A_{12}E|}{|A_{12}P|}$$  \(11\)
\[
\tan \frac{\gamma}{2} = \frac{|A_{12}E|}{r}
\] (12)

By combining Equations (10)–(12), output variable \( \gamma \) can be expressed as

\[
\gamma = 2\arctan(\tan \alpha \sin \theta)
\] (13)

According to the decoupled kinematic equations (Equations (9) and (13)), the proposed mechanism can be seen as a serial chain composed of a translation joint and an arc translation joint.

For the proposed 8R mechanism, singular configurations can be derived through the analysis of the corresponding twist system \( \{S_{A11}, S_{C11}, S_{A12}, S_{B12}, S_{A22}, S_{C21}, S_{A21}, S_{B21}\} \). As a result, there are two singular cases where \( \phi \) and \( \theta \) both belong to \((0, \pi/2)\).

Singular case I: When \( \phi \) equals \( \pi/2 \), \( S_{A11}, S_{C11} \) and \( S_{A12} \) are linearly dependent. In addition, \( S_{A21}, S_{C21} \) and \( S_{A22} \) are linearly dependent. Meanwhile, \( A_{11}C_{11} \) and \( C_{11}A_{12} \) are collinear without overlapping, and \( A_{21}C_{21} \) and \( C_{21}A_{22} \) are collinear without overlapping, as shown in Figure 7a.

Singular case II: When \( \theta \) equals \( \pi/2 \), \( S_{A11}, S_{B11} \) and \( S_{A21} \) are linearly dependent, and \( S_{A12}, S_{B12} \) and \( S_{A22} \) are linearly dependent. Meanwhile, \( A_{11}B_{11} \) and \( B_{11}A_{21} \) are coplanar without overlapping, and \( A_{12}B_{12} \) and \( B_{12}A_{22} \) are coplanar without overlapping, as shown in Figure 7b.

The fully deployed configuration of the mechanism (Figure 7c) is the intersection of singular case I and singular case II.

3. 2-DOF Cylindrical DMs Composed of 8R DUs

In this section, the proposed 8R mechanisms are used as DUs to construct 2-DOF cylindrical DMs. Due to the rotational symmetries and translational symmetries of the link poses in the 8R DU, here, we consider using the linear pattern combination and circular pattern combination of the 8R mechanisms to construct 2-DOF cylindrical DMs. For the linear pattern combination, the neighboring DUs share two circular links directly and share two planar links through a scissor connection. For the circular pattern combination, the neighboring DUs share two planar links directly and share two circular links through a scissor connection. For instance, Figure 8 shows a cylindrical DM composed of four DUs. \( \text{DU}_{11} \) and \( \text{DU}_{12} \) share circular links \( A_{12}B_{12} \) and \( B_{12}A_{22} \) directly and share planar links \( C_{11}C_{12} \) and \( C_{21}C_{22} \) through a scissor connection, while \( \text{DU}_{11} \) and \( \text{DU}_{21} \) share planar links \( A_{21}C_{21} \) and \( C_{21}A_{22} \) directly and share circular links \( B_{11}B_{21} \) and \( B_{12}B_{22} \) through a scissor connection. Furthermore, the mobility of the DM is briefly verified as follows.
Figure 9. The topological constraint graph of the cylindrical DM.

For the combination of DU_{11} and DU_{12}, if the twists are considered as six-dimension column vectors, Equations (15) and (17) can be rewritten as

\[ \mathbf{S}_{12} \mathbf{\omega}_1 = \mathbf{0}_{12 \times 1} \]  \hspace{1cm} (18)

