The Dual Faces of String Theory

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ABSTRACT

Duality symmetries for strings moving in non-trivial spacetime backgrounds are analysed. It is shown that, for backgrounds generated from compact WZW and coset models, such duality symmetries are exact to all orders in string perturbation theory. A global treatment of duality symmetries is given, by associating them to the known symmetries of affine current algebras (affine-Weyl group and external automorphisms). It is argued that self-duality symmetries of WZW and coset models generate the duality symmetries of their moduli space. Some remarks are presented, concerning the survival of such symmetries in the non-compact case. The implications of duality symmetries for string dynamics in non-trivial/singular spacetimes are discussed.

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1 Introduction

Strings, being extended objects, sense the target space into which they are embedded, in a different way than point particles. In a compact space this difference appears because, strings, except from their local excitations that mimic point particle behavior (“momentum” modes), have “winding” excitations where the string wraps around non-contractible cycles of the manifold. The masses of momentum modes are inversely proportional to the volume of the manifold, whereas those of the winding modes are proportional to the volume, since it costs energy in order to stretch the string. Moreover, the string contains oscillating modes that respond to background fields differently than the center of mass of the string. In certain cases, the physics of string propagation remains invariant under a reorganization of the one string Hilbert space and a specific change in the background. This symmetry is known as duality. In the simplest possible example, that of a string moving on a circle, it was observed that the spectrum of the theory with radius $R$ and that with radius $1/R$ are identical, once we interchange winding and momentum modes, [1].

It turns out that such duality symmetries exist (semi-classically) for all backgrounds with isometries, [2]. In CFT, some of these symmetries were identified as different abelian gaugings of a WZW theory, [3], and this was generalized to abelian gaugings of arbitrary theories with chiral currents, [4], and organized into (semi-classical) O(d,d,$\mathbb{Z}$) type symmetries, [5], mimicking the situation for flat backgrounds. Moreover, for coset models, such duality symmetries exist also for backgrounds without any isometries, [3,6]. Non-abelian duality received more attention recently, [7] but its status is not yet clear. A careful analysis of the underlying CFT structure, revealed that most of these semiclassical symmetries, pertaining to compact cosets, are indeed exact in string theory, [3], and they are intimately related to the affine Weyl symmetries of the “parent” theory, the WZW model.

In the non-compact case, the affine Weyl group is not a manifest symmetry but it can be shown that a particular kind of duality, axial-vector duality, [3] is still a symmetry, [8].

At the semiclassical level, provided there is an abelian isometry, the duality transformation can be effected by gauging this isometry and adding also a langrange multiplier coupled to the field strength of the gauge field, [2]. Integrating out the langrange multiplier, forces the gauge field to be pure gauge which can, subsequently, be gauged away, giving back the original model. On the other hand, one can gauge fix to a unitary gauge and then integrate out the gauge field (which appears quadratically in the action). In this way, a different (dual) sigma model action is obtained (the measure can be also taken care off, effectively changing the dilaton). Modulo global properties, [3], the original and the dual action describe the same theory.

Duality has important implications for string theory. It can be thought of as an

\* See also A. Giveon’s talk in this volume.
unbroken part of the full string symmetry in a particular background, and it can provide important hints concerning the string physics around that background. In particular, all possible dual backgrounds are relevant for string propagation as each determines the response of some of the string modes. It is also interesting that duality seems to preserves some general relativistic notions as that of a Hawking temperature and entropy of black holes.\[10\].

Explicit studies in specific models, [6] (although the result seems to be general) indicate that, in the non-flat case, duality maps zero modes to oscillator modes of the string. In some coordinate system, such oscillator modes resemble winding modes, the only difference being that they are not “topological” (the target manifold has no non-contractible cycles). I will comment more on the possible implications of duality for string physics in non-compact curved backgrounds in the last section.

2 WZW Models

In this section, we will analyze in detail the duality symmetries of WZW models, both from the $\sigma$-model and the CFT (affine current algebra) point of view.

It turns out that understanding the simplest group, SU(2), will suffice. In the case of non-simply laced simple groups there are some minor changes due to the short roots that will be dealt with latter on. The case of non-simple groups has further complications that we will not consider here.

