I. INTRODUCTION

The perceived accelerating expansion of the universe is the most mysterious puzzle of contemporary theoretical physics. The standard interpretation of this acceleration requires the presence of a substance with negative pressure, i.e. vacuum energy or dark energy. The origin and nature of matter with such exotic equation of state is presently not understood. Recent measurements of the temperature fluctuations in the cosmic microwave background \[1\] and the Hubble diagram of supernovae \[2, 3\] indicate that the present average cosmological vacuum energy problems with gravity and quantum physics (for recent reviews see for example \[2]\) or \[3]\) and references therein). A possible avenue to solve the first problem is the utilization of a fundamental symmetry which readily renders the vacuum energy zero. To explain the size of the vacuum energy some landscape or multiverse scenarios derive it from anthropic considerations \[7, 8\]. Another class of proposed solutions introduces a scalar field that couples to gravity such that the quantum fluctuations are cancelled when the scalar field reaches its equilibrium value. Recently, much attention has been focused on quintessence models \[9\].

In this work, we are only concerned with the first problem. What explains the naively unnatural smallness of the critical density? Why does quantum field theory overestimate the vacuum energy density by 123 orders of magnitude? What protects the vacuum energy from becoming the order of the Planck scale? We argue that in holographic theories \[10\] the entropy bound sets an upper limit on the average energy density. Based on this, we derive the energy density of the universe from the Fischler-Susskind entropy limit \[11\]. We show that the energy density of the universe is bound by the inverse of its entropy when the holographic bound is saturated. Thus in the framework of holographic theories the solution of the first vacuum energy problem is profoundly simple: the amount of quantum fluctuations (and matter energy) is capped by the entropy limit of the universe.

General relativity is the prime example of a holographic theory \[12\]. As a consequence of this, Friedmann’s equation governing the evolution of the universe is known to saturate the holographic bound \[13\] for a wide range of cosmological parameters \[14\]. We show that holography is the primary reason that the dynamic argument based on Friedmann’s equation yields the smallness of the total energy density and thereby the vacuum energy. On the contrary, quantum field theories in their present form are not holographic. This is why they overestimate the vacuum energy by an enormous magnitude, ...
related to the ratio of the horizon area to the Planck length squared. As quantum fluctuations fill the available quantum degrees of freedom, the observed increase of the vacuum energy is natural in an expanding universe.

II. THE VACUUM ENERGY PROBLEM

To quantify the vacuum energy problem, we seek the average energy density, \( \rho = E/V \), of a homogeneous, spherical system occupying a volume \( V = 4\pi R^3/3 \). We assume that the total angular momentum and charge of the system are negligible. In any extensive theory, such as quantum field theories, we can stack (real or virtual) energy quanta of Planck size \( M_P \) at Planck length \( L_P = 1/M_P \) distances from each other to fill a volume \( V \), such that the total energy is \( E \sim (V/L_P^3)M_P \). This idea can be formalized as a Planck scale ultraviolet regularization, and leads to an enormous energy density

\[
\rho \sim M_P^4.
\]

This result is typically contrasted with the critical density \( \Omega \) which is, indeed, about 123 orders of magnitude smaller than the quantum field theory estimate. Friedmann’s equation (1), relating the energy density to the Hubble constant, follows from general relativity. This begs the question: why does general relativity relate the energy density to a fundamental parameter of the universe ‘correctly’, while quantum field theory grossly fails?

III. THE VACUUM ENERGY: NO PROBLEM

We argue that the smallness of the observed cosmological vacuum energy density is a simple and natural consequence of two known facts:

- the universe is a holographic system, i.e. its entropy is limited by a holographic entropy bound, and
- the universe is large (compared to the Planck length).

To illustrate this, we consider the following example. While trying to pile Planck energy quantum oscillators at Planck distances, it is easy to realize that gravity will strongly limit the total energy in a given volume. Since the radius of a Planck mass Schwarzschild black hole is \( L_P M_P^2/2 \), when placing two of them close enough to each other they form a black hole with radius \( L_P M_P^2 \). Any extra energy in the \( 4\pi L_P^3/3 \) volume is gravitationally unstable. Thus, the energy of the gravitationally stable configurations in a given volume is limited by the radius, rather than the volume.

When assuming that an \( R^3 \) volume can be filled by \( M_P \) quanta at \( L_P \) distances, we overestimate the vacuum energy density roughly by a factor of \( r \sim (2R/L_P)^3/(2R/L_P) \). In the case of a black hole with the present size of the universe this factor is

\[
r \sim \frac{4M_P^2}{H_0^2} \sim 3 \times 10^{122},
\]

where we equated the radius with the apparent horizon of the universe.