where \( \mathbf{\omega}_1 = (\omega_{A1}, \omega_{B1}, \omega_{A21}, \omega_{C11}, \omega_{A22}, \omega_{B12}, \omega_{A12}, \omega_{B13}, \omega_{A13}, \omega_{B23}, \omega_{A23}, \omega_{C23})^T \) and \( \mathbf{S}_{12} \) is listed in the Appendix A. According to the mobility analysis method of closed-loop mechanisms [30], the mobility of the combination of DU_{11} and DU_{12} equals \( n_{\text{col}} \cdot r(S_{c1}) \), where \( n_{\text{col}} \) and \( r(S_{c1}) \) denote the number of columns in matrix \( \mathbf{S}_{c1} \) and the rank of \( \mathbf{S}_{c1} \), respectively. By substituting the parameters and joint twists into Equation (18), it is discovered that the rank of \( \mathbf{S}_{c1} \) equals 11 when \( \theta \) and \( \varphi \) change within (0, \( \pi/2 \)). Therefore,
the mobility of the combination of DU_{11} and DU_{12} is 2. Since the DM is constructed with 2-DOF DUs, the linear pattern combination does not change the motion pattern.

For the combination of DU_{11} and DU_{21}, Equations (14) and (16) can be rewritten as

$$S_{c2} \omega_2 = 0_{12 \times 1}$$  \hspace{1cm} (19)

where \( \omega_2 = (\omega_{A11}, \omega_{B11}, \omega_{A21}, \omega_{B21}, \omega_{A31}, \omega_{C11}, \omega_{C21}, \omega_{C31}, \omega_{A12}, \omega_{B12}, \omega_{A22}, \omega_{B22}, \omega_{A32})^T \) and \( S_{c2} \) is listed in the Appendix A. By substituting the parameters and joint twists into Equation (19), the rank of \( S_{c2} \) is also 11 when \( \theta \) and \( \phi \) change within \((0, \pi/2)\), and the mobility of the DM is also 2. Therefore, the circular pattern combination does not change the motion pattern.

Similarly, for the DM composed of four DUs, Equations (14)–(17) can be rewritten as

$$S_{c3} \omega_3 = 0_{24 \times 1}$$  \hspace{1cm} (20)

where \( \omega_3 = (\omega_{A11}, \omega_{B11}, \omega_{A21}, \omega_{C11}, \omega_{C21}, \omega_{A12}, \omega_{B12}, \omega_{A22}, \omega_{B22}, \omega_{A23}, \omega_{C12}, \omega_{C22}, \omega_{B21}, \omega_{A31}, \omega_{B22}, \omega_{A32}, \omega_{B23}, \omega_{A33}, \omega_{C31}, \omega_{C32})^T \) and \( S_{c3} \) is listed in the Appendix A. It is discovered that the mobility is also two. Consequently, the DM has the same motion pattern with the DU. The above mobility verification processes can be extended to DMs composed of any number of DUs. Similar to the prior DMs \([2,3,15,18]\), the proposing DMs are overconstrained multi-loop mechanisms, which require high precision during manufacturing and assembly.

### 4. Magnification Ratio Analysis for the 2-DOF DMs

#### 4.1. Definition of Circumferential and Axial Magnification Ratios

For a deployable mechanism, one of the most important performance indicators is the magnification ratio, which is usually defined as the ratio of the key parameter (length, area or volume) in the fully deployed configuration and the fully folded configuration. For cylindrical DMs, the circumferential magnification ratio \( k_W \) and axial magnification ratio \( k_H \) are defined as follows. Since points \( B_{ij} \) and \( C_{ij} \) are not on the cylindrical surface, first, these points are projected onto the cylindrical surface \( \pi_S \) along the radial direction, obtaining points \( B'_{ij} \) and \( C'_{ij} \). The magnification ratio can be defined based on the circumscribed cylindrical rectangle of the points \( \{A_{ij}, B'_{ij}, C'_{ij}\} \). The length and height of the cylindrical rectangle are denoted by \( W \) and \( H \), respectively. Figure 10 shows the cylindrical circumscribed rectangle of the simplest DM, composed of only one DU. Under the fully deployed and the fully folded configurations, the lengths and heights are denoted by \( W_d, H_d \), \( W_f \) and \( H_f \), respectively, and the circumferential magnification ratio \( k_W \) and axial magnification ratio \( k_H \) are defined as

\[
\begin{align*}
    k_W &= \frac{W_f}{W_d} \\
    k_H &= \frac{H_f}{H_d}
\end{align*}
\]  \hspace{1cm} (21)