The action of the WZW model is

$$I(g) = \frac{k}{4\pi} I_{NS}(g) + \frac{ik}{6\pi} \Gamma_{WZ}(g)$$

$$I_{NS}(g) = \int d^2 x Tr[U_\mu U_\mu] , \quad \Gamma_{WZ}(g) = \int_{\partial B = S^2} d^3 y \varepsilon^{\mu\nu\rho} Tr[U_\mu U_\nu U_\rho]$$

where

$$U_\mu = g^{-1} \partial_\mu g , \quad V_\mu = \partial_\mu gg^{-1}$$

$g$ is a matrix in the fundamental representation of $G$, and $Tr$ is a properly normalized trace such that

$$\frac{1}{12\pi^2} \int_{S^3} Tr[U \wedge U \wedge U] \in Z .$$

The action $I(g)$ is invariant under the group $G_R \otimes G_L$, generated by left and right group transformations, $g \rightarrow h_1 gh_2$, with associated conserved currents

$$J_\mu^R = \frac{k}{2\pi} P_-^{\mu\nu} U_\nu , \quad J_\mu^L = \frac{k}{2\pi} P_+^{\mu\nu} V_\nu$$

with $P_\pm^{\mu\nu} \equiv \delta^{\mu\nu} \pm i\varepsilon^{\mu\nu}$. These currents are conserved and chirally conserved and they generate two copies of the affine $\hat{G}$ current algebra. An important property of the WZW
action is that it satisfies the Polyakov-Wiegman formula

\[ I(gh) = I(g) + I(h) - \frac{k}{2\pi} \int d^2x P_{+\mu} Tr[U_\mu(g) V_\nu(h)] \]  (2.6)

To generate duality transformations in the WZW model, we pick a generator of the Lie algebra of \( G \), \( T^0 \), normalized as \( Tr[(T^0)^2] = 1 \). We can then parametrize \( g = e^{i\phi T^0} h \). Using (1.6), the action \( I(g) \) takes the form

\[ I(g) = I(h) + \frac{k}{4\pi} \int \partial_\mu \phi \partial^\mu \phi - \frac{ik}{2\pi} \int P_{+\mu} \partial_\mu V^0_\nu(h) \]  (2.7)

where \( V^0_\mu(h) = Tr[T^0 U_\mu(h)] \). We can now apply the duality map, \( \mathbb{R} \) to obtain

\[ I^{\text{dual}}(g) = I(h) + \frac{1}{4\pi k} \int \partial_\mu \phi \partial^\mu \phi - \frac{i}{2\pi} \int P_{+\mu} \partial_\mu V^0_\nu(h) \]  (2.8)

The angle \( \phi \) was originally normalized to take values in \([0, 2\pi]\). It is obvious from (2.8) that the effect of the duality transformation is to change the range of values to \([0, 2\pi/k]\). To see how many independent duality transformations exist, we have to explicitly parametrize the Cartan torus dependence of the WZW model. Pick a basis in the Cartan algebra, \( T_i \), \( i = 1, 2, \ldots, r \), \( [T_i, T_j] = 0, Tr[T_i T_j] = \delta^{ij} \) and parametrize,

\[ g = e^{i \sum_{i=1}^r \alpha_i^i T_i} h e^{i \sum_{i=1}^r \gamma_i^i T_i}. \]  (2.9)

Then using (2.6) the WZW action becomes

\[ I(g) = I(h) + \frac{k}{4\pi} \int (\partial_\mu \alpha_i^i \partial^\mu \alpha_i^i + \partial_\mu \gamma_i^j \partial^\mu \gamma_j^i) - \frac{ik}{2\pi} \int (P_{+\mu} \partial_\mu \alpha_i^i V^0_\nu(h) + P_{-\mu} \partial_\mu \gamma_i^j U^i_{\nu}(h)) + \frac{k}{2\pi} \int P_{+\mu} \partial_\mu \alpha_i^i \partial_\nu \gamma^j M^{ij}(h) \]  (2.10)

where

\[ U^i_{\mu}(h) = Tr[T^i U_\mu(h)], V^i_\mu(h) = Tr[T^i V_\mu(h)], M^{ij}(h) = Tr[T^i h T^j h^{-1}] \]  (2.11)

It is obvious from (2.10) that we can apply the duality transformation using any of the \( \alpha_i \), \( \gamma^i \). Thus, there are \( 2^r - 1 \) non-trivial duality transformations. A duality transformation on \( \alpha_i \) effectively makes the substitution \( \alpha_i \to \alpha_i/k \) in the action whereas a duality transformation on \( \gamma^i \) makes the substitution \( \gamma^i \to -\gamma^i/k \). The new manifold has a similar metric to the group manifold but due to the different angle periodicities, has Taub-NUT type singularities.