While it is obvious for a Schwarzschild black hole, we argue that it is a generic property of holographic systems that their total energy scales with linear size as a straightforward consequence of holography. More precisely, a system that saturates the holographic bound also satisfies the Schwarzschild condition, i.e. its maximal mass is the half of its radius in Planck units. To show this, we consider a system with total energy \( E \). The system is to saturate the holographic entropy bound, so we are looking for its maximum entropy. Since the entropy of the system is bound from above by the entropy of a black hole, its maximal entropy is

\[
S = \pi R^2 M_P^2.
\]

If the system saturates Bekenstein’s entropy bound

\[
S \leq 2\pi ER,
\]

then

\[
E = \frac{R}{2} M_P^2,
\]

that is the Schwarzschild condition is satisfied by the system.

Let us calculate the average energy density \( \rho \) of a homogeneous, spherical system that saturates the holographic entropy bound. Since the mass of any gravitationally stable holographic system is bound from above by the Schwarzschild condition, the maximal energy density of the system

\[
\rho = \frac{3}{8\pi R^2} M_P^2
\]

is the inverse function of its size. This implies that a small system with \( R = L_P \) has an upper limit \( \rho = 3M_P^2/8\pi \) on its energy density, as naively expected in field theory. On the other hand, a larger system will necessarily have a smaller energy density. This is the simple consequence of holography: energy scales with linear size, so the energy density decreases with the area. This scaling is consistent with the smallness of the measured cosmological energy density: a system the size of the universe has a rather stringent upper limit on its energy density. The holographic principle reconciles the field theory Planck scale cutoff with the smallness of the cosmological vacuum energy density in a remarkably simple manner.

To show that the above limit on the energy density will also include the energy density of the vacuum, consider a Schwarzschild-de Sitter black hole with mass \( M \) and cosmological constant \( \Lambda \). The energy contained inside its horizons is

\[
E = M + \Lambda \frac{R^3}{3} = \frac{R}{2} M_P^2.
\]

We introduce the matter and vacuum energy densities as \( \rho_M = M/V \) and \( \rho_V = \Lambda/(4\pi) \), which leads to

\[
\rho = \rho_M + \rho_V = \frac{3}{8\pi R^2} M_P^2.
\]
Since any gravitating system with the same radius and higher energy content is gravitationally unstable, we consider the right hand side of this equation as an upper limit on the average total energy density of a physical system in de Sitter space. Our result is strengthened by the observation that quantum corrections in holographic theories follow exactly the same \(1/R^2\) scaling \([16,17,18]\).

These results suggest that in gravitating quantum systems the vacuum contribution to the total energy density is strongly limited by gravity. Assuming that this holographic limit holds for the entire universe, the worst fine tuning problem in the history of physics appears to be naturally eliminated.

**IV. HOLOGRAPHIC LIMIT ON THE ENERGY DENSITY OF THE UNIVERSE**

While the previous results are intriguing, they are not obviously applicable to the universe: the corresponding Friedman-Robertson-Walker metric of the universe is different from that of a Schwarzschild (de Sitter) black hole. Moreover, an argument based on gravity would not be independent of the dynamic argument based on Friedmann’s equation. In this section, we show is that the holographic conjecture combined with thermodynamics sets the same limit on the vacuum energy that follows from the Einstein-Friedmann equations. Furthermore, the present value saturates this limit, without any fine tuning.

We argue that the smallness of the energy density for large black holes and the universe are all the consequence of the general holographic entropy limit. To clarify this, we sketch the derivation of the cosmological energy density from the holographic conjecture without relying on Einstein’s equation. We start with the Fischler-Susskind cosmic holographic conjecture \([11]\): the entropy of the universe is limited by its ‘surface’ measured in Planck units

\[
S \leq \frac{A}{4} M_P^2, \tag{10}
\]

where the surface area \(A = 4\pi R^2\), motivated by causality, is defined in terms of the apparent (Hubble) horizon

\[
R = \frac{1}{\sqrt{H^2 + k/a^2}}, \tag{11}
\]

with curvature and scale factors \(k\) and \(a\), respectively. According to equation \(10\) the average entropy density is limited by

\[
\frac{S}{V} \leq \frac{\pi R^2}{V} M_P^2. \tag{12}
\]

The first law of holographic thermodynamics relates the entropy and energy of a holographic system, \([12,13,21]\)

\[
E = TS, \tag{13}
\]

where we assumed that the universe has no net angular momentum or electric charge. From equations \(12\) and \(13\), we obtain

\[
\rho \leq \frac{3T}{4R} M_P^2. \tag{14}
\]

This is the holographic limit on the energy density of the universe in terms of the temperature and horizon.

