Calculation of Dynamic Hysteresis Loss of Power Transformer under Harmonic Background

Shaoxin Meng¹, Xue Cui², Zhexu Li², Jianben Liu¹, Qionglin Li³, Zhiye Du²

¹China Electric Power Research Institute Wuhan Branch, State Key laboratory of Power Grid Environmental Protection, Wuhan, Hubei Province, 430074, China
²College of Electrical and Automation Chemistry Wuhan University, Wuhan, Hubei Province, 430072, China
³State Grid HAEPC Electric Power Research Institute, Zhengzhou, Henan Province, 450052, China

*Corresponding author’s e-mail: dqcx@whu.edu.cn

Abstract: The construction of smart grid uses a large number of power electronic equipment, which leads to the increase of harmonic content of transformer core winding, and brings some difficulties to the calculation of core loss. In order to study the influence of harmonics on core loss, a dynamic hysteresis loss model of transformer core under harmonics is established. On the basis of J-A static hysteresis model, the calculation methods of eddy current loss and abnormal loss under harmonic wave are deduced, and the calculation method of core loss under harmonic wave is established according to the principle of energy conservation. The core loss of three-phase transformer is simulated and verified by experiments under different harmonic frequency and harmonic voltage percentage. The results verify the validity of the model.

1. Introduction

With the rapid development of power system and the wide application of power electronic equipment, the harmonic problem in power grid becomes more and more serious. The long-term operation of transformer in harmonic environment will increase the temperature rise noise of transformer and affect the service life of transformer. Accurate core loss measurement is the key to core loss analysis and optimization [1]. The core mainly consists of ferromagnetic materials, usually silicon steel sheets. The magnetization and loss of silicon steel sheet are non-linear. Compared with sinusoidal excitation, the magnetic properties of silicon steel sheet under complex excitation including harmonics have changed obviously, and the loss problem under complex excitation can not be solved according to the magnetic properties under sinusoidal excitation. How to accurately calculate the core loss of ferromagnetic materials under harmonic conditions has attracted wide attention [2].

At present, for the study of core loss under complex excitation conditions, reference [3] measured core loss under DC bias excitation based on a small transformer; reference [4] studied core loss under distorted flux, and investigated the influence of harmonic phase difference on core loss; reference [5] adopted improved Be. The core loss is calculated by rtotti loss model. The core loss under complex excitation is measured by building a product-level laminated core model. The simulation is carried out by Mag-Net, a magnetic field finite element software. Reference [6] Based on the hysteresis loop measurement system, the core under DC bias is predicted by parameter discrimination and correction. The prediction method of core loss under non-sinusoidal excitation is studied in reference...
[7]. It is found that the square of magnetic loss and waveform coefficients has some relationship with the product of frequency, and then the core loss under non-sinusoidal excitation is predicted by experiment. Although the above-mentioned references have all involved the measurement of core loss under harmonic excitation, they have not been used for iron loss measurement. The action law of heart loss was analyzed in detail.

In this paper, the factors affecting the hysteresis loss of ferrocardia under harmonic wave are analyzed. By using J-A static hysteresis model and considering eddy current loss and abnormal loss under harmonic wave, a method for calculating the hysteresis loss of ferrocardia under harmonic wave is established. The correctness of the model is verified by experiment and simulation.

2. Establishment of dynamic hysteresis model of iron core

Under the action of alternating magnetic field, the hysteresis loop formed by periodic repeated magnetization of ferromagnetic materials is called dynamic hysteresis loop, and its area is core loss. Therefore, the foundation of core loss research is to establish an accurate core hysteresis model and calculate the hysteresis loop. According to the theory of core loss separation, core loss consists of hysteresis loss, eddy current loss and abnormal loss. Hysteresis loss is independent of frequency, while eddy current loss and abnormal loss are related to frequency. Considering that JA static core hysteresis theory can only accurately obtain frequency-independent hysteresis loss, this paper further establishes a ferrodynamic hysteresis model considering eddy current loss and abnormal loss.