In order to identify the underlying property of the WZW model, responsible for the invariance under these duality transformations, we have delve a bit into such elements of the the representation theory of the affine Lie algebras as the affine Weyl group and external automorphisms.

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The affine Weyl group $\hat{W}$ is a semidirect product of the Lie algebra Weyl group $W$ times a translation group, $\hat{W} = W \triangleright T$. Apart from the action of finite Weyl group elements, there are Weyl transformations associated to roots which have a component in the direction of the imaginary simple root. The action of such an element $\hat{W}_{\alpha}$ on a finite Lie algebra weight $\lambda$ and on the grade $n$ is

\[ \hat{W}_{\alpha}(\lambda) = W_{\alpha}(\lambda) - k\beta \]  \hspace{1cm} (2.12a)
\[ \hat{W}_{\alpha}(n) = n - \lambda \cdot \beta - \frac{k}{2} \beta \cdot \beta \]  \hspace{1cm} (2.12b)

where $\beta = 2\alpha / \alpha \cdot \alpha$ is the coroot associated to the finite Lie algebra root $\alpha$, the grade $n$ is basically the mode number\(^5\) and $W_{\alpha}(\lambda) = \lambda - \alpha(\lambda \cdot \beta)$ is a finite Weyl transformation. It is important to note that affine Weyl transformations, in general, map states inside a representation at different levels.

There are also external automorphisms of the affine algebra which are essentially associated to symmetries of the affine Dynkin diagram. For the $SU(n)$ case, the affine Dynkin diagram consists of $n$ nodes connected around a circle. The external automorphisms are generated by a basic rotation, and a reflection which corresponds to the finite Lie algebra external automorphism (that maps a representation to its complex conjugate). When we write a highest weight $\lambda = \sum_{i=1}^{n-1} m_i \lambda_i$ in terms of the fundamental weights $\lambda_i$, ($m_i$ are non-negative integers), the action of the generating rotation of the affine Dynkin diagram is as follows

\[ \sigma(\lambda) = (k - \sum_{i=1}^{n-1} m_i)\lambda_1 + m_1\lambda_2 + \cdots + m_{n-2}\lambda_{n-1} . \]  \hspace{1cm} (2.13)

$\sigma$ generates a $Z_n$ group\(^6\) where $\sigma^n = 1$ on the highest weights, but acts as an affine Weyl transformation in the representation. Specializing to $SU(2)$, let $m \in Z/2$ be the weight, and $j \in Z/2$ the highest weight (spin of a representation). Then the finite Weyl group acts as $m \rightarrow -m$, and combined with the affine translation $m \rightarrow m + k$ they generate the affine Weyl group. The only nontrivial outer automorphism $\sigma$ acts as $j \rightarrow k - j$ and $\sigma^2$ is a Weyl translation.

The non-trivial statement now is: For compact groups, integer level and integrable highest weight representations, both the affine Weyl group and the external automorphisms are symmetries. In particular, in a WZW model the Hilbert space is constructed by tying together (in a modular invariant way) two copies of representations of the affine algebra. Thus, we have invariance under independent affine Weyl transformations acting on left or right representations. Moreover, since the modular transformation properties of the affine characters reflect the external automorphism symmetries, the theory is invariant under external automorphisms that act at the same time on left and right representations. These invariance properties can be verified for correlation functions on the sphere and the torus. This then implies that they hold on an arbitrary Riemann

\(^5\) In a highest weight representation where the affine primaries have $L_0$ eigenvalue $\Delta$, the grade $n$ of a state is the eigenvalue of $L_0 - \Delta$ on that state.

\(^6\) In general this group is isomorphic to the center of the finite Lie group.
surface since the sphere and torus data are sufficient in order to construct the correlators at higher genus.