As stated in Section 2.3, \( \varphi \) and \( \theta \) are input variables of the mechanism \( \varphi, \theta \in [0, \pi/2] \) which determine the deploying state. According to Figures 8 and 10, as long as the number of units in both directions is more than one, the width and height of the cylindrical circumscribed rectangle in any state can be calculated by

\[
\begin{align*}
    W &= mw + 2w' \\
    H &= nh + 2h'
\end{align*}
\]  \hspace{1cm} (22)

where \( w \) and \( w' \) are the arc lengths, \( h \) and \( h' \) are the axial lengths and \( m \) and \( n \) are the numbers of the DUs in the circumferential direction and the axial direction, respectively. The detailed deduction of these variables is as follows.
The detailed deduction of these variables is as follows.

The numbers of the DUs in the circumferential direction and the axial direction, respectively. According to Figures 8 and 10, as long as the number of units in both directions is more than one, the width and height of the cylindrical circumscribed rectangle can be calculated by

\[ W = r \cdot \tan \alpha \cdot \sin \theta + r \cdot \arctan \left( \frac{\cos \varphi}{r} \right) \]

\[ H = 2m \cdot \sin \varphi + r \cdot \sin \alpha \cdot \cos \theta \]

For the 2-DOF DM, if the input variables \( \theta \) and \( \varphi \) are determined, \( W \) and \( H \) can be calculated utilizing Equation (31).

In practical applications, each link of the mechanism has a certain shape with a certain width and thickness, which may influence the interference condition and furthermore influence the magnification ratios. The shape of the links can be customized to postpone certain

Figure 10. The cylindrical circumscribed rectangle.

In \( \Delta A_{11}C_{11}A_{12} \) on plane \( \pi_3 \), there are

\[ h = |A_{11}A_{12}| = 2l \sin \varphi \]

\[ |A_{11}C_{11}''| = l \cos \varphi \]

In \( \Delta A_{11}C_{11}''A_{12} \) on the bottom plane of the cylinder, there is

\[ w' = r \gamma' = r \cdot \arctan \left( \frac{|A_{11}C_{11}''|}{r} \right) + r \cdot \arctan \frac{l \cos \varphi}{r} \]

Based on the kinematic equation (Equation (18)), variable \( w \) can be obtained by

\[ w = r \gamma = 2r \cdot \arctan(\tan \alpha \sin \theta) \]

In Figure 6, by applying trigonometric functions to \( \Delta NA_{12}P, \Delta EA_{12}P \) and \( \Delta PNE \), we obtain

\[ \sin \alpha = \frac{|A_{12}P|}{|PN|} \]

\[ \cos \theta = \frac{|PE|}{|A_{12}P|} \]

\[ \sin \angle PNE = \frac{|PE|}{|PN|} = \frac{h'}{r} \]

Combining Equations (27)–(29) yields

\[ h' = \frac{r|PE|}{|PN|} = r \sin \alpha \cos \theta \]

Substituting Equations (23), (25), (26) and (30) into Equation (22) yields

\[
\begin{cases}
W = 2mr \cdot \arctan(\tan \alpha \sin \theta) + r \cdot \arctan \left( \frac{\cos \varphi}{r} \right) \\
H = 2m \cdot \sin \varphi + r \cdot \sin \alpha \cdot \cos \theta
\end{cases}
\]
interferences during the folding processes and improve the magnification ratio. However, to illustrate the feature of the mechanism, only regular-shaped links are considered in this paper. Due to the rotational and translational symmetries of the link poses, the interference conditions are also symmetric. Therefore, we only need to consider interference conditions in one or two DUs.

