Phantom scalar emission in the Kerr black hole spacetime

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Abstract

We study the absorption probability and Hawking radiation spectra of a phantom scalar field in the Kerr black hole spacetime. We find that the presence of negative kinetic energy terms modifies the standard results in the graybody factor, super-radiance and Hawking radiation. Comparing with the usual scalar particle, the phantom scalar emission is enhanced in the black hole spacetime.

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1. Introduction

Our universe is assumed to be filled with dark energy because it can explain the accelerated expansion of the universe, which is strongly supported by many cosmological observations [1, 2]. Dark energy is an exotic energy component with negative pressure and constitutes about 72% of present total cosmic energy. The leading interpretation of such a dark energy is the cosmological constant with an equation of state $\omega_x = -1$ (for a classic review see [3], for a recent good review see [4] and for a recent discussion see [5–7]). The energy density of this dark energy is associated with quantum vacuum [3–8]. Although this explanation is consistent with the observational data, it is plagued with the so-called coincidence problem, namely ‘why are the vacuum and matter energy densities of precisely the same order today?’.

Therefore, the dynamical scalar fields, such as quintessence [9], $k$-essence [10] and phantom field [11], are proposed as possible alternatives of dark energy.

Compared with other dynamical scalar fields, the phantom field model is more interesting because it has a negative kinetic energy and the super negative equation of state $\omega_x < -1$. Although the null energy condition is violated, this dark energy model is not ruled out by recent precise observational data involving CMB, the Hubble Space Telescope and type Ia Supernova [12]. The dynamical evolution of the phantom field in cosmology has been investigated in the
last few years [13–20]. It shows that the energy density increases with time and approaches to infinity in a finite time [13]. This implies that in standard Einstein cosmology the flat universe dominated by phantom energy will blow up incessantly and arrive at a future singularity finally named big rip which has such a strong exclusive force that anything in the universe including the large galaxies will be torn up. Recently, many efforts have been made to avoid the big rip [21]. It has been argued that this future singularity could vanish in the universe if one considers the effects from loop quantum gravity [22–25].

The presence of negative kinetic energy results in many exotic properties of the phantom field in the black hole spacetime. Babichev et al [26] considered the phantom energy accretion of the black hole and found that the mass of the black hole is decreased. This can be explained by the fact that the kinetic energy of the phantom field is negative which yields the super negative equation of state $\omega_x < -1$. The decrease of mass of the black hole in the phantom energy accretion will lead to the ratio between charge and mass of the black hole possibly being larger than 1 ($Q/M > 1$) and there may exist a naked singularity [27], which implies that the cosmological censorship is violated. The negative kinetic energy also yields that the dynamical evolution of phantom scalar perturbations possesses some unique characteristics in the black hole spacetime [28]. One of them is that it grows with an exponential rate in the late-time evolution rather than decays as in the usual scalar perturbations. These new results will excite more efforts to be devoted to the study of phantom energy in the background of a black hole. In this paper we will focus on the Hawking radiation of the phantom scalar particles in the Kerr black hole spacetime and see what is the effect of the negative kinetic energy on the power and angular momentum emission spectra of the Hawking radiation.

2. Phantom scalar emission in the Kerr black hole spacetime

In the curve spacetime, the action of the phantom scalar field with the negative kinetic energy term is

$$S = \int d^4x \sqrt{-g} \left[ -\frac{R}{16\pi G} - \frac{1}{2} \partial_{\mu}\psi \partial^{\mu}\psi + V(\psi) \right]. \quad (1)$$

Here, we take metric signature $(+−−−)$ and the potential $V(\psi) = -\frac{1}{2}\mu^2\psi^2$, where $\mu$ is the mass of the scalar field. Varying the action with respect to $\psi$, we obtain the Klein–Gordon equation for a phantom scalar field in the curve spacetime

$$\frac{1}{\sqrt{-g}} \partial_{\mu}(\sqrt{-g} g^{\mu\nu} \partial_{\nu})\psi - \mu^2 \psi = 0. \quad (2)$$

The presence of negative kinetic energy leads to the negative sign of the mass term $\mu^2$ in the wave equation, which will yield the peculiar properties of the Hawking radiation of the phantom scalar particle in the black hole spacetime.