2.1 Traditional J-A Hysteresis Model

The expression of hysteresis-free magnetization curve is [8]:

\[
\frac{dM}{dH} = \frac{k_0 - \alpha M_s - M}{1 - \alpha c} + \frac{c M_s}{dH} \quad (1)
\]

In the formula, \( k \approx H \) (\( H \) is the coercive force), \( M_s \) is the hysteresis-free magnetization, \( M_a \) is the saturated magnetization, \( c \) is the parameter characterizing the reversible domain wall motion, \( a = \frac{k_B T}{\mu_0 m} \), \( k_B \) is the Boltzmann constant, \( T \) is the thermodynamic temperature, \( \mu_0 \) is the permeability, \( m \) is the magnetic moment, and \( \alpha \) is the minimum field coefficient. The precision of parameters is very high. The J-A model obtained by experiments and optimized by parameters has better convergence.

The harmonic loss of transformer core mainly includes hysteresis loss, eddy current loss and abnormal loss. Hysteresis loss is the loss caused by hysteresis during repeated magnetization of ferromagnetic materials, which is related to the crystalline purity, orientation and distortion of internal grains in silicon steel sheets. The area around the hysteresis loop is the magnitude of hysteresis loss. The hysteresis loss of J-A hysteresis loop can be obtained by integrating the loop directly [9]:

\[
P_h = f \cdot v \cdot w_h \quad (2)
\]

In the formula, \( f \) is the frequency of magnetization, \( v \) is the volume of core, and \( w_h \) is the hysteresis loss of magnetic material per cycle of magnetization in unit volume, which is equal to the area of the closed curve surrounded by the hysteresis loop B-H of core.

In order to facilitate the analysis of the relationship between hysteresis loss and harmonic frequency, Steinmetz hysteresis loss formula is introduced:
In the formula, $\eta$ is the Steinmetz coefficient, whose value is constant depending on the material and the unit system adopted; $S$ is the Steinmetz index, which is usually taken from 1.5 to 2.5 according to the different materials.

Because:

$$\max_{h} 4.44 \nu f N A B = (4.44)^{n} S v K N A \nu \eta =$$

(4)

In the formula: $N$ is the number of winding turns; $A$ is the effective cross-section area of the core; $V$ is the induction voltage.

The simultaneous type 2, type 3 and type 4 are available:

$$P_{h} = \frac{K_{h}(\nu)^{s}}{f^{s-1}}$$

(5)

$K_{h} = \frac{\eta \nu}{(4.44 NA)^{s}}$ is a constant in the formula. When there are multiple harmonics and the maximum harmonic number is $M$, the hysteresis loss can be expressed as:

$$P_{n} = P_{1} + P_{2} + \ldots + P_{n} = \sum_{n=1}^{M} (K_{h}^{n}(\nu_{n})^{s})$$

(6)

In the formula, $P_{n}$ is the hysteresis loss of the $n$th harmonic, $\nu_{n}$ is the effective value of the induced voltage of the $n$th harmonic, and $f_{n}$ is the frequency of the $n$th harmonic.

It can be seen that the hysteresis loss increases with the increase of harmonic voltage content and decreases with the increase of harmonic frequency.

2.2 Eddy Current Model

Considering a single silicon steel sheet, the instantaneous power generated by eddy current in a unit volume can be expressed as [10]:

$$P_{e}(t) = \frac{w^{3}}{12 \rho} \left(\frac{dB}{dt}\right)^{2}$$

(7)

Among them, $\rho$ is the resistance coefficient of ferromagnetic material.

Based on the orthogonality of trigonometric function, the calculation formula of eddy current loss per unit volume in the presence of harmonics can be deduced from literature:

$$P_{e} = \sum_{n=1}^{M} \frac{1}{R_{e}} \frac{1}{T} \int_{0}^{T} [V_{n}(t)]^{2} dt$$

(8)

In the formula, $R_{e} = \frac{12 \rho (NA)^{2}}{w^{2}}$ is the resistance parameter to characterize the core loss; $\rho$ is the resistivity of silicon steel sheet; $A$ is the effective cross-sectional area of silicon steel sheet; $N$ is the number of transformer excitation windings turns; $w$ is the thickness of silicon steel sheet.