As an example, we will present the SU(2) case and focus on the spectrum. We introduce the (affine) SU(2)$_k$ characters

$$\chi_l(q = e^{2\pi i \tau}, w) = Tr_{l}[q^{L_0} e^{2\pi i w J^0_0}] = \sum_{m=-k+1}^{k} c^l_m(q) \vartheta_{m,k}(q, w)$$

where $l$ is twice the spin (a non-negative integer) and $m$ is twice the $J^3_0$ eigenvalue. The trace is in the affine hw representation of spin $l$,

$$\vartheta_{m,k}(q, w) = \sum_{n \in \mathbb{Z}} q^{k(n+\frac{m}{2k})^2} e^{2\pi i w (kn+\frac{m}{2})}$$

and $c^l_m$ are the standard string functions which satisfy $c^l_m = 0$ when $l - m = 1 \mod 2$ (which means that the spin is increased or decreased in units of 1). For integrable representations ($k$ is a positive integer and $0 \leq l \leq k$), invariance under the affine Weyl group is equivalent to

$$c^l_m = c^{-l}_{-m}, \quad c^l_m = c^l_{m+2k}$$

The first relation is due to the Weyl group of SU(2) while the second is the generating translation in the affine Weyl group. There is another important relation

$$c^l_m = c^{k-l}_{k-m}$$

which is a consequence of the external affine automorphism.

The duality transformation on $\alpha^i$ amounts to replacing $\bar{J}^i \rightarrow -\bar{J}^i$, where $\bar{J}^i$ is the right Cartan current in the $T^i$ basis of the Cartan subalgebra. Similarly the duality transformation on $\gamma^i$ amounts to the replacement $J^i \rightarrow -J^i$ at the level of the Cartan subalgebra. This is not the whole story however. With a bit more effort one can see that they act as Weyl transformations on the left or right SU(2) currents. This identification can be seen clearly by coupling the WZW action to external gauge fields and monitoring the effect of the duality transformation on the currents. It can also be recovered from the twisted partition function via the action of the duality transformation on the gauge field moduli (for the Cartan).

The duality transformations

$$D_i : \bar{J}^i \rightarrow -\bar{J}^i$$

$$\tilde{D}_i : J^i \rightarrow -J^i$$

are exact symmetries of the model. This can be verified explicitly, since characters are invariant under the finite Weyl group.

This invariance is similar, but qualitatively different than that present in flat backgrounds. There, one has a family of theories parametrized by $G, B$ and duality is the statement that two theories are equivalent for different values of the parameters. Here,
there is no parameter present and, in this sense, this is what we could call self-duality. It corresponds to the self duality (in the flat case) of level 1 WZW models appearing at special values of the moduli. Now we are in a position to discuss the general WZW model for a simple group $G$. Let $M$ be the root lattice, $M_L$ the long root lattice and $M^*$ the weight lattice. The character of a hw representation of $\hat{G}$ with hw $\vec{\Lambda}$ is defined as

$$
\chi_{\vec{\Lambda}}(q, \vec{w}) = Tr[q^{L_0}e^{2\pi i \vec{w} \cdot \vec{J}_0}] 
$$

where $\vec{J}_0$ generates the cartan subalgebra of $G$. The character admits the string function decomposition, $\chi_{\vec{\Lambda}}\equiv \sum_{\vec{\lambda} \in M^*/kM_L} c_{\vec{\lambda}} c_{\vec{\lambda}}(q) \Theta_{\vec{\lambda}}(\vec{w}, q)$ (2.20)

with $\Theta_{\vec{\lambda}}$ being the classical $\vartheta$-function of level $k$ of the Lie algebra of $G$

$$
\Theta_{\vec{\lambda}}(\vec{w}, q) = \sum_{\vec{\gamma} \in M_L} q^{\frac{k}{2}(\vec{\gamma} + \vec{\lambda})^2} e^{2\pi i \vec{w} \cdot (k\vec{\gamma} + \vec{\lambda})}. 
$$

The string functions are invariant under the Weyl group and Weyl translations

$$
c_{w(\vec{\lambda})} = c_{\vec{\lambda}} , \ c_{\vec{\lambda} + k\vec{\beta}} = c_{\vec{\lambda}} (2.22)
$$

where $w$ is a Weyl transformation and $\vec{\beta} \in M_L$.

The (left) generating duality transformations $D_i$ correspond to Weyl reflections generated by the simple roots $\vec{\alpha}_i$ which implement the transformations (2.18a). The invariance of the spectrum (and partition function) is encoded in the fact, obvious from (2.21,22), that $\chi_{\vec{\Lambda}}$ is invariant under $w_i \rightarrow -w_i$. Although $w_{\vec{\alpha}_i}$ do not commute, they do so when applied to the character, thus at the level of the partition function they generate a group isomorphic to $Z_2^r$. However, at the level of correlation functions the (left) duality group is larger and in fact isomorphic to $W_L \times W_R/W_D$, where $W_{L,R}$ are the left(right) Weyl groups of the (finite) Lie algebra of the WZW model and $W_D$ is the diagonal Weyl group (whose action corresponds to reparametrizations of the action).