Before the magnification ratio analysis, the fully deployed and fully folded configurations should be defined. According to Section 2, the DU is fully deployed in the axial direction when link \( A_{11}C_{11} \) and link \( C_{11}A_{12} \) are collinear without overlapping \( (\varphi = \pi/2) \), and it is fully deployed in the circumferential direction when link \( A_{11}B_{11} \) and link \( B_{11}A_{21} \) are coplanar without overlapping \( (\theta = \pi/2) \). Therefore, the fully deployed configuration of the DM is defined as the configuration corresponding to \( \theta = \varphi = \pi/2 \). Although this configuration is singular, the mechanism can approach this configuration as closely as possible to maximize the magnification ratios. Under this configuration, we have

\[
\begin{align*}
W_d &= 2mr \\
H_d &= 2nl
\end{align*}
\]  

(32)

The fully folded configuration of the DM is the most compact configuration without interference. Given the symmetry in the DM and the links, two interference cases are considered: interference between the circular links and interference between the planar links.

4.2. Interference Case I: The Interference between Circular Links

In this subsection, the interference between circular links in DU\( _{11} \) is discussed, which is obviously related to the folding condition in the axial direction, particularly to angle \( \varphi \). Since the circular links have the same parameters, when links \( A_{11}B_{11} \) and \( A_{12}B_{12} \) are in contact, links \( A_{21}B_{11} \) and \( A_{22}B_{12} \) are also in contact. Thus, only one contact needs to be considered. According to Section 2, links \( A_{11}B_{11} \) and \( A_{12}B_{12} \) are translationally symmetrical in \( z_0 \)-axis. Consequently, when links \( A_{11}B_{11} \) and \( A_{12}B_{12} \) are just starting to contact each other, the lower side face of link \( A_{11}B_{11} \) coincides with the upper side face of link \( A_{12}B_{12} \). Herein, the thickness of a link, which is denoted by \( t \), is defined as the distance between the contralateral outer surface of the link along the joint axis. It can be seen that the thickness of the links had no influence on interference case I, and therefore, the thickness was set to zero for convenience in derivation, as shown in Figure 11. The projections of circular arcs \( A_{11}B_{11} \) and \( A_{12}B_{12} \) on plane \( \pi_3 \) were two parallel lines. We draw a line on plane \( \pi_3 \), which was perpendicular to the projection line of \( A_{11}B_{11} \) and passed \( A_{12} \). In triangle \( A_{11}QA_{12} \), we have

\[
\sin \theta_{f1} = \frac{|A_{12}Q|}{|A_{11}A_{12}|}
\]

(33)

where \( |A_{12}Q| \) equals the width of the circular links (s). Substituting Equation (23) yields

\[
\sin \theta_{f1} \sin \varphi_{f1} = \frac{s}{2l}
\]

(34)

This suggests that there are infinite critical interference conditions. The interference conditions relate to the folding conditions in the axial direction as well as the circumferential direction. Aside from that, interference case I was influenced by the width of the circular links.
4.3. Interference Case II: The Interference between Planar Links

The interference between planar links directly related to the folding condition in the circumferential direction, which means it directly related to angle $\theta$. For each planar link, the rear side face was tangential to the cylindrical surface $\pi_5$ with radius $r$, while the front side face was tangential to the coaxial cylindrical surface with radius $(r + l)$. Based on the geometric relationship, when planar links $C_{11}C_{12}$ and $C_{21}C_{22}$ approached each other, the interference started on the side faces tangential to $\pi_5$. Therefore, the thickness of the links was also set to zero. It is noteworthy that the interference point was located on both $\pi_3$ and $\pi_4$, which implies that it was on the intersection line of $\pi_3$ and $\pi_4$. Furthermore, the interference point was located at the same cross-section of the cylinder with $A_{12}A_{22}$. Therefore, it coincided with point $E$ (Figure 10). The interference case is shown in Figure 12. In $\Delta EA_{12}G$, the angle $\angle A_{12}EG$ equals $\varphi_{f2}$, and we have

$$\sin \varphi_{f2} = \frac{|A_{12}G|}{|A_{12}E|}$$

(35)

where $|A_{12}G|$ equals half of the width of the planar links ($s/2$). Substituting Equations (10) and (11) into Equation (35) yields

$$\sin \theta_{f2} \sin \varphi_{f2} = \frac{s}{2r \tan \alpha}$$

(36)