The natural value of the temperature in an adiabatically expanding system is \(T \simeq 1/R\), perhaps with a constant of proportionality of order unity. We fix the constant by using the Gibbons-Hawking temperature \([21]\) of the horizon of the universe \([22,23]\)

\[
T = \frac{1}{2\pi R}. \tag{15}
\]

which sets the \(\mathcal{O}(1)\) constant to be \(1/2\pi\). In this context, naive usage of the measured temperature of the cosmic microwave background (or neutrinos) would constitute a fine tuning of an order \(10^{30}\). Moreover, this substitution would yield a much weaker limit than the natural value (although it would be still \(10^{30}\) times tighter than the traditional field theory estimate). Using equation \(15\), we obtain

\[
\rho \leq \frac{3}{8\pi R^2} M_P^2 = \frac{3}{2A} M_P^2, \tag{16}
\]

that is the energy density of the universe is bound by the inverse area of its horizon.

From general relativity, we know that the universe saturates the holographic bound \([13]\). This is a natural expectation from quantum physics: if matter contributes less, quantum fluctuations will maximize the entropy of the universe. Then, using the equal sign, equation \(10\) yields Friedmann’s equation

\[
\rho = \frac{3}{8S} M_P^4, \tag{17}
\]

stating that the energy density of the universe is bound by its entropy limit

\[
S = \frac{\pi}{H^2 + k/a^2}. \tag{18}
\]

The derivation of Friedmann’s equation from thermodynamics and the cosmological holographic limit can be performed more rigorously for various cosmological models with results identical to ours \([19,24]\).

The presence and otherwise mysterious increase of the vacuum energy density is also consistent with the holographic picture. Due to the expansion of the universe the total energy density dilutes as \(1/R^3\), while the upper limit set by gravity decreases only as \(1/R^2\). The new quantum degrees of freedom created by the increase of the horizon area are continuously filled by vacuum energy.

Connecting the energy density and the number of quantum states of a gravitating system, relation \(17\) can
be reinterpreted as an equation of quantum gravity, perhaps signaling the importance of the holographic principle for the quantized description of gravitation. Details of such a future theory are unknown at this time, but it is most likely to obey the holographic conjecture. The covariant form of (17), possibly a modification of Einstein’s equations, might be a step toward an effective field theory of quantum gravity. We also speculate that the appearance of the inverse entropy (or inverse mass squared) might also signal a (self-)duality of quantum gravity.

Finally, we make it explicit where the apparent $10^{123}$ factor comes from. It is simply the ratio $r$ of the holographic entropy limit of the universe to the entropy of a Plank mass and length size Schwarzschild black hole:

$$r = M_P^4 \left( \frac{3M_P^2}{8\pi R^2} \right)^{-1} = \frac{8\pi R^2}{3} \frac{S}{L_P^3} = \frac{8\pi S}{3 S_P} \propto \log(N),$$

where $N$ is the available (micro) quantum states of the universe. Indeed, substituting numerical values, we obtain

$$r = 5.44 \times 10^{122}. \quad (19)$$

### V. CONCLUSIONS

Holographic thermodynamics imposes an upper limit on the energy density of the universe by its entropy bound. Friedmann’s equation saturates this limit, therefore the inferred value of the vacuum energy from cosmological observations of the expansion rate is naturally consistent with the holographic context. In fact, this saturation has an intuitive meaning of the vacuum energy filling up all available quantum degrees of freedom as the universe expands. While holography is a conjecture, it provides general framework which future quantum gravity theories are likely to fill with microphysical content. Thus our simple thermodynamic estimates are expected to remain valid in quantum gravity, although the technology of the calculation is likely to be more complex.

The simplicity of the holographic limit on the vacuum energy is contrasted with the mysterious $10^{123}$ discrepancy between the quantum field theory estimate and the measured cosmological energy density. This is yet another argument strengthening the case for the holographic conjecture, and motivating the search for holographic theories. Since holography has its roots in quantum theory, our considerations suggest that Friedmann’s equation, although originates from a classical field theory, has a corresponding quantum interpretation connecting the energy density with the available quantum states of the universe.

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[25] Throughout this paper, we equate the Boltzmann and reduced Planck constants to the speed of light, as $k = h = c = 1$, and express dimensional quantities in $M_P$ or GeV units.