We have seen already (from affine algebra representation theory) that the (non-local) symmetry of the (compact and unitary) $g$-WZW model is larger: It is generated by the left and right affine Weyl groups $\hat{W}_{L,R}^g$ as well as the external affine automorphisms $A^g$. I conjecture that $\hat{W}_{L}^g \times \hat{W}_R^g \times A^g/W_D$ is the full self-duality group of the WZW. Affine Weyl transformations act semi-classically as $GL(r,\mathbb{Z})$ rotations, that is, as particular changes of basis in the lattice of weights. This implies, that although semi-classically, for curved backgrounds $GL(r, Z)$ is a symmetry, $\mathbb{E}$, in the exact theory only some part of it survives. Concerning the external affine automorphisms, their action in $\sigma$-model language is not known.

### 3 Compact Coset Models

From the WZW theory, we can built other CFTs by projections. The simplest such projection corresponds to constraining the affine currents of a subalgebra, known as
The \( \sigma \)-model action of coset models is obtained by gauging the appropriate subgroup of the WZW model. Gauging different dual versions of the WZW model, dual versions of the coset model are obtained. It can be shown that the Killing symmetries of a coset model \( G/H \) are of two types. Chiral \( H'_L \times H'_R \) isometries, when there is a subgroup \( H' \) such that \([H, H'] = 0\) and non-chiral abelian isometries in one to one correspondence with \( U(1) \) factors of \( H \). In many cases, the aforementioned duality exists for coset actions without isometries. 

A special form of duality is obtained for coset models where the gauged subgroup contains a \( U(1) \) factor. In such a case, one has the option of gauging either the axial or the vector subgroup of the original \( U(1)_L \times U(1)_R \) subalgebra. The \( \sigma \)-model actions of these two gauged models are generically different but it can be shown that the models are dual to each other. This type of duality is known as axial-vector duality and at the semiclassical level is powerful in generating different types of backgrounds. It turns out that the underlying symmetry of current algebra responsible for axial-vector duality is affine-Weyl symmetry, \( \hat{W}_L \times \hat{W}_R \). For integrable (unitary) representations of (compact) affine algebras the affine Weyl group is a symmetry and so is axial-vector duality.

The full duality symmetry of a coset model \( g/h \) is generated by the full duality symmetry of the “parent” \( g \)-WZW theory. Let \( H \) be a reductive subgroup of \( G \) and \( H^{na} \) be its non-abelian component, (discard \( U(1) \) factors). Denote the Lie algebra of \( H^{na} \) by \( h \). Then, \( D^h = \hat{W}_L^h \times \hat{W}_R^h \times A^h \) is a normal subgroup of \( D^g = \hat{W}_L^g \times \hat{W}_R^g \times A^g \). The full self-duality group of \( G/H \) is \( D^g/D^h \). The need to factor \( D^h \) comes since it acts only on the gauge degrees of freedom and it is thus, invisible in the \( G/H \) theory.

### 4 Duality Symmetries in the Moduli Space of WZW and Coset Models.

So far we have been concerned with the self duality symmetries of WZW and coset models. We will now show how this self duality generates duality symmetries of their moduli space.

The primary example of this is the moduli space of (flat) D-dimensional toroidal models. At special points of the moduli, one finds WZW models (at level one). It can be shown in this case that, the full duality group of the moduli space, \( O(D, D, Z) \) can be generated from the self duality symmetries of the WZW points.

A similar phenomenon happens in the general case. Consider a WZW (or coset) model and its neighborhood in moduli space which is generated via marginal perturbations by integrable (1,1) operators \( O_{1,1}^i \). This set of operators contains dual pairs, that is for each

\[ \text{There are more general projections though, preserving conformal invariance.} \]
there is a $\tilde{O}^i_{1,1}$ in the set related by a self-duality of the WZW or coset theory. At the level of the (abstract) conformal field theory, $O^i$ and $\tilde{O}^i$ correspond to the same operator. In a $\sigma$-model (field theoretic) realization, they are different fields.