The well-known Kerr metric in the Boyer–Lindquist coordinate is

$$ds^2 = \left(1 - \frac{2Mr}{\Sigma}\right)dt^2 + \frac{4Mar\sin^2\theta}{\Sigma} dt d\phi$$

$$-\frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 - \left(r^2 + a^2 + \frac{2Mar^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2, \quad (3)$$

with

$$\Delta = r^2 - 2Mr + a^2, \quad \Sigma = r^2 + a^2 \cos^2\theta, \quad (4)$$

where $M$ is the mass and $a$ is the angular momentum of the black hole. Equation (2) is separable in terms of the spheroidal harmonics $\psi(t, r, \theta, \phi) = e^{(-\omega t + \alpha r + \phi)} R(r) S(\theta)$. The angular and the
The power and angular momentum emission spectra of the phantom scalar particle is then numerically.

Radial functions $S(\theta)$, $R(r)$ obey to

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left[ \sin \theta \frac{dS(\theta)}{d\theta} \right] + \left[ (\omega^2 + \mu^2) \cos^2 \theta - \frac{m^2}{\sin^2 \theta} + \lambda \right] S(\theta) = 0,$$

(5)

and

$$\frac{d}{dr} \left[ \Delta \frac{dR(r)}{dr} \right] + \left\{ \frac{(r^2 + a^2) \omega - ma}{\Delta} + \mu^2 r^2 - E_{\text{Im}} \right\} R(r) = 0,$$

(6)

respectively. Where $\lambda$ is the eigenvalue and the function $E_{\text{Im}} = \lambda + \omega^2 a^2 - 2ma\omega$. In order to calculate the absorption probability $|A_{lm}|^2$ and the luminosity of the Hawking radiation for a phantom scalar particle, we must solve the radial equation (6) above. Following the standard matching techniques [29–38], we can create a smooth analytical solution of equation (6) in the low-energy and low-angular momentum limit. Near the horizon ($r \sim r_+$) regime and at infinity, it has the form

$$r \to r_+, \quad R(r = r_+) = A^{(r = r_+)}_{\text{in}} e^{-(\omega - m\Omega_0) r_+} + A^{(r = r_+)}_{\text{out}} e^{(\omega - m\Omega_0) r_+},$$

(7)

$$r \to \infty, \quad R(r = \infty) = A^{(r = \infty)}_{\text{in}} e^{-\sqrt{\omega^2 + \mu^2} r} + A^{(r = \infty)}_{\text{out}} e^{\sqrt{\omega^2 + \mu^2} r},$$

(8)

respectively. Unlike the usual scalar particle, we find that for the phantom particle with an arbitrary value of $\mu$ the solution above denotes incoming and outgoing spherical waves at large distances from the black hole. From this solution, we can calculate the absorption probability

$$|A_{lm}|^2 = 1 - \frac{|A_{\text{out}}^{(r = \infty)}|^2}{|A_{\text{in}}^{(r = \infty)}|^2}.$$  

(9)

The power and angular momentum emission spectra of the phantom scalar particle is then written as

$$\frac{d^2 E}{dt \, d\omega} = \sum_{l,m} \frac{\omega^2}{\sin \mu \Omega_0 t} \frac{N_l|A_{lm}|^2}{2\pi \sqrt{\omega^2 + \mu^2}},$$

(10)

$$\frac{d^2 J}{dt \, d\omega} = \sum_{l,m} \frac{m \omega}{\sin \mu \Omega_0 t} \frac{N_l|A_{lm}|^2}{2\pi \sqrt{\omega^2 + \mu^2}},$$

(11)

where $T_H$ is the Hawking temperature of the Kerr black hole. These equations can be integrated numerically.

Here we present the numerical results about the absorption probability $|A_{lm}|^2$ and the Hawking radiation of a phantom scalar field in the background of a Kerr black hole.

In figure 1, we fix $a = 0.2$ and examine the dependence of the absorption probability of the phantom scalar particle on its mass $\mu$ for the first partial waves ($l = 0, m = 0$) in the background of a Kerr black hole. Comparing with the usual scalar particle, the absorption probability of the phantom increases rather than decreasing as the mass $\mu$ increases. This is not surprising because in the wave equation of phantom field (2) the negative sign of the mass term gives rise to the absorption probability in the form $|A_{00}|^2 \sim 4\omega \sqrt{\omega^2 + \mu^2} (r_s^2 - a^2)$ in the low-energy and low-angular momentum limit. For other values of $l$, we also find that the absorption probability of the phantom scalar particle increases with the mass $\mu$.