Similar to interference case I, there were infinite critical interference conditions for interference case II. The interference conditions related to the folding conditions in the circumferential direction as well as the axial direction. Interference case II was influenced by the width of the planar links.
4.4. Magnification Ratio

During the folding process, one of these two interference cases will happen before the other. Assume the planar links and the circular links have the same width (s). Then, the comparison of \( s/(2l) \) and \( s/(2r\tan\alpha) \) determines the order, where \( s, l, r, \) and \( \alpha \) are link parameters. To analyze the magnification ratios in two directions, the folding performance of a half-circular cylindrical shelter was studied. The values of the link parameters are listed in Table 2.

**Table 2.** Link parameter values.

| Parameters                              | Values     |
|-----------------------------------------|------------|
| Radius of the circular links (\( r \))  | 1000 mm    |
| Length of the planar links (l)          | 250 mm     |
| Twist angle of the circular links (\( \alpha \)) | \( \pi/12 \) |
| Width of the planar links and circular links (s) | 16 mm |
| Thickness of the planar links and circular links (l) | 8 mm |
| The number of DUs in the circumferential direction (m) | 6 |
| The number of DUs in the axial direction (n) | 6 |

Obviously, \( s/(2l) \) was larger than \( s/(2r\tan\alpha) \). According to Equations (34) and (36), the folding performance in the circumferential direction and axial direction were coupled; when the DM was extremely compact in one direction, it may have reached a low folding efficiency in the other direction. A natural choice for the indicator of the overall folding performance was the product of magnification ratios in both directions, namely the magnification ratio of the area of the cylindrical rectangle:

\[
k = kWk_H = \frac{4mlr\alpha}{(2mr \cdot \arctan(\tan\alpha \sin \theta) + 2r \cdot \arctan(\cos \frac{\phi}{r})(2nl \sin \varphi + 2r \sin \alpha \cos \theta)}
\]  

(37)

Figure 13 shows the influence of the circumferential magnification ratio \( k_W \) on the axial magnification ratio \( k_H \) and the area magnification ratio \( k \). It can be seen that the magnification ratios in the two directions were negatively correlated. When the area magnification ratio reached the maximum value, the DM was highly folded in the axial direction and entirely unfolded in the circumferential direction, which is not the ideal fully folded configuration. In different applications, more practical and customized indicators need to be considered. As an example, here, we define the indicator of folding performance as the area magnification ratio when the magnification ratios in two directions are equal.

![Figure 13. The relationship of magnification ratios.](image-url)
4.5. Optimization of the Magnification Ratio

For the task of forming a $W_0 \times H_0$ cylindrical surface, some of the parameters can be optimized to obtain the largest area magnification ratio. Normally, parameters $r$, $s$ and $t$ can be determined according to the application requirements. The area magnification ratio and corresponding parameters can be obtained utilizing the following two optimizations.

Optimization I: Find $X = \{\theta, \varphi, \alpha, l, m, n\}$, which maximizes Equation (37) and meets the constraints of $2mr\alpha = W_0$; $2nl = H_0$; $\sin \theta \sin \varphi = s/(2l)$; $s/(2l) > s/(2\tan \alpha)$; and $k_W = k_H$.