Self-duality at the WZW or coset point implies that the line generated by $O^i$ is equivalent as a CFT to that generated by $\tilde{O}^i$. The backgrounds ($\sigma$-models) corresponding to the two lines though are different. Thus, in the full moduli space, duality symmetries can be generated by the self-duality symmetries at special points (WZW and cosets)

In the case of a G-WZW model for simple G, at a generic level the only integrable perturbations are generated by $J^i\bar{J}^j$ constructed out of the left and right Cartan currents. Thus (except at special points) the dimension of the moduli space is $D^2$ where $D = \text{rank}_G$. The duality group here is similar to O(D,D,Z), since in such deformations the generalized G-parafermion theory does not change along the moduli space and duality acts on the cartan-torus bosons. When G is semi-simple, then one has extra generic (1,1) operators, however their integrability is an open question.

Similar remarks apply to perturbations by relevant operators. In this case self-duality of the WZW or coset theory implies Krammers-Wanier duality for the off-critical theory. The archetype of this duality exists in the $Z_N$ parafermion models perturbed by the first energy operator, which upon a self-duality transformation in the fixed point theory, changes sign.

It is tempting to conjecture that all duality symmetries of the full moduli space of WZW and coset models are reflections of self-duality at these special points, as it happens in the flat case.

5 Duality in the Non-compact Case.

So far we have dealt with compact WZW models and their cosets. In the non-compact case things can, a priori, be different. At the level of duality symmetries of the G-WZW model itself, those associated with the finite Weyl group are still symmetries (if G is reductive). However the affine Weyl acts differently. For a generic affine representation, affine Weyl transformations map it to different representations. We have seen that affine Weyl symmetry is important for axial-vector duality. The only way that axial-vector duality can survive in the non-compact case is, if the spectrum is organized into complete orbits of the affine Weyl group.

There is a different way of showing that axial-vector duality remains an exact symmetry in the non-compact case, [8]. This, in retrospect implies the organization of the spectrum as mentioned above. Important information about the spectrum can be retrieved that way since, (at least for the case of SL(2,R)) non-compact string functions are known, [15], as well as their transformation properties under the affine Weyl group. It is an interesting problem to try to obtain the spectrum this way.
6 Comments on the Physics of Duality.

The main lesson from duality and related symmetries is that the background fields do not determine uniquely the spectrum and physics of string theory. Duality can be viewed as a tiny (unbroken) part of the huge string gauge symmetry, whose glory remains obscure to our days. Another way to state this, is, that different modes of the string feel different geometry. Thus, the background interpretation of string vacua should be used with care in order to ascertain the physics. The only cases were the description is reliable is the large volume limit of compact manifolds, and at the asymptotically flat region of non-compact manifolds. Once at a region of finite curvature, the geometrical description of string theory breaks down. Even topology is not preserved under duality and there are continuous families of ground states in string theory where topology changes without the occurrence of anything catastrophic, [17, 16, 8].

An example of how this type of symmetry can affect string propagation, can be given (heuristically) as follows. Consider a string background which is highly curved or even singular (semi-classically) in a certain region, (the 2-d black hole, [18], is such an example but one needs to add a few extra dimensions in order to have non-trivial massive states). In the asymptotic region, (which is obtained by some spacetime-depended radius becoming very large), one has quantum numbers for asymptotic states that correspond roughly to windings and momenta. Momentum states are the only low energy states in this region. Consider a momentum mode travelling towards the high curvature region. Its effective mass starts growing as it approaches large curvatures. At some point it becomes energetically possible for it to decay to winding states which, in this region, start having effective masses that are lower than momentum modes. In such backgrounds (unlike flat ones) winding and momentum are not separately conserved so that such a transition is possible. The reason for this is that there is a non-trivial dilaton field and thus, winding and momentum conservation is broken by the screening operators which transfer it to discrete states localized at the high curvature region. An alternative interpretation of this, is that particles interact with such localized states loosing momentum (in discrete steps) and gaining winding number.

Once such a momentum to winding mode transition happens in the strongly curved region, the winding state sees a different geometry, namely the dual one and thus continues to propagate further into the strong curvature region since it feels only the (weak) dual curvature. This phenomenon, implies the need of a novel treatment of the underlying effective field theory approach, which would resemble the effective treatment of particles in condensed matter physics, with position dependent masses.

This type of picture indicates that the physics of string black holes will be qualitatively different that their classical general relativity counterparts.

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