Thermodynamic wetness loss calculation in a steam turbine rotor tip section: nucleating steam flow

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Abstract: Rapid expansion of steam in the last stages of a steam turbine causes condensation. The formation of liquid droplets due to condensation results in wetness losses, which include aerodynamic losses (due to friction between liquid droplets and the vapour), thermodynamic losses (due to irreversible heat addition), and braking losses (due to the impact of liquid droplets on the blade). The thermodynamic loss contributes up to 80% to the wetness losses when the diameter of the droplets formed is less than 1 μm. In this study, the thermodynamic loss in a two-dimensional steam turbine rotor tip section is numerically investigated for various operating and off-design conditions. A pressure based, Eulerian-Eulerian approach is used to model the non-equilibrium condensation process. The entropy change due to condensation is used to compute the thermodynamic losses.

1. Introduction

During power generation in a steam turbine, steam undergoes rapid expansion across each stage of the turbine. In the last stages, the high-speed steam crosses the vapor saturation line and reaches a meta-stable state before condensing. At this state, fine liquid droplets of less than one micron are formed, resulting in wetness losses in the turbine. The total wetness loss in a steam turbine due to condensation has two major contributors, the aerodynamic and the thermodynamic losses. For droplets of size less than 1 μm in diameter, the aerodynamic loss is less than 1 % of the total work output [1]. The present study investigates the thermodynamic loss in the last stage of a steam turbine, numerically, based on evaluating the specific entropy rise in terms of the interphase mass and heat flux, an approach suggested originally by Young [2]. Further, the variation of this loss with pressure ratio, inlet subcooling and inlet angle is also predicted.

Although early modeling methods of non-equilibrium condensation were proposed in one-dimensional nozzles, e.g. [1], Eulerian-Lagrangian approach was suggested for complex two-dimensional analysis in turbine cascades in the 1990s [3-4]. As the Lagrangian approach is computationally expensive, the Eulerian-Eulerian approach is followed for studying the condensing wet steam flows in a one-dimensional nozzle [5] by assuming the liquid droplets to be in continuum. Ishazaki et al. [6] followed the same approach for transonic non-equilibrium condensation flows through a steam turbine cascade. Gerber and Kermani [7] used a pressure based

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Eulerian-Eulerian model to predict non-equilibrium condensation in transonic flow. The paper estimates the pressure distribution on a typical cascade of a turbine rotor tip section and validated the results against the experimental data of Bakhtar et al. [8]. A similar approach is used in this paper to model non-equilibrium condensation and from the predicted results; thermodynamic wetness loss is calculated using post processing techniques.

Nomenclature

| Symbol | Description |
|--------|-------------|
| $C_p$ | isobaric Specific Heat (J/kg K) |
| $E$ | total Energy per mass (J/kg) |
| $h$ | specific enthalpy (J/kg) |
| $I$ | nucleation rate (No. of droplets/m$^3$s) |
| $N$ | number of droplets per volume (No. of droplets/m$^3$) |
| $P$ | pressure (Pa) |
| $q$ | heat flux (W/m$^2$) |
| $Q_{dis}$ | rate of heat dissipation due to irreversibility (W) |
| $R$ | characteristic gas constant of steam (461.5 J/kg K) |
| $\bar{r}$ | average radius (m) |
| $T$ | temperature (K) |
| $u$ | velocity component (m/s) |
| $V$ | resultant velocity vector (m/s) |
| $x$ | spatial variable (m) |
| $\alpha$ | liquid mass fraction |
| $\gamma$ | ratio of specific heats |
| $\Delta S$ | change in specific entropy (J/kgK) |
| $dV$ | volume of a control volume (m$^3$) |
| $\zeta$ | enthalpy loss coefficient |
| $\lambda$ | heat transfer coefficient (W/m$^2$K) |
| $\rho$ | density (kg/m$^3$) |
| $\sigma$ | liquid surface tension (N/m) |
| $\tau$ | shear stress tensor (N/m$^2$) |

Subscripts

- $0$ total properties
- $1$ state at inlet
- $2$ state at outlet
- $f$ liquid
- $g$ gas
- $i$ x-direction component
- $j$ y-direction component
- $p$ droplet
- $sc$ sub-cooled
- $sat$ saturation
- $sh$ superheat

Greek alphabets

- $\Delta$ change in specific entropy (J/kgK)

2. Computational Model

In order to model the non-equilibrium condensation process, two additional conservation equations (for liquid mass fraction and droplet density) are solved along with the compressible Navier-Stokes equations for the single flow mixture; an approach similar to [7] is used for solving the governing set of equations.