In figure 2, we plot the absorption probability of the phantom particle for the mode ($l = 1, m = 1$) and find that the super-radiance possesses some peculiar properties in this case. First, for the phantom particle the range of $\omega$ for the super-radiance to happen is independent of $\mu$, which means that the phantom scalar particle can be radiated away without any constraint of the particle’s energy. This can be explained by the fact that the
wavefunction of the phantom particle at the spatial infinity has the form $e^{i \sqrt{\omega^2 + \mu^2} r_*}$. It is different from that of the usual scalar particles. It is well known that the usual scalar particle with the energy $\omega^2 < \mu^2$ cannot be radiated away. These particles are forced to be trapped in the system and the amplified wave can accumulate in the potential well, which will yield a black hole bomb in which the mass $\mu$ of a usual scalar particle plays the role of a natural mirror [39]. While in the super-radiance of the phantom scalar particle, it cannot yield a black hole bomb since all phantom scalar particles can be radiated away and their wavefunctions are not in the bounded state. Second, the magnitude of the super-radiance of phantom particle increases with the mass $\mu$. While for a usual scalar particle, it decreases. The new properties of super-radiance imply that the phantom field will enhance the Hawking radiation of the black hole.

Let us now study the luminosity of the Hawking radiation of phantom particles in the Kerr black hole spacetimes. We first focus on the first partial waves ($l = 0, m = 0$), which play a dominant role in the graybody factor. The luminosity of the Hawking radiation for the phantom scalar particle can be expressed as

$$L = \int_{\omega_o}^{\infty} \frac{|A_{00}|^2}{\sqrt{\omega^2 + \mu^2}} \frac{\omega^2}{e^{(\omega - \Omega)/\Gamma} - 1} \frac{d\omega}{2\pi}.$$  (12)
Here, the low bound of the integral is zero rather than $\mu$ since the emission of the phantom scalar particle is not constrained by the value of the particle mass $\mu$. In general, this integral cannot be computed analytically. In figure 3, we present some numerical results. For the fixed $\mu$, the Hawking radiation of the phantom scalar particles decreases as the angular momentum increases. This is the same as that of the usual scalar particles in the Kerr black hole. But for the fixed $a$, with an increase in the mass $\mu$ the Hawking radiation of the black hole increases for the phantom scalar field and decreases for the usual scalar field. Therefore, the presence of negative kinetic energy leads to that the emission of the phantom field increases and the black hole evaporates more quickly. Figure 4 also tells us that with the increase of the mass $\mu$ the difference between the emission of the phantom and the usual scalar particle increases rapidly. When $\mu = 0.1$, the luminosity of the Hawking radiation of the phantom field is almost 1.5 times that of the usual scalar one for all $a$. Thus, for the larger $\mu$, the Hawking radiation of the
black hole is dominated by the phantom scalar field. In figures 5 and 6, we summed up to \( l = 4 \) modes in calculating the energy and angular momentum emission rates. We find that the mass \( \mu \) of phantom scalar particle enhances the power and angular momentum emission spectra. Our results are consistent with those obtained in the phantom energy accretion of the black hole where the black hole mass is decreased. This leads to the increase of the temperature of the black hole and enhances the luminosity of the Hawking radiation.

3. Summary

In this paper, we have studied the graybody factor and Hawking radiation for a phantom scalar field in the background of the Kerr black hole in the low-energy and low-angular momentum approximation. We have found that the absorption probability and Hawking radiation contain the imprint of the phantom scalar field. The effects of \( \mu \) on the super-radiance and the luminosity of the Hawking radiation for the phantom scalar field are different from that for the usual scalar field since the phantom scalar particle possesses a negative kinetic energy. Our results may help us to detect whether our universe is filled with the phantom field or not. It would be of interest to generalize our study to other black hole spacetimes, such as stringy black holes etc. Work in this direction will be reported in the future.
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