2.3 Establishment of Abnormal Loss Transient Model

This part of the loss belongs to the micro category, and the current technical means are difficult to simulate according to its physical mechanism. Therefore, according to the statistical law, a mathematical model of abnormal loss is established [11]:

$$P_{an} = \sqrt{GSV \sigma \left|\frac{dB}{dt}\right|^{3}} = k_{a} \left|\frac{dB}{dt}\right|^{3}$$

(9)

The calculation of abnormal loss, in the case of harmonics, takes into account:
The average value of abnormal loss in the case of multiple harmonics can be obtained:

\[
P_{av} = \sum_{n=1}^{N} \frac{k_s}{(NA)^2} \int_{0}^{T} |V_n(t)|^2 dt
\]

Among them, \( k_s = \sqrt{GSV_0 \sigma} \) is the abnormal loss coefficient, \( S \) is the cross-sectional area of the laminate, \( G \) is the coupling constant without unit \( G=0.1356 \), \( V_0 \) is the statistical coupling field parameter, and its magnitude is related to the AC peak magnetic density \( B_m \). It can be solved by subtracting the total loss from the eddy current loss in a linear relationship with the root mean square of the frequency. In order to simplify the calculation, we set the \( V_0 \) unchanged on the basis of referring to the relevant literature at home and abroad, thus we can get that the value of \( k_s \) is unchanged.

### 2.4 Core dynamic hysteresis loss and calculation method

Based on the loss separation theory, the total hysteresis loss can be divided into static hysteresis loss, eddy current loss and abnormal loss \[12]\:

\[
W = W_h + W_e + W_{av}
\]

The energy balance equation of dynamic J-A hysteresis model for eddy current loss and residual loss is as follows:

\[
\mu_s \int M_{an}(H) dH_s = \mu_s k_c(1-c) \int \left( \frac{dM_{an}}{dH_s} \right) dH_s + k_s \int \left( \frac{dB}{dt} \right)^2 dt + k_h \int \left( \frac{dV}{dt} \right)^2 dt + \mu_s \int M(H) dH_s
\]

Substituting Equation 6, Equation 8, and Equation 11 into Equation 12 can obtain the total core harmonic loss under multiple harmonics:

\[
P_r = \sum_{k=1}^{M} \left( HRU_k \right)^2 P^1 + \sum_{k=1}^{M} \left( HRU_k \right)^2 P^1 + \sum_{k=1}^{M} \left( HRU_k \right)^2 P_{an}^1
\]

When we know that each harmonic voltage contains \( HRU_k \), we can estimate the eddy current loss, hysteresis loss and abnormal loss under harmonic wave respectively, which are multiple of the respective loss under fundamental wave, and obtain a simple expression of iron loss as follows:

\[
P_r = \sum_{k=1}^{M} \left( HRU_k \right)^2 P^1 + \sum_{k=1}^{M} \left( HRU_k \right)^2 P^1 + \sum_{k=1}^{M} \left( HRU_k \right)^2 P_{an}^1
\]

In the formula, \( HRU_k \) is the percentage of \( K \) harmonic voltage; \( M \) is the maximum harmonic frequency; \( P^1 \) is the eddy current loss of iron core under fundamental wave; \( P^1 \) is the magnetic hysteresis loss of iron core under fundamental wave; \( P_{an}^1 \) is the abnormal loss of iron core under fundamental wave.

### 3. Model validation

#### 3.1 Transformer Parameters

According to the harmonic loss model of transformer core, the core loss can be calculated by MATLAB programming under different harmonic frequency and harmonic voltage percentage. The relationship between harmonic voltage and iron loss is studied by setting harmonic voltage of different times and different contents on the primary side of no-load secondary side of transformer. For the three-phase transformer used in experiment and simulation, the model is ZSFG (H) - 400/10; the connection group number is Yy0, and the specific parameters are shown in Table 1.
### Table 1 Transformer parameters