2.1. Scalar transport equation for liquid mass conservation

This scalar equation is solved for conserving the liquid mass fraction $\alpha$, defined as the mass of liquid to the mass of vapor, in each control volume.

$$\frac{\partial (\rho_s \alpha)}{\partial t} + \frac{\partial (\rho_s c u_j)}{\partial x_j} = S_\alpha \tag{1}$$

The source term of this equation $S_\alpha$ - the liquid mass generated, is the sum of mass increase due to nucleation (the formation of critically sized droplets) and the growth/demise of these droplets, based on the classical nucleation theory [9], is given by

$$S_\alpha = \frac{4}{3} \pi \rho_f \bar{r}_f \bar{r}^3 + 4 \pi \bar{r}^2 N \frac{\partial \bar{r}}{\partial t} \rho_f \rho_s \tag{2}$$

where, $'I'$ is the droplet nucleation rate, discussed in the subsequent section. The term $\bar{r}_f$ gives the critical radius defined as the threshold value of radius, only above which the droplet would
sustain in the liquid phase. It is derived based on the Kelvin-Helmholtz equation and is simplified as given in Young [2].

\[ r_s = \left( \frac{2\sigma}{\rho_f \Delta G} \right) \approx \left( \frac{2\sigma}{\rho_f RT \ln S} \right) \tag{3} \]

Large clusters of very fine droplets are formed when the free-energy barrier for the formation of droplets is overcome. This occurs at high degrees of subcooling, in the range of 30-40 K. ’S’ is the supersaturation ratio defined as the ratio of vapor pressure to the saturation pressure at the corresponding vapor temperature. It takes a value greater than unity when the vapor is in the sub-cooled region. Supersaturation gives the degree of subcooling attained by the vapor before nucleation and is a measure of the deviation of vapor from its thermodynamic equilibrium.

The second term of \( S_n \) in equation (2) is the increase/decrease in liquid mass fraction due to the growth of the droplet, given as the product of the interfacial surface area of all the droplets and the droplet growth rate \( \left( \frac{\partial \mathcal{A}}{\partial t} \right) \). To compute the surface area of the droplet, we use the average radius \( r \) of a droplet \( \mathcal{F} \), which is derived based on the assumption of a spherical droplet and is given by

\[ m_p = \frac{\alpha}{N} \Rightarrow r = \left( \frac{3\alpha}{4\rho_f \pi N} \right)^{1/3} \tag{4} \]

The liquid droplets formed grow when the vapor surrounding it condenses on its surface. The rate of droplet growth is obtained by performing an energy balance over the spherical droplet. The heat released by a condensing droplet, i) increases the droplet temperature and ii) heats up the surrounding sub-cooled vapor. The former component is neglected since the size of the droplet is very small and a uniform droplet temperature is assumed. Therefore, the remaining terms give

\[ 4\rho_f \pi r^2 (h_g - h_p) \frac{\partial \mathcal{F}}{\partial t} = 4\pi r^2 \lambda_s \left( T_p - T_g \right) \tag{5} \]

The heat transfer coefficient \( \lambda_s \) accounts for the Knudsen number effects. The growth rate is a strong function of the heat convected away from the droplet and based on this, a simplified equation available in Young [10] is used.

\[ \frac{\partial \mathcal{F}}{\partial t} = \frac{P}{h_g \rho_f \sqrt{2\pi RT}} \frac{\gamma + 1}{2\gamma} C \rho (T_p - T_g) \tag{6} \]

The term \( T_p \) is the droplet temperature, calculated based on the capillarity effect described in [11]. For a droplet having a size \( \mathcal{F} < 1\mu m \), its temperature is calculated from

\[ T_p = T_{sat}(P) - T_{sc} \frac{r_s}{\mathcal{F}} \tag{7} \]