Optimization II: Find $X = \{\theta, \varphi, \alpha, l, m, n\}$, which maximizes Equation (37) and meets the constraints of: $2mr\alpha = W_0$; $2nl = H_0$; $\sin \theta \sin \varphi = s/(2\tan \alpha)$; $s/(2l) \leq s/(2\tan \alpha)$; and $k_W = k_H$.

The larger one of these two optimization results is the overall optimized area magnification ratio.

A case study is presented as follows. The required surface was a half-circular cylindrical surface with a radius of 1000 mm and height of 3000 mm. Here, parameters $r$, $s$ and $t$ were determined to be 1000 mm, 16 mm and 8 mm, respectively. Aside from that, $m$ and $n$ were constrained within 16 for the consideration of complexity avoidance. Utilizing the optimization toolbox of Matlab, it was discovered that the optimized area magnification ratio was 11.2673, and the corresponding parameters were as follows: $m$ equalled 11, $n$ equalled 10, $\alpha$ equalled $\pi/22$ and $l$ equalled 150 mm. The influences of $m$ and $n$ on the area magnification ratio are shown in Figure 14. For the resultant DM composed of 11 × 10 DUs, the simulation of the deployment is shown in Figure 15. It can be seen that the fully deployed configuration formed an exact half-circular cylindrical surface.

Figure 16 shows the deployment of a 3D-printed prototype, which verified the feasibility of the proposed design. It can be seen that the radial dimensions of the DM were kept at a relatively small level during deployment. In comparison with the cylindrical DMs based on Bennett linkages, the joints of the proposed design were more convenient to install. Furthermore, every partially deployed configuration formed a part of the same cylindrical surface.
Figure 14. Influences of \( m \) and \( n \) on the area magnification ratio. ... was equivalent to the 8R DU under the constraint of \( \theta = \phi \), and the configuration space of this mechanism was a sub-manifold of that of the 2-DOF 8R DU. Similarly, coordinate frames \( \{F_0\} \) and \( \{F_P\} \) could also be established, and the motion pattern of \( \{F_P\} \) relative to \( \{F_0\} \) was a special case of

5. 1-DOF DMs Based on 6R DUs

The fully deployed configuration of the above 2-DOF DM is a singular configuration, which is inconvenient to fold. To avoid the singularity, a single DOF DU is proposed through the following processes. For the proposed 2-DOF 8R DU, adjust the relative pose of link \( A_{11}C_{11} \) and link \( A_{11}B_{11} \) so that line \( A_{11}C_{11} \) is perpendicular to the projection of circular arc \( A_{11}B_{11} \) on plane \( \pi_3 \), and combine the two links into one rigid body. Based on the geometric analysis, under this condition, line \( A_{22}C_{21} \) is perpendicular to the projection of circular arc \( A_{22}B_{12} \) on plane \( \pi_4 \). Then, combine link \( A_{22}C_{21} \) and link \( A_{22}B_{12} \) into one rigid body. As a result, we obtained a new mechanism composed of six links and six revolute joints as shown in Figure 17. It can be seen that this 6R mechanism was equivalent to the 8R DU under the constraint of \( \theta = \phi \), and the configuration space of this mechanism was a sub-manifold of that of the 2-DOF 8R DU.
2-DOF cylindrical motion, namely 1-DOF screw motion with variable pitch. The twist system of the 6R mechanism was as follows:

\[
\begin{align*}
&\mathbf{s}_{C11} = (1,0,0,0,l \sin \theta, l \cos \theta)^T \\
&\mathbf{s}_{A12} = (1,0,0,0,2l \sin \theta,0)^T \\
&\mathbf{s}_{B12} = (\cos \alpha, \sin \alpha \sin \theta, \sin \alpha \cos \theta, -2l \sin \alpha \sin^2 \theta, 2l \cos \alpha \sin \theta,0)^T \\
&\mathbf{s}_{C21} = (\cos \gamma, \sin \gamma, 0, -l \sin \gamma \sin \theta, l \sin \theta \cos \gamma, -l \cos \theta)^T \\
&\mathbf{s}_{A21} = (\cos \gamma, \sin \gamma, 0, 0,0,0)^T \\
&\mathbf{s}_{B11} = (\cos \alpha, \sin \alpha \sin \theta, \sin \alpha \cos \theta, 0,0,0)^T
\end{align*}
\]  