| Parameter                          | Value       |
|------------------------------------|-------------|
| Rated capacity /kVA                | 400         |
| Rated current /A                   | 23.1/577.4  |
| High Voltage Winding Resistance /Ω | 5.249       |
| Rated no-load loss /W              | 968         |
| Short circuit loss /W              | 8 951       |
| Turn Number of Primary Winding     | 600         |
| Equivalent length of magnetic circuit /m | 1.687 5   |
| Winding layer thickness /m         | 0.001 2     |
| Rated voltage /V                   | 10 000/400  |
| Rated Frequency /Hz                | 50          |
| Low Voltage Winding Resistance /Ω  | 0.006       |
| No-load current /%                 | 0.60        |
| Impedance voltage /%               | 5.18        |
| Turn Number of Secondary Side Winding | 24       |
| Steinmez coefficient S              | 2           |

In the simulation calculation, it is difficult to obtain some parameters, such as the statistical coupling field parameters, the specific parameters of silicon steel sheet, etc. which are set on the basis of reference to the relevant literature. Therefore, there will be some errors between the simulation results and the actual results.

### 3.2 Experimental Platform

Relevant experiments are mainly carried out on the 10 kV power quality integrated experimental platform of the "Power Harmonic Characteristic Analysis and Evaluation Technology Laboratory" of the State Grid Corporation. The overall structure of the experimental platform is shown in Figure 1. The platform is based on the chain bidirectional converter and adopts the phase-shifted pulse width modulation (PWM) technology with voltage. The two modes of disturbance and current disturbance can realize the real simulation of all typical power quality problems in medium voltage distribution network [13].

### 3.3 Experimental Method

Because the original no-load current of transformer is very small (about 0.13 A) and the secondary no-load current is relatively large (about 3.5 A), no-load experiment is carried out on the secondary side of transformer in order to improve the accuracy of current measurement. The experimental scheme is as follows:

1) The rated fundamental voltage of 400 V is applied to the auxiliary side of the transformer.
2) Then the 5th harmonic voltage is superimposed to measure and record the no-load loss values of the 5th harmonic voltage containing 1%, 3%, 5%, 7%, 9%, 11%, 13% and 15% respectively.
3) Repeat step 2 to measure and record the no-load losses at 7, 11, 13, 17, 19 and 23 harmonic voltages.
3.4 Experiments and simulation results

Measuring and recording the average temperature of the surrounding air, drawing and fitting the relationship curves of "loss-harmonic voltage percentage" and "loss-harmonic voltage number" according to the no-load loss values of three-phase transformer under different harmonic frequency and different harmonic voltage percentage, and simulating the core loss. The comparison results are shown in Fig. 2-5.

Fig. 1 10 kV power quality comprehensive experimental platform

Fig. 2 5-23 Harmonic Iron Loss Varies with Harmonic Voltage Inclusion Rate (Simulation Results)

Fig. 3 5-23. Variation of harmonic iron loss with harmonic voltage percentage (experimental results)

Fig. 4 The variation curve of iron loss with harmonic number at different harmonic voltage percentage (simulation results)

Fig. 5 The curve of iron loss with the number of harmonics at different harmonic voltage containing rates (experimental results)
When analyzing the spectrum of the waveform recorded data, it is found that the 5th, 7th and 11th harmonic voltages applied in the experiment contain different degrees of 5th, 7th and 11th harmonics, and the corresponding harmonic contents are shown in the table below.

| harmonic frequency | 5th harmonic content (%) | 7th harmonic content (%) | 11th harmonic content (%) |
|-------------------|-------------------------|--------------------------|--------------------------|
| 5                 | 69.71                   | 18.89                    | 11.4                     |
| 7                 | 15.32                   | 77.98                    | 6.7                      |
| 11                | 9.23                    | 3.85                     | 86.92                    |

According to the calculation formula of harmonic core loss and harmonic voltage percentage, the approximate estimates of 5, 7 and 11 harmonic core loss errors caused by harmonic impurity are 4.57%, 3.20% and 1.40% respectively (compared with the experimental data).

The variation trend of transformer core loss curve obtained by theoretical calculation is the same as that obtained by practical experiment. There are several main reasons for the errors between calculated and measured data.