2.2. Scalar transport equation for droplet density conservation

The second transport equation to obtain the number of droplets per unit mass of vapour (N) is

\[ \frac{\partial (\rho_s N)}{\partial t} + \frac{\partial (\rho_s u_j N)}{\partial x_j} = S_N \tag{8} \]

The variable in the source term \( S_N = I \), is the droplet nucleation rate per volume, based on the classical nucleation [12] given by

\[ I = \frac{q_s}{1 + \eta} \left( \frac{2\sigma}{\pi m^2} \right)^{1/2} \frac{\rho_s^2}{\rho_f^2} \exp \left( \frac{-4\pi \rho_s^2 \sigma}{3KT_g} \right) \tag{9} \]
here, the term $q_c$ is the condensation coefficient defined as the fraction of vapour molecules incident on the surface of the droplet that condenses on it. Its value is taken to be unity [11] in this model. 'm' is the mass of one molecule of water and 'K' is the Boltzmann constant. A correction factor for non-isothermal effects ($\eta$) is used for non-equilibrium nucleation, as proposed by Kantrowitz [13].

$$\eta = \frac{2^{\gamma-1} h_{fg}}{\gamma + 1 \frac{RT}{h_{fg}} \frac{RT - h_{fg}}{2}}$$  \hspace{1cm} (10)$$

where ($h_{fg}$) is the equilibrium latent heat. $\sigma$ and $\rho_f$ are the liquid surface tension and liquid density obtained as a function of temperature. The other terms used in this section are given in the nomenclature. The effects of liquid formation and its growth in the flow physics are given as additional source terms in the flow equations. These source terms are discussed by Gerber [7]. The equations (1) to (10) along with the Navier-Stokes equation are used to predict the condensation of expanding steam.

For rapidly expanding steam flows, the vapor properties have to be evaluated at the sub-cooled region since it crosses the saturation line before condensing. The equation-of-state (EOS) of steam, the virial coefficients and its thermodynamic properties are evaluated based on the equations by Young [10, 14].

3. Validation

The developed numerical model and the property functions are integrated to the commercial code ANSYS Fluent using User-Defined Scalars (UDS) and User-Defined Functions (UDF) respectively. The finite volume discretization method is used to solve the conservation equations. A pressure based solver, with non-segregated coupled approach, is used for pressure-velocity coupling. A complex case of condensation in a steam turbine rotor-tip cascade [8] is used for validation.

The same blade profile and computational domain is used in this study to compute the thermodynamic losses. The blade profile is that of a low-pressure (penultimate) stage of a steam turbine where the condensation is maximum. The geometric details of the cascade are given in [8].

![Figure 1: Validation of the numerical model.](image)

A single blade passage (figure 1) with supersonic exit flow condition is used. A case with a pressure ratio of 2.34 and an inlet subcooling of 12K as operating conditions is studied. The choice of turbulence model used is not of much significance in predicting condensation, therefore, the high Reynolds number K-ω SST model appropriate for flows with separation is adopted in this study.
The pressure ratio \((P/P_o)\) on both the suction and pressure sides of the blade are compared (figure 2). It is observed that values on the pressure side of the blade are in agreement with that of the experiments while there is a slight deviation in the computational result in the suction side. The location of the condensation shock is almost same while its strength is slightly under-predicted. This said, the deviation on the suction side is very minimal which is acceptable. With this validation, the above model is used for predicting the thermodynamic losses for nucleating flows.

4. Computation of Thermodynamic Loss

The major source of thermodynamic loss in a steam turbine stage is due to the non-equilibrium between the vapour and the liquid phases. The difference in the rate of latent heat released (from the liquid droplet) and the nucleation rate is responsible for the thermal non-equilibrium. This latent heat transfer leads to an increase in entropy in the system which leads to the thermodynamic loss.