(38)

![Figure 17. The 6R DU.](image)

It can be proven that the rank of the twist system was always five when \( \varphi \) changed within the range \((0, \pi/2)\). Thus, there was an overconstraint in the mechanism. By substituting the values of the parameters into the mobility formula (Equation (8)), we obtained the mobility of the 6R mechanism, namely one. Since the rank of the twist system was invariant when \( \varphi \) changed within \((0, \pi/2)\), there was no singularity in the mechanism.

Through linear pattern combination and circular pattern combination of the 6R DUs, new cylindrical DMs could be constructed. As an example, a 1-DOF DM composed of four 6R DUs is shown in Figure 18, where fixed connections were located at \(A_{11}, A_{22}, A_{13}, A_{31}\) and \(A_{33}\). Using the approach adopted in Section 3, it can be proven that the DM had one DOF.

![Figure 18. The 1-DOF DM composed of four 6R DUs.](image)
Due to the 1-DOF feature of the DM, the critical interference case was certain, and therefore, the fully folded configuration was certain. If the parameters of the DU met $s/(2l) > s/(2rtan\alpha)$, the fully folded configuration corresponded to

$$\varphi = \theta = \arcsin\left(\frac{s}{2l}\right)$$  \hspace{1cm} (39)

If the parameters of the DU met $s/(2l) \leq s/(2rtan\alpha)$, the fully folded configuration corresponded to

$$\varphi = \theta = \arcsin\left(\frac{s}{2r tan\alpha}\right)$$  \hspace{1cm} (40)

If a DM was constructed by the above 6R DUs with parameters $m = 11$, $n = 10$, $\alpha = \pi/22$ and $l = 150$ mm, the area magnification ratio was 10.834, which was slightly smaller than that of the 8R DUs.

6. Conclusions

This paper presents a new type of 2-DOF single-loop DU consisting of eight links and eight revolute joints. The proposed DU can be deployed and retracted on cylindrical surfaces, and the axial deployment and circumferential deployment are independent. Utilizing screw theory, the motion pattern of the DU was verified, and the singular configurations were obtained. Through the linear pattern combination and circular pattern combination of DUs, the cylindrical DM was constructed, which had the same motion pattern as the DUs.

As one of the key performance indicators of the DM, the area magnification ratio was defined as the ratio of the area of the DM’s circumscribed cylindrical rectangle in the fully deployed configuration and the fully folded configuration. Based on the geometrical analysis, two interference cases were analyzed and applied to the optimization of deployable mechanisms, forming an example surface. A simplified 1-DOF single-loop DU was derived from the 2-DOF DU, which was used to construct 1-DOF cylindrical DMs with singular-free workspaces.

In comparison with prior DMs, the proposed DU and DM form a scalable cylindrical rectangle during deployment and retraction. This feature enables many potential applications of the DU and DM, including retractable rooftops, emergency shelters and reflectors. For example, if the parameters of the links are specially customized, the proposed 2-DOF and 1-DOF DUs can be deployed to form parabolic cylindrical surfaces, which is required in antennas and reflectors.

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Appendix A

$$S_{c1} = \begin{pmatrix} s_{A_{11}} & s_{B_{11}} & s_{A_{21}} & -s_{C_{11}} & -s_{A_{12}} & -s_{B_{12}} & -s_{A_{22}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{A_{12}} & s_{B_{12}} & s_{A_{22}} & -s_{A_{13}} & -s_{B_{13}} & -s_{A_{23}} & -s_{C_{12}} & s_{C_{22}} \end{pmatrix}$$
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