1) The data used in theoretical calculation are not accurate enough.
2) Compared with the actual situation of transformer iron loss, the theoretical calculation model still has some differences and needs further improvement.
3) In the process of calculating abnormal loss, the treatment of abnormal loss coefficient is not perfect enough and needs to be further studied.
4) The applied 5th, 7th and 11th harmonic voltages are not pure, resulting in relatively large errors.

4. Conclusion
(1) In this paper, a core loss calculation model considering dynamic hysteresis loss in harmonic case is established, and the calculation method of the model is optimized. The accuracy of the core loss model is verified by simulation and experiment.

2) For transformer core loss, when the harmonic voltage content increases from 3% to 9%, the iron loss increases by nearly 7.5 times, while when the harmonic voltage content increases from 9% to 15%, the iron loss increases by nearly 2.54 times. This shows that the core loss of transformer increases greatly when the harmonic voltage content ratio is low.

3) The harmonic core loss of transformer decreases slightly with the increase of harmonic frequency, and increases rapidly with the increase of harmonic voltage content. Generally speaking, the number of harmonics has little effect on core loss, and the main factor affecting core loss is the harmonic voltage percentage. Therefore, in actual operation, efforts should be made to reduce the harmonic content of transformers.

Acknowledgment
This work was supported by Open Fund of State Key Laboratory of Power Grid Environmental Protection (No. GYW51201801171).

Reference
[1] S. M. Yang, Z. G. Cheng, X. Q. Zhu, et al. (2007) Measurement technology of stray loss in steel[J]. Journal of Instruments and) Instruments, 28(11): 2039-2044.
[2] L. F. Zhu, J. G. Zhu, W. M. Tong, etc. (2017) Analytical calculation of harmonic losses of axial flux amorphous motors fed by PWM inverters [J]. Journal of Electrical Technology, 32 (16): 115-123.
[3] L. Cao, J. L. He, B. Zhang. (2008) Model and validation of hysteresis loss of power transformer under DC bias [J]. Journal of Electrical Engineering, 28 (24): 141-146.
[4] Y. Liu, F. Y. Yang, Y. N. Fan, et al. (2017) Experimental study and simulation analysis of transformer core model loss under distorted flux effect [J]. New Electrical and Electrical Power Technology, 65-69.
[5] G. Liu, L. P. Sun, X. G. Wang, et al. (2018) Improvement and simulation application of core loss calculation method under sinusoidal and harmonic excitation [J]. Journal of Electrical Technology, 33(21): 13-22.

[6] Y. Wang, Z. Z. Liu, (2017) Core loss prediction under DC bias based on Jiles Atherton hysteresis theory [J]. Journal of Electrical Engineering, 37(1): 313-322.

[7] S. Yanase, H. Kimata, Y. Okazaki, et al. (2005) A simple predicting method for magnetic losses of electrical steel sheets under arbitrary induction waveform[J]. IEEE Transactions on Magnetics, 41(11):4365-4367.

[8] Z. Li, Q.M. Li, et al. (2011) J-A. Research on the Questioning and Correction Method of Magnetization Modeling Theory [J]. Chinese Journal of Electrical Engineering, 31 (3): 124-131.

[9] Gao Lixinka V, Kelly D H. (1982) Electromechanical Energy Conversion [M]. 2 Edition. Beijing: National Defense Industry Press, 47-50.

[10] Jiles D C. (1994) Frequency dependence of hysteresis curves in conducting magnetic materials[J]. IEEE Transactions on Magnetics,75(10):5511-5511.

[11] D. C. Jiles, (1994) “Modelling the effects of eddy current losses on frequency dependent hysteresis in electrically conducting media,” IEEE Trans.Magn., vol. 30, no. 6, pp. 4326–4328, Nov.

[12] Baghel A P S, Kulkarni S V. (2014) Dynamic Loss Inclusion in the Jiles–Atherton (JA) Hysteresis Model Using the Original JA Approach and the Field Separation Approach[J]. IEEE Transactions on Magnetics, 50(2):369-372.

[13] Q. L. Li, L. Zou, et al. (2013) Simulation Calculation and Experimental Study on Harmonic Loss of Power Transformers[J]. Power System Technology, v.37; No.361(12):3521-3527.