Consider a droplet at temperature \(T_p\) that releases latent heat during condensation to the surrounding vapor which is at temperature \(T_g\). Due to this irreversible heat transfer, there is an increase in entropy in the flow field. This increase in entropy is a direct measure of the thermodynamic loss, which is calculated using the second law of thermodynamics under the assumption that each phase is in thermal equilibrium experiencing an internally reversible heat transfer. The entropy production rate is given by

\[
\Delta S = Q \left( \frac{1}{T_g} - \frac{1}{T_p} \right) d\varphi,
\]

(11)

The rate of heat release \(Q\) (in W) is given as the product of liquid mass generated \(S_a\) and the latent heat \((h_{fg})\) released during generation. The heat dissipation rate due to irreversibility in a control volume can be expressed as

\[
Q_{dis} = \int \int \int \int S_a h_{fg} \left( \frac{1}{T_g} - \frac{1}{T_p} \right) T_{mean} d\varphi
\]

(12)

For droplets of size less than 1μm, the mean temperature can be taken to be equal to either the sub-cooled vapour temperature \(T_g\) or liquid droplet temperature \(T_p\), with a small loss in accuracy [15]. The heat dissipated in a control volume, integrated over the entire domain gives the total energy dissipated.

During transonic flows in the blade passage, shockwaves are observed downstream, near the trailing edge. Across the shock, temperature and pressure of the vapour increases which leads to the evaporation of droplets at these regions. This results in droplets gaining latent heat from the vapour which again increases the entropy. This phenomenon is accounted in equation (12), (by taking the absolute value of the inverse of temperature difference).

To study the losses only due to condensation (and evaporation), it is appropriate to compare the condensing steam flow conditions with the dry exit case as reference. In this study, the thermodynamic loss is quantitatively expressed in terms of enthalpy loss coefficient \((\zeta)\), defined as the ratio of change in static enthalpy due to condensation \((h_{2, wet} - h_{2, dry})\) to the total to static enthalpy change at the exit for dry steam \((h_{0, dry} - h_{2, dry})\).

\[
\zeta = \frac{h_{2, wet} - h_{2, dry}}{h_{0, dry} - h_{2, dry}} \approx \frac{Q_{dis}}{\dot{m}_{dry} \sqrt{V_{2, dry}}}
\]

(13)
where \( m_{\text{dry}} \) and \( V_{\text{2,dry}} \) are the mass flow rate of steam and the average velocity at the exit plane respectively for a dry steam condition. Using the equation (13), the enthalpy loss coefficient is obtained for various operating conditions.

4.1. Effect of pressure ratio

During operation, the pressure ratio varies in these turbine stages with varying load. A prediction of change in thermodynamic loss with change in pressure ratio is carried out in this section. The exit pressure is varied by maintaining a constant inlet sub-cooling of 12 K. The flow is maintained transonic/supersonic as the exit Mach number is varied from 1.04 to 1.45.

The numerical results show that the nucleation rate is lower in the low pressure ratio cases, which implies that equilibrium conditions may not have reached at the exit of the cascade. Due to the higher rate of expansion in the higher pressure ratio conditions, the nucleation rate is high at upstream locations (figure 2.1 and 2.2), where the pressure and temperature are high. The values of latent heat at these upstream locations are higher, since latent heat is a strong function of temperature. The high value of latent heat results in the increased thermodynamic loss (equation (12)). The liquid mass generated is also high for higher pressure ratio, contributing further to the loss (Table 1). Downstream of the cascade, negative values of liquid mass generation are noticed, these are due to the presence of oblique shock, which result in evaporation (figure 2.3 and 2.4).

![Figure 2.1 Liquid mass fraction.](image1.png)

![Figure 2.2 Liquid mass fraction.](image2.png)

![Figure 2.3 Liquid mass generation rate.](image3.png)

![Figure 2.4 Liquid mass generation rate.](image4.png)
Across the shock, the temperature and pressure of steam increases, which leads to local zones where the droplets are surrounded by higher temperature steam. Liquid droplets receive heat from the surrounding steam, which is at a higher temperature to vapourize. This irreversible process also leads to a rise in entropy.

4.2. Effect of inlet sub-cooling
In multi-stage steam turbines operating at high speeds, the state of inlet steam to the last stages is subcooled in most cases. To examine the effect of inlet temperature on condensation and thermodynamic loss, different inlet subcooling (6K, 12K and 15K) conditions are analyzed with the same rate of expansion i.e. at constant pressure ratio.

The exit liquid mass fraction for 6K inlet subcooling scenario is about 5 % whereas for 15K subcooling case its value increased to 6 % (figure 3.1 and 3.2). Also for the lower inlet subcooling (6K) case, the condensation commences later in the bulk steam compared to the higher inlet subcooling condition (figure 3.3). Therefore, the non-equilibrium and heat dissipation is more with higher inlet sub-cooling.
4.3. Effect of incidence angle

The incidence angle of steam, (measured from the tangent to the camber line at the leading edge of the blade) varies with change in mass flow rate of steam, blade speed and during off-design conditions. This variation in incidence affects the location of onset of condensation and thereby losses. To account for these variations, a range of incidence angle between -25 to +25 degree is investigated. It is observed that, as the flow incidence angle increases from -25 to +25 degree, the onset of condensation moves upstream on the suction side (figure 4). For the 0 degree incidence, the nucleation commences at about 40 % of the chord, near the throat of the blade passage (figure 2), whereas for cases with higher positive incidence, the flow turning near the leading edge is more which results in higher expansion rate. This leads to higher degrees of subcooling and, therefore, early onset of nucleation and higher values of heat dissipation. However, this change in location of onset does not significantly affect the value of maximum outlet liquid mass fraction (α) in the domain. When the incidence becomes negative, the velocity component in the horizontal direction is lesser and the flow is directed more towards the suction surface of the blade. As a result, the expansion rate of the vapor is lower, resulting in a low value of heat dissipation and, therefore, thermodynamic loss. This argument is exclusively valid only for loss due to condensation as the other components of losses (aerodynamic) may vary differently.

![-25 degree](image1)

![+25 degree](image2)

**Figure 4.1** Liquid mass fraction.

**Figure 4.2** Liquid mass fraction.

| Case No. | Pressure ratio | Subcooling (K) | Incidence angle (degree) | Exit mass flow(kg/s) (dry case) | Average exit Velocity(m/s) (dry case) | Thermodynamic Loss Coefficient |
|----------|----------------|----------------|-------------------------|-------------------------------|-------------------------------------|-------------------------------|
| 1        | 1.81           | 12             | 0                       | 2.01                          | 411.03                              | 0.053                         |
| 2        | 2.34           | 12             | 0                       | 1.93                          | 484.39                              | 0.064                         |
| 3        | 2.80           | 12             | 0                       | 1.93                          | 527.16                              | 0.067                         |
| 4        | 2.34           | 6              | 0                       | 1.91                          | 489.00                              | 0.055                         |
| 5        | 2.34           | 12             | 0                       | 1.93                          | 484.39                              | 0.064                         |
| 6        | 2.34           | 15             | 0                       | 1.94                          | 482.40                              | 0.066                         |
| 7        | 2.34           | 12             | -25                     | 1.89                          | 478.88                              | 0.061                         |
| 8        | 2.34           | 12             | -15                     | 1.92                          | 483.51                              | 0.063                         |
| 9        | 2.34           | 12             | +15                     | 1.92                          | 483.45                              | 0.065                         |
| 10       | 2.34           | 12             | +25                     | 1.91                          | 482.52                              | 0.066                         |
5. Conclusion
This paper utilizes an Eulerian-Eulerian approach to predict the non-equilibrium condensation based on classical nucleation theory. A numerical approach is used to compute the thermodynamic loss, associated with non-equilibrium condensation, based on entropy generation. The thermodynamic loss component is a significant factor and must be accounted in the estimation of losses in a wet steam flow. This method computes losses associated with condensation as well as evaporation. The various operating conditions that alter the location of onset of condensation and thereby heat dissipation are investigated. From this study, it can be inferred that:

a) When subcooled steam enters the LP stage, the thermodynamic losses are higher for high pressure ratio (due to early large expansion).
b) Higher the degree of subcooling, higher the loss.
c) During higher positive incidence, the flow turning near the leading edge is more leading to greater expansion rates and therefore losses.

6